We discuss a holographic soft-wall model that consider several Fock states in nucleon description. In our approach nucleon structure is a superposition of a three-valence quark state with high Fock states including an adjustable number of partons (quarks, antiquarks and gluons). With a minimal number of free parameters (dilaton scale parameter, mixing parameters of partial contributions of Fock states, coupling constants in the effective Lagrangian) we achieve a reasonable agreement with data.

1 Introduction

Based on the gauge/gravity duality, a new class of approaches which model QCD by using methods of extra dimensional fields theories formulated in anti de Sitter (AdS) space, was recently successfully developed for describing some hadronic properties.

A popular model of this case is the soft wall model, which uses a soft IR cutoff in the fifth dimension to break conformal invariance. Applications of the Gauge/Gravity dualities in hadron physics have been worked by several authors, and you can find in [1,2] a couple of recent reviews about this topic.

Here we present a summary of a model treated in detail and with examples in [3,4], where the fermion bulk fields are characterized by the 5D mass $\mu$ (related with scaling dimension) which is holographically dual to $N$ (the number of partons in baryons), but at difference of most of applications where AdS fermion field is identified with lowest dimension with baryons containing three quarks, we do not restrict to three valence quark picture of baryons and include higher Fock states involving non valence degrees of freedom. The latter are dual to the AdS fermion fields on higher dimension (Fig. 1; Table 1).
Table 1 Helicity amplitudes $A^{N/2}_{1/2}(0)$ and $S^{N/2}_{1/2}(0)$, $N = p, n$

| Quantity | Our results | Data [7] |
|----------|-------------|----------|
| $A^{p/2}_{1/2}(0)$ (GeV$^{-1/2}$) | $-0.065(-0.065)$ | $-0.065 \pm 0.004$ |
| $A^{n/2}_{1/2}(0)$ (GeV$^{-1/2}$) | $0.040 (0.040)$ | $0.040 \pm 0.010$ |
| $S^{p/2}_{1/2}(0)$ (GeV$^{-1/2}$) | $0.047 (0.048)$ | |
| $S^{n/2}_{1/2}(0)$ (GeV$^{-1/2}$) | $-0.044(-0.045)$ | |

We present results for some nucleon properties in a holographic soft wall model where high Fock contributions are holographically incorporated in the nucleon. In cases discussed here we restrict ourselves to the contribution of 3, 4 and 5 parton components in the nucleon Fock state. Notice that the role of the higher Fock components in Pion, in the context of holographic QCD, was considered before in [5].

2 Model

We consider the propagation of a fermion field with spin 1/2 in five dimensional AdS space, which contains the contribution of different twist dimension, that according to AdS/QCD dictionary it corresponds to the inclusion of the three quark and higher parton state in the nucleon. Here we just consider contribution of $3q$, $3q + g$, $3q + q\bar{q}$ and $3q + 2g$ Fock states, where $q, \bar{q}$ and $g$ denote quark, antiquark and gluon respectively.

The model consider an AdS metric specified by

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

where $\eta_{\mu\nu} = diag(1, -1, -1, -1)$ and $\mu, \nu = 0, 1, 2, 3, z$ is the holographic coordinate, $A(z) = \ln(R/z)$ and $R$ is the AdS radius.

The relevant AdS/QCD action for examples considered here are:

$$S = \int d^4x dz \sqrt{g}e^{-\Phi(z)}[\mathcal{L}_\psi(x, z) + \mathcal{L}_{V+A}(x, z) + \mathcal{L}_{int}(x, z)]$$

$$\mathcal{L}_\psi(x, z) = \sum_{i=+,-} \sum_{\tau} c_\tau \bar{\Psi}_{i,\tau}(x, z) D_{i,\tau} \Psi_{i,\tau}(x, z)$$

$$\mathcal{L}_{V+A} = \frac{1}{4} V_{MN} V^{MN} - \frac{1}{4} A_{MN} A^{MN}$$

$$\mathcal{L}_{int}(x, z) = \sum_{i=+,-} \sum_{\tau} c_\tau \bar{\Psi}_{i,\tau}(x, z) (\hat{V}_i + \hat{A}_i) \Psi_{i,\tau}(x, z)$$
where $M$ and $N = 0, 1, 2, 3, 4, \Psi(x, z)$ is fermion field with spin 1/2 and scaling dimension $\tau$, $V_M(x, z)$ is a vector field with spin 1 (holographic analogue of the electromagnetic field) and $A_M(x, z)$ is an axial field (holographic analogue of the axial isovector field). Additionally,

$$\hat{D}_i = i \frac{\Gamma_M}{2} \hat{\partial} M \mp (\mu + U_F(z)) \tag{6}$$

$$\hat{\gamma}_{\pm} = Q \Gamma^M V_M \pm \frac{i}{4} \eta_V [\Gamma^M, \Gamma^N] V_{MN} \pm g_{V} \tau_3 \Gamma^M i \Gamma^z V_M. \tag{7}$$

$$A_{\pm} = \frac{\tau_3}{2} (\mp \Gamma^M A_M + \frac{i}{4} \eta_A [\Gamma^M, \Gamma^N] A_{MN} + g_A \Gamma^M i \Gamma^z A_M). \tag{8}$$

where $F_{MN} = \partial_M F_N - \partial_N F_M$ ($F = V, A$), $Q = \text{diag}(1, 0)$ is the nucleon charge matrix, $\tau_3 = (1, -1)$, $A \hat{\partial} B = A(\partial B) - (\partial A) B, \phi(z) = \kappa^2 z^2$ is the dilaton field with $\kappa$ being a free scale parameter, $\Gamma^M = \epsilon^M_a \Gamma^a$, $\Gamma^a = (\gamma^\mu, -i\gamma^5)$ and $\mu$ is given by $m = \mu R = \tau - 3/2$. Notice that the scaling dimension of the AdS fermion field is holographically identified with the scaling dimension of the baryon interpolating operator $\tau = N + L$, where $N$ is the number of partons in the baryon and $L = \text{max} |L_z|$ is the maximal value of the $z$ component of the quark orbital angular momentum in the light front wave function.

### 3 Mass Spectrum

Considering the fermionic part in the action, we rescale the fermionic field as

$$\Psi_{\lambda, \tau}(x, z) = e^{\phi(z)/2} \psi_{\lambda, \tau}(x, z) \tag{9}$$

we get the EOM for $\psi_{\lambda, \tau}(x, z)$

$$\left[ i \gamma^\mu \partial_\mu + 2A'(z) \gamma^5 + \frac{e^{A(z)}}{R} (m + \phi(z)) \right] \psi_{\lambda, \tau}(x, z) = 0. \tag{10}$$

Next we split the fermion field into left and right chiral components

$$\psi_{\lambda, \tau}(x, z) = \psi_{\lambda, \tau}^{L}(x, z) + \psi_{\lambda, \tau}^{R}(x, z), \tag{11}$$

and perform a KK expansion for the $\psi_{\lambda, \tau}^{L/R}(x, z)$ fields:

$$\psi_{\lambda, \tau}^{L/R}(x, z) = \frac{1}{\sqrt{2}} \sum_n \psi_n^{L/R}(x) e^{-2A(z)} f_{\tau, n}^{L/R}(z), \tag{12}$$

where $\psi_n^{L/R}(x)$ are the four-dimensional boundary fields. After straightforward algebra one can obtain the decoupled EOMs:

$$\left[ -\frac{\partial_z^2}{z^2} + 2\kappa^2 \left( m \mp \frac{1}{2} \right) \right] f_{\tau, n}^{L/R}(z) = M_{\tau n}^2 f_{\tau, n}^{L/R}(z). \tag{13}$$

The nucleon mass is identified with the expression

$$M_n = \sum_{\tau} c_{\tau} M_{\tau n} = 2\kappa \sum_{\tau} c_{\tau} \sqrt{n + \tau - 1}. \tag{14}$$

Integration over the holographic coordinate $z$, gives a four dimensional action for the fermion field $\psi_n = \psi_n^{L}(x) + \psi_n^{R}(x)$:

$$S_0 = \sum_n \int d^4 x \bar{\psi}_n(x) [i \gamma^\mu \partial_\mu - M_n] \psi_n, \tag{15}$$

this explicitly demonstrates that effective actions for conventional hadrons in four dimensions can be generated from actions for bulk fields propagating in five dimension is encoded in the hadronic mass squared. Notice that the constraint $\sum_{\tau} c_{\tau} = 1$ for the mixing parameters $c_{\tau}$ is essential to get the correct normalization of the kinetic term of the four dimensional spinor field.
4 Electromagnetic Structure of Nucleons

The nucleon electromagnetic form factors $F_N^1$ and $F_N^2$ ($N = p, n$ correspond to proton and neutron) are conventionally defined by matrix element of the electromagnetic current as

$$\langle p' | J^\mu(0) | p \rangle = \bar{u}(p') [\gamma^\mu F_N^1(t) + \frac{i}{2m_N} \sigma^\mu \nu q_\nu F_N^2(t)] u(p),$$

where $q = p' - p$ is momentum transfer and $t = q^2$; $m_N$ is the nucleon mass; and $F_N^1$ and $F_N^2$ are the Dirac and Pauli form factors, which are normalized to the electric charge $e_N$ and anomalous magnetic moment $k_N$ of the corresponding nucleon: $F_N^1(0) = e_N$ and $F_N^2(0) = k_N$.

In our approach the nucleon form factor are generated by the action

$$S_{int}^V = \int d^4x d\mathbf{z} \sqrt{g} e^{-\phi(z)} L_{int}^V$$

and with this we can get Dirac and Pauli form factors by protons given by

$$F_p^1(Q^2) = C_1(Q^2) + g_v C_2(Q^2) + \eta_p^V$$
$$F_p^2(Q^2) = \eta_p^V C_4(Q^2)$$

where $Q^2 = -t$ and $C_i(Q^2)$ are given by

$$C_1(Q^2) = \sum_\tau c_\tau B(a + 1, \tau + \frac{a}{2})$$
$$C_2(Q^2) = \frac{a}{2} \sum_\tau c_\tau B(a + 1, \tau)$$
$$C_3(Q^2) = a \sum_\tau c_\tau B(a + 1, \tau + 1) \frac{a(\tau - 1) - 1}{\tau}$$
$$C_4(Q^2) = \frac{2m_N}{\kappa} \sum_\tau c_\tau (a + 1 + \tau) B(a + 1, \tau + 1) \sqrt{\tau - 1}$$

where $a = Q^2/(4\kappa^2)$ and

$$B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m + n)}$$

is the Beta function.

5 Axial Isovector Form Factor of Nucleon

The nucleon isovector axial form factor $G_A(t)$ is conventionally defined by the matrix element of the axial isovector current as

$$\langle p' | A_3^\mu(0) | p \rangle = \bar{u}(p') [\gamma^\mu G_A(t) + \frac{q^\mu}{2m_N} G_P(t)] y^5 T_3 u(p),$$

which is normalized to the nucleon axial charge $G_A(0) = g_A$.

Here the $G_A(Q^2)$ form factor is generated by the action

$$S_{int}^V = \int d^4x d\mathbf{z} \sqrt{g} e^{-\phi(z)} L_{int}^V.$$
6 Summary

We presented a soft wall model which allows us to include higher Fock states in the analysis of the nucleon structure [3,4]. In this case we restrict ourselves to the contribution of 3, 4 and 5 parton components in the nucleon Fock states. With a reduce set of parameters we achieved a reasonable agreement with data.

In our previous models we fix the mix parameters using spectroscopy data, but condition $\sum_{c} c_\tau = 1$ suggest a possibility of consider such parameters in a probabilistic way and use it as an input extracted from models that considers Fock expansions for nucleons as example in cloud mesons models. We have some advances in this direction that we hope to publish soon.

Finally, we like to remember that in [3,4] you can find more detail in calculations and additional interesting applications of this model, as for example a study about electroproduction of the N(1440) Roper resonance AdS/QCD models.

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