Abstract—We present a 3D zigzag rafter -first in literature- which allows us to obtain the exact sequence of spectral components after application of Discrete Cosine Transform 3D (DCT-2D) over a cube. Such cube represents part of a video or eventually a group of images such as multi-slicing (e.g., Magnetic Resonance or Computed Tomography imaging) and multi or hyperspectral imagery (optical satellites). Besides, we present a new version of the traditional 2D zigzag, including the case of rectangular blocks. Finally, all the attached code is done in MATLAB®, and that code serves both blocks of pixels or blocks of blocks.

Keywords—Multi-slicing – Video processing – Zigzag sequences.
1 Introduction

In the past decades it has highlighted the importance of 2D zigzag scanning for such diverse applications as: the compression algorithm for graphic file format known as Joint Photographic Experts Group (JPEG) [1,2], medical imaging [3], multispectral [4-7] and hyperspectral imagery [8-10].

There is no doubt that a correct 3D zigzag scanning will be very welcome in areas as diverse as those representing part of a video [11] or eventually a simple group of images [12] such as multi-slicing (of Magnetic Resonance and Computed Tomography imaging) [3] and multi [4-7] or hyperspectral imagery (for optical satellites) [8-10].

Video codecs seek to represent a fundamentally analog data set in a digital format. Because of the design of analog video signals, which represent luma and color information separately, a common first step in image compression in codec design is to represent and store the image in a YCbCr color space. The conversion to YCbCr provides two benefits: first, it improves compressibility by providing decorrelation of the color signals; and second, it separates the luma signal, which is perceptually much more important, from the chroma signal, which is less perceptually important and which can be represented at lower resolution to achieve more efficient data compression. It is common to represent the ratios of information stored in these different channels in the following way Y:Cb:Cr. Refer to the following article for more information: Chroma subsampling [13].

Different codecs use different chroma subsampling ratios as appropriate to their compression needs. Video compression schemes for Web and DVD make use of a 4:2:0 color sampling pattern, and the DV standard uses 4:1:1 sampling ratios. Professional video codecs designed to function at much higher bitrates and to record a greater amount of color information for post-production manipulation sample in 3:1:1 (uncommon), 4:2:2 and 4:4:4 ratios. Examples of these codecs include Panasonic's DVCPRO50 and DVCPROHD codecs (4:2:2), and then Sony's HDCAM-SR (4:4:4) or Panasonic's HDD5 (4:2:2). Apple's Prores HQ 422 codec also samples in 4:2:2 color space. More codecs that sample in 4:4:4 patterns exist as well, but are less common, and tend to be used internally in post-production houses. It is also worth noting that video codecs can operate in RGB space as well. These codecs tend not to sample the red, green, and blue channels in different ratios, since there is less perceptual motivation for doing so—just the blue channel could be undersampled [13].

Some amount of spatial and temporal downsampling may also be used to reduce the raw data rate before the basic encoding process. The most popular such transform is the 8x8 discrete cosine transform (DCT). Codecs which make use of a wavelet transform are also entering the market, especially in camera workflows which involve dealing with RAW image formatting in motion sequences. The output of the transform is first quantized, then entropy encoding is applied to the quantized values. When a DCT has been used, the coefficients are typically scanned using a zig-zag scan order, and the entropy coding typically combines a number of consecutive zero-valued quantized coefficients with the value of the next non-zero quantized coefficient into a single symbol, and also has special ways of indicating when all of the remaining quantized coefficient values are equal to zero. The entropy coding method typically uses variable-length coding tables. Some encoders can compress the video in a multiple step process called n-pass encoding (e.g. 2-pass), which performs a slower but potentially better quality compression [13].

The decoding process consists of performing, to the extent possible, an inversion of each stage of the encoding process. The one stage that cannot be exactly inverted is the quantization stage. There, a best-effort approximation of inversion is performed. This part of the process is often called "inverse quantization" or "dequantization", although quantization is an inherently non-invertible process [13].

This process involves representing the video image as a set of macroblocks. For more information about this critical facet of video codec design, see B-frames [13].

Video codec designs are usually standardized or eventually become standardized—i.e., specified precisely in a published document. However, only the decoding process need be standardized to enable interoperability.
The encoding process is typically not specified at all in a standard, and implementers are free to design their encoder however they want, as long as the video can be decoded in the specified manner. For this reason, the quality of the video produced by decoding the results of different encoders that use the same video codec standard can vary dramatically from one encoder implementation to another [13].

Notwithstanding this, it is known for decades that the ideal codec involves using a 3D version of JPEG. The problem to date 3D version of zigzag scanning is unknown. Therefore, this version will be very welcome throughout the 3D image processing, which obviously includes video [11-13].

A new 2D zigzag version (for squared and rectangular blocks) is outlined in Section 2. 3D zigzag is presented in Section 3. In Section 4, we discuss the more appropriate and comparative experimental results. Finally, Section 5 provides a conclusion and future works proposal of the paper.

2 New 2D zigzag scanning

As can be seen in the literature, current versions of 2D zigzag scanning use too many if-else [1]. Therefore, it is imperative to find a version are computationally more efficient. This is one of the main reasons of this paper, which we present here: a) a new 2D version for squared blocks, b) a new version 2D for rectangular blocks, and c) in Section 3, a 3D version for cubical blocks for the first time in literature, both video and in all versions of Digital Image Processing [1-17].

2.1 For squared blocks

The left side of Fig.1 shows the zig-zag spatial scanning method [1], which is fundamental for JPEG compression algorithm [1]. On the other hand, the right side of Fig.1 shows each numbering cell represent a sub-block (inside spectral domain) which may be spatially ordered (in upward order).

As can be seen from Fig.1, pixels (or blocks), which have to be treated or not with a DCT, are concentrated in blocks. Block clusters of 2×2, 4×4, 8×8 … pixels, can be easily extracted, since pixels in these blocks are transmitted one after another (zig-zag ordering, the same ordering employed in JPEG image compression format [1]). This feature can be handy for spatial image processing, such as resolution reduction. In order to reduce image resolution by a factor of two, the mean of four pixels (a 2×2 block) has to be calculated. With this ordering (zig-zag), it can be done in a simple, straightforward way, without requiring multiple storage elements. This calculation can be expanded to blocks of sizes 4×4, 8×8, etc.

Fig. 1 2D zigzag allows the transition from 8-by-8 spectral structure (left side of figure) to 64-by-1 spectral structure (right side of figure) after DCT-2D. Each block represents a pixel of a block itself composed of several pixels (e.g., 8-by-8).
Here, we present both direct and reverse version of the new 2D zigzag scanning in MATLAB® code [18].

```matlab
function v = zigzag2d(M)
    N = length(M(:,1));
    fmax = [(1:N-1) (N*ones(1,N))];
    fmin = [ones(1,N) (2:N)];
    k = 0;
    v = [];
    for u = 2:N+N
        for r = fmin(u-1):fmax(u-1)
            c = u-r;
            k = k+1;
            if rem(u,2) == 0,
                v(k) = M(r,c);
            else
                v(k) = M(c,r);
            end
        end
    end
    v = v';
end

function M = izigzag2d(v)
    N = round(sqrt(length(v)));
    fmax = [(1:N-1) (N*ones(1,N))];
    fmin = [ones(1,N) (2:N)];
    k = 0;
    M = [];
    for u = 2:N+N
        for r = fmin(u-1):fmax(u-1)
            c = u-r;
            k = k+1;
            if rem(u,2) == 0,
                M(r,c) = v(k);
            else
                M(c,r) = v(k);
            end
        end
    end
end
```

### 2.2 For rectangular blocks

The rectangular version is equal to the square with one difference, the number of rows "R" is different from columns "C". Here, we present both direct and reverse version of the new 2D zigzag scanning in MATLAB® code [18].

```matlab
function [v,R,C] = zigzag2d(M)
    [R,C] = size(M);
    fmin = [ones(1,C) (2:R)];
    fmax = [(1:R-1) (R*ones(1,C))];
    v = [];
    for t = 1:R+C-1
        acu = [];
        for r = fmin(t):fmax(t)
            c = t+1-r;
            acu = [ acu M(r,c) ];
        end
        v = v';
    end
end
```
end
if C >= R,
if rem(t,2) == 1,
v = [ v acu ];
else
v = [ v rot90(rot90(acu)) ];
end
else
if rem(t,2) == 0,
v = [ v acu ];
else
v = [ v rot90(rot90(acu)) ];
end
end
end

function M = izigzag2d(v,R,C)
fmin = [ ones(1,C) (2:R) ];
fmax = [ (1:R-1) (R*ones(1,C)) ];
k = 0;
v2 = [];
for t = 1:R+C-1
acu = [];
for r = fmin(t):fmax(t)
k = k+1;
acu = [ acu v(k) ];
end
if C >= R,
if rem(t,2) == 1,
v2 = [ v2 acu ];
else
v2 = [ v2 rot90(rot90(acu)) ];
end
else
if rem(t,2) == 0,
v2 = [ v2 acu ];
else
v2 = [ v2 rot90(rot90(acu)) ];
end
end
end
end
k = 0;
for t = 1:R+C-1
for r = fmin(t):fmax(t)
k = k+1;
c = t+1-r;
M(f,c) = v2(k);
end
end

3 Finally, 3D zigzag scanning

Figure 2 shows the passage from a cubical matrix to an unidimensional vector thanks to 3D zigzag scanning. Scanning (which is not shown to avoid complicating the drawing) is in a downward spiral, if we see it on perpendicular planes to the 3D diagonal (which is the sub-block 1-1-1 to 8-8-8) the result would look like a 2D zigzag, which is logical, considering the projection with a dimension loss.
Here, we present both direct and reverse version of the 3D zigzag scanning in MATLAB® code [18].

```matlab
function vector = zigzag3d(matrix)
    N = length(matrix(:,:,1)); % Number of characters, or cube side
    D = 3; % Dimensions
    T = (N-1)*D+1;
    % First column:
    Vc = [ 1:N N-1:-1:1 ]; lVc = length(Vc);
    Vs = 1:1:N;
    for t = 1:T
        % First column:
        if t <= N,
            Lic(T-t+1) = 1;
        else
            Lic(T-t+1) = t-N+1;
        end
        if t <= N,
            Lsc(t) = lVc;
        else
            Lsc(t) = lVc-t+N;
        end
        if t <= T-(N-1),
            Lis(t) = 1;
        else
            Lis(t) = t-(T-N);
        end
        if t <= N,
            Lss(t) = t;
        else
            Lss(t) = N;
        end
    end
    % First column:
```

**Fig. 2** Passage from a cubical matrix to an unidimensional vector thanks to 3D zigzag scanning.
Wc = []; 
Ws = []; 
% Second column: 
Av = [ 1:N N*ones(1,N-2) N:-1:1 ]; 
Ci = [ ones(1,N-1) 1:N ]; lCi = length(Ci); 
Cd = [ 1:N N*ones(1,N-1) ]; 
d = 1; 
Ci2 = []; 
Cd2 = []; 
Sig = []; 
for t = 1:T 
if rem(t,2) == 0, 
% First column: 
Hc = Vc(Lic(t):Lsc(t)); 
Wc = [ Wc Hc ]; 
Hs = Vs(Lis(t):Lss(t)); 
Ws = [ Ws Hs ]; 
% Second column: 
if t <= N, 
Ci2 = [ Ci2 rot90(rot90(Ci(d:d+Av(t)-1)))); 
Cd2 = [ Cd2 rot90(rot90(Cd(d:d+Av(t)-1)))); 
else 
d = d+1; 
Ci2 = [ Ci2 rot90(rot90(Ci(d:d+Av(t)-1)))); 
Cd2 = [ Cd2 rot90(rot90(Cd(d:d+Av(t)-1)))); 
end 
else 
% First column: 
Hc = rot90(rot90(Vc(Lic(t):Lsc(t)))); 
Wc = [ Wc Hc ]; 
Hs = rot90(rot90(Vs(Lis(t):Lss(t)))); 
Ws = [ Ws Hs ]; 
% Second column: 
if t <= N, 
Ci2 = [ Ci2 Ci(d:d+Av(t)-1)]; 
Cd2 = [ Cd2 Cd(d:d+Av(t)-1)]; 
else 
d = d+1; 
Ci2 = [ Ci2 Ci(d:d+Av(t)-1)]; 
Cd2 = [ Cd2 Cd(d:d+Av(t)-1)]; 
end 
end 
% Second column: 
Sig = [ Sig (-1)^t*ones(1,Av(t)) ]; 
end 
L = length(Ws); 
W = []; 
X = []; 
Z = []; 
for l = 1:L 
% First column: 
W = [ W Ws(l)*ones(1,Wc(l)) ]; 
% Second column: 
if Sig(l) > 0, 
X = [ X Ci2(l):Cd2(l) ]; 
Z = [ Z Cd2(l):-1:Cl2(l) ]; 
else
function matrix = izigzag3d(vector)

D = 3; % Dimensions
N = length(vector)^(1/D); % Number of characters, or cube side
N = round(N);
T = (N-1)^D+1;

% First column:
Vc = [ 1:N N-1:-1:1 ]; lVc = length(Vc);
Vs = 1:1:N;
for t = 1:T
    % First column:
    if t <= N,
        Lic(t) = 1;
    else
        Lic(t) = t-N+1;
    end
    if t <= N,
        Lsc(t) = lVc;
    else
        Lsc(t) = lVc-t+N;
    end
    if t <= T-(N-1),
        Lis(t) = 1;
    else
        Lis(t) = t-(T-N);
    end
    if t <= N,
        Lss(t) = t;
    else
        Lss(t) = N;
    end
end

% First column:
Wc = [];
Ws = [];

% Second column:
Av = [ 1:N N*ones(1,N-2) N:-1:1 ];
Ci = [ ones(1,N-1) 1:N ]; lCi = length(Ci);
Cd = [ 1:N N*ones(1,N-1) ];
d = 1;
Ci2 = [];
Cd2 = [];
Sig = [];
for t = 1:T
    if rem(t,2) == 0,
        % First column:
        Hc = Vc(Lic(t):Lsc(t));
        Wc = [ Wc Hc ];
        Hs = Vs(Lis(t):Lss(t));
    end
end
Ws = [ Ws Hs ];
% Second column:
if t <= N,
    Ci2 = [ Ci2 rot90(rot90(Ci(d:d+Av(t)-1)))) ];
    Cd2 = [ Cd2 rot90(rot90(Cd(d:d+Av(t)-1)))) ];
else
    d = d+1;
    Ci2 = [ Ci2 rot90(rot90(Ci(d:d+Av(t)-1)))) ];
    Cd2 = [ Cd2 rot90(rot90(Cd(d:d+Av(t)-1)))) ];
end
else
% First column:
    Hc = rot90(rot90(Vc(Lic(t):Lsc(t))));
    Wc = [ Wc Hc ];
    Hs = rot90(rot90(Vs(Lis(t):Lss(t))));
    Ws = [ Ws Hs ];
% Second column:
    if t <= N,
        Ci2 = [ Ci2 Ci(d:d+Av(t)-1)];
        Cd2 = [ Cd2 Cd(d:d+Av(t)-1)];
    else
        d = d+1;
        Ci2 = [ Ci2 Ci(d:d+Av(t)-1)];
        Cd2 = [ Cd2 Cd(d:d+Av(t)-1)];
    end
end
% Second column:
    Sig = [ Sig (-1)^t*ones(1,Av(t)) ];
end
L = length(Ws);
W = [];
X = [];
Z = [];
for l = 1:L
    % First column:
    W = [ W Ws(l)*ones(1,Wc(l)) ];
    % Second column:
    if Sig(l) > 0,
        X = [ X Ci2(l):Cd2(l) ];
        Z = [ Z Cd2(l):-1:Ci2(l) ];
    else
        X = [ X Cd2(l):-1:Ci2(l) ];
        Z = [ Z Ci2(l):Cd2(l) ];
    end
end
LL = length(W);
for ll = 1:LL
    matrix(W(ll),X(ll),Z(ll)) = vector(ll);
end

4 Simulations

Figure 3 shows the result of 2D zigzag scanning after 2D Discrete Cosine Transform (DCT2) [19-36] for blocks of 64x64 elements (pixels o sub-blocks). Instead, Fig.4 shows the result of 3D zigzag scanning after 3D Discrete Cosine Transform (DCT3) for blocks of 16x16x16 elements (pixels o sub-blocks). This shows the consistency of 3D zigzag scanning regarding the 2D version. Identical results were achieved with other transforms (e.g., Discrete Fourier Transform, Karhunen-Loève Transform, etc) [37-43].
Then, we present both, the code in MATLAB® [18] of Fig.3, and Fig.4, including the code of DCT3 because it is not a MATLAB® built-in function.

**Code for Fig.3**

```matlab
N = input('N = ');
matrix = round(255*rand(N,N));
matrix = dct2(matrix); % this is a built-in MATLAB function
vector = zigzag2d(matrix); % this is the code of Sub-section 2.1
T = 0:1:N*N-1;
plot(T,vector,'r')
axis([0 N*N-1 min(vector) max(vector)])
```

**Fig. 3** 2D zigzag after DCT2 for blocks of 64x64 elements.

**Fig. 4** 3D zigzag after DCT3 for blocks of 16x16x16 elements.
Code for Fig.4
N = input('N = ');
matrix = round(255*rand(N,N,N));
matrix = dct3(matrix); % this function is below this code
vector = zigzag3d(matrix); % this is the code of Section 3
t = 0:1:N*N*N-1;
plot(t,vector,'r')
axis([ 0 N*N*N-1 min(vector) max(vector) ])

function Mout = dct3(Min)
[R,C,B] = size(Min);
Mout = zeros(R,C,B);
for b = 1:B
    mid2dmtx = Min(:,:,b);
dctcoef = dct2(mid2dmtx); % this is a built-in MATLAB function
    Mout(:,:,b) = dctcoef;
end
for r = 1:R
    for c = 1:C
        midvec = [];
        for b = 1:B
            midvec = [midvec, Mout(r,c,b)];
        end
        coefdct1 = dct(midvec); % this is a built-in MATLAB function
        for b = 1:B
            Mout(r,c,b) = coefdct1(1,b);
        end
    end
end
end

5 Conclusions

We have presented here an unprecedented 3D zigzag scanning (among two new versions of 2D) which allow
the development of a new family of video codecs much more efficient than those currently in use, namely:
VP9 [44], VP10 [45], H.264 [46], and H.265 [47] (among others), with consequent benefits that this brings
for the transmission of video on Internet and on mobile channels in almost any resolution and frame per second rate.

References

JPEG
1. Pennebaker, W.B., and, Mitchell, J.L.: JPEG – Still Image Data Compression Standard. International
    Thomsan Publishing, New York, 1993.
2. Miano J., Compressed Image File Formats: JPEG, PNG, GIF, XBM, BPM. Addison Wesley, 1999.

Medical imaging
3. Semmlow, J.L.: Biosignal and biomedical image processing: MATLAB-Based applications, Marcel
    Dekker, Inc., New York, 2004.

Multispectral
4. Epstein, B.R., et al: Multispectral KLT-wavelet data compression for landsat thematic mapper images.
    In Data Compression Conference, pp. 200-208, Snowbird, UT, March 1992.
5. Lee, J.: Optimized quadtree for Karhunen-Loève Transform in multispectral image coding. IEEE
    Transactions on Image Processing, 8(4), pp.453-461, 1999.
6. Saghri, J.A., et al: Practical Transform coding of multispectral imagery. *IEEE Signal Processing Magazine*, 12, pp.32-43, 1995.

7. Kim, T-S., et al: Multispectral image data compression using classified prediction and KLT in wavelet transform domain. *IEICE Transactions on Fundam Electron Commun Comput Sci*, Vol. E86-A; No.6, pp.1492-1497, 2003.

**Hyperspectral**

8. Christophe, E., et al: Hyperspectral image compression: adapting SPIHT and EZW to anisotropic 3D wavelet coding. Submitted to IEEE Transactions on Image processing, pp.1-13, 2006.

9. Rodríguez del Río, L.S.: Fast piecewise linear predictors for lossless compression of hyperspectral imagery. Thesis for Degree in Master of Science in Electrical Engineering, University of Puerto Rico, Mayaguez Campus, 2003.

10. -. *Hyperspectral Data Compression*. Edited by Giovanni Motta, Francesco Rizzo and James A. Storer, Chapter 3, Springer, New York, 2006.

**Video**

11. Wien, M.: Variable Block-Size Transforms for Hybrid Video Coding, Degree Thesis, Institut für Nachrichtentechnik der Rheinisch-Westfälischen Technischen Hochschule Aachen, February 2004.

**Image sequence processing**

12. Borman, S., and, Stevenson, R.: Image sequence processing. Department, Ed. Marcel Dekker, New York, 2003. pp. 840-879.

**Video codec**

13. https://en.wikipedia.org/wiki/Video_codec

**DIP**

14. Jain, A.K.: Fundamentals of Digital Image Processing. Prentice Hall Inc., Englewood Cliffs, NJ (1989)

15. Gonzalez, R.C., Woods, R.E.: Digital Image Processing, 2nd edn. Prentice-Hall, Englewood Cliffs (2002)

16. Gonzalez, R.C., Woods, R.E., Eddins, S.L.: Digital Image Processing Using Matlab. Pearson Prentice Hall, Upper Saddle River (2004)

17. Schalkoff, R.J.: Digital Image Processing and Computer Vision. Wiley, New York (1989)

**MATLAB**

18. MATLAB® R2015a (Mathworks, Natick, MA). [http://www.mathworks.com/](http://www.mathworks.com/)

**DCT**

19. Khayam, S.A.: The Discrete Cosine Transform (DCT): Theory and Application Technical Report, WAVES-TR-ECE802.602, 2003.

20. Strang, G.: The Discrete Cosine Transform. SIAM Review, Volume 41, Number 1, pp.135-147, 1999.

21. Clark, R.J.: Transform Coding of Images. New York: Academic Press, 1985.

22. Hung, A.C., and Meng, TH-Y: A Comparison of fast DCT algorithms. Multimedia Systems, No. 5 Vol. 2, Dec 1994.

23. Aggarwal, G., and, Gajski, D.D.: Exploring DCT Implementations. UC Irvine, Technical Report ICS-TR-98-10, March 1998.

24. Blinn, J.F.: What's the Deal with the DCT. IEEE Computer Graphics and Applications, July 1993, pp.78-83.

25. Chiu, C.T., and, Liu, K.J.R.: Real-Time Parallel and Fully Pipelined 2-D DCT Lattice Structures with Application to HDTV Systems. *IEEE Trans. on Circuits and Systems for Video Technology*, vol. 2 pp. 25-37, March 1992.

26. Haque, M.A.: A Two-Dimensional Fast Cosine Transform. *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. ASSP-33 pp. 1532-1539, December 1985.

27. Vetterli, M.: Fast 2-D Discrete Cosine Transform. ICASSP ’85, p. 1538.
28. Kamangar, F.A., and Rao, K.R.: Fast Algorithms for the 2-D Discrete Cosine Transform. IEEE Transactions on Computers, v C-31 p. 899.
29. Linzer, E.N., and, Feig, E.: New Scaled DCT Algorithms for Fused Multiply/Add Architectures. ICASSP ’91, p. 2201.
30. Loeffler, C., Ligtenberg, A., and Moschytz, G.: Practical Fast 1-D DCT Algorithms with 11 Multiplications. ICASSP ’89, p. 988.
31. Duhamel, P., Guillemot, C., and Carlush, J.C.: A DCT Chip based on a new Structured and Computationally Efficient DCT Algorithm. ICCAS ’90, p. 77.
32. Cho, N.I., and, Lee, S.U.: Fast Algorithm and Implementation of 2-D DCT. IEEE Transactions on Circuits and Systems, vol. 38 p. 297, March 1991.
33. Cho, N.I., Yun, I.D., and, Lee, S.U.: A Fast Algorithm for 2-D DCT. ICASSP ’91, p. 2197-2220.
34. McMillan, L., and, Westover, L.: A Forward-Mapping Realization of the Inverse DCT. DCC ’92, p. 219.
35. Duhamel, P., and, Guillemot, C.: Polynomial Transform Computation of the 2-D DCT. ICASSP ’90, p. 1515.
36. Britanak, B., Yip, P., and, Rao, K.R.: Discrete cosine and sine transforms: General properties, fast algorithms and integer approximations. Academic Press, N.Y., 2006.

DFT/FFT
37. Briggs, W.L., and, Van Emden, H.: The DFT: An Owner’s Manual for the Discrete Fourier Transform. SIAM, 1995, Philadelphia.
38. Hsu, H.P.: Fourier Analysis. Simon & Schuster, 1970, New York.
39. Tolimieri, R., M., An, and, Lu C.: Algorithms for Discrete Fourier Transform and convolution, Springer Verlag, 1997, New York.
40. Tolimieri, R., An, M., and, Lu C.: Mathematics of multidimensional Fourier Transform Algorithms, Springer Verlag, 1997, New York.
41. Claveau, F., and, Poirier, M.: Real time FFT based cross-covariance method for vehicle speed and length measurement using an optical sensor. ICSPAT 96, pp.1831-1835, Boston, MA, Oct.1996.

Transforms
42. -, The transform and data compression handbook, Edited by K.R. Rao, and P.C. Yip, CRC Press Inc., Boca Raton, FL, USA, 2001.
43. Tjoa, S., et al: Transform coder classification for digital image forensics. IEEE Int. Conf. Image Processing, September 2007.

Video codecs
44. https://en.wikipedia.org/wiki/VP9
45. https://en.wikipedia.org/wiki/VP9#VP10
46. https://en.wikipedia.org/wiki/H.264/MPEG-4_AVC
47. https://en.wikipedia.org/wiki/High_Efficiency_Video_Coding