Muon Anomalous Magnetic Moment and Lepton Flavor Violation

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Abstract

A non-universal interaction, which involves only the third family leptons induces lepton flavor violating couplings and contributes to the anomalous magnetic moment of muon. In this paper, we study the effects of non-universal interaction on muon \( (g-2) \) and rare decay \( \tau \rightarrow \mu \gamma \) by using an effective Lagrangian technique, and a phenomenological \( Z' \) model where \( Z' \) couples only to the third family lepton. We find that the deviation from the theory can be explained and the induced \( \tau \rightarrow \mu \gamma \) rate could be very close to the current experimental limit. In the \( Z' \) model, \( M_{Z'} \) has to be lighter than 2.6 TeV.

Recently the Brookhaven AGS experiment 821 group has announced a new measurement of the anomalous magnetic moment with three times higher accuracy than it was previously known [1]. The experimental data now is in conflict with the theoretical prediction of the Standard Model (SM) with an excess of 2.6 \( \sigma \). This may be an indication of new physics if it is more than a statistical fluctuation. Many authors have considered various possibilities of interpreting the discrepancy in models beyond the Standard Model, such as, supersymmetric version of the standard model, muon with substructure in composite model and anomalous gauge boson couplings [2].

In this paper we consider a class of models where non-universal interactions exist and study the effects of new physics on muon \( (g-2) \) and lepton rare decay. We first take a model independent approach (effective Lagrangian) to new physics, then consider a phenomenolog-
ical $Z'$ model. We find that the non-universal interaction can account for the deviation of muon (g-2) from the SM prediction. Furthermore, our models predict the rare decay rate $\tau \rightarrow \mu \gamma$ which can be tested in the planned $\tau$-charm factories and in $Z'$ model $M_{Z'}$ has to be lighter than 2.6 TeV which results in many interesting phenomena \[3,4\].

The tau lepton is the heaviest one in the lepton sector and especially the top quark is heavier than all other fermions, with mass close to the electroweak symmetry breaking scale, the new physics, if exists, is expected to become manifest in the effective interaction of the third family fermions \[5\]. There are also theoretical and phenomenological motivations that there may exist interactions with generation non-universal couplings \[3,4\]. Furthermore, the possible anomalies in the $Z$- pole $b\bar{b}$ asymmetries may suggest a non-universal $Z'$ \[6,7\].

In terms of an effective lagrangian, the new physics is described by higher dimensional operators. At dimension-six, there are two operators \[8,9\] which contribute directly to the anomalous magnetic moment of leptons,

$$O_{TB} = \bar{L}\sigma^{\mu\nu}\tau_R \Phi B_{\mu\nu},$$

$$O_{TW} = \bar{L}\sigma^{\mu\nu}\vec{\sigma}\tau_R \Phi \vec{W}_{\mu\nu},$$

(1)

where $L = (\nu_\tau, \tau_L)$, $\Phi$ is the Higgs scalar, $B_{\mu\nu}$ and $W_{\mu\nu}$ are field strengths of $U_Y(1)$ and $SU_L(2)$, $\vec{\sigma}$ are Pauli matrices.

In the presence of operators $O_{TB}$ and $O_{TW}$, the effective Lagrangian $\mathcal{L}_{eff}$ can be written as,

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \frac{1}{\Lambda^2}(C_{TB}O_{TB} + C_{TW}O_{TW} + h.c.),$$

(2)

where $\mathcal{L}_0$ is the standard model lagrangian and $C_{TB}, C_{TW}$ are constants which represent the coupling strengths of $O_{TB}$ and $O_{TW}$. $C_{TB}, C_{TW}$ are expected to be $O\left(\frac{1}{16\pi^2}\right)$ in theories with weakly interaction \[10\], however could be large if the fundamental theory is strongly interaction such as the models of composite tau lepton.

After the electroweak symmetry is broken and the mass matrices of the fermions and the gauge bosons are diagonalized, the magnetic moment-type couplings of the leptons to gauge boson $Z$ and the photon $\gamma$ are
\[ \mathcal{L}_{\text{eff}}^{Z,\gamma} = e g_{Z,\gamma} \left( \begin{pmatrix} \tau \\ \mu \\ \tau \end{pmatrix} \right)^T \left\{ \left(-\frac{1}{2m_\tau}\right)(i k_\nu \sigma^{\mu\nu})S_{Z,\gamma} \right\} \times U_l \left( \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} U_l^\dagger \right) \left( \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \right) V_\mu, \]

where \( g^Z = 1/(4s_W c_W) \), \( g^\gamma = 1 \), and

\[ S^Z = -g^Z \frac{2\sqrt{2} m_\tau v}{e} \frac{1}{\Lambda^2} \left[ C_{\tau W} c_W - C_{\tau B} s_W \right], \]

\[ S^\gamma = \frac{2\sqrt{2} m_\tau v}{e} \frac{1}{\Lambda^2} \left[ C_{\tau W} s_W - C_{\tau B} c_W \right]. \]

The matrix \( U_l \) in Eq. (3) is the unitary matrix which diagonalizes the mass matrix of the charged lepton. In the SM, which corresponds to \( \mathcal{L}_{\text{eff}} \) in the limit of \( \Lambda \to \infty \), the matrix \( U_l \) is not measurable because of the zero neutrino masses. Furthermore, the universality of the gauge interaction guarantees the absence of the flavor changing neutral current and the lepton flavor violation in the lepton sector.

The lagrangian \( \mathcal{L}_{\text{eff}}^{Z,\gamma} \) in Eq. (3) gives rise to the anomalous magnetic moment of muon \((g-2)\) and the rare decays \( \tau \to \mu \gamma \) etc, whose relative size, however, depends on the rotation matrix \( U_l \). In Ref. [8], we have shown that \( \mu \to e \gamma \) puts a very stringent constraint on the product of the tau-electron and muon-electron mixing angles. So in the following discussion we assume that the tau mixes only with the muon, but not with the electron and for simplicity we have assumed that rotation matrices of left-handed leptons and the right-handed leptons are equal \([1]\).

\[ ^1 \text{We have also considered the case with different left-handed and the right-handed rotation matrices and find our conclusions unchanged.} \]
\[
U_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix},
\]
(5)

where \( s = \sin \theta \) and \( c = \cos \theta \) with \( \theta \) being mixing angle. The decay width of \( \tau \to \mu \gamma \) is given by

\[
\Gamma(\tau \to \mu \gamma) = \frac{\alpha}{8} m_\tau (s c S^\gamma)^2,
\]
(6)

and the new contribution to the tau and muon anomalous magnetic moments are given by

\[
|\delta a_\tau| = |c^2 S^\gamma|; \quad |\delta a_\mu| = |s^2 S^\gamma| \frac{m_\mu}{m_\tau}.
\]
(7)

Within the parameter space of \( C_{\tau B}, C_{\tau W}, \Lambda \) and \( \sin \theta \), we have quantitatively studied the new physics effects on the \( \tau \to \mu \gamma, Z \to \tau \mu, \delta a_\tau \) and \( \delta a_\mu \). We find that the predicted \( Br(\tau \to \mu \gamma) \) is correlated to \( \delta a_\mu \), which is shown in Fig. 1. The figure also shows that \( Br(Z \to \tau \mu) \) could be quite close to the experimental limit. On the other hand, given the current limits on \( Br(\tau \to \mu \gamma) \) and \( \delta a_\mu \), the predicted \( Br(Z \to \tau \mu) \) and \( \delta a_\tau \) are well below the experimental limits. Numerically to fit \( \delta a_\mu \) to be within 90% C.L. of the new experimental data, we obtain that \( s^2 \) has to be bigger than 0.46, which is quite reasonable in some models\cite{11}. And \( Br(\tau \to \mu \gamma) \) could be close to the current experimental limit. For example, taking \( s^2 \sim 0.5 \), \( C_{\tau B}, C_{\tau W} \sim 1/(4\pi)^2 \)\cite{10} and \( \Lambda \sim 17.5 \) TeV, the new physics contribution to muon (g-2) is,

\[
\delta a_\mu = 221 \times 10^{-11},
\]
(8)

which is within 90% C.L. of the experimental data

\[
a_\mu^{exp} - a_\mu^{SM} = 426 \pm 165 \times 10^{-11},
\]
(9)

and \( Br(\tau \to \mu \gamma) = 1 \times 10^{-6} \) which is close to the current experimental limit \( Br^{exp}(\tau \to \mu \gamma) = 1.1 \times 10^{-6} \)\cite{12}. From Fig.1, one can see that increasing the mixing angle \( \theta \) will decrease the value of predicted \( Br(\tau \to \mu \gamma) \), however for a large range of the mixing angle it remains to be close to the current experimental limit.
FIG. 1. Plot of the predicted $\text{Br}(\tau \rightarrow \mu \gamma)$ versus $\delta a_\mu$: the five solid curves correspond to mixing angles $\sin^2 \theta = 0.1, 0.5, 0.7, 0.9, 0.95$, and between the dashed lines is the region of experimental value of $\delta a_\mu$ with 90% C.L..

Now we consider a phenomenological model where a heavy neutral vector boson $Z'_\mu$ is introduced and coupled to only the tau lepton,

$$\mathcal{L}^{\text{int}} = -g Z'_\mu \bar{\tau} \gamma^\mu \tau,$$

where $g$ is the coupling constant. In Eq. (10) we have considered only the vectorial coupling for simplicity. The couplings of $Z'$ to other fermions are not given since they are irrelevant for the purpose of this paper.

After diagonalizing the lepton mass matrix by $U_l$ in Eq. (3), Eq. (10) becomes

$$\mathcal{L}^{\text{int}} = -g Z'_\mu \left[ c^2 \bar{\tau} \gamma^\mu \tau + s c \bar{\tau} \gamma^\mu \mu + s c \bar{\mu} \gamma^\mu \tau + s^2 \bar{\mu} \gamma^\mu \mu \right].$$

At one-loop level, the anomalous magnetic moment of the muon is generated as shown in Fig. 2(a) and Fig. 2(b). However compared with Fig. 2(a), contribution to muon $(g-2)$ from Fig. 2(b) is less suppressed by the mixing angles in the vertices and is enhanced by the tau lepton mass in the internal propagator, which gives rise to

$$\delta a_\mu = \frac{g^2}{(4\pi)^2} \frac{8 s^2 c^2 m_\tau m_\mu}{M_{Z'}^2} \int \left( \frac{m_\tau^2}{M_{Z'}^2} \frac{m_\mu^2}{M_{Z'}^2} \right),$$

$$I = \int_0^1 \frac{y(1-y)dy}{y^2 m_\mu^2/M_{Z'}^2 - y(1 - m_\tau^2/M_{Z'}^2 + m_\mu^2/M_{Z'}^2) + 1}.$$
\[ = \frac{1}{2} + O \left( \frac{m_{\tau}^2}{M_{Z'}^2}, \frac{m_{\mu}^2}{M_{Z'}^2} \right). \]  

(12)

Taking \( g^2 \leq 4\pi \) to make the perturbative calculation reliable and \( s^2 \sim 0.5 \) to maximize \( s^2c^2 \), we find that to fit muon (g-2) to be within 90\% C.L. of the experimental data, \( M_{Z'} \) has to be lighter than 2.6 TeV. With these input parameters of \( g, s \) and \( M_{Z'} \), the predicted rate of \( \tau \rightarrow \mu\gamma \) is \( 9.3 \times 10^{-7} \). The result holds unchanged for weakly interaction theory if the ratio of \( g^2/M_{Z'}^2 \) remains the same as of \( 4\pi/(2.6\text{TeV})^2 \). In this type of models, there are in general two sources which contribute to \( Z \rightarrow \tau\mu \). One is the radiative vertex correction by the \( Z' \) vector boson with the result given by

\[ Br(Z \rightarrow \tau\bar{\mu}) = Br(Z \rightarrow l\bar{l}) \times \left( \frac{g^2sc^2}{12\pi^2} \frac{M_Z^2}{M_{Z'}^2} \ln \left( \frac{M_{Z'}^2}{M_Z^2} \right) \right)^2. \]  

(13)

Taking \( g^2 \leq 4\pi, s^2 \sim 0.5 \) and \( M_{Z'} = 2.6 \text{ TeV} \), we find that the branching ratio of \( Z \rightarrow \tau\bar{\mu} \) is about \( 1.6 \times 10^{-9} \), which is well below the experiment limit \( 1.2 \times 10^{-5} \) \[12\]. If a \( Z-Z' \) mixing \( \theta_{Z'} \) is explicitly introduced, we will have \[4\]

\[ Br(Z \rightarrow \tau\bar{\mu}) = Br(Z \rightarrow l\bar{l}) \times \frac{2g^2\sin^2 \theta_{Z'} s^2 c^2}{g_Z^2(g_{ZL}^2 + g_{ZR}^2)}, \]  

(14)

where \( \theta_{Z'} \) is the \( Z-Z' \) mixing angle, \( g_Z, g_{(L,R)} \) are the \( Z \) coupling constants in the SM. Given the experimental limit on \( \theta_{Z'} \), \( Br(Z \rightarrow \tau\mu) \) is quite safe with the experimental constraint \[12\]. For example, taking \( g^2 \leq 4\pi, s^2 \sim 0.5 \) and \( \sin \theta_{Z'} \sim 10^{-3} \), Eq. (14) shows that the branching ratio of \( Z \rightarrow \tau\bar{\mu} \) is about \( 3 \times 10^{-6} \). A full study of the \( Z' \) physics on the electroweak observables depends on the detail of the model parameters. This goes beyond the scope of this paper.

FIG. 2. (a) One-loop \( Z' \) radiative correction to \( a_\mu \); (b) One-loop correction to \( a_\mu \) from lepton flavor violating couplings.
In summary, we have proposed in this paper a possible solution to the discrepancy of the muon \((g - 2)\) between the standard model prediction and the new experimental data, by introducing the new physics with the third family. Compared to other approaches to \(\delta a_\mu\) [2], ours is the only one which relates the new physics responsible for \(\delta a_\mu\) to another exciting possibility, \textit{i.e.} lepton flavor violation. Furthermore, in model with \(Z'\), our results show that \(M_{Z'}\) has to be lighter than 2.6 TeV.

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