Eigenmodes of the Dirac Operator and Chiral Properties of QCD with Sea Quarks

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I describe a mechanism to understand the relation between chiral-symmetry breaking and eigenmodes of the Dirac operator in lattice QCD with Kogut-Susskind sea quarks. It can be shown that if chiral symmetry is spontaneously broken, the eigenvalues $\lambda_i$ should behave as $z(i)/V$ for large volume $V$, where $z(i)$ is the $i$-th zero of the Bessel function. With neither chiral nor $\lambda$ extrapolation, one can precisely calculate the chiral condensate using only a small set of eigenvalues. Therefore, it is economical and free of systematic uncertainties. I present the first QCD data to support this mechanism and encourage the lattice community to test and use it.

One can not completely understand physics of hadrons without understanding the QCD vacuum, which main properties are confinement and chiral-symmetry breaking.

The chiral order parameter is $\langle \bar{\psi} \psi \rangle$ in the chiral limit. Suppose $\langle \bar{\chi} \chi(m, V) \rangle$ is the one calculated at bare mass $m$ and lattice volume $V$. Since $\langle \bar{\chi}(0, V) \rangle = 0$, conventionally one has to assume

$$\langle \bar{\psi} \psi \rangle = \lim_{m \to 0} \lim_{V \to \infty} \langle \bar{\chi}(m, V) \rangle.$$ (1)

Practically, most people do extrapolation only on finite $V$ and very small set of $m$ by means of some unjustified fitting function (linear, quadratic, polynomial or logarithm ...). Unfortunately, such an process is arbitrary and sometimes gives wrong results. Here we provide two known evidences.

(i) QED in 4 dimensions. Non-compact lattice QED experiences second order chiral phase transition at finite some bare coupling constant $g$, where the chiral condensate vanishes. The critical coupling determined by naive $m$ extrapolation of $\langle \bar{\chi}(m, V) \rangle$ is not necessary correct. For detailed discussions and current status, see [1].

(ii) The one-flavor massless Schwinger model. In the continuum, as is well known, $\langle \bar{\psi} \psi \rangle_{cont} = 0.16e$, where $e$ is the electric charge. As indicated in Ref. [2], the exact result can not be reached at finite bare coupling $g = ae$ by the any extrapolation mentioned above.

In Ref. [3], we proposed a new mechanism to get reliable quantitative information on chiral-symmetry breaking, based on the computation of the probability distribution function of $\langle \bar{\psi} \psi \rangle$:

$$P(c) = \lim_{N \to \infty} \langle \delta(c - \frac{1}{N} \sum_x \bar{\psi}(x) \psi(x)) \rangle.$$ (2)

Here $N$ is the degrees of freedom, being proportional to $V$. $P(c)$ is the probability to get the value $c$ for the chiral condensate. If there is exact chiral symmetry in the ground state, $P(c) = \delta(c)$. If chiral symmetry is spontaneously broken, $P(c)$ will be a more complex function.

For Kogut-Susskind fermions, we derive several relations between the chiral condensate and the eigenmodes of the Dirac operator.

(a) The first formula is

$$\langle \bar{\psi} \psi \rangle = \lim_{m \to 0} \lim_{V \to \infty} \frac{1}{N} \sum_{i} \frac{2m}{\lambda_i^2 + m^2}.$$ (3)

where $\lambda_i$ is the $i$-th positive eigenvalue of the massless Dirac matrix. This is the formula many people are currently using. It requires $m$ extrapolation, and may not produce reliable result.

(b) The second formula is

$$\langle \bar{\psi} \psi \rangle = \lim_{\lambda \to 0} \lim_{V \to \infty} \frac{\pi}{N} \langle \rho(\lambda) \rangle,$$ (4)

where $\rho(\lambda)$ is the eigenmode density. This formula is also known. Since $\rho(0) = 0$ at finite $V$, and it requires $\lambda$ extrapolation, which is again arbitrary.
The most interesting application is QCD. Here I would like to present the first data for QCD with various numbers of quark flavors $N_f$. In Ref. [3], the configurations were generated and eigenvalues of the fermionic matrix were calculated. With these, it is very easy to evaluate the chiral condensate and compare different methods.

Figure 1 shows the quenched result for $\beta = 5.5088$ and $V = 6^4$, using the first formula: Eq. (3). Only a small set of data can be fitted by a linear function. If one fits only the data for $m \in [0.04, 0.08]$ to the chiral limit, the result is 0.060. But outside this range, either for larger or smaller $m$, this slope changes and global fitting function can not be easily found. For smaller $m$, the change is very rapid. We attribute it to finite size effects.

Figure 3 shows the quenched result for $\beta = 5.5088$ and $V = 6^4$, but using the fourth formula: Eq. (6). Even on such a small lattice, there is a nice plateau for $i \in [10, 25]$, from which one can reliably get $C(i) = 0.06$. No fitting function is necessary. Of course, detailed finite size analysis remains to be done.

Figures 4, 5, 6 are the results for $N_f = 1, 2, 3, 4$. All these satisfy well Eq. (6).

In conclusion, I have shown a reliable method for investigating the chiral properties and obtaining the chiral condensate from only a small set of $\lambda$, without $m$ or $\lambda$ extrapolation. This method was previously tested in the Schwinger model. In this paper I have presented the first data for QCD. Therefore, I encourage other people in the lattice community to test and use Eq. (6).

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Figure 1. $\langle \bar{\chi}(m, V) \rangle$ for $N_f = 0$, $\beta = 5.5088$ and $V = 6^4$ using Eq. (3).

Figure 2. $C(i)$ as a function of $i$ for $N_f = 0$, $\beta = 5.3469$ and $V = 6^4$ using Eq. (6).

Figure 3. $C(i)$ as a function of $i$ for $N_f = 1$, $\beta = 5.3469$ and $V = 6^4$ using Eq. (6).

Figure 4. $C(i)$ as a function of $i$ for $N_f = 2$, $\beta = 5.3136$ and $V = 6^4$ using Eq. (6).

Figure 5. $C(i)$ as a function of $i$ for $N_f = 3$, $\beta = 5.2814$ and $V = 6^4$ using Eq. (6).
Figure 6. $C(i)$ as a function of $i$ for $N_f = 4$, $\beta = 5.2500$ and $V = 6^4$ using Eq. (6).