Electromagnetic Polarizabilities and Charge Radii
of the Nucleons in the Diquark-model

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Abstract

The diquark model is used to calculate the electromagnetic polarizabilities and charge radii of the nucleons for three different potentials. Making the scalar diquark lower in mass introduces a mixing angle $\theta$ between the $|56\rangle$ and $|70\rangle$ states, which allows an improvement in value of all 6 properties. Generalizing the Gamov-Teller matrix and the magnetic moment operator to the diquark model gives constraints on this mixing. We obtain for the Richardson potential

$\theta = 23.2^\circ$, $\alpha_p = 7.9^{+1.0}_{-0.9} \times 10^{-4} fm^3$, $\alpha_n = 7.7^{+0.3}_{-0.6} \times 10^{-4} fm^3$, $\beta_p = 5.4^{+1.6}_{-0.4} \times 10^{-4} fm^3$, $\beta_n = 6.7^{+1.3}_{-0.7} \times 10^{-4} fm^3$, $\langle r^2 \rangle_p = 0.37^{+0.02}_{-0.03} fm^2$, $\langle r^2 \rangle_n = -0.07^{+0.03}_{-0.02} fm^2$. Additional pion cloud contributions could improve on all six results.

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The electric and magnetic polarizabilities, labeled $\alpha$ and $\beta$, respectively, have been measured recently\cite{1,2}, yielding the results $\alpha_p = (10.9 \pm 2.2 \pm 1.3) \times 10^{-4} fm^3$, $\beta_p = (3.3 \mp 2.2 \mp 1.3) \times 10^{-4} fm^3$, $\alpha_n = (12.3 \pm 1.5 \pm 2.0) \times 10^{-4} fm^3$ and $\beta_n = (3.5 \mp 1.5 \mp 2.0) \times 10^{-4} fm^3$. The experimental values are obtained by fitting the Compton scattering data to relations obtained from low energy theorems\cite{3}. In addition to the polarizabilities we will be concerned here with the charge radii $\langle r^2 \rangle = \sum e_i x_i^2$, whose values were measured to be $\langle r^2 \rangle_p = 0.708 fm^2$ and $\langle r^2 \rangle_n = -0.113 \pm 0.003 fm^2$. Theoretical studies, using the 3 symmetric quark model\cite{4}, Skyrme models\cite{4}, MIT-bag models\cite{4} or nonlinear meson theories\cite{4} have not obtained satisfactory results for all these quantities. A recent paper using chiral perturbation theory\cite{9} gives good agreement for $\alpha$, but predicts negative values for the magnetic polarizabilities.

In this paper we will use another approach, the diquark model. This model, which provides improved agreement with a wide range of data\cite{10}, reduces the three body problem of the quark picture to a two body problem and results, therefore, in a considerable simplification of the calculation of baryonic properties. To review the salient features, the proton and neutron
wavefunctions have spin-flavour forms

\[ |p, \uparrow\rangle = \frac{1}{\sqrt{1 + a^2}} \left( \sqrt{\frac{2}{3}} S_{uu} d - \sqrt{\frac{2}{3}} S_{ud} u \right) \left( \sqrt{\frac{2}{3}} |1, \downarrow\rangle - \sqrt{\frac{1}{3}} |0, \uparrow\rangle \right) + at_{ud}u|0, \uparrow\rangle \]

(1)

\[ |n, \uparrow\rangle = \frac{1}{\sqrt{1 + a^2}} \left( \sqrt{\frac{2}{3}} S_{dd} u - \sqrt{\frac{2}{3}} S_{ud} d \right) \left( \sqrt{\frac{2}{3}} |1, \downarrow\rangle - \sqrt{\frac{1}{3}} |0, \uparrow\rangle \right) + at_{du}d|0, \uparrow\rangle \]

(2)

where S and t are two different diquark states (S for sextet and t for triplet of SU(3) flavour) with spin S=1 and S=0, respectively. The parameter a=1 for the fully SU(6) symmetric scheme. The spin interaction between the two quarks, which form the diquark, is assumed to yield a mass splitting between the S- and the t-state and breaks SU(6) symmetry. From the QCD hyperfine interaction \( \Delta m \propto \vec{S}_1 \cdot \vec{S}_2 \) it follows that the S-state is heavier than the t-diquark. Hence a will differ from 1, as we discuss below. In addition there will be another mass splitting \( \Delta m_2 \) coming from the spin interaction between the quark and the diquark, which increases the spin \( \frac{3}{2} \) mass relative to the spin \( \frac{1}{2} \) state.

To determine the spatial dependence of the wavefunctions we use three different forms of scalar potentials, a logarithmic one \( V_L (r) = C \cdot \log \left( \frac{r}{r_0} \right) \), a superposition of linear and Coulomb-potential \( V_{CL} (r) = -\frac{4\alpha_s}{3r} + Cr \) and the Richardson potential[11] \( V_R \), which interpolates the Coulombic potential.
for small r and the linear potential for large r.

The expressions for the polarizabilities, derived for the nuclei of atoms\textsuperscript{3} can be modified for the nucleons, yielding

\[ \alpha = \frac{\alpha}{3M} \left( \sum e_i^2 \right) + \frac{2\alpha}{3} \sum \frac{|\langle 0| \vec{D} |n \rangle|^2}{E_n - E_o} \]  

(3)

\[ \beta = 2\alpha \sum \frac{|\langle 0| \sum \mu_i^z |n \rangle|^2}{E_n - E_o} - \frac{\alpha}{6} \left( \sum e_i^2 x_i^2 \right) - \frac{\alpha}{2M} \langle D^2 \rangle \]  

(4)

where \( \alpha \) is the fine structure constant, \( M \) the mass of the nucleon, \( m_i \) the mass of its constituents, \( \vec{D} \) the electric dipole operator and \( \mu_i \) the magnetic moment operator.

The nonrelativistic wavefunctions for all three potentials were solved numerically. To get the parameters of the different potentials we fitted the eigenenergies to different baryon states, the N(938), N(1710), \( \Delta(1232) \) and \( \Delta(1600) \). In the diquark picture\textsuperscript{12} they correspond to the (56,0), (56,0)** (56,0) and (56,0)*, respectively, where the \( \Delta \) wavefunction

\[ |\Delta^+, \uparrow\rangle = \left| \left( \sqrt{\frac{T}{3}} S_{ud}d + \sqrt{\frac{2}{3}} S_{uu}u \right) \left( \sqrt{\frac{T}{3}} |1, \uparrow\rangle + \sqrt{\frac{2}{3}} |0, \uparrow\rangle \right) \right| \]

has to be used for the two latter states. The dipole operator requires that the intermediate states of the electric polarizabilities eq.(3) are all \( L=1 \) states. Therefore we have to take (56,1) and (70,1) states as well as their excitations.

The |56\rangle states have the same form as eqs.(1) (2) and the |70\rangle states are
\[ |70^+\rangle = \left( \sqrt{\frac{7}{3}} S_{ud} - \sqrt{\frac{7}{6}} S_{ud^*} \right) \left( \sqrt{\frac{7}{3}} |1, \downarrow\rangle - \sqrt{\frac{7}{3}} |0, \uparrow\rangle \right) - \sqrt{\frac{7}{2}} t_{ud} |0, \uparrow\rangle \]

Excited states soon give a negligible contribution to \( \alpha \), since the denominators \( E_n - E_0 \) increase as the numerator decreases. It is sufficient to take the 2p and 3p wavefunctions into account. As we will see below it is necessary to introduce a mixing between the \( |56\rangle \) and the \( |70\rangle \) states. To see the essential features of a "symmetric diquark model" in what follows we would treat the nucleons as if they were pure \( |56\rangle \) states (i.e. set \( a = 1 \) in the wavefunctions). Defining \( C_{1t,s} \equiv \frac{m}{m+m_{t,s}} \) and \( C_{2t,s} \equiv \frac{m_{t,s}}{m+m_{t,s}} \) we get for the electric polarizabilities

\[
\underline{\alpha}_p = \frac{\alpha}{9M (1 + a^2)} \left( \langle r_t^2 \rangle a^2 (C_{1t}^2 + 2C_{2t}^2) + 3 \langle r_s^2 \rangle C_{1s}^2 \right) + \sum_{n=2,3} \frac{2\alpha}{3 (1 + a^2)^2} \left( \frac{a^2 \langle 56, np \mid r_t \mid 1s \rangle}{\sqrt{E_{np,t} - E_{1s,t}}} \left( \frac{1}{3} C_{1t} - \frac{2}{3} C_{2t} \right) + \frac{\langle 56, np \mid r_s \mid 1s \rangle}{\sqrt{E_{np,s} - E_{1s,s}}} C_{2s} \right)^2
\]

\[
\underline{\alpha}_n = \sum_{n=2,3} \frac{2\alpha}{27 (1 + a^2)^2} \left( \frac{a^2 \langle 56, np \mid r_t \mid 1s \rangle}{\sqrt{E_{np,t} - E_{1s,t}}} - \frac{\langle 56, np \mid r_s \mid 1s \rangle}{\sqrt{E_{np,s} - E_{1s,s}}} \right)^2 + \sum_{n=2,3} \frac{2\alpha}{9 (1 + a^2)^2} \left( \frac{-a^2 \langle 70, np \mid r_t \mid 1s \rangle}{\sqrt{E_{np,t} - E_{1s,t}}} - \frac{\langle 70, np \mid r_s \mid 1s \rangle}{\sqrt{E_{np,s} - E_{1s,s}}} \right)^2
\]
where $r_{s,t}$ are the coordinates of the reduced mass.

For the magnetic polarizabilities the first term of eq. (4) needs some modification. The magnetic moment operator has to be generalized for the quark-diquark states to

$$
\mu_z = g \frac{e_d}{2m_d} S^z + \frac{e_3}{2m_3} \sigma^z
$$

However this alone leads to problems when calculating the ratio of the magnetic moments $\frac{\mu_p}{\mu_n}$. Only a value of $g=0$ would reproduce the successful predictions of the symmetric quark model of $-1.5$ (compared to the experimental value of $-1.46$). Analyzing the difference between the diquark picture and the latter reveals the need for a contribution coming from $\langle S_{ud} | \mu_z | t_{ud} \rangle$, which is non-zero for a composite diquark. Therefore we add $\langle S_{ud} | \frac{e_1}{2m_1} \sigma^z | t_{ud} \rangle$ to eq. (7). If we include this term we get exactly the quark result again, in the limit $g \to 2$ and $m_d \to 2m$. This shows how important it is not to see the diquark naively as a single point particle but rather as a composite of two quarks. To continue, it is sufficient to take the $\Delta$ as the dominant intermediate state in eq. (4) and neglect the higher mass states. The second and third terms of eq. (4) are straightforward to calculate and so the magnetic polarizabilities become
\[ \beta_P = \frac{2\alpha}{81(M_\Delta - M_p)(1 + a^2)} \left( \frac{2 + 3a}{m} + \frac{g}{m_\alpha} \right)^2 \]
\[ - \frac{\alpha}{54(1 + a^2)} \left( a^2 \langle r_t^2 \rangle \left( \frac{C_{1t}^2}{m_t} + \frac{4C_{2t}^2}{m} \right) + \langle r_s^2 \rangle \left( \frac{11m_sC_{1s}^2 + 2C_{2s}^2}{m} \right) \right) \]
\[ - \frac{\alpha}{54M(1 + a^2)} \left( 3a^2 \langle r_t^2 \rangle \left( C_{1t} - 2C_{2t} \right)^2 + \langle r_s^2 \rangle \left( 2(4C_{1s} + C_{2s})^2 + (C_{1s} - 2C_{2s})^2 \right) \right) \]  
\[ (8) \]

\[ \beta_n = \frac{2\alpha}{81(M_\Delta - M_p)(1 + a^2)} \left( \frac{2 + 3a}{m} + \frac{g}{m_\alpha} \right)^2 \]
\[ - \frac{\alpha}{54(1 + a^2)} \left( a^2 \langle r_t^2 \rangle \left( \frac{C_{1t}^2}{m_t} + \frac{C_{2t}^2}{m} \right) + 3 \langle r_s^2 \rangle \left( \frac{C_{1s}^2}{m_s} + \frac{C_{2s}^2}{m} \right) \right) \]
\[ - \frac{\alpha}{54M(1 + a^2)} \left( 3a^2 \langle r_t^2 \rangle \left( C_{1t} + C_{2t} \right)^2 + \langle r_s^2 \rangle \left( 8(C_{1s} + C_{2s})^2 + (C_{1s} + C_{2s})^2 \right) \right) \]  
\[ (9) \]

The charge radii in the diquark picture are

\[ \langle r_P^2 \rangle = \frac{a^2 \langle r_t^2 \rangle}{3(1 + a^2)} \left( C_{1t}^2 + 2C_{2t}^2 \right) + \frac{\langle r_s^2 \rangle}{1 + a^2}C_{1s}^2 \]  
\[ (10) \]

\[ \langle r_N^2 \rangle = \frac{1}{3(1 + a^2)} \left( a^2 \langle r_t^2 \rangle \left( C_{1t}^2 - C_{2t}^2 \right) + \langle r_s^2 \rangle \left( C_{1s}^2 - C_{2s}^2 \right) \right) \]  
\[ (11) \]

Using these formulas with \( a = 1 \) we obtain the results shown in table 1. The values of the electric polarizabilities are too small, the magnetic quantities too big and the charge radii too small. Note especially that the neutron charge radius has the wrong sign and is very close to zero.

So far we have treated the nucleons as pure \( |56\rangle \) states. The situation
\[ \alpha \approx 1 \pm 0.45, \quad 0 = 0.25 \pm 0.02, \quad \langle r^2 \rangle = 0.005 \pm 0.004 \]

|               | Logarithmic | Lin.-Coulomb. | Richardson |
|---------------|-------------|---------------|------------|
| \( \alpha_p [fm^3] \times 10^{-4} \) | 4.1 ± 0.4   | 5.0 ± 0.4     | 5.2 ± 0.5  |
| \( \alpha_n [fm^3] \times 10^{-4} \) | 3.3 ± 0.1   | 4.5 ± 0.1     | 4.4 ± 0.2  |
| \( \beta_p [fm^3] \times 10^{-4} \) | 3.9 ± 0.2   | 7.8 ± 0.4     | 7.6 ± 0.2  |
| \( \beta_n [fm^3] \times 10^{-4} \) | 4.1 ± 0.2   | 8.4 ± 0.4     | 9.0 ± 0.4  |
| \( \langle r^2 \rangle_p [fm^2] \) | 0.20 ± 0.02 | 0.25 ± 0.01   | 0.25 ± 0.02|
| \( \langle r^2 \rangle_n [fm^2] \) | 0.005 ± 0.004 | 0.012 ± 0.006 | 0.015 ± 0.006 |

Table 1: Results for the polarizabilities and charge radii for different forms of potentials with no mixing, \( a=1 \)

turns out quite favourable if we introduce a mixing angle \( \theta \) between the \( |56\rangle \) and the \( |70\rangle \) states to form the nucleons

\[ |N\rangle = \cos \theta |56\rangle + \sin \theta |70\rangle \] (12)

which is equivalent to introducing a mixing parameter \( a \), for which \( \cos \theta = \frac{a+1}{\sqrt{2a^2+2}} \). The QCD hyperfine splitting as well as phenomenological studies\[15\] suggest that the S=0 state has lower mass and thus is a larger component of the nucleon. We will use values \( a \geq 1 \) in the above equations. To get an upper constraint on \( a \), we fit our model to the ratio of the nucleon magnetic moments \( \frac{\mu_p}{\mu_n} \) as well as to the axial to vector current coupling constant ratio \( \frac{g_A}{g_V} \).
Using the above magnetic moment operator for the diquark model and
the wavefunctions eqs.(1),(2) we obtain
\[ \mu_p = -3 \left( \frac{a^2 + a}{m} + \frac{g}{m_s} \right) \]
\[ \mu_n = \frac{3}{2} \left( \frac{3a^2 + 6a + 1}{2m} + \frac{g}{m_s} \right) \] (13)
The operator for \( \frac{g_a}{g_v} \), also called the Gamov-Teller matrix \[17\] is
\[ \frac{\langle p, \uparrow| \sum_i \tau_i^+ S_i^Z | n, \uparrow \rangle}{\langle p, \uparrow| \sum_i \tau_i^+ | n, \uparrow \rangle} \] (14)
As we generalized the magnetic moment operator to the diquark model we
will also have to generalize the Gamov-Teller matrix. Analogously we start
with
\[ \sum_i \tau_i^+ S_i^z = \tau_3^+ \sigma_3^z + hT_d^+ S_d^z \]
h can be factorized as \( h = t \ast g \) and comparing the terms to the quark picture
shows that \( t = \frac{1}{\sqrt{2}} \). As in the case of the magnetic moments we have to add
terms from the quark picture,
\[ \left( \tau_1^+ \sigma_3^+ \tau_2^+ \sigma_3^2 \right) | t_{ud}, 0 \rangle = \sqrt{2} | S_{uu}, 0 \rangle \]
\[ \left( \tau_1^+ \sigma_1^+ \tau_2^+ \sigma_2^2 \right) | S_{dd}, 0 \rangle = \sqrt{2} | t_{ud}, 0 \rangle \]
The denominator of eq.(14) remains \(-1\) also for the case of mixing and
so we get
\[ \frac{g_a}{g_v} = - \langle p, \uparrow| \sum_i \tau_i^+ S_i^Z | n, \uparrow \rangle = \frac{1 + 12a + 9a^2 + 4g}{9 (1 + a^2)} \] (15)
Now we take the ratio of the magnetic moments fixed at its experimental value $\frac{\mu_p}{\mu_n} = -1.46^{[18]}$ and use eq.(13) to calculate $g$ as a function of $a$ and $\frac{m_d}{m}$, which in turn (eq.(13)) gives $\frac{g}{g_v}$ as another function of those parameters. The dependence on $\frac{m_d}{m}$ is weak compared to the $a$-dependence. A value of $a = 2.5 \pm 0.1$ fits the experimental value of $\frac{g}{g_v} = 1.25^{[18]}$. Since taking the relativistic nucleon wavefunctions would have a lowering effect anyway on $\frac{g}{g_v}^{[19]}$ we will take $a = 2.5$ as the upper limit and take $\Delta a = -0.3$ for a theoretical error calculation. In terms of the mixing angle of eq.(12) $a = 2.5$ means $\theta = 23.2^\circ$ and therefore gives a small $|70\rangle$ state contribution to the nucleons.

Figure 1 shows the results for our quantities as a function of the mixing factor $a$ for the Richardson potential. The situation for the other potentials looks qualitatively similar. We see immediately that a large mixing factor improves the data for all 6 quantities and justifies our approach.

Table 2 shows the results for all 3 forms of potentials, where the diquark mass splitting was taken as $\Delta m = 100 \pm 50 MeV$. Although the absolute values for the charge radii are still too small they acquired the right sign. The results for the superposition of the linear & Coulombic potential and the Richardson potential lie within the error bars of the the experimental results. The magnetic polarizabilities are on the upper limit and the
Table 2: Results for the polarizabilities and charge radii for different forms of potentials

electric quantities on the lower edge. The magnitudes of the charge radii are too small by about 50%. Overall, the ability of the diquark model to give reasonable values for all the static properties of the nucleon are very encouraging.

Analyzing the contributions of the different terms shows us that an increase of the terms containing the spatial wavefunction would lower the electric and raise the magnetic values of the polarizabilities. A pion cloud around the nucleon core gives exactly such a contribution. Such sources were already included by Weiner and Weise[16] and had considerable effects on the electric polarizabilities. Pion clouds will also effect the charge radii.
The transitions $p \rightarrow n + \pi^+$ give positive contribution and $n \rightarrow p + \pi^-$ yields the desired negative part. Therefore we have begun to consider the influence the pion cloud will have on the constraints $\frac{\mu_p}{Y_e}$ and $\frac{\mu_p}{\mu_n}$, and to find the corresponding values for the mixing $a$. The core values can be directly taken from this paper - the pion cloud terms will have to be added.

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Figure Captions

1. Electric (a), magnetic (b) polarizabilities and charge radii (c) for neutron (solid line) and proton (dashed line) as a function of the mixing parameter $a$, which is explained in the text.
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