Case study regarding the use of three different fuels in a multi-stage hybrid rocket engine

S Predoi¹, S Grigorean¹* and G Dumitrascu¹

¹Faculty of Mechanics, “Gheorghe Asachi” Technical University of Iasi, D. Mangeron Boulevard 46, Iasi, Romania

*E-mail: stefan.grigorean@tuiasi.ro

Abstract. Based on previous researches carried in the field on hybrid rocket engines, the authors propose the idea of three different fuel used in a multi-stage hybrid rocket engine. The aim of this paper is make a comparative analysis of rockets hybrid propulsion systems, considering a three stage hybrid rocket engine using one single type of fuel, and a three stage hybrid rocket engine using three different fuels for each stage. Numerical modelling is assuming a similar design mathematical model for both rockets. It is assumed that a concept like this, using different fuel types could lead to a reduction of pollutants and the improvement of combustion efficiency. For both design solution, the comparative analysis is using one imposed criteria, respectively the payload. The calculations are made in order to determine the required thrust force based on the required mass flows of fuel and oxidizer, and next, it is resulting the length and critical diameter of the flue gases nozzle. It is to be mentioned that the mathematical modelling is based on estimated values regarding combustion processes from different studies available in this research field. The numerical model can be used in loop cycle, to obtain a proper design of combustion system for each stage, and thus to get the performance of rocket engine.

1. Introduction

In the domain of space transport, especially when it came to shipment outside the atmosphere, there is more interest regarding the hybrid rocket engines. For this purpose until now there were analysed various solutions for fuels and oxidizers, regarding solid and liquid fuels, design solutions which aims to improve the fuel regression rate with high impact on entire assembly design, and pollutant emissions. As we presented in our previous work regarding hybrid rocket engines [1] at actual stage there were many advantages and disadvantages regarding solid and liquid fuels. In this paper our interest is focused on the use of hybrid rocket engines with solid fuels based on the main advantages regarding fuel storage system, fuel feeding system and overall safety of the rocket engine. Until now many researchers carried on lab scale simulators or even real scale prototypes which use only one type of solid fuel, even if the rocket is designed to use a multi-stage propulsion system. The aim of this paper is to prove the opportunity of using a multi-stage propulsion system using different fuels in each stage. Are taken into account different solid fuels already used in single stage propulsion systems for hybrid rockets, and calculations start from these known resources. Most of the disadvantages of these fuels for hybrid rocket engine are related to low solid fuel regression rates. To overcome related issues to fuel regression rates, researchers tried to enhance the fuel regression rate using different fuels and oxidizers paid and different various techniques to direct the oxidizer flows through combustion chamber in such a manner that would lead to a swirl or turbulent flow.
2. Fundamentals and mathematical model

As mentioned before, in this study the analysis is based on known fuels and oxidizers, which have been studied before regarding combustion process, regression rate and even pollutant emissions. Based on known literature there will be used HTPB, Paraffin, [2] cryogenic CH4 and HTPB additivated with aluminium [3]. Properties for various pairs for fuel/oxidizer are presented in table 1, and in table 2 are presented values regarding regression rate for different combustion chamber design. Based on these available data, the mathematical algorithm aims to create a comparative analysis for a three-stage hybrid rocket engine using three different fuels.

As main input parameter it is imposed a payload for the rocket design with value of 1000 kg, and starting from this it will be proved the gain of a multi-stage hybrid rocket engine regarding pollutant emissions reduction.

Table 1. Values for a and n used for calculation of regression rate on different fuel oxidizer mixtures [4].

| System                  | Propellant   | \(a(m^{1+2n}kg^{-n}s^{-n})\) | n  | \(G_0(\text{kg/m}^2\text{s})\) |
|-------------------------|--------------|-------------------------------|----|-----------------------------|
| Pure HTPB               | GOx/HTPB     | 2.85x10^{-5}                 | 0.681 | 35-280                     |
| Paraffin                | GOx/wax      | 9.1x10^{-5}                  | 0.690 | 20-120                     |
| Paraffin/13%Silbal      | GOx/fuel     | 9.4x10^{-5}                  | 0.766 | 150-300                    |
| Cryo                    | GOx/CH4      | 4.14x10^{-5}                 | 0.830 | 3-30                       |
| Pure HTPB               | GOx/HTPB     | 8.7x10^{-5}                  | 0.530 | 50-400                     |
| HTPB/Al                 | GOx/fuel     | 1.4x10^{-5}                  | 0.930 | 50-400                     |
| HTPB/AP                 | GOx/fuel     | 3.8x10^{-5}                  | 0.710 | 50-400                     |
| HTPB/AL/AP              | GOx/fuel     | 1.2x10^{-5}                  | 0.97  | 50-400                     |

After previous studies in the field of hybrid rocket engines appeared the idea of a case study regarding the design of a multi-stage rocket using different fuel – oxidizer pair for each stage. This paper presents the study regarding the overall rocket dimensions and weight for the possible design solution based on a three stage rocket using HTPB – GOx for the first stage, HTPB – LOx for the second stage and Paraffin – GOx in comparison with the solution based on a three stage rocket using HTPB – LOx for all hybrid engines.

In next lines are mentioned the main equations used for mathematical model. Based on these equations and using Mathcad were determined main theoretical parameters for both analysed design solutions [5]. Equation (1) is used to determine the regression rate, based on experimental data from table 1.

\[
r = a \cdot G_0^n. \tag{1}
\]

Using the equation (2), where \(R\) - radius for a circular port [m] (in this case is 1), \(mmo\) - oxidizer flow rate [kg/s], \(A_p\) - combustion port area [m²], \(N\) - number of combustion port of radius. Replacing \(G_0\) in equation (1), the regression rate can be written as equation (3) [6]

\[
G_0 = mmo \cdot A_p^{-1} = mmo \cdot \left(\pi \cdot R^2\right)^{-1}, \tag{2}
\]

\[
r = a \cdot \left(\frac{mm\alpha}{\pi \cdot R^2}\right)^n. \tag{3}
\]

With equation (4) is calculated the length of propellant surface:

\[
L = \frac{4 \cdot V_f}{\pi \cdot \left(D_{\text{ext}}^2 - D_{\text{int}}^2 \left(0\right)\right)}, \tag{4}
\]
where $D_{extg}$ and $D_{intg}$ are the external and internal diameters of fuel grain.

Instantaneous fuel flow rate is defined using equation (5):

$$mmf = \dot{m}_{propj} \cdot (1 + OF_j)^{-1},$$

(5)

where $\dot{m}_{propj}$ represents the mass flow rate of propellant and $OF_j$ is the oxidizer/fuel mass flow rate.

Thrust is calculated using equation (6):

$$F = (mmo + mmf) \cdot Is \cdot g_o,$$

(6)

where: $F$ - thrust of a hybrid rocket motor [N], $mmo$ - oxidizer flow rate [kg/s], $mmf$ - fuel flow rate [kg/s], $Is$ - specific impulse [s], $g_o$ - acceleration of gravity [m/s$^2$] [7].

Replacing the fuel flow rate, thrust can be calculated using next equation:

$$F = \left[ mmo + \left( 2\pi \cdot N \cdot \rho_f \cdot L \cdot a \cdot \left( \frac{mmo}{\pi \cdot N} \right)^n \right) \left[ a \cdot (2n + 1) \left( \frac{mmo}{\pi \cdot N} \right)^n t + R_i^{2n+1} \right] \left( \frac{1}{Tr_i^{2n+1}} \right) \right] \cdot Is \cdot g_o,$$

(7)

Table 2. Values for different parameter used to compare the proposed design solutions.

| Parameter                          | Stage 1   | Stage 2   | Stage 3   |
|------------------------------------|-----------|-----------|-----------|
| Oxidizer                           | GOx       | LOx       | GOx       |
| Fuel                               | HTPB      | HTPB      | Paraffin  |
| Oxidizer density [kg/m$^3$]        | 1141      | 1141      | 1141      |
| Fuel density [kg/m$^3$]            | 919       | 919       | 940       |
| Oxidizer-Fuel ratio (XO2=1.05)     | 3.2       | 3.2       | 3.61      |
| Specific Impulse [s]               | 300       | 300       | 300       |
| delta V [m/s]                      | 2634      | 2634      | 2634      |
| $g$ [m/s$^2$]                      | 9.8       | 9.8       | 9.8       |
| Combustion chamber pressure [N/m$^2$] | 4.5*10$^6$ | 4.5*10$^6$ | 4.5*10$^6$ |
| a – regression constant            | 0.000087  | 0.000039  | 0.000091  |
| n – mass flux component            | 0.53      | 0.681     | 0.69      |
| Oxidizer flux [kg/(m$^2$xs)]       | 2500      | 2500      | 250       |
| Burning time [s]                   | 56.81     | 56.81     | 56.81     |
| Burning velocity [m/s]             | 0.006     | 0.008     | 0.004     |
| Outer diameter of oxidizer tank [m]| 3.187     | 1.97      | 1.177     |
| Length of oxidizer tank [m]        | 3.187     | 1.97      | 1.177     |
| Outer diameter of fuel tank [m]    | 1.027     | 1.109     | 0.719     |
| Inner diameter of fuel [m]         | 0.402     | 0.196     | 0.319     |
| Length of fuel [m]                 | 8.747     | 1.5       | 0.82      |
| Outer diameter of stage engine [m] | 3.193     | 1.976     | 1.3       |
| Total length of stage [m]          | 22.94     | 11.299    | 8.3       |
| Load [kg]                          | 20430     | 4827      | 1000      |
| Traction [N]                       | 1.2*10$^6$ | 2.8*10$^5$ | 6.0*10$^4$ |
| Fuel flow [kg/s]                   | 99.296    | 23.461    | 4.428     |
| Oxidizer flow [kg/s]               | 317.746   | 75.074    | 15.985    |
| Fuel mass [kg]                     | 5641      | 1333      | 251.555   |
| Oxidizer mass [kg]                 | 18050     | 4265      | 908.113   |
In table 2 are represented the determined parameters for each stage of the rocket based on the equations (1) to (7).

The design of rocket staging use a simplified assumption that each of the stages of the system use the same specific impulse, structural ratio, and payload ratio, with the difference between stages that the total mass of each stage is less than that of the previous stage. Starting from this point and using the equations presented in appendices there are determined many other parameters like burning velocities, burning time, mass of oxidizer and fuel, together with estimation of components mass. Using this model it is a way to compare the characteristics of a multi-stage hybrid rocket using various fuels.

Another aspect with great influence through the functioning of a multi-stage rocket is the influence of burning velocity related to the number of stages. It is to mention that only keeping a ratio for payload, structural design, additional equipment and specific impulse will lead to an increase of burning velocity compared to an identical system with a lower number of stages.

In table 3 are represented the determined parameters for each stage of the rocket using HTPB – LOx, using equations (1) to (7).

| Table 3. Different parameter used to compare the proposed design solutions for three stage HTPB – LOx hybrid rocket engines. |
|---------------------------------------------------------------|
| Stage 1 | Stage 2 | Stage 3 |
| Oxidizer density [kg/m$^3$] | 1141 | 1141 | 1141 |
| Fuel density [kg/m$^3$] | 919 | 919 | 940 |
| Oxidizer-Fuel ratio (XO2=1.05) | 3.2 | 3.2 | 3.61 |
| Specific Impulse [s] | 300 | 300 | 300 |
| delta V [m/s] | 2634 | 2634 | 2634 |
| $g$ [m/s$^2$] | 9.8 | 9.8 | 9.8 |
| Combustion chamber pressure [N/m$^2$] | 4.5×10$^6$ | 4.5×10$^6$ | 4.5×10$^6$ |
| a – regression constant | 0.000039 | 0.000039 | 0.000091 |
| n – mass flux component | 0.681 | 0.681 | 0.69 |
| Oxidizer flux [kg/(m$^2$xs)] | 2500 | 2500 | 250 |
| Burning time [s] | 56.81 | 56.81 | 56.81 |
| Burning velocity [m/s] | 0.008 | 0.008 | 0.008 |
| Outer diameter of oxidizer tank [m] | 3.659 | 2.345 | 1.166 |
| Length of oxidizer tank [m] | 3.659 | 2.345 | 1.166 |
| Outer diameter of fuel tank [m] | 1.408 | 1.167 | 1.002 |
| Inner diameter of fuel [m] | 0.495 | 0.254 | 0.089 |
| Length of fuel [m] | 6.806 | 2.4 | 0.384 |
| Outer diameter of stage engine [m] | 3.665 | 2.351 | 1.172 |
| Total length of stage [m] | 23.988 | 13.655 | 7.222 |
| Load [kg] | 30910 | 8139 | 1000 |
| Traction [N] | 1.8×10$^6$ | 4.8×10$^5$ | 6.0×10$^4$ |
| Fuel flow [kg/s] | 150.231 | 39.558 | 4.86 |
| Oxidizer flow [kg/s] | 480.741 | 126.585 | 15.553 |
| Fuel mass [kg] | 8535 | 2247 | 276.111 |
| Oxidizer mass [kg] | 27310 | 7191 | 883.556 |

In the domain of multi-stage rockets, usually each stage is manufactured in special factories around the world, and then shipped and assembled to the launching site. So there is an influence of the hybrid rocket engine on the overall price of the launch, and it is a good opportunity to analyse the use of
different fuels for each stage with a comparison of fuel and oxidizers costs and overall costs of multistage rocket.

3. Discussion

As stated before, the aim of this paper is to realise a case study based on the comparison between two design solutions related to the multistage rockets using one type of fuel – oxidizer for all stages, or different pairs for each stage. A benefit for the use of different fuel – oxidizer for all stages is the possibility to manufacture each stage in different areas around the world, where production costs can be improved using the most favourable technology available locally.

Many hybrid rocket engines used until now are working with LOx as oxidizer, thus the costs regarding safety and transportation are bigger compared to the GOx - HTPB or GOx - paraffin solution.

Based on the results shown in previous chapter, it is to mention that for the proposed input parameters, the calculated length of the stages of the three stage hybrid rocket using different fuels it is 40 m, smaller with 10% compared to the length of the stages for LOx - HTPB hybrid rocket which is 44 m. A more important difference is related to the overall weight of the rocket, and in this case the weight of three stage hybrid rocket using different fuels is about 30% less compared to the case using one fuel-oxidizer pair for all stages.

4. Conclusion

In this case study the comparison for both design solution started from a imposed payload, and aiming a way to obtain a design with reduced weight and dimensions, this way gaining the possibility to produce the rocket stages in areas where it is cheaper to produce related to the components manufacture, but also to fuel and oxidizer costs. These mathematical modelling proved that the idea of using different fuel – oxidizer pair for each stage may lead to a significant reduction in overall system weight and dimensions, regarding a reduction of required fuel and oxidizer quantity, this influencing directly the total length and diameters for all the stages.

5. Nomenclature

- $r$ - regression rate [m/s];
- $a$ - regression constant (experimental data) [$m^{1+2n}kg^{-n}s^{-1}$];
- $n$ - mass flux exponent (experimental data);
- $G_0$ - oxidizer mass flux [kg / (m$^2$·s)];
- $R$ - radius for a circular port [m];
- $mmo$ - oxidizer flow rate [kg/s];
- $A_p$ - combustion port area [m$^2$];
- $N$ - number of combustion port of radius $R$;
- $R_i$ - initial port radius (circular port) [m];
- $mmf$ - fuel flow rate [kg/s];
- $\rho_f$ - fuel density [kg/m$^3$];
- $N_i$ - number of combustion port of radius $R_i$;
- $r_i$ - initial regression rate [m/s];
- $L$ - fuel grain length [m];
- $t$ - burn time [s];
- $F$ - thrust of a hybrid rocket motor [N];
- $Is$ - specific impulse [s];
- $g_o$ - acceleration of gravity [m/s$^2$].

6. References

[1] Predoi S, Grigorean S and Dumitrescu G 2019 Comparative analysis regarding burning process for different fuels in hybrid rocket engines AIP Conference Proceedings 2190 020099
[2] Karabeyoglu M A, Altman D and Cantwell B J 2002, Combustion of Liquefying Hybrid
Propellants: Part1, General Theory *Journal of Propulsion and Power* 18(3) 35-42

[3] Sutton G P and Biblarz O 2001 *Rocket Propulsion Elements* Seventh Edition (New York: John Wiley & Sons) pp 579-608

[4] Larson C W, Pfeil K L, DeRose M E and Carrick P G 1996 High pressure combustion of cryogenic solid fuels for hybrid rockets *32nd AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit* (Lake Buena Vista) 96-2594

[5] Costa F and Vieira R 2010 Preliminary Analysis of Hybrid Rockets for Launching Nanosats into LEO *Journal of the Brazilian Society of Mechanical Sciences and Engineering* XXXII(4) 502-9

[6] Lee T S and Cho S M 2006 Internal Ballistics of a Hybrid Rocket Burning with Paraffin-Based Fuels *AASRC/CCAS Joint Conference* (Changhua)

[7] Potapkin A and Lee T S 2004 Experimental Study of Thrust Performance of a Hybrid Rocket Motor with Various Methods of Oxidizer Injection *Combustion, Explosion, and Shock Waves* 40(4) 386-92

**Acknowledgments**

This research was funded by UEFISCDI Romania—research grant PCCDI 32/2018