ON THE COEFFICIENT OF THE $n^{th}$ CESARO MEAN OF ORDER $\alpha$ OF BI-UNIVALENT FUNCTIONS

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ABSTRACT. The purpose of the present paper is to introduce a new subclasses of the function class of bi-univalent functions defined in the open unit disc. Furthermore, we obtain estimates on the coefficients $|a_2|$ and $|a_3|$ for functions of this class. Some results related to this work will be briefly indicated.

1. INTRODUCTION

Let $A$ denote the class of the functions $f$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are analytic in the open unit disc $U = \{ z \in \mathbb{C} : |z| < 1 \}$ and satisfy the normalization condition $f(0) = f'(0) - 1 = 0$. Let $S$ be the subclass of $A$ consisting of functions of the form (1) which are also univalent in $U$.

A function $f \in A$ is said to be in the class of strongly bi-starlike functions of order $\alpha$ ($0 < \alpha \leq 1$), denoted by $S^*_\Sigma(\alpha)$, if each of the following conditions is satisfied:

$$\left| \arg \left\{ \frac{zf'(z)}{f(z)} \right\} \right| < \frac{\alpha \pi}{2}, \,(|z| < 1, 0 < \alpha \leq 1),$$

and

$$\left| \arg \left\{ \frac{zg'(w)}{g(w)} \right\} \right| < \frac{\alpha \pi}{2}, \,(|w| < 1, 0 < \alpha \leq 1),$$

where $g$ is the extension of $f^{-1}$ to $U$ (for details see [Brannan and Taha]). And is said to be in the class of strongly bi-convex functions of order $\alpha$ ($0 < \alpha \leq 1$), denoted by $K^*_\Sigma(\alpha)$, if it satisfies the following inequality

$$\left| \arg \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} \right| < \frac{\alpha \pi}{2}, \,(|z| < 1, 0 < \alpha \leq 1).$$

and

$$\left| \arg \left\{ 1 + \frac{wg''(w)}{g'(w)} \right\} \right| < \frac{\alpha \pi}{2}, \,(|w| < 1, 0 < \alpha \leq 1).$$

Where $g$ is the extension of $f^{-1}$ to $U$. Recall that the Koebe one-quarter theorem [2] ensures that the image of $D$ under every univalent function $f \in S$ contains a disk of radius $\frac{1}{4}$. Thus every univalent function $f$ has an inverse $f^{-1}$ satisfying $f^{-1}(f(z)) = z, (z \in D)$ and

$$f^{-1}(f(w)) = w, (|w| < r_0 f, r_0 f \geq \frac{1}{4}).$$

$$g(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^2 - 5a_2a_3 + a_4)w^4 + \ldots \quad (2)$$

In recent years, many authors discussed estimate on the coefficients $|a_2|$ and $|a_3|$ for subclasses of bi-univalent function (see for example [3], [4], [5], [6], [7], [8]).
Let \( f : D \to C \) be an analytic function on \( D \) having Taylor expansion \( f(z) = \sum_{n=1}^{\infty} a_n z^n, z \in D \), with \( a_n \in C, a_1 = 1, n = 1, 2, 3, \ldots \). A function \( f \in S \) is bi-univalent in \( D \) if both \( f \) and \( f^{-1} \) are univalent in \( D \).

The object of the present paper is to introduce a new subclass \( \Sigma \) of the function class \( \Sigma \) and to find estimates on the coefficients \( |a_2| \) and \( |a_3| \) for new functions in these new subclasses of the function class \( \Sigma \).

We say that \( \delta_n^\alpha f(z) \) is the \( n^\text{th} \) Cesaro mean of order \( \alpha \geq 0 \) of \( f \) is defined by

\[
\delta_n^\alpha f(z) = z + \sum_{n=2}^{\infty} A_n a_n z^n
\]

where

\[
A_n = \frac{\left( k + \alpha - n \right)}{\left( k - n \right)} - \frac{\left( k + \alpha - 1 \right)}{\left( k - n \right)}, \quad a_1 = 1.
\]

Let \( D \) denote the open unit disk in \( C \). It is well known that outer functions are zero-free on the unit disk. Outer functions, which play an important role in \( H_p \) theory to find a suitable finite (polynomial) approximation for the outer infinite series \( f \) so that the approximant reduces the zero-free property of \( f \), arise in the characteristic equation which determines the stability of certain nonlinear systems of differential equations. Recall that an outer function is a function \( f \in H_p \) of the form

\[
f(z) = e^{i\gamma} e^{\int_{-\pi}^{\pi} \frac{1}{1 - e^{it} z} \log \psi(t) dt}
\]

where \( \psi(t) \geq 0 \), \( \log \psi(t) \) is in \( L^1 \) and \( \psi(t) \) is in \( L^p \). See [9] for the definitions and classical properties of outer functions. Since any function \( f \) in \( H^1 \) which has \( 1/f \) in \( H^1 \) is an outer function, then typical examples of outer functions can be generated by functions of the form \( \prod_{k=1}^{\infty} (1 - e^{i\theta_k z})^{\alpha_k} \) for \(-1 < \alpha_k < 1\).

We observe for outer functions that the standard Taylor approximants do not, in general, retain the zero-free property of \( f \). It was shown in [10] that the Taylor approximating polynomials to outer functions can vanish in the unit disk. By using convolution methods that the classical Cesaro means, retains the zero-free property of the derivatives of bounded convex functions in the unit disk. The classical Cesaro means play an important role in geometric function theory (see [11],[12]).

**Lemma 1.1.** If \( h \in p \) then \( |c_k| < 1 \), for each \( k \), where \( p \) is the family of all functions \( h \) analytic in \( U \) for which \( \Re\{h(z)\} > 0 \), then

\[
h(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \ldots, z \in U.
\]

**2. COEFFICIENT BOUNDS FOR THE FUNCTION CLASS \( H_{\Sigma}(\psi) \)**

In the sequel, it is assumed that \( \varphi \) is an analytic function with positive real part in the unit disk \( D \), satisfying \( \psi(0) = 1, \psi'(0) > 0, \) and \( \psi(D) \) is symmetric with respect to the real axis. Such a function has a Taylor series of the form

\[
\psi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots, (B_1 > 0).
\]
Suppose that \( u(z) \) and \( v(z) \) are analytic in the unit disk \( D \) with \( u(0) = v(0) = 0, |u(z)| < 1, |v(z)| < 1 \), and suppose that

\[
u(z) = b_1z + \sum_{n=2}^\infty b_n z^n, \quad v(z) = c_1z + \sum_{n=2}^\infty c_n z^n, \quad (|z| < 1).
\]

(4)

It is well known that

\[
|b_1| \leq 1, \quad |b_2| \leq 1 - |b_1|^2, \quad |c_1| \leq 1, \quad |c_2| \leq 1 - |c_1|^2.
\]

(5)

By a simple calculation, we have

\[
\psi(u(z)) = 1 + B_1b_1z + (B_1b_2 + B_2b_1^2)z^2 + \ldots, \quad |z| < 1
\]

(6)

and

\[
\psi(v(w)) = 1 + B_1c_1w + (B_1c_2 + B_2c_1^2)w^2 + \ldots, \quad |w| < 1.
\]

(7)

**Definition 2.1.** [13] A function \( f \in \Sigma \) is said to be in the class \( H_{\Sigma}(\psi) \) if and only if

\[
f'(z) < \psi(z), \quad g'(z) < \psi(w),
\]

where \( g(w) = f^{-1}(w) \).

**Theorem 2.2.** If \( f \) given by (1) is in the class \( H_{\Sigma}(k, \psi) \), then

\[
|a_2| \leq \left| \frac{k + \alpha - 1}{k - 2} \right| \frac{B_1\sqrt{B_1}}{\sqrt{[3B_1^2 - 4B_2] + 4B_1}}
\]

(8)

and

\[
|a_3| \leq \left[ \frac{k + \alpha - 1}{k - 3} \right] \left[ (1 - \frac{4}{3B_1}) \frac{B_1^3}{[3B_1^2 - 4B_2] + 4B_1} + \frac{B_1}{3} \right].
\]

(9)

**Proof.** Let \( f \in H_{\Sigma}(k, \psi) \) and \( g = f^{-1} \). Where \( a_1 = 1 \). Then there are analytic functions \( u, v : D \to D \) given by (4) such that

\[
[\delta_n^a f(z)]' = \psi(u(z)), \quad [\delta_n^a g(w)]' = \psi(v(w)),
\]

(10)

since

\[
[\delta_n^a f(z)]' = 1 + 2A_2a_2z + 3A_3a_3z^3 + \ldots,
\]

\[
[\delta_n^a g(w)]' = 1 - 2A_2a_2w + 3[2A_2^2a_2^2 - A_3a_3]w^3 + \ldots,
\]

(11)

it follows from (6), (7), (10) and (11) that
\[2A_2a_2 = B_1b_1, \quad (12)\]
\[3A_3a_3 = B_1b_2 + B_2b_1^2, \quad (13)\]
\[-2A_2a_2 = B_1c_1, \quad (14)\]
\[3[2A_2a_2^2 - A_3a_3] = B_1c_2 + B_2c_1^2. \quad (15)\]

From (12) and (14), we get
\[b_1 = -c_1. \quad (16)\]

By adding (15) to (13), further computations using (12) and (16) lead to
\[A_2^2a_2^2[3B_1^2 - 8B_2] = B_1^3(b_2 + c_2). \quad (17)\]

Also, from (16) and (17), together with (5), we obtain
\[|A_2^2a_2^2[3B_1^2 - 8B_2]| \leq 2B_1^3(1 - |b_1|^2). \quad (18)\]

From (12) and (18) we get
\[|a_2| \leq \left| \frac{\binom{k + \alpha - 1}{k - 2}}{\binom{k + \alpha - 2}{k - 2}} \right| \frac{B_1\sqrt{B_1}}{\sqrt{\|3B_1^2 - 4B_2\| + 3B_1}}. \]

Which, in view of the well-known inequalities \(|b_2| \leq 2\) and \(|c_2| \leq 2\) for functions with positive real part, gives us the desired estimate on \(|a_2|\) as asserted in (8). By subtracting (15) from (13), further computations using (12) and (16) lead to
\[6A_3a_3 = 6A_2^2a_2^2 + B_1(b_2 - c_2). \quad (19)\]

From (5), (12), (16) and (19), it follows that
\[|a_3| \leq \frac{6A_3^2|a_2|^2 + B_3(|b_2| + |c_2|)}{6A_3} \]
\[\leq \frac{6A_3^2|a_2|^2 + B_1(1 - |b_1|^2) + (1 - |c_1|^2)}{6A_3} \]
\[\leq \frac{1}{A_3} \cdot \left| \frac{4}{3B_1} \right|^2 + \frac{B_1}{3A_3} \]
\[|a_3| \leq \left( \frac{k + \alpha - 1}{k - 3} \right) \left( \frac{k + \alpha - 3}{k - 3} \right) \left[ (1 - \frac{4}{3B_1}) \frac{B_1^3}{\|3B_1^2 - 4B_2\| + 3B_1} + \frac{B_1}{3} \right]. \]
3. COEFFICIENT BOUNDS FOR THE FUNCTION CLASS $Q_{\Sigma}(\alpha, \mu, \lambda)$

**Definition 3.1.** A function $f(z)$ given by (1) is said to be in the class $Q_{\Sigma}(\alpha, \mu, \lambda)$ if the following conditions are satisfied: For $f \in \Sigma$,

$$\left| \arg \left\{ \frac{(1 - \lambda)\delta_n^\alpha f(z) + \lambda z[\delta_n^\alpha f(z)]'}{z} \right\} \right| < \frac{\pi\alpha}{2}, \alpha(0 < \alpha \leq 1, \lambda \geq 1, z \in U),$$

(20)

and

$$\left| \arg \left\{ \frac{(1 - \lambda)\delta_n^\alpha g(w) + \lambda w[\delta_n^\alpha g(w)]'}{w} \right\} \right| < \frac{\pi\alpha}{2}, \alpha(0 < \alpha \leq 1, \lambda \geq 1, w \in U),$$

(21)

where the function $g$ defined by (2).

**Theorem 3.2.** Let the function $f(z)$ given by (1) be in the class $Q_{\Sigma}(\alpha, \mu, \lambda)$, $n \in N_0, 0 \leq \beta < 1, \lambda \geq 1$. Then

$$|a_2| \leq 2\alpha \left| \frac{\left( \frac{k + \alpha - 1}{k - 2} \right) - \left( \frac{k + \alpha - 2}{k - 2} \right)}{\sqrt{4} \left( 1 + \lambda \right)^2 + \alpha \left[ 2.3k \left( 1 + \lambda \right) - 4k \left( 1 + \lambda \right)^2 \right]} \right|,$$

where $p(z)$ and $q(w)$ in $P$ and $a_1 = 1$, and have the forms

$$p(z) = 1 + p_1z + p_2z^2 + p_3z^3 + \ldots,$$

(26)

and

$$q(w) = 1 + p_1w + q_2w^2 + q_3w^3 + \ldots.$$

(27)

Now, equating the coefficients in (24) and (25), we obtain

$$(1 + \lambda)A_2a_2 = \alpha p_1,$$

(28)

$$(1 + 2\lambda)A_3a_3 = \frac{1}{2}[2\alpha p_2 + \alpha(\alpha - 1)p_1^2],$$

(29)

and

$$-(1 + \lambda)A_2a_2 = \alpha q_1,$$

(30)

$$(1 + 2\lambda)[2A_2a_2^2 - A_3a_3] = \frac{1}{2}[2\alpha q_2 + \alpha(\alpha - 1)q_1^2].$$

(31)
From (28) and (30), we obtain

\[ p_1 = -q_1 \]  

(32)

and

\[ 2(1 + \lambda)^2 A_2^2 a_2^2 = \alpha^2(p_1^2 + q_1^2). \]  

(33)

Now, from (29), (31) and (33), we obtain

\[ 2(1 + 2\lambda) A_2^2 a_2^2 = \alpha(p_2 + q_2) + \frac{\alpha(\alpha - 1)}{2} \frac{2(1 + \lambda)^2 A_2^2 a_2^2}{\alpha^2}. \]

Therefore we have

\[ a_2^2 = \frac{\alpha^2(p_2 + q_2)}{(1 + \lambda)^2 + \alpha(1 + 2\lambda) - \lambda^2} A_2. \]

Applying Lemma 1.1 for the coefficients \( p_2 \) and \( q_2 \), we immediately have

\[ |a_2| \leq 2\alpha \left| \left( \frac{k + \alpha - 1}{k - 2} \right) \left( \frac{k + \alpha - 2}{k - 2} \right) \right| \frac{1}{\sqrt{4^\epsilon(1 + \lambda)^2 + \alpha[2.3^\epsilon (1 + \lambda) - 4^\epsilon (1 + \lambda)^2]}}. \]

This gives the bound as asserted in (22).

Next, in order to find the bound on \( |a_3| \), we subtract (29) from (31) and obtain

\[ 2[(1 + 2\lambda)(A_3 a_3 - A_2^2 a_2^2) = \frac{1}{2} \left( 2\alpha(p_2 - q_2) + \alpha(\alpha - 1)(p_1^2 - q_1^2) \right) \]

\[ = \frac{\alpha(p_2 - q_2)}{2(1 + 2\lambda) A_3} + \frac{\alpha^2(p_1^2 + q_1^2)}{2(1 + \lambda)^2 A_3}, \]

\[ a_3 = \left[ \begin{array}{c} \frac{k + \alpha - 1}{k - 3} \\ \frac{k + \alpha - 3}{k - 3} \end{array} \right] \left[ \frac{\alpha(p_2 - q_2)}{2(1 + 2\lambda) A_3} + \frac{\alpha^2(p_1^2 + q_1^2)}{2(1 + \lambda)^2 A_3} \right]. \]

Applying Lemma 1.1 for the coefficients \( p_2 \) and \( q_2 \), we immediately have

\[ |a_3| \leq \left[ \begin{array}{c} \frac{k + \alpha - 1}{k - 3} \\ \frac{k + \alpha - 3}{k - 3} \end{array} \right] \left[ \frac{2\alpha}{(1 + 2\lambda)} + \frac{4\alpha^2}{(1 + \lambda)^2} \right]. \]

This completes the proof of Theorem 3.2. \( \square \)
4. COEFFICIENT BOUNDS FOR THE FUNCTION CLASS $H_{\Sigma}(\beta, \mu, \lambda)$

**Definition 4.1.** A function $f(z)$ given by (1) is said to be in the class $H_{\Sigma}(\beta, \mu, \lambda)$ if the following conditions are satisfied: For $f \in \Sigma$,

$$\Re \left\{ \frac{(1 - \lambda) \delta_{\alpha} f(z) + \lambda z [\delta_{\alpha} f(z)]'}{z} \right\} > \beta, z \in U, n \in N_0, 0 \leq \beta < 1, \lambda \geq 1.$$

(34)

and

$$\Re \left\{ \frac{(1 - \lambda) \delta_{\alpha} g(w) + \lambda w [\delta_{\alpha} g(w)]'}{w} \right\} > \beta, w \in U, n \in N_0, 0 \leq \beta < 1, \lambda \geq 1,$$

(35)

where the function $g(z)$ defined by (2).

**Theorem 4.2.** Let $f(z)$ given by (1) be in the class $H_{\Sigma}(\beta, \mu, \lambda), 0 \leq \beta < 1, \mu \geq 0, \text{ and } \lambda \geq 1$. Then

$$|a_2| \leq \left[ \frac{k + \alpha - 1}{k - 2} \right] \left[ \frac{k - 2}{k + \alpha - 2} \right] \sqrt{\frac{2(1 - \beta)}{1 + 2\lambda}}.$$  

(36)

and

$$|a_3| \leq \left[ \frac{k + \alpha - 1}{k - 3} \right] \left[ \frac{k - 3}{k + \alpha - 3} \right] \left[ \frac{4(1 - \beta)^2 + 2(1 - \beta)}{(1 + \lambda)^2 + (1 + 2\lambda)} \right].$$

(37)

**Proof.** It follows from (34) and (35) that there exists $p, q \in P$ such that

$$\frac{(1 - \lambda) \delta_{\alpha} f(z) + \lambda z [\delta_{\alpha} f(z)]'}{z} = \beta + (1 - \beta)p(z),$$

(38)

and

$$\frac{(1 - \lambda) \delta_{\alpha} g(w) + \lambda w [\delta_{\alpha} g(w)]'}{w} = \beta + (1 - \beta)q(w),$$

(39)

where $a_1 = 1$, and have the forms

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots,$$

(40)

and

$$q(w) = 1 + q_1 w + q_2 w^2 + q_3 w^3 + \ldots,$$

(41)

respectively. Equating coefficients in (38) and (39) yields

$$[(1 + \lambda) A_2 a_2 = (1 - \beta)p_1,$$

(42)

$$[(1 + 2\lambda) A_3 a_3 = (1 - \beta)p_2,$$

(43)

$$- [(1 + \lambda) A_2 a_2 = (1 - \beta)q_1,$$

(44)

and

$$(1 + 2\lambda)[2 A_2^2 a_2^2 - A_3 a_3] = (1 - \beta)q_2.$$

(45)
From (42) and (44), we have

\[-p_1 = q_1 \tag{46}\]

and

\[2(1 + \lambda)^2 A_2^2 a_2^2 = (1 - \beta)^2(p_1^2 + q_1^2) \tag{47}\]

Also, from (43) and (45), we find that

\[2(1 + 2\lambda) A_2^2 a_2^2 = (1 - \beta)(p_2 + q_2), \tag{48}\]

\[|a_2|^2 \leq \begin{bmatrix} k + \alpha - 1 \\ k - 2 \end{bmatrix} \begin{bmatrix} k + \alpha - 2 \\ k - 2 \end{bmatrix} \frac{(1 - \beta)(|p_2| + |q_2|)}{2(1 + 2\lambda)}, \tag{49}\]

\[|a_2| \leq \sqrt{\frac{2(1 - \beta)}{1 + 2\lambda}}, \tag{50}\]

which is the bound on $|a_2|$ as given in (36).

Next, in order to find the bound on $|a_3|$ by subtracting (45) from (43), we obtain

\[2A_3(1 + 2\lambda) a_3 = 2(1 + 2\lambda) A_2^2 a_2^2 + (1 - \beta)(p_2 - q_2) \]

or, equivalently

\[a_3 = \frac{2(1 + 2\lambda) A_2^2 a_2^2 + (1 - \beta)(p_2 - q_2)}{2A_3(1 + 2\lambda)}. \]

Upon substituting the value of $a_2^2$ from (47), we obtain

\[a_3 = A_3 \left[ \frac{(1 - \beta)^2(p_1^2 + q_1^2)}{2(1 + \lambda)^2} + \frac{(1 - \beta)(p_2 - q_2)}{2(1 + 2\lambda)} \right]. \]

Applying Lemma 1.1 for the coefficients $p_1, p_2, q_1$ and $q_2$ we obtain

\[a_3 = A_3 \left[ \frac{4(1 - \beta)^2}{(1 + \lambda)^2} + \frac{2(1 - \beta)(p_2 - q_2)}{(1 + 2\lambda)^2} \right]. \]

\[|a_3| \leq \begin{bmatrix} k + \alpha - 1 \\ k - 3 \end{bmatrix} \begin{bmatrix} k + \alpha - 3 \\ k - 3 \end{bmatrix} \left[ \frac{4(1 - \beta)^2}{(1 + \lambda)^2} + \frac{2(1 - \beta)}{(1 + 2\lambda)^2} \right]. \]

which is the bound on $|a_3|$ as asserted in (37). \qed

**Remark.** 1. For all $\alpha \geq 0$, and $k = n$ in Theorems 2.2, we obtain the corresponding results due to Zhigang and Qiuqiu [14].
Remark. 2. For all $\alpha \geq 0$, and $k = n$ in Theorems 3.2 and 4.2, we obtain the corresponding results due to Frasin and Aouf [4].

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