Transport through Andreev bound states in a Weyl semimetal quantum dot

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We study transport through a Weyl semimetal quantum dot sandwiched between an s-wave superconductor and a normal lead. The conductance peaks at regular intervals and exhibits double periodicity with respect to two characteristic frequencies of the system, one that originates from Klein tunneling in the system and the other coming from the chiral nature of the excitations. Using a scattering matrix approach as well as a lattice simulation, we demonstrate the universal features of the conductance through the system and discuss the feasibility of observing them in experiments.

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Introduction.—Weyl semimetals (WSMs) are 3D topological systems with an even number of Weyl nodes in the bulk, with low-energy excitations having a definite chirality when the Fermi energy is near the Weyl nodes. The study of such systems has exploded in recent times both in the theoretical [21,31] as well as the experimental [33,35] front. The reason for this excitement is the non-trivial physics that can arise in Weyl systems, such as broken chiral symmetry (or the chiral anomaly) and Fermi arcs where the Fermi vector at one surface is a discontinuous arc that connects to the other surface through the bulk, giving rise to exotic physical properties and transport signatures.

A quantum dot made of WSM material in the presence of superconductors is of particular interest due to the distinctive nature of transport at a WSM-Superconductor (SC) interface [24,25,26,27] and provides the possibility of capturing the otherwise elusive physics associated with the chiral excitation in the WSM [28]. In this manuscript we study transport through the Andreev states of a WSM quantum dot in a simple setup where we sandwich the dot in between a superconductor and a normal lead (see Fig. 1). Bound levels will form in the dot due to multiple reflections from the two boundaries and these levels will strongly depend on the Fermi-energy mismatch between the dot and the SC, as well as on the size of the dot. One expects some of the physics of a graphene quantum dot [29] to carry over to this case, since the WSMs also have a linear dispersion; however there are differences as well. One of the features of the Dirac dispersion is that the Andreev bound states carry current that oscillates as a function of $\chi = V_0 L/v_F$, where $V_0$ is the chemical potential of the dot, $L$ its size and $v_F$ is the Fermi velocity. A second oscillation appears as a function of $\delta k L$, where $\delta k$ is the momentum separation of the nodes that are connected by superconducting pairing. In graphene, an s-wave superconductor couples electrons at one valley with holes at the other valley and the Andreev bound states are hence also dependent on the matching of the valley polarizations, with $\delta k = K - K'$ as the separation of the valleys in momentum space. On the other hand in a WSM-SC interface, the s-wave superconductor is required to couple the electrons at one node with the holes at the other node. Hence, reflection processes couple one chiral node to another node of opposite chirality [21,22] and $\delta k = 2k_0$ where $2k_0$ is the distance between the nodes in momentum space. Coupling between nodes is otherwise forbidden, irrespective of their positions in momentum space. Further, the inter-valley length scale $K - K'$ in graphene is quite large, whereas in WSMs, $2k_0$ is a relevant length scale, because the nodes are typically quite close to each other. At finite bias, however, as we shall see below, the relevant parameter changes from $2k_0 L$, and the nature of the bands becomes important. In the rest of this paper, our focus is to study and predict the behavior of the current through the Andreev bound states of the WSM quantum dot at a finite bias.

The central result of our work is to disentangle the periodicity of the conduction peaks of the WSM quantum dot (with a superconductor on one edge) due to the chirality of the nodes from the periodicity due to the finite bias and Klein tunneling. We find that at finite bias the conduction peaks follow a periodic pattern of the form $(q_\pm) L = 2n\pi$, with $n$ being integers. Here $q_\pm$ are the Fermi momenta in the quantum dot at finite bias, along the direction of conduction, and are the analogs of $k_0$ at zero bias. Their values can be determined from the band structure of the system and the bias $V_0$ present in the dot. At small enough bias, the periodicity reduces to the expected $2k_0 L = n\pi$ oscillations [24,25].

Model and setup.—The simplest model of a WSM with

FIG. 1. Setup of the system. A time-reversal broken Weyl semimetal WSM of length $L$ has been sandwiched between a superconductor (SC) with a gap $\Delta$ and a normal/WSM metal lead (N). The momentum separation between the Weyl nodes in the WSM dot is $2k_0$ and the WSM has a bias $V_0$. 
broken time-reversal (TR) symmetry requires two chiral nodes in momentum space, whereas the simplest WSM with broken inversion symmetry requires the presence of four chiral nodes. In the main text we restrict ourselves to using the simplest model of a TR-broken WSM, having two nodes, for analytic simplicity. We consider an inversion symmetry broken model, which also has some new aspects beyond what is present in the two node model, in the appendix.

A two-band TR-broken WSM model can be obtained by starting from a four-band Hamiltonian describing a 3D TI in the Bi$_2$Se$_3$ family and including a time-reversal breaking perturbation $b$ [2].

$$H_0 = \epsilon_k \tau_x - \lambda_z \sin k_z \tau_y - \lambda \tau_z (\sigma_x \sin k_y - \sigma_y \sin k_z) + b_z \sigma_z + V_0,$$  \hspace{1cm} (1)

where $\epsilon_k = \epsilon - 2t \sum_i \cos k_i$ is the kinetic energy and $\lambda, \lambda_z$ are spin-orbit coupling strengths. In the limit $\lambda_z \ll \epsilon - 6t \ll b_z$, this gives a WSM phase, where the gap closes at momentum points $(0, 0, \pm k_0)$, where $ik_0^2 \approx b_z - \epsilon + 6t$. A gate potential $V_0$ is applied to the dot region which spans a distance $L$. For sufficiently small $V_0$, the low energy excitations can be described by the two-band Hamiltonian

$$H_{WSM} = \hat{\epsilon}_k \sigma_z + \lambda (k_x \sigma_x + k_y \sigma_y) + V_0,$$  \hspace{1cm} (2)

with $\hat{\epsilon}_k \approx i(\tau^2 + k_z^2 - k_0^2)$ and with $\tau^2 = k_x^2 + k_y^2$. In the rest of the paper, all parameters are scaled with respect to $\hat{\epsilon}$ which is the energy scale. The eigenvalues of Eq. (2) are $E_{\pm}(k) = \pm \sqrt{\epsilon^2 + \lambda^2 \tau^2 + V_0}$. This implies that the Fermi velocity is anisotropic - the velocity in the $z$ direction is different from that in the $x$, $y$ direction. Close to the Weyl nodes $k_z = \pm k_0$, the Fermi velocity along the $z$-direction $v_z = 2k_0$.

We construct a WSM dot by sandwiching the dot region (with a finite $V_0$) in between a normal-metal (N) and an $s$-wave superconductor (S). We then study transport through the quantum dot, first using a scattering matrix approach, where the N region is chosen to be an unbiased WSM ($V_0 = 0$) and we use Eq. (2) to solve for the wavefunctions. Next, we further study and verify our findings using a lattice simulation where we model the normal metal using a flat band approximation, i.e. by considering a uniform density of states within the relevant energy scales.

The superconducting region can be described in terms of the Bogoliobov-de Gennes (BdG) Hamiltonian:

$$H_{SC} = \left( \begin{array}{cc} \xi_k \tau_{2x2} & \Delta i \sigma_y \\ -\Delta i \sigma_y & -\xi_k \tau_{2x2} \end{array} \right),$$  \hspace{1cm} (3)

where $\Delta$ is the pairing potential in the superconductor and $\xi_k = (\hbar^2(k_x^2 + k_y^2 + k_z^2)/2m_S - \mu_S)$. $m_S$ is the effective mass of the electron in the superconductor (we take $m_S \approx m$ for simplicity) and $\mu_S$ is the chemical potential. The parameter $\mu_S$ depends on the details of the superconducting material. In the numerical results shown, we take $\mu_S \gg \Delta$, which is the realistic limit.

**Scattering matrix approach.**—Using familiar methods of solving for the wavefunction and matching them at the two boundaries, we obtain the net reflection matrix of the form

$$S(E, \mathbf{p}) = \left( \begin{array}{cc} r_{ee}(E, \mathbf{p}) & r_{he}(E, \mathbf{p}) \\ r_{eh}(E, \mathbf{p}) & r_{hh}(E, \mathbf{p}) \end{array} \right),$$  \hspace{1cm} (4)

where, $r_{ee}$ and $r_{hh}$ are the reflection matrices, and $r_{eh}$ and $r_{he}$ are the Andreev reflection matrices, in the basis of excitations near the two nodes with $\pm$ chirality [1321]. $E$ is the incident energy and $\mathbf{p} = (p_x, p_y, 0)$ is the momentum in the transverse direction. The differential conductance is then written as

$$G_{ee}(E) = \frac{e^2}{h} Tr |I_2 - R_{ee}(E, \mathbf{p}) R_{ee}(E, \mathbf{p})^\dagger| + R_{he}(E, \mathbf{p}) R_{he}(E, \mathbf{p})^\dagger,$$  \hspace{1cm} (5)

where,

$$R_{ee(he)} = \left( \begin{array}{cc} v^+_{e(h)} & 0 \\ 0 & v^+_{e(h)} \end{array} \right) R_{ee(he)} \left( \begin{array}{cc} \frac{1}{\sqrt{v^+_{e}}} & 0 \\ 0 & \frac{1}{\sqrt{v^+_{h}}} \end{array} \right).$$

where $v^+_{e(h)}$ is the velocity of the electron (hole) channel of the $j$th node. The nature of processes at the WSM-SC boundary is depicted in Fig. 2. The relation in Eq. 5 is true for each momentum $\mathbf{p}$ in the transverse direction. Finally, we integrate over the transverse momentum to obtain the current $G(E) = \sum_{\mathbf{p}} G_{ee}(E)$. 

![FIG. 2. Diagrammatic representation of the possible scattering processes showing all the relevant aspects. The Weyl nodes are located at $k_z = \pm k_0$ and $q_{\pm}$ are the two possible momenta of electronic excitations above the finite potential barrier $V_0$ of the WSM. $\tan \theta$ denotes the Fermi velocity $v_F$ of such excitations. $I$ describes an incident electron and $R$ and $AR$ describe normal and Andreev reflected electrons and holes respectively.](image-url)
multiple reflections in the dot region, similar to those of a graphene dot, when $2k_0$ and $\nu_F$ are respectively quantum mechanical double barrier problem. But for a WSM, such reflections can only take place from one chiral node to the other chiral node of opposite chirality (c.f. Fig. 3), with inter-nodal distance $2k_0$. At finite bias, due to the presence of $V_0$, the relevant length scale depends on a combined function of $k_0$ and $V_0$, i.e., they depend on $q_{\pm} = \sqrt{k_0^2 \pm 2m_0\nu_FV_0}$, which are momenta along the direction of propagation at the Fermi energy in the dot-region. This allows us to predict the oscillation frequencies depending on the symmetry, the positions of the Weyl nodes, the bias, etc. In the present model, the conductance can be fitted well with the functional dependence of the form

$$G = \alpha + \beta \sin [(q_+ + q_-)L] \sin [(q_+ - q_-)L],$$

where, $\alpha, \beta$ are independent of the length $L$, and can, in principle, be obtained analytically, as shown in the appendix. In Fig. 3(a), we show the pattern of the conductance obtained at normal incidence, $G_0$, fitted with a function of the form given in Eq. (6). The close correspondence shows that the theoretically obtained function can predict all the peaks in the conductance $G$. In Fig. 3(b), we show the full conductance, after integrating over the transverse momenta. The conductance continues to peak at values of $L$ where $(q_+ + q_-)L/\pi$ is an integer. Finally, in Fig. 3(c) we show the variation of $G_0$ as functions of both the barrier height $V_0$ and $k_0$. This pattern can be fully predicted from the functional dependence in Eq. (6).

Note that for $V_0 \ll k_0^2$, $q_{\pm} \approx k_0 \pm (m_0 V_0/k_0)$. We also note that the amplitude of the velocity at the Fermi energy in the dot-region is $\nu_F = k_0/m_0$. So, the conductance oscillations have a slow frequency envelope whose period is $L/v_F = n\pi$ and a faster oscillation characterized by $k_0L = m\pi$, (where $n, m$ are integers), allowing us to write the conductance as

$$G \approx \alpha + \beta \sin (2k_0L) \sin (2V_0L/v_F),$$

with corrections to the above equation appearing only at the order $O(V_0^2/k_0^2)$. Note however, that in Fig. 3 we have specifically chosen a value of $V_0$, such that condition for Eq. (7) is not satisfied. In the regime, when the condition for Eq. (7) is satisfied, we find that the periodicity for the conductance shows peaks as a function of $L$ and $V_0$ whenever $k_0L = \pi n$ and $V_0L/v_F = n\pi$ as expected.

Finally, we also note that the amplitude $\beta$ of conductance oscillations depends strongly on the ratio $k_F/k_0$ and increases with increasing $V_0$. On the other hand, $\beta$ decreases with increasing incident energy $E$ and the conductance reaches a maximum value of $4e^2/h$, and becomes independent of the barrier height $V_0$ in the limit $E \to \Delta$, matching earlier results in similar systems like graphene. We discuss the dependence of $\beta$ on $E$ and other parameters in the appendix. In passing, we also note that a similar functional dependence (as shown in Eq. (6)), of the conductance oscillations would be true for a graphene dot, when $2k_0$ and $\nu_F$ are respectively
replaced by the momentum separation between the two valleys of graphene \( K - K' \) and the Fermi velocity near the Fermi energy.

Lattice simulation.—In order to study transport in our geometry, we implement a slight modification of the standard Landauer-Buttiker formalism to suit our purpose. We write the Fourier transformed Hamiltonian of Eq. (1)

\[
\begin{align*}
H & = H_L + H_W + \sqrt{\rho}S \\
\mathbf{\Sigma} & = \mathbf{\Sigma}_L = (\omega - H_L - \mathbf{\Sigma}_L)^{-1} \\
\mathbf{G} & = (\omega - H_W - \mathbf{\Sigma}_SC - \mathbf{\Sigma}_L)^{-1}
\end{align*}
\]

where \( \mathbf{\Sigma}_L \) is the self-energy matrix for the leads, \( \mathbf{\Sigma}_SC \) is the self-energy matrix for the superconducting dots, and \( \mathbf{\Sigma}_W \) is the self-energy matrix for the WSM dots. Further, we implement a flatband approximation for the self-energy matrix of the leads, where the density of states of the lead, \( \rho_L(\omega) \), is taken to be a constant independent of the energy, and so \( \mathbf{\Sigma}_L = -i\pi\rho_L \). The Green’s function for the superconducting lead \( \mathbf{G}_SC \) is obtained by recursively solving for the surface Green’s function of the s-wave superconductor. A schematic diagram that represents this process is presented in Fig. 3(a).

We then compute the current flowing from a site \( z \) to \( z + 1 \) in the WSM dot given by

\[
J_z(t) = -\frac{2e}{h} (\tilde{t} + \lambda_z \tau) \text{Im}(\Psi^\dagger_{z+1,\tilde{\tau},\sigma}(t)\Psi_{z,\tau,\sigma}(t))
\]

where \( \Psi_{i,\tau,\sigma} \) is a column matrix representing the annihilation operator at site \( i \) in the basis of orbital index \( \tau = -\tilde{\tau} \pm \) and spin index \( \sigma = \pm \). The information about the chemical potential of the leads (and the temperature, in principle) is included when averaging over the lead states. We show in the appendix C that this current can be written in terms of the Green’s function of the full system \( \mathbf{G}(\omega) \), at zero temperature, as

\[
J_z = \sum_{P=SC,L} \epsilon \text{Im} \left\{ \int d\omega Tr [ A \mathbf{G}_z,1(\omega) \zeta^P(\omega) \mathbf{G}^I_{z+1,1}(\omega) ] \right\}
\]

where \( A_{31} = A_{42} = -\tilde{t} + \lambda_z \) and \( A_{33} = A_{44} = -\tilde{t} - \lambda_z \) and \( A_{32} = 0 \) otherwise. Here, \( \zeta^{SC}(\omega) = \sqrt{\rho_{SC}(\omega)} \Psi^{\dagger} \) and \( \zeta^{L}(\omega) = \sqrt{\rho_{L}(\omega)} \Psi^{\dagger} \). We further consider the simplest case when the system is translation invariant in the transverse direction, so that the transverse momentum is just a parameter.

We obtain the current as a function of \( k_0 \) and \( V_0 \) with the chemical potential on the left lead kept fixed at \( \Delta = 0 \), and summarize the results in Figs. 3(b) and (c), where we have also taken the transverse momentum to be zero. As in the scattering matrix calculation, here again, the current oscillates as a function of both \( k_0L/\pi \) and \( V_0L/\pi v_F \), which clearly confirms the central result of our paper that the inter-node Andreev reflection, if not prohibited by additional symmetries of the problem, plays a crucial role in determining transport properties of the Weyl semimetal-superconducting interface.

The distinct unambiguous signatures of WSM systems can be further clarified if one takes an inversion symmetry broken WSM. An inversion broken WSM requires the presence of at least four chiral nodes in the Brillouin zone. In the simplest situation, the nodes can be co-linear in momentum space, and without loss of generality, can be placed at momentum \( \mathbf{k}_1 = (-k^+, 0, 0) \), \( \mathbf{k}_2 = (-k^-, 0, 0) \), \( \mathbf{k}_3 = (k^-, 0, 0) \), \( \mathbf{k}_4 = (k^+, 0, 0) \). Time reversal symmetry requires the first and last nodes to have the same chirality, and the two nodes in the middle to have opposite chirality. If the chirality of the nodes were not relevant -i.e., if we were working with a 3 dimensional...
Dirac metal, then proximity to an s-wave superconductor would couple nodes of opposite momenta through Andreev processes. So we would expect the relevant momentum scales to be $2k_F$. But in a WSM the coupling is only allowed between nodes 1-2, and 3-4, giving the relevant momentum scale $k^+ - k^-$ and between nodes 1-3, and 2-4, giving the relevant momentum scale $k^+ + k^-$. Thus the relevant scales of the conductance oscillations strongly distinguishes between a dot made of a Dirac metal from a dot made of a WSM. However, working with a 4-band model is cumbersome in the scattering matrix framework. We discuss the lattice results of such a WSM dot in the appendix.

Summary.—To summarise, we have discussed transport through a normal-metal-WSM-superconductor geometry, that captures a number of features unique to the presence of chiral nodes in the WSM. We took a simple time-reversal broken WSM and studied it in the scattering matrix approach as well as by using tight-binding simulations. The key result of our work, Eq. (6), differentiates the effect of Klein tunneling in the Dirac system from that due to the presence of chiral nodes in the WSM. An experimental setup should be similar in essence to that shown in Ref. [40], but the details of the prediction would depend on the material used.

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APPENDIX

A. Solving the scattering problem

In this section, we describe the derivation of the scattering matrix in a Normal-WSM dot-SC system. As described in the main text, the normal Hamiltonian is modelled by a WSM Hamiltonian without any chemical potential whereas the WSM dot is modelled by the same WSM Hamiltonian along with a barrier potential \( V_0 \). We define \( V(z) = V_0(\Theta(z) - \Theta(z - L)) \) where \( \Theta(z) \) is the Heaviside function. The locations of the Normal-WSM dot junction and the WSM dot-SC junctions are at \( z = 0 \) and \( z = L \) respectively.

The wavefunction corresponding to energy \( E \) in the normal system for \( z < L \) is given by the following energy eigenstates of Eq. (2) of the main text in the Nambu-Gor’kov space (with the Hamiltonian in the hole space written as \( -H_{\text{WSM}}^{\text{SC}}(-k) \)).

\[
\psi_N(z < 0) = \sum_{\sigma = \pm} \left\{ \mathcal{E}^\sigma \left( a_R^\sigma e^{\sigma ikz} + a_L^\sigma e^{-\sigma ikz} \right) + \mathcal{H}^\sigma \left( b_R^\sigma e^{-\sigma ikz} + b_L^\sigma e^{\sigma ikz} \right) \right\}, \tag{11}
\]

and similarly, the wavefunction in the WSM dot corresponding to the same energy is given by:

\[
\psi_{\text{WSM}}(0 < z < L) = \sum_{\sigma = \pm} \left\{ \mathcal{E}^\sigma \left( c_R^\sigma e^{\sigma ikz} + c_L^\sigma e^{-\sigma ikz} \right) + \mathcal{H}^\sigma \left( d_R^\sigma e^{-\sigma ikz} + d_L^\sigma e^{\sigma ikz} \right) \right\}. \tag{12}
\]

Here \( \sigma = \pm \) is the band index, \( a_i, c_i(b_i, d_i) \) denote the electron (hole) amplitudes with \( i \in \{ L, R \} \) denoting the left or right moving solution. \( \mathcal{E}^\sigma(\mathcal{H}^\sigma) \) are normalized eigenvectors, which are non-zero in electron (hole) sector of the Hamiltonian. In each sector \( \mathcal{E}(\mathcal{H})^+ \propto (f_{\pm\pm}(h), (-\lambda_{\pm\pm})^T) \), and \( \mathcal{E}(\mathcal{H})^- \propto ((-\lambda_{\pm\pm}), f_{\pm\pm}(h))^T \), with \( f_{\pm\pm}(h) = \mu_W + V(x) + (-E_i + \sqrt{\mu_W + V(x) + (-E_i)^2 - (\lambda p)^2}, \lambda_{\pm} = \lambda(k_x + ik_y) \).

In the superconductor, the solutions of Eq. (3) of the main text are:

\[
\psi_{\text{SC}}(z > L) = \begin{pmatrix} u_+ & u_- & v_+ & v_- \end{pmatrix} e^{iqz} + \begin{pmatrix} v_+ & v_- & u_+ & u_- \end{pmatrix} e^{-iqz},
\]

where, with \( \Omega = \sqrt{\Delta^2 - E_i^2} \),

\[
u(v) = \sqrt{(E_i + (-i\Omega))/2E_i}.
\]

and \( q_e \) and \( -q_h \) are, respectively, the outgoing electron and hole momenta in the superconductor, defined as

\[
q_e(h) = \sqrt{k_F^2 - p^2} - (-)^m_s\Omega/\hbar^2.
\]

The boundary conditions at \( z = \{ 0, L \} \) are given by the continuity of the wavefunction and its derivative at that point:

\[
\begin{align*}
\psi_N(z = 0) &= \psi_{\text{WSM}}(z = L)
\end{align*}
\]

\[
\begin{align*}
\partial_z \psi_N(z) &= \partial_z \psi_{\text{WSM}}(z) \bigg|_{z = 0} = \partial_z \psi_{\text{WSM}}(z) \bigg|_{z = L},
\end{align*}
\]

with \( \sigma_z \) being the Pauli matrix. As was mentioned in the main text, we take \( m_s \approx m_W \) for simplicity. By solving these equations, we get the reflection matrices,

\[
\begin{pmatrix} a_R^+ \\ a_L^+ \\ b_R^- \\ b_L^- \end{pmatrix} = \begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{hh} \end{pmatrix} \begin{pmatrix} a_R^+ \\ a_L^+ \\ b_R^- \\ b_L^- \end{pmatrix}, \tag{13}
\]

which were used in the main text.

![FIG. 5. (a) Variation of the amplitude of oscillation (\( \beta \)) of the zero-bias conductance with the barrier height along the \( x \) axis and the ratio of the Fermi momentum and the separation of Weyl nodes along the \( y \) axis. The parameters used are \( k_0 = 1, m_S = m_W = 0.5, \lambda = 0.5, \Delta = 0.01 \). (Here we only consider normal incidence). (b) Conductance as a function of the length of the barrier along the \( x \) axis and the incident energy along the \( y \) axis at fixed \( V_0 = 0.2 \)](image-url)
to the Fermi energy mismatch. We show the numerical fitting of $\beta$ in the phase space of $k_F - V_0$ in Fig. 5(a).

With increasing incident energy $E$, the net conductance reaches a universal value of $4e^2/h$ when $E/\Delta$ reaches unity as depicted in Fig. 5(b).

### C. Details of the tight-binding simulation

Here we briefly describe how we arrive at Eq. (10) of the main text. For the TR symmetry broken Weyl semimetal, the Hamiltonian is written as $H = H_C + H_{SO} + H_E$, where,

$$H_C = -\tilde{t} \sum_{\langle \alpha, \alpha' \rangle} \psi^\dagger_{\alpha z} \tau^z \sigma I_\alpha \psi_{\alpha' z'} + \epsilon \sum_{\alpha} \psi^\dagger_{\alpha z} \eta \tau^z I_\alpha \psi_{\alpha z} + h.c.$$  

$$H_{SO} = i\lambda \sum_{\alpha} \left( \psi^\dagger_{\alpha z} \eta \tau^z \sigma I_\alpha \psi_{\alpha z} + \psi^\dagger_{\alpha z} \eta \tau^z \sigma I_\alpha \psi_{\alpha z} + h.c. \right)$$  

$$H_E = \sum_{\alpha} \left( \sum_{\alpha} b_{\alpha} \eta \tau^z \sigma I_\alpha \psi_{\alpha z} + b_\alpha \psi_{\alpha z} \sigma \psi_{\alpha z} + h.c. \right).$$

Here $\psi^\dagger_{\alpha \sigma n}$ is the creation operator of electron with spin $\sigma$ and orbital index $n$ at site $i$ of the WSM. We consider the $x, y$ directions to be translationally invariant, so that the momenta $k_x, k_y$ appear as parameters. After Fourier transforming in the $x, y$ directions, our next step is to rewrite the Hamiltonian in the Nambu-Gorkov form -

$$H^W = \frac{1}{2} \sum_{\langle \alpha, \alpha' \rangle} \Psi^\dagger_{\alpha z} h_{ij}(k_x, k_y) \Psi_{\alpha' z', j},$$  

using the basis

$$\Psi^\dagger_{\alpha z} = \left( \psi^\dagger_{\alpha z, 1}, \psi^\dagger_{\alpha z, 2}, \psi^\dagger_{\alpha z, 3}, \psi^\dagger_{\alpha z, 4}, \psi^\dagger_{\alpha z, 5}, \psi^\dagger_{\alpha z, 6}, \psi^\dagger_{\alpha z, 7}, \psi^\dagger_{\alpha z, 8} \right).$$

For each site $z$, the basis $\Psi_{\alpha z}$ has 8 components for $\alpha = 1, \ldots, 8$. The superconductor is modeled as a 1D superconductor:

$$H^S = \sum_{\alpha} \Phi^\dagger_{\alpha z} (\epsilon_{SC} \eta^2 + \Delta \eta^2) \Phi_{\alpha z}$$  

$$- t_{SC} \sum_{\langle \alpha, \alpha' \rangle} \Phi^\dagger_{\alpha z} \eta \tau^z I_\alpha \Phi_{\alpha' z'} + h.c. \equiv \frac{1}{2} \sum_{\langle \alpha, \alpha' \rangle} \Phi^\dagger_{\alpha z} h_{ij} \Phi_{\alpha' z', j},$$

where $\Phi^\dagger_{\alpha z} = (\phi^\dagger_{\alpha z, 1}, \phi^\dagger_{\alpha z, 2}, \phi^\dagger_{\alpha z, 3}, \phi^\dagger_{\alpha z, 4}, \phi^\dagger_{\alpha z, 5}, \phi^\dagger_{\alpha z, 6}, \phi^\dagger_{\alpha z, 7}, \phi^\dagger_{\alpha z, 8})$. The normal leads Hamiltonian is the written as:

$$H^L = \frac{1}{2} \sum_{\langle \alpha, \alpha' \rangle} a^\dagger_{\alpha z} h_{Lij} a_{\alpha' z', j},$$

in the basis $a^\dagger_{\alpha z} = (\alpha^+_z, \alpha^+_z, \alpha^-_z, \alpha^-_z)$. The tunneling Hamiltonian between the WSM and the superconductor and between the WSM and the normal leads are given respectively by:

$$H^{WS} = \frac{1}{2} \sum_{\langle \alpha, \alpha' \rangle} \Psi^\dagger_{\alpha i} \Psi_{\alpha' j} \phi_{\alpha i} + \frac{1}{2} \sum_{\langle \alpha, \alpha' \rangle} \Psi^\dagger_{\alpha i} \Psi_{\alpha' j} \phi_{\alpha' j} \Psi_{\alpha' j} \psi_{\alpha' j}$$

and

$$H^{WL} = \frac{1}{2} \sum_{\langle \alpha, \alpha' \rangle} \Psi^\dagger_{\alpha i} \psi_{\alpha' j} \phi_{\alpha i} + \frac{1}{2} \sum_{\langle \alpha, \alpha' \rangle} \Psi^\dagger_{\alpha i} \psi_{\alpha' j} \phi_{\alpha' j} \Psi_{\alpha' j} \psi_{\alpha' j}.$$

Here, $\phi^\dagger_{\alpha}$ and $a^\dagger_{\alpha}$ are, respectively, the creation operators at the superconductor and the normal lead, without any orbital index. Also note that we couple both orbitals equally to the superconducting site, which, albeit not the most generic case, represents the simplest coupling.

With this choice of basis,

$$\Psi^\dagger_{i=SC/L} = \begin{pmatrix} t^i & 0 & t^i & 0 & 0 & 0 & 0 & 0 \\ 0 & t^i & 0 & t^i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -t^i & 0 & -t^i & 0 \\ 0 & 0 & 0 & 0 & 0 & -t^i & 0 & -t^i \end{pmatrix}^T,$$

where $t^i = t^{SC/L}$ are the hopping matrix elements between the leads and the WSM. The Hamiltonian has an explicit particle-hole symmetry under

$$\Phi^\dagger_{\alpha i} = C_{ij} \Phi_{\alpha j}, \quad a^\dagger_{\alpha i} = C_{ij} a_{\alpha j} \quad \Psi^\dagger_{\alpha i} = C^{W}_{ij} \Psi_{\alpha j}$$

where $C = \sigma^y \otimes \sigma^x$, and $C^{W} = \sigma^y \otimes \mathbb{I} \otimes \sigma^y$.

Now, we wish to compute how the field operators evolve in time. Starting from the Heisenberg equation of motion

$$a_{\alpha i} = \frac{i}{\hbar} \left[ H^L + H^{WL}, a_{\alpha i} \right],$$

we obtain

$$\dot{a}_{\alpha} = \frac{i}{\hbar} (\partial_t - h_L) a_{\alpha} = \nabla^L \Psi_{\alpha i}. $$

The solution for the operator is given by

$$a(t) = \int_{t_0}^{t} dt' \Psi_{\alpha i}(t') + \int_{t_0}^{t} dt' \nabla^L \Psi_{\alpha i}(t'),$$

where the Green’s function $\mathcal{G}_L$ of the uncoupled lead is the solution of the equation

$$(i \hbar \partial_t - h_L) \mathcal{G}_L(t - t') = \mathbb{I} \delta(t - t').$$

Similarly, for the superconducting lead, one obtains

$$\Phi(t) = \int_{t_0}^{t} dt' \mathcal{G}_S(t - t') \Psi(t') + \int_{t_0}^{t} dt' \mathcal{G}_S(t - t') \nabla^S \Psi(t'),$$

where $\mathcal{G}_S(t - t') = \mathbb{I} \delta(t - t')$. 

$$\Phi(t) = \frac{1}{2} \sum_{\langle \alpha, \alpha' \rangle} \Phi^\dagger_{\alpha i} \Phi_{\alpha' j}.$$
Finally, for the operators in the Weyl semi-metal, we write:

$$\dot{\Psi} = \frac{i}{\hbar} (\Psi_{\text{t}} - hW_{\text{t}} \Psi - \nabla^L a - \nabla^S \Phi).$$

(25)

In the above equation, we need to substitute the solutions of $a(t)$ and $\Phi(t)$. We define the self energy operators as

$$\Sigma_L(t) = \int_0^t dt' \Psi^\dagger(t') \mathcal{G}_L(t - t') \nabla^L \Psi$$

and

$$\Sigma_S(t) = \int_0^t dt' \Psi^\dagger(t') \mathcal{G}_S(t - t') \nabla^S \Psi.$$ 

(26)

Fourier transforming the equation for $\Psi(t)$, we obtain

$$\Psi(\omega) = \mathcal{G}_W(\omega) \Gamma(\omega)$$

where $\mathcal{G}_W = (\omega - hW/\hbar - \Sigma_L(\omega)/\hbar - \Sigma_S(\omega)/\hbar)^{-1}$ is the Green’s function of the whole system and $\Gamma(\omega) = \frac{1}{\tau} (\nabla^S \eta S(\omega) + \nabla^L \eta L(\omega)).$

When the system is finite along the $z$ direction and periodic along $x, y$:

$$\hat{N}_z = \frac{i}{\hbar} [\hat{H}, \hat{N}_z]$$

$$= \frac{i}{\hbar} (-i + \lambda_z \tau) \left( \langle \Psi^\dagger_{z+1, \tau, \sigma}(t) \Psi_{z, \tau, \sigma}(t) \right)$$

$$- \langle \Psi^\dagger_{z, \tau, \sigma}(t) \Psi_{z+1, \tau, \sigma}(t) \rangle.$$ 

(27)

Here we have used the explicit form of the Hamiltonian in the main text of the paper. So, the current along $z$ from a given site $z$ to $z+1$:

$$J_z(t) = \frac{ie}{\hbar} (-i + \lambda_z \tau) \left( \langle \Psi^\dagger_{z+1, \tau, \sigma}(t) \Psi_{z, \tau, \sigma}(t) \right)$$

$$- \langle \Psi^\dagger_{z, \tau, \sigma}(t) \Psi_{z+1, \tau, \sigma}(t) \rangle.$$ 

(28)

Now, Fourier transforming the field operators, we have,

$$\langle \Psi^\dagger_{z,i}(t) \Psi_{z+1,j}(t) \rangle = \int_{\omega, \omega'} \langle \Psi^\dagger_{z,i}(\omega) \Psi_{z+1,j}(\omega') \rangle e^{i(\omega - \omega')t},$$ 

(29)

with

$$\langle \Psi^\dagger_{z,i}(\omega) \Psi_{z+1,j}(\omega') \rangle$$

$$= \sum_{P, P'} G_{W_{z+1, I, j, m}^P(\omega)} \zeta_{m, i}^{P'}(\omega) G_{W_{z, i+1, l}^P(\omega)} \delta(\omega - \omega').$$ 

Here $\zeta_{m, i}^{P'}(\omega) = (\nabla^P_{I, j, m} P'_{I, j} P')_{m, i}$ where $\{I, P\}$ is either $\{1, L\}$ or $\{N, SC\}$ denoting either the normal or the superconducting lead respectively.

Putting everything back in, we can finally evaluate the current

$$J_z(t) = \text{eIm} \int d\omega \text{Tr} \left[ \mathcal{A} G_{W_{z, j}^P(\omega)} \sigma^P(\omega) G_{W_{z+1, l}^P(\omega)} \right],$$

(30)

where $\mathcal{A}_{31} = \mathcal{A}_{42} = -i + \lambda_z$ and $\mathcal{A}_{33} = \mathcal{A}_{44} = -i - \lambda_z$ and $\mathcal{A}_{4j} = 0$ otherwise. For the superconducting part, we obtained the Greens function by recursively solving for the surface of an $s$-wave superconductor. Also, we imposed the flatband approximation for the normal lead. Hence, $\Sigma_L(\omega) = \nabla^L \mathcal{G}_L(\omega) \nabla^L = -i \tau \mathcal{G}_L \mathcal{G}_L^\dagger = -i \tau \mathcal{G}_L.$

For this calculation, we have used $t^{SC} = t_L = 0.25$. The values of the other parameters are given in the main text.
D. Inversion symmetry broken WSM

The Hamiltonian used to describe an inversion symmetry broken WSM is

$$H^W = \sum_r \left( \Psi_r^\dagger (t_h \eta^z I_z \sigma^y) \Psi_{r+x} + \Psi_r^\dagger (t_h \eta^z I_z \sigma^y) \Psi_{r+y} \right. $$
$$+ \Psi_r^\dagger (t_h \eta^z I_z \sigma^y) \Psi_{r+z} + (m + 2) \Psi_r^\dagger \eta^z \tau^y \sigma^y \Psi_r \right)$$
$$- \frac{1}{2} \sum_{\langle rr' \rangle} \Psi_r^\dagger \eta^z \tau^y \sigma^y \Psi_{r'}, \tag{31}$$

where $t_h$ is the hopping element inside the WSM and $m$ is the mass parameter. (For simplicity, we have combined spin-orbit couplings and hoppings and used $t_h$ to denote it). This model describes a normal insulator when $m > t_h$ and a Dirac semi-metal when $m = t_h$ with two nodes at $k_y = \pm \pi/2$. When $m < t_h$, each of the nodes split into two Weyl nodes forming a Weyl semi-metal with 4 nodes. The 4 Weyl nodes are located at $\pm k^+ = \pm \sin^{-1}(m/t_h)$ and $\pm k^- = \pm (\pi - \sin^{-1}(m/t_h))$. Here, $k^\pm$ are defined in congruence with the main text. Note that for this model, $k^+ + k^- = \pi$ is fixed. The relevant inter-nodal distance is $k^+ - k^- = k_0$. We keep the $x$ and $z$ directions periodic and the $y$ direction finite. Repeating the calculations for this setup, we end up with the same expression for the current (i.e, Eq.30) with $A$ redefined such that $A_{21} = A_{43} = t_h$ and $A_{12} = A_{34} = -t_h$ and $A_{ij} = 0$ otherwise. The results are summarized in Fig. 6 and clearly, the two basic periodicities of the current as emphasized in the main text are seen here as well.