Dynamically Favored Chiral Symmetry Breakings in Supersymmetric Quantum Chromodynamics *

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Using an effective superpotential in supersymmetric quantum chromodynamics (SQCD) with $N_f$ flavors and $N_c$ colors of quarks for $N_f \geq N_c + 2$, the influence of soft supersymmetry (SUSY) breakings is examined to clarify dynamics of chiral symmetry breakings near the SUSY limit. In the case that SQCD triggers spontaneous chiral symmetry breakings, it is possible to show that our superpotential dynamically favors the successive formation of condensates, leaving either $SU(N_f - N_c)$ or $SU(N_f - N_c + 1)$ unbroken as a chiral nonabelian symmetry.

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I. INTRODUCTION

It has been argued that $N=1$ supersymmetric quantum chromodynamics (SQCD) with $N_f$ flavors and $N_c$ colors of massless quarks exhibits chiral $SU(N_f) \times U(1)$ symmetry which, in the confining phase, will be spontaneously broken for $N_f \leq N_c$ and will remain unbroken for $N_f \geq N_c + 1$. The existence of unbroken chiral symmetries are dynamically supported by non-perturbative superpotential \cite{1,2} and algebraically by the t’Hooft anomaly-matching conditions \cite{3}. However, the suggested dynamics of SQCD with $N_f \geq N_c + 2$ is based on a plausible extrapolation from those respecting the marvelous duality in $N=2$ supersymmetric (SUSY) gauge theories \cite{4}. To utilize this duality in SQCD, “magnetic” quarks are set to contribute in the physics of SQCD. The dynamics of “magnetic” quarks is not derived, but arranged by our convenience so as to fulfill the algebraic requirement from the anomaly-matching conditions. Once such “magnetic” quarks are admitted to participate in SQCD, every other physical consequences correctly follows and a massive number of consistency checks are realized \cite{5}.

However, what is the “magnetic” quark? The answer can be found in SQCD embedded in a softly broken $N=2$ SQCD \cite{6,7}, where the scale invariance of the $N=2$ theory plays a crucial role. The “magnetic” quarks can be identified with those appearing in the $N=2$ SQCD. Such a description of the $N=1$ duality in terms of the $N=2$ duality is expected to be consistent with the correct physics in SQCD with $3N_c/2 < N_f < 3N_c$, where the phase is characterized as an interacting Coulomb phase \cite{6}. On the other hand, in SQCD with $N_f \leq 3N_c/2$, where confinement will take place, no direct convincing derivation of the desired physics of “magnetic” quarks has been made. It is thus reasonable to start seeking possibilities other than SQCD with “magnetic” quarks, in other words, the possibility of spontaneously broken chiral $SU(N_f)$ symmetry \cite{8}.

In SQCD with $N_f \geq N_c + 2$, low-energy chiral symmetry of “electric” quarks cannot sit on the origin of moduli space, where no vacuum expectation values are generated, since no set of composite superfields that perfectly satisfies the anomaly-matching conditions has been found. To maintain full chiral symmetry as “quantum” chiral symmetry, it has been conjectured \cite{1} that the theory experiences phase transition to its “magnetic” phase, whose weak coupling regime corresponds to the confined SQCD in the “electric” phase. The anomalies in the “electric” theory turn out to be balanced by those from hypothetical “magnetic” quarks. However, in spite of the well-motivated physical view, one may consider a conventional option that the theory undergoes successive spontaneous symmetry breakdown until the anomaly-matching is realized by Nambu-Goldstone superfields together with other chiral composite superfields.
The dynamical requirement on the phase transition from the “electric” phase to the “magnetic” phase is replaced by an alternative requirement on the spontaneous chiral symmetry breakdown in the “electric” phase. Both types of dynamics in SQCD is equally possible to be realized.

In the present article, the dynamics of chiral symmetry breaking is examined to gain more insight into physics of SQCD. Since SUSY is broken in the real physics, the smooth SUSY limit of softly broken SQCD [10,11] is also emphasized in our study. Our analyses are performed by the use of an effective superpotential of the Veneziano-Yankielowicz type [12,14], which, in the SUSY vacuum, becomes equivalent to Seiberg’s superpotential [13]. It will then be demonstrated that there is a solution that indicates spontaneous chiral symmetry breaking. Once one vacuum expectation value (VEV) is set to be non-vanishing, the dynamics forces other VEV to be generated so that at most either $SU(N_f - N_c)$ or $SU(N_f - N_c + 1)$ becomes a residual nonabelian chiral symmetry. In the SUSY breaking phase, all remaining chiral symmetries will be broken. In §2, our superpotential is formulated. In §3, the properties of the superpotential together with soft SUSY breaking terms are examined. The last section is devoted to a summary.

II. SUPERPOTENTIAL

Our general strategy is to utilize an arbitrariness appearing in the effective superpotential that cannot be eliminated by a symmetry principle only [13,14] just as in Seiberg’s superpotential for SQCD with $N_f = N_c + 1$. The superfields describing low-energy massless spectra are assumed to come from chiral meson superfields $(T)$ composed of a quark-antiquark pair and chiral (anti)baryon superfields $(B(\bar{B}))$ by $N_c$-quarks $(N_c$-antiquarks). Furthermore, chiral exotic meson superfields $(U)$ composed of $(N_c - 1)$-quarks and $(N_c - 1)$-antiquarks are allowed to participate in our analyses. Combinations of color-singlet states such as (with abbreviated notations), $BT^{N_f - N_c} B/\det(T) \ (= Z_B)$ and $U^{N_f - N_c + 1}/\det(T) \ (= Z_U)$, are totally neutral under the entire set of chiral symmetries as well as an anomalous $\bar{U}(1)$ symmetry. Therefore, an effective superpotential determined by a symmetry principle can involve any function of $Z_B$ and $Z_U$ as $f(Z_B, Z_U)$. The formation of $(0)Z_B|0 \ (\neq 0)$ (or $(0)Z_U|0)$ leads to the unbroken chiral $SU(N_f - N_c)$ (or $SU(N_f - N_c + 1)$) symmetry. For $SU(N_f - N_c)$, massless fields are contained in $T, B$ and $\bar{B}$ while for $SU(N_f - N_c + 1)$ they are contained in $U$ as well.

Our effective superpotential, $W_{\text{eff}}$, for $N_c > 2$ is defined as

$$W_{\text{eff}} = S \left\{ \ln \left[ \frac{S^{N_c - N_f} \det(T) f(Z_B, Z_U)}{A^{3N_c - N_f}} \right] + N_f - N_c \right\},$$

(1)

where $\Lambda$ is the scale of SQCD and $S$ is a color-singlet bilinear in a chiral gauge superfield, whose scalar component is $\lambda \bar{\lambda}$, with $\lambda$ a gaugino. The decoupling property is manifest, as discussed in Ref. [12]. Because the anomaly-matching conditions are not satisfied in the full chiral symmetry, it must spontaneously break down to the next stage where all anomalies present in residual chiral symmetries are consistently generated by massless composite superfields. The SUSY vacuum characterized by $\pi_i (i = 1 \sim N_f) = \pi_b = \pi_b = \pi_u = \pi_\lambda = 0$ is not dynamically allowed. The $\pi$ denote scalar components of superfields defined by

$$\pi_i = \langle 0 | T_i^a | 0 \rangle, \quad \pi_b = \langle 0 | B^{1[N_c - 1]} | 0 \rangle, \quad \pi_b = \langle 0 | \bar{B}^{1[N_c - 1]} | 0 \rangle,$$

$$\pi_u = \langle 0 | U^{1[N_c - 1]} | 0 \rangle, \quad \pi_\lambda = \langle 0 | S | 0 \rangle$$

(2)

with

$$T_j^i = \sum_{A=1}^{N_c} Q_A^i \tilde{Q}_j^A, \quad S = \frac{1}{32\pi^2} \sum_{A,B=1}^{N_c} W_A^B W_B^A,$$

$$B_{[i_1 \cdots i_{N_c}]^{[A_1 \cdots A_{N_c}}} = \sum_{A_1 \cdots A_{N_c} = 1}^{N_c} \frac{1}{N_c!} \varepsilon_{A_1 A_2 \cdots A_{N_c}} Q_{A_1}^{i_1} \cdots Q_{A_{N_c}}^{i_{N_c}},$$

$$\bar{B}_{[i_1 \cdots i_{N_c}]_{[A_1 \cdots A_{N_c}]}^{(1)}} = \sum_{A_1 \cdots A_{N_c} = 1}^{N_c} \frac{1}{N_c!} \varepsilon_{A_1 A_2 \cdots A_{N_c}} Q_{A_1}^{i_1} \cdots Q_{A_{N_c}}^{i_{N_c}},$$

$$U_{[i_1 \cdots i_{N_c - 1}] [j_1 \cdots j_{N_c - 1}]^{[A_1 \cdots A_{N_c - 1}}} = \sum_{A=1}^{N_c} C_A^{A_1 [i_1 \cdots i_{N_c - 1}] [j_1 \cdots j_{N_c - 1}]},$$

(3)

where $C_{[i_1 \cdots i_{N_c - 1}]}^{[A_1 A_2 \cdots A_{N_c - 1}] v^{A_1 A_2 \cdots A_{N_c - 1}} Q_{A_1}^{i_1} \cdots Q_{A_{N_c - 1}}^{i_{N_c - 1}} / (N_c - 1)!$, and the similarly of $\bar{C}$ and $Q_A^i$, and $\bar{Q}_i^A$ and $W_A^B$, respectively, represent chiral superfields of quarks, antiquarks, and gluons. The behavior of $W_{\text{eff}}$ in the limit
of vanishing gauge coupling $g$ is readily found, by applying the rescaling $S \rightarrow g^2 S$ and invoking the definition $\Lambda = \mu \exp(-8\pi^2/(3N_c - N_f))$, to be $W_{\text{eff}} \rightarrow WW/4$, which is the tree superpotential for the gauge kinetic term.

To proceed to discussing effects from soft SUSY breakings, let us include breaking terms in the potential $V = V_{\text{SUSY}} + V_{\text{soft}}$. The SUSY-invariant $V_{\text{SUSY}}$ is defined by

$$V_{\text{SUSY}} = G_T \left( \sum_{i=1}^{N_f} |W_{\text{eff};i}|^2 \right) + G_B \left( \sum_{i=b,b} |W_{\text{eff};i}|^2 \right) + G_U |W_{\text{eff};u}|^2 + G_S |W_{\text{eff};s}|^2,$$

where $W_{\text{eff};i} = \partial W_{\text{eff}} / \partial \pi_i$, etc., and $G_T = G_T(T^i T)$ characterizes the kinetic term for $T$ defined by the Kähler potential, $K$, which is assumed to be diagonal, $\partial^2 K / \partial T_{ik} \partial T_{jl} = \delta_{ij} \delta_{k,l} G_T^{-1}$, and similarly for $G_B = G_B(B^i B + B^i B)$, $G_U = G_U(U^i U)$ and $G_S = G_S(S^i S)$. The SUSY-breaking is induced by soft breaking masses denoted by $\mu_{Li}$, $\mu_{Ri}$, $\mu_i$ and $m_\lambda$, through $\mathcal{L}_{\text{mass}}$ for scalar quarks $\phi_i^A$, scalar antiquarks $\bar{\phi}_i^A$, and gluinos $\lambda_i^B$ with $\text{Tr}(\lambda) = 0$ ($A, B = 1 \sim N_c$) generally expressed as

$$- \mathcal{L}_{\text{mass}} = \sum_{i=1}^{N_f} \left[ \mu_{Li}^2 |\phi_i|^2 + \mu_{Ri}^2 |\bar{\phi}_i|^2 + \mu_i^2 (\phi_i \bar{\phi}_i + \phi_i^\dagger \bar{\phi}_i^\dagger) \right] + m_\lambda (\lambda \lambda + \bar{\lambda} \bar{\lambda}).$$

It is not necessary for subsequent discussion that all of the $\mu$ be effective. In contrast to $\mu_{Li}, \mu_{Ri}$, the $\mu_i$ explicitly break chiral $SU(N_f)$ symmetry and $m_\lambda$ breaks chiral $U(1)$ symmetry. However, once the breaking masses are generated, their mass scales are expected to be of the same order. Since physics very near the SUSY-invariant vacua is our main concern, all breaking masses are kept much smaller than $\Lambda$. For composite superfields, $\mathcal{L}_{\text{mass}}$ is translated into $V_{\text{soft}}$, which can be cast into the following form (with higher orders in scalar masses neglected):

$$V_{\text{soft}} = \left\{ \sum_{i,j=1}^{N_f} \left[ (\mu_{Li}^2 + \mu_{Ri}^2) \left( \Lambda^{-2} |T_j|^2 + \Lambda^{-2} |U_j|^2 \right) + \mu_i^2 (T_j^i + T_i^j) \right] + \Lambda^{-2} \left( \sum_{i=1}^{N_f} |B_i|^2 \right)^2 \right\}_{\theta = \bar{\theta} = 0}.$$

Here $|U_j|^2 \sim \sum_{j=1}^{N_f} |\pi_j|^2$, $|B_i|^2 \sim \sum_{i=1}^{N_f} |\pi_i|^2$ and similarly for $|\bar{B}_i|^2$.

**III. SPONTANEOUS BREAKING**

Now, let us consider a confining phase where $T, B$ and $\bar{B}$ serve as massless composites. It will be shown that the favored number of $\pi_i$ with VEV $= \mathcal{O}(\Lambda^2)$ is $N_c$. Since the dynamics requires that some of the $\pi_i$ acquire non-vanishing VEV’s, suppose that one of the $\pi_i$ ($i=1 \sim N_f$) develops a VEV, and let this be labeled by $i = 1$: $|\pi_1|^2 = \Lambda_1^2 \sim \Lambda^2$. The conditions on $\pi_i$, including $\langle 0 | Z_B | 0 \rangle = z_B$ are simply given by

$$G_T W_{\text{eff};i}^* \frac{\pi_1}{\pi_i} (1 - \alpha_B) = G_S W_{\text{eff};i}^* (1 - \alpha_B) + M_i^2 + \beta_B X_B,$$

for $i = 1 \sim N_c$ and by (8), with $\alpha_B = \beta_B = 0$, for $i = N_c + 1 \sim N_f$, where $\alpha_B = z_B f'(z_B) / f(z_B)$, $\beta_B = z_B \alpha_B'$, and

$$M_i^2 = (\mu_{Li}^2 + \mu_{Ri}^2) \left| \frac{\pi_1}{\pi_i} \right|^2 + \mu_i^2 \pi_i + G_T |\pi_i|^2 \sum_{j=1}^{N_f} |W_{\text{eff};j}|^2,$$

$$X_B = G_T \sum_{i=1}^{N_c} W_{\text{eff};i}^* \frac{\pi_1}{\pi_i} - G_B \sum_{x=b,b} W_{\text{eff};x}^* \frac{\pi_1}{\pi_x}.$$

It should be noted that the effects of field-dependent kinetic terms can be regarded as extra sources of soft SUSY breakings as in (8). Suppose that some VEV other than $\pi_1$ are zero (or much smaller than $\pi_1$). Then $M_i^2 \gg M_j^2$ ($i \neq 1$) because the SUSY breaking soft masses of the same order. Equation (8) yields
for \(i = 2 \sim N_c\), from which \(|\pi_i| \sim |\pi_1| = \Lambda_T^2\) is derived in the SUSY limit defined by \(M_i^2 \to 0\) as long as \(M_i^2 \sim GSW^*_{\text{eff};\lambda}(1 - \alpha_B) + \beta_B X_B\), which will be the case. This behavior implies that, in the SUSY limit, \(f(z_B)\) forces all \(\pi_i (i = 1 \sim N_c)\) be of the same order; i.e. \(G_T W^*_{\text{eff};i} \pi_{\lambda}/\pi_1 = \cdots = G_T W^*_{\text{eff};N_c} \pi_{\lambda}/\pi_{N_c}\), giving \(\pi_i \sim \Lambda^2\), although \(\pi_{\lambda}/\pi_1 = 0\) is satisfied for any values of \(\pi_i\) including \(\pi_i = 0\) as long as \(\pi_1 = 0\). On the other hand, in the extreme case, where \(\mu L_1 R_1 = 0\), there is a solution for which \(\pi_i = 2 \sim N_f = \pi_{b(b)} = \pi_1 = 0\) in the SUSY limit. However, this is not dynamically allowed, since the anomaly-matching conditions for the residual chiral symmetries are not fulfilled. Therefore, in the general case, where all soft breaking masses are of the same order, the SUSY vacuum is characterized by

\[
|\pi_{i=1 \sim N_c}| = \Lambda_T^2,
\]

thus yielding \(SU(N_c)_{L+R}\). In other words, once the spontaneous breaking is triggered, then \(|\pi_{i=1 \sim N_c}| = \Lambda_T^2\) is a natural solution of SQCD, where the soft SUSY breakings can be consistently introduced.

From the constraint \(W_{\text{eff};\lambda} = 0\), for the exact SUSY

\[
f(z_B) = \prod_{i=N_c+1}^{N_f} \left( \frac{\pi_{\lambda}}{\lambda N_i} \right) \cdot \prod_{i=1}^{N_c} \left( \frac{\Lambda^2}{\pi_a} \right)
\]

can be derived. The function \(f(z_B)\) should satisfy \(f(z_B) = 0\), because of \(\pi_{\lambda}/\pi_{i=N_c+1 \sim N_f} = 0\) from \(W_{\text{eff};i} = 0\) and \(|\pi_{a=1 \sim N_c}| = \Lambda_T^2\). It is consistent to demand that \(f(z_B) = 0\) provides the classical constraint of \(\det(T) = BT^{N_f-N_c} B\). The simplest form of \(f(Z_B)\) is then taken to be

\[
f(Z_B) = (1 - Z_B)^\rho \quad (\rho > 0)
\]

from which, due to \(z_B = \pi_b (\prod_{i=N_c+1}^{N_f} \pi_i) \pi_b/ \prod_{i=1}^{N_c} \pi_i = \pi_b/ \prod_{i=1}^{N_c} \pi_i = 1,\)

\[
|\pi_b| \sim |\pi_b| \sim \Lambda_T^N
\]

is derived. The influence of the SUSY breaking arises through \(f(z_B)\) as a tiny deviation from zero: \(f(z_b) = (1 - z_b)^\rho = \xi^\rho\) for \(\xi \ll 1\). This behavior of \(f(z_B)\) allows us to employ \(\alpha_B \sim -\rho/\xi\) and \(\beta_B \sim -\rho/\xi^2\), whose magnitude is much larger than unity.

With this in mind, we further calculate the constraints (14) by inserting \(W_{\text{eff};i=1 \sim N_c} = (1 - \alpha_B)\pi_{\lambda}/\pi_i\) and similar constraints on \(\pi_{i=N_c+1 \sim N_f}\) and \(\pi_{b(b)}\), which turn out to be

\[
G_T \left| \frac{\pi_{\lambda}}{\pi_{i=N_c+1 \sim N_f}} \right|^2 = GSW^*_{\text{eff};\lambda} + M_i^2,
\]

\[
G_T \left| \frac{\rho \pi_{\lambda}}{\xi \pi_{i=1 \sim N_c}} \right|^2 = \frac{1}{N_c-2} \left[ (\rho - 2)GSW^*_{\text{eff};\lambda} - M^2 \right] + M_i^2,
\]

\[
G_B(B) \left| \frac{\rho \pi_{\lambda}}{\xi \pi_{b(b)}} \right|^2 = \frac{1}{N_c-2} \left[ (N_c - \rho)GSW^*_{\text{eff};\lambda} + M^2 \right] + M_{b(b)}^2,
\]

where \(M^2 = \sum_{i=1}^{N_c} M_i^2 + \sum_{x=b,b} M_x^2\) with

\[
M_x^2 = \sum_{i=1}^{N_c} \rho_{L_i(R_i)} \left| \frac{\pi_x}{\Lambda_{N_c-1}} \right|^2 + G_B' \left| \pi_x \right|^2 \sum_{y=b,b} \left| W_{\text{eff};y} \right|^2.
\]

The relations (13)- (17) show that

\[
G_T \left| \pi_{b(b)} \right|^2 \sim G_B(B) \left| \pi_{i=1 \sim N_c} \right|^2, \quad \left| \pi_{i=N_c+1 \sim N_f} \right| \sim \xi \left| \pi_{i=1 \sim N_c} \right|,
\]

which are also consistent with (14). From the relation (12), \(\pi_{\lambda}\) is calculated to be

\[
|\pi_{\lambda}| \sim \Lambda^3 \xi^{N_f-N_c}.
\]
These solutions indicate, in softly broken SQCD, the breakdown of all chiral symmetries that is in agreement with the result of the dynamics of ordinary QCD [13]. In the SUSY limit of $\pi_{i=N_c+1\sim N_f} \to 0$ owing to $\xi \to 0$, chiral $SU(N_f-N_c)$ symmetry is preserved. The constraint on the form of the $W_{\text{eff}}$ will arise to ensure the positivity of the right-hand side of (13) - (17). For instance, (14) and (17) using the definition of $M^2$ give $W_{\text{eff};i}^* > 0$. It should be noted that $M^2_b$ can take any values, depending upon the explicit form of $G_B$, because the term $G_B^\prime |\pi_b|^2 |W_{\text{eff};b}|^2$ turns out to be $\sim \mu^2_{\text{det}(T)} A^2$ as implied by (17) since $W_{\text{eff};b} = \rho |\pi_b| \xi |\pi_b|$, and similarly for $M^2_b$ and $M^2_{i=1\sim N_c}$.

For the case in which $U$ as well as $T$, $B$ and $\bar{B}$ are sources of massless const"{a}tes, constraints on $\pi_{i=1\sim N_c-1}$ are given by

$$G_T W_{\text{eff};i}^* \frac{\pi_{\lambda}^i}{\pi_i} (1 - \alpha_B - \alpha_U) = G_S W_{\text{eff};\lambda}^* (1 - \alpha_B - \alpha_U) + M^2_i$$

$$+ (\beta_B X_B + \beta_U X_U),$$

where $\alpha_U = z_U f_U^i (z_U) / f_U (z_U)$, $\beta_U = z_U \alpha_U$ and

$$X_U = \sum_{i=1}^{N_c-1} G_T W_{\text{eff};i}^* \frac{\pi_{\lambda}^i}{\pi_i} - G_U W_{\text{eff};u}^* \frac{\pi_{\lambda}^u}{\pi_u}.$$  

(21)

Here, $f(Z_B, Z_U) = f_B(Z_B) f_U(Z_U)$ is assumed for simplicity. It can be proved that the relation

$$f_B(Z_B) = \exp(Z_B),$$

(23)

giving $Z_B$ in $W_{\text{eff}}$, ensures $\pi_0 = \pi_b = 0$, even in the SUSY-breaking phase. This form of $f_B$ simply gives $\alpha_B = \beta_B = z_B = 0$ and further reduces (21) to

$$G_T W_{\text{eff};i}^* \frac{\pi_{\lambda}^i}{\pi_i} (1 - \alpha_U) = G_S W_{\text{eff};\lambda}^* (1 - \alpha_U) + M^2_i + \beta_U X_U.$$  

(24)

By the same reasoning as in the previous case, one concludes that

$$|\pi_{i=1\sim N_c-1}| = \Lambda_T^2, \  |\pi_u| = \Lambda_T^{N_c-1}, \  |\pi_{i=N_c\sim N_f}| = \xi \Lambda_T^2, \  |\pi_{\lambda}| \sim \Lambda^3 \xi^{N_f - N_c + 1},$$

(25)

which indicate that $SU(N_c-1)_L \times SU(N_f-N_c+1)_L \times SU(N_f-N_c+1)_R$ remains unbroken at $\xi = 0$. The arbitrary function, $f(Z_B, Z_U)$, is described by

$$f(Z_B, Z_U) = \exp(Z_B) (1 - Z_U)^\rho,$$

(26)

yielding the classical constraint of $\det(T) = UT^{N_f-N_c+1}$.

**IV. SUMMARY**

Summarizing our discussions, we have shown that the dynamical symmetry breaking of SQCD with $N_f \geq N_c + 2 \ (N_c > 2)$ in the “electric” phase can be handled by either

$$W_{\text{eff}} = S \left\{ \frac{S^{N_c-N_f \det(T)} (1 - B T^{N_f-N_c+1})^\rho}{\Lambda^{3N_c-N_f} \det(T)} + N_f - N_c \right\},$$

(27)

or

$$W_{\text{eff}} = S \left\{ \frac{S^{N_c-N_f \det(T)} (1 - T^{N_f-N_c+1})^\rho}{\Lambda^{3N_c-N_f} \det(T)} + B T^{N_f-N_c+1} \right\},$$

(28)

for $\rho > 0$. Seiberg’s superpotential for SQCD with $N_f = N_c + 1$ that corresponds to the $\rho=1$ case is not unique in the sense that the former case with determined parameter, $\rho$, also describes the same physical properties including the decoupling property. Our main finding is that $W_{\text{eff}}$ in the present form dynamically triggers the successive formation of condensates once one VEV such as $\langle 0/T_1^1 | 0 \rangle_{\theta=0}$ is made non-vanishing. Such successive formation can be made
visible by watching the behavior of SQCD with soft SUSY breakings in its SUSY limit. It has been demonstrated that, to be consistent, soft SUSY breakings are constrained to include terms of the scalar components with non-vanishing VEVs. This is reasonable since $M_f^2$ (i.e., fields with VEV ($\sim \Lambda$)) $\gg M_f^2$ (i.e., fields without VEV) as long as the SUSY-breaking masses are of the same order. In the SUSY limit, the residual symmetry turns out to include either $SU(N_c)_L \times SU(N_f - N_c)_R$ or $SU(N_c - 1)_L \times SU(N_f - N_c + 1)_L \times SU(N_f - N_c + 1)_R$. The SUSY breaking further induces spontaneous breakdown of the residual nonabelian chiral symmetry as in (19), which is in accord with the result in ordinary QCD physics that all chiral symmetries are spontaneously broken. The details of the anomaly-matching conditions as well as the possible application to physics of composite quarks and leptons have been discussed in Ref. [16].

It should be noted that the present breakings include a spontaneous breakdown of vector symmetries such as $SU(N_c)_L \times SU(N_f - N_c)_R$ to $SU(N_c + 1)_L \times SU(N_f - N_c + 1)_R$ [13]. A similar breakdown of a vector symmetry has already been found to occur in SQCD with $N_f = N_c$, which permits the breaking of $U(1)$ of the baryon number. These breakings are precisely determined by the dynamics regulated by a relevant effective superpotential, where the anomaly-matching problem can be translated. Whether these physics can be addressed through future lattice calculations that reveal the "real" physics of SQCD [24], although the possibility of testing our proposed superpotential remains quite remote.

Note Added: There is a work done by N. Arkani-Hamed and R. Rattazzi [21] who have discussed the similar subject on an instability in the magnetic phase of SQCD near the SUSY limit and also have pointed out the possibility of the spontaneous breaking of $SU(N_f)_{L+R}$.

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