Metastable Vacua
in Superconformal SQCD-like Theories

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Abstract: We study dynamical supersymmetry breaking in vector-like superconformal $\mathcal{N} = 1$ gauge theories. We find appropriate deformations of the superpotential to overcome the problem of the instability of the non supersymmetric vacuum. The request for long lifetime translates into constraints on the physical couplings which in this regime can be controlled through efficient RG analysis.
1. Introduction

In the last few years many models of metastable dynamical supersymmetry breaking (DSB) based on the ISS breakthrough [1] have been proposed (see [2] and references therein). Usually in DSB the strong dynamics jeopardizes the calculability of the model. The novelty of the approach of ISS relies in describing the low energy theory by the Seiberg dual phase [3, 4] which is weakly coupled in the IR. For a $\mathcal{N} = 1$ $SU(N_c)$ supersymmetric gauge theory with $N_f > N_c + 1$ flavors the low energy physics can be equivalently described by a different magnetic $SU(N_f - N_c)$ gauge group with $N_f$ flavors and a singlet. Furthermore if $N_f < 2N_c$, the $SU(N_c)$ gauge group is strongly coupled whereas the dual magnetic gauge group $SU(N_f - N_c)$ is weakly coupled in the IR.

The ISS model is based on SQCD with $N_c + 1 < N_f < 3/2N_c$ and small masses for the quarks. In this window the dual gauge theory at low energy flows to an IR free fixed point. This theory breaks supersymmetry at tree level in the small field region. In this region the strong dynamics effect are safely negligible and perturbation theory is reliable. The supersymmetric vacua are recovered in another region of the field space, namely at large vevs. The analysis shows that the supersymmetry breaking vacuum is metastable, and the lifetime of this state can be made parametrically large by tuning the scales of the theory.

In principle the same mechanism is applicable in the conformal window if $3/2N_f < N_c < 2N_f$, where there is a weakly interacting fixed point. In [1] the authors showed that in such window the non supersymmetric vacuum is unstable to decay because the strong dynamics effects are relevant and not negligible around the origin of the field space. Indeed the bounce action between the non supersymmetric vacuum and the supersymmetric one is
not parametrically large, and the lifetime is short. Recent studies for realizing metastable
vacua in the conformal window has been done in [5].

In this paper we investigate this problem more deeply, and we find a viable model of
metastable supersymmetry breaking in the conformal regime of a SQCD like theory.

We start our analysis by revisiting the ISS model in the conformal window, studying
the RG evolution of the couplings and of the bounce action. The lifetime of the non super-
symmetric vacuum is proportional to the ratio between the IR supersymmetry breaking
scale and the IR holomorphic scale. We find that this ratio depends only on the gauge
coupling calculated at the conformal fixed point. This shows that the lifetime of the vac-
um cannot be parametrically large below the IR scale at which the theory exits from the
conformal regime.

Nevertheless, we argue that by adding some deformations the metastable vacua can
still exist in the conformal window. We propose a deformation of the ISS model, by
adding a small number of massive quarks and some new singlets in the dual description of
massive SQCD. This model is a $SU(N)$ SCFT dual to the SSQCD defined in [6] with some
relevant deformations. When these deformations are small, the theory is approximately a
CFT. In this approximate CFT regime this theory is interacting, and we restrict to the
weakly coupled window such that the perturbative analysis is reliable. This model can
evade the argument of ISS because the new massive fields modify the non perturbative
superpotential and thus the supersymmetric vacuum. As a consequence the bounce action
has a parametrical behavior in terms of the relevant deformations. The lifetime can be
large if we impose some constraints on the physical couplings at the CFT exit scale.

Differently from the IR free case, in which the low energy theory is free, in this case the
model is interacting. The anomalous dimensions of the fields are not zero, and the Kahler
potential is renormalized. This implies that the physical couplings undergo RG evolution
in the approximate CFT regime. The constraints for the stability of the non supersym-
metric vacuum have to be imposed on the physical IR couplings after RG evolution. These
translate in conditions for the UV couplings and for the duration of the approximate CFT
regime. We then look for the allowed region of UV couplings such that the bounds on the
lifetime of the vacuum, imposed in the IR, are satisfied.

We argue that metastable vacua are common in the conformal window, and we give a
procedure to find other models. The basic requirement is that there must be a regime of
parameters and ranks such that the supersymmetric vacua are far away in the field space,
and that the bounce action is a function of the relevant deformations. As in SSQCD,
which is the simplest example, a richer set of relevant deformations than in massive SQCD
is necessary.

The paper is organized as follows. In Section 2 we discuss the obstructions to the
existence of metastable vacua in SQCD in the conformal window, and we introduce the
analysis of the RG evolution for the couplings and the holomorphic scale. In the main Sec-
tion 3 we outline our strategy for the search of metastable vacua by studying the SSQCD
model appropriately deformed. The key point just relies on the features of super CFT,
where RG analysis and determination of anomalous dimensions are feasible. In Section 4
we discuss the generalization of our analysis to $\mathcal{N} = 1$ SCFTs. In Section 5 we conclude.
In the Appendix A we study the RG flow associated to the bounce action. In the Appendix B we review the Seiberg duality in SSQCD and discuss the origin of the relevant couplings.

While we were completing this paper, the work [7] appeared which has some overlap with our results.

2. The case of SQCD

In the original paper [1] the authors studied a $SU(N_c)$ gauge theory with $N_f$ flavors of quarks charged under an $SU(N_f)^2$ flavor symmetry broken to $SU(N_f)$ by the superpotential

$$W = mQar{Q}$$

(2.1)

where the mass $m$ is much smaller than the holomorphic scale of the theory $\Lambda$. In the window $N_c + 1 < N_f$ this theory admits a dual description in term of a magnetic gauge group $SU(\tilde{N}) = SU(N_f - N_c)$, $N_f$ magnetic quarks $q$ and $\bar{q}$ and the electric meson $N = Q\bar{Q}$ normalized to have mass dimension one. The dual superpotential reads

$$W_m = -h\mu^2 N + hNq\bar{q} + \tilde{N} \left( \tilde{\Lambda}^b h^{N_f \text{ det } N} \right)^{\frac{1}{b}}$$

(2.2)

where we introduced the marginal coupling $h$ and the holomorphic scale of the dual theory $\tilde{\Lambda}$, and we added the non perturbative contribution due to gaugino condensation. From now on we set $h = 1$. The holomorphic scales $\Lambda$ and $\tilde{\Lambda}$ are related by a scale matching relation [4]. The one loop beta function coefficient is $\tilde{b} = 3\tilde{N} - N_f = 2N_f - 3N_c$.

In the range $N_c + 1 < N_f < \frac{3}{2}N_c$, this theory has a supersymmetry breaking vacuum at $N = 0$, with non zero vev for the quarks. The supersymmetric vacuum is recovered in the large field region for $N$. The parametrically long distance between the two vacua guarantees the long life time of the non supersymmetric one.

The metastable non supersymmetric vacua found in the magnetic free window of massive SQCD are destabilized in the conformal window $\frac{3}{2}N_C < N_f < 3N_C$. This fact is based on the observation that the non perturbative superpotential in (2.2) is not negligible in the small field region, as instead it happens in the magnetic free window.

Here we study more deeply this problem. In general, in the presence of relevant deformations the conformal regime is only approximated. If these deformation are small enough there is a large regime of scales in which the theory flows to lower energies remaining at the conformal fixed point. The physical couplings vary along the RG flow because of the wave function renormalization of the fields, until the theory exits from the conformal regime. Below this scale the theory is IR free and the renormalization effects are negligible.

We study the RG properties of the ISS model in the conformal window by using a canonical basis for the fields. Flowing from a UV scale $E_{UV}$ to an IR scale $E_{IR}$ the fields are not canonically normalized anymore, and we have to renormalize them by the wave function renormalization $Z_i(E_{IR}, E_{UV})$, namely $\phi_i^{IR} = \sqrt{Z_i} \phi_i^{UV}$. In terms of the renormalized fields the Kahler potential is canonical. The couplings appearing in the superpotential undergo
RG evolution, and are the physical couplings. In this way the coupling \( \mu_{IR} \) of the IR superpotential becomes
\[
\mu_{IR} = \mu_{UV} Z_N(E_{IR}, E_{UV})^{-\frac{1}{4}} \tag{2.3}
\]
The holomorphic scale that appears in the superpotential is unphysical in the conformal window and it is defined as
\[
\tilde{\Lambda} = E e^{-\frac{8\pi^2}{g^2}} \tag{2.4}
\]
where \( E \) is the RG running scale, and \( g_* \) is the gauge coupling at the superconformal fixed point. In the canonical basis \( \tilde{\Lambda} \) is rescaled as well during the RG conformal evolution as \[8, 9, 10]\]
\[
\tilde{\Lambda}_{IR} = \tilde{\Lambda}_{UV} \frac{E_{IR}}{E_{UV}} \tag{2.5}
\]
In the ISS model the two possible sources of breaking of the conformal invariance are the masses of the fields at the non supersymmetric vacuum and the masses of the fields at the supersymmetric vacuum. We define the CFT exit scale as \( E_{IR} = \Lambda_c \). In this model this scale is necessarily set by the masses of the fields at the supersymmetric vacuum, which are proportional to the vev of the field \( N \). In fact by setting
\[
\Lambda_c \equiv \langle N \rangle_{susy} = \mu_{IR} \left( \frac{\mu_{IR}}{\Lambda_{IR}} \right)^{\frac{i}{N_f-N}} \tag{2.6}
\]
the physical mass at this scale results
\[
\mu_{IR} = \Lambda_c e^{-\frac{4\pi^2}{g^2 N}} \ll \Lambda_c \tag{2.7}
\]
Hence the assumption that \( \langle N \rangle_{susy} \) stops the conformal regime is consistent. The opposite case, with \( \Lambda_c \equiv \mu_{IR} \gg \langle N \rangle_{susy} \) cannot be consistently realized.

The bounce action at the scale \( \Lambda_c \) is
\[
S_B \sim \left( \frac{\mu_{IR}}{\Lambda_{IR}} \right)^{\frac{i}{N_f-N}} e^{\frac{16\pi^2}{N_f N}} \sim e^{\frac{16\pi^2}{N_f N}} \tag{2.8}
\]
This bounce is not parametrically large and it depends only on the coupling constant \( g_* \) at the fixed point. In general, as we shall see in the appendix A, the bounce action is not RG invariant, but it runs during the RG flow. In this case \( S_B \) at the CFT exit scale only depends on the ratio of the two relevant scales in the theory which is the RG invariant coupling constant.

In general, by adding other deformations, the bounce action is not RG invariant anymore and we have to take care about its flow. In some cases, the lifetime of a vacuum decreases as we flow towards the infrared. In the next section, by adding new massive quarks to the ISS model, we show that long living metastable vacua exist in the conformal window.
3. Metastable vacua by adding relevant deformations

In this section we describe our proposal for realizing metastable supersymmetry breaking in the conformal window of $N = 1$ SQCD-like theories. The key point is the addition of massive quarks in the dual magnetic description. This introduces a new mass scale that controls the distance in the field space of the supersymmetric vacuum.

We consider the magnetic description of the ISS model of the previous section. We add a new set of massive fields $p$ and $\tilde{p}$ charged under a new $SU(N_f^{(2)})$ flavor symmetry. We also add new bifundamental fields $K$ and $L$ charged under $SU(N_f^{(1)}) \times SU(N_f^{(2)})$. The added number of flavors is such that $3/2 \tilde{N} < N_f^{(1)} + N_f^{(2)} < 3 \tilde{N}$. The superpotential of the model is

$$W = Kp\bar{q} + L\tilde{p}q + \rho p\tilde{p} - \mu^2 N$$

(3.1)

and the field content is summarized in the Table 1. This model is the dual description of the SSQCD studied in [6], deformed by two relevant operators. In the appendix B we show the Seiberg dual electric description of this theory, and we discuss a mechanism to dynamically generate the mass term for the new quarks.

In the rest of this section we show that in the case of $N_f^{(1)} > \tilde{N}$ there are ISS like metastable supersymmetry breaking vacua if we are near the IR free border of the conformal window, i.e. $N_f^{(1)} + N_f^{(2)} \sim 3\tilde{N}$.

We shall work in the window between the number of flavor and the number of colors

$$2\tilde{N} < N_f^{(1)} + N_f^{(2)} < 3\tilde{N}$$

(3.2)

such that the gauge group is weakly coupled and we can rely on the perturbative analysis.

The non supersymmetric vacuum

The non supersymmetric vacuum is located near the origin of the field space where the superpotential (3.1) can be studied perturbatively. Neglecting the non perturbative dynamics requires some bounds on the parameters $\rho$ and $\mu$. In the rest of the paper we will see that these bounds can be consistent with the running of the coupling constants.

Tree level supersymmetry breaking is possible if

$$N_f^{(1)} > \tilde{N} \quad \Rightarrow \quad 2\tilde{N} > N_f^{(2)}$$

(3.3)
where the second inequality follows from (3.2). The equation of motion for the field \( N \) breaks supersymmetry through the rank condition mechanism. We solve the other equations of motion and we find the non supersymmetric vacuum

\[
\begin{align*}
q &= \left( \mu + \sigma_1 \phi_1 \right) \\
\tilde{q} &= \left( \mu + \sigma_2 \phi_2 \right) \\
N &= \left( \sigma_3 \phi_3 X \right) \\
p &= \phi_5 \\
\tilde{p} &= \phi_6 \\
L &= \left( \phi_7 \tilde{Y} \right) \\
K &= \left( \phi_8 Y \right)
\end{align*}
\]

(3.4)

where we have also inserted the fluctuations around the minimum, \( \sigma_i \) and \( \phi_i \). The fields \( X, Y \) and \( \tilde{Y} \) are pseudomoduli. The infrared superpotential is

\[
W_{\text{IR}} = X\phi_1\phi_2 - \mu^2 X + \mu(\phi_1\phi_4 + \phi_2\phi_3) + \mu(\phi_5\phi_8 + \phi_6\phi_7) + Y\phi_2\phi_5 + \tilde{Y}\phi_1\phi_6 + \rho\phi_5\phi_6
\]

(3.5)

In the limit of small \( \rho \), this is the same superpotential studied in [11]. This superpotential corresponds to the one studied in [12] in the R symmetric limit. The fields \( X, Y \) and \( \tilde{Y} \) are stabilized by one loop corrections at the origin with positive squared masses.

**The supersymmetric vacuum**

We derive here the low energy effective action for the field \( N \), and we recover the supersymmetric vacuum in the large field region. The supersymmetric vacuum is characterized by a large expectation value for \( N \). This vev gives mass to the quarks \( q \) and \( \tilde{q} \) and we can integrate them out at zero vev. Also the quarks \( p \) and \( \tilde{p} \) are massive and are integrated out at low energy. The scale of the low energy theory \( \Lambda_L \) is related to the holomorphic scale \( \tilde{\Lambda} \) via the scale matching relation

\[
\Lambda_L^{2N} = \tilde{\Lambda}^{3N - N_f (1) - N_f (2)} \det \det \rho \det N
\]

(3.6)

The resulting low energy theory is \( N = 1 \) SYM plus a singlet, with effective superpotential

\[
W = -\mu^2 N + \tilde{N}(\tilde{\Lambda}^{3N - N_f (1) - N_f (2)} \det \det N)^{1/N}
\]

(3.7)

where the last term is the gaugino condensate. By solving the equation of motion for \( N \) we find the supersymmetric vacuum

\[
\langle N \rangle_{\text{susy}} = \frac{\mu^{3N - N_f (1) - N_f (2)} \rho^{N_f (2)}}{\tilde{\Lambda}^{N_f (1) - N_f (2)} \rho^{N_f (1) - N_f (2)}}
\]

(3.8)

**Lifetime**

The lifetime of the non supersymmetric vacuum is controlled by the bounce action to the supersymmetric vacuum. In this case, the triangular approximation [13] is valid and the
action can be approximated as \( S_B \simeq (\Delta \Phi)^4/(\Delta V) \). If we estimate \( \Delta \Phi \sim \langle N \rangle_{\text{susy}} \) and \( \Delta V \sim \mu^4 \) we obtain

\[
S_B = \left( \frac{\hat{\Lambda}}{\rho} \right)^{4N_f^{(2)}} \frac{\mu}{\hat{\Lambda}}^{\frac{12N_f^{(1)}-4N_f^{(2)}}{N_f^{(1)}-N}}
\]  
(3.9)

This expression is not automatically very large since \( \mu \ll \hat{\Lambda} \). However, we can impose the following bound on \( \rho \)

\[
\langle N \rangle_{\text{susy}} \gg \mu \quad \rightarrow \quad \rho \ll \hat{\Lambda} \left( \frac{\mu}{\hat{\Lambda}} \right)^{\frac{3N_f^{(1)}-N_f^{(2)}}{N_f^{(1)}-N}}
\]  
(3.10)

If this bound is satisfied, the supersymmetric and the non supersymmetric vacua are far away apart in the field space and the non perturbative terms can be neglected at the supersymmetry breaking scale. This differs from the ISS model in the conformal window. In that case the non-perturbative effects became important at the supersymmetry breaking scale. The bounce action was proportional to the gauge coupling constant at the fixed point and it was impossible to make it parametrically long. The introduction of the new mass scale \( \rho \) allows a solution to this problem.

The bound (3.10) should be imposed on the IR couplings at the CFT exit scale \( E_{IR} = \Lambda_c \). In this case we have a new possible source of CFT breaking, namely the relevant deformation \( \rho \). However we look for a regime of couplings such that the CFT exit scale is set by the supersymmetric vacuum scale, i.e. \( \Lambda_c = \langle N \rangle_{\text{susy}} \gg \mu_{IR}, \rho_{IR} \). The scale \( \Lambda_c \) is

\[
\Lambda_c = \langle N \rangle_{\text{susy}} = \tilde{\Lambda}_{IR} \left( \frac{\mu_{IR}}{\tilde{\Lambda}_{IR}} \right)^{\frac{2N_f^{(2)}}{N_f^{(1)}-N_f^{(2)}}} \left( \frac{\rho_{IR}}{\tilde{\rho}_{IR}} \right)^{\frac{N_f^{(2)}}{N_f^{(1)}-N_f^{(2)}}}
\]  
(3.11)

At this scale we define \( \epsilon_{IR} \) as the ratio between the IR masses \( \rho_{IR} \) and \( \mu_{IR} \) and we demand that

\[
\epsilon_{IR} = \frac{\rho_{IR}}{\mu_{IR}} \ll 1
\]  
(3.12)

Rearranging (3.11) for \( \mu_{IR} \) and \( \rho_{IR} \) we have

\[
\mu_{IR} = \Lambda_c e^{-\frac{8s_e^2}{s_e^2(2N_f-N_f^{(2)})}} \epsilon_{IR}^{N_f^{(2)}} \left( \frac{\hat{\Lambda}_{IR}}{\rho_{IR}} \right)^{\frac{N_f^{(2)}}{N_f^{(1)}-N_f^{(2)}}} \ll \Lambda_c
\]

\[
\rho_{IR} = \Lambda_c e^{-\frac{8s_e^2}{s_e^2(2N_f-N_f^{(2)})}} \epsilon_{IR}^{2N_f^{(2)}} \ll \Lambda_c
\]  
(3.13)

This shows that requiring \( \epsilon_{IR} \ll 1 \) is consistent with the CFT exit scale to be \( \langle N \rangle_{\text{susy}} \).

By substituting (3.11) and (3.13) in (3.9), the bounce action becomes

\[
S_B = e^{\frac{32s_e^2}{s_e^2(2N_f-N_f^{(2)})}} \frac{4N_f^{(2)}}{\epsilon_{IR}^{2N_f^{(2)}}}
\]

(3.14)
and in the limit $N_f^{(2)} \to 0$ it reduces to the one computed in the (2.8). Here the bounce is not only proportional to a numerical factor depending on $g_s^2$, but there is also a parameter, relating the ratios of the physical masses $\rho_{IR}$ and $\mu_{IR}$ at the CFT exit scale. The bounce action can be large if $\epsilon_{IR} \ll 1$, providing a parametrically large lifetime for the non supersymmetric vacuum.

Using the RG evolution equations the bound $\epsilon_{IR} \ll 1$ translates in constraints on the UV masses $\rho_{UV}$ and $\mu_{UV}$ at the UV scale. These masses are relevant perturbations and their ratio must be small along the RG flow.

**RG flow in the approximate conformal regime**

The relevant coupling constants run from $E_{UV}$ to $E_{IR} = \Lambda_c$. We require that these terms are so small in the UV to be considered as perturbations of the CFT, i.e. $\rho_{UV}, \mu_{UV} \ll \Lambda_{UV}$.

The ratio $\epsilon_{UV}$ given at the scale $E_{UV}$ runs as the coupling constants down to $\Lambda_c$. We now study the evolution of this ratio. The requirement of long lifetime of the metastable vacuum (3.10) corresponds to $\epsilon_{IR} \ll 1$ and it constrains both $\epsilon_{UV}$ and the duration of the approximate conformal regime, $\Lambda_c/E_{UV}$.

The running of the relevant couplings in the conformal windows is parameterized by the equations

$$\begin{align*}
\rho_{IR} &= \rho_{UV} Z_p(\Lambda_c, E_{UV})^{-1/2} Z_p(\Lambda_c, E_{UV})^{-1/2} \\
\mu_{IR} &= \mu_{UV} Z_N(\Lambda_c, E_{UV})^{-1/4}
\end{align*}
$$

(3.15) (3.16)

The wave function renormalization $Z$ is obtained by integrating the equation

$$\frac{d \log Z_i}{d \log E} = -\gamma_i$$

(3.17)

from $E_{UV}$ to $\Lambda_c$, where $\gamma_i$ is constant in the conformal regime, and it reads

$$Z_\phi(\Lambda_c, E_{UV}) = \left( \frac{\Lambda_c}{E_{UV}} \right)^{-\gamma_\phi_i}$$

(3.18)

The physical couplings at the CFT exit scale are

$$\begin{align*}
\rho_{IR} &= \rho_{UV} \left( \frac{\Lambda_c}{E_{UV}} \right)^{\gamma_p} \\
\mu_{IR} &= \mu_{UV} \left( \frac{\Lambda_c}{E_{UV}} \right)^{\gamma_N/4}
\end{align*}
$$

(3.19)

where we have used the relation $\gamma_p = \gamma_p \phi_i$. Along the flow from $E_{UV}$ to $\Lambda_c$ the coupling $\mu_{IR}$ is suppressed, because $\gamma_N > 0$, while $\rho_{IR}$ becomes larger, because $\gamma_p < 0$.

The ratio $\epsilon$ evolves as

$$\epsilon_{IR} = \epsilon_{UV} \left( \frac{\Lambda_c}{E_{UV}} \right)^{\gamma_p - \gamma_N/4}$$

(3.20)

and we demand that it is $\epsilon_{IR} \ll 1$ in order to satisfy the stability constraint for the non supersymmetric vacuum. The flow from $\epsilon_{UV}$ to $\epsilon_{IR}$ depends on $\Lambda_c/E_{UV}$ and on the anomalous dimensions. The precise relation between $\epsilon_{UV}$ and $\epsilon_{IR}$ is found by calculating the exact value of $\gamma_p$ and $\gamma_N$. The anomalous dimensions of the fields $\phi_i$ are obtained from
the relation $\Delta_i = 1 + \gamma_i/2$ where $\Delta_i = \frac{3}{2} R_i$. The $R$ charges can be computed by using a-maximization.

The a-maximization procedure, defined in [14], shows that in SCFT the correct $R$-charge at the fixed point is found by maximizing the function

$$a_{\text{trial}}(R) = \frac{3}{32} (3 \text{Tr} R^3 - \text{Tr} R)$$  \hspace{1cm} (3.21)$$

The $R$-charges in (3.21) are all the non anomalous combinations of the $R_0$ charges under which the supersymmetry generators have charge $-1$ and all the other flavor symmetries commuting with the supersymmetry generators. The $\text{Tr}(R^3)$ and $\text{Tr}(R)$ are the coefficients of the gauge anomaly and gravitational anomaly. The $R$-charges that maximize (3.21) are the $R$ charges appearing in the superconformal algebra.

The $R$ charge assignment has to satisfy the anomaly free condition and the constraint that the superpotential couplings should be marginal. These conditions are

$$\tilde{N} + N_f^{(1)}(N[q] − 1) + N_f^{(2)}(N[p] − 1) = 0, \quad R[p] + R[q] + R[L] = 2, \quad R[N] + 2R[q] = 2 \quad (3.22)$$

where the symmetry enforces $R[q] = R[q], R[p] = R[p]$ and $R[K] = R[L]$. The $a_{\text{trial}}$ function that has to be maximized is

$$a_{\text{trial}} = \frac{3}{32} \left( 2N_f^{(1)} N (3(R[q] − 1)^3 − R[q] + 1) + 2N_f^{(2)} \tilde{N} (3(R[p] − 1)^3 − R[q] + 1) 
+ 2N_f^{(1)} N_f^{(2)} (3(R[L] − 1)^3 − R[L] + 1) + N_f^{(1)2} (3(R[N] − 1)^3 − R[N] + 1) + 2\tilde{N}^2 \right)$$  \hspace{1cm} (3.23)$$

By defining $R[N] = 2y$ we have $R[q] = 1 − y$. The other $R$ charges are

$$R[p] = \frac{1}{n}(n − x + y), \quad R[L] = y + \frac{x − y}{n}$$  \hspace{1cm} (3.24)$$

where $n = \frac{N_f^{(2)}}{N_f^{(1)}}$ and $x = \frac{\tilde{N}}{N_f^{(1)}}$. We can simplify the $a$ maximization in terms of the only variable $y$, obtaining

$$y_{\text{max}} = \frac{-3(n^3 + 3(-1 + n)^2x - 3x^2 + \sqrt{n^2}[(8n(x - 1)^3 + 8n^3(x - 1)^4 + 8n(x - 1)^3 + 8n^2(x - 1)^4 - 6n^2(1 + 3(x - 2)x))]}{3(1 - n(3 + n + n^2) + (-1 + n^2)x)}$$  \hspace{1cm} (3.25)$$

Once we know the anomalous dimensions and once we fix the duration of the approximate conformal regime we can see what is the bound to impose on the UV ratio $\epsilon_{UV} = \rho_{UV}/\mu_{UV}$ such that

$$\epsilon_{IR} = \epsilon_{UV} \left( \frac{\Lambda_c}{E_{UV}} \right)^{\frac{1}{8\pi}(n - 2x + 2y - ym)} \ll 1$$  \hspace{1cm} (3.26)$$

In the Figures 1-6 we have plotted some region of the ranks $x$ and $n$ by fixing $\epsilon_{UV}$ and $\Lambda_c/E_{UV}$. The colored part of the figures represent the allowed region, where all the constraints are satisfied. We also plotted two lines delimiting the weakly coupled regime of the conformal window $(2\tilde{N} > (N_f^{(1)} + N_f^{(2)}))$ and the IR free window $(3\tilde{N} < (N_f^{(1)} + N_f^{(2)}))$. 

- 9 -
From the figures we see that smaller values of the ratio $\epsilon_{UV}$ guarantees that the running can be longer in the CFT window. The red region shaded in the figures, near $N_f^{(1)} + N_f^{(2)} = 3\tilde{N}$, is filled also if the running is extended over a large regime of scales. At the lower edge

**Figure 1:** $\frac{\rho_{UV}}{\mu_{UV}} = 10^{-2}, \frac{\Lambda_{c}}{\mu_{UV}} = 10^{-4}$

**Figure 2:** $\frac{\rho_{UV}}{\mu_{UV}} = 10^{-4}, \frac{\Lambda_{c}}{\mu_{UV}} = 10^{-4}$

**Figure 3:** $\frac{\rho_{UV}}{\mu_{UV}} = 10^{-2}, \frac{\Lambda_{c}}{\mu_{UV}} = 10^{-6}$

**Figure 4:** $\frac{\rho_{UV}}{\mu_{UV}} = 10^{-4}, \frac{\Lambda_{c}}{\mu_{UV}} = 10^{-6}$

**Figure 5:** $\frac{\rho_{UV}}{\mu_{UV}} = 10^{-2}, \frac{\Lambda_{c}}{\mu_{UV}} = 10^{-8}$

**Figure 6:** $\frac{\rho_{UV}}{\mu_{UV}} = 10^{-4}, \frac{\Lambda_{c}}{\mu_{UV}} = 10^{-8}$
of this region the anomalous dimensions are close to zero, the UV hierarchy imposed on the relevant deformations is preserved during the flow, and $\epsilon_{UV} \sim \epsilon_{IR}$. As we approach the strongly coupled region of the conformal window the anomalous dimensions get larger. In this case $\epsilon_{IR}$ approaches to one and we represented this behavior by changing the color of the shaded region from red to orange and then to yellow. The white part of the figures represents the region in which $\epsilon_{IR} > 1$.

In conclusion we have found regions in the parameter space where the theory possesses metastable non supersymmetric vacua. The RG flow analysis gives non trivial constraints on the relevant deformations and on the duration of the approximate conformal regime.

4. General strategy

We discuss here the generalization of the mechanism of supersymmetry breaking in SCFTs deformed by relevant operators. As in SSQCD, the lifetime of the metastable vacuum can be long in the conformal window of other models, with opportune choices of the parameters. Consider a $SU(N_c)$ gauge theory with $N^{(1)}_f$ flavors of quarks in the magnetic IR free window and with a metastable supersymmetry breaking vacuum in the dual phase. In the magnetic phase a new set of $N^{(2)}_f$ massive quarks must be added to reach the conformal window. If there is some gauge invariant operator $O$ that hits the unitary bounds, $R(O) < 2/3$, it is necessary to add other singlets and also marginal couplings in the superpotential between the quarks and these new singlets. The mass term for the new quarks is a relevant perturbation which grows in the infrared, and it has to be very small with respect to the other scales of the theory, down to the CFT exit scale. This mass term modifies the non perturbative superpotential and the supersymmetric vacuum, which sets the CFT exit scale. One must inspect a regime of couplings such that the supersymmetric vacuum is far away in the field space. This regime corresponds to a bound on the parameters of the theory, which have to be consistent with the RG running of the physical coupling constants. In the canonical basis the running of the physical couplings can be absorbed in the superpotential by the wave function renormalization of the fields. If there is a relevant operator $\Delta W = \eta O$, with classical dimension $\text{dim}(O) = d$, the physical coupling $\eta$ runs from the UV scale $E_{UV}$ to the IR scale $E_{IR}$ as

$$\eta(E_{IR}) = \eta(E_{UV}) Z_O(E_{IR}, E_{UV})^{-\frac{1}{2}} = \eta(E_{UV}) \left( \frac{E_{IR}}{E_{UV}} \right)^{\gamma/2} \quad (4.1)$$

We require that the running in this approximate conformal regime stops at the energy scale $\Lambda_c$ set by the masses at the supersymmetric vacuum. The bounds on the parameters that ensure the stability of the metastable vacuum have to hold at this IR CFT exit scale. The equation (4.1) translates these bounds in some requirements on the UV deformations. The metastable vacua have long lifetime if there is some regime of UV couplings in which the stability requirements are satisfied in the weakly coupled conformal window.

Here we have shown that in SSQCD there are some regions in the conformal window in which a large hierarchy among the couplings allows the existence of long living metastable vacua. We expect other models with this behavior.
5. Discussion

In this paper we discussed the realization of the ISS mechanism in the conformal window of SQCD-like theory. In [1] the metastable vacua disappeared if $3/2N_c < N_f < 3N_c$ because the non perturbative dynamics was not negligible in the small field region, and this destabilized the non supersymmetric vacua.

We have reformulated this problem in terms of the RG flow from the $UV$ cut-off of the theory down to the CFT exit scale. In the ISS model the CFT exit scale and the supersymmetry breaking scale are proportional because of the equation of motion of the meson. Their ratio depends only on the gauge coupling constant at the fixed point. The bounce action is proportional to this ratio and cannot be parametrically long.

This behavior suggests a mechanism to evade the problem and to build models with long living metastable vacua in the conformal window of SQCD-like theories. A richer structure of relevant deformations than in the ISS model is necessary. Metastable vacua with a long lifetime can exist if the bounce action at the CFT exit scale depends on the relevant deformations and it is not RG invariant. We have studied this mechanism in an explicit model, the SSQCD, and we have found that in this case, by adding a new mass term for some of the quarks, the bounce action has a parametrical dependence on the relevant couplings. The RG flow of these couplings for different regimes of scales sets the desired regions of $UV$ parameter that gives a large bounce action in the $IR$. We restricted the analysis to a region of ranks in which the model is interacting but weakly coupled, and the perturbative analysis at the non supersymmetric state is applicable. It is possible to extend this example to other SCFT theories as we explained in Section 4.

It would be interesting to find some dynamical mechanism to explain the hierarchy among the different relevant perturbations, that are necessary for the stability of the metastable vacua. For example in the appendix we see that in quiver gauge theories the mass of the new quarks can be generated with a stringy instanton as in [15, 16]. The supersymmetry breaking metastable vacua that we have found in the conformal window might be used in conformally sequestered scenarios, along the lines of [17]. Another application is the study of Yukawa interactions along the lines of [18, 19]. Superconformal field theories naturally explain the suppression of the Yukawa couplings if some of the gauge singlet fields are identified with the $T_i = 10_i$ and $F_i = \bar{5}_i$ generations of the $SU(5)$ GUT group. Here we have shown that supersymmetry breaking in superconformal sectors is viable. It is in principle possible to build a supersymmetry breaking SCFT where some of the generation of the MSSM are gauge singlets, marginally interacting with the fundamentals of the SCFT group. In this case the Yukawa arising from these generations can be suppressed as in [18, 19]. Since supersymmetry is broken one can imagine a mechanism of flavor blind mediation, like gauge mediation, to generate the soft masses for the rest of the multiplets of the MSSM. Closely related ideas has recently appeared in [10] and [20].

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A. The renormalization of the bounce action

In the paper we analyzed the bounce action at the CFT exit scale. We distinguished the infrared bounce action $S_{B,IR}$ from $S_{B,UV}$, the action evaluated at the UV scale. Indeed in a supersymmetric field theory in the holomorphic basis the bounce action is obtained from the Lagrangian

$$\mathcal{L} = Z_\phi \dot{\phi}^2 + Z_\phi^{-1} V(\phi) \quad (A.1)$$

and we have

$$S_{B,IR} = S_{B,UV} Z_\phi^3 \quad (A.2)$$

Hence the bounce action undergoes non trivial renormalization. Here we show that our analysis, performed in the canonical basis, is consistent with (A.2), both for the ISS model and for the model in Section 3.

The ISS bounce action in the UV is

$$S_{B,UV} = \left( \frac{\mu_{UV}}{\tilde{\Lambda}_{UV}} \right)^{\frac{4b}{N_f - N}} \quad (A.3)$$

In the IR this action is renormalized because of the wave function renormalization of the fields. In the paper we computed the action in the canonical basis and renormalization effects have been absorbed into the couplings. From (A.2) the IR renormalized action $S_{B,IR}$ is

$$S_{B,IR} = S_{B,UV} Z_\phi^3 \quad (A.4)$$

where the wave function renormalization is

$$Z_M = \left( \frac{E_{IR}}{E_{UV}} \right)^{-\gamma N} \quad (A.5)$$

We now compute $S_{B,IR}$ and show that indeed it is (A.4). The coupling $\mu_{IR}$ and the scale $\tilde{\Lambda}_{IR}$ are given as functions of their UV values

$$\mu_{IR} = \mu_{UV} Z_N^{-1/4}, \quad \tilde{\Lambda}_{IR} = \tilde{\Lambda}_{UV} \frac{E_{IR}}{E_{UV}} = \tilde{\Lambda}_{UV} Z_N^{-1/7N} \quad (A.6)$$

By substituting on the l.h.s. of (A.4) we have

$$S_{B,IR} = \left( \frac{\mu_{IR}}{\tilde{\Lambda}_{IR}} \right)^{\frac{4b}{N_f - N}} = \left( \frac{\mu_{UV} Z_N^{-1/4}}{\tilde{\Lambda}_{UV} Z_N^{-1/7N}} \right)^{\frac{4b}{N_f - N}} = S_{B,UV} Z_N^3 \quad (A.7)$$
where the last equality is obtained by substituting $\tilde{b} = 2N_f - 3N_c$ and $\gamma_N = 2\tilde{b}/N_f$. Nevertheless the bounce action in SQCD at the CFT exit scale results RG invariant. This is because the mass scales of the theory are related by the equation of motion of $N$. The relation between these scales is proportional to the gauge coupling which is constant during the running in the conformal window. For this reason the lifetime of the metastable vacuum cannot be parametrically large in SQCD.

In the model discussed in section 3 instead the bounce action depends non trivially on the relevant deformations

$$S_{B,IR} = \left(\frac{\bar{\Lambda}_{IR}}{\rho_{IR}}\right)^{4N_f^{(2)}} N_f^{(1)} - \tilde{N} \left(\frac{\mu_{IR}}{\Lambda_{IR}}\right)^{6N - 4N_f^{(1)}} N_f^{(1)} - \tilde{N} \gamma_N \right) \gamma_N \right) \gamma_N \right)$$

(A.8)

The UV bounce action has the same expression but in term of the UV couplings and scale. The IR coupling and scale are related to the UV values as

$$\mu_{IR} = \mu_{UV} Z_N^{-1/4} \quad \rho_{IR} = \rho_{UV} Z_N^{-\gamma_N} \quad \tilde{\Lambda}_{IR} = \tilde{\Lambda}_{UV} Z_N^{-\gamma_N}$$

(A.9)

The infrared bounce action is then

$$S_{B,IR} = S_{B,UV} Z_N^A$$

(A.10)

where

$$A = \frac{4N_f^{(2)}(\gamma_p - 1)}{(N_f^{(1)} - \tilde{N})\gamma_N} + \frac{(3\tilde{N} - N_f^{(1)})(4 - \gamma_N)}{(N_f^{(1)} - \tilde{N})\gamma_N} = 3$$

(A.11)

The last equality can be obtained by substituting the relations $\gamma_{\phi_i} = 3R[\phi_i] - 2$, with $R[N] = 2y$ and $R[p] = (n - x + y)/n$. Hence we verified the general result (A.2) concerning the renormalization of the bounce action.

B. The SSQCD

In this appendix we review the SSQCD defined in [6] and its behavior under Seiberg duality. The model is a $SU(N_c)$ gauge theory with quarks charged under the $SU(N_f^{(1)}) \times SU(N_f^{(2)})$ flavor symmetry and a singlet in the bifundamental of $SU(N_f^{(2)})$. The matter content is given in Table 2. The superpotential is

$$W = SP\tilde{P}$$

(B.1)

| $Q + Q$ | $\bar{N}_f^{(1)} \oplus N_f^{(1)}$ | $N_c$ |
|--------|------------------------------|-------|
| $P + P$ | $\bar{N}_f^{(1)} \oplus N_f^{(1)}$ | $N_c \oplus \bar{N}_c$ |
| $S$    | $\bar{N}_f^{(2)} \oplus N_f^{(2)}$ | $N_c \oplus \bar{N}_c$ |

| $\bar{N}_f^{(2)} \oplus N_f^{(2)}$ | $1$ |

Table 2: Matter content of the SSQCD
In the conformal window, $3/2N_c < N_f^{(1)} + N_f^{(2)} < 3N_c$ there is a Seiberg dual description, with $SU(N_f^{(1)} + N_f^{(2)} - N_c)$ magnetic gauge group with matter content given in Table 1, where the mass term for the field $S$ and the meson $M = P\tilde{P}$ is integrated out. The dual superpotential is

$$W = Kp\tilde{q} + L\tilde{pq} + Nq\tilde{q}$$

(B.2)

In the conformal window these theories are dual if there are no accidental symmetry, not manifest in the UV Lagrangian, that emerges in the IR.

If some accidental symmetry arise, some gauge invariant operator, $O$, in the chiral ring, violates the unitary bound and we have $R(O) < 2/3$ from the a-maximization.

The marginal term in the superpotential associated to this operator becomes irrelevant and can be neglected in the IR.

In SSQCD the first operator that hits the unitary bound is $N = Q\tilde{Q}$. By using the $y_{\text{max}}$ that we calculated in (3.25) we see that the unitary bound is hit at

$$x = \frac{1}{3} \left( 2 - 2n + \sqrt{1 - 14n + 13n^2} \right)$$

(B.3)

where $x = \frac{N}{N_f^{(1)}}$ and $n = \frac{N_f^{(2)}}{N_f^{(1)}}$. For higher values of $x$ the dual superpotential becomes

$$W = Kp\tilde{q} + L\tilde{pq}$$

(B.4)

In the paper we have studied a region were this meson does not hit the unitary bounds, and we can trust the duality without adding new operators.

**Figure 7:** The Stringy instanton contribution

**Relevant deformations**

Some deformations must be added to (B.2) to recover (3.1). The linear term for $M$ can be generated in the electric gauge theory by adding a mass term for the quarks $Q$ and $\tilde{Q}$, while the mass term for the field $p$ and $\tilde{p}$ can be generated by adding a linear deformation $k^2S$. When we integrate out the mass term $mMS$ in the magnetic theory the fields $p$ and $\tilde{p}$ acquire a mass term proportional to $\rho = k^2/m$.

However a large hierarchy is required between the scale $\mu$ and the mass $\rho$ for the existence of the metastable vacua. We can impose this hierarchy at hand or find a dynamical mechanism. For example, when $N_f^{(2)} = 1$ we can think to embed the magnetic theory in a quiver and couple the fields $p$ and $\tilde{p}$ with an $SP(0)$ node as in Figure 7.
In the instantonic action an interaction $S \sim \alpha p \tilde{p} \beta$ between the instanton moduli and the fields is present. By integrating over the instantonic zero modes we are left with the desired suppressed mass term

$$\Delta W = \int d\alpha d\beta e^{S_{\text{inst}}} = \Lambda e^{-A p \tilde{p}}$$

for the $p$ and $\tilde{p}$ quarks, where $\Lambda$ represents the area of curve associated to the $SP(0)$ node and $\Lambda$ is associated to a string scale.

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