BOTTOM PRODUCTION IN HADRONIC COLLISIONS

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I review the status of the comparison between theoretical predictions and experimental results for bottom production in hadronic collisions, and discuss the possible sources of the discrepancies found. The study of jets containing bottom quarks is proposed as a promising tool to investigate the $b$ production mechanism. I present next-to-leading order QCD predictions for this process, and compare them with data.

1 Open bottom at fixed-target and collider experiments

Bottom production constitutes a challenging testing ground for perturbative QCD. The quark mass, which sets the scale of the hard process, is such that $\alpha_s \simeq 0.2$. Therefore, the bottom rates cannot be predicted with full reliability at NLO in QCD, the radiative corrections being of the same size of the leading-order contribution. On the other hand, non-perturbative phenomena are expected to play a less important rôle than in the case, for example, of charm.

Fixed-target experiments have in general too a low energy to perform a statistically significant study of bottom production. Most of the available data have been obtained in $\pi N$ collisions, and allow for a measurement of the total cross section. A measurement of the total cross section has also been performed in $p N$ collisions. Unfortunately, due to the limited coverage of the detectors, the results are somewhat model-dependent. Taking into account the large theoretical uncertainties, the data and the theory are in reasonable agreement. There is no value of the bottom mass which allows to describe all the experimental results; at variance with the measurements at colliders, some results favour large values of the bottom mass. The E653 collaboration also presented a measurement for single-inclusive and double differential distributions, which turn out to be consistent with QCD predictions.

Bottom quarks are copiously produced at colliders. Although the rejection of the background in the low-$p_T$ region, where most of the $b$'s are produced,
is difficult, a large set of data is available for distributions measured in the central region in rapidity.

In figure 1 I present the comparison between the NLO QCD predictions and the experimental results for the bottom cross section $\sigma (p_T > p_{T\text{min}})$ at $p\bar{p}$ colliders, as a function of $p_{T\text{min}}$. The blobs have been obtained by dividing the data by the central theoretical curve (the default values of the parameters entering the calculations are: $m_b = 4.75$ GeV, $\mu_R = \mu_F = \mu_0$, $\Lambda_{\overline{MS}}^5 = 152$ MeV, where $\mu_0 = \sqrt{p_T^2 + m_b^2}$ is the transverse mass of the bottom quark). The boxes are on the other hand obtained by considering quite an extreme choice of the parameters, namely $m_b = 4.5$ GeV, $\mu_R = \mu_F = \mu_0/2$, $\Lambda_{\overline{MS}}^5 = 300$ MeV. This choice gives a result which can be considered as the upper limit of the theoretical predictions. I used the MRSA set for the partonic densities; the dependence of the result upon the choice of the densities is small, since they are known with a good accuracy in the $x$ range probed in $b$ production at SppS and Tevatron. A result quite close to the upper limit can also be obtained by choosing $m_b = 4.5$ GeV, $\mu_R = \mu_F = \mu_0/4$, $\Lambda_{\overline{MS}}^5 = 152$ MeV. Further details can be found elsewhere. The average value of the data/theory points (represented in the figure by the dotted and dot-dashed lines) has been calculated weighting
these points with the inverse of their relative error.

It is apparent from the figure that the results at \( \sqrt{S} = 630 \text{ GeV} \) and at \( \sqrt{S} = 1800 \text{ GeV} \) are consistent with each other. The CDF results are about 30\% higher in normalization than the D0 ones, while the shape of the theoretical curves is well reproduced by all the three collaborations. The comparison with the theory is quite satisfactory if one is willing to accept an extreme choice of the parameters: collider measurements favour small values of the bottom mass and of factorization/renormalization scales, and \( \alpha_s \) values compatible with LEP measurements. On the other hand, the data are higher than the default theory prediction by a factor of 2 or more.

The fixed-order perturbative QCD calculations I used above to compare with the experimental data may become unreliable in certain kinematical regions, due to the appearance of potentially large logarithms which spoil the convergence of the perturbative expansion. In this case, a resummation to all orders of these large logarithms has to be performed.

When the available center-of-mass energy gets large, the effective expansion parameter of the perturbative series becomes \( \alpha_s \log(S/m_b^2) \). The problem of resumming these terms (small-\(x\) effects) has been tackled by several authors. In the Tevatron energy regime, it was shown that the total cross section can increase by a factor of 30\% at most with respect to the NLO prediction. Furthermore, the resummation should have a negligible effect in the tail of the \( p_T \) distribution, where the effective scale is not the quark mass, but the transverse mass, and the ratio \( S/\mu_0^2 \) is not that large.

The transverse momentum distribution is in principle more affected by the presence of \( \log(p_T/m_b) \) terms. These logarithms can be resummed by observing that, at high \( p_T \), the bottom mass is negligible, and by using perturbative fragmentation functions. It turns out that the resummation only slightly changes the shape of the fixed-order prediction, but improves the perturbative stability of the result.

Finally, multiple soft gluon emission makes the perturbative expansion unreliable close to the threshold or to the borders of the phase space, like for example the regions \( p_{Tb}^bb \approx 0 \) and \( \Delta \phi^{bb} \approx \pi \). A lot of theoretical work has been performed in this field; at currently probed energy, these effects are not affecting the total rate or single inclusive distributions, while they may be relevant when investigating more exclusive quantities, like the correlations between the quark and the antiquark.

One is therefore led to conclude that the resummation of large logarithms cannot improve the comparison between theory and data for the \( p_T \) spectrum at the SpS and Tevatron colliders. It has to be observed that experimental results for bottom quarks depend on the assumptions made for the hadroniza-
Figure 2: Transverse momentum spectrum of the $B$ mesons: CDF data versus theoretical predictions.

The production process, since only $B$ hadrons are experimentally accessible. On the other hand, to compare with data on $B$ mesons, the QCD prediction for bare quarks has to be convoluted with a fragmentation function. Usually, the fragmentation function is determined by fitting $e^+e^-$ data. In this way, it is conceivable that the fraction of $B$ mesons coming from the splitting $g \rightarrow b\bar{b}$ is underestimated in hadron collisions, since this mechanism is much more important in this case than in the case of $e^+e^-$ annihilations. I will show later that indeed the gluon splitting is a key feature for bottom production at Tevatron. In figure 2 the CDF data on the $p_T$ spectrum of $B$ mesons are compared with the QCD predictions obtained with and without the Peterson fragmentation. The experimental measurements are close to the upper limit of the theoretical curve with fragmentation. On the other hand, they stay inside the band obtained without fragmentation, displaying a slightly softer behaviour. Notice that, although the fragmentation cannot affect the total rate, it however does affect quantities like $\sigma (p_T > p_T^{\text{min}})$ for moderate and large $p_T^{\text{min}}$ values, since the degradation of the momentum is sizeable in the tail of the transverse momentum spectrum.

For this reason, it would be useful to have data on the transverse momentum of the $B$ mesons for $p_T$ values larger than those displayed in figure 2. This would help in clarifying the issue whether in hadronic collisions the experimental results favour a fragmentation function more peaked towards the region $x \simeq 1$ than suggested by the Peterson parameterization.
The very same considerations enter into play when the comparison between
theory and experiments is made for more exclusive quantities. As mentioned
before, in this case the importance of soft gluons effect has to be taken into ac-
count. The CDF collaboration recently presented a study on \( b\bar{b} \) correlations
at the Tevatron, and the results appear to be at variance with QCD. Since
correlations provide us with the most complete information on the production
mechanism, further studies should be devoted to this topic.

2 Heavy-quark jets

An interesting way of understanding the production mechanism of heavy flavours
is to consider the cross section of jets which contain a heavy quark (briefly:
heavy-quark jets). The main difference between the study of a heavy quark and
a heavy-quark jet is that in the former case one is interested in the momentum
of the quark itself, regardless of the properties of the event in which the quark
is embedded, while in the latter case one is interested in the properties of a
jet containing one or more heavy quarks, regardless of the momentum fraction
of the jet carried by the quark. A priori it is expected that variables such as
the \( E_T \) distribution of a heavy-quark jet should be described by a finite-order
QCD calculation more precisely than the \( p_T \) distribution of open quarks, since
the jet \( E_T \) does not depend on whether the energy is carried all by the quark or
is shared among the quark and collinear gluons, and therefore large collinear
logarithms \( \log(p_T/m_b) \) do not appear in the cross section. The experimental
measurement of the \( E_T \) distribution of heavy-quark jets does not depend on
the knowledge of the heavy-quark fragmentation functions, contrary to the
case of the \( p_T \) distribution of open heavy quarks. Experimental systematics,
such as the knowledge of decay branching ratios for heavy hadrons or of their
decay spectra, are also largely reduced.

The calculation of the heavy-quark jet rate is very similar to the one of
the generic jet cross section. Two important differences have nevertheless to
be stressed: by its very definition, a heavy-quark jet is not flavour-blind; one
has to look for those jets containing a heavy flavour. Furthermore, the mass of
the heavy flavour is acting as a cutoff against final state collinear divergences.
This in turn implies that the structure of the singularities of the heavy-quark
jet cross section is identical to the one of the open-heavy-quark cross section
(a proof of this statement, and a detailed derivation of all the steps needed
to build a NLO heavy-quark jet cross section in perturbative QCD, can be
found elsewhere). The heavy-quark jet cross section at NLO can therefore
be written in the following way:

\[
d\sigma = d\sigma^{(open)} + d\Delta, \tag{1}
\]
where \( d\sigma^{(open)} \) is the open-heavy-quark cross section, and \( d\Delta \) is implicitly defined in eq. (1). The key feature of this equation is that all the subtractions needed to get an infrared-safe result are contained in the term \( d\sigma^{(open)} \). By construction, at NLO in QCD a heavy-quark jet can coincide with the heavy quark itself, or it can contain a heavy quark and a light parton, or the heavy quark-antiquark pair. The latter two possibilities are peculiar of the heavy-quark jet cross section, and are not present in the open-heavy-flavour one; formally, they are described by the \( \Delta \) term in eq. (1), which contribution I will call from now on as “jet-like component” of the cross section.

To present some results of interest for measurements at the Tevatron, I will consider jets produced within \( |\eta| < 1 \), in order to simulate a realistic geometrical acceptance of the Tevatron detectors. The jets will be defined using the Snowmass convention, whereby particles are clustered in cones of radius \( R \) in the pseudorapidity-azimuthal angle plane. The default parameters are the same as before, but now \( \mu_0 = \sqrt{E_T^2 + m_b^2} \), where \( E_T \) is the transverse energy of the \( b \)-jet (notice that \( E_T \) is not equal to \( p_T \) since the bottom is massive; the difference is however almost negligible in the energy range interesting for current phenomenological studies).

Figure 3 shows the prediction for the \( E_T \) distribution of \( b \)-jets at the Tevatron.
Figure 4: Left: $b$-jet inclusive $E_T$ rate, as a function of the cone size $R$, at $E_T = 50$ GeV and for various scale choices ($\mu_R = \mu_F \equiv \mu$). Right: Scale dependence of the $b$-jet $E_T$ distribution ($R = 0.4$, solid lines) and of the open-quark inclusive $E_T$ distribution (dashed lines).
	ron for $R = 0.7$. For the purpose of illustration, the open-quark component is separately presented. It is apparent that the jet-like component becomes dominant as soon as $E_T$ becomes larger than 50 GeV. It can be shown\textsuperscript{13} that this value actually depends significantly on the cone size, being equal to 25 and 100 GeV for $R = 1$ and 0.4 respectively. I also show the part of the jet-like component due to jets that include the $b\bar{b}$ pair (I will call these $b\bar{b}$-jets). The figure suggests that, for this $E_T$ range and with $R = 0.7$, this is the dominant part of the jet-like component. This is consistent with the expectation that, for large enough $E_T$ and provided that the majority of the final-state generic jets are composed of primary gluons, heavy-quark jets are dominated by the process of gluon splitting, with the jet formed by the heavy-quark pair.

The left side of figure 4 presents the theoretical prediction for the absolute heavy-quark jet rate at $E_T = 50$ GeV versus the cone size, for different choices of the factorization/renormalization scale. In this case, the cross section at $R = 0$ is well defined, and it is equal to the open-heavy-quark cross section. This should be contrasted with the case of generic jets, in which the cross section at $R = 0$ is not well defined, being negative at any fixed order in perturbation theory.\textsuperscript{16} The right side of figure 4 shows the scale dependence of the $b$-jet cross section (for $R = 0.4$) as a function of $E_T$, for values up to 450 GeV.

The strong scale dependence exhibited by the absolute rates at low and moderate $E_T$ values is of the same size as the one present in the inclusive $p_T$ distribution of open bottom quarks. This scale dependence is usually attributed to the importance of the gluon splitting contribution. One expects therefore
that in a regime in which the gluon splitting contribution is suppressed by the dynamics the scale dependence should be milder. I will show later that this suppression is indeed taking place for high transverse energies. This explains why in the high-$E_T$ region the scale dependence is indeed reduced to the value of 20% when the scales are varied in the range $\mu_0/2 < \mu < 2\mu_0$, a result consistent with the limited scale dependence of the NLO inclusive-jet cross sections.

The high-energy behaviour of $b$-jet cross section is presented in figure 5, for a given choice of scales and cone size. In fig. 5a, the separate contribution to the $b$-jet cross section of the three possible initial states, $gg$, $q\bar{q}$ and $qg$, is displayed. Notice that the $q\bar{q}$ contribution becomes dominant for $E_T > 250$ GeV. Figures 5b–d show, for each individual channel, the separate contribution of the open-quark and $b\bar{b}$-jet components. For $E_T$ large enough, the dominant component of the $gg$ and $qg$ channels is given by the $b\bar{b}$-jet contribution, because of the gluon-splitting dominance. In the case of the $q\bar{q}$ channel, on the contrary, the $b\bar{b}$-jet term is always suppressed, and most of the $b$-jets are composed of a single $b$ quark, often accompanied by a nearby gluon.

Coming finally to the comparison with data, preliminary results are avail-
Figure 6: Ratio of the $b(\bar{b})$-jet to inclusive-jet $E_T$ distributions, for different choices of renormalization and factorization scales ($\mu_R = \mu_F \equiv \mu$), for $R = 0.4$ (top) and $R = 0.7$ (bottom). The data points for $R = 0.4$ represent preliminary results from the CDF experiment, for which only the statistical uncertainty is shown.

The data for the fraction of heavy-quark jets relative to generic jets. I present in fig. 6 the ratio of the $b$-jet to inclusive-jet $E_T$ distributions. The inclusive-jet $E_T$ cross section has been calculated with the JETRAD program. For consistency with CDF prescriptions, the $b$-jets are defined here as jets containing either a $b$ or a $\bar{b}$ quark, jets containing both being counted only once. I will call these $b(\bar{b})$-jets. It is interesting to notice that there is a good agreement between the CDF data and the theoretical prediction obtained with $\mu_R = \mu_F = \mu_0/2$; this choice of scale is also supported by inclusive-jet $E_T$ spectrum data. This is particularly significant since the choice of scale for the heavy-quark jet cross section is not independent from the scale chosen to predict the open-heavy-quark one (see eq. (6)). But, as I stressed before, to get a satisfactory description of the data for the open-bottom $p_T$ spectrum, a more extreme choice of the parameters has to be done. Should this situation persist when additional data on $b$-jets will become available, it would indicate an inconsistency in describing two phenomena due to the same underlying physics. The poor understanding of the fragmentation mechanism is very likely a source of this inconsistency. In this sense, the study of $b$-jet production is a very promising tool, since theoretical predictions are in this case independent from a detailed knowledge of the final state long-distance physics.
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