Nonlinear $N = 2$ Supersymmetry, 
Effective Actions and Moduli Stabilization

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Abstract

Nonlinear supersymmetry is used to compute the general form of the effective $D$-brane action in type I string theory compactified to four dimensions in the presence of internal magnetic fields. In particular, the scalar potential receives three contributions: (1) a nonlinear part of the $D$-auxiliary component, associated to the Dirac-Born-Infeld action; (2) a Fayet-Iliopoulos (FI) $D$-term with a moduli-dependent coefficient; (3) a $D$-auxiliary independent (but moduli dependent) piece from the $D$-brane tension. Minimization of this potential leads to three general classes of vacua with moduli stabilization: (i) supersymmetric vacua allowing in general FI terms to be cancelled by non-trivial vacuum expectation values (VEV’s) of charged scalar fields; (ii) anti-de Sitter vacua of broken supersymmetry in the presence of a non-critical dilaton potential that can be tuned at arbitrarily weak string coupling; (iii) if the dilaton is fixed in a supersymmetric way by three-form fluxes and in the absence of charged scalar VEV’s, one obtains non supersymmetric vacua with positive vacuum energy.
1 Introduction

$D$-branes break dynamically half of the bulk supersymmetries which are realized on their world-volume in a nonlinear way. This nonlinear supersymmetry can provide a powerful tool for computing the off-shell brane effective action [1]. This is particularly important for moduli stabilization in the presence of internal magnetic fluxes. Indeed, for generic Calabi-Yau compactifications of type I string theory to four dimensions, magnetic fluxes generate a potential for closed string Kähler class moduli and play complementary role to three-form fluxes that generate a potential for the complex structure moduli and the dilaton [2, 3], in order to stabilize all closed string moduli [4] without introducing non-perturbative effects [5, 6, 7, 8]. In this work, we explore the tool of nonlinear $\mathcal{N} = 2$ supersymmetry to determine the brane effective action and in particular the scalar potential.

We first rederive the Dirac-Born-Infeld (DBI) action from the $\mathcal{N} = 2$ free Maxwell action by imposing the standard nonlinear constraint [9, 10], that relates its $\mathcal{N} = 1$ chiral and vector multiplet components. Eliminating the former, one obtains the DBI action for the latter which is identified with the Goldstino multiplet of the second supersymmetry that becomes nonlinearly realized [1]. Indeed, the corresponding transformations are modified by an additive constant piece. We can thus compute the off-shell scalar potential which is a nonlinear function of the $\mathcal{N} = 1$ D-auxiliary field.

We then demonstrate that the ordinary $\mathcal{N} = 1$ Fayet-Iliopoulos (FI) term is also invariant under the second nonlinear supersymmetry. Therefore, when added to the DBI potential, upon elimination of the D-auxiliary field, it yields an expression that depends nonlinearly on the coefficient of the FI-term which is a function of the closed string Kähler class moduli.

We finally show that the full potential contains an additive constant piece (from the point of view of global supersymmetry on the brane) arising from the brane tension, which we compute by taking into account the Ramond-Ramond (R-R) tadpole cancellation condition. Analyzing the resulting potential, including the dilaton factor, one finds three possible generic classes of minima:

1. A supersymmetric vacuum with Kähler class moduli stabilized by the vanishing condition for the coefficients of the FI-terms. An example of such complete stabilization is provided by the toroidal models of Refs. [6, 7, 11], in which case the complex structure moduli can also be stabilized by turning on magnetic fluxes on holomorphic two-cycles (which are absent in Calabi-Yau manifolds), leaving only the dilaton unfixed. However, these examples also show that sometimes the supersymmetry conditions are incompatible with R-R tadpole cancellations,
unless non-vanishing vacuum expectation values (VEV’s) for charged scalar fields on the branes are turned on, as well. Obviously, these break partly the gauge symmetry on the branes.

2. Alternatively, in the absence of charged scalar VEV’s, supersymmetry breaking vacua can be found. If the dilaton is already fixed in a supersymmetric way, for instance by three-form fluxes, these vacua are of de Sitter (dS) type with positive vacuum energy.

3. Otherwise, by going off criticality in less than ten dimensions, one brings to the scalar potential an extra dilaton-dependent piece proportional to the central charge deficit, resulting to a supersymmetry breaking anti-de Sitter (AdS) vacuum with negative energy. This non-critical dilaton potential corresponds to a particular gauging of \( \mathcal{N} = 2 \) effective supergravity associated to a shift isometry of the kinetic term of the universal (dilaton) hypermultiplet. The dilaton is then fixed at a value that can lead to an arbitrarily weak coupling, by making the central charge deficit infinitesimally small. A simple example is provided by replacing one free string coordinate with a conformal model from the minimal series.

The organization of this paper is as follows. In Section 2, we review the main properties of \( \mathcal{N} = 2 \) linearly-realized supersymmetry. In particular, we describe the single tensor multiplet and the abelian vector multiplet in terms of two real \( \mathcal{N} = 1 \) vector superfields. We then construct the \( \mathcal{N} = 2 \) superspace and derive the general form of an \( \mathcal{N} = 2 \) supersymmetric action with arbitrary prepotential and FI terms of both electric and magnetic type, as well as the vector-tensor multiplet couplings. In Section 3, we describe the algorithm to obtain nonlinear \( \mathcal{N} = 2 \) supersymmetric actions. By imposing a nonlinear constraint, we derive the modification in the supersymmetry transformations, the DBI action and the corresponding off-shell scalar potential as a function of the D-auxiliary field. We also show that the ordinary FI term is invariant under the \( \mathcal{N} = 2 \) nonlinear supersymmetry. In Section 4, we discuss the \( \mathcal{N} = 2 \) supergravity coupling and describe in particular the gauging that corresponds to adding a non-critical dilaton potential to be used in the following section. In Section 5, we apply our formalism to compute the scalar potential in four-dimensional type I string compactifications in the presence of internal magnetic fields. We then analyze this potential and find its supersymmetric and supersymmetry breaking minima. In addition, for self-consistency, there are three appendices. Appendix A contains our conventions for \( \mathcal{N} = 1 \) superspace, some useful identities involving super-covariant derivatives are listed in Appendix B and, finally, Appendix C presents the solution of the constraint for the nonlinear supersymmetry used in Section 3, following Ref. [1].
2 Linear $\mathcal{N} = 2$ supersymmetry

The $D$-brane configurations we are interested in have linear $\mathcal{N} = 1$ supersymmetry and a second nonlinearly-realized supersymmetry. A convenient and simple formulation is then to use $\mathcal{N} = 1$ superspace and construct linear $\mathcal{N} = 2$ supersymmetry in this superspace. The nonlinear realization is obtained by imposing constraints.

In this context, there is a very simple way to realize linearly $\mathcal{N} = 2$ supersymmetry on $\mathcal{N} = 1$ superspace. Our conventions for $\mathcal{N} = 1$ superspace are presented in Appendix A. Start with two $\mathcal{N} = 1$ superfields $V_i$, $i = 1, 2$. Under $\mathcal{N} = 1$ supersymmetry, 

$$\delta V_i = (\epsilon Q + \tau \overline{Q}) V_i, \quad (i = 1, 2),$$

with spinor parameter $\epsilon$. Then since 

$$\{Q_{\alpha}, \overline{Q}_\dot{\alpha}\} = \{D_\alpha, \overline{D}_\dot{\alpha}\} = -2i(\sigma^\mu)_{\alpha\dot{\alpha}} \partial_\mu$$

while all other anticommutators vanish, one can define a second supersymmetry with spinor parameter $\eta$ by the transformations

$$\delta^* V_1 = (a \eta D - \bar{a} \eta \overline{D}) V_2, \quad \delta^* V_2 = -(b \eta D - \bar{b} \eta \overline{D}) V_1,$$

with two complex numbers $a$ and $b$ such that $ab = 1$. We will use the convention $a = -i/\sqrt{2}$ and $b = -\sqrt{2}i$. Notice that this procedure explicitly eliminates the $SU(2)$ covariance of the $\mathcal{N} = 2$ supersymmetry algebra.

Transformations (2.1) provide an off-shell, linear realization of $\mathcal{N} = 2$ supersymmetry on $\mathcal{N} = 1$ superfields $V_1, V_2$. In order to describe an irreducible $\mathcal{N} = 2$ multiplet, constraints compatible with both supersymmetries must be applied on $V_1$ and $V_2$. For instance, if $V_1$ is chiral, then, since $\delta^* V_2 = -b\eta DV_1$, $V_2$ cannot be chiral (or antichiral): this procedure cannot be used to construct for instance the hypermultiplet (which does not admit an off-shell realization) using two chiral superfields. It can however be used to describe the single tensor multiplet and the abelian vector multiplet which are of direct interest for us. Since it will be needed in abelian gauge transformations, we begin with the single-tensor multiplet.

2.1 The (single) tensor multiplet

The $\mathcal{N} = 2$ tensor multiplet [12, 13, 14] describes an antisymmetric tensor, three real scalars and two Majorana spinors. These are the physical states of a chiral and a linear $\mathcal{N} = 1$ superfields.
Suppose that $V_1$ is a real linear superfield, $V_1 = L$ with $D\bar{D}L = \bar{D}D = 0$. Since $D_\alpha (\eta DL) = \bar{D}_\alpha (\bar{\eta}D L) = 0$, the natural partner of $V_1 = L$ is then $V_2 = \phi \pm \bar{\phi}$, with $\phi$ chiral and transformations

$$\delta^* L = -\frac{i}{\sqrt{2}} [\eta D \phi + \bar{\eta} D \bar{\phi}] = -\frac{i}{\sqrt{2}} [\eta D + \bar{\eta} D] (\phi + \bar{\phi}),$$

$$\delta^* \phi = \sqrt{2i} \bar{\eta} D L, \quad \delta^* \bar{\phi} = \sqrt{2i} \eta D L.$$  \hfill (2.2)

A linear multiplet can be expressed in terms of a spinor chiral superfield $\chi_\alpha$, $D \bar{\chi}_\alpha = 0$:

$$L = D_\alpha \chi_\alpha - \bar{D}_\alpha \bar{\chi}_\dot{\alpha}.$$  

It is defined up to the gauge transformation $\chi_\alpha \rightarrow \chi_\alpha + \bar{D}_\alpha U_\chi$, for any real vector superfield $U_\chi$. Instead of transformations (2.2), we may as well write

$$\delta^* \chi_\alpha = -\frac{i}{\sqrt{2}} \phi \eta_\alpha, \quad \delta^* \bar{\chi}_\dot{\alpha} = \frac{i}{\sqrt{2}} \bar{\phi} \bar{\eta}_\dot{\alpha}.$$  

$$\delta^* \phi = 2\sqrt{2i} \left[ \frac{i}{4} D D \eta \chi + i \partial_\mu \chi \sigma^\mu \eta \right], \quad \delta^* \bar{\phi} = -2\sqrt{2i} \left[ \frac{i}{4} D D \eta \bar{\chi} - i \eta \sigma^\mu \partial_\mu \bar{\chi} \right].$$  \hfill (2.3)

Transformations (2.2) and (2.3) of the single-tensor $\mathcal{N} = 2$ supermultiplet were given by Lindström and Röček [13].

### 2.2 The abelian vector multiplet

In $\mathcal{N} = 1$ superspace, the $\mathcal{N} = 2$ vector multiplet [15] is commonly realized using the gauge curvature chiral superfield $W_\alpha = -\frac{i}{4} \bar{D}D D_\alpha V$ and another chiral superfield $X$. The superfield $V$ is real (and dimensionless). In the Wess-Zumino gauge, it contains the gaugino spinor $\lambda$, the real auxiliary scalar field $d$ and the gauge field:

$$V = \theta \sigma^\mu \bar{\theta} A_\mu + i \theta \theta \bar{\theta} \lambda - i \bar{\theta} \bar{\theta} \theta \lambda + \frac{i}{2} \bar{\theta} \theta \theta \theta d,$$

$$W_\alpha = -i \lambda_\alpha + \theta_\alpha d + \frac{i}{2} (\theta \sigma^\mu \bar{\sigma}^\nu) A_{\mu \nu} + \ldots, \quad F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$  

The superfield Bianchi identity is $D^\alpha W_\alpha = \bar{D}_\alpha \bar{W}$. The chiral $X$ has (mass) dimension one and since we consider the abelian theory, it is gauge invariant. Its components are the second gaugino $\psi_\alpha$, the complex scalar $x$ and a complex auxiliary scalar $f$:

$$X = x + \sqrt{2} \theta \psi - \theta \theta f.$$  

The physical fields of the $N = 2$ vector multiplets are then $(A_\mu, \psi, \lambda, x)$. Under $\mathcal{N} = 1$ supersymmetry,

$$\delta x = \sqrt{2} \epsilon \psi, \quad \delta \psi_\alpha = -\sqrt{2} i (\sigma^\mu \epsilon) A_\mu x + \ldots$$

\footnote{While $\chi_\alpha$ includes an antisymmetric tensor $b_{\mu \nu}$, $L$ includes its curl $\partial_\mu b_{\nu \mu}$}.
(the auxiliary \( f \) vanishes in the \( \mathcal{N} = 2 \) super-Maxwell theory). We then also expect
\[
\delta^* x = \sqrt{2} \eta \lambda, \quad \delta^* \lambda_{\alpha} = -\sqrt{2} i (\sigma^{\mu} \bar{\eta})_{\alpha} \partial_{\mu} x + \ldots
\]
for the second supersymmetry. This suggests to consider superfield variations of the form 
\[
\delta^* X = \sqrt{2} i \eta^a W_a + \ldots \quad \text{and} \quad \delta^* W_a = -\sqrt{2} (\sigma^{\mu} \bar{\eta})_{\alpha} \partial_{\mu} X + \ldots
\]
to realize the \( \mathcal{N} = 2 \) superalgebra.

To actually derive the second supersymmetry variations, suppose that \( V_1 \) and \( V_2 \) are two real vector superfields and reduce their field content by imposing the following abelian gauge invariance:
\[
\begin{align*}
\delta_{\text{gauge}} V_1 &= \Lambda_\ell, \quad DD \Lambda_\ell = D D \Lambda_\ell = 0, \\
\delta_{\text{gauge}} V_2 &= \Lambda_c + \bar{\Lambda}_c, \quad D \dot{\alpha} \Lambda_c = D \alpha \bar{\Lambda}_c = 0.
\end{align*}
\]
(2.4)

While the gauge transformation of \( V_2 \) is as expected for a \( \mathcal{N} = 1 \) abelian gauge superfield, \( V_1 \) transforms with a linear gauge parameter \( \Lambda_\ell \). The second supersymmetry transformations
\[
\delta^* V_1 = -\frac{i}{\sqrt{2}} \left[ \eta D + \eta \bar{D} \right] V_2, \quad \delta^* V_2 = \sqrt{2} i \left[ \eta D + \eta \bar{D} \right] V_1
\]
(2.5)
are compatible with the gauge transformations since \( \Lambda_c \) and \( \Lambda_\ell \) form a tensor multiplet under \( \delta^* \), with second supersymmetry transformations \( \delta^* \Lambda_\ell \) and \( \delta^* \Lambda_c \) as in eqs. (2.2). Some useful identities involving covariant derivatives are given in Appendix B. Define then the two gauge invariant superfields
\[
W_\alpha = -\frac{1}{4} D D D \alpha V_2, \quad X = \frac{1}{2} \bar{D} \bar{D} V_1.
\]
(2.6)

Their variations under the second supersymmetry are
\[
\begin{align*}
\delta^* X &= \sqrt{2} i \eta^a W_a, \quad \delta^* \bar{X} = \sqrt{2} i \bar{\eta}_a \bar{W}^a, \\
\delta^* W_\alpha &= \sqrt{2} i \left[ \frac{1}{4} \eta_\alpha \bar{D} \bar{D} X + i (\sigma^{\mu} \bar{\eta})_{\alpha} \partial_{\mu} X \right], \\
\delta^* \bar{W}_\dot{\alpha} &= \sqrt{2} i \left[ \frac{1}{4} \bar{\eta}_{\dot{\alpha}} D D X - i (\eta \sigma^{\mu})_{\dot{\alpha}} \partial_{\mu} X \right].
\end{align*}
\]
(2.7)

As a consequence of the definition of \( W_\alpha \), they leave invariant the Bianchi identity 
\( D^a W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^\dot{\alpha} \).

Then, if \( (W_\alpha, X) \) is the \( \mathcal{N} = 2 \) supermultiplet of the abelian gauge curvature, \( (V_1, V_2) \) with gauge invariance (2.4) gives the same multiplet in terms of gauge fields (or gauge potentials). In the (generalized to \( \mathcal{N} = 2 \)) Wess-Zumino gauge, the physical degrees of freedom of the supermultiplet are the gauge potential in the \( \theta \sigma^\mu \theta \) component of \( V_2 \), the two gauginos in the \( \theta \theta \theta \) and \( \bar{\theta} \theta \theta \) components of \( V_1 \) and \( V_2 \) and the complex
scalar in the $\theta \theta$ and $\bar{\theta} \bar{\theta}$ components of $V_1$. The $\theta \theta \bar{\theta} \bar{\theta}$ components $d_1, d_2$ of $V_1$ and $V_2$ and the longitudinal vector in the $\theta \sigma^\mu \theta$ component of $V_1$ are expected to be auxiliary.

More precisely, choosing the Wess-Zumino gauge amounts to remove in the real vector superfield $V_1$ a linear superfield $\Lambda_\ell$. In this gauge, $V_1$ reduces then to a Majorana fermion $\tilde{\lambda}$ (the second gaugino), a complex scalar $x$, the real (auxiliary) scalar $d_1$ and the longitudinal component of a vector field $\tilde{v}_\mu$: 

$$V_1|_{W.Z.} = \frac{1}{2} \theta \bar{\theta} x - \frac{1}{2} \theta \bar{\theta} \lambda - \frac{i}{\sqrt{2}} \theta \bar{\theta} \bar{\theta} \bar{\lambda} + \frac{1}{2} \theta \bar{\theta} \bar{\theta} \bar{\theta} d_1,$$

with residual gauge invariance $\delta \tilde{v}_\mu = \epsilon^{\mu \nu \rho \sigma} \partial_\nu \Lambda_\rho \sigma$ to eliminate the transverse part of $\tilde{v}_\mu$. With relation $X = \frac{1}{2} \bar{D} D V_1$ (and in chiral variables),

$$X = x + \sqrt{2} \theta \bar{\lambda} - \theta \theta f_X, \quad f_X = d_1 + i \partial^\mu \tilde{v}_\mu.$$

In a theory depending on $X$ only, as in the super-Maxwell theory, replacing $\text{Im} f_X$ by the field $\partial^\mu \tilde{v}_\mu$ has a single implication: a linear term $c X$ ($c$ complex) in the superpotential, which potentially breaks supersymmetry, reduces now to a term Re $c \text{Re} X$, with the same supersymmetry breaking pattern. In other words, replacing $\text{Im} f_X$ by $\partial^\mu \tilde{v}_\mu$ is equivalent to a choice of phase of the term linear in $X$ in the superpotential.

### 2.3 The $\mathcal{N} = 2$ super-Maxwell theory

The $\mathcal{N} = 2$ super-Maxwell theory with Lagrangian

$$\mathcal{L}_{\text{Max.}} = \int d^2 \theta d^2 \bar{\theta} X X + \frac{1}{4} \int d^2 \theta \bar{\theta} W \bar{W} + \frac{1}{4} \int d^2 \bar{\theta} W \bar{W}$$

$$= \frac{1}{4} \int d^2 \theta \left[ WW - \frac{1}{2} X \bar{D} D X \right] + \text{h.c.} + \text{total derivative}$$

(2.8)

is invariant under the second supersymmetry: from variations (2.7), one obtains

$$\delta^* \left[ WW - \frac{1}{2} X \bar{D} D X \right] = -2 \sqrt{2} \partial_\mu (W \sigma^\mu \bar{\eta} X),$$

(2.9)

a total derivative leading to an invariant action.

With two vector superfields $V_1$ and $V_2$ and a chiral $X$, the following supplementary terms are $\mathcal{N} = 2$ and gauge invariant:

$$\mathcal{L}_{F.I.} = \int d^2 \theta d^2 \bar{\theta} \left[ \xi_1 V_1 + \xi_2 V_2 \right] + \zeta \int d^2 \theta X + \text{h.c.},$$

(2.10)

with $\xi_1$ and $\xi_2$ real and $\zeta$ complex. Since the $\theta \theta$ component of $W_\alpha$ is a total derivative,

$$\delta^* \int d^2 \theta \zeta X = \sqrt{2} i \zeta \int d^2 \theta W \bar{\eta}.$$
is a total derivative, but a more complicated superpotential in $X$ is forbidden by the second supersymmetry. However, since $X = \frac{1}{2} \overline{DD} V_1$,

$$\zeta \int d^2 \theta X + \text{h.c.} = -4(\text{Re} \zeta) \int d^2 \theta d^2 \bar{\theta} V_1 + \text{total derivative},$$

which indicates that the imaginary part of $\zeta$ is irrelevant while $\text{Re} \zeta$ and $\xi_1$ are redundant. The theory has then two Fayet-Iliopoulos terms

$$\mathcal{L}_{F.I.} = \int d^2 \theta d^2 \bar{\theta} [\xi_1 V_1 + \xi_2 V_2] = -\frac{1}{2} \xi_1 \text{Re} \int d^2 \theta X + \xi_2 \int d^2 \theta d^2 \bar{\theta} V_2,$$

(2.11)

with two real arbitrary parameters. In the Wess-Zumino gauge,

$$\mathcal{L}_{F.I.} = \frac{1}{2} [\xi_1 d_1 + \xi_2 d_2]$$

(2.12)

which, if added to an interacting theory with a non-trivial prepotential (see eq. (2.15) below), generates a positive scalar potential breaking both supersymmetries with the same order parameter $\sqrt{\xi_1^2 + \xi_2^2}$.

### 2.4 $\mathcal{N} = 2$ superspace construction and prepotential

For completeness, we relate the above derivation of the supermultiplet $(W_\alpha, X)$ with its familiar construction in $\mathcal{N} = 2$ superspace [16]. Start with a chiral $\mathcal{N} = 2$ superfield in chiral coordinates $(y, \theta, \bar{\theta})$ and expand in $\bar{\theta}$:

$$\Phi(y, \theta, \bar{\theta}) = X(y, \theta) + i \sqrt{2} \bar{\theta} W(y, \theta) - \bar{\theta} \bar{\theta} F(y, \theta).$$

We know from $\mathcal{N} = 1$ superspace that

$$\delta^* X = \sqrt{2} i \eta W,$$

$$\delta^* W_\alpha = \sqrt{2} i [F_\eta_\alpha + i (\sigma^\mu \bar{\eta})_\alpha \partial_\mu X],$$

$$\delta^* F = \sqrt{2} \partial_\mu W \sigma^\mu \bar{\eta}.$$  

(2.13)

Since

$$\frac{1}{4} \int d^2 \bar{\theta} \Phi^2 = \frac{1}{4} [WW - 2XF],$$

the $\mathcal{N} = 2$ super-Maxwell system (2.8) is recovered if we impose

$$F = \frac{1}{4} \overline{DD} X.$$  

(2.14)

This condition is actually compatible with the supersymmetry variations (2.13) and eq. (2.14), when inserted into the first two eqs. (2.13), leads again to the transformations (2.7).
This derivation of the free $\mathcal{N} = 2$ Maxwell theory easily generalizes to an interacting model using the holomorphic prepotential $\mathcal{F}(\Phi)$:

$$
\mathcal{L}_F = \frac{m^2}{2} \int d^2\theta \int d^2\bar{\theta} \mathcal{F}(\Phi/m) + h.c.
$$

$$
= \frac{1}{4} \int d^2\theta \left[ \mathcal{F}''(X/m) \bar{W}W - \frac{m}{2} \mathcal{F}'(X/m) \bar{D}D \bar{X} \right] + h.c.
$$

$$
= \frac{m}{2} \int d^2\theta d^2\bar{\theta} \left[ \mathcal{F}'(X/m) \bar{X} + \mathcal{F}(\bar{X}/m) X \right] + \frac{1}{4} \int d^2\theta \mathcal{F}''(X/m) \bar{W}W + h.c.,
$$

(2.15)

where $m$ is an arbitrary (real) mass scale$^2$. Invariance under the second supersymmetry, with variations (2.7), follows from

$$
\delta^* \left[ \mathcal{F}''(X/m) \bar{W}W - \frac{m}{2} \mathcal{F}'(X/m) \bar{D}D \bar{X} \right] = -2\sqrt{2} \partial_{\mu} \left[ \mathcal{F}'(X/m) \omega^{\mu}\bar{\eta} \right].
$$

(2.16)

The Fayet-Iliopoulos terms (2.11), which break spontaneously both supersymmetries, can be added to Lagrangian (2.15). In addition, as demonstrated in Ref. [17], this combined theory admits a deformation in which one supersymmetry is nonlinearily realized (in the “Goldstino mode”) and supersymmetry partially breaks.$^3$

### 2.5 Vector–tensor multiplet couplings

To couple a tensor multiplet $(L, \phi)$ to the vector multiplet $(V_1, V_2)$, we may postulate gauge variations

$$
\delta_{\text{gauge}} L = h\Lambda_{\ell}, \quad \delta_{\text{gauge}} \phi = h\Lambda_c,
$$

(2.17)

where the real number $h$ plays the role of a charge. The gauge invariant combinations

$$
\hat{V}_1 = L - hV_1, \quad \hat{V}_2 = \phi + \overline{\phi} - hV_2
$$

(2.18)

verify then

$$
\delta^* \hat{V}_1 = -\frac{i}{\sqrt{2}} \left[ \eta D + \overline{\eta D} \right] \hat{V}_2, \quad \delta^* \hat{V}_2 = \sqrt{2} i \left[ \eta D + \overline{\eta D} \right] \hat{V}_1.
$$

(2.19)

Since

$$
\delta^* \left( \hat{V}_1 \pm \frac{i}{\sqrt{2}} \hat{V}_2 \right) = \mp \left[ \eta D + \overline{\eta D} \right] \left( \hat{V}_1 \pm \frac{i}{\sqrt{2}} \hat{V}_2 \right),
$$

one immediately infers that

$$
\int d^2\theta d^2\bar{\theta} \left[ \mathcal{H}(\hat{V}_1 + i\hat{V}_2/\sqrt{2}) + \mathcal{H}^*(\hat{V}_1 - i\hat{V}_2/\sqrt{2}) \right]
$$

(2.20)

$^2$The scale $m$ disappears in the free, scale-invariant, case with quadratic prepotential, $\mathcal{F} = X^2/2m^2$.

$^3$For a generalization to the non-abelian case see Ref. [18].
has $\mathcal{N} = 2$ supersymmetry for any function $\mathcal{H}$ since
\[
\delta^* \mathcal{H}(\hat{V}_1 + i\hat{V}_2/\sqrt{2}) = \mathcal{H}' \delta^*(\hat{V}_1 + i\hat{V}_2/\sqrt{2}) = -\left[\eta D + \eta D\right] \mathcal{H}.
\]
As found by Lindström and Roček [13], these non-trivial couplings are generated by solutions of the Laplace equation for variables $\hat{V}_1$ and $\hat{V}_2/\sqrt{2}$.

Expression (2.20) is sufficient to propagate the physical fields of the tensor multiplet. If $h \neq 0$ however, it is inconsistent in itself since the highest components of both $V_1$ and $V_2$ imply $\mathcal{H}' = 0$ by their field equations. Consistency and propagation terms for the fields in the vector multiplet require to add the vector multiplet Lagrangian (2.15) to eq. (2.20).

This method for coupling a vector and a tensor supermultiplet corresponds to a $\mathcal{N} = 2$ “Stückelberg gauging”. The simplest example is $\mathcal{H}(x) = -\frac{1}{2} x^2$, for which the Lagrangian (2.20) is
\[
\int d^2\theta d^2\bar{\theta} \left[ \frac{1}{2} (\phi + \bar{\phi} - h V_2)^2 - (L - h V_1)^2 \right].
\]
It includes the free tensor multiplet Lagrangian $\int d^2\theta d^2\bar{\theta} \left[ \phi \overline{\phi} - L^2 \right]$ and mass terms for $V_1$ and $V_2$: there is a gauge where $L$ and $\phi$ are eliminated and $V_1$ and $V_2$ acquire a mass proportional to $h$.

Two particular choices for the function $\mathcal{H}$ lead to terms of special interest. Firstly, the Fayet-Iliopoulos terms (2.11) are obtained from a linear function $\mathcal{H}(x) = \xi x/h$, with $\xi$ complex. Secondly, choosing $\mathcal{H}(x) = \frac{i}{2\sqrt{2} h} \zeta x^2$ ($\zeta$ real) leads to the action contribution
\[
\mathcal{L}_{BF} = \zeta \int d^4 x \int d^2\theta d^2\bar{\theta} \left[ L V_2 + (\phi + \bar{\phi}) V_1 - h V_1 V_2 \right] \]
\[
= -\frac{1}{2} \zeta \int d^2\theta [\phi X + 2 \chi^\alpha W_\alpha] - \frac{1}{2} \zeta \int d^2\bar{\theta} [\bar{\phi} X + 2 \bar{\chi}_\dot{\alpha} \bar{W}^\dot{\alpha}] - \zeta h \int d^2\theta d^2\bar{\theta} V_1 V_2.
\]
It is important to notice that this expression has a smooth $h \to 0$ limit: the resulting contribution
\[
\zeta \int d^4 x \int d^2\theta d^2\bar{\theta} \left[ L V_2 + (\phi + \bar{\phi}) V_1 \right],
\]
which is the $\mathcal{N} = 2$ extension of the Chern-Simons coupling $\epsilon^{\mu\nu\rho} B_{\mu
u} F_{\rho}$, also exists if the tensor multiplet is not charged under the abelian gauge symmetry.

Hence a gauge-invariant coupling of a tensor multiplet $(L, \phi)$ to the vector multiplet $(V_1, V_2)$ can be described by the Lagrangian
\[
\mathcal{L}_{\text{vect.-tens.}} = \int d^2\theta d^2\bar{\theta} \left[ \mathcal{H}(\hat{V}_1 + i\hat{V}_2/\sqrt{2}) + \mathcal{H}^*(\hat{V}_1 - i\hat{V}_2/\sqrt{2}) \right] + \mathcal{L}_F,
\]
(2.23)
where $L_F$ is the vector multiplet action with prepotential $F$. The Chern-Simons and Fayet-Iliopoulos terms can be added if the tensor multiplet does not transform under the gauge symmetry. On the other hand, they arise from $\mathcal{H}$ if the tensor multiplet is charged.

3 Nonlinear $\mathcal{N} = 2$ supersymmetry and the Born-Infeld theory

3.1 Partially broken supersymmetry and a nonlinear deformation

In our formulation of $\mathcal{N} = 2$ supersymmetry on $\mathcal{N} = 1$ superspace, partial supersymmetry breaking corresponds to a simple nonlinear deformation of the linear supersymmetry transformations. Suppose then that instead of transformations (2.7) we use

$$
\delta^* X = \sqrt{2} i \eta^a W_a, \quad \delta^* \bar{X} = \sqrt{2} i \bar{\eta}_\dot{a} \bar{W}^{\dot{a}},
$$

$$
\delta W_a = \sqrt{2} i \left[ \frac{1}{2\kappa} \eta_\alpha + \frac{1}{4} \eta_\alpha \bar{D} \bar{D} X + i (\sigma^\mu)_\alpha \partial_\mu X \right],
$$

$$
\delta \bar{W}^{\dot{a}} = \sqrt{2} i \left[ \frac{1}{2\kappa} \bar{\eta}_{\dot{a}} + \frac{1}{4} \bar{\eta}_{\dot{a}} D \bar{D} X - i (\bar{\eta}_\sigma)^{\dot{a}} \partial_\sigma X \right],
$$

with an arbitrary (nonzero) constant $\kappa$ with dimension $(\text{length})^2$ and a complex phase $u$, $|u| = 1$. This modification does not affect the second supersymmetry algebra or the Bianchi identity verified by $W_a$. In this nonlinear variation, the gaugino in $W_a$ transforms like a Goldstino.

With modified transformations (3.1), the second supersymmetry variation of the Lagrangian (2.15) acquires the new contribution

$$
\frac{\sqrt{2} i u}{4\kappa} \int d^2 \theta \int d^2 \theta F''(X/m) W \eta + \text{h.c.} = \frac{m}{4\kappa} u \delta^* \int d^2 \theta F'(X/m) + \text{h.c.}
$$

Hence, the modified Lagrangian

$$
L_F = \frac{1}{4} \int d^2 \theta \left[ F''(X/m) WW - \frac{m}{2} F'(X/m) \bar{D} \bar{D} X - \frac{m}{\kappa} u F'(X/m) - \frac{1}{2} \xi_1 X \right] + \text{h.c.}
$$

$$
+ \xi_2 \int d^2 \theta d^2 \bar{\theta} V_2
$$

(3.2)

has linear $\mathcal{N} = 1$ supersymmetry and a second, nonlinearly-realized, supersymmetry with variations (3.1). The introduction of the terms with coefficient $\kappa^{-1}$ breaks then spontaneously $\mathcal{N} = 2$. The resulting superpotential is

$$
w = - \frac{m}{4\kappa} u F'(X/m) - \frac{1}{8} \xi_1 X.
$$

(3.3)
It includes a new “magnetic” term proportional to the first derivative of the prepotential [17, 18]. Together with the Fayet-Iliopoulos term for $V_2$, this superpotential leads to the scalar potential

$$V = (\text{Re } \mathcal{F}''^{-1}) \left( \frac{1}{8} \xi_2^2 + \frac{1}{16} \left| \frac{1}{2} \xi_1 + \frac{1}{\kappa} u \mathcal{F}'' \right|^2 \right). \quad (3.4)$$

It has a stationary point with

$$\text{Re } \mathcal{F}'' = \left[ 2 \xi_2^2 \kappa^2 + \frac{1}{4} \xi_1^2 \kappa^2 (\text{Re } u)^2 \right]^{1/2}, \quad \text{Im } \mathcal{F}'' = \frac{1}{2} \xi_1 \kappa \ \text{Im } u,$$

$$V = \frac{1}{8 \kappa^2} \text{Re } \mathcal{F}'' + \frac{\xi_1}{10 \kappa} \text{Re } u.$$  

The nonlinear second supersymmetry transformations (3.1) of $W_\alpha$ translates into a modified variation of the vector superfield $V_2$:

$$\delta^\ast V_2 = \sqrt{2} i \left[ \eta D + \eta D \right] V_1 + \frac{1}{\sqrt{2} \kappa} u \theta \bar{\theta} \theta \bar{\eta} - \frac{1}{\sqrt{2} \kappa} \pi \theta \bar{\theta} \theta \bar{\eta}$$

$$= \sqrt{2} i \left[ \eta D + \eta D \right] \left( V_1 + \frac{1}{4 \kappa} \text{Re } u \theta \bar{\theta} \theta \bar{\eta} \right) - \frac{1}{\sqrt{2} \kappa} \text{Im } u \left( \theta \bar{\theta} \theta \bar{\eta} + \theta \bar{\theta} \theta \bar{\eta} \right). \quad (3.5)$$

Notice that the nonlinear deformation does not affect the existence of Fayet-Iliopoulos terms: only fermion variations are modified.

Note that the phase $u$ is in principle an arbitrary parameter. By imposing the exchange symmetry $\lambda \leftrightarrow \psi$ of the two gaugino mass terms\(^4\), $u$ is found to be purely imaginary, $u = \pm i$ [17].

### 3.2 The nonlinear constraint

The Born-Infeld theory with linear $\mathcal{N} = 1$ supersymmetry can be nicely derived as a nonlinear realization of $\mathcal{N} = 2$ supersymmetry on $\mathcal{N} = 1$ superspace. This nonlinear realization can be obtained by imposing a nonlinear constraint on the linear $\mathcal{N} = 2$ vector multiplet introduced in the previous section.

Following Roček and Tseytlin [9, 10], we construct the nonlinear realization by imposing the constraint

$$\frac{1}{\kappa} X = WW - \frac{1}{2} X DDX,$$  

where $\kappa$ describes the scale of supersymmetry breaking and has dimension (length)$^2$. Since

$$X = \frac{WW}{\kappa^{-1} + \frac{1}{2} DDX}$$

\(^4\)The four-fermion interactions do not depend on the superpotential.
and $W_\alpha W_\beta W_\gamma = 0$, the constraint implies $W_\alpha X = 0$, and the derivative in the variation (2.9) disappears. The right-hand side of constraint (3.6) is then invariant under the linear second supersymmetry (2.7) and covariance of the constraint requires a nonlinear modification of $\delta^* W_\alpha$, given by eq. (3.1) with the phase $u = 1$:

$$
\delta^* X = \sqrt{2} i \eta^\alpha W_\alpha, \quad \delta^* X = \sqrt{2} i \eta_\alpha \bar{W}^\alpha,
$$

$$
\delta^* W_\alpha = \sqrt{2} i \left[ \frac{1}{2\kappa} \eta_\alpha + \frac{1}{4} \eta_\alpha DD \bar{X} + i(\sigma^\mu \eta)_\alpha \partial_\mu X \right], \quad (3.7)
$$

$$
\delta^* W_\alpha = \sqrt{2} i \left[ \frac{1}{2\kappa} \eta_\alpha + \frac{1}{4} \eta_\alpha DD X - i(\eta \sigma^\mu)_\alpha \partial_\mu \bar{X} \right].
$$

The superfield $W_\alpha$ includes then the Goldstino of the nonlinear supersymmetry with transformations $\delta^*$. With this modification,

$$
\delta^* \left[ WW - \frac{1}{2} X DD \bar{X} \right] = \frac{\sqrt{2} i}{\kappa} \eta W = \frac{1}{\kappa} \delta^* X, \quad (3.8)
$$

as required by the constraint (3.6). Since $\delta^* \eta_\alpha$ and $D_\alpha \eta_\beta$ vanish, the modification does not affect the algebra and the Bianchi identity. Eqs. (3.7) and (3.6) provide a nonlinear realization of $\mathcal{N} = 2$ supersymmetry, with linearly-realized $\mathcal{N} = 1$.

The constraint (3.6) allows to express $X$ as a function of $WW$ and of its supersymmetric derivatives (see below), and the $\mathcal{N} = 2$ super-Maxwell theory reduces simply to the Fayet-Iliopoulos term

$$
\mathcal{L}_{\text{Max.}} = \frac{1}{4\kappa g^2} \int d^2 \theta X + \frac{1}{4\kappa g^2} \int d^2 \bar{\theta} \bar{X}, \quad (3.9)
$$

adding a constant gauge coupling $g$ (which is useless in the abelian theory without charged fields). The invariance of this action follows from

$$
\int d^4 x \delta^* \mathcal{L}_{\text{Max.}} = \frac{-\sqrt{2} i}{16\kappa g^2} \int d^4 x \int d^2 \theta \ DD(\eta W) + \text{h.c.}
$$

$$
= \frac{\sqrt{2}}{4\kappa g^2} \int d^4 x \int d^2 \theta \partial_\mu (\eta \sigma^\mu W) + \text{h.c.}
$$

Solving the constraint and expanding the action in components shows that theory (3.9) provides an extension of the Born-Infeld Lagrangian with linear $\mathcal{N} = 1$ supersymmetry and a nonlinearly-realized second supersymmetry with variations (3.7) [1].

### 3.3 The $\mathcal{N} = 2$ Born-Infeld theory

Bagger and Galperin [1] have shown how to solve the constraint (3.6). The result is

$$
X = \kappa W^2 - \kappa^3 DD \left[ \frac{W^2 \bar{W}^2}{1 + A + \sqrt{1 + 2A + B^2}} \right], \quad (3.10)
$$
where
\[ A = \frac{\kappa^2}{2} (DD W^2 + \overline{DD} \overline{W}^2) = A^*, \quad B = \frac{\kappa^2}{2} (DD W^2 - \overline{DD} \overline{W}^2) = -B^*. \]
The derivation is also given in Appendix C. Notice that
\[ A|_{\theta=0} = -8\kappa^2 \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{8} d^2 + \frac{i}{2} \sigma^\mu \partial_\mu \overline{\lambda} - \frac{i}{2} \overline{\partial}_\mu \sigma^\mu \lambda \right], \]
\[ B|_{\theta=0} = 4i\kappa^2 \left[ \frac{1}{2} F_{\mu\nu} \tilde{F}^{\mu\nu} - \partial_\mu (\lambda \sigma^\mu \overline{\lambda}) \right], \quad F_{\mu\nu} \tilde{F}^{\mu\nu} = \partial_\mu (\epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}). \]

(3.11)
The expression (3.10) has been obtained by Cecotti and Ferrara [19] in the context of \( \mathcal{N}=1 \) superspace, with however an ambiguity in the \( \mathcal{N}=1 \) supersymmetrization which is removed when the second (nonlinear) supersymmetry is imposed [1]. Notice also that, as expected, the solution (3.10) is compatible with \( X = \frac{1}{2} \overline{DD} V_1 \), but it only defines \( V_1 \) up to a linear superfield.

The Lagrangian (3.9) includes the following gauge kinetic terms (see Appendix C):
\[ \mathcal{L}_{\text{Max.}} \rightarrow \frac{1}{8\kappa^2 g^2} \left[ 1 - \sqrt{1 + 4\kappa^2 F_{\mu\nu} F^{\mu\nu} - 4\kappa^4 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} \right] = -\frac{1}{8g^2} F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(\kappa^4 F_{\mu\nu}^4) \]
\[ = \frac{1}{8\kappa^2 g^2} \left[ 1 - \sqrt{-\det(\eta_{\mu\nu} + 2\sqrt{2} \kappa F_{\mu\nu})} \right]. \]

(3.12)
It also includes terms with derivatives of \( F_{\mu\nu} \), fermionic contributions and the following auxiliary contribution:
\[ \mathcal{L}_{\text{Max.}} \rightarrow \frac{1}{8\kappa^2 g^2} \left[ 1 - \sqrt{1 - 8\kappa^2 d^2} \right] = \frac{1}{2g^2} d^2 + \mathcal{O}(d^4). \]

(3.13)
The usual Born-Infeld Lagrangian for a \( D3 \) brane is
\[ \mathcal{L}_{\text{BI}} = -T_3 \frac{2\pi}{g^2} \sqrt{-\det(\eta_{\mu\nu} + 2\pi \alpha' F_{\mu\nu})}, \]

(3.14)
where \( g \) is the gauge coupling and \( T_3 \) is the brane tension:
\[ T_3 = \frac{1}{(2\pi)^3 (\alpha')^2}. \]

(3.15)
Comparing then eqs. (3.12) and (3.14) leads to the identifications [20]:
\[ T_3 = \frac{1}{16\pi \kappa^2} ; \quad \kappa = \frac{\pi \alpha'}{\sqrt{2}}. \]

(3.16)
Note that, upon imposing the nonlinear constraint (3.6), the DBI action is identical to the first FI–term proportional to \( \xi_1 \) in eq. (2.11). On the other hand, the second FI–term proportional to \( \xi_2 \) is obviously invariant under the nonlinear supersymmetry transformation (3.5) (with \( u = 1 \)), compatible with the constraint, and can be also added to the action. This leads to an additional contribution, linear in the D-term auxiliary \( d \).
4 The $\mathcal{N} = 2$ dilaton

In string theory, the gauge coupling is related to the VEV of the dilaton field and the contribution (3.13) provides a dilaton potential at the level of the disk world-sheet topology. On the other hand, a tree-level dilaton potential at the level of spherical topology can be generated by going off-criticality, away from ten dimensions. This will be used in the next section, where we study supersymmetry breaking vacua with dilaton stabilization. In the context of the effective field theory, a non-critical dilaton potential can be described as a gauging of the $\mathcal{N} = 2$ supergravity of the closed string sector, that we present in this section.

In type IIB superstrings compactified to four dimensions on a Calabi-Yau threefold, the dilaton belongs to a hypermultiplet of four-dimensional $\mathcal{N} = 2$ supergravity. This multiplet can be dualized into a single-tensor or a double-tensor multiplet since two of its four scalar components are actually modes of the NS-NS (Neveu-Schwarz) and R-R (Ramond-Ramond) two-form fields. Taking the orientifold (with $D9$ branes) of this theory leads to type I strings compactified on a Calabi-Yau space, the axion partner of the dilaton being a mode of the R-R two-form field of the original type IIB theory.

To describe the non-critical dilaton potential which we will use in the next section, we use $\mathcal{N} = 2$ supergravity with a single hypermultiplet and we gauge the axion shift symmetry using the graviphoton as gauge field. The quaternionic scalar manifold is $SU(2,1)/SU(2) \times U(1)$, which is also a Kähler manifold. The terms of the $\mathcal{N} = 2$ theory relevant to our purposes are simply [21]:

$$e^{-1} \mathcal{L}_{N=2} = -\frac{M_P^2}{2} R + M_P^2 h_{ab}(q) \left( \partial_\mu q^a \right) \left( \partial^\mu q^b \right) - \mathcal{V}(q) + \ldots,$$

where $q^a$, $a = 0, 1, 2, 3$, are the four hypermultiplet real scalar fields and $h_{ab}(q)$ is the $SU(2,1)/SU(2) \times U(1)$ metric. Here, $R$ is the scalar curvature and $M_P$ the reduced Planck mass. Defining precisely the scalar potential $\mathcal{V}(q)$ requires some preliminaries.

The metric $h_{ab}(q)$ of a quaternionic manifold is hermitian with respect to a triplet of complex structures $J^x$ verifying the quaternionic algebra

$$J^x J^y = -\delta^{xy} I + \epsilon^{xyz} J^z; \quad (x, y, z = 1, 2, 3).$$

The three hyper-Kähler forms $K'_{ab} = h_{ac}(J^c)_b$ are then covariantly closed with respect to a $SU(2)$ connection $\omega^x$:

$$d K^x + \epsilon^{xyz} \omega^y \wedge K^z = 0.$$  \hspace{1cm} (4.2)

\footnote{We follow the conventions of Ref. [22]. See in particular the Appendix.}

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In the case of a quaternionic manifold, the $SU(2)$ curvatures are proportional to the hyper-Kähler forms,

$$
\Omega^x = \frac{1}{2} \Omega^x_{ab} dq^a \wedge dq^b \equiv d \omega^x + \frac{1}{2} \epsilon^{xyz} \omega^y \wedge \omega^z = \lambda K^x, \quad (\lambda \neq 0).
$$

(4.3)

This is the case relevant to hypermultiplets coupled to $\mathcal{N} = 2$ supergravity. Eqs. (4.2) and (4.3) can be viewed as a set of equations for the connection $\omega^x_{ab}$. The definitions of $J^x$ and $K^x$ and the proportionality equation (4.3) lead to

$$
h^{cd} \Omega^x_{ac} \Omega^y_{db} = -\lambda^2 \delta^{xy} h_{ab} + \lambda \epsilon^{xyz} \Omega^z_{ab}, \quad (4.4)
$$

where $h^{cd}(q)$ is the inverse quaternionic metric. This equation defines $\lambda h_{ab}$ in terms of the curvatures $\Omega^x$, or in terms of the connections $\omega^x$. Supersymmetry leads in general to kinetic terms (in $M_P$ mass units):

$$
-\frac{1}{2} eR - e \lambda h_{ab}(q) (\partial_\mu q^a) (\partial_\mu q^b) + \ldots
$$

and canonical normalization is obtained with the choice $\lambda = -1$.

With one hypermultiplet only, the theory has a single vector field, the graviphoton, and we can gauge one isometry of the quaternionic metric. The Killing vector $k^a$ for the gauged symmetry is defined from the symmetry action

$$
q^a \rightarrow q^a + \epsilon k^a(q), \quad (4.5)
$$

where $\epsilon$ is the (infinitesimal) parameter of the transformation. Each Killing vector of a quaternionic manifold can be expressed in terms of a triplet of prepotentials $\mathcal{P}^x$:

$$
2 k^a \Omega^x_{ab} = - (\partial_b \mathcal{P}^x + \epsilon \omega^y_b \mathcal{P}^y) \equiv -\nabla_b \mathcal{P}^x. \quad (4.6)
$$

Eq. (4.4) leads then to:

$$
k^a = \frac{1}{6 \lambda^2} \sum_{x=1}^3 h^{bc} \partial_b (\mathcal{P}^x \Omega^x_{cd}) h^{da}. \quad (4.7)
$$

For our $SU(2,1)/SU(2) \times U(1)$ manifold, an appropriate triplet of $SU(2)$ connections with closed curvatures for any value of the real parameter $r$ is [23]

$$
\omega^1 = -r \frac{d\tau}{\sqrt{V}}, \quad \omega^2 = r \frac{d\theta}{\sqrt{V}}, \quad \omega^3 = -r \frac{d\sigma}{4V}(d\sigma - 2\tau d\theta + 2\theta d\tau), \quad (4.8)
$$

In the case of global $\mathcal{N} = 2$ supersymmetry, the $SU(2)$ curvature $\Omega^x$ vanishes and the manifold is hyper-Kähler.
using the basis \( q^a = (V, \sigma, \theta, \tau) \). In this basis, gauging the axionic shift symmetry on \( \sigma \) means that we choose a Killing vector (4.5)

\[
k^a = (0, 1, 0, 0),
\]

i.e. we gauge the transformation \( \sigma \to \sigma + \epsilon \). With these connections, we choose the prepotential triplet

\[
\vec{P} = (0, 0, -\frac{1}{V})
\]

and equation (4.6) is verified if \( r = 2 \) in expressions (4.8). We can then use eq. (4.7) or eq. (4.4) to obtain the quaternionic metric, choosing \( \lambda = -1 \) as required by supersymmetry and canonical normalization of kinetic terms. The result is then

\[
ds^2 = \frac{dV^2}{2V^2} + \frac{1}{2V^2}(d\sigma - 2\tau d\theta + 2\theta d\tau)^2 + \frac{2}{V}(d\theta^2 + d\tau^2) \equiv h_{ab}(q) dq^a dq^b .
\]

Since the scalar manifold is also Kähler, it can be derived from the Kähler potential

\[
K = -2 \ln(S + \overline{S} - 2CC),
\]

with definitions

\[
S = V + \theta^2 + \tau^2 + i\sigma,
\]

\[
C = \theta - i\tau.
\]

Since \( S + \overline{S} - 2CC = 2V \), \( V \) is the four-dimensional dilaton field associated to the four-dimensional type II string coupling \( e^{\phi_4} \): \( V = e^{-\phi_4} \) (see next section). Moreover, the shift isometry on \( \sigma \) (4.5) follows by its (Poincaré) dualization from the NS-NS antisymmetric tensor.

Gauging symmetries generates in particular a scalar potential. For a single hypermultiplet, this potential receives two contributions [21]:

\[
\mathcal{V}(q) = g_s^2 M_P^4 \left[ 4 h_{ab}(q) k^a k^b - 3 \sum_{x=1}^{3} \mathcal{P}^x \mathcal{P}^x \right] L^0 \overline{L}^0 ,
\]

where \( g_s \) is the gauge coupling. The “section” \( L^0 \) can be chosen \( L^0 = 1 \) since vector multiplets are absent. With our Killing vector \( k^a \), our prepotentials \( \mathcal{P}^x \) and our metric (in particular with \( h_{11} = (2V^2)^{-1} \)) the scalar potential is

\[
\mathcal{V}(q) = g_s^2 M_P^4 \left[ 4 h_{11}(q) - \frac{3}{V^2} \right] = -\frac{g_s^2 M_P^4}{V^2} .
\]
5 Scalar potential

We are now interested in the disk contributions to the four-dimensional scalar potential induced by the presence of magnetized branes in a type I orientifold compactification. It will be shown that it receives three independent contributions. First, the uncancelled NS-NS tadpole contribution is encoded in the “branes tension deficit” \( \delta T \) which gives the tree-level dilaton tadpole. The second contribution comes from the “anomalous” FI-term proportional to \( \xi \), while the last term arises from the supersymmetrization of the DBI action, as presented in the previous section. The only consistent vacuum formed by magnetic fluxes turns out to be supersymmetric, where at least one of the closed string moduli (the dilaton) remains unfixed. The situation may however be different when strings propagate in a non-trivial background: either in the presence of three-form closed string fluxes, or in non-critical dimensionality. In this case, an extra contribution to the scalar potential from the sphere world-sheet changes the equation of motion for the dilaton and may lead to different vacua with broken supersymmetry in curved space-time.

5.1 FI-terms from magnetic fluxes

Let us consider type I string theory compactified on a Calabi-Yau threefold \( \mathcal{M}_6 \). The \( \mathcal{N} = 1, d = 4 \) action contains a number of scalar fields describing the size \( J_\alpha \) and shape \( \tau_k \) moduli of the internal manifold as well as the dilaton field \( \varphi \). In addition to them, there exist \( h_{1,1} + 1 \) axionic fields \( C_\alpha \) and \( C_0 \) which arise from the compactification of the ten-dimensional two-form \( C_2 \). Their kinetic terms may be diagonalized in terms of the chiral superfields \( T_\alpha, S_I \) and \( U_k \) as \[ S_I = e^{-\varphi} \frac{V_6}{(4\pi^2\alpha')^3} + i c, \quad T_\alpha = -e^{-\varphi} \frac{J_\alpha}{4\pi^2\alpha'} + i c_\alpha, \] where the overall volume \( V_6 \) is defined by the integral of the Kähler moduli \( J = J_\alpha\omega^\alpha \) over the internal manifold \( \mathcal{M}_6 \): \[ V_6 = \int_{\mathcal{M}_6} J \wedge J \wedge J, \] and \( \{\omega_\alpha\} \) is a basis of the two-forms on \( \mathcal{M}_6 \). Note that the type I dilaton superfield \( S_I \) differs from the one that appears in the universal type II hypermultiplet (4.13). Their Kähler potential in the absence of fluxes is

\[
K = -M_P^2 \left[ \ln (S_I + \overline{S}_I) + \ln \int_{\mathcal{M}_6} (T + \overline{T}) \wedge (T + \overline{T}) \right], \quad T = T_\alpha \omega^\alpha.
\]

For simplicity, we omit the complex structure moduli \( U_k \) from our discussion.

Let us now consider \( K U(1)_a \) magnetized D9 branes, with \( a = 1, \ldots, K \). These give rise to \( K \) gauge fields with non-trivial gauge bundle on \( \mathcal{M}_6 \). Let us denote the
corresponding gauge superfields by $V_a$. The gauge bundles on the internal manifold manifest themselves by topological couplings $Q^a_0$ and $Q^\alpha_a$ of the axionic fields $c_0$ and $c_\alpha$ to the corresponding $U(1)_a$ field strengths. These can be determined by the dimensional reduction of the Wess-Zumino action:

$$Q^a_0 = \frac{1}{(2\pi)^3} \int_{M_6} F^a \wedge F^a \wedge F^a, \quad Q^\alpha_a = \frac{1}{2\pi} F^a_\alpha,$$

(5.3)

where the $F^a_\alpha$'s are the (quantized) components of the $U(1)_a$ field strengths along the two-cycle $\alpha$. In other words, these fluxes modify the Kähler potential (5.2) to

$$\frac{1}{M_P^2} \mathcal{K} = -\log \left( S_I + \bar{S}_I + \sum_a Q^a_0 V_a \right) - \log \int_{M_6} \left( T + \bar{T} + \sum_a Q^a V_a \right)^3,$$

(5.4)

where $Q^a = Q^a_0 \omega^a$ and the power 3 is defined in terms of the wedge product as in (5.2).

In addition to the topological coupling, one is able to extract from the Kähler potential $\mathcal{K}$ the FI-term $\xi_a$ induced by the magnetic fluxes for each $U(1)$ gauge component [26, 27, 28]:

$$\int d^2 \theta d^2 \bar{\theta} \mathcal{K} = \left( \frac{\partial \mathcal{K}}{\partial V_a} \right)_{V_a=0, \theta=0} \int d^2 \theta d^2 \bar{\theta} V_a + \cdots$$

(5.5)

or, equivalently,

$$\frac{\xi_a}{g^2_a} \equiv -\left( \frac{\partial \mathcal{K}}{\partial V_a} \right)_{V_a=0, \theta=0} = M_P^2 \left( \frac{Q^a_0}{S_I + \bar{S}_I} + \frac{1}{2 \int_{M_6} (\text{Re} T)^3} \int_{M_6} Q^a \wedge \text{Re} T \wedge \text{Re} T \right)$$

$$\equiv \frac{e^{-\varphi} M_s^2}{2\pi} \left( \frac{1}{(4\pi^2 \alpha')^3} \int (\mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} - J \wedge J \wedge \mathcal{F}) \right)$$

$$\equiv \frac{e^{-\varphi} M_s^2}{2\pi} \frac{\xi_a}{g^2_a} = \frac{1}{(4\pi^2 \alpha')^3} \int (\mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} - J \wedge J \wedge \mathcal{F}),$$

(5.6)

where $M_s = (\alpha')^{-1/2}$, $\mathcal{F}^a = 2\pi \alpha' F^a$ and we have used the definitions (5.1) and the relation for the reduced Planck mass:

$$M_P^2 = \frac{1}{\pi} M_s^2 e^{-2\varphi} \frac{V_6}{(4\pi^2 \alpha')^3}.$$

(5.7)

### 5.2 Scalar potential

On the world-volume of each D9 brane stack lives a $U(N_a)$ gauge theory. Let us restrict to its $U(1)_a$ subgroup whose NS-NS sector is described at low energy by the ten-dimensional DBI action

$$S_{BI,a} = -T_9 \int_{\Sigma_a} d^{10}x \ e^{-\varphi} \sqrt{-\det(G + 2\pi \alpha' F_a)}, \quad T_p = \frac{1}{(2\pi)^p \alpha'^{p+1}},$$

(5.8)
where $\Sigma_a$ is the ten-dimensional world-volume of the a-th $D9$ brane with metric $G$ and $F_a$ is the field strength of the $U(1)$ gauge theory. In the reduction relevant for us, the action (5.8) simplifies to

$$S_{BI,a} = -T_9 \int_{\mathcal{M}_4} d^4x e^{-\varphi} \sqrt{-\det(G_4 + 2\pi\alpha' F_{4,a})} \cdot \int_{\mathcal{M}_6} d^6y \sqrt{\det(G_6 + 2\pi\alpha' F_{6,a})},$$

(5.9)

where $G_4$, $G_6$ and $F_{4,a}$, $F_{6,a}$ are the metric and $U(1)$ field strength on $\mathcal{M}_4$ and $\mathcal{M}_6$ respectively. Let us consider the case where the metric moduli of $G_6$ are stabilized at specific points on their moduli space. The last factor of the action (5.9) can then be considered as constant from a four-dimensional viewpoint:

$$S_{BI,a} = -T_9 \frac{2\pi}{g_a^2} \int_{\mathcal{M}_4} d^4x e^{-\varphi} \sqrt{-\det(G_4 + 2\pi\alpha' F_{4,a})},$$

(5.10)

$$\frac{2\pi}{g_a^2} = \frac{1}{(4\pi^2\alpha')^3} \int_{\mathcal{M}_6} d^6y \sqrt{\det(G_6 + 2\pi\alpha' F_{6,a})}.$$  

As shown in Section 3, the supersymmetrization of the action (5.10) leads to a potential for the auxiliary component $d_a$ of the a-th $U(1)$ gauge superfield. Using eqs. (3.13) and (5.10), this reads:

$$S_{SP,a} = -T_3 \frac{2\pi}{g_a^2} \int_{\mathcal{M}_4} d^4x \sqrt{-\det G_4} e^{-\varphi} \left( 1 - 1 + \sqrt{1 - (2\pi\alpha'd_a)^2} \right).$$

(5.11)

The first term in the bracket comes from the dilaton tadpole contribution, whereas the factor $1 - \sqrt{1 - (2\pi\alpha'd_a)^2}$ comes from the DBI action (3.13).

Let us now introduce an $O9$ orientifold plane\(^7\). It is defined as the set of fixed points of the orientifold projection $\mathcal{O} = \Omega$, where $\Omega$ is the world-sheet parity. The $O9$ plane is a ten-dimensional object whose effective action is

$$S_{O9} = -32 T_9 \int_{\mathcal{M}_{10}} d^{10}x e^{-\varphi} \sqrt{-\det G}.$$  

(5.12)

Note that the integral is over the ten-dimensional space-time $\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6$. After compactification to four dimensions, the action (5.12) reads

$$S_{O9} = -32 T_3 \frac{V_6}{(4\pi^2\alpha')^3} \int_{\mathcal{M}_4} d^4x e^{-\varphi} \sqrt{-\det G_4},$$

(5.13)

where the volume $V_6$ is taken in a particular point on its moduli space determined by the stabilization procedure.

The various contributions of the K stacks of branes and of the orientifold planes to the scalar potential arise from their tensions, the supersymmetrization of the DBI

\(^7\)Note that the expressions presented here are also valid in the case of $D3/D7$ magnetized branes [7].
action and the FI-terms. Using eqs. (5.6), (5.11) and (5.13), the potential then reads (in the string frame)

\[ V(\phi, J_a, d_a) = T_3 e^{-\phi} \left[ \left( \sum_a N_a \frac{2\pi}{g_a^2} - 32 \frac{V_6}{(4\pi^2\alpha')^3} \right) - \sum_a N_a \frac{2\pi}{g_a^2} \left( 1 - \sqrt{1 - (2\pi\alpha'd_a)^2} \right) + \sum_a \frac{2\pi}{g_a^2} (2\pi\alpha'd_a) \xi_a \right] . \quad (5.14) \]

The supersymmetric vacua correspond to points of the moduli space where the VEV's of the auxiliary fields \( d_a \) are zero. The expression (5.14) of the potential indicates that \( d_a = 0 \) is only possible if all FI-terms \( \xi_a \) vanish, \( \xi_a = 0, \forall a = 1, \ldots, K \). Using eq. (5.6), one then obtains the condition:

\[ \int (F_a \wedge F_a \wedge F_a - J \wedge J \wedge F_a) = 0, \quad \forall a = 1, \ldots, K. \quad (5.15) \]

When these equations are satisfied, the gauge coupling constants \( g_a \) defined in eq. (5.10) reduces to the polynomial form

\[ g_a^{-2} \sim \frac{1}{(4\pi^2\alpha')^3} \int_{M_6} (J \wedge J \wedge J - J \wedge F^a \wedge F^a) > 0. \quad (5.16) \]

Note that the dilaton factor \( e^{-\phi} \) was omitted from the definition of the gauge couplings for simplicity. The physical gauge couplings are given by \( g_a e^{\langle \phi \rangle / 2} \).

By unitarity, these couplings must be positive. The conditions (5.15) and (5.16) are equivalent to the geometrical conditions found in Ref. [29]. They ensure that the magnetized branes preserve a common supersymmetry with the orientifold projection. These D-flatness conditions restrict regions of the moduli space where supersymmetry is restored. For given magnetic fluxes \( F_a \), only particular regions of the Kähler moduli space give rise to supersymmetric vacua. It should be kept in mind that the above D-flatness conditions are only valid at the point of the open string moduli space where all open string charge states have zero VEV’s. Strictly speaking, only a combination of the Kähler moduli and charged Higgs-type fields are stabilized by magnetic fluxes.

### 5.3 Supersymmetric vacua

In addition to these necessary conditions (5.15) and (5.16), a consistent supersymmetric vacuum exists only if the sum of the contributions to the R-R tadpoles vanishes. In the type I compactification, these tadpole conditions read

\[ \sum_a N_a = 32 \quad \text{and} \quad \sum_a N_a \int_{\Pi_\alpha} F_a \wedge F_a = 0, \quad \forall \alpha = 1, \ldots, h_4(M_6), \quad (5.17) \]
where $h_4(\mathcal{M}_6)$ is the number of four-cycles $\Pi_\alpha$ (dual to the two-cycle $\alpha$) of the manifold $\mathcal{M}_6$.

When the necessary and sufficient conditions (5.15), (5.16) and (5.17) are satisfied, the sum over the brane tensions is exactly compensated by the one of the orientifold planes. The auxiliary fields and the FI-parameters vanish. As it should be, the value of the scalar potential at a supersymmetric vacuum is zero. Note however that the supersymmetric conditions and the R-R tadpole cancellation with only $O9$ planes seem to be incompatible as they stand. One way out to find solutions consists of considering compactifications with orientifold five-planes $O5$. Alternatively, as shown in Ref. [7], there exists a second possibility without five-planes. Indeed, in the quadratic approximation, the D-flatness condition (5.15), which is equivalent to the vanishing of the FI parameter $\xi$ in eq. (5.6), is modified in the presence of charged fields $\phi_i$ to

$$0 = \langle d_a \rangle = \sum_i q_i^a \langle |\phi_i|^2 \rangle + \xi_a, \quad (5.18)$$

while the tadpole conditions (5.17) remain intact. It is then possible to obtain consistent supersymmetric vacua ($d_a = 0, \forall a$) with non-vanishing FI-parameters $\xi_a \neq 0$. The presence of small non-vanishing VEV’s, $\langle |\phi_i|^2 \rangle = v_i^2 \ll M_s^2$, for some charged fields compensates the contribution of the FI-parameter to the D-term.\(^8\) The sum over the tensions and the scalar potential vanishes. In this way, it is possible to fix combinations of Kähler and open string moduli in a Minkowski vacuum.

Since the computation of the FI-term is restricted to the disk amplitude, the dilaton enters only as an overall factor in the scalar potential. It is not constrained by the magnetic fluxes and remains therefore a flat direction. The same conclusion may be drawn from an analysis of the Stückelberg couplings. In fact, the stabilized Kähler moduli must enter in massive $\mathcal{N} = 1$ vector supermultiplets. This is achieved due to the Stückelberg couplings which allow the corresponding R-R axions to become the longitudinal polarizations of the magnetized $U(1)$ gauge bosons. The massive scalar modulus and vector then form the bosonic content of a massive $\mathcal{N} = 1$ vector multiplet. However, in a configuration where all branes satisfy the supersymmetry condition (5.15), the maximal rank of the matrix of topological couplings $M_{a\bar{\alpha}} = Q_{\bar{\alpha}}^a$ is given by the number of Kähler moduli $h_{1,1}(\mathcal{M}_6)$, (for $\bar{\alpha} = 0, \ldots, h_{1,1}(\mathcal{M}_6)$). Therefore, there always remains at least one linear combination of the dilaton and Kähler moduli which does not couple to the (anomalous) $U(1)_s$’s. A full stabilization of the closed string moduli in a supersymmetric vacuum can therefore not be achieved by magnetic fluxes only. It may however be achieved by the introduction of three-form fluxes [7].

\(^8\)The smallness condition for the charged field VEV’s guarantees the validity of the perturbative in $\alpha'$ approach.
5.4 Non-supersymmetric vacua

Let us now study the existence of consistent non-supersymmetric vacua $\langle d_a \rangle \neq 0$ induced by magnetized branes. Here, the cancellation of all R-R tadpoles does not anymore imply the cancellation of all tension contributions. On the contrary, a positive contribution $\delta T$ to the scalar potential arises from the sum over all tensions. Moreover, the FI-parameter $\xi$ must be different than zero for the non-supersymmetric branes. Altogether, the net contribution to the scalar potential on the disk is positive and the equation of motion for the dilaton cannot be satisfied. As it stands, non-supersymmetric magnetized branes do not lead to a consistent vacuum.

Here, we propose two solutions to this problem. On the one hand, the closed string background may contain three-form fluxes. A Gukov-Vafa-Witten superpotential is then generated [2, 30]. In this case, assuming that minimal $\mathcal{N} = 1$ supersymmetry is preserved in the bulk, the dilaton is determined in terms of the NS-NS and R-R flux quanta [3]. In a second step, supersymmetry is broken on some branes\(^9\). This generates a disk-level contribution to the scalar potential. The dilaton’s equation of motion is nevertheless satisfied perturbatively if a small hierarchy exists between the brane (disk) and the flux contributions (tree).

This however forms a consistent scenario under some strong constraints. First, the Freed-Witten anomaly drastically restricts the configuration of branes [32, 33]. For instance, $D9$ branes are forbidden. Second, three-form fluxes preserve the same supercharge as the $O3$ planes, but do not form a supersymmetric configuration with $O9$ planes. Consistent scenarios must then involve magnetized $D7$ branes in an orientifold compactification with $O3$ planes [7]. Third, one may wonder if the D-term breaking considered here is consistent with $\mathcal{N} = 1$ supergravity constraints [34]. Contrary to the standard case of global supersymmetry, local supersymmetry forbids pure D-term supersymmetry breaking. An uplift from an original AdS supersymmetric vacuum by pure D-term is then impossible. Combined effects of F- and D-term breaking must then be considered [35]. The scenario presented here is different. Indeed, the stabilization of Kähler moduli is achieved without any non-perturbative effects. Imaginary self-dual (ISD) three-form fluxes and constant internal magnetic fields can stabilize the vacuum in Minkowski space. Unlike in Ref. [36], the value of the superpotential before uplifting vanishes, $\langle W \rangle = 0$, and the argument of Ref. [34] does not apply anymore. A dS uplifting by pure D-terms can then be achieved.

The above constructions must satisfy a last constraint: non-supersymmetric branes usually contain tachyonic modes in their open string spectra that signal instabilities.

\(^9\)This can also be applied to models where the moduli are stabilized in a Minkowski vacuum [31].
For instance, magnetized branes that do not preserve the same supersymmetry may contain tachyons in their twisted sectors \[37\]. These instabilities must be absent of consistent vacua. Examples of such models were presented in Ref. \[38\]. It was shown that in particular regions of the closed string moduli space, the squared-masses of all twisted open string scalars are non-negative\(^{10}\).

The second solution involves non-critical strings. In addition to the disk contribution, the scalar potential acquires a contribution on the sphere arising from the central charge deficit \(\delta c\) of the conformal field theory (CFT). Non-critical strings allow the presence of AdS vacua where the dilaton field is fixed at a perturbative regime\(^{11}\).

Let us now be more precise and derive explicitly the above statements. We start by considering the disk contribution to the scalar potential for a consistent set of \(K + 1\) magnetized branes. Let us assume that the first \(K\) stacks stabilize the metric moduli by supersymmetric conditions. The only remaining massless scalar field from the closed string sector is the dilaton and the corresponding R-R axion. There is also a single massless \(U(1)\) vector boson from the last magnetized brane with flux \(\mathcal{F}_{K+1} \equiv \mathcal{F}\).

The scalar potential (5.14) is then the sum of three different terms. Upon the R-R tadpole cancellation conditions, the first term is the tension deficit \(\delta T\) of the last non-supersymmetric brane, the second term comes from the DBI-action (3.13) of the remaining massless \(U(1)\), and the last contribution arises from its FI-term proportional to the parameter \(\bar{\xi}\). Together, they can be written as

\[
\mathcal{V}(\varphi, d) = T_3 e^{-\varphi} \frac{2\pi}{g_{K+1}^2} \left[ \delta T - \left(1 - \sqrt{1 - (2\pi\alpha' d)^2}\right) + \bar{\xi}(2\pi\alpha' d) \right] \equiv T_3 e^{-\varphi} \delta \bar{T},
\]

where

\[
\delta T = 1 - \sin x, \quad \bar{\xi} = \cos x, \quad \sin x = \frac{A}{\sqrt{A^2 + B^2}} \quad (5.20)
\]

and

\[
A = \frac{1}{\left(4\pi^2\alpha'\right)^3} \int \left( J^3 - \mathcal{F}^2 J \right) \quad \text{and} \quad B = \frac{1}{\left(4\pi^2\alpha'\right)^3} \int \left( \mathcal{F}^3 - J^2 \mathcal{F} \right). \quad (5.21)
\]

After elimination of the auxiliary field \(d\), one easily realizes that the potential is positive semi-definite

\[
\delta T = \frac{2\pi}{g_{K+1}^2} \left(1 - \sin x - 1 + \sqrt{1 + \cos x} \right) = \sqrt{A^2 + B^2} \left( \sqrt{1 + \bar{\xi}^2} - \sqrt{1 - \bar{\xi}^2} \right) > 0. \quad (5.22)
\]

The only solution to the dilaton’s equation of motion corresponds to the supersymmetric configuration where \(\langle d \rangle = \bar{\xi} = \delta T = 0\) as in the quadratic approximation.

\(^{10}\)A similar analysis including non-perturbative effects has been done in Ref. [39].

\(^{11}\)A similar phenomenon has been studied in non-critical type 0B string by Ref. [40].
possibility is however excluded in our example with vanishing VEV’s for open string charged states. Note that even in the case where the four-dimensional background is not Minkowski, but has a constant curvature, the equation of motion for the dilaton and the Einstein equations for the metric are incompatible. It is therefore not possible to obtain consistent non-supersymmetric configurations of magnetized branes, neither in the Minkowski nor in the (A)dS space.

In the presence of three-form closed string fluxes, the dilaton can be stabilized in a supersymmetric way by minimizing the tree-level potential induced by F-terms. In this case, the disk contribution (5.19) arising from the FI D-term is a positive constant and the Einstein equations for the metric are satisfied in a dS space-time with positive curvature, setup by

\[
\delta \bar{T} = \frac{2}{T^3} e^{3\varphi_0} \delta \bar{T} \leq M_p^2 > 0; \quad v_6 \equiv \frac{V_6}{(4\pi^2 \alpha')^3},
\]

where \(\varphi_0\) is the VEV of the dilaton and we used the expression (5.7) for the Planck mass. Supersymmetry is broken by a D-term of the \((K + 1)\)-th brane, given by:

\[
\langle d \rangle = \frac{1}{2\pi \alpha'} \frac{\xi}{\sqrt{1 + \xi^2}},
\]

which in principle can be made small compared to the string scale by tuning the fluxes. This mechanism provides a solution to the so-called vacuum uplifting problem in the KKLT context [4].

In the absence of three-form fluxes, a different vacuum can be found by going off-criticality. Indeed, for non-critical strings, an additional contribution to the scalar potential appears at the sphere-level, proportional to the central charge deficit \(\delta c\). Together with the disk contribution, the scalar potential acquires the form

\[
V_{nc}(\varphi) = e^{-2\varphi_0} v_6 \delta c + e^{-\varphi_4} T_3 \delta \bar{T}
\]

\[
= e^{-2\varphi_4} \delta c + e^{-\varphi_4} v_6^{-1/2} T_3 \delta \bar{T},
\]

in the string frame \((M_s = 1)\), or equivalently,

\[
V_{nc}(\varphi) = e^{2\varphi_4} \delta c + \frac{e^{3\varphi_4}}{(2\pi)^3 v_6^{1/2}} \delta \bar{T}
\]

in the Einstein frame \((M_P = 1)\). Here \(\varphi_4\) is the four-dimensional dilaton related to the ten-dimensional dilaton \(\varphi\) by \(e^{-2\varphi_4} = e^{-2\varphi} v_6\), with \(v_6\) the six-dimensional volume given in eq. (5.23). The potential (5.25) has a minimum for a positive string coupling \(g_s = e^{\varphi_0}\), with \(\varphi_0 = \langle \varphi \rangle\), only if \(\delta c\) is negative: \(\delta c < 0\). In this case, the value of the potential at the minimum is also negative. We have

\[
V(\varphi = \varphi_0) = \frac{4}{27} \frac{\delta c^3}{\delta \bar{T}^2} (2\pi)^6 v_6 M_P^4 < 0 \quad \text{and} \quad e^{\varphi_0} = -\frac{2 \delta c}{3 \delta \bar{T}} (2\pi)^3 v_6 > 0.
\]
The scalar potential has therefore a non-supersymmetric minimum with a D-term supersymmetry breaking given by eq. (5.24) and a negative vacuum energy. This solution corresponds to an AdS vacuum whose curvature may be given in terms of the fluxes and $\delta c$ as [41]:

$$
R = \frac{8}{27} \frac{\delta c^3}{\delta \bar{T}^2} (2\pi)^6 v_6 M_P^2 = \frac{2}{3\pi} \delta c M_P^2, \quad g_s = e^{\phi_0} = -\frac{2\delta c}{3\delta \bar{T}} (2\pi)^3 v_6, \quad (5.28)
$$

where $\delta \bar{T}$ is given in eq. (5.22), in terms of volume moduli and fluxes of the non-supersymmetric brane. One sees that the string coupling $g_s$ can be made arbitrarily small by an adequate choice of CFT with small negative central charge deficit $\delta c$. Similarly, for fluxes and values of the moduli at their minimum such that $\delta \bar{T}$ is large, the string coupling can be fixed at a perturbative regime. This may be achieved together with a perturbatively small gauge coupling $g^2_{K+1} e^{\phi_0} / \sqrt{A^2 + B^2}$ for the last non-supersymmetric brane. The AdS curvature can also be tuned in the same way. In the perturbative regime where $g_s \ll 1$, this is also small provided the one-loop potential contribution $\delta \bar{T}$ is not too large.

Note that $\delta c$ can be done infinitesimally small only for negative values that are required here. The reason is that unitary CFT’s can have accumulation points for their central charge only from below. It is then expected that $\delta c$ is quantized but can take infinitesimally small values. One simple example is provided by replacing a free compactified coordinate with a CFT from the minimal series. It would be of course very interesting to study explicitly the (closed and open) string quantization in this setup.

As shown in Section 4, a non-critical dilaton potential, proportional to $\delta c$ in eq. (5.25), is described by a gauging of the effective $N = 2$ supergravity of the closed string sector. Indeed, by considering the single dilaton hypermultiplet (in a vacuum where all other closed string moduli are fixed) and gauging the isometry associated to the shift of the NS-NS four-dimensional antisymmetric tensor using the graviphoton, one obtains the scalar potential (4.15). Identifying $V = e^{-\varphi_4}$ and the coupling constant of the gauging with $\delta c$, $g^2_s = -\delta c$, one obtains that the dilaton is stabilized at

$$
g_s = \frac{2}{3} g^2_s \delta \bar{T}^{-1} (2\pi)^3 v_6 = \sqrt{\frac{2}{3} (2\pi)^3/2 v_6^{1/2}} g\, g \left( \sqrt{1 + \tilde{\xi}^2} - \sqrt{1 - \tilde{\xi}^2} \right)^{-1/2}, \quad (5.29)
$$

where $g$ is the physical gauge coupling of the non-supersymmetric brane.\footnote{Note that $g$ differs in general from the gauge couplings of the Standard Model which may arise on different set of branes.} The $\tilde{\xi}$-dependent term in eq. (5.29) is bounded, since $\tilde{\xi} \in [0,1]$. In the limit of vanishing FI-parameter $\tilde{\xi} \to 0$, supersymmetry is restored on the entire set of branes, the disk
amplitude vanishes and we end up with the sphere contribution which leads to a run-
away behaviour for the dilaton field. At finite values of \( \xi \) however, supersymmetry is
broken. A perturbative regime can then be found when the gauge coupling \( g \) is small,
or equivalently from eq. (5.16), when the volume of the internal manifold is stabilized
at a relatively large value.

It is important to notice that the validity of the approximation which allows us to
fix the dilaton VEV by the method presented above relies on a perturbative expansion
around the critical dimension for \( \delta c \) small, together with the string loop expansion for
\( g_s \) small, in a way that \( g_s \) and \( \delta c \) are the same order. Higher-order corrections can then
be consistently neglected in the solution (5.27), under the usual assumption that there
are no large numerical coefficients involved.

The supersymmetry breaking solutions described above, with all closed string mod-
uli stabilized (even in toroidal type I string compactifications), may be used for building
simple models with interesting phenomenology. Indeed, Ref. [11] provides an exam-
ple of a supersymmetric \( SU(5) \) grand unified gauge group with three generations of
quarks and leptons. As was pointed out in this work, the set of branes with VEV’s for
charged scalars needed to restore supersymmetry may be replaced by a brane sector
where supersymmetry is broken by D-terms, while the dilaton is stabilized in a dS or
AdS vacuum. This sector can be used as a source of supersymmetry breaking, mediated
to the observable world by gauge interactions [42]. An obvious advantage of this
framework is its calculability at the string level.

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A Conventions for $\mathcal{N} = 1$ superspace

The $\mathcal{N} = 1$ supersymmetry variation of a superfield $V$ is

$$\delta V = (\epsilon Q + \bar{\epsilon} \bar{Q})V,$$

with supercharges

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} + i(\sigma^\mu \bar{\theta})_\alpha \partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^\dot{\alpha}} - i(\theta \sigma^\mu)_{\dot{\alpha}} \partial_\mu,$$

where $\theta, \bar{\theta}$ are the Weyl spinor coordinates of the $\mathcal{N} = 1$ superspace and $\sigma^\mu = (\mathbb{1}, \sigma^i)$ with $\mathbb{1}$ the identity and $\sigma^i$ the three Pauli matrices. Since

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = -2i(\sigma^\mu)_{a\dot{a}} \partial_\mu,$$

the supersymmetry algebra is

$$[\delta_1, \delta_2]V = -2i(\epsilon_1 \sigma^\mu \epsilon_2 - \epsilon_2 \sigma^\mu \epsilon_1) \partial_\mu V.$$

The covariant derivatives

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\sigma^\mu \bar{\theta})_\alpha \partial_\mu, \quad \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^\dot{\alpha}} - i(\theta \sigma^\mu)_{\dot{\alpha}} \partial_\mu,$$

anticommutate with the supercharges and verify

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i(\sigma^\mu)_{a\dot{a}} \partial_\mu$$

as well. The identities

$$DD \theta \theta = D\bar{D} \bar{\theta} \bar{\theta} = -4, \quad \int d^2 \theta d^2 \bar{\theta} = -\frac{1}{4} \int d^2 \theta D\bar{D} = -\frac{1}{4} \int d^2 \bar{\theta} D\bar{D},$$

valid under a space-time integral $\int d^4 x$, are commonly used.

The super-Maxwell Lagrangian is

$$\mathcal{L}_{Max} = \frac{1}{4} \int d^2 \theta WW + \frac{1}{4} \int d^2 \bar{\theta} \bar{W} \bar{W},$$

with

$$W_\alpha = -\frac{1}{4} D\bar{D} D_\alpha A, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4} D\bar{D} \bar{D}_{\dot{\alpha}} A,$$

and $A$ is real. In this convention, $\bar{W}_{\dot{\alpha}}$ is minus the conjugate of $W_\alpha$:

$$W_\alpha = -i \lambda_\alpha + \ldots \quad \bar{W}_{\dot{\alpha}} = -i \bar{\lambda}_{\dot{\alpha}} + \ldots$$

where $\lambda$ is the gaugino spinor. Then

$$WW = -\lambda \lambda + \ldots + \theta \bar{\theta}[d^2 - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} + 2i \lambda \sigma^\mu \partial_\mu \bar{\lambda}],$$

$$\bar{W} \bar{W} = -\bar{\lambda} \bar{\lambda} + \ldots + \bar{\theta} \theta[d^2 - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} - 2i \partial_\mu \lambda \sigma^\mu \bar{\lambda}],$$

(A.10)
and
\[ L_{\text{Max.}} = \frac{1}{2} d^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \lambda \sigma^\mu \partial_\mu \bar{\lambda} - \frac{i}{2} \partial_\mu \lambda \sigma^\mu \bar{\lambda}. \]  
(A.11)

For a chiral superfield \( \phi(y, \theta) = z(y) + \sqrt{2} \theta \psi(y) - \theta \theta f(y) \), the \( \mathcal{N} = 1 \) supersymmetry variations are
\[ \delta z = \sqrt{2} \epsilon \psi, \]
\[ \delta \psi_\alpha = -\sqrt{2} [f_\epsilon + i(\sigma^\mu)_{\alpha} \partial_\mu z], \]  
(A.12)
\[ \delta f = -\sqrt{2} i \partial_\mu \psi \sigma^\mu \bar{\tau}. \]

**B Useful identities**

With \( 1 = \epsilon^{12} = \epsilon^{i\bar{j}} = -\epsilon_{12} = -\epsilon_{\bar{i}\bar{j}} \),
\[ D_\alpha D_\beta = \frac{1}{2} \epsilon_{\alpha\beta} D D, \quad \bar{D}_{\dot{\alpha}} \bar{D}_{\dot{\beta}} = -\frac{1}{2} \epsilon_{\dot{\alpha}\dot{\beta}} \bar{D} \bar{D}, \]
\[ [D_\alpha, \bar{D}_{\dot{\alpha}}] = -4i(\sigma^\mu \bar{D})_{\alpha} \partial_\mu, \quad [\bar{D}_{\dot{\alpha}}, D_\alpha] = +4i(D\sigma^\mu)_{\dot{\alpha}} \partial_\mu, \]
\[ DD W_\alpha = 4i(\sigma^\mu \partial_\mu W)_{\alpha}, \quad \bar{D} D \bar{W}_{\dot{\alpha}} = -4i(\partial_\mu W \sigma^\mu)_{\dot{\alpha}}, \]
\[ [DD, \bar{D} \bar{D}] = -8i (D\sigma^\mu \bar{D}) \partial_\mu + 16 \Box = 8i (D\sigma^\mu D) \partial_\mu - 16 \Box. \]

Since
\[ D_\alpha \bar{D}_{\dot{\alpha}} D_\beta - D_\beta \bar{D}_{\dot{\alpha}} D_\alpha = \frac{1}{2} \epsilon_{\alpha\beta} (D_\alpha \bar{D} D + D D \bar{D}_{\dot{\alpha}}), \]
\[ \bar{D}_{\dot{\alpha}} D_\alpha \bar{D}_{\dot{\beta}} - \bar{D}_{\dot{\beta}} D_\alpha \bar{D}_{\dot{\alpha}} = \frac{1}{2} \epsilon_{\dot{\alpha}\dot{\beta}} (D_\alpha \bar{D} \bar{D} + \bar{D} \bar{D} D_\alpha), \]
we also have
\[ D^\alpha \bar{D}_{\dot{\alpha}} D_\alpha = -\frac{1}{2} (\bar{D}_{\dot{\alpha}} D D + DD \bar{D}_{\dot{\alpha}}), \]
\[ \bar{D}_{\dot{\alpha}} D_\alpha \bar{D}^\dot{\alpha} = -\frac{1}{2} (D_\alpha \bar{D} \bar{D} + \bar{D} \bar{D} D_\alpha). \]

On a chiral superfield,
\[ D^\alpha \bar{D}_{\dot{\alpha}} D_\alpha \phi = -\frac{1}{2} \bar{D}_{\dot{\alpha}} D D \phi, \]
\[ \bar{D}_{\dot{\alpha}} D_\alpha \bar{D}^\dot{\alpha} \phi = -\frac{1}{2} D_\alpha \bar{D} \bar{D} \phi. \]

For any chiral spinor superfield \( \psi \),
\[ (\psi \sigma^\mu \bar{\eta}) \partial_\mu (\psi \psi) = -(\partial_\mu \psi \sigma^\mu \bar{\eta}) \psi \psi. \]

It is useful to notice that
\[ [\bar{D} \bar{D}, \eta D + \bar{\eta} \bar{D}] = \eta^\alpha [\bar{D} \bar{D}, D_\alpha] = 4i (\eta \sigma^\mu \bar{D}) \partial_\mu. \]

Hence, applying \( \bar{D} \bar{D} \) on the variation \( \delta^* \) of a superfield is not the same as the variation \( \delta^* \) of \( \bar{D} \bar{D} \) applied on the same superfield, except if the superfield is chiral or under a space-time integral \( \int d^4 x \).
C Solution of the constraint (3.6)

The nonlinear constraint (3.6) can be rewritten as:

$$\kappa^2 X = WW - \frac{1}{2} DD \frac{WW}{(\kappa^2 + \frac{1}{2} DD X)(\kappa^2 + \frac{1}{2} DD X)}.$$  \hfill (C.1)

To find $X$, we need to find an expression for $DDX$ in the denominator. In general

$$DDX = DD \frac{WW}{\kappa^2 + \frac{1}{2} DD X},$$

but we know that in the denominator of expression (C.1), the derivatives must act on $WW$: any other choice would lead to a factor $W_\alpha$ or $\overline{W}_\dot{\alpha}$ in the expansion of the denominator and then to a vanishing contribution since $W_\alpha W_\beta W_\gamma = 0$. It is then sufficient to solve the simple equation

$$DDX = \frac{1}{\kappa^2 + \frac{1}{2} DD X} DDWW. \hfill (C.2)$$

The solution is

$$\overline{DDX} = -\kappa^2 \left[ 1 + B - \sqrt{1 + 2A + B^2} \right], \hfill (C.3)$$

with

$$A = \frac{1}{2\kappa^4} (DDW^2 + DWD^2) = A^*, \quad B = \frac{1}{2\kappa^4} (DDW^2 - DWD^2) = -B^*.$$  

This solution can then be inserted in the denominator of eq. (C.1), to obtain the final expression (3.10).

In order to derive the component expression of eq. (3.12) of the Lagrangian (3.10), one uses the identities:

$$\left( F_{\mu\nu} \hat{F}^{\mu\nu} \right)^2 = \frac{1}{4} \left( \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \right)^2 = 4 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu} - 2 (F_{\mu\nu} F^{\mu\nu})^2,$$

$$-\det (\eta_{\mu\nu} + AF_{\mu\nu}) = 1 + \frac{A^2}{2} F_{\mu\nu} F^{\mu\nu} - \frac{A^4}{16} (F_{\mu\nu} \hat{F}^{\mu\nu})^2.$$
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