Tuning resonant interaction of orthogonally polarized solitons and dispersive waves with the soliton power

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Abstract: We demonstrate that the relatively small power induced changes in the soliton wavenumber comparable with splitting of the effective indexes of the orthogonally polarized waveguide modes result in significant changes of the efficiency of the interaction between solitons and dispersive waves and can be used to control energy transfer between the soliton and newly generated waves and to delay or accelerate solitons.

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Interaction of optical solitons with dispersive waves (DWs) is an active research topic, which besides its fundamental significance plays an important role in understanding of supercontinuum (SC) generation \[1\]. In particular, it has been demonstrated that at advanced stages of SC generation interaction of solitons with strong DWs can lead to significant changes of the soliton and DW frequencies and energies \[2–8\]. A related effect is the long range soliton-soliton interaction mediated by DWs trapped between the solitons and leading to either acceleration or deceleration of solitons and significant spectral reshaping happening in the course of propagation \[9, 10\]. Interaction of solitons with a strong DW has been also proposed for use in all optical switching \[11\] and for SC generation \[12, 13\].

A fundamental process behind the effects observed in the interaction of solitons with DWs is the four-wave mixing (FWM) of DWs with the selected frequency components of the soliton spectrum \[1\]. This type of FWM is mediated either by the nonlinear refractive index change induced by the soliton intensity (cross-phase modulation (XPM) effect) or by the FWM term (square of the soliton field times the complex conjugated DW field) \[1,14,15\]. The DW-soliton interaction through the XPM and FWM processes can be referred to as the phase insensitive and phase sensitive processes, respectively. Efficiency of the frequency conversion happening in the soliton-DW interaction depends on many factors and potential avenues of its enhancement, possible novel effects and practical applications have not been explored in sufficient details.

Efficiency of the soliton-DW interaction can be trivially boosted through the increase of the amplitudes of the involved waves, so that the nonlinear mixing terms become larger. Less obvious, but more sensitive mechanism is taking an advantage of the power dependence of the phase matching (PM) conditions, so that, e.g., the signal waves generated through the DW-soliton interaction can either appear or disappear together with PM itself. The nonlinear shift of the soliton wavenumber is not a significant factor in the PM condition for the soliton to emit a dispersive wave (Cherenkov radiation) and can be disregarded \[1\]. PM condition for the phase insensitive (XPM induced) scattering of an externally applied DW on a soliton simply does not depend on the nonlinear shift of the soliton phase \[2,16,17\]. However, the situation is different if one considers the phase sensitive interaction \[1,15\]. In this case the nonlinear phase shifts can become comparable with the linear wavenumber splitting of, e.g., orthogonally polarized modes, and hence the soliton-DW scattering processes with the PM conditions critically depending on the soliton power become feasible. While the power dependence has been previously reported \[1,15\], its impact on the dispersive wave scattering and amplification has not been addressed so far.

This work aims to target several problems related to the above conjecture. In particular, we will demonstrate that the presence or absence of the frequency conversion achieved in the phase sensitive interaction of the orthogonally polarized solitons and DWs critically depend on the soliton intensity and can be either enhanced or completely suppressed through relatively small
changes in the soliton power. The signs of the soliton acceleration and of the frequency drift induced by DWs also can be controlled through the soliton power.

Propagation of polarized light in optical waveguides can be described using the following dimensionless equations \cite{13}:

\[
[i \partial_z + \hat{D}_{x,y}] A_{x,y} + \left[|A_{x,y}|^2 + \frac{2}{3}|A_{x,y}|^2 \right] A_{x,y} + \frac{1}{3} A_{x,y}^* A_{x,y} = 0, \tag{1}
\]

where \( A_{x,y} \) are the amplitudes of the orthogonally polarized modes. \( \hat{D}_x(i \partial_t) = \frac{i}{2} \partial^2_t \) and \( \hat{D}_y(i \partial_t) = \beta_0 + i \beta_1 \partial_t + i \frac{\beta_2}{2} \partial^2_t \) are the dispersion operators. \( \beta_0 = (\beta_0^{(0)} - \beta_0^{(1)})L_d \) is the normalised difference of the propagation constants of the two modes, \( \beta_1 = (\beta_1^{(0)} - \beta_1^{(1)})L_d \) is the difference of their inverse group velocities, and \( \beta_2 = \beta_2^{(0)} / \beta_2^{(1)} \) is the ratio of their group velocity dispersion (GVD) coefficients. We will assume hereafter that \( \beta_2 < 0 \), so that GVD is anomalous in the x-component and normal in the y-component. Thus x- and y-components are natural hosts for the soliton and dispersive wave pulses, respectively. Time \( t \) is normalized to the pump pulse duration, propagation distance \( z \) is measured in units of the GVD length, \( L_d \), of the x-mode, \( |A_{x,y}|^2 \) are normalized to the soliton power, making the nonlinear length equal to the GVD length. We have not included Raman nonlinearity, assuming that our results can be applied for the sufficiently long pulses in optical fibers and in the context of semiconductor waveguides \cite{19,20}, where the Raman effect on even ultrashort pulses is negligible.

Input (\( z = 0 \)) conditions used through most of our work is the soliton in the x-polarization and DW in the y-polarization: \( A_3 = \sqrt{2q} \text{sech} \left( \sqrt{2q}t \right) \), \( A_y = B \text{sech} \left( (t - t_0) / T \right) e^{i q t} \), where \( q \) is the soliton parameter proportional to its peak power and giving the nonlinear shift of the soliton wavenumber \( e^{iqz} \), \( B \) is the amplitude and \( T \) is the duration of the DW pulse, \( t_0 \) is the delay and \( \omega_1 \) is the frequency offset between the interacting pulses. Through out this work we consider \( \beta_0 > 0 \), which corresponds to the phase velocity in the soliton component being greater than the one in the dispersive wave component, so that soliton is the 'fast wave'. Note that we restrict our considerations to the parameter values where the fast wave soliton is stable with respect to the exponential growth of the orthogonally polarised small amplitude perturbations; see Fig. 2(c), Ref. \cite{21} and references therein. PM condition governing DW-soliton mixing are \cite{15-17}:

\[
\beta(\omega) = \beta(\omega_k) \tag{2}
\]

for the phase insensitive process and

\[
\beta(\omega) = 2q - \beta(\omega_k) \tag{3}
\]

give frequencies of the DWs transmitted through, \( \omega_k \), and reflected by, \( \omega_k \), a soliton. Note, that the multiple transmitted and reflected waves are allowed to coexist.

Figs. 1(a-c) illustrate a typical scattering event. An incident DW with \( \omega_k = 5 \) propagates to the right showing some dispersion. When it overlaps with the soliton \( (q = 6 \) it gets reflected into a wave with \( \omega_k = -14 \) and splits on transmission into two waves with \( \omega_k = \omega_h \) and \( \omega_k = -0.9 \), see Figs. 1(a,c). Fig. 1(b) shows the associated phase matching diagram. The phase insensitive resonances, Eq. (2), are given by the green line. These do not depend on \( q \) and hence on the soliton power. One of these resonances corresponds to the incident wave and the other one is negligibly weak in this instance. The phase sensitive resonances, Eq. (3), are given by the red line and they correspond to the one reflected and one transmitted waves. Reducing the soliton power, i.e. reducing \( q \), the phase sensitive resonances tends towards degeneracy at the bottom of the parabolic dispersion of the linear waves, simultaneously, the power of the scattered waves.
drops down. The point of the exact degeneracy corresponds to the propagation of the transmitted and reflected waves parallel to the soliton interface. The phase sensitive processes are phase matched for \( q > q_{cr} \), where \( q_{cr} = \beta_0 + \left( \beta_1 \omega_i - \frac{\beta_2 \omega_i^2}{2} + \frac{\beta_2^2}{2} \right) / 2 \). The cases of \( q < q_{cr} \) and \( q \) greater than, but close to, \( q_{cr} \) are illustrated in Figs. 1 (d-f) and (g-i), respectively. Phase sensitive resonances as functions of \( q \) for several values of \( \omega_i \) are shown in Fig. 2(a), where one can see that \( q_{cr} \) increases with \( \omega_i \). Thus, tuning the soliton power one can not only shift the resonant frequencies of the phase sensitive FWM-mediated scattering, but also suppress the resonance completely. Boundaries separating the areas in the \((\beta_0, \beta_1)\)-plane where phase matching is possible from the ones where it is not are shown in Fig. 2(c), together with the threshold of the polarization instability of the soliton. One can see that there is a limit on how large \( \beta_0 \) is allowed to be for the phase matching to be realizable.

One should also note significant amplification of the wave transmitted without any change in frequency, see Fig. 1(c). While theoretical understanding of this potentially useful effect has not been developed yet, we can quantify it numerically calculating relative change in the photon number (amplification coefficient) \( K \) of the pulse launched at the \( \omega_i \) frequency: \( K = \frac{E(z \to \infty)}{E(z = 0)} \), where \( E(t) = \int |A_t|^2 dt \). Plots of \( K \) vs the soliton parameter \( q \) are shown in Fig. 2(b) revealing an order of magnitude energy change, thereby suggesting to look for the possible role of this effect in shaping polarization dependent supercontinuum spectra [18]. In this panel the amplification coefficient is defined as a ratio of the total photon number in both transmitted waves to the photon number in the incident wave is shown by dashed curves.

Alongside with the generation of DWs with new frequencies, the DW-soliton interaction
Fig. 2. (a) Resonance frequencies for the phase sensitive process as a function of the soliton parameter $q$. Analytical solutions are given by the solid lines, the black dots show the frequencies obtained from numerical simulations. (b) The amplification coefficient $K$ defined as a ratio of the photon numbers in the transmitted waves at frequency $\omega_i$ to the photon number in the incident wave; $\omega_i = 3$. (c) Domains on the left from the parabolas give the values of $\beta_0$ and $\beta_1$ where the phase sensitive conditions can be satisfied for the suitable resonance frequencies (see Fig. 2(a)). The red-orange color indicates the region of the polarization instability of the soliton solutions for $q = 5$.

\begin{figure}[h]
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\includegraphics[width=\textwidth]{fig2}
\caption{(a) Resonance frequencies for the phase sensitive process as a function of the soliton parameter $q$. Analytical solutions are given by the solid lines, the black dots show the frequencies obtained from numerical simulations. (b) The amplification coefficient $K$ defined as a ratio of the photon numbers in the transmitted waves at frequency $\omega_i$ to the photon number in the incident wave; $\omega_i = 3$. (c) Domains on the left from the parabolas give the values of $\beta_0$ and $\beta_1$ where the phase sensitive conditions can be satisfied for the suitable resonance frequencies (see Fig. 2(a)). The red-orange color indicates the region of the polarization instability of the soliton solutions for $q = 5$.}
\end{figure}

Fig. 3. Evolution of the total intensity $|A_x|^2$ (a-c) and $|A_y|^2$ (d-f) resulting from the soliton collision with a DW: (a,d) $B = 0.14$, $\omega_i = 3$, $q = 3.5$; (b,e) $B = 0.14$, $\omega_i = 3$, $q = 4.35$; (c,f) $B = 0.14$, $\omega_i = 3$, $q = 5$. $\omega_i$ leads to the appreciable impact of DWs on the soliton itself, resulting in the change of the soliton momentum associated with the shift of the soliton frequency and bending of its spatiotemporal trajectory \cite{11,17}. If the phase insensitive scattering associated with the DW reflection dominates over the other scattering channels, then the associated changes of the soliton frequency and velocity are such that the soliton trajectory bends towards the incident DW \cite{9}. For example, if DW hits a soliton from the right then the soliton trajectory bends to the left. If one takes two solitons and arranges for the reflected DW to bounce between them, the net force exerted on the solitons results in attraction \cite{4,5,9,11}. However, when the phase sensitive scattering processes dominates, which happens for some $q$ above $q = q_{cr}$, we observe that for the sufficiently large soliton amplitudes the soliton trajectory bends away from the incident wave, see Fig. 3(c,f). If the phase sensitive resonances are eliminated ($q < q_{cr}$), then the soliton trajectory bends towards the incident wave as in the scalar case, see Figs. 3(a,d). Naturally, there exist parameters, when the phase sensitive and phase insensitive processes balance each other so that the soliton frequency does not change, see Figs. 3(b,e). Dependencies of the soliton velocity shift induced by the scattering of DW vs the soliton parameter $q$ and vs frequency, $\omega_i$, of the...
incident wave containing intervals of positive and negative velocities are shown in Fig. 4.

To conclude we briefly summarize the main results reported in the paper. It is shown that in vector case the frequencies of the radiation generated by four wave mixing of DW with solitons can be very sensitive not only to the frequency but also to the intensity of the solitons. It was demonstrated that the phase sensitive resonant scattering can be completely suppressed in case of solitons of low intensity. Studying the recoil from the scattering of the DW on solitons we demonstrated that the soliton trajectory can bend either to the left or to the right depending on the intensity of the soliton. In other words the resonant scattering changes the frequency of solitons and the sign of the frequency shift can depend on the soliton intensity. Another interesting effect considered in the paper is the amplification of DW happening due to four-wave mixing between the solitons and the DW. The reported effects open new possibilities to control optical soliton by the dispersive waves and can potentially be important for practical applications.

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