Abstract: Mathematical model of an electric discharge in a fluid is developed in the paper. Cylindrical problem was solved for the active and post-discharge stages of electric discharge in water with an explicit specification of the boundary condition for pressure on the contact surface. A transition to other variables has been made. The moving boundary in new variables corresponds to the origin of coordinates. A uniform rectangular mesh was used in numerical solution. Some results of computational experiments are presented.

Key words: Electric discharge in liquid, numerical simulation, hydrodynamic parameters.

Language: English

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Introduction

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Impulse energy technology is widely used in various fields of production and processing of useful materials. The brief review presented in [1] showed the advantage of the high-voltage discharge current method in production of porous materials from titanium, niobium and tantalum powders. The recycling of useful materials, such as metals and plastics, is rated as an important process in terms of conserving resources and protecting the environment. In this regard, the use of impulse energy technology [2] in the field of recycling attracts considerable attention.
Experimental studies of the phenomena occurring in droplets of various liquids under the action of nanosecond spark discharges [3] showed the need to pay attention to hydrodynamic and physicochemical phenomena in droplets. Experimental and theoretical studies concerning partial discharges in liquids [4] showed that an additional source of ionizing radiation (x-rays) should be used for the occurrence of partial discharges under these conditions.

The use of this process in fluids has its application in the processing of various materials, in stamping, crushing, in food industry [5]. Mathematical simulation of hydrodynamic processes during an electric discharge in a liquid is a way of finding the optimal parameters of this process.

From a hydrodynamic point of view, an electric discharge in a fluid is considered as a process of cavity expansion in a fluid. The cavity dynamics can be described by an analytical solution [6] or determined by numerical methods of gas dynamics problems [7, 8]. The process of fluid flow behind the cavity can be determined by the equations of hydrodynamics [9].

Experimental studies of electric discharge in a fluid to determine the hydrodynamic parameters were carried out at a considerable distance from the source. Theoretical studies also related to the areas remote from the discharge channel.

A mathematical model of electric discharge in a fluid was proposed; it determines the change in hydrodynamic parameters, both in remote areas and close to the source of discharge.

**Statement of problem.** Consider a cylindrical model. Let the cylindrical cavity formed in a fluid as a result of the break-down of the interelectrode space expands at high velocity and sets the fluid in motion. The fluid flow is described by a system of unsteady-state equations [9] in a conservative form

\[
\begin{align*}
\frac{\partial (rp)}{\partial t} + \frac{\partial (rp\nu)}{\partial r} &= 0, \\
\frac{\partial (rp\nu)}{\partial t} + \frac{\partial \left[ r(P + \rho \nu^2) \right]}{\partial r} &= 0,
\end{align*}
\]

(1)

where \( r \) is the radial coordinate, \( \nu \) is the flow velocity, \( P \) is the pressure, \( \rho \) is the density of the medium, \( t \) is time. The system of equations (1) is closed by the Tait equation of state of the fluid in the form

\[
P = A(\rho/\rho_0)^{\chi} - B,
\]

(2)

where \( \chi = 7, A = 3.04 \times 10^9 Pa, B = A - P_0, \rho_0 = 10^3 \text{kg/m}^3, P_0 = 1.04 \times 10^5 \text{Pa} \) – is the hydrostatic pressure.

The initial conditions at \( t = 0 \) are:

\[
P(0, r) = P_0, \nu(0, r) = 0
\]

(3)

where \( a_0 \) is the initial radius of the plasma cavity or the lower boundary of fluid \( (\leq 0.1-0.15 \text{ mm}), r_b \) is the upper boundary of computational domain. These conditions correspond to the fluid at rest; the conditions imposed on the lower boundary of fluid are:

\[
P(t, a) = P_a(t), \nu(t, a) = \frac{da}{dt},
\]

(4)

To solve the problem, it is necessary to know the law of expansion of the discharge channel \( a(t) \) or the pressure at the channel boundary \( P_a(t) \). The equation of energy balance in the channel [10] is

\[
\frac{dP_a}{dt} + 2\gamma a P_a \frac{da}{dt} = \frac{\gamma - 1}{\pi l} \frac{dE}{dt},
\]

(5)

where \( dE/dt \) is the power released in the channel, \( V \) is the channel volume, \( \gamma = 1.26 \) is the effective adiabatic index.

To determine the value of \( P_a \), we substitute the expression for the volume \( V = \pi a^2 \) in (5) and obtain:

\[
a^2 \frac{dP_a}{dt} + 2\gamma a P_a \frac{da}{dt} = \frac{\gamma - 1}{\pi l} \frac{dE}{dt}.
\]

(6)

Taking into account the boundary condition (4), the latter takes the form:

\[
a^2 \frac{dP_a}{dt} + 2\gamma a P_a \frac{da}{dt} = \frac{\gamma - 1}{\pi l} \frac{dE}{dt}.
\]

(7)

The law of energy input into the discharge channel is taken in the form [10]:

\[
E(t) = \left( 1.9 \frac{t^2}{\tau_0} + 1.3 \frac{t^3}{\tau_0} - 2.2 \frac{t^4}{\tau_0^4} \right) E_0,
\]

(8)

where \( \tau_0 \) is the discharge duration, \( E_0 \) is the total energy released in the channel.

Equations (1), (2) and (5) with initial (3) and boundary (4) conditions represent a closed system of equations of electric discharge in a fluid, but they cannot be solved analytically.

**The solution of the problem.** Numerical solution of such problems is performed in a moving mesh. Numerical calculation can be carried out in a normalized interval from 0 to 1, introducing a new coordinate system:

\[
\tau = t, \eta = \frac{r - a(t)}{r_b - a(t)},
\]

where the cavity boundary corresponds to zero value of the newly introduced coordinate \( \eta \), and the upper boundary corresponds to \( \eta = 1 \).

The system of equations of fluid flow in new coordinate systems takes the form:
\[
\frac{\partial}{\partial \tau} \left[ \rho \left(a + \eta \left(r_b - a\right)\right) \right] - \frac{1 - \eta}{r_b - a} \frac{\partial}{\partial \eta} \left[ \rho \left(a + \eta \left(r_b - a\right)\right) \right] + \frac{1}{r_b - a} \frac{\partial}{\partial \eta} \left[ \rho v \left(a + \eta \left(r_b - a\right)\right) \right] = 0,
\]

\[
\frac{\partial}{\partial \tau} \left[ \rho v \left(a + \eta \left(r_b - a\right)\right) \right] - \frac{1 - \eta}{r_b - a} \frac{\partial}{\partial \eta} \left[ \rho v \left(a + \eta \left(r_b - a\right)\right) \right] + \frac{1}{r_b - a} \frac{\partial}{\partial \eta} \left[ \left(\eta \left(r_b - a\right) + a\right)(P + \rho v^2) \right] = P.
\]

The initial and boundary conditions in the newly introduced coordinates have the form:

\[
P(0, \eta) = P_b, v(0, \eta) = 0, \quad 0 \leq \eta \leq 1;
\]

\[
P(\tau, a) = P_b(\tau), v(\tau, a) = \frac{da}{dt}.
\]

The presented equations were nondimensionalized using scaled quantities — the density of a fluid at rest, the size of computational domain, and the velocity of sound in water. In the numerical solution, a uniform rectangular mesh \( t^n = nh_t, \ n = 0, 1, 2, \ldots; \eta_m = mh_{\eta}, \ m = 0, 1, 2, \ldots \) was used. For the stability of the calculation, the time derivatives were approximated by the forward differences, i.e. derivatives of \( \partial v/\partial t \) type at the point \((n h_t, m h_{\eta})\) were replaced by the difference relation:

\[
\frac{\partial v}{\partial t} = \frac{v_{m+1}^n - 0.5 \left( v_{m+1}^n - v_{m-1}^n \right)}{\eta_t}.
\]

The derivatives with respect to spatial variables \( \partial v/\partial \eta \) type are approximated by central differences

\[
\frac{\partial v}{\partial \eta} = \frac{v_{m+1}^n - v_{m-1}^n}{2h_{\eta}}.
\]

The initial condition \( v(0, \eta) = 0 \) involved in the problem generates a mesh initial condition \( v_{0}^n = 0 \).

As a result, from the continuity equation we obtain the mesh equation of density to determine the fluid density at the interior points of the computational domain,

\[
\rho_{m+1}^n = \rho_{m+1}^n \left( a^n + (m+1)h_{\eta} \left(1 - a^n\right)\right) \left( \frac{1 - mh_{\eta} a_{m+1}^n - a^n}{2 - a^n} h_{\eta} \right) + \rho_{m-1}^n \left( a^n + (m-1)h_{\eta} \left(1 - a^n\right)\right) \left( \frac{1 - mh_{\eta} a_{m-1}^n - a^n}{2 - a^n} h_{\eta} \right).
\]

In the same way, from the equations of momentum and energy balance in the discharge channel, the fluid rate and pressure between the cavity and the fluid boundary are determined. The value of \( a \) is calculated by numerical integration of the velocity \( v \) at \( \eta = 0 \).

The steps in the numerical calculations were taken as \( h_t = 0.06; \eta_t = 0.006 \). The parameters are determined by the marching method in time.

The pressure at the cavity boundary is determined from the equation of energy balance in the discharge channel. In the same way the fluid density and velocity at \( \eta = 0 \) are determined from the corresponding equations.

At the upper boundary of cylindrical domain, the no-fluid-loss condition was posed for the flow velocity, and the density and pressure were determined by the extrapolation method.

**Results of computational experiment.** The results of numerical calculations for different discharges showed that at the initial time instances the maximum values of hydrodynamic parameters are at the lower boundary. Over time, the disturbance reaches the end of computational domain. After a certain time, the disturbance moves from the end to the beginning of fluid volume. Then the pressure drops everywhere. The overall pattern of the process
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The difference lies in the parameter values and the process time. List the results obtained for one discharge. The electrical parameters of the circuit used in calculations for this discharge had the following values: 

\[ u = 6 \text{ kV}, l = 3 \text{ cm}, \tau_0 = 30 \mu s, E_0 = 2480 J, \frac{C_u}{2} = 2970 J, R_0 = 0.89 \text{ cm} \]

Figure 1 shows the changes in dimensionless fluid pressure depending on dimensionless radius for this discharge at time instants \( \tau/\tau_0 = 0.3 \) (1); 1,0 (2); 1,3 (3); 1,7 (4). As seen from the figure, at the beginning of the process the maximum pressure value is at the origin of calculated spatial coordinates, which corresponds to the cavity boundary and is 1.35. In the upper boundary of the calculation domain, the pressure at this time still retains its original value, which means that at time \( \tau/\tau_0 = 0.3 \) at points close to the upper boundary the fluid is still at rest. Further, at \( \eta = 0 \) a decrease in pressure is observed and its value gradually grows in the upper domain.

This is noticeable on the curve at \( \tau/\tau_0 = 1 \) and at this time, the maximum pressure is still in the lower boundary. This means that the fluid is still receiving force from the discharge channel. At \( \tau/\tau_0 = 1.3 \) the maximum pressure is in the middle zone, and its value is greater at \( \eta = 1 \) than at \( \eta = 0 \). After a certain time, the disturbance moves from the upper boundary to the lower one and this is clearly seen on the curve at \( \tau/\tau_0 = 1.3 \). The calculations showed that at further calculated time instants a decrease in pressure is observed in the calculation domain.

Figure 1. Change in fluid pressure

The change in fluid velocity for this discharge is shown in Fig. 2 at four consecutive time instants: 

\[ \tau/\tau_0 = 0.3 \) (1); 1,0 (2); 1,3 (3); 1,7 (4) \]. At the first fixed point in time, the value of dimensionless velocity reaches 0.5. Then, its value decreases and a reverse flow forms, explained by the collapse of the cavity.
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The change in fluid density corresponding to time instants is shown in Fig. 3. The highest density value is 1.39 and it is reached on the channel boundary at $\tau_0 = 0$. Then, the density disturbance moves to the upper boundary of computational domain.

The calculations on the electric discharge in the fluid for the cylindrical expansion of the channel showed that the key parameters of this process are the values of the energy supplied to the unit of the interelectrode distance and of the time of energy release. When developing various devices based on the phenomenon of electric discharge in a fluid, a mode can be chosen (varying these values), in which rational characteristics are obtained for the defined structure.
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Conclusion

Computational experiments have confirmed the correctness of the presented model by describing the ongoing physical processes of electric discharge in a fluid. The advantage of this model is the determination of hydrodynamic parameters close to the discharge source.

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