The neutrino pair annihilation
\((\nu\nu \rightarrow e^- e^+)\) around a massive source with an
\(f(R)\) global monopole

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Abstract

In this work we investigate the neutrino pair annihilation around a gravitational object involving an \(f(R)\) global monopole. We derive and calculate the ratio \(\frac{\dot{Q}}{Q_{\text{Newton}}}\) meaning that the energy deposition per unit time is over that in the Newtonian case. It is found that the more deviation from general relativity leads more energy to set free from the annihilation with greater ratio value. It should also be pointed out that the existence of global monopole makes a sharp increase in the ratio \(\frac{\dot{Q}}{Q_{\text{Newton}}}\), causing heavier gamma-ray burst. We also discuss the derivative \(\frac{d\dot{Q}}{dr}\) as a function of radius \(r\) of star to show the similar characters that the considerable modification of Einstein’s gravity and the global monopole with unified theory order will raise the amount of \(\frac{d\dot{Q}}{dr}\) greatly. The stellar body with \(f(R)\) global monopole can be well qualified as a source of gamma-ray bursts. Moreover, we can select the factor \(\psi_0\) to be comparable with the accelerating universe while regulate the parameter \(\eta\) for the global monopole in order to make the ratio curves to coincide with the results from astronomy. It is possible to probe the monopole from astrophysical observations.

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I. Introduction

The temperature decreases during the evolution of universe [1, 2]. Various topological defects such as domain walls, cosmic strings and monopoles may have been formed in the process of the vacuum phase transition in the early stage of the universe [1, 2]. These topological defects generated due to a breakdown of local or global gauge symmetries. A global monopole is a spherically symmetric topological defect arose in the phase transition of a system composed of a self-coupling triplet of a scalar field whose original global \( O(3) \) symmetry is spontaneously broken to \( U(1) \) [3]. There should exist a kind of massive sources involving global monopoles and the metric of the source has a solid deficit angle [4]. The spacetime metric with \( f(R) \) modification is subject to the fact of the accelerated expansion of the universe. The theory of \( f(R) \) gravity first proposed by Buchdahl [5] has been applied to explain the accelerated-inflation problem instead of adding dark energy or dark matter [6-8]. The \( f(R) \) theory as a generalization of the general relativity can be used to describe the gravitational field of the massive object with a global monopole [9]. It was discovered that the presence of the parameter associated with the modification of gravity is indispensable in providing stable circular orbits for particles [10]. The metric of the gravitational object containing a global monopole within the frame of \( f(R) \) gravity has its own property [9-11]. The factor from monopole for a typical grand unified theory changes its event horizon [9, 10]. The non-vanishing modified parameter \( \psi_0 \) brings a cosmological horizon as a boundary of the universe to the spacetime outside the source [9-11]. Certainly this kind of metrics are composed of terms subject to the \( f(R) \) model and global monopoles [9-11]. Recently more investigations have been paid for the black holes shown with this type of metric. We studied the gravitational lensing of the Schwarzschild-like source swallowing the \( f(R) \) global monopole in the strong field limit [12, 13]. We also calculated the thermodynamic quantities of this kind of the black hole to examine the black hole’s stability [14]. We derive the greybody factor for scalar fields in the Schwarzschild spacetime with \( f(R) \) global monopole [15]. The timelike naked singularities of the black hole were discussed [16]. The Hawking radiations of the \( f(R) \) global monopole corrected black hole was considered based on the Heisenberg uncertainty principle or generalized uncertainty principle respectively [17, 18]. Further, the absorption and scattering of an \( f(R) \)-global-monopole black hole have been investigated [19]. It was found that the generalization of Heisenberg’s uncertainty principle leads the fragmentation of the \( f(R) \)-corrected black hole involving global monopole [20]. We generalize the consideration on the Hawking radiations and fragmentation of the black hole to the case under the extended uncertainty principle to show that the generalized uncertainty encourages the black hole instability refer to the Parikh-Kraus-Wilczek tunneling radiation and division [21]. The observational appearances illuminated by three simple models of accretions of an \( f(R) \) global monopole black hole were studied [22].

A lot of attentions from the astrophysical community have been paid to an understanding of an energy source great enough to explain the gamma-ray burst phenomenon. The hot accretion disk emits neutrinos and antineutrinos [23-36]. Further the neutrino-antineutrino annihilation into
electrons and positrons can become an energy source of gamma-ray bursts, so the gamma-ray bursts may be thought as systems powered by newborn, stellar-mass black holes accreting matter at hyper-critical rates [23-36]. The processes $\nu + \bar{\nu} \rightarrow e^- + e^+$ augment the neutrino heating of the envelope leading a supernova explosion, further the pair $e^- e^+$ near the surface of collapsing neutron star sets the gamma ray that may provide a possible explanation of the observed bursts [37]. It was found that the efficiency of neutrino-antineutrino annihilation into electron-positron pair is enhanced over the Newtonian values up to a factor of more than 4 in the case of Type II supernovae and by up to a factor of 30 near the surface of collapsing neutron stars [37]. The relativistic effects on the energy deposition rate according to the neutrino pair annihilation near the rotation axis of a Kerr black hole with a thin accretion disk was discussed [38, 39]. As the extension of works of Ref. [38, 39], the off-axis contributions to the energy-momentum deposition rate from the $\nu - \bar{\nu}$ pair collisions above a Kerr black hole within thin accretion disk were probed [40]. The results of Ref. [40] indicated that the off-axis energy deposition rate is larger by a factor of 10-20 than the values in the on-axis cases. The authors of Ref. [41] researched on the deposition of energy and momentum by neutrino-antineutrino ($\nu \bar{\nu}$) annihilation in the vicinity of an accretion disk or torus around a central stellar-mass black hole to exhibit that the general relativity effects and the rotation of Kerr spacetime both increase the energy deposition rate by $\nu \bar{\nu}$-annihilation.

The neutrino-antineutrino annihilation into electron-positron pairs near the surface of a neutron star within the frame of modified gravity theories has been analyzed [42]. It was shown that the energy deposition processes will increase significantly in the neutron star and supernova envelope for charged Galileon, Einstein dilation Gauss-Bonnet, Brans-Dicke, Eddington-inspired Born-Infeld, Born-Infeld generalization of Reissner-Nordstrom solution and higher derivative gravity, as various models of gravity beyond the general relativity [42]. The authors of Ref. [43] studied the influence of the presence of the quintessence field around a gravitational object belonging to the black hole on the neutrino pair annihilation efficiency like $\nu + \bar{\nu} \rightarrow e^- + e^=$ to show that the quintessence powers the emitted energy rate ratio, so the enhancement could be a source for the gamma-ray burst. Here we are going to consider the neutrino pairs annihilation into electron-positron pair ($\nu \bar{\nu} \rightarrow e^- e^+$) near the surface of neutron star or supernova including a global monopole governed by $f(R)$ theory.

In this paper, we plan to discuss the energy deposition rate by the neutrino annihilation process around the massive source containing a global monopole in the context of $f(R)$ gravity. We derive the integral form of the neutrino pair annihilation efficiency based on the metric considered here. Secondly, we calculate the ratio of total energy deposition to total Newtonian energy deposition for factors $8\pi G\eta^2$ and $\psi_0$ from global monopole and the correction to the general relativity respectively. Our numerical estimation will reveal the influences from global monopole and $f(R)$ approach on the possibility that the astrophysical bodies attract the annihilation process generating the gamma-ray burst. The results are listed in the end.
II. The energy deposition rate by the neutrino annihilation process in the spacetime of massive source with an $f(R)$ global monopole

We adopt the spherically symmetric line element as follow,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (1)$$

In the $f(R)$ gravity theory, the action is given by [9-11],

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S_m \quad (2)$$

where $f(R)$ is an analytical function of Ricci scalar $R$, $\kappa = 8\pi G$ with $G$ Newton constant, and $g$ the determinant of the metric tensor (1). Here $S_m$ is the action associated with the matter fields as global monopole [5],

$$S_m = \int d^4x \sqrt{-g} \left[ \frac{1}{2}(\partial_\mu \phi^a)(\partial^\mu \phi^a) - \frac{1}{4}\lambda(\phi^a \phi^a - \eta^2)^2 \right] \quad (3)$$

where the triplet of the field configuration showing a monopole is $[2, 3]$,

$$\phi^a = \eta h(r)^x^a_r \quad (4)$$

with $x^a x^a = r^2$. Here, $\lambda$ and $\eta$ are model parameters. This model has a global $O(3)$ symmetry, which is spontaneously broken to $U(1)$ [2, 3].

With help of the Einstein Equation, the field equation reads [5, 44-46],

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu}F(R) = \kappa T_{\mu\nu} \quad (5)$$

where $F(R) = \frac{df(R)}{dR}$, and $T_{\mu\nu}$ is the minimally coupled energy-momentum tensor from action (3). Under the weak field approximation, the components of metric tensor can be chosen as $A(r) = 1 + a(r)$ and $B(r) = 1 + b(r)$ with $|a(r)|$ and $|b(r)|$ being smaller than unity [9-11]. The field equation is solved and the metric is found [9-11],

$$A(r) = B^{-1}(r) = 1 - 8\pi G\eta^2 - \frac{2GM}{r} - \psi_0 r \quad (6)$$

Here, the generalized theory of gravity corresponds to a tiny correction to the general relativity like $F(R(r)) = 1 + \psi(r)$ with $\psi(r) << 1$ [9-11]. The deviation of standard general relativity can be taken as the simplest analytical function of the radial coordinate $\psi(r) = \psi_0 r$ [9-11]. It should be pointed that the correction $\psi_0 r$ in the metric (6) is linear, which is different from those in cases such as de Sitter spacetime and the Reissner-Nordstrom metric, etc.. In addition, the monopole
parameter $\eta$ is of the order $10^{16} GeV$ according to the typical grand unified theory, which means $8\pi G\eta^2 \approx 10^{-5}$ [3]. For metric (6), as roots of $A(r) = 0$, a cosmological horizon [9-11],

$$r_c = \frac{1}{\psi_0}(1 - 8\pi G\eta^2 + \sqrt{(1 - 8\pi G\eta^2)^2 - 8GM\psi_0})$$  \hspace{1cm} (7)

appears beside an event horizon,

$$r_h = \frac{1}{\psi_0}(1 - 8\pi G\eta^2 - \sqrt{(1 - 8\pi G\eta^2)^2 - 8GM\psi_0})$$  \hspace{1cm} (8)

The presence of a nonzero $\psi_0$ bring a cosmological horizon as a boundary of the universe to the spacetime described by the $f(R)$ monopole metric, but the spacetime without gravity modification is asymptotically flat.

We plan to treat the energy deposition in the spacetime with the descriptions of (1) and (6). The energy deposition per unit time and per volume for the neutrino annihilation process is given by [37, 47],

$$\frac{dE(r)}{dt dV} = 2K G_F^2 F(r) \int \int n(\varepsilon_{\nu})n(\varepsilon_{\bar{\nu}})(\varepsilon_{\nu} + \varepsilon_{\bar{\nu}})\varepsilon_{\nu}^2\varepsilon_{\bar{\nu}}^2 d\varepsilon_{\nu} d\varepsilon_{\bar{\nu}}$$  \hspace{1cm} (9)

where

$$K = \frac{1}{6\pi}(1 \pm 4 \sin^2 \theta_w + 8 \sin^4 \theta_w)$$  \hspace{1cm} (10)

with the Weinberg angle $\sin^2 \theta_w = 0.23$ and,

$$K(\nu_{\mu}, \bar{\nu}_{\mu}) = K(\nu_{\tau}, \bar{\nu}_{\tau}) = \frac{1}{6\pi}(1 - 4 \sin^2 \theta_w + 8 \sin^4 \theta_w)$$  \hspace{1cm} (11)

$$K(\nu_e, \bar{\nu}_e) = \frac{1}{6\pi}(1 + 4 \sin^2 \theta_w + 8 \sin^4 \theta_w)$$  \hspace{1cm} (12)

and the Fermi constant $G_F = 5.29 \times 10^{-44} cm^2 MeV^{-2}$. The angular integration factor is represented by [37],

$$F(r) = \int \int (1 - \Omega_{\nu}\Omega_{\bar{\nu}})^2 d\Omega_{\nu} d\Omega_{\bar{\nu}}$$

$$= \frac{2\pi^2}{3}(1 - x)^4(x^2 + 4x + 5)$$  \hspace{1cm} (13)

where

$$x = \sin \theta_r$$  \hspace{1cm} (14)

The angle $\theta_r$ is between the particle trajectory and the tangent vector to a circular orbit at radius $r$. $\Omega_{\nu}(\Omega_{\bar{\nu}})$ is the unit direction vector and $d\Omega_{\nu}(d\Omega_{\bar{\nu}})$ is a solid angle. Here $n(\varepsilon_{\nu})$ and $n(\varepsilon_{\bar{\nu}})$ are number densities in phase space [37].
\[ n(\nu) = \frac{2}{h^3} \frac{1}{e^{\frac{\nu}{kT}} + 1} \]  

(15)

where \( h \) is Planck constant. The integration of Eq.(9) can be performed and the expression of rate per unit time and unit volume is given by [37],

\[
\frac{dE}{dtdV} = \frac{21\zeta(5)\pi^4}{h^6} KG^2 F(r)(kT)^9
\]  

(16)

The local temperature measured by a local observer is defined as [37, 47],

\[ T(r)\sqrt{g_{00}(r)} = \text{constant} \]  

(17)

The neutrino temperature at the neutrinosphere reads [37],

\[ T(r)\sqrt{g_{00}(r)} = T(R)\sqrt{g_{00}(R)} \]  

(18)

where \( g_{00} \) is a component of spacetime metric. The luminosity relating to the redshift can be selected as [37],

\[ L_\infty = g_{00}(R)L(R) \]  

(19)

where the luminosity for a single neutrino species at the neutrinosphere is [37],

\[
L(R) = L_\nu + L_\bar{\nu} = 4\pi R^2 \frac{7ac}{4} T^4(R)
\]  

(20)

where \( a \) is the radiation constant and \( c \) is the speed of light in vacuum. The Eq.(18), Eq.(19) and Eq.(20) are substituted into the Eq.(16) to obtain [37],

\[
\frac{dE(r)}{dtdV} = \frac{21\zeta(5)\pi^4}{h^6} KG^2 k^9 \left( \frac{7ac}{4} \right)^{-\frac{9}{4}} L_\frac{2}{3} F(r) \frac{\left( g_{00}(R) \right)^{\frac{2}{7}}}{\left( g_{00}(r) \right)^{\frac{2}{7}}} R^{-\frac{9}{2}}
\]  

(21)

In order to calculate the angular integration \( F(r) \), we should further the discussion on the variable \( x \). We follow the procedure of Ref.[37],

\[ \tan \theta = \sqrt{\frac{|g_{11}|}{|g_{33}|}} \frac{dr}{d\varphi} \]  

(22)

The null geodesic in the spacetime of a spherically symmetric gravitational object is given by [44-47],

\[ \left( \frac{1}{r^2} \frac{dr}{d\varphi} \right)^2 + A(r) \frac{A(r)}{r^2} = \frac{1}{b^2} \]  

(23)

where \( b \) is the impact parameter. The expression (22) can be substituted into Eq.(23) and simplified to give [37],
\[ \frac{r^2}{A(r)} \cos^2 \theta = b^2 \]  

(24)

The components \( g_{11} \) and \( g_{33} \) are from metric (1) and (6). According to Eq.(24), it can be rewritten \[42, 48\],

\[ \frac{r^2}{A(r)} \cos^2 \theta_r = \frac{R^2}{A(R)} \cos^2 \theta_R = b^2 \]

\[ \cos^2 \theta_r = \frac{R^2}{r^2} \frac{A(r)}{A(R)} \cos^2 \theta_R \]  

(25)

Combining Eq.(14) and Eq.(25), it can be inserted \[42, 48\],

\[ x^2 = \sin^2 \theta_r |_{\theta_R=0} \]

\[ = 1 - \frac{R^2}{r^2} \frac{A(r)}{A(R)} \]  

(26)

We can proceed the integration of rate per unit time and unit volume shown in Eq.(21) \[43\],

\[ \frac{d}{dt} \frac{dE}{\sqrt{g_{00}}} = 4\pi \int_R^{R_{CH}} r^2 \sqrt{-g_{11}} \frac{dE}{dt} dV \]

\[ = 84\zeta(5) \pi^5 \frac{h^6}{K_G^2 k^9} \left( \frac{7}{4} \pi ac \right)^{-2} L_\infty \left( g_{00}(R) \right)^\frac{9}{2} R^{-\frac{5}{2}} \int_R^{R_{CH}} \frac{r^2 \sqrt{-g_{11} F(r)}}{(g_{00}(r))^{\frac{9}{2}}} dr \]  

(27)

where \( R_{CH} \) is the cosmological horizon. Here \( \dot{Q} \) can reflect the total amount of energy converted from neutrinos to electron-positron pairs at any radius \[37\]. According to the energy deposition rate (27) and the quantities from Ref.[37, 42, 43], it is possible to write \[42\],

\[ \frac{\dot{Q}}{\dot{Q}_{Net}} = 3 \left( g_{00}(R) \right)^\frac{9}{2} \int_1^{R_{CH}} (x - 1)^4 (x^2 + 4x + 5) g_{00}^2 \sqrt{-g_{11}(Ry)} \frac{y^2}{(g_{00}(Ry))^{\frac{9}{2}}} dy \]  

(28)

with dimensionless variable \( y = \frac{r}{R} \).

We can also obtain the function of radius \( r \) like \( \frac{d\dot{Q}}{dr} \) to reflect the enhancement according to the issue of Ref.[37],

\[ \frac{d\dot{Q}}{dr} = 4\pi \frac{dE}{dt} \sqrt{-g_{11}} r^2 \]

\[ = \frac{168\zeta(5) \pi^7}{3h^6} \frac{K_G^2 k^9}{(7/4) \pi ac} \left( \frac{7}{4} \pi ac \right)^{-2} L_\infty \]

\[ \times (x - 1)^4 (x^2 + 4x + 5) \left( \frac{g_{00}(R)}{g_{00}(r)} \right)^\frac{9}{2} R^{-\frac{5}{2}} \sqrt{-g_{11} \left( \frac{r}{R} \right)^2} \]  

(29)
It is significant to discuss the ratio \((28)\) in the background described by metric components \((6)\). Comparing the metric \((1)\) with the component function \((6)\), we can relate that 
\[
g_{00}(r) = \frac{1}{g_{11}(r)} = 1 - 8\pi G\eta^2 - \frac{2GM}{r} - \psi_0r
\]
(30)
We rewrite the variable \((26)\) around the gravitational source with solid deficit angle in the context of \(f(R)\) theory,
\[
x^2 = 1 - \frac{1}{y^2} \frac{1 - 8\pi G\eta^2 - \frac{2GM}{R}}{1 - 8\pi G\eta^2 - \frac{2GM}{R}} - \psi_0R
\]
(31)
We substitute the metric \((30)\) into the Eq.(28) to obtain the energy deposition rate for the neutrino annihilation surrounding the source involving the \(f(R)\) global monopole,
\[
\dot{Q}_{\text{Newt}} = 3(g_{00}(R))\frac{2}{4} \int_1^{\frac{R}{M}} \left(x - 1\right)^4 \left(x^2 + 4x + 5\right) \frac{y^2}{(g_{00}(Ry))^{5/4}} dy
\]
(32)
It should be emphasized that,
\[
g_{00}(R) = 1 - 8\pi G\eta^2 - \frac{2GM}{R} - \psi_0R
\]
(33)
and
\[
g_{00}(Ry) = 1 - 8\pi G\eta^2 - \frac{2GM}{R} - \psi_0Ry
\]
(34)

It is necessary to quantify the integral expression of ratio \(\frac{\dot{Q}}{\dot{Q}_{\text{Newt}}}\) by Eq.(32) and depict the dependence on \(\frac{R}{M}\) in the Figures. We compare the curves under the conditions to find that more considerable deviations from the general relativity give rise to larger ratio \(\frac{\dot{Q}}{\dot{Q}_{\text{Newt}}}\) shown in Figure 1. Meanwhile it is found that the larger values of the parameter like \(\psi_0\) result in much more amount of the ratio, which is similar to the conclusions from Ref.[42, 43]. It is obvious that the existence of global monopoles also augment the quotient of \(\dot{Q}\) and \(\dot{Q}_{\text{Newt}}\) according to the Figure 2. It should be pointed out that the larger values of parameters describing the monopoles cause more ratio. As an interesting example, we plot the emitted energy ratio \(\frac{\dot{Q}}{\dot{Q}_{\text{Newt}}}\) which is relevant for the generation of gamma-ray burst due to the monopole parameter \(8\pi G\eta^2 \approx 10^{-5}\) from typical grand unified theory [3] and the generalization of general relativity \(\psi_0 \approx 10^{-3}\) [9-11, 22] in the Figure 3. It is indicated that the converting energy per unit time for the annihilation is made larger by a factor almost up to 2 at about \(R \approx 10M\). In the case of Type II supernova with nearly \(R \approx 5M\), the factor grows to be more than 5, which is enough to explain the observed GRBs. Within the range down to \(R \approx 3M\) corresponding to collapsing neutron stars, the factor for the promotion becomes to be more than 10. According to Ref. [49], We can also explore this kind of massive source supporting the burst in another direction. At first, the value of \(\psi_0\) can be chosen under the limit of the observing data for accelerating universe. Adjusting the global monopole variable \(\eta\) to enable the curves of \(\frac{\dot{Q}}{\dot{Q}_{\text{Newt}}\)
to coincide the ones from observation, we can estimate $\eta$ by comparing with the order from unified theory [3]. It is clear that the annihilations surrounding the celestial body swallowing the $f(R)$ global monopole will emit much greater energy per unit time, which is the distinct feature of this kind of gamma-ray burst.

For the sake of stressing the promotion of $e^-e^+$ pair energy from the neutrino annihilation, we start to elaborate the $\frac{dQ}{dr}$ as a function of radius for several stellar masses $\frac{M}{R}$. We illustrate the dependence of the derivative on the dimensionless variable $\frac{r}{R}$ because of $8\pi G \eta^2 \approx 10^{-5}$ and $\psi_0 \approx 10^{-3}$ as mentioned above [9-11, 22] in Figure 4. It is similar that the structure with smaller values of $\frac{R}{M}$ result in the greater $\frac{dQ}{dr}$. Certainly the augment is much stronger near the surface of the neutron star like Ref.[37]. It should be emphasized that the annihilation of neutrino-antineutrino pairs owing to the Schwarzschild-like spacetime modified by the presence of global monopole in view of $f(R)$ theory may provide the much more energy than the needed to efficiently originate the gamma-ray bursts. It is not necessary for the burst to occur near the surface of the specific source from Figure 4.

III. Conclusion

Our investigations are performed in this paper refer to the neutrino pair annihilation $\nu\bar{\nu} \rightarrow e^-e^+$ around the gravitational source involving the global monopole in the frame of $f(R)$ approach. We calculate the emitted energy rate ratio $\frac{Q}{Q_{\text{Newt}}}$ in the allowed range of parameters for $f(R)$ gravity and global monopole in the background of a massive source with an $f(R)$ global monopole to find that the $f(R)$ corrections to the general relativity can also stimulate the annihilation process to produce excessive energy per unit time, while exhibit that the existence of global monopole can significantly increase much more energy deposition. The neutrino pair annihilation subject to this kind of gravitational bodies emits much greater energy per unit time to be proposed as a well-qualified candidate of gamma-ray bursts no matter the central objects are Type II supernova or collapsing neutron stars. By comparison of our calculations with the observational results, we can open a new window to estimate the global monopole parameter.

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Figure 1: The solid, dotted and dashed curves of the ratio $\frac{\dot{Q}}{Q_{\text{Newt}}}$ as functions of radius of gravitational sources for $f(R)$ factors $\psi_0 = 0.001, 0.005, 0.01$ respectively with global monopole variable $8\pi G \eta^2 = 10^{-5}$.
Figure 2: The solid, dotted and dashed curves of the ratio $\frac{\dot{Q}}{Q_{\text{Newt}}}$ as functions of radius of gravitational sources for global monopole variable $8\pi G\eta^2 = 0.1, 0.2, 0.3$ respectively with $f(R)$ factors $\psi_0 = 0.001$.
Figure 3: The curve of the ratio $\frac{\dot{Q}}{Q_{\text{Newt}}}$ as functions of radius of gravitational source for $f(R)$ factors $\psi_0 = 0.001$ and global monopole variable $8\pi G\eta^2 = 10^{-5} f(R)$
Figure 4: The dotted, dashed and solid curves of the derivative $\frac{dQ}{dr}$ as function of radius of various gravitational source for $f(R)$ factor $\psi_0 = 0.001$ and global monopole variable $8\pi G\eta^2 = 10^{-5}$ under $\frac{R}{M} = 3, 5, 10$ respectively.