Mathematical model of biological tissue surface based on one-dimensional stochastic process of fractional Brownian motion

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Abstract. In this work the model for the profiling of biological tissue surfaces is introduced. The surface was considered as the realization of the stochastic process of the fractional Brownian motion (fBm) with the scale parameter σ and Hurst index H. The contour of the epidermal surface of the normal cut banana peel was studied. The magnified digital photos of the investigated specimens were made using the microscope (spatial resolution 1 µm) with the built-in camera. The function that linearly interpolated the surface contour was derived. The dispersion law of the differences between the values of the interpolating function in the adjacent knots was in good agreement with that one of the fBm stochastic process. The values of σ=0.1 and H=0.806 were obtained.

1. Introduction

Study of the surface reliefs of biological tissues can provide information about the state of tissues and the body as a whole. The surface spatial structures of wholesome and injured tissues will be different, e.g. in the papers [1, 2] it was demonstrated that the surface structures of healthy and damaged epithelial tissues considerably differ. As follows, the study of the surface reliefs of biological tissues can facilitate the detection of various pathologies.

In some methods used for studying surfaces, e.g. when studying the reflection of optical radiation, the model that characterizes the surface relief is required [3]. Conventionally, natural surfaces are described as the realizations of Gaussian stationary stochastic processes [3, 4, 5]. However, real biological tissues have its own characteristics, due to which the use of Gaussian stationary processes is challenging.

All multicellular organisms have specific cell structures. Since most cells at the tissue surface are the same in their functions and origin, in a certain area the sizes (volumes for example) of the cells will differ slightly from the sizes of its neighbors. The variation of the relief height in such area cannot be much larger than the size of the cell itself. Also, as all cells are originated by the division from some maternal ones the spatial location of any two cells will be...
interdependent. As follows, the process must be Gaussian, non-stationary and must have memory. The process of the fractional Brownian motion (fBm) satisfies these conditions. This is the Gaussian, non-stationary, continuous process \( Z(x) \) [6], which increments \( \Delta Z = Z(x_2) - Z(x_1) \) are Gaussian random variables with zero expected value \( E \) and the dispersion:

\[
D = \sigma^2 (x_2 - x_1)^{2H}
\]

(1)

The fBm has two parameters: the Hurst index \( H \) and the scale parameter \( \sigma \). The processes are correlated positively and negatively when \( H > \frac{1}{2} \) and \( H < \frac{1}{2} \) respectively:

\[
\begin{align*}
E &\left[ (Z(x_2) - Z(x_1))(Z(x_2 + \Delta x) - Z(x)) \right] > 0, & H > \frac{1}{2} \\
E &\left[ (Z(x_2) - Z(x_1))(Z(x_2 + \Delta x) - Z(x)) \right] < 0, & H < \frac{1}{2}
\end{align*}
\]

(2)

In the case \( H = \frac{1}{2} \) the fBm becomes the classical Brownian motion. As follows, the parameter \( H \) characterizes the persistence: the larger it is, the less likely that the process will change the propagation direction. So that, the larger is \( H \) the smoother is the resulting graph (see fig. 1)

![fBm implementation graphs with different H parameters and the same σ](image)

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The fBm process is non-Markov, which means that the conditional probability of \( Z(x_2) \) to reach a certain value for the given \( Z(x_1) \) should depend on the behavior of \( Z(x) \) at \( x < x_1 \).

Also, the fBm increments have the property of a statistical self-similarity:

\[
Z(x + \Delta x) - Z(x) \approx \frac{1}{r^H} (Z(x + r\Delta x) - Z(x))
\]

(3)

where \( r \) is constant. It means that the random variables of the left and right parts of the equation (3) have the same distribution, namely the same expected value and the variance. The fractality lies in this property.

The models based on the implementations of fBm are widely used for the simulations of natural [7] and, in particular, biological structures [8]. A biological tissue surface can be modeled also using fBm [2, 9]. In the paper [2] the human bone surface was studied using X-ray radiation.
and modeled using the implementation of fBm. The image resolution was 400 microns, which is not suitable for studying the relief at the cellular level (typical cell sizes are 10-50 microns), and the relief of the cell is necessary for optical applications. Also, X-rays are not suitable for studying soft tissue surfaces. So, the purpose of the current work is to build a surface profile model with the spatial resolution less than the typical cell size and without the use of ionizing radiation.

2. Object of study
In this work, the epidermis of the banana peel was studied, due to its availability and ease of the sample preparation. The crosscut of the banana peel (see fig. 2a) was prepared using the microtome. Seventeen successive surface areas were photographed using the optical microscope with the built-in camera. Using these images, the surface profile contour was reconstructed as an array of the profile function $\zeta$ values at the nodes located at the distance of one micron from each other. The graph in fig. 2c shows the digitalized profile of the surface area obtained from the image shown in fig. 2a. In fig. 2b the graph of the consistently consolidated profiles is shown.

Fig. 2 a) The magnified image of the crosscut of the banana peel and its profile; b) The digitalized surface profile; c) The digitalized contour of the surface profile from one image

3. Results
The algorithm for finding of the Hurst index was represented in the reference [6]. It is based on the analysis of the logarithmic dispersion law:

$$\log std(\Delta Z) = c + H \log \Delta x$$

Where $std(\Delta Z)$ is the standard deviation of the increments $\Delta Z$ corresponding to the interval $\Delta x$. The algorithm calculates $\log std(\Delta Z)$ for several $\Delta x$ values. The fig. 3 shows the graph of the dependence $\log std(\Delta Z)$ on $\Delta x$ for the banana peel profile. The slope of the linear approximation equals to the Hurst index.
Fig. 3 Dependence of log std (ΔZ) on log (Δx) and its linear approximation.

The scale parameter σ of the biological tissue can be chosen as the order of the difference between the maximum and minimum heights of the surface.

For the banana peel profile, the Hurst index $H = 0.803 \pm 0.002$ and the scale parameter $\sigma = 0.1$ were obtained. The fig. 4b shows the computer generated random surface profile as the realization of the fBm process with $H = 0.803$ and $\sigma = 0.1$. In the fig. 4a the real profile is given for the comparison.

Fig. 4 a) Digitalized banana peel profile; b) Surface profile generated using the fBm process with $H = 0.803$ and $\sigma = 0.1$.

4. Conclusion
In this work, it was proposed to use the implementation of the fBm process for the characterization of the surface profiles of biological tissues with the spatial resolution less than the cell size. As it can be seen from fig. 4 (a) and (b), the standard deviations of the increments ΔZ corresponding to the interval Δx of the banana tissue profile function are well described by the fBm dispersion law. Also, it can be seen that the graph of the process obtained using the found parameters is remarkably alike the real profile. As a result, the fBm process can be used to describe the profiles of real tissues, and its parameters can be used as the quantitative characteristics of tissue surfaces and its conditions.

The introduced model also provides the functional description of the biological tissue surface profiles. In particular, using this model it is possible to link the structure of the surface profile with its influence on the propagation of optical radiation.
5. References

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