Coulomb coupling effects in the gigahertz complex admittance of a quantum $R-L$ circuit

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Keywords: low-dimensional, quantum transport, quantum phase transitions in quantum Hall effects

Abstract

We report on the gigahertz admittance measurements of a quantum conductor, i.e. a quantum $R-L$ circuit, to probe the intrinsic dynamic of the conductor. The magnetic field dependence of the admittance phase provides us with an effective way to study the role of Coulomb interaction between counterpropagating edge channels. In addition, there is a small jump in the admittance phase when the transmitted modes are changed. This is because the gate voltage leads to a static potential shift of the quantum channel, then a quantum capacitance related to the density of states of the edge channels are influenced. Our study has made new discoveries of the dynamic transport in a quantum conductor, finding evidence for the deviations from quantum chiral transport associated with Coulomb interactions.

1. Introduction

The dc quantum transport properties of a mesoscopic phase-coherent conductor was well studied both theoretically and experimentally [1, 2]. This coherent dc transport is nonlocal over the electronic coherence length. Thus, the conductance can be expressed in terms of the transmission probabilities of channels. And normally, the effects of Coulomb interaction can be disregarded for dc quantum transport. Recently, time dependent quantum transport is gaining considerable attention from the recent advances in quantum electron optics and quantum information processing [3–9]. The theoretical study of the dynamical properties of time dependent transport in mesoscopic phase-coherent conductor has been vastly done [10–14]. And a manifestation of quantum coherence has been investigated experimentally at gigahertz frequency and milliKelvin temperatures [15, 16]. One major concept of time dependent transport is the charge relaxation resistance (or Büttiker’s resistance) in a mesoscopic capacitor, i.e. a quantum $R-C$ circuit. In a single electronic mode, the Büttiker’s resistance is equal to the half-resistance quantum $h/2e^2$, which differs from the standard dc Landauer resistance [11–18]. For mesoscopic capacitor device, Nigg et al and Zohar et al performed a theoretical investigation on the Coulomb interactions [19–21]. In these works, Coulomb interactions are treated within the Hartree–Fock approximation. As a result, the dot density of states is modified by the Coulomb interaction. And the quantum capacitance is still periodic along with the density of states. Another essential investigation related to time dependent transport is the quantum chirality effects in a quantum conductor, i.e. the quantum $R-L$ circuit [22, 23]. The main result presented in [22, 23] is that the relaxation time of the quantum $R-L$ circuit is resistance independent. In such one-dimensional electronic systems, Coulomb interactions plays a central role. Their effects have been confirmed experimentally. For example, decoherence [24] and energy relaxation [25] are induced by the Coulomb interaction between copropagating edge channels. These studies show that a quantum $R-Cor R-L$ circuit displays discrepancies with its classical counterpart [15, 16, 22, 23], and the Coulomb interaction plays a significant role in contrast to that of dc case.

The purpose of this paper is to study the time dependent electronic transport more specifically by investigating the effects of magnetic field and gate voltage on the gigahertz admittance of a quantum $R-L$ circuit,
focusing on the role of Coulomb coupling effects. The phase of the admittance is transmission-independent when the magnetic fields are low. This behavior is non-classical and is purely due to chirality, which provides us with an effective way to directly probe the intrinsic time scales of the conductor. We also observed a superlinear transmission dependence of the admittance phase as soon as the magnetic fields are high enough, which is a clear evidence of deviations from quantum chiral transport. The magnetic field effects can be well described by a current and charge conserving scattering theory, which takes into account both counterpropagating edge coupling and coupling to side gates. In addition, it is of particular interest to investigate how the density of states of the edge channels affects the dynamic transport properties of the quantum \( R-L \) circuit. This is because the electrostatic potential of each channel is not determined by the geometry of the edge channel arrangement alone but contains quantum corrections due to the finite density of states of the edge channels \([10]\). In the experiment, there is a small jump in the admittance phase when the transmitted modes of the quantum point contacts (QPC) are changed. The effect of gate voltage \( V_g \) to linearly change the static potential of the edge channels by a capacitive coupling from the QPC gate to the channel is responsible for the phase jump. A proper account of such capacitive coupling would modify the density of states of edge channels, and hence the quantum capacitance. The new result of phase jump has opened the way to get information on the density of states of the edge channels, which will affect the dynamic transport properties of the electrons dramatically. Our study demonstrates that the role of Coulomb interaction is crucial for electron dynamics in realistic one-dimensional systems. It is a central issue in developing our understanding of electronic transport in quantum conductors.

2. Experimental setup

As shown in figure 1 (a), the sample is realized by using a two-dimensional electron gas (2DEG) of a GaAs/AlGaAs heterostructure. The 2DEG density and mobility are \( 1.3 \times 10^{11} \) cm\(^{-2} \) and \( 2 \times 10^6 \) cm\(^2\) V\(^{-1}\) s\(^{-1} \), respectively. A QPC is defined in the 2DEG mesa using a split gate technique which has a length of about 7 \( \mu \)m and a width of 300 nm. The resistor is a QPC linking two wide 2DEG reservoirs. By varying voltage applied on the split gate, the number of transmitted channels can be controlled. Two Ohmic contacts (contact 1 and 2) provide electrical connection for electron reservoirs on both sides of the constriction. A time dependent voltage is applied to one of the Ohmic contact electrodes. Our results are obtained by measuring the transmission...
3. Experimental results and discussions

3.1. Magnetic field and gate voltage dependence of the admittance phase in the experiment

Figures 2(a) and (b) show the real (Re(G)) and imaginary (Im(G)) parts of the admittance as a function of the gate voltage at $B = 0.1$ T and $B = 0.7$ T, respectively ($P = -80$ dBm, $f = 2.2$ GHz). Here, a perpendicular magnetic field is applied to the sample so as to measure the conductance of a QPC in quantum Hall regime, where the electrons propagate ballistically along the chiral edge channels. Note that Im(G) < 0 demonstrates an inductive contribution of the sample. Both the real and imaginary parts of the complex admittance exhibit steps when gate voltage is varied. Here, calibration is performed by assigning a purely capacitive admittance to the sample at the pinchoff. And then we can assign the value $2e^2/h$ to the Re(G) steps after the calibration. For the imaginary part shown in figures 2(a) and (b), the height is no longer equal to $2e^2/h$. There are three noteworthy phenomena presented in figures 2(a) and (b). First, the number of the quantized steps decreases for $B = 0.7$ T comparing to that of $B = 0.1$ T. This is because of the relatively large filling factor at $B = 0.1$ T, allowing the QPC to control the transmission of a relatively large number of edge states. Let N be the number of filled Landau levels which can be calculated by $n_e h/(eB)$. Here, the 2DEG density is $n_e = 1.3 \times 10^{11}$ cm$^{-2}$ for our device. For $B = 0.7$ T, then $n_e h/(eB) = 7.6$, indicating that seven Landau levels are fully occupied and the eighth one is partially occupied. Thus, $N = 7$ for $B = 0.7$ T. The $V_g$ is scanned between gate voltage of $-0.55$ and $-0.25$ V and only three steps is shown in figure 2(b). With positively sweeping the voltage, the number of seven steps should be observed.

Similarly, for $B = 0.1$ T, the number of fully occupied landau levels is $N = 53$. Then, the length of quantized plateaus increases when the magnetic field is equal to 0.7 T in figure 2(b). And the increase in the energy level spacing between two edge states induced by the magnetic field increasing is responsible for the quantized plateau lengthening. Also, the real part of the admittance is observed to be quantized at Re(G) = $n(2e^2/h)$ for $B = 0.1$ T ($n$ is a fixed integer), while the quantized plateau at Re(G) = $2.5(2e^2/h)$ appears for $B = 0.7$ T. It is because the spin degeneracy is lifted at a relatively high magnetic field, and a single spin-polarized channel yields the quantized plateau at Re(G) = $2.5(2e^2/h)$ for $B = 0.7$ T.

Figure 2(c) shows the Im(G)/Re(G) as a function of gate voltage $V_g$ and magnetic field $B$ ($P = -80$ dBm, $f = 2.2$ GHz). In figure 2(c), the phase of the admittance is transmission-independent when the magnetic fields are below about 0.2 T. Meanwhile, as marked by the red arrow in figure 2(c), the color changes obviously along the $V_g$ axis, indicating that the admittance phase is transmission-dependent when the magnetic field are relatively high. In order to show the deviations more clearly, the Nyquist representation of the admittance (imaginary part versus real part) at high magnetic field is also shown in the insert of figure 2(b). It is obvious that a superlinear transmission dependence of the admittance phase at high magnetic field is shown.

Figures 3(a) and (b) show the Nyquist representation of the admittance for different magnetic fields and frequency, respectively. In figure (3), the absolute values of the phase increase when the magnetic field and frequency increase. In addition, there is a small jump in the admittance phase when the transmitted modes are changed (marked by red arrows in figure 3). And this jump behavior is robust to the changing of magnetic field and frequency. The effects of magnetic field and frequency on the admittance and the generation mechanism for admittance phase jump will be given in the next part.

3.2. Theoretical modeling and discussions

In a time dependent situation, a voltage oscillation $V_\alpha(t)$ is applied at a contact $\beta$ of a sample. And the ac current response $I_\alpha(t)$ flowing from contact $\alpha$ is the difference between $I_{\alpha,+}$ and $I_{\alpha,-}$ [22],

$$I_{\alpha,+} = \sum_k \Delta_{k_0} h_{k_+} = \frac{e^2}{h} \sum_k \Delta_{k_0} V_\alpha,$$

$$I_{\alpha,-} = \sum_k \Delta_{k_0} h_{k_-} = \sum_k \Delta_{k_0} (h_{k_+} - \partial t Q_\alpha).$$

(1)

(2)
As shown in figure 1(a), $\alpha$ and $\beta$ are the current contacts labeled by 2 and 1, respectively. $I_{\alpha+}$ and $I_{\alpha-}$ is the current carried by the outgoing and incoming channels, respectively. The notation $\Delta_{k\alpha} = 1$ indicates that the edge channel $k$ emerges from contact $\alpha$ (0 otherwise), while $\Delta_{nk} = 1$ indicates the channel $k$ injects to contact $\alpha$ (0 otherwise).

Figure 1(b) shows the schematic diagram of a quantum R–L circuit with $n$ fully transmitted channels and one partially transmitted channel (transmission $T$). Here, the number of edge channels is equal to the filling factor $N$. Therefore, the number of totally reflected channel is $(N - 1 - n)$. For our experiment, the time dependent voltage is applied to contact 1 ($V_2 = 0$). Thus, the current $I_2$ is given by

![Figure 2](image-url)

**Figure 2.** Experiment results of the real part Re($G$) (black curve) and imaginary part Im($G$) (red curve) of the ac admittance as a function of the gate voltage $V_g$ for (a) $B = 0.1$ T and (b) $B = 0.7$ T. The insert in figure 2(a) shows the Nyquist representation of the admittance at $B = 0.1$ T (black dot). The green line is a linear fit of the data. And the insert in figure 2(b) shows the Nyquist representation of the admittance at high magnetic field. (c) Im($G$)/Re($G$) on a gray scale as a function of magnetic field $B$ and gate voltage $V_g$. The measurements are taken at $P = -80$ dBm, $f = 2.2$ GHz.
In order to describe the admittance $G_{21}(\omega) = \frac{I_2}{V_1}$, the charges $Q$ accumulated in the channels which inject to contact 2 can be obtained through

$$Q = (n + T)\left[\frac{D_0}{2}(V_1 - U_1) + \frac{D_0}{2}(V_1 - U_3)\right] + (N - n - T)\left[\frac{D_0}{2}(V_2 - U_2) + \frac{D_0}{2}(V_3 - U_4)\right].$$

The first term of equation (4) represents the charge accumulated in the channels which emerge from contact 1 and inject to contact 2. And the second term is that of emerging from contact 2 and inject to contact 2. Here, we emphasize that the edge states on the left upper side of the sample experience the same electrostatic potential labeled $U_1$ as a result of the strong interchannel interaction [23]. In a similar way, $U_2, U_3, \text{ and } U_4$ are labeled for those of the left lower, right upper and right lower side, respectively. First, let us consider a simple case that only $V_1$ changes with time $t$ in equation (4).

The admittance is

$$G_{21}(\omega) = \frac{I_2}{V_1} = \frac{e^2}{\hbar}(n + T)V_1 - \frac{\partial Q}{\partial t},$$

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The phase of the ac conductance is

$$\tan \phi = \frac{\text{Im}(G_{21})}{\text{Re}(G_{21})} = \frac{\omega l}{v_D \tau} = \omega \tau.$$
upper and lower edge channels are capacitatively coupled to a grounded metallic gate with equal capacitance $C_g$. Counterpropagating edge channels are coupled by the mutual capacitance $C_{gh}$. Considering $C_g$ and $C_{gh}$, we get

\begin{align}
Q_1 &= N \frac{D_0}{2} (V_1 - U_1) = C_g U_1 + C_{gh} (U_1 - U_2), \\
Q_2 &= (n + T) \frac{D_0}{2} (V_2 - U_2) + (N - n - T) \frac{D_0}{2} (V_1 - U_2) \\
&= C_g U_2 + C_{gh} (U_2 - U_1), \\
Q_3 &= (n + T) \frac{D_0}{2} (V_1 - U_3) + (N - n - T) \frac{D_0}{2} (V_2 - U_3) \\
&= C_g U_3 + C_{gh} (U_3 - U_4), \\
Q_4 &= N \frac{D_0}{2} (V_2 - U_4) = C_g U_4 + C_{gh} (U_4 - U_3).
\end{align}

Substituting $U_1$, $U_2$, $U_3$ and $U_4$ into equation (4), we obtain the $Q$, then the time dependent conductance response is described as

$$G_{21}(\omega) = G_{21}^{(dc)} - i\omega E_{21} = \frac{e^2(n + T)}{h} - i\omega \left[ \frac{(n + T)D_0 C_g}{ND_0 + C_g} + \frac{(n + T)^2 D_0^2 C_{gh}}{(ND_0 + C_g + 2C_{gh})(ND_0 + C_g)} \right].$$

In equation (9), $G_{21}^{(dc)}$ is the Landauer dc conductance $e^2(n + T)/h$. The emittance $E_{21}$ is positive, indicating an inductive behavior of the sample. From equation (9), the absolute values of the phase increase when the frequency increase, which is the case in our experiment. For $C_{gh} \gg C_g$, the value of the inductance $L$ does not vary when the resistance $R$ is modified, corresponding to a behavior of a classical $R$-$L$ circuit. For $C_g \gg C_{gh}$, we get $E_{21} \approx (n + T)/N|C_{gh}|$ and $L = |N/(n + T)| C_{gh}/(Ne^2/h)^2$. Here, $C_{gh} = D C_{gh}/(D + C_g)$ is the electrochemical capacitance to the gate which consists of the quantum capacitance $D$ in series with the geometrical capacitance $C_g$. Here, we assume equal density of states on each channels: $D = ND_0$. In this case, the phase of the ac conductance $\tan \varphi = \text{Im}(G_{21})/\text{Re}(G_{21}) = -\omega C_{gh}/(Ne^2/h)$ is resistance independent and only depends on the number of edge channels $N$. This transmission-independent phase allows us to determine the electronic transit time directly, which differs observably from the time constant of $L/R$ in the classical circuit [22]. The insert of figure 2(a) shows the Nyquist representation of the admittance at $B = 0.1$ T. The transit time of charges through the channel can be estimated from the constant phase observed in the Nyquist representation: $\tan \varphi = -\omega \tau_e$. We define a charge velocity $v_p = 1/\tau_e$ by considering the Coulomb interaction. We can get $\tau_e = 1/v_p = C_{gh}/(Ne^2/h) = 1/\{V_0 + Ne^2/(hC_g)\}$. Here, the geometrical capacitance per unit of channel length can be expressed as $C_g = C_{gh}/l$. The charge velocity $v_p$ can be obtained from $\tau_e = 0.007$ ns extracted from a linear fit. The channel length for our QPC device is about 7 $\mu$m. The charge velocity $v_p$ extracted from our measurements is in agreement with the result in [27], while larger than that of [23]. There are two reasons for this difference: first, the charge velocity is inversely proportional to the magnetic field. And there is a relatively large magnetic field in [23]. Second, a larger value of $C_g$ leads to a smaller charge velocity $v_p$ in [23].

In our experiment, both counterpropagating edge coupling and coupling to side gates need to be considered. As shown in equation (9), the first term of $E_{21}$ predicts a constant phase of the complex admittance. The second term of $E_{21}$ can be expressed as $[(n + T)^2/Ne^2](ND_0 C_g) \approx [(N^2D_0 + C_g + 2C_{gh})(ND_0 + C_g)], which is the reason for the superlinear transmission dependence of the admittance phase at high magnetic fields in our experiment. Note that $V_0 \propto 1/B$, $D_0 \propto B$, so that $ND_0$ do not depend on $N$. In addition, $C_{ph} = ND_0 C_{gh}/(ND_0 + 2C_{gh})$ and $C_{gh} = ND_0 C_{gh}/(ND_0 + C_g)$ do not depend on $N$, either [22]. And $1/N^2$ in the second term of $E_{21}$ is the only factor dependent on the magnetic field. Therefore, the phase of the admittance is transmission-dependent at relatively high magnetic fields because the second term of $E_{21}$, has a relatively obvious effect on the admittance phase. And a superlinear transmission dependence of the admittance phase at high magnetic field shown in figure 2 is the clear evidence of the interedge coupling.

Now, we will discuss the generation mechanism for admittance phase jump presented in figure 3, which cannot be explained by equation (9). The theoretical description shown above disregards an important effect of gate voltage in electronic transport. Let us consider the general case that the conductor is uniform in the $x$-direction and has a transverse confining potential $U(y)$. And a non-zero magnetic field is applied perpendicular to the 2DEG plane ($z$ direction). Using lowest order perturbation theory to include the effect of the confining potential, we can obtain the energy eigenvalues $E(n, k)$ [28]

$$E(n, k) = E_0 + (n + \frac{1}{2}) \hbar \omega_c + U(y_k).$$
Here, the relation between the coordinate $y$ and the wavenumber $k$ can be expressed as $y_k = \hbar k/(eB)$. $E_r$ indicates the band-edge energy of the conductor. $\omega_c$ is the cyclotron frequency. The energy dispersion relation of $E(n, k)$ versus $k$ for an edge state is not known and depends on the shape of the confinement potential $U(y_k)$, which varies in the direction perpendicular to the edge channel and plays the role of the kinetic energy.

The theoretical works of Christen et al and Aronov et al showed that with the opening of channels the emittance decreased in a steplike manner in synchronism with the conductance steps [29, 30]. The droop of the emittance along conductance steps predicted in [29, 30] differs from the phase jump observed in the Nyquist representation of the admittance. The gate voltage $V_g$ has two effects on the edge channels. First and most apparently, it changes the QPC transmission $T$, which is the reason for the emittance droop in [29, 30]. Then, the gate voltage $V_g$ also leads to a shift of the static potential $U_{dc}$ of the edge channel [15], which is the reason for the phase jump observed in our experiment. Such effect of $V_g$ is not considered in equation (10). Because there is a non-negligible capacitive coupling between the QPC gates and the edge channel, changing $V_g$ would shift the position of the energy levels in the channel with a lever arm $\gamma$ given by $U_{dc} = \gamma V_g$, eventually causing a shift on the kinetic energy of electrons. At that point, the electron kinetic energy can be described as

$$U(y_k) = E(n, k) - E_r = (n + \frac{1}{2}) \hbar \omega_c - e\gamma V_g.$$  

(11)

In fact, parabolic potential often provides a good description of narrow quantum wires which allows us to obtain the dispersion relation. And the kinetic energy of electrons in equation (10) can be described as $\hbar^2 k^2/(2M)$ when we assume a parabolic confining potential. Here, $M$ is the effective mass with a non-zero magnetic field. Considering a static potential shifting $U_{dc}$ induced by $V_g$, the following equation can be obtained in the case of a parabolic confining potential,

$$\frac{1}{2} M v_D^2 = \frac{\hbar^2 k^2}{2M} - e\gamma V_g.$$  

(12)

As can be clearly seen from equation (12), the drift velocity $v_D$ is changed due to the static potential shifting $\gamma V_g$ induced by the capacitive coupling from the QPC gate to the channel. And then the density of states of the edge channels ($\nu = 1/\hbar v_D$) is changed. Hence, a quantum capacitance ($D_{g} = e^2 \nu$) variation can be achieved attributed to the change of the density of state. The phase jump corresponds to the modulation of the density of states of edge channels when the static potential $U_{dc}$ is varied. At low temperature, the conductance is determined entirely by electrons with energy close to the Fermi energy. From this point of view, the current is carried by a small fraction of the total electrons which move with the Fermi velocity. Thus, the wavenumber $k$ of such electrons in equation (12) is referred as the Fermi wavenumber ($k_f$), which can be expressed in terms of the electron density $n$: $k_f = \sqrt{2\pi n_i}$. And $y_k$ plays the role of the Fermi momentum. Thus the drift velocity $v_D$ in equation (9) can be replaced by the Fermi velocity $v_F$. From equation (12), the Fermi velocity $v_F$ can be obtained as a function of $\gamma$. As the $\gamma$ is increased, the Fermi velocity gets smaller. By tuning the static potential $U_{dc}$, the energy of the bottom of the subband can be shifted. Theoretically, the $\gamma$ can be increased to a maximum as the bottom of the subband is above the Fermi level, in which state there is no net current flows along the channel. Substituting equation (12) into equation (9), a quantitative description of the gate voltage effects on admittance can be performed. The theoretical result of the Nyquist representation at different values of $\gamma$ is shown in figure 4(a). The admittance phase jump becomes more and more apparent when $\gamma$ is increased from 0 to 0.002, while the jump disappears for the case of $\gamma = 0$. Figure 4(b) (red dot line) shows the experimental result measured at $P = -80$ dBm, $f = 2.2$ GHz, and $B = 0.1$ T. The Nyquist representation of the conductance for such conditions can be simulated by the theory mentioned above also. The theoretical result can be quantitatively compared with our experimental data with three adjustable parameters, $C_p$, $C_0$, and $\gamma$. The method for determining the value of $C_p$, $C_0$, and $\gamma$: the measured transmission-independent phase at low magnetic fields allows us to get the value of $C_p$ since the first term of $E_{12}$ (see equation (9)) predicts a constant phase of the complex admittance. And the superlinear transmission dependence of the admittance phase at high magnetic field can be achieved by adjusting the values of $C_0$. Finally, by appropriately adjusting the value of $\gamma$, the simulated result is in agreement with the experimental result. Here, $\gamma$ is equal to 0.002 235 for our experiment. The $\gamma$ describes the coupling strength between the QPC gate to the channel, which can be influenced by the QPC configuration.

4. Conclusions

In conclusion, our work addresses specifically electronic transport along the chiral edge channels of a quantum conductor in time dependent situations. The signature of quantum chiral transport is a transmission-independent admittance phase, given by the transit time of the electrons in the conductor. Our experiment results show clear deviations from quantum chiral transport associated with Coulomb interactions. First, we found clear evidence for Coulomb coupling between counterpropagating edge channels from the superlinear transmission dependence of the admittance phase at high magnetic field. In addition, an admittance phase jump...
is observed when the transmitted modes are changed. The density of state of edge channels changes when the capacitive coupling from the QPC gate to the channel is not negligible, causing the phase jumps in the experiment. A current and charge conserving scattering theory can give a quantitative description of the experiment behavior. Our observations clarified that Coulomb interaction plays a significant role in time dependent situations in contrast to that of the dc case. And the experiment has opened the way for obtaining information on the density of states of edge channels and elucidating interesting physics of high-frequency quantum transport in quantum conductor.

**Acknowledgments**

The authors would like to thank Professor D C Glattli for providing the devices and many fruitful discussions. The authors would like to thank the support of the Key Program of National Natural Science Foundation of China under Grant No. 11234009, National Key Research and Development Program of China under Grant No. 2016YFF0200403.

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