Is there np pairing in odd-odd N=Z nuclei?

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The binding energies of even-even and odd-odd N = Z nuclei are compared. After correcting for the symmetry energy we find that the lowest T = 1 state in odd-odd N = Z nuclei is as bound as the ground state in the neighboring even-even nucleus, thus providing evidence for isovector np pairing. However, T = 0 states in odd-odd N = Z nuclei are several MeV less bound than the even-even ground states. We associate this difference with a pair gap and conclude that there is no evidence for isoscalar correlated pairs in N = Z nuclei.

Soon after the interpretation of superconductivity in terms of a condensate of strongly correlated electron pairs (Cooper pairs) by Bardeen, Cooper and Schrieffer (BCS) [1] a similar pairing mechanism was invoked for the nucleus [2] to explain, for example, the energy gap in even-even nuclei and the magnitudes of moments of inertia. For almost all known nuclei, i.e. those with N > Z, the “superfluid” state consists of neutron (nn) and/or proton (pp) pairs coupled to angular momentum zero and isospin T=1. However, for nuclei with N = Z the near degeneracy of the proton and neutron Fermi surfaces (protons and neutrons occupy the same orbitals) leads to a second class of Cooper pairs consisting of a neutron and a proton (np). The np pair can couple to angular momentum zero and isospin T = 1 (isovector), or, since they are no longer restricted by the Pauli exclusion principle, they can couple to T = 0 (isoscalar) and the angular momentum is J = 1 or J = J_{max} [3], but most commonly the maximum value [4]. Charge independence of the nuclear force implies that for N = Z nuclei, T = 1 np pairing should exist on an equal footing with T = 1 nn and pp pairing. Whether there also exists strongly correlated T = 0 np pairs, has remained an open question. Early theoretical works [5] discussed the competition between T = 0 and T = 1 pairing within the BCS framework. Recent works have focussed on the solutions of schematic (or algebraic) [6][7][8] and realistic shell models [9], as well as on the properties of heavier N = Z nuclei [10], and the effects of rotation [11][12].

To date, there exists a wealth of experimental evidence in support of the existence of nn and pp pairs, but little or no evidence for np pairing mainly because of the experimental difficulties in studying N = Z nuclei. Nevertheless, following recent advances in the experimental techniques and considering the new possibilities that will become available with radioactive beams, there has been a revival of nuclear structure studies along the N = Z line. In this letter we present an analysis of experimental binding energies (BE) of nuclei along the N = Z line and the relative excitation energies of the lowest T = 0 and T = 1 states in self-conjugate (N = Z, T_z = 0) odd-odd nuclei. We have found evidence for the existence of strong T = 1 np pairing in N = Z nuclei, but find no such evidence for T = 0 np pairing.

Let us start by recalling that pairing effects can be isolated by studying differences in binding energies [2]. Particularly, the difference

\[ BE_{\text{even-even}} - BE_{\text{odd-odd}} \approx \Delta_p + \Delta_n \approx 2\Delta \]  \hspace{1cm} (1)

is used as a measure of the pair gap, \( \Delta \), for both protons and neutrons.\(^\dagger\)

Implicit in Eq. (1) is the assumption that the ground states have the same isospin, which is the case for nuclei with N \( \neq \) Z since they are maximally aligned in isospace, i.e. T = T_z = \( \frac{1}{2}(N - Z) \) [12]. Equation (1) is also true when comparing T = 0 states in even-even and odd-odd N = Z nuclei, and the difference in binding energy, given by

\[ BE_{ee}(N,Z) - \frac{(BE_{oo}(N-1,Z-1) + BE_{oo}(N+1,Z+1))}{2} \]  \hspace{1cm} (2)

is shown in Fig. 1, where the binding energies are from Ref. [3]. For comparison, the same quantities are given for nuclei with N = Z + 4. Taking the average in Eq. (2) removes the smooth variations due to volume, surface, and Coulomb energies, and any remaining differences are then attributed to shell or pairing effects. The extra binding of the even-even systems is clearly seen in Fig. 1 and it follows the known 1/A\(^{1/2}\) dependence [12].

This result shows that the T = 0 states in odd-odd N = Z nuclei behave like those in any other odd-odd

\(^\dagger\)There is usually a correction term due to the residual np interaction. This term is of order 20 MeV/A and we will not consider it here.
nuclei. Assuming that the binding energy differences reflect differences in pairing energy then the extra $n$ and $p$ block the pairing to the same degree as any “standard” 2-quasiparticle state. Note, if the ground states of $N = Z$ even-even nuclei contained $T = 0$ correlated pairs, the addition of a $T = 0$ $np$ pair would not give a gap, and the average binding energy of the two odd-odd nuclei would be the same as the even-even neighbor. This suggests that correlated $T = 0$ pairs do not contribute significantly to the pairing energy in $N = Z$ nuclei.

Is it possible that only $T = 1$ pairing is important for these $N = Z$ nuclei? If $np$ $T = 1$ pairs form a correlated state, the lowest $T = 1$ state in self-conjugate odd-odd nuclei should be as bound as that of the neighboring even-even ground state. An analysis similar to that used for $T = 0$ states should provide the answer. However, in applying Eq. (1) or (2) to determine the binding energy difference we need to include a symmetry energy term because of the different isospins (i.e. $T = 1$ in odd-odd $N = Z$ nuclei and $T = 0$ in neighboring even-even $N = Z$ nuclei). A discussion on the symmetry term is given in refs. [12–14]. To extract the symmetry energy ($E_{\text{sym}} = -BE_{\text{sym}}$) the experimental binding energies of several nuclei in the range $A = 10 – 64$ were plotted, as shown in Fig. 3 after subtracting volume, surface, and Coulomb terms. (The surface, Coulomb, and symmetry terms have the opposite sign to the volume term.) They are plotted as a function of $T(T + x)$, for three cases: 1) $x = 4$, corresponding to the SU(4) Wigner supermultiplet expression [15], 2) $x = 1$, i.e. $T(T + 1)$, and 3) $x = 0$, giving a $T^2$ approximation. While any of these choices can be used, the $T(T + 1)$ expression provides a better account of the experimental data, as discussed in Ref. [10]. In our analysis we use a symmetry energy given by $E_{\text{sym}} = \frac{75.1}{A} T(T + 1)$ which represents an average neglecting the effects of shell structure and pairing. The binding energy difference for $T = 1$ states in odd-odd $N = Z$ nuclei compared with $T = 0$ ground states in neighboring even-even $N = Z$ nuclei is presented in Fig. 3 (squares). If the only difference between the even-even ground state and the odd-odd $T = 1$ state were the symmetry term, then the difference in binding energy is given by the upper solid line. That is, the symmetry energy of the $T = 1$ state ($\frac{75.1}{A} T(T + 1)$) subtracted from the binding energy of the even-even nucleus provides the correct reference to which the odd-odd $T = 1$ states should be compared. It is also possible to use the even-even $T = 1$ ($T_z = -1, 1$) isobaric analog states as a reference, rather than the global expression $\frac{75.1}{A} T(T + 1)$. After correcting for the Coulomb energy, the binding energies of the isospin triplet are very similar, often within a few hundred keV. The average binding energies of the even-even $T = 1$ ($T_z = -1, 1$) isobaric analog states, relative to the even-even $T = 0$ ground state, are also shown in Fig. 3 (dotted line). These values are extremely close to those of the corresponding $T = 1$, $T_z = 0$ state in the odd-odd nucleus. Since, (i) the binding energy difference between the $T = 1$, $T_z = 0$, (odd-odd) and $T = 0$, $T_z = 0$ (even-even) states is described by the symmetry energy term only, and (ii) the $T = 1$ ($T_z = -1, 1$) state is the ground state of the even-even isobaric analog, then the binding energy difference ($BE_{oo}(T = 0) – BE_{oo}(T = 1)$) cannot be associated with a difference in pairing. Rather, it is due to the difference in isospin for which the smooth overall behavior is given by the symmetry energy.

These results indicate that the lowest $T = 1$ state in a
Self conjugate odd-odd nucleus is as bound as the neighboring even-even $N = Z$ ground state (after correcting for the symmetry energy). In other words, there is no difference in pairing, and just as the addition of an $nn$ or $pp$ pair to an even-even nucleus does not block pair correlations, neither does the addition of an $np$ $T = 1$ pair in $N = Z$ nuclei. However, as expected, adding a single $n$ or $p$ to the even-even core does reduce the pair energy and results in a binding energy difference in excess of the symmetry energy, as seen by the fact that the data points (stars in Fig. 3) for an odd nucleus ($N = Z + 1$) lie higher than the symmetry energy expected for a $T = 1/2$ nucleus (lower solid curve in Fig. 3). In view of the charge-independence of the nuclear force these results may not be too surprising; nevertheless they provide a strong argument in favor of the existence of full (i.e. $nn$, $pp$, and $np$) isovector pairing correlations in $N = Z$ nuclei.

Finally, we consider the relative energies of the $T = 0$ and $T = 1$ states in odd-odd $N = Z$ nuclei. If there were no $np$ pairing of any type ($T = 0$ or $T = 1$) the $T = 1$ state should lie above the $T = 0$ state at an excitation energy given by the symmetry term. However, the analysis of the experimental data presented above shows strong evidence for the existence of $T = 1$ $np$ pair correlations, and at the same time no evidence for $T = 0$ correlated pairs. The $T = 1$ states should then lie at a lower energy than that given by the symmetry term, and if the $T = 1$ pairing energy were sufficiently large, the $T = 1$ state may lie lower than the $T = 0$ state. The experimental energy differences are shown in Fig. 3 along with the expected contribution from the symmetry energy. The energy separation between the states of different isospin is clearly less than that predicted by the symmetry term. This is consistent with the pairing arguments presented above, and suggests that whether the $T = 0$ or $T = 1$ state is lower depends largely on the relative magnitudes of the symmetry and pairing energies. We further note that while the near cancellation of the symmetry and pairing terms (for $T = 1$ compared with $T = 0$) appears to be accidental we can not rule out, at this time, a deeper physical origin.

Assuming the reduced separation is only due to the effects of pairing then, in the language of the BCS model and taking the symmetry term into account, the $T = 0$ state in the odd-odd $N = Z$ nucleus can be interpreted as a 2-quasiparticle excitation (“broken-pair” with seniority 2) relative to the $T = 1$ correlated pair state. In complete analogy with Eq. (1) we have

$$BE_{T=1} - BE_{sym} - BE_{T=0} \approx 2\Delta_{np},$$

or, in terms of excitation energies,

$$E_{sym} - (ET_{T=1} - ET_{T=0}) \approx 2\Delta_{np}.$$  

(Note, this is the difference between the lowest $T = 1$ and $T = 0$ state in the same $N=Z$ odd-odd nucleus.)
The effective gap ($\Delta_{np}$), thus extracted, is presented in the insert to Fig. 4, where for comparison the result of a BCS calculation that includes nn, pp, and np $T = 1$ pairs is also shown. In this calculation we adopted standard single-particle levels from a spherical Nilsson potential and a pairing strength of 20MeV/A. This figure illustrates that the magnitude of 2$\Delta_{np}$ extracted from experiment using Eq. (4) compares favorably with that obtained from the spectrum of single-particle levels. While the gap (difference in binding energy) is not necessarily related only to a pairing interaction [17], the agreement is remarkable.

Due to the presence of shell gaps the simple BCS model gives a characteristic oscillation in $\Delta$. In this calculation, the single-particle levels were truncated at $N = Z = 50$, which led to an artificial quenching of $\Delta$ at $A = 100$. The reversal of the favored isospin from $T = 1$ to $T = 0$ at $^{58}$Cu coincides with it being one np pair above the $N = Z = 28$ and, within the pairing interpretation given here, occurs because the shell gap reduces the magnitude of the $T = 1$ pair gap. For heavier nuclei, $A > 60$, the $T = 1$ state is favored and we would expect that this is likely to remain the case until the $N = Z = 50$ shell gap is reached, where for $^{90}$In ($N = Z = 49$) the ground state may well revert to $T = 0$ once more. The competition between pairing and symmetry energy was also discussed in Ref. [17]. In this work, semi-empirical fits to the binding energies suggested that for odd-odd $N = Z$ nuclei beyond the $1f_{7/2}$ shell, pairing correlations will result in $T = 1$ ground states.

In conclusion, we have argued that binding energy differences indicate that the lowest $T = 1$ states in odd-odd $N = Z$ nuclei are as bound as their even-even neighbors, which provides strong evidence for the presence of isovector np pairing. There is, however, no similar evidence to support the existence of np isoscalar pair correlations. The intriguing switch from $T = 0$ to $T = 1$ ground states in odd-odd $N = Z$ nuclei arises from a subtle competition between the symmetry energy and isovector pairing. For $A > 40$, $T = 1$ pairing wins over the symmetry energy and the $J = 0^+$ state becomes the ground state, except, possibly, near closed shells where the “collective” effects of pairing are expected to be reduced. Future experiments on $N = Z$ nuclei to determine the binding energies and the relative excitation energies of $T = 1$ and $T = 0$ states (in odd-odd nuclei), as well as studies of their high-spin rotational properties are necessary and will provide further tests of the role of pairing in $N = Z$ nuclei.

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