Experimental and theoretical evidences for the ice regime in planar artificial spin ices

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Abstract
In this work, we explore a kind of geometrical effect in the thermodynamics of artificial spin ices (ASI). In general, such artificial materials are athermal. Here, we demonstrate that geometrically driven dynamics in ASI can open up the panorama of exploring distinct ground states and thermally magnetic monopole excitations. It is shown that a particular ASI lattice will provide a richer thermodynamics with nanomagnet spins experiencing less restriction to flip precisely in a kind of rhombic lattice. This can be observed by analysis of only three types of rectangular artificial spin ices (RASI). Denoting the horizontal and vertical lattice spacings by \(a\) and \(b\), respectively, then, a RASI material can be described by its aspect ratio \(\gamma \equiv a/b\). The rhombic lattice emerges when \(\gamma = \sqrt{3}\). So, by comparing the impact of thermal effects on the spin flips in these three appropriate different RASI arrays, it is possible to find a system very close to the ice regime.

Keywords: nanomagnetism, spin ices, magnetic monopoles

(Some figures may appear in colour only in the online journal)

Introduction

Arrays of nanomagnets designed to resemble the spin ice materials (disordered magnetic states) are known as artificial spin ices (ASI). Nowadays, with the advances of the nanotechnology and nanofabrication, ASI systems have become so famous as well as their natural counterparts, with the advantage that they can be constructed with desirable geometries and properties. The first ASI was built in 2006 and it consists of a two-dimensional (2D) square array of 80 000 elongated magnetic nanoislands, each a few hundred nanometers long [1]. The net magnetic moment (spin) of each individual nanoisland is aligned parallel to its longest axis (like in a bar magnet), and is coupled to all other nanoislands of the planar array by the ubiquitous dipolar interaction. Then, in its original configuration, ASI tiles a square lattice of vertices, with four nanoislands meeting at each vertex.

The ground state of the artificial square ice obeys the ice rule, which remains the familiar two-in, two-out (two spins must point in, while the other two must point out in each vertex). However, in two dimensions, the standard ice rule is no longer degenerate[1–3]. In addition, ASI systems have long been athermal (these compounds were almost always found in frozen at room temperature), until the most recent investigations on patterned ultrathin magnetic films could pave the way to explore and visualize the real-time dynamics of all kinds of different frustrated geometries. Indeed, recently, several works have given attention to certain thermal properties of ASI compounds in diverse types of planar lattices[4–12]. However, the 2D lattice obeying the usual two-in, two-out ice
rule with a degenerated ground state did not deserve yet an adequate treatment. In this paper, we would like to do this by comparing arrays that exhibit different ground states, when they are heated to a high temperature regime (we mean by high temperature, the practical values in the range 300 K–800 K). Some of their magnetic properties (as a function of temperature \( T \)) are observed by photoemission electron microscopy (PEEM) combined with x-ray magnetic circular dichroism (XMCD) measurements. MOKE signals are also analyzed. It is done by deforming continuously the square lattice in a rectangular one. Therefore, we experimentally focus on rectangular artificial spin ices (RASI) with horizontal and vertical lattice parameters given by \( a \) and \( b \) respectively (\( b \) is kept constant while \( a \) is varied). Our samples match in three classes of RASI, appropriately designed to illustrate how a particular planar system can approximate of the ice regime. They are heated from room temperature until a temperature near 800 K (just below the Curie Temperature of the permalloy, which is \( T_{\text{C}} \approx 873 \) K).

Theoretical calculations [13] concerning the rectangular artificial systems suggest that the ice regime (with the required degenerate ground state), could be observed when the aspect ratio of the lattice (\( \gamma \equiv a/b \)) is equal to \( \sqrt{3} \) (the rhombic lattice). On the other hand, like the artificial square array, the ground states of RASI compounds with \( 1 < \gamma < \sqrt{3} \) and \( \gamma > \sqrt{3} \) are not degenerate, but they have very distinct magnetic properties: in the first case (\( 1 < \gamma < \sqrt{3} \)), there are residual magnetic charges at each vertex, alternating from positive to negative along the neighbor vertices (these charges are not cylindrically symmetric exhibiting a strong quadrupole moment). This system can be characterized as antiferromagnetic (along the vertical and horizontal lines of spins). In the second case (\( \gamma > \sqrt{3} \)), there are alternating residual magnetic moments along the neighbor vertices. Again, looking the vertical and horizontal lines of spins, it can be characterized as a ferromagnetic state. At \( \gamma = \gamma_{\text{R}} = \sqrt{3} \), these two distinct types of states have the same energy and the ground state becomes degenerated. This special case separates the antiferromagnetic state (\( 1 < \gamma < \sqrt{3} \)) from the ferromagnetic one (\( \gamma > \sqrt{3} \)) and can be distinguished as a more realistic spin ice state in these artificial systems (later, the difficulty of obtaining an accurate ice state, theoretically and experimentally, will be discussed).

Figure 1(a) shows an image of a RASI with \( \gamma = \sqrt{3} \) obtained by PEEM measurements combined with XMCD technique (see also figure 1(b)), displaying the magnetic monodroms of the nanoislands. Figure 1(c) shows the five possible vertex types of these structures. Vertex types \( t_1 \) and \( t_2 \) obey the ice rule (two-in, two-out) while the other three (\( t_3, t_3^* \) and \( t_4 \)) represent excited states singly (\( t_3 \) and \( t_3^* \)) and doubly (\( t_4 \)) charged magnetic monopoles [13]. The energy of all these vertex types depends on \( \gamma \). For \( \gamma = \gamma_{\text{R}} \), the \( t_1 \) and \( t_2 \) types have the same energy, yielding to a degenerate ground state. To easily see such an array, it is better to replace the net magnetic moment of the nanoislands by a point-like dipole at their centers. Then, the four dipoles (spins) for the case \( \gamma = \sqrt{3} \) are located at the vertices of a rhombus with short and long diagonals \( b \) and \( a = \sqrt{3}b \), respectively and, therefore, they are equidistant (in this case, the distance of each pair of spins in a rhombus is \( b \)). Then, we can say that the spins are placed at the vertices of a kind of rhombic lattice with rhombi having spins pointing out parallel to their diagonals alternating (along the diagonal of the rectangles) with rhombi having spins pointing out perpendicular to their diagonals. A rhombic lattice is one of the five 2D lattice types as given by the crystallographic restriction theorem.

Here, we show theoretically and experimentally that the geometry distortion of the planar arrays may be an additional ingredient able to cause important physical phenomena in ASI materials. In fact, by studying three RASI systems with different aspect ratios \( \gamma \), we demonstrate that the geometrical influence goes beyond the simple effect of the variation of the lattice parameters. To explain the aims of this work, we organize the sequence of the paper as follows: firstly, we use Monte Carlo (MC) simulations to calculate the specific heat and topologies density as functions of temperature \( T \). In principle, based on the square lattice, these thermodynamic quantities have important features only when the temperature scale is on the order of \( 10^3 \) K. If it is true, most of these theoretical results could not be experimentally verified but they will give us, at least, interesting insights about ASI and RASI. On the other hand, our MC simulations also suggest that the critical temperatures for the specific heat fall rapidly as \( \gamma \) increases till \( \sqrt{3} \), guiding some possibilities and questions about RASI that only experiments could answer. Hence, an experimental investigation is the second natural step of this paper. We then experimentally measure the magnetization and topology density as functions of \( T \) by heating the samples from room temperature (300 K) to \( \sim 750 \) K. Although this range of temperature is about (10–100) times smaller than the typical temperature scale necessary to easily flip the large spin of the magnetic nanoislands, we can directly observe very interesting thermal effects in some artificial materials, when the geometry is stretched from the square to the rectangular lattice. The main conclusion here is that arrays with \( \gamma \approx 1 \) (almost square) are essentially athermal systems as already mentioned in several works, while arrays with \( \gamma \approx \sqrt{3} \) present important thermal and dynamical effects even at a temperature scale on the order of \( 10^2–10^3 \) K. We will see that this effect does not have to do with the fact that the rectangular arrays (with \( \gamma > 1 \)) are more weakly coupled than the square array (of side \( b \)). Really, although the temperature-induced onset of magnetic fluctuations increases with the lattice spacing and related interaction strength between nanoislands [8], our results show that an extra factor will be more fundamental here: lattice geometry. A purely geometrical effect will make the net magnetic moments to appear flexible enough to rather flip with a smaller amount of effort (for temperatures compatible with artificial magnets), allowing the dipolar interaction to have more protagonism in determining the dynamical properties of the system than usually it does. For investigating the two aims cited above, RASI materials with aspect ratios equal to \( \gamma = \sqrt{2} \), \( \gamma = \gamma_{\text{R}} = \sqrt{3} \) and \( \gamma = \sqrt{4} \) were appropriately chosen and fabricated. So the effects proposed here will be exposed by comparing some thermodynamic quantities of these three rectangular arrays, observing the roles of their geometries.
Figure 1. (a) 30 µm × 30 µm PEEM image of γ = √3 RASI lattice. The Permalloy nanoislands have 2800 nm × 400nm × 10nm. It would be useful to say that those nanomagnets, larger then usual ASI systems, were used in order to achieve higher image contrast in our PEEM, after careful tests performed by micromagnetic simulations and Magnetic Force Measurements, which ensured that it is inside the limit to present monodoms (not supporting vortices or multidomains). (b) Typical XMCD measurement for the same area of (a) with the clear and dark contrast representing the orientation of the islands monodoms, in each direction. The measurement was taken with the array rotated 45 degrees from the x-ray sensitivity to resolve x and y direction at the same time. (c) Possible vertex types for each vertex represented by different colors, where arrows represent the orientation of the island related to XMCD pattern.

Results

We start by reporting the thermodynamic results for RASI as obtained by MC simulations. The calculations presented here use the point-dipole approximation but we have also done calculations using the dumbbell model. We did not find any relevant differences between the two methods. The model is implemented by standard MC techniques on a system with N = 364 spins. Here, the spin–spin interaction is assumed to be purely dipole-dipole such that the Hamiltonian is given by $H = D \sum_{\langle i, j \rangle} \left( \frac{\hat{e}_i \cdot \hat{e}_j}{r_{ij}^3} - \frac{3(\hat{e}_i \cdot \hat{r}_{ij})(\hat{e}_j \cdot \hat{r}_{ij})}{r_{ij}^5} \right) s_i s_j$, where $D \approx 2.1 \times 10^{-19}$ J is the coupling constant of the dipolar interaction, $\hat{e}_i$ is the local Ising axes of the rectangular lattice, $r_{ij}$ is the distance between spins and $s_i = \pm 1$ represents the two states (up/down) of the Ising spin. In our procedure, Monte Carlo step consists of $N$ single-spin flips and we have used $10^6$ Monte Carlo steps to reach equilibrium configurations and $10^5$ steps to get thermodynamics averages. Samples are first prepared in a disordered state and then cooled to low temperatures; this annealing protocol in general drives the spin configuration to the ground state. In this process, the specific heat is calculated by the fluctuations in the total energy of the system, $c = (\Delta E)^2/k_B T^2 N$.

For the usual artificial square ice, previous calculations by Silva et al [5] have suggested the existence of a phase transition at a critical temperature $T_p \sim 7.2D/k_B$, where $D$ is the coupling constant of the dipolar interaction and $k_B$ is the Boltzmann constant. Really, the specific heat exhibits a sharp peak at $T_p$, whereas the amplitude diverges logarithmically with the system size $L$. Such a phase transition was speculated to be attributed to the vanishing of the string tension joining monopoles of opposite charges: below $T_p$, the monopoles are joined by an energetic (and observable) string (Nambu monopoles [15]); above $T_p$, the string tension should vanish and some monopoles become free to move (actually, they may not be completely free because a monopole pair is subjected to an entropic force that exhibits, in two dimensions, a logarithmic distance dependence [5, 16]). Concerning the specific heat, our MC calculations lead to similar behaviors for the RASI materials. Figure 2 shows the specific heat as a function of temperature for the three different cases considered here. As expected, the critical temperature ($T_{c, n}, n = 2, 3, 4$), at which the peaks occur, is a function of the aspect ratio $\gamma$. Initially, as $\gamma$ is increased from 1 (square ice), the critical temperature decreases. It is expected because the coupling among nanoislands becomes weaker as the array becomes more stretched. However, interestingly, this critical temperature has a minimum for $\gamma = \sqrt{3}$, in such a way that $T_p > T_{c, 2} > T_{c, 3} < T_{c, 4} < T_{c, 2}$. Except for $\gamma = \sqrt{3}$, the reasonable idea that the critical temperature decreases as $\gamma$ increases works very well ($T_{c, 2} = 2.7D/k_B, T_{c, 3} = 0.5D/k_B$...)
and $T_{\sqrt{\gamma}} = 0.8D/k_B$) see figure 2(a)). It reinforces the fact that a rhombic ice has special properties as demonstrated by previous calculations [13]. Indeed, to our knowledge, it is the only one planar case, obeying the familiar two-in, two-out ice rule, that has a degenerate ground state (topologies $t_1$ and $t_2$ possess the same energy). In these circumstances, the string tension should be zero for any temperature (including the absolute zero). However, an entropic effect generates an attractive interaction potential $\left(T \ln R, \text{where} \ R \text{is the distance between a monopole and its antimonopole in a pair} \right)$ in such a way that monopoles should become free only at $T = 0$. For finite temperatures, they may be found apart but not completely free. To try to observe some vestiges of the geometry features, we have also calculated the topology densities as functions of temperature. These results are shown in figures 2(b)-(d).

The rectangular ice magnets with $\gamma = \sqrt{2}$ and $\gamma = \sqrt{4}$ start (at $T = 0$) with all vertices in their respective natural ground states (the vertex types $t_1$ and $t_2$, respectively). On the other hand, the rhombic ice ($\gamma_R = \sqrt{3}$) starts with all vertices in the vertex type $t_1$, which may be merely one of its possible ground states. In all situations, as $T$ is increased from zero, the other vertex types start slowly to arise around the lattice. It occurs in different manners as $\gamma$ changes. For $\gamma = \sqrt{2}$, the system practically remains entirely in its ground state for a large range of temperatures, in such a way that, the the vertex type $t_1$ begins to really decrease only for $T > 3D/k_B$ (figure 2(b)). Like the square ice, this high temperature value for the materialization of excitations indicates that systems with relatively small aspect ratio ($\gamma \approx 1$) tend to be athermal (in the context frequently used in the literature for ASI). Conversely, for the special case of a rhombic lattice, the initial state (with all vertices in $t_1$) reduces rapidly as $T$ increases and, simultaneously, the density of the $t_2$ vertex type increases considerably (similar results are obtained if the initial state would have all vertices in $t_2$, which is also a possible ground state in this case). It suggests an immediate activity in the spin fluctuations, even at relatively low temperatures. Note that the density of the ground state vertex types ($t_1$ and $t_2$) becomes exactly equal just at the critical temperature $T_{\sqrt{\gamma}}$ (see figure 2(c)). Only these two vertex types are essentially fluctuating for low $T$. It means that, for $0 < T < T_{\sqrt{\gamma}}$, the system still persists almost entirely in its ground states ($t_1$ and $t_2$ have the same energy when $\gamma = \sqrt{3}$), since the vertex types $t_3$, $t_4$ and $t_2$ were not significantly excited yet. Indeed, for this particular case, any mixture of the states $t_1$ and $t_2$, if it is possible to occur, would also be a ground state. Therefore, as the density of $t_2$ increases (and the density of the excited states $t_1$, $t_3$, $t_4$ and $t_2$ remains close to zero), this degenerate system seems to be accessing several of its different ground states. For $T > T_{\sqrt{\gamma}}$, the vertex types $t_3$ and $t_4$ start to be excited and then, this upper-temperature phase rather signalizes the emergence of monopoles with tensionless strings. Finally, figure 2(d) shows that the case $\gamma = \sqrt{4}$ has a behavior qualitatively similar to the first example studied here ($\gamma = \sqrt{2}$). The basic differences are the ground states and the fact that the $t_3$ monopoles (red circles) are more easily excited in the array with $\gamma = \sqrt{4}$, producing a lower transition temperature. The expressive presence of the $t_1$ monopoles in comparison with the $t_4$ monopoles (in all cases) is justified because they have smaller energy (actually, the difference between the energy of $t'_i$ and $t_i$ vertex types increases as $\gamma$ increases [13]). The above theoretical calculations are very suggestive: $T_{\sqrt{\gamma}}$ is about 15 times smaller than $T_p$, which means that the rhombic ice exhibits, at much lower temperatures, a richer dynamics than the square ice. Moreover, it is not only due to a larger mean distance among the magnetic nanoislands, but mainly, due to the lattice geometry itself since the RASI with $\gamma = \sqrt{4}$ presents less fluctuations than the artificial rhombic lattice. Therefore, RASI materials deserve a deeper investigation from the experimental point of view. This is a natural next step: the study of high temperature RASI. Here, high temperature means the practical range in the interval $300 \text{K} < T < 800 \text{K}$.

With these indicative theoretical results in mind, we now consider realistic experimental arrangements in which the temperature of the three kinds of RASI systems varies from 300 K to 750 K. This range of $T$ is, in principle, about 10–100 times smaller than the temperature scale of the most
theoretical calculations presented above, but it is below the Curie temperature ($T_C$) of the permalloy nanoislands.

For the fabrication of Permalloy nanoislands, a multi-layer with composition Si/Ta 3 nm/Ni$_{80}$Fe$_{20}$ 10 nm/Ta 3 nm was previously prepared by sputtering from tantalum (seed and cap layer) and alloyed permalloy target, on silicon substrate. Then, the samples were covered with a 85 nm layer of AR-N 7520.18 negative tone photoresist and patterned by electron lithography at 100 kV of acceleration voltage. After development, the samples were etched by ion milling at 20° from normal incidence, using secondary ion mass spectrometry to detect the end of the process. An ashing in oxygen plasma was subsequently performed to remove the photoresist. The nanoislands dimensions of $l = 2800$ nm and $w = 400$ nm were conceived in order to present magnetic monodomain in each island. The $y$-axis lattice spacing of $b = 3550$ nm was kept in all samples and the $x$-axis lattice constant $a$ ranged from 5017–7100 nm in such a way that we have investigated by (PEEM − XMCD), RASI arrangements with aspect ratios $a/b = \sqrt{2}$, $\sqrt{3}$ and $\sqrt{4}$ (see figure 3). For the PEEM − XMCD measurements the samples were heated at different temperatures and images were taken just after temperature switch-off to prevent different sample holder dilatation and beam defocusing. The MOKE signal was obtained during sample heating. These systems were built on an area of 100 $\mu$m$^2$, which enabled topologies density analysis in arrays of up 10 × 14 unit cell (280 islands). We have also done some Monte Carlo numerical calculations to compare with experimental results.

The XMCD measurements were performed at PGM beamline of the Brazilian Synchrotron Light Laboratory$^6$. They result from the core-level absorption of circularly polarized soft x-ray by a magnetic element and the transfer of right (RCP) or left (LCP) circularly polarized angular momentum of the photons to the excited photoelectrons. The spin and orbital moments can be determined from linear combinations of the dichroic difference intensities of RCP and LCP. The images were taken on the Nickel $L_{2,3}$ edge with a photon energy of 850 eV. The islands array was placed rotated $45^\circ$ related to the x-ray sensitivity in order to fully resolve both $x$ and $y$ directions. MOKE measurements were made using a $p$-polarized source. The islands array were placed rotated

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Figure 3. (a) 100 $\mu$m × 100 $\mu$m PEEM − XMCD images of $\gamma = \sqrt{4}$ RASI for different temperatures (up) and mapped respective magnetic configurations (down). (b) PEEM − XMCD images for $\gamma = \sqrt{2}$, $\gamma = \sqrt{3}$ and $\gamma = \sqrt{4}$ for same temperature of 573 K.

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$^6$ lns.cnem.br/en/
45° related to the scattering plane to fully resolve both $x$ and $y$ directions. The array was saturated by an external magnetic field and the samples were heated in a low vacuum environment to prevent oxidation. The MOKE signal was recorded by varying the temperature in the sample.

Figure 3(a) shows a sequence of snapshots of a RASI with $\gamma = \sqrt{3}$. The images were obtained with samples initially at room temperature (RT) and heated to 473 K, 573 K, 623 K and 673 K. The ground state of this array [13] should have all vertices in the vertex type $t_1$. Figure 3(a) shows, more explicitly, how the temperature can actuate to generate fluctuations, creating and destroying all possible low energy excitations into the system (including monopole pairs). Note also that the density of these low energy excitations increases with $T$.

These pictures also demonstrate that most of these excitations are displayed along the vertical direction, since this is the case with less energy [13]. To emphasize and illustrate the role of the thermal effects on the creation of excitations in RASI materials, we investigate experimentally how magnetized samples will be affected as the temperature is increased. Figure 3(b) presents a comparison of the vertex types for different RASI ratio at the same temperature. Those measurements corroborate the prediction of higher density of high energetic vertex types for the orthorhombic $\gamma = \sqrt{3}$ geometry [14].

All samples were initially prepared (at RT) with normalized magnetization along lattice diagonal. It was used four values for $T$. As the temperature is increased, $M_{\text{tot}}$ decreases for the three cases but with different behaviors. For instance, figure 4(a) shows the Magnetization (left) and the vertex types density (right) for the case $\gamma = \sqrt{2}$ as the temperature is varied from 300 K to $\sim$750 K. Note that the magnetization decreases very slowly as $T$ increases. Meanwhile, there is a slow increasing of the monopoles number ($t_2$ and $t_3$). Such a behavior reveals that this system, like the square ice, is essentially athermal in the range of temperatures compatible with experiments in artificial spin ices. The magnetic moments of the nanoislands do not flip easily and the magnetization remains practically unchanged during all interval of temperatures studied. On the other hand, for the special case of $\gamma = \sqrt{3}$, the magnetization decreases very rapidly as $T$ increases (left side of figure 4(b)) and seems to vanish even at $T < T_c$, while the monopoles population (including also doubly charged monopoles) increases considerably as $T$ increases, mainly for $T > 550$ K (right side of figure 4(b)). This example indicates that the geometry may produce favorable conditions to flip the magnetic moments, making the system to exhibit its thermodynamic properties at lower temperatures. To confirm this fact, we have also investigated the case $\gamma = \sqrt{4}$. It suggests that, for $\gamma > \sqrt{8}$, the magnetization decreases somewhat slower again as $T$ increases (see the left side of figure 4(c)), but it falls much more rapidly as compared to the array with $\gamma = \sqrt{2}$. The different temperature range was chosen to better show the magnetization decay region, which is slightly different from the two other previous samples, once in this particular sample the ground state is predicted to be in the ferromagnetic regime. We then notice that the experimental data obtained for the magnetization can be connected with the theoretical calculations for the specific heat: the temperature in which $M_{\text{tot}}$ goes to zero and the specific heat presents a peak is a minimum for $\gamma = \sqrt{3}$. These thermodynamical features are caused mainly by geometrical effects, reinforcing the idea that the string tension tends to vanish for the rhombic ice. This lattice is singular since it changes the tendencies in rectangular arrays as $\gamma$ is increased from 1 (square ice). Indeed, if the lattice were stretched continuously, one expects that the critical temperature should decrease monotonically as $\gamma$ increases because the mean distance among the nanoislands increases (and as a consequence, the mean coupling among the spins decreases). Of course, it is not the case here. Figure 5 shows the in situ MOKE signal measured as a function of temperature in the lattice diagonal direction, just after magnetization saturation in the same direction. These results also confirm the geometrical deformation effect observed in the MC simulations and XMCD measurements. The difference in the slope of $\gamma = \sqrt{3}$ curve comes from the fact that the ground state is degenerated and then the saturation, which imply in $t_2$ vertex
type, remains at low temperatures in comparison with the other geometries that favors vertex configuration to ground state or formation of high energy vertex configuration with no residual magnetization.

Discussion and conclusion

Important questions about the systems investigated here remain to be explained. They are mainly related to the doubts about whether it is really possible to achieve the perfect ice regime in these artificial materials. We must say that if the system with aspect ratio \( \gamma = \sqrt{3} \) were really in the ice state, then there would be no peak in the specific heat (see figure 2) but just a smooth bump, because there is no phase transition to the ice state, just a crossover. If it is so, one should not say that this configuration is really degenerate, but just that it is effectively degenerate for \( T > 0 \). However, some details of the theoretical approach and experimental measurements must be observed before any conclusion about a desirable complete theoretical approach and experimental measurements must be indicated (theoretically as well as experimentally) that geom-

Figure 5. MOKE signal as function of temperature for the three lattices compared. Here \( T_{\sqrt{2}} \approx 707 \) K, \( T_{\sqrt{3}} \approx 630 \) K and \( T_{\sqrt{4}} \approx 657 \) K.


doing. For the simulations, the use of any rational number \( r \) very close to \( \sqrt{3} \) will imply that the system is only near to the ice regime, but it still keeps some features of the antiferromagnetic (if \( r \) is immediately below \( \sqrt{3} \)) or ferromagnetic phase (if \( r \) is immediately above \( \sqrt{3} \)). Experimentally, things are still more complicated because the measurements of distances between islands have the errors of the instruments and, in addition, the nanoislands are not point dipoles. So, all results concerning the lattices analyzed are only approximations. Despite these difficulties, our results indicate (theoretically as well as experimentally) that geometry may induce some roles on the thermodynamic properties of these systems.

In conclusion, we have demonstrated experimentally that the lattice geometry can be an important ingredient to transform the thermodynamic properties of artificial spin ice compounds. By stretching the lattice, the fluctuations do not decrease monotonically as expected when the mean distances among the nanoislands increase. Indeed, normally, ASI ma-

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