Pre-inflation and Trans-Planckian Censorship

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Abstract

We investigate the implication of Trans-Planckian Censorship Conjecture (TCC) for the initial state of primordial perturbations. It is possible to set the state of perturbation modes in the infinite past as the Minkowski vacuum, only if the pre-inflationary era is past-complete. We calculate the evolution of the perturbation modes in such a pre-inflationary era, and show that at the beginning of inflation the perturbation modes with wavelengths much shorter than the Hubble scale (but still larger than the Planck length scale) will behave as they are in the Bunch-Davis state. Therefore, a past-complete pre-inflationary evolution may automatically prepare the initial state required for the inflationary perturbations at the CMB window while obey the TCC.

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I. INTRODUCTION

Inflation [1–4] is the standard paradigm of the early universe. It predicted the nearly Gaussian and scale-invariant primordial perturbations, which are consistent with the observations of cosmic microwave background (CMB) anisotropies [5, 6]. However, inflation is not the final story of the early universe, since it is past-incomplete, as showed in Refs. [7, 8].

Usually, the initial state of the inflationary perturbations is set as the Bunch-Davis (BD) vacuum [9], which is the lowest-energy state of de Sitter spacetime. However, due to the past-incompleteness of inflation [8], initially the background and fluctuations of spacetime will be inevitably involved in the cosmological singularity, where the length scale is smaller than the Planck scale or the cutoff scale of the effective field theory (EFT). This is the so-called “trans-Planckian” problem of inflation [10, 11]. In this situation, inflation does not
automatically pick out the BD state as the initial state of perturbation modes.\footnote{Non-BD initial states may significantly affect the power spectrum and Non-Gaussianity of primordial perturbations, e.g., \cite{12–14}.}

Recently, Bedroya and Vafa proposed the Trans-Planckian Censorship Conjecture (TCC) \cite{15}, which states that “a field theory consistent with a quantum theory of gravity does not lead to a cosmological expansion where any perturbation with length scale greater than the Hubble radius trace back to trans-Planckian scales at an earlier time”, see also its application to inflation models \cite{16}. Actually, since inflation is past-incomplete, the inflationary perturbations inevitably originated from the sub-Planckian scale fluctuations. It is well-known that the BD initial state of perturbations is quite essential for acquiring the nearly scale-invariant power spectrum of density perturbations. Therefore, how to set the BD state at the beginning of observable inflationary era (i.e., the CMB window) is a question to be answered, see also \cite{17}. In certain sense, the TCC indicates that the existence of a pre-inflationary era might be required.

One possibility of such pre-inflationary scenarios is a nonsingular bounce, i.e., the so-called bounce inflation \cite{18–20}, which not only avoids the past-incompleteness problem of inflation but also reserves the advantages of inflation. Recently, the instability problems of perturbations in flat nonsingular cosmologies (which is inevitable \cite{21, 22} in Horndeski theory, see also \cite{23, 24}) have been solved in the “beyond Horndeski” theory, as proved within the EFT of cosmological perturbations \cite{25–29}, see also \cite{30–37} (see \cite{38, 39} for reviews).\footnote{The earlier attempts (but with the instabilities of perturbations) on nonsingular bounce, as well as G-bounce, can be found in, e.g., Refs. \cite{40–52}.}

In the pre-inflationary era, initially the universe is slowly contracting with $\epsilon = -\dot{H}/H^2 \gg 1$. Thus, in the infinite past, the spacetime is nearly Minkowskian.\footnote{See also \cite{53–55} for the scenario with a slow expansion ($\epsilon \ll -1$) preceding inflation.} In this case, it is natural to pick out the Minkowski vacuum state as the initial state of the primordial perturbations. However, at present the pre-inflationary physics is still in speculation and exploration. It is possible that the pre-inflationary era might consist of multiple different phases. The question is: provided the primordial perturbation modes originate from the Minkowski vacuum in the infinite past and obey the TCC throughout, whether the sub-horizon modes at the beginning of inflation can be in the BD state? In this paper, we will investigate the relevant issues.

The outline of this paper is as follows: In Sec. II, we briefly revisit the “trans-Planckian”
problem within the EFT of cosmological perturbations. In Sec. III, we discuss the TCC and its implication. We point out that it is possible to set the state of perturbations in the infinite past as the Minkowski vacuum only if the spacetime is past-complete. In Sec. IV, we calculate the evolution of the perturbation modes in past-complete pre-inflationary era to see whether the corresponding modes approximate to the BD state at the beginning of inflation. Sec. V is the conclusion.

II. TRANS-PLANCKIAN PROBLEM

In this section, we will focus on the evolution of the primordial perturbations. In unitary gauge, the quadratic action of scalar perturbation $\zeta$ is

$$S^{(2)}_\zeta = \int a^3 Q_s \left[ \dot{\zeta}^2 - c_s^2 \left( \frac{\partial \zeta}{a^2} \right)^2 \right] d^3 x dt. \quad (1)$$

It is convenient to define $u_k = z\zeta_k$ with $z = a\sqrt{2Q_s}$, where $\zeta_k$ is the scalar perturbation mode. The Mukhanov-Sasaki equation for $u_k$ is

$$u_k'' + \left( c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0, \quad (2)$$

where $' = d/d\tau$, $c_s$ is the sound speed. In general relativity (GR), $c_s^2 = 1$ and $Q_s = M_P^2 \epsilon$ with $\epsilon = -\dot{H}/H^2$.

In the slow-roll inflation, $z''/z \sim a''/a \simeq (2 + \mathcal{O}(\epsilon))/\tau^2$. Thus the solution of Eq. (2) is

$$u_k(\tau) = \frac{\sqrt{-\pi \tau}}{2} \left[ \alpha(k) H^{(1)}_{\nu}(-k\tau) + \beta(k) H^{(2)}_{\nu}(-k\tau) \right], \quad (3)$$

where the $k$-dependent coefficients $\alpha(k)$ and $\beta(k)$ obey the Wronskian condition $|\alpha|^2 - |\beta|^2 = 1$, $\nu \approx 3/2$, $H^{(1)}_{\nu}$ and $H^{(2)}_{\nu}$ are the first and second kind Hankel functions of the $\nu$-th order, respectively.

One must select the initial state of perturbations to calculate the coefficients $\alpha(k)$ and $\beta(k)$. Requiring the perturbation modes coincide with the Minkowski solution ($u_k \simeq \frac{1}{\sqrt{2k}} e^{-ik\tau}$) in the infinite past gives

$$|\alpha| = 1, \quad |\beta| = 0, \quad (4)$$

Note that there could be an undetermined phase difference in $\alpha$ (see, e.g., [56] for a review).
where we have used that
\[
H^{(1)}_{\nu}(-k\tau) \approx \sqrt{\frac{2}{-\pi k\tau}} e^{-i k\tau} e^{-i\left(\frac{\nu}{2} + \frac{1}{4}\right)\pi}
\] (5)
in the limit \(-k\tau \gg 1\). Particularly, we have \(\nu = 3/2\) for de Sitter inflation. Using
\[
H^{(1)}_{3/2}(-k\tau) = -\sqrt{\frac{2}{-\pi k\tau}} e^{-i k\tau} (1 - \frac{i}{k\tau}),
\]
one gets
\[
u_k = \frac{1}{\sqrt{2k}} e^{-i k\tau} \left(1 - \frac{i}{k\tau}\right),
\] (6)
which is just the lowest-energy state in de Sitter spacetime (i.e., the BD vacuum).

Generally, setting the initial state of perturbation modes as the BD state is essential for inflation to predict the nearly Gaussian, scale-invariant primordial perturbations. As for the non-BD states, we will have \(|\alpha| > 1\) and \(|\beta| > 0\), where \(|\beta|^2\) could be interpreted as the number density of particle excitations [57].

The choice of the initial BD state seems natural. However, it is not the case due to the well-known “trans-Planckian” problem [10, 11]. In the following, based on the EFT of cosmological perturbations, we will revisit this problem.

As mentioned, one will get the BD state by requiring the perturbation modes \(u_k\) at \(-k\tau \gg 1\) to coincide with the Minkowski solution. Usually, it is thought that in the infinite past, the perturbation modes are deep inside the horizon, so that Eq. (2) equals to the equation of free wavepacket in flat Minkowski space. However, the so-called Minkowski spacetime is only “conformal” Minkowski, which is actually expanding. In such a spacetime, since the wavelength of perturbation modes \(\lambda \sim a\), which will blueshift back to the past. We will inevitably have \(\lambda \lesssim 1/\Lambda\) before some \(\tau = \tau_\Lambda\), where \(\Lambda \lesssim M_P\) is the cutoff scale below which the higher-order derivative operators are negligible.

When \(\lambda \simeq O(1/\Lambda)\), the EFT responsible for (1) is no longer robust, and the neglected higher-order derivative operators, such as (see, e.g., [25]),
\[
L/M_P^2 \sim \frac{\lambda_4}{\Lambda^2} (R^{(3)})^2 + \frac{\lambda_6}{\Lambda^4} (\nabla_i R^{(3)})^2 + \cdots,
\] (7)
will be no longer negligible, where \(R^{(3)}\) is the Ricci scalar on the 3-dimensional spacelike hypersurface and \(\lambda_{4,6} \simeq O(1)\) are constants. One is also allowed to take account of the operators \(R^{(3)}_{ij} R^{(3)ij}\) and \(\nabla_i R^{(3)}_{jk} \nabla^i R^{(3)jk}\), which we have omitted for simplicity. We have put such operators and the higher-order spatial derivative operators of \(R^{(3)}\) into the ellipsis.
Generally,\(^5\)
\[
(R^{(3)})^2 \sim k^4 \zeta_k^2,
\]
\[
(\nabla_i R^{(3)})^2 \sim k^6 \zeta_k^2.
\]

The higher-order derivative operators in (7) do not affect the background. Hence, the effective theory of inflation may be still valid at the background level when the perturbation mode satisfies \(\lambda \lesssim 1/\Lambda\). However, in this situation, Eq. (2) will be modified to
\[
u''_k + \left( \sim \sum_p \frac{k^{2p}}{\Lambda^{2p}} \right) k^2 \nu_k = 0,
\]
where \(p \geq 0\) is some integer. In this case, what the perturbation \(u_k\) feels will be a regime dominated by the higher-order momentum operators instead of a Minkowski space, since Eq. (2) is no longer the equation of free wavepacket in flat Minkowski space. Therefore, it is not sufficiently reasonable to set the state of the perturbation modes in the infinite past as the Minkowski vacuum.

III. IMPLICATIONS OF TCC FOR PRE-INFLATION

A. On TCC

Recently, Bedroya and Vafa proposed the Trans-Planckian Censorship Conjecture (TCC) \cite{15}, which requires that the sub-Planckian fluctuations should never cross their Hubble scale to become classical, otherwise the corresponding EFT belongs to the swampland \cite{59–61}. The TCC suggests that inflation can only last for a limited efolding number, i.e.,
\[
\int_{t_i}^{t_f} H dt < \ln \frac{M_p}{H_f} ,
\]
where the subscript “\(i\)” and “\(f\)” represent the beginning and ending of inflation, respectively. The TCC might be also argued as follows.

Generally, neglecting the higher-order operators in (7), we have \(|u_k| = 1/\sqrt{2k}\) for the sub-horizon perturbation modes, (noting that it is also approximately valid for \(\lambda \lesssim 1/H\), see also Sec. IV A). Thus one will have \(|\zeta_k| \sim 1/a\) for such a perturbation mode, since \(|\zeta_k| \sim |u_k|/a\) on sub-horizon scale. It can be found that at the beginning of inflation, the

\(^5\) One also has considered the operators \(\sim \nabla_\mu \delta K_{\nu\sigma} \nabla^\mu \delta K^{\nu\sigma}\), which contribute \(k^8 \zeta_k^2\) as well \cite{58}.
spectrum of sub-horizon perturbation \((\lambda \ll 1/H)\) is

\[
P^{1/2}_\zeta(t_i, k) = P^{1/2}_\zeta(t_f, k) \frac{a_f}{a_i} \sim \frac{H_f}{\sqrt{\epsilon_f M_p}} \frac{a_f}{a_i},
\]

provided this perturbation mode cross the horizon \((k \simeq a_f H_f)\) at \(t_f\). The validity of perturbation theory requires that \(P^{1/2}_\zeta(t_i, k) < 1\), otherwise the spacetime will be distorted by lots of black holes on sub-Hubble scale. According to (11), we have

\[
\int_{t_i}^{t_f} H dt < \ln \frac{\sqrt{\epsilon_f M_p}}{H_f},
\]

which, for \(\epsilon_f \lesssim 1\), is consistent with the TCC constraint (10).

The TCC might suggest the existence of a pre-inflationary evolution. The question is whether the pre-inflationary evolution can prepare the required initial condition for observational (i.e., at the CMB window) inflationary perturbations. The relevant issue is closely related to the past-incompletion (or singularity) of inflation [8]. In inflationary spacetime, the affine length \(\int d\eta \zeta \sim \int c_s^2 Q_s a dt\) of scalar perturbation mode must be finite. As a result, back to the past the perturbation modes will inevitably hit the singularity, as depicted in Fig. 1. Although it seems acceptable to set the BD state as the initial state of perturbation modes below the scale \(\Lambda \lesssim M_p\), it is not justified either.

### B. Minkowski in past-complete pre-inflation

Recalling the Raychaudhuri equation for the null geodesics, which is simplified as (assuming the null vector \(k^\mu\) is orthogonal to the hypersurfaces, see e.g., [65])

\[
\frac{d\theta}{d\eta} \leq -\frac{\theta^2}{2} - R_{\mu\nu} k^\mu k^\nu,
\]

where \(\eta\) is the affine parameter of the null geodesics and \(\theta = \nabla_\mu k^\mu\) is the expansion parameter. When the null convergence condition (NCC) \(R_{\mu\nu} k^\mu k^\nu \geq 0\) is satisfied, one has \(d\theta/d\eta + \theta^2/2 < 0\). In this case, back along \(\eta\) the null rays will converge, which indicates that the wavelengths of perturbation modes will be blueshifted, hence \(\lambda \lesssim 1/\Lambda\) is inevitable. In Ref. [7], the NCC has been applied to argue the past-incompletion of inflation. Thus,

\[\footnote{It is interesting that for the tensor perturbation, (12) is exactly same as the TCC constraint (10).}\]

\[\footnote{The decay of non-BD state into BD state through the self-interaction of the perturbations \(\sim \epsilon^2 \zeta (\zeta')^2\) is very inefficient [62], since \(\epsilon \ll 1\). Nevertheless, see also [63, 64] for further discussions.}\]
FIG. 1: Causal patch of the FLRW universe with inflation, where $T$ and $R$ are the timelike and radial coordinates, respectively. The dashed line represents the initial cosmological singularity, the red solid curve represents the comoving horizon (i.e., $1/(aH_{\text{per}}) = |z/z''|^{1/2}$) for the perturbation modes, $T_{RH}$ and $T_0$ represent the reheating surface and present, respectively. Initially, all perturbation modes are deep inside the horizon. As time goes by, perturbation modes with sufficiently large wavelength will exit horizon and eventually reenter the horizon. We have assumed that the universe enters into the radiation or matter phase after reheating.

only if the pre-inflationary era is past-complete,\footnote{Necessary condition but not sufficient, see e.g., [66].} it is possible to set the initial state of perturbations in the infinite past (as the Minkowski vacuum).

In pre-inflationary era, the past-completion requires that the converging null rays must be reversed (i.e., $d\theta/d\eta > 0$) at least for some period. According to (13), during this period the NCC must be violated, i.e.,

$$R_{\mu\nu}k^\mu k^\nu < 0,$$

or equivalently, $\dot{H} > 0$, since $R_{\mu\nu}k^\mu k^\nu = -2\dot{H}(k^0)^2$ for the spatially flat FLRW metric.

One possibility that satisfies (14) is the pre-inflationary bounce, where $\dot{H} > 0$ regardless of whether the gravity is GR or not. In the corresponding bounce inflation scenario, before the nonsingular bounce, one may have

$$a \sim \frac{1}{1 + \left(\frac{\tau_c}{\tau}\right)^n},$$

for $0 < n \ll 1$, where $\tau_c$ is negative. In the infinite past $\tau \to -\infty$, $a \sim 1$, we have an asymptotically Minkowski spacetime. When $|\tau| \ll |\tau_c|$, $a \sim (\tau/\tau_c)^n$, the pre-inflationary
universe is slowly contracting (i.e., the ekpyrotic phase [67]) until a nonsingular bounce occurred. Hereafter it will begin to inflate. The corresponding causal patch is displayed in Fig. 2.

In such a bounce inflation scenario, all perturbation modes are in a flat Minkowski space in the infinite past. The wavelengths of the perturbation modes that are interested for observations satisfy $\lambda \gg 1/\Lambda$, so that the $k/\Lambda$-suppressed higher-order derivative operators in (7) are negligible. In particular, such a scenario is also consistent with the TCC, since the sub-Planckian perturbation modes may never have the opportunity to cross their horizon, as long as $a_f/a_b < M_p/H_f$ and $a_b$ is sufficiently large (see also [16]), where the subscript “b” corresponds to the bounce point.

FIG. 2: Causal patch of the FLRW universe in the bounce inflation scenario, where $T_b$ represents the time of bounce.

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9 Strictly, the so-called “horizon” is actually “the horizon of the perturbations”, which is defined by $1/H_{\text{per}} = a\sqrt{|z/z''|}$. It will be significantly different from $1/H$, if $Q_s$ is rapidly changing, see also [68].
IV. BUNCH-DAVIS AT THE BEGINNING OF INFLATION

A. Analytical evaluation

In this section, we will focus on the bounce inflation scenario and evaluate the states of perturbation modes at the beginning of inflation. The result is also applicable for other scenarios with past-complete pre-inflationary era.

Generally, the evolution of the perturbation modes satisfies Eq. (2) with $c_s^2$ replaced by $C_s^2$, where

$$C_s^2 = c_s^2 + \left( \sum_p \frac{k^{2p}}{\Lambda^{2p}} \right).$$

In pre-inflationary era, we can safely neglect higher-order derivative operators and have $C_s^2 = c_s^2$, provided the wavelengths of the perturbation modes $\lambda \ll 1/\Lambda$. One can always set $Q_s > 0$ and $c_s^2 = 1$, as in Refs. [25–28], so that the ghost or gradient instabilities are avoided.

Considering the pre-inflationary era consists of different phases with $n = n_j$ ($j \geq 0$) in Eq. (15), we have

$$z_j = \sqrt{2a^2Q_s} \sim (\tau_{R,j} - \tau)^{n_j}$$

for the $j$-th phase, where $n_j \simeq \text{const}$. During the bounce, $a$ is almost constant. However, since $Q_s$ is rapidly changing, we still could have Eq. (17), see Ref. [69]. Hence, in the phase $j$, $z_j''/z_j$ can be written as

$$\frac{z_j''}{z_j} = \frac{\nu_j^2 - 1/4}{(\tau - \tau_{R,j})^2},$$

where $\nu_j = |n_j - 1/2|$. Regarding the phases $j$ and $j + 1$ as adjacent phases, we can write down the solutions to Eq. (2) in the corresponding phases as

$$u_{k,j}(\tau) = \frac{\sqrt{\pi(\tau_{R,j} - \tau)}}{2} \left\{ \alpha_j H^{(1)}_{\nu_j} [k(\tau_{R,j} - \tau)] + \beta_j H^{(2)}_{\nu_j} [k(\tau_{R,j} - \tau)] \right\}, \quad (\tau < \tau_{j+1}),$$

$$u_{k,j+1}(\tau) = \frac{\sqrt{\pi(\tau_{R,j+1} - \tau)}}{2} \left\{ \alpha_{j+1} H^{(1)}_{\nu_{j+1}} [k(\tau_{R,j+1} - \tau)] + \beta_{j+1} H^{(2)}_{\nu_{j+1}} [k(\tau_{R,j+1} - \tau)] \right\}, \quad (\tau > \tau_{j+1}),$$

where $\alpha_{j(j+1)}$ and $\beta_{j(j+1)}$ are $k$-dependent coefficients, $\tau_{j+1}$ denotes the conformal time of the juncture of these two phases.
Using the matching conditions \( u_{k,j}(\tau_{j+1}) = u_{k,j+1}(\tau_{j+1}) \) and \( u'_{k,j}(\tau_{j+1}) = u'_{k,j+1}(\tau_{j+1}) \), we have

\[
\begin{pmatrix}
\alpha_{j+1} \\
\beta_{j+1}
\end{pmatrix}
= \begin{pmatrix}
\mathcal{M}_{11} & \mathcal{M}_{12} \\
\mathcal{M}_{21} & \mathcal{M}_{22}
\end{pmatrix}
\begin{pmatrix}
\alpha_j \\
\beta_j
\end{pmatrix},
\]

with

\[
\mathcal{M}_{11} = \frac{i k \pi \sqrt{\bar{\tau}_{j+1,1} \bar{\tau}_{j+1,2}}}{4} \left[ H^{(1)}_{\nu_j}(k \bar{\tau}_{j+1,1}) H^{(2)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) - H^{(1)}_{\nu_j-1}(k \bar{\tau}_{j+1,1}) H^{(2)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) \right] + \left( \frac{2 \nu_j - 1}{2 k \bar{\tau}_{j+1,1}} - \frac{2 \nu_j + 1 - 1}{2 k \bar{\tau}_{j+1,2}} \right) H^{(1)}_{\nu_j}(k \bar{\tau}_{j+1,1}) H^{(2)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) ,
\]

\[
\mathcal{M}_{12} = \frac{i k \pi \sqrt{\bar{\tau}_{j+1,1} \bar{\tau}_{j+1,2}}}{4} \left[ H^{(2)}_{\nu_j}(k \bar{\tau}_{j+1,1}) H^{(2)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) - H^{(2)}_{\nu_j-1}(k \bar{\tau}_{j+1,1}) H^{(2)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) \right] + \left( \frac{2 \nu_j - 1}{2 k \bar{\tau}_{j+1,1}} - \frac{2 \nu_j + 1 - 1}{2 k \bar{\tau}_{j+1,2}} \right) H^{(2)}_{\nu_j}(k \bar{\tau}_{j+1,1}) H^{(2)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) ,
\]

\[
\mathcal{M}_{21} = -\frac{i k \pi \sqrt{\bar{\tau}_{j+1,1} \bar{\tau}_{j+1,2}}}{4} \left[ H^{(1)}_{\nu_j}(k \bar{\tau}_{j+1,1}) H^{(1)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) - H^{(1)}_{\nu_j-1}(k \bar{\tau}_{j+1,1}) H^{(1)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) \right] + \left( \frac{2 \nu_j - 1}{2 k \bar{\tau}_{j+1,1}} - \frac{2 \nu_j + 1 - 1}{2 k \bar{\tau}_{j+1,2}} \right) H^{(1)}_{\nu_j}(k \bar{\tau}_{j+1,1}) H^{(1)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) ,
\]

\[
\mathcal{M}_{22} = -\frac{i k \pi \sqrt{\bar{\tau}_{j+1,1} \bar{\tau}_{j+1,2}}}{4} \left[ H^{(2)}_{\nu_j}(k \bar{\tau}_{j+1,1}) H^{(1)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) - H^{(2)}_{\nu_j-1}(k \bar{\tau}_{j+1,1}) H^{(1)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) \right] + \left( \frac{2 \nu_j - 1}{2 k \bar{\tau}_{j+1,1}} - \frac{2 \nu_j + 1 - 1}{2 k \bar{\tau}_{j+1,2}} \right) H^{(2)}_{\nu_j}(k \bar{\tau}_{j+1,1}) H^{(1)}_{\nu_j+1}(k \bar{\tau}_{j+1,2}) ,
\]

where we have defined \( \bar{\tau}_{j+1,1} = \tau_{R,j} - \tau_{j+1} \) and \( \bar{\tau}_{j+1,2} = \tau_{R,j+1} - \tau_{j+1} \).

When \( k \bar{\tau}_{j+1,1} \) and \( k \bar{\tau}_{j+1,2} \gg 1 \), we have

\[
\mathcal{M}_{11} \approx (1 - i x) e^{i \theta_{j+1}} , \quad \mathcal{M}_{12} \approx (1 + i x) e^{-i \theta_{j+1}} , \quad \mathcal{M}_{21} \approx (1 + i x) e^{i \theta_{j+1}} , \quad \mathcal{M}_{22} \approx (1 + i x) e^{-i \theta_{j+1}} ,
\]

where \( x = \frac{1}{4 k \bar{\tau}_{j+1,1}} - \frac{1}{4 k \bar{\tau}_{j+1,2}} \), \( \theta_{j+1} = k \bar{\tau}_{j+1,1} - \bar{\tau}_{j+1,2} - \frac{\pi}{2} (\nu_j - \nu_{j+1}) \), \( \tilde{\theta}_{j+1} = k (\bar{\tau}_{j+1,1} + \bar{\tau}_{j+1,2} - \frac{\pi}{2} (\nu_j + \nu_{j+1}) ) \). Thus, considering the perturbation modes what we concern satisfy \( k \bar{\tau}_{j+1,1} \) and \( k \bar{\tau}_{j+1,2} \gg 1 \), we have

\[
\begin{pmatrix}
\alpha_{j+1} \\
\beta_{j+1}
\end{pmatrix}
\approx \begin{pmatrix}
 e^{i \theta_{j+1}} [1 + \mathcal{O}(\frac{1}{k \bar{\tau}_{j+1}})] \\
\mathcal{O}(\frac{1}{k \bar{\tau}_{j+1}}) e^{i \theta_{j+1}} [1 + \mathcal{O}(\frac{1}{k \bar{\tau}_{j+1}})]
\end{pmatrix}
\begin{pmatrix}
\alpha_j \\
\beta_j
\end{pmatrix},
\]

where \( \tilde{\tau} \) correspond to \( \bar{\tau}_{j+1,1} \) or \( \bar{\tau}_{j+1,2} \) and \( j \geq 0 \).

Define the past-infinite phase and the inflation as the phases \( j = 0 \) and \( j = i \), respectively, with Eq. (27), we straightforwardly find that

\[
\begin{pmatrix}
\alpha_i \\
\beta_i
\end{pmatrix}
\approx \begin{pmatrix}
 e^{i \sum_{j=1}^{i} \theta_j} [1 + \mathcal{O}(\frac{1}{k \bar{\tau}_{j}})] \\
\mathcal{O}(\frac{1}{k \bar{\tau}_{j}}) e^{-i \sum_{j=1}^{i} \theta_j} [1 + \mathcal{O}(\frac{1}{k \bar{\tau}_{j}})]
\end{pmatrix}
\begin{pmatrix}
\alpha_0 \\
\beta_0
\end{pmatrix},
\]
where
\[
\sum_{j=1}^{i} \theta_j = -\frac{\pi}{2}(\nu_0 - \nu_i) + k \sum_{j=1}^{i} (\tilde{\tau}_{j,1} - \tilde{\tau}_{j,2}),
\]
(29)
for the perturbation modes satisfying \(k \tilde{\tau} \gg 1\) (i.e., \(k \tilde{\tau}_{j,1} \gg 1\) and \(k \tilde{\tau}_{j,2} \gg 1\) for \(j = 1, 2, \ldots, i\)), which actually equals to requiring that the corresponding perturbation modes have not ever exited its horizon \(1/H_{\text{per}}\) in the pre-inflationary era. Therefore, for the perturbation modes with \(\lambda \ll 1/H_i\) at the beginning of inflation, \(\alpha_i(k)\) and \(\beta_i(k)\) acquire only some phase shift, compared with their initial values \(\alpha_0(k)\) and \(\beta_0(k)\) in the infinite past.

In the past infinity, if the universe is flat and Minkowskian, naturally the perturbation mode is in its Minkowski vacuum, i.e., \(|\alpha_0| = 1\) and \(\beta_0 = 0\). Hence,
\[
|\alpha_i| \approx 1 + \mathcal{O}\left(\frac{1}{k^{\tilde{\tau}}}\right), \quad |\beta_i| \approx \mathcal{O}\left(\frac{1}{k^{\tilde{\tau}}}\right)
\]
(30)
for \(k \tilde{\tau} \gg 1\). Therefore, at the beginning of inflation, the perturbation modes with the wavelengths \(\lambda \ll 1/H_i\) (the Hubble scale of inflation) behave themselves as they are in the BD state, which will be responsible for the nearly scale-invariant density perturbations. As for the modes with \(\lambda \gtrsim 1/H_i\), they are in the non-BD state at the beginning of inflation, hence possibly explain the power deficit of the CMB TT-spectrum at low multipoles and the dip at multipole \(l \sim 20\) (see e.g., [69] and references therein).

### B. Numerical simulation

Defining
\[
C(k) = \frac{k}{2} \left( u_k u_k^* + \frac{1}{k^2} u_k' u_k'^* \right) + \frac{1}{2}, \quad (\tau \approx \tau_i),
\]
(31)
we have
\[
C(k) = |\alpha_i|^2 + \mathcal{O}\left(\frac{1}{(-k/H_i)^2}\right) \simeq |\alpha_i|^2
\]
(32)
for \(\nu_i \approx 3/2\) and \(-k\tau_i \gg 1\), where \(H_i = aH = \tau_i\) and the Wronskian condition has been used. We will use \(|C(k)| = 1\) as the criterion to check whether the perturbation modes with \(\lambda \ll 1/H_i\) at the beginning of inflation are in the BD state.

We consider the bounce inflation model proposed in Ref. [69] (see Appendix A), where around the bounce we have \(Q_s^2 = c_s^2 = 1\) and
\[
Q_s = A_Q \left[ B - \tanh\left(\frac{t}{t_s}\right) \right]
\]
(33)
with constant $A_Q$ and $B$. As mentioned, in Eq. (2), $z = \sqrt{2a^2Q}$ depends not only on $a$ but also on $Q_s$. During the ekpyrotic contraction, $z''/z \approx 0/\tau^2$, since both $a$ and $Q_s$ are nearly constant. During the inflation, $z''/z \approx 2/\tau^2$, which is well-known. Around $t_*$, $Q_s$ has a rapid change, but $H \simeq H_b^2(t - t_b)$ around the bounce point $t_b$, hence

$$a \sim e^{H_b^2(t-t_b)^2/2} \simeq 1 + H_b^2(t-t_b)^2/2$$

(34)

is nearly constant for $t - t_b \ll 1/H_b$. As a result, we have $z''/z \approx -\left(\tau - \tau_R\right)^2/4$ with $\tau_R$ being some reference time. Therefore, all phases are actually consistent with (18). As expected, small deviation will not make any qualitative difference for our result.

With the background of bounce inflation in Ref. [69] (see Fig. 4 in Appendix A), we plot $|C(k)|$ in Fig. 3 while $A_Q = 3$ and $B = 2$. We can see that for sufficiently large $k$ (i.e., $k^2 \gg z''/z \approx a^2H_i^2$), we will have $|C(k)| = 1$. Thus the perturbation modes with wavelengths $\lambda \ll 1/H_i$ at the beginning of inflation will be naturally in the BD state.

![FIG. 3: $|C(k)| \rightarrow 1$ for $k \gg a_iH_i$. Thus, the perturbation modes with wavelengths much shorter than the Hubble scale at the beginning of inflation behave as they are in the BD vacuum.](image)

C. Trans-Planckian effect

So far we have neglected the higher-order momentum corrections in (16). However, it is interesting to see what will happen if they are not negligible.
Generally, with (16), we have the solution to Eq. (2) as

$$u_{k,j}(\tau) = \sqrt{\pi/(\tau_{R,j} - \tau)} \left\{ \alpha_j H^{(1)}_{\nu_j} \left[ C_{s,j} k (\tau_{R,j} - \tau) \right] + \beta_j H^{(2)}_{\nu_j} \left[ C_{s,j} k (\tau_{R,j} - \tau) \right] \right\}, \quad (\tau < \tau_{j+1}). \quad (35)$$

By matching phase $j$ with phase $j + 1$, we can get (21) with

$$\mathcal{M}_{11} = \frac{ik\pi}{4} \sqrt{\tau_{j+1,1} \tau_{j+1,2}} \left[ \frac{2\nu_j - 1}{2k\tau_{j+1,1}} - \frac{2\nu_j + 1 - 1}{2k\tau_{j+1,2}} \right] H_{\nu_j}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(2)}(C_{s,j+1} k \tilde{\tau}_{j+1,2})$$

$$+ C_{s,j+1} H_{\nu_{j+1}}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(2)}(C_{s,j+1} k \tilde{\tau}_{j+1,2}) - C_{s,j} H_{\nu_{j+1}}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(2)}(C_{s,j+1} k \tilde{\tau}_{j+1,2}), \quad (36)$$

$$\mathcal{M}_{12} = \frac{ik\pi}{4} \sqrt{\tau_{j+1,1} \tau_{j+1,2}} \left[ \frac{2\nu_j - 1}{2k\tau_{j+1,1}} - \frac{2\nu_j + 1 - 1}{2k\tau_{j+1,2}} \right] H_{\nu_j}^{(2)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(2)}(C_{s,j+1} k \tilde{\tau}_{j+1,2})$$

$$+ C_{s,j+1} H_{\nu_{j+1}}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(2)}(C_{s,j+1} k \tilde{\tau}_{j+1,2}) - C_{s,j} H_{\nu_{j+1}}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(2)}(C_{s,j+1} k \tilde{\tau}_{j+1,2}), \quad (37)$$

$$\mathcal{M}_{21} = -\frac{ik\pi}{4} \sqrt{\tau_{j+1,1} \tau_{j+1,2}} \left[ \frac{2\nu_j - 1}{2k\tau_{j+1,1}} - \frac{2\nu_j + 1 - 1}{2k\tau_{j+1,2}} \right] H_{\nu_j}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(1)}(C_{s,j+1} k \tilde{\tau}_{j+1,2})$$

$$+ C_{s,j+1} H_{\nu_{j+1}}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(1)}(C_{s,j+1} k \tilde{\tau}_{j+1,2}) - C_{s,j} H_{\nu_{j+1}}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(1)}(C_{s,j+1} k \tilde{\tau}_{j+1,2}), \quad (38)$$

$$\mathcal{M}_{22} = -\frac{ik\pi}{4} \sqrt{\tau_{j+1,1} \tau_{j+1,2}} \left[ \frac{2\nu_j - 1}{2k\tau_{j+1,1}} - \frac{2\nu_j + 1 - 1}{2k\tau_{j+1,2}} \right] H_{\nu_j}^{(2)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(1)}(C_{s,j+1} k \tilde{\tau}_{j+1,2})$$

$$+ C_{s,j+1} H_{\nu_{j+1}}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(1)}(C_{s,j+1} k \tilde{\tau}_{j+1,2}) - C_{s,j} H_{\nu_{j+1}}^{(1)}(C_{s,j} k \tilde{\tau}_{j+1,1}) H_{\nu_{j+1}}^{(1)}(C_{s,j+1} k \tilde{\tau}_{j+1,2}). \quad (39)$$

Thus for the perturbation modes satisfying $k\tilde{\tau}_{j+1,1}$ and $k\tilde{\tau}_{j+1,2} \gg 1$, we have

$$\begin{pmatrix} \alpha_{j+1} \\ \beta_{j+1} \end{pmatrix} \approx \begin{pmatrix} e^{i\theta_{j+1}} \frac{C_{s,j} + C_{s,j+1} + O(1/\tau)}{2\sqrt{C_{s,j}C_{s,j+1}}} & -e^{-i\theta_{j+1}} \frac{i(C_{s,j} - C_{s,j+1}) + O(1/\tau)}{2\sqrt{C_{s,j}C_{s,j+1}}} \\ e^{i\bar{\theta}_{j+1}} \frac{i(C_{s,j} - C_{s,j+1}) + O(1/\tau)}{2\sqrt{C_{s,j}C_{s,j+1}}} & e^{-i\bar{\theta}_{j+1}} \frac{i(C_{s,j} + C_{s,j+1} + O(1/\tau))}{2\sqrt{C_{s,j}C_{s,j+1}}} \end{pmatrix} \begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix}, \quad (40)$$

which reduces to (27) when $C_s^2 = 1$, where $\theta_{j+1} = k(C_{s,j+1} \tilde{\tau}_{j+1,1} - C_{s,j+1} \tilde{\tau}_{j+1,2}) - \frac{\pi}{2}(\nu_j - \nu_{j+1})$ and $\bar{\theta}_{j+1} = k(C_{s,j+1} \tilde{\tau}_{j+1,1} + C_{s,j+1} \tilde{\tau}_{j+1,2}) - \frac{\pi}{2}(\nu_j + \nu_{j+1})$.

Consider such a pre-inflationary era, in which $C_s^2 = (k/\Lambda)^{2p} \gg 1$ for the phase $j$, while $C_s^2 = 1$ in other phases. According to (40), we straightforwardly have

$$\begin{pmatrix} \alpha_{j+1} \\ \beta_{j+1} \end{pmatrix} \approx \frac{k^p}{2\Lambda^p} \begin{pmatrix} e^{i\theta_{j+1}} & -ie^{-i\bar{\theta}_{j+1}} \\ ie^{i\bar{\theta}_{j+1}} & e^{-i\theta_{j+1}} \end{pmatrix} \begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix}. \quad (41)$$
This suggests that even if we have $|\alpha_0| = 1$ and $\beta_0 = 0$ initially or in the infinite past, both $\alpha_i$ and $\beta_i$ will be altered significantly as

$$|\alpha_i| \sim \frac{k^p}{2\Lambda^p}, \quad |\beta_i| \sim \frac{k^p}{2\Lambda^p}$$

(42)

for $k\tilde{\tau} \gg 1$, after passing through one Planckian regime ($k \gg \Lambda$). Thus in this case, the initial state of perturbation modes on sub-horizon scale will be no longer (actually far away from) the BD state at the beginning of inflation. This is actually equivalent to the “trans-Planckian problem” of inflation. The initial state of perturbation modes at the beginning of inflation being the BD state requires that the wavelengths of the perturbation modes (at the CMB window) must satisfy $\lambda \gg 1/\Lambda$ at any moment in the pre-inflationary era, so that the higher-order spatial derivative operators may be neglected. This is consistent with the condition of TCC. In certain sense, such a pre-inflationary era should be past-complete, otherwise the phase with $\lambda < 1/\Lambda$ is inevitable.

V. CONCLUSION

Due to the past-incompleteness of inflation, the pre-inflationary physics must be considered to set the initial states for the inflationary perturbations. We discussed the implication of TCC for the initial states of the primordial perturbations, and pointed out that it is possible to consistently set the initial state of perturbation modes in the infinite past, only if a past-complete pre-inflationary era exists.

In the past-complete pre-inflationary era, the spacetime may be Minkowskian in the infinite past. Hence, it is natural to pick out the Minkowski vacuum state as the initial state of perturbations. Moreover, in such a scenario, the sub-Planckian perturbation modes can never cross their horizon, which is also consistent with the TCC. We calculate the evolution of the perturbation modes in such a pre-inflationary era, and show that the perturbation modes will behave as they are in the BD state at the beginning of inflation, as long as their wavelengths $\lambda \ll 1/H$ at that time. Therefore, a past-complete pre-inflationary evolution may automatically prepare the initial state required for the inflationary perturbations.

The past-complete pre-inflationary era might have left the imprint on CMB. One possibility of such scenarios is the bounce inflation [18–20]. As showed recently in [69], the pre-inflationary bounce might be responsible for the power deficit of the CMB TT-spectrum.
at low multipoles and the dip at \( l \sim 20 \). Relevant issue is interesting for further study. In addition, it is also possible that the universe, as well as the initial state of perturbation modes, was created quantumly \([70, 71]\). In such a scenario, how the perturbation is in its lowest-energy state after the creation is still under study, see recent Refs. \([72–75]\).

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**Appendix A: Bounce inflation model**

The stable nonsingular bounce inflation model has been implemented in the beyond Horndeski theory, as inspired by the EFT of nonsingular cosmologies \([25–29]\). In this Appendix, we briefly review it, see \([69]\) for the details.

Our effective Lagrangian for the bounce inflation is

\[
L \sim \frac{M_p^2}{2} R - \frac{M_p^2}{2} X - V(\phi) + \frac{\tilde{P}(\phi, X)}{\text{Contraction + Inflation}} + \frac{L_{\delta g^{(3)}}}{\text{(Ghost free) Bounce Removing } c_s^2 < 0} + L_{\delta K \delta g^{(0)}},
\]

where

\[
L_{\delta g^{(3)}} = \frac{f_1(\phi)}{2} \delta g^{(0)} R^{(3)}
\]

\[
= \frac{f}{2} R - \frac{X}{2} \int f \phi d \ln X - \left( f \phi + \int f \phi d \ln X \right) \Box \phi
\]

\[
+ \frac{f}{2X} \left[ \phi \mu \nu \phi \rho \sigma - \Box \phi \right] - f\frac{2XfX}{X^2} \left[ \phi \mu \nu \phi \rho \sigma \phi \nu - \Box \phi \phi \mu \nu \phi \rho \sigma \right] ,
\]

\[
L_{\delta K \delta g^{(0)}} = \frac{g_1(\phi)}{2} \delta K \delta g^{(0)}
\]

\[
= \frac{g}{2 \sqrt{-X}} \left( \frac{\phi \mu \nu \phi \rho \sigma}{X} - \Box \phi \right) - \frac{3}{2} g \cdot h ,
\]

\( R^{(3)} \delta g^{(0)} \) and \( \delta K \delta g^{(0)} \) are the EFT operators which will modify \( c_s^2 \) and \( Q_s \) in Eq. (2), \( \phi_\mu = \nabla_\mu \phi, \; \phi_G = \nabla^\mu \phi, \; \phi_{\mu \nu} = \nabla_\nu \nabla_\mu \phi, \; X = \phi_\mu \phi^\mu \) and \( \Box \phi = \phi_G^\mu \). Here, the coefficients \( f \) and \( g \)
depend on $\phi$ and $X$, while $h$ and $f_2$ depend only on $\phi$. In certain conditions (see [69] for details), $f$, $g$ and $h$ can be set to be vanishing at the background level.

As showed in Ref. [69], with the operators $R^{(3)} \delta g^{00}$ and $\delta K \delta g^{00}$, we can have $c_s^2 = 1$ and $Q_s$ satisfying Eq. (33). In [69], the background solution of bounce inflation has been displayed. Here, we only show the evolution of the background (the Hubble parameter $H$) in Fig. 4, which will be used in Sec. IV.

![Hubble parameter evolution](image)

FIG. 4: The evolutions of the Hubble parameter $H$ in bounce inflation model, where we have set $M_p = 1$.

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