Bottleneck transportation problem with a fuzzy random constraint

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Abstract

This paper considers a multi-objective bottleneck transportation problem with fuzzy random constraint about transportation time and chance constraint about the total cost. There exist m supply points with flexible supply quantity and n demand points with flexible demand quantity. For each route, that is, each pair of a supply and a demand, the transportation time is an independent random variable according to a normal distribution with an uncertain mean denoted by an L-fuzzy number and a crisp variance. Further for each route, existence possibility denoting the preference choosing the route is attached. Satisfaction degrees about the supply and demand quantity are attached to each supply and demand point, respectively. These satisfaction degrees are denoted by membership functions of corresponding fuzzy sets. Moreover transportation cost on each route is a random variable and these random variables are assumed to be independent each other. We consider four criteria, given as follows.

(1) Minimize the transportation time target under the possibility that the satisfaction probability with respect to transportation time below the target is over a certain threshold should be over a given level. (2) Maximize the minimal preference among used routes in transportation. (3) Maximize the minimal satisfaction degree among all supply and demand points. (4) Minimizing the budget under the chance constraint, the probability that the total cost is below the budget should be over a certain level.

Above setting, we formulate the model with three objective and fuzzy random constraint with respect to random transportation time of each route. This is a multi-objective bottleneck transportation problem. First this problem is transformed into deterministic equivalent problem with four objectives. Usually an optimal transportation pattern optimizing four objectives at a time and so we propose solution algorithm to seek some non-dominated transportation pattern after non-domination with introducing threshold about preference of route. Finally, we summarize the result of the paper and discuss future research problems.

Key words: Preference of route, Flexible supply and demand quantity, Fuzzy random transportation time of route, Fuzzy random constraint, Non-dominated transportation patterns.

1. Introduction

This paper considers a multi-objective bottleneck transportation problem with fuzzy random constraint about transportation time as an extension of our previous papers ([1-3]). Since big earthquake occurred on 3.11 2011 at Tohoku, Japan, how to deliver necessary goods to accident places as quick as possible by the reliable routes becomes very important. We consider the transportation model to be applicable to these types of accidents in n t his paper. There exist huge amount of research on various types of transportation problems and its solution procedures such as [4-7]. Basic stochastic version of usual transportation problem are appeared in [8-13]. The bottleneck type transportation problem, that is, objective function is not a sum of transportation cost but a bottleneck route with respect to transportation is considered in [14]. While time minimization transportation problems minimizing maximum delivery time of all transported route are considered in [15], [16]. By dividing demand points into some groups, competitive transportation problem is considered in [17] since in some cases transportation costs are covered by some groups and each group of demands minimizes corresponding transportation total cost.

This paper is organized as follows. Section 2 formulates our models and transforms our model to deterministic equivalent problem with four objectives. Usually an optimal transportation pattern optimizing four objectives at a time and so we propose solution algorithm to seek some non-dominated transportation patterns after non-domination with introducing threshold about preference of route. Finally, we summarize the result of the paper and discuss future research problems.

Key words: Preference of route, Flexible supply and demand quantity, Fuzzy random transportation time of route, Fuzzy random constraint, Non-dominated transportation patterns.

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2. Problem formulation

We consider the following model:

(1) There exist a set of $m$ supply points $S = (S_1, S_2, \ldots, S_m)$ and a set of $n$ demand points $T = (T_1, T_2, \ldots, T_n)$. Define set $A$ as a set of routes connecting each supply point $S_i$ to each demand point $T_j$. Let $s_i, t_j$ be the flow value from $S_i$ and to $T_j$, respectively, and non-negative integers. Different to the usual transportation problem, upper limit of supply quantity for each supply point and lower limit of demand quantity for each demand point are flexible. They are expressed by the following two kinds of membership functions $\mu_{S_i}(s_i), \mu_{T_j}(t_j)$ for fuzzy supply quantity from $S_i$ and fuzzy demand quantity to $T_j$, respectively, which characterize the satisfaction degrees of supply points and demand points.

$$
\mu_{S_i}(s_i) = \begin{cases} 
1 & (s_i \leq a_i) \\
\frac{b_i - s_i}{b_i - a_i} & (a_i < s_i < b_i) \\
0 & (s_i \geq b_i)
\end{cases}
$$

$$
\mu_{T_j}(t_j) = \begin{cases} 
0 & (t_j \leq d_j) \\
\frac{t_j - d_j}{e_j - d_j} & (d_j < t_j < e_j) \\
1 & (t_j \geq e_j)
\end{cases}
$$

where $a_i < b_i, d_j < e_j$ and $a_i, b_i, d_j, e_j$ are positive integers. We assume that

$$
\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} e_j
$$
since otherwise our problem becomes trivial, that is, all values of membership functions can be set to 1; besides, we assume that

$$
\sum_{i=1}^{m} b_i > \sum_{j=1}^{n} d_j
$$

Otherwise the value of the second objective of our Problem is 0. We denote the transportation quantity using the route $(i, j)$ by $f_{ij}$, and assume that these $f_{ij}$ are nonnegative integers and decision variables. Note that

$$
s_i = \sum_{j=1}^{n} f_{ij}, i = 1, 2, \ldots, m, t_j = \sum_{i=1}^{m} f_{ij}, j = 1, 2, \ldots, n
$$

Then we set $(f_{ij})$ to maximize

Min $\{\mu_{S_i}(s_i), \mu_{T_j}(t_j), i = 1, 2, \ldots, m, j = 1, 2, \ldots, n\}$.

(2) For each route $(i, j)$, the transportation time $t_{ij}$ is an independent random variable according to a normal distribution $N(M_{ij}, \sigma_{ij}^2)$ with mean $M_{ij}$ and variance $\sigma_{ij}^2$. The following chance constraint is attached:

$$
\Pr[t_{ij} \leq f] \geq \alpha
$$

where $\alpha > 1/2$ and $f$ is also a decision variable to be minimized. Further we assume that mean $M_{ij}$ is an $L$-fuzzy number with membership function $\mu_{M_{ij}}(t)$ as the following shape function $L\left(\frac{t - m_{ij}}{\alpha_{ij}}\right)$ where

$$
L(t) = L(-t), L(0) = 1, L(t) = 0 (t \geq t_0)
$$

and $L(t)$ is non-increasing for $t \geq 0$. If $L(t)$, $m_{ij}, \alpha_{ij}$ is fixed, $\mu_{M_{ij}}(t)$ is denoted as $(m_{ij}, \alpha_{ij})$. For this case, we extend the chance constraint (2.1) to possibility that the above chance constraint (2.1) holds should be less than $\beta$; that is, the possibility constraint

$$
\Pr[\Pr[t_{ij} \leq f] \geq \alpha] \geq \beta
$$

for every $f_{ij} > 0$ $(0 < \alpha \leq 1)$ and it is transformed as follows:

$$
\Pr[\Pr[t_{ij} \leq f] \geq \alpha] \geq \beta \Leftrightarrow \Pr[f \geq M_{ij} + K_\alpha \sigma_{ij}] \geq \beta
$$

since

$$
\Pr[t_{ij} \leq f] \geq \alpha \Leftrightarrow \Pr\left\{\frac{t_{ij} - M_{ij}}{\sigma_{ij}} \leq \frac{f - M_{ij}}{\sigma_{ij}}\right\} \geq \alpha,
$$

$t_{ij} - M_{ij}$ is the random variable according to the standard normal distribution and so

$$
\Pr\left\{\frac{t_{ij} - M_{ij}}{\sigma_{ij}} \leq \frac{f - M_{ij}}{\sigma_{ij}}\right\} \geq \alpha \Rightarrow f - M_{ij} \geq K_\alpha
$$

$K_\alpha$ is the $\alpha$ percentile point of cumulative distribution with respect to the standard normal distribution $N(0, 1)$. Further

$$
\Pr[f \geq M_{ij} + K_\alpha \sigma_{ij}] \geq \beta \Leftrightarrow m_{ij} - L^*(\beta)\alpha_{ij} \leq f - \sigma_{ij} \Leftrightarrow m_{ij} - L^*(\beta)\alpha_{ij} + K_\alpha \sigma_{ij} \leq f
$$

and so we set

$$
f = \max \{m_{ij} - L^*(\beta)\alpha_{ij} + K_\alpha \sigma_{ij} \mid f_{ij} > 0\}
$$

where $L^*(\beta)$ is a pseudo inverse of $L(\cdot)$, that is,

$$
L^*(\beta) = \sup \{t \mid L(t) \geq \beta\}.
$$

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(3) Preference of the route is attached and it is denoted by \( \mu_i \in (0,1] \). It reflects on the satisfaction degree using this route. Then we should maximize \( \min \{ \mu_i \} \).

(4) For each route \((i,j)\), we assume that cost \( c_{ij} \) for unit transportation of a good is a random variable according to the normal distribution \( N(u_{ij}, v_{ij}^2) \) and they are independent each other. Then to tal transportation cost becomes a random variables according to the normal distribution \( N(\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} f_{ij}, \sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij}^2 f_{ij}^2) \). Then we consider the following chance constraint (2.2) about budget \( F \) of the transportation cost

That is, probability that total transportation cost is not over the budget should be not less the fixed level \( \gamma \).

\[
\Pr \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} f_{ij} \leq F \right\} \geq \gamma \quad (2.2)
\]

\[
\Rightarrow \Pr \left\{ \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} f_{ij} - \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} f_{ij}}{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij}^2 f_{ij}^2}} \leq \frac{F - \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} f_{ij}}{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij}^2 f_{ij}^2}} \geq K_{\gamma} \right\}
\]

\[
\Rightarrow F \geq \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} f_{ij} + K_{\gamma} \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij}^2 f_{ij}^2}
\]

where \( \frac{1}{2} \leq \gamma < 1 \), \( K_{\gamma} \) is the \( \gamma \) percentile point of the cumulative distribution with respect to the standard normal distribution. Then since the budget should be minimized, we set \( F = \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} f_{ij} + K_{\gamma} \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij}^2 f_{ij}^2} \) and it should be minimized.

(5) From above setting from (1)-(4), we consider the following problem \( P_1 \):

\[ P_1 \text{ : maximize } \left[ \min \{ \mu_i \} \right]_{i=1}^{n} \]

minimize \( \max \left\{ m_y - L^* (\beta) \alpha_y + K_u \sigma_y \right\} \mid f_{ij} > 0 \}

maximize \( \min \{ \mu_i \} \mid f_{ij} > 0 \}

minimizes \( \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} f_{ij} + K_{\gamma} \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij}^2 f_{ij}^2} \)

subject to \( f_{ij} \geq 0 \), integer, \( i=1,2,..,n \), \( j=1,2,..,n \), \( s_j = \sum_{j=1}^{n} f_{ij}, i=1,2,..,n \), \( t_j = \sum_{i=1}^{n} f_{ij} \), \( s_j \geq t_j \)

Note that \( P_1 \) is a multi-objective optimization problem with four criteria. Usually there exist infeasible solution optimizing four criteria at a time and so we seek some no-dominated solution after defining non-dominations. Let \( v(f) = (v^1(f), v^2(f), v^3(f), v^4(f)) \) be a vector with 4 components

\[
v^1(f) = \min \{ \mu_i \} \mid f_{ij} > 0 \}
\]

\[
v^2(f) = \max \left\{ m_y - L^* (\beta) \alpha_y + K_u \sigma_y \right\} \mid f_{ij} > 0 \}
\]

\[
v^3(f) = \min \{ \mu_i \} \mid f_{ij} > 0 \}
\]

\[
v^4(f) = \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} f_{ij} + K_{\gamma} \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij}^2 f_{ij}^2}
\]

where \( ^t \) denote transpose. The \( n \) we define fine non-dominated solution.

(Non-dominated solution)

For transportation flow patterns \( f^1 = (f_{ij}^1), f^2 = (f_{ij}^2) \), if

\[
v^1(f^1) \geq v^1(f^2), v^2(f^1) \leq v^2(f^2), v^3(f^1) \geq v^3(f^2), v^4(f^1) \leq v^4(f^2),
\]

then we call \( f^1 \) dominates \( f^2 \). If there exists no transportation flow pattern dominates, we call \( f \) is a non-dominated transportation flow pattern or simply \( n \)-on-dominated flow.

3. Solution Procedure

In order to find non-dominated flows, we set thresholds with respect to

\[
v^1(f) = \min \{ \mu_i \} \mid f_{ij} > 0 \}
\]

\[
v^2(f) = \max \left\{ m_y - L^* (\beta) \alpha_y + K_u \sigma_y \right\} \mid f_{ij} > 0 \}
\]

as follows \( v^1(f) \leq l' \) and \( v^2(f) \geq l' \). the delivery time to the accident place should be less than a limit time \( l' \) and safety limit \( l' \) of delivery should be assured over a fixed level. We seek some non-dominated flows among transportation flow patterns satisfying thresholds since safety limit is very important for the survival. Another reason is too complicated and time consuming to seek all

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non-dominated solutions. That is, we use only routes belonging to a set
\[ RT(l', l) = \{(i, j) \mid m_y - L^* (\beta \alpha_y + K_u \sigma_y) \leq l', \mu_y \geq l', i = 1, 2, \ldots, m, j = 1, 2, \ldots, n\} \]
Further
\[ RS(j) = \{(i) \mid m_y - L^* (\beta \alpha_y + K_u \sigma_y) \leq l', \mu_y \geq l', i = 1, 2, \ldots, m\} \]
for each \( j \) with some \( i \) such that \((i, j) \in RT(l', l)\)
\[ RT(i) = \{(j) \mid m_y - L^* (\beta \alpha_y + K_u \sigma_y) \leq l', \mu_y \geq l', j = 1, 2, \ldots, n\} \]
for each \( i \) with some \( j \) such that \((i, j) \in RT(l', l)\)
where \( RS(j) \neq \phi, j = 1, 2, \ldots, n, RT(i) \neq \phi, i = 1, 2, \ldots, m \)
without loss of generality.
As for \( v^1(f) \), since \( s_i, t_j \) are nonnegative integers, we can denote ranges of \( \mu_S (s_i) \) and \( \mu_T (t_j) \) with \( \{\mu_S, 1, \ldots, \mu_S, k, \ldots \} \) and \( \{\mu_T, 1, \ldots, \mu_T, j, \ldots \} \), respectively, \( i = 1, \ldots, m, j = 1, \ldots, n \). Now sorting them, let the result be
\[ 0 < \mu^1 < \cdots < \mu^k \leq 1 \] with \( g \) the number of different values. For fixed \( k \in \{1, \ldots, g\} \), we only consider
\[ \mu_S (s_i) \geq \mu^k, \mu_T (t_j) \geq \mu^k \Leftrightarrow s_i \leq b_i - \mu^k (b_i - a_i), t_j \geq d_j + \mu^k (e_j - d_j), i = 1, \ldots, m, j = 1, \ldots, n \]
As it is easily seen, the total supply quantity should not be less than the total demand quantity since otherwise, the problem becomes feasible. So we assume
\[ \sum_{i=1}^{m} s_i - \mu^k (b_i - a_i) \geq \sum_{j=1}^{n} t_j - \mu^k (e_j - d_j) \]
\[ 
\Rightarrow \mu^k \leq \frac{\sum_{i=1}^{m} b_i - \sum_{j=1}^{n} d_j}{\sum_{i=1}^{m} (b_i - a_i) + \sum_{j=1}^{n} (e_j - d_j)} = \bar{\mu}. 
\]
Denote \( k_0 = \operatorname{arg}\{u \in \{1, \ldots, g\} \mid \mu^u \leq \bar{\mu} < \mu^{u+1}\} \), then we set for \( k^u, k = 1, 2, \ldots, k_0 \),
\[ s_i^k = \left[b_i - \mu^k (b_i - a_i)\right], t_j^k = \left[d_j + \mu^k (e_j - d_j)\right] \]
where \( \lceil x \rceil \) is the greatest integer not over \( x \) and \( \lfloor x \rfloor \) the least integer not less than \( x \). Note that this setting satisfies the feasibility condition \( \sum_{i=1}^{m} s_i^k \geq \sum_{j=1}^{n} t_j^k \), and consider the following sub-problem \( P_k (l', l) \):
\[ P_k (l', l) : \]
\[ \text{minimize} \quad \sum_{(i, j) \in RT(l', l')} u_{ij} f_{ij} + K_f \sqrt{\sum_{(i, j) \in RT(l', l')} v_{ij}^2 f_{ij}^2} \]
\[ \text{subject to} \quad f_{ij} \geq 0, \text{integer}, (i, j) \in RT(l', l'), \sum_{(i, j) \in RT(l')} f_{ij} \leq s_i^k, i = 1, 2, \ldots, m, \sum_{(i, j) \in RS(j)} f_{ij} \geq t_j^k, j = 1, 2, \ldots, n, \]
Based on [3], we consider how to solve \( P_k (l', l) \). In order to solve \( P_k (l', l) \), we introduce the following auxiliary problem \( P_k^* (l', l) \) with positive parameter \( R \):
\[ P_k^* (l', l) : \]
\[ \text{minimize} \quad \sum_{(i, j) \in RT(l', l')} u_{ij} f_{ij} + K_f \sum_{(i, j) \in RT(l', l')} v_{ij}^2 f_{ij}^2 \]
\[ \text{subject to} \quad f_{ij} \geq 0, \text{integer}, (i, j) \in RT(l', l'), \sum_{(i, j) \in RT(l')} f_{ij} \leq s_i^k, i = 1, 2, \ldots, m, \sum_{(i, j) \in RS(j)} f_{ij} \geq t_j^k, j = 1, 2, \ldots, n \]
Then we have the following relation between \( P_k (l', l) \) and \( P_k^* (l', l) \) as is easily shown by the very similar manner in [18].

**Theorem 1.** An optimal transportation flow pattern \( f (R) = (f_{ij} (R)) \) for \( P_k^* (l', l) \) is also optimal for \( P_k (l', l) \) if
\[ R = 2 \sqrt{\sum_{(i, j) \in RT(l', l')} v_{ij}^2 f_{ij}^2 (R)} \] Further for \( R^* = 2 \sqrt{\sum_{(i, j) \in RT(l', l')} v_{ij}^2 f_{ij}^2 (R)} \) corresponding to the optimal transportation flow pattern of \( P_k^* (l', l) \), following relation holds with respect to \( R \):
\[ R > 2 \sqrt{\sum_{(i, j) \in RT(l', l')} v_{ij}^2 f_{ij}^2 (R)} \Leftrightarrow R < R^* \]
\[ R < 2 \sqrt{\sum_{(i, j) \in RT(l', l')} v_{ij}^2 f_{ij}^2 (R)} \Leftrightarrow R > R^* \]
So in order to find an optimal transportation flow pattern for \( P_k (l', l) \), we consider optimal transportation flow patterns of \( P_k^* (l', l) \) by changing parameter \( R \) by a binary method. Since the objective function \( R \sum_{(i, j) \in RT(l', l')} u_{ij} f_{ij} + K_f \sum_{(i, j) \in RT(l', l')} v_{ij}^2 f_{ij}^2 \) of \( P_k^* (l', l) \) is convex as is easily shown and \( f_{ij} \) is nonnegative integer, \( P_k^* (l', l) \) is equivalent to the following problem \( P_k^* (l', l) \):
\[ P_k^* (l', l) : \text{minimize} \quad \sum_{(i, j) \in RT(l', l')} C_{ij}^R f_{ij} \]
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\[ P_{k}^{f}(l', l^{'}) : \text{minimize } \left( \sum_{(i,j) \in RT(l', l^{'})} C_{ij}^{f} f_{ij} \right) \]

subject to \( f_{ij} \geq 0, \text{integer } (i, j) \in RT(l', l^{'}) \),

\[ \sum_{(i,j) \in RT(l', l^{'})} f_{ij} \leq s_{i}, \ i=1,2,\ldots,m, \]

\[ \sum_{(i,j) \in RT(l', l^{'})} f_{ij} \geq t_{j}^{l}, 1,2,\ldots,n \]

where \( C_{ij}^{f}(f_{ij}) \) is defined as follows:

\[ C_{ij}^{f}(f_{ij}) = (R_{ij}q + K_{ij}y_{ij}^{2}) + (R_{ij} + K_{ij}y_{ij}^{2})/2 \]

Note that \( C_{ij}^{f}(f_{ij}) \) is a piecewise linear convex function. In order to solve \( P_{k}^{f}(l', l^{'}) \), we consider the following problem \( P_{k}^{f}(l', l^{'}) \) given as follows where the integer condition is relaxed from \( P_{k}^{f}(l', l^{'}) \):

\[ \tilde{P}_{k}^{f}(l', l^{'}) : \text{minimize } \left( \sum_{(i,j) \in RT(l', l^{'})} C_{ij}^{f} f_{ij} \right) \]

subject to \( f_{ij} \geq 0, (i, j) \in RT(l', l^{'}) \),

\[ \sum_{(i,j) \in RT(l', l^{'})} f_{ij} \leq s_{i}, i=1,2,\ldots,m, \]

\[ \sum_{(i,j) \in RT(l', l^{'})} f_{ij} \geq t_{j}^{l}, 1,2,\ldots,n \]

\( \tilde{P}_{k}^{f}(l', l^{'}) \) can be solved by using the algorithm described in [3]. Let a non-optimal transportation flow pattern of \( \tilde{P}_{k}^{f}(l', l^{'}) \) be \( \tilde{f}(R) = (\tilde{f}_{ij}(R)) \). Then we have the following theorem.

**Theorem 2.** \( \tilde{f}(R) = (\tilde{f}_{ij}(R)) \) becomes an integer transportation flow pattern and so it is an optimal solution of \( \tilde{P}_{k}^{f}(l', l^{'}) \).

**Proof:** It can be shown by the very similar manner to Theorem 3.2 of [3]. Let a non-optimal transportation flow pattern \( \tilde{f}(R) = (\tilde{f}_{ij}(R)) \) of \( P_{k}^{f}(l', l^{'}) \). Algorithm 1 is a method to find some non-dominated transportation flow patterns.

**Algorithm 1**

Step 1: Set \( k = 1, DS = \phi \) and go to step 2.

Step 2: Solve \( P_{k}^{f}(l', l^{'}) \) and find an optimal transportation flow pattern \( f_{ij}^{k}(l', l^{'}) = (f_{ij}^{k}(l', l^{'})^{'}) \) of \( P_{k}^{f}(l', l^{'}) \). Go to step 3.

Step 3: Check whether \( f_{ij}^{k}(l', l^{'}) = (f_{ij}^{k}(l', l^{'})^{'}) \) dominates transportation flow patterns among \( DS \) or not. If so, then delete dominated one from \( DS \) and update \( DS \). Then, check \( f_{ij}^{k}(l', l^{'}) = (f_{ij}^{k}(l', l^{'})^{'}) \) is dominated by some member among \( DS \) or not. If it is not dominated, update \( DS \) by adding \( \tilde{f}_{ij}^{k}(l', l^{'}) = (\tilde{f}_{ij}^{k}(l', l^{'})^{'}) \) and go to step 4. Otherwise, that is, if \( DS \) is dominated, go to step 4 directly.

Step 4: Set \( k = k + 1 \). If \( k = k_{0} \), then terminate (current \( DS \) is a set of some non-dominated transportation flow patterns and it is denoted with \( DS(l', l^{'}) \)). Otherwise, that is, if \( k < k_{0} \), return to Step 2.

Computational order of algorithm 1 depends on that of algorithm of [19] mainly. Note that the algorithm of [19] is polynomial and so applying this algorithm to solve \( P_{k}^{f}(l', l^{'}) \) results at most \( O(\min \{ \sum_{i=1}^{n} k_{i} \} ) \) complexity.

Therefore complexity of algorithm 1 is due to changing number of \( R \) greatly.

Now we seek other non-dominated flow pattern by changing the threshold \( l' \) though in the above discussion it is fixed. Sorting \( \tau_{ij} \equiv m_{i}/L_{i}(\beta_{ij}+\mu)_{ij} \) in descending order for all \( (i,j), l=1,2,\ldots,m, j=1,2,\ldots,n \), let the result be \( \tau_{i} > \tau_{2} > \cdots > \tau_{r} \) where \( r \) is the number of different \( \tau_{ij} \). The we see that \( l' = \tau_{r} \), \( w=1,2,\ldots,\tau \) sequentially. Some non-dominated transportation flow patterns of \( P_{k}^{f}(l', l^{'}) \) using algorithm 1 and obtain some non-dominated transportation flow patterns where \( F = \min \{ \lambda_{i}^{l'} l_{i}, RT l_{i} = \phi \} \) and or \( j=\lambda_{i}^{l'} l_{i} \), \( RS(j) = \phi_{i}, (i,j) \in RT(\tau^{l'} l', l') \) \( \omega_{i} l_{i} \). That is, we have following algorithm 2.

**Algorithm 2**

Step 1: Set \( \lambda = 1, l' = \tau^{l'} l', DT = \phi \) and go to step 2.

Step 2: Solve \( P_{l}^{f}(l', l^{'}) \), \( k = 1,2,\ldots,k_{0} \) using algorithm 1 and find \( DS(l', l^{'}) \). Go to step 3.

Step 3: Set \( DT \leftarrow DS(l', l^{'}) \cup DT \) and update by deleting some dominated ones from \( DT \). Go to step 4.

Step 4: Set \( \lambda = \lambda + 1 \). If \( \lambda = \tau + 1 \), then terminate (current \( DT \) is set of some non-dominated transportation flow patterns). Otherwise set \( \lambda = \tau + 1 \) and return to Step 2.

4. Conclusion

We have proposed multi-objective transportation problem with four criteria and a solution procedure to find some non-dominated transportation flow patterns. Since threshold is introduced to minimal satisfaction of used route, only three criteria is considered. Further our solution method is straightforward though
sensitivity an alys i s may b e app lied b y selecting an algorithm to s ol ve transportation problems. Eve n more solution procedure t o piecewise lin ear tran sportation problem should be refined since an algorithm i n [19] i s general one for a convex cost. Therefore many further research problems are left. O n e i s ex tension to a capacitated transportation problem such as [20]. Another is [21] to solve fuzzy transportation problem.

Further R ecently related wo rk but extended model is found in [22] where truck purchase cost is the dominant criteria for fleet acquisition-related decisions but other cost factors such as loss due to the number of each route truck stoppages based on a truck type and recovery cost associated with a route choice decision, also are considered for deciding the fleet mix and minimizing the overall costs for long-haul shipments.

Anyway, we hope our model becomes useful to a big accident such as Tohoku earthquake occurred on 3.11 2011. Fur ther if we change which values are suitable tho ugh it may be difficult.

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