Supersymmetry at Large Distance Scales

Herman Verlinde

Physics Department, Princeton University, Princeton, NJ 08544

Abstract

We propose that the UV/IR relation that underlies the AdS/CFT duality may provide a natural mechanism by which high energy supersymmetry can have large distance consequences. We motivate this idea via (a string realization of) the Randall-Sundrum scenario, in which the observable matter is localized on a matter brane separate from the Planck brane. As suggested via the holographic interpretation of this scenario, we argue that the local dynamics of the Planck brane – which determines the large scale 4-d geometry – is protected by the high energy supersymmetry of the dual 4-d theory. With this assumption, we show that the total vacuum energy naturally cancels in the effective 4-d Einstein equation. This cancellation is robust against changes in the low energy dynamics on the matter brane, which gets stabilized via the holographic RG without any additional fine-tuning.
1. Introduction

The observed smallness of the cosmological constant requires a remarkable cancellation of all vacuum energy contributions [1]. Supersymmetry seems at present the only known symmetry that could naturally explain this cancellation, but thus far no mechanism for supersymmetry breaking is known that would not destroy this property. Nonetheless, in searching for a possible resolution of the cosmological constant problem, it seems natural to include supersymmetry as a central ingredient. What would then be needed, however, is a mechanism – some UV/IR correspondence – by which the short distance cancellations of supersymmetry can somehow be translated into a long distance stability of the cosmological evolution equations. In this paper we would like to propose a possible candidate for such a mechanism. The basic idea is as follows.

Consider a string realization [3][4] of the Randall-Sundrum compactification scenario [2]. In such a compactification, space-time consists of a slice of $AdS_5$ (times some small compact 5-manifold) bounded by two brane-like structures, which we will call the Planck and matter brane, respectively. Both branes must be thought of as effective descriptions of more elaborate structures, defined by the full high energy string theory [4] and the low energy quantum field theory [8], respectively. Now, via the holographic UV/IR duality [9][10] of the AdS/CFT correspondence [5][6][7], we may identify the 5-d physics at different radial locations in the bulk region with 4-d physics at corresponding intermediate energy scales. The fact that the matter degrees of freedom are localized on a matter brane separate from the Planck brane, is therefore just a reflection of the fact that they spend most of their time at much lower energy scales than the Planck scale. This 4-d matter still feels ordinary long distance gravity via its interaction with the 5-d bulk [2].

The two ingredients that we would like to add to this scenario are the following:

- Since all observable (non-supersymmetric) matter is localized at the matter brane, it seems an allowed assumption that the physics on and near the Planck brane is supersymmetric, at least to a very good approximation. In particular, the classical action that describes the effective dynamics of the Planck brane could reasonably be taken to be of a particular supersymmetric form. Via the AdS/CFT dictionary, this assumption can be thought of as the holographic image of requiring 4-d high energy supersymmetry.

- Since almost all of the 5-d volume is contained in the Planck region, the large distance structure of the effective 4-d geometry will be determined by the shape of the Planck brane. Locality of the 5-d supergravity in turn implies that this shape is determined by local 5-d equations of motion, satisfied by the 5-d supergravity fields in the direct neighborhood of the Planck brane.

Both these assumptions, if indeed realizable, are most likely valid only within a certain level of approximation. It is clear however that, if both are truly valid, combined they imply that the equations of motion that determine the large scale 4-d geometry can indeed
be protected by supersymmetry. This possible reappearance of supersymmetry at large distance scales can be seen as related to the fact that – unlike in the conventional non-compact set-up – the UV/IR mapping of the AdS/CFT correspondence now acts on one single space-time that combines both the 4-d boundary field theory and the 5-d bulk gravity. Via this UV/IR duality, the infra-red bulk region of the AdS-space near the Planck brane becomes the natural home base for both the shortest and longest distance physics.

In the following, we will try to test the consistency of these two assumptions. To this end, we will address the following obvious and most serious counter-argument. Intuitively, one would expect that the low energy matter sector on the matter brane will produce some quite arbitrary effective tension, that (without some unnatural or non-local fine-tuning) is expected to induce a non-zero cosmological constant for the total 4-d effective field theory. If indeed present, its backreaction would curve the Planck brane and consequently break its supersymmetry.

A different version of the same objection is that the AdS/CFT dictionary tells us that the normal variations of the local supergravity fields near the Planck brane in fact know about low energy quantities of the dual field theory, such as vacuum expectation values, etc. In particular, the normal variation of the bulk metric (or more precisely, the extrinsic curvature at the Planck brane) knows about the full vacuum energy produced by the low energy field theory. It would seem quite unnatural to expect that the Planck brane dynamics could be chosen such that, without any pre-knowledge of the IR dynamics, it exactly cancels this matter contribution to the vacuum energy.

In the following sections we will describe a mechanism that will neutralize this counter-argument. In the final section we address some other aspects of our proposal, and discuss its relation with other recently proposed scenarios.
2. Set-up

We first briefly state our assumptions about the physics inside the three different regions – the bulk region, the matter brane, and the Planck brane.

The bulk supergravity

The 5-dimensional bulk region is a negatively curved space, with a varying 5-d cosmological constant $V(\phi)$, typically of planckian magnitude and dependent on various bulk matter fields $\phi$. The metric thus takes a warped form, such that the ratio between the warp factor on the Planck and matter brane matches the hierarchy between the Planck and typical matter scale [3]. The classical equations of motion for the bulk region are prescribed by a 5-d gauge supergravity action, of the schematic form (omitting fermionic fields)

$$S_{\text{bulk}} = \int d^5x \sqrt{-G} \left( V(\phi) + \kappa R - \frac{1}{2} (\nabla \phi)^2 \right).$$

(1)

The matter brane

The matter brane hosts all visible matter. It typically represents a strongly curved or even singular region of the 5-d geometry. Because the supergravity approximation breaks down inside this region, we will introduce an artificial line of separation between the matter brane and the bulk, located at some arbitrary scale (see fig 1). On this line, we consider the values of the 5-d supergravity fields $(g, \phi)$. Here $g$ is short-hand for the 4-d metric $g_{\mu\nu}$ and $\phi$ determine the couplings and expectation values of the matter theory. The matter brane effective action

$$\Gamma_{\text{matter}}(g, \phi)$$

(2)

is obtained by integrating out all degrees of freedom to the left of the dividing line, with fixed boundary values for $g$ and $\phi$. Since supersymmetry is broken in this sector, we will not require any special symmetries of this effective action. Moreover, we will in principle allow the matter brane to have a finite temperature and/or matter density, which may vary with time and position; $\Gamma$ will then contain specific terms whose variation with respect to $g_{\mu\nu}$ will equal the corresponding stress-energy contributions.

The effective action $\Gamma_{\text{matter}}$ will not be completely arbitrary, however: consistency of the geometric set-up requires that the total partition function of the bulk and matter region does not depend on the location of the artificial dividing line between the two regions. In AdS/CFT dual terms, this means that we imagine that around the energy scale corresponding to this line, the matter theory has become equivalent to the exact holographic dual of the 5-d bulk supergravity. Shifting the position of the line then corresponds to changing some arbitrarily chosen RG scale [15]. We will formulate this invariance requirement in more detail in section 5.
The Planck brane

The region near the Planck brane represents the internal compactification geometry of the full string theory outside the AdS-like region [3]. In accordance with the holographic interpretation of the 5-d geometry, its internal structure is directly probed only by Planck scale 4-d physics. In our scenario, this physics is assumed to be supersymmetric. To make this symmetry manifest, we will now postulate, as an effective description of the Planck brane brane dynamics, the following special boundary action

\[ S_{\text{planck}}(g, \phi) = \int d^4x \sqrt{-g} W(\phi). \]  

Here \( W(\phi) \) is the superpotential of the 5-d supergravity, related to \( V(\phi) \) via

\[ V(\phi) = \frac{1}{2} (\partial_\phi W(\phi))^2 - \frac{1}{3} W(\phi)^2. \]  

In particular, we will be assuming that the Planck brane will not carry any independent matter density on its own world volume. As we will see shortly, the choice of action (3) implies that the Planck brane geometry satisfies an effective 4-d Einstein equation with vanishing cosmological constant. Ultimately, it will therefore be important to determine how well protected (3)-(4) are against quantum corrections induced via the presence of the non-supersymmetric matter brane. For now, however, we will simply define the classical action of our model via (3)-(4).

3. Equation of Motion I

The boundary conditions that follow from (3) are

\[ \theta_{\mu\nu} - \theta g_{\mu\nu} = W(\phi) g_{\mu\nu}, \quad \partial_n \phi = \partial_\phi W(\phi), \]  

where \( g_{\mu\nu} \) denotes the induced metric and \( \theta_{\mu\nu} \) the extrinsic curvature of the Planck brane; \( \partial_n \) the derivative normal to its world-volume. Now the normal component of the 5-d bulk Einstein equation, when written in terms of 4-d geometric quantities, reads

\[ \frac{1}{4}(\theta^2 - (\theta_{\mu\nu})^2) - \frac{1}{2}(\partial_n \phi)^2 + V(\phi) + \kappa R - \frac{1}{2}(\nabla \phi)^2 = 0. \]  

Here \( \nabla \) and \( R \) are the 4-d gradient and curvature scalar. When combined with (5) and (4), we deduce from (3) that on the Planck brane world volume

\[ \kappa R - \frac{1}{2}(\nabla \phi)^2 = 0. \]  

This is a simplification, that in effect amounts to a restriction on the 4-d physics, namely that it is always sub-Planckian. This in particular means that we will be ignoring the possible stress-energy contributions of gravitationally collapsed matter inside 4-d black holes.
The classical supersymmetric Planck brane geometry thus solves the trace of the 4-d Einstein equations, with possibly non-zero matter density, but always with zero cosmological constant. This result holds independently of what happens inside the bulk: possible corrections to (7) can only arise from local physics that happens at or near the location of the Planck brane. In the following sections we will present a rederivation of (7) within the quantum context.

4. Partition Function

Consider the total quantum mechanical partition function of our system. We can first divide it into three parts, corresponding to the three sub-regions. The bulk supergravity partition function can best be thought of as an evolution operator $\hat{U}$ that describes the propagation through the bulk from the matter to the Planck brane. Its matrix element between eigen states $|g, \phi\rangle$ of the boundary fields has the formal path integral expression

$$\langle g_1, \phi_1 | \hat{U} | g_2, \phi_2 \rangle = \int \left[ dG \right] \left[ d\phi \right] e^{i \frac{\hbar}{\kappa} S_{\text{bulk}}(G, \phi)}. \quad (8)$$

Since in our set-up the matter brane is defined as the region left to some quite arbitrary line of separation with the bulk (see fig 1), its partition function is indeed most appropriately thought of as a wave-function

$$\langle \Psi_{\text{matter}} | g_1, \phi_1 \rangle = e^{i \frac{\kappa}{\hbar} \Gamma_{\text{matter}}(g_1, \phi_1)}. \quad (9)$$

Similarly we can define

$$\langle g_2, \phi_2 | \Psi_{\text{planck}} \rangle = e^{i \frac{\kappa}{\hbar} S_{\text{planck}}(g_2, \phi_2)}. \quad (10)$$

The total partition function is obtained by taking the inner-product of these two wave-functions, with the bulk evolution operator inserted in between

$$Z = \langle \Psi_{\text{matter}} | \hat{U} | \Psi_{\text{planck}} \rangle. \quad (11)$$

Inserting the above definitions, this matrix element represents the complete functional integral over all matter and gravity fields. We will imagine that it can be given a well-defined definition via the underlying fundamental string theory.
5. Holographic RG

Now let us consider the evolution of the matter state under local variations of the position of the dashed line in fig 1. Thinking about the radial direction as a Euclidean time direction, we may identify the corresponding Hamilton operator $\hat{\mathcal{H}}$ of the bulk supergravity as the generator of these variations. However, as in any gravity theory, physical wave functions should be invariant under local variations of the particular time-slicing. We may write this condition as

$$\hat{\mathcal{H}} \hat{U} |\Psi_{\text{matter}}\rangle = 0.$$  (12)

The Hamilton operator associated with the bulk action (11) reads

$$\hat{\mathcal{H}} = \hbar^2 \sqrt{-g} \left( \frac{1}{3} \left( \frac{\delta}{\delta g^\lambda} \right)^2 - \left( \frac{\delta}{\delta g^{\mu\nu}} \right)^2 - \frac{1}{2} \left( \frac{\delta}{\delta \phi} \right)^2 \right) \left( \sqrt{-g} \left( \kappa R - \frac{1}{2} (\nabla \phi)^2 \right) \right).$$  (13)

Classically, the condition $\hat{\mathcal{H}} = 0$ is simply the 5-d bulk Einstein equation (6). Indeed, we emphasize that eqn (12) is not some special symmetry requirement on the matter sector, but a property of any state that has traveled through the bulk supergravity region.

After applying the AdS/CFT dictionary, our geometric set-up is equivalent to assuming that the complete matter theory above some energy scale – corresponding roughly to the line of separation between the matter brane and the bulk – becomes equivalent to the exact holographic dual of the bulk supergravity. From this 4-d perspective, the relation (12) acquires a new meaning as the evolution of the matter partition function under the holographic RG flow [15, 16]. Note however, that for finite rank $N$ and gauge coupling, the bulk theory is a complete string theory with finite string length and string coupling. The constraint (12) for finite $\hbar$ includes $1/N$ effects, but the Hamiltonian (13) will receive various string corrections. We expect however that these will not significantly alter the basic form of the RG equation [18].

6. Equation of Motion II

In our set-up, the boundary state at the Planck brane has been chosen to be invariant under 4-d global supersymmetry transformations, $Q_\alpha |\Psi_{\text{planck}}\rangle = 0$. Using the explicit form (11) of $|\Psi_{\text{planck}}\rangle$ and the classical relation (3), we find that

$$\hat{\mathcal{H}} |\Psi_{\text{planck}}\rangle = \sqrt{-g}(\kappa R - \frac{1}{2} (\nabla \phi)^2) |\Psi_{\text{planck}}\rangle.$$  (14)

Here we used that the one-loop term, proportional to $\hbar \left( \frac{4}{3} W - \frac{1}{2} \partial_\phi^2 W \right)$, is cancelled by the corresponding fermionic contribution, which is true provided the fermionic part of the
wavefunction is chosen such that $|\Psi_{\text{planck}}\rangle$ indeed represents a supersymmetric ground state. Combined with the $\hat{H} = 0$ condition (12), eqn (14) implies

$$\langle \kappa R - \frac{1}{2} (\nabla \phi)^2 \rangle = \frac{1}{Z} \langle \Psi_{\text{matter}} | \hat{U} \hat{H} | \Psi_{\text{planck}} \rangle = 0. \quad (15)$$

This equation generalizes the classical result of section 3. As emphasized before, possible corrections to this identity can only arise from matter and/or supersymmetry breaking terms at the direct location of the Planck brane.

7. Supersymmetry breaking

Suppose we insist on maintaining the Planck brane as supersymmetric as needed to make the cosmological constant as small as observed today. The question then arises as to whether the evolution into the 5-d bulk, that describes the holographic RG flow towards the IR, can still lead to a realistic non-supersymmetric low energy field theory on the matter brane. A possible scenario by which this may happen is when the RG flow at some scale enters a strongly coupled region (coupling $g$ of order 1 or larger) from both the 5-d and 4-d point of view. The dynamics in this region could then be sufficiently non-trivial to induce dynamical breaking of 4-d supersymmetry.

To gain some insight into how this may happen, suppose we define the matter state by means of some non-supersymmetric low energy matter sector, so that

$$Q_{\alpha} |\Psi_{\text{matter}}\rangle \neq 0. \quad (16)$$

The vacuum amplitude is then still supersymmetric, since the Planck state via (11) projects out the supersymmetric part of the matter wave-function. The violation of supersymmetry becomes visible, however, as soon as we consider non-trivial expectation values. Generalizing our prescription of section 4, we can formally represent these via

$$\langle \Psi_{\text{matter}} | \mathcal{O}_{\text{phys}} \hat{U} | \Psi_{\text{planck}} \rangle \quad (17)$$

where $\mathcal{O}_{\text{phys}}$ represents some physical observable inside the matter brane region. Suppose we now try to derive a supersymmetry Ward identity. Because of (16), it will be violated

$$\langle \Psi_{\text{matter}} | [Q_{\alpha}, \mathcal{O}_{\text{phys}}] \hat{U} | \Psi_{\text{planck}} \rangle \neq 0. \quad (18)$$

The 4-d low energy observer would thus conclude that supersymmetry is broken. On the other hand, since physical operators $\mathcal{O}_{\text{phys}}$ must commute with the generator $\hat{H}$, the derivation of eqn (13) as given in the previous section still goes through (see also Appendix A).
8. 4-d Effective Field Theory

The reasoning in the above sections is rather formal. To make things somewhat more concrete, let us from now on assume that the 4-d high energy gauge theory is at large \( N \) and strong 't Hooft coupling, so that the 5-d supergravity is in its classical regime.

In terms of pure 4-d language the situation is then summarized as follows. Consider the complete matter effective action, computed in some fixed 4-d background \((g, \phi)\), obtained by integrating out both the low energy energy matter, as well as the high energy matter dual to the 5-d bulk supergravity. We will still call this effective action \( \Gamma_{\text{matter}} \), since indeed it is just the same action \((3)\) evolved via the RG flow \((12)\) to a larger scale factor for \( g_{\mu\nu} \). Note that \( \Gamma_{\text{matter}} \) now contains the full Einstein term of the 4-d gravity. The total 4-d action is obtained by just adding the Planck brane action \((8)\)

\[
\Gamma_{\text{total}}(g, \phi) = \Gamma_{\text{matter}}(g, \phi) + \int d^4x \sqrt{-g} W(\phi). \tag{19}
\]

The claim is that, regardless of the details of the low energy matter theory, the classical equations of motion of this action

\[
\frac{1}{\sqrt{-g}} \frac{\delta \Gamma_{\text{matter}}}{\delta g^{\mu\nu}} = \frac{1}{2} W(\phi) g_{\mu\nu}
\]

\[
\frac{1}{\sqrt{-g}} \frac{\delta \Gamma_{\text{matter}}}{\delta \phi} = \partial_\phi W(\phi) \tag{20}
\]

are such that the trace of the Einstein equation always takes the form \((7)\). This is possible since \( \Gamma_{\text{matter}} \) is not completely arbitrary, but due to the high energy equivalence with the 5-d supergravity satisfies \((12)-(13)\). In the classical approximation, this identity reduces to the Hamilton-Jacobi relation, which looks identical to eqn. \((6)\) with the replacement

\[
\theta_{\mu\nu} - \theta g_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta \Gamma_{\text{matter}}}{\delta g^{\mu\nu}}
\]

\[
\partial_n \phi = \frac{1}{\sqrt{-g}} \frac{\delta \Gamma_{\text{matter}}}{\delta \phi}. \tag{21}
\]

Via this same replacement, the classical equations of motion \((20)\) just amount to the boundary conditions \((3)\). Combining these two results gives \((7)\), just as before.

9. Stabilizing the Matter Brane

Finally, let us address how to stabilize the matter brane. This is an important issue, since its location relative to the Planck brane represents an invariant physical scale \([2]\). As expected, some subtleties will arise here.
A first subtlety is that the 5-d physics in the matter brane region is likely to be singular. Thus far, however, we did not need to worry about this issue, because we chose to describe the matter brane region by means of the dual low energy field theory. No special assumptions about the properties of the singularity were needed, except that at some distance away from it we can use the geometric supergravity language to derive the constraint (12). It is not important for our argument, for example, whether there are discrete or continuously many solutions to this constraint.

Regardless of this issue, it is clear that the matter brane effective action should be allowed to be as arbitrary as possible. Nonetheless one would hope to naturally stabilize its location by means of its interaction with the Planck brane. We will first describe two possible mechanisms, which however both will fall short in that they either remain unstable or need unnatural fine-tuning. We will then describe a third scenario that will resolve these problems.

**Attempt 1: Goldberger-Wise mechanism [17]**

Suppose that, in spite of the fact that it describes a singular space-time region, we would assume that the matter brane dynamics is well approximated by that of some classical brane with some arbitrary tension \( \Lambda(\phi) \). We can then look for a classically stable location for matter brane as follows [19]. A static bulk solution, when matched onto the supersymmetric boundary conditions set by the Planck brane, is described by scalar fields \( \phi(r) \) and a metric

\[
\mathrm{d}s^2 = a^2(r) \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2,
\]

satisfying the supersymmetric flow equations

\[
\frac{a'}{a} = -\frac{1}{6} W(\phi) \quad \quad \phi' = \partial_\phi W.
\]

From this one finds that the matching relations at the matter brane, that are required for there to be a flat solution, are that (cf. eqn (20))

\[
\partial_\phi \Lambda(\phi_c) = \partial_\phi W(\phi_c) \quad \quad \Lambda(\phi_c) = W(\phi_c)
\]

for some critical value of \( \phi_c \) for the scalar fields. The first equation generically gives at most a discrete set of solutions for \( \phi_c \). The second relation, however, is then valid only if the value of \( \Lambda \) at such a critical point \( \phi_c \) is exactly equal to that of the superpotential \( W \). This amounts to an unnatural fine-tuning of the matter brane action. However, even if we would choose \( \Lambda(\phi) \) such that this condition is satisfied, the equations of motion (24) still fail to stabilize the relative location of the Planck and matter brane, as they do not pick out one particular preferred relative ratio for the scale factors \( a \) at the two branes.
Because of both these problems, we will instead choose a different route.

**Attempt 2: Hamilton-Jacobi action**

In accordance with our definition of the matter brane as the region behind the dashed line in fig 1, we will now reinstate the condition that its effective action satisfies (12). Let us further assume that for slowly varying fields it takes the general form

\[ \Gamma_{\text{matter}} = \int \sqrt{-g} \left( \Lambda(\phi) + \Phi(\phi) R + \ldots \right) \]  

(25)

Using the classical supergravity approximation, (12) then reduces to the Hamilton-Jacobi relation [16]

\[ \frac{1}{2} \left( \partial_\phi \Lambda \right)^2 - \frac{1}{3} \Lambda^2 + \left( \partial_\phi \Lambda \partial_\phi \Phi - \frac{1}{3} \Lambda \Phi \right) R + \ldots = V + \kappa R + \ldots \]  

(26)

with \( V \) and \( \kappa \) the 5-d potential and Newton constant. Apart from this constraint, the functions \( \Lambda(\phi) \) and \( \Phi(\phi) \) will be allowed to be arbitrary.

In principle, one could read (26) as a condition on classical field configurations \( (g_c, \phi_c) \), that determines the matter brane curvature \( R \) for given tension \( \Lambda(\phi_c) \), etc. In our set-up, however, (25) defines a true Hamilton-Jacobi action, for which (26) in fact amounts to a functional identity valid for all field configurations \( (g, \phi) \). So in particular the relation

\[ \frac{1}{2} \left( \partial_\phi \Lambda \right)^2 - \frac{1}{3} \Lambda^2 = V \]  

(27)

represents a relation between \( \Lambda(\phi) \) and \( V(\phi) \) that holds for all values of \( \phi \).

Next let us again look for a consistent flat matter brane solution in the presence of the Planck brane. Following the same derivation as before, we again arrive at the condition (24) for \( \Lambda(\phi) \). Now the situation is somewhat different: if we can find a solution to the first relation in (24) then via (27) we automatically satisfy the second relation. At first this seems like good news, but in fact the situation is now worse than before: for generic \( V(\phi) \) the only solution to the H-J relation (27) that will ever be tangent to \( W(\phi) \) is \( W(\phi) \) itself.

Hence if we require that (24) is satisfied, we must have that \( \Lambda(\phi) = W(\phi) \). But this is a hyper fine-tuned situation, since it says that the matter brane is just as supersymmetric as the Planck brane. This is not what we want.

**Attempt 3: Holographic effective action**

The problem we just encountered is in essence Weinberg’s counter-argument against the use of various adjustment mechanisms for cancelling the cosmological constant [1]. We now propose a mechanism that will evade this counter-argument. See also [13].
There are many indications that the matter brane action \( \Gamma_{\text{matter}}(\phi, g) \), at finite values for the scale factor of \( g_{\mu\nu} \), in fact corresponds to a Wilsonian effective action of the dual 4-d field theory, defined with a finite UV cut-off \( \epsilon \). Therefore, a more accurate formula for the matter effective action is

\[
\Gamma_{\text{matter}} = \int \sqrt{-g} (\Lambda(\phi, \epsilon) + \Phi(\phi, \epsilon) R + \ldots).
\]  

This cut-off dependence of \( \Gamma_{\text{matter}} \) will turn out to be crucial. The holographic dictionary furthermore relates variations in \( \epsilon \) to variations in the scale factor of \( g_{\mu\nu} \) via

\[
e \frac{\partial}{\partial \epsilon} = 2 \int g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}} = -a \frac{\partial}{\partial a}
\]

with \( a \) the overall warp factor in (22). We can use this relation to replace the short-distance cut-off \( \epsilon \) in (28) by \( a \). The effective action (28) thus acquires a new and unexpected dependence on \( g_{\mu\nu} \).

The holographic relation (29) between RG transformations and constant Weyl rescalings, and the resulting extra dependence of \( \Gamma_{\text{matter}} \) on \( g_{\mu\nu} \), has quite non-trivial implications. In particular, it implies that the Hamilton-Jacobi relation (27) and matching relations (24) now take the more general form:

\[
\frac{1}{2} (\partial_\phi \Lambda)^2 - \frac{1}{48} (4\Lambda - a \frac{\partial \Lambda}{\partial a})^2 = V,
\]

\[
\partial_\phi \Lambda(\phi_c, a) = \partial_\phi W(\phi_c),
\]

\[
(4 - a \frac{\partial}{\partial a}) \Lambda(\phi_c, a) = 4W(\phi_c).
\]

In fact, now we are in business: it is easy to show that these equations in general do have solutions. The reason is that, via the non-trivial dependence on \( a \), we now have an extra parameter that we can adjust.

Things are however still less trivial than one might expect: the above equations have an RG invariance and generically allow for a whole critical line of solutions \( \phi_c(a) \), generated by the holographic RG-flow (see eqn (23)):

\[
a \frac{\partial}{\partial a} \phi_c(a) = \beta_\phi(\phi_c(a))
\]

\[
\beta_\phi = \frac{6 \partial_\phi W}{W}.
\]
Eqn (31) does therefore not pick out one particular preferred value for the scale \( a \) at which the matter brane is localized: \( a \) is still left free. This is in accordance with the definition of the matter brane as the region behind a relatively arbitrary line of separation with the bulk region.

Nonetheless, we have achieved a stabilization of the matter brane location. For a given matter brane effective action (28), eqn (31) selects out a particular RG trajectory. This trajectory will generically become unstable around a particular scale \( a_{\text{crit}} \), where one or more of the scalar fields starts to run off a steep slope and induce a large 5-d curvature. This critical scale \( a_{\text{crit}} \) is the natural scale of the matter brane, and also the scale where non-supersymmetric physics can start to take place.

10. An Explicit Example

To illustrate the above stabilizing mechanism, we now describe a specific example. Consider an effective potential of the matter brane of the form

\[
\Lambda(\phi, a) = -W(\phi) + \omega(\phi, a)
\]  

(35)

where \( \omega(\phi, a) \) can be treated as a small perturbation. Eqns (30)-(32) then reduce to

\[
\left( a \frac{\partial}{\partial a} - \beta_\phi \partial_\phi - 4 \right)\omega(\phi, a) = 0
\]

(36)

\[
\partial_\phi \omega(\phi_c, a) = 0
\]

(37)

\[
\left( a \frac{\partial}{\partial a} - 4 \right)\omega(\phi_c, a) = 0,
\]

(38)

with \( \beta_\phi \) as defined in (34). The first equation states that \( \omega \) is invariant under the holographic RG flow generated by \( W \). The last two equations further show that \( \omega \) is the potential that needs to be minimized to obtain the classical vacuum values of the fields. We can thus identify \( \omega(\phi, a) \) with the total effective potential.

Now as a simple example, let us consider the case with just one scalar field \( \phi \) – the dilaton – and assume that

\[
W(\phi) = e^{\frac{1}{\sqrt{3}} \phi}.
\]

(39)

is

\[
\beta_\phi = 2\sqrt{3}.
\]

(40)
Now suppose that $\omega(\phi, a)$ takes the form

$$a^4 \omega(\phi, a) = \frac{1}{4} \lambda a^4 e^{\frac{2}{\sqrt{3}} \phi} \left( \log\left( \mu a^2 e^{\frac{1}{\sqrt{3}} \phi} \right) - \frac{1}{2} \right)$$

(41)

with $\lambda$ and $\mu$ some free parameters. It is easily verified that this expression, for arbitrary $\lambda$ and $\mu$, solves the linearized RG flow equation \cite{36}.

The minimum of the effective potential lies at

$$\mu a^2 e^{\frac{1}{\sqrt{3}} \phi_c} = 1$$

(42)

This minimum describes an RG trajectory

$$\phi_c(a) = -\sqrt{3} \log(\mu a^2).$$

(43)

Via (23) this corresponds to a classical bulk geometry (22), with warp factor

$$a(r) = \sqrt{1 - \frac{r}{3\mu}} \quad 0 \leq r \leq 3\mu.$$  

(44)

Here we normalized $a$ such that the Planck brane is located at $a = 1$. We see that the $r$-direction describes a compact interval: the geometry closes off by means of a naked singularity, located at the critical distance $r_{\text{crit}} = 3\mu$ from the Planck brane.

The matter brane is the strongly curved neighborhood of the naked singularity. Its location is therefore indeed dynamically stabilized by the presence of the Planck brane. Furthermore, by virtue of our presumption that the Planck brane remains supersymmetric, both branes are flat. This result holds for arbitrary values of the free parameters $\mu$ and $\lambda$. Note that the effective 4-d cosmological constant of the matter brane is indeed not equal to the minimum value of the effective potential

$$a^4 \omega(\phi_c, a) = -\frac{\lambda}{8\mu^2}.$$

(45)

Rather, it is equal to its variation with respect to the overall scale $a$

$$\Lambda_{\text{eff}} = a \frac{d}{da} \left( a^4 \omega(\phi_c, a) \right) = 0.$$  

(46)

\footnote{This potential has been deliberately chosen to take the typical form of a one-loop correction, obtained by integrating out a 4-d matter field, in this particular case with a mass proportional to $e^{\frac{1}{\sqrt{3}} \phi}$. The extra $a$-dependence of the potential can be derived for example by computing the one-loop determinant via dimensional regularization, in the 4-d induced background metric $g_{\mu\nu} = a^2 \eta_{\mu\nu}$ on the matter brane.}
This last equality of course follows by inspection from (45), but more fundamentally, arises as a direct consequence of the (linearized) RG constraint (36).

The RG trajectory (43) and geometry (44) describe a supersymmetric bulk solution. This is the classical version of the result that in the vacuum amplitude (11), the Planck state always projects out the supersymmetric component of the matter state. We expect, however, that for general non-supersymmetric observables, the non-supersymmetric form (35)-(41) of the matter brane effective action will become manifest. Supersymmetry remains intact only for the BPS-type limit $\lambda \to 0$ and $\mu \to 0$ (in some fixed ratio).

11. Discussion

We have explored some physical consequences of a Randall-Sundrum scenario [2], in which the Planck brane region is assumed to be supersymmetric. Via the UV/IR mapping of the AdS/CFT duality, this condition appears equivalent to that of 4-d high energy supersymmetry. As we have shown, however, the same supersymmetry also stabilizes the large scale 4-d geometry. It appears therefore that our set-up implies a stronger restriction on the 4-d effective field theory than ordinary high energy supersymmetry. From the higher dimensional viewpoint this stronger restriction seems natural, however, as it pertains to just one single region of the 5-d geometry.

Our set-up differs in a number of aspects from other recent proposals [12] [13]. In these papers, the effective 4-d flatness appears as a consequence of a self-tuning mechanism of a single 4-d brane-world, embedded inside a fine-tuned 5-d bulk. This mechanism however relies on rather strong assumptions about the physics of the IR singularity of the 5-d geometry. In our case, on the other hand, the 4-d stability is the consequence of a fine-tuned Planck brane action. This fine-tuning is justified by our assumption of supersymmetry, and furthermore local, both from the 5-d and the 4-d RG perspective.

Given our observation that the 4-d flatness is proportional to the amount of supersymmetry breaking at the Planck brane, it will clearly be important to look for possible mechanisms that may prevent or suppress the mediation of supersymmetry breaking via the bulk. It seems clear that holography (both 5-d and 4-d) will be an important ingredient in this discussion.

First, it will be important to know how best to describe the local degrees of freedom on the Planck brane. Via the string theory description [3] [4], much is known in principle about its internal structure, but its effective dynamics still seems hard to extract. Let us nonetheless imagine computing some supersymmetry violating one loop correction to the Plank brane physics, induced by a bulk graviton traveling back and forth to the matter brane region. A priori, one would not expect any large suppression from this propagator,

\[^3\text{In [20], a somewhat similar suggestion was made for a scenario in which our 4-d world might be stabilized via the embedding into an unobservable 5-d supersymmetric world.}\]
The geometric distinction between the matter and Planck regions in fig 1, when translated in terms of the shape of string world-sheets, suggests that while large open string loops are non-supersymmetric, long closed string propagators can still be protected by supersymmetry.

and it would thus appear that supersymmetry will indeed be violated on the Planck brane at roughly the same length scale as that set by the matter brane location.

This conclusion, however, disregards the implications of 5-d holography, which suggest that the local quantum fluctuations at the Planck brane – in as far present – all have roughly Planckian frequencies and wave-lengths. If this is the case, the 5-d graviton modes emitted by these fluctuations must also have Planckian wavelengths along the brane direction. Such modes, however, decay exponentially along the \( r \)-direction with a Planckian length. Hence, if this physical picture is correct, this would indeed give a mechanism by which the bulk mediation of supersymmetry breaking gets exponentially suppressed via the separation between the two branes.

Additional restrictions on the use of naive effective field theory may come from 4-d holography. It is clear, for example, that a direct AdS/CFT interpretation of the 5-d bulk region, as dual to a 4-d quantum field theory with Planckian cut-off, still introduces far too many degrees of freedom. It is tempting to speculate that supersymmetry near the Planck region could be instrumental in hiding this excess in degrees of freedom, thus protecting the 4-d holographic bound.

Finally: the UV/IR connection explored here, just like all other ones pointed out earlier in [21] [22] [18], seems directly connected with the fundamental equivalence between open string loops and closed string propagators. Roughly, our assumption here has been that long closed string propagators and short open string loops are protected by supersymmetry, even though short closed string propagators and long open string loops are not. It is reassuring that these two types of diagrams represent clearly separated regions in the parameter space of open string Feynman diagrams.

**Acknowledgements**
Appendix: A Small Matter Perturbation

Here we consider the effect of a small matter perturbation on the geometry of the Planck brane. The perturbation induces a small extra contribution $\delta \Gamma(g, \phi)$ to the matter+bulk effective action. The variation of this extra contribution with respect to the metric and field $\phi$ represent the expectation values of the corresponding dual operators

$$\frac{1}{\sqrt{-g}} \frac{\delta (\delta \Gamma)}{\delta g_{\mu\nu}} = \langle T_{\mu\nu} \rangle \quad \frac{1}{\sqrt{-g}} \frac{\delta (\delta \Gamma)}{\delta \phi} = \langle O_\phi \rangle$$

which represent the physical effect of the matter perturbation. The RG flow relation (12) must still hold for the total effective action, including the extra term. (This is true since we assume that all matter is located on the matter brane.) This RG condition implies that the above two expectation values cannot be independent. Working to linearized order in $\langle T_{\mu\nu} \rangle$, and within the classical supergravity approximation, one finds that

$$\langle T \rangle = \beta_\phi \langle O_\phi \rangle + \frac{6}{W} \langle O_\phi \rangle \langle O_\phi \rangle$$

Here $T$ is the trace of the stress energy tensor and $\beta_\phi$ the holographic beta-function defined in (34). Now if we consider the linearized equations of motion of the small metric variation $h_{\mu\nu} = \delta g_{\mu\nu}$ induced by the matter perturbation, then because of the relation (A.2) it is possible to combine the trace of the $h$ equation with the $\phi$ equation such that the source term $\langle T \rangle$ cancels. The resulting equation takes the form

$$\kappa \Box h + \frac{1}{2} (\nabla \phi)^2 = 0.$$
The trace of the stress energy tensor of matter away from the Planck brane thus always gets represented by means of variations in the $\phi$ field.

References

[1] S. Weinberg, “The cosmological constant problem”, Rev.Mod.Phys.61 (1989), 1-23.

[2] L. Randall and R. Sundrum, “A Large Mass Hierarchy from a Small Extra Dimension”, hep-ph/9905221.

[3] H. Verlinde, “Holography and Compactification”, hep-th/9906182.

[4] C.S. Chan, P.L. Paul and H. Verlinde, “A Note on Warped String Compactification,” hep-th/0003230.

[5] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200.

[6] S. Gubser, I. Klebanov and A. Polyakov, “Gauge Theory Correlators from Non-Critical String Theory,” Phys.Lett.B428 (1998) 105, hep-th/9802109.

[7] E. Witten, “Anti-de Sitter Space and Holography,” Adv.Theor.Math.Phys.2 (1998) 253, hep-th/9802150.

[8] For recent discussions of the physics near the IR region see e.g.: S.S. Gubser, “Curvature Singularities: the Good, the Bad, and the Naked,” hep-th/0002160, and J. Polchinski and M. Strassler, “The String Dual of a Confining Four-Dimensional Gauge Theory,” hep-th/0003136.

[9] L. Susskind and E. Witten, “The Holographic Bound in Anti-de Sitter Space,” hep-th/9905114.

[10] A. Peet and J. Polchinski, “UV/IR Relations in AdS Dynamics,” hep-th/9809022.

[11] V. Balasubramanian and P. Kraus, “A Stress Tensor for Anti-de Sitter Gravity,” hep-th/9902124, Commun.Math.Phys. 208 (1999) 413-428.

[12] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, “A Small Cosmological Constant from a Large Extra Dimension,” hep-th/0001197.

[13] S. Kachru, M. Schulz and E. Silverstein, “Self-tuning flat domain walls in 5d gravity and string theory,” hep-th/0001206. “Bounds on curved domain walls in 5d gravity,” hep-th/0002123.

[14] V.A. Rubakov, M.E. Shaposhnikov, “Extra Space-Time Dimensions: Towards a Solution of the Cosmological Constant” Phys.Lett. 125B (1985) 372. “Do we live inside a Domain Wall?” Phys. Lett. 125B (1983) 139.
[15] E. Verlinde, and H. Verlinde, “RG flow, Gravity and the Cosmological Constant,” hep-th/9912018.

[16] J. de Boer, E. Verlinde, and H. Verlinde, “On the Holographic Renormalization Group,” hep-th/9912012.

[17] W. Goldberger and M. Wise, “Modulus Stabilization with Bulk Fields,” hep-ph/9907447.

[18] J. Khoury and H. Verlinde, “On Open/Closed String Duality,” hep-th/0001056; M. Li, “A Note On Relation Between Holographic RG Equation And Polchinski’s RG Equation,” hep-th/0001193.

[19] K. Skenderis, P. Townsend, “Gravitational Stability and Renormalization-Group Flow,” hep-th/9909070; U. Ellwanger, “Constraints on a Brane-World from the Vanishing of the Cosmological Constant,” hep-th/9909103; O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, “Modeling the fifth dimension with scalars and gravity,” hep-th/9909134.

[20] E. Witten, “The Cosmological Constant From The Viewpoint Of String Theory,” hep-ph/0002297.

[21] M. R. Douglas, D. Kabat, P. Pouliot, and S. H. Shenker, “D-branes and Short Distances in String Theory,” Nucl. Phys. B485 (1997), hep-th/9608024.

[22] S. Minwalla, M. Van Raamsdonk and N. Seiberg, “Noncommutative Perturbative Dynamics,” hep-th/9912072.