De Sitter Vacua from Heterotic M-Theory

Melanie Becker \textsuperscript{a,1}, Gottfried Curio \textsuperscript{b,2}, Axel Krause \textsuperscript{a,3}

\textsuperscript{a} Department of Physics, University of Maryland, College Park, MD 20742, USA

\textsuperscript{b} Humboldt-Universität zu Berlin, Institut für Physik, D-12489 Berlin, Germany

Abstract

It is shown how metastable de Sitter vacua might arise from heterotic M-theory. The balancing of its two non-perturbative effects, open membrane instantons against gaugino condensation on the hidden boundary, which act with opposing forces on the interval length, is used to stabilize the orbifold modulus (dilaton) and other moduli. The non-perturbative effects break supersymmetry spontaneously through F-terms which leads to a positive vacuum energy density. In contrast to the situation for the weakly coupled heterotic string, the charged scalar matter fields receive non-vanishing vacuum expectation values and therefore masses in a phenomenologically relevant regime. It is important that in order to obtain these de Sitter vacua we are not relying on exotic effects or fine-tuning of parameters. Vacua with more realistic supersymmetry breaking scales and gravitino masses are obtained by breaking the hidden $E_8$ gauge group down to groups of smaller rank. Also small values for the open membrane instanton Pfaffian are favored in this respect. Finally we outline how the incorporation of additional flux superpotentials can be used to stabilize the remaining moduli.

PACS: 04.65.+e, 11.25.Mj, 11.25.Yb

Keywords: De Sitter Vacua, Heterotic M-Theory

\texttt{hep-th/0403027}

\texttt{1melanieb@physics.umd.edu}
\texttt{2curio@physik.hu-berlin.de}
\texttt{3krause@physics.umd.edu}
1 Introduction and Conclusions

In view of current astronomical data [1] which are in nice agreement with a dark energy component generated by a cosmological constant, modern theoretical physics faces the challenge of finding realistic non-supersymmetric vacua with positive vacuum energy. Indeed these seem to be the right class of vacua not only today but also during an early epoch of inflation with however vastly different vacuum energies. The search for de Sitter vacua in ordinary supergravity theories dates back to some rather early papers (see e.g. [2] and references therein). More recently the connection between supergravity theories and de Sitter vacua was discussed in [3], [4], [5] and other interesting ideas on how to obtain de Sitter vacua from string-theory appeared in [6], [7], [8].

Progress towards a straightforward derivation of 4d de Sitter spaces or more generally 4d accelerated cosmologies from a standard reduction of 10d effective string-theories was hampered by the No-Go theorems presented in [9] and [10]. However, ways around these theorems were found – either in the way of incorporating instantons [11], [6] or time-dependent hyperbolic compact internal spaces [12] (which arise as special S-brane [13] solutions with vanishing flux [14]; S-branes were connected to accelerated cosmologies e.g. in [13]). While the former case leads to 4d de Sitter vacua the latter situation leads to accelerating 4d cosmologies with an equation of state \( p = w \rho \), where \( w = (4 - n)/(3n) \) with \( n \) the number of internal compact dimensions, \( p \) being pressure and \( \rho \) the density. Unfortunately, the tight observational constraint of \( w < -0.78 \) [1] resulting from the combined WMAP (Wilkinson Microwave Anisotropy Probe) data, supernova observations, HST (Hubble Space Telescope) data and 2dFGRS (2dF Galaxy Redshift Survey) large scale structure data evaluated under the premise that \( w > -1 \) cannot be satisfied by any \( n \in \mathbb{N} \). Moreover without this premise a value of \( w = -0.98 \pm 0.12 \) is favored [1] by the combined data which clearly points towards a cosmological constant which gives \( w = -1 \) (which again cannot be obtained for any positive integer \( n \)). We are therefore motivated to address in this paper the question of how 4d de Sitter spaces can arise from string/M-theory leaving the question of a realistic cosmology with these spaces for future work.\(^4\)

The interesting question on how to embed de Sitter solutions into string theory has been lately addressed in the literature in the context of string theory and M-theory compactifications with non-vanishing fluxes. In fact, de Sitter spaces in IIB string/F-theory in

\(^4\)For a recent interesting proposal to realize inflation in M-theory see e.g. [16].
the presence of NSNS and RR fluxes were found recently in the work of [6] and [8] building on results of [17]. In order to stabilize the moduli fields the work of [6] used NSNS and RR fluxes while anti-D3-branes were introduced to achieve the necessary supersymmetry breaking.

From the phenomenological point of view (see e.g. [18], [19]) it is, of course, more interesting to consider heterotic M-theory instead of Type IIB theory. It has been known for quite a while that de Sitter vacua can occur in Calabi-Yau (CY) flux compactifications of heterotic M-theory when non-perturbative effects are included [11]. Our goal in this paper is to establish the de Sitter vacua found in [11] more rigorously by using instead of the linearized background of [18] the full non-linear background of [20], [21] and moreover by including charged matter fields, which is of interest for particle phenomenology as well as cosmology. Recall that in heterotic M-theory the stabilization of the orbifold length and the corresponding axion requires a four-form $G$-flux component $G_{(2,2,0)}$ of M-theory which is projected out in the weakly coupled string-theory limit. Thus the stabilization mechanism of [11] is a truly strongly coupled one. This mechanism is in a sense complemental to the moduli stabilization mechanism of the weakly coupled heterotic string that has been recently proposed in the literature [22], [23] and [24] based on a non-trivial $G_{(1,2,1)} = H_{1,2}$ component. We will comment towards the end of the paper on the relation between our paper and the results obtained in the previous references in the context of the weakly coupled heterotic string. In fact, we shall see that the stabilization of the radial modulus obtained in [22] expands very nicely the stabilization of the moduli fields achieved in heterotic M-theory and we will borrow the results from [22] in order to fix most (and maybe all) of the moduli fields. Another way to stabilize at least some of the remaining moduli is to consider alternatively a balancing of the $H_{0,3}$ component of the Neveu-Schwarz $H$-flux against gaugino condensation as was done for the weakly coupled heterotic string in [25]. In fact, this might be an easier starting point to incorporate further moduli, as the internal CY manifold still remains Kähler. Work in this direction is in progress.

Getting back to the strongly coupled theory, we shall see that non-perturbative open membrane instanton effects contributing to the superpotential break supersymmetry spontaneously, leading to a positive scalar potential. Therefore, our primary goal here will be to establish 4d de Sitter vacua in heterotic M-theory in the presence of charged matter fields $C$ and to explore the global potential landscape beyond what was possible in [11] due to the limitations of the linearized background to show that (in agreement with general expectations [26], [27], [28], [6]) the arising de Sitter minima are actually false
metastable vacua. The possibility to study the global potential profile arose only recently after it was understood [21] (based on [20]) how to extend the linearized background for flux compactifications of heterotic M-theory to the full non-linear background and still preserve supersymmetry. Only with this extension will it be possible to study the potential for arbitrary large orbifold-modulus $\mathcal{L}$ and thus to establish the expected runaway behaviour in the decompactification limit (see also the discussion in [29]). The linearized background has been used recently in [30] to find supersymmetric Anti de Sitter vacua in heterotic M-theory.

Let us summarize briefly some of the results obtained in this paper. In order to find 4d de Sitter vacua in our context the orbifold length and the volume modulus have to be stabilized. We will find that one can naturally, i.e. without invoking hierarchically large or small values for some parameters stabilize the orbifold modulus $\mathcal{L}_0$ close to the maximally allowed value $\mathcal{L}_{\text{max}}$, which is quite satisfying as this position leads to the correct 4d Newton’s Constant once the GUT-scale and the GUT gauge coupling attain their usual values.

The stabilization of the orbifold length modulus is exemplified in figure 3 appearing later on in the paper. The logarithmic plot of figure 3, displaying the full behavior of the potential in the whole interval $\mathcal{L} \in [0, \mathcal{L}_{\text{max}}]$ (whose bound arises from a true physical singularity not from an artifact of the linear approximation), shows nicely that the minimum is caused by a balancing between two distinct non-perturbative effects possessing different monotony behavior: open membrane instantons and gaugino condensation. The precise equation for this balancing will be derived in section 4, but we will show there also that it is well approximated by the simplified condition

$$e^{-ReT} = e^{-\frac{1}{c_H}ReS + \frac{2}{c_H}ReT}$$  (1.1)

which expresses clearly the balance between open membrane instantons ($\sim e^{-ReT}$) and gaugino condensation ($\sim e^{-\frac{1}{c_H}ReS + \frac{2}{c_H}ReT}$). Moreover one finds that at leading order in $1/V$ and $1/V_{\text{OM}}$ e.g.

$$DSW = -\frac{W_{GC}}{C_H} \neq 0,$$  (1.2)

which shows that at the location of the minimum in particular $DSW$ is non-vanishing, i.e. supersymmetry is broken spontaneously through F-term expectation values. Moreover we find that the resulting supersymmetry breaking scale and gravitino mass can be brought close to the phenomenologically relevant regime when the hidden $E_8$ is broken down
to smaller gauge groups. Moreover small values for the Pfaffian $h$ arising in the open membrane superpotential are favored in this respect.

Until now for this class of vacua not all moduli were stabilized explicitly thus the de Sitter vacua could potentially be unstable in some directions. Nevertheless the vacua would still have positive vacuum energy. The stabilization of these remaining moduli might however be achieved by incorporating in addition $H$-flux superpotentials which we will outline in the last part of the paper. We will concentrate on the simplest case possible, namely on the standard vacua without additional M5 branes but including the most dominant non-perturbative effect coming from open membrane instantons stretching between the two boundaries. We will treat the case with an additional M5 brane elsewhere. We will show that when additional non-perturbative effects coming from gaugino condensation on the hidden boundary are taken into account it is possible to stabilize in addition the charged matter fields. This is a rather promising result for particle phenomenology as for a long time one of the major drawbacks of heterotic string theory was the presence of massless charged matter fields (see e.g. [31] and [32]). We will see that the vacuum expectation values (vev) for these scalars lie in a phenomenologically interesting range.

The organization of the paper is as follows. In section 2 we compute the effective potential resulting from compactifications of heterotic M-theory on a manifold $X \times S_1/\mathbb{Z}_2$, where $X$ is the internal CY three-fold taking non-trivial fluxes, open membrane instantons and gaugino condensation into account. In section 3 we analyze the scalar potential obtained in the previous section without taking gaugino condensation into account and show that while the charged scalars obtain a non-trivial vev the orbifold length does not get stabilized. In section 4 we include the effects of gaugino condensation and show that the axions and both the orbifold length and the charged matter field obtain a non-trivial vev. We finish in section 5 with a discussion on the connection to the moduli stabilization mechanisms for the weakly coupled heterotic string recently proposed in the literature [22], [23], [24] and [25]. In fact we borrow some results of these papers e.g. the stabilization of the radial modulus achieved in [22] to fix many (and maybe all) of the moduli fields appearing in our compactifications. Clearly a more direct analysis has to be done to clarify which moduli fields are precisely stabilized. This question shall not be addressed in this paper and will be left for future work.
2 The Effective 4D Heterotic M-Theory

Our starting point are compactifications of heterotic M-theory \[33\], \[34\] on an internal seven-space

\[ X \times S^1/\mathbb{Z}_2 \] (2.1)

where \( X \) is a Calabi-Yau (CY) threefold. The resulting effective 4d theory is described by an N=1 supergravity \[35\] which is completely determined by the gauge kinetic function, the Kähler- and the superpotential for the occurring moduli. As long as one switches on only the \( G_{(2,2,0)} \) (the first number counts holomorphic, the second antiholomorphic CY indices and the third the orbifold index) component of the 11d supergravity four-form field-strength the resulting 7d flux compactification background is given by a warped geometry whose 6d piece is a conformal deformation of \( X \) \[18\], \[20\], \[21\]. One therefore has \( h^{(1,1)} \) complex moduli \( T^i \) which correspond to deformations of the Kähler class \( \omega \) of \( X \) and \( h^{(1,2)} \) complex moduli \( Z^\alpha \) describing the deformations of the complex structure of \( X \).

Notice, however, that when switching on a \( G_{(1,2,1)} \) component which corresponds to a Neveu-Schwarz background \( H_{NS} \) in the 10d limit where the orbifold length shrinks to zero, it is known that then \( X \) becomes a non-Kähler manifold which is no longer conformal to a CY \[9\] and whose moduli are not explicitly known. We will therefore in the main part assume that \( G_{(1,2,1)} = H_{1,2} = 0 \) and will comment towards the end on the consequences of including it. The fact that in heterotic M-theory one can set \( G_{(1,2,1)} \) to zero therefore allows us to avoid the complications which arise in the weakly coupled heterotic string but nevertheless study the implications of the \( G_{(2,2,0)} \) component for the stabilization of the universal moduli. The situation is therefore similar to the type IIB case, where a nontrivial \( H_{NS} \) also merely leads to a conformal deformation of the Calabi-Yau.

In addition to the moduli described above one has the volume modulus \( S \) and the charged matter \( C^I \). Here \( I \) represents a multi-index \((R, i)\) running over the representations \( R \) of the unbroken visible gauge group \( G \) (where \( G \times G_{hol} \subset E_8 \) and \( G_{hol} \) includes the gauge instanton’s holonomy), \( i = 1, \ldots, \dim H^1(X, V_S) \) (respectively also over \( n = 1, \ldots, \dim R \) when we are referring to the charged scalar components inside the representation; they are not to be confused with the \( h^{(1,1)} \) neutral scalars given by the Kähler moduli) where the \( V_S \) are those vector bundles into which the gauge bundle decomposes when the \( 248 \) of \( E_8 \) is decomposed under \( G \times G_{hol} \). In this paper we will take a CY compactification with \( h^{(1,1)} = 1 \) for simplicity.
The moduli are then combined into the following 4d, N=1 chiral superfields

\[ S = V(L) + i\sigma_S \]  

\[ T = V_{OM}(L) + i\sigma_T \]  

\[ C^I, \quad Z^\alpha \]  

Here \( V_{OM} \) describes the normalized volume of an open membrane instanton stretching between both boundaries and wrapping a holomorphic curve \( \Sigma \) inside the CY (for simplicity we will assume that the instanton wraps \( \Sigma \) only once)

\[ V_{OM}(L) = L \left( \frac{6V(L)}{d} \right)^{1/3}, \]  

where \( d \) is the CY intersection number. The dimensionless moduli \( V(L), \mathcal{L} \) are related to the dimensionful CY volume \( V(x^{11}) \) and the orbifold length \( L \) through \( (x^{11} \in [0, L] \) is the 11d orbifold coordinate)

\[ V(L) = \langle \frac{V(x^{11})}{v} \rangle = \frac{1}{vL} \int_0^L dx^{11} V(x^{11}) \, , \quad \mathcal{L} = \frac{L}{l} \]  

where \( v \) and \( l \) are two conveniently chosen dimensionful reference values

\[ v = 8\pi^5 l_{11}^6 \, , \quad l = 2\pi^{1/3} l_{11} \]  

given in terms of the 11d Planck-length \( l_{11} \) which itself is related to the 11d gravitational coupling constant \( \kappa \) through

\[ 2\kappa^2 = (2\pi)^8 l_{11}^9 \]  

The axions \( \sigma_S \) and \( \sigma_T \) arise from two different components of the 11d three-form potential \( C_{AB11} \). While \( \sigma_S \) comes from the usual 4d dualization of \( C_{\mu\nu11} \), one obtains \( \sigma_T \) as the coefficient from the expansion of \( C_{lm11} \) in terms of the single base element of \( H^{1,1}(X) \) (remember that \( h^{(1,1)} = 1 \)).

### 2.1 The Kähler-Potential

We won’t need the gauge kinetic functions in the following. So let us specify first the Kähler-potential for the above moduli. For the \( S \) and \( T \) moduli the Kähler-potential reads

\[ K_S = -\ln(S + \overline{S}) \, , \quad K_T = -\ln \left( \frac{d}{6} (T + \overline{T})^3 \right) \]  

\[ (2.9) \]
while for the C’s it is given by

\[ K(C) = \left( \frac{3}{T + \overline{T}} + \frac{2\beta_v}{S + \overline{S}} \right) H_{IJ} C^I \overline{C}^J \]  

(2.10)

at leading order in the \( C^I \). The positive-definite metric \( H_{IJ} \) depends only on the complex structure and bundle moduli [36, 37] while the instanton number \( \beta_v \in \mathbb{Z} \) of the visible boundary is given by the expansion coefficient of the visible boundary second Chern-classes

\[ c_2(F_v) - \frac{1}{2} c_2(R) = \frac{-\text{tr} F_v \wedge F_v + \frac{1}{2} \text{tr} R \wedge R}{8\pi^2} = \beta_v \left[ \Sigma_1 \right]. \]  

(2.11)

Because \( h^{(1,1)} = 1 \), there is only one basis element \( \Sigma_1 \) of the second homology group \( H_2(X, \mathbb{Z}) \) whose Poincaré dual four-form is \( [\Sigma_1] \). An analogous expansion involving the hidden boundary gauge field defines the hidden sector instanton number \( \beta_h \in \mathbb{Z} \). Anomaly cancelation demands that

\[ \beta_v + \beta_h = 0 \]  

(2.12)

which means that either \( \beta_v \) or \( \beta_h \) has to be negative. In the phenomenologically relevant case where the volume of the deformed CY decreases from the visible towards the hidden boundary, \( \beta_v \) is positive [18]

\[ \beta_v > 0 \]  

(2.13)

which is the case we will consider henceforth.

Besides these contributions to the Kähler-potential, there is also the contribution from the complex structure moduli

\[ K(Z) = -\ln \left( -i \int_X \Omega \wedge \overline{\Omega} \right) \]  

(2.14)

and the Kähler-potential \( K(A) \) for the vector bundle moduli. We will consider the complex structure moduli as ‘frozen’ in this paper and address their stabilization in a separate publication. The contribution from the vector bundle moduli, \( K(A) \), is considerably suppressed. In [30] it was shown to be generically smaller by a factor \( 10^{-5} \) as compared to \( K(S), K(T) \).

The Kähler-potential described so far, in particular the contributions (2.9), (2.10) which we will employ later on, comprises those terms which are universally present in all heterotic compactifications. As such they would also occur in compactifications over
the background derived in [20], [21] (see also [38]) which generalizes and extends the
flux compactification background of [18] in such a way that the background geometry
becomes trustworthy until a naked singularity is hit at some finite critical value of the
orbifold coordinate

\[ x_{11}^0 = \frac{l}{G_v} \]  

(2.15)
determined by the dimensionless visible boundary flux-parameter \( G_v \), which will be defined
below in (2.25). At \( x^{11} = x_{11}^0 \) the ‘classical’ CY volume vanishes. The derivation of the
effective 4d heterotic M-theory action as presented in the literature [35] took as a starting
point the flux compactification background of [18]. This background was obtained as
a solution to the 11d gravitino Killing-spinor equation under the assumption that the
warp-factor of the background geometry stays small and could be used as a dimensionless
expansion parameter. Consequently the solution is a perturbative one which is linear in
the warp-factor and neglects all higher powers in the warp-factor. The imposition of the
smallness of the parameter

\[ \epsilon = \frac{2\pi L}{3V_v^{2/3}} \left( \frac{\kappa}{4\pi} \right)^{1/2} = \frac{2L}{3V_v^{2/3}} = \frac{2}{3} \left( \frac{d}{6} \right)^{1/3} \frac{V_{OM}}{(V_v^2 V)^{1/3}} \simeq \frac{V_{OM}}{V} \left( \frac{d}{6} \right)^{1/3} \]

(2.16)
on the effective 4d theory, where

\[ V_v = \frac{V_v}{v} \]

(2.17)
is the dimensionless, normalized CY volume \( V_v \) on the visible boundary, stems directly
from this linearized background solution. Namely to ensure that the warp-factor stays
small one has to guarantee that its absolute value stays small which is upper-bounded by \( \epsilon \).
Therefore, though phenomenology requires an \( \epsilon = \mathcal{O}(1) \) [39], the perturbative background
demands a perturbatively small \( \epsilon \).

It is however known how to extend the linear background solution to incorporate the
required higher order (in the warp-factor) correction terms demanded by the full non-
linear 11d gravitino Killing-spinor equation, hence by supersymmetry [20, 21]. As this
extended full background leads to a manifest positive Riemannian metric and positive CY
volume over the full moduli space (which is not the case for the linearized background),
it is the better starting point for an analysis of the effective potential. This even more so
because the extended background can be used reliably until one hits the singularity at \( x_{11}^0 \)
and therefore stabilization of the orbifold modulus \( L \) in the phenomenologically favoured
regime, where \( \epsilon = \mathcal{O}(1) \), becomes feasible.
Though the complete reduction of heterotic M-theory over the extended full background to obtain the corresponding 4d effective theory hasn’t been carried out yet [10], it is nevertheless clear that the universal structure of the Kähler-potential (2.9), (2.10) or standard composition of moduli into $N = 1$, 4d chiral superfields won’t change. What will change, is the functional dependence of $\mathcal{V}(\mathcal{L})$ on $\mathcal{L}$ which becomes quadratic instead of linear as we will see below. The extended full background of [20], [21] with its manifestly positive metric and CY volume will be used in the subsequent derivation of de Sitter vacua.

2.2 The Superpotential

The second ingredient needed to determine the potential is the superpotential which consists of a perturbative and a non-perturbative piece

$$W = W_{\text{tree}} + W_{\text{non-pert}}.$$  (2.18)

The perturbative piece is given by the standard cubic superpotential [41]

$$W_{\text{tree}} = \Lambda_{IJK}C^I C^J C^K = \frac{4\pi\sqrt{2}}{3} \lambda_{IJK}C^I C^J C^K$$  (2.19)

where $\lambda_{IJK}$ are the Yukawa couplings. In general the Yukawa couplings are quasi-topological, i.e. they depend on the complex structure and bundle moduli only [37].

Leaving aside M5 branes in this paper, the non-perturbative contribution to the superpotential comes from open membrane instantons which stretch between the two boundaries and gaugino condensation on the hidden boundary

$$W_{\text{non-pert}} = W_{\text{OM}} + W_{\text{GC}}.$$  (2.20)

The former is described by a superpotential [36, 42]

$$W_{\text{OM}} = he^{-T}$$  (2.21)

where the Pfaffian $h$ is a holomorphic section of a line bundle over the complex structure moduli space. To preserve supersymmetry the open membrane has to wrap the holomorphic 2-cycle $\Sigma_1$ of the CY. Though in the case of standard embedding without M5 branes the sum of (2.21) over all curves $\Sigma_1$ in a fixed homology class vanishes and likewise in the special cases of non-standard embeddings which arise from weakly coupled heterotic
(0, 2) vacua related to linear sigma models, this is not the case for a generic heterotic (0, 2) compactification with non-standard embedding which we will assume in this paper.

Gaugino condensation occurs naturally on the hidden boundary [43] where the gauge theory becomes strongly coupled due to the decrease of the deformed CY volume from visible to hidden boundary (for $\beta_v > 0$). It leads to a superpotential [44]

$$W_{GC} = g e^{-\frac{1}{3\pi^2}(S - \gamma(L))}, \quad (2.22)$$

where

$$\gamma(L) = \mathcal{G}_v \mathcal{L}, \quad g = -C_H \mu^3 = -\frac{C_H}{32\pi^2} \left( \frac{2M_{GUT}}{M} \right)^3 = -5.0 \times 10^{-8} C_H \quad (2.23)$$

and $C_H$ stands for the dual Coxeter number of the hidden gauge group $H$. For instance for $H = E_8, E_6, SO(10), SU(5)$ one has $C_H = 30, 12, 8, 5$. The exponent receives an additive contribution from the $T$ modulus as a result of the modified gauge kinetic function for the hidden gauge group which is caused by the non-trivial CV volume dependence on $L$. Note that the fundamental 11d scale of heterotic M-theory is twice the grand unified scale $M_{GUT} = 3 \times 10^{16}$ GeV (‘lowering of the string scale’) [45] and therefore lower than the conventional weakly coupled string scale. It is therefore $2M_{GUT}$ which we have used above as an ultraviolet cut-off for the gauge-theory. Note further that the appearance of small numbers like $10^{-8}$ for $g$ is in part due to our conventions whereby all scalar fields and superpotentials are dimensionless and obtain their conventional mass dimensions through multiplication by the appropriate power of $M = M_{Pl}/\sqrt{8\pi}$, the reduced Planck mass. Likewise we expect similar small numbers for $h$. As the canonical mass dimension of a superpotential is mass$^3$ and we know that the highest available scale is $2M_{GUT}$, the fundamental 11d scale, the absolute value for $h$ should have an upper bound of

$$|h| \leq (2M_{GUT}/M)^3 = 1.6 \times 10^{-5} \quad (2.24)$$

in our conventions where $h$ becomes dimensionless by dividing through $M^3$.

The ‘slope’ $\gamma$ is controlled by the visible boundary flux-parameter $\mathcal{G}_v$ which is given through the integral

$$\mathcal{G}_v = -\frac{l}{V_v} \left( \frac{\kappa}{4\pi} \right)^{2/3} \int_{CY} \omega \wedge \frac{(trF \wedge F - \frac{1}{2}trR \wedge R)_v}{8\pi} \quad (2.25)$$

over the visible boundary CY and $\omega$ is the Kähler-form of the undeformed CY. Using (2.11), this parameter is related to the visible instanton number $\beta_v$ in the following way

$$\mathcal{G}_v = \frac{\pi l}{V_v} \left( \frac{\kappa}{4\pi} \right)^{2/3} \beta_v \int_{\Sigma_1} \omega. \quad (2.26)$$
For positive $\beta_v$ also $\mathcal{G}_v$ will therefore be positive. For $h^{(1,1)} = 1$ the Kähler-form $\omega$ can be written in terms of the single $H^{(1,1)}(X)$ basis-element $\omega_1$

$$\omega = \left(\frac{6\mathcal{V}_v}{d}\right)^{1/3} \omega_1$$

(2.27)

with the prefactor representing the single Kähler modulus. With the estimate for the integral

$$\int_{\Sigma_1} \omega_1 \simeq V^{1/3}_v$$

(2.28)

which should be quite accurate for our case of $h^{(1,1)} = 1$, and the values (2.7) plus (2.8) we obtain

$$\mathcal{G}_v = \beta_v \left(\frac{d}{6\mathcal{V}_v}\right)^{1/3}.$$  

(2.29)

### 2.3 The Effective Potential

The validity of the effective potential requires both $\mathcal{V}$ and $\mathcal{V}_{OM}$ to be sufficiently larger than one such that multiply wrapped instantons can be neglected. Furthermore, the derivation of the Kähler-potential $K_C$ requires the expansion parameter

$$\delta = \frac{H_I C^I \overline{C}^J}{\mathcal{V}_{OM}}$$

(2.30)

to be small. Notice that due to the employment of the full background of [20, 21] we are not restricted to work in the regime of $\epsilon \ll 1$ which via (2.16) would generically imply a hierarchy $\mathcal{V} \gg \mathcal{V}_{OM}$. Note, that this would mean that gaugino condensation which is of $\mathcal{O}(e^{-V/C_H})$ would be drastically exponentially suppressed against the open membrane instantons which go like $\mathcal{O}(e^{-\mathcal{V}_{OM}})$. However, in the regime where $\epsilon = \mathcal{O}(1)$ the contributions of gaugino condensation and open membrane instantons are generically of same size and therefore the possibility arises to balance both effects against each other as we will see later.

As the 4d effective heterotic M-theory is a supergravity theory with $N = 1$ supersymmetry its potential for the moduli is given by the standard formula

$$U = M^4 e^K \left(K^{ij} D_i \overline{W} D_j W - 3|W|^2\right) + U_D$$

(2.31)
where \( D_i W = \partial_i W + K_i W \) are the Kähler covariant derivatives, \( i, j \) run over all chiral fields and \( M \) denotes the reduced Planck mass. Moreover

\[
U_D = M^4 \frac{18\pi^2}{\sqrt{\nu_{OM}^2}} \sum_a (\overline{CT^a} C)^2 = M^4 \frac{18\pi^2}{\sqrt{\nu_{OM}^2}} \sum_a \left( H_{T^a} \overline{C} (T^a)^I K^I C^K \right)^2. \tag{2.32}
\]

is the D-term for the charged scalars. The \( T^a, a = 1, \ldots, \dim \) are the generators of the unbroken visible gauge group \( H \).

Since \( \delta, 1/\mathcal{V}, 1/\mathcal{V}_{OM} \) have to be considered as small, a hierarchy is introduced among the terms appearing in the potential. We will therefore find dominant contributions and terms which are suppressed by at least one of these small entities. Let us first concentrate on the open membranes alone and omit gaugino condensation in (2.20). Using the expressions collected in the appendix one sees that the dominant terms come from the \( K^{ij} \partial_i \overline{W} \partial_j W \) piece while the \( K^{ij} \partial_i \overline{W} K_j W \) piece contributes only via \( K^{\overline{T^I}} \partial_{\overline{T^I}} \overline{W} K_T W \) to the dominant term cubic in \( C \). Moreover, the D-term contributes at this order. All other terms which we will suppress in the following are smaller by at least one additional factor of \( \delta, 1/\mathcal{V}, 1/\mathcal{V}_{OM} \). The dominant terms of the effective potential are

\[
U = M^4 e^K \left( K^{ij} \partial_i \overline{W} \partial_j W + 2 \text{Re}(K^{\overline{T^I}} \partial_{\overline{T^I}} \overline{W} K_T W) \right) + U_D \tag{2.33}
\]

\[
= \frac{M^4 e^{K(\lambda) + K(\xi)}}{d\sqrt{\nu_{OM}^2}} \left( \frac{|h|^2}{2} \nu_{OM} e^{-2\nu_{OM}} + \frac{3}{2} (1 - \mathcal{N}) \text{Re}(\overline{h} e^{-\overline{T} \Lambda C^3}) + \left( \frac{3}{2} \right)^2 \mathcal{N} |\Lambda C^2|^2 \right) + \frac{18\pi^2 M^4}{\sqrt{\nu_{OM}^2}} \sum_a (\overline{CT^a} C)^2. \tag{2.34}
\]

where

\[
\mathcal{N}(\mathcal{L}) = \frac{1}{(1 + \beta_v \lambda_{OM}/3\nu)} < 1 \tag{2.35}
\]

and we have used the compact notation

\[
\Lambda C^3 = \Lambda_{IJK} C^I C^J C^K \tag{2.36}
\]

\[
|\Lambda C^2|^2 = H^{\overline{T^I}} \Lambda_{TJK} \Lambda_{LMN} \overline{C}^{\overline{T^I}} C^K C^M C^N. \tag{2.37}
\]

The inequality (2.35) stems from the fact that we consider \( \beta_v > 0 \) as explained before. The single negative contribution to the potential, \(-3|W|^2\), is among the neglected suppressed terms which means that the potential has to be positive. That this is indeed true irrespective of the values of \( \delta, 1/\mathcal{V}, 1/\mathcal{V}_{OM} \) can be easily shown. Namely, by investigating
the suppressed terms, one finds that a $+|W|^2$ term resp. a $+3|W|^2$ term come from

$$K^{\overline{SS}}K_{\overline{S}}W K_{\overline{S}}W = \left(1 + \beta_v \frac{H \overline{C}^TC}{V}\right)^2 |W|^2, \quad (2.38)$$

$$K^{\overline{TT}}K_{\overline{T}}W K_{\overline{T}}W = 3 \left(1 + \frac{H \overline{C}^TC}{2V_{OM}}\right)^2 |W|^2. \quad (2.39)$$

which together overcompensate the negative $-3|W|^2$ contribution and prove the positivity of the potential on the whole moduli space. Actually this was to be expected in view of the ‘no-scale’ structure, $K_{(T)} = -3 \ln (T + \overline{T}) + \ldots$, of the Kähler-potential.

Let us next consider the full non-perturbative contribution to (2.20) comprising both open membrane instantons and gaugino condensation. Again we will keep only the dominant terms in view of the smallness of $\delta, 1/V, 1/V_{OM}$. With help of the technical expressions of the appendix one finds that

$$K^{ij} \partial_i \overline{W} \partial_j W = \frac{4V^2}{C_H^2} |W_{GC}|^2 + \frac{4}{3} \frac{\nu^2_{OM}}{C_H} |W_{OM}| - \frac{\gamma}{C_H} |W_{GC}|^2 \quad (2.40)$$

and

$$\text{Re} (K^{\overline{SS}} \partial_{\overline{S}} \overline{W} K_{\overline{S}}W + K^{\overline{TT}} \partial_{\overline{T}} \overline{W} K_{\overline{T}}W) = \text{Re}\left(\left[2\nu_{OM} \overline{W}_{OM} + \frac{4}{C_H} (\chi - \gamma \nu_{OM}) \overline{W}_{GC}\right] \Lambda C^3\right) + \ldots \quad (2.41)$$

are the dominant terms while all other terms and those indicated by dots are suppressed by at least a further factor $\delta, 1/V, 1/V_{OM}$. Once more, one can verify that among the suppressed terms, $K^{\overline{TT}} K_{\overline{T}}K_T|W|^2$, contributes a $+3|W|^2$ which cancels the negative $-3|W|^2$ piece in (2.31). Since all other terms are manifestly positive, it is clear that the effective potential has to be positive. Again, positivity was to be expected from the fact that $K_{(T)}$ is of the form $-3 \ln (T + \overline{T}) + \ldots$ which implies a cancelation of the negative $-3|W|^2$ contribution. If furthermore $W$ would be independent of $T$ then the resulting tree level cosmological constant coming from the $T$ sector would be zero. Here, however, $W$ depends on $T$ which therefore generates a further positive contribution to the potential.
At leading order in \( \delta, 1/V, 1/V_{OM} \) the potential becomes

\[
U = M^4 e^K \left( K^\dagger \partial W \partial_j W + 2 \text{Re}(K S^S \partial S W K S W + K^T T \partial T W K_T W) \right) + U_D \quad (2.42)
\]

\[
= 3M^4 e^{K(A) + K(z)} \left( \frac{4V^2}{C_H V_{OM}} |W_{GC}|^2 + \frac{4}{3} V_{OM} |W_{OM} - \frac{\gamma}{C_H} W_{GC}|^2 \right.
\]
\[
+ \text{Re} \left( 4(1 - N) W_{OM} A C^3 + \frac{4}{C_H} \left( \frac{V}{V_{OM}} - \gamma + N(\gamma - 2 \beta v)) W_{GC} A C^3 \right) \right)
\]
\[
+ 6N |A C^2|^2 \right) + \frac{18 \pi^2 M^4}{V_{OM}^2} \sum_a \left( C T^a C \right)^2 + \ldots,
\]

where the dots indicate the omitted suppressed terms. Notice the squares in the second line. Obviously, a trivial minimization of the potential, in view of its positivity, is given by \( W_{GC} = W_{OM} = C = 0 \) which corresponds to the decompactification vacuum characterized by \( V_{OM}, V \rightarrow \infty \). Our main goal in this paper will be to show that there are further non-trivial local minima of the potential with positive energy corresponding to metastable de Sitter vacua.

### 2.4 Volume Modulus and Parameter Range

We also need to specify the dependence of \( \mathcal{V}(\mathcal{L}) \) on \( \mathcal{L} \). In the full non-linear background of [20], which we use here, the deformed CY volume depends quadratically on the orbifold coordinate \( x^{11} \)

\[
V(x^{11}) = V_v \left( 1 - G_v \frac{x^{11}}{l} \right)^2.
\quad (2.44)
\]

One then infers, using (2.6), the following average CY volume

\[
\mathcal{V}(\mathcal{L}) = V_v \left( 1 - G_v \mathcal{L} + \frac{1}{3}(G_v \mathcal{L})^2 \right). \quad (2.45)
\]

\( \mathcal{V}(\mathcal{L}) \) is monotonously decreasing throughout the interval \( \mathcal{L} \in [0, \frac{3}{2 G_v}] \). Since the moduli space for \( \mathcal{L} \) is restricted to the smaller interval \( [0, \frac{1}{G_v}] \), as we will see shortly, \( \mathcal{V}(\mathcal{L}) \) is monotonously decreasing with \( \mathcal{L} \). Via (2.5) also the complete \( \mathcal{L} \) dependence of \( V_{OM}(\mathcal{L}) \) is now determined. In contrast to \( \mathcal{V}(\mathcal{L}) \) one finds that \( V_{OM}(\mathcal{L}) \) is monotonously increasing for all values of \( \mathcal{L} \).

An interesting constraint which is imposed by the 11-dimensional theory, results from the fact that at \( x_{a}^{11} = l/G_v \) a naked singularity appears in the geometric background [20],
This necessitates the following upper bound on the orbifold length modulus $\mathcal{L}$

$$\mathcal{L} \leq \mathcal{L}_{\text{max}} = \frac{1}{\mathcal{G}_v}.$$  \hspace{1cm} (2.46)

A stabilization of $\mathcal{L}$ should therefore occur below or at $\mathcal{L}_{\text{max}}$. Indeed a stabilization close to $\mathcal{L}_{\text{max}}$ would result in a very precise prediction of the 4d Newton’s Constant $\mathcal{G}$ and is therefore clearly desirable. Notice that the upper bound on $\mathcal{L}$ leads directly to an upper bound on $\gamma(\mathcal{L})$

$$\gamma(\mathcal{L}) \leq 1.$$  \hspace{1cm} (2.47)

Let us finally give for orientation the phenomenologically favored visible boundary CY volume. The issue of how one might stabilize this modulus will be addressed in the final section. In the phenomenological regime the CY radius on the visible boundary is given by $\mathcal{G}$

$$V_v^{1/6} \simeq 2\kappa^{2/9}.$$  \hspace{1cm} (2.48)

Together with (2.7) and (2.8) this implies a value $V_v \simeq 300$ for the dimensionless normalized volume. Note however that this is merely a first order estimate and we should therefore just take the order of magnitude for granted.

### 3 Analysis of the Potential Including Open Membranes

The prime idea which we will pursue here and in the following sections is whether non-perturbative effects in conjunction with $G_{lmnp}$ fluxes might generate a boundary-boundary potential\(^5\) which will cause a stabilization in particular of the orbifold length (‘dilaton’). The influence of the fluxes at this stage comes from a deformation of the geometric background which in turn enters the 4d effective potential through the Kähler-potential and the geometric exponents of open membrane instantons and gaugino condensation. Let us first investigate the simplest case where gaugino condensation is absent but open membrane instantons stretching from boundary to boundary are included. The potential is given by $\mathcal{G}$ and we will now look for its extrema to see whether further de Sitter vacua are present beyond the global minimum with vanishing energy describing the decompactified vacuum.

\(^5\)For earlier investigations of boundary-boundary potentials see $\mathcal{G}$.\(^6\)
3.1 The Axion Sector

To start the analysis of the potential (2.34) for local minima, it is most convenient to consider the $T$-modulus axion $\sigma_T$ first. To this aim let us define

$$
V_1 = \text{Re} \left( \bar{h} \Lambda C^3 \right), \quad V_2 = \text{Im} \left( \bar{h} \Lambda C^3 \right)
$$

(3.1) such that we can write

$$
\text{Re} \left( \bar{h} e^{-\bar{T} \Lambda C^3} \right) = e^{-\nu_{OM}} (V_1 \cos \sigma_T - V_2 \sin \sigma_T).
$$

(3.2)

Extremization of the potential (2.34) w.r.t. the axion $\sigma_T$, i.e. setting $\partial U / \partial \sigma_T = 0$, implies its fixation at

$$
\sigma_T = - \arctan \left( \frac{V_2}{V_1} \right) + n \pi,
$$

(3.3)

where $n \in \mathbb{Z}$. From this we get

$$
e^{i\sigma_T} = (-1)^n \frac{V_1 - iV_2}{|V_1 - iV_2|}
$$

(3.4)

and finally obtain that at the extremal point

$$
\text{Re} \left( \bar{h} e^{-\bar{T} \Lambda C^3} \right) = (-1)^n e^{-\nu_{OM}} |V_1 + iV_2| = (-1)^n |h| e^{-\nu_{OM}} |\Lambda C^3|.
$$

(3.5)

Having fixed $\sigma_T$ the potential at the extremum becomes

$$
U = M^4 e^{K_1 + K_2} \left( \frac{|h|^2}{2} \nu_{OM} e^{-2\nu_{OM}} + \frac{3}{2} (-1)^n (1 - \mathcal{N}) |h| e^{-\nu_{OM}} |\Lambda C^3| + \left( \frac{3}{2} \right)^2 \mathcal{N} |\Lambda C^2|^2 \right) \\
+ M^4 \frac{18 \pi^2}{V_{OM}^2} \sum_a (\mathcal{C} T^a C)^2.
$$

(3.6)

Notice that open membrane instantons alone cannot fix the axion $\sigma_S$. This will become possible once we include gaugino condensation in the next section. Note further, the general feature that fixing an axion still allows for an arbitrary discrete parameter choice, that of $n$. It is however only the class of even or odd values for $n$ which lead to distinct physics. Obviously, in view of the inequality $1 \geq \mathcal{N}$ (2.35), it is for non-vanishing $C$ the odd sector

$$
n \in 2\mathbb{Z} + 1
$$

(3.7)

which leads to lower potential energy.
3.2 The Charged Matter Sector

The general extremization condition which follows in the charged matter sector from $\partial U/\partial C^N = 0$ is

$$\frac{e^{K(A)+K(Z)}}{8d} \left( 2NH^{IL}LA_{JK}C^JCKMNC^M + (1 - N)W_{OM}A_{NJ}C^IC^J \right) + 2\pi^2 \sum_a \left( H_{IJ}C^I(T^a)_{JKC^J} \right) H_{LM}C^M(T^a)_{LN} = 0 .$$

(3.8)

These are as many complex equations as there are unknown complex components of $C$. Therefore the solution to these equations will fix $C$ completely. Obviously

$$C_0 = 0$$

(3.9)

is one solution. We will however see soon that any non-trivial solution for $C$ gives a lower potential energy than $C_0 = 0$. Consequently the extremal value $C_0 = 0$ must correspond to a maximum of the potential while one of the non-trivial solutions $C_0 \neq 0$ gives the minimum of lowest energy. This is good news as it was apparently one of the unsolved open problems of the weakly-coupled heterotic string to stabilize the $C$’s at some non-vanishing values once supersymmetry was broken e.g. by gaugino condensation (see e.g. [47]). Therefore the weakly coupled heterotic string led to charged massless scalars which are experimentally ruled out.

Let us now concentrate on the $C_0 \neq 0$ solution. Since a non-trivial vev for $C$ will be of direct phenomenological relevance, we are now aiming at determining its size. For this purpose, let us contract equation (3.8) with $C^N$ and analyze the resulting single equation

$$C^N \partial U/\partial C^N = 0$$

$$\frac{e^{K(A)+K(Z)}}{8d} \left( 2N|AC|^2 + (1 - N)W_{OM}AC^3 \right) + 2\pi^2 \sum_a (CT^a C)^2 = 0 .$$

(3.10)

Its imaginary part

$$\text{Im}(W_{OM}AC^3) = 0$$

(3.11)

turns out to be equivalent to the condition (3.3) resulting from the axion sector thus showing that the conditions coming from the axion and the charged matter sectors are compatible. By noticing that due to the fixing of the $\sigma_T$ axion in (3.3) one has

$$\text{Re}(W_{OM}AC^3) = (-1)^n e^{-\nu_{OM}|hAC^3|} ,$$

(3.12)
Figure 1: The dependence of $N(L)$ on $L$ is plotted for the values $\beta_v = 1, d = 1, V_v = 300$ which imply $G_v = 0.3$. Notice that the physical range of $L$ is limited and reaches from zero to $L_{\text{max}} = 1/G_v = 3.3$.

we obtain for its real part

$$\frac{e^{K(A)+K(Z)}}{8d} \left( 2N|\Lambda C^2|^2 + (-1)^n(1-N)e^{-V_{OM}}|\Lambda C^3| \right) + 2\pi^2 \sum_a (\overline{C}^a C)^2 = 0 \ . \ (3.13)$$

In conformity with what we said before about the preference of the odd $n$ axion sector, we see here that because $1 \geq N$, we won’t get a non-trivial solution for $C$ if $n$ is even since then all terms are individually positive. A non-trivial solution $C_0$ arises only in the $n$ odd sector when the cubic term becomes negative. Henceforth we will concentrate on this sector.

To determine the size of the non-trivial $C_0$, we now focus on the most influential factor, the exponential $e^{-V_{OM}(L)}$, which amounts to study the $L$ dependence of $C_0$. Note that the only $L$ dependence enters through $N(L)$ and $V_{OM}(L)$. It is however clear that the polynomial $L$ dependence of $N(L)$ is much milder than the exponential $L$ dependence of $e^{-V_{OM}(L)}$. Indeed as illustrated in figure 1, $N(L)$ differs only little from 1 so that subsequently we will regard it as being constant $N = O(1)$. Then only the cubic $C$ term exhibits a dependence on $L$ through the exponential and it is thus clear that satisfying (3.13) requires

$$|C_0| \sim f e^{-V_{OM}(L)} \quad (3.14)$$

where $f$ does not depend on $L$. The size of the extremal $C_0$ will therefore be exponentially sensitive to $V_{OM}$ which is a satisfactory result as it allows to bring $|C_0|$ easily down to phenomenologically relevant values given that $f$ takes its natural values at the reduced Planck scale (after reinstating dimensions).
Let us now see how the potential looks like at the extremal point when the extremization condition (3.13) is applied. With (3.13) we can either eliminate all quartic $C$ terms in (3.6) and replace them by a cubic term or vice versa. The first choice leads to

$$U = \frac{M^4 e^{K_0 + K(x)}}{dV^2} \left( \frac{|h|^2}{2} V_{OM} e^{-2V_{OM}} - \frac{3}{8} (1 - N)^2 e^{-V_{OM}} |h| |\Lambda C_0^3| \right),$$

which clearly shows that the non-trivial solutions $C_0 \neq 0$ lead to a lower potential energy than the trivial solution $C_0 = 0$ which gives maximal potential energy. It furthermore shows that because the $C_0$ is proportional to $e^{-V_{OM}}$, that the charged matter dependent part of the potential is suppressed at the critical point by four times this exponential factor instead of just two times like the $C_0$ independent open membrane part. Furthermore the charged matter part is of lower order in $1/V_{OM}$. We can therefore drop the $C_0$ dependent part for the subsequent investigation of the critical point. The potential at the critical point then becomes

$$U = \frac{M^4 e^{K_0 + K(x)} |h|^2}{2d} \left( e^{-2V_{OM}(L)} \right).$$

(3.16)

### 3.3 The Orbifold Length

Let us finally see whether the potential exhibits a minimum along the $L$ direction in moduli space and therefore allows for a stabilization of the orbifold length. At the locus determined by (3.8) and (3.9) where the potential becomes extremal, the value of the potential is given by (3.16). Here, only the bracket in (3.16) depends on $L$ such that the extremization condition $U'' = 0$ (a prime denotes the derivative w.r.t. $L$) amounts to solving

$$(2V_{OM} + \ln(V_{OM} V))' = \frac{1}{L} (2V_{OM} + 1) + \frac{2V'}{3V} (V_{OM} + 2) = 0,$$

(3.17)

where to arrive at the second equation we have employed (2.5). Since we are working in the $V_{OM} \gg 1$ regime, we can neglect both constants in brackets and obtain

$$3V + V'L = 0 .$$

(3.18)

This simplified equation is equivalent to setting $V'_{OM} = 0$. Substituting (2.45) for $V$, this equation becomes a simple quadratic equation in $L$

$$L^2 - \frac{12}{5G_v} L + \frac{9}{5G_v^2} = 0 .$$

(3.19)
This equation, however, has no real solution for $L$. Therefore, for $V_{OM} \gg 1$ the minimization problem does not have a solution which means that there is no further local vacuum in addition to the global decompactification vacuum.

Let us note that for $V_{OM} \ll 1$ one can find non-trivial solutions $L_0$ to the full equation (3.17). For example for an intersection number $d = 1$ and value $V_v = 300$ one obtains solutions $L_0 \ll 1$ provided that $\beta_v \geq O(10^3)$. However, in this regime the hitherto neglected terms with higher powers of $1/V_{OM}$ would have to be added to the potential besides contributions of multiple instanton wrappings which would then no longer be suppressed. To avoid such complications and because length values $L \ll 1$ seem to be outside the regime of validity of a local field theory description, we will not consider this case further but instead combine the open membrane instantons with the second naturally appearing non-perturbative effect, gaugino condensation.

But before proceeding, a final comment on the background dependence. It is evident from (3.18) that a background which leaves (3.18) intact but provides a sufficiently more negative derivative $V'$ than the one arising from (2.45) would lead to a solution of (3.18). This is actually the case for the linearized background of [18] whose linear CY volume drops faster with $x^{11}$ than the full quadratic volume (the former represents the tangent to the latter at the location of the visible boundary) [20, 21]. With the linearized background one finds indeed a minimum of the potential at $L = L_{\text{max}} = 1/G_v$ and therefore a stabilization at maximal orbifold length. This analysis was carried out in [11] for the case with an additional 4d spacetime-filling M5 brane (the reason to include an M5 brane is to avoid the negative CY volume problem arising within the linearized background, see [20]). One then obtains from open membrane instantons an $L$ dependence $U \sim e^{-V_{OM}/(V V_{OM})}$ instead of $U \sim e^{-2V_{OM}/(VV_{OM})}$ for the case without M5 brane which we face in this paper. In the large $V_{OM}$ limit, however, any $U \sim e^{-aV_{OM}/(VV_{OM})}$, $a = \text{const}$, leads to the same constraint (3.18). Therefore with or without additional M5 brane, one obtains for the linearized background a stabilization of $L$ at the maximally allowed length because $V'$ is more negative in this background. However, by going from the approximative (the approximation works best close to the visible boundary and becomes worse the farther one moves into the bulk) linearized background to the exact full non-linear background, $V'$ becomes less negative and we cannot find a solution to (3.18) any longer. Hence, open membrane instantons aren’t enough to stabilize $L$ in the regime of large $V_{OM}$.
4 Analysis of the Potential Including Open Membranes And Gaugino Condensation

We will now include gaugino condensation (see [48] for a recent review) as the second non-perturbative effect which arises naturally in the strongly coupled gauge theory on the hidden boundary. The main idea for the stabilization of $\mathcal{L}$ is now the following. From the absolute values of the superpotentials

$$|W_{OM}| \sim |h| e^{-\nu_{OM}}, \quad |W_{GC}| \sim |g| e^{-\frac{1}{\beta v} (\nu - \nu_{OM})}$$

and the monotony properties of $\nu$ and $\nu_{OM}$, as pointed out after (2.45), it is clear that $W_{OM}$ decreases with $L$ while $W_{GC}$ increases in the relevant regime $L \in [0, \frac{1}{G_v}]$ (see figure 2). Therefore one might expect that also at the level of the potential (2.43) the combination of both contributions could lead to a non-trivial minimum due to the opposite monotony properties of the two non-perturbative effects. We will now show that this balancing of open membrane instantons against gaugino condensation indeed works.

Figure 2: The dependence of the absolute values of the open membrane and gaugino condensation superpotentials, $|W_{OM}|$ (left curve) and $|W_{GC}|$ (right curve) on $L$. $|W_{OM}|$ decreases with $L$ while $|W_{GC}|$ increases steeply. For the plot we took the values $\beta v = d = 1, |h| = 10^{-7}, \nu_v = 300$ and hidden gauge group $SO(10)$, i.e. $C_H = 8$. The upper bound $L_{max} = 1/G_v$ on $L$ lies at 3.7.
4.1 The Axion Sector

Starting with the potential (2.4.3) let us first derive its minimization constraints w.r.t. the axions $\sigma_S, \sigma_T$. In order to focus on the $\sigma_S, \sigma_T$ dependence of the potential, it proves useful to define the following complex valued quantities

$$X_1 + iX_2 = e^{-\frac{V}{C_H} + \left(\frac{\gamma}{C_H} - 1\right)\nu_{OM} g\tilde{h}}$$

$$Y_1 + iY_2 = e^{-\nu_{OM} \tilde{h} \Lambda C^3}$$

$$Z_1 + iZ_2 = e^{-\frac{V}{C_H} + \frac{\gamma}{C_H} \nu_{OM} g\Lambda C^3}.$$ 

The portion of the potential which depends on $\sigma_S, \sigma_T$ is captured by the three mixed terms

$$\text{Re}(\overline{W}_{OM} W_{GC}) = X_1 \cos \left(\frac{\sigma_S}{C_H} - \left(\frac{\gamma}{C_H} + 1\right)\sigma_T\right) + X_2 \sin \left(\frac{\sigma_S}{C_H} - \left(\frac{\gamma}{C_H} + 1\right)\sigma_T\right)$$

$$\text{Re}(\overline{W}_{OM} \Lambda C^3) = Y_1 \cos \sigma_T - Y_2 \sin \sigma_T$$

$$\text{Re}(\overline{W}_{GC} \Lambda C^3) = Z_1 \cos \left(\frac{\sigma_S}{C_H} - \frac{\gamma}{C_H} \sigma_T\right) - Z_2 \sin \left(\frac{\sigma_S}{C_H} - \frac{\gamma}{C_H} \sigma_T\right),$$

where the first term arises from expanding the complete square $|W_{OM} - \frac{\gamma}{C_H} W_{GC}|^2$. The extremization conditions of the potential w.r.t. $\sigma_S$ and $\sigma_T$ are $\partial U/\partial \sigma_S = \partial U/\partial \sigma_T = 0$. However more succinct and easier to work with are the equivalent conditions, $\partial U/\partial \sigma_S - \partial U/\partial \sigma_T = 0$ which reads

$$2\frac{\gamma}{C_H} \nu_{OM} \left(X_1 \sin \left(\frac{\sigma_S}{C_H} - \left(\frac{\gamma}{C_H} + 1\right)\sigma_T\right) - X_2 \cos \left(\frac{\sigma_S}{C_H} - \left(\frac{\gamma}{C_H} + 1\right)\sigma_T\right)\right)$$

$$= 3(N - 1) \left(Y_1 \sin \sigma_T + Y_2 \cos \sigma_T\right),$$

and $(\gamma + C_H)\partial U/\partial \sigma_S - \gamma \partial U/\partial \sigma_T$ which is

$$\frac{1}{C_H} \left(\frac{V}{\nu_{OM}} - \gamma + N(\gamma - 2\beta_v)\right) \left(Z_1 \sin \left(\frac{\sigma_S}{C_H} - \frac{\gamma}{C_H} \sigma_T\right) + Z_2 \cos \left(\frac{\sigma_S}{C_H} - \frac{\gamma}{C_H} \sigma_T\right)\right)$$

$$= (N - 1) \left(Y_1 \sin \sigma_T + Y_2 \cos \sigma_T\right).$$

These two equations fix the axions $\sigma_S, \sigma_T$ in terms of $\mathcal{L}$ and the charged scalars. Alternatively they can also be expressed in terms of the original variables as

$$-\frac{2}{3} \frac{\gamma}{C_H} \nu_{OM} \text{Im}(\overline{W}_{OM} W_{GC}) = (N - 1) \text{Im}(\overline{W}_{OM} \Lambda C^3)$$

$$= \frac{1}{C_H} \left(\frac{V}{\nu_{OM}} - \gamma + N(\gamma - 2\beta_v)\right) \text{Im}(\overline{W}_{GC} \Lambda C^3).$$ (4.10)
4.2 The Charged Matter Sector

In the charged scalar sector the extremization constraint, \( \partial U / \partial C^N = 0 \), can straightforwardly be evaluated to give

\[
\frac{\epsilon^{K(A)+K(Z)}}{8d} \left( 2N H^T L^I J K C^J C^K \Lambda_{LMN} C^M + \frac{1}{C_H} \left( \frac{V}{V_{OM}} - \gamma + N(\gamma - 2\beta_v) \right) W_{GC} \Lambda_{N1JC^I} C^J \\
+ (1 - N) W_{OM} \Lambda_{N1JC^I} C^J \right) + 2\pi^2 \sum_a \left( H_{I}^{\bar{J}} C I C^K (T^a)_k C^K \right) H_{LM}^{\bar{M}} (T^a)_N L \right) = 0 .
\] (4.11)

These are as many complex equations as there are unknown complex components of \( C \).

The solution to this system of equations will therefore fix all components of \( C \).

A more handy, and for our purposes sufficient, implied constraint consists of the index-free contracted equation \( C^N \partial U / \partial C^N = 0 \) which reads

\[
\frac{\epsilon^{K(A)+K(Z)}}{8d} \left( 2N |\Lambda C|^2 + (1 - N) W_{OM} \Lambda C^3 + \frac{1}{C_H} \left( \frac{V}{V_{OM}} - \gamma + N(\gamma - 2\beta_v) \right) W_{GC} \Lambda C^3 \right) \\
+ 2\pi^2 \sum_a (\bar{C} T^a C)^2 = 0 .
\] (4.12)

It is obvious that

\[
C_0 = 0
\] (4.13)

constitutes a solution not only to the contracted equation but also to the full set of constraints, \( \partial U / \partial C^N = 0 \). There is however, as pointed out, also a nontrivial solution \( C_0 \). As already indicated before, this is interesting as it solves the notorious problem of vanishing \( C \)'s, and therefore massless charged scalars, of the weakly coupled heterotic string after supersymmetry breaking through gaugino condensation \[47\]. Let us now investigate further the constraint (4.12) for the non-trivial \( C \).

Observe first that in the complex valued equation (4.12) only the second and third term possess an imaginary part, so that the real valued equation resulting from just the imaginary part of the complex equation (4.12) simply becomes equal to the second equation in (4.11) or equivalently equal to (4.9). It therefore represents no further constraint beyond those already obtained from the axion sector but shows that the extremization conditions coming from the axion sector and the charged matter sector are compatible.

On the other hand the real part of the equation \( C^N \partial U / \partial C^N = 0 \) gives us the inde-
dependent constraint

$$e^{K_{(A)} + K_{(Z)}} \left( 2N|\Lambda C|^2 + (1 - N)\text{Re} (\overline{W}_{OM} \Lambda C^3) + \frac{1}{C_H} \left( \frac{\mathcal{V}}{\mathcal{V}_{OM}} - \gamma + N(\gamma - 2\beta_v) \right) \right) \times \text{Re} (\overline{W}_{GC} \Lambda C^3) + 2\pi^2 \sum_a |\mathcal{C}T^a C|^2 = 0.$$  \hspace{1cm} (4.14)

This is an important equation because it controls the size of $C_0$ and therefore the size of the $C$ dependent terms in the potential at the extremal point. This in turn will show whether the $C$ dependent terms are of the same order (or bigger) than the sofar included $C$ independent dominant terms and therefore have to be kept in the analysis or whether the $C$ dependent terms are of the same small size as the suppressed terms which would mean that the $C$ terms likewise would have to be discarded.

Since we have used $1/\mathcal{V}$ and $1/\mathcal{V}_{OM}$ as expansion parameters (only sufficiently large $\mathcal{V}$ and $\mathcal{V}_{OM}$ allow us to neglect contributions from multiple instanton wrappings which are currently not well understood) to determine the leading dominant part of the potential, the question will therefore be how does $C_0$ depend on these two parameters. Let us remark that though $\mathcal{N}(\mathcal{L})$ has a dependence on these two parameters it is with sufficient accuracy constant, $\mathcal{N} = \mathcal{O}(1)$, (see figure 1) and is therefore of no concern in the sequel. If we furthermore approximate the real parts in (4.14) through absolute values then the magnitude of $C_0$ is determined by

$$f_1 |C_0|^4 + f_2 |h| e^{-V_{OM}} |C_0|^3 + f_3 \frac{|g|}{C_H} \left( \frac{\mathcal{V}}{\mathcal{V}_{OM}} - \gamma + N(\gamma - 2\beta_v) \right) e^{-\frac{1}{C_H} (\mathcal{V} - \gamma \mathcal{V}_{OM}) |C_0|^3} \sim 0$$ \hspace{1cm} (4.15)

where the coefficients $f_i$ do not depend on $\mathcal{V}$ and $\mathcal{V}_{OM}$. The size of $C_0$ is therefore estimated as

$$|C_0| \sim \frac{f_2}{f_1} |h| e^{-V_{OM}} + \frac{f_3 |g|}{f_1 C_H} \left( \frac{\mathcal{V}}{\mathcal{V}_{OM}} - \gamma + N(\gamma - 2\beta_v) \right) e^{-\frac{1}{C_H} (\mathcal{V} - \gamma \mathcal{V}_{OM})}.$$ \hspace{1cm} (4.16)

The non-perturbative exponential factors are phenomenologically interesting as they allow to bring $C$ down to relevant values far below the reduced Planck scale (after reinstating dimensions for $C$ by multiplying with $M$).

After having estimated the $C$ vev, let us now look at the resulting vacuum energy. The condition (4.14) implies, by using it to eliminate the two cubic $C^3$ terms, that the
potential \(2.43\) at the critical point becomes

\[
U = \frac{3M^4e^{K_{(A)}+K_{(Z)}}}{2dV_{OM}^2} \left( \frac{\mathcal{V}^2}{C_H^2\mathcal{V}_{OM}}|W_{GC}|^2 + \frac{1}{3}\mathcal{V}_{OM}|W_{OM} - \frac{\gamma}{C_H}W_{GC}|^2 - \frac{1}{2}\mathcal{N}^2|\Lambda C^2|^2 + \ldots \right) \\
- \frac{6M^4\pi^2}{VV_{OM}^2} \sum_a (\mathcal{C}^a\mathcal{C})^2, \tag{4.17}
\]

where the dots comprise the neglected suppressed terms. It is therefore clear from the negative definiteness of the quartic \(C^4\) terms that the solution with non-vanishing \(C_0\) is the one with the lower energy density and hence preferred over the \(C_0 = 0\) solution. Notice however that though the quartic terms enter with a negative sign, the total potential energy density must still be non-negative in view of the cancelation of the \(-3|W|^2\) term, as has been shown earlier.

It is easy to see that for the further analysis we can now drop the \(C\) contribution. The \(C^4\) terms of the potential at the extremum are of magnitude

\[
|C_0|^4 \sim \left( |h|e^{-\mathcal{V}_{OM}} + \frac{|g|\mathcal{V}}{C_H\mathcal{V}_{OM}}e^{-\frac{1}{C_H}(\mathcal{V} - \gamma\mathcal{V}_{OM})} \right)^4. \tag{4.18}
\]

They are therefore strongly suppressed against the pure non-perturbative contributions not only because they are of lower order in \(1/\mathcal{V}, 1/\mathcal{V}_{OM}\) but also because they are suppressed by the fourth power of the exponentials while the non-perturbative contributions go like the second power of the exponentials. We can therefore conclude that though a non-trivial vev for \(C\) is a general outcome of our analysis, the \(C\) dependent terms can safely be neglected at the critical point in comparison to the non-perturbative contributions. They will consequently be omitted henceforth.

Before doing so, however, let us briefly comment on the by now ubiquitous exponential factors and their virtues. With the \(C^4\) terms exponentially suppressed as well, the whole potential exhibits an exponential suppression at the critical point. Therefore, at least at tree level, the resulting positive vacuum energy might be brought down to a phenomenologically acceptable value in the range of \(U \sim \text{meV}^4\) triggered by values for \(\mathcal{V}, \mathcal{V}_{OM}\) of \(O(100)\). In this respect, a hidden gauge group of small rank with smaller value for the dual Coxeter number \(C_H\) will be favored. This idea of obtaining a small cosmological constant through an exponential suppression was introduced earlier in the string-theory brane-world context [49]. However while for those brane-worlds an exponential warp-factor was exploited it is here the non-perturbative effects which give the exponential suppression. It would of course be interesting to investigate quantum corrections to our effective potential and to see whether they likewise come out exponentially suppressed. On the other
hand with values of $V/C_H, V_{OM} \sim O(10)$ one would get $C$ vev’s and therefore masses at the TeV scale (see e.g. [50]). We will see later when we are studying the values of $V, V_{OM}$ for our vacua that it is actually the latter possibility which can be realized here. An explanation of the tiny value of the observed cosmological constant is however out of reach. Presumably the solution to this most puzzling enigma seems to require radical new ideas about the structure of spacetime itself which might perhaps be based on theories with just a finite number of degrees of freedom as suggested by [26], [28], [51].

4.3 The Orbifold Length

As explained we will henceforth omit the $C$ terms. As far as the derivation of the axion constraints is concerned omitting the $C$ terms amounts to setting $Y_1 = Y_2 = Z_1 = Z_2 = 0$ which means that the initially two constraints (4.8), (4.9) collapse to just one constraint which fixes the linear combination $a_{\sigma}S - (a\gamma + 1)\sigma_T$ to be

$$\frac{\sigma_S}{C_H} - (\gamma + 1)\sigma_T = \arctan\left(\frac{\text{Im}h}{\text{Re}h}\right) + n\pi, \quad n \in \mathbb{Z}. \quad (4.19)$$

We will see below that its real complement $\frac{1}{C_H}\text{Re}S - (\frac{\gamma}{C_H} + 1)\text{Re}T$ determines the extremal value of $L$ by becoming approximately zero. It is therefore the combination of chiral fields $\frac{1}{C_H}S - (\frac{\gamma}{C_H} + 1)T$ which becomes fixed at the extremum at leading order.

Let us now actually perform the extremization w.r.t. $L$. Without $C$’s the leading potential simplifies to

$$U = \frac{M^4 e^{K(A) + K(z)}}{2dVV_{OM}} \left(3\left(\frac{V}{C_HV_{OM}}|W_{GC}|\right)^2 + |W_{OM} - \gamma W_{GC}|^2\right) \quad (4.20)$$

Expanding the second square gives a mixed term which, with help of (4.19) and the axion fixation (4.19), simplifies to

$$\text{Re}(\overline{W}_{OM}W_{GC}) = (-1)^n|W_{OM}||W_{GC}|. \quad (4.21)$$

Consequently the potential can be written as

$$U = \frac{M^4 e^{K(A) + K(z)}}{2dVV_{OM}} \left(3\left(\frac{V}{C_HV_{OM}}|W_{GC}|\right)^2 + \left(|W_{OM}| - (-1)^n\frac{\gamma}{C_H}|W_{GC}|\right)^2\right). \quad (4.22)$$

Obviously the axion sector with even $n$

$$n \in 2\mathbb{Z} \quad (4.23)$$
Figure 3: The left picture shows the leading order effective 4-dim. potential in Planck units as a function of the orbifold length $\mathcal{L}$. To give an impression of the global behavior of the potential over the complete interval $[0, \mathcal{L}_{\text{max}}]$, we display in the right picture the logarithm of the potential. A de Sitter minimum appears at $\mathcal{L}_0 = 6.9$. Towards its left open membrane instantons dominate the potential, towards its right it’s the gaugino condensation. For the assumed values $\beta_v = 1, d = 10, |h| = 10^{-8}, \mathcal{V}_v = 800$ and a hidden gauge group $SO(10)$ the upper bound on $\mathcal{L}$ is $\mathcal{L}_{\text{max}} = 11.0$.

results in the lower potential energy density. Hence we will concentrate on this even sector hereafter. The extremization condition $\partial U/\partial \mathcal{L} = 0$ leads to the transcendental equation

$$
\frac{\partial}{\partial \mathcal{L}} \left[ \frac{1}{\mathcal{V}_0 \mathcal{V}_M} \left( 3 \left( \frac{|g| \mathcal{V}_M}{C_H \mathcal{V}_M} e^{-\frac{\mathcal{V}_M}{C_H}} + \frac{\gamma \mathcal{V}_0}{C_H} \right)^2 + \left( |h| e^{-\mathcal{V}_0} - \frac{|g| \gamma \mathcal{V}_0}{C_H} e^{-\frac{\mathcal{V}_0}{C_H}} \right)^2 \right) \right] = 0 \quad (4.24)
$$

which we have to solve numerically. We will now discuss its solutions.

### 4.4 De Sitter Minima

The first important result is that without the need for a fine-tuning of the parameters generically an extremal value $\mathcal{L}_0$, i.e. a real valued solution to this equation, is found below the upper bound $\mathcal{L}_{\text{max}} = 1/\mathcal{G}_\nu$ (see tables 1-4 in appendix B). That this extremal value indeed corresponds to a minimum of the potential in $\mathcal{L}$ direction becomes apparent once the potential (4.22) for even $n$ is plotted. Figure 3 shows the potential for the case of a hidden $SO(10)$. Notice once more that the smallness of $|h|$ is natural in our conventions, cf. (2.24). In order to capture the global behavior of the potential in the whole admissible interval $\mathcal{L} \in [0, \mathcal{L}_{\text{max}}]$ we have plotted in addition the logarithm of the potential. Characteristic in this second plot is the nearly linear dependence both to the left and to the right of the minimum. It clearly reveals the dominance of the open
membrane instantons to the left and that of gaugino condensation to the right with their different monotonicity properties. To directly check that the minimum lies in a regime where the theory is under control, we have plotted in figure 4 the values for $V, V_{OM}$ around the minimum. Indeed both volumes are sufficiently larger than 1 so that one can trust the leading order terms included in the potential and moreover can be sure that multiply wrapped instanton contributions are considerably suppressed and need not be considered.

As we see from figure 3 and was also clear from the fact that the $-3|W|^2$ term canceled out of the potential, the minimum comes with a positive vacuum energy density and represents hence a local de Sitter vacuum. It is easy to see that at leading order in $1/V$ and $1/V_{OM}$

$$D_S W = -\frac{W_{GC}}{C_H} \neq 0,$$

thus showing that at the location of the minimum in particular $D_S W$ is non-vanishing. Consequently *supersymmetry is broken spontaneously in these vacua* through F-term expectation values.

One might wonder whether our potential can be directly extrapolated to the weakly coupled regime. This is however not possible as it stands because we derived the potential in the regime where both $V \gg 1, V_{OM} \gg 1$, in order to have the supergravity theory under control. To connect to the weakly coupled heterotic string one would have to go instead to the limit where $V_v \sim V \gg 1$ but $V_{OM} \to 0$. In this limit some terms which we have suppressed would become dominant while some of those terms which we included would

Figure 4: *To check that the minimum lies in a controllable regime we present here $V$ (upper curve) and $V_{OM}$ (lower curve) for the same parameter values as in the previous figure. Both are substantially larger than 1 at the minimum’s position $L_0 = 6.9$ to guarantee that the neglected terms or multiply wrapped instantons are adequately suppressed.*
become suppressed. Therefore the potential plotted in figure 3 should not be trusted far below the minimum $L_0$ where one reaches the weakly coupled limit $L_0 \gg L \rightarrow 0$.

### 4.5 How Robust is the Minimum?

Let us now analyze how robust the mechanism to generate these de Sitter vacua is. To this end we have to study the influence of the choice of the hidden gauge group $H$, the choice of the intersection number $d$ and therefore the choice of the CY. Furthermore, since the numerical value of $|h|$ cannot be calculated precisely with current technology, we will have to investigate its influence as well. Finally, it will be important to check that both volumes $V_0 = V(L_0)$ and $V_{OM,0} = V_{OM}(L_0)$ are substantially bigger than 1 at the minimum’s position as this secures that the minimum lies in a region of moduli space which is under control in the sense that we can trust the leading terms considered in the potential and moreover do not have to worry that multiply wrapped instantons would generate substantial corrections. Moreover from a phenomenological perspective it will be interesting to see whether we can achieve to bring the minimum’s position $L_0$ close to the maximum $L_{max}$ as this position is distinguished by leading to the correct 4-dimensional Newton’s Constant once the GUT-scale and the GUT gauge coupling attain their traditional values $[18, 21]$.

Finally we aim to show that the minimum’s position $L_0$ which is obtained as a solution to the fairly complicated transcendental equation (4.24) can reasonably well be approximated by $L_{prox}$ which is obtained from the much simpler equation

$$\left(\frac{1}{C_H} Re S - \frac{\gamma}{C_H} + 1) Re T\right)\big|_{L=L_{prox}} = 0.$$  \hspace{1cm} (4.26)

This condition can be obtained from the full extremization condition (4.24) by neglect of the first term against the second and by neglect of the derivative of a polynomial in $V, V_{OM}$ against the derivative of an exponential in $V, V_{OM}$. The former is justified as it is the second term whose derivative changes sign near the minimum while the first term’s derivative doesn’t change sign. The physics which is captured by the simplified condition (4.20) is the balance between open membrane instantons ($\sim e^{-Re T}$) and gaugino condensation ($\sim e^{-\frac{1}{\tau_H} Re S + \frac{\gamma}{\tau_H} Re T}$) since it is equivalent to setting

$$e^{-Re T} = e^{-\frac{1}{\tau_H} Re S + \frac{\gamma}{\tau_H} Re T}$$  \hspace{1cm} (4.27)

at $L = L_{prox}$. The so obtained approximate minimum’s position $L_{prox}$ does not depend on $|h|, |g|$ and represents the true position most faithfully if $|h| \simeq |g|$. 

29
Tables 1-4 of appendix B display the dependence of $L_0$, $L_{\text{prox}}$, $L_{\text{max}}$, $V_0$, $V_{OM,0}$ on the hidden gauge group $H$ and $d$, $|h|$, $V_v$. Table 1 which investigates the influence of the hidden gauge group $H$ clearly shows that in order to bring $L_0$ close to $L_{\text{max}}$ one would like to take a non-trivial hidden gauge bundle which breaks the hidden $E_8$ gauge symmetry down to a group with considerably smaller dual Coxeter number $C_H$. Furthermore, we discern from table 1 that a hidden gauge group with low $C_H$ will bring $V_{OM,0}$ up while maintaining rather large values for $V_0$ thereby lowering both the vev for the charged matter $C$ and the positive vacuum energy at the minimum. Table 2 studies the influence of varying the CY intersection number $d$. A larger $d$ drives $L_0$ to larger values. This can be quantified by observing that a changing $d$ does not alter $V_0$ or $V_{OM,0}$. It is then clear from (2.25) that

$$L_0 \propto d^{1/3}. \quad (4.28)$$

Finally table 3 and table 4 which study the influence of $|h|$ and $V_v$ show that the influence of $|h|$ on the position $L_0$ of the critical point is rather mild while increasing values of $V_v$ let $L_0$ and $V_0$, $V_{OM,0}$ grow.

Moreover, we see from the tables of appendix B that no fine-tuning needs to be invoked to get both $V_0 \gg 1$ and $V_{OM,0} \gg 1$ in order to have the minimum located in a controllable regime. Besides that we also see that $L_{\text{prox}}$ approximates $L_0$ rather well. In conjunction with the stabilization of the axions (4.19) we hence see that at leading order and under the approximation leading to (4.26) it is

$$\frac{1}{C_H}S - \left(\frac{\gamma}{C_H} + 1\right)T = i\left(\arctan\left(\frac{\text{Im}h}{\text{Re}h}\right) + 2\pi n\right) \quad (4.29)$$

which gets stabilized. Since $\partial U/\partial L = 0$ is a scalar equation it is of course clear that it cannot stabilize both $\text{Re}S$ and $\text{Re}T$ individually. To stabilize both $\text{Re}S$ and $\text{Re}T$ individually requires a stabilization of $V_v$ as well for which we will have to take into account also the $H$-flux superpotential as will be outlined in the final section. Note, however, that at subleading order we know that there are two independent conditions (4.8) and (4.9) of the axion sector which stabilize both $\text{Im}S$ and $\text{Im}T$.

### 4.6 Properties of the De Sitter Vacua

Let us now look at some of the properties of our de Sitter vacua. As mentioned before one can easily verify that the $N = 1, D = 4$ supersymmetry is broken spontaneously in
them. At leading order in $1/V$ and $1/V_{OM}$ one finds
\[
D_{SW} = -\frac{1}{C_H}W_{GC} \neq 0
\] (4.30)
\[
D_{TW} = -W_{OM} + \frac{\gamma}{C_H}W_{GC}
\] (4.31)
\[
D_{I}W \equiv D_{CI}W = 3\Lambda_{JKL}C^J C^K + \frac{3}{2V_{OM}}N_{HI}C^I .
\] (4.32)

Because of (4.29) we can write for these vacua
\[
D_{TW} = \left( \frac{\gamma}{C_H} \left| \frac{g}{H} \right| - 1 \right)W_{OM} .
\] (4.33)

Due to the balancing of open membrane instantons and gaugino condensation at the potential’s minimum, $D_{SW}$ and $D_{TW}$ will be roughly of the same order. As to $D_{I}W$, its first term will in view of (4.16) be exponentially smaller than the second term and is therefore negligible while the second term is suppressed by an additional $1/V_{OM}$ as compared to $D_{SW}$ and $D_{TW}$. One therefore finds in these vacua (except for the fine-tuned case where the bracket in (4.33) vanishes and one has $D_{TW} = 0$)
\[
|D_{SW}| \sim |D_{TW}| \gg |D_{I}W| > 0 ,
\] (4.34)
and therefore an $F$-term supersymmetry breaking. The last inequality holds because of the non-vanishing $C$’s in these vacua. Note that for $C \neq 0$ also the D-term might be non-vanishing. However because it contains four $C$’s it would be strongly exponentially suppressed due to (4.16) against the F-terms.

The next important characterization of our vacua will then be the supersymmetry breaking scale $M_{SUSY}$. It is given by the largest vev of the F-terms
\[
M_{SUSY} = M \max \{|F^{\bar{I}}|^{1/2}\} .
\] (4.35)
Evaluated for our vacua the F-terms at leading order in $1/V_0$, $1/V_{OM,0}$ are
\[
F^\bar{S} = -\frac{e^{\frac{1}{2}(K_{(A)}+K_{(z)})}}{\sqrt{6V_0}}\sqrt{\frac{dV_{OM,0}}{C_H}W_{GC}}
\] (4.36)
\[
F^\bar{T} = \frac{e^{\frac{1}{2}(K_{(A)}+K_{(z)})}}{\sqrt{3dV_0}}\left( \frac{\gamma}{C_H}W_{GC} - W_{OM} \right)
\] (4.37)
\[
F^\bar{T} \equiv F^{\bar{T}j} = N e^{\frac{1}{2}(K_{(A)}+K_{(z)})}\left( \frac{1}{6dV_0V_{OM,0}} \right)^{1/2}\left( C^T \left[ \frac{(\gamma - 2\beta_v)}{C_H}W_{GC} - W_{OM} \right] + 3H^{Tj} \Lambda_{JKL}C^K C^L \right) .
\] (4.38)
Here we have discarded contributions of higher powers in $C$ to the first two F-terms as they are due to (4.16) exponentially suppressed w.r.t. the terms given here. As is clearly recognizable from tables 1-4 our vacua exhibit next to $V_0 \gg 1, \nu_{OM,0} \gg 1$ the inequality $V_0 > \nu_{OM,0}$. This means that

$$F^S > F^T \gg F^I,$$

(4.40)

the last inequality once more because of (4.16) which renders $F^T$ exponentially suppressed as compared to the former two F-terms. We therefore conclude that

$$M_{\text{SUSY}} = M |F^S|^{1/2} = M e^{\frac{1}{2}(K(A)+K(Z))} \left( \frac{6V_0^3}{dV_{OM,0}^3} \right)^{1/4} \left( \frac{|W_{GC}|}{C_H} \right)^{1/2}.$$  

(4.41)

The next important entity characterizing the de Sitter vacua is the gravitino mass. Its value is given by

$$m_{3/2}^2 = M^2 e^{K/2} |W|.$$  

(4.42)

For its evaluation we will have to determine the absolute value of the complete superpotential. Note first that because of (4.16) the cubic $C$ superpotential is again exponentially smaller as compared to the two non-perturbative superpotentials and will therefore be discarded. To evaluate the remaining $|W_{OM} + W_{GC}|$ we use (4.21) in the axion sector with even $n$. The gravitino mass then becomes

$$m_{3/2} = M e^{\frac{1}{4}(K(A)+K(Z))} \left( \frac{3}{8dV_0V_{OM,0}^3} \right)^{1/4} (|W_{OM}| + |W_{GC}|)^{1/2}.$$  

(4.43)

Hence the ratio of supersymmetry breaking scale to gravitino mass is

$$\frac{M_{\text{SUSY}}}{m_{3/2}} = \frac{2V_0}{C_H^{1/2}} \left( \frac{|W_{GC}|}{|W_{OM}| + |W_{GC}|} \right)^{1/2}.$$  

(4.44)

As can be seen from the tables in appendix $H$ the vacuum energy, supersymmetry breaking scale and gravitino mass can be brought into the phenomenological regime for rather small unbroken hidden gauge groups, not too small $d$, small absolute value for the Pfaffian $|h|$ and CY volumes $V_v \geq 300$. In particular it will be interesting to better understand the Pfaffians $h$. Progress in this direction was made in [42], [52].

### 4.7 The Decompactification Limit

We have stressed before that our effective 4d potential is valid in the strongly-coupled regime $\mathcal{V} \gg 1, \nu_{OM} \gg 1$ but looses validity towards weak-coupling where $\mathcal{L} \to 0$ and
therefore $\mathcal{V} \to \mathcal{V}_v, \mathcal{V}_{OM} \to 0$. In this regime some terms which were neglected here are no longer suppressed and need to be included while on the other hand some terms which we included here will become negligible. The prime reason for us to consider the 4d effective potential in the limit $\mathcal{V} \gg 1, \mathcal{V}_{OM} \gg 1$ was to ensure that volumes (in Planck units) are big enough to trust the field theory framework and moreover to be able to neglect multiply wrapped open membrane instantons about which less is known at present.

However, we are certainly able to study the potential in its decompactification limit in which $\mathcal{V}_v \to \infty, \mathcal{L} < \mathcal{L}_{\text{max}} \to \infty$ as this amounts to sending $\mathcal{V} \to \infty, \mathcal{V}_{OM} \to \infty$. Here a flat background geometry is recovered with vanishing vacuum energy. In accordance with this we will now verify that our potential indeed fulfills this expectation and declines towards zero in this limit. First note that since $\mathcal{V}$ decreases from $\mathcal{V}(\mathcal{L} = 0) = \mathcal{V}_v$ towards $\mathcal{V}(\mathcal{L} = \mathcal{L}_{\text{max}}) = \mathcal{V}_v/3$ we have in the decompactification limit $\mathcal{V} \simeq \mathcal{V}_v$. Hence the flux parameter (cf. (2.29)) vanishes

$$ G_v \sim \frac{1}{\mathcal{V}_v^{1/3}} \to 0 , \quad (4.45) $$

which implies that

$$ \mathcal{L}_{\text{max}} \sim \mathcal{V}_v^{1/3} \to \infty . \quad (4.46) $$

It is therefore indeed consistent to study the potential in the decompactification limit at arbitrarily large orbifold lengths which is not possible for finite $\mathcal{V}_v$ since it implies a finite largest $\mathcal{L}_{\text{max}}$ (the situation of figure 3). Now

$$ \gamma \mathcal{V}_{OM} \leq \mathcal{V}_{OM} \sim \mathcal{L} \mathcal{V}^{1/3} \leq \mathcal{L}_{\text{max}} \mathcal{V}^{1/3} \sim \mathcal{V}^{2/3} . \quad (4.47) $$

Therefore for the gaugino condensation exponent we get

$$ \frac{1}{C_H} (-\mathcal{V} + \gamma \mathcal{V}_{OM}) \sim -\frac{\mathcal{V}}{C_H} + \mathcal{O}(\mathcal{V}^{2/3}) , \quad (4.48) $$

which shows that

$$ |W_{GC}| \sim e^{-\mathcal{V}_v/C_H} \to 0 , \quad |W_{OM}| \sim e^{-\mathcal{L} \mathcal{V}_v^{1/3}} \to 0 \quad (4.49) $$

and therefore up to polynomial factors the potential energy density $U$ (4.22) is exponentially declining towards zero in the decompactification limit as expected. The importance in showing this smooth connection towards the decompactification with vanishing energy density lies in the fact that it proves our de Sitter vacua to be metastable.
5 Stabilization of the Non-Universal Moduli

Up to now we have found a stabilization of the model-independent moduli through the inclusion of non-perturbative open membrane instantons and gaugino condensation on the hidden boundary. For this it was essential that the average CY volume $V(L)$ and the orbifold length $L$ are not independent. Otherwise one would have obtained a minimum only in the decompactification limit where $V$ and $V_{OM}$ become infinite. This non-trivial volume dependence was caused by a non-vanishing vev for $G_{lmnp}$. A non-trivial vev for the four-form $G$ is generically required due to the boundaries which represent magnetic sources for $G$ and therefore render the Bianchi-identity for $G$ non-trivial. However, in general one could also have a $G_{tmn11}$ component compatible with the boundary sources. Namely the general solution to the Bianchi identity

$$dG = \delta(x^{11} - x_{i}^{11})S_i(y) \wedge dx^{11} \quad (5.1)$$

is given by

$$G = p\Theta(x^{11} - x_{i}^{11})S_i(y) + q\delta(x^{11} - x_{i}^{11})\omega_i(y) \wedge dx^{11} \quad (5.2)$$

where $p + q = 1$, the Chern-Simons three-forms $\omega_i(y)$ satisfy $d\omega_i(y) = S_i$ and $S_i$ are the possible magnetic four-form sources. $G$ therefore contains both types of field-strengths. Whereas the first part ($G_{lmnp}$) exists through (part of) the bulk, the second part ($G_{tmn11}$) is localized on the boundaries or on the M5 branes in case that these are present. This component of the eleven dimensional flux corresponds to the $H$-flux appearing in the weakly coupled heterotic string. It was shown in [9], and the second reference of [10] that this flux acts as a torsion on the internal geometry so that the six dimensional internal manifold $X$ is no longer Kähler and satisfies $d\omega \neq 0$, where $\omega$ is the fundamental two-form of the internal manifold. As we already mentioned at the beginning of this paper, this component of the flux is orthogonal to the component taken into account in this paper and originates from the second term appearing in the previous Bianchi identity.

Recall that in [22], [23] and [24] it was argued that most of the moduli fields of the non-Kähler internal manifold can be stabilized once the $G_{tmn11}$ component of the flux has a non-trivial vev, even though the question on how the moduli fields for this class of manifolds look like very concretely still remains open. Work in this direction is in progress. Let us recapitulate the main results for the moduli stabilization mechanism of the weakly coupled heterotic string proposed in the previous references as the results obtained therein very nicely supplement the results obtained in this paper.
The supersymmetry constraints that are present for compactifications of the weakly heterotic string were first derived in [9] and the second reference of [10]. One of these constraints is the torsional equation which relates the fundamental two-form of the internal manifold to the $H$-flux

$$H = i(\partial - \bar{\partial})\omega .$$

(5.3)

The expression for the $H$-flux that is gauge invariant and anomaly free is written in terms of the Chern Simons three-forms of the non abelian gauge field $A$ and the spin connection $\omega_0$

$$H = dB - \alpha' [\Omega_3(A) - \Omega_3(\omega_0 - \frac{1}{2}H)].$$

(5.4)

This torsion is not closed because we cannot embed the gauge connection into the torsional spin connection, as in that case the $H$-flux would be vanishing. In order to obtain a solution for the $H$-flux for a specific background geometry one has to solve the previous equation iteratively in $H$, as the flux appears on both sides of the previous equation. This has in done in [22] for a particular background of the $SO(32)$ heterotic string that can be obtained from duality chasing a particular model of the general class of models describing M-theory compactifications with non-vanishing fluxes [53]. What becomes clear from the previous two equations is that under a rescaling of the fundamental form with an overall factor ‘t’ which represents the radial modulus and determines the volume of the internal manifold

$$\omega \rightarrow t\omega ,$$

(5.5)

the $H$-flux does not transform in any simple way, so that it is expected that the radial modulus can be stabilized for this type of compactifications at tree level. In fact, it is expected that all the complex structure moduli and at least some of the Kähler moduli can be stabilized in these compactifications. Notice that we are using a rather lax wording at this point, because as explained before, the actual moduli for this type of compactifications are still under investigation. The superpotential responsible for the stabilization of these fields takes the form (cf. [22] fourth reference and [24] second reference)

$$W = \int (H + id\omega) \wedge \Omega .$$

(5.6)

In the ordinary CY case we know that the fundamental two-form is closed $d\omega = 0$, so that we recover the superpotential for the heterotic string compactified on a CY three fold conjectured in [54] and checked in [55]. From this formula of the superpotential we
observe that the dynamics of the weakly coupled theory can be described in terms of a complex three-form

$$\mathcal{H} = H + i d \omega ,$$

which is anomaly free and gauge invariant. From the kinetic term of this three-form $\int |\mathcal{H}|^2$ we can build the scalar potential $V$ for the moduli fields (cf. also [21] second reference). For the simplest example in which we only have a radial modulus this potential was written down explicitly in the last two references of [22]

$$V(t) = \frac{t^3}{\alpha'} - \frac{2\alpha' f^4}{t^3} + \frac{7\alpha'^2 f^6}{t^6} + \ldots ,$$

where $f$ is the flux density. One can easily see that this potential has a minimum for

$$t = 1.288(\alpha' |f|^2)^{1/3} ;$$

which stabilizes the radial modulus and thus the volume of the internal manifold in terms of the flux density.

**Acknowledgements**

We would like to thank F. Denef, S. Kachru, J. Maldacena, G. Moore, J. Pati, E. Poppitz, Q. Shafi, G. Shiu, L. Susskind, H. Verlinde for interesting discussions related to this work. The work of M.B. is supported by NSF grant PHY-01-5-23911 and an Alfred Sloan Fellowship, that of A.K. by NSF grant PHY-0099544.

**A Kähler-Potential and Derivatives**

Let’s consider the following part of the Kähler-potential which neglects the complex structure and bundle moduli contributions

$$\tilde{K} := K(S) + K(T) + K(C) = - \ln \left( \frac{d}{6} (S + \overline{S})(T + \overline{T})^3 \right) + \left( \frac{3}{T + \overline{T}} + \frac{2\beta_v}{S + \overline{S}} \right) H_{IJ} C^I \overline{C}^J$$

$$= - \ln \left( \frac{8}{3} dV_{OM}^3 \right) + \left( \frac{3}{2V_{OM}} + \frac{\beta_v}{V} \right) H_{IJ} C^I \overline{C}^J$$

It follows that

$$e^{\tilde{K}} = \frac{3}{8dV_{OM}^3} \left( 1 + \left( \frac{3}{2V_{OM}} + \frac{\beta_v}{V} \right) H_{IJ} C^I \overline{C}^J \right).$$
First derivatives of $\tilde{K}$ with respect to the moduli appear in the Kähler covariant derivative $D_i W$. They are given by (notation: $\tilde{K}_I = \partial \tilde{K} / \partial C^I$, $\tilde{K}_T = \partial \tilde{K} / \partial \overline{C}^T$)

\[
\tilde{K}_S = \tilde{K}_{\overline{S}} = -\frac{1}{2V} \left( 1 + \beta_v \frac{H_I \overline{C}^I \overline{C}^T}{V} \right) \quad (A.3)
\]

\[
\tilde{K}_T = \tilde{K}_{\overline{T}} = -\frac{3}{2V_{OM}} \left( 1 + \frac{H_I \overline{C}^I \overline{C}^T}{2V_{OM}} \right) \quad (A.4)
\]

\[
\tilde{K}_I = \left( \frac{3}{2V_{OM}} + \frac{\beta_v}{V} \right) H_I \overline{C}^T \quad (A.5)
\]

Second derivatives:

\[
\tilde{K}_{S\overline{S}} = \frac{1}{4V^2} \left( 1 + 2\beta_v \frac{H_I \overline{C}^I \overline{C}^T}{V} \right) \quad (A.6)
\]

\[
\tilde{K}_{S\overline{T}} = 0 \quad (A.7)
\]

\[
\tilde{K}_{S\overline{J}} = -\frac{\beta_v H_I \overline{C}^I}{2V^2} \quad (A.8)
\]

\[
\tilde{K}_{T\overline{T}} = \frac{3}{4V_{OM}^2} \left( 1 + \frac{H_I \overline{C}^I \overline{C}^T}{V_{OM}} \right) \quad (A.9)
\]

\[
\tilde{K}_{T\overline{J}} = -\frac{3H_I \overline{C}^I}{4V_{OM}^2} \quad (A.10)
\]

\[
\tilde{K}_{I\overline{J}} = \left( \frac{3}{2V_{OM}} + \frac{\beta_v}{V} \right) H_I \overline{C}^T \quad (A.11)
\]

These expressions contain leading and subleading terms in $\delta$. The Kähler-matrix $\tilde{K}$ of these second derivatives of $K$ then reads

\[
\tilde{K} = K_0 + \delta K_1 . \quad (A.12)
\]

Its inversion is given by

\[
\tilde{K}^{-1} = (1 - \delta K_0^{-1} K_1) K_0^{-1} + \mathcal{O}(\delta^2) \quad (A.13)
\]
and reads in components at leading order (we have dropped all $\delta^2$ contributions or higher)

\[
\tilde{K}^{SS} = 4V^2 \sim V^2 \quad (A.14)
\]

\[
\tilde{K}^{ST} = \frac{4}{3} \beta_v N V_{OM} H^T C^T \sim V_{OM}^2 \delta \quad (A.15)
\]

\[
\tilde{K}^{SI} = \frac{4}{3} \beta_v N V_{OM} C^I \sim V_{OM}^{3/2} \delta^{1/2} \quad (A.16)
\]

\[
\tilde{K}^{TT} = \frac{4V_{OM}^2}{3} \sim V_{OM}^2 \quad (A.17)
\]

\[
\tilde{K}^{TI} = \frac{2}{3} N V_{OM} C^I \sim V_{OM}^{3/2} \delta^{1/2} \quad (A.18)
\]

\[
\tilde{K}^{JJ} = \frac{2}{3} N V_{OM} H^{IJ} \sim V_{OM} \quad (A.19)
\]

where we have defined

\[
\mathcal{N} = \frac{1}{1 + \beta_v \frac{2V_{OM}}{3V}}. \quad (A.20)
\]


B Dependence of Minimum’s Position on Parameters

We present in the following tables the influence of varying the hidden gauge group and $d, |h|, \mathcal{V}_v$ on our de Sitter vacua. The visible boundary CY volume $\mathcal{V}_v$ has not been fixed explicitly in this paper, so we keep it as a parameter, though its stabilization might be achieved along the lines outlined in chapter five. More specifically we give the position of the critical point $\mathcal{L}_0$, its approximation $\mathcal{L}_{prox}$ (see (4.26)), the maximal length $\mathcal{L}_{max}$ for comparison. Beyond these the values $\mathcal{V}_0 = \mathcal{V}(\mathcal{L}_0)$, $\mathcal{V}_{OM,0} = \mathcal{V}_{OM}(\mathcal{L}_0)$ will be important as one has to check that they are substantially bigger than 1 to have our vacua in a controllable regime. Moreover, of clear phenomenological interest will be the value for the positive vacuum energy $E$, the supersymmetry breaking scale $\tilde{M}_{SUSY}$ and the mass of the gravitino $\tilde{m}_{3/2}$ defined as

\[
E = (U/e^{K_{(A)} + K_{(z)})}^{1/4}, \quad \tilde{M}_{SUSY} = M_{SUSY}/e^{K_{(A)} + K_{(z)}}, \quad \tilde{m}_{3/2} = m_{3/2}/e^{K_{(A)} + K_{(z)}} \quad (B.1)
\]

at the critical point. Notice that these values bear also a factor $e^{-K_{(z)}}$ (the $K_{(A)}$ being negligible) as compared to the actual physical parameters $U^{1/4}, M_{SUSY}, m_{3/2}$. This might therefore change the values $U^{1/4}, M_{SUSY}, m_{3/2}$ still by a few orders of magnitude. In all of the following tables we will keep the discrete parameter

\[
\beta_v = 1 \quad (B.2)
\]
Table 1: The dependence on the hidden gauge group $H$ which enters through its dual Coxeter number $C_H$. The other parameters are set to $V_v = 400, d = 10, |h| = 10^{-8}$.

| $H, C_H$ | $E_8, 30$ | $E_6, 12$ | $SO(10), 8$ | $SU(5), 5$ | $SU(3), 3$ | $SU(2), 2$ |
|---------|---------|---------|---------|---------|---------|---------|
| $\mathcal{L}_0$ | 1.1 | 3.0 | 4.1 | 5.5 | 7.1 | 8.3 |
| $\mathcal{L}_{\text{prox}}$ | 1.8 | 3.6 | 4.6 | 6.0 | 7.5 | 8.6 |
| $\mathcal{L}_{\text{max}}$ | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 |
| $\mathcal{V}_0$ | 341 | 256 | 215 | 171 | 135 | 116 |
| $\mathcal{V}_{\text{OM,0}}$ | 7 | 16 | 21 | 26 | 31 | 34 |
| $E/TeV$ | $1.2 \times 10^9$ | $5.7 \times 10^6$ | 495213 | 36455 | 3185 | 564 |
| $M_{\text{SUSY}}/TeV$ | $3.1 \times 10^{10}$ | $1.1 \times 10^8$ | $8.5 \times 10^6$ | 527697 | 39158 | 6406 |
| $\tilde{m}_{3/2}/TeV$ | $3.4 \times 10^8$ | $1.3 \times 10^6$ | 103752 | 6991 | 567 | 96 |

Table 2: The dependence on the intersection number $d$. The other parameters are set to $H = SU(3), V_v = 400, |h| = 10^{-8}$.
| $|h|$ | $10^{-14}$ | $10^{-12}$ | $10^{-10}$ | $10^{-8}$ | $10^{-6}$ |
|-----|-----------|-----------|-----------|----------|-----------|
| $L_0$ | 12.9 | 13.6 | 14.4 | 15.3 | 16.2 |
| $L_{prox}$ | 16.1 | 16.1 | 16.1 | 16.1 | 16.1 |
| $L_{max}$ | 15.5 | 15.5 | 15.5 | 15.5 | 15.5 |
| $V_0$ | 160 | 151 | 143 | 135 | 127 |
| $V_{OM,0}$ | 27 | 28 | 30 | 31 | 32 |
| $E/TeV$ | 10 | 56 | 318 | 1791 | 9852 |
| $M_{SUSY}/TeV$ | 125 | 706 | 3969 | 22020 | 119823 |
| $\tilde{m}_{3/2}/TeV$ | 2 | 10 | 57 | 319 | 1744 |

Table 3: The dependence on the modulus $|h|$ of the open membrane Pfaffian. The remaining parameters are set to $H = SU(3), d = 100, V_v = 400$.

Since $V_0, V_{OM,0}$ are independent of $d$, it follows from (2.5) that

$$L_0 = 3.075 d^{1/3}.$$  \hfill (B.3)

With this relation one reproduces the results for $L_0$ in table 2. One sees that larger values for $d$ bring $E, M_{SUSY}, \tilde{m}_{3/2}$ closer towards the phenomenologically relevant regime. As to the dependence on the modulus $|h|$ of the Pfaffian we see from table 3 that clearly small values are favored. Notice again, that such small values are not too surprising in view of the bound (2.21) on $|h|$. Finally table 4 shows that too small values for $V_v$ are phenomenologically disfavored.

| $V_v$ | 50 | 100 | 200 | 300 | 400 | 500 |
|-----|----|----|----|----|----|----|
| $L_0$ | 2.6 | 3.9 | 5.4 | 6.4 | 7.1 | 7.7 |
| $L_{prox}$ | 3.3 | 4.5 | 5.8 | 6.8 | 7.5 | 8.1 |
| $L_{max}$ | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 |
| $V_0$ | 34 | 56 | 88 | 113 | 135 | 155 |
| $V_{OM,0}$ | 7 | 13 | 20 | 26 | 31 | 35 |
| $E/TeV$ | $1 \times 10^9$ | $5 \times 10^7$ | 789773 | 38817 | 3185 | 348 |
| $M_{SUSY}/TeV$ | $8 \times 10^9$ | $4 \times 10^8$ | $8 \times 10^7$ | 440333 | 39158 | 4600 |
| $\tilde{m}_{3/2}/TeV$ | $4 \times 10^8$ | $1 \times 10^7$ | 171633 | 7495 | 567 | 58 |

Table 4: The dependence on the visible CY volume $V_v$. The remaining parameters are $H = SU(3), d = 10, |h| = 10^{-8}$.  

40
(notice however the implicit further factor $e^{-K(z)}$ which we haven’t evaluated explicitly and which could help to lower e.g. the supersymmetry breaking scale further) while values $\mathcal{V}_v \gtrsim 400$ bring the supersymmetry breaking scale and gravitino mass closer to the TeV regime. However a $\mathcal{V}_v = 500$ would already be too large and bring $\mathcal{L}_0$ beyond its upper bound $\mathcal{L}_{\text{max}}$. Moreover, we see that $\mathcal{L}_{\text{prox}}$ in all these cases gives a rather reliable estimate of the true position $\mathcal{L}_0$. This is remarkable in view of the drastic simplification which reduces (4.24) to (4.20).

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