On Type IIA Penrose Limit and $\mathcal{N} = 6$ Chern-Simons Theories

Tatsuma Nishioka$^1$ and Tadashi Takayanagi$^2$

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

Abstract

Recently, Aharony, Bergman, Jafferis and Maldacena proposed that the $\mathcal{N} = 6$ Chern-Simons gauge theories are holographically dual to the M-theory backgrounds with multiple M2-branes on orbifolds $C^4/Z_k$. When $k$ is large, they have the type IIA string description. In this paper we analyze the Penrose limit of this IIA background and express the string spectrum as the conformal dimensions of operators in the gauge theories. For BPS operators, we can confirm the agreements between the IIA string on plane waves and the gauge theories. We point out that there exist BMN-like operators in the gauge theories, though their holographic interpretation does not seem to be simple. Also we analyze the weak coupling limit of this theory and show that the Hagedorn/deconfinement transition occurs as expected.

$^1$e-mail:nishioka@gauge.scphys.kyoto-u.ac.jp
$^2$e-mail:takayana@gauge.scphys.kyoto-u.ac.jp
1 Introduction

Recently, there have been very interesting progresses on the world-volume theory of multiple M2-branes in M-theory. Since we expect that this theory is dual to $AdS_4 \times S^7$ in the decoupling limit, it should be described by a three dimensional $\mathcal{N} = 8$ superconformal theory. The superconformal Chern-Simons theories have been constructed in [1, 2, 3], though they are less supersymmetric. Bagger, Lambert and Gustavsson constructed the three dimensional $\mathcal{N} = 8$ supersymmetric theory based on the Lie 3-algebra structure [4, 5]. For subsequent developments refer to [6]-[43].

If we assume the positive metric of the Lie 3-algebra, the algebra constraints the Lagrangian strongly [14, 18, 19, 20]. This only allows us to construct a $\mathcal{N} = 8$ supersymmetric theory which is dual to two M2-branes. If we allow the non-degenerate metric, we can find $\mathcal{N} = 8$ supersymmetric theories where we can take the number of branes $N$ arbitrary large [22, 23, 24, 33, 34, 37, 40]. However, this theory can be reduced to the well-known $\mathcal{N} = 8$ super Yang-Mills theory on $N$ D2-branes.

Aharony, Bergman, Jafferis and Maldacena have constructed very interesting $\mathcal{N} = 6$ $U(N) \times U(N)$ Chern-Simons theories (ABJM theory) and proposed that they are dual to the world-volume theory of $N$ M2-branes for the arbitrary number of $N$ [44] (see also [45, 46] for further study). This theory is parameterized by the level $k$ of the Chern-Simons gauge theory. The ABJM theory with level $k$ and the gauge group $U(N) \times U(N)$ is argued to describe the world-volume theory of M2-branes on the orbifold $C^4/Z_k$. When the gauge group is $SU(2) \times SU(2)$, this theory becomes equivalent to the BLG theory [4, 5].

In the paper [44], it was also pointed out that the ABJM theory at large $k$ is dual to the type IIA string theory and the Chern-Simons gauge theory side. In this paper we would like to report a modest progress in this direction. Namely, we would like to consider the Penrose limit of the type IIA background because in this limit the string theory becomes solvable even in the presence of $\alpha'$ corrections and RR-fluxes. It has been well-known that the Penrose limit of type IIB string on $AdS_5 \times S^5$ successfully reproduces the results of BMN operators in the $\mathcal{N} = 4$ super Yang-Mills theory [47].

As a next step, it will be very intriguing to check this proposed duality from both the IIA string theory and the Chern-Simons gauge theory side. In this paper we would like to report a modest progress in this direction. Namely, we would like to consider the Penrose limit of the type IIA background because in this limit the string theory becomes solvable even in the presence of $\alpha'$ corrections and RR-fluxes. It has been well-known that the Penrose limit of type IIB string on $AdS_5 \times S^5$ successfully reproduces the results of BMN operators in the $\mathcal{N} = 4$ super Yang-Mills theory [47].

We will show that the Penrose limit of this IIA string background becomes the plane wave background with 24 supersymmetries studied in [48] after an appropriate coordinate transformation. We find the exact string spectrum and express the results as the anomalous dimensions of operators in the ABJM theory. We will also notice that in the ABJM theory, we can define a BMN-like operator and we will compute its anomalous dimension to leading order of the effective 't Hooft coupling.

We will also study the weak coupling limit $k \to \infty$ of the ABJM theory. Since we can neglect
the non-singlet flux contributions in this limit, we can analyze the partition function of the
ABJM theory compactified on $S^1 \times S^2$ analytically and show that the deconfinement/confinement
transition occurs at a specific temperature as expected from the Hagedorn transition in the string
theory side.

This paper is organized as follows. In section 2, we review and analyze in the detail the
reduction of $AdS_4 \times S^7$ background in M-theory to the type IIA string. We will also compute
the holographic entanglement entropy of the ABJM theory. In section 3, we take the Penrose
limit of the IIA background. We compute the string spectrum and express the results from
the gauge theoretic viewpoint. In section 4, we define BMN-like operators in ABJM theory
and compute their anomalous dimensions. In section 5, we evaluate the partition of free ABJM
theory and confirm the Hagedorn transition at a specific temperature. In section 6 we summarize
our conclusions.

After we finished this paper, we noticed a very interesting preprint [46] by Bhattacharya and
Minwalla, where an agreement of the supersymmetric index between the $N = 6$ Chern-Simons
theory and its dual supergravity was shown. The section 5 of our paper has some overlap with
their calculations.

2 M2-branes on $C^4/Z_k$ and Reduction to IIA String

2.1 M2-brane Solution

We start with the 11 dimensional supergravity action

$$S = \frac{1}{2\kappa_{11}^2} \int dx^{11} \sqrt{-g} \left( R - \frac{1}{2 \cdot 4!} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} \right) - \frac{1}{12\kappa_{11}^2} \int C^{(3)} \wedge F^{(4)} \wedge F^{(4)},$$

where $\kappa_{11}^2 = \frac{2\pi^4 l_p^9}{\sqrt{\kappa^2}}$. The equations of motions read

$$R^{\mu}_{\nu} = \frac{1}{2} \left( \frac{1}{3!} F^{\mu\alpha\beta\gamma} F_{\nu\alpha\beta\gamma} - \frac{1}{3 \cdot 4!} \delta^{\mu}_{\nu} F_{\alpha\beta\rho\sigma} F^{\alpha\beta\rho\sigma} \right),$$

and

$$\partial_\sigma (\sqrt{-g} F^{\sigma \mu \nu \xi}) = \frac{1}{2 \cdot (4!)^2} \epsilon^{\mu \nu \xi \alpha_1 \ldots \alpha_8} F_{\alpha_1 \ldots \alpha_4} F_{\alpha_5 \ldots \alpha_8}.$$  \hspace{1cm} (2.3)

Then the near horizon limit of M2-brane solution becomes $AdS_4 \times S^7$

$$ds^2 = \frac{R^2}{4} ds^2_{AdS_4} + R^2 d\Omega_7^2,$$

where the radius $R$ is given by $R = l_p (2^5 N' \pi^2)^{1/6}$ ($N'$ is the number of the M2-branes). The four
form flux is found to be

$$F^{(4)} = \frac{3R^3}{8} \epsilon_{AdS_4},$$

where $\epsilon_{AdS_4}$ is the unit volume form of the $AdS_4$ space. If we assume the Poincare metric
$ds^2_{AdS_4} = \frac{dr^2}{r^2} + r^2 \sum_{\mu=0}^2 dx^\mu dx_\mu$, we have $\epsilon_{AdS_4} = r^2$ or equally $F_{012r} = \frac{3R^3 r^2}{8}$. 

2
2.2 The Reduction to IIA

We take the $Z_k$ orbifold of $S^7$ and reduce the M-theory background $AdS_4 \times S^7/Z_k$ to the type IIA string background following [44]. We can express $S^7$ by the complex coordinate $X_1, X_2, X_3$ and $X_4$ with the constraint $|X_1|^2 + |X_2|^2 + |X_3|^2 + |X_4|^2 = 1$. We can parameterize $S^7$ by

\[
X_1 = \cos \xi \cos \frac{\theta_1}{2} e^{\frac{i \chi_1 + \varphi_1}{2}}, \\
X_2 = \cos \xi \sin \frac{\theta_1}{2} e^{\frac{i \chi_1 - \varphi_1}{2}}, \\
X_3 = \sin \xi \cos \frac{\theta_2}{2} e^{\frac{i \chi_2 + \varphi_2}{2}}, \\
X_4 = \sin \xi \sin \frac{\theta_2}{2} e^{\frac{i \chi_2 - \varphi_2}{2}},
\]

(2.6)

where the angular valuables run the values $0 \leq \xi < \frac{\pi}{2}$, $0 \leq \chi_i < 4\pi$, $0 \leq \varphi_i < 2\pi$ and $0 \leq \theta_i < \pi$. Then the metric of $S^7$ can be written as

\[
ds_{S^7}^2 = d\xi^2 + \cos \xi^2 \sin^2 \xi \left[(d\chi_1 + \cos \theta_1 d\varphi_1)^2 + d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2\right] \\
+ \frac{\sin^2 \xi}{4} \left[(d\chi_2 + \cos \theta_2 d\varphi_2)^2 + d\theta_2^2 + \sin^2 \theta_2 d\varphi_2^2\right].
\]

(2.7)

Now we define new coordinates

\[
\chi_1 = 2y + \psi, \quad \chi_2 = 2y - \psi.
\]

(2.8)

The $Z_k$ orbifold action is now given by $y \sim y + \frac{2\pi}{k}$. Then the metric of $S^7$ can be rewritten as follows

\[
ds_{S^7}^2 = ds_{CP^3}^2 + (dy + A)^2,
\]

(2.9)

where

\[
A = \frac{1}{2}(\cos^2 \xi - \sin^2 \xi)dy + \frac{1}{2} \cos^2 \xi \cos \theta_1 d\varphi_1 + \frac{1}{2} \sin^2 \xi \cos \theta_2 d\varphi_2,
\]

(2.10)

and

\[
ds_{CP^3}^2 = d\xi^2 + \cos \xi^2 \sin^2 \xi \left(d\psi + \frac{\cos \theta_1}{2} d\varphi_1 - \frac{\cos \theta_2}{2} d\varphi_2\right)^2 \\
+ \frac{1}{4} \cos^2 \xi \left(d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2\right) + \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\varphi_2^2).
\]

(2.11)

This expression (2.11) of $CP^3$ can be found in e.g. [49].

By comparing the above result with the conventional reduction formula (below we always work with the string frame metric setting $\alpha' = 1$)

\[
ds_{11D}^2 = e^{-2\phi/3} ds_{IA}^2 + e^{\frac{4\phi}{3}} (d\tilde{y} + \tilde{A})^2,
\]

(2.12)
where $\tilde{y}$ is compactified as $\tilde{y} \sim \tilde{y} + 2\pi$. Since we are taking the $Z_k$ orbifold, we identify $\tilde{y} = ky$, which leads to the value of dilaton

$$e^{2\phi} = \frac{R^3}{k^3} = 2\frac{\tilde{k}^5}{\sqrt{\lambda}} \sqrt{\frac{N}{k^5}}. \quad (2.13)$$

The RR 2-form $F^{(2)} = d\tilde{A}$ in the type IIA string is explicitly given by

$$F^{(2)} = k\left(-\cos \xi \sin \xi d\xi \wedge (2d\psi + \cos \theta_1 d\varphi_1 - \cos \theta_2 d\varphi_2) - \frac{1}{2} \cos^2 \xi \sin \theta_1 d\theta_1 \wedge d\varphi_1 - \frac{1}{2} \sin^2 \xi \sin \theta_2 d\theta_2 \wedge d\varphi_2\right), \quad (2.14)$$

while the RR 4-form remains the same

$$F^{(4)} = \frac{3R^3}{8} \epsilon_{AdS_4}. \quad (2.15)$$

The string frame metric now becomes

$$ds^2_{IIA} = \tilde{R}^2(ds^2_{AdS_4} + 4ds^2_{CP^3}), \quad (2.16)$$

where $\tilde{R}^2 = \frac{R^3}{2k^3} = \sqrt{\frac{2N}{k}}$. In this way we obtain the $AdS_4 \times CP^3$ IIA background \[44, 50\]. This background preserves the 24 supersymmetries including the near horizon enhancement as it is dual to three dimensional $\mathcal{N} = 6$ superconformal symmetry.

### 2.3 Holographic Entanglement Entropy

To measure the degrees of freedom in a given conformal field theory, a useful quantity is known as the entanglement entropy $S_A$ in addition to the ordinary thermodynamical entropy. We expect that it becomes more important in $CFT_3$ since in odd dimensions we do not have a precise definition of the central charges. We trace out the subsystem $A$ which is defined by an infinite strip with the width $l$. Then the holographic area law formula in \[51\] leads to the following result\[\textsuperscript{3}\] from the analysis of minimal surfaces in the Poincare $AdS_4$

$$S_A = \frac{\sqrt{2}}{6\pi} N^2 \sqrt{\frac{k}{N}} \left( \frac{L}{a} - 2\pi \frac{\Gamma(3/4)^2}{\Gamma(1/4)^2} \cdot \frac{L}{l} \right), \quad (2.17)$$

where $L$ represents the infinitely large length of the strip and $a$ denotes the ultraviolet cutoff (or the lattice spacing). Since $S_A$ is proportional to $\frac{1}{\sqrt{\lambda}}$ in addition to the leading factor $N^2$ in the planar limit, we cannot explain this result from the free field theory approximation. Therefore we can say that this system is a more interacting theory than the $\mathcal{N} = 4$ Yang-Mills, where we can qualitatively reproduce the supergravity result of $S_A$ from the free Yang-Mills \[51\].

\[\textsuperscript{3}\]Here we employed the explicit value $\text{Vol}(CP^3) = \pi^2$ of the volume of $CP^3$ in the coordinate \[24,11\].
3 Penrose Limit of Type IIA on $AdS_4 \times CP^3$

3.1 Penrose Limit and Plane Wave Solution

We would like to take the Penrose limit \cite{47} of type IIA background $AdS_4 \times CP^3$. We express the metric of $AdS_4$ by

$$ds^2 = -\cosh^2 \rho dt^2 + d\rho^2 + \sinh \rho^2 d\Omega_2^2. \quad (3.1)$$

The metric of $CP^3$ is given by (2.11). We are focusing on the null geodesic defined by

$$\rho = 0, \quad \theta_1 = \theta_2 = 0, \quad \xi = \frac{\pi}{4}. \quad (3.2)$$

We introduce a new angular coordinate

$$\tilde{\psi} = \psi + \frac{\varphi_1 - \varphi_2}{2}. \quad (3.3)$$

The Penrose limit is defined by the following coordinate transformation

$$\frac{t + \tilde{\psi}}{2} = x^+ , \quad \tilde{R} \frac{t - \tilde{\psi}}{2} = x^- , \quad \rho = \frac{r}{R} , \quad \theta_i = \frac{\sqrt{2} y_i}{R} , \quad \xi = \frac{\pi}{4} + \frac{y_3}{2 \tilde{R}}. \quad (3.4)$$

setting $\tilde{R}$ to infinity with $x^\pm , r , y_1 , y_2$ and $y_3$ kept finite.

In the end we find the following metric in this limit $\tilde{R} \to \infty$

$$ds^2_{IIA} = -4 dx^+ dx^- - (r^2 + y_3^2)(dx^+)^2 + dx^+ (-y_3^2 d\varphi_1 + y_2^2 d\varphi_2) + dr^2 + r^2 d\Omega_3^2 + (dy_1^2 + y_1^2 d\tilde{\varphi}_1^2) + (dy_2^2 + y_2^2 d\varphi_2^2) + dy_3^2. \quad (3.5)$$

At the same time, the RR fluxes becomes

$$F_{+y_3} = \frac{k}{2 \tilde{R}} , \quad F_{+r \Omega_2} = \frac{3k}{2 \tilde{R}} r^2. \quad (3.6)$$

If we define

$$\tilde{\varphi}_1 = \varphi_1 - \frac{x^+}{2} , \quad \tilde{\varphi}_2 = \varphi_2 + \frac{x^+}{2}, \quad (3.7)$$

we can rewrite the metric as

$$ds^2_{IIA} = -4 dx^+ dx^- - \left( r^2 + y_3^2 + \frac{y_1^2 + y_2^2}{4} \right) (dx^+)^2 + dr^2 + r^2 d\Omega_3^2 + (dy_1^2 + y_1^2 d\tilde{\varphi}_1^2) + (dy_2^2 + y_2^2 d\varphi_2^2) + dy_3^2, \quad (3.8)$$

which is a familiar form of the plane wave.

If we introduce the Cartesian coordinate $(x_1, \cdots, x_8)$ in an obvious way we get

$$ds^2 = -4 dx^+ dx^- - \left( \sum_{i=1}^{4} x_i^2 + \frac{1}{4} \sum_{i=5}^{8} x_i^2 \right) (dx^+)^2 + \sum_{i=1}^{8} (dx^i)^2. \quad (3.9)$$
with
\[ F_{+4} = \frac{k}{2R}, \quad F_{+123} = \frac{3k}{2R}. \] (3.10)

Since the dilaton is expressed as \( e^{\phi} = \frac{2}{k} \), we can rewrite the values of RR-fluxes as \( e^{\phi} F_{+4} = 1 \) and \( e^{\phi} F_{+123} = 3 \), which will be useful later.

This plane-wave background (3.10) in IIA string has been known in the literature [48] and has been shown to have 24 supersymmetries as we expect.

### 3.2 Gauge Theory Interpretation

It is argued that the type IIA on \( AdS_4 \times CP^3 \) is dual to the ’t Hooft limit of \( \mathcal{N} = 6 \) superconformal Chern-Simons theory with the level \( k \) and the gauge group \( U(N) \times U(N) \) in [44]. Since the gauge theory coupling in Chern-Simons theories is proportional to \( \frac{1}{k} \), the ’t Hooft coupling is identified with \( \lambda = \frac{N}{k} \). Thus the ’t Hooft limit is defined as the large \( N \) limit with \( \lambda = \frac{N}{k} \) kept finite. It is natural to expect that our Penrose limit should correspond to a certain limit of this gauge theory.

The ABJM theory consists of the Chern-Simons \( U(N) \times U(N) \) gauge potentials at level \( k \) coupled to the four chiral superfields \( A_1, A_2, B_1, B_2 \), whose structure is very similar to the Klebanov-Witten theory [52]. The fields \((A_1, A_2, B_1, B_2)\) belong to the \((\mathbb{N}, \bar{\mathbb{N}})\) representation under the \( SU(4) \) R-symmetry of this \( \mathcal{N} = 6 \) Chern-Simons theory.

First we relate the transverse scalars in the directions of \((X^1, X^2, X^3, X^4)\) in (2.6) with the scalar fields \((\tilde{\psi}, \varphi_1, \varphi_2, \varphi_3)\) in the ABJM theory, following the \( SU(4) \) R-symmetry. We denote the conformal dimension \( \Delta \) and define \( U(1) \) parts of R-charges \( J_1, J_2, J_3 \) as follows (here we still did not perform the shift \( \varphi_i \rightarrow \tilde{\varphi}_i \) in (3.7))

\[
J_1 = -i \frac{\partial}{\partial \varphi_1} \bigg| \tilde{\psi} = -i \left( \frac{\partial}{\partial \varphi_1} - \frac{1}{2} \frac{\partial}{\partial \tilde{\psi}} \right), \\
J_2 = -i \frac{\partial}{\partial \varphi_2} \bigg| \tilde{\psi} = -i \left( \frac{\partial}{\partial \varphi_2} + \frac{1}{2} \frac{\partial}{\partial \tilde{\psi}} \right), \\
J_3 = -i \frac{\partial}{\partial \tilde{\psi}}. \] (3.11)

Notice that in the final forms of three R-charges we fixed \( \tilde{\psi}, \varphi_1 \) and \( \varphi_2 \) to be constant. Using the dependence of the angles in (2.6) we find

\[
J_1(A_1) = \frac{1}{4}, \quad J_1(A_2) = -\frac{3}{4}, \quad J_1(B_1) = -\frac{1}{4}, \quad J_1(B_2) = -\frac{1}{4}, \quad J_2(A_1) = \frac{1}{4}, \quad J_2(A_2) = \frac{1}{4}, \quad J_2(B_1) = -\frac{1}{4}, \quad J_2(B_2) = \frac{3}{4}, \quad J_3(A_1) = \frac{1}{2}, \quad J_3(A_2) = \frac{1}{2}, \quad J_3(B_1) = \frac{1}{2}, \quad J_3(B_2) = \frac{1}{2}. \] (3.12)

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4 The Penrose limit of the Klebanov-Witten theory \( AdS_5 \times T^{1,1} \) has been studied in [53].

5 In this paper we also express the scalar field part of the chiral superfield \( A_1 \) and \( B_i \).
Now we would like to relate the light-cone momenta $p^+$ and $p^-$ in the type IIA string to the
gauge theoretic quantities assuming the $AdS_4/CFT_3$ duality. To do this we need to rewrite the
metric in terms of $\tilde{\phi}_i$ instead of $\phi_i$ as in (3.8). In this process, we regard any derivative as the
one with $\tilde{\psi}, \tilde{\phi}_1$ and $\tilde{\phi}_2$ fixed to be a constant. In the end, we find

$$2p^- = i \frac{\partial}{\partial x^+} = \Delta - J, \quad 2p^+ = i \frac{\partial}{\partial x^-} = \frac{\Delta + J}{R^2},$$

(3.13)

where $J$ is defined by

$$J = J_3 + \frac{1}{2} J_1 - \frac{1}{2} J_2.$$

(3.14)

Explicitly, we get

$$J(A_1) = \frac{1}{2}, \quad J(A_2) = 0, \quad J(B_1) = \frac{1}{2}, \quad J(B_2) = 0.$$

(3.15)

### 3.3 World-Sheet Analysis

First we analyze the bosonic sector. The world-sheet action in the light-cone gauge $X^+ = 2p^+ \tau$
looks like (notice $0 \leq \sigma \leq \pi$)

$$S_B = \frac{1}{4\pi\alpha'} \int d\sigma d\tau \partial_a X^\mu \partial_a X^\nu g_{\mu\nu}(X),$$

$$= \frac{1}{4\pi\alpha'} \int d\sigma d\tau \left[ \sum_{i=1}^{8} ((\partial_\tau X^i)^2 - (\partial_\sigma X^i)^2) - 4(p^+)^2 \sum_{i=1}^{4} (X^i)^2 - (p^+)^2 \sum_{i=5}^{8} (X^i)^2 \right].$$

(3.16)

Then we easily find that the spectrum is given by (setting $\alpha' = 1$)

$$2p_B^- = \sum_{n=-\infty}^{\infty} N_n^{(1)} \sqrt{1 + \frac{n^2}{(p^+)^2}} + \sum_{n=-\infty}^{\infty} N_n^{(2)} \sqrt{\frac{1}{4} + \frac{n^2}{(p^+)^2}},$$

(3.17)

where $N^{(1)}$ (and $N^{(2)}$) denote the total occupation number of $n$-th string modes with respect to
the oscillators $\alpha_n^{1,2,3,4}$ (and $\alpha_n^{5,6,7,8}$). We always need to impose the level matching condition

$$\sum_{n=-\infty}^{\infty} n(N_n^{(1)} + N_n^{(2)}) = 0.$$

(3.18)

Using the relation $p^+ = \frac{\Delta + J}{2R^2} \simeq \frac{J}{R^2}$, we can rewrite the above formula in term of the gauge
theory quantities

$$\Delta - J = \sum_{n=-\infty}^{\infty} N_n^{(1)} \sqrt{1 + \frac{2\pi^2 n^2}{J^2} \cdot \frac{N}{k}} + \sum_{n=-\infty}^{\infty} N_n^{(2)} \sqrt{\frac{1}{4} + \frac{2\pi^2 n^2}{J^2} \cdot \frac{N}{k}}.$$

(3.19)

As in the BMN case [17], we expect that the insertion of the string oscillators corresponds
to that of the impurity operators in $\text{Tr}(A_1 B_1)^J$. Indeed, $A_1 B_1$ and their powers are the unique
operators which satisfy $\Delta - J = 0$, as is clear from \((3.15)\). Also notice that $\text{Tr}(A_1 B_1)^J$ is the chiral primary operator. By inspecting the $R$-charge of impurities we can easily identify (assuming the zero mode $n = 0$) the 4 oscillators $c_0^{5,6,7,8}$ with

$$A_1 B_2, \quad A_1 \bar{A}_2, \quad A_2 B_1, \quad \bar{B}_2 B_1.$$ \(3.20\)

Indeed these four operators satisfy $\Delta - J = \frac{1}{2}$. Therefore we argue that the oscillators $(\alpha_0^5 - i\alpha_0^6, \alpha_0^5 + i\alpha_0^6, \alpha_0^7 - i\alpha_0^8, \alpha_0^7 + i\alpha_0^8)$ are dual to the replacement procedures

$$A_1 \rightarrow A_2, \quad B_1 \rightarrow \bar{A}_2, \quad A_1 \rightarrow \bar{B}_2, \quad B_1 \rightarrow B_2.$$ \(3.21\)

On the other hand, we expect that the three oscillators $\alpha_1^{1,2,3}$ should be dual to the covariant derivative $D_\mu$, where $\mu = 0, 1, 2$. We still need to identify one more. There are six other operators which satisfy $\Delta - J = 1$:

$$A_1 \bar{A}_1, \quad \bar{A}_2 B_2, \quad A_2 \bar{A}_2, \quad \bar{B}_1 B_1, \quad \bar{B}_2 B_2, \quad \bar{B}_2 \bar{A}_2.$$ \(3.22\)

Among them only $A_1 \bar{A}_1$ and $\bar{B}_2 B_2$ are independent from the double excitations of the previous operations in \((3.20)\). Therefore, $\alpha_4^0$ is expected to be dual to a linear combination of these operators.

Next we turn to the fermionic sector (we follow the convention in \[54\]). The fermion part in the light cone gauge $\Gamma^+ S = 0$ in the Green-Schwartz formalism looks like

$$S_F = \frac{1}{4\pi\alpha'} \int d\tau d\sigma \partial_a X^\mu S \Gamma_\mu (g^{ab} - \epsilon^{ab} \Gamma^{11}) D_b S,$$ \(3.23\)

where $S$ is a ten dimensional Majorana spinor and the covariant derivative $D_b$ is the pullback to the world-sheet of the supercovariant derivative in IIA supergravity

$$D_\mu \epsilon = \nabla_\mu \epsilon + \frac{e^\phi}{4} F_{\mu \nu} \Gamma^{11} \epsilon - \frac{e^\phi}{(4!)^2} (3F_{\alpha \beta \gamma \delta} \Gamma^{\alpha \beta \gamma \delta} \Gamma_\mu - F_{\alpha \beta \gamma} \Gamma_{\mu \nu} \Gamma^{\alpha \beta \gamma \delta}) \epsilon.$$ \(3.24\)

Then the action is simplified up to a constant

$$S_F = \int d\tau d\sigma \left[ \bar{S} \Gamma_+ (\partial_\tau + \Gamma^{11} \partial_\sigma) S + \frac{p^+}{2} \bar{S} \Gamma_+ (\Gamma^4 \Gamma^{11} + 3 \Gamma^{123}) S \right].$$ \(3.25\)

The equation of motion becomes

$$(\partial_\tau + \Gamma^{11} \partial_\sigma) S = -\frac{p^+}{2} (\Gamma^4 \Gamma^{11} + 3 \Gamma^{123}) S.$$ \(3.26\)

By multiplying $\partial_\tau - \Gamma^{11} \partial_\sigma$ we obtain

$$(\partial_\tau^2 - \partial_\sigma^2) S = (2p^+)^2 \mu^2 S.$$ \(3.27\)

\(^6\)Here we neglect the contribution from the operators with Wilson line attached \[\text{\textsuperscript{44}}\] since we are assuming $k$ is large.
where
\[ \mu^2 \equiv - \left( \frac{\Gamma^4 \Gamma^{11} + 3 \Gamma^{123 \downarrow}}{4} \right)^2. \] (3.28)

Since \(16 \mu^2 = 10 - 6 \Gamma^{12341 \downarrow}\), we can conclude that among eight physical fermions, half of them have \(\mu^2 = 1\), while other half do \(\mu^2 = \frac{1}{4}\). This mass spectrum is exactly the same as the bosonic one. Thus we find the fermion spectrum
\[
2p_F = \sum_{n=-\infty}^{\infty} N_n^{(1)} \sqrt{1 + \frac{n^2}{(p^+)^2}} + \sum_{n=-\infty}^{\infty} N_n^{(2)} \sqrt{\frac{1}{4} + \frac{n^2}{(p^+)^2}},
\] (3.29)
and the values of \(\Delta - J\) for fermions are given by the same formula \[\text{[31,19]}\]. Among totally sixteen fermions in the dual gauge theory, four fermions satisfy \(\Delta - J = \frac{1}{2}\) and other four fermions do \(\Delta - J = \frac{3}{2}\), while the rest eight fermions have \(\Delta - J = 1\). Therefore we find that the string spectrum is consistent with this gauge theory fermionic operators at least for the zero modes.

In this way have shown a nice matching between the zero modes of the IIA string theory in the Penrose limit and the gauge theory operators. This is of course expected since the operators dual to zero modes (or KK modes) are protected under the change of the coupling constant.

4 BMN like Operators

Motivated by the analysis of Penrose limit in the previous section we would like to examine non-BPS operators in the ABJM theory. Especially, we are interested in the BMN-like operators\(^7\) (almost BPS operators). Indeed it is not difficult to find analogous operators (refer also to [2] for similar operators in less supersymmetric Chern-Simons theory.).

We would like to concentrate on the following operators assuming \(J\) is very large
\[
\mathcal{O}_n = \frac{1}{\sqrt{2J}} \sum_{i=0}^{J} e^{2\pi i n_i} \text{Tr}[(A_1 B_1)^i A_1 B_2 (A_1 B_1)^{J-i}(A_1 B_2)].
\] (4.1)
Notice that \(\mathcal{O}_0\) is chiral primary since the index \(i\) and \(j\) of \(A_i\) and \(B_j\) are both symmetrized independently. We can treat the impurity of the form \(A_2 B_1\) exactly in the same way.

4.1 Anomalous Dimension

We would like to compute two point functions of these operators and obtain the anomalous dimensions to lowest order. Since we know that the operator \(\mathcal{O}_0\) is chiral primary and its anomalous dimension is vanishing, we have only to consider the Feynman diagrams whose results depend on \(n\). Then the relevant part of the Lagrangian looks like (we follow the convention in [43])
\[
\mathcal{L} = \sum_{i=1}^{2} \left( \partial_\mu A^i \partial^\mu \bar{A}^i + \partial_\mu B^i \partial^\mu \bar{B}^i \right) + \frac{16 \pi^2}{k^2} \text{Tr}[B_2 A_1 B_1 \bar{A}_2 \bar{B}_1] + \frac{16 \pi^2}{k^2} \text{Tr}[B_1 A_1 B_2 \bar{B}_1 \bar{A}_2].
\] (4.2)
\(^7\)The existence of the spin chain like structure was already suggested in [2,11].
Then the propagator is normalized as follows

\[
\langle A^i_{ab}(x)\bar{A}^j_{cd}(0) \rangle = \langle B^i_{ba}(x)\bar{B}^j_{dc}(0) \rangle = \frac{\delta_{ij}\delta_{da}\delta_{bc}}{4\pi|x|},
\]

(4.3)

where we neglect any interactions which do not affect our leading computation of the anomalous dimension.

Since the two interactions in (4.2) exchange \(B_2\) with the two nearest \(B_1\)s, respectively, they produce the phase factors \(e^{\pm 2\pi i n}\), which is very similar to the BMN analysis [47]. Also notice that the insertion of either of these interactions adds two loops in the fat diagram and leads to \(N^2\) factor. Therefore we obtain

\[
\langle O_n(x)\bar{O}_n(0) \rangle = \frac{N}{|x|^{2(J+2)}} \left(1 + \frac{1}{(4\pi)^2} \cdot \frac{32\pi^2 N^2}{k^2} (\cos \frac{2\pi n}{J} - 1) \cdot I(x) \right),
\]

(4.4)

where \(I(x) = |x|^3 \int \frac{dy^3}{|y|^{|x|_y}}\) and \(N\) is a normalization factor. Since \(I(x) \sim 8\pi \log x\Lambda\) (\(\Lambda\) is the cutoff), we can conclude that the leading anomalous dimension \(\delta^{CS}_n\) of \(O_n\) is given by

\[
\delta^{CS}_n = 4\pi^2 N^2 n^2 \frac{k^2}{k^2 J^2} + \cdots,
\]

(4.5)

where \(\cdots\) denotes the higher order terms with respect to \(\frac{NkJ}{k}\).

It is also useful to remember that in the original analysis of BMN operators there are two different coupling constants: one is the rescaled 't Hooft coupling \(\lambda J^2\) and the other is the effective string coupling \(J^2 N\) [55]. The latter appears when we consider the non-planar diagrams, which we neglected in the above. In our Chern-Simons gauge theory, we can see that the non-planar corrections come with the same factor \(\frac{J^2 N}{k}\). Since in our argument which derives (4.5), we keep the rescaled 't Hooft coupling \(\frac{NkJ}{k}\) a small value, the non-planar correction is negligible if \(N \ll k^2\).

### 4.2 Comparison with IIA Plane Wave

One may naively guess that the Penrose limit of the IIA string studied in section 3 corresponds to the BMN-like limit assumed in the previous subsection

\[
\frac{N}{kJ} = \text{finite}, \quad N \ll k^2,
\]

(4.6)

as the analogous relation was true in the duality between \(AdS_5 \times S^5\) and the four dimensional \(N = 4\) Yang-Mills theory [47]. However, this does not seem to be the case here, even though about the chiral primary operators there is a nice matching between them as we have seen in the previous section. In fact, the anomalous dimension found in the Penrose limit reads (see (3.19))

\[
\delta^{IIA}_n = \frac{2\pi^2 N n^2}{kJ^2} + \cdots,
\]

(4.7)

for the impurities of \(A_1 B_2\) and \(A_2 B_1\). This is different from (4.5) by the factor \(\frac{2N}{k}\).
We would like to suggest that this is not a contradiction and that this occurs because the Penrose limit and the BMN-like limit are different in our case. Indeed, to find the spectrum of the type IIA string in the Penrose limit, we need to keep the string coupling $e^{2\phi}$ small and $p^+$ finite, which requires

$$\frac{1}{(p^+)^2} \sim \frac{N}{k^2} \text{ finite,} \quad e^{2\phi} \sim \sqrt{\frac{N}{k^2}} \ll 1.$$  \hspace{1cm} (4.8)

Even though in both limits we take the large $J$ limit, we scale it differently: it scaled as $J \sim \sqrt{N/k}$ in the Penrose limit, while the BMN-like limit $J \sim \frac{N}{k}$. In this sense the former corresponds to a smaller operator than the latter.

If we take the limit $1 \ll k \ll N \ll k^5$ with $J \sim \frac{N}{k}$, the $\mathcal{N} = 6$ Chern-Simons gauge theory should be dual to the Penrose limit of type IIA string as follows from (4.8). Therefore our results of anomalous dimensions in the Penrose limit should be reproduced from the analysis of the gauge theory by studying the corresponding subsectors. Since the string theory side leads to an analytical expression (3.19), one may find an interesting structure which is different from the BMN operators. This will be an interesting future problem.

5 Free $\mathcal{N} = 6$ Chern-Simons Theory on $S^1 \times S^2$

Obviously, another limit which we can take to make a given theory simpler and more tractable is the weak coupling limit. We would like to finish this paper by studying the weak coupling limit $k \to \infty$ of the $\mathcal{N} = 6$ Chern-Simons theory on $S^1 \times S^2$. We will show that the Hagedorn/deconfinement transition will occur in almost the same way as in the $\mathcal{N} = 4$ free Yang-Mills on $S^1 \times S^3$ \cite{56,57}.

The original ABJM action in this limit becomes

$$S_{ABJM} = \int d^3x \frac{1}{4\pi} \text{Tr} \left[ \left( A_{(1)} \wedge dA_{(1)} + \frac{2}{3\sqrt{k}} A^3_{(1)} \right) - \left( A_{(2)} \wedge dA_{(2)} + \frac{2}{3\sqrt{k}} A^3_{(2)} \right) \right]$$

$$+ \text{Tr} \sum_{i=1,2} \left[ |D^{(+)}_{\mu} A_i|^2 + |D^{(-)}_{\mu} B_i|^2 + i\bar{\psi}_i D^{(+)}_{\mu} \psi_i + i\bar{\chi}_i D^{(-)}_{\mu} \chi_i \right] + \mathcal{O}\left( \frac{1}{k} \right),$$ \hspace{1cm} (5.1)

where we define the covariant derivatives

$$D^{(\pm)}_{\mu} = \nabla_{\mu} \pm \frac{i}{\sqrt{k}} \left( A_{(1)\mu} \otimes \mathbf{1} - \mathbf{1} \otimes A_{(2)\mu} \right).$$ \hspace{1cm} (5.2)

We must be careful in taking $k$ infinity since a naive treatment spoils the Gauss’ law constraint \cite{57}. We must choose the gauge fixing by the temporal gauge

$$A_{(a)\mu}(x) = \sqrt{k} \ a_{(a)}, \quad (a = 1, 2).$$ \hspace{1cm} (5.3)
Under this gauge fixing, the action (5.1) on $S^1 \times S^2$ becomes

$$S_{free} = iS_{CS}(A_{(1)}; S^1 \times S^2) - iS_{CS}(A_{(2)}; S^1 \times S^2)$$

$$+ \text{Tr} \sum_{i=1,2} \left[ \hat{A}_i \left( -(D_{\mu}^{(\pm)})^2 + \frac{R}{8} \right) \hat{A}_i + \hat{B}_i \left( -(D_{\mu}^{(-)})^2 + \frac{R}{8} \right) \hat{B}_i + i\bar{\psi}_i \bar{D}_{\mu}^{(\pm)} \psi_i + i\bar{\chi}_i \bar{D}_{\mu}^{(-)} \chi_i \right],$$

(5.4)

where we included $\frac{R}{8}$ term which arises from a conformal coupling of the scalar field and defined $D_{\mu}^{(\pm)} = (D_{\mu}^{(\pm)})_\mu \equiv \partial_\mu \pm i(a_{(1)} \otimes 1 - 1 \otimes a_{(2)}), \nabla_1, \nabla_2$. The Ricci scalar is $R = 2$ for the unit two-sphere, and integrating the matter fields out gives the one-loop effective action

$$\text{Tr} \ln \left( -(D_0^{(\pm)})^2 - \nabla^2 + \frac{R}{8} \right) = -\sum_{n=1}^{\infty} \frac{1}{n} z_B(x^n) (\text{tr} U^n \text{tr} V^{-n} + \text{tr} U^{-n} \text{tr} V^n),$$

$$\text{Tr} \ln(-\langle \bar{D}_{\mu}^{(\pm)} \rangle^2) = \text{Tr} \ln(-(D_0^{(\pm)})^2 - \nabla^2 + \frac{R}{8}) = \sum_{n=1}^{\infty} \left[ \frac{(-1)^{n+1}}{n} z_F(x^n) (\text{tr} U^n \text{tr} V^{-n} + \text{tr} U^{-n} \text{tr} V^n) \right],$$

(5.5)

where we denote $x = e^{-\beta}$ and introduce $U = e^{i\beta a_{(1)}}, V = e^{i\beta a_{(2)}}$ as Wilson loops along $S^1$. We omit the irrelevant terms independent of $\alpha$ and define the single-particle partition function of bosons and fermions as

$$z_B(x) = \frac{x^2 (1 + x)}{(1 - x)^2}, \quad z_F(x) = \frac{2x}{(1 - x)^2}. \quad (5.6)$$

After all, the partition function becomes the expectation value of the Wilson loops of $U(N) \times U(N)$ Chern-Simons gauge theory

$$Z = \int [DA_{(1)}] [DA_{(2)}] \exp \left[ iS_{CS}(A_{(1)}; S^1 \times S^2) - iS_{CS}(A_{(2)}; S^1 \times S^2) + \sum_{n=1}^{\infty} \frac{1}{n} (4z_B(x^n) + (-1)^{n+1} 4z_F(x^n)) (\text{tr} U^n \text{tr} V^{-n} + \text{tr} U^{-n} \text{tr} V^n) \right],$$

$$= \left\langle \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} z_n(x) (\text{tr} U^n \text{tr} V^{-n} + \text{tr} U^{-n} \text{tr} V^n) \right] \right\rangle_{S^1 \times S^2},$$

(5.7)

where, in the second equality, we define $z_n(x) = 4z_B(x^n) + (-1)^{n+1} 4z_F(x^n)$. It is known that only singlet representation of the Wilson loops takes non-zero expectation value in Chern-Simons gauge theory on $S^1 \times S^2$ \[58\], then we can rewrite the above expression as the matrix model

$$Z = \int [dU] [dV] \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} z_n(x) (\text{tr} U^n \text{tr} V^{-n} + \text{tr} U^{-n} \text{tr} V^n) \right]. \quad (5.8)$$

Once taking the large-$N$ limit, we can obtain the effective action

$$I_{eff} = N^2 \sum_{n=1}^{\infty} \frac{1}{n} \left( |u_n|^2 + |v_n|^2 - z_n(x) (u_n v_{-n} + u_{-n} v_n) \right), \quad (5.9)$$
where $u_n \equiv \text{tr} U^n / N$, $v_n \equiv \text{tr} V^n / N$ and the first two terms in the right hand side come from the measure.

We now consider the saddle point of the matrix model action (5.9). The eigenvalues $\lambda$ of the quadratic form in (5.9) with respect to $(u_n, v_n)$, read $\lambda = 1 \pm z_n(x)$. Thus the trivial saddle point $u_n = v_n = 0$ is dominated if $z_1(x) < 1$ since $z_n(x)$ is monotonically decreasing function of $n$. For $z_1(x) > 1$, one of the eigenvalues becomes negative and the action is dominated by another saddle point which gives order $N^2$ free energy. Then, there is a deconfinement transition at $z_1(x) = 1$ and the Hagedorn temperature is calculated using (5.6) as $T_H = \frac{1}{\log(17 + 12\sqrt{2})} \sim 0.283648$.

In this way we have shown that in the large $k$ limit (free limit), a Hagedorn/deconfinement transition occurs in the ABJM theory. In the strong coupling region $\frac{N}{k} \gg 1$, this is expected from supergravities [59]; both the IIA string on $CP^3$ and the M-theory on $S^7/Z_k$ have the $AdS_4$ black hole solution. To understand the finite $k$ region in the gauge theory side, which is dual to the M-theory, we need to take the non-singlet flux contributions [44] into account and this will be an interesting future problem.

6 Conclusion

In this paper we examined the Penrose limit of the type IIA string on $AdS_4 \times CP^3$, which is argued to be dual to the $\mathcal{N} = 6$ Chern-Simons gauge theory (ABJM theory) in the ’t Hooft limit. We obtained the resulting plane wave background and compute the string spectrum in terms of gauge theoretic quantities. For BPS operators, we find the agreement between the IIA string and the ABJM theory. Also the string spectrum in the plane wave limit provides us with an important prediction of the anomalous dimensions in certain sectors which satisfy $J \sim \sqrt{\frac{N}{k}}$ in the ABJM theory. We also analyzed the gauge theory sides and argued that we can define BMN-like (almost BPS) operators when the R-charge $J$ is large. However, the relation between the spectrum of IIA string theory in the plane wave background and that of the BMN-like operators does not seem to be straightforward. This issue definitely deserves future studies.

We also examined the weak coupling limit $k \to \infty$ of the ABJM theory on $S^1 \times S^2$ and evaluated the partition function at finite temperature. We showed that the Hagedorn/deconfinement transition occurs in this limit of the ABJM theory as naturally expected.

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