Research Article

Fuzzy Adaptive Control for a Class of Nonlinear System with Prescribed Performance and Unknown Dead-Zone Inputs

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Based on fuzzy logic systems and prescribed performance technology, this paper investigated the stable tracking problem of a class of nonlinear systems with unknown dead-zone inputs. Firstly, the original system is transformed into an equivalent system. Then, a Nussbaum-type function is added in the predefined performance controller to estimate the unknown control direction, and the boundedness of all signals in the closed-loop system is guaranteed. In addition, tracking errors will always remain within the prescribed performance boundaries. Finally, a comparative simulation example shows the effectiveness of the method proposed in this paper.

1. Introduction

As is known to all, the dead zone may badly limit the system performance, such as electronic circuits and electric servomotors [1–5]. So, many researchers used different control methods (i.e., backstepping technique, fuzzy systems, neural networks, sliding mode control, and so on) to research the stability of nonlinear systems with unknown control directions [6–9]. For example, based on the terminal sliding mode control, Chen and Ren in [6] proposed an adaptive neural control scheme for permanent magnet synchronous motor (PMSM) servo system with dead-zone input. For uncertain discrete-time systems with nonaffine dead-zone input, Liu and Tong [7] used adaptive neural networks tracking control such that the tracking error converges to a small neighborhood of zero. By using high-dimensional integral Lyapunov–Krasovskii functional theory and direct adaptive control scheme, control of uncertain MIMO systems with time-varying delay and dead-zone input was considered in [8]. In [9], the authors investigated an adaptive backstepping control method for an SISO system with unknown dead-zone input. For eliminating the influence of unknown control directions, one of the commonly applied methods is the Nussbaum-type function technique [10–12].

Liu et al. [10] constructed an adaptive neural controller for discrete-time system, which made all signals of the closed-loop system bounded in the condition of unknown dead-zone input. For a class of MIMO stochastic systems in [11], Li and Tong put forward an adaptive fuzzy output-feedback control method by using Nussbaum gain function and fuzzy logic systems. Xu [12] applied the adaptive neural control scheme to guarantee that the flexible hypersonic flight vehicle system is bounded stable with dead-zone input.

However, control methods in aforementioned literature can only guarantee that the tracking error converges to a small residual set, rather than a small residual set with the prescribed performance bounds. For an uncertain nonlinear system [4, 13–15], in order to solve the above problem, the prescribed performance control (PPC) strategy has been proposed. The idea of this method is converting the original system to an equivalent system and ensuring the boundedness of the states of the equivalent system. For example, based on the backstepping technique and the PPC, Wang and Hu [13] proposed an improved PPC for the longitudinal model of an air-breathing hypersonic vehicle (AHV) with unknown dead-zone input nonlinearity. Yang and Chen [14] developed an adaptive neural PPC for near space vehicles (NSVs) with input nonlinearities including saturation and
dead zone. It can be seen from the literature [13, 14] that it has certain practical significance to adopt the PPC method in the practical system. For MIMO nonlinear systems with unknown dead-zone inputs, Shi et al. [4] proposed an adaptive fuzzy PPC scheme such that only three parametric adaptive rules were needed. Cao et al. [16] studied a fractional neural network model with impulse amplitude, time-delay time-varying, and reaction diffusion terms. Many other interesting results about PPC of nonlinear systems can be seen in [17–19].

Inspired by the work of [4, 20], this paper explores the stability problem that the tracking error meets prescribed performance boundary (PPB) with unknown dead-zone input. Compared with related works, the main contributions of this paper are listed as follows: (1) compared with the work of [14, 17, 19], the introduced error transformation variable $z_i$ can be directly differentiated without the participation of the transformation function. (2) The Nussbaum-type function is incorporated into the control method so that the influence of unknown control directions is eliminated. (3) The singular problem in the PPC for nonlinear systems can be avoided by using the proposed method.

The structure of this paper is as follows. In Section 2, the uncertain system model with unknown dead-zone inputs is raised and provides assumption and lemmas. Section 3 includes the designation of fuzzy adaptive prescribed performance control method. The results of simulation are presented in Section 4. In Section 5, a brief conclusion is given.

2. Preliminaries

Consider the following uncertain nonlinear system, which described as

$$
\begin{align*}
\dot{x}_1 &= f_1(t, x) + d_1(t) + b_1 \phi_1(u_1), \\
\dot{x}_2 &= f_2(t, x) + d_2(t) + b_2 \phi_2(u_2), \\
&\vdots \\
\dot{x}_n &= f_n(t, x) + d_n(t) + b_n \phi_n(u_n),
\end{align*}
$$

(1)

where $x = [x_1, x_2, \ldots, x_n]^T$ is the system state vector, $f_i(t, x)$ is an unknown smooth nonlinear function, $d_i(t)$ is an external disturbance, $\phi_i(u_i)$ denotes the control input subject to dead-zone type nonlinear, and $b_i$ is an unknown control coefficient, $i = 1, 2, \ldots, n$. Here, $\phi_i(u_i)$ is described as follows:

$$
\phi_i(u_i) = \begin{cases} 
  m_i (u_i - b_i), & \text{if } u_i \geq b_i, \\
  0, & \text{if } b_i < u_i < b_i, \\
  m_i (u_i + b_i), & \text{if } u_i \leq -b_i,
\end{cases}
$$

(2)

where $m_i, b_i$, and $b_i$ are unknown positive constants, $i = 1, 2, \ldots, n$. Then, we rewrite (2) as

$$
\phi_i(u_i) = m_i u_i + \Delta u_i,
$$

(3)

where

$$
\Delta u_i = \begin{cases} 
  -m_i b_i, & \text{if } u_i \geq b_i, \\
  -m_i u_i, & \text{if } b_i < u_i < b_i, \\
  m_i b_i, & \text{if } u_i \leq -b_i.
\end{cases}
$$

(4)

From (3), system (1) is described as

$$
\begin{align*}
\dot{x}_1 &= f_1(t, x) + d_1(t) + b_1 \phi_1(u_1), \\
\dot{x}_2 &= f_2(t, x) + d_2(t) + b_2 \phi_2(u_2), \\
&\vdots \\
\dot{x}_n &= f_n(t, x) + d_n(t) + b_n \phi_n(u_n),
\end{align*}
$$

(5)

Designing two adaptive prescribed performance control methods is the control objective of this paper. The goal of this paper is that the tracking error $e_i$ satisfies PPB, and all closed-loop system signals are bounded.

In order to meet the objective, we proposed the following assumptions.

**Assumption 1.** The system state $x$, the reference signal $x_d$, and its derivative $\dot{x}_d$ are available and bounded.

**Assumption 2.** The nonlinear function $f_i(t, x)$, the control coefficient $b_i$, and the external disturbance $d_i(t)$ are unknown but bounded, $i = 1, 2, \ldots, n$.

**Remark 1.** Assumptions 1 and 2 are commonly used to address the problem of unknown control direction [21–23]. And owing to the boundedness of $b_i, \Delta u_i$, and $d_i(t)$, we know that there exists an unknown positive constant $D_i^*$, such that $|b_i \Delta u_i + d_i(t)| \leq D_i^*, i = 1, 2, \ldots, n$.

Let $\bar{b}_i = b_i m_i$ and $D_i(t) = b_i \Delta u_i + d_i(t)$, then $|D_i(t)| \leq D_i^*$. The error state is defined as $e = [e_1, e_2, \ldots, e_n]^T = [x_1 - x_{1d}, x_2 - x_{2d}, \ldots, x_n - x_{nd}]^T$, and the following error system is acquired:

$$
\begin{align*}
\dot{e}_1 &= f_1(t, x) + D_1(t) + \bar{b}_1 u_1(t) - \dot{x}_{1d}, \\
\dot{e}_2 &= f_2(t, x) + D_2(t) + \bar{b}_2 u_2(t) - \dot{x}_{2d}, \\
&\vdots \\
\dot{e}_n &= f_n(t, x) + D_n(t) + \bar{b}_n u_n(t) - \dot{x}_{nd}.
\end{align*}
$$

(6)

The following definition and lemmas used in this paper are to overcome the problem of unknown control directions.

**Definition 1.** A continuous function $N(\cdot)$ is called a Nussbaum function, if $N(\cdot)$ satisfies

$$
\lim_{t \to +\infty} \sup \frac{1}{t} \int_0^t N(s)ds = -\infty,
$$

(7)

**Remark 2.** Generally, the selection of the Nussbaum functions for the controller design with unknown control direction is $\xi^2 \cos(\xi)$ or $\xi^2 \cos((\Pi/2)\xi)$ or $\ln(\xi + 1)$
cos(\sqrt[x]{\ln(\xi + 1)}) and so on. In this paper, we choose the even Nussbaum function \(\xi^2 \cos(\xi)\) to design the PPC controller.

**Lemma 1** (see [15]). Let \(V(t)\) and \(\xi(t)\) be smooth functions, \(V(t) \geq 0\) for all \(t \in [0, t_f]\), and \(N(\cdot)\) be an even smooth Nussbaum function. If the following inequality holds:

\[
V(t) \leq r_0 + e^{-r_1 t} \int_0^t (a(x(s))N(\xi(s)) + 1) \cdot \xi(s)e^{-r_1 s} ds, \\
\forall t \in [0, t_f],
\]

where \(r_0\) and \(r_1\) are constants with \(r_1 > 0\), \(a(x(t))\) is a time-varying parameter which takes values in the unknown closed intervals \(I = : [I^-, I^+]\) with \(0 \in I\), then \(V(t), \xi(t)\), and \(\int_0^t (a(x(s))N(\xi(s)) + 1) \xi(s)e^{-r_1 s} ds\) must be bounded on \([0, t_f]\).

**Lemma 2** (see [4]). Let \(A \in \mathbb{R}^{n \times n}\) be a symmetric matrix and \(x \in \mathbb{R}^n\) be a nonzero vector. Let \(\alpha = (x^T A x / x^T x)\). Then, there exist eigenvalues of \(A\) such that \(\lambda_1 \in (-\infty, \alpha]\) and \(\lambda_2 \in [\alpha, +\infty)\).

**Remark 3.** In this paper, we will construct the time-varying parameter \(a(t)\) by using Lemma 2. For example, let \(A = \text{diag}(a_1, a_2, \ldots, a_n)\) and \(z\) is a nonzero vector; we can design \(a(t) = \alpha(t) = (z^T A z / z^T z)\), and then there exist two constants \(I^-\) and \(I^+\) such that

\[
I^- \leq \min_i [a_i] \leq \alpha(t) \leq \max_i [a_i] \leq I^+.
\]

In this paper, fuzzy logic systems will be used to approximate unknown nonlinear functions. Generally, the output of fuzzy logic system can be expressed as [24–32]

\[
\tilde{f} = \sum_{i=1}^{N} u_i w_i(t) \mu_i(x(t)),
\]

where \(x = [x_1(t), \ldots, x_n(t)]^T \in C^1 [\mathcal{F}, \Omega] (\forall t \in \mathcal{F})\) is the input vector. \(\tilde{f}: \Omega \rightarrow \mathcal{F}\) is a Lipschitz continuous mapping, \(\Omega \subseteq \mathbb{R}^m\) is a compact subset, and \(\mathcal{F}\) is a real line. \(I = \prod_{i=1}^n \mathcal{F}_i\), where \(\mathcal{F}_i\) consists of \(N_i\) fuzzy sets \((1 \leq j \leq n)\). The membership function of the \(i\)-th rule is \(\mu_i(i \in I)\), where \(\mu_1: \mathcal{F} \rightarrow [0, 1], w_i: \mathcal{F} \rightarrow (i \in 1, 2, \ldots, N)\), which represents the centroid of the \(i\)-th consequent set. Let \(\theta_i = [\theta_{i1}(t), \ldots, \theta_{in}(t)]^T\) and \(\varphi_i(t) = [\varphi_{i1}(x(t)), \varphi_{i2}(x(t)), \ldots, \varphi_{in}(x(t))]^T\), where \(\varphi_{ii}(i \in I)\) is the \(i\)-th fuzzy basis function and \(\varphi_{i}\) is a continuous mapping, then \(\varphi_{i}\) can be defined as

\[
\varphi_{i}(x(t)) = \frac{\mu_i(t)}{\sum_{i=1}^{N} \mu_i(x(t))}.
\]

Thus, (10) can be rearranged as

\[
\tilde{f}(x(t)) = \theta^T(t)\varphi(t).
\]

### 3. Synchronization Controller Design

#### 3.1. Prescribed Performance

The steady-state performance of \(e_i\) can be preserved by setting the performance constraint condition as

\[
-\lambda_2 \mu(t) < e_i < \lambda_1 \mu(t), \quad i = 1, 2, \ldots, n,
\]

where \(\lambda_1\) and \(\lambda_2\) are positive constants, which can regulate the error boundaries. \(\mu(t)\) is denoted by

\[
\mu(t) = (\mu_0 - \mu_{\infty})e^{-\tau t} + \mu_{\infty},
\]

where \(\tau > 0\) and \(\mu_0, \mu_{\infty} > 0\). Obviously, \(\mu(t)\) is a decreasing function and \(\mu(0) = \mu_0, \lim_{t \rightarrow +\infty} \mu(t) = \mu_{\infty}\).

We abbreviate \(\mu(t)\) to \(\mu\) and give an error transformation as

\[
z_i = \frac{\lambda_1 e_i}{\lambda_1 \mu - e_i} l(e_i) + \frac{\lambda_2 e_i}{\lambda_2 \mu + e_i} (1 - l(e_i)),
\]

where \(l(e_i)\) satisfies

\[
l(e_i) = \begin{cases} 1, & e_i \geq 0, \\ 0, & e_i < 0. \end{cases}
\]

Obviously, if we choose the initial condition \(e_i(0)\) as \(-\lambda_2 \mu(0) < e_i(0) < \lambda_1 \mu(0)\) and \(z_i\) is bounded, then the error state \(e_i\) will satisfy the inequality that \(-\lambda_2 \mu < e_i < \lambda_1 \mu\). Because when \(e_i\) approaches \(-\lambda_2 \mu(0)\) or \(\lambda_1 \mu(0)\), \(z_i\) will approach infinity, which contradicts the boundedness of \(z_i\). So, here, the work for us is to prove that the error transformation variable \(z_i\) is bounded, \(z_{ij} = 1, 2, \ldots, n\).

#### 3.2. Control Design and Stability Analysis

The time derivative of \(z_i\) is

\[
\dot{z}_i = \mu \Pi e_i - \mu \Pi e_i,
\]

where \(\Pi_i = (\lambda_1^2/(\lambda_1 \mu - e_i)^2)l(e_i) + (\lambda_2^2/(\lambda_2 \mu + e_i)^2)(1 - l(e_i))\).

So, we obtain the following error transformation dynamic system.

Due to the fact that \(f_i(t, x)\) is unknown, we need to employ fuzzy logic system to approximate \(f_i(t, x)\). So, the approximate of function \(f_i(t, x)\) can be expressed as

\[
\hat{f}_i(x, \tilde{\theta}_i) = \tilde{\theta}_i^T \varphi_i(x), \quad i = 1, 2, \ldots, n,
\]

where \(\tilde{\theta}_i\) is the parameter vector and the fuzzy function vector is \(\varphi_i(x)\). For \(f_i(t, x)\), there is an optimal parameter vector \(\theta_i^{*}\), such that

\[
f_i(t, x) = \theta_i^{*T} \varphi_i(x) + e_i(x),
\]
where $\theta^*_f = \arg \min_{\theta_i} \left\{ \sup_{x} |f_i(t, x) - \overline{D}_i^T \phi_f(x)| \right\}$ and $\varepsilon_f(x)$ is the approximation error, which is bounded as $|\varepsilon_f(x)| \leq \varepsilon_f^*$, where $\varepsilon_f^*$ is a positive constant, $i = 1, 2, \ldots, n$.

Let $\overline{D}_i(t) = \varepsilon_f^*(x) + D_i(t)$, then $|\overline{D}(t)| \leq \varepsilon_f^* + D^*_i \leq \overline{D}_i^*$. The error transformation dynamic system is rewritten as follows:

\begin{align}
\dot{z}_1 &= \mu \Pi_1 f_1(t, x) + D_1(t) + \overline{D}_1 u_1(t) - \dot{x}_1, \\
\dot{z}_2 &= \mu \Pi_2 f_2(t, x) + D_2(t) + \overline{D}_2 u_2(t) - \dot{x}_2, \\
&\vdots \\
\dot{z}_n &= \mu \Pi_n f_n(t, x) + D_n(t) + \overline{D}_n u_n(t) - \dot{x}_n.
\end{align}

Next, we design the controller $u_i$ to guarantee the boundedness of $z_i$. Let $z = [z_1, z_2, \ldots, z_n]^T, \Pi = [\Pi_1, \Pi_2, \ldots, \Pi_n]^T$, and another controller as follows:

\begin{align}
\dot{u}_i &= \frac{z_i \Pi (\xi \overline{m})}{\mu \Pi_i}, \\
\overline{m} &= \frac{\Delta^2}{\Delta \|z\|^2 + \sigma}, \\
\xi &= \|z\|^2 \overline{m},
\end{align}

where $\Delta = \Delta_1 + \Delta_2$, $\Delta_1 = \mu \hat{\theta} \|z\| \sqrt{\sum_{i=1}^n \|\phi_f(x)\|^2}, \Delta_2 = \mu \|z\|^2$, $\dot{\theta} = \sum_{i=1}^n \|\phi_f(x)\|^2 \cdot ||z||$, $\theta^* = \sqrt{\sum_{i=1}^n \|\phi_f(x)\|^2}, D^* = \sum_{i=1}^n D_i^2$, $\overline{m} = \frac{\Delta^2}{\Delta \|z\|^2 + \sigma}$.

Let $\overline{D}_i(t) = \varepsilon_f^*(x) + D_i(t)$, then $|\overline{D}(t)| \leq \varepsilon_f^* + D^*_i \leq \overline{D}_i^*$. The error transformation dynamic system is rewritten as follows:

\begin{align}
\dot{z}_1 &= \mu \Pi_1 f_1(t, x) + D_1(t) + \overline{D}_1 u_1(t) - \dot{x}_1, \\
\dot{z}_2 &= \mu \Pi_2 f_2(t, x) + D_2(t) + \overline{D}_2 u_2(t) - \dot{x}_2, \\
&\vdots \\
\dot{z}_n &= \mu \Pi_n f_n(t, x) + D_n(t) + \overline{D}_n u_n(t) - \dot{x}_n.
\end{align}

Next, we design the controller $u_i$ to guarantee the boundedness of $z_i$. Let $z = [z_1, z_2, \ldots, z_n]^T, \Pi = [\Pi_1, \Pi_2, \ldots, \Pi_n]^T$, and another controller as follows:

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\overline{m} &= \frac{\Delta^2}{\Delta \|z\|^2 + \sigma}, \\
\xi &= \|z\|^2 \overline{m},
\end{align}

where $\Delta = \Delta_1 + \Delta_2$, $\Delta_1 = \mu \hat{\theta} \|z\| \sqrt{\sum_{i=1}^n \|\phi_f(x)\|^2}, \Delta_2 = \mu \|z\|^2$, $\dot{\theta} = \sum_{i=1}^n \|\phi_f(x)\|^2 \cdot ||z||$, $\theta^* = \sqrt{\sum_{i=1}^n \|\phi_f(x)\|^2}, D^* = \sum_{i=1}^n D_i^2$, $\overline{m} = \frac{\Delta^2}{\Delta \|z\|^2 + \sigma}$.

\begin{align}
\dot{V} &= -\frac{1}{2} z^T z + z^T \left( z + \frac{z}{2} \right) \theta \theta^* + \overline{D} \overline{D} \\
&= -\frac{1}{2} z^T z + \sum_{i=1}^n \left( z_i \mu \Pi_i \theta^*_f \phi_f(x) + z_i \mu \Pi_i \overline{D}_i (t) - \mu \|z_i \| \Pi_i \|e_i\| - z_i \dot{x}_i \mu \Pi_i + \frac{z_i^2}{2} \right) + \sum_{i=1}^n \left( z_i \mu \Pi_i \overline{D}_i u_i + \dot{\theta} \theta + \overline{D} \overline{D} \right) \\
&\leq -\frac{1}{2} z^T z + \sum_{i=1}^n \left( z_i \mu \Pi_i \|\theta^*_f\| \|\phi_f(x)\| + \|z_i \| \mu \Pi_i |\overline{D}_i (t)| + \|z_i \| \mu \Pi_i \|e_i\| + z_i \| \dot{x}_i \| \mu \Pi_i + \frac{z_i^2}{2} \right) + \sum_{i=1}^n \left( z_i \mu \Pi_i \overline{D}_i u_i + \dot{\theta} \theta + \overline{D} \overline{D} \right) \\
&\leq -\frac{1}{2} z^T z + \mu \|z\| \Pi \sum_{i=1}^n \|\theta^*_f\| \|\phi_f(x)\| + \|z\| \|\Pi\| \sum_{i=1}^n \overline{D}_i^2 + \|z\| \sum_{i=1}^n |\mu \Pi_i \|e_i\| + \|z\| \sum_{i=1}^n \| \dot{x}_i \| \mu \Pi_i + \frac{1}{2} \|z\|^2 \\
&+ \sum_{i=1}^n \left( z_i \mu \Pi_i \overline{D}_i u_i + \dot{\theta} \theta + \overline{D} \overline{D} \right).
\end{align}
According to Cauchy–Schwarz inequality,
\[
\sum_{i=1}^{n} ||\tau_i|| \cdot ||\varphi_f(x)|| \leq \left( \sum_{i=1}^{n} ||\tau_i||^2 \cdot \sum_{i=1}^{n} ||\varphi_f(x)||^2 \right)^{1/2}
\]
\[
= \theta^* \sqrt{\sum_{i=1}^{n} ||\varphi_f(x)||^2}
\]
\[
= (\bar{\theta} + \hat{\theta}) \sqrt{\sum_{i=1}^{n} ||\varphi_f(x)||^2}.
\]
Considering its adaptive law (21), it is concluded that
\[
\hat{\theta} = -\tilde{\theta}
\]
\[
= -\hat{\theta} \left( \mu||\varphi_f(x)|| \sqrt{\sum_{i=1}^{n} ||\varphi_f(x)||^2} \cdot ||z|| - \gamma_1 \tilde{\theta} \right)
\]
\[
= -\hat{\theta} \mu ||\varphi_f(x)|| \sqrt{\sum_{i=1}^{n} ||\varphi_f(x)||^2} \cdot ||z|| + \gamma_1 \hat{\theta}
\]
\[
\leq -\frac{\gamma_1}{2} \theta^2 - \hat{\theta} \mu ||\varphi_f(x)|| \sqrt{\sum_{i=1}^{n} ||\varphi_f(x)||^2} \cdot ||z|| + \frac{\gamma_1}{2} \theta^2.
\]
Similarly,
\[
\hat{D} \hat{D} \leq -\frac{\gamma_2}{2} \hat{D}^2 - \hat{\theta} \mu ||\varphi_f(x)|| ||z|| + \frac{\gamma_2}{2} \hat{D}^2.
\]
So, one gets
\[
\dot{V} \leq -\varphi V + \varphi \left( \mu \hat{\theta} \mu \sqrt{\sum_{i=1}^{n} ||\varphi_f(x)||^2} \right)
\]
\[
+ \mu ||\varphi_f(x)|| \hat{D} \sum_{i=1}^{n} |\hat{\theta}| ||\varphi_f(x)|| + \sum_{i=1}^{n} x_i \theta_i \theta_i
\]
\[
+ \frac{||z||}{2} + \sum_{i=1}^{n} z_i b_i z_i N(\xi) \mu + \tilde{\theta} \tilde{\theta},
\]
where \( \varphi = \min\{1, \gamma_1, \gamma_2\} \) and \( \tilde{\theta} = (\gamma_1/2) \theta^2 + (\gamma_2/2) \hat{D}^2. \)

For \( z_i \neq 0, \) we define \( \alpha(t) = \{(\sum_{i=1}^{n} z_i \beta_i z_i)/(\sum_{i=1}^{n} z_i^2)\} = (z^T U z/z^T z), \) where \( U = \text{diag}(\beta_1, \beta_2, \ldots, \beta_n). \) By using Lemma 2, there are two constants \( l^* \) and \( l^* \) such that
\[
\alpha(t) \leq \beta(t) \leq \max b_i \mu.
\]
So,
\[
\sum_{i=1}^{n} z_i \beta_i z_i = ||z||^2 \alpha(t).
\]

Obviously, if \( z_i = 0, (28) \) also holds. By substituting \( u_i(t) \) in (20) into (27), it yields
\[
\dot{V} \leq -\varphi V + \alpha(t) N(\xi) + 1 \hat{\xi} + \tilde{\theta} \tilde{\theta},
\]
where \( \tilde{\theta} = \bar{\theta} + \sigma. \) Integrating (29) over \([0, t], \) one obtains
\[
V \leq V(0) + \frac{\tilde{\theta} \tilde{\theta}}{\varphi} + \int_0^t (1 + \alpha(s) N(\xi)) \hat{\xi} \dot{\xi} ds.
\]

According to Lemma 2, \( V(t) \) is bounded into \([0, t_j]. \) So, \( z_i \) is bounded for any given positive constant \( t_j, \) and then the boundedness of \( z_i \) is true for \( t_j = +\infty \) (see [20]).

Through the above analysis and discussion, the following theorem is obtained.

**Theorem 1.** Consider the uncertain nonlinear system (1) with dead-zone inputs and unknown control directions. The proposed control method (20) can guarantee the boundedness of all the signals in the closed-loop system. Furthermore, for any \( \varepsilon_i(0) \) satisfies \(-\lambda \mu(0) < \varepsilon_i(0) < \lambda \mu(0), \) the error \( \varepsilon_i \) will satisfy the prescribed performance (13).

**4. Simulation Studies**

Consider the Chen chaotic system as a simulation example:
\[
\begin{align*}
\dot{x}_1 &= 35x_2 - 35x_1 + d_1(t) + b_1 \phi_1(u_1), \\
\dot{x}_2 &= -x_1 x_3 - 7x_1 + 28x_2 + d_2(t) + b_2 \phi_2(u_2), \\
\dot{x}_3 &= -3x_3 + x_1 x_3 + d_3(t) + b_3 \phi_3(u_3),
\end{align*}
\]
where \( d_1(t) = d_2(t) = d_3(t) = 2 \sin(2t), \) \( b_1 = 2, b_2 = -2, b_3 = 2, \) and \( \phi_i(u_i) \) is described as
\[
\phi_i(u_i) = \begin{cases}
3(u_i - 2), & u_i \geq 2, \\
0, & -2 < u_i < 2, \\
3(u_i + 2), & u_i \leq -2,
\end{cases}
\]
\( i = 1, 2, 3. \) One reference signal \( x_i = [\sin(t) \sin(t) \sin(t)]^T. \) Choose the fuzzy membership function as.

Firstly, we use the method in [15] to control Chen chaotic system (31), and controller is designed as follows:
\[
\begin{align*}
u_i &= \xi_i \cos(\xi_i) \nu_i, \\
\hat{\nu}_i &= \tilde{\theta}_i \phi_f(x) + \tilde{\theta} \theta_i \theta_i, \\
\hat{\xi}_i &= \epsilon_i \nu_i, \\
\hat{\theta}_i &= \epsilon_i \nu_i, \quad i = 1, 2, 3,
\end{align*}
\]
where \( \tilde{\eta}_f \), \( \varphi_f \) \( (x) \) is the estimate of \( f_i(x) \) and \( f_i(x) = \eta f_i^T \varphi_f(x) + \varepsilon f_i(x) \), where \( \varepsilon f_i(x) \) is the bounded approximation error. Let \( |d_i(t) + \varepsilon f_i(x) + \dot{x}_d| \leq \rho_i \), where \( \rho_i \) is the estimate of \( \rho_i \). The parameters are \( \varepsilon = 0.5 \), \( y_{i1} = 3 \), \( y_{i2} = 2 \), \( i = 1, 2, 3 \). The initial values are \( x_1(0) = 2 \), \( x_2(0) = 1 \), \( x_3(0) = -2 \), \( \xi_1(0) = 0.01 \), \( \xi_2(0) = 0.1 \), \( \xi_3(0) = 0.1 \), \( \tilde{\rho}_1(0) = 0.05 \), \( \tilde{\rho}_2(0) = -0.3 \), \( \tilde{\rho}_3(0) = 0.4 \), \( \tilde{\eta}_{f_1}(0) = \tilde{\eta}_{f_2}(0) = \tilde{\eta}_{f_3}(0) = 0 \), \( \mu(t) = 3 e^{-2t} + 0.1 \). By using control scheme (33), the trajectories of \( x_1, x_2, \) and \( x_3 \) are shown in Figures 1–3. The corresponding error trajectories are shown in Figure 4. It can be seen that, in the initial control stage, \( e_1 \) and \( e_2 \) will have a jump beyond the prescribed performance boundary (PPB) \([-\mu(t), \mu(t)]\) and then remain within the BBP \([-\mu(t), \mu(t)]\), while \( e_3 \), on the contrary, is within the PPB \([-\mu(t), \mu(t)]\) at the initial stage and stays outside the PPB \([-\mu(t), \mu(t)]\) after 3 seconds.

In order to compare with the control scheme (33), we employ our proposed control scheme (20). The parameters are \( \lambda_1 = \lambda_2 = 1 \) and \( \sigma = 0.5 \), and the initial values are \( \tilde{D}_1 = 0.2 \), \( \tilde{D}_2 = 0.3 \), and \( \tilde{D}_3 = 0.4 \). Other parameters and initial values in this example are same as those of the control scheme (33). The simulation results are shown in Figures 5–8. The states \( x_1, x_2, \) and \( x_3 \) can track the reference signal \( x_1_{d}, x_2_{d}, \) and \( x_3_{d} \) quickly after a transient response process in Figures 5–7. From Figure 8, errors \( e_1, e_2, \) and \( e_3 \) are not beyond the PPB \([-\mu(t), \mu(t)]\). In Figure 9, two
These results verify that the proposed control scheme (20) has a better control effect than control scheme (33).

5. Conclusions

The problem of robust fuzzy adaptive prescribed performance control for uncertain nonlinear system is investigated. The proposed control approach is to make sure that all signals in closed-loop system are bounded and tracking errors converge to a small neighborhood of zero with the prescribed performance bounds. The Nussbaum-type function is incorporated into two control methods such that the influence of unknown control directions is eliminated. It is unnecessary to use the inverse operation in the proposed methods. Simulation results verify the effectiveness and robustness of the proposed method. How to effectively estimate the unknown function of the nonlinear system is the future research direction.

Data Availability

All datasets generated for this study are included within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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