Implications of CTEQ PDF analysis for collider observables

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In the context of a recent CTEQ6.6 global analysis, we review a new technique for studying correlated theoretical uncertainties in hadronic observables associated with imperfect knowledge of parton distribution functions (PDFs). The technique is based on the computation of correlations between the predicted values of physical observables in the Hessian matrix method. It can be used, for example, to link the dominant PDF uncertainty in a hadronic cross section to PDFs for individual parton flavors at well-defined \((x, \mu)\) values. As an illustration, we apply the PDF correlation analysis to study regularities in the PDF dependence of \(Z, W\), and Higgs boson production cross sections at the Tevatron and LHC.

Theoretical description of LHC observables requires accurate parton distribution functions (PDFs), determined from a comprehensive fit of theoretical cross sections to a diverse range of experimental data. Reliability of the existing PDF parametrizations depends on our understanding of rich connections existing between PDFs of different flavors and in different kinematical ranges, arising as a consequence of physical symmetries (such as scale invariance or parton sum rules) and experimental constraints implemented in the global fit. This talk addresses the need to explore such connections effectively with the help of new quantitative tools provided by global PDF analyses. I will refer to the slides of the talk available at [1].

A convenient measure of the relation between the PDF dependence of two physical quantities \(X\) and \(Y\) is the angle \(\varphi\) formed by the gradient vectors \(\nabla X\) and \(\nabla Y\) in the space of \(N\) PDF parameters \(a_i\). The correlation angle was originally introduced in [2, 3] and systematically explored in the context of the recent CTEQ6.6 NLO PDF analysis [4]. Together with the usual PDF uncertainties \(\Delta X\) and \(\Delta Y\), \(\varphi\) can be derived using the Hessian matrix method [2] from the values \(X_i^+, Y_i^+\) \((i = 1, N)\) taken by \(X\) and \(Y\) for maximal positive (+) and negative (−) displacements for each PDF eigenparameter \(a_i\) within the fit’s tolerance region. The cosine of \(\varphi\) is explicitly given by

\[
\cos \varphi = \frac{1}{4\Delta X \Delta Y} \sum_{i=1}^{N} \left( X_i^+ - X_i^- \right) \left( Y_i^+ - Y_i^- \right).
\]

The usefulness of \(\cos \varphi\) can be appreciated by noticing that \(\Delta X\), \(\Delta Y\), and \(\cos \varphi\) are sufficient to establish a Gaussian probability distribution \(P(X, Y|\text{data})\) for finding certain values of \(X\) and \(Y\) based on the experimental data sets included in the global analysis. Hence, the three parameters come in handy in certain statistical estimates. For example, they determine joint confidence regions for the \(X-Y\) pair (error ellipses in the \(X-Y\) coordinate plane), such as the error ellipses for \(W\) and \(Z\) production cross sections discussed below.

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They are also sufficient for the estimation of the PDF uncertainty $\Delta f$ of any function $f(X,Y)$ according to the formulas presented in [4].

The value of $\cos \varphi$ is a quantitative measure of our ability to reduce the PDF uncertainty in $Y$ by precisely measuring $X$. The measurement of $X$ would constrain $Y$ substantially if $X$ and $Y$ are strongly correlated ($\cos \varphi \approx 1$) or anticorrelated ($\cos \varphi \approx -1$). Conversely, if $\cos \varphi \approx 0$, the measurement of $X$ is not likely to constrain $Y$.

Some applications of the correlation analysis were considered in [4]. We focus, in particular, on correlations between PDFs of specific flavors and physical cross sections. The recent CTEQ6.6 PDFs with more precise treatment of $s$, $c$, and $b$ quarks (summarized in [5]) allow us to assess the flavor dependence of the PDF-induced correlations as reliably as possible.

An instructive example of PDF-induced correlations is provided by total cross sections for $Z$, $W$, and $t\bar{t}$ production ($\sigma_Z$, $\sigma_W$, and $\sigma_{t\bar{t}}$) at the LHC. These cross sections are plotted pairwise as dots for 41 CTEQ6.1 PDF sets in two figures on slide 3. In the upper figure, the dots for 41 pairs of $Z$ and $W$ cross sections lie within a narrow ellipse, with the center corresponding to the best-fit CTEQ6.1M PDF set. For each extreme PDF set, variations in $\sigma_Z$ and $\sigma_W$ tend to be of the same sign and of similar relative magnitudes, indicating a strong correlation in their PDF dependence.

On the other hand, variations due to the PDFs in the $t\bar{t}$ total cross section (lower figure) tend to be opposite in sign to those of $W/Z$ cross sections, indicating a substantial anticorrelation [6]. The two figures or simple arguments do not explain what drives the (anti-)correlation, for instance, why the $W$ and $Z$ cross sections (dominated by light-quark scattering) are anticorrelated with the $t\bar{t}$ cross section (dominated by $g-g$ scattering).

In order to reveal the underlying physics mechanism, on slides 14a,b we plot $\cos \varphi$ between the $Z$ boson production cross sections and PDFs $f_a(x,Q)$ of different flavors, evaluated for the Tevatron Run-2 and LHC as a function of the momentum fraction $x$ at an energy scale $Q = 85$ GeV. The results for $W$ boson production are qualitatively the same [4]. A PDF flavor having a strong correlation with $\sigma_Z$ contributes a large part of the PDF uncertainty $\Delta \sigma_Z$ in $\sigma_Z$. Additional constraints on this flavor would help reduce $\Delta \sigma_Z$. In $Z$ boson production, the largest correlations occur at momentum fractions $x$ of order $M_Z/\sqrt{s}$, i.e., at $x \sim 0.05$ at the Tevatron and 0.007 at the LHC, corresponding to central rapidity production.

According to the figures, correlations in $\sigma_Z$ at the LHC are not the same as at the Tevatron. At the Tevatron (slide 14a), large correlations ($\cos \varphi \approx 0.95$) exist with $u$, $\bar{u}$, $d$, and $\bar{d}$ PDFs, while no tangible correlation occurs with the PDFs of other flavors. At the LHC (slide 14b), the largest correlations are driven by charm, bottom, and gluon PDFs, followed by smaller correlations with $u$, $d$, and $s$ quarks. This feature may come across as surprising, as $Z$ bosons are mostly produced in $u$ and $d$ quark-antiquark scattering at both colliders. However, this dominant channel contributes little to the PDF uncertainty at the LHC because of tight constraints imposed on the $u$ and $d$ PDFs at relevant $x$ by the DIS and Drell-Yan data. Rather, the bulk of the PDF uncertainty comes from the less constrained $s$, $c$, $b$, and $g$ scattering channels.

In the LHC case, a large positive correlation of $W$, $Z$ cross sections with $g$, $c$, and $b$ PDFs at $x \sim 0.005$ is accompanied by a large anticorrelation ($\cos \varphi \sim -0.8$) with the same PDFs at $x \sim 0.1 - 0.2$. The anticorrelation reflects the nucleon’s momentum sum rule, which demands that variations in the gluon PDF at small $x$ are compensated by opposite

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variations at large $x$ in order to satisfy

$$\int_0^1 x f_g(x, Q) dx + \sum_{\text{quark flavors}} \int_0^1 x [f_q(x, Q) + f_{\bar{q}}(x, Q)] dx = 1.$$ 

The anticorrelation can be viewed directly at the level of the underlying PDFs by examining $\cos \varphi$ between $f_g(x_1, Q)$ and $f_g(x_2, Q)$ as a function of $x_1$ and $x_2$. A contour plot of such dependence is shown for $Q = 85$ GeV in slide 13, the upper right figure. The colors of the contours correspond to the value of $\cos \varphi$ according to the palette included in the slide. In this plot, the red area along the diagonal reflects a trivial perfect correlation of $f_g(x, Q)$ with itself at the same $x$ ($\cos \varphi = 1$ if $x_1 = x_2$). The anticorrelation due to the momentum sum rule produces dark-blue areas near $(x_1, x_2) = (0.2, 0.01)$. The same anticorrelation also appears in the case of the $c$ and $b$ PDFs (lower figures), the distributions generated radiatively from $f_g(x, Q)$ via DGLAP evolution. It does not occur in the case of light (anti-)quarks. For instance, the $u - u$ contour plot in the upper left figure only shows a weak anticorrelation at $(x_1, x_2) \approx (0.1, 0.7)$ associated with the valence sum rule, $\int_0^1 |u(x, Q) - \overline{u}(x, Q)| dx = 2$.

PDF-PDF correlation plots for other parton flavors or different $Q$ are posted at [7].

Since the gluon anticorrelation originates from a basic sum rule, it universally affects processes involving gluon scattering. In particular, $tt$ production at the LHC is strongly correlated with $f_g(x, Q)$ at $x \sim 0.1$, and, therefore, anticorrelated with $f_g(x, Q)$ at $x$ of a few $10^{-3}$ (slide 15). This explains why the LHC $tt$ cross sections are anticorrelated with the $W, Z$ cross sections. For the same reason, the PDF uncertainty for Higgs boson production in gluon-gluon fusion at the LHC is correlated with that for production of $Z$ bosons if the Higgs boson is relatively light ($M_H = 100 - 150$ GeV) and strongly anticorrelated if it is heavy ($M_H \approx 500$ GeV); cf. slide 17.

These findings can be of relevance for various aspects of the LHC physics program, in view that $\sigma_Z, \sigma_W,$ and $\sigma_{t\bar{t}}$ will be measured with high precision in order to calibrate the LHC experimental equipment and accurately determine standard-model parameters (particularly measure the $W$ boson and top-quark masses). Many LHC analyses deal with ratios of two cross sections $\sigma_1/\sigma_2$, such as those introduced to normalize an LHC cross section to a well-known “standard candle” cross section or deduce statistical significance from the signal and background event rates. Ratios of correlated (but not anticorrelated) cross sections have greatly reduced PDF uncertainty. The correlation analysis identifies straightforwardly such pairs of correlated cross sections. Altogether, the results in Ref. [4] demonstrate that the correlation analysis is a simple, yet informative, technique helping to clarify counterintuitive aspects of the PDF dependence of collider observables.

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