Sum rules for isospin centroids in pick-up reactions on general multishell target states

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Abstract

Sum Rules equations for pick-up reactions are presented for the first time for the energy centroids of states both for the isospin $T_<(\equiv T_0 - \frac{1}{2})$ and $T_>(\equiv T_0 + \frac{1}{2})$ of the final nucleus when a nucleon is picked up from a general multishell target state with isospin $T_0$. These equations contain two-body correlation terms, $< H^{01} >$, which, at the present moment, are difficult to handle analytically. These terms are managed by combining these equations with the known stripping reactions equations. Sample applications of these equations to experimental data are presented.
1. Introduction

Sum rule methods have long been in use for analysing experimental data obtained from direct transfer reactions. The linear energy-weighted sum rules [1,2] relate the strengths and energies of residual nucleus states to multipole moments of the target state and the effective nucleon-nucleon interaction. In particular, the monopole energy weighted sum rule may be used to obtain an expression for the energy centroid of states of the residual nucleus. For inequivalent transfer involving a general multishell target state, the expression for centroid energy turns out to be fairly simple [3].

For equivalent transfer, the expressions for isospin centroids of states obtained via single particle addition to a target state, having only neutrons in the transfer orbit [4], though a bit more involved, are still amenable to useful applications [5,6].

However, when no restriction is imposed on the occupancy of orbits in the target state, the equations for isospin centroids, obtained by equivalent single particle stripping [7], contain an isovector two-body correlation term which defies simple analytical evaluation unlike the other terms appearing in these equations. This difficulty could be overcome, if the same stripping experiment provides complete information about the states of the residual nucleus having $T_<$ and $T_>$ values of the isospin, because in that case the ‘problem term’ could be eliminated. However, usually it is not feasible to extract this complete information from a single stripping reaction and this limits the scope of application of sum rules to generalized stripping situations.

The aim of this article is to present, for the first time, explicit algebraic expressions for isospin centroids of states of residual nucleus obtained via single nucleon pick-up from a general multishell target state. As expected, these expressions also involve the same isovector two-body correlation term, $\langle H^{01} \rangle$, as found in the case of
stripping situation for the same target state. It is, therefore, possible to eliminate this term by combining the equations for isospin centroids of the states of two residual nuclei obtained from the same target state, one via stripping and the other via pick-up of one nucleon. These combinations will prove much more useful as these correspond to the actual experimental situation, namely, that states belonging to one particular isospin (\(T_\leq\) or \(T_\geq\)) are usually populated by single particle transfer reactions. Sample applications are also presented to illustrate this approach.

2. Sum Rules and Energy Centroids

The general linear energy weighted sum rule for single particle pick-up may be written as \([2,7]\)

\[
\sum_{\Gamma_x} (-1)^{\Gamma_0 + \rho_i - \Gamma} (2\Gamma_0 + 1)(2\Gamma + 1)^{-1/2}U(\Gamma_0 \rho_i \Gamma_0 \rho_i; \Gamma \Lambda) S_{\Gamma_x} E_{\Gamma_x}^{-} = (-1)^\Lambda \langle n\Gamma_0 x_0 | (A^\rho_i \times H \times B^\rho_i)\Lambda | n\Gamma_0 x_0 \rangle
\]

(1)

where we use the familiar notion of the product space with each Greek letter representing two quantities—one in the angular momentum space and the other in the isospin space. Thus \(\Gamma_0\) stands for \(J_0\) and \(T_0\); \(\Gamma\) for \(J\) and \(T\); \(\rho\) for \(j\) and \(\frac{1}{2}\); \(\Lambda\) for \(\kappa\) and \(\tau\); and the factors \((2\Gamma_0 + 1) \equiv (2J_0 + 1)(2T_0 + 1)\) etc. Here \(J_0, j, J\) refer to the angular momentum of the target state, a nucleon in orbit \(\rho\) and a residual nucleus state respectively. \(T_0, \frac{1}{2}, T\) are the corresponding isospins. The target state is \(| n\Gamma_0 x_0 \rangle\) with \(n\) being the number of nucleons and \(x_0\) standing for all non angular momentum quantum numbers required to specify the target state uniquely. \(A^\rho\) and \(B^\rho\) are properly symmetrized and normalized creation and destruction operators respectively for a nucleon in orbit \(\rho\). The subscript \(i\) is used here for the transfer orbit. \(E_{\Gamma_x}^{-}\) and \(S_{\Gamma_x}^{-}\) are the energy and spectroscopic factor, respectively, of the state \(| n - 1, \Gamma x \rangle\) of the residual nucleus with superscript “-” reminding us that this state is obtained by pick-up (removal) of a nucleon from the target state. \(\Lambda \equiv \kappa\tau\)
represents the rank of the operator \((A^\rho_i \times H_0 \times B^\rho_i)^\Lambda\) in both angular momentum space \((\kappa)\) and isospin space \((\tau)\). \(\kappa\) can take the values \(0, 1, 2, \ldots, 2j_0\) \((j_0\) being the smaller of \(j_i\) and \(J_0\)) while \(\tau = 0\) or 1. The Hamiltonian \(H\) can be written as

\[
H = H(c.s.) + \sum_r \epsilon_r n_{op}(\rho_r) + H^{(2)}
\]

(2)

where \(H(c.s.)\) is the contribution from closed shells (inert core) and the second term is the effective one-body part of the Hamiltonian. The two-body part is given by

\[
H^{(2)} = -\sum_{rstu, \gamma; r \leq s, t \leq u} \frac{(2\gamma + 1)^{1/2}}{(1 + \delta_{rs})(1 + \delta_{tu})^{1/2}} W_{rstu}^{\gamma} \{ (A^{\rho_r} \times A^{\rho_s})^{\gamma} \times (B^{\rho_t} \times B^{\rho_u})^{\gamma} \}^0
\]

(3)

In this expression \(W_{rstu}^{\gamma}\) represents the antisymmetrized matrix element,

\[
\langle (\rho_r \rho_s)^{\gamma} | H^{(2)} | (\rho_t \rho_u)^{\gamma} \rangle,
\]

of the effective two-body nucleon nucleon interaction. The summation indices, \(r, s, t, u\) run over all the active orbits and are supposed to be ordered in some definite fashion.

Substituting equations (2) and (3) in equation (1), performing standard algebraic simplifications, we obtain, for the nucleon pick-up case from the \(\rho_i\)–orbit of the target state, the following expression for the isospin centroid of the residual nucleus states.

\[
E_T^- - E^-(riz) = \frac{\sum_J x \left( \frac{2T_0 + 1}{2T_1 + 1} \right) S_{JT_x}^- E_{JT_x}^-}{\sum_J x \left( \frac{2T_0 + 1}{2T_1 + 1} \right) S_{JT_x}^-} - E^-(riz)
\]

\[
= \sum_k \left\{ <H_{00}^{0k}>_{T_{tar}} - \frac{f(T)}{T_0} <H_{01}^{0k}>_{T_{tar}} \right\}
\]

(4)

It may be mentioned here that while evaluating various matrix elements during the algebraic process, we have assumed a pure multishell configuration for the target state, where, in the spirit of the low energy approximation, the isospin,

\[T_0 = T_{0z} = (N - Z)/2.\]

In the equation (4), \(E^-(riz)\) is the energy of the “Residual Interaction Zero" state of the final nucleus, that is, the state obtained by assuming that the picked-
up particle had no interaction with the active nucleons in the target state. Thus
\[ E^-(riz) = E_0 - \epsilon_{\rho_i}, \]
where \( E_0 \) is the target state energy and \( \epsilon_{\rho_i} \) is the single-particle energy of a particle in \( \rho_i \) orbit with respect to the inert core. The factor \( f(T) \) in the equation is given by
\[ f(T) = T(T + 1) - \frac{3}{4} T_0(T_0 + 1) = \begin{cases} T_0 & \text{for } T = T_\geq(\equiv T_0 + \frac{1}{2}) \\ -(T_0 + 1) & \text{for } T = T_\leq(\equiv T_0 - \frac{1}{2}) \end{cases} \] (5)
The summation index \( k \) on the right hand side of equation (4) runs over all the active orbits in the target state and \( \langle H_{ik}^\Lambda >_{Tar} \) is the expectation value, in the target state, of the two-body correlation operator given by
\[ H_{ik}^\Lambda = \frac{1}{2} \sum \gamma (2\gamma + 1)^\frac{1}{2} W_{ikik}^{\gamma} [(A^{\rho_k} \times A^{\rho_i})^\gamma \times B^{\rho_k} \rho_i \times B^{\rho_i}]^\Lambda \] (6)
For the isoscalar correlation operator, the expectation value turns out to be
\[ \langle H_{ik}^{00} >_{Tar} = -\frac{1}{2} (1 + \delta_{ik}) E^{(2)}_{Tar}(i - k) \] (7)
where \( E^{(2)}_{Tar}(i - k) \) is the contribution to the target state energy from two-body interaction of active nucleons in the \( i \)th orbit with those in the \( k \)th orbit.
\[ \langle H_{ik}^{01} >_{Tar} \] in equation (4) is the expectation value of the isovector two-body correlation operator, in the target state, which cannot be evaluated analytically or simply in the same manner as \( \langle H_{ik}^{00} >_{Tar} \).

The denominator on the right hand side in equation (4) is given by the non-energy weighted sum rules [8] in terms of the occupancy \( n_i \) of the transfer orbit \( \rho_i \) in the target state, and the partial isospin contribution, \( T_{0i} \), of the \( \rho_i \) orbit; using these results, we can write for the \( T_\geq(\equiv T_0 + \frac{1}{2}) \) and \( T_\leq(\equiv T_0 - \frac{1}{2}) \) centroids
\[ E_{T_\geq} - E^-(riz) = \sum_k \{ \langle H_{ik}^{00} >_{Tar} - \langle H_{ik}^{01} >_{Tar} \rangle_{\rho_i} \} \] (8)
and
\[ E_{T_\leq} - E^-(riz) = \sum_k \{ \langle H_{ik}^{00} >_{Tar} + (\frac{T_0 + 1}{T_0}) \langle H_{ik}^{01} >_{Tar} \rangle_{\rho_i} \} \] (9)
The term $< H_{ik}^{01} >_{Tar}$ occurring in these equations defies simple analytical evaluation, thus limiting the scope of application of any of these individually. If data are available for pick-up reactions, on the same target leading to both $T_<$ and $T_>$ states, then the equations (8) & (9) may be suitably combined to eliminate this term. However, in the experimental situation, usually, such is not the case. More often, we find one stripping and one pick-up reaction on the same target, in each case providing reliable information about states having a particular $T$-value. Therefore, it is often convenient to eliminate the term $< H_{01}^{01} >$, of course, depending on the particular situation, by combining one of the pick up equations (8) & (9) with one of the equations, for $T$-centroids from single particle stripping reactions[7]. For convenience we reproduce these below:

$$E_{T_>}^+ - E^+(riz) = \sum_k \{ p_{T_>}^+(i-k) + (N_i - \delta_{ik}) q_{T_>}^+(k) W_{ik}^{T=1} + (N_i + \delta_{ik}) r_{T_>}^+(k) W_{ik}^{T=0} \} <\text{neutron}(\text{holes})>_{\rho_i}$$

(10)

and

$$E_{T_<}^+ - E^+(riz) = \frac{\sum_k \{ p_{T_<}^+(i-k) + (N_i - \delta_{ik}) q_{T_<}^+(k) W_{ik}^{T=1} + (N_i + \delta_{ik}) r_{T_<}^+(k) W_{ik}^{T=0} \}}{<\text{proton}(\text{holes})>_{\rho_i} + \frac{1}{2T_0} \{ <\text{proton}(\text{holes})>_{\rho_i} - <\text{neutron}(\text{holes})>_{\rho_i} \}}$$

(11)

In these equations

$$p_T^+(i-k) = < H_{ik}^{00} >_{Tar} + \frac{f(T)}{T_0} < H_{ik}^{01} >_{Tar}$$

(12)

$$q_T^+(k) = \frac{3n_k}{4} + \frac{f(T)T_{0k}}{2T_0}$$

(13)

$$r_T^+(k) = \frac{n_k}{4} - \frac{f(T)T_{0k}}{2T_0}$$

(14)

$$N_i = 2j_i + 1$$

(15)

and

$$E^+(riz) = E_0 + \epsilon_{\rho_i}$$

(16)
$W_{ik}^{T=1}$ and $W_{ik}^{T=0}$ appearing in equations (10) & (11) are the average two-body interaction energies in the isotriplet and isosinglet states, respectively, of one nucleon in the $i$th orbit and the other in the $k$th orbit. These are given by

$$W_{ik}^{T} = \frac{\sum_{J}(2J+1)W_{ikik}^{JT}}{\sum_{J}(2J+1)}$$

(17)

The symbol $n_k$ in equations (13) & (14) is the number of nucleons in the $k$th active orbit of the target state whereas $T_{0k}$ is, in some sense, the partial contribution of these nucleons towards the isospin of the target state.

### 3. Applications and Discussion

As discussed in the previous section, a combination of any of the equations (8) & (9) with one of (10) and (11) helps us to eliminate the term $< H_{ik}^{01} >_{T_{ar}}$. Assuming a pure multishell configuration for the target state with $T_0 = T_{0z} = (N - Z)/2$, we can easily evaluate the factors $q^+_{T}$ and $r^+_{T}$. The values of $E^- (r_{iz})$, $E^+ (r_{iz})$ and $< H_{ik}^{00} >_{T_{ar}}$ can be computed using the Binding Energy Tables [9]. Obtaining the values of $E^-_T$ and $E^+_T$ from experimentally measured energies and spectroscopic factors in one pick-up and one stripping reaction experiments on the same target state, we can extract the values of average effective interaction parameters $W_{f_7}^{T=1}$ and $W_{f_7}^{T=0}$ by making least-squares fits.

In the present study we have limited ourselves to reactions involving the transfer (both pick-up and stripping) of a nucleon to the $1f_7$ orbit with targets having $1f_7$ as the only active shell outside inert core $^{40}$Ca. The reaction data used in setting up the equations for $W_{f_7}^{T=1}$ and $W_{f_7}^{T=0}$ are listed in Table 1. The seventeen linear equations, so formed, have been used to obtain the best fitted values. The values of $W_{f_7}^{T=1}$ and $W_{f_7}^{T=0}$ so obtained are compared with the previous results in Table 2.

In our previous calculations of average effective interaction parameters, we had to restrict ourselves to targets having only active neutrons because in that case, the
isovector two-body correlation term \(< H_{ik}^{01} >_{Tar}\) happens to be equal to isoscalar correlation \(< H_{ik}^{00} >_{Tar}\) due to certain specific isospin constraints. The equations for isospin centroids obtained via single nucleon pick-up, presented in this article, along with similar equations for stripping reactions reported earlier [7], complete the set for direct transfer reactions involving general multishell target states. It is hoped that the sum rule analysis can now be extended to much larger experimental data to extract useful information about effective two-body interaction.
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Table 1. List of experiments from which the centroids have been obtained for the present study and the various combinations of stripping and pick-up reactions used in calculations.

| Target | Stripping Reaction | Centroid Isospin | Value (MeV) | Pick-up Reaction | Centroid Isospin | Value (MeV) |
|--------|--------------------|------------------|-------------|------------------|------------------|-------------|
| 41Ca   | \((d, p)\) [10]    | \(T_\rangle\)    | 2.819       | \((d, t)\) [11] | \(T_\langle\)    | 0.083       |
| 42Ca   | \((d, p)\) [15]    | \(T_\rangle\)    | 0.066       | \((d, t)\) [12] | \(T_\langle\)    | 0.000       |
| 42Ca   | \(\left(^3\text{He}, d\right)\) [13] | \(T_\langle\) | 1.079      | \((p, d)\) [16] | \(T_\langle\)    | 0.567       |
| 42Ca   | \(\left(^3\text{He}, d\right)\) [13] | \(T_\langle\) | 1.079      | \((d, t)\) [12] | \(T_\langle\)    | 0.000       |
| 42Ca   | \(\left(^3\text{He}, d\right)\) [13] | \(T_\rangle\) | 4.234      | \((d, t)\) [12] | \(T_\langle\)    | 0.000       |
| 44Ca   | \((d, p)\) [15]    | \(T_\langle\)    | 0.352       | \((p, d)\) [14] | \(T_\langle\)    | 0.000       |
| 44Ca   | \(\left(^3\text{He}, d\right)\) [17] | \(T_\langle\) | 0.453      | \((p, d)\) [16] | \(T_\langle\)    | 0.000       |
| 45Sc   | \((d, p)\) [18]    | \(T_\rangle\)    | 0.383       | \(\left(d,\text{He}\right)\) [19] | \(T_\rangle\) | 1.086       |
| 46Ca   | \((d, p)\) [20]    | \(T_\rangle\)    | 0.000       | \((d, t)\) [12] | \(T_\langle\)    | 0.000       |
| 46Ti   | \(\left(^3\text{He}, d\right)\) [21] | \(T_\langle\) | 0.150    | \((p, d)\) [22] | \(T_\langle\)    | 0.434       |
| 46Ti   | \((d, p)\) [23]    | \(T_\rangle\)    | 0.555       | \((p, d)\) [22] | \(T_\rangle\) | 4.760       |
| 48Ca   | \(\left(^3\text{He}, d\right)\) [24] | \(T_\langle\) | 0.000      | \((p, d)\) [16] | \(T_\langle\)    | 0.050       |
| 48Ti   | \(\left(^3\text{He}, d\right)\) [25] | \(T_\langle\) | 0.151      | \(\left(^3\text{He}, \alpha\right)\) [26] | \(T_\langle\) | 0.819       |
| 48Ti   | \((d, p)\) [27]    | \(T_\rangle\)    | 0.583       | \(\left(d,\text{He}\right)\) [28] | \(T_\rangle\) | 0.000       |
| 50Cr   | \((d, p)\) [29]    | \(T_\rangle\)    | 0.302       | \(\left(t, \alpha\right)\) [25] | \(T_\rangle\) | 0.490       |
| 50Cr   | \(\left(^3\text{He}, d\right)\) [21] | \(T_\langle\) | 0.312      | \(\left(^3\text{He}, \alpha\right)\) [30] | \(T_\langle\) | 0.257       |
| 51V    | \(\left(^3\text{He}, d\right)\) [31] | \(T_\langle\) | 2.279      | \((p, d)\) [32] | \(T_\langle\)    | 1.769       |

Table 2. Average two-body interaction parameters for \(1f_2\) shell nuclei.

|                      | \(W_{f_2/f_2}^{\text{present}} (\text{MeV})\) | \(W_{f_2/f_2}^{\text{previous}} (\text{MeV})\) | rms deviation |
|----------------------|-----------------------------------------------|-----------------------------------------------|---------------|
| Present calc.        | -0.212                                        | -1.662                                        | 0.542         |
| Previous calc.[7]    | -0.215                                        | -1.714                                        | 0.191         |
| Kuo-Brown[33]        | -0.128                                        | -1.154                                        | -             |
| Lips & McEllistren[34]| -0.240                                        | -                                             | -             |
| Federman & Pittel[35]| -0.228                                        | -                                             | -             |
| Schiffer & True[36]  | -                                             | -1.739\(^a\),-1.594\(^b\)                  | -             |

\(^a\) Derived from a potential fitted to the experimental data
\(^b\) Determined from a direct fit to experimental data