Joule-Thomson expansion of higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity

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In this paper, the Joule-Thomson expansion of the higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity is investigated by considering the cosmological constant as the thermodynamic pressure. First, according to the thermodynamic quantities of the black hole in extended phase space, we derive the Joule-Thomson coefficient $\mu_{BH}$, the equation of the inversion curve, the ratio $\eta_{BH}$ between the minimum of inversion temperature and the critical temperature, and the equation of the isenthalpic curves. Then, the influence of various parameters of the black hole on the aspects characteristic of the Joule-Thomson expansion are analyzed via numerical method. The results show the $\mu_{BH}$ has a zero point and a divergent point in the $\mu - r_+$ plane, which are coincide the inversion temperature $T_i$ and the zero point of Hawking temperature, respectively.

I. INTRODUCTION

By introducing the quantum mechanism into general relativity, Hawking and Bekenstein demonstrated that black holes have temperature and entropy [1-3]. After that, people have established the complete theories of black hole thermodynamics. According to those theories, the thermodynamic properties of a variety of complicated black holes were explored in the past few decades [4-11]. Among all, the anti-de Sitter (AdS) black holes have attracted people’s attention for their strange thermodynamic properties. Hawking and Page first pointed out the existence of a thermodynamic phase transition in Schwarzschild-AdS spacetime [12]. This heuristic work shows the deeper-seated relation between confinement and deconfinement phase transition of gauge field in the AdS/CFT correspondence, and aroused people’s interest in the study of the phase transition of black holes. Within this context, a series of works have been done to investigate the common properties between AdS spacetimes and general thermodynamic system. In Refs. [13, 14], Chamblin et al. proposed that the phase structures of Reissner-Nördstrom(R-N) AdS black hole is similar as that of van der Waals system. Furthermore, by interpreting the cosmological constant $\Lambda$ and mass $M$ as the thermodynamic pressure $P$ and chemical enthalpy $H$, receptively [15-17], the implications of black hole thermodynamics have been investigated in many contexts. In the extended phase space, Kubizňák and Mann obtained the $P - V$ critical behavior of R-N AdS black hole and demonstrate that they also coincide with those of the Van der Waals fluid [18]. In Ref. [19], Johnson showed the AdS black holes can be considered as holographic heat engines, which outputs work from the cycle in a pressure-volume space. Besides, Dolan found that the black hole with the non-positive cosmological constant has no adiabatic compressibility [20, 21].

Apart from works that we mention in above, the black hole thermodynamics recently have been extended to the regime of Joule-Thomson expansion [22, 23]. In the van der Waals system, the Joule-Thomson expansion occurs when the temperature change of non-ideal gas from the high-pressure zone through the porous plug into the low-pressure zone, and the enthalpy, which can be used to defined the non-equilibrium states of expansion, remains same in this process. Based on the viewpoints of Refs. [16, 24, 25], Okci and Aydner showed that the R-N AdS black hole and Kerr AdS have the similar Joule-Thomson expansion process with the van der Waals fluid [26, 27]. Subsequently, the Joule-Thomson expansion of D-dimensional charged AdS spacetimes, charged Gauss-Bonnet black hole, Bardeen-AdS black hole were obtained in Refs. [28-34].

On the other hand, an interest in the subject of higher dimensional nonlinearly black hole has been growing. It is well known that the linear electrodynamics would fails when the electromagnetic field is too strong. To crack down this problem, Born and Infeld introduced a class of non-linear electrodynamics by eliminating the infinite self-energy. Moreover, for keeping the conformal symmetry of Maxwell action in higher dimensions, one proposed another non-linear electrodynamics model, i. e. the power Maxwell invariant (PMI) theory [35, 36]. In this theory, the PMI field Lagrangian density can be ex-
pressed as \( L_{\text{PMI}} = (-F)^s = (-F_{\mu \nu} F^{\mu \nu})^s \) with an arbitrary rational number \( s \). By introducing the PMI with the astrophysical frameworks, many new results are obtained [37–40]. Recently, the black hole solutions Einstein gravity coupled to the PMI theory have attracted people’s attention. In Refs. [41–43], the thermodynamic properties of those black holes have been analyzed in detail. In particular, in the extended phase space, it is found that the phase structure and the critical behavior of higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity is similar to those of Van der Waals fluid [44]. Due to the above discussion, it is believe that the higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity has Joule-Thomson expansion process. Therefore, we calculate Joule-Thomson coefficient, the equation of the inversion curve and the equation of the isenthalpic curves in this paper. Meanwhile, we also use the numerical method to analyze the influence of various parameters of the black hole on Joule-Thomson expansion.

The paper is organized as follows. In the next section, we review the metric of properties of nonlinearly charged AdS black hole and its thermodynamic properties. Section III is devoted to derive the Joule-Thomson coefficient, the ratio between the minimum of inversion temperature and the critical temperature, the equation of the inversion curve and the equation of the isenthalpic curves. Then, we investigate the influence of various parameters of the black hole on Joule-Thomson expansion via numerical method. The conclusion and discussion are contained in Section IV. This research takes the units \( G = c = k_B = 1 \).

II. THE HIGHER DIMENSIONAL NONLINEARLY CHARGED ADS BLACK HOLE IN EINSTEIN-PMI GRAVITY IN EINSTEIN-PMI GRAVITY

A. The metric of higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity

To begin with, it is necessary to review the properties of nonlinearly charged AdS black hole in Einstein-PMI gravity. In the \( D \)-dimensional \((D \geq 4)\) AdS spacetimes, the bulk action of Einstein-PMI gravity can be written in the form [35, 36]:

\[
I = -\frac{1}{16\pi} \int d^D x \sqrt{-g} \left[ R - 2\Lambda + (-F_{\mu \nu} F^{\mu \nu})^s \right],
\]

where \( R \) is the Ricci scalar, \( \Lambda = -(D - 1) (D - 2)/2L^2 \) is the cosmological constant with the radius of the AdS space \( L \), \( F_{\mu \nu} = \partial_{\mu} A_\nu - \partial_{\nu} A_\mu \) represents the strength of the electromagnetic field, and \( s \) is the nonlinearity parameter. Based on the Eq. (1), the line element of a \( D \)-dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity takes the form [44]

\[
ds^2 = -f(r) \, dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2}^2,
\]

where \( d\Omega_{D-2} \) denotes the line element of a \((D - 2)\)-dimensional space. When considering the field equations arising from the variation of the bulk action with the metric (2), the metric function \( f(r) \) and gauge potential \( A \) are given by:

\[
f(r) = 1 - \frac{m}{r^{D-3}} + \frac{r^2}{f} - \frac{(2s - 1)^2 ((D-2)(2s-D+1)q^2)^s}{(D-2)(2s-D+1)r^2(\frac{2s-D+1}{2s-D+1})},
\]

\[
A = -\sqrt{\frac{D-2}{2(D-3)}} q^r_{2s-1} \frac{d}{f} dt,
\]

where the electromagnetic field 1-form is \( F = dA \). Notably, in order to keep the nonlinear term of the source, it requires \( s > 1/2 \) and \( s \neq (D - 1)/2 \). The parameters \( m \) and \( q \) are related to the ADM mass \( M \) and total electric charge \( Q \) of black hole respectively, which reads

\[
M = \frac{\omega_{D-2}(D-2) m}{16\pi},
\]

\[
Q = \frac{\omega_{D-2}\sqrt{2}(2s - 1)s}{8\pi} \left( \frac{D-2}{D-3} \right)^{s-\frac{1}{2}} \left[ \frac{(D-1-2s)q}{2s-1} \right]^{2s-1},
\]

with the volume of a unit \((D - 2)\) sphere \( \omega_{D-2} = 2\pi^{\frac{D-1}{2}} \Gamma [(D - 1)/2] \). According to Eq. (5), metric function (3) can be rewritten as follows:

\[
f(r) = 1 - \frac{\omega_{D-2}(D-2) m}{r^{D-3} \omega_{D-2}} + \frac{16P r^2}{(D-1)(D-2)}
\]

\[
+ \frac{(1-2s)^2}{(D-2)(1-D+2s)} \right)^{\frac{2s-D+1}{2s-D+1}} \Theta,
\]

where

\[
\Theta = \left\{ \begin{array}{cl}
\frac{2^s}{2s} - \frac{1}{2s} & \frac{s}{2s} \left( \frac{(D-2)}{D-3} \right) + \left( \frac{1}{D-3} \right) d \frac{Q}{s (2s-1) \omega_{D-2}} \right)^{\frac{s}{2s}}
\end{array} \right\},
\]

and \( P = -\Lambda/8\pi = -(D-1)(D-2)/16\pi L^2 \) is a thermodynamic pressure, which is a key definition in the framework of black hole chemistry.

B. The thermodynamics of higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity

On the event horizon \( r_+ \), one can rewrite the ADM mass in terms of the \( P \) with the condition \( f(r) \big|_{r=r_+} = 0 \),
which read as follows:
\[
M = -\frac{r_+^{D-3} \omega_{D-2}}{8\pi} \left[ 1 - \frac{D}{2} - \frac{8P\pi r_+^2}{D - 1} \right]
+ \frac{(1 - 2s)^2 \Theta}{2(1 - D + 2s)} r_+^{2 + 2s(2 - D)/D - 1}. \tag{8}
\]
Furthermore, the Hawking temperature of the black hole can be easily obtained as
\[
T = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi} \left[ \frac{D - 3}{r} + \frac{16P\pi r}{D - 2} \right]
- \frac{(2s - 1) \Theta}{D - 2} r_+^{1 + 2s(2 - D)/D - 1}, \tag{9}
\]
and the entropy is
\[
S = \int_0^{r_+} \frac{1}{T} \left( \frac{\partial M}{\partial r} \right) d\phi = \frac{\omega_{D-2}, D-2}{4r_+^{D-2}}. \tag{10}
\]
It is clear that the entropy of black hole obeys the area formula \( S = A/4 \). Based on above thermodynamics quantities, the first law of black hole thermodynamics in the extended phase space is given by
\[
dM = TdS + VdP + \Phi dQ, \tag{11}
\]
where \( \Phi \) is the electric potential, and \( V = (\partial M/\partial P)_S = \omega_{D-2} r_+^{D-1}/(D - 1) \) is the thermodynamic volume, which corresponding conjugate quantity is the thermodynamic pressure \( P \) [45]. The connected Smarr relation becomes
\[
(D - 3) M = (D - 2) TS - 2PV + (D - 3) \Phi Q. \tag{12}
\]
Next, substituting Eq. (8) into Eq. (9), one get the equation of state
\[
P = -\frac{(D - 2)(D - 3)}{16\pi r_+^2} + \frac{(D - 2) T}{4r_+} + \frac{(2s - 1) \Theta}{16\pi} r_+^{1 + 2s(2 - D)/D - 1}. \tag{13}
\]
According to the viewpoints in Ref. [18], the critical points of this thermodynamic system can be obtained by the conditions \( (\partial P/\partial r_)+)_{T=Tcr} = (\partial^2 P/\partial r_+^2)_{T=Tcr} = 0 \), which gives the critical temperature
\[
T_{cr} = \frac{4(D - 3)(D - 4s + 1)}{\pi(D - 2)(2Ds - 6s + 1)} \times \left[ \frac{ks(2D - 6s + 1) q_s^2}{16(D - 3)(2s - 1)^2} \right]^{1/2 + 2s(2 - D)/D - 1}, \tag{14}
\]
where
\[
k = \frac{16^{(D - 2)} (2s - 1)^{(D - 2)(2s - 1)}/(2s - 1)^2}{(D - 2)^{2 + 2s(2 - D)/D - 1}}. \tag{15}
\]
Here we only focus on the critical temperature since it will be used to analyze the Joule-Thomson expansion of the black hole in the next section. The expressions critical pressure and the critical radius can be found in Ref. [44] if needed. From Eq. (9)-Eq. (15), one can see that the thermodynamic quantities of the higher dimensional nonlinearly charged AdS black hole sensitively depends on the event horizon radius \( r_+ \), the spacetime dimension \( D \), the ADM mass \( M \), the total electric charge \( Q \) and the nonlinearity parameter \( s \). However, those thermodynamic quantities can easily go back to the higher dimensional R-N AdS case when \( s = 1 \).

### III. JOULE-THOMSON EXPANSION OF HIGHER DIMENSIONAL NONLINEARLY CHARGED ADS BLACK HOLE IN EINSTEIN-PMI GRAVITY

In this section, the Joule-Thomson expansion of higher dimensional nonlinearly charged AdS black hole in the extended phase space is investigated. In the Joule-Thomson expansion, the enthalpy \( H \) of the van der Waals system can be used to defined the non-equilibrium states. Meanwhile, the enthalpy is related the temperature and pressure of the system [22, 23]. Hence, one can find a slope of an isentropic curve in the \( T - P \) plane, that is, the Joule-Thomson coefficient. According to Ref. [26], the Joule-Thomson coefficient is denoted as follows:
\[
\mu = \left( \frac{\partial T}{\partial P} \right)_H = \frac{1}{C_p} \left[ P \left( \frac{\partial V}{\partial T} \right)_P - V \right], \tag{16}
\]
where \( C_p = P(\partial S/\partial T)_P \) is the heat capacity at constant pressure. For \( \mu > 0 \), one has a cooling region in the \( T - P \) plane, whereas a heating region appears for \( \mu < 0 \). Moreover, when Joule-Thomson coefficient vanishes, one can obtain the inversion temperature \( T_i = V(\partial T/\partial V)_p \). In Ref. [44], the phase structure of higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity is analogous to that of van der Waals system. Thus, it is interesting to investigate the throttling process of the higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity. Now, substituting the thermodynamic quantities of the black hole into Eq. (16), one yields
\[
\mu_{BH} = \frac{4r_+ \left[ 16P\pi r_+^2 + (D - 3) D - \Theta (4s - 1) r_+^{2 + 2s(2 - D)/D - 1} \right]}{(D - 1) \left[ 6 + (D - 5) D + 16P\pi r_+^2 - \Theta (1 - 2s) r_+^{2 + 2s(2 - D)/D - 1} \right]. \tag{17}
\]
It is obvious that the Joule-Thomson coefficient (17) is sensitive to the properties of spacetime of the black hole. By fixing the pressure \( P \), the behaviors of Joule-Thomson coefficient \( \mu_{BH} \) and Hawking temperature \( T \) for...
different dimension $D$, the charge $Q$ and the nonlinearity parameter $s$ are shown in Fig. 1.

From Fig. 1(a)-Fig. 1(c), one can see that the general behavior of $\mu_{\text{BH}}$ changing with $r_+$ as follows: when event horizon enough large, $\mu_{\text{BH}}$ is greater than zero. However, by decreasing the event horizon of the black hole, the Joule-Thomson coefficient reaches zero (i.e. $T_i$), and then goes to negative. When comparing Fig. 1(a)-Fig. 1(c) with Fig. 1(d)-Fig. 1(e), it is found that the Joule-Thomson coefficient diverges at the points where the Hawking temperature becomes zero. Finally, the $\mu_{\text{BH}}$ decreases to zero as $r_+ \to 0$. Besides that, the curves of Joule-Thomson coefficient move to the right as the dimension or the charge increases, while the curves move to the left as the nonlinearity parameter $s$ increases. Thus, the inversion temperature $T_i$ can be raised by increasing $D$ and $Q$, or reducing $s$.

Next, the definition of $T_i$ with aid of Eq. (9) at $P = P_i$ directly lead to the parameter equation of the inversion curve as follows: [46]

$$
\begin{align*}
T_i &= \frac{1}{2\pi r_+(D-2)} \left[ 3 - D + s r_+^{2+2/(D-4)} \frac{3+2s}{(4s-1)} \right], \\
P_i &= \frac{1}{10\pi r_+^2} \left[ (3-D) + r_+^{2+2/(D-4)} (4s-1) \right] .
\end{align*}
$$ (18)

For further investigation of the inversion curve of higher dimensional nonlinearly charged AdS black hole, we plot Fig. 2.

Fig. 2 illustrates the inversion temperature $T_i$ associated with inversion pressure $P_i$ for different dimension spacetimes $D$, the charge $Q$ and the nonlinearity parameter $s$, respectively. One can see that $T_i$ increases monotonously with $P_i$. It leads to each curve in diagram only exist one minimum value of inversion temperature $T_{i\text{min}}$, and the cooling region and the heating region are located above and below these curves, respectively, which are different from that of van der Waals fluids system. Moreover, the behaviors of inversion curves for high pressures are reverse to those for low pressure (as shown in the lower box). At high pressures, they increase as the dimension spacetimes and the nonlinearity param-
eter decrease, or the charge increase. However, for low pressures, $T_i$ increase as the spacetimes dimension and the nonlinearity parameter increase, or the charge decrease. Therefore, the spacetimes dimension, the charge and the nonlinearity parameter are all affect the slope of inversion curves.

Now, by demanding $P_i$ of Eq. (18) equals to zero, the roots are given by

$$r_{min} = \left[ \frac{(4s - 1) \Theta}{D(D - 3)} \right]^{2^{2s-1} (D^2 - 4s)} , r'_{min} = \left[ -\frac{(4s - 1) \Theta}{D(D - 3)} \right]^{2^{2s-1} (D^2 - 4s)} .$$

It worth note that $r'_{min}$ should be neglected since it always negative, and also avoid $1 - (4 - D) s = 0$. Substituting

FIG. 2. The inversion curves for various combinations of $D, Q$ and $s$. 

(a) $D = 4, s = 1.1$
(b) $D = 5, s = 1.1$
(c) $D = 6, s = 1.1$

(d) $D = 4, s = 1.2$
(e) $D = 5, s = 1.2$
(f) $D = 6, s = 1.2$

(g) $D = 4, s = 1.3$
(h) $D = 5, s = 1.3$
(i) $D = 6, s = 1.3$
show that the numerator of the critical temperature decrease as $s$ and $D$ increase, which means that the denominator $T_{cr}$ is grows faster than the numerator $T_{min}$. Furthermore, as seen from Fig. 3, one can find that the ratio difference between adjacent $s$ values is getting smaller and smaller, and the curve of $\eta$ approaches an constant as $D \to \infty$.

Finally, considering the Joule-Thomson expansion is an isenthalpic process, it is interesting to investigate the isenthalpic curves in $T-P$ plane. According to Eq. (8) and Eq. (13), the equation of the isenthalpic curves can be expressed as follows:

\[
\begin{align*}
T &= \frac{1}{2\pi r_+} + \frac{4M (D-1)}{(D-2) \omega_{D-2} r_+^{D-2}} + \frac{\frac{1+2(D-3)s}{2}}{2\pi (1-D+2s)} (2s-1)^{1+2s} \\
P &= \frac{(D-1)(D-2)}{16 \pi r_+^2} + \frac{(D-1)M}{\omega_{D-2} r_+^{D-2}} - \frac{(D-1)(1-2s)^2 r_+^{2s(D-2)}}{16 \pi (D-2s-1)} \Theta.
\end{align*}
\]

By using Eq. (21) and considering the ADM mass of the black hole is equals to its enthalpy in the extended phase space, that is $H = M$, the isenthalpic curves for various combinations of $D, M, Q$ and $s$ are plotted in Fig. 4.

In Fig. 4, each diagram has three isenthalpic curve for different mass, and the gray dot dash curve represents the inversion curve, which is consistent with that in Fig. 2. The inversion curve always intersects the maximum point of the temperature curve, it naturally leads to the LHS of isenthalpic curve has a positive slope, while the slope of isenthalpic curve becomes negative at the RHS. Therefore, in the throttling process, the inversion curve is regarded as dividing line between heating region (RHS) and cooling region (LHS). Meanwhile, by analyzing and comparing, it is easy find that increase of mass and $s$, or reduce the charge and $D$ can enhance the isenthalpic curve. Furthermore, Fig. 4(a) and Fig. 4(e) show that isenthalpic curve still expand when increase the $D, Q$ and $s$ at the same time. This indicates that the effect of $s$ on the isenthalpic curve is much greater than other parameters.

IV. CONCLUSION AND DISCUSSION

In this paper, by considering cosmological constant as the pressure, we investigated Joule-Thomson expansion of the higher dimensional nonlinearly charged AdS black hole in Einstein-PMI gravity. First, according to the
Meanwhile, it has a zero point and a divergent point in the temperature $T_i$ and the zero point of Hawking temperature, respectively. The curves of Joule-Thomson coefficient move to the right as the spacetimes dimension or the charge increases, while they move to the left as the nonlinearity parameter $s$ increases. Second, we analyzed inversion curve via Eq. (18) and Fig. 2. It is found that $T_i$ increases monotonously with $P_i$, and leads to only one minimum value of inversion temperature $T_i^{\text{min}}$ in the black hole system. The inversion curves for high pressure area increase as the spacetimes dimension and the
nonlinearity parameter decrease, or the charge increase, whereas $T_1$ at low pressure area increase as the space-times dimension and the nonlinearity parameter increase, or the charge decrease. Next, we derived expression of $T_1^{\text{min}}$ and calculate the ratio between the minimum of inversion temperature and the critical temperature $\eta$. Both $T_1^{\text{min}}$ and $T_{cr}$ are all contain the charge, but the ratio $\eta$ between them has nothing to do with $Q$. For $s = 1$, $\eta$ recovers the characteristic of higher R-N AdS black hole system. If $s > 1$, it becomes smaller and smaller as $D$ increase and approaches, which means that the denominator $T_{cr}$ is grows faster than the numerator $T_{\text{min}}$. In Fig. 3, it is obvious that the ratio difference between adjacent $s$ values is getting smaller and smaller, and the curve of $\eta$ approaches a constant as $D \to \infty$. Finally, according to Eq. (21) and considering $M = H$ in the extended phase space, we plot the isenthalpic curves for various combinations of $D$, $M$, $Q$ and $s$ in Fig. 4. It is easy find that increase of mass and $s$, or reduce the charge and $D$ can enhance the isenthalpic curve, and effect of $s$ on the isenthalpic curve is much greater than other parameters.

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