An actuarial multi-state transition modelling in long-term care study

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Abstract. With the growing numbers of elderly and numbers of disability among the elderly in the population, long-term care services are at great demand now. Long-term care provides a variety of services by helping the elderly or non-elderly with medical illnesses or disabilities who are unable to foster themselves for prolonged terms. Since Malaysia’s aged care and health care industry still has ways to go in contrast to other countries, such as Australia, therefore, understand the long-term care services could be helpful to provide quality lifestyle for the elderly in the future. We study the multi-state transition model which is the common model used on health-related data. Four levels of core activity restrictions are included in the model to allow the probability of disability to be included in the long-term care cost projection. Estimation of transition intensities are provided by fitting the Australian data with Gompertz and Weibull models. The graphs of actual and fitted transition probabilities are presented for better illustration. We concluded that Gompertz model has the better fit for both male and female in our study. Some limitations and recommendations have been outlined for future studies.

1. Introduction

Long-term care provides variety of personal care services to elderly or non-elderly with medical illness or disability to perform daily activities, while maintaining independency for prolonged terms [1]. Long-term care is crucial to many countries with the growing number of elderlies in the population. In Australia, 12.9% of the population are 65 years old and above in year 2005, 15.0% in year 2017 and is expected to reach 18.0% in year 2020 [2][3]. With the growing segment of elderly in the population and the increase in life expectancy [4], more people are expected to use long-term care support and services in the future. Not to forget the group of people with disability who almost always be in need of assistance with their daily activities. By knowing how many are in special needs condition, we can build inclusive communities and services through policies and practices, allowing elderly or non-elderly with disability to improve the quality of their lives [2]. Based on the report given by Australian Institute of Health and Welfare [5], approximately 4.3 million or 18% of the Australian population are with disability in year 2015 and the numbers have increased over years. Among the 4.3 million Australians with disability, 1.4 million or 5.8% of it are either severe or profound core activity limitations. In other words, this group of people are always, if not almost always need assistance with their daily activities. Furthermore, based on the percentages shown in Figure 1, the chances of having disability, severe or profound core activity limitation increase when the age increases. This is another sign of worrying as we are expecting the growing ageing population with disability in the near future.
Figure 1. The Australia population with disability, severe or profound core activity limitation by age group in percentage for year 2015 [5].

Apart from the statistics given in Australia, the increase in Malaysia’s ageing population is also undeniable. It is expected that 10% of the Malaysia’s population are 60 years old and above by year 2020 and is expected to achieve 15% by year 2030 [6]. Moreover, the number of persons with disabilities in Malaysia has increased from 453,258 to 488,948 (1.5% of Malaysia’ population), from year 2017 to year 2018 [7]. Even though the statistics for disability is below the benchmark of the World Health Organisation, both statistics are still worrying as Malaysia’s health care and aged care industry have ways to go in contrast to other countries, such as Australia [8]. Though Malaysia has initiated the public long-term care services under the Department of Social Welfare, but Malaysians are still preferring to take care of the elderlies by themselves due to their cultural values and there are rooms of improvement on long-term care services in Malaysia [9]. On the other hand, Australia has more than 20 years of experience in long-term care services since the establishment of Aged Care Act 1997. Since the quality of the Australia long-term care services is perceived as good [2], additionally with the limitation of data available in Malaysia, the Australian data will be used in our study to get insight in the long-term care area. The purpose of this study is to determine the transition intensities for a 6 states multi-state transition model, including the 4 levels of core activity restrictions (CAR). Since less attention has been given to the estimation of CAR levels as compared to estimation of mortality [10], by including the CAR levels information in the model, it will allow a more accurate projection of long-term care cost. Descriptions of the model and its states will be outlined in the next section. To estimate the transition intensities, the equations of the transition intensities and transition probabilities for the Gompertz and Weibull mortality models will be introduced. The results of the fitted transition probabilities are discussed. Finally, we conclude our study.

2. Model and Data
A multi-state model is a probabilistic approach that can be used to model health related data, including disability, long-term disease and critical illness data [11]. Leung has proposed a multi-state transition model which allows one to transit from one state to another state within a given time as below [10]. There are 6 states in the model which are:

State 1: Able
State 2: Profound CAR
State 3: Severe CAR
State 4: Moderate CAR
State 5: Mild CAR
State 6: Dead
Leung applied this model to obtain the number of Australian acquire long-term care in the future. The CAR refers here are related to self-care, mobility and communication restrictions as stated in Leung [10]. 4 levels of CAR, namely profound, severe, moderate and mild are identified and defined in Table 1. The model proposed by Leung is modified and displayed in Figure 2. This model is applicable for both male and female where State 1 is the only starting state, and State 2 to State 6 are considered as the exit states. In short, we assume that once the person leaves State 1, there is no chance of returning back to State 1. The modified model will ease the calculation in long-term care cost projection. The transition intensities, $\mu_{ij}$ are included in the model as presented in Figure 2. $\mu_{ij}$ refers to transition intensity between states $i$ and $j$ at age $x$. It also refers to the instantaneous rate of the process leaving states $i$ to state $j$ for someone age $x$ [10]. Mathematically, $\mu_{ij}$ can be written in equation (1):

$$\mu_{ij} = \lim_{t \to \infty} \frac{P_{ij}}{t} \quad \text{for } x, t \geq 0, \quad i, j \in S, \quad i \neq j$$

where $P_{ij}$ is the probability of a person age $x$ currently at state $i$ transit to state $j$ within the next $t$ years.

By following the model in Figure 2, the set of states, $S = \{1, 2, 3, 4, 5, 6\}$ and $i < j$.

**Figure 2.** Multi-state Transition Model.

**Table 1.** Descriptions of the state in the multi-state transition model.

| State | Descriptions |
|-------|---------------|
| 1: Able | The individual is able to perform a core activity task without the assistance of others. |
| 2: Mild | The individual does not require any help and could perform core activity tasks quite smoothly but uses aiding equipment and is in need of others’ supervision. |
| 3: Moderate | The individual requires limited assistance and has slight struggle in performing a core activity errand. |
| 4: Severe | The individual is frequently requiring assistance with his or her core activity tasks and has trouble understanding or being understood by others. |
| 5: Profound | The individual is unable to perform a core activity task without the assistance of others. |
| 6: Dead | The individual is dead due to any causes. |
The data used are the transition probabilities obtained by Leung [12] based on the “Survey of Disability, Ageing and Carers” conducted by the Australian Bureau of Statistics in year 1998. Table 2 and 3 present the transition probabilities at 10-yearly age intervals for males and females respectively. We define the transition probability notation shown in Table 2 and 3 as \( P_{ij}^{xt} \) and \( P_{ij}^{ty} \), where \( x \) denotes the age for males and \( y \) for the age of female. If we refer to \( P_{ij}^{xt} \), this is the probability of a male age \( x \) currently at state \( i \) transit to state \( j \) within the next \( t \) years. \( P_{ij}^{xt} \) can be written in equation as shown in (2):

\[
P_{ij}^{xt} = \mathbb{P}(S_{x+t} = j | S_x = i) \quad \text{for} \quad x, t \geq 0, \ i, j \in S
\]

(2)

\( S_{x+t} \) represents the state at time \( x+t \). Similar to equation (1), the set of states, \( S = \{1, 2, 3, 4, 5, 6\} \) and \( i < j \). A few observations can be obtained based on the data in Table 2 and 3. As age increases, the probability of death increases for both male and female. Male has higher chances of having mild and moderate CARs as compared to female. On the other hand, female has higher chance of having severe and profound CARs as compared to male.

| Age | \( P_{12}^{10} \) | \( P_{13}^{10} \) | \( P_{14}^{10} \) | \( P_{15}^{10} \) | \( P_{16}^{10} \) |
|-----|----------------|----------------|----------------|----------------|----------------|
| 20  | 0.005229       | 0.001793       | 0.000926       | 0.000808       | 0.001199       |
| 30  | 0.007353       | 0.002137       | 0.001104       | 0.000963       | 0.001313       |
| 40  | 0.008726       | 0.002992       | 0.001545       | 0.001348       | 0.001742       |
| 50  | 0.014860       | 0.005095       | 0.002632       | 0.002295       | 0.003562       |
| 60  | 0.029544       | 0.0010130      | 0.005234       | 0.004564       | 0.010245       |
| 70  | 0.043650       | 0.015034       | 0.007801       | 0.006833       | 0.029305       |
| 80  | 0.119224       | 0.046477       | 0.027297       | 0.027065       | 0.077222       |

Table 2. The transition probabilities for male.

| Age | \( P_{12}^{10} \) | \( P_{13}^{10} \) | \( P_{14}^{10} \) | \( P_{15}^{10} \) | \( P_{16}^{10} \) |
|-----|----------------|----------------|----------------|----------------|----------------|
| 20  | 0.004906       | 0.001299       | 0.0009463      | 0.000929       | 0.000417       |
| 30  | 0.005612       | 0.001486       | 0.0018026      | 0.001063       | 0.000492       |
| 40  | 0.007486       | 0.001983       | 0.0014443      | 0.001418       | 0.000893       |
| 50  | 0.012414       | 0.003290       | 0.0023981      | 0.002357       | 0.002231       |
| 60  | 0.024985       | 0.006659       | 0.0048804      | 0.004823       | 0.005772       |
| 70  | 0.037427       | 0.010481       | 0.0080711      | 0.008381       | 0.015529       |
| 80  | 0.088349       | 0.033458       | 0.0348423      | 0.048925       | 0.048903       |

Table 3. The transition probabilities for female.

3. Methodology
The purpose of this study is to obtain the transition intensities for the model shown in Figure 2. We carried out the model fitting for transition probabilities by using Gompertz and Weibull models, to obtain the best fitting. Once the best fitting is obtained, we proceed with the estimation of transition intensities. In the study of Leung [12], he determined the estimation of parameters of the model by graduating the transition intensities. In our study, we consider two models, i.e. Gompertz and Weibull for the fitting due to its popularity in fitting health and survival data [13]. Under Gompertz model, the transition intensity can be written as equation (3) and the transition probability can be obtained as equation (4). On the hand, the transition intensity and transition probability for Weibull model can be expressed in equation (5) and (6) respectively. Both are two-parameter models.
\[ \mu_x = Bc^x \quad \text{for} \quad B > 0, \ c > 1, x \geq 0 \]  
(3)

\[ p_x = e^{\frac{-e^{x/c}}{B \cdot c}} \quad \text{for} \quad B > 0, \ c > 1, \ x \geq 0, \ t \geq 0 \]  
(4)

where \( B \) and \( c \) are shape parameters, \( x \) represents the age of a person, \( t \) represents the years.

\[ \mu_x = kx^n \quad \text{for} \quad n > 0, \ k > 0, x \geq 0 \]  
(5)

\[ p_x = e^{-\frac{k^{(1-t/e^{x/k})}}{n \cdot x}} \quad \text{for} \quad n > 0, \ k > 0, \ x \geq 0, \ t \geq 0 \]  
(6)

where \( k \) is the scale parameter, \( n \) is the shape parameter, \( x \) represents the age of a person, \( t \) represents the years.

The parameters of the models are obtained by using equation (4) and (6), through minimizing the sum of squared residuals (SSR) via the Solver add-in available in Microsoft Excel spreadsheet. A best model is when one has the smallest SSR compare to another model. The estimated parameters and minimized SSR for both models are presented in Table 4 in the next section.

4. Results and Discussion

After we performed the data fitting with Gompertz and Weibull models, we presented the results in Table 4 for male and female. Besides, the graphs of actual and fitted transition probabilities from State 1 to any exit states (State 2 to State 6), for both Gompertz and Weibull models are illustrated in Figures 3 and 4 for male and female respectively.

**Table 4. The estimated parameters under Gompertz and Weibull models for male and female.**

| State | Gender | Model      | Estimated parameters | SSR   |
|-------|--------|------------|----------------------|-------|
| 1 → 2 | Male   | Gompertz   | B = 0.00001113, c = 1.07701 | 0.000339 |
|       |        | Weibull    | k = 0.00000412, n = 1.28216 | 0.002965 |
|       | Female | Gompertz   | B = 0.00004821, c = 1.07552 | 0.000097 |
|       |        | Weibull    | k = 0.00000621, n = 1.13973 | 0.001489 |
| 1 → 3 | Male   | Gompertz   | B = 0.00000695, c = 1.07008 | 0.000066 |
|       |        | Weibull    | k = 0.00000153, n = 1.28216 | 0.000507 |
|       | Female | Gompertz   | B = 0.00000602, c = 1.06538 | 0.000066 |
|       |        | Weibull    | k = 0.00000343, n = 0.98417  | 0.000331 |
| 1 → 4 | Male   | Gompertz   | B = 0.00000340, c = 1.10639 | 0.00012  |
|       |        | Weibull    | k = 0.00000430, n = 0.92584  | 0.000241 |
|       | Female | Gompertz   | B = 0.00000377, c = 1.07090  | 0.000134 |
|       |        | Weibull    | k = 0.00000441, n = 0.96934  | 0.000449 |
| 1 → 5 | Male   | Gompertz   | B = 0.00000433, c = 1.06540 | 0.000077 |
|       |        | Weibull    | k = 0.00000675, n = 0.83012  | 0.000268 |
|       | Female | Gompertz   | B = 0.00000417, c = 1.07465  | 0.000297 |
|       |        | Weibull    | k = 0.00000469, n = 1.01090  | 0.001009 |
| 1 → 6 | Male   | Gompertz   | B = 0.00000612, c = 1.07821 | 0.000208 |
|       |        | Weibull    | k = 0.00000405, n = 1.17343  | 0.001885 |
|       | Female | Gompertz   | B = 0.00000482, c = 1.07552  | 0.000098 |
|       |        | Weibull    | k = 0.00000508, n = 1.01538  | 0.000879 |
Figure 3. Actual and fitted transition probabilities for male from “able” to (a) mild CAR; (b) moderate CAR; (c) severe CAR; (d) profound CAR; (e) dead.
Figure 4. Actual and fitted transition probabilities for female from “able” to (a) mild CAR; (b) moderate CAR; (c) severe CAR; (d) profound CAR; (e) dead.
Based on the SSR in Table 4 and the graphs presented in Figures 3 and 4, it is evident that the Gompertz model is better fitted for our data, for both male and female, as it always provides lower SSR than the Weibull model. Our results are consistent with the study by Li [13] where Gompertz model is stood out compare to Weibull model. Therefore, the best estimation of the transition intensities in this study can be easily determined by substituting the estimated values of \( B \) and \( c \) obtained in Table 4 into equation (3).

5. Conclusion

This study presented the modified multi-state transition model in long-term care study. Besides, we are able to develop the equations for transition intensities under Gompertz model and provide an insight that Gompertz would be suitable to model long-term care data in a multi-state transition model. We can focus on the long-term care cost estimation in our future research. Long-term care cost is deemed to be important especially when it is largely unfunded and require self-pay from the policyholders. However, there are some limitations found in our study. The data applied in this study are based on Australia in year 1998. Next, we assumed that the recovery of the disability is not allowed in our model. Also, we only considered two models in our data fitting. Malaysia has a comprehensive health care system that focuses more on the short-term care and hospitalisation [4]. The present health care system is inadequate to accommodate the elderly with critical illness and disabilities and is insufficient to overcome the ageing population and disabilities issues. The way forward is to learn from other countries which already have the long-term care system in place, such as Australia. With the growing ageing population and given the urgency to address on Malaysia’s need for long-term care, it provides a new challenge on Malaysia health care and social services in the future. The government, non-government agencies, private sectors and community must go hand-in-hand to overcome this challenge together.

6. References

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