Color symmetrical superconductivity in a schematic nuclear quark model

H. Bohr\(^1\), C. Providência\(^2\) and J. da Providência\(^2(a)\)

\(^1\) Department of Physics, B.307, Danish Technical University - DK-2800 Lyngby, Denmark, EU
\(^2\) Departamento de Física, Universidade de Coimbra - P-3004-516 Coimbra, Portugal, EU

received 4 December 2009; accepted in final form 26 January 2010
published online 26 February 2010

PACS 21.60.Fw – Models based on group theory
PACS 21.65.Qr – Quark matter
PACS 25.75.Jq – Quark deconfinement, quark-gluon plasma production, and phase transitions

Abstract – In this letter, a novel BCS-type formalism is constructed in the framework of a schematic QCD inspired quark model, having in mind the description of color symmetrical superconducting states. In the usual approach to color superconductivity, the pairing correlations affect only the quasi-particle states of two colors, the single-particle states of the third color remaining unaffected by the pairing correlations. In the theory of color symmetrical superconductivity here proposed, the pairing correlations affect symmetrically the quasi-particle states of the three colors and vanishing net color charge is automatically insured. We stress that the present note is concerned with the description of quark matter in terms of effective models, such as the NJL model, which are solely expressed in terms of fermion operators, so that in them the gluonic gauge fields are not present.

Introduction. – At present, it is generally accepted that QCD matter at high densities exhibits color superconductivity induced by the familiar phenomenon of Cooper instability [1]. For a recent review, see [2]. Quarks are free in the deconfined phase, but the deconfined phase itself should be a color singlet, since gauge symmetries cannot be spontaneously broken [3]. More recently, this question has again been considerably discussed in the literature[4,5].

Our aim is to develop a version of the BCS theory which is appropriate to describe a color singlet\(^1\) superconducting phase, in the framework of effective models which involve only fermion operators. The physical properties of the BCS vacuum which is here constructed (for instance, the average numbers of quarks of different colors) remain unchanged under an arbitrary color rotation. The problem of reconciling color superconductivity with color symmetry has been addressed by some authors [6] who introduce color chemical potentials which ensure equal average numbers of quarks for any color.

One may also impose color singletness with the help of the Peierls-Yoccoz projection method (see [7], pp. 460–462, for a description of the method). This approach has been considered in [8]. However, the projection method is not entirely reliable since it leads to results which depend strongly on the choice of the mean-field state under consideration. For instance, it is well known that the Peierls-Yoccoz method may lead to the wrong moment of inertia of a deformed nucleus.

For a superconductor in condensed matter, it is often stated that the global \(U(1)\) gauge invariance is spontaneously broken [9]. Superconductivity is, according to Peierls [9], a case of symmetry-breaking approximation, it being the BCS wave function that breaks the \(U(1)\) symmetry in order to take into account important correlations, but not the physical reality itself. A given BCS state \(|\Phi\rangle\) does not have an exact number \(N\) of electrons. However, the state \(|\Phi\rangle\) and any other state \(|\Phi\rangle'\) which is obtained from \(|\Phi\rangle\) by a \(U(1)\) transformation have precisely the same average number \(\langle N\rangle\) of electrons. In this letter, we are concerned with the description of the so-called color superconducting quark phase of hadronic matter in the framework of color \(SU(3)\) symmetric effective models involving only fermion operators (it being assumed, in these models, that the gluonic gauge fields have been somehow integrated over). Such effective models contain sectors belonging to the \(SU(3)\) singlet representation,
which we regard as the physical sectors, but, unavoidably, they also contain sectors belonging to other representations which, in view of [3–5], must be regarded as unphysical. For instance, suppose that for a given BCS state \( \Phi \) the numbers of green quarks, red quarks and blue quarks were \( |N_1\rangle = |N_2\rangle = |N_3\rangle = |N\rangle/3 \). Under a \( SU(3) \) transformation a new state \( \Phi' \) will be obtained having \( |N_1\rangle' \) green, \( |N_2\rangle' \) red and \( |N_3\rangle' \) blue quarks, being, in general, \( |N_1\rangle', |N_2\rangle', |N_3\rangle', \langle N\rangle'/3 \) distinct. This happens if, and only if, the state \( \Phi \) describes an unphysical state which does not belong to a singlet representation of \( SU(3) \), so that it is not \( SU(3) \) symmetric. Our point is precisely that it is not enough to insure \( |N_1\rangle = |N_2\rangle = |N_3\rangle = |N\rangle/3 \), which may be achieved by using appropriate Lagrange multipliers, as is done in [6] and also in [10]. It should, however, be recognized that in [10] it has already been pointed out that such a procedure is not sufficient. Such a state must be discarded as unphysical because it belongs to a \( SU(3) \) representation other than the singlet one.

A BCS wave function \( |\Phi\rangle \) describes a state with average zero net color charge if \( |N_1\rangle = |N_2\rangle = |N_3\rangle = |N\rangle/3 \), so that, denoting by \( S_{kl} \) the color \( U(3) \) generator which will be defined in the next section, we have

\[
\langle \Phi | S_{11} | \Phi \rangle = \langle \Phi | S_{22} | \Phi \rangle = \langle \Phi | S_{33} | \Phi \rangle = 0.
\]

We will argue that the requirement (1), which is implemented in [6,10], is not sufficient to insure that \( |\Phi\rangle \) describes a color singlet, which is the condition for physical acceptability. As has also been observed in [10], a stronger condition must then be imposed, since the \( SU(3) \) symmetry cannot be broken, according to [3–5]. Therefore, color rotated BCS states must be equivalent in the sense of the physics they describe. Let \( U_c \) denote an arbitrary color rotation, i.e.,

\[
U_c = \exp \sum_{k,l=1}^3 i x_{kl} S_{kl}, \quad x_{kl} = x_{lk}^*.
\]

The BCS state \( |\Phi\rangle \) must be equivalent to the state \( U_c |\Phi\rangle \), for any \( U_c \), as far as expectation values of physical observables are concerned. Thus, the condition (1) must be replaced by

\[
\langle \Phi | U_c^\dagger S_{11} U_c | \Phi \rangle = \langle \Phi | U_c^\dagger S_{22} U_c | \Phi \rangle = \langle \Phi | U_c^\dagger S_{33} U_c | \Phi \rangle = 0.
\]

This implies (1) and, in addition,

\[
\langle \Phi | S_{kl} | \Phi \rangle = 0, \quad \text{for } k \neq l.
\]

This is the condition the BCS state \( |\Phi\rangle \) must necessarily satisfy in order to be physically meaningful. If only the condition (1) is implemented, as in [6], and not the condition (2), the BCS state \( |\Phi\rangle \) is, in general, not equivalent to the state \( U_c |\Phi\rangle \), so that it describes a state belonging to a representation of \( SU(3) \) other than the singlet one, which is physically unacceptable. Buballa and Shovkovy [10], have discussed the implementation of (1) and they have recognized that the full condition (2) should also be imposed. Here, we show how this condition may be easily implemented in the framework of effective models which involve only fermion operators.

In [11], an appropriate generalization of the Bogoliubov transformation that treats all colors on the same footing has already been applied to the NJL model. By applying it to a simpler model, one can find exact solutions which are compared with those of the generalized BCS ansatz, so that the present study complements the previous one. We consider a standard nuclear quark model exemplified by the Bonn model which was proposed by H. R. Petry et al. in 1985 [12]. In ref. [13], it was shown that the model’s ground state is not color symmetrical, although it admits an important color symmetrical sector. Using a representation of \( SU(4) \) of the Schwinger type which is formulated in terms of appropriate boson operators and is due to Yamamura et al. [14], it has been possible to characterize the referred color symmetrical sector [15]. In ref. [13], it has been shown that conventional BCS states describe adequately the ground state of the model. In this letter we focus on the description of the color symmetrical sector of the model, by a generalized BCS ansatz.

A schematic pairing model with \( SU(3) \) symmetry. – The quark model proposed by H. R. Petry et al. [12] is defined by the Hamiltonian

\[
H = G \sum_{j=1}^3 A_j^\dagger A_j,
\]

where

\[
A_j^\dagger = \sum_{m>0} \left( c_{1m}^j c_{3m}^j + c_{2m}^j c_{3m}^j \right), \quad \bar{m} = m.
\]

Moreover, \( G < 0 \) is the coupling constant and the expressions for \( A_1^\dagger, A_3^\dagger \), are obtained by circular permutation of the indices 1,2,3. In eq. (4), \( c_{1m}^i \) are quark creation operators and the indices \( i \) and \( m \) denote, respectively, the color and the remaining single-particle quantum numbers, namely, momentum \( p \), spin \( \sigma \) and isospin \( \tau \). By \( \bar{m} \) we mean the state obtained from \( m \) by time reversal in an extended sense, namely, through the replacement \((p, \sigma, \tau) \rightarrow (\bar{p}, -\sigma, -\tau)\), so that double-time reversal operation retrieves the Fermionic state, and not, as in condensed-matter superconductivity, the Fermionic state with the minus sign. A double minus sign, one coming from spin and the other one from isospin, is equivalent to plus sign.

Color superconductivity has been applied in [13] to the description of the ground state of the Bonn model, which in general is not color symmetric. Indeed, although \( H \) has \( SU(3) \) symmetry, its eigenstates are not necessarily invariant under color \( SU(3) \) rotations, that is, they are not necessarily color singlets. The study of the color symmetrical sector is particularly interesting. The generators of color \( U(3) \) read

\[
S_{kl} = \sum_m c_{km}^l c_{lm}^k = \sum_{m>0} \left( c_{nm}^l c_{lm}^k + c_{mn}^l c_{lm}^k \right).
\]
A state $|\Phi\rangle$ is a color singlet if it satisfies the following condition:

$$S_k(\Phi) = 0, \quad k \neq l, \quad S_{kk}(\Phi) = \lambda(\Phi), \quad k = 1, 2, 3.$$  \hspace{1cm} (5)

Let us consider the general BCS state.

$$|\Phi\rangle = e^{-\sum_{j=1}^{3}(K \sum_{0 < m \leq \Omega'} A_j^\dagger m + \tilde{K} \sum_{\Omega' < m \leq \Omega} A_j m)} |0\rangle,$$

where

$$|0\rangle' = \left( \prod_{j=1}^{3} \prod_{\Omega' < m \leq \Omega} c_j^\dagger m c_j^m \right) |0\rangle,$$

and

$$A_j^\dagger m = (c_{2m}^\dagger c_{3m}^m + c_{3m}^\dagger c_{2m}^m).$$

This state is color neutral in the sense of vanishing net color charge. The expressions for $A_{1m}^\dagger, A_{3m}^\dagger$, are obtained by circular permutation of the indices 1, 2, 3. The parameters $K, \tilde{K}$ are real. We denote by $2\Omega$ the level degeneracy for a fixed color, that is, the totality of eigenstates pertaining to all quantum numbers beyond color. If $\Omega = \Omega'$, the quark number $N$ satisfies $0 \leq N \leq 4\Omega$. If $\Omega' = 0$, the quark number $N$ satisfies $4\Omega \leq N \leq 6\Omega$. The state vector $|\Phi\rangle$ has obviously zero net color charge, but it is not color symmetrical, since it cannot satisfy (5). This is so because $[S_{12}, (A_i^\dagger A_j^\dagger)] = -A_j^\dagger \neq 0$. However, $K, \tilde{K}$ may be chosen so that $|\Phi\rangle$ is color symmetrical in the average, that is, so that (2) is satisfied. We observe that

$$c_{1m} |\Phi\rangle = K \left( c_{2m}^\dagger - c_{3m}^\dagger \right) |\Phi\rangle,$$

$$c_{1m}^\dagger |\Phi\rangle = K \left( c_{2m}^\dagger - c_{3m}^\dagger \right) |\Phi\rangle, \quad 0 < m \leq \Omega';$$

$$c_{1m}^\dagger |\Phi\rangle = -\tilde{K} \left( c_{2m}^\dagger - c_{3m}^\dagger \right) |\Phi\rangle, \quad \Omega' < m \leq \Omega.$$  \hspace{1cm} (6)

These relations are crucial. They are straightforward consequences of the commutation relations

$$\left[ c_{1p}^\dagger, \left( K \sum_{0 < m \leq \Omega'} A_j^\dagger m + \tilde{K} \sum_{\Omega' < m \leq \Omega} A_j m \right) \right] =$$

$$K \left( c_{2p}^\dagger - c_{3p}^\dagger \right), \quad 0 < p \leq \Omega',$$

$$\left[ c_{1p}^\dagger, \left( K \sum_{0 < m \leq \Omega'} A_j^\dagger m + \tilde{K} \sum_{\Omega' < m \leq \Omega} A_j m \right) \right] =$$

$$-\tilde{K} \left( c_{2p}^\dagger - c_{3p}^\dagger \right), \quad \Omega' < p \leq \Omega.$$

From (6) it follows that the BCS vacuum $|\Phi\rangle$ is annihilated by the operators

$$d_{1m} = c_{1m} - K \left( c_{2m}^\dagger - c_{3m}^\dagger \right),$$

$$d_{1m}^\dagger = c_{1m}^\dagger - K \left( c_{2m}^\dagger - c_{3m}^\dagger \right), \quad 0 < m \leq \Omega';$$

$$d_{1m} = c_{1m}^\dagger + \tilde{K} \left( c_{2m}^\dagger - c_{3m}^\dagger \right),$$

$$d_{1m}^\dagger = c_{1m} - \tilde{K} \left( c_{2m}^\dagger - c_{3m}^\dagger \right), \quad \Omega' < m \leq \Omega.$$  \hspace{1cm} (7)

The expressions for $d_{2m}, d_{3m}, d_{2m}^\dagger, d_{3m}^\dagger$, are obtained by circular permutation of the indices 1, 2, 3. These operators characterize the so-called Bogoliubov quasi-particles. The transformation in eq. (7) is not canonical, since $(d_{im}, d_{jm}^\dagger) \neq \delta_{ij}$, but the corresponding canonical transformation, which is not needed for the present purpose, may be easily obtained.\footnote{A generalized Bogoliubov transformation has been proposed in eq. (25) of [13]. However, the associated BCS state does not satisfy (2). The transformation (7) replaces eq. (25) of [13].}

We introduce the notation

$$\langle W \rangle = \langle \Phi | W |\Phi \rangle / \langle \Phi |\Phi \rangle.$$

We observe that the contractions $\langle c_{im}^\dagger c_{jm} \rangle, i \neq j$, are independent of $i, j$. Similarly the contractions $\langle c_{im}^\dagger c_{jm} \rangle$, are independent of $j$. We easily find, for $0 < m \leq \Omega'$,

$$\langle c_{im}^\dagger c_{jm} \rangle = -\frac{K^2}{1 + 3K^2}, \quad i \neq j,$$

$$\langle c_{im}^\dagger c_{jm} \rangle = \frac{2K^2}{1 + 3K^2}, \quad i \neq j.$$

On the other hand, for $\Omega' < m \leq \Omega$,

$$\langle c_{im}^\dagger c_{jm} \rangle = \frac{K^2}{1 + 3K^2}, \quad i \neq j,$$

$$\langle c_{im}^\dagger c_{jm} \rangle = \frac{1 - 2K^2}{1 + 3K^2}.$$  \hspace{1cm} (8)

Then, we obtain

$$\langle S_{ij} \rangle = -2\Omega' \frac{K^2}{1 + 3K^2} + 2(\Omega - \Omega') \frac{\tilde{K}^2}{1 + 3K^2}, \quad i \neq j.$$  \hspace{1cm} (9)

By conveniently choosing $K, \tilde{K}$, we may insure that the condition (2) for the BCS vacuum $|\Phi\rangle$ to be a color singlet, is satisfied. The state $|\Phi\rangle$ satisfies automatically (1). However, (1) remains valid when we replace $|\Phi\rangle$ by $U_c|\Phi\rangle$, for an arbitrary color rotation $U_c$, only if (2) is further imposed.

Next we compute the contractions $\langle c_{2m}^\dagger c_{1m} \rangle = \langle c_{3m} c_{2m} \rangle = \langle c_{1m} c_{3m} \rangle = \langle c_{2m} c_{1m} \rangle = \langle c_{3m} c_{2m} \rangle = \langle c_{1m} c_{3m} \rangle =: D_m$, where $D_m$ is real. We find

$$D_m = \frac{K}{1 + 3K^2}, \quad 0 < m \leq \Omega';$$

$$D_m = \frac{\tilde{K}}{1 + 3K^2}, \quad \Omega' < m \leq \Omega.$$  \hspace{1cm} (10)

We are now able to compute the energy expectation value

$$\mathcal{E} = \frac{1}{\Omega} \sum_{j=1}^{3} \frac{\langle \Phi | A_j^\dagger A_j |\Phi \rangle}{\langle \Phi |\Phi \rangle}.$$

Let

$$p = \frac{\Omega'}{\Omega}, \quad q = 1 - p = \frac{\Omega - \Omega'}{\Omega}.$$
Define $\theta, \hat{\theta}$ such that
\[
\frac{\sqrt{3}K}{\sqrt{1+3K^2}} = \sin \theta, \quad \frac{1}{\sqrt{1+3K^2}} = \cos \theta, \\
\frac{\sqrt{3}K}{\sqrt{1+3K^2}} = \sin \hat{\theta}, \quad \frac{1}{\sqrt{1+3K^2}} = \cos \hat{\theta}.
\]

Then, the color symmetry constraint reads
\[
p \cos 2\theta - q \cos 2\hat{\theta} = p - q.
\]

The main contribution to the energy expectation value comes from the square of the expectation values
\[
\langle A_1 \rangle = \langle A_2 \rangle = \langle A_3 \rangle = \frac{\Omega}{\sqrt{3}} \left( p \sin 2\theta + q \sin 2\hat{\theta} \right),
\]
which involve contractions of the form $\langle cc \rangle$. The corresponding constrained extremum occurs for
\[
\cos 2\theta = -\cos 2\hat{\theta} = p - q, \\
\sin 2\theta = \sin 2\hat{\theta} = \sqrt{1 - (p - q)^2},
\]
so that, in the leading order,
\[
\frac{\mathcal{E}}{G} \approx \Omega^2 (1 - (p - q)^2).
\]

In terms of the variables $\theta, \hat{\theta}$, the number of quarks reads
\[
N = 6\Omega \left[ p(1 - \cos 2\theta)/3 + q \left( 1 - \left( 1 - \cos 2\hat{\theta} \right)/3 \right) \right].
\]

At the extremum, $N = 6\Omega q$, and
\[
\frac{\mathcal{E}}{G} \approx \Omega^2 [1 - (1 - N/(3\Omega))^2].
\]

To complete the calculation of $\mathcal{E}/G$ we must add the small corrections coming from the neglected contractions of the form $\langle c^\dagger c \rangle$.

In terms of $\theta, \hat{\theta}$, we have
\[
\langle c_{jm}^\dagger c_{jm} \rangle = \langle c_{j\bar{m}}^\dagger c_{j\bar{m}} \rangle = \frac{1}{3} (1 - \cos 2\theta),
\]
for $0 < m \leq \Omega$, and
\[
\langle c_{jm}^\dagger c_{j\bar{m}} \rangle = \langle c_{j\bar{m}}^\dagger c_{jm} \rangle = 1 - \frac{1}{3} (1 - \cos 2\hat{\theta}),
\]
for $\Omega' < m \leq \Omega$. At the extremum,
\[
\langle c_{jm}^\dagger c_{j\bar{m}} \rangle = \langle c_{j\bar{m}}^\dagger c_{jm} \rangle = \frac{2}{3} q,
\]
for $0 < m \leq \Omega'$, and
\[
\langle c_{jm}^\dagger c_{j\bar{m}} \rangle = \langle c_{j\bar{m}}^\dagger c_{jm} \rangle = \frac{1}{3} (1 + 2q),
\]
for $\Omega' < m \leq \Omega$. Finally, the ground-state energy of the color symmetrical superconducting phase reads
\[
\frac{\mathcal{E}}{G} = \frac{N}{9} \left( 6\Omega - N + 1 + \frac{4N}{3\Omega} \right), \quad 0 \leq N \leq 6\Omega. \tag{8}
\]

Fig. 1: Ground-state energy of the color symmetrical sector vs. the quark number, for $\Omega = 6$. Thick line: exact result according to (9); thin line: color symmetrical BCS estimate according to (8). Since $G < 0$, the upper curve lies below in energy, in agreement with Ritz theorem.

Although eq. (8) is close to eq. (29) of ref. [13], showing the same qualitative behavior, this agreement is, to some extent, accidental\(^3\).

Using the Schwinger-type representation of $SU(4)$, formulated in terms of appropriate boson operators, which was developed by Yamamura et al. [14], the color symmetrical sector of the Bonn model has been characterized in [15]. There, the exact ground-state energy of the color symmetrical sector was found to read
\[
\frac{\mathcal{E}}{G} = \frac{N}{3} \left( 2\Omega + 3 - \frac{N}{3} \right), \quad 0 \leq N \leq 6\Omega. \tag{9}
\]

It is interesting to compare eqs. (8) and (9). This is done, in fig. 1.

**Conclusions.** – We have constructed a BCS-type formalism, based on a conveniently generalized Bogoliubov transformation, which is appropriate to describe color symmetrical superconducting states of quark matter. It is found that the ground-state energy of the color symmetrical sector of the Bonn model is well approximated by the average energy of the color symmetrical superconducting state proposed here and it is easily seen that the color symmetric BCS result becomes closer to the exact one while $\Omega$ increases.

It should be emphasized that the present approach automatically ensures vanishing average net color charge of $|\Phi\rangle$, even if (2) is not imposed. Condition (2) is important because most effective models contain sectors which belong to other color $SU(3)$ representations than the color singlet one. Such sectors are therefore unphysical and should be discarded. According to the procedure here

\[^3\text{It is due to an accidental error that the curve labeled “color neutral BCS estimate” in fig. 2 of [13] shows a very similar behavior to the corresponding curve in fig. 1. This similarity is extremely misleading because eq. (25) of [13] describes an unacceptable BCS transformation which actually leads to a vanishing gap. See appendix.}\]
described, (1) is implemented with the help of a single chemical potential, which is the same for all colors, while in [6,10], condition (1) is implemented with the help of different chemical potentials for different colors. We also show that in order to implement (2), which is rather simple, only one extra Lagrange multiplier is needed.

The two-flavor superconductivity phase is correctly described by our formalism. The existing treatments either describe an unphysical phase which is not compatible with color singletness, or, at most provide a deficient treatment of the true physical phase. A reference is in order to the mechanism of color-flavor-locking (CFL) which was introduced in [1] and does lead to color symmetric superconductivity, in the average. In CFL the color and flavor degrees of freedom are locked according to the scheme \((ud-br), (ds-rg), (su-gb)\). Our approach allows for an unlocked treatment of \(u, d, s\) from \(b, r, g\). A convenient extension of our ansatz is appropriate to describe a phase with unlocked color and flavor indices, which is both a color and flavor singlet, so that the number of quarks is the same, for each color and flavor. Alternatively, color symmetric but flavor asymmetric phases may also be considered.

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The authors are grateful to Prof. V. Zelevinsky and to the Referees for valuable comments. One of the authors (JP) wishes to acknowledge most valuable discussions with Prof. M. Yamamura. The present research was partially supported by Projects CERN/FP/109316/2009 and PTDC/FIS/64707/2006.

APPENDIX

It happens that eq. (25) of [13] is not suitable to investigate the color symmetric sector of the Hamiltonian in (3). However, it is suitable to study the related Hamiltonian

\[ H' = \mathcal{G} \sum_{j=1}^{3} B_{j}^{1} B_{j}, \]

where

\[ B_{j}^{1} = \sum_{m>0} (c_{jm}^{\dagger} c_{3m}^{\dagger} - c_{2m}^{\dagger} c_{3m}^{\dagger}), \]

the expressions for \(B_{j}^{1}, B_{j}^{2}\), being obtained by circular permutation of the indices 1, 2, 3. The curve labeled “color neutral BCS estimate” in fig. 2 of [13], is based on eq. (25) of [13]. It would be correct for \(H'\) but is wrong for the Hamiltonian in (3).

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