Some recent developments in the physics of photon induced reactions are discussed. My presentation is biased towards HERA physics with David Miller’s talk being biased towards the $\gamma\gamma$ topics. Within the context of the data which were presented, I shall concentrate upon the following topics: diffraction; jets; prompt photons; open charm and charmonium.

1 Diffraction

I’m going to restrict myself to diffractive phenomena in $\gamma p$ interactions (where the photon can be real or virtual). For the present purposes “diffraction” means that there is a rapidity gap in the final state (I’ll have more to say on this later). Let’s start by recalling some results on total rates. In particular, I want to discuss the $W$-dependence of total rates ($W$ is the $\gamma p$ invariant mass). A convenient way to parameterize the data on the $W$-dependence is to extract an effective pomeron intercept, $\alpha_P(0)$. It is to be understood that this value would be the true pomeron intercept if the physics were solely due to exchange of a single Regge pole. Recall that total hadronic cross-sections and exclusive photoproduction of light vector mesons (e.g. $\gamma p \rightarrow Vp$ where $V$ is a vector meson and the photon is close to its mass-shell, i.e. $Q^2 \approx 0$) can be described with a single value of $\alpha_P(0) \approx 1.08$. However, there are processes which do not follow this trend. The dissociation process, $\gamma^* p \rightarrow Xp$ ($X$ denotes the dissociation products which are distant in rapidity from the outgoing proton), is characterized by a rather large effective intercept, i.e. $\alpha_P(0) = 1.18 \pm 0.02 \pm 0.04$. There is also a tendency for the intercept to grow as $Q^2$ increases. In addition, the growth for $\gamma p \rightarrow J/\psi p$ is larger still, $\alpha_P(0) \approx 1.3$ (even for photoproduction, i.e. $Q^2 \approx 0$). The situation for light meson production off virtual photons is less clear. There is a suggestion that the $W$ dependence is much steeper than it would be for on-shell photons, and that it steepens as $Q^2$ rises. However the conclusion relies upon extrapolating from low energy data, where the recent E665 measurement (of a high cross-section value at $W \approx 11$ GeV) confuses the issue. Data from Hermes should help sort this out. It seems that a large photon virtuality or a large quark mass is correlated with a more rapidly rising cross-section (recall also that the deep inelastic total cross-section rises rapidly with increasing $W$, i.e. at small

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$^a$Talk presented at the International Conference on the Structure and Interactions of the Photon, including the 11th International Workshop of Photon-Photon Collisions, Egmond aan Zee, The Netherlands, May 1997.
x). Michele Arneodo asked “just what is the scale which determines how steep the rise is?” To gain some insight into the physics which determines the answer to this question is the next part of my talk.

1.1 A physical picture of diffraction

Consider shining a coherent beam of partons onto a target at rest and let $z_i$ & $b_i$ be the fraction of the total beam energy carried by parton $i$ and its position in the plane transverse to its direction of travel respectively. We expect that, as the beam energy increases, so too does the probability that the parton passes through the target undeflected (i.e. any momentum transfer it receives is too small to deflect it appreciably). Of course the target itself can be broken up by the momentum transfer (or scattered into some excited state or scattered elastically). This type of event can have a big rapidity gap between the final state partons and the products of the target. It also follows that the eigenstates of this kind of reaction will be states of fixed $z_i$ and $b_i$. The target simply alters the profile of the incoming beam. The coherent sum over the final state partons will lead to a state which has some overlap with the initial state (elastic scattering) and also will lead to states which have non-zero overlap with other final states. The analogy with optical diffraction is clear (the parton states play the analogous role the the Huygens wavelets) and hence the name. It’s now time to turn to diffraction in photon induced reactions.

To describe the hadronic interactions of the photon we need to consider its fluctuation into a $q\bar{q}$ pair and the subsequent interaction with the target (at high beam energy, the pair will typically be produced way upstream of the target). We describe the $q\bar{q}$ fluctuations with the wavefunction, $\psi_{\gamma}(z, r)$, i.e.

$$|\gamma\rangle = \int dz d^2r \, \psi_{\gamma}(z, r) |z, r\rangle.$$  \hspace{1cm} (1)

The defining statement that the eigenstates of diffraction are states of fixed $z$ and $r$ can be written

$$\hat{T}|z, r\rangle = i\tau(s, b; z, r) |z, r\rangle$$  \hspace{1cm} (2)

where $\hat{T}$ is the operator which determines how the $q\bar{q}$ state scatters off the target and the eigenvalue $i\tau(s, b; z, r)$ is the associated amplitude for scattering the partons elastically (I just chose to take out a factor of $i$ since it turns out that the amplitude is dominated by its imaginary part, i.e. $\tau$ is real). The $\gamma$–Target invariant mass is denoted $s = W^2$ and $b$ is the impact parameter for the collision.
The **elastic scattering** amplitude,

\[ A^{el}(s, t) = \int d^2 b \ e^{i q \cdot b} \langle \gamma | \hat{T} | \gamma \rangle, \]  

(3)

(we have taken the Fourier transform so as to get the amplitude in terms of the momentum transfer, \(-t = q^2 > 0\)) can then be written:

\[ \frac{\text{Im} A^{el}(s, t = 0)}{s} = \int dz d^2 r \ |\psi_\gamma(z, r)|^2 \sigma(s, r) \]

\[ = \sigma_\gamma^{T}(s), \]  

(4)

where \(\sigma_\gamma^{T}(s)\) is the total \(\gamma\)-Target cross-section and

\[ \int d^2 b \ \tau(s, b; z, r) \equiv \sigma(s, r) \]

is the total cross-section for scattering the \(q\bar{q}\) pair off the target (for convenience we suppress any dependence on \(z\)). In writing these last two formulae, we have made use of the optical theorem.

Similarly, we can write the cross-section for **vector meson production**:

\[ \frac{d\sigma}{dt} \bigg|_{t=0} = \frac{1}{16\pi} \left[ \int dz d^2 r \ \psi_\gamma^*(z, r) \psi_\gamma(z, r) \sigma(z, r) \right]^2, \]  

(5)

where \(\psi_\gamma\) is the meson wavefunction.

For **photon dissociation** processes, we want to sum incoherently over the cross-sections to scatter into all possible final states, i.e.

\[ \frac{d\sigma}{dt} \bigg|_{t=0} = \frac{1}{16\pi} \int dz d^2 r \ |\psi_\gamma(z, r)|^2 \sigma(s, r)^2. \]  

(6)

So with nothing more than a bit of quantum mechanics and our definition of diffraction we have arrived at these useful formulae. In particular note that the elastic scattering amplitude and the photon dissociation cross-section only involve the photon wavefunction (calculable in QED) and the universal cross-section, \(\sigma(s, r)\). The essential physics of diffraction lives in \(\sigma(s, r)\) (e.g. pomeron exchange, gluon ladders,...). In order to proceed further, and gain some insight into the aforementioned \(W\)-dependencies, we need to input a bit more physics.

The photon wavefunction is calculated from the vacuum polarization graph and possesses the following properties: (1) an exponential suppression sets in...
for \( r^2 > 1/[Q^2 \bar{z} + m^2] \); (2) \( |\psi^L_\gamma|^2 \sim Q^2 \bar{z}^2 \); (3) \( |\psi^T_\gamma|^2 \sim Q^2 \bar{z} \). The superscripts label the mode of polarization, \( \bar{z} \equiv z(1 - z) \) and \( m \) is the quark mass. Gousset discussed the large size behaviour of the photon wavefunction.\(^{[4]}\)

The dipole cross-section, \( \sigma(s, r) \), must vanish in proportion to \( r^2 \) as \( r^2 \rightarrow 0 \). This is the colour transparency property which follows directly from QCD. For large \( r \) we expect \( \sigma(s, r) \) to saturate to some typical hadronic size, \( R^2 \) (due to confinement). We are now ready to make some qualitative statements about photon induced diffraction phenomena.

We’ll start with the diffraction dissociation cross-section, (6), and look separately at the contributions from large size \( q\bar{q} \) pairs (i.e. \( r > R \)) and small size pairs (i.e. \( r < 1/Q \)). For the large size pairs the important range of the \( z \) integral comes from the end-points, where \( \bar{z} < 1/[Q^2 R^2] \) (these are the only regions which don’t feel the exponential suppression from the tail of the wavefunction), i.e. the \( q\bar{q} \) pair is produced with a highly asymmetric partitioning of the photon energy. No such restriction is present for scattering small size pairs. We can get a feel for what is going on without having to go into too much detail.

For the large size pairs we can write

\[
\frac{d\sigma}{dt}\bigg|_{t=0} \sim \frac{1}{Q^2 R^2} \cdot R^2 \cdot Q^2 \left( \frac{1}{Q^2 R^2} \right)^n \cdot R^4.
\]

(7)

The first factor on the rhs (\( 1/[Q^2 R^2] \)) is from the volume of the \( z \) integral, the second (\( R^2 \)) is from the \( r \) integral, the third is the wavefunction factor (\( n = 1 \) for transverse photons and \( n = 2 \) for longitudinal photons) and the final factor is \( \sigma^2 \sim R^4 \). Thus the rate induced by transverse photons is \( \sim R^2/Q^2 \) whilst that by longitudinal photons is suppressed by an additional factor of \( Q^2 \), i.e. \( \sim 1/Q^4 \). The additional factor of \( \bar{z} \) in the longitudinal photon wavefunction makes all the difference by suppressing the \( z \) end-point contribution.

For the small size pairs, similar reasoning gives

\[
\frac{d\sigma}{dt}\bigg|_{t=0} \sim 1 \cdot \frac{1}{Q^2} \cdot Q^2 \cdot \frac{1}{Q^4} \sim \frac{1}{Q^4}.
\]

(8)

The \( z \) volume gives the factor unity, the \( r \) volume is now \( \sim 1/Q^2 \) and the photon wavefunction simply gives a factor \( \sim Q^2 \) (regardless of the polarization). The final factor is from \( \sigma^2 \sim r^4 \sim 1/Q^4 \). The contribution is therefore higher twist.

We have arrived at the interesting conclusion that there is a leading twist contribution to the diffraction dissociation rate and that it is a result of scattering large size \( q\bar{q} \) pairs produced by transversely polarized photons. The HERA data support this picture, except perhaps for the fact that the qualitative picture I’ve just presented suggests that \( \alpha_P(0) \approx 1.08 \) should be observed (since
the dominant contribution comes from scattering large size $q\bar{q}$ pairs). The fact that a larger value is seen is interesting and presumably arises because of QCD corrections which build up an anomalous dimension which leads to an enhancement of the short distance contribution.

Now let’s turn to vector meson production (5). For “heavy” mesons (e.g. $J/\psi$), the non-relativistic approximation leads us to assume that $|\psi_V|^2 \sim \delta(z - 1/2)$ (or else the meson could not be bound together). There is no end-point contribution and the quark mass is large therefore the contribution from large size pairs is exponentially suppressed. The rate for producing $J/\psi$ mesons off on-shell photons rises rapidly with increasing $W$. This is a characteristic of perturbation theory and is in accord with our conclusion that only small size $q\bar{q}$ pairs need be considered.

For light mesons the situation is much more complicated and, not surprisingly, depends critically on the end-point behaviour of the meson wavefunction. For example, if we assume that $|\psi_V^T|^2 \sim \bar{z}^{m+1/2}$ and that $|\psi_V^L|^2 \sim \bar{z}^{m+1}$ then it follows (following precisely the same reasoning that led to the estimates for the dissociation rate) from (5) that

$$\frac{\sigma_T(r > R)}{\sigma_T(r < 1/Q)} \sim (Q^2 R^2)^{1-2m}$$

and

$$\frac{\sigma_L(r > R)}{\sigma_L(r < 1/Q)} \sim (Q^2 R^2)^{-2m}.$$  

Putting $n = 1/2$ (which seems reasonable), means that the production rate off transverse photons is sensitive to all sizes (i.e. both perturbative and non-perturbative configurations) whilst the rate off longitudinal photons is dominated by scattering of small size pairs (perturbative). Light meson production is thus a potentially very interesting mix of soft and hard physics. Information which will help untangle what is going on comes in the form of measurements of $\sigma_L/\sigma_T$ and the variation of the total rate with $Q^2$ and $W$.

1.2 Pomeron parton densities

Regge theory inspires the factorization of the structure function, $F_2^{D(3)}$, extracted from high $Q^2$ photon dissociation, i.e.

$$F_2^{D(3)}(\beta, Q^2, x_P) = f_P(x_P)F_2^P(\beta, Q^2) + \text{secondary exchanges.}$$

This really is an educated guess, it is a real challenge to understand the light meson wavefunction.
The data support this picture and are moving into the domain where they can really test the notion of universal pomeron parton distribution functions. At present, a model with DGLAP evolution describes the ZEUS data on diffractive dijet production in photoproduction and on $F_2^{D(3)}$. Also, H1 results on the hadronic final state (high $p_t$ particle production, energy flows and charm production in DIS dijets in both DIS and photoproduction) are all consistent with the DGLAP approach.

The universality of the pomeron parton densities is intimately connected to the notion of the gap survival probability. Comparison between data on direct and resolved processes, and from the Tevatron, will certainly provide essential information in helping unravel the nature of diffraction.

1.3 Squeezing the pomeron

There are some rare diffractive processes whose rates can be calculated purely using perturbative QCD.

Hautmann presented results on the $\gamma^*\gamma^*$ total cross-section. It is extracted (using the optical theorem) from the elastic $\gamma^*\gamma^*$ amplitude at $t = 0$, so is concerned with the physics of diffraction. Since the photons are way off shell, they scatter perturbatively via exchange of “reggeized gluons” between their respective $q\bar{q}$ pairs. There are no hadrons to worry about, so the calculation is very clean and worth looking for at LEP2 and beyond.

An even better way of keeping things perturbative is to look at high-$t$ diffraction. For example, one can look for a pair of high $p_t$ jets which are separated by a big rapidity gap. Presumably one is looking at parton-parton elastic scattering at $-t \approx p_t^2$ and, since there’s a gap, without exchange of colour. The fraction of dijet events with a gap to all dijet events as a function of increasing gap size has been presented by ZEUS out to gaps of $\approx 4$ units and by D0 out to gaps of $\approx 6$ units. To really unravel the important physics behind these data requires an understanding of the gap survival probability. A very similar process that can be studied at HERA and which avoids the issue of gap survival is high-$t$ vector meson production (the proton dissociates to produce a forward jet, which, since it need not be seen, means bigger gaps are admitted). Both H1 and ZEUS are starting to accumulate good data on this process.

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$^c$There is no need to worry about diffusion effects
2 Jets: rates and shapes

For an introduction to jet photoproduction, I refer to Patrick Aurenche’s presentation.

The structure of the virtual photon is starting to be examined at HERA, and Rick presented results which showed that the $\gamma^{\ast}$ has a significant “resolved” component for $Q^2$ as big as 50 GeV$^2$ in those events where jets are produced with $E_T^2 > Q^2$.

2.1 Dijets

Data on two or more jets provides us with further options to test QCD and understand the nature of the “strongly interacting” photon. ZEUS has defined direct enriched and resolved enriched samples by separating events according to a cut at $x_\gamma = 0.75$. The direct enriched sample is very sensitive to the small-$x$ gluon content of the proton: the more backward the dijets, the lower the $x$ values in the proton that are probed. Conversely, the resolved enriched sample is sensitive to the gluon content in the photon. In addition, NLO calculations for the dijet rates are now available for comparison with the data. Let’s summarize the situation as it stands right now.

For $x_\gamma > 0.75$, the NLO theory does a good job. However there remains quite a large contamination from the large-$x$ part of the photon quark distribution functions. This arises because of the harder form of the photon quark densities. To unravel the effects of the low-$x$ gluons in the proton from the large-$x$ quarks in the photon requires a tighter cut on $x_\gamma$. To facilitate a clean comparison between data and theory, the ZEUS collaboration has started to use the $k_t$-cluster algorithm.

For $x_\gamma < 0.75$ the theory falls well below the data for the lowest $E_T$ forward dijets. The effect exhibits a strong dependence upon the $E_T$ cut, which suggests that it cannot be explained by modifying the parton distribution functions of the photon in any sensible way. Presumably, this is the same problem as that which has been encountered for the single jets i.e. H1 and ZEUS both see an excess of single jet events in the forward direction for $E_T < 15$ GeV (see ). We really need to understand what is going on before we can extract the gluon density of the photon. Furthermore, these forward jets are fatter than might naively be expected.

A likely explanation for this effect could be due to the presence of a large soft underlying event. Multiple parton interactions (MI) simulate (at least part of) this physics. MI are anticipated on the grounds that forward jets at low $E_T$ are produced as a result of interactions between slow partons in the colliding particles. We know that QCD predicts a proliferation of these
slow partons, and as such it may well be that more than one pair of them can interact in each $\gamma p$ interaction. MI can describe the broader nature of the forward jets and also increase the cross-sections for forward jet production, e.g. see [27]. One way of unambiguously identifying MI might be to look at higher (3 or 4) jet rates.

B"urgin presented the OPAL results of jets in $\gamma\gamma$ [28]. Separation of events into classes involving direct and/or resolved photons was performed via the $x_\gamma$ variable and good agreement with the NLO calculations of Kleinwort & Kramer [29] were found. However, the error bars are still too large to allow much discrimination between different parton distribution function parameterizations.

In conclusion, the dijets provide information which complements the single jet measurements. The data are now reaching a high level of precision, and comparison with NLO theory has revealed a number of pressing issues. In particular, we need to understand better the forward jets and use the most appropriate jet algorithm. Once these issues have been addressed, we can expect to gain further insight into the gluon content of both the photon and proton.

2.2 Soft gluons

The ZEUS dijet measurements have been made with a cut on the minimum $E_T$ of the jets and the cut is the same for both jets. This introduces a further theoretical problem. This arises because most of the jets will be produced around the minimum allowable $E_T$, i.e. the typical difference between the jet transverse momenta, $\Delta p_T$, will be small. So, the 3 parton final state (which is present in the NLO calculation) must have one of the partons either collinear with another, or very soft. The collinear configuration is easy to deal with (it is factorized) but the soft parton emission leads to a $\ln \Delta p_T$ contribution. This large logarithm signals that multiple soft parton emission is important. These soft parton effects can be studied by looking explicitly at the $\Delta p_T$ distribution of the dijets or they can be avoided by making a cut which keeps away from $\Delta p_T \approx 0$.

Similar effects need to be considered in double prompt photon production which is being observed at the Tevatron [30]. D0 cut on the photon transverse energies, i.e. $E_{T1} > 14$ GeV and $E_{T2} > 13$ GeV. The need to sum the soft gluon effects can be seen in that the theory overshoots data for $\Delta p_T < 3$ GeV.
3 Prompt Photon Production

The rates for prompt photon production seen by D0 and CDF suggest a possible excess of events at low $E_T/\sqrt{s}$\textsuperscript{[9]}. However, Andreas Vogt pointed out that the data are within the theoretical uncertainty.

Gordon presented results on prompt $\gamma$ plus jet at HERA\textsuperscript{[31]}. The NLO calculation is coded into Monte Carlo. This process is sensitive to the gluon density of the photon (for low $x$, where there are no data yet) and isolation cuts kill off the fragmentation contribution. We saw the good agreement between these NLO calculations and the ZEUS data\textsuperscript{[32]}. At present the data are statistics limited.

4 Open Charm

The photoproduction of charm quarks at large $p_T$ is a process which involves two large scales, $p_T, m_c \gg \Lambda$ and as such, makes life more complicated from the theoretical point of view. Good data, which can be expected in the future (especially if the charm can be tagged using a microvertex detector), will surely shed light on this intriguing area. At present, there are two main routes used in theoretical calculations.

Massive charm: The charm quark mass is considered to provide the hard scale, as such charm only ever appears in the hard subprocess and there is no notion of radiatively generated charm in the sense of parton evolution. This means that terms $\sim \alpha_s \ln(p_T^2/m_c^2)$ are not summed to all orders (in the parton distribution and fragmentation functions). As such, we might expect this approach to become less accurate when $p_T^2 \gg m_c^2$. However, it does provide a systematic way of accounting for charm quark mass effects, which will be important for $m_c \sim p_T$.

Massless charm: In this approach, the charm quark is treated as massless (above threshold), and as such is treated like any other light quark in the parton evolution equations and hard subprocess cross-sections. The $\ln(p_T/m_c)$ terms are now summed to all orders, but charm quark mass effects are ignored. So, this approach should get better as $p_T/m_c$ increases.

Gladilin presented new results from ZEUS\textsuperscript{[33]}. At present the data lie in the intermediate region where $p_T < 10$ GeV, i.e. it is not clear which, if any, of the two approaches should be used. In order to compute the inclusive $D^*$ rate, one needs the appropriate fragmentation function. Either the Peterson\textsuperscript{[34]} form or the $x^a(1-x)^b$ form do a good job, and can be well constrained by $e^+e^-$ data.

Initial comparisons between theory and data suggest that the massive charm calculation is too low, e.g see\textsuperscript{[33]}. However, the full NLO calculations re-
quire that the fragmentation function be consistently extracted from the $e^+e^-$ data. When this is done, Cacciari and Greco find that the theoretical predictions are increased significantly relative to what is found using the softer (LO) fragmentation functions. This is true for both massless and massive charm calculations and, within theoretical uncertainties, both are now consistent with the present data.

More data at large $p_T$ and increased statistics at intermediate $p_T$ will certainly help in our study of the interplay of $\ln(p_T^2/m_c^2)$ and $m_c^2/p_T^2$ effects. In addition, for $p_T \gg m_c$ we have the possibility to study the “intrinsic” charm within the photon (charm in dijets offers good prospects here).

In $e^+e^-$, Andreev presented new results from L3. The open charm total cross-section agrees with the NLO calculation of Drees et al. We can look forward to more statistics which will allow comparison with differential distributions.

5 Charmonium

Originally, inelastic photoproduction of charmonium, e.g. $J/\psi$, was advertised as an ideal way to extract the gluon density in the proton (since it is driven by photon-gluon fusion into a $Q\bar{Q}$ pair). More recently, NLO calculations have put a dampener on this goal. However, there has been a great deal of recent interest in the non-relativistic QCD (NRQCD) approach to heavy quarkonium production, and the inelastic photoproduction of heavy quarkonia provides the ideal opportunity to test NRQCD.

Bodwin, Braaten and Lepage derived a factorization formula which describes the inclusive production (and decay) of a heavy quarkonium state. In the case of photoproduction, Fig. shows the lowest order contribution. The NRQCD factorization formula for the corresponding cross-section reads

$$d\sigma(H + X) = \sum_n d\hat{\sigma}(Q\bar{Q}[n] + X)(O_n^H).$$

(12)

$X$ denotes that the process is inclusive, $d\hat{\sigma}(Q\bar{Q}[n] + X)$ is the perturbatively
calculable cross-section for $\gamma p \rightarrow Q\bar{Q} + X$ and it can be written as a series expansion in $\alpha_s(m_Q)$. The $Q\bar{Q}$ pair is produced with quantum numbers $n$. The matrix element, $\langle O^H_n \rangle$, contains the long distance physics associated with the formation of the quarkonium state $H$ from the $Q\bar{Q}$ state – it is essentially the probability that the pointlike $Q\bar{Q}$ pair forms $H$ inclusively. The typical scale associated with this part of the process is $\sim m_Q v$ which is much smaller than $m_Q$ ($v$ is the relative velocity of the $Q\bar{Q}$ pair, and is small for heavy enough quarks). This hierarchy of scales underlies the NRQCD factorization. Note that the $Q\bar{Q}$ state is not restricted to having the same quantum numbers as the meson. Fortunately, there exist “velocity scaling rules” which allow us to identify which states, $n$, are the most important. More precisely, the “velocity scaling rules” order the operators $\langle O^H_n \rangle$ according to how many powers of $v$ they contain, i.e. relativistic corrections can be computed systematically.

The NRQCD approach therefore provides us with a systematic way of computing inclusive heavy quarkonium production (modulo corrections which are suppressed by powers of $\sim \Lambda/m_Q$). The strategy is first to organise the sum over $n$ into an expansion in $v$ and then to systematically compute $d\sigma$ order-by-order in $\alpha_s(m_Q)$. Technically, we do not a priori know where our efforts are best placed, i.e. do we work at lowest order in $v$ and to NLO in $\alpha_s$ or do we attempt to work at higher orders in $v$, but computing each hard subprocess to lowest order? We need to know $v$ in order to judge better what to do.

One final word before moving on to discuss $J/\psi$ photoproduction. For small $p_T$, NRQCD factorization is likely to break down, due to contamination from higher twist effects. Also, one expects breakdown of the NRQCD approach in the vicinity of the elastic scattering region, i.e. $z \rightarrow 1$ where $z$ is the fraction of the photon energy carried by the quarkonium (see later).

Inelastic photoproduction of $J/\psi$ is something which has already been measured at HERA. Let’s see how the theory shapes up. To lowest order in the velocity expansion, $|n| = [1,3S_1]$. The first entry in the square brackets tells us that the $c\bar{c}$ is in a colour singlet state, whilst the second entry tells us the spin and angular momentum of the state. Not surprisingly, to lowest order in the velocity expansion, the $c\bar{c}$ must be produced with the same quantum numbers as the $J/\psi$. This is just the colour singlet model (CSM) of old. The lowest order diagram which can contribute is shown in Fig.[3] and

\[
\langle O^{J/\psi}[1,3S_1] \rangle \sim |\phi(0)|^2
\]

where $\phi(0)$ is the wavefunction at the origin (it can be extracted from the electronic width of the $J/\psi$). NLO($\alpha_s$) corrections have been computed[23] and shown to be large. The NLO corrections enhance the LO calculation and lead
to a reduced sensitivity to the gluon density in the proton.

One might well ask as to the significance of the resolved photon contribution. It is important at small enough $z$\[^{38,41,42}\]. In addition, for mesons produced at high enough $p_T$ we have an additional scale to consider and terms which are leading in $\alpha_s$ can be suppressed by powers of $\sim m_c^2/p_T^2$. This is true for example of the diagram shown in Fig.1 relative to that shown in Fig.2. The latter fragmentation contribution is higher order in $\alpha_s$, however there is one less hard quark propagator and so it will dominate for large enough $p_T$. Fragmentation contributions and resolved photon contributions are not important in computing the total rate for $z > 0.4$ (which is essentially where the data are).

![Diagram](image)

Figure 2: Fragmentation contribution to the production of the $J/\psi$.

Going to NLO in the velocity expansion means moving away from the CSM. For the first time we encounter colour octet contributions. In particular, the LO($\alpha_s$) diagram is again that of Fig.1, now the $c\bar{c}$ can be formed in one of 5 states, i.e. $[n] = [8,^1S_0], [8,^3S_1], [8,^3P_{0,2}]$. The price one pays for having to convert this state into the $J/\psi$ is an extra power of $v^4$ relative to the colour singlet matrix element, i.e.

$$\langle O^{J/\psi}[n] \rangle \sim v^4 \langle O^{J/\psi}[1,^3S_1] \rangle.$$  

It turns out that the $v^4 \sim 0.01$ suppression of the long distance matrix elements is partially compensated for by a strong enhancement of the corresponding short distance cross-section. In particular, this is so for the $[8,^1S_0]$ and $[8,^3P_{0,2}]$ states. The colour singlet matrix element can be extracted from data, e.g. the leptonic width of the $J/\psi$. Likewise, we need to fit these new matrix elements to data (or extract them from lattice calculations). It is therefore clear that a test of the NRQD framework requires data from different sources – the challenge being to find a consistent description. This is a particularly topical issue, since an explanation of the Tevatron excess of direct $J/\psi$ and $\psi'$ production needs, in addition to fragmentation contributions, colour octet

\[^{d}\text{There is a lower order contribution in which no gluon is radiated off, however this would give a contribution only at } z = 1 \text{ which lies outside our region of interest.}\]
contributions. One can use the Tevatron data to fit the relevant matrix elements. The validity of this explanation can be checked on comparing to data which can be obtained from HERA. Unfortunately, the matrix elements which are important at the Tevatron are not so important in $J/\psi$ photoproduction for $z > 0.4$. However, the key matrix element for the Tevatron ([8,3 $S_1$]) does play a key role in the region of lower $z$, where (for large enough $p_T$) the dominant contribution comes from the fragmentation mechanism via resolved photons. Another process which is sensitive to the [8,3 $S_1$] state is the photoproduction of $J/\psi + \gamma$ (where the photon is produced in the hard subprocess, i.e. not via the radiative decay of a $P$-wave quarkonium). So, with the anticipated increase in statistics, we can really expect to test NRQCD at HERA. Going back to the $J/\psi$, there are some weak constraints on the important matrix elements from the Tevatron data and these have been used in the theoretical calculations of. The HERA data on the $z$ distribution compare very well with the colour singlet calculation. The colour octet contribution however, is much too large at large $z$. Thus the HERA data is not supporting a large colour octet contribution at large $z$. However, one must be careful in interpreting this as evidence against the NRQCD approach, since the $z \to 1$ region is sensitive to higher order non-perturbative contributions which lead to the breakdown of the NRQCD expansion.

In addition to the processes just discussed, increased statistics will allow measurement of other meson states, e.g. $\psi'$ and $\Upsilon$, which will certainly further test our understanding of QCD.

6 Outlook

We really need to improve our understanding of the $e\gamma$ final state if we are to reduce the systematic uncertainty which presently dominates the experimental measurements of $F_2^\gamma$.

An improved understanding of the soft underlying event and of multiple interactions is needed in order to understand the gap survival probability in diffractive events. It is also needed for a better understanding of forward jets at HERA (which can then be used to extract the gluon density of the photon).

We can look forward to the accumulation of data on prompt photon production, $\gamma^*\gamma^*$ reactions and virtual photon structure, high $t$ diffraction and diffractive meson production at high $Q^2$. Comparison of diffraction data from deep inelastic scattering with that from photo- (and hadro-) production will play a central role in developing our understanding of diffraction. It would be great to see data on diffraction in $\gamma\gamma$ collisions. Meanwhile, the search for the odderon will continue.
Charm production will provide tests of NRQCD and allow us to unravel the subtle issues associated with open charm production.

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References

1. D.J. Miller, these proceedings.
2. M.Arneodo, these proceedings.
3. A.Domachie and P.V.Landshoff, Phys. Lett. B 296, 227 (1992).
4. T.Ebert, these proceedings.
5. K.Piotrzkowski, these proceedings.
6. J.H.Köhne, these proceedings.
7. J.A.Crittenden, these proceedings.
8. E665 collaboration, M.R.Adams et al., MPI-PHE-97-03, FERMILAB-PUB-97-103-E.
9. M.Kolstein, these proceedings.
10. T.Gousset, these proceedings.
11. S.J.Brodsky, et al., Phys. Rev. D 50, 3134 (1994).
12. J.Puga, these proceedings.
13. R.Engel, these proceedings.
14. F.Hautmann, these proceedings.
15. M.Hayes, these proceedings.
16. B.May, these proceedings.
17. J.R.Forschaw and M.G.Ryskin, Z. Phys. C 68, 137 (1995); J.Bartels, J.R. Forshaw, H.Lotter and M.Wusthoff, Phys. Lett. B 375, 301 (1996).
18. P.Aurenche, these proceedings.
19. H.Rick, these proceedings.
20. E.Strickland, these proceedings.
21. M.Klasen and G.Kramer, Phys. Lett. B 366, 385 (1996); DESY-96-246, hep-ph/9611450.
22. B.W.Harris and J.F.Owens, presented at the Annual Divisional Meeting of the Division of Particles and Fields of the APS, Minneapolis, USA (1996), hep-ph/9608378, FSU-HEP-970411, hep-ph/9704324.
23. S.Catani, Yu.L.Dokshitzer, M.H.Seymour and B.R.Webber, Nucl. Phys. B 406, 187 (1993).
24. H1 collaboration, I.Abt et al., Phys. Lett. B 314, 436 (1993); Phys. Lett. B 328, 176 (1994); Z. Phys. C 70, 17 (1996); ZEUS collaboration, M.Derrick, et al., Phys. Lett. B 322, 287 (1994); Phys. Lett. B 342, 417 (1995); contribution pa02-041 to 28th ICHEP, Warsaw (1996).
25. J.Chyla, these proceedings.
26. G.Pancheri, these proceedings.
27. J.M.Butterworth, J.R.Forshaw and M.H.Seymour, Z. Phys. C 72, 637 (1996).
28. R.Burgin, these proceedings.
29. T.Kleinwort and G.Kramer, DESY-96-223, hep-ph/9610489.
30. W.Chen, these proceedings.
31. L.E.Gordon, these proceedings.
32. T.Vaiciulis, these proceedings.
33. L.Gladilin, these proceedings.
34. C.Peterson, D.Schlatter, I.Schmitt and P.M.Zerwas, Phys. Rev. D 27, 105 (1983).
35. M.Cacciari and M.Greco, DESY 97-029, hep-ph/9702389.
36. V.Andreev, these proceedings.
37. M. Drees, M. Kramer, J. Zunft and P.M.Zerwas, Phys. Lett. B 306, 371 (1993).
38. M.Cacciari and M.Krämer, in Future Physics at HERA, Proceedings of the Workshop 1995/96, Volume 1, hep-ph/9609500.
39. G.T.Bodwin, E.Braaten and G.P.Lepage, Phys. Rev. D 51, 1125 (1995).
40. M.Krämer, J.Zunft, J.Steegborn and P.Zerwas, Phys. Lett. B 348, 657 (1995); M.Krämer, Nucl. Phys. B 459, 3 (1996).
41. R.Godbole, D.P.Roy and K.Sridhar, Phys. Lett. B 373, 328 (1996).
42. B.A.Kniehl and G.Kramer, DESY 97-036, hep-ph/9703280.
43. E.Braaten and S.Fleming, Phys. Rev. Lett. 74, 3327 (1995); M.Cacciari, M.Greco, M.L.Mangano and A.Petrelli, Phys. Lett. B 356, 560 (1995); P.Cho and A.K.Leibovich, Phys. Rev. D 53, 150 (1996); Phys. Rev. D 53, 6203 (1996).
44. M.Cacciari, M.Greco and M.Krämer, Phys. Rev. D 55, 7126 (1997).
45. M.Cacciari and M.Krämer, Phys. Rev. Lett. 76, 4128 (1996).
46. M.Beneke, I.Z.Rothstein and M.B.Wise, CERN-TH/97-86, hep-ph/9705280.
47. S.Tapprogge, these proceedings.