Minimal Direct Gauge Mediation

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We propose a simple model of gauge mediation where supersymmetry is broken by a strong 
dynamics at $O(100) \text{TeV}$. 

A. Introduction

One of the simplest and the most natural scenarios for supersymmetry breaking is to assume dynamical supersymmetry breaking at an energy scale of $O(100) \text{TeV}$. The electroweak scale comes out as a one-loop factor lower scale than $O(100) \text{TeV}$ via gauge mediation\textsuperscript{[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]}. The scenario contains only a single scale as opposed to other scenarios where the supersymmetry breaking scale and the messenger scale are generated by different mechanisms. However, it has been known that a concrete model building of such a one-scale scenario is theoretically challenging (see Refs.\textsuperscript{[8, 9, 10, 11, 12, 13, 14, 15] for earlier attempts). In this letter, we propose a simple (and possibly the simplest) model with such low scale dynamics where not only the supersymmetry breaking but also the masses of the messenger particles are generated by the effects of the strong dynamics of a gauge interaction.

B. Dynamical supersymmetry breaking/messenger sector

A model of the dynamics of the supersymmetry breaking is based on a supersymmetric SU(5)$_H$ gauge theory with five flavors ($F$ and $\bar{F}$)\textsuperscript{[16]}, where the subgroup of a global SU(5)$_F$ symmetry ($F : 5$ and $\bar{F} : 5$) is identified with the gauge groups of the standard model. As we see later, dynamical supersymmetry breaking is therefore directly communicated to the standard model sector directly\textsuperscript{[16, 17]}. The only other ingredient of the model is a singlet superfield $S$ which couples to $F$ and $\bar{F}$ in the superpotential,

$$W = kSF\bar{F},$$

where $k$ is a coupling constant. $F$ and $\bar{F}$ (or their composite particles) serve as the messenger fields once both the scalar- and the F-term of the singlet obtain non-vanishing vacuum expectation values; $\langle S \rangle \neq 0$ and $F_S \neq 0$\textsuperscript{[31]}. Recently, models of the sweet spot supersymmetry\textsuperscript{[17]} based on this dynamical supersymmetry breaking/messenger model have been analyzed in the elementary messenger regime\textsuperscript{[18]} and in the composite messenger regime\textsuperscript{[19, 20]}. In the sweet spot supersymmetry, the supersymmetry breaking local minimum at $\langle S \rangle \neq 0$ is realized by the gravitational stabilization mechanism\textsuperscript{[20]}. In this letter, we seek another possibility of making $\langle S \rangle \neq 0$.

To see how the supersymmetry breaking occurs, let us consider the region where the “messenger mass”, $M_{\text{mess}} \equiv kS$, is smaller than the scale $\Lambda$ around which the SU(5)$_H$ gauge interaction becomes strong. In this region, the model can be described by using mesons, $M \sim F\bar{F}$ and baryons $B \sim F^5$ and $\bar{B} \sim F\bar{F}^5$. Here, we omit the indices of the flavor SU(5)$_F$ symmetry. In terms of the mesons and baryons, the full effective superpotential is given by

$$W_{\text{eff}} = kS \cdot \text{Tr}M + X(\det M - B\bar{B} - (\Lambda_{\text{dyn}}^2/5)^5),$$

where $X$ is a Lagrange multiplier field which guarantees the quantum modified constraint between the mesons and baryons\textsuperscript{[21]}. $\Lambda_{\text{dyn}}$ denotes the dynamical scale of SU(5)$_H$ gauge interaction and it is naively related to the scale $\Lambda$ by $\Lambda_{\text{dyn}} \sim \sqrt{N_cL/(4\pi)}$ with $N_c = 5$\textsuperscript{[22, 23]}. By expanding the meson and baryon fields around a solution to the above constraint: $M = \Lambda_{\text{dyn}}(\Lambda_{\text{dyn}}\delta_{ij}/5 + \delta M)$, $(\text{Tr}\tilde{M}=0)$, $B \sim B/\Lambda_{\text{dyn}}$ and $\bar{B} \sim B/\Lambda_{\text{dyn}}^4$, we obtain a superpotential,

$$W_{\text{eff}} \sim k\Lambda_{\text{dyn}}^2S + S \left(\frac{k}{2}\text{Tr}\delta M^2 + k\tilde{B}\bar{B} + \cdots\right),$$

which has a linear term of the singlet $S$. Here, the ellipses denote the higher dimensional operators of mesons and baryons, and we have neglected non-calculable corrections to the coupling constants which are naively expected to be $O(1)$. This superpotential shows that there is a supersymmetric minimum at $S = 0$ and $\delta M^2 \neq 0$ or $\bar{B}\bar{B} \neq 0$. However, if the singlet $S$ has a local minimum at $S = \langle S \rangle \gtrsim \Lambda_{\text{dyn}}$, the mesons and baryons have positive masses squared and the spontaneous supersymmetry breaking is achieved by $F_S \sim k\Lambda_{\text{dyn}}^4$.

Now, the question is: is there a possibility for the singlet $S$ to have a local minimum at $S = \langle S \rangle > \Lambda_{\text{dyn}}$? To address this question, notice that there are non-calculable contributions to the effective Kähler potential of the singlet $S$ from the strong interactions below the scale $\Lambda$,

$$K_{\text{eff}} = S^4 + \frac{25\Lambda^2}{(4\pi)^2} \delta K \left(\frac{kS}{\Lambda}\right).$$

Here, we have used the “naive dimensional analysis”\textsuperscript{[22, 23]}, and we expect that the non-calculable contribution
\( \delta K(x) \) has no small parameter. From this Kähler potential, we obtain a scalar potential,

\[
V(S) \sim \frac{|F_S|^2}{1 + 25(k/4\pi)^2 \delta K^{(2)}(kS/\Lambda)},
\]

where \( \delta K^{(2)} \) denotes the second derivative of \( \delta K \) with respect to \( kS/\Lambda \). This potential shows that there is a possibility that the potential has a local minimum around \( \Lambda/k \) which is larger than \( \Lambda_{\text{dyn}} \) for \( k \lesssim 4\pi/\sqrt{5} \). Therefore, if we take this possibility positively, it is not hopeless to expect that the singlet \( S \) has a local minimum around \( \Lambda/k \) by the effect of the non-calculable contributions from the strong dynamics (see Fig. 1).

Note that calculable radiative corrections to the Kähler potential through the diagrams in which the mesons and baryons circulate dominate over the non-calculable contribution \( \delta K \) for a small value of the singlet \( S/k \). The potential curves up by this contribution \[32\]. The masses of the mesons and baryons, however, become comparable to the scale \( \Lambda \) around \( S \sim \Lambda/k \). Hence, their effects can be overwhelmed by the non-calculable contribution in the region of \( S \sim \Lambda/k \). Therefore, the possibility of the local minimum around \( S \sim \Lambda/k \) cannot be excluded by these effects (see Ref. \[25\] for a similar discussion).

For \( M_{\text{mess}} = ks \gg \Lambda \), the dynamics can be described by using \( F \) and \( \bar{F} \) as elementary fields. In this region, it can be shown that the potential curves up in the \( S \) direction by radiative corrections to the Kahler potential \[26\]. Therefore, the singlet \( S \) cannot have a local minimum at \( S \gg \Lambda/k \).

The above discussion does not exclude the possibility of having a local minimum around \( S \sim \Lambda/k \) where all the calculable contributions are comparable to the non-calculable ones. Put it all together positively, we here assume that there is a local and supersymmetry breaking minimum around \( \Lambda/k \), aside from the supersymmetric minimum at \( S = 0 \) \[32\]. In the following, we consider a model with gauge mediation around the local minimum at

\[
\langle S \rangle \sim \Lambda/k \sim 4\pi \Lambda_{\text{dyn}}/(\sqrt{5}k) \gtrsim \Lambda_{\text{dyn}},
\]

\[
F_S \sim k\Lambda_{\text{dyn}}^2,
\]

\[
\left\langle \delta \bar{M} \right\rangle = \left\langle \bar{B} \right\rangle = \left\langle \bar{\phi} \right\rangle = 0.
\]

C. Spectrum of supersymmetric standard model particles

The mesons \( \delta \bar{M} \) transform as the adjoint representation under SU(5)\(_F\) whose subgroup is identified with the standard model gauge group, i.e. \( (8, 1)_0, (1, 3)_0, (3, 2)_{-5/6} \) and \( (3, 2)_{5/6} \) under the standard group. Thus, the mesons mediate the effects of the supersymmetry breaking to the standard model sector via gauge interactions. Through loop diagrams of the mesons, we obtain masses of the gauginos and scalar particles in the standard model sector,

\[
m_{\text{gaugino}} = \frac{N_m}{2} \left( \frac{g^2}{4\pi^2} \right) \left( \frac{F_S}{\langle S \rangle} \right) \left( 1 + O \left( \frac{\sqrt{5}k/4\pi)^4} \right) \right),
\]

\[
m^2_{\text{scalar}} = 2N_m C_2 \eta \left( \frac{g^2}{4\pi^2} \right)^2 \frac{|F_S|^4}{\langle S \rangle^2},
\]

where \( N_m = 5 \) is the sum of the Dynkin index of the mesons, \( C_2 \) is the quadratic Casimir invariant of the scalar particles, and \( g \) denotes the gauge coupling constant of the standard model gauge group. As we discussed above, \( F_S/\langle S \rangle \) is given by \( F_S/\langle S \rangle \sim \sqrt{5}k^2\Lambda_{\text{dyn}}/4\pi \). The \( O(1) \) coefficient \( \eta \) for the scalar masses denotes non-calculable \( O(F_S^2/\langle S \rangle^2) \) contributions from the heavy hadrons which are charged under the standard model gauge groups, while gaugino masses do not get \( O(F_S/\langle S \rangle) \) contributions from them. The \( O((\sqrt{5}k/4\pi)^4) \) contribution comes from the diagrams with more \( S \) inserted. As a result, we achieve a model with gauge mediation with a low dynamical scale,

\[
\Lambda_{\text{dyn}} \sim 10^5 \text{GeV} \times k^{-2} \left( \frac{M_{\text{grainino}}}{1 \text{TeV}} \right),
\]

for \( k = O(1) \).

As an interesting prediction, the gravitino mass can be as small as \( O(1) \) eV,

\[
m_{3/2} \sim 3 eV \times k^{-3} \left( \frac{M_{\text{grainino}}}{1 \text{TeV}} \right)^2,
\]

for \( \Lambda_{\text{dyn}} \sim 10^5 \text{GeV} \) \[34\]. Here, we have used the definition of the gravitino mass,

\[
m_{3/2} = \frac{F_S}{\sqrt{3}M_{\text{pl}}} \sim \frac{k\Lambda_{\text{dyn}}}{\sqrt{3}M_{\text{pl}}},
\]

where \( M_{\text{pl}} \simeq 2.4 \times 10^{18} \text{GeV} \) denotes the reduced Planck scale.

Now several comments are in order. The perturbativity of the standard model gauge interactions up to
the scale of the Grand Unification Theory (GUT) put a bound on the sum of the Dynkin index of the messenger field \( N_m \) as

\[
N_m \lesssim 150 / \ln(M_{\text{GUT}}/M_{\text{mess}}),
\]

where \( M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV} \) denotes the scale of GUT. For \( M_{\text{mess}} \approx \Lambda \approx 10^6 \text{ GeV} \), this condition requires \( N_m \) as \( N_m \leq 6 \). Therefore, the Dynkin index of the present model, \( N_m = 5 \), satisfies the perturbative condition of the standard model gauge interactions up to the GUT scale.

We should also mention the perturbativity of the coupling constant \( k \). The coupling constant \( k \) becomes small at the high energy scale as a result of the large renormalization effect from the strong gauge interaction of SU(5). Thus, we can expect the coupling constant \( k \) stays perturbative up to around the GUT scale, although it is not necessarily required \([35]\).

The tunneling rate to the supersymmetric vacuum at \( S = 0 \) per unit volume is roughly given by \( \Gamma / V \sim \langle S \rangle^4 e^{-S_E} \), where \( S_E \) is estimated by \( S_E \sim 2\pi^2 \langle S \rangle^4 / V(S) \sim 2\pi^2(4\pi)^4/(5^2k^6) \sim 10^4 \) for \( k \sim 1 \). On the other hand, the vacuum stability condition within the observable volume and over the age of the universe only requires \( S_E \gtrsim 400 \). Thus, although our vacuum is not stable quantum mechanically, it has a lifetime much longer than the age of the universe.

Finally, we comment on the effects of the supergravity to the scalar potential of \( S \). The leading effect of the supergravity comes from the linear term of the singlet \( S \) in the superpotential which leads to a linear term in the scalar potential.

\[
V(S)_{\text{linear}} = 2m_{3/2}k\Lambda_{\text{dyn}}^2 S + h.c.
\]

The linear term, however, is negligible compared with the scalar potential in Eq. (14) around \( S \sim \Lambda / k \) as long as,

\[
k \gtrsim 4 \left( \frac{\Lambda_{\text{dyn}}}{M_{\text{pl}}} \right)^{1/3}.
\]

Here we have used \( \partial V(S)/\partial S \sim 5(\sqrt{5}k/4\pi)^3|F_S|^2/\Lambda_{\text{dyn}} \) for \( S \sim 4\pi\Lambda_{\text{dyn}}/(\sqrt{2}k) \). The condition is easily satisfied for \( \Lambda_{\text{dyn}} \sim 10^5 \text{ GeV} \) (\( k = O(1) \)) (Eq. (14)), and hence, the local minimum we chose is stable against the supergravity effects.

On the other hand, the linear term plays an important role to generate the mass of the \( R \)-axion which corresponds to the spontaneous breaking of the \( R \)-symmetry by \( S \neq 0 \). Since the \( R \)-symmetry is broken explicitly by the linear term, the \( R \)-axion obtains a mass \([28]\),

\[
m_a \sim 2m_{3/2} \left( \frac{M_{\text{pl}}}{S} \right)^{1/2} \sim 10 \text{ MeV} \times k^{-3/2} \left( \frac{M_{\text{gluino}}}{1 \text{ TeV}} \right)^{3/2}.
\]

The \( R \)-axion couples to the standard model particles through the loop diagrams of mesons which are relevant for the gauge mediation. As a result, it decays mainly into photons at the cosmic temperature \( T \sim O(10) \text{ MeV} \). Note that for the axion with mass \( m_a \sim O(10) \text{ MeV} \), final states with hadrons or electroweak gauge bosons are kinematically forbidden. The cosmic abundance of the \( R \)-axion before the decay (both from thermal and non-thermal production) is estimated to be small enough that the decay does not cause a large entropy production \([36]\). The decay temperature is also high enough not to spoil the success of the Big-Bang Nucleosynthesis. Besides, the above \( R \)-axion marginally satisfies an astrophysical constraint based on stellar cooling rate and supernova dynamics: \( m_a \gtrsim O(10) \text{ MeV} \) \([29]\). Therefore, we find that the \( R \)-axion in our model does not cause any cosmological and astrophysical problems.

### D. Conclusions

We find a very simple model with gauge mediation where the supersymmetry breaking/mediation is realized by a dynamics at \( \Lambda_{\text{dyn}} \sim 10^5 \text{ GeV} \). Furthermore, the model predicts a very small gravitino mass \( (m_{3/2} \sim O(1) \text{ eV}) \) for \( \Lambda_{\text{dyn}} \sim 10^5 \text{ GeV} \), which can be measured at the future collider experiments such as LHC/ILC, by measuring the branching ratio of the decay rate of the next to lightest superparticle \([30]\).

Finally, it should be noted that the present model is also applicable for a wide range of the dynamical scale up to \( \Lambda_{\text{dyn}} \sim 10^{10} \text{ GeV} \) (\( k \sim 10^{-2} \), \( m_{3/2} \sim O(10) \text{ MeV} \), where the condition in Eq. (14) is saturated.

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[31] We use the same notation for chiral superfields and their scalar parts interchangeably.
[32] To ensure the origin of the singlet is not a local minimum in models with $R$-symmetry like our model, we need to introduce fields with $R$-charges other than 0 or 2 [24], although we do not require the instability of the origin of the singlet in our model.
[33] Here, we are also assuming that the Kähler metrics of $\delta \hat{M}$, $\hat{B}$ and $\bar{B}$ are positive definite around the local minimum, although the composite description is not quite well for $k (S) \sim \Lambda$.
[34] With such a small gravitino mass, it has been known that the overproduction problem by thermal scattering processes is absent [27]. However, in this model in order to avoid destabilization of the meta-stable vacuum by thermal effects, we need to assume a low reheating temperature and/or a low scale inflation. We thank T. Yanagida for pointing this out.
[35] We cannot trace the renormalization group evolution from the value in Eq. (11) since there is non-calculable $O(1)$ threshold corrections between the value of $k$ at the scale $\Lambda$ and the one at the scale higher than $\Lambda$.
[36] The decay mode into gravitinos are suppressed by a helicity suppression factor of $(m_{3/2}/m_a)^2$, and hence, the gravitino abundance produced by the decay of the $R$-axion is negligible compared with the one from the thermal bath [28].