Analysis of the vector form factors $f^+_{K\pi}(Q^2)$ and $f^-_{K\pi}(Q^2)$ with light-cone QCD sum rules

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Abstract

In this article, we calculate the vector form factors $f^+_{K\pi}(Q^2)$ and $f^-_{K\pi}(Q^2)$ within the framework of the light-cone QCD sum rules approach. The numerical values of the $f^+_{K\pi}(Q^2)$ are compatible with the existing theoretical calculations, the central value of the $f^+_{K\pi}(0)$ ($f^+_{K\pi}(0) = 0.97$) is in excellent agreement with the values from the chiral perturbation theory and lattice QCD. The values of the $|f^-_{K\pi}(0)|$ are very large comparing with the theoretical calculations and experimental data, and can not give any reliable prediction. At large momentum transfers with $Q^2 > 5 GeV^2$, the form factors $f^+_{K\pi}(Q^2)$ and $|f^-_{K\pi}(Q^2)|$ can either take up the asymptotic behavior of $1/Q^2$ or decrease more quickly than $1/Q^2$, more experimental data are needed to select the ideal sum rules.

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Key Words: Vector form factor, CKM matrix element, light-cone QCD sum rules

1 Introduction

Semileptonic $K \to \pi \ell \nu$ ($K_{\ell 3}$) decays provide the most precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{us}|$ \(^1\). The experimental input parameters are the semileptonic decay widths and the vector form factors $f^+_{K\pi}(q^2)$ and $f^-_{K\pi}(q^2)$, which are necessary in calculating the phase space integrals. The main uncertainty in the quantity $|V_{us}f^+_{K\pi}(0)|$ comes from the unknown shape of the hadronic form factor $f^+_{K\pi}(q^2)$, which is measurable at $m_\tau^2 < q^2 < (m_K - m_\pi)^2$ in the $K_{\ell 3}$ decays or $(m_K + m_\pi)^2 < q^2 < m_\tau^2$ in the $\tau \to K\pi\nu$ decays. The experimental data can be fitted to the functions with either pole models or series expansions, however, systematic errors are introduced due to the different parameterizations. The conservation of the vector current implies $f^+_{K\pi}(0) = 1$ at zero momentum transfer \(^2\), another powerful theoretical constraint on the $f^+_{K\pi}(0)$ is provided by the $SU(3)$ symmetry of the light pseudoscalar mesons and the Ademollo-Gatto theorem \(^3\), the $SU(3)$ symmetry breaking effects $f^+_{K\pi}(0) - 1$ start at second order in $m_s - m_q$. Chiral perturbation theory (ChPT) provides a natural and powerful tool to take

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into account the $SU(3)$ symmetry breaking effects due to the masses of the light quarks, the $f_{K\pi}^+(q^2)$ is usually calculated by the ChPT. Presently, the comparison between theory and experiment, and among different experiments, is complicated by the uncertainties in the form factor $f_{K\pi}^+(q^2)$ with zero momentum transfer.

In this article, we calculate the values of the vector form factors $f_{K\pi}^+(Q^2)$ and $f_{K\pi}^-(Q^2)$ within the framework of the light-cone QCD sum rules approach. The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$ while the non-perturbative matrix elements are parameterized by the light-cone distribution amplitudes which classified according to their twists instead of the vacuum condensates. The non-perturbative parameters in the light-cone distribution amplitudes are calculated by the conventional QCD sum rules and the values are universal.

The article is arranged as: in Section 2, we derive the vector form factors $f_{K\pi}^+(Q^2)$ and $f_{K\pi}^-(Q^2)$ with the light-cone QCD sum rules approach; in Section 3, the numerical results and discussions; and in Section 4, conclusion.

### 2 Vector form factors $f_{K\pi}^+(Q^2)$ and $f_{K\pi}^-(Q^2)$ with light-cone QCD sum rules

In the following, we write down the definitions for the vector form factors $f_{K\pi}^+(q^2)$ and $f_{K\pi}^-(q^2)$,

$$\langle \pi(q+p) | J_\mu(0) | K(p) \rangle = 2 f_{K\pi}^+(q^2) p_\mu + \{ f_{K\pi}^+(q^2) - f_{K\pi}^-(q^2) \} q_\mu, \quad (1)$$

where the $J_\mu(x)$ is the vector current. We study the vector form factors $f_{K\pi}^+(q^2)$ and $f_{K\pi}^-(q^2)$ with the two-point correlation functions $\Pi^A_\mu(p,q)$ and $\Pi^B_\mu(p,q)$,

$$\Pi^A_\mu(p,q) = i \int d^4x e^{-i(q+p)\cdot x} \langle 0 | T \{ J_\mu(0) J_\mu^A(x) \} | \pi(p) \rangle, \quad (2)$$

$$\Pi^B_\mu(p,q) = i \int d^4x e^{-i(q+p)\cdot x} \langle 0 | T \{ J_\mu(0) J_\mu^B(x) \} | K(p) \rangle, \quad (3)$$

$$J_\mu^A(x) = \bar{s}(x) \gamma_\mu u(x),$$

$$J_\mu^B(x) = \bar{d}(x) \gamma_\mu s(x),$$

$$J_K(x) = \bar{d}(x) i\gamma_5 s(x),$$

$$J_\pi(x) = \bar{u}(x) i\gamma_5 d(x), \quad (4)$$

where the $J_K(x)$ and $J_\pi(x)$ interpolate the $K$ and $\pi$ mesons respectively, we choose the pseudoscalar currents to avoid the possible contaminations from the axial-vector mesons. The correlation functions $\Pi^{A(B)}_\mu(p,q)$ can be decomposed as

$$\Pi^{A(B)}_\mu(p,q) = \Pi^{A(B)}_\mu(q^2,(q+p)^2) p_\mu + \Pi^{A(B)}_q(q^2,(q+p)^2) q_\mu, \quad (5)$$

due to the Lorentz covariance. In this article, we derive the sum rules with the tensor structures $p_\mu$ and $q_\mu$ respectively.
According to the basic assumption of current-hadron duality in the QCD sum rules approach [7], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_K(x)$ and $J_\pi(x)$ into the correlation functions $\Pi^A_\mu$ and $\Pi^B_\mu$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the $K$ and $\pi$ mesons, the correlation functions $\Pi^A_\mu$ and $\Pi^B_\mu$ can be expressed in the following forms,

$$
\Pi^A_\mu(p, q) = \frac{2f_Km_K^2f_K^+(q^2)}{(m_d + m_s)\{m_K^2 - (q - p)^2\}}p_\mu \\
+ \frac{f_Km_K^2\{f_K^+(q^2) - f_K^-(q^2)\}}{(m_d + m_s)\{m_K^2 - (q - p)^2\}}q_\mu + \cdots , 
$$

(6)

$$
\Pi^B_\mu(p, q) = \frac{2f_\pi m_\pi^2f_\pi^+(q^2)}{(m_d + m_u)\{m_\pi^2 - (q - p)^2\}}p_\mu \\
+ \frac{f_\pi m_\pi^2\{f_\pi^+(q^2) - f_\pi^-(q^2)\}}{(m_d + m_u)\{m_\pi^2 - (q - p)^2\}}q_\mu + \cdots ,
$$

(7)

here we have not shown the contributions from the high resonances and continuum states explicitly, they are suppressed after the Borel transformation and subtraction.

We use the standard definitions for the weak decay constants $f_K$ and $f_\pi$,

$$
\langle 0|J_K(0)|K(q)\rangle = \frac{f_Km_K^2}{m_s + m_d} \\
\langle 0|J_\pi(0)|\pi(q)\rangle = \frac{f_\pi m_\pi^2}{m_u + m_d}.
$$

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi^A_\mu$ and $\Pi^B_\mu$ in perturbative QCD theory. The calculations are performed at the large space-like momentum regions $P^2 = -(q + p)^2 \gg 0$ and $Q^2 = -q^2 \gg 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the operator product expansion approach. We write down the propagator of a massive quark in the external gluon field in the Fock-Schwinger gauge firstly [8],

$$
\langle 0|T\{q_i(x_1)\bar{q}_j(x_2)\}|0\rangle = i \int \frac{d^4k}{(2\pi)^4} e^{-ik(x_1-x_2)} \\
\left\{ \frac{k^2 + m^2}{k^2 - m^2} \delta_{ij} - \int_0^1 dv g_s G_{ij}^{\mu\nu} (vx_1 + (1-v)x_2) \\
\left[ \frac{1}{2} \frac{k^2 + m^2}{k^2 - m^2} \sigma_{\mu\nu} - \frac{1}{k^2 - m^2} v(x_1 - x_2)\mu\gamma_\nu \right] \right\},
$$

(8)

where the $G_{\mu\nu}$ is the gluonic field strength, the $g_s$ denotes the strong coupling constant. Substituting the above $s$, $d$ quark propagators and the corresponding $\pi$,
$K$ mesons light-cone distribution amplitudes into the correlation functions $\Pi^A_\mu$ and $\Pi^B_\mu$ in Eqs. (2-3) and completing the integrals over the variables $x$ and $k$, finally we obtain the representations at the level of quark-gluon degrees of freedom,

$$
\Pi^A_\mu = -\frac{f_\pi m^2_\pi}{m_u + m_d} \int_0^1 du \frac{\nu \phi_\nu(u)}{m^2_u - (q + up)^2} - m_s f_\pi m^2_\pi \int_0^1 du \int_0^u dt \frac{uB(t)}{m^2_u - (q + up)^2} + \frac{1}{6 m_u + m_d} \int_0^1 du \phi_\nu(u) \left\{ 1 - \frac{d}{du} \frac{u}{m^2_u - (q + up)^2} + \frac{1}{2 m^2_u} \right\} + m_s f_\pi \int_0^1 du \left\{ \phi_\pi(u) \frac{m^2_u}{m^2_u - (q + up)^2} - \frac{2}{2 m^2_u} \right\} - f_\pi \int_0^1 dv \int_0^1 dv \alpha_g \int_0^{1-\alpha_g} d\alpha_g T(\alpha_d, \alpha_g, \alpha_u) \left\{ \frac{(1 + 2v)um^2_u}{m^2_u - (q + up)^2} - 2(1 - v) \frac{d}{du} \frac{1}{m^2_u - (q + up)^2} \right\} \bigg|_{u=(1-v)\alpha_g + \alpha_u} + 4 m_s f_\pi \int_0^1 dv \int_0^1 dv \alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\alpha_g} d\alpha u \Phi(1 - \alpha - \beta, \beta, \alpha) \frac{1}{m^2_u - (q + up)^2} \left\{ (1 - \alpha - \beta, \beta, \alpha) \right\} |_{u=(1-v)\alpha_g + \alpha_u} - 4 m_s f_\pi \int_0^1 dv \int_0^1 dv \alpha_g \int_0^{1-\alpha_g} d\alpha u \Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha) \frac{1}{m^2_u - (q + up)^2} \left\{ (1 - \alpha - \alpha_g, \alpha_g, \alpha) \right\} |_{u=(1-v)\alpha_g + \alpha_u} + m_s f_\pi \int_0^1 dv \int_0^1 dv \alpha_g \int_0^{1-\alpha_g} d\alpha u \Psi(\alpha_d, \alpha_g, \alpha_u) \frac{1}{m^2_u - (q + up)^2} \left\{ (1 - \alpha - \alpha_g, \alpha_g, \alpha) \right\} |_{u=(1-v)\alpha_g + \alpha_u}
$$

$$
\Pi^B_\mu = -\frac{f_K m^2_K}{m_u + m_s} \int_0^1 du \frac{\nu \phi_\nu(u)}{m^2_u - (q + up)^2} - m_d f_K m^2_K \int_0^1 du \int_0^u dt \frac{uB(t)}{m^2_u - (q + up)^2} + \frac{1}{6 m_u + m_s} \int_0^1 du \phi_\nu(u) \left\{ 1 - \frac{d}{du} \frac{u}{m^2_u - (q + up)^2} + \frac{2}{2 m^2_u} \right\} + m_d f_K \int_0^1 du \left\{ \phi_K(u) \frac{m^2_u}{m^2_u - (q + up)^2} - \frac{2}{2 m^2_u} \right\} - f_K \int_0^1 dv \int_0^1 dv \alpha_g \int_0^{1-\alpha_g} d\alpha_g T(\alpha_u, \alpha_g, \alpha_s) \left\{ \frac{(1 + 2v)um^2_u}{m^2_u - (q + up)^2} - 2(1 - v) \frac{d}{du} \frac{1}{m^2_u - (q + up)^2} \right\} \bigg|_{u=(1-v)\alpha_g + \alpha_s} + 4 m_d f_K m^4_K \int_0^1 dv \int_0^1 dv \alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\alpha_g} d\alpha u \Phi(1 - \alpha - \beta, \beta, \alpha) \frac{1}{m^2_u - (q + up)^2} \left\{ (1 - \alpha - \beta, \beta, \alpha) \right\} |_{u=(1-v)\alpha_g + \alpha_s} - 4 m_d f_K m^4_K \int_0^1 dv \int_0^1 dv \alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{1-\alpha_g} d\alpha u \Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha) \frac{1}{m^2_u - (q + up)^2} \left\{ (1 - \alpha - \alpha_g, \alpha_g, \alpha) \right\} |_{u=(1-v)\alpha_g + \alpha_s} + m_d f_K m^2_K \int_0^1 dv \int_0^1 dv \alpha_g \int_0^{1-\alpha_g} d\alpha_s \Psi(\alpha_u, \alpha_g, \alpha_s) \frac{1}{m^2_u - (q + up)^2} \left\{ (1 - \alpha - \alpha_g, \alpha_g, \alpha) \right\} |_{u=(1-v)\alpha_g + \alpha_s}
$$

(9)
The correlation functions $\Pi^A$ and $\Pi^B$ can be estimated with the QCD sum rules approach [5, 6, 8, 9, 10]. In this article, the parameters in the light-cone distribution amplitudes are scale dependent and meson can be obtained by simple substitution of the non-perturbative parameters. We have used the two-particle and three-particle $K\pi$ amplitudes [5, 6, 8, 9, 10], the explicit expressions of the $K\pi$ meson light-cone distribution amplitudes are presented in the appendix, the corresponding ones for the $\pi$ meson can be obtained by simple substitution of the non-perturbative parameters. The parameters in the light-cone distribution amplitudes are scale dependent and can be estimated with the QCD sum rules approach [5, 6, 8, 9, 10]. In this article, the energy scale $\mu$ is chosen to be $\mu = 1 GeV$.

We take the Borel transformation with respect to the variable $P^2 = -(q + p)^2$ for the correlation functions $\Pi^A$ and $\Pi^B$, and obtain the analytical expressions for those invariant functions. After matching with the hadronic representations below the thresholds, we obtain the following four sum rules for the form factors $f^+_K(q^2)$ and $f^-_{K\pi}(q^2)$.

\begin{align}
\Pi^A &= \frac{f_\pi m^2_{\pi\pi} }{m_u + m_d} \int_0^1 du \frac{\phi_p (u)}{m^2_u - (q + up)^2} - m_s f_\pi m^2_{\pi\pi} \int_0^1 du \int_0^u dt \frac{B(t)}{\{m^2_s - (q + up)^2\}^2} \\
&- \frac{1}{6} \frac{f_\pi m^2_{\pi\pi} }{m_u + m_d} \int_0^1 du \phi_\sigma (u) \frac{d}{du} \frac{1}{m^2_s - (q + up)^2} \\
&- f_3 m^2_{\pi\pi} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s T(\alpha_d, \alpha_g, \alpha_u) \frac{1 + 2v}{\{m^2_s - (q + up)^2\}^2} |_{u = (1 - v)\alpha_g + \alpha_u} \\
&+ 4m_s f_\pi m^4_{\pi\pi} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \frac{\Phi (1 - \alpha - \beta, \beta, \alpha)}{\{m^2_s - (q + up)^2\}^3} |_{u = (1 - v)\alpha_g + \alpha_u} \\
&- 4m_s f_\pi m^4_{\pi\pi} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_u \int_0^{\alpha_u} d\alpha \frac{\Phi (1 - \alpha - \alpha_g, \alpha_g, \alpha)}{\{m^2_s - (q + up)^2\}^3} |_{u = (1 - v)\alpha_g + \alpha_u} ,
\end{align}

\begin{align}
\Pi^B &= \frac{f_K m^2_K }{m_u + m_s} \int_0^1 du \frac{\phi_p (u)}{m^2_u - (q + up)^2} - m_d f_K m^2_K \int_0^1 du \int_0^u dt \frac{B(t)}{\{m^2_d - (q + up)^2\}^2} \\
&- \frac{1}{6} \frac{f_K m^2_K }{m_u + m_s} \int_0^1 du \phi_\sigma (u) \frac{d}{du} \frac{1}{m^2_d - (q + up)^2} \\
&- f_3 m^2_K \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s T(\alpha_u, \alpha_g, \alpha_s) \frac{1 + 2v}{\{m^2_d - (q + up)^2\}^2} |_{u = (1 - v)\alpha_g + \alpha_s} \\
&+ 4m_d f_K m^4_K \int_0^1 dv \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \frac{\Phi (1 - \alpha - \beta, \beta, \alpha)}{\{m^2_d - (q + up)^2\}^3} |_{u = (1 - v)\alpha_g + \alpha_s} \\
&- 4m_d f_K m^4_K \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \frac{\Phi (1 - \alpha - \alpha_g, \alpha_g, \alpha)}{\{m^2_d - (q + up)^2\}^3} |_{u = (1 - v)\alpha_g + \alpha_s} ,
\end{align}

where $\Phi = A_{||} + A_{\perp} - V_{||} - V_{\perp}$ and $\Psi = 2A_{\perp} - 2V_{\perp} - A_{||} + V_{\parallel}$. In calculation, we have used the two-particle and three-particle $K\pi$ mesons light-cone distribution amplitudes [5, 6, 8, 9, 10], the explicit expressions of the $K\pi$ meson light-cone distribution amplitudes are presented in the appendix, the corresponding ones for the $\pi$ meson can be obtained by simple substitution of the non-perturbative parameters.
\[
\begin{align*}
\frac{2f_Km_K^2}{m_d + m_s}f_K^+(q^2)e^{-\frac{m_K^2}{M^2}} &= \\
\frac{f_s m_s^2}{m_u + m_d} \int_{\Delta_A}^1 du \phi_p(u) e^{-DD} - m_s f \frac{m_s^2}{4u^2 M^4} \int_{\Delta_A}^1 du \int_0^u \frac{B(t)}{u M^2} e^{-DD} & \\
+ \frac{1}{6} \frac{f \pi m_s^2}{m_u + m_d} \int_{\Delta_A}^1 du \phi_g(u) \left( \left[ 1 - u \frac{d}{du} \right] \frac{1}{u} + \frac{2m_s^2}{u^2 M^2} \right) e^{-DD} & \\
+ m_s f \pi \int_{\Delta_A}^1 du \left\{ \frac{\phi_p(u)}{u} - \frac{m_s^2}{4u^3 M^4} A(u) \right\} e^{-DD} & \\
-f_3 \pi \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_u T(\alpha_d, \alpha_g, \alpha_u) \Theta(u - \Delta_A) & \\
\left\{ \frac{(1 + 2v)m_\pi^2}{u M^2} - 2(1 - v) \frac{d}{du} \left[ \frac{1}{u} \right] \right\} e^{-DD} & \bigg| u = (1 - v) \alpha_g + \alpha_u \\
+ 2m_s f \pi m_s^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\beta} d\alpha & \\
\Phi(1 - \alpha - \beta, \beta, \alpha) \Theta(u - \Delta_A) e^{-DD} & \bigg| u = (1 - v) \alpha_u + \alpha_u \\
- 2m_s f \pi m_s^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_u \int_0^{1-\alpha_g} d\alpha_u & \\
\Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha) \Theta(u - \Delta_A) e^{-DD} & \bigg| u = (1 - v) \alpha_g + \alpha_u \\
+ m_s f \pi m_s^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_u & \\
\Psi(\alpha_d, \alpha_g, \alpha_u) \Theta(u - \Delta_A) e^{-DD} & \bigg| u = (1 - v) \alpha_g + \alpha_u ,
\end{align*}
\]
\[
\frac{2f_{\pi}m_{\pi}^{2}}{m_{u} + m_{d}} f_{K}\left(q^{2}\right) e^{-\frac{m_{\pi}^{2}}{M^{2}}} \\
= \frac{f_{K}m_{K}^{2}}{m_{u} + m_{s}} \int_{\Delta B}^{1} du \phi_{\rho}(u) e^{-EE} - m_{d} f_{K}m_{K}^{2} \int_{\Delta B}^{1} du \int_{0}^{u} dt \frac{B(t)}{uM^{2}} e^{-EE} \\
+ \frac{1}{6} m_{u} + m_{s} \int_{\Delta B}^{1} du \phi_{\sigma}(u) \left\{ \left[ 1 - u \frac{d}{du} \right] \frac{1}{u} + \frac{2m_{d}^{2}}{u^{2}M^{2}} \right\} e^{-EE} \\
+ m_{d} f_{K} \int_{\Delta B}^{1} du \left\{ \frac{\phi_{K}(u)}{u} - \frac{m_{K}^{2}m_{d}^{2}A(u)}{4u^{3}M^{4}} \right\} e^{-EE} \\
- f_{3K} \int_{0}^{1} dv \int_{0}^{1} d\alpha_{g} \int_{0}^{1-\alpha_{g}} d\alpha_{s} T(\alpha_{u}, \alpha_{g}, \alpha_{s}) \Theta(u - \Delta B) \\
\left\{ \frac{(1 + 2v)m_{K}^{2}}{uM^{2}} - 2(1 - v) \frac{d}{du} \frac{1}{u} \right\} e^{-EE} \bigg|_{u=(1-v)\alpha_{g}+\alpha_{s}} \\
+ 2m_{d} f_{K}m_{K}^{4} \int_{0}^{1} dv \int_{0}^{1} d\alpha_{g} \int_{0}^{\alpha_{g}} d\beta \int_{0}^{1-\beta} d\alpha \\
\Phi(1 - \alpha - \beta, \beta, \alpha) \Theta(u - \Delta B) e^{-EE} \bigg|_{u=(1-v)\alpha_{g}} \\
- 2m_{d} f_{K}m_{K}^{4} \int_{0}^{1} dv \int_{0}^{1} d\alpha_{g} \int_{0}^{1-\alpha_{g}} d\alpha_{s} \int_{0}^{\alpha_{s}} d\alpha \\
\Phi(1 - \alpha - \alpha_{g}, \alpha_{g}, \alpha) \Theta(u - \Delta B) e^{-EE} \bigg|_{u=(1-v)\alpha_{g}+\alpha_{s}} \\
+ m_{d} f_{K}m_{K}^{2} \int_{0}^{1} dv \int_{0}^{1} d\alpha_{g} \int_{0}^{1-\alpha_{g}} d\alpha_{s} \\
\Theta(\alpha_{u}, \alpha_{g}, \alpha_{s}) \Theta(u - \Delta B) e^{-EE} \bigg|_{u=(1-v)\alpha_{g}+\alpha_{s}} \\n\tag{14}
\]
\[
\begin{align*}
&= \frac{f_K m_K^2}{m_d + m_s} \left\{ f_K^+ (q^2) - f_K^- (q^2) \right\} e^{-\frac{m_K^2}{M^2}} \\
&- \frac{f_K m_K^2}{m_u + m_d} \int_{\Delta_A}^1 du \phi_p(u) e^{-DD} - m_s f_K m_K^2 \int_{\Delta_A}^1 du \int_0^u dt B(t) e^{-DD} \\
&- \frac{1}{6} \frac{f_K m_K^2}{m_u + m_d} \int_{\Delta_A}^1 du \phi_p(u) \frac{d}{du} e^{-DD} \\
&- f_{3K} m_K^2 \int_0^1 dv \int_0^1 \int_{\alpha_g}^{1-\alpha_g} d\alpha d\beta \int_0^{1-\beta} d\alpha \\
&\Theta(u - \Delta_A) \left[ 1 + 2v \right] e^{-DD} \mid_{u = (1-v)\alpha_g + \alpha_u} \\
&\Phi(1 - \alpha - \beta, \beta, \alpha) \Theta(u - \Delta_A) e^{-DD} \mid_{1 - \alpha_g} \\
&\Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha) \Theta(u - \Delta_A) e^{-DD} \mid_{u = (1-v)\alpha_g + \alpha_u},
\end{align*}
\]

\[
\begin{align*}
&= \frac{f_K m_K^2}{m_u + m_d} \left\{ f_K^+ (q^2) - f_K^- (q^2) \right\} e^{-\frac{m_K^2}{M^2}} \\
&- \frac{f_K m_K^2}{m_u + m_s} \int_{\Delta_B}^1 du \phi_p(u) e^{-EE} - m_d f_K m_K^2 \int_{\Delta_B}^1 du \int_0^u dt B(t) e^{-EE} \\
&- \frac{1}{6} \frac{f_K m_K^2}{m_u + m_s} \int_{\Delta_B}^1 du \phi_p(u) \frac{d}{du} e^{-EE} \\
&- f_{3K} m_K^2 \int_0^1 dv \int_0^1 d\alpha g \int_0^{1-\alpha_g} d\alpha_s T(\alpha_u, \alpha_g, \alpha_s) \\
&\Theta(u - \Delta_B) \left[ 1 + 2v \right] e^{-EE} \mid_{u = (1-v)\alpha_g + \alpha_s} \\
&\Phi(1 - \alpha - \beta, \beta, \alpha) \Theta(u - \Delta_B) e^{-EE} \mid_{1 - \alpha_g} \\
&\Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha) \Theta(u - \Delta_B) e^{-EE} \mid_{u = (1-v)\alpha_g + \alpha_s},
\end{align*}
\]

(15)
where
\[ DD = \frac{m_s^2 + u(1-u)m_\pi^2 - (1-u)q^2}{uM^2}, \]
\[ EE = \frac{m_d^2 + u(1-u)m_K^2 - (1-u)q^2}{uM^2}, \]
\[ \Delta_A = \frac{m_s^2 - q^2}{s_K^0 - q^2}, \]
\[ \Delta_B = \frac{m_d^2 - q^2}{s_\pi^0 - q^2}, \]

(17)

here the \( s_K^0 \) and \( s_\pi^0 \) are threshold parameters for the interpolating currents \( J_K(x) \) and \( J_\pi(x) \) respectively.

3 Numerical results and discussions

The input parameters of the light-cone distribution amplitudes are taken as \( \lambda_3 = 1.6 \pm 0.4, f_{3K} = (0.45 \pm 0.15) \times 10^{-2} GeV^2, \omega_3 = -1.2 \pm 0.7, \omega_4 = 0.2 \pm 0.1, a_2 = 0.25 \pm 0.15, a_1 = 0.06 \pm 0.03, \eta_4 = 0.6 \pm 0.2 \) for the \( K \) meson; \( \lambda_3 = 0.0, f_{3\pi} = (0.45 \pm 0.15) \times 10^{-2} GeV^2, \omega_3 = -1.5 \pm 0.7, \omega_4 = 0.2 \pm 0.1, a_2 = 0.25 \pm 0.15, a_1 = 0.0, \eta_4 = 10.0 \pm 3.0 \) for the \( \pi \) meson \([5, 6, 8, 9, 10]\); and \( m_s = (137 \pm 27) MeV, m_u = m_d = (5.6 \pm 1.6) MeV, f_K = 0.160 GeV, f_\pi = 0.130 GeV, m_K = 498 MeV, m_\pi = 135 MeV \). The threshold parameters are chosen to be \( s_K^0 = 1.1 GeV^2 \) and \( s_\pi^0 = 0.8 GeV^2 \), which can reproduce the values of the decay constants \( f_K = 160 MeV \) and \( f_\pi = 130 MeV \) in the QCD sum rules.

The Borel parameters in the four sum rules (see Eqs.(13-16)) are taken as \( M^2 = (1 - 2) GeV^2 \), in this region, the values of the form factors \( f_{K\pi}(Q^2) \) from Eqs.(13-14) and the \( f_{K\pi}^+(Q^2) - f_{K\pi}^-(Q^2) \) from Eq.(15) are rather stable, the values of the \( f_{K\pi}^+(Q^2) - f_{K\pi}^-(Q^2) \) from Eq.(16) are not as stable as the ones from Eqs.(13-15), which are shown, for example, in Fig.1 and Fig.2 respectively. In this article, we take the special value \( M^2 = 1.5 GeV^2 \) in numerical calculations, although such a definite Borel parameter can not take into account some uncertainties, the predictive power can not be impaired qualitatively.

The uncertainties of the seven parameters \( f_{3K}(f_{3\pi}), a_2, a_1, \lambda_3, \omega_3, \omega_4 \) and \( \eta_4 \) can only result in small uncertainties for the numerical values. The main uncertainties come from the two parameters \( m_s \) and \( m_q (= m_u = m_d) \), the variations of those parameters can lead to large changes for the numerical values, which are shown, for example, in Fig.3 and Fig.4, respectively.

From the two sum rules in Eqs.(13-14), we can see that due to the pseudoscalar currents we choose to interpolate the \( K \) and \( \pi \) mesons, the main contributions come from the two-particle twist-3 light-cone distribution amplitudes, not the twist-2 light-cone distribution amplitudes, those channels can be used to evaluate the non-perturbative parameters in the twist-3 light-cone distribution amplitudes with the
Figure 1: The $f_{K\pi}^+(Q^2)$ with the parameter $M^2$, A from Eq.(13) and B from Eq.(14).

Figure 2: The $f_{K\pi}^+(Q^2) - f_{K\pi}^-(Q^2)$ with the parameter $M^2$, A from Eq.(15) and B from Eq.(16).
Figure 3: The $f_{K\pi}^+(Q^2)$ with the parameter $m_s$, A from Eq.(13) and B from Eq.(14).

Figure 4: The $f_{K\pi}^+(Q^2)$ with the parameter $m_q$, A from Eq.(13) and B from Eq.(14).
experimental data. The dominating contributions to the nucleons light-cone distribution amplitudes come from the three valence quarks, additional contributions from the gluons and quark-antiquark pairs are very small \[11\], the main contributions to the pseudoscalar mesons light-cone distribution amplitudes come from the two valence quarks, the two cases are analogous. In the light-cone QCD sum rules, we can neglect the contributions from the light-cone distribution amplitudes with additional valence gluon (or quark-antiquark pair) and make relatively rough estimations. For the heavy-light form factors $B \to \pi, K$, we use the pseudoscalar current to interpolate the $B$ meson in the framework of the light-cone QCD sum rules, the current mass of the $b$ quark is very large, we can take the chiral limit for the masses of the $K$ and $\pi$ mesons \[12\], the contributions from the two-particle twist-3 light-cone distribution amplitudes are very small and can be safely neglected, the analytical expressions are simple. If we use the axial-vector currents to interpolate the $K$ and $\pi$ mesons, the tensor structures are more complex, some structures will get dominating contributions from the twist-2 light-cone distribution amplitudes \[2\].

We obtain the values of the $f^+_{K\pi}(Q^2)$ from the two sum rules in Eqs.(13-14), then take those values as input parameters, we can obtain the $f^-_{K\pi}(Q^2)$ from the two sum rules in Eqs.(15-16).

Taking into account all the uncertainties, finally we obtain the numerical values of the form factors $f^+_{K\pi}(Q^2)$ and $f^-_{K\pi}(Q^2)$, which are shown in the Fig.5, at zero momentum transfer,

\begin{equation}
\begin{align*}
 f^+_{K\pi}(0) &= 0.69^{+0.30}_{-0.20}, \\
 f^-_{K\pi}(0) &= 0.97^{+0.35}_{-0.31}, \\
 |f^-_{K\pi}(0)| &= 5.15^{+1.59}_{-0.91}, \\
 |f^-_{K\pi}(0)| &= 11.47^{+4.98}_{-4.44},
\end{align*}
\end{equation}

from Eq.(13), Eq.(14), Eq.(15) and Eq.(16) respectively. From the Fig.5, we can see that the uncertainties are rather large, we should refine the input parameters $m_s$ and $m_q$ especially the $m_q$ to improve the predictive ability.

The form factors $f^+_{K\pi}(q^2)$ and $f^0_{K\pi}(q^2)$ \[3\] are measured in the $K_{l3}$ decays with the squared momentum $q^2 > m_l^2$ transfer to the leptons. The curves (or shapes) of the form factors are always parameterized by the linear model, quadratic model and pole models to carry out the integrals in the phase space, the normalization is always chosen to be $f^+_{K\pi}(0)$, i.e. $f^+_{K\pi}(q^2) = f^+_{K\pi}(0) \{1 + \lambda_1 q^2 + \lambda_2 q^4 + \cdots\}$, etc, the parameters $\lambda_1, \lambda_2, \cdots$ can be fitted by the $\chi^2$, etc \[14\]. From the experimental data, we can obtain the values of the $f^+_{K\pi}(0)|V_{us}|$, the basic parameter $f^+_{K\pi}(0)$ has to be calculated with some theoretical approaches to extract the CKM matrix element $|V_{us}|$.

\[2\] The results with the axial-vector currents will be presented elsewhere.

\[3\] $f^0_{K\pi}(q^2) = f^0_{K\pi}(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f^+_{K\pi}(q^2)$, the current algebra predicts the value of the scalar form factor $f^0_{K\pi}(\Delta)$ be $f^0_{K\pi}(\Delta) = -f_K/f_\pi$ at the Callan-Treiman point $\Delta = m_K^2 - m_\pi^2$ \[13\].
Figure 5: The values of the $f_{K\pi}(Q^2)$ and $f_{K\pi}(Q^2)$, A from Eq.(13), B from Eq.(14), C from Eq.(15) and Eq.(13), D from Eq.(16) and Eq.(14).
Comparing with the theoretical calculations from the ChPT [4] and lattice QCD [13], the central value of the $f_{K\pi}^+(0)$ ($f_{K\pi}^+(0) = 0.97$) from Eq.(14) is excellent while the value $f_{K\pi}^+(0) = 0.70$ from Eq.(13) is somewhat smaller. The vector form factor $f_{K\pi}^+(Q^2)$ has been calculated by the ChPT [4], lattice QCD [13], QCD sum rules [16], the Bethe-Salpeter equation [17], etc. The numerical values $f_{K\pi}^+(0) = 0.97^{+0.35}_{-0.31}$ from Eq.(14) are more reasonable than the ones from Eq.(13).

In Figs.6-7, we plot the form factors $f_{K\pi}^+(Q^2)$ and $f_{K\pi}^-(Q^2)$ at the momentum range $Q^2 = (0 - 12) GeV^2$, from the figures, we can see that the curve (or shape) of the $Q^2 f_{K\pi}^+(Q^2)$ from Eq.(13) is rather flat at $Q^2 > 5 GeV^2$, which means that at large momentum transfers, the $f_{K\pi}^+(Q^2)$ takes the asymptotic behavior $f_{K\pi}^+(Q^2) \sim \frac{1}{Q^2}$, it is expected from the naive power counting rules [18], the terms proportional to $\frac{1}{Q^2}$ with $n \geq 2$ are canceled out with each other. The scalar form factor, axial form factor and induced pseudoscalar form factor of the nucleons take up the behavior $\frac{1}{Q^2}$ at large $Q^2$ [19], which are also expected from the naive power counting rules [18].

The curve (or shape) of the $Q^2 f_{K\pi}^+(Q^2)$ from Eq.(14) at $Q^2 < 5 GeV^2$ is analogous to the electromagnetic form factors of the $K$ and $\pi$ mesons [20]. Because of the $SU(3)$ symmetry of the light flavor quarks, we expect the vector form factor $f_{K\pi}^+(Q^2)$ will not have much difference from the electromagnetic form factors of the $K$ and $\pi$ mesons [21, 22], the results from Eq.(14) at low $Q^2$ are more reasonable than the ones from Eq.(13). The electromagnetic form factors of the $K$ and $\pi$ mesons have been calculated with the Bethe-Salpeter equation [22], ChPT [24], QCD sum rules [16, 21], perturbative QCD [23, 25], etc, our numerical values are compatible with those theoretical calculations. At large momentum transfers with $Q^2 > 5 GeV^2$, the terms of the $f_{K\pi}^+(Q^2)$ proportional to $\frac{1}{Q^2}$ with $n \geq 2$ from Eq.(14) manifest themselves, which result in the curve (or shape) of the $Q^2 f_{K\pi}^+(Q^2)$ decreases with the increase of the $Q^2$. The curve (or shape) of the form factor $f_{K\pi}^+(Q^2)$ from Eq.(14) decreases more quickly than the one from Eq.(13) with the increase of the $Q^2$.

The numerical values $|f_{K\pi}^-(0)| = 5.15$ from Eq.(15) and $|f_{K\pi}^-(0)| = 11.47$ from Eq.(16) are very large comparing with the values from the experimental data [14], ChPT [4], lattice QCD [13] and the Bethe-Salpeter equation [17], they can not give any reliable predictions. It is not un-expected, from the sum rules in Eqs.(15-16), we can see the terms like

$$\int_{\Delta A}^{1} \frac{du \phi_p(u)}{u} e^{-m^2_{\pi} + u(1-u)m^2_{\pi} - (1-u)q^2_{\pi}} \frac{1}{u M^2},$$

$$\int_{\Delta B}^{1} \frac{du \phi_p(u)}{u} e^{-m^2_{\pi} + u(1-u)m^2_{\pi} - (1-u)q^2_{\pi}} \frac{1}{u M^2}.$$

The $f_{K\pi}^-(Q^2)$ are greatly enhanced in the region of small $Q^2$ due to the extra $\frac{1}{u^2}$ comparing with the corresponding $f_{K\pi}^+(Q^2)$ in Eqs.(13-14), in the limit $Q^2 = 0$.

\footnote{One can consult the thesis [20] for more literatures on the present states of experimentally determined electromagnetic form factors of the $\pi$, $K$ and the proton.}
Figure 6: The central values of the $f_{K\pi}^+(Q^2)$ and $f_{K\pi}^-(Q^2)$ at $Q^2 = (0 - 12) GeV^2$, A from Eq.(13), B from Eq.(14), C from Eq.(15) and Eq.(13) , D from Eq.(16) and Eq.(14).

Figure 7: The central values of the $Q^2 f_{K\pi}^+(Q^2)$ and $Q^2 f_{K\pi}^-(Q^2)$ at $Q^2 = (0 - 12) GeV^2$, A from Eq.(13), B from Eq.(14), C from Eq.(15) and Eq.(13) , D from Eq.(16) and Eq.(14).
\( \Delta_A \approx 0.017 \) and \( \Delta_B \approx 0.00004 \), the dominant contributions come from the endpoint of the light-cone distribution amplitudes, such a infrared behavior can spoil the extrapolation; we should introduce extra phenomenological form-factors (for example, the Sudakov factor \([25]\)) to suppress the contribution from the end-point, that may be our next work. Our numerical values of \( f_{K\pi}(Q^2) \) have a negative sign to the ones of the \( f_{K\pi}(Q^2) \), which is consistent with the existing theoretical calculations and experimental data. In Figs. 6-7, we plot the form factor \( f_{K\pi}(Q^2) \) at the momentum range \( Q^2 = (0 - 12) GeV^2 \), from the figures, we can see that the form factor \( Q^2 f_{K\pi}(Q^2) \) from Eq.(15), just like the \( Q^2 f_{K\pi}(Q^2) \) from Eq.(13), is rather flat at \( Q^2 > 5 GeV^2 \), which means that at large momentum transfers, the \( f_{K\pi}(Q^2) \sim \frac{1}{Q^2} \), it is also expected from the naive power counting rules \([15]\), the terms proportional to \( \frac{1}{Q^{2n}} \) with \( n \geq 2 \) canceled out with each other. At momentum transfers with \( Q^2 > 5 GeV^2 \), the terms of the \( f_{K\pi}(Q^2) \) from Eq.(16) proportional to \( \frac{1}{Q^{2n}} \) with \( n \geq 2 \) manifest themselves, which results in the values of the \( Q^2 f_{K\pi}(Q^2) \) (just like the \( Q^2 f_{K\pi}(Q^2) \) from Eq.(14)) decrease with the increase of the \( Q^2 \).

In the light-cone QCD sum rules, we carry out the operator product expansion near the light-cone \( x^2 \approx 0 \), which corresponds to the \( Q^2 \gg 0 \) and \( P^2 \gg 0 \). The four sum rules in Eqs.(13-16) can be taken as some functions which model the vector form factors \( f_{K\pi}^+(Q^2) \) and \( f_{K\pi}^-(Q^2) \) at large momentum transfers, we extrapolate the \( f_{K\pi}^+(Q^2) \) and \( f_{K\pi}^-(Q^2) \) to the zero momentum transfer or beyond with the analytical continuation. The chosen functions may have good or bad lower \( Q^2 \) behaviors,

\( \Delta_A \approx 0.017 \) and \( \Delta_B \approx 0.00004 \), the dominant contributions come from the endpoint of the light-cone distribution amplitudes, such a infrared behavior can spoil the extrapolation; we should introduce extra phenomenological form-factors (for example, the Sudakov factor \([25]\)) to suppress the contribution from the end-point, that may be our next work. Our numerical values of \( f_{K\pi}(Q^2) \) have a negative sign to the ones of the \( f_{K\pi}(Q^2) \), which is consistent with the existing theoretical calculations and experimental data. In Figs. 6-7, we plot the form factor \( f_{K\pi}(Q^2) \) at the momentum range \( Q^2 = (0 - 12) GeV^2 \), from the figures, we can see that the form factor \( Q^2 f_{K\pi}(Q^2) \) from Eq.(15), just like the \( Q^2 f_{K\pi}(Q^2) \) from Eq.(13), is rather flat at \( Q^2 > 5 GeV^2 \), which means that at large momentum transfers, the \( f_{K\pi}(Q^2) \sim \frac{1}{Q^2} \), it is also expected from the naive power counting rules \([15]\), the terms proportional to \( \frac{1}{Q^{2n}} \) with \( n \geq 2 \) canceled out with each other. At momentum transfers with \( Q^2 > 5 GeV^2 \), the terms of the \( f_{K\pi}(Q^2) \) from Eq.(16) proportional to \( \frac{1}{Q^{2n}} \) with \( n \geq 2 \) manifest themselves, which results in the values of the \( Q^2 f_{K\pi}(Q^2) \) (just like the \( Q^2 f_{K\pi}(Q^2) \) from Eq.(14)) decrease with the increase of the \( Q^2 \).

In the light-cone QCD sum rules, we carry out the operator product expansion near the light-cone \( x^2 \approx 0 \), which corresponds to the \( Q^2 \gg 0 \) and \( P^2 \gg 0 \). The four sum rules in Eqs.(13-16) can be taken as some functions which model the vector form factors \( f_{K\pi}^+(Q^2) \) and \( f_{K\pi}^-(Q^2) \) at large momentum transfers, we extrapolate the \( f_{K\pi}^+(Q^2) \) and \( f_{K\pi}^-(Q^2) \) to the zero momentum transfer or beyond with the analytical continuation. The chosen functions may have good or bad lower \( Q^2 \) behaviors,

\[ f_{\gamma^*\pi^0}(Q^2) = \frac{1}{\pi f_\pi \left[1 + Q^2/(4\pi^2 f_\pi^2)\right]} = \frac{1}{\pi f_\pi \left(1 + Q^2/s_0\right)} \]

can reproduce both the value of \( Q^2 = 0 \) and the behavior of large-\( Q^2 \), the energy scale \( s_0 = \frac{4\pi^2 f_\pi^2}{\pi f_\pi} \approx 0.67 \text{GeV}^2 \) is numerically close to the squared mass of the \( \rho \) meson, \( m_\rho^2 \approx 0.6 \text{GeV}^2 \). The Brodsky-Lepage interpolation formula is similar to the result of the vector meson dominance, \( f_{\gamma^*\pi^0}(Q^2) = 1/\{\pi f_\pi \left(1 + Q^2/m_\rho^2\right)\} \). In the vector meson dominance approach, the calculation is performed at the time-like energy scale \( q^2 < 1 \text{GeV}^2 \) and the electromagnetic current is saturated by the vector meson \( \rho \), where the mass \( m_\rho \) serves as a parameter determining the pion charge radius. With a slight modification of the mass parameter, \( m_\rho = \Lambda_\pi = 776 \text{MeV} \), the experimental data can be well described by the single-pole formula at the interval \( Q^2 = (0 - 10) \text{GeV}^2 \). In Ref.\([27]\), the four form-factors of the \( \Sigma \to n \) have satisfactory behaviors at large \( Q^2 \) which are expected by the naive power counting rules, and have finite values at \( Q^2 = 0 \), the analytical expressions of the four form factors \( f_1(Q^2), f_2(Q^2), g_1(Q^2) \) and \( g_2(Q^2) \) are taken as some Brodsky-Lepage type interpolation formulae, although they are calculated at rather large \( Q^2 \), the extrapolation to the lower energy transfers has no solid theoretical foundation. The numerical values of the \( f_1(0), f_2(0), g_1(0) \) and \( g_2(0) \) are compatible with the experimental data and theoretical calculations (in magnitude). In this article, the vector form factors \( f_{K\pi}^+(Q^2) \) and \( f_{K\pi}^-(Q^2) \) can also be taken as some Brodsky-Lepage type interpolation formulae, the low momentum transfer \( Q^2 \) behaviors may be good or bad.
which correspond to the systematic errors, more experimental data are needed to select the ideal ones.

4 Conclusions

In this article, we calculate the vector form factors \( f_{K\pi}^+(Q^2) \) and \( f_{K\pi}^-(Q^2) \) within the framework of the light-cone QCD sum rules approach. The \( f_{K\pi}^+(0) \) is the basic input parameter in extracting the CKM matrix element \( |V_{us}| \) from the \( K_{\ell3} \) decays. The numerical values of the \( f_{K\pi}^+(Q^2) \) are compatible with the existing theoretical calculations, the central value \( f_{K\pi}^+(0) = 0.97 \) is in excellent agreement with the values from the ChPT and lattice QCD. The values of the \( |f_{K\pi}^-(0)| \) are very large comparing with the theoretical calculations and experimental data, and can not give any reliable predictions. At large momentum transfers with \( Q^2 > 5 GeV^2 \), the form factors \( f_{K\pi}^+(Q^2) \) and \( |f_{K\pi}^-(Q^2)| \) can either take up the asymptotic behavior of \( \frac{1}{Q^2} \) or decrease more quickly than \( \frac{1}{Q^2} \), more experimental data are needed to select the ideal sum rules.
Appendix

The light-cone distribution amplitudes of the $K$ meson,

$$
\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 s(x)|K(p)\rangle = i f_{Kp} \int_0^1 du e^{-iupx} \left\{ \varphi_K(u) + \frac{m_K^2 x^2}{16} A(u) \right\}
+ if_K m_K^2 \frac{x^\mu}{2p \cdot x} \int_0^1 du e^{-iupx} B(u),
$$

$$
\langle 0|\bar{u}(0)i\gamma_5 s(x)|K(p)\rangle = \frac{f_K m_K^2}{m_s + m_u} \int_0^1 du e^{-iupx} \varphi_p(u),
$$

$$
\langle 0|\bar{u}(0)\sigma_{\mu\nu}\gamma_5 s(x)|K(p)\rangle = i(p_{\mu}x_{\nu} - p_{\nu}x_{\mu}) \frac{f_K m_K^2}{6(m_s + m_u)} \int_0^1 du e^{-iupx} \varphi_s(u),
$$

$$
\langle 0|\bar{u}(0)\sigma_{\alpha\beta}\gamma_5 g_s G_{\mu\nu}(vx)s(x)|K(p)\rangle = f_{3K} \left\{ (p_{\mu}p_{\alpha}g_{\nu\beta}^\perp - p_{\nu}p_{\alpha}g_{\mu\beta}^\perp) - (p_{\mu}p_{\beta}g_{\nu\alpha}^\perp - p_{\nu}p_{\beta}g_{\mu\alpha}^\perp) \right\} \int D\alpha_i \varphi_{3K}(\alpha_i)e^{-i p x(x_s + v x_0)},
$$

$$
\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)s(x)|K(p)\rangle = \frac{p_{\mu}p_{\alpha}x_{\beta} - p_{\beta}x_{\alpha}}{p \cdot x} f_K m_K^2 \int D\alpha_i A_{||}(\alpha_i)e^{-i p x(x_s + v x_0)}
+ f_K m_K^2 (p_{\beta}g_{\alpha\mu} - p_{\alpha}g_{\beta\mu}) \int D\alpha_i A_{\perp}(\alpha_i)e^{-i p x(x_s + v x_0)},
$$

$$
\langle 0|\bar{u}(0)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)s(x)|K(p)\rangle = \frac{p_{\mu}p_{\alpha}x_{\beta} - p_{\beta}x_{\alpha}}{p \cdot x} f_K m_K^2 \int D\alpha_i V_{||}(\alpha_i)e^{-i p x(x_s + v x_0)}
+ f_K m_K^2 (p_{\beta}g_{\alpha\mu} - p_{\alpha}g_{\beta\mu}) \int D\alpha_i V_{\perp}(\alpha_i)e^{-i p x(x_s + v x_0)},
$$

where the operator $\tilde{G}_{\alpha\beta}$ is the dual of the $G_{\alpha\beta}$, $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\mu\nu} G^{\mu\nu}$, $D\alpha_i$ is defined as $D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. The light-cone distribution amplitudes are
parameterized as

\[
\phi_K(u, \mu) = 6u(1-u) \left\{ 1 + a_1 C_1^3 (2u - 1) + a_2 C_2^3 (2u - 1) + a_4 C_4^3 (2u - 1) \right\},
\]

\[
\varphi_p(u, \mu) = 1 + \left\{ 30\eta_3 - \frac{5}{2} \rho^2 \right\} C_2^3 (2u - 1)
+ \left\{ -3\eta_3\omega_3 - \frac{27}{20} \rho^2 - \frac{81}{10} \rho^2 a_2 \right\} C_4^3 (2u - 1),
\]

\[
\varphi_\sigma(u, \mu) = 6u(1-u) \left\{ 1 + \left[ 5\eta_3 - \frac{1}{2} \eta_3\omega_3 - \frac{7}{20} \rho^2 - \frac{3}{5} \rho^2 a_2 \right] C_2^3 (2u - 1) \right\},
\]

\[
T(\alpha_i, \mu) = 360\alpha_u\alpha_s a_1 g \left\{ 1 + \lambda_3 (\alpha_u - \alpha_s) + \omega_3 \frac{1}{2} (7\alpha_g - 3) \right\},
\]

\[
V_{\parallel}(\alpha_i, \mu) = 120\alpha_u\alpha_s\alpha_g (v_{00} + v_{10} (3\alpha_g - 1)),
\]

\[
A_{\parallel}(\alpha_i, \mu) = 120\alpha_u\alpha_s\alpha_g a_{10} (\alpha_s - \alpha_u),
\]

\[
V_{\perp}(\alpha_i, \mu) = -30\alpha_g^2 \left\{ h_{00} (1 - \alpha_g) + h_{01} [\alpha_g (1 - \alpha_g) - 6\alpha_u\alpha_s]
+ h_{10} \left[ \alpha_g (1 - \alpha_g) - \frac{3}{2} (\alpha_u^2 + \alpha_s^2) \right] \right\},
\]

\[
A_{\parallel}(\alpha_i, \mu) = 30\alpha_g^2 (\alpha_u - \alpha_s) \left\{ h_{00} + h_{01} \alpha_g + \frac{1}{2} h_{10} (5\alpha_g - 3) \right\},
\]

\[
A(u, \mu) = 6u(1-u) \left\{ \frac{16}{15} + \frac{24}{35} a_2 + 20\eta_3 + \frac{20}{9} \eta_4
+ \left[ -\frac{1}{15} \right. \right. + \left. \frac{1}{16} - \frac{7}{27} \eta_3\omega_3 - \frac{10}{27} \eta_4 \right] C_2^3 (2u - 1)
+ \left[ \frac{11}{210} a_2 - \frac{4}{135} \eta_3\omega_3 \right] C_4^3 (2u - 1) \right\}
+ \left\{ -\frac{18}{5} a_2 + 21 \eta_4 \right\}
\left\{ 2u^3 (10 - 15u + 6u^2) \log u + 2\bar{u}^3 (10 - 15\bar{u} + 6\bar{u}^2) \log \bar{u}
+ u\bar{u}(2 + 13u\bar{u}) \right\},
\]

\[
g_K(u, \mu) = 1 + g_2 C_2^3 (2u - 1) + g_4 C_4^3 (2u - 1),
\]

\[
B(u, \mu) = g_K(u, \mu) - \phi_K(u, \mu),
\]

(20)
where

\[ h_{00} = v_{00} = -\frac{\eta_4}{3}, \]
\[ a_{10} = \frac{21}{8} \eta_4 \omega_4 - \frac{9}{20} a_2, \]
\[ v_{10} = \frac{21}{8} \eta_4 \omega_4, \]
\[ h_{01} = \frac{7}{4} \eta_4 \omega_4 - \frac{3}{20} a_2, \]
\[ h_{10} = \frac{7}{2} \eta_4 \omega_4 + \frac{3}{20} a_2, \]
\[ g_2 = 1 + \frac{18}{7} a_2 + 60 \eta_3 + \frac{20}{3} \eta_4, \]
\[ g_4 = -\frac{9}{28} a_2 - 6 \eta_3 \omega_3, \] (21)

here \( C_2^1 \), \( C_4^3 \) and \( C_2^3 \) are Gegenbauer polynomials, \( \eta_3 = \frac{f_{K} m_a + m_x}{M_K} \) and \( \rho^2 = \frac{m^2}{M_K^2} \).

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