Individual Estimates of the Virial Factor in 10 Quasars: Implications on the Kinematics of the Broad-line Region

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Abstract

Assuming a gravitational origin for the Fe III λ2039-2113 redshift and using microlensing based estimates of the size of the region emitting this feature, we obtain individual measurements of the virial factor, f, in 10 quasars. The average values for the Balmer lines, \( f_{\text{Balmer}} \), is 0.43 ± 0.20 and \( f_{\text{FeIII}} \) = 0.50 ± 0.24, are in good agreement with the results of previous studies for objects with lines of comparable widths. In the case of Mg II, consistent results, \( f_{\text{MgII}} \), can be also obtained accepting a reasonable scaling for the size of the emitting region. The modeling of the cumulative histograms of individual measurements, CDF(\( f \)), indicates a relatively high value for the ratio between isotropic and cylindrical motions, a ~ 0.4–0.7. On the contrary, we find very large values of the virial factor associated to the Fe III λ2039-2113 blend, \( f_{\text{FeIII}} \) = 14.3 ± 2.4, which can be explained if this feature arises from a flattened nearly face-on structure, similar to the accretion disk.

1 Unified Astronomy Thesaurus concepts: Active galaxies (17); Quasars (1319); Supermassive black holes (1663)

1. Introduction

The measurement of the mass of central super massive black holes (SMBH) in quasars is mainly based on the application of the virial theorem to the emitters that give rise to the observed broad emission lines (BELs). The basic idea is to use the Doppler broadening of these lines, \( \Delta V \), related to the motion of the emitters under the gravitational pull of the SMBH, to estimate its mass using the virial equation,

\[
M_{\text{BH}} = f \frac{\Delta V^2 R}{G}.
\]

A measurement of the radius of the emitting region, \( R \), obtained from reverberation mapping (RM) or other alternative method is also needed (see a summary of techniques in Campitiello et al. 2019). This equation is affected by a scaling factor, \( f \), which encompasses all the information about the unknown geometry and dynamics (which could include nongravitational forces) of the BLR. Usually we have only statistical information about this factor which, in principle, may be very different from object to object and for different emission lines. For these reasons, \( f \) is commonly considered the major source of uncertainty in virial mass determinations. However, the large experimental uncertainties in the estimate of SMBH masses from primary methods, the complexity of the BELs, and the lack of agreement in the definition and systematics of the procedures to estimate the line widths, make this a non-obvious conclusion. In any case, the knowledge of the range of values spanned by \( f \) is very important, not only to analyze the limitations and possibilities of individual and average mass determinations using the virial, but also to understand the physics of the BLR.

To estimate the virial factor, \( f \), an alternative way 11 to measure the mass of the SMBH is needed. This has been done in a number of previous works assuming that active galactic nuclei (AGNs) follow the \( M_{\text{BH}} \sim f_{\text{ Stokes}} \) relation of inactive galaxies (Ferrarese & Merritt 2000; Gebhardt et al. 2000). The resulting measurements span a large range of values (see the compilation by Campitiello et al. 2019) from \( f_{\text{ Stokes}} = 2.8 \pm 0.6 \) (Onken et al. 2004) to \( f_{\text{ Stokes}} = 5.5 \pm 1.8 \) (Graham et al. 2011), with a large scatter. Part of this scatter may be intrinsic. In fact, some studies (Collin et al. 2006; Ho & Kim 2014) indicate that AGNs can be separated in populations with different values of \( f \). Using mass estimates based on accretion disk fitting, Campitiello et al. (2019) derive \( f_r \) for a large sample of objects, obtaining values \( \langle \log f_r \rangle = 0.63 \pm 0.49 \), consistent to within uncertainties with other values 12 in the literature. However, the large scatter (\( f_r \) spans a range of ~2 orders of magnitude), likely due to the uncertainties in \( f \) measurements, makes difficult the interpretation of the results.

A new method to infer SMBH masses (Mediavilla et al. 2018, 2019) based on the measurement of the redshift of the Fe III λ2039-2113 blend is now available. This method is free

11 i.e., independent from the virial.

12 The value of \( f \) is different if we take \( \sigma \) or FWHM as the line width indicator. For Gaussian line profiles, FWHM = 2.35\( \sigma \), but in many applications of the virial using emission line profiles, \( \sigma \) is the second moment of the experimental line profile, and the FWHM/\( \sigma \) ratio depends on the profile shape (Collin et al. 2006).

13 This value can change with the BH spin range considered in the models.
from geometrical effects and largely insensitive to nongravitational forces. We propose to use it, in combination with microlensing based estimates of the size of the region emitting the Fe IIIλ2039-2113 blend, to determine f. We will apply the method to the sample of 10 quasars of Capellupo et al. (2015, 2016) for which redshifts of the Fe IIIλ2039-2113 blend have been measured (Mediavilla et al. 2019), and to the composite quasar spectra of the Baryon Oscillation Spectroscopic Survey (BOSS; Jensen et al. 2016).

The paper is organized as follows. In Section 2.1, we derive virial factors for the sample of 10 quasars and composite BOSS spectra. In Section 2.2, we analyze the experimental correlation between the redshift of the Fe IIIλ2039-2113 blend and the squared widths of several emission lines. Section 3 is devoted to discuss the geometry and the kinematics of the BLR. Finally, the main conclusions are summarized in Section 4.

2. Results

2.1. Virial Factor Determinations

2.1.1. Individual Quasar Spectra from Mejía-Restrepo et al. (2016)

Using the virial theorem,

\[ M_{\text{BH}} \simeq f_{\beta} \frac{\text{FWHM}_{\beta}^{2} R_{\beta}}{G}, \]

and the equation relating (under the gravitational redshift hypothesis) the SMBH mass with the redshift of the Fe III UV lines (see Equation (3) in Mediavilla et al. 2018),

\[ M_{\text{BH}} \simeq \frac{2 e^{2}}{3 G} \left( \frac{\Delta \lambda}{\lambda} \right)_{\text{Fe III}} R_{\text{Fe III}}, \]

we can obtain the virial factor in terms of the squared widths, the redshifts, and the ratio between sizes:

\[ f_{\beta} \simeq 2 \frac{R_{\text{Fe III}}}{3 R_{\beta}} \left( \frac{\Delta \lambda}{\lambda} \right)_{\text{Fe III}} \text{FWHM}_{\beta}^{2}(c)^{2}, \]

(notice that here and hereafter we calculate f taking the FWHM as the line width indicator). To estimate \( R_{\beta} \), we can use the size versus luminosity, R-L, scaling adopted by Mejía-Restrepo et al. (2016),

\[ R_{\beta} = 538 \left( \frac{\lambda L_{\lambda 250}}{10^{46} \text{erg s}^{-1}} \right)^{0.65} \text{lt-day}. \]

In the case of \( R_{\text{Fe III}} \), we can use the average microlensing size estimated by Fian et al. (2018) rescaling it by applying the \( R \propto \sqrt{\lambda \lambda} \) relationship from the photoionization theory. Inserting the value of \( \langle R \rangle \) from Fian et al. (2018) and the average of the square root of the luminosities of the quasars, \( \langle \sqrt{\lambda \lambda} \rangle \), used by these authors to infer \( \langle R \rangle \), we obtain,

\[ R_{\text{Fe III}} = 13.3^{+6}_{-5} \left( \frac{\lambda L_{\lambda 250}}{10^{45.79} \text{erg s}^{-1}} \right)^{0.5} \text{lt-day}. \]

Using the luminosities from Mejía-Restrepo et al. (2016) and Equations (4)–(6), we compute the virial factors, \( f_{\beta} \) (see Table 1), for each of the 10 quasars from Mejía-Restrepo et al. (2016) considered in Mediavilla et al. (2019). In Table 1 we also include the virial factors corresponding to Hα obtained using the same R-L relationship as for Hβ. The cumulative histograms of values are represented in Figures 1 and 2. The mean values of the distributions are (1σ uncertainties), \( \langle f_{\beta} \rangle = 0.43 \pm 0.20 \) and \( \langle f_{\text{Hα}} \rangle = 0.50 \pm 0.24 \).

To analyze the impact of the choice of the R-L slope in Equations (5) and (6), we repeat the calculations applying the widely used relationship, with slope 0.53, found for the entire luminosity range by Bentz et al. (2013), to \( R_{\beta} \) and the same slope to \( R_{\text{Fe III}} \). The results imply a ~30% systematic shift of the virial factors toward higher values. In general, a shift of the \( R_{\beta} \) (\( R_{\text{Fe III}} \)) slope toward smaller (higher) values would result in a systematic increase of the virial factor estimates.

We lack on a reliable R-L scaling relationship for Mg II. However, recent results from the Sloan Digital Sky Survey (SDSS) RM project (Shen et al. 2019) indicate that the Mg II emitting region is ~2 and ~1.4 times greater than the regions corresponding to C IV and C III], respectively. The larger size of the region emitting the C III] line seems reasonable as this

\[ 14 \text{ The slope of 0.65 adopted for Hβ is motivated by the trend of the slope at the high-luminosity end of Bentz et al. (2013) data, which is suitable for our sample of objects taken from Mejía-Restrepo et al. (2016).} \]

\[ 15 \text{ Instead of using this relationship directly inferred from microlensing and photoionization theory, we could have used the R-L relationships by Mediavilla et al. (2018), which is quite similar, or by Mediavilla et al. (2019), indirectly derived to match the redshift-based and virial masses obtained using emission lines arising from much larger regions.} \]
line shows less variability and is less prone to microlensing than C IV, which may exhibit a variability comparable to that of the Balmer lines (i.e., $R_{\text{C IV}} \sim R_{\text{H}\beta}$). Assuming, $R_{\text{Mg II}} \sim 2R_{\text{C IV}} \sim 2R_{\text{H}\beta}$, we obtain the virial factors for Mg II, $f_{\text{Mg II}}$, (see Table 1). The mean value for the virial factor $\langle f_{\text{Mg II}} \rangle \sim 0.44 \pm 0.21$, is consistent with the ones obtained for the Balmer lines.

2.1.2. Composite Quasar Spectra from BOSS

From the values of FWHM$_{\text{Mg II}}$ and $(\Delta\lambda/\lambda)_{\text{Fe III}}$ of BOSS composite spectra (Mediavilla et al. 2018) and Equation (4), we estimate $f_{\text{Mg II}}$ (see Table 2). We use the same assumption

\[ f_{\text{Mg II}} \]

\[ \text{For the BOSS composite spectra that include the Mg II emission line within the wavelength coverage.} \]
about the size of the Mg II region, \( R_{\text{Mg II}} \sim 2 R_{\text{CIV}} \sim 2 R_{\text{H}\beta} \). The average virial factor inferred from BOSS composites \((f_{\text{Mg II}}) = 0.35 \pm 0.15\) is consistent with the ones inferred in the previous section from the individual spectra. In Mediavilla et al. (2018), we have also derived values of FWHM\(_{\text{Fe III}}\) for the BOSS quasar composite spectra. Inserting these values in Equation (4) (in this case the ratio between sizes is trivially 1) we obtain very high values for the virial factor, \( f_{\text{Fe III}} \) (see Table 2), with mean, \( \langle f_{\text{Fe III}} \rangle = 14.3 \pm 2.4 \).

### 2.2. Correlation between the Fe III λ2039-2113 Redshifts and the Squared Widths of the BELs

If, as it seems to be in our case, the virial factor does not change very much from object to object, and there is a proportionality between H\(\beta\) and Fe III λ2039-2113 sizes, \( R_{\text{Fe III}} \propto R_{\text{H}\beta} \), Equation (4) can be rewritten to express a correlation\(^{17}\) between FWHM\(_{\text{H}\beta}\) and \( \frac{\Delta \lambda}{\lambda} \) \( \text{Fe III} \),

\[
\left( \frac{\text{FWHM}_{\text{H}\beta}}{c} \right)^2 \simeq 2 \frac{1}{3} R_{\text{Fe III}} \frac{R_{\text{H}\beta}}{R_{\text{H}\beta}} \left( \frac{\Delta \lambda}{\lambda} \right) \text{Fe III}. \tag{7}
\]

This correlation has been experimentally found for the H\(\beta\), H\(\alpha\), and Mg II emission lines (Mediavilla et al. 2019 and for the Fe III λ2039-2113 blend (Mediavilla et al. 2018). We can directly fit these correlations to derive statistical estimates of the virial factors for the different lines and compare them with the individual determinations. We take the average value for the ratio between sizes, \( R_{\text{Fe III}}/R_{\text{H}\beta} \), obtained using Equations (5) and (6). We also take, \( R_{\text{H}\alpha} = R_{\text{H}\beta} \) and \( R_{\text{Mg II}} = 2 R_{\text{H}\beta} \).

#### 2.2.1. Individual Quasar Spectra from Mejia-Restrepo et al. (2016)

Using the sample of quasars of Capellupo et al. (2015, 2016), Mediavilla et al. (2019) found a correlation between the redshift of the Fe III λ2039-2113 blend and the squared widths of H\(\beta\), H\(\alpha\), and Mg II (see Figure 1 of Mediavilla et al. 2019). A linear fit to Equation (7) results in values very similar to the ones inferred from the average of the individual estimates: \( f_{\text{H}\beta} = 0.43^{+0.06}_{-0.05} \), \( f_{\text{H}\alpha} = 0.46^{+0.07}_{-0.05} \), \( f_{\text{Mg II}} = 0.45^{+0.06}_{-0.06} \).

#### 2.2.2. Composite Quasar Spectra from BOSS

In Mediavilla et al. (2019) it is found (see their Figure 2) that the redshifts versus squared widths correlation derived from the sample of individual quasar spectra is also very well matched by the Fe III λ2039-2113 redshifts and FWHM\(_{\text{H}\alpha}\) obtained from the high signal-to-noise ratio composite quasar spectra of BOSS. In principle, for composite spectra, the good correlation between redshifts and widths do not necessarily imply an invariance of the virial factor, as it may be averaged during the taking process to obtain the composites. However, the good quantitative matching of the relationships corresponding to the individual quasar spectra from Mejia-Restrepo et al. (2016) and to the BOSS composites would be virtually impossible if the virial factors were very different among quasars. A linear fit to Equation (7) results in \( f_{\text{Fe III}}^{\text{BOSS}} = 0.37^{+0.02}_{-0.01} \).

Finally, a correlation was also found between the redshifts and squared widths of the Fe III λ2039-2113 blend (Mediavilla et al. 2018). This experimental correlation reproduces the expected linear relationship between squared widths and redshifts but does not match the \( (\text{FWHM}^2/\Delta \lambda/\lambda)) \) ratio corresponding to the relationship based on the FWHM\(_{\text{Fe III}}\). In this case the linear fit to Equation (7) gives \( f_{\text{Fe III}}^{\text{BOSS}} = 13.89^{+0.60}_{-0.56} \). This is, likely, evidence of a different origin for the Mg II and Fe III emissions.

### 3. Discussion

The mean values obtained for the Balmer lines, \( \langle f_{\text{H}\beta} \rangle = 0.43 \pm 0.20 \) and \( \langle f_{\text{H}\alpha} \rangle = 0.50 \pm 0.24 \), and the estimate for Mg II, \( \langle f_{\text{Mg II}} \rangle \sim 0.44 \pm 0.21 \), match well the \( f \sim 0.52 \) value found by Collin et al. (2006) for objects (Population B according to Sulentic et al. 2000) with FWHM \( \geq 4000 \text{ km s}^{-1} \) when the mean spectrum is used to measure the FWHM.\(^{18}\) These values are also in agreement with the results by Ho & Kim (2014) for pseudobulges (0.5 \pm 0.2).

\(^{17}\) We write the correlation for the case of H\(\beta\). Analogous relationships can be written for H\(\alpha\) and Mg II.

\(^{18}\) The average widths of the Balmer lines for the quasars in our sample are: \( \langle \text{FWHM}_{\text{H}\beta} \rangle = 4805.7 \text{ km s}^{-1} \) and \( \langle \text{FWHM}_{\text{H}\alpha} \rangle = 4514.0 \text{ km s}^{-1} \).

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Table 2

| Composite # | log(\(\lambda_{\text{L}3300}/\text{erg s}^{-1}\)) | log(\(\lambda_{\text{L}5100}/\text{erg s}^{-1}\)) | \(f_{\text{Mg II}}\) | \(f_{\text{Fe III}}\) |
|-------------|-----------------|-----------------|-----------------|-----------------|
| 2           | 45.35           | 45.05           | 0.32 ± 0.04     | 13.71 ± 1.97    |
| 4           | 45.66           | 45.09           | 0.55 ± 0.06     | 11.58 ± 1.38    |
| 5           | 45.68           | 45.38           | 0.33 ± 0.04     | 14.71 ± 2.02    |
| 6           | 45.64           | 45.62           | 0.21 ± 0.02     | 11.73 ± 1.64    |
| 7           | 46.04           | 45.51           | 0.54 ± 0.06     | 12.71 ± 1.60    |
| 8           | 46.06           | 45.76           | 0.32 ± 0.04     | 12.20 ± 1.61    |
| 9           | 46.02           | 45.99           | 0.21 ± 0.02     | 12.11 ± 1.58    |
| 11          | 45.66           | 45.09           | 0.61 ± 0.06     | 12.91 ± 1.47    |
| 14          | 45.68           | 45.38           | 0.33 ± 0.04     | 15.06 ± 2.09    |
| 15          | 45.65           | 45.63           | 0.20 ± 0.03     | 16.87 ± 3.28    |
| 16          | 46.04           | 45.51           | 0.44 ± 0.04     | 17.48 ± 2.31    |
| 17          | 46.06           | 45.76           | 0.26 ± 0.03     | 11.28 ± 1.55    |
| 18          | 46.02           | 45.99           | 0.16 ± 0.02     | 13.79 ± 2.25    |
| 23          | 45.68           | 45.38           | ...             | 18.04 ± 2.76    |
| 25          | 46.05           | 45.52           | ...             | 18.39 ± 2.48    |
| 26          | 46.06           | 45.76           | ...             | 16.16 ± 2.42    |

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Our measurements are consistent with the results of Campitiello et al. (2019), $f = 0.77^{+1.61}_{-0.52}$, although the large scatter lessens the interest of the comparison. In any case, in the compilation by Campitiello et al. (2019), our results are in good agreement with the estimates obtained from Grier et al. (2019) data.

3.1. Restrictions on the Range of Virial Factor Estimates

As far as the individual error estimates (Table 1) are comparable with the standard deviations of the samples, we cannot be sure that the range spanned by the virial factors is related to intrinsic variations. On the other hand, assuming normality for the parent distribution, a t-test rejects the null hypothesis that the mean of the $f_{H\beta}$ population is greater (smaller) than 0.63 (0.23) with a p-value equal to 0.006 (0.006), i.e., the statistical restriction of values imposed by the data is rather tight. Similar results are derived for $f_{H\alpha}$ and $f_{Mg\ II}$.

The range of values spanned by the $f_{H\beta}$, $f_{H\alpha}$, and $f_{Mg\ II}$ virial factors is notably small if we compare it with the scatter of the experimental results from Campitiello et al. (2019), $f = 0.77^{+1.61}_{-0.52}$. However, any comparison between both results should be taken with care. While Campitiello et al. (2019) gather virial estimates from several sources, which use a rather heterogeneous sample of objects and methods, we apply a single method to a quasar sample of limited size, covering a relatively small range in luminosity. Thus, the comparatively narrow range of intrinsic variability that can be inferred from our data, cannot be extrapolated, in general.

Regarding the BLR structure of our sample of quasars, the relative invariance of $f_{H\beta}$, $f_{H\alpha}$, and $f_{Mg\ II}$ indicates that the geometry and the dynamics of the distribution of emitters are in some way regular among them. Thinking in two extreme cases, the small range spanned by the virial factors can be achieved if the kinematics of the emitters is more or less isotropic (hence insensitive to inclination) or, alternatively, if the emitters lie in a flattened structure whose orientation is limited to a narrow range of values (possibly due to the observational bias limiting quasars identification to objects with low axial inclination). On the other hand, the relative invariance of $f$ also implies that the impact of nongravitational forces is small or restricted to a relatively narrow range of values.

3.2. Isotropic versus Cylindrical Kinematics in the BLR

According to the previous discussion, it is interesting to compare our results with the expectations corresponding to isotropic and cylindrical kinematics. In the case of an idealized isotropic configuration of orbits (a shell of circular orbits with all the possible orientations, for instance) the resulting emission line profile (see Appendix) is a rectangular function of FWHM$ = 2 \frac{G M_{d}}{R} = 2V_{Kep}$. Thus, according to Equation (2), the associated virial factor would be $f_{iso} = 1/4$. In the cylindrical case, the Doppler broadening is conservatively bound by the maximum projected Keplerian velocity so that FWHM $< 2V_{Kep} \sin i$, where $i$ is the inclination (0° is face-on). Consequently, $f_{cyl} > 1/(4 \sin^{2}i)$. According to the unified scheme (Antonucci & Miller 1985), quasars cannot be seen edge on and, in fact, it seems that their inclinations are confined to a narrow range of values not far from 0. If we take, conservatively, $i \leq i_{90} = 30$, we obtain $f_{cyl} > 1$.

None of the individual virial factors obtained from the Balmer lines is greater than 1 and, on average, they are relatively close to $f_{iso}$. Thus, our results support that the kinematics of the Balmer line emitters has a significant contribution from isotropic motion. In the case of $Mg\ II$ we obtain similar results, provided that $R_{Mg\ II}/R_{H\beta} \sim 2$ (see Section 2.1.1).

We can also use the simple parameterization proposed by Collin et al. (2006, Equation (11) of these authors; see also Decarli et al. 2008) to discuss the degree of isotropy of the emitters kinematics. According to this model the line width is given by,

$$FWHM = (a^{2} + \sin^{2}i)1/2 2V_{Kep},$$

and the virial factor is,

$$f = \frac{1}{4(a^{2} + \sin^{2}i)},$$

where $a$ can be interpreted as the $V_{\text{turbulent}}/V_{Kep}$ ratio (Collin et al. 2006), although we prefer to interpret it as the $V_{\text{isotropic}}/V_{\text{cylindrical}}$ Ratio. In Figure 3 we have represented the virial factor corresponding to different values of $a$ and $i$.

According to this figure, to obtain values of the virial factor $f \sim 0.4$ we need to consider relatively large values of $a$ if $i \leq 40$. From Equation (9) it is possible to derive the probability density function of virial factors, $p(f)$, corresponding to a random inclination of the quasars,

$$p(f) = \frac{1}{8f^{2}} \frac{1}{\sqrt{1 - \frac{1}{4f} + a^{2}}}.$$  

for $1/(4a^{2}) \geq f > 1/(4(1 + a^{2})$. From Equation (10) we can obtain the cumulative probability density function of virial
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for $f \leq 1/(4a^2)$. Using a $\chi^2$ criterion, we compare the observed and theoretical cumulative histograms to obtain best-fit estimates of $a$ and $\iota$. The resulting best-fit models are plotted in Figure 1 for H$\beta$, H$\alpha$, and Mg II. The best-fit values for the inclination, $\iota$, and the isotropy parameter, $a$, are: $\iota_{H\beta} \sim 59^\circ$, $\iota_{H\alpha} \sim 57^\circ$, $\iota_{Mg\,II} \sim 57^\circ$, $a_{H\beta} \sim 0.50$, $a_{H\alpha} \sim 0.44$ and $a_{Mg\,II} \sim 0.49$ (in the case of Mg II we assume $R_{Mg\,II}/R_{H\beta} \sim 2$, see Section 2.1.1). If we limit the inclination to $\iota \lesssim 30^\circ$, which is more likely for quasars, we obtain (see Figure 2): $\iota_{H\beta} \sim 30^\circ$, $\iota_{H\alpha} \sim 28^\circ$, $\iota_{Mg\,II} \sim 25^\circ$, $a_{H\beta} \sim 0.68$, $a_{H\alpha} \sim 0.62$ and $a_{Mg\,II} \sim 0.67$. In principle the fits look better when the restriction in the inclination is not applied. However, the improvement in the fits may be only apparent as the spreading of the values of $f$ around the mean value, which is easier to fit when the inclination plays a role, is likely due to experimental errors. In any case, the CDF analysis favors a rather high value for the isotropy parameter, $a \sim 0.4-0.7$.

The virial factors associated to the Fe $\text{III}\lambda\lambda2039-2113$ blend, $\langle f_{\text{Fe}\,\text{III}} \rangle = 14.3 \pm 2.4$, are really high as compared with the results for the Balmer lines, although Liu et al. (2017) find values in the $f \sim 8$ to $\sim 16$ range using redshifts (interpreted as gravitational) and widths of several emission lines in Mrk 110. The distribution of the Fe III emitters in a flattened structure with an inclination of a few degrees can explain the large observed values of $f_{\text{Fe}\,\text{III}}$, which imply $a \lesssim 0.13$. This is in agreement (Guerras et al. 2013a, 2013b; Fian et al. 2018) with the large microlensing magnifications experimented by the Fe III, comparable to those of the continuum and much larger than those of other emission lines (like H$\beta$ or C IV), supporting that the Fe III emission arises from a region related to the accretion disk, while the emitters of H$\beta$ and H$\alpha$ belong to a larger, less flattened structure.

4. Conclusions

Using a new method to measure SMBH masses, based on the redshift of the Fe $\text{III}\lambda\lambda2039-2113$ blend, we estimate the virial factor in quasars and study the kinematics of the emission line emitters. The main conclusions are the following:

1. We have obtained individual measurements of the virial factor in 10 quasars corresponding to the Balmer lines, H$\beta$, H$\alpha$, and to Mg II. The mean values for the Balmer lines, $\langle f_{H\beta} \rangle = 0.43 \pm 0.20$ and $\langle f_{H\alpha} \rangle = 0.50 \pm 0.24$, agree with previous estimates for lines of comparable widths when the FWHM are derived from the mean spectrum as it is our case. We obtain similar results for Mg II, $\langle f_{Mg\,II} \rangle \sim 0.44 \pm 0.21$, scaling the Mg II to C IV according to Shen et al. (2019). $R_{Mg\,II} \sim 2R_{C\,IV}$, and C IV to H$\beta$, $R_{C\,IV} \sim R_{H\beta}$. We have also measured the virial factors associated to the Fe $\text{III}\lambda\lambda2039-2113$ emitters obtaining very high values, $\langle f_{\text{Fe}\,\text{III}} \rangle = 14.3 \pm 2.4$, only comparable to other measurements based on gravitationally redshifted lines (Liu et al. 2017).

2. We have also measured the virial factors for H$\beta$, H$\alpha$, and Mg II on one side, and Fe III on the other. This fact is reflected in the correlations between the Fe $\text{III}\lambda\lambda2039-2113$ redshift and the squared emission line widths. The virial factors directly derived fitting these relationships are very similar to the average of the individual measurements.

3. The relatively small scatter in virial factors not only supports a significant amount of regularity in the geometry and dynamics of the emission line regions among the quasars of the sample (not necessarily the same geometry and dynamics for different emission line regions), but also that the virial factor might not be the main cause of the scatter found in virial mass determinations.

5. According to the virial measurements, the BLR region associated to the Balmer lines may be a 3D structure, with a significant contribution from isotropic motion. Comparing the observed cumulative histograms of virial factors, CDF($f$), with the predictions of a simple model, we find that the ratio between isotropic and cylindrical contributions to the kinematics is rather high, $a \sim 0.4-0.7$. On the contrary, the Fe $\text{III}\lambda\lambda2039-2113$ emitters likely belong to an almost face-on flattened region, which may be identified with the accretion disk.

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Appendix

Line Profile for an Isotropic Spherical Shell of Emitters

We suppose that the emitters are confined to a spherical shell and move tangent to the surface with velocity,

$$v = |v| \cos \alpha \, e_\theta + |v| \sin \alpha \, e_\phi,$$  \hspace{1cm} (A1)

where $\alpha$ is uniformly distributed between 0 and $2\pi$. The line profile for an observer located at $z = \infty$ can be obtained from,

$$F_\lambda \propto \int_0^{2\pi} d\phi \int_S dS \, \delta \left( \lambda - \lambda_0 \left( 1 + \frac{v \cdot e}{c} \right) \right),$$  \hspace{1cm} (A2)

where $S$ is the surface of the spherical shell. Taking into account that $v \cdot e = -|v| \sin \theta$, the integral over $S$ can be written as,

$$\int_0^{2\pi} d\phi \int_{-\pi/2}^{\pi/2} d\theta \sin \theta \, \delta \left( \lambda - \lambda_0 \left( 1 - \frac{|v|}{c} \cos \alpha \sin \theta \right) \right).$$  \hspace{1cm} (A3)
Defining, \( f = \lambda - \lambda_0 \left( 1 - \frac{|v|}{\lambda} \cos \alpha \sin \theta \right) \) and performing the integration over \( \phi \), Equation \((A3)\) can be written as,

\[
2\pi \int_{f}^{v} \frac{df}{\lambda_0 \left| \frac{|v|}{c} \cos \alpha \cos \theta \right|} \sin \theta \delta(f).
\]

(A4)

This integral is null except when \( f = 0 \), that is, \( \sin \theta = \frac{x}{\cos \alpha} \), which implies, \( \frac{c}{\cos \alpha} \lambda - \lambda_0 \). Thus we can integrate Equation \((A4)\) to obtain,

\[
\frac{2\pi}{\lambda_0 \left| \frac{|v|}{c} \right|} \frac{x}{\cos \alpha} \sqrt{\cos^2 \alpha - x^2},
\]

with the condition, \( \frac{x}{\cos \alpha} \leq 1 \). Now, Equation \((A2)\) can be written as,

\[
F_x \propto \int_{0}^{\arccos x} d\alpha \frac{2\pi}{\lambda_0 \left| \frac{|v|}{c} \right|} \frac{x}{\cos \alpha} \sqrt{\cos^2 \alpha - x^2}.
\]

(A6)

with \( x \leq 1 \) and \( F_x = F_t \). The result of this integral is a constant,

\[
F_x \propto \frac{\pi^2}{\lambda_0 \left| \frac{|v|}{c} \right|^3},
\]

(A7)

for \(-1 \leq x \leq 1\). Thus, the resulting profile is a rectangular function of width \( 2|v| \).