Polar Broad Absorption Line Quasars: An Open Question

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ABSTRACT

It has been argued that certain broad absorption line quasars are viewed within 35\degree of the axis of a relativistic radio jet, based on two-epoch radio flux density variability. It is true if the surface brightness of a radio source is observed to change by a sufficiently large amount, the inferred brightness temperature will exceed 10\textsuperscript{12} K and Doppler beaming in our direction must be invoked to avoid a Compton cooling catastrophe. However, flux density changes cannot be linked to surface brightness changes without knowledge of the size of the source. If an optically thick source changes in projected area but not surface brightness, its brightness temperature is constant and its flux variability yields no constraint on its orientation. Moreover, as pointed out by Rees, spherical expansion of an emission source at relativistic speeds yields an apparently superluminal increase in its projected area, which can explain short-timescale flux density variability without requiring a relativistic jet oriented near to our line of sight. Therefore, two-epoch radio flux density variability by itself cannot unambiguously identify sources with jets directed toward us. Only VLBI imaging can robustly determine the fraction of broad absorption line quasars which are polar.

Key words: radio continuum: general - quasars: general - quasars: absorption lines

1 INTRODUCTION

Broad absorption line (BAL) quasars are those quasars which show ultraviolet absorption troughs thousands of \text{km s\textsuperscript{-1}} in width and outflow velocity (e.g., Allen et al. 2011). Models for BAL quasars have often relied on winds from accretion disks, in which BAL quasars are more likely to be observed at low latitudes above the disk than at high latitudes (e.g., de Kool & Begelman 1995; Murray & Chiang 1995; Proga & Kallman 2004; Fukumura et al. 2010). It was therefore notable when Zhou et al. (2006, hereafter Z+06) reported that six BAL quasars from the Sloan Digital Sky Survey (York et al. 2000) were “polar”, constrained by radio flux density variability to lie within 35\degree of a relativistic jet (presumably located at 90\degree latitude above the accretion disk). Shortly thereafter, Ghosh & Punsly (2007, hereafter GP07) presented nine additional candidate polar BAL quasars, Montenegro-Montes et al. (2008) and Doi et al. (2009) noted two new candidates each, and Reynolds et al. (2009) suggested Mrk 231 as a candidate.

The approach of using radio flux density variability to constrain a quasar’s orientation is based upon the existence of a limiting brightness temperature \(T_b\). If \(T_b > 10^{12}\) K in a radio source, a Compton cooling catastrophe occurs. The timescale for the electrons producing the radio synchrotron emission to lose all their energy through inverse Compton scattering of ambient radio photons decreases to just a few days, and the brightness temperature falls back below \(10^{12}\) K (Kellermann & Pauliny-Toth 1969). (In fact, the upper limit brightness temperature may be closer to \(T_b \approx 10^{11}\) K; see Readhead 1994.) To explain cases where \(T_b > 10^{12}\) K is observed, Doppler boosting in our direction is invoked. Relativistic motion of the radio-emitting plasma in our direction, e.g. in a relativistic jet oriented close to our line of sight, will boost the flux and brightness temperature we infer (Lind & Blandford 1985). The brightness temperature is related to the surface brightness; therefore, a sufficiently large change in the surface brightness of a radio source indicates an emission component with a brightness temperature \(T_b > 10^{12}\) K and Doppler boosting in our direction.

As knowledge of the incidence of BALs as a function of latitude would be a useful constraint on models of BAL outflows, close analysis of the assumptions leading to the designation of some BAL quasars as “polar” is worthwhile. We present a brief such analysis in \(\S\) 3. First, in \(\S\) 2 we correct an error in the literature that led to overestimates of the significance of variability in some “polar” BAL quasars.

2 DATA

The data used in Z+06, GP07, Montenegro-Montes et al. (2008), Doi et al. (2009) and herein comes from the...
FIRST ([Becker et al. 1995, White et al. 1997] and NVSS (Condon et al. 1998) catalogs. In Table 1 we report peak flux densities and their uncertainties from both surveys for all 21 polar BAL quasars reported to date. For ease of calculation later in the paper, these measurements are reported as Epochs 1 and 2 for each quasar, with a code to indicate which survey supplied that epoch of data. We report the date of each epoch as a Modified Julian Date (MJD). For Mrk 231, we include on a second row the data of McCutcheon & Gregory [1978].

A word about NVSS flux densities is in order. Z+06 used integrated flux densities to be conservative in selecting variable objects which were brighter in the FIRST survey. GP07 used peak flux densities to obtain a more accurate measurement of the variable radio flux density given the different resolutions of the two surveys. However, for objects unresolved in NVSS, the integrated and peak flux densities are the same (Eq. 42 of Condon et al. 1998). All targets in Table 1 except Mrk 231 are unresolved by NVSS, and the NVSS peak flux densities therein match the NVSS integrated flux densities reported by Z+06 for their objects. The flux densities reported by GP07 are the raw flux densities, uncorrected for known biases, which “should not normally be used unless corrections are applied” (§5.2.1 of Condon et al. 1998).

The significance of the variability for each quasar is calculated following Eq. (5) of GP07:

\[ \sigma_{\text{var}} = \frac{F_F - F_N}{\sqrt{\sigma_F^2 + \sigma_N^2}} \]  

where the \( F \) and \( N \) subscripts denote the FIRST and NVSS surveys, respectively. Six of the twenty-two BAL candidates reported in the literature do not meet the \( \sigma_{\text{var}} > 3 \) criterion proposed by GP07 when correct NVSS peak flux densities are used. These objects are kept in the table for completeness, but it should be kept in mind that their variability is not formally statistically significant. The variability of Mrk 231 between the NVSS and FIRST epochs was negative and could be due to extended flux missed by FIRST. For Mrk 231 we use only the fluxes from McCutcheon & Gregory (1978) in our subsequent analysis.

The other entries in Table 1 are discussed in the relevant sections below.

3 ANALYSIS

3.1 Brightness Temperature & Surface Brightness

Following GP07, the brightness temperature in the quasar’s cosmological reference frame, \( T_{b,q} \), can be expressed in terms of observable quantities on Earth, designated by the subscript \( o \). Consider a source in which the monochromatic intensity (surface brightness) has changed by an amount \( \Delta I_q(\nu_o) \) between epochs 1 and 2 separated by a time \( \Delta t \). In the Planck regime where \( h \nu \ll k_B T \), the change in brightness temperature associated with the change in intensity is

\[ T_{b,2} - T_{b,1} \equiv \Delta T_{b,q} = \frac{c^2 \Delta I_q(\nu_o)}{2k_B \nu_o^2} \]  

Intensity is equivalent to surface brightness. To relate the change in the intrinsic surface brightness to the observed flux densities, we assume a uniform source and use the monochromatic version of Liouville’s theorem (Eq. 30 of Gunn, 1978):

\[ \Delta I_q(\nu_q) = (1 + z)^3 \Delta I_o(\nu_o) \equiv (1 + z)^3 \left( \frac{F_{\nu,o,2}}{\Omega_2} - \frac{F_{\nu,o,1}}{\Omega_1} \right) \]  

where \( F_{\nu,o,1} \) and \( F_{\nu,o,2} \) are the flux densities observed on Earth at frequency \( \nu_o = \nu_q/(1 + z) \) at epochs 1 and 2, separated by time interval \( \Delta t = \Delta t_q(1 + z) \). The source subtends solid angle \( \Omega_1 \) at epoch 1 and \( \Omega_2 \) at epoch 2, where the solid angle is the proper area of the source on the sky divided by the angular diameter distance squared.

Equation 3 shows that flux density changes can occur without changes in brightness temperature, if the emitting area changes in the same proportion as the flux density. That is, the flux density can change just because of a change in the emitting area of a source of constant surface brightness.

3.2 Minimum Radii for a Source with \( T_{b,q} < 10^{12} \) K

We can place a lower limit on the sizes of the emitting regions in these quasars by finding the minimum sizes required for their brightness temperatures to be \( < 10^{12} \) K. We assume a static source in this section, and discuss the limitations of that assumption in later sections.

For a circular source of proper radius \( r \) and observed flux density \( F_{\nu,o} \) at a given epoch, Equations 2 and 3 can be combined to write its brightness temperature at that epoch as

\[ T_{b,q} = \frac{c^2}{2k_B \nu_q^2} \times (1 + z)^3 \frac{F_{\nu,o} d_q^2}{\pi r^2} \]  

where \( d_q \) is the angular diameter distance, which is related to the luminosity distance \( d_L \) as \( d_A = d_L/(1 + z)^2 \). GP07 adopt the expression \( d_L = (1+z)cZ/H_0 \) given by Penz (1994). For \( H_0=70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \Omega_\Lambda = 0.7 \) and \( \Omega_m = 0.3 \), the quantity \( Z \) is defined by

\[ Z = 3.31 - 3.65 \left[ \frac{1}{z_1^4} - 0.203 z_1^3 + 0.749 z_1^2 + 0.444 z_1 + 0.205 \right]^{1/8} \]  

where \( z_1 \equiv 1 + z \).

Combining all the above, \( T_{b,q} \) can be written as

\[ T_{b,q} = \frac{c^2(1+z)^3}{2k_B(1+z)^2} \frac{F_{\nu,o} (1+z)^2(cZ/H_0)^2}{\pi r^2(1+z)^4} \]  

\[ T_{b,q} \pm \delta T = \frac{2.0 \times 10^{12}(1+z)2^2}{(\nu_o/1 \text{ GHz})^2([1+(1+z)r/c]/1 \text{ yr})^2} \left( \frac{F_{\nu,o}}{1 \text{ mJy}} \right) \text{ K} \]  

(7)

where \( (1+z)r/c \) replaces \( \Delta t_q/2 \) in GP07 Eq. (3) and (4), explaining the factor of four smaller normalization above. The uncertainty on \( T_{b,q} \) follows from assuming that the uncertainty on the flux density is dominant.

By setting \( T_{b,q} = 10^{12} \) K in Eq. (7) above, one can solve for \( r \equiv r_{\text{min}} \) given the FIRST or NVSS \( F_{\nu,o} \) measurement at \( \nu_o = 1.4 \text{ GHz} \). The uncertainty on \( r_{\text{min}} \) is given by \( \sigma_{\text{min}} = r_{\text{min}} \sigma_{\nu} / 2F \). Table 1 gives the resulting values for both epochs 1 and 2, and \( r_{\text{min}} \) in proper light-years. The sizes are reasonable for radio-emitting structures at the cores of luminous AGN (e.g., Gallimore & Beswick 2004).
3.3 Expansion of an Emitting Source in the Plane of the Sky

To explain these quasars using emission from plasma with \( T_{b,q} < 10^{12} \text{ K} \) at both epochs requires reasonable minimum radii. However, we must also determine whether the required expansion between the two epochs is reasonable.

We first consider whether sources expanding only in the plane of the sky can explain the observations. The minimum expansion velocity in the plane of the sky needed to explain the observed variability by a source of constant brightness temperature \( T_{b,q} \leq 10^{12} \text{ K} \) is

\[
\beta_{\text{min}}^{\perp} = \frac{r_{\text{min}} - r_{\text{min}}^{\perp}}{c} \Delta t_q. \tag{8}
\]

Table 1 gives values of \( \beta_{\text{min}}^{\perp} \) and associated uncertainties, calculated from the uncertainties on the input \( r_{\text{min}} \). There are four cases of contraction (\( \beta_{\text{min}}^{\perp} < 0 \)) — one apparently superluminal — which we consider in the next section, plus two cases for which \( \beta_{\text{min}}^{\perp} > 1 \). In all other cases of expansion, the values of \( \beta_{\text{min}}^{\perp} \) are large but physically possible.

However, the values of \( \beta_{\text{min}}^{\perp} \) in Table 1 are for uniform planar expansion in all directions in the plane of the sky. A more likely scenario is a symmetric jet expanding along two directions in the plane of the sky at \( \beta \cong 1 \) with opening angle \( 180^\circ \), instead of expanding at \( \beta_{\text{min}}^{\perp} \) as a filled disk; i.e., opening angle \( 180^\circ \) in two opposite directions. As the emitting area at the first and second epochs must be the same in both cases, because such a jet covers only a fraction \( f \) of a filled disk it must start from a radius a factor of \( f^{-1/2} \) larger. By assuming an expansion velocity of \( c \) from such a radius, we find \( f^{\text{min}} = (\beta_{\text{min}}^{\perp})^2 \). Leaving out cases of contraction, \( \beta_{\text{min}}^{\perp} > 1 \), and \( \sigma_{\text{ext}} < 3 \), we find minimum opening angles of \( 4^\circ \) to \( 10^\circ \). Such values are larger than those seen in quasars with comparable radio luminosities, although they are similar to those found in weak radio galaxies [Bridle & Perley 1984]. Therefore, expansion close to the plane of the sky seems an unlikely explanation for the observed flux variability in these quasars, even though it is physically possible.

3.4 Spherical Expansion of an Emitting Source

Spherical expansion of a radio-emitting source seems a more likely explanation. Equation 7 suggests that for a given source and \( \nu_{\text{c},b} \), an observed flux density variability episode obeys \( \Delta F_{\nu,o}(t) \propto T_{b,q}(t)|r(t)/r(0)|^2 \), where \( r(t) \) is the apparent projected radius of the source on the sky at observed time \( t \). Z+06 and GP07 assumed \( r(t) = r(0) + ct \). However, as pointed out by Rees (1966, 1967), an optically thick sphere expanding in all directions at \( \beta = v/c \) increases its projected area on the sky at a rate proportional to \( \gamma \beta^2 \), where \( \gamma = (1 - \beta^2)^{-1/2} \). Such roughly spherical, expanding blobs of magnetized plasma could be launched by a relativistic jet close to the plane of the sky in these objects.

Light travel time effects explain why a spherical emission source which is observed to expand between time \( t = 0 \) to \( t = t_q \) in our frame (time \( t = 0 \) to \( t = t_q = t_o/(1 + z) \) in the quasar frame) can appear to cover a larger area than \( \pi \beta\Delta t_q^{\perp} \), where we take \( r(0) = 0 \) for simplicity in this example. Consider the light from such a source which reaches us at time \( t_o \) in our frame: not all of it was emitted at time \( t_q \) in the quasar frame. Some of that light was emitted after time \( t_q \) in the quasar frame from material which had travelled toward us and decreased the light-travel time to reach us from it, thus affording us a glimpse of a larger emitting source after a given observed time interval than in the case of expansion solely in the plane of the sky. The only requirement for that to happen is that the emitting source must expand for a time \( \gamma^2 t_q \) in the quasar frame to be seen to expand to an area \( \pi (\beta \Delta t_q)^2 \) over time \( t_o \) in the observer frame. (To see this note, that the part of the sphere which yields the largest apparent transverse motion is that located at an angle \( \epsilon = \arccos \beta \) to our line of sight. During a time \( t \) in the quasar frame, that material travels a distance \( \beta \cos \epsilon t = \beta^2 ct \) toward Earth and light from the projected center of the sphere travels a distance \( ct \) toward Earth. Equating the difference in those distances to \( c \Delta t_q/(1 + z) = c \Delta t_q \) yields \( t = \Delta t_q/(1 - \beta^2) = \gamma^2 \Delta t_q \).

Because \( r(t) - r(0) \propto \gamma \beta t \) for spherical expansion with \( v = \beta c \), extremely large flux density increases can be reproduced with \( \beta \) sufficiently close to 1 without an intrinsic \( T_{b,q} > 10^{12} \text{ K} \). In fact, the above equation is an underestimate for a source expanding with \( v \approx c \) because it does not account for all relativistic effects, such as Doppler boosting in the part of the source expanding toward us. Full accounting of such effects in a source whose outer edge expands with velocity \( \beta c \) shows that \( \Delta F_{\nu,o}(t) \propto T_{b,q}(0) \gamma^{7/2} \beta^2 t^3 \) (Eq. (8) of Rees 1967) in the first phase of the expansion. As the source starts to become optically thin to synchrotron self-absorption, \( \Delta F_{\nu,o}(t) \) peaks and then declines, initially roughly as \( \gamma \beta^7 \beta^2 t^{-1.5} < 0.5 \) (Rees 1967). Therefore, extremely large flux density decreases can also be reproduced with \( \beta \) close to 1 without an intrinsic \( T_{b,q} > 10^{12} \text{ K} \).

To compute the required \( \beta \) values, we compare to Z+06 and GP07, who in effect assumed \( v = c \) in Eq. 7 of Rees (1967) to (incorrectly) estimate the brightness temperature required for a source expanding at \( \beta \) in the plane of the sky to match the observed flux variations. We denote those brightness temperatures with an asterisk (\( \Delta T_{b,q}^\ast \)) and give them in Table 1 in units of \( 10^{12} \text{ K} \). They are found by setting \( r = c \Delta t_q \) in our Eq. 7 and using \( \Delta F_{\nu,o} = |F_{\nu,o,2} - F_{\nu,o,1}| \) and its uncertainty in place of \( F_{\nu,o} \) and its uncertainty. (The lower values of \( \Delta T_{b,q}^\ast \) as compared to GP07 arise mainly because the correct formula for \( \Delta T_{b,q}^\ast \) given in Eq. 7 has a factor of 4 smaller normalization than given in GP07.)

The minimum \( \beta \) required for spherical expansion to explain the observed flux increases in these quasars depends on whether the source responsible for the flux increase is pre-existing or new. In the former case, the required \( \beta \) is close to \( \beta_{\text{min}}^{\perp} \) except that it never exceeds unity. In the latter case, the required \( \beta \) can differ considerably from \( \beta_{\text{min}}^{\perp} \). We therefore compute the values \( \beta_{\text{min}}^{\perp} \) for the latter case; these are the minimum \( \beta \) required if spherical expansion from \( r = 0 \) is to explain the observations without requiring an intrinsic \( T_{b,q} > 10^{12} \text{ K} \). They are obtained by noting that in the first phase of the expansion, the apparent \( T_{b,q} \propto \Delta F_{\nu,o}(t)/|r(t)|^2 = \gamma^{3/2} t \). Therefore, including rela-

1 The flux density grows more rapidly than \( t^2 \) even for nonrelativistic expansion because the source is assumed to be initially synchrotron self-absorbed at \( \nu_q \), so that the intensity of the emission at that frequency increases as the magnetic field weakens in the expanding plasma; see Eq. (4) of Rees 1967.
tivistic effects means the intrinsic $T_{b,q}$ can be lower than the value $\Delta T_{b,q}$ calculated under the assumptions of $Z+06$ and GP07 by a factor $X(\beta)$ of about $3^{1/2}$, assuming a fixed fiducial magnetic field strength in the sphere.

Therefore, the observed flux variations can be matched by a source with $T_{b,q} = 10^{14}$ K expanding spherically at $\beta = \beta_{\gamma < 0}$, where $\beta_{\gamma < 0}$ is the solution to $X(\beta_{\gamma < 0}) = \Delta T_{b,q}$. Values of $\beta_{\gamma < 0}$ are given in Table 1 for all cases where $\Delta T_{b,q} \gtrsim 10^{12}$ K.

The values of $\beta_{\gamma < 0}$ in Table 1 correspond to $\gamma < 4.5$ in all but a few cases: $\gamma \simeq 5.5$ for J1346+3924, $\gamma \simeq 9$ for J0828+3718, $\gamma \simeq 20$ for Mrk 231 and $\gamma \simeq 43$ for J0756+3714. Values of up to $\gamma = 50$ have been inferred for extragalactic jets (Lister et al. 2007), but whether such values occur in cases of spherical expansion of radio-emitting plasma is not clear.

Nonetheless, spherical expansion of an emitting source at $\beta_{\gamma < 0}$ or larger can explain all the radio variability in these objects while still maintaining an intrinsic $T_{b,q} < 10^{12}$ K. Values of $\beta$ above the minimum required would allow for shorter episodes of variability to explain the same flux density changes, which eases the requirement on how long the source must expand and remain optically thick.

Larger values of $\beta$ could also maintain an intrinsic $T_{b,q} < 10^{13}$ K. For the three objects mentioned above, the required $\beta$ values correspond to $\gamma \simeq 25$ for J1346+3924, $\gamma \simeq 40$ for J0828+3718, $\gamma \simeq 94$ for Mrk 231 and $\gamma \simeq 200$ for J0756+3714.

Those quasars appear to require the most extreme parameters to explain their variability without resorting to a jet directed close to our line of sight. However, underestimated flux errors could reduce the significance of the observed radio variability of J1346+3924 ($\sigma_{var} = 3.1$) or J0756+3714 or Mrk 231 ($\sigma_{var} = 3.6$) below a true 3$\sigma$ (99.7% confidence) threshold, although that is extremely unlikely for J0828+3718 ($\sigma_{var} = 10.4$).

Furthermore, in the case of Mrk 231, if there is a jet oriented near our line of sight then the lack of apparent superluminal motion in the secondary VLBI component (Reynolds et al. 2009) means must that it be a near-stationary shock in that jet. Otherwise, the jet would need to be oriented at $i < 2^{\circ}$ of our line of sight for the $\gamma$ values considered in Reynolds et al. 2009. Such small angles are extremely unlikely a priori (<0.1% chance if quasars are seen at all $i < 60^{\circ}$). They are also potentially ruled out in Mrk 231 by the detection of a rotating nuclear gas disk producing velocity gradients on the sky of $\pm 110$ km s$^{-1}$ and $\pm 70$ km s$^{-1}$ in H$\alpha$ (Carilli et al. 1998) and CO (Bryant & Scoville 1998) at radii of $< 55 h_{70}$ pc and $< 255 h_{70}$ pc, respectively. Such large observed gradients would imply unprecedentedly large intrinsic velocity gradients for $i < 2^{\circ}$, except in the case where the detected outer disk is misaligned with the innermost disk that sets the orientation of the jet (see, e.g., Kondratko et al. 2005).

We briefly consider additional effects that might affect the likelihood of jets in these objects. Varying the fiducial magnetic field in the radio-emitting region cannot increase the limiting intrinsic $T_{b,q}$. The model of Rees (1967) which we have used does not include the additional surface brightness boost that would occur for a relativistically expanding sphere that was also moving toward us at a significant fraction of $c$. However, the speed in our direction required to keep $T_{b,q} < 10^{11}$ K in J0756+3714 (e.g.) is 0.91c (for expansion at $\beta_{\gamma < 0}$), which would require a relativistic jet oriented within $\pm 24^{\circ}$ of our line of sight. Postulating two new radio-emitting regions, each responsible for half the observed flux increase, can reduce $\Delta T_{b,q}$ by a factor of two at most. That could be significant for J0756+3714, Mrk 231 or J1346+3924, given the large uncertainties on their inferred brightness temperatures, but not for J0828+3718. Overall, J0828+3718 is the best candidate for a true polar BAL quasar.

4 CONCLUSION

Of the twenty-two candidate polar BAL quasars previously reported in the literature, only sixteen have statistically significant variability. Those sixteen can be explained without requiring $T_{b,q} > 10^{12}$ K, either by expansion of a pre-existing source at velocities of a few tenths of lightspeed in most cases, or by a newly appeared spherical emitting source expanding at a lower limit $\beta_{\gamma < 0}$ of 0.35c or greater. To ensure $T_{b,q} < 10^{11}$ K would require larger velocities, but only in one or two cases would the required velocity be unprecedentedly large.

Although a relativistic jet oriented close to our line of sight is not required to explain the observed flux variability in any of these quasars, we have not ruled out such a jet in any of them. (However, note that a further observation of the candidate polar BAL quasar SDSS J025625.56−011912.1 by Montenegro-Montes et al. 2008 did not reveal the continued variability which is expected if a relativistic jet is oriented along our line of sight to that object.) As two-flux density variability is unable to unambiguously identify cases of Doppler boosting, determining the relative incidence of relativistic jets oriented along our line of sight in normal and BAL quasars will require VLBI imaging to directly measure brightness temperatures. In cases with $T_{b,q} > 10^{12}$ K, repeated imaging will be required to determine whether such $T_{b,q}$ values are temporary (due to beaming from relativistic expansion of plasma structures in intermittent flares) or persistent (due to jet beaming) and to constrain apparent transverse jet velocities in the latter case. SDSS J082817.25+371853.7 is the best candidate for such imaging.

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2 The exact factor $X(\beta)$ is given by the ratio of $(F/\Omega)_{el}$ to $(F/\Omega)_{GP07}$, where $(F/\Omega)_{el}$ (surface brightness in the relativistic case) is given by Eq. 6 of Rees (1967) divided by $\pi(\beta c)^2$, and $(F/\Omega)_{GP07}$ (surface brightness under the assumptions made by GP07) is given by Eq. 7 of Rees (1967), with $\sigma_0$ set to $c$ and a missing fraction of $\frac{1}{2}$ multiplied in, divided by $\pi(c t)^2$. 

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Table 1. Reported Candidate Polar BAL Quasars

| Name (SDSS J) | z | Epoch 1 information | Epoch 2 information | σ_{var} | Z | Ref | r_{\text{min}}^1 ± σ_{r_{\text{min}}^1} | r_{\text{min}}^2 ± σ_{r_{\text{min}}^2} | Δt_{q} | β_{\text{min}}^q ± σ_{β_{\text{min}}^q} | ΔT_{\nu,q}^{\text{qmin}} | ΔT_{\nu,q}^{\text{qmin}} | ΔT_{\nu,q}^{\text{qmin}} | ΔT_{\nu,q}^{\text{qmin}} |
|---------------|---|-------------------|-------------------|---------|---|-----|----------------|----------------|------|----------------|----------------|----------------|----------------|----------------|----------------|
| 025625.63−011119.6 | b | 49306 | N | 22.3 ± 0.8 | 50052 | F | 25.78 ± 0.15 | 4.3 | 1.357 | 3 | 3.47 ± 0.06 | 3.73 ± 0.01 | 0.585 | 0.45 ± 0.11 | 0.9120 | 5.47 ± 1.28 |
| 075310.42+210244.3 | b | 49292 | A | 14.4 ± 0.6 | 51071 | F | 16.78 ± 0.15 | 3.9 | 1.301 | 1 | 2.75 ± 0.06 | 2.97 ± 0.01 | 1.480 | 0.15 ± 0.04 | ... | 0.57 ± 0.15 |
| 075628.25+371455.6 | b | 49336 | N | 216.2 ± 6.5 | 49556 | F | 239.36 ± 0.17 | 3.6 | 1.362 | 4 | 10.80 ± 0.16 | 11.36 ± 0.01 | 0.172 | 3.28 ± 0.04 | 0.9997 | 422 ± 119. |
| 081102.91+500724.2 | b | 49306 | N | 19.5 ± 0.7 | 50091 | F | 23.07 ± 0.19 | 4.9 | 1.153 | 1.2 | 3.05 ± 0.05 | 3.32 ± 0.01 | 1.239 | 0.22 ± 0.04 | 0.3823 | 1.11 ± 0.23 |
| 081618.99+483282.8 | b | 49306 | N | 69.3 ± 20.0 | 50569 | F | 68.32 ± 1.73 | 1.7 | 1.590 | 2 | 6.04 ± 0.09 | 6.25 ± 0.08 | 0.757 | 0.27 ± 0.16 | 0.8834 | 4.40 ± 2.63 |
| 081839.00+311000.2 | b | 49374 | N | 76.0 ± 0.5 | 50013 | F | 8.82 ± 0.14 | 2.4 | 1.323 | 2 | 2.01 ± 0.07 | 2.16 ± 0.02 | 0.550 | 0.28 ± 0.12 | 0.7119 | 2.14 ± 0.91 |
| 08217.25+371513.7 | b | 49336 | N | 14.8 ± 0.6 | 49556 | F | 21.18 ± 0.13 | 10.4 | 0.952 | 1.2 | 2.41 ± 0.05 | 2.89 ± 0.01 | 0.256 | 1.85 ± 0.18 | 0.9933 | 38.31 ± 3.69 |
| 090552.40+205811.4 | b | 49306 | N | 36.4 ± 1.2 | 51099 | F | 43.54 ± 0.14 | 5.9 | 1.145 | 1 | 4.16 ± 0.07 | 4.54 ± 0.01 | 1.653 | 0.24 ± 0.04 | 0.4537 | 1.24 ± 0.21 |
| 093348.37+313335.2 | b | 49336 | N | 163.6 ± 0.6 | 50013 | F | 18.35 ± 0.14 | 3.3 | 1.386 | 2 | 2.98 ± 0.05 | 3.16 ± 0.01 | 0.515 | 0.35 ± 0.11 | 0.8766 | 4.21 ± 1.26 |
| 104106.04+441417.4 | b | 49327 | N | 19.2 ± 0.7 | 51027 | F | 27.46 ± 0.15 | 11.5 | 1.485 | 2 | 3.28 ± 0.06 | 3.93 ± 0.01 | 1.502 | 0.43 ± 0.04 | 0.6964 | 2.05 ± 0.18 |
| 113415.83+431857.9 | b | 49306 | N | 25.2 ± 0.9 | 50499 | F | 27.38 ± 0.20 | 2.4 | 1.267 | 2 | 3.60 ± 0.06 | 3.76 ± 0.01 | 1.027 | 0.15 ± 0.06 | 0.3505 | 1.06 ± 0.45 |
| 121323.94+041014.7 | b | 49775 | N | 27.5 ± 0.9 | 50400 | F | 21.54 ± 0.14 | 6.5 | 1.445 | 3 | 3.91 ± 0.06 | 3.46 ± 0.01 | 0.902 | −0.50 ± 0.07 | 0.8708 | 4.06 ± 0.62 |
| 122836.92−030439.2 | b | 49775 | N | 136.3 ± 4.1 | 51071 | F | 143.88 ± 0.15 | 1.8 | 1.138 | 4 | 8.02 ± 0.12 | 8.24 ± 0.01 | 1.267 | 0.17 ± 0.10 | 0.7260 | 2.23 ± 1.21 |
| 125613.25+562205.3 | b | 49314 | N | 271.8 ± 8.2 | 50583 | F | 235.3 ± 0.2 | −4.4 | 0.043 | 5 | ... | ... | ... | ... | ... | ... |
| 125614.25+562253.6 | b | 49314 | N | 271.8 ± 8.2 | 50583 | F | 235.3 ± 0.2 | −4.4 | 0.043 | 5 | ... | ... | ... | ... | ... | ... |
| 125614.25+562253.6 | b | 49314 | N | 271.8 ± 8.2 | 50583 | F | 235.3 ± 0.2 | −4.4 | 0.043 | 5 | ... | ... | ... | ... | ... | ... |

a) The names for each object are taken from the SDSS Data Release Seven Quasar Catalog (Schneider et al. 2010) if an SDSS spectrum exists for that object; otherwise, they are taken from the SDSS Catalog Archive Server (http://cas.sdss.org/dr7/). Due to improvements in the astrometric solution over the course of the SDSS, the names given may differ slightly from those in earlier references. The information for each epoch of observation includes a Modified Julian Date (MJD), a survey code S (N = NVSS, F = FIRST, M = McCutcheon & Gregory 1978), and a peak flux density measurement and uncertainty. The NVSS and FIRST measurements are at 1.4 GHz; the McCutcheon & Gregory (1978) measurements at 22.2 GHz. Where only the month of observation was available, the MJD for the 15th of that month was used. The quantity σ_{var} is the significance of the variability, based on the observed fluxes and uncertainties. The quantity Z (Pen 1999) is related to the luminosity distance. The Ref column indicates the reference(s) in which the objects were discussed as polar BAL quasars: 1) Zhou et al. (2006); 2) Ghosh & Punsly (2007); 3) Montenegro-Montes et al. (2008); 4) Doi et al. (2009); 5) Reynolds et al. (2009). The sizes r_{\text{min}}^1 and r_{\text{min}}^2 have units of light-years, Δt_{q} units of years in the quasar rest frame, and ΔT_{\nu,q}^\text{qmin} has units of 10^{12} K. Negative β_{\text{min}}^q indicates apparent contraction of an optically thick emitting region (i.e., greater flux in the first epoch). In such cases, ΔT_{\nu,q}^\text{qmin} is calculated using the absolute value of the flux difference. β_{\text{min}}^q is not calculated for cases of ΔT_{\nu,q}^\text{qmin} < 10^{12} K.

b) Mkn 231.
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