The Ultimate Future of the Universe,
Black Hole Event Horizon Topologies,
Holography, and
The Value of the Cosmological Constant

by

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Abstract. Hawking has shown that if black holes were to exist in a universe that expands forever, black holes would completely evaporate, violating unitarity. I argue this means unitarity requires that the universe exist for only a finite future proper time. I develop this argument, showing that unitarity also requires the boundaries of all future sets to be Cauchy surfaces, and so no event horizons can exist. Thus, the null generators of the surfaces of astrophysical black holes must leave the surface in both time directions, allowing non-spherical topologies for black hole surfaces. Since all information eventually escapes astrophysical black holes, and since the null surfaces defining astrophysical black holes are Cauchy surfaces, holography automatically holds. I further show that unitarity requires the effective cosmological constant to be zero eventually, since otherwise the universe would expand forever.

INTRODUCTION

Hawking showed a quarter century ago that if a black hole were to exist in a spacetime that exists for infinite proper time, it would completely evaporate, destroying the information inside the BH, thereby violating unitarity. Hawking argued that this result demonstrated that unitarity was indeed violated, but since unitarity is absolutely fundamental to quantum mechanics, I shall explore the implications of assuming that Hawking’s result and unitarity are BOTH correct. This assumption will be shown to imply: (1) the universe must be closed, with its future c-boundary being a single point, which means that there are no event horizons, and sets of the form ∂I^+ (p) for any event p are Cauchy surfaces, so the information inside a black hole both gets out in the far future and is also coded entirely on the surface of an astrophysical black hole: holography automatically holds; (2) the fact that the generators of any set ∂I^+ (p) eventually leave in the future direction, in contrast to event horizon null generators, means that higher genus black hole surface topologies MAY be possible; (3) the value of the cosmological constant is required to be near zero; in particular the “natural value” Λ = 8πc^5/Gℏ, corresponding the the Planck energy density would violate unitarity.

THE ULTIMATE FUTURE OF THE UNIVERSE

I shall now show that unitarity strongly constrains the future of the universe. Astrophysical black holes exist, but Hawking has shown that if black holes are allowed to exist for unlimited proper time, then they will completely evaporate, and unitarity will be violated. Thus unitarity requires that the universe must cease to exist after finite proper time, which implies that the universe has spatial topology S^3. (All other recollapse topologies, e.g. S^2 × S^1 [9] and negative Λ universes can be eliminated as possibilities by arguments which will be published elsewhere.) The Second Law of Thermodynamics says the amount of
entropy in the universe cannot decrease, but it can be shown ([1], p. 410) that the amount of entropy already in the CBR will eventually contradict the Bekenstein Bound [8] near the final singularity unless there are no event horizons, since in the presence of horizons the Bekenstein Bound implies the universal entropy $S \leq constant \times R^2$, where $R$ is the radius of the universe, and general relativity requires $R \to 0$ at the final singularity. The absence of event horizons by definition means that the universe’s future c-boundary is a single point, call it the Omega Point. MacCallum has shown that an $S^3$ closed universe with a single point future c-boundary is of measure zero in initial data space. Barrow [6] has shown that the evolution of an $S^3$ closed universe into its final singularity is chaotic. Yorke [7] has shown that a chaotic physical system is likely to evolve into a measure zero state if and only if its control parameters are intelligently manipulated. Thus life (≡ intelligent computers) almost certainly must be present arbitrarily close to the final singularity in order for the known laws of physics to be mutually consistent at all times. Misner has shown in effect that event horizon elimination requires an infinite number of distinct manipulations, so an infinite amount of information must be processed between now and the final singularity. The amount of information stored at any time diverges to infinity as the Omega Point is approached, since $S \to +\infty$ there, implying divergence of the complexity of the system that must be understood to be controlled.

**THE TOPOLOGY OF BLACK HOLE EVENT HORIZONS**

If event horizons do not exist, then strictly speaking neither do black holes. However, astrophysical black holes exist, and I have constructed [10] a spherically symmetric spacetime satisfying all the energy conditions which shows how this is possible: the spacetime is identical to a dust-filled closed universe with black holes in the expanding phase, but it has no event horizons. So the non-existence of event horizons does not contradict any observation on astrophysical black holes. However, this model does show that we have to take the theorems (e.g. [3], [4], [5]) proving black hole horizons to be 2-spheres with a grain of salt. These theorems must in effect make an assumption about the topology of scri [2], and use the fact that event horizon generators cannot leave the horizons in the future direction. However, if the universe is closed with the future c-boundary a single point, then it is easy to show that the boundaries of all future sets which lie in the future of some Cauchy surface must be Cauchy surfaces. Thus the null generators of an astrophysical black hole pseudo-horizon [10] must leave the black hole surface both in the future and the past, since I have argued that the universe must have $S^3$ Cauchy surfaces. The toroidal horizons of Hughes et al [11] are a slicing phenomenon in spacetimes with the standard scri [12], and can exist for only period $\sim M$ because in standard scri spacetimes, eventually horizon null generators must cease to enter the horizon, and once on the horizon, can never leave it. But neither is necessarily true if only pseudo-horizons exist. Thus if unitarity and standard quantum gravity are both true, higher genus black holes MAY exist.

**THE VALUE OF THE COSMOLOGICAL CONSTANT**

Current observations show an accelerating universe with $\Omega_\Lambda = 2/3$, and if the universe were to continue to accelerate, black holes would evaporate, violating unitarity. Hence, unitarity requires that the acceleration will eventually return to a de-acceleration, followed by a recollapse. Now Gibbons and Hawking [13] have shown that the vacuum energy in de Sitter space generates thermal Hawking radiation with a temperature of $T_{deS} = (\hbar/2\pi k_B)^{1/3} = 3.9397 \times 10^{-30}h\sqrt{\Omega_\Lambda} \text{ K}$, or $T_{deS} = 2.25 \times 10^{-30}$ degrees Kelvin with $h = 0.70$ and $\Omega_\Lambda = 2/3$. So if the cosmological constant were never to be canceled, any black hole with a Hawking temperature greater than this would eventually evaporate and violate unitarity. Thus only black holes with a mass greater than $\sim 10^{25} M_\odot$ — more mass than there is in the visible universe — could avoid evaporation.

In fact, we can use the same argument to show why the cosmological constant must be near zero, instead of being its expected value of $\Lambda = 8\pi G \rho_{vac} = 8\pi G(c^5/hG^2) = 8\pi c^3/hG$ given by the Planck density. If the cosmological constant were this large, in a universe that expands forever, there would be a finite (though extremely small) probability that vacuum or other density fluctuations would give rise to a black hole smaller than the de Sitter horizon $R_{deS} = c\sqrt{3/\Lambda}$, which would have a higher Hawking temperature than the de Sitter background temperature, and hence evaporate, violating unitarity. So ultimately, the cosmological constant is near zero because a large cosmological constant would violate unitarity.
References

[1] Tipler F J 1994 *The Physics of Immortality* (New York: Doubleday).
[2] Brill, D. R. et al Phys. Rev. D56 (1997), 3600.
[3] Galloway, G. Comm. Math. Phys. 151 (1993), 53.
[4] Chrusciel, P. T. and R. M. Wald, Class. Quan. Grav. 11 (1994), L147.
[5] Jacobson, T. and S. Venkataramani, Class. Quan. Grav. 12 (1995), 1055.
[6] Barrow, J. D. Phys. Reports 85 (1982).
[7] Yorke, J. A. et al Phys. Rev. Lett. 68 (1992), 2863.
[8] Schiffer, M. and J. D. Bekenstein Phys. Rev. D39 (1989), 1109.
[9] Barrow, J. D. and F. J. Tipler, Mon. Not. R. astr. Soc. 216 (1985), 395.
[10] Tipler, F. J. et al, gr-qc/0003082
[11] Hughes, S. A. et al Phys. Rev. D49 (1994), 4004
[12] Galloway, G. J. et al gr-qc/9902081
[13] Gibbons, G. W. and S. W. Hawking, Phys. Rev. 15 (1977), 2738.
[14] Weinberg, S. Rev. Mod. Phys. 61 (1989), 1.