Simple patterns and Symmetries in Nuclei: Correlations of Mass and Spectroscopic Observables

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Abstract. The relation of collective observables in deformed nuclei is discussed with particular emphasis on the correlations between masses and spectroscopic observables. The role of valence nucleon number in modulating such correlations is introduced and exemplified by data on the Pt isotopes.

1. Introduction

With the accumulation of extensive data on nuclei in the last decades, it is now possible to study correlations between various observables in ways, and to an extent, never before possible. It is also possible in many cases to inspect nuclear data over large stretches of isotopic sequences, often spanning significant fractions of a major neutron shell. In this way, one can study the evolution of structure simultaneously from several perspectives.

2. Correlations of observables

The purpose of this short discussion is to outline some current ideas along these lines. We will focus on a few key observables – namely, nuclear charge radii, two-neutron separation energies, and observables that selectively reveal collective correlations such as \(2_{1}^{+}, R_{4/2}\) and \(B(E2: 2_{1}^{+} \rightarrow 0_{1}^{+})\) -- as well as on how nuclear structure varies with nucleon number. We start with a simple example, shown in Fig. 1, illustrating the data on \(R_{4/2}\) and \(S_{2n}\) for the rare earth region, centered on the quantum phase transition around \(N \approx 90\). The left panel shows a clear shape change from spherical (\(R_{4/2}\) values well below 3.0) to axial deformed (\(R_{4/2}\) values approaching 3.33) as a function of proton number. There is a sudden change from concave to convex between \(N = 88\) and 90 that forms a kind of bubble pattern \(^[1]\) characteristic of the dissolution of a subshell at \(Z = 64\) \(^[2]\), driven by the valence \(p-n\) interaction. The important point for the present discussion is that this structural change is reflected as well in the \(S_{2n}\) values (Fig. 1(right)), which interrupt their characteristic nearly linear downtrend against \(N\) near \(N = 90\) when the nuclei gain binding energy due to the onset of deformation. We will now extend the discussion of such correlations to a wider set of observables.

Figure 2, taken from Ref. \(^3\), shows the data for the five observables mentioned earlier on the left. They each behave very smoothly and regularly across the region shown. This is typical of nuclear data and reflects the striking simplicity of nuclear phenomenology. This simplicity is all the more remarkable when one considers that heavy nuclei consist of up to a couple hundred nucleons, of two types, interacting with both the strong force and the electromagnetic force, and filling about 60% of the nuclear volume in which they orbit about \(10^{21}\) times per sec. Despite this remarkably regular behavior, the patterns on the left are each different from each other and it is not easy to use the behavior of one observable to predict or anticipate the behavior of another. However, if one takes the differentials of these observables, one obtains the panels on the right. Now, we again see regular behavior, but now each of the patterns resembles every other one. Previously, occasional examples of pairwise correlations have been noted (Refs. \(^4,5\), see Ref. \(^3\) for a summary of these) but such an extensive set of correlations as in Fig. 2 (and, for other regions, shown in Ref. \(^3\)) has never
These correlations are so similar that they have a special use in nuclei far from stability where data will be sparse: data on one or two of them [e.g., \( S_{2n} \) and \( E(2_1^+) \)] can be used to estimate values or ranges for the others, facilitating experiments, guiding theory, and providing an expectation against which deviations can be assessed (such deviations may occur if new physical effects enter in exotic nuclei far from stability).

![Spectroscopic Mass](image)

**Figure 1.** Experimental \( R_{4/2} \) and \( S_{2n} \) values in the rare earth region showing the correlation of behavior in the \( N\sim90 \) quantum phase transitional region.

### 3. Sensitivity of masses to collectivity

The correlation of \( S_{2n} \) values with collective observables brings up an important point – namely the sensitivity of masses to structure and vice versa. That is, the collective correlations that give rise to the data on observables such as \( R_{4/2} \) or \( E(2_1^+) \) or \( B(E2) \) values also affect the nuclear binding and therefore \( S_{2n} \). Exploiting this linkage can give new insights. The dependence of collectivity on the number of valence nucleons [6,7] brings in a third ingredient that can be extremely important. We have explored this in the past [7] but it has again become significant with new data that is emerging [8]. To probe the sensitivity to valence nucleon number, one needs a collective model that depends explicitly on that number. The IBA [9] is the obvious choice. The most common IBA Hamiltonian uses two parameters (plus a scale factor). The Hamiltonian can be written as [10]:

\[
H = a[(1 - \zeta) m_d - (\zeta / 4N_B) Q \cdot Q]
\]

Here, the first term gives a spherical structure and dominates for \( \zeta = 0 \), while the second term gives collective, deformed structures and includes a parameter \( \chi \) in the \( Q \) operator. The parameter \( \zeta \) varies from zero to 1 and \( \chi \) from zero (\( \gamma \)-soft nucleus) to \( -\sqrt{7/2} = -1.32 \) for an axially deformed nucleus. Clearly, the Hamiltonian reflects a competition between spherical-driving and deformation-driving terms. An important point is that the first term of Eq. 1 goes roughly as the number of valence nucleons \( 2 \times N_B \) where \( N_B \) is the boson number) while the second term goes as the square of that number. Therefore, even for constant parameters, the resulting structure will tend towards deformed as the number of valence nucleons increases. Since the essence of collectivity lies in a mixing of non-collective states, with the lowest perturbed state decreasing in energy and exhibiting collective, in-phase, correlations, it is also clear that \( H(\text{IBA}) \) will involve collective contributions to binding. Thus, in general, as the number of valence nucleons increases, collectivity will increase and so will collective binding. Only the second term in \( H \) contributes to this binding, which, therefore, scales as \( N_B^2 \). This latter effect is well known but often forgotten.
Figure 2. Normal (left) and differential (right) plots for five observables representing integral, collective and single particle aspects of structure (taken from Ref. [3]).

The dependence of collective binding on the boson number and the parameters of the Hamiltonian is illustrated in Fig 3. Note the very strong, nearly quadratic, rise in binding energy with $N_B$ and the strong dependence on $\zeta$ and (slightly weaker) on $\chi$ as well. It is useful to illustrate these effects in a synoptic way. Since the IBA has three dynamical symmetries, $U(5)$ – a vibrator, $SU(3)$ – an axial
rotor, and O(6) – a $\gamma$-soft rotor, it is convenient to represent its structures in terms of a symmetry triangle. The one shown in Fig. 4, for 16 bosons, is shaded according to the legend to indicate the collective binding energies. Note that they increase rapidly towards SU(3) from all directions. Similar triangles for lower boson numbers would exhibit similar shading but with a lower scale of binding.

Figure 3. Dependence of the collective contributions to binding energies on boson number, and on $\zeta$ and $\chi$ in the IBA model. Based on Ref. [7].

Figure 4. Symmetry triangle of the IBA showing the collective contributions to binding for calculations with 16 bosons. Based on Ref. [7].

These results have an important implication for the correlations between $S_{2n}$ and the other observables in Fig. 2, namely the effect on $S_{2n}$ of changes in structure depends on the number of valence nucleons. This is illustrated in Fig. 5 for the Pt isotopes. Let us first consider the $2^+$ energies on the left. They show the characteristic behavior one typically sees across a major shell [see also Fig. 2 (left)], namely a drop in $E(2^+_1)$ after the magic number as collectivity begins to set in, then a minimizing near mid-shell and a reversal of trend in the second half of the shell. Notice that rate of the initial decline in energy abruptly changes at $N \sim 100$, then flattens, then abruptly changes again at $N = 110$, and continues upward after that, exhibiting another abrupt change in upward movement at $N = 120$. So, we have three distinct sudden changes in pattern. Now consider the plot of $S_{2n}$. Two of these kinks in $2^+$ energies are reflected in the binding energies, namely those at $N = 98$ and 110. The one at $N = 120$ is accompanied by an almost perfectly smooth behavior in $S_{2n}$. 
Why does this happen? The answer seems to be the following. The collective binding depends more sensitively on changes in structure for larger boson numbers than for smaller ones, and the increased sensitivity is quadratic in $N_B$. For $N = 98$, 110 and 120, the Pt isotopes have boson numbers $N_B = 10$, 10, and 5, respectively. So, roughly speaking, this effect alone would make $S_{2n}$ roughly four times more sensitive to a given change in $E(2^{+})$ for the lighter two Pt isotopes than for the heavier. In addition, Pt with $N=120$ is far less deformed than the Pt isotopes with either $N = 98$ or 110 (as can be seen from the $R_{4/2}$ values, for example) and thus will be positioned further from the SU(3) vertex, still further reducing the collective binding and hence the sensitivity of $S_{2n}$ to structural changes. The combination of these effects will be considerably larger than a factor of four in reduced sensitivity for $N=120$ and hence it is natural that one would not see the structural change evidenced by the $2^{+}$ energy reflected in $S_{2n}$ in that case.

This type of analysis of the correlations, including explicitly the effects of valence nucleon numbers, is new and would seem to add an important new tool in our arsenal with which to understand the systematic behavior of nuclear data and the role of collectivity in various observables.

Acknowledgments
We are grateful to F. Iachello, K. Blaum, and Yu.A. Litvinov for useful and enlightening discussions. Work supported by the US DOE under Grant Number DE-FG02-91ER-40609, and the Alexander von Humboldt Foundation.

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