The Influence of the Area Size and the Method of Transverse Load Application to the Ice Cover Surface on the Ice Failure Efficiency

Yu A Dvoichenko\textsuperscript{1a}, N M Semenova\textsuperscript{1b}

\textsuperscript{1}“Shipbuilding and aviation technology” department, Nizhny Novgorod State Technical University n.a. R. E. Alekseev, 603950, Minin str., 24, Nizhny Novgorod, Russia

\textsuperscript{a}E-mail: edbas2@yandex.ru
\textsuperscript{b}E-mail: ShaNaMix@yandex.ru

Abstract. The vertical load application to the ice cover for the purpose of its failure for the passage of the transport vessels in the ice is considered. The energy comparison consumed by the technical vehicles with the volume of the destroyed ice is made when the area size of the application of the failure load increases. The specific work of the external forces (per 1 m\textsuperscript{3} of the destroyed ice) is shown to significantly decrease with the increasing area of the imposed load. In addition, the additional reduction mechanism of the energy expenditure is shown in case when the failure is performed by the moving distributed load compared with the impact of the load applied on a rigid contour.

1. Introduction
The ice cover failure in its interaction with the icebreaking vehicles is associated with the energy transformation of the technical vehicle propulsion system into the strain energy of the ice cover. When the limit value of this energy is reached, the ice failure occurs.

The mechanism of the ice deformation and failure under the load consists in the consecutive formation of the radial and circumferential cracks [1]. The ice plate is divided into many blocks by these cracks. And the blocks are tightly pressed by the crack faces presenting some kind of the “arched” structure. Such structures are used in the arches, domes, bridges building etc. The final failure occurs when the most stressed block of the ice structure breaks under the compressive force, and the remaining blocks that rested on it go to pieces [2].

Thus, the energy of the technical vehicles turns into the elastic strain energy of the ice, the surface energy of the crack formation and the stain energy of the elastic foundation, i.e. water.

The failure of the solid infinite ice cover by the vertical load is considered (the case of the central bend [3, 7], figure 2 (a)). The load shape is square in the plan, and the side of a square is \(a\). Under this load the ice deflection in the radius direction is extending from the point of force application, taking the form of the exponentially damped wave, as it shown in figure 1, a.

The area around the load, bounded by the first apex of the bending wave, is called “the deflection bowl”. This bowl contains approximately 90% of the total energy of the deformed ice cover and its elastic base.
Figure 1. The deflection bowl produced by the load:
a – small width of the load \(a\); b – the deflection bowl increasing \(r_1 > r\) during the load application area increasing \(a_1 > a\)

Under load with small area its radius \(r_0\) is determined by the characteristic linear dimension \(l_0\):

\[
r = \frac{3\pi \cdot l_0}{4},
\]

here \(l_0 = \sqrt[3]{E \cdot h^3 / [12(1 - \mu^2) \cdot \rho g]}\); \(E\) – elastic modulus of the ice; \(h\) – ice thickness; \(\mu\) – Poisson's ratio; \(\rho\) – density of water; \(g\) – gravitational acceleration.

With the application area of the same total vertical force \(P_3\) increasing, the radius of the deflection bowl increases, the deflection and the curvature decreases (figure 1, b). Therefore the greater total force \(P_{31}\) will be required to break the ice, so it may seem unfavourable. However, the aim of the icebreaker vehicles is to create a channel of the predetermined breadth. Therefore, to estimate the effectiveness of the ice failure, it is necessary to correlate the expenditure of energy of the technical vehicles with the volume of the destructible ice.

2. Calculations and dependences

The diagrams of the ice cover failure in the \(P_{S-w}\) axes take the form close to a triangular [3]. Therefore, the load work \(A\) on the ice failure can approximately be estimated as a half of the product of the maximum value of the total load \(P_3\) on the critical deflection \(w_{kp}\) at the destruction point:

\[
A = \frac{1}{2} P_3 \cdot w_{kp}
\]

The natural ice cover failure investigations [3, 4, 5, 6] allowed to come to dependence [3]:

\[
P_3 = k_p \cdot \left(1 + 1.9 \sqrt{S_k / l_0}\right) \cdot h^2,
\]

here \(k_p = 1.9\) MPa is empirically determined coefficient; \(S_k\) – area of support contour.

The critical deflection \(w_{kp}\), by which \(P_3\) is a maximum, is also expressed by the empirical dependence [4]:

\[
w_{kp} = k_w \cdot \sqrt{h},
\]

here \(k_w = 0.43\) m\(^{1/2}\) – empirically determined coefficient [3,4].

With the substituting (3) and (4) in (2) the result is obtained:

\[
A = \frac{1}{2} \cdot k_p \cdot k_w \cdot \left(1 + 1.9 \cdot \sqrt{S_k / l_0}\right) \cdot h^{5/2}.
\]

The failure scheme under the central loading is shown in figure 2, a.
The width of the destructible ice area, as it shown by the observations of the failure in the natural ice cover [3], exceeds the width of the applied load by 5-8 ice thicknesses $h$. This value averages $0.4l_0$ (0.2$l_0$ on each side). If the support contour of the load is a square with the side $a=\sqrt{S_{kc}}$, then the volume of the destructible ice can be approximated by the formula:

$$V = l_0^2 \left( \frac{a}{l_0} + 0.4 \right)^2 \cdot h.$$  \hfill (6)

Deformation of the ice cover and the elastic foundation under it are written as:

$$A_0 = \frac{A}{V} = \frac{1}{2} \cdot k_p \cdot k_w \cdot \sqrt{\frac{12 \cdot (1 - \mu^2) \cdot \rho \cdot g}{E}} \cdot \left( \frac{1 + 1.9 \cdot a \cdot l_0}{a/l_0 + 0.4} \right)^2.$$  \hfill (7)

According to the formula, it follows that the specific energy does not clearly depend on the ice thickness $h$.

Using the values of the empirically determined coefficient for formulas (3) and (4), as well as the average values of the ice elastic characteristics $E=5 \cdot 10^9$ Pa, $m=0.35$, the dependence inversely proportional to the size of the load application site is obtained (curve 1, figure 6). A closer approach to practice is to lay a channel in the ice cover. In this case the load is placed on the edge of the ice broken in the previous ice loading (figure 2, b).

Based on the experimental data [3], the dependence similar to (3) was obtained at the value of the empirically determined coefficient $k_p^1=1.1$ MPa and the maximum deflection at the moment of the failure $l_0^1=0.59$ m$^{1/2}$. As it follows from the diagram in figure 2, h, the volume of the destructible ice will decrease due to the lack of the compact ice in the channel during the load in comparison with the value given in (6).

$$V = l_0^2 \left( \frac{a^2}{l_0^2} + 0.6a/l_0 - 0.08 \right) \cdot h.$$  \hfill (8)

In addition, after the next break, the load must be lifted from water and transferred to the new edge being laid. The question of the expenditure of energy of the load transfer in the horizontal direction was left open, so only the work on the load rise before the new break was considered. And it was assumed that the value of the lifting load is equal to the value of the breaking load. Therefore (2) is taken on form: $A = (0.5+1) P_k \cdot w_{kp}$. And the specific energy (7) is shown as:

$$A_0 = \frac{A}{V} = 1.5 \cdot \left( k_p^1 \cdot k_w^1 \right) \cdot \sqrt{\frac{12 \cdot (1 - \mu^2) \cdot \rho \cdot g}{E}} \cdot \frac{1 + 1.9 \cdot a/l_0}{\left( a^2/l_0^2 + 0.6a/l_0 - 0.08 \right)}. $$  \hfill (9)

In figure 4 this dependence has the form of curve 2. Thus, the channel laying with the consecutive setting of the load on the edge is more power-consuming than the central break. The inefficient reason
of using the load with a rigid contour is shown in figure 3. When setting the load on ice, the ice plate and the elastic foundation are deformed in the area of the deflection bowl (position 1), far above the contour of the application of force. After the break, ice and water return to their initial unstrained condition (position 2), so the energy outside the ice failure zone is irreversibly lost and should be spent again at the new break.

Figure 3. The return of the spun surface 1 of the ice cover to its initial position 2 after the break and the fragments formation 3.

There is a different picture when the channel is destroyed by moving a distributed load of the intensity \( p \) (Pa) along it and without a rigid contour. It is also considered that the load shape is square in plan, and the side of the square is \( a \). This process is shown schematically in figure 4.

Figure 4. The ruptured zones moving when moving a distributed load.

The deflection bowl moves with the load. In the movement direction the bowl moves with five typical deformation areas and the ice cover failure. The unbroken ice in the zone 1 under the load begins to bend and crack passing into the zone 2. In the zone 3 the ice structure is deformed. This structure is made up of ice blocks pressed together which resist the load due to the so-called “the arch action”. In the zone 4, the most strained blocks go to pieces from the compression. It causes adjacent blocks that rested on it to separate from the ice cover. The view of this strain and destroying structure is given in figure 5.

Figure 5. The mechanism of the ice cover failure under the distributed load in the zone 3 and 4 (see figure 4), the load is not shown for convenience.

At the same time, the deformed area in the movement holds its shape without the slip-stick power waste. The load work on the ice cover failure is determined by the product of the lateral force which moves the load by the amount of displacement. The force required for the movement is determined by the simplest model, i.e. by the upward motion on the oblique surface of ice, as it shown in figure 4. For simplicity the ice deflection in front of the load is not taken into account. The height \( h_p \) of the
contour \( h \cdot a \), that is affected by the intensity of the distributed load \( p \) is \( h = \rho / (\rho \cdot g) \). The resistance \( R \) to its upward motion on the oblique plane is approximately

\[
R = p^2 \cdot a / (\rho \cdot g)
\]

The work of the load moving on the length \( a \) is written as:

\[
A = a \cdot R = p^2 \cdot l_0^2 \cdot (a^2 / l_0^2) / (\rho \cdot g)
\]

Taking into account that the channel breadth is 0.4 \( l_0 \) larger than the size \( a \) the volume of the destructible ice on the length \( a \) is written as:

\[
V = h \cdot l_0^2 \cdot \left( a^2 / l_0^2 + 0.4a / l_0 \right)
\]

The load intensity \( p \) on the area of reference dimensions \( a \times b \) is found by the theoretical and experimental method in the paper [2]. The reference dimensions are necessary for the failure of the ice thickness \( h \). And the load intensity is determined as:

\[
p = (1/(2\sigma_i)) \cdot \sqrt{W^* \cdot \rho \cdot g \cdot h (1 + \mu)},
\]

here \( \sigma_i = f(a/l_0, b/l_0) \) is the theoretical dependence of intensity on dimensionless load length and width [2]; \( W^* = 0.93 \text{kJ/m}^3 \) – theoretical and experimental values of the specific critical energy of forming, that causes the failure of the most loaded block from figure 5.This value is calculated from data on the failure of the natural ice cover [3].

With substituting (13) in (11) the result is finally obtained:

\[
A_0 = \frac{A}{V} = \frac{W^* \cdot (a/l_0)^2}{4\sigma_i^2 \left[ (a/l_0)^2 + 0.4(a/l_0) \right] \cdot (1 + \mu)}.
\]

The function values \( \sigma_i \) from the data [2] for \( a/l_0 = b/l_0 \) are shown in the table 1.

| \( a/l_0 \) | \( \sigma_i \) |
|---------|---------|
| 0.5     | 0.038   |
| 0.8     | 0.078   |
| 1.2     | 0.112   |
| 2.0     | 0.160   |
| 2.5     | 0.162   |

The calculation results are shown in curve 3 in figure 6.

**Figure 6.** The dependence of the specific fracture energy of the ice cover on the relative sizes of the load action area and the methods of its application:

a – full view of dependencies in the logarithmic dependence; b in natural axes when \( a/l_0 > 1.2 \);

1– central load; 2 – channel laying with the changeable load; 3 – channel laying with the moving distributed load.
For instance, the ice cover thickness $h=0.5\text{m}$ is considered. According to formula (1) $l_0=8.5\text{ m}$ is calculated (with the value of the characteristics in formula (7)). The intensity value of the distributed load required for the ice failure is calculated by formula (12) and is obtained $p=5.75\ \text{kPa}$. The lateral force required to move this pressure area (see figure 4) according to formula (10) is $R=2.9\ \text{kN}$. It is not comparable to the lateral force of a conventional icebreaker, that reaches 0.5-1.5 $\text{mN}$ for such ice thickness and the same width [8].

Since the distributed pressure is usually created by air [2], the frictional force of the area of such pressure is negligibly small compared with the force that moves this area along the ice oblique surface.

3. Main conclusions

1. From the data in figure 6 it follows that it is energetically advantageous to destroy the ice cover as much as possible with the larger area of its application. In some ways it contradicts the well-established idea that the smaller amount of force required to break the ice of the given thickness, the more effective are would be the technical vehicles. It is confirmed by the data on the greater efficiency of the icebreakers with a spoon-shaped design waterline compared with the traditional pointed design waterline [8].

2. Channel laying with the load is characterized by the greater specific work than the single acts of the central break.

3. The dependence 3 is the most interesting and has a sharp decline in the area $a/l_0>1.6$. According to this data, it follows that the use of the distributed load moving across the ice for the channel laying is energetically advantageous at $a/l_0>1.8$.

4. The obtained results indicate that the technical vehicles destroying the ice cover by the system of the distributed pressures are undoubtedly promising.

5. The proposed approach to evaluate the effectiveness of the ways of the load application to the ice cover should be recommended for evaluating traditional technical vehicles of laying channels in the ice cover.

4. References

[1] Dvouchenko Yu A 1980 The ice field deformation and break Theory and Strength of an Icebreaking Ship (Gorkiy: Gorkiy Polytechnical Institute)

[2] Dvouchenko Yu A 1984 Numerical model of ice cover failure when the ACV moves at a low speed Theory, Strength and Design issues of an Icebreaking Ship (Gorkiy: Gorkiy Polytechnical Institute) pp 81–8

[3] Zuev V A, Gramuzov E M and Dvouchenko Yu A 1984 Experimental investigations of ice cover failure Theory, Strength and Design issues of an Icebreaking Ship (Gorkiy: Gorkiy Polytechnical Institute) pp 4–12

[4] Panfilov D F 1960 Experimental investigations of ice cover capacity Proceedings of the Vedeneev VNIIG (Moscow: Gosenergoizdat) 65 pp 101–6

[5] Klyacharev V and Izyumov S 1943 Determination of ica crossing capacity Military engineering magazine 2–3 pp 30–4

[6] Kobeko P P, Shishkin N I, Marey F I and Ivanov N S 1946 Ice deflection and capacity Journal of Applied Physics 16 273–6

[7] Zuev V A, Gramuzov E M, Dvouchenko Yu A and Sebin A S 2017 The criterion of the ice cover modeling Proc. of the forth int. conf. “Polar Mechanics-2017” pp 134-40

[8] Ionov B P and Gramuzov E M 2001 Ice propulsive quality of ships (Saint-Petersburg: Sudostroenie) 512

Acknowledgments

The reported study was funded by RFBR and NSFC according to the research project №20-58-53049.