ABSTRACT: Since the advent of the internal combustion engine, knock has been a vital issue limiting the thermal efficiency of spark ignition engines under heavy load conditions. The occurrence of knock is also directly influenced by several operating parameters simultaneously. In order to investigate the effects of multiple variables on economic performance and power performance under knock limits, this study adopts single-objective optimization and multi-objective optimization methods to optimize the engine operating parameters, including exhaust gas recirculation rate, exhaust valve timing, spark timing, and intake valve timing. The optimization aims to obtain maximum volumetric efficiency, brake mean effective pressure, and minimum brake specific fuel consumption on the knock limit. First, based on the bench test data at the operation point 2800 rpm and 11.42 bar, a one-dimensional simulation engine model is established in GT-power software and verified. Second, four engine operating parameters are input into the GT-power model as controlled parameters. The epsilon-constrained differential evolution algorithm and the multi-objective differential evolution algorithm are employed to optimize the above four parameters to minimize the knock index and the damage to engine performance due to knock suppression, respectively. Finally, the results show that the two optimization algorithms optimize four parameters. The results of the epsilon-constrained differential evolution algorithm indicate that the decreasing extent of the knock index is 73.3%. In addition, the decreasing extent of brake mean effective pressure is 10.2%. What is more, the increased brake specific fuel consumption is only 0.07%. The multi-objective differential evolution algorithm gives a set of nondominated Pareto optimal solution sets. The optimal solution has a 64.4% decrease in the knock index, a 5.78% decrease in brake mean effective pressure, and a 1.45% decrease in brake specific fuel consumption.

1. INTRODUCTION

Since the advent of the spark ignition internal combustion engine, knock has been one of the adverse abnormal combustions of the internal combustion engine. Knock is induced by auto-ignition of the gas mixture near the edge of the combustion chamber. When a knock occurs, the knock could cause damage to the valves, piston, and combustion chamber, which negatively impact the indicated thermal efficiency and power output of the internal combustion engine. Because of energy shortages and tightening emission regulations, more and more scholars are devoting their efforts to improving thermal efficiency and reducing emissions of internal combustion engines. Increasing the compression ratio can improve the internal combustion engine’s thermal efficiency and power performance. Nevertheless, the risk of engine knock increases with increasing compression ratio. Therefore, knock has become one of the vital obstructions hindering the SI engine’s thermal efficiency and power performance. Aiming to reduce knock and improve performance, people invent and employ
more and more technologies and methods to deal with this problem, such as retarding the spark timing, Exhaust Gas Recirculation (EGR) technology, variable valve timing (VVT) technology, and other technologies.\(^5\) The simultaneous use of these technologies often results in a trade-off between engine power and economic performance, especially under heavy load conditions with a high risk of knock.\(^6\) The engine is even more limited by knock not to achieve maximum power and fuel economy, which leaves an opportunity to engine performance improvement. This opportunity is to reduce the damage to engine performance from knock by coordinating the adjustment of several parameters that affect the engine performance.

Because of the cost and time consumption of the bench experiment, it is almost impossible to achieve multiparameter optimization on the experimental bench in studying the impact of multiparameter optimization on knock and engine performance.\(^7\) In addition, with the enhancement of computer computing power, the simulation technology of the internal combustion engine has also been greatly improved, making internal combustion engine simulation technology a powerful tool for optimizing internal combustion engine operating parameters.\(^8\) Mahrous et al. used a 1D fluid-dynamic simulation model to optimize intake and exhaust valve strategies. The results show that optimizing intake and exhaust strategies can reduce fuel consumption by reducing the power loss caused by pumping internal combustion engines.\(^9\) In a study by De Bellis, they used the internal combustion engine simulation technology to optimize the internal combustion engine’s intake and exhaust valve strategy under medium load. The optimized EVIC strategy can reduce the knock and the brake specific fuel consumption.\(^10\) According to Tornatore et al., a GT-power internal combustion engine simulation model was adopted to explore the influence of the EGR rate on knock and economy performance. Under high load, EGR has a slight improvement in fuel consumption, but the improvement in antiknock is apparent.\(^11\) Teodosio et al. presented a 1D model to optimize the intake and exhaust valve strategy, compression ratio, water injection, and EGR to obtain the best fuel economy performance. The simulation results show that applying these technologies to engines can improve economic performance under different operating conditions.\(^12\) In the GT-power simulation software, Kakee et al. established a four-cylinder gasoline engine model to optimize inlet and exhaust valve timing and spark timing to reduce fuel consumption and increase engine torque. After parameter optimization, fuel consumption is reduced by 5%, and torque increases by 5.65% under full load conditions.\(^13\) It can be concluded that engine knock is influenced by multiple engine operating parameters.

This problem of exploring the influence of multi-parameter optimization on engine performances could be extracted as a single-objective or multi-objective optimization task. The evolutionary algorithm has attracted the attention of researchers because of its excellent global optimization ability in solving single-objective and multi-objective optimization problems.\(^14\) In recent years, many scholars have also adopted evolutionary optimization algorithms combined with engine simulation models to make optimization of multiple operating parameters for optimum engine performance.\(^15\) Jabbor et al. showed a multi-objective evolutionary algorithm to explore the influence of several operating parameters on internal combustion engine power and NOx based on a neural network model. The combination of optimization algorithms and neural network models can quickly search for the best parameters of an engine to maximize power and minimize emissions.\(^16\) According to Menzel, an engine model established in the GT-power software was employed to evaluate a SI engine’s volumetric and thermal efficiency by changing intake valve opening (IVO), intake valve closing (IVC), exhaust valve opening (EVO), and exhaust valve closing (EVC). A MODE algorithm and the NSGA-II algorithm were used to optimize valve timing to obtain the maximum volumetric and thermal efficiency.\(^17\) The results indicate that the MODE algorithm finds a better optimal solution for the volumetric and thermal efficiency than the NSGA-II algorithm. In the study by Guan et al., a digital twin engine model was developed to evaluate spark timing, EGR rate, VVT_I, VVT_E, and compression ratio for fuel consumption and emissions.\(^18\) The five parameters were optimized by the NSGA-II algorithm. The optimized results revealed a 16.37% reduction in the engine fuel consumption rate and a 74.18% reduction in NOx emissions. In another investigation, Shirvani et al. used a multi-objective evolutionary algorithm to search for the optimal fuel injection strategy to reach the EURO6 emission regulations.\(^19\) After optimization, the emission of the internal combustion engine meets the EURO6 emission regulations, and the thermal efficiency is also increased by 2%.

Based on the above existing research literature, scholars focus on the influence of multiple parameters on the economy, power, and emissions of internal combustion engines. Although knock was considered in the study, the authors only explored the impact of knock limits under different EIVC strategies on the optimization of BMEP and BSFC without considering these parameters such as spark timing, EGR rate, and EVO time.\(^20\) Parameters such as spark time, EGR rate, VVT_I, VVT_E, and compression ratio were controlled to optimize engine economy and emission performance.\(^21\) However, the research focuses on optimizing engine BSFC and NOx emissions with the NSGA-II algorithm. This study did not investigate the effect of multiparameter optimization on BSFC and BMEP under knock limitation. Therefore, a few studies have applied the differential evolution algorithm to the impact of multiparameter optimization on the power and economy of engines under the restriction of knock. To make up for the above research gaps, this study explores the influence of four-parameter optimization (intake valve timing, IVT; exhaust valve timing, EVT; spark timing; and EGR rate) on BMEP, BSFC, and volumetric efficiency under the knock limitation condition. The primary purpose of this study is to search for the optimal multiparameter combination that maximizes BMEP and volumetric efficiency and minimizes BSFC under the knock limitation.

A three-cylinder SI engine was built in the GT-power software. The engine model was then verified depending on the dates of the experimental bench. On the one hand, the multiparameter optimization of the three-cylinder engine’s performance is transformed into a multi-constrained one-objective optimization problem with the objective of the knock index. The ε-constrained differential evolution algorithm is utilized to optimize multiple parameters with the knock index as the objective optimization function, the BMEP, BSFC, and volumetric efficiency as the inequality constraints. On the other hand, this engine optimization problem is regarded as a multi-objective optimization problem. The knock index, BMEP, BSFC, and volumetric efficiency are all regarded as optimization objectives. The multiobjective differential evolution algorithm is applied to seek the best parameter combination to obtain the Pareto nondominated solution set.
2. EXPERIMENTAL SETUP AND PROCEDURES

The three-cylinder SI engine was tested at the operation point 2800 rpm and 11.42 bar. The operating conditions for the bench test are filled in Table 1. The engine used for the experiment is a naturally aspirated water-cooled spark ignition engine equipped with a VVT device. Table 2 demonstrates the key parameters of the internal combustion engine. The structure diagram of the bench experiment can be found in Figure 1. An AC electric dynamometer is used to control the engine to run at a stable operating point. The cylinder pressure sensor and crank angle encoder cooperate to send the measured cylinder pressure signal to the combustion analyzer to collect and store data at a frequency of 0.5CAD. The fuel flow meter and air flow meter are used to measure fuel consumption and air intake. Specific information on the equipment used above is presented in Table 3. The oil and cooling water temperatures were controlled at 30 and 85 °C, respectively.

Table 1. Test Operation Condition

| boundary condition | value |
|--------------------|-------|
| speed (rpm)        | 2800  |
| throttle opening (%) | 85.6 |
| fuels (−)          | E10   |
| EGR ratio (−)      | 0     |
| air−fuel ratio (−) | 14.2  |

Table 2. Main Engine Parameters and Conditions

| engine type          | DAM10E |
|----------------------|--------|
| number of cylinders  | 3, in-line |
| displacement (L)     | 1.0    |
| bore (mm)            | 74     |
| stroke (mm)          | 77.4   |
| speed (rpm)          | 2800   |
| compression ratio (−) | 10.5  |
| top dead center (CAD) | 720   |
| fuel injection time (CAD) | 600   |
| cooling water temperature (K) | 358 ± 2 |
| fuel/air mixture equivalence ratio (−) | 1.0 ± 0.01 |
| maximum torque at 4500 rpm (N*m) | 94     |
| maximum power at 6000 rpm (kW) | 55.4   |
| aspiration mode      | naturally aspirated |

Table 3. Information on Equipment Used for the Bench Experiment

| equipment                     | type                        | accuracy | manufacturers           |
|-------------------------------|-----------------------------|----------|-------------------------|
| AC electric dynamometer       | 2SB-3-18.B-BOLV4-1C5N       | 0.1% F.S | AVL LIST GMBH           |
| cylinder pressure sensor      | 615AFD36Q03                 | 0.8% F.S | Kistler China Ltd       |
| crank angle encoder           | AVL.364C04/000.00           | 0.1CAD   | AVL LIST GMBH           |
| combustion analyzer           | AVL 6162 616620             | 0.01% F.S| AVL LIST GMBH           |
| fuel flow meter               | AVL735C                     | 0.1% F.S | AVL LIST GMBH           |
| air flow meter                | SENSIFLOW                   | 1% F.S   | AVL LIST GMBH           |

3. MODELING APPROACH AND MODEL VALIDATION

3.1. Combustion Model. In this study, the three-cylinder SI engine model established in GT-power 2014 is shown in Figure 2. Because this article aims to optimize the effects of multiple parameters on knock and engine performance, both the combustion and knock models used in GT power should be predictive instead of nonpredictive. 26 Therefore, the SITur combustion model is utilized to simulate in-cylinder combustion, which considers the influence of operation parameters such as spark timing, EGR rate, and air−fuel mixtures on combustion and flows in the cylinder. Furthermore, in the SITur combustion model, the flame kernel growth multiplier, turbulent flame speed multiplier, and Taylor length scale multiplier are all functions of speed and load. The SITur combustion model has higher adaptability in describing the combustion in the SI internal combustion engine cylinder. 27,28

Figure 1. Diagram of the engine test bed. 1-air cleaner, 2-air flow meter, 3-surge tank, 4-throttle, 5-fuel injectors, 6-fuel flow meter, 7-fuel tank, 8-crank angle encoder, 9-engine, 10-ignition system, 11-in-cylinder pressure sensor, 12-dynamometer, 13-EGR cooler, 14-three-way catalytic, 15-electronic control unit, 16-EGR valve, 17-PC control, 18-combustion analyzer.
The rate at which the unburned mixture turns into the front of the flame is positively related to the velocity of the sum of the laminar flame and the turbulent flame, as defined by eq 1. The laminar flame velocity and turbulent flame velocity are determined by eqs 2 and 3, respectively.

\[
\frac{dM_e}{dt} = \rho_u A_e (S_T + S_L) \tag{1}
\]

\[
S_L = (B_m + B_\Phi (\Phi - \Phi_m)^2) \left( \frac{T_u}{298} \right)^\alpha \left( \frac{p}{101325} \right)^\beta
(1 - 2.06(\text{Dilution}))^{0.57 \times \text{DEM}} \tag{2}
\]

\[
S_T = C_s \times u' \left( 1 - \frac{1}{1 + C_i \frac{R_f}{L_t}} \right) \tag{3}
\]

where \( M_e \) is unburnt mixture mass into the flame front, \( \rho_u \) and \( A_e \) represent the density of the unburned gas and the region’s size in front of the flame surface, respectively. \( S_T \) and \( S_L \) mean the laminar flame velocity and turbulent flame velocity, respectively. \( B_m \) is the maximum laminar speed equal to 0.35, and \( B_\Phi \) is the laminar speed roll-off value equal to \(-0.549\). \( \Phi \) and \( \Phi_m \) are the equivalence ratio in the cylinder and the equivalence ratio at the maximum laminar flame speed, respectively. \( T_u \) and \( p \) are the temperature and pressure of the unburned mixture, respectively. \( ^\beta \) Dilution is the mass fraction of the residuals in the unburned zone. \( \alpha \) and \( \beta \) are the temperature exponent and pressure exponent, respectively. \( \text{DEM} \) represents the dilution exponent multiplier. \( u' \) means the turbulence intensity. \( R_f \) is the flame radius. Turbulent length scale \( L_t \) depends on the turbulence induced by the valve flows, swirl, squish, injection flows, combustion, and effects of compression. \( C_s \) scales the flame front evolution from an initially smooth surface (corresponding to complete laminar combustion) to a fully developed turbulent wrinkled flame. \( C_i \) is a scaling factor for the turbulent flame speed.

3.2. Knock Model. The kinetics-fit knock model specially built for SI engine knock was used to describe the knock in the combustion process of the three-cylinder SI engine.\(^{26,47}\) In the kinetics-fit knock model, knock prediction is based on empirical induction time correlations. The induction time integral is defined by eq 4.

\[
I(t) = \int_0^{t_{\text{Knock}}} \frac{dt}{\tau} \tag{4}
\]

where \( I \) means the induction time integral, \( t \) and \( \tau \) represent the time after inlet valve closure and the induction time, which is the inverse of the reaction rate of the end-gas, respectively.\(^{33,44}\) \( t_{\text{Knock}} \) is the time of knock.

In order to capture the different chemistry of knocklover a wide range of temperatures, the kinetics-fit model uses three different induction times to describe the knock in the low-, intermediate-, and high-temperature regions, respectively.\(^{35,46}\) The overall induction time of this model comprises three different induction times.\(^{37-49}\) Each induction time is defined by eq 5.

\[
\tau_i = M_i \delta \left( \frac{\text{ON}}{100} \right)^\delta \left( \text{Fuel} \right)^\gamma \left( \text{O}_2 \right)^\delta \left( \text{Diluent} \right)^\delta \exp \left( \frac{f}{M_i T} \right) \tag{5}
\]

\[
= 1, 2, 3
\]
where \( M_i \) is the induction time multiplier, \( a_i \) through \( f \) are the constant parameters of the model, \( ON \) represents the octane number of the fuel used to run the engine, \([\text{Diluent}]\) means the mass fraction of the residuals in the unburned zone, mainly including \( N_2, CO_2, \) and \( H_2O \) and \( M_i \) is the activation energy multiplier. \(^{30}\)

The overall induction time integral is defined by eq 6. When the overall induction time is integrated to 1, the knock occurs at \( t_{\text{knock}} \):

\[
\frac{1}{\tau} = \frac{1}{\tau_1 + \tau_2 + \tau_3} \tag{6}
\]

Where \( \tau_1, \tau_2, \) and \( \tau_3 \) are the induction time for low-, intermediate-, and high-temperature regions, respectively.

When knock occurs, the knock index, which indicates the intensity of knock, is determined by eq 7.

\[
KI = \frac{10000M_{\text{ub}} V_{\text{TDC}}}{V} \exp \left( -\frac{6000}{T_u} \right) \max(0, 1 - (\Phi^2))
\]

\[
\frac{I_{\text{ave}}}{I_{K_{\text{ref}}}} = \frac{K_{\text{corr}}}{K_{\text{corr}}} \tag{7}
\]

where \( K_I \) means the knock index, \( M \) is the knock index multiplier, \( M_{\text{ub}} \) is the unburned fraction of the mixture when auto-ignition occurs, \( V \) is the in-cylinder volume when auto-ignition occurs, \( V_{\text{TDC}} \) is the in-cylinder volume at TDC, \( T_u \) is the average temperature of the unburned zone, \( \Phi \) is the equivalence ratio of the unburned zone, \( I_{\text{ave}} \) is the mean induction time integral of the unburned mixture at the end, \( I_{K_{\text{ref}}} \) represents the induction time integration threshold set by the occurrence of a knock, and \( I_{K_{\text{corr}}} \) is a correction factor.

3.3. Model Validation. For verifying the GT-power SI engine models, engine bench experiments data were used to measure the main parameters required to validate the engine model, including BMEP, air mass flow, fuel consumption rate, the in-cylinder pressure, and so forth. The validation of the key parameters is listed in Table 4. The error between the simulation and the experimental test value of the key parameters was controlled within 5%. Figure 3 illustrates pressure curves in the cylinder obtained from the GT-power and test bench of the three-cylinder SI engine at 2800 rpm and 11.42 bar opening point. The error of the maximum in-cylinder pressure between the simulation and test bench does not exceed 0.2%. Moreover, the crankshaft phase of the peak pressure in the cylinder is just 1.5 CAD away from the test phase. According to Table 4 and Figure 3, it can be inferred that the key parameters and the simulated pressure in the cylinder are in high match with the test results. The knock model was calibrated by running virtually at knock boundaries. \(^{37}\) The initial knock index of the knock model used in the paper is set to 75.

4. OPTIMIZATION TECHNIQUES

Differential evolution algorithms with strong global search capabilities are used to search for optimal optimization results. \(^{31}\) This article employs two variants of the differential evolutionary algorithm to deal with the engine optimization problem. On the one hand, if this engineering problem is viewed as one objective function with constraints, the \( \varepsilon \)-constrained evolutionary difference algorithm is applied to solve this single-objective optimization problem. On the other hand, if the engine optimization problem is considered as multiple objective functions with constraints, the multiple objective optimization difference algorithm is applied to deal with this optimization problem. The following is a detailed description of how the two algorithms are applied to optimize engine parameters. Both \( \varepsilon \)-constrained DE and MODE algorithms are based on a simple differential evolution algorithm. \(^{32,33}\) The general differential evolutionary algorithm procedure includes the following parts: initialization, mutation, recombination, and selection. \(^{34,35}\)

4.1. \( \varepsilon \)-Constrained DE Algorithm. Takahama et al. first introduced the \( \varepsilon \)-constrained method to find a better solution between two solutions. \(^{36} \) The \( \varepsilon \)-constrained method is then applied to the general DE algorithm to form the \( \varepsilon \)-constrained DE algorithm search for the optimal solution. \(^{36}\) The key distinction between the \( \varepsilon \)-constrained DE algorithm and the general differential evolution algorithm is reflected in the “selection”. Except for the “selection” procedure, the rest of the \( \varepsilon \)-constrained DE algorithm is identical to the general differential evolution algorithm.

Usually, the minimum problem under the constraint is described in eq 8.

\[
\min f(x)
\]

s. t. \( h_i(x) = 0 \ i = 1, 2, \ldots, n_e \)

\( g_j(x) \leq 0 \ j = 1, 2, \ldots, n_i \)

\( x_{e,\min} \leq x \leq x_{e,\max} \ k = 1, 2, \ldots, n_e \) \tag{8}

where \( f(x) \) is the objective function. \( h(x) \) is an equality constraint. There are a total of \( n_e \) \( g(x) \) is an inequality constraint, and there is \( n_i \) in total.

To quantify the constraints of the problem, eq 9 is used to define a constraint violation value function \( \varphi(X) \) to describe the value of constraint violation. In the \( \varepsilon \)-constrained method, the
constraint violation function is employed to deal with constraint problems. The solution is judged to be feasible or infeasible based on whether the value is greater than zero.

\[ \varphi(X) = \sum_{j=1}^{n} \max(0, g_j(X)) + \sum_{i} (h_j(X)) - \delta, 0) \]

The \( \varepsilon \)-constrained method is defined by eq 10. The \( \varepsilon \)-constrained method uses the value of the objective function and the constraint violation function value to compare the pros and cons of two solutions \( X_1 \) and \( X_2 \).

\[
(f_X(X_1), \varphi(X_1)) < \varepsilon (f_X(X_2), \varphi(X_2)) \]
\[
\Leftrightarrow (f(X_1) < f(X_2) \text{ if } \varphi(X_1), \varphi(X_2) \leq \varepsilon)
\]
\[
(\varepsilon(X_1) \leq \varphi(X_2) \text{ otherwise}) \]

The parameter \( \varepsilon \) restricting the constraint violation value is determined using the adaptive control method in the study.\(^{18}\) The process schematic of the \( \varepsilon \)-constrained DE algorithm is demonstrated in Figure 4. The fake code of the \( \varepsilon \)-constrained DE algorithm can be found in the study conducted by Zhang et al.\(^{37}\)

### Table 5. Variation Ranges of the Four Parameters

| parameters     | lower limit | initial value | upper limit |
|----------------|-------------|---------------|-------------|
| EGR ratio (%)  | 0           | 0             | 15          |
| EVT (CAD)      | 220         | 260           | 300         |
| IVT (CAD)      | 413         | 453           | 493         |
| sparking time (CAD) | −10.6     | −15.6         | −20.6       |

\[ \text{MinKnock Index}(x) \]
\[ \text{BMEP}(x) \geq 5.5 \]
\[ \text{VE}(x) \geq 0.75 \]
\[ x_{k,\min} \leq x \leq x_{k,\max}, k = 1, 2, \ldots, n \]

\[ \varphi(x) = \max(0, 9.5 - \text{BMEP}(x)) \]
\[ + \max(0, \text{BSFC}(x) - 290) \]
\[ + \max(0, 0.75 - \text{VE}(x)) \]

where \( x = (x_{IVT}, x_{EVT}, x_{EGR}, x_{ST}) \). \( x_{IVT}, x_{EVT}, x_{EGR} \text{ and } x_{ST} \) represent the intake valve timing, exhaust valve timing, EGR rate, and
spark timing. The definitions of IVT and EVT in Table 5 are shown in Figure 5.

![Figure 5. Intake and exhaust valve lift vs crankshaft angle.](image)

### 4.2. MODE Algorithm

The MODE algorithm is a combination of the NSGA-II algorithm and the general evolution differential algorithm. Using the NSGA algorithm as a framework, the MODE algorithm employs differential evolution algorithm operators to replace the original genetic algorithm mutation and recombination operators in the NSGA-II algorithm. The MODE algorithm still uses tournament selection, the fast nondominated sorting method, and the crowded distance sorting method to search for the best individual to produce the next generation and adopts the elite-preservation method to keep the population diversity. These methods allow the algorithm not to converge too quickly to lose the optimal solution. The process schematic of the MODE algorithm is shown in Figure 6. The constraints of the parameters have been displayed in Table 5. Knock index, BSFC, BMEP, and volumetric efficiency are all considered object functions. This problem can be transformed into a mathematical form for the engineering multi-objective optimization problem, as shown in eq 13. In order to express it as a general case to the minimum value, a minus sign is added in front of BMEP and volumetric efficiency.

![Figure 6. Flow chart of the MODE algorithm.](image)
Figure 7. Optimization results of the ε-constrained DE algorithm. (a) knock index, (b) values of the constraint violation, (c) BSFC, (d) volumetric efficiency, (e) BMEP, (f) IVT, (g) EVT, (h) spark timing, (i) EGR rate, (j) BMEP-KI correspondence in the 50th generation population, and (k) BSFC-KI correspondence in the 50th generation population.
Min F(x) = [KI(x), -BMEP(x), BSFC(x), -VE(x)]

s. t. x_{k,min} \leq x_k \leq x_{k,max} \quad k = 1, 2, \ldots, m

(13)

where m represents the number of parameters with a value of 4. x has been described in Section 4.1.

5. RESULTS AND DISCUSSION

The mutation and cross-over probability of the general DE algorithm used in the two methods are 0.85 and 0.9, respectively. The number of individuals in the population was set to 30 per generation, and the population cycled through 50 generations.

5.1. Results of the ε-Constrained DE Algorithm. Figure 7a,b shows the knock index and constraint violation values for all individuals in each generation of the optimization process, respectively. The knock index was reduced by 93.3% from 75 to 5. The constraint violation value decreases from greater than
zero down to zero, and as the number of evolutionary generations increases, more and more individuals in the population satisfy the constraint. All individuals in the population after the 8th generation are meeting the \( \epsilon \)-constraint method. Figure 7c provides the BSFC of all individuals in 50 generations of the population. Although some individuals with smaller BSFC appeared in all the first 15 generations, due to the limitation of the knock index, the BSFC of the best individual in the 50th generation population was finally 229.76 g/(kw*th), as shown in Figure 7k, which increased by 0.18 g/(kw*th) or 0.07% compared to the initial model BSFC of 229.58 g/(kw*th). As shown in Figure 7d, the volumetric efficiency corresponding to the best individual in the 50th generation population was 77% due to the limitation of knock. It can be inferred that the BMEP of all individuals decreased in all 50 generation populations in Figure 7e. Based on Figures 7e,j, the BMEP of the individual with a knock index of 5 in the 50th generation population dropped from 11.462 bar to 10.293 bar, a decrease of 10.2%. In order to suppress knock, retarding the spark timing and introducing EGR into the cylinder is an inevitable choice. However, the increase of BSFC and the decrease of BMEP are also mainly caused by the retarded spark timing and EGR use. The late closing of the intake valve and early opening of the exhaust valve reduces the pumping loss at the point of operation at 2800 rpm and 11.42 bar, which partially compensates for the adverse effects on fuel economy and power performance because of the retarding spark timing and the use of EGR. Minimizing damage to engine dynamics and economy while suppressing knock is achieved.\(^{50,51}\)

Figure 7f–i show the changes in the values of the four parameters for all individuals in each generation of the population. From Figure 7f, we can see that the EVT values of all the individuals in the 50th generation are smaller than the initial value of 260 CAD. The reduced EVT value means that the exhaust valves open earlier, and the combustion exhaust gases in the cylinder are expelled quickly under cylinder pressure, lowering the cylinder temperature and contributing to a lower knock index. The EVT value for the best individual is 252.7 CAD. In Figure 7g, it can be obtained that the IVT for all individuals in the 50th generation takes a value greater than the initial value of 453 CAD, which means that the intake valve closes later. The late closing angle of the intake valve is conducive to the intake process using the inertia of the intake air to reduce intake pressure loss. A fresh charge can reduce the cylinder temperature and reduce the knock. The value of IVT corresponding to the best individual is 473.08 CAD. Figure 7h shows that in the 50th generation of the population, the spark timing value for most individuals is greater than the initial value of \( -15.1 \) CAD, which means that the spark timing is retarded. The retard of the spark timing leads to the retard of the combustion phase and allows less fuel to be burned before the TDC to reduce the average temperature and pressure in the cylinder. The best individual’s spark timing value is \( -13.6 \) CAD. Figure 7i shows that the use of EGR decreases gradually in the 50th generation of the population. It is due to the fact that the use of EGR in large amounts causes a drastic reduction in BMEP, considering the power performance. The optimal individual corresponds to an EGR rate of 1%.

5.2. Results of the MODE Algorithm. Figure 8a illustrates the relationship between the knock index, BMEP, BSFC, and volumetric efficiency. As displayed in Figure 8a, the set of nondominated Pareto optimal solutions is not clustered together. It is due to the fact that this study is an optimization of four performance indicators of the engine at one operating point. Any solution whose one performance dominates that performance of other solutions will become a part of the set of nondominated Pareto solutions. While there is a trade-off between these four optimization objectives, which inevitably leads to multiple solutions in the solution set, the optimal solution set is scattered, as shown in Figure 8a. Figure 8a is projected in the three coordinate system directions to form Figures 8b–d, respectively. In Figures 8b–d, it is evident that the optimal solutions are widely distributed without any apparent aggregation phenomenon in the Pareto optimal solution set. Among the solutions with a knock index less than 25, although these solutions have a volumetric efficiency greater than 80%, the BMEP is significantly less than 10.8 bar, or the BSFC is greater than 226.25 g/(kw*th). Based on the engine knock limit, economy, and dynamics performance, the optimized solution was chosen to have a knock index of 26.7, a 64.4% decrease compared to the initial knock index of 75. The BMEP of this solution is 10.8 bar, which is 5.78% lower than the initial value of 11.462 bar. The BSFC of this solution is 226.25 g/(kw*th), which is 3.33 g/(kw*th) lower compared to the initial value of 229.58 g/(kw*th) of the original model. The volumetric efficiency of this solution is 79.3%.

The relationship between the four parameters and the four performances in the 50th generation population is represented in Figures 8e–h. Figure 8e represents the EVT corresponding to each solution in the Pareto optimal solution set. It can be seen that the EVT corresponding to the best solution with a knock index of 26.7 is 245.77 CAD. Figure 8f shows the IVT corresponding to all solutions in the Pareto optimal solution set. The IVT corresponding to the chosen optimal solution is 469.7 CAD. According to Figures 8e,f, the optimal solution has a delayed intake time and an earlier exhaust valve time, which reduces the pumping loss at the operation point of 2800 rpm and 11.42 bar and also reduces the in-cylinder temperature to suppress detonation.\(^{50,51}\) Figure 8g shows the spark timing for each solution in the Pareto optimal solution set. The spark timing for the optimal solution chosen in the study is \( -15.9 \) CAD. Compared to the original engine, the spark timing is retarded because of the limitation of the knock. Figure 8h shows the EGR rate corresponding to all Pareto optimal solution set solutions. The EGR rate corresponding to the optimal solution is 1%.

In addition, there is another optimal solution that has attracted the interest of the study. The solution has a knock index of 23.73, a BMEP of 11.11 bar, a BSFC of 250.62 g/(kw*th), and volumetric efficiency of 88.65%. Except for BSFC, which is inferior to the optimal solution chosen above, the other three performances of this solution are superior to the optimal solution. It brings another perspective to the study. If engine power performance is a high-priority objective, this solution will be more effective in suppressing the occurrence of knock with less impact on engine power performance.

6. CONCLUSIONS

In this study, a model of a three-cylinder gasoline engine equipped with a VVT mechanism was built in GT-power and verified based on test data. Two optimization methods are employed to optimize four parameters to obtain the minimum knock index, the minimum BSFC, the maximum BMEP, and the maximum volumetric efficiency. Both optimization methods are effective in reducing the knock index and finding the best
combination of the four parameters. The main conclusions from the study are summarized as follows:

1. Adopting the $\varepsilon$-constrained DE algorithm to optimize engine performance by adjusting control parameters. The engine knock index was suppressed from 75 to 5, a 93.3% reduction in the knock index. The corresponding BMEP decreased from 11.462 to 10.293 bar, a decrease of 10.2%, with negligible reduction in BSFC.

2. The MODE algorithm gave a Pareto optimal solution set for the engine performance optimization problem. On the one hand, taking into account the trade-off between the knock index, BMEP, BSFC, and volumetric efficiency, the solution with a knock index of 26.7 was considered optimal, 64.4% lower than the base knock index. The BMEP was decreased by 5.78%, and BSFC was reduced by 1.45% compared to the base engine model. On the other hand, the results of the nondominated Pareto solution set show that it was also possible to reduce the knock index and damage caused by the suppression of knock to the BMEP by sacrificing the economic performance of the engine.

3. By comparing the results of the two optimization algorithms, the knock index, BMEP, BSFC, and volumetric efficiency obtained by the $\varepsilon$-constrained DE algorithm and the MODE algorithm are 5, 10.293 bar, 229.76 g/(kw*h), and 77% as well as 26, 10.8 bar, 226.25 g/(kw*h), and 79.3%. The $\varepsilon$-constrained DE algorithm obtains a smaller knock index than the MODE algorithm. However, the BMEP, BSFC, and volumetric efficiency of the former results are inferior to those of the latter.

4. The results of both optimization methods show that the suppression of knock occurrence can be achieved by using only the VVT mechanism with the spark timing adjustment, which may not require using EGR. However, there is a price to pay for suppressing knock, which, in the case of this study, is a reduction in BMEP or an increase in BSFC.

According to the study results, the $\varepsilon$-constraint DE algorithm and the MODE algorithm effectively optimize multiple operating parameters to suppress the occurrence of knock. However, some points still need to be further explored in the study. For instance, only the valve timing was considered in the study parameters without a detailed analysis of the intake and exhaust valves early opening and late closing angles. On the one hand, because of the complexity of the model and the limited computing capability, only the optimization of the knock under a specific operating point is studied in this study. On the other hand, it would be better if the results of the optimized parameters could be verified on the test stand. Therefore, future research aims to deepen the study of the effect of early opening and late closing angles of intake and exhaust valves on engine performance and then carry out multiparameter optimization research under the knock limitation on an experimental bench.

**NOMENCLATURE**

- BMEP: brake mean effective pressure
- BSFC: brake specific fuel consumption
- CAD: crankshaft angle degrees
- DE: different evolutionary
- EGR: exhaust gas recirculation
- EVIC: early intake-valve closing
- EVC: exhaust valve closing
- EVO: exhaust valve opening
- EVT: exhaust valve timing
- IVC: intake valve closing
- IVO: intake valve opening
- IVT: intake valve timing
- VVT: variable valve timing
- KI: knock index
- MODE: multi-objective different evolutionary
- NSGA-II: nondominated sorting genetic algorithm-ii
- VE: volumetric efficiency
- VVT_I: variable valve of intake valves timing
- VVT_E: variable valve of exhaust valves timing
- SI: spark ignition
- TDC: the top dead center
- $a_i$: constant parameters
- $m$: the number of parameters.
- $n_e$: the number of equation constraints.
- $n_i$: the number of inequality constraints.

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**Author Contributions**

Y.K. designed the study and carried out definition of intellectual content, literature search, data acquisition, data analysis, and manuscript preparation. Y.G. provided funding assistance for the study. Y.W. and Y.Y. revised the manuscript. Y.Y. carried out grammar modification and manuscript editing.

**Notes**

The authors declare no competing financial interest. The authors confirm that the data supporting the findings of this study are available within the article.

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$p$ the pressure of the unburnt mixture
$t_{\text{knocx}}$ time of knock
$\mu'$ turbulence intensity
$A_r$ the region in front of the flame surface
$B_{\text{m}}$ maximum laminar speed
$B_{\phi}$ laminar speed roll-off value
$C_k$ flame kernel growth multiplier
$C_t$ turbulent flame speed multiplier
[Di]luent mass fraction of the residuals
DEM dilution exponent multiplier
$I$ induction time integral
$I_{\text{ave}}$ mean induction time integral
$I_{\text{k-corr}}$ a correction factor
$I_{\text{k-ref}}$ induction time integration threshold
$L_t$ turbulent length scale
$M$ the knock index multiplier
$M_{\text{u}}$ unburnt mixture mass into the flame front
$M_{\text{i}}$ induction time multiplier
$M_{\phi}$ activation energy multiplier
$R_f$ flame radius
$S_L$ laminar flame velocity
$S_T$ turbulent flame velocity
$T$ average temperature of the cylinder
$T_u$ average temperature of the unburned zone
$V$ in-cylinder volume when auto-ignition starts
$V_{\text{TDC}}$ the cylinder volume at the top dead center
$\alpha$ temperature exponent
$\beta$ pressure exponent
$\tau$ overall induction time
$\rho_{\text{u}}$ the density of the unburned gas
$\phi$ equivalence ratio
$\phi_{\text{m}}$ equivalence ratio at the maximum laminar flame speed

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