Physically Motivated Approximation to a Parton Distribution Function in QCD

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Abstract:
It has been suggested that parton distributions in coordinate space, so called Ioffe-time distributions, provide a more natural object for non-perturbative methods compared to the usual momentum distributions. In this paper we argue that the shape of experimentally determined Ioffe-time distributions of quarks in a nucleon target clearly indicates separation of longitudinal scales, which is not easily recognizable in terms of conventional longitudinal momentum space considerations. We demonstrate how to use this observation to determine parton distributions, using non-perturbative information about the first few moments and the Regge asymptotics at small $x$.

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Deep inelastic scattering provides one of the cleanest applications of perturbative QCD. According to factorization theorems [1], the entire $Q^2$ dependence of the cross section can be calculated perturbatively, while all dynamical effects of large distances can be parametrised by a set of one-particle parton distribution functions given at a certain reference scale. Although these parton distributions are determined experimentally with a good accuracy, their calculation from first principles remains a challenge for non-perturbative QCD methods.

In the past decade a remarkable progress has been made at the experimental side, and apart from the region of very small Bjorken $x$, there is now not much controversy regarding the existing parametrizations of parton distributions [2]. The theoretical progress has been much slower. Apart from several quark-model or MIT bag model calculations, there have been relatively few attempts to determine parton distributions from QCD. The problem has proved to be very difficult for the theory. QCD sum rules calculations of properties of parton distributions have been moderately successful [3, 4]. The state-of-art lattice QCD calculation of the lowest moments of quark distribution functions has appeared just recently [5, 6]. In the present paper we propose an approximate scheme to compute bulk of momentum space quark distributions starting from relatively modest theoretical information. The approximation can be systematically improved when new information becomes available. Note that we are not primarily concerned here with an effective mathematical method of reconstruction of a parton distribution from its (many) moments. Such methods exist [7, 8, 9] and are very useful in the perturbative QCD analysis of experimental data. In this paper we consider a different problem of computation of structure function from a non-perturbative theoretical input. Hence, anticipating technical difficulties in calculation of higher moments from QCD, we have been looking for a method which can give a satisfactory description using a minimal amount of theoretical information.

Parton distribution functions arise from long-distance QCD physics which is still perhaps the least understood domain of strong interactions. Given the intrinsic complexity of the problem it is clear that an approximation scheme is necessary. The main problem is to understand what information is required to compute a parton distribution function in QCD with a reasonable accuracy. Usually modern parametrizations of parton densities [10, 11, 12] rely on input distributions assumed to be valid at some low normalization scale. Parton distributions at any larger scale can be uniquely determined thanks to the QCD evolution equations. As very little is understood at present about the low scale input distributions, they are usually assumed to follow a simple shape which is adjusted iteratively to reproduce the experimental data. Alternatively one can calculate them in a certain QCD motivated model, but the significance of such calculation is not obvious. Thus, the question we address in this paper can be formulated as follows: how to determine input distributions making the best use of the information available from state-of-art non-perturbative QCD calculations.

It is a textbook statement that twist-2 parton distribution functions describe probability densities of partons longitudinal momenta. Furthermore it is usually assumed that
the connection between moments of parton distributions and matrix elements of local twist-2 operators, provided by OPE \cite{13, 14}, should make it possible to compute them, say, in the lattice QCD approach. Our experience with QCD sum rules and understanding of the present status of the lattice QCD technology tells us however that in reality only the lowest moments are computable with a reasonable accuracy, assuming that available resources will not suddenly increase by many orders of magnitude. While we realize that such assumption has proved wrong many times in the history, we have found it nevertheless justified to look for a non-standard scheme which results in a good approximation to the parton densities starting from a very modest amount of information. As we shall show below, such a scheme can be derived from the analysis of experimental properties of parton distributions. We shall argue that the amount of information relevant to quark distribution functions is surprisingly small, even though a crucial piece cannot be easily obtained using the standard QCD methods.

In this context it turns out to be advantageous to analyze longitudinal distance - or Ioffe-time - distributions rather than more common longitudinal momentum distributions. Longitudinal distance distributions were introduced many years ago \cite{15}, but until very recently \cite{16, 17} their significance has not been fully understood. Mathematically Ioffe-time distributions are just Fourier transformed longitudinal momentum distributions. Let $q^+(u, \mu^2) = q(u, \mu^2) + \bar{q}(u, \mu^2)$ denote the positive C-parity quark longitudinal momentum distribution at a scale $\mu^2$. The corresponding Ioffe-time distribution can be defined as

$$Q^+(z, \mu^2) = \int_0^1 du q^+(u, \mu^2) \sin(uz).$$  \hspace{1cm} (1)

Because in this paper we concentrate on positive C-parity distributions the superscript $^+$ will be neglected in the following. As we are primarily concerned with low-scale distributions, $\mu^2$ is always equal to $4 \text{ GeV}^2$, unless it is explicitly indicated. Although it is not possible to gain any information by transforming a momentum distribution into coordinate space, we shall show that certain phenomenological properties of quark distributions are easier to grasp in the Ioffe-time representation. The coordinate space variable $z$ is the invariant measure of the longitudinal light-cone distance between the points where the hard probe was absorbed and emitted by the target. In the leading logarithmic approximation $Q(z)$ has a very simple and transparent interpretation as being related to the target matrix element of a non-local QCD string operator \cite{13, 14}. Indeed, let $\Delta$ denote a light-like vector, $\Delta^2 = 0$. If $P$ denotes a target momentum, then the Ioffe-time $z = P \cdot \Delta$ and $Q(z, \mu^2)$ can be defined as

$$\langle P \mid \bar{\Psi}(\Delta) \Delta [\Delta; 0] \Psi(0) \mid P \rangle_{\mu^2} - (\Delta \rightarrow -\Delta) = 4i(P \cdot \Delta) Q(z, \mu^2).$$  \hspace{1cm} (2)

where $[\Delta; 0]$ is the path-ordered exponential necessary to insure gauge invariance. The evolution equations for Ioffe-time distributions were considered in \cite{14, 17}. It is important to realize that derivatives of Ioffe-time distribution $Q(z)$ at the origin are given by moments of corresponding structure functions $q(u)$, or equivalently \cite{13, 14} by matrix elements of twist-2 operators of increasing dimension. It has been argued \cite{14, 17} that from
the theoretical point of view this connection makes Ioffe-time distributions much easier to analyze than parton distributions in momentum space.

Typical shapes of u and d-quark Ioffe-time distributions $Q_u(z)$ and $Q_d(z)$ obtained with the help of existing parton parametrizations [10, 11, 12] are shown in figures 1 and 2. As expected different parametrizations [10, 12] produce very similar Ioffe-time distributions, although, somewhat surprisingly, parametrization [11] results in different behavior for large $z$. The value of $Q(z)$ at $z = 0$ equals zero by definition if the momentum distribution $q(u)$ is less singular than $u^{-2}$ at $u = 0$. A much less trivial observation concerns the shape of $Q(z)$. In the region of small $z$, say up to $z \sim 3 - 4$, the distribution is smooth, to a good approximation linear function of $z$. Note that its slope at the origin is equal to the longitudinal momentum fraction carried by quarks. The behavior of $Q(z)$ in this region is determined by average properties of the corresponding longitudinal momentum distribution which are encoded in its few lowest moments. A change of the shape of $Q(z)$ in this region results in a change of the bulk properties of the momentum distribution. Note that $z = 10$, i.e. the onset of the asymptotic behavior, corresponds in the nucleon rest frame to a longitudinal distance of the order of 2 fm, or the nucleon diameter. Having in mind a simple geometrical picture one can argue that for larger values of $z$ absorption and emission of the virtual photon by the target occurs outside the space-time volume occupied by the nucleon. For larger values of $z$ absorption respectively emission of the virtual photon by the target occurs outside the space-time volume occupied by the nucleon.

In this asymptotic domain $Q(z)$ behaves according to small-$u$ behavior of the longitudinal momentum distribution. Indeed, if $q(u) \sim u^\alpha$ at small $u$, then from (1) it follows that $Q(z) \sim 1/z^{1+\alpha}$ for large $z$. A change of shape of $Q(z)$ in this region would influence the small-$u$ behavior of the momentum distribution.

Physically the behavior at large-$z$ reflects properties of wee partons, while the small-$z$ region is sensitive to the distribution of hard partons i.e., those which carry finite longitudinal momenta. The presence of a clearly recognizable transition between these two regions, see figures 1 and 2, suggests that they should be treated separately. From the mathematical point of view the Taylor expansion of $Q(z)$ around point $z = 0$ has infinite radius of convergence, so formally the asymptotic region can be reached from the origin. It corresponds to the fact that the inverse Mellin transformation always allows us to reconstruct the momentum distribution from its moments. In practice it requires to know a lot of them - many more than we can perhaps ever hope to have computed on the lattice. To illustrate this point on figure 3 we show the convergence of the short-distance expansion of $Q_u(z)$. A line labeled Tn denotes the Taylor approximation constructed from the first n non-vanishing moments. It can be seen that while T1 is a good approximation to $Q(z)$ in the small-$z$ domain, it takes a lot of moments to reach the asymptotics. Hence the idea, first proposed in [17], to consider each of these two regions separately, and then match them in the transition region. Lattice QCD is certainly the best approach to calculate $Q(z)$ for small-$z$. For the asymptotic domain one has to consider other methods. At present one can resort either to the Regge phenomenology [18], or to the perturbative QCD analysis [19, 20]. While the former has purely phenomenological character, the
latter is still subject to an ongoing research and hence to some controversy [21].

Consider now the problem of construction of the quark momentum distribution from the available theoretical input. For definiteness we shall consider only the u-quark distribution, postponing the detailed discussion of the d-quark distribution to a more detailed publication [22]. We choose the MRS(A) parametrization [10] to represent the data. Our goal is to develop an approximation which allows to reproduce, with a reasonable accuracy, the MRS(A) u-quark distribution at $\mu^2 = 4 \text{ GeV}^2$ using modest amount of theoretical information.

As the first step we consider the Ioffe-time distribution. As discussed above, it is natural to consider the small-$z$ and large-$z$ regimes separately. The information necessary to determine $Q(z)$ in the the first region is contained in a few lowest moments of the momentum distribution. We assume that they are computable on the lattice, but of course the smaller is the required number of moments the better. We shall assume that lattice QCD is capable of computing these moments with a high accuracy and calculate their numerical values using the MRS(A) parametrization. In the large-$z$ region we assume that the standard, naive Regge argument is valid, namely that $q(u) \sim u^{-1}$ at small $u$, which corresponds to $Q(z) \sim \text{const.}$ at large $z$. If the normalization point $\mu^2$ is relatively low, it may not be a bad approximation and, as it is shown below, it will influence only the small-$u$ behavior. Now, we have to match the small-$z$ behavior, given by an almost straight line with the slope given by the first moment of $q(u)$ with the asymptotic behavior given by the line $y = \text{const.}$, and for that we need additional information about the behavior of $Q(z)$ in the transition region. In the first approximation one can argue that the transition region occurs when $z$ reaches the value which corresponds to the confinement radius $\sim 1 \text{ fm}$ in the nucleon rest frame, or $z \approx 5$. Assuming that the transition is infinitely sharp we have obtained the approximate Ioffe-time distribution which is depicted by the dot-dashed line on figure 4. After transformation to longitudinal momentum space it results in the approximation to $uq(u)$ denoted by the dot-dashed line on figure 5. Note that this approximation, while certainly not satisfactory, relies only on the momentum fraction carried by quarks, which is nowadays computable in lattice QCD, and arguments about the Regge behavior and confinement radius.

It is clear that in the next step one has to take into account more accurately the magnitude of $Q(z)$ in the asymptotic domain. This is crucial, see figure 4, for a successful reconstruction of the shape of the experimental distribution. Two numbers are required to predict gross features of u-quark distribution function. The first one is the u-quark momentum fraction, the second is the large-$z$ magnitude of its Ioffe-time distribution. It can be nicely demonstrated by matching the small-$z$ behavior given by the first moment with the correct large-$z$ magnitude. The result is denoted by the dotted line in figure 5. When the negative large-$u$ tail, which contradicts the positivity requirement of a probability distribution, is neglected, the approximation is quite satisfactory and probably much better than what one can hope to obtain from many phenomenological models. While the momentum fraction is computable on the lattice, we are not aware about any analysis which could provide us with reliable information about the large-$z$
magnitude of $Q(z)$. To proceed still further we have chosen to approximate $Q(z)$ more accurately in the transition region using information about the higher moments of the parton distribution. It can be done very efficiently using Padé approximation technique. Note that the shape of the u-quark Ioffe-time distribution exhibits a maximum in the transition region which is a natural candidate for a point where the curves determined by the small-$z$ and the asymptotic behavior should match each other. The improved description of $Q(z)$ in the transition region based on the [3,2] Padé approximation results in the dashed curve on figure 4. Note that it has been obtained using the information about the first three non-vanishing moments and the Regge argument about flat large-$z$ behavior. The corresponding approximation to the longitudinal momentum distribution $uq(u)$ is denoted by a dashed line in figure 5. The agreement with the exact result, solid line, is satisfactory, except for the small-$u$ region, where the true behavior is markedly different from the assumed naive Regge asymptotics.

Calculation of the first three moments of the $C = +1$ quark longitudinal momentum distribution is equivalent to a lattice measurement of matrix elements of operators built from a one gamma matrix and one, three, or five derivatives between two quark fields. The first lattice QCD results on the first two moments in the quenched approximation have been already reported \[6\]. Unfortunately, due to technical difficulties it is not yet possible to measure the third moment.

When the Ioffe-time momentum distribution has been obtained, it has to be inverted according to the formula:

$$ q(u) = \frac{2}{\pi} \int_{0}^{\infty} dz \sin(uz)Q(z), \tag{3} $$

to produce the experimentally measurable longitudinal momentum distribution. Note that we are dealing with the positive C-parity combination of quark and antiquark longitudinal momentum densities. Because $Q(z)$ is not square integrable, the integral (3) converges very slowly. A fast and accurate calculational method is available if the the asymptotic behavior of $Q(z)$ for large $z$ is known. Indeed, let us assume that $Q(z) \sim Cz^{\alpha} + D$ when $z \to \infty$. Then, subtracting the asymptotic behavior of $Q(z)$ from the integrand in (3) one easily obtains

$$ q(u) = \frac{2}{\pi} \int_{0}^{\infty} dz \sin(uz) [Q(z) - Cz^{\alpha} - D] 
+ \frac{2}{\pi} C \cos\left(\frac{\pi \alpha}{2}\right) \frac{\Gamma(1 + \alpha)}{u^{1+\alpha}} + \frac{2}{\pi} D \frac{1}{u}. \tag{4} $$

Now the integral converges fastly and can be easily computed. We have used this method to invert consecutive approximations of Ioffe-time distributions discussed above.

Having in mind the problem of determination of the input distributions for parametrizations of parton densities, we have compared the present method with a fit to the first three moments which assumes some simple functional form. In the full analogy with the actual shape of the input distribution to the MRS(A) parametrization we have tested the
Equation (5) as it stands depends on four parameters $A$, $b$, $C$, and $d$. As it is not possible to determine all of them from the information about three moments we decided to fix $d$ and to find three remaining parameters. The best agreement with the original distribution is found for $d \sim 1.5$. Because the MRS(A) parametrization at $\mu^2 = 4 \text{ GeV}^2$ follows at large $u$ exactly the shape (5), the agreement with the approximate form is perfect in this domain. The Regge assumption about small-$u$ behavior leads to a discrepancy in this region. Note however that the method based on the consideration of Ioffe-time distribution allows us actually to \textit{derive} the shape (5) from first principles augmented by a Regge argument about the large-$z$/small-$u$ behavior. Hence we have been able to show that the input distribution can be strongly constrained by the information stemming from non-perturbative QCD without having to rely on purely phenomenological models. On the other hand it is tempting to conclude from our discussion that traditional quark models of hadronic structure should attempt to compute first few moments of parton distributions rather than their full Bjorken $x$ dependence.

Approximate determination of the $d$-quark distribution can be done using essentially the same technique [22]. As the MRS(A) parametrization in the transition region shows no maximum, there is no obvious choice of the matching point between small-$z$ and large-$z$ behavior. As a consequence, a reasonable approximation requires knowledge of the first four moments i.e., one more than in the $u$-quark case. This problem can be circumvented in the case of the CTEQ3 parametrization which is almost flat in the asymptotic region and therefore fits exactly into our scheme.

Can the accuracy be further improved to reach, for example, the level of techniques developed in [7, 8, 9]? The answer is yes, despite of the fact that the question has, from our point of view, purely academical character. We have found [22] that Padé approximation taking into account the lowest six moments of the momentum distribution allows to reconstruct $Q(z)$ reliably in the $z$ region which extents so far away from the origin that the naive Regge argument is not more necessary and the true asymptotic behavior can be found by inspection. The resulting approximations of Ioffe-time and longitudinal momentum distributions are almost perfect. We argue, however, that such procedure, although mathematically consistent, is not sensible from the physical point of view. Calculation of the sixth moment requires e.g., a lattice measurement of the matrix element of a operator with twelve Lorentz indices i.e., one gamma matrix and eleven derivatives. Note that on the lattice a derivative is replaced by a finite difference. Thus, because of the size of such an operator and because of its mixing with operators with lower dimension which occurs in the discretized lattice formulation [23], it would be prohibitively difficult to perform such a measurement using the present technology. In such situation it is natural to suggest that one should develop a different technique to understand the shape and the magnitude of a Ioffe-time distribution at large $z$. In this domain the Compton scattering amplitude in the target rest frame is dominated by the photon splitting into a quark-antiquark pair, which subsequently scatters off the background color field of the target [16]. While it
seems that the resulting shape of $Q(z)$ can be understood within perturbative QCD [20], the magnitude is certainly a problem of a non-perturbative character [24].

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Figure captions

**Fig. 1** $Q_u(z)$, the u-quark Ioffe-time distribution at the scale $\mu^2 = 4$ GeV$^2$. The solid line denotes MRS(A) [10], the dashed line denotes CTEQ3 [11], and the dotted line denotes the Glück, Reya, Vogt [12] parametrizations.

**Fig. 2** $Q_d(z)$, the d-quark Ioffe-time distribution at the scale $\mu^2 = 4$ GeV$^2$. The solid line denotes MRS(A) [10], the dashed line denotes CTEQ3 [11], and the dotted line denotes the Glück, Reya, Vogt [12] parametrizations.

**Fig. 3** Convergence of the Taylor expansion of $Q_u(z)$. The line labeled $T_n$ denotes an approximation which requires the knowledge of the first $n$ nonvanishing derivatives at the origin, or equivalently the moments of quark longitudinal momentum distribution.

**Fig. 4** Consecutive approximations of $Q_u(z)$, solid line labeled MRS(A). The first approximation, dot-dashed line labeled $T_1$, assumes a sharp transition between small-$z$ and asymptotic regimes at the value $z = 5$, which corresponds to the confinement radius $\sim 1$ fm. The next approximation, dashed line labeled $P[3,2]$, takes into account the behavior of $Q(u)$ in the transition region more accurately by employing a Padé approximation based on the first three moments.

**Fig. 5** Consecutive approximations to the u-quark momentum distribution $uq(u)$, solid line labeled MRS(A). The dot-dashed line, labeled $T_1$, is based on the approximation which relies on the value of the first moment of $q(u)$, the Regge behavior at small-$u$, and the magnitude of the confinement radius. The dotted line, labeled $T_1'$ results from matching of small-$z$ behavior given by the first moment with the correct large-$z$ magnitude. The dashed line, labeled $P[3,2]$, requires the values of the first three moments of $q(u)$. 
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Fig. 1 $Q_u(z)$, the $u$-quark Ioffe-time distribution at the scale $\mu^2 = 4 \text{ GeV}^2$. The solid line denotes MRS(A) [10], the dashed line denotes CTEQ3 [11], and the dotted line denotes the Glück,Reya,Vogt [12] parametrizations.
Fig. 2 \( Q_d(z) \), the d-quark Ioffe-time distribution at the scale \( \mu^2 = 4 \text{ GeV}^2 \). The solid line denotes MRS(A) [10], the dashed line denotes CTEQ3 [11], and the dotted line denotes the Glück,Reya,Vogt [12] parametrizations.
Fig. 3 Convergence of the Taylor expansion of $Q_u(z)$. The line labeled $T_n$ denotes an approximation which requires the knowledge of the first $n$ non-vanishing derivatives at the origin, or equivalently the moments of quark longitudinal momentum distribution.
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Fig. 5 Consecutive approximations to the u-quark momentum distribution $uq(u)$, solid line labeled MRS(A). The dot-dashed line, labeled T1, is based on the approximation which relies on the value of the first moment of $q(u)$, the Regge behavior at small-$u$, and the magnitude of the confinement radius. The dotted line, labeled T1’ results from matching of the small-$z$ behavior given by the first moment with the correct large-$z$ magnitude. The dashed line, labeled P[3,2], requires the values of the first three moments of $q(u)$. Please note that the corresponding dotted curve in our published version of the paper is wrong due to an error in our computer program.