Tetraquark state candidates: $Y(4140)$, $Y(4274)$ and $X(4350)$

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Abstract

In this article, we tentatively assign the $Y(4140)$, $Y(4274)$ and $X(4350)$ to be the scalar and tensor $c\bar{s}c\bar{s}$ tetraquark states, respectively, and study them with the QCD sum rules. In the operator product expansion, we take into account the vacuum condensates up to dimension-10. In calculations, we use the formula $\mu = \sqrt{M^2_{X/Y/Z} - (2M_c)^2}$ to determine the energy scales of the QCD spectral densities. The numerical results favor assigning the $Y(4140)$ to be the $J^{PC} = 2^{++}$ diquark-antidiquark type tetraquark state, and disfavor assigning the $Y(4274)$ and $X(4350)$ to be the $0^{++}$ or $2^{++}$ tetraquark states.

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1 Introduction

In 2009, the CDF collaboration observed a narrow structure ($Y(4140)$) near the $J/\psi \phi$ threshold with statistical significance in excess of $3.8 \sigma$ in exclusive $B^+ \to J/\psi \phi K^+$ decays produced in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV [1]. The measured mass and width are $(4143.0 \pm 2.9 \pm 1.2)$ MeV and $(11.7^{+8.3}_{-5.0} \pm 3.7)$ MeV, respectively [1]. There have been several assignments, such as the molecular state [2] [3] [4] [5] [6] [7] [8] [9], charmonium hybrid [10], rescattering effect [11], tetraquark state [12], etc.

Later, the Belle collaboration measured the process $\gamma \gamma \to \phi J/\psi$ for the $\phi J/\psi$ invariant mass distributions between the threshold and 5 GeV, and observed no signal for the decay $Y(4140) \to \phi J/\psi$, however, they observed a narrow peak ($X(4350)$) of $8.8^{+1.4}_{-1.2}$ events with an significance of $3.2 \sigma$ [13]. The measured mass and width are $(4350.6^{+4.5}_{-4.1} \pm 0.7)$ MeV and $(13.3^{+17.9}_{-9.1} \pm 4.1)$ MeV, respectively [13]. There also have been several assignments, such as the molecular state [14] [15] [16] [17], conventional charmonium [18] [19], charmonium-molecule mixing state [20], etc.

In 2011, the CMS collaboration confirmed the $Y(4140)$ in the $B^\pm \to J/\psi \phi K^\pm$ decays with a statistical significance greater than $5\sigma$, the measured mass and width are $(4143.4^{+2.9}_{-3.0} \pm 0.6)$ MeV and $(15.3^{+10.4}_{-6.1} \pm 2.5)$ MeV, respectively [21]. Furthermore, the CDF collaboration observed an evidence for a second structure ($Y(4274)$) with approximate significance of $3.1 \sigma$. The measured mass and width are $(4274.4^{+8.2}_{-6.4} \pm 1.9)$ MeV and $(32.3^{+21.9}_{-15.3} \pm 7.6)$ MeV, respectively [21]. The $Y(4274)$ maybe (or maybe not) a molecular state [22] [23] [24] or a $0^{-+}$ tetraquark state [25].

In 2013, the CMS collaboration confirmed the $Y(4140)$ in the $J/\psi \phi$ mass spectrum in the $B^\pm \to J/\psi \phi K^\pm$ decays produced in $p\bar{p}$ collisions at $\sqrt{s} = 7$ TeV collected with the CMS detector at the Large Hadron Collider, and fitted the structure to a $S$-wave relativistic Breit-Wigner line-shape with the statistical significance exceeding $5\sigma$ [26]. Also in 2013, the D0 collaboration confirmed the $Y(4140)$ in the $B^+ \to J/\psi \phi K^+$ decays in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV collected by the D0 experiment at the Fermilab Tevatron collider with the statistical significance of $3.1 \sigma$ [27]. The $X(4350)$ and $Y(4274)$ have not been confirmed yet. For detailed discussions on this subject, one can consult Ref. [25].

The $S$-wave $J/\psi \phi$ systems have the quantum numbers $J^{PC} = 0^{++}, 1^{++}, 2^{++}$, while the $P$-wave $J/\psi \phi$ systems have the quantum numbers $0^{--}, 1^{--}, 2^{--}, 3^{--}$. The $X(4350)$ is observed in the $\gamma \gamma$ fusion, the $J^{PC} = 1^{++}, 1^{--}, 3^{--}$ assignments are excluded due to Yang’s Theorem [28]. The possible assignments are $J^{PC} = 0^{++}, 0^{--}, 2^{++}, 2^{--}$. In the scenario of tetraquark states, the

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masses of the $0^{-+}$ and $2^{-+}$ states are much larger than that of the $0^{++}$ and $2^{++}$ states [29]. The $Y(4140)$, $X(4350)$ and $Y(4274)$ are observed in the $J/\psi \phi$ invariant mass distribution, if they are tetraquark states, their quark constituents must be $cs\bar{c}s$. So in this article, we study the masses of the $0^{++}$ and $2^{++}$ $cs\bar{c}s$ tetraquark states with the QCD sum rules, and try to identify the $Y(4140)$, $X(4350)$ and $Y(4274)$.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the scalar and tensor tetraquark states in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

2 QCD sum rules for the scalar and tensor tetraquark states

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ and $\Pi(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu\alpha\beta}(p) = i \int d^4x e^{ip\cdot x}\langle 0| \{ J_{\mu\nu}(x) J_{\alpha\beta}^\dagger(0) \} |0\rangle, \quad (1)$$

$$\Pi(p) = i \int d^4x e^{ip\cdot x}\langle 0| \{ J(x) J^\dagger(0) \} |0\rangle, \quad (2)$$

where

$$J_{\mu\nu}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}} \{ s^i(x) C\gamma^k(x) s^m(x) \gamma_\mu \bar{c}c^\alpha(x) + s^i(x) C\gamma^k(x) s^m(x) \gamma_\nu \bar{c}c^\alpha(x) \}, \quad (3)$$

$$J(x) = \epsilon^{ijk}\epsilon^{imn}s^j(x) C\gamma^k(x) s^m(x) \gamma^\lambda \bar{c}c^\alpha(x), \quad (4)$$

the $i, j, k, m, n$ are color indexes, the $C$ is the charge conjugation matrix. The currents $J_{\mu\nu}(x)$ and $J(x)$ have positive parity and charge conjugation. We take the currents $J(x)$ and $J_{\mu\nu}(x)$ to interpolate the scalar and tensor tetraquark states, respectively.

At the hadronic side, we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{\mu\nu}(x)$ and $J(x)$ into the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ and $\Pi(p)$ to obtain the hadronic representation [30, 31]. After isolating the ground state contributions of the scalar and tensor tetraquark states (denoted by $X, Y$ and $Z$), we get the following results,

$$\Pi_{\mu\nu\alpha\beta}(p) = \frac{\lambda^2_{X/Y/Z}}{M^2_{X/Y/Z} - p^2} \left( \frac{g_{\mu\alpha}\bar{g}_{\nu\beta} + g_{\mu\beta}\bar{g}_{\nu\alpha}}{2} - \frac{\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta}}{3} \right) + \cdots , \quad (5)$$

$$\Pi(p) = \frac{\lambda^2_{X/Y/Z}}{M^2_{X/Y/Z} - p^2} + \cdots , \quad (6)$$

where $g_{\mu\nu} = g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{m^2}$, the pole residues $\lambda_{X/Y/Z}$ are defined by

$$\langle 0| J_{\mu\nu}(0) |X/Y/Z(p)\rangle = \lambda_{X/Y/Z} \varepsilon_{\mu\nu}, \quad (7)$$

the summation of the polarization vector $\varepsilon_{\mu\nu}$ results in the following formula,

$$\sum_{\lambda} \varepsilon^*_{\alpha\beta}(\lambda, p) \varepsilon_{\mu\nu}(\lambda, p) = \frac{g_{\alpha\mu}\bar{g}_{\beta\nu} + g_{\alpha\nu}\bar{g}_{\beta\mu}}{2} - \frac{\bar{g}_{\alpha\beta}\bar{g}_{\mu\nu}}{3}. \quad (8)$$

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ and $\Pi(p)$ in perturbative QCD. We contract the $s$ and $c$ quark fields in the correlation
functions $\Pi_{\mu\nu\alpha\beta}(p)$ and $\Pi(p)$ with Wick theorem, and obtain the results:

$$
\Pi_{\mu\nu\alpha\beta}(p) = \frac{i\epsilon^{ijk}\epsilon^{imn}\epsilon^{j'k'}\epsilon^{m'n'}}{2} \int d^4x e^{ip\cdot x} \left\{ \text{Tr} \left[ \gamma_\mu C^{k'k}(x)\gamma_\alpha CS^{ij'T}(x)C \right] \text{Tr} \left[ \gamma_\beta C^{m'n}(x)\gamma_\nu CS^{m'm'T}(x)C \right] 
+ \text{Tr} \left[ \gamma_\mu C^{k'k}(x)\gamma_\beta CS^{ij'T}(x)C \right] \text{Tr} \left[ \gamma_\alpha C^{m'n}(x)\gamma_\nu CS^{m'm'T}(x)C \right] 
+ \text{Tr} \left[ \gamma_\mu C^{k'k}(x)\gamma_\alpha CS^{ij'T}(x)C \right] \text{Tr} \left[ \gamma_\beta C^{m'n}(x)\gamma_\nu CS^{m'm'T}(x)C \right] 
+ \text{Tr} \left[ \gamma_\nu C^{k'k}(x)\gamma_\alpha CS^{ij'T}(x)C \right] \text{Tr} \left[ \gamma_\beta C^{m'n}(x)\gamma_\mu CS^{m'm'T}(x)C \right] \right\},
$$

$$
\Pi(p) = \frac{i\epsilon^{ijk}\epsilon^{imn}\epsilon^{j'k'}\epsilon^{m'n'}}{2} \int d^4x e^{ip\cdot x} \left\{ \text{Tr} \left[ \gamma_\mu C^{k'k}(x)\gamma_\alpha CS^{ij'T}(x)C \right] \text{Tr} \left[ \gamma_\alpha C^{m'n}(x)\gamma_\nu CS^{m'm'T}(x)C \right] \right\},
$$

where the $S_{ij}(x)$ and $C_{ij}(x)$ are the full $s$ and $c$ quark propagators respectively,

$$
S_{ij}(x) = \frac{i\delta_{ij}}{2\pi^2 x^4} - \delta_{ij} m_s \frac{12}{4\pi^2 x^2} - \frac{i\delta_{ij}}{32\pi^2 x^2} \left( \frac{1152}{192} \right) + \frac{i\delta_{ij} x^2 g_s x (s\bar{s})}{7776} - \frac{1}{4} \left( \bar{s}_i \gamma^\mu s_i \right) \gamma_\mu + \cdots,
$$

$$
C_{ij}(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik\cdot x} \left\{ \frac{\delta_{ij}}{k - m_c} - \frac{g_s G_{\alpha\beta} G_{ij}^{\alpha\beta} (k + m_c)}{4 (k^2 - m_c^2)^2} + \frac{g_s D_{\alpha\beta} G_{ij}^{\alpha\beta} (f^{\lambda\mu\nu} + f^{\lambda\nu\mu})}{4 (k^2 - m_c^2)^5} + \cdots \right\},
$$

$$
\lambda^X_{X/Y/Z} = \frac{M^2_{X/Y/Z}}{T^2} \exp \left( -\frac{M^2_{X/Y/Z}}{T^2} \right) = \int_{4m_c^2}^{s_0} ds \rho(s) \exp \left( -\frac{s}{T^2} \right),
$$

where

$$
\rho(s) = \rho_0(s) + \rho_3(s) + \rho_4(s) + \rho_5(s) + \rho_6(s) + \rho_7(s) + \rho_8(s) + \rho_{10}(s),
$$

the explicit expressions of the $\rho_i(s)$ are given in the appendix.

We differentiate Eq.(12) with respect to $\frac{1}{T^2}$, then eliminate the pole residues $\lambda^X_{X/Y/Z}$, and obtain the QCD sum rules for the masses of the scalar and tensor tetraquark states,

$$
M^2_{X/Y/Z} = \frac{\int_{4m_c^2}^{s_0} ds \frac{d}{ds} \rho(s) \exp \left( -\frac{s}{T^2} \right)}{\int_{4m_c^2}^{s_0} ds \rho(s) \exp \left( -\frac{s}{T^2} \right)}.
$$
3 Numerical results and discussions

The vacuum condensates are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1)(\bar{q}q)$, $\langle \bar{q}g \sigma G \rangle = m_0^2 \langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \alpha_s G \rangle = (0.33 \text{ GeV})^4$ at the energy scale $\mu = 1 \text{ GeV}$ \cite{30 31 32 33}. The quark condensates and mixed quark condensates evolve with the renormalization group equation, $\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(Q) \left[ \frac{\alpha_s(\mu)}{\alpha_s(Q)} \right]^{\frac{1}{n_f}}$, $\langle \bar{s}s \rangle(\mu) = \langle \bar{s}s \rangle(Q) \left[ \frac{\alpha_s(\mu)}{\alpha_s(Q)} \right]^{\frac{1}{n_f}}$.

$$\langle \bar{s}g \sigma G \rangle(\mu) = \langle \bar{s}g \sigma G \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(Q)} \right]^{\frac{1}{n_f}},$$

we take into account the energy scale dependence.

In the article, we take the $MS$ masses $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ and $m_s(\mu = 2 \text{ GeV}) = (0.905 \pm 0.005) \text{ GeV}$ from the Particle Data Group \cite{34}, and take into account the energy-scale dependence of the $MS$ masses from the renormalization group equation, $m_s(\mu) = m_s(2 \text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{\frac{1}{n_f}}$, $m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{3}{2}}$, $\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2} + \frac{b_2}{b_0^2 t^2} \right], \quad (15)$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33 - 2n_f}{12\pi}$, $b_1 = \frac{153 - 19 n_f}{24\pi}$, $b_2 = \frac{2857 - 563 n_f + 24\pi n_f^2}{128\pi}$, $\Lambda = 213 \text{ MeV}$, 296 MeV and 339 MeV for the flavors $n_f = 5, 4$ and 3, respectively \cite{34}.

In Refs.\cite{39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59}, we study the acceptable energy scales of the QCD spectral densities for the hidden charmed (bottom) tetraquark states and molecular states in the QCD sum rules in details for the first time, and suggest a formula,

$$\mu = \sqrt{M_{X/Y/Z}^2 - (2M_Q)^2}, \quad (16)$$

with the effective $Q$-quark masses $M_Q$ to determine the energy scales of the QCD spectral densities. In Refs.\cite{55 56 57 58 59}, we focus on the scenario of tetraquark states, study the dipquark-antidipquark type scalar, vector, axial-vector, tensor hidden charmed tetraquark states and axial-vector hidden bottom tetraquark states systematically with the QCD sum rules, and try to make possible assignments of the $X(3872)$, $Z_c(3900)$, $Z_c(3885)$, $Z_c(4020)$, $Z_c(4025)$, $Z(4050)$, $Z(4250)$, $Y(4430)$, $Z(4430)$, $Y(4630)$, $Y(4660)$, $Z_{bs}(10610)$ and $Z_{bs}(10650)$. In the operator product expansion, we calculate the vacuum condensates up to dimension-10, just like in the present case; the energy scale formula works very well.

In the conventional QCD sum rules \cite{30 31}, we usually take the energy gap between the ground states and the first radial excited states to be $(0.4 - 0.6) \text{ GeV}$. Such relation survives in the tetraquark sector, for example, the $Z(4430)$ is tentatively assigned to be the first radial excitation of the $Z_c(3900)$ according to the analogous decays, $Z_c(3900) \rightarrow J/\psi \pi^\pm$; $Z(4430) \rightarrow \psi' \pi^\pm$, and the mass differences $M_{Z(4430)} - M_{Z_c(3900)} = 576 \text{ MeV}$, $M_{\psi'} - M_{J/\psi} = 589 \text{ MeV}$ \cite{38 41 42}.

Firstly, we take the $Y(4140)$, $Y(4274)$ and $X(4350)$ as the scalar and tensor $cs\bar{s}$ tetraquark states, respectively, and choose the continuum threshold parameters as $s_{Y(4140)}^0 = (4.70 \text{ GeV})^2$, $s_{Y(4274)}^0 = (4.80 \text{ GeV})^2$ and $s_{X(4350)}^0 = (4.85 \text{ GeV})^2$. In Fig.1, the masses of the scalar and tensor tetraquark states are plotted with variations of the Borel parameters $T^2$ and energy scales $\mu$. From the figure, we can see that the masses decrease monotonously with increase of the energy scales, and we can also obtain the allowed energy scales to reproduce the experimental values of the masses.

In Table 1, we denote the allowed energy scales which can reproduce the experimental values of the masses as $\mu_A$, and denote the resulting energy scales from the energy scale formula as $\mu_T$. From the table, we can see that the $\mu_A$ and $\mu_T$ are compatible only in the case of the $Y(4140)$ with the assignment $J^{PC} = 2^{++}$. 


Table 1: The continuum threshold parameters $s_0$, allowed energy scales $\mu_A$, theoretical energy scales $\mu_T$, and the energy scale formula of the QCD spectral densities, pole contributions, masses and pole residues of the scalar and tensor tetraquark states.

| $J^{PC}$ | $T^2(\text{GeV}^2)$ | $\sqrt{s_0}(\text{GeV})$ | $\mu(\text{GeV})$ | pole | $M_{X/Y/Z}(\text{GeV})$ | $\lambda_{X/Y/Z}$ |
|----------|-------------------|------------------|------------------|------|------------------|------------------|
| $2^{++}$ | $3.0 - 3.4$       | $4.7 \pm 0.1$    | $2.0$            | $(49 - 69)\%$ | $4.13^{+0.08}_{-0.08}$ | $5.34^{+0.08}_{-0.08} \times 10^{-2}\text{GeV}^2$ |
| $0^{++}$ | $2.5 - 2.9$       | $4.5 \pm 0.1$    | $1.7$            | $(46 - 70)\%$ | $3.98^{+0.08}_{-0.08}$ | $4.87^{+0.08}_{-0.08} \times 10^{-2}\text{GeV}^2$ |

Table 2: The Borel parameters, continuum threshold parameters, energy scales of the QCD spectral densities, pole contributions, masses and pole residues of the scalar and tensor tetraquark states.

Now, we assume the $Y(4140)$ to be the tensor tetraquark state, take the continuum threshold parameter as $s_0^0 = (4.7 \pm 0.1)^2 \text{GeV}^2$ and the energy scale as $\mu = 2.0 \text{GeV}$ to search for the Borel parameter $T^2$ to satisfy the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. Furthermore, we study the scalar tetraquark state in the same way, i.e. we search for the optimal Borel parameter $T^2$ and threshold parameter $s_0$ to satisfy the two criteria of the QCD sum rules and the energy scale formula of the QCD spectral densities. The resulting Borel parameters, continuum threshold parameters and the pole contributions are shown explicitly in Table 2.

In Fig.2, we plot the contributions of different terms in the operator product expansion with variations of the Borel parameters $T^2$ for the threshold parameters $s_0^0 = (4.7 \text{GeV})^2$ and $s_0^0 = (4.5 \text{GeV})^2$, respectively. In the Borel windows, the $D_0$, $D_3$ and $D_5$ play an important role, the $D_6$ and $D_8$ play a minor important role, while the $D_4$, $D_7$ and $D_{10}$ are tiny, where the $D_i$ denote the contributions of the vacuum condensates of dimensions $D = i$. The operator product expansion is well convergent. It is obvious that the two criteria of the QCD sum rules are fully satisfied, so we expect to make reasonable predictions.

We take into account all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the scalar and tensor tetraquark states, which are shown explicitly in Figs.3-4 and Table 2. The prediction $M_{J=2} = (4.13^{+0.08}_{-0.08}) \text{GeV}$ is consistent with the experimental value $M_{Y(4140)} = (4143.0 \pm 2.9 \pm 1.2) \text{MeV}$ [34]. The present predictions favor assigning the $Y(4140)$ to be the $J^{PC} = 2^{++}$ diquark-antidiquark type tetraquark states, and disfavor assigning the $Y(4274)$ and $X(4350)$ to be the $J^{PC} = 0^{++}$ or $2^{++}$ diquark-antidiquark type tetraquark states. At the present time, there is no experimental candidate for the scalar $cs\bar{s}$ tetraquark state, we can search for the scalar tetraquark state at the BESIII, LHCb and Belle-II in the futures.

Recently, Mo et al study the $X(4350)$ as a $cs\bar{s}$ tetraquark state with the assignment $J^{PC} = 1^{--}$ using the QCD sum rules, and obtain the mass $M_{J=1} = (4.82 \pm 0.19) \text{GeV}$, which is not compatible with the $X(4350)$ as a $1^{--}$ tetraquark state [43]. So the $X(4350)$ is unlikely to be a $cs\bar{s}$ tetraquark state. Furthermore, the $X(4350)$ and $Y(4274)$ are still need confirmation.
Figure 1: The masses of the $Y(4140)$, $Y(4274)$ and $X(4350)$ with the assignments $J^{PC} = 0^{++}$ and $2^{++}$ respectively vary with the Borel parameters $T^2$ and the energy scales $\mu$, where the horizontal lines denote the experimental values of the masses of the $Y(4140)$, $Y(4274)$ and $X(4350)$, respectively.
Figure 2: The contributions of different terms in the operator product expansion for the $J^{PC} = 0^{++}$ and $2^{++}$ tetraquark states with variations of the Borel parameters $T^2$, where the 0, 3, 4, 5, 6, 7, 8, 10 denote the dimensions of the vacuum condensates.

Figure 3: The masses of the $J^{PC} = 0^{++}$ and $2^{++}$ tetraquark states with variations of the Borel parameters $T^2$, where the horizontal lines denote the experimental value of the mass of the $Y(4140)$. 
4 Conclusion

In this article, we tentatively assign the $Y(4140)$, $Y(4274)$ and $X(4350)$ to be the scalar and tensor $cs\bar{c}\bar{s}$ tetraquark states, respectively, and study them with the QCD sum rules. In the operator product expansion, we calculate the contributions of the vacuum condensates up to dimension-10. Furthermore, we use the formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$ to determine the energy scales of the QCD spectral densities. The numerical results of the masses $M_{X/Y/Z}$ favor assigning the $Y(4140)$ to be the $J^{PC} = 2^{++}$ $cs\bar{c}\bar{s}$ tetraquark state, and disfavor assigning the $Y(4274)$ and $X(4350)$ to be the $0^{++}$ or $2^{++}$ tetraquark states. There is no candidate for the scalar $cs\bar{c}\bar{s}$ tetraquark state, we can search for it at the BESIII, LHCb and Belle-II in the futures.

Appendix

The spectral densities $\rho_i(s)$ with $i = 0, 3, 4, 5, 6, 7, 8, 10$ at the level of the quark-gluon degrees of freedom,

$$\rho_0(s) = \frac{1}{15360\pi^6} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \frac{yz (1-y-z)^3 (s-m_c^2)^2 (293 s^2 - 190 s m_c^2 + 17 m_c^4)}{s-m_c^2}$$

$$+ \frac{1}{5120\pi^6} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \frac{(1-y-z)^2 (s-m_c^2)^4}{s-m_c^2}$$

$$+ \frac{m_s m_c}{128 \pi^6} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \frac{(y+z) (1-y-z)^2 (s-m_c^2)^2 (4s-m_c^2)}{s-m_c^2},$$

(17)
\begin{align}
\rho_3^2(s) &= -\frac{m_c\langle ss\rangle}{16\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(y + z\right) \left(1 - y - z\right) \left(s - m_c^2\right) \left(3s - m_c^2\right) \\
&\quad + \frac{m_s\langle ss\rangle}{160\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(yz\right) \left(1 - y - z\right) \left(115s^3 - 112sm_c^2 + 17m_c^4\right) \\
&\quad + \frac{m_s\langle ss\rangle}{160\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(yz\right) \left(s - m_c^2\right)^2 \\
&\quad - \frac{m_s m_c^2 \langle ss\rangle}{4\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(s - m_c^2\right), \\
\rho_4^2(s) &= -\frac{m_c^2\alpha_{GG}}{11520\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(1 - y - z\right)^3 \left(56s - 17m_c^2 + 10m_c^4\delta\left(s - m_c^2\right)\right) \\
&\quad - \frac{m_c^2}{3840\pi^4} \frac{\alpha_{GG}}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(\frac{z}{y^2} + \frac{y}{z^2}\right) \left(1 - y - z\right)^2 \left(s - m_c^2\right) \\
&\quad - \frac{1}{15360\pi^4} \frac{\alpha_{GG}}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(y + z\right) \left(1 - y - z\right)^2 \left(185s^2 - 208sm_c^2 + 43m_c^4\right) \\
&\quad + \frac{1}{7680\pi^4} \frac{\alpha_{GG}}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(y + z\right) \left(1 - y - z\right) \left(s - m_c^2\right)^2 \\
&\quad - \frac{1}{2304\pi^4} \frac{\alpha_{GG}}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(y + z\right) \left(1 - y - z\right)^2 \left(15s^2 - 16sm_c^2 + 3m_c^4\right) \\
&\quad - \frac{1}{13824\pi^4} \frac{\alpha_{GG}}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(1 - y - z\right)^3 \left(25s^2 - 24sm_c^2 + 3m_c^4\right) \\
&\quad - \frac{1}{6912\pi^4} \frac{\alpha_{GG}}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(yz\right) \left(1 - y - z\right) \left(25s^2 - 24sm_c^2 + 3m_c^4\right) \\
&\quad - \frac{1}{4608\pi^4} \frac{\alpha_{GG}}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(1 - y - z\right)^2 \left(s - m_c^2\right)^2 \\
&\quad - \frac{1}{6912\pi^4} \frac{\alpha_{GG}}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(yz\right) \left(s - m_c^2\right) \left(13s - 5m_c^2\right), \\
\rho_5^2(s) &= \frac{m_c \langle sg, \sigma Gs \rangle}{32\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(y + z\right) \left(2s - m_c^2\right) \\
&\quad + \frac{m_c \langle sg, \sigma Gs \rangle}{144\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(1 - y - z\right) \left(2s - m_c^2\right) \\
&\quad - \frac{m_c \langle sg, \sigma Gs \rangle}{480\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(yz\right) \left(56s - 17m_c^2 + 10m_c^4\delta\left(s - m_c^2\right)\right) \\
&\quad - \frac{m_c \langle sg, \sigma Gs \rangle}{480\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(1 - y\right) \left(s - m_c^2\right) + \frac{m_s m_c^2 \langle sg, \sigma Gs \rangle}{16\pi^4} \int_{y_i}^{y_f} dy \\
&\quad + \frac{m_s m_c^2 \langle sg, \sigma Gs \rangle}{288\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left(\frac{1}{y} + \frac{1}{z}\right), \\
\end{align}
\[ \rho_0^2(s) = \frac{m_c^2(\bar{s}s)^2}{6\pi^2} \int_{y_i}^{y_f} dy + \frac{g_c^2(\bar{s}s)^2}{240\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \{ 56s - 17\bar{m}_c^2 + 10\bar{m}_c^4 \delta (s - \bar{m}_c^2) \} \\
+ \frac{g_s^2(\bar{s}s)^2}{320\pi^2} \int_{y_i}^{y_f} dy y(1-y) (s - \bar{m}_c^2) \\
- \frac{g_s^2(\bar{s}s)^2}{9720\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (1-y-z) \left\{ 45 \left( \frac{z}{y} + \frac{y}{z} \right) (2s - \bar{m}_c^2) + \left( \frac{z}{y^2} + \frac{y}{z^2} \right) \right\} m_c^2 \left[ 19 + 20\bar{m}_c^2 \delta (s - \bar{m}_c^2) \right] + (y+z) \left[ 18(3s - \bar{m}_c^2) + 10\bar{m}_c^4 \delta (s - \bar{m}_c^2) \right] \right\} \\
- \frac{g_s^2(\bar{s}s)^2}{9720\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (1-y-z) \left\{ 15 \left( \frac{z}{y} + \frac{y}{z} \right) (2s - \bar{m}_c^2) + \left( \frac{z}{y^2} + \frac{y}{z^2} \right) \right\} m_c^2 \left[ 6 + 5\bar{m}_c^2 \delta (s - \bar{m}_c^2) \right] + (y+z) \left[ 56s - 17\bar{m}_c^2 + 10\bar{m}_c^4 \delta (s - \bar{m}_c^2) \right] \right\} \\
- \frac{m_s m_c(\bar{s}s)^2}{12\pi^2} \int_{y_i}^{y_f} dy \{ 1 + \bar{m}_c^2 \delta (s - \bar{m}_c^2) \} , \tag{21} \]

\[ \rho_1^2(s) = \frac{m_c^2(\bar{s}s)^2 (144\pi^2 T^2)}{144\pi^2 T^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left\{ \frac{1}{y^3} + \frac{1}{y^2} + \frac{1}{y} + \frac{1}{z^3} \right\} (1-y-z) m_c^2 \delta (s - \bar{m}_c^2) \\
- \frac{m_c(\bar{s}s)^2 (48\pi^2) \alpha_s GG}{48\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left\{ \frac{1}{y^2} + \frac{1}{y} \right\} (1-y-z) \{ 1 + \bar{m}_c^2 \delta (s - \bar{m}_c^2) \} \\
+ \frac{m_c(\bar{s}s)^2 (144\pi^2 T^2)}{48\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \{ 1 + \bar{m}_c^2 \delta (s - \bar{m}_c^2) \} \\
+ \frac{m_c(\bar{s}s)^2 (48\pi^2) \alpha_s GG}{48\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left\{ \frac{1}{y} + \frac{1}{z} \right\} \{ 1 + \bar{m}_c^2 \delta (s - \bar{m}_c^2) \} \\
- \frac{m_s m_c(\bar{s}s)^2}{288\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{y_i}^{y_f} dy \{ 1 + \bar{m}_c^2 \delta (s - \bar{m}_c^2) \} , \tag{22} \]

\[ \rho_8^2(s) = -\frac{m_c^2(\bar{s}s)(\bar{s}g_s Gs)}{12\pi^2} \int_{0}^{1} dy \left[ \frac{1}{T^2} \right] \delta (s - \bar{m}_c^2) \\
- \frac{m_c^2(\bar{s}s)(\bar{s}g_s Gs)}{216\pi^2} \int_{0}^{1} dy \frac{1}{y(1-y)} \delta (s - \bar{m}_c^2) , \tag{23} \]

\[ \rho_{10}^2(s) = \frac{m_c^2(\bar{s}s)(\bar{s}g_s Gs)^2}{96\pi^2 T^6} \int_{0}^{1} dy \bar{m}_c^2 \delta (s - \bar{m}_c^2) \\
- \frac{m_c^2(\bar{s}s)^2 (108T)^4}{108T^4} \int_{0}^{1} dy \left\{ \frac{1}{y^3} + \frac{1}{y^2} \right\} \delta (s - \bar{m}_c^2) \\
+ \frac{m_c^2(\bar{s}s)^2 (36T)^2}{36T^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{0}^{1} dy \left\{ \frac{1}{y^2} + \frac{1}{y} \right\} \delta (s - \bar{m}_c^2) \\
- \frac{m_c^2(\bar{s}s)^2 (324T^2)}{324T^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{0}^{1} dy \frac{1}{y^2} \delta (s - \bar{m}_c^2) \\
+ \frac{m_s m_c(\bar{s}s)(\bar{s}g_s Gs)^2}{864\pi^2 T^4} \int_{0}^{1} dy \frac{1}{y} \delta (s - \bar{m}_c^2) \\
+ \frac{m_s m_c(\bar{s}s)(\bar{s}g_s Gs)^2}{576\pi^2 T^2} \int_{0}^{1} dy \frac{1}{y} \delta (s - \bar{m}_c^2) \\
+ \frac{m_c^2(\bar{s}s)^2 (\bar{s}g_s Gs)^2}{108T^6} \left( \frac{\alpha_s GG}{\pi} \right) \int_{0}^{1} dy \bar{m}_c^2 \delta (s - \bar{m}_c^2) , \tag{24} \]
\[ \rho_0^0(s) = \frac{1}{256\pi^6} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, yz \, (1-y-z)^3 \, (s-m_c^2)^2 \, (7s^2 - 6sm_c + m_c^4) \]
\[ + \frac{1}{256\pi^6} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, yz \, (1-y-z)^2 \, (s-m_c^2)^3 \, (3s-m_c^2) \]
\[ + \frac{m_am_c}{128\pi^6} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, (y+z) \, (1-y-z)^2 \, (s-m_c^2)^2 \, (5s-2m_c^2) \]  \( , \) (25)

\[ \rho_3^0(s) = -\frac{m_c \langle \bar{s}s \rangle}{8\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, (y+z) \, (1-y-z) \, (s-m_c^2) (2s-m_c^2) \]
\[ + \frac{m_s \langle \bar{s}s \rangle}{8\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, yz \, (1-y-z) \, (10s^2 - 12sm_c^2 + 3m_c^4) \]
\[ + \frac{m_s \langle \bar{s}s \rangle}{8\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, yz \, (s-m_c^2) (2s-m_c^2) \]
\[ - \frac{m_s m_c^2 \langle \bar{s}s \rangle}{2\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, (s-m_c^2) \]  \( , \) (26)

\[ \rho_3^0(s) = -\frac{m_c^2}{192\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, \left( \frac{z}{y^2} + \frac{y}{z^2} \right) \, (1-y-z)^3 \]
\[ \left\{ 2s-m_c^2 + \frac{m_c^4}{6} (s-m_c^2) \right\} \]
\[ - \frac{m_c^2}{384\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, \left( \frac{z}{y^2} + \frac{y}{z^2} \right) \, (1-y-z)^2 \, (3s-2m_c^2) \]
\[ - \frac{1}{768\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, (y+z) \, (1-y-z)^2 \, (10s^2 - 12sm_c^2 + 3m_c^4) \]
\[ + \frac{1}{384\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, (y+z) \, (1-y-z) \, (s-m_c^2) \, (2s-m_c^2) \]
\[ + \frac{1}{384\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, (y+z) \, (1-y-z)^2 \, (10s^2 - 12sm_c^2 + 3m_c^4) \]
\[ + \frac{1}{3456\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, (1-y-z)^3 \, (10s^2 - 12sm_c^2 + 3m_c^4) \]
\[ + \frac{1}{576\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, yz \, (1-y-z) \, (10s^2 - 12sm_c^2 + 3m_c^4) \]
\[ + \frac{1}{576\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, (1-y-z) \, (s-m_c^2) \, (2s-m_c^2) \]
\[ + \frac{1}{288\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \, yz \, (s-m_c^2) \, (2s-m_c^2) \]  \( , \) (27)
$$\rho^0_\pi(s) = \frac{m_c \langle \bar{s}g_s \sigma G \rangle}{32\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \ (y + z) \ (3s - 2m_c^2)$$
$$- \frac{m_c \langle \bar{s}g_s \sigma G \rangle}{48\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \ (1 - y - z) \ (3s - 2m_c^2)$$
$$- \frac{m_c \langle \bar{s}g_s \sigma G \rangle}{8\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \ yz \left\{ 2s - m_c^2 + \frac{m_c^2}{6} (s - \bar{m}_c^2) \right\}$$
$$- \frac{m_c \langle \bar{s}g_s \sigma G \rangle}{48\pi^4} \int_{y_i}^{y_f} dy \ y(1 - y) \ (3s - 2\bar{m}_c^2)$$
$$+ \frac{m_s m_c^2 \langle \bar{s}g_s \sigma G \rangle}{8\pi^4} \int_{y_i}^{y_f} dy$$
$$- \frac{m_s m_c^2 \langle \bar{s}g_s \sigma G \rangle}{48\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \ (1 + \frac{1}{y}) \ , \quad (28)$$

$$\rho^0_\pi(s) = \frac{m_c^2 \langle \bar{s}s \rangle^2}{3\pi^2} \int_{y_i}^{y_f} dy \ y(1 - y) \ (3s - 2\bar{m}_c^2)$$
$$+ \frac{m_s^2 \langle \bar{s}s \rangle^2}{324\pi^4} \int_{y_i}^{y_f} dy \ y(1 - y) \ (3s - 2\bar{m}_c^2)$$
$$\frac{g_c^2 \langle \bar{s}s \rangle^2}{648\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \ (1 - y - z) \left\{ 3 \left( \frac{y}{y + z} \right) (3s - 2\bar{m}_c^2) + 3 \left( \frac{z}{y + z} \right) \right\}$$
$$m_c^2 \left[ 2 + m_c^2 \delta (s - \bar{m}_c^2) \right] + (y + z) \left[ 12 (2s - \bar{m}_c^2) + 2m_c^2 \delta (s - \bar{m}_c^2) \right] \}$$
$$- \frac{g_c^2 \langle \bar{s}s \rangle^2}{1944\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \ (1 - y - z) \left\{ 15 \left( \frac{y}{y + z} \right) (3s - 2\bar{m}_c^2) + 7 \left( \frac{z}{y + z} \right) \right\}$$
$$m_c^2 \left[ 2 + m_c^2 \delta (s - \bar{m}_c^2) \right] + (y + z) \left[ 12 (2s - \bar{m}_c^2) + 2m_c^2 \delta (s - \bar{m}_c^2) \right] \}$$
$$- \frac{m_s m_c^2 \langle \bar{s}s \rangle^2}{12\pi^2} \int_{y_i}^{y_f} dy \left\{ 2 + m_c^2 \delta (s - \bar{m}_c^2) \right\} \ , \quad (29)$$

$$\rho^0_\pi(s) = \frac{m_c^2 \langle \bar{s}s \rangle}{144\pi^4} \left( \frac{\alpha_s GG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{y^2} + \frac{z}{y^2} + \frac{1}{y^2} + \frac{1}{z^2} \right) (1 - y - z)$$
$$\left( 1 + \frac{m_c^2}{T^2} \right) \delta (s - \bar{m}_c^2)$$
$$- \frac{m_c \langle \bar{s}s \rangle}{48\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{y^2} + \frac{z}{y^2} \right) (1 - y - z) \left\{ 2 + m_c^2 \delta (s - \bar{m}_c^2) \right\}$$
$$+ \frac{m_c \langle \bar{s}s \rangle}{48\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left\{ 2 + m_c^2 \delta (s - \bar{m}_c^2) \right\}$$
$$- \frac{m_c \langle \bar{s}s \rangle}{144\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y} + \frac{1}{z} \right) (1 - y - z) \left\{ 2 + m_c^2 \delta (s - \bar{m}_c^2) \right\}$$
$$- \frac{m_c \langle \bar{s}s \rangle}{288\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_{y_i}^{y_f} dy \left\{ 2 + m_c^2 \delta (s - \bar{m}_c^2) \right\} \ , \quad (30)$$
\[ \rho_0^0(s) = -\frac{m_2^2 \langle \bar{s}s \rangle \langle \bar{s}g_s Gs \rangle}{6\pi^2} \int_0^1 dy \left( 1 + \frac{\bar{m}_c^2}{T^2} \right) \delta \left( s - \bar{m}_c^2 \right) + \frac{m_2^2 \langle \bar{s}s \rangle \langle \bar{s}g_s Gs \rangle}{36\pi^2} \int_0^1 dy \frac{1}{y(1-y)} \delta \left( s - \bar{m}_c^2 \right), \]  

(31)

\[ \rho_1^0(s) = \frac{m_2^2 \langle \bar{s}s \rangle \langle \bar{s}g_s Gs \rangle^2}{48\pi^2 T^6} \int_0^1 dy \bar{m}_c^4 \delta \left( s - \bar{m}_c^2 \right) - \frac{m_2^4 \langle \bar{s}s \rangle^2}{54T^4} \left( \frac{\alpha_s G}{\pi} \right) \int_0^1 dy \left\{ \frac{1}{y^3} + \frac{1}{1 - y)^3} \right\} \delta \left( s - \bar{m}_c^2 \right)
+ \frac{m_2^4 \langle \bar{s}s \rangle^2}{18T^2} \left( \frac{\alpha_s G}{\pi} \right) \int_0^1 dy \left\{ \frac{1}{y^2} + \frac{1}{1 - y)^2} \right\} \delta \left( s - \bar{m}_c^2 \right)
+ \frac{m_2^4 \langle \bar{s}s \rangle^2}{54T^2} \left( \frac{\alpha_s G}{\pi} \right) \int_0^1 dy \left\{ \frac{1}{y(1-y)} \right\} \delta \left( s - \bar{m}_c^2 \right)
- \frac{m_2^4 \langle \bar{s}s \rangle^2}{144\pi^2 T^4} \int_0^1 dy \frac{1}{y(1-y) \bar{m}_c^2} \delta \left( s - \bar{m}_c^2 \right)
+ \frac{m_2^4 \langle \bar{s}s \rangle^2}{32\pi^2 T^2} \int_0^1 dy \frac{1}{y(1-y)} \delta \left( s - \bar{m}_c^2 \right)
+ \frac{m_2^4 \langle \bar{s}s \rangle^2}{54T^6} \left( \frac{\alpha_s G}{\pi} \right) \int_0^1 dy \bar{m}_c^4 \delta \left( s - \bar{m}_c^2 \right), \]  

(32)

the subscripts 0, 3, 4, 5, 6, 7, 8 and 10 denote the dimensions of the vacuum condensates, the superscripts 0 and 2 denote the spin the tetraquark states, the \( T^2 \) denotes the Borel parameter; \( y_f = \frac{1+\sqrt{1-4m_{c}^{2}/s}}{2}, y_i = \frac{1-\sqrt{1-4m_{c}^{2}/s}}{2} \), \( z_i = \frac{\gamma m_{c}^{2}}{y_{s}-m_{c}^{2}} \), \( \bar{m}_{c}^{2} = \frac{(y+z)m_{c}^{2}}{y_{s}} \), \( \bar{m}_{c}^{2} = \frac{m_{c}^{2}}{y_{s}(1-y)} \), \( \int_{y_{f}}^{y_{i}} dy \rightarrow \int_{0}^{1} dy, \) \( \int_{z_{i}}^{1-y} dz \rightarrow \int_{0}^{1-y} dz \), when the \( \delta \) functions \( \delta \left( s - \bar{m}_{c}^{2} \right) \) and \( \delta \left( s - \bar{m}_{c}^{2} \right) \) appear.

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