Research on analysis method for lateral displacement of buried pipeline

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Abstract. Oil and gas pipeline will inevitably pass through the area such as active fault, mine goaf and soft soil where ground movement is likely to happen. In such area, the pipeline is easy to move laterally along the direction vertical to axial direction with the movement of ground. Displacement will occur at the interface of external constraint section and section with large displacement due to continuity of deformation of pipeline, i.e. there is a deflection at the end of external constraint section. In this paper, deflection equation for lateral movement of buried pipeline is derived depending on fundamental principle in structural mechanics for elastic foundation beam. It is verified by means of finite element method that the equation has high accuracy and can reflect the deformation of buried pipeline. The equation presented in this paper lays a foundation for establishing analysis method for lateral displacement of pipeline and its engineering application.

1 Introduction
The pipeline needs to pass inevitably pass through the area such as active fault, mine goaf and soft soil where ground movement is likely to happen[1][2][3]. As the ground moves, the pipeline laid in the ground will move along the direction vertical to the axial direction of pipeline, resulting to increased stress on pipeline that may threaten to the safety of pipeline. Currently, deformation calculation and stress analysis of pipeline can only by means of finite element method[4]. This method is complex and not convenient for the application by personnel of pipeline design, construction and operation. A simple and easy-to-use deflection curve equation for lateral displacement of buried pipeline is derived in this paper on the basis of mechanical theory to facilitate engineering application.

2 Simplified mechanical model
Displacement will occur at the interface of external constraint section and section with large displacement due to continuity of pipeline deformation, i.e. there is a deflection at the end of external constraint section. When the section with large displacement of buried pipeline deflects, an inverse force proportional to the deflection is imposed on pipeline by soil. This force can be represented with elastic foundational beam [5] in structural mechanics. The mechanical model of buried section and lateral displacement for lateral movement of pipeline is shown in figure 1.
The equivalent model can be established by breaking a point in mechanical model and adding moment, \( M_0 \), and support force, \( R_0 \), equal in size and opposite in direction at both sides. It is known from the condition of continuous displacement \(^{5,6}\) that both buried constraint section and laterally deformed section will produce the same rotational angle, \( \theta \), and displacement, \( \Delta \) at the broken location. A deflection curve is derived in this paper on this basis of simplified model.

3 Analysis on deflection curve for buried section of pipeline

3.1 Basic assumptions
Following basic assumptions are made for deriving the deflection curve for buried constraint section of pipeline: (1) The buried section of pipeline appears very small deformation and the deformation follows geometric linear relationship; (2) The pipe material does not yield, conforming to Hook’s law; (3) The soil does not yield. Displacement of soil and inverse force follows linear relationship; (4) soil-pipeline interaction is assumed as the linear zone of horizontal lateral soil spring in GB50470-2008 \(^3\).

3.2 Mechanical model and coordinate system
The coordinate system defining buried constraint section of pipeline is shown in figure 3.

3.3 Deflection curve for buried pipeline
Equation 1\(^{3,4}\) is a differential equation for bend beam:

\[
EI \frac{d^4v}{dx^4} = q_v
\]  

(equation 1)

where, \( E \) = elasticity modulus, in Pa; \( I \) = inertial moment, in \( m^4 \); \( v \) = deflection, in \( m \); \( x \) = the distance along axial direction of pipeline, in \( m \); \( q_v \) = distributed payload in unit length vertical to axis of pipeline, in N/m.

For buried pipeline, the distributed payload \( q_v = kv \) satisfies when the pipeline deflects. Substitute \( q_v = -kv \) into equation 1, then
\[ EI \frac{d^4v}{dx^4} + kv = 0 \]  \hspace{1cm} \text{(equation 2)}

\[ k = \frac{p_u}{X_u} \]  \hspace{1cm} \text{(equation 3)}

Where, \( k \) = stiffness/ stiffness coefficient of soil surrounding the pipe, in N/m²; \( p_u \) = lateral action force between pipeline and soil (unit length), in N/m; \( X_u \) = yield displacement of horizontal lateral soil spring, in m. \( p_u \) and \( X_u \) can be calculated as per GB50470-2008[3].

The general solution of equation 2 is

\[ v = e^{i\alpha x} (B_1 \cos \alpha x + B_2 \sin \alpha x) + e^{-i\alpha x} (B_3 \cos \alpha x + B_4 \sin \alpha x) \]  \hspace{1cm} \text{(equation 4)}

Where, \( B_1, B_2, B_3 \) and \( B_4 \) are undetermined coefficients; \( \alpha \) is characteristic value of pipeline, in 1/m.

The expression for \( \alpha \) is as below

\[ \alpha = \sqrt{\frac{k}{4EI}} \]  \hspace{1cm} \text{(equation 5)}

The relationships respectively between moment, \( M \), and shield force, \( N \), at the cross section of any point in pipeline is as follows:

\[ M = EI \frac{d^2v}{dx^2} \]  \hspace{1cm} \text{(equation 6)}

\[ N = EI \frac{d^3v}{dx^3} \]  \hspace{1cm} \text{(equation 7)}

The buried pipeline is supposed as a structure with infinite length. If \( x \) tends to infinite, the moment and shield force of the cross section, \( M \) and \( N \), created by the moment and support force, \( M_0 \) and \( R_0 \), at point O tends to 0. Combining equation 6 and 7, the undetermined coefficients in equation 3 and 4, \( B_1 \) and \( B_2 \), are all equal to zero, i.e. \( B_1 = B_2 = 0 \). Then, equation can be transformed into equation 8:

\[ v = e^{-i\alpha x} (B_3 \cos \alpha x + B_4 \sin \alpha x) \]  \hspace{1cm} \text{(equation 8)}

Suppose that the moment and support force at cross section be \( M_0 \) and \( R_0 \), then:

\[ M_{x=0} = EI \frac{d^2v}{dx^2} \bigg|_{x=0} = -2EI\alpha^2 B_3 = M_0 \]  \hspace{1cm} \text{(equation 9)}

\[ N_{x=0} = EI \frac{d^3v}{dx^3} \bigg|_{x=0} = 2EI\alpha^3 (B_3 + B_4) = R_0 \]  \hspace{1cm} \text{(equation 10)}

Combining equation 9 and 10, following equations can be derived:

\[ B_3 = \frac{R_0 + \alpha M_0}{2EI\alpha^2} \]  \hspace{1cm} \text{(equation 11)}

\[ B_4 = -\frac{M_0}{2EI\alpha^2} \]  \hspace{1cm} \text{(equation 12)}

Now, the expression of deflection curve for buried pipeline is derived.

4 Precision verification of deflection curve

| parameter                                      | symbol | unit | value  |
|-----------------------------------------------|--------|------|--------|
| diameter                                      | D      | m    | 1.219  |
| wall thickness                                | t      | m    | 0.0184 |
| elasticity modulus                            | E      | Pa   | \( 2.10 \times 10^{11} \) |
| Horizontal lateral action force between soil and pipeline | p_u   | kN/m | 185.103 |
Yield displacement of horizontal lateral soil spring | X_0 | m | 0.096
--- | --- | --- | ---
Pipe tip lateral force | H0 | N | 400000
Pipe tip moment | M0 | N.m | 260000

Table 2 summarizes the displacements of all points along axis of pipe calculated with the analytical equations derived in this paper and the results calculated with finite element method. The displacement curve for all points along the axis of pipeline is plotted in figure 4.

| x (m) | Displacement (m) analytical | Displacement (m) Finite element | Displacement (m) analytical | Displacement (m) Finite element | Displacement (m) analytical | Displacement (m) Finite element |
|-------|-----------------------------|---------------------------------|-----------------------------|---------------------------------|-----------------------------|---------------------------------|
|       | x (m)                        |                                 | x (m)                       |                                 | x (m)                       |                                 |
| 0     | -0.0519                      | -0.0508                         | 16                          | 0.0029                          | 0.0028                       | 32                              | 0.0010                          | 0.0010                          | 48                              | -0.0002                         | -0.0001                         |
| 2     | -0.0394                      | -0.0383                         | 18                          | 0.0036                          | 0.0035                       | 34                              | 0.0006                          | 0.0006                          | 50                              | -0.0001                         | -0.0001                         |
| 4     | -0.0281                      | -0.0272                         | 20                          | 0.0037                          | 0.0036                       | 36                              | 0.0003                          | 0.0003                          | 52                              | -0.0001                         | -0.0001                         |
| 6     | -0.0186                      | -0.0180                         | 22                          | 0.0035                          | 0.0034                       | 38                              | 0.0001                          | 0.0001                          | 54                              | -0.0001                         | -0.0001                         |
| 8     | -0.0111                      | -0.0106                         | 24                          | 0.0031                          | 0.0029                       | 40                              | 0.0000                          | 0.0000                          | 56                              | -0.0001                         | -0.0001                         |
| 10    | -0.0054                      | -0.0051                         | 26                          | 0.0025                          | 0.0024                       | 42                              | -0.0001                         | -0.0001                         | 58                              | -0.0001                         | 0.0000                          |
| 12    | -0.0013                      | -0.0012                         | 28                          | 0.0020                          | 0.0019                       | 44                              | -0.0001                         | -0.0001                         | 60                              | 0.0000                          | 0.0000                          |
| 14    | 0.0013                       | 0.0013                          | 30                          | 0.0014                          | 0.0014                       | 46                              | -0.0002                         | -0.0001                         |                                 |                                 |                                 |

Figure 4 Displacement curve for all points along the axis of pipeline

It is indicated from the calculated results that the tendency of displacement and rotation angle calculated from analytical method derived in this paper is consistent with those by finite element method. The maximum error ratio is 2.17%, which is less than 5%, indicating better precision.

5 Conclusion
Equations of deflection curve for displacement of buried pipeline are derived in this paper on the basis of fundamental principle of elastic foundation beam in structural mechanics. The precision of these equations is verified by means of finite element method. The results show that the equations deflection curve derived in this paper have high precision. The deformation tendency is consistent with the results from finite element method, which can reflect the deformation of buried pipeline.

References
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