Short-duration lensing events: I. wide-orbit planets? free-floating low-mass objects? or high-velocity stars?

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ABSTRACT

Short duration lensing events tend to be generated by low-mass lenses or by lenses with high transverse velocities. Furthermore, for any given lens mass and speed, events of short duration are preferentially caused by nearby lenses (mesolenses) that can be studied in detail, or else by lenses so close to the source star that finite-source-size effects may be detected, yielding information about both the Einstein ring radius and the surface of the lensed star.

Planets causing short-duration events may be in orbits with any orientation, and may have semimajor axes smaller than an AU, or they may reach the outer limits of their planetary systems, in the region corresponding to the Solar System’s Oort Cloud. They can have masses larger than Jupiter’s or smaller than Pluto’s. Lensing therefore has a unique potential to expand our understanding of planetary systems. A particular advantage of lensing is that it can provide precision measurements of system parameters, including the masses of and projected separation between star and planet. We demonstrate how the parameters can be extracted and show that a great deal can be learned. For example, it is remarkable that the gravitational mass of nearby free-floating planet-mass lenses can be measured by complementing observations of a photometric event with deep images that detect the planet itself.

A fraction of short events may be caused by high-velocity stars located within a kpc. Many high-velocity lenses are likely to be neutron stars that received large natal kicks. Other high-speed stars may be members of the halo population. Still others may be hypervelocity stars that have been ejected from the Galactic Center, or runaway stars escaped from close binaries, possibly including the progenitor binaries of Type Ia supernovae.

1. Introduction

Typical lensing events are highly degenerate: a given event may correspond to lensing by any of a wide range of masses. (See Dominik 2009 and references therein.) We demonstrate that, when the Einstein-diameter crossing time, $\tau_E$, of an event is short enough, only a narrow range of physical models are possible. For values of $\tau_E$ on the order of days the lens can be a planet, a brown dwarf, or a high-velocity star.

The lens location is also constrained by the event duration. If we consider a given type of lens, characterized by its mass and a range of spatial velocities, the shortest events are associated the lenses that lie very close to the observer, or else very close to the lensed source. Nearby lenses are called mesolenses because the astrometric and photometric effects of lensing both may be detectable, and because the possibility of observing the lens...
system directly opens new avenues of study (Di Stefano 2008a, 2008b). This is particularly intriguing for planetary systems, because the full range of available techniques (lensing, radial velocity studies, transits, and direct detection of planets) may be applied to individual systems, making them among the best-studied planetary systems. Similarly, lensing by nearby brown dwarfs can produce high-precision mass measurements. In fact, two short events caused by nearby brown-dwarf lenses have already been observed. (See Fukui et al. 2007; Gaudi et al. 2008; Gould et al. 2009).

Through the study of short events we can also learn more about high-velocity lenses, especially those that are nearby. Because the rate of events is proportional to the angular speed, high-speed lenses should produce more events per unit mass than slower-moving lenses. High velocities are expected for some classes of objects, such as halo stars. Neutron stars are also promising because they can receive natal kicks. We know that, within a kpc of Earth, there is a large population \((10^6-10^7)\) of non-pulsing neutron stars, but only a relative handful have been discovered (see, e.g., Haberl 2005). A significant fraction (a few percent; Di Stefano 2008b) of detected lensing events are likely caused by nearby neutron stars, making lensing events a potentially very productive way of discovering neutron stars and conducting mass measurements. Among short events, a significant fraction not generated by planets are likely to be generated by high-speed neutron stars (Di Stefano 2009).

Hypervelocity stars, apparently ejected from the Galactic center, are an exciting and relatively recent discovery (see Brown et al. 2009 and references therein). While many, including the prototype, SDSS J090745.0+024507 (Brown et al. 2005) are unbound, there is also a bound component (Brown et al. 2007). While the hypervelocity stars appear to have been ejected from the Galaxy center (see, e.g., Ginsburg & Loeb 2007; cf. Abadi et al. 2009), there is also a population of high velocity “runaway” stars that appear to have originated in the Galactic disk (Blauw 1954, 1993; Bromley et al. 2009). Lensing events are most likely to be detected when the lens is dim; hypervelocity and runaway stars are therefore most likely to produce detectable events after becoming stellar remnants. For example, some high-speed white dwarfs may have started their lives in binaries that later produced Type Ia supernovae. Unlike core collapse supernovae, Type Ia supernovae do not leave a long-lasting remnant: the matter from a white dwarf that achieves the Chandrasekhar mass undergoes explosive nuclear burning and is dispersed. In a very promising class of models, however, the white dwarf gains mass from a non-degenerate companion. The companion is not destroyed by the explosion, and is free after it occurs. If the binary was a close one, the speed of the companion star may be \(\sim 200\) km s\(^{-1}\). Should close binaries constitute the major class of Type Ia supernova progenitors, then the Galaxy could house \(10^7\) high-speed stars, most of them old white dwarfs descended from companions to accreting white dwarfs. Although these are only a small fraction of all white dwarfs, the rate at which they cause lensing events can be significant because they have higher angular speeds than other white dwarfs. In addition, the initial masses of the companion stars in Type Ia supernova progenitors may be large enough to produce white dwarfs with masses \(> 0.6\) M\(_\odot\), thereby also ensuring that these products of binary progenitors produce lensing events at rates higher than other white dwarfs. The failure to find fast moving white dwarfs that are part of a disk population could place constraints on Type Ia progenitor models.

Whatever the nature of the lens, if it is not nearby, but is instead close to the lensed
source star, finite-source-size effects are more likely to be detectable. Finite-source-size can be used to measure the Einstein angle, $\theta_E$, of the lens, thus providing a relationship between the lens mass and its distance, $D_L$ from us. Together with the value of $\tau_E$ derived from a fit to the light curve, $\theta_E$ can be used to compute the value of the angular speed $\omega = v/D_L$. We can then test models in which both $v$ and $D_L$ are relatively small, or relatively large. Finite-source-size effects also allow us to explore features of the source star’s surface.

In §2 we establish that an event with short Einstein-diameter crossing time can be caused only by a limited set of lenses. Section 3 is an overview of the types of measurements that can be used to identify the correct physical model for the lens producing a short-duration event. We focus on effects other than caustic crossings, which were studied for wide-orbit planets in Han 2005. We show that in many cases the lens mass and its distance from us can be determined through a combination of (a) studying the light curve, (b) determining the source size, (c) measuring the astrometric effects of lensing, and (d) detecting the lens or placing limits on the flux we receive from it. These same procedures can determine if a planet orbits a star and, if it does, can measure the mass ratio and projected orbital separation. Such a rich set of tests is available that independent measurements of key quantities, such as the lens mass, can be made in some cases. In §4, we explicitly consider the case in which the lens is a free-floating planet and demonstrate that, especially for nearby planets, the model can be well tested and that mass measurements may be possible in some cases. §5 sketches the advantages of focusing attention on events of short duration.

2. Short-duration Events: What Can the Lenses Be?

2.1. Relevant Equations

If the mass of the lens is $M$ and its distance from us is $D_L$, then the Einstein angle is:

$$\theta_E = 9.025 \text{ milliarcsec} \left( \frac{100 \text{ pc}}{D_L} \right)^{\frac{1}{2}} \left[ \left( \frac{M}{M_\odot} \right) \left( 1 - x \right) \right]^{\frac{1}{2}}$$

$$= 0.2788 \text{ milliarcsec} \left( \frac{100 \text{ pc}}{D_L} \right)^{\frac{1}{2}} \left[ \left( \frac{M}{M_{\text{Jupiter}}} \right) \left( 1 - x \right) \right]^{\frac{1}{2}}$$

(1)

Here, $x = D_L/D_S$, and $D_S$ is the distance to the lensed source. When the angular separation between the source and lens is $\theta_E$ ($2\theta_E$, $3.5\theta_E$), the magnification is 34% (6%, 1%).

The relative angular speed between source and lens is $\omega$. For nearby lenses, we have

$$\omega = \left( \frac{0.0527''}{\text{yr}} \right) \left( \frac{100 \text{ pc}}{D_L} \right) \left( \frac{v}{25 \text{ km/s}} \right).$$

(2)

For lenses closer to the lensed source, the relative motion may not be dominated by the motion of the lens. The expression for $\omega$ is more properly represented as a sum of terms, but we can use Equation 2 to derive an approximate value by setting $v$ equal to the relative transverse speed.
Let $\tau_E$ represent the Einstein diameter crossing time.

\[
\tau_E = \frac{2 \theta_E}{\omega}
\]

\[
= 3.86 \text{ days} \left( \frac{25 \text{ km/s}}{v} \right) \left[ \left( \frac{D_L}{100 \text{ pc}} \right) \left( \frac{M}{M_{\text{Jupiter}}} \right) \left( 1 - x \right) \right]^{1/2}
\]  

(3)

The actual duration of the deviation detected in any realistic case depends on the distance of closest approach between the source and lens and also on the photometric sensitivity. OGLE and MOA are both sensitive to deviations on the order of 1%, so that the duration of the detectable portion of events is generally longer than $\tau_E$.

Events in which light from the lensed source is heavily blended with light from other stars can mimic short-duration events, even when the genuine Einstein diameter crossing times are weeks or months (Di Stefano & Esin 1995). The hypothesis that each short-duration event is simply a highly blended event with a larger value of $\tau_E$ must therefore be considered (see Di Stefano & Esin 1995; Dominik 2009). By tracking the event in several wavebands we can measure the fraction of the baseline light contributed by the lensed source in each, determining its color and spectral type. In addition to quantifying the amount of blending, this facilitates the identification of the lensed star in high-resolution images and allows us to estimate its radius.

The mass of the lens can be expressed as follows.

\[
M = \frac{21.3 M_{\oplus}}{(1 - x)} \left( \frac{D_L}{100 \text{ pc}} \right) \left[ \left( \frac{\tau_E}{1 \text{ d}} \right) \left( \frac{\omega}{0.0527''/\text{yr}} \right) \right]^2
\]

\[
= \frac{21.3 M_{\oplus}}{(1 - x)} \left( \frac{D_L}{100 \text{ pc}} \right) \left[ \frac{\theta_E}{0.072 \text{ mas}} \right]^2
\]

(4)

The value of $\tau_E$ can be determined from fitting the light curve. In many cases, the values of $D_L$ and $\omega$ (or $D_L$ and $\theta_E$) can be measured, producing a high-precision measurement of the lens mass. When $D_L$ and $\omega$ cannot be measured, models of the Galaxy can be used to derive probability distributions for them which can, in turn, be used to construct a probability distribution for the lens mass.

2.2. Lenses Producing Short-Duration Events

2.2.1. Low-Mass Lenses

Here we will make an empirical distinction between brown dwarfs and planets. Let $M_J$ represent the mass of Jupiter. Then, objects with $10 M_J < M < 0.08 M_{\odot}$ will be referred to as brown dwarfs and objects of lower mass will be referred to as planets. Those planet-mass objects that are not bound to stars will be referred to as free-floating planets.

Consider lenses that are part of the Milky Way’s disk or halo population. Values of the transverse speed may range from roughly 10 km s$^{-1}$ to 200 km s$^{-1}$. Figure 1 illustrates that an event with small $\tau_E$ can only be generated by a low-mass lens, either a planet or a brown dwarf. The left panel was generated for a transverse speed of 75 km s$^{-1}$. Consider
a short event with a light-curve fit yielding a value of $\tau_E$. The swath corresponding to this value of $\tau_E$ shows the range of possible lens masses, $M$, and distances from us, $D_L$. For example, given a transverse speed of 75 km s$^{-1}$, events with $\tau_E < 1$ day can only be generated by planet-mass lenses. Even events with durations near 4 days can be produced by objects as massive as brown dwarfs only if the lens lies within a roughly a hundred pc of us, or a similar distance from the Galactic center. Because relatively few brown dwarfs lie within these small volumes, the rate of very short brown-dwarf-lens events is limited. Those very short events that are generated by brown dwarfs are very interesting, however, since the lens will be detectable if it is nearby, and will produce detectable finite-source-size effects if it is in the source system.

Many nearby lenses have smaller transverse speeds (see, e.g. Lépine 2005), while halo stars would likely have larger transverse speeds (see, e.g., Dominik 2006). The left panel of Figure 1 can be easily adapted for any value of $v$: when the transverse speed is 75 km s$^{-1}$/N, then the values of $\tau_E$ for each swath should be multiplied by $N$. For the slower (faster) speeds, any given time duration corresponds to a lens of lower (higher) mass.

2.2.2. High-velocity Lenses

A combination of small (or large) $D_L$ and large $v$ can conspire to produce short lensing events. (See the right panel of Figure 1.) Stars moving with velocities above several hundred km s$^{-1}$ are rare, but they do exist ($\S$1). The two most well-known examples are neutron stars which receive strong natal kicks (see, e.g., Chatterjee et al. 2009), and hypervelocity stars (see Brown et al. 2009 and references therein). In addition, “runaway stars” which may have been ejected from clusters, supernova-progenitor binaries or other close binaries, can have velocities in excess of 200 km s$^{-1}$. (See, e.g., Hoogerwerf, de Bruijne, & de Zeeuw 2001, and references therein.)

The lenses used to generate the points in the right panel of Figure 1 have masses of $1.4 M_\odot$, comparable to typical neutron star masses. They differ from each other in their distances $D_L$ from us and in their transverse velocities, $v$. Each swath plotted in Figure 1 corresponds to a specific value of $\tau_E$. The right panel of Figure 1 shows that with a lens mass of $1.4 M_\odot$, $v$ must be greater than roughly 1000 km s$^{-1}$ (300 km s$^{-1}$) in order for the Einstein-diameter crossing time to be 1 day (2 days) or shorter. Even 4-day events generated by high-velocity stellar-mass lenses are likely to be rare, since for $\sim v = 300$ km s$^{-1}$, the lens must be within $\sim 20$ pc of Earth, or comparably close to the lensed source.

The right panel of Figure 1 can be easily adapted for any value of $M$: when the lens mass is $N \times 1.4 M_\odot$, then the values of $\tau_E$ for each swath should be multiplied by $\sqrt{N}$.

2.2.3. Summary

In summary, Figure 1 illustrates that selecting for small values of $\tau_E$ is guaranteed to select events with interesting lenses. The shorter the event, the lower the mass of the lens and/or the faster its transverse speed.

Assuming that the local population of lenses and the population in the source field is
similar, roughly half of the short events are likely to be generated by lenses close enough to us to allow detailed model tests. A significant portion of the more distant lenses may produce finite-source-size effects, providing constraints on the lens and/or the lensed source. On the other hand, the nearby lenses can be studied by a variety of complementary observations that can combine with measurements of the photometric event to constrain the lens model and in some cases to obtain a measurement of the gravitational mass of the lens and/or of any stellar companion it may orbit.

3. Overview of Model Tests: Breaking the Degeneracy

For each short-duration event, we would like to answer several key questions. What is the nature of the lens? Is it a planet? If so, is it free-floating or bound to a star? Is the lens a brown dwarf? Is the lens a high-velocity object? If it is a high-velocity object, what is it and from where did it come? These questions can be answered by constraining the lens mass, testing for evidence that the lens has a companion, and in some cases, through the direct detection of the lens system. Observing teams are already sampling some short-duration events with the photometric sensitivity and cadence needed to allow meaningful model fits. Especially if alerts are called to encourage almost continuous monitoring, the light curves will quantify the effects of blending, and identify those events with genuinely short Einstein diameter crossing times. Fits of the light curve will provide values of $\tau_E$ and $b$, the angle of closest approach, expressed in units of $\theta_E$. They will also determine if there are signatures of finite-source-size effects or other deviations from the standard point-source/point-lens form.

3.1. Determining $\theta_E$

When the angle of closest approach between the source and lens, $b \theta_E$, is comparable to the angular dimensions of the star, the light curve shape is altered (Witt & Mao 1994; Lee et al. 2009 and references therein). Thus, when the Einstein angle is small, as is the case for low-mass lenses at moderate to large distances, finite-source-size effects are most likely to be detectable through their alteration of the light curve shape. To set the scale, note that $\theta_E$ for an Earth-mass planet at 100 pc is $1.58 \times 10^{-5}$ arcsec, only $\sim 2.7$ times larger than the angular size of a 10 $R_\odot$ star at 8 kpc.

When, on the other hand, the Einstein angle is large it may be possible to measure the astrometric effects of lensing. The Einstein angle of a neutron star, for example, is 0.01" at $D_L = 100$ pc. Under ideal circumstances, this is measurable, because the maximum shift in the source’s centroid of light is nearly equal to the size of the Einstein ring. The maximum shift is attained when the lens-source separation is approximately equal to $\theta_E$. The magnitude of the shift falls off as the inverse of the separation. (See Miyamoto & Yoshii 1995; Hog et al. 1995; Miralda-Escude 1996; Dominik & Sahu 2000; An & Han 2002; Han

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1Neutron stars at roughly this distance are known (Haberl 2005), but, with $\sim 7$ out of roughly 10$^7$ nearby isolated non-pulsing neutron stars discovered so far, our census of nearby neutron stars is woefully incomplete.
To detect the shift, we need good angular resolution, a bright source, and a lens that does not dominate in the wavelength range in which the source is most luminous. Among short-duration events, those caused by high-speed stellar remnants provide the greatest opportunity to measure centroid shifts. Not only are their Einstein rings typically bigger, but in addition, isolated compact objects can be dim (see, e.g., Haberl 2005), making it easier to measure the position of the source star’s centroid of light.

An interesting case to consider is one in which a short-duration event is caused by a planet lens that happens to be in orbit around a star. At the time of the short-duration photometric event caused by the planet, the magnification of the lensed source is dominated by the gravitational influence of the planet; the central star has only a small influence on the magnification if the angular separation, \( \phi \), between the lensed source and the central star of the planetary system is larger than \( \sim 1.5 \theta_{E,*} \). The Einstein ring of the planet is small enough that present-day observations would not be able to detect the astrometric shift in the source position produced by the planet. It is possible, however, that at the time of the short event, the central star can produce a detectable astrometric shift in the lensed source’s light centroid. Depending on the size of its Einstein ring and its luminosity relative to that of the lensed star, its astrometric influence may be detected for \( a \) as large as \( \sim 10 \theta_{E,*} \) using HST, and much farther using astrometry possible in future missions (Han 2006). The detection of the astrometric effects associated with lensing by the central star can measure the value of its Einstein angle, \( \theta_{E,*} \).

### 3.2. Determining \( \omega \) and \( D_L \)

As a larger portion of the sky is monitored with good photometric sensitivity, and as monitoring programs establish longer baselines, the proper motions and parallaxes of a large fraction of nearby stars and brown dwarfs will be measured and published. It will increasingly become the case that no new observations are needed to establish that the lens is likely to be a nearby low-mass object, and to measure its proper motion, and and its photometric and geometric parallax. In such cases, the mass of the lens can be determined simply by using the light curve to measure \( \tau_E \).

Below we concentrate on determining the values of \( D_L \) and \( \omega \) in the more difficult cases in which little or nothing is known about the lens at the time of the event. An example is a case in which the lens system is extremely dim. Lenses that may not be detected even with deep follow-up observations include low-mass free-floating planets and high-velocity isolated black holes. For other dim lens systems, there may be enough flux for detection, but high-precision astrometry to measure proper motion and parallax may be difficult. Fortunately, even when the lens is not detectable, there are situations in which its parallax and/or proper motion can be measured.

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2 There is a complication that must be addressed when the lens is a planet orbiting a star, because the catalogued proper motion is that of the star. In such cases, however, it may be possible to determine or constrain the planet’s orbit (§3.3, §5.3), allowing us to determine the angular speed of the planet at the time of the event.
First, parallax effects can influence the shape of the light curve, even for short events. This was illustrated by a recent short-duration event in which the lens was a brown dwarf located $525 \pm 40$ pc away (Gould et al. 2009). The lens mass is $M = 0.056 \pm 0.004 M_\odot$, and its transverse velocity is $113 \pm 21$ km s$^{-1}$. The degeneracy was broken through a combination of finite-source-size and parallax effects. The reason parallax effects were detectable in this short event was that the magnification was extremely high; in fact it was the high magnification that caused an alert to be called (Griest & Safizadeh 1998). Even with a less extreme magnification, however, a combination of other effects can sometimes accomplish the same goal.

In some cases the proper motion can be measured even without direct detection of the lens. When, for example, the lens is a high-proper-motion object moving across a dense stellar field, it may generate a sequence of independent events. In each, a different source star is lensed. The angular distance between sources, combined with the time between events, then measures the angular speed of the lens. This method relies only on measuring the angular separation between source stars that were lensed at known times. For some nearby lenses, i.e., mesolenses, the angular speed can be high enough, and the Einstein rings large enough that sequential events are expected. A free-floating planet with $D_L = 10$ pc with a transverse speed of $50$ km s$^{-1}$ covers an arcsecond per year; a neutron star at $100$ pc with a speed of $200$ km s$^{-1}$ covers $0.4$ arcsecond per year.\(^3\)

When the lens itself can be detected, as may be the case for nearby brown dwarfs, then we can measure $\omega$ directly. Because the lens and source are coincident at the time of the event, a single image taken after they can be resolved will determine the value of $\omega$ and also the photometric parallax of the lens, yielding the value of $D_L$. A single WFPC2 image taken $\sim 6$ years after the MACHO LMC-5 event obtained a very good measurement of both $\omega$ and $D_L$ (Alcock et al. 2001).\(^4\) Additional measurements and analyses confirmed this, and provided a high-precision mass measurement of an isolated M dwarf (Gould 2004; Gould, Bennett, & Alves 2004; Drake, Cook, & Keller 2004; Nguyen et al. 2004).

Consider the case in which the lens is a planet orbiting a star that can be detected. In this case, it may be possible to measure the distance to the star and its angular speed. The distance $D_L$ to the planet is approximately the same as the distance to the star. The angular speed of the planet is likely to be roughly equal to the angular speed of the star for wide orbits. To refine the measurement of the planet’s proper motion at the time of the event, we must estimate the amount by which the two angular speeds differ. One way to estimate the difference is to measure the projected separation, $a$, between the planet and star (see §3.3 and §5.1).

Finally, even if the lens system is not detected and even if it generates just one event, we can determine the value of $\omega$ if both $\tau_E$ and $\theta_E$ have been measured:

$$\omega = \frac{2 \theta_E}{\tau_E} = \frac{0.0114''}{\text{year}} \left( \frac{\theta_E}{1.56 \times 10^{-5}''} \right) \left( \frac{1 \text{ day}}{\tau_E} \right).$$  \(5\)

\(^3\)The rates generally depend on the background density of sources and the value of $\theta_E$. For some nearby lenses, sequences of low-magnification events are expected to occur over an interval of years to decades (Di Stefano 2008a.)

\(^4\)The lens in this case was an M dwarf, not a brown dwarf.
3.3. Determining System Parameters

We would like to determine the intrinsic properties of the lens, including its mass, and whether or not it has companions. As the example of MACHO-LMC-5 illustrates, the mass of a lens can be measured if $\tau_E, D_L,$ and $\omega$ are measured. The mass of the lens can be the most significant clue to its nature. If, for example, we find that the mass of a nearby lens is $0.001 M_\odot, 1.4 M_\odot, 7.0 M_\odot$, and there is no obvious optical counterpart, then the lens is likely to be, respectively, a free-floating planet, neutron star, black hole.

Determining whether or not the lens generating a short-duration event has a companion can be simple or challenging, depending on the situation. Here we will consider the case in which the lens is a planet which orbits a star. The Einstein angle of the star can be written as follows.

$$R_{E,*} = 0.4513 \text{ AU} \times \left[ \frac{M}{0.25 M_\odot} \left( \frac{D_L}{100 \text{ pc}} \right) \left( 1 - x \right) \right]$$

(6)

In the “classical” events which have been used to discover planets with lensing, the projected orbital separation, $a$, lies between $0.5 R_{E,*}$ and $1.5 R_{E,*}$. This is the zone for resonant lensing, so named because the planet-induced deviations are often associated with caustic crossings and can be dramatic. For planets in the zone for resonant lensing the event is not short. Instead, the time duration of the event is determined by the Einstein-diameter crossing time of the central star, while evidence of the planet is provided by a short-lived deviation from the stellar-lens event. For both smaller (Di Stefano & Night 2008) and larger separations (Di Stefano & Scalzo 1999a), isolated events of short-duration are expected. When $a < 0.5 R_{E,*}$, the short-duration events should all bear the signature of the star’s influence. This range of separations is interesting, corresponding to the habitable zones of many nearby stars (Di Stefano & Night 2008). Nevertheless, in the remainder of the paper we focus on planets in wide orbits ($a > 1.5 R_{E,*}$). These wide-orbit planets alone they are likely to produce more events than planets in the zone for resonant lensing (see also Di Stefano & Scalzo 1999a, 1999b). We return to the short-duration events expected when the orbital separation is small in the companion paper (Di Stefano 2009).

For separations in the range between roughly $1.5 \theta_{E,*}$ and $3.5 \theta_{E,*}$, the gravitational influence of the central star is significant enough to perturb the short-duration planet-lens light curve from the point-lens form that would have been created by a free-floating planet. (See Figure 2.) The star-induced deviation is long-lasting, because its duration is comparable to the Einstein-diameter crossing time of the star. It therefore frames the planet-lens light curve. This star-induced deviation (a) can be used as a clue that the lens is a planetary system; when a shorter higher-magnification event occurs on top of it, observers can call alerts with a high degree of confidence that the lens is a planetary system; (b) can be used to obtain the possible values of the mass ratio ($q = m_{\text{planet}}/M_*$), and also the orbital separation, $a$, expressed in units of the Einstein radius of either the star or planet. As mentioned in section 3.1, it may be possible to measure astrometric shifts associated with lensing by the central star and to thereby measure the star’s Einstein angle. Generally such measurements would require HST. It is therefore fortunate that, when the separation is in the range above, there is also a photometric signature that provides clear evidence for the possible value of HST observations.
In addition, for all values of $a$ large enough that the planet causes an independent event, there are sets of source tracks that pass behind the lensing regions of both the planet and the star it orbits. If the track of the source passes near both the planet and star, two independent events are generated, one of short duration and the other of longer duration. For such repeating events, the values of $q$ and $a$ can be derived (Di Stefano & Mao 1996; Di Stefano & Scalzo 1999b).

Note that none of the information mentioned in the paragraphs above relies on detecting the central star. Therefore, even in the case in which the central star is a dark compact object, a neutron star or black hole, it may be possible to derive a great deal of information about the planetary system. In many cases, however, particularly if the system is nearby, the star can be detected. This makes it possible to learn about the system in a variety of other ways. For very wide orbits, the separation between the central star and lensed star (hence the planet) may be measurable at the time of the event. Whatever the orbital separation, detection of the central star can provide an opportunity to directly measure $\omega$ and $D_L$.

The combination of information derived through lensing and direct detection of the central star can provide a detailed model of the planet-star system. In addition, if the system is close enough, subsequent studies can be carried out to search for radial velocity variations, transits, and even to conduct imaging studies (Marois et al. 2008, Kalas et al. 2008). In some cases these could detect evidence of the planet that produced the short-duration event. In others, additional planets can be discovered.

4. Planets: Bound or Free-Floating?

Our understanding of the formation of low-mass objects is still evolving. In particular we don’t know the mass distribution of low-mass objects bound to stars, or the frequency with which such objects are ejected. Nor do we know the low-mass limit for objects that can form in isolation. (See, e.g., Kroupa & Bouvier 2003; Gahm & Kristen 2007; Hurley & Shara 2002). It is therefore difficult to predict the numbers and characteristics of free-floating planets. Observations are difficult as well, because the planets would be small, cool, and dim. As lenses, however, free-floating planets produce effects that are the same as the effects produced by planets orbiting stars in wide orbits.

For planets orbiting stars, the detection of the star can provide important information such as a value for $D_L$ and an estimate of the transverse speed. For free-floating planets, no such source of additional information is available. In fact, the absence of a stellar companion can be taken as indirect evidence that a lens producing a short-duration event is free-floating, but only if we can establish that the lens is close enough that any stellar companion would be detectable. Fortunately, the combination of short event duration and the ability to detect or place limits on finite-source-size effects is very powerful.

Suppose that a combination of studying the light curve and identifying the lensed source has allowed us to either measure the Einstein angle or to place a strong upper limit: $\theta_E \geq \theta_{E}^{\lim}$. Combining equations (2) and (5), we can express $D_L$ as follows.

$$D_L \leq 463 \text{pc} \left( \frac{v}{25 \text{ km/s}} \right) \left( \frac{\tau_E}{1 \text{ day}} \right) \left( \frac{1.56 \times 10^{-5\prime\prime}}{\theta_{E}^{\lim}} \right). \tag{7}$$
The measurement of $\tau_E$ and $\theta^\text{lim}_E$, with values in the range considered above, accomplishes three things. First, as the relation above shows, these measurements can establish that the lens is nearby. The only caveat is that the transverse velocity should be in the range of velocities typical of either disk stars or halo stars. Even for extreme stellar velocities, the derived limit can be important. For example if the lens is a halo object with a transverse velocity of $250 \text{ km s}^{-1}$, it must lie within 5 kpc. The smaller the value of $D_L$, the more likely it is that we can detect a stellar companion, if one exists. Second, measurement of $\tau_E$ and $\theta^\text{lim}_E$ establishes that the mass of the lens is likely to be in the planet range. This can be seen by combining Equations (4) and (7). Third, measurement of $\tau_E$ and $\theta^\text{lim}_E$ places a lower limit on the value of $\omega$ [Equation (5)]: $\omega \geq 0.0114''/\text{yr} \left[\theta^\text{lim}_E/(1.56 \times 10^{-5})''\right]\left[(\text{day})/\tau_E\right]$.

### 4.1. Eliminating Bound-Planet Models

Suppose that the lens is a planet orbiting a star. Let the angular separation between the planet lens and the star it orbits be $\eta''$. The projected separation between them is

$$a = 100 \text{ AU} \frac{\eta'' D_L}{100 \text{ pc}}$$

(8)

For large separations, the angular speeds of the planet and its central star will be almost identical, so that $\omega = (2 \theta_E)/\tau_E$ should be a good approximation to the value of the star’s proper motion. By looking for all high-proper motion stars within any specified angle of the event location, we can check for the presence of central stars with angular separations within a corresponding range of values of $a$. If such a star is catalogued or found through new observations, the probability is large that it is associated with the planet lens. In this case, the distance to the lens can be estimated by the photometric or geometric parallax of the associated star, and the planet’s mass can be estimated.

If no such star is discovered, then it is still possible that the planet orbits a star that is close enough to it that it cannot be resolved from the lensed source at the time of the event. In this case the light from the central star would be blended with light from the lensed source. Testing for blending in different wavebands during the event would then determine the amount of light that could possibly emanate from any star associated with the lens. A high-resolution image taken soon after the event would then both identify the lensed source and any other stars in the field. Additional images would determine if any stars in the region have the requisite proper motion. (In addition, as discussed in §3, high-resolution images may also be able to measure astrometric shifts in the source position caused by a central star.) Even if high-resolution images cannot be taken, however, images at later times can test for the presence of a central star as the lens system moves away from the event position.

The bottom line is that it is possible to place quantifiable limits on the presence of a central star. These limits can be expressed in terms of the mass of the star, the orbital separation of the planet from it, and the transverse velocity of the lens.
4.2. Direct Verification

At the time of a planet-lens event, the planet and lensed star are nearly coincident. After a time that can be as short as several years, the planet may have moved far enough from the event location, that, it may be possible to detect it. If the majority of free-floating planets have masses comparable to the Solar System’s terrestrial planets, it is unlikely that they will be detected in the foreseeable future. If they are more massive, however, direct detection will be possible for some of them. The feasibility of direct detection has been established by recent detections of wide-orbit planets. For example, three planets with masses between 5 and 13 times the mass of Jupiter have been discovered orbiting the star HR 8799, which is 39.4 pc away (Marois et al. 2008). These planets have projected distances from the star ranging from 24 to 68 AU. At these distances the radiation they receive from the star contributes little to their luminosity, which is instead dominated by cooling. In addition, a planet with mass estimated to be no greater than a few times that of Jupiter has been discovered in a wide orbit (119 AU) around a star, Formalhaut, which is 7.7 pc away (Kalas et al. 2008). Planets with radii comparable to or larger than that of Jupiter, and temperatures of \( \sim 1000 \) K, can be imaged if their distances from us are not too great. Imaging planets, even in these wide orbits, requires deep observations which can only be undertaken if there is a good chance of successfully finding a planet or else placing meaningful limits. For planets orbiting stars, such as those around HR 8799 or Formalhaut, the star itself provides a well-defined region within which to conduct a search. For free-floating planets, the occurrence of a lensing event provides a location about which to search. If \( \tau_E \) and \( \theta_E \) are both measured, then \( \omega \) is known, hence the angular separation of the lens from the lensed source can be computed for all later times. Observers can then plan observations to occur several years after the event, in order to image the lens. This procedure will image a subset of nearby free-floating planets.

If enough photons are collected to estimate the planet’s temperature, then the range of possible radii will provide a range of possible luminosities, allowing the distance \( D_L \) (and the uncertainty in its value) to be estimated. In addition, a sequence of images may be able to establish the geometric parallax. Armed with an estimate of \( D_L \), the lens mass can be determined in those cases in which \( \theta_E \) has been measured through finite-source-size effects.\(^5\)

4.3. Eliminating High-Velocity-Star Models

To distinguish between models in which the lens is a planet bound to a star or else is a free-floating planet, we search for evidence of a star that could be associated with the lens. Even if a star is not detected, however, it is possible that the event was caused by a high-velocity stellar-mass object rather than by a free-floating planet. Consider a short event (say \( \tau_E = 1 \) day), and a measured small Einstein angle (say \( \theta_E = 1.56 \times 10^{-5}'' \)). If the lens is not a planet, how fast must the lens be moving and how far away must it be? For

\(^5\) Note that the analogous procedure is simpler for planet lenses which are in wide orbits around a star that is close enough to be detected. In the latter case, the photometric and or geometric parallax of the star provides the distance to the star, which is, to high precision, equal to \( D_L \).
example, with $D_S = 8\ \text{kpc}$, a neutron star would have to have $v = 435\ \text{km/s}$, and would have to be located within roughly a pc of the source star. This is so unlikely, compared to the probability of a stellar-speed planet-mass object within a few hundred pc, that it can be effectively ruled out. In fact, if free-floating planets are common, we will have dozens to hundreds of similar cases, and the hypothesis that the majority of the events they generate were actually generated by high-velocity stellar remnants can be eliminated.

Note that the arguments above apply to the case in which the Einstein angle has been measured and is small. If the light curve shape provides only an upper limit on the value of $\theta_E$, then it is still possible that the lens was a high-velocity object. High-resolution images that check for a shift in the source’s light centroid can measure $\theta_E$, if the lens is indeed a nearby stellar remnant. Alternatively, they can place an upper limit on the value of $\theta_E$, thereby at least providing a constraint on stellar-mass models for the lens.

5. Focus on Short Events

When an event with a small measured value of $\tau_E$ occurs, we are guaranteed that the lens is of special interest. It could be a low-mass object such as a brown dwarf, a planet bound to a star, or a free-floating planet. Or it could be a high-speed object, either a dim member of the Galactic halo or else a high-speed stellar remnant, a product of an earlier high-energy event.

Because short events are being discovered by current monitoring teams (Udalski 2003; Bond et al. 2001), it is possible to initiate programs designed to learn about both low-mass and high-speed lenses. The science returns will be high, because each class of lenses generating short events is important, each for its own reasons.

Furthermore, the lenses will be nearby (within about a kpc) for a large fraction of short events. This means that they are amenable to a variety of complementary studies. Short events could, for example, provide a unique way to discover local fast-moving neutron stars which would otherwise be known only as dim, unexceptional sources. Short events will almost certainly identify planetary systems which can be studied in a variety of ways. Some of these will become the “gold standards” of their class, because key quantities such as the gravitational mass of the star and planet can be measured, sometimes in complementary ways (Di Stefano 2009).

We have shown that even short events generated by free-floating planets can be correctly identified, and planet properties can be measured. It is significant that this can be accomplished without sophisticated astrometry missions (cf. Han 2006). Our result relies on careful study of short event light curves, including multiwavelength monitoring to identify the effects of blending and finite-source size. We also require comprehensive catalogs and/or additional high-quality observations of the region in which the event occurred to search for counterparts, sometimes during and sometimes years after the event. Lensing will even provide gravitational mass measurements for some free-floating planets.

In the companion paper we demonstrate that the expected rate of short events is high enough that programs designed to study them will be productive. We also suggest a set of procedures designed to optimize their efficacy. These procedures will not only identify individual interesting lenses, but will also allow population properties to be measured. We
will be able to, for example, estimate the frequency of free-floating planets, both in absolute terms and also relative to the frequency of planets orbiting stars. We will quantify the size of the population local neutron stars and their velocity distribution. We will discover or place limits on the companions to accreting white dwarfs, flung from the vicinity of Type Ia explosions, and we will discover or place limits on the number of remnants of hypervelocity and runaway stars still bound to the Milky Way.

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Left panel: Distance to the lens, $D_L$, versus lens mass $M$. Each distinct swath corresponds to events within a narrow range of Einstein diameter crossing times, $\tau_E$. The values of $\tau_E$ are within 10% of a central value. The left-most cyan swath corresponds to $\tau_E = 0.5$ days, with the central value of $\tau_E$ doubling in each successive swath. The transverse speed was taken to be 75 km s$^{-1}$. Right panel: Distance to the lens, $D_L$, versus the relative transverse velocity, $v$. The lens mass $M$ is taken to be $1.4 M_\odot$, comparable to the mass of a neutron star. Neutron stars with high kick velocities may be the most numerous high-velocity objects. Each distinct swath corresponds to events within a narrow range (10%) of Einstein diameter crossing times, $\tau_E$: 16 days for the leftmost swath, decreasing by a factor of 2 for each swath to the right.
Fig. 2.— Magnification patterns associated with a planet lens with $q = 0.001$. Red: the magnification lies within 1% of the value expected for a point lens when $u = 2 \theta_E$. Blue: the magnification lies within 1% of the value expected for a point lens when $u = 1 \theta_E$. Cyan: the magnification lies within 1% of the value expected for a point lens when $u = 0.5 \theta_E$.

**Bottom:** the planet is far enough from the star ($a = 5 R_{E,*}$) that the isomagnification contours are symmetric and show no evidence of the presence of the central star. **Middle:** the planet is closer to the star and the isomagnification contours are distorted; many short events generated by the planet would require fit parameters modeling the relative mass and separation of the central star. **Top:** The planet-star separation is $1.5 R_{E,*}$ and the isomagnification contours are highly distorted; almost every event generated by the planet would provide evidence of the central star.