Lattice effective theory and the phase transition at finite densities

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Abstract

The transition from the hadronic phase to the phase of color-superconductivity at large densities is addressed by an effective theory which incorporates the Yang-Mills dynamics in addition to the di-quark degree of freedom. A toy version of this theory is studied by lattice simulations. A first order phase transition separates the regime of broken color-electric flux tubes from the color superconducting phase. My findings suggest that the quark and gluon liberation occurs at the same critical chemical potential.

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Introduction. While the QCD phase diagram is well understood as function of the temperature $T$ for small baryon chemical potential $\mu$, very little is known about the phase diagram as function of $\mu$ and small temperatures. At very large densities, one might expect that a Fermi surface of quarks exists by virtue of asymptotic freedom. The residual weak gluonic interactions therefore lead to the formation of a condensate in certain di-quark channels \[1, 2\]. For a baryon chemical potential of the order of several GeVs, the Fermi surfaces smears out due to the strong interactions, and when confinement effects become important at small $\mu$, the quark matter is re-arranged forming a Fermi surface of baryons. In my talk, I will address this transient regime.

The effective theory. For this purpose, I will develop a lattice effective theory for the degrees of freedom which are relevant at the transition: the Yang-Mills degrees of freedom, and the composite scalar field representing the 2-flavor di-quark field, $\phi_A = \epsilon_{ABC} \epsilon^{ik} \bar{q}_c^B \gamma^5 q^C_k$, where $A, B, C = 1 \ldots 3$ denote the color index of the SU(3) fundamental representation and $i, k$ represent the flavor index. $q$ and $q_c$ are the quark spinor and its charge conjugated.

I am assuming the following evolution of scales: even if one encounters a first order transition, one expects that the relevant correlation length grows when the chemical potential approaches the critical values from below, $\mu \to \mu_c^-$. In the low density phase, di-quarks are confined within the baryon. The crucial assumption is that e.g. the electro-magnetic mean square radius increases less rapidly than the correlation length. In this case, one might treat the composite di-quark field as a point-like Higgs field belonging to the fundamental representation of the Yang-Mills color group. The effective theory is determined by gauge invariance and renormalizability. The Higgs Lagrangian with a non-vanishing di-quark chemical potential included is therefore given by

$$L_{\text{Higgs}} = [D_\mu \phi(x)]^\dagger [D_\mu \phi(x)] + [m^2 - \mu^2] \phi^\dagger(x) \phi(x) + \lambda [\phi(x)^\dagger \phi(x)]^2 - i \mu Q_0(x),$$  

where

$$Q_0(x) = \frac{1}{i} \left\{ \phi^\dagger(x) D_0 \phi(x) - [D_0 \phi(x)]^\dagger \phi(x) \right\}$$

is proportional to the Baryon density, which, in the present effective theory, is entirely provided by the scalar currents. One observes that for $\mu \approx m$ the theory is pushed towards the Higgs, i.e., the color-superconducting, phase. The aim of the present investigation is to study the competition between the confining forces and the di-quark correlations of the superconducting phase.
**Bose-Einstein condensation versus confinement.** To my knowledge, lattice Yang-Mills theory is the only possibility to put rigor to an effective theory designed for a description of confinement as well as condensation effects. In this first investigation, I will adopt a coarse graining point of view: I will study an SU(2) gauge group with a fundamental Higgs field rather than the realistic gauge group SU(3). I will drop the imaginary part of the action which gives rise to severe sign problem in practical simulations. With the latter simplification, one looses the interpretation of $\mu$ as di-quark chemical potential. An inspection of (1), however, shows that this parameter can still be used to change the sign of the mass squared term of the Landau theory and to drive the theory towards the Higgs phase. The complete lattice effective action, which is subject of the simulations below is given by

$$S_{\text{latt}} = \beta \sum_p \frac{1}{2} \text{tr} U_p$$

$$+ \kappa \sum_{x \nu} \rho_x \rho_{x+\nu} \left[ 1 + (\cosh(\mu a) - 1) \delta_{\nu 0} \right] \text{tr} \{\alpha(x) U_{\nu}(x) \alpha^+(x + \nu) \}$$

$$- \sum_x \left\{ \lambda [\rho_x^2 - 1]^2 + \rho_x^2 \right\}$$

where $p$ denotes the plaquettes of the discretized space-time, and $U_p$ is the plaquette matrix constructed in the usual way from the link variables. $\kappa$ is the Higgs hopping parameter, and $\lambda$ parameterizes the strength of the quartic Higgs coupling. The $SU(2)$ matrix $\alpha$ represents the angular part of the Higgs field, and $\rho(x)$ its modulus.

**Litmus papers.** For a detection of the Higgs phase, one usually uses the expectation values of the spatial Higgs hopping term, i.e., $E = \langle \text{tr} \phi^+_f U_{x,k} \phi_{x+k} \rangle$, $k \in \{1, 2, 3\}$. One observes that $E$ jumps when the first order transition line (see figure 1) is crossed. Alternatively, one might use that fact that, after gauge fixing, the residual global color symmetry is spontaneously broken in the Higgs phase. Here, I use Landau gauge for these purposes. If $\phi^f$ denotes the gauge fixed Higgs field, I use $\langle \phi^f \rangle = \left\langle \left( \frac{1}{N} \sum_x \phi^f(x) \right)^2 \right\rangle^{1/2}$ as an indicator for color-superconductivity.

The properties of the so-called center vortices are ideal candidates to understand quark confinement. Using the so-called Maximal Center Projection, these vortex matter was observed to extrapolate properly to continuum theory. A percolating vortex cluster signals the formation of color-electric...
Figure 1: The phase diagram of the SU(2) Yang-Mills theory with a fundamental Higgs field; schematic sketch (left panel); $E$ and the vortex density as function of $\kappa$ in the crossover regime.

flux tubes: at zero density, this leads to quark confinement, while at non-zero densities short flux tubes are formed until the color-electric string breaks (see figure 1). It will turn out that the (planar) vortex density is the interesting observable in the present case.

In order to address the question whether quark- and gluon-deconfinement occurs at the same $\mu_c$, a litmus paper for the liberation of gluons is required. The vacuum energy density is the ideal quantity, since it rapidly rises when gluonic black body radiation becomes possible due to de-confinement. Here, I use the expectation value of the difference between the spatial and time-like plaquettes, which is know to contribute the major part to the energy density [8].

Results. Let us first study the phase-diagram of the effective lattice theory as function of $\kappa$ and $\beta$ for a vanishing chemical potential. I find that the crossover at small values $\beta$ as function of $\kappa$ is accompanied by a drastic reduction of the vortex density. These results are in agreement with the recent achievements reported in [9]. With the help of the vortex density, it
Figure 2: The transition from the confinement to the color-superconducting phase is possible to extend the line of the phase transitions (green lines) into the crossover regime. Figure 1 (left panel) illustrates my findings concerning the phase diagram. The right panel shows the result of lattice simulation using a $6^4$ lattice, $\beta = 0.4$. Hence, the line provided by the vortex suppression (red line in figure 1 left panel) is able to separate between the regime of short color electric flux tubes and the regime which has lost is capability to form color-electric strings. This line can be viewed as the analog of the “Kertész-line” $[10, 11]$ of the Ising model. The center vortices in the present case can be used in the same manner as the Polyakov lines in the context of Yang-Mills theory with dynamical quarks: in the latter case, the “Kertész-line” of percolating Polyakov lines separates the phase with a broken color electric string from the Coulomb type phase $[12]$.

Subsequently, I have chosen the parameters $\beta = 2.3$, $\kappa = 0.1$, $\lambda = 0.01$, $12^4$ lattice to lie within the regime of the broken flux tubes. I have then increased the parameter $\mu$. At small values $\mu$, the confining effects dominate and the dependence of $E$ with $\mu$ is small. If $\mu$ exceeds a critical value, the onset of color-superconductivity is signaled by the rapid increase of $E$ and a value
of $\langle \phi^4 \rangle$ significant from zero (see figure 2 left panel). At the same time, the vortex density rapidly drops indicating that the regime of short flux tubes was abandoned. The drop of the vortex density is accompanied by the rise of the difference between the spatial and time-like plaquettes. This indicates that liberation of gluons and quarks takes place at the same critical value for $\mu$.

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