Leggett inequalities and the completeness of quantum mechanics

Ramon Lapiedra

Departament d’Astronomia i Astrofísica
Universitat de València, Campus de Burjassot, 46100 Burjassot (València), Spain.

Miguel Socolovsky

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México,
Circuito Exterior, Cd Universitaria, 04510 México D. F., México.
(On sabbatical year at the IAFE, Universidad de Buenos Aires, Argentina)
(Dated: June 12, 2008)

Abstract

We consider the so called Leggett inequalities which are deduced from the assumption of general (local or non-local) realism plus the arrow of time preservation. Then, instead of assuming crypto-nonlocal hidden variables, we assume any (local or non-local) realism compatible with the joint and non-joint expected values dictated by quantum mechanics. Hence, we prove that this double assumption is not consistent, since the corresponding general Leggett inequalities are violated by quantum mechanics. Thus, realism plus arrow of time preservation and quantum mechanics are not compatible. In other words, quantum mechanics cannot be completed with any (local or non-local) hidden variables, provide we assume the common sense of the arrow of time. The result would deserve to be experimentally tested and we discuss why it is not invalidated by hidden variables theories as the one from Bohm.

PACS numbers: 03.65.Ud, 03.65.Ta

*Electronic address: ramon.lapiedra@uv.es
†Electronic address: socolovs@iafe.uba.ar
I. INTRODUCTION

Several years ago, Leggett [1], assuming a class of hidden variable theories which he called cripto-nonlocal, found a new type of Bell’s inequalities, presently known as Leggett’s inequalities, which were violated when the joint expected values inserted in these inequalities were the ones dictated by quantum mechanics (QM). More recently, in [2] [3] [4], these inequalities have been adapted to the experimental requirements and it has been shown that they are violated in the laboratory according to the above predictions. In all these papers one considers a source which produces pairs of entangled polarized photons, with mixtures of different polarizations according to some given probability distribution. In [5], assuming again crypto non-local realism, some other Leggett inequalities are proved for well definite polarizations, which are also violated by experiments when the inserted joint expected values are the ones dictated by QM.

In the present paper we consider Leggett inequalities for well definite polarizations too. We follow [2] in order to show that these inequalities, in its general form, which henceforth we will call ‘basic Leggett inequalities’, result from the assumption of realism, i.e., local or non-local realism, plus the preservation of the arrow of time. In particular, to this general level you do not need to assume the above crypto-nonlocality. Then we show how these basic Leggett inequalities can be violated by QM. This violation means, modulus this arrow of time preservation, that QM cannot be completed, that is, the state of a quantum system plus the measurement ‘direction’ cannot be supplemented with any, local or non local, hidden variables in order to complete the statistical predictions of QM with sure predictions for individual measurements. Since it is widely claimed that non local hidden variable theories giving the same results as QM exist, as for example the one from Bohm [6], we will have to explain why this claim is basically a non justified one.

In Sec. II following [1], we precise the assumptions involved in the construction of a non-local hidden variable theory and recall the derivation of the basic Leggett inequalities, which deal with subensembles of entangled pairs of photons with given polarizations (and not with an ensemble of mixtures of different polarizations, as considered in [1] - [4]).

Then, in Sec. III we consider a particular quantum system involving two entangled polarized photons. We use quantum mechanics to calculate the joint and non-joint expected values entering in the corresponding basic Leggett inequalities, which have been deduced
assuming only a general realism which preserves the arrow of time. Inserting all the calculated quantum expected values in the inequalities derived in Sec. II, we show how the latter are violated for certain quantum states and some ranges of the polarizer angles involved. This means that, when assuming the common sense concerning the arrow of time, realism, local or not local, is incompatible with quantum mechanics. In other words, quantum mechanics cannot be completed with any (local or non-local) hidden variables. We stress the differences among our conclusion and the results from the recent papers quoted above, and remark the interest of looking for a polarized entangled system of two photons that, like the one considered in the present paper, leads to a violation of Leggett inequalities, while fulfilling the requirements of experimental testing.

Finally, in Sec. IV we consider the Bohm-like hidden variable theories [6] and we show why it is not obvious, from the very beginning, that the realism of this theory can be consistent with all experiments. Thus, at the end, the incompatibility that we find here between realism and quantum mechanics becomes also an incompatibility with Bohm theory itself, against the expectations raised in [7] of still leaving an open door to this theory, or against the uncritical confidence put on it in [8].

II. THE BASIC LEGGETT INEQUALITIES

Following [2], let a source $S$ emit pairs of photons with entangled polarization directions, in a given global quantum state, $\Psi_{AB}$, towards the corresponding analyzers 1 and 2, placed, respectively, at the localities $A$ and $B$, with orientations given by real unit vectors $\vec{a}$ and $\vec{b}$. Notice the difference with [1] - [4], where there is a source which emits not a well given polarized entangled global state, $\Psi_{AB}$, as in our case, but mixtures of different entangled polarizations according to some given probability distribution.

Let us go to our case and assume general (local or non-local) realism. Then, when each photon of each pair is detected, the results of the polarization measurements are given, respectively, by functions $A(\vec{a}, \vec{b}; \lambda)$ and $B(\vec{b}, \vec{a}; \lambda)$ which, at detection, take the values +1 or -1. Here, $\lambda$ is a supplementary (“hidden”) variable taking values in a real domain $\Lambda$, such that, for the ensemble of pairs of polarized entangled photons in the state $\Psi_{AB}$, has a probability distribution $\rho_{\Psi}(\lambda)$ obeying
\[ \rho_\Psi(\lambda) \geq 0 \quad \text{and} \quad \int_\Lambda d\lambda \rho_\Psi(\lambda) = 1. \tag{1} \]

Then one has the following three expected values over the ensemble:

\[ \bar{A} = \int_\Lambda d\lambda \rho_\Psi(\lambda) A(\vec{a}, \vec{b}; \lambda), \tag{2} \]

\[ \bar{B} = \int_\Lambda d\lambda \rho_\Psi(\lambda) B(\vec{b}, \vec{a}; \lambda), \tag{3} \]

and

\[ \bar{A}\bar{B} = \int_\Lambda d\lambda \rho_\Psi(\lambda) A(\vec{a}, \vec{b}; \lambda)B(\vec{b}, \vec{a}; \lambda). \tag{4} \]

In principle, \( \bar{A} \), \( \bar{B} \) and \( \bar{A}\bar{B} \) depend on \( \Psi \) and on the set of variables \((\vec{a}, \vec{b})\). Non locality is allowed by the possible dependence of \( A \) on \( \vec{b} \) and of \( B \) on \( \vec{a} \), and realism is represented by the supplementary variable \( \lambda \).

Since the quantities \( A \) and \( B \) only take the values \( \pm 1 \), one has

\[ 1 - |A - B| = AB = -1 + |A + B|. \tag{5} \]

Then, averaging on the different values of \( \lambda \), and using the obvious inequality \( \int_\Lambda d\lambda \rho_\Psi |A - B| \geq | \int_\Lambda d\lambda \rho_\Psi (A - B)| \), one obtains the basic Leggett inequalities

\[ 1 - | \int_\Lambda d\lambda \rho_\Psi (A(\vec{a}, \vec{b}; \lambda) - B(\vec{b}, \vec{a}; \lambda)) | \geq | \int_\Lambda d\lambda \rho_\Psi (A(\vec{a}, \vec{b}; \lambda)B(\vec{b}, \vec{a}; \lambda)) | \]

\[ \geq -1 + | \int_\Lambda d\lambda \rho_\Psi (A(\vec{a}, \vec{b}; \lambda) + B(\vec{b}, \vec{a}; \lambda)) |, \tag{6} \]

or in a more compact notation

\[ 1 - |\bar{A} - \bar{B}| \geq \bar{A}\bar{B} \geq -1 + |\bar{A} + \bar{B}|. \tag{7} \]

It must be stressed that, unlike what it happens with the proof of Bell’s inequalities \cite{9,10}, one does not need the assumption of local realism in order to derive the above Leggett inequalities. It has been enough to assume realism as such, local or non-local. The reason for this is that now we perform all measurements along the same directions \( \vec{a} \) and \( \vec{b} \). Furthermore, each time, both measurements, along \( \vec{a} \) and \( \vec{b} \), respectively, are jointly...
performed. This is why we have been able to use a unique probability distribution $\rho_\Psi(\lambda)$ in the calculation of the mean value of the equations (5).

However, notice that the above reasoning cannot lead to the conclusion (7) unless we assume, as we have done, the ‘outcome independence’ assumption [11], that is, unless we assume that $A$ does not depend on $B$ and reciprocally. Actually, let us write $A = A(\bar{a}, \bar{b}; \lambda, B)$ and $B = B(\bar{a}, \bar{b}; \lambda, A)$, such that these conditions on $A$ and $B$ do not define two functions $A = A(\bar{a}, \bar{b}; \lambda)$ and $B = B(\bar{a}, \bar{b}; \lambda)$. That is, given the arguments $\bar{a}, \bar{b}$ and $\lambda$, assume that more than a couple of $A$ and $B$ values can exist that satisfy the conditions $A = A(\bar{a}, \bar{b}; \lambda, B)$ and $B = B(\bar{a}, \bar{b}; \lambda, A)$. In this case, the above probability distribution, $\rho_\Psi$, could depend of the different couples of $A$ and $B$ values, i.e., the four couples of values $(1, 1), (1, -1, (-1, 1)$ and $(-1, -1)$. Then, we could not be sure that we can use a unique probability distribution, $\rho_\Psi$, in order to conclude (7) from (5). This could be an example of absence of a ‘common probability space’, as it has been argued in [12] on different grounds.

Nevertheless, it can be easily seen that the above outcome dependence can only be maintained if we give up the arrow of time: in plain words, only if we accept that the future could affect the past. Actually, if we preserve the arrow of time, and assume, for example, that measurements at $A$ always precede measurements at $B$, we could have at most $A = A(\bar{a}, \bar{b}; \lambda)$ and $B = B(\bar{a}, \bar{b}; \lambda, A)$. But then, by inserting $A = A(\bar{a}, \bar{b}; \lambda)$ in $B = B(\bar{a}, \bar{b}; \lambda, A)$, we finally have $B$ as a function of $\bar{a}, \bar{b}$ and $\lambda$, that is, we are in fact in the original case of outcome independence.

III. VIOLATION OF THE INEQUALITIES BY QUANTUM MECHANICS

Let us consider the quantum system of two entangled polarized photons whose state, $\Psi_{AB}$, is

$$\Psi_{AB} = (1 - c^2)^{1/2}u_A u_B + cv_A v_B,$$

where $u$ and $v$ stand for the entangled states (the kets) of two entangled photons. The states refer to two linear polarizations in the directions given by two unit orthogonal 3-space vectors $\vec{u}$ and $\vec{v}$, respectively. The coefficient $c$ is a real positive quantity such that $1 \geq c$. The $A$ and $B$ index denote the corresponding two separated entangled localities.

As explained in Sec. [11] we place at the localities $A$ and $B$ the polarization analyzers whose orientations are given by the unit vectors $\vec{a}$ and $\vec{b}$. In the present Section, we easily show...
how the basic Leggett inequalities (7) are violated for some configurations of the vectors $\vec{a}$ and $\vec{b}$, for some values of the coefficient $c$, according to QM, i. e., when we insert in these inequalities the expecting values, $\bar{A}$, $\bar{B}$ and $\overline{AB}$, dictated by QM.

Let us consider the corresponding expected values $\bar{A}$, $\bar{B}$ and $\overline{AB}$ which appear in the basic Leggett inequalities (7).

To begin with, we will have

$$\bar{A} = 2P_{A}(a, +) - 1,$$

(9)

where $P_{A}(a, +)$ stands for the probability of one of the two polarization outcomes at $A$, the one to which we have conventionally assigned the value +1. In an analogous way we have

$$\bar{B} = 2P_{B}(b, +) - 1,$$

(10)

where the meaning of $P_{B}(b, +)$ is now obvious. To obtain (9) and (10) we have used the completeness relations $P_{A}(a, +) + P_{A}(a, -) = 1$, $P_{B}(b, +) + P_{B}(b, -) = 1$, where the meaning of the non defined terms should be obvious.

For the expected value $\overline{AB}$ we will have

$$\overline{AB} = 2[P_{AB}(a, b, ++) + P_{AB}(a, b, --)] - 1,$$

(11)

Here $P_{AB}(a, b, ++)$ stands for the joint probability of having the same kind of polarization outcome (the one to which it has been conventionally assigned the value +1) at both analyzers. Similarly for $P_{AB}(a, b, --)$. When getting (11), we have used the completeness relation $P_{AB}(a, b, ++) + P_{AB}(a, b, --) + P_{AB}(a, b, +--) + P_{AB}(a, b, --) = 1$, where again the notation should be obvious.

Inserting the above expressions, (9), (10) and (11), for $\bar{A}$, $\bar{B}$ and $\overline{AB}$, in the left hand side of the Leggett’s inequalities (7), we find the equivalent inequality

$$1 \geq |P_{A}(a, +) - P_{B}(b, +)| + P_{AB}(a, b, ++) + P_{AB}(a, b, --).$$

(12)

So, let us calculate the different probabilities which appear in this version of the basic Leggett inequalities. To begin with, we have:

$$P_{A}(a, +) = |\Psi_{AB}.a_{A}|^{2},$$

(13)

where $\Psi_{AB}.a_{A}$ stands for the Hilbert scalar product of the kets $\Psi_{AB}$ and $a_{A}$. The last one refers to a polarized photon which is at the place $A$, with the linear polarization correspond-
ing to \( \vec{a} \). We easily find

\[
P_A(a, +) = (1 - c^2) \cos^2 \alpha + c^2 \sin^2 \alpha,
\]

(14)

where \( \alpha \) is the angle between the vectors \( \vec{u} \) and \( \vec{a} \).

Similarly:

\[
P_B(b, +) = (1 - c^2) \cos^2 \beta + c^2 \sin^2 \beta,
\]

(15)

with \( \beta \) the angle between the vectors \( \vec{u} \) and \( \vec{b} \).

For the first joint probability, \( P_{AB}(a, b, ++) \), we easily find

\[
P_{AB}(a, b, ++) = |\Psi_{AB} \cdot a_A b_B|^2 = (1 - c^2) \cos^2 \alpha \cos^2 \beta + c^2 \sin^2 \alpha \sin^2 \beta + \frac{1}{2} c(1 - c^2)^{1/2} \sin 2\alpha \sin 2\beta.
\]

(16)

For the other joint probability \( P_{AB}(a, b, ---) \), we must calculate the expression

\[
|\Psi_{AB} \cdot a_A \perp b_B \perp|^2,
\]

where the kets \( a_A \perp \) and \( b_B \perp \) refer, respectively, to two unit vectors, \( \vec{a} \perp \) and \( \vec{b} \perp \), which are orthogonal to \( \vec{a} \) and \( \vec{b} \), respectively. The result is

\[
P_{AB}(a, b, ---) = (1 - c^2) \sin^2 \alpha \sin^2 \beta + c^2 \cos^2 \alpha \cos^2 \beta + \frac{1}{2} c(1 - c^2)^{1/2} \sin 2\alpha \sin 2\beta. \]

(17)

Inserting the above probabilities (14), (15), (16) and (17), in (12), the left hand side of the basic Leggett inequality becomes

\[
1 \geq |1 - 2c^2||\cos^2 \alpha - \cos^2 \beta| + \cos^2 \alpha \cos^2 \beta + \sin^2 \alpha \sin^2 \beta + c(1 - c^2)^{1/2} \sin 2\alpha \sin 2\beta. \]

(18)

Furthermore, for \( \alpha = \epsilon^{1/2} \) and \( \beta = \frac{\pi}{2} - \epsilon^{1/2} \), where \( \epsilon \) is a real positive infinitesimal quantity, the above inequality reduces to

\[
1 \geq |1 - 2c^2|(1 - 2\epsilon) + 2\epsilon + 4c(1 - c^2)^{1/2}\epsilon,
\]

(19)

to first order in \( \epsilon \).

Now, let us assume that \( 1 > 2c^2 \). Hence, (19) becomes

\[
c \geq 2\epsilon + 2(1 - c^2)^{1/2}\epsilon,
\]

(20)

to first order in \( \epsilon \). Then, it is straightforward to see that this inequality becomes slightly violated for any value of \( c \) of order \( \epsilon \) but such that \( c < 2\epsilon \). Notice that this infinitesimal value for \( c \) fulfills the initial condition \( 1 > 2c^2 \). In other words, the basic Leggett inequalities (7) are in contradiction with QM. But, as we have already noticed at the end of Sec. II.
these inequalities are deduced by only assuming any sort of (local or non-local) realism plus the arrow of time (in particular, we do not use the crypto non-local realism assumption, used in [1] - [4]). Then, as it has been announced in the Introduction, if the arrow of time is preserved, QM and realism, local or non-local, are incompatible: QM cannot be completed, i.e., the given quantum state and the measurement ‘direction’ cannot be supplemented with any, local or non local, hidden variables in order to complete the statistical predictions of the QM with sure predictions for individual measurements, unless we allow for an unphysical violation of the arrow of time.

As we have mentioned in the Introduction, in [5] some Leggett inequalities have been deduced for well definite polarizations too, under the assumption of crypto non-local realism. These inequalities are experimentally violated when the inserted joint expected value, $\overline{AB}$, is the one calculated according to QM, for an entangled pair of two polarized photons in the negative parity state, i.e., in the singlet state. Nevertheless, in our case, where we assume no other condition that realism, local or non-local, jointly with the arrow of time, plus QM (that is plus the expected values $\overline{A}$, $\overline{B}$ and $\overline{AB}$, dictated by QM), it can be seen that there is no contradiction between the corresponding Leggett inequalities applied to the above singlet state and QM (likewise, there is neither contradiction for the positive parity state, or for the singlet state of two entangled 1/2 spin particles: the details of the calculation will be given elsewhere). This is why we have considered a state like state (8), which is more general that the ones considered above for entangled photons, in order to display the above contradiction between realism and QM. Now, in the framework of a certain realism, the assumption of crypto non-local realism may sound reasonable when we have a source of entangled polarized photons which produces a mixture of different polarizations, according to some probability distribution. This is the case in [1] - [4]. Nevertheless, if we consider that the source produces definite polarizations, we must leave the crypto non-local realism hypothesis and simply accept the testable quantum expected values for $\overline{A}$ and $\overline{B}$. This is just what we have done in the present Section.

Since in the present paper we only want to prove that QM, on one hand, and realism as such plus the arrow of time preservation, on the other hand, are incompatible, at the end we have chosen a very special case of the more general quantum state (8), in order to accomplish the proof in the most simple way. Nevertheless, the importance of the result deserves that some quantum state be found (perhaps the same considered in Sec. III) such
that, while still leading to the violation of the basic Leggett inequalities (7), be able to fulfill the requirements of an experimental test.

IV. NON-LOCAL REALISM AND BOHM THEORY. CONCLUSIONS

It seems at first sight that saying that QM cannot be completed, as we say, should be erroneous since it has been largely claimed that a hidden variable theory (HVT) exists, the Bohm’s theory (6), which is, at the same time, a non local realistic theory and one which reproduces all the predictions of QM. Furthermore, the kind of realism assumed in Bohm’s theory seems to be the most general kind of realism one can conceive. To begin with, in this HVT theory, the assumed hidden variables ”depend both on the state of the measuring apparatus and the observed system” (contextual realism). On the other hand, even if implicitly, Bohm only places his hidden variables behind any actually obtained outcome measurement (actual realism), and not behind a merely obtainable outcome (joint realism) (13). This kind of contextual actual realism is also the one considered here, for example when proving (7), and the one implicitly considered in [1] - [4].

Now, let us consider in detail whether is it true that Bohm’s HVT is always consistent with this actual, apparatus dependent, non local realism. Bohm proves that his theory gives the same probability of finding a particle in a given position that QM does. From this, he reasonably concludes that his ”interpretation is capable of leading in all possible experiments to identical predictions to those obtained from the usual interpretation”, that is to say, from those obtained from QM. Then, when considering an entangled extended system, as in Einstein-Podolsky-Rosen experiments (similar to the ones considered by Bell in his seminal papers), Bohm assumes that his realism is non local. In this way, his non local HVT can explain the observed violation of the ordinary Bell inequalities, in agreement with QM, without having to give up realism (see [9] for example).

But is it always this way? Is it true that we can devise actual non-local HVT that lead to the same predictions that QM, for all conceivable experiments? Let us make some considerations in order to show why, from the very beginning, it is not evident that HVT can always agree with QM and at the same time with non local realism.

First of all, in these theories, each time that one performs a measurement on the particle position, if one wants to complete, beyond the obtained outcome, the precedent particle
trajectory with a new trajectory piece, one must provide the probability density of the particle position just after this outcome. The provided probability becomes the new initial probability. Then, this initial probability must be taken the same as the one dictated by standard QM if we want the HVT to agree henceforth with QM. After this, in the HVT framework, one does not need to worry about how this initial probability evolves in time until one performs a subsequent measurement, since HVT are just designed to predict the same probability evolution as the predicted by Schrödinger equation. Hence, when some consecutive different measurements are performed on the same particle [13], or better on the different particles of an entangled system, one expects to find some well definite correlations among the corresponding outcomes: the correlations dictated by QM and observed in Bell type experiments.

More precisely: let it be a system of two entangled polarized photons. Assume, for mere sake of simplicity, that we measure both polarizations at a simultaneous time, i. e., both measurements are space-like events. Either in QM or in HVT, the probabilities of the two simultaneous measurements outcomes are given by the corresponding initial quantum entangled state, just the previous one to both measurements: in the notation of the precedent sections, states $u$ and $v$ conveniently entangled. In HVT, these initial quantum states are supplemented with the (uncritically) assumed initial values of some non-local hidden variables, $\lambda$ (actually, $\lambda$ plus direction $\vec{b}$, for example, in the notation of Sec. II), whose deterministic time evolution, in absence of measurement, preserves, as it must be, the quantum evolution of the outcome probabilities. This evolution of the $\lambda$ values can always be established and this is the great triumph of Bohm theory. Nevertheless, the point here is that this evolution of $\lambda$, which mimics so perfectly well the quantum evolution of the above probabilities, has nothing to do with the explanation of the quantum correlations, as for instance the ones which are behind the reported quantum violation of inequalities (7). It has nothing to do since these correlations have only to do with the initial $\lambda$ values, which are uncritically assumed to exist, plus the quantum state entangling $u$ and $v$, which actually exist. Then, is it sure from the very beginning that these correlations will always be compatible with some non local realism, that is, with the assumption that some initial values of non local hidden variables, $\lambda$, are behind all these outcome probabilities? No, we cannot be sure of this compatibility, unless we be able to prove it. But, as we have seen, HVT, though uncritically assuming it, do not actually prove it, while the result in the last Section saying
that QM cannot be completed can be seen as a counter example showing that this cannot be proved, since such non local hidden variables do not always exist. It is true nevertheless that, according to the end of Sec. III, one could still argue that a consistent Bohm theory could exist by allowing it to violate the arrow of time. But such a strange addition to a full realistic theory, like Bohm theory, would really be an unnatural addition.

Thus, it seems that there is no room left ”for models that force Nature to mimic the concept of trajectory” as it is still expected in [7].

To summarize: according to the above discussions, either Quantum Mechanics, or a realism that preserves the arrow of time, must be false. So, if on the ground of its general success we accept QM, plus the arrow of time, we must conclude that realism as such, i. e., local or nonlocal, should contradict experiments, an statement that would deserve being tested. Then the answer to the Leggett question [8] of ”it is indeed realism rather than locality which has to be sacrificed?” would be ‘yes’. All in all: against Einstein’s old dream, it seems that QM cannot be completed.

Acknowledgments

This work was partially supported by the project PAPIIT IN113607, DGAPA-UNAM, México (M. S) and by the Spanish Ministerio de Educación y Ciencia, MEC-FEDER project FIS2006-06062 (R. L.). M. S. thanks for hospitality at the Facultad de Astronomía y Astrofísica de la Universidad de Valencia, Spain, where part of this work was performed. R. L. recognizes fruitful discussions with Eugenio Roldan and Arcadi Santamaria.

[1] A. J. Leggett, Foundations of Physics 33, 1469 (2003).
[2] S. Gröblacher, T. Paterek, R. Kaltenbaek, C. Brukner, M. Zukowski, M. Aspelmayer, and A. Zeilinger, Nature 446, 871 (2007).
[3] T. Paterek, A. Fedrizzi, S. Gröblacher, T. Jennewein, M. Zukowski, M. Aspelmeyer, A. Zeilinger, Phys. Rev. Lett. 99, 210406 (2007).
[4] C. Branciard, A. Ling, N. Gisin, Ch. Kurtsiefer, A. Lamas-linares, V. Scarini, Phys. Rev. Lett. 99, 210407 (2007).
[5] C. Branciard, N. Brunner, N. Gisin, arXiv:0801.2241v1 [quant-ph].
[6] D. Bohm, Phys. Rev. 85, 166 (1952).

[7] A. Suarez, arXive:0801.3050v1 [quant-ph].

[8] A. J. Leggett, Rep. Prog. Phys. 71, 022001 (2008).

[9] J. S. Bell, Physics 1, 195 (1965).

[10] J. F. Clauser, M. A. Horne, A. Shimony, P. A. Hold, Phys. Rev. Lett. 23, 880 (1969).

[11] J. P. Jarrett, Noûs 18, 569 (1984)

[12] K. Hess, W. Phillip, Found. Phys. 35, 1749 (2005)

[13] R. Lapiedra, Europhys. Lett. 75, 202 (2006).