A Note on Tsallis Holographic Dark Energy

M. Abdollahi Zadeh1, A. Sheykhi1, 2, H. Moradpour2† and Kazuharu Bamba3†
1 Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran
2 Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran
3 Division of Human Support System, Faculty of Symbiotic Systems Science, Fukushima University, Fukushima 960-1296, Japan

We explore the effects of considering various infrared (IR) cutoffs, including the particle horizon, Ricci horizon and Granda-Oliveros (GO) cutoffs, on the properties of Tsallis holographic dark energy (THDE) model, proposed inspired by Tsallis generalized entropy formalism [1]. Interestingly enough, we find that for the particle horizon as IR cutoff, the obtained THDE model can describe the accelerated universe. This is in contrast to the usual HDE model which cannot lead to an accelerated universe, if one consider the particle horizon as IR cutoff. We also investigate the cosmological consequences of THDE under the assumption of a mutual interaction between the dark sectors of the Universe. It is shown that the evolution history of the Universe can be described by these IR cutoffs and thus the current cosmic acceleration can also been realized. The sound instability of THDE models for each cutoff are also explored, separately.

I. INTRODUCTION

There are various cosmological observations which indicate that our Universe is now experiencing an accelerated expansion phase [2–10]. The origin of this phase is attributed to a mysterious matter which is called dark energy (for reviews on the dark energy problem and the modification of gravity, which is called geometric dark energy, to account for the late-time cosmic acceleration, see, e.g., [17]). In this regard, holographic dark energy (HDE) is an interesting attempt to solve this bizarre problem in the framework of quantum gravity by using the holographic hypothesis [18–20]. This model is in agreement with various astronomical observations [21,22], and some of its various scenarios can be found in [23–38]. Horizon entropy is the backbone of the HDE models, and hence, any change to the horizon entropy affects the HDE model. Another important player in these models is the IR cutoff, and indeed, various IR cutoffs lead to different HDE models [26,29,30].

Since gravity is a long-range interaction, one can also use generalized statistical mechanics to study the gravitational systems [39–44]. In this regard, due to this fact that the Bekenstein entropy can be obtained by applying the Tsallis statistics to the system [39,40], three new HDE models with titles THDE, SMHDE and RHDE have recently been proposed [1,41,42]. Among these three models, in the absence of an interaction between the cosmos sectors, RHDE, based on the Renyi entropy and the first law of thermodynamics, shows more stability from itself [42]. In fact, in a non-interacting universe, while SMHDE is classically stable whenever SMHDE is dominant in the universe, THDE, built using the Tsallis generalized entropy [44], is never stable at the classical level [1,41]. It is also worth mentioning that a THDE model whose IR cutoff is the future event horizon has been studied in a non-interacting universe showing satisfactory results [45].

Observations admit an interaction between the dark sectors of cosmos including dark energy and dark matter [46–56]. The existence of such mutual interaction may provide a solution for the coincidence problem [54–62]. Here, we are interested in studying the dynamics of a flat FRW universe filled with a pressureless source and THDE in both interacting and non-interacting cases. In our setups, various IR cutoffs, including the apparent and particle horizons together with the GO and Ricci cutoffs, have been used in order to build THDE.

The organization of this paper is as follows. In the next section, considering an interaction between dark matter and THDE whose IR cutoff is the apparent horizon, the universe evolution has been studied. Thereinafter, new THDE is built by employing the particle horizon as the IR cutoff, and then, the cosmic evolution has been investigated for both interacting and non-interacting universes in section III. The cases of the GO and Ricci cutoffs have also been studied in Secs. IV and V, respectively. The last section is devoted to a summary and concluding remarks.

II. INTERACTING THDE WITH HUBBLE CUTOFF

The energy density of THDE model is as follows [41]$
\rho_D = BL^{2n-4},$
where $B$ is an unknown parameter. We consider a homogeneous and isotropic flat Friedmann-Robertson-Walker (FRW) universe described by the line element

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2 d\Omega^2],$$
in which $a(t)$ is scale factor, and hence, the first Friedmann equation takes the form

$$H^2 = \frac{1}{3m_p^2} (\rho_D + \rho_m).$$

(3)

Here, $\rho_m$ and $\rho_D$ denote the energy density of pressure less matter (DM) and the THDE density, respectively. Defining, as usual, the dimensionless density parameters as

$$\Omega_D = \frac{\rho_D}{\rho_c} = \frac{B}{3m_p^2} H^{-2\delta + 2}, \quad \Omega_m = \frac{\rho_m}{\rho_c},$$

(4)

where $\rho_c = 3m_p^2 H^2$ is called the critical energy density, we can easily rewrite the first Friedmann equation in the form

$$\Omega_m + \Omega_D = 1.$$  

(5)

Moreover, we assume that DM and DE interact with each other meaning that the conservation law is decomposed as

$$\dot{\rho}_m + 3H\rho_m = Q,$$

(6)

$$\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = -Q,$$

(7)

in which $\omega_D \equiv p_D/\rho_D$ is the equation of state (EoS) parameter of THDE and $Q$ denotes the interaction term between DE and DM. Throughout this paper, $Q = 3b^2 H (\rho_m + \rho_D)$, where $b^2$ is a coupling constant, is considered as the mutual interaction between the cosmos sectors [55, 56]. The ratio of the energy densities is also evaluated as

$$r = \frac{\Omega_m}{\Omega_D} = \frac{1 - \Omega_D}{\Omega_D}.$$  

(8)

Taking the time derivative of Eq. (3), and by using Eqs. (6), (7) and (8), we can obtain

$$\frac{\dot{H}}{H^2} = \frac{3}{2} (1 + \omega_D + r)\Omega_D.$$  

(9)

In addition, by considering the Hubble horizon as the IR cutoff, $L = H^{-1}$, the energy density [1] takes form

$$\rho_D = BH^{-2\delta + 4},$$

(10)

The time derivative of above equation, combined with Eqs. (7) and (9), also leads to

$$\omega_D = \frac{\delta - 1 + \frac{b^2}{\Omega_D}}{(2 - \delta)\Omega_D - 1}.$$  

(11)

Simple calculations for the deceleration parameter, defined as

$$q = -1 - \frac{\dot{H}}{H^2},$$

(12)

yield

$$q = \left[ \frac{(1 - 2\delta)\Omega_D + 1 - 3b^2}{2(1 - (2 - \delta)\Omega_D)} \right],$$

(13)

where we used Eqs. (11) and (9) to obtain this result. Combining the time derivative of Eq. (4) with Eqs. (9) and (11), and defining $\Omega'_D = d\Omega_D/d(\ln a)$, we get

$$\Omega'_D = 3(\delta - 1)\Omega_D \left( \frac{1 - \Omega_D - b^2}{1 - (2 - \delta)\Omega_D} \right),$$

(14)

for the $L = H^{-1}$ case.

![FIG. 1: Evolution of $\Omega_D$ versus redshift parameter $z$ for interacting THDE with Hubble horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $\delta = 1.4$.](image)

For $\delta = 1.4$ case and the initial condition $\Omega_D^0 = 0 = 0.73$, the evolutions of $\Omega_D$, $\omega_D$ and $q$ versus $(1 + z)$ have been plotted in Figs. 1, 2 and 3. From these figures, one can see that $\omega_D$ can cross the phantom line, and moreover, the value of the transition redshift is increased as a function of $b^2$. Finally, we explore the stability of

![FIG. 2: Evolution of $\omega_D$ versus redshift parameter $z$ for interacting THDE with Hubble horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $\delta = 1.4$.](image)
FIG. 3: Evolution of $q$ versus redshift parameter $z$ for interacting THDE with Hubble horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $\delta = 1.4$.

the THDE model as

$$v_s^2 = \frac{dP_D}{d\rho_D} = \frac{\dot{P}_D}{\dot{\rho}_D} = \frac{\rho_D}{\dot{\rho}_D} \omega_D + \omega_D,$$

(15)

Combining time derivative of Eq. (10) with Eq. (9), we have

$$\dot{\rho}_D = 3\rho_D(\delta - 2)H(1 + \Omega_D\omega_D),$$

(16)

which finally leads to

$$v_s^2 = \frac{(\delta - 1)(\Omega_D - 1) + b^2[\delta + \frac{1}{(3\Omega_D - 2)H_D}]}{[1 - (2 - \delta)\Omega_D]^2},$$

(17)

where Eq. (15) and the time derivative of Eq. (11) have been employed to get the above result. It is also useful to note here that in the absence of interaction term ($b^2 = 0$), Eqs. (11), (13), (14) and (17) are reduced to relations obtained in Ref [1]. Figs. 4 and 5 show that the interacting THDE model with Hubble cutoff is stable neither for a fixed $\delta$ nor for a fixed $b^2$ meaning that the model is unstable, a result the same as that of the non-interacting case [1].

III. THDE WITH PARTICLE HORIZON CUTOFF

A. Non-interacting

It is well-known that HDE model with particle horizon as IR cutoff cannot lead to an accelerated universe and it is impossible to obtain an accelerated expansion [26]. Indeed, with this cutoff, one always arrives at $\omega_D > -1/3$, which is in contradiction with recent cosmological observations [26]. As we shall see in this section, for the THDE with particle horizon as IR cutoff, it is quite possible to reproduce an accelerating universe which is one of the main advantages of THDE in compared to the usual HDE model. The particle horizon is defined as [26]

$$R_p = a(t) \int_0^t \frac{dt}{a(t)},$$

(18)

and meets the condition

$$\dot{R}_p = HR_p + 1.$$  

(19)

Therefore, bearing Eq. (11) in mind, the energy density of THDE is obtained as

$$\rho_D = BR_p^{2\delta - 4},$$

(20)

where its time derivative leads to

$$\dot{\rho}_D = \rho_D(2\delta - 4)H(1 + F),$$

(21)

in which

$$F = \left( \frac{3\Omega_D H^{2\delta - 2}}{B} \right)^{1/(4 - 2\delta)}.$$  

(22)
By substituting Eq. (21) into the conservation law,
\[ \dot{\rho}_D + 3H(1 + \omega_D)\rho_D = 0, \]  
(23)
one finds the EoS parameter of THDE as
\[ \omega_D = -1 - \left( \frac{2\delta - 4}{3} \right)(1 + F). \]  
(24)
Additionally, if we combine the time derivative of \( \Omega_D = \rho_D/(3m_p^2H^2) \) with Eqs. (9), (21) and (24), then one may arrive at
\[ \Omega_D' = \Omega_D(\Omega_D - 1)[1 + 2F(\delta - 2) - 2\delta]. \]  
(25)
The deceleration parameter \( q \) and the squared speed of sound (defined in Eq. (15)), are also founded out as
\[ q = \frac{1 + (1 - 2F(\delta - 2) - 2\delta)\Omega_D}{2}, \]  
(26)
and
\[ v_s^2 = -\frac{2 - 9F - 10F^2 + 4\delta(F + 1)^2}{6(F + 1)} + \frac{F[-1 + 2F(-2 + \delta) + 2\delta\Omega_D]}{6(F + 1)}, \]  
(27)
respectively. Here, in order to obtain Eq. (26), we employed Eq. (24) in writing Eq. (9), and then we used relation (12). In Figs. 6-12, the system parameters have been plotted for some values of the system unknowns and the initial condition \( \Omega_D^0 = 0.73 \), \( B = 2.4 \) and \( H(a = 1) = 67 \). As it is apparent, although this cutoff leads to a model can provide acceptable behavior for \( \Omega_D, q \) and \( \omega_D \), the model is not stable.
FIG. 10: Evolution of $v_s^2$ and $\omega_D$ versus redshift parameter $z$ for non-interacting THDE with particle horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$, $B = 2.4$, $\delta = 2.4$ and $H(a = 1) = 67$.

B. Interacting

Using Eq. (21) and $Q = 3b^2H(\rho_m + \rho_D)$ in the conservation equation (1), the EoS parameter is found as

$$\omega_D = -1 - \frac{b^2}{\Omega_D} - \left(\frac{2\delta - 4}{3}\right)(1 + F). \quad (28)$$

For this choice of interaction, the evolution of density parameter, the deceleration parameter $q$ and stability for the model are calculated as

$$\Omega_D' = -3b^2\Omega_D + \Omega_D(\Omega_D - 1)[1 + 2F(\delta - 2) - 2\delta], \quad (29)$$

$$q = \frac{[1 - 3b^2 + (1 - 2F(\delta - 2) - 2\delta)\Omega_D]}{2}, \quad (30)$$

and
FIG. 15: Evolution of $q$ versus redshift parameter $z$ for interacting THDE with particle horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$, $B = 2.4$, $\delta = 2.4$ and $H(a = 1) = 67$.

\[
v_s^2 = \frac{9b^2(b^2 - 1) + \Omega_D [A - FB]}{-6(F + 1)(\delta - 2)\Omega_D}, \tag{31}
\]

\[A = (\delta - 2)(-2 - 9F - 10F^2 + 4(F + 1)^2\delta)
+ b^2(-3 - 6F + 6\delta + 3F\delta),
\]

\[B = [-1 + 2F(-2 + \delta) + 2\delta(\delta - 2)\Omega_D],
\]

respectively. They have also been plotted in Figs. 13-17 for some values of the model’s parameters. Fig. 17 and 16 show that, unlike the non-interacting case, the model is stable for some values of $z$. In addition, comparing Figs. 13 and 6 with each other, it is obtained that the changes in the density parameter of interacting case is slower than that of the non-interacting case. Moreover, Figs. 14 and 15 indicate that the model behaves as the phantom source, and thus, the model eventually enters the supper accelerated phase (or equally $q < -1$).

IV. THDE WITH GO HORIZON CUTOFF

A. Non-interacting

In order to solve the causality and coincidence problems, Granda and Oliveros (GO) \cite{29, 30} suggested a new cutoff, usually known as GO cutoff in the literatures, which is defined as

\[L = (\gamma H^2 + \zeta \dot{H})^{-1/2}.
\]

In this case the energy density of THDE becomes

\[\rho_D = (\alpha H^2 + \beta \dot{H})^{-\delta + 2}, \tag{32}\]

which leads to

\[
\frac{\dot{H}}{H^2} = \frac{1}{\beta} \left( \frac{(3m^2\Omega_D)^{1/2}}{H^{2/3}} - \alpha \right). \tag{33}\]

Simple calculations for the deceleration and density parameters yield

\[q = -1 - \frac{1}{\beta} \left( \frac{(3m^2\Omega_D)^{1/2}}{H^{2/3}} - \alpha \right), \tag{34}\]

and

\[
\dot{\Omega}_D = \frac{\dot{\rho}_D}{3m_p^2H^2} - 2\Omega_D \frac{\dot{H}}{H}, \tag{35}\]

respectively. For the $Q = 0$ case, inserting the time derivative of Eq. (3) into Eq. (6), we obtain

\[
\frac{\dot{\rho}_D}{3m_p^2H^3} = \frac{2H}{H^2} + 3(1 - \Omega_D), \tag{36}\]

combined with Eqs. (35) and (33) to reach at

\[\Omega_D^0 = (1 - \Omega_D) \left[ 3 + \frac{2}{\beta} \left( \frac{(3m^2\Omega_D)^{1/2}}{H^{2/3}} - \alpha \right) \right]. \tag{37}\]
In this manner, the EoS parameter $\omega_D$ of THDE is given by

$$\omega_D = -1 - \frac{1}{3\Omega_D} \left[ \frac{2}{\beta} \left( \frac{(3m_p^2\Omega_D)^{\frac{1}{3}}}{H^{\frac{1}{3}}} \right)^{\frac{1}{2}} - 1 - \beta \right] + 3(1 - \Omega_D),$$

(38)

where we inserted $\dot{\rho}_D$ from Eq. (36) in the energy conservation law (37). Finally by taking time derivative from Eq. (38), and using it in rewriting Eq. (15), one easily finds

$$v_s^2 = -1 + \frac{2\alpha}{3\beta} + \frac{3\frac{1+\delta}{1+\frac{1}{\beta}} H^{\frac{1}{2+\frac{1}{\beta}}} \Omega_D^{\frac{1}{2+\frac{1}{\beta}}} (2\alpha - 3\beta)}{-2 \times 3^{\frac{1}{2+\frac{1}{\beta}}} H^{\frac{1}{2+\frac{1}{\beta}}} \Omega_D^{\frac{1}{2+\frac{1}{\beta}}} + H^{\frac{2}{2+\frac{1}{\beta}}} [2\alpha - 3\beta + 3\beta \Omega_D]}$$

(39)

and $\delta$ by considering the initial conditions $\Omega_D^{0} = 0.73$ and $H(a = 1) = 67$. It is interesting to note here that the model begin to show stability from itself whenever $q \rightarrow \frac{1}{2}$. Moreover, the depicted curves are some of those which do not cross the phantom line for $z \geq -1$.

### B. Interacting

One can check that $q$ and $\omega_D$ have the same form as those of the non-interacting case meaning that the mutual interaction does not affect them. Thus, we only need the $\omega_D$ and $v_s^2$ parameters evaluated as

$$\omega_D = -3\delta^2 + (1 - \Omega_D) \left[ 3 + \frac{2}{\beta} \left( \frac{(3m_p^2\Omega_D)^{\frac{1}{3}}}{H^{\frac{1}{3}}} \right)^{\frac{1}{2}} - \alpha \right],$$

(40)
and

\[ v_s^2 = -1 + \frac{2\alpha}{3\beta} + 2 \frac{3^{1+\delta}}{\beta(-2+\delta)} H \frac{2^{2+\gamma}}{D} \Omega D \frac{2^{1+\delta}}{\beta(-2+\delta)} \]

respectively. These parameters have been plotted in Figs. 22-23 for some values of the system constants. As it is apparent from Figs. 22-23, this case can model the current accelerated universe. The \( \lambda = 1 \) case is interesting, because unlike SMHDE, this model is stable (unstable) for \( q > 0 \) (\( q < 0 \)). Hence, since the Ricci horizon is a special case of the GO cutoff, the GO cutoff can also produce the same results if proper values for the system unknown constants have been chosen.

\[ \Omega_D = (1 - \Omega_D) \left[ 3 + 2 \left( \frac{3^{-1}m^2 D \Omega_D}{H} \right) - 2 \right]. \]

In this manner, we have

\[ \omega_D = -1 - \frac{1 - \Omega_D}{\Omega_D} \left[ \frac{2}{3(1 - \Omega_D)} \frac{(3^{-1}m^2 D \Omega_D)}{H} - 2 \right] + 1, \]

where Eqs. (36), (41) and (43) have been used to get the above result. Moreover, by taking time derivative from Eq. (46) we find

\[ v_s^2 = \frac{1}{3} + 2 \frac{3^{1+\delta}}{\beta(-2+\delta)} H \frac{2^{2+\gamma}}{D} \left( \lambda^{-1} \Omega_D \right) \frac{2^{1+\delta}}{\beta(-2+\delta)} \]

\[ (-1 + \Omega_D) \]

As it is apparent from Figs. 24-27, this case can model the current accelerated universe. The \( \lambda = 1 \) case is interesting, because unlike SMHDE, this model is stable (unstable) for \( q > 0 \) (\( q < 0 \)). Hence, since the Ricci horizon is a special case of the GO cutoff, the GO cutoff can also produce the same results if proper values for the system unknown constants have been chosen.

\[ \varphi(z) = \Lambda(z) = 0 \]

\[ \frac{H}{H^2} = \left( \frac{(3^{-1}m^2 D \Omega_D)}{H} \right) - 2 \right), \]

and use Eq. (12) to get

\[ q = -1 - \left( \frac{(3^{-1}m^2 D \Omega_D)}{H} \right) - 2 \right). \]

It is also a matter of calculation to combine Eqs. (35) and (46) with (43) to reach at

\[ \Omega_D = (1 - \Omega_D) \left[ 3 + 2 \left( \frac{3^{-1}m^2 D \Omega_D}{3^{1+\delta}} - 2 \right) \right]. \]
FIG. 24: Evolution of $\Omega_D$ versus redshift parameter $z$ for non-interacting THDE with Ricci horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $H(a = 1) = 67$ and $\delta = 1$.

FIG. 25: Evolution of $\omega_D$ versus redshift parameter $z$ for non-interacting THDE with Ricci horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $H(a = 1) = 67$ and $\delta = 1$.

FIG. 26: Evolution of $q$ versus redshift parameter $z$ for non-interacting THDE with Ricci horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $H(a = 1) = 67$ and $\delta = 1$.

FIG. 27: Evolution of $v_s^2$ versus redshift parameter $z$ for non-interacting THDE with Ricci horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $H(a = 1) = 67$ and $\delta = 1$.

In this case, a result due to the fact that the Ricci cutoff is a special case of the GO cutoff. The calculations lead to

$$\Omega_D' = -3b^2 + (1 - \Omega_D)[3 + 2\left(\frac{3(3\lambda - 1)\Omega_D}{H^2} - 2\right)], \quad (48)$$

and

$$v_s^2 = \frac{1}{3} + \frac{2\Omega_D(-2 + \delta)}{\Omega_D(-2 + \delta) + \lambda \Omega_D(-2 + \delta)} \left(\frac{\frac{\lambda - 1}{\Omega_D}}{\Omega_D(-2 + \delta)}\right)^{\frac{1}{\Omega_D}}.$$ 

(49)

In Figs. 28-31 the system parameters have been plotted versus $z$ for some values of the unknown constants. It is obvious that the system parameters affected by the mutual interaction. It is also interesting to note that while $v_s^2$ was not negative for $\lambda = 1 \cdot 5$ in the non-interacting case, here, we always have it is not stable for all values of $v_s^2 < 0$.

VI. CLOSING REMARKS

In the shadow of the holographic principle and based on the non-additive generalized Tsallis entropy expression [44], a new holographic dark energy model called THDE was recently proposed [1]. In this paper, by considering various IR cutoffs, including the particle horizon, Ricci horizon and GO cutoff in the background of FRW universe, we investigated the evolution of THDE models and studied their cosmological consequences. We found out that when the particle horizon is considered as IR cutoff, then the THDE model can explain the current acceleration of the universe expansion. This is in contrast to the usual HDE model which cannot lead to an accelerated universe, if one consider the particle horizon as IR cutoff [26]. We also explored the sound stability...
FIG. 28: Evolution of $\Omega_D$ versus redshift parameter $z$ for interacting THDE with Ricci horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$, $H(a = 1) = 67$, $b^2 = 0.01$ and $\delta = 1$.

FIG. 29: Evolution of $\omega_D$ versus redshift parameter $z$ for interacting THDE with Ricci horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $H(a = 1) = 67$, $b^2 = 0.01$ and $\delta = 1$.

FIG. 30: Evolution of $q$ versus redshift parameter $z$ for interacting THDE with Ricci horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$ and $H(a = 1) = 67$, $b^2 = 0.01$ and $\delta = 1$.

FIG. 31: Evolution of $v_s^2$ versus redshift parameter $z$ for interacting THDE with Ricci horizon as IR cutoff. Here, we have taken $\Omega_D^0 = 0.73$, $H(a = 1) = 67$, $b^2 = 0.01$ and $\delta = 1$.

of the THDE models with various cutoffs. In this manner, the assumed mutual interaction between the cosmos sectors makes the model to be stable for some values of the redshift parameter $z$. For the GO and Ricci horizon cutoffs, we found out that although acceptable behavior for some parameters of system, including $q$, density parameter and $\omega_D$, are achievable, the model is not always stable. In addition, the effects of considering a mutual interaction between the cosmos sectors on the behavior of the solutions have also been studied.

Acknowledgments

We thank Shiraz University Research Council. This work has been supported financially by Research Institute for Astronomy & Astrophysics of Maragha (RI-AAM), Iran. The work of KB was supported in part by the JSPS KAKENHI Grant Number JP 25800136 and Competitive Research Funds for Fukushima University Faculty (18RI009).
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