Quantum-Mechanical Non-Perturbative Response of Driven Chaotic Mesoscopic Systems

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Consider a time-dependent Hamiltonian \(\mathcal{H}(Q, P; x(t))\) with periodic driving \(x(t) = A \sin(\Omega t)\). It is assumed that the classical dynamics is chaotic, and that its power-spectrum extends over some frequency range \(|\omega| < \omega_c\). Both classical and quantum-mechanical (QM) linear response theory (LRT) predict a relatively large response for \(\Omega < \omega_c\), and a relatively small response otherwise, independently of the driving amplitude \(A\). We define a non-perturbative regime in the \((\Omega, A)\) space, where LRT fails, and demonstrate this failure numerically. For \(A > A_{\text{pert}}\), where \(A_{\text{pert}} \propto \hbar\), the system may have a relatively strong response for \(\Omega > \omega_c\) due to QM non-perturbative effect. The shape of the response function becomes \(A\) dependent.

The \textit{wall formula} for the calculation of friction in nuclear physics \([1]\), and the \textit{Drude formula} for the calculation of conductivity in mesoscopic physics, are just two special results of a much more general formulation of ‘dissipation theory’ \([2,3]\). The general formulation of the ‘dissipation’ problem \([1]\) is as follows: Assume a time-dependent chaotic Hamiltonian \(\mathcal{H}(Q, P; x(t))\). For \(x = \text{const}\) the energy is constant of the motion. For non-zero \(V \equiv \dot{x}\) the energy distribution evolves, and the average energy increases with time. This effect is known as \textit{dissipation}. Ohmic dissipation means that the rate of energy absorption (‘heating’) is \(d\langle H \rangle/ dt = \mu V^2\), where \(\mu\) is defined as the dissipation coefficient. In case of periodic driving \(x(t) = A \sin(\Omega t)\), one should replace \(V^2\) by the mean square value \(\frac{1}{2} (A\Omega)^2\), and the dissipation coefficient \(\mu(\Omega)\) becomes frequency dependent. For simplicity we assume that conservative work is not involved in changing \(x\).

In case of the wall formula, \((Q, P)\) is a particle moving inside a chaotic ‘cavity’, and \(x\) controls the deformation of the boundary. Ohmic dissipation (in the sense defined above) implies a friction force which is proportional to the velocity, where \(\mu\) is the ‘friction coefficient’, and \(\mu V^2\) is the ‘heating’ rate. A mesoscopic realization of such system would be a quantum dot whose shape is controlled by electric gates. In case of the mesoscopic Drude formula, \((Q, P)\) is a charged particle moving inside a chaotic ‘ring’, and \(x\) is the magnetic flux through the hole in the ring. Ohmic dissipation implies Ohm law, where \(V \equiv \dot{x}\) is the electro-motive-force, \(\mu\) is the conductance, and \(\mu V^2\) is the ‘heating’ rate. For a mesoscopic realization of such system note that ring geometry is not important. One may consider a simple two dimensional quantum dot driven by a time-dependent homogeneous perpendicular magnetic field \([2]\). For the latter geometry it is better not to use the term conductance while referring to the dissipation coefficient \(\mu\).

In the general analysis of the ‘dissipation’ problem one argues that due to the driving there is diffusion in energy space. This diffusion process is biased because of its \(E\) dependence, leading to systematic increase of the average energy. This is the reason for having dissipation. Therefore we find convenient from now on to consider the diffusion coefficient \(D_{\text{e}}\) as the object of our study. The relation between \(d\langle H \rangle/ dt\) and \(D_{\text{e}}\) constitutes a generalization of the so called fluctuation-dissipation relation.

\[
\begin{align*}
\text{QM-adiabatic} & \quad \text{linear-response} & \quad \text{non-perturbative} \\
\text{non-perturbative regime} & \quad \text{linear-response regime} & \\
A_{\text{pert}} & \quad A_{\text{c}} & \\
\Delta/h & \quad \omega_c \Delta & \\Omega
\end{align*}
\]

Fig. 1. Upper diagram: The various \(V\) regimes in the theory of quantum dissipation for linear driving \(x(t) = V t\). Lower diagram: The various \((\Omega, A)\) regimes for periodic driving \(x(t) = A \sin(\Omega t)\). Note the analogy with Fig. 5 of Ref. \([5]\) with \(x \leftrightarrow A\) and \(V \leftrightarrow A\Omega\). The QM-adiabatic regime (including the regime \(A < A_{\text{c}}\), but excluding the narrow stripes of QM-resonances) is defined by having \textit{vanishing} first-order probability to go to other levels. See the text for further explanations and definitions of \(A_{\text{c}}\) and \(A_{\text{pert}}\).

Ohmic dissipation is implied if \(D_{\text{e}} \propto V^2\), or \(D_{\text{e}} \propto A^2\) in case of periodic driving. Such behavior can be established within the framework of classical mechanics \([3]\) using general classical considerations \([4]\). The classical formulation of the dissipation problem \([2]\) can be regarded as a systematic scheme that justifies the use of classical ‘linear response theory’ (LRT). The precise conditions for the applicability of the classical LRT result are further discussed in \([4]\), and we are going to mention later on what we call ‘the trivial slowness condition’. We are interested in the \textit{quantum mechanical} (QM) theory of dissipation. The traditional derivation \([5]\) of QM LRT leads formally to the same result as in the classical analysis \([3]\). Therefore from now on we no longer distinguish
between the ‘classical’ LRT result and the ‘quantal’ LRT result and use just the term ‘LRT result’. As a matter of terminology, it should be noted that the QM formulation of LRT, also known as Kubo-Greenwood formalism, is completely equivalent to the well known Fermi golden rule (FGR) picture.

So let us assume that the obvious classical conditions for the validity of the LRT result are satisfied. Now the question is whether, upon quantization, there are additional $\hbar$-dependent conditions for the applicability of the LRT result. In the traditional quantum mechanical literature, as well as in the recent mesoscopic literature, the focus is on the consequences of having finite mean level spacing $\Delta$. This leads to the identification of the QM-adiabatic regime (extremely slow driving), and to the discussion of either the Landau-Zener mechanism or else the Debye relaxation absorption mechanism for dissipation, as well as to the discussion of QM-resonances. The main observation of \cite{12} is that there is another regime, the non-perturbative regime (see Fig.1), where QM LRT is not valid. As strange as it sounds, this does not imply a failure of the LRT result. On the contrary, another observation of \cite{12} is that the regime where the classical approximation applies, is well-contained in the non-perturbative regime, hence the LRT result becomes valid again because of quantal-classical correspondence (QCC) considerations. However, if the system does not have a good classical limit (as in RMT models) this ‘recovery’ of the LRT result is not guaranteed. Moreover, as we are going to discuss later, QCC consideration cannot exclude the possibility of having a relatively large quantal non-perturbative response whenever the LRT result is small in comparison.

The outline of this letter is as follows: (1) We extend the theoretical considerations of \cite{12} to the case of periodic driving. (2) We give a specific example where the LRT result fails because of a quantal non-perturbative effect. (3) We comment on the issue of localization. (4) We discuss the role of QCC considerations in the theory. Based on the theoretical considerations, the reader should realize that the existence of the non-perturbative regime is not related to having finite mean level spacing $\Delta$, but rather to having finite bandwidth $\Delta_b = \hbar \omega_{cl}$, where $\omega_{cl}$ is the dropoff frequency of the LRT response.

In the context of mesoscopic physics this bandwidth is known as the Thouless energy.

Given $H(Q,P;x)$ with $x = \text{const}$, we can define a fluctuating quantity $F(t) = -\partial H/\partial x$. The autocorrelation function of $F(t)$ will be denoted by $C(\tau)$. The power spectrum $\tilde{C}(\omega)$ is defined as its Fourier transform. The intensity of fluctuations is defined as $\nu = \tilde{C}(0)$, and it is convenient to define the correlation time as $\tau_c = \tilde{C}(0)/\tilde{C}(0)$. We assume for simplicity of presentation that the single time scale $\tau_c$ completely characterizes the chaotic dynamics of the system: The power spectrum of the chaotic motion is assumed to be continuous, and it is non-vanishing up to the cutoff frequency $\omega_{cl} = 2\pi/\tau_c$. We assume that $\tilde{C}(\omega)$ is vanishingly small for $\omega > \omega_{cl}$.

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**Fig.2.** The response of a quantum mechanical system is displayed as a function of $A$ and $\Omega$. The evolution is determined by the WBRM model Eq.\cite{11}. The units of energy and time and amplitude are chosen such that $\Delta = 0.5$ and $\hbar = 1$ and $\sigma = 1$ respectively. **Left:** Plots of $D_E/A^2$ versus $\Omega/\omega_{cl}$ for few values of $A$. For small $\omega$ the plots coincide as expected from Eq.\cite{11}. As $A$ becomes larger the deviations from Eq.\cite{11} become more pronounced, and we get response also for $\Omega > \omega_{cl}$. **Right:** Plots of $D_E/D_0$ versus $A/\sqrt{b}$ for few values of $\Omega/\omega_{cl}$. The LRT result Eq.\cite{11} implies $D_E/D_0 = 1$ for $\Omega/\omega_{cl} < 1$ and $D_E/D_0 = 0$ for $\Omega/\omega_{cl} > 1$. The purpose of the horizontal scaling is to demonstrate that $A_{\text{min}}$, rather than $A_c$, is responsible for the deviation from this LRT expectation. Each ‘point’ in the above plots is determined by a simulation that involves typically 35 realizations of the evolution until we start to see saturation due to dynamical localization effect. The typical time step is $dt = 10^{-4}$. In each step we verify that the normalization is preserved to an accuracy of 0.01%. In order to eliminate finite size effects we have used a self-expanding algorithm. Namely, additional 10$b$ sites are added to each edge whenever the probability in the edge sites exceeds $10^{-15}$. The diffusion coefficient is determined from the fitting $\delta E(t)^2 = \text{const} \times t^\beta$. For sub-diffusive behavior ($\beta < 0.86$) we set $D_E = 0$. The preparation of each ‘point’ in the above plots requires $\sim 4$ CPU days on Alpha XP1000 machine.
Consider the time dependent case \( x(t) = A \sin(\Omega t) \). The LRT result for the diffusion in energy is

\[
D_E = \frac{1}{2} \tilde{C}(\Omega) \times \frac{1}{2}(A\Omega)^2
\]

and we shall use the notation \( D_0 = \frac{1}{2} \nu(\Omega)^2 \). The most transparent QM derivation of this result is based on the FGR picture. The energy levels of the systems are \( E_n \), and the mean level spacing is \( \Delta \). The Heisenberg time is \( t_{\text{hi}} = 2\pi \hbar/\Delta \). The transitions between levels are determined by the coupling matrix elements \( (\partial H/\partial x)_{nm} \). It is well known [3] that for reasonably small \( n \) this matrix is a banded matrix. The bandwidth is \( \Delta_b = 2\pi \hbar/\tau_{\text{ci}} \), and the variance of the in-band elements is \( \sigma = \nu/\tau_{\text{hi}} \). It is common to define the QM system using the four parameters \((\Delta, b, \sigma, \hbar)\) where \( b = \Delta_b/\Delta \). It is also useful to regard the semiclassical relations \( \tau_{\text{ci}} = 2\pi \hbar/(b\Delta) \) and \( \nu = (2\pi \hbar/\Delta)^2 \) as definitions, whenever the classical limit is not explicitly specified [as in Random Matrix Theory (RMT) models]. The FGR picture implies strong response if and only if \( \hbar \Omega < \Delta_b \), leading to \( \Omega < \omega_{\text{ci}} \), as in the classical case. Using the FGR picture it is straightforward to recover \( D_0 = (\pi/2)(\hbar/\Delta)(\sigma A\Omega)^2 \) in agreement with Eq. (1).

The \textit{trivial slowness condition} for the applicability of classical LRT [6] is \( V \tau_{\text{ci}} < \delta x_{\text{ci}}^2 \). Here \( \delta x_{\text{ci}}^2 \) is the parametric change that leads to the breakdown of the linearization of \( H(Q, P; x + \delta x) \) with respect to \( \delta x \). Upon quantization there are two other parametric scales that become important [6], namely \( \delta x_{\text{cm}} \) and \( \delta x_{\text{prt}} \). The former is the parametric change which is required in order to mix neighboring levels, while the latter is the parametric change required in order to mix all the levels within the band. Hence we define

\[
A_c \equiv \delta x_{\text{cm}} = \frac{\Delta}{\sigma} \propto \hbar^{(1+d)/2}
\]

\[
A_{\text{prt}} \equiv \delta x_{\text{prt}} = \frac{\sqrt{\Delta}}{\sigma} = \frac{2\pi \hbar}{\sqrt{\hbar^2 \tau_{\text{ci}}}}
\]

The above parametric scales, of time-independent first-order perturbation theory (FOPT), manifest themselves also in the time-dependent analysis. FOPT gives the following result for the probability to make a transition from an initial level \( m \) to some other level \( n \),

\[
P(t|m) = \left| \frac{1}{\hbar} \left( \frac{\partial H}{\partial x} \right)_{nm} \int_0^t x(t') \exp \left( i \left( E_n - E_m \right) t' \right) \frac{dt'}{\hbar} \right|^2
\]

The total transition probability is \( p(t) = \sum_n P(t|m) \) where the prime imply omission of the \( n = m \) term. In the regime \( \Omega < \omega_{\text{ci}} \) one obtains

\[
p(t) = \frac{1}{\hbar^2} \nu A^2 \times \left\{ \begin{array}{ll}
(\Omega^2/\tau_{\text{ci}}) \frac{t^4}{4} & \text{for } 0 < t < \tau_{\text{ci}} \\
(\Omega/\tau_{\text{ci}})^2 \frac{t^3}{3} & \text{for } \tau_{\text{ci}} < t < 1/\Omega \\
\frac{1}{2}t^2 & \text{for } 1/\Omega < t < t_{\text{hi}}
\end{array} \right.
\]

while in the regime \( \Omega > \omega_{\text{ci}} \) one obtains

\[
p(t) = \frac{1}{\hbar^2} \nu A^2 \times \left\{ \begin{array}{ll}
(1 - \cos(\Omega t))^2 & \text{for } 0 < t < \tau_{\text{ci}} \\
3/2 & \text{for } \tau_{\text{ci}} < t < t_{\text{hi}}
\end{array} \right.
\]

In both cases for \( t > t_{\text{hi}} \) we have recurrences, and therefore \( p(t) < p(t_{\text{hi}}) \). [One should be more careful near resonances: There \( t_{\text{hi}} \) should be replaced by \( 2\pi \hbar/\delta \) where \( \delta \) is the detuning]. The necessary condition for applicability of FOPT at time \( t \) is that \( p(t') < 1 \) for any \( t' < t \), which can be written as \( p([0, t]) < 1 \). The necessary condition for the applicability of the FGR picture is \( p([0, \tau_{\text{ci}}]) < 1 \). This is the FGR condition [6] which guarantees the separation of time scales \( \tau_{\text{ci}} \ll \tau_{\text{prt}} \). The FOPT breake time \( \tau_{\text{prt}} \) is defined as the maximal \( t \) for which \( p([0, t]) < 1 \). Now we can define a \textit{non-perturbative regime} by the requirement \( p([0, \tau_{\text{ci}}]) > 1 \). It is straightforward to observe that the non-perturbative regime is contained in the region \( A > A_{\text{prt}} \), where

\[
\left( \frac{A_{\text{prt}}}{A} \right) \omega_{\text{ci}} < \Omega < \left( \frac{A}{A_{\text{prt}}} \right) \omega_{\text{ci}}
\]

The location of the non-perturbative regime is illustrated in Fig.1. For completeness of presentation we have also indicated the subregion in \( \Omega < \omega_{\text{ci}} \) where we have first-order response equal to zero. The condition is \( p([0, \infty]) < 1 \) or equivalently \( p(t_{\text{hi}}) < 1 \). This region contains the QM adiabatic regime, including the region \( A < A_{\text{ci}} \), but excluding the narrow stripes of resonances.

We have defined the location of the non-perturbative regime, but we did not yet give a suggestion how Eq. (4) should be modified. Using RMT assumptions with regard to \( (\partial H/\partial x)_{nm} \), and inspired by related studies of wavepacket dynamics [8], we expect the result

\[
D_R = (C/\sqrt{v_{\text{PR}}}) \times D_0
\]

where \( v_{\text{PR}} = V/(\delta x_{\text{prt}}/\tau_{\text{ci}}) \), and \( C \) is a numerical constant. A detailed derivation of Eq. (4) will be presented in the future. (The crucial step is to argue that at \( \tau_{\text{ci}} \), there is a crossover from ballistic behavior to diffusion in the sense of Eq. (4)). For periodic driving this result should be averaged over a period leading to \( D_R \propto A^{2-\alpha} \) with \( \alpha = 1/2 \).

We wanted to give a numerical example that demonstrate the non-perturbative response effect. Evidently, the simplest is to consider a time-dependent version of Wigner’s banded random matrix (WBRM) model,

\[
H = E_0 + x(t) \mathbf{B}
\]

where \( E_0 \) is an ordered diagonal matrix, and \( \mathbf{B} \) is a banded matrix. This model [4,6] is characterized by the parameters \((\Delta, b, \sigma, \hbar)\) which we have defined previously. For the numerical experiment we have assumed rectangular band profile such that all the elements \( 0 < |n - m| \leq b \) are taken from the \textit{same} distribution, and outside the band all the elements are identically zero. The results of the simulations are summarized in Fig.2. It should be realized that WBRM model Eq. (4) has a big disadvantage. Namely, unlike the physical examples of the introduction, the statistical properties of
the model are not invariant for \( x(t) \mapsto x(t) + \text{const.} \) One may wonder why we do not use one of the two other popular variations of Wigner model \([10,11]\). eg \( \mathcal{H} = E + (\cos x)B_1 + (\sin x)B_2 \). The problem is that for these models one obtains \( \delta x_\text{pert} \sim \delta x_\text{cl}^0 \sim 2\pi \). Therefore there is no regime there where LRT fails because of quantal non-perturbative effect\(^{\dagger}\). (Such failure requires the generic separation of scales \( \delta x_\text{pert} \ll \delta x_\text{cl}^0 \).) Thus it seems that the only way to make a RMT model \( x\text{-invariant} \), is to keep it ‘perturbative’ in nature. The lack of \( x\)-invariance in the standard WBRM model complicates the calculation of the period-averaged \( D_\text{cr} \) and leads to \( D_\text{cr} \propto A^{2-\alpha} \) with \( \alpha \) changing gradually from \( 1/2 \) to \( 1 \). The numerical analysis of Fig.2b fits well to \( \alpha \sim 3/4 \).

There are two types of localization effects that we had to consider in our numerical experiments. The “WBRM model localization” can be avoided by using amplitudes \( A < b^{1/2}(\Delta/\sigma) \) in order to guarantee that the instantaneous eigenstates of Eq.\([\bar{h}]\) are not localized at any time. The “dynamical localization effect” on the other hand cannot be avoided. It is associated with the periodic nature of the driving. Extending standard argumentation one observes that the eigenstates of the (one period) Floquet operator have localization length \( \xi \times \Delta \), and that the associated breaktime is \( t^* = \xi \times (2\pi/\Omega) \). The two must be related by \( 2D_\text{cr}t^* = (\xi \Delta)^2 \) leading in the (LRT regime) to the result \( t^* = 2\pi^2(A/A_\text{cr})^2 \times t_0 \). In all our numerical experiments the diffusion has been determined for times when dynamical localization is not yet apparent. Another possibility, which we have not used, is to add a small noisy component to the driving, such as to mimic the typical experimental situation of having dephasing time much shorter than \( t^* \).

It is not obvious that the non-perturbative behavior that is implied by RMT assumptions, and applies to RMT models, should apply also to Hamiltonians that possess a well defined classical limit. On the contrary, the same considerations as in \([10,11]\) can be applied in order to argue that RMT considerations are not compatible with the QCC principle. Here we are going to explain the main idea, and to define our expectations.

Taking \( \hbar \) to be very small, it is obvious that eventually we shall find ourselves in the regime where \( A \gg A_\text{cr} \). Let us consider the dynamics during a specified time interval \( 0 < t' < t \). The time \( t \) is chosen to be much larger than \( \tau_\text{cl} \). On the basis of QCC considerations, we should be able to make \( \hbar \) sufficiently small such that the quantum evolution becomes similar to the classical evolution up to the time \( t \). The classical analysis implies that during this time the stochastic behavior is established. Therefore having detailed QCC during the time \( t \) implies that the quantal \( D_\text{cr} \) can be approximated by the classical result. This leads to a contradiction with the RMT prediction Eq.\([\bar{h}]\) in the domain \( \Omega < \omega_\text{cl} \), but \( \text{not} \) in the domain \( \Omega > \omega_\text{cl} \). We are going to further explain this last point.

Denote the energy dispersion by \( \delta E_\text{qm}(t) \), and the corresponding classical result by \( \delta E_\text{cl}(t) \). For sufficiently small \( \hbar \) it should be possible to make a leading order approximation \( \delta E_\text{qm}(t) \approx \delta E_\text{cl}(t) + \hbar^\gamma g(t) \), with \( \gamma > 0 \). In the \( \Omega < \omega_\text{cl} \) regime the first term in this approximation is dominant. On the other hand for \( \Omega > \omega_\text{cl} \) the first term gives a vanishingly small result for \( D_\text{cr} \). Therefore, without any contradiction with QCC considerations, the second term becomes important. Therefore we may have in principle an enhanced quantal response for \( \Omega > \omega_\text{cl} \).

In conclusion, we have defined a non-perturbative regime in the \((\Omega, A)\) plane, where LRT cannot be trusted. We have demonstrated an actual failure of LRT for a particular (RMT) Hamiltonian. We believe that for generic chaotic systems the RMT mechanism for diffusion competes with the classical mechanism. The actual response of the system is expected to be determined by the predominant mechanism. The study of this conjecture is the theme of our future studies.

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\(^{\dagger}\) The most serious attempt to challenge LRT has been done in \([\bar{h}]\). However, in view of a later work \([\bar{h}]\), and as explained in \([\bar{h}]\), the suppression of diffusion for large \( V \) in those studies follows from the violation of the trivial slowness condition \( V \tau_\text{cl} \ll \delta x_\text{cl}^0 \).

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