Islamic economy and capitalism: Comparison using econophysics models of exchange and redistribution

Takeshi Kato

1 Hitachi Kyoto University Laboratory, Open Innovation Institute, Kyoto University, Kyoto, Japan

* Corresponding author

E-mail: kato.takeshi.3u@kyoto-u.ac.jp (TK)
Abstract

What are the fundamental differences between an Islamic economy and capitalism? The Islamic economy is characterized by the prohibition of Riba (interest) and the enforcement of Mudaraba (joint venture) and Waqf (donation). We propose new econophysics models of wealth exchange and redistribution to quantitatively compare these characteristics with the differences in capitalism, and evaluate wealth distribution and disparities by simulation. Specifically, we propose a loan interest model representing finance capitalism and Riba, a joint venture model representing shareholder capitalism and Mudaraba with respect to exchange, and a transfer model representing inheritance tax and Waqf with respect to redistribution. As exchanges are repeated from an initial uniform distribution of wealth, the distribution of wealth approaches a power-law distribution more quickly in the loan than in the joint, and the Gini index, which represents disparity, rapidly increases. The joint has a slower increase in the Gini index, but eventually the wealth distribution in both models becomes a delta distribution, and the Gini index gradually approaches 1. Next, when both models are combined with the transfer model to redistribute wealth every given period, the loan has a larger Gini index than the joint, but both models converge to a value with a Gini index less than 1. These results quantitatively reveal that in the Islamic economy, disparities are restrained by the prohibition of Riba, reciprocal exchange in Mudaraba, and redistribution through Waqf. These correspond to the three economic modes Polanyi presented: reciprocity, redistribution, and exchange. The insights gained from the comparison of the Islamic economy and capitalism encourage an economy that embraces the morals of mutual aid as described in Mauss's theory of gifts, Kropotkin's theory of mutual aid, Graeber's theory of debt,
Sarthou-Lajus's repayment to third parties, and Karatani's mode of exchange D, and provide guidelines for the next alternative to capitalism.

**Introduction**

The unequal distribution of wealth has created disparities in income and assets and has become a major social issue worldwide. The Gini index, which represents disparity, is gradually rising in the major developed countries of the OECD [1]. A Gini index of 0.4 is considered a warning level against social unrest [2], but there are many countries in the world where the coefficient exceeds 0.4 [3]. Reduced Inequalities is listed in Goal 10 of the United Nations Sustainable Development Goals [4]. Goal 10 is closely related to Goal 1: No Poverty, Goal 2: Zero Hunger, Goal 3: Good Well-being, Goal 8: Inclusive Economics, and Goal 16: Justice.

Looking back at human history, according to David Graeber, the past 5,000 years have alternated between cycles of a bullion-based monetary economy and a virtual money-based credit economy [5]. The age of the agriculture of the credit economy, the axial age of the monetary economy, the Middle Ages of the credit economy, and the age of the great capitalist empire of the monetary economy, culminating in the modern era with the transition from the gold standard to a floating currency system. The era of the money economy is characterized by interest-bearing debt, war, and slavery, while the era of the credit economy tends to create a peaceful society with moral. Today, we are in the transition from a money economy to a credit economy, but we are still stuck in debt and war and unable to create the next alternative to capitalism.
In the Middle Ages, an era of credit economy one before the modern era, moral and financial innovations emerged from the Islamic world [5]. Islamic codes outlawed interest-bearing loans that made money a self-object and encouraged bankers and merchants to engage in credit operations. Joint management was favored in finance as an extension of mutual aid, and labor arrangement was organized on a profit-sharing basis.

The Islamic economy is characterized by the prohibition of the Riba (interest) and the Gharar (speculation) as an "economy based on real transactions," the Mudaraba (joint venture) and the Murabaha (profit sharing through agreed contracts) as a "face-to-face economy," and Zakat (charity) and Waqf (donation) as an "economy embedded in religion" in the Ummah (community) [6, 7, 8]. The Islamic economy can be considered as a reference in transforming the modern world from capitalism to the next credit economy and creating an equal and free society with reduced disparity.

In econophysics, which aims to elucidate economic phenomena from a physical approach and perspective, the distribution of wealth has been investigated using a multi-agent exchange model based on the analogy of the exchange of kinetic energy between two ideal gas particles [9, 10, 11]. In these exchange models, various wealth distributions appear, including exponential, power-law, and delta distributions, depending on parameters such as the amount of exchange between the two agents, the savings rate, and the stock contribution rate. For example, Pareto principle [12] for the distribution of income and Zipf's law [13] for the distribution of city size are known as empirical laws, and exchange models in econophysics explain well the basic causes of these distributions. Therefore, if we apply an econophysics approach to the Islamic
economy and capitalism, we can see fundamental differences between the two.

The aim of this study is to identify the fundamental differences between the Islamic economy and capitalism from the perspective of econophysics and to obtain guidelines for the credit economy, the next alternative to capitalism. Therefore, our goal is to establish new econophysics models representing the Islamic economy and capitalism, and to compare the two by simulating wealth distribution and disparity based on these models.

The new model we propose introduces the concept of Karl Polanyi’s economic modes (reciprocity, redistribution, and market exchange) [14], not just the exchange of wealth in traditional models. With regard to exchange, we focus on the loan interest (Riba), which is a characteristic of the monetary economy and finance capitalism, and the reciprocal joint venture (Mudaraba), which is said to have its roots in shareholder capitalism, and model it as the distribution of profits and losses in exchange. With regard to redistribution, we model a compulsory inheritance tax based on the centrality of power in capitalism shown by Polanyi and a reciprocal donation (Waqf) based on the non-centrality of the community as transfers every predetermined period (although the actual Waqf is a relinquishment of ownership and deposit with a trustee, this study will consider it a transfer of wealth to the public, i.e., to the community as a whole).

The current model is novel in that it incorporates loan interest and profit/loss distribution in joint ventures into the mathematical model, while the previous model only incorporated the exchange of wealth, as well as redistribution through transfers, which were not taken into account in the previous model. Moreover, there have been
no previous studies comparing Islamic economies and capitalism from the perspective of econophysics, and this study provides new insights into the Middle Ages and the credit economy that came next.

The remainder of this paper is organized as follows: the next section presents a literature review on exchange models in econophysics, the "Methods" section proposes a new exchange and redistribution model, and the "Results" section presents simulation results of wealth distribution and computation of the Gini index for disparity, together with a comparison of the Islamic economy and capitalism. The "Discussion" section revisits the contemporary significance of the Islamic economy in light of the "Results" and discusses how the credit economy should be the next alternative to capitalism. The last section presents conclusions and future challenges.

**Literature Review**

This section provides a brief literature review on exchange models in econophysics based on Chakrabarti et al [10] and Kato et al [11].

Economist Vilfredo Pareto discovered in 1906 that the distribution of income follows a power law [12]. This is called Pareto principle, where income is concentrated in a few wealthy people [15]. Champernowne explained Pareto principle in 1953 using a model in which the income distribution changes over time through a stochastic process [16]. Based on a review of a number of studies in 2009, Yakovenko and Rosser suggested that the distribution of income and wealth is consistent with a lognormal or gamma distribution and that the hem of the distribution follows a power law [9].
Sociologist John Angle showed in 1986 that a gamma distribution arises from a stochastic process in which two economic agents contribute to each other's wealth surplus, excluding savings, and one randomly chosen agent gets all of the amount contributed [17]. Hayes and Chakraborti showed in 2002 and 2002, respectively, using a model of kinetic energy exchange in collisions of ideal gas particles, that a delta distribution arises in which wealth is concentrated in a single economic agent [18, 19]. This concentration of wealth is due to the fact that the amount of wealth contributed is determined according to the poorer of the two economic agents.

Subsequently, several exchange models were proposed, extending the model of Angle, Hayes, and Chakraborti. For example, in Chakrabarti et al.'s model [10], the exponential distribution is obtained by having both agents divide the contribution with random probability, rather than one agent getting all of the contribution. In Chatterjee et al.'s model [20], a gamma distribution is obtained by randomly dividing contributions with a constant savings rate for all economic agents. In another Chatterjee's model [21], a power-law distribution is obtained in a model where the savings rate for all economic agents follows a uniform distribution.

In order to restrain wealth disparities, models have been proposed that introduce the concepts of taxation and insurance. In Guala's taxation model [22], a fixed tax rate is imposed and the total tax is distributed equally among all economic agents after a random division of the exchange. As the tax rate increases, the distribution shifts from an exponential distribution to a gamma distribution and back to an exponential distribution again. In Chakrabarti et al.'s insurance model [23], economic agents insure against risk and after an exchange, the winner transfers a portion of the surplus
to the loser at a constant rate. As the transfer rate increases, the distribution shifts from an exponential distribution through a gamma distribution to a delta distribution.

In addition to these, other models have been proposed that take into account regions and surplus stocks. The regional model of Kato et al [24] introduces a spatial exchange range and a regional support bias (giving an advantageous probability for poorer regions) in addition to the regional economic circulation rate (savings rate). The narrower the exchange range and the larger the bias, the closer to a normal distribution. In Kato et al.'s surplus stock model [11], the wealthy contribute surplus stock in addition to the matching contribution to the poor, which is divided randomly. As the surplus stock contribution rate increases from 0 to 1, the distribution changes from a delta distribution to a gamma distribution (as in Chatterjee et al.'s model [20]). The model also shows that the contribution of surplus stock by the wealthy is necessary to both stimulate the economy and reduce disparity.

So far, we have outlined the exchange models of econophysics. All of the conventional models assume a random exchange of wealth. The purpose of this study is not to improve upon the conventional models, but to construct new models for the Islamic economy and capitalism. Thus, for the first time, we are modeling loan interest (borrower's interest and lender's interest, borrower's profit/loss burden) and joint venture (profit/loss allocation between two agents) in wealth exchange and transfers (redistribution every predetermined period) in wealth redistribution, rather than random exchange as conventional models.
Methods

Exchange model

First, we cite the traditional basic Chatterjee et al model [20] as the exchange model for reference. We refer to it here as the random exchange model (R-model) for convenience. In the R model, two agents $i, j (= 1, 2, \cdots, N)$ are randomly selected from among $N$ economic agents. Let $m_i(t)$ denote the wealth of agent $i$ at time $t$ and $m_j(t)$ denote the wealth of agent $j$. As shown in Fig 1B, two agents $i, j$ save a portion of their wealth at time $t$ at a common savings rate $\lambda$ and exchange the remaining wealth $(1 - \lambda) \cdot \left( m_i(t) + m_j(t) \right)$, excluding savings, with random division probability $\epsilon$. $\epsilon$ is a uniform random number defined in the range $0 \leq \epsilon \leq 1$. The

Fig 1. Exchange models.
(A) before exchange, (B) R model (random exchange), (C) L model (loan interest), and (D) J model (joint venture). $\lambda$ is savings rate, $\epsilon$ is division probability, $\rho$ is interest rate, and $\delta$ is profit/loss rate.
wealth \( m_i(t + 1) \), \( m_j(t + 1) \) of two agents \( i, j \) at time \( t + 1 \) is expressed as Eq (1).

\[
\begin{align*}
m_i(t + 1) &= \lambda \cdot m_i(t) + \epsilon \cdot (1 - \lambda) \cdot (m_i(t) + m_j(t)); \\
m_j(t + 1) &= \lambda \cdot m_j(t) + (1 - \epsilon) \cdot (1 - \lambda) \cdot (m_i(t) + m_j(t)).
\end{align*}
\]  

(1)

Based on the R model, our original method is to construct a loan interest model corresponding to financial capitalism. We refer to it here as the L model for convenience. In the L model, the interest rate \( \rho \) and the profit/loss rate \( \delta \) are newly set in addition to the savings rate \( \lambda \). \( \delta \) is a uniform random number defined in the range \( -\delta_w \leq \delta \leq \delta_w \) (\( \delta_w \geq 0 \)). As shown in Fig 1C, one agent \( i \) of the two agents \( i, j \), randomly selected from \( N \), is the borrower and the other agent \( j \) is the lender. Borrower \( i \) pays interest \( (1 - \lambda) \cdot \rho \cdot m_j(t) \) and bears profit/loss \( (1 - \lambda) \cdot \delta \cdot (m_i(t) + m_j(t)) \), while lender \( j \) earns interest \( (1 - \lambda) \cdot \rho \cdot m_j(t) \). The wealth \( m_i(t + 1) \), \( m_j(t + 1) \) of two agents \( i, j \) at time \( t + 1 \) is expressed as Eq (2).

\[
\begin{align*}
m_i(t + 1) &= \lambda \cdot m_i(t) + (1 - \lambda) \\
&\quad \cdot \left( m_i(t) - \rho \cdot m_j(t) + \delta \cdot (m_i(t) + m_j(t)) \right); \\
m_j(t + 1) &= \lambda \cdot m_j(t) + (1 - \lambda) \cdot (1 + \rho) \cdot m_j(t).
\end{align*}
\]  

(2)

Furthermore, our original approach is to build a joint venture model that addresses shareholder capitalism and the Mudaraba of the Islamic economy. We refer to it here as the J model for convenience. Compared to the relationship between shareholders and operators, in Mudaraba the joint operators are actively involved with each other in
a partnership, but both are represented by the same model mathematically. In the J model, the interest rate $\rho$ in the L model is not used, only the profit/loss rate $\delta$. As shown in Fig 1D, two agents $i, j$, randomly selected from $N$, obtain profit/loss 

$$(1 - \lambda) \cdot \delta \cdot m_i(t)$$ 

and $(1 - \lambda) \cdot \delta \cdot m_j(t)$, respectively, depending on the profit/loss rate $\delta$. The wealth $m_i(t + 1), m_j(t + 1)$ of two agents $i, j$ at time $t + 1$ is expressed as Eq (3).

$$m_i(t + 1) = \lambda \cdot m_i(t) + (1 - \lambda) \cdot (1 + \delta) \cdot m_i(t);$$

$$m_j(t + 1) = \lambda \cdot m_j(t) + (1 - \lambda) \cdot (1 + \delta) \cdot m_j(t).$$

(3)

**Redistribution model**

Then, with respect to redistribution, which is not considered in the traditional exchange model, we construct a transfer model that corresponds to the inheritance tax of capitalism and the Waqf of the Islamic economy. We refer to it here as the T model

\[ T \text{ model (transfer)} \]

\[ m_i(t) \]

\[ m_1(t + \Delta) \quad m_2(t + \Delta) \quad m_i(t + \Delta) \quad m_j(t + \Delta) \quad m_N(t + \Delta) \]

\[ \xi \cdot m_i(t) \]

**Fig 2. Redistribution models.**

T model (transfer), where $\xi$ is the transfer rate. The wealth $\xi \cdot m_i(t)$ is equally distributed from one agent to the other $N$ agents, and this distribution is carried out by all $N$ agents every period $t$. Then, with respect to redistribution, which is not considered in the traditional exchange model, we construct a transfer model that corresponds to the inheritance tax of capitalism and the Waqf of the Islamic economy. We refer to it here as the T model.
for convenience. While inheritance taxes are levied based on the centrality of power, Waqf is self-initiated based on the non-centrality of community and the Zakat, but both are mathematically equivalent in terms of redistributing wealth. As shown in Fig 2, the T model sets a new transfer rate $\xi$ and period $t_p$. Although the timing of inheritance and donation in and after life differs from agent to agent, the T model assumes that $N$ agents simultaneously distribute the wealth $\xi \cdot m_i(t)$ corresponding to the transfer rate $\xi$ to all others equally in each period $t_p$. This is because, in estimating the effect of redistribution on reducing disparity, it is considered sufficient to establish an average time period and an average amount of redistribution. The wealth $m_i(t + \Delta)$ of agent $i$ at time $t + \Delta$ immediately after period $t_p$ is expressed as Eq (4).

$$m_i(t + \Delta) = (1 - \xi) \cdot m_i(t) + \xi \cdot \frac{\sum_{j \neq i} m_j(t)}{N - 1}.$$  \hfill (4)

In addition, the concept of quantile is newly introduced for redistribution. We refer to this as the Q model for convenience. Quantiles are sometimes used to assess disparities [25], and are used here to estimate what level of wealth redistribution is saving the poor. In the Q model, Eq (5) is used to first calculate the number of agents $N_q$ whose wealth $m_i(t)$ is less than or equal to the $1/q$ quantile of the wealth maximum $m_{MAX}(t)$, and then the total wealth $S_q$ of agents whose wealth $m_i(t)$ exceeds the $1/q$ quantile.

$$m_{MAX}(t) = \text{Max}(m_i(t), i \in \{1, 2, \ldots, N\}),$$

$$N_q = \text{Count} \left( m_i(t) \leq \frac{m_{MAX}(t)}{q}, i \in \{1, 2, \ldots, N\} \right),$$  \hfill (5)
\[ S_q = \frac{1}{q} \sum_{m_i(t) > \frac{m_{\text{MAX}}(t)}{q}} m_i(t). \]

Then, agent \( i \) above the \( 1/q \) quantile transfers wealth \( \xi \cdot m_i(t) \) corresponding to the transfer rate \( \xi \), and agent \( i \) below the \( 1/q \) quantile receives a redistribution of wealth \( \xi \cdot S_q / N_q \). The wealth \( m_i(t + \Delta) \) of agent \( i \) at time \( t + \Delta \) immediately after period \( t_p \) is expressed as Eq (6) according to the wealth \( m_i(t) \) at time \( t \).

\[ \begin{align*}
    & \text{if } m_i(t) > \frac{m_{\text{MAX}}(t)}{q}, \\
    & \quad m_i(t + \Delta) = (1 - \xi) \cdot m_i(t); \\
    & \text{if } m_i(t) \leq \frac{m_{\text{MAX}}(t)}{q}, \\
    & \quad m_i(t + \Delta) = m_i(t) + \xi \cdot S_q / N_q. \\
\end{align*} \tag{6} \]

**Gini index**

The Gini index is used as a method to evaluate disparities due to exchange and redistribution. The Gini index is obtained by drawing the Lorenz curve and the equal distribution line [26]. Mathematically, the wealth \( m_i(t) \) of the \( N \) agents at time \( t \) is ordered from smallest to largest and the Gini index \( g \) is calculated using Eq (7). When the wealth of \( N \) agents is perfectly equal (uniform distribution), the Gini index \( g \) is 0. When all wealth is concentrated in a single agent (delta distribution), \( g \) is 1. In other words, \( g \) ranges from 0 to 1, and the larger the disparity, the larger \( g \) becomes.
\[ r_i(t) = \text{Sort}(m_i(t)), \]
\[ g = \frac{2 \cdot \sum_{i=1}^{N} t \cdot r_i(t)}{N \cdot \sum_{i=1}^{N} r_i(t)} - \frac{N + 1}{N}. \]  

(7)

Results

Exchange

First, we examine the wealth distribution for the R model represented by Eq (1), the L model represented by Eq (2), and the J model represented by Eq (3). Fig 3 shows the simulation results. The R model for the reference shown in Fig 3A shows that the wealth distribution approaches a gamma distribution as time \( t \) elapses, as shown in the literature [20]. The L model shown in Fig 3B approaches a power-law distribution with extreme disparity as time \( t \) elapses. The J model shown in Fig 3C has the same width \( \delta_w \) of the profit/loss rate \( \delta \) as the L model, but it is looser exponential distribution than a power-law distribution, and the disparity is suppressed compared to the L model. The J model shown in Fig 3D approaches a power-law distribution similar to the L model because the width \( \delta_w \) of the profit/loss rate \( \delta \) is larger than in Fig 3C.
Next, we examine the Gini indexes $g$ for the R, L, and J models using Eq (7). Fig 4 shows the simulation results. The R model for reference shows that the Gini index $g$ generally converges to 0.4 as time $t$ elapses, as expected from the literature [20]. The L model has a Gini index $g$ approaching 1 regardless of the interest rate $\rho = 0$ and 0.05. The J model shows that the Gini index $g$ approaches 1 slowly when the width of the profit/loss rate $\delta$ is $\delta_w = 0.1$, but when $\delta_w = 0.2$, the Gini index $g$ approaches 1.
more quickly.

\[ N = 1,000 \]
\[ \lambda = 0.5 \]

**Fig 4. Gini indexes for R, L and J models.**
The horizontal axis represents time \( t \) and the vertical axis represents the Gini index \( g \). In all models, the number of agents is \( N = 1000 \), the initial values of wealth are \( m_i(0) = 1 \) \( (i = 1, 2, \ldots, N) \), and the savings rate is \( \lambda = 0.25 \). The interest rates of the L model are \( \rho = 0 \) and 0.05, and the width of the profit/loss rate \( \delta \) is \( \delta_w = 0.1 \). The widths of the J model are \( \delta_w = 0.1 \) and 0.2.

The reason why the Gini index \( g \) converges to a small value in the R model is that wealth is distributed between the poor and the wealthy with a random division probability \( \varepsilon \). Comparing the L and J models with the same width \( \delta_w = 0.1 \) and interest rate \( \rho = 0 \), we can see that the reason for the larger disparity earlier in the L model than in the J model is the structure where only one of the agents bears the profit and loss in the L model. This suggests that joint ventures in shareholder capitalism and Mudaraba are more effective in restraining disparities than financial capitalism, as well as the prohibition of Riba. The fact that the Gini index \( g \) approaches 1 sooner or
later in the L and J models suggests that exchange alone cannot avoid widening disparity and that redistribution is essential. The fact that the Gini index $g$ is larger for the case of $\delta_w = 0.2$ than for the case of 0.1, the width of the profit/loss ratio $\delta$ in the J model, suggests that speculative projects increase the disparity, i.e., the prohibition of the Gharar is effective, and moreover, a face-to-face Mudaraba with partnership is more effective than shareholder capitalism suggesting that it would restrain speculation.

**Redistribution**

We combine the R model in Eq (1), the L model in Eq (2), and the J model in Eq (3) with the T model in Eq (4) to examine the Gini index $g$ using Eq (7) when wealth is redistributed by transfer. Fig 5 shows the simulation results. As the period $t_p$ for the transfer, we take $10^4$, when the Gini index $g$ begins to rise in Fig 4, and $10^5$, when $g$ exceeds the alert level of 0.4 for social disturbances. The results of the R-T model are almost identical to those of the R model, regardless of redistribution or period $t_p$. This is because the R model itself distributes wealth between the poor and the wealthy. In the R-T, L-T, and J-T models, the Gini index $g$ converges from 0.4 to 0.9, which is smaller than 1, for the period $t_p = 10^5$, and $g$ converges from 0.1 to 0.3, which is even smaller, for $t_p = 10^4$. The dependence of the Gini index $g$ on the period $t_p$ is inferred to be approximately logarithmic. These results indicate that redistribution through transfers suppresses disparities, and that repeated transfers over a shorter period of time further suppress disparities.
We then investigate the dependence of the Gini index $g$ on the transfer rate $\xi$. Fig 6 shows the simulation results. In all of the R-T, L-T, and J-T models, the Gini index $g$ decreases rapidly until the transfer rate $\xi$ increases from 0 to roughly 0.2, but $g$ does not fall over roughly $\xi = 0.5$ or higher. In other words, it does not make sense to make $\xi$ unnecessarily large, since it is difficult to obtain the effect of suppressing disparities when $\xi$ is greater than 0.5. Although the actual inheritance tax rate varies with the amount of inheritance and the heirs, $\xi = 0.5$ is the lower saturation point of $g$, which may correspond to the fact that the maximum tax rate in OECD countries is generally 0.5 [28, 29]. For Waqf, we could not find any statistical data that would
correspond to an inheritance tax, but given the very large role of Waqf for public facilities and welfare in Islamic societies [30, 31], we can infer that the value corresponding to the transfer rate $\xi$ is quite large. In addition, voluntary Waqf in the Islamic economy is considered more meaningful for the wellbeing of both the individual and the community (Ummah) than inheritance tax, which is forced by the authorities in a capitalist society.

![Gini indexes for R-T, L-T and J-T models.](image)

The horizontal axis represents the transfer rate $\xi$ and the vertical axis represents the Gini index $g$. In the R-T, L-T and J-T models, the number of agents is $N = 1000$, the initial values of wealth are $m_i(0) = 1$ ($i = 1,2,\cdots,N$) and the savings rate is $\lambda = 0.25$. The interest rate in the L-T model is $\rho = 0.05$ and the width of the profit/loss rate $\delta$ is $\delta_w = 0.1$. The widths for the J-T model are $\delta_w = 0.1$ and 0.2. The time periods for the transfers are $t_p = 10^4$ and $10^5$.

We combine the R, L, and J models with the Q model expressed in Eq (5) and (6) to investigate the Gini index $g$ when wealth is redistributed according to quantile $1/q$.

Fig 7 shows the simulation results. For $1/1$ quantiles, the R-Q, L-Q, and J-Q models are equivalent to the R, L, and T models, respectively. The R-Q, L-Q, and J-Q models
all generally approach the Gini index $g$ of the R-T, L-T, and J-T models between the 1/4 and 1/6 quantiles, respectively. This corresponds to the quintile axiom in welfare economics that places welfare to the poor in the 1/5 quantile as the objective [32]. The reason why the Gini index $g$ of the J-Q model does not fall fully to the level of the J-T model when the width of the profit/loss rate $\delta$ is $\delta_w = 0.2$ is that the wealth distribution is not stable due to wealth fluctuations across quantiles because $\delta_w$ is large. The reason why $g$ is larger inversely above the 1/8 quantile is thought to be that redistribution causes a reversal of wealth between the $1/q$ quantile, the poorest quantile, and the $2/q$ quantile, the next poorest quantile.

![Fig 7. Gini indexes for R-Q, L-Q and J-Q models.](image)

The horizontal axis represents the quantile $1/q$ and the vertical axis represents the Gini coefficient $g$. The Q model is combined for the R, L, and J models. In the R-Q, L-Q, and J-Q models, the number of agents is $N = 1000$, the initial values of wealth are $m_i(0) = 1 \ (i = 1, 2, \ldots, N)$, the savings rate is $\lambda = 0.25$, the transfer rate is $\xi = 0.5$. The L-Q model’s interest rate is $\rho = 0.05$, and the width of the profit/loss rate $\delta$ is $\delta_w = 0.1$. The widths of the J-Q model are $\delta_w = 0.1$ and $0.2$. The period over which the transfer takes place is $t_p = 10^5$. 

$N = 1000, \lambda = 0.25, \xi = 0.5, t_p = 10^5$
An overview of Figs 5 through 7 shows that exchange alone, as in the L and J models, cannot help but widen the disparity, and that a combination with redistribution, as in the T and Q models, is essential. Using the Gini index \( g = 0.4 \) as a guide [2], which is a warning level against social disturbances, we can show that interest rate \( \rho \) should be avoided as in the L model of finance capitalism, that speculation with a large profit/loss width \( \delta_w \) should be avoided even in the J model of joint ventures, that the inheritance tax and Waqf's T model should be redistributed with a transfer ratio \( \xi \) in the range of 0.2 to 0.5, and that the interval \( t_p \) of redistribution should be shortened from \( 10^5 \) to \( 10^4 \) as much as possible, and according to the Q model, redistribution should be made to the poor in the 1/4 to 1/6 quantile, as indicated by the quintile axiom [32].

These findings are derived from an econophysics model that simplifies only the essence of the phenomenon, but therefore shows the inevitability of its occurrence under physical laws. It is astonishing that in the absence of econophysics, the Islamic world, based on the millennium-order rule of thumb, was able to compile the above-mentioned findings into a legal system. Incidentally, if we were to correspond to the time \( t \) in this econophysics model, the millennium would be on the order of \( 10^{4-5} \) (12 months - 365 days x 1000 years), and a human lifetime would correspond to \( 10^{3-4} \) (12 months - 365 days x 10 - 100 years). The population of the medieval Islamic world is estimated to be of the order of \( 10^{6-7} \) [33], which, divided by the medieval population size of cities of the order of \( 10^{2-3} \) [34], is roughly of the order of \( 10^{3-4} \). Thus, combining time and population, the number of economic trials would have been repeated on the order of \( 10^{4-5} \) times during the millennium. The Islamic world has
experienced many social disturbances through such trials, and as a rule of thumb, it has developed a legal system to control disparities.

**Discussion**

A comparison of the Islamic economy and capitalism based on the econophysics model quantitatively supports that the advantages of controlling disparity in the Islamic economy are the prohibition of the Riba, which unevenly distributes wealth, reciprocal joint ventures by the Mudaraba and the redistribution of wealth by the Waqf. From the perspective of long-term human history, the modern era is in transition from a monetary economy to a credit economy, and the Islamic economy, which brought about moral and financial innovations in the Middle Ages, provides guidelines for how the credit economy should be the next alternative to capitalism.

These guidelines include a prohibition on financial transactions (interest and speculation), a return to an "economy based on real transactions" rooted in nature and local communities, the promotion of a "face-to-face association economy" based on joint ventures and cooperatives, and the revival of an "economy embedded in the moral of mutual aid" that replaces power-based taxes and specific religions.

In his theory of gifts [35], Marcel Mauss showed that the moral of gifts is a relationship that does not weigh how much we give and are given to each other based on the assumption that we will help each other in our time of need, and that it is the system of giving across generations that ensures the establishment of a community. In his theory of mutual aid [36, 37], Pyotr Kropotkin also showed that moral has its origin in social instincts and that the combination of mutual aid-justice-moral makes
human society progress, and that in modern times the state absorbed society, but in
de-modern times society needs to regain the state.

In his theory of debt [5], David Graeber presented the three main moral principles of
economic relationships (baseline communism, exchange, and hierarchy). Baseline
communism is a relationship in which each person contributes according to his or her
ability and each person is given according to his or her needs. Exchange is a process
of exchange toward equivalence, often with an element of competition, with
calculating profit and loss, and the awareness that you can always dissolve the entire
relationship. Hierarchical relationships are governed by a web of customs and
precedents and have no tendency to operate through reciprocity.

Graeber then showed that baseline communism would be the basis for the next
alternative to capitalism, that market relations require the norms of community and
mutual aid that typify the human economy, and that debt in exchange becomes
problematic because its quantity is rigorously calculated, equivalence is demanded,
and people are disconnected from their own social context.

Nathalie Sarthou-Lajus, in her philosophy of debt [38], positions Mauss's gift,
Kropotkin's mutual aid, and Graeber's communism as debts that need not be paid off
that exist outside of equivalence, debts that are not debts, and repayments to third
parties that span generations. And both Graeber and Sarthou-Lajus state that the
distribution of wealth should be done as repayment to someone else in the free
manner they desire.

In the structure of world history [39], Kojin Karatani presented four main modes of
exchange as the various stages of the world system. Mode of exchange A is reciprocity in civil society (gift-return), B is plunder-redistribution in empire (submission-protection), C is commodity exchange in the capitalist economy (money-commodity), and D is the restoration of the reciprocal and mutual-aid relationships of mode of exchange A at a higher dimension in the coming world republic. The reason exchange mode of exchange D is of a higher dimension is that it erases the negative aspect of communal constraints in A and retains the positive aspect of individual freedom and self-interest in C. In other words, in mode of exchange D, self-interest based on individual freedom and altruism based on individual freedom, as Graeber and Sarthou-Lajus say, are compatible.

Anarchism is an ideology that believes that individual freedom and communal solidarity are not contradictory and seeks to build a free and equal society through mutual agreement. Graeber and Andrej Grubacic define anarchism by four qualities: decentralization, voluntary association, mutual aid, and the network model [40]. The above-mentioned discussions by Mauss, Kropotkin, Graeber, Sarthou-Lajus, and Karatani are in tune with anarchism in that they aim at a human economy of free exchange and redistribution while incorporating the moral of gift and mutual aid [41].

The Islamic economy, with its legal system encompassing politics, economics, and society, successfully balances self-interest as the pursuit of self-interest through Mudaraba (joint ventures) and Murabaha (consensual contracts) among individuals, while prohibiting Riba (interest) and Gharar (speculation), which cause disparities, and altruism as mutual aid through Waqf and Zakat in the equal and non-centered Ummah (community) under God [6, 7, 8]. The Islamic economy provides meaningful
guidelines in pursuit of anarchism not yet realized, although it is based on religion. In other words, the Islamic economy has the potential to transform from capitalism to the next credit economy and bring about the next alternative to capitalism in modern times, just as it brought about moral and financial innovation in the Middle Ages.

The challenge in the non-Islamic world, however, is not redistribution through taxes collected under the centrality of power, but redistribution through one's own free choice under the non-centrality of community, and how to reconstruct the social norm of mutual aid, that is, to make it possible for redistribution to occur without a specific religion.

Walter Scheidel's human history of violence and inequality [42] shows that four things have reduced economic inequality: mass-mobilization warfare, transformative revolutions, state collapse, and catastrophic plagues. Currently, the world is suffering from the epidemic COVID-19, war in Ukraine, and natural disasters and conflicts caused by the effects of global warming. While these are extremely unfortunate, it cannot be denied that they also have the aspect of strengthening social connections and mutual aid in the community. Can we not turn these crises into opportunities to rebuild the moral of giving and mutual aid?

ESG investments [43, 44] and social enterprises [45, 46] are becoming popular trends in the economic world. The former encourages repayment and mutual aid to third parties as discussed above, while the latter aims at an association economy linking communal reciprocity, public redistribution, and private market exchange. In the information industry, there is a move toward digital democracy [47, 48] and platform corporativism [49, 50]. The former aims for citizen participation in policy consensus
and government services, and the latter for joint ownership, fair profit sharing, and
democratic governance. These movements may be a preliminary step toward moral
restructuring.

**Conclusions**

In order to compare the Islamic economy with capitalism and to obtain guidelines for
a credit economy, the next alternative to capitalism, we proposed a new econophysics
model of wealth exchange and redistribution.

For exchange, we took as parameters the savings rate \( \lambda \), the interest rate \( \rho \), and the
profit/loss rate \( \delta \), and developed a loan interest model to represent financial capitalism
and a joint venture model to represent shareholder capitalism and the Mudaraba of the
Islamic economy. In the loan-interest model, one of the economic agents earns
interest on the amount exchanged excluding savings, and the other pays interest and
bears all profits and losses on the amount exchanged by both combined. In the joint
venture model, profits and losses are distributed in proportion to the amount
exchanged by each economic agent.

For redistribution, we take as parameters the transfer rate \( \xi \) and period \( t_p \), and set up a
transfer model that represents the inheritance tax of capitalism and the Waqf of the
Islamic economy. In this model, the amount of wealth corresponding to the transfer
rate \( \xi \) is equally distributed to all others in each period \( t_p \).

Simulations using the exchange model show that for the same savings rate \( \lambda \) and
profit/loss rate \( \delta \), when the exchange is repeated from an initial uniform distribution
of wealth, the loan interest model approaches a power-law distribution of wealth
faster than the joint venture model, and the Gini index $g$ increases rapidly. It was also shown that even if the interest rate $\rho$ were 0, the Gini index $g$ would still be large. In other words, we found that the prohibition of interest in the Islamic economy is effective in reducing disparity, and that the structure in which only one of the economic agents bears the profit and loss is the cause of the widening of disparity in the loan interest model.

The joint venture model shows a gradual increase in the Gini index $g$, but after further exchanges, the wealth distribution eventually becomes a delta distribution in both models, and the Gini index $g$ gradually approaches 1. In other words, exchange alone, even a reciprocal joint venture, cannot avoid widening disparities, and redistribution is found to be essential. As for the difference in the profit/loss ratio $\delta$, it is shown that the Gini index $g$ is larger when the profit/loss ratio $\delta$ is larger, indicating that the prohibition of Gharar in the Islamic economy and the active involvement of Mudaraba in each other's affairs deter speculation that increases disparities.

Simulations combining the exchange and transfer models show that both models converge to Gini indexes $g$ smaller than 1, although the loan interest model has a larger Gini index $g$ than the joint venture model when the transfer rate $\xi$ and period $t_p$ are the same. As for the period $t_p$, the shorter this period is, the logarithmically smaller the Gini index $g$ becomes. As for the transfer rate $\xi$, as this goes from 0 to 0.2, the Gini index $g$ decreases rapidly, and when $\xi$ goes above 0.5, there is not much effect in reducing disparity. In other words, these results show that the more the transfer rate $\xi$ is raised from 0.2 to 0.5 and the period $t_p$ is shortened in the redistribution of wealth, the more the effect of controlling disparities increases.
In the quantile model, in which transfers are made to the $1/q$ quantile of the wealth distribution, it was shown that a transfer model that redistributes equally to all others is roughly equivalent to redistribution to the 1/4 to 1/6 quantiles of the poor, supporting the quintile axiom in welfare economics.

Note that although shareholder capitalism and Mudaraba are mathematically represented by the same model, the latter is qualitatively more desirable in that there is more face-to-face active involvement than the former. Also, although the inheritance tax and Waqf are mathematically identical, they are qualitatively more desirable in that the former is a compulsion based on the centrality of power, while the latter is voluntary based on the non-centrality of the Ummah and Zakat.

This research has quantitatively confirmed once again the superiority of the Islamic economy over capitalism. The next alternative to capitalism should be a return to an "economy based on real transactions," a promotion of a "face-to-face association economy" through joint ventures and cooperatives, and a revival of an "economy embedded in the moral of mutual aid" as an alternative to the power of the state and specific religions. These insights continue the lineage of Mauss’s gift, Kropotkin’s mutual aid, Graeber’s baseline communism, Sarthou-Lajus’s repayment to third parties, and Karatani’s mode of exchange D, all leading toward the ideal of anarchism.

Although the present study modeled only the essence of exchange and redistribution in order to clarify the fundamental differences between the Islamic economy and capitalism, we recommend a detailed analytical study in the future by setting, for example, the savings rate $\lambda$, interest rate $\rho$, and profit/loss rate $\delta$ according to the
actual conditions of various states and communities, and by setting the transfer rate $\xi$ and the period $t_p$ according to the various asset management system and social security system. Moreover, while the present study, based on an econophysics approach, provides some indication of the underlying causes of disparities in exchange and redistribution, it does not provide practical policy recommendations or effective policy implementation in the various nations and communities. Such attempts remain for future empirical studies in economics, political science, and sociology.

Acknowledgements

The author received valuable advice from the Hitachi Kyoto University Laboratory of the Kyoto University Open Innovation Institute regarding how to pursue this research; the author would like to express their deepest gratitude.

References

1. Levy H. Income and wealth inequality in OECD countries. Wirtschaftsdienst. 2016; 96(13): 19-23. doi: 10.1007/s10273-016-1946-8.

2. UN-Habitat. State of the world’s cities 2008/2009 –harmonious cities. London: Earthscan; 2018. Available from: https://unhabitat.org/state-of-the-worlds-cities-20082009-harmonious-cities-2

3. The World Bank. Gini index (World Bank estimate). n.d. [cited 2022 Jun 6]. Available from: https://data.worldbank.org/indicator/SI.POV.GINI
4. United Nations. The 17 goals. n.d. [cited 2022 Jun 6]. In United Nations Department of Economic and Social Affairs [Internet]. Available from: https://sdgs.un.org/goals.

5. Graeber D. Debt: The first 5000 years. New York: Melville House; 2011.

6. Nagaoka S. The future of capitalism and the modern Islamic economy. Japanese edition. Tokyo: Shisousha; 2020.

7. Nagaoka S. The Future of capitalism and the Islamic economy. In: Yamash’ta S, Yagi T, Hill S, editors. The Kyoto Manifesto for Global Economics: The Platform of Community, Humanity, and Spirituality. Singapore: Springer; 2018. pp. 395-415. doi: 10.1007/978-981-10-6478-4_22.

8. Kato H. Social order in the Islamic world: Another "market and fairness." Japanese edition. Tokyo: Shisousha; 2020.

9. Yakovenko VM, Rosser JB. Colloquium: Statistical mechanics of money, wealth and income. Rev Mod Phys. 2009; 81: 1703-1725. doi: 10.1103/RevModPhys.81.1703.

10. Chakrabarti AS, Chakrabarti BK. Statistical theories of income and wealth distribution. Economics. 2010; 4: 1-31. doi: 10.5018/economics-ejournal.ja.2010-4.

11. Kato T, Hiroi Y. Wealth disparities and economic flow: Assessment using an asset exchange model with the surplus stock of the wealthy. PLoS ONE. 2021; 16(11): e0259323. doi: 10.1371/journal.pone.0259323.
12. Pareto V. Manuel d’économie politique. Paris: Giard & Brière; 1909.

13. Zipf GK. Human behavior and the principle of least effort: An introduction to human ecology. Cambridge: Addison-Wesley Press; 1949.

14. Polanyi K. The livelihood of man. New York: Academic Press; 1977.

15. Newman ME. Power laws, Pareto distributions and Zipf’s law. Contemp Phys. 2005; 46(5): 323–351. doi: 10.1080/00107510500052444.

16. Champernowne DG. A model of income distribution. Econ J. 1953; 63(250): 318–351. doi: 10.2307/2227127.

17. Angle J. The surplus theory of social stratification and the size distribution of personal wealth. Soc Forces. 1986; 65: 293–326. doi: 10.2307/2578675.

18. Hayes B. Follow the money. Am Sci. 2002; 90(5): 400–405. doi: 10.1511/2002.33.3291.

19. Chakraborti A. Distributions of money in model markets of economy. Int J Mod Phys C. 2002; 13: 1315–1321. doi: 10.1142/S0129183102003905.

20. Chatterjee A, Chakrabarti BK. Kinetic exchange models for income and wealth distributions. Eur Phys J B. 2007; 60: 135–149. doi: 10.1140/epjb/e2007-00343-8.

21. Chatterjee A, Chakrabarti BK, Manna SS. Pareto law in a kinetic model of market with random saving propensity. Physica. 2004; 335: 155–163. doi: 10.1016/j.physa.2003.11.014.

22. Guala S. Taxes in a wealth distribution model by inelastically scattering of particles.
23. Chakrabarti AS, Chakrabarti BK. Microeconomics of the ideal gas like market models. Physica A. 2009; 388(19): 4151–4158. doi: 10.1016/j.physa.2009.06.038.

24. Kato T, Kudo Y, Mizuno H, Hiroi Y. Regional inequality simulations based on asset exchange models with exchange range and local support bias. Appl Economics Finance. 2020; 7: 10–23. doi: 10.11114/aef.v7i5.4945.

25. Belz E. Estimating inequality measures from quantile data. Working paper, Center for Research in Economics and Management, University of Rennes 1, University of Caen Normandie. 2019. Available from: https://halshs.archives-ouvertes.fr/halshs-02320110/file/Belz_Estimating-Inequality-Measures-from-Quantile-Data_2019.pdf

26. Xu K. How has the literature on Gini’s index evolved in the past 80 years? Dalhousie University, Economics Working Paper. 2003. Available from: http://dx.doi.org/10.2139/ssrn.423200

27. The World Bank. Gross savings (% of GDP). n.d. [cited 2022 Jun 6]. Available from: https://data.worldbank.org/indicator/NY.GNS.ICTR.ZS

28. OECD. Inheritance taxation in OECD countries. 2021. Paris: OECD Publishing; 2021. doi: 10.1787/e2879a7d-en.
29. Cole A. Estate and inheritance taxes around the world. 2015 March 17. [cited 2022 Jun 6]. Available from: https://taxfoundation.org/estate-and-inheritance-taxes-around-world/

30. Budiman MA. The Significance of waqf for economic development. Equilibrium. 2014: 2(1); 19-34. doi: 10.21043/equilibrium.v2i1.718.

31. Adıgüzel FS, Kuran T. The Islamic waqf: instrument of personal security, worldly and otherworldly. Economic Research Initiatives at Duke (ERID) Working Paper No. 305. 2021. Available from: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3836060

32. Basu K. Beyond the invisible hand: Groundwork for a new economics. Princeton: Princeton University Press; 2010.

33. Shatzmiller M. Why did the early Islamic middle east have the highest standards of living? Prices, Wages and Population Levels. n.d. [cited 2022 Jun 6]. Available from: Why did the Middle East.pdf (uwo.ca)

34. Stone N. Notes on medieval population geography. 2016 Jul 6. [cited 2022 Jun 6]. Available from: Notes on Medieval Population Geography | by Lyman Stone | In a State of Migration | Medium

35. Mauss M. Essai sur le don: Forme et raison de l'échange dans les sociétés archaïques. Paris: Presses Universitaires de France; 1923-1924.

36. Kropotkin P. Mutual aid: A factor of evolution. New York: McClure, Philips & Company; 1902.
37. Kropotkin P. On mutual aid, again. Japanese edition. Tokyo: Doujidaisya; 2012.

38. Sarthou-Lajus N. Éloge de la dette. Paris: Presses Universitaires de France; 2012.

39. Karatani K. The structure of world history: From modes of production to modes of exchange. Durham: Duke University Press; 2014.

40. Grubacic A, Graeber D. Anarchism, or the revolutionary movement of the twenty-first century. 2014. [cited 2022 Jun 6]. Available from: https://theanarchistlibrary.org/library/andrej-grubacic-david-graeber-anarchism-or-the-revolutionary-movement-of-the-twenty-first-centu

41. Yamada H. Possible anarchism: Marcel Mauss and the moral of gift. Japanese edition. Tokyo: Inscript; 2020.

42. Sheidel W. The great leveler: Violence and the history of inequality from the stone age to the twenty-first century. Princeton: Princeton University Press; 2017.

43. Mizuguchi T. ESG investment: The shape of new capitalism. Japanese edition. Tokyo: Nikkei Business Publications, Inc.; 2017.

44. Fuma K. ESG thinking. Japanese edition. Tokyo: Kodansya; 2020.

45. Fujii A, Harada K, Ootaka K. Social enterprise tracking social exclusion. Japanese edition. Tokyo: Keisoshobo; 2013.

46. Takahashi M, Kimura T, Ishiguro T. Theorizing social innovation: To discover new practice of social entrepreneurship. Japanese edition. Tokyo: Bunshindo Publishing; 2018.
47. Hacker KL, Dijk J. What is digital democracy? In: Hacker KL, Dijk J, editors. Digital Democracy: Issues of Theory and Practice. Los Angeles: SAGE Publications; 2000. pp. 1-9. doi: 10.4135/9781446218891.n1.

48. Berg S, Hofmann J. Digital democracy. Internet Policy Review. 2021; 10(4). doi: 10.14763/2021.4.1612.

49. Scholz T. Platform cooperativism vs. the sharing economy. Medium. 2014 Dec 5. [cited 2022 Jun 6]. Available from: Platform Cooperativism vs. the Sharing Economy | by Trebor Scholz | Medium

50. Schneider N. Everything for everyone: The radical tradition that is shaping the next economy. New York: Bold Type Books; 2018.