Thermal conductivity of cylindrical tanks for backup fuel of boiler rooms

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Abstract. In this paper, the method of determining the level of heat losses through structures of a cylindrical tank for backup fuel of boiler rooms and steady-state non-uniform temperature distribution in a multilayer tank have been considered, given that the heat transfer coefficient depends on \( r \) and \( z \) coordinates.

1. Introduction

A boiler system is a complex technical installation. It consists of a boiler unit and the boiler auxiliary equipment, hosted in the building of the boiler room or outside, and is designed for generation of steam of the required parameters or hot water preparation, or both [1]. For the smooth operation of boiler rooms, backup fuel is presupposed (diesel oil, fuel oil, etc.) which is stored in cylindrical tanks. For fuel oil storage, steel or reinforced concrete drain fill tanks or underground tanks should be provided for. To store light heating volume and liquid additives, steel tanks, as a rule, should be provided for. Use of tanks made of special plastic materials that meet the climatic conditions of the construction area, and fire safety requirements is permitted, provided that the compliance is confirmed through the certificate of compliance with fire regulations. For terrestrial metal tanks to be installed in areas with an annual average outside air temperature up to +9\(^{\circ}\)C, thermal insulation from non-combustible materials should be envisaged [2]. The tanks are equipped with heated devices for warming up the fuel. Fuel heating in the fuel storage units, needed to ensure the normal operation of the fuel pumps feeding fuel to the boiler nozzles, is effective with coil-tube heaters located in the tanks. [1]

Currently, energy saving issues are sharing much attention, focused on reduction of the amount of energy resources consumed, and simultaneous conservation of the appropriate useful effect from their use [3]. Calculation of power losses of energy resources, including thermal energy, is an important task hereat.

Due to thermal conductivity, thermal losses occur through constructions of the tanks. The heat losses refer to the amount of heat transferred from the internal heating source through construction of the tanks into the environment. Having calculated the temperature distribution in each layer of the tank, significant reduction of heat losses can be achieved through selection of the optimum thickness of the thermal insulation layer.
Within the scope of this work, we will consider the method of determining the level of heat losses through the structures of tanks and steady-state non-uniform temperature distribution in a multilayer tank, given that the heat transfer coefficient depends on coordinates \( r \) and \( z \).

2. Methods

According to Fourier law, the total amount of heat transferred through the pipeline surface is calculated by formula [4]:

\[
Q = -\sum_{j=1}^{\infty} \int_{\sigma_j} \int \int \text{div} \left[ \lambda_j \nabla T_j \right] d\sigma_j, \quad (j = 1, m)
\]

here \( \text{grad} T_j \) is temperature gradient in layer \( j \); \( \lambda_j \) is heat transfer coefficient of the non-uniform layer \( j \); \( \sigma_j \) is surface of layer \( j \) in structures of the heat networks; \( m \) is number of layers.

Conduction heat transfer in cylindrical coordinates is defined by formula [5]:

\[
div \left[ \lambda_j \text{grad} T_j \right] = \frac{\partial \lambda_j}{\partial r} \frac{\partial T_j}{\partial r} + \frac{1}{r} \frac{\partial \lambda_j}{\partial \theta} \frac{\partial T_j}{\partial \theta} + \frac{\partial \lambda_j}{\partial z} \frac{\partial T_j}{\partial z} + \lambda_j \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_j}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T_j}{\partial \theta^2} + \frac{\partial^2 T_j}{\partial z^2} \right),
\]

\( (j = 1, m) \)

To determine the level of heat losses through the tank constructions, the concept of a perfect tank will be introduced. The perfect tank is to be interpreted as the tank whose constructions are heat-proof. Accordingly, no heat losses will occur in such a tank.

The level of heat losses through constructions of the tank may be defined as the difference between the amount of heat in ideal conditions and the amount of heat inclusive of thermal losses occurring through the constructions of the tank.

Research paper [6] considers an approximate solution of the steady-state heat conduction problem at a section of a multilayer heat network pipeline.

To determine heat losses as per formula (1), it is required to specify the temperature pattern.

Heat conduction of uniform cylindrical constructions is widely considered in works [4], [5], [7] – [10]. Heat transfer coefficient in these solutions is treated as a constant value. In non-uniform layers of heat network constructions are to be considered, then heat transfer coefficient is generally dependent on coordinates. One-dimensional steady-state problems of heat conduction have been considered in research [11].

Within the scope of this work, let us consider a two-dimensional steady-state heat conduction problem for cylindrical tanks, with the heat carrier medium flowing along \( z \)-coordinate, given that each of the layers has its own physical properties and heat transfer coefficients depend on \( r \) and \( z \) coordinates (figure 1).
Let us consider the steady-state problem of identifying the temperature field in a multilayer tank with heating sources, given that the temperature at all points of space is time-invariant, and the heat transfer coefficient depends on the coordinate $r$. In this case, the equation takes the form [5]:

$$
\frac{\partial \lambda_j \frac{\partial T_j}{\partial r}}{\partial r} + \lambda_j \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_j}{\partial r} \right) + \frac{\partial^2 T_j}{\partial z^2} \right) + \frac{\partial \lambda_j \frac{\partial T_j}{\partial z}}{\partial z} + Q(r) = 0
$$

(2)

Here $Q(r)$ is the internal heating source, being $r$ is coordinate dependent.

3. Results

Let us present equation (2) in the form:

$$
\left[ \frac{\partial \lambda_j \frac{\partial T_j}{\partial r}}{\partial r} + \lambda_j \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_j}{\partial r} \right) + Q(r) \right] + \left[ \frac{\partial \lambda_j \frac{\partial T_j}{\partial z}}{\partial z} + \lambda_j \frac{\partial^2 T_j}{\partial z^2} \right] = 0
$$

(3)

Here $T_j = T_j(r, z)$ is temperature pattern of layer $j$.

$T_j(r, z)$ will be presented as a product of two functions, one of which is $r$ -coordinate dependent, another one depends on $z$ -coordinate.

$$
T_j(r, z) = \psi_j(r) \varphi_j(z)
$$

(4)

Thus, we get two equations:

$$
\frac{\partial \lambda_j \frac{\partial \psi_j}{\partial r}}{\partial r} + \lambda_j \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi_j}{\partial r} \right) + Q(r) = 0, \quad (5)
$$

$$
\frac{\partial \lambda_j \frac{\partial \varphi_j}{\partial z}}{\partial z} + \lambda_j \frac{\partial^2 \varphi_j}{\partial z^2} = 0 \quad (6)
$$

Solving equation (5), we obtain:
\[
\frac{\partial \lambda_j}{\partial r} \frac{\partial \psi_j}{\partial r} + \lambda_j \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi_j}{\partial r} \right) + Q(r) = 0
\]
\[
\lambda_j'(r)r\psi'(r) + \lambda_j(r)(r\psi'(r))' = -Q(r)r
\]

After substitution \( r\psi'_j(r) = U_j(r) \):
\[
\lambda_j'(r)U_j(r) + \lambda_j(r)U'_j(r) = -Q(r)r
\]
\[
\left( \lambda_j(r)U_j(r) \right)' = -Q(r)r \quad (7)
\]

Upon integrating over the range of \( r_{j-1} \) to \( r \):
\[
\lambda_j(r)r \left. \frac{d\psi_j(r)}{dr} \right|_{r_{j-1}}^{r} = - \int_{r_{j-1}}^{r} Q(r) r dr
\]
\[
\lambda_j(r)r \left. \frac{d\psi_j(r)}{dr} \right|_{r_{j-1}}^{r} = \lambda_j(r_{j-1})r_{j-1} \left. \frac{d\psi_j(r_{j-1})}{dr} \right|_{r_{j-1}}^{r} - \int_{r_{j-1}}^{r} Q(r) r dr
\]
\[
\left. \frac{d\psi_j(r)}{dr} \right|_{r_{j-1}}^{r} = \frac{\lambda_j(r_{j-1})r_{j-1} - 1}{\lambda_j(r)r} A_{j-1}
\]

here \( F'_{j-1} = \left. \frac{d\psi_j(r_{j-1})}{dr} \right|_{r_{j-1}}^{r} \), \( A_{j-1} = \int_{r_{j-1}}^{r} Q(r) r dr \).
\[
\psi_j(r) = F'_{j-1}A_{j-1}(r_{j-1}) \left. r \int_{r_{j-1}}^{r} \frac{dr}{\lambda_j(r)r} \right|_{r_{j-1}}^{r} - \left. \frac{A_{j-1}(r)}{\lambda_j(r)r} dr \right|_{r_{j-1}}^{r} + D'_{j-1}
\]

and for the layer with no heating source:
\[
\psi_j(r) = F'_{j-1}A_{j-1}(r_{j-1}) \left. r \int_{r_{j-1}}^{r} \frac{dr}{\lambda_j(r)r} \right|_{r_{j-1}}^{r} - \left. \frac{A_{j-1}(r)}{\lambda_j(r)r} dr \right|_{r_{j-1}}^{r} + D'_{j-1}
\]

The relationship between the permanent \( F'_{j}, \) \( F'_{j-1}, \) \( D'_{j} \) and \( D'_{j-1} \) is calculated through a matching condition:
The remaining constants are determined depending on the problem setting, based on the conditions at the internal layer of the cylindrical tank at \( r = R_i \) and at the outside layer \( R_o \).

Similarly, we get the solution to equation (6): \[
\varphi_j(z) = F^z_{j-1} \lambda_j (z_{j-1}) \int_{z_{j-1}}^{z} \frac{dz}{\lambda_j (z)} + D^z_{j-1}, \quad z_{j-1} \leq z \leq z_j
\] (11)

here \( D^z_{j-1} = \varphi_j(z_{j-1}) \).

And for layer \((j+1)\), we will get the formula:

\[
\varphi_j(z) = F^z_j \lambda_j (z_j) \int_{z_j}^{z} \frac{dz}{\lambda_j (z)} + D^z_j
\] (12)

here \( D^z_j = \varphi_j(z_j) \).

Application of the obtained solutions (9) and (11) to (4) will provide:

\[
T_j(r,z) = \left[ F^r_{j-1} \lambda_j (r_{j-1}) \int_{r_{j-1}}^{r} \frac{dr}{\lambda_j (r)} - \int_{r_{j-1}}^{r} \frac{\lambda_{j-1} (r) dr}{\lambda_j (r)} + D^r_{j-1} \right] - \left[ F^z_{j-1} \lambda_j (z_{j-1}) \int_{z_{j-1}}^{z} \frac{dz}{\lambda_j (z)} + D^z_{j-1} \right]
\] (13)

Having determined the temperature pattern (13), we further calculate the rate of heat losses through constructions of cylindrical tank by formula (1) as the difference between the amount of heat under ideal conditions and the amount of heat, in view of heat losses through constructions of cylindrical tank.

**Conclusions**

A method of calculating heat losses through constructions of cylindrical tank has been defined. A solution of the two-dimensional steady-state heat conduction problem for multilayer structures of cylindrical tanks, with any number of layers and any non-uniformity of thermal-physical properties, has been obtained. Using simple transformations, solutions for various boundary conditions can be obtained. The provided solution is common for any non-uniform heat transfer coefficients. The provided solution is convenient, since it is possible to obtain a set of solutions for any type of materials, through setting individual curves of heat transfer coefficients, which is significant for non-uniform materials.

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