Open P-Branes

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Abstract
It is shown that many of the $p$-branes of type II string theory and $d = 11$ supergravity can have boundaries on other $p$-branes. The rules for when this can and cannot occur are derived from charge conservation. For example it is found that membranes in $d = 11$ supergravity and IIA string theory can have boundaries on fivebranes. The boundary dynamics are governed by the self-dual $d = 6$ string. A collection of $N$ parallel fivebranes contains $\frac{1}{2}N(N-1)$ self-dual strings which become tensionless as the fivebranes approach one another.
Type II string theories contain a variety of BPS-saturated $p$-brane solitons carrying a variety of charges $Q^i$. All of these are extended extremal black holes. This means that they are extremal members of a one-parameter family of $M^i \geq Q^i$ solutions which, for $M^i > Q^i$, have regular event horizons and geodesically complete, nonsingular spacelike slices with a second asymptotic region. Furthermore the $M^i > Q^i$ solutions decay via Hawking emission to the BPS-saturated $M^i = Q^i$ states. Recently there has been spectacular progress, initiated by Polchinski, in describing the dynamics of those $p$-branes which carry RR charge by representing them as D-branes in a type I theory. In this paper we will rederive some of these recent results from low-energy reasoning in a manner that will generalize to all $p$-branes and uncover new phenomena.

Viewing $p$-branes as extended holes in spacetime naturally leads one to consider configurations in which one $p$-brane threads through the hole at the core of the second $p$-brane. For example consider a static configuration consisting of two like-charged, parallel NS-NS (i.e. symmetric) fivebranes in the IIB theory. The metric is given by

$$ ds_{10}^2 = \eta_{\mu\nu}dy^\mu dy^\nu + (1 + \alpha' \|x - x_1\|^2 + \alpha' \|x - x_2\|^2)\delta_{jk}dx^j dx^k, $$  \hspace{1cm} (1)

where $\mu, \nu = 0, ...5$ and $j, k = 6, ...9$. This has two infinite throats located at $x = x_1$ and $x = x_2$. Next consider a RR closed string which comes out one throat and goes in the next:

$$ X^0 = \tau, $$
$$ X^1 = x_1 + (x_2 - x_1)\sigma. $$ \hspace{1cm} (2)

The existence of such a configuration may be obstructed by charge conservation. In particular an $S^7$ which surrounds a RR string has a non-zero integral $Q^{RR} = \int *H^{RR}$ where $H^{RR}$ is the RR 3-form field strength. This would seem to prevent strings from ending, since in that case the $S^7$ may be contracted to a point by slipping it off the end. However in so doing one must first pass it through the fivebrane. Using the explicit construction of

\footnote{At the extremal limit, many of the $p$-brane solutions are singular or strongly coupled at the core, so the spacetime metric is not a reliable guide to the geometry. One is still however led to consider the fate of a $p$-brane which threads a large, smooth non-extremal $p$-brane which subsequently evaporates down to extremality.}
it may be seen that the low-energy effective field theory on the fivebrane worldvolume contains a coupling

$$\int d^6\sigma B_{\mu\nu}^{RR} F^{\mu\nu},$$

(3)

where $B^{RR}$ is the spacetime RR Kalb-Ramond field and $F$ is the worldbrane $U(1)$ gauge field strength. This leads to the equation of motion

$$d \ast H^{RR} = Q^{RR} \delta^8 + \ast F \wedge \delta^4,$$

(4)

where $\delta^4 (\delta^8)$ is a transverse 4-form (8-form) delta function on the fivebrane (RR string) and $\ast F$ denotes the Hodge dual within the worldvolume. The total integral of $d \ast H$ over any $S^8$ must vanish. Consider an $S^8$ which intersects the string at only one point. Such an $S^8$ point must intersect the fivebrane in an $S^4$. Integrating (4) over the $S^8$ we find

$$0 = Q^{RR} + \int_{S^4} \ast F.$$

(5)

We conclude that $H^{RR}$ charge conservation can be maintained if an electric flux associated to the fivebrane $U(1)$ charge emanates from the point at which the string enters the fivebrane. In other words the end of the string looks like a charged particle on the worldbrane.

As most easily seen from the Green-Schwarz form of the string action, the stretched string preserves those supersymmetries generated by spinors $\epsilon$ obeying

$$\Gamma_{MN} \partial_+ X^M \partial_- X^N \epsilon = \epsilon.$$

(6)

(2) and (1) together preserves one quarter of the supersymmetries so this configuration is BPS-saturated to leading order. At next order one must include the back reaction of the string on the spacetime geometry and fields. Since there is no obstruction from charge conservation we presume that a fully supersymmetric configuration describing a RR string stretched between two NS-NS fivebranes exists and corresponds to a BPS state.

There is no coupling of the form (3) involving the NS-NS $B$ field. Charge conservation therefore prohibits fundamental IIB strings from ending at NS-NS fivebranes. However $SL(2, Z)$ interchanges NS-NS and RR onebranes and fivebranes. Hence $SL(2, Z)$ invariance implies that a fundamental IIB string can end at a RR fivebrane. The latter (like all the

2 A correction to the zero mode wave function may be found in [16].
RR solitons) can be realized as a D-brane. So this is not a surprise: we have reproduced results of [4,3].

Next let us consider what happens as the two fivebranes approach one another. The mass of the stretched string is given by a BPS bound and is a function on the two-fivebrane moduli space. It decreases with the string length. When the fivebranes become coincident, the stretched string has zero length and becomes a massless state carrying the $U(1)$ charges of both fivebranes. The result is therefore an $N = 4$ $U(2)$ gauge theory on the fivebrane [4,6]. Note that the dual relation to open string theory is not required for this conclusion. From this perspective the source of massless gauge bosons is similar to that in [17,18,19]: they arise from a degenerating one-cycle which threads two horizons.

A similar story applies to the RR threebrane. Reduction of the formulae in [2,20] leads to spacetime-worldbrane couplings of the form (3) for both the NS-NS and RR $B$ fields. This is required by $SL(2, Z)$ invariance because the threebrane acts as a source for the self-dual 5-form and hence is itself self-dual. In [4] it was shown that fundamental strings can end on D-branes but here we see that D-strings may in some cases also end on D-branes. This dovetails nicely with S-duality of the $N = 4$, $d = 4$ gauge theory which lives on the threebrane: The ends of fundamental strings are electrically charged particles while the ends of D-strings are magnetically charged particles.

There may seem to be a puzzle for example for the RR 0-brane. Clearly charge conservation will prevent (except when there is a RR background [21]) a fundamental string from ending at a 0-brane. This may seem to conflict with the picture in [4] which involves an $SU(N)$ gauge theory for $N$ 0-branes coming from strings ending at the 0-branes. However there is not really a conflict because our reasoning applies only to BPS states, and charge confinement in 0+1 $SU(N)$ gauge theories indeed eliminates the charged BPS states.

So far we have reproduced from a different perspective results previously obtained either directly in [4,3], as well as some $SL(2, Z)$ duals of those results. Our point of view gives the leading low-energy dynamics, but probably cannot easily reproduce the detailed prescription given in [1,8] for computing e.g. finite momentum string-D-brane scattering as in [12]. However in considering higher $p$-branes this low-energy perspective will lead us to new phenomena.

As a further example we consider a membrane stretched between two fivebranes of eleven-dimensional supergravity\(^3\). (Of course reduction of this leads to examples in the

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\(^3\) Preliminary observations on open membranes are made in [22].
IIA theory.) Unlike its IIB partner, the $d = 11$ (and IIA) fivebrane has chiral dynamics governed by the $d = 6$ tensor multiplet containing 5 scalars and a self-dual antisymmetric tensor field strength $\mathcal{A}$. The membrane worldvolume condition for unbroken supersymmetry is

$$\Gamma_{MNP}\epsilon^{\alpha\beta\gamma}\partial_\alpha X^M \partial_\beta X^N \partial_\gamma X^P \epsilon = \epsilon.$$  \hspace{1cm} (7)

Again it is easily seen that a membrane stretched between two fivebranes preserves one quarter of the supersymmetries at leading order. For appropriate brane orientations the unbroken supersymmetries are generated by spinors obeying the two chirality conditions $\Gamma^{016}\epsilon = \Gamma^{012345}\epsilon = \epsilon$. The membrane can be surrounded by an $S^7$ for which there is a nonzero value of the charge $Q^M = \int_{S^7} * F$, where $F$ here is the spacetime 4-form field strength. In the presence of a membrane and a fivebrane the equation of motion for $F$, as follows from formulae in [22,24], is

$$d * F = Q^M \delta^8 + A \wedge \delta^5.$$  \hspace{1cm} (8)

We see that the boundary of the membrane - which is a string lying in the fivebrane - must carry self-dual antisymmetric tensor charge $\int_{S^3} A = -Q^M$. This string is the self-dual string of Duff and Lu [27].

Further insight into this construction can be gained by considering S- and T-duality. Polchinski [4] has shown that the worldbrane dynamics of the IIB RR fivebrane are described by open fundamental Dirichlet strings. $SL(2,\mathbb{Z})$ invariance then implies that worldbrane dynamics of the IIB NS-NS fivebrane are described by open RR strings (although this description is weakly coupled only at large $g_s$). Now periodically identify and T-dualize along one direction of the fivebrane. This gives a IIA theory [4]. The NS-NS (i.e. symmetric) fivebrane solution is represented by a conformal field theory involving only the transverse coordinates, and hence is unaffected by longitudinal T-duality (This is in contrast to RR $p$-branes, which lose (gain) a dimension under longitudinal (transverse) T-duality.). However the zero modes which propagate parallel to the fivebrane are affected, and the $N = 4 U(1)$ vector multiplet is transformed into an $N = 4$ antisymmetric tensor multiplet. At the same time the open strings which govern the IIB fivebrane dynamics are T-transformed into open membranes which govern the IIA fivebrane dynamics.

Next we consider the dynamics of $N$ parallel $d = 11$ or IIA fivebranes. When the fivebranes are separated the low energy dynamics is governed by a globally supersymmetric
(0, 2) $d = 6$ theory with $N$ tensor multiplets. The moduli space of the $5N$ scalars is uniquely
determined to be locally the symmetric space $T(5, N) \equiv SO(5, N)/(SO(5) \times SO(N))$.
Since this is a chiral theory it is not possible for extra massless fields to appear when
the fivebrane positions coincide. However tensionless strings can and do arise, because the
tension of a BPS string which arises as the boundary of a membrane stretched between two
fivebranes vanishes when the fivebrane coincides. These strings transform in the adjoint of
the global $SO(N)$ which acts on the $N$ tensor multiplets. Upon $S^1$ compactification along
the fivebranes, winding states of the tensionless strings lead to the appearance of extra
massless gauge bosons which - together with the reduced tensor multiplets which dualize
to vector multiplets - fill out a $U(N)$ gauge theory [28], as predicted by T-duality.

Aspects of the preceding are quite similar to Witten’s discussion [28] of K3 compact-
ification of IIB string theory, whose moduli space is locally $T(5, 21)$ and which contains
(5) $5+16$ (anti) self-dual antisymmetric tensor fields. In this case tensionless strings arise
from threebranes wrapping degenerating 2-cycles. Indeed there is a dual IIA description of
this IIB compactification, in the spirit of [29-31], as 16 toroidally compactified symmetric
fivebranes and NS-NS orientifolds, where the extra $5+5$ antisymmetric tensor fields arise
from the supergravity multiplet [30]. In [29] it was shown that IIA on K3 is equivalent
to IIB on a D-manifold with 16 RR orientifolds and 16 RR fivebranes. IIB S-duality con-
verts NS-NS to RR fields, so this is S-equivalent to a IIB configuration with 16 NS-NS
orientifolds and 16 NS-NS fivebranes. Next T-dualize this last representation of IIA on
K3 (yielding IIB on K3) along one of the noncompact directions. This will not affect the
4-geometry which involves only NS-NS fields. Hence IIB on K3 is equivalent to IIA on a
“$p$-manifold” with 16 NS-NS orientifolds and 16 symmetric fivebranes. This provides the
concrete connection to [28].

As pointed out in [28] the fact that self-dual strings (or open membranes$^4$ ) become
light as the fivebranes approach one another suggests that supergravity might be decoupled
and the dynamics of self-dual strings and symmetric fivebranes consistently studied in
isolation from the rest of string theory. This is also suggested by superconformal invariance
of the tensor multiplet in $d = 6$ [23]. The relation by compactification to superconformal
d = 4 $N = 4$ Yang-Mills makes this a particularly fascinating problem.

Further examples of $p$-branes with boundaries can be found. It may be directly checked
in the IIB theory that charge conservation allows a threebrane to end on a membrane in

$^4$ The relation in the infrared between the self-dual open membranes and self-dual strings may
involve Chern Simons theory as in [31].
the RR fivebrane. The membrane carries magnetic charge with respect to the fivebrane $U(1)$ gauge field. In general every RR $p$-brane has a $U(1)$ gauge field. Electric charges are always carried by zerobranes and arise from fundamental strings which terminate at the $p$-brane. Magnetic charges are carried by a $(p - 3)$-brane, which can arise as the boundary of a $(p - 2)$-brane. It is difficult to check charge conservation directly for $p > 5$ because the zero mode wave functions have not been worked out. However T-duality along a dimension transverse to an configuration of RR $p$-branes increases $p$, so we presume it is always possible (in IIA or IIB) for a RR $(p - 2)$-brane to end at a RR $p$-brane. All of these new multi-$p$-brane configurations can be used to construct $p$-manifold generalizations of the D-manifolds introduced in [8], and may arise in the process of dualization.

In conclusion string theory contains a rich variety of extended objects which interact in an intricate and beautiful fashion. Higher $p$-branes provide endpoints for branes of lower $p$, which latter in turn govern the dynamics of the former.

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