CYCLIC TRANSIT PROBABILITIES OF LONG-PERIOD ECCENTRIC PLANETS DUE TO PERIASTRON PRECESSION

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ABSTRACT

The observed properties of transiting exoplanets are an exceptionally rich source of information that allows us to understand and characterize their physical properties. Unfortunately, only a relatively small fraction of the known exoplanets discovered using the radial velocity technique are known to transit their host due to the stringent orbital geometry requirements. For each target, the transit probability and predicted transit time can be calculated to great accuracy with refinement of the orbital parameters. However, the transit probability of short period and eccentric orbits can have a reasonable time dependence due to the effects of apsidal and nodal precession, thus altering their transit potential and predicted transit time. Here we investigate the magnitude of these precession effects on transit probabilities and apply this to the known radial velocity exoplanets. We assess the refinement of orbital parameters as a path to measuring these precessions and cyclic transit probabilities.

Key words: celestial mechanics – ephemerides – planetary systems – techniques: photometric

1. INTRODUCTION

The realization that we have crossed a technology threshold that allows transiting planets to be detected sparked a flurry of activity in this direction after the historic detection of HD 209458 b’s transits (Charbonneau et al. 2000; Henry et al. 2000). This has resulted in an enormous expansion of exoplanetary science such that we can now explore the mass–radius relationship (Burrows et al. 2007; Fortney et al. 2007; Seager et al. 2007) and atmospheres (Agol et al. 2010; Deming et al. 2007; Knutson et al. 2009a, 2009b) of planets outside of our solar system. Most of the known transiting planets were discovered using the transit method, but some were later found to transit after first being detected using the radial velocity technique. Two notable examples are HD 17156 b (Barbieri et al. 2009) and HD 80606 b (Laughlin et al. 2009), both of which are found to transit after first being detected using the radial velocity technique.

Planets in eccentric orbits are particularly interesting because of their enhanced transit probabilities (Kane & von Braun 2008, 2009). This orbital eccentricity also makes those planets prone to orbital precession. In celestial mechanics, there are several kinds of precession which can affect the orbital properties, spin rotation, and equatorial plane of a planet. These have been studied in detail in reference to known transiting planets, particularly in the context of the precession effects on transit times and duration (Carter & Winn 2010; Damiani & Lanza 2011; Heyl & Gladman 2007; Jordán & Bakos 2008; Miralda-Escudé 2002; Pál & Kocsis 2008; Ragazzine & Wolf 2009). One consequence of these precession effects is that a planet that exhibits visible transits now may not do so at a different epoch and vice versa.

Here we present a study of some precession effects on known exoplanets. The aspect which sets this apart from previous studies is that we are primarily interested in planets not currently known to transit, particularly long-period eccentric planets which have enhanced transit probabilities and larger precession effects. We investigate the subsequent rate of change of the transit probability to show how they drift in and out of a transiting orientation. We calculate the timescales and rates of change for the precession and subsequent transit probabilities and discuss implications for the timescales on which radial velocity planets will enter into a transiting configuration, based upon assumptions regarding their orbital inclinations. We finally compare periastron argument uncertainties to the expected precession timescales and suggest orbital refinement as a means to measure this effect.

2. TRANSIT PROBABILITY

Here we briefly describe the fundamentals of the geometric transit probability for both circular and eccentric orbits. For a detailed description we refer the reader to Kane & von Braun (2008).

In the case of a circular orbit, the geometric transit probability is defined as follows:

\[ P_t = \frac{R_p + R_*}{a}, \]

where \( a \) is the semi-major axis and \( R_p \) and \( R_* \) are the radii of the planet and host star, respectively.

The star–planet separation as a function of \( \omega \) can be given by

\[ r = \frac{a(1 - e^2)}{1 + e \cos f}, \]

where \( f \) is the true anomaly, which describes the location of the planet in its orbit, and so is a time-dependent variable as the planet orbits the star. For a transit event to occur the condition of \( \omega + f = \pi/2 \) must be fulfilled (Kane 2007), where \( \omega \) is the argument of periastron, and so we evaluate the above equations with this condition in place. The geometric transit probability
may thus be re-expressed as

$$P_t = \frac{(R_p + R_\star)(1 + e \cos(\pi/2 - \omega))}{a(1 - e^2)},$$

(3)

which is valid for any orbital eccentricity. Note that these equations are independent of the true inclination of the planet’s orbital plane.

Figure 1. Transit probability for a sample of the known exoplanets as a function of orbital period. In cases where a change in $\omega$ from current to 90° results in a transit probability improvement >1%, a vertical arrow indicates the improvement.

Given the sensitivity of transit probability to the argument of periastron, it is useful to assess how the probabilities for the known exoplanets would alter if their orientation was that most favorable for transit detection: $\omega = 90^\circ$. We extracted data from the Exoplanet Data Explorer3 (Wright et al. 2011) which include the orbital parameters and host star properties for 592 planets and are current as of 2012 June 30. For each planet, we calculate transit probabilities for two cases: (1) using the current value of $\omega$, and (2) using $\omega = 90^\circ$. The transit probabilities for case (1) are shown in Figure 1. Those planets whose case (2) probabilities are improved by >1% are indicated by a vertical arrow to the improved probability. There are several features of note in this figure. The relatively high transit probabilities between 100 and 1000 days are due to giant host stars whose large radii dominates the probabilities (see Equation (3)). There are several cases of substantially improved transit probability, most particularly HD 80606 b, which is labeled in the figure. The following sections investigate the periastron precession required to produce such an increase in transit probability.

3. AMPLITUDE OF PERIASTRON
(APSIDAL) PRECESSION

Periastron (or apsidal) precession is the gradual rotation of the major axis which joins the orbital apsides within the orbital plane. The result of this precession is that the argument of periastron becomes a time-dependent quantity. There are a variety of factors which can lead to periastron precession, such as general relativity (GR), stellar quadrupole moments, mutual star–planet tidal deformations, and perturbations from other planets (Jordán & Bakos 2008). For Mercury, the perihelion precession rate due to general relativistic effects is $43^\prime$ century$^{-1}$.

$\omega_{\text{GR}} = \frac{7.78}{(1 - e^2)} \left( \frac{M_\star}{M_\odot} \right) \left( \frac{a}{0.05/\text{AU}} \right)^{-1} \left( \frac{P}{\text{day}} \right)^{-1},$

(6)

where $G$ is the gravitational constant, $M_\star$ is the mass of the host star, and $P$ is the orbital period of the planet. The total periastron precession is the sum of the individual effects as follows:

$$\dot{\omega}_{\text{total}} = \dot{\omega}_{\text{GR}} + \dot{\omega}_{\text{quad}} + \dot{\omega}_{\text{tide}} + \dot{\omega}_{\text{pert}},$$

(5)

where the precession components consist of the precession due to GR, stellar quadrupole moment, tidal deformations, and planetary perturbations, respectively. Jordán & Bakos (2008) conveniently express these components in units of degrees per century. The components of $\dot{\omega}_{\text{quad}}$ and $\dot{\omega}_{\text{tide}}$ have $a^{-5}$ and $a^{-3}$ dependencies, respectively. Since we are mostly concerned with long-period planets in single-planet systems, we consider here only the precession due to GR since this is the dominant component in such cases. This imposes a lower limit on the total precession of the system, particularly for multi-planet systems. This precession is given by the following equation:

$\omega_{\text{GR}} = \frac{7.78}{(1 - e^2)} \left( \frac{M_\star}{M_\odot} \right) \left( \frac{a}{0.05/\text{AU}} \right)^{-1} \left( \frac{P}{\text{day}} \right)^{-1},$

(6)

with units in degrees per century.

Figure 2. Calculated GR periastron precession rates plotted as a function of eccentricity for the known exoplanets with Keplerian orbital solutions. The radius of the points is logarithmically proportional to the orbital period of the planet. The symbol for Mercury is used to indicate its position on the plot.

(0.0119 century$^{-1}$). By comparison, the precession due to perturbations from the other solar system planets is $532^\prime$ century$^{-1}$ ($0.148$ century$^{-1}$) while the oblateness of the Sun (quadrupole moment) causes a negligible contribution of $0.025$ century$^{-1}$ ($0.000007$ century$^{-1}$; Clemence 1947; Iorio 2005).

Here we adopt the formalism of Jordán & Bakos (2008) in evaluating the amplitude of the periastron precession. We first define the orbital angular frequency as

$$n \equiv \sqrt{\frac{GM_\star}{a^3}} = \frac{2\pi}{P},$$

(4)

where $G$ is the gravitational constant, $M_\star$ is the mass of the host star, and $P$ is the orbital period of the planet. The total periastron precession is the sum of the individual effects as follows:

$$\dot{\omega}_{\text{GR}} = \frac{7.78}{(1 - e^2)} \left( \frac{M_\star}{M_\odot} \right) \left( \frac{a}{0.05/\text{AU}} \right)^{-1} \left( \frac{P}{\text{day}} \right)^{-1},$$

(6)
divide occurs at a periastron precession of $\sim 0.1$ century$^{-1}$. It is no coincidence that this divide corresponds to the known relative dearth of planets in the semi-major axis range of 0.1–0.6 AU (Burkert & Ida 2007; Cumming et al. 2008; Currie 2009).

As expected from Equation (6), the amplitude of the precession is dominated by the orbital period rather than the orbital eccentricity. Thus, even planets in eccentric orbits do not exhibit significant GR precession at longer periods. This is further demonstrated in Figure 3 where we show lines of constant precession as a function of orbital period and eccentricity for a solar-mass host star. This shows that the GR periastron precession is almost independent of orbital eccentricity except at extreme values of $e > 0.8$. Once again, the location of Mercury on the plot is indicated using the appropriate symbol.

As noted by Miralda-Escudé (2002) and Jordán & Bakos (2008), the total precession timescales are large. Thus, what really matters is the rate of change of the periastron argument and quantifying when it is worth returning to a particular target for re-investigation. This is the context of our analysis in Section 4.

3.1. Nodal (Orbital Plane) Precession

For completeness, we briefly consider the effects of nodal precession. Nodal precession occurs when the orbital plane precesses around the total angular momentum vector, which is usually aligned with the rotation axis of the host star. The precession is caused by the oblateness of the star which results in a non-zero gravitational quadrupole field. This has the potential to be the dominant source of precession when the orbit is polar. For example, the nodal precession for the near-polar retrograde orbit of WASP-33 b has been calculated by Iorio (2011) to be $9 \times 10^9$ times larger than that induced on the orbit of Mercury by the oblateness of the Sun.

A description of nodal precession and its effect on transit durations has been provided by Miralda-Escudé (2002). The frequency of nodal precession can be expressed as

$$
\Omega = n \frac{R^2}{a^2} \frac{3J_2}{4} \sin 2i,
$$

where $n$ is the orbital angular frequency described in Equation (4), $J_2$ is the quadrupole moment, and $i$ is the orbital inclination relative to the stellar equatorial plane. A typical quadrupole moment for the star may be approximated as $J_2 \sim 10^{-6}$ and one may expect a relatively aligned orbit such that $\sin 2i \sim 0.1$. For a typical hot Jupiter, values for $a$ are $10 R_\star$, whereas for Mercury $a = 83 R_\star$. Since the nodal precession is in units of the orbital angular frequency, one can see that the resulting precession rate is typically several orders of magnitude smaller than that of a hot Jupiter, even at the orbital distance of Mercury. This effect is generally only considered for circular orbits, most notably for short-period orbits that are the most frequently encountered nature of known transiting planets. Here, we are considering longer period eccentric orbits where this is a much smaller effect on the orbital dynamics of the planet.

4. CYCLIC TRANSIT EFFECTS

As discussed in Section 2, the transit probability for a given planet is a function of the periastron argument for orbits with non-zero eccentricity (Kane & von Braun 2008). The precession of the periastron argument thus leads to a cyclic change in the transit probability. Here we quantify this cyclic behavior and determine rates of change and total timescales.

Using the periastron precession rates calculated in Section 3 and combining these with the transit probability equations of Section 2 allows us to compute the time-dependent transit probability for each planet. Recall also that this cyclic behavior will only occur for planets which have non-zero eccentricities. Shown in Figure 4 are three examples of this time dependence.
# Table 1
Exoplanet Periastron Precession, Transit Probabilities, and Timescales

| Planet          | $P$  | $e$  | $\omega$ | $\omega_{GR}$ | $P_t$ | $P_t^*$ | $\Delta t$ | $\frac{dP_t}{dt}$ |
|-----------------|------|------|----------|---------------|-------|---------|------------|-----------------|
| HD 88133 b      | 3.42 | 0.13 | 349.0    | 2.9490        | 14.6  | 17.0    | 34.2       | 0.101368        |
| HD 76700 b      | 3.97 | 0.09 | 30.0     | 2.1838        | 12.9  | 13.5    | 27.5       | 0.036099        |
| HD 73256 b      | 2.55 | 0.03 | 337.3    | 4.3194        | 16.1  | 16.8    | 26.1       | 0.033421        |
| HD 108147 b     | 10.90| 0.53 | 308.0    | 0.5732        | 5.1   | 13.4    | 247.7      | 0.028841        |
| HD 102956 b     | 0.49 | 0.05 | 12.0     | 1.2451        | 22.8  | 23.6    | 62.6       | 0.022932        |
| BD –08 2823 b   | 0.56 | 0.15 | 30.0     | 0.9420        | 11.6  | 12.4    | 63.7       | 0.022925        |
| HD 7924 b       | 5.40 | 0.17 | 25.0     | 1.0901        | 7.2   | 7.9     | 59.6       | 0.019663        |
| HD 68988 b      | 6.28 | 0.12 | 31.4     | 1.0214        | 8.7   | 9.1     | 57.4       | 0.015372        |
| HD 1461 b       | 5.77 | 0.14 | 58.0     | 1.1102        | 9.4   | 9.6     | 28.8       | 0.011871        |
| HD 217107 b     | 7.13 | 0.13 | 24.4     | 0.8192        | 6.9   | 7.4     | 80.1       | 0.010737        |

Notes.

- $P_t^*$ refers to the transit probability where $\omega = 90^\circ$.
- $\Delta t$ refers to the time until $P_t'$ occurs.
- $\frac{dP_t}{dt}$ is calculated over the coming century but is a time-dependent quantity.
over a period of 100,000 years. When viewing such a plot one is tempted to interpret the cyclic variability in terms of the orbital period, however this variation is caused by the periastron precession, not by the orbital period. There is, of course, some period dependency involved, in that shorter period orbits will tend to have a higher cyclic frequency. The planets shown here (HD 88133 b, HD 108147 b, and HD 190360 c) have orbital periods of 3.4, 10.9, and 17.1 days, respectively (Butler et al. 2006; Wright et al. 2009). HD 108147 b, in particular, displays very large amplitude variations due to the relatively high eccentricity of its orbit ($e = 0.53$). HD 190360 c has a smaller eccentricity and periastron precession rate, which leads to a cyclic timescale much greater than 100,000 years.

We have performed these calculations for a subset of the known exoplanets using the data extracted from the Exoplanet Data Explorer, described in Section 2. We restrict our sample to those planets which are not known to transit and have non-zero eccentricities. The results of these calculations are shown in Table 1 for 60 of the planets. The calculated values include the periastron precession rate ($\omega_{\Delta t}$), transit probability ($P_t$), maximum transit probability at $\omega = 90^\circ$ ($P_t^\circ$), time from the current epoch until maximum transit probability ($\Delta t$), and the transit probability rate of change ($dP_t/dt$). The table has been sorted according to $dP_t/dt$ which is presented in units of % century$^{-1}$. The $dP_t/dt$ values have been calculated from the current epoch over the coming century and thus represent the present rate of change. The importance of this is that $dP_t/dt$ is not constant and indeed can have negative values as the periastron argument rotates past $\omega = 90^\circ$. Specifically, $dP_t/dt$ will be negative for $90^\circ < \omega < 270^\circ$ and positive elsewhere. This further restricts the planets considered to those whose current $\omega$ falls in this range such that $dP_t/dt > 0$.

It can be clearly seen that the time required to reach maximum transit probability is immense, certainly beyond the lifetime of anyone reading this work. However, the rate of change can yield an improved idea of which planets may have a measurable change in configuration. Consider the case of HD 156846 b, whose orbital parameters and transit potential have been studied in detail by Kane et al. (2011). This is one of the planets in the table with the longest period and also has one of the highest orbital eccentricities. The transit probability is relatively high for this planet and is close to the maximum probability since $\omega$ only needs to change by $38^\circ$. Even so, observations of the periastron precession are unlikely for the timescales involved. By contrast, the hot Saturn HD 88133 b discovered by Fischer et al. (2005) has the highest transit probability rate of change.

5. CONCLUSIONS

Transiting planets have become an essential component of exoplanetary science due to the exceptional opportunities they present for characterization of these planets. Many of the known exoplanets discovered through the radial velocity technique are currently not known to transit. However, transit probabilities can be substantially improved if the periastron argument approaches $\omega = 90^\circ$. Since, for eccentric orbits, the periastron argument is time dependent as a result of their precession, planets which do not transit at the present epoch may transit in the future and vice versa. The planet Mercury falls quite central to the current distribution of calculated periastron precessions for the known exoplanets. This distribution has an eccentricity dependence but is most strongly affected by the orbital period. If a precession rate for a given planet is found to be markedly different from our calculations then this could be indicative of further as yet undiscovered planets in that system. These additional planets would normally be detected from the radial velocity data unless insufficient observations allow them to remain hidden.

The periastron precession leads to a cyclic transit probability variation for all exoplanets with non-zero eccentricities. Timescales vary enormously but will likely lead to many of these planets transiting their host stars at some point in the future. A reasonable question to ask at this point is if the periastron arguments of the known planets are known with sufficient precision to detect precession in any acceptable time frame. Once again, we exploit the data extracted from the Exoplanet Data Explorer, described in Section 2. The uncertainties associated with the values of $\omega$ for all these planets have a mean of 28$^\circ$ and a median of 15$^\circ$. This is much higher than the precession effects shown in Table 1. A program of refining the orbits of the known exoplanets, such as that described by Kane et al. (2009), would result in many of these precession effects being detectable in reasonable time frames. For example, the first planet in the table, HD 88133 b, has a precession rate that will cause a shift of $\sim 0.3$ per decade. Uncertainties on $\omega$ of less than 1$^\circ$ are not unusual and can certainly be achieved for those planets in particularly eccentric orbits. The exoplanet HD 156846 b has a current $\omega$ uncertainty of 0.16 (Kane et al. 2011) which demonstrates that such refinement is possible even for relatively long-period planets. More data and longer time baselines will produce subsequent improvements for many more planets which can result in the detection of the precession for high-precession cases.

The relevance of this work may be extended to the Kepler mission which has detected many candidate multi-planet systems (Borucki et al. 2011a, 2011b; Batalha et al. 2012), most of which are likely to be real exoplanets (Lissauer et al. 2012). Due to simply geometric transit probabilities, most of these systems will certainly have planets which are not transiting the host star at present. The known transiting multi-planet systems are largely in circular orbits, but may have periastron precession due to perturbations from other planets leading to an eventual transit from currently non-transiting planets in the system. For example, Kepler-19 c is known to exist from transit timing variations of the inner planet, but does not currently have a detectable transit signature. Similarly, some of these planets will cease exhibiting an observable transit signature. Issues such as these are important for considering the completeness of these surveys in determining multi-planetary system architectures.

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