Alpha-Alpha scattering, chiral symmetry and 
$^8$Be lifetime$^1$

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Abstract. Alpha-alpha scattering is discussed in terms of a chiral two pion exchange potential (TPE) which turns out to be attractive and singular at the origin, hence demanding renormalization. When $^8$Be is treated as a resonance state a model independent correlation between the Q-factor and lifetime $1/\Gamma$ for the decay into two alpha particles arises. For a wide range of parameters compatible with potential model analyses of low energy $\pi\alpha$ scattering it is found $\Gamma = 4.4(4)$eV in fairly good agreement with the experimental value $\Gamma_{\text{exp}} = 5.57(25)$eV. The remaining discrepancy as well as the phase shift up to $E_{\text{LAB}} = 15$MeV could be accommodated by the leading nuclear peripheral contributions due to the $^3\text{H}+\text{p}$ and $^3\text{He}+\text{n}$ continuum.

Keywords: Alpha-alpha scattering, Two-Pion Exchange, $^8$Be-lifetime, Renormalization

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1. The scattering of $\alpha$-particles has been one of the most studied nuclear reactions both theoretically and experimentally [1] and reveals the existence of $^8$Be as a narrow $(J^P, T) = (0^+, 0)$ threshold resonance at CM energy $Q = 91.84(4)$KeV and a very small (Breit-Wigner) width $\Gamma_{BW} = 5.57(25)$eV (see [2] for a review) which corresponds to a life-time of $\tau = 1.18^{-16}$s. The $^8$Be state is so close to the $\alpha\alpha$-threshold that the scattering length is $a_0 = -1600$fm, a huge scale in nuclear physics. This requires a great deal of fine tuning in the interaction, an issue relevant for the invokers of the Anthropic Principle. At the resonant energy the corresponding de Broglie wavelength is $1/p = 1/\sqrt{M_\alpha Q} \sim 10$fm $^2$ much larger than the size of the $\alpha$-particle so one would not expect internal structure playing a crucial role. We illustrate this for the s-wave state below the inelastic $^7\text{Li}+\text{p}$ and $^7\text{B}+\text{n}$ thresholds which satisfies

$$-u''_p(r) + \left[ M_\alpha V_{\alpha\alpha}(r) + \frac{2}{a_B r} \right] u_p(r) = p^2 u_p(r), \quad (1)$$

with $a_B = 2/(M_\alpha Z_\alpha^2 e^2) = 3.63$fm the Bohr radius and $p = \sqrt{M_\alpha E}$ the CM momentum. If we implement the strong interaction by an energy independent short distance boundary condition a fit of the s-wave scattering phase shifts [1] up to $E_{\text{LAB}} = 15$MeV becomes

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2 We take $M_\alpha = 2(M_p + M_n) - B = 3727.37$MeV, corresponding to a binding energy $B = 28.2957$MeV.
possible (see Fig. 3) yielding the result

$$u_p'(r_c)/u_p(r_c) = -0.357(3)\text{fm}^{-1} \quad r_c = 2.88(3)\text{fm} \quad \chi^2/DOF = 0.5 \quad (2)$$

The value of the Coulomb potential at $r_c = 3\text{fm}$ is $V_C(r_c) \sim 1\text{MeV}$. Actually, if we switch it off down to $r_c$ we get a would-be $^8\text{Be}$ bound state, $u_B(r) = A_B e^{-\gamma_B r}$ where $B = -\gamma_B^2/M_\alpha = 1.3\text{MeV}$. So, the nuclear attraction must cancel the $\sim 1\text{MeV}$ electromagnetic energy to allow the $\alpha\alpha$ system to tunnel through the barrier and eventually form $^8\text{Be}$.

The $\alpha$-particle m.s.r. as measured in electron scattering [3] is $r^\text{em}_\alpha = 1.668(5)\text{fm}$ and due to isospin symmetry the matter density is $\rho_B(r) = 2\rho^\text{em}(r)$, so at relative distances of $r \sim 3\text{fm}$ we do not expect finite size effects to be crucial, as the boundary condition model fit, Eq.(2), explicitly shows.

2. Besides the Coulomb interaction it seems unclear what does effectively make two alpha particles attract each other. In addition to the simple boundary condition described above there are various potentials describing the data successfully up to similar or higher energies; an attractive square well potential plus hard core [4], a Woods-Saxon potential [5] as well as a Gaussian potential (see e.g.[6] for a review on microscopic approaches). Here we analyze model independent attraction mechanisms based on the assumption of a marginal influence of internal $\alpha$-particle structure.

The longest possible particle exchange corresponds to two pions (one pion exchange is forbidden because of parity and isospin), which yields an exponential tail $\sim e^{-2m_\pi r}$ with the scale $1/(2m_\pi) = 0.7\text{fm}$. This looks like too short as compared to the $\alpha$-particle size, but note that it is only necessary that such a contribution cancels the Coulomb barrier when the two alphas are nearly touching or slightly overlapping. Obviously we do not expect to see TPE inside the nucleus. A shorter range scale is provided by the $t+p$ and $^3\text{He}+n$ continuum. These and other scales generate left cuts in the $\alpha\alpha$ scattering partial wave amplitudes for the complex LAB energy plane as illustrated in Fig. 1 where also the right cuts due to opening of $^7\text{Li}+n$ and $^7\text{B}+n$ channels are displayed.

3. The idea of associating elementary fields to nuclei at low energies complies to a manifest irrelevance of internal structure [7]. The role of chiral symmetry, essential to describe long range pion exchanges, has been addressed only recently [8]. We take a scalar-isoscalar Klein-Gordon charged as well as chirally invariant under $SU(2)_R \otimes SU(2)_L$ transformations field for the $^4\text{He}$ nucleus. The effective Lagrangean will include pions [9] and $\alpha$ particles which being much heavier, $M_\alpha \gg m_\pi$, are better treated by transforming the Klein-Gordon field as $\alpha(x) = e^{-iM_\alpha v^\nu x} \alpha_v(x)$ with $\alpha_v(x)$ the heavy field
and $\nu^\mu$ a four-vector fulfilling $\nu^2 = 1$, eliminating the heavy mass term [10]. Keeping the leading $M_\alpha$ term, the effective Lagrangean reads

$$L = 2iM_\alpha \bar{\alpha}_\nu \gamma^\mu \partial_\mu \alpha + \frac{f_\pi^2}{4} \left[ \langle \partial^\mu U^\dagger \partial_\mu U \rangle + \langle \chi U^\dagger + \chi^\dagger U \rangle \right]$$

$$+ g_0 \bar{\alpha}_\nu \alpha_\nu \langle \partial^\mu U^\dagger \partial_\mu U \rangle + g_1 \bar{\alpha}_\nu \alpha_\nu \langle \chi U^\dagger + \chi^\dagger U \rangle + g_2 \bar{\alpha}_\nu \alpha_\nu \langle \nu^\cdot \partial U^\dagger \nu \cdot \partial U \rangle$$

(3)

where the pion field in the non-linear representation is written as a $SU(2)$-matrix, $U = e^{i\vec{\tau}\cdot \vec{\pi}/f_\pi}$, with $\vec{\tau}$ the isospin Pauli matrices, $f_\pi$ the pion weak decay constant $f_\pi = 92.6$MeV, $\chi = m_\pi^2/2$ and $\langle , \rangle$ means trace in isospin space. Here $g_0$, $g_1$, $g_2$ are dimensionless coupling constants which are not fixed by chiral symmetry. This Lagrangean is the analog of the Weinberg-Tomozawa Lagrangean and EFT extensions for $\pi N$ interactions [11] to the case of the $\pi\alpha$ system. Photons are included by standard minimal coupling $\partial^\mu \alpha \to D^\mu \alpha = \partial^\mu \alpha + Z_\alpha \epsilon A^\mu \alpha$.

4. The leading direct t-channel TPE contribution to the scattering amplitude is depicted in Fig. 2. The final result reads [8]

$$V_{2\pi}^{2\pi}(r) = -\frac{3m_\pi^2 [K_0(2x) f(x) + K_1(2x) g(x)]}{32\pi^3 M_\alpha^2 f_\pi^4 x^6}$$

(4)

where $x = m_\pi r$, $K_0(x)$ and $K_1(x)$ are modified Bessel functions and $f(x) = 4(g_0 + g_1)^2 x^4 + 10(2g_0^2 + 4g_2 g_0 + 3g_2^2) + [84g_0^2 + 24(g_1 + g_2)g_0 + g_2^2(4g_1 + 15g_2)] x^2$, $g(x) = 4(g_0 + g_1)(6g_0 + g_2)x^3 + 10(12g_0^2 + 4g_0g_2 + 3g_2^2)x$. The TPE potential is attractive everywhere. In fact, the TPE potential becomes singular at short distances,

$$V_{2\pi}^{2\pi}(x) = -\frac{15(12g_0^2 + 4g_0g_2 + 3g_2^2)}{32\pi^3 M_\alpha^2 f_\pi^4} \frac{1}{r^7} + \ldots$$

(5)

This is a relativistic and attractive Van der Waals interaction which is explicitly independent on the pion mass. In the opposite limit of long distances we have

$$V_{2\pi}^{2\pi}(x) \to -\frac{3(g_0 + g_1)^2 m_\pi^{9/2}}{16\pi^{5/2} M_\alpha^2 f_\pi^4} \frac{e^{-2m_\pi r}}{r^{5/2}}$$

(6)

The couplings $g_0$, $g_1$ and $g_2$ may be estimated from the analysis of low energy $\pi\alpha$ scattering after pion-absorption and Coulomb effects are eliminated [12] yielding $g_0 = \ldots$. 

FIGURE 2. Diagrams contributing to the $\alpha - \alpha$ potential at long distances (from left to right) : One-Photon Exchange, Two-Pion Exchange loop, $p$ and $n$ exchange loops.
−82(11), \(g_1 = -5.3(3)\) and \(g_2 = 77(12)\). Pion exchange interactions between \(\alpha\) particles have been treated in the past in a variety of ways. A resonating group method approach was used in Ref. [13] with an approximation for the TPE in the mid-range. Forward dispersion relations for \(\alpha\) scattering have been discussed [14] with the TPE cut replaced by a pole. A folding model from a NN potential was used in Ref. [15] and the \(I\)-wave phase shift was computed in first order perturbation theory. This corresponds to our potential but missing the term with \(g_2\).

5. Singular potentials are commonplace in effective theories. Fortunately, one may use renormalization to actually minimize our lack of knowledge at short distances, which we expect to be irrelevant. This viewpoint in fact refuses to explain the fine tunings, but makes use of their existence to insure short distance insensitivity given some reliable long distance physics. In our case, the problem is to solve Eq. (1) with suitable boundary conditions. For the singular power-like potential (5) written as \(M_{\alpha}V(r) \sim -R^5/r^7\) (\(R \sim 1.5\)fm) one can show that at short distances the wave function goes as

\[
 u_p(r) \rightarrow C \left( \frac{R}{r} \right)^{7/4} \sin \left[ \frac{2}{5} \left( \frac{R}{r} \right)^{5/2} + \varphi \right].
\]  

(7)

where the arbitrary phase \(\varphi\) must be real and energy independent. As we see, the regularity condition, \(u_p(0) = 0\), does not fix the solution uniquely [16, 17], thus some information must be provided in addition to the potential. For instance, if we choose the \(Q\) we can determine \(\Gamma\) and the full phase shift. This is the non-perturbative renormalization program with one counterterm described in [18] for singular potentials within the NN context. In practice, it is more convenient to introduce a short distance cut-off \(r_c \ll 1\)fm and to use a real and energy independent boundary condition

\[
 \text{Re} \left[ \frac{u_p'(r_c)}{u_p(r_c)} \right] = \frac{u_p'(r_c)}{u_p(r_c)} = \frac{u_0'(r_c)}{u_0(r_c)}.
\]  

(8)

which implements self-adjointness in the domain \(r \geq r_c\).

6. The experimentally determined [2] Breit-Wigner small width involves the s-wave phase shift [19, 20] \(\Gamma_{BW} = 2/\delta_0'(E_R)\) for \(\delta_0(E_R) = \pi/2\). We get

\[
 \Gamma_{BW}^{(8\text{Be} \rightarrow \alpha\alpha)} = 4.3(3)\text{eV}, (\exp.5.57(25)\text{eV}),
\]  

(9)

for \(E_R = 91.8\)KeV and the couplings \(g_0, g_1\) and \(g_2\) with their uncertainties obtained from low energy \(\pi\alpha\) scattering 3.

Nevertheless, a rigorous treatment of \(8\)Be as a exponentially time-decaying state requires finding a pole of the S-matrix in the second Riemann sheet of the complex energy plane, corresponding to solve Eq. (1) for a spherically outgoing Coulomb wave. For complex momenta \(p = p_R + ip_I\) the energy also becomes complex \(E = Q - i\Gamma/2\). The boundary condition, Eq. (7) provides a correlation between \(\Gamma\) and \(Q\) through the TPE potential. We get \(\Gamma_{\text{pole}} = 3.4(2)\text{eV}\) for the S-matrix pole width, fairly independently of the cut-off radius for \(r_c \ll r_{\text{min}} \sim 3\)fm.

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3 A WKB analysis was also done [8] yielding \(\Gamma_{WKB} = 8.6(4)\text{eV}\) for the experimental \(Q\).
Furthermore, for $p \ll 2m_{\pi}$ we have the effective range expansion \footnote{Recent work \cite{21} provides a one-to-one mapping from a Lagrangean with alpha and di-alpha fields to Eq. (10) but disregarding pion exchanges.}

$$
\frac{2\pi \cot \delta_0(p)}{a_B(e^{2\pi\eta} - 1)} + \frac{2}{a_B} h(\eta) = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 p^2 + \ldots
$$

(10)

with $h(x)$ the Landau-Smorodinsky function \cite{19, 20} where the scattering length $\alpha_0$ and the effective range $r_0$ are defined. From the universal low energy theorem of Ref. \cite{18} in the Coulomb case we obtain (in fm)

$$
r_0 = 1.03(1) - \frac{5.3(3)}{\alpha_0} + \frac{29(4)}{\alpha_0^2} + \ldots
$$

(11)

The numerical coefficients depend on the TPE potential, Eq. (4), plus Coulomb potential only. Using Eq. (8) we find $\alpha_0^{\text{th}} = -1210(70)$fm and $r_0^{\text{th}} = 1.03(1)$fm, in reasonable agreement with $\alpha_0 = -1630(150)$fm and $r_0 = 1.08(1)$fm from a low energy analysis of the data \cite{19}. The correlation between $(Q, \Gamma)$ and $(\alpha_0, r_0)$ was pointed out long ago \cite{20, 22} but has no predictive power. The underlying chiral TPE potential correlates, in addition, $r_0$ with $\alpha_0$ and $\Gamma$ with $Q$. At higher energies, however, further ingredients are needed since for $E_{\text{LAB}} = 3$MeV, we get $\delta_0 = 141 \pm 2^0$ whereas $\delta_0 = 128.4 \pm 1^0$ from data \cite{1}. The wavelength $1/p \sim 2.5$fm corresponds to the two $\alpha$’s overlapping.

7. The leading nuclear structure contribution is given by the lowest pionic excited $\alpha^*$-state which, however, lies above the $t + p$ and $^3\text{He} + n$ continuum ($M_{\alpha^*} - M_\alpha = 25.28$MeV whereas $M_t + M_p - M_{\alpha^*} = S_p = 19.8$MeV and $M_{^3\text{He}} + M_n - M_{\alpha^*} = S_n = 20.5$MeV). So, at long distances the continuum states will dominate and thus we consider the triangle diagrams of Fig. 2 which involve three fermion propagators. Taking the heavy particle limit $M_N, M_{^3\text{N}}, M_{\alpha} \to \infty$ with fixed separation energy $S_N = M_{^3\text{N}} + M_N - M_{\alpha}$ the calculation reduces to the anomalous threshold contribution. If we
further take \( g_{aaam} = g_{aap} = g_4 \) and also \( g_{ap} = g_{ct}H_e = g_3 \) we get

\[
V_{\text{nucl}}(r) = -\frac{g^2}{4\pi M_\alpha} \frac{e^{-2\gamma r}}{r^2}
\]

where \( g^2 \sim g_3^2 g_4 \) (up to numerical factors) and the scale is \( 1/(2\gamma) = 1/2\sqrt{2\mu_{N,3N}S_{N,3N}} \sim 0.58\text{fm} \) and \( S \sim 20\text{MeV} \) is the separation energy. In fact, this potential corresponds to the amplitude that one peripheral nucleon of one \( \alpha \)-particle scatters with the other \( \alpha \)-particle as a whole. Note the high level of degeneracy with the long distance TPE potential, Eq. (6) given the similarity of both scales. A more complete study would require fixing these couplings from nucleon-alpha scattering. Note that for \( g^2 > \pi \) this potential is also singular at short distances, and similarly to the TPE case either \( Q \) or \( a_0 \), cannot be fixed from the potential. We find that taking \( g \sim 8 \) the missing 1eV for the width as well as the needed 0.04fm in the effective range are obtained. Actually, a satisfactory description of phase-shifts can be achieved (see Fig. 3).

In conclusion, the present analysis suggests that TPE as determined from a chirally symmetric effective Lagrangean may indeed provide the bulk of the \( ^8\text{Be} \) lifetime, but competes with leading nuclear structure effects at higher energies. A more precise statement requires a better determination of the couplings which implies a thorough study both of \( \pi \alpha \) and \( N \alpha \) scattering.

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