The Momentum Conserving Scheme Implementation for Simulating Dambreak Flow in a Channel with Various Contractions

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Abstract. Rapid flow downstream due to dambreak has a detrimental effect on the surrounding environment or, more dangerously, can be life-threatening. From a practical point of view, these flows are important to studies due to the limited dambreak real case data. This paper discusses the numerical modelling of the dambreak flow through a channel with three different contractions. Our goal here is to investigate the performance of a numerical model for solving the Saint-Venant equations using a momentum conserving staggered grid scheme (MCS). The scheme is the conservative formulation of the governing equations. Flows across channels of various widths and depths have been successfully simulated using a version of this scheme. In this work, we extend our previous work by simulating dambreak flow in a wave tank through several forms of contraction; trapezoidal and triangular. Our simulation results show good agreement with the experimental data in the literature. This assessment shows the merit of the scheme, which is suitable for dambreak flows in channels of varying width.

1. Introduction
Dams around the world are always haunted by the potential threat of a dambreak that can cause rapid downstream flooding, with catastrophic implications in terms of loss of life and property and destruction of natural habitats. This potential threat can be reduced by disaster mitigation, one of which is by modelling and simulating a dambreak using a mathematical model that has been done by many researchers [1,2,3].

The mathematical model commonly used to describe dambreak phenomena is the Saint Venant equations [4,5]. The equations consist of mass conservation and momentum balance. The exact solution of dambreak problems is certainly not simple, especially when the fluid passes through an inhomogeneous channel, with contraction or expansion. The presence of contraction in the channel influences the dambreak wave [6,7,8], also triggers the formation of hydraulic jumps and negative surges, thus complicating the evaluation of flood risk management. Because of the scarcity of field data on the collapsed dam [11,12,13] numerical modelling and laboratory research are necessary for understanding these complicated flow problems [9,10].
Laboratory experiments were carried out to examine the propagation and evolution of the dambreak flow over irregular channels, as discussed in the literature [1,7,8,13,14]. For the numerical approach, an accurate approximation of the Saint Venant equations requires a suitable approximation of the advection term. Here, we adopt the momentum conservative approach applied on a staggered grid, as originally proposed by [15]. This scheme has been further developed by several authors [4,16,18], and the numerical scheme is referred to as the momentum conserving staggered-grid (MCS) scheme. In contrast to [17], this paper uses the simple hydrostatic model, the Saint Venant model and the upwind approximation for the variable cross-sectional area. We also extend the work of [16,17] with the three types of contractions.

In this paper, we focus on the implementation of a momentum conserving scheme (MCS) for simulating dambreak flow through various contractions. The organization of this paper is as follows; governing equations are discussed in Section 2. The numerical scheme and its discretization are given in Section 3. Comparison of numerical results and experimental data are elaborated in Section 4. Finally, the conclusion is given in Section 5.

2. Material and Methods

2.1 Governing Equations

The motion of free surface from a layer of fluids flowing through a varying channel can be governed by Saint-Venant equations,

\[ \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \]  
\[ \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + gA \left( \frac{\partial h}{\partial x} \right) = -gA \frac{d\bar{d}}{dx}, \]  

where \( x \) and \( t \) represents spatial and time coordinates respectively. The notations \( A(x, t) \) is the wet cross-sectional area of the channel, \( Q(x, t) \) is the momentum, and \( d(x) \) is the topography, see Figure 1. For a regular channel, the cross-sectional area \( A(x, t) \) depends on the water depth \( h(x, t) \), e.g. for a rectangular cross-section with the channel width \( b(x) \), the cross-sectional area is \( A(x, t) = h(x, t)b(x) \). Furthermore, \( Q(x, t) = A(x, t)u(x, t) \), with \( u(x, t) \) represent the horizontal velocity of the fluids.

Figure 1. Sketch of the problem and notations; (left) channel side view and (right) channel top view.

Suppose that

\[ uu_x = \frac{1}{A}((Qu)_x - uQ_x), \]  

Then using (3), the governing equations (1) and (2) can be rewritten into

\[ \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \]  

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \left( \frac{\partial h}{\partial x} + \frac{d \hat{d}}{dx} \right) = 0. \]  
\hspace{1cm} (5)

Equations (4) and (5) will be used for simulation the dambreak phenomena. For this purpose, we need the numerical approach discussed in Section 3.

2.2 Numerical Scheme

In this paper, we focus on the implementation of our momentum conserving scheme (MCS) for simulating dambreak flows in channels with contraction. In our previous article [16], we developed the MCS scheme for flow in channels with arbitrary cross-sections. For clarity reason, in the following we re-write the MCS scheme.

Consider the governing equations on the spatial domain \( x \in [a, b] \) and uniformly discretized with a spatial step size \( \Delta x / 2 \), with \( a, b \in \mathbb{R} \). The resulting partition point from this configuration is:

\( a = x_{1/2}, x_1, \ldots, x_{j-1/2}, x_j, x_{j+1/2}, \ldots, x_{N_x+1/2} = b, \) where \( N_x = \frac{b-a}{\Delta x} \) represent number of partition points. In this staggered configuration, we approximate the mass conservation (4) at cell \([x_{j-1/2}, x_{j+1/2}]\) whereas the momentum (5) is approximated at cell \([x_j, x_{j+1}]\), so we have discrete numerical approximation scheme read as follows,

\[ \frac{dA^n_j}{dt} + \frac{Q^n_{j+1/2} - Q^n_{j-1/2}}{\Delta x} = 0, \]  
\hspace{1cm} (6)

\[ \frac{du^n_{j+1/2}}{dt} + (u u_x)^n_{j+1/2} + g \frac{h^n_{j+1} - h^n_{j+1}}{\Delta x} = 0, \]  
\hspace{1cm} (7)

where most discussion in this paper will consider flat bottom \( \hat{d}(x) = 0 \). Next, we compute the following variables consistently in a staggered grid, for example for the rectangular cross-sectional channel \( A^n_j = h^n_j b_j \), while

\[ Q^n_{j+1/2} = v A^n_{j+1/2}, \quad A^n_{j+1/2} = \begin{cases} A^n_{j+1}, & u^n_{j+1/2} < 0, \\ A^n_j, & u^n_{j+1/2} \geq 0. \end{cases} \]  
\hspace{1cm} (8)

Meanwhile adopting (3), the consistent approximation for advection term read as

\[ (u u_x)^n_{j+1/2} = \frac{1}{A^n_{j+1/2}} \left( \frac{Q^n_{j+1} + u^n_{j+1} - Q^n_j - u^n_j}{\Delta x} - u^n_{j+1/2} \frac{Q^n_{j+1} + Q^n_j}{\Delta x} \right), \]  
\hspace{1cm} (9)

whereas

\[ \tilde{A}^n_{j+1/2} = \frac{A^n_{j+1} + A^n_j}{2}, \quad \tilde{Q}^n_{j} = \frac{Q^n_{j+1/2} + Q^n_{j-1/2}}{2}, \]  
\hspace{1cm} (10)

and the first-order upwind approximation for horizontal velocity is

\[ v^n_j = \begin{cases} u^n_{j-1/2}, & \hat{Q}^n_j \geq 0, \\ u^n_{j+1/2}, & \hat{Q}^n_j < 0. \end{cases} \]  
\hspace{1cm} (11)

In this paper we will use Momentum Conserving Scheme (6) - (11) for simulating dambreak problem. Dry area conditions are often encountered when simulating the dambreak phenomenon. To overcome this, we will adopt a simple wet–dry procedure [19,20] together with thin layer technique [21] to avoid the instability condition. At the initial condition, in the dry area, if the total water depth is less than the prescribed threshold value \( h_{\text{thin}} \), the water depth is replaced by a thin-layer of water with depth \( h_{\text{thin}} \), explicitly
\[ h = \begin{cases} h_j^n, & \text{for } h_j^n \geq h_{\text{thin}}, \\ h_{\text{thin}}, & \text{for } h_j^n < h_{\text{thin}}. \end{cases} \] (12)

Furthermore, the momentum discrete scheme (7) is deactivated in the dry area. Whereas, a location is considered to be dry if the total cross section area \( A_j^n \) is less than positive threshold value \( A_{\text{thres}} \) for \( j = 1, 2, \ldots, N_x \).

3 Simulation and Discussion

In this section, to test the performance and validate the numerical method of MCS, we simulate the dambreak on a laboratory scale [1] for three types of contractions. Numerical simulation was performed using the same spatial step size \( \Delta x = 0.01 \text{ m} \), gravity \( g = 9.81 \) and the threshold number \( A_{\text{thres}} = 0.01 \text{ m}^2 \) in wet dry procedure and \( h_{\text{thin}} = 0.0001 \text{ m} \) in thin film technique.

3.1 Laboratory experiment scale benchmarking

The laboratory experiments were carried out by [1] in a rectangular horizontal channel with the following dimensions: 8.90 m long, 0.30 m wide and 0.34 m high, see Figure. 2a. The reservoir upstream was formed by a vertical gate representing the dam, located at a distance of 4.65 m from the left boundary, and filled with water up to a depth of \( h_0 = 0.25 \) m in the initial state. The downstream section of the channel, 4.25 m long, was initially dry and right boundary is open so that the flow would fall freely without reflection, see Figure. 2b. Three types of contraction, two trapezoidal and one triangular, are placed in the contraction zone in such a way represent transition from smooth (represent by trapezoidal A and B contractions) to sudden contraction (represent by triangular), see Figure. 2c. The lengths of the contraction zone 0.95 m, the maximum contraction width 0.10 m and the distance from the gate 1.52 m were chosen equally in each experiment.

Figure 2. Experiment configuration; (a) side view, (b) top view of the channel along with locations of the four gauges are indicated by crosses: G1 (blue), G2 (green), G3 (magenta), and G4 (brown). (c) Top view of the three contraction models and their dimension (in meter).
In this section, we observed the capability of the MCS scheme for simulating dambreak through three different contraction channels. For simulation purposes, the initial condition is $u(x, 0) = 0$, $Q(x, 0) = 0$, whereas the initial water level is 0.25 m on the upstream part of the reservoir, and $h_{\text{thin}}$ on the downstream part of the channel to represent the dry situation.

Figure 3 presents snapshots of the fluid surface obtained from the numerical calculation of a dambreak in the canal with trapezoidal A. All results are presented in the non-dimensional water level $H = h/h_0$ and the non-dimensional time $T = t\sqrt{g/h_0}$. In the experiment [1], the gate representing the dam is lifted suddenly causing the water level in the upstream reservoir will gradually decrease and

![Figure 3](image)

**Figure 3.** The non-dimensional water levels at subsequent times of the dambreak simulation in the channel with trapezoidal A contraction. The red vertical line indicates the gate location, whereas the grey rectangular indicates the contraction area.
propagate downstream through contraction area as a flood wave (see Figure 3a). When $T = 0.5$ (see Figure 3b), this flood wave reaches the contraction area causing the water level to rise and reaches a maximum height i.e. $H = 0.81$ when $T = 1.05$ (see Figure 3c). After reaching the maximum height, the flood wave develops into two parts (see Figure 3d), a transmission wave (which moves downstream and is absorbed by the right boundary) and a reflection wave (moves upstream). The reflection wave reaches the left boundary at $T = 6.25$ and then hits the hard wall boundary (see Figure 3e) and propagates downstream again, reaching the area of contraction when $T = 10$ (see Figure 3f). These propagating waves upstream and downstream continuously occurs until the water in the channel drained due to the right absorbing boundary. Similar phenomena also occurred in the other two cases (trapezoidal B and triangular).

Further, along the canal, there are four observation locations, where the water level were recorded at all time. In Figure 2b, the wave gauge as observation locations are indicated by crosses; right upstream of the dam (G1), right downstream of the dam (G2), at half distance between the dam location and the contraction area (G3), and at the contraction starting point (G4). The comparison between the results of numerical calculations and experimental data are presented through the wave signal plot of non-dimensional water level at wave gauges G1-G4 for the three experiments; Figure 4 and Figure 5 are for the case of trapezoidal A and trapezoidal B, respectively, while Figure 6 is for the triangular. The results of the comparative study between numerical results and experimental data generally show a satisfactory agreement. Slight differences arise in the early stages of the appearance of the reflected wave. Although there are discrepancies in the negative spike prediction, the MCS scheme generally predicts hydraulic jumps well in all three channel types.

When the dam break wave propagates through contraction channel where its encounters a cross-sectional change, some of the flow passes through the existing opening, another wave is reflected in the contracted area and forms a reflected wave that moves upward between the contraction and the end of the upstream channel until the water is completely exhausted. In a real situation when the rapid downstream flow generated by dambreak encounters very abrupt transitions along its path induce a stronger reflection, higher water levels and mixed flow conditions occur upstream of the narrowing section.

![Figure 4](image_url) Time varying water levels at Gauges G1-G4 from numerical versus experimental for channel with trapezoidal A contraction.
**Figure 5.** Time varying water levels at Gauges G1-G4 from numerical versus experimental for channel with trapezoidal B contraction.

**Figure 6.** Time varying water levels at Gauges G1-G4 from numerical versus experimental for channel with triangular contraction.
4 Conclusion
We have studied numerically, the dambreak flow in channels with three different contractions (one triangular and two trapezoidal). The simulation results show good agreement with the experimental data in the literature. This assessment shows the merit of the scheme, which is suitable for dambreak flows in channels of varying width. It should be noted that for simulation of these 2-dimensional dambreak flow events, the MCS numerical scheme that we use is relatively cheap because it is based on a quasi-1d model, but it has been able to provide fairly good accuracy.

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