On the separation of two meteoroid fragments of different shapes

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Abstract. Separation of two meteoroid bodies is studied using modelling method based on a system of grids and solving adjoint aerodynamic and ballistic problems. Bodies of the same mass but different shapes were considered. Calculations were done for different combinations of the cylindrical bodies with their cross sections having a shape of square, elongated rectangular and circle. Initially the bodies were placed near each other on the line perpendicular to the flight direction. The lowest value of separation velocity was obtained for the bodies with circular cross section; the separation velocity is significantly increased for the bodies with elongated rectangular cross section due to rotation. Stable aerodynamic position of the bodies with elongated cross sections occurs when the bodies fly with a large side perpendicular to the flight direction. The lateral velocity acquired by each body appears to be primary determined by its own geometry and show no dependency on the shape of a neighboring body.

1. Introduction
During the flight in the Earth’s atmosphere, meteor body is slowly destroyed under the influence of increasing velocity head and thermal heating [1, 2]. One of the possible and widely acknowledge mechanisms of meteoroid destruction is its fragmentation into a group of bodies that are similar in mass and structural parameters [3, 4]. Such fragments are initially located near each other and continue to move together with significant aerodynamic interaction with each other. The simulation of the dynamics of such systems should reproduce the flow patterns under conditions of significant interference.

The common methods to study the flight dynamics of a system of bodies are based on determining aerodynamic characteristics of the bodies at their various relative positions [2, 5]. Such approach is quite convenient for systems with a few bodies as the number of their potential positions is limited and can be estimated. The behavior of more complex systems is often assumed to follow the similar patterns and described by summarizing the characteristics of the bodies and introducing the overall expansion speed of the group. It is also important to note that in most cases the shape of meteoroid and its fragments is unknown, thus the bodies are usually represented by bodies of small elongation approximately corresponding to the sphere. In order to estimate the possible influence of the non-spherical shapes on the value of the drag coefficient, the shapes of a rectangular beam and plate are used [6, 7]. However, when considering a rectangular shape, the number of possible positions for the system increases significantly due to the fact that rectangular shape has a pitching moment, i.e. a
moment is formed on surface of the body of rectangular shape when it turns at the angle of attack. Pitching moment leads to the rotation of the body during its flight in the atmosphere so, in order to study the flight dynamics of a system of bodies of rectangular shape taking into account rotation, it is necessary to determine the aerodynamic properties of bodies not only at different relative positions but also at the different initial angles of the bodies [8]. Due to the high complexity the effects of rotation on the expansion and separation of meteoroid fragments are usually not considered [2, 9, 10].

Alternative approach in studying the dynamics of a system of bodies is to consider an adjoint problem when aerodynamic and ballistic problems are solved in parallel. In this case, the most time-consuming aerodynamic problem is solved only for those states of the system that are realized during the flight. An example of a study of the dynamics of two bodies, that have spherical shape, using the solution of the adjoint problem can be found in [11]. At first the flow field is calculated based on the current configuration of the bodies and aerodynamic properties of each individual body are determined. Then the speeds and coordinates of the bodies are changed in accordance with the forces and moments acting on each body, the flow field is recalculated, and the process is repeated.

In paper [12] authors developed and tested a method for solving the adjoint problem that is based on the system of grids to simulate the flow field. This method allows to easily consider different placement of the bodies in the system and provides the possibility to calculate the flow around bodies that have various sizes and shapes. The method was tested on the problem of separation of two identical cylindrical bodies with circle cross sections and it was shown that obtained values for the separation velocity are consistent with theoretical estimates. It was considered in [13] how the shape of bodies can affect the separation velocity of identical fragments without taking into account rotation, and the necessary coefficients were obtained for calculation of the separation velocity for cylindrical bodies of elliptical and rectangular cross sections depending on their elongation. It was found that bodies with an elongated shape acquire a higher separation speed. In [14] the algorithm for solving the adjoint problem was implemented taking into account the possible rotation of bodies. Calculations showed that, when two identical bodies of rectangular shape fly apart while rotating, the bodies acquire lateral speed not only under the influence of lateral force associated with the interference between the bodies, but also due to rotation at the flight section without interference. If bodies are initially placed at the angle of attack to the flight direction or turned at the angle of attack before reaching a stable aerodynamic position, the rotation leads to an additional increase of the separation velocity. As a result, the acquired separation velocity may increase up to five times in comparison with spherical bodies under the assumption that the fragments have elongated shape and rotate during the flight.

Calculations performed in [12, 13, 14] assumed the separation occurred for two identical bodies with a symmetrical arrangement in the flow. It was possible to calculate half of the flow region with one body using the symmetry. This paper presents the results of calculations for the separation of two bodies that have the same mass but different shape. The entire flow field was considered and all calculations were done for two bodies placed in the flow field. Using three different configurations of bodies, we studied how a change in the shape of a neighboring body can affect the separation process and compared how the lateral velocity has been changing.

2. Methodology
A complete mathematical formulation for the problem and method for solving the equations are given in [12, 14]. A system of grids is used to model the flow field. A uniform grid with rectangular cells is used to describe an external non viscous flow field. This grid is not adapted to the shape of the surfaces of the bodies and has rather limited amount of nods so it is not able to simulate dissipative processes in the trail of the bodies. The Euler equations are used to simulate the flow field on this grid and artificial viscosity is used to smooth the solution. A set of smaller grids is placed on top of it. These smaller grids are connected with the surfaces of the bodies and are built only within a certain proximity to them. The set of grids is designed to take into account the shape of the surfaces of the corresponding bodies, and Navier-Stokes equations in the approximation of a thin layer are used on
these grids to simulate the viscous flow field. In the framework of this model all terms of the Euler equations are taken into account and only the second derivatives normal to the surface of the body are taken into account when calculating the dissipative term.

Due to high complexity of the calculations, a two-dimensional plane problem is considered in this work. Navier-Stokes equations in the thin-layer approximation for a two-dimensional plane flow in a curvilinear coordinate system $\xi = \xi(x, y)$, $\eta = \eta(x, y)$ have the following form:

$$
\frac{\partial}{\partial \tau} \mathbf{U} + \frac{\partial}{\partial \xi} \mathbf{E} + \frac{\partial}{\partial \eta} \mathbf{F} + \frac{\partial}{\partial \eta} \mathbf{E} + \eta \mathbf{F} = \frac{\partial}{\partial \eta} \mathbf{S},
$$

$$
\mathbf{U} = \begin{cases}
\rho \\
\rho u \\
\rho v \\
e
\end{cases}, \quad \mathbf{E} = \begin{cases}
\rho u \\
\rho u^2 + p \\
\rho uv \\
(e + p)u
\end{cases}, \quad \mathbf{F} = \begin{cases}
\rho v \\
\rho uv \\
\rho v^2 + p \\
(e + p)v
\end{cases}, \quad \mathbf{S} = \mu \begin{cases}
0 \\
\frac{m_i \partial u}{\partial \eta} + m_s \eta_i \\
\frac{m_i \partial v}{\partial \eta} + m_s \eta_v \\
m_i
\end{cases}.
$$

$$
m_i = \eta_i^2 + \eta_v^2; \quad m_s = \frac{1}{3} \left[ \eta_i \frac{\partial \xi}{\partial \eta} + \eta_v \frac{\partial v}{\partial \eta} \right]; \quad m_i = m \left[ \frac{\gamma}{\gamma - 1} \frac{1}{\text{Pr}} \frac{\partial T}{\partial \eta} + \frac{\partial u^2 + v^2}{2} \right] + m_s \left[ \eta_i \eta_i + \eta_v \eta_v \right].
$$

Here $t$ is the time; $\rho$ – air density; $u$, $v$ – projections of the body’s velocities on the corresponding coordinate axes $Ox$ and $Oy$; $V$ – magnitude of the velocity; $p$ – air pressure; $e$ – total energy of a unit volume of gas; Pr is the Prandtl number that is assumed to be constant; $\Re$ is the Reynolds number. The coefficients of the transformation matrix can be calculated using the following formulas:

$$
\xi_s = J \frac{\partial \xi}{\partial \eta}, \quad \xi_s = -J \frac{\partial \xi}{\partial \eta}, \quad \eta_s = -J \frac{\partial \eta}{\partial \xi}, \quad \eta_s = J \frac{\partial \eta}{\partial \xi},
$$

where $J$ is Jacobian of the transformation, $J^{-1} = \frac{\partial \xi}{\partial \eta} \frac{\partial \eta}{\partial \xi} - \frac{\partial \xi}{\partial \xi} \frac{\partial \eta}{\partial \eta}$. When deriving the given system of equations it is assumed that the coordinate lines $\xi = \text{const}$ come at the perpendicular direction to the surface of the body. The directional derivatives actually correspond to the derivatives along the local normals to the surface of the body. This ensures that second derivatives normal to the surface of the body are taken into account when calculating the dissipative term. The ability to move and rotate the grids together with their corresponding bodies relative to an external rectangular grid allows performing calculation in the adjoint formulation of the problem.

The initial solution of aerodynamic problem is obtained with the help of relaxation method. An explicit second-order approximation scheme of TVD-type is used. A time integration step is selected according to stability condition for external flow field and a local time step is used when integrating on the grids built around the bodies, i.e. for each computational grid the time step is selected from local conditions. In order to connect the solutions, the values of gas-dynamic characteristics on the outer boundaries of the grids near the bodies are determined from the solution obtained on the external grid. At the same time, the solution at all nodes of the external grid that appear to be inside the area of the grids constructed near bodies is replaced by the solution obtained on the smaller grids. Lineal interpolation is used to recalculate the values of gas-dynamic characteristics since the grid nodes do not match with each other.

At the initial stage the configuration of the system of bodies is set, and the flow field is calculated. The position of the bodies relative to each other is taken into account but the possible difference in the velocities of the bodies is neglected similar to the experiments in wind tunnels. Then aerodynamic forces and moments acting on each individual body are calculated from the pressure distribution. In
the case of plane problem the motion of the center of mass of each individual body is described by the system of equations:

\[
\begin{align*}
\frac{dx}{dt} &= u, \quad m \frac{du}{dt} = -c_x \cdot 0.5 \rho V^2 S, \\
\frac{dy}{dt} &= v, \quad m \frac{dv}{dt} = c_y \cdot 0.5 \rho V^2 S,
\end{align*}
\]

(1)

Here \(t\) is time; \(x, y\) – coordinates of the center of mass; \(u, v\) – projections of the body’s velocities on the corresponding coordinate axes \(Ox\) and \(Oy\); \(V\) – magnitude of the velocity; \(m\) – mass of the body; \(\rho\) – air density; \(S\) – characteristic area of the body; \(c_x\) – coefficient of aerodynamic drag; \(c_y\) – coefficient of lateral force (or coefficient of aerodynamic lift in the chosen system of coordinates). The rotation of the body around its center of mass is described by the equations:

\[
\begin{align*}
\frac{d\alpha}{dt} &= \omega, \\
I_{zz} \frac{d\omega}{dt} &= m_\alpha \cdot 0.5 \rho V^2 SL,
\end{align*}
\]

(2)

Here \(\alpha\) – angle of attack of the body in regards to the initial flight direction; \(\omega\) – angular velocity of the body relative to the axis \(Oz\); \(I_{zz}\) – moment of inertia of the body; \(m_\alpha\) – coefficient of the pitching moment; \(L\) – characteristic size of the body.

The system of equations (1), (2) for each of the bodies determines the dynamics of the system as a whole. The aerodynamic coefficients \(c_x, c_y\) and \(m_\alpha\) are necessary in order to determine the solution and are calculated from the pressure distribution on the surface of the bodies. After this the state of the system (coordinates and velocities of the bodies, rotation angle and angular velocity) is recalculated after a small time interval \(\Delta t\). The aerodynamic coefficients in this case are determined from the solution of the gas-dynamic problem, and the values of the ballistic parameters are determined from the current values for each of the body. The bodies are moved over the selected time step \(\Delta t\) – each body at a distance corresponding to the relative movement of that body in the system. At the same time, restrictions on the maximum possible value of the movement and rotation of each body in the system are imposed. After moving the bodies the flow field is recalculated for a give time step \(\Delta t\) with a new pressure distribution forming on the surfaces of the bodies. The new values for aerodynamic coefficients \(c_x, c_y\) and \(m_\alpha\) are determined, and the system is integrated over time again. With the help of this iterative method it is possible to observe the dynamic of the system at large time intervals.

The main assumption of this method is the initial state of the system as the bodies are place in certain configuration and fly along the same direction at the same speed. This assumption allows to reduce the number of unknown parameters, and it seems reasonable to study the influence of physical phenomena on the separation of bodies such as the effects of shock waves from neighboring bodies or movements in the trail of another body. The magnitude and direction of the velocity of each individual body in the system changes over time, however its deviation from the average velocity of the system will be insignificant:

\[
\sqrt{(u - u_{av})^2 + v^2} < u_{av},
\]

(3)

Here \(u_{av}\) is the average velocity of the system along the initial flight direction. If any of the body has a significantly higher velocity and condition (3) is not satisfied then that body will quickly move away from the system, and then its movement can be considered independently from the other bodies of the system.
3. Results and discussion
The basic cylindrical body was taken with a cross section having a cube shape of an edge size of 1 m (body No.1). The mass of such body is 3300 kg, assuming that characteristic density of meteor bodies of stone composition $\rho_m = 3.3 \text{ g/cm}^3$ [15]. In order to construct other bodies, the cross section of this body was deformed so that its volume remained constant.

Table 1 shows the dimensions and inertial-mass characteristics of all the bodies that were used in calculations. We constructed a body with rectangular cross section of elongation $\lambda = 2.0$, having dimensions $a = 0.707 \text{ m}$ along the $Oy$ axis, $b = 1.414 \text{ m}$ along the $Ox$ axis and $c = 1 \text{ m}$ along the $Oz$ axis (body No.2) to study the possible effect of elongation on the separation of fragments. Also a body with circular cross section was constructed in order to compare the results of calculations (body No.3). The moment of inertia of homogeneous bodies with rectangular cross section in relation to the $Oz$ axis is determined by the expression $I_{zz} = m(a^2 + b^2)/12$, and the moment of inertia of a homogeneous body with circular cross section is determined by the expression $I_{zz} = mR^2/2$.

| No | Shape of the cross section | $a$ (m) | $b$ (m) | $c$ (m) | elongation $\lambda$ | $I_{zz}$ (kg\cdot m$^2$) |
|----|-----------------------------|---------|---------|---------|----------------------|-------------------------|
| 1  | Square                      | 1       | 1       | 1       | 1                    | 550                     |
| 2  | Rectangle                   | 0.707   | 1.414   | 1       | 2                    | 687                     |
| 3  | Circle                      | 1.128   | 1.128   | 1       | 1                    | 525                     |

The presented three variants of the bodies geometry define three possible combinations for the problem of separation of two bodies when they have different shapes. In addition the calculations were performed for the separation of these bodies with identical shape. The bodies were initially placed on a line perpendicular to the flight direction with a distance between their centers of masses of 1.5 m. All calculations were performed for the flight conditions at a speed of 15 km/s at an altitude of 20 km. Calculations were carried out for the Mach number $M = 6$, since aerodynamic properties of the bodies change little at large Mach numbers according to the theory of flows with high supersonic speed [16]. The laminar viscosity was set corresponding to the Reynolds number $Re = 10^5$, determined by the length chosen as unit size. Taking viscosity into account allows to obtain physically plausible flow pattern in the trail of the body and provide the possible to make stable simulations in a wide range of conditions during the movement of the bodies. At the same time the real effects of viscosity at high supersonic flight speed have little effect on the aerodynamic drag of the bodies and do not have any effect on the lateral force characteristics that determine the separation of the bodies that are the primal study of this paper.

The corresponding pressure patterns in comparison to the atmospheric pressure $p_a$ for these three initial combinations of bodies are presented in Figure 1. A collective head shock wave is formed in front of the bodies; due to asymmetry in pressure distribution there is lateral force acting on the bodies that will push them apart. It is also important to note that bodies with rectangular section were placed at the zero initial angle of attack, which corresponds to the state of aerodynamic equilibrium if those bodies were flying alone. However due to interference a moment is formed on their surface that should lead to the rotation of the bodies.
Figures 2-4 show the pressure patterns for the separation of bodies from the beginning up to a time \( t=0.05s \) for all three considered combinations of the bodies. By the time \( t=0.04s \) the presence of a neighboring body no longer affects the flight of each body. Bodies with rectangular cross section rotate during the separation. Body No.2 has a significantly larger angle of attack at time \( t=0.05s \) in contrast to the body No.1, which leads to additional lateral force acting on the body. A stable aerodynamic position of the body with elongated rectangular cross section occurs when body flies with a large side perpendicular to the direction of the flight which is consistent with results obtained in [8]. The body with square cross section remains in a relatively stable position after the separation of bodies. The rotation of the body with circular cross section does not affect its flight at all.

![Pressure patterns for three combinations of initial positions of the bodies.](image1)

**Figure 1.** Pressure patterns for three combinations of initial positions of the bodies.

![Pressure patterns for the separation of two bodies with square and elongated rectangular cross sections (bodies No.1 and No.2).](image2)

**Figure 2.** Pressure patterns for the separation of two bodies with square and elongated rectangular cross sections (bodies No.1 and No.2).
Figure 3. Pressure patterns for the separation of two bodies with circular and square cross sections (bodies No.1 and No.3).

Figure 5 shows the absolute value of the lateral velocity \( v \) as a function of time \( t \) for the three calculated combinations of the initial positions of the bodies. The first digit in the designation corresponds to the number of the body for which the function of changing the speed is given, and the second digit corresponds to the number of the neighboring body. It is important to note that dependency lines for the same body appear close to each other and reflect the same behavior during the separation process. For body No.2 the lateral velocity is still oscillating after \( t = 0.15 \)s but will eventually stabilize at the same value.

Figure 4. Pressure patterns for the separation of two bodies with circular and elongated rectangular cross sections (bodies No.2 and No.3).
Figure 5. Lateral velocity $v$ of the bodies depending on the time $t$. The first digit corresponds to the shape of cross section of the body and the second digit corresponds to the shape of cross section of neighboring body: 1 – square, 2 – elongated rectangle, 3 – circle.

Despite the fact that separation of the bodies of different shapes occurs under influence of the bodies on each other, the change in lateral velocity is primarily determined by the geometry of the body itself. The lowest value of the separation velocity was obtained for the body with circular cross section (body No.3) – about 40 m/s. For body No.1 with square cross section the separation velocity was increased to 65-70 m/s, which is mainly associated with increased interference of the bodies due to the difference in the geometry of their cross sections [13]. A significant increase in separation velocity for body No.2 is caused not only by interference (the separation phase until $t = 0.035s$) but also by the presence of the flight phase when body was turned at the angle of attack (time interval $0.035s < t < 0.065s$). The flight of body No.2 after $t = 0.035s$ takes place without interaction of bodies with one another. In this case the body has a rather large value of rotation speed and turns around through the position of aerodynamic equilibrium – accordingly, the rotation speed decreases at the time interval $0.065s < t < 0.13s$. Subsequently, due to damping of the angular velocity, the body No.2 will move to a position with a larger side against the direction of the flight, and the lateral velocity will then be set to approximately 150 m/s.

Figure 6 shows how the coefficient of lateral force $c_y$ depends on the distance $y$ between the center of mass of the bodies and the axis $Ox$ located between bodies. Three sets of lines are shown – 1, 2 and 3 that correspond to the bodies from Table 1. Lines 1 are shown as solid, lines 2 and 3 are shows as dashed lines. The line sets for each body appear to be near each other and group well. The lines are marked to indicate the type of body with which the separation was considered. Marker in the form of a square corresponds to the body No.1, rhombus – to the body No.2 and circle – to the body No.3. The separation velocity for each of the bodies is determined by the integral of the function $c_y(y)$ and the fact, that corresponding lines are grouped together for each of the bodies, means that their lateral velocities have similar values (Figure 5). This behavior of dependencies can also be noted in similar pressure patterns in Figures 2-4 and allows to conclude that bodies of the same shape follow the same pattern during the process of separation. In other words, the acquired separation velocity of the body depends only on its shape and independent from the shape of a neighboring body. This is important conclusion as the dynamics of the system of bodies may heavily depend on the initial conditions, how it was shown in [14], however it appears to be possible to consider and study the process of separation of meteoroid fragment with c certain shape without the regard to the shape of its neighboring body.
Figure 6. Change off the coefficient of lateral force $c_y$ depending on $y$. A set of lines corresponds to the shape of the body: 1 – square, 2 – rectangle, 3 – circle. Marker indicates the shape of the neighboring body.

4. Conclusion
Calculations are done for the dynamics of the supersonic separation of two bodies of the same mass but different shape. Initially the bodies were positioned so that their centers of masses were on a line perpendicular to the direction of motion. We considered bodies with cross sections having the shape of square, elongated rectangle and circle. The lowest value of separation velocity was obtained for the bodies with circular cross section while the separation velocity is significantly increased for the bodies with elongated shape due to the rotation in the similar manner to [14]. Stable aerodynamic position of the bodies with elongated cross sections occurs when the body flies with a large side perpendicular to the fight direction in accordance with [8]. A completely new result provided by the calculations is that the lateral velocity acquired by each individual body seems to be determined by its own geometry and has no dependency on the shape of a neighboring body. It allows to conclude that bodies of the same shape follow the same pattern during their separation from the group of fragments regardless of structure or shape of that group. That said, it is important to note that in the case if there is a significant mass or size disparity of the bodies then the separation dynamics may be different and requires further investigation.

References
[1] Grigorian S S 1979 On the motion and destruction of meteorites in the atmosphere of planets Cosmic Res+ 17 875
[2] Stulov V P, Mirskii V N, Vislyi A I 1995 Fireball Aerodynamics (Moscow: Nauka) p 240
[3] Borovicka J, Kalenda P 2003 The Moravka meteorite fall: 4. Meteoroid dynamics and fragmentation in the atmosphere Meteorit. Planet. Sci. 38 1023–43
[4] Borovicka J et al. 2013 The Kosice meteorite fall: Atmospheric trajectory, fragmentation, and orbit Meteorit. Planet. Sci. 48 1757–79
[5] Marwege A, Willems S, Gulhan A, Aftosmis M J and Stern E C 2018 Superposition Method for Force Estimation on bodies in Supersonic and Hypersonic Flows J. Spacecraft. Rockets 55 1166–80
[6] Zhdan I A, Stulov V P, Stulov P V and Turchak L I 2007 Drag coefficients for bodies of meteorite-like shapes Sol. Syst. Res. 41 505–8
[7] Gritsevich M I 2008 Estimating the terminal mass of large meteoroids Dokl. Phys. 53 588–594
[8] Zhdan I A, Stulov V P, Stulov P V and Turchak L I 2009 Motion of meteor form bodies at an
arbitrary angle of attack Sol. Syst. Res. 43 434–7
[9] Artemieva N A and Shuvalov V V 2001 Motion of a fragmented meteoroid through the planetary atmosphere J. Geophys. Res. 106 3297–309
[10] Syzranova N G and Andrushchenko V A 2016 Simulation of the motion and destruction of bolides in the Earth’s atmosphere High Temp. 54 308–15
[11] Barri N G 2010 Dynamics of two spherical objects in supersonic flow Dokl. Phys. 55 516–518
[12] Lukashenko V T and Maksimov F A 2017 Numerically simulated model of meteor body fragments distribution after destruction Eng. J. Sci. Innov. 69 1–13
[13] Lukashenko V T and Maksimov F A 2017 Proc. of the XX Int. Conf. on Computational Mechanics and Modern Applied Software Systems (Moscow: MAI) pp 500–3
[14] Lukashenko V T and Maksimov F A 2019 Modeling the flight of meteoroid fragments with accounting for rotation Computer Research and Modeling 11 593–612
[15] Britt D T, Yeomans D, Housen K and Consolmagno G 2002 Asteroids III ed V F Bottke, A Celino, P Paolicchi and R P Binzel (Tucson: Univ. Arizona Press) 485–500
[16] Chernyj G G 2004 Gas flows at high supersonic speed (Moscow: Fizmatlit) p 220