Tensor Polarization and spectral properties of vector meson in QCD medium

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Under the framework of thermal field theory, we calculate the tensor polarization of a generic vector meson in local equilibrium, which is closely related to the spin alignment measured in the heavy-ion collision experiments, up to the first order in the hydrodynamics gradient. At the zeroth order of the hydrodynamics gradient, the spin alignment of the vector meson emerges due to the divergence between the transverse and longitudinal spectral functions. At the first order, the tensor polarization can be induced by all the hydrodynamic gradients allowed by symmetry, including the shear strength tensor. The coefficient of the shear-induced tensor polarization (SITP) is calculated using linear response theory up to the order of one loop, and turns out related to the in-medium spectral properties of vector mesons, e.g. width and the in-medium mass-shift. Our results indicate that spin alignment can be utilized as a new probe revealing the in-medium spectral properties of vector mesons.

Introduction.—Tensor polarization of spin-1 Bosons plays vital roles in many branches of physics [1]. For example, tensor polarization related observables in electron-deuteron scattering experiments are utilized to probe the features of nuclear interaction [2–5]. In the context of the high energy $e^+e^-$ and pp collisions, the measurement of vector meson’s tensor polarization is employed for investigating the spin-dependent fragmentation functions and hence the hadronization mechanism in QCD [6, 7]. Motivated by pioneering works [8–11], tensor polarization of vector meson produced in heavy-ion collisions (HIC), including $K^+$, $\phi$ and $J/\Psi$ have been explored at both RHIC and LHC via analyzing spin alignment observables $\delta \rho_{00}$ [12–16], which is the deviation of the “00” components of the vector meson’s spin density matrix from 1/3. Those studies parallel the experimental [17–21] and theoretical investigation of the polarization of spin-1/2 hyperons (e.g. Refs. [9, 22–33] and see Refs. [34–37] for reviews), deepening our understandings on the relativistic spin physics and making spin related observables as new tools to study the properties of the hot and dense matter created in heavy-ion collisions.

While substantial spin alignment has been observed for vector mesons mentioned above, the theoretical descriptions for $\phi$, $K$ are much smaller than the data and are difficult to accommodate the sign changes of $\delta \rho_{00}$ at different transverse momenta [11, 38–40], since the predicted spin alignment is proportional to the square of the vorticity. Efforts have been made from the other perspectives [41–44] to reveal the mechanism of the spin alignment of the vector mesons as well. But the origin of the spin alignment is still considered as an open question [40]. Besides the theoretical descriptions mentioned above, a systematic calculation under the framework of thermal field theory, which has achieved great success in solving the “spin sign puzzles” of $\Lambda$-hyperon [20, 45], seems to be missing in studying the spin alignment at this moment.

In this work, we will study the tensor polarization and the resulted spin alignment of the vector mesons near local thermal equilibrium using thermal field theory and the linear response theory. Under such a framework, the tensor polarization of vector mesons emerges naturally in exact thermal equilibrium due to the breaking of the degeneracy between the longitudinal and the transverse modes of the vector meson. Furthermore, once the dissipative processes are taken into consideration, the spin alignment is found induced by the hydrodynamics gradients, such as the shear stress tensor, as well, with the corresponding coefficients depending on the width and the mass shift of the spectral function. Both the mechanism might contribute with opposite signs and be responsible for the sign shift of $\delta \rho_{00}$. For the first time, we relate the tensor polarization and the spin alignment of the vector mesons to their in-medium spectral properties and interactions.

Wigner function.— The phase-space distribution of the polarized massive vector meson with mass $m$ is related to the field operator $V^\mu$ via the Wigner functions defined as

$$W^{\mu\nu}(t, x, p) \equiv \varepsilon_\mu \int \frac{dp^0}{2\pi} \int d^4ye^{ipy}(V^\mu(x_--)V^{\nu}(x_+))$$

$$= W^{\mu\nu}_+(x, p) + W^{\mu\nu}_-(x, p) \quad (1)$$
where \( x_\pm = x \pm y/2, \varepsilon_p = \sqrt{p^2 + m^2}, \langle \ldots \rangle \) denotes the thermal ensemble average, and \( W_\perp \) denote the integration over the positive and negative \( p^0 \) respectively, representing the Wigner function of the particle (in-coming mode) and anti-particle (out-going mode). [22]

Wigner function contains all the information necessary for describing the polarization status of the vector mesons. The differential spin-density matrix \( \varrho \) is embedded in Wigner function as

\[
W_{\mu\nu}^+(x, p) = \sum_{s, s'} \epsilon_{s}^{\mu}(p)\epsilon_{s'}^{\nu}(p)\varrho_{ss'}(x, p) + \delta W_{\mu\nu}^+(x, p)
\]

\[
\equiv W_{\mu\nu}^+(x, p) + \delta W_{\mu\nu}^+(x, p),
\]

(2)

where \( \epsilon_s(p) \) represents the polarization vector of the vector meson moving with momentum \( p \) and occupying the spin state \( s \), and satisfies \( \hat{p} \cdot \epsilon_s(p) = 0 \) with \( \hat{p} = (\varepsilon_p, \mathbf{p}) \) being the on-shell 4-momentum. So, the projected Wigner function

\[
W_{\mu\nu}(x, p) \equiv \sum_{s, s'} \epsilon_{s}^{\mu}(p)\epsilon_{s'}^{\nu}(p)\varrho_{ss'}(x, p)
\]

(3)

is perpendicular to the on-shell 4-momentum \( \hat{p} \) as well, and can therefore be expressed as

\[
W_{\mu\nu}(x, p) = \hat{\Delta}_{\mu\nu}^\perp \hat{W}_{\perp}^{\mu\nu}(x, p)
\]

(4)

where \( \hat{\Delta} \) is the shorthand of \( \hat{\Delta}(\hat{p}) \) with \( \Delta^{\mu\nu}(p) \equiv -\eta^{\mu\nu} + p^\mu p^\nu/p^2 \) being the projector with respect to a 4-momentum \( p \). Conversely, the differential spin-density matrix can be evaluated via the projected Wigner function as [46]

\[
\varrho_{ss'}(x, p) = \epsilon_{s}^{\mu}(p)\epsilon_{s'}^{\nu}(p)W_{\mu\nu}(x, p).
\]

(5)

The projected Wigner function can be further decomposed, according to the representation of the rotational symmetry, into three parts as

\[
W_{\mu\nu} = \frac{1}{3} \hat{\Delta}_{\mu\nu}^\perp \hat{S} + W_{[\mu\nu]} + T_{\mu\nu}
\]

(6)

where \( S \equiv W_{\mu\nu} \hat{\Delta}_{\mu\nu}^\perp \) is the 3D trace of \( W_{\mu\nu} \) and related to spin-summed phase space density, \( W_{[\mu\nu]} \equiv (W_{\mu\nu} - W_{\nu\mu})/2 \) corresponds to the vector polarization of the vector meson, and \( T_{\mu\nu} \) defined as

\[
T_{\mu\nu} \equiv W_{(\mu\nu)} \equiv W_{(\mu\nu)} - \frac{1}{3} \hat{\Delta}_{\mu\nu}^\perp \hat{S}
\]

(7)

corresponds to the tensor polarization of the vector field, which is of major interest in this work. Here, the round bracket \( "(...)" \) stands for symmetrizing the included space-time indices, i.e., \( W_{(\mu\nu)} = (W_{\mu\nu} + W_{\nu\mu})/2 \), while the angle bracket \( "(...)" \) stands for further making the tensor traceless about the included indices.

Near thermal equilibrium, the projected Wigner function can be expressed as the function of the thermodynamics parameters such as temperatures and chemical potentials, and can be expanded accounting to the gradients of the hydrodynamic variables. The zero and first order of the expansion can be expressed schematically as

\[
W_{\mu\nu}(t, x, p) = W_{\mu\nu}^{(0)}(t, x, p) + C_{\mu\nu}^{(1)} A_\alpha A_\beta + C_{\mu\nu}^{(2)} \beta^{-1} \beta_A \beta + C_{\mu\nu}^{(3)} \theta.
\]

(8)

The zeroth order term \( W_{\mu\nu}^{(0)} \) is the projected Wigner function in the absence hydrodynamic gradients, and characterizes the polarization status and the phase-space distribution of the vector mesons in exact thermal equilibrium. \( \partial_{\mu}^\perp \equiv \Delta_{\mu\nu}^\perp \partial_{\nu} \) is the projected derivative, where \( \Delta_{\mu\nu} \equiv \eta_{\mu\nu} - u_{\mu} u_{\nu} \) is the projector with respect to the flow velocity \( u \). The first order gradients listed in Eq.(8) include the shear-stress tensor \( \sigma_{\mu\nu} \equiv \frac{1}{2} \left[ \partial_{\mu}^\perp u_{\nu} + \partial_{\nu}^\perp u_{\mu} - (1/3) \Delta_{\mu\nu} \partial \cdot u \right] \), the gradient of inverse temperature \( \beta \equiv T^{-1} \) as \( \beta^{-1} \partial_{\alpha} \beta \), and the bulk stress \( \theta = \partial \cdot u \).

In the following sections, we shall evaluate \( W_{\mu\nu}^{(0)} \) and the coefficients \( C_{S/T/B} \) using thermal field theory and the linear response theory, and further estimate magnitude of the tensor polarization and the induced spin alignment order by order in the narrow width limit.

Zeroth-order of gradient.— In this section, we shall look into the tensor polarization of the vector mesons in exact thermal equilibrium, and show that such a tensor polarization is induced by the difference of the spectral functions of the longitudinal and the transverse modes of the vector mesons.

In thermal equilibrium, the two point Green’s function \( \langle V_{\nu}(x) V_{\nu}(x') \rangle \) can be expressed using the spectral function \( A_{\mu\nu} \) as

\[
\int \text{d}^4 y e^{ip'y} \langle V_{\nu}(x) V_{\nu}(x') \rangle = n(\omega) A_{\mu\nu}(p),
\]

(9)

where \( n(\omega) = 1/(e^{\beta\omega} + 1) \) is the Bose-Einstein distribution. In the thermal medium, the longitudinal and transverse modes of the vector meson should diverge, so that the spectral function can be decomposed as [47, 48]

\[
A_{\mu\nu}(p) = \sum_{a=L,T} \Delta_{\mu\nu}^a A_a,
\]

(10)

\[
A_a = \frac{1}{\pi} \text{Im} \left[ \frac{-1}{p^2 - m^2 - \Pi_a} \right].
\]

(11)

The longitudinal and transverse projector \( \Delta_{T,L} \) are \( \Delta_{T}^\mu = v^\mu v^\nu, \Delta_{L}^\mu = \Delta^\mu - \Delta_{T}^\mu \), where the \( v^\mu = \hat{u}^\mu/\sqrt{-\hat{u}^2} \) where \( \hat{u}^\mu = \Delta_{L}^\mu u_\mu \) is the projected velocity with respect to \( p \).

The projected Wigner function \( W_{\mu\nu}^{(0)} \) in thermal equilibrium can thus be expressed as

\[
W_{\mu\nu}^{(0)} \approx \Delta_{\mu}^a \Delta_{\nu}^a \int_0^\infty d\omega n(\omega) A_{\mu\nu}(\omega, p),
\]

(12)

and its tensor polarized sector in the narrow width limit, where all the projectors are assumed to be with respect
to the on-shell momentum, is

$$\mathcal{T}^{\mu\nu} = \mathcal{W}^{(\mu\nu)}_{(0)} \equiv \alpha_0 \Delta_{L}^{\mu\nu} n(\epsilon_p)$$

(13)

$$\alpha_0 \approx 2 \varepsilon_p \int_0^{\infty} \frac{d\omega}{n(\epsilon_p)} \left[ (A_L - A_T) - \frac{\Delta \omega^2 v^2}{p^2} A_L \right]$$

(14)

where the $\Delta \omega = \omega - \epsilon_p$. The $A_L$ usually have a factor proportional to $p^2$ to cancel the $1/p^2$ to make the integral convergent. In quasi-particle limits, one just need to do the integral around the peak, so that one can expands with small parameter $\Delta \omega/T$, $\Delta \omega/m$ to get some analytical formula. However, as we check numerically with some ansatz for spectral density, we find that the value of $\alpha_0$ seems to be sensitive to the subtleties of spectral functions. Therefore, we do not proceeds to further simplify expressions at the moment, but an example of simple formulation will be discussed later.

In medium rest frame, nonzero momentum $p$ specifies a particular direction that breaks the rotation invariance. Therefore, a particle can have different polarizations in different directions even without any hydrodynamic gradient, leading to the splitting in longitudinal and transverse modes. This splitting leads to the first term in $\alpha_0$ in Eq. (13). In addition, the projection of an off-shell in-medium particle will mismatch the on-shell final states particle, which leads to the second term in $\alpha_0$ in Eq. (13). The manifestation of these zeroth order effects on spin alignment in heavy-ion collisions will be discussed later.

**First-order of gradients.**— We now turn to $\mathcal{W}^{\mu\nu}$ at first order in gradient, accounting for the off-equilibrium contribution induced by the inhomogeneity of the system. Our strategy is to first inspect all possible first order gradient terms that are allowed by symmetry, including a term proportional to shear strength tensor with T-odd coefficients, indicating the dissipative nature of this physics. While the corresponding coefficients are given by the Kubo formula derived either via the Luttinger’s method [49] with metric variation in Ref [50] or using the Zubarev’s formalism [51, 52]. Such a general procedure has recently applied to study the spin polarization of fermions, and has uncovered a long overlooked effect, namely the shear-induced polarization [28, 29] (see also Refs. [30, 31]). Here, we simply report the final form, leaving the detailed derivation to our long paper [53].

$$\mathcal{W}^{\mu\nu}_{(1)}(t, p, x) = \epsilon_p \lim_{\nu, q \rightarrow 0} \frac{\partial}{\partial \nu^{\mu}} \left[ -\text{Im} \mathcal{G}_{R}^{\mu\nu}(\nu, q, p) \right] \sigma_{\lambda \gamma}\ |\sigma_{\lambda \gamma}\rangle$$

(15)

where the retarded Green function is

$$\mathcal{G}_{R}^{\mu\nu}(t - t', x, y, z) \equiv \int \frac{d\nu}{2\pi} \frac{d^3q}{(2\pi)^3} e^{-i\nu(t-t')} \times e^{i\nu(x-z)} e^{i\nu p} y G^{\mu\nu}(t, \nu, p)$$

(16)

$$= (-i)\Theta(t - t') \langle [V^{\mu}(t, x^-) V^{\nu}(t, x^+), T^{\lambda\gamma}(t', z)] \rangle$$

For the energy momentum tensor, we shall use the expression for sourceless Proca field $\mathcal{T}^{\mu\nu} = F^{\mu\nu}_{\mu\nu} + m^2 V^{\mu\nu} V^{\nu\mu} - \eta^{\mu\nu} (F^{\mu\nu}/4 + m^2 V^{\mu\nu}/2)$ where $F^{\mu\nu} = i\partial^{\mu} V^{\nu} - i\partial^{\nu} V^{\mu}$. In leading order skeleton expansion in the imaginary time, $\mathcal{G}_{R}^{\mu\nu}(\nu, x, y, z)$ is composed by propagator $D^{\mu\nu} \sim \theta(\tau - \tau') |(V^{\nu}(\tau), V^{\mu}(\tau'))|$ and propagators with derivatives $E^{\mu\alpha\nu} \sim \theta(\tau - \tau') |(V^{\mu}(\tau), \partial^\alpha V^{\nu}(\tau'))|$ and $\partial^\mu D^{\nu\alpha}(\tau - \tau')$ [54].

With the relations mentioned above, all the derivatives involved can be transferred to real momenta in momentum space and we can express the $\mathcal{G}_{R}^{\mu\nu}(\nu, p, q)$ as:

$$\mathcal{G}_{R}^{\mu\nu}(\nu, p, q) = \int_{0}^{\infty} dk_0 \int_{0}^{\infty} dk_0' \frac{n(k_0) - n(k_0')}{k_0 - k_0' + i0^+} \times \sum_{a,b=L,T} A_a(k) A_b(k') I_{a,b}^{\mu\nu\alpha\beta}(k, k')$$

(17)

In this work, the integral limits excluding the negative energy region allow us to select out the modes that are related to the physical modes in the Wigner functions. With the projectors, the $I_{a,b}^{\mu\nu\alpha\beta}(k, k')$ in Eq. (17) can be explicitly written as

$$I_{a,b}^{\mu\nu\alpha\beta}(k, k') = [k^\lambda k'^\gamma + k^' k^\gamma \Lambda(\nu, \mu)] \Delta_{a^\mu}(k) \Delta_{b^\nu}(k')$$

$$- [k_a k_a^\nu \Delta_{a^\lambda}(k) \Delta_{b^\mu}(k') + k' k_a' \Delta_{a^\nu}(k) \Delta_{b^\mu}(k')]$$

$$- [k^\lambda k'^\mu \Delta_{b^\lambda}(k) \Delta_{a^\nu}(k') + k' k^\mu \Delta_{b^\nu}(k) \Delta_{a^\lambda}(k')]$$

$$- [k_a k_a^\nu - m^2] \Delta_{a^\nu}(k) \Delta_{b^\mu}(k') + \Delta_{a^\nu}(k) \Delta_{b^\nu}(k')$$

+ $\eta^{\alpha\beta}[m^2 k^\alpha k'^\beta - \eta_{\alpha\beta}]$ and propagators $\tau_{a,b}^{\mu\nu}(\nu, \mu)$ with derivatives $\Delta_{a,\mu}(\nu, \mu)$ and $\partial^\mu D^{\nu\alpha}(\nu, \mu)$ have been neglected. In the following, we simply denote these quantities as $\Pi$ and $\Pi_{\alpha\beta}$ without labelling the subscripts, one can understand them as either $\Pi$ or $\Pi_{\alpha\beta}$, respectively. In the last line, the relation $\eta_{\mu\nu} = 0$ will set these contributions to zero. However, second and third line contribute to the polarization in non-dissipative processes. The resulting polarization of spin-1 particle is 4/3 of the spin-1/2 particle with the same form as expected, which will be discussed in our long paper [53]. The first and the last line contribute to the tensor polarization through some more subtle way, we will not discuss this in detail, but will just show the results in the end of the section. The most straight forward contribution to tensor polarization is the fourth line, which will be discussed below.

To proceed, we first take the limit, $|\Pi_{L} - \Pi_{L}|/|\Pi_{T} + \Pi_{L}| \ll 1$ so that difference of $L$ and $T$ modes can be temperately neglected. In the following, we simply denote these quantities as $\Pi$ and $A(\omega, p)$ without labelling the subscripts, one can understand them as either $\Pi$ or $\Pi_{\alpha\beta}$, respectively. Also, in the calculation, we take the quasi-particle limits as $\Pi_{\alpha\beta}$, $\epsilon_p \ll 1$, while all the integrals over the $k_0$ and $k'_0$ can be done in the vicinity of the pole of the propagator. The in-medium dispersion relation $\omega = \omega_p$ is given by $\omega^2 - \omega_p^2 - \epsilon p = 0$, while the energy shift is defined as the difference between $\omega_p$.
and the on-shell energy \(\varepsilon_p\), i.e., \(\Delta\varepsilon_p = \omega_p - \varepsilon_p\). The width around the pole is \(\Gamma_p = \Im\Pi(\omega_p, p)/\varepsilon_p\).

With the approximations discussed above, the tensor polarization can be obtained from the Eq. (17), Eq. (15), and Eq. (18) as

\[
T^{\mu\nu} \left( t, \omega, \mathbf{p} \right) = \alpha_{\text{sh}} \Delta^{(\mu \nu)} \frac{n(\varepsilon_p)}{T}.
\]

where only the fourth line of Eq. (18) contribute to Eq. (19). The dimensionless coefficients \(\alpha_{\text{sh}}\) is

\[
\alpha_{\text{sh}} = \frac{\pi}{\varepsilon_p} \int_0^\infty \frac{\partial n(\omega)}{\partial \omega} d\omega (\omega^2 - \varepsilon_p^2) A^2(\omega, \mathbf{p}) \\
\approx - (1 + n(\varepsilon_p)) \frac{2\Delta\varepsilon_p}{\Gamma_p}. 
\]

From the first line to the second line, we need to expand the \(\omega\) around the \(\varepsilon_p\) with \(\Delta\varepsilon_p = (\omega - \varepsilon_p)\) keep the results to the leading order of \(\Delta\omega/\varepsilon_p\) or \(\Delta\omega/T\). Notice the integrand in Eq. (20) is the first order in \(\Delta\omega\) since \(\omega^2 - \varepsilon_p^2 \approx 2\Delta\omega\varepsilon_p\) in this limits. After the integral, it can be organized as an expansion for \(\Gamma_p\), \(\Delta\varepsilon_p\) weighted by various quantities with energy dimension, such as, \(\varepsilon_p\), \(T\). As we check numerically, the results of the integral have a simple form independent from the details of the spectral functions, which is \(4\pi^2 \int d\omega (2\Delta\omega\varepsilon_p) A^2(\omega_0, \mathbf{p}) \approx 2\varepsilon_p\Delta\varepsilon_p/\Gamma_p\). With the generic integral \(\int dx (c/(x^2 + c^2))^2 = \pi/(2c), \int dx x^2 (c/(x^2 + c^2))^2 = \pi e/2\), one can expand the \(\Delta\omega\) to the order \(\Delta\omega^2\) and calculate the analytical results. There are many terms, including some terms from the off-shell effects in projector, which we will probably delay these to our long paper [53].

Without explicit derivation, we just list the results for bulk stress and other contributions at the one loop level, which they can combined into a compact form as

\[
T^{\mu\nu} \left( t, \omega, \mathbf{p} \right) = \alpha_{\text{sh}} \Delta^{(\mu \nu)} \frac{n(\varepsilon_p)}{T} \\
- \frac{\varepsilon_p}{\Gamma_p} \Gamma_p^{\Delta} \frac{\varepsilon_p}{\Gamma_p} \left( 1 + n(\varepsilon_p) \right) \Delta^L \frac{\varepsilon_p}{\varepsilon_p} (1 + \sigma(\beta u)) \frac{\varepsilon_p}{\varepsilon_p} n(\varepsilon_p). 
\]

The \(\Gamma_p^{\Delta} \equiv \Gamma_p^{T, L} - \Gamma_p^{T, L}\) is the difference between of width of the transverse and longitudinal modes. The above formula zeroth order terms for the small parameters \(\Delta\varepsilon_p/\varepsilon_p\), \(\Gamma_p/\varepsilon_p\), \(\Gamma_p^{\Delta}/\Gamma_p\), if we assume all of them are around the same order of magnitude. Note that unlike in the non-dissipative processes [28], the longitudinal derivatives with respect to \(u\) appear as well in a dissipative process following the formalism in Ref [51]. However, there are cancellations if we consider the constrains from the hydrodynamic equations. Finally, to make the formula manifest covariant, we need to replace the \(\mathbf{p}\) with \(p_L^\mu\) and \(\varepsilon_p\) with \(\varepsilon_0 = p \cdot u\).

**The estimate of effects.**—We use the following prescription to connect \(T^{\mu\nu}\) and the symmetric part of differential spin density matrix \(\delta \rho_{ss'}\)

\[
\delta \rho_{ss'} \left( t, \mathbf{x}, \mathbf{p} \right) = \epsilon_s^\mu (\hat{\mathbf{p}}) \left( \epsilon_s'^\nu (\hat{\mathbf{p}}) \right)^* T^{\mu\nu} \left( t, \mathbf{x}, \mathbf{p} \right),
\]

see our upcoming paper for further details [53]. Here, the polarization states are labelled by \(s = 0, \pm\). For a given quantization axis \(\hat{n}\), the polarization vectors are eigenstate of spin operator \(\hat{J}\) projected along \(\hat{n}\), i.e., \((\hat{n} \cdot \hat{J})\epsilon_s = s \epsilon_s^\mu\).

By definition, spin alignment is determined by density matrix distribution integrated over space and momentum \(\mathbf{x}, \mathbf{p}\) and been normalized by the density, which is \(\delta \rho_{00} (\hat{n}) = \int \mathbf{x}, \mathbf{p} (\hat{n} | u)_{\text{PRF}}^\dagger (\hat{n} | u)_{\text{PRF}}^i \int \mathbf{x}, \mathbf{p} \rightarrow 1/3\) where \(\hat{n}\) is evaluated at particle rest-frame (PRF). Noticing \((\epsilon_s^\mu (\hat{\mathbf{p}}))_{\text{PRF}} = (0, \hat{n})\), together with Eq. (22), we finally arrive at:

\[
\delta \rho_{00} (\hat{n}) = \frac{\int \mathbf{x}, \mathbf{p} \hat{n}_i \hat{n}_j (\hat{T}) \int \mathbf{x}, \mathbf{p} \hat{n}_i \hat{n}_j (\hat{T})_{\text{PRF}}}{\int \mathbf{x}, \mathbf{p} \hat{T}},
\]

Eq. (23) is the relation to be used to connect spin alignment to the tensor polarizations \(T^{\mu\nu}\).

For zero order effects, appearing at zeroth order in gradient, is described by Eq (13) and induces spin alignment according to Eq. (23):

\[
\delta \rho_{00, (0)} (\hat{n}) \approx \frac{\alpha_0}{9} \left( \langle 3 (\mathbf{v} \cdot \mathbf{p}) \cdot \mathbf{n} \rangle - 1 \right).
\]

The average is defined as \(\langle (\ldots) \rangle \equiv \int \mathbf{p} \cdot \mathbf{n} (\mathbf{P} \cdot \mathbf{u})/(\int \mathbf{p} \cdot \mathbf{n} (\mathbf{P} \cdot \mathbf{u})\ldots).\) Here, \((X)_{\text{eff}}\) refers to the characteristic value of \(X\) in the phase space domain where the average is taken, and \(\mathbf{v}_{\text{PRF}}\) is the direction of relative velocity between the medium and particle in PRF (see Appendix for explicit expressions). To estimate Eq. (24), we assume that the self-energy of Eq. (14) is dominated by a non-analytical energy shift, similar to those scalar thermal mass \(\alpha g T\) in hard thermal loops [55], a leading contribution could be \(\alpha_0 = \langle \omega_p^T - \omega_p^T \rangle / T\), where the \(\omega_p^{T,L}\) is the in-medium pole mass for “L” and “T” modes defined below Eq. (18).

Since \(\langle (\ldots) \rangle\) in (24) essentially measures the difference of \(\mathbf{v}_{\text{PRF}}\) projected along different direction, so we expect such a factor is of the order \(v_2 \sim 0.1\). This implies that \(\delta \rho_{00, (0)} (\hat{n})\) to be of the order 0.01 if \(\alpha_0 \sim 0.1\). This value of \(\alpha_0\) require the split of energy between L and T modes at the order of 10 MeV around the thermal momentum, if we use the simple formula above. Definitely, a more quantitative studies should be based on microscopic calculations [47]. We also expect that the sign of \(\langle (\ldots) \rangle\) in Eq. (24) can be sensitive \(p_T\) and centrality, see Appendix where we illustrate this point with Bjorken flow. It is interesting to test those anticipations with quantitative modeling.

As for the gradient effects, we concentrate on SITP (19) for illustrative purpose. The final expression is:

\[
\delta \rho_{00, (1), \text{sh}} (\hat{n}) \approx \frac{1}{3} \left( \alpha_{\text{sh}} (\partial_T \sigma_{\text{PRF}} \hat{n}_i \hat{n}_j)_{\text{eff}} \right),
\]

where the ball-mark estimate of \(\alpha_{\text{sh}}\) is given by Eq. (20). The mass-shift and width in a microscopic calculation has
significant uncertainties for our purpose [56–61]. The ratio can be at the order of 1 in early study [56], while it can be as an order of 0.1 in other calculations [60]. The sign of the mass shift can be negative in some study [57, 62], but be positive in others [56, 60]. Therefore, it is reasonable to expect that shear strength can give a signature of the order of $\pm 0.01$ for $\rho_{00}$ if $|\sigma|_{\text{eff}} \sim 10$ MeV. Meanwhile, the $a_{sh}$ may provides extra $p_T$ dependence that help to make the sign flip. The sign and magnitude can of the spin alignment could allow us to understand the microscopic physics in medium. The above estimate is admittedly crude, but it illustrates the feasibility of extracting shear polarizability experimentally.

To this point, our discussion is quite general and applies to a generic massive vector meson. To adapt our results to a specific vector meson, we need to input its mass shift and decay width, or more precisely speaking its spectral density; all of those in-medium properties can vary species by species, and may change non-trivially with varying $\mu_B$ [60, 63].

**Summary.—** We have studied the near-equilibrium behavior of the tensor polarization and spin alignment of vector meson up to first order in gradient using thermal field theory. We find that tensor polarization and the spin alignment are nonzero at the zero-order in gradient expansion, which is different from the case of the vector polarization. Its value is significantly affected by the splitting of the longitudinal and transverse spectral functions. Also, we find the the tensor polarization of the vector boson is simply proportional to shear stress tensor with some projections. The coefficients is related to the medium induced mass-shift and width, where the sign of the results is mostly affected by the direction of the mass-shift. We should notice, all this spectral properties are closely related to the microscopic interactions between the vector mesons and the mediums.

It is interesting and important that those near-equilibrium expectations are studied quantitatively with detailed modeling and confront with current and future spin alignment data. Doing so would be a crucial step towards making spin alignment measurement as a new way to probe to the spectral properties and the microscopic physics for hot QCD medium.

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The relative velocity in particle rest-frame (PRF)

For the convenience of applying our results in the future, we write down the explicit expression for the normalized relative velocity in PRF where the on-shell particle four-momentum $P_{\text{PRF}} = (m, 0)$. From the definition $N^\mu = \Delta^{\mu\nu}u_\nu$, we have

$$ N^\mu_{\text{PRF}} = (0, u_{\text{PRF}}), $$

where the fluid velocity in PRF is given by

$$ u^0_{\text{PRF}} = -\frac{E_u}{m}, $$

$$ u^i_{\text{PRF}} = -\frac{m u^0 + E_u}{m(E + m)} \vec{p}^i + u^i. $$

with $E_u \equiv -\vec{P} \cdot \vec{u}$. Consequently, the relative velocity reads

$$ v^i_{\text{PRF}} = \frac{u^i_{\text{PRF}}}{\sqrt{u^0_{\text{PRF}}^2}}. $$

An illustration of "energy shift effect"

To illustrate ESE, we apply Eq. (24) to the specific case that $\hat{n} = \hat{e}_y$, the direction of the angular momentum in a collision. We adopt the convention $\hat{e}_x = \hat{e}_y \times \hat{e}_z$, with $\hat{e}_z$ being the beam axis. To characterize the difference in the magnitude of relative velocity in $x − y$ plane and that of along $z$-direction, we can define

$$ r_{\perp} \equiv \langle \frac{1}{2} v^2_{\perp,\text{PRF}} - v^2_{z,\text{PRF}} \rangle, $$

where $v^2_{\perp,\text{PRF}} = v_{x,\text{PRF}}^2 + v_{y,\text{PRF}}^2$ and the definition of $\langle \cdots \rangle$ is specified in the paragraph below Eq. (??). We
may use the approximation
\[
\frac{\langle v_{x,\mathrm{PRF}}^2 \rangle - \langle v_{y,\mathrm{PRF}}^2 \rangle}{\langle v_{y,\mathrm{PRF}}^2 \rangle} \approx v_{2,\mathrm{flow}}
\]
(30)
where \(v_{2,\mathrm{flow}}\) is the standard second Fourier coefficient of the azimuthal distribution of produced particles relative to the even plane. We then have
\[
\delta \rho_{00}(0) \langle \hat{c}_y \rangle \approx \frac{\Delta E}{9T} \langle \langle r_{\perp} - v_{2,\mathrm{flow}} \rangle \rangle,
\]
(31)
where we have dropped a term \(\propto r_{\perp} v_{2,\mathrm{flow}}\) which is expected to be small compared with the terms we keep in Eq. (31).

To illustrate the behavior of \(r_{\perp}\), we shall consider Bjorken expansion in which case the flow velocity in the lab frame reads
\[
u_{l} = (\cosh(\eta), 0, 0, \sinh(\eta)) .
\]
(32)
Using the standard parametrization
\[
p(\eta) = (m T \cosh Y, p_T \cos \phi, p_T \sin \phi, m T \sinh Y) ,
\]
(33)
where \(Y\) is rapidity and \(\phi, p_T\) are azimuthal angle and momentum in \(x-y\) plane respectively, and where \(m_T = \sqrt{m^2 + p_T^2}\). The relative velocity in PRF can be read from (27) and we have:
\[
v_{y,\mathrm{PRF}} = -\frac{(m \cosh \eta + E_u) p_T \sin(\phi)}{\sqrt{E_u^2 - m^2 (m + m_T \cosh Y)}} .
\]
(34)
where we have used \(E_u = -p \cdot u = m_T \cosh(Y - \eta)\). The average \(v_{y,\mathrm{PRF}}\) for a fixed \(p_T\) can now be written as
\[
\langle \langle v_{y,\mathrm{PRF}}^2 \rangle \rangle = \int_{n_B(\phi, \eta)} e^{-\beta E_u} v_{y,\mathrm{PRF}}^2 \int_{n_B(\phi, \eta)} e^{-\beta E_u} \]
(35)
where we have replaced \(n_B\) with Boltzmann distribution for simplicity. The integration in Eq. (35) can be written more explicitly as
\[
\int_{n_B(\phi, \eta)} \equiv \int_{-Y_0}^{Y_0} dY \int_{-\infty}^{\infty} d\eta \int_{0}^{2\pi} d\phi / 2\pi .
\]
(36)
Here \(Y_0\) specifies the range for the integration over momentum rapidity. Then, we can compute \(r_{\perp}\) using (29).

In Fig. 1, we show \(r_{\perp}\) as a function of \(p_T/T\) with set-up specified above and use \(m/T = 5, Y_0 = 1\). The resulting \(r_{\perp}\) ranges from \(-0.5\) at very low \(p_T\) to \(0.3\) at very high \(p_T\). This also confirms our anticipation that \(r_{\perp}\) to be of the same order as \(v_2\). An important observation is that the sign can be sensitive to \(p_T\). This makes sense in physical ground since at very low \(p_T\), \(v_{x,\mathrm{PRF}}^2 < 2v_{x,\mathrm{PRF}}\) whereas \(v_{x,\mathrm{PRF}}^2 > 2v_{x,\mathrm{PRF}}^2\) when \(p_T\) becomes large. We anticipate that the sensitivity to the kinematic range is a qualitative feature of ESE.

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