Solar System Tests of a New Class of $f(z)$ Theory

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Recently, a new kind of $f(z)$ theory is proposed to provide a different perspective for the development of reliable alternative models of gravity in which the $f(R)$ Lagrangian terms are reformulated as a polynomial parameterizations $f(z)$. In the previous study, the parameters in the $f(z)$ models have been constrained by using cosmological data. In this paper, these models will be tested by the observations in the solar system. After solving the Ricci scalar as a function of the redshift, one could obtain $f(R)$ that could be used to calculate the standard Parameterized-Post-Newtonian (PPN) parameters. We find that some models are consistent with or favored by the tests, while other ones are not.

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I. INTRODUCTION

Einstein’s general relativity (GR) has been successful in predicting many phenomenologies in the universe and the solar system. In the past 20 years, more and more astronomical observations have strongly confirmed that the universe is under accelerating expansion[1–7]. However, ordinary matters can only drive a decelerating universe. To explain the accelerating, a kind of exotic component in the universe is needed called the dark energy.

Another way to drive the accelerating expansion of the universe is to modify Einstein’s gravity theory. $f(R)$ theory is a kind of such modified gravity theories. In the $f(R)$ theory, the Einstein-Hilbert action is replaced by a function of $f(R)$. When $f(R) = R$, it is just the Einstein’ gravity. Such a modified theory gives a geometrical explanation for the accelerating expansion of the universe [8–10]. Some famous $f(R)$ models have been deeply studied, such as $R + \alpha R^2$ [11], $R + \mu/R$ [12], see also [13]. Recently, a new kind of $f(R)$ theory is proposed[14], in which the $f(R)$ Lagrangian terms are reformulated as a polynomial parameterizations $f(z)$. It provides a new and different perspective for the development of reliable alternative models of gravity. Cosmological data have been used to constrain the parameters in the $f(z)$ models.

There are many experiments that could be used to test gravity theories in a relatively high accurate level, including those in the Solar system[15–17], such as the gravitational redshift[18], the perihelion advance of Mercury[19], the Shapiro time delay[20] and the Nordervert Effect[21]. As is known to all, general relativity is well consistent with the solar system tests. The parametric post Newtonian[22] limit measures the deviations of modified theories of gravity with respect from the general relativity, and it connects the observations with some parameters in the gravitational potential, i.e. the Parameterized-Post-Newtonian (PPN) parameters. Therefore, it has become a useful framework to test the theories of gravity in the solar system.

In this paper, the $f(z)$ theory will be tested in the solar system. After solving the Ricci scalar as a function of the redshift, one could obtain $f(R)$ that could be used to calculate the standard PPN parameters. We find that some models are with or favored by the tests, while some ones are not, which may need the help of some mechanisms like the chameleon mechanism[23] to pass the solar system tests.

The structure of this paper is as follows. In Section II, we obtain the equation for the Ricci scalar as a function of the redshift. And then, we solve this equation for each model proposed in Ref.[14], in which every model has an explicit formalism of $f(z)$. In Section III, we perform the solar system test on these models. The influence of the variations of parameters in the models is also discussed. Finally, discussions and conclusions will be given in Section IV.

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II. FROM \( f(z) \) MODEL TO \( f(R) \)

The most general \( f(R) \) modified gravity theory is described by the following action:

\[
S = \int d^4 x \sqrt{-g} \left[ f(R) + \mathcal{L}_m \right] \tag{1}
\]

where \( g \) is the metric determinant and \( \mathcal{L}_m \) is the Lagrangian of matter component. Here we use the units \( 8\pi G = 1 \).

By varying the action with respect to the metric \( g_{\mu\nu} \), one obtains the equations of motion as

\[
R_{\mu\nu}f_R - \frac{1}{2}g_{\mu\nu}f + (g_{\mu\nu}\nabla_\alpha - \nabla_\mu \nabla_\nu)f_R = T^m_{\mu\nu} \tag{2}
\]

where \( f_R = df/dR \) and \( T^m_{\mu\nu} \) is the stress energy tensor of the matter. The FRW metric that describes a homogeneous and isotropic flat universe is given by

\[
 ds^2 = -dt^2 + a(t)^2 \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \tag{3}
\]

where \( a(t) \) is the scale factor. From Eq. (2) with the FRW background, one can obtain the modified Friedmann equations:

\[
 H^2 = \frac{1}{3f_R} \left( \rho_m + \frac{Rf_R - f}{2} - 3H\dot{R}f_{2R} \right), \tag{4}
\]

\[
 -3H^2 - 2\dot{H} = \frac{1}{f_R} \left[ \dot{R}^2 f_{3R} + (2H\dot{R} + \ddot{R})f_{2R} + \frac{1}{2}(f - Rf_R) \right]. \tag{5}
\]

In Ref. [14], the authors have expressed \( f(R) \) as a function of the redshift \( z \), with \( 1 + z = 1/a \). The derivatives of \( f(R) \) with respect to \( R \), and of \( R \) with respect to time are provided in terms of derivatives with respect to the redshift \( z \). Therefore, one can obtain the Hubble parameter \( H(z) \) from a given \( f(z) \) model, and use cosmological observational data such as the Type Ia Supernovae to constrain the parameters in these \( f(z) \) models. In Ref. [14], the authors have suggested eight ansatze for \( f(z) \):

\[
 f(z)_{\text{Model}1} = f_0 + f_3 (1 + z)^3, \tag{6}
\]

\[
 f(z)_{\text{Model}2} = f_0 + f_1 (1 + z) + f_2 (1 + z)^2 + f_3 (1 + z)^3, \tag{7}
\]

\[
 f(z)_{\text{Model}3} = f_0 + f_2 (1 + z)^2 + f_3 (1 + z)^3, \tag{8}
\]

\[
 f(z)_{\text{Model}4} = f_0 + f_1 (1 + z) + f_3 (1 + z)^3, \tag{9}
\]

\[
 f(z)_{\text{Model}5} = f_0 (1 + z)^{1/2} + f_3 (1 + z)^3, \tag{10}
\]

\[
 f(z)_{\text{Model}6} = f_0 (1 + z)^{1/4} + f_1 (1 + z) + f_2 (1 + z)^2 + f_3 (1 + z)^3, \tag{11}
\]

\[
 f(z)_{\text{Model}7} = f_0 (1 + z)^{1/4} + f_3 (1 + z)^3, \tag{12}
\]

\[
 f(z)_{\text{Model}8} = f_0 (1 + z)^{1/4} + f_1 (1 + z) + f_2 (1 + z)^2 + f_3 (1 + z)^3, \tag{13}
\]

where \( f_i, i \in \{0, 1, 2, 3, 12, 14\} \) are constant coefficients determined by observations. In the following, we call the above as Model 1 ~ 8.

In this paper, we would like to test these \( f(z) \) models in the solar system observations. So the function of \( f(R) \) should be solved for a given \( f(z) \) model. Then, by using the Eqs. (4) and (5), we eliminate the Hubble parameter \( H(z) \) and get the equation of \( R \) as the following:

\[
 D_0 (R_{3z} R_z^2 - 2 R_{2z}^2 R_z) + R_{3zz} R_z R - 3 R_{2zz}^2 R + D_3 R_{2z} R_z^2 + D_5 R_{2z} R_z R + D_6 R_z^3 + D_7 R_z^2 R = 0, \tag{14}
\]
where

\[ D_0 = \frac{(2f_m - f)}{f_z}, \]
\[ D_3 = \frac{4f z \rho_m}{f_z} + \frac{2f m}{f_z(1 + z)} - \frac{4f}{(1 + z)f_z} - \frac{2ff z}{f_z}, \]
\[ D_5 = \frac{4f z}{f_z} + \frac{1}{1 + z}, \]
\[ D_6 = \frac{2f m}{(1 + z)^2 f_z} - \frac{2f z \rho_m}{f_z^2} - \frac{2f z \rho m}{(1 + z)f_z^2} + \frac{4f f z}{(1 + z)f_z^2} + \frac{f z}{f_z} - \frac{4f}{(1 + z)^2 f_z} , \]
\[ D_7 = \frac{f z^2}{f_z^2} - \frac{f z}{f_z(1 + z)} - \frac{f z}{f_z} + \frac{2}{(1 + z)^2}. \]

The subscript \( z \) denotes the derivatives with respect to the redshift \( z \), i.e. \( f_z = df/df z \), \( f_{zz} = d^2f/dz^2 \), \( f_{3z} = d^3f/dz^3 \) and \( R_z = dR/dz, R_{zz} = d^2R/dz^2, R_{3z} = d^3R/dz^3 \). Therefore, for a given \( f(z) \) model, one obtains \( R(z) \) from Eq. (14), then equations \( (f, R) = (f(z), R(z)) \) form a parametric representation of the function \( f(R) \). Here we have used the following relations:

\[ R = -3(H^2)z z(1 + z) + 12H^2, \]  
\[ R_z = 9(H^2)z - 3(1 + z)(H^2)z z, \]  
\[ R_{2z} = 6(H^2)z z - 3(1 + z)(H^2)z z z. \]  

and

\[ f_R = R_z^{-1} f_z, \]  
\[ f_{2R} = (f_{2z} R_z - f_z R_{zz}) R_z^{-3}, \]  
\[ f_{3R} = \frac{f_{3z}}{R_z^2} f_z R_{zz} + 3f_z R_{2z} + 3f_z R_{zz} R_z^{-1}. \]

The Friedmann equation becomes:

\[ H^2 = \frac{D_0 R_z + R}{6} \left[ 1 - (1 + z) \left( \frac{f_{2z}}{f_z} - \frac{R_{2z}}{R_z} \right) \right]^{-1}. \]  

III. OBSERVATIONAL TESTS FROM THE SOLAR SYSTEM

In this section, we will perform some observational tests on Model 2-8 from Eqs. (6) - (13). Model 1 has an exact solution with \( f_0 = 6(1 - \Omega m) \), \( f_3 = 3\Omega m \):

\[ R(z) = 12(1 - \Omega m) + 3\Omega m(1 + z)^3, \]  

where \( \Omega_i \) are the relative densities of the components and hereafter the subscript \( m \) denotes the dust matter. So the function of \( f(R) \) is

\[ f(R) = R - 6(1 - \Omega m), \]  

which is just the \( \Lambda \)CDM Model. For Model 2 - 8, one usually can not obtain the exact solution of \( R(z) \) through Eq.(14), then the numerical approach is needed to solve this equation. To numerically solve Eq.(14), we take the same initial conditions as those in Ref.[14]. Once the solution of \( R(z) \) is found, one can obtain \( f(R) \) immediately. To clearly see the differences between each model, we plot \( \log f_i/\log f_1 \) (i.e. \( i = 2 \cdots 8 \)) as a function of \( \log R \) in Fig.1.

From Fig.1, one can clearly see that the differences of each model are obvious at \( z \to 0 \), but different models reach a same point at high red shift except model 1, as the modified terms works.

The GR theory is very successful in predicting the behavior of the gravitational phenomena in the Solar System, so every kind of generation of GR proposed to explain the accelerating expansion of the universe, such as the \( f(R) \) theories, should be tested in the Solar System.
FIG. 1: The comparison of each solution to Model 1 is represented as $\log f_i / \log f_1, (i \in 2 \cdots 8)$ v.s. $\log R$.

Usually, one expands about the GR solutions up to some perturbation orders when taking into account deviation from GR. In the following, we take the standard PPN expansion of the Schwarzschild metric:

$$ds^2 = - \left[ 1 - 2 \frac{GM}{r} + 2(\beta - \gamma) \left( \frac{GM}{r} \right)^2 \right] dt^2 + \left[ 1 + 2\gamma \frac{GM}{r} \right] dr^2 + r^2 d\Omega^2,$$

where $\alpha, \beta$ and $\gamma$ are dimensionless parameters known as the Eddington parameters, which describe the deviations from GR. It is evident that the standard GR solution corresponds to the case $\beta = \gamma = 1$. The parameter $\gamma$ measures how the space is curved by unit mass and it is also connected with time delay or the effect of light deflection, while the parameter $\beta$ measures how much the non-linearity is in gravitational superposition, which can be measured through Nordtvedt effect and the perihelion shift.

The expression of PPN-parameters can be extended from the definitions in the scalar-tensor theories, since they could be rigorously compared:

$$\gamma - 1 = - \frac{\xi^2}{f_z R_z^2 + 2\xi^2},$$

$$\beta - 1 = \frac{1}{4} \left[ - f_z R_z \xi \right] \gamma_z,$$

$$\gamma_z \equiv \frac{d\gamma}{dz} = - \frac{2\xi \xi_z}{f_z R_z^2 + 2\xi^2} + \frac{R_z^4 \xi_z^2 (\xi + 6f_z R_z) + 4\xi^3 \xi_z}{(f_z R_z^2 + 2\xi^2)^2},$$

where we have used Eqs.(19) and (20). Here the function $\xi$ is defined by

$$\xi(z) \equiv f_{2z} R_z - f_z R_{2z},$$

and then we have $\xi_z = f_{3z} R_z - f_z R_{3z}$. As usual, the uncertainties of these parameters are given by

$$\sigma_\gamma = \sqrt{\sum_i \left( \frac{\delta \gamma}{\delta f_i} \right)^2 \sigma_{f_i}^2}, \quad \sigma_\beta = \sqrt{\sum_i \left( \frac{\delta \beta}{\delta f_i} \right)^2 \sigma_{f_i}^2}, \quad i \in \{0, 1, 2, 3, 12, 14\}.$$
where the variations of $\gamma, \beta$ can be obtained by using the following equations:

$$
\delta \gamma = \frac{R^4_\xi}{(f_z R_z^2 + 2z^2)^2} \left[ -2f_z R_z^2 \delta f_{z2} + R_z f_z (2f_z R_{z2} + \xi) \delta f_z - f_z (2f_z R_z - 5\xi) \delta R_z - 2f_z R_z \delta R_{z2} \right],
$$

$$
\delta \beta = \frac{\gamma z}{4(2z^2 R_z^2 + 3z^2)^2} \left[ f_z R_z (2f_z R_z^2 - 3z^2)(R_z \delta f_{z2} - f_z \delta R_{z2}) + R_z^2 (3z^2 f_z 2 - 2f_z R_z^2) \delta f_z 
+ f_z^2 (2 R_z^4 f_{z2} - 3z^2 R_z - 8R_z^5) \delta R_z 
+ \frac{1}{4} \left( \frac{f_z R_z \xi z}{2f_z R_z^2 + 3z^2} \right) \delta \gamma_z, \right.
$$

$$
\delta \gamma_z = \frac{1}{(f_z R_z^2 + 2z^2)^2} \left[ \left( 2 \xi_z (f_z R_z^2 + 2z^2) + 8 \xi_z^2 z - 8 \xi_z^4 z^2 (\xi + 6f_z R_z) + 4z^4 \xi^2 z 
+ (R_z^4 z^2 + 2R_z^4 z (\xi + 6f_z R_z) + 12z^2 \xi_z) \right) \delta \xi + \left( 4z^3 + 2z \right) \delta \xi 
+ \left( 2 \xi_z R_z^5 + 6R_z^4 z R_z + 2z \right) \left( \xi + 6f_z R_z + 4z^3 \xi_z \right) f_z R_z^4 + 4z^4 \xi^2 (\xi + 6f_z R_z) \right) \delta R_z + 6z^4 \xi z^2 \delta R_{z2} \right],
$$

where the variations of $f, f_z, f_{z2}$ and $f_{z3}$ could be easily obtained by using Eqs. (31). For instance,

$$
\delta f = \delta f_0 + \delta f_1 (1 + z) + \delta f_2 (1 + z)^2,
$$

$$
\delta f_z = \delta f_1 + 3 \delta f_2 (1 + z)^2,
$$

$$
\delta f_{z2} = 6 \delta f_3 (1 + z),
$$

$$
\delta f_{z3} = 6 \delta f_3,
$$

for Model 4. However, to get the variations of $R, R_z, R_{z2}$ and $R_{z3}$, one needs to solve the following equation for $\delta R$:

$$
A_0 + A_1 \delta R_{z2} + A_2 \delta R_{z3} + A_3 \delta R_z + A_4 \delta R = 0,
$$

with the coefficients

$$
A_0 = - \left[ (R_{z3} R_z^2 - 2R_{z2}^2 \hat R_z) \delta D_0 + R_{z2} R_z^2 \delta D_3 + R_{z2} \hat R_z \delta D_5 + R_z^3 \delta D_6 + R_z \hat R \delta D_7 \right],
$$

$$
A_1 = \hat D_0 R_z^2 + \hat R_z \hat R_z,
$$

$$
A_2 = -4 \hat D_0 R_z R_{z2} + \hat D_3 R_z^2 + \hat D_5 R_z \hat R_z - 6 R_{z2} \hat R_z,
$$

$$
A_3 = -2 \hat D_0 R_{z2}^2 + 2 \hat D_3 R_{z2} \hat R_z + \hat D_5 \hat R_z + 3 \hat D_6 \hat R_z^2 + 2 \hat D_7 \hat R_z + \hat R_{z2} \hat R_z,
$$

$$
A_4 = \hat D_0 R_{z2} R_z + \hat D_7 R_z^2 + \hat R_{z2} \hat R_z - 3 \hat R_{z2},
$$

where

$$
\delta D_0 = \left( \hat D_0 \delta f_z + \delta f \right) / \hat f_z,
$$

$$
\delta D_3 = 2 \hat f_z \delta f + 4 \hat f \delta f \left( 4 \rho_m - 2 \hat f \right) \left( \delta f_z - \frac{\delta f_{z2}}{\hat f_z} \right) + \hat D_3 \delta f_z / \hat f_z,
$$

$$
\delta D_5 = 4 \hat f_z \delta f_z - 4 \hat f \delta f z / \hat f_z,
$$

$$
\delta D_6 = (2 \rho_m - \hat f) \left( \delta f_{z3} - \frac{\delta f_{z2}}{\hat f_z} \delta f_z \right) + \delta f \left( \frac{4 \hat f}{\hat f_z} \left( \hat f_z^2 (1 + z) \right) - \frac{4 \hat f_{z2}}{\hat f_z^2 (1 + z)} \right) + \hat D_6 \delta f_z / \hat f_z,
$$

$$
\delta D_7 = \frac{\delta f_{z3}}{\hat f_z} + \frac{\delta f_{z2}}{\hat f_z (1 + z)} - \frac{\delta f_{z2}}{\hat f_z (1 + z)} \delta f_z + \frac{2 \hat f_z \delta f_{z2}}{\hat f_z} - \frac{2 \hat f_z \delta f_z}{\hat f_z}.
$$

(38)
| PPN Parameters | Related Phenomenon | Experiment | Result          |
|----------------|--------------------|------------|-----------------|
| $\gamma - 1$   | Time Delay         | Cassini mission | $(2.1 \pm 2.3) \times 10^{-5} \ [25]$ |
| $\beta - 1$    | Perihelion Advance of Mercury | VLBA | $\pm 2 \times 10^{-4} \ [26]$ |
| $\eta_N$       | Nordtvert Effect   | LLT        | $(-0.2 \pm 1.1) \times 10^{-4} \ [28]$ |

**TABLE I:** The observational values of PPN parameters.

It is hardly to solve Eq.(33) exactly, however, for Model 1, one could get the asymptotic solutions. In the limit of $z \to 0$, the coefficients $A_0 \sim A_4$ all become constants, so we have a constant solution

$$\delta R|_{z \to 0} = \frac{A_0}{A_4} = \frac{4 + 15 \Omega_m}{\Omega_m} \delta f_3 + 6(\delta f_0 + \delta f_3). \quad (39)$$

In the limit of $z \to \infty$, $A_1$ is the most important coefficient, then Eq.(40) becomes

$$\delta f_3 + \delta R_{3z} = 0, \quad (40)$$

then we get

$$\delta R|_{z \to \infty} = -\delta f. \quad (41)$$

Therefore, the asymptotic behavior of $\delta R$ is regular in Model 1. In fact, this conclusion is also valid in Model 2-8.

**A. Data Description**

To test the $f(z)$ models Eqs.(7)-(13) in the solar system, we use the data from the Very Long Baseline Array (VLBA) at 43, 23 and 15 GHz, in which the gravitational bending of radio waves is observed and then the Eddington parameter $\gamma - 1$ is constrained by $[26]$

$$|\gamma - 1| \leq 2 \times 10^{-4}. \quad (42)$$

From the observations of the the perihelion advance of Mercury, $\beta - 1$ is constrained by $[27]$

$$|\beta - 1| \leq 0.0023. \quad (43)$$

As is known, the Nordtvert effect $[21]$, as an effect that relates to the difference between the inertial mass $M(I)$ and the gravitational mass $M(G)$,

$$\frac{M(G)}{M(I)} = 1 - \eta_N \frac{1}{M_e c^2} \int \frac{G \rho(r) \rho(r') d^3r d^3r'}{2 |r - r'|}, \quad (44)$$

can be described by the combination of $\gamma$ and $\beta$ $[29]$

$$\eta_N = 4 \beta - \gamma - 3, \quad (45)$$

which could be observed by the Lunar Laser Ranging Tests (LLT). This parameter $\eta_N$ could be regarded as as another PPN parameter, which is constrained by $[29]$

$$- 1.3 \times 10^{-4} \leq \eta_N \leq 0.9 \times 10^{-4}. \quad (46)$$

We summarized these data in Table I.

**B. Test Results**

By taking the values of $f_i$ in Table 1 of Ref.$[14]$, one can obtain the values of PPN parameters with their uncertainty through Eqs.(42)-(32). We summarized the results in Table II.
Within the second term in Eq.(9) becomes much more important than the third one, i.e., favored. According to the fitting values of the parameters in Model 4 in Ref.[14], one could see that Model 4 in Eq.(9) is much more favored by the solar system observations, while Model 5-8 are hardly to be favored. As the authors of Ref.[14] stressed that theories may not be the definitive answer to explain why the universe is under accelerating expansion, but it provides a different and interesting perspective on how to relate the modified gravity with observations. We also believe that even Model 5-8 can hardly be favored by the solar system test, there is under accelerating expansion, but it provides a different and interesting perspective on how to relate the modified gravity with observations. We also believe that even Model 5-8 can hardly be favored by the solar system test, there

\[ f(z) \approx 2 \times 10^{-4} \]

However, the constant term \( f_0 = 4.43 \) is already much more larger than the second one at the same time. So the \( \Delta \) denotes that the model is in consistent with the solar system observations, while the \( \Delta \) denotes that the model is not. The "–" sign means that the 1σ error is not necessarily considered while the center value is favored.

In Table II the central values of \( \gamma - 1, \beta - 1, \eta_N \) are obtained by using the best fitting values of \( f_1 \) in Ref.[14]. If the center value of one parameter, such as the \( |\gamma - 1| \), falls within the range given by Eq. (42) (43) or (46), the corresponding model is regarded as being consistent with observations in respect of that parameter. From Table III one can see that Model 2-8 could be hardly favored by observations in respect of \( \gamma - 1 \) and \( \eta_N \). When the 1σ uncertainty of these parameters are taken into account, Model 4 is most favored by observations, while Model 5-8 are not consistent with the solar system observations. We summarized the results in Table III.

From Eqs. (7)-(13), one can see that all models have the parameter \( f_3 \). Therefore, we also plot the changes of the PPN parameters with respect to \( \delta f_3 \) in Fig.2 for some models. From Fig.2 one can see that the values of the PPN parameters changed little while \( |\delta f_3| \) is larger than its 1σ error. Therefore, the uncertainty of \( \delta f_3 \) can hardly change our results. We also checked other parameters in Model 2-8, and got the same conclusion.

### IV. CONCLUSION AND DISCUSSION

In this paper, we have performed the solar system tests to the \( f(z) \) models, which are proposed to explain the accelerating expansion of the universe in Ref.[14]. After solving the equation for the Ricci scalar (14) numerically with \( f(z) \) given by Model 2-8 in Eqs. (7)-(13), we calculate the PPN parameters and compare them to recent data. We find that Model 4 in Eq.(9) is much more favored by the solar system observations, while Model 5-8 are hardly to be favored. According to the fitting values of the parameters in Model 4 in Ref.[14], one could see that \( f_1 = 2.0 \times 10^{-4} \) is much smaller than \( f_3 = 0.94 \), so Model 4 has a slightly difference to Model 1, i.e., the \( \Lambda \)CDM model. In the future, the second term in Eq.(9) becomes much more important than the third one, i.e.

\[ 1 + z < \sqrt{\frac{f_1}{f_3}} - 1 \approx 0.015. \] (47)

TABLE III: The test results of \( f(z) \) parametric models. The \( \checkmark \) denotes that the model is in consistent with the solar system observations, while the \( \Delta \) denotes that the model is not. The "–" sign means that the 1σ error is not necessarily considered while the center value is favored.
FIG. 2: The pictures show how the perturbations work on the PPN parameters of \( f(z) \) parametric models. The upper ones show how the perturbations work in Model 2 and the below ones are those in Model 4.

are some mechanisms like the chameleon mechanism that could help the theory to pass the solar system tests. And the future data of BepiColombo Mission\[34\] will improve the precision of PPN parameter and may help us to test the theories of gravitation.

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