Quantum field theory of photon–Dirac fermion interacting system in graphene monolayer

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Abstract
The purpose of the present work is to elaborate quantum field theory of interacting systems comprising Dirac fermion fields in a graphene monolayer and the electromagnetic field. Since the Dirac fermions are confined in a two-dimensional plane, the interaction Hamiltonian of this system contains the projection of the electromagnetic field operator onto the plane of a graphene monolayer. Following the quantization procedure in traditional quantum electrodynamics we chose to work in the gauge determined by the weak Lorentz condition imposed on the state vectors of all physical states of the system. The explicit expression of the two-point Green function of the projection onto a graphene monolayer of a free electromagnetic field is derived. This two-point Green function and the expression of the interaction Hamiltonian together with the two-point Green functions of free Dirac fermion fields established in our previous work form the basics of the perturbation theory of the above-mentioned interacting field system. As an example, the perturbation theory is applied to the study of two-point Green functions of this interacting system of quantum fields.

Keywords: quantum field, Dirac fermion, electromagnetic field, Green function, perturbation theory
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1. Introduction

After the discovery of graphene by Novoselov et al [1, 2], a new extremely promising interdisciplinary scientific area—the physics, chemistry and technology of graphene and similar two-dimensional hexagonal semiconductors—has emerged and strongly developed as ‘a rapidly rising star on the horizon of materials science and condensed-matter physics, having already revealed a cornucopia of new physics and potential applications’, as Geim et al stated [3]. The quantum motion of electrons as spinless point particles in graphene is essentially governed by Dirac’s (relativistic) equations [4] in the $(2+1)$-dimensional Minkowski space-time.

It is known that in the terminology of quantum field theory the spinless Dirac fermions in graphene monolayers are described by two spinor quantum fields $\psi^K(\mathbf{r}, t)$ and $\psi^{K'}(\mathbf{r}, t)$, $\mathbf{r} = \{r_x, r_y\} = \{x, y\}$ [5]. The points $K$ and $K'$ are the two nearest corners of the first Brillouin zone in the reciprocal lattice of the hexagonal graphene structure. They are called Dirac points.

Since the Dirac fermions are considered as the spinless fermions, the quantum fields $\psi^K(\mathbf{r}, t)$ and $\psi^{K'}(\mathbf{r}, t)$ are the two-component spinors realizing the fundamental representation of the $\text{SU}(2)$ group of rotations in some fictive three-dimensional Euclidean space. Let us call them the quasi-
spinors or pseudospinors in the analogy with the notion of isospinor used in the theory of elementary particles [6–9].

Let us denote $\tau_\mu$, $\mu = 1, 2, 3$ three generators of the SU(2) group of rotations in the fictive three-dimensional Euclidean space. We call them the quasi-spin or pseudospin operators acting on the quantum fields of Dirac fermions as two-component spinors. They are similar in the matrix form but have a quite different physical meaning compared to the Pauli matrices $\sigma_i$, $i = 1, 2, 3$, representing conventional spin operators of spin 1/2 fermions and being generators of the SU(2) group of rotations in the physical three-dimensional space. In the unit system with $\hbar = c = 1$ (c being the light speed in the vacuum) and the approximation assuming the linear dispersion law for the Dirac fermions, the Hamiltonian of the system of free Dirac fermions in the graphene monolayer has the following expression [5]

$$H_G = \int \mathrm{d}r \{ \psi^K(r, t)^* \tau(\pm \nabla) \psi^K(r, t) + \psi^K(r, t)^* \tau(\pm \nabla) \psi^K(r, t) \} A_m(r, t).$$

Let us chose the Cartesian coordinate system as follows: the plane of a graphene monolayer is the coordinate plane $xOy$ and, therefore, the Oz-axis is perpendicular to this plane. The coordinate of a point in the three-dimensional physical space is denoted $[r, z] = \{x, y, z\}$. In conventional quantum electrodynamics it is known [6–11] that three components $A_\mu(x, z, t)$ of the vector potential field $A(x, z, t)$ together with the scalar potential field $\phi(x, z, t) = A_0(x, z, t)$ form a four-component vector field $A_\mu(x, z, t)$, $\mu = 1, 2, 3, 4$, $A_4(x, z, t) = iA_0(x, z, t)$, in the $(3 + 1)$-dimensional Minkowski space-time. In order to take into account the interaction between Dirac fermion fields $\psi^K(r, t)$ and $\psi^K(r, t)$ with the vector potential field $A(x, z, t)$, we must perform the substitution $-i\nabla \to -i\nabla + eA(r, o, t)$ in the Hamiltonian (1), $e$ being the absolute value of the electron charge [6–12]. Then we obtain the following expression of the Hamiltonian of the interaction between the vector potential field $A(x, z, t)$ and Dirac fermion field $\psi^K(r, t)$ and $\psi^K(r, t)$

$$H_{\text{int}}^V = e \int \mathrm{d}r \{ \psi^K(r, t)^* \tau_+ \psi^K(r, t) + \psi^K(r, t)^* \tau_+ \psi^K(r, t) \} A(r, o, t).$$

Dirac fermions interact also with the scalar potential field $\phi(x, z, t)$. The corresponding part of the interaction Hamiltonian is

$$H_{\text{int}}^S = e \int \mathrm{d}r \{ \psi^K(r, t)^* \psi^K(r, t) + \psi^K(r, t)^* \psi^K(r, t) \} \phi(r, o, t).$$

The interaction between the electromagnetic field and Dirac fermion fields is completely described by the following total interaction Hamiltonian

$$H_{\text{int}} = H_{\text{int}}^V + H_{\text{int}}^S.$$
of the Dirac fermion gas

\[ \langle \cdots \rangle = \langle G \cdots | G \rangle. \]  

This ground state \( | G \rangle \) can be considered as the vacuum state of the free electromagnetic field.

Since the theory of the electromagnetic field is invariant under a class of gauge transformations

\[ A_{\mu}(x) \rightarrow A_{\mu}(x) + \frac{\partial \chi(x)}{\partial x_{\mu}}, \]  

the vector field \( A_{\mu}(x) \) is not uniquely determined. In classical electrodynamics [12] to simplify equations and calculations the vector field \( A_{\mu}(x) \) satisfying the following Lorentz condition

\[ \frac{\partial A_{\mu}(x)}{\partial x_{\mu}} = 0 \]  

was frequently used.

However, in quantum electrodynamics this condition cannot hold for the quantum vector field \( A_{\mu}(x) \). Instead of condition (11) it was reasonably proposed to assume another similar but weaker condition imposed on the state vector of all physical states of the electromagnetic field:

\[ \langle \Phi_1 | \frac{\partial A_{\mu}(x)}{\partial x_{\mu}} | \Phi_2 \rangle = 0. \]  

In the fundamental research works on quantum electrodynamics [10, 11] it was demonstrated that due to condition (12) the electromagnetic waves in the states with longitudinal and scalar polarizations play no role in any physical processes. Therefore in the Hilbert space of state vectors of all elements

\[ \expansion{A}(x) = A(\mathbf{r}, z, t) \]  

has the following effective Fourier expansion formula

\[ \begin{aligned}
A(\mathbf{r}, z, t) &= \frac{1}{(2\pi)^{3/2}} \int \! \! d\mathbf{k} \int \! \! dl \\
&\times \sum_{\sigma = \pm 1} \frac{\xi_{\sigma \mathbf{k} l}}{\sqrt{2\Omega(\mathbf{k}, l)}} \{ e^{i(kr + zl - \Omega(\mathbf{k}, l)t)} c_{\sigma \mathbf{k} l}^\dagger \\
&+ e^{-i(kr + zl - \Omega(\mathbf{k}, l)t)} c_{\sigma \mathbf{k} l} \} , \end{aligned} \]  

where \( \xi_{\sigma \mathbf{k} l} \), with \( \sigma = \pm 1 \) are two three-component complex unit vectors characterizing two transversely polarized states of the electromagnetic plane waves with the wave vector \( \{ \mathbf{k}, l \} \). Let us represent each vector \( \xi_{\sigma \mathbf{k} l} \) as a column with three elements

\[ \xi_{\sigma \mathbf{k} l} \rightarrow \begin{pmatrix} (\xi_{\sigma \mathbf{k} l})_1 \\ (\xi_{\sigma \mathbf{k} l})_2 \\ (\xi_{\sigma \mathbf{k} l})_3 \end{pmatrix} \]  

For two plane waves propagating along the direction of the Oz-axis we have

\[ \xi_{+1 \alpha} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \xi_{-1 \alpha} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \]  

It is straightforward to project the vector field (13) onto the graphene plane to obtain the vector field \( A_{\parallel}(\mathbf{r}, t) \) with two components

\[ A_{\parallel}(\mathbf{r}, t) = \frac{1}{2\pi} \int \! \! d\mathbf{k} A_{\parallel}(\mathbf{r}, t), \]  

\[ A_{\parallel}(\mathbf{r}, t) = \frac{1}{2\pi} \int \! \! d\mathbf{k} \\
\times \sum_{\sigma = \pm 1} \frac{1}{\sqrt{2\Omega(\mathbf{k}, l)}} \{ (\xi_{\sigma \mathbf{k} l})_2 e^{i(kr + zl - \Omega(\mathbf{k}, l)t)} c_{\sigma \mathbf{k} l}^\dagger \\
+ (\xi_{\sigma \mathbf{k} l})_1 e^{-i(kr + zl - \Omega(\mathbf{k}, l)t)} c_{\sigma \mathbf{k} l} \}. \]  

The field \( A_{\parallel}(\mathbf{r}, t) \) with an index \( l \neq 0 \) looks like the conventional free vector field with transverse polarizations of a massive particle with the mass \( |l| \) and the helicities \( \sigma = \pm 1 \) in the \((2 + 1)\)-dimensional Minkowski space-time.

Note that the electromagnetic waves with the scalar polarization play no role in any physical processes. Therefore the free scalar field \( \phi(\mathbf{r}, z, t) \) effectively does not have the non-vanishing projection onto the graphene plane.

Now we consider the projection of the two-point Green function

\[ D_{\mu\nu}(x) = D_{\mu\nu}(\mathbf{k}, z, t) \]  

of the free electromagnetic field onto the graphene monolayer. In the relativistic quantum electrodynamics [10, 11] it was shown that \( D_{\mu\nu}(x) \) has following general expression

\[ D_{\mu\nu}(x) = \frac{1}{(2\pi)^4} \int \! \! d^4k e^{ikx} D_{\mu\nu}(k), \]  

\[ D_{\mu\nu}(k) = \begin{pmatrix} \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + \delta(k^2) \frac{k_\mu k_\nu}{k^2} \frac{1}{i(k^2 - i0)} \end{pmatrix}, \]  

where \( k \) denotes a four-momentum vector with the components \( k_\mu, \mu = 1, 2, 3, 4 \), \( k_4 = ik_0 \), in the Minkowski space-time,

\[ k = (\mathbf{k}, i k_0) \]  

\[ k^2 = k_0^2 + \mathbf{k}_0^2. \]
\[ kx = kr + lz - k_0 t, \]

\[ \int d^4k = \int dk \int df \int dk_0 \]

and \( d(k^2) \) is a scalar function depending on the choice of the gauge for the free electromagnetic field.

Since the theory is invariant under gauge transformations of the whole system of all interacting quantum fields, for simplifying the calculations in certain cases one often chose to work in such a gauge that

\[ d(k^2) = 1. \]  \hspace{1cm} (21)

In this case formula (20) becomes

\[ \tilde{D}^{(0)}_{\nu\mu}(k) = \delta_{\nu\mu} \frac{1}{i(k^2 - io)} \]  \hspace{1cm} (22)

i.e.

\[ \tilde{D}^{(0)}_{\nu\mu}(k, l, k_0) = \delta_{\nu\mu} \frac{1}{i(k^2 + l^2 - k_0^2 - io)}. \]  \hspace{1cm} (23)

The projection \( \tilde{D}^{(0)}_{mn}(r, t) \) of the Green function \( D^{(0)}_{mn}(r, z, t) \) onto a graphene monolayer is determined by the following definition

\[ \tilde{D}^{(0)}_{mn}(r, t) \overset{\text{def.}}{=} D^{(0)}_{mn}(r, o, t), \]  \hspace{1cm} (24)

where \( m, n = 0, 1, 2 \). From the above presented formulae it is easy to show that

\[ D^{(0)}_{mn}(r, t) = \frac{1}{(2\pi)^3} \int dk \int df \int dk_0 e^{ikr-k_0 t} \tilde{D}^{(0)}_{mn}(k, l, k_0) \]  \hspace{1cm} (25)

with

\[ \tilde{D}^{(0)}_{mn}(k, l, k_0) = \delta_{mn} \frac{1}{i(k^2 + l^2 - k_0^2)}. \]  \hspace{1cm} (26)

Formula (25) shows that \( D^{(0)}_{mn}(r, t) \) is a linear combination

\[ D^{(0)}_{mn}(r, t) = \frac{1}{2\pi} \int df \ D^{(0)}_{mn}(r, t) \]

of an innumerable set of functions \( D^{(0)}_{mn}(r, t) \) labeled by the index \( l \) running all integer values from \(-\infty\) to \(+\infty\):

\[ D^{(0)}_{mn}(r, t) \overset{\text{def.}}{=} \delta_{mn} \frac{i}{(2\pi)^3} \int dk \int dk_0 \int df e^{ikr-k_0 t} \]

\[ \times \frac{1}{k^2 + l^2 - k_0^2 - io}. \]  \hspace{1cm} (28)

Each function \( D^{(0)}_{mn}(r, t) \) is the two-point Green function of a massive relativistic particle with the mass \( |l| \) (in two dimensions).

However, if we impose on the state vectors of all physical states of the system the weak Lorentz condition (12), then \( D^{(0)}_{\mu\nu}(x) \) must satisfy the transversality condition

\[ \frac{\partial D^{(0)}_{\mu\nu}(x)}{\partial x^\nu} = 0, \]  \hspace{1cm} (29)

and instead of equation (21) we have the relation

\[ d(k^2) = 0, \]  \hspace{1cm} (30)

meaning that the tensor \( \tilde{D}^{(0)}_{mn}(k, l, k_0) \) has the following components:

\[ \tilde{D}^{(0)}_{ij}(k, l, k_0) = \left( \delta_{ij} - \frac{k_l k_i}{k^2 + l^2 - k_0^2} \right) \frac{1}{i(k^2 + l^2 - k_0^2 - io)}. \]  \hspace{1cm} (31)

\[ \tilde{D}^{(0)}_{ij}(k, l, k_0) = \tilde{D}^{(0)}_{ij}(k, l, k_0) = \left( 1 - \frac{k_l^2}{k^2 + l^2 - k_0^2} \right) \frac{1}{i(k^2 + l^2 - k_0^2 - io)}. \]  \hspace{1cm} (32)

with \( i, j = 1, 2 \),

\[ \tilde{D}^{(0)}_{33}(k, l, k_0) = \tilde{D}^{(0)}_{33}(k, l, k_0) = \left( \frac{1}{k^2 + l^2 - k_0^2} \right) \frac{1}{i(k^2 + l^2 - k_0^2 - io)}. \]  \hspace{1cm} (33)

and

\[ \tilde{D}^{(0)}_{00}(k, l, k_0) = - \left( 1 + \frac{k_l^2}{k^2 + l^2 - k_0^2} \right) \frac{1}{i(k^2 + l^2 - k_0^2 - io)}. \]  \hspace{1cm} (35)

### 3. Interacting Dirac fermion fields and electromagnetic field

In order to apply perturbation theory to the study of interacting system comprising the Dirac fermion fields and the electromagnetic field it is necessary to use explicit expressions of following physical quantities:

- Dirac fermion fields \( \psi^K(r, t) \) and \( \psi^{K'}(r, t) \).
- Two-point Green functions \( \Delta^K_{\alpha\beta}(r, t)^{(0)} \) and \( \Delta^{K'}_{\alpha\beta}(r, t)^{(0)} \) of free Dirac fermion fields,
- Projection \( A_i(r, t) \) with the two-component \( A_i(r, t), i = 1, 2, \) of the electromagnetic field onto the graphene monolayer,
- Projection \( D_{mn}(r, t)^{(0)}, m, n = 0, 1, 2, \) of the two-point Green function of the free electromagnetic field onto the graphene monolayer, and
- Interaction Hamiltonian \( H_{int}(t) \) of the system.
The interaction Hamiltonian \( H_{\text{int}}(t) \) was determined by formula (7). The projection \( \mathcal{A}_i(t) \) of the electromagnetic field and the projection \( D_{\text{int}}(t) \), \( m, n = 0, 1, 2 \), of the two-point Green function of the free electromagnetic field were investigated in the preceding section 2. It remains to establish the explicit expressions of Dirac fermion fields \( \psi^K(r, t) \), \( \psi^{K'}(r, t) \) and two-point Green functions \( \Delta_{0,\beta}^{K,K'}(r, t) \), \( \Delta_{0,\beta}^{K,K'}(r, t, t') \) of free Dirac fermions.

In our previous work [13] we derived explicit expressions of two-point Green functions \( \Delta_{0,\beta}^{K,K'}(r, t) \), \( \Delta_{0,\beta}^{K,K'}(r, t, t') \) of free Dirac fermions in a free Dirac fermion gas at \( T = 0 \). They depend on the value \( E_F \) of the Dirac fermion gas. For simplicity let us consider the case with \( E_F = 0 \). The extension to other cases is straightforward.

In the simple case with \( E_F = 0 \) the Dirac fermion fields \( \psi^K(r, t) \) and \( \psi^{K'}(r, t) \) have the following Fourier expansion formula

\[
\psi^{K,K'}(r, t) = \frac{1}{2\pi} \int \text{d}k [ e^{i(kr - E(k)t)} a_{K,k}^{K,K'} + e^{-i(kr - E(k)t)} b_{K,k}^{K,K'} ] , \tag{36}
\]

where \( a_{K,k}^{K,K'} \) and \( b_{K,k}^{K,K'} \) are the destruction operators of the Dirac fermion and Dirac hole, respectively, with wave functions being plane waves, \( k \) is the wave vector to be considered also as the momentum of the Dirac fermion or Dirac hole, \( a_{K,k}^{K,K'} \) and \( b_{K,k}^{K,K'} \) are corresponding creation operators, \( E_{\text{e,h}}(k) \) and \( E_{\text{h,e}}(k) \) are energies of the Dirac fermion and Dirac hole, respectively, with momentum \( k \),

\[
E_{\text{e,h}}(k) = v_F k, \quad k = |k| = \sqrt{k_1^2 + k_2^2} , \tag{37}
\]

\( v_F \) is the speed of the relativistic Dirac fermion in the unit system with \( c = 1 \),

\[
u_K(k) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta(k)/2} \\ e^{i\theta(k)/2} \end{pmatrix} \xi , \tag{38}
\]

\[
\nu^{K'}(k) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta(k)/2} \\ -e^{i\theta(k)/2} \end{pmatrix} \eta' ,
\]

and

\[
u^{K'}(k) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta(k)/2} \\ e^{-i\theta(k)/2} \end{pmatrix} \eta , \tag{39}
\]

\[
\theta(k) = \arctan \frac{k_1}{k_2} , \tag{40}
\]

\( \eta \) and \( \eta' \) being two arbitrary phase factors \( |\eta| = |\eta'| = 1 \).

Two-point Green functions of Dirac fermions in free Dirac fermion gas at \( T = 0 \) have the following definition

\[
\Delta_{0,\alpha}^{K,K'}(r - r', t - t') = -i \left\langle T \left[ \tilde{\psi}_{\alpha}^{K,K'}(r, t) \tilde{\psi}_{\beta}^{K,K'}(r', t') \right] \right\rangle = -i \left\langle \theta(t - t') \left( \psi_{\alpha}^{K,K'}(r, t) \psi_{\beta}^{K,K'}(r, t') \right) \rightangle - \theta(t' - t) \left( \psi_{\alpha}^{K,K'}(r, t) \psi_{\beta}^{K,K'}(r, t') \right) \right\rangle , \tag{41}
\]

Introducing their Fourier transformations

\[
\Delta_{0,\alpha}^{K,K'}(r, t) = \frac{1}{(2\pi)^3} \int \text{d}k \frac{1}{2\pi} \int \text{d}\omega \, e^{i(kr - \omega t)} \Delta_{0,\alpha}^{K,K'}(k, \omega) , \tag{42}
\]

we have

\[
\Delta_{0,\alpha}^{K,K'}(k, \omega) = u_{\alpha}^{K,K'}(k) \nu_{\beta}^{K,K'}(k) + \omega - E_{\text{e,h}}(k) + io \]

\[
+ \nu_{\alpha}^{K,K'}(-k) \nu_{\beta}^{K,K'}(-k) + \omega + E_{\text{e,h}}(k) - io . \tag{43}
\]

Thus the basics for elaborating the perturbation theory of an interacting system comprising Dirac fermion fields and an electromagnetic field were established.

4. Perturbation theory

The most efficient tool for the theoretical study of interaction processes between quanta of any interacting system of quantum fields is the scattering matrix \( S \), briefly called the \( S \)-matrix. In the perturbation theory the \( S \)-matrix is expressed in terms of the interaction Hamiltonian \( H_{\text{int}}(t) \) of the system as follows

\[
S = T \left\{ \exp \left[ -i \int \text{d}t \, H_{\text{int}}(t) \right] \right\} , \tag{44}
\]

where the integration with respect to the time variable \( t \) is performed over the whole real axis from \(-\infty\) to \(+\infty\). By expanding the exponential function on the right-hand side of formula (44) into power series, we write the \( S \)-matrix in the form of a series

\[
S = 1 + \sum_{n=1}^{\infty} S^{(n)} , \tag{45}
\]

the term \( S^{(n)} \) of \( n \)th order is

\[
S^{(n)} = \frac{(-i)^n}{n!} \int \text{d}t_1 \int \text{d}t_2 \ldots \times \int \text{d}t_n \left( H_{\text{int}}(t_1) H_{\text{int}}(t_2) \ldots H_{\text{int}}(t_n) \right) . \tag{46}
\]

As an example of the application of perturbation theory let us study two-point Green functions of an interacting system comprising Dirac fermion fields and the projection of the electromagnetic field onto the graphene monolayer at \( T = 0 \). They are expressed in terms of free Dirac fermion fields.
$\psi_K(r, t)$ and $\psi^K(r, t)$, components $A_i(r, t)$ of the projection $A_{ij}(r, t)$ of a free electromagnetic field onto the graphene monolayer and S-matrix as follows:

$$D_{ij}(r - r', t - t') = -i \langle T \left\{ S A_i(r, t) A_j(r', t') \right\} \rangle / \langle S \rangle$$

(47)

and

$$\Delta_{ij}^{K'}(r - r', t - t') = -i \langle T \left\{ S \psi_i^K(r, t) \psi_{j}^{K'}(r', t') \right\} \rangle / \langle S \rangle$$

(48)

Using expansion formula (45) of the S-matrix, we write each of the Green functions (47) and (48) in the form of a series:

$$D_{ij}(r - r', t - t') = \sum_{n=0}^{\infty} D_{ij}(r - r', t - t')^{(2n)}$$

(49)

$$\Delta^{K'}_{ij}(r - r', t - t') = \sum_{n=0}^{\infty} \Delta^{K'}_{ij}(r - r', t - t')^{(2n)}$$

(50)

$n$ running all non-negative integers $n = 0, 1, 2 \ldots$

We have calculated $D_{ij}(r - r', t - t')^{(0)}$ and $\Delta^{K'}_{ij}(r - r', t - t')^{(0)}$. In order to calculate $D_{ij}(r - r', t - t')^{(2)}$, let us consider matrix element

$$\langle T \left\{ S^2 A_i(r, t) A_j(r', t') \right\} \rangle = \frac{(-i)^2}{2!} \times \int \text{d}n \int \text{d}t' \int \text{d}t \int \text{d}t' \int \text{d}t' \sum_{n=0}^{2} \sum_{m=0}^{2} \langle T \left\{ \psi_i^K(r, t) \psi_{j}^{K'}(r, t') \right\} \rangle$$

(51)

Similarly, in order to calculate $\Delta^{K'}_{ij}(r - r', t - t')^{(2)}$, for example, we consider matrix element

$$\langle T \left\{ S^2 \psi^K_i(r, t) \psi^{K'}_j(r', t') \right\} \rangle = \frac{(-i)^2}{2!} \times \int \text{d}n \int \text{d}t' \int \text{d}t \int \text{d}t' \sum_{n=0}^{2} \sum_{m=0}^{2} \langle T \left\{ \psi^K_i(r, t) \psi^{K'}_j(r', t') \right\} \rangle$$

(52)

The matrix elements on the right-hand side of equations (51) and (52) can be calculated by applying the Wick theorem in quantum field theory. They are expressed in terms of two-point Green functions $\Delta^{KK'}_{ij}(r - r', t - t')^{(0)}$ of free Dirac fermion fields $\psi^{K'}_{i}(r, t)$ and the projection $D_{ij}^{(0)}(r, t)$ of two-point Green functions of the free electromagnetic field onto a graphene monolayer.

By using derived expressions of the above-mentioned matrix elements it is straightforward to calculate second-order terms in the series (49) and (50). We obtain the following result:

$$\Delta^{K}_{ij}(r - r', t - t')^{(2)} = \int \text{d}n \int \text{d}t' \int \text{d}r_1 \int \text{d}r_2 \times \sum_{n=0}^{K} \Delta_{11}^{K_{ij}}(r_1 - r_2, 0) \Delta_{11}^{K_{ij}}(r_2 - r_1, 0) \pi^{(2)}_{ij}(r_1 - r_2, t_1 - t_2)$$

(53)

where

$$\pi^{(2)}_{ij}(r_1 - r_2, t_1 - t_2) = \int \text{d}n \int \text{d}t' \sum_{n=0}^{K} \Delta_{11}^{K_{ij}}(r_1 - r_2, t_1 - t_2) \pi^{(2)}_{ij}(r_1 - r_2, t_1 - t_2)$$

(54)

is the self-energy part of the Dirac fermion field $\psi^K(r, t)$, and

$$D_{ij}(r - r', t - t')^{(2)} = \int \text{d}n \int \text{d}t' \int \text{d}r_1 \int \text{d}r_2 \times \sum_{n=0}^{N} D_{nm}(r - r_1, t_1 - t_2) D_{nm}(r_2 - r', t_2 - t')$$

(55)

where

$$\pi^{(2)}_{nm}(r_1 - r_2, t_1 - t_2) = -i \text{Tr} \left\{ \pi_{nm} \Delta^{K}(r_1 - r_2, t_1 - t_2) \right\} \pi^{(2)}_{nm}(r_1 - r_2, t_1 - t_2)$$

(56)

can be considered as the self-energy part of the projection of the electromagnetic field onto a graphene plane, $\Delta^K(r, t)^{(0)}$ and $\Delta^{K}_{ij}(r, t)^{(0)}$ being $2 \times 2$ matrices with elements $\Delta^{KK'}_{ij}(r, t)^{(0)}$ and $\Delta^{KK'}_{ij}(r, t)^{(0)}$.

All higher-order terms in the series (49) and (50) can be calculated analogously. Summing them up, we obtain the Dyson equations for the whole Green functions (49) and (50) of interacting quantum fields in the ladder approximation:

$$D_{ij}(r - r', t - t') = \int \text{d}n \int \text{d}t' \int \text{d}r_1 \int \text{d}r_2 \times \sum_{n=0}^{N} D_{nm}(r - r_1, t - t_2) \pi^{(2)}_{nm}(r_1 - r_2, t_2 - t')$$

(57)

$$\pi^{(2)}_{nm}(r_1 - r_2, t_1 - t_2) = \int \text{d}n \int \text{d}t' \sum_{n=0}^{K} \Delta_{11}^{K_{ij}}(r_1 - r_2, t_1 - t_2) \pi^{(2)}_{nm}(r_1 - r_2, t_2 - t')$$

(58)
and
\[
\Delta_{\alpha_1}^{K,K'}(r - r', t - t') = \int dr_1 \int dr_2 \int dt_2 \times \int dr_2 \Delta_{\alpha_1}^{K,K'}(r - r_1, t - t_1)^{(0)}
\]
\[
\times \sum_{\alpha_2, t_2} \Delta_{\alpha_2}^{K,K'}(r_2 - r', t_2 - t')^{(0)}.
\]

(58)

5. Conclusion and discussions

In the present work we have developed the quantum theory of an interacting system comprising Dirac fermion fields and the projection onto a graphene monolayer of an electromagnetic field. The explicit expressions of these fields, the interaction Hamiltonian of the system and the two-point Green functions of free fields as well as the integral equation determining the two-point Green functions of interacting fields in the ladder approximation were established.

We have not yet investigated the electromagnetic scattering processes taking place in the graphene monolayer. In our subsequent works the presented expressions and equations will be applied to the study of various interaction processes with the participation of photon and Dirac fermions. In particular, the application of the whole theoretical tool elaborated in the present work is necessary and also sufficient for the study of physical processes taking place completely inside the graphene monolayer. This would be also useful for the study of electromagnetic properties of graphene-based optoelectronic and photonic nanostructures and nanocomposites.

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