The $b - \tau$ unification in GUTs with non-chiral matter

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Abstract

It is shown, that the presently accepted value for $b$-quark mass can be obtained from the requirement of the exact $b - \tau$ unification in the both non-SUSY and SUSY non-chiral extended GUTs.

Introduction. In due time the minimal SU(5) grand unification proposal [1] had ignited interest to the Grand Unified Theory (GUT) business by the calculation of the unification scale $M_{GUT}$ and mixing angle $\sin^2\theta_W$ [2] and also by successful prediction of the $b$-quark and $\tau$-lepton mass ratio, $R = m_b/m_\tau \approx 3$ [3,4]. It had been also shown, that the predicted mass ratio depends on the number of chiral quark-lepton families and that the observed ratio seems to require three such families [4,5]. At present we know that further prediction of three light families was confirmed by determination of the number of neutrino species in Z- boson decays at LEP and SLC [6]. However, we also know from the present accurate data on the Standard Model (SM) gauge couplings $\alpha_S$, $\alpha_W$ and $\alpha_Y$, that the minimal SU(5) GUT, in which the first such predictions were made, is ruled out due to the actual non-unification of the SM running gauge couplings. This initiates one to go beyond the minimal SU(5) model.

One such way is a supersymmetric (SUSY) extension of the minimal SU(5) GUT, in which gauge couplings meet each other at a single point around $10^{16}$ GeV [7,8]. In addition to the unification of gauge couplings, the unification of the $b$-quark and $\tau$-lepton Yukawa couplings appears naturally [9]. Namely, in the small $\tan\beta$ ($\tan\beta = v_2/v_1$ is the ratio of the two Higgs vacuum expectation values (VEVs)) regime one obtains, that for the presently allowed values of the electroweak parameters and the $b$-quark mass $b - \tau$ unification demands for the large values of the $t$-quark Yukawa coupling $Y_t = y_t^2/4\pi \approx 0.1 - 1$ at the $M_{GUT}$ scale. These large values are exactly those that ensure the attraction towards the infrared fixed point solution [10] of the $t$-quark mass, providing an explanation for the heaviness of the $t$-quark, $M_t = 180 \pm 12$ GeV [11]. From the other hand, although there is exact $b - \tau$ unification in the large $\tan\beta$ regime, but as it was shown in [12], in this case the predicted $b$-quark mass strongly depends on the SUSY particle spectra due to the importance of the corrections induced by the SUSY breaking sector.

However, it is well known by now that the simple single scale canonical SUSY GUTs predict the value of the strong gauge coupling ($\alpha_S(M_Z) \approx 0.125$ or so) higher than the values extracted from low-energy experiments ($\alpha_S(M_Z) \approx 0.11$) and the inclusion of threshold corrections do not change this situation [13]. This discrepancy initiates many authors to consider various extensions of the minimal SUSY GUT [14] or an alternative string type unification [15].

Another way to achieve gauge coupling unification is the split-multiplet non-SUSY SU(5) models first proposed by Frampton and Glashow and extensively studied in refs [7,17]. It was shown, that despite the gauge coupling unification most of such models with one electroweak Higgs doublet predict too large value for the $b$-quark mass, while in the case of two Higgs doublets the correct $b$-quark mass could be achieved by setting $t$-quark Yukawa coupling near its infrared fixed point.

Needless to say, that the split-multiplet non-SUSY models are less motivated than those of SUSY extended, mostly due to the gauge hierarchy problem. However, until the experimental confirmation of the SUSY they are real alternatives at least in the observable aspects of unification.

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In refs.\cite{18,19} we have proposed a mechanism for the natural (without any "fine tunings") explanation of the splitting of non-chiral (vectorlike) multiplets in the frames of extended SU(2N) GUTs. In the minimal non-SUSY SU(6) version of such GUTs the natural unification of the SM gauge couplings does appear assuming three families of split fermions belonging to \(15 + 1\) SU(6) reps with radiative induced masses \cite{18}. In the case of SUSY SU(6) the intermediate \(G_{I} \equiv SU(3)_{S} \otimes SU(3)_{W} \otimes U(1)_{Y}\) symmetry scale does exists naturally in the "Grand desert" region and one family of such split superfields is required in order to achieve gauge coupling unification \cite{19}. In the sharp contrast with the minimal SUSY SU(5) case, this unification appears for the low values of the strong gauge coupling as well and is close to the string (or even Planck) unification which is welcome feature from the point of view of the string theories \cite{20}.

These attractive features of the non-chiral extended GUTs initiate us to their further investigation. In the present paper we examine non-SUSY and SUSY non-chiral extended SU(6) models of refs \cite{18,19} by the calculation of the \(b - \tau\) mass ratio.

**The \(b - \tau\) unification in non-SUSY SU(6).** Let us consider non-SUSY SU(6) GUT of ref. \cite{18} with the fermion content

\[
3 \cdot (2 \cdot \overline{5} + 15) + N_{nc} \cdot (15 + \overline{15}) \ ,
\]

where the first term includes three chiral families of ordinary quarks and leptons while the second one corresponds to the \(N_{nc}\) non-chiral families of the complementary fermions with the naturally light split submultiplets. This later on the language of \(G_{SM} \equiv SU(3)_{S} \otimes SU(2)_{W} \otimes U(1)_{Y}\) decomposition are (see \cite{18,19} for more details):

\[
N_{nc} \cdot \left[ (3, 2, 1/3) + (\overline{3}, 2, -1/3) + (3, 1, -2/3) + (\overline{3}, 1, 2/3) \right]
\]

The mass \(M_{SF}\) of split submultiplets \((2)\) strongly depends on a number of the starting non-chiral families of the complementary fermions in order to provide the final unification. This easily can be seen at Table 1 obtained by numerical integration of the two-loop renormalization group equation (RGE) system for the running gauge couplings\cite{18}. As pointed in \cite{18}, the case of three \((N_{nc} = 3)\) families of the complementary fermions looks quite natural if one assumes radiative origin of the masses of split multiplets \((2)\), \(M_{SF} \sim \alpha_{GUT}^{2} \cdot M_{GUT}\).

The scalar sector besides the two heavy scalars \(\Sigma \sim 35\) and \(\varphi \sim 6\) (which break SU(6) down to the SM and provide splitting between submultiplets of the complementary fermions) includes also two Salam-Weinberg doublets from \(H_{1} \sim 6\) and \(H_{2} \sim 15\) as it is usually in SU(6) in order to give masses to down and up quarks, respectively, from the Yukawa interactions:

\[
y^{(down)\overline{6}}_{15} H_{1} + y^{(up)15}_{15} H_{2} + h.c. \quad (3)
\]

From (3), ignoring any mixing between families, we can write down the masses of the top, bottom and tau, which are given by VEVs of \(H_{1}\) and \(H_{2}\) as:

\[
m_{t} = y_{t}v \sin \beta, \quad m_{b} = y_{b}v \cos \beta, \quad m_{\tau} = y_{\tau}v \cos \beta , \quad (4)
\]

where \(v = 1/\sqrt{2} (H_{1}^{2} + H_{2}^{2})^{1/2}\) and \(\tan \beta = \frac{y_{H_{1}}^{2}}{y_{H_{2}}^{2}}\). The appropriate one-loop RGEs for \(Y_{t}, Y_{b}\) and \(Y_{\tau}\) (\(Y_{a} = y_{a}^{2}/4\pi, a = t, b, \tau\)) have the form:

\[
\mu \frac{d \ln Y_{a}}{d \mu} = \frac{1}{2\pi} \left( C_{ab} Y_{b} - D_{ai} \alpha_{i} \right) , \quad (5)
\]

where \(i\) runs over the \(Y, W, S\) indices; \(\mu\) denotes the renormalization scale and the numerical factors \(C_{ab}, D_{ai}\) are those as in two Higgs doublet extended SM \cite{21}.

Taking the experimental data for gauge couplings and the physical mass \(t\)-quark \cite{11,22}

\[
\begin{align*}
\alpha_{s}(M_{Z}) &= 0.118 \pm 0.005 \\
\sin^{2}\theta_{W}(M_{Z}) &= 0.2312 \pm 0.0003 \\
\alpha_{EM}^{-1}(M_{Z}) &= 127.9 \pm 0.2 \\
M_{t} &= 180 \pm 12 GeV
\end{align*}
\]

\footnote{The \(\beta\) functions below the energy \(M_{SF}\) are those of two Higgs doublet SM model while those above the \(M_{SF}\) are modified by inclusion of the split fermion contributions \cite{18}.}
and using $b - \tau$ unification condition, $R(M_{GUT}) = 1$, we calculate the $b$-quark pole mass\footnote{Two-loop QCD dressed pole mass relates with running mass $m(\mu)$ (4) by the well known formulae : $M = m(M) \left(1 + \frac{4}{3\pi} \alpha_S(M) + 12.4 \cdot \left(\frac{\alpha_S(M)}{\pi}\right)^{1/2}\right)$} by the numerical integration of the RGEs system (5). We have presented the results of our calculations as a dependence of the $b$-quark pole mass $M_b$ on the $\tan\beta$ for small $\tan\beta$ regime (Fig.1) as well as for large $\tan\beta$ regime (Fig.2), using the values of strong gauge coupling $\alpha_S = 0.11, 0.117$ and $M_t = 180\, GeV, M_\tau = 1.778\, GeV$ for the $t$-quark and $\tau$-lepton masses, respectively. Dashed lines in Figs.1,2 correspond to the case of one family ($N_{nc} = 1$) of the non-chiral fermions while solid lines to the three family case ($N_{nc} = 3$).

One can see from these Figs. that the predicted $b$-quark mass decreases with increasing of the number of non-chiral families and with decreasing of $\alpha_S(M_Z)$. This tendency is quite favorable because, as it was mentioned above, natural values for the split fermion masses ($M_{SF} > \alpha^2_{GUT}\cdot M_{GUT} \sim 10^{12}\, GeV$) can be obtained only for $N_{nc} \geq 3$. So, we can conclude that the predicted $b$-quark mass in non-chiral extended non-SUSY SU(6) is in a good agreement with the observed value $M_b = 4.9 - 5.2\, GeV$ [22] for the small $\tan\beta \simeq O(1 - 3)$ as well as for the large $\tan\beta \simeq O(40 - 60)$ and for the values of the strong coupling constant $\alpha_S(M_Z) \leq 0.12$.

The $b - \tau$ unification in SUSY SU(6). Now let us consider the SUSY extension of the model considered in the previous section. The essential point related with SUSY extension seems to be that the general superpotential of the Higgs superfields $\Sigma \sim 35$, and $\varphi \sim 6 + \overline{6} \sim 5$ allows SU(6) breaking mainly along the foregoing $G_I = SU(3)_S \otimes SU(3)_W \otimes U(1)$ channel providing the splitting of the $15 + \overline{15}$ matter superfields just as in non-SUSY case. The $G_I$ intermediate symmetry scale $M_I$ given by VEVs of the $\varphi(\overline{5})$ can be expressed through the basic parameters of the model – the unification scale $M_{GUT}$ and the masses of split states (2) (now chiral superfields) $M_{SF}$ as :

$$M_I = \left[M_{GUT}\cdot M_{SF}\right]^{1/2} \eta,$$

where $\eta \sim O(1)$ is the dimensionless parameter expressed through the coupling constants of $\Sigma$, $\varphi(\overline{5})$ and $15 + \overline{15}$ superfield interactions [19]. While in non-SUSY case the natural gap between masses $M_{SF}$ and $M_{GUT}$ would be at most the radiative one, in SUSY case the mass scale $M_{SF}$ in principal could be much lower and even down to the SUSY breaking scale. In Table 2 we present the predictions of the $M_{SF}$ for the one family of $15 + \overline{15}$. These predictions are based on the requirement of the gauge coupling unification at two-loop level, using as an effective SUSY scale $T_{SUSY} = M_Z$ [23], $M_t = 180\, GeV$, $\sin\theta_W = 0.2312$, $\alpha^{-1}_{em} = 127.9$ and $\alpha_S = 0.11$ and 0.125.

One can see from Table 2, that in a sharp contrast with the canonical SUSY GUT situation the influence of the new gauge interactions below the intermediate scale $M_I$ together with the contribution coming from the split states (2) provide the increasing of the unification scale up to the string $M_{string} \simeq 5.5 \cdot 10^{17}\, GeV$ and even the Planck scale. This is a welcomed feature from the point of view of string theories [20] and on the other hand such large unification scale gives the sufficient suppression of the $d = 5$ operator induced proton decay for the whole range of the $\tan\beta$ parameter.

Now let us look what happens with $b - \tau$ unification. Besides the Higgs superfields $\Sigma$ and $\varphi(\overline{5})$ as in non-SUSY case we introduced the additional Higgs superfields $H_1 \sim 6 + \overline{15}$ and $H_2 \sim 15 + \overline{15}$ and by appropriate fine tuning of the tree level superpotential parameters extract Salam-Weinberg doublet and antidoublet from $H_2$ and $\overline{H_1}$ respectively. Assuming effective gauge theory for any given energy region one can calculate $\beta$-functions for gauge coupling RGEs as well as numerical factors $C_{ab}$ and $D_{ai}$ for the Yukawa couplings (5) [21]. For the energy region below the intermediate scale $M_I$ the numerical factors $C_{ab}$ and $D_{ai}$ are exactly those as in minimal SUSY model [21] while are significantly modified above the $M_I$ :

$$C_{ab} = \begin{pmatrix} 7 & 1 & 0 \\ 1 & 7 & 1 \\ 0 & 3 & 3 \end{pmatrix}, \quad D_{ai} = \begin{pmatrix} \frac{16}{3} & \frac{16}{3} & \frac{4}{3} \\ \frac{16}{3} & \frac{16}{3} & \frac{4}{3} \\ 0 & 8 & 1 \end{pmatrix} \quad (8)$$

The results of numerical integration of RGEs (5) with appropriate numerical factors as in the previous section are presented graphically in the $M_b - \tan\beta$ plane for the gauge coupling unification solutions from Table 2 and for small and large $\tan\beta$ regimes respectively at Fig.3 and Fig.4. Note that the influence of new gauge and Yukawa interactions leads to the decreasing of $M_b$ in the both small and large $\tan\beta$ cases. As it is well known, the decreasing of the strong gauge coupling works in the same direction, so to obtain the correct value for $b$-quark mass one can take the values of $t$-quark Yukawa coupling at $M_{GUT}$ significantly lower than in the case of canonical SUSY SU(5). This shifts the prediction of the $t$-quark mass from its infrared fixed point.
To conclude, I have shown that the presently accepted value for $b$-quark mass can be obtained from the requirement of the exact $b-\tau$ unification in the both non-SUSY and SUSY non-chiral extended GUTs. Namely, in non-SUSY case the small and large tan$\beta$ regimes demand low values of the strong gauge coupling $\alpha_s < 0.12$ for the tree non-chiral families and $\alpha_s < 0.116$ for one family of such fermions. In the SUSY case, the influence of new gauge and Yukawa interactions decrease the $b$-quark mass value relative to the standard SU(5) situation and shift the $t$-quark mass from its infrared fixed point.

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Table 1: The dependence of the mass of split submultiplets $M_{SF}$ on the number of non-chiral families $N_{nc}$ and $\alpha_s(M_Z)$ obtained from the requirement of gauge coupling unification at two-loop level in non-SUSY SU(6).

| $N_{nc}$ | $\alpha_s(M_Z)$ | $M_{SF}$ [GeV] | $M_{GUT}$ [GeV] | $\alpha_{GUT}^{-1}$ |
|----------|-----------------|----------------|----------------|-------------------|
| 1        | 0.11            | $10^{5.19}$    | $10^{15.55}$   | 36.35             |
| 1        | 0.117           | $10^{3.87}$    | $10^{15.95}$   | 35.46             |
| 1        | 0.125           | $10^{2.60}$    | $10^{16.35}$   | 34.56             |
| 3        | 0.11            | $10^{12.05}$   | $10^{15.55}$   | 36.35             |
| 3        | 0.117           | $10^{11.91}$   | $10^{15.90}$   | 35.59             |
| 3        | 0.125           | $10^{11.76}$   | $10^{16.25}$   | 34.80             |

Table 2: The representative solutions of the two-loop RGEs for gauge couplings in non-chiral extended SUSY SU(6).

| $\alpha_s(M_Z)$ | $M_{SF}$ [GeV] | $M_I$ [GeV] | $M_{GUT}$ [GeV] | $\alpha_{GUT}^{-1}$ |
|-----------------|----------------|-------------|----------------|-------------------|
| 0.11            | A              | $10^{6.45}$ | $10^{12.30}$   | $10^{18.70}$      | 3.35             |
|                 | B              | $10^{9.08}$ | $10^{13.30}$   | $10^{17.80}$      | 10.18            |
|                 | C              | $10^{12.42}$| $10^{14.30}$   | $10^{16.85}$      | 16.87            |
| 0.125           | A              | $10^{6.68}$ | $10^{13.30}$   | $10^{19.15}$      | 2.92             |
|                 | B              | $10^{9.24}$ | $10^{14.30}$   | $10^{18.25}$      | 9.80             |
|                 | C              | $10^{12.55}$| $10^{15.30}$   | $10^{17.30}$      | 16.49            |
Figure 1: The $b$-quark mass as a function of $\tan\beta$ (small $\tan\beta$ regime) in non-SUSY SU(6) model with non-chiral split fermions. The solid lines correspond to the case of three family ($N_{nc} = 3$) of non-chiral fermions, while those of dotted to the one family case ($N_{nc} = 1$). Lines denoted by A, B, C correspond to the values of $\alpha_s(M_Z) = 0.11, 0.117$ and 0.122, respectively.
Figure 2: The same as in Fig.1 in the large $\tan \beta$ regime.
Figure 3: The $b$-quark mass as a function of $\tan\beta$ (small $\tan\beta$ regime) in SUSY SU(6) model. The solid lines correspond to the value of $\alpha_S(M_Z) = 0.11$, while those of dotted to the value of $\alpha_S(M_Z) = 0.122$. Lines denoted by A, B, C correspond to A, B, C cases of gauge coupling unification (see Table 2).
Figure 4: The same as in Fig. 3 in the large $\tan\beta$ regime.