We investigate the one pion decay of the Roper resonance \( N^*(1440) \rightarrow N\pi \) in the Sakai-Sugimoto model of the holographic QCD. The nucleon and Roper resonance emerge as ground and first excited states of the collective radial motion of the instanton in the four dimensional space with one extra dimension. It is found that the ratio of the \( \pi NN^* \) and \( \pi NN^* \) couplings, and hence the ratio of \( g^{NN^*}_{A} \) and \( g^{NN^*}_{A} \), is well reproduced in comparison with the experimental data. The mechanism of this result is due to the collective nature of excitations, which is very different from that of the single particle nature of the constituent quark model. Our results are obtained in the large-\( N_c \) and large \( \lambda \) (‘t Hooft coupling) limit which are useful to test how baryon resonances share what are expected in these limits.

I. INTRODUCTION

The Roper resonance \( N^*(1440) \) is the first excited state of the nucleon with the spin and parity \( J^P = 1/2^+ \). Its mass smaller than the negative parity nucleon \( N(1535) \) has attracted great amount of interests because the naive quark model predicts the mass of the Roper resonance much higher than that of the negative parity state. To resolve this problem, and also to reproduce the electromagnetic transitions, the importance of the meson cloud has been emphasized \[2,3\]. Turning to strong decays, an almost vanishing partial decay width of one pion emission when computed by the leading order terms of non-relativistic expansion of the pion-quark interaction disagree with the large value of the experimental data. While it has been pointed out recently that higher order corrections can improve this significantly \[4\], this problem should be further investigated.

The relatively low mass has lead to the idea of collective vibrational mode along the radial direction \[5\]. Extensive discussions were made in the Skyrme model in 1980’s, where the soliton’s radial vibrations were investigated in various context \[6\]. Later the solitonic picture of baryons has been further strengthened by the holographic QCD. The Sakai-Sugimoto model is one of successful descriptions of hadrons in the holographic QCD based on the D4-D8 brane construction \[10,11\]. They have derived an effective action of the flavor gauge field in the five dimensional space (four space-time and one extra dimension), implementing the spontaneous breaking of chiral symmetry leading to the successful low-energy effective action of hadrons. Moreover the extra dimension of the model naturally accommodates various excited states of hadrons.

In the holographic model, baryons emerge as instantons of the five-dimensional space \[12\], which is very much the same as the Skyrme mode, baryons as chiral solitons \[13,14\]. Such a baryon structure looks very different from the one of the quark model. Baryon dynamics is dominated by the collective motions of instantons/solitons, while that of the quark model by single-particle motions of quarks. Interestingly, it was found that the resulting Roper and the negative parity resonance \[12\] are degenerate and appear very close to the observed masses. This is one of good features of the holographic QCD for baryons.

The holographic baryons have been further studied by Hata et al \[15\] and by Hashimoto et al \[16\] for various static properties of the nucleon including electromagnetic and weak coupling constants. Inspired by these works, we would like to further study the properties of the Roper resonance in the holographic model. In this paper, we investigate the one pion emission decay. It is the axial transition between the Roper resonance and the nucleon, and is dictated by the transition matrix element of the axial current. Following Ref \[16\], we define chiral currents by introducing the external gauge field that couples to the currents. By calculating the matrix elements of the obtained axial current, the axial coupling and hence decay width are calculated. The results are compared with the experimental data. The model and computation procedures are realized in the large-\( N_c \) and large ‘t Hooft coupling \( \lambda \) limits. Hence our study provides the measure to what extend hadron properties share the features of these limits.

This paper is organized as follows. In section II we present the actions used in this paper and derive the solutions of the equations of motion. Then we define the chiral currents and obtain their concrete expressions by the solutions. In section III we compute matrix elements of the axial currents for the nucleon and that of the Roper to the nucleon transitions. The resulting decay width is compared with the experimental data. We discuss the ratio of \( g^{NN^*}_{A} \) and \( g^{NN^*}_{A} \), and compare with the data carefully. Final section IV is for some discussions.
and summary of the present work.

II. AXIAL CURRENT

A. Classical solutions and collective quantization

Let us start by briefly summarizing how the baryon states are obtained in the Sakai-Sugimoto model by collectively quantizing the instanton solution. The action of hadron effective theory is composed of the Yang-Mills term $S_{YM}$ and the Chern-Simons term $S_{CS}$,

\[ S = S_{YM} + S_{CS} \]

(1)

where

\[ S_{YM} = -\kappa \int d^4x dx z \left[ \frac{1}{2} h(z) F_{\mu \nu} + k(z) F_{\mu \nu}^2 \right] \]
\[ S_{CS} = \frac{N_c}{24\pi^2} \int_{M+\infty}^{\infty} \omega_5(A), \]
\[ \kappa = \frac{\lambda N_c}{216\pi^2} = a\lambda N_c. \]

(2)

In these equations $N_c$ is the number of colors, $\lambda$ the 't Hooft coupling, and the indices $\mu, \nu = 0, 1, 2, 3$ are for the 4-dimensional space-time. The curvatures along the extra dimension $z$ are defined by

\[ h(z) = (1 + z^2)^{-1/3}, \quad k(z) = 1 + z^2. \]

(3)

The 1-form $A$ expresses $A = A_0 dx^0 + A_\alpha dx^\alpha$ which consists of the flavor SU(2) part $A_0$ and the U(1) part $A_\alpha$ with $\alpha = 0, 1, 2, 3, z$. The Chern-Simons 5-form is given by

\[ \omega_5(A) = \text{tr} \left( \frac{i}{2} A^\dagger F - \frac{1}{10} A^5 \right). \]

(4)

In general, it is difficult to analytically solve the equations of motion in the presence of the curvatures $h(z)$ and $k(z)$. However, it can be simplified in the large $\lambda$ limit since the instanton profile is localized around $z \sim 0$ as proportional to $\lambda^{-1/2}$, where we can set $h(z) = k(z) = 1$. Therefore, the following instanton solution is available, with $M = 1, 2, 3, z$,

\[ A_0 = 0, \]
\[ A_\dagger = \frac{1}{8\pi^2 a} \left[ 1 - \frac{\rho^4}{(\xi^2 + \rho^2)^2} \right], \]

(5)

where

\[ g(x, z) = \frac{(z - Z) - i (x - X) \cdot \tau}{\xi}, \]

with $(X, Z)$ and $\rho$ the location and size of the instanton, respectively. The profile function $f(\xi)$ is given by

\[ f(\xi) = \frac{\xi^2}{(\xi^2 + \rho^2)}, \]
\[ \xi = \sqrt{(x - X)^2 + (z - Z)^2}. \]

The classical instanton solution needs to be quantized for the physical nucleon and Roper resonances. This can be done by the collective coordinate method, where the relevant time dependent dynamical variables are, $X, Z, \rho$ and the rotational variable in the isospin and spin space. As shown in Ref. [12], the time dependent gauge field is given by

\[ A_M(t, x^N) = V A_M^{cl}(x^N; X^N(t), \rho(t)) V^{-1} - iV \partial_M V^{-1}, \]
\[ \Phi(t, x^M) = -iV^{-1} \dot{V} \]
\[ = -\dot{X}^M(t) A_\dagger + \chi^a f(\xi) g_{\alpha}^a g^{-1}, \]
\[ \chi^a = -\text{itr} (\tau^a a^{-1} \dot{a}), \]

where $a = a_4 + ia_5 \tau^a$ is for the spin and isospin rotation. By using this gauge field, we find the collective Hamiltonian as

\[ H = -\frac{1}{2M_0} \left( \dot{\bar{\psi}} X^2 + \dot{\bar{\psi}} Z^2 \right) - \frac{1}{4M_0} \partial^2 \bar{\psi} \right)_t \frac{1}{U(\rho, Z)} \]
\[ U(\rho, Z) = M_0 + \frac{M_0}{6} \rho^2 + \frac{N_c^2}{5M_0} \frac{1}{\rho^2} + \frac{M_0}{3} \rho^2, \]

(8)

where $M_0 = 8\pi^2 \kappa$ is the classical soliton mass [12], and $y_t$ is related to the orientation coordinates by $y_t = pa_t$. The baryon states are labeled by its momentum $\vec{p}$ and quantum numbers $(l, I_3, s_3, n_\rho, n_z)$, where $l/2$ is the equal isospin and spin values; $I_3, s_3$ are the third components of the isospin and spin; and $n_\rho, n_z$ are the quanta for oscillations along the radial and $z$-directions. For the spin up proton $(I_3 = 1/2, s_3 = 1/2)$ with a finite momentum $\vec{p}$, the wave functions of ground and Roper resonance are given as [12, 16]

\[ \psi_N \propto e^{i\vec{p} \cdot \vec{X}} R_N(\rho) \psi_Z(Z) (a_1 + ia_2), \]
\[ \psi_N'_{(1440)} \propto e^{i\vec{p} \cdot \vec{X}} R_N'(\rho) \psi_Z(Z) (a_1 + ia_2), \]

(9)

where

\[ R_N(\rho) = \rho^{-1+2\sqrt{1+\frac{N_c^2}{5}}} e^{-\frac{M_0}{\sqrt{\rho}}} \]
\[ R_N'(\rho) = \left( \frac{2M_0}{\sqrt{6}} \rho^2 - 1 - 2 \sqrt{1 + \frac{N_c^2}{5}} \right) R_N(\rho), \]
\[ \psi_Z(Z) = e^{-\frac{M_0}{\sqrt{6}} Z^2}. \]

(10)

We note that the wave function for the $z$ oscillation is the lowest ($n_z = 0$) for both the nucleon and Roper resonance. Thus the only difference between them is in the radial part, $R_N(\rho)$ and $R_N'(\rho)$.

B. The asymptotic solution of the instanton

The BPST instanton that we have summarized in the previous subsection is only an approximate solution in the large $\lambda$ limit where the instanton size is small. This
can be used for the computation of baryon masses. However, for the computation of currents which are defined at \(|z| \to \infty\) such a solution is not suited. As shown in Ref. [10] we need to find the solution that is properly extended to the large \(|z|\) region to obtain the well-defined currents. In this paper, we simply summarize the final result of such a solution;

\[
\begin{align*}
\hat{A}_0 &= -\frac{1}{2a\lambda} G(\vec{x}, z; \vec{X}, Z), \\
\hat{A}_i &= \frac{1}{2a\lambda} \left[ X^i + \frac{\rho^2}{2} \left( \frac{\chi^a}{2} \left( \frac{\partial}{\partial \chi^a} - \delta^a_{\beta} \frac{\partial}{\partial \beta} \right) - \frac{\rho}{\rho} \frac{\partial}{\partial \chi^a} \right) \right] G(\vec{x}, z; \vec{X}, Z), \\
\hat{A}_z &= \frac{1}{2a\lambda} \left[ Z + \frac{\rho^2}{2} \left( \frac{\chi^a}{2} \frac{\partial}{\partial \chi^a} + \frac{\rho}{\rho} \frac{\partial}{\partial \chi^a} \right) \right] H(\vec{x}, z; \vec{X}, Z), \\
A_0 &= 4\pi^2 \rho^2 i a \hat{a}^{-1} G(\vec{x}, z; \vec{X}, Z) + 2\pi^2 \rho^2 a \tau^a a^{-1} \left( \hat{X}^i \left( \frac{\epsilon^{iaj}}{\partial} - \delta^a_{\beta} \frac{\partial}{\partial \beta} \right) \right) G(\vec{x}, z; \vec{X}, Z), \\
A_i &= -2\pi^2 \rho^2 a \tau^a a^{-1} \left( \epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^a_{\beta} \frac{\partial}{\partial \beta} \right) G(\vec{x}, z; \vec{X}, Z), \\
A_z &= -2\pi^2 \rho^2 a \tau^a a^{-1} \frac{\partial}{\partial \chi^a} H(\vec{x}, z; \vec{X}, Z).
\end{align*}
\]

where the index \(i\) runs 1 – 3. In these equations, \(G\) and \(H\) are given by

\[
\begin{align*}
G(\vec{x}, z; \vec{X}, Z) &= \kappa \sum_{n=1}^{\infty} \psi_n(z) \psi_n(Z) Y_n(|\vec{x} - \vec{X}|), \\
H(\vec{x}, z; \vec{X}, Z) &= \kappa \sum_{n=1}^{\infty} \phi_n(z) \phi_n(Z) Y_n(|\vec{x} - \vec{X}|).
\end{align*}
\]

The function \(\psi_n(z)\) are the solutions of the eigenvalue equation

\[
-h(z)^{-1} \partial_z (k(z) \partial_z \psi_n) = \lambda_n \psi_n(z),
\]

with the eigenvalue \(\lambda_n\), and

\[
\begin{align*}
\phi_0(z) &= \frac{1}{\sqrt{\kappa \kappa}} \frac{1}{k(z)}, \\
\phi_n(z) &= \frac{1}{\sqrt{\lambda_n}} \partial_z \psi_n(z), \\
Y_n(r) &= -\frac{1}{4\pi} \frac{e^{-\sqrt{\lambda_n} r}}{r}, \quad r = |\vec{x}|.
\end{align*}
\]

C. Currents

Now we are ready to calculate the axial current. Following Ref. [10], the chiral current is derived from the coupling with the external gauge field \(\delta A_\mu\), which is defined by

\[
\begin{align*}
A_\mu(x^\mu, z) &= A_\mu^{\text{cl}}(x^\mu, z) + \delta A_\mu(x^\mu, z),
\end{align*}
\]

They are related to the left and right gauge fields in the four dimensional space at \(z \to \pm \infty\),

\[
\begin{align*}
\delta A_\mu(x^\mu, z \to +\infty) &= A_{L\mu}(x^\mu), \\
\delta A_\mu(x^\mu, z \to -\infty) &= A_{R\mu}(x^\mu).
\end{align*}
\]

Substituting this field into the action, the coefficients of the first order in \(A_{L\mu}, A_{R\mu}\) is identified with the left and right currents \(J_L^\mu, J_R^\mu\) with the sign properly taken into account,

\[
\begin{align*}
\kappa \int d^4 x \left[ 2 \text{tr} \left( \delta A^\mu \kappa (z) (F_{\mu z})^\text{cl} \right) \right]_{z = \pm \infty} &
= 2 \int d^4 x \text{tr} \left( A_{L\mu} J_{L}^\mu + A_{R\mu} J_{R}^\mu \right), \quad (18)
\end{align*}
\]

The vector and axial currents are then obtained by

\[
\begin{align*}
J_{L}^\mu &= J_{L}^{\mu} + J_{R}^{\mu}, \\
J_{A}^{\mu} &= J_{L}^{\mu} - J_{R}^{\mu} = -\kappa \left[ \psi_0(z) \kappa (z) (F_{\mu z})^\text{cl} \right]_{z = \pm \infty}, \quad (20)
\end{align*}
\]

with \(\psi_0(z) = (2/\pi) \arctan z\).

When the instanton oscillates along the \(z\) direction in a narrow range in the large \(\lambda\) limit, the metrics are approximated as \(h(Z) \simeq k(Z) \simeq 1\). Then, substituting \([13]\) for \([20]\) gives the following form (\(r \equiv |\vec{x} - \vec{X}|\))

\[
\begin{align*}
J_{A}^{\mu}(r; \vec{X}, Z, \rho, \hat{a}) &= -2\pi^2 \kappa \rho^2 a \tau^a a^{-1} \\
&\times \left( (\partial_\rho \partial_\rho - \delta^a_{\beta} \partial^2_\beta) H^A - \epsilon^{iaj} \partial_j G^A \right) \quad (21)
\end{align*}
\]
where
\[ G^A (r; \vec{X}, Z) = [\psi_0 (z) k (z) \partial_z G]_{z=+\infty}^{z=-\infty} \]
\[ = - \sum_{n=1}^{\infty} g_a^n \psi_{2n} (Z) Y_{2n} (r), \quad (22) \]
\[ H^A (r; \vec{X}, Z) = [\psi_0 (z) k (z) H]_{z=+\infty}^{z=-\infty} \]
\[ = - \frac{1}{2 \pi^2} k (Z) \frac{1}{r} \sum_{n=1}^{\infty} \frac{g_a^n}{\lambda_{2n}} \partial_Z \psi_{2n} (Z) Y_{2n} (r), \quad (23) \]
\[ g_a^n = \lambda_{2n} \kappa \int dz h (z) \psi_{2n} \psi_0. \quad (24) \]

To go further, it is convenient to present the Fourier transform in the momentum space, (in what follows the dependence on the collective coordinates \( \vec{X}, Z, \rho, \vec{a} \) are suppressed)
\[ \tilde{J}_A^\mu (\vec{q}) = \int d^3 x e^{-i \vec{q} \cdot \vec{r}} J_A^\mu (r). \quad (25) \]
We obtain the following form:
\[ \tilde{J}_A^\mu (\vec{q}) = e^{-i \vec{q} \cdot \vec{X}} 2 \pi^2 k^2 2 \pi^2 \left( \tau^a \tau^b \tau^{-1} \right) \]
\[ \times \left( \delta_{a \bar{a}} - \frac{g_a(q)}{q^2} \right) \sum_{n \geq 1} \frac{g_a^n \partial_Z \psi_{2n} (Z)}{q^2 + \lambda_{2n}}. \quad (26) \]

This current is regarded as an operator in terms of the dynamical variable \( \vec{X}, Z, \rho \) and \( \vec{a} \), which is used when taking the matrix elements by the corresponding wave functions.

### III. DECAY PROPERTIES OF ROPER RESONANCE

Now, we investigate the decay properties of the Roper resonance, in particular the one pion emission decay \( N^* (1440) \rightarrow N + \pi \). Because the Roper resonance has a very large width causing uncertainties in the Breit-Wigner fitting, we refer to the results of the pole analysis. Following the PDG table \( [17] \), we quote the following nominal values
\[ M_{N^*} = 1360 - 1380 \ (\sim 1370) \ \text{MeV}, \]
\[ \Gamma_{\text{total}} = 160 - 190 \ (\sim 175) \ \text{MeV}, \]
and the branching ratio of the one pion decay
\[ N^* \rightarrow N \pi : \ 55 - 75 \%. \quad (28) \]

Using the lower and upper bounds for the total decay width and branching ratio, we find the partial decay width of the one pion decay
\[ \Gamma_{N^* \rightarrow N \pi} \sim 90 - 140 \ \text{MeV}. \quad (29) \]

### A. Axial coupling \( g_A \)

The axial coupling \( g_A^{NN^*} \) for the transition \( N^* (1440) \rightarrow N + \pi \) is defined as follows:
\[ \int d^3 x \langle N, s' i | J_A | N^*, s_3, I_3 \rangle \times 2 \]
\[ = - \frac{2}{3} g_A^{NN^*} (\sigma^a s'_{i3} (\tau^a)_{i3})_{i3}. \quad (30) \]
The factor \( 2/3 \) on the right hand side is needed in the chiral limit \( [14] \). Using \( (26) \) and \( (29) \), we obtain
\[ g_A^{NN^*} (\vec{q}) = \frac{8 \pi^2 \kappa}{3} \langle R_{NN^*} | \rho^2 | R_N \rangle \sum_{n=1}^{\infty} \frac{g_a^n (\partial_Z \psi_{2n} (Z))}{q^2 + \lambda_{2n}} \quad (31) \]
where \( \langle \partial_Z \psi_{2n} (Z) \rangle \) stands for the expectation value using the wave functions of \( Z \). The matrix element of \( \rho^2 \) can be computed and the result is
\[ \langle R_{NN^*} | \rho^2 | R_N \rangle = \left( 1 + 2 \sqrt{1 + \frac{N_c^2}{5}} \right)^{-1/2} \langle R_N | \rho^2 | R_N \rangle \]
\[ = \frac{\sqrt{5}}{2 \sqrt{5}} \left( 1 + 2 \sqrt{1 + \frac{N_c^2}{5}} \right)^{1/2} \rho_{cl}^2 \quad (32) \]
with \( \rho_{cl} \) being the classical instanton size given by
\[ \rho_{cl} = \frac{N_c}{8 \pi^2} \sqrt{\frac{6}{5}}. \quad (33) \]

We note that the transition matrix element for \( N^* (1440) \rightarrow N + \pi \) is related to the nucleon matrix element, an interesting feature of the present model associated with the collective nature of baryons. The axial coupling constant is then defined at \( \vec{q} = 0 \), \( g_A^{NN^*} = g_A^{NN^*} (0) \).

Using the relation
\[ \sum_{n=1}^{\infty} \frac{g_a^n \partial_Z \psi_{2n} (Z)}{\lambda_{2n}} = \frac{2}{\pi} \frac{1}{k (Z)}. \quad (34) \]
\( g_A^{NN^*} \) can be expressed in a compact form:
\[ g_A^{NN^*} = \frac{16 \pi \kappa}{3} \langle R_{NN^*} | \rho^2 | R_N \rangle \left( \frac{1}{k (Z)} \right). \quad (35) \]
In the above equations, \( \langle \cdots \rangle \) stands for the expectation value using the wave functions of \( Z \).

There are two parameters of this model, \( M_{KK} \) and \( \kappa \). Following Adkins et al \( [14] \), they are determined to reproduce the mass splitting of the nucleon and delta, and the pion decay constant \( f_\pi = 64.5 \ \text{MeV} \),
\[ M_{KK} = 488 \ \text{MeV}, \quad \kappa = 0.0137. \quad (36) \]

Then, the prediction of the present model for \( g_A^{NN^*} \) is
\[ g_A^{NN^*} = 0.402. \quad (37) \]
B. Decay width

The decay width of $N^*(1440) \to N + \pi$ can be computed by the formula

$$
\Gamma_{N^*(1440)\to N+\pi} = \frac{1}{2M_N} \int \frac{d^3p_N}{(2\pi)^3} \frac{d^3p_\pi}{(2\pi)^3} \frac{3}{2E_N} \frac{3}{2E_\pi} \times \frac{1}{(2\pi)^4} \delta^4(p_N + p_\pi - \vec{q}) \left| t_{fi} \right|^2,
$$

where the amplitude $t_{fi}$ is given by the Lagrangian

$$
L = \frac{i}{2f_\pi} \frac{M_N + M_{N^*}}{2f_\pi} g_A^{NN^*} \bar{\psi}_N \gamma_5 \vec{\tau} \cdot \vec{\sigma} \psi_N + h.c.,
$$

as follows

$$
t_{fi} = \langle N(-\vec{q}) \pi(\vec{q}) | L | N^*(\vec{0}) \rangle = \sqrt{2M_N} \sqrt{M_N + E_N} \times \frac{M_N + M_{N^*}}{2f_\pi} g_A^{NN^*} \frac{q}{E_N + M_N} \langle s_1 | \vec{\sigma} \cdot \vec{q} | s_3 \rangle.
$$

Here we have expressed the effective $\pi NN^*$ coupling $g_{\pi NN^*}$ in terms of the axial coupling by using the Goldberger-Treiman relation,

$$
g_A^{NN^*} = \frac{f_\pi g_{\pi NN^*}}{(M_N + M_{N^*})/2}.
$$

Hence we obtain

$$
\Gamma_{N^*(1440)\to N+\pi} = \frac{q}{4\pi} \frac{M_N + E_N}{M_{N^*}} \left( \frac{M_N + M_{N^*}}{2f_\pi} g_A^{NN^*} q \right)^2.
$$

Using $M_N = 940$ MeV, $M_{N^*} = 1370$ MeV, $q = 342$ MeV, we find

$$
\Gamma_{N^*(1440)\to N+\pi} = 64$ MeV.
$$

In this computation the value of $g_A^{NN^*}$ at $\vec{q} = 0$ is used. By considering the form factor effect, the $g_A^{NN^*}$ value at $\vec{q} = 342$ MeV becomes about 15 % smaller, and hence $\Gamma_{N^*(1440)\to N+\pi} \sim 55$ MeV.

These values are smaller than the experimental value $25$. This is because the axial coupling $g_A^{NN^*}$ is small, which is a common feature of the solitonic picture of baryons. In fact, the nucleon $g_A^{NN}$ is computed in a similar manner as for $g_A^{NN^*}$ by using the nucleon wave function $R_N(\rho)$. The result is

$$
g_A^{NN} = 0.837.
$$

Though small, it is interesting to observe that the value is somewhat larger than that of $11$. One possible resolution to recover the experimental value $g_A^{NN} = 1.25$ is to take into account $1/N_c$ corrections (Ref. 18 and references there). Here, however, we do not discuss this anymore. On the other hand, it is interesting to observe the relation between the axial couplings of the nucleon and that of the Roper-nucleon transition. Inspection of Eq. (32), we find

$$
g_A^{NN^*} : g_A^{NN} = 1 : \left( 1 + 2\sqrt{1 + \frac{N_c^2}{5}} \right)^{1/2}
$$

$$
= 1 : 2.08.
$$

We emphasize that this relation does not include any model parameters (except for $N_c = 3$), and so a model independent relation. Experimentally, if we use the partial decay width $\Gamma_{N^*\to \pi N} \sim 110$ MeV, we find the ratio

$$
g_A^{NN^*} : g_A^{NN} = 0.77 : 1.25 \sim 1 : 1.62,
$$

which agrees well with the present model prediction within $\sim 20 \%$ accuracy.

IV. DISCUSSIONS AND SUMMARY

In this paper, we have studied one pion emission decay of the Roper resonance, $N^*(1440) \to N + \pi$ in the Sakai-Sugimoto model of the Holographic QCD. Baryons are described as collective states of instantons of the five-dimensional Yang-Mills theory. We have then employed the currents as defined in Ref. 16, and computed the matrix elements. Resulting axial coupling has turned out to be too small as compared to what is expected from the experimental data. This is a rather common feature of the solitonic model for baryons. However, an important finding has been made for the ratio of $g_A$’s of the nucleon and Roper-nucleon transition in a model independent manner.

The present picture of baryons as instantons with collective dynamics is very much the same as the Skyrmion picture, baryons as chiral solitons. In contrast, it is very much different from the conventional quark model one, where baryons are described by single particle states of the constituent quarks. As anticipated, the quark model gave only a tiny decay rate for the Roper resonance when the leading term in $1/m^2$ expansion of the quark-pion interaction is used, which has been the widely adopted prescription. For this problem a resolution has been recently proposed by including higher order terms of $1/m^2$ [2].

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