Searching and identifying leptoquarks through low-energy polarized scattering processes $e^- p \rightarrow e^- \Lambda_c$

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We investigate the potential for discovering and identifying the different leptoquark (LQ) models in the charm sector through the low-energy polarized scattering processes $e^- p \rightarrow e^- \Lambda_c$, $e^- p \rightarrow e^- \Lambda_c$, and $e^- p \rightarrow e^- \Lambda_c$. Considering only the longitudinally polarized processes, we show that the LQ models can be disentangled from each other by measuring the four spin asymmetries, $A_1^L$, $A_2^L$, $A_{1T}^P$, and $A_{1L}^P$, constructed in terms of the polarized cross sections. Although it is challenging to accomplish the same goal with the transversely polarized processes, we find that investing them in the future experiments is especially beneficial, since they can directly probe into the imaginary part of the Wilson coefficients in the general low-energy effective Lagrangian. With our properly designed experimental setups, it is also demonstrated that promising event rates can be expected for all these processes and, even in the worst-case scenario—no LQ signals are observed at all, they can still provide competitive potential for constraining the new physics, compared with those from the conventional charmed-hadron weak decays and the high-$p_T$ dilepton invariant mass tails at high-energy colliders.

I. INTRODUCTION

Many extensions of the Standard Model (SM), such as the grand unified theories [1–10], predict the existence of a particular type of bosons named leptoquarks (LQs). These hypothetical particles can convert a quark into a lepton and vice versa and, due to such a distinctive character, have very rich phenomenology in precision experiments and at particle colliders [11, 12]. Particularly, several anomalies observed recently in the semileptonic charged- and neutral-current $B$-meson decays [13–16], as well as in the muon anomalous magnetic moment [17] have attracted extensive studies of the LQ interactions, due to their abilities to address the anomalies simultaneously (see, e.g., Refs. [18–44]). Often these analyses focus on the LQ couplings to the heavy quarks, but growing interest in the LQ interactions involving the light quarks has also been ignited (see, e.g., Refs. [45–57]).

Among the various processes used to probe the LQ interactions, the flavor-changing neutral-current (FCNC) ones in the charm sector are the ideal searching ground, due to their absence at tree level and being strongly suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism at one loop in the SM. The known FCNC processes in the charm sector consist of the rare weak decays of the charmed hadrons [59–79], the high-$p_T$ dilepton invariant mass tails of the processes $pp \rightarrow \ell\ell(\nu)\ell(\nu)$ at high-energy colliders [80, 81], as well as the low-energy scattering processes $e^- p \rightarrow e^- (\mu^-)\Lambda_c$ proposed recently by us [82]. Interestingly enough, we found that, based on a set of high-intensity electron beam and liquid hydrogen target—both have been used to search for sub-GeV dark vector bosons [83–86]—and taking into account the most stringent constraints on the corresponding LQ interactions from other processes, very promising event rates can be expected for both the scattering processes in some specific LQ models [82]. Motivated by such a promising prospect of the LQ searches at the low-energy scattering processes (as well as the tenacious hunting for the LQs at the Large Hadron Collider (LHC) [87–94] and other future facilities [95, 96]), we will address another interesting question: can the different LQs be systematically disentangled from each other through the low-energy FCNC scattering processes?

A partial answer has been provided in Ref. [82], where we have demonstrated that a combined analysis of the experimental signals of the low-energy scattering processes and the semileptonic $D$-meson decays can distinguish certain scalar LQs from the vector ones. Unfortunately, only the LQs that would not induce tree-level proton decays have been considered there, and the vector LQs are found to be unable to be disentangled from each other [82]. Such a deficiency, as will be shown in this work, can be amended by considering the low-energy polarized scattering processes $e^- p \rightarrow e^- \Lambda_c$. To be specific, we will consider the FCNC production of $\Lambda_c$ baryons through the low-energy $ep$ scattering processes with only the electron beam polarized ($e^- p \rightarrow e^- \Lambda_c$), with only the proton target polarized ($e^- p \rightarrow e^- \Lambda_c$), and with both of them polarized ($e^- p \rightarrow e^- \Lambda_c$). Performing a simple analysis of the four spin asymmetries, $A_1^L$, $A_2^L$, $A_{1T}^P$, and $A_{1L}^P$, constructed in terms of the polarized cross sections, we will

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show that the different LQs mediating these processes at tree level can be distinguished from each other.

It is also interesting to note that a comprehensive study that worked in a different regime and with a much stricter condition has already existed [97]. In particular, focusing on the polarized $ep$ deep inelastic scatterings mediated by the LQs that couple only to the first-generation fermions, the authors of Ref. [97] have shown that the commonly discussed LQ models [12] can be disentangled from each other through the precise measurements of some observables, such as the polarized cross sections and spin asymmetries, provided that the high-intensity polarized lepton (both electron and positron) and proton beams are available. By contrast, our proposal will involve much less demanding (both electron and positron) and proton beams are available. Our proposal will involve much less demanding) and include the case where the LQ couples to quarks belonging to more than a single generation in the mass eigenstate, and can be easily adapted to other kinds of new physics (NP) models beyond the SM.

Of course, all the proposals above cannot be carried out if no LQ signals were observed at all. In this paper, we will show that, thanks to recent advances in the technologies of polarized electron beams and proton targets—both have been resourcefully exploited for studying the nucleon structure (see, e.g., Refs. [98–100] and references therein)—promising event rates can be expected for the polarized $ep$ scattering processes, if they are measured with properly designed experimental setups, together with the constraints from the rare charmed-hadron weak decays and the high-$p_T$ dilepton invariant mass tails as input. On the other hand, even in the worst-case scenario—no LQ signals are observed at all, the polarized $ep$ scattering processes can still yield competitive constraints with respect to those obtained from the rare charmed-hadron weak decays and the high-$p_T$ dilepton invariant mass tails. In particular, direct access to the chiral structure of the lepton current in the effective four-fermion operators offers these polarized scattering processes a unique advantage in constraining individually the effective Wilson coefficients (WCs) of the LQ models, which is, otherwise, not available from other FCNC processes in the charm sector without any additional implements.

The paper is organized as follows. In Sec. II, we start with a brief introduction of our theoretical framework, including the most general effective Lagrangian (also the LQ models), the polarized cross sections, and the various spin asymmetries for the polarized scattering processes $e^- p \rightarrow e^- \Lambda_c$. In such a framework, we firstly consider in Sec. III the longitudinally polarized scattering processes and then in Sec. IV the transversely polarized ones, focusing mainly on their possible applications—the identification of LQ models in particular. With the properly designed experimental setups and the currently existing constraints, we evaluate in Sec. V the prospect for discovering the LQ effects in these low-energy polarized scattering processes. Our conclusions are finally made in Sec. VI. For convenience, the polarized cross sections, together with their relations to the experimentally measurable quantities, are discussed in Appendix A, while explicit expressions of the amplitudes squared for both the longitudinally and transversely polarized scattering processes are given in Appendices B and C, respectively.

II. THEORETICAL FRAMEWORK

A. Effective Lagrangian

The most general effective Lagrangian responsible for the polarized scattering processes $lp \rightarrow l\Lambda_c$ (or $lu \rightarrow lc$ at the partonic level) can be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^{\text{SM}} + \mathcal{L}_{\text{eff}}^{\text{LQ}},$$

where $\mathcal{L}_{\text{eff}}^{\text{SM}}$ and $\mathcal{L}_{\text{eff}}^{\text{LQ}}$ represent the SM and the LQ contribution, respectively. Since the SM long-distance effects do not contribute to the scattering processes [82], while the short-distance effects are strongly GIM suppressed [59–75], here we can safely neglect the contribution from $\mathcal{L}_{\text{eff}}^{\text{SM}}$.

The most general low-energy effective Lagrangian $\mathcal{L}_{\text{eff}}^{\text{LQ}}$ induced by tree-level exchanges of LQs is given by [53]

$$\mathcal{L}_{\text{eff}}^{\text{LQ}} = \sum_{i,j,m,n} \left\{ [g_{V}^{LL}]^{ij,mn} (j_{V}^{L})^{ij} (J_{V}^{L})_{mn}^{\alpha\beta} + [g_{V}^{LR}]^{ij,mn} (j_{V}^{L})^{ij} (J_{V}^{R})_{mn}^{\alpha\beta} + [g_{V}^{RL}]^{ij,mn} (j_{V}^{R})^{ij} (J_{V}^{L})_{mn}^{\alpha\beta} + [g_{V}^{RR}]^{ij,mn} (j_{V}^{R})^{ij} (J_{V}^{R})_{mn}^{\alpha\beta} + [g_{T}^{L}]^{ij,mn} (j_{T}^{L})^{ij} (J_{T}^{L})_{mn}^{\alpha\beta} + [g_{T}^{R}]^{ij,mn} (j_{T}^{R})^{ij} (J_{T}^{R})_{mn}^{\alpha\beta} + [g_{S}^{L}]^{ij,mn} (j_{S}^{L})^{ij} (J_{S}^{L})_{mn}^{\alpha\beta} + [g_{S}^{R}]^{ij,mn} (j_{S}^{R})^{ij} (J_{S}^{R})_{mn}^{\alpha\beta} \right\},$$

(2)

with

$$\begin{align*}
(j_{S}^{L,R})^{ij} &= \bar{P} P_{R,L} \ell^i, \\
(j_{V}^{L,R})^{ij} &= \bar{q} P_{R,L} \gamma^i, \\
(j_{T}^{L,R})^{ij} &= \bar{q} \gamma^5 P_{R,L} \ell^i,
\end{align*}$$

(3)

where $P_{R,L} = (1 \pm \gamma_5)/2$, and $i,j$ and $m,n$ represent the flavor indices of leptons and quarks, respectively. The effective WCs $g_{ij}$ with their explicit expressions given in terms of the masses and the couplings of the LQs to the SM fermions in a specific model, are obtained by integrating out the heavy LQs, together with proper chiral
Table I. Scalar and vector LQ interactions with the SM fermions, together with their representations under the SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, where $\mathbf{\tau}_a$, with $a = 1, 2, 3$, denote the Pauli matrices, and the hypercharge $Y$ is given by $Q_{em} = T_3 + Y$.

Fierz transformations (see, e.g., Ref. [101]) of the resulting four-fermion operators to the ones given by Eq. (2).

There are 7 LQs (3 scalar and 4 vector ones) that can mediate the polarized scattering processes at tree level. Following the same notation as used commonly in literature [11, 12], we present in Table I their interactions with the SM fermions, where the left-handed lepton (quark) doublets are denoted by $L_L = (\nu_L^i, \ell_L^i)^T$ ($Q_L = (u_L^i, d_L^i)^T$), while the right-handed up-(down-)type quark and lepton singlets by $u_R^i$ ($d_R^i$) and $e_R^i$, respectively. Note that, for simplicity, their hermitian conjugation is not shown explicitly.

Following the aforementioned procedure, we obtain the effective WCs for each LQ model in Table I as follows:

$$[g_V^{LL(LR)}]_{ij,mn} = k_V^{LL(LR)} (\lambda_L)^{il} (\lambda_L)^{jm} M^2,$$

$$[g_V^{RR(RL)}]_{ij,mn} = k_V^{RR(RL)} (\lambda_R)^{il} (\lambda_R)^{jm} M^2,$$

$$[g_S^{LL}]_{ij,mn} = k_S (\lambda_L)^{il} (\lambda_L)^{jm} M^2,$$

$$[g_S^{RR}]_{ij,mn} = k_S (\lambda_R)^{il} (\lambda_R)^{jm} M^2,$$

$$[g_T^{LL}]_{ij,mn} = k_T (\lambda_L)^{il} (\lambda_L)^{jm} M^2,$$

$$[g_T^{RR}]_{ij,mn} = k_T (\lambda_R)^{il} (\lambda_R)^{jm} M^2,$$

where, as done in Ref. [82], we have coincided the mass-eigenstate basis of the left-handed up-type quarks and charged leptons with their flavor basis, and uniformly denoted the LQ masses by $M$, but bearing in mind that the coefficients $k$ can be directly read out from Table II, and so are the handy relations

$$g_S^{LL} (\lambda_L)^{il} (\lambda_L)^{jm} = \pm 4 |g_T^{RR}]_{ij,mn},$$

where the $-$ and + signs apply to the LQ models $\lambda_L$ and $\lambda_R$, respectively. It is important to note that the WCs in Eq. (4) are all given at the matching scale $\mu = M$. To connect the SM coupling constants $\lambda_L$ and $\lambda_R$ to the low-energy polarized scattering processes $e^+p \rightarrow e^-\Lambda_p$, they must be evolved to the corresponding low-energy scale through the renormalization group (RG) equation. Moreover, since large mixings of the tensor operators into the scalar ones can arise due to QED and electroweak (EW) one-loop effects [102, 103], both the QCD and EW/QED effects must be taken into account. Taking $M = 1$ TeV as the benchmark for the LQ mass, and performing the RG running from the benchmark scale down to the characteristic scale $\mu = 2$ GeV (we refer to Ref. [82] for more details), we eventually obtain

$$g_S (2 \text{ GeV}) \approx 2.0 g_S (1 \text{ TeV}) - 0.5 g_T (1 \text{ TeV}),$$

$$g_T (2 \text{ GeV}) \approx 0.8 g_T (1 \text{ TeV}),$$

$$g_V (2 \text{ GeV}) \approx g_V (1 \text{ TeV}),$$

where $\chi = L, R$, and the RG running effects of the vector operators have been neglected, since these operators do not get renormalized under QCD while their RG running effects under EW/QED are only at percent level.
the results in Eq. (6), the scalar-tensor WC relations, $g_5^R(\text{1 TeV}) = \pm 4g_7^R(\text{1 TeV})$, in Eq. (5) are modified as

$$g_5^R(2\text{ GeV}) \approx \mp 9.4\ g_7^R(2\text{ GeV}),$$

at the scale $\mu = 2$ GeV for the LQ models $S_1$ and $R_2$, respectively. Note that, for convenience, we will denote the WCs $g(2\text{ GeV})$ simply by $g$ hereafter.

B. Polarized cross sections

From the kinematic point of view, the differential cross sections of the polarized scattering processes $e^-(k) + p(P) \rightarrow e^-(k') + \Lambda_c(P')$ are identical to that of the unpolarized one. In what follows, we will thus briefly introduce the unpolarized cross section, and point out the necessary adjustments for the polarized ones.

The unpolarized differential cross section, with the kinematics $P = (m_p, 0)$, $P' = (E_{\Lambda_c}, p')$, $k = (E, k)$, and $k' = (E', k')$, is given by

$$d\sigma = \frac{1}{4[(Pk)^2 - m_p^2 m_{p}^{1/2}]^3/2} \frac{d^3k'}{2E'} \frac{d^3p'}{2E_{\Lambda_c}} \times |\mathcal{M}|^2 \delta^4(P + k - P' - k'),$$

where the amplitude $\mathcal{M}$ can be written as

$$\mathcal{M} = \sum g_{\alpha} (e^-(k', r')|j_{\alpha}|e^-(k, r)|\Lambda_c(P', s')|J_{\alpha}|p(P, s)),$$

with $r$ and $s$ ($r'$ and $s'$) denoting the spins of initial (final) electron and baryon, respectively. The hadronic matrix elements $\langle \Lambda_c(P', s')|J_{\alpha}|p(P, s)\rangle$ are given by the complex conjugate of $\langle p(P, s)|J_{\alpha}^\dagger|\Lambda_c(P', s')\rangle$, which are further parametrized by the $\Lambda_c \rightarrow p$ transition form factors [65, 104, 105]. However, since a scattering process generally occupies a different kinematic region from that of a decay, to extend the form factors that are commonly convenient for studying the $\Lambda_c$ weak decays to the scattering process, their parametrization must be analytic in the proper $q^2$ region. Interestingly, there already exists such a parametrization scheme, which was initially proposed to parametrize the $B \rightarrow \pi$ vector form factor [106], and has been recently utilized in the lattice QCD calculation of the $\Lambda_c \rightarrow N$ (nucleon) form factors [65]. We thus adopt the latest lattice QCD results [65], given that they provide also an error estimation.

To obtain the polarized amplitudes, one just replaces the spinor $u(k, s)$ of a polarized, massive spin-$1/2$ particle by $(1 + \gamma_5\slashed{\xi})u(k, s)/2$ in Eq. (9), where $\xi^\mu$ is the spin (or polarization) four-vector of the particle, which can be in general written as

$$\xi^\mu = \left( \frac{k}{m}\hat{n} + \frac{k \cdot \hat{n} k}{m(m + k^0)} \right),$$

where the unit vector $\hat{n}$ signifies the direction of the particle spin in its rest system [107, 108]. If $\hat{n}$ of the initial electron and/or proton is parallel (or anti-parallel) to the electron beam direction, the electron beam and/or proton target are longitudinally polarized, and otherwise transversely polarized if orthogonal to the beam direction. In this work, only longitudinally polarized electron beam will be considered, whereas both the longitudinally and transversely polarized proton targets will be utilized in exploring the low-energy polarized scattering processes.

For the longitudinally polarized electron beam, the polarization four-vector $\xi^\mu_p$ in the proton target rest frame (lab frame) is given by

$$\xi^\mu_p = \pm \frac{1}{m_e} \left( \frac{|k|}{m_e} k^0 k \right),$$

where the $+$ ($-$) sign corresponds to the case when the unit vector $\hat{n}$ of the incoming electron is parallel (antiparallel) to its direction of motion, or when the electron beam is right-handed (left-handed) polarized. Note that in the relativistic limit $m_e/E \rightarrow 0$, the projection operator $(1 + \gamma_5\xi^\mu_e)/2$ becomes $P_{R,L}$. On the other hand, the generic spin four-vector of the proton target in the lab frame can be parametrized as [108]

$$\xi^\mu_p = (0, \sin \alpha \cos \beta, \sin \alpha \sin \beta, \cos \alpha),$$

where $\alpha$ is the angle of $\hat{n}$ with respect to the z axis, and $\beta$ is the azimuthal angle between $\hat{n}$ and the x axis. Taking the beam direction to be the z axis, one can see that $\alpha = 0, \pi$ and $\xi^\mu_p = \pm (0, 0, 0, 1)$ for a longitudinally polarized proton target, while $\alpha = \pi/2$ and $\xi^\mu_p = (0, \cos \beta, \sin \beta, 0)$ for a transversely polarized proton target—if the target is transversely polarized along the x axis (i.e., $\beta = 0, \pi$), then $\xi^\mu_p = \pm (0, 1, 0, 0)$.

In addition to the spinor replacement, it is important to note that the unpolarized cross section is obtained by averaging over the initial electron and proton spins, whereas the polarized ones are not over the spins of the initial, polarized particles. More detailed discussions and explicit expressions of the cross sections of the two specific polarized $ep$ scattering processes can be found in Appendices A, B, and C.

C. Spin asymmetries

In terms of the polarized (differential) cross sections, several spin asymmetries can be defined, some of which, as will be shown later, are very useful for disentangling the NP models.

If only the incoming electron beam is polarized, we can define the single-spin parity-violating (PV) longitudinal asymmetry

$$A^L_L = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+},$$

where the unit vector $\hat{n}$ signifies the direction of the particle spin in its rest system [107, 108]. If $\hat{n}$ of the initial electron and/or proton is parallel (or anti-parallel) to the electron beam direction, the electron beam and/or proton target are longitudinally polarized, and otherwise transversely polarized if orthogonal to the beam direction. In this work, only longitudinally polarized electron beam will be considered, whereas both the longitudinally and transversely polarized proton targets will be utilized in exploring the low-energy polarized scattering processes.

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$$A^L_L = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+},$$

where $\sigma^-$ and $\sigma^+$ are the cross sections for the electron beam being left-handed polarized and right-handed polarized, respectively.
where \( \sigma^- \) and \( \sigma^+ \) denote the scattering cross sections with the incoming electron beam being right-handed (\( e_L^+ \)) and left-handed (\( e_L^- \)) polarized, respectively. Here the cross sections can be the total (\( \sigma^{\pm} \)) or the differential (\( d\sigma^\pm / dQ^2 \) with \( Q^2 = -q^2 \)) ones. Similarly, we can also define the single-spin PV asymmetries

\[
A_L^p = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+}, \quad A_T^p = \frac{\tilde{\sigma}^- - \tilde{\sigma}^+}{\tilde{\sigma}^- + \tilde{\sigma}^+},
\]

while only the proton target is longitudinally (\( p_L \)) and transversely (\( p_T \)) polarized, respectively. Here \( \sigma_p \) and \( \tilde{\sigma}_p \) are the corresponding scattering cross sections.

Concerning the case when the incoming electron beam and the proton target are both longitudinally polarized, we can construct six double-spin asymmetries \([97, 109]\).

\[
A_{L1}^p = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+}, \quad A_{L2}^p = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+},
\]

while four parity-conserving (PC) ones are defined, respectively, as

\[
A_{L3}^p = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+}, \quad A_{L4}^p = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+},
\]

where the first superscript of \( \sigma \) indicates the polarization direction of the incoming electron beam, whereas the second one denotes that of the proton target. Concerning the case when the proton target is transversely polarized, on the other hand, we can also build six double-spin asymmetries in the same way \([97, 109]\), with the PV double-spin asymmetries given, respectively, by

\[
A_{T1}^p = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+}, \quad A_{T2}^p = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+},
\]

and the PC double-spin asymmetries by

\[
A_{T3}^p = \frac{\tilde{\sigma}^- - \tilde{\sigma}^+}{\tilde{\sigma}^- + \tilde{\sigma}^+}, \quad A_{T4}^p = \frac{\tilde{\sigma}^- - \tilde{\sigma}^+}{\tilde{\sigma}^- + \tilde{\sigma}^+},
\]

where \( \tilde{\sigma} \) represents the scattering cross section with the proton target being transversely polarized.

### III. LONGITUDINALLY POLARIZED SCATTERING PROCESSES \( e^- p \to e^- \Lambda_c \)

#### A. Observable analyses

The first observable associated with the longitudinally polarized scattering processes \( e^- p \to e^- \Lambda_c \) is the single-spin PV asymmetry \( A_L^p \), which is defined in terms of the polarized cross sections \( \sigma_{\pm} \) (cf. Eq. (13)). Since multiple operators in Eq. (2) can contribute to the polarized scattering processes, we can generally write \( \sigma_{\pm} \) as

\[
d\sigma_{\pm} = \sum g_{\alpha} g_{\beta}^* |M_{\alpha-\beta}|^2 \times g_{\alpha} g_{\beta}^* |M_{\alpha-\beta}|^2,
\]

where \( g_{\alpha} \) and \( g_{\beta}^* \) go through all the WC s in Eq. (2), and \( |M_{\alpha-\beta}|^2 \) with a subscript, e.g., \( V_{RL} - V_{RR} \), represents the reduced amplitude squared that is induced by the interference between the operators \( j^R J^L_5 \) and \( j^L J^R_5 \).

Now a few comments about the reduced amplitude squared \( |M_{\alpha-\beta}|^2 \) in Eq. (19) are in order. First, as shown in Table III, the polarization direction of the electron beam selects the proper \((\alpha, \beta)\) combinations. This can be verified by noting that, for the scattering processes with \( e_L^- \), only the operators with left-handed lepton current \( j_L^\pm \) are at work, since the projection operator \((1 + \gamma_5 \gamma_\mu)/(1 + \gamma_5 \gamma_\mu)\) becomes \( P_L \) in the relativistic limit \( m_e / E \to 0 \). The same conclusion also holds for the operators with \( j^R \) when the electron beam is right-handed polarized. Second, the amplitude squared \( |M_{\alpha-\beta}|^2 \) is obtained by averaging over the initial electron spins, while \( |M_{\alpha-\beta}|^2 \) is not—hence differing from the former by a factor 2. Third, with our convention in Eq. (19), the reduced amplitudes squared are all real, and thus \( |M_{\alpha-\beta}|^2 \) is identical to \( |M_{\beta-\alpha}|^2 \). Finally, due to the chiral structures of the lepton and quark currents involved, certain reduced amplitudes squared with different subscripts are identical to each other, e.g., \( |M_{\nu_{eL} - \nu_{eR}}|^2 = |M_{\nu_{eR} - \nu_{eL}}|^2 \).

In this case, only one of them is preserved. It is now clear from Table III that, if only one operator (or more in certain cases) contributes to the scattering processes, as commonly happens in several LQ models in Table I, its associated \( A_L^p \) is always equal to \pm 1 or \pm 1. Such a feature, as will be shown later, can help to disentangle the different LQ models.

Features of other spin asymmetries, like \( A_T^p \) in Eq. (14) and \( A_{L1-L6}^p \) in Eqs. (15) and (16), are even richer. Given that they are all defined in terms of the double-spin polarized cross sections (note that \( \sigma_{\pm} = (\sigma^{\pm} + \sigma^{\pm}) / 2 \)), it is more convenient to focus on these cross sections. In the framework of the general effective Lagrangian \( \mathcal{L}_{eff} \), we can schematically write the double-spin polarized cross section \( \sigma^{ab} \), with \( a, b = \pm \), as

\[
d\sigma^{ab} = \sum g_{\alpha} g_{\beta}^* \left( 2 |M_{\alpha-\beta}|^2 + c |M'_{\alpha-\beta}|^2 \right),
\]

where the coefficient \( c = \pm \) depends on the subscripts \( \alpha \) and \( \beta \), as well as the superscripts \( a \) and \( b \). Here \( |M_{\alpha-\beta}|^2 \) and \( |M'_{\alpha-\beta}|^2 \) represent, respectively, the \( \xi_p \)-independent and \( \xi_p \)-dependent reduced amplitudes squared—we refer the readers to Appendix B for further details. Note that \( |M_{\alpha-\beta}|^2 \) is identical to that in \( \sigma_{\pm} \) due to the fact that \( \sigma_{\pm} = (\sigma^{\pm} + \sigma^{\pm}) / 2 \). The possible \((\alpha, \beta)\) combinations, together with the non-zero reduced amplitudes squared associated with \( \sigma^{\pm} \), are shown in Table IV. Clearly,
As can be seen from Table IV, the overall reduced amplitude squared for \( \sigma_{V_{LL}-V_{LR}}^{\alpha-\beta} \) is \( 2|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} - |\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{LR}} \), which is found to be identical to that for \( \sigma_{V_{LR}-V_{RL}}^{\alpha-\beta} \).

The presence of \( \xi_{\rho} \)-dependent \( |\mathcal{M}|_{\alpha-\beta}^{2} \) indicates that, contrary to \( A_{L}^{L} \) and \( A_{L}^{R} \), will not take a trivial form, even if only one operator contributes to the scattering processes. It is, however, by no means useless in distinguishing the LQ models. Note that the observations above also lead to some interesting relations between \( A_{L}^{P} \) and \( A_{L}^{P} \) for certain operators. For instance, \( A_{L}^{P} \) for the operator \( j_{V}^{P} J_{V}^{P} \) is identical to \( A_{L}^{P} \) for the operator \( j_{V}^{P} J_{V}^{P} \). These relations will be very handy in disentangling the LQ models later. Finally, it is easy to check that if only one operator is at work, \( A_{L}^{P}, A_{L}^{P}, A_{L}^{P}, \) and \( A_{L}^{P} \) can only be ±1, just

\[
\begin{align*}
\sigma_{V_{LL}-V_{LR}}^{\alpha-\beta} &= 2|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} - |\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{LR}} \\
\sigma_{V_{LR}-V_{RL}}^{\alpha-\beta} &= 2|\mathcal{M}|_{V_{LR}-V_{RL}}^{V_{RR}} - |\mathcal{M}|_{V_{RL}-V_{RL}}^{V_{LR}} \\
\end{align*}
\]

more \((\alpha, \beta)\) combinations are now available for \( \sigma_{\rho}^{\pm} \), but \( \sigma_{\rho}^{\pm} \) and \( \sigma_{\rho}^{\pm} \) still share the same set of \((\alpha, \beta)\) combinations as for \( \sigma_{\rho}^{\pm} \). It is also interesting to observe that, if certain \( \xi_{\rho} \)-independent reduced amplitudes squared with different subscripts are identical to each other, so are the \( \xi_{\rho} \)-dependent ones accordingly—as an example, we have \(|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} = |\mathcal{M}|_{V_{RR}-V_{RL}}^{V_{LR}} \) and \(|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} = |\mathcal{M}|_{V_{RL}-V_{RL}}^{V_{LR}} \). This leads to another interesting observation that the overall reduced amplitude squared associated with \( \sigma_{\alpha-\beta}^{\pm} \) is identical to that with \( \sigma_{\alpha-\beta}^{\pm} \), where the indices \( \alpha' (\beta') \) and \( \alpha (\beta) \) must be matched so that their associated operators have opposite chiral structures in both the lepton and quark currents. Let us take \( V_{LL} - V_{LR} \) again as a demonstration.

As can be seen from Table IV, the overall reduced amplitude squared for \( \sigma_{V_{LL}-V_{LR}}^{\alpha-\beta} \) is

\[
\begin{align*}
\sigma_{V_{LL}-V_{LR}}^{\alpha-\beta} &= 2|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} - |\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{LR}} \\
\sigma_{V_{LR}-V_{RL}}^{\alpha-\beta} &= 2|\mathcal{M}|_{V_{LR}-V_{RL}}^{V_{RR}} - |\mathcal{M}|_{V_{RL}-V_{RL}}^{V_{LR}} \\
\end{align*}
\]

more \((\alpha, \beta)\) combinations are now available for \( \sigma_{\rho}^{\pm} \), but \( \sigma_{\rho}^{\pm} \) and \( \sigma_{\rho}^{\pm} \) still share the same set of \((\alpha, \beta)\) combinations as for \( \sigma_{\rho}^{\pm} \). It is also interesting to observe that, if certain \( \xi_{\rho} \)-independent reduced amplitudes squared with different subscripts are identical to each other, so are the \( \xi_{\rho} \)-dependent ones accordingly—as an example, we have \(|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} = |\mathcal{M}|_{V_{RR}-V_{RL}}^{V_{LR}} \) and \(|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} = |\mathcal{M}|_{V_{RL}-V_{RL}}^{V_{LR}} \). This leads to another interesting observation that the overall reduced amplitude squared associated with \( \sigma_{\alpha-\beta}^{\pm} \) is identical to that with \( \sigma_{\alpha-\beta}^{\pm} \), where the indices \( \alpha' (\beta') \) and \( \alpha (\beta) \) must be matched so that their associated operators have opposite chiral structures in both the lepton and quark currents. Let us take \( V_{LL} - V_{LR} \) again as a demonstration.

As can be seen from Table IV, the overall reduced amplitude squared for \( \sigma_{V_{LL}-V_{LR}}^{\alpha-\beta} \) is \( 2|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} - |\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{LR}} \), which is found to be identical to that for \( \sigma_{V_{LR}-V_{RL}}^{\alpha-\beta} \).

The presence of \( \xi_{\rho} \)-dependent \( |\mathcal{M}|_{\alpha-\beta}^{2} \) indicates that, contrary to \( A_{L}^{L} \) and \( A_{L}^{R} \), will not take a trivial form, even if only one operator contributes to the scattering processes. It is, however, by no means useless in distinguishing the LQ models. Note that the observations above also lead to some interesting relations between \( A_{L}^{P} \) and \( A_{L}^{P} \) for certain operators. For instance, \( A_{L}^{P} \) for the operator \( j_{V}^{P} J_{V}^{P} \) is identical to \( A_{L}^{P} \) for the operator \( j_{V}^{P} J_{V}^{P} \). These relations will be very handy in disentangling the LQ models later. Finally, it is easy to check that if only one operator is at work, \( A_{L}^{P}, A_{L}^{P}, A_{L}^{P}, \) and \( A_{L}^{P} \) can only be ±1, just

\[
\begin{align*}
\sigma_{V_{LL}-V_{LR}}^{\alpha-\beta} &= 2|\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{RR}} - |\mathcal{M}|_{V_{LL}-V_{LR}}^{V_{LR}} \\
\sigma_{V_{LR}-V_{RL}}^{\alpha-\beta} &= 2|\mathcal{M}|_{V_{LR}-V_{RL}}^{V_{RR}} - |\mathcal{M}|_{V_{RL}-V_{RL}}^{V_{LR}} \\
\end{align*}
\]
like $A_L^e$, making them less interesting to this work.

**B. NP model identification**

In this subsection, we will use the 4 spin asymmetries, $A_L^e$, $A_L^p$, $A_L^{pp}$, and $A_L^{ppp}$, to identify the various NP models. To make our following analyses as general as possible, we will consider all the 4 vector operators $j_V^L J^L_V$, $j_V^R J^R_V$, $j_L^L J^L_L$, and $j_R^L J^R_L$, as well as their possible combinations:

1. cases with one vector operator: (I) $j_V^L J^L_V$, (II) $j_V^R J^R_V$, (III) $j_L^L J^L_L$, and (IV) $j_R^L J^R_L$;

2. cases with two vector operators: (V) $j_V^L J^L_V$ and $j_V^R J^R_V$, (VI) $j_V^L J^L_V$ and $j_L^L J^L_L$, (VII) $j_V^R J^R_V$ and $j_L^L J^L_L$, (VIII) $j_L^L J^L_L$ and $j_R^L J^R_L$;

3. cases with three vector operators: (IX) $j_V^L J^L_V$, $j_V^R J^R_V$ and $j_L^L J^L_L$, (X) $j_V^L J^L_V$, $j_V^R J^R_V$ and $j_R^L J^R_L$, (XI) $j_V^L J^L_V$, $j_L^L J^L_L$ and $j_R^L J^R_L$, (XII) $j_V^R J^R_V$, $j_L^L J^L_L$ and $j_R^L J^R_L$;

4. cases with four vector operators: (XIII) $j_V^L J^L_V$, $j_V^R J^R_V$ and $j_L^L J^L_L$, $j_R^L J^R_L$, (XIV) $j_V^L J^L_V$, $j_V^R J^R_V$ and $j_L^L J^L_L$, $j_R^L J^R_R$, (XV) $j_V^L J^L_V$, $j_V^R J^R_V$ and $j_R^L J^R_L$, $j_R^L J^R_R$.

In addition, we will neglect contributions from the scalar and tensor operators in these NP models, due to the stringent constraints on them from the leptonic $D$-meson decays [82, 110]. Note that we here have made an implicit assumption that the WCs of the scalar and tensor operators are connected in the NP models through, say, Eq. (5) as in the LQ models $S_1$ and $R_2$. Otherwise, only contributions from the scalar operators can be neglected, and our analyses must be modified accordingly.

For a simple demonstration, let us consider as a benchmark the beam energy $E = 3$ GeV and compute $A_L^e$, $A_L^p$, $A_L^{pp}$, and $A_L^{ppp}$ in these different cases. Clearly, $A_L^e$ and $A_L^p$ in the cases I–IV are independent of the WCs, since only one operator is involved, whereas those in other cases involving multiple vector operators do depend on their associated WCs. We present in Table V the resulting $A_L^e$ and $A_L^p$ for the cases I–IV. By contrast, $A_L^p$ and $A_L^{pp}$ in other cases can take any number within the range $[-1, 1]$. Here we expect that they should not be close to ±1 in general—an exception can happen if an extreme hierarchy exists among the WCs. Thus, one can separate the four cases I–IV from others by measuring $A_L^e$ and $A_L^p$. Meanwhile, based on the combined results of $A_L^e$ and $A_L^p$, these four cases are already disentangled from each other too.

To further disentangle the remaining 11 cases, one can exploit $A_L^{pp}$ and $A_L^{ppp}$. The complete results for the cases V–XV are presented in Table VI, where the constant $c_1$, depending on $|g_{LLV}^L|^2$, $|g_{R}^{LRV}|^2$, $|g_{LLV}^R|^2$, and $|g_{R}^{LRL}|^2$, is within $[-1, 1]$, but not expected to be close to ±1 or 0 in general. It is now clear from Table VI that measurements of $A_L^{pp}$ in the cases I–IV, $A_L^{pp} = 1, 0, -1, c_1$ can divide the remaining 11 cases into 4 subgroups, i.e., (VIII, IX, XIV), (X), (VI, VII, XIII), and (V, XI, XII, XV), accordingly. Except the already distinguished case X with $A_L^{pp} = 0$, all the other subgroups consist of at least 3 cases, which can be further disentangled by measurements of $A_L^{pp} = 1, 0, -1, c_1$. Thus, all the 15 scenarios are fully identified by measuring the four spin asymmetries, $A_L^e$, $A_L^p$, $A_L^{pp}$, and $A_L^{ppp}$. It is also interesting to note that certain repeated entries appear in both Tables VI and V. This is because the electron polarization in $A_L^{pp}$ (or $A_L^{ppp}$) remains intact, so that the operators it automatically selects in the cases V–XV can be identical to the ones in the cases I–IV. Let us illustrate this point with the case VI. Although the two operators $j_V^R J^R_V$ and $j_R^R J^R_R$ emerge in this case, only the former contributes to $A_L^{pp}$, rendering $A_L^{pp}$ to be equivalent to $A_L^p$ in the case I.

We are now ready to apply the model-independent results to the 7 concrete LQ models listed in Table I, $S_1$, $R_2$, $S_3$, $U_3$, $V_2$, $U_1$, and $V_3$, which correspond to the cases VII, VIII, I, II, IV, and III, respectively. Obviously, all the LQ models can be fully identified by the four spin asymmetries, except $S_3$ and $U_3$ since they generate the same effective operator (see Table II). Thus, other distinct features between them are necessary to distinguish $S_3$ from $U_3$. To this end, an interesting feature is that $S_3$ can mediate proton decays at tree level while $U_3$ does not [111–113]. Given the strong constraints from proton stability (or generally the $|\Delta B| = 1$ processes) [94], the LQ model $S_3$ could be, therefore, severely constrained, if no additional symmetry is invoked.

Thus far, our analyses have been carried out with the polarized total cross sections. It may be, however, more convenient to work with the polarized differential cross sections. As can be inferred from the discussions above, $A_L^p$ in the cases I–IV is always equal to ±1, irrespective of whether it is formulated in terms of the total or the differential cross sections. Besides, the repeated entries in Tables V and VI can be actually understood from $A_L^{pp}$, which in turn indicates that only $A_L^{pp}$ in the cases I–IV needs a special attention—certainly, $c_1$ changes as well, but is expected to be still not too close to 0 or ±1. In what follows, we will thus explore how $A_L^{pp}$ and its associated polarized differential cross sections vary within the available kinematic region in the cases I–IV.

As shown in the left two panels of Fig. 1, when the proton target is left-handed polarized (blue (dashed) curves), the longitudinally polarized differential cross sec-
Table VI. Spin asymmetries $A_{L3}^{ep}$ and $A_{L6}^{ep}$ in the cases V–XV, where the constant $c_1$ takes any value within the range $[-1,1]$, but is not expected to be close to $\pm 1$ or 0 in general; see text for details.

| V   | VI  | VII | VIII | IX  | XI  | XII | XIII | XIV | XV |
|-----|-----|-----|------|-----|-----|-----|------|-----|-----|
| $A_{L3}^{ep}$ | $c_1$ | $-0.998$ | $-0.998$ | $0.975$ | $0.975$ | $0$  | $c_1$ | $-0.998$ | $0.975$ | $c_1$ |
| $A_{L6}^{ep}$ | $0$  | $0.975$ | $-0.998$ | $0.975$ | $-0.998$ | $c_1$ | $0.975$ | $-0.998$ | $c_1$ | $c_1$ |

Figure 1. Longitudinally polarized differential cross sections $\frac{d\sigma}{dQ^2}g_R^L$ in the cases I–IV, with $d\sigma = (256\pi m_e^2 E^2)\frac{d\sigma}{dQ^2}$. Red (solid) and blue (dashed) curves are obtained with a right- and a left-handed polarized proton target, respectively.

Figure 2. Longitudinally polarized differential cross sections $\frac{d\sigma}{dQ^2}g_V^L$ in the case II, with three different electron beam energies, where $d\bar{\sigma} = (256\pi m_e^2)\frac{d\sigma}{dQ^2} = d\sigma'/E^2$.

C. Identification of the scalar and vector contributions in the LQ models

In the analyses above, we have neglected contributions to the polarized scattering processes from the scalar and tensor operators, because they are severely constrained by the leptonic $D$-meson decays [82, 110]. Nevertheless, it is still interesting to ask if the constraints on both the vector and scalar contributions reach a similar level in the future—so that both matter—is it possible to fully distinguish them through the low-energy polarized scattering processes $e^- p \rightarrow e^- \Lambda_c$ in the LQ models? In what follows, we will demonstrate explicitly that the answer is
as well as certain LQ models.

As shown in Table II, there are only two LQ models, $S_1$ and $R_2$, that can generate both scalar and vector operators—since contributions from the tensor and scalar operators are connected through Eq. (5) at high energy or through Eq. (7) at low energy, we will present them universally in terms of the scalar ones. Let us start with the model $R_2$. The resulting differential cross sections $d\sigma/(dQ^2|y|^2)$ of the longitudinally polarized scattering processes mediated by the operators $j^L_0 J^L_0$, $j^R_0 J^L_0$, $j^S_0 J^S_0$ ($j^L_0 J^L_0$), and $j^S_0 J^S_0$ ($j^R_0 J^L_0$) are shown in Fig. 3. It can be seen that if the electron beam is left-handed polarized, only the operators $j^L_0 J^L_0$ and $j^S_0 J^S_0$ ($j^R_0 J^L_0$) contribute to the polarized scattering processes, with the corresponding differential cross sections $d\sigma/(dQ^2|y|^2)$ depicted by the two plots on the left; if the electron beam is right-handed polarized, on the other hand, only the operators $j^R_0 J^L_0$ and $j^S_0 J^S_0$ ($j^L_0 J^L_0$) are at work, whose $d\sigma/(dQ^2|y|^2)$ are presented by the two plots on the right.

From the two plots on the left of Fig. 3, one can see that if the proton target is left-handed polarized (blue dashed curves), contribution from the operator $j^L_0 J^L_0$ ($j^R_0 J^L_0$) can be neglected, and thus one actually probes the contribution from the operator $j^L_0 J^L_0$. Then, flipping the polarization direction of the proton target but keeping the electron beam intact, one probes the contribution from the operator $j^L_0 J^L_0$ ($j^R_0 J^L_0$), since the contribution from the operator $j^R_0 J^L_0$ in this case becomes trivial. In this way, the contributions from the two operators $j^L_0 J^L_0$ ($j^R_0 J^L_0$) and $j^R_0 J^L_0$ are distinguished from each other. Following the same procedure but with the polarization direction of the electron beam flipped, one can also identify the contributions from the other two operators $j^S_0 J^S_0$ ($j^R_0 J^L_0$) and $j^R_0 J^L_0$ efficiently.

Such a scheme, however, cannot be applied to the LQ model $S_1$. This is due to the observation that both the operators $j^L_0 J^L_0$ and $j^S_0 J^S_0$ ($j^R_0 J^L_0$) contribute significantly to the longitudinally polarized scattering process with $e_L p_L^T$ (red solid curves), as can be inferred from Figs. 1 and 3. On the other hand, both of their contributions become trivial, when the electron beam and proton target are both left-handed polarized (blue dashed curves).

Clearly, this scheme also fails if an extreme hierarchy arises between the scalar (tensor) and vector contributions. Unfortunately, the current experimental constraints on them seem to imply so [82]. Thus, to ensure this procedure to be at work, it would be better that the constraints on both the vector and scalar contributions can reach a similar level in the future.

IV. TRANSVERSELY POLARIZED SCATTERING PROCESSES $e^- p \rightarrow e^- \Lambda_c$

We now turn to discussing the low-energy scattering processes $e^- p \rightarrow e^- \Lambda_c$ with the proton target transversely polarized—the electron beam will always be assumed to be longitudinally polarized throughout this work. As explained already in Sec. II, the polarization vector $\xi_p^\mu$ of the transversely polarized proton depends on $\beta$, the azimuthal angle between the unit vector $\hat{n}$ and the x axis (cf. Eq. (10)). In what follows, we will, for simplicity, set that the proton target is polarized along the x axis, so that $\beta = 0, \pi$, and $\xi_p^\mu = (\pm 0, 1, 0, 0)$ accordingly.

A. Observable analyses

Similar to the longitudinally polarized case presented in the previous section, it is also more convenient to discuss the double-spin polarized cross sections $\bar{\sigma}^{ab}$ with $a, b = \pm$, since $A_T^p$ in Eq. (14) and $A_T^{\perp p}$ in Eqs. (17) and (18) are all built in terms of them. In the framework of the general low-energy effective Lagrangian $L_{\text{eff}}$, $\bar{\sigma}^{ab}$ can be conveniently defined in the same way as for $\sigma^{ab}$ in Eq. (20), except that $|\bar{M}|^2$ should be now replaced by $|\bar{M}|^2$, where the latter represents the $\xi_p$-dependent reduced amplitude squared. Thus, we will dive into the reduced amplitudes squared immediately.

Compared with the reduced amplitudes squared in Table IV, these in Table VII exhibit certain similarities. For instance, the $\xi_p$-independent parts of the reduced amplitudes squared induced by the operators $j^L_0 J^L_0$ and $j^S_0 J^S_0$ are the same, whereas the $\xi_p$-dependent parts are opposite in sign, rendering that the observable $A_T^p$ induced by the operator $j^L_0 J^L_0$ is opposite in sign to that by the operator $j^R_0 J^L_0$. The same conclusion also applies to other pairs, like $(j^L_0 J^L_0, j^R_0 J^L_0)$, $(j^S_0 J^S_0, j^S_0 J^S_0)$, and $(j^L_0 J^L_0, j^R_0 J^L_0)$. Besides, the $\xi_p$-independent parts induced by the same operators in both cases are identical to each other, due to $\sigma_{\perp a} = (\sigma^{+ \perp} + \sigma^{- \perp})/2 = (\sigma^{+ \perp} + \sigma^{- \perp})/2$, with $a = \pm$. 

![Figure 3. Longitudinally polarized differential cross sections](image-url)
on the differential cross sections, since integrating over and other spin asymmetries, as well as their associated operators \( j \), following the same procedure as in the longitudinally energy transversely polarized prospect for identifying NP models by using the low-energy transversely polarized proton scattering processes. We can now explore the difference is that the \( \xi_\nu \)-dependent \( |M|_{\nu-\beta}^2 \) yields the corresponding amplitudes squared for the processes with \( e_L^+ p_T^- \) and \( e_L^- p_T^+ \).

Table VII. Non-zero reduced amplitudes squared of the polarized scattering processes with \( e_L^+ p_T^- \) (the upper two tables) and \( e_L^- p_T^+ \) (the lower two tables). Flipping the sign of the \( \xi_\nu \)-dependent \( |M|_{\nu-\beta}^2 \) yields the corresponding amplitudes squared for the processes with \( e_L^+ p_T^- \) and \( e_L^- p_T^+ \).

\[
\begin{array}{c|cc}
\hline
& (g_{LL}^L)^* & (g_{LR}^L)^* \\
\hline
\xi_\nu & 2|\mathcal{M}|_{LL-LL}^2 + |\mathcal{M}|_{LL-RL}^2 & 2|\mathcal{M}|_{RL-LL}^2 + |\mathcal{M}|_{RL-RL}^2 \\
\hline
\end{array}
\]

\[
\begin{array}{c|cc}
\hline
& (g_{S}^L)^* & (g_{T}^L)^* \\
\hline
\xi_\nu & 2|\mathcal{M}|_{SL-SL}^2 + |\mathcal{M}|_{SL-TL}^2 & 2|\mathcal{M}|_{TL-TL}^2 + |\mathcal{M}|_{TL-TL}^2 \\
\hline
\end{array}
\]

\[
\begin{array}{c|cc}
\hline
& (g_{SR}^R)^* & (g_{TR}^R)^* \\
\hline
\xi_\nu & 2|\mathcal{M}|_{RL-RL}^2 - |\mathcal{M}|_{RL-LL}^2 & 2|\mathcal{M}|_{LL-RL}^2 - |\mathcal{M}|_{LL-LL}^2 \\
\hline
\end{array}
\]

\[
\begin{array}{c|cc}
\hline
& (g_{J}^R)^* & (g_{T}^R)^* \\
\hline
\xi_\nu & 2|\mathcal{M}|_{SL-SL}^2 - |\mathcal{M}|_{SL-TL}^2 & 2|\mathcal{M}|_{TL-TL}^2 - |\mathcal{M}|_{TL-TL}^2 \\
\hline
\end{array}
\]

Figure 4. Transversely polarized spin asymmetry \( A_T^p \) (the left pannel) and differential cross sections \( d\tilde{\sigma}/dQ^2|g|^2 \) (the right pannel) for the operators \( j_L^L J_V^L \) (the upper plots) and \( j_L^R J_V^R \) (the lower plots), with \( d\tilde{\sigma} = (256\pi m_p^2 E^2) d\tilde{\sigma}_p \).

I) and \( j_L^L J_V^R \) (case II) for an illustration. It should be noted that contributions from the scalar and tensor operators will not be considered due to the same reason as explained in the longitudinally polarized case.

The variations of \( A_T^p \) in the cases I and II with respect to \( Q^2 \) are shown, respectively, by the upper and the lower plot in the left panel of Fig. 4. From these two figures, one can immediately make the following two in-
teresting observations. First, $A_T^p$ in both cases exhibits a periodic pattern with respect to $\phi$. This is because the $\xi_p$-dependent $|M|^2$ is proportional to $\cos \phi$, as shown in Appendix C. Such a periodic characteristic indicates that one cannot explore $A_T^p$ by exploiting the polarized total cross sections, as already pointed out above. Second, $A_T^p$ in both cases are generally suppressed, particularly in the former. It can be seen that $|A_T^p|$ is always trivial, practically irrespective of $\phi$ and $Q^2$ in the case II, while is below 0.5 in most of the available kinematic region in the case I, even with $\phi = 0, \pi$ where the polarization effects are maximal.

Given that the polarization effects are far less prominent in the transversely polarized $ep$ scattering processes in the cases I–IV, the mechanism previously used to distinguish the NP models (the cases I–XV) becomes less applicable now. One may argue about this by pointing out that $|A_T^p|$ in the case II can reach a relatively high value in the high-$Q^2$ region. However, such a region could easily exceed the upper limit of $Q^2$, where the theoretical framework fails—particularly for the high electron beam energy $E$ [82]. Indeed, as shown in Fig. 5, high $|A_T^p|$ can no longer be reached in the available $Q^2$ region as $E$ increases. Furthermore, the transversely polarized differential cross sections $d\sigma'/d\phi$ decrease rapidly as $Q^2$ increases, as explicitly demonstrated (especially the lower plot) in the right panel of Fig. 4. Thus, measuring a non-zero $A_T^p$ becomes very demanding, which, nevertheless, makes the mechanism less appealing to this work.

**B. Direct probe of the imaginary part of the WCs**

Although the transversely polarized $ep$ scattering processes may be less convenient to distinguish the NP models, they can directly probe into the imaginary part of the WCs, which is currently not possible (or at least not optimal) for the rare FCNC decays of charmed hadrons, the resonant searches in the high-$p_T$ dilepton invariant mass tails, as well as the low-energy unpolarized (and longitudinally polarized) $ep$ scattering processes, since their associated reduced amplitudes squared are all real.

As mentioned before and also explicitly shown in Appendix C, the $\xi_p$-dependent reduced amplitudes squared in Table VII induced by the interference between a pair of different operators, $(j^p_0 J^p_0, j^p_0 J^p_0)$, $(j^p_0 J^p_0, j^p_0 J^p_0)$, $(j^p_0 J^p_0, j^p_0 J^p_0)$, and $(j^p_0 J^p_0, j^p_0 J^p_0)$, consist of both the real and imaginary parts, with the former proportional to $\cos \phi$, while the latter to $\sin \phi$. To be as general as possible, we will consider the transversely polarized $ep$ scattering processes in the framework of the general low-energy effective Lagrangian $\mathcal{L}_{\text{eff}}$, but with contributions from the scalar operators neglected due to the stringent constraints from the leptonic $D$-meson decays [82, 110].

We will firstly consider the polarized $ep$ scattering process with $e^-_L p_T$. Its amplitude squared $|M|^2$ can be written as

$$|M|^2 = |M_{V_{LL}}|^2 + |M_{V_{LR}}|^2 + |M_{T_{LL}}|^2 + |M_{T_{LR}}|^2,$$

where the first three non-interference terms are given, respectively, by

$$|M_{V_{LL}}|^2 = |g_{VLL}|^2 \left(2 \left|\tilde{M}_{V_{LL}}\right|^2 + \left|\tilde{M}_{V_{LL}}\right|^2\right),$$

$$|M_{V_{LR}}|^2 = |g_{VLR}|^2 \left(2 \left|\tilde{M}_{V_{LR}}\right|^2 + \left|\tilde{M}_{V_{LR}}\right|^2\right),$$

$$|M_{T_{LL}}|^2 = |g_{TLL}|^2 \left(2 \left|\tilde{M}_{T_{LL}}\right|^2 + \left|\tilde{M}_{T_{LL}}\right|^2\right),$$

which depend on $q^2$ and $\cos \phi$. The last two interference terms, on the other hand, can be written as

$$|M_{V_{LL}}|^2 = |M_{V_{LL}}|^2 + \tilde{M}_{V_{LL}} + M_{V_{LR}} |M_{T_{LL}}|^2 + |M_{T_{LR}}|^2$$

$$= 2 \text{Re}[g_{VLL}^L g_{VLR}^R]\mathcal{A} - 2 \text{Im}[g_{VLL}^L g_{VLR}^R]\mathcal{A}' \sin \phi,$$

where $\mathcal{A}$, which consists of $2|\tilde{M}_{V_{LL}}|_L - |M_{V_{LL}}|_R$ and the real part of $|\tilde{M}_{V_{LL}}|_L - |M_{V_{LL}}|_R$, is also dependent on $q^2$ and $\cos \phi$, whereas $\mathcal{A}'$, the imaginary part of $|\tilde{M}_{V_{LL}}|_L - |M_{V_{LL}}|_R$ with $\sin \phi$ being factored out, depends exclusively on $q^2$. Focusing on the small $\phi$ region, we can then write the amplitude squared $|M|^2$ approximately as

$$|M|^2 \approx A_1(q^2) - A_2(q^2) \phi,$$

with

$$A_1 = |M_{V_{LL}}|^2 + |M_{V_{LR}}|^2 + |M_{T_{LL}}|^2 + 2 \text{Re}[g_{VLL}^L g_{VLR}^R] \mathcal{A},$$

$$A_2 = 2 \text{Im}[g_{VLL}^L g_{VLR}^R] \mathcal{A}' $$

where $A_{1,2}$ depend only on $q^2$ due to $\cos \phi \approx 1$. Now it can be seen that measuring $A_2$, the slope of $|M|^2$ in Eq. (24), directly probes into the imaginary part of
$g^{LL}_{V}g^{RR}_{V^*}$. Similarly, the imaginary part of $g^{RR}_{V}g^{LL^*}_{V^*}$ can be measured by flipping the polarization direction of the electron beam. Certainly, the same procedure can be again applied to $g^{LL}_{V}g^{RR}_{V}$ and $g^{RR}_{V}g^{LL^*}_{V^*}$ after flipping the polarization direction of the proton target. This is, however, unnecessary, since neither $A_1$ nor $A_2$ will change too much, as indicated by Fig. 4, and only the sign before $A_2$ in Eq. (24) will be flipped.

Given that the differential cross section $d^2\tilde{\sigma}/dQ^2d\phi$ can be conveniently connected to $|\mathcal{M}|^2$ through $d^2\tilde{\sigma}/dQ^2d\phi = |\mathcal{M}|^2/(128\pi^2m_e^2E^2)$ (see Appendix A), we thus explore in the following how the slopes $A'$ and $A'/E^2$ vary with respect to the kinematics $Q^2$ and the beam energy $E$. It can be seen from Fig. 6 that relatively low $Q^2$ is certainly favored. In particular, $Q^2 \approx 0.4$ GeV$^2$ and $Q^2 \approx 1$ GeV$^2$ are the two best kinematic regimes to measure $A'/E^2$ with $E = 3$ GeV and $E = 5$, 8 GeV, respectively. It is also interesting to note that increasing $E$ will not be of as much benefit to $A'/E^2$ as to $A'$.

Let us finally conclude this section by making the following comment. Since none of the LQ models, as shown in Table II, can produce the interference between two different vector operators, a clear non-zero measurement of $A'/E^2$ will certainly exclude all the LQ models, unless at least two of them arise simultaneously in a given model.

V. PROSPECT AND CONSTRAINTS

All the mechanisms proposed in the previous two sections will be less appealing, if the polarized scattering processes $e^-p \rightarrow e^-\Lambda_c$ cannot be observed at all. In this section, we will show that, with our properly designed experimental setups, very promising event rates associated with the polarized scattering processes can be expected in the LQ models. Even if, unfortunately, no event was observed with the designed experimental setups in the future, the polarized $e^-p$ scattering processes would still yield competitive constraints, compared with the charmed-hadron weak decays and high-$p_T$ dilepton invariant mass tails.

A. Experimental setups

We will consider the fixed-target polarized scattering experiments. For the single-spin polarized scattering process, $e^-p \rightarrow e^-\Lambda_c$, its event rate $dN/dt$ can be defined as [114, 115]

$$\frac{dN}{dt} = \mathcal{L}a(\sigma_0 + hP\sigma_{e,p}) = \mathcal{L}a(1 + hPA),$$

where $\sigma_0$ and $\sigma_{e,p}$ denote the spin-independent and the spin-dependent part of the cross section of this process, respectively. For $e^-p \rightarrow e^-\Lambda_c$, $h = h_e$, $P = P_p$, and $A = \sigma_e/\sigma_0 = -\tilde{A}_{L}^p$, whereas $h = h_p$, $P = P_p$, and $A = \sigma_p/\sigma_0 = -\tilde{A}_{L}^h$ for $e^-\bar{p} \rightarrow e^-\Lambda_c$, where $f$ is the width factor of the target [108, 116], and $P_e$ and $P_p$ denote the beam energy and target polarizations, respectively. The luminosity $\mathcal{L}$ is given by $\mathcal{L} = IL\kappa PT$, where $I$ is the beam intensity, while $\kappa$, $L$, and $\rho_T$ are the packing factor, the length, and the number density of the proton target accordingly. $^2$ The apparatus acceptance $a$ will be assumed to be 1 in our later analyses for simplicity. Based on the event rate defined by Eq. (26), the measured (or the raw) spin asymmetries $A_{\exp}$ can be connected to the single-spin asymmetries $A$ in Eqs. (13) and (14) through the relations $A_{\exp} = PA$.

On the other hand, the event rate $dN/dt$ of the double-spin polarized scattering process $e^-\bar{p} \rightarrow e^-\Lambda_c$ is given by [114, 115]

$$\frac{dN}{dt} = \mathcal{L}a_0\left(1 - h_eP_eA_L^p - h_hP_hA_L^h + h_eP_eA_C^p\right),$$

where $\sigma_{e,p}$ denotes the terms that are dependent on both $\xi_e$ and $\xi_p$, and $C_{\exp} = \sigma_{e,p}/\sigma_0$ represents the analyzing power of the reaction (we refer the readers to Appendix A for further details). In contrast to the single-spin asymmetries, the measured double-spin asymmetries $A_{\exp}$ are connected to $A_{L1}^p$, $A_{L2}^p$, $A_{L3}^p$, and $A_{L5}^p$ in a non-trivial way. Let us illustrate this point with the observable $A_{L4}^p$.

---

$^2$ Note that $\rho_T = \rho/m_p$, where $\rho$ is the density of the proton target.
It is, by definition, given by

\[ A_{L3}^{ep} = \frac{\sigma_{ep} - \sigma_p}{\sigma_0 - \sigma_e}, \]  

(28)

whereas the corresponding measured one reads

\[ (A_{L3}^{ep})_{\text{exp}} = f_P p_\epsilon \frac{\sigma_{ep} - \sigma_p}{\sigma_0 - \sigma_e}. \]  

(29)

It can be seen that \( A_{L3}^{ep} = f_P p_\epsilon A^{ep} \), if both \( \sigma_e \) and \( \sigma_p \) vanish, as happens in the well-known polarized deep-inelastic lepton-nucleon inclusive scattering cross section in the one-photon-exchange approximation (see, e.g., the review [117] and references therein). It is also interesting to note that \( A_{L3}^{ep} = f_P p_\epsilon A^{ep} \) in the limit of \( P_e = 1 \), irrespective of whether \( \sigma_e \) and \( \sigma_p \) vanish or not. Since a high degree of the electron beam polarization with \( P_e \approx 0.9 \) has been achieved [99], we expect that \( A^{ep} \) can be extracted from \( A_{L3}^{ep} \) in this way within a tolerable accuracy.

We now move on to a brief introduction of the experimental setups designed for the polarized \( ep \) scattering processes. Our desired longitudinally polarized electron beam is with a beam energy \( E = 3.48 \text{ GeV} \), an intensity up to 100 \( \mu A \), and a polarization of 89.4\% approximately. This has been used to constrain the strange vector form factors of the proton by the HAPPEX collaboration at Jefferson Laboratory (JLab) [99].

For the unpolarized proton target, we choose a liquid hydrogen (LH) with density \( \rho = 71.3 \times 10^{-3} \text{g/cm}^3 \) at 20K and 207-228 kPa. Such a target has been utilized in the Qweak experiment at JLab with an electron beam of \( E = 1.16 \text{ GeV} \) and \( I = 180 \mu A \) [86]. To break its length limit induced by the heating problem, a 3-kW cooling power was applied, which in turn rendered its maximal length to be around 35 cm [86]. Applying the same cooling system to our case, i.e., exposing the LH target to our desired beam energy \( (E = 1.16 \text{ GeV} \) and \( I = 180 \mu A \)), we find that the maximal LH target length is about 70 cm. For the polarized proton target, on the other hand, we favor the UVa/JLab polarized solid ammonia (SNH\(_3\)) target, which has been extensively used in many experiments, such as E08-007 [118] and E08-027 [119] at JLab. The target is operating at a 1-K temperature and a 5-T magnetic field, and can be polarized both longitudinally and transversely. With a polarized beam of 10–100 \( \mu A \) on the target, a luminosity of \( 10^{35} \text{ cm}^{-2} \text{s}^{-1} \), together with an average in-beam polarization of 80\% for NH\(_3\), has been reached [120]. Interestingly, a much higher luminosity of \( 10^{38} \text{ cm}^{-2} \text{s}^{-1} \) to \( 10^{39} \text{ cm}^{-2} \text{s}^{-1} \) is expected at JLab in the upcoming era [100], indicating that SNH\(_3\) target can tolerate much intense beam (\( I > 100 \mu A \)) will be available. For a simple event-rate estimation of the polarized \( ep \) scattering processes, we here assume the same target parameters, i.e., its polarization and packing factor, can be achieved in the future. Finally, we summarize in Table VIII our preferred experimental parameters of the polarized proton targets for the low-energy polarized \( ep \) scattering processes.

| Packing | Length | Density | Dilution | Polarization |
|---------|--------|---------|----------|-------------|
| LH      | 1      | 70      | 1        | 0           |
| SNH\(_3\) | 0.5    | 6       | 0.917    | 0.15        | 80\%        |

Table VIII. Experimental parameters of the proton targets for the low-energy polarized \( ep \) scattering processes.

B. Polarized e\(^-\)p → e\(^-\)Λ\(_c\) in the LQ models

With the aforementioned experimental setups, let us firstly evaluate the modified total cross sections with the degrees of beam and target polarizations, \( P_{e,p} \), in Eqs. (26) and (27) taken into account, in the framework of the general low-energy effective Lagrangian \( \mathcal{L}_{\text{eff}} \).

We will start with the longitudinally polarized scattering process \( e^−p \rightarrow e^−\Lambda_c \). In the \( e^+_L \) case, the modified total cross section reads

\[ \sigma^+_L = \{ 82.95 |g_{V}^{RR}|^2 + 36.40 |g_{V}^{RL}|^2 + 4.68 \Re [g_{V}^{RR}g_{V}^{RL *}] \]  

\[ + 4.64 |g_{V}^{LL}|^2 + 2.04 |g_{V}^{LR}|^2 + 0.26 \Re [g_{V}^{LL}g_{V}^{LR *}] \]  

\[ + 16.80 |g_{S}^{LL}|^2 + 0.94 |g_{S}^{LR}|^2 + 533.39 |g_{T}^{L}|^2 \]  

\[ + 29.85 |g_{T}^{L}|^2 - 109.42 \Re [g_{S}^{L}g_{L}^{R *}] \]  

\[ - 6.12 \Re [g_{S}^{L}g_{L}^{R *}] \} \times 10^{-4} \text{ GeV}^2, \]  

(30)

while in the \( e^-_L \) case,

\[ \sigma^-_L = \{ 82.95 |g_{V}^{LL}|^2 + 36.40 |g_{V}^{LR}|^2 + 4.68 \Re [g_{V}^{LL}g_{V}^{LR *}] \]  

\[ + 4.64 |g_{V}^{RR}|^2 + 2.04 |g_{V}^{RL}|^2 + 0.26 \Re [g_{V}^{RR}g_{V}^{RL *}] \]  

\[ + 16.80 |g_{S}^{LL}|^2 + 0.94 |g_{S}^{LR}|^2 + 533.39 |g_{T}^{L}|^2 \]  

\[ + 29.85 |g_{T}^{L}|^2 - 109.42 \Re [g_{S}^{L}g_{L}^{R *}] \]  

\[ - 6.12 \Re [g_{S}^{L}g_{L}^{R *}] \} \times 10^{-4} \text{ GeV}^2. \]  

(31)

Note that the WCs associated with the operators involving the lepton current \( j^L \) (\( j^R \)) appear in Eq. (30) (Eq. (31)) due to the imperfect \( P_e \), i.e., \( P_e \neq 1 \).

For the longitudinally polarized scattering process \( e^−p \rightarrow e^−\Lambda_c \), we have the modified total cross sections

\[ \sigma^+_p = \{ 49.04 |g_{V}^{LL}|^2 + 16.96 |g_{V}^{LR}|^2 + 2.35 \Re [g_{V}^{LL}g_{V}^{LR *}] \]  

\[ + 38.56 |g_{V}^{RR}|^2 + 21.48 |g_{V}^{RL}|^2 + 2.60 \Re [g_{V}^{RR}g_{V}^{RL *}] \]  

\[ + 9.91 |g_{S}^{L}|^2 + 7.83 |g_{S}^{L}|^2 + 315.07 |g_{T}^{L}|^2 \]  

\[ + 109.42 \Re [g_{S}^{L}g_{L}^{R *}] \} \times 10^{-4} \text{ GeV}^2, \]  

(32)

(33)
while the latter two read

\[
\sigma_{p} = \left\{ 49.04 |g_{V}^{RR}|^2 + 16.96 |g_{V}^{RL}|^2 + 2.35 \text{ Re} [g_{V}^{RR} g_{V}^{RL*}] \right. \\
+ 38.56 |g_{S}^{LL}|^2 + 21.48 |g_{S}^{LR}|^2 + 2.60 \text{ Re} [g_{S}^{LL} g_{S}^{LR*}] \\
+ 9.91 |g_{S}^{R}|^2 + 7.83 |g_{S}^{R*}|^2 + 315.07 |g_{T}^{R}|^2 \\
\left. + 248.17 |g_{T}^{R*}|^2 - 64.81 \text{ Re} [g_{S}^{R} g_{T}^{R*}] \right\} \times 10^{-4} \text{ GeV}^2, 
\]

(32)

in the cases of $p_{L}^{+}$ and $p_{L}^{-}$, respectively. It should be noted that we have, for simplicity, used the experimental parameters ($E$ and $I$) of the polarized electron beam to give a simple estimation of this process, although an unpolarized electron beam with similar $E$ and $I$ is already available, and has been utilized in the APEX experiment at JLab for hunting sub-GeV dark vector bosons [83, 84].

We also compute the modified total cross section of the double-spin polarized scattering process $e^{-}p \rightarrow e^{-}\Lambda_{c}$. As an illustration, let us consider the two sets of modified cross sections, $(\sigma^{++}, \sigma^{--})$ and $(d\sigma^{--}/d\phi, d\sigma^{--}/d\phi)$, which are related to the measured double-spin asymmetries ($A_{L3}^{exp}$ and $A_{P3}^{exp}$, respectively. Numerically, the former two are given by

\[
\sigma^{++} = \left\{ 92.88 |g_{V}^{LL}|^2 + 32.13 |g_{V}^{LR}|^2 + 4.44 \text{ Re} [g_{V}^{LL} g_{V}^{LR*}] \right. \\
+ 4.09 |g_{V}^{RR}|^2 + 2.28 |g_{V}^{RL}|^2 + 0.27 \text{ Re} [g_{V}^{RR} g_{V}^{RL*}] \\
+ 18.77 |g_{S}^{LL}|^2 + 0.83 |g_{S}^{LR}|^2 + 596.74 |g_{T}^{L}|^2 \\
+ 26.31 |g_{T}^{R}|^2 - 122.76 \text{ Re} [g_{S}^{L} g_{T}^{L*}] \\
\left. - 5.38 \text{ Re} [g_{S}^{R} g_{T}^{R*}] \right\} \times 10^{-4} \text{ GeV}^2, 
\]

\[
\sigma^{--} = \left\{ 73.03 |g_{V}^{LL}|^2 + 40.68 |g_{V}^{LR}|^2 + 4.91 \text{ Re} [g_{V}^{LL} g_{V}^{LR*}] \right. \\
+ 5.20 |g_{V}^{RR}|^2 + 1.80 |g_{V}^{RL}|^2 + 0.25 \text{ Re} [g_{V}^{RR} g_{V}^{RL*}] \\
+ 14.83 |g_{S}^{LL}|^2 + 1.05 |g_{S}^{LR}|^2 + 470.04 |g_{T}^{L}|^2 \\
+ 33.40 |g_{T}^{R}|^2 - 96.09 \text{ Re} [g_{S}^{L} g_{T}^{L*}] \\
\left. - 6.87 \text{ Re} [g_{S}^{R} g_{T}^{R*}] \right\} \times 10^{-4} \text{ GeV}^2, 
\]

(33)

while the latter two read

\[
\frac{d\sigma^{--}}{d\phi} \bigg|_{\phi=0} = \left\{ 13.20 |g_{V}^{LL}|^2 + 5.73 |g_{V}^{LR}|^2 + 0.74 |g_{V}^{RR}|^2 \right. \\
+ 0.33 |g_{V}^{RL}|^2 + 1.76 \text{ Re} [g_{V}^{LL} g_{V}^{LR*}] - 0.02 \text{ Re} [g_{V}^{RR} g_{V}^{RL*}] \\
+ 2.64 |g_{S}^{LL}|^2 + 0.15 |g_{S}^{LR}|^2 + 85.06 |g_{T}^{L}|^2 \\
+ 4.74 |g_{T}^{R}|^2 - 17.04 \text{ Re} [g_{S}^{L} g_{T}^{L*}] \\
\left. - 1.00 \text{ Re} [g_{S}^{R} g_{T}^{R*}] \right\} \times 10^{-4} \text{ GeV}^2,
\]

where the azimuthal angle $\phi$ has been set to 0 for simplicity.

Before moving on to the event-rate estimation in different LQ models, as did in Ref. [82], let us briefly summarize in Table IX the most relevant and stringent constraints on the effective coefficients $|g|^{re, cu}$ from the charm-hadron weak decays and the high-$p_{T}$ dilepton invariant mass tails. It can be seen that contributions from the scalar operators are stringently constrained by the measurement of the branching ratio $B(D^{0} \rightarrow e^{+}e^{-})$, which, through the low-energy WC relation in Eq. (7), in turn leads to severe constraints on the tensor contributions in the LQ models $S_{1}$ and $R_{2}$, the only two LQ models in Table I that can generate non-vanishing scalar and tensor effective operators. We thus neglect these contributions and focus on the vector ones, whose most severe upper bounds clearly come from the high-$p_{T}$ dilepton invariant mass tails, as shown in Table IX.

Taking now the upper limits on $q_{V}$ from Table IX and supposing a running time of 1 year, we evaluate the expected event rates of the various polarized $ep$ scattering processes in units of number per year ($N$/yr) in the LQ models. The final results are presented in Table X, from which the following interesting observations can be made immediately. First, the expected $(A_{L3}^{\mu})_{\exp}$ and $(A_{P3}^{\mu})_{\exp}$ in the models $S_{1}$ and $R_{2}$ will be equal to 0, because the same upper limits on $g_{V}^{LL}$ and $g_{V}^{RR}$ have been taken. Due to the same reason, the expected event rate of the single-spin polarized scattering process with $e_{L}^{+}$ ($p_{L}^{+}$) in the model $V_{2}$ is identical to that with $e_{L}^{-}$ ($p_{L}^{-}$) in the model $V_{2}$ (see

| Processes | $|g_{V}^{LL, RR}|^2$ | $|g_{V}^{LR, RL}|^2$ | $|g_{S}^{L, R}|^2$ | $|g_{T}^{L, R}|^2$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| $D^{0} \rightarrow e^{+}e^{-}$ [110] | \( \setminus \) | \( \setminus \) | 0.062 | \( \setminus \) |
| $D^{+} \rightarrow \pi^{+}e^{+}e^{-}$ [76] | 14 | 14 | 6.3 | 13 |
| $pp(qq) \rightarrow e^{+}e^{-}$ [121] | 3.6 | 3.6 | 22 | 0.57 |
M

Table X. Summary of the event-rate estimations for the polarized $e^+p \to e^-\Lambda_c$ scattering processes in different LQ models, where the event rate is given in units of $N/\text{yr}$, and $\Delta \phi = 1$ has been taken for the cases $e_L p^+_L$ and $e_L p^-_T$.

| $S_1$ | $S_3$ | $R_2$ | $U_3$ | $\bar{U}_1$ | $V_2$ | $\bar{V}_2$ |
|-------|-------|-------|-------|-----------|-------|-----------|
| $e^+_L$ | 532 | 28 | 234 | 28 | 504 | 221 | 12 |
| $e^-_L$ | 532 | 504 | 234 | 504 | 28 | 12 | 221 |
| $p^+_L$ | 293 | 164 | 129 | 164 | 129 | 72 | 57 |
| $p^-_L$ | 293 | 129 | 129 | 129 | 164 | 57 | 72 |
| $e^-_L p^+_L$ | 325 | 311 | 115 | 311 | 14 | 8 | 108 |
| $e^-_L p^-_T$ | 262 | 245 | 142 | 245 | 17 | 6 | 136 |
| $e^-_L p^-_T$ | 47 | 44 | 20 | 44 | 2 | 1 | 19 |

Eqs. (30), (31), and (32)). Second, identical event rates will always be expected in the LQ models $S_3$ and $U_3$ for any polarized scattering processes listed in Table X, since these two models are indistinguishable from each other in this work. Third, processes with $e_L p^+_T$ and $e_L p^-_T$ will have the same (or very close) event rates, mainly because the $\xi_p$-dependent $|\tilde{M}|^2$ is much smaller than the $\xi_p$-independent $|\bar{M}|^2$, as has been explicitly demonstrated in Fig. 4. Such an observation once again demonstrates that it becomes less applicable to use the transversely polarized $ep$ scattering processes to distinguish the NP models.

C. Competitive constraints from the polarized scattering process $\vec{e}^-p \to e^-\Lambda_c$

The various event rates in Table X are what one can expect in the best scenario, because they are just carried out with the currently available upper limits of the WCs. Thus, it is still possible that no event of the polarized $ep$ scattering processes is observed with our properly designed experimental setups in the future. In what follows, we will show that, even in such a worst-case scenario, investigating experimentally the various polarized $ep$ scattering processes in the future will not be in vain.

From the numerical coefficients of the various WCs $g$ in Eqs. (30)–(34), one can expect that the polarized scattering process $\vec{e}^-p \to e^-\Lambda_c$ with the designed experimental setups will set the most severe constraints on them. Thus, let us concentrate on this process. Assuming that only one WC contributes to the scattering process at a time, with the 100% detection efficiency of the final particles and the running time of 1 yr, we obtain the resulting constraints on the WCs $g$ in Table XI. Note that the constraints on $|g^{L|L}_V|$, $|g^{L|L}_U|$, $|g^{R|L}_S|$, and $|g^{L|L}_T|$ in the $e^-_L$ case have not been given, mainly because their contributions are much smaller in comparison with those from $|g^{L|L}_V|$, $|g^{R|L}_U|$, $|g^{R|L}_S|$, and $|g^{L|L}_T|$. Besides, the presence of $|g^{L|L}_V|$, $|g^{R|L}_U|$, $|g^{R|L}_S|$, and $|g^{L|L}_T|$ in the $e^-_L$ case, as has been pointed out before, results from $P_c \neq 1$, and is expected to disappear as $P_c \to 1$. The same arguments have been applied to the $e^-_L$ case too.

From the results presented in Tables XI and IX, it can be seen that, compared with other processes except the leptonic D-meson decay, significant improvements in constraining the effective WCs can be made through the low-energy polarized scattering process $\vec{e}^-p \to e^-\Lambda_c$, and the constraint on $g_S$ is even comparable with that from the leptonic D-meson decay.

Let us conclude this section by pointing out another merit of the low-energy polarized scattering processes. As shown in Table II, two pairs of effective vector operators, $(j^L_V J^L_V, J^R_V J^R_V)$ and $(j^L_T J^L_T, J^R_T J^L_T)$, are generated in the LQ modes $S_1$ and $R_2$, respectively. When setting constraints on the corresponding effective WCs $g$ from the charmed-hadron weak decays, the high-$p_T$ dilepton invariant mass tails, as well as the unpolarized $ep$ scattering processes [82], one meets the following constraining formulas:

$$a|g^{L|L}_V|^2 + b|g^{R|L}_U|^2 \leq c', \quad a'|g^{R|L}_U|^2 + b'|g^{RL}_S|^2 \leq c' \tag{35}$$

for the LQ models $S_1$ and $R_2$, respectively. Here contributions from the scalar and tensor operators have been neglected, and the non-zero constants $a$, $b$, $c$, $a'$, $b'$, and $c'$ are in general distinct for different processes. It is interesting to note that $a = b$ and $a' = b'$ always hold for the charmed-hadron weak decays and the high-$p_T$ dilepton invariant mass tails, while $a \neq b$ and $a' \neq b'$ for the unpolarized $ep$ scattering processes [82]. Now it is clear from Eq. (35) that setting constraints on the individual WCs $g_V$ from these processes becomes very nontrivial without adopting any special treatments—the most common one is probably by saturating the process with one nonvanishing WC at a time. For the process $\vec{e}^-p \to e^-\Lambda_c$, on the other hand, such a treatment becomes not necessary, since only one $g_V$ appears in each constraining.
VI. CONCLUSION

In this paper, we have investigated the potential for discovering and identifying the different LQ models in the charm sector through the low-energy polarized scattering processes $e^-p \to e^-\Lambda_c$. Specifically, we have considered the single-spin polarized scattering processes, $e^-p \to e^-\Lambda_c$ and $e^-\bar{p} \to e^-\Lambda_c$, as well as the double-spin polarized one $e^-\bar{p} \to e^-\Lambda_c$. Based on these polarized processes, together with their associated polarized cross sections, we have defined several spin asymmetries, which are found to be very efficient in disentangling the LQ models from each other.

Focusing firstly on the longitudinal polarized scattering processes, we have shown in an almost model-independent way that the 15 NP scenarios, including the 7 concrete LQ models, can be effectively disentangled from each other by measuring the 4 spin asymmetries, $A_{hL}$, $A_{cL}$, $A_{hR}$, and $A_{cR}$. To be thorough, we have also examined this mechanism by considering the polarized differential cross sections and exploring how these spin asymmetries behave with respect to the electron beam energy $E$ and the kinematics $Q^2$. It is found that, to make the procedure most efficient, both the low $Q^2$ and the high $E$ regime are favored. Besides this appealing application, we have discovered that the scalar (tensor) and vector contributions in the LQ model $R_2$ can be distinguished through the longitudinally polarized scattering processes as well.

We have then investigated the transversely polarized scattering processes in various aspects. In contrast to the longitudinal case, it would be very challenging to identify the NP models through these transversely polarized scattering processes, mainly because the polarization effects are far less prominent. Far from being in vain, however, measurements of these transversely polarized scattering processes offer us a unique opportunity to probe directly into the imaginary part of the effective WCs.

Given that all the mechanisms we have proposed are based on the premise that the LQ signals can be detected, we have finally performed a simple event-rate estimation for all the polarized scattering processes with the properly designed experimental setups, demonstrating that promising even rates can be expected for these processes. On the other hand, even in the worst-case scenario—no LQ signals are observed at all, we have shown in a model-independent way that the low-energy polarized scattering processes can provide more competitive constraints, in comparison with the charmed-hadron weak decays and the high-$p_T$ dilepton invariant mass tails. Furthermore, we have pointed out that, by maneuvering the electron beam polarization in the process $e^-p \to e^-\Lambda_c$, one can directly set constraints on the WCs $g_{V}$ in the LQ models $S_1$ and $R_2$, which, by contrast, will be tricky for other processes without taking any special treatments.

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Appendix A: Polarized cross sections and experimental quantities

In this appendix, we briefly clarify the relations among the polarized cross sections, the spin asymmetries, as well as the experimentally measurable quantities that are relevant to this work.

Starting with the double-spin longitudinally polarized scattering process $e^- (k) + \bar{p} (P) \to e^- (k') + \Lambda_c (P')$ mediated by the general low-energy effective Lagrangian $\mathcal{L}_{\text{eff}}$, we can write its cross section in the lab frame as

$$\frac{d\sigma_{h_1^L h_2^L}}{dq^2} = \frac{1}{64\pi m_p^2 E^2} \left( A_0 + h_c A_c + h_p A_p + h_c h_p A_{c+p} \right)$$

$$= \frac{d\sigma_0 + h_c d\sigma_c + h_p d\sigma_p + h_c h_p d\sigma_{c+p}}{dq^2}, \quad (A1)$$

where $h_c = \pm 1$ and $h_p = \pm 1$ are originated from the polarization four-vector $\xi^{\mu}_{c,p}$, such that $\xi^{\mu}_{c,p}$ now take the following forms

$$\xi^{\mu}_{c} = h_c \left( \frac{|k|}{m_c}, \frac{k^0 k}{m_c |k|} \right) \quad \xi^{\mu}_{p} = h_p (0, 0, 0, 1). \quad (A2)$$

As can be seen, the whole amplitude squared $|M|^2$ in Eq. (A1) is divided into 4 pieces, among which $A_0$ denotes the $\xi^{\mu}_{c,p}$-independent one, $h_c A_c (h_p A_p)$ the $\xi^{\mu}_{c}$ ($\xi^{\mu}_{p}$)-dependent one, and $h_c h_p A_{c+p}$ clearly the $\xi^{\mu}_{c} \xi^{\mu}_{p}$-dependent one. Here it is important to remind that all these amplitudes squared are not averaged over the spins of the initial electron and proton.

Based on the double-spin cross section in Eq. (A1), the cross sections of other longitudinally polarized scattering processes can be obtained straightforwardly. For the single-spin polarized scattering process $e^- p \to e^- \Lambda_c$, its cross section is given by

$$\frac{d\sigma^{h, h'}_{\mu}}{dq^2} = \frac{1}{2} \sum h_{h'} \frac{d\sigma^{h, h'}}{dq^2} = \frac{d\sigma_0 + h_c d\sigma_c}{dq^2}, \quad (A3)$$

where the factor $1/2$ arises from the spin average of the initial proton—since it is unpolarized. Similarly, one can
get the cross section of $e^-\bar{p} \to e^-\Lambda_c$.

$$
\frac{d\sigma_p^{kp}}{dq^2} = \frac{1}{2} \sum_{h_e} \frac{d\sigma_h^{h,p}}{dq^2} = \frac{d\sigma_0 + h_p d\sigma_p}{dq^2}.
$$  \hfill (A4)

It is also easy to verify that the cross section of the unpolarized scattering process $e^- p \to e^-\Lambda_c$ is given by

$$
\frac{d\sigma}{dq^2} = \frac{1}{4} \sum_{h_e} \frac{d\sigma_h^{h,p}}{dq^2} = \frac{d\sigma_0}{dq^2}.
$$  \hfill (A5)

The cross section of the double-spin transversely polarized process can be written in a similar way as

$$
\frac{d^2\sigma_{h_e,h_p}^{h,p}}{dq^2 dq'0} = \frac{1}{128\pi^2 m_P^2 E^2} (A_0 + h_e A'_0 + h_p A'_p + h_e h_p A''_{ep})
\nonumber
= d\sigma_0 + h_e d\sigma_e + h_p d\sigma_p + h_e h_p d\sigma_{ep}. \quad (A6)
$$

Then, following the same procedure as above, one can easily obtain the cross sections of other transversely polarized processes.

From Eqs. (A3) and (A4), one can see that

$$
A'_L = \frac{d\sigma_e}{d\sigma_0}, \quad A'_p = \frac{d\sigma_p}{d\sigma_0},
$$  \hfill (A7)

which in turn lead to

$$
\sigma_{h_e,h_p} = \sigma_0 + h_e \sigma_e + h_p \sigma_p + h_e h_p \sigma_{ep}
\nonumber
= \sigma_0 (1 - h_e A'_L - h_p A'_p + h_e h_p C_{ep}) , \quad (A8)
$$

where $C_{ep} = \sigma_{ep}/\sigma_0$ represents the analyzing power of the reaction [114]. With the replacements $L \to T$ and $\sigma \to \bar{\sigma}$, all the above formulas hold for the transversely polarized processes too.

Thus far, we have been focusing on the theoretical analyses. In reality, given that neither the degree of the electron beam polarization ($P_e$) nor the degree of the proton target polarization ($P_p$) can reach 100%, one must take such a deficiency into account during the event-rate estimations or the data analyses in experiment. To this end, we can conveniently replace the quantities $h_e,p$ by $P_{ep} h_e,p$ in the polarized cross sections accordingly (see, e.g., Refs. [108, 116] for explicit examples).

With the modified cross sections at hand, it can be clearly seen that the measured single-spin asymmetries $A_{exp}$ are related to $A'_L$ and $A'_p$ through $A_{exp} = PA$. For the measured double-spin asymmetries, on the other hand, no simple relations are available in general. For example, the measured $(A'_{L3})_{exp}$ is given by

$$
(A'_{L3})_{exp} = P_P \frac{P_p \sigma_{ep} - \sigma_p}{\sigma_0 - P_e \sigma_e} , \quad (A9)
$$

which is non-trivially connected to $A'_{L3}$.

Finally, we make a comment about the proton target polarization. Usually, the proton polarization is realized through the polarization of nucleus. However, since only a fraction of the target nucleons can be polarized, a dilution factor $f$ is often introduced to take account of this fact. Thus, $P_p$ in various differential cross sections is often accompanied by the factor $f$ [108, 116].

### Appendix B: Amplitude squared of longitudinally polarized scattering process $e^-\bar{p} \to e^-\Lambda_c$

For the convenience of future discussions, we here provide the explicit expression of the amplitude squared $|\mathcal{M}|^2$ of the longitudinally polarized scattering process $e^- (k) + \bar{p}(P) \to e^- (k') + \Lambda_c (P')$ mediated by the general low-energy effective Lagrangian $\mathcal{L}_{\text{eff}}$. With all the operators in Eq. (2) taken into account, the amplitude squared $|\mathcal{M}|^2$ with a left-handed polarized proton target ($p_L$) is given by

$$
|\mathcal{M}|^2 = (|g_{LL}|^2 + |g_{RR}|^2) |\mathcal{M}|^2_{V_{L-L} - V_{L-L}} + \frac{1}{2} (|g_{LL}|^2 - |g_{RR}|^2) |\mathcal{M}|^2_{V_{L-L} - V_{L-L}} + \frac{1}{2} (|g_{LL}|^2 - |g_{RR}|^2) |\mathcal{M}|^2_{V_{L-L} - V_{L-L}}
\nonumber
+ \frac{1}{2} (|g_{LL}|^2 + |g_{RR}|^2) |\mathcal{M}|^2_{S_{L-S} - S_{L-S}} + \frac{1}{2} (|g_{LL}|^2 + |g_{RR}|^2) |\mathcal{M}|^2_{T_{L-T} - T_{L-T}}
\nonumber
+ 2 \text{Re}[g_{LL}^* g_{RR}] |\mathcal{M}|^2_{V_{L-L} - V_{L-L}} + \text{Re}[g_{LL}^* g_{RR}] |\mathcal{M}|^2_{V_{L-L} - V_{L-L}} + \text{Re}[g_{LL}^* g_{RR}] |\mathcal{M}|^2_{V_{L-L} - V_{L-L}}
\nonumber
+ 2 \text{Re}[g_{LL}^* g_{RR}] |\mathcal{M}|^2_{S_{L-S} - S_{L-S}} + \text{Re}[g_{LL}^* g_{RR}] |\mathcal{M}|^2_{T_{L-T} - T_{L-T}} , \quad (B1)
$$
where a factor 2 accounting for the average over the initial electron spins has been taken care of, and $|\mathcal{M}|^2_{\alpha-\beta}^2$ and $|\mathcal{M}|^2_{\alpha-\beta}$ on the right-hand side represent the reduced $\xi_p$-independent and $\xi_p$-dependent amplitudes squared, respectively. One can easily obtain from Eq. (B1) the amplitude squared $|\mathcal{M}|^2$ with a right-handed polarized proton target ($q^2_L$) by flipping the sign of $|\mathcal{M}|^2_{\alpha-\beta}^2$ while keeping $|\mathcal{M}|^2_{\alpha-\beta}$ intact. Since the explicit expressions of the $\xi_p$-dependent $|\mathcal{M}|^2_{\alpha-\beta}$ have already been given in Ref. [82], here we only present the $\xi_p$-dependent $|\mathcal{M}|^2_{\alpha-\beta}$, which read, respectively, as

\begin{align}
|\mathcal{M}|^2_{\nu_{eL}^2 - \nu_e^2} &= \left\{-\frac{q^2 f^2_L}{2E[(m_{\Lambda_c} + m_p)^2 - q^2]} + \frac{q^2 g^2_L}{2E[(m_{\Lambda_c} - m_p)^2 - q^2]} \right\} \\
&\times \left( m_p^2 - m_{\Lambda_c}^2 + 4Em_p + q^2 \right) \left[ m_p q^2 + E(m_{\Lambda_c}^2 - m_p^2 + q^2) \right] \\
&\times \left[ 4E^2 m_p + m_p q^2 + 2E(m_{\Lambda_c}^2 - q^2) \right] \\
&\times \left[ (m_{\Lambda_c} - m_p) f_+ g_+ - (m_{\Lambda_c} + m_p) f g_\perp \right] \\
&\times \left\{ q^2 m_p(m_{\Lambda_c} + m_p) f_+ f_\perp \right\} - \left\{ q^2 m_p(m_{\Lambda_c} - m_p) g_+ g_\perp \right\} \\
&\times \left[ 4E^2 m_p + m_p q^2 + 2E(m_{\Lambda_c}^2 - m_p^2 + q^2) \right] \\
&\times \left[ 2m_p(m_{\Lambda_c}^2 - m_p^2) \right] \\
&\times \left[ 4E^2 m_p + m_p q^2 + 2E(m_{\Lambda_c}^2 - m_p^2 + q^2) \right] \left[ f_+ g_+ - q^2 [m_{\Lambda_c}^2 + 8E^2 m_p^2 \right. \\
&\left. + m_p^2 + q^4 + 4Em_p(m_{\Lambda_c}^2 + q^2) - 2m_{\Lambda_c}^2 (2Em_p + m_p^2 + q^2)] f g_\perp \right), \tag{B2}
\end{align}

\begin{align}
|\mathcal{M}|^2_{\nu_{\mu L}^2 - \nu_{\mu L}^2} &= \left\{-\frac{q^2 f^2_L}{2E[(m_{\Lambda_c} + m_p)^2 - q^2]} + \frac{q^2 g^2_L}{2E[(m_{\Lambda_c} - m_p)^2 - q^2]} \right\} \\
&\times \left( m_p^2 - m_{\Lambda_c}^2 + 4Em_p + q^2 \right) \left[ m_p q^2 + E(m_{\Lambda_c}^2 - m_p^2 + q^2) \right] \\
&\times \left[ 4E^2 m_p + m_p q^2 + 2E(m_{\Lambda_c}^2 - m_p^2 + q^2) \right] \\
&\times \left[ (m_{\Lambda_c} - m_p) f g_+ - (m_{\Lambda_c} + m_p) f_+ g_\perp \right] \\
&\times \left\{ q^2 m_p(m_{\Lambda_c} + m_p) f_+ f_\perp \right\} - \left\{ q^2 m_p(m_{\Lambda_c} - m_p) g_+ g_\perp \right\} \\
&\times \left[ 4E^2 m_p + m_p q^2 + 2E(m_{\Lambda_c}^2 - m_p^2 + q^2) \right] \\
&\times \left[ 2m_p(m_{\Lambda_c}^2 - m_p^2) \right] \\
&\times \left[ 4E^2 m_p + m_p q^2 + 2E(m_{\Lambda_c}^2 - m_p^2 + q^2) \right] \left[ f g_+ - q^2 [m_{\Lambda_c}^2 + 8E^2 m_p^2 \right. \\
&\left. + m_p^2 + q^4 + 4Em_p(m_{\Lambda_c}^2 + q^2) - 2m_{\Lambda_c}^2 (2Em_p + m_p^2 + q^2)] f_\perp g_\perp \right), \tag{B3}
\end{align}

\begin{align}
|\mathcal{M}|^2_{\nu_{eL}^2 - \nu_e^2} &= \frac{q^2 f_0 g_0(m_{\Lambda_c}^2 - m_p^2)[E(m_{\Lambda_c}^2 - m_p^2 - q^2) - m_p q^2]}{2Em_e^2} \tag{B4},
\end{align}
of the transversely polarized scattering process
+ 2E(m_p^2 - m_\Lambda^2 + q^2) - 2h_0 h_+ q^2(m_p - m_\Lambda^2 + 4E_m + q^2)}
\times \frac{4E(m_\Lambda^2 - m_p^2 - q^2) - m_p q^2}{E((m_\Lambda^2 - m_p^2 - q^2)[(m_\Lambda^2 + m_p^2)^2 - q^2])^2},
\end{equation}

|\mathcal{M}|^2_{\pi_L - \pi_L} = -\frac{16m_pe^2[(m_\Lambda^2 + m_p^2)h_0 h_+ - (m_\Lambda^2 - m_p^2)h_0 h_+]}{E[(m_\Lambda^2 - m_p^2 - q^2)[(m_\Lambda^2 + m_p^2)^2 - q^2]]}
\times [4E^2m_p + m_p q^2 - 2E(m_\Lambda^2 - m_p^2 - q^2) - (8h_0 h_+ m_\Lambda^2 - m_p^2) m_p [4E^2m_p
+ m_p q^2 + 2E(m_p^2 - m_\Lambda^2 + q^2)] - 2h_0 h_+ q^2(m_p - m_\Lambda^2 + 4E_m + q^2)^2}
\times \frac{4E(m_\Lambda^2 - m_p^2 - q^2) - m_p q^2}{E((m_\Lambda^2 - m_p^2 - q^2)[(m_\Lambda^2 + m_p^2)^2 - q^2])^2},
\end{equation}

\begin{equation}
|\mathcal{M}|^2_{\pi_L - \pi_L} = -\frac{q^2f_\perp^2}{2E[(m_\Lambda^2 + m_p^2)^2 - q^2]} - \frac{g_\perp^2}{2E[(m_\Lambda^2 - m_p^2)^2 - q^2]}
\times (m_p^2 - 2m_\Lambda^2 + 4E_m + q^2)[m_p q^2 + E(m_\Lambda^2 - m_p^2 + q^2)]
\times \frac{q^2m_p(m_\Lambda^2 + m_p^2) f_\perp f_\perp}{E_m[(m_\Lambda^2 + m_p^2)^2 - q^2]} + q^2m_p(m_\Lambda^2 - m_p^2) g_\perp g_\perp
\times \frac{4E^2m_p + m_p q^2 + 2E(m_p^2 - m_\Lambda^2 + q^2)}{(m_\Lambda^2 - m_p^2)^2 - q^2}],
\end{equation}

**Appendix C: Amplitude squared of transversely polarized scattering process $e^- \pi^0 \rightarrow e^- \Lambda_c$**

Let us now present the explicit amplitude squared $|\mathcal{M}|^2$ of the transversely polarized scattering process
\begin{equation}
|\mathcal{M}|^2 = |g_\perp^L|^2 + |g_\perp^R|^2 + \frac{1}{2}(|g_\perp^L|^2 - |g_\perp^R|^2)|\tilde{\mathcal{M}}|^2_{\pi_L - \pi_L}
\end{equation}

where the average over the initial electron spins has been taken into account, and $|\tilde{\mathcal{M}}|_{\alpha - \beta}^2$ on the right-hand side.
represents the reduced $\xi_\rho$-dependent amplitude squared. Same as for the previous case discussed in Appendix B, flipping the sign of $|\mathcal{M}|_{\alpha-\beta}^2$ while keeping $|\mathcal{M}|_{\alpha-\beta}^2$ intact in Eq. (C1) yields the amplitude squared $|\mathcal{M}|^2$ of the scattering process with a right-handed polarized proton target ($p_T^+$). For convenience, we give the $\xi_\rho$-dependent $|\mathcal{M}|^2_{\alpha-\beta}$, respectively, as

\begin{align}
|\mathcal{M}|_{\nu_{LL-}\nu_{LL}}^2 &= \left\{ \frac{2m_p}{2E[(m_{\lambda_c} + m_p)^2 - q^2]} \left[ f_+ g_+ (m_p + m_{\lambda_c}) + f_+ g_+ (m_p - m_{\lambda_c}) \right] \right. \\
&\quad - \left. f_+^2 \frac{4Em_p + m_p^2 - m_{\lambda_c}^2 + q^2}{2E[(m_{\lambda_c} + m_p)^2 - q^2]} \left[ E \left( m_{\lambda_c} - m_p \right)^2 - q^2 \right] \right\} \sqrt{-m_p q^2} \left( 2E \left( m_p^2 - m_{\lambda_c}^2 + q^2 \right) + m_p q^2 \right) \cos(\phi), \quad (C2) \\
|\mathcal{M}|_{\nu_{LR-}\nu_{LR}}^2 &= \left\{ \frac{2m_p}{2E[(m_{\lambda_c} + m_p)^2 - q^2]} \left[ f_+ g_+ (m_p + m_{\lambda_c}) + f_+ g_+ (m_p - m_{\lambda_c}) \right] \right. \\
&\quad - \left. f_+^2 \frac{4Em_p + m_p^2 - m_{\lambda_c}^2 + q^2}{2E[(m_{\lambda_c} + m_p)^2 - q^2]} \left[ E \left( m_{\lambda_c} - m_p \right)^2 - q^2 \right] \right\} \sqrt{-m_p q^2} \left( 2E \left( m_p^2 - m_{\lambda_c}^2 + q^2 \right) + m_p q^2 \right) \cos(\phi), \quad (C3) \\
|\mathcal{M}|_{\xi_L-\xi_L}^2 &= -\frac{q^2 f_0 g_0 (m_{\lambda_c} - m_p^2)}{2Em_p^2} \sqrt{-m_p q^2} \left[ 2E \left( m_p^2 - m_{\lambda_c}^2 + q^2 \right) + m_p q^2 \right] \cos(\phi), \quad (C4) \\
|\mathcal{M}|_{\xi_L-\xi_L}^2 &= 4 \left\{ \frac{8 \left( m_{\lambda_c}^2 - m_p^2 \right) m_p \left[ 4E^2 m_p^2 + 2E \left( m_p^2 - m_{\lambda_c}^2 + q^2 \right) + m_p q^2 \right]}{2Em_p^2} \left[ h_+ \tilde{h}_- + 4 \left[ h_+ \tilde{h}_- \left( m_p - m_{\lambda_c} \right) + h_+ \tilde{h}_+ \left( m_p + m_{\lambda_c} \right) \right] \right. \\
&\quad - \left. 2q^2 \left( 4Em_p + m_p^2 - m_{\lambda_c}^2 + q^2 \right) h_+ \tilde{h}_+ + 4 \left[ h_+ \tilde{h}_- \left( m_p - m_{\lambda_c} \right) + h_+ \tilde{h}_+ \left( m_p + m_{\lambda_c} \right) \right] \left[ 4Em_p + m_p^2 - m_{\lambda_c}^2 + q^2 \right] \left[ E \left( m_p^2 - m_{\lambda_c}^2 + q^2 \right) + m_p q^2 \right] \right\}.
\end{align}
\[ \begin{aligned}
&\sqrt{-m_p q^2 \left( 2E \left( 2E m_p + m_p^2 - m_\Lambda^2 - q^2 \right) + m_p q^2 \right) \cos(\phi)} \\
&\frac{E \left( [m_\Lambda - m_p]^2 - q^2 \right) \left( m_\Lambda + m_p^2 - q^2 \right)}{E \left( [m_\Lambda - m_p]^2 - q^2 \right) \left( m_\Lambda + m_p^2 - q^2 \right)} ,
\end{aligned} \] (C5)

\[ |\vec{M}|_{\LL \rightarrow \LL}^{x_{LR}} = \left\{ \begin{array}{l}
\frac{q^2 f_+^2}{2E \left( [m_\Lambda - m_p]^2 - q^2 \right)} - \frac{q^2 f_0^2}{2E \left( [m_\Lambda + m_p^2 - q^2 \right) (4E m_p + m_p^2 - m_\Lambda^2 - q^2) \cos(\phi) \\
- \frac{(m_p - m_\Lambda)g_{+} g_{+}}{E \left( [m_\Lambda - m_p]^2 - q^2 \right)} - \frac{(m_p + m_\Lambda) f_{+} f_{+}}{E \left( [m_\Lambda + m_p^2 - q^2 \right) \left( m_p q^2 + E \left( m_p^2 - m_\Lambda^2 + q^2 \right) \right) \cos(\phi) \\
- i \left( f_{+} g_{+} (m_p - m_\Lambda) - f_{+} g_{+} (m_p + m_\Lambda) \right) \sin(\phi) \right\} \sqrt{-m_p q^2 \left( 2E \left( 2E m_p + m_p^2 - m_\Lambda^2 - q^2 \right) + m_p q^2 \right) ,
\end{array} \right. \] (C6)

\[ |\vec{M}|_{\SS \rightarrow \LL}^{x_{LR}} = \left\{ \begin{array}{l}
\frac{(m_\Lambda - m_p) f_0 h_+}{Emc \left( [m_\Lambda - m_p]^2 - q^2 \right)} + \frac{(m_p + m_\Lambda) g_0 h_+}{Emc \left( [m_\Lambda + m_p^2 - q^2 \right) \left( 4Em_p + m_p^2 - m_\Lambda^2 - q^2 \right) \cos(\phi) \\
+ 2 \left\{ \frac{(m_p - m_\Lambda)^2 f_0 h_+}{Emc \left( [m_\Lambda - m_p]^2 - q^2 \right)} \right\}} - \frac{(m_p + m_\Lambda)^2 g_0 h_+}{Emc \left( [m_\Lambda + m_p^2 - q^2 \right) \left( m_\Lambda + m_p^2 - q^2 \right) \left( m_\Lambda + m_p^2 - q^2 \right) \cos(\phi) \\
+ 2i \left( m_p^2 - m_\Lambda^2 \right) \left( f_0 h_+ - g_0 h_+ \right) \sin(\phi) \right\} \sqrt{-m_p q^2 \left( 2E \left( 2E m_p + m_p^2 - m_\Lambda^2 - q^2 \right) + m_p q^2 \right) .
\end{array} \right. \] (C7)

It can be seen that $|\vec{M}|_{\alpha - \beta}$ induced by the interference between two different operators consists of both the real and imaginary terms. Moreover, the real terms are proportional to $\cos(\phi)$, while the imaginary ones to $\sin(\phi)$.

By contrast, $|\vec{M}|_{\alpha - \beta}$ induced by the same operator only has the real terms, which are proportional to $\cos(\phi)$.

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