This work has been submitted to the IEEE for possible publication. Copyright may be transferred without notice, after which this version may no longer be accessible.
Abstract

We consider a scheduler for the downlink of a wireless channel when only partial channel-state information is available at the scheduler. We characterize the network stability region and provide two throughput-optimal scheduling policies. We also derive a deterministic bound on the mean packet delay in the network. Finally, we provide a throughput-optimal policy for the network under QoS constraints when real-time and rate-guaranteed data traffic may be present.

1 Introduction

Scheduling has always been an indispensable part of resource allocation in wireless networks. The seminal work of Tassiulas et al. [22], and later [23], [24] considered the case where both channel states and queue lengths are fully available to the scheduler. It was shown that the MaxWeight algorithm, which serves the longest connected queue, is throughput-optimal. Subsequently, the MaxWeight algorithm was found to be throughput-optimal in many other settings as well ([1]-[12] and the references therein) using tools from Lyapunov optimization. Some other works (see [25], [26], [28]) also approach the scheduling problem using convex optimization and dual decomposition techniques. [5] even considers the role of imperfect queue length information on network throughput, showing that the stability region does not reduce. But, in all these cases, accurate information about channel-state is assumed as a modeling simplification.

In a real-life network, e.g., Long Term Evolution (LTE) [21] or IEEE 802.16e WiMAX, the channel-state information fed back to the transmitter can have uncertainty. The primary reason being that although resource-allocation is done at the finer granularity of a Physical Resource Block (PRB), channel-state information is still fed back at the coarser granularity of a subband, which is a group of PRBs. This is done to reduce the feedback traffic from the users to the Base Station (BS). However, this averaging causes information loss and hence, the resulting uncertainty at the scheduler. Moreover, uncertainty might be present in the channel-estimates because of the very process of estimation.

Some recent works have, hence, tried to model this uncertainty in the channel-estimate. In [7], the authors show that infrequent channel-state measurement, unlike infrequent queue length measurement, reduces the maximum attainable
throughput. [14] considers the effect of inaccuracy of channel estimation on throughput, but does so assuming a specific probability distribution of the channel-state and does not study the stability of the data queues either. [12] attempts at modeling channel- and queue-state uncertainty by considering the case where only heterogeneously delayed information is available at the scheduler. They however assume knowledge of the channel-state transition probabilities. In [20], the authors study scheduling with rate adaptation in a single-hop network with a single channel under channel uncertainty. They consider cases when the channel estimates are inaccurate but complete or incomplete knowledge of the channel-estimator joint statistics is available at the scheduler. The authors, however, assume that the channel-estimates are independent across the channels for each user.

Delay performance of various wireless systems has also been investigated by many researchers recently. Among previous work in the area, the authors in [29] study the problem of opportunistic scheduling of a wireless channel while also trying to minimize the mean delay. Neely [19] has given a $O(1/(1 - \rho))$ delay bound in the case of ON/OFF channels and a $O(N/(1 - \rho))$ bound for multi-rate channels for the classical MaxWeight algorithm for a network of size $N$ and any traffic input-rate vector within a $\rho$-scaled version of the stability region (where $0 < \rho < 1$). Subsequent work in [17] established $O(1/(1 - \rho))$ and $O(N/(1 - \rho))$ delay bounds for the case of single-hop and multi-hop networks, respectively, for ON/OFF channels and under both i.i.d. and Markov modulated arrival traffic scenarios. [16] derives lower and upper bounds on the delay in a wireless system with single-hop traffic and general interference constraints.

Our contributions of this paper are as follows:

• Firstly, we model the channel-estimate inaccuracy and characterize the network stability region. Compared to [20], we allow the channel estimates to have dependence among themselves, which is a more realistic situation in a modern LTE or WiMax network. Besides, we study a multi-channel setup whereas they consider a single channel.

• Secondly, we propose two simple MaxWeight based scheduling schemes that achieve any rate in the interior of the stability region.

• Thirdly, we derive an $O(N/(1 - \rho))$ delay bound for our system under one of the throughput-optimal policies we propose.

• Lastly, we propose a throughput-optimal policy for the network under traffic with heterogeneous Quality of Service (QoS) constraints and present some numerical results studying its performance.

The remainder of the paper is organized as follows. In Section II, we describe the system model and the assumptions made on the arrival and channel processes. Section III provides the network stability region. We also discuss an example that illustrates that partial channel information may lead to a loss of throughput. In Section IV, we propose two throughput-optimal policies for the network and prove their optimality. We also present some simulation results in this section studying their performance. Section V provides a bound on the mean packet delay in the network. Section VI gives a throughput-optimal policy for the network under QoS constraints and studies its performance. We conclude with Section VII.

3
2 System Model

Consider a multi-user cellular downlink system with \( N \) users and \( M \) orthogonal channels. The system operates with fixed-size data packets and in synchronized time slots denoted by \( t \in \{0, 1, 2, \ldots \} \). It may, for example, be a cellular downlink Orthogonal Frequency-Division Multiple Access (OFDMA) system. Each user has a separate queue for its data and \( Q_i(t) \) denotes the queue-length (in terms of packets) for user \( i \) in slot \( t \) where \( i \in \{1, \ldots, N\} \). We assume an infinite buffer at each queue. \( A_i(t) \) denotes the number of exogenous packet arrivals for user \( i \) in slot \( t \). It is assumed that \( \{A_i(t)\} \) is i.i.d. from slot to slot with \( \mathbb{E}\{A_i(0)\} = \lambda_i \) and with \( \mathbb{E}\{A_i(0)^2\} < \infty \) for all \( i \). However, in a particular slot \( t \), \( A_i(t) \) may be dependent among themselves.

The channel-state for user \( i \) and channel \( j \) in slot \( t \) is denoted by \( X_{ij}(t) \). We assume that \( X_{ij}(t) \) is i.i.d. from slot to slot and independent across users. Such an assumption holds, for example, in a wireless system like LTE where our channel corresponds to a PRB in the LTE system. A PRB has 180 kHz bandwidth which is close to the coherence bandwidth of the channel for a typical delay spread of 4-5 \( \mu \) (Sec. 5.3.2 in [21]). We define channel-state as the maximum number of packets that can be sent over the channel successfully without suffering an outage. We assume that \( X_{ij}(t) \in \mathcal{X} \) where \( \mathcal{X} \) is a discrete state-space and \( \mathbb{E}\{X_{ij}^2(t)\} < \infty \). The scheduler has access to only estimates \( \hat{S}_{ij}(t) \) of the channel-state in slot \( t \) where \( i \in \{1, \ldots, N\} \) and \( j \in \{1, \ldots, M\} \). These estimates are used by the scheduler to schedule different channels to the users in slot \( t \). The estimates for a particular user may be dependent on each other. This can be used to model the effect of averaging (or even calculating any deterministic function, for that matter) of the channel-gains as done in LTE systems which are sent by the users to the BS to reduce the feedback traffic. The only constraint we impose on \( S_{ij}(t) \) is that \( S_{ij}(t) \in \mathcal{S} \), where \( \mathcal{S} \) is a discrete set, and that it is i.i.d. from slot to slot. As a shorthand, we shall use \( Q(t), A(t) \) and \( S(t) \) to denote the queue-length vector, arrival vector and channel-estimate matrix, respectively, at slot \( t \). We use \( \mathbb{P}_S(\cdot) \) to denote the probability mass function of the random variable \( S \). We also assume that the channel/estimator statistics given by the set of probabilities \( \mathbb{P}(X_{ij} = x \mid S = s), \forall x \in \mathcal{X} \) and \( s \in \mathcal{S} \) is available at the scheduler. This can be achieved, possibly, using a mechanism that learns the statistics on-the-fly. We assume that a channel can be allocated to at most one user in a particular slot. We use the notation \( I_j(t) \) and \( R_j(t) \), \( j \in \{1, \ldots, M\} \) to denote the user scheduled on channel \( j \) and the corresponding rate allocated to it, respectively, in the slot \( t \).

We can then write the queue evolution equation as:

\[
Q_i(t + 1) = (Q_i(t) - \mu_i(t))^+ + A_i(t)
\]  \hspace{1cm} (1)

where \( a^+ = \max\{a, 0\} \) and

\[
\mu_i(t) \triangleq \sum_{j=1}^{M} 1(I_j(t) = i)R_j(t)1(R_j(t) \leq X_{ij}(t)).
\]  \hspace{1cm} (2)

In this equation, we have assumed that the packets sent on channel \( j \) are received successfully if and only if \( R_j(t) \leq X_{ij}(t) \), i.e., probability of error is negligible.\(^3\)

\(^3\)To be precise, we can transmit data at rate \( X_{ij}(t) \) with any arbitrarily small probability of
if \( i \) transmits at a rate less than or equal to \( X_{ij}(t) \) on channel \( j \). Under these conditions, \( \{Q(t)\} \) is a countable state Markov chain. For simplicity, we will assume it to be irreducible. But the general case can be easily handled.

We will use the following notation: \( C[A] \) is the convex hull and \( \text{int}[A] \) is the interior of set \( A \). \( \vec{1}_i \) is the \( i \)th coordinate vector and \( \vec{0} \) and \( \vec{1} \) denote \( N \)-dimensional vectors of zeroes and ones, respectively.

Let \( L(Q(t)) \) be a Lyapunov function. We define one-slot conditional Lyapunov drift as

\[
\Delta(Q(t)) \triangleq \mathbb{E}\{L(Q(t + 1)) - L(Q(t)) \mid Q(t)\},
\]

In the following, we provide an upper bound on \( \Delta(Q(t)) \) which will be used later on.

**Lemma 1.** For the quadratic Lyapunov function \( L(Q(t)) = \sum_{i=1}^{N} Q_i^2(t) \),

\[
\Delta(Q(t)) \leq \mathbb{E}\{U(t) \mid Q(t)\} + 2 \sum_{i=1}^{N} Q_i(t)(\lambda_i - \mathbb{E}\{\mu_i(t) \mid Q(t)\})
\]

where

\[
U(t) \triangleq \sum_{i=1}^{N} (A_i^2(t) + \mu_i^2(t)).
\]

Further, if \( \mathbb{E}\{A_i^2(t)\} < \infty \) and \( \mathbb{E}\{X_{ij}^2(t)\} < \infty \) for each \( i \) and \( j \), there exists \( B < \infty \) such that

\[
\Delta(Q(t)) \leq B + 2 \sum_{i=1}^{N} Q_i(t)(\lambda_i - \mathbb{E}\{\mu_i(t) \mid Q(t)\}).
\]

### 3 The Network Stability Region

We first define the notion of stability we use in the paper.

**Definition 1.** A queue \( Q_i(t) \) is strongly stable if

\[
\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{Q_i(\tau)\} < \infty.
\]

The network of queues is strongly stable if each individual queue is strongly stable.

Strong stability implies positive recurrence of the Markov chain \( \{Q(t)\} \). In general, it is a stronger notion than positive recurrence. Throughout the paper, we shall use the term “stability” to refer to strong stability.

We characterize the network stability region of the system now. Consider the set of stationary policies \( G \) that base their scheduling decisions at time \( t \) only on \( (Q(t), S(t)) \) and the channel-estimator statistics. The network stability region is defined to be the closure of the arrival rates that can be stably supported by the policies in \( G \).

---

error (provided \( X_{ij}(t) \) is less than the Shannon capacity) assuming the physical-layer coding scheme supports it. For example, we can use an appropriate Turbo code or LDPC code for our purpose.
Theorem 1. The network stability region $\Lambda$ is given by

$$\Lambda = \sum_{s \in S} \mathbb{P}(s) \mathcal{C}\left[ \bar{0}, \sum_{j=1}^{M} \mathbb{P}(X_{ij} \geq r_{ij}^*(s) \mid S = s) r_{ij}^*(s) \bar{1}_i; \forall i \in \{1, \ldots, N\} \right],$$

where

$$r_{ij}^*(s) \triangleq \arg \max_{x \in \mathcal{X}} \{ \mathbb{P}(X_{ij} \geq x \mid S = s) \}.$$

ties being broken lexicographically.

Proof The proof goes along the lines of the proof of Proposition 1 in [20].

We have included it in Appendix A for the sake of completeness.

3.1 An example

We illustrate the loss in throughput caused by partial channel information using a simple example. We show that scheduling schemes that naively trust the channel-estimate fed back as the true channel-state may perform much worse than the policies that don’t.

Consider a system with 1 user and 2 channels. The 2 channel are assumed independent. Also, $\mathbb{P}\{X_{11}(t) = 0\} = 0.5 = \mathbb{P}\{X_{12}(t) = 0\} = \mathbb{P}\{X_{12}(t) = 6\}$ and $\mathbb{P}\{S_{11}(t) = 0\} = 0.25 = \mathbb{P}\{S_{12}(t) = s\}$, for $s \in \{0, 1, 3, 4\}$. Now, if we take the $S$ value to be the true channel-state, it can be easily shown that the mean service provided will be $\frac{1}{4} \times (0 + 1 + 3 + 4) = 2$ packets per slot. However, due to the special choice of the support set, the $S$ values give us complete information about the channel-state of both the channels. Then, it is easy to see that we can provide mean service of $\frac{1}{4} \times (0 + 2 + 6 + 8) = 4$ packets per slot.

We note that a careful scheduling decision can even double the mean service rate as shown in the example. Though the example may appear a bit contrived, numerical studies in the next section show that performance gains due to clever scheduling may indeed be substantial in many realistic situations.

4 Throughput-optimal policies

In this section, we describe two throughput-optimal policies and also prove their optimality. Even though the STAT policy described in the proof of Theorem 1 is throughput-optimal, we require knowledge of the arrival rates $\lambda$ for STAT to be able to perform the channel-allocation. The throughput-optimal policies described here, in contrast, just require the arrival rate vector to lie within the stability region (without knowing $\lambda$) and need only knowledge of the current queue-lengths. This will be available to a downlink scheduler used at a BS. Both, of course, require knowledge of the channel-estimator statistics. Moreover, as will be seen later in the section, scheduling schemes that naively trust the channel-estimate fed back perform worse than the policies in this section. For notational simplicity, we shall drop all the slot indices in this section. Lyapunov drift analysis techniques are used to prove the throughput-optimality of the policies in this section.
4.1 MaxWeight policy

We consider the MaxWeight (MW) type policy described below.

At each slot $t$, the channel-estimate $S = s$ is observed, and the decisions $I_j$ and $R_j$ are computed separately, for each $j$, as follows:

1. To each user $i$, assign rate $R_{ij}$ such that,

   $R_{ij} = \arg \max_{x \in X} \{ \mathbb{P}(X_{ij} \geq x \mid S = s) \cdot x \}.$

2. Schedule the user $I_j$ that maximizes the rate-backlog-success-probability product:

   $I_j = \arg \max_{1 \leq i \leq N} \{ Q_i \mathbb{P}(X_{ij} \geq R_{ij} \mid S = s) \cdot R_{ij} \}.$

For the sake of completeness, we assume that all ties here are broken lexicographically.

**Theorem 2.** The MaxWeight policy is throughput-optimal.

**Proof** See Appendix B.

4.2 Iterative MaxWeight policy

We now analyses an iterative version of the above MaxWeight policy. These policies have also been studied in [3] and [4]. The new policy will be referred to as $i$MW. We study this policy because we find that it can give a lower mean delay than $MW$ in some networks. In the $i$MW policy, we allocate the channels sequentially from 1 to $M$ in $M$ rounds taking into account the channels allocated so far. The virtual queue-lengths at the beginning of round $j$ are considered for the allocation in round $j$. To aid the analysis, we use $Q_i^{(j)}$ to denote the virtual queue-length of queue $i$ at the beginning of round $j$ of the allocation. $Q_i^{(1)}$ is defined to be $Q_i$. We also assume that the set $X$ contains a largest element denoted by $x_{\max}$. We can formulate the $i$MW policy now as follows.

At each slot $t$, the channel-estimate $S = s$ is observed. The decisions $I_j$ and $R_j$ are computed sequentially from channel 1 to $M$, as follows:

1. Start with $j = 1$.
2. For each $j$, do the following:
   
   (a) To each user $i$, assign rate $R_{ij}$ such that,

   $R_{ij} = \arg \max_{x \in X} \{ \mathbb{P}(X_{ij} \geq x \mid S = s) \cdot x \}.$

   (b) Schedule the user $I_j$ that maximizes the rate-backlog-success-probability product:

   $I_j = \arg \max_{1 \leq i \leq N} \{ Q_i^{(j)} \mathbb{P}(X_{ij} \geq R_{ij} \mid S = s) \cdot R_{ij} \}.$

3. If $j = M$, stop.
   
   Else, put $Q_i^{(j+1)} = (Q_i^{(j)} - 1(I_j = i)R_{ij})^+$, for $1 \leq i \leq N$, increment $j$ by 1 and continue.
Here, as before, ties are broken lexicographically.

**Theorem 3.** The iterative MaxWeight policy is throughput-optimal.

**Proof** See Appendix C.

### 4.3 Simulations

We present numerical simulation results to compare MW with $iMW$ and also show the advantage of using channel estimators instead of the average channel gains. Firstly, we consider an ON/OFF system with $P\{X_{ij}(t) = 0\} = 1/2 = P\{X_{ij}(t) = 1\}$, for all $i \in \{1, \ldots, 10\}$ and $j \in \{1, \ldots, 6\}$. We assume that each user estimates its channel-state correctly all the time but feeds back only the sum of the six channel-estimates to the scheduler. We do this to study the effect of averaging the channel-estimates as in LTE on the stability region. The naive scheduling schemes (MaxWeight and SSG) calculate the average channel-estimate from the sum fed back, round it down to nearest integer and use that for scheduling, taking it to be the true channel-state for each of the six channels. We consider symmetric Binomial$(10, \lambda)$ arrivals with equal rates for all users. We have simulated the system for $10^6$ slots for values of $\lambda$ from 0.01 to 0.5. The resulting simulated queue backlogs are shown in Fig. 1. We see a huge gain in the stability region compared to the naive algorithms. Similar results are obtained when the channel-estimates are rounded up to nearest integer instead of down.

![Figure 1: Simulation for the ON/OFF system with symmetric traffic](image)

In the second set of simulations, we consider a multi-rate system with $P\{X_{ij}(t) = x\} = 1/4$, for all $i \in \{1, \ldots, 10\}$, $j \in \{1, \ldots, 6\}$ and $x \in \{0, 1, 2, 3\}$. Here too, we study the averaging effect by making the naive scheduling schemes (MaxWeight and SSG) calculate the average channel-estimate from the sum fed back, round

\[\text{We note that this is somewhat different from an LTE setup. Arithmetic mean or EESM is usually used in that case.}\]
Figure 2: Simulation for the multi-rate system with symmetric traffic. The inset shows the queue backlog performance of iMW versus MW under moderate traffic conditions.

it up to the nearest integer and use that for scheduling. We simulate the system for $10^7$ slots with symmetric Binomial$(10, \lambda)$ arrivals for values of $\lambda$ from 0.05 to 1. Fig. 2 shows the resulting mean queue backlogs. We again see a decrease in the stability region when we use the naive schedulers. In this figure, we have also expanded the graph to show that iMW performs better than MW at least under moderate traffic conditions. Similar results are obtained when the channel-estimates are rounded down instead of up.

We also simulated asymmetric systems and a similar behavior of the corresponding stability regions was observed. The results are not reported here for lack of space.

5 A delay bound

In this section, we derive an upper bound on the mean delay for the MW policy.

**Theorem 4.** Assume $\lambda \in \text{int}[\Lambda]$. Then, the average delay in the system, denoted by $\overline{D}$, satisfies the following bound:

$$
\overline{D} \leq \frac{N(1 + K) \sum_{i=1}^{N} \mathbb{E}\{A_i^2(\tau)\}}{2\mu(1 - \rho) \sum_{j=1}^{N} \mathbb{E}\{A_j(\tau)\}}
$$

(7)

where

$$
\mu \triangleq \min_{1 \leq i \leq N} \left( \sum_{s \in \mathcal{S}} \mathbb{P}(s) \sum_{j=1}^{M} \mathbb{P}(X_{ij} \geq r_{ij}^*(s) \mid S = s) r_{ij}^*(s) \right)
$$

(8)

and $0 < \rho < 1$ and $K < \infty$.

**Proof** See Appendix D.
Note that the delay bound derived is $O(N/(1 - \rho))$ since $\sum_{j=1}^{\infty} \frac{\mathbb{E}(A^2(\tau))}{\mathbb{E}(A_j(\tau))} = O(1)$, and $K$ and $\mu$ are fixed constants.

6 QoS constraints

In the previous sections, there were no QoS constraints on the data. We only had to ensure that all the queues were stable. In this section, we consider three types of traffic:

- **Real-time (RT) traffic:** Demands an upper bound on packet dropping ratio and delay deadline guarantees.
- **Rate-guarantee (RG) traffic:** Demands minimum rate guarantees.
- **Best-effort (BE) traffic:** Demands only queue stability.

Let $\mathcal{R}$, $\mathcal{G}$ and $\mathcal{B}$ denote the set of RT, RG and BE users, respectively, in the system. The set of all users is denoted by $\mathcal{N}$. We have a slotted system as before. However, the slots are grouped into frames and each frame consists of $T$ consecutive slots. We assume that the RT packets have a deadline of $T$ slots. These RT packets might be coming, for example, from a voice or a video source. RG packets coming into the system only require guarantees on minimum rate. These packets might be coming from a source with flow-control, such as by TCP protocol. Thus, the RG users may be treated, for our purposes, as packet sources with infinite backlogs. The minimum-rate guarantee sought by a RG user $g$ is denoted by $\beta_g$. The channel model is the same as before except that we assume $X_{bj}(t) \leq x_{\text{max}} < \infty$, $\forall b \in \mathcal{B}$ and for each channel $j$. All exogenous packets arriving into the system arrive only at the start of the frame and $A_n[k]$ denotes the number of packet arrivals in frame $k$, $k \in \{0, 1, 2, \ldots\}$ for user $n \in \mathcal{N}$. As before, $\{A_n[k]\}$ is i.i.d. from frame to frame with $\mathbb{E}(A_n[0]) = \lambda_n$ and with $\mathbb{E}(A_n[0]^2) < \infty$ for all $n \in \mathcal{N}$. In case, some of the RT packets arriving in a frame could not be served by the end of the frame, they are simply dropped. We denote the packet dropping ratio for user $r$ by $\alpha_r$, i.e., at most $\alpha_r$ fraction of the packets arriving for user $r$ can be dropped in the long term.

In order to satisfy the QoS constraints of the RT and RG users, we use the concept of virtual queues (see [15], [27]) that evolve from frame to frame. Corresponding to each RT user $r$ and RG user $g$, we have virtual queue-length processes $\{Y_r[k]\}_{k=0}^{\infty}$ and $\{Z_g[k]\}_{k=0}^{\infty}$, respectively. We can then write the queue evolution equations for the RT users as,

$$Y_r[k + 1] = (Y_r[k] - \mu_r[k] + A_r[k](1 - \alpha_r))^+, \forall r \in \mathcal{R}, \quad (9)$$

where $Y_r[0] = 0$, $\mu_r[k] \triangleq \sum_{t=kT}^{(k+1)T-1} \mu_r(t)$, and $\mu_r(t)$ is as defined in (2). Similarly, the virtual queues of the RG users are updated as follows:

$$Z_g[k + 1] = (Z_g[k] - \mu_g[k] + \beta_g)^+, \quad (10)$$

$\forall g \in \mathcal{G}$, where $Z_g[0] = 0$ and $\mu_g[k]$ is defined just like $\mu_r[k]$. Unlike the RT and RG users, the queues of the BE users evolve from slot to slot. Besides, BE users only maintain real queues since they do not have any QoS constraints.
whatevsoever. \( Q_b(t) \) denotes the queue-length of BE user \( b \) at slot \( t \) and its dynamics are governed by the equations,

\[
\begin{align*}
Q_b(kT + 1) &= (Q_b(kT) - \mu_b(kT) + A_b[k])^+, \\
Q_b(kT + t + 1) &= (Q_b(kT + t) - \mu_b(kT + t))^+, 
\end{align*}
\] (11a)(11b)

\( \forall t \in \{1, 2, \ldots, T - 1\} \) in frame \( k \). We also define:

\[
\Phi_n[k] = \begin{cases} 
Y_n[k], & \text{if } n \in \mathcal{R}, \\
Z_n[k], & \text{if } n \in \mathcal{G}, \text{ and} \\
Q_n(kT), & \text{if } n \in \mathcal{B}.
\end{cases}
\] (12)

### 6.1 Throughput-optimal policy

We consider a modified version of the MaxWeight type policy described in the previous sections, which we call QoS-MaxWeight (QMW).

At each slot \( t \) in frame \( k \), the channel-estimate \( S(t) = s \) is observed, and the decisions \( I_j(t) \) and \( R_j(t) \) are computed separately for each channel as follows:

1. To each user \( n \) and channel \( j \), assign rate \( R_{nj} \) such that,

\[
R_{nj} = \arg \max_{x \in \mathcal{X}} \{ P(X_{nj}(t) \geq x | S(t) = s) x \}.
\]

2. Schedule the user \( I_j(t) \), on channel \( j \), that maximizes the rate-backlog-success-probability product below:

\[
I_j(t) = \arg \max_{n \in \mathcal{N}} \{ \Phi_n[k] P(X_{nj}(t) \geq R_{nj} | S(t) = s) R_{nj} \}.
\]

For the sake of completeness, we assume that all ties here are broken lexicographically.

**Theorem 5.** The QoS-MaxWeight policy is throughput-optimal.

**Proof** See Appendix E.

### 6.2 Simulation results

We investigate the effect of partial information on the network stability region using numerical simulation. We consider a multi-rate system with \( P(X_{ij}(t) = x) = 1/4 \) for all \( i \in \{1, \ldots, 15\}, j \in \{1, \ldots, 6\} \) and \( x \in \{0, 1, 2, 3\} \). There are 2 RT users (with the maximum dropping ratios and deadlines being 0.01 and 10 slots for user 1, and 0.02 and 10 slots for user 2), 3 RG users (with minimum rate guarantees being 5, 2 and 1 packets-per-frame for users 3, 4 and 5, respectively) and the rest 10 are BE users. RT users have Binomial(10, 2.75) arrivals. RG users have Binomial arrivals with parameters 10 and the corresponding minimum rate. We have simulated the system for \( 5 \times 10^6 \) slots with symmetric Binomial(10, \( \lambda \)) arrivals for the BE users with the value of \( \lambda \) varying from 0.18 to 10. Fig. 3 shows the resulting mean sum queue backlogs.
7 Conclusions

We have considered a multi-rate wireless downlink with multiple channels and users. The base station schedules the traffic based on partial channel information. We have obtained its network stability region. We have then proposed two throughput-optimal schemes to achieve the stability region and proved their optimality. We have also derived a bound on the mean packet delay in the network. Finally, we have proposed a throughput-optimal policy for the network under QoS constraints. A natural extension of our work would be to investigate the effect of uncertainty in the queue-length information. Also, more complicated traffic models are left to be studied in greater detail.

Appendix A

Proof of Theorem 1:

Sufficiency: We show that $\lambda \in \text{int}[\Lambda]$ is a sufficient condition for stability. Now, since $\Lambda$ is convex, for each $s \in S$, there exists a scaling vector $\gamma^s$ and a scalar $\epsilon > 0$ such that

$$\lambda_i + \epsilon < \sum_{s \in S} \gamma_i^s \mathbb{P}(s) \sum_{j=1}^{M} \mathbb{P}(X_{ij} \geq r_{ij}^*(s) \mid S = s) \ r_{ij}^*(s)$$

for any user $i$, and where $\sum_{i=1}^{N} \gamma_i^s = 1, \forall s \in S$.

Consider the following stationary randomized policy, henceforth referred to as $\text{STAT}$: for channel estimate $S(t) = s$, allocate all channels to user $i$ with probability $\gamma_i^s$ and set the rate allocated on channel $j$ to it to be $r_{ij}^*(s)$. In that
case, the service rate of user $i$, call it $u_i$, is given by

$$u_i = \mathbb{E}\{\mu_i(t) \mid Q(t)\} = \sum_{s \in S} \gamma_i^s \mathbb{P}_S(s) \sum_{j=1}^{M} \mathbb{P}(X_{ij} \geq r_{ij}^*(s) \mid S = s) r_{ij}^*(s).$$ \hfill (14)

Considering the quadratic Lyapunov function defined before and using (13) and (14) in (6), we can write the Lyapunov drift inequality as

$$\Delta(Q(t)) < B - 2\epsilon \sum_{i=1}^{N} Q_i(t).$$ \hfill (15)

Now, we note that $Q(t)$ evolves as a Markov chain with $L(Q(t))$ bounded below by zero and $\mathbb{E}\{L(Q(t+1)) \mid Q(t)\} < \infty$. Also, notice that the conditional drift in (15) is negative outside the finite set $\{Q(t) : \sum_{i=1}^{N} Q_i(t) \leq \frac{B}{\epsilon}\}$. Thus, using Theorem 2 from [18], we can say that the queue-length process is stable.

**Necessity:** Here we show that $\lambda \in \Lambda$ is a necessary condition for stability. For if not so, i.e., if $\lambda \notin \Lambda$, we have a vector $\alpha$ and a scalar $\delta > 0$, such that for any $v \in \Lambda$, we have

$$\sum_{i=1}^{N} \alpha_i (\lambda_i - v_i) \geq \delta,$$

from the Strict Separation Theorem (see, for example, Proposition B.14 in [13]). Let us define the linear Lyapunov function $L(Q(t)) = \sum_{i=1}^{N} \alpha_i Q_i(t)$. Now, using definition (3) and the queue evolution equation (1), for any stationary policy in $G$, we can write

$$\Delta(Q(t)) \geq \sum_{i=1}^{N} \alpha_i \mathbb{E}\{A_i(t) - \mu_i(t) \mid Q(t)\} = \sum_{i=1}^{N} \alpha_i (\lambda_i - \mathbb{E}\{\mu_i(t) \mid Q(t)\}).$$ \hfill (16)

Let $u_i \triangleq \mathbb{E}\{\mu_i(t) \mid Q(t)\}$. We now show that $u \in \Lambda$.

$$\mathbb{E}\{\mu_i(t) \mid Q(t)\} = \sum_{s \in S} \mathbb{P}_S(s) \mathbb{E}\{\mu_i(t) \mid Q(t); S(t) = s\}$$

$$= \sum_{s \in S} \mathbb{P}_S(s) \left( \sum_{j=1}^{M} \mathbb{P}(R_j(t) \leq X_{ij}(t) \mid S(t) = s) 1(I_j(t) = i) R_j(t) \right)$$

$$\leq \sum_{s \in S} \mathbb{P}_S(s) \left( \sum_{j=1}^{M} \mathbb{P}(r_{ij}^*(s) \leq X_{ij}(t) \mid S(t) = s) 1(I_j(t) = i) r_{ij}^*(s) \right).$$

The second equality holds since the policy decisions $I_j(t)$ and $R_j(t)$ are completely determined by $Q(t)$ and $S(t)$ within the class of stationary policies $G$.  

13
The inequality at the end holds because of the way \( r^*_ij(s) \) is defined. Thus, \( u \in \Lambda \).

Thus, from the Strict Separation Theorem and (16), we get

\[
\Delta(Q(t)) \geq \sum_{i=1}^{N} \alpha_i(\lambda_i - E[\mu_i(t) \mid Q(t)])
\]
\[
= \sum_{i=1}^{N} \alpha_i(\lambda_i - u_i)
\]
\[
\geq \delta.
\]

It can be shown that \( E[|L(Q(t+1)) - L(Q(t))||Q(t)|] < \infty \). Taking the finite set as \( \{0\} \) and noting that \( L(Q(t)) > 0 \) for at least some \( Q(t) \), we see that the DTMC will not be positive recurrent, and hence, not strongly stable either. Thus, we conclude that the network will be unstable.

**Appendix B**

**Proof of Theorem 2:**

Assume that we are working with \( \lambda \in \text{int}[\Lambda] \). Let the MW policy make decisions \( I_j \) and \( R_j \), and the STAT policy make decisions \( I'_j \) and \( R'_j \). We then have,

\[
Q_i \ 1(I_j = i) \ 1(R_j \geq R_j \mid S = s) \geq Q_i \ 1(I'_j = i) \ 1(R'_j \geq R'_j \mid S = s)
\]

\( \forall i \in \{1, \ldots, N\} \) and \( \forall j \in \{1, \ldots, M\} \), since MW maximizes the expression in the LHS over the class \( G \). Thus, summing over all \( i \) and \( j \), we get

\[
\sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \ 1(I_j = i) \ 1(R_j \geq R_j \mid S = s)
\]
\[
\geq \sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \ 1(I'_j = i) \ 1(R'_j \geq R'_j \mid S = s).
\]

Therefore,

\[
\sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \ E\{1(I_j = i) \ 1(R_j \geq R_j) \mid Q, S = s\}
\]
\[
\geq \sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \ E\{1(I'_j = i) \ 1(R'_j \geq R'_j) \mid Q, S = s\}
\]

where we have used the fact that the policy decisions are completely determined by \( Q \) and \( S \) within the class \( G \). Taking expectation w.r.t. \( S \),

\[
\sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \ E\{1(I_j = i) \ 1(R_j \geq R_j) \mid Q\}
\]
\[
\geq \sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \ E\{1(I'_j = i) \ 1(R'_j \geq R'_j) \mid Q\}.
\]  (17)
Note that STAT stabilizes the system since $\lambda \in \text{int}([\Lambda])$. Thus, for some $\epsilon > 0$, we can write

$$\lambda_i + \epsilon < \sum_{j=1}^{M} \mathbb{E}\{1(I_j' = i)R_j'1(R_j' \leqslant X_{ij})\}$$

$$= \sum_{j=1}^{M} \mathbb{E}\{1(I_j' = i)R_j'1(R_j' \leqslant X_{ij}) \mid Q\}$$

where the equality holds because STAT makes its decisions independent of the queue-lengths. Multiplying $Q_i$ on both sides and summing over all $i \in \{1, \ldots, N\}$, we get

$$\sum_{i=1}^{N} Q_i(\lambda_i + \epsilon) < \sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \mathbb{E}\{1(I_j' = i)R_j'1(R_j' \leqslant X_{ij}) \mid Q\}$$

$$\leq \sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \mathbb{E}\{1(I_j = i)R_j1(R_j \leqslant X_{ij}) \mid Q\}$$

where we have used (17) in the second inequality. Therefore,

$$\sum_{i=1}^{N} Q_i \left(\lambda_i - \sum_{j=1}^{M} \mathbb{E}\{1(I_j = i)R_j1(R_j \leqslant X_{ij}) \mid Q\}\right) < -\epsilon \sum_{i=1}^{N} Q_i$$

(19)

Finally, using (19) in the drift inequality (6), we get

$$\Delta(Q(t)) < B - 2\epsilon \sum_{i=1}^{N} Q_i(t).$$

Now the queue evolves as a Markov chain since the scheduling and rate allocation decisions are taken based on the current queue-lengths and channel-estimate. The drift inequality above gives negative drift but for a finite set of queue-lengths. Hence, using Theorem 2 from [18], the network is stable.

**Appendix C**

**Proof of Theorem 3:**

Assume that we are working with $\lambda \in \text{int}([\Lambda])$. Let the $iMW$ policy make decisions $I_j$ and $R_j$, the $MW$ policy make decisions $I_j'$ and $R_j'$ and the STAT policy make decisions $I_j''$ and $R_j''$. For ease of exposition, we use $R_{ij}$ and $R_{ij}'$ as the intermediate allocation variables for $iMW$ and $MW$ policies, respectively. We then have,

$$Q_{I_j} R_{I_j} \mathbb{P}(X_{ij} \geqslant R_{I_j} \mid S = s)$$

$$\geq Q_{I_j'}^{(j-1)} R_{I_j'} \mathbb{P}(X_{ij} \geqslant R_{I_j'} \mid S = s)$$

$$\geq Q_{I_j'}^{(j-1)} R_{I_j'} \mathbb{P}(X_{ij} \geqslant R_{I_j'} \mid S = s)$$

$$\geq (Q_{I_j'} - Mx_{max}) R_{I_j'} \mathbb{P}(X_{ij} \geqslant R_{I_j'} \mid S = s)$$

$$\geq Q_{I_j'} R_{I_j'}^{(j-1)} \mathbb{P}(X_{ij} \geqslant R_{I_j'} \mid S = s) - Mx_{max}^2.$$
In the first inequality, we have used the fact that the virtual queue-lengths cannot increase with allocation rounds. The second inequality is due to the way $iMW$ does the allocation at each round. Summing both sides over all $j \in \{1, \ldots, M\}$, we get

$$\sum_{j=1}^{M} Q_{I_j} R_{I_j} \mathbb{P}(X_{ij} \geq R_{I_j} | S = s) \geq \sum_{j=1}^{M} Q_{I'_j} R'_{I'_j} \mathbb{P}(X_{ij} \geq R'_{I'_j} | S = s) - M^2 x_{\text{max}}^2.$$ 

Therefore,

$$\sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \mathbf{1}(I_j = i) R_{ij} \mathbb{P}(X_{ij} \geq R_{ij} | S = s) \geq \sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \mathbf{1}(I'_j = i) R'_{ij} \mathbb{P}(X_{ij} \geq R'_{ij} | S = s) - M^2 x_{\text{max}}^2,$$

where we have used the property of indicator functions. Now, using the fact that $\mathbf{1}(I_j = i) R_{ij} = \mathbf{1}(I'_j = i) R'_j$, for all $i$ for the $iMW$ policy (and a similar thing for $MW$), and proceeding in a way similar to the previous proof, we will get

$$\sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \mathbb{E}\{\mathbf{1}(I'_j = i) R'_j 1(X_{ij} \geq R'_j) | Q\} \leq \sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \mathbb{E}\{\mathbf{1}(I_j = i) R_j 1(X_{ij} \geq R_j) | Q\} + M^2 x_{\text{max}}^2 \quad (20)$$

Now, using a $STAT$ policy, as in the previous proof, to stabilize $\lambda + \tilde{\epsilon} \tilde{1} \in \text{int}[\Lambda]$ for some $\epsilon > 0$, and using the property of $MW$ for the expression obtained, as done in (18), we get

$$\sum_{i=1}^{N} Q_i (\lambda_i + \epsilon) \leq M^2 x_{\text{max}}^2 + \sum_{i=1}^{N} \sum_{j=1}^{M} Q_i \mathbb{E}\{\mathbf{1}(I_j = i) R_j 1(R_j \leq X_{ij}) | Q\}$$

where we have used the inequality in (20). Finally, rearranging the terms in the above inequality and using it in (10), we get

$$\Delta(Q(t)) < (B + 2M^2 x_{\text{max}}^2) - 2\epsilon \sum_{i=1}^{N} Q_i(t)$$

Now, as before, we know that the queue evolves as a Markov chain. As before, observe that the drift is negative but for a finite set of queue-lengths. Hence, the network is stable.
Appendix D

Proof of Theorem 4:
Here too we drop the slot indices for the sake of brevity. Let the $MW$ policy make decisions $I_j$ and $R_j$, and any other policy $G'$ in class $G$ make decisions $I'_j$ and $R'_j$. Then, we can write

$$\sum_{i=1}^N \sum_{j=1}^M Q_i \mathbb{E}\{1(I_j = i) R_j 1(X_{ij} \geq R_j) \mid Q\}$$

$$\geq \sum_{i=1}^N \sum_{j=1}^M Q_i \mathbb{E}\{1(I'_j = i) R'_j 1(X_{ij} \geq R'_j) \mid Q\}.$$  \hfill (21)

Since $\lambda \in int[\Lambda]$, we can say that $\lambda \in \rho \Lambda$ for some $0 < \rho < 1$. Consequently, $\lambda' \rho \in \Lambda$. Furthermore, defining $\mu$ as in (8), we can say that $\mu' \vec{1} \in \Lambda$ where $\mu' = \frac{\mu}{N}$. Thus, owing to the convexity of $\Lambda$, we have

$$\lambda + (1 - \rho)\mu' \vec{1} \in \Lambda.$$  

Hence, we can find a $STAT$ policy such that

$$\mathbb{E}\left\{ \sum_{j=1}^M 1(I'_j = i) R'_j 1(X_{ij} \geq R'_j) \mid Q \right\} = \lambda_i + (1 - \rho)\mu'.$$  \hfill (22)

Substituting (22) into (21), and then using (4), we get

$$\Delta(Q(t)) \leq \mathbb{E}\{U(t) \mid Q(t)\} - 2\mu'(1 - \rho) \sum_{i=1}^N Q_i(t).$$  \hfill (23)

Finally, using Lemma 4.1 of [15] and the previous inequality, we can write

$$\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{i=1}^N \mathbb{E}\{Q_i(\tau)\} \leq \frac{N}{2\mu(1 - \rho)} \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{U(\tau)\}.$$  \hfill (24)

Now, we notice that since the system evolves as an ergodic Markov chain with countable state-space, the time-averages are well defined, and hence, lim can be used instead of limsup. Also, note that since $\mathbb{E}\{X_{ij}^2(t)\} < \infty$ and $N$ is finite, we can write $\mathbb{E}\{\mu_i^2(t)\} < K\mathbb{E}\{A_i^2(t)\}$ for all $i$ and $K < \infty$. So, $\mathbb{E}\{U(\tau)\} < (1 + K) \sum_{i=1}^N \mathbb{E}\{A_i^2(\tau)\} < \infty$. Using this fact and Little’s Theorem, we thus get the delay bound in (7).

Appendix E

Proof of Theorem 5:

Squaring both sides of (9) and proceeding as in Appendix F, for each RT user $r$, we get

$$Y_r[k+1]^2 - Y_r[k]^2 \leq (A_r[k](1-\alpha_r))^2 + \mu_r[k]^2 + 2Y_r[k](A_r[k](1-\alpha_r) - \mu_r[k]).$$  \hfill (25)
Similarly, for each RG user $g$, from (10), we get

$$Z_g[k+1]^2 - Z_g[k]^2 \leq \beta_g^2 + \mu_g[k]^2 + 2Z_g[k](\beta_g - \mu_g[k]). \tag{26}$$

Squaring both sides of (11), for each BE user $b$, we get

$$Q_b(kT + 1)^2 - Q_b(kT)^2 \leq A_b[k]^2 + \mu_b(kT)^2 + 2Q_b(kT)(A_b[k] - \mu_b(kT)), \tag{27}$$

$Q_b(kT + t + 1)^2 - Q_b(kT + t)^2 \leq \mu_b(kT + t)^2 - 2Q_b(kT + t)\mu_b(kT + t), \tag{28}$

for $t = \{1, \ldots, T-1\}$. Now, observe that

$$Q_b(kT + t) \geq Q_b(kT) - t\mu_{max},$$

where a finite $\mu_{max}$ exists since we assumed that $X_{bj}(t) \leq x_{max} < \infty$, $\forall b \in B$ and for each channel $j$. So, we can write

$$-Q_b(kT + t)\mu_b(kT + t) \leq -Q_b(kT)\mu_b(kT + t) + t\mu_{max}\mu_b(kT + t)$$

$$\leq -Q_b(kT)\mu_b(kT + t) + t\mu_{max}^2.$$

Using the above fact, we can rewrite (25) as

$$Q_b(kT + t + 1)^2 - Q_b(kT + t)^2 \leq \mu_b(kT + t)^2 + 2t\mu_{max}^2 - 2Q_b(kT)\mu_b(kT + t).$$

Summing both sides of the previous inequality over $t = \{1, \ldots, T-1\}$, we get

$$Q_b(kT + T)^2 - Q_b(kT + 1)^2 \leq (T^2 - 1)\mu_{max}^2 - 2Q_b(kT)\sum_{t=1}^{T-1} \mu_b(kT + t). \tag{29}$$

Using (27) and the above inequality, we can write

$$Q_b(kT + T)^2 - Q_b(kT)^2 \leq A_b[k]^2 + T^2\mu_{max}^2 + 2Q_b(kT)(A_b[k] - \mu_b[k]), \tag{30}$$

where we have used the fact that $\mu_b(kT)^2 \leq \mu_{max}^2$ and $\mu_b[k] \triangleq \sum_{t=0}^{T-1} \mu_b(kT+t)$.

Consider the quadratic Lyapunov function $L(\Phi[k]) = \sum_{n \in N} \Phi_n[k]^2$. We now define the one-frame conditional Lyapunov drift as

$$\Delta(\Phi[k]) \triangleq \mathbb{E}(L(\Phi[k] + 1) - L(\Phi[k]))|\Phi[k]).$$

Using (25), (26) and (30), we can express the drift for the QMW policy as

$$\Delta(\Phi[k]) \leq \mathbb{E}\left\{ \sum_{r \in \mathcal{R}} (A_r[k]^2(1 - \alpha_r)^2 + \mu_r[k]^2) + \sum_{b \in \mathcal{B}} (A_b[k]^2 + T^2\mu_{max}^2) \right\}$$

$$+ \sum_{g \in \mathcal{G}} \left( \beta_g^2 + \mu_g[k]^2 \right) |\Phi[k]) + 2 \mathbb{E}\left\{ \sum_{b \in \mathcal{B}} (Q_b(kT)(A_b[k] - \mu_b[k])) \right\}$$

$$+ \sum_{r \in \mathcal{R}} (Y_r[k](A_r[k](1 - \alpha_r) - \mu_r[k])) + \sum_{g \in \mathcal{G}} (Z_g[k](\beta_g - \mu_g[k])) |\Phi[k]). \tag{31}$$
Now, we can find a $B < \infty$ such that

$$B \geq E\left\{ \sum_{r \in R} (A_r[k]^2(1 - \alpha_r)^2 + \mu_r[k]^2) + \sum_{b \in B} (A_b[k]^2 + T^2 \mu_{max}^2) + \sum_{g \in G} (\beta_g^2 + \mu_g[k]^2) \mid \Phi[k]\right\}$$

since $E[A_n^2[k]] < \infty$ and $E[X_n^2(t)] < \infty$ for each $n \in \mathcal{N}$ and channel $j$. Also, $\{A_n[k]\}$ is i.i.d. and $E[A_n[0]] = \lambda_n$. So, we can rewrite (31) as

$$\Delta(\Phi[k]) \leq B + 2 E\left\{ \sum_{r \in R} (Y_r[k](1 - \alpha_r) - \mu_r[k]) + \sum_{g \in G} (Z_g[k](\beta_g - \mu_g[k])) + \sum_{b \in B} (Q_b(kT)(\lambda_b - \mu_b[k])) \mid \Phi[k]\right\}$$

Rearranging the terms and using the definition (12), we get

$$\Delta(\Phi[k]) \leq B + 2 E\left\{ \sum_{r \in R} Y_r[k]\lambda_r(1 - \alpha_r) + \sum_{g \in G} Z_g[k]\beta_g + \sum_{b \in B} Q_b(kT)\lambda_b \mid \Phi[k]\right\} + 2 E\left\{ \sum_{n \in \mathcal{N}} \Phi_n[k]\mu_n[k] \mid \Phi[k]\right\}.$$  

From the definition of the QMW algorithm, it can be easily shown that

$$E\left\{ \sum_{n \in \mathcal{N}} \Phi_n[k]\mu_n[k] \mid \Phi[k], S(kT), \ldots, S(kT + T - 1)\right\}$$

$$\geq E\left\{ \sum_{n \in \mathcal{N}} \Phi_n[k]\mu'_n[k] \mid \Phi[k], S(kT), \ldots, S(kT + T - 1)\right\}$$

where $\mu'_n[k]$ corresponds to any other stationary randomized policy. Taking expectation w.r.t. the channel-estimates in the frame, we get

$$E\left\{ \sum_{n \in \mathcal{N}} \Phi_n[k]\mu_n[k] \mid \Phi[k]\right\} \geq E\left\{ \sum_{n \in \mathcal{N}} \Phi_n[k]\mu'_n[k] \mid \Phi[k]\right\}.$$

Thus, using the above fact, we can rewrite (32) as

$$\Delta(\Phi[k]) \leq B + 2 E\left\{ \sum_{r \in R} Y_r[k]\lambda_r(1 - \alpha_r) + \sum_{g \in G} Z_g[k]\beta_g + \sum_{b \in B} Q_b(kT)\lambda_b \mid \Phi[k]\right\}$$

$$- 2 E\left\{ \sum_{n \in \mathcal{N}} \Phi_n[k]\mu'_n[k] \mid \Phi[k]\right\}.$$  

(34)
Rearranging the terms again and using definition [12], we then get

$$\Delta(\Phi[k]) \leq B + 2 \sum_{r \in R} Y_r[k]E\{(\lambda_r(1 - \alpha_r) - \mu'_r[k])|\Phi[k]\}$$

$$+ 2 \sum_{g \in G} Z_g[k]E\{(\beta_g - \mu'_g[k])|\Phi[k]\} + 2 \sum_{b \in B} Q_b(kT)E\{(\lambda_b - \mu'_b[k])|\Phi[k]\}.$$ 

Now, if the system is stabilizable, i.e., if $\lambda \in int[\Lambda]$, then a stationary randomized policy exists that stabilizes the system. Besides, this policy makes decisions independent of the queue-lengths. Taking the $\mu'_n[k]$ decisions to be the service decisions corresponding to this stationary randomized policy, we have

$$E\{\lambda_r(1 - \alpha_r) - \mu'_r[k]|\Phi[k]\} < -\epsilon,$$

$$E\{\beta_g - \mu'_g[k]|\Phi[k]\} < -\epsilon,$$

$$E\{\lambda_b - \mu'_b[k]|\Phi[k]\} < -\epsilon,$$

where $\epsilon > 0$. Thus, the previous drift inequality can be simplified as

$$\Delta(\Phi[k]) \leq B - 2\epsilon \sum_{n \in N} \Phi_n[k],$$

where we have used definition [12]. Now, as before, the queue evolves as a Markov chain. The drift inequality above gives negative drift but for a finite set of queue-lengths. Hence, using Theorem 2 from [15], the network is stable.

**Appendix F**

**Proof of Lemma 1:**

We are given the Lyapunov function $L(Q(t)) = \sum_{i=1}^{N} Q_i^2(t)$. Squaring both sides of equation (1), and using the fact that $(\max\{a, 0\})^2 \leq a^2$, we have

$$Q_i^2(t + 1) - Q_i^2(t) \leq \mu_i^2(t) + A_i^2(t) + 2Q_i(t)(A_i(t) - \mu_i(t)).$$

Summing over all $i$ and taking $E\{\cdot | Q(t)\}$,

$$\Delta(Q(t)) \leq E\{U(t) | Q(t)\} + 2 \sum_{i=1}^{N} Q_i(t)(\lambda_i - E\{\mu_i(t) | Q(t)\})$$

(36)

where $U(t)$ is as defined in (5). Since, $E\{A_i^2(t)\} < \infty$ and $E\{X_j^2(t)\} < \infty$ for each $i$ and $j$,

$$B \triangleq \sum_{i=1}^{N} E\{U(t) | Q(t)\}$$

is bounded and independent of $Q(t)$. Hence, we are done.

**References**

[1] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, R. Vijayakumar, and P. Whiting, “Scheduling in a queuing system with asynchronously varying service rates,” Probability in the Engineering and Informational Sciences, vol. 18, no. 2, 2004
[2] S. Bodas, S. Shakkottai, L. Ying, and R. Srikant, “Scheduling in multi-channel wireless networks: Rate function optimality in the small-buffer regime,” In Proc. Ann. ACM SIGMETRICS Conf., Jun. 2009.

[3] S. Bodas, S. Shakkottai, L. Ying, and R. Srikant, “Low-complexity scheduling algorithms for multi-channel downlink wireless networks,” In Proc. IEEE INFOCOM, Mar. 2010.

[4] S. Bodas, S. Shakkottai, L. Ying, and R. Srikant, “Scheduling for small delay in multi-rate multi-channel wireless networks,” In Proc. IEEE INFOCOM, Apr. 2011.

[5] A. Eryilmaz, R. Srikant, and J.R. Perkins, “Stable scheduling policies for fading wireless channels,” IEEE/ACM Transactions on Networking, vol. 13, no. 2, pp. 411-424, April 2005.

[6] A. Eryilmaz and R. Srikant, “Joint congestion control, routing and MAC for stability and fairness in wireless networks,” IEEE Journal on Selected Areas in Communications, vol. 24, no. 8, pp. 1514-1524, Aug. 2006.

[7] K. Kar, X. Luo, S. Sarkar, “Throughput-optimal scheduling in multichannel access point networks under infrequent channel measurements,” IEEE Trans. Wireless Commun., vol. 7, no. 7, pp. 2619-2629, Jul. 2008.

[8] X. Lin and N. B. Shroff, “Joint rate control and scheduling in multihop wireless networks,” 43rd IEEE Conference on Decision and Control, vol.2, pp. 1484-1489, Dec. 2004.

[9] M. J. Neely, E. Modiano, and C. E. Rohrs, “Power allocation and routing in multibeam satellites with time-varying channels,” IEEE/ACM Transactions on Networking, vol. 11, no. 1, pp. 138-152, Feb. 2003.

[10] M. J. Neely, E. Modiano and C. Li, “Fairness and optimal stochastic control for heterogeneous networks,” IEEE Transactions on Information Theory, vol. 52, no. 7, pp. 2915-2934, Jul. 2006.

[11] A. Stolyar, “Maximizing queueing network utility subject to stability: Greedy primal-dual algorithm,” Queueing Systems, vol. 50, no.4, pp. 401-457, Aug. 2005.

[12] L. Ying and S. Shakkottai, “On throughput optimality with delayed network-state information,” IEEE Transactions on Information Theory, vol 57, no. 8, pp. 5116-5132, Aug. 2011.

[13] D. Bertsekas, “Nonlinear Programming,” Belmont, MA: Athena Scientific, 1995.

[14] S. Donthi and N. B. Mehta, “Joint performance analysis of channel quality indicator feedback schemes and frequency-domain scheduling for LTE,” IEEE Transactions on Vehicular Technology, vol. 60, no. 7, pp. 3096-3109, Sept. 2011.

[15] L. Georgiadis, M. J. Neely, and L. Tassiulas, “Resource allocation and cross-layer control in wireless networks,” Foundations and Trends in Networking, NOW Publishers, vol. 1, no. 1, pp. 1-149, 2006.
[16] G. R. Gupta and N. B. Shroff, “Delay analysis for wireless networks with single hop traffic and general interference constraints,” IEEE/ACM Transactions on Networking, vol. 18, no. 2, pp. 393-405, Apr. 2010.

[17] L. B. Le, K. Jagannathan and E. Modiano, “Delay analysis of maximum weight scheduling in wireless Ad Hoc networks,” 43rd Annual Conference on Information Sciences and Systems, pp. 389-394, Mar. 2009.

[18] E. Leonardi, M. Mellia, F. Neri and M. Ajmone Marsan, “Bounds on average delays and queue size averages and variances in input-queued cell-based switches,” INFOCOM 2001, Anchorage, USA, vol.2, pp. 1095-1103, 2001.

[19] M. J. Neely, “Delay analysis for max weight opportunistic scheduling in wireless systems,” IEEE Transactions on Automatic Control, vol. 54, no. 9, pp. 2137-2150, Sept. 2009.

[20] W. Ouyang, S. Murugesan, A. Eryilmaz, and N. B. Shroff, “Scheduling with rate adaptation under incomplete knowledge of channel/estimator statistics”, arXiv:1008.1828v2, Oct. 2010.

[21] S. Sesia, I. Toufik, and M. Baker, “LTE – The UMTS Long Term Evolution: From Theory to Practice,” John Wiley and Sons, 2009.

[22] L. Tassiulas and A. Ephremides, “Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multi-hop radio networks,” IEEE Trans. Automat. Contr., vol. 37, no. 12, pp. 1936-1948, Dec. 1992.

[23] L. Tassiulas and A. Ephremides, “Dynamic server allocation to parallel queues with randomly varying connectivity,” IEEE Transactions on Information Theory, vol. 39, no. 2, pp. 466-478, Mar. 1993.

[24] L. Tassiulas, “Scheduling and performance limits of networks with constantly changing topology,” IEEE Transactions on Information Theory, Vol. 43, No. 3, pp. 1067-1073, May 1997.

[25] L. M. Lopez-Ramos, A. G. Marques, J. Ramos, and A. Caamano, “Cross-Layer Resource Allocation for Downlink Access Using Instantaneous Fading and Queue Length Information,” IEEE Proc. MCECN at Global Communications Conf., Miami, USA, Dec. 6-10, 2010.

[26] A. G. Marques, L. M. Lopez-Ramos, G. B. Giannakis, J. Ramos, and A. Caamano, “Optimal Cross-Layer Resource Allocation in Cellular Networks Using Channel and Queue State Information,” IEEE Trans. on Vehic. Tech, vol. 61, no. 6, pp. 2789 - 2807, July 2012.

[27] M. J. Neely, E. Modiano, and C. E. Rohrs, “Dynamic power allocation and routing for time-varying wireless networks,” IEEE Journal on Selected Areas in Communications, vol. 23, issue 1, pp. 89-103, 2005.

[28] X. Lin and N. B. Shroff, “The impact of imperfect scheduling on cross-layer congestion control in wireless networks,” IEEE/ACM Trans. Netw., vol. 14, no. 2, pp. 30-31, Apr. 2006.
[29] V. Sharma, D. K. Prasad and E. Altman, “Opportunistic scheduling of wireless links,” *20th International Teletraffic Congress*, Ottawa, Canada, Jun. 2007.