Does Optical Anisotropy Lead to Negative Refraction at an Interface?

Ivan Biaggio
Center for Optical Technologies and Department of Physics,
Lehigh University, Bethlehem, PA 18015
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This is a comment inspired by recently published results [Y. Zhang et al., Phys. Rev. Lett. 91, 157404 (2003)] that introduced the name “amphoteric refraction” for the fact that at the interface with an optically anisotropic material there can be a range of incidence angles for which the component of the Poynting vector parallel to the interface changes sign upon refraction. The latter effect is a well-known consequence of optical anisotropy, but it was described as a new negative refraction phenomenon that can be put in the same class as the negative refraction observed at an interface with a left-handed material with negative refractive index.

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In a recent letter (“Total Negative Refraction in Real Crystals for Ballistic Electrons and Light”) [1], Zhang et al. studied light propagation through the plane boundary between two different orientations of the same birefringent crystal (“twin boundary”), with the two optic axes parallel to the plane of incidence but tilted by the same amount in opposite directions with respect to the boundary. They pointed out the range of incidence angles for which the components of the incident and refracted Poynting vectors parallel to the interface have a different sign. They called this effect “amphoteric refraction”, claiming that it was “unusual” and a case of negative refraction.

However, the refraction effect described in Ref. [1] is simply an expression of the difference between Poynting vector and wavevector in an electromagnetic wave [2]. It is misleading to put it in the same class as the negative refraction effects that affect the wavevector of the wave, as in the presence of a negative refractive index

![FIG. 1] A twin boundary between two different orientations of an optically anisotropic material (anisotropy ratio \(n_2/n_1 = 1.5\)). The indicatrix is tilted by \(-45^\circ\) and by \(+45^\circ\) with respect to the boundary. An optical wave that is polarized in the plane of the figure is propagating with a wavevector \(k_i\) perpendicular to the interface. The angle between wavevector and Poynting vector \(S_i\) is \(\sim 21^\circ\) and the latter has two different orientations in the two materials.

![FIG. 2] Refraction vs. incidence angles in the same representation as in Fig. 4 of Ref. [1] and calculated using the same material parameters. The dashed and solid curves refer to Poynting vectors and wavevectors, respectively. The thick curves are for the twin boundary, the thin curves are the result when the crystal on the incidence side is replaced by air. The inset shows the full range of angles.

3 4. What has been described as a new “amphoteric” refraction phenomenon in Ref. [1] always occurs at the interface with any anisotropic material when the main axes of the indicatrix are not perpendicular to the interface, and leads to such well-known effects as double-refraction in calcite and conical refraction [2]. A sketch of the interface in question is given in Fig. [1].

Fig. [2] plots the incidence and refraction angles in the same way used in Ref. [1] with its data and theory given by the thick dashed curves. The solid curves give the corresponding wavevector angles, i.e. the incidence and refraction angles used in Snell’s law. The longitudinal and the transverse components of the wavevectors of incident and refractive wave, and the corresponding incident and refracted angles, always have the same sign, the usual refraction effect expected for positive refractive in-
indices. The fact that the dashed curves describing the Poynting vectors do not go through the origin — called “amphoteric refraction” in Ref. \(^1\) — is a normal effect in anisotropic materials which is also seen at an air/crystal interface (thinner curves). Expressions for calculating refraction and reflectivity in this case have been given, e.g., in Ref. \(^5\).

These are basic facts of wave propagation in anisotropic crystals, but it is important to point this out in reference to the claim of Ref. \(^1\) of having obtained negative refraction in the sense of Ref. \(^4\) “despite all components of \(\varepsilon\) and \(\mu\) being positive” \(^1\), when what they showed is just the usual effect of anisotropic (positive) refractive indices.

I think that in order to allow future discussions that focus on the physics it highlights, the expression “negative refraction” should be used with some care. Simply applying the label to all effects that cause the lateral component of the Poynting vector to switch sign at an interface, thus including standard double-refraction effects, is a generalization that does not appear to be very helpful in order to connect to the underlying physical phenomena. It seems more logical to apply the “negative refraction” label in a more restrictive way, only to those cases where unusual effects are observed in terms of the wavefronts and wavevectors that are used in Snell’s refraction law.

Since wave propagation is described in terms of phase-changes and wavevectors — which we have seen behave normally at this interface — the discussion of experimental opportunities and possible applications of the refraction at the twin-boundary in Ref. \(^1\) should also be re-examined.

As a final note, it must be pointed out that the present comment should not overshadow another result derived in Ref. \(^1\), namely that the reflection from the symmetric twin boundary is exactly zero for all angles of incidence and despite the presence, at non-vanishing incidence angles, of a finite refraction! \(^1\)

**Note:** Please see Refs. \(^7\) and \(^8\) for previous publications that support the present comment, and Ref. \(^9\) for a complementary, more general discussion of this topic.

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