Collapse of Geometrically Imperfect Stainless Steel Tubes under External Hydrostatic Pressure

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Abstract
This paper reports on an investigation into the buckling behaviour of 5 geometrically imperfect duplex stainless steel tube models, subjected to external hydrostatic pressure. The research was partly experimental and partly theoretical, where in the latter case; both analytical & numerical theoretical analyses were carried out.

The experiments were carried out on five stainless steel tube models of different lengths, using two mild steel end bungs to seal the models. The experimental results showed that the Duplex stainless steel specimens behaved similarly to other isotropic materials tested by other researchers.

The theoretical calculations and analyses were made by MisesNP, a DOS-based computer program, together with the ANSYS finite element structural analysis software.

Combining the results of the present series of models, together with the results of other experimenters, a design chart was produced, which can be used for designing full-scale vessels. It should be emphasised that this design chart has been extended to those from previous studies, so that shorter and thicker vessels can now be designed.

Keywords: buckling, shell instability, stainless steel, design chart, ANSYS.

1. Introduction.
According to Dickens et al [1], there are about 10,000 billion tonnes of methane lying in the deep seas in the form of methane hydrates, from some 3000m to 11000m underwater. The value of this methane is about 536 times the GDP of the USA or about $1,250,000 per person on Earth. The main reason why this methane has not been exploited before is because of the very high hydrostatic pressures in the great depths of the oceans, where in the case of the Mariana Trench, it is about 1,100 bar Ross [2].

Today’s materials of construction of submersibles consist in the main, of materials similar to those used over a century ago. Over this period, however, technology has advanced and this is why the present study has been made.

The challenge is that larger Human Occupied Vehicles could be designed and built, with the ability to carry a greater complement at greater depths of water. The Royal Naval submarines can have a displacement of about 16,000 tonnes and a length of about 149.5
metres, which are substantial sizes. However, their operational depth is limited to about 400 m, while the maximum ocean depth is some 29 times this value!
There are not many materials that can withstand high external hydrostatic pressure and this is why this research has been conducted. Titanium, for example, has the highest strength-weight ratio of structural metals; additionally, it has a good corrosion resistance, making it a desirable material for building submersibles; however, it is a very expensive material to use. When a pressure vessel collapses under external hydrostatic pressure, it usually collapses through non-symmetric bifurcation buckling (lobar buckling) or shell instability [2], as it is sometimes called. When this mode of failure occurs, the buckling pressure is often a small fraction of that for the same vessel to fail under uniform internal pressure. Another mode of failure for pressure vessels is through axisymmetric yield, with the vessel imploding axisymmetrically, keeping its circular form while collapsing.
To improve the buckling resistance of these vessels to the shell instability mode of collapse, it is usual to stiffen these vessels with ring stiffeners. If the ring stiffeners are not strong enough, the entire ring-stiffened combination can fail by buckling bodily in its flank; this failure is called general instability [2].
In this paper, only collapse by shell instability is studied, because this is the most common mode of failure of such vessels. Both the well-known analytical formula by von Mises [3], as shown by equation (1) and the commercial computer package ANSYS [4] are utilised in the analysis. In the case of the von Mises formula, the number of lobes (n) has to be changed, while calculating the buckling pressure corresponding to each value of ‘n’, until the lowest buckling pressure is found, namely $P_{cr}$.

$$p = \frac{Eh}{R} \left[ \frac{1}{n^2 + \left(\frac{n\pi}{L}\right)^2} \right] + \left[ \frac{\left(\frac{n\pi}{L}\right)^4}{n^2 + \left(\frac{n\pi}{L}\right)^2} \right] + \left[ \frac{\left(\frac{h}{R}\right)^2}{12(1-\mu^2)} \right] \left[ \frac{\left(\frac{n\pi}{L}\right)^2}{n^2 + \left(\frac{n\pi}{L}\right)^2} \right] \right]^{0.25} \times \sqrt{\frac{\sigma_{yp}}{E}}$$

where,

p = external pressure
n = the number of circumferential waves or lobes that the vessel buckles into.
R = mean radius
H = wall thickness
L = unsupported length of the circular cylinder
E = Young’s modulus
μ = Poisson’s ratio

Additionally, the thinness ratio, produced by Windenburg and Trilling [5] as shown by equation (2), has to be determined to allow for the effects of initial out-of-circularity, which further decreases the buckling pressure if the vessel fails through inelastic shell instability, where

$$\lambda = \frac{E}{d} \left(\frac{L}{d}\right)^2 \left(\frac{t}{d}\right)^3 \sqrt{\frac{\sigma_{yp}}{E}}$$

where

$L$ = unsupported length of the circular cylinder
$D$ = diameter
$t$ = Shell thickness
Using the above, it is possible to create a design chart by plotting the reciprocal of ‘λ’ against the experimentally determined Plastic Knockdown Factors (PKD). Once the design chart is obtained it is possible to predict the theoretical inelastic buckling pressure, by calculating the theoretical elastic buckling pressure ($P_{cr}$) for a perfect vessel, together with its ‘λ’, and from the Design Chart to determine PKD for that particular vessel. Once PKD is determined, the predicted collapse pressure ($P_{EXP}$) for that vessel can be obtained by dividing $P_{cr}$ by PKD, where:

$$ PKD = \frac{P_{cr}}{P_{EXP}} $$

$P_{cr}$ = theoretical elastic instability buckling pressure

$P_{EXP}$ = experimental buckling pressure

2. Test Models and Material Properties

2.1 Test Models

The models used for the tests were made from a Duplex stainless steel tube, which was cut into the required five lengths, as shown in Fig. 1.

The end bungs are shown in Fig. 1. The major dimensions for specimens can be seen in Table 1. The dimensions shown in Table 1 are later optimised for use with ANSYS and MisesNP. The unsupported length was taken as the overall length $L_0$ of the model, minus the depth of the mid-location of each O-ring from the ‘edge’. For both MisesNp and ANSYS, the boundary conditions were assumed to be simply-supported for all the models. The initial out-of-roundness was measured on a Mitutoya Co-ordinate Measuring by an experienced senior technician.
Table 1: Specimens’ Dimensions

| Model | Overall Length $L_o$ [mm] | Unsupported length $L$ [mm] | Mean Radius [mm] | Thickness [mm] | Out-of-Roundness [mm] |
|-------|---------------------------|-----------------------------|------------------|--------------|---------------------|
| TB1   | 50                        | 40                          | 34.04            | 1.62         | 0.0808              |
| TB2   | 60                        | 50                          | 34.04            | 1.62         | 0.0700              |
| TB3   | 70                        | 60                          | 34.04            | 1.62         | 0.1433              |
| TB4   | 80                        | 70                          | 34.04            | 1.62         | 0.0868              |
| TB5   | 100                       | 90                          | 34.04            | 1.62         | 0.0440              |

2.2 Material properties of the ‘Duplex’ stainless steel.
The Young’s Modulus was determined by testing a thin-walled circular ring under diametric compression, as shown in Fig. 2.

![Image of a circular ring](image)

Figure 2: Method of determining the experimental value for Young’s Modulus

The Load-Deflection relationship for the diametrically loaded circular ring of Fig. 2 is shown in Fig. 3.
The calculation for the Young’s Modulus was made by using Roark’s formulae [6], where flexural, axial and shear deflections were accounted for, but not anticlastic curvature. However, the error in not accounting for anticlastic curvature would have been very small as the decrease in curvature for a diametric deflection of about 1 mm was only about 1.5%.
The measured value of ‘E’ was as follows:

\[ E = \text{Young’s modulus} = 146.2 \, \text{GPa} \]

The yield stress and Ultimate Tensile Strength (UTS) were obtained from two ‘flat’ tensile test specimens, which were cut along the meridian of an unused length of the tube. The following values of UTS and yield point were found:

\[ \text{UTS1} = 527.9 \, \text{MPa} \]
\[ \text{Yield Stress1} = 243.7 \, \text{MPa} \]
\[ \text{UTS2} = 544.1 \, \text{MPa} \]
\[ \text{Yield Stress2} = 251.0 \, \text{MPa} \]
The experimentally determined values for UTS and Yield stress agreed with ‘book’ values and the measured value for ‘E’ agreed with values obtained by other experiments on different sized rings made from Duplex stainless steel.

3. Experimental Testing

This section records the various steps in the experimental testing of the buckling behaviour of the five Duplex stainless steel models when the external hydrostatic pressure was gradually increased until failure of each specimen took place.

3.1 Equipment

- Test tank; capable of withstanding pressures up to 210 bar
- Hydraulic pump; capable of exerting a pressure of up to 410 bar
- Bourdon tube pressure gauge
- Connecting hose
- Water

3.2 Methodology

The hose of the hand-operated pressure pump was connected to a two-part heavy stable cylinder container (“the tank”) in which the models were placed, one at a time, and surrounded with water and covered by the tank top or closure plate. The closure plate was placed on the top of the tank and was sealed by tightening the retaining bolts through the 16 holes distributed equally around the circumference of the closure plate and the top of the tank. The pressure pump was able to exert a maximum pressure of up to 410 bar.

It was important to be able to identify each model and its buckling effects individually throughout the experimental testing and thus, the five models were named TB1 to TB5 respectively, in increasing values of length, as shown in Fig. 1. The reservoir of the pressure
pump was filled with water. The first specimen to be tested, namely, TB5, was a cylindrical shape of stainless steel 100mm length, with an outer diameter of 69.82mm and thickness of 1.62mm. A mild steel bung was inserted firmly into each end of the cylindrical model, to ensure that it remained watertight throughout the experimental testing.

3.2.1 Preparation. Model TB5 was placed gently and vertically into the tank. The tank was then filled with water to about 2cm below its rim. The ‘O’ ring was then fitted into the retaining groove around the circumference of the inside surface of the closure plate. The closure plate was then placed on top of the tank and firmly screwed down by 16 bolts, to seal the system, apart from the bleed screw, which was not in its normal place. With the bleed screw removed, water was pumped into the tank until all the trapped air had escaped through the bleed hole. Each model was then tested to destruction. Each model collapsed with a distinct ‘bang’ and/or an associated pressure drop. Each model was inspected after collapse and the collapse pressure noted from the reading of a Bourdon tube pressure gauge. Each model was photographed after its collapse, as shown in Fig. 4.

![Figure 4: TB5 to TB1 Models (left to right), vertical view after failure](image)

3.3 Experimental Results
The experimental results are given in Table 2, where it can be seen that the shorter models had higher buckling pressures than the longer ones. This is especially true in the case for Specimen TB1, which had a much higher buckling pressure than TB2; this was because its unsupported length was relatively much shorter than that of TB2.

4. Theoretical Investigation
Two theoretical investigations were carried out; one was using MisesNP, a DOS computer program, based on a closed form trivial solution of von Mises. The other theoretical investigation was via the commercial computer package, namely ANSYS; this was based on the Finite Element Method. The circular cylinders were modelled with about 1000 elements for each circular cylinder, where each cylinder was assumed to be perfect. The elements used were the well-known 8-node thin-walled Shell 93 ANSYS quadrilateral elements, which had 6 degrees of freedom per node. The ANSYS analysis was based on a linear Eigen-buckling elastic analysis.

4.1 MisesNP
For this program the input data consisted of the unsupported length of the cylindrical shell, the mean radius of the model, the material thickness, the Young’s Modulus, the Poisson’s Ratio and the yield stress; it was simple to use. The program outputted the buckling pressures, together with the associated number of circumferential waves or lobes, namely ‘n’. The program also outputted the Windenburg thinness ratio of Equation (2).
Table 2: Experimental results.

| Model | $L_0$ (mm) | $L$ (mm) | $P_{EXP}$ (bar) | Comments     |
|-------|------------|----------|----------------|--------------|
| TB5   | 100        | 90       | 144.79         | Biggest Noise|
| TB4   | 80         | 70       | 150.31         | Big Noise    |
| TB3   | 70         | 60       | 151.69         | Big Noise    |
| TB2   | 60         | 50       | 159.96         | Low Noise    |
| TB1   | 50         | 40       | 191.67         | No Noise     |

4.2 ANSYS
ANSYS has many finite element analysis capabilities, enabling it to solve many complex problems and structures in various fields. In this case, the 8-node quadrilateral Shell 93 finite element was used in a linear ‘Eigen buckling’ analysis. The buckled form of a model is shown in Fig. 5.

Figure 5: Buckled form of the cross-section of a model, showing its lobar pattern.

5. Theoretical and Experimental Results.
The results from the experimental and theoretical investigations are shown in Table 3, where the symbols ‘DTMB’ represent the David Taylor Model Basin formula; this is a simplified version of that of the von Mises formula of Equation (1). From Table 3, it must be noted that the theoretical buckling pressures ($P_{cr}$) were all based on elastic theory for perfect vessels and this is why the plastic knockdown factor (PKD), from the Design Chart, must be divided into these theoretical values to give the predicted or experimental buckling pressures ($P_{EXP}$). From Table 3, it should also be noted that ANSYS predicted slightly higher buckling pressures than von Mises and the DTMB formulae; this was because in the ANSYS analysis, the eight node quadrilateral element was used and these could not truly represent the circumferences of the vessels as perfect circles. The problem was worsened because the vessels collapsed around their circumferences. Once the Design Chart is available, it is used as follows: Calculate $P_{cr}$ and $\lambda$ from MisesNp and using the value of $\lambda$, determine PKD from the design Chart. The predicted buckling pressure, namely $P_{EXP}$, can then be obtained by dividing $P_{cr}$ by PKD.
Table 3: Theoretical and Experimental buckling pressures.

| Model | Length L [mm] | $P_{cr}$ [MPa] | $\lambda$ | $P_{EXP}$ [MPa] | PKD | DTMB [MPa] |
|-------|---------------|----------------|---------|-----------------|------|-------------|
| TB1   | 40            | 63.163         | 0.516   | 19.167          | 3.295| 5           |
| TB2   | 50            | 51.087         | 0.577   | 15.996          | 3.194| 6           |
| TB3   | 60            | 40.198         | 0.636   | 15.168          | 2.650| 5           |
| TB4   | 70            | 33.748         | 0.683   | 15.031          | 2.245| 4           |
| TB5   | 90            | 27.624         | 0.774   | 14.479          | 1.908| 4           |

5.1 Final Design Chart

From Table 3, the Plastic Knockdown Factors (PKD) can now be calculated with the values of the experimentally ($P_{EXP}$) and theoretically ($P_{cr}$) obtained buckling pressures and the Design Chart can now be produced, where:

$$PKD = \frac{P_{cr}}{P_{EXP}}$$

Table 4 gives the PKD’s and reciprocal thinness ratios for the present series of specimens, which together with the results from other experimenters [2], was used to produce the Design Chart of Fig.6. This Design Chart allows shorter and thicker vessels to be analysed, which was not possible with previous Design Charts [2].

Table 4: Results of plastic knockdown factor, thinness ratio and out-of-roundness

| Model | Length L [mm] | $P_{cr}$ [MPa] | $\lambda$ | $P_{EXP}$ [MPa] | PKD | 1/$\lambda$ | Out of Roundness [mm] |
|-------|---------------|----------------|----------|-----------------|------|-------------|-----------------------|
| TB1   | 40            | 63.163         | 0.516    | 19.167          | 3.295| 1.937       | 0.0808                |
| TB2   | 50            | 51.087         | 0.577    | 15.996          | 3.194| 1.733       | 0.0700                |
| TB3   | 60            | 40.198         | 0.636    | 15.168          | 2.650| 1.572       | 0.1433                |
| TB4   | 70            | 33.748         | 0.683    | 15.031          | 2.245| 1.465       | 0.0868                |
| TB5   | 90            | 27.624         | 0.774    | 14.479          | 1.908| 1.292       | 0.0440                |

6. Conclusions

Both the experimental and theoretical investigations of the five Duplex stainless steel tube specimens were performed successfully. The main findings are listed as follows;

- The experimentally obtained Ultimate Tensile Strength and Yield Stress agreed with ‘book’ values for Duplex stainless steel.
Figure 6: Design Chart using the von Mises results.

- The experimentally obtained Young’s Modulus agreed with values obtained by other experimenters, who used circular rings with different dimensions to those used in the present study.
- The design chart was created successfully for use with MisesNP; it appears to be of similar form to other design charts determined at the University of Portsmouth.
- TB1 buckled symmetrically about its cross-section, leaving a near-perfect lobar buckled model, where \( n = 6 \).
- The results of MisesNP have been favoured over those of ANSYS as they were generally more accurate when compared with the experimental results.
- ANSYS predicted slightly higher buckling pressures than von Mises; this was because ANSYS used eight node quadrilateral elements, which did not model the circumferences of the specimens as being perfectly circular and it must be emphasised that the vessels collapsed around their circumferences.
- The time and effort needed to complete an ANSYS analysis was significantly greater than when MisesNP was used.
- The design chart created here should prove useful to designers, if the results for new structures have the similar characteristics as those investigated here.
- The design chart also allows shorter and thicker vessel to be analysed, which was not possible before.
- Designers in this field have found BS 5500 to be too conservative in the design of circular cylinders suffering shell instability under external hydrostatic pressure; moreover they find BS 5500 more difficult to use than the method presented here.
- The initial out-of-circularity of the vessels should not exceed 0.16t, where \( t \) = the wall thickness of the vessel under consideration.
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