A two-stage robust optimization framework for land-water-environment nexus management

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Abstract: On the premise of rational utilization of soil and water resources, systematic engineering method is widely considered to balance agriculture and ecology. Regular planning is carried out by the authority based on historical data and future expectation, however, uncertain environment cannot be ignored. Hence, we propose an adjusted dynamic two-stage robust optimization framework to explore comprehensive managerial insights of irrigative areas and forest expansion. Dynamic planning is adopted to reflect the long-period planning, and two-stage is to differ “wait-and-see” decision from “right-and-now” decision. Considering the whole system’ benefit, economic and ecological benefits are maximized and water scarcity, water/soil erosion penalties are minimized. To solve the proposed two-stage robust model, we propose two structural properties to obtain the global solution. After transformation, a C&CG algorithm is designed to calculate the proposed model.

1. Introduction
Water/soil erosion and pollution are widespread environmental problems, which is harmful to sustainable development all over the world [1]. Land is limited and then the land use conflict between ecological recovery use and crop planting use are obvious severe in many developing countries. Agriculture production is regarded as one of the significant contributors to the land degradation and environmental pollution, while forest and grazing land protect the nature ecology [2]. UN (2015) reported the target that protecting and restoring water related ecosystems as many as possible by 2020, including mountains, forests, wetlands, rivers, aquifers and lakes [3]. Terrestrial ecosystems play a critical role in climate change mitigation [4]. Hence, balancing a trade-off among farmland use and forest land use is conducive to achieving sustainable development.

Restoring the reclaimed land to forest and less-disturbed ecosystem has been regarded as water and soil conservation project for a long time [5]. Some scholars found that it is beneficial to achieve ecological restoration in some ecologically fragile regions. However, some scholars considered that ecological restoration was gained at the cost of farmers’ economic incomes. They argued that this characterization cedes too much to large-scale plantation forestry, revegetation, reclamation and rehabilitation approaches, which include little ambition for securing the ability of an ecosystem to support and maintain ecological processes and a diversity of organisms [6]. Hence, identifying environmental thresholds at which they will require different levels of intervention [7]. To guide governor optimally allocate lands to grain, economic crops and ecological restoration (economic forest, shelter forest, and ecological park constructions) during a given planning horizon, this study plans to design an optimization model.

Many countries have paid more attention to the nexus of land, water and environment regulation [8]. Sustainable management of water resources have been emphasized, such as the U.S. Clean Water Act,
the Québec Water Policy and the European Water Framework Directive [9]. Generally, effective use and water conservation are tools to save water resources [10]. Further, O'Hara (1997) proposed the relation between soils quality and water quality when conducting agricultural management [11]. Baskaran, Colombo and Cullen (2013) studied the efficient resource allocation decision-making problem between irrigation and conservation development at basin level [12]. Li et al. (2018) extended the exploration of solving contradiction between environmental protection and economic development [13]. The above researches gave the inspiration of nexus analyses (such as water and soil; economy and environment), however, there is less article on land-water-environment system optimization. Hence, systematic thinking for any actions such as water allocation, land planning and ecological protection should be considered together.

Climate change and its potential impacts on water resources and water environment capacity have attracted global attentions [14]. However, projections of future climate change (e.g., temperature and precipitation) are plagued with uncertainties, causing difficulties for planners taking decisions on adaptation measures. Meanwhile, water resources and environment capacity are found to be sensitive to these uncertainties and difficult to be exactly predicted in changing climate conditions. Consequently, as the long-term outlook on climate change is uncertain, water management strategies in times of global change need to be developed within a complex and uncertain environment. Hence, researches focus on adaptive solutions to climate change. Xie et al. (2018) applied an interval-stochastic programming approach to identify the water resources management strategies in adaptation to climate change [15]. For the both researches, a precondition is needed, that is knowing the discrete intervals or probability distributions of uncertain variables. Unlike the stochastic programming and fuzzy programming theories, robust optimization doesn’t need the certain distribution or membership functions about the variables [16].

In this paper, we plan to propose a two-stage robust optimization model to provide decision makers with feasible and flexible solutions with the promise of satisfying environmental sustainability, and socio-economic development requirement. Unlike the traditional robust optimization model, two-stage robust programming theory is known as robust adjustable or adaptable optimization [17]. To be specific, the second-stage decision making process is optimized after the first-stage process are optimized and the uncertainty is revealed at the second stage. Two-stage robust optimization is applied to many areas, such as logistics network design problem [18], location-transportation problem [19], elective surgery and downstream capacity planning problem [20]. Here, we examine the effects of available water and environmental capacity simultaneously. We do this by changing the values of the budget of uncertainty, where the budget of uncertainty can be defined by decision makers, which relatively reflects the attitude of them towards future uncertainty.

Above all, we focus on the two questions in this study: 1) how should the governor manage the land-water-environment nexus system to reduce wastewater and water/soil erosion in the future? 2) how would be decision makers’ response to the uncertainty caused by climate change. Hence, we first use random variables to characterize the uncertainty in available water and environment capacity considering the future changing climate conditions. For the uncertain problem, we propose a two-stage robust model to optimize the land planning and water irrigation. The objective is to maximize the sum of system net benefit, including economic benefits, ecological benefits. Thus, the contributions of this study are given as follows:

1. This is the first two-stage robust optimization model that integrates economic and environmental influences of land use with varying functions simultaneously under uncertain environment.
2. A general C&CG algorithm is designed considering more than one uncertainty.
3. Corresponding trade-offs between farmland and forestland use are analyzed in this paper.

2. Methodology

2.1. Conceptual framework and data
In this study, three main irrigative plants ($i = 1, 2, 3$ denoted as grain plant, oil plant and vegetable) and
three types of forest \( j = 1, 2, 3 \) denoted as economic forest, shelter forest, and ecological park) are competitors in land and water demands in three planning period \( t = 1, 2, 3 \) denoted as periods 1, 2 and 3, and five years is one planning period, and three planning periods are considered).

2.2. Modelling

2.2.1. Notations

**Subscript**
- \( i \in I = \{1, 2, 3\} \) : the indicator set of main irrigative plants,
- \( j \in J = \{1, 2, 3\} \) : the indicator set of forest,
- \( t \in T = \{1, 2, 3\} \) : the indicator set of planning years,

**Decision variables**

**First-stage decision variables**
- \( x_t^i \) : planted land area for agricultural crop \( i \) at period \( t \), ha
- \( y_t^j \) : planted land area for forest recovery \( j \) at period \( t \), ha

**Second-stage decision variables**
- \( s_t^x_i \) : land area lacking of irrigation for agricultural crop \( i \) at period \( t \), ha
- \( s_t^y_j \) : land area lacking of irrigation for forest recovery \( j \) at period \( t \), ha

**Parameters**
- \( L_t \) : available land at period \( t \), ha
- \( bx_t^i \) : revenue benefit per thousand tons of water consumption for agricultural crops, USD/10^3 m^3
- \( px_t^i \) : penalty cost because of water shortage in farm land, USD/10^3 m^3
- \( by_t^j \) : revenue benefit per 10^3 t of water consumption for forest recovery-use, USD/10^3 m^3
- \( py_t^j \) : penalty cost because of water shortage in forest land, USD/10^3 m^3
- \( cx_t^i \) : pollution disposal cost, USD/ton
- \( ey_t^{N_j}, ey_t^{P_j} \) : the net ecological benefit of nutrients accumulation (TN and TP) from forest recovering, USD/10^3 ton

Figure 1 Geographical location of Xi xian, Henan
$e^{SN}_{j}, e^{WP}_{j}$: the net ecological benefit for soil conservation and water storage, USD/ 10^3 ton, USD/ 10^3 m^3

$\alpha, \beta$: water consumption per unit area for irrigative activity and forestry recovering in period $t$, m^3/ha

$\eta^N, \eta^P$: per volume of waste water for irrigative activity in period $t$, 10^3 ton/ha

$\theta^N, \theta^P$: Nutrients accumulation (TN and TP), ton/ha

$\theta^S, \theta^W$: soil conservation factor and water storage factor, ton/ha, 10^2 m^3/ha

Uncertain Parameters

$\bar{A}^t = A^t - \xi^{hi}_{t}, \bar{A}, \xi^{hi}_{t} \in [0,1]$: available water at period $t$, m^3

$\bar{TN}^t = TN^t - \xi^{Nh}_{t}, \bar{TN}, \xi^{Nh}_{t} \in [0,1], \bar{TP}^t = TP^t - \xi^{Ph}_{t}, \bar{TP}, \xi^{Ph}_{t} \in [0,1]$: total N and P emission limit, 10^3 ton/year

Auxiliary variables

$F$: objective function.

$Q$: second-stage objective function when using KKT condition and Big-M methods.

$R$: second-stage objective function when using C&CG algorithm.

2.2.2. Model 1: Deterministic model for the target problem

Deterministic model 1 is developed first to optimize the water-land-environment nexus problem without considering uncertain environment.

(1) Objective function

This study aims to maximize system’s net economic benefit by make a trade-off among land, water, and environment. In general, after the land are divided and pre-planned in terms of each user, if the pre-planned land is not irrigated, and then water deficit occurs, which directly leading to economic losses; hence losses from water deficit areas’ penalty are incorporated when calculating the net economic benefit. In this way, non-irrigation penalties ($\sum_{i \in T} \sum_{i \in I} px'_i \times \alpha \times sx'_i, \sum_{i \in T} \sum_{j \in J} by'_j \times \beta \times sy'_j$) are incorporated in $F_1$ and $F_2$ besides the economic benefits ($\sum_{i \in T} \sum_{i \in I} bx'_i \times \alpha \times x'_i, \sum_{i \in T} \sum_{j \in J} by'_j \times \beta \times y'_j$).

Further, $F_3$ presents the treatment costs for environmental pollution caused by agricultural fertilizer. $F_4$ and $F_5$ are ecological benefits because of ecological restoration.

$$\max F = F_1 + F_2 - F_3 + F_4 + F_5$$  \hspace{1cm} (1)

$$F_1 = \sum_{i \in T} \sum_{i \in I} bx'_i \times \alpha \times x'_i - \sum_{i \in T} \sum_{i \in I} px'_i \times \alpha \times sx'_i$$ \hspace{1cm} (2)

$$F_2 = \sum_{i \in T} \sum_{j \in J} by'_j \times \beta \times y'_j - \sum_{i \in T} \sum_{j \in J} by'_j \times \beta \times sy'_j$$ \hspace{1cm} (3)

$$F_3 = \sum_{i \in T} \sum_{i \in I} (x'_i - sx'_i) (\eta^N + \eta^P) cx'_i$$ \hspace{1cm} (4)

$$F_4 = \sum_{i \in T} \sum_{j \in J} \theta^N \times ey^N_{ji} (y'_j - sy'_j) + \sum_{i \in T} \sum_{j \in J} \theta^P \times ey^P_{ji} (y'_j - sy'_j)$$ \hspace{1cm} (5)

$$F_5 = \sum_{i \in T} \sum_{j \in J} \theta^S \times ey^S_{ji} (y'_j - sy'_j) + \sum_{i \in T} \sum_{j \in J} \theta^W \times ey^W_{ji} (y'_j - sy'_j)$$ \hspace{1cm} (6)

(2) Constraints

Constraints are developed from four aspects: land, water, environment and technic.

Land utilization constraint: maximum land utilization area would be smaller than available land.
\[
\sum x_i^t + \sum y_j^t \leq L^t, \forall t. (7)
\]

**Water supply constraint:** maximum supply capacity would be smaller than available water.

\[
\sum \alpha (x_i^t - sx_i^t) + \sum \beta (y_j^t - sy_j^t) \leq A^t, \forall t. (8)
\]

**Total nitrogen and phosphorus allowance constraint:** With high runoff and soil loss, excess total nitrogen (TN) and total phosphorus (TP) from crop irrigation are discharged into water body directly, leading to pollution. And maximum total nitrogen (TN) and total phosphorus (TP) poured into water should be smaller than the limits.

\[
\sum (x_i^t - sx_i^t) \eta^N - \sum (y_j^t - sy_j^t) \theta^N \leq TN^t, \forall t, (9)
\]

\[
\sum (x_i^t - sx_i^t) \eta^P - \sum (y_j^t - sy_j^t) \theta^P \leq TP^t, \forall t. (10)
\]

**Technical and non-negative constraints:**

\[
x_i^t \geq sx_i^t, \forall t, i, (11)
\]

\[
y_j^t \geq sy_j^t, \forall t, j, (12)
\]

\[
sx_i^t, sy_j^t \geq 0, \forall t, j, i, (13)
\]

\[
x_i^t, y_j^t \geq 0, \forall t, j, i. (14)
\]

2.2.3. Model 2: Two stage robust model for the target model

In deterministic **model 1**, all parameters are assumed to be known with certainty. However, in real-world practice, it is difficult to predict the water availability and environmental capacity considering the changing natural environment. In this study, \( A^t \), \( TN^t \) and \( TP^t \) are characterized as uncertain variables with polyhedral uncertainty sets. In this way, the process can be regarded as a two-stage process in which the land planning is initially decided in the first stage without uncovering the uncertainty. In the second stage, we aim to minimize the worst-case scenario for non-irrigation penalty after being exposed to an uncertain environment.

**Definition 2.1:**

1. Let \( \hat{A}^t = [A^t, A^t - \xi^A A^t], \xi^A \in [0, 1] \), \( A^t \) is the normal value for available water, and \( \hat{A}^t \) is the maximum deviation from the normal value. \( \Gamma^A \) is set as budge of uncertainty, and \( \sum \xi^A \leq \Gamma^A \).

2. Let \( TN^t = [TN^t, TN^t - \xi^TN TN^t], \xi^TN \in [0, 1] \), \( TN^t \) is the normal value for environmental capacity for total nitrogen, and \( \xi^TN \) is the maximum deviation from the normal value. \( \Gamma^N \) is set as budge of uncertainty, and \( \sum \xi^TN \leq \Gamma^N \).

3. Let \( TP^t = [TP^t, TP^t - \xi^TP TP^t], \xi^TP \in [0, 1] \), \( TP^t \) is the normal value for environmental capacity for total phosphorus, and \( \xi^TP \) is the maximum deviation from the normal value. \( \Gamma^P \) is set as budge of uncertainty, and \( \sum \xi^TP \leq \Gamma^P \).

Based on **Definition 2.1**, optimization **Model 2** for the target problem can be written as follows:
\[
\max_{x', y', \xi'} F = \sum_{i \in I} \sum_{j \in J} h_{ij} x'_{ij} + \sum_{i \in I} \sum_{j \in J} b_{ij} y'_{ij} - \sum_{i \in I} \sum_{j \in J} x'_{ij} \left( \eta^N + \eta^P \right) c_{ij} + \sum_{i \in I} \sum_{j \in J} \theta^N \times e_{ij} \times y'_{ij} + \sum_{i \in I} \sum_{j \in J} \theta^P \times e_{ij} \times y'_{ij} + \sum_{i \in I} \sum_{j \in J} \theta^N \times e_{ij} \times y'_{ij} + \text{opt} \left[ R \left( x'_{ij}, y'_{ij}, \xi^A, \xi^N, \xi^P \right) \right]
\]

Subject to (7),
\[
\sum_{i \in I} \alpha \times (x_{ij} - x'_{ij}) + \sum_{j \in J} \beta \times (y_{ij} - y'_{ij}) \leq A' - \xi^A \hat{A} + \xi^e \in [0, 1],
\]
\[
\sum_{i \in I} (x_{ij} - x'_{ij}) \times \eta^N - \sum_{j \in J} (y_{ij} - y'_{ij}) \times \theta^N \leq TN' - \xi^N \hat{T} N' + \xi^N \in [0, 1],
\]
\[
\sum_{i \in I} (x_{ij} - x'_{ij}) \times \eta^P - \sum_{j \in J} (y_{ij} - y'_{ij}) \times \theta^P \leq TP' - \xi^P \hat{T} P' + \xi^P \in [0, 1].
\]

(11)-(14),
\[
\sum_{i \in I} \xi^i \leq \Gamma^I, \sum_{i \in I} \xi^N \leq \Gamma^N, \sum_{i \in I} \xi^P \leq \Gamma^P.
\]

Note from Model 2 aims to solve the minimum economic cost under the expected condition.
\[
\text{opt} \left[ R \left( x'_{ij}, y'_{ij}, \xi^A, \xi^N, \xi^P \right) \right] \text{ is the optimal solution to the second stage problem (denoted as } R \left( x'_{ij}, y'_{ij}, \xi^A, \xi^N, \xi^P \right) \text{). Because we aim to optimize the problem under worsened conditions, the second stage objective function is shown in Eq. (20).}
\]
\[
R \left( x'_{ij}, y'_{ij}, \xi^A, \xi^N, \xi^P \right) = \min_{\xi^A, \xi^N, \xi^P} \max_{f(x', y')} \sum_{i \in I} \sum_{j \in J} \left( \eta^N + \eta^P \right) c_{ij} \times x_{ij} + \sum_{i \in I} \sum_{j \in J} \left( \theta^N \times e_{ij} \times y_{ij} + \theta^P \times e_{ij} \times y_{ij} + \theta^N \times e_{ij} \times y_{ij} \right) \times y_{ij}
\]

3. Structural properties

In this section, structural properties of the land-water-environment nexus problem are analyzed. To be specific, models with uncertainty have a bi-level property, in which the outer-minimization seeks the worst-case scenario for a given land planning. And the inner-maximization problem aims to maximize the economic benefits, of which \( sx_{ij}, sy_{ij} \) are decision variables. Two methods are given to transform the bi-level problem into a single-level problem, which can be solved directly. It is noted that fewer variables and constraints will be generated by a strong duality based inner algorithm compared to those generated by KKT condition based inner algorithm [21].

3.1. Strong duality

Definition 3.1. Given a bi-level min-max optimization model (21)-- \( R' \left( sx_{ij} \right) \). If the inner maximization is a linear program, strong duality can be used to substitute the inner-maximization problem with its dual formulation. Thus a transformed form (22) derived from strong duality approach is shown as:

\[
R' \left( sx_{ij} \right) = \min_{\xi^A, \xi^N, \xi^P} \max_{f(x', y')} \sum_{i \in I} \sum_{j \in J} c_{ij} \times x'_{ij} \left[ \sum_{i \in I} ax_{ij} \times sx_{ij} \leq b_i \right. \] 
\[
\left. sx_{ij} \geq 0 \right]
\]
\[ R'(s_x') = \min_{x \in \mathcal{X}} \sum_{i \in I} \sum_{t \in T} a_{i,t} x_{i,t} \]

s.t. \[
  \begin{align*}
    & b_{i,t} x_{i,t} \geq c_{i,t}, \forall i, t \\
    & u_{i,t} x_{i,t} \geq 0
  \end{align*}
\]

where \( u_{i,t} \) is the dual variable for constraint \( \sum_{i \in I} a_{i,t} x_{i,t} \leq b_{i,t} \).

Based on Definition 3.1, to solve the land-water-environment nexus problem, the min-max objective function (20) is transformed into a single minimization problem.

\[
\begin{align*}
\min_{u', v', y', \xi, \xi', \eta, \eta', \alpha, \beta} & \left[ \left( A' - \sum_{i \in I} \alpha \times x_{i,t}' + \sum_{j \in J} \beta \times y_{j,t}' \right) u' + \left( TN' - \sum_{i \in I} \eta_{i,t} x_{i,t}' + \sum_{j \in J} \theta_{j,t} \times y_{j,t}' \right) \right] \\
& + \left( TP' - \sum_{i \in I} \eta'_{i,t} x_{i,t}' + \sum_{j \in J} \theta'_{j,t} \times y_{j,t}' \right) y' + \Gamma' \xi u + \Gamma' \xi' v + \Gamma' \eta y + \sum_{i \in I} \sum_{j \in J} \left( \alpha_{i,j} x_{i,j} + \alpha'_{i,j} x_{i,j}' \right) \xi_{i,j} + \sum_{i \in I} \sum_{j \in J} \left( \beta_{i,j} y_{i,j} + \beta'_{i,j} y_{i,j}' \right) \xi_{i,j}' \\
\text{s.t.} & \quad \hat{A}' u' \geq 0, \forall t \\
& \quad \hat{T} N' v' \geq 0, \forall t \\
& \quad \hat{T} P' y' \geq 0, \forall t \\
& \quad u', v', y', \xi, \xi', \eta, \eta', \alpha, \beta \geq 0, \forall t, i, j
\end{align*}
\]

where \( u', v', y', \xi, \xi', \eta, \eta', \alpha, \beta \) are dual variables associated with constraints in Model 2.

Thus Model 3 is constructed, where an inner-minimization and an outer-maximization in objective function \( F \) is shown in model (23). And constraints are shown in inequation (24).

\[
\max_{x(x')} F = \frac{1}{2} \sum_{i \in I} \sum_{j \in J} \left( \sum_{p \in P} \sigma_{i,j,p} x_{i,j,p}' + \sum_{q \in Q} \eta_{i,j,q} x_{i,j,q}' \right) + \sum_{i \in I} \sum_{j \in J} \left( \sum_{p \in P} \sigma_{i,j,p} y_{i,j,p}' + \sum_{q \in Q} \eta_{i,j,q} y_{i,j,q}' \right)
\]

\[
\begin{align*}
& \min_{u', v', y', \xi, \xi', \eta, \eta', \alpha, \beta} \left[ \left( A' - \sum_{i \in I} \alpha \times x_{i,t}' + \sum_{j \in J} \beta \times y_{j,t}' \right) u' + \left( TN' - \sum_{i \in I} \eta_{i,t} x_{i,t}' + \sum_{j \in J} \theta_{j,t} \times y_{j,t}' \right) \right] \\
& + \left( TP' - \sum_{i \in I} \eta'_{i,t} x_{i,t}' + \sum_{j \in J} \theta'_{j,t} \times y_{j,t}' \right) y' + \Gamma' \xi u + \Gamma' \xi' v + \Gamma' \eta y + \sum_{i \in I} \sum_{j \in J} \left( \alpha_{i,j} x_{i,j} + \alpha'_{i,j} x_{i,j}' \right) \xi_{i,j} + \sum_{i \in I} \sum_{j \in J} \left( \beta_{i,j} y_{i,j} + \beta'_{i,j} y_{i,j}' \right) \xi_{i,j}' \\
& \text{s.t.} \quad \hat{A}' u' \geq 0, \forall t \\
& \quad \hat{T} N' v' \geq 0, \forall t \\
& \quad \hat{T} P' y' \geq 0, \forall t \\
& \quad u', v', y', \xi, \xi', \eta, \eta', \alpha, \beta \geq 0, \forall t, i, j
\end{align*}
\]

Model 3:
### 3.2. KKT condition

**Definition 3.2** [22]. If the inner maximization is a linear program, KKT can be used to substitute the inner-maximization problem on condition that all the constraints are linear. Thus a form (26) of general bi-level min-max optimization model (21) using KKT is shown as:

\[ R^n(s x^i_j) = \min_{\lambda \in \mathbb{U}^t} \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} a x^i_j \times u x^i_f \]

\[\begin{align*}
& cx^i_j - \lambda^i a x^i_j = 0, \forall i, t \\
& \lambda^i \left( \sum_{i \in I_{ct}} a x^i_j \times s x^i_j - b^i_j \right) = 0, \forall t \\
& \sum_{i \in I_{ct}} a x^i_j \times s x^i_j \leq b^i_j \\
& s x^i_j \geq 0
\end{align*}\]

(26)

Based on **Definition 3.2**, the min-max objective function (20) is transformed into a single minimization problem by using KKT.

\[
\begin{align*}
\max F = & \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} b x^i_j \times \alpha^i x^i_f + \sum_{i \in I_{ct}} \sum_{j \in J_{ct}} b y^i_j \times \beta^i y^i_f - \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} x_i^j \left( \eta^i + \eta^j \right) c x^i_j + \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} \theta^i \times e y^i_j \times y^i_f \\
& + \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} \theta^i \times e y^i_j \times y^i_f + \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} \theta^i \times e y^i_j \times y^i_f + \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} \theta^i \times e y^i_j \times y^i_f + \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} \left( \eta^i + \eta^j \right) c x^i_j - \sum_{i \in I_{ct}} \sum_{i \in I_{ct}} \left( \theta^i \times e y^i_j + \theta^i \times e y^i_j + \theta^i \times e y^i_j + \theta^i \times e y^i_j \right) \times s y^i_j \\
\text{s.t.} & (7), (14),
\end{align*}
\]

(27)
\[(\eta^N + \eta^p)cx_j - px_j + \alpha - \alpha \times \lambda_i^j - \alpha \times \eta^N \times \lambda_2^i - \alpha \times \eta^p \times \lambda_1^i - \lambda_{a_i} + \lambda_{m_i} = 0\]

\[\theta^N \times ey_j^N + \theta^p \times ey_j^P + by_j \times \beta + \theta^N \times ey_j^N + \beta \times \lambda_i^j - \beta \times \eta^N \times \lambda_2^i - \beta \times \eta^p \times \lambda_1^i - \lambda_{a_{ij}} + \lambda_{m_{ij}} = 0\]

\[0 - \lambda_i^j - \lambda_a = 0\]

\[0 - \lambda_i^j - \lambda_r = 0\]

\[0 - \lambda_i^j - \lambda_s = 0\]

\[\lambda_i^j \left( \sum_{k \in I} \alpha^*(x^i_j - sx^i_j) + \sum_{j \in J} \beta^*(y^j_i - sy^j_i) - A^i + \xi^{d^i} \hat{A}^i \right) = 0, \xi^i \in [0, 1]\]

\[\lambda_j^i \left( \sum_{i \in I} \alpha^*(x^i_j - sx^i_j) \eta^N - \sum_{j \in J} \beta^*(y^j_i - sy^j_i) \eta^N - TN^i + \xi^{N^i} \hat{TN}^i \right) = 0, \xi^{N^i} \in [0, 1]\]

\[\lambda_j^i \left( \sum_{i \in I} \alpha^*(x^i_j - sx^i_j) \eta^p - \sum_{j \in J} \beta^*(y^j_i - sy^j_i) \eta^p - TP^i + \xi^{P^i} \hat{TP}^i \right) = 0, \xi^{P^i} \in [0, 1]\]

\[\lambda_i^j (sx^i_j - x^i_j) = 0\]

\[\lambda_j^i (sy^i_j - y^j_i) = 0\]

\[\lambda_a \left( \sum_{i \in I} \xi^{d^i} - \Gamma^d \right) = 0\]

\[\lambda_r \left( \sum_{i \in I} \xi^{N^i} - \Gamma^N \right) = 0\]

\[\lambda_s \left( \sum_{i \in I} \xi^{P^i} - \Gamma^p \right) = 0\]

\[\lambda_{a_{ij}} (-sx^i_j) = 0\]

\[\lambda_{m_{ij}} (-sy^j_i) = 0\]
\[
\sum_{i \in I} \alpha \times (x_i' - sx_i') + \sum_{j \in J} \beta \times (y_j' - sy_j') - A' + \xi^{At} A' \leq 0
\]
\[
\sum_{i \in I} \alpha \times (x_i' - sx_i') \eta^N - \sum_{j \in J} \beta \times (y_j' - sy_j') \times \theta^N - TN^t + \xi^{Ni} TN^t \leq 0
\]
\[
\sum_{i \in I} \alpha \times (x_i' - sx_i') \eta^P - \sum_{j \in J} \beta \times (y_j' - sy_j') \times \theta^P - TP^p + \xi^{Pi} TP^p \leq 0
\]
\[
sx_i' - x_i' \leq 0
\]
\[
sy_j' - y_j' \leq 0
\]
\[
\sum_{i \in I} \xi^{At} - \Gamma^A \leq 0
\]
\[
\sum_{i \in I} \xi^{Ni} - \Gamma^N \leq 0
\]
\[
\sum_{i \in I} \xi^{Pi} - \Gamma^P \leq 0
\]
\[
-sx_i' - sy_j' \leq 0
\]
\[
\xi^{At} \in [0,1], \xi^{Ni} \in [0,1], \xi^{Pi} \in [0,1]
\]

It is obvious to find nonlinear constraints in (29), hence big M method is applied to linearize nonlinear constraints. In all, the transformed model is subject to (7), (14), (28), (30), (31).
4. Solution method: Column and constraint generation algorithm

It is said that C&CG algorithm could be a solution strategy to solve a model with multi-level programs [21]. C&CG algorithm is implemented as a master problem (MP) and sub-problem (SP) scheme similar to Benders’ decomposition method (Ding et al., 2015). **Model 2** is decomposed into MP and SP as follows.

4.1. MP: in the outer-level algorithm

Given that $A', TN', TP', \forall t$ is the mean value. And the outer-level model is as follows:

\[
\begin{align*}
\mu_i' - Mq_i' &\leq 0 \\
-\left(\sum_{i \in l} \alpha \times (x_i' - sx_i') + \sum_{j \in J} \beta \times (y_j' - sy_j') - A' + \xi^{\mu'} A'\right) - M \left(1 - q_i'\right) &\leq 0, \xi' \in [0, 1] \\
\mu_j' - Mq_j' &\leq 0 \\
-\left(\sum_{i \in l} \alpha \times (x_i' - sx_i') \eta^N - \sum_{j \in J} \beta \times (y_j' - sy_j') \times \theta^N - TN' + \xi^{\eta N} \hat{T}N'\right) - M \left(1 - q_j'\right) &\leq 0, \xi^N \in [0, 1] \\
\mu_k' - Mq_k' &\leq 0 \\
-\left(\sum_{i \in l} \alpha^* \times (x_i' - sx_i') \eta^P - \sum_{j \in J} \beta^* \times (y_j' - sy_j') \times \theta^P - TP' + \xi^{\eta P} \hat{T}P'\right) - M \left(1 - q_k'\right) &\leq 0, \xi^P \in [0, 1] \\
\lambda_{i_0}' - Mq_{i_0}' &\leq 0 \\
-\left(sx_j' - x_j'\right) - M \left(1 - q_{i_0}'\right) &\leq 0 \\
\lambda_{j_1}' - Mq_{j_1}' &\leq 0 \\
-\left(sy_j' - y_j'\right) - M \left(1 - q_{s_1}'\right) &\leq 0 \\
\lambda_{s_2}' - Mq_{s_2}' &\leq 0 \\
-\left(\sum_{i \in l} \xi^{\mu'} - \Gamma^{\mu}\right) - M \left(1 - q_{s_2}\right) &\leq 0 \\
\lambda_{s_3}' - Mq_{s_3}' &\leq 0 \\
-\left(\sum_{i \in l} \xi^{\eta N} - \Gamma^{N}\right) - M \left(1 - q_{s_3}\right) &\leq 0 \\
\lambda_{s_4}' - Mq_{s_4}' &\leq 0 \\
-\left(\sum_{i \in l} \xi^{\eta P} - \Gamma^{P}\right) - M \left(1 - q_{s_4}\right) &\leq 0 \\
\lambda_{s_5}' - Mq_{s_5}' &\leq 0 \\
-\left(-sx_j'\right) - M \left(1 - q_{s_5}'\right) &\leq 0 \\
\lambda_{s_6}' - Mq_{s_6}' &\leq 0 \\
-\left(-sy_j'\right) - M \left(1 - q_{s_6}'\right) &\leq 0 \\
\end{align*}
\]
max $F = \sum_{i, j, \sigma} h_i' \times x_i' + \sum_{i, j, \sigma} b_j' \times y_j' - \sum_{i, j, \sigma} c_{ij} \left( \eta^+ + \eta^- \right) x_i' + \sum_{i, j, \sigma} \theta^N \times e_{ij}^N \times y_j' + \sum_{i, j, \sigma} \theta^W \times e_{ij}^W \times y_j' + \sigma$

\[ \quad s.t. \quad \sigma \leq \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' + \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' \times \theta^N \times e_{ij}^N \times y_j' + \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' \times \theta^W \times e_{ij}^W \times y_j' + \sigma \]

\[ \quad \sum_{i, j} x_i' + \sum_{i, j} y_j' \leq L, \forall t \]

\[ \quad x_i', y_j' \geq 0, \forall i, j, t \]

\[ \quad \alpha \times x_i' - \alpha \times s_j' + \sum_{j, \sigma} \beta \times y_j' - \sum_{j, \sigma} \beta \times s_j' \leq A', \forall t \]

\[ \quad \alpha \times x_i' - \alpha \times s_j' + \sum_{j, \sigma} \beta \times y_j' - \sum_{j, \sigma} \beta \times s_j' \leq T N', \forall t \]

\[ \quad \alpha \times x_i' - \alpha \times s_j' + \sum_{j, \sigma} \beta \times y_j' - \sum_{j, \sigma} \beta \times s_j' \leq T P', \forall t \]

\[ \quad x_i' \geq \alpha \times x_i', \forall i, t \]

\[ \quad y_j' \geq \alpha \times y_j', \forall j, t \]

\[ \quad s x_j', s y_j' \geq 0, \forall i, j, t \]

\[ \quad (32) \]

4.2. SP: in the inner-level algorithm

For a given solution \((x_i', y_j')\), the inner-level model is as follows:

(a) Strong duality

\[ \left[ \left( A' - \alpha \times x_i' - \sum_{j, \sigma} \beta \times y_j' \right) u' + \left( T N' - \alpha \times y_j' - \sum_{j, \sigma} \beta \times x_i' \right) v' + \left( T P' - \alpha \times y_j' - \sum_{j, \sigma} \beta \times x_i' \right) w' \right] \]

s.t. \[ \sigma \leq \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' + \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' \times \theta^N \times e_{ij}^N \times y_j' + \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' \times \theta^W \times e_{ij}^W \times y_j' + \sigma \]

(7), (14), (24).

(b) KKT and big M method

s.t. \[ \sigma \leq \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' + \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' \times \theta^N \times e_{ij}^N \times y_j' + \sum_{i, j, \sigma} \left( \left( \eta^+ + \eta^- \right) c_{ij} \times x_i' - px_i' \times \alpha \right) \times s_j' \times \theta^W \times e_{ij}^W \times y_j' + \sigma \]

(7), (14), (28), (30), (31).

4.3. A general C&C Algorithm

Step 1. Set $LB = -\infty$ and $UB = \infty$, and iteration begins with $k = 0$ and $O = \emptyset$.

Step 2. Solve MP, derive an optimal solution \( \left( x_{i, l}, y_{j, l}, s x_{i, l}, s y_{j, l} \right), \forall l \leq k \), and update $UB = \sum_{i, l} \left( h_i' \times x_{i, l} + \sum_{j, l} b_j' \times y_{j, l} \right) - \sum_{i, l} \sum_{j, l} \left( \eta^+ + \eta^- \right) x_{i, l} + \sum_{i, l} \sum_{j, l} \theta^N \times e_{ij}^N \times y_{j, l} + \sigma_{k+1}^*$

Step 3. input the \( \left( x_{i, l}, y_{j, l} \right), \forall l \leq k \), and solve the SP, and update $LB = \max \left\{ LB, R^*_{s, i} \left( \ldots \right) \right\}$.

Step 4. if $UB - LB \leq \varepsilon$, return the optimal values of decision variables and terminate. Otherwise do.
Step 4.1. if $\sigma^*_{i=1} \left( u^i, v^i, y^i, u^j, u^j_y \right) > -\infty$, create variables $u^i, v^i, y^i, u^j, u^j_y$, and add the following either constraints to MP based on the chosen method transforming the inner-level algorithm:

$$
\sigma \leq \sum_{i=1}^{\beta} \left( \left( A' - \sum_{j \in J} \alpha x^j - \sum_{j \in J} \beta y^j \right) u^i + \left( T P' - \sum_{i=1}^{\eta} \alpha \eta x^j + \sum_{j \in J} \beta \theta y^j \right) y^i \right) v^i + \left( \Gamma^0 \xi u^i + \Gamma^0 \xi y^i + \sum_{i=1}^{\mu} x^i x^i + \sum_{j \in J} y^j y^j \right)
$$

(7), (14), (24) (dual),

$$
\sigma \leq \sum_{i=1}^{\beta} \left( \left( \eta x^j - px^j \alpha \right) x^j + \sum_{j \in J} \left( \theta x^j + \theta y^j + by^j + \beta \theta y^j + \theta y^j \right) x^j y^j \right) y^i + \left( \Gamma^0 \xi u^i + \Gamma^0 \xi y^i + \sum_{i=1}^{\mu} x^i x^i + \sum_{j \in J} y^j y^j \right)
$$

(7), (14), (28), (30), (31) (KKT),

update $k = k + 1$, $O = O \cup \{k + 1\}$ and go to step 2.

Step 4.2. if $\sigma^*_{i=1} \left( u^i, v^i, y^i, u^j, u^j_y \right) = -\infty$, namely the sub-problem is non-feasibility. And create variables $u^i, v^i, y^i, u^j, u^j_y$, and add the following constraints to MP:

$$
\left( 14, (24) \right) \text{(dual), }
$$

$$
\left( 7, (14), (28), (30), (31) \right) \text{(KKT),}
$$

and go to step 2.

5. Conclusion

In line with the necessity of solving uncertain environment and dynamic problem, this paper proposes a dynamic two-stage robust optimization model for planning the land-water-environment nexus problem, where the crop irrigation and forest expansion are considered in a coupled framework. Considering the structural properties of the two-stage robust model under uncertain environment, strong duality and KKT condition are comprehensively analyzed. In all, the contributions of this study include (1) it is the first time to propose a two-stage robust optimization framework to solve the land use problem under uncertain environment. (2) considering the dynamic property and bi-level property, we transfer the proposed two-stage robust model into a master problem and sub-problem. Later, a general C&CG algorithm is designed considering these uncertainties. Finally, managerial insights on trade-offs between farmland and forestland use can be analyzed in the future.

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