THE QUASILINEAR PREMISE FOR THE MODELING OF PLASMA TURBULENCE

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ABSTRACT

The quasilinear premise is a hypothesis for the modeling of plasma turbulence in which the turbulent fluctuations are represented by a superposition of randomly-phased linear wave modes, and energy is transferred among these wave modes via nonlinear interactions. We define specifically what constitutes the quasilinear premise, and present a range of theoretical arguments in support of the relevance of linear wave properties even in a strongly turbulent plasma. We review evidence both in support of and in conflict with the quasilinear premise from numerical simulations and measurements of plasma turbulence in the solar wind. Although the question of the validity of the quasilinear premise remains to be settled, we suggest that the evidence largely supports the value of the quasilinear premise in modeling plasma turbulence and that its usefulness may also be judged by the insights gained from such an approach, with the ultimate goal to develop the capability to predict the evolution of any turbulent plasma system, including the spectrum of turbulent fluctuations, their dissipation, and the resulting plasma heating.

Subject headings: turbulence — solar wind

1. INTRODUCTION

The presence of turbulence impacts the evolution of a wide variety of plasma environments, from galaxy clusters to accretion disks around compact objects, to the solar corona and solar wind, and to the laboratory plasmas of the magnetic confinement fusion program. Establishing a thorough understanding of plasma turbulence is a grand challenge that has the potential to impact this wide range of research frontiers in plasma physics, space physics, and astrophysics. Ultimately, such efforts are aimed at developing the capability to predict the evolution of any turbulent plasma system. The quasilinear premise (Klein et al. 2012) is a hypothesis for the modeling of plasma turbulence with the potential to lead to a quantitative, predictive theory of plasma turbulence.

The quasilinear premise states simply that some characteristics of the turbulent fluctuations in a magnetized plasma may be usefully modeled by a superposition of randomly-phased, linear wave modes. The nonlinear interactions inherent to the turbulent dynamics may be considered to transfer energy among these linear wave modes—therefore, the model is quasilinear.

This premise is hotly debated at present, with significant questions raised by the heliospheric physics community about the validity of using the theory of linear plasma waves to analyze and interpret the turbulent fluctuations measured in the solar wind plasma. On one hand, a large body of work on plasma turbulence either explicitly or implicitly assumes the relevance of some linear plasma wave properties. On the other hand, the nonlinearity inherent in turbulent interactions raises obvious questions about the relevance of linear theory.

In this paper, we define precisely the concepts encapsulated by the quasilinear premise and identify the limitations of such an approach. We outline the theoretical arguments that justify the application of linear plasma wave theory to the study of plasma turbulence, and review supporting and conflicting evidence from theory, simulation, and observation.

2. DEFINITION OF THE QUASILINEAR PREMISE

The quasilinear premise proposes a fundamental picture of plasma turbulence in which the turbulent fluctuations are represented by a superposition of randomly-phased linear wave modes, and energy is transferred among these wave modes via nonlinear interactions. Although this simple model cannot capture all of the known characteristics of turbulence, it does provide a foundation upon which a quantitatively predictive model of turbulent nonlinear energy transfer and plasma heating may be constructed. Below we describe the features of a model for turbulence in a magnetized plasma based on the quasilinear premise, focusing in particular on the aspects of turbulence that can and cannot be described by such an approach.

Several important properties of plasma turbulence are described by a model adopting the quasilinear premise. First, and most important, is that the characteristic eigenfunctions of the turbulent fluctuations (the amplitude and phase relationships among the electric, magnetic, velocity, and density fluctuations of a single spatial Fourier mode) are given by the linear wave eigenfunctions of the background plasma. Above all, this premise provides a simple and transparent framework for understanding the role of nonlinearity in plasma turbulence.

1 A small sample of these studies includes Coleman (1968); Belcher & Davis (1971); Tu et al. (1978a); Matthaeus et al. (1990); Tu & Marsch (1994); Verma et al. (1993); Leamon et al. (1998); Quataert (1996); Stawicki et al. (2001); Bale et al. (2006); Markovskii et al. (2006); Hamilton et al. (2008); Howes et al. (2008); Sahraoui et al. (2006); Schekochihin et al. (2008); Chandran et al. (2010); Chen et al. (2010); Podesta et al. (2010); Saito et al. (2010); Rudakov et al. (2012).
functions. Second, in a weakly collisional plasma such as the solar wind, the fluctuations associated with each wave mode are damped at the appropriate instantaneous collisionless damping rate given by linear kinetic theory. Third, the nonlinear energy transfer is described by a phenomenological prescription derived from modern theories for anisotropic plasma turbulence [Sridhar & Goldreich 1993; Goldreich & Sridhar 1995; Boldyrev 2000; Howes et al. 2008a; Schekochihin et al. 2003; Howes et al. 2011]. Finally, the distribution of the power of the turbulent fluctuations in three-dimensional wavevector space is guided by theory and numerical simulations of plasma turbulence, and is chosen to satisfy key constraints, such as the observed wavenumber spectrum of magnetic energy. Thus, the quasilinear premise is a marriage of the quantitative linear physics of waves in a kinetic plasma with a phenomenological prescription for the nonlinear turbulent dynamics given by modern turbulence theories.

It is important to note that the quasilinear premise is not the same as quasilinear theory in plasma physics, the rigorous application of perturbation theory to explore the long-time evolution of weakly nonlinear systems. Rather, the quasilinear premise employs a phenomenological prescription for the nonlinear energy transfer in strong turbulence, rather than calculating it rigorously from first principles. As described below in 3, the mathematical properties of the equations that describe turbulence in a magnetized plasma, in conjunction with a phenomenological understanding of the properties of the turbulence, provide the motivation for this quasilinear approach.

A model based on the quasilinear premise can be used to explore the second-order statistical properties of plasma turbulence, such as the energy spectrum of the turbulence or the correlations between two different fields, for example the cross correlation between the density and parallel magnetic field fluctuations (Howes et al. 2012a; Klein et al. 2012). The synthetic spacecraft data method (Klein et al. 2012) is an application of the quasilinear premise for the interpretation of spacecraft measurements of plasma turbulence. In this method, a synthetic plasma volume is filled with turbulent fluctuations in the form of a distribution of randomly-phased, linear wave modes. This synthetic plasma volume is then sampled along a single trajectory, generating single-point time series of turbulent plasma and electromagnetic fluctuations that may be analyzed with the same procedures as spacecraft measurements. A large ensemble of such synthetic data sets has proven useful in gaining new understanding of the compressible fluctuations in the inertial range of solar wind turbulence (Klein et al. 2012) and of the magnetic helicity of solar wind turbulent fluctuations as a function of the angle between the solar wind flow and the local mean magnetic field (Klein et al. 2014b). In addition, one may also use the quasilinear premise to construct a turbulent cascade model to predict the nonlinear transfer of energy from large to small scales, enabling predictions of the resulting energy spectra of the turbulent fields and of the plasma heating resulting from the dissipation of the turbulent energy via kinetic mechanisms at small scales (Howes et al. 2008a; Podesta et al. 2010; Howes 2010; Howes et al. 2011; Howes 2011).

Since the nonlinear interactions under the quasilinear premise are not computed from the nonlinear terms in the governing equations, but rather are given by a phenomenological prescription, the third-order and higher order correlations observed in solar wind turbulence (Tu & Marsch 1993; Bruno & Carbone 2003; Haas et al. 2003, 2007; Kiyani et al. 2009) cannot be investigated using a turbulence model based on the quasilinear premise. Such higher order statistics depend critically on the phase relationships between different linear wave modes, and these phase relationships are determined by the nonlinear interactions responsible for the turbulent cascade of energy from large to small scales. For a collection of randomly phased linear waves, such higher order statistics will average to zero, yielding no useful information. In other words, the random phases between the constituent wave modes cannot capture the intermittency and coherent structures, such as current sheets, that are observed in plasma turbulence simulations (Wan et al. 2012; Karimabadi et al. 2013) and inferred from observations of solar wind turbulence (Kiyani et al. 2009; Osman et al. 2012a,b; Wu et al. 2013).

In addition, inherently nonlinear fluctuations—fluctuations that cannot be expressed as a superposition of linear eigenfunctions—cannot be described by a model based on the quasilinear premise. Such an inherently nonlinear fluctuation has recently been derived in the asymptotic analytical solution of the nonlinear interaction between counterpropagating Alfvén waves (Howes & Nielson 2013); this mode has been shown to play an important role in the nonlinear transfer of Alfvén wave energy to smaller scales. However, if a phenomenological picture can be devised to describe the distribution of such nonlinear modes, such as that suggested by Schekochihin et al. (2012), it may be possible to produce refined turbulence model by extending the quasilinear premise to incorporate such modes.

In addition to specifying which properties of the turbulence this approach can and cannot describe, it is also important to state which conditions are necessary, and which are not, for the quasilinear premise to be applicable. First, the quasilinear premise is expected to become valid at length scales significantly smaller than the scale of turbulent energy injection, where the amplitude of the magnetic field fluctuations becomes small relative to the local mean magnetic field, $\delta B \ll B_0$. It remains valid at all smaller scales, including the scales at which dissipation mechanisms act to terminate the cascade. Second, the application of the quasilinear premise requires both guidance from turbulence theories and constraints from simulations and observations. The most important element required from turbulence theory is the prescription for the nonlinear transfer of energy among wave modes that underlies the turbulent cascade from large to small scales. Third, the division of turbulent power among the possible linear wave modes, and the possibly anisotropic distribution in wavevector space of the turbulent power for each wave mode, also must be guided by theoreti-

2 Note that, in principle, if the observed development of intermittency is taken into account in the phenomenological prescription chosen to describe nonlinear energy transfer, for example in Chandran et al. (2012), the resulting model will thereby partly account for intermittency in terms of the energy transfer, although the turbulent fields themselves will not be intermittent, so third-order statistics still cannot be explored.
cal considerations as well as numerical and observational constraints. The amplitude of the spectrum of linear modes as a function of wavenumber can be specified such that the scaling of the turbulent power in the model is consistent with the observed turbulent power spectrum. Finally, we emphasize that the fruitful investigation of the nature of plasma turbulence using the quasilinear premise does not require the turbulent fluctuations derived from the linear wave mode properties to persist for many wave periods, nor does it require evidence of a “linear dispersion relation” to be apparent in the measured turbulent power spectrum, as will discussed in more detail in 3.3 below.

3. THEORETICAL MOTIVATION FOR THE QUASILINEAR PREMISE

The quasilinear premise, as defined above, is motivated by the mathematical properties of the equations that describe turbulence in a magnetized plasma. The key concepts are most easily explained for the simplified case of turbulence in an incompressible MHD plasma, the minimal model containing the essential physics describing the dissipation range of turbulence in the solar wind typically satisfies 

$$\chi \equiv \left| \frac{(|z|^\perp \cdot \nabla)z^\perp}{|v_A \cdot \nabla|z^\perp} \right| \sim k_L v_L \sim k_L \delta B_\perp \sim \frac{k_L B_\perp}{k_L B_0}$$

Therefore, in the limit of weak nonlinearity, $\chi \ll 1$, it is possible to identify a regime of weak MHD turbulence, motivating a quasilinear approach using perturbation theory. Note that, in the case of incompressible hydrodynamic turbulence (Euler or Navier-Stokes), the absence of a linear term disallows the possibility of a perturbative approach, a fundamental distinction between turbulent hydrodynamic and magnetohydrodynamic systems.

In the absence of the nonlinear terms (setting the right-hand side of equation (1) to zero), the behavior of the plasma is entirely determined by the linear term. If the right-hand side of the equation is considered to be an arbitrary perturbing source term, the linear term determines the instantaneous response of the plasma to the imposed perturbation. In the case of weak turbulence, $\chi \ll 1$, it is possible to identify a regime of weak MHD turbulence (Sridhar & Goldreich 1994), motivating a quasilinear approach using perturbation theory. Perturbation theory may be applied to the study of the turbulent dynamics in this limit (Galtier et al. 2000, Howes et al. 2013, Nielson et al. 2013), so the quasilinear premise is clearly valid for the case of weak turbulence.

But the turbulent magnetic fluctuations at large scales in the solar wind typically satisfy $\delta B/B_0 \sim 1$, so, at the scale of energy injection, solar wind turbulence is believed to be in a state of strong turbulence, $\chi \sim 1$. The theory of strong incompressible MHD turbulence (Goldreich & Sridhar 1993, Boldyrev 2006) suggests that the turbulent fluctuations at small scales become anisotropic, where the nonlinear cascade of energy generates turbulent fluctuations with smaller scales in the perpendicular direction than in the parallel direction, $k_L \ll k_L$. This inherent anisotropy of magnetized plasma turbulence has long been recognized in laboratory plasmas (Robinson & Rusbridge 1971, Zweben et al. 1979, Montgomery & Turner 1981) and in the solar wind (Belcher & Davis 1971), as well as in early numerical
analogy of a damped simple harmonic oscillator illustrates the concept, central to the quasilinear premise, that the linear terms of an equation contribute significantly to the evolution of the system, even in the absence of oscillatory (or wave-like) behavior. The equation of evolution for a damped harmonic oscillator is given by

$$\frac{d^2 x}{dt^2} + \omega_0^2 x + \nu \frac{dx}{dt} = 0,$$

(3)

where $\omega_0$ is the undamped frequency of the harmonic oscillator and $\nu$ is a frictional damping coefficient. The dimensionless parameter that characterizes the strength of the damping relative to the linear restoring term responsible for oscillatory behavior is $\zeta = \nu/(2\omega_0)$. This term is analogous to the nonlinearity parameter $\chi$ in plasma turbulence which characterizes the strength of the nonlinear to the linear term, given by equation (2). The system is underdamped for $\zeta < 1$, critically damped for $\zeta = 1$, and overdamped for $\zeta > 1$.

Specifying the initial conditions at $t = 0$ to be $x = 0$ and $dx/dt = v_0$, we consider the evolution of the system in time, as shown in panel (a) of Figure 1. In the underdamped $\zeta < 1$ case (dashed), the oscillatory behavior of the system is evident. For either critically damped $\zeta = 1$ case (solid) or overdamped $\zeta > 1$ case (dotted), the system does not demonstrate oscillatory behavior. In either of these non-oscillatory cases, however, the linear restoring term still plays an important role in governing the evolution of the system.

If the restoring term is eliminated by setting $v_0 = 0$ in equation (3), the initial velocity (not shown) monotonically decreases to zero and the position asymptotes to a value $x_\infty = v_0/\nu$, as shown in panel (b) of Figure 1. But when the linear restoring term is present, the evolution is qualitatively different, as seen by comparing the left and right panels of Figure 1. For the critically damped case (right panel, solid), the position reaches maximum value $x < x_\infty$ before returning monotonically to zero, while the velocity (not shown) decreases from the initial velocity $v_0$ to a minimum $v < 0$, before returning monotonically to zero. Therefore, even though the damped simple harmonic oscillator for $\zeta \geq 1$ exhibits no oscillatory behavior, the presence of the linear restoring term still significantly influences the evolution of the system.

In a turbulent magnetized plasma, the balance is between the nonlinear and linear response terms, rather than linear damping and linear restoring terms, but the lesson is analogous: even in a strongly turbulent plasma in which the nonlinear term is of the same order as the linear term, the linear terms continue to play a significant role in governing the turbulent dynamics. The phenomenon of critical damping in this harmonic oscillator system is analogous to the concept of critical balance.
Fig. 1.— (a) The position $x$ vs. time $t$ for a damped simple harmonic oscillator, given by equation (3), for an underdamped $\zeta < 1$ case (dashed), a critically damped $\zeta = 1$ case (solid), and an overdamped $\zeta > 1$ case (dotted). (b) For a system in which restoring force is eliminated ($\omega_0 = 0$), the position $x$ vs. time $t$ is qualitatively different.

3.3. Is Evidence of a Linear Dispersion Relation Necessary?

The concept central to the quasilinear premise that the turbulence consists of a broadband spectrum of Alfvén waves is frequently criticized with the argument that, if waves are indeed present in the turbulence, one should be able to see clear evidence of the linear dispersion relation (meaning a plot of $\omega$ vs. $k$) in the analysis of turbulence simulations. We argue here that there are two good reasons why one should not necessarily be able to see clearly such a dispersion relation. The first reason is related to the semantic difference between an Alfvén wave and an Alfvénic fluctuation. The second reason is that changes in the amplitude or phase of a wave with a constant frequency will significantly broaden the frequency content determined by a Fourier transform of a time series.

In the weak turbulence limit, $\chi \ll 1$, it requires nonlinear interactions with many counterpropagating Alfvén waves before the bulk of the energy of a given Alfvén wave is cascaded to a smaller scale (Sridhar & Goldreich 1993; Howes & Nielsen 2013). In this limit, therefore, it is likely that the signature of wave-like motions may be relatively easy to observe. But for a case of strong turbulence, in which the nonlinearity parameter $\chi \sim 1$, the energy of an Alfvén wave is completely cascaded to smaller scale through a collision with a single counterpropagating Alfvén wave, so that the timescale for nonlinear energy transfer is the same order as the linear wave period. In this case, if the interacting turbulent fluctuations persist for only a single wave period, does it make sense to refer to the interacting fluctuations as waves? In our view, this point is largely semantic, for the following reasons.

In the simplified context of incompressible MHD, any finite amplitude fluctuation with $z^+ \neq 0$ and $z^- = 0$ (or with $z^+ = 0$ and $z^- \neq 0$), is an exact nonlinear solution of the equations of evolution, corresponding to an arbitrary waveform propagating unchanged in one direction along the mean magnetic field at the Alfvén velocity. This solution is clearly a finite amplitude Alfvén wave, but it need not oscillate sinusoidally as one typically envisions when discussing a wave-like motion. A Fourier transform of the time series measured at a fixed Eulerian point as the wave passes by may yield a broad frequency response; for example, an isolated “wavepacket” consisting of a single parallel wavelength $\lambda_{||}$ of a perfectly sinusoidal signal will indeed return a frequency response that is significantly broadened about the “linear” frequency $\omega = 2\pi v_A/\lambda_{||}$. Some may prefer the more general term “Alfvénic fluctuation” in place of “Alfvén wave” in this case, be the difference is merely semantic. That an Alfvén wave may not be purely sinusoidal, and thus not have a well defined frequency $\omega$, does not change the nature of the fluctuation, which is determined by the linear response of the plasma to an applied perturbation, as dictated by the linear term in the equation of motion equation (1). It is in this sense that we speak of turbu-
lence supported by Alfvén waves. Therefore, we hereby clarify that our use of the term Alfvén wave pertains to any fluctuation whose linear response is the same as that of an Alfvén wave—thus, we refer to any Alfvén fluctuation as an Alfvén wave.

The second reason that evidence of a linear dispersion may not be apparent in measurements or simulations of strong plasma turbulence, even for turbulence consisting precisely of a sum of linear wave modes, is that changes in the amplitude or phase of a wave will broaden the frequency response substantially, as shown in Figure 2. Here we plot the frequency retrieved from a Fourier transform of the time series for a pure sinusoidal signal (black), sinusoidal signal with linearly increasing amplitude (red), and a sinusoidal signal with shifting phase (blue). It is clear that the frequency response when the amplitude or phase of a mode changes, one obtains a very broadened response in frequency, even when the basic signal has a well-defined frequency. Therefore, it is likely to be very difficult to recover a clean ω vs. k dispersion relation, even for turbulence simulation data.

To make matters more complicated, for spacecraft measurements, the presence of a broad spectrum of Alfvén wave power in three-dimensional wavevector space further complicates the interpretation of a time series of measurements at a fixed point in space in terms of the frequency of the underlying fluctuations.

It is worthwhile to raise one final point regarding the frequency spectrum of the fluctuations in plasma turbulence. One may view the dynamical evolution of plasma according to equation 1 as the linear response of the plasma (given by the left-hand side of the equation) to perturbations generated by the nonlinear terms (given by the right-hand side of the equation). For a particular wavevector k, the characteristic frequency of the linear plasma response is given by ω0 = k||vA, where the component of the wavevector parallel to the mean magnetic field is k|| = k•B0/B0. As in the case of a driven, damped single harmonic oscillator, one may obtain a response at a frequency ω ≠ ω0 only by sustained driving at another frequency. But since the nonlinear terms perturbing a given wavevector mode are dominated by the interactions between nearby wavevectors, the frequency driving a particular spatial Fourier mode in plasma turbulence is likely to be very near the linear frequency of the mode being driven. The power generated by the nonlinear interaction between two counterpropagating Alfvén waves cannot have a frequency higher than the sum of the frequency of those two modes (Howes & Nielson 2013), so the turbulent power will not contain frequencies significantly higher than the characteristic linear Alfvén wave frequency for a particular wavevector. In addition, there would be a significant impedance mismatch if the plasma is driven at a frequency well above the linear Alfvén wave frequency, as seen in simulations (Parashar et al. 2011; TenBarge & Howes 2012; TenBarge et al. 2014).

3.4 Collisionless Damping of Turbulent Fluctuations

An important component of the quasilinear premise that enables the construction of predictive models of the turbulent dissipation and resulting plasma heating is that the turbulent fluctuations associated with each wave mode are damped at the appropriate instantaneous collisionless damping rate given by linear kinetic theory. The applicability of these linear kinetic damping rates, of course, depends on the central concept of the quasilinear premise that the turbulent fluctuations have the properties of the linear waves supported by the weakly collisional plasma. It is important to note that the terms in the kinetic equation for weakly collisional plasmas that are responsible for collisionless wave-particle interactions with the equilibrium distribution function are linear, and therefore the same arguments for the relevance linear wave modes in a turbulent plasma also apply to the linear collisionless damping. Here we discuss specifically two important issues regarding the collisionless dissipation: (i) possible nonlinear saturation or inhibition of the collisionless damping and (ii) the spatial distribution of damping via wave-particle interactions.

There are two possible ways that the linear collisionless damping rate may be altered: (i) quasilinear or nonlinear evolution of the equilibrium distribution function may flatten the slope at the resonant velocity for a particular wave, thus reducing or entirely inhibiting collisionless damping of that wave (Rudakov et al. 2011, 2012); and (ii) forcing by strong turbulence in a stochastic manner may break the coherence between a particular wave mode and a particle necessary for collisionless damping to extract significant energy from the wave (Plunk 2013; Kanekar et al. 2014). It has been suggested that electron Landau damping is the dominant collisionless mechanism for the damping of fluctuations in the dissipation range of solar wind turbulence (Leamon et al. 1998a, 1999; Leamon et al. 2000; Howes et al. 2008a; Schekochihin et al. 2009; Howes 2009; TenBarge & Howes 2013; TenBarge et al. 2013). In this regime, it has been proposed that the turbulent fluctuations are kinetic Alfvén waves, a hypothesis now strongly supported by a number of numerical simulations (Howes et al. 2008b, 2013).
turbulent kinetic Alfvén waves parallel velocity of electrons that are resonant with the over the entire range, without flattening the electron velocity distribution completely suppress Landau damping for quasilinear flattening of the distribution function could be explained, with no fitting parameters, by Landau damping in the solar wind plasma. See Podesta (2013) for a recent review of evidence for kinetic Alfvén waves in the solar wind. Flattening of the distribution function, and the associated reduction in collisionless damping rates, due to quasilinear evolution of the equilibrium is a well-understood process in plasma physics. The key question in the context of the turbulent solar wind plasma is whether this mechanism operates effectively to quench collisionless damping, as recently suggested (Rudakov et al. 2011, 2012).

An argument against this quasilinear quenching of the Landau damping of kinetic Alfvén waves arises from the variation in the resonant velocity in the electron distribution function. Since the kinetic Alfvén wave is a dispersive wave mode, its phase velocity (which is nearly parallel to the magnetic field direction) increases linearly with \( k_\perp \) in the dissipation range at \( k_\perp \rho_i \gtrsim 1 \). Therefore, the parallel velocity of electrons that are resonant with the turbulent kinetic Alfvén wave increases from a velocity \( v_{\|} \ll v_{te} \) at \( k_\perp \rho_i \sim 1 \) to \( v_{\|} \sim v_{te} \) at \( k_\perp \rho_i \sim 1 \). Therefore, without flattening the electron velocity distribution over the entire range \( v_{\|} \lesssim v_{te} \), it seems unlikely that a quasilinear flattening of the distribution function could completely suppress Landau damping for \( k_\perp \rho_i \lesssim 1 \).

A second possible way to suppress collisionless wave-particle interactions is to interfere with the coherence time between electromagnetic waves and particles so that the particles are unable to experience a net gain of energy. For example, in strongly collisional plasmas, such as MHD plasmas, electromagnetic waves are undamped by collisionless wave-particle interactions because frequent collisions prevent a single particle from maintaining a sufficiently long coherence time to interact resonantly with the waves. A recent study has shown that, when the linearized Vlasov equation is perturbed by a stationary random force, the effective Landau damping rate can be significantly reduced under appropriate conditions (Plunk 2013). Work continues to understand the effect of Landau damping in a turbulent setting (Kanekar et al. 2014). Nonlinear kinetic simulations of plasma turbulence will hopefully be able to quantify any suppression of the linear kinetic damping rate.

Another issue associated with the collisionless damping of plasma turbulence is the spatial distribution of the plasma heating resulting from the dissipation of the turbulent energy. Based on the decomposition of turbulent fluctuations into component plane waves, it has frequently been asserted that Landau damping must lead to spatially uniform heating. Recently, there has been significant work investigating non-uniform distribution of heating in the solar wind plasma, especially focusing on enhanced heating in the neighborhood of current sheets

\[ T_i \sim T_e. \]

\[ k_\perp \rho_i \gtrsim 1 \]
Another proposed technique for evaluating the importance of linear wave modes is to compute the frequency spectrum for particular Fourier modes or the Eulerian frequency spectrum at a particular point in space from numerical simulations (Dmitruk & Matthaeus 2009). As discussed above in §3.3 except for the case of weak turbulence, however, it has not been established that one should indeed expect to see evidence of a “linear dispersion relation” in the frequency spectra of strong, driven plasma turbulence since an individual wave mode is not expected to persist for more than a single wave period. A study of driven turbulence in 3D incompressible MHD simulations found little evidence that linear waves play a significant role in MHD turbulence (Dmitruk & Matthaeus 2009), but it has been pointed out that the method of driving used in the simulations may have had a dominant impact on the measured frequency spectrum (TenBarge & Howes 2012, TenBarge et al. 2014). A particle-in-cell (PIC) simulation of decaying 2D magnetosonic turbulence over the \( k_{i}\parallel k_{L} \) plane found a cascade consistent with the properties of fast magnetosonic waves, and that little energy appeared to be nonlinearly transferred to the slow magnetosonic or ion Bernstein waves (Svidzinski et al. 2009). Two studies of driven, 2D hybrid kinetic ion and fluid electron simulations of turbulence over the perpendicular plane found a low level of wave activity, with the dynamics dominated by nonlinear activity (Parashar et al. 2010, 2011). The 2D geometry of these hybrid simulations, however, admits only linear wave modes with \( k_{i} = 0 \), and therefore these simulations cannot support Alfvén waves, kinetic Alfvén waves, or whistler waves, calling into question their relevance to the study of solar wind turbulence. Finally, 3D decaying simulations of turbulence at the high-frequency end of the inertial range using both the Hall MHD and Landau fluid theory (Passot & Sulem 2007) find that the peak of the turbulent magnetic and kinetic energy frequency spectra for particular Fourier modes show excellent agreement with the linear wave mode frequencies for the fast, Alfvén, and slow modes (Humana et al. 2011). This collection of apparently contradictory findings suggests that this line of investigation of the relevance of linear wave properties in numerical simulations of turbulence will remain an active area of research.

Given that the idea of critical balance (Goldreich & Sridhar 1993) in strong plasma turbulence is essentially a quasi linear concept—that the frequency of the nonlinear energy transfer remains of the same order as the linear wave frequency—numerical evidence in support of critical balance indirectly supports the quasi linear premise. Although the concept of critical balance was initially applied only to the case of MHD turbulence, the theory has been extended to the dispersive wave regime of the dissipation range (Biskamp et al. 1999; Cho & Lazarian 2004; Krishan & Mahajan 2004; Shaikh & Zank 2005; Galtier 2006; Howes et al. 2008a; Schekochihin et al. 2004; Howes et al. 2011). In numerical simulations, critical balance is generally tested by a magnitude of the turbulent power on the \( k_{L}\parallel k_{L} \) plane in wavevector space in numerical simulations (Cho & Vishniac 2000; Maron & Goldreich 2001; Cho & Lazarian 2004; Grappin & Müller 2010). The results of these numerous studies are contradictory, with some claiming to support critical balance, and others, to refute it. The lines of conflict, however, coincide with the method used to determine the direction of the magnetic field: studies using a local mean magnetic field (Cho & Vishniac 2000; Maron & Goldreich 2001; Cho & Lazarian 2004) are consistent with the predictions of critical balance, while studies employing a global magnetic field (Grappin & Müller 2010) are inconsistent with critical balance. It appears that, as long as the direction of the magnetic field is determined locally, there exists significant evidence in support of critical balance.

A recently proposed alternative method for testing critical balance eliminates the need to define the direction of the magnetic field by noting that the linear wave frequency for Alfvénic plasma waves is proportional to the parallel wavenumber, \( \omega \propto k_{L} \) (TenBarge & Howes 2012). In this case, one may compute the distribution of turbulent power on the \( \omega\times k_{L} \) plane and look for the predicted scaling \( \omega \propto k_{L} \times k_{L}^{2} \), where \( \alpha = 2/3 \) (Goldreich & Sridhar 1993) or \( \alpha = 1/2 \) (Boldyrev 2006) for the MHD Alfvén wave cascade and \( \alpha = 1/3 \) for the kinetic Alfvén wave cascade (Cho & Lazarian 2004; Howes et al. 2008a; Schekochihin et al. 2009). Gyrokinetic simulations of driven, 3D kinetic Alfvén wave turbulence support the predicted critical balance scaling of \( \alpha = 1/3 \) in this regime (TenBarge & Howes 2012, TenBarge et al. 2013).

Finally, a recent study of ion temperature gradient driven turbulence in gyrokinetic simulations of magnetic confinement fusion plasmas has found support for the turbulent fluctuations satisfying critical balance (Barnes et al. 2011).

5. SUPPORTING EVIDENCE FROM SOLAR WIND OBSERVATIONS

Spacecraft measurements in the turbulent wind plasma have been used to evaluate the characteristic nature of the turbulent fluctuations, seeking agreement with the typical frequencies of Alfvénic and kinetic Alfvén wave modes using a k-filtering analysis of multi-spacecraft data and searching for evidence of critical balance through the spectral indices measured at different angles with respect to the direction of magnetic field. In addition, laboratory experiments of plasma turbulence have begun to uncover evidence suggesting the turbulence indeed satisfies critical balance.

Multi-spacecraft observations of solar wind turbulence may also be used to determine the fluctuation frequency in the solar wind plasma frame for each spatial Fourier mode (Sahraoui et al. 2010, Narita et al. 2011, Roberts et al. 2013). The first k-filtering analysis of Cluster multi-spacecraft data in the unperturbed solar wind showed that the turbulent fluctuations in the inertial and transition range, \( 0.04 \leq k_{L} \rho_{L} \leq 2 \), are consistent with the dispersion relation of the Alfvén/kinetic Alfvén wave branch, and are inconsistent with the fast/whistler branch (Sahraoui et al. 2010). A subsequent study that performed a similar k-filtering analysis of Cluster multi-spacecraft data obtained contradictory results, finding little agreement with any particular linear wave mode (Narita et al. 2011). A number of issues, however, cast serious doubts on the validity of the latter study: the error in the plasma-frame frequency is not estimated,
the results of this study at large scales are inconsistent with the observations that demonstrate the largely incompressible, Alfvénic nature of large scale fluctuations in the solar wind (Belcher & Davis 1971; Tu & Marsch 1993; Bruno & Carbone 2005). Inspection of the CIS and PEACE data show that all of the intervals studied suffer either electron or ion foreshock contamination, and the spacecraft configuration for each of these intervals shows significant levels of planarity and elongation, indicating a poor tetrahedron. A subsequent k-filtering study found further evidence in support of low frequency kinetic Alfvén waves in Cluster multispacecraft data (Roberts et al. 2013). It is clear that new multi-spacecraft studies will provide valuable guidance in assessing the relevance of linear wave modes in the solar wind plasma.

In measurements of solar wind turbulence, one can test the idea of critical balance (Goldreich & Sridhar 1995) by measuring the 1D magnetic energy spectrum as a function of the angle $\theta_{\|}^\perp$ with respect to the magnetic field (Horbury et al. 2008; Podesta 2009; Tessein et al. 2009; Chen et al. 2010; Wicks et al. 2011; Forman et al. 2011). As in the case for numerical simulations, there exist contradictory findings, with support for critical balance found when the magnetic field direction is determined locally (Horbury et al. 2008; Podesta 2009; Chen et al. 2010; Wicks et al. 2011; Forman et al. 2011) and conflict with critical balance when the study employs a global magnetic field to determine the parallel direction (Tessein et al. 2009).

Finally, recent experimental measurements of strongly magnetized plasma turbulence driven by the ion temperature gradient in the Mega Amp Spherical Tokamak (MAST) have been found to be consistent with the predictions of critical balance (Ghim et al. 2013).

6. DISCUSSION

The question of the validity of the quasilinear premise—that linear wave properties are relevant to the dissipation range of solar wind turbulence—clearly remains to be settled. On balance, however, the bulk of the evidence appears to argue in favor of its validity. The utility of the quasilinear premise for the study of plasma turbulence, however, may also be judged a posteriori by the insights gained from such an approach. Below we briefly discuss a number of applications of the quasilinear premise to the study of plasma turbulence.

The synthetic spacecraft data method (Klein et al. 2012) is a technique for producing artificial plasma turbulence measurements that can be directly compared to in situ measurements of turbulence in the solar wind. The turbulence in the synthetic plasma volume is constructed assuming the quasilinear premise. Such an approach has proven to be very successful, discovering that the compressible fluctuations in the inertial range of solar wind turbulence are anisotropically distributed slow waves (Howes et al. 2012a; Klein et al. 2012), and illuminating the nature of the fluctuations responsible for the observed magnetic helicity signature of turbulent fluctuations as a function of the angle between the magnetic field and the solar wind flow (Klein et al. 2014).

Additionally, based on the fundamental concepts embodied by the quasilinear premise, a number of simple models for the turbulent cascade of energy in weakly collisional plasma turbulence have been devised (Howes et al. 2008a; Podesta et al. 2010; Howes et al. 2011), with the ability to model the dissipation of the turbulent cascade based on kinetic damping mechanisms (TenBarge et al. 2012). In addition to successfully modeling the energy spectra in nonlinear kinetic simulations of turbulence (Howes et al. 2008b, 2011), such cascade models can be applied to develop a predictive capability for the plasma heating resulting from the dissipation of turbulence in weakly collisional space and astrophysical plasmas (Howes 2010, 2011).

Finally, the characteristic eigenfunctions of the linear kinetic plasma waves can be exploited in an attempt to identify the characteristic nature of the turbulent solar wind fluctuations (Sahraoui et al. 2009; Howes & Quataert 2010; Gary et al. 2010; Podesta & Gary 2011; He et al. 2011; Salem et al. 2012; Smith et al. 2012; Chen et al. 2013a,b). There are also a couple of other lines of argument that support the relevance of linear wave properties to the strongly turbulent solar wind plasma. First, an important feature of solar wind turbulence is the observation that linear kinetic temperature anisotropy instabilities appear to constrain the limits of the temperature anisotropy $T_\| / T_\perp$ of ions in the solar wind (Kasper et al. 2002; Hellinger et al. 2006; Bale et al. 2009). Second, from fundamental theorems about dimensional analysis, i.e. the Pi Theorem, the dimensionless parameters upon which the linear theory depends will necessarily also be parameters upon which the nonlinear theory depends (since linearization involves dropping terms, never adding), although the nonlinear theory undoubtedly depends on additional dimensionless parameters.

7. CONCLUSION

The quasilinear premise is a hypothesis for the modeling of plasma turbulence in which the turbulent fluctuations are represented by a superposition of randomly phased linear wave modes, and energy is transferred among these wave modes via nonlinear interactions. Although a large body of work on plasma turbulence either explicitly or implicitly assumes the relevance of some linear plasma wave properties, the nonlinearity inherent in turbulent interactions raises obvious questions about the relevance of linear theory. This paper attempts to present a broad range of theoretical, numerical, and observational evidence in the attempt to evaluate the validity of the quasilinear premise. After defining the quasilinear premise precisely, we highlight the the aspects of turbulence that can and cannot be described by such an approach: turbulent fluctuation properties such as the eigenfunction, frequency, and collisionless damping rate as well as second-order statistics such as the energy spectra or magnetic helicity of the turbulence can be described by a model of turbulence based on the quasilinear premise; third-order and higher order statistics, such as intermittency and coherent structures, such as current sheets, cannot be investigated using such an approach.

We present a wide range of theoretical arguments in support of the relevance of linear wave properties even in a strongly turbulent plasma, motivated by the mathematical properties of the nonlinear equation of evolution for an incompressible MHD plasma. We present an anal-
magnetic field are consistent with the predictions of critical balance. Simulations again yield contradictory results, but the line dividing these conflicting results appears to coincide with the method used to determine the frequency of the turbulent dynamics employing all three dimensions in space appear to largely find the frequency of the turbulent fluctuations consistent with Alfvén and kinetic Alfvén waves. Measurements of the the magnetic energy spectrum as a function of the angle of the solar wind flow with respect to the magnetic field also find support for critical balance found when the magnetic field direction is determined locally.

The question of the validity of the quasilinear premise—that linear wave properties are relevant to strong plasma turbulence—clearly remains to be settled. On balance, however, we argue here that the bulk of the evidence appears to support it as a valuable means of modeling turbulence. The utility of the quasilinear premise for the study of plasma turbulence, however, may also be judged a posteriori by the insights gained from such an approach, and we review a number of studies, including those using the novel synthetic spacecraft data method, that have succeeded in more strongly constraining the fundamental nature of plasma turbulence.

The ultimate goal of turbulence models based on the quasilinear premise is to develop the capability to predict the spectrum of turbulent fluctuations, their dissipation, and the resulting plasma heating.

Finally, we discuss observational evidence from turbulence in the solar wind that supports the quasilinear premise, including multi-spacecraft k-filtering analyses that find plasma-frame frequencies of the turbulent fluctuations consistent with Alfvén and kinetic Alfvén waves. This work was supported by NSF CAREER AGS-1054061 and NASA NNX10AC91G.

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