Reflection beamshifts of visible light due to graphene

Nathaniel Hermosa

National Institute of Physics, University of the Philippines, Diliman, Quezon City 1101 Philippines

E-mail: nhermosa@nip.upd.edu.ph

Received 25 October 2015, revised 4 December 2015
Accepted for publication 7 December 2015
Published 19 January 2016

Abstract

I present calculations of reflection beamshifts, Goos-Hänchen and Imbert-Fedorov shifts, due to the presence of a graphene monolayer on a dielectric medium when using a beam with wavelength in the visible range. Measuring the Goos-Hänchen and Imbert-Fedorov shifts is an alternative method to determine graphene’s conductivity. I look at beamshifts for different polarization states (p, s, 45°, σ+) and I discuss other possible experimental routes to determine these beamshifts and consequently, the graphene’s optical conductivity. The Goos-Hänchen shifts for visible light I calculated are in good agreement with results of a recent experiment.

Keywords: graphene, Goos-Hänchen shift, Imbert-Fedorov shift

1. Introduction

A beam of light with finite waist experiences minute angular and spatial deviations from what is expected from the law of reflection when it strikes an index gradient. Known collectively as beamshifts, the Goos-Hänchen (GH) and Imbert-Fedorov (IF) shifts are corrections to the lateral and the transverse position and skew of the reflection, respectively [1, 2]. These shifts may occur simultaneously or separately, and their presence depends on the polarization of the incident beam [2–6], the index gradient seen by the beam [7–11], the divergence of the beam [12] and the modal structure of the beam [13–19]. Bliok and Aiello offer an excellent review of beamshifts in [1].

Measurement of beamshifts can offer an alternative method to determine optical properties of materials. It is nondestructive and dynamic (i.e. it can instantaneously change values with a change in the properties of the material). Several experiments have reported beamshifts in different materials such as metals [8, 9], semiconductors [10] and materials with negative indices of refraction [11].

Recently, Li et al measured a giant GH shift in total internal reflection in a graphene-dielectric interface [20]. Their model does not completely describe the experimental results they obtained. In this paper, the cause of this large GH shift is explained and some ways to determine the other beamshifts are presented. A measurement of these shifts can provide alternative ways of determining material properties of graphene.

Graphene, an atom-thick sheet of carbon, has been identified as an adaptable material in photonics and optoelectronic applications due to its remarkable absorption and its highly flexible optical properties, which can change drastically when it is electrically gated (see for example [21]). The problem, however, is that there is still major debate on the optical properties of graphene, especially since those properties are sensitive to its immediate environment [22]. As yet, there is no universally accepted experimental value of graphene’s optical conductivity other than the predicted theoretical value for a clean graphene monolayer at half-filling at zero temperature, \( \sigma = e^2/4\hbar \) where \( e \) is the charge of the electron and \( \hbar \) is Planck’s constant [23, 24]. This fact makes it imperative to study the optical conductivity of graphene. An alternative route that is dynamic and non-invasive will contribute significantly to this end.

In this paper, I calculate reflection beamshifts due to graphene for light sources in the visible range. I also provide experimental insights into how to measure these shifts.
2. Theoretical framework

2.1. Reflection beamshifts

When a beam of light of finite transverse extent impinges on an index gradient, the beam may experience four beam shifts: two spatial shifts ($\Delta_{GH}$ and $\Delta_{IF}$) and two angular shifts ($\Theta_{GH}$ and $\Theta_{IF}$). The derivation of these expressions is too long to be presented here and has been extensively discussed elsewhere [1]. However, it follows the general procedure of decomposition of the incident beam into plane wave components and the action of the Fresnel reflection coefficients to the s and p polarization components of these waves. Summing all the reflected plane wave components and taking the centroid of the intensity distribution gives the following dimensionless beamshifts:

$$\Delta_{GH} = w_p \text{Im} \left( \frac{\partial \ln r_p}{\partial \theta} \right) + w_i \text{Im} \left( \frac{\partial \ln r_i}{\partial \theta} \right)$$

$$-\Theta_{GH} = w_p \text{Re} \left( \frac{\partial \ln r_p}{\partial \theta} \right) + w_i \text{Re} \left( \frac{\partial \ln r_i}{\partial \theta} \right)$$

$$\Delta_{IF} = -\frac{a_p a_i \cot \theta}{R_i^2 a_p^2 + R_p^2 a_i^2} \times [(R_p^2 + R_i^2) \sin \eta + 2R_p R_i \sin (\eta - \varphi_p + \varphi_i)]$$

$$\Theta_{IF} = \frac{a_p a_i \cot \theta}{R_i^2 a_p^2 + R_p^2 a_i^2} [(R_p^2 - R_i^2) \cos \eta],$$

where $\Delta_{GH}$ and $\Theta_{GH}$ are the dimensionless spatial and angular GH shifts, respectively, and $\Delta_{IF}$ and $\Theta_{IF}$ are the dimensionless spatial and angular IF shifts, respectively, with $w_{i/p} = R_{i/p}^2 a_{i/p}^2 / (R_i^2 a_i^2 + R_p^2 a_p^2)$, and $r_{i/p} = R_{i/p} \exp (i \varphi_{i/p})$, the Fresnel reflection coefficient evaluated at the incident angle $\theta$, the $\varphi_{i/p}$ is the phase gain after reflection, $a_{i/p}$ are the electric field components and $\eta$ is the relative phase difference between these components. The factors in the dimensionless beamshifts are independent of the wavelength $\lambda$ except for the materials’ permittivity incorporated in the Fresnel coefficients. Since the beamshifts are dependent on the Fresnel coefficient, a change in it will change the beamshifts.

The physical beamshifts $\Gamma_X$ and $\Gamma_Y$ are the sum of the contribution of the spatial $\Delta_{GH,IF}$ and angular $\Theta_{GH,IF}$ given by

$$k_0 \Gamma_X = \Delta_{GH} + (z/L) \Theta_{GH}$$

$$k_0 \Gamma_Y = \Delta_{IF} + (z/L) \Theta_{IF},$$

respectively, where $k_0 = 2\pi/\lambda$, $z$ is the distance of the detector from the minimum beam waist, and $L = k_0 \omega_0^2/2$ is the Rayleigh length.

One can use the scheme developed by Woerdman et al to detect these shifts (see for example [8, 9, 12]). In that method, the polarization differential beamshift is measured with a quadrant detector while toggling between two orthogonal polarizations and reducing technical noise with a lock-in amplifier. The difference in the intensity of the signal as detected by the quadrant detector gives the value of the differential beamshifts between the two polarization states.

On the other hand, weak measurements can also be used in determining beam shifts (see for example [23–25]). As a matter of fact, weak measurements can amplify these shifts via weak amplification. However, the amplification factor has been found to not be constant [24]. Hence an a priori knowledge is necessary and this method may not be as useful when the material properties are unknown.

2.2. Fresnel coefficients of a graphene monolayer

The Fresnel coefficient for a dielectric-graphene-dielectric interface can be solved by imposing the following boundary conditions: $n \times (E_{i+1} - E_i)\mid_{\eta=0} = 0$ and $n \times (H_{i+1} - H_i)\mid_{\eta=0} = J$ where $n$ is the unit surface normal, $E_{i+1}$ and $H_{i+1}$ are the electric and magnetic fields at the interface and $J$ is the surface current density of the graphene, which is proportional to its conductivity $\sigma$ [26]. The resulting reflection coefficients are

$$r_i = \frac{\sqrt{r_1} \cos \theta_1 - \sqrt{r_2} \cos \theta_2 + \sigma / \epsilon_0 c}{\sqrt{r_1} \cos \theta_1 + \sqrt{r_2} \cos \theta_2 + \sigma / \epsilon_0 c},$$

$$r_p = \frac{\sqrt{r_2} \cos \theta_2 - \sqrt{r_1} \cos \theta_1 + \sigma / \epsilon_0 c}{\sqrt{r_2} \cos \theta_2 + \sqrt{r_1} \cos \theta_1 + \sigma / \epsilon_0 c},$$

where $r_{1,2}$ are the dielectric constant of the material above and below the graphene, $\epsilon_0$ is the permittivity of free space and $c$ is the speed of light. The $r_{i/p}$ equations are derived in [21, 27], independently.

The conductivity of graphene I use to calculate the GH and IF shifts in the visible range is $\sigma = e^2/4\hbar$ even though the conductivity of real graphene depends on the temperature and the wavelength of the incident light, and on the Fermi level or the amount of doping. This value, however, is not implausible experimentally, as Kuzmenko et al measured a conductivity that is close to the predicted conductivity of a clean graphene monolayer [28] and at high photon energies the conductivity of a clean graphene approaches this value even at finite temperatures [29]. Moreover, Mak observed that at visible-light photon energies, the conductivity approaches this value even with a scattering rate of 20 eV and chemical potential of 100 eV [30]. For calculations of the beamshifts in the visible-range, I assume a graphene monolayer on a BK7 substrate ($n_{BK7} = 1.515$) with air ($n_{air}^2 = \epsilon_{air} = 1$) on the other surface.

3. Results and discussion

Figure 1 shows the reflectance and the amount of phase jumps after the incident beam is reflected at an angle $\theta$ on an air-graphene-glass/glass-graphene-air interface. When light strikes from air (figure 1(a) and (c)), the reflectance is very similar to that of the no-graphene sample. There is a minute difference in the $R_p$, while it acquires the same amount of phase jumps as in the no-graphene interface. These reflectances and phase jumps indicate that there are differences between the beamshifts for a graphene-on glass substrate and a bare substrate. Also, Brewster’s angle has been shifted by $\approx 0.6^\circ$ due to the presence of the graphene. This is particularly
helpful when measuring $\Theta_{\text{GH}}$ as it increases rapidly near Brewster’s angle.

The $R_s^2$ and $R_p^2$ with the graphene do not immediately reach the value of 1 after the critical angle (figure 1(b)) in contrast to the no-graphene sample, where $R_s^2 = 1$ at and beyond the critical angle. There is a slowly sloping $R_s^2$. This fact is important since angular GH may be present after the critical angle for a glass substrate with graphene. The amounts of phase jump after reflection, however, are almost the same (figure 1(d)).

Figure 2 shows the dimensionless spatial GH shifts ((a) and (b)) and angular GH shifts ((c) and (d)). The main results here are the observation of a nonzero $\Theta_{\text{GH}}$ for $p$ and $s$ linear polarization polarization states (figure 2(d)) when light strikes the graphene film from the glass side. These do not occur for a bare dielectric. This fact, which has not been reported previously, means that by focusing the beam and letting it propagate further, the beamshifts can be amplified [2].

In [20], Li et al report a giant GH shift which decays more slowly with a focused beam than with a nonfocused beam. In their experiments, they measured the GH shifts for focused beams as a position differential between the shift of $p$ and $s$ with and without graphene, using two balanced amplified photodetectors. Their beam is a HeNe laser (wavelength $\lambda = 632.8$ nm) with an initial waist of 1 mm that is focused by a lens with a 2 mm focal length. The researchers needed to focus the beam to achieve high measurement accuracy. Aiello et al describing the role of a lens in the measurement of the shifts [2], note that the lens changes the nature of the shift. It becomes either a skew or a spatial deflection, or a combination of the two, depending on the placement of the lens. However, the general behavior of the shift, with respect to the incident angle will not be affected. Although my calculations did not consider a lens after reflection, I obtained a similar trend: a gradual roll-off behavior with respect to the incident angle. Since the experimental parameters in [20] are not complete, a full comparison cannot be made. The order of magnitude and the behavior strongly indicate that the cause of the experimental results is indeed $\Theta_{\text{GH}}$.

Moreover, I calculated small spatial shifts $\Delta_{\text{GH}}$ at polarization states $p$ and $s$ (figure 2(c)) that follow the Artmann formula [31]. The values however, are too close to those of the bare sample to be distinguishable in experiments (i.e. $\Gamma_X \sim$ fraction of a nanometer).
In the vicinity of Brewster’s angle, the $\theta_{\text{GH}}$ magnitude is huge (figure 2(c)) when light strikes from air. This can be exploited when determining graphene’s conductivity using beamshift with a $p$-polarized beam. However, at very near Brewster’s angle, the intensity will be diminished which will make it difficult to detect. There is a parasitic signal from cross-polarization [32], which will be troublesome as the theoretical equation derived in [1] is not sufficient. The $\Gamma_{\text{GH}}$ values when using $s$-polarized light also give easily detectable beamshifts (figure 2(c)).

The dimensionless spatial IF shifts (figures 3(a) and (b)) and angular IF shifts (figure 3(c)) are expected for all polarization states, again, except for the angular IF shifts for 45° polarization (figure 3(d)). The $\Delta_{\text{IF}}$ occurs only for $\sigma^+$ (figure 3(a)) and the $\Theta_{\text{IF}}$ occurs only for 45° (figure 3(c)) for external reflection. Measuring $\Delta_{\text{IF}}$ could be daunting because at maximum difference, it is only about 2.5 nm greater than in the case of $\Delta_{\text{IF}}$ with a bare substrate. The calculation of non-zero $\Theta_{\text{IF}}$ similar to $\theta_{\text{GH}}$ in internal reflection has not been reported in the literature. This could be exploited in determining the optical conductivity of graphene.

It will be instructive to give the differential beamshifts in physical units with and without the graphene ($\Delta_{\text{IF,x,y}} = \Gamma_{\text{GH,x,y}} - \Gamma_{\text{bare}}$). Here, I calculated the shifts using a beam with a HeNe laser wavelength ($\lambda = 632.8$ nm), a detection distance of 23 cm from the focus of a lens ($f = 70$ mm) and a beam waist of $\omega_0 = 20$ μm. The beamshifts will add up as the $\Delta_{\text{GH}}$ ($\Delta_{\text{IF}}$) and $\Theta_{\text{GH}}$ ($\Theta_{\text{IF}}$) both manifest as minute movements of the beam. The difference in the beamshifts due to the $\Delta$’s and $\Theta$’s is that the latter grows linearly with propagation. For internal reflection, the maximum differential shift in the longitudinal direction $\Delta_{\text{IF,x}}$ occurs in $p$-polarized light and is on the order of micrometers (as measured by [20]), while the maximum differential shift in transverse direction $\Delta_{\text{IF,y}}$ occurs in 45°-polarized light and is on the order of hundreds of nanometers. In the case of external reflection, $\Delta_{\text{IF,x}}$ happens with a $p$-polarized beam (also on the order of micrometers). In all these beamshifts, the $\Theta$ shifts dominate the $\Delta$ shifts. These can easily be detected with a quadrant cell.

The $\Theta_{\text{GH}}$ and the $\Theta_{\text{IF}}$ during internal reflection can be exploited to measure the $\tilde{\sigma}$. Near the critical angle, I have

Figure 2. Dimensionless GH shifts for polarization states $p$ and $s$. Spatial shifts $\Delta_{\text{GH}}$ for (a) external and (b) internal reflection and angular shifts $\Theta_{\text{GH}}$ for (c) external and (d) internal reflection. (See text for details.)
calculated the expression for $\Theta_{\text{GH}}$ from equation (2) as

$$\Theta_{\text{GH}} \approx \frac{\sigma}{n \varepsilon_0 c} \frac{2 \cos^3(\theta) \sin(2\theta)}{\left( n^2 \sin^2(\theta) - 1 + \left( \frac{\cos^2(\theta)}{n} \right)^2 \right)^2},$$

(9)

where $n$ is the index of refraction of the substrate. In this expression, $\sigma$ is just a constant factor which can be used as a parameter. Equation (9) gives values within 10% of the values without the approximation, within $\sim 8^\circ$ after the critical angle. As an order of magnitude comparison, the physical beam shift due to $\Theta_{\text{GH}}$ given the parameters above is one to two orders of magnitude greater than the correction due to the approximation.

The simplified expression for $\Theta_{\text{IF}}$ from equation (4) is given by

$$\Theta_{\text{IF}} \approx \frac{\sigma}{\varepsilon_0 c} \frac{n \sin(2\theta) \sin(\theta(n^2 - 1))}{(n^4 - n^2 - n^2 + 1) \sin^2(\theta) - n^2 + 2n^2 - 1}.$$

(10)

Again, the $\sigma$ is just a factor in $\Theta_{\text{IF}}$. Equation (10) is within 5% of the value with the approximation for all angles greater than the critical angle. With the experimental parameters given above, the correction to the physical beam shift due to the approximated $\Theta_{\text{IF}}$ given in equation (10) is on the order of a few nanometers.

The values of the physical beamshifts due to $\Theta_{\text{GH}}$ and $\Theta_{\text{IF}}$ are within experimental resolution even without the need to use weak amplification. The values in [9, 12, 20] have similar orders of magnitude to my calculations here.

4. Conclusion

In this paper, I have presented calculations for reflection beamshifts, Goos–Hänchen and Imbert-Fedorov shifts, for visible light when a monolayer of graphene is placed on a dielectric surface. The four beamshifts can be present depending on the polarization of the incident beam. The spatial Goos–Hänchen and Imbert-Fedorov shifts, $\Delta_{\text{GH}}$ and $\Delta_{\text{IF}}$ for internal reflection, as well as the angular Goos–Hänchen and Imbert-Fedorov shifts, $\Theta_{\text{GH}}$ and $\Theta_{\text{IF}}$ for external reflection are quite similar in behavior to their counterpart shifts with a bare substrate. The main results here are the nonzero $\Theta_{\text{GH}}$ and $\Theta_{\text{IF}}$ even at angles beyond the critical angle under internal reflection. I calculated shifts that can be

Figure 3. Dimensionless IF shifts for polarization states $\sigma^+$ and $45^\circ$. Spatial shifts $\Delta_{\text{IF}}$ for (a) external and (b) internal reflection and angular shifts $\Theta_{\text{IF}}$ for (c) external and (d) internal reflection. (See text for detail.)
measured relatively easily in experiments. I also derived expressions for the $\Theta_{GH}$ and $\Theta_{IF}$ shifts that isolate the optical conductivity $\sigma$ from known factors such as the incident angle, $\theta$, and the index of refraction of the substrate. These approximations are well within 10% of the non-approximated value. Measuring beam shifts could be an alternative way to determine the optical conductivity of graphene.

Acknowledgments

N Hermosa is a University of the Philippines Office of the Vice President for Academic Affairs Balik PhD program recipient (UP OVPAA BPhD 2015-06).

References

[1] Bliokh K Y and Aiello A 2014 J. of Opt. 15 014001
[2] Aiello A and Woerdman J P 2008 Opt. Lett. 33 1437–9
[3] Li C-F 2006 Phys. Rev. A 76 013811
[4] Goos F and Hänchen H 1947 Ann. Phys. 436 1–1
[5] Fedorov F I 1955 Dokl. Akad. Nauk SSSR 105 465
[6] Imbert C 1972 Phys. Rev. D 5 787–96
[7] Merano M, Aiello A, ’tHooft G W, Van Exter M P, Eliez E R and Woerdman J P 2007 Opt. Express 15 15928–34
[8] Aiello A, Merano M and Woerdman J P 2009 Phys. Rev. A 80 4
[9] Hermosa N, Nugrowati A M, Aiello A and Woerdman J P 2011 Opt. Lett. 36 3200–2
[10] Ménard J-M, Mattacchione A, Van Driel H, Hautmann C and Betz M 2010 Phys. Rev. B 82 1–0
[11] Berman P R 2002 Phys. Rev. E 66 067603
[12] Merano M, Aiello A, Van Exter M P and Woerdman J P 2009 Nat. Photonics 3 337–40
[13] Bliokh K Y, Shadrivov I V and Kivshar Y S 2009 Opt. Lett. 34 389–91
[14] Merano M, Hermosa N, Woerdman J P and Aiello A 2010 Phys. Rev. A 82 023817
[15] Aiello A and Woerdman J P 2011 Opt. Lett. 36 543–5
[16] Dasgupta R and Gupta P 2006 Opt. Commun. 257 91–6
[17] Golla D and Gupta S D 2011 Pramana J. Phys. 76 603–12
[18] Hermosa N, Merano M, Aiello A and Woerdman J P 2011 Proc. SPIE 7950 79500F
[19] Hermosa N, Aiello A and Woerdman J P 2012 Opt. Lett. 37 1044–6
[20] Li X, Wang P, Xing F, Chen X-D, Liu Z-B and Tian J-G 2014 Opt. Lett. 39 5574–7
[21] Koppens F H L, Chang D E and Javier Garcia de A F 2011 Nano Lett. 11 3370–7
[22] Stauber T, Peres N M R and Geim A K 2008 Phys. Rev. B 78 085432
[23] Zhou X, Ling X, Luo H and Wen S 2012 Appl. Phys. Lett. 101 251602
[24] Luo H, Zhou X, Shu W, Wen S and Fan D 2011 Phys. Rev. A 84 043806
[25] Zhou X, Xiao Z, Luo H and Wen S 2012 Phys. Rev. A 85 043809
[26] Jackson J D 2001 Classical Electrodynamics (New York: Wiley) III Ed
[27] Zhan T, Shi X, Dai Y, Liu X and Zi J 2013 J. Phys.: Cond. Matter 25 215301
[28] Kuzmenko A B, van Heumen E, Carbone F and van der Marel D 2008 Phys. Rev. Lett. 100 117401
[29] Peres N M R and Stauber T 2008 Int. J. Mod. Phys. B 22 2529–36
[30] Mak K F, Steir M Y, Wu Y, Lui C H, Misewich J A and Heinz T F 2008 Phys. Rev. Lett. 101 196405
[31] Artmann K 1948 Ann. Phys., Lpz. 2 87
[32] Aiello A, Merano M and Woerdman J P 2009 Opt. Lett. 34 1207–9