Pressure-actuated cellular structures

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Received 1 August 2011
Accepted for publication 1 November 2011
Published 26 January 2012
Online at stacks.iop.org/BB/7/016007

Abstract
Shape changing structures will play an important role in future engineering designs since rigid structures are usually only optimal for a small range of service conditions. Hence, a concept for reliable and energy-efficient morphing structures that possess a large strength to self-weight ratio would be widely applicable. We propose a novel concept for morphing structures that is inspired by the nastic movement of plants. The idea is to connect prismatic cells with tailored pentagonal and/or hexagonal cross sections such that the resulting cellular structure morphs into given target shapes for certain cell pressures. An efficient algorithm for computing equilibrium shapes as well as cross-sectional geometries is presented. The potential of this novel concept is demonstrated by several examples that range from a flagellum like propulsion device to a morphing aircraft wing.

(Some figures in this article are in colour only in the electronic version)

Main notation

| Symbol | Description |
|--------|-------------|
| $\alpha_{Pn,1}, \alpha_{Pn,2}, \alpha_{Hn}$ | rotational degrees of freedom that are expressed with respect to the $n$th pentagonal cell |
| $\beta_{Pn,1}, \beta_{Pn,2}$ | internal angles of a pentagonal cell |
| $\gamma_{Pn,1}, \gamma_{Pn,2}, \gamma_{Hn}$ | angles between pentagonal and hexagonal cells |
| $\Delta \alpha_{Pn,1,2}, \Delta \beta_{Pn,1,2}$ | discrete angles of first and second target shapes at nodes between two pentagonal cells |
| $\lambda$ | step length during form finding |
| $\sigma$ | normal stresses of 1 mm thick cell sides |
| $A_{Pn}, A_{Hn}$ | cross-sectional area of pentagonal and hexagonal cells |
| $A_{Hn,1}, A_{Hn,2}$ | cross-sectional subareas of a divided hexagonal cell |
| $a_{Pn}, b_{Pn}, c_{Pn,1}, c_{Pn,2}$ | length of pentagonal and hexagonal cell sides |
| $a_{Hn}$ | length between two pentagonal and hexagonal nodes |
| $b_{Hn,1}, c_{Hn,1}, c_{Hn,2}$ | length of hexagonal cell sides |
| $f, f_{Pn}, f_{Hn}$ | global, pentagonal and hexagonal force vector |
| $g$ | gradient vector for form finding |
| $h_{Pn}$ | altitude of a triangle inside a pentagonal cell |
| $K, K_{Pn}, K_{Hn}$ | global, pentagonal and hexagonal stiffness matrices |
| $l_{Pn,1}, l_{Pn,2}$ | fractions of a triangle inside a pentagonal cell |
| $n_p, n_e, n_c$ | integers that refer to the $n$th node and the $n$th cell |
| $n_z, n_d$ | number of displacement constraints |
| $p_{P,1}, p_{H}$ | number of displacement dof |
| $p_{P,1}, p_{P,2}$ | number of cell sides, nodes |
| $p_{P,1}$ | number of zero energy modes |
| $p_{P,2}$ | pressure of pentagonal and hexagonal cells |
| $r$ | pressure of pentagonal, hexagonal cells for first and second target shapes |
| $S$ | residual shape vector |
| $S_{1}, S_{2}$ | global sensitivity matrix for one pressure set |
| $S_{1}, S_{2}$ | global sensitivity matrix for cell pressures of first, second target shape |

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S_{h-t_2} \quad \text{global sensitivity matrix for cell pressures of both target shapes}

u, u_{P_n}, u_{H_n} \quad \text{global, pentagonal, hexagonal displacement vector}

u_{m}, v_{m} \quad \text{horizontal and vertical displacement of a node}

v_{P_n}, v_{H_n} \quad \text{cell side lengths of a pentagonal, hexagonal cell}

2. Literature review
The accessible literature about pressure-actuated cellular structures is sparse and can be grouped into two categories. The first category comprises beam-like structures where cross-sectional dimensions are small compared to the longitudinal dimension. The second category consists of plate-like structures where in-plane dimensions are large compared to the out-of-plane dimension. A brief review of some major publications about both beam- and plate-like morphing structures is given in the following.

2.1. Beam-like morphing structures
A market-ready robot arm by Festo that won the ‘Deutscher Zukunftspreis’ (German Future Prize) in 2010 is shown in figure 3 (Wilson 2011). The robot arm consists of several cells that are each made from three mechanically connected and independently controlled bellows actuators. Hence, the length and number of degrees of freedom (dof) of the robot arm can potentially be changed by adding or removing cells. A similar structure for catheters that uses bellows actuators was proposed by Ikuta et al (2007). To reduce the cross-sectional area of catheters, Ikuta et al used a single bellows actuator per cell that is connected via a band pass valve to a single driving tube that connects all valves.

A different actuation principle for cellular, pressure-actuated beam-like structures was published by Tsukagoshi et al (2001). They created a single cell by winding several strands of equally spaced inflatable segments around a flexible cylinder with a spine-like core (figure 4). Hence it is possible to control the cell shape by altering the pressures in the single strands. Compared to bellows actuated cells, this concept results in larger actuation forces for a given cross-sectional area.

2.2. Plate-like morphing structures
The demand for shape changing aircraft components is the main driving force behind the development of plate-like morphing structures. Hence it is not surprising that current research efforts are mainly focused on wings and horizontal and vertical stabilizers. To increase clarity, plate-like morphing structures are subsequently divided into two categories. The first category comprises single-acting morphing structures and the second category comprises double-acting morphing structures. Pressurized cells in a single-acting structure exert a force onto an elastic part of the structure such that it deforms into a desired shape. Therefore, shape and cell pressures are coupled. On the other hand, double-acting morphing structures do not require the storage of strain energy. Hence, they can alter their shape and cell pressures independently.

2.2.1. Single-acting morphing structures. Vos and Barrett (2010) proposed a concept for morphing wings that is based on a large number of identical prismatic cells and an additional energy-storing structure that can undergo large displacements. A leading edge that is made from an elastic skin and a large number of hexagonal cells is shown in figure 5. Pressurizing
all cells increases the strain energy in the skin and deforms the leading edge such that flight performance is maximized. Reducing the cell pressures relaxes the strain energy in the skin which leads to an overall wing shape that increases the performance during low-speed flights.

A method for twisting a wing to change its angle of attack was patented by Kiceniuk (1964). He used several internally placed Bourdon tubes to control the wing’s lift by altering the tube pressures. A more general approach to create single-acting morphing structures was published by Shan et al (2009). They embedded tailored fluidic muscles into a soft matrix to create a multicellular solid (Chou and Hannaford 1994). The shape and anisotropy of such a structure can be tuned, steered by controlling the inlet valves of each actuator.

2.2.2. Double-acting morphing structures. An actuator that is made from flexible pressure tubes that are pairwise enclosed between two connected cells was developed by Menon and Lira (2006). The rendering of two cells and simulated displacements of an actuated structure that is made from seven cells are shown in figure 6. Note that the stiffness and shape of such a structure can be independently controlled by changing the magnitude and pressure ratio of each pair of tubes.
A number of pressure-actuated cellular structures that can be used instead of conventional double-acting pneumatic or hydraulic cylinders were patented by Dittrich (2005). He created plane symmetry groups from two different convex and concave prismatic cells as shown in figure 7. Independently pressurizing both kinds of cells leads to deformations in at least one working direction. The advantage compared to an actuation with pneumatic or hydraulic cylinders is that the exerted force is distributed over a larger area. Furthermore, these symmetry groups can be projected onto curved surfaces which extends the application range of this concept.

A concept for morphing aircraft wings that is based on a computer-controlled pressurization of inflatable cells was patented by Dorsett and Stewart (2000). They propose the placement of several cells with varying cross sections along the upper and lower surfaces or the leading and trailing edges. A similar approach that uses cellular modules in front and behind a box-shaped spar was patented by To and Kammer (2002). Pressurizing the modules causes the plates between them to move parallel so that desired wing profiles can be obtained. Finally, good starting points for fundamental numerical research about morphing liquid cell materials are the publications by Freeman and Weiland (2009) and Zhang et al (2010).

The remainder of the paper is organized as follows. Section 3 introduces a novel concept for pressure-actuated cellular structures that is inspired by the nastic movement of plants. It is shown that this concept leads to plate-like,
double-acting morphing structures that are lightweight, strong and capable of large shape changes. Section 4 presents a rotation-based kinematic description for a certain class of cellular structures as well as corresponding force, stiffness and sensitivity matrices. An efficient algorithm to compute cell geometries for given target shapes and cell pressures is described in section 5. Furthermore, the potential of the proposed concept is demonstrated by several examples. Section 6 concludes the paper.

3. Working principle

Two cantilevers that are assembled from a number of prismatic cells with identical pentagonal and hexagonal cross sections
are shown in figure 8. The cells are made from rigid plates that are connected to each other at the cell corners via hinges. Pressurizing the cells of a cantilever by an arbitrary amount results in an equilibrium configuration that reassembles a circular arc since each cell maximizes its volume. Hence, the radius of the circular arc is solely a function of the cell geometry and does not depend on the cell pressures.

Both cantilevers can be connected if the opposite cell sides are of equal length. The equilibrium shape of the resulting structure is again a circular arc if the geometry and pressure of all cells in each row are identical. However, the radius of the circular arc is now not only a function of the pentagonal and hexagonal cell geometries but also of the pressure ratio \( \frac{p_P}{p_H} \) where \( p_P \) is the pentagonal and \( p_H \) is the hexagonal cell pressure. Equilibrium shapes of the connected cantilevers for pressure ratios of \( \frac{p_P}{p_H} = 5/1 \) and \( \frac{p_P}{p_H} = 1/5 \) are shown in figure 9. The pictured upper and lower limits are the equilibrium shapes for \( \frac{p_P}{p_H} = \infty \) and \( \frac{p_P}{p_H} = 0 \).

The strength of pressure-actuated cellular structures is demonstrated with the help of a 1 m long cantilever (figure 10). The cantilever is subjected to a tip load of 5 and 10 kN m\(^{-1}\) for cell pressures \( p_P \) and \( p_H \) of 0.4 and 2.0 MPa, respectively. It can be seen that the cantilever is capable of moving a tip load of 10 kN m\(^{-1}\) over a distance that nearly equals its own length. The corresponding normal stresses for both tip loads are shown in figure 11 for the cell.

3 The unit kN m\(^{-1}\) of the tip load refers to a unit width, depth of the cantilever.

4 Sub-cellular osmotic hydration motors in plants can create cell pressures of up to 5 MPa (Stahlberg and Taya 2006).
side thicknesses of 1 mm. A remarkable result is that the normal stresses in the cell sides are mainly caused by the cell pressures and therefore nearly independent of the tip load. Furthermore, the resulting axial forces are relatively small and the additional bending stresses due to cell differential pressures can be minimized by using a sandwich-type construction of the cell sides.

Although the previous example is based on a cantilever that is made from two rows of identical pentagonal and hexagonal cells, the underlying physics is by no means restricted to such a geometry. Individually tailoring the length of each cell side makes it possible to design structures that morph between any two given target shapes as a function of the pressure ratio $p_P/p_H$. Furthermore, adding additional rows increases the number of independent pressure ratios and therefore target shapes into which a structure could morph. A few feasible combinations of pentagonal and hexagonal cells that can be used for pressure-actuated cellular structures are shown in figure 12. However, for the sake of brevity, in the following we will focus on structures that are made from two rows of pentagonal and hexagonal cells.

4. Numerical simulation
4.1. Displacement-based approach
A three-dimensional cellular structure is represented in the following by a two-dimensional cross-sectional plane that
Figure 13. Clamped cantilever that is made from one row of \( n_P \) pentagons and one row of \( n_P - 1 \) hexagons.

Figure 14. Degrees of freedom (dof) of a patch of pentagonal and hexagonal cells. (a) Twenty-two displacement dof. (b) Six rotational dof (zoom-in view).

Figure 15. Notation for the geometry of a single pentagonal cell: (a) cell sides, lengths and (b) angles.

is perpendicular to all prismatic cells. Furthermore, it is assumed that cell sides possess an infinite Young’s and shear modulus as well as frictionless hinges. Hence it is possible to model each cell side by a single two-dimensional bar finite element that is connected to the neighboring elements at the cell corners via a hinge. Therefore, the pressure on each cell side is equally lumped to both nodes of the corresponding bar element. Additional stiffness terms that originate from the cell pressurization can be taken into account by means of load following stiffness matrices. For example, the equilibrium shapes and axial stresses of figures 8–11 were computed on the basis of such a model. This approach is well suited to determine the equilibrium shape and normal stresses of a cellular structure for certain cell pressures. However, it cannot be used to determine the cell side lengths for given target shapes and cell pressures. This is due to the fact that a displacement-based approach generates more dof than required to describe the inherent structural zero energy modes. Since cell sides are usually rigid, these additional dof can lead to numerical problems during the form-finding process. The ratio between displacement dof and zero energy modes of a cantilever that is made from \( n_P \) pentagonal and \( n_p - 1 \) hexagonal cells is subsequently derived (figure 13).

The total number of nodes \( n_n \), elements \( n_e \) and constraints \( n_c \) are

\[
n_n = 5n_P + 1, \quad n_e = 7n_P - 1, \quad n_c = 3
\]

so that the number of displacement dof is \( n_d = 2n_n - n_c = 10n_P - 1 \) and the number of zero energy modes is \( n_z = 2n_n - n_e - n_c = 3n_P \). Therefore, the limit ratio between displacement dof and zero energy modes is

\[
n_d \approx \frac{10}{3} \frac{1}{n_P} \quad \text{as} \quad n_P \to \infty, \quad n_z \approx \frac{10}{3}.
\]
4.2. Rotation-based approach

Since the limit ratio between displacement dof and zero energy modes is 10/3, it is best to describe the kinematics of pressurized cellular structures by means of rotations rather than displacements. The dof of a small patch of two pentagonal and one hexagonal cells are shown in figure 14 for both the displacement- and rotation-based approaches. It can be seen that the patch possesses 22 displacements but only six rotational dof. Furthermore, the displacement dof are expressed in a global coordinate system, whereas each pentagonal cell side $a_{Pn}$ is the basis for three rotational dof. Two of the three rotational dof, namely $\alpha_{Pn,1}$ and $\alpha_{Pn,2}$, describe the geometry of the $n$th pentagonal and the neighboring hexagonal cells, whereas the third angle $\alpha_{Hn}$ describes only the geometry of the neighboring hexagonal cells.

The notation for the geometry of a single pentagon is summarized in figure 15. The vector $l_{Pn} = l_{Pn,1} + l_{Pn,2}$ is

$$l_{Pn} = \begin{bmatrix} a_{Pn} + \sin (\alpha_{Pn,2}) b_{Pn+1} - \sin (\alpha_{Pn,1}) b_{Pn} \\ \cos (\alpha_{Pn,2}) b_{Pn+1} - \cos (\alpha_{Pn,1}) b_{Pn} \end{bmatrix}$$

and the fractions $l_{Pn,1}, l_{Pn,2}$ and altitude $h_{Pn}$ are

$$l_{Pn,1} = \frac{1}{2l_{Pn}} (l_{Pn}^2 + c_{Pn,1}^2 - c_{Pn,2}^2)$$

$$l_{Pn,2} = \frac{1}{2l_{Pn}} (l_{Pn}^2 - c_{Pn,1}^2 + c_{Pn,2}^2)$$

$$h_{Pn} = \sqrt{c_{Pn,1}^2 - l_{Pn,1}^2}$$

The two pentagonal side vectors $c_{Pn,1}$ and $c_{Pn,2}$ can be expressed as

$$c_{Pn,1} = \begin{bmatrix} l_{Pn,1} \frac{\partial l_{Pn}}{\partial x} - h_{Pn} \frac{\partial l_{Pn}}{\partial y} \\ l_{Pn,1} \frac{\partial l_{Pn}}{\partial y} + h_{Pn} \frac{\partial l_{Pn}}{\partial x} \end{bmatrix}$$

$$c_{Pn,2} = \begin{bmatrix} l_{Pn,2} \frac{\partial l_{Pn}}{\partial x} + h_{Pn} \frac{\partial l_{Pn}}{\partial y} \\ -l_{Pn,2} \frac{\partial l_{Pn}}{\partial y} + h_{Pn} \frac{\partial l_{Pn}}{\partial x} \end{bmatrix}$$

$$c_{Pn,1} = \sqrt{c_{Pn,1,x}^2 + c_{Pn,1,y}^2}$$

$$c_{Pn,2} = \sqrt{c_{Pn,2,x}^2 + c_{Pn,2,y}^2}$$

$$l_{Pn} = \sqrt{l_{Pn,x}^2 + l_{Pn,y}^2}$$

9
so that the angles \( \beta_{Pn,1} \) and \( \beta_{Pn,2} \) result in

\[
\beta_{Pn,1} = \arccos \left( \frac{c_{Pn,1,1}}{c_{Pn,1}} \right); \quad \beta_{Pn,2} = \arccos \left( \frac{c_{Pn,2,1}}{c_{Pn,2}} \right).
\]

(6)

Equations for the area, force vector and stiffness matrix of a pentagonal cell are subsequently derived. This can be done on the basis of a single cell since the relevant rotational dof are expressed with respect to the pentagonal cell side \( a_{Pn} \). Hence, the area \( A_{Pn} \) of the \( n \)th pentagon is

\[
A_{Pn} = \frac{1}{2} l_{Pn} h_{Pn} + \frac{1}{2} a_{Pn} [\cos(\alpha_{Pn,1})b_{Pn} + \cos(\alpha_{Pn,2})b_{Pn+1}] + b_{Pn}b_{Pn+1} \sin(\alpha_{Pn,2} - \alpha_{Pn,1}).
\]

(7)

The corresponding force vector \( f_{Pn} \) is obtained by computing the weighted derivatives of the pentagonal area \( A_{Pn} \) with respect to the rotational dof \( u_{Pn} \):

\[
f_{Pn} = p_{P} \frac{\partial A_{Pn}}{\partial u_{Pn}} = p_{P} \begin{bmatrix} \frac{\partial A_{Pn}}{\partial \alpha_{Pn,1}} & \frac{\partial A_{Pn}}{\partial \alpha_{Pn,2}} \end{bmatrix}^T.
\]

(8)

so that the stiffness matrix \( K_{Pn} \) results in

\[
K_{Pn} = \frac{\partial f_{Pn}}{\partial u_{Pn}} = p_{P} \begin{bmatrix} \frac{\partial^2 A_{Pn}}{\partial \alpha_{Pn,1}^2} & \frac{\partial^2 A_{Pn}}{\partial \alpha_{Pn,1} \partial \alpha_{Pn,2}} & \frac{\partial^2 A_{Pn}}{\partial \alpha_{Pn,2}^2} \end{bmatrix}.
\]

(9)

The matrix \( H_{Pn} \) that contains second-order mixed derivatives is used to compute the sensitivity matrix that relates the change of rotational dof to a change of cell side lengths. It can be derived by differentiating the force vector \( f_{Pn} \) with respect to the pentagonal side lengths \( v_{Pn} \) such that

\[
H_{Pn} = \frac{\partial f_{Pn}}{\partial v_{Pn}} = p_{P} \frac{\partial^2 A_{Pn}}{\partial u_{Pn} \partial v_{Pn}} = p_{P} \begin{bmatrix} \frac{\partial^2 A_{Pn}}{\partial v_{Pn,1}^2} & \frac{\partial^2 A_{Pn}}{\partial v_{Pn,1} \partial v_{Pn,2}} & \frac{\partial^2 A_{Pn}}{\partial v_{Pn,2}^2} \end{bmatrix}.
\]

(10)
Figure 20. Structure that morphs between concave and convex circular arcs with identical radii ($\Delta \alpha_{Pn} = \pm 5.0^\circ$): (a) $p_{P1}/p_{H1} = 1/5$, (b) $p_{P2}/p_{H2} = 5/1$.

Figure 21. Structure that morphs between a sinus and cosinus: (a) $p_{P1}/p_{H1} = 1/5$, (b) $p_{P2}/p_{H2} = 5/1$ and (c) flagellum-like propulsion device.

Note that $\mathbf{H}_{Pn}$ lacks terms related to the pentagonal cell side $a_{Pn}$. This is because the base lengths $a_{Pn}$ are determined during the discretization of the target shapes prior to computing the cell side lengths.

Unlike previous derivations, expressions for the area, force vector and stiffness matrix of a hexagonal cell can only be obtained on the basis of a cell patch. The notation for two neighboring pentagonal and one hexagonal cells is
and the force vector results in $p_{H,1} = 5/1$ and $p_{P,2} = 1/5$.

The triangular area $A_{H,n}$, that is a part of the hexagon is

$$A_{H,n} = \frac{1}{2} \sin(\gamma_{P,n} + \gamma_{P,n+1}) c_{P,n} c_{P,n+1}$$

and the remaining pentagonal area $A_{H,n}$ can be computed by replacing the angles $\alpha_{P,n}$ and $\alpha_{P,n+1}$ in equation (7) with

$$\alpha_{H,n} = \alpha_{H,n} - \alpha_{P,n} - \gamma_{H,n}$$

$$\alpha_{H,n+1} = \alpha_{H,n+1} - \alpha_{P,n+1} - \gamma_{H,n}$$

Hence, the hexagonal area is

$$A_{H} = A_{H,1} + A_{H,2}$$

$$= A_{H,1} + \alpha_{P,n} (\alpha_{P,n} = \alpha_{H,1}, \alpha_{P,n+1} = \alpha_{H,2},\ldots)$$

and the force vector results in

$$f_{H,n} = p_H \frac{\partial A_{H,n}}{\partial u_{H,n}}$$

$$= p_H \left[ \frac{\partial A_{H,n}}{\partial \alpha_{P,n}} \frac{\partial A_{H,n}}{\partial \alpha_{P,n+1}} \frac{\partial A_{H,n}}{\partial \alpha_{P,n+1}} \frac{\partial A_{H,n}}{\partial \alpha_{H,n+1}} \frac{\partial A_{H,n}}{\partial \alpha_{H,n+1}} \right]^T$$

The corresponding stiffness matrix $K_{H,n}$ is

$$K_{H,n} = p_H \frac{\partial^2 A_{H,n}}{\partial u_{H,n}^2}$$
can be used to find an equilibrium configuration by iteratively computing an increment \( \Delta \mathbf{u} \) for the current state variables \( \mathbf{u} \) such that

\[
\Delta \mathbf{u} = -K(\mathbf{u})^{-1} \mathbf{f}(\mathbf{u}).
\]

(22)

The sensitivity matrix \( \mathbf{S} = \partial \mathbf{u} / \partial \mathbf{v} \) is needed to determine the cell side lengths during form finding. It can be computed at an equilibrium configuration by evaluating

\[
\mathbf{S} = -K^{-1} \mathbf{H},
\]

(23)

where

\[
\mathbf{H} = \sum_{n=1}^{n_p} \mathbf{H}_{pq} + \sum_{n=1}^{n_p} \mathbf{H}_{nq}.
\]

(24)

Note that the size of \( \mathbf{H} \) and \( \mathbf{S} \) is \( [3n_p \times 6n_p - 1] \) so that there are about twice as many cell sides of unknown length than rotational dof. Nodal coordinates and element stresses of some example structures that were computed by the displacement- and rotation-based approaches can be found in the appendix. It is shown, at single nodes, that the example structures are in equilibrium.

5. Form finding of cellular structures

5.1. Approach

This section introduces an efficient algorithm that can be used to determine the cell side lengths of a cellular structure for a given set of target shapes and cell pressures. The discrete angles that represent the first \( \Delta \alpha_{Pn,t_1} \) and second \( \Delta \alpha_{Pn,t_2} \) target shapes at the nodes between two pentagonal cells are illustrated in figure 18. A cellular structure morphs into given target shapes for certain cell pressures if the following relationships hold for all \( n \in [1, n_p - 1] \):

\[
\Delta \alpha_{Pn,t_1} = \alpha_{Pn,2} - \alpha_{Pn+1,1} \quad \text{for} \quad p_{P,t_1} / p_{H,t_1}
\]

\[
\Delta \alpha_{Pn,t_2} = \alpha_{Pn,2} - \alpha_{Pn+1,1} \quad \text{for} \quad p_{P,t_2} / p_{H,t_2}.
\]

(25)

Hence, the residual shape vector \( \mathbf{r} \) of a cellular structure can be defined as the difference between the angles of the current and target shapes for given cell pressures and side lengths such that

\[
\mathbf{r} = \begin{bmatrix} \Delta \alpha_{P1,t_1} \\ \vdots \\ \Delta \alpha_{Pn,t_1} \\ \Delta \alpha_{P1,t_2} \\ \vdots \\ \Delta \alpha_{Pn,t_2} \end{bmatrix} = \begin{bmatrix} \alpha_{P1,2} - \alpha_{P2,1} \\ \vdots \\ \alpha_{Pn-1,2} - \alpha_{Pn,1} \\ \alpha_{P1,2} - \alpha_{P2,1} \\ \vdots \\ \alpha_{Pn-1,2} - \alpha_{Pn,1} \end{bmatrix} - \begin{bmatrix} \alpha_{P1,2} - \alpha_{P2,1} \\ \vdots \\ \alpha_{Pn-1,2} - \alpha_{Pn,1} \\ \alpha_{P1,2} - \alpha_{P2,1} \\ \vdots \\ \alpha_{Pn-1,2} - \alpha_{Pn,1} \end{bmatrix}
\]

(26)

The sensitivity matrix \( \mathbf{S}_{t_1,t_2} \) that relates the change of angles \( \alpha_{Pn-1,2} - \alpha_{Pn,1} \) to the change of cell side lengths \( \mathbf{v} \) is

\[
\mathbf{S}_{t_1,t_2} = \begin{bmatrix}
\frac{\partial \alpha_{P1,2}}{\partial b_{P1}} & \frac{\partial \alpha_{P1,2}}{\partial b_{P1}} & \cdots & \frac{\partial \alpha_{P1,2}}{\partial c_{Hn,1}} & \frac{\partial \alpha_{P1,2}}{\partial c_{Hn,1}} \\
\frac{\partial \alpha_{Pn-1,2}}{\partial b_{P1}} & \frac{\partial \alpha_{Pn-1,2}}{\partial b_{P1}} & \cdots & \frac{\partial \alpha_{Pn-1,2}}{\partial c_{Hn,1}} & \frac{\partial \alpha_{Pn-1,2}}{\partial c_{Hn,1}} \\
\frac{\partial \alpha_{P1,2}}{\partial b_{P1}} & \frac{\partial \alpha_{P1,2}}{\partial b_{P1}} & \cdots & \frac{\partial \alpha_{P1,2}}{\partial c_{Hn,1}} & \frac{\partial \alpha_{P1,2}}{\partial c_{Hn,1}} \\
\frac{\partial \alpha_{Pn-1,2}}{\partial b_{P1}} & \frac{\partial \alpha_{Pn-1,2}}{\partial b_{P1}} & \cdots & \frac{\alpha_{Pn-1,2}}{\partial c_{Hn,1}} & \frac{\partial \alpha_{Pn-1,2}}{\partial c_{Hn,1}} \\
\frac{\partial \alpha_{P1,2}}{\partial b_{P1}} & \frac{\partial \alpha_{P1,2}}{\partial b_{P1}} & \cdots & \frac{\partial \alpha_{P1,2}}{\partial c_{Hn,1}} & \frac{\partial \alpha_{P1,2}}{\partial c_{Hn,1}} \\
\frac{\partial \alpha_{Pn-1,2}}{\partial b_{P1}} & \frac{\partial \alpha_{Pn-1,2}}{\partial b_{P1}} & \cdots & \frac{\partial \alpha_{Pn-1,2}}{\partial c_{Hn,1}} & \frac{\partial \alpha_{Pn-1,2}}{\partial c_{Hn,1}} \\
\end{bmatrix}
\]

(27)

Note that \( \mathbf{S}_{t_1,t_2} \) is assembled from the sensitivity matrices \( \mathbf{S}_{t_1} \) and \( \mathbf{S}_{t_2} \) that are computed at the equilibrium configurations for the current cell side lengths and cell pressures of both target shapes. Since the size of \( \mathbf{S}_{t_1,t_2} \) is \( [2n_p - 2 \times 6n_p - 1] \) there are more than three times as many unknown than known variables. Therefore, the geometry of a cellular structure that morphs between two given target shapes for certain cell pressures is not unique. The advantage of this non-uniqueness is that a wide range of additional constraints can be enforced.

The gradient vector \( \mathbf{g} \) that relates the change of the residual shape vector \( \mathbf{r} \) to the change of the cell side lengths \( \mathbf{v} \) is

\[
\mathbf{g} = \mathbf{S}_{t_1,t_2}^T \mathbf{r},
\]

so that an increment \( \Delta \mathbf{v} \) for the current cell side lengths \( \mathbf{v} \) can be computed by evaluating

\[
\Delta \mathbf{v} = \lambda \mathbf{g}.
\]

(29)

Note that \( \lambda \) is determined by a line search algorithm such that the \( \ell^2 \)-norm of the residual shape vector \( \mathbf{r} \) is minimized.

5.2. First example—circular arcs

The first example is a structure that morphs between a concave and convex circular arc with identical radii. The form-finding result for

\[
\Delta \alpha_{Pn,t_1} = +2.5^\circ \quad \text{for} \quad p_{P,t_1} / p_{H,t_1} = 5/1
\]

\[
\Delta \alpha_{Pn,t_2} = -2.5^\circ \quad \text{for} \quad p_{P,t_2} / p_{H,t_2} = 1/5
\]

(30)
**Figure 23.** Integration of a morphing leading edge into an aircraft wing: (a) flight configuration and (b) low-speed configuration.

**Figure 24.** Left part of a morphing leading edge with integrated rigid bodies: (a) flight \( \frac{p_{P,t_1}}{p_{H,t_1}} = \frac{5}{1} \) and (b) low-speed configuration \( \frac{p_{P,t_2}}{p_{H,t_2}} = \frac{1}{5} \).

**Figure 25.** Structural implementation of the proposed concept. (a) Stiff plates are connected via hinges at cell corners; (b) monolithic sandwich construction with a low-bending stiffness at cell corners.

is shown in figure 19 and a corresponding result for

\[
\begin{align*}
\Delta \alpha_{P,n,t_1} &= +5.0^\circ \quad \text{for} \quad \frac{p_{P,t_1}}{p_{H,t_1}} = \frac{1}{5} \\
\Delta \alpha_{P,n,t_2} &= -5.0^\circ \quad \text{for} \quad \frac{p_{P,t_2}}{p_{H,t_2}} = \frac{5}{1}
\end{align*}
\]  

is shown in figure 20. It can be seen that the cell geometries change gradually toward the ends of both structures. This boundary effect, that decays relatively fast, is due to the missing hexagonal cells at the ends. Another noteworthy point
is that the realizable curvature change depends on the pressure ratios of the concave and convex configuration. Pressure ratios of, for example, \( p_{P,1}/p_{H,1} = 1/5 \) for the concave and \( p_{P,1}/p_{H,1} = 5/1 \) for the convex target shape allow approximately twice the curvature change compared to inverse pressure ratios of \( p_{P,1}/p_{H,1} = 5/1 \) and \( p_{P,1}/p_{H,1} = 1/5 \). This dependence of the shape changing capability on the cell pressures is due to the pentagonal cell sides \( a_{Pt} \) that form an inextensional outer boundary of the structure. Hence, a structure that is made from, for example, two rows of hexagons does not exhibit such a behavior.

5.3. Second example—flagellum

The second example is a structure that changes its shape between two trigonometric functions. The form-finding result for

\[
\Delta a_{P,H} = \sin \left( \frac{6\pi n}{n_p - 1} \right) 5.0^\circ \quad \text{for} \quad p_{P,1}/p_{H,1} = 1/5
\]

![Figure A1. Element and node numbering of structure from figure 10: (a) nodes and (b) elements.](image-url)
### Table A2. Nodes and stresses of elements for pressure sets $p_1$ and $p_2$. Note that all cell side thicknesses are $t = 1$ mm.

| No | Node 1 | Node 2 | $\sigma_{t_1}$ (MPa) | $\sigma_{t_2}$ (MPa) | No | Node 1 | Node 2 | $\sigma_{t_1}$ (MPa) | $\sigma_{t_2}$ (MPa) |
|----|---------|---------|-----------------------|-----------------------|----|---------|---------|-----------------------|-----------------------|
| 1  | 4       | 1       | 5.6967                | 101.1813              | 38 | 28     | 29      | 33.7915               | 204.0862              |
| 2  | 1       | 2       | 25.1299               | 97.0876               | 39 | 29     | 30      | 192.5062              | 133.5500              |
| 3  | 2       | 3       | 1.5200                | 95.6071               | 40 | 30     | 31      | 235.6901              | 33.7887               |
| 4  | 3       | 5       | 40.5616               | 192.2466              | 41 | 31     | 32      | 71.2196               | 10.0724               |
| 5  | 4       | 5       | 178.6629              | 92.2729               | 42 | 32     | 33      | 71.2196               | 10.0724               |
| 6  | 5       | 6       | 17.8448               | 98.2794               | 43 | 33     | 34      | 71.2196               | 10.0724               |
| 7  | 6       | 8       | 174.8691              | 95.1198               | 44 | 34     | 35      | 71.2196               | 10.0724               |
| 8  | 7       | 9       | −4.1023               | 89.7046               | 45 | 35     | 36      | 72.7398               | 10.5494               |
| 9  | 10      | 7       | −4.1023               | 195.0859              | 46 | 36     | 37      | 72.7398               | 10.5494               |

$\Delta \alpha_{p_{1,2}} = \cos \left( \frac{6 \pi n}{n_p - 1} \right) \cdot 5.0^\circ$ for $p_{P,2}/p_{H,2} = 5/1$

and $n_p = 100$ is shown in figure 21. It can be seen that the geometry of the cellular structure is, like the target shapes, periodic. The only constraint for the form finding is the previously discussed maximum change of curvature. The latter could be increased by using a topology that is based on two rows of hexagons or two rows of pentagons. This would allow a greater amplitude of the first and second target shapes for a given wavelength and total number of elements. Such a structure could be used as a propulsion device that mimics, for example, the movement of a swimming snake or a flagellum.

#### 5.4. Third example—leading edge

The third form-finding example is a leading edge for an aircraft wing where the pentagonal cell side lengths $a_{P,n}$ are not constant (figure 22). The use of adaptive base lengths reduces the required number of cells for given target shapes and cell pressures so that the overall stiffness of the structure increases due to larger average cell sizes. Although the base lengths in this example were chosen prior to the form-finding, it would be possible to use sophisticated algorithms instead to iteratively determine each base length during the form-finding process according to certain criteria. An integration of such a leading edge into an aircraft wing is shown in figure 23.

A great advantage of morphing cellular structures is the non-uniqueness of the form-finding process. For example, given a set of target shapes, cell pressures and pentagonal base lengths, there exists a large number of feasible solutions. Hence it is possible to use this redundancy to incorporate additional constraints. The enforcement of constraints that allow the attachment of rigid bodies into the left part of the leading edge is shown in figure 24. The advantage of these passive mechanisms is that they stiffen the structure by rigidly connecting both the top and bottom rows to the airframe.
to other technologies in terms of energy efficiency (Huber et al. 2012). Pressure-actuated cellular structures score well compared to other lattice structures due to their large strength to self-weight ratio as well as an increased bending stiffness at the cell corners (figure 25). Since all prismatic cells change their cross-sectional shape during actuation, each cell has to be sealed by a tailored membrane or a pouch. Such a structure possesses a low self-weight and therefore a large strength to self-weight ratio as well as an increased reliability due to a small part count. Furthermore, pressure-actuated cellular structures score well compared to other technologies in terms of energy efficiency (Huber et al. 2012). These properties and their ability to undergo large shape changes make them an attractive option for future developments in a wide range of fields. An efficient algorithm for computing equilibrium shapes and cross-sectional geometries of cellular structures for given target shapes and cell pressures was presented. The algorithm is based on rotation rather than displacement dof which improves the numerical efficiency. Finally, the potential of this novel concept was demonstrated by several examples that range from two previous examples.

Acknowledgments

The authors want to thank Stanley van Kempen for validating simulation results from the displacement- and rotation-based approach.

6. Conclusions

This paper introduced a novel concept for shape changing structures that is inspired by the nastic movement of plants. A great advantage of this concept is its possible implementation by means of a monolithic structure that possesses a low bending stiffness at the cell corners (figure 25). Since all prismatic cells change their cross-sectional shape during actuation, each cell has to be sealed by a tailored membrane or a pouch. Such a structure possesses a low self-weight and therefore a large strength to self-weight ratio as well as an increased reliability due to a small part count. Furthermore, pressure-actuated cellular structures score well compared to other technologies in terms of energy efficiency (Huber et al. 2012). These properties and their ability to undergo large shape changes make them an attractive option for future developments in a wide range of fields. An efficient algorithm for computing equilibrium shapes and cross-sectional geometries of cellular structures for given target shapes and cell pressures was presented. The algorithm is based on rotation rather than displacement dof which improves the numerical efficiency. Finally, the potential of this novel concept was demonstrated by several examples that range from a flagellum-like propulsion device to a morphing aircraft wing.

Appendix

In the following we summarize data and validate simulation results from two previous examples.

A.1. Loaded cantilever

The node and element numbering of the structure from figure 10 is shown in figure A1. The nodal coordinates for two pressure sets \( p_{f_1} = \{p_{P,f_1} = 0.4 \text{ MPa, } p_{H,f_1} = 2.0 \text{ MPa}\} \) and \( p_{f_2} = \{p_{P,f_2} = 2.0 \text{ MPa, } p_{H,f_2} = 0.4 \text{ MPa}\} \) are listed in table A1. The corresponding start and end nodes of each element as well as the element stresses are listed in table A2.

The structure is in equilibrium if the residual force at each node for both sets of pressures vanishes. The residual force of

| Table A3. Nodal coordinates for pressure sets \( p_{f_1} \) and \( p_{f_2} \). |
|-----------------|-----------------|-----------------|-----------------|
| No | \( x_{f_1} \) (mm) | \( y_{f_1} \) (mm) | \( x_{f_2} \) (mm) | \( y_{f_2} \) (mm) |
|-----|-----------------|-----------------|-----------------|-----------------|
| 1   | 0               | 0               | 0               | 0               |
| 2   | 0               | -59.0858        | 0               | -59.0858        |
| 3   | 51.7390         | -99.5815        | 47.2077         | -104.7833       |
| 4   | 99.9762         | 2.1815          | 99.9762         | -2.1815         |
| 5   | 102.2410        | -57.1003        | 96.9630         | -61.4300        |
| 6   | 155.2741        | -95.9070        | 141.5268        | -109.7263       |
| 7   | 199.7621        | 8.7218          | 199.7621        | -8.7218         |
| 8   | 204.2960        | -51.1480        | 193.5040        | -68.4360        |
| 9   | 259.1184        | -88.6116        | 235.6896        | -119.7135       |
| 10  | 299.1677        | 19.6085         | 299.1677        | -19.6085        |
| 11  | 306.0683        | -41.1130        | 289.3289        | -79.9236        |
| 12  | 363.6082        | -77.4668        | 329.8823        | -134.5849       |
| 13  | 398.0039        | 34.8208         | 398.0039        | -34.8208        |
| 14  | 407.6492        | -26.5643        | 384.6945        | -95.5170        |
| 15  | 468.7754        | -61.4447        | 424.4645        | -153.5807       |
| 16  | 496.0824        | 54.3299         | 496.0824        | -54.3299        |
| 17  | 508.8893        | -6.4523         | 480.0928        | -114.3534       |
| 18  | 573.7048        | -38.9345        | 519.6734        | -175.0948       |
| 19  | 593.2166        | 78.0984         | 593.2166        | -78.0984        |
| 20  | 609.4604        | 19.1943         | 576.1388        | -136.7662       |
| 21  | 676.4513        | -9.0893         | 615.1382        | -198.1404       |
| 22  | 689.2216        | 106.0813        | 689.2216        | -106.0813       |
| 23  | 708.4194        | 50.5579         | 672.5238        | -162.6042       |
| 24  | 776.6039        | 27.8717         | 711.5578        | -222.8629       |
| 25  | 783.9146        | 138.2253        | 783.9146        | -138.2253       |
| 26  | 805.9271        | 83.5571         | 767.9685        | -194.9605       |

\[ f_{P,T_1} = 2.0 \text{ MPa} \]
\[ p_{H,T_1} = 0.4 \text{ MPa} \]

Figure A2. Forces at node 4 of structure from figure 10.
**Table A4.** Nodes and stresses of elements for pressure sets $p_{t1}$ and $p_{t2}$. Note that all cell side thicknesses are $t = 1 \text{ mm}$.

| No | Node 1 | Node 2 | $\sigma_{t1}$ (MPa) | $\sigma_{t2}$ (MPa) | Node | Node 1 | Node 2 | $\sigma_{t1}$ (MPa) | $\sigma_{t2}$ (MPa) |
|----|--------|--------|---------------------|---------------------|-------|--------|--------|---------------------|---------------------|
| 1  | 4      | 1      | 197.2833           | 46.6494            | 37    | 31     | 28     | 156.9827           | 34.7630             |
| 2  | 1      | 2      | 254.2443           | 48.9705            | 38    | 28     | 29     | 509.0388           | 99.7716             |
| 3  | 2      | 3      | 244.6118           | 206.1560           | 39    | 29     | 30     | 242.3220           | 256.0505            |
| 4  | 3      | 5      | 242.8243           | 205.2496           | 40    | 30     | 32     | 257.6116           | 53.6402             |
| 5  | 7      | 4      | 200.0596           | 47.3526            | 41    | 31     | 32     | 273.6545           | 56.7292             |
| 6  | 4      | 5      | 508.5567           | 97.9281            | 42    | 3      | 33     | 102.6344           | 472.2832            |
| 7  | 5      | 6      | 245.2115           | 206.2819           | 43    | 33     | 34     | 67.6001            | 207.0346            |
| 8  | 6      | 8      | 240.3460           | 204.8335           | 44    | 34     | 35     | 67.6001            | 207.0346            |
| 9  | 10     | 7      | 206.0041           | 49.0321            | 45    | 6      | 35     | 103.1028           | 473.3550            |
| 10 | 7      | 8      | 508.7740           | 97.8880            | 46    | 35     | 36     | 70.0943            | 208.9688            |
| 11 | 8      | 9      | 243.8377           | 204.9487           | 47    | 36     | 37     | 70.0943            | 208.9688            |
| 12 | 9      | 11     | 237.7376           | 206.3026           | 48    | 9      | 37     | 104.0731           | 476.4318            |
| 13 | 10     | 10     | 215.0395           | 52.0452            | 49    | 37     | 38     | 74.7213            | 213.8930            |
| 14 | 10     | 11     | 509.1462           | 97.8176            | 50    | 38     | 39     | 74.7213            | 213.8930            |
| 15 | 11     | 12     | 239.3261           | 201.7358           | 51    | 12     | 39     | 104.6298           | 481.2880            |
| 16 | 12     | 14     | 235.6593           | 209.4697           | 52    | 39     | 40     | 80.8899            | 222.5601            |
| 17 | 16     | 13     | 224.5596           | 56.0828            | 53    | 40     | 41     | 80.8899            | 222.5601            |
| 18 | 13     | 14     | 509.5597           | 97.7008            | 54    | 15     | 41     | 104.4456           | 486.6941            |
| 19 | 14     | 15     | 231.7016           | 197.5915           | 55    | 41     | 42     | 86.6842            | 234.3015            |
| 20 | 15     | 17     | 234.6692           | 212.6331           | 56    | 42     | 43     | 86.6842            | 234.3015            |
| 21 | 19     | 16     | 229.9146           | 60.1959            | 57    | 18     | 43     | 102.5314           | 488.7875            |
| 22 | 16     | 17     | 509.8234           | 97.5263            | 58    | 43     | 44     | 88.1702            | 244.6123            |
| 23 | 17     | 18     | 221.9063           | 193.4999           | 59    | 44     | 45     | 88.1702            | 244.6123            |
| 24 | 18     | 20     | 236.7107           | 214.4762           | 60    | 21     | 45     | 102.3763           | 491.4979            |
| 25 | 22     | 19     | 226.2025           | 62.2856            | 61    | 45     | 46     | 84.3539            | 252.5524            |
| 26 | 19     | 20     | 509.8447           | 97.3267            | 62    | 46     | 47     | 84.3539            | 252.5524            |
| 27 | 20     | 21     | 216.9512           | 194.2161           | 63    | 47     | 48     | 74.5037            | 251.3588            |
| 28 | 21     | 23     | 242.2245           | 220.2649           | 64    | 47     | 48     | 74.5037            | 251.3588            |
| 29 | 25     | 22     | 212.7714           | 60.5155            | 65    | 48     | 49     | 74.5037            | 251.3588            |
| 30 | 22     | 23     | 509.6341           | 97.3134            | 66    | 27     | 49     | 104.6360           | 514.7914            |
| 31 | 23     | 24     | 217.7584           | 202.7894           | 67    | 49     | 50     | 61.8931            | 254.3356            |
| 32 | 24     | 26     | 256.4748           | 235.9580           | 68    | 50     | 51     | 61.8931            | 254.3356            |
| 33 | 28     | 25     | 195.6002           | 53.1806            | 69    | 30     | 51     | 64.6870            | 271.1605            |
| 34 | 25     | 26     | 509.0791           | 97.7713            | 70    | 2      | 52     | 0                  | 0                   |
| 35 | 26     | 27     | 233.3520           | 220.8611           | 71    | 52     | 33     | 66.8362            | 206.5875            |
| 36 | 27     | 29     | 273.0320           | 266.4469           |       |        |        |                    |                     |
node 4 is subsequently computed for the structural properties that are summarized in figure A2. The normalized vectors of the three elements that meet at node 4 are

\[ \mathbf{v}_{1,t_2} = \begin{bmatrix} \frac{x_1}{y_1} - \frac{x_4}{y_4} \\ \frac{x_1}{y_1} - \frac{x_3}{y_3} \end{bmatrix}, \]

\[ \mathbf{v}_{5,t_2} = \begin{bmatrix} \frac{x_5}{y_5} - \frac{x_4}{y_4} \\ \frac{x_5}{y_5} - \frac{x_2}{y_2} \end{bmatrix}, \]

\[ \mathbf{v}_{6,t_2} = \begin{bmatrix} \frac{x_6}{y_6} - \frac{x_4}{y_4} \\ \frac{x_6}{y_6} - \frac{x_2}{y_2} \end{bmatrix}. \] (A.1)

and the normalized normal vectors of elements 1 and 5 are

\[ \mathbf{n}_{1,t_2} = \begin{bmatrix} v_{1,y,t_2} \\ -v_{1,x,t_2} \end{bmatrix}, \quad \mathbf{n}_{5,t_2} = \begin{bmatrix} -v_{5,y,t_2} \\ v_{5,x,t_2} \end{bmatrix}. \] (A.2)

Hence, the residual force vector \( f_{l,t_2} \) of node 4 is

\[ f_{l,t_2} = f_{l,t_1} + f_{n,t_1} + f_{pH,t_1} + f_{pP,t_1} + f_{pP,t_2} = \mathbf{v}_{1,t_2} \sigma_{1,t_2} + \mathbf{v}_{5,t_2} \sigma_{5,t_2} + \mathbf{v}_{6,t_2} \sigma_{6,t_2} + \frac{1}{2} \left( \mathbf{n}_{1,t_2} l_1 + \mathbf{n}_{5,t_2} l_5 \right) p_{p,l_2} \approx \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \] (A.3)

where the cell side lengths are \( l_1 = l_5 = 100 \) mm and the pentagonal pressure is \( p_{p,l_2} = 2.0 \) MPa.

### A.2. Circular arc

The structure that is shown in figure 19 is used in the following to validate the rotation-based approach. The reflection symmetry of the structure is taken into account as shown in figure A3 to reduce the amount of data. The node and element numbering is identical to the one used in figure A1. The only exception is that node 53 and elements 72 and 73 do not exist. The nodal coordinates for both pressure sets \( p_t = \{ p_{p,t_1} = 5 \) MPa, \( p_{l,t_1} = 1 \) MPa \} and \( p_t = \{ p_{p,t_2} = 1 \) MPa, \( p_{l,t_2} = 5 \) MPa \} are summarized in table A3. The corresponding start and end nodes of each element as well as the element stresses are listed in table A4.

Equilibrium of the structure can be verified by computing the equilibrium of each node as it was done in the previous example. In the following, it is shown that the angle between elements 1 and 5 satisfies the form-finding criteria for the second pressure set \( p_{l,t_2} \). The normalized vectors \( \mathbf{v}_{1,t_2} \) and \( \mathbf{v}_{5,t_2} \) of both elements are described by equation (A.1) so that the angle between both vectors is

\[ \Delta \alpha_{p_{t_2}} = \arccos \left( \mathbf{v}_{1,t_2}^T \cdot \mathbf{v}_{5,t_2} \right) \approx 2.5^\circ. \] (A.4)

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