Targeting Precision in Imperfect Targeted Advertising: Implications for the Regulation of Market Structure and Efficiency

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Abstract
With the rapid development of e-commerce and related information technologies, consumers’ individual information can be accurately collected and analyzed according to their previous purchase histories or browsing behaviors. Then the targeted advertising can be directly sent to the heterogeneous consumers in the segmented market by the firms. This study aims to explore the possible role of targeting precision on the market structure. This assay set up a game-theoretical model with duopolistic firms. Both firms compete in prices and imperfect targeted advertising. In this setup, the targeting precision is variable. The results indicate that each competitive firm’s equilibrium profit varies with the targeting accuracy of targeted advertising and shows a U-shape variation with precision. Moreover, both firms’ profits reveal monotonic properties, improving targeting precision while using distinct advertising strategies. Then, the model is extended into n different competitive firms to seek the possible market capacity. The results prove that the equilibrium free-entry numbers of firms also exhibit a U-shape variation as a function of targeting precision while sending the imperfect targeted advertising to the consumers. This conclusion implies that the firm conducting imperfect targeted advertising with a moderate targeting precision is beneficial for the most efficient free-entry of new rival competitive firms and shows the most effective efficiency for the whole market.

Keywords
imperfect targeted advertising, U-shape, quality of information, targeting precision, market structure

Introduction
Advertising is deemed one of the most effective marketing strategies for the firm while informing the firm’s potential consumers about the specific existence, prices, and other characteristics of new products. In the view of informative advertising, the fundamental role of advertising is to transmit the product’s relative messages to uninformed consumers. In response to controlling an advertising cost, equilibrium prices of the product fall, and the market converges to the socially optimal resource allocation for brands that are sufficiently close substituted (Hamilton, 2009). However, due to asymmetry between the firm and consumers, some consumers might conceal their personal details. It is pretty difficult for the firm to acquire accurate, complete information of consumers, even adopting advanced information tracking technologies. Therefore, it is always a challenge for a firm to send the advertisements (ads) to the targeted consumers in the segmented market who might purchase the product.

Consequently, some consumers might never purchase the products even receiving the related ads. Quite many ads using traditional media, including TV channels and newspapers, are wasted for the firm. Recent advances in information technologies, especially IP-tracking technology and data mining with the application of database, allow e-commerce firms to collect, analyze and share the consumers’ detailed information, thereby making promotional strategies including targeted advertising and pricing (Athey & Gans, 2010; Esteves, 2014). Therefore, consumer-specific data can be facilitated to gather, store, and process. More accurate advertising messages can be sent to the segmented consumers who might purchase the products.

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According to the advertisers’ perspectives, targeted advertising is considered a potent marketing tool (Iyer et al., 2005). For one thing, the wasted ads could be greatly reduced, and the direct information of products could be sent to specific consumer groups. For another, the firms could directly measure the specific value of advertising on the consumers’ profiles, which could adjust their advertising strategies more effectively. Consequently, targeted advertising can be used with price discrimination simultaneously. Iyer et al. (2005) explored the advertising strategies within a market when competitive firms can send targeted advertising to distinct groups of consumers. Their results indicated that the firms’ equilibrium profits could be increased with the targeting of advertising, no matter whether or not the firms have the ability to set targeted prices. Likewise, Esteban et al. (2003) demonstrated that the equilibrium market price of the product was likely to be enhanced, and the level of advertising was reduced while using targeted advertising. Additionally, the socially optimal degree of media specialization chosen might be exceeded by the monopolist. Even in the early studies, Hernández-García (1997) established a horizontal differentiation model in a monopoly performing at a specific type of targeting, in which potential consumers with a high valuation of products can be reached, and revealed that both consumers’ surplus and social welfare might be lower by the targeting in such monopolistic framework. However, in Bergemann and Bonatti’s (2011) previous report, the multiple advertising messages can be redundant. Their model assumed that online media showed better target ability than offline media, and the enhanced competition from online sources showed ambiguous effects on advertising prices. Likewise, Johnson (2013) proved that when the firm’s targeting ability increased and could target their particular individuals in the market, the targeting might always be beneficial for the consumers even the consumers could obtain the advertising avoidance tools. Obviously, these studies have confirmed the positive role of targeted advertising on the firm’s equilibrium profit. However, some other studies argued the role of targeted advertising in a specific market on a firm’s equilibrium profit. For instance, Gal-Or et al. (2006) established a Hotelling model with two specific measures, including accuracy and recognition, which aims to explore the conventional measurement of targeting quality. They demonstrated that the improvement of recognition induces intensified price competition in between and reduces the total market profitability. Later, Anand and Shachar (2009) constructed a signaling model of advertising while selling products with horizontal differentiation. They confirmed that there was no equilibrium of targeting when advertisements contained none of the information. Additionally, the choice of ads as a signal is inherently arbitrary. Likewise, Ben Elhadj-Ben Brahim et al. (2011) assumed that duopolistic firms have perfect targeting ability and demonstrated that the competing firms’ equilibrium profits might be lower with targeted advertising than mass advertising. Subsequently, Zhang and Katona (2012) considered an intermediary accessible to a content base and sell advertising space to advertisers competing in the product market while using targeting technology. Their results revealed that the intermediaries intentionally reduced the targeting accuracy at the expense of the advertisers. Bimpikis et al. (2016) prove that firms invest inefficiently high at equilibrium in targeted advertising. The extent of the inefficiency is increasing in the centralities of the agents they target. In a recent study, the targeting extent on the firm’s equilibrium profit is investigated (Zhao et al., 2018). The authors prove that the targeting extent plays a bidirectional regulatory role in a firm’s equilibrium profit. Likewise, Jiang et al. (2020) starts by setting up a model with two horizontally differentiated firms competing in prices and targeted advertising at a variable targeting precision in an initially uninformed heterogeneous consumers market. The results indicate that both e-business firms should target only their advantage markets with optimal targeting precision. Consequently, these previous studies explore the possible role of targeted advertising on the firm’s equilibrium profit and competition and compare targeted advertising and mass advertising. Due to their different assumptions on the firm’s competing market structure in their specific models, the opposite results of the effect of targeted advertising on the equilibrium profit of the firm occur. In fact, in most conditions, the firms might only obtain imperfect information about the consumers due to asymmetric properties between the firms and consumers, thereby sending the imperfect targeted advertising towards the most potential customers at distinct precisions. Therefore, two imperative simplifying assumptions are considered in this article to reduce the complexities of the model: (1) Both firms have equal size of consumers in all segments according to their preferences; (2) The distribution of consumers is uniform in a linear city at a length of one. In addition, the three-stage game is established as follows: (1) Stage 1, Acquisition decisions on targeting precision. The information quality is deemed as an integer parameter \( k \), and the targeting precision \( (R) \) is considered as \( R = 1 - 2^{-k} \). After acquiring the information simultaneously and independently, the firms decide whether to acquire it or not. (2) Stage 2, Regular pricing decisions. The regular prices of the products are chosen simultaneously and independently by both firms. (3) Stage 3, Advertising decisions. Firms choose different advertising strategies for the sales of the products.

Therefore, a game-theoretical model of duopolistic firms competing with distinct advertising strategies is formulated. The following questions are addressed in this article. (1) Does a high targeting precision mean a relative higher profit for the competitive firm while sending imperfect targeted advertising to the consumers? (2) How does the precision of targeted advertising affect competitive firms’ equilibrium profits? (3) Whether both firms have incentives for information acquisition, profits and welfare evolve as targeting precision improves? (3) In a complete competitive marketing
condition, how does the new entry firm regulate the targeting precision of imperfect targeted advertising to acquire the equilibrium optimal profit?

Related Literature

Targeted advertising in this study is related to previous literature on informative advertising (Butters, 1977; Celik, 2007; Grossman & Shapiro, 1984; Soberman, 2004). In recent years, more and more attention has been paid to exploring the firm’s targeting ability. Athey and Gans (2010) identified a supply-side impact on targeting: it allowed more efficient allocation of scarce advertising space, resulting in an increase in the supply of ads space that might push down the advertising price. Then, Taylor (2013) mainly addressed targeting accuracy on competition in product markets and media choices regarding content differentiation. However, in other two empirical studies, Chandra (2009) on newspapers and Chandra and Kaiser (2010) on magazines showed that the ad prices are higher in markets with more homogeneous subscribers. Later, Chen and Stallaeart (2014) compared the price competition between advertisers using targeted and mass advertising. They proved that the advertiser’s competition could be softened by behavior targeting. Zhao and Xue (2012) explored the possible relationship between competitive targeted advertising and consumer data sharing. They revealed that the better-informed firm might not save the advertising expenditure, enabling it to reap a higher expected profit in competition. Zhang and He (2019) studies advertising competition between duopoly firms with asymmetric costs, proving that imperfect targeting restores the low-cost firm’s advantage and thus makes it better off.

In addition, this paper is closely related to a previous study from Rutt (2012), who proved that in competitive media markets, the entry of excessive media market and under-provision of advertising might be contributed by targeted advertising. Likewise, Gong et al. (2019) investigated the targeted advertising in the two-sided markets. They considered that more consumers and advertising firms might be attracted by the platform with a higher targeting ability. With the increase of the targeting ability of either platform, all consumers obtained benefits as they might incur lower nuisance costs from advertising. Besides applying targeted advertising in the platform, Boerman et al. (2017) overviewed the theoretical positioning of targeted advertising by placing the theories used to explain consumers’ responses to the advertising in the framework. However, considering two platforms with different targeting abilities, Gong et al. (2019) prove that the advantaged platform will have more advertising firms, attract more consumers, and become more profitable.

Additionally, previous studies on targeted price discrimination also reveal some hints. Chen et al. (2001) considered that when individual marketing is feasible but imperfect, improvements in targetability by either or both competitive firms can lead to win-win competition. Otherwise, when the cost of acquiring targetability is high, the firm with a large number of loyal customers tends to invest more in targetability. Later, Chen and Iyer (2002) examined the strategic implications of consumer addressability on competition between database/direct market firms. They considered that consumer addressability creates two effects called surplus extraction effect and competitive effect. Likewise, Liu and Serfes (2004) examined price competition between two symmetric firms (i.e., pure horizontal differentiation). As in the present paper, the level of segmentation depends directly on the underlying precision of customer information. Liu and Serfes (2005a) introduced a flexible third-degree price discrimination framework by modeling firms’ information about consumers’ locations (preferences) on the Salop circle as a partition. Later, based on consumer information of varying degrees of “precision,” Liu and Serfes (2005b) explored the competitive implications of third-degree price discrimination in a vertical differentiation duopoly model. Their results proved that the high-quality firm could only offer them targeted promotions. However, the opportunity costs of targeting to optimize promoted product sales are poorly understood. A series of randomized field experiments with a large e-book platform shows that although targeted promotions increase promoted product sales and purchases of similar products, they can crowd out different products (i.e., e-books from nontargeted genres) by decreasing search activities of nontargeted goods on the same platform (Fong et al., 2019). Otherwise, Media and advertising practices will continue to evolve, and advertising using traditional media in combination with the consumer-directed search will likely become the dominant form of advertising (Koslow & Stewart, 2021).

In contrast, the low-quality firm might commit a uniform price for any degree of consumer information precision. A previous study on targeting pricing (Chen & Zhang, 2009) set up a model with dynamic targeted pricing with strategic consumers. They showed that the perfect targeting might depend on the choice of the consumer. This result might induce the firms to adopt strategically targeted advertising (Galeotti & Moraga-Gonzalez, 2003). In a recent study, Shin and Yu (2021) find that the increase in consumer search creates an advertising spillover beyond the level of the mere awareness effects of advertising and that firms’ equilibrium level of targeted advertising can be non-monotonic in targeting accuracy. However, in our model, we consider the myopic consumers who only care about their current profit.

The rest of the paper organizes as follows. The model and four sub-games are presented in Section 2. The role of targeting precision of targeted advertising on each firm’s equilibrium profit in the game is analyzed in Section 3. Section 4 contains the model extension on the regulatory role of targeted advertising in regulating the heterogeneous market with n competitive firms. Section 5 concludes the main results.
The Description of the Model

Assumptions of the Model

Two competitive firms are located at a linear city of the endpoints of a unit interval selling the competitive brands to a continuum of customers who have only one unit demand. It assumes that each consumer derives a utility of \( V \) while purchasing a product from either one of the firms. Let \( p_i \) and \( p_j \) considered as the prices that firm \( i \) and \( j \) charge for the product, respectively. In addition, each consumer incurs a linear unit transportation cost of \( t \). Therefore, a consumer who is located at \( x \in [0,1] \) and purchases a product from firm\( i \) has a surplus of \( V - tx - p_i \). Likewise, a buyer at this location enjoys a surplus of \( V - t(1-x) - p_j \) while purchasing from the firm \( j \). All consumers belong to myopic consumers. They will never search for any information about the product. As reported in a previous study, a potential consumer could not be an actual buyer unless the firm invested in advertising for sales (Stahl, 1994). Each consumer will purchase the product at a certain probability as long as he receives the advertisement from the firm. Otherwise, when the consumer gets advertisements from both firms, he will purchase the product at a lower price from one firm. Without loss of generality, the marginal cost of the product in each firm is considered to be zero.

This setup assumes that all the firms can recognize the heterogeneous consumers with current information technologies uniformly distributed on the unit interval. Therefore, firms can acquire consumers’ information accurately by data mining, IP tracking technologies, and previous purchasing histories. According to the correlated information from consumers’ repeated past transactions, the consumers purchasing interest or preference can be segmented into distinct segmentation. As long as the firm obtains more detailed information about the consumer, the firm could segment the consumers more accurately. Therefore, the information partitions the \([0,1]\) intervals into \( N \) subintervals (indexed by \( m, m = 1, \cdots, N \)) of equal length in our setup. Then the competitive firms can send the targeted advertising to each subinterval of consumers at different advertising intensities \( (\phi_{im},i = 1,2 \text{ and } m,m = 1,\cdots, N) \). In this condition, the advertising intensity is the same within each subinterval (Figure 1).

The cost of reaching a fraction of \( \phi_{im} \) \((i = 1,2) \) is assumed to be \( A(\phi_{im}) = a\phi_{im}^2 / 2 \) with a maximum advertising expenditure of \( a / 2. \) Here, the parameter \( a \) is the cost parameter for advertising. The FOC condition on the advertising intensity is higher than zero, similar to previous reports (Ben Elhadj-Ben Brahim et al., 2011; Iyer et al., 2005).

We further assume that \( N = 2^k \) \((k = 0,1,2,\cdots) \). Hence, the parameter \( k \) represents the targeting quality, which determines the accurately recognized segments of consumers in a linear city. A higher \( k \) means a higher targeting precision of targeted advertising, which is an exogenous variable. Otherwise, we carry out two other assumptions to simplify the model: (1) Length of all segments is equal to \( 1/2^k \). (2) Distribution of consumers is uniform across the linear city. Therefore, we construct a game-theoretic model within competitive firms that include stage 1 on the regular pricing decisions and stage 2 on the competition of the advertising strategies with distinct targeting precision of targeted advertising.

The Four Sub-Games Between the Oligopolistic Firms Competing in Targeted Advertising With Distinct Precision

Subgame 1. Neither Firm Can Conduct Targeted Advertising (MA, MA)

The standard Hotelling model is considered with two firms competing in prices and mass advertising (MA) simultaneously. Each consumer is separately located in a city line with a distinct preference for each firm’s product, which is considered as \( x \). Both firms choose their regular prices in stage 2. We define \( \hat{x} \in [0,1] \) as the marginal consumer when totally informed of the existence of both products with advertising, enjoys equal utility between purchasing from the firm \( i \) and \( j \).

\[
V - tx - p_i = V - t(1-x) - p_j
\]

Therefore, we can get the marginal consumer of \( \hat{x} = \frac{p_j - p_i + t}{2t} \). In our assumptions above, all the consumers will purchase the product only after receiving the ads from competitive firms. As neither firm can conduct targeted advertising to the segmented consumers, they send the mass advertising (MA) to all the consumers in a line city. The firm’s profit can be expressed as follows:

\[
\pi_i = (p_i - c) \frac{p_j - p_i + t}{2t} \phi_i - a \phi_i^2 = \frac{p_i(p_j - p_i + t)}{2t} \phi_i - a \phi_i^2
\]

In equilibrium, both firms will perform price competition in stage1. Therefore, the price game indicates that each firm’s
first-order derivative equals zero, whereas the second-order derivative is less than zero.

\[
\frac{\partial \pi_i}{\partial p_i} = \frac{(-2p_i + p_j + t)\phi_i}{2t} = 0; \frac{\partial^2 \pi_i}{\partial p_i^2} = -2\phi_i < 0
\]  
(3)

\[
\frac{\partial \pi_j}{\partial p_j} = \frac{(-2p_j + p_i + t)\phi_j}{2t} = 0; \frac{\partial^2 \pi_j}{\partial p_j^2} = -2\phi_j < 0
\]  
(4)

According to the equations above (3) and (4), each competitive firm’s equilibrium price can be expressed as follows.

\[p_i^* = p_j^* = t\]  
(5)

Meanwhile, the FOC (first-order-condition) condition of each firm’s profit on each firm’s advertising intensity is shown as follows.

\[
\frac{\partial \pi_i}{\partial \phi_i} = p_i - p_j + t - a\phi_i = 0; \frac{\partial^2 \pi_i}{\partial \phi_i^2} < 0
\]  
(6)

\[
\frac{\partial \pi_j}{\partial \phi_j} = p_j - p_i + t - a\phi_j = 0; \frac{\partial^2 \pi_j}{\partial \phi_j^2} < 0
\]  
(7)

That means both firms’ equilibrium advertising intensity can be obtained below.

\[\phi_i^* = \phi_j^* = \min\left(\frac{t}{2a},1\right)\]  
(8)

Therefore, the firm's equilibrium profit are equal while providing mass advertising for the sales of products.

\[\pi_i^{MA,MA} = \pi_j^{MA,MA} = \begin{cases} 
\frac{t^2}{8a}; & t \leq 2a \\
\frac{t-a}{2}; & t > 2a 
\end{cases}
\]  
(9)

**Proposition 1.** When the two symmetry-competitive firms compete with mass advertising, their advertising intensities and profits are equal, showing a positive correlation with transportation cost \(t\) and a negative correlation with advertising parameter \(a\).

From proposition 1, it is easy to obtain that when the firm’s transportation cost is less than the typical advertising cost, its equilibrium profit shows a positive quadratic relationship with its transportation cost. However, suppose the transportation cost of the product is higher than that of the advertising cost. In that case, each firm’s equilibrium profit is correlated with the discrepancy between transportation cost and advertising cost.

**Subgame 2. Both Firms Can Conduct Targeted Advertising (TA, TA)**

Due to the rapid development of information technologies, both firms can acquire the consumers’ relative accurate information, thereby sending the targeted advertising in each consumer segment of \(N = 2^k\). Otherwise, the targeting intensity in each segment is equal to \(1/2^k\). The segment \(m\) can be expressed in the interval of \((N - 1)/2^k, N/2^k\), where \(m\) is an integer between 1 and \(2^k\) (Figure 1).

In the segment \(m\), firms 1 and 2 charge the advertising at an intensity of \(\Phi_{1N}\) and \(\Phi_{2N}\), respectively. The demands of their products are \(d_{1N} = p_{2N} - p_{1N} + t - \frac{N-1}{2^k}\) and \(d_{2N} = \frac{N}{2^k} - p_{2N} - p_{1N} + t\). Therefore, when both firms can conduct targeted advertising and price discrimination, their profits at segment \(m\) can be expressed as follows:

\[\pi_{1N} = p_{1N}\left(p_{2N} - p_{1N} + t - \frac{N-1}{2^k}\right)\Phi_{1N} - \frac{a}{2}\Phi_{1N}^2 \frac{1}{2^k}\]  
(10)

\[\pi_{2N} = p_{2N}\left(N - p_{2N} - p_{1N} + t\right)\Phi_{2N} - \frac{a}{2}\Phi_{2N}^2 \frac{1}{2^k}\]  
(11)

According to FOC (first-order-condition) on each firm’s price, the pricing game and advertising game between the competitive firms can be expressed as follows:

\[
\frac{\partial \pi_i}{\partial p_{1N}} = 0; \frac{\partial \pi_j}{\partial p_{2N}} = 0 \frac{\partial^2 \pi_i}{\partial p_{2N}^2} < 0
\]

Consequently, both firms’ equilibrium prices are shown as follows.

\[p_{1N} = \frac{t(-2N + 4 + 2^k)}{3 \times 2^k}; p_{2N} = \frac{t(2N + 2 - 2^k)}{3 \times 2^k}\]  
(12)

\[
\frac{\partial \pi_i}{\partial \phi_{1N}} = 0; \frac{\partial \pi_j}{\partial \phi_{2N}} = 0 \frac{\partial^2 \pi_i}{\partial \phi_{2N}^2} < 0
\]

Therefore, both firms’ equilibrium advertising intensities in each segment are shown as follows.

\[\Phi_{1N}^* = \frac{p_{2N} \times 2^k}{a}\left(p_{2N} - p_{1N} + t - \frac{N-1}{2^k}\right) = \frac{t(-2N + 4 + 2^k)^2}{18a \times 2^k}\]  
(13)

\[\Phi_{2N}^* = \frac{p_{2N} \times 2^k}{a}\left(N - p_{2N} - p_{1N} + t\right) = \frac{t(2N + 2 - 2^k)^2}{18a \times 2^k}\]
(i) Obviously, the inequality of $-2N + 4 + 2^k \geq 0$ and $2N + 2 - 2^k \geq 0$ are satisfied with each integer of $N$. Here, we define $N_1 = 2^{k-1} - 1$ and $N_2 = 2^{k-1} + 2$, for those $N = N_1 + 1, \ldots, N_2 - 1$, the firms’ equilibrium price and market demands show a positive value. Finally, in the segment $N$, with targeted advertising, both equilibrium profits can be shown in the following expressions.

$$
\pi^*_{1,N} = \frac{t^2(-2N + 4 + 2^k)^4}{648 \times 8^6 a}; \pi^*_{2,N} = \frac{t^2(2N + 2 - 2^k)^4}{648 \times 8^6 a} \quad (14)
$$

It is easy to obtain that $-2N + 4 + 2^k \geq 0$; $2N + 2 - 2^k \geq 0$. That means $2^{k-1} + 2 \geq N \geq 2^{k-1} - 1$. Obviously, $N_1 = 2^{k-1} - 1$ and $N_2 = 2^{k-1} + 2$.

Hence, for any integer of $m = m_1, \ldots, m_2 - 1$, both firms send the targeted advertising to the consumers in each segment for the marketing and have strictly positive segment demands.

(ii) When $N \leq N_i$ (left segment) the firm cannot perform price discrimination in our assumption. That means $\frac{N}{2^k} - \frac{N_2 - N_1}{2^k} + t = \frac{1}{2^k}$. Therefore, firm 1’s price in the left segment (advantage market) is $p_{1,N} = \frac{t(2^k - 2N)}{2^k a}$. In this segment, firm 1 forms the local monopolistic market. Then, firm 1’s profit in the segment $N$ using targeted advertising at a specific precision can be expressed as follows:

$$
\pi_{1,N} = \frac{1}{2^k} \phi_{1,N} - \frac{a}{2} \phi_{2,N} \frac{1}{2^k} \quad (15)
$$

Therefore, the equilibrium advertising intensity and profit of firm 1 are shown as follows.

$$
\phi^*_{1,N} = \frac{p_{1,N}}{a} = \frac{t(2^k - 2N)}{2^k a}, \pi^*_{1,N} = \frac{t^2(2^k - 2N)^2}{2 \times 8^k a}
$$

Meanwhile, the equilibrium advertising intensity and profit of firm 2 are shown as follows.

$$
\phi^*_{2,N} = \frac{p_{2,N}}{a} = \frac{t(2N - 2^k - 2)}{2^k a}
$$

$$
\pi^*_{2,N} = \frac{t^2(2N - 2^k - 2)^2}{2 \times 8^k a}
$$

In all, both firm’s equilibrium profits using targeted advertising in the segment $k$ are considered in the following expressions (Appendix 1):

$$
\pi^T_{1,k}(k) = \sum_{N=1}^{N_i-1} \frac{t^2(2^k - 2N)^2}{2 \times 8^k a} + \sum_{N=N_i}^{N_i-1} \frac{t^2(-2N + 4 + 2^k)^4}{648 \times 8^6 a}
$$

That means the equilibrium profit of firm $i$ can be obtained in the following equation with distinct parameter $k$ (Appendix 1. I)

$$
\pi^T_{i,k}(k) = \frac{t^2(27 - 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})}{324 a} \quad (16)
$$

To make this equation more convenient to understand, we define the firm’s targeting precision of targeted advertising as $R = 1 - 2^{-k}$. This condition means that the targeting precision is an exogenous variable that is less than one and higher than zero. Obviously, it is easy to explore the possible correlation between targeting accuracy and a firm’s equilibrium profit in this situation. Therefore, the firm’s equilibrium profit with the imperfect targeted advertising is shown as follows (Appendix 1. II).

$$
\pi^T_{i,k}(R) = \frac{t^2(27 - 81 \times (1-R) + 54 \times (1-R)^2 + 136 \times (1-R)^3)}{324 a} \quad (17)
$$

**Proposition 2.** When the two firms compete with imperfect targeted advertising with the improvement of parameter $k$, the firm’s equilibrium profit is decreased when $k \in \left[0, -\frac{1}{\ln 2} \ln \left(-\frac{9}{68} + \frac{3}{\sqrt{111}}\right)\right]$, whereas the firm’s profit increases in the condition of $k \in \left(-\frac{1}{\ln 2} \ln \left(-\frac{9}{68} + \frac{3}{\sqrt{111}}\right), +\infty\right)$. When the targeting precision ($R$) is defined as $R = 1 - 2^{-k}$, the firm’s minimal profits generated at an optimal equilibrium value of $R^* = \frac{77}{68} - \frac{3\sqrt{111}}{68}$. Otherwise, the firm’s equilibrium profit with perfect targeted advertising is lower than mass advertising ($\frac{t^2}{12a}$ vs $\frac{34t^2}{81a}$).

Obviously, according to proposition 2, targeted advertising might show a worse effect in a specific condition than that of mass advertising.

**Subgame 3. Only One Firm Can Perform With Targeted Advertising ($T,M$) or ($M,T$)**

Subsequently, consider the condition when only one firm can perform marketing with targeted advertising, whereas the other performs with mass advertising. This phenomenon often occurs when an e-commerce firm competes with a traditional firm using different advertising strategies. Here, we assume that only firm one can send targeted advertising to the consumers for the sales of products, whereas firm two...
can forward mass advertising for the selling, and then firm 1 chooses the targeted price called $p_{1N}(N = 1, \cdots, 2^k)$. Thus both firms’ profits are shown as follows:

(i) Considering $N < N_1$ (left segment), firm 1 can acquire the total monopolistic market while using the targeted advertising. That means the following equation can be obtained:

$$\frac{p_1 - p_{1N} + t}{2t} - \frac{N-1}{2^k} = \frac{1}{2^k}$$

According to the previous study (Liu & Serfes, 2004), it is easy to know that $p_{1N} = p_2 + t - \frac{2Nt}{2^k}$.

Therefore, firm 1’s profit shows as follows:

$$\pi_{1NL}^M = p_{1N} \phi_{1NL} - a \phi_{1NL}^2 \frac{1}{2^k}$$  \hspace{1cm} (18)

According to the FOC condition on a firm’s price and advertising intensity, in equilibrium, firm 1’s optimal profit can be expressed in the following equation

$$\pi_{1NL}^* = \left(p_2 + t - \frac{2Nt}{2^k}\right) / 2^{k+1} a.$$  

Likewise, firm 2’s profit is shown in the following equation.

$$\pi_{2NR}^M = p_2 \left[\frac{N}{2^k} - \frac{p_2 - p_{1N} + t}{2t}\right] \phi_2 - a \phi_2^2 \frac{1}{2^k}$$  \hspace{1cm} (19)

According to the FOC condition on a firm’s price and advertising intensity, in equilibrium, Firm 2’s optimal profit can be shown in the following expression.

$$\pi_{2NR}^* = \frac{2^k p_2^2}{2a} \left(\frac{N}{2^k} - \frac{p_2 - p_{1N} + t}{2t}\right)^2$$  \hspace{1cm} (20)

(ii) Considering the middle segment when firm 1 and firm 2 compete with prices and advertisements simultaneously in the partial market. According to the FOC condition on firm 1’s price,

$$\frac{\partial \pi_1}{\partial p_{1N}} = \frac{p_2 - 2p_{1N} + t}{2t} - \frac{N-1}{2^k} = \frac{p_2 - 2\left(p_2 + t - \frac{2Nt}{2^k}\right) + t}{2t} - \frac{N-1}{2^k} = 0$$

It is required to get the results of $p_{1N} = \frac{2^k p_2^2}{2a} + 2^k t - 2Nt + 2t$ and $N^* = \frac{2^k p_2^2}{2a} + 2^k t - 2t$.

Therefore, Firm 1’s equilibrium profit in the middle segment reveals as follows.

$$\pi_{1NL}^M = p_{1N} \left(\frac{p_2 - p_{1N} + t}{2t} - \frac{N-1}{2^k}\right) \phi_{1NL} - a \phi_{1NL}^2 \frac{1}{2^k}$$  \hspace{1cm} (22)

According to the FOC condition on firm 1’s advertising intensity, it is easy to obtain its equilibrium advertising intensity.

$$\frac{\partial \pi_1}{\partial \phi_{1NL}} = 0; \phi_{1NL}^* = \frac{4^k p_{1N}}{a} \left(\frac{p_2 - p_{1N} + t}{2t} - \frac{N-1}{2^k}\right)$$

Thus, in equilibrium, firm 1’s optimal profit in this segment is considered as

$$\pi_{1NL}^* = \frac{4^k p_{1N}^2}{2a} \left(\frac{p_2 - p_{1N} + t}{2t} - \frac{N-1}{2^k}\right)^2$$  \hspace{1cm} (23)

Likewise, the following equations can be obtained in the right segment for firm 2’s utility while purchasing the product.

$$0 + \frac{(N-1)t}{2^k} = p_2 + t \left[1 - \frac{(N-1)}{2^k}\right]$$

That means $N^* = \frac{2^k p_2 + 2^k t + 2t}{2t}$.

In addition, firm 2’s monopolistic market in each segment can be expressed as $\pi_{2NR}^M = \frac{2^k p_2^2}{2a} - \frac{a \phi_2^2}{2^k}$.

According to the FOC condition on firm 2’s advertising intensity of $\phi_{2N}$, firm 2’s equilibrium advertising intensity ($\phi_2$) shows as follows:

$$\frac{\partial \pi_2}{\partial \phi_{2N}} = 0; \phi_{2N}^* = \frac{p_2}{a}$$

After that, firm 2’s optimal profit in each segment is considered in the following expression.

$$\pi_{2NR}^M = \frac{p_2^2}{2^{k+1} a}$$  \hspace{1cm} (24)

According to (i) and (ii), firm 1’s profit can be expressed as

$$\pi_1(p_1, k) = \sum_{N=1}^{N_1} \frac{1}{2^{k+1} a} \left(\frac{p_2 + t - \frac{2Nt}{2^k}}{2^k}\right)^2 + \sum_{N=\frac{N_1}{2}}^{N_{\frac{N_1}{2}}} \frac{2^k p_{1N}^2}{2a} \left(\frac{p_2 - p_{1N} + t}{2t} - \frac{N-1}{2^k}\right)^2$$  \hspace{1cm} (25)
Likewise, firm 2’s profit can be expressed as the following equation (26).

\[
\pi_2(p_2, k) = \sum_{N=N_1(p,k)+1}^{N_1(p,k)} \frac{2^k p_2^2}{2a} \left( \frac{N - p_2 - p_{1N} + t}{2^2} \right)^2 + \sum_{N=N_1(p,k)+1}^{2^k} \frac{p_2^2}{2^{k+1}a}.
\]

(26)

According to a previous study from Liu and Serfes (2005b), it is easy to know that \( N_1 = 3 \times 2^{k-2} - 1 \) and \( N_2 = 3 \times 2^{k-2} + 2 \), \( p_2 = t \left( \frac{1}{2} + 2^{-(k+1)} \right) \).

\[
p_{1N} = \frac{2^k p_2 + 2^k t - 2Nt + 2t}{2 \times 2^k} = \frac{t \left(2^{k-1} + 2^{-1}\right) + 2^k t - 2Nt + 2t}{2 \times 2^k}.
\]

In all, firm 1’s equilibrium profit is considered as follows (Appendix 2):

\[
\pi_1(p_1, k) = \sum_{N=N_1(p,k)}^{N_1(p,k)} \frac{1}{2^{k+1}a} \left( p_2 + t - \frac{2Nt}{2^2} \right)^2 + \sum_{N=N_1(p,k)+1}^{N_2(p,k)+1} \frac{2^k p_{1N}^2}{2a} \left( \frac{p_2 - p_{1N} + t}{2t} - \frac{N-1}{2^2} \right)^2.
\]

(27)

\[
= \frac{t^2}{a} \left[ \frac{9}{32} - \frac{1}{32} \times 2^{-2k} + \frac{185}{1024} \times 2^{-3k} - \frac{9}{32} \times 2^{-k} \right].
\]

However, firm 2’s equilibrium profit can be expressed in the following expression:

\[
\pi_2(p_2, k) = \sum_{N=N_1(p,k)}^{N_1(p,k)+1} \frac{2^k p_2^2}{2a} \left( \frac{N - p_2 - p_{1N} + t}{2^2} \right)^2 + \sum_{N=N_1(p,k)+1}^{2^k} \frac{p_2^2}{2^{k+1}a}.
\]

(28)

\[
= \frac{t^2}{a} \left[ \frac{1}{32} - \frac{19}{256} \times 2^{-k} + \frac{31}{128} \times 2^{-2k} - \frac{35}{256} \times 2^{-3k} \right].
\]

To make it clear, we define the targeting precision of targeted advertising as \( R = 1 - 2^k \). The firm’s equilibrium profit with targeting accuracy can be expressed as follows.

\[
\pi_1(p_1, R) = \frac{t^2}{a} \left[ \frac{9}{32} - \frac{1}{32} \times (1-R)^2 + \frac{185}{1024} \times (1-R)^3 - \frac{9}{32} \times (1-R) \right].
\]

(29)

\[
\pi_2(p_2, R) = \frac{t^2}{a} \left[ \frac{1}{32} - \frac{19}{256} \times (1-R) - \frac{31}{128} \times (1-R)^2 - \frac{35}{256} \times (1-R)^3 \right].
\]

(30)

**Proposition 3.** In equilibrium, the two competitive firms might adopt distinct advertising strategies for sales. (1) In equilibrium, a firm’s equilibrium profit with imperfect targeted advertising increases when \( k \in \left[ -\frac{1}{\ln 2} \ln\left( \frac{32}{555} + \frac{4}{555} \sqrt{10054} \right), +\infty \right] \).

However, the rival firm’s equilibrium profit increases with a parameter \( k \) when using mass advertising.

(2) With the improvement of targeting precision, the firm using targeted advertising will capture its market size while the rival adopts mass advertising, and its profit will also increase. Meanwhile, the rival’s profit also increases with the improvement of the targeting precision. That is, the improvement of targeting precision reduces the competition between the firms using different targeting strategies. The firm’s minimal profits generate at an equilibrium targeting precision of \( R^* = \frac{523}{555} - \frac{4\sqrt{10054}}{555} = 0.2197 \) . (3) Otherwise, the firm’s equilibrium profit with perfect targeted advertising is higher than that using mass advertising \( \left( \frac{9}{32} \frac{a^2}{t} \right. \right) vs \ \left. \frac{t^2}{32a} \right).$

From proposition 3, increasing a firm’s targeting precision of targeted advertising might lead to a higher profit. This result might be due to the reduced wasted advertising cost. Then we consider the choice of competitive firms on imperfect targeted advertising or mass advertising. Then the payoff matrix of both firms using different advertising strategies is expressed in the following Table 1.

---

**Table 1.** Payoff matrix of competitive duopolistic firms with distinct advertising strategies.

| Firm 1 | Mass advertising(M) | Targeted advertising(T) |
|--------|----------------------|------------------------|
|        | \( \pi_{1M}, \pi_{2M} \) | \( \pi_{1T}, \pi_{2T} \) |
|        | \( \pi_{1M}, \pi_{2T} \) | \( \pi_{1T}, \pi_{2M} \) |

---
Model Extension

Here, we extend our model into $n$ competitive firms to explore the role of targeting precision of targeted advertising on the market’s capacity. In this new setup, there is a continuum of consumers uniformly distributed on the unit circle (Salop). This setup assumes that each consumer derives a benefit equal to $V$ if he buys a firm’s production. Suppose there are already $n$ firms, equidistantly located in the market, which implies that the distance between any two adjacent firms is $1/n$. Likewise, all consumers are divided into $2^k$ segments due to their interests or preferences, with a parameter of $k = 0, 1, 2, \cdots$. Hence, $k$ will parameterize the targeting precision of targeted advertising. In practice, the firms can choose different advertising strategies in the competition. We consider that all the firms can perform with targeted advertising while adopting the marketing strategies. This condition often occurs when many electronic commerce firms simultaneously compete in prices and targeted advertising for sales.

First, considering the middle segment of the market, due to the limit of the targeting precision, both firms can send targeted advertising in the $N$th segment of $[(N - 1)/2^k, N/2^k]$. Let $p_{Nn}$ denote the firm’s price in the $N$th segment. According to the Hotelling model, the consumer’s utilities can be equal with distinct transportation costs while purchasing each firm’s product. The following results can be obtained.

$$V - tx - p_{N} = V - t\left(\frac{1}{n} - x\right) - p_{2N} \quad (31)$$

That means the segmented consumers can be directly recognized.

$$x_{N} = \frac{p_{2Nn} - p_{Nn} + t}{2nt} \quad (32)$$

Consequently, both firms’ profits can be expressed as follows:

$$\pi_{1N} = p_{1N}\left(\frac{p_{2Nn} - p_{1Nn} + t}{2nt} - \frac{N - 1}{2^k n}\right) - \frac{a}{2} \phi_{1N}^2 = 1 - \frac{1}{2^k n} \quad (33)$$

$$\pi_{2N} = p_{2N}\left(\frac{N}{2^k n} - \frac{p_{2Nn} - p_{1Nn} + t}{2nt}\right) - \frac{a}{2} \phi_{2N}^2 = 1 - \frac{1}{2^k n} \quad (34)$$

According to FOC condition on both firms’ prices and advertising intensities,

$$\frac{\partial \pi_1}{\partial p_1} = 0, \quad \frac{\partial \pi_2}{\partial p_2} = 0, \quad \frac{\partial \pi_1}{\partial \phi_{1N}} = 0, \quad \frac{\partial \pi_2}{\partial \phi_{2N}} = 0 \quad (35)$$

Therefore, firm 1 and 2’s equilibrium prices are considered as $p_{1N} = t\left(-2N + 4 + 2^k\right)/3\times2^k n$, $p_{2N} = t\left(2N + 2 - 2^k\right)/3\times2^k n$

Meanwhile, it is easy to get firm 1 and 2’s equilibrium targeting intensity as follows.

$$\phi_{1N} = \frac{p_{1N}}{a} \left(\frac{p_{2Nn} - p_{1Nn} + t}{2nt} - \frac{N - 1}{2^k n}\right) 2^k n = t\left(-2N + 4 + 2^k\right)^2/18a \times 2^k n$$

$$\phi_{2N} = \frac{p_{2N}}{a} \left(\frac{N}{2^k n} - \frac{p_{2Nn} - p_{1Nn} + t}{2nt}\right) 2^k n = t\left(2N + 2 - 2^k\right)^2/18a \times 2^k n$$

Obviously, as $p_{1N} \geq 0$ and $p_{2N} \geq 0$, we can get $-2N + 4 + 2^k \geq 0$ and $2N + 2 - 2^k \geq 0$. Here, we also define $N_1 = 2^{k-1} - 1$ and $N_2 = 2^{k-1} + 2$, for those any $N = N_1 + 1, \cdots, N_2 - 1$. Both firms’ prices and marketing demands are positive.

Accordingly, the firms’ equilibrium profits are considered in the following expressions.

$$\pi_{1N}^* = \frac{t^2 (-2N + 4 + 2^k)^4}{648 \times 8^4 a^2 n^2}, \quad \pi_{2N}^* = \frac{t^2 (2N + 2 - 2^k)^4}{648 \times 8^4 a^2 n^2}$$

However, when $N \leq N_1$, $\frac{N}{2^k n} - \frac{p_{2Nn} - p_{1Nn} + t}{2nt} \leq 0$. Therefore, it is easy to get the following expression of $\pi_{2N} = t(2^k - 2N)$. Firm 1’s profit in this segment can be expressed as follows:

$$\pi_{1N} = p_{1N} \frac{1}{2^k n} - \frac{a}{2} \phi_{1N}^2 = 1 - \frac{1}{2^k n} \quad (36)$$

Therefore, $\phi_{1N} = \frac{p_{1N}}{a} = \frac{t(2^k - 2N)}{2^k a n}$ and

$$\pi_{1N}^* = \frac{t^2 (2^k - 2N)^2}{2 \times 8^2 a n^2}$$

Likewise, $\phi_{2N} = \frac{p_{2N}}{a} = \frac{t(2N - 2^k - 2)}{2^k a n}$ and

$$\pi_{2N}^* = \frac{t^2 (2N - 2^k - 2)^2}{2 \times 8^2 a n^2}$$

In all, both firms’ profits can be expressed as follows (see Appendix 3):

$$\pi_{1N}^* (k) = \sum_{N=1}^{N_1} \frac{t^2 (2^k - 2N)^2}{2 \times 8^2 a n^2} + \sum_{N=N_1+1}^{N_2} \frac{t^2 (-2N + 4 + 2^k)^4}{648 \times 8^4 a^2 n^2}$$

$$\pi_{2N}^* (k) = \sum_{N=N_1+1}^{N_2} \frac{t^2 (2N + 2 - 2^k)^4}{648 \times 8^4 a^2 n^2} + \sum_{N=N_1}^{N_2-1} \frac{t^2 (2N - 2^k - 2)^2}{2 \times 8^2 a n^2}$$

(37)
That means firm $i$'s equilibrium profit using targeted advertising can be considered as follows.

$$\pi_{i,T}^T(k) = \frac{t^2(27 - 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})}{324an^2}$$

(39)

According to the induction of the equation mentioned above, it is easy to obtain the following proposition 4.

**Proposition 4.** (1) Both firms send targeted advertising in the segment from $N_i + 1$ to $N_i - 1$. In equilibrium, firm 1's price is $p_{1N} = \frac{t(-2N + 4 + 2^k)}{3 \times 2^n}$, and firm 1's advertising intensity is deemed as $\phi_{1N} = \frac{t(-2N + 4 + 2^k)^2}{18a \times 2^n}$. Firm 2's price is considered as $p_{2N} = \frac{t(2N + 2 - 2^k)}{3 \times 2^n}$, whereas firm 2's targeting advertising intensity is shown as $\phi_{2N} = \frac{t(2N + 2 - 2^k)^2}{18a \times 2^n}$. (2) When $N \leq N_i$, firm 1's price is $p_{iN} = \frac{t(2^k - 2N)}{2^n}$, and firm 1's advertising intensity is considered as $\phi_{iN} = \frac{p_{iN}}{a} = \frac{t(2^k - 2N)}{2^n}$. Firm 2's price is $p_{2N} = \frac{t(2N - 2^k - 2)}{2^n}$, whereas firm 2's advertising intensity is considered to be $\phi_{2N} = \frac{t(2N - 2^k - 2)}{2^n}$.

At this time, both firms will form the local monopolistic market in their strong market. (3) In equilibrium, the competitive firm's profit is considered to be $\pi_{i,T}(k) = \frac{t^2(27 - 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})}{324an^2}$.

From the proposition mentioned above, the competitive firm's equilibrium profit shows a negative correlation with the square of firm numbers. Therefore, the following corollary can be obtained below.

**Corollary 1.** When $n$ different firms compete with targeted advertising, the firm's equilibrium profit while adopting targeted advertising also shows U-shape with parameter $k$. Moreover, when $n$ increases, they show an inconspicuous downturn U-shape. (2) With improving the firm's targeting precision of targeted advertising, the firm's equilibrium profit increases. However, in different firms, the firm's equilibrium profit decreases with augmenting entry firm numbers.

**Free Entry of the Firms Compete With Targeted Advertising**

Here, we assume, when one firm enters the market, there will be a constant cost $F$ for the entry. These costs often consist of some fixed house rental costs and some personal costs. Since each firm makes sales to its two adjacents (symmetric) intervals at a length of $1/n$, and the firm's total (net) profits are considered in the following expression.

$$\prod_{i} (k,n) = 2\pi_{i}(k,n) - F = \frac{t^2(27 - 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k}) - F}{162an^2}$$

(40)

By setting $\prod_{i} (k,n) = 2\pi_{i}(k,n) - F = 0$, we can derive the free-entry equilibrium number of firms in the market as a function of the targeting precision (see Figure 2).

$$n^*(k) = \frac{t\sqrt{2(27 - 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})} - F}{18\sqrt{Fa}}$$

(41)

From this equation, we can easily see that the equilibrium number of firms entering the market follows a U-shape variation pattern as a function $k$ (see Figure 3).

Therefore, we can get the equilibrium number of firms under perfect targeted advertising with a function that tends to infinity. When $k = 0$, that means the firm is adopting mass advertising. The equilibrium number of firms is considered as $n^*(0)$. Therefore, the following equations can be obtained.

$$\lim_{k \to \infty} n^*(k) = \frac{t}{\sqrt{6Fa}}; n^*(0) = \frac{2\sqrt{177}}{9\sqrt{Fa}}$$

**Proposition 5.** When the firms have imperfect targeting precision, the number of new entry firms can be shown as the following expression of $n^*(k) = t\sqrt{2(27 - 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})}/18\sqrt{Fa}$.

In comparison, the number of equilibrium firms is considered
to be \( n^*(0) = \frac{2\sqrt{17t}}{9F_a} \). When the firms have perfect targeting precision, the numbers of equilibrium firms are regarded as \( \lim_{k \to \infty} n^*(k) = \frac{t}{\sqrt{6F_a}} \).

According to proposition 5, when the parameter \( k \) equals zero, that means the firms there are no new entry firms, and the numbers of equilibrium firms are correlated with transportation cost and advertising cost, and the fixed cost.

**Corollary 2.** When the targeting precision of targeted advertising improves, the free-entry numbers will show U-shape variation with the changes of targeting accuracy. That means the numbers of firms might first reduce and then increase. When both firms can send perfect targeted advertising to the consumers, the firm numbers in the market are less than those sending mass advertising.

From the corollary mentioned above, it is easy to understand that the targeting precision of targeted advertising could affect the structure of the market due to the targeting effect. Only in a particular condition, the firm can obtain the optimal profit with targeted advertising. In this setup, the entry of the other firm is assumed to be free. However, it is not common in specific marketing. In most conditions, the market is controlled by an outside power, especially government regulation. Then, we consider the regulatory effect of society on entry as previously reported (Liu & Serfes, 2005b). When the firms can perform imperfect targeted advertising, the imperfect targeted advertising will cause additional social costs. The second item of \(-81 \times 2^{4k}\) shows no effect on social costs. Therefore, the equilibrium numbers of firms can be expressed as follows (see Figure 4):

\[
n^*(k) = \frac{t\sqrt{2(27+54 \times 2^{-2k} +136 \times 2^{-3k})}}{18\sqrt{F_a}}
\]

According to equation (42), it is easy to obtain the relationship between specific entry numbers of the firm and the targeting precision. Then, the following proposition 6 can be accepted.

**Proposition 6.** Considering some social regulators in the market, the regulators can control the entry of the firms but cannot stop imperfect targeted advertising. Then the social regulator expects more firms to enter the market to reduce the stealing effect of imperfect targeted advertising for the consumers.

The intuition behind the U-shape of the free-entry number of firms as a function of targeting precision of the targeted advertising is as follows. Two opposite effects govern market interaction and are responsible for the non-monotonicity of firms’ profits (see Figure 5). The ability to classify consumers into different segments allows firms to send targeted advertising to loyal consumers. This result leads to an all-out competition (intensified competition effect). On the other hand, better targeting precision allows firms to extract more surpluses (surplus extraction effect). With a low targeting precision, the first effect is dominant than the second, making profits fall. After reaching a specific targeting precision, the surplus extraction effect becomes more critical, and the profit rebounds.

To make our results more transparent, we perform some numerical simulations in the following part.

**Simulation**

Without loss of generality, we assume \( t = a = 1 \) when two competitive firms adopt targeted advertising or distinct advertising strategies. Likewise, we also assume \( t = a = 1 \) and \( n = 3, 4, 5 \) when \( n \) competitive firms use targeted advertising.
Simulation of Two Competitive Firms With Targeted Advertising

Simulation 1. The correlation between the parameter $k$ and the firm's equilibrium profit can be simulated as follows (Figure 6):

Simulation 2. When the targeting precision belongs to the range of $(0,1)$, the correlation between targeting accuracy and firm's equilibrium is shown as Figure 7:

Simulation of Two Competitive Firms With Distinct Advertising Strategies

Simulation 3. The firms' profit with targeted advertising $\pi(T)$ and mass advertising $\pi(M)$ can be simulated in Figure 8.

Simulation 4. When the targeting precision belongs to the range of $(0,1)$, the correlation between the targeting precision and firm's equilibrium can be expressed in Figure 9:
Simulation 5. The correlation between parameter $k$ and the firm’s profit in different entrants ($n = 3, 4, 5$) can be expressed in Figure 10.

Simulation 6. When the targeting precision belongs to the range of $(0,1)$, the correlation between targeting accuracy and firm’s equilibrium in different numbers of firms ($n = 3, 4, 5$) can be expressed as follows:

Simulation 7. We assume that $t / \sqrt{Fa} = 10$, in this condition, the correlation between the parameter $k$ and numbers of entrants is becoming evident.

Simulation 8. The correlation between targeting precision and the number of entrants can be shown in Figure 4.

Simulation 9. To explore the regulatory effect on the numbers of entrants, we also assume $t / \sqrt{Fa} = 10$ and target precision as $R = 1 - 2^{-k}$. Figure 5a shows the comparison between the free entry and regulatory controls entry with distinct targeting precision. In contrast, Figure 5b shows a correlation between parameter $k$ and numbers of entrants with regulatory controls.

**Conclusion**

The main result of this paper is that firms competing with a moderate targeting precision of imperfect targeted advertising yields the most efficient outcome in a duopolistic competitive location model. In our model, both firms provide inadequate targeted advertising for the sales of products. The results indicate that the competitive firm’s equilibrium profit shows a U-shape with targeting precision of targeted advertising. This result is theoretical prove for Dogruel (2019). Their study showed that ad explanations with a medium level of detail led to more favorable advertising evaluations among users than ad explanations with a high level of detail.

This result partially explains that the perfect targeting might not always be beneficial than mass advertising. It is because the ideal targeting might induce fierce competition.
in a specific segment. Our conclusion is similar to what Ben Elhadj-Ben Brahim et al. (2011) reported, while firms are given more options with targeted advertising. Otherwise, if one firm adopts targeted advertising, whereas the other firm competes with mass advertising, the firm using targeted advertising with distinct targeting precision will acquire more profits than those using mass advertising. That means the firm using targeted advertising might steal more profits from those using mass advertising due to the reduced cost by targeting. In our extension model, we also show similar results in $n$ different competitive firms using targeted advertising. In addition, we also prove that when the firm improves the targeting precision of targeted advertising, the free-entry numbers of firms will show a U-shape with targeting precision. Our study reveals that the targeting precision of targeted advertising plays a critical role in regulating the market structure. Finally, when both firms can send perfect targeted advertising to the consumers, the entry numbers of firms in the market are less than those sending mass advertising. This result indicates that the targeted advertising might induce intensified competition. In all, we show that the most efficient free-entry outcome occurs when the targeting precision is moderate. In other words, neither mass advertising nor perfect advertising leads to a less efficient outcome. This result partially explained that some e-business firms are eager to obtain the optimal targeted precision, if not the perfect targeting, due to the high cost of the targeting. In addition, it is difficult for the new entry of the e-business firm to live well in fierce marketing competition due to targeting precision on its efficiency. Further studies should focus on how targeting affects a firm’s efficiency in the supply chain or two-sided market.

**Appendix**

**Proof of Proposition 2**

(1) In equilibrium, the firm’s profit can be expressed as follows: 

$$\pi_{1, T}^T(k) = \sum_{N=1}^{N_i} \frac{t^2(2k - 2N)^2}{2 \times 8^2a} + \sum_{N=N_i+1}^{N+1} \frac{t^2(-2N + 4 + 2^k)^4}{648 \times 8^4a}$$

Here, we define $N_i = 2^{k-1} - 1$ and $N_2 = 2^{k-1} + 2$, for those $N = N_1 + 1, \ldots, N_2 - 1$. Therefore,

$$\pi_{1, T}^T(k) = \frac{t^2}{2 \times 8^4a} \left( 2^{2k} - 4N \times 2^k + 4N^2 \right) + \frac{t^2(-2 \times 2^{2k-1} + 4 + 2^k)^4}{648 \times 8^4a} + \frac{t^2(-2 \times (2^{k-1} + 1) + 4 + 2^k)^4}{648 \times 8^4a}$$

$$= \frac{t^2}{2 \times 8^4a} \left( 2^{2k} \times (2^{k-1} - 1) - 2^{2k} \times (2^{k-1} - 1) + 4 \times \frac{(2^{k-1} - 1) \times 2^{k-1} \times (2^4 - 1)}{6} \right) + \frac{t^2 \times (4^4 + 2^4)}{648 \times 8^4a}$$

$$= \frac{t^2 \times (27 - 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})}{324a}$$

It is assumed that 

$$\frac{\partial \pi_{1, T}^T(k)}{\partial k} = \frac{1}{4} \times 2^{(-k)} \times ln2 - \frac{1}{3} \times 2^{(-2k)} \times ln2 - \frac{34}{27} \times 2^{(-3k)} \times ln2 = 0$$

That means: $k = -\frac{1}{\ln 2} \ln \left( -\frac{9}{68} + \frac{3}{68} \sqrt{111} \right) \approx 1.588654073$

When $k \in \left[ 0, -\frac{1}{\ln 2} \ln \left( -\frac{9}{68} + \frac{3}{68} \sqrt{111} \right) \right)$, 

$$\frac{\partial \pi_{1, T}^T(k)}{\partial k} < 0; \text{ However, When}$$

$$k \in \left( -\frac{1}{\ln 2} \ln \left( -\frac{9}{68} + \frac{3}{68} \sqrt{111} \right), +\infty \right), \frac{\partial \pi_{1, T}^T(k)}{\partial k} > 0.$$ 

$$\lim_{k \to \infty} \pi_{1, T}^T(k) = \lim_{k \to \infty} \left[ \frac{t^2}{a} \left( \frac{1}{12} - \frac{1}{4} \times 2^{-k} + \frac{1}{6} \times 2^{-2k} + \frac{34}{81} \times 2^{-3k} \right) \right] = \frac{t^2}{12a}$$

$$\lim_{k \to 0} \pi_{1, T}^T(k) = \lim_{k \to 0} \left[ \frac{t^2}{a} \left( \frac{1}{12} - \frac{1}{4} \times 2^{-k} + \frac{1}{6} \times 2^{-2k} + \frac{34}{81} \times 2^{-3k} \right) \right] = \frac{34}{81a};$$

When $k = 2$, it is easy to know that 

$$\lim_{k \to 0} \pi_{1, T}^T(k) = \frac{341t^2}{10368a} \div \frac{0.03289t^2}{a}$$
(II) When the targeting precision \( R \) is defined as \( R = 1 - 2^{-k} \),
\[
\pi_i^{T,T}(R) = \frac{t^2 \left[ 27 - 81 \times (1-R) + 54 \times (1-R)^2 + 136 \times (1-R)^3 \right]}{324 a}
\]
\[
\frac{\partial \pi_i^{T,T}(R)}{\partial R} = \frac{t^2}{a} \left( -1 + \frac{1}{3} R - \frac{34}{27} (1-R)^2 \right) = 0, R \in (0,1); \\
R^* = \frac{77}{68} = 0.66757
\]
\[
\frac{\partial^2 \pi_i^{T,T}(R)}{\partial R^2} = \frac{77}{27} - 68 R, \text{ It is evident that when } R \in (R^*, 1), \frac{\partial^2 \pi_i^{T,T}(R)}{\partial R^2} > 0
\]

Proof of Proposition 3

(I) While both firms perform distinct targeting strategies, the firm’s profit can be expressed as follows:
\[
\pi_2(p_2, k) = \sum_{N=N_i(p_2, k)+1}^{N_f} \frac{2^k}{2a} \left( \frac{N}{2^k} - \frac{p_2 - P_{2N} + t}{2t} \right)^2 + \sum_{N=N_i(p_2, k)}^{N_f} \frac{p_2^2}{2^{k+1} a}
\]

Here, according to previous studies (Liu & Serfes, 2004, 2005b), it is easy to get
\[
N_i = 3 \times 2^{-k-2} - 1; N_f = 3 \times 2^{-k+2} + 2; p_2 = t \left( \frac{1}{2} + 2^{-k+1} \right), \text{ for those } N = N_i + 1, \ldots, N_f - 1. \text{ Therefore, firm 2’s profit can be expressed as follows.}
\]
\[
\pi_2(p_2, k) = \frac{2^k}{2a} \left( \frac{1}{2} + 2^{-k+1} \right)^2 \times \left( \frac{58}{64} \times 2^{-2k} \right) + \left( \frac{1}{2} + 2^{-k+1} \right)^2 \times \left( \frac{2^k - 3 \times 2^{-k+2} - 2}{2^{k+1} a} \right)
\]
\[
= \frac{t^2}{a} \left( \frac{1}{32} - \frac{19}{256} \times 2^{-k} - \frac{31}{128} \times 2^{-2k} - \frac{35}{256} \times 2^{-3k} \right)
\]

Therefore,
\[
\frac{\partial \pi_2(p_k,k)}{\partial k} = \frac{19}{256} 2^{-k} \ln(2) - \frac{31}{64} 2^{-2k} \ln(2) + \frac{105}{256} 2^{-3k} \ln(2)
\]
\[
\frac{\partial^2 \pi_2(p_k,k)}{\partial k^2} = -\frac{19}{256} 2^{-k} \ln(2)^2 + \frac{31}{32} 2^{-2k} \ln(2)^2 - \frac{315}{256} 2^{-3k} \ln(2)^2 > 0
\]
\[
\lim_{k \to \infty} \frac{a^2}{t} \left( \frac{1}{32} - \frac{19}{256} 2^{-k} + \frac{31}{128} 2^{-2k} - \frac{35}{256} 2^{-3k} \right) = \frac{1}{32} \frac{a^2}{t}
\]
\[
\lim_{k \to 0} \frac{a^2}{t} \left( \frac{1}{32} - \frac{19}{256} 2^{-k} + \frac{31}{128} 2^{-2k} - \frac{35}{256} 2^{-3k} \right) = -\frac{27}{64} \frac{a^2}{t}
\]

(II) Likewise, firm 1’s profit can be expressed as follows:
\[
\pi_1(p_2, k) = \sum_{N=1}^{3 \times 2^{k+1} - 1} \left( \frac{p_2 + t}{2^k a} \right)^2 + \sum_{N=1}^{3 \times 2^{k+1} - 1} \frac{2^k p_2^2}{2a} \left( \frac{p_2 - P_{1N} + t - N - 1}{2t} \right)^2
\]
That means:
\[
P_{1N} = \frac{2^k p_2 + 2^k t - 2N + 2t}{2^k} = \frac{t (2^{k+1} + 1)^2 + 2^k t - 2N + 2t}{2^k} = \frac{3}{4} + \frac{5 \times 2^{-k}}{2^k}
\]
\[
p_2 - P_{1N} + t - N - 1 = \left( \frac{1}{2} + \frac{1}{2} 2^{-k} - \frac{3}{4} + \frac{5 \times 2^{-k}}{2^k} + \frac{N}{2^k} + 1 \right) - \frac{N}{2^k} + \frac{1}{2} = \frac{3}{8} + \frac{5 \times 2^{-k}}{2^k} - \frac{N}{2^{k+1}} = \frac{1}{2} P_{1N}
\]
When \( N = 3 \times 2^{k-2} \), \( p_{kN} = \frac{5}{4} \times 2^{-k} \), that means \( \frac{P_2 - p_{kN} + t}{2t} - N-1 = \frac{5}{8} \times 2^{-k} \).

\[
\pi_1(p_2, k) = \frac{t^2}{2^{k+1}a} \sum_{N=1}^{N(p_2)} \left( \frac{3}{2} + 2^{-k+1} \right)^2 - 2 \left( \frac{3}{2} + 2^{-k+1} \right) 2N \frac{2^k}{2^k} + \left( \frac{N^2}{2^{k-1}} \right)
\]

\[
= \frac{t^2}{2^{k+1}a} \left[ \left( \frac{3}{2} + 2^{-k+1} \right)^2 \left( 3 \times 2^{k-2} - 1 \right) - 2 \left( \frac{3}{2} + 2^{-k+1} \right) \left( 1 + 3 \times 2^{k-2} - 1 \right) \right] + \frac{1}{2^{2k-2}}
\]

\[
= \frac{t^2}{a} \left[ \frac{9}{32} \times 2^{-2k} - \frac{9}{32} \times 2^{-k} + \frac{185}{1024} \times 2^{-3k} \right]
\]

Therefore,

\[
\frac{\partial \pi_1(p_2, k)}{\partial k} = \frac{1}{16} \times 2^{-2k} \ln 2 - \frac{555}{1024} \times 2^{-3k} \ln 2 + \frac{9}{32} \times 2^{-k} \ln 2
\]

When \( \frac{\partial \pi_1(p_2, k)}{\partial k} = 0 \), it is easy to get that \( k = -\frac{1}{\ln 2} \ln \left( \frac{32}{555} + \frac{4}{555} \sqrt{10054} \right) \) is the root. However, when \( k \in \left( 0, -\frac{1}{\ln 2} \ln \left( \frac{32}{555} + \frac{4}{555} \sqrt{10054} \right) \right) \), \( \frac{\partial^2 \pi_1(p_2, k)}{\partial k^2} < 0 \); and when \( k \in \left( -\frac{1}{\ln 2} \ln \left( \frac{32}{555} + \frac{4}{555} \sqrt{10054} \right), +\infty \right) \), \( \frac{\partial^2 \pi_1(p_2, k)}{\partial k^2} > 0 \).

\[
\lim_{k \to +\infty} \frac{a^2}{t} \left( \frac{9}{32} \times 2^{-2k} - \frac{9}{32} \times 2^{-k} - \frac{185}{1024} \times 2^{-3k} \right) = \frac{9}{32} \times \frac{a^2}{t}
\]

\[
\lim_{k \to 0} \frac{a^2}{t} \left( \frac{9}{32} \times 2^{-2k} - \frac{9}{32} \times 2^{-k} - \frac{185}{1024} \times 2^{-3k} \right) = \frac{153}{1024} \times \frac{a^2}{t}
\]

**Proof of Proposition 4**

When \( n \) different firms compete with imperfect targeted advertising

\[
\pi_{1,2}^n(t) = \sum_{N=1}^{N(p_2)} \frac{t^2 (2^k - N)}{2 \times 8^k an^2} + \sum_{N=1}^{N(p_2)} \frac{t^2 (-2N + 4 + 2^k) \times 10^2}{648 \times 8^k an^2}
\]

Likewise, we define \( N_1 = 2^{k-1} - 1 \) and \( N_2 = 2^{k-1} + 2 \), for those \( N = N_1 + 1, \ldots, N_2 - 1 \). Therefore,

\[
\pi_{1,2}^n(t) = \frac{t^2}{2 \times 8^k an^2} \left( 2^{2k} - 4N \times 2^k + 4N^2 \right) + \frac{t^2 (-2 \times 2^{k-1} + 4 + 2^k) \times 10^2}{648 \times 8^k an^2} + \frac{t^2 (-2 \times 2^{k-1} + 1) + 4 + 2^k \times 10^2}{648 \times 8^k an^2}
\]

\[
= \frac{t^2 (27 \times 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})}{324 an^2}
\]

**Proof of Proposition 5**

(1) When \( \prod_{k=1}^n (k, n) = 2n! - F = \frac{t^2 (27 \times 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})}{324 an^2} - F = 0 \)

Therefore, \( n^2 = \frac{t^2 (27 \times 81 \times 2^{-k} + 54 \times 2^{-2k} + 136 \times 2^{-3k})}{162an^2} \)
(2) When \( k = \infty \), \( \lim_{k \to \infty} n^2 = \frac{t^2(27 - 81 \lim_{k \to \infty} 2^{-k} + 54 \lim_{k \to \infty} 2^{-2k} + 136 \lim_{k \to \infty} 2^{-3k})}{162aF} = \frac{t^2}{6aF} \\
(3) When \( k = 0 \), \( \lim_{k \to 0} n^2 = \frac{t^2(27 - 81 \lim_{k \to 0} 2^{-k} + 54 \lim_{k \to 0} 2^{-2k} + 136 \lim_{k \to 0} 2^{-3k})}{162aF} = \frac{68t^2}{81aF} \\

Proof of Proposition 6

(1) When \( k = \infty \), \( \lim_{k \to \infty} n^2 = \frac{t^2(27 + 54 \lim_{k \to \infty} 2^{-2k} + 136 \lim_{k \to \infty} 2^{-3k})}{162aF} = \frac{t^2}{6aF} \\
(2) When \( k = 0 \), \( \lim_{k \to 0} n^2 = \frac{t^2(27 + 54 \lim_{k \to 0} 2^{-2k} + 136 \lim_{k \to 0} 2^{-3k})}{162aF} = \frac{217t^2}{162aF} \geq \frac{68t^2}{81aF} \\

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References

Anand, B. N., & Shachar, R. (2009). Targeted advertising as a signal. Quantitative Marketing and Economics, 7(3), 237–266.
Athey, S., & Gans, J. S. (2010). The impact of targeting technology on advertising markets and media competition. American Economic Review, 100(2), 608–613.
Ben Elhadj-Ben Brahim, N., Lahmandi-Ayed, R., & Laussel, D. (2011). Is targeted advertising always beneficial? International Journal of Industrial Organization, 29(6), 678–689.
Bergemann, D., & Bonatti, A. (2011). Targeting in advertising markets: implications for offline versus online media. The RAND Journal of Economics, 42(3), 417–443.
Bimpikis, K., Ozdaglar, A., & Yildiz, E. (2016). Competitive targeted advertising over networks. Operational Research, 64(3), 705–720.
Boerman, S. C., Kruikemeier, S., & Zuiderveen Borgesius, F. J. (2017). Online behavioral advertising: A literature review and research agenda. Journal of Advertising, 46(3), 363–376.
Butters, G. R. (1977). Equilibrium distributions of sales and advertising prices. The Review of Economic Studies, 44, 465–491.
Celik, L. (2007). Strategic informative advertising in a horizontally differentiated duopoly. Mimeo, CERGE-EI 2008, Working Paper series (ISSN 1211-3298). Center for Economic Research and Graduate Education, Academy of Sciences of the Czech Republic, Economics Institute.

Chandra, A. (2009). Targeted advertising: the role of subscriber characteristics in media markets. Journal of Industrial Economics, 57(1), 58–84.
Chandra, A., & Kaiser, U. (2010). Targeted advertising in magazine markets (Discussion Paper No. 10-063). ZEW.
Chen, Y., & Iyer, G. (2002). Research Note Consumer addressability and customized pricing. Marketing Science, 21, 197–208.
Chen, Y., Narasimhan, C., & Zhang, Z. J. (2001). Individual marketing with imperfect targetability. Marketing Science, 20(1), 23–41.

Chen, J., & Stallaert, J. (2014). An economic analysis of online advertising using behavioral targeting. MIS Quarterly, 38(2), 429–449.
Chen, Y., & Zhang, Z. J. (2009). Dynamic targeted pricing with strategic consumers. International Journal of Industrial Organization, 27, 43–50.
Dognel, L. (2019). Too much information!? Examining the impact of different levels of transparency on consumers’ evaluations of targeted advertising. Communication Research Reports, 36(5), 383–392.
Esteban, L., Gil, A., & Hernández, J. M. (2003). Informative advertising and optimal targeting in a monopoly. Journal of Industrial Economics, 49(2), 161–180.
Esteves, R. B. (2014). Price discrimination with private and imperfect information. The Scandinavian Journal of Economics, 116(3), 766–796.
Fong, N., Zhang, Y., Luo, X., & Wang, X. (2019). Targeted promotions on an e-book platform: Crowding out, heterogeneity, and opportunity costs. JMR, Journal of Marketing Research, 56(2), 310–323.
Galeotti, A., & Moraga-Gonzalez, J. L. (2003). Strategic targeted advertising (Discussion Paper TI 2003-0351/1). Tinbergen Institute.
Gal-Or, E., Gal-Or, M., May, J. H., & Spangler, W. E. (2006). Targeted advertising strategies on television. Management Science, 52(5), 713–725.
Gong, Q., Pan, S., & Yang, H. (2019). Targeted Advertising on Competing Platforms. The BE Journal of Theoretical Economics, 19(1), 1–20. https://doi.org/10.1515/bejte-2017-0126
Grossman, G. M., & Shapiro, C. (1984). Informative advertising with differentiated products. *The Review of Economic Studies, 51*, 63–82.

Hamilton, S. F. (2009). Informative advertising in differentiated oligopoly markets. *International Journal of Industrial Organization, 27*(1), 60–69.

Hernández-García, J. M. (1997). Informative advertising, imperfect targeting and welfare. *Economics Letters, 55*(1), 131–137.

Iyer, G., Soberman, D., & Villas-Boas, J. M. (2005). The targeting of advertising. *Marketing Science, 24*(3), 461–476.

Jiang, Z., Dan, W., & Jie, L. (2020). Distinct role of targeting precision of Internet-based targeted advertising in duopolistic e-business firms’ heterogeneous consumers market. *Electronic Commerce Research, 20*, 453–474.

Johnson, J. P. (2013). Targeted advertising and advertising avoidance. *The RAND Journal of Economics, 44*, 128–144.

Koslow, S., & Stewart, D. W. (2021). Message and media: the future of advertising research and practice in a digital environment. *International Journal of Advertising. Advance online publication. https://doi.org/10.1080/02650487.2021.1954804*

Liu, Q., & Serfes, K. (2004). Quality of information and oligopolistic price discrimination. *Journal of Economics & Management Strategy, 13*(4), 671–702.

Liu, Q., & Serfes, K. (2005a). Imperfect price discrimination in a vertical differentiation model. *International Journal of Industrial Organization, 23*(5–6), 341–354.

Liu, Q., & Serfes, K. (2005b). Imperfect price discrimination, market structure, and efficiency. *Canadian Journal of Economics/Revue canadienne d’économique, 38*(4), 1191–1203.

Rutt, J. (2012). Targeted Advertising and Media Market Competition. SSRN 2103061. https://doi.org/10.2139/ssrn.2103061

Shin, J., & Yu, J. (2021). Targeted Advertising and Consumer Inference. *Marketing Science, 40*(5), 900–922.

Soberman, D. A. (2004). Research Note: Additional learning and implications on the role of informative advertising. *Management Science, 50*(12), 1744–1750.

Stahl, D. O. (1994). Oligopolistic pricing and advertising. *Journal of Economic Theory, 64*, 162–177.

Taylor, C. R. (2013). Customised communications: Relevance vs privacy in targeted messaging. *International Journal of Advertising, 32*(4), 483–485.

Zhang, J., & He, X. (2019). Targeted advertising by asymmetric firms. *Omega, 89*, 136–150.

Zhang, K., & Katona, Z. (2012). Contextual advertising. *Marketing Science, 31*(6), 980–994.

Zhao, J., Mei, S. E., & Zhong, W. J. (2018). Bidirectional regulatory effect of targeting extent in targeted advertising. *Journal of Industrial Engineering and Engineering Management, 32*(1), 161–170.

Zhao, X., & Xue, L. (2012). Competitive Target Advertising and Consumer Data Sharing. *Journal of Management Information Systems, 29*(3), 189–222.