Identification of the Shape of the Yarn Balloon Using MATHEMATICA

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Abstract: The balloon of yarn in parallel spinning process is studied in this work. In parallel spinning process, a filament yarn was unwound from a package at high rotating speed and twisted on a staple core yarn to form the wrapped yarn. The forces acting on the unwinding yarn made the yarn move as a 3-D curve, which has a great influence on the quality of the final yarn. A nonlinear mathematical model is employed, and a computer software, MATHEMATICA, is applied to identification of the balloon shape and the distribution of the yarn tension.

1. Introduction

A yarn is whirled around the spindle axis at a considerably high angular velocity, which forms the moving trajectory called ‘spinning balloon’ in spinning process. Spinning balloon is a common phenomenon in many processes (e.g., winding, doubling, twisting, warping, etc.) for it permits high speed at low energy input in textile industry [1], and it has been one of widely discussed focuses in academic and industry circle.

Researchers have been paying much attention to this phenomenon since the first study by Lüdicke in 1881[2]. Since then many open literatures on this subject have appeared. Lindneran [3] proposed an elementary mathematical description of the shape of balloon thread. Mack [4] gave a detailed mathematical model of the 3D balloon thread. Kothari et al.[5-7] analyzed the theory of yarn-unwinding. Batra and his colleagues [8, 9] investigated the spinning balloon in the cylindrical polar coordinate. He [10] have developed an effective homotopy perturbation method in solving the nonlinear equation arising in ring-spinning. Recently, Ghosh et al.[1, 11] have published a series of papers to explore the role of several critical parameters and reveal the influence of package parameters on unwinding tension and balloon shapes.

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The parallel spinning, also called the hollow-spindle spinning, is one of the novel spinning processes. This process has hitherto mainly been designed as fabrication of fancy yarn, and it is becoming an important system because it is advantageous for processing speeds, costs, and ability to spin the wrapped yarn. In parallel spinning, a yarn balloon was formed during the filament yarn unwound from the package, shown in Figure 1(a). Then the filament yarn meets the staple core yarn at the hollow spindle apex and forms a final wrapped yarn. There are many forces, such as yarn tension, centrifugal force, Coriolis force, air-drag and gravity, acting on the yarn between the unwinding-point on the package and the spindle apex in this spinning process. The balloon shape becomes a 3D curve from the balloon apex to the unwinding-point because it is mainly subjected to Coriolis force and air-drag especially at a high angular velocity.

Fundamental research in this spinning process is especially necessary to design optimum process. The accurate shape of spinning balloon is of great practical interest in designing new apparatus, improving the efficiency of spinning and controlling the yarn quality. The balloon yarn tension can greatly affect the ultimate quality of the wrapped yarn in this spinning process. However, very few works have been published on simulating the shape of spinning balloon and predicting the distribution of the yarn tension in parallel spinning process.

In this study, we simulated the real shape of the spinning balloon and then predicted the yarn tension along the balloon thread employing a computer software MATHEMATIC. We adopted Kothari’s equations as the model in numerical analysis.

2. The filament motion model and numerical simulation

Figure 1(b) shows the basic motion locus of the filament yarn from the unwinding point on the package to the spindle apex in parallel spinning. For simplification and clarity, we assumed the filament yarn has the following properties: 1) The yarn is perfectly flexible and inextensible; 2) The yarn mass per unit length is a constant. 3) The air-drag could be divided into the tangential air-drag and the normal air-drag.

![Figure 1](image)

**Figure 1.** (a) Schematic Diagram of parallel-spinning process. 1-core yarn, 2-drafting roller, 3-balloon yarn, 4-package, 5-hollow spindle, 6-false twister, 7-suction tube, 8-guide roller, 9-bobbin; (b) Diagram of balloon thread unwinding from a package.

In Figure 1(b), $OXYZ$ is a rotation frame relative to the referential Cartesian frame $OXYZ$ with angular velocity $\omega$ about $Oz$, $O$ is at the spindle apex. $Oz$-axis and $OZ$-axis are coincident with the rotation axis. The equations below were done by Kothari [5-7].
where $T$ is yarn tension, $m_o$ is a yarn mass per unit length, $v_i$ is a component of yarn velocity normal to the yarn, $v_t$ is a component of yarn velocity tangential to the yarn, $(-l_i, -m_i, -n_i)$ is the direction cosine of the normal velocity component, $(-l_t, -m_t, -n_t)$ is the direction cosine of the tangential velocity component, $C_n$ and $C_T$ are drag coefficients, $\rho$ is the density of air, $d$ is the diameter of filament yarn.

There are three conditions: $z = 0, r = 0, T = T_0$. $T_0$ is the yarn tension at the top spindle.

Padfield[12] has proven that time derivatives were small compared with the space derivatives in case of filament yarn withdrawn from a package. So $\dot{x}$ and $\ddot{x}$ can be simply expressed as:

$$\begin{align*}
\dot{x} &= -Vx' \\
\ddot{x} &= V^2 x''
\end{align*}$$

where $x'$ donates differentiation with respect to $s$, and $\dot{x}$ donates differentiation with respect to $t$.

Kothari[5-7] proved that the tangential air-drag and gravity were so little that it could be ignored, compared with the normal air-drag by numerical calculations.

Suppose $a$ is an instantaneous radius when filament yarn left the package, equations (1-3) can be reduced to a dimensionless form. Let $T_i = (T - m_oV^2)/m_o a^2 \omega^2$, $k = V/a \omega$, $x_i = x/a$, $y_i = y/a$, $s_i = s/a$, $V_k = |v_k|/a \omega$, $p_n = 16a P_n/m_o$, thus the original equations have the normalized form:

$$\begin{align*}
x'' &= \frac{T_i x_i' + x_i - 2k y_i' + \frac{P_n}{16} \left[ x_i^2 + y_i^2 - (x_i y_i' - y_i x_i')^2 \right]^{1/2} \left[ (x_i y_i' - y_i x_i') x_i' + y_i \right]}{-T_i} \\
y'' &= \frac{T_i y_i' + y_i + 2k x_i' + \frac{P_n}{16} \left[ x_i^2 + y_i^2 - (x_i y_i' - y_i x_i')^2 \right]^{1/2} \left[ (x_i y_i' - y_i x_i') y_i' - x_i \right]}{-T_i} \\
z_i' &= 1 - x_i' - y_i' \\
T_i'' &= -(x_i x_i' + y_i y_i')
\end{align*}$$

where the normal air-drag parameter $p_n = 8a C_n \rho d / m_o$. $C_n$ is not a constant, and varies with the logarithm of the Reynolds number $Re[13,14]$. And

$$\begin{align*}
Re &= \frac{v_n d}{\eta}
\end{align*}$$

where $\eta$ is the kinematics viscosity of air, $v_n$ is the air speed.
Equations (6-10) are with respect to $s_i$. The equation set can be solved with initial values. Here the initial values are: $x_i = 0, y_i = 0, z_i = 0, x_i' = \sin \gamma_0$ ($\gamma_0$ is the indication of filament at the apex of balloon to the vertical), $y_i' = 0$. According to the initial values $\omega = \frac{53.47 r}{s}, V = 69 cm/s, a = 2.65 cm, \gamma_0 = 38^\circ, \eta = 15 \times 10^{-2} cm^2/s, d = 1.66 \times 10^{-2} cm, v_n \approx 240.6 cm/s, \rho = 1.2 \times 10^{-3} g/cm^2$, therefore $\sin \gamma_0 = 0.616, k = 0.5, p_n/16 = 5.28$.

Numerical solution of the equation set (6-10) could be obtained by employing a commercial computer software, MATHEMATIC, and the shape of the spinning balloon was simulated from the numerical solution with above given conditions, shown in Fig.2(a). It is obvious that the balloon thread is a 3D curve. A digital camera was used to capture the filament’s motion. The real filament shape shown in the picture (see Fig.2(b)) is in agreement with the shape of the curve simulated by MATHEMATIC, which shows the model can be employed to compute and simulate the real shape of the balloon yarn in this and other similar spinning process. Additionally, the distribution of the yarn tension along the balloon thread was also calculated as shown in Fig.2(c). The result illustrates that the minimum yarn tension locates at the largest radius point and the maximum one at the smallest radius point.

![Figure 2](image)

**Figure 2.** (a) Simulation of balloon curve from numerical solution; (b) The picture of the real balloon yarn captured by a digital camera; (c) Distribution of yarn tension along the balloon thread.

### 3. Conclusions

A commercial computer software, MATHEMATIC, was employed to simulate the shape of balloon yarn and the yarn tension distribution in parallel spinning process. Kothari’s nonlinear equations were adopted as the model of filament motion in this process. To verify the validity of the model, a digital camera was used to capture images of the movement of the filament yarn. The result shows that this model can be adopted to predict the yarn balloon shape and the tension on each element alone the balloon yarn. The simulate curve is a 3-D curve and can accurately depict the real balloon shape. The yarn tension curve shows that the minimum yarn tension is at the largest radius point and the maximum one is at the smallest radius point. The computation results coincide well with the experimental results.

Our computational simulation has provided a useful insight into filament yarn motion and the distribution of yarn tension in parallel spinning process, which has not previously been reported in the
published literature. The results also have wider applications for helping to optimize spinning parameters to improve the quality of the wrapped yarn in parallel spinning and others.

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