Mimicking the LCDM model with Steaths

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We present a new cosmological model that mimics the Lambda Cold Dark Matter by using a stealth field. This kind of field is characterized as not coupling directly to gravity; however, it is connected to the underlying matter content of the universe model. As is known, stealth fields do not back-react on the space-time; however, their mimicry skills show how this field and its self-interaction potential determines the cosmic evolution. We show the study of the simplest model that can be developed with the stealth field.

I. Introduction. Precise astronomical measurements of the universe indicate that nearly 25% of its content is in the form of dark matter (DM), the key ingredient necessary to explain large scale structure formation, and 70% dark energy (DE), the unknown component driving the recent cosmic acceleration.

In this context, the best model to describe almost all the observational data is a mixture of elements from the standard cosmological model plus a cosmological constant, the so-called “concordance” ΛCDM model. Although successful in fitting the observational data, from a theoretical point of view the model seems too arbitrary. First, there is no clue about where this cosmological constant came from, and with it, why its value is so close to the critical energy density, and second, why we live in a very special epoch where the contributions from DM and DE are of the same order of magnitude, the well-known “cosmic coincidence problem” (CCP).

Physicists have proposed different ways to overcome this dilemma. The first was to adopt a dynamical cosmological constant, trying to adjust the dynamics of it to alleviate the CCP. This is the idea behind quintessence [1–6], where a scalar field is responsible for driving the current cosmic acceleration. The second was to modify the left-hand side of Einstein’s equations, trying to explain the presence of a cosmological constant as a non-standard geometric effect [7–9]. The third was to violate the Copernican Principle, i.e., by assuming we live in an inhomogeneous universe [10–13]. Although some successes have been obtained in each one of these alternatives scenarios, there is so far no clear evidence of a preference compared to the ΛCDM model.

Since Einstein’s field equations link the geometric properties of the universe with its total content, there is a well-known degeneracy between these two components: DM and DE. This is in fact one good reason to consider unified dark models. Of course the simplicity of considering a single component acting as both DM and DE is also a good reason.

In this Letter, we show that a new class of scalar field model that exhibits a non-trivial response to geometry, dubbed stealth, which serves as a unified model of the dark sector.

The idea of considering unified scalar field models (see [14] for a review) to describe DM and DE emerges as a natural way to alleviate the so-called “coincidence problem”, namely, to explain why the energy densities of these two dark components are of the same order of magnitude today. Models of this type have been proposed in the past, assuming the stress energy tensor of the scalar field back-react to the geometry according to Einstein’s equations. Among them we can mention the model of [15]; where a potential $V(\phi) = V_0(\cosh\lambda\phi - 1)^p$ is considered, the Chaplygin gas [16], the generalized Chaplygin gas model [17], and models with a non-canonical kinetic term called k-essence models [18].

On the other hand, it is well-known that in The General theory of Relativity gravity is understood as a manifestation of the curvature of space-time and the latter is caused by the presence of matter. This fundamental principle is codified in the equations proposed by Einstein. So the slightest presence of matter on the right hand side of Einstein’s equations is sufficient to alter the geometry of space-time. However, the stealth is a kind of matter that remains present in the space-time without altering or changing its geometry. The stealth appears only for a scalar field non-minimally coupled to gravity, and its origins date back to the improved energy-momentum tensor considered first in [19], where the authors showed the possibility of this new tensor becoming the source of the gravitational field; meanwhile, the dynamic of the scalar field is dictated by the Klein-Gordon equation.

The original proposal of the stealth was reported for a three-dimensional BTZ black hole in [20], in higher dimensions in Minkowski space-time [21] and (anti-)de Sitter [(A)dS] space [22]. Also in Lifshitz space-time in [21], for a four dimensional black hole [23], for an AdS black hole in Lovelock gravity [24], in Einstein-Gauss-Bonnet gravity for topological black hole [25], for a rotating AdS black hole in new massive gravity [26], and finally for a BTZ rotating black hole present two solutions in [27]. Lately, as was shown in [30], there are stealth fields dur-
ing the cosmological evolution and some cosmological solutions are given in order to have a LCDM cosmology, in particular those with polynomials and power-law evolution are analyzed. Also, the general solutions for de Sitter cosmologies and inhomogeneous stealths have been studied, concluding that only for de Sitter backgrounds allow a full dependence on the space-time coordinates. In this letter, we examine the case of a cosmological model coming from a non-minimal coupling with a stealth scalar field as a unified component describing both DM and DE, and thus mimicking the ΛCDM model.

II. Stealths as a Unified dark model.

In the present work we study a cosmological model coming from a non-minimal coupling with a stealth scalar field, described by the action:

\[ S = \int d^4x\sqrt{-g} \left[ \frac{R}{2k} + L_m - \frac{1}{2} \zeta R\phi^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] . \]  
\[(1)\]

Here, \( L_m \) is the Lagrangian matter. Clearly, for \( \zeta = 0 \) the scalar field stress tensor reduces to the usual case of a minimally coupled field. By varying the action (1), the scalar field stress tensor reduces to the usual case of a perfect fluid with zero pressure, i.e., the dust case, plus a cosmological constant, and when the scalar field depend only on time.

As was demonstrated in [22], there is a stealth solution in the context of a Friedmann-Lemaître-Robertson-Walker (FLRW) space-time. Now we obtain our cosmological model by coupling

\[ ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] , \]
\[(4)\]
to a perfect fluid with zero pressure, i.e., the dust case, plus a cosmological constant, and when the scalar field depend only on time.

On the left-hand side of (4) we use the stress-energy tensor for a perfect fluid as the DM contribution, leading to the usual ΛCDM model, where the DM density \( \rho \) and the cosmological constant \( \Lambda \) determine the cosmic evolution \( a(t) \). At the same time, from the right-hand side of (4), the stealth field \( \phi \) and its self interacting potential \( V(\phi) \) determines completely the cosmic evolution \( a(t) \). As a consequence of this – the evolution must be the same as that of the ΛCDM model – the stealth here works as a unifying field simultaneously describing the effects of the action of both DM and DE. We get:

\[ 3 \left( \frac{\dot{a}}{a} \right)^2 + 3k = -6\frac{\ddot{a}}{a} - 1\frac{1}{2\zeta} \left( \frac{\dot{\phi}}{\phi} \right)^2 - \frac{V}{\zeta \phi^2} , \]  
\[(5)\]

and

\[ \frac{2V}{3\zeta \phi^2} + \frac{6\zeta - 2}{3\zeta} \left( \frac{\dot{\phi}}{\phi} \right)^2 + 2\frac{\ddot{a}}{a} + 3\frac{k}{a} = 0 . \]  
\[(6)\]

III. Specific Stealth realizations

As is well-known, the ΛCDM model is so far the best fit model for a large set of astronomical observations, such as, type Ia supernovae (SNIa), baryon acoustic oscillations (BAO), cosmic microwave background radiation (CMBR), growth of structure, etc. [37]. In this setup the cosmological constant \( \Lambda \) drives the current accelerated expansion of the universe, detected for the first time using type Ia supernovae [39, 38].

As we mentioned in the introduction, although the stealth field does not back-react to the geometry, the existence of a non-zero coupling \( \zeta \) enables the stealth to appear dynamically coupled to the matter content. In this section we characterize the stealth field associated with this cosmological model.

In order to give a complete description of the model we are presenting, we display the features of the ΛCDM model. The Friedmann equation is

\[ H^2 + \frac{k}{a^2} = \frac{\kappa}{3} \rho + \frac{\Lambda}{3} , \]  
\[(7)\]

and the stress-energy conservation equation implies

\[ \dot{\rho} + 3H\rho = 0 , \]  
\[(8)\]

where we have assumed explicitly an equation of state \( p = 0 \) for the matter content (cold dark matter). From
equation (8) we obtain the energy density, which evolves as \( \rho = \rho_0 a^{-3} \). This simple model fits several observational probes quite well. The best fit parameters so far, assuming a curved FRW metric, are those from the Planck Collaboration [40]: \( \Omega_L = 0.685 \pm 0.018 \), \( \Omega_m = 0.315 \pm 0.018 \), and \( H_0 = 67.3 \pm 1.2 \), where \( \Omega_L = \Lambda/(3H_0^2) \), \( \Omega_k = -k/H_0^2 \), and \( \Omega_m = \kappa \rho_0/(3H_0^2) \).

On the other hand, from the vanishing of the stealth stress-energy tensor, Eqs. (5, 6), we can read the equivalence relations between the set \([\rho, \Lambda]\) for the \( \Lambda \)CDM model, and the set \([\phi, V(\phi)]\) for the stealths. It is easy to show that an equivalence can be met by proposing the following relation:

\[
-\frac{V}{\zeta^2} = \Lambda. \tag{9}
\]

This means that the self interacting potential is related only algebraically to the cosmological constant. Using this relation and after some manipulations, the equivalence is complete after we impose

\[
-\frac{\dot{\phi}}{\phi} \frac{d}{dt} \ln[\phi^{1/2}\kappa a^6] = \kappa \rho, \tag{10}
\]

as well as

\[
\frac{\dot{\phi}}{\phi} = -\frac{c}{\phi_0 a^2}. \tag{11}
\]

where \( \beta = (8\zeta - 1)/4\zeta \), and \( c \) is an integration constant.

What we have obtained here is a stealth field \( \phi \) – given by the solution of (11) – with self-interaction (9) that (by construction) generates an evolution – \( a(t) \) – totally indistinguishable from that obtained from the \( \Lambda \)CDM model.

The equivalence enables us to use cosmological observations to fix the values (and the uncertainties) of the model parameters. Using (11) in the equation for \( \rho \) we get for the Hubble function:

\[
H = \frac{\kappa \rho_0}{6a} \frac{\phi^\delta}{c} + \frac{1}{12\zeta a^2} \frac{c}{\phi^2}. \tag{12}
\]

Before testing the model, we have to write it in terms of the redshift \( z \). Recalling that \( a = (1 + z)^{-1} \) the relation (11) can be written as

\[
E(z) = \frac{(1 + z)\Omega_m}{\Xi}, \tag{13}
\]

where \( E(z) = H(z)/H_0 \), \( \varphi = \phi/\phi_0 \), \( \delta = \beta - 1 \) and \( \Xi = 1 + \sqrt{1 - \Omega_m/6\zeta} \). On the other hand, Eq. (12) can be written as

\[
E(z) = \frac{(1 + z)}{\frac{2}{3}} \varphi^\delta \Xi + \frac{(1 + z)^2}{12\zeta} \varphi^2 \Xi. \tag{14}
\]

The free parameters to constrain are clearly \( \zeta \) and \( \Omega_m \). There is no way to constrain \( H_0 \) based on sets (13) and (14). However, if we test the model using \( H(z) \) measurements, we can get a number for \( H_0 \) just by minimizing the residuals of

\[
[H^{obs}(z_i) - H_0 E(z_i|\zeta, \Omega_m)]. \tag{15}
\]

In practice we solve differential equation (13) numerically with the initial condition \( \varphi(z = 0) = 1 \), and by making use of (14) to get \( E(z) \). Then, we compute the residuals. In what follows, we use observational measurements of \( H(z) \) extracted from [41] – consisting of 30 data points – to constrain the free parameters in the model.

In addition to the parameters \( \zeta, \Omega_m \), and \( H_0 \), we must also consider the parameters associated with the stealth potential \( V(\phi) \). Given our choice of (9), the potential can be described by just one parameter that is fixed by relation (9). In fact, by writing \( V = V_0 \phi^2 \), from (9) we find that \( V_0 = -3H_0^2 \zeta \Omega_L \). In this way, it is not necessary to fit it along with the other three, because it depends on the best fit value of \( \zeta, H_0 \) and should be consistent with the known value of \( \Omega_L \).

Along these lines, it is clear that our Stealth have more free parameters than the original \( \Lambda \)CDM model (the former has four and the latter three). However, as we just mentioned, the only free parameters that we can fix using (13) and (14) are \( \zeta, \Omega_m \) and \( H_0 \). After the fit (assuming the plus sign in \( \Xi \), we get \( h = 0.59 \pm 0.02 \), \( \Omega_m = 0.10 \pm 0.05 \) and \( \zeta = 0.10 \pm 0.12 \). In Fig. (11) we show the 1\( \sigma \) and 2\( \sigma \) C.L. among the free parameters. In Fig. (2) we display the data points used to constrain the model along with the best theoretical curve.

The reduced Hubble parameter (\( h = H_0/100 \)) is the most sensitive parameter in the fit. As we mentioned in the last paragraph, this parameter essentially controls the amplitude of the theoretical curve displayed in Fig. (2). By contrast parameters \( \Omega_m \) and \( \zeta \) are not very sensitive to changes, so it was very difficult to find a best fit set based on the \( H(z) \) data. A close study of the system of Eqs. (13) and (14) enables us to understand this behavior. In fact, the use of observational data for each value of \( E(z) \) – instead of using an analytical expression for it – renders these two parameters highly correlated.

IV. Conclusions

In this paper we have shown an example of how the Stealth can operate during the cosmological evolution describing the \( \Lambda \)CDM model. This example make use of an explicit quadratic potential for the Stealth, and opens the possibility of extends this finding using other forms of \( V(\phi) \). We have also re-write the Stealth equations in a way to find explicitly the equivalence between the \( \Lambda \)CDM content – non-relativistic matter and a cosmological constant – and the Stealth field and its potential. It was through this philosophy – working with the Stealth equivalent of the \( \Lambda \)CDM – that we have found the example we studied.

To put our model to the test, considering the special features of the current cosmological model, we opt not to
FIG. 1. Here we display the confidence level contours, at 1σ and 2σ, for the parameters of the model.

FIG. 2. We display the theoretical curve together with the data points for the $H(z)$ measurements obtained from [41], i.e., the function $E(z)$ extracted from (14) – together with the data is shown in Fig. 2, showing the capacity of the stealth to describe the observational data directly.

Although by construction the stealth mimics the evolution of the ΛCDM model, we have performed a direct test of the model against observational data, and we have found that in this case the best fit modifies the value for $\Omega_m$ (instead of the typical $\simeq 0.27$, by our value $\simeq 0.1$) at the expense of fixing an extra parameter, $\zeta$, which is absent in the ΛCDM model.

Finally, we would like to emphasize the importance of understanding the potential role the stealth may play in cosmic evolution. As we have shown, although stealth does not back-react to the space-time, it can describe both dark contributions at once, accounting for almost 95% of the matter content of the universe. There is no doubt that we must continue to explore the consequences of the stealth in the recent cosmic evolution as well as in the early ages of the universe.

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REFERENCES

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[1] C. Wetterich, Nucl. Phys. B 302, 668, 1988
[2] B. Ratra and P.J.E. Peebles, Phys. Rev. D 37, 3406, 1988
[3] J.A. Frieman , C.T. Hill , A. Stebbins and I. Waga, Phys. Rev. Lett. 75, 2077, 1995
[4] M.S. Turner and M. White, Phys. Rev. D 56, R4439, 1997
[5] R.R. Caldwell , R. Dave and P.J. Steinhardt, Phys. Rev. Lett. 76, 1582, 1998
[6] P.J. Steinhardt, L. Wang , and I. Zlatev, Phys. Rev. D 59, 123504, 1999
[7] S. Tsujikawa, Lect. Notes Phys, 800, 99, 2010
[8] S. Capozziello and M. De Laurentis, Phys. Rept., 509, 167, 2011
[9] G. D. Starkman, Phil. Trans. Roy. Soc. Lond. A, 369, 5018, 2011
[10] M. P. Dabrowski, Astrophys. J. 447, 43, (1995); M. P. Dabrowski and M. A. Hendry, Astrophys. J. 498, 67, (1998).
[11] J. F. Pascual-Sanchez, Mod. Phys. Lett. A14, 1539 (1999).
[12] M.-N. Celerier, Astron. and Astrophys. 353, 63 (2000).
[13] K. Tomita, Astrophys. J. 529, 382011 (2000); MNRAS 326, 287 (2001)
[14] D. Bertacca, N. Bartolo, & S. Matarrese, Advances in Astronomy, 2010, 904379 (2010).
[15] V. Sahni and L.-M. Wang, Phys. Rev. D62 103517, (2000).
[16] A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B511, 265-268,(2001).
[17] M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D66, 043507, (2002).
[18] R. J. Scherrer, Phys. Rev. Lett. 93, 011301, (2004).
[19] C.G. Jr. Callan, S. Coleman, & R. Jackiw, R., Annals of Physics, 59, 42 (1970).
[20] E. Ayón-Beato, C. Martínez and J. Zanelli, Gen. Rel. Grav. 38, 145 (2006) [hep-th/0403228].
[21] E. Ayón-Beato, C. Martínez, R. Troncoso and J. Zanelli, Phys. Rev. D 71, 104037 (2005) [hep-th/0505086].
[22] E. Ayón-Beato, A. A. García, P. I. Ramírez-Baca and C. A. Terrero-Escalante, Phys. Rev. D 88, no. 6, 063523 (2013) [arXiv:1307.6534 [gr-qc]].
[23] E. Ayón-Beato, C. Martínez, R. Troncoso, and J. Zanelli, Stealths overflying (A)DSs, in preparation.
[24] E. Ayón-Beato, M. Hassaine, M. M. Juárez-Aubry. [arXiv:1506.03545 [gr-qc]].
[25] M. M. Caldarelli, C. Charmousis, M. Hassaine. JHEP 1310, 015 (2013).
[26] M. Bravo-Gaete, M. Hassaine. JHEP 1311, 177 (2013).
[27] M. Bravo-Gaete, M. Hassaine. Phys.Rev. D88, 104011 (2013).
[28] M. Hassaine. Phys.Rev. D89, 4, 044009 (2014)
[29] M. Bravo-Gaete, M. Hassaine. Phys.Rev. D90 2, 024008 (2014).
[30] E. Ayón-Beato, P. I. Ramírez-Baca and C. A. Terrero-Escalante. (2015) [arXiv:1512.09375[gr-qc]].
[31] Sokolowski, L. M., Acta Phys.Polon. B35, 587, (2004).
[32] M. Bravo-Gaete, M. Hassaine. JHEP 10, 015 (2015).
[33] A. Alvarez, C. Campuzano, V. Cardenas, R. Herrera and E. Rojas in preparation.
[34] N. Banerjee, R. K. Jain and D. P. Jatkar, Gen. Rel. Grav. 40, 93 (2008) [arXiv:hep-th/0610109].
[35] H. Maeda and K.I. Maeda, Phys. Rev. D 86, 124045 (2012) [arXiv:1208.5777 [gr-qc]].
[36] Ayón-Beato, Eloy et al. Phys.Rev. D88, 6, 063523 (2013) [arXiv:1307.6534]
[37] L. Perivolaropoulos, Journal of Physics: Conference Series, Volume 222, Issue 1, id. 012024 (2010).
[38] S. Perlmutter, et al., Astrophys. J. 517, 565 (1999)
[39] A.G. Riess, et al., Astron. J. 116, 1009 (1998).
[40] P. Ade, et al. [Planck Collaboration] Astron. and Astrophys. (2014).
[41] M. Moresco et al., JCAP 1605, no. 05, 014 (2016) doi:10.1088/1475-7516/2016/05/014 [arXiv:1601.01701 [astro-ph.CO]].