STELLAR PROPER MOTIONS IN THE GALACTIC BULGE FROM DEEP HUBBLE SPACE TELESCOPE ACS WFC PHOTOMETRY

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ABSTRACT

We present stellar proper motions in the Galactic bulge from the Sagittarius Window Eclipsing Extrasolar Search (SWEEPS) project using ACS WFC on HST. Proper motions are extracted for more than 180,000 objects, with >81,000 measured to accuracy better than 0.3 mas yr⁻¹ in both coordinates. We report several results based on these measurements: (1) Kinematic separation of bulge from disk allows a sample of >15,000 bulge objects to be extracted based on >6σ detections of proper motion, with <0.2% contamination from the disk. This includes the first detection of a candidate bulge blue straggler population. (2) Armed with a photometric distance modulus on a star-by-star basis, and using the large number of stars with high-quality proper-motion measurements to overcome intrinsic scatter, we dissect the kinematic properties of the bulge as a function of distance along the line of sight. This allows us to extract the stellar circular speed curve from proper motions alone, which we compare with the circular speed curve obtained from radial velocities. (3) We trace the variation of the (l, b) velocity ellipse as a function of depth. (4) Finally, we use the density-weighted (l, b) proper-motion ellipse produced from the tracer stars to assess the kinematic membership of the 16 transiting planetary candidates discovered in the Sagittarius Window; the kinematic distribution of the planet candidates is consistent with that of the disk and bulge stellar populations.

Subject headings: Galaxy: bulge — Galaxy: disk — Galaxy: kinematics and dynamics — instrumentation: high angular resolution — methods: data analysis — techniques: photometric

Online material: color figures

1. INTRODUCTION

The formation and evolution of merger-built bulges and secularly grown pseudobulges and bars are crucial to the evolution of spiral galaxies, and indeed their formation history is used to test models for the formation of structure in the universe such as the ΛCDM framework (for a review see Kormendy & Kennicutt 2004). The inner region of our own Milky Way shows evidence for a bulge (e.g., Blitz & Spergel 1991) and at least one barlike structure (e.g., Benjamin et al. 2005; López-Corredoira et al. 2007). Furthermore, for our own bulge the stars are close enough that detailed stellar data may be obtained on a star-by-star basis, such as radial velocities and proper motions, which are simply not yet available for most external galaxies.

These studies of our own bulge show it to be a distinct stellar population from the disk of the Milky Way with a wide range of abundances (Rich 1988). High [α/Fe] compared to the disk suggests rapid enrichment; Type II supernovae (SNe) produce the α-elements and result from short-lived, high-mass stars, while iron enrichment requires the rather slower buildup of a significant number of Type I SNe (McWilliam & Rich 1994; Zoccali et al. 2006; Lebreuille et al. 2007; Fulbright et al. 2007). A recent spectroscopic comparison with halo objects shows the majority of bulge stars to be α-enhanced compared to the halo; thus, the majority bulge stellar population cannot have formed from the halo (Fulbright et al. 2006, 2007).

The detailed balance of populations of the bulge and its current kinematics remain rather poorly constrained, and thus the picture of its step-by-step formation and evolution is far from complete. Strong, variable extinction caused by gas and dust in the intervening spiral arms (e.g., Sumi 2004), contamination of the test population by stars in the foreground disk, and the significant spatial depth of the bulge along the line of sight combine to make the separation of a pure-bulge sample for further study a challenging task. While one can identify a candidate main sequence, the main-sequence turnoff (MSTO) usually used as an age/metallicity diagnostic is broadened by the age, metallicity, and depth range of the bulge and confused by the foreground disk population. Force-fitting the horizontal branch in the color-magnitude diagram (CMD) and comparison with globular cluster sequences suggest a majority population ~10 Gyr in age, but with as much as 30% of the stars belonging to a young population (Holtzman et al. 1993; Ortolani et al. 1995). Furthermore, a population of OH masers (Sevenster et al. 1997) and a few hundred mass-losing asymptotic giant branch (AGB) stars (van Loon et al. 2003) has been detected in the inner Milky Way. These objects are claimed to represent an intermediate-age (~1 to a few Gyr) population within the inner Milky Way. These objects populate the circumnuclear molecular zone ([l] < 1.5°, [b] < 0.5°) and are thought to be tracers of a larger population that is difficult to separate from the bulk stellar population in the CMD (Lindqvist et al. 1992; Messineo et al. 2002; Haring et al. 2006). In optical CMDs, young to intermediate-age populations overlap significantly.
with the main sequence of a young foreground disk population; separation of the bulge from the disk population is therefore critical if the detailed population distribution of the bulge is to be obtained.

Feltzing & Gilmore (2000) compare number counts along the CMD between populous clusters at a range of Galactic latitudes to estimate the foreground disk contamination, producing a prediction of disk contamination in two bulge fields and thus a differential estimate of the bulge population itself. They find a bulge population almost exclusively older than \( \sim 10 \) Gyr. This conclusion was reinforced by a later study employing statistical subtraction to take off a scaled contamination from a comparison field in the foreground disk; globular cluster comparison suggested an old, metal-rich population to fit the bulge well, but without a precise age distribution (Zoccali et al. 2003). While a metallicity gradient with height \( z \) above the Galactic midplane probably does exist (Zoccali et al. 2003), a radial metallicity gradient has yet to be conclusively demonstrated (Rich & Origlia 2005; Minniti & Zoccali 2008).

No evidence has yet been found for blue stragglers in the bulge. Blue stragglers are hydrogen-burning stars with apparent age younger than that of the parent population, most likely the result of significant mass deposition onto the star (e.g., Stryker 1993; Bailyn 1995). These objects are found in clusters of all ages (e.g., Stryker 1993) and would thus be expected to be present in the bulge. However, they occupy a region of the CMD that overlaps with the foreground disk and so are difficult to discriminate.

Stellar kinematics offers the cleanest method to separate bulge from disk. The disk shows apparent streaming motion in front of the bulge due to the motion of the Sun about the Galactic center, while the proper-motion dispersion of the two populations should differ because the two populations have widely different ages and likely differing relaxation times (Binney & Tremaine 1994). While statistical subtraction tells us that some fraction of objects in a region of the CMD may be bulge objects, kinematic constraints allow us to assign likely bulge/disk membership to individual objects.

Beyond bulge-disk separation, stellar proper motions are valuable in their own right as a probe of the present-day kinematics of the bulge. Proper motions have suggested that the bulge stars show net rotation in the same sense as the rest of the disk, although a smaller retrograde population may also be required (Spaenhauer et al. 1992; Zhao et al. 1994). Most previous studies converge on the model that the bulge apparently rotates as a solid body, with circular velocity \( \langle v_\phi \rangle \) rising linearly with distance from the Galactic center until the stellar population becomes dominated by disk stars (which show roughly constant \( \langle v_\phi \rangle \)); broadly similar behavior is reproduced by bulge models including a rapidly rotating bar (e.g., Sellwood 1981; Zhao 1996). This consensus has recently been called into question by the radial velocity study of Rich et al. (2007), which (with velocities consistent with the PN results of Beaulieu et al. 2000) suggests an inflection point in the \( \langle v_\phi \rangle \) curve roughly \( \sim 0.6 \) kpc from the Galactic center, outside which \( \langle v_\phi \rangle \) is constant at \( \sim 50 \) km s\(^{-1}\). This suggests that the solid-body–like rotation may only persist over the inner regions of the bulge/bar system. A change in the general character of stellar orbits within the inner region of the bulge may relate to an inner resonance (e.g., Sevener 1999). Clearly, an independent measure of the stellar \( \langle v_\phi \rangle \) curve is required. This is best constructed from data sets for which systematics at different galactocentric radii are similar, in particular the correction due to solar motion about the Galactic center; as we demonstrate below, proper motions along a single sight line provide just such a data set provided that the observations are at sufficient depth to assemble large numbers of objects in each distance bin to overcome intrinsic variation.

The bulge has now been the subject of a number of proper-motion studies. Early ground-based studies of 427 K and M giants\(^7\) in Baade’s window \((l, b = 1.02^\circ, -3.93^\circ)\), using photographic plates separated by nearly 33 yr, showed a proper-motion distribution with small anisotropy, \(\sigma_l/\sigma_b = 1.15 \pm 0.06\) (Spaenhauer et al. 1992). On the bulge minor axis, plate scans over an area \(25^\prime \times 25^\prime\) in Plaut’s window \((l, b = 0^\circ, -8^\circ)\) across a 21 yr time interval were used to extract proper motions from 5088 objects \((14 < V < 18)\), finding similar dispersion and anisotropy to the Baade’s window sample (Mendez et al. 1996).

From the ground, 5 yr of MACHO photometry allowed stars with proper motions \(\geq 18\) mas yr\(^{-1}\) to be isolated, to search for future microlensing events toward the bulge \((50 \times 0.5^\prime \times 0.5^\prime\) fields at \(-1^\circ < l < -10^\circ\) and \(-11^\circ < b < -1.5^\circ)\) and Magellanic Clouds (Alcock et al. 2001). The OGLE-II experiment yielded proper motions of some \(5 \times 10^3\) stars from \(49 \times 0.24^\prime \times 0.95^\prime\) fields within \(-11^\circ < l < +11^\circ\) and \(-6^\circ < b < +3^\circ\), with proper motions measured to a precision of 0.8–3.5 mas yr\(^{-1}\) (Eyer & Wozniak 2001; Sumi et al. 2004), which was recently used as the basis for a study of the trends in proper motion with location in the bulge (Rattenbury et al. 2007a, 2007b). Most recently, Plaut’s window has been the subject of a second plate-based study of some 21,000 stars within a \(25^\prime \times 25^\prime\) region, producing proper motions to \(\sim 1\) mas yr\(^{-1}\) and using \(\sim 8700\) cross matches with the Two Micron All Sky Survey to carefully select bulge giants (Vieira et al. 2007).

Turning to space-based proper-motion studies, a number of studies of clusters that use proper motions to discriminate the cluster from the field produce bulge proper motions if the field includes a significant number of bulge stars. Zoccali et al. (2001) reported the velocity dispersion of about \(10^4\) bulge stars at \((l, b) = (5.25^\circ, -3.02^\circ)\), as a by-product of their WFPC2 study of the cluster NGC 6553, finding proper-motion dispersion consistent with previous ground-based studies of Baade’s window. Similar results were obtained from the bulge stars in the field of a WFPC2 study of the metal-rich cluster NGC 6528, at \((l, b) = (1.14^\circ, -4.12^\circ)\) (Feltzing & Johnson 2002). However, Kuijken & Rich (2002) were the first to dedicate Hubble Space Telescope (HST) observations to bulge proper-motion studies, returning to Baade’s window \([(l, b) = (1.13^\circ, -3.77^\circ)]\) and the Sagittarius low-extinction window \([(l, b) = (1.25^\circ, -2.65^\circ)\)] with WFPC2. This isolated 3252 bulge stars in Baade’s window and 3867 in the Sagittarius window, concluding that the bulge can be best fitted with a 10 Gyr old open cluster–type sequence. No detection of \(\{l, b\}\) covariance was reported, nor was any detection of blue straggler candidates. An Advanced Camera for Surveys (ACS) WFC and WFPC2 program is ongoing to reimage fields for which observations are already present in the HST archive (Kuijken 2004; for preliminary results see Soto et al. 2007). Recently, ACS HRC was used to survey \(35 \times 35\)” \(\times 35\)” fields near Baade’s window across a \(5\)” \(\times 2.5\)” region in the vicinity of Baade’s window, in order to compare the bulk kinematic properties of bulge fields as a function of location relative to the Galactic center (Kozlowski et al. 2006). As pointed out by Vieira et al. (2007), the results of the ACS HRC study and the literature differ quite strongly from those of Rattenbury et al. (2007a; particularly the dispersions \(\sigma_l\) and \(\sigma_b\)). This appears to be due to differences in the selection of tracers used; the sample of Rattenbury et al. (2007a) may be contaminated by evolved disk objects (Vieira et al. 2007; see also Rattenbury et al. 2007c). Clearly, some care

\(^7\) The Spaenhauer et al. (1992) figure of 429 stars includes two repeated entries.
is required in the selection of bulge tracer objects, particularly above the MSTO.

We report on the use of ACS WFC to extract precise proper motions from a large number of stars in the Sagittarius window toward the bulge. The Sagittarius Window Eclipsing Extrasolar Planet Search (SWEEPS project; Sahu et al. 2006) obtained an extremely well sampled photometric data set with ACS WFC of the Sagittarius window toward the bulge; this forms our first epoch. A repeat visit just over 2 yr later forms the second epoch, from which we extract proper motions. At \((l, b) = (1.25\degree, -2.65\degree)\), the line of sight passes within \(\sim 300\) pc of the Galactic center. This is far enough from the center that the claimed population of young to intermediate-age objects traced by the OH masers and AGB stars is likely to be an insignificant contributor to the observed field; such objects are likely confined to within \(\sim 100\) pc of the Galactic midplane and are found preferentially near the Galactic center (Sevenster et al. 1999; Frogel et al. 1999). Thus, the stellar population of our field of view consists of bulge, disk, and halo objects.

Our first epoch has a total integration time \(\sim 86\) ks each in F814W and F606W and is the deepest ever observation of the Galactic bulge. With 6 \(\sigma\) detections of proper motions toward the wings of the proper-motion distributions where disk/bulge separation is greatest, we push the disk contamination below the 0.3% level and extract the purest, largest sample of bulge stars yet assembled.

This report is organized as follows: We provide the particulars of the observations used in § 2. We report the procedures used to produce precise position measurements and the resulting proper motions in § 3. Section 4 outlines the broad features of the proper-motion distribution of the population as a whole. To correct for disk contamination in the sample of kinematically selected bulge objects, we must first estimate the fraction of disk and bulge objects in the observed sample; to do so requires tracing the proper-motion distribution as a function of distance along the line of sight. We do so in § 5; before returning to the bulge/disk population distinction, we use the distance dependence of the proper motions to extract some kinematic properties of the bulge in § 6. We select a likely bulge population in § 7 and briefly examine its properties. Finally, in § 8 we assess the likely kinematic membership of the 16 SWEEPS planet candidates from Sahu et al. (2006).

2. OBSERVATIONS

The target field, at \((l, b) = (1.25\degree, -2.65\degree)\), has been observed a total of five times with \(HST\). The field was first observed on 1994 August 21 with WFPC2, then again with WFPC2 on 2000 August 8. Proper motions based on these observations have been reported by Kuijken & Rich (2002). ACS WFC first observed this field and four nearby regions on 2003 June 9 to allow optimal target selection for the SWEEPS planet search. We focus on the final two epochs: the ACS epoch 1 and ACS epoch 2. The ACS epoch 1 is February 2004, and the ACS epoch 2 is March 2006; hereafter the “first” and “second” epochs in this text refer to the 2004 and 2006 epochs.

Our field is the farthest from the minor axis in which deep proper-motion studies of the Galactic bulge have been performed. That said, our line of sight passes within \(\sim 200\) pc of the center of the Milky Way. With the Sun located \(\sim 12 - 20\) pc above the Galactic midplane (Joshi 2007), our line of sight reaches \(\sim 0.5\) kpc beneath the Galactic midplane before intercepting the innermost spiral arm (assuming that the logarithmic spiral of, e.g., Cordes [2004] accurately describes the Norma spiral arm on the far side of the Galactic center). This is \(\sim 1\) disk scale height beneath the middisk (slightly more with the Milky Way disk warp; e.g., Momany et al. 2006); thus, the disk contribution to the field on the far side of the bulge should be rather lower than the near side.

2.1. ACS Epoch 1: 2004 February

ACS WFC epoch 1 observations took place between 2004 February 22 and 29. A total of 254 exposures in F606W and 265 in F814W were taken, each with integration time 339 s. The target field was specifically chosen to maximize the yield of potential host stars in a single WFC pointing and at the same time minimize the number of bright objects that would black out regions of the chip through charge bleeding (and thus reduce the efficiency of the survey to planet detection). Subpixel dithers were set to well and redundantly sample intrapixel sensitivity variations. The integration times chosen provide per-observation photometric accuracy of about 0.04 mag at F814W = 23, with saturation point just above the MSTO at F814W = 18.6. To measure bright objects, three integrations each in F814W and F606W were taken at 20 s integration time, providing unsaturated measurements over the 18.5 \(\leq\) F814W \(\leq\) 13.8 range, aiding isochrone fits to the mixture of stellar populations present. Information about these observations can be found in Sahu et al. (2006), with further detail forthcoming (K. Sahu et al. 2008, in preparation).

An additional few observations were taken offset by 3′ in detector Y to cover the interchip gap; we do not use these bridging observations in this work.

2.2. ACS Epoch 2: 2006 March

Repeat observations of the SWEEPS field were taken with ACS WFC on 2006 March 9 in F814W. Ten deep and two shallow integrations were taken; a slightly longer visibility interval per \(HST\) orbit led to 349 s integration times for each deep observation. Subpixel dithers were programmed in pairs, nominally at \(\pm 0.25\) pixels in detector \(x, y\) from pixel center, in reality \(\pm 0.05 - 0.1\) pixels from the programmed values. The two shallow observations were taken at 20 s integration times to provide proper-motion estimates for bright objects.

The two deepest ACS epoch images are quite well aligned: not including subpixel dithers, image centers from the two epochs are shifted with respect to each other by (4.78, 8.48) pixels (0.24′′, 0.424′′) along the detector, with mutual rotation \(\leq 8′′\), corresponding to a pixel offset \(\sim 0.1\) at the corners of the detector from rotation alone.

3. ANALYSIS AND REDUCTION

The epoch 1 ACS data set is among the deepest set of observations ever taken in the optical with \(HST\), with a strategy specifically set to well sample the intrapixel sensitivity variations with redundancy, thus allowing an optimal combination of images into an oversampled representation of the image scene. The SWEEPS project achieved this using an extension of the Gilliland techniques (Gilliland et al. 1999, 2000), in which an image model of the scene is produced for each filter. The flux at each pixel is represented by a Legendre polynomial in the subpixel offsets \(\Delta x, \Delta y\) for each input frame (the polynomial coefficients and number of terms to retain being determined by the counts in each pixel). For the SWEEPS data set this process was augmented through the fitting to each image of a convolution kernel that maps each input frame onto the master representation and thus accounts for focus breathing. The resulting continuous

\[ M_{\text{inst}} = -2.5 \log e^{-} \]
The well-documented geometric distortion due to the optical layout of ACS WFC (e.g., Anderson & King 2006) presents challenges when attempting precise position measurements. Residuals that vary in a complex way across the chips persist at the 0.05–0.1 pixel level after the standard fourth-order polynomial distortion correction is applied during the drizzle process; this alone makes the standard drizzled image stacks supplied by the pipeline problematic for our science goals. We must use the raw images (or a combination thereof) to preserve positional accuracy. After some experimentation (Appendix A), it was found that position scatter was minimized by using each image within an epoch for a separate estimate of the position and flux of each object. This produces superior results to positions measured from a stack of images. This approach rests on the existence of a highly super-sampled model for the “effective PSF” (the instrumental PSF as recorded by the detector, hereafter ePSF; for discussion see Anderson & King 2000). For the ACS WFC, focus breathing is taken into account by adding a perturbation PSF to the ePSF; this perturbation PSF is fitted from each image separately (Anderson & King 2006).

3.2. Multipass Position Estimates

With at least 246,793 objects in the frame (on average one every ~8 × 8 pixels), the field is crowded but not pathologically so. To measure positions on the frame, we thus used an improved version of the Anderson & King (2006) fitting routine img2xy-m.F, with a 3 × 2 perturbation PSF grid rather than a single perturbation PSF, which provides an improved measurement, as well as the capability to subtract neighbors from each object before measuring its flux and position, should similar estimates for the neighbors be available. To minimize error, the ePSF was constructed from frames that have been flat-fielded and bias-subtracted but not resampled or corrected for distortion. Thus, measurements are performed in the raw coordinate system of the detector (_flt space). The existence of extensive globular cluster observations allowed distortion in the camera to be constrained (see Anderson & King 2003, 2006). This distortion correction is used by the Anderson & King (2006) routines to produce output in both the _flt system and a transformed coordinate set with higher order distortions removed to the level of 0.005–0.01 pixels. We refer to this frame as the “distortion-free” frame, although it should be remembered that residual distortion at the levels just quoted may still be present. In addition to position and flux measurements \( \{x, y, m, q\} \), the Anderson & King (2006) routines output a quality factor \( q \), which measures the difference between the ePSF-fitted flux and the aperture flux (sum of pixel values) within a 5 × 5 pixel region centered on the position of the star, scaled by the ePSF-fitted flux. This ratio \( q \) is seen to correlate with the total flux from detected crowding objects in the master photometry catalog (Fig. 1), so we adopt it as a measure of crowding due to neighboring objects (and thus including crowding objects too close to have been isolated and measured in the photometry).

We use multiple passes of stellar position and flux measurement to take account of the tendency of neighbor subtraction to build in a dependency of the measurement of each star on those surrounding it. Each pass requires a master list of mean estimates for each star from the previous pass and a matchup list for each frame giving the coordinates of a sample of well-measured, isolated objects in the input frame and master list, so that the transformation between the master list and the frame itself can be properly taken into account when subtracting neighbors.

The first pass with the Anderson & King (2006) routines produces a set of \( \{x, y, m, q\} \) estimates for each image without neighbor subtraction, and with nonuniform row ordering. We match these measurements to the master catalog of 246,793 objects from photometry of the optimal superimage from Sahu et al. (2006) to recast each of the pass 1 estimates with the same row ordering as the Sahu et al. (2006) catalog. Note that we never use the position and flux estimates from Sahu et al. (2006) again in this analysis due to the shifting of flux that takes place when constructing the stack (Appendix A); this catalog is used purely to enforce a uniform row ordering in the object catalogs for proper-motion extraction. We now construct a sigma-clipped median set of \( \{x, y, m, q\} \) measurements of each star from pass 1, accounting for residual trends in the manner described below. This forms the input catalog to the multipass photometry.

In subsequent passes, this input catalog is transformed back into each individual _flt frame using the matchup lists just produced. Neighbors are now subtracted from each object before measurement; to avoid measurement instability and dependence on the ordering in which objects are processed, the position and
magnitude measurements from the previous pass are used when subtracting neighbors. The individual measurements are now used to produce a sigma-clipped mean measurement for \(\{x, y, m, \phi\}\), which updates the input list for the next pass, as well as the matchup lists. As a matter of record, photometry using the modified Anderson & King (2006) routines with neighbor subtraction takes over an hour per frame on a 3 GHz Linux CPU with 1 GB RAM, so we split this step across multiple CPUs to reduce the time necessary to process 265 images in the first epoch. The collation, accounting for trends and refinement of the input list to the next pass, requires the complete measurement list for each star and can take up to 8 hr per pass. This process is repeated until convergence; there is little improvement between the third and fourth pass of the photometry except at the faintest end of the star list, so we stop after four passes. Although there are only 10 deep images in the second epoch, the production of a clipped master list does improve the photometry for neighbor subtraction so we follow a similar process for the 2006 epoch.

### 3.3. Reference Frames and Accounting for Residual Trends

A reference image is chosen for each epoch with focus within a few percent of the maximum focus sharpness and commanded orientation at the middle of the dither pattern (j8q632er was used for the 2004 epoch, and frame j9ev01ntq was used for the 2006 epoch). The Anderson & King (2006) estimates in \(\_fft\) space are related to those in distortion-free space by a well-constrained transformation; thus, we are free to choose which frame we use for fitting the transformation that maps each individual set of measurements onto the master set (essential for producing the refined matchup lists); we work in distortion-corrected space so that linear transformations can be used, reducing the transformation error.

The distortion correction transformation itself is claimed to be accurate to the 0.005–0.01 pixel level (Anderson & King 2006); this tolerance introduces 0.25–0.5 mas uncertainty in position measurement. In addition, observatory-level systematics are present that cause the position and flux to vary with time: this is clearly seen in a plot of the amplitude of the perturbation PSF that must be added to the library PSF to optimize the measurements in the first pass (a proxy for focus; Fig. 2). These trends are not an entirely smooth function of time and thus can add scatter of up to \(\sim0.5\) mas to the position estimates. We account for and subtract trends in \(\Delta x, \Delta y, \) and \(\Delta m\) for each chip independently, assessing the trends from the \(\sim4000\) best-measured objects in the frame. Experimentation with polynomial surface fitting and a lookup table approach suggested that for our data a polynomial with cross terms up to \(x^3y^3\) provided the most robust subtraction of trends. In cases where subtraction of the polynomial surface increased the scatter across the frame (an indication that residual trends were not significant for the frame in question), the subtraction was not used.

When positions and fluxes are measured with neighbor subtraction, the variance of measurement within an epoch is generally lower than from a single pass alone and the distribution is rather tighter. However, after the second pass (i.e., the first with neighbor subtraction) a small population appears that is fainter than instrumental magnitude of roughly \(-13.2\) and with roughly constant position rms 0.08 pixels or so (Fig. 3). These objects are apparently due to the influence of neighboring objects and were dealt with appropriately (Appendix B). In addition, the effects of differential charge transfer efficiency (CTE) are noticeable both in position measurements taken at differing integration times within the same epoch and in magnitude measurements taken at similar integration times in two epochs, due to degradation of the detector in the low Earth environment. However, any differential

![Amplitude of the perturbation PSF](image.png)

**Fig. 2.**—Amplitude of the perturbation PSF that must be added to the library PSF to best represent the scene in each image. The history of this amplitude variation is a proxy for focus history. This measurement is from the first pass at position measurement in which a single perturbation is fitted to the entire frame; pass 2 onward uses a spatially dependent perturbation PSF.

### 3.4. Positions to Proper Motions

The result is two star lists, where mean positions in the 2004 epoch are in the distortion-free frame of the most representative 2004 image and the mean positions in the 2006 list are in the distortion-free frame of the most representative 2006 image. The distortion correction of Anderson & King (2006) was applied to both lists of coordinates (and both sets of fluxes through the variation in pixel area on the sky). Residual imperfections in the distortion remaining after subtracting trends will lead to distortion remaining between the two position lists.

The ACS WFC distortion is now known to change monotonically with time, as can be seen, for example, in the evolution of the skew terms in the distortion solution used here. This change is rather small for most purposes, roughly 0.3 pixels in the 2002–2006 interval at the corners of the image, and appears to be confined to the linear terms (Anderson 2007). However, evolution in distortion as traced by the first six terms of the transformation is automatically accounted for in our approach as we are fitting six-term transformations between the epochs; no evidence is seen for evolution in the higher terms of the distortion correction.

The 2006 epoch is particularly prone to residual distortion due to the comparatively low number of frames available to characterize and remove the frame-by-frame residual trends. We therefore return to the individual 2006 epoch position lists, recomputing the proper motions on an image-by-image basis. We use local transformations to best map the frame near each target star onto the frame of the 2004 star list. For each target star in each 2006 image, we use an AMOEBA fit (Press et al. 1992) to find the six-term transformation that maps the positions of a set of nearby tracer stars from 2006 onto their positions in the 2004 star list (Fig. 4). This transformation is then used to predict the position of the target star in the 2004 epoch. The offset of this position from the true position
in 2004 is then the proper motion of the target star, estimated from the individual 2006 star list with reference to the 2004 master list. The set of 3–10 proper-motion estimates (at least three good measurements are needed) is then averaged together with outlier removal to estimate the true proper motion of the target star. Finally, the proper motions in image coordinates are transformed into proper motions in Galactic coordinates. This process is repeated for every object in the master photometry list, which is computationally accomplished most conveniently through an overnight run.

Care must be taken when selecting frame-to-frame reference stars in this process. As we lack an extragalactic reference point, we assume that all the stars in the frame are in motion and adopt as the zero point the average motion of the bulge population as estimated from the CMD. Of likely nondisk objects we include only those unsaturated objects for which the combined $x$- and $y$-coordinate rms from the 2004 epoch is less than 0.007 pixels, the crowding measure $q$ is less than 0.025, and at least 200 measurements were taken in the 2004 epoch (so the positions are well characterized). As a precaution, the 2006 mean-position list produced above is used to remove tracer stars that show discrepancy above 200 mas (4 pixels) between mean position in the two epochs, indicative of unusually high proper motion, a problem with the transformation between frames, or an object mismatch; this culling is needed for a handful of objects. We do not include the target star itself in the transformation (see Anderson et al. 2006) and are careful to ensure that tracer stars only on the same chip as the target star are used, to guard against any trends over time in the distortion of the two chips relative to each other. These considerations leave $\sim$8000 reference objects over both chips. Of these objects, the nearest 100 to the target star are selected to fit the local transformation, with five-pass sigma clipping removing typically

![Diagram](https://example.com/diagram.png)

**Fig. 3.**—Scatter in position measurement as a function of object flux, stepping through the photometry passes. A single pass (top row) shows a significant cloud of fainter objects with high coordinate dispersion. After a second pass using neighbor subtraction (second row), a significant population is visible with $\sim$0.1 pixels rms. Each object in this population shows at least two clusters of position estimates. When this is accounted for, these objects show rather less scatter in their position estimates (third row). A total of 5% of stars are plotted for clarity, and the same stars are plotted in all panels.
8–20 objects (Fig. 4). The area covered by the reference stars is typically 500 × 500 pixels, or \(\sim 4\times 4\) the field of view.

This approach allows the proper-motion error to be fully characterized on a star-by-star basis. For each star, this error contains three terms: the internal scatter \(\sigma_{2004}\) accompanying \(N_{2004}\) repeated position measurements in the 2004 epoch, the rms variation in position difference \(\epsilon_2\) over the 2.04 yr interval between the \(N_{2006}\) set of position pairs, and finally the relative position scatter that will be built up when \(n_{tr}\) tracer stars are allowed to move in random directions with intrinsic scatter \(\sigma_{pm}\) mas yr\(^{-1}\) over a 2.04 yr time interval. Per coordinate, then, the proper-motion error is

\[
\epsilon^2 = \frac{1}{2.04^2} \left[ \frac{\epsilon^2_{2004}}{N_{2004}} + \frac{\epsilon^2_2}{N_{2006}} + \frac{(2.04\sigma_{pm})^2}{n_{tr} - 2} \right].
\]

For well-measured stars the first two terms are rather small; in 2004 we find that the brightest nonsaturated objects show positional scatter \(\epsilon_{2004}\) in the 2–3 millipixel regime (Fig. 3); the intrinsic scatter \(\sigma_{pm}\) over the 2.04 yr interval dominates the error estimate for these objects. However, the advantage to this approach is that the additional systematic introduced when two average position lists are used under the same distortion correction has been fully accounted for. At the faint end the measurement error dominates the error budget (Fig. 5).

3.5. Proper Motions from Saturated Objects

We will want to compare proper motions above the MSTO with those below it. With 265 + 10 exposures total at 339 s or longer, we might expect better statistics when producing average proper motions from the long exposures compared to the five available short exposures. However, the integration time for the long exposures was chosen so that saturation coincides roughly with the turnoff; thus, position measurements and therefore proper motions may be subject to greater error than the variance of position measurements would indicate. Indeed, we note a CTE-like effect when comparing the positions measured for highly saturated objects compared to the same objects measured with much shorter integration times (Fig. 6; see also Appendix C).

We thus extracted proper motions from the shorter exposures to compare with the long exposures, to search for any saturation effects. The process is similar to that used for the deeper exposures; however, this time too few integrations are available to make possible the trend fitting and removal procedures used for the deeper exposures. Instead, proper motions are computed pairwise between each of the six possible pairs of measurements between the three short integrations in 2004 and the two in 2006. The results are then robustly averaged with outlier removal to produce proper-motion estimates from the short integrations. The measured magnitude for each star is the sigma-clipped mean of all five brightness measurements for each star, to mitigate the effects of cosmic rays when subtracting neighbors for object measurement. The resulting proper-motion distribution is statistically indistinguishable from that from the deeper integrations, as we show in § 4.

3.6. Multiepoch Proper Motions

To illustrate that we are indeed measuring proper motions, and not spurious events, we construct multiepoch proper motions, using all five epochs at which this field has been observed (§ 2). Position measurements from the WFPC2 observations and the relatively poorly temporally sampled 2003 ACS WFC observations have rather larger error than our 2004 and 2006 measurements, while the transformation from WFPC2 to ACS WFC systems is not yet fully refined. To extract science from our proper motions, we therefore content ourselves with the 2004 and 2006 epochs. However, as can be seen in Figure 7, we clearly are measuring motion of the stars across the field of view, with the distance traveled between measurements proportional to the time interval between the measurements (Fig. 7).

4. STELLAR PROPER MOTIONS

The result of the procedures in the previous section is a set of proper motions for 187,346 objects in our frame, 81,140 to better than 0.3 mas yr\(^{-1}\) in both coordinates. From the shorter observations, a list of >42,000 objects is produced with proper motions at better than 1 mas yr\(^{-1}\) accuracy. We aim to estimate proper motions in the disk as separate from the bulge, so we first examine the proper motions above the MSTO.

4.1. Stellar Proper Motions above the MSTO

As noted by several previous authors, the region above the MSTO, where the CMD splits relatively cleanly into disk-dominated and evolved bulge-dominated populations, provides a convenient tracer that can be used to estimate the kinematics of disk and bulge populations. We use this as a first estimate when we attempt to divide the populations further by apparent distance below.

The longitudinal proper motions clearly separate the disk and bulge populations, with the latitudinal proper motions also providing separation but to a lesser extent. Fitting a single Gaussian to each of the proper-motion histograms of the two populations suggests separations (\(\Delta \mu_l, \Delta \mu_b\)) = (3.24 ± 0.16, 0.76 ± 0.13) mas yr\(^{-1}\) between the two populations. There is thus an average tilt of order 13° between the centroid proper motions of the two populations (Fig. 8). The proper motions in this region from the short exposures are statistically identical to those from the long exposures:
(\(\Delta \mu_l, \Delta \mu_b\)) = (3.21 \pm 0.15, 0.81 \pm 0.13) \text{ mas yr}^{-1} \) between the two populations (Fig. 9). However, the scatter in the resulting proper distribution is lower, implying superior measurement. Because proper motions above the MSTO show different systematics between the long and short exposures, we cannot simply add the two sets of measurements in quadrature. Instead, for objects that (1) are saturated in the long exposures and for which (2) at least five proper-motion measurements are retained after sigma clipping, we simply substitute proper motions from long exposures with those from the short exposures. Because of different selection biases for bulge objects above and below the MSTO (which define the reference transformation between epochs), we are careful to first take into account the difference between proper-motion zero points between long and short exposures; this difference amounts to (\(\Delta \mu_l, \Delta \mu_b\)) = (−0.2, 0.01) \text{ mas yr}^{-1}.

4.2. Stellar Proper Motions below the MSTO

Kinematic distinction between bulge and disk is most vividly illustrated by color-coding the CMD by the mean proper motions \(\mu_l, \mu_b\) and their dispersions \(\sigma_l, \sigma_b\). As noted previously by Kuijken & Rich (2002), \(\mu_l\) and particularly \(\sigma_b\) show clear association with the bulge population (Fig. 10), the latter expected for a dynamically older population. The conclusion that the bulge is mostly an old stellar population is thus independent of the details of any isochrone fits. The longitudinal proper-motion scatter is also a tracer of the bulge stellar population, although less obviously than the latitudinal scatter, as expected for a foreground disk with multiple populations participating in streaming motion of various speeds around the Galactic plane. There is also a hint of a trend in \(\sigma_b\); however, from proper motions alone the significance of this trend is not clear.

The motion of the Sun with respect to the local standard of rest (LSR) causes a trend in \(\mu_l\) and \(\mu_b\) that is strongest for foreground disk objects in off-axis fields; indeed, with multifold observations to estimate distance and extinction directly, this can be used to constrain stellar motions in the disk (Vieira et al. 2007). This trend increases as \(1/d\); any intrinsic disk-to-bulge proper-motion trends are superimposed on this trend. Because the Sun orbits the
Galactic center slightly faster than the LSR, this trend will be in the opposite sense to the $\mu_t$ trend we observe, in that the solar reflex contribution to proper motions will be negative with respect to our tracer bulge objects. For our field, the size of this correction to bulge objects will be $\sim 0.1 - 0.2$ mas yr$^{-1}$ from the near to far side of the bulge. With two-filter photometry for our field, we do not have distance estimates for most of our objects. We therefore restrict detailed kinematic interpretation to those objects for which distances can be estimated (see below); for these objects the solar motion drops out as a constant velocity term. We note in passing that the size of the solar motion trend is the same size as the apparent trend in $\mu_b$ across the CMD (Fig. 10).

5. PHOTOMETRIC PARALLAX

We need to assess the contamination from foreground disk stars in our kinematically selected bulge sample; this requires us to estimate the number of disk and bulge stars in our field. This exercise in turn requires that we determine the kinematics as a function of distance along the line of sight, so we now turn to dissection of proper motion as a function of distance.

We estimate distances to each star by computing its distance modulus relative to the isochrone of the mean-bulge population (from Sahu et al. 2006). This isochrone was fitted by dividing the CMD into strips of equal color, taking the median in each strip and metallicity that best fits the resulting ridgeline for a canonical bulge age of 10 Gyr. Isochrones from VandenBerg et al. (2006) were used, transformed to the ACS bandpasses as reported in (Sumi 2004) and corresponds to a mean-bulge distance of 7.24 kpc.

With the geometrically determined Sun–Galactic center distance $R_S = 7.62 \pm 0.2$ kpc (Eisenhauer et al. 2005), this bulge distance is consistent with the distance of intersection of the bulge major axis with the line of sight for bulge orientation angle $\beta \gtrsim 14^\circ$.

The majority of the interstellar extinction takes place in the foreground screen of the Galactic spiral arms (Stanek 1996; Cordes & Lazio 2003). For objects within about 3.3 kpc of the Galactic center (i.e., interior to the Norma spiral arm), this approximation is adequate for detailed constraints on the bulge (Cordes 2004; Wainscoat et al. 1992). Farther out from the Galactic center, the line of sight passes through the spiral arms Carina-Sagittarius (at approximately 6.7 and 6.4 kpc from the Galactic center on the near and far side of the galaxy, respectively), Crux-Scutum (at 4.8 and 4.8 kpc), and Norma (3.3 and 2.9 kpc). The interstellar dust distribution of the Galactic disk shows scale height $\sim 0.14$ kpc (Bienaymé et al. 1987), which means that our line of sight passes more than one dust scale height from the Galactic midplane just before intercepting the Norma spiral arm on the near side. Thus, distances inferred for objects closer than $\sim 4$ kpc will be somewhat uncertain.

Some care must be taken interpreting the resultant distance modulus, as the magnitude scatter is not due to distance effects alone. This is most clearly seen by comparing the distance estimates obtained from the main sequence to those of HB clump+RCG bump objects (which we denote together as red clump giant or RCG objects; the small contamination from evolved disk objects referred to in Zoccali et al. [2003] and Vieira et al. [2007] is not an important contamination for this estimate). Figure 11 shows the result: while the maximum distance spread (1 $\sigma$) of RCG objects corresponds to 0.17 mag, below the main sequence the spread is twice this amount.

Were the disk to be highly overrepresented in the main-sequence sample, this would bias the distance modulus distribution toward the bright end; this is not seen. The two measures should therefore be identical. The fact that the scatter of the main-sequence...
distance measure is a factor of 2 higher than that from the RCG measure suggests that the distance estimate for the main sequence is contaminated by other effects, which would include metallicity variations, the presence of stellar binaries, etc., as described in § 5.1. When using photometric distance estimates to draw conclusions about bulge kinematics, we use the scaling between the RCG sample and the sample below the MSTO to correct all apparent photometric distance moduli to estimated true distance moduli.

5.1. Can We Recover Bulge Kinematics from Our Data?

To assess the impact of this pollution of our distance estimate on our ability to recover bulge kinematics, we first attempt to reproduce the distance discrepancy and then compare the kinematics we would recover with those simulated.

The CMD of a trial bulge population was simulated using the mean-bulge isochrone from Sahu et al. (2006). First, a stellar population was produced along the mean-bulge isochrone (a Salpeter initial mass function [IMF] was assumed for simplicity; the results turn out to be rather insensitive to the IMF form adopted). The population was then perturbed by a distance modulus distribution matching that measured from the RCG objects (Fig. 11, top right panel). The population was then further perturbed due to a binary population. Because equal-mass binaries are rather unlikely above the MSTO, the binary effect was only added to objects beneath the turnoff. A metallicity distribution to the magnitudes was then

![Graph showing multiepoch positions on sky of high-proper-motion stars.](image)

**Fig. 7.**—Multiepoch positions on sky of a random selection of high-proper-motion stars. Detector positions are given in ACS WFC pixel coordinates (1 pixel = 50 mas), with the F814W magnitude listed in the panel for each star. Epochs may be identified from the position separation in most cases; the top left panel explicitly shows the epochs for one example. Epochs 1994 and 2000 represent WFPC2 measurements; all the others are ACS WFC. See § 3.6.
added using the Rich et al. (2007) spectroscopic results to estimate
the distribution of metallicity and the VandenBerg isochrones to
translate this into a magnitude perturbation (see Brown et al.
2005); beneath the turnoff this perturbs the magnitudes and (to
a lesser extent) colors; above the turnoff the main effect is on
color.

The binary fraction of the bulge is unknown, as is the distribu-
tion of relative brightness \( f(\Delta I) \) of binary components. We first
used the solar neighborhood as a proxy, using the tabulation from
Hipparcos of Söderhjelm (2007), although repeated experiments
suggest that the final result is somewhat insensitive to both the
binary fraction and \( f(\Delta I) \), providing that equal-light binaries
are not dominant, and the binary fraction is of order \( \sim 0.3 \)–0.6.
The difference in spreads in apparent distances between the clump
and main sequence is reliably reproduced (Fig. 12).

We now assess our ability to recover input kinematics. For
each trial, stellar velocities were simulated following a simple
model that qualitatively matches the radial velocity results of
Rich et al. (2007); the bulge is assumed to participate in solid-
body rotation up to some cutoff radius \( R_{\text{cut}} = 0.4R_S \) with a flat
mean circular speed curve \( \langle v_c \rangle = 50 \text{ km s}^{-1} \) exterior to this ra-
dius. Velocity dispersion of 75 km s\(^{-1}\) was added to the canonical
curve, and the resulting stellar velocities translated into proper
motions. Finally, the proper motions are translated back into
observed" velocities, this time without any information as to the metallicity or binarity of each star. For reference, the relations between the mean inferred transverse velocities $v_{l}$, $v_{b}$ and the intrinsic radial, azimuthal, and vertical components $v_{R}$, $v_{φ}$, and $v_{z}$ are, for our pointing,

$$v_{l} = (v_{φ} \sin(α) + (v_{R}) \cos(α) - (v_{l})_{0},$$
$$v_{b} = 0.9989(v_{φ}) - 0.0462(v_{φ}) \cos(α) - 0.0462(v_{φ}) \sin(α) - (v_{b})_{0},$$

where the line of sight to the star and the velocity vector of the star in a circular orbit make angle $α$ to each other. [The projected star--Galactic center distance $R$ is fixed by the position of the field and the Sun--Galactic center distance $R_S$, allowing easy estimation of $α$ through the relation $R \cos(α) = R_S \sin(β).$] Error in the distance inferred thus propagates through to error in the inferred circular speed curve.

The circular speed curve extracted from the simulated proper motions is then fitted to the simple model for the trial, and the result is compared to the input model. This process is repeated for many trials; Figure 13 shows the resulting distribution of recovered parameters over 300,000 trials. Aside from cases in which the fit fails to find a solution (both $R_{cut}$ and $v_{φ}$ zero), the cutoff radius $R_{cut}$ tends to be about a factor of 2 higher than that input, and with a high scatter (0.2$R_S$). The recovered tangential circular speed $v_{φ}$ is systematically lower than that input (50 km s$^{-1}$) by a factor of $\sqrt{2}$, the most frequently recovered value being $25 \pm 6$ km s$^{-1}$. Thus, the form and approximate cutoff of the input circular speed curve can be recovered, but with a constant
6. STELLAR KINEMATICS VERSUS DISTANCE FROM THE GALACTIC CENTER

We group objects by distance modulus to estimate the space motion of the average star in each bin as a function of distance \( d \) (Fig. 14; we remind the reader that we have corrected apparent distance modulii to estimates of the true distance modulii; \S 5.1). Within 3 kpc of the mean bulge, we use bins of equal \( \sqrt{d^{2n+1} + d^{2n}} \), so that the number of objects in each bin traces the mean volume density of stars in that bin. We assume that the bulge stars in our selected sample are members of the same population, so that the absolute magnitude distribution is the same for each bin across the bulge. We also assume that the kinematic properties of the bulge do not vary strongly with absolute magnitude, so that our tendency to preferentially detect intrinsically fainter objects when they are closer does not bias the kinematics we produce. At \( 20 < F814W < 23 \), our tracer objects are well above the brightness at which photometric completeness becomes a significant systematic (e.g., Piotto et al. 2007). Note that the proper-motion dispersion decreases once we probe the far side of the bulge, an indication that photometric errors are not dominating our kinematics in the far side of the bulge. Furthermore, while the proper-motion dispersion decreases on the bulge far side, the corresponding velocity dispersion does not. Our field of view encompasses an ever larger area on the sky with distance, so the far distance bins may sample a wider range of velocities than those close by.

6.1. Circular Speed Curve

Without the line-of-sight counterpart \( \langle v_r \rangle \) for this field, equation (2) represents two conditions in three unknown velocities; to break the degeneracy, we must either observe \( \langle v_r \rangle \) as a function of line-of-sight distance or adopt a relationship between the three components of streaming velocity; such a relationship is best obtained through numerical modeling in a realistic potential (see, e.g., Zhao 1996). We adopt a simple prescription for \( \langle v_z \rangle \) and set the other two components to zero. The ongoing radial velocity survey of Rich et al. (2007) finds no evidence for minor-axis rotation, so \( \langle v_z \rangle \sim 0 \) is at this stage reasonable. Observations are rather more ambiguous for \( \langle v_R \rangle \) within a few degrees of the Galactic rotation axis (Rich et al. 2007); we set this term to zero to determine if it is required to fit the observed velocities. We assume that \( \langle v_{\phi} \rangle \) follows the broad pattern of the interstellar medium (ISM; e.g., Clemens 1985), rising monotonically with galactocentric radius interior to some cutoff radius \( R_{\text{cut}} \), exterior to which \( \langle v_{\phi} \rangle \) is roughly constant at \( \langle v_\phi \rangle \) km s\(^{-1}\). In this respect our prescription for \( \langle v_{\phi} \rangle \) is similar to that of a rapidly rotating bar, producing an apparently solid-body--type rotation curve (e.g., Zhao 1996). We search for the combination of \( R_{\text{cut}}, \langle v_\phi \rangle \), and \( R_{\text{c}} \) that best reproduces the observed \( \langle v_\phi \rangle \) variation. The sensitivity of proper motion to intrinsic circular velocity decreases when motion is largely along the line of sight, as occurs near the meridional
plane for our line of sight. In addition, the most dramatic change in the balance of components occurs over a relatively narrow range in line-of-sight distance $d$, so that our chosen binning scheme (which preserves information on the intrinsic stellar density) results in relatively few bins over a complicated velocity variation and formally precise fitting is difficult. We thus impose the additional condition that the $h_V/C_30$ trend on near and far sides of the bulge be similar. Finally, we restrict ourselves to $R_S$ within $1 \sigma$ of the value determined geometrically by Eisenhauer et al. (2005). This artificially simple model does reproduce the observed variation of $\langle v_\phi \rangle$ with distance reasonably well, adopting $\langle v_\phi \rangle = 25 \text{ km s}^{-1}$, $R_{\text{cut}} = 0.3$–$0.4 \text{ kpc}$, and $R_S = 7.7 \text{ kpc}$ (Fig. 15). In agreement with Zhao et al. (1994), we find that the mean streaming motion must be prograde.

To our knowledge, this is the first measurement of the circular speed curve $h_V/C_30$ of the inner Milky Way to be determined purely from proper motions and is thus an independent check of the stellar circular speed curve determined by other means. Our stellar rotation curve from proper motions agrees qualitatively with that recovered from a sample of radial velocities of K and M giants across a swath at $b = -4^\circ$ (Rich et al. 2007). The circular speed inflection of Rich et al. (2007) takes place $0.6 \text{ kpc}$ from the Galactic center, which for the $b = -4^\circ$ swath corresponds to radial coordinate parallel to the Galactic plane $R_{\text{cut}} \sim 0.3 \text{ kpc}$. Thus, our turning point at $R_{\text{cut}} = 0.3$–$0.4 \text{ kpc}$ is entirely consistent with that of Rich et al. (2007), suggesting cylindrical, non–solid-body rotation.

While the apparent departure points from solid-body–like rotation agree with radial velocity and proper-motion studies, the amplitudes of $\langle v_\phi \rangle$ toward which both circular speed measurements trend at high galactocentric radii differ from each other by about a factor of 2. We found in § 5.1 that the metallicity, age, and binarity uncertainties tend to produce $\langle v_\phi \rangle$ roughly half that of the intrinsic value. Therefore, our circular speed curve is fully
consistent with that of the Rich et al. (2007) radial velocity survey in both turning point and velocity amplitude. Because of the high intrinsic velocity scatter, we are not able to set fine limits on the degree of any disparity; however, it seems clear that highly eccentric average stellar orbits are not consistent with our data.

We note in passing that the circular speed curve of the ISM does not provide a check of this discrepancy, as it measures the circular speed of a population with a definite, well-defined pattern speed of \( \sigma_{c24} \) at the galactocentric radii of interest (e.g., Burton & Liszt 1992). This is in stark contrast to the stellar circular speed curve, which measures the average of a population with high intrinsic velocity dispersion and likely several sub-populations (which may include a population on retrograde orbits; e.g., Zhao et al. 1994). The importance of measured mean velocity on the mixing of populations is also evident in the behavior of \( \langle v_b \rangle \) at nearby distances, which rises to rather more modest values than the \( \sigma_{c27} \) pattern speed of the disk (Fig. 15).

### 6.2. Minor-Axis Rotation?

When binned to increase signal, \( \langle v_b \rangle \) exhibits an apparent change in sign about either the \( \alpha = 0^\circ \) distance or the distance of maximum density (Fig. 14, top right panel), with peak-to-peak amplitude \( \sim 0.2 \) mas yr\(^{-1}\) roughly 10 times lower than the switch in \( \langle v_t \rangle \). Such a trend is in line with that expected due to solar reflex motion (Vieira et al. 2007); discrimination of any intrinsic minor-axis rotation from the signature of the Sun’s own motion with respect to the LSR is deferred to future work.

### 6.3. Velocity Ellipse

The \( \{l, b\} \) velocity ellipse is of great interest to bulge kinematics as (1) it may provide a further discriminant between bulge stellar populations and (2) within a given population it carries information about the structure of the potential in the \( z \)-direction (e.g., Kuijken 2004). Previous studies have been inconclusive on this matter; Zhao et al. (1994) found no relative orientation between the two metallicity populations, although rather few objects were used with which to detect such an offset significantly; the follow-up study of Soto et al. (2007) used 66 and 227 objects at low and high metallicity, respectively, and did not detect significant discrepancy between the two populations. However, the recovery of intrinsic velocity components from those observed is a rather strong function of distance, so a depth-integrated study such as theirs can smear out intrinsic variation.

We constructed the velocity ellipse as a function of apparent distance using the tracer population beneath the MSTO (Fig. 16 and 17). There is a clear signature of changing \( \sigma_{lb} \) with line-of-sight distance (also Fig. 14). Interpreting this in terms of stellar velocity components \( \sigma_{l0}, \sigma_{b0}, \) and \( \sigma_z \) is not trivial. Although there

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**Fig. 12.** Example comparison of inferred distance moduli from a synthetic bulge CMD, for the bulge RGB (top) and below the MSTO (BMSTO; bottom). Regions identical to those in Fig. 11 were used. [See the electronic edition of the Journal for a color version of this figure.]
is a clear resemblance to the prograde family of orbits in the Zhao et al. (1994) models, those models give the projected ve-
locities for the Baade’s window field, which lies closer to the assumed bulge rotation axis. We apply the Zhao et al. (1994) mod-
els to our field to predict the $\sigma_{r}^2$, $\sigma_{\phi}^2$, and $\sigma_{z}^2$ in future work; however, we may gain some insight by considering the relationship between the observed and intrinsic velocity dispersions. For our line of sight, the observed variances will in general relate to the intrinsic dispersions as

$$\sigma_{r}^2 = \sigma_{\phi}^2 \sin^2(\alpha) + \sigma_{\phi}^2 \cos^2(\alpha) + 2\sigma_{R}^2 \sin(\alpha) \cos(\alpha),$$

$$\sigma_{\phi}^2 = 0.9978\sigma_{z}^2 - 0.1282\left[\sigma_{\phi z}^2 \cos(\alpha) + \sigma_{Rz}^2 \sin(\alpha)\right] + 2.13 \times 10^{-3} \times \left[\sigma_{\phi}^2 \cos^2(\alpha) + \sigma_{\phi z}^2 \sin^2(\alpha) + 2\sigma_{Rz}^2 \cos(\alpha) \sin(\alpha)\right],$$

$$\sigma_{z}^2 = 0.9989 \left[\sigma_{\phi z}^2 \sin(\alpha) + \sigma_{Rz}^2 \cos(\alpha)\right] - 0.0462 \left[\sin(\alpha) \cos(\alpha) \left(\sigma_{R}^2 + \sigma_{\phi}^2\right) + \sigma_{Rz}^2\right].$$

which relates six unknowns to three observables (for assumed $R_S$). Proper-motion observations of two fields together would [if the fields can be assumed to feel similar potential $\Phi(R, \phi, z)$] enable this system to be solved; alternatively, a similar depth-sensitive set of observations including the line-of-sight dispersions $\sigma_{r}^2$, $\sigma_{\phi}^2$, and $\sigma_{z}^2$ would allow this system to be solved to measure the intrinsic dispersions in this field. Because the line of sight passes close to the Galactic center, the projection angle $\alpha$ is a strong function of distance. For $|d - R_S| \gtrsim 0.4$ kpc, $\cos(\alpha) \approx 0$, while $|d - R_S| \leq 0.1$ kpc corresponds to $\sin(\alpha) \approx 0$. At both extremes, we expect $\sigma_{z}^2 \approx \sigma_{\phi}^2$, while at $d \approx R_S$

$$\sigma_{R}^2 \approx \sigma_{\phi}^2, \quad \sigma_{r}^2 \approx \sigma_{\phi}^2,$$

and for $|d - R_S| \gtrsim 0.4$ kpc, the subscript $R$ is replaced by subscript $\phi$ in equation (4). Between the two regimes the velocity variation is complex; in the triaxial bulge potential we may well have $\sigma_{Rz}^2 \approx \sigma_{\phi z}^2 \approx \sigma_{\phi z}^2$ (see, e.g., Sellwood 1981).
In a complementary manner, Kozłowski et al. (2006) have also detected a nonzero tilt in the velocity ellipsoid from their study of 300–500 stars in each of 35 ACS HRC fields. They find the tilt to be $\theta = 34^\circ \pm 8^\circ$, which is likely consistent with their measurement errors. Curiously, their proper-motion covariance is roughly constant within measurement error across a range of different sight lines, which might argue against projection effects. However, because the ACS HRC was used in their survey and the exposures were comparatively short, tracing the tilt of the velocity ellipse as a function of distance along the line of sight is difficult from their data because far fewer objects are traced per apparent distance bin than with ACS WFC. We remind the reader that a nontilted stellar population can easily exhibit a tilted velocity ellipsoid if stars are being selected in a small region of space, even if the potential is spherical (e.g., Binney & Tremaine 1994). The best way to use the tilt of the velocity ellipsoid is probably to select objects closest to the meridional plane from multiple sight lines; this requires data sets of sufficient depth to be confident that enough objects in the meridional plane can indeed be selected. The Baade’s window data sets of Kuijken & Rich (2002), newly complemented with the ACS WFC observations of the ChaMPPlane project (e.g., van den Berg et al. 2006), constitute a good example of the type of data set useful for such an approach (see also the discussion in Kuijken 2004).

Existing models compare the shape of the depth-integrated velocity ellipse at different lines of sight (e.g., Kozłowski et al. 2006; Rattenbury et al. 2007a, 2007b, 2007c; see also Vieira et al. 2007). It would be of great interest to trace the tilt variation with depth of the velocity ellipsoid as a function of metallicity; if there is indeed a separate stellar population exhibiting a tilt with respect to the majority of the bulge objects, perhaps accompanying the tilted gas (Burton & Liszt 1992), such a population may be of a newer generation and thus differ in metallicity from the rest of the bulge stellar population. The study of Soto et al. (2007) showed...
We take a simpler approach: we first assume that the superposition of populations along the line of sight can be approximated with a pair of two-dimensional (2D) Gaussians and rely on the symmetry of these distributions to correct the bulge population for the contamination induced by the disk. The number of points within the 0.7 \( \sigma \) ellipse of each population is evaluated to avoid counting objects twice due to overlap of regions in proper-motion space. Because of the high overlap, the estimated population within each region is corrected by the expected contribution from the other population (Fig. 18); the result is a “raw” disk fraction of 11.5%. The discrepancy between this value and the intrinsic value is assessed through simulation; using the distance-dependent proper-motion ellipses constrained previously, we simulate populations with input disk fraction and assess the difference between the value returned by this process and the value input (Fig. 18); this leads to a true value 14% ± 1% for the disk fraction among our population with proper-motion measurements.

### 7.2. A Clean-Bulge Sample

To select a clean-bulge sample, we modify slightly the kinematic selection criteria of Kuijken & Rich (2002); we use a cut on longitudinal proper motion \( \mu_l \) and on proper-motion measurement errors \( \epsilon_l, \epsilon_b \) but we discard cuts on \( \mu_b \) because bulge and disk show similar latitudinal motion (Figs. 8 and 10). The proper-motion error is an estimate of the measurement scatter; this estimate does not take into account any bias in the measurement due to nearby brighter objects. For this reason, we further impose a cut on the crowding measure \( q \) (§ 3.2 and Fig. 1); we choose to keep objects with \( q < 0.15 \).

To set the proper-motion cutoff, we return to the population above the MSTO to find the value of \( \mu_l \) at which the disk contribution is low but the cut not so severe that too few objects are retained to be useful. We use the same selection regions as Figure 8 to produce nominal “disk” and “bulge” populations. The “disk” population is markedly asymmetric in \( \mu_l \). Indeed, this population is best fitted with a two-component model, of which the one with high positive centroid proper motion (\( \mu_{l0} = +4.17 \, \text{mas yr}^{-1} \)) is identified with the disk (hereafter the “true-disk” population). The component with centroid proper motion consistent with zero (Fig. 19, top panels) is indistinguishable from the single component that is needed to fit the \( \mu_l \) distribution of the bulge population above the MSTO (Fig. 19, bottom panels), indicating some bulge/disk overlap in the “disk” region of the CMD. The proper-motion cut \( \mu_l < -2.0 \, \text{mas yr}^{-1} \) of Kuijken & Rich (2002) lies approximately 2.9 \( \sigma \) from the center of the true-disk distribution, so only about 0.19% of disk objects have \( \mu_l < -2.0 \, \text{mas yr}^{-1} \). We thus adopt \( \mu_l < -2.0 \, \text{mas yr}^{-1} \) as our proper-motion cutoff. To ensure that proper motions in the clean-bulge sample are measured to at least 6 \( \sigma \) significance, we impose the proper-motion error cutoff \( \epsilon_l, \epsilon_b < 0.3 \, \text{mas yr}^{-1} \). This leaves 15,323 objects kinematically associated with the bulge (Fig. 20). Because we have set our \( \mu_l \) cutoff with reference to disk objects above the MSTO, our cutoff is not affected by the small contribution of evolved disk stars to the bulge red giant branch (RGB) in the CMD (Vieira et al. 2007).

We now examine the contamination due to nonbulge stars that would pass these kinematic cuts. The disk makes up approximately 14% of the stars with measured proper motions, while 81,140 of the 187,346 objects with proper-motion measurements show errors \( \epsilon_l, \epsilon_b < 0.3 \, \text{mas yr}^{-1} \). The likely disk contamination to our proper-motion–selected sample is therefore 81,140 × 0.14 × 0.0019 = 22 objects. Thus, perhaps 0.14% of the “clean-bulge” sample may in reality be disk objects. The Galactic halo also provides a small contaminant; integrating the Bahcall & Soneira (1984) models...
along the line of sight predicts ~23 halo objects in the ACS WFC field of view. The halo shows proper-motion dispersion $\sigma_l \simeq 22/d$ mas yr$^{-1}$ (Binney & Merrifield 1998; with distance $d$ in kpc), about a proper-motion zero point similar to the bulge. We thus expect perhaps 40% of halo objects to pass our proper-motion cuts, or ~9 objects. Thus, the handful of objects high above the main sequence (visible in Fig. 20) are probably halo stars and can later be easily excised from the sample by position in the CMD. So the total contamination due to nonbulge stars in our clean-bulge sample is of order $22 + 9 = 31$ of 15,323 objects.

Isochrone fitting to the bulge population is complicated by several factors. The extinction curve along the line of sight is not well constrained, may vary spatially over the ACS WFC field of view, and may be anomalous (Sumi et al. 2004), making dereddening challenging. The binary fraction of the bulge is unknown, causing an unknown systematic between single-star isochrones and the observed population. In addition, very few $\alpha$-enhanced isochrones exist for metallicities greater than +0.5. We therefore defer detailed fitting of the bulge isochrone for future work and content ourselves with a visual comparison to VandenBerg isochrones that have been transformed using the adopted bulge extinction and distance in the manner described by Brown et al. (2005).

To estimate the best-fitting bulge sequence, a median bulge population was estimated by binning the clean-bulge population along the isochrone (Fig. 20). Because the uncertain binary fraction is not significant above the MSTO, we use the bulge-only CMD in this region to assess the spread in age and metallicity that overlap the observed range of colors. The 10 Gyr, $\alpha$-enhanced solar-metallicity isochrone of Sahu et al. (2006) is replaced by a slightly older population at 11 Gyr; this better reproduces the median population. Isochrones corresponding approximately to $\pm 1 \sigma$ in both metallicity and age encompass most of the color variation along the post-MSTO bulge population. A very young, very metal-poor isochrone ([Fe/H] = −1.009, 5 Gyr) overlaps the outliers along the bulge RGB; younger, more metal-poor isochrones

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**Fig. 16.**—Observed transverse velocity distribution as a function of line-of-sight distance. Distance is marked in the right panel for each pair and increases reading left to right. The left panels denote the observed velocity distribution, while the right panels give the 2D histograms and $1 \sigma$, $2 \sigma$ best-fitting ellipses. This figure shows 5.2 kpc $\leq d$ $\leq$ 7.6 kpc.
are not consistent with the clean-bulge population (Fig. 20). The transformed isochrones do not reproduce an apparently metal-rich, old population visible at the red edge of the post-MSTO bulge sequence; this might be due to regions with higher extinction than our mean adopted value of $E(B-V) = 0.64$.

Cohen et al. (2008) recently compiled metallicities of a selection of main-sequence objects in the bulge that had been sufficiently amplified by microlensing to permit abundance analysis with comparable precision to the more traditionally used giants (see also Minniti et al. 1998). The three objects in their sample show $\frac{\text{[Fe/H]}}{\text{[C]}} < 0.3$, leading to the claim that a random sample of bulge giants should be less metal-rich than a similar sample from the main sequence, perhaps due to highly metal-rich objects losing so much mass that they never undergo the helium flash and evolve along a different track to the rest of the tracer giants (Cohen et al. 2008). We find no support for this hypothesis; allowing for the uncertain binary frequency that affects the main sequence far more than the bulge RGB, the metallicities of the giants and main-sequence samples are consistent with each other. More quantitative exploitation of the clean-bulge sample to constrain the star formation history of the bulge is beyond the scope of this article and will be reported elsewhere.

7.3. Blue Straggler Candidates

Within a few magnitudes above saturation, a population is visible with a disklike location in the CMD (blueward of and brighter than the bulge MSTO) but with bulgelike proper motions (Fig. 10; particularly obvious in $\mu_e$, but also visible in $\sigma_e$). When we apply the kinematic cuts to extract a likely clean-bulge sample, 72 objects remain in this region of the cleaned CMD (Fig. 19). As the halo is a somewhat evolved population, its objects are not expected to lie in this region of the CMD, and with an estimated 22 disk contaminants within the entire clean-bulge sample, at least 69% of this population must consist of bulge objects.

This population may consist of very young, very metal-poor objects, although how such objects would form is not clear, and indeed isochrones younger than about 5 Gyr, which would be required to describe the brightest, bluest objects in this population, are not consistent with the CMD below the MSTO (Fig. 20). The alternative is that a significant fraction of these objects may be
bulge blue stragglers; their location in the CMD overlaps with
the region such objects are expected to occupy (e.g., Sarajedini
1992). Further details of these objects will be reported in a sep-
arate paper (W. Clarkson et al. 2008, in preparation).

8. KINEMATICS OF THE SWEEPS
PLANET CANDIDATES

We are now in a position to examine the 16 SWEEPS transit
planet candidates for membership of disk or bulge populations;
this distinction will inform the history of their formation and evo-
lution. We construct a mean bulge proper-motion best-fit ellipse
by taking a population-weighted average of the best-fit proper
motion (not velocity) ellipses using the kinematic tracer objects of
the previous section. We produce a mean (foreground) disk proper-
motion ellipse using stellar tracers in the nearest distance bin.

When we overplot the best-fitting mean-bulge and mean-disk
proper-motion contours, we find an apparent grouping of four
objects within the 1/C27 contour of the disk, and all but two of the
rest within the 2/C27 ellipse of the bulge population. Furthermore, the object SWEEPS-04, which lies well within the 1/C27 ellipse of the disk population, resides on the upper disk sequence (Sahu et al. 2006), where disk stars are expected to dominate.9 However, all objects are also within the 2/C27 ellipse of the bulge population.

8.1. SWEEPS Candidates as Disk/Bulge Objects

We use the angular distribution of candidates in proper-motion
space to assess kinematic membership of the SWEEPS candi-
dates. In \(\{\mu_l, \mu_b\}\)-space, let \(\Theta_i\) be the counterclockwise angle
between the major axis of the best-fit bulge ellipse and the line
joining the center of the best-fit bulge ellipse to the \(i\)th candidate. The cumulative distribution function (CDF) of \(\Theta_i\) is then
used as an indicator of the angular distribution of the SWEEPS
candidates in proper-motion space. Should a large number of can-
didates reside in the disk, one would expect a sharp steepening in
the CDF near \(\Theta_d\), the angle between the major axis of the bulge
ellipse and the center of the disk distribution (Fig. 22). Altern-
atively, if all 16 candidates were bulge objects, then the CDF
would be a straight line; no angle \(\Theta_i\) would be preferred. We
compare the observed CDF of the SWEEPS candidates to a large
number of trial artificial data sets, in which 16 objects are gen-
erated under the best-fit bulge and disk proper-motion distribu-
tions. For each trial, the two-sided Kolmogorov-Smirnov (K-S)
statistic is computed between the trial and the observed distribu-
tion, yielding the associated formal probability that the SWEEPS
candidates and the trial data set are both realizations of the same
probability distribution. This process is repeated for 10^5 trial data
sets. This test is repeated for differing sizes of disk contribu-
tion \(N_d\) to the total population (for 0 \(\leq N_d \leq 16\), and the formal
probability that the SWEEPS sample matches the distribution
using each \(N_d\) is recovered.

To maximize use of available information, we have also
applied the 2D K-S test to the set of positions in \(\{\mu_l, \mu_b\}\)-space of all the candidates. We use the implementation in Numerical
Recipes (Press et al. 1992; see also Metchev & Grindlay 2002).
In 2D the equivalent K-S statistic \(D_2\) is a function of the input
distribution. We thus evaluate the significance of the maximum
\(D_2\) at each disk fraction \(N_d\) using Monte Carlo simulations. This

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9 Assuming that its host is not a blue straggler, which would seem unlikely.
produces an equivalent significance curve as a function of \( N_d \) (Fig. 22).

8.2. The Bulge and Disk Planet Fractions

Although the most probable disk population \( N_d \) differs slightly between the two tests, both are consistent (at 1 \( \sigma \)) with a disk population in the range \( 1 \leq N_d \leq 8 \). If the fraction of stars hosting Jovian planets with periods less than 4.2 days were identical between disk and bulge, we would expect the planet candidates to follow the same disk/bulge distribution as the stars in general. Our kinematic analysis would then suggest that 14\% of planet candidates (two candidates) would reside in the disk. This is entirely consistent with the actual distribution of candidate kinematics. However, the sample of SWEEPS transit planet candidates is too small to draw meaningful conclusions about the fraction of planets in the disk versus that of the bulge. Because we cannot state that the fractions of planet candidates in disk and bulge are inconsistent with each other, we cannot make any claims about the consistency or otherwise of the fraction of stars hosting planets between the disk and bulge.

Any inference on the true planet host fraction in the bulge then reflects our ignorance of the true ratio of astrophysical false positives to true planets; we restrict ourselves to upper and lower bounds. Considering the likely population of stellar triples, grazing-incidence stellar binaries, and low-mass stellar companions, Sahu et al. (2006) estimate the maximum rate of astrophysical false positives among the candidates of 9/16, or 56\%. We remind the reader that this is probably a conservative upper limit, as no such false positives were found in the similar 47 Tuc analysis (Gilliland et al. 2000; for further discussion of this issue see Sahu et al. 2006). Thus, the lower bound on the fraction of true planets among the candidates is 44\%; the upper bound is 100\%, predicting 6–14 true detected planets in the bulge. Taking the detection efficiency, period distribution, and the probability of transit due to random orbital inclinations into account, the frequency of stars hosting Jovian planets with periods shorter than 4.2 days was...
estimated to be 0.42% (Sahu et al. 2006). The 6–14 true detected planets then imply an extra uncertainty of perhaps a factor of 2, since the planet frequency consistent with observations depends on not only the fraction of true planets but also the actual period, radius, transit phasing, and host brightness of each planet.

We ask if the subpopulation of five planet host candidates with periods less than 1 day (the "ultra-short-period planets," or USPPs; Sahu et al. 2006) themselves are preferentially located in the disk or bulge. Here there is no obvious correlation between period and membership: two USPPs fall within the 1/√2 ellipse of the best-fit disk, three fall within the 1/√2 ellipse of the best-fit bulge, and all are within 2/√2 of the best-fit bulge. Thus, the USPPs do not show any preferred kinematic association compared to the non-USPP candidates; the best that can be said is that the USPPs as a family are unlikely to all be disk objects.

9. SUMMARY AND CONCLUSIONS

We have measured proper motions for >180,000 objects within the Sagittarius low-reddening window toward the bulge ([l, b] = (1.65°, −2.65°)) and used them to extract a clean-bulge sample of 15,323 objects, a sample roughly a factor of 4 larger than that afforded by WFPC2 across a 6 yr interval (Kuijken & Rich 2002). This clean-bulge sample contains perhaps 31 contaminants from both disk and halo, making it the purest bulge population ever isolated. Constructing a median stellar sequence from this bulge sample, we find that an 11 Gyr isochrone best represents the bulge population, with most of the variation along the bulge subgiant branch falling within the range [Fe/H] = 0.0 ± 0.4 and age 11 ± 3 Gyr. Use of this sample to inform bulge age studies, in conjunction with extensive completeness tests, will be the subject of future reports. Work along these lines is particularly exciting when we consider the parallel NICMOS observations we have undertaken, for which similar selection should be possible and which brings the possibility of tracing the bulge IMF to the neighborhood of the hydrogen-burning limit.
The large number of stars with proper-motion measurements allows kinematic features to be resolved in the CMD. We construct the $l, b$ velocity ellipse as a function of line-of-sight distance, demonstrating that its properties are quite sensitive to the distance to the objects under consideration. Finally, we use its proper-motion analog to attempt to classify the SWEEPS planetary candidates by kinematic membership with bulge or disk. The proper-motion distribution of the candidates is consistent with the division of the stellar population between bulge and disk, but the candidate population is too small to draw further conclusions. We find no evidence that USPP candidates are preferentially associated with the disk; instead, we can only claim that the distribution is entirely inconsistent with all the USPPs originating in the disk.

Although the metallicities of the majority of our targets are unknown (with the exception of the bright objects, largely above the MSTO, for which VLT spectroscopy has been possible), the large number of objects with accurate proper motions allows the distance uncertainty due to metallicity to be overcome by virtue of a sample containing $>500$ objects bin$^{-1}$. Armed with photometric distances and therefore mean transverse velocities for our distance bins, we produce an independent determination of the stellar circular speed curve $v_\text{circ}$. Although the sampling of this rotation curve is relatively sparse, we clearly detect a transition away from solid-body–like rotation at galactocentric radius $R_\text{cut} = 0.3$–$0.4$ kpc, in line with evidence from radial velocities. Within the uncertainties of the distance estimates we have used, our limiting circular speed at large radius is consistent with that from radial velocity measurements.

With position-estimate dispersions reaching the 2–3 milliarcsecond level ($\sigma$), we are near the limit of position measurement with current $HST$ instrumentation. The intrinsic velocity dispersion of the bulge and disk nevertheless makes kinematic classification of object groups with few members difficult. As the bulge is likely a superposition of multiple stellar populations (e.g., Zoccali et al. 2003), the intrinsic dispersion problem might be overcome if populations can be separated by metallicity on a star-by-star basis (see Soto et al. 2007), which will require further, multifilter observations of this field.

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APPENDIX A

PROPER MOTIONS FROM DEEP PHOTOMETRY

Two approaches have in the past been taken to obtain proper motions from two different-depth epochs of observations of crowded stellar fields (at least one of which is dithered), both with some success. The key difference lies in the production of the master list of positions from the deeper epoch. The first approach is to stack the deepest epoch images and build the master list from the image stack. This takes advantage of the subpixel dithers to produce a (usually) twice-oversampled superimage, from which deep positions can be obtained; the PSF is well sampled in the resulting superimage and thus the object centers are better constrained than the individual input images. This approach was used quite successfully by Richer et al. (2004a, 2004b) when separating members of M4 from the rest of the field. The second approach is to measure positions on each image individually and combine the measurements with sigma clipping to produce the master list. Positions are measured in the raw frame of each image (the _flt frame), using a highly supersampled model for the “effective PSF” (the instrumental PSF as recorded by the detector, or ePSF), with a perturbation scaled to the data to account for focus breathing. This usually allows the measurement of positions from the individual images to higher accuracy than possible from an ePSF constructed from the data themselves (Anderson & King 2006) because the observations used to produce the library ePSF are rather better sampled than most program observations. Our data set is exceptionally well sampled, so whether this advantage should hold here was less clear when embarking on the reductions for this project. We thus tried both approaches for this data set to allow a side-by-side comparison of the two methods and picked the winner for further evaluation of proper motions.

A1. METHOD 1: MASTER LIST FROM STACKED IMAGES

Positions in the second epoch are mapped onto positions in the master list built from the epoch 1 image stack. Mutual misalignment between images in the two epochs means that the epoch 2 images cannot be directly evaluated onto the superimage for comparison. In this approach, images in the less well-populated epochs are not stacked together before position determination as there are too few images in each epoch to optimize the stacking; instead, each individual input image provides a separate estimate of the proper motion for each object. Because frame-to-master offsets are likely to be of a high order, image regions of size 1600 × 1600 twice-oversampled pixels (so 40” × 40") are used to simplify the transformation required (including a 200 pixel buffer at the frame edges where transformations will be least well fitted); each region contains ~7000 unsaturated stars in the master list, of which ~60% are at F814W < 25.5.

The optimal transformation from the master image to each individual second-epoch image is determined from an AMOEBA fit (Press et al. 1992) to the positions of marker stars; typically 250 bulge marker stars are used as candidates to the transformation, which is cut down to ~150–200 after sigma clipping of contaminating outliers. The stand-alone f77 implementation of STSDAS BLOT is then used to transform the master image to the individual second-epoch image, so that the less well-determined input image is never resampled. Positions are then found in both the input and the transformed master image and compared directly to produce proper motions. The result is a separate estimate of the proper motion for each input image, for each postage stamp subregion. Cosmic rays and other artifacts are weeded out at the stage of proper-motion estimate collation by sigma clipping (see Sahu et al. 2002).

A2. METHOD 2: MASTER LIST FROM COLLATED MEASUREMENTS

The alternative approach, producing the master list by combining measurements from each individual image within an epoch, is preferable if the ePSF in _flt space is well enough constrained that its errors are smaller than those introduced due to the small spatial shifting of flux in the production of the epoch 1 superimage. The observations used for the ePSF of Anderson & King (2006; see also Anderson & King 2003) provided a 4 times supersampled, spatially dependent supersampled PSF model in _flt space, as well as a distortion solution that transforms raw positions into positions on the sky. The distortion is accurate to ~1% of a pixel (Anderson & King 2006). Thus, for the epoch of the calibration observations, positions and fluxes can be measured to high accuracy and transformed to a distortion-free frame using this solution.

A3. METHOD COMPARISON

We compare the two methods of proper-motion determination by the proper-motion distributions each produces over the same region of the image, in this case the 80" × 80" surrounding the exoplanet candidate host SWEEPS-13. The resulting proper-motion distribution along pixel-x coordinate shows width ~0.2 mas yr^{-1} (0.008 ACS WFC pixels over 2 yr) greater when determined using the epoch 1 stack than from the image-to-image estimates, with the latter showing closer agreement to the previously published WFPC2-based proper motions determined by Kuijken & Rich (2002). The stack-based approach thus provides proper-motion estimates roughly 7% less precise than the image-to-image approach (Fig. 23).

The cause of this discrepancy is most likely the combination of deep-epoch images into the image model, which has caused flux to be slightly rearranged. While the number of electrons associated with a given object is preserved (so the image model is still ideal for photometry), their distribution within the region occupied by the star has been altered (Fig. 24). The magnitude of this systematic is

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10 When fitting coordinate transforms, we try both AMOEBA (Press et al. 1992) and Levenberg-Marquardt (using the Markwardt MPFITFUN IDL implementation, available at http://cow.physics.wisc.edu/~craigm/idl/fitting.html) approaches and select the approach that fits the transformation most robustly.
Fig. 23.—Comparison of proper-motion measurements (in mas yr\(^{-1}\)) when computed by comparison to positions using the optimal stack for photometry (top) and using an image-by-image approach for both epochs (bottom). [See the electronic edition of the Journal for a color version of this figure.]

Fig. 24.—Alteration of the ePSF in the production of the twice-oversampled superimage used for SWEEPS photometry. The image model computed in Sahu et al. (2006) provides an estimate of the full pixel flux at each position \(x, y\) in this oversampled space; thus, we may deinterlace the superimage into four \_flt\_type images. When the Anderson & King (2006) techniques are used to measure positions on each of the four images, a difference in position as a function of pixel phase between pairs of deinterlaced images becomes apparent. This illustrates that the combination of images into the superimage has subtly changed the ePSF of the scene. While optimal for photometry, use of the SWEEPS superimage can lead to a position systematic at the 0.02 pixel level.
reduced somewhat by the blot process; while it can be mitigated in principle by refitting the ePSF to the postage stamp for each region in each image, there is no guarantee that any spatial variation of this shifting can be swept into the ePSF fit. For this reason, all proper motions we report here are based on the image-to-image approach.

APPENDIX B
OBJECTS WITHOUT A UNIQUE MEAN POSITION ESTIMATE

For roughly 6% of objects, the techniques of § 3.2 fail to converge on a single solution for mean position \((X_c, Y_c)\). Objects for which the fitted peaks separate by <0.15 pixels show a main clustering and a trail, or in some cases another island of points (top left). Objects with wider fitted separation show clearer separation between two clusters (bottom left). Measurements within the island with the greatest number of points are selected for further passes; this usually coincides with the island with lowest scatter (right panels).

Thus, we conclude that the trailing in these position estimates is due to the influence of a bright neighbor with slightly incorrect subtraction.

Objects with separation between peaks greater than 0.15 pixels make up the remaining 14% of the bimodal objects (or 0.8% of the total population) and generally show more well-separated clusters of position estimates. In contrast to the low-separation bimodal objects, the reported separation between peaks is close to the apparent separation between islands on visual inspection (Fig. 25, bottom left panel). Also unlike the low-separation bimodal objects, the position angle between the peaks shows no relation to the...
However, for this population, the nearest bright neighbor is on average closer than for the population as a whole. It thus appears clear that a significant fraction of both classes of object with bimodal position estimates represent different manifestations of the effects of crowding by stellar neighbors. The former class of bimodal object is found all throughout the sample diagram, while the high-separation class of bimodal objects coincides with the population showing anomalously high position scatter during the second pass of the photometry (Fig. 3). Where two clouds of points are produced for an object, we select the cloud with the highest number of measurements; visual inspection suggests that in most cases this clump shows the lower scatter of the two clouds and is interpreted as the true position of the star (Fig. 25, right panels). This selection takes place at the same time as sigma clipping when the master list is updated after each pass of the photometry (§ 3.2). The resulting master position list now shows rather small internal variation (Fig. 3). We remind the reader that when selecting a clean-bulge sample, our selection on the crowding index $q$ will remove most of the bimodal objects from consideration.

APPENDIX C
TIME VARIATION IN CHARGE TRANSFER EFFICIENCY

ACS WFC has its horizontal shift registers along the detector $X$ direction at the top and bottom of the detector, so that during readout charge is transferred in the positive $Y$-direction (WFC1) and negative $Y$-direction (WFC2). It thus incorporates a CTE effect; the signal measured from a star is dependent on the detector $Y$ position of that star due to the number of transfers of the signal during...
readout. This effect may be observed in a number of ways; the same scene may be observed at two different integration times or pointings and the change in magnitude or position observed plotted against detector Y position (e.g., Riess & Mack 2004). Or, a sharply defined feature in the CMD may be measured and its variation over the CCD tracked if there are enough stars in the frame (e.g., Brown et al. 2005). The effect on measured magnitudes for ACS WFC is a V-shaped pattern in instrumental magnitude versus detector Y; objects nearest the chip gap are farthest from the shift registers and thus suffer the greatest signal decrease during readout. Recently it has been pointed out that CTE can also affect the detected position of stars on the detector; the signature behavior is a trend in measured position with detector Y, with a discontinuity at the interchip gap (Kozhurina-Platais et al. 2007).

We thus searched for evidence of any CTE effects in our data to assess its impact on proper-motion measurements. Brown et al. (2005) used the ~3 mag difference between the horizontal branch and subgiant branch to probe this distribution and concluded that the tendency of the high stellar background to fill pixel wells was reducing CTE effects to beneath the 5 mmag level. This process is difficult to apply to our data because similar features with low intrinsic dispersion are not available in the faint magnitude regime at which CTE is expected to be significant; when following a similar procedure to assess the magnitude spread as a function of position, we see largely random variation with detector Y, which we attribute to intrinsic variation in the scene. Instead, we establish that CTE effects are present in the SWEEPS data by measuring the difference in instrumental magnitude between 339 and 20 s exposures within the 2004 epoch. Because the CTE effect is more pronounced for fainter objects, it will affect measurements in the short exposures more than the long; thus, the V-shaped CTE fingerprint is expected. Indeed, it is clearly observed (Fig. 27) at the 20–30 mmag level and with an amplitude $\Delta$.

**Fig. 27.**—Differential CTE. Within the 2004 epoch, comparison of extracted instrumental magnitudes at 339 and 20 s shows position dependence typical of CTE effects, where the apparent brightness depends on the distance over the chip the flux must travel at readout. Titles give the instrumental magnitude range in the 339 s exposures, the dashed line the expected magnitude difference from the differences in exposure time alone. Inset numbers give variation amplitude across each chip and the scatter $\sigma$ in the fit due to measurement errors.
that increases with object faintness. Next, we compare instrumental magnitude measurements of objects observed at nearly identical exposure times between the two observation epochs (339 s in 2004 and 345 s in 2006). The CTE signal is again clearly visible at the $2 \mu$mag level and also increases in amplitude with increasing object faintness. This is a differential CTE measurement, in that the CTE of the detector has degraded somewhat in the 2 yr between measurements (Fig. 28).

To estimate the CTE contribution to astrometry and thus proper-motion measurements, we compare position measurements between 339 and 20 s exposures in the 2004 epoch. This is then scaled by the size of the interepoch CTE magnitude effect to estimate the contribution of CTE to the proper-motion measurements. Positions from the 20 s exposures were transformed to the median frame of the 339 s exposures using local transformations ($x$), with objects in likely bulge regions of the CMD and at instrumental magnitude ($M_{\text{inst}} < -12$) in the 339 s epoch used as tracers for the transformation. A linear trend was fitted to the position difference $\Delta Y$ as a function of detector $Y$ (polynomials were also tried but found to give no advantage over the linear trends) and the range of $\Delta Y$ across each chip measured. Errors on the trends were estimated by simulating a large number of trials assuming no intrinsic variation with detector $Y$ and computing the standard deviation of recovered $\Delta Y$ ranges. The astrometric CTE effect noted by Kozhurina-Platais et al. (2007) is clearly present in the 2004 epoch (Fig. 29). The size of the trend increases with target faintness compared to the tracer stars. No
Fig. 29.— Astrometric CTE effects. The position-dependent CTE shift $\Delta Y$ is given as a function of position on the detector, when comparing the same stars at exposure time 339 and 20 s. The titles give the instrumental magnitude range in the 339 s exposures. The amplitude of variation across the chip $\Delta$ is given, as well as the expected error $\sigma$ on these fits due to the measurement scatter.
trend is detected in the tracer-star magnitude range, while the trend reaches 10 millipixels in the $-10.7 \leq M_{\text{inst}} < -9$ range. Assuming the astrometric CTE signal to scale with the magnitude CTE signal, we thus expect an astrometric CTE signal between the deep observations in the 2004 and 2006 epochs of perhaps 0.5–3 millipixels at the faintest instrumental magnitudes. However, when we search for such a signal in the position differences between epochs, such a signal is not detectable above the scatter caused by intrinsic motion of the stars between the two epochs (Fig. 30). Thus, the predicted astrometric effect due to differential CTE is less than 0.2 mas and thus not a significant source of error.

Fig. 30.—Same as Fig. 29, this time giving the position discrepancy between the 339 s exposures in 2004 and the 349 s exposures in 2006. Here the scatter $\sigma$ in fitted trends is much larger due to the intrinsic motion of the stars; no differential CTE effect is detectable.

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