The equivalent elastic parameters: A lozenge grid structure with negative Poisson’s ratio

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Abstract. A new kind of chiral stent was designed by three-dimensional lozenge grid structure structure with four ligaments. Based on the conventional beam theory and energy conservation principle, in-plane Poisson’s ratio and equivalent Young’s modulus are derived, and finite element analysis was employed to validate the analytical estimates of the elastic module. Then the FEM models are used to investigate the elastic deformation behaviours of lozenge grid structure. The calculated results of equivalent elastic modulus based on beam theory are in consistent with those obtained by finite element method. The theoretical and numerical results show that lozenge grid structure stent proposed in this paper is negative Poisson’s ratio material. The presented theory could be used in designing new hierarchical honeycomb structures for multifunctional applications.

1. Introduction

Negative Poisson’s ratio (NPR) material has long been widespread concern of scholars due to the unique mechanical features. An auxetic structure with enhanced mechanical properties is designed and manufactured through the tailored negative Poisson’s ratio, which aims to treat oesophageal cancer and prevent dysphagia [1]. Two types of innovative chiral stents, lozenge grid structure stent with circular and elliptical nodes and hierarchical lozenge grid structure stents with circular and elliptical nodes, are proposed, which exhibit remarkable radial expansion ability while maintaining axial stability and has a good application prospect in clinical application [2]. Besides, the blood flow in the initial and dilated artery models are simulated by computational fluid dynamics (CFD) [3], which provides personalized design of stent and interventions for the actual situation of patients. Also, a semi-interpenetrating network with better biological activity is developed. The repeated unit cell model is presented based on its repeatability, which is used to research the stent’s global deformation [4]. Finite element method is also used to study flexibility and fatigue performance of stent besides its expansion deformation behaviour. Methods for measurement of stent flexibility are proposed based on finite element method [5]. Dumoulin believes that fatigue of stainless steel stent is a small probability event according to the result of simulation [6]. Stolpmann takes the influence of residual stress after inflation and micro-structure on the fatigue of stent into consideration, and thinks general fatigue theory is not suitable for study of stent [7]. Perry [8-9] researches the fatigue of stent by using finite element method and considers finite element method is vital for studying it. Two navel structures are proposed, one is combining re-entrant honeycomb and star-shape honeycomb named as re-entrant star-shaped honeycomb, and another...
is star-arrowhead honeycomb by combining double arrowhead honeycomb cells and star-shaped honeycomb [10], both of which have enhanced impact resistance.

Based on the research of scholars, a lozenge grid structure with negative passion’s ratio aiming to get the large deformation is proposed in this paper.

At the same time, the equivalent Young's modulus and Poisson's ratio of the structure are deduced based on the principle of energy conservation, and the analytical formulas in the linear elastic range are obtained. Finally, the relationship between geometric variables and equivalent elastic parameters is studied.

2. Lozenge grid structure geometry models and linear elastic behaviour

2.1. Lozenge grid structure geometry

Figure 1 shows a typical unit cell of the lozenge grid structure. The parameters $L_x/2$, $L_y/2$, $r$, and $t$ represent the length along the $x$ and $y$ directions, the circle radius and the rod thickness, respectively. The letter $b$ is the cell depth. Structure parameters in detail are shown in Table 1.

![Figure 1. Cell unit.](image1)

| Length $l$ (mm) | Angle $\alpha$ ($^\circ$) | Width $t$ (mm) | Outer Radius $R$ (mm) | Inner Radius $r$ (mm) | Thickness $b$ (mm) |
|----------------|--------------------------|----------------|-----------------------|----------------------|--------------------|
| $\pi/6$        | arctan(1/2)              | 0.01           | 0.05                  | 0.03                 | 0.01               |

2.2. Analytical formulation

In this paper, the new negative Poisson's ratio structure- lozenge grid structure- is proposed. The schematic diagram of its configuration is shown in figure 1.

![Figure 2. Equivalent beam model.](image2)

According to beam theory, the equivalent beam model of structure is established (figure 2). The model uses the cantilever beam calculation method. First, the relationship between curvature and bending moment is obtained. Then, the relationship between curvature and deformation angle is
deduced by deflection calculation formula. Finally, the strain and angle formulas are established (equation (1)). The equivalent process is shown in figure 1. The formula is as follows:

$$\varepsilon = \frac{2t}{L} \varphi$$  \hspace{1cm} (1)

The equivalent elastic parameters of the proposed structure are deducted in theoretical ways, and the following assumptions are used to derive the model formula.

1. There are only small deformation structures in the rod, and the nodes in the configuration are completely rigid rotating elements.
2. The traditional beam theory is employed to derived equivalent Young’s modulus and Poisson’s ratio of lozenge grid structure.
3. The circle is considered as rigid body.

Assuming that the unit cell is subjected to an angle along x and y direction and the strain of the unit cell is analysed. Based on assumptions (1) and (2), the x direction strain is $\varepsilon_x$ and y direction is $\varepsilon_y$, and they can be expressed as (equations (2) and (3)):

$$\varepsilon_x = \frac{2(R-r-t/2)}{L_1} (i=x,y)$$  \hspace{1cm} (2)

$$\varepsilon_y = \frac{2(R-r-t/2)}{L_2} (i=x,y)$$  \hspace{1cm} (3)

According to the definition of material Poisson’s ratio, the ratio $\nu_{xy}$ is expressed as:

$$\nu_{xy} = \frac{\varepsilon_x}{\varepsilon_y}$$  \hspace{1cm} (4)

Meanwhile, substituting equations (2) and (3) into equation (4), the Poisson’s ratio is reduced to $\nu_{xy}=-1$. It’s demonstrates that the lozenge grid structure is a negative Poisson’s ratio structure.

According to the principle of energy conservation, the linear elastic formula of the lozenge grid is deduced. We assume that the energy generated by the small strain $\varepsilon_i$ $(i=x,y)$ is equated to the equivalent beam energy $W_i$ stored in the unit cell (figure 3). Equation (4) is established according to the principle of conservation of energy (equation (4)) and Hooke’s law of linear elasticity material (equation (6)). And the volume of equivalent beam is expressed by equation (8).

Figure 3. Stent deformation versus radius.

$$\int_0^1 \int_D \frac{1}{2} \sigma \varepsilon_i dV = U_{\varepsilon_i}$$  \hspace{1cm} (5)
\[
\sigma_i = E_i \epsilon_i \quad (6)
\]
\[
\frac{1}{2} E_i \epsilon_i^2 = \frac{4}{V} W_x + \frac{4}{V} W_y \quad (7)
\]
\[
V = 4L_e L_o b \quad (8)
\]

The edge length of the equivalent beam can be equal to the rod length of lozenge grid structure minus the diameter of the circular hole in the model. The effective length is expressed by \( L_{ex} \) and \( L_{ey} \).

According to the geometric relationship of the rod in Figure 1, the effective length \( L_{ex} \) and \( L_{ey} \) are equal to:

\[
L_{ex} = L_x - 2R - 2r \quad (9)
\]
\[
L_{ey} = L_x - 2R - 2r \quad (10)
\]

According to beam theory, the equal moments \( M \) which act on each end of the rod will produce an angular deflection \( \theta \). The strain energy of the beam can be written as:

\[
W = \int_0^\theta M d\theta \quad (11)
\]

According to the effective length \( L_{ei} \) (i=x,y) and the equivalent rectangular section of the rod, the rod’s second moment of inertia is:

\[
\theta = \frac{ML_{ei}}{2E_i I} \quad (12)
\]
\[
I = \frac{bt^3}{12} \quad (13)
\]

Where, \( E_i \) is the material elastic module of equivalent beam. Substituting equations (12) and (13) into equation (11), a new formula for equivalent beam energy \( W \) is derived:

\[
W = \frac{E_i bt^3 \theta^2}{6L_{ei}} \quad (14)
\]

The small strains \( \epsilon_x \) and \( \epsilon_y \) are applied to the unit cell. Substituting equations (2), (3), and (14) into equation (7), a new energy expression is obtained by:

\[
\frac{1}{2} E_i \epsilon_i^2 = \frac{4E_i bt^3 \theta^2}{6L_{ei}V} \quad (15)
\]

Where, \( E_x \) and \( E_y \) are the Young’s moduli of lozenge grid structure. Equation (16) and equation (17) can be rewritten as:

\[
E_x = \frac{E_i t^3 L_x^2}{3L_{ex}^2 L_{ey} (R - r - \frac{t}{2})^2} \quad (16)
\]
\[ E_s = \frac{E_t l^2}{3L_y^2L_y (R - r - \frac{t}{2})^2} \]  \hspace{1cm} (17)

3. FEM simulations and discussion

3.1. FEM simulations and material parameters

In this paper, COMSOL Multiphysics computing platform is used for finite element simulation. In order to simulate the real working conditions, the material adopts self-defined material and its parameters are in Table 2. And the depth of stent is 0.03 mm,

| Table 2. Test parameters. |
|---------------------------|
| item         | Parameters (mm) |
| Radius       | 0.05  | 0.08  | 0.10  | 0.15  | 0.20  |
| Length       | π/8   | π/10  | π/12  | π/14  | π/16  |

Tetrahedron element is used in the process of mesh generation. The the minimum cell size is 0.01 mm, and the maximum cell growth rate is 1.5. The total number of cells is 1128. Finally, the non-linear large plastic strain solver is used to calculate the plastic strain of the stent during expansion. The data are convergent in the process of solving.

3.2. Comparison between theory and FEM

The pictures are obtained by the FEM results. The elastic variation of the lozenge grid structure along with the cells geometry parameters are showed in Figure 3 and 4. Tensile deformation displacement is obvious and the expansion phenomenon is occurred in the lozenge grid structure. Negative Poisson’s ratio structure expands when it is stretched. Figure 3 shows that the bracket expands when it is stretched, this phenomenon shows that the structure is negative Poisson’s ratio structure.

![Figure 4. Stent stress versus length.](image)

The Young’s modulus errors between FEM and Theory with the geometry parameters are compared in tables 3 and 4. The equivalent Young’s modulus is obtained by numerical calculation. Comparing the results in Table 3, the results of equivalent elastic modulus equation based on energy principle are consistent with those of numerical simulation, and the error between them is less than 10% except a few points, and the Young’s modulus error raise with the radius growing. The FEM results show a discrepancy of 4.9% over the analytical equation along with the length changing in table 4. The difference between experiment and analysis can be attributed to the use of equivalent elastic modulus Ec. The theoretical calculation is basically consistent with the results of the finite element model.
Table 3. The Young’s modulus error between FEM and theory with the radius variation.

| Radius | FEM(GPa) | Theory(GPa) | Error(%) |
|--------|----------|-------------|----------|
| 0.005  | 555      | 595         | 6.72     |
| 0.008  | 558      | 619         | 9.85     |
| 0.01   | 505      | 544         | 7.17     |
| 0.015  | 586      | 458         | 21.84    |
| 0.018  | 314      | 434         | 27.65    |

Table 4. The young’s modulus error between FEM and theory with the length variation.

| Length | FEM(GPa) | Theory(GPa) | Error(%) |
|--------|----------|-------------|----------|
| Pi/8   | 540      | 511         | 5.37     |
| Pi/10  | 525      | 515         | 1.90     |
| Pi/12  | 505      | 475         | 5.94     |
| Pi/14  | 500      | 583         | 14.23    |
| Pi/16  | 662      | 653         | 1.35     |

4. Conclusions
In this work, a lozenge grid structure is proposed and material equivalent isotropic elastic modulus has been deducted with beam theory. Then structural deformation is carried out by finite element method by COMSOL Multiphysics. By comparing the theoretical equations with the finite element simulation results, it is found that:

- The theoretical equation deduces the Poisson’s ratio equation and its value is -1, which shows that the lozenge grid structure is a typical negative Poisson’s ratio structure.
- Finite element analysis of the lozenge grid structure tension deformation and the error between the simulation results and theoretical results is less than 10% mostly, the equivalent elastic modulus is good associated with the Finite element results.
- The lozenge grid structure is a new sandwich structures which can be applied in a lot of engineering structures.

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