Flavor mixing inspired by flipped SU(5) GUT

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We obtain a phenomenologically acceptable Cabibbo-Kobayashi-Maskawa matrix in a flipped SU(5) model inspired by the compactification of heterotic string $E_8 \times E_8$.

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I. INTRODUCTION

“How is the current allocation of flavors realized?” is the most urgent and also interesting one in the theoretical problems of the standard model (SM) of particle physics. Advocates of string theory for the heterotic string argue that string compactification is the most complete answer to this problem [1–6]. Along this line, we study a phenomenological Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [7, 8] from the recent R-parity model [9] obtained from [10]. Other phenomenological aspects are included in [9].

String compactifications aim at obtaining (i) large 3D space, (ii) standard-like models with three families, and (iii) no exotics at low energy (or vectorlike representations if they exist). Regarding a solution to item (i), the string landscape scenario is suggested [11], predicting about $10^{500}$ vacua for a reasonable cosmological constant (CC). Regarding item (ii), the standard-like models from heterotic string has been suggested from early days [12, 13] until recently [14–32]. Model constructions are discussed in detail in [33–35]. It has been suggested that by exploring the entire string landscape one might obtain statistical data which could lead to probabilistic experimental statements [36, 37]. Yet the clearest statement to date is that standard-like models are exceedingly rare [38, 39]. In addition, the flavor problem asks for a detailed model producing the observed CKM and Pontecorvo-Maki-Nakagawa-Sakada (PMNS) matrices [40, 41]. In this paper, we study the flavor problem analytically in the simplest orbifold compactification based on $Z_{12-1}$. Since the number of fields are over hundred in these standard-like models, we simplify further by choosing GUT models to ease the analytical study. Therefore, in addition we require supersymmetry (SUSY) and simple or semi-simple group grand unification (GUT) [42–48]. SUSY models have been widely used to introduce a mechanism for generating a hierarchically small electroweak (EW) scale compared to the GUT scale. Above the EW scale, SUSY must be broken since no superpartner has been observed up to a TeV scale [49]. In the model, therefore, SUSY breaking mechanism must be present. The gauge group at the GUT scale is taken as $G_{\text{GUT}} \times G_{\text{cond}}$ where the most probable $G_{\text{cond}}$ is SU(4) [50]. In SUSY models, R-parity $P_R = (-1)^{B(L) + 2S}$ dictates proton stability, where $B$ is baryon number, $L$ is lepton number, and $S$ is spin. For a conserved R-parity, it is usually assigned to a subgroup of $B - L$. From string compactification, R-parity was calculated before in this framework [28, 51, 52]. Because of dangerous dimension-5 operators, leading to proton decay, $Z_{4R}$ has been proposed in contrast to $Z_{2R}$ [53–57]. In this paper, we work for the model of [3] which introduced $Z_{4R}$ from a string GUT.

GUTs from string compactification favor the flipped SU(5) semi-simple GUTs [10, 58, 59] and anti-SU(7) [60]. For the simple group GUTs, SU(5), SO(10), and $E_6$, we need an adjoint representation to break the GUT groups down to the SM gauge group and it is impossible to obtain adjoint representation at the level 1 [33]. [Note, however, an adjoint representation of SO(10) was obtained in Ref. [62] at the level 3.] So, for simple group models and level 1, anti-SU(N) GUTs are relevant for phenomenological studies. The SUSY flipped SU(5) can allow a real and symmetric $Q_{em} = -\frac{1}{3}$ quark mass matrix, and hence the CP phase in the CKM matrix can be introduced from the $Q_{em} = \frac{2}{3}$ quark mass matrix. This observation makes it possible to obtain the form of the CKM matrix from the string GUT.

In Sec. [11] we point out key features on the mass matrices of Ref. [9], and in Sec. [111] we diagonalize the mass matrices suggested in Sec. [11]. Three real angles of the CKM matrix are determined dominantly by the diagonalization
of the $Q_{em} = -\frac{1}{3}$ quark mass matrix, and the weak CP phase is provided from the diagonalization of the $Q_{em} = +\frac{2}{3}$ quark mass matrix. Neutral singlets attached to have appropriate matrix elements are listed in [9]. Sec. IV is a conclusion.

II. MASS MATRICES INSPIRED BY FLIPPED SU(5)

In Ref. [9] based on the flipped SU(5) model of [10], a possible identification $Z_{4R}$ has been achieved, forbidding dimension-5 $B$ violating operators but allowing the electroweak scale $\mu$ term and dimension-5 $L$ violating Weinberg operator. The $Z_{4R}$ quantum numbers, $Q_{4R}$, of the SM fields and neutral singlets ($\sigma$’s), are presented in Ref. [9]. In the flipped SU(5), the $Q_{em} = -\frac{1}{3}$ quarks obtain masses by the coupling

$$-L_d^{ij} = f_{d}^{(d)}(d) \epsilon_{ijklm}^{10} T_{-1}^{ij} T_{-1}^{kklm} 5_{+2} + h.c.,$$

(1)

where the couplings $f_{d}^{(d)}$ are real parameters, $I$ and $J$ are flavor indices and $i, j, k, l, m$ are SU(5) indices, and the subscript is the U(1)$_X$ quantum number of SU(5) flip. $5_{+2}$ is usually denoted as $H_{dL}$ whose quantum numbers in SU(5) flip are given in Ref. [9]. Since interchange of the first $10$ and the second $10$ in Eq. (1) is possible, the $d$-type quark mass matrix is symmetric in the weak interaction basis. $f_{d}^{(d)}$ can include vacuum expectation values (VEVs) of SU(5) flip singlet fields presented in Ref. [9], and we choose these VEVs to be real. So, the $d$-type quark mass matrix is real and symmetric which can be diagonalized by an orthogonal matrix.

On the other hand, the $Q_{em} = +\frac{2}{3}$ quarks obtain masses by the coupling

$$-L_u^{ij} = f_{u}^{(u)}(u) \epsilon_{ijklm}^{10} T_{-1}^{ij} T_{-1}^{klm} 5_{-2} + h.c.,$$

(2)

which need not be symmetric under the exchange $I \leftrightarrow J$. So, the couplings $f_{u}^{(u)}$ need not be real parameters, and we will assume that SU(5) flip singlet VEVs in $f_{u}^{(u)}(u)$ can provide complex couplings. $5_{-2}$ is usually denoted as $H_{uL}$ whose quantum numbers in SU(5) flip are given in Ref. [9]. So, Eq. (2) can be diagonalized by a bi-unitary transformation.

The effective operators of neutrino masses in the SU(5) flip arise from

$$5_{+3,5_{+3}}, 5_{-2}, 5_{+3,5_{+3}}, 5_{+3,5_{+3}}, 5_{-2}, 5_{-2,5_{-2}}, 5_{-2,5_{-2}},$$

(3)

which were discussed in [9]. For the PMNS matrix, the observed data are not accurate enough to analyze it here at the level of the CKM matrix.

III. DIAGONALIZATION OF MASS MATRICES AND MIXING ANGLES

With the above guidelines from string compactification, let us parametrize the mass matrices to obtain the successful mixing matrices $V_{CKM}$ and $V_{PMNS}$.

$^3$ If there is a permutation symmetry $S_2$, for the exchange $1 \leftrightarrow 2$ (identifying $1 \rightarrow \Phi$ and $2 \rightarrow \Psi$), we consider the symmetric $S$ and the antisymmetric $A$ as

$$S = \frac{1}{\sqrt{2}}(\Phi + \Psi),$$

$$A = \frac{1}{\sqrt{2}}(\Phi - \Psi).$$

(4)

Products of these two singlets are

$$AA = \frac{1}{2}(\Phi \Phi + \Psi \Psi - \Phi \Psi - \Psi \Phi),$$

$$SS = \frac{1}{2}(\Phi \Phi + \Psi \Psi + \Phi \Psi + \Psi \Phi),$$

$$AS = \frac{1}{2}(\Phi \Phi - \Psi \Psi),$$

$$SA = \frac{1}{2}(\Phi \Phi - \Psi \Psi).$$

$^3$ $V_{PMNS}$ can be discussed in parallel but we postpone the study until more accurate data on the PMNS matrix elements are determined experimentally.
The following parametrization of mass matrices for $Q_{em} = -\frac{1}{3}$ and $+\frac{2}{3}$ quarks are used,

$$\tilde{M}_b \equiv \frac{M_{\text{weak}}^{(d)}}{m_b} = \begin{pmatrix} a_1, & c_1, & x_d \\ c_1, & b_1, & x_s \\ x_d, & x_s, & 1 + O(\varepsilon^3) \end{pmatrix},$$

$$\tilde{M}_t \equiv \frac{M_{\text{weak}}^{(u)}}{m_t} = \begin{pmatrix} a_2, & c_2, & x_u \\ c_2, & b_2, & x_c \\ x_u, & x_c, & 1 + O(\eta^3) \end{pmatrix}.$$  \(6\)

The CKM matrix is given by

$$V_{\text{CKM}} = V_u^\dagger V_d$$  \(7\)

where $V_u$ and $V_d$ are diagonalizing unitary matrices of L-handed $Q_{em} = +\frac{2}{3}$ and $Q_{em} = -\frac{1}{3}$ quark fields. The data for the CKM matrix is \([6]\)

$$|V_{\text{CKM}}| \simeq \begin{pmatrix} 0.97446, & 0.22452, & 0.00365 \\ 0.22438, & 0.97359, & 0.04214 \\ 0.00896, & 0.04133, & 0.999105 \end{pmatrix}.$$  \(8\)

$$|V_{\text{CKM}}|_{\text{err}} \simeq \begin{pmatrix} 0.00010, & 0.00044, & 0.00012 \\ 0.00044, & 0.00011, & 0.00076 \\ 0.00024, & 0.00074, & 0.000032 \end{pmatrix}.$$  \(9\)

The Wolfenstein parametrization is written as

$$V^W \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2}, & \lambda, & A\lambda^3(\rho - i\eta) \\ -\lambda, & 1 - \frac{\lambda^2}{2}, & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta), & -A\lambda^2, & 1 \end{pmatrix} + O(\lambda^3),$$  \(10\)

from which we take the following signs of the CKM elements

$$V_{\text{sign}} = \begin{pmatrix} + & + & + \\ - & + & + \\ + & - & + \end{pmatrix}.$$  \(11\)

A unitary matrix close to Eqs. \((8)\) and \((9)\), consistent with \((11)\), is

$$V_{\text{CKM}}^{\text{try}} = \begin{pmatrix} 0.974395 + 8.6794 \times 10^{-5}i, & 0.22481 + 5.66 \times 10^{-6}i, & 1.41 \times 10^{-3} - 3.33 \times 10^{-3}i, \\ -0.224672 - 1.416 \times 10^{-4}i, & 0.97352 - 7.46 \times 10^{-5}i, & 4.23 \times 10^{-2} + 5.32 \times 10^{-6}i \\ 8.132 \times 10^{-3} - 3.24 \times 10^{-3}i, & -4.151 \times 10^{-2} - 7.42 \times 10^{-4}i, & 0.99910 - 4.502 \times 10^{-5}i \end{pmatrix}.$$  \(12\)

To apply the Kim-Seo(KS) form \([63]\) of the Jarlskog determinant, we check the reality of the determinant of the CKM matrix. Indeed, it is almost real: $\det V_{\text{CKM}}^{\text{try}} = 1 - 1.35525 \times 10^{-20}i$. Also, it is almost unitary, i.e. $V_{\text{CKM}}^{\text{try}} V_{\text{CKM}}^{\text{try}\dagger}$ is

$$\begin{pmatrix} 1, & -2.535 \times 10^{-9} - 8.45 \times 10^{-12}i, & -3.63782 \times 10^{-9} - 6.7422 \times 10^{-12}i \\ -2.535 \times 10^{-9} + 8.45 \times 10^{-12}i, & 1, & 4.922 \times 10^{-9} - 3.42 \times 10^{-6}i \\ -3.63782 \times 10^{-9} + 6.7422 \times 10^{-12}i, & 4.922 \times 10^{-9} + 3.42 \times 10^{-6}i, & 1 \end{pmatrix}.$$  \(13\)

Note that the following mass ratios

$$Q_{em} = -\frac{1}{3} \text{ quarks : \(m_d/m_b \simeq 1.25 \times 10^{-3}, \ m_s/m_b \simeq 2.5 \times 10^{-2}\)},$$

$$Q_{em} = +\frac{2}{3} \text{ quarks : \(m_u/m_t \simeq 1.4 \times 10^{-5}, \ m_c/m_t \simeq 0.7 \times 10^{-2}\)}.$$  \(14\)
The mass hierarchy of \( Q_{em} = +\frac{2}{3} \) quarks is more pronounced than that of \( Q_{em} = -\frac{1}{3} \) quarks, and hence the mixing matrix of \( Q_{em} = +\frac{2}{3} \) quarks is closer to the identity than that of \( Q_{em} = -\frac{1}{3} \) quarks. Therefore, the first approximation of the CKM matrix, \( V_{CKM}^{O} \), is set from the diagonalization of \( Q_{em} = -\frac{1}{3} \) quark masses. An orthogonal matrix close to \( V_{CKM}^{O} \) of Eq. (12) is

\[
V_{CKM}^{O} = \begin{pmatrix}
0.974034, & 0.26385, & 0.00293448 \\
-0.226277, & 0.972973, & 0.0460661 \\
0.0075732, & -0.0455326, & 0.998934
\end{pmatrix}
\]

which gives \( \text{Det}V_{CKM}^{O} = 1 \) and

\[
V_{CKM}^{O}V_{CKM}^{O\dagger} = \begin{pmatrix}
1, & 7.343 \times 10^{-7}, & -7.494 \times 10^{-9} \\
7.343 \times 10^{-7}, & 1, & 1.355 \times 10^{-6} \\
-7.494 \times 10^{-9}, & 1.355 \times 10^{-6}, & 1
\end{pmatrix}.
\]

For the \( Q_{em} = -\frac{1}{3} \) quark fields, the mass eigenstate basis is related to the weak eigenstate basis by

\[
q_{dL}^{\text{weak}} = V_{dL}^{\text{mass}}, \quad q_{dR}^{\text{weak}} = U_{dR}^{\text{mass}}.
\]

Inspired from SU(5)\_flipped, let \( V_{d} \) parametrize (approximately) the real angles and \( V_{u} \) determine the CP phase in the CKM matrix. Along this strategy, using the real matrix \( V_{CKM}^{O} \) of Eq. (13), we determine

\[
\hat{M}_{d} \approx V_{CKM}^{O} \begin{pmatrix}
\frac{m_{d}}{m_{u}}, & 0, & 0 \\
0, & \frac{m_{s}}{m_{d}}, & 0 \\
0, & 0, & 1
\end{pmatrix} (V_{CKM}^{O})^{-1} \approx \begin{pmatrix}
2.47579 \times 10^{-3}, & 5.36634 \times 10^{-3}, & 2.68287 \times 10^{-3} \\
5.36632 \times 10^{-3}, & 2.58529 \times 10^{-2}, & 4.49073 \times 10^{-2} \\
2.68285 \times 10^{-3}, & 4.4906 \times 10^{-2}, & 1 - 2.07873 \times 10^{-3}
\end{pmatrix}.
\]

Small parameters \( \varepsilon \) (for \( Q_{em} = -\frac{1}{3} \) quarks) and \( \eta \) (for \( Q_{em} = +\frac{2}{3} \) quarks) are introduced for the matrices in Eq. (13). For \( \varepsilon \) from the ratio of (1,3) and (2,3) elements of Eq. (18), let us parametrize \( \hat{M}_{d} \) in terms of \( \varepsilon \) as

\[
\hat{M}_{d} \propto \begin{pmatrix}
\alpha_{1,1} \varepsilon^{3}, & \alpha_{1,2} \varepsilon^{3}, & \alpha_{1,3} \varepsilon^{3} \\
\alpha_{2,1} \varepsilon^{3}, & \alpha_{2,2} \varepsilon^{2}, & \alpha_{2,3} \varepsilon^{2} \\
\alpha_{3,1} \varepsilon^{3}, & \alpha_{3,2} \varepsilon^{2}, & 1 + \alpha_{3,3} \varepsilon^{3}
\end{pmatrix}.
\]

We chose the hierarchy \( \hat{M}_{d}(1,3) < \hat{M}_{d}(2,3) \) and \( \hat{M}_{d}(2,1) < \hat{M}_{d}(3,1) \) due to the symmetry properties of \( A \) and \( S \). To apply to the flipped SU(5) model discussed in Sec. [11] we use the following symmetrized \( \alpha_{i,j} \): \(^4\)

\[
\text{For } \varepsilon \approx 0.16078: \quad \alpha_{1,1} \rightarrow +0.595592, \quad \alpha_{1,2} \rightarrow +1.29096, \quad \alpha_{1,3} \rightarrow +0.645408, \\
\alpha_{2,1} \rightarrow +1.29096, \quad \alpha_{2,2} \rightarrow +1, \quad \alpha_{2,3} \rightarrow +1.73703 \approx \sqrt{3}, \\
\alpha_{3,1} \rightarrow +0.645403, \quad \alpha_{3,2} \rightarrow +1.73698 \approx \sqrt{3}, \quad \alpha_{3,3} \rightarrow -0.500073 \approx -\frac{1}{2}.
\]

We proceed to obtain a CKM matrix consistent with the above symmetric matrix. The hierarchical structure of Eq. (13) can be obtained as discussed in [8], which however is not pursued in this paper. Then, the mass eigenvalues of \( Q_{em} = -\frac{1}{3} \) quarks are

\[
e_{1} \approx 0.595592 \varepsilon^{3} - 1.66658 \varepsilon^{4} - 0.9926 \varepsilon^{5} + O(\varepsilon^{6}), \\
e_{2} \approx \varepsilon^{2} - 1.3506 \varepsilon^{4} + 0.9926 \varepsilon^{5} + O(\varepsilon^{6}), \\
e_{3} \approx 1 - 0.500073 \varepsilon^{3} + 3.01718 \varepsilon^{4} + O(\varepsilon^{6}),
\]

and, in terms of \( \varepsilon \) the diagonalizing matrix \( V_{d} \) is given by

\[
V_{d} \approx \begin{pmatrix}
1 - 0.833285 \varepsilon^{2}, & +1.29096 \varepsilon + 0.768885 \varepsilon^{2}, & +0.64541 \varepsilon^{3} \\
-0.992596 \varepsilon^{3} - 0.648831 \varepsilon^{4}, & +0.00471 \varepsilon^{4} - 1.5216 \varepsilon^{5}, & +0.64541 \varepsilon^{3} \\
-1.29096 - 0.768885 \varepsilon^{2}, & 1 - 0.83329 \varepsilon^{2}, & +1.73703 \varepsilon^{3} \\
-0.004693 \varepsilon^{3} + 1.52155 \varepsilon^{4}, & -0.9926 \varepsilon^{5} - 2.15741 \varepsilon^{4}, & +1.73703 \varepsilon^{3} \\
+1.59696 \varepsilon^{3} + 1.33553 \varepsilon^{4}, & -1.73698 \varepsilon^{2} - 1.2276 \varepsilon^{4}, & 1 - 1.50863 \varepsilon^{4}
\end{pmatrix}.
\]

\(^4\) Since \( \frac{m_{s}}{m_{u}} \approx 0.1 \), we vary \( \varepsilon^{2} \) near \( \frac{1}{40} \) such that \( \alpha_{2,2}, \alpha_{2,3}, \alpha_{3,2} \) and \( \alpha_{3,3} \) turn out to be simple numbers.
Similarly, the $Q_{em} = \pm \frac{2}{3}$ quark eigenstate bases can be related. However, it is more complicated than the $Q_{em} = -\frac{1}{3}$ quark mass matrix in two aspects. Firstly, the $Q_{em} = \frac{2}{3}$ quark mass matrix $M_u$ need not be symmetric and hence we need two unitary matrices, the L unitary matrix $V_u$ and the R unitary matrix $U_u$, and second the phase $e^{i\delta}$ is introduced through $V_u$. For the diagonalization, we need $U_u$ which does not appear in the CKM matrix. For the CKM matrix, therefore, an explicit form of $U_u$ is not needed in this paper. To place the CP phase, we study phenomenologically the form of $M_u$ when expanded in terms of the small parametr $\eta$. To minimize the effect of the phases, let us consider a hermitian matrix

$$\tilde{M}_u \tilde{M}_u^\dagger = V_u \begin{pmatrix} (m_u/m_t)^2 & 0 & 0 \\ 0 & (m_c/m_t)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_u^\dagger. \quad (23)$$

Because the R-unitary matrix does not appear explicitly in the CKM matrix, we employ the freedom on $U_u$. Namely, we use the hierarchy for the hermitian matrix $M_u \tilde{M}_u^\dagger$, but $\tilde{M}_u$ may not have the hierarchical form, as we will see later. Namely, we use the freedom of $\tilde{M}_u$ in obtaining the observed $V_{CKM}$. Using

$$V_u = V_d V_{CKM}, \quad (24)$$

with $V_d$ of (22) for $\epsilon = 0.16$, $V_u$ can be written as

$$\tilde{M}_u \tilde{M}_u^\dagger = \begin{pmatrix} 1.31795 \cdot 10^{-5} & 1.38244 \cdot 10^{-5} \cdot e^{i(\frac{\pi}{2} - 0.4153)} & 3.63026 \cdot 10^{-3} \cdot e^{i(\frac{\pi}{2} - 0.4153)} \\ 1.38244 \cdot 10^{-5} \cdot e^{i(-\frac{\pi}{2} + 0.4153)} & 6.34168 \cdot 10^{-5} & 3.79682 \cdot 10^{-3} \cdot e^{i(-0.00248)} \\ 3.63026 \cdot 10^{-3} \cdot e^{i(-\frac{\pi}{2} + 0.4125)} & 3.79682 \cdot 10^{-3} \cdot e^{i(0.00248)} & 1 - 2.7465 \cdot 10^{-5} \end{pmatrix} \quad (25)$$

from which we can calculate the matrix multiplication of two $(\tilde{M}_u \tilde{M}_u^\dagger)$’s

$$(\tilde{M}_u \tilde{M}_u^\dagger)_{1,1} - (\tilde{M}_u \tilde{M}_u^\dagger)_{1,3}(\tilde{M}_u \tilde{M}_u^\dagger)_{1,3}^\ast = 6.97823 \cdot 10^{-10},$$

$$(\tilde{M}_u \tilde{M}_u^\dagger)_{1,2} - (\tilde{M}_u \tilde{M}_u^\dagger)_{1,3}(\tilde{M}_u \tilde{M}_u^\dagger)_{1,3}^\ast = 8.25259 \cdot 10^{-8} - 8.548 \cdot 10^{-8}i,$$

$$(\tilde{M}_u \tilde{M}_u^\dagger)_{2,2} - (\tilde{M}_u \tilde{M}_u^\dagger)_{2,3}(\tilde{M}_u \tilde{M}_u^\dagger)_{2,3}^\ast = 4.9 \cdot 10^{-5}. \quad (26)$$

Thus, we obtain the following approximate relations

$$\tilde{M}_u(1,3) \approx (\tilde{M}_u \tilde{M}_u^\dagger)_{1,3} \text{ (from the assumption } \tilde{M}_u(3,3) \approx 1),$$

$$\tilde{M}_u(2,3) \approx (\tilde{M}_u \tilde{M}_u^\dagger)_{2,3},$$

$$\tilde{M}_u(2,1) \sim \tilde{M}_u(2,2) \sim \tilde{M}_u(2,3). \quad (27)$$

Parametrizing $\tilde{M}_u$ in terms of $\eta$ as

$$\tilde{M}_u \propto \begin{pmatrix} \eta^4 \beta_{1,1} & \eta^4 \beta_{1,2} e^{i\delta_{1,2}} & \eta^2 \beta_{1,3} e^{i\delta_{1,3}} \\ \eta^2 \beta_{2,1} e^{i\delta_{2,1}} & \eta^2 \beta_{2,2} & \eta^2 \beta_{2,3} e^{i\delta_{2,3}} \\ \eta^4 \beta_{3,1} e^{i\delta_{3,1}} & \eta^4 \beta_{3,2} e^{i\delta_{3,2}} & 1 + \eta^4 \beta_{3,3} \end{pmatrix}, \quad \tilde{M}_u^\dagger \propto \begin{pmatrix} \eta^4 \beta_{1,1} & \eta^2 \beta_{1,2} e^{-i\delta_{2,1}} & \eta^4 \beta_{1,3} e^{-i\delta_{1,3}} \\ \eta^2 \beta_{2,1} e^{-i\delta_{2,1}} & \eta^2 \beta_{2,2} & \eta^4 \beta_{2,3} e^{-i\delta_{2,3}} \\ \eta^4 \beta_{3,1} e^{-i\delta_{3,1}} & \eta^2 \beta_{3,2} e^{-i\delta_{3,2}} & 1 + \eta^4 \beta_{3,3} \end{pmatrix}, \quad (28)$$

we obtain

$$\tilde{M}_u \tilde{M}_u^\dagger = \begin{pmatrix} \eta^4 \beta_{1,1} & \eta^4 \beta_{1,2} e^{i\delta_{1,2}} & \eta^2 \beta_{1,3} e^{i\delta_{1,3}} e^{-i\delta_{2,1}} \\ \eta^4 \beta_{1,1} & \eta^4 \beta_{1,2} e^{-i\delta_{1,2}} & \eta^2 \beta_{1,3} e^{-i\delta_{1,3}} e^{i\delta_{2,1}} \\ \eta^2 \beta_{1,3} e^{-i\delta_{2,1}} & \eta^2 \beta_{2,3} e^{-i\delta_{2,3}} & 1 \end{pmatrix} \quad (29)$$

From Eqs. (22), (25) and (28), we obtain

$$\tilde{M}_u \approx \begin{pmatrix} \eta^4 \beta_{1,1} & \eta^4 \beta_{1,2} e^{i\delta_{1,2}} & 3.63026 \cdot 10^{-3} \cdot e^{i(\frac{\pi}{2} - 0.4153)} \\ \eta^2 \beta_{2,1} e^{i\delta_{2,1}} & \eta^2 \beta_{2,2} & 3.79682 \cdot 10^{-3} \cdot e^{i(-0.00248)} \\ \eta^4 \beta_{3,1} e^{i\delta_{3,1}} & \eta^4 \beta_{3,2} e^{i\delta_{3,2}} & 1 + \eta^4 \beta_{3,3} \end{pmatrix} \quad (30)$$
with $\eta^2 \sqrt{(\beta_{2,1})^2 + (\beta_{2,2})^2} = 0.700 \times 10^{-2}$ such that $\beta_{2,1}^2 + \beta_{2,2}^2 \approx 1$ and $\eta \approx 0.0837$. Thus, we obtain
\begin{align*}
\beta_{1,3} &\simeq 0.518604, \\
\beta_{2,3} &\simeq 0.542398.
\end{align*}

(31)

Now, let us obtain $\tilde{M}_u$ from
\begin{equation}
\tilde{M}_u \simeq V_u \begin{pmatrix}
\frac{m_1}{m_t}, & 0, & 0 \\
0, & \frac{m_2}{m_t}, & 0 \\
0, & 0, & 1
\end{pmatrix} \ U_u^\dagger,
\end{equation}

(32)

where $V_u$ is given in Eq. (25) and $U_u$ is
\begin{equation}
\begin{pmatrix}
R_{1,1}, & R_{1,2}, & R_{1,3} \\
R_{2,1}, & R_{2,2}, & R_{2,3} \\
R_{3,1}, & R_{3,2}, & R_{3,3}
\end{pmatrix}.
\end{equation}

(33)

Thus, $\tilde{M}_u$ becomes
\begin{equation}
\begin{pmatrix}
1.40 \times 10^{-5}, & 1.178 \times 10^{-5} \cdot e^{i(0.09211)}, & 3.6303 \times 10^{-3} \cdot e^{i(-0.4125)} \\
2.36 \times 10^{-8} \cdot e^{i(\pi - 0.099)}, & 7.00 \times 10^{-3} \cdot e^{i(0.0000)}, & 3.80 \times 10^{-3} \cdot e^{i(-0.0024)} \\
5.97810 \times 10^{-8} \cdot e^{i(\frac{\pi}{2} + 0.4109)}, & 2.66 \times 10^{-5} \cdot e^{i(\pi + 0.0020)}, & 0.99999
\end{pmatrix}
\end{equation}

(34)

\begin{equation}
= (u_1, u_2, u_3)
\end{equation}

where $u_i$ is
\begin{equation}
[3.6303 \cdot e^{i(\frac{\pi}{2} - 0.4125)}(R_{4,3})^*] \times 10^{-3} \\
[7.00(R_{4,2})^* + 3.80 \cdot e^{i(-0.0024)}(R_{4,3})^*] \times 10^{-3} \\
0.9999(R_{4,3})^* + 2.66 \cdot e^{i(\pi + 0.0020)}(R_{4,2})^* \times 10^{-5}.
\end{equation}

(35)

Existence of $U_u$ is sufficient for our study of the CKM matrix. In this regard, note that an R-hand unitary matrix $U_u$ close to
\begin{equation}
U_u \simeq \begin{pmatrix}
\frac{1}{2}, & -\frac{\sqrt{3}}{2}, & 0 \\
\frac{\sqrt{3}}{2}, & \frac{1}{2}, & 0 \\
0, & 0, & 1
\end{pmatrix}
\end{equation}

(36)

gives a solution [34] consistent with the data [30]. Later, we will obtain the CP phase of the CKM matrix is close to $\frac{\pi}{2}$. Let us suppose that $U_u$ has 0 entries as shown in [36]. The first line of Eq. (35) is useful since there is only one term and a statement on the phase is clear. From [35], then if $(R_{4,3})^*$ has a phase close to $\frac{\pi}{2} \approx 0.393$, then the element $\tilde{M}_u(1,3)$ has a phase $\frac{\pi}{2} - 0.02$. So, the $Q_{em} = \frac{2}{3}$ quark mass matrix can have a phase close to the observed CP phase of the CKM matrix, which can help constructing a field theoretic model.

What we obtain from $V_d$ of [22] and $V_u$ given in [25] is the same as Eq. (12),
\begin{equation}
V_{CKM} = V_u V_d =
\begin{pmatrix}
+0.974395 \cdot e^{i(8.90745 \times 10^{-5})}, & +0.224814 \cdot e^{i(2.51923 \times 10^{-5})}, & +0.003615 \cdot e^{i(-\frac{\pi}{2} + 0.4005)} \\
-0.224672 \cdot e^{i(6.302 \times 10^{-5})}, & +0.973517 \cdot e^{i(-7.666 \times 10^{-5})}, & +0.042275 \cdot e^{i(1.258 \times 10^{-4})} \\
+0.008754 \cdot e^{i(-0.37945)}, & -0.041516 \cdot e^{i(1.788 \times 10^{-2})}, & 0.99910 \cdot e^{i(-4.506 \times 10^{-5})}
\end{pmatrix}.
\end{equation}

(37)

Then, the KS form [63] for the Jarlskog determinant is
\begin{equation}
J = |\text{Im} \ V_{31} V_{22} V_{13}| = |(0.008754) \cdot (0.973517) \cdot (0.003615) \cdot \sin(-0.37945 - 7.666 \times 10^{-5} - \frac{\pi}{2} + 0.4005)|
s \simeq |3.081 \times 10^{-5} \cdot \sin(-88.8^\circ)| \simeq 3.08 \times 10^{-5}.
\end{equation}

(38)

which is consistent with the value of the Particle Data Group, $J_{PDG} = (3.18 \pm 0.15) \times 10^{-5}$ [61]. Here, $\alpha$ of the unitarity triangle is $88.8^\circ$. 

IV. CONCLUSION

Starting from a SU(5)_{6p} inspired mass matrices \[Q\], we obtained the CKM matrix which can be made consistent with the observed data \[Q\]. The model presented in Ref. \[Q\] allows a \(Z_{4R}\) discrete symmetry such that it forbids the dimension-5 B violating operators but allows the needed electroweak scale \(\mu\) term and dimension-5 lepton number violating Weinberg operators.

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[1] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Vacuum configurations for superstrings, Nucl. Phys. B 258 (1985) 46 [doi:10.1016/0550-3213(85)90602-9].
[2] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Strings on orbifolds. 2., Nucl. Phys. B 274 (1986) 285 [doi:10.1016/0550-3213(86)90287-7].
[3] L. E. Ibanez, H. P. Nilles, and F. Quevedo, Orbifolds and Wilson lines, Phys. Lett. B 187 (1987) 25 [doi:10.1016/0370-2693(87)90066-9].
[4] H. Kawai, D. C. Lewellen, and S. H. H. Tye, Construction of fermionic string models in four-dimensions, Nucl. Phys. B 288 (1987) 1 [doi:10.1016/0550-3213(87)90208-2].
[5] I. Antoniadis, C. P. Bachas, and C. Kounnas, Four-dimensional superstrings, Nucl. Phys. B 289 (1987) 87 [doi:10.1016/0550-3213(87)90372-5].
[6] D. Gepner, Space-time supersymmetry in compactified string theory and superconformal models, Nucl. Phys. B 296 (1988) 757 [doi:10.1016/0550-3213(88)90397-5].
[7] N. Cabibbo, Unitary symmetry and leptonic decays, Phys. Rev. Lett. 10 (1963) 531 [doi: 10.1103/PhysRevLett.10.531].
[8] M. Kobayashi and T. Maskawa, CP violation in the renormalizable theory of weak interaction, Prog. Theor. Phys. 49 (1973) 652 [10.1143/PTP.49.652].
[9] J. E. Kim, R-parity from string compactification, arXiv:1810.10796.
[10] J. H. Huh, J. E. Kim, and B. Kyae, SU(5)_{6p} x SU(5)' from \(Z_{12-1}\), Phys. Rev. D 80 (2009) 115012 [arXiv: 0904.1108 [hep-ph]].
[11] L. Susskind, The anthropic landscape of string theory, arXiv:hep-th/0302219.
[12] L. E. Ibanez, J. E. Kim, H. P. Nilles, and F. Quevedo, Orbifold compactifications with three families of SU(3)×SU(2)×U(1)', Phys. Lett. B 191 (1987) 292 [doi:10.1016/0370-2693(87)90255-3].
[13] C. Casas and C. Munoz, Three generation SU(3)×SU(2)×U(1)_Y models from orbifolds, Phys. Lett. B 214 (1988) 63 [doi:10.1016/0370-2693(88)90452-2].
[14] S. Chaudhuri, G. Hockney, and J. D. Lykken, Three generations in the fermionic construction, Nucl. Phys. B 469 (1996) 357 [arXiv:hep-th/9510241].
[15] W. Pokorski and G. G. Ross, Flat directions, string compactification and three generation models, Nucl. Phys. B 551 (1999) 515 [arXiv:hep-ph/9809537].
[16] G. B. Cleaver, A. E. Faraggi, and D. V. Nanopoulos, String derived MSSM and M theory unification, Phys. Lett. B 455 (1999) 135 [arXiv:hep-ph/9811427].
[17] G. B. Cleaver, A. E. Faraggi, and D. V. Nanopoulos, A minimal superstring standard model I: Flat directions, Int. J. Mod. Phys. A 16 (2001) 425 [arXiv:hep-ph/9904301].
[18] G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos, and J. W. Walker, Phenomenological study of a minimal superstring standard model, Nucl. Phys. B 593 (2001) 471 [arXiv:hep-ph/9910230].
[19] R. Donagi, B. A. Ovrut, T. Pantev, and D. Waldram, Spectral involutions on rational elliptic surfaces, Adv. Theor. Math. Phys. 5 (2002) 93 [arXiv:math/0008011].
[20] T. Kobayashi, S. Kaly, R-J. Zhang, Searching for realistic 4d string models with a Pati-Salam symmetry: Orbifold grand unified theories from heterotic string compactification on a \(Z_6\) orbifold, Nucl. Phys. B 704 (2005) 3, [arXiv:hep-ph/0409098].
[21] R. Donagi, Y-H. He, B. A. Ovrut, and T. Pantev, A heterotic standard model, Phys. Lett. B 618 (2005) 252 [arXiv:hep-th/0501070].
[22] R. Donagi, Y-H. He, B. A. Ovrut, and R. Reinbacher, The spectra of heterotic standard model vacua, JHEP 06 (2005) 070 [arXiv:hep-th/0411156].
[23] V. Bouchard and R. Donagi, An SU(5) heterotic standard model, Phys. Lett. B 633 (2006) 783 [arXiv:hep-th/0512149].
[24] V. Braun, Y-H. He, B. A. Ovrut, and T. Pantev, The Exact MSSM spectrum from string theory, JHEP 05 (2006) 043 [arXiv:hep-th/0512177].
[25] R. Blumenhagen, S. Moster, and T. Weigand, Heterotic GUT and standard model vacua from simply connected Calabi-Yau manifolds, Nucl. Phys. B 751 (2006) 186 [arXiv: hep-th/0603015].
[26] V. Bouchard, M. Cvetic, and R. Donagi, *Tri-linear couplings in an heterotic minimal supersymmetric standard model*, Nucl. Phys. B 745 (2006) 62 [arXiv: hep-th/0602096].

[27] R. Blumenhagen, S. Moster, R. Reinbacher, and T. Weigand, *Massless spectra of three generation U(N) heterotic string vacua*, JHEP 0705 (2007) 041 [arXiv: hep-th/0612039].

[28] J. E. Kim, J-H. Kim, and B. Kyae, *Superstring standard model from Z(12-1) orbifold compactification with and without exotics, and effective R-parity*, JHEP 0706 (2007) 034 [arXiv: hep-ph/0702278].

[29] A. E. Faraggi, C. Koumas, and J. Rizos, *Chiral family classification of fermionic $\mathbb{Z}_2 \times \mathbb{Z}_2$ heterotic orbifold models*, Phys. Lett. B 648 (2007) 84 [arXiv: hep-th/0606144].

[30] G. B. Cleaver, *In search of the (minimal supersymmetric) standard model string*, [arXiv: hep-th/0703027].

[31] C. Munoz, *A kind of prediction from string phenomenology: Extra matter at low energy*, Mod. Phys. Lett. A 22 (2007) 989 [arXiv:0704.0957 [hep-ph]].

[32] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange, and A. Wingerter, *Heterotic road to the MSSM with R parity*, Phys. Rev. D 77 (2008) 046013 [arXiv:0708.2691 [hep-th]].

[33] J.-S. Choi and J. E. Kim, *Quarks and Leptons from Orbifolded Superstring*, Lecture Notes in Physics Vol. 696 (Springer-Verlag, 2006).

[34] L. E. Ibanez and A. M. Uranga, *String Theory and Particle Physics* (Cambridge Univ. Press, 2012).

[35] S. Raby, *Supersymmetric Grand Unified Theories : From Quarks to Strings via SUSY GUTs*, Lecture Notes in Physics Vol. 939 (Springer-Verlag, 2017).

[36] T. P. T. Dijkstra, L. R. Huiszoon, A. N. Schellekens, *Orientifolds, hypercharge embeddings and the standard model*, Nucl. Phys. B 759 (2006) 83 [arXiv:hep-th/0605220].

[37] F. Gmeiner, R. Blumenhagen, G. Honecker, D. Lust, and T. Weigand, *One in a billion: MSSM-like D-brane statistics*, JHEP 0001 (2006) 004 [arXiv:hep-th/0510170].

[38] M. R. Douglas and W. Taylor, *The landscape of intersecting brane models*, JHEP 0701 (2007) 031 [arXiv: hep-th/0606109].

[39] B. Pontecorvo, *Inverse beta processes and nonconservation of lepton charge*, Phys. JETP 7 (1957) 172 [Zh. Eksp. Teor. Fiz. 34, 247 (1957)].

[40] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, Prog. Theor. Phys. 28 (1962) 870 [doi: 10.1143/PTP.28.870].

[41] H. Georgi and S. L. Glashow, *Unity of all elementary particle forces*, Phys. Rev. Lett. 32 (1974) 438 [doi:10.1103/PhysRevLett.32.438].

[42] H. Georgi, *The state of the art-Gauge theories*, AIP Conf. Proc. 23 (1975) 575 [doi:10.1063/1.2947450].

[43] H. Fritzsch and P. Minkowski, *Unified interactions of leptons and hadrons*, Annals Phys. 93 (1975) 193 [doi:10.1016/0003-4916(75)90211-0].

[44] F. Garsasay, P. Ramond, and P. Sikivie, *A universal gauge theory model based on $E_6$*, Phys. Lett. B 60 (1976) 177 [doi:10.1016/0370-2693(76)90417-2].

[45] J. C. Pati and Abdus Salam, *Unified lepton-hadron symmetry and a gauge theory of the basic interactions*, Phys. Rev. D 8 (1973) 1240 [doi: 10.1103/PhysRevD.8.1240].

[46] S. M. Barr, *A new symmetry breaking pattern for SO(10) and proton decay*, Phys. Lett. B 112 (1982) 219 [ doi:10.1016/0370-2693(82)90966-2].

[47] J. P. Derendinger, J. E. Kim, and D. V. Nanopoulos, *Anti-SU(5)*, Phys. Lett. B 139 (1984) 170 [doi:10.1016/0370-2693(84)91238-3].

[48] T. Carli, Talk presented at ICHEP 2018, Seoul, Korea, 9 July 2018.

[49] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange, and A. Wingerter, *Low energy supersymmetry from the heterotic landscape*, Phys. Rev. Lett. 98 (2007) 181602 [arXiv: hep-th/0611203].

[50] J-W Kim, J. E. Kim and B. Kyae, *Harmless R-parity violation from Z(12-1) compactification of $E_6 \times E_6$ heterotic string*, Phys. Lett. B 647 (2007) 275 [arXiv: hep-ph/0612365].

[51] R. Kappl, H. P. Nilles, S. Ramos-Sanchez, M. Ratz, K. Schmidt-Hoberg, and P. K. S. Vaudrevange, *Large hierarchies from approximate R symmetries*, Phys. Rev. Lett. 102 (2009) 121602 [arXiv:0812.2120 [hep-th]].

[52] K. S. Babu, I. Gogoladze, and K. Wang, *Natural R-parity, $\mu$-term, and fermion mass hierarchy from discrete gauge symmetries*, Nucl. Phys. B 660 (2003) 322 [arXiv:hep-ph/0212245].

[53] K. Kurosawa, N. Maru, and T. Yanagida, *Nonanomalous R-symmetry in supersymmetric unified theories and leptons*, Phys. Lett. B 512 (2001) 203 [arXiv:hep-ph/0105136].

[54] H. M. Lee, S. Raby, M. Ratz, G. R. Ross, R. Schieren, K. Schmidt-Hoberg, P. K. S. Vaudrevange, *Discrete R symmetries for the MSSM and its singlet extensions*, Nucl. Phys. B 850 (2011) 1 [arXiv:1102.3595 [hep-ph]].

[55] M. Paraskevas and K. Tamvakis, *On discrete R-symmetries in MSSM and its extensions*, Phys. Rev. D 86 (2012) 015009 [arXiv:1205.1391 [hep-ph]].

[56] J. E. Kim, *Abelian discrete symmetries $\Sigma_N$ and $\Sigma_{N\Gamma}$ from string orbifolds*, Phys. Lett. B 726 (2013) 450 [arXiv:1308.0343 [hep-th]].

[57] I. Antoniadis, J. R. Ellis, J. S. Hagelin, and D. V. Nanopoulos, *The flipped SU(5)$\times U(1)$ string model revamped*, Phys. Lett. B 231 (1989) 65 [doi: 10.1016/0370-2693(89)90115-9].

[58] J. E. Kim and B. Kyae, *Flipped SU(5) from $Z_{12-1}$ orbifold with Wilson line*, Nucl. Phys. B 770 (2007) 47 [arXiv: hep-th/0608080].
A. Ceccucci, Z. Ligeti, and Y. Sakai, in M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98 (2018) 030001, The CKM quark-mixing matrix [doi:10.1103/PhysRevD.98.030001].

Z. Kakushadze, S. H. H. Tye, Three family SO(10) grand unification in string theory, Phys. Rev. Lett. 77 (1996) 2612 [arXiv:hep-th/9605221].

Z. Kakushadze, S. H. H. Tye, Three family SO(10) grand unification in string theory, Phys. Rev. Lett. 77 (1996) 2612 [arXiv:hep-th/9605221].

J. E. Kim and M-S. Seo, Azino mass, PoS (DSU2012) 009 [arXiv:1211.0357 [hep-ph]]; J. E. Kim, D. Y. Mo, and S. Nam, Final state interaction phases obtained by data from CP asymmetries, J. Korean Phys. Soc. 66 (2015) 894 [arXiv:1402.2978 [hep-ph]].

C. Jarlskog, Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal CP nonconservation, Phys. Rev. Lett. 55 (1985) 1039 [doi: 10.1103/PhysRevLett.55.1039].

C. D. Froggatt and H. B. Nielsen, Hierarchy of quark masses, Cabibbo angles and CP violation, Nucl. Phys. B 147 (1979) 277 [doi:10.1016/0550-3213(79)90316-X].

T2K Collaboration (K. Abe et al.), Combined analysis of neutrino and antineutrino oscillations at T2K, Phys. Rev. Lett. 118 (2017) 151801 [arXiv:1701.00432 [hep-ex]].

J.E. Kim and S. Nam, Unifying CP violations of quark and lepton sectors, Euro. Phys. J. C 75 (2015) 619 [arXiv:1506.08494 [hep-ph]].

H. W. Zaglauer and K. H. Schwarzer, The mixing angles in matter for three generations of neutrinos ZaglauerZaglauerand the MSW mechanism, Z. Phys. C 40 (1988) 273 [doi:10.1007/BF01555889].