ABSTRACT. The numerical integration of the motion equations of a passive high-orbit fragment with large surface area was performed accounting for the main gravitational perturbations of the Earth, Moon, and Sun, as well as the solar radiation pressure.

Based on the numerical model of motion in the near-Earth space that accounts for only the most powerful perturbations, a new method for de-orbiting artificial celestial bodies from high altitudes is suggested.

Key words: methods: numerical – celestial mechanics.

1. Introduction

Today there are tens of thousands of artificial celestial bodies in the near-Earth space. Most of them belong to the space debris as such worn-out artificial satellites or their fragments. Such celestial bodies can remain in high orbits essentially indefinitely. Their motion is subjected to the perturbations by the Moon and Sun, as well as by the asymmetry of the Earth’s gravitational field. The high-orbit objects are monitored using optical telescopes.

This paper describes a new method for de-orbiting of worn-out artificial satellites from the geostationary orbits in the near-Earth space to lower altitudes.

2. Observations

We selected fragment 43096, which was detected with the ESA Space Debris 1-m Telescope located on the island of Tenerife, Spain, by Thomas Schildknecht’s team during their cooperation with the ISON project (Volvach et al., 2006). The fragment’s reference corresponds to the number in the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences database. This fragment is interesting by its high area-to-mass ratio (HAMR). When describing changes in its orbiting, it is necessary to account for significant perturbations due to the radiation pressure in addition to those by gravity. Perturbations due to the solar radiation pressure tend to be periodic. We processed observation data for the indicated fragment, which had been obtained by the ISON network during 2006-2012 within the framework of the Pulkovo Cooperation of Optical Observers (PulCOO) programme.

3. Results

A total of 226 series of observations conducted from 18 November 2006 to 16 June 2012 were processed. Each series averaged to 20-30 measurements of the topocentric right ascensions, declinations and UTC time references. The Keplerian elements were determined for each series by Laplace’s method with the subsequent refining by the 6-parameter iteration method. The accuracy of the orbital elements was estimated using the residual errors representing differences of the observed positions of the fragment from the predicted ones. The computation procedure is specified in Bazey et al. (2005) and Escobal (1970). The least errors in orbital element determination were obtained for the following series of observations:

24 November 2006 01:09:56.87 (Tenerife)
\( p = (6.4666 \pm 0.0002) \) equatorial radius
\( e = (0.06681 \pm 0.00002) \)
\( \omega = (261.16 \pm 0.02) ^\circ \)
\( \Pi = (321.588 \pm 0.002)^\circ \)
\( i = (9.0212 \pm 0.0003)^\circ \)
\( M_0 = (242.48 \pm 0.03)^\circ \).

The state vector was determined as of 24 November 2006 01:55:50.76 UTC:
\( x = -2.28181 \) equatorial radius
\( y = 6.21066 \) equatorial radius
\( z = 0.54755 \) equatorial radius
\( V_x = -0.0250860 \) equatorial radius per minute
\( V_y = -0.0111238 \) equatorial radius per minute
\( V_z = -0.00394734 \) equatorial radius per minute.

The indicated values were assumed to be the initial conditions for the fragment’s orbit integration. The
area-to-mass ratio was assumed equal to $\frac{S}{m} = 2.56$ sq.m/kg (Früh & Schildknecht, 2011). The acceleration due to the direct solar radiation was estimated as follows:

$$a = C\frac{S}{m} \left( \frac{r}{r_0} \right) \left( \frac{r-r_s}{r} \right)$$

with $C = P_0(1 + A)$, $P_0 = 0.0000045606$ N/sq.m the solar radiation pressure at the Earth’s orbit, $A$ – the electro-magnetic radiation reflection coefficient ($0 < A < 1$), $r_0$ the average radius of the Earth’s orbit, $r$ and $r_s$ – the fragment’s and the Sun’s positions in the Earth-centered coordinate system.

As fragment 43096 is referred as a high-orbit artificial Earth’s satellites, the perturbations of its motion due to the Moon and Sun are comparable to those by the Earth’s flattening (Borodovitsyna & Avdyushev, 2007). We accounted for the perturbations by the second zonal harmonic of the Earth’s gravitational field, the Moon and Sun, as well as the solar radiation pressure. The integration was performed by the Runge-Kutta methods of the 10th order (Bazyey & Kara, 2005).

During the whole period the orbit’s semi-major axis has not been subjected to the secular perturbations. The eccentricity and argument of perigee are exposed to the periodic perturbations with duration of some 370 days. The eccentricity varies from 0.017 to 0.071. The apse line oscillates with an amplitude of about 80° and slowly rotates with the angular velocity of 0.020°/day. The longitude of the ascending node and inclination of the orbit decrease at the rate of 0.0028°/day and 0.0016°/day, respectively, within the whole observation interval.

Therefore, the 43096 fragment orbit periodically changes the shape and position of the apse line, leaving its size unaltered. Besides, the apse line, longitude of the ascending node and inclination change monotonically. The numerical integration shows that the periodic perturbations in the eccentricity and argument of perigee are due to the solar radiation pressure: assuming $P_0 = 0$, those perturbations disappear.

That fact can be used to purposely change orbits of the geostationary objects and their de-orbiting to lower altitudes as down as the Earth’s atmosphere.

Let us explain that by exemplifying simulation of the 43096 fragment motion. Using the eccentricity variation curve, it is easy to detect that the eccentricity was increasing from 05 May 2007 to 13 November 2007, from 08 May 2008 to 19 November 2008, from 15 May 2009 to 26 November 2009, from 22 May 2010 to 03 December 2010, and from 29 May 2011 to 04 December 2011. During those periods the perigee distance decreases due to the radiation pressure with the semi-major axis remaining altered. The eccentricity was decreasing from 13 November 2007 to 08 May 2008, from 19 November 2008 to 15 May 2009, from 26 November 2009 to 22 May 2010 and from 03 December 2010 to 29 May 2011. If the solar radiation pressure force is stronger while the eccentricity increases comparing to those time intervals when it decreases, then it is possible to determine general secular increase of the orbital eccentricity. The same effect can be reached, for instance, by increasing the area-to mass ratio of the fragment while the eccentricity increases. We conducted the numerical experiment on the simulation of the 43096 fragment orbit with the same initial conditions as of 24 November 2006 01:55:50.76 UTC, but with alternating radiation pressure. It was assumed that $P_0 = 0.0000045606$ N/sq.m for the increasing eccentricity and $P_0 = 0$ for the decreasing eccentricity. The result is shown in Figure 1. The eccentricity increased from 0.02 as of 08 May 2007 to 0.30 as of 31 July 2012. And the semi-major axis remained unaltered at that. At the end of the integration interval the perigee distance decreased down to 29000 km (the Earth’s equatorial radius 4.54). Such a considerable

![Figure 1: The change in the orbital elements of the celestial body (eccentricity and semi-major axis) subjected to the alternating radiation pressure](image-url)
change in the fragment’s orbit was successfully attained just by changing the solar radiation pressure force two times per year.

Colligating the result obtained, it should be noted that such a method of changing the celestial body orbit in the near-Earth space can be applied to solve problems of the near-space ecology. Provided the capabilities to control the changes in the area-to-mass ratios of worn-out satellites, it is possible to solve the problem of cleaning up the near-Earth space from the space debris of artificial origin using the solar radiation pressure exclusively.

Based on the numerical model of motion in the near-Earth space that accounts for only the most powerful perturbations, a new method for de-orbiting artificial celestial bodies from high altitudes is suggested.

Conclusion

For the first time such a considerable amount of data over long time intervals was gathered for the objects with high area-to-mass ratios that enabled us to determine and estimate their observation and orbital characteristics.

The method of the celestial body orbit changing in the near-Earth space which is described in this paper can be useful in solution of the near-space ecology problem, particularly in the cleaning up the near-Earth space from the artificial space debris using the solar radiation pressure only.

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