Radiation pressure in the Coupled Nonlinear Schrödinger Equation exerted on the solitons.

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In the paper an interaction between a soliton and a traveling wave in the Coupled Nonlinear Schrödinger Equation (CNLSE) is considered. The force acting on the soliton can both push or pull the soliton depending on parameters of the equation, structure of the soliton and the incident wave. In special cases analytical results are presented and in general numerical solutions of both linearized and the full problem are shown.

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Introduction. The coupled nonlinear Schrödinger equation (CNLSE) appears in many physical problems starting from propagation of wave packets in wave guides in nonlinear optics [1–4] to description of the Bose-Einstein condensates [5–8]. One of the most interesting solutions of the equations are solitons, studied extensively mostly in the NLSE due to its integrability. The coupling breaks the integrability opening new possibilities such as for example resonant multi bounce collisions [1, 2]. In our previous papers we have found that solitons in various models reveal a surprising feature which we call negative radiation pressure (NRP). When small waves (radiation) hit a large object, such as a soliton, they scatter and exchange momentum and energy. Usually this leads to a force pushing the object. However there are situations when the force exerted by radiation pulls the object. This feature is present in many well known and extensively studied models including kinks in \( \phi^4 \) model [9–10], kinks (and domain walls) in three vacuum \( \phi^6 \) model [11–12], vortices in abelian, nonabelian Higgs models [13] and in the Gross-Pitaevskii (GP) model [14]. This feature requires some mechanism which transforms energy from slower modes to faster ones. In the \( \phi^4 \) model nonlinear coupling creates a double frequency wave which effectively exerts NRP on the kink. This is possible due to a very rare feature of reflectionless of the \( \phi^4 \) kink in the first order. However we have found that there exists a second mechanism which can reveal in many important physically models. We have shown that sometimes it is enough that there exist two modes with different masses or, in more general, different dispersion relations. As an example asymmetric kinks and domain walls in \( \phi^6 \) model can serve, which we discuss in a separate paper [12]. Waves coming from one side exert negative radiation pressure and from the other side positive. Similar mechanism can be observed in a theory of more than one component field. Global U(1) theory exhibits a vortex which is a topological defect with a winding number as a topological charge [13]. Moreover in such theory there exist two types of small excitation of a vacuum: a massless Goldstone’s mode and a massive mode responsible for excitation of the amplitude of the field. A scattering on a vortex can lead to a mixture of the two modes. The mass difference can lead to NRP in case when the amplitude wave hits the vortex. Similar phenomena are recently described in other fields and sometimes referred to as optical pulling force but originated from a different mechanism [15–18]. In the present paper investigate NRP in case of CNLSE. The mechanism responsible for NRP presented in the present paper is different from the mechanism presented in our previous papers.

To the best of our knowledge it is possible to prepare an experiment which can test our predictions.

The model. We discuss a two component system described by coupled nonlinear Schrödinger equation (CNLSE) (symmetric interaction)

\[
i \partial_t \psi_i + \frac{1}{2} \partial_{xx} \psi_i + \left( \alpha_i^2 \psi_i^2 \right) \left( \psi_i^2 \right) \psi_i = 0, \quad (1)
\]
or after rescaling \( (\alpha_1 \psi_1 = \psi_1 \text{ and } \beta_i = \gamma / \alpha_i^2) \)

\[
i \partial_t \psi_i + \frac{1}{2} \partial_{xx} \psi_i + \left( \text{sgn}(\alpha_i^2) \right) \left( \psi_i^2 \psi_j^* \right) \psi_i = 0, \quad (2)
\]
where \( i, j = 1, 2 \) but \( i \neq j \) and \( \alpha_i^2 > 0 \). Following [2] we will use the second form. For special choice of \( \beta_1 = \beta_2 = 1 \) or when one of the components is absent everywhere the system is integrable. There exists a particular solution, the soliton, which can be parametrized by \( (\psi_1, \psi_2) = (e^{i \Omega t} \Phi(x), e^{i \Omega t} \Phi(x)) \). We will consider initially stationary solitons at \( x = 0 \) described by even profiles \( \Phi_i(-x) = \Phi_i(x) \) with amplitude \( A_i = \Phi_i(0) \). The equation possesses scaling symmetry \( \psi_i(x, t) \rightarrow \lambda \psi_i(\lambda^2 t, \lambda x) \). We can use it to set \( \gamma = 1 \). The solutions can be also multiplied by an arbitrary constant phase \( \psi_i(x, t) \rightarrow e^{i \phi_i} \psi_i \).

Effective mass. Momentum density in the model is defined as:

\[
\mathcal{P} = \frac{i}{2} \left( \Psi_x^\dagger \Psi - \Psi^\dagger \Psi_x \right), \quad \Psi = (
\tilde{\psi}_1, \tilde{\psi}_2)^T
\]
or

\[
\mathcal{P} = \frac{i}{2} \sum_{k=1}^2 \frac{1}{\beta_k} (\partial_x \psi_k^* \psi_k - \psi_k^* \partial_x \psi_k).
\]
The moving soliton can be written as
\[
\psi_i(x, t) = e^{i\Omega_i t} \Phi_i(x - vt)e^{i\theta(x-t/2)}.
\] (5)

It’s total momentum is equal to
\[
P = v \int_R \left( |\Phi_1^2| / \beta_1 + |\Phi_2^2| / \beta_2 \right).
\] (6)

Another important property is the continuity equation \( \partial_t |\psi_i|^2 / \beta_i + \partial_x P_i = 0. \)

A wave. The second type of solutions we discuss is the wave. In the absence of the soliton the wave with small amplitude in a single component can be written as \( \psi_i = a_i \exp(i(\omega_i t + k_i x)) \), where \( k_i^2 / 2 = -\omega_i \).

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\[
\psi^{(1)}_i(x, t) \underset{x \to \infty}{\longrightarrow} a_i e^{i\omega_i t} \left( \delta_i e^{ik_i x} + R_i e^{-ik_i x} \right),
\] (7)

where \( R_i \) is a reflection coefficient in the \( i \)-th channel and

\[
\psi^{(1)}_i(x, t) \underset{x \to \infty}{\longrightarrow} a_i T_{ii} e^{i\omega_i t + ik_i x},
\] (8)

with \( T_{ii} \) being the transition coefficient. There is only one wave which brings in the momentum and the other waves carry away the momentum. The surplus which is left is the force exerted on the soliton

\[
P_i = F = a_i^2 \sum_{i=1}^{2} \left( \delta_{ii} + |R_{ii}|^2 - |T_{ii}|^2 \right) |\omega_i| / \beta_i.
\] (9)

Perturbation series. Assuming the wave has a small amplitude we can apply the perturbation theory with the wave amplitude as an expansion parameter. Let us substitute

\[
\psi_i = e^{i\Omega_i t} \sum_{n=0}^{\infty} a_i^n \psi^{(n)}_i \approx e^{i\Omega_i t} \left( \Phi_i + a_i \xi_i + \cdots \right),
\] (10)

where \( \Phi_i \) is an \( i \)-th component of the stationary solution with frequency \( \Omega_i \). All the equations in \( n \)-th order will have the following form

\[
L_\Psi^{(n)} = g^{(n)} \left( \Psi^{(0)}, \ldots, \Psi^{(n-1)} \right),
\] (11)

where \( \Psi^{(n)} = \left( \psi_1^{(n)}, \psi_1^{(n)*}, \psi_2^{(n)}, \psi_2^{(n)*} \right)^T \) and

\[
L = \left( i\partial_t + \frac{1}{2} \partial_{xx} \right) \mathbb{1}_4 - \left( \begin{array}{cc} \Omega_1 & \Omega_2 \end{array} \right) \otimes \mathbb{1}_2 + \left( \begin{array}{cccc} 2\Phi_1^2 + \beta_1 \Phi_2^2 & \Phi_1^2 & \beta_1 \Phi_1 \Phi_2 & \beta_1 \Phi_1 \Phi_2 \\
\Phi_1^2 & 2\Phi_1^2 + \beta_1 \Phi_2^2 & \beta_1 \Phi_1 \Phi_2 & \beta_1 \Phi_1 \Phi_2 \\
\beta_2 \Phi_1 \Phi_2 & \beta_2 \Phi_1 \Phi_2 & 2\Phi_2^2 + \beta_2 \Phi_1^2 & \Phi_2^2 \\
\beta_2 \Phi_1 \Phi_2 & \beta_2 \Phi_1 \Phi_2 & \Phi_2^2 & 2\Phi_2^2 + \beta_2 \Phi_1^2 \end{array} \right).
\] (12)

Concerning a monochromatic wave in one of the components we can consistently decompose the first order solution into Fourier modes

\[
\psi^{(1)}_i(x, t) = \xi_i(x, t) = e^{i\omega x} \xi_i^+(x) + e^{-i\omega x} \xi_i^-(x).
\] (13)

and rewrite the set of equations as

\[
(K + M) \Xi = 0, \quad \Xi = \left[ \xi_i^+, \xi_i^-, \xi_j^+, \xi_j^- \right]^T
\] (14)

where

\[
K = \frac{1}{2} \partial_{xx} \mathbb{1}_4 - \text{diag} \left( \Omega_1 + \tilde{\omega}, \Omega_1 - \tilde{\omega}, \Omega_2 + \tilde{\omega}, \Omega_2 - \tilde{\omega} \right).
\] (15)

and \( M \) is the large matrix from (12). For a wave moving in the first component \( \omega = \Omega_1 - \tilde{\omega} < 0 \) and for the wave in the second component we will assume \( \omega_2 = \Omega_2 - \tilde{\omega} < 0 \). The other two frequencies are positive and cannot propagate. We will solve the above system of equations with appropriate boundary conditions (7) and (8). In general the above system has to be solved numerically but first we discuss two special cases of a single component soliton and a wave in one of the components.

Scalar soliton. Let us consider the scalar soliton in the first sector \( (\Phi_1(x) = A_1 \text{sech}(A_1 x), \Phi_2(x) = 0) \) with the phase rotating with the frequency \( \Omega_1 = A_1^2 / 2 \). The momentum of such soliton moving with velocity \( v \) is equal to \( P = 2A_1 v / \beta_1 \) therefore we can identify its mass as \( m = 2A_1 / \beta_1 \). First let us assume that the wave is present only in the first component \( (\xi_2 = 0) \) or in more general we seek solutions with \( \psi_2 \equiv 0 \). The equation in the linear order can be reduced and written in a matrix form as

\[
B \left( \begin{array}{c} \xi_1^+ \\ \xi_1^- \end{array} \right) = 0
\]

with

\[
B = \left( -\Omega + \frac{1}{2} \partial_{xx} + 2\Phi^2 \right) \mathbb{1} + \left( \tilde{\omega} \Phi^2 - \tilde{\omega} \right).
\] (16)
Surprisingly, because of the integrability, the solution can be found quite easily in a closed form
\[ \psi_1^{(1)}(x,t) = e^{i\Omega t} \left[ e^{i(\Omega t + \xi)} \cosh^2(x) + e^{-i(\Omega t + \xi)}(k - i \tanh(x))^2 \right]. \] (17)

The first term, oscillating with the frequency \( \Omega_1 + \dot{\omega} > 0 \), describes localized, nonpropagating correction. The second term with frequency \( \omega_1 = \Omega_1 - \dot{\omega} < 0 \) describes a moving wave. It is worth noting the wave moves in one and only in one direction and has no reflected part. We believe this is the feature of all integrable models. There is no dynamical interaction (no energy exchange) between the soliton and the wave. The soliton is transparent to the radiation. One can only observe a phase shift. The same feature was also seen in sine-Gordon (sG) model. sG kinks are transparent in all orders of the perturbation series. We believe this is also the case for CNLSE. Our numerical simulations of the full nonlinear equation (2) confirmed that the soliton does not feel any force exerted by the wave.

**Scalar soliton and a wave in the second component.** Let us consider again the scalar soliton in the first component but with the incident wave propagating in the second sector. Still \( \Phi_2(x) = 0 \) but \( \psi_1^{(1)} = 0 \). Equations for positive \( \psi_2^{(1)} \) and negative frequency \( \psi_2^{(-1)} \) decouple. We can set \( \psi_2^{(1)} = 0 \) as positive frequencies cannot propagate. Equation for \( \psi_2^{(-1)} = e^{-i\omega t} \xi_2(x) \) now can be reduced to a single ordinary differential equation
\[ -\omega \xi_2 + \frac{1}{2} \partial_{xx} \xi_2 + \beta_2 \text{sech}^2(x) \xi_2 = 0. \] (18)

Soliton from the first component is a source of a potential over which the wave in the second component scatters. The potential is a well known Pöschl-Teller potential and has known solutions which in terms of of Legendre functions which can be written as
\[ \psi_1^{(-1)} = e^{-i\omega t} P^\lambda_k(\tanh x), \ \lambda = \pm \frac{1}{2} \sqrt{8\beta_2 + 1} - \frac{1}{2}. \] (19)

Note that for \( \beta_2 = \lambda(\lambda + 1)/2 \) for integer \( \lambda \) the potential is reflectionless (F=0).

The asymptotics of the Legendre functions as \( |x| \to \infty \) are well known in terms of gamma function. Imposing the boundary condition of incoming wave from +\( \infty \) with amplitude equal to 1 \( (\xi_2(x) \to -\infty) e^{ikx} + Re^{-ikx} \) we read the reflection coefficient which is needed to calculate the momentum balance and hence the force acting on the soliton \( F = -k^2|\mathbf{R}| a_2^2/\beta_2 \) from which we calculate the acceleration
\[ \ddot{x} = \frac{a_2^2 k^2 \sin^2(\lambda \pi)}{\sin^2(\lambda \pi) \cos^2(\lambda \pi) + \cos^2(\lambda \pi) \sin^2(\lambda \pi) \beta_1} \frac{\beta_1}{\beta_2}. \] (20)

Note that the direction of the acceleration is determined by the sign of the quotient \( \beta_1/\beta_2 \). If both has the same sign the soliton experiences Positive Radiation Pressure (PRP) and if \( \beta_2 \) have opposite signs the soliton undergoes Negative Radiation Pressure (NRP). This works perfectly, see Fig[4]. By changing the sign of \( \beta_1 \) we observed numerically that the radiation pressure changes the sign as well. However, for \( \beta_2 \)s with the different signs the equation (2) cannot be transformed into (1) and it is possible that this NRP result has no physical meaning in this context.

**Vector soliton** Let us now consider a monochromatic wave in the first sector hitting the two component vector soliton. \( A_2 = 0 \) corresponds to the previous case and when \( A_2 \) tends to infinity it formally reproduces the first, integrable case. In a special case, when \( \beta_1 = \beta_2 = \beta \) and the solitons have the same amplitude we can also find their profiles and frequencies analytically \( \Phi_1 = \Phi_2 = A \text{sech}(\sqrt{1 + \beta} x) \), \( \Omega = \sqrt{1 + \beta} A^2/2 \). The total momentum in this case equals \( P = \frac{4A \sqrt{\beta}}{\sqrt{1 + \beta}} \) so the mass of the vector soliton is equal to \( 4A/(\sqrt{1 + \beta}) \). In the case when the second component is much smaller \( A_2 \ll A_1 = 1 \) it can be treated as a linear perturbation to the large soliton and is often referred to as a shadow soliton. In such a case the amplitude of the second soliton is arbitrary but small, and the frequency of the companion soliton does not depend on the amplitude (in the first approximation) \( \Phi_1 = \text{sech}(x), \Omega_1 = 1/2, \Phi_2 = A_2 \text{sech}^\lambda(x), \Omega_2 = \lambda^2/2 \). Formally \( \Phi_2 \) satisfies equation (18) but with positive eigenfrequency and vanishing boundary conditions as \( |x| \to \infty \). There can be other higher bound states of the potential (depending on \( \beta_2 \)). When \( A_2 > 0 \) they couple with the scattering modes of the field \( \psi_1 \) and become resonance modes.

In general the profiles and the frequencies of the soliton counterparts are to be determined numerically by solving the system of two nonlinear ODEs with appropriate boundary conditions, but the frequencies for \( \beta_1 = \beta_2 = \beta \) change monotonically from \( (\Omega_1, \Omega_2) = (1/2, \lambda^2/2) \) as \( A_2 \to 0 \) to \( (1/\sqrt{1 + \beta}, 1/\sqrt{1 + \beta}) \) for \( A_2 =
Let us first consider a wave $\xi_1^+$ propagating in the first sector with the frequency $\omega_1 = \Omega_1 - \omega$. It is accompanied by the nonpropagating localized perturbation $\xi_1^-$ oscillating with $\Omega + \omega = 2\Omega_1 - \omega$. In the second sector the nonpropagating perturbation $\xi_2^-$ has the frequency $\Omega_2 + \omega = \Omega_2 + \Omega_1 - \omega$. The second perturbation $\xi_2^+$ oscillates with $\omega_2 = \Omega_2 + \omega = \Omega_2 - \Omega_1 + \omega$. The most important dynamical properties of this scattering problem depend on this frequency. For simplicity let us assume that the reflection is negligible. From the numerical solution we can see that the reflection is much smaller that the transition for wide range of parameters. We can now discuss only the simplified scattering problem from $\xi_1^+$ to $\xi_1^-$ and $\xi_2^+$. If $|\omega_2| > |\omega_1|$ (which is true when $\Omega_2 < \Omega_1$) the wave excited in the second component moves faster than initial wave and hence carries more momentum. This surplus has to be balance by additional force acting on the soliton. In this case the force pushes the soliton backwards which can be interpreted as the Negative Radiation Pressure. Let us stress again that this is true only when the reflection is sufficiently small. However this feature was confirmed by both solving the linearized system of ODEs (12) and the full system of nonlinear PDEs (2). When $|\omega_2| < |\omega_1|$ meaning $\Omega_2 > \Omega_1$ the scattered wave in the second component carries less momentum, and the soliton is pushed by the wave revealing Positive Radiation Pressure. Moreover there is a critical value of $\omega_1 = \omega_2$ when $\omega_2 = 0$. Above this frequency, the second companion cannot propagate, and only one channel of scattering remains open. At $\omega_1 = \omega_c$ the kinetic term in the linearized system has a secular term meaning that the Fourier decomposition doesn't work in this case. Near that critical value the force acting on the soliton becomes large, showing resonant behavior. These predictions were verified by solving the linearized equation (11) for some set of parameters. For constant amplitudes $A_1 = 1$ and $A_2$ we varied $\beta_1 = \beta_2 = \beta$ and frequency $\omega_1$.

For $\beta < 1$ ($\Omega_2 < \Omega_1$) we generally obtained small but negative force and for $\beta > 1$ ($\Omega_2 > \Omega_1$) the force was always positive (Fig. 2a). For constant frequency $\omega_1 = -0.5$ we also varied $\beta_1 = \beta_2 = \beta$ and the polarization of the soliton but keeping $A_1^2 + A_2^2 = 1$ constant. For $A_1 < A_2$ and $\beta < 1$ or $A_1 > A_2$ but with $\beta > 1$ the frequency of the second companion was larger $\Omega_2 > \Omega_1$ and the force was positive. For $A_1 > A_2$ and $\beta < 1$ or $A_1 < A_2$ but with $\beta > 1$ ($\Omega_2 < \Omega_1$) and when the reflection was small enough negative force was obtained (Fig. 2b). Especially on this figure but to some extent also on the Fig. 2a one can see two lines along which the force changes very rapidly. One of them is easily to identify as the critical frequency. The second line corresponds to the first resonance or quasi-normal mode which we believe should be present for $\beta > 1$ and small values of $A_2$. We have also performed a full numerical simulation of the PDE. Example comparison between the acceleration measured from the full nonlinear numerics and the linearized analysis is shown on Fig. 2. We have found that the agreement is sufficient only for large frequencies. Near the critical frequency the linear approximation breaks.

Long time behavior. Note that the solution to the linear equation satisfying the boundary conditions (1) and (2) violates the continuity equation. It is especially obvious in the sector without the incoming wave. The continuity equation states that $|R_{12}^2| + |T_{12}^2| = 0$. In fact this is not a contradiction because the continuity equation can be satisfied using higher order terms. However one important conclusion can be drown here already. Physically the wave created by the scattering carries away particles and energy from the soliton (forced emission). In order to create a wave in the sector without incoming wave the amplitude of the soliton must decrease. This is also what can be observed in the full numerics (Fig. 3). Probably higher order analysis would reveal this feature from the perturbation series. It is worth noting that similar secular terms to that near the critical frequency are present also in higher orders of the perturbation series for any frequency. The source term $g^{(2)}$ contains terms like $\psi_1^{(1)} \psi_1^{(1)}$ which oscillate with the frequency of the soliton. This term does not depend on the frequency $\omega$ because the complex conjugation kills that dependence. Therefore it cannot be canceled by any cancellation of the resonant terms procedure. The meaning of this feature is that the Fourier decomposition may work only for a very short time, before the resonant terms become important. We believe that these resonant terms are responsible for the changes of amplitude of the soliton.

Conclusions. In the paper we have shown that solitons in CNLSE show very rich and sometimes unexpected interaction with small waves. The most important conclusion is that the solitons can experience both positive and negative radiation pressure depending on the sector in which the incident wave travels. Scattering from faster rotating counterpart can result in NRP. Scattering from slower to faster always results PRP. We have also found that the wave can take away particles from the soliton leading to the decrease of its amplitude very similar to the forced emission known from atomic physics. This effect is very different from interactions between topological solitons and waves studied in our earlier papers. Although CNLSE is nonrelativistic equation we expect to see similar phenomenon also in case of the multicompont Q-Balls which can have importance in cosmological context. Scattering on Q-balls can also happen between modes with different frequencies and momenta.

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FIG. 2: a) Force acting on soliton calculated from the solutions of \[ \text{(12)} \] for \( A_2 = 0.5 \) and \( A_1 = 1 \) as a function of \( \beta_1 \) and \( \omega_1 \). NRP visible as blue for \( \beta_1 < 1 \). Critical frequency is shown with dashed line.
b) Force acting on soliton calculated from the solutions of \[ \text{(12)} \] for \( \omega = -0.5 \) as a function of \( \beta_1 \) and \( A_1 = \cos \phi, A_2 = \sin \phi \). NRP visible as blue for \( \beta_1 < 1 \) and \( A_2 < A_1 \) and for \( \beta_1 > 1 \) and \( A_2 > A_1 \).
c) Acceleration calculated from linearized problem (ODE) compared with the full PDE solution for \( \beta_1 = \beta_2 = \beta \), \( A_1 = 1 \), \( A_2 = 0.5 \) and \( \omega_0 = 0.01 \) as a function of wave frequency \( \omega \). Solid line shows the acceleration for the wave in the first sector, dashed in the second.
d) Acceleration calculated from linearized problem (ODE) compared with the full PDE solution for \( \omega = -0.5 \) and \( \omega_1 = 0.01 \) as a function of wave frequency \( \beta_1 = \beta_2 \).

FIG. 3: Path of the soliton and the evolution of its amplitude under the influence of a large wave \( a_2 = 0.1 \) for \( \omega = 2, \beta = 2, A_1 = 1, A_2 = 0.5 \). Initially the soliton is pulled by the wave. But in time the amplitude change and when \( A_1 \) becomes smaller than \( A_2 \) the acceleration changes its direction.

**Supplementary material.** The paper has additional material in form of four short animations depicting the problems discussed.

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