An Alternative Method of Determining the Neutrino Mass Ordering in Reactor Neutrino Experiments

S.M. Bilenky\textsuperscript{a,b}, F. Capozzi\textsuperscript{c,d} and S. T. Petcov\textsuperscript{e,f}\textsuperscript{1}

\textsuperscript{a)} Joint Institute for Nuclear Research, Dubna, R-141980, Russia.
\textsuperscript{b)} TRIUMF 4004, Wesbrook Mall, Vancouver BC, V6T 2A3 Canada.
\textsuperscript{c)} Dipartimento di Fisica e Astronomia “Galileo Galilei”, Università di Padova, Via F. Marzolo 8, I-35131 Padova, Italy.
\textsuperscript{d)} Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Padova, Via F. Marzolo 8, I-35131 Padova, Italy.
\textsuperscript{e)} SISSA/INFN, Via Bonomea 265, 34136 Trieste, Italy.
\textsuperscript{f)} Kavli IPMU (WPI), The University of Tokyo, Kashiwa, Chiba 277-8583, Japan.

Abstract

We discuss a novel alternative method of determining the neutrino mass ordering in medium baseline experiments with reactor antineutrinos. Results on the potential sensitivity of the new method are also presented.

1 Introduction

In the present article we consider a complementary method to the one proposed in \cite{1} of determination of the neutrino mass ordering, i.e., the type of spectrum the neutrino masses obey, in medium baseline experiments with reactor antineutrinos \cite{2}. The neutrino mass ordering, as is well known, is one of the fundamental characteristics of the reference 3-neutrino mixing scheme that still remains undetermined experimentally at present (see, e.g., \cite{3}). Many basic neutrino physics observables which are planned to be measured in currently running and/or upcoming neutrino experiments, depend critically on the neutrino mass ordering. These include the CP violation asymmetry in long baseline neutrino oscillation experiments \cite{4, 5, 6}, the effective Majorana mass in neutrinoless double beta decay experiments \cite{7}, the sum of neutrino masses in the case of hierarchical neutrino mass spectrum, etc. Without the knowledge of what is the neutrino mass ordering, or the spectrum of neutrino masses, it is impossible to make progress in understanding the mechanism giving rise to nonzero neutrino masses and neutrino mixing. Determining the type of neutrino mass spectrum is one of the principal goals of the program of future research in neutrino physics (see, e.g., \cite{3, 5, 6, 8, 9, 10, 11, 12}).

Within the reference 3-neutrino mixing scheme we are going to consider, the neutrino mass spectrum is known to be of two varieties: with normal ordering (NO) and with inverted ordering (IO). The two possible types of neutrino mass spectrum are related to the two possible signs of the neutrino mass squared difference $\Delta m^2_{31(32)} \equiv m_3^2 - m_1^2(2)$,
which is associated, e.g., with the dominant oscillations of the atmospheric muon neutrinos and anti-neutrinos (see, e.g., [3]). The sign of $\Delta m^2_{31(32)}$ cannot be determined from the existing global neutrino oscillation data. In a widely used convention of numbering the neutrinos with definite mass in the two cases of $\Delta m^2_{31(32)} > 0$ and $\Delta m^2_{31(32)} < 0$ we are going to employ (see, e.g., [3]), the two possible neutrino mass spectra are defined as follows.

\textbf{i) Spectrum with normal ordering (NO):}

$$m_1 < m_2 < m_3, \quad \Delta m^2_{31(32)} > 0, \quad \Delta m^2_{21} > 0,$$

$$m_{2(3)} = (m_i^2 + \Delta m^2_{21(31)})^{\frac{1}{2}}.$$  \hspace{1cm} (1)

\textbf{ii) Spectrum with inverted ordering (IO):}

$$m_3 < m_1 < m_2, \quad \Delta m^2_{32(31)} < 0, \quad \Delta m^2_{21} > 0,$$

$$m_2 = (m_3^2 - \Delta m^2_{32})^{\frac{1}{2}}, \quad m_1 = (m_3^2 - \Delta m^2_{32} - \Delta m^2_{21})^{\frac{1}{2}}.$$ \hspace{1cm} (2)

In eqs. (1) - (4), $\Delta m^2_{31(32)}$ is the neutrino mass squared difference which is responsible for the flavour conversion of the solar electron neutrinos (see, e.g., [3]) and we have expressed the two heavier neutrino masses in terms of the lightest neutrino mass and the two neutrino mass squared differences measured in neutrino oscillation and solar neutrino experiments.

The existing neutrino oscillation data allow to determine the values of $\Delta m^2_{21}$ and $|\Delta m^2_{31(32)}|$, as well as the values of the three neutrino mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix $U_{PMNS} \equiv U$, with impressively high precision [10, 17]. The best fit values (b.f.v.) and the $3\sigma$ allowed ranges of $\Delta m^2_{21}$, $\sin^2 \theta_{12} \equiv s_{12}^2$, $|\Delta m^2_{31(32)}|$ and $\sin^2 \theta_{13} \equiv s_{13}^2$, which are relevant for our further analysis, read [10]:

$$(\Delta m^2_{21})_{BF} = 7.37 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{21} = (6.93 - 7.97) \times 10^{-5} \text{ eV}^2,$$

$$|\Delta m^2_{31(32)}|_{BF} = 2.56 (2.54) \times 10^{-3} \text{ eV}^2,$$

\begin{align}
\sin^2 \theta_{12} & = 0.297, \quad 0.250 \leq \sin^2 \theta_{12} \leq 0.354, \\
|\Delta m^2_{31(32)}| & = 2.45 (2.43) - 2.69 (2.66) \times 10^{-3} \text{ eV}^2, \\
0.0215 & \leq \sin^2 \theta_{13} \leq 0.0240 (0.0242),
\end{align} \hspace{1cm} (5, 6, 7, 8, 9)

where the value (the value in brackets) corresponds to $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$). The quoted values of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ are obtained using the standard parametrisation of the PMNS matrix (see, e.g., [3]). We have, in general: $\sin^2 \theta_{12} = |U_{e2}|^2/(1 - |U_{e3}|^2)$, $\sin^2 \theta_{13} = |U_{e3}|^2$, where $U_{e2}$ and $U_{e3}$ are elements of the first row of the PMNS matrix.

The possibility to determine the neutrino mass ordering in experiments with reactor neutrinos was discussed first in [1]. It was based on the observation made in [18] that, given the energy $E$ and the distance $L$ travelled by the reactor $\bar{\nu}_e$, the 3-neutrino mixing probabilities of $\bar{\nu}_e$ survival in the cases of NO and IO spectra, $P^{NO}(\bar{\nu}_e \to \bar{\nu}_e)$ and $P^{IO}(\bar{\nu}_e \to \bar{\nu}_e)$, differ provided $\sin^2 \theta_{12} \neq \cos^2 \theta_{12}$ and $\sin^2 \theta_{13} \neq 0$. From the data on $\sin^2 \theta_{12}$ available already in the second half of 2001 it followed that the first inequality is fulfilled. The two different expressions of $P^{NO}(\bar{\nu}_e \to \bar{\nu}_e)$ and $P^{IO}(\bar{\nu}_e \to \bar{\nu}_e)$ were used in [18] to perform a 3-neutrino mixing analysis of the data of the CHOOZ reactor neutrino experiment [19], in which the first significant constraint on $\sin^2 2\theta_{13}$ was obtained.
As is well known, $\bar{\nu}_e$ are detected in reactor neutrino experiments of interest via the inverse $\beta$-decay reaction $\bar{\nu}_e + p \to e^+ + n$. For protons at rest, the $\bar{\nu}_e$ energy $E_\nu$ is related to the $e^+$ energy $E_e$ to a good approximation via

$$E_\nu = E_e + (m_e - m_p), \quad E_\nu^{th} = m_e + (m_n - m_p) \approx 1.8 \text{ MeV},$$

where $m_e$, $m_n$ and $m_p$ are the masses of the positron, neutron and proton and $E_\nu^{th}$ is the threshold neutrino energy.

In [1] it was realised that for not exceedingly small $\sin^2 2\theta_{13}$ the difference between $P^{\text{NO}}(\bar{\nu}_e \to \bar{\nu}_e)$ and $P^{\text{IO}}(\bar{\nu}_e \to \bar{\nu}_e)$ leads for medium baselines $L$ to a noticeable difference between the NO and IO spectra of $\bar{\nu}_e$, and thus of $e^+$, measured in reactor neutrino experiments. This led to the conclusion [1] that a high precision measurement of the $\bar{\nu}_e$ (or $e^+$) spectrum in a medium baseline reactor neutrino experiment can provide unique information on the type of spectrum neutrino masses obey. In [1] most of the numerical results were obtained for the value of $\Delta m_{21}^2 = 2.0 \times 10^{-4} \text{ eV}^2$ from the existing in 2001 “high-LMA” region of solutions of the solar neutrino problem. It was found, in particular, that the optimal source-detector distance for determining the neutrino mass ordering (i.e., the sign of $\Delta m_{31}^2$) in the discussed experiment for the quoted “high” LMA value of $\Delta m_{21}^2$ is $L \sim 20 \text{ km}$. For the current best fit value of $\Delta m_{21}^2 \approx 7.4 \times 10^{-5} \text{ eV}^2$, the optimal distance is by a factor $2.0 \times 10^{-4}/7.4 \times 10^{-5} \approx 2.7$ bigger: $L \sim 54 \text{ km}$. Already in [1] it was realised that the determination of the neutrino mass ordering in the proposed reactor neutrino experiment would be very challenging from experimental point of view (see the “Conclusion” section in [1]).

A more general and detailed analysis performed in [20] revealed that a reactor neutrino experiment with a medium baseline tuned to achieve highest sensitivity in the determination of the neutrino mass ordering has a remarkable physics potential. It was found, in particular, that for $\sin^2 2\theta_{13} \lesssim 0.02$ it would be possible to measure in such an experiment also i) $\sin^2 \theta_{12}$, ii) $\Delta m_{12}^2$ and iii) $|\Delta m_{31}^2|$ with an exceptionally high precision. It was concluded that the precision on $\sin^2 \theta_{12}$ and $|\Delta m_{31}^2|$ that can be achieved in the discussed experiment cannot be reached in any of the other currently running or proposed experiments in which these oscillation parameters can be measured.

Subsequently, the possibility to determine the type of spectrum neutrino masses obey, discussed in [1] [20], was further investigated by a number of authors (see, e.g., [21, 22, 23, 24]). It was further scrutinised, e.g., in the studies [25, 26, 27, 28, 29, 30, 31, 32, 33], after the high precision measurements of $\sin^2 2\theta_{13}$ in 2012 in the Daya Bay and RENO experiments [34, 35], which demonstrated that $\sin^2 2\theta_{13} \approx 0.025$. These studies showed that the determination of the neutrino mass ordering in the experiment proposed in [1] is indeed very challenging. It requires: i) an energy resolution $\sigma/E_{\text{vis}} \lesssim 3\%/\sqrt{E_{\text{vis}}/\text{MeV}}$, where $E_{\text{vis}} = E_e + m_e$ is the “visible energy”, i.e., the energy of the photons emitted when the positron produced in the reaction $\bar{\nu}_e + p \to e^+ + n$ annihilates with an electron in the detector; ii) a relatively small energy scale uncertainty; iii) a relatively large statistics ($\sim (300 - 1000) \text{ GW}\times\text{kton}\times\text{yr}$); iv) relatively small systematic errors; v) subtle optimisations (e.g., of distances to the reactors providing the $\bar{\nu}_e$ flux, of the number of bins, etc.).

Two reactor neutrino experiments (employing liquid scintillator detectors) with medium baseline of $L \approx 50 \text{ km}$, aiming to determine the neutrino mass ordering, have been proposed: JUNO (20 kton) [3], which is approved and is under construction, and RENO50 (18 kton) [9]. In addition of the potential to measure $\sin^2 \theta_{12}$, $\Delta m_{21}^2$ and $|\Delta m_{31}^2|$ with remarkably high precision, these experiments can be used also for detection and studies of geo, solar and supernovae neutrinos.
2 Determining the Neutrino Mass Ordering in Medium Baseline Reactor Neutrino Experiments

It follows from eqs. (11) - (14) that, given the unique value of the lightest neutrino mass, \( \text{min}(m_i) \), the following relations hold:

\[
\Delta m_{31}^2 (NO) = - \Delta m_{32}^2 (IO), \\
\Delta m_{32}^2 (NO) = - \Delta m_{31}^2 (IO),
\]

where \( \Delta m_{31(32)}^2 (NO) \) (\( \Delta m_{32(31)}^2 (IO) \)) is the “atmospheric” neutrino mass squared difference in the case of NO (IO) neutrino mass spectrum. One can use either of the two sets of neutrino mass squared differences given in eq. (11) and eq. (12) in the analyses of the neutrino oscillation data\(^3\). In what follows we will use the set in eq. (11), which we will denote as \( \Delta m_{\text{atm}}^2 \):

\[
\Delta m_{31}^2 (NO) = - \Delta m_{32}^2 (IO) = \Delta m_{\text{atm}}^2 .
\]

The 3-neutrino mixing probabilities of \( \bar{\nu}_e \) survival in the cases of NO and IO neutrino mass spectra of interest, \( P^{NO}(\bar{\nu}_e \to \bar{\nu}_e) \) and \( P^{IO}(\bar{\nu}_e \to \bar{\nu}_e) \), have the following form\(^3\)\(^4\)\(^5\):

\[
P^{NO}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m_{\text{atm}}^2 L}{2 E_\nu} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_{12} \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E_\nu} \right) + \frac{1}{2} \sin^2 2\theta_{13} \sin^2 \theta_{12} \left( \cos \left( \frac{\Delta m_{\text{atm}}^2 L}{2 E_\nu} - \frac{\Delta m_\odot^2 L}{2 E_\nu} \right) - \cos \frac{\Delta m_{\text{atm}}^2 L}{2 E_\nu} \right),
\]

\[
P^{IO}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m_{\text{atm}}^2 L}{2 E_\nu} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_{12} \left( 1 - \cos \frac{\Delta m_{32}^2 L}{2 E_\nu} \right) + \frac{1}{2} \sin^2 2\theta_{13} \cos^2 \theta_{12} \left( \cos \left( \frac{\Delta m_{\text{atm}}^2 L}{2 E_\nu} - \frac{\Delta m_\odot^2 L}{2 E_\nu} \right) - \cos \frac{\Delta m_{\text{atm}}^2 L}{2 E_\nu} \right).
\]

As it follows from eqs. (14) and (15), the only difference between the expressions for \( P^{NO}(\bar{\nu}_e \to \bar{\nu}_e) \) and \( P^{IO}(\bar{\nu}_e \to \bar{\nu}_e) \) is in the coefficient of the last terms: it is \( \sin^2 \theta_{12} \) in

\(^3\)In what concerns the values of \( \Delta m_{31}^2 (NO) \) and \( \Delta m_{32}^2 (IO) \) (or of \( \Delta m_{32}^2 (NO) \) and \( \Delta m_{31}^2 (IO) \)) extracted from the data on neutrino oscillations, it should be clear that the equalities given in eqs. (11) and (12) would be valid in the cases when the oscillations take place in vacuum. If the matter effects in neutrino oscillations are non-negligible, the values of \( \Delta m_{31}^2 (NO) \) and \( |\Delta m_{32}^2 (IO)| \) (or of \( \Delta m_{32}^2 (NO) \) and \( |\Delta m_{31}^2 (IO)| \)) obtained from the data will differ, in general, since the corresponding flavour neutrino (antineutrino) oscillation probabilities depend on the neutrino mass ordering.

\(^4\)\(^5\)
the NO case and $\cos^2 \theta_{12}$ in the IO case. For the current best fit value of $\sin^2 \theta_{12} = 0.297$ quoted in eq. (6) we have $\cos^2 \theta_{12} \cong 0.703$, i.e., the coefficient under discussion in $P^{\text{IO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ is is approximately by a factor of 2.3 larger than that in $P^{\text{NO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$.

In the standard approach of evaluation of sensitivity of a medium baseline reactor neutrino experiment to the type of spectrum neutrino masses obey, apart of the specific characteristics of the detector, the $\bar{\nu}_e$ flux and energy spectrum and their uncertainties, etc., one uses as input the data on the oscillation parameter $s$ and $ar{\nu}_e$ characteristics of the detector, the $\bar{\nu}_e$ experiment to the type of spectrum neutrino masses obey, apart of the specific characteristics of the detector, the $\bar{\nu}_e$ flux and energy spectrum and their uncertainties, etc., one uses as input the data on the oscillation parameters $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ $\Delta m^2_{\text{atm}}$ and $\Delta m^2_{\nu}$, including the errors with which they are determined (see, e.g., [20, 26, 28, 29, 30]).

With the indicated inputs one performs effectively two statistical analyses of prospective data. First, employing expression (14) for $P^{\text{NO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ and thus testing statistically the hypothesis of NO spectrum, and second - using the expression (15) for $P^{\text{IO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ and testing the possibility of IO spectrum. The results of these analyses are utilised to determine the statistical sensitivity of the experiment to the neutrino mass ordering. In this standard approach the data are used to determine an observable - the neutrino mass ordering or the sign of $\Delta m^2_{\nu}$ - which is not continuous but can assume just two discrete values.

Extracting information about such a discrete observable requires somewhat non-standard statistical methods of treatment and interpretation of the data [33].

The alternative method of determining the neutrino mass ordering we are going to discussed next is based on the observation [2] that the two different expressions (14) and (15) for $P^{\text{NO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ and $P^{\text{IO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ can be written as:

$$P^{(X)}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m^2_{\text{atm}} L}{2 E_{\nu}} \right) - 2 \cos^4 \theta_{13} X^2 (1 - X^2) \left( 1 - \cos \frac{\Delta m^2_{\nu}}{2 E_{\nu}} \right) + \frac{1}{2} \sin^2 2\theta_{13} X^2 \left( \cos \left( \frac{\Delta m^2_{\text{atm}} L}{2 E_{\nu}} \right) - \frac{\Delta m^2_{\nu}}{2 E_{\nu}} \right) - \cos \left( \frac{\Delta m^2_{\text{atm}} L}{2 E_{\nu}} \right) \right), \quad (16)$$

where

$$X^2 = \sin^2 \theta_{12}, \quad \text{NO spectrum,} \quad (17)$$

$$X^2 = \cos^2 \theta_{12}, \quad \text{IO spectrum.} \quad (18)$$

The determination of the neutrino mass ordering is then equivalent to the determination of the value of the continuous parameter $X^2$ and comparing the result with the value of $\sin^2 \theta_{12}$, including its uncertainty, determined, e.g., in the solar neutrino and in KamLAND experiments. Given the fact that, according to the current data, the best fit values of $\sin^2 \theta_{12}$ and $\cos^2 \theta_{12}$ differ by a factor of 2.3, and that $\sin^2 \theta_{12}$ is determined with a $1\sigma$ uncertainty of approximately 5.5%, the proposed method of determining the neutrino mass ordering seems feasible. Moreover, since $X^2$ is a continuous parameter, one can use standard statistical methods of extracting the value of $X^2$ and its respective uncertainty from the data. The values of $\sin^2 \theta_{13}$ and $\Delta m^2_{\nu}$ measured in independent experiments can be used as input in the proposed alternative analysis of the relevant (prospective) reactor

\footnote{To simulate prospective reactor neutrino data one has to choose one of the two neutrino mass orderings (spectra) to be the “true” one.}
neutrino data. However, $\Delta m^2_{\text{atm}}$ has to be determined together with the parameter $X^2$ in the corresponding statistical analysis.

We have performed a statistical analysis to evaluate the potential sensitivity of the proposed alternative method of neutrino mass ordering determination. The analysis has the following characteristics. The “true” spectrum of events $S^*(E_{\text{vis}})$ is calculated for the best fit values of the oscillation parameters given in eqs. (5) - (9) and for either normal or inverted ordering, i.e., for $X^2 = (\sin^2 \theta_{12})_{\text{BF}}$ or $X^2 = 1 - (\sin^2 \theta_{12})_{\text{BF}}$. The statistical component of the $\chi^2$ is obtained comparing the “true” spectrum $S^*$ with a family of spectra $S(E_{\text{vis}})$ obtained by varying the parameters $(\Delta m^2_{\odot}, \Delta m^2_{\text{atm}}, \theta_{13}, X^2)$, through the equation

$$\chi^2_{\text{stat}} = \int dE_{\text{vis}} \left( \frac{S^*(E_{\text{vis}}) - S(E_{\text{vis}})}{\sqrt{S^*(E_{\text{vis}})}} \right)^2,$$

where we are taking the limit of an infinite number of bins, which is practically valid numerically with $\gtrsim 250$ bins. As a case study we consider the JUNO experiment [8], with a total exposition of $3.6 \times 10^3$ GW$\times$kton$\times$yr and an energy resolution

$$\sigma_{e}(E_{e}) = \frac{2.57 \times 10^{-2}}{E_{e} + m_{e}} + 0.18 \times 10^{-2}.$$ 

(20)

The calculation of $S$ and $S^*$ follows the approach employed in [30] [31] including a background from geo-neutrinos and two far reactors. For simplicity we assumed neutrino

Figure 1: The 1$\sigma$, 2$\sigma$ and 3$\sigma$ C.L. contours in the $\Delta m^2_{\text{atm}} - X^2$ plane obtained in a statistical analysis of prospective (simulated) reactor neutrino data from JUNO-like detector. The prospective data was generated assuming NO neutrino mass spectrum and statistics corresponding to $3.6 \times 10^3$ GW$\times$kton$\times$yr. See text for further details.
oscillations in vacuum and the presence of only one nuclear reactor contributing to the respective event rate. As systematic uncertainties we take into account three normalisation errors: one regarding the reactor flux uncertainty (3%), and two related to the normalization of the Uranium (20%) and Thorium (27%) components of the geo-neutrino flux, respectively. We are not considering energy scale [24] and flux shape uncertainties, as done, e.g., in [31], which is beyond the scope of the present study. We also include normalization errors: one regarding the reactor flux uncertainty (3%), and two related to the respective event rate. As systematic uncertainties we take into account three normalization parameters, one regarding the reactor flux uncertainty (3%), and two related to the respective event rate. As systematic uncertainties we take into account three normalization parameters, one regarding the reactor flux uncertainty (3%), and two related to the respective event rate. 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In Fig. 1 we show the 1σ, 2σ and 3σ allowed regions (for 1 degree of freedom) in the plane $\Delta m_{23}^2 - X^2$ assuming normal ordering. The red point represents the best fit point, which corresponds by construction to the values reported in eqs. (4) and (7). The value $X^2 = 1 - (\sin^2 \theta_{12})_{BP} = 0.703$ lies outside the 3σ allowed region, indicating that the type of neutrino mass ordering can be established at least at the 3σ C.L. We have checked also that the sensitivity to the neutrino mass ordering that can be achieved employing the proposed alternative method of determination of the ordering is approximately the same, and in any case not worse, than the sensitivity that can achieved using the “standard” approach [1]. The precision with which $\sin^2 \theta_{12}$ can be determined using the proposed alternative approach, as Fig. 1 indicates, is worse than the precision on $\sin^2 \theta_{12}$ that can be reached utilising the “standard” method [20]. We have found out further that the requisite detector energy resolution for successful determination of the neutrino mass ordering at least at the 3σ C.L. using the discussed alternative method is the same as that required when employing the standard approach [1, 20, 24]. These results suggest that the considered novel method of determination of the neutrino mass ordering (spectrum) can be used as a complementary method of the ordering (spectrum) determination independently of the standard method.

3 Summary

In the present article we have investigated an alternative method of determination the type of spectrum neutrino masses obey in medium baseline experiments with reactor neutrinos. The study was performed within the reference 3-neutrino mixing scheme. Within this scheme the neutrino mass spectrum, as well known, can be of two varieties: with normal ordering (NO) and with inverted ordering (IO). The two possible types of neutrino mass spectrum are related to the two possible signs of the atmospheric neutrino mass squared difference $\Delta m_{31(32)}^2$, allowed by the existing global neutrino oscillation data. The new method is based on the observation that the expressions for the relevant $\bar{\nu}_e$ survival probability in the cases of NO and IO spectra, $P^{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ and $P^{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, differ only by one factor $X^2$ in the interference term involving the solar neutrino and the atmospheric neutrino mass squared differences, $\Delta m_{\odot}^2 = \Delta m_{21}^2$ and $\Delta m_{\text{atm}}^2$ (see eqs. (13) - (16)): $X^2 = \sin^2 \theta_{12}$ in the NO case and $X^2 = \cos^2 \theta_{12}$ in the IO one, $\theta_{12}$ being the solar neutrino mixing angle. In the standard approach of determining the neutrino mass ordering (spectrum), one is supposed to use the data on the relevant neutrino oscillation parameters, $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$, $\Delta m_{\text{atm}}^2$ and $\Delta m_{\odot}^2$, including the errors with which they are determined, and the two different expressions for the $\bar{\nu}_e$ survival probability, $P^{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ and $P^{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, in the analysis of the data of the corresponding experiment
(JUNO, RENO50) with reactor $\bar{\nu}_e$. The alternative method discussed in the present article consists instead of treating $X^2$ as a free parameter, to be determined in the data analysis and the result compared with the value of $\sin^2 \theta_{12}$ found in the solar neutrino and KamLAND experiments. Since, according to the current data, the best fit values of $\sin^2 \theta_{12}$ and $\cos^2 \theta_{12}$ differ by a factor of 2.3, and $\sin^2 \theta_{12}$ is determined with a $1\sigma$ uncertainty of approximately 5.5%, the proposed new method of establishing the neutrino mass ordering appears to be feasible. Moreover, since $X^2$ is a continuous parameter, one can use standard statistical methods of extracting the value of $X^2$ and its respective uncertainty from the data.

To test the feasibility of the proposed alternative method of neutrino mass ordering (spectrum) determination we have performed a statistical analysis of prospective (simulated) reactor neutrino data obtained with JUNO-like detector. The prospective data was generated assuming NO neutrino mass spectrum and statistics corresponding to $3.6 \times 10^3$ GW $\times$ kton $\times$ yr. In the analysis both $X^2$ and $\Delta m^2_{\text{atm}}$ were determined using the simulated data. The results of our analysis show that with the “data” used as input the type of neutrino mass ordering can be established at least at the 3$\sigma$ C.L. (see Fig. 1). We have checked also that the sensitivity to the neutrino mass ordering that can be achieved employing the proposed alternative method of determination of the ordering is approximately the same, and in any case not worse, than the sensitivity that can achieved using the standard approach proposed in [1] [20]. The precision with which $\sin^2 \theta_{12}$ can be determined using the alternative method considered in the present article (as Fig. 1 indicates) is worse than the precision on $\sin^2 \theta_{12}$ that can be reached utilising the standard method (see, e.g., [20]). The analyses performed by us show also that the requisite detector energy resolution for successful determination of the neutrino mass ordering at least at the 3$\sigma$ C.L. using the discussed alternative method is approximately the same as that required when employing the standard approach, i.e., it should be not worse than approximately $3\%/\sqrt{E_{\text{vis}}/\text{MeV}}$.

The results obtained in the present study suggest that the discussed alternative method of determination of the neutrino mass ordering (spectrum) in medium baseline reactor neutrino experiments can be used as an additional independent method of neutrino mass ordering determination as well as a complementary method to the standard one discussed in [1] [20] [21] [22] [23] [24] [25] [26] [27] [28] [29] [30] [31] [32] [33].

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