Non-conformal asymptotic behavior of the time-dependent field-field correlators of 1D anyons

O. I. Pătu$^{1,2}$, V. E. Korepin$^1$ and D. V. Averin$^3$

$^1$C.N. Yang Institute for Theoretical Physics, State Stony Brook University - Stony Brook, NY 11794-3840, USA
$^2$Institute for Space Sciences - Bucharest-Măgurele, R 077125, Romania, EU
$^3$Department of Physics and Astronomy, Stony Brook University - Stony Brook, NY 11794-3800, USA

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Abstract – The exact large time and distance behavior of the field-field correlators has been computed for one-dimensional impenetrable anyons at finite temperatures. The result reproduces known asymptotics for impenetrable bosons and free fermions in the appropriate limits of the statistics parameter. The obtained asymptotic behavior of the correlators is dominated by the singularity in the spectral density of the quasiparticle states at the bottom of the band, and differs from the predictions of the conformal field theory. One can argue, however, that the anyonic response to the low-energy probes is still determined by the subleading conformal terms in the asymptotic expansion.

Introduction. – When confined to move in one dimension, hard-core impenetrable particles cannot be exchanged directly. This means that the symmetry of the wave function with respect to exchanges of coordinates can be defined arbitrarily, e.g., independently of the dynamic particle-particle interaction (see, e.g., [1,2]). The wave function describes then the exchange statistics with, in general, fractional statistics parameter $\kappa \in [0,1]$ interpolating between Bose ($\kappa = 0$) and Fermi ($\kappa = 1$) particles. Quantitatively, this “anyonic” exchange statistics implies that the multi-particle wave functions behave as [3–5]:

$$\psi(\cdots, x_i+1, x_i, \cdots) = e^{i\pi \kappa \epsilon(x_i-x_{i+1})} \psi(\cdots, x_i, x_{i+1}, \cdots),$$

upon exchange of the two nearest-neighbor coordinates, where $\epsilon(x) \equiv x/|x|$, $\epsilon(0) = 0$. While the wave functions $\psi$ depend on the statistics parameter $\kappa$, the absence of real exchanges means that thermodynamic properties of the hard-core particles are independent of statistics. In particular, in the simplest case of only hard-core interaction of zero radius, which is considered in this work, the thermodynamics coincides [6,7] with that of free fermions.

The exchange statistics manifests itself, however, in the correlation functions of the field operators, which describe processes with changing number of particles. In the general case of 1D anyons, only “static” same-time correlators of the field operators have been calculated [8–12] so far. The purpose of this work is to present the exact calculation of the large time and distance asymptotics of the time-dependent field-field correlators of impenetrable anyons. The time-dependent correlators are more relevant experimentally, as they determine, e.g., the tunneling density of states of the system. For anyons, tunneling characteristics can be measured, e.g., in FQHE structures (see, e.g., [5,13]). The new qualitative feature of the time-dependent anyonic correlators obtained in this work is that, in contrast to previously found [14,15] asymptotics for impenetrable bosons, the correlators for the general statistics parameter $\kappa$ do not exhibit conformal behavior in the low-temperature limit. This result challenges the accepted notion that the large-time asymptotic behavior of the correlators of the 1D systems is determined by the low-energy excitations. Our exact calculation shows that the large-time asymptotic behavior can actually be dominated by the singularity of the density of the quasiparticle states at the bottom of the energy band which is characteristic for one dimension. This conclusion adds another limitation on the applicability of the conformal field theory to the large space and time asymptotic description of the 1D particle systems. Previously known limitations were related to the effects of the strong interaction on particles.
with spin, which leads to vanishing of the exchange energy scale, making the spin dynamics non-conformal [16–19], and the effects of finite nonlinearity of the quasiparticle spectrum at the Fermi energy [20].

The model and main result. – In details, the model considered in this work describes one-dimensional hard-core anyons interacting via a δ-function potential, which simulates the hard-core repulsion, and are free otherwise. The second quantized Hamiltonian of such anyons is given by

$$H = \int dx \left[ \frac{\partial_x \psi(x) \partial_x \psi(x) + e \psi(x) \psi(x) \psi(x)}{2} \right] - \hbar \psi(x) \psi(x),$$

(1)

in the $c \to \infty$ limit. Here $h$ is the chemical potential, and the anyonic fields satisfy the following commutation relations

$$\psi(x) \psi(x') = e^{-i\pi \kappa} \psi(x) \psi(x') + \delta(x - x'),$$

$$\psi(x) \psi(x') = e^{i\pi \kappa} \psi(x) \psi(x').$$

(2)

The focus of this work is on the calculation of the large time and distance asymptotic behavior of the field-field correlation function at finite temperature defined as

$$\langle \psi(x_0) \psi(x_1) \rangle = \text{Tr}(e^{-\beta H/T} \psi(x_0) \psi(x_1)) / \text{Tr}(e^{-\beta H/T}).$$

As a function of the statistics parameter $\kappa$, this correlator should interpolate between the case of impenetrable bosons ($\kappa = 0$) and free fermions ($\kappa = 1$).

To state first our main result, we introduce the rescaled variables $x = (x_1 - x_2) \sqrt{T/2} > 0$, $t = (t_2 - t_1) T/2 > 0$, $\beta = h/T$. In terms of these variables, the field correlator can be written as

$$\langle \psi(x_0) \psi(x_1) \rangle = \sqrt{T} g(x_0, x_1, \beta, \kappa),$$

(3)

where the function $g$ is defined below. Due to the fact that $g(x_0, x_1, \beta, \kappa) = g(-x_0, -x_1, \beta, -\kappa)$ and $g(x_0, x_1, \beta, \kappa) = g^*(x_0, -x_1, \beta, -\kappa)$ it is sufficient to consider only the case $x_0 > 0, t > 0$. One needs to distinguish the space-like, $x/2t > \sqrt{\beta}$, and time-like, $x/2t < \sqrt{\beta}$, regions. While the leading term in the asymptotics is the same in both regions, the next-to-leading term behaves in them differently. The large time and distance asymptotic form of the function $g$, for $x, t \to \infty$, with $x/t = \text{const}$, is

$$g(x, t, \beta, \kappa) \simeq e^{\nu^2/2} e^{C(x, t, \beta, \kappa) + i \pi \kappa} \int e^{i(x_0 - x_1) \sqrt{T / 2}} \left[ c_0 e^{-\nu x / \sqrt{T}} + c_1 e^{i(x_0 - x_1) \sqrt{T / 2}} + o(1 / \sqrt{T}) \right],$$

(4)

where

$$\lambda_0^\pm = [-\alpha^{1/2} \mp i (\alpha - 2 \beta)^{1/2}] / \sqrt{2}, \quad \alpha = \beta + \sqrt{\beta^2 + \pi^2 \kappa^2},$$

and the upper (lower) sign corresponds to the space-like (time-like) regions. Also, $\lambda_0 = -x/2t$, $\nu = -i \ln |\varphi(\lambda_0^+, \beta, \kappa)|$, the constants $c_0, c_1$ are some undetermined amplitudes, $I(\beta, \kappa) = 3 \int e^{C(x, t, \beta, \kappa)} d\lambda / \pi$, and

$$C(x, t, \beta, \kappa) = \frac{1}{\pi} \int e^{-x^2 - 2t \lambda |\ln |\varphi(\lambda_0^+, \beta, \kappa)| |} d\lambda,$$

(5)

with

$$\varphi(\lambda_0^+, \beta, \kappa) \equiv (e^{x^2 - \beta} - e^{-i \pi \kappa}) / (e^{x^2 - \beta} + 1).$$

(6)

Even though it seems that the second term (with amplitude $c_1$) in the expansion (4) is superfluous, because it is in general exponentially decreasing and smaller than the error term, one can see that it gives the dominant contribution in the bosonic limit, thus justifying its presence. Another remark is that the precise analytic expression for the second term in the expansion (4) depends on the analytical structure of the solution of the Riemann-Hilbert problem. While there is no additional difficulties in analyzing all regimes, eq. (4) is written out specifically in the situation when this term is determined by the pole at $\lambda_0^0$, which is the case when

$$|\Re \sqrt{\beta + i \pi \kappa} - x / 2t| > 3 \sqrt{\beta + i \pi \kappa}. (7)$$

The positive branch of the square root should be taken in this realization. This condition is always satisfied, in particular, in the more interesting low-temperature regime, $\beta \gg 1$. An observation should be made regarding the amplitudes of the leading terms in eq. (4). Even though we were not able to compute them here for the time-dependent correlators, they were determined in the static case. At zero temperature, the amplitudes of the leading terms were obtained in [8] using the Fisher-Hartwig conjecture and in [10] with the help of the replica method generalized to the anyonic case. For finite temperatures, the leading amplitudes were computed in [21].

Analysis of the result. – We begin the analysis of eq. (4) by demonstrating that it reproduces the known asymptotics for bosons and fermions. In the bosonic limit $\kappa \to 0$, we have $\varphi(\lambda_0^+, \beta, 0) = (e^{x^2 - \beta} - 1) / (e^{x^2 - \beta} + 1), \nu = -(1/\pi) \ln |\varphi(\lambda_0^+, \beta, 0)|$, and

$$C(x, t, \beta, 0) = \frac{1}{\pi} \int e^{-x^2 - 2t \lambda |\ln |\varphi(\lambda_0^+, \beta, 0)| |} d\lambda.$$  

Also, $I(\beta, 0) = -2 \sqrt{\beta}$ for $\beta > 0$, and $I(\beta, 0) = 0$ for $\beta < 0$. In the case of negative chemical potential, $\beta < 0$, condition (7) implies that the second term in the expansion (4) is exponentially small and the asymptotic behavior is determined by the first term, which reduces for $\kappa = 0$ to:

$$g(x, t, \beta, 0) \simeq c_0 e^{-(\nu - 1)^2 / 2} e^{x^2 - 2t \lambda_0^+ \beta}.$$  

For the positive chemical potential, however, $\lambda_0^+ = -\sqrt{\beta}$, and the leading contribution in both space-like and
time-like regions is given by the second term in the expansion (4), thus giving for $\beta = h/T > 0$:

$$g(x, t, \beta) \simeq c_1 t^{\nu}/2e^{C(x, t, \beta, 0)}.$$  

Both of these expressions agree with the previous results [14,15] for the impenetrable bosons.

In the case of free fermions, the equivalent $g$-function in the rescaled variables is

$$g(x, t, \beta) = \frac{e^{2it\beta}}{2\pi} \int_{-\infty}^{+\infty} e^{2i\pi \lambda^2 - 2i\pi \lambda} \, d\lambda.$$  

In the large $t$ and $x$ limit with $t/x = \text{const}$, the steepest descent method gives the following asymptotic behavior under the condition (7):

$$g(x, t, \beta) \simeq c_0 t^{-1/2} e^{2it(\beta + \lambda_1)} + c_1 e^{2(\pm i\pi - i\lambda^2)},$$

where $\lambda_1^2$ are given by the same expression as $\lambda_0^2$ with $\kappa = 1$, and as before, the upper (lower) sign corresponds to the space-like (time-like) region. Comparing this expression to eq. (4), which is simplified for $\kappa \rightarrow 1$ by the relations $C(x, t, \beta, 1) = I(\beta, 1) = \nu = 0$, we see that eq. (4) indeed reproduces the correct behavior of the asymptotic correlators for free fermions.

At positive chemical potential and low temperatures the system is expected to exhibit conformal behavior. One of the simplest ways to calculate the asymptotic correlators in this regime is to use the standard bosonization approach [22], in which the field operators are expressed through two bosonic fields: the integral $\theta(x)$ of the long-wave part of the density fluctuations $\rho(x)$, i.e., $\partial \theta(x)/\partial x = \rho(x)/\pi$, and the conjugate field $\phi(x)$ defined by the commutation relation $[\phi(x), \theta(x')] = i\pi \delta(x - x')/2$. The fact that the operator $e^{i\phi}$ reduces density by the amount that corresponds to one particle, and $\theta$ produces the appropriate phase changes of the wave function across each particle, implies then that the anyonic operators (2) can be written as [23]

$$\Psi(x, t) \sim \sum_m e^{-i(\kappa + 2m) [k_F x + \theta(x, t)]/e^{i\phi(x, t)}}, \quad (8)$$

The sign $\sim$ in eq. (8) is the reminder that the relative amplitudes of different components are not defined in the sum. The commutation relations between $\theta$ and $\phi$ shows directly that the individual terms in (8) satisfy the appropriate exchange relations (2) away from $x_1 = x_2$. The standard average over the equilibrium fluctuations of $\theta$ and $\phi$ gives then the large time and distance behavior of the correlator of the anyonic field operators (8) [7,23]. The two leading asymptotic terms are

$$\langle \Psi(x_2, t_2)\Psi^\dagger(x_1, t_1) \rangle = \begin{cases} b_0 e^{ik_F x_{12}} \left[ u_{-1}(1-\kappa)^{1/4} [u_{-1}(1+\kappa)^{1/4}]^2 \right] & \text{for } x_1 < x_2, \\ b_1 e^{ik_F x_{12}} \left[ u_{-1}(1+\kappa)^{1/4} [u_{-1}(1-\kappa)^{1/4}]^2 \right] & \text{for } x_1 > x_2 \end{cases},$$

where $u_{\pm} \equiv \sinh[\pi T(t_{21} - i0 \pm x_{12}/v_F)]$, the constants $b$ are some undetermined amplitudes of different components of the correlator, and $t_{21} = t_2 - t_1$, $x_{12} = x_1 - x_2$. The Fermi vector and the Fermi velocity are, respectively, $k_F = \sqrt{\kappa}$ and $v_F = 2\sqrt{\kappa}$.

In eq. (9), we kept the two leading terms to facilitate comparison to the case of the static correlators [12], for which both terms are important in the vicinity of the Fermi point $\kappa = 1$. For the dynamic correlators considered in this work, it is assumed that $t_{21} \gg 0$. In this case, only the first term is relevant for our discussion. In the $T \rightarrow 0$ limit, the criterion used above to separate the space-time and time-like regions reduces to the comparison between $t_{21}$ and $x_{12}/v_F$. Keeping only the leading term in eq. (9) we see that this equation reduces to

$$\langle \Psi(x_2, t_2)\Psi^\dagger(x_1, t_1) \rangle \simeq e^{ik_F x_{12}} e^{\pi T t_{21}} e^{-\frac{x_{12}}{v_F} \left( \frac{1}{2} + \frac{\pi}{2} \right)},$$

in the space-like region, and to

$$\langle \Psi(x_2, t_2)\Psi^\dagger(x_1, t_1) \rangle \simeq e^{ik_F x_{12}} e^{\pi T t_{21}} e^{-\pi T t_{21}} e^{-\frac{x_{12}}{v_F} \left( \frac{1}{2} + \frac{\pi}{2} \right)},$$

in the time-like region. In the same low-temperature limit, one can also simplify our main expression (4). Indeed, for $T \rightarrow 0$ one has $i\pi I(\beta, \kappa) \approx 2\pi \sqrt{\beta}(\kappa - 1)$, and $\lambda_0^2 \approx -\sqrt{\beta} + \frac{x_{12} \pi}{2\sqrt{\beta}}$. Also,

$$C(x, t, \beta, \kappa) = -\pi(1-\kappa)^2 \left\{ \begin{array}{ll} x/(2\sqrt{\beta}), & \text{for } x/2t > \sqrt{\beta}, \\ t, & \text{for } x/2t < \sqrt{\beta}. \end{array} \right.$$  

Using these equations one can see directly that the exponential parts of the next-to-leading term in eq. (4) combine to give exactly the same result as predicted by CFT. This means that although the leading asymptotic term in the correlator (4) is not conformal, the next-to-leading term is. The exponential factors in the leading term of the correlator (4) imply that in the low-temperature limit, $\beta \gg 1$, this term can manifest itself in the response of the anyonic liquid to external perturbation only in the case of large energy $\omega$ of the perturbation $\omega \simeq \epsilon_F = h$. Therefore, at small energies, $\omega \ll \epsilon_F$, the response is determined by the second asymptotic term, and coincides with the prediction of the CFT.

To see this more explicitly, one can consider the simplest example of weak single-point tunneling into the anyonic liquid. As usual, the tunneling density of states is given in this case by the Fourier transform of the field correlator with $x = 0$. Equation (4) for the anyonic correlator reduces for $x = 0$ (as before, $\beta \gg 1$) to

$$g(0, t, \beta, \kappa) \simeq e^{C(0, t, \beta, \kappa)} \left[ c_0 e^{2it\beta/\sqrt{T}} + c_1 e^{-2it\kappa} \right].$$  

Taking Fourier transform of this correlator, one can see that the contribution of the first term to the tunneling density of states $A(\omega)$ is $A(\omega) \propto (h - \omega)^{-1/2}$. Although the large-time asymptotics (10) is not sufficient to establish all features of $A(\omega)$, this estimate illustrates the fact that
this term in the asymptotic correlator is produced by the singularity of the 1D density of states at the bottom of the band. This implies that at small energies this contribution to tunneling will be suppressed, and the tunneling density of states will exhibit the power law nonlinearities with the power \((1 + \kappa^2)/2\) produced by the leading term in the CFT correlator (9).

As the last part of the discussion of our main result (4), we provide a simple heuristic interpretation of the exponential factors in the leading term in this equation. The qualitative statement proven by this interpretation is that in the \(T \neq 0\) regime considered in this work, the exponential decay of the correlator is caused mainly by the thermal fluctuations of the number of particles which create the fluctuations of the accumulated statistical phase. The average taken over these phase fluctuations leads to the exponential suppression of the correlator.

To see this, one starts with the standard Wigner-Jordan transformation expressing the anyonic fields (2) in terms of (in our case free) fermions \(\xi\):

\[
\Psi(x, t) = e^{i\pi(1 - \kappa)n(x, t)} \xi(x, t), \quad n(x, t) = \int x' \rho(x', t),
\]

where \(\rho = \xi^\dagger \xi\) is the operator of the particle density. This transformation results in the following expression for the anyonic field-field correlator:

\[
\langle \xi(x_1, t_1) \xi(x_2, t_2) \rangle = \langle \xi(x_1, t_1) \xi(x_2, t_2) e^{i\pi(\kappa - 1)n(x_1, t_1)} \rangle,
\]

where \(n_{d} \equiv n(x_1, t_1) - n(x_2, t_2)\). For large distances \(x_1 - x_2\) and time differences \(t_2 - t_1\), the fluctuations of \(n_{d}\) can be calculated from the following quasiclassical considerations. Contribution to these fluctuation with particles which differ from each other by different momenta \(k\) can be treated independently from each other. For given \(k\), the particle fluctuations at the initial moment \(t_1\) are governed by the Fermi distribution \(\tau(k) = 1/e^{(k^2 - h)/T} + 1\), which gives the probability density \(\tau(k) dk/2\pi\) for momentum-\(k\) particles to be present in any space interval. The resulting random distribution of particles moves with time with velocity \(2k\) (in our notations) \(2k\). The difference \(n_{d}\) can then take values 1, 0, or -1, producing the phase factors in the correlator \(-e^{i\pi \kappa}, 1, -e^{-i\pi \kappa}\). Analyzing the shifts of the distributions with time, and averaging over the probabilities to have or not to have a particle in the initial distribution, one sees that

\[
\langle e^{i\pi (\kappa - 1)n_{d}} \rangle = e^{G(x, t; \beta, \kappa) + i\pi I(\beta, \kappa)},
\]

where

\[
G(x, t; \beta, \kappa) = \int_0^x dx' \rho(x', t),
\]

\[
I(\beta, \kappa) = \int \frac{d\lambda}{\pi} \cot (\beta \lambda)
\]

and \(\beta = \pi \kappa/2\). The auxiliary potential \(b_{\pm} = \alpha + \beta - G\) with

\[
\langle \xi(x_1, t_1) \xi(x_2, t_2) e^{i\pi (\kappa - 1)n_{d}} \rangle = e^{G(x, t; \beta, \kappa) + i\pi I(\beta, \kappa)}
\]

and \(\beta = \pi \kappa/2\). The auxiliary potential \(b_{0} = \alpha + \beta - G\) with

\[
\langle \xi(x_1, t_1) \xi(x_2, t_2) e^{-i\pi (\kappa - 1)n_{d}} \rangle = e^{G(x, t; \beta, \kappa) - i\pi I(\beta, \kappa)}
\]

and \(\beta = \pi \kappa/2\). The auxiliary potential \(b_{\pm} = \alpha + \beta - G\) with

\[
\langle \xi(x_1, t_1) \xi(x_2, t_2) e^{i\pi (\kappa - 1)n_{d}} \rangle = e^{G(x, t; \beta, \kappa) + i\pi I(\beta, \kappa)}
\]

and \(\beta = \pi \kappa/2\). The auxiliary potential \(b_{0} = \alpha + \beta - G\) with

\[
\langle \xi(x_1, t_1) \xi(x_2, t_2) e^{-i\pi (\kappa - 1)n_{d}} \rangle = e^{G(x, t; \beta, \kappa) - i\pi I(\beta, \kappa)}
\]

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Outline of the derivation. – Finally, we present a brief outline of the derivation of the asymptotic behavior of the correlator (4). The starting point of our calculations is the determinant representation of the space-, time-, and temperature-dependent field-field correlator for anyons obtained in [11]. In rescaled variables, the correlator is given by eq. (3), with

\[
g(x, t, \beta, \kappa) = -\frac{1}{2\pi} e^{2it\beta} b_{\pm} \det(1 + \hat{V}_T).
\]

In eq. (12), \(\hat{V}_T\) is an integral operator acting on the entire real axis with the kernel given by

\[
V_T(\lambda, \mu) = e_+ (\lambda) e_- (\mu) - e_- (\lambda) e_+ (\mu),
\]

where

\[
e_- (\lambda) = \frac{\cos (\pi \kappa/2)}{\pi} \sqrt{\delta(\lambda)} \exp (\lambda),
\]

\[
e_+ (\lambda) = e_- (\lambda) E(\lambda),
\]

\[
E(\lambda) = \text{P.V.} \int_{-\infty}^{+\infty} d\mu \frac{e^{-2i\phi(\mu)}}{\mu - \lambda} + \pi \tan (\pi \kappa/2) e^{-2i\phi(\lambda)},
\]

and \(\phi(\lambda) = t\lambda^2 + x\lambda\). The auxiliary potential \(b_{\pm}\) is defined as

\[
b_+ = \alpha + \beta - G\]

and \(b_{-} = \alpha + \beta - G\). The integral operator appearing in eq. (12) is of the special kind called “integrable” operators [15,26]. This type of operators appears frequently in the investigations of correlation functions of integrable models and distribution of eigenvalues of random matrices [26]. The specific factorization properties of the kernel \(V_T\) allow one to obtain differential equations for the correlators. These differential equations do not depend on the statistics parameter \(\kappa\) and are the same as the ones obtained for impenetrable bosons. The statistics parameter enters in the initial conditions [9,12]. The large time and distance asymptotic behavior of the correlation functions is obtained from the analysis of a matrix Riemann-Hilbert problem associated with these differential equations using the “Manakov ansatz” [14,15,27,28]. The details of the computations will be presented elsewhere.

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