Nonequidistant antenna arrays synthesis based on genetic algorithm

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Abstract. A new approach to the synthesis of nonequidistant antenna arrays with uniform distribution based on the Genetic Algorithm of the global extrema search is introduced and analyzed. The technique is demonstrated for various uniform antenna array structure. The radiation pattern of the nonequidistant antenna array is formed using the Genetic Algorithm to reduce a side lobe level. A mathematical formula is presented to highlight the operation of objective function minimization.

1. Introduction

In radar applications, the antenna has to be designed and manufactured to fulfil all requirements of size, weight, manufacturability, and of signal-processing performance. The low sidelobe level (SSL) of antenna array radiation pattern is one of most important requirements.

As is known [1], equidistant spaced uniformly exited arrays have a side lobe level of about -13 dB. In order to reduce the side lobe levels, the array must be designed to radiate more power towards the center, and much less at the edges. This can be achieved through tapering (windowing) the current distribution over the face of the array. There are many possible tapering sequences that can be used for this purpose. However, as known from spectral analysis, windowing reduces side lobe levels at the expense of widening the main beam. Thus, for a given radar application, the choice of the tapering sequence must be based on the trade-off between side lobe reduction and main beam widening. For example, Dolph-Tschebyscheff tapering leads to an antenna aperture efficiency decreasing, respectively, a directivity decreases, and the efficiency of the antenna decreases at the same aperture dimensions [1].

Many analytical techniques exist for synthesizing low-sidelobe amplitude tapers for arrays. Among the optimization methods, the Genetic Algorithm (GA) is one of the most popular optimization techniques used for SSL reduction. A binary GA is appropriate to use when the amplitude weights are quantized. The GA method is a global and random search algorithm that simulates natural selection and evolution. It searches through the total solution space and can find the optimal solution globally over a domain. However, the global nature of the algorithms and the lack of derivative information cause them to converge very slowly compared to other nonglobal methods [2].

The analysis of non-uniform linear arrays based on GA optimization techniques of elements locations and the SSL suppression is presented in [3–5]. The synthesis of linear and planar arrays was considered in [6]. The results show the flexibility of GAs to solve complex problems of classical optimization techniques.

GA optimization of large aperture nonuniform array antenna element’s positions was proposed in [7], there are three parameters that can be changed in the development process, one is the spacing element...
arrangement, the other is the excitation weight of the array elements, and the third is the phase. Note, that SSL reduction doesn’t cause the mainlobe width broadening.

Planar sparse array antennas with square shapes and variable number of elements are analyzed and synthesized using GA [8]. The radiation pattern of non-uniform elements distances and non-uniform element currents distribution optimization process is discussed. The GA technique was applied for distance and current optimization, the results show the advantage of this approach such as doubled SSL seduction and lower number of elements for similar performance.

This article discusses the possibility to reduce the SLL of linear array with uniform current distribution by interelement spacing GA optimization.

2. Mathematical formulation
A linear array far field and a normalized far field can be written as follows:

\[ U(\theta) = F(\theta) \cdot \sum_{n=1}^{N} I_n e^{j\frac{2\pi}{\lambda}(n-1)\cos\theta - \cos\theta_n) \]

\[ U(\theta) = 20 \log \left| \frac{U(\theta)}{U_{\text{max}}(\theta_{p})} \right| \]  \hspace{1cm} (1)

Where \( F(\theta) \) is the far field of an element, \( i \) is the imaginary unit, \( k \) is the wave number \( (k = \frac{2\pi}{\lambda}) \), \( N \) is the element’s number, \( I_n \) is \( n^{th} \) element excitation current, and \( I_1=I_2=\ldots=I_N=I \), \( d_n \) is the distance to the \( n^{th} \) element from the array origin or interelement spacing.

It can be seen from Eq. (1) that the array far field is related to the position of the array element, so the optimization of the array elements location can improve the antenna radiation pattern and reduce the SLL. So the objective function that minimized the maximum SLL can be determined from array factor.

The main goal is to find \( d_n \) to minimized a SLL. Herewith, the first and last radiation elements remain in their own place. That is, the ‘N’ element’s array length is invariable. To solve the task, the functional is minimized:

\[ f = c_1 \left| U_{1p} - U_{p} \right| + c_2 \left| U_{2p} - U_{1p} \right| \] \hspace{1cm} (2)

Where \( U_{1p}, U_{p} \) are assigned and calculated SLL in the front radiation zone, \( U_{2p}, U_{1p} \) are assigned and calculated SLL in the backward radiation zone, \( c_1, c_2 \) are cost coefficients (Lagrange coefficients).

In the cost function given in Eq. (2) interelement spacings \( d_n \) are unknown, and GA global extremum search is applied for their minimization [2].

The linear antenna schematic radiation zone arrangement is showed on Figure 1. The front radiation zone lies in angle range of 0-90 degrees, and the backward radiation zone lies in angle range of 90-180 degrees.

\[ \text{Figure 1. Radiation zones.} \]

If the required SLL in the front and back radiation zones is the same as in cophasal array, the sum in Eq. (2) can be replaced by one term with the maximum calculated SLL.

Linear arrays directivity can be reduced and written as follows
The proposed algorithm
The main goal of this study is to design a low sidelobe radiation pattern for nonequidistant uniform linear antenna arrays with a fixed halfpower beamwidth. A considerable amount of discussion in the literature has been devoted to the problem of using GAs to optimize array element spacing. This is primarily because this class of array synthesis problems has been extremely challenging for traditional optimization methods due to the large number of variables that are typically involved. Moreover, conventional aperiodic array synthesis techniques based on statistical methods fall far short of optimal configurations.

Assume the array has an even number of elements. With this assumptions, the objective function is written as a function of the amplitude weights [9]. The GA is one of the evolutionary algorithms used to find the global extrema of the function of many variables. The principle of GA operation is based on modeling of some mechanisms of population genetics: manipulation of chromosomal set in the formation of a new biological individual by inheritance of sites of chromosomal sets of parents, accidental change of genotype (mutation). Another important mechanism is the natural selection procedure aimed to improve the fitness of the population members from generation to generation by making them more adaptable to the "survival" of individuals with certain characteristics. The function update population assigns ranks to individuals in the population generated by the union of parents and offspring. This is in order to hold the best individuals in each generation.

A GA population-based stochastic procedure was applied. The procedure for the used GA technique is described as follows.

The implementation of the basic GA can be represented as an iterative process that includes several stages:
1. The generation of the initial population.
2. The reproduction of offspring:
   (a) choice of parent,
   (b) selection and implementation of one of the crossover operators,
   (c) mutation.
3. The establishment of reproduction of the group.
4. The procedure for the selection and formation on its basis of a new generation.
5. If the stop condition is not met, the transition to paragraph 2 is made.

According to GA terminology, a specific antenna array with a given amplitude current distribution over the radiation elements is called an individual, and the set of the latter is represented as a finite population of individuals. Each individual in the population has a measure of adaptation to the environment, that is, has a certain value of the objective function. The process of finding the optimal solution is described by the process of simulated "evolution", the purpose of which is to find an individual (or a set of individuals) with the maximum fitness, that is, an individual corresponding to the optimal value of the controlled parameters \(d_n\).

For a more convenient GA application, we present these parameters as
\[
d_e = d + d_m \mathbf{X}_n
\]

Where \(d\) is the equidistant array spacing, \(d_m\) is the maximum element spacing, \(\mathbf{X}_n\) is the parametric vector whose components must be found in the optimization process. These components lie within the range [-1;1]. In fact, the objective function depends on the parametric vector, and the main goal is to find its optimal value.
At the first stage, ‘m’ antenna arrays with definite $d_n$ are formed randomly by the Monte Carlo method. Then, also by accident, ‘m1’ of antenna pairs is selected from ‘m’ antenna arrays. In the GA language, at this stage the initial population is formed, and then parent pairs are formed too. At the next stage, the numerical values of $d_n$ are converted from the decimal system to binary (transition to the genotype) and then GA algorithms are applied, that is, using the crossover operator (recombination of the bits of the corresponding pairs) and the mutation operator (replacement with some probability of each bit by its opposite value), six ‘m1’ new sets of currents are obtained. Then these currents are transferred back from the binary system to the decimal (the transition from the genotype to the phenotype), for each variant the objective function is calculated and a rank objective vector is formed, the components of which are arranged by the numerical value (‘m’ best options). If none of the selected options does not give the required value of the objective function ($f \geq 0.99$), then the cycle repeats. ‘m’ best variants of the previous cycle are used only as an initial population. If the condition ($f \geq 0.99$) is satisfied for at least one set of $d_n$, the selection of the optimal variant is completed.

4. Optimization results
The results of the objective function optimization given in Eq. (2) for three cases (N = 4, 8, 10) are shown in table 1. Interelement spacings $d_n$ are given in fractions of wavelength. Initial spacing is equal 0.5, and arrays are cophased. Consider that the linear array has N antenna elements distributed on it randomly. The element was set at each end of the array to guarantee the fixed array aperture; the remaining ‘N-2’ elements were randomly assigned to the other locations.

For N=4 SLL is reduced to -23 dB, and for N=8, 10 SLL is reduced to -21 dB only. Since the length of the antennas remains invariant, the halfpower beamwidth does not change during optimization. The Figures 2–4 depict radiation patterns of optimized arrays compared with equidistant arrays one. The pattern peaks are normalized to 0 dB for easy comparison of SLL. There is no cross-polarized radiation in the principal planes. GA patterns generally exhibit lower levels of sidelobe radiation.

The table 1 shows that some radiation elements are shifted to the right (from the array origin), and some of them are shifted to the left.

**Table 1.** The interelement spacing for cophased arrays.

| n  | Interelement spacing of n$^{th}$ element, $d_n$ |
|----|-----------------------------------------------|
|    | N = 4  | N = 8  | N =10  |
| 1  | 0      | 0      | 0      |
| 2  | 0.62683| 0.78099| 0.73037|
| 3  | 0.90835| 1.2648 | 1.2472 |
| 4  | 1.5    | 1.7132 | 1.7689 |
| 5  | –      | 2.0005 | 2.2166 |
| 6  | –      | 2.5267 | 2.5294 |
| 7  | –      | 2.8179 | 3.0332 |
| 8  | –      | 3.5    | 3.2317 |
| 9  | –      | –      | 3.8991 |
| 10 | –      | –      | 4.5    |
Figure 2. Radiation pattern of 4-elements arrays.

Figure 3. Radiation pattern of 8-elements arrays.

Figure 4. Radiation pattern of 10-elements arrays.

For linear phase arrays, SLL can also be reduced by varying the spacing of the elements. The element spacing and radiation pattern of the array that has a mainlobe position of 60 degrees are shown in table 2 and Figure 5 correspondently. The GA radiation pattern is compared with the equidistant array one. It can be seen that the mainlobe doesn’t change its width.
Figure 5. Radiation pattern of 10-elements linear phase arrays.

Table 2. The interelement spacing for linear phase array.

| n  | Interelement spacing of n$^{th}$ element, $d_n$ |
|----|-----------------------------------------------|
| 1  | 0                                             |
| 2  | 0.68298                                       |
| 3  | 1.2444                                        |
| 4  | 1.7444                                        |
| 5  | 2.1199                                        |
| 6  | 2.5645                                        |
| 7  | 2.9866                                        |
| 8  | 3.3853                                        |
| 9  | 3.9326                                        |
| 10 | 4.5                                            |

Note that the last radiation element will not be fixed, and the SLL can be further lowered. But the length of the array is reduced, respectively, directivity and antenna aperture efficiency are reduced too.

For example, table 3 shows the interelement spacing $d_n$ for 3-element arrays that has -25 dB and -30 dB of SLL, and halfpower beamwidth of 42 and 44 degrees accordingly. The Figure 6 depict their radiation patterns. For $d_n=0.5$, the halfpower beamwidth is equal 34 degrees (a dotted line).

Tables should have only horizontal rules and no vertical ones. Generally, only three rules should be used: one at the top of the table, one at the bottom, and one to separate the entries from the column headings. Table rules should be 0.5 points wide.

Table 3. The interelement spacing for 3-element array

| n  | Interelement spacing of n$^{th}$ element, $d_n$ |
|----|-----------------------------------------------|
|    | $SLL=-25\,\text{dB}$                      | $SLL=-30\,\text{dB}$ |
| 1  | 0                                          | 0                     |
| 2  | 0.41009                                     | 0.39848               |
| 3  | 0.81714                                     | 0.75697               |
The directivity of nonequidistant and equidistant antenna arrays with same radiator number can be compared by Eq. (3) MATLAB evaluation. The results are showed in table 4. The directivities difference is negligible.

Table 4. The array’s directivity.

| N  | Nonequidistant array | Equidistant array |
|----|----------------------|-------------------|
| 4  | 4.2961               | 4.2326            |
| 8  | 8.2907               | 7.8966            |
| 10 | 10.2896              | 9.7724            |

5. Conclusion
In this paper, we showed that a low SLL nonequidistant arrays can be well evaluated in MATLAB using the GA. It is possible to significantly reduce the SLL by nonequidistance introduced in the uniform array’s element positions without compromising the energy efficiency of the antenna, and optimal interelement intervals were obtained using GA. It should be noted that the optimization time for small arrays (N < 15) is not large enough. For arrays with a large number of elements, this time increases significantly, which makes the method less suitable.

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