p+\(^{4,6,8}\)He elastic scattering at intermediate energies

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Abstract

Using a relativistic nuclear optical potential consisting of a Lorentz scalar, \(V_s\), and the time-like component of a four-vector potential, \(V_0\), we calculate elastic scattering differential cross sections and polarizations for \(p+^{4}\text{He}\) at intermediate energies for which experimental data are available. We also calculate the differential cross sections and analyzing powers for \(p+^{6,8}\text{He}\) at intermediate energies and compare with the few available experimental data.

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I. INTRODUCTION

The scattering of medium-energy nucleons from nuclei can provide information about both nuclear structure and the NN-interaction. In particular, the study of proton-nucleus scattering at intermediate energies is a useful method for determining accurate nuclear matter distributions in stable nuclei [1]. Recently, it has also been used extensively to determine the extension of the nuclear matter density of exotic nuclei. In particular, in a recent set of experiments, inverse kinematics was used to study the elastic scattering of $p+^{4,6,8}$He and determine their matter densities [2, 3].

Elastic scattering at intermediate energies can be calculated theoretically using either a non-relativistic [4, 5] or a relativistic optical [6, 7] model, usually with about the same good results. An advantage of a relativistic optical model is that its two potentials simultaneously determine both the central and spin-orbit interactions. When the relativistic impulse approximation (RIA) is used [8], these interactions may be determined in terms of the corresponding scalar and vector nuclear densities [9].

In this work we use a relativistic nuclear optical potential constructed from a Lorentz scalar, $V_s$, and the time-like component of the four-vector potential, $V_0$, to calculate elastic scattering angular distributions and analyzing powers for $p+^{4}$He at intermediate energies and compare two different fits to the $p+^{4}$He data and also to the RIA results. We also calculate angular distributions and analyzing powers of the elastic scattering of $p+^{6,8}$He and compare an extension of the more physical of our two fits with the RIA and the few available experimental data.

II. THE DIRAC OPTICAL POTENTIAL

In relativistic optical model analyses of intermediate energy scattering [6], the Dirac equation is usually used in the form,

$$\{\vec{\alpha} \cdot \vec{p} + \beta [m + V_s(r)] + [V_0(r) + V_c(r)]\} \Psi(\vec{r}) = E \Psi(\vec{r}),$$

(1)

where $V_s$ is an attractive Lorentz scalar potential, $V_0$ is the repulsive time-like component of a four-vector potential and $V_c$ is the Coulomb potential. The choice of the potentials is motivated by meson exchange considerations and simplicity. The simplest meson exchange interaction possessing a certain justification on physical grounds, which is also capable of
providing nuclear saturation properties, takes into account the exchange of an attractive intermediate range isoscalar scalar meson and a repulsive shorter range isoscalar vector meson \[9, 10, 11, 12\]. The two corresponding potentials \(V_s\) and \(V_0\) play an essential role in the description of both elastic scattering and polarization data at intermediate energies since their sum is the principal contribution to the central potential while their difference determines the spin-orbit interaction \[13\]. The scalar, \(V_s\), and vector, \(V_0\), optical potentials used in the analyses here can be written as

\[
V_s = U_s f_{U_s}(r) + i W_s f_{W_s}(r)
\]

\[
V_0 = U_0 f_{U_0}(r) + i W_0 f_{W_0}(r)
\] (2)

with each of the potentials possessing both real and imaginary parts with possibly different radial dependences.

Based on nuclear structure calculations, we take the radial dependence \(f(r)\), for the case of \(p+^4\text{He}\), to have a Gaussian form

\[
f(r) = \exp[-r^2/r_0^2].
\] (3)

III. RESULTS

A. \(p+^4\text{He}\) elastic scattering

Relativistic Hartree calculations of the \(^4\text{He}\) nucleus yield almost identical scalar and vector densities but an rms radius about 30% larger than the experimental one \[14\]. To obtain a physically reasonable RIA potential for this system, we use equal scalar and vector matter densities having the rms radius of 1.49 fm found in Ref. \[2\] together with the relativistic Love-Franey NN t-matrix of Ref. \[15\]. The results, represented by full lines in Figs. (1)-(2), show that the RIA agrees well with the experimental data at low momentum transfer but deviates substantially from the experimental results as the momentum transfer increases. We also note that the RIA angular distributions do not reproduce the oscilatory structure seen in the experimental data, decreasing monotonically instead. The polarizations are not reproduced either, except at very low values of the momentum transfer. Due to these differences, we have tried to fit the experimental data of Ref. \[6\] using a Dirac optical potential of the form given in equations \([2]-[3]\).
In a first attempt, labeled with the dashed lines in the figures, the parameters were left free to vary so as to obtain the simultaneous best fit to the experimental angular distributions and polarizations at the three values of the laboratory energy, $E_{lab}$. The same geometrical parameters were used at the three energies, while the strengths were assumed to vary linearly with the laboratory energy as $V = V_0 + V_1 \times E_{lab}$, a common parametrization of optical model strengths. The best fit parameters are given in Table I. As can be seen in the figures, we indeed obtain a fairly reasonable fit to the angular distribution and polarization at all three energies. However, an interpretation of the imaginary part of the potential in the context of a RIA potential would require a $NN$ total cross section about 2.5 times the physical one, which makes the fit unsatisfactory on physical grounds. The radii of the imaginary potentials are in agreement with the matter radius of Ref. [2], while the radii of the real potentials are found to be about 10% bigger. These also disagree with what one would expect from a RIA potential, which usually yields real radii close to the matter radius and imaginary ones slightly larger.

To obtain a more physically reasonable fit to the $p+^4$He data, we reduced the imaginary strengths of the first fit by a factor of 3, reset the radii to the value of $r_0 = 1.22$ fm, corresponding to an rms radius of 1.49 fm [2], and then let the parameters vary freely once again. The results, labeled with dotted lines in the figures, present slightly poorer agreement with the experimental data, but continue to reproduce the oscillations in the angular distributions that are not obtained in the RIA calculation (full lines). The imaginary potentials of the second fit are consistent with those of the RIA potential, corresponding to a physically reasonable $NN$ total cross section. The radii are also closer to those of a RIA potential.
TABLE I: Optical parameters for fits to the $p+^4$He experimental data.

| $U_s^1$  | $r_0$ (fm) | $V_1$   |
|----------|------------|---------|
| -325.50  | 1.354      | 0.1051  |
| $W_s^1$  | 208.65     | 1.209   | 0.0147  |
| $U_0^1$  | 271.84     | 1.346   | -0.0160 |
| $W_0^1$  | -300.00    | 1.186   | -0.0538 |

| $U_s^2$  | $r_0$ (fm) | $V_1$   |
|----------|------------|---------|
| -325.50  | 1.309      | 0.1162  |
| $W_s^2$  | 79.65      | 1.295   | 0.0653  |
| $U_0^2$  | 271.84     | 1.252   | -0.0540 |
| $W_0^2$  | -100.00    | 1.257   | -0.1658 |

FIG. 1: Differential cross section of $p+^4$He, as a function of the momentum transfer, calculated for $E_{lab} = 0.561, 0.800$ and $1.029$ GeV. The solid lines represent the RIA calculation while the dashed and dotted ones stand for the first and second fits. The experimental data are labeled with diamonds.

B. $p+^6,^8$He elastic scattering

We next considered the elastic scattering of $p+^6,^8$He at laboratory energies of $E_{lab} = 0.721$ GeV and $E_{lab} = 0.678$ GeV, respectively. To perform the RIA calculation for $^6,^8$He, we
FIG. 2: Polarization of $p+^{4}\text{He}$, as a function of the momentum transfer, calculated for $E_{lab} = 0.561, 0.800, 1.029$ GeV. The solid lines represents the RIA approximation while the dashed and dotted ones stand for the first and second fits. The experimental data are labeled with diamonds.

followed the same procedure used for $^{4}\text{He}$. We generated equal scalar and vector matter densities using the appropriate Gaussian harmonic oscillator (HO) geometrical parameters of Ref. [2], together with the relativistic Love-Franey NN t-matrix of Ref. [15]. The results of the RIA calculation in Figs. (3) and (4) agree well with the experimental data in the range over which the data exist. We emphasise that the RIA also describes the $p+^{4}\text{He}$ data well in this range of momentum transfers but, at larger momentum transfers, deviates substantially from the data in form and absolute values.

For the cases of $p+^{6,8}\text{He}$, the data is insufficient to attempt a fit. We have thus attempted to extend our fits to the $^{4}\text{He}$ data by interpreting them in the context of a simple RIA calculation. To do so, we extract the effective strengths, $\tilde{V}_{0i} = \frac{1}{4}(U_i, W_i)[\pi r^2_{0,i}]^{3/2}$, that, when multiplied by the $^{4}\text{He}$ density, would yield the potentials obtained in our fits to the $p+^{4}\text{He}$ data. We have multiplied these strengths by the appropriate $^{6,8}\text{He}$ densities, obtained using the Gaussian HO parametrization of Ref. [2] to obtain effective Dirac potentials for these systems,

$$V_{i}(r) = \tilde{V}_{0i} \left( \frac{4}{(\pi r^2_{0c})^{3/2}} \exp \left[ -r^2/r^2_{0c} \right] \right) +$$
\[ \zeta \frac{2}{3} \left( \frac{1}{(\pi r_{0v}^2)^{3/2}} \right) \frac{r^2}{r_{0v}^2} \exp \left( -\frac{r^2}{r_{0v}^2} \right) \],

where \( \zeta = 2, 4 \) for \(^6\)He and \(^8\)He, respectively. The radii were modified slightly to take into account the differences between the experimental \(^4\)He radius of Ref. \(^{[2]}\) and those obtained in the fits.

Our calculations for these two nuclei do not reproduce the experimental data as well as the RIA calculation does, as can be seen in Figs. (3) and (4), which leads us to conclude that the parameters obtained in the \( p + ^4\)He fit cannot be so easily extended to the \( p + ^6,^8\)He elastic scattering. However, we feel that they can transmit an idea of the discrepancies with an RIA calculation that the experimental values could show. In Figure (4) we show the polarizations expected for proton scattering from these nuclei, for both the RIA and our extension of the \(^4\)He fits. The differences between the two are again quite large. However, no data exist in this case. By comparison with the results for \(^4\)He it is difficult to say what one could expect of the experimental angular distribution and polarization in \( p + ^6,^8\)He scattering at larger values of the momentum transfer.

FIG. 3: Differential cross sections of \( p + ^6,^8\)He at \( E_{Lab} = 0.721, 0.678 \) GeV, respectively. The dashed line corresponds to the RIA approximation while the dotted one represents the second fit. The experimental data are labeled with diamonds.
IV. CONCLUSION

We have calculated $p+^{4,6,8}$He elastic scattering differential cross sections and polarizations using the relativistic impulse approximation and an adjusted Dirac optical potential. We have shown that the RIA results obtained using the parameters of Ref. [2] describe all three systems well at low values of the momentum transfer. However, in the case of $p+^{4}$He, for which more data exist, the RIA deviates substantially from the data at higher momentum transfers. As an alternative to the RIA we have adjusted the parameters of a Dirac optical potential and then attempted to extend the parameters obtained to the cases of $p+^{6,8}$He, but with unsatisfactory results. Based on our analyses it is difficult to say what one could expect at higher values of transferred momenta in the cases of $p+^{6,8}$He. More data are needed to elucidate this situation.

In closing, we should mention that a possible reason for the failure of RIA in describing the $p+^{4}$He data is that it has been pushed beyond the limit of its applicability in the simple manner in which it is used here. The decomposition of the effective Dirac potential into a scalar potential and the fourth component of vector is valid in the target rest frame. The scattering calculation, however, is performed in the CM frame and the boost that carries one frame to the other will convert the vector fourth component potential into a full vector...
potential. For proton scattering on a system such as $^{40}$Ca the vector components introduced are small and can be neglected. For extremely light systems, such as those studied here, this is not the case. We plan to examine the importance of this effect in the future.

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