ACCELERATING COMPACT OBJECT MERGERS IN TRIPLE SYSTEMS WITH THE KOZAI RESONANCE: A MECHANISM FOR “PROMPT” TYPE Ia SUPERNOVAE, GAMMA-RAY BURSTS, AND OTHER EXOTICA

TODD A. THOMPSON

Department of Astronomy and Center for Cosmology & Astro-Particle Physics, The Ohio State University, Columbus, OH 43210, USA; thompson@astronomy.ohio-state.edu

Received 2010 November 23; accepted 2011 August 3; published 2011 October 20

ABSTRACT

White dwarf–white dwarf (WD–WD) and neutron star–neutron star (NS–NS) mergers may produce Type Ia supernovae and gamma-ray bursts (GRBs), respectively. A general problem is how to produce binaries with semi-major axes small enough to merge in significantly less than the Hubble time (t_H), and thus accommodate the observation that these events closely follow episodes of star formation. I explore the possibility that such systems are not binaries at all, but actually coeval, or dynamical formed, triple systems. The tertiary induces Kozai oscillations in the inner binary, driving it to high eccentricity, and reducing its gravitational wave (GW) merger timescale. This effect significantly increases the allowed range of binary period \( P \) such that the merger time is \( t_{\text{merge}} < t_H \). In principle, Chandrasekhar-mass binaries with \( P \sim 300 \) days can merge in \( t_{\text{merge}} \lesssim t_H \) if they contain a prograde solar-mass tertiary at high enough inclination. For retrograde secondaries, the maximum \( P \) such that \( t_{\text{merge}} \lesssim t_H \) is yet larger. In contrast, \( P \lesssim 0.3 \) days is required in the absence of a tertiary. I discuss implications of these findings for the production of transients formed via compact object binary mergers. Based on the statistics of solar-type binaries, I argue that many such binaries should be in triple systems affected by the Kozai resonance. If true, expectations for the mHz GW signal from individual sources, the diffuse background, and the foreground for GW experiments like LISA are modified. This work motivates future studies of triples systems of A, B, and O stars, and new types of searches for WD–WD binaries in triple systems.

Key words: binaries: close – celestial mechanics – gravitational waves – stars: kinematics and dynamics – stars: neutron – supernovae: general – white dwarfs

Online-only material: color figure

1. INTRODUCTION

Stellar-mass compact object mergers driven by gravitational wave (GW) radiation may power many types of observed astrophysical transients and produce rare stellar exotica. Notably, the merger of white dwarf–white dwarf (WD–WD) binaries has been suggested as a mechanism for producing Type Ia supernovae (Iben & Tutukov 1984; Webbink 1984), and neutron star–neutron star (NS–NS) or NS–black hole (BH) mergers are a leading model for the central engine of short-duration gamma-ray bursts (SGRBs; Ruffert & Janka 1999; Janka et al. 1999). Other types of compact object mergers such as those of BH–helium star or BH/NS–WD binaries have been identified as possible mechanisms for long-duration GRBs (Fryer & Woosley 1998; Fryer et al. 1999a, 1999b), or peculiar supernovae (Metzger 2011). WD–WD mergers may also produce companionless millisecond pulsars (Saio & Nomoto 1985) and R CrB stars (Webbink 1984; Nelemans et al. 2001). Finally, close compact object binaries brought near enough for mass transfer power a variety of astrophysical phenomena. One example is AM CVn stars (Warner 1995), which may produce faint “Ia” supernovae (Bildsten et al. 2007).

Estimates suggest that \(~2\%\) of the all the stars born with zero-age main-sequence masses in the range \(~2.5–8\ M_\odot\) become Ia supernovae (e.g., Hiraiuchi & Beacom 2010; but, see Maoz 2010). If sub-Chandrasekhar-mass binaries can contribute to the Ia rate (e.g., van Kerkwijk et al. 2010), the initial mass function implies that a still smaller fraction of all stars produce Ia’s. Similarly, the massive star birthrate, which is approximately equal to the Type II supernova rate, \(~10^2\ \text{Gpc}^{-3} \text{yr}^{-1}\) (e.g., Horiuchi et al. 2009), is ~20–600 times larger than the NS–NS merger rate (Kalogera et al. 2001, 2004a, 2004b; O’Shaughnessy et al. 2008).

These comparisons imply that although a large fraction of all stars are in binaries, only a small fraction have the requisite characteristics to produce compact object binaries that can merge via GW emission on a timescale short compared to the Hubble time. This issue is particularly important since there is evidence for a “prompt” subset of Ia supernovae that track the star formation rates of galaxies (Scannapieco & Bildsten 2005; Mannucci et al. 2006). “Prompt” may mean \(<1\ \text{Gyr}\) after a burst of star formation, or potentially even \(<200\ \text{Myr}\) (Aubourg et al. 2008; Brandt et al. 2010; Maoz & Badenes 2010; Maoz et al. 2010, 2011). Similarly, SGRBs occur in both quiescent and actively star-forming galaxies (e.g., Berger 2009). In order to solve this problem, most studies focus on binary evolution channels that make more compact object binaries with small enough semi-major axis that a merger via GW radiation occurs “promptly” (e.g., Belczynski & Kalogera 2001; Ruiter et al. 2009). Indeed, common envelope (CE) evolution of compact stellar binary systems can lead to compact object binaries whose GW merger time is significantly less than the Hubble time (Iben & Tutukov 1984, 1985, 1987).

In this paper, I forward an alternate hypothesis: the rare binary systems that produce compact object mergers are not binaries at all, but instead hierarchical triple systems. In such systems, Kozai (1962) showed that the tertiary induces oscillations in the orbital eccentricity of the inner binary via a secular resonance. In the absence of general relativistic effects, the maximum
eccentricity attained by the inner binary is
\[ e_{\text{max}} = \left(1 - \frac{5}{3} \cos^2 i\right)^{1/2}, \]
where \( i \) is the inclination of the outer orbit relative to the plane of the inner binary. Because the GW merger timescale is a very strong function of eccentricity \((t_{\text{GW}} \propto (1 - e^2)^{7/2}/2; \) Peters 1964; Equation (2)), the addition of a tertiary at high inclination to a binary system can decrease \( t_{\text{GW}} \) dramatically. Blaes et al. (2002, hereafter BLS02) showed this explicitly for the case of triple systems of super-massive BHs, which would be formed by successive galaxy–galaxy mergers during structure formation. Miller & Hamilton (2002) independently applied this same idea to four-body interactions (binary–binary scattering) involving stellar-mass BH binaries in globular clusters to accelerate the assembly of intermediate-mass BHs in these systems.

Here, I explore the possibility that WD–WD, NS–WD, and NS–NS mergers driven by GW radiation, of relevance particularly for Ia supernovae, GRBs, and other transients, are accelerated by the presence of a tertiary at high inclination. I consider both coeval triple systems and triples formed by binary–single and binary–binary scatterings in dense stellar environments. I show explicitly that the GW merger timescale for a subset of compact object binaries in triple systems, whether coeval or dynamically formed, is significantly decreased from the expectation for the binary alone. This effect dramatically increases the range of semi-major axes for which a merger will occur in a single Hubble time \( (t_H) \). Put another way, some systems which one might think have no hope of merging in \( t_H \), actually merge in \( \ll t_H \) with a suitably placed tertiary. This will affect the population synthesis of Ia- and GRB-producing compact object binaries, and may significantly affect the overall rate, even though there are fewer triple systems than lone binaries.

These statements quickly raise the question of whether or not the parent population of triples that produce binary compact objects can accommodate the observed Ia or GRB rates. Although a complete discussion of this issue is beyond the scope of this paper, there are several reasons to believe that in the case of WD–WD mergers particularly, all such systems are (or were) triple. First, although only \( \sim 5\%-10\% \) of all solar-type stars are thought to be in triple systems (Tokovinin et al. 2006; Raghavan et al. 2010), fully \( \sim 50\% \) and \( \sim 100\% \) of all close solar-type binaries with orbital period \( P \lesssim 10 \) and \( \lesssim 3 \) days, respectively, are in triple systems (Tokovinin et al. 2006; Pribulla & Rucinski 2006). Second, Fabrycky & Tremaine (2007, hereafter FT07) have shown that the peak in the observed binary period distribution at \( \sim 3 \) days can be accounted for by the combined action of tidal friction between the two stars in the inner binary and Kozai oscillations induced by a hierarchical tertiary (see also Mazeh & Shaham 1979; Wu & Murray 2003; Wu et al. 2007; Perets & Fabrycky 2009). Thus, the closest binaries—those most likely to merge in less than a Hubble time—are precisely those that are most likely to be triples. Additionally, although little is known about the triple fraction of the A/B stars that give rise to WDs sufficiently massive to be plausible double-degenerate Ia progenitors and the massive O/B stars \( (\gtrsim 10 M_\odot) \) that produce NSs, there is evidence that the multiplicity of stars increases as a function of stellar mass (Lada 2006; Raghavan et al. 2010). Since the intermediate-mass and massive stars that produce WD and NS binaries become Cepheids during their post-main-sequence evolution, the high occurrence of triples among such systems (Evans et al. 2005) further motivates consideration of their eventual compact objects.  

The high percentage of close solar-type binaries containing secondaries motivates this paper in part. The (albeit limited) statistics on such systems implies that the rate of Ia supernovae and GRBs compared to their overall parent population might be accommodated by the triple fraction (see Section 5). There is an additional motivation for considering triple systems as the progenitors for Ia supernovae. The delay-time distribution of Ia supernovae implies that many systems explode on 1–10 Gyr timescales (e.g., Totani et al. 2008), and this fact implies a certain number density of progenitor systems in the Galaxy. Simple estimates suggest that the nearest progenitor system is of order just \( \sim 50 \) pc away. So far, there have been no confirmed identifications of WD–WD binaries with combined mass greater than the Chandrasekhar mass discovered (see Mullally et al. 2009 for a recent compilation). Although there are a number of potential explanations for this fact—e.g., sub-Chandrasekhar-mass mergers or single-degenerate systems dominate the Ia rate—an additional possibility is that the reason no such systems have been found is that they are in triple systems. Since many such systems would be composed of a close WD–WD binary with a hierarchical main-sequence tertiary, such systems would not have been found by searches that color-select WD binaries (Napiwotzki et al. 2001; Badenes et al. 2009; Brown et al. 2010). Additionally, since the massive WD binaries that produce Ia supernovae may often have a lower-mass tertiary that subsequently evolves, triple WD systems may appear as a single WD (the young, low-mass tertiary), and without a strong radial velocity signal from the distant, dimmer, but more massive WD–WD binary. Although a detailed discussion of these effects on the selection of triple systems is beyond the scope of this work, I consider the lack of observed WD–WD progenitors to be an additional motivation for exploring the strong triple hypothesis that all Ia progenitors are triple systems.

The remainder of this paper is organized as follows. In Section 2, I discuss the types of systems that will be affected by Kozai oscillations. Although I outline some of the evolutionary processes that will affect triple-star evolution (e.g., CE evolution and mass loss), for the purposes of this paper, I simply assume that binary compact objects are formed with a range of semi-major axes and masses, and with tertiaries with a variety of masses, semi-major axes, and inclinations relative to the inner binary. I then calculate the evolution of these representative systems using the methods described in Section 3 and the Appendix. The results are presented in Section 4. In Section 5, I discuss the results and the implications for transients produced from these mergers, and the overall rate. I also discuss implications of these results for the GW foreground and background, as well as the signal from individual sources, for LISA (see also Gould 2011).

2. SCENARIOS

2.1. WD–WD Binaries and Triples

Consider a system of two \( \sim 2–8 M_\odot \) (A5–B3) main-sequence stars (e.g., most commonly 2+2, 3+2 \( M_\odot \), etc., and perhaps preferentially “twins”; Pinsonneault & Stanek 2006), \( m_0 = m_1 \), in a close binary with semi-major axis \( a_1 \), and a coeval

\[ 2 \] Typically, a Cepheid with a \( \pm 1 \) day period corresponds to a \( \sim 3 M_\odot \) main-sequence A0 star, which produces a \( \lesssim 0.7 M_\odot \), WD (e.g., Kalirai et al. 2008), the minimum required for a Chandrasekhar mass, equal mass ratio binary.
hierarchical tertiary of mass $m_{2}$ and semi-major axis $a_{2}$. The minimum value of $a_{2}$ is set by stability of the triple system: $a_{2} \gtrsim 3a_{1}$ (Eggleton & Kiseleva 1995; Mardling & Aarseth 2001)

If $a_{1}$ is less than a few AU, then the binary will undergo one or more mass transfer and CE evolutionary phases that decrease $a_{1}$ from its initial value by a factor of $\sim 10$–100 (Iben & Tutukov 1984, 1985; Iben & Livio 1993; Ruiter et al. 2009). Although the physics of the CE phase(s) is uncertain, it is the primary mechanism for producing close WD–WD binaries that will merge in substantially less than $t_{\text{K}}$ (see Equation (2)).

In the triple system considered, mass loss from the inner binary during the CE phase(s) will in general increase $a_{2}$ from its initial value, typically by a factor of $\sim 2–10$, depending on the masses of system’s constituents, and the final WD masses of the inner binary. Mass loss from $m_{2}$ as it evolves to a WD will also increase $a_{2}$. If $a_{2}$ is less than a few AU either before or after the inner binary evolves, then there will be a complicated phase of triple CE evolution where the binary is engulfed by the tertiary, potentially decreasing both $a_{2}$ and $a_{1}$. A number of related evolutionary channels for triples were sketched by Iben & Tutukov (1999).

In this paper, I show that for a range of parameters the timescale for the merger of WD–WD binaries via GWs is dramatically decreased by the presence of a tertiary. As implied by Equation (1), and as discussed in detail in Sections 3 and 4, this mechanism requires (1) that the tertiary be at high relative inclination with respect to the inner binary and (2) that $a_{2}/a_{1}$ must be in the range of $\sim 3$–100. Both are expected to be modified by stellar evolution preceding WD formation. The latter is important since the combination of mass loss and $a_{1}$ evolution during the CE phase of a close binary will in general increase $a_{2}/a_{1}$ from its initial value by a factor of 20–1000. However, the phase of triple CE evolution that may occur as the tertiary evolves can in principle decrease $a_{2}$. The evolution in inclination is also important since Equation (1) and the fact that $t_{\text{GW}} \propto (1 - e^{2})^{7/2}$ imply that $t_{\text{GW}}$ will be a very strong function of $i$. For a random distribution of inclination angles between 0° and 90° (prograde tertiaries), the probability of having $i$ > 70°, 80°, 85°, and 89° is $\cos i \simeq 0.3$, 0.2, 0.09, and 0.02, respectively. Although the distribution of tertiary inclinations is unknown, the results of FT07 imply that if a uniform distribution in $\cos i$ is assumed when the stars first form, the resulting distribution, after many Kozai times (Equation (4)), is quite flat (their Figure 7), but with peaks near the critical Kozai angles ($i \sim 39°$ and 141°). Because tides detune the Kozai resonance, if the tertiary is at high $i$, the binary’s semi-major axis and eccentricity will be affected by the combination of Kozai oscillations and tidal friction before WD formation (FT07). In particular, the inner binary will be driven to lower eccentricity than if Kozai acted alone (Equation (1)). If there is a CE evolutionary phase as the stars evolve, and mass transfer, the inner binary may well be nearly circular at the time of WD formation since the Kozai resonance will be terminated by such strong interaction between the inner components. However, subsequent to this complicated evolution, one expects Kozai to again operate after compact object formation. The system will then be driven to high eccentricity, and rapidly coalesce via GW radiation.

Given the complications posed by triple-star evolution, and in particular triple CE evolution, I simply assume that such systems lead to binary WDs with a range of $a_{1}$, together with a tertiary WD or main-sequence star with a range of inclinations $a_{2}$. The combination of triple-star evolution with dynamics, mass transfer, and CE evolution, together with population synthesis is crucial from a theoretical perspective for evaluating the mechanism proposed here for rapid coalescence of WDs via gravity waves. However, these theoretical complications aside, the results of Tokovinin et al. (2006) and Pribulla & Rucinski (2006) strongly motivate an observational search for WD–WD binaries with main-sequence tertaries. The discovery and characterization of such systems—in particular the quantities $i$ and $a_{2}/a_{1}$—will be the ultimate arbiter in establishing the importance of the Kozai mechanism for rapid compact object mergers.

Although I have made no attempt to model specific systems, recent work on the exotic planetary nebula SuWt 2 provides further motivation since it is thought to contain a close A-star binary with a (possibly WD) $\sim 0.7 M_{\odot}$ tertiary (Exter et al. 2010). One can also imagine an important role for the Kozai mechanism in driving binaries composed of a WD and a main-sequence star to contact, as in the single-degenerate scenario for Ia supernovae (Whelan & Iben 1973).

Similar scenarios with different mass components may produce AM CVn or R CrB stars, depending on the stability of mass transfer as the WDs interact (Webbink 1984; Nelemans et al. 2001), or millisecond pulsars (Saio & Nomoto 1985).

In globular clusters, close WD–WD binaries may pick up a tertiary from the dense stellar field via either binary–single or binary–binary scattering. The latter should dominate the rate. This mechanism for the formation of triple systems that lead to Kozai-induced mass transfer has been studied by Ivanova (2008) and Ivanova et al. (2008, 2010), and has been found to be important for producing the observed X-ray binary populations of globular clusters (see also Fregeau et al. 2004, 2009). As I show in Section 4, if the tertiary is captured into a high-inclination prograde orbit, or a retrograde orbit with $i \lesssim 110°–120°$, the GW merger timescale for the inner binary can be very short. Since such systems will have very high eccentricity, their GW signals will be peaked at high frequencies (Section 4), and for individual systems LISA will see GW “pulses” at periastron (Gould 2011).

2.2. BH, NS–BH, NS, WD Scenarios

For binaries with a BH or NS in triple systems, the supernova explosions that accompany NS (and possibly BH) formation may in some cases unbind the inner binary or the tertiary, depending on the binary mass ratio, the tertiary mass, the eccentricities of the inner and outer orbits, and the magnitude and direction of the “kick” velocity given to the NS at birth. For NS–NS-producing binaries of relevance for SGRBs, one imagines an inner binary with components of $\sim 9–12 M_{\odot}$ (e.g., 9+10, 9+9, 11+10 $M_{\odot}$, etc.), forming via standard scenarios (Bhattacharya & van den Heuvel 1991), and with a relatively distant tertiary with $m_{2} \sim 5–8 M_{\odot}$, which may be either a main-sequence B star, or perhaps a BH formed by a more massive star. Depending on the mass of the system expelled before and during the supernovae that produce the NSs and the magnitude and direction of their kicks, the tertiary may remain bound, but with small $a_{2}$ preferred (see, e.g., Figure 10 of Kalogera 1996 for the binary case; Hills 1983).

After the supernovae, the resulting NS–NS binary will be subject to Kozai oscillations by the tertiary, depending on its semi-major axis and inclination. If the tertiary is a
main-sequence star, it will become a massive WD with \( \sim 1-1.4 \, M_\odot \). As in the WD–WD case, because the timescale for mass loss during this transformation is long compared with the orbital times in the system, it should remain bound, albeit with larger \( a_2/a_1 \). In addition, the system may also undergo triple CE evolution as the tertiary evolves. After this is complete, if the system persists, Kozai cycles will then resume.

Many similar scenarios can be considered for NS–WD binaries or BH–NS/WD binaries in coeval triple systems, although the survival of the triple through the formation of the compact object binary must be evaluated case by case. A discussion of one such system in the context of Kozai oscillations in a hierarchical triple system is given in Champion et al. (2008) and Freire et al. (2011) for the system PSR J1903+0327.

As discussed in Section 2.1 in the WD–WD case, NS–NS/WD binaries that undergo Kozai oscillations can also be formed dynamically in dense stellar environments via binary–single and binary–binary scattering. A significant fraction of such interactions would be expected to produce a stable triple system (see MH02; Wen 2003; Ivanova 2008; Ivanova et al. 2008, 2010).

3. METHOD

3.1. Timescales

I consider an inner compact object binary with semi-major axis \( a_1 \), eccentricity \( e_1 \), argument of periastron \( g_1 \), and masses \( m_0 \) and \( m_1 \). The hierarchical tertiary has mass \( m_2 \), semi-major axis \( a_2 \), argument of periastron \( g_2 \), and mutual inclination with respect to the inner binary of \( i \). Subscripts of “0” are used to indicate initial values (e.g., \( e_1 \), \( a_1 \)).

There is a strong hierarchy of timescales in the problem of triple systems consisting of a compact object binary. First, in the limit of high eccentricity, the GW merger timescale of the inner binary is (Peters 1964)

\[
t_{GW} = \frac{3}{85} \frac{a_1}{c} \left( \frac{a_1 c^6}{G^2 m_0 m_1 M} \right) \left( 1 - e_1^2 \right)^{7/2}
\]

\[
\simeq 1.6 \times 10^{13} \text{yr} \left( \frac{2 M_\odot}{m_0 m_1 M} \right) \left( \frac{a_1}{0.1 \text{AU}} \right)^4 \left( 1 - e_1^2 \right)^{7/2}, \tag{2}
\]

where \( M = m_0 + m_1 \). The low-eccentricity version of Equation (2) is \( t_{GW} \times (425/768)(1 - e_1^2)^{-7/2} \) (Peters 1964). Throughout this paper, I focus on stellar-mass binaries whose initial semi-major axis is large enough that the nominal value of \( t_{GW} \), in the absence of a tertiary, is greater than the Hubble time, \( t_H \simeq 14 \text{Gyr} \). This implies values for \( a_{1,0} \) larger than \( \sim 0.017 \text{AU} \), depending on the masses of the binary components considered.

As discussed by BLS02 and MH02 in the context of supermassive BH mergers and the formation of intermediate-mass BHs, respectively, GR periastron precession “detunes” the secular Kozai resonance. This effect in general decreases the maximum eccentricity attainable at fixed tertiary inclination (see the Appendix of BLS02 and the discussion in FT07). The timescale (period) for GR precession (e.g., Equation (23) of FT07) is

\[
t_{GRp} = \frac{1}{3} \frac{a_1}{c} \left( \frac{a_1 c^6}{G M} \right)^{3/2} (1 - e_1^2)
\]

\[
\simeq 3.7 \times 10^4 \text{yr} \left( \frac{2 M_\odot}{M} \right)^{3/2} \left( \frac{a_1}{0.1 \text{AU}} \right)^{5/2} (1 - e_1^2), \tag{3}
\]

and the Kozai timescale is (Innanen et al. 1997; Holman et al. 1997)

\[
t_K = \frac{4}{3} \left( \frac{a_1^3 M}{G m_2^2} \right)^{1/2} \left( \frac{b_2}{a_1} \right)^3
\]

\[
\simeq 77 \text{yr} \left( \frac{a_1}{0.1 \text{AU}} \right)^{3/2} \left( \frac{M}{2 M_\odot} \right)^{1/2} \left( \frac{M_2}{m_2} \right)^{3/2} \left( \frac{b_2/a_1}{20} \right)^3, \tag{4}
\]

where \( b_2 = a_2(1 - e_2^2)^{1/2} \). As discussed in BLS02, Kozai oscillations only operate if \( t_K < t_{GRp} \). The strong dependence of \( t_K \) on \( a_2/a_1 \) implies that there is a maximum \( a_2 \), beyond which Kozai oscillations are ineffective. Additionally, since

\[
t_{GRp}/t_K \propto a_1^4,
\]

as the binary evolves to smaller \( a_1 \) because of GW radiation, \( t_K \) eventually becomes larger than \( t_{GRp} \). This can cause the binary to circularize before coalescence, and it is the competition between \( t_{GRp} \) and \( t_K \) that determines much of the time evolution of the system as it evolves toward merger.

Momentarily ignoring the complications of GR precession, one can combine Equation (1) with Equation (2) to get a rough order-of-magnitude sense of the importance of Kozai oscillations in triple systems for the rapid merger of compact objects. Taking the maximum eccentricity the system reaches to be \( e_{\text{max}} \), the roughest estimate of the merger time is simply

\[
t_{\text{merge}} \sim t_{GW}(a_1, e_{\text{max}})(1 - e_{\text{max}}^2)^{-1/2}, \tag{5}
\]

where the factor \((1 - e_{\text{max}}^2)^{-1/2}\) corrects for the small relative amount of time the system spends at high eccentricity (e.g., MH02). Substituting, one finds that

\[
t_{\text{merge}} \sim \frac{25}{153} a_1 \left( \frac{a_1 c^6}{G^3 m_0 m_1 M} \right) \cos^6 i
\]

\[
\sim 8.7 \times 10^9 \text{yr} \left( \frac{2 M_\odot}{m_0 m_1 M} \right) \left( \frac{a_1}{0.1 \text{AU}} \right)^4 \left( \frac{\cos i}{0.2} \right)^6 \tag{6}
\]

which shows the very strong expected dependence on the inclination angle. The estimate of Equation (6) is only valid for angles in the critical Kozai range between \( 39^\circ \leq i \leq 141^\circ \), and fails to account for important corrections to \( e_{\text{max}} \) from GR precession (BLS02; MH02; Wen 2003; FT07). In particular, it has no dependence on \( a_2/a_1 \), and its dependence on \( a_1 \) is only approximate. In some regions of parameter space Equation (6) thus grossly underestimates \( t_{\text{merge}} \) (see Section 4). A much more accurate, but less simply stated, estimate of \( t_{\text{merge}} \) can be made using the method of Wen (2003) that is accurate to a factor of a few over many decades in \( t_{\text{merge}} \) (see the Appendix).

3.2. Equations, Assumptions, and the Merger Time

I solve the octopole-order equations for the secular evolution of the orbital elements of the system, as given in BLS02 (their Equations (11)–(17)), which are based on the expressions derived in Ford et al. (2000, 2004; see also Krymolowski & Mazeh 1999; Marchal 1990). These equations include both GR periastron precession and GW radiation (MH02; Wen 2003), but neglect tidal forces and treat the masses as point particles. This amounts in part to neglecting terms in the equation for the time evolution of the longitude of periastron of the inner binary associated with both tidal and rotational bulges. Both can
suppress Kozai oscillations in a manner similar to GR precession (Equation (3)). In particular, the timescale associated with the apsidal motion induced by a tidal bulge is (e.g., FT07)

\[ t_{\text{tide}} = \frac{4}{15k} \left( \frac{a_1}{R} \right)^5 \left( \frac{a_1}{GM} \right)^{1/2} \left( 1 - e_1^2 \right)^5 \frac{8 + 12e_1^2 + e_1^4}{5000 \text{ km} R} \]

\[ \simeq 2.35 \times 10^{15} \text{ yr} \left( \frac{a_1}{0.1 \text{ AU}} \right)^{13/2} \left( \frac{5000 \text{ km}}{R} \right)^5 \times \left( \frac{2M_\odot}{M} \right)^{1/2} \left( 1 - e_1^2 \right)^5 \frac{8 + 12e_1^2 + e_1^4}{5000 \text{ km} R} \]  

(7)

where \( R \) is the radius of the compact objects (assumed equal), \( m_0 = m_1 \) has been assumed, and \( k \) is the classical apsidal motion constant (for the numerical estimate \( k = 0.1 \)). For sufficiently high eccentricity (e.g., \( e_1 \gtrsim 0.9986 \) for the parameters of Equation (7)), \( t_{\text{tide}} \) becomes less than \( t_{\text{GRP}} \), and one expects tides to become important to the evolution. In some cases, the eccentricity can become large enough that periapsis approaches the WD radius. In such cases, one expects strong tidal heating and circularization. Although these effects are of interest in their own right, and they might be especially important for the resulting transients from such mergers, they are not captured in the current study, and will be the subject of a future work. For the present purposes, it is sufficient to note that GW merger timescale on a scale comparable to the compact object radius (whether NS or WD) is ultrashort compared with the merger time of circular binaries without a tertiary at the semi-major axes of interest (>\( t_{\text{tide}} \)). Thus, if the binary does circularize on such scales, the merger timescale \( t_{\text{merge}} \), as estimated by numerical solution of the time-dependent equations, is not dramatically affected.

While the merger time \( t_{\text{merge}} \) reported for binaries calculated in Section 4 is the time required for the semi-major axis to reach the radius of the compact object \( R_{\text{CO}} \) (I take \( R_{\text{WD}} = 5000 \text{ km} \) and \( R_{\text{NS}} = 10 \text{ km} \)), in some extreme cases the periapsis of the orbit reaches \( \sim R_{\text{CO}} \). In such cases, I explicitly note that such solutions will be strongly affected by tides. The combined action of tidal friction and Kozai oscillations on the period distribution of solar-type binaries in triple systems has been considered in detail by FT07 (see also Mazeh & Shaham 1979; Kiseleva et al. 1998; Eggleton & Kiseleva-Eggleton 2001; Wu & Murray 2003; Wu et al. 2007; Perets & Fabrycky 2009). A similar calculation of WD and NS binaries in triple systems is saved for a future paper.

Finally, all of the initial configurations described below are chosen to be stable when compared to the empirical three-body stability criterion of Mardling & Aarseth (2001; see also Eggleton & Kiseleva 1995). For a discussion in the context of Kozai oscillations, see BLS02.

3.3. Numerics

Standard methods (e.g., Bulirsch-Stoer; Press et al. 1992) are employed to solve the system of equations, and I have done several checks to ensure the fidelity of the results presented. First, I have varied the numerical tolerance of the algorithm systematically and found that the results presented here are converged. As an additional check, I have verified that as \( a_2 \) becomes large and \( f_k \gg t_{\text{GRP}} \), the solution for \( t_{\text{merge}} \) of the inner binary approaches the result of Peters (1964) for an isolated binary. Finally, I have spot checked my calculations directly against the code of BLS02 and the numerical results of Wen (2003), and find excellent agreement.

Some of the results presented make use of the simple approximate method of Wen (2003), described in the Appendix. Comparisons between this approximation and the actual solution of the time-dependent problem are provided there.

4. RESULTS

The parameter space of possible masses and orbits is very large. To restrict the total model space, I take the initial values of the orbital eccentricities and arguments of periastron to be
$e_{i_0} = e_{2_0} = 0.1$, $g_{i_0} = 0^\circ$, and $g_{2_0} = 90^\circ$ throughout this paper. One expects the assumption of low initial eccentricity to be reasonable except in two cases of particular interest: (1) in dynamically formed triple systems the eccentricity of the tertiary may be large $e_2 \sim 0.9$ (Ivanova 2008), and (2) in NS–NS binaries the NS kicks at birth may cause $e_1$ to be large. Higher $e_1$ and $e_2$ generically lead to faster mergers, and thus the assumption of low initial eccentricities is conservative.

Figure 1 shows the time evolution of a selection of orbital elements for the fiducial WD–WD case with $m_0 = 0.8 M_\odot$, $m_1 = 0.6 M_\odot$, and $m_2 = 1.0 M_\odot$. The masses of $m_0$ and $m_1$ are chosen to sum to 1.4 $M_\odot$ for illustrative purposes, and $m_0 \neq m_1$ so that the octupole-order terms in the dynamical equations operate. Results for $m_0 = m_1 = 0.7 M_\odot$ are not significantly different. The initial semi-major axis ($a_{i_0}/a_{1_0}$) is chosen so that the nominal GW merger timescale in the absence of the tertiary is $\sim 200 Gyr$. The initial mutual inclination of the system is $i = 85^\circ$ ($\cos i \simeq 0.09$). The left panel shows the early-time evolution of $i$ (dotted), $g_1$ (dashed), and $e_1$ (solid). The periodic changes in $e_1$ correspond to $f_\perp$ (Equation (4)). The right panel shows the late-time evolution of $e_1$ and the semi-major axis, scaled by its initial value ($a_{i_0}/a_{1_0}$). The system merges in $\sim 2.6 \times 10^6$ yr, approximately $10^2$ times faster than without the tertiary. At no time before the very end of the calculation does the periastron of the orbit reach $2 \times R_{WD}$.

The results of many such calculations are presented in Figure 2. The left panel shows $t_{\text{merge}}$ as a function of the initial value of $a_{2_0}/a_{1_0}$, for $a_{i_0}/a_{1_0} = 0.01$ AU (thin solid), 0.1 AU (dashed), and 0.5 AU (heavy solid), for several different values of $i_0$ as labeled. The nominal (no tertiary) binary merger timescales via GW radiation are $\sim 5 \times 10^9$ yr ($\sim 0.3$ $t_H$), $\sim 5 \times 10^{13}$ yr ($3 \times 10^3$ $t_H$), and $\sim 3 \times 10^{16}$ yr ($2 \times 10^9$ $t_H$), for $a_{1_0}/a_{1_0} = 0.01$ AU, 0.1 AU, and 0.5 AU, respectively.

In the models with $a_{i_0}/a_{1_0} = 0.01$ AU, as $a_{2_0}/a_{1_0}$ becomes greater than $\sim 50–60$, there is essentially no decrease in $t_{\text{merge}}$ with respect to the case without the tertiary, and all models approach $t_{\text{merge}} \sim 5 \times 10^9$ yr. For $a_{2_0}/a_{1_0} \sim 20$, there is a minimum in $t_{\text{merge}}$. The retrograde cases shown with $i = 95^\circ$ have very short $t_{\text{merge}}$ for small $a_{2_0}/a_{1_0}$, and typically merge in a single Kozai timescale $f_\perp$.

Setting the WD radius to be $R_{WD} = 5000$ km, for the cases with $a_{2_0}/a_{1_0} = 10$ and 20 and $i = 89^\circ$, the periastron of the inner binary orbit becomes less than the $R_{WD}$ in the first Kozai oscillation, at time $t_H$. However, for $R_{WD} = 1000$ km it does not, and the evolution is qualitatively similar to that presented in the right panel of Figure 1. Clearly, for these cases a more complete model with tidal dissipation and circularization is required to capture the dynamics and to make an accurate calculation of $t_{\text{merge}}$. For the purposes of constructing this figure, I have assumed that $R_{WD}$ is small enough that a “collision” (periastron $< R_{WD}$) does not occur, and thus the results presented may be an upper limit to $t_{\text{merge}}$.

The right panel of Figure 2 shows $t_{\text{merge}}$ as a function of $\cos i$ for $a_{i_0}/a_{1_0} = 0.01$ AU and $a_{2_0}/a_{1_0} = 20, 30, 40, 50, 60$, and 70. The estimate of Equation (6), which fails to capture the very strong dependence on $a_{2_0}/a_{1_0}$, is shown as the dashed line. Again, for $a_{2_0}/a_{1_0} \leq 20$ and $89 \lesssim i_0 \lesssim 96$, strong tidal interactions in a single $f_\perp$ are expected; for these models, as $e_1$ reaches its first maximum, the periastron is less than the fiducial WD radius of 5000 km. However, as in the left panel, the results shown assume $R_{WD}$ small enough that a “collision” never occurs. All other models have the same qualitative behavior as shown in Figure 1.

In order to make a broad exploration of parameter space for many models, instead of calculating the detailed time evolution of each system, as in Figures 1 and 2, I use the approximate
The Astrophysical Journal, 741:82 (14pp), 2011 November 10

Thompson

Figure 3. Range of allowed initial inner binary semi-major axis $a_{1,0}$ vs. initial mutual inclination $i_0$, such that the inner binary merges in $t_H$, $t_H/10$, $t_H/100$, and $t_H/1000$ (darkest to lightest), for outer tertiary semi-major axis $a_{2,0}/a_{1,0} = 10$ (top left), 40 (top right), 60 (bottom left), and 80 (bottom right), computed by estimating $t_{\text{merge}}$ using the algorithm described in the Appendix for $m_0 = m_1 = 0.7$, $m_2 = 1.0$, $e_{1,0} = e_{2,0} = 0.1$, $g_{1,0} = 0^\circ$, and $g_{2,0} = 90^\circ$. Note that for these inner binary masses, only those with $a_{1,0} \lesssim 0.015$ AU (off the bottom of the range shown) would be expected to merge in $t_H$ in the absence of a tertiary companion. Note further that for terriaries randomly distributed in $i$, cos$(i)$ is proportional to the probability of having such a system.

Using this approximate method, Figure 3 shows results analogous to those presented in Figure 2, but for $m_0 = m_1 = 0.7$, and $m_2 = 1.0$. As a function of the initial value of the inner binary semi-major axis, $a_{1,0}$, and as a function of $\cos i_0$, I survey this parameter space for regions where $t_{\text{merge}}$ is $< t_H$ (black), $< 0.1t_H$ (dark gray), $< 0.01t_H$ (light gray), and $< 0.001t_H$ (interior white) for initial values of $a_{2,0}/a_{1,0} = 20$ (upper left), 40 (upper right), 60 (lower left), and 80 (lower right). Only regimes where the nominal GW merger timescale without the tertiary is larger than $t_H$ are explored ($a_{1,0} \gtrsim 0.015$ AU; see Equation (2)). The right vertical axis gives the associated no-tertiary GW merger time; considered values for $a_{1,0}$ run up to 10 AU, equivalent to $t_{GW} \sim 10^{22}$ yr without the tertiary. The basic trends presented in the dynamical calculations shown in Figure 2 are reproduced. Exceedingly rapid mergers can be...
induced for a narrow range of retrograde values of $i$ near $95^\circ$, and for many prograde tertiary orbits. As $a_{2,0}/a_{1,0}$ increases, the allowed area in the $a_{1,0}-\cos i_0$ plane decreases, and for $a_{2,0}/a_{1,0} \gtrsim 100$, Kozai oscillations do not accelerate mergers; this follows from the strong dependence of $t_K$ on $a_2/a_1$ and the effects of GR precession. Nevertheless, it is clear from Figure 3 that for a significant region of parameter space, $t_{\text{merge}}$ can be less than 10, 1, 0.1, or even 0.01 Gyr, even for systems that have nominal (no tertiary) merger times of $\gg t_H$.

Using these estimates of $t_{\text{merge}}$, it is possible to calculate the maximum possible value of the inner binary semi-major axis $a_{1,0}^{\text{max}}$ such that merger occurs in a single Hubble time, given a tertiary of mass $m_2 = 1.0 M_\odot$ at any $i$. Figure 3 implies that for retrograde orbits $a_{1,0}^{\text{max}}$ is extremely large near $i_0 \sim 95^\circ$. If we restrict our attention to prograde orbits, there is a unique value of $a_{1,0}^{\text{max}}$ for each initial $i_0$ such that $t_{\text{merge}} \leq t_H$. As implied by the left panel of Figure 2, since $t_{\text{merge}}$ exhibits a minimum as a function of $a_{2,0}/a_{1,0}$, $a_{1,0}^{\text{max}}$, will occur not at the smallest

Figure 4. Total mass of compact object binaries $M = m_0 + m_1$ vs. orbital period (lower axis) and semi-major axis (upper axis, assuming $M = 1.4 M_\odot$). The heavy solid lines show the critical value of the inner binary period ($P_1^{\text{max}}$) such that $t_{\text{merge}} = t_H$, including a tertiary with $i_0 = 70^\circ$, $80^\circ$, $85^\circ$, $89^\circ$, and $90^\circ$, assuming $m_0 = m_1$, $m_2 = 1.0 M_\odot$, and the same parameters used in Figure 3. Each line is labeled with the values of the critical tertiary period $P_2$ (yr) for which $P_1^{\text{max}}$ occurs (small numbers). Dotted lines are of constant no-tertiary GW merger time of $t_{GW} = t_H$, $10^3 t_H$, $10^6 t_H$, and $10^9 t_H$, assuming $m_0 = m_1$, and $e_1 = 0$. Thus, for example, triple systems consisting of binaries with $M = 1.4 M_\odot$ (along the horizontal dashed line) and $P_1 \lesssim 500$ days can in principle merge in $t_H$. For larger $m_2$, the allowed range of $P_1$ increases. For some retrograde orbits ($i_0 \sim 95^\circ$), it increases dramatically (see Figure 3). Data on WD+WD (filled squares), NS+WD (filled+open squares), and NS+NS (filled+open circles) binaries are shown (Mullally et al. 2009; Kulkarni & van Kerkwijk 2010; Nelemans et al. 2005; Kilic et al. 2010a, 2010b, Stairs 2004). The Chandrasekhar mass (dashed line) and the maximum NS mass (gray shaded) are indicated for reference.
for \( m_{1} = 1.4 \, M_{\odot} \), an intermediate-mass tertiary can stay bound, and perhaps acquire more rapid mergers than implied by Figure 5. As discussed in Section 2.2, in some cases the tertiary might also be a massive BH, in which case the allowed region for which \( t_{\text{merge}} \) is less than \( t_{\text{H}} \) grows. Similar to the WD–WD case, Figure 4 shows that for a 1 \( M_{\odot} \) tertiary with \( i \gtrsim 85^\circ \) and \( P_{2} \lesssim 10 \, \text{yr} \), all the observed NS–NS systems merge in \( t_{\text{H}} \) given an appropriately placed tertiary.

The parameter space for which the tertiary remains bound to the system through both of the supernovae that produce the NS–NS binary may not be large. In particular, the tertiary can neither be too close, because of dynamical stability, nor too far away, because of the binding energy of the orbit relative to the kick imparted to the center of mass during each of the inner binary component’s supernovae. However, inspection of the plots by Kalogera (1996) for the binary case indicates that for relatively small tertiary semi-major axis (\( a_{2} \gtrsim \text{few } M_{\odot} \)), an intermediate-mass tertiary can stay bound, and perhaps acquire high eccentricity during the supernovae for kick velocities of \( \sim 100 \, \text{km s}^{-1} \) in some cases. A high value of \( e_{2} \) could lead to more rapid mergers than implied by Figure 5. As discussed in Sections 2.1 and 2.2, \( a_{2} \sim \text{AU} \) likely guarantees strong (CE) interaction between all components as the primary and then the secondary evolve off the main sequence.

\[
a_{2,0}/a_{1,0}=20 \\
m_2=6 \, M_{\odot} \\
t_{\text{merge}} < t_{\text{H}}/300
\]

Figure 5. Same as Figure 3, but for NS–NS binaries with \( m_{1} = 1.4 \, M_{\odot}, a_{2,0}/a_{1,0} = 20, \) and \( m_{2} = 6.0 \, M_{\odot} \) (left panel), and \( m_{2} = 1.0 \, M_{\odot} \) (right panel). In the left panel, only \( t_{\text{merge}} \lesssim t_{\text{H}}/300 \) is shown since this is approximately the main-sequence lifetime of the tertiary.
5. DISCUSSION

5.1. Rates

The fact that essentially all close solar-type binaries are in triple systems argues that their subsequent WD–WD binaries will also be in triple systems. Depending on the semi-major axis and inclination distribution of the tertiaries at the time of formation of the compact objects, one expects Kozai oscillations to speed up the process of coalescence of the inner binary significantly, as shown in Figures 1–3. The work of FT07, together with Figure 3, serves as a guide to an estimation of the rate, but the estimate is complicated by the expectation that the distribution of inclinations may be biased toward co-planar orbits for coeval systems.

Nevertheless, to make a simple rate estimate, assume that the distribution of binary and tertiary semi-major axes is flat (equal numbers in log semi-major axis), and that the probability of having a system at inclination \( i \) is \( dp/di = \sin i \). Importantly, Figure 3 shows that the range of \( a_1 \) strongly affected by Kozai oscillations is comparable to the relevant range of close binary semi-major axes. In addition, the Kozai mechanism operates over \( \sim 1 \) decade in \( a_2/a_1 \sim 3–100 \), which corresponds to \( \sim 2 \) decades in \( a_2 \). Thus, one then expects many of the triples to be strongly affected. Finally, although the area in the \( a_1-\cos i \) plane is smaller for more rapid mergers, the systems at large \( i \) dominate the rate. For example, comparing the prograde regions of the upper left panel of Figure 3, one sees that the total area of the shaded regions (with \( a_1 \) measured in log units) measures the fraction of all systems with that merger time. Although the area of the black region is roughly 10 times that of the white, the latter fraction of all systems with that merger time. Thus, the average rate is dominated by \( a_1 \) if the later merger 1000 times faster. Thus, the average rate is dominated by the highest inclination systems, a consequence in part of the very steep scaling of the time to merger with \( \cos i \) (e.g., Equation (6), Figure 2).

This can be shown explicitly by using the very crude estimate of Equation (6). Momentarily ignoring the dependence on \( a_2 \), one may write the total merger rate as

\[
\frac{dN}{dt} \sim \frac{153}{100} \left( \frac{M_{\text{mom}} G^3}{a_1^5 \cos^6 i} \right) \int \frac{dN}{d \ln a_1} + \frac{2}{3} \frac{dN}{d \ln \cos i},
\]

where \( N \) is the total number of systems. As in Equation (6), this expression neglects the very important \( m_2 \) dependence of the Kozai mechanism and the \( a_2/a_1 \) dependence, and thus Equation (8) should be considered schematic. For practical purposes, the dependence on \( a_2/a_1 \) can be considered nearly a step function since for all relevant parameters Kozai is ineffective for \( a_2/a_1 \gtrsim 100 \), as shown in Figures 2 and 3. As discussed in Section 2, the latter is particularly important since the inner binary will likely undergo two CE events and significant mass loss, thus increasing \( a_2/a_1 \) from its initial value. Nevertheless, if triple systems are prevalent, one expects them to contribute both to the very prompt rate \( (< 10^8 \text{ yr}; \text{Section 5.2}) \), and to the delayed rate \( (> \text{ Gyr}) \), since for \( i \lesssim 39.2^\circ \) (and/or \( a_2/a_1 > 100 \) and/or \( m_2 < 1 M_\odot \)), Kozai does not affect the merger time of the inner binary. Thus, for low-\( i \) or large \( a_2/a_1 \) systems, one should obtain a delay-time distribution identical to the binary-only case. A more complete calculation of the delay-time distribution of triple systems is saved for a future work.

5.2. How “Prompt” is Prompt?

A prompt component to the supernova Ia rate has been claimed by a number of authors (e.g., Scannapieco & Bildsten 2005; Mannucci et al. 2006; Aubourg et al. 2008; Brandt et al. 2010; Maoz & Badenes 2010; Maoz et al. 2010, 2011). Various estimates suggest that \( \sim 50\% \) of Ia’s are “prompt,” with a characteristic short timescale of \( \sim 0.1–1 \text{ Gyr} \), and potentially with a two-component power-law or bimodal delay-time distribution (see Mannucci et al. 2006 and the recent review by Maoz 2010).

In the picture presented here, a fraction of intermediate-mass stars are born in triple systems. The fraction with high-inclination tertiaries can merge extremely rapidly as soon as both stars in the inner binary are WDs. Because the calculated merger times are in many cases much less than even the post-main-sequence timescales of these stars, one expects the fastest WD–WD mergers to be limited only by the main-sequence lifetime of their progenitors. Thus, \( \sim 8 + 8 M_\odot \) binaries in triple systems merge first, immediately after WD birth, and thus the minimum delay time is \( \sim 3 \times 10^7 \text{ yr} \). In this case, one expects a delay-time distribution of \( \sim \tau^{-1/2} \) (Pritchet et al. 2008). These prompt supernovae would preferentially be super-Chandrasekhar-mass binaries (see Pakmor et al. 2010).

Many of these statements are equally applicable to NS–NS mergers. For example, Figure 5 shows that the merger timescale for NS–NS binaries in triple systems can be very short. Again, there is a region of parameter space where the limiting factor is the time required to produce the NS–NS binary, again implying that the fastest NS–NS mergers can come just \( \sim 10^7 \text{ yr} \) after the last star formation episode. This may explain the fact that many SGRBs are seen in star-forming galaxies (e.g., Berger 2009). By extension, one expects similar arguments to hold for BH–WD/NS or NS–WD systems that in some cases might give rise to long-duration GRBs (Fryer et al. 1999a, 1999b).

These considerations are amplified if the multiplicity of stars increases with zero-age main-sequence mass (Lada 2006; Raghavan et al. 2010).

5.3. Progenitors

Even if they constitute much of overall merger rate, the progenitor systems for the shortest-lived compact object mergers will be very rare in a given galaxy simply because the time a given system spends as a progenitor is very short. Nevertheless, as in single-degenerate models of Ia supernovae, for WD–WD systems with Gyr merger time (with the tertiary), one could look for close binaries with evidence of a tertiary companion. This would work if the tertiary is a WD or NS, but the problem is that in many cases one expects the companion to be a main-sequence star of \( \sim 1 M_\odot \) (Section 2). The latter would necessitate a new search strategy, since most searches for close WD–WD binaries are color selected (Napiwotzki et al. 2001; Badenes et al. 2009; Brown et al. 2010). One example would be to target intermediate-mass main-sequence stars for radial velocity measurements and search for the signal of a more massive, but unseen companion, which might be an old, high-mass WD–WD (or NS/BH) binary.

The same is true of the coeval NS–NS systems described in Section 2. In that case, one might look for NS–NS binaries in pulsar searches, with timing characteristics that suggest the presence of a tertiary component. However, in many cases the NS components may not be observed as pulsars, and the tertiary may be a main sequence \( \sim 6–8 M_\odot \) star. In both the WD–WD and NS–NS cases, the only visible progenitor at the sight of the subsequent explosion (the compact object merger) in pre-explosion imaging may be a bright main-sequence star (similar in spirit and conclusion to the recent work by Kochanek 2009).
5.4. Transients

Recently, several authors have discussed the possibility that WD–WD collisions, at small impact parameter, may be a way of producing Ia-like supernovae (e.g., Rosswog et al. 2009; Raskin et al. 2010). For some of the calculations presented in Figures 2 and 3, the merger occurs in a single Kozai timescale, at very high eccentricity. I have neglected tidal forces on the compact objects throughout this work (Section 3), but these findings suggest that in some cases something akin to a “collision,” or at least a very strong tidal interaction, may be induced by the Kozai mechanism. This is particularly promising for retrograde orbits with \( i \sim 90^\circ \) (Figure 3). Such tertiaries may be captured in binary–binary collisions in dense stellar environments (Ivanova et al. 2008, 2010), or perhaps in some cases may be coeval (see Figure 7 of FT07).

Simple numerical experiments with the evolutionary equations described in Section 3, but including apsidal motion of the inner binary as a result of tides (Equation (7)), still allow for a region of parameter space where the merger occurs at very high eccentricity.

5.5. Interactions and the Tertiary

**WD–WD mergers.** If WD–WD mergers driven by a hierarchical tertiary produce Type Ia supernovae, the explosion should interact with and overtake the tertiary. Thus, just as in searches for the remaining star in the single-degenerate scenario (e.g., Kerzendorf et al. 2009), in the triple scenario proposed here, the tertiary in historical Galactic supernovae should be currently inside the Ia supernova remnant. In addition, depending on the structure of the tertiary, some amount of mass might be expected to be lost by the shockwave interaction (Marekta et al. 2007).

However, in contrast to the single-degenerate picture (Whelan & Iben 1973), one expects the tertiary to be in many cases more than an AU distant from the explosion and would only very rarely be a giant. Using the simple analytic model of mass stripping and ablation in supernova explosions with binary companions developed by Wheeler et al. (1975), I find that even in the most compact cases considered here, with \( a_2 \sim 0.1 \) AU, the total mass stripped and ablated from a main-sequence solar-mass tertiary is just \( \sim 0.01 \, M_\odot \). For \( a_2 \gtrsim 1 \) AU, the expected mass lost from the tertiary will be minimal. Similarly, based on the work of Kasen (2010), one expects a soft X-ray flash as the Ia shockwave overtakes the tertiary on a timescale of \( \sim 10^4 \) s (\( a_2/\text{AU} \)) for a shockwave of \( \sim 10^4 \) km s\(^{-1} \) and a very small change to the early-time optical and UV light curve.

**NS–NS mergers.** If NS–NS mergers in triple systems produce SGRBs, then there is a chance for interaction of the relativistic blastwave with the tertiary companion because the systems most likely to be affected by Kozai oscillations are at high mutual inclination relative to the inner binary, and because the inferred opening angle for SGRBs is relatively large (e.g., Nakar 2007). One then envisions an interaction similar to that calculated in MacFadyen et al. (2005), but with either a main-sequence, WD, or BH companion to the SGRB explosion (see Section 2.2, Figure 5). The timescale for interaction is \( \sim a_2/c \sim 500 (a_2/\text{AU}) \) s. Moreover, because we are presumably looking roughly down the jet axis, the tertiary may in some cases be roughly along the line of sight. In the case of a main-sequence companion, some material may be stripped and ablated. However, as in the WD–WD case, \( a_2 \) is expected to be in the range of \( \sim 0.1–10 \) AU, and this may preclude a large effect on the tertiary, or the SGRB light curve.

5.6. Gravity Waves

**The diffuse background.** If most close WD–WD binaries were born in triple systems, predictions for the GW background may be modified from fiducial predictions (e.g., Farmer & Phinney 2003). For highly eccentric orbits, the total power in GWs, and the peak frequency of GWs, increases significantly. The peak GW frequency is well approximated by Wen (2003)

\[
f_{GW}^{max} = \frac{1}{\pi} \left( \frac{GM}{a_1^4 (1-e_1^2)^{3/2}} \right)^{1/2} (1+e_1)^{-1.1954}.
\]

As an example, Figure 6 shows the early (left panel) and late (right panel) time evolution of the GW frequency for the triple system shown in Figure 1. Although the inner binary, has \( f_{GW} \approx 0.008 \) mHz in the absence of a tertiary, with the addition of the tertiary \( f_{GW} \) increases strongly, and periodically, on a timescale \( \tau_K \) (Equation (4)).

As shown in the Appendix, the maximum eccentricity attained by the inner binary can be estimated semi-analytically. It is these estimates that are used in the calculations of the merger time shown in Figures 3 and 5. Substituting this maximum eccentricity into Equation (9) yields \( f_{GW}^{max} \), the maximum GW frequency produced by the binary during inspiral. The horizontal solid line in Figure 6 shows this estimate for \( f_{GW}^{max} \).

Figure 7 shows a summary of results for \( f_{GW}^{max} \) computed from the results for two panels of Figure 3 for WD–WD binaries \( (a_{2,0}/a_{1,0} = 10, 60) \). For the models that merge in less than \( t_{1/2} \), regions of different peak \( f_{GW}^{max} \) are shaded in the \( a_{1,0}, \cos i_0 \) plane. All frequencies shaded are in the range detectable by LISA. Note that the WD–WD binaries that attain the highest frequencies (interior white) have the highest peak eccentricities, the smallest periapses, and will interact tidally, thus modifying the results presented here (Section 3.2). This can be seen since \( f_{GW}^{max} \) approaches the inverse dynamical time for an individual WD for some regions of parameter space.

Since the extragalactic GW background is dominated by close WD–WD binaries (Farmer & Phinney 2003), which are generally assumed to be circular as a result of the preceding CE evolution, if the fraction of close binaries that are actually in triple systems is fairly large, as implied by the statistics on solar-type binaries (Tokovinin et al. 2006; FT07), then the expected GW background will be modified. Recent studies of the LISA GW foreground focus exclusively on circular WD–WD binaries (Ruiter et al. 2010), and these results too would need to be revisited if the triple fraction of close WD–WD binaries is high. The only cases where eccentric WD–WD binaries have been considered are in globular clusters where high eccentricity can be imparted to binaries during binary–single and binary–binary interactions (Benacquista 2001; Ivanova et al. 2006; Willems et al. 2007).

**Sources and foreground.** Whether NS–NS, WD–WD, or otherwise, the most eccentric systems will be short lived, and this decreases the probability that they can be seen. The competing effect, of course, is that their overall GW luminosity is larger. The right panel of Figure 6 indicates that a
Figure 6. Time evolution of the peak GW frequency $f_{GW}(\text{mHz})$ (Equation (9)) for the system shown in Figure 1. The gray line shows the maximum GW frequency, obtained by combining Equation (9) with the estimate of the maximum eccentricity given in the Appendix.

Figure 7. Same as the top and bottom left panels of Figure 3, respectively, but showing regimes for which the maximum value of the GW frequency $f_{\text{max}}^{GW}$ (as computed from Equation (9) and the Appendix) is in the range 0.1–1 (black), 1–10 (dark gray), 10–10^2 (light gray), and 10^2–10^3 mHz (white). The latter will be affected by tidal interaction between the WDs, which is not captured by the calculations presented.

$a_1 = 0.05$ AU WD–WD binary in a triple system with a $1 M_\odot$ tertiary at $a_2/a_1 = 20$ would go through periodic decade-long enhancements in $e_1$ that are potentially detectable by LISA for nearby systems. During the mission itself, many such binaries might be seen, both Galactic WD–WD systems with larger $a_1$ and NS–NS systems with higher $f_{\text{max}}^{GW}$. The effect of the Kozai mechanism is to make otherwise unobservable Galactic WD–WD binaries observable during pericenter passage, since it is at these times at high $e_1$ that $f_{GW}$ is maximized. This may cause individual sources to be observable at pericenter passage, and it may cause an added degree of complexity to the GW foreground from all local sources (see Gould 2011).

I am grateful to Omer Blaes for sharing the code from Blaes et al. (2002) for the purposes of initially validating the results of the code presented here. In addition, I thank Andy Gould, Chris Kochanek, and Kris Stanek for discussions and encouragement. Additional discussions with Ondřej Pejcha, Benjamin Shappee,
and Brian C. Lacki are acknowledged. This work is supported in part by an Alfred P. Sloan Foundation Fellowship and NSF grant AST-0908816.

APPENDIX

APPROXIMATION TO THE MERGER TIME

I use the methods discussed in MH02 and Wen (2003) to make simple, but accurate, estimates of $t_{\text{merge}}$ to supplement the direct calculation of the time dependence of the orbital elements, as described in Section 3. Since this scheme provides an efficient way to estimate the merger timescale for a very wide range of system parameters, in Figures 3 and 4, I repeat the steps here.

The goal is to estimate $t_{\text{merge}}$ from the initial conditions of the triple system. Following Wen (2003), I use the fact that (neglecting gravitational radiation), the quadrupole-level Hamiltonian is conserved throughout the evolution. It can be written in terms of $\epsilon = 1 - e_1^2$, $i$, and $g_1$ as (MH02)

$$W(\epsilon, g_1) = -2\epsilon + \epsilon \cos^2 \epsilon i + 5(1 - \epsilon) \sin^2 g_1(\cos^2 \epsilon i - 1)$$

$$+ \frac{4}{\sqrt{\epsilon}} (\frac{m_0 + m_1}{m_2}) (\frac{b_2}{a_1})^3 \left(\frac{2G(m_0 + m_1)}{a_1 c^2}\right), \quad (A1)$$

where $b_2 = a_2(1 - e_2^2)^{1/2}$, and the last term is the first-order post-Newtonian correction that accounts for GR precession.

Starting with initial values of the eccentricity of the inner binary and its argument of periastron, $\epsilon_{1,0}$ ($\epsilon_0$) and $g_{1,0}$, as well as the initial inclination $i_0$, the system evolves to a maximum eccentricity $\epsilon_{1,\text{max}}$ and thus minimum $\epsilon_{\text{min}}$, at a critical $g_{1,\text{crit}}$ and $i_{\text{crit}}$. Taking $d\epsilon/dt = 0$ at the moment $\epsilon = \epsilon_{\text{min}}$ provides a relationship between $\epsilon_{\text{min}}$, $g_{1,\text{crit}}$, and $i_{\text{crit}}$ (Wen 2003):

$$\sin(2g_{1,\text{crit}}) = \frac{8}{225} \left(\frac{G(m_0 + m_1)}{a_1 c^2}\right)^{3/2} \frac{Gm_0 m_1}{a_1 c^2 m_2}$$

Using the fact that (Wen 2003)

$$\cos i_{\text{crit}} = \cos i_0 \left(\epsilon_0 \epsilon_{\text{min}}\right)^{1/2} + \frac{a_1}{2\beta} \frac{1}{\sqrt{\epsilon_{\text{min}}}} \left(\epsilon_0 - \epsilon_{\text{min}}\right), \quad (A3)$$

where $\beta = (m_2/m_0 + m_1)((m_0 + m_1)\beta_0 a_2)/((m_0 + m_1 + m_2)(1 - e_2^2))^{1/2}$, allows one to write $\sin(2g_{1,\text{crit}})$ (Equation (A2)) in terms of just $\epsilon_{\text{min}}$ and the initial parameters of the system. In calculating $g_{1,\text{crit}}$ and $i_{\text{crit}}$, I assume that $a_1$, $a_2$, and $e_2$ are unchanged from their initial values. Solving the implicit equation $\Delta W = W(\epsilon_0, g_{1,0}) - W(\epsilon_{\text{min}}, g_{1,\text{crit}}) = 0$ for $\epsilon_{\text{min}}$, then allows for an accurate estimate of the merger time (MH02):

$$t_{\text{merge}} = t_GW(a_1, \epsilon_{\text{min}}) \epsilon_{\text{min}}^{-1/2}, \quad (A4)$$

where $t_GW$ is computed from the formalism of Peters (1964). In practice, I solve the equation $\Delta W = 0$ using Newton–Raphson iteration with an initial guess for $\epsilon_{\text{min}}$ using the approximate expressions from Wen (2003) and MH02.

A comparison between the numerically calculated value of $t_{\text{merge}}$ and this approximate scheme is shown in Figure 8. Typically, $t_{\text{merge}}$ as calculated from Equation (A4) is underestimated by a factor of $\sim 1.5–2$. There are systematic deviations as a function of both $i$ and $a_2/a_1$. These deviations are not due to the fact that the numerical solution is to the octopole-level equations, whereas Equation (A4) is only at quadrupole order, since for $m_0 = m_1$, the equations reduce from octopole to quadrupole and this hypothesis can be explicitly tested. The right panel shows that for retrograde orbits and small $a_2/a_1$, the estimate of Equation (A4) can overpredict $t_{\text{merge}}$ by a factor of $\sim 10$. Note, however, that it is precisely in these cases where tidal effects
Ininan, K. A., Zheng, J. Q., Mikkola, S., & Valtonen, M. J. 1997, AJ, 113, 1072
Iben, I., Jr., & Tutukov, A. V. 1999, ApJ, 511, 324
Iben, I., Jr., & Tutukov, A. V. 1985, ApJS, 58, 661
Iben, I., Jr., & Livio, M. 1993, PASP, 105, 1373
Hills, J. G. 1983, ApJ, 267, 322
Ivanova, N., Heinke, C. O., & Rasio, F. A., et al. 2006, MNRAS, 372, 1043
Ivanova, N., Heinke, C. O., Rasio, F. A., Belczynski, K., & Fregeau, J. M. 2008, MNRAS, 386, 553
Janka, H.-T., Eberl, T., Ruffert, M., & Fryer, C. L. 1999, ApJ, 527, L39
Kalirai, J. S., Hansen, B. M. S., Kelson, D. D., et al. 2008, ApJ, 676, 594
Kalogera, V. 1996, ApJ, 471, 352
Kalogera, V., Kim, C., Lorimer, D. R., et al. 2004a, ApJ, 601, L179
Kalogera, V., Kim, C., Lorimer, D. R., et al. 2004b, ApJ, 614, L137
Kalogera, V., Narayan, R., Spiegel, D. N., & Taylor, J. H. 2001, ApJ, 556, 340
Kasen, D. 2010, ApJ, 708, 1025
Kerzendorf, W. E., Schmidt, B. P., Asplund, M., et al. 2009, ApJ, 701, 1665
Kilic, M., Allende Prieto, C., Brown, W. R., et al. 2010a, ApJ, 721, L158
Kilic, M., Brown, W. R., Allende Prieto, C., Kenyon, S. J., & Panei, J. A. 2010b, ApJ, 716, 122
Kiseleva, L. G., Eggleton, P. P., & Mikolla, S. 1998, MNRAS, 300, 292
Kochanek, C. S. 2009, ApJ, 707, 1578
Kozai, Y. 1962, AJ, 67, 591
Krymolowsky, Y., & Mazeh, T. 1999, MNRAS, 304, 720
Kulkarni, S. R., & van Kerkwijk, M. H. 2010, ApJ, 719, 1123
Lada, C. J. 2006, ApJ, 640, L63
MacFadyen, A. I., Ramirez-Ruiz, E., & Zhang, W. 2005, arXiv:astro-ph/0510192
Mannucci, F., Della Valle, M., & Panagia, N. 2006, MNRAS, 370, 773
Maoz, D. 2010, in AIP Conf. Ser. 1314, International Conference on Binaries, ed. V. Kologera & M. van der Sluys (Melville, NY: AIP), 223
Maoz, D., & Badenes, C. 2010, MNRAS, 407, 1314
Maoz, D., Mannucci, F., Li, W., et al. 2011, MNRAS, 412, 1508
Maoz, D., Sharon, K., & Gal-Yam, A. 2010, ApJ, 722, 1879
Marchal, C. 1990, Studies in Astronautics, Studies in Aeronautics, 4 (Amsterdam: Elsevier)
Mardling, R. A., & Aarseth, S. J. 2001, MNRAS, 321, 398
Marietta, E., Burrows, A., & Fryxell, B. 2000, ApJS, 128, 615
Mazeh, T., & Shaham, J. 1979, A&A, 77, 145
Metzger, B. D. 2011, arXiv:1105.6096
Miller, M. C., & Hamilton, D. P. 2002, ApJ, 576, 894 (M02)
Mullally, F., Badenes, C., Thompson, S. E., & Lupton, R. 2009, ApJ, 707, L51
Nakar, E. 2007, Phys. Rep., 442, 166
Napiwotzki, R., Christlieb, N., Drechsel, H., et al. 2001, Astron. Nachr., 322, 411
Nelemans, G., Napiwotzki, R., Karl, C., et al. 2005, A&A, 440, 1087
Nelemans, G., Portegies Zwart, S. F., Verbunt, F., & Yungelson, L. R. 2001, A&A, 368, 939
O’Shaughnessy, R., Belczynski, K., & Kalogera, V. 2008, ApJ, 675, 566
Packmor, R., Kromer, M., Röpke, F. K., et al. 2010, Nature, 463, 61
Perets, H. B., & Fabrycky, D. C. 2009, ApJ, 697, 1048
Peters, P. C. 1964, Phys. Rev. B, 136, 1224
Pinsonneault, M. H., & Stanek, K. Z. 2006, ApJ, 639, L67
Press, W. H., Teukolsky, S. A., Yetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in FORTRAN: The Art of Scientific Computing (2nd ed.; Cambridge: Cambridge Univ. Press)
Pribulla, T., & Rucinski, S. M. 2006, AJ, 131, 2986
Pritchet, C. J., Howell, D. A., & Sullivan, M. 2008, ApJ, 683, L25
Raghavan, D., McAlister, H. A., Henry, T. J., et al. 2010, ApJS, 190, 1
Raskin, C., Scannapieco, E., Rockefeller, G., et al. 2010, ApJ, 724, 111
Rosswog, S., Kasen, D., Guillochon, J., & Ramirez-Ruiz, E. 2009, ApJ, 705, L128
Ruffert, M., & Janka, H.-T. 1999, A&A, 344, 573
Rutier, A. J., Belczynski, K., Benaquacita, M., Larson, S. L., & Williams, G. 2010, ApJ, 717, 1006
Rutier, A. J., Belczynski, K., & Fryer, C. 2009, ApJ, 699, 2026
Saio, H., & Nomoto, K. 1985, A&A, 150, 241
Scannapieco, E., & Bildsten, L. 2005, ApJ, 629, L85
Stairs, I. H. 2004, Science, 304, 547
Tauris, O. M., & Rampp, M. 2006, A&A, 450, 681
Totani, T., Morokuma, T., Oda, T., Doi, M., & Yasuda, N. 2008, PASJ, 60, 1327
Willems, B., Kalogera, V., Vecchio, A., et al. 2007, ApJ, 665, L85
Wstation: Research in Science, 304, 547
Takahashi, K., & Kawai, N. 2006, ApJ, 645, 619
Tauris, O. M., & Rampp, M. 2006, A&A, 450, 681
Webbink, R. F. 1984, ApJ, 277, 355
Wenzel, J., & Barenghi, F. 1998, Earth, Moon, and Planets, 78, 335
Willems, B., Kalogera, V., Vecchio, A., et al. 2007, ApJ, 665, L85
Wstation: Research in Science, 304, 547
Takahashi, K., & Kawai, N. 2006, ApJ, 645, 619
Webbink, R. F. 1984, ApJ, 277, 355
