Optimal location of cutout within a cross-ply laminated cantilever beam for maximum lateral buckling load

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Abstract. This paper deals with the location optimization of a cutout within a laminated cantilever beam for maximizing the lateral buckling load. Various shapes of cutout, such as circular, elliptical, triangular, are generally used in structural components as a design requirement or sometime to reduce the overall weight of the structures. In this study, a laminated cantilever beam, with a single cutout, is considered. The beam is subjected to a concentrated load at the free end. The objective is to obtain the optimal location of the cutout which resist maximum lateral buckling load. For this an optimization routine has been used in which a finite element calculation of critical lateral buckling load in ANSYS is coupled with an in-built global optimization tool (Genetic Algorithm) in MATLAB. The optimal results are reported for various cases arising from change of geometry and material properties of the plate. It has been concluded that the position of cutout in the laminated beam plays a significant role while designing against lateral buckling load.

1. Introduction

Composite materials, due to their desirable characteristics such as low weight and high strength, are widely used in engineering applications such as aerospace component, shipping industries, automotive parts and defence application [1]. It has been also found that thin beam made up of composite materials are used as structural components in these engineering structures. Often, these structural components are subjected to various types of loading e.g. lateral, axial, torsional loads. Due to these loads, beams are likely to fail due to buckling instability. When a laminated beam is subjected to loads that are perpendicular to the beam’s axis, the beam will have lateral displacement in the downward direction and it might become unstable in a twisting mode. In this twisting mode the cross-section rotate about the beam axis. This is termed as the lateral buckling of the beam. The load at which buckling occurs is called the critical lateral buckling load. For design requirement, these structures are often provided with cutout or hole to provide access for inspection, electric lines and fuel lines etc. Therefore, study of lateral buckling characteristics of composite beam with cutout is necessary in the design process of the structure.

In the literature, many research papers can be found that investigate the effect of cutout on
the buckling of laminated composite beam due to uniaxial or biaxial type of loading [2, 3, 4]. However, only few research papers are found which discuss the lateral buckling of beam with cutout. Among them, Shanmugam and Thevendran [5] investigated, numerically using energy approach, the lateral buckling of rectangular beam with central opening. The authors considered both cantilever and simply supported beam. Eryigit et al. [6] studied the effect of lateral buckling on the central circular cutout within a laminated beams using both numerical and experimental methods. Eryklig et al. [7] determined the lateral buckling load of laminated beam with different cutouts e.g. square, elliptical, circular, triangular and rectangular using experimental and finite element analysis approach. Pasinli [8] studied the effect of lateral buckling with double circular and square hole in the laminated cantilever beam. Pasinli [9] also investigated the effect of single square cutout dimension and location in composite beam using finite element software.

It has been observed from above literature review that most of the authors used finite element software to study the effect of lateral buckling on the laminated beam with different cutout size and shape. In the finite element software, authors first selected the cutout position and then critical buckling load is determined at that particular position. The limitation of these studies is that it might become very difficult to search all possible locations of the cutout manually for a longer beam in order to find the optimal location for which the lateral buckling load will be maximum. It may also be very time consuming and thereby impractical. No literature discussed the approach of selecting the cutout location automatically by some optimization routine. Therefore, the aim of this present study is to select the optimal cutout location in the laminated beam automatically by using a suitable optimization technique. A similar type of automated approach was developed by the authors earlier [10, 11] for the uniaxial loading for the plates. In this paper, optimization study is performed for finding the location of cutout in beam where lateral buckling load is maximum. The optimal cutout location results reported here will be useful while designing the composite beam with different cutout and subjected to lateral load.

2. Problem definition

In this study, a cross-ply $(0/90)_{4s}$ laminated beam of length $a$ and width $b$ with circular (or elliptical) cutout is taken at an arbitrary position $x_c$ and $y_c$ from the center of beam $(0,0)$ as shown in Fig. 1. The laminated beam is fixed at one end and at the free end a vertical load $(P)$ is applied. The circular cutout of radius $r$ is considered. The objective here is to obtain the cutout position $(x_c, y_c)$ in the cantilever beam which sustain maximum lateral buckling load.

![Figure 1: Schematic of the laminated cantilever beam with circular cutout of radius (r) placed at position (x_c, y_c).](image)
$P_{cr}$. For this we have used ANSYS parametric design language (APDLs) to calculate the critical lateral buckling load ($P_{cr}$). And to obtain the optimal position ($x_c$ and $y_c$) of the cutout, this buckling computation is coupled with a MATLAB based optimization routine (see Section 4 for the details).

3. Lateral buckling analysis
For the buckling analysis, a geometrical 2D model is created in ANSYS and then meshed with SHELL181 element. This SHELL181 element has six degree of freedom at each node: 3 translational and 3 rotations [12]. It is suitable for composite layered structure. A mesh convergence study has been done to keep in mind the accuracy of result and relatively finer mesh at the cutout edges has been used. The laminated beam with a circular hole is meshed in the finite element software ANSYS as shown in Fig. 2.

To determine the stiffness matrices in the eigenvalue buckling analysis, ANSYS uses strain of first and second order [13]. The first order strain term generate stiffness matrix ($k_0$) corresponding to no stress state. The second order strain is for initial stress stiffness matrix ($k_i$). This matrix account for effect of existing initial stress. Therefore, total stiffness matrix of the beam is: $K = k_0 + k_i$. At the beginning of analysis, initial stress ($\sigma_0$) is unknown. When we apply $P$ load to the beam, stress reach the level of ($\alpha\sigma_0$), where $\alpha$ is scaler multiplier.

$$\Delta P = [k_0 + \alpha k_i] \Delta u,$$

Where $\Delta u$ and $\Delta P$ are incremental nodal displacement vector and corresponding incremental force vector respectively. When buckling occurs, external load do not changes with increase in its displacements. Mathematically, it is written as:

$$[k_0 + \alpha k_i] \Delta u = 0,$$

Since $\Delta u \neq 0$. Therefore, for non-trivial solution taking determinant of above Eq.(2)

$$det [k_0 + \alpha k_i] = 0,$$

The solution of Eq.(3) provide $N$-different eigenvalues ($\alpha_i$) where $N$ is the dimension of stiffness matrix. For the first buckling mode $\alpha_1$ corresponds to the critical lateral load $P_{cr} = \alpha_1 \times P$, where buckling occurs and $\Delta u$ becomes the eigenvectors defining the buckling mode.

3.1. Validation of lateral buckling load
For the verification of lateral buckling load computation, we compare our results with the published literature. For this we had taken a cantilever beam of same dimension, materials properties, and fiber orientations as given in reference [7]. We validated the critical lateral buckling loads result for both solid beam and beam with circular cutout. The result obtained are shown in Table 1. It is observed that approximately same result for both present study and references [7] are obtained. Therefore, it is noted that the results obtain from present study are correct and can be used for further studies.

4. Optimization routine
An optimization routine has been created in MATLAB to obtain the position of cutout where lateral buckling load is maximum in composite cantilever beam. For this we coupled the critical lateral buckling load computation in ANSYS with the MATLAB optimization routine. This principle is described in three stages as shown in Fig. 3. Firstly, For a certain geometry of beam, ANSYS parametric design language (APDLs) accepts input parameter $n = (x_c$ and $y_c$) and return
critical lateral buckling load \( (P_{cr}) \). Secondly, with the help of input parameter \( (n) \) and critical lateral buckling load \( (P_{cr}) \) an objective function is defined by MATLAB code. Here, objective function is to maximize the critical lateral buckling load. Thirdly, by selecting a suitable in-built optimization method such as genetic algorithm(GA), fminsearch etc. in MATLAB, optimal location is obtained.

In this paper, we selected global search optimization tool Genetic algorithm (GAs) for our analysis. This optimization can also be done by using gradient based optimization such as fminsearch [10] but the restriction might be the number of design variables. A large no of design parameter might become a problem for gradient based optimization, as it will not give converged solution to the global optimum. Gradient based optimization [14], make use of differential calculus in locating the optimum solution. In that there is a high chance that optimum solution get stuck in a first local optima because slope at any local optima is zero. This will not be the case for GA, as it is considered to be a global optimization tool. It is used to find global optimum solution, in presence of multiple local optimum. The only limitation is with time taken for solution to converge. This can be fixed by proper selection of GA parameter like number of generations, population size etc. The main advantage of using Genetic algorithm [15] is that it searches a population of point in parallel not only at one point. In GA it is also not necessary that the function to be differentiable or continuous. With the help of objective function, it determines the fitness level which influence the direction of optimal search solution. As MATLAB in-built GA optimization tool is coded for minimization problem only, we convert the maximization problem into a minimization problem by suitably multiplying the objective function with -1.
function with negative sign i.e \( P_{cr}^* = -(P_{cr}) \).

The optimization problem is defined as:
Maximize: \( P_{cr}(x_k) \)
with respect to: \( x_k = [x_c, y_c] \)
Subject to: \( 0 \leq x_k \leq x_{kcR} \) where \( x_{kcR} = [x_{cR}, y_{cR}] \)
Here, \( x_{kcR} \) is the permissible range i.e a minimum margin between edge of cutout and the edge of beam. They are calculated by the formula below.
\[
\begin{align*}
x_{cR} &= \frac{a}{2} - 0.05 \times b - r \\
y_{cR} &= \frac{b}{2} - 0.05 \times b - r
\end{align*}
\]

\( a, b \) are the length and width of beam respectively and \( r \) is the radius of cutout.

\[ P_{xcr} = P_{cr} - P_{0cr}, \] (4)

where \( P_{cr} \) and \( P_{0cr} \) are the critical lateral buckling load of beam with cutout and without cutout, respectively.

In the subsequent sections, we first discuss the results for cross-ply laminate beam with a circular cutout and then cross-ply laminate beam with an elliptical cutout is discussed. At the end, the effect of optimal location for different circular cutout size is discussed.

5.1 Circular cutout
Figure 4 show the optimal location of circular cutout of radius \( r = 0.1 \text{ m} \) in a cantilever beam. It is observed that for beam with aspect ratio \( (a/b) = 1 \) to 1.3, optimal cutout location is at center of the beam. As we increase the aspect ratio \( (a/b) = 1.4 \) to 1.7, optimal cutout location shift
above the center point but remain at the center line of the y-axis. For aspect ratio \((a/b) = 1.8\) to 4 , cutout location shift towards the edge of the beam but remain on the \(x\)-axis.

![Figure 4: Optimal center locations \((x_c, y_c)\) of the circular cutout within the cross-ply \((0^\circ/90^\circ)_{4s}\) laminated cantilever beam.](image)

Figure 4: Optimal center locations \((x_c, y_c)\) of the circular cutout within the cross-ply \((0^\circ/90^\circ)_{4s}\) laminated cantilever beam.

![Figure 5: Nondimensional buckling load \((P_{xcr})\) at three different location of cutout.](image)

Figure 5: Nondimensional buckling load \((P_{xcr})\) at three different location of cutout.

Figure 5 show the plot between Nondimensional lateral buckling load \((P_{xcr})\) vs aspect ratio \((a/b)\) of the beam for three different position of cutout. First position of cutout is at the center \((0, 0)\) of the beam. Second position of cutout is at the \((a/4, 0)\) of the beam. Third position of the cutout is taken at as obtained in Fig. 4. From this plot it is indicated that critical lateral buckling load is maximum when the cutout is at the optimal position.

5.2. Elliptical cutout
In this, we considered elliptical cutout in a cross-ply \((0^\circ/90^\circ)_{4s}\) laminated cantilever beam. The area of elliptical cutout is same as circular cutout of radius \((r = 0.1)\). The optimal location \((x_c, y_c)\) of the elliptical cutout are shown in figure 6. In the figure 7 we have taken five different position of elliptical cutout including one optimal position as obtained in Fig. 6. It is seen that at different cutout position, buckling load is different. There is one optimal position in the laminated beam where lateral buckling load is maximum.

6. Effect of circular cutout size
In this section, we considered circular cutouts of various size (e.g. \(r = 0.1, 0.15, 0.2\)) within cross-ply \((0^\circ/90^\circ)_{4s}\) laminated cantilever beam. The optimal \(xc\) and \(yc\) location of cutout for maximum lateral buckling load are shown in Fig. 8 and 9. It is observed that for different size of cutout, optimal \(xc\) and \(yc\) location would be different. For small beam, cutout location is at the center and for longer beam, position varies from center to the edge of the beam. Therefore, while designing laminated cantilever beam with cutout for maximum lateral buckling load, both size and location of cutout play a significant role.

7. Conclusion
In this paper, optimal position of circular and elliptical cutout in a laminated cantilever beam for maximum lateral buckling load has been studied. For this we couple critical lateral buckling load computation in ANSYS with a MATLAB based optimization scheme. Optimal results for a given radius of circular and elliptical cutout have been presented. From our study, the following
Important conclusion are made.

- Position of circular or elliptical cutout has significant effect on the laminated beam for maximum lateral buckling load.

- Optimal location of cutout depends on shape of the cutout. For circular cutout, optimal location for maximum buckling load is at the centre, for smaller beam i.e aspect ratio $(a/b)=1$ to $1.3$. For larger beam optimal cutout location shift from center to towards the edge of the beam. For elliptical cutout, optimal location is at the centre of the beam for aspect ratio $(a/b)=1$ to $1.3$. For aspect ratio $(a/b)=1.4$ to $1.6$ and $1.8$, optimal location shift above the centre point but remain at the centre line of the y-axis. For aspect ratio
$(a/b) = 1.9$ to $4$ and $1.7$, optimal location is at the edge of the beam. This shows that optimal location vary with shape of cutout.

- As size of cutout in the laminated beam vary, optimal location changes. This indicates that optimal location of cutout depends on the size of cutout.

Hence it is concluded that while designing laminated cantilever beam with cutout, knowledge of optimal position of the cutout will play an important role.

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References

[1] Jones, R.M. and Bert, C.W., 1975. Mechanics of composite materials.
[2] Larsson, P.L., 1987. On buckling of orthotropic compressed plates with circular holes. Composite Structures, 7(2), pp.103-121.
[3] Jain, P. and Kumar, A., 2004. Postbuckling response of square laminates with a central circular/elliptical cutout. Composite Structures, 65(2), pp.179-185.
[4] Ghannadpour, S.A.M., Najafi, A. and Mohammadi, B., 2006. On the buckling behavior of cross- ply laminated composite plates due to circular/elliptical cutouts. Composite Structures, 75(1-4), pp.3-6.
[5] Thevendran, V. and Shanmugam, N.E., 1992. Lateral buckling of narrow rectangular beams containing openings. Computers structures, 43(2), pp.247-254.
[6] Eryigit, E., Zor, M. and Arman, Y., 2009. Hole effects on lateral buckling of laminated cantilever beams. Composites Part B: Engineering, 40(2), pp.174-179.
[7] Erklig, A., Yeter, E. and Bulut, M., 2013. The effects of cut-outs on lateral buckling behavior of laminated composite beams. Composite Structures, 104, pp.54-59.
[8] Pasinli, A., 2013. Shape and position effects of double holes on lateral buckling of cantilever composite beams. Composites Part B: Engineering, 55, pp.433-439.
[9] Pasinli, A., 2014. Lateral buckling analysis of composite cantilevers with square hole using the finite element method. Materials Testing, 56(3), pp.255-260.
[10] Jana, P., 2016. Optimal design of uniaxially compressed perforated rectangular plate for maximum buckling load. Thin-Walled Structures, 103, pp.225-230.
[11] Choudhary, P.K. and Jana, P., 2018. Position optimization of circular/elliptical cutout within an orthotropical rectangular plate for maximum buckling load. Steel and composite structures, 29(1), pp.39-51.
[12] ANSYS Inc. (2015). “ANSYS Reference Manual.”. Release 15.0 Documentation for ANSYS.
[13] Peter, K. (1994). ANSYS Theory Reference Manual, Release 5.6. Ansys Inc.
[14] Kalyannoy, D. (2001). Multi objective optimization using evolutionary algorithms (pp. 124-124). John Wiley and Sons.
[15] Callahan, K. J., Weeks, G. E. (1992). Optimum design of composite laminates using genetic algorithms. Composites Engineering, 2(3), 149-160.