Abstract

This note analyzes the asymptotic performance of two popular affirmative action policies, majority quota and minority reserve, under the top trading cycles mechanism (TTCM). These two affirmative actions will induce different matching outcomes with non-negligible probability under the TTCM even if the number of reserved seats for minorities grows relatively slow in large markets.

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1 Introduction

Together with the public school choice reform, affirmative action policies were implemented in many school districts which offer students from socioeconomically disadvantaged groups (i.e., minority students) preferential treatments to mitigate the ethnic and social-economic desegregation in schools. The quota-based affirmative action (majority quota, henceforth) and the reserve-based affirmative action (minority reserve, henceforth) are two common affirmative action policy designs. [1] formalize the majority quota policy which limits the number of admissible students from socioeconomically advantaged groups (i.e., majority students) and leaves the difference to minority students. The minority reserve policy proposed by [3], on the other hand, gives higher priority to minority students up to the point that all reserved seats have been assigned to minorities. [3] imply that unlike the student optimal stable mechanism (SOSM, henceforth), the majority quota and its corresponding minority reserve do not present a clear Pareto dominance relationship for minorities under the top trading cycles mechanism (TTCM, henceforth).

This note further compares these two affirmative action policies in a sequence of random markets of different sizes. Our Theorem 1 reveals that the two affirmative actions produce different matching outcomes under the TTCM with non-negligible probability, even if the number of reserved seats grows at a slower rate of $O(n^a)$ in a sequence of random markets, where $0 \leq a < 1/2$ and $n$ is the number of schools in a random market (see Condition (3) of Definition 1). This result differs from the asymptotic outcome equivalence of the majority
quota and its corresponding minority reserve under the SOSM in [10]. The distinct asymptotic performance of the TTCM and the SOSM essentially comes from the priority trade nature of the TTCM, as blocking possible priority trades under the TTCM with affirmative actions requires that it is unlikely for any students list a school with affirmative actions in random markets with arbitrary sizes, which cannot be satisfied even if the share of the schools with affirmative actions vanishes with market size. By contrast, the convergence process under the SOSM, as illustrated by [10], only demands that no two particular students (either majority or minority) will list the same school implemented with affirmative actions. An important policy implication of our result is that the SOSM is more cost-effective compared to the TTCM in school choice with affirmative actions, as it is unnecessary to identify the different welfare effects of these two affirmative actions under the SOSM if the policymaker can assure a sufficient supply of popular schools to the matching markets.

The literature on large matching markets has been growing rapidly in recent years. Most studies nevertheless indicate that many existing impossibility results, ranging from incentives to existence and efficiency in finite matching markets, disappear if we admit the approximate version of these properties in large market environments. Two exceptions include [4] and [2]: [4] show that all stable mechanisms approximately respect improvements of school quality, while neither the Boston nor the TTC mechanism satisfies this approximate property; [2] suggest that the inefficiency of the SOSM and instability of the TTCM remain significant even when the market grows large. Given the fading of many impossibility results in large markets, some researchers have criticized using approximation properties in market design problems in the sense that the large market analytic framework may be too permissive to make market design irrelevant (i.e., all problems go away; see [7] for more detailed discussions.) Our result thus also supports the validity of large market analytic approach, as it still enables us to capture the subtle difference between the mechanisms that can approximately satisfy some desirable properties from those that cannot.

2 Model

2.1 School Choice

Let $S$ and $C$ be two finite and disjoint sets of students and schools, $|S| \geq 2$. There are two types of students, majority and minority. $S$ is partitioned into two subsets of students based on their types. Denote $S^M$ the set of majority students, and $S^m$ the set of minority students, $S = S^M \cup S^m$ and $S^M \cap S^m = \emptyset$. Each student $s \in S$ has a strict preference order $P_s$ over the set of schools and being unmatched (denoted by $s$). All students prefer to be matched with some school instead of herself, $c P_s s$, for all $s \in S$. Each school $c \in C$ has a total capacity of $q_c$ seats, $q_c \geq 1$, and a strict priority order $>$ over the set of students which is complete, transitive, and antisymmetric. Student $s$ is unacceptable by a school if $e > c s$, where $e$ represents an empty seat in school $c$.

For each school $c$ implemented with majority quota affirmative action policy, it cannot admit more majority students than its type-specific majority quota $q_c^M \leq q_c$, $q^M = (q^M_c)_{c \in C}$, for all $c \in C$. Accordingly, the minority reserve policy gives priority to the minority applicants of school $c$ up to its minority reserve $r^m_c \leq q_c$, $r^m = (r^m_c)_{c \in C}$, and allows $c$ to accept majority students up to its capacity $q_c$ if there are not enough minority applicants to fill the reserves.

A school choice market with affirmative actions is a tuple $\Gamma = (S, C, P, >, (q^M, r^m))$ when we compare the effects of a majority quota policy with its corresponding minority reserve policy in market $\Gamma$, where $P = (P_i)_{i \in S}$, $> = (>)_{c \in C}$, $q = (q_c)_{c \in C}$, and $r^m_c + q^M_c = q_c$, for all $c \in C$. 

2
A matching $\mu$ is a mapping from $S \cup C$ to the subsets of $S \cup C$ such that, for all $s \in S$ and $c \in C$: (i) $\mu(s) \in C \cup \{s\}$; (ii) $\mu(s) = c$ if and only if $s \in \mu(c)$; (iii) $\mu(c) \subseteq S$ and $|\mu(c)| \leq q_c$; and (iv) $|\mu(c) \cap S^M| \leq q^M_c$. That is, a matching specifies the school where each student is assigned or matched with herself, and the set of students assigned to each school; no school admits more students than its capacity, and no school admits more majority students than its majority quota. A mechanism $f$ is a function that produces a matching $f(\Gamma)$ for each market $\Gamma$.

### 2.2 Top Trading Cycles Mechanism

For each market $(S, C, P, (q^M, r^m))$, the top trading cycles mechanism with affirmative actions algorithm runs as follows:

**Step 1**: Start with a matching in which no student is matched. For each school $c$, set its capacity counter at $q_c$. If $c$ is implemented with majority quota policy, set its quota counter at its majority quota $q^M_c$, if $c$ is implemented with minority reserve policy, set its reserve counter at its minority reserve $r^m_c$. If the reserve counter of school $c$ is positive, then it points to its most preferred minority student; otherwise it points to its most preferred student. Each student $s$ points to her most preferred acceptable school that still has a seat for her, and otherwise points to herself; that is, an acceptable school $c$ whose capacity counter is strictly positive and, if $s \in S^M$, its quota counter is strictly positive. There exists at least one cycle (if a student points to herself, it is regarded as a cycle). Every student in a cycle is assigned a seat at the school she points to (if she points to herself, then she gets her outside option) and is removed. The capacity counter of each school in a cycle is reduced by one and, if: (i) the assigned student $s$ is a majority student and the school matched to $s$ is implemented with majority quota policy, then reduces the quota counter of the matched school by one; (ii) the assigned student $s$ is a minority student and the school matched to $s$ is implemented with minority reserve policy, then reduces the reserve counter of the matched school by one. If no student remains, terminate. Otherwise, proceed to the next step.

**Step $k$**: Start with the matching and counter profile reached at the end of Step $k - 1$. For each remaining school $c$, if its reserve counter is positive, then $c$ points to its most preferred minority student among all remaining minority students; otherwise it points to its most preferred student among all remaining students. Each remaining student $s$ points to her most preferred acceptable school that still has a seat for her, and otherwise points to herself; that is, an acceptable school $c$ whose capacity counter is strictly positive and, if $s \in S^M$, its quota counter is strictly positive. There exists at least one cycle (if a student points to herself, it is regarded as a cycle). Every student in a cycle is assigned a seat at the school she points to (if she points to herself, then she gets her outside option) and is removed. The capacity counter of each school in a cycle is reduced by one and, if: (i) the assigned student $s$ is a majority student and the school matched to $s$ is implemented with majority quota policy, then reduces the quota counter of the matched school by one; (ii) the assigned student $s$ is a minority student and the school matched to $s$ is implemented with minority reserve policy, then reduces the reserve counter of the matched school by one. If no student remains, terminate. Otherwise, proceed to the next step.

The algorithm terminates in a finite number of steps since there is at least one student matched and removed in any step of the algorithm. For a market $(S, C, P, (q^M, r^m))$, if $r^m_c = 0, \forall c \in C$, i.e., a market with only majority quota, then the above algorithm reduces to the top trading cycles mechanism with majority quota (TTCM-Q henceforth) proposed by [1]
and [6]; accordingly, if \( q_c^M = q_c, \forall c \in C \), i.e., a market with only minority reserve, then the above algorithm reduces to the top trading cycles mechanism with minority reserve (TTCM-R henceforth) proposed by [3]. When comparing the effects of a majority quota policy with its minority reserve counterpart in a market \( \Gamma \), we assume \( \Gamma \) is either with only majority quota or with only minority reserve, such that \( r_c^m + q_c^M = q_c, \forall c \in C \).

2.3 Large Markets

We follow the large markets framework of [10], which generalizes the analytic framework studied by [5, 8, 9, 4] with concerns on affirmative actions. A random market is a tuple \( \tilde{\Gamma} = ((S^M, S^m), C, >, (q^M, r^m), k, (\mathcal{A}, \mathcal{B})) \), where \( k \) is a positive integer, \( \mathcal{A} = (a_c)_{c \in C} \) and \( \mathcal{B} = (\beta_c)_{c \in C} \) are the respective probability distributions on \( C \), with \( a_c, \beta_c > 0 \) for each \( c \in C \). We assume that \( \mathcal{A} \) for majorities to be different from \( \mathcal{B} \) for minorities to reflect their distinct favors for schools. Each random market induces a market by randomly generated preference orders of each student \( s \) according to the following procedure introduced by [5]:

**Step 1:** Select a school independently from the distribution \( \mathcal{A} \) (resp. \( \mathcal{B} \)). List this school as the top ranked school of a majority student \( s \in S^M \) (resp. minority student \( s \in S^m \)).

**Step 1 ≤ k:** Select a school independently from \( \mathcal{A} \) (resp. \( \mathcal{B} \)) which has not been drawn from steps 1 to step \( l - 1 \). List this school as the \( l^{th} \) most preferred school of a majority student \( s \in S^M \) (resp. minority student \( s \in S^m \)).

Each majority (resp. minority) student iteratively chooses \( k \) schools from \( \mathcal{A} \) and \( \mathcal{B} \) without replacement. Each student \( s \) only lists these \( k \) schools in her preference order.

A sequence of random markets is denoted by \( (\tilde{\Gamma}^1, \tilde{\Gamma}^2, \ldots) \), where \( \tilde{\Gamma}^n = ((S^{M,n}, S^{m,n}), C^n, >_n, (q^{M,n}, r^{m,n}), k^n, (\mathcal{A}^n, \mathcal{B}^n)) \) is a random market of size \( n \), with \( |C^n| = n \) as the number of schools and \( |r^{m,n}| \) the number of seats reserved for minorities. We introduce the following regularity conditions as in [10] for the sake of comparing the asymptotic performance of the TTCM with the SOSM in random markets with affirmative actions.

**Definition 1.** Consider majority quotas \( q^M \) and minority reserves \( r^m \) such that \( r^m + q^M = q \). A sequence of random markets \( (\tilde{\Gamma}^1, \tilde{\Gamma}^2, \ldots) \) is regular, if these exist \( a \in [0, \frac{1}{2}) \), \( \lambda, \theta > 0 \), \( r \geq 1 \), and positive integers \( k \) and \( \bar{q} \), such that for all \( n \):

1. \( k^n \leq k \);
2. \( q_c \leq \bar{q} \) for all \( c \in C^n \);
3. \( |S^n| \leq \lambda n, |r^{m,n}| \leq \theta n^a \);
4. \( \frac{a_c}{a'_c}, \frac{\beta_c}{\beta'_c} \in [\frac{1}{r}, r] \), for all \( c, c' \in C^n \);
5. \( \alpha_c = 0 \), for all \( c \in C^n \) with \( q_c^M = 0 \).

Condition (1) and (2) assume that the length of students’ preferences and the capacity of each school are bounded across schools and markets. Condition (3) requires that the number of students does not grow much faster than the number of schools; in particular, the number of seats reserved for minority students grows at a slower rate of \( O(n^a) \), where \( a \in [0, \frac{1}{2}) \). Note that we do not distinguish the growth rate between majority and minority students, as minority students are generically treated as the intended beneficial student.
groups from affirmative action policies rather than race or other single social-economic status; therefore, the number of minority students is not necessarily less than majorities. Also, we do not require that the sequence of random markets has an excess supply of school capacities for all \( n \), i.e., \( \sum_{c \in C} q_c < |S^n| \) for some \( n \) is allowed. Condition (4) requires that the popularity of different schools, as measured by the probability of being selected by students from \( \mathcal{A} \) for majorities and \( \mathcal{B} \) for minorities, does not vary too much. Condition (5) requires that a majority student will not select a school that can only accept minority students after implementing the quota \( q^M_c = 0 \), as these two affirmative actions trivially induce disparate matching outcomes in any arbitrarily large markets when a majority student applies to a school with zero majority quota.

We formally define the asymptotic outcome equivalence condition of these two affirmative actions in a sequence of random markets of different sizes as follows.

**Definition 2.** For any random market \( \tilde{\Gamma} \), let \( \eta_c(\tilde{\Gamma}; f, f') \) be the probability that school \( c \in C^n \) matched with different sets of students which induces \( f(\tilde{\Gamma}) \neq f'(\tilde{\Gamma}) \). We say two mechanisms are outcome equivalence in large markets, if for any sequence of random markets \( (\tilde{\Gamma}^1, \tilde{\Gamma}^2, \ldots) \) that is regular, \( \max_{c \in C^n} \eta_c(\tilde{\Gamma}^n; f, f') \to 0 \), as \( n \to \infty \); that is, for any \( \varepsilon > 0 \), there exists an integer \( m \) such that for any random market \( \tilde{\Gamma}^n \) in the sequence with \( n > m \) and any \( c \in C^n \), we have \( \max_{c \in C^n} \eta_c(\tilde{\Gamma}^n; f, f') < \varepsilon \).

### 3 Results

We are now ready to present our main argument on the asymptotic performance of the TTCM with affirmative actions, which implies that the majority quota and its corresponding minority reserve will induce different matching outcomes with non-negligible probability under the TTCM, even in arbitrarily large markets with sufficiently many schools and a relatively slow growth of reserved seats.

**Theorem 1.** The TTCM-Q and its corresponding TTCM-R are not outcome equivalence in large markets.

**Proof.** Consider a sequence of random markets \( (\tilde{\Gamma}^1, \tilde{\Gamma}^2, \ldots) \), where there are \( n \) schools and \( \lambda n \) students, \( \lambda \geq 1 \), in each random market \( \tilde{\Gamma}^n \). Assume that the preferences of all students are generated according to the preference generation procedure defined in Section 2.3, with uniform distribution over all schools and preference length \( k = 1 \). Also, assume that school priorities are drawn identically and independently from the uniform distribution over students such that all students are acceptable. For each random market \( \tilde{\Gamma}^n \), denote \( t_n \in (0, 1) \) the portion of majority students, while \( 1 - t_n \) the corresponding portion of minority students. Also, assume that \( q_c = 1 \) or 2 for every school \( c \) in \( \tilde{\Gamma}^n \), denote \( \delta_n \in (0, 1) \) the portion of schools with one seat, while \( 1 - \delta_n \) the corresponding portion of schools with two seats. The preceding assumptions guarantee that the regularity conditions of Definition 1 are satisfied.

Let \( p_n \) be the probability that the two affirmative actions produce different outcomes under the TTCM in market \( \tilde{\Gamma}^n \). We construct the proof by showing that the probability that \( p_n \) is strictly bounded away from zero in a sequence of random markets of different sizes \( (\tilde{\Gamma}^1, \tilde{\Gamma}^2, \ldots) \).

\( p_1 > 0 \) is trivially satisfied when \( \tilde{\Gamma}^1 \) contains one majority student \( M \) and one minority student \( m \), while the exact school \( c \) has one seat, \( \delta_1 \in (0, 1) \), and \( M \succ c m \). For \( n \geq 2 \). Let \( c \) be an arbitrary school, \( q_c = 2 \), implemented with either a majority quota \( q^M_c = 1 \) or its corresponding minority reserve policy \( r_c^m = 1 \). Let Event 1 be the event that there are exactly
one student, who is a majority student denoted by $M$, ranks $c$ first. The probability of Event 1 is

$$\left(\frac{nt_n}{1}\right) \times \frac{1}{n} \times \left(1 - \frac{1}{n}\right)^{n-1},$$

where $t_n \in (0, 1)$ for any arbitrarily large $n$. We can derive its limit when $n$ approaches $\infty$ as

$$\lim_{n \to \infty} t_n \times \left(1 - \frac{1}{n}\right)^{n-1} = \lim_{n \to \infty} t_n \times \left(1 - \frac{1}{n}\right)^{n} \times \left(1 - \frac{1}{n}\right)^{-1} = t_n \times \frac{1}{e} \times 1 = \frac{t_n}{e}.$$

Therefore, for any sufficiently large $n$, the probability of Event 1 is at least, say, $\frac{t_n}{2e} > 0$. Given Event 1, consider Event 2 such that except school $c$, there is exactly one school $c'$, $q_{c'} = 1$, lists $M$ over all the rest students in its priority order. The conditional probability of Event 2 is given by

$$\left(\frac{n \delta_n}{1}\right) \times \frac{1}{n} \times \left(1 - \frac{1}{n}\right)^{n-2},$$

where $\delta_n \in (0, 1)$ for any arbitrarily large $n$. The above expression converges to $\frac{\delta_n}{e}$, as $n$ approaches $\infty$. Thus, for any sufficiently large $n$, the conditional probability of Event 2 given Event 1 is at least, say, $\frac{\delta_n}{2e} > 0$. Given Event 1 and 2, consider Event 3 such that except the majority student $M$, there is exactly one majority student $M'$ and one minority student $m$ rank $c'$ first. The conditional probability of Event 3 is

$$\left(\frac{(n-1) t_n}{1}\right) \times \left(\frac{n(1-t_n)}{1}\right) \times \frac{1}{n^2} \times \left(1 - \frac{1}{n}\right)^{n-3}.$$

The limit of this expression when $n$ approaches $\infty$ is $\frac{t_n(1-t_n)}{e}$. For any sufficiently large $n$, the conditional probability of Event 3 given Event 1 and 2 is at least, say, $\frac{t_n(1-t_n)}{2e}$. Given Events 1, 2, and 3, let Event 4 be the event that apart from other students in $\tilde{\Gamma}^n$, $c$ ranks $M$, $M'$, and $m$, as

$$M' >_c m, \quad \text{and} \quad m >_c M.$$

Since Events 1-3 do not impose any restrictions on the rankings of $M'$, $m$, and $M$ in $c$’s priorities, the conditional probability of Event 4 is $\frac{1}{e}$. Given Events 1-4 and the assumption that $k = 1$, the event that school $c'$ is matched with $M'$ or $m$ while being contained in a cycle involving another agent than $c'$, $M'$, and $m$ occurs with conditional probability 1. Thus, the unconditional probability that $c'$ is matched with $M'$ when $c$ is implemented with the majority quota $q_c^M = 1$ while matched with $m$ when $c$ has the corresponding minority reserve $r_c^m = 1$, is at least $\frac{t_n(1-t_n)}{48e^3}. \delta_n > 0$. Therefore, for any sufficiently large $n$, there is an $\tilde{n}$ such that $p_{\tilde{n}} \geq \frac{t_n(1-t_n)}{48e^3}, \delta_n$, for any $n \geq \tilde{n}$. Together with $p_1 > 0$, we can see that $p_n$ is at least $\min \left\{ p_1, p_2, \ldots, p_{\tilde{n}-1}, \frac{t_n(1-t_n)}{48e^3} \delta_n \right\}$, which is bounded away from below by 0. This completes the proof. \hfill \Box

The intuition for the proof is as follows. Different from its majority quota counterpart, the minority reserve policy requires a school to first consider the minority student with highest priority. For a school $c$ implemented with minority reserve policy, we cannot rule out the possibility that a minority student (call it $m$) with lower priority compared to a majority student (call it $M'$) in $c$ (with affirmative actions) matched to her most preferred school $c'$ (without affirmative actions), if $m$ can trade priorities with another majority student (call it...
who has a higher priority over \( m \) and \( M' \) in \( c' \). Such distinct priority trade orders under the majority quota and its minority reserve counterpart remain with non-negligible probability regardless of the market size.

Theorem 1 differs from the asymptotic outcome equivalence of the majority quota and its corresponding minority reserve under the SOSM illustrated by [10]. This is because the SOSM does not permit students to trade their priorities throughout the matching process, the convergence process under the SOSM thus only demands that no two particular students (either majority or minority) will list the same school implemented with affirmative actions. By contrast, in order to block the possible priority trades under the TTCM, the asymptotic convergence of the TTCM-Q and its corresponding TTCM-R requires that it is very unlikely for any majority students list a school with affirmative actions, which cannot be satisfied in large markets even under our relatively strong regularity conditions of Definition 1.

Remark 1. We can further state that the asymptotic convergence of the TTCM-Q and its corresponding TTCM-R essentially requires that it is very unlikely for any students list a school with affirmative actions. To see this, we first preserve assumptions in the preceding proof and restate Event 1 as there are exactly one student, who is a minority student denoted by \( m' \), ranks \( c \) first. The probability of this alternative of Event 1 is

\[
\left( \frac{n(1-t_n)}{1} \right) \times \frac{1}{n} \times \left( \frac{1-\frac{1}{n}}{n} \right)^{n-1},
\]

which converges to \( \frac{1-t_n}{e} \) as \( n \) approaches \( \infty \). Keep Event 2 and 3 as in the proof with their corresponding conditional probabilities, we restate Event 4 as the event that apart from other students, \( c \) ranks \( m' \), \( M' \), and \( m \), as

\[ M' \succ_c m, \quad \text{and} \quad m \succ_c m'. \]

which has the same conditional probability \( \frac{1}{6} \) as the original Event 4. Thus, when a minority student \( m' \)—instead of a majority student \( M \) as in the preceding proof—lists a school with affirmative actions, the unconditional probability that the TTCM-Q and its corresponding TTCM-R generate disparate matching outcomes remains strictly positive for any arbitrarily large \( n \), \( p_n \geq \frac{t_n(1-t_n)^2\delta_n}{48e^{3}} \). Also, given our restrictive regularity conditions, the impossibility theorem is robust to other alternative approximations of large school choice markets; e.g., correlations in preferences, preference distributions with geographic heterogeneity, unbounded (but sufficiently slow growth of) preference length, as discussed in [9] and [4].

4 Concluding Remarks

This note analyzes the asymptotic performance of two commonly used affirmative action policies, majority quota and minority reserve, under the TTCM. Our result implies that these two affirmative actions induce different matching outcomes under the TTCM in large markets even with a relatively slow growth of reserved seats for minorities. Compared with the asymptotic equivalence of the SOSM with affirmative actions in [10], an important policy implication of our result is that the SOSM is more cost-effective against the TTCM if the policymaker considers a transition from one affirmative action policy to the other, as it is unnecessary to compare the welfare effects of these two affirmative actions under the SOSM when the matching market contains sufficiently many schools. Last, since we can treat the affirmative actions as a generic type-specific (capacity) constraint, questions like how to identify
the asymptotic equivalence conditions of different type-specific constraints in other large market design problems may deserve further research attention.

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