The application of game theory principles for the increase in the productivity of coal industry workers

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Abstract. In the Russian Federation the main factor for the reduced labor productivity in the coal mining industry, with almost the same level of technological equipment as in other countries, is human: non-compliance with safety regulations, low motivation, lack of labor discipline among employees of enterprises. The solution to the problem of increasing the labor productivity of coal industry workers largely depends on how effective the fight against these factors is. By choosing the mechanisms of economic and administrative incentives for employees, as well as by creating a hierarchical control system, it is possible to ensure that the behavior optimal for employees is exactly what management needs. This work is devoted to the discussion of methodology for constructing such mechanisms, based on the analysis of game-theoretic models of organizing a safety management system at a coal mining enterprise, proposed in earlier works by the authors of the paper.

1. Introduction
The importance of the coal mining industry for the economy is difficult to overestimate. Coal accounts for 41% of global electricity production. Coal is used to produce more than 70% of steel in the world. In mining this mineral Russia ranks sixth in the world with a volume of 366 million tonnes in 2016, and China is the largest coal producer (3243 million tonnes) [1]. In coal mining assurance of safe and cost-effective working conditions is of particular importance. This is due to the fact that accidents in mines cause deaths, lead to serious social consequences and disrupt the production rhythm. The problems of relatively low labor productivity in general and high risks for the Russian coal mining industry are closely related. While annual mortality risks per miner in the USA and Russia differ insignificantly from each other and labor productivity directly at the mining sites gradually approaches American as well, the indicators of total productivity and the death toll per 100 million tonnes of products are about 7.5 and 14 times worse than the USA (table 1).

Up to 60% of work carried out at coal mining enterprises of the Russian Federation is accompanied by deviations from technical and technological regulations and violations of labor protection and industrial safety requirements [3-7]. In recent years progress has been made in this issue due to the joint efforts of the state, science and mining industry: the number of fatal injuries per 1 million tonnes of coal production has been reduced from 1 case in 1993 to 0.07 in recent years [8].
Table 1. Comparative safety indicators for the coal mining industry of Russia and the USA in 1991-2009 [2].

| Indicator                                      | Russia   | USA       |
|------------------------------------------------|----------|-----------|
| Average annual production                      | 0.26-0.3 | 1.05-1.1  |
| Fatal workplace injuries                       | 115-180  | 25-49     |
| Fatal workplace injuries per tonne of output   | 42-66    | 2.81-3.91 |
| Fatal workplace injuries per 10 thousand employees | 6.9-8.9 | 3.8 (5.1 for underground ) |

However, a high level of risk of large-scale accidents remains. Researchers note the cyclical nature of large-scale accidents with a large number of fatalities in past injury statistics [8]. Most of these accidents occurred mainly because of organizational reasons: due to inefficiency or lack of production control over compliance with industrial safety requirements, violation of safety rules, technological and labor discipline, careless or unauthorized actions of work executors. Such conclusions are typical not only for Russia, but also for a number of other countries with coal mining industry (for example, Australia [9]). Therefore, a decisive role in the increase of the reliability of modern mines belongs to the level of professional specialization and motivation of people employed in production.

2. Game-theoretic approach to modeling of coal enterprise activities

For the correct organization of the work process, mathematically sound models are required to control the implementation of the requirements of technological discipline, safety and labor protection. The models considered in the work are based on game-theoretic principles of designing economic and social mechanisms [10-12].

We assume that the management team can organize a sudden inspection and identify violations, however, the organization of such inspection requires significant costs. The costs of organizing an external check are denoted by $c_{ex}$. Let the inspection consists of two levels. In the model of such inspection, there are three generalized agents: the foreman (audited agent), the first-level inspector and the external inspector – trusted representative of the managers. The first level inspector is in the face and can always check that the team follows the safety rules. The costs of checking the first level consist only of the inspector’s salary, which is equal to $C_{ins}$ and does not depend on whether the level of methane concentration allows the work to be carried out.

All working hours are divided into periods, in each period the methane concentration exceeds the permissible safety level with probability $h$ and a mine can be inspected by an external inspector. The probability of such check $p$ is a manager’s strategy. In case of violations the first level inspector and the team are fined $f_{ins}$ and $f_{br}$, respectively. The external inspector is a trustee of the managers and never conspires with the foreman and the first level inspector. In the case of an acceptable methane level, the foreman receives an increased income $V_H$, otherwise (when the work is stopped) $V_L$: $V_H > V_L$. The expected value of the team’s income is $S_{br} = (1-h)V_H + hV_L$. In case of dismissal, the team foreman can expect that he will find an alternative source of income providing a salary of $S_{alt} < S_{br}$, and the first-level inspector in the event of dismissal will be able to secure an income of $S_{ins}^{alt}$.

We will call the strategy stable with respect rejecting a coalition if the total gain of the team and the first-level inspector reaches its maximum with the right behavior. These strategies of the managers and their optimality in terms of the organization costs are the main object of our study. Using the assertions and theorems formulated and proved in [13], we will discuss the qualitative features of coalition-stable strategies and illustrate them with numerical examples.

3. Results and discussion

Let the team undergo an external check directly, without involving inspectors of the first level. In this case, the optimal strategy of checks, ensuring the unprofitableness of violations, is given by the
probability \( p_1^* = \frac{V_H - V_L}{f_{br}} \) in the case of violations only at a high methane concentration and \( p_2^* = \frac{h(V_H - V_L)}{f_{br} + f_{ins}} \) in case of constant violations. However, such organizational scheme of checks may turn out to be less effective than a two-level one with an external inspector.

3.1. Calculation of costs for the organization of one- and two-level control schemes.

The first factor affecting the optimal inspection strategy is the team’s ability to quickly assess the current level of methane concentration and to turn off sensors in a timely manner. Two options are possible – violations can occur only with increased concentration, or workers always prefer to turn off or block the gas sensors.

**Statement 1.** In case of safety rules violation only at a high methane level, the strategy is stable with respect to coalition deviations if and only when \( p \geq \frac{V_H - V_L}{f_{br} + f_{ins}} \). In case of constant safety violations, if and only if \( p \geq \frac{h(V_H - V_L)}{f_{br} + f_{ins}} \).

Given the costs for one check, the minimization the total costs for the checks is achieved by their number reduction.

**Statement 2.** An optimal verification strategy that is stable with respect to coalition deviations is defined by the probabilities \( p_1^* = \frac{V_H - V_L}{f_{br} + f_{ins}} \) in case of violations only at a high methane level and \( p_2^* = \frac{h(V_H - V_L)}{f_{br} + f_{ins}} \) in case of constant violations.

Let us consider not a fixed fine as the main disciplinary sanction, but dismissal. Instead of the total income of the team, we will take only the income of its head, since it is he who has the greatest influence on the decision to stop or resume works. Each subject of the production process is aware that in case of dismissal, he will receive a lower salary and estimates the amount of the damage, taking into account the loss of future earnings. From this point of view, dismissal can also be considered as a kind of fine. In particular, for the foreman a one-time fine equivalent to dismissal is \( \alpha(S_{br} - S_{alt}) \), where \( \alpha = (1 - \delta)/\delta \) is the reduction coefficient; \( \delta \) is the discount coefficient related to the period of one inspection, used to recalculate future revenue streams in a single value of the current value [13]. Similarly, for the inspector, such a one-time fine is equal to \( \alpha(S_{ins} - S_{alt}) \), where \( S_{alt}^{ins} \) is the expected salary of the inspector after dismissal under this article.

Let us denote \( k_i, i = 1, 2 \) the remuneration coefficients of the labour of foreman and inspector, respectively, determined from the conditions \( V_H - V_L = k_1 S_{br}, h(V_H - V_L) = k_2 S_{br} \). The optimal probability of inspection by an external inspector is: \( p_i^* = \frac{f_{br} k_i S_{br}}{\alpha(S_{br} + S_{ins} - S_{alt} - S_{alt}^{ins})}, i = 1, 2 \). The total costs for the optimal check probability are \( C(S_{ins}) = S_{ins} + \frac{c_{ex} k S_{br}}{f_{br} + f_{ins}} = S_{ins} + \frac{c_{ex} k S_{br}}{\alpha(S_{br} + S_{ins} - S_{alt} - S_{alt}^{ins})} \).

Let us consider the task of cost function minimization the by choosing the salary of the first level inspector:

\[
C(S_{ins}) \rightarrow \min \\
\text{s.t. } S_{ins} \geq S_{alt}
\]

**Statement 3.** The optimal salary of a first-level inspector is:

\[
S_{ins}^* = \begin{cases} 
\frac{c_{ex} k S_{br}}{\alpha} + S_{alt} + S_{ins}^{alt} - S_{br} & \text{if } \frac{c_{ex} k S_{br}}{\alpha} + S_{alt} \geq S_{br} \\
S_{alt}^{ins} & \text{otherwise}
\end{cases}
\]

The corresponding minimum costs for a two-level inspection are:
We introduce the quantity $\Delta = S_{br} - S_{alt}$, which characterizes the net losses of the foreman from dismissal. The minimum cost for organization of a single-level inspection is $C_1 = \frac{c_{ex} k S_{br}}{\alpha \Delta}$. From statement 3 for a two-level inspection it follows:

$$
\begin{cases}
\frac{c_{ex} k S_{br}}{\alpha \Delta} \geq \frac{2}{\Delta} \quad \text{if} \quad \frac{c_{ex} k S_{br}}{\alpha} + S_{alt} \geq \Delta \\
\frac{c_{ex} k S_{br}}{\alpha \Delta} \geq \frac{S_{alt}^ins}{\alpha} + \frac{c_{ex} k S_{br}}{\alpha \Delta} \quad \text{otherwise.}
\end{cases}
$$

The second system inequality is obviously not satisfied under the indicated condition, that is, it makes sense to compare options only if the costs of one external check exceed $c_{ex, min} = \frac{\Delta^2 \alpha}{k S_{br}}$. If $c_{ex} \leq \frac{\Delta^2 \alpha}{k S_{br}}$, then a single-level check is more profitable than a two-level check.

Let us consider the first inequality in more detail. We denote $a = \frac{c_{ex} k S_{br}}{\alpha}$, then:

$$
a^2 \Delta - 2 a - S_{alt}^ins + \Delta \geq 0 \quad \Leftrightarrow \quad (a - \Delta)^2 \geq S_{alt}^ins \Delta, \quad a \geq \Delta
$$

Thus, we finally obtain the following statement.

Statement 4. Participation in the check of a first-level inspector is more profitable than a direct inspection of a team by an external inspector, if

$$
\frac{c_{ex} k S_{br}}{\alpha \Delta} \geq 1 + \sqrt{\frac{S_{alt}^ins}{\Delta}} \quad \text{if} \quad \frac{c_{ex} k S_{br}}{\alpha} \geq \Delta
$$

(2)

The team head can expect that in case of dismissal due to a violation, his level of training and the universality of his competencies will allow him quickly to find a job with a lower, but still comparable level of earnings. The first level inspector, as a rule, is a representative of a middle managerial level, whose competencies are more specific, and earnings are much higher than that of a team head. In case of dismissal, it is problematic for the inspector to get the same job, therefore, his reduction factor is higher (let us denote it by $\beta$): after being fined his relative loss in income is more significant than that of the foreman ($\alpha$). Then the one-time fine for the inspector is $\beta (S_{ins} - S_{alt}^ins)$ and the costs for organization of the inspection are of the form $C(S_{ins}) = S_{ins} + \frac{c_{ex} k S_{br}}{a \Delta + \beta (S_{ins} - S_{alt}^ins)}$.

Statement 5. The optimal salary of the first-level inspector is

$$
S_{ins}^* = \left\{ \begin{array}{ll}
\frac{\sqrt{\frac{c_{ex} k S_{br} \beta}{\alpha \Delta}} - \alpha \Delta}{\beta} + S_{alt}^ins & \text{if} \quad \frac{c_{ex} k S_{br} \beta}{\alpha} \geq \Delta \\
S_{alt}^ins & \text{otherwise}.
\end{array} \right.
$$

The associated costs for inspection organization are equal
\[ C_2 = \begin{cases} S_{alt_1} + \frac{2\sqrt{c_{ex}kS_{br}\beta} - \alpha\Delta}{\beta} & \text{if } \frac{\sqrt{c_{ex}kS_{br}\beta}}{\alpha} \geq \Delta, \\ S_{alt_1} + \frac{c_{ex}kS_{br}}{\alpha\Delta} & \text{otherwise}. \end{cases} \]

Participation in the safety control of the first-level inspector is more profitable than a direct inspection of a team by an external inspector, if and only if
\[ \frac{\sqrt{c_{ex}kS_{br}}}{\Delta\sqrt{\alpha}} \geq \frac{\alpha}{\beta} + \sqrt{\frac{S_{alt_1}}{\Delta}} \]  

(3)

3.2. An example of calculation of the inspection optimal structure

We will use the obtained results to draw some practical conclusions. We will take the value \( S_{br} = 88\,000 \) as the estimated value of the foreman’s salary. The salary of the foreman after dismissal will significantly decrease, since for some time he will not be able to hold a similar position. Let us assume \( S_{alt} = 0.5S_{br} = 44\,000 \), which corresponds to the average monthly salary in Russia in 2018. Let \( V_L = \frac{1}{2}V_H \), and violations occur only at a high methane concentration. The share of non-routine downtime in the structure of the coal mine working time is estimated from 10% ([14]) to 17% ([15]).

For simplicity of calculations we assume that the average downtime is \( \frac{1}{6} \) of the total working time.

Then \( k_1 = \frac{V_H - V_L}{S_{br}} = \frac{V_H - V_L}{\frac{1}{2}V_H + \frac{1}{6}V_L} = \frac{6}{11} \).

Let the team head estimate the amount of lost income in case of dismissal as his income for 5 years, then \( \alpha = 60 \) and \( c_{ex, \text{min}} = \frac{(0.688\,000)^2\times 60}{5 \times 88\,000} = 3484\,800 \). Let us consider the minimum costs \( c_{ex} \), starting with which a two-level inspection is more profitable than a single-level inspection, as a function from the inspector’s salary after leaving. After data substitution, we get
\[ c_{ex} = \left(1556 + \sqrt{55 \cdot \frac{S_{ins}}{S_{alt}}}\right)^2. \]

For example, if \( S_{alt} = 25\,000 \), then \( c_{ex} = 7443\,100 \). In the practically significant cost range \( c_{ex} \geq 5 \) million rubles the relationship between \( S_{ins} \) and \( c_{ex} \) is close to linear one.

The optimal salary for the first-level inspector is \( S_{ins}^\star = S_{alt} + Q \) where the bonus to the alternative salary is \( \sqrt{\frac{c_{ex}kS_{br}}{\alpha}} + S_{alt} - S_{br} \). In this example at \( Q=32\,116 \) (figure 1).

Let us find the optimal probabilities of inspections for both inspection options. For external inspection costs below \( c_{ex} \) a single-level inspection is the best option. In this case it is necessary to carry out sudden checks with a probability of
\[ p_1^\star = \frac{k_1S_{br}}{a(S_{br} + S_{ins} - S_{alt})} = \frac{6/11 \cdot 88\,000}{60 \cdot 44\,000} = 1.8\%. \]

If we assume that the check takes one business day, then in one year it is necessary to perform 4-5 checks on randomly selected dates. If the costs for external checks exceed the threshold value, then a two-level inspection is necessary, and the probability of external inspection is determined from the condition
\[ p_2^\star = \frac{k_1S_{br}}{a(S_{br} + S_{ins} - S_{alt})} = \frac{k_1S_{br}}{\sqrt{c_{ex}kS_{br}}} = \frac{6/11 \cdot 88\,000}{60 \cdot c_{ex}} = \frac{800}{c_{ex}}. \]

The equilibrium behavior of all participants in the inspection model – with different reduction factors for the inspector and the foreman can be calculated. We estimate the reduction factors of the first level inspector as \( 4\alpha \), and take the same values of other parameters as before; \( \beta = 4\alpha = 240 \).

The minimum value of the total cost of external inspection, which makes sense to consider a two-level inspection option, is
\[ \overline{c_{ex, \text{min}}} = \frac{a^2\Delta^2}{kS_{br}\beta} = 605000. \]
From relation (2) let us find the dependence of the boundary value of costs $c_{ex}$ on the alternative salary of the inspector $S_{alt}$: 

$$\bar{c}_{ex} = \frac{\alpha \Delta}{\sqrt{kS_{br}}} + \frac{S_{alt} \alpha \Delta}{\sqrt{kS_{br}}} = \left(778 + \sqrt{55 \cdot S_{alt}}\right).$$

Figure 2 shows the dependences of the boundary value of costs on $S_{alt}$ for both considered examples ($\beta = \alpha$ and $\beta = 4\alpha$).

**Figure 1.** Dependence of the optimal salary of a first-level inspector on the costs of external inspection ($Q$ – allowance to the inspector’s alternative salary, $c_{ex}$ – inspection costs).

**Figure 2.** Dependence of the minimal costs of external inspection for two- (blue line) and single-level (red line) inspections on the alternative salary of a first level inspector.

In the second case, it is more profitable to organize a two-level inspection. For example, if $S_{alt}^{ins} = 25000$, then $c_{ex} = 4082097, c_{ex} = 8665040$. The optimal salary of the first level inspector is $S_{ins}^* = Q + S_{alt}^{ins}$, where the bonus to the alternative salary $Q = \frac{c_{ex} R S_{br}^H - \alpha \Delta}{\beta} \approx 15458$. This allows us to draw an unexpected conclusion and recommend the need for increased inspection activity with an increase in the salaries of the foreman, that is, an increase in the income of the foreman does not reduce his motivation to commit violations.
4. Conclusion
As a conclusion, we can formulate a number of practical recommendations for the organization of safety control and prevention of emergency incidents. In particular, it is recommended that the control of safety rules observation should be imposed on a two-level inspection. The first level of inspection consists of workers responsible for observation of safety rules at the mining sites themselves, continuously monitoring the workers.

The top level of inspection is represented by trustees of managers. To increase the efficiency of their work it is recommended that they use various control tools that are not under the control of junior managers. In particular, we can talk about the secret installation of portable control devices in places inaccessible for representatives of the mid- and junior-level managers and workers.

The wages of employees involved into the internal inspection of workers is determined according to the formulas for $S_{ins}$. At the same time, an increase in wages for workers directly engaged in production will adversely affect their behavior: an increase in income increases the likelihood of their rules violation in conducting mining operations and the likelihood of their collusion with an internal inspector. As a result, it is necessary to increase the intensity of inspections.

One of the most promising areas for further research is to study the dynamic modification of the model, taking into account the advanced management strategy (in addition to organization of external inspection, it also implies rotation of teams). The preliminary analysis of such model showed great practical possibilities in improving the manageability of the situation in the field of labor protection. A decrease in costs for possibly applicable control organization schemes and, as a result, an increase in its effectiveness was noted. Another important direction in the development of the considered model is the consideration of the remuneration scheme for all employees as a strategy of company management.

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