Is Gravity Quantum?

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What gravitational field is generated by a massive quantum system in a spatial superposition? This is one of the most important questions in modern physics, and after decades of intensive theoretical and experimental research, we still do not know the answer. On the experimental side, the difficulty lies in the fact that gravity is weak and requires large masses to be detectable. But for large masses, it becomes increasingly difficult to generate spatial quantum superpositions, which live sufficiently long to be detected. A delicate balance between opposite quantum and gravitational demands is needed. Here we show that this can be achieved in an optomechanics scenario. We propose an experimental setup, which allows to decide whether the gravitational field generated by a quantum system in a spatial superposition is the superposition of the two alternatives, or not. We estimate the magnitude of the effect and show that it offers good perspectives for observability. Performing the experiment will mark a breakthrough in our understanding of the relationship between gravity and quantum theory.

Quantum field theory is one of the most successful theories ever formulated. All matter fields, together with the electromagnetic and nuclear forces, have been successfully embodied in the quantum framework. They form the much celebrated standard model of elementary particles, which not only has been confirmed in all advanced accelerator facilities, but has also become an essential ingredient for the description of the universe and its evolution.

In light of this, it becomes obvious to seek a quantum formulation of gravity as well. Yet, the straightforward procedure for promoting the classical field as described by general relativity into a quantum field, does not work. Over the decades, several strategies have been put forward, which turned into very sophisticated theories of gravity, perhaps the most advanced being string theory and loop quantum gravity. Yet, none of them has reached the goal of providing a fully consistent quantum theory of gravity.

At this point, one might wonder whether the very idea of quantizing gravity is ill-posed [1,2]. At the end of the day, according to general relativity, gravity is rather different from all other forces. Actually, it is not a force at all, but a mere manifestation of the curvature of spacetime, and there is no obvious reason why the standard approach to the quantisation of fields should work for spacetime as well. A future unified theory of quantum and gravitational phenomena might require a radical revision not only of our notions of space and time, but also of (quantum) matter. This scenario is growing in likelihood.

From the experimental point of view, it has now been ascertained that quantum matter (i.e. matter in a genuine quantum state, such as a coherent superposition state) couples to the Earth’s gravity in the most obvious way. This has been confirmed in neutron [3], atom [4] interferometers and used for velocity selection in molecular interferometry [5]. However, in all cases, the gravitational field is classical, i.e. it is generated by a distribution of matter (the Earth) in a fully classical state. Therefore, the plethora of successful experiments mentioned above does not provide hints, unfortunately, on whether gravity is quantum or not.

The large attention and media coverage about the BICEP2 experiment having shown the quantum origin of primordial gravitational fluctuations [6], subsequently disproved by Planck’s data analysis [7], testifies the importance and urgency of a pragmatic assessment of the question of whether gravity is quantum or not.

In this paper, we propose an approach where a macroscopic system is forced in the superposition of two different positions in space, and its gravitational field is explored by a probe (Fig. 1). Using the exquisite potential for transduction offered by optomechanics, we can in principle determine whether the gravitational field is the superposition of the two gravitational fields associated to the two different states of the system, or not. The first case amounts to a quantum behavior of gravity, the second to a classical-like one. We show that the sensitivity necessary to appreciate the difference between such behaviors is close to the current state of the art in specific optomechanical configuration, although quite demanding.

Framework. - We consider a system $S_1$ (with mass $m_1$) prepared in a superposition of two different positions in space. The wave function is $\psi(r_1) = \frac{1}{\sqrt{2}} (\alpha(r_1) + \beta(r_1))$ with $\gamma(r_1) = \langle r_1 | \gamma \rangle$ ($\gamma = \alpha, \beta$) and $\langle \alpha | \beta \rangle = 0$, stating the distinguishability (in a macroscopic sense) of the two states. $S_1$ generates a gravitational field that can be
A displaced slightly away from equilibrium, alternatively monic trap, a position measurement will reveal it being zero. If the motion of $S_2$ is constrained within an harmonic trap, the (square modulus of the) wave function acts as some field $S_1$ generates. However, a natural answer is that in this case, no one really knows which gravitational gravity, for whatever reason, is fundamentally classical. In this case, the gravitational field is not the quantum superposition of the two quantum fields associated to $\alpha(r_1)$ and $\beta(r_1)$, but the classical sum of the two classical fields generated by their square modulus, respectively. In such conditions, $S_2$ feels a force which pulls it in between region $A$ and $B$. Quite evidently, the two cases imply two different motions for the probe $S_2$, such difference being the way to discriminate between classical-like and quantum treatment.

**Theoretical modelling.**– We refer again to the situation illustrated in Fig. 1. In what follows, when no explicit time dependence is reported, we imply $t = 0$. We let the two systems interact gravitationally for a time $\tau$, and then measure the position of $S_2$ along the $x$-axis. The timescale $\tau$ can be, at most, the lifetime of the superposition state $\psi$. Also, experimental parameters are adjusted such that all interactions, except gravity, are negligible, for all practical purposes.

In the quantum approach, the total Hamiltonian is given by $H = H_1 + H_2 + V_3(r_1 - r_2)$, where $H_{1,2}$ are the Hamiltonians of $S_1$ and $S_2$, respectively, and $V_3(r_1 - r_2)$ is the Newtonian interaction. The final state of the overall system is given by $\Psi(r_1, r_2, \tau) = \Psi(\alpha, \beta, r_1, r_2, \tau) / \sqrt{2}$, where each term is the solution of the equation $i\hbar \partial_t \Psi_{\alpha,\beta} = H \Psi_{\alpha,\beta}$ with initial conditions $\Psi_{\alpha,\beta}(r_1, r_2) = \alpha(r_1)\phi(r_2)$ and $\Psi_{\beta,\beta}(r_1, r_2) = \beta(r_1)\phi(r_2)$. We assume $m_1 \gg m_2$, implying an adiabatic approximation in which the degrees of freedom of the two systems can be separated as

$$\Psi_{\gamma}(r_1, r_2, t) = \gamma(r_1, t)\phi_\gamma(r_2, t),$$

(1)

where the motion of $\alpha(r_1, t)$ and $\beta(r_1, t)$ are determined by $H_1$, while $\phi_\gamma(r_2, t)$ evolves with the Hamiltonian $H_\gamma = H_2 + V_3$ with

$$V_\gamma = -Gm_1 m_2 \int d^3r_1 \frac{|\gamma(r_1, t)|^2}{|r_1 - r_2|},$$

(2)

In this quantum scenario, the initial superposition state of $S_1$ generates a superposition of gravitational fields, which in turn generates a superposition of motions for $S_2$.

On the other hand, in the semiclassical treatment of gravity, and under the same approximations discussed above, the evolution of $S_2$ is determined by $H_3 = H_2 + V_3$, where the gravitational potential now reads

$$V_3 = -Gm_1 m_2 \int d^3r_1 \frac{\psi(r_1, t)^2}{|r_1 - r_2|},$$

(3)

Here, the evolution of $\psi(r_1, t)$ is determined by $H_3$. Eq. (3) shows the signatures of a classical treatment of
gravitational field acting on S2 is the one produced by a total mass $m_1$ with density $\frac{1}{\pi} (|a(x)|^2 + |\beta(x)|^2)$.

One can further approximate the above mentioned gravitational potentials, which will be quite useful when we compute the motion of optomechanical systems. Henceforth we will work in the Heisenberg picture. We assume that the quantum fluctuations around the mean values for S1 are small. Therefore, $V_{\gamma}$ in Eq. (2) can be approximated as

$$V_{\gamma} \approx -\frac{Gm_1m_2}{|\langle r_1(t)\rangle_\gamma - \langle r_2(t)\rangle_\phi|}, \quad (\gamma = \alpha, \beta). \quad (4)$$

Assuming that the quantum fluctuations around the mean values for S2 are also small, $V_{\gamma}$ can be expanded in Taylor series as

$$V_{\gamma} \approx -\frac{Gm_1m_2}{|\langle r_1(t)\rangle_\gamma - \langle r_2(t)\rangle_\phi|} + \delta r_2(t) \cdot \frac{Gm_1m_2(\langle r_1(t)\rangle_\gamma - \langle r_2(t)\rangle_\phi)}{|\langle r_1(t)\rangle_\gamma - \langle r_2(t)\rangle_\phi|^3}, \quad (5)$$

where $\delta r_2(t) = r_2(t) - \langle r_2(t)\rangle$.

The same procedure can be applied to $V_{a_l}$. As for the quantum case, assuming that the fluctuations in the motions of S1 and S2 are small, we find

$$V_{a_l} \approx \sum_{\gamma=\alpha,\beta} \left[ -\frac{Gm_1m_2}{2|\langle r_1(t)\rangle_\gamma - \langle r_2(t)\rangle_\phi|} + \delta r_2(t) \cdot \frac{Gm_1m_2(\langle r_1(t)\rangle_\gamma - \langle r_2(t)\rangle_\phi)}{2|\langle r_1(t)\rangle_\gamma - \langle r_2(t)\rangle_\phi|^3} \right]. \quad (6)$$

**Optomechanical test.** We now consider the exquisite potential for motional transduction offered by optomechanics and let system S2 be the movable end-mirror of an optomechanical cavity. On the other hand, we shall not specify explicitly what is the chosen embodiment for S1, which could well be a second vibrating mechanical structure. Explicit configurations will be described elsewhere [17].

The transduction cavity is pumped by an external laser field and S2 is in contact with a bath of phononic modes. The axis of the optomechanical cavity, as well as the localization axis of S1, is assumed to lay along the $x$-axis of a reference frame. Denoting the position operator of S2 along the cavity axis by $x_2$, and its momentum by $p_2$, moving to a frame rotating at the frequency of the pumping field, we find the Hamiltonian model [15]

$$H_2 = \hbar(\omega_c - \omega_0)a^\dagger a - \hbar \chi a^\dagger a x_2 + \frac{1}{2} m_2 \omega_2^2 x_2^2 + \frac{p_2^2}{2m_2} + i\hbar \mathcal{E}(a^\dagger - a), \quad (7)$$

where $\omega_0$ is the frequency of the external laser, $\omega_c$ is the frequency of the cavity mode derived by the laser, $\omega_2$ is the harmonic frequency of the mechanical oscillator, $\chi = \omega_c/L$ is the optomechanical coupling constant between the cavity and the mechanical oscillator with $L$ the size of the cavity, and $\mathcal{E} = \sqrt{2\kappa \mathcal{P}}/\hbar \omega_0$ with $\mathcal{P}$ the laser power and $\kappa$ the cavity photon decay rate. Following conventional approach, we expand each operator as $\hat{O} = \hat{O} + \delta \hat{O}$ with $\hat{O}$ the steady-state mean value and $\delta \hat{O}$ small quantum fluctuation around $\hat{O}$. Accordingly, one finds: $\hat{p}_2 = 0$, $\hat{x}_2 = \hbar \chi |a|^2/m_2 \omega_2^2$, and $\bar{a} = \mathcal{E}/(\kappa + i\Delta)$ with $\Delta = \omega_c - \omega_0 - \chi \bar{x}_2$. We now assume that the mean-value of the position of system S2, $\langle r_2(t)\rangle$, takes a steady-state value. This implies that the coordinates of S2 in the Cartesian reference frame that we have chosen are $\langle r_2(t)\rangle \approx (\bar{x}_2, \bar{d}_y, 0)$. Also, as $m_1 \gg m_2$, within the aforementioned adiabatic approximation we have $\langle r_1(t)\rangle_\alpha \approx (d_x, 0, 0)$ and $\langle r_2(t)\rangle_\beta \approx (-d_x, 0, 0)$ [cf. Fig. 1]. We introduce these approximations into equa-

Fig. 2: (a) The gravitational field acting on S2 is a linear combination of gravitational fields produced by S1 being in a superposed state. (b) The semi-classical treatment of gravity, where the gravitational field acting on S2 is the one produced by a total mass $m_1$ with density $\frac{1}{\pi} (|a(x)|^2 + |\beta(x)|^2)$.

Fig. 3: The proposed set-up for the optomechanical falsification of quantum/classical gravity. System S1 is prepared in a superposition of two localised states at $\pm d_x$ along the $x$ axis. An optomechanical cavity acts as transducer and a probe of (potentially quantum) gravity effects S2: the effect of the gravitational coupling between S1 and the mechanical oscillator of an optomechanical cavity induces an effect on the variance of the position fluctuations of the oscillator. The mean position of the latter along the $x$ axis is $\bar{x}_2$. The cavity is pumped by an external field (frequency $\omega_0$ and coupling rate $\mathcal{E}$).
tions of $V_\gamma$ and $V_\delta$ and, by taking $d_y \gg d_x, \bar{x}_2$, we find
\begin{equation}
V_\gamma \approx -\frac{G_m m_2}{d_y} \left( 1 + \frac{\bar{x}_2 + s_\alpha d_x}{d_y^2} \right) \delta x_2 - \frac{1}{d_y} \delta y_2 , \tag{8}
\end{equation}
with $\gamma = \alpha, \beta$, $s_\alpha = -s_\beta = -1$, and
\begin{equation}
V_\delta \approx -\frac{G_m m_2}{d_y} \left( 1 + \frac{\bar{x}_2}{d_y^2} \right) \delta x_2 - \frac{1}{d_y} \delta y_2 . \tag{9}
\end{equation}
The derivative of the potentials above with respect to $\delta x_2$ contributes a term in the quantum Langevin equation of the momentum. Accordingly, the quantum Langevin equations read
\begin{equation}
\frac{d}{dt} \delta x_2(t) = \delta p_2(t)/m_2 , \tag{10}
\end{equation}
\begin{equation}
\frac{d}{dt} \delta a(t) = -(i\Delta + \kappa) \delta a(t) + i\chi \delta a^\dagger(t) + \sqrt{2\kappa} \delta a_{in}(t) , \tag{11}
\end{equation}
where $\nu = \gamma, \mu$. Notice that $\partial V_\gamma / \partial \delta x_2$ is not an operator-valued function. We shall denote $f = -\partial V / \partial \delta x_2$. Solving the above equations in the frequency domain gives us
\begin{equation}
\delta x_2(\omega) = -\frac{1}{D(\omega)} \left[ (\Delta^2 + (\kappa - i\omega)^2) [\xi(\omega) + 2\pi f_\nu \delta(\omega)] + i\hbar \sqrt{2\kappa} \left[ \delta a_{in}(\omega) + \delta a_{in}^\dagger(\omega) \right] \right] , \tag{12}
\end{equation}
where $D(\omega) = m(\Delta^2 + (\kappa - i\omega)^2)/\omega^2 - \omega_x^2 + i\gamma_m \omega + 2\hbar \lambda^2 \Delta$. The correlation functions of the noise operators are
\begin{equation}
\langle \delta a_{in}(\omega) \delta a_{in}^\dagger(\Omega) \rangle = 2\pi \delta(\omega - \Omega) \tag{13}
\end{equation}
\begin{equation}
\langle \xi(\omega) \xi(\Omega) \rangle = 2\pi h \gamma_m m_\omega \left[ 1 + \coth(\mu \omega) \right] \delta(\omega - \Omega) \tag{14}
\end{equation}
where $\mu = \hbar/k_B T$. All other correlators are zero. Therefore, one finds the spectrum of fluctuations in the position of the mechanical oscillator $S_{x_2}$ as
\begin{equation}
S_{x_2}(\omega) = \frac{1}{|D(\omega)|^2} \left[ 2\hbar \chi^2 \kappa |a|^2 \left( \Delta^2 + \kappa^2 + \omega^2 \right) + h m \omega \gamma_m \coth(\mu \omega) \left[ \Delta^2 + (\kappa^2 - \omega^2) + 4\kappa^2 \omega^2 \right] + \frac{2\pi f_\nu}{D(\omega)|D(0)|} \delta(\omega)(\Delta^2 + \kappa^2) \left[ \Delta^2 + (\kappa - i\omega)^2 \right] \right] . \tag{15}
\end{equation}
The first two lines in this expression reproduce the standard density noise spectrum of an optomechanical system. On the other hand, the term proportional to $f_\nu^2$ in Eq. (15) is the result of the gravitational interaction. This contribution can be directly observed in the variance of the position of $S_2$. The variance of the fluctuations in the position of mechanical oscillator $S_2$ is given by
\begin{equation}
\langle (\delta x_2)^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega S_{x_2}(\omega) . \tag{16}
\end{equation}
Introducing Eq. (13) into the above expression yields
\begin{equation}
\langle (\delta x_2)^2 \rangle = \langle (\delta x_2)^2 \rangle_0 + f_\nu^2 \frac{(\Delta^2 + \kappa^2)^2}{D(0)} , \tag{17}
\end{equation}
where $\langle (\delta x_2)^2 \rangle_0$ denotes the variance of the position fluctuations of the mechanical oscillator when there is no gravitational interaction. We also used $f_\nu = -\partial V_\gamma / \partial \delta x_2$ where $V$ can be either $V_\gamma$, or $V_\delta$ [cf. Eqs. (8) and (9)]. Explicitly
\begin{equation}
\frac{\partial V_\gamma}{\partial \delta x_2} \approx -\frac{G_m m_2}{d_y^3} (\bar{x}_2 + s_\alpha d_x) ; \quad \frac{\partial V_\delta}{\partial \delta x_2} \approx -\frac{G_m m_2 \bar{x}_2}{d_y^3} , \tag{18}
\end{equation}
where $\gamma = \alpha, \beta$, and $s_\alpha = -s_\beta = -1$. As one can appreciate from Eq. (15), the gravitational interaction between $S_1$ and $S_2$ manifests as an extra widening in the position distribution of $S_2$. Eqs. (15) and (18) allow to evaluate the difference between a classical and a quantum treatment of gravity. As a figure of merit we can indeed take
\begin{equation}
\Theta = G_m m_2 \frac{(\Delta^2 + \kappa^2)^2}{d_y^3 |D(0)|} , \tag{19}
\end{equation}
which is the difference between the standard deviation in classical and quantum cases, and has the dimension of a length. Our goal now is to achieve the largest possible deviation. Upon inspection, one can see that $D(0)$ is minimized for $\Delta = 0$ and $\omega = \omega_x$. Moreover, by assuming the (experimentally undemanding) sideband-not-resolved limit given by the condition $\kappa \gg \omega_x$, we find the optimal expression
\begin{equation}
\Theta^* = G_m \sqrt{d_x(d_x + 2x_2) d_y^3 \omega_\gamma m} \approx G_m d_x d_y^3 \omega_\gamma m , \tag{20}
\end{equation}
where the last expression is valid by assuming $d_x \gg x_2$. This result shows that a high mechanical quality factor of a low-frequency oscillator would bring $\Theta^*$ to values close to observability, provided that the distance $d_y$ between the centres of mass of $S_1$ and $S_2$ is larger than the linear dimension of the objects along the $y$ axis. An estimate is as follows: we take $E = 6 \times 10^{12} \text{Hz}$, $\gamma_m / 2\pi = 100 \text{Hz}$, $\kappa = 9 \times 10^7 \text{Hz}$, $\omega_x / 2\pi = 3.7 \times 10^{13} \text{Hz}$, $\omega_\gamma / 2\pi = 10^7 \text{Hz}$, $m_1 = 100 \text{ng}$, and a cavity of $1 \text{mm}$ length. Consistently with the formal approach above, we work under the assumptions $d_y \gg d_x, x_2$. A suitable range of values for $d_y$ is from $10^{-6} \text{m}$, which would be suited for micromechanical systems, to $10^{-5} \text{m}$, which would imply the use of a nanomechanical oscillator (possibly embodied by a carbon nanotube or a graphene sheet, such as in Ref. [18]). For $m_2 = 1 \text{ng}$ ($m_2 = 3 \times 10^{-12} \text{g}$), we have $x_2 \approx 7.7 \times 10^{-10} \text{m}$ ($x_2 \approx 2.6 \times 10^{-9} \text{m}$). For $d_y = 10^{-6} \text{m}$ ($d_y = 10^{-8} \text{m}$) and $d_x = 5 \times 10^{-7} \text{m}$ ($d_x = 5 \times 10^{-9} \text{m}$), we find $\Theta^* \approx 1.3 \times 10^{-9} \text{G} \; (\Theta^* \approx 1.3 \times 10^{-5} \text{G})$. The optimality of such value is assessed in Fig. 3 where we
and quantum gravity can be revealed, in principle, in an optomechanical experiment, which showcases all the necessary ingredients to falsify one of the two treatments of gravity.

Conclusions

We have illustrated a method to infer the nature of the gravitational interaction between two massive objects, in principle capable of discerning between a quantum and classical approach to gravity. Our approach is based on the fundamental differences occurring in light of the possibility to prepare quantum coherent states of a system, within the quantum mechanical framework, which in turn gets manifested in the possibility to achieve coherent superpositions of distinguishable gravitational fields. Such a crucial difference between classical and quantum gravity can be revealed, in principle, in an optomechanical experiment, which showcases all the necessary ingredients to falsify one of the two treatments of gravity.

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