Multi-Objective Constrained Optimizations Based Fuzzy Fault Tolerance Algorithm of Quarter Vehicle Suspensions

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ABSTRACT In this paper, a novel fuzzy fault tolerance algorithm is proposed for quarter vehicle active suspensions based on multi-objective constrained optimizations, which improves the riding comfort and driving safety. The objective function approach the optimal solutions via integrating multi-objective constraints as cost functions. By introducing the barrier function, the nonlinear active suspensions with multi-objective constraints are transformed into pure feedback systems without constraints. The problem is rather complicated yet challenging if the actuator faults are taken into account in quarter vehicle. The actuator faults are solved by the utilization of proportional actuation method. The unknown smooth dynamics based on physical truth are identified by exploiting the fuzzy logic knowledge. Meanwhile, the signals received in the suspensions do not violate the constraint boundary. Finally, the simulation results show that the algorithm is effective.

INDEX TERMS Multi-objective constraints, active suspensions, constrained optimization, fault-tolerance algorithm, fuzzy logic knowledge

I. INTRODUCTION

Now, along with social progress, economic growth, and urban development, from the perspective of travel efficiency and comfort, vehicles have become a necessity for people [1]-[3]. Vehicle suspension system is the general name of car frame and vehicle-bridge or wheels between all the connections, which is composed of shock absorber, spring and guide system. The function of the suspension system is to transmit the force and moment between the wheels and the frame, which cushions the impact force caused by the uneven road surface in the process of driving. Under the circumstances, it ensures the smooth driving and riding comfort. The combination of advanced control algorithm and vibration control mechanism is an efficient manner to enhance the quality of suspension systems. For this reason, a variety of suspension systems have been studied, such as passive suspension systems [4], semi-active suspension systems [5] and active suspension systems [6]. In these studies, the research of active suspension system lays a solid theoretical foundation for the realization, so as to realize its practical application in modern vehicles.

In the past few years, with the development of information science, many control technologies, such as adaptive neural network (NN) control and adaptive fuzzy control, have been put forward to discuss the approximate problem of unknown nonlinear function in active suspension, see [7-9] for details. For example, the fuzzy control and NN control are proposed for nonlinear systems with unmodeled dynamic or unknown control direction [10]-[12]. Besides, the adaptive intelligent control is also applied to pure-feedback systems [13], stochastic systems [14]-[15] and nonlinear delayed systems [16]. Up to present, lots of controller design methods based on fuzzy logic systems (FLSs) or NNs have been proposed in [17] and [18]. At the same time, domestic and foreign scholars put forward various methods of suspension system control under different conditions. More precisely, a genetic algorithm based fuzzy proportion integration control and proportional derivative control approaches is developed in [19] to improve passenger comfort of active suspension system. On this basis, a method based on sliding mode control is applied to active suspension systems in [20]. Sliding mode control is a special kind of nonlinear control in essence, and the nonlinearity is the discontinuity of control. The difference between this control strategy and other controls is that the "structure" of the system is not fixed. Instead, it can change purposefully according to the current state of the system (such as deviation and its derivatives) in the dynamic process, forcing the system to move
The components of suspensions are a complex of mechanical, systems, actuator failure is inevitable. The fault in the actuator instability for many practical systems. In active suspension suspension system controller. the vehicle, it cannot be ignored in the design of the active problem of actuator failure is very important to the stability of forward model predictive control, [24] gives reference the problem of system control under constraint, a lot of many practical systems have different constraints. Aiming at ignored. Otherwise, it may generate undesirable consequences.

Generally speaking, because of the influencing factors of structural which leads to the existence of various constraints, many practical systems have different constraints. Aiming at the problem of system control under constraint, a lot of research works have been put forward. For example, [23] puts forward model predictive control, [24] gives reference regulator, and [25] proposes extreme-seeking control. At the same time, the barrier Lyapunov function (BLF) is introduced to guarantee that all states of the system are kept within the bounds of constraints [29]-[34]. It is worth noting that many BLF based methods for nonlinear systems with complete constraints use the feasibility conditions [26]-[28] on the virtual controllers, which bring great difficulties to the design and implementation of the controller with parameter determination. In practice, violating constraints may result in performance degradation or even lead to catastrophic accidents. Moreover, for active suspension systems, body displacement or acceleration constraints are important factors to ensure vehicle safety and stability. Hence, it is necessary to impose corresponding constraints on the performance of the active suspension systems. Until now, few studies on constraints at home and abroad have focused on active suspension systems. This prompts us to research control methods for active suspension systems with constraints. Meanwhile, because of the problem of actuator failure is very important to the stability of the vehicle, it cannot be ignored in the design of the active suspension system controller.

In practice, the actuator failure has limited efficacy for the desired control performance and even leads to closed-loop instability for many practical systems. In active suspension systems, actuator failure is inevitable. The fault in the actuator is unknown, which may cause great economic and human loss. The components of suspensions are a complex of mechanical, electronic, software language and others. For the active suspension systems with actuator delay and actuator failure, an effective control strategy is proposed by using adaptive fault tolerant control (FTC) approaches [35]-[36], and the effectiveness of the method is verified. Then, [37-40] propose FTC methods for the quarter active suspension systems. In addition, these schemes are extended to half-active and fully active suspension systems in [41] and [42]. By means of the intelligent FTC of active suspension systems, the control reliability of suspensions is effectively improved, which is conducive to the overall vehicle riding comfort and handling stability. To enhance the vehicle control quality, the actuator failure of car suspension design is turning out to be increasingly imperative. Therefore, it is meaningful to put forward a fault tolerant method with multi-objective constraint control for suspension systems.

Motivated by the above analysis, we put forward a fuzzy adaptive tolerant control strategy for active suspension systems with multiple objective constraints. Firstly, the considered suspension system with multi-objective constraints is transformed into pure feedback systems without constraints by introducing a barrier function. Then, the unknown smooth dynamics existed in the considered system are approximated by fuzzy logic systems. Moreover, the fault tolerant controller is designed based on backstepping technique. Finally, the simulation results show the effectiveness of the proposed approach.

The main innovations of this paper compared with the existing results are as follows.

1) In this paper, the problem of constraints and actuator failure in active suspension systems is considered. And an adaptive FTC approach based on FLSs is proposed. Combined with the two kinds of problems that often appear in active suspension systems, and by choosing the appropriate fault parameters, the result is more general, this kind of actuator fault processing method has great practical significance.

2) Considering the importance of body displacement in active suspension system, many existing results have studied [22] [34]. In this paper, a different constraint method is adopted to restrict the body displacement, which makes the body displacement fluctuate in a small range all the time. And thus, it better ensures the ride comfort.

3) The backstepping method is used to design the controller, which avoids the problem that the parameters are too large to calculate. Furthermore, for the suspension system, reduce on the amount of calculation means the decrease of the energy consumption in the vehicle driving, which has great practical significance.

The rest of the article is organized as follows. In the second part, the description of active suspension systems and the formulation problem is presented. In the third section, a Lyapunov function is introduced for the quarter active
suspension systems with multi-objective constraints. On this basis, a fuzzy adaptive fault-tolerant controller is designed for the active suspension system. In the fourth part, the simulation example is given to verify the effectiveness of the scheme. The fifth part summarizes the survey results.

II. SYSTEM DESCRIPTIONS

The car model of active suspensions is shown in Fig. 1. Through mechanical analysis [43], the following kinematic formulas are obtained

\[
\begin{align*}
    m_1 z_1' + l_1 (\dot{z}_1 - \dot{z}_u) + k_s (z_1 - z_u) &= u \\
    m_2 z_2' - l_2 (\dot{z}_2 - \dot{z}_u) + l_1 (\dot{z}_u - \dot{z}_1) + k_s (z_2 - z_u) &= u \\
    + k_s (z_u - z_1) - k_u (z_u - z_u) &= u
\end{align*}
\]

where \( u \) is the control input of active suspension systems, \( m_s \) represents the sprung mass and \( m_u \) represents the unsprung mass, respectively; \( z_s \) and \( z_u \) are the displacement of the sprung and unsprung, respectively; \( z_i \) indicates interference on the road; \( k_s \) and \( k_u \) stand for the stiffness coefficients, \( l_s \) and \( l_i \) describe the damping coefficients.

According to the stress analysis, these forces are expressed as

\[
\begin{align*}
    F_c &= k_s (z_s - z_u) \\
    F_s &= c_s (\dot{z}_s - \dot{z}_u)
\end{align*}
\]

Define the following state variables \( x_1 = z_s \), \( x_2 = \dot{z}_s \), \( x_3 = z_u \), \( x_4 = \dot{z}_u \). We know the state space form of quarter car active suspension systems is

\[
\begin{align*}
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= \frac{1}{m_s} (F_c + F_d + u) \\
    \dot{x}_3 &= x_4 \\
    \dot{x}_4 &= \frac{1}{m_u} (-F_s - F_d + F_c + F_b - u)
\end{align*}
\]

Because of changes in the number of passengers or other qualities, the mass of the car \( m_s \) is an unknown. And the assumption about the mass is introduced as in the following.

**Assumption 1:** Because the limitation of car structure and actual physical performance, the body mass is affected by \( m_{s,\text{min}} < m_s < m_{s,\text{max}} \).

**Control Objectives:** From a practical perspective, the ride comfort, safety and suspension limitations are three important performance indexes. In a word, the following three tasks will be realized.

1. **The ride comfort:** Based on the designed controller \( u \), it should ensure that the displacement \( z_s \) and acceleration \( \ddot{z}_s \) are moderated as soon as possible. Therefore, comfort from the suspension to passenger is necessary.

2. **The driving safety:** The dynamic tire load should be limited in a certain range \( |F_c + F_d| \leq (m_s + m_u) g \).

3. **The suspension space:** Considering the stability of suspension physical structure, the suspension space must be smaller than their maximums: \( |z_s - z_u| \leq z_{\text{max}} \) where \( z_{\text{max}} \) is a constant.

In this paper, the actuator faults included deviation coefficient is given [44]

\[
    u = \delta u_d + \sigma_d, t \in [t_1, t_2]
\]

where \( 0 \leq \delta \leq 1 \), \( \sigma_d \) is an unknown constant and \( \sigma_{d,\text{max}} = V_n \), \( t_1 \) and \( t_2 \) are the time instants when the actuator fault begin and end. The actuator fault model includes the following situations:

1) when \( \delta = 1 \) and \( \sigma_d = 0 \), it means that there is not any actuator failure.
2) when \( 0 < \delta < \delta > 0 \) and \( \sigma_d = 0 \), it shows partial actuator failure.
3) when \( \delta = 0 \) and \( \sigma_d = 1 \), it implies \( u_d \) can no longer be influenced by the control inputs \( u \).

The multiple-objective constraints are as follows

\[
\begin{align*}
    G_1 &= \min (\xi_1) \\
    G_2 &= \max (\xi_2) \\
    \vdots \\
    G_n &= \max (\xi_n)
\end{align*}
\]

where \( G_i \) is the minimum optimization value, which can be used as the minimum production cost and as the cost function. \( G_2 \) represents the maximum optimization value. As a rate function, this optimization value can be used as the highest rate. \( G_3 \) to \( G_n \) have similar physical significance.

In the actual environment, the case that all objective functions are optimal is almost nonexistent. We use the following inequality to make the objective function approach the optimal solution.
The fuzzy logic systems are used to approximate nonlinear functions $H(X)$, which are shown in the below

$$H(X) = W^T \varphi(X) + \varepsilon(X)$$  \hspace{1cm} (13)$$

where $\varphi(X) : \Omega \rightarrow R$ is the radial basis function, $\varepsilon(X)$ is the minimum approximation error. The optimal parameter $W^*$ is defined as follows

$$W^* = \arg \min_{w \in \mathbb{R}^d} \left\{ \sup_{x \in \Omega} |H(X) - W^T \varphi(X)| \right\}$$  \hspace{1cm} (14)$$

**Lemma 2** ([48][49]): For any $x, y \in R^n$, there are following inequalities

$$x^T y \leq \frac{a^p}{p} \|x\|^p + \frac{1}{qa^p} \|y\|^q$$  \hspace{1cm} (15)$$

where $a > 0$, $p > 1$, $q > 1$ and $(p-1)(q-1) = 1$.

### III. ADAPTIVE CONTROLLER DESIGN

#### A. BARRIER FUNCTION

Here, we introduce a barrier function for the convenience of control design. For the sake of realizing constraints better, the barrier function [50] is defined

$$\zeta = \frac{J - F_1}{J - F_1} + \frac{J - F_2}{F_2 - J}$$  \hspace{1cm} (16)$$

where $J(0) \in \Omega$ is defined in the open region $\Omega$. There are upper bound and lower bound denoted as $F_1$ and $F_2$, i.e., it can be known that $F_1 < F_1$ and $F_2 \geq F_2$.

**Remark 1**: For any initial value $J(0) \in \Omega$ and $\zeta \rightarrow \infty$, it deduces that $J \rightarrow F_1^*(t)$ or $J \rightarrow F_2^*(t)$. That is to say, for any initial value $J(0) \in \Omega$, $J$ obeys the asymmetric constraints as long as the boundedness of $\zeta$ is guaranteed. Therefore, the problem that the objective function satisfies the asymmetric constraints can be transformed into the guaranteed boundedness problem of $\zeta$.

According (16), we can obtain

$$\zeta = \zeta_1 \mu + \zeta_2$$  \hspace{1cm} (17)$$

and

$$\zeta_1 = \frac{F_1 - F_1 + F_2 - F_2}{J - F_1}$$  \hspace{1cm} (18)$$

$$\zeta_2 = \frac{F_1 F_2 - F_1 F_2}{F_2 - J}$$  \hspace{1cm} (19)$$

which are well defined in $\Omega$ [51]. Thus, it can be obtained that $J = \zeta_1 / \zeta_1 - \zeta_2$, where $\zeta_1$ is defined as

$$\zeta_1 = \frac{F_1 - F_1 + F_2 - F_2}{F_1 - F_1 + F_2 - F_2}$$  \hspace{1cm} (19)$$

with

$$J = \frac{F_1 F_2 - F_1 F_2}{F_2 - J}$$  \hspace{1cm} (18)$$

In terms of singleton function, center average defuzzification, and product inference, the FLSs are presented as follows:

$$\varphi(x) = \frac{\sum_{i=1}^{n} \prod_{j=1}^{m} \mu A^j_i(x)}{\sum_{i=1}^{n} \prod_{j=1}^{m} \mu A^j_i(x)}$$  \hspace{1cm} (11)$$

Define the fuzzy basis functions as follows:

$$\varphi(x) = \frac{\sum_{i=1}^{n} \prod_{j=1}^{m} \mu A^j_i(x)}{\sum_{i=1}^{n} \prod_{j=1}^{m} \mu A^j_i(x)}$$  \hspace{1cm} (11)$$

Let $W^T = [\bar{W}_1, \bar{W}_2, \ldots, \bar{W}_n]$, then the FLSs are expressed as

$$H(x) = W^T \varphi(x)$$  \hspace{1cm} (12)$$

The fuzzy logic systems can be used for function approximation. The approximation Lemma is as follows:

**Lemma 1** [47]: Given a compact set $\Omega$, the fuzzy logic systems are used to approximate nonlinear functions $H(X)$, which are shown in the below

$$H(X) = W^T \varphi(X) + \varepsilon(X)$$  \hspace{1cm} (13)$$

where $\varphi(X) : \Omega \rightarrow R$ is the radial basis function, $\varepsilon(X)$ is the minimum approximation error. The optimal parameter $W^*$ is defined as follows

$$W^* = \arg \min_{w \in \mathbb{R}^d} \left\{ \sup_{x \in \Omega} |H(X) - W^T \varphi(X)| \right\}$$  \hspace{1cm} (14)$$

**Lemma 2** ([48][49]): For any $x, y \in R^n$, there are following inequalities

$$x^T y \leq \frac{a^p}{p} \|x\|^p + \frac{1}{qa^p} \|y\|^q$$  \hspace{1cm} (15)$$

where $a > 0$, $p > 1$, $q > 1$ and $(p-1)(q-1) = 1$.
\begin{equation}
\eta_t = \frac{F_i - F_d}{(J - F_i)} + \frac{F_i - F_{d_i}}{(J - F_i)} \quad (20)
\end{equation}

\begin{equation}
\eta_s = \frac{(J - F_i)F_i}{(J - F_i)^2} - \frac{(J - F_i)F_d}{(F_d - J)^2}.
\end{equation}

Further, (20) is rewritten as
\begin{equation}
\dot{\zeta} = \eta_t \frac{\partial J}{\partial c_1} \xi_1 + \eta_s \quad (21)
\end{equation}
\begin{equation}
\dot{\zeta} = \eta_t \xi_1 + \eta_s \quad (22)
\end{equation}
where \( \eta_t = \eta_t (\hat{c} J / \partial c_1) \). From (18) and (\( \hat{c} J / \partial c_1 \)) \neq 0, it implies that \( \eta_t \neq 0 \).

### B. ADAPTIVE CONTROLLER DESIGN WITH ACTUATOR FAILURE

**Step 1:** For the convenience of calculation, let \( f_2 = F_i + F_{d_i} \).

It holds that
\begin{equation}
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m} \left(-f_2 + \delta u_d + \sigma_d \right)
\end{aligned}
\end{equation}

The controller design is implemented by the backstepping technology. Define the tracking error and the error variable as follows:
\begin{equation}
\begin{aligned}
z_1 &= \zeta_1 - y_d (t) \\
z_2 &= \dot{x}_2 - \alpha_1
\end{aligned}
\end{equation}
where \( \alpha_1 \) is the virtual controller, \( y_d (t) \) is the desired trajectory and it is continuous and derivable. In the meantime, it has \( |\dot{y}_d| \leq D_1 \), in which \( D_1 > 0 \) is a constant.

Note that \( z_1 = \zeta_1 - y_d \), then it follows:
\begin{equation}
\begin{aligned}
\dot{z}_1 &= \eta_t \dot{x}_1 + \eta_s - \dot{y}_d \\
&= \eta_t \dot{x}_1 + \eta_s - \dot{y}_d
\end{aligned}
\end{equation}

Based on the above-mentioned fuzzy logic systems, let
\begin{equation}
H_t (X_t) = W_{1i}^T S_i + e_i = \eta_2 - y_d.
\end{equation}
Then, we get
\begin{equation}
\begin{aligned}
z_1 \dot{z}_1 &= \zeta_1 \dot{x}_1 + \eta_s - \dot{y}_d \\
&= \zeta_1 \dot{x}_1 + \eta_2 - \dot{y}_d \\
&= \zeta_1 \dot{x}_1 + \alpha_1 + \Xi_1 \\
&= \zeta_1 \dot{x}_1 + \alpha_1 + \Xi_1 + \dot{z}_1 \\
&= \zeta_1 \dot{x}_1 + \alpha_1 + \Xi_1 + \dot{z}_1 (W_1^{-1} S_i + e_i)
\end{aligned}
\end{equation}
where \( \Pi_1 = \zeta_1 \eta_2 \) and \( \Xi_1 = \dot{z}_1 \alpha_1 \), with \( \Lambda_1 = \eta_2 - y_d \), which is a continuous unknown nonlinear function.

According to Lemma 2, \( p \), \( q \) and \( a \) are chosen as \( 2 \), \( 2 \) and \( \sqrt{2} \), respectively. Then, it yields
\begin{equation}
\begin{aligned}
z_1 \dot{z}_1 &\leq \eta_2^2 z_1^2 z_1^2 + \frac{1}{4} \\
z_1 W_{1i}^T S_i &\leq \dot{S}_i^2 z_1^2 + \frac{1}{4}
\end{aligned}
\end{equation}
where \( \dot{\theta} = \max \{m_i W_i^T \}, i = 1, 2, \ldots, n \}. In addition, \( W_i^* \) and \( \theta \) are unknown. According to (27), it obtains
\begin{equation}
\begin{aligned}
\Pi_1 + \Xi_1 &\leq \eta_2^2 z_1^2 z_1^2 + \frac{1}{2} \\
&+ \delta S_i^2 z_1^2 + \frac{e_2^2}{4}
\end{aligned}
\end{equation}

Therefore, it deduces
\begin{equation}
\begin{aligned}
z_1 \dot{z}_1 &\leq \eta_2 z_1 \alpha_1 + z_1^2 \\
&+ \delta S_i^2 z_1^2 + \eta_2 z_1 \alpha_1 + \Delta_i
\end{aligned}
\end{equation}
where \( \Delta_i = 1/2 + e_2^2 / 4 \).

Select the Lyapunov function as
\begin{equation}
V_1 = \frac{1}{2} z_1^2
\end{equation}
The derivative of \( V_1 \) is as follows
\begin{equation}
\dot{V}_1 = z_1 \dot{z}_1
\end{equation}
Then, we get
\begin{equation}
\dot{V}_1 \leq \frac{1}{m} (\alpha_1 \dot{x}_1) - \hat{\theta}_i
\end{equation}
The virtual control is designed as
\begin{equation}
\alpha_1 = \frac{1}{m} (c_i z_1 + \dot{S}_1^2 z_1)
\end{equation}
where \( \hat{\theta}_i \) is the estimation of \( \theta \) with \( \hat{\theta} = \theta - \hat{\theta} \) and \( c_i \) is a positive design parameter.

From (34), it deduces
\begin{equation}
\dot{V}_1 \leq (c_i - 1) z_1^2 + \eta_2^2 z_1 \alpha_1 + \Delta_i + \dot{\theta} S_1^2 z_1
\end{equation}
**Step 2:** Consider active suspension system with actuator failure, the time derivative of \( z_1 \) is written as
\begin{equation}
\dot{z}_2 = \dot{x}_2 - \hat{\alpha}_1
\end{equation}
where \( \hat{\alpha}_1 \) is an unknown continuous function, which is approximated by the FLSs as \( H_t (X_t) = W_{1i}^T S_i + e_i = \alpha_1 - f_2 \).

From (35), further results in
\begin{equation}
\dot{z}_2 \dot{z}_2 = \frac{1}{m} \left(\delta u_d + \alpha_2 - f_2 \right) - z_1 \hat{\alpha}_1
\end{equation}
Next, it causes
\begin{equation}
m_1 z_2 \dot{z}_2 = z_2 (\delta u_d + \alpha_2) + \Xi_2
\end{equation}
where \( \Xi_2 = z_2 m_1 \alpha_1 - z_2 f_2 = z_2 \Lambda_1 \), and \( \Lambda_1 = -m_1 \alpha_1 - f_2 \) is an unknown continuous function, which is approximated by the FLSs as \( H_t (X_t) = W_{1i}^T S_i + e_i = \alpha_1 - f_2 \).

From (35)-(37), the following result is got
\begin{equation}
\begin{aligned}
\frac{m_1}{\delta} z_2 \dot{z}_2 &\leq \frac{1}{2} z_2 \dot{z}_2 + \eta_1 z_1^2 z_1^2 + \delta S_i^2 z_1^2 \frac{1}{4} \\
&\leq \frac{1}{2} \Phi_2 z_2 - \eta_1 z_1^2 z_1^2 + \delta S_i^2 z_1^2 + \Delta_2
\end{aligned}
\end{equation}
where \( \Phi_2 = 1 + \eta_1 z_1^2 \) and \( \Delta_2 = \frac{e_2^2}{4} + \frac{1}{4} \).

Select the Lyapunov Function as
\begin{equation}
V_2 = V_1 + \frac{m_1}{2} z_2^2 + \frac{1}{2} \beta \dot{\theta}^2
\end{equation}
where \( \beta \) is a positive design parameter.

The controller of active suspension systems and adaptive law are designed as
\[ u_d = -(c_1 z_1 + z_2 \Phi_2 + \theta S_z^2 z_1) \] (40)

\[ \dot{\theta} = \beta S_z^2 z_1^2 + \beta z_2^2 z_1^2 - k_1 \theta \]

with \( c_1 > 0 \) and \( k_1 > 0 \) being assigned by designers.

Taking the time derivative of \( V_z \) turns out

\[ \dot{V}_z = \dot{V}_1 + \frac{m}{\delta} z_2 \dot{z}_2 - \frac{\dot{\theta}^3}{\beta} \]

Then, it leads to

\[ \dot{V}_z = \dot{V}_1 + \frac{m}{\delta} z_2 \dot{z}_2 - \frac{\dot{\theta}^3}{\beta} \]

(43)

Moreover, it holds that

\[ \dot{V}_z \leq -(c_1 - 1) z_1^2 - c_2 z_2^2 - \frac{k_2}{2} \theta^2 + z_1^2 + \frac{k_2}{2} \theta^2 + \Delta_1 + \Delta_2 \]

(44)

Theorem 1: Take the active suspension systems (1) into account. By designing the virtual control signal as shown in (33), the actual controller \( u \) (40), and adaptive laws (41), then, all the signals in the active suspension systems are bounded, the feasibility condition is eliminated, and the multiple target constraints are guaranteed.

Proof: Based on (24), (28) and (34), the fact in (39) can be further rewritten as

\[ \dot{V}_z \leq -BV + O \]

(45)

where

\[ O = \frac{k_2}{2} \theta^2 + \Delta_1 + \Delta_2 \]

and

\[ B = \min \{2(c_1 - 1), 2c_1/m_{max}, \beta\} \]

Thus, by integrating \( \dot{V}_z \), one has

\[ V_z(t) \leq \left( V_z(0) - \frac{O}{B} \right) e^{-\frac{B}{O}} + \frac{O}{B} \]

(46)

Therefore, the errors satisfy

\[ |z_i| \leq \sqrt{2 \left( \left( V_z(0) - \frac{O}{B} \right) e^{-\frac{B}{O}} + \frac{O}{B} \right)} \quad i = 1, 2 \]

(47)

From the above analysis, the \( \gamma_i(t) \) can be limited to a small neighborhood of zero. At the same time, we obviously know that \( |\alpha| \leq (c_1 - 1)|z_1| + D_3 = D_3 \) where \( D_3 > 0 \) and \( D_3 \) is given below (24). Since the state variables satisfy \( |x_2| \leq |z_2| + |\gamma_2| \) and \( |x_2| \leq |z_2| + |\gamma_2| = |z_2| + |D_3| \), one gets that the vertical displacement and velocity of the spring load in the active suspension systems are bounded.

C. ZERO DYNAMICS

In the above work, according to the proposed control scheme, the stability of system (3) is proved and the first control objective is achieved. By selecting appropriate design parameters in this section, other two objectives can be discussed.

At first, the boundedness of \( x_1 \) and \( x_4 \) (i.e., \( z_w \) and \( z_w \)) in system (3) will be considered. In general, the errors \( z_1 \) and \( z_2 \), the car displacement \( x_1 \) and speed \( x_2 \) converge to any small neighborhood of zero in Theorem 1. Then, introducing (42) into (3), one gets

\[ \dot{X} = AX + \Gamma \]

(48)

where

\[ A = \begin{bmatrix} 0 & 1 \\ -k_1 & -c_1/m_{max} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 \\ k_1 z_1 + c_1 z_1 + Y \end{bmatrix}, \quad X = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \]

and \( Y = \frac{1}{m_{max}} \left( c_1 z_1 + z_2 \Phi_2 + \dot{\theta} S_z^2 z_2 \right) \).

Therefore, \( Y \) is bounded as \( 0 < \|Y\| \leq \bar{T} \). Define the Lyapunov function as \( V = X^T K X \), where \( K \) is a positive definite symmetric matrix. Further, one gets

\[ \dot{V} = X^T \left( A' K + PK \right) X + 2X^T K \dot{X} \]

Obviously, the matrix \( A \) has the eigenvalues whose real part is negative. Thus, it is established that \( A' K + PK = -L \), where \( L \) is a positive definite symmetric matrix.

Then, it holds that

\[ 2X^T K \dot{X} \leq \left( X^T K X \right)/\psi + \psi \Gamma^T \Gamma \]

Next, we obtain that

\[ \dot{V} \leq -X^T LX + \psi^{-1} X^T K^T + \psi \Gamma^T \Gamma \]

\[ \leq \left[ -\lambda_{min} \left(K^{-\frac{1}{2}} L K^{-\frac{1}{2}} \right) + \psi^{-1} \lambda_{max}(K) \right] V + \psi \Gamma^T \Gamma \]

(49)

Furthermore, one gets

\[ \lambda \leq -\lambda_{min} \left(K^{-\frac{1}{2}} L K^{-\frac{1}{2}} \right) + \psi^{-1} \lambda_{max}(K) \]

(50)

Define \( \mu \geq \psi \Gamma^T \Gamma \), and then, the derivative of \( V \) is further obtained as \( \dot{V} \leq -\lambda V + \mu \). Thus, one has the following fact

\[ \dot{V} \leq \left( V(0) - \frac{\mu}{\lambda} \right) e^{-\mu/\lambda} + \frac{\mu}{\lambda} \leq \zeta \]

(51)

where

\[ \zeta = \begin{cases} V(0), & V(0) \geq \frac{\mu}{\lambda} \\ \frac{2\mu}{\lambda} - V(0), & V(0) < \frac{\mu}{\lambda} \end{cases} \]

Based on the above facts, it gets

\[ |x_i| \leq \sqrt{2 \zeta / \lambda_{min}(P)}, \quad i = 3, 4 \]

(52)

Hence, we know that the state variables \( x_3 \) and \( x_4 \) are bounded. Meanwhile, we can also know that
The objective of driving safety is achieved by choosing appropriate parameters. Next, the suspension space is expressed as follows

\[
[F_x] = k_x |x_o - z_o| \leq k_x \left( \alpha_x + \sqrt{\frac{z_o}{\lambda_{\text{min}}}} (P) \right)
\]

\[
[F_z] = l_z |z_o - \dot{z}_o| \leq l_z \left( \alpha_z + \sqrt{\frac{z_o}{\lambda_{\text{min}}}} (P) \right)
\]

(53)

The objective of driving safety is achieved by choosing appropriate parameters. Next, the suspension space is expressed as follows

\[
[x_i - x_j] \leq [x_i] + [x_j] \leq \sqrt{\frac{z_o}{\lambda_{\text{min}}}} (P)
\]

(54)

We can clearly see that there is a limit to the suspension space. Therefore, all signals in the active suspension systems are bounded and the control objectives are achieved.

IV. SIMULATION VERIFICATION

In order to prove the effectiveness of the proposed method, a quarter car example is given in this part. The \(y_o\) is given as \(y_o = 0\), and the initial values are \(x_1(0) = 0.03\), \(x_2(0) = 0.03\), \(x_3(0) = 0\), \(x_4(0) = 0\) and \(x_5(0) = 0.02\).

Then, the interference on the road is given as \(z_o = 0.1 \sin(2\pi t)\). The parameters of the vehicle are described by Table 1.

Table 1: 1/4 car model parameters

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \(k_x\)   | 18000 N/m | \(m_w\)   | 59 kg |
| \(c_x\)   | 2400 Ns/m | \(m_i\)   | 590 kg |
| \(k_i\)   | 15000 N/m | \(m_{w\text{max}}\) | 700 kg |
| \(c_i\)   | 1200 Ns/m | \(m_{i\text{max}}\) | 520 kg |

Fig. 4 The suspension space \(x_i - x_j\)

The design parameters are got as \(c_1 = 11\), \(c_2 = 12\), \(k_2 = 100\), \(\beta = 0.01\) and the constraints are \(F_1 = 0.5\) and \(F_2 = -0.5\). The problem of actuator failure is considered in this paper. Then, the deviation coefficient \(\delta\) of actuator faults and the unknown constant \(\sigma\) are designed as \(\delta = 0.8\) and \(\sigma = 0.5\). Meanwhile, consider \(j_1 = x_1\) and \(a_1 = 1\). Only \(x_1\) is under the multiple-objective constraints.

Fig. 5 The error signal of \(z_o\)
The vertical displacement of the quarter car is near zero in Fig. 2. At the same time, $x_1$ does not overstep the boundary bounds. Fig. 3 shows that the vertical speed is stabilized in the neighborhood of zero. We can also obtain that the suspension space is small enough around zero in Fig. 4. On the side, the curves of $z_1$ and $z_2$ are given in Fig. 5 and Fig. 6. Obviously, these signals are bounded. The adaptive law $\hat{\theta}$ is plotted in Fig. 7. It is fluctuated in the neighborhood of zero. The control inputs $u$ is plotted in Fig. 8, which can be got that the input force is stable. In Fig. 9 and Fig. 10, the signals of $x_3$ and $x_4$ are described, which also proves the effectiveness of the proposed control method.
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V. CONCLUSION

In this paper, a new adaptive fault tolerant control scheme is put forward to achieve the goals of safe driving, ride comfort and handling stability of the active suspension systems. Considering the importance of body displacement in active suspension system, a different constraint method is adopted to restrict it in a small range all the time. And this better ensures the ride comfort. Besides, the actuator faults are dealt with here. The design of fault tolerant controller is based on the method of backstepping and fuzzy logic systems. The simulation results show that the developed scheme is effective in the end. The main advantage of the method proposed in this paper is that both constraints and actuator failure problems are solved for quarter vehicle active suspension systems. However, the approach to these problems is relatively traditional. In our future work, some advanced methods will be considered in vehicle systems, such as second-order sliding mode control design subject to an asymmetric output constraint, HOSM controller design with asymmetric output constraints, sliding mode direct yaw-moment control design for in-wheel electric vehicles, and so on.

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