Multi-Message Broadcast in Dynamic Radio Networks

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Abstract

We continue the recent line of research studying information dissemination problems in adversarial dynamic radio networks. We give two generic algorithms which allow to transform generalized version of single-message broadcast algorithms into multi-message broadcast algorithms. Based on these generic algorithms, we obtain multi-message broadcast algorithms for dynamic radio networks for a number of different dynamic network settings. For one of the modeling assumptions, our algorithms are complemented by a lower bound which shows that the upper bound is close to optimal.

1 Introduction

When developing algorithms for wireless networks, one has to deal with unique challenges which are not or much less present in the context of wired networks. All nodes share a single communication medium and whether a transmitted signal can be received by a given node might therefore depend on the behavior of all other nodes in the network. Moreover, the reception of a wireless signal can be influenced by additional wireless devices, multi-path propagation effects, other electric appliances, or also various additional environmental conditions, see e.g., [25, 35, 37, 39, 38]. As a consequence, wireless connections often tend to be unstable and unreliable. Moreover, wireless devices might be mobile, in which case connectivity changes even when ignoring all the above effects. We therefore believe that in order to study such dynamic and unpredictable communication behavior, it is important to also study unreliable and dynamic variants of classic wireless communication models such as, e.g., the classic radio network model introduced in [9, 5] or the more complex but also more realistic models like the SINR model [22, 33] or the affectance model [26].

In recent years, there has been a considerable effort in investigating basic communication and distributed computation problems in radio network models which exhibit adversarial dynamic, nondeterministic behavior. In [16], Clementi et al. study the problem of broadcasting a single message in a synchronous dynamic radio network where in each round a subset of the links might fail adversarially. Communication is modeled using the standard graph-based radio network model [5]. In each round, a node can either transmit or listen and a node \( u \) receives a message transmitted by a neighbor \( v \) in a given round \( r \) if and only if \( v \) is the only round-\( r \) neighbor of \( u \) transmitting in round \( r \). The paper studies deterministic algorithms and it in particular shows that if \( D \) is the diameter of the fault-free part of the network, deterministic single-message broadcast requires time \( \Theta(Dn) \), where \( n \) is the number of nodes of the network. In [14], Clementi et al. study an even more dynamic network model where the network topology can completely change from round to round. It is in particular shown that the single-message broadcast problem can be solved in time \( O(n^2/\log n) \) by a simple randomized algorithm where in each round, each node knowing the broadcast message, transmits it with probability \( \ln(n)/n \). It is

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also shown that the asymptotic time complexity of this algorithm is optimal. A similar model to the one in [16] has been studied in [29]. In the dual graph model of [29], it is assumed that the set of nodes is fixed and the set of edges consists of two parts, a connected set of reliable edges and a set of unreliable edges. In each round, the communication graph consists of all reliable and an arbitrary (adversarially chosen) subset of the unreliable edges. Among other results, it is shown that there is a randomized algorithm which achieves single-message broadcast in time $O(n \log^2 n)$. The algorithm in [29] works in the presence of a strongly adaptive adversary which determines the set of edges of a given round $r$ after the distributed algorithm decides what messages are transmitted by which nodes in round $r$. In [20], the same problem is considered for weaker adversaries. A weakly adaptive adversary has to determine the topology of round $r$ before a randomized distributed algorithm determines the randomness of round $r$ and an oblivious adversary has to determine the whole sequence of network topologies at the very beginning of the execution of a distributed algorithm. Additional problems in the dual graph model of [29, 20] have also been studied in [8, 21, 32].

The dynamic network models of the above papers can (mostly) be seen as the extreme cases of the $T$-interval connected dynamic graph model of [30]. For a positive integer $T$, a dynamic network is called $T$-interval connected, if for any time interval of $T$ consecutive rounds the graph which is induced by the set of edges present throughout the whole time interval is connected. Hence, for $T = 1$, the network graph has to be connected in every round, whereas for $T = \infty$, we obtain the dual graph model. In [1], the single-message broadcast problem has been studied for general $T$-interval connected dynamic radio networks and for a more fine-grained adversary definition. More specifically, for an integer $\tau \geq 0$, an adversary is called $\tau$-oblivious if the network topology of round $r$ is determined based on the knowledge of all random decisions of the distributed algorithm of rounds $1, \ldots, r - \tau$. Hence, an oblivious adversary is $\infty$-oblivious, a weakly adaptive adversary is $1$-oblivious, and a strongly adaptive adversary is $0$-oblivious.

**Additional Related Work.** Information dissemination and specifically broadcasting is a fundamental problem in networks and therefore there exists a rich literature on distributed algorithm to solve broadcast in various communication settings. In particular, the problem is well-studied in static wireless networks (see, e.g., [5, 4, 9, 10, 15] and many more). More recently, the multi-message problem has been studied quite extensively in the context of wireless networks, see e.g., [13, 11, 12, 18, 24, 19, 28].

We also note that several dynamic network models similar to the one in [30] have been studied prior to [30], for example in [3, 6, 7, 14, 36]. For a recent survey, we also refer to [31]. In addition to the works already mentioned, information dissemination and other problems in faulty and dynamic radio network models have also been studied in, e.g., [2, 16, 27].

**1.1 Contributions**

In the present work, we extend [1] and more generally the above line of research and we study the multi-message broadcast problem in $T$-interval connected dynamic radio network against a $\tau$-oblivious adversary. For some $s \geq 1$, we assume that there are $s$ broadcast messages, each initially given to a single source node. A randomized distributed algorithm solves the $s$-multi-message broadcast problem if with high probability (w.h.p.), it disseminates all $s$ broadcast messages to all nodes of the network. We say that the communication capacity of a network is $c \geq 1$ if every message sent by a node can contain up to $c$ different broadcast messages.

**1.1.1 Upper Bounds**

Most of our upper bounds are achieved by store-and-forward algorithms, i.e., by algorithms which treat the broadcast messages as single units of data that cannot be altered or split into several pieces. All our store-and-forward protocols are based on two generic algorithms which allow to turn more general variants of single-message broadcast algorithms into multi-message broadcast algorithms. In addition, Theorem 1.4 proves an upper bound that can be achieved using random linear network coding. When dealing with linear network coding algorithms, we use the common convention and disregard any overhead created by the message headers
Theorem 1.1. Assume that we are given an infinite-interval connected dynamic $n$-node network controlled by a 0-oblivious adversary. Using store-and-forward algorithms, for communication capacity $c = 1$, $s$-multi-message broadcast can be solved, w.h.p., in time $O(ns \log^2 n)$, whereas for arbitrary $c \geq 1$, it can be solved, w.h.p., in time $O((1 + \frac{\tau}{c})n \log^4 n)$.

Theorem 1.2. Consider the $s$-multi-message broadcast problem in 1-interval connected dynamic $n$-node networks controlled by a 0-oblivious adversary. Using store-and-forward algorithms, for communication capacity $c = 1$, the problem can be solved, w.h.p., in time $O((1 + \frac{\tau}{\log n})n^2)$, and for arbitrary $c \geq 1$, it can be solved, w.h.p., in time $O((1 + \frac{\tau}{c})n^2 \log n)$.

Theorem 1.3. Let $T \geq 1$ and $\tau \geq 1$ be positive integer parameters. In $T$-interval connected dynamic networks controlled by a $\tau$-oblivious adversary, for communication capacity $c = 1$, the $s$-multi-message broadcast problem can be solved, w.h.p., in time

$$O \left( \left(1 + \frac{n}{\min\{\tau, T\}} \right) \cdot (s + \log n) \cdot n \cdot \log^3 n \right),$$

and for an arbitrary $c$, it can be solved, w.h.p., in time

$$O \left( \left(1 + \frac{n}{\min\{\tau, T\}} \right) \cdot \frac{ns}{c} \cdot \log^4 n \right).$$

Theorem 1.4. Using linear network coding, in 1-interval connected dynamic networks with communication capacity 1 and a 0-oblivious adversary, $s$-multi-message broadcast can be solved in time $O(n^2 + ns)$, w.h.p.

1.1.2 Lower Bound

Theorem 1.5. In infinite-interval connected dynamic networks with communication capacity $c \geq 1$ and being controlled by a 0-oblivious adversary, any $s$-multi-message broadcast algorithm requires at least time $\Omega(ns/c)$, even when using network coding. Further, there is a constant-diameter infinite-interval connected network with communication capacity 1 such that any store-and-forward algorithm requires at least $\Omega((ns - s^2)/c)$ rounds to solve $s$-multi-message broadcast against a 0-oblivious adversary.

2 Model and Problem Definition

Dynamic Networks. We model dynamic radio networks using the synchronous dynamic network model of [30]. A dynamic network is represented by a fixed set of nodes $V$ of size $n$ and a sequence of undirected graphs $\langle G_1, G_2, \ldots \rangle$, where $G_i = (V, E_i)$ is the communication graph in round $i$. Hence, while the set of nodes remains the same throughout an execution, the set of edges can potentially change from round to round. A dynamic graph $\langle G_1, G_2, \ldots \rangle$ is called $T$-interval connected for an integer parameter $T \geq 1$ if the graph

$$\overline{G}_{r,T} = (V, \overline{E}_{r,T}), \quad \text{where} \quad \overline{E}_{r,T} := \bigcap_{r' = r}^{r+T-1} E_{r'},$$

is connected for all $r \geq 1$.

Communication Model. We define an $n$-node distributed algorithm $\mathcal{A}$ as a collection of $n$ processes which are assigned to the nodes of an $n$-node network. Thus, at the beginning of an execution, a bijection from $V$ to $\mathcal{A}$ is defined by an adversary. In the following, we will use “node” to refer to a node $u \in V$ and to the process

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1Note that this assumption is reasonable as long as the number of broadcast messages which are combined with each other is at most the length of a single broadcast message (in bits).

2We note that many other, similar dynamic network models have appeared in the literature (cf. Section 1).
running at node \( u \). We assume that each node has a unique ID of \( O(\log n) \) bits. In each round of an execution, each node \( u \) decides to either transmit a message or listen to the wireless channel. When a node decides to transmit a message in round \( r \), the message reaches all of its neighbors in \( G_r \). A node \( u \) in round \( r \) successfully receives a message from a neighbor \( v \) if and only if \( v \) is the only neighbor of \( u \) in \( G_r \), transmitting in round \( r \). Otherwise, if zero or multiple messages reach a node \( u \), \( u \) receives silence, i.e., nodes cannot detect collisions.

**Adversary.** We assume that the dynamic graph \( \langle G_1, G_2, \ldots \rangle \) is determined by an adversary. Classically, in this context, three types of adversaries have been considered (see, e.g., [20]). An oblivious adversary has to determine the whole sequence of graphs at the beginning of an execution, independently of any randomness used in the algorithm. An adaptive adversary can construct the graph of round \( r \) depending on the history and thus in particular the randomness up to round \( r \). Typically, two different adaptive adversaries are considered. A strongly adaptive adversary can choose graph \( G_r \) dependent on the history up to round \( r \) including the randomness of the algorithm in round \( r \), whereas a weakly adaptive adversary can only use the randomness up to round \( r - 1 \) to determine \( G_r \). In the present paper, we use a more fine-grained adversary definition which was in this form introduced in [1]. For an integer \( \tau \geq 0 \), we call an adversary \( \tau \)-oblivious if for any \( r \geq 1 \), the adversary constructs \( G_r \) based on the knowledge of the algorithm description and the algorithm’s random choices of the first \( r - \tau \) rounds. Note that the three classic adversaries described above are all special cases of a \( \tau \)-oblivious adversary, where \( \tau = \infty \) corresponds to an oblivious adversary, \( \tau = 0 \) to a strongly adaptive adversary and \( \tau = 1 \) to a weakly adaptive adversary.

**Multi-message Broadcast Problem.** For some positive integers \( s \) and \( B \), we define the \( s \)-multi-message broadcast problem as follows. We assume that there are \( s \) distinct broadcast messages of size \( B \) bits, each of which is given to some node in the network, called a source node. Then the problem requires the dissemination of these \( s \) broadcast messages to all nodes in the network. It can be solved by two types of algorithms; store-and-forward algorithms and network coding algorithms. In store-and-forward algorithms, each broadcast message is considered as a black box, and nodes can only store and forward them. In contrast to store-and-forward algorithms, in network coding algorithms, each node can send a message which can be any function of the messages it has received so far. In the following we define two types of problems and later we show that a procedure that can solve each of these problems can be used as a subroutine for solving the multi-message broadcast problem.

**Communication Capacity.** To solve the multi-message broadcast problem we consider a restriction on the amount of data which can be transmitted in a single message of a distributed protocol. The communication capacity \( c \geq 1 \) is defined as the maximum number of broadcast messages that can be sent in a single message of a distributed algorithm. In addition, an algorithm can send some additional control information of the same (asymptotic) size, i.e., an algorithm is allowed to send messages of \( O(cB) \) bits.

**Limited Single-Message Broadcast Problem.** Let us assume that there is a single broadcast message initially given to some node in an \( n \)-node network. For some integer parameter \( k \leq n \), \( k \)-limited broadcast problem requires the successful receipt of the broadcast message by at least \( k \) nodes (including the source) in the network with probability at least \( 1/2 \).

**Concurrency-Resistant Single-Message Broadcast Problem.** In this problem we assume that there can be 0, 1, or more source nodes in the network, each of which is given a broadcast message. If there exists only one source node, then its broadcast message is required to be successfully received by all nodes in the network, and thus this execution is considered a successful broadcast. Otherwise, we have an unsuccessful broadcast, where there are no source nodes or more than one source nodes. The problem requires that all nodes detect whether the broadcast was successful or not by the end of the execution. This broadcast with success detection is actually simulating a single-hop communication network with collision detection, where if only one node broadcasts in a round, all nodes receive the message and otherwise all nodes detect collision/silence.
3 Multi-message Broadcast Algorithms

In this section we present upper bounds for the multi-message broadcast problem in different scenarios depending on the communication capacity, the interval connectivity of the communication network, the adversary strength, and also the ability to use network coding for disseminating information. We start by describing generic techniques for broadcasting multiple messages in dynamic radio networks. Further, the discussion of how to use network coding to solve multi-message broadcast in dynamic radio networks appears in Section 4.

3.1 Generic Algorithm for Large Communication Capacity $c$

We start by describing a generic method for coping with a general communication capacity parameter $c$ (see Section 2). If we want to design store-and-forward algorithms which exploit the fact that in a given time slot a node can transmit $c \gg 1$ source messages to its neighbors, we have to deal with the problem that initially, all broadcast messages might start at distinct source nodes. In order to collect sufficiently many broadcast messages at single nodes and to nevertheless avoid too many redundant retransmission of the same broadcast message by several nodes, we adapt a technique introduced by Chrobak et al. in [13]. Their algorithm runs in phases and it is based on iterative applications of a $k$-limited single-message broadcast routine for exponentially growing values of $k$. In each phase, for each broadcast message $M$, the minimum number of nodes which know $M$ doubles. Typically, the maximum time for reaching at least $k$ nodes with a single-message broadcast algorithm grows linearly in $k$. If $k$ is doubled in each phase, the time for each $k$-limited single-message broadcast instance therefore also doubles. However, since in each phase, the number of nodes which know each source message $M$ at the beginning of a phase also doubles, the number of source nodes from which we need to start a $k$-limited single-message broadcast instance gets divided by 2 and overall, the time complexity of each phase will be about the same.

**Distributed Coupon Collection.** Formally, in [13] this idea is modeled by a distributed coupon collection process which we generalize here. The distributed coupon collection problem is characterized by four positive integer parameters $n$, $s$, $\ell$, and $c$ (in [13]), the parameters $s$ and $c$ are both equal to $n$). There are $n$ bins and $s$ distinct coupons (in the application, the bins will correspond to the nodes of the communication network and the distinct coupons will be the broadcast messages). There are at least $\ell$ copies of each coupon and the at least $s\ell$ coupons are distributed among the $n$ bins such that each bin contains at most one copy of each of the $s$ distinct coupons. The coupon collection proceeds in discrete time steps. In each time step, a bin is chosen uniformly at random. If a bin is chosen, it is opened with probability 1/2. If the bin is opened, at most $c$ coupons of it are collected as follows. If the bin has at most $c$ coupons, all coupons of the bin are collected, otherwise a randomly chosen subset of size $c$ of the coupons in the bin is collected. The coupon collection ends as soon as each of the $s$ distinct coupons has been collected at least once. The following lemma upper bounds the total number of time steps needed to collect all $s$ coupons.

**Lemma 3.1 (Distributed Coupon Collection).** With high probability, the described distributed coupon collection process ends after $O\left(\frac{ns}{c} \cdot \log(n+s)\right)$ steps.

**Proof.** Consider a particular coupon $x$ and one step of the distributed coupon collection process. Since there are at least $\ell$ copies of $x$, each in a different bin, there are at least $\ell$ bins containing $x$. Thus, by picking a random bin, we pick a bin having $x$ with probability at least $\ell/n$. Moreover, as there are $s$ distinct coupons and each bin can hold at most one copy of each coupon, when collecting a random subset of $c$ of the coupons in a bin containing $x$, the probability for collecting $x$ is at least $c/s$.

Overall, in each step of the process, the probability for collecting $x$ is therefore at least $c\ell/ns$. To collect $x$ with probability at least $1 - \delta$ for some $\delta > 0$, we therefore need at most $\frac{c\ell}{n}\cdot \ln(1/\delta)$ steps. The lemma then follows by setting $\delta = \frac{1}{sn\gamma}$ for a constant $\gamma > 0$ and by applying a union bound over all the $s$ distinct coupons. \hfill $\square$

**Coupon-Collection-Based Generic Multi-message Broadcast Algorithm.** We now discuss how to use the above abstract distributed coupon collection process to efficiently broadcast multiple messages in a dynamic
Our algorithm is a generalization of the idea of Chrobak et al. [13]. The algorithm consists of \( \lceil \log_2 n \rceil \) phases. By the end of phase \( i \), for each broadcast message \( M \), we want to guarantee that at least \( \ell_i := 2^i \) nodes know \( M \). We can therefore assume that at the beginning of a phase, each source message \( M \) is known by at least \( \ell_{i-1} = 2^{i-1} \) nodes (note that this is trivially true for \( i = 1 \)). We can achieve that each broadcast message is known by \( \ell_i \) nodes by running sufficiently many instances of \( \ell_i \)-limited broadcast such that each source message \( M \) is disseminated in \( O(\log n) \) of these \( \ell_i \)-limited broadcast instances. The details of the algorithm are given by Algorithm 1. In the pseudocode of Algorithm 1, \( \alpha > 0 \) is a constant which is chosen sufficiently large.

**Algorithm 1** Generic Multi-message Broadcast Algorithm Based on Distributed Coupon Collection.

1: for each \( v \in V \) do \( S_v \leftarrow \) set of broadcast messages known by \( v \)
2: for \( i \leftarrow 1 \) to \( \lceil \log_2 n \rceil \) do
3: \( \ell_i \leftarrow 2^i \)
4: for \( j \leftarrow 1 \) to \( \alpha \cdot \frac{ns}{c} \cdot \ln n \) do
5: for each \( v \in V \) do
6: (independently) mark \( v \) with probability \( 1/n \)
7: if \( v \) is marked then
8: \( R_v \leftarrow \) random subset of \( S_v \) of size \( \min \{ |S_v|, c \} \)
9: \( v \) initiates \( \ell_i \)-limited broadcast with message \( R_v \)

The following lemma shows that if the cost of \( k \)-limited broadcast depends at most linearly on \( k \), Algorithm 1 achieves \( s \)-multi-message broadcast in essentially the time needed to perform \( s/c \) single-message broadcasts (i.e., \( n \)-limited broadcasts).

**Lemma 3.2.** Let \( t(k, n) \) be the time needed to run one instance of \( k \)-limited broadcast in an \( n \)-node network. Then, when using a sufficiently large constant \( \alpha > 0 \) in Algorithm 1, the algorithm w.h.p. solves \( s \)-multi-message broadcast in time

\[
O \left( \sum_{i=1}^{\lceil \log_2 n \rceil} \frac{ns}{c2^i} \cdot t(2^i, n) \cdot \log n \right).
\]

**Proof.** Recall that if only one node in the network initiates the given \( k \)-limited broadcast routine with broadcast message \( M \), then with probability at least \( 1/2 \), at least \( k \) nodes receive \( M \) successfully. Fix some broadcast message \( \zeta \) as one of the \( s \) broadcast messages initially given to source nodes. We show by induction that, w.h.p., at the beginning of the \( i \)th iteration (and thus at the end of the \( (i-1) \)th iteration) of the outer for loop, there are at least \( \ell_{i-1} \) nodes which know \( \zeta \). Clearly, this is true for \( i = 1 \). For \( i \geq 1 \), consider the \( i \)th iteration of the outer for loop of the algorithm. Assuming that there exist at least \( \ell_{i-1} \) nodes having \( \zeta \), any node which has \( \zeta \) chooses \( \zeta \) to pack it in its sent message with probability at least \( c/s \) in each iteration of the inner for loop. Furthermore, each node is the only marked node in the network in each iteration of the inner for loop with probability at least \( \frac{1}{2} \left( 1 - \frac{1}{2} \right)^{\ell_{i-1}} \geq \frac{1}{e^{\ell_{i-1}}} \). Therefore, \( \zeta \) will be packed into the sent message of a node which is the only marked node in that round with probability at least \( \frac{c\ell_{i-1}}{en^s} \) and in this case, it will reach at least \( \ell_i \) nodes with probability at least \( 1/2 \). Thus, \( \zeta \) will be known by at least \( \ell_i \) nodes after \( \frac{\alpha ns}{\ell_i} \ln n = \frac{\alpha ns}{2\ell_{i-1}} \ln n \) repetitions of the inner for loop with probability at least \( 1 - n^{-\alpha/4e} \).

Typically, the time to run one instance of \( k \)-limited broadcast depends linearly on \( k \). The following corollary simplifies the statement of Lemma 3.2 for this particular case.

**Corollary 3.3.** Assume that the time for running one instance of \( k \)-limited broadcast in an \( n \)-node network is given by \( t(k, n) \leq k \cdot t(n) \). Then, when using a sufficiently large constant \( \alpha > 0 \) in Algorithm 1, the algorithm
w.h.p. solves $s$-multi-message broadcast in time

$$O \left( t(n) \cdot \frac{ns}{c} \cdot \log^2 n \right).$$

Proof. Follows directly from Lemma 3.2.

3.2 Generic Algorithm for Constant Communication Capacity $c$

We next describe a more efficient generic algorithm for the case $c = 1$ (or any constant $c$). Hence, we assume that each message sent by the algorithm can only contain a single broadcast message. In this case, we do not need to care about collecting different broadcast messages at a single node and Algorithm 1 is therefore too costly. Instead, we use an algorithm which is based on a more standard single-message broadcast algorithm. In the following, we assume that for some given setting we are given a concurrency-resistant single-message broadcast algorithm $B$. Recall that if there exists only one broadcast message while running $B$, all nodes receive the broadcast message and $B$ returns 1, otherwise it returns 0. We use this algorithm as a subroutine in designing the generic multi-message broadcast algorithm. We assume that initially, the number of broadcast messages $s$ is known. The generic algorithm runs in phases and in each phase we run one instance of $B$. Note that if the instance returns 1, all nodes know the broadcast message which has been disseminated to all nodes. Therefore at all times, all nodes know how many messages still need to be broadcast. If at the beginning of a phase, there are $x \leq s$ broadcast messages which still need to be broadcast, for each broadcast message $M$, the source node of $M$ decides to start an instance of $B$ with broadcast message $M$ with probability $1/x$.

**Algorithm 2** Generic Multi-Message Broadcast Algorithm Based on Concurrency-Resistant Single-Message Broadcast Algorithm.

1: **for each** $v \in V$ **do** $S_v \leftarrow$ set of broadcast messages given initially to $v$
2: $x \leftarrow s$
3: **while** $x \neq 0$ **do**
4:  **for each** $v \in V$ **do**
5:  **if** $S_v \neq \emptyset$ **then**
6:    (independently) mark each of the broadcast messages in $S_v$ with probability $1/x$
7:  **if** $v$ has a marked broadcast message **then**
8:    $v$ initiates $B$ with its marked broadcast messages
9:  **if** $B$ returns 1 **then**
10:    $x \leftarrow x - 1$
11:    remove the broadcast message of node $u$ which is delivered to all nodes from $S_u$
12: **unmark** all broadcast messages

**Lemma 3.4.** If $B$ is a concurrency-resistant single-message broadcast algorithm with running time $t(n)$, then Algorithm 2 solves the $s$-multi-message broadcast problem in time $O \left( (s + \log n) t(n) \right)$.

Proof. In each phase for some $1 \leq x \leq s$, there are $x$ broadcast messages left to be received by all. Each of these broadcast messages is marked to be broadcasted with probability $1/x$. Therefore, the probability of each broadcast message to be the only marked broadcast message in the phase is $p$, given by

$$p = \frac{1}{x} \left( 1 - \frac{1}{x} \right)^{(x-1)} \geq \frac{1}{ex}.$$ 

Let us call a phase which has exactly one marked broadcast message a *successful* phase. Therefore, the probability that a phase with $x$ broadcast messages is a successful phase is at least $1/e$. Hence if the number of phases is at least $c' \log n$ for some large constant $c'$, we have $s$ successful phases with high probability, which concludes the proof.
In the sequel, we apply this generic broadcast algorithm to solve the multi-message broadcast problem in three different settings.

### 3.3 Application of the Generic Algorithms in Different Settings

In this section we intend to show how to apply the generic multi-message broadcast techniques introduced in Section 3.1 and Section 3.2 in different settings. In the following we consider the problem in networks with different interval connectivity $T$. In each of these settings we investigate the two cases of large and constant communication capacities separately. For the first case the generic algorithm requires the existence of a $k$-limited single-message broadcast algorithm and in the second case the generic algorithm requires the existence of a concurrency-resistant single-message broadcast algorithm. We therefore need to show that the existing single-message broadcast algorithms in the considered dynamic radio network settings can be turned into $k$-limited and concurrency-resistant variants of these algorithms.

#### 3.3.1 (Setting I) $\infty$-Interval Connected Dynamic Networks

We consider the $s$-multi-message broadcast problem in an $\infty$-interval connected dynamic network against a 0-oblivious adversary. To obtain a $k$-limited and a concurrency-resistant single-message broadcast algorithm, we adapt the harmonic broadcast algorithm introduced in [29]. In the harmonic broadcast algorithm, in the first round, the source node transmits its broadcast message to its neighbors. From the second round on, any node which receives the broadcast message in round $r_v$, transmits the message in any round $r > r_v$ with probability $p_v(r)$, given by

$$p_v(r) := \frac{1}{1 + \lceil \frac{r-r_v-1}{\epsilon} \rceil},$$

where $T = \Theta(\log n)$ is a parameter. That is, for the first $T$ rounds immediately after some node receives the message, it transmits the message with probability 1, for the next $T$ rounds it transmits with probability 1/2, then with probability 1/3 and so on.

**Procedure Harmonic $k$-Limited Broadcast.**

By adapting the harmonic broadcast algorithm in [29] we limit the number of nodes that receive the message successfully to $k$. For this purpose we only run the harmonic broadcast algorithm for $4kT(\ln(n) + 1)$ rounds, where $T = \lceil 12 \ln(n/\epsilon) \rceil$ and $\epsilon$ is a small value which will be fixed later.

We start with some definitions. For any $r \geq 1$ we define $P(r)$ as the sum of transmitting probabilities of all nodes in round $r$. Any round $r$ with $P(r) < 1$ is called free and any other round is called a busy round. If some node $v$ is the only node that is transmitting in some round $r$, then we say that node $v$ is isolated in round $r$.

**Lemma 3.5.** (Lemma 13 in [29]) Consider a node $v$ and let $r_v$ be the round when $v$ first receives the broadcast message. Further, let $r > r_v$ be such that at least half of the rounds $r_v + 1, \ldots, r$ are free. If $T \geq 12 \ln(n/\epsilon)$ for some $\epsilon > 0$, then with probability larger than $1 - \epsilon/n$ there exists a round $r' \in [r_v + 1, r]$ such that $v$ is isolated in round $r'$.

**Lemma 3.6.** Let us assume that a source node, by transmitting message $M$, initiates the Harmonic $k$-Limited Broadcast procedure in an $\infty$-interval connected dynamic network controlled by a 0-oblivious adversary. Then with probability larger than $1 - \epsilon$, by the end of the procedure at least $\min \{k, n\}$ nodes receive $M$ successfully.

**Proof.** We prove this lemma based on the analysis in [1]. For the sake of contradiction let us assume that only $k' < \min \{k, n\}$ nodes receive $M$ by time $t := 4kT(\ln(n) + 1)$. Therefore, the sum of all transmitting probabilities of these $k'$ nodes until time $t$ is at most

$$k' T \sum_{i=1}^{[t/T]} \frac{1}{i} \leq kT \left( \ln \left( \left\lceil \frac{t}{T} \right\rceil \right) + 1 \right) \leq \frac{t-1}{2}.$$

We can conclude that the number of free rounds is more than the number of busy rounds until time $t$. Since in the first round one source node initiates the execution by transmitting with probability 1, the first round is busy.
We define $\theta_0 := 0$, and for $i > 0$, $\theta_i$ is the first time after time $\theta_{i-1}$ that the number of free and busy rounds in the time interval $[\theta_{i-1}, \theta_i]$ are equal. Moreover, let $m$ be a positive integer such that $\theta_m$ is the last such time in the time interval $[0, \hat{t}]$. For each $i \in \{1, \ldots, m\}$, let us assume that the number of nodes that get informed until time $\theta_i$ is $k_i$ (note that $k_i \leq k'$ for all $i \leq m$). In order to complete the proof of the lemma, we next proof three helper claims.

**Claim 1.** If a node $v$ gets informed in round $t_v$, where $\theta_{i-1} \leq t_v < \theta_i$, and round $\theta_{i-1} + 1$ is busy, then $v$ gets isolated by time $\theta_i$ with high probability.

**Proof of Claim 1.** Let $\hat{t}$ be the first time after $t_v$ such that the number of free and busy rounds in $[t_v, \hat{t}]$ are equal. We show that if round $\theta_{i-1} + 1$ is busy, it holds that $\hat{t} \leq \theta_i$. The claim that we need to proof then directly follows from Lemma 3.5. For the sake of contradiction, let us assume that $\hat{t} > \theta_i$. Note that node $v$ transmits with probability 1 in round $t_v + 1$ and therefore round $t_v + 1$ is busy. Assuming that $\hat{t} > \theta_i$ therefore implies that the number of free rounds is less than the number of busy rounds in $[t_v, \theta_i]$. From Claim 2, it follows that a concurrency-resistant single-message broadcast algorithm to be used in the second generic algorithm.

**Lemma 3.7.** There exists a concurrency-resistant algorithm which solves the single-message broadcast problem in $\infty$-interval connected dynamic $n$-node networks against a 0-oblivious adversary in $O(n \log^2 n)$ rounds.

**Proof.** Consider an algorithm which runs in two phases. Each phase is a single run of the harmonic broadcast algorithm in [29]. In the first phase all the source nodes initiate the broadcast algorithm. In the second phase, each node that has received at least two messages in the first phase initiates the broadcast algorithm with the broadcast message $\bot$. At the end of the second phase, each node that receives only one broadcast message
in the first phase and nothing in the second phase, detects a successful broadcast. Otherwise, each node that receives nothing in the first phase or receives ⊥ in the second phase, detects an unsuccessful broadcast.

If there exists no source nodes, then in the first phase all nodes receive nothing and therefore they detect the unsuccessful broadcast correctly. Hence, for this case even the first phase is enough for all the nodes to detect the failure of the broadcast. Now, let us consider the case when there are more than one source nodes. Based on the analysis of the algorithm in [29], due to the transmitting probability of the nodes in the network all the nodes will be informed by a broadcast message until the end of the first phase. Since we have a connected network there exists at least two nodes which are connected in the stable backbone of the network and have received different broadcast messages. Hence, after their isolation at least one of them knows at least two different broadcast messages. Then in the second phase of the execution, there exists at least one node that initiates the broadcast with ⊥, and hence all the nodes receive ⊥ by the end of the second phase.

**Theorem 1.1 (restated).** Assume that we are given an ∞-interval connected dynamic n-node network controlled by a 0-oblivious adversary. Using store-and-forward algorithms, for communication capacity c = 1, s-multi-message broadcast can be solved, w.h.p., in time $O(ns \log^2 n)$, whereas for arbitrary $c \geq 1$, it can be solved, w.h.p., in time $O\left((1 + \frac{c}{n})n \log^4 n\right)$.

**Proof.** Let us assume Algorithm 1 performs the k-limited broadcasting by running the Harmonic $k$-Limited Broadcast procedure. Considering the selection of $\alpha$ in Algorithm 1, it is enough to set $\epsilon$ in the Harmonic $k$-Limited Broadcast procedure to $1/2$. Then it takes $t(n, k) \leq ak \ln^2 n$ rounds for a sufficiently large constant $a$. Therefore, $t(n, k) = kt(n)$, where $t(n) = O(\log^2 n)$. Due to Corollary 3.3, Algorithm 1 solves the $s$-multi-message broadcast problem in this setting in time $O\left((1 + \frac{c}{n}) n \log^4 n\right)$.

On the other hand, Lemma 3.7 shows the existence of a concurrency-resistant single-message broadcast algorithm with running time of $O(n \log^2 n)$. Hence, by embedding the concurrency-resistant single-message broadcast algorithm in Algorithm 2, the problem in this setting, based on Lemma 3.4, can be solved in time

$O\left((s + \log n)n \log^2 n\right)$.

3.3.2 (Setting II) 1-Interval Connected Dynamic Networks

For 1-interval connected networks, we assume that the adversary is 1-oblivious. Note that in [1], it is shown that even the single-message problem cannot be solved with a 0-oblivious adversary in 1-interval connected networks. We adapt the homogenous algorithm by Clementi et al. [14] to obtain a $k$-limited and a concurrency-resistant single-message broadcast procedure. The single-message broadcast algorithm of [14] is a simple randomized algorithm where every node that knows the broadcast message, broadcasts the message with probability $\ln(n)/n$. In [14], it is shown that in every round, the probability that a node knowing the broadcast message succeeds in sending it to a neighboring node which does not know the message is at least $\ln(n)/n$. In order to have a $k$-limited broadcast algorithm which succeeds in reaching at least $k$ nodes with probability at least 1/2, we therefore need to run the algorithm of [14] for $\Theta(kn/\ln n)$ rounds. Hence, the second statement of Theorem 1.2 follows directly from Corollary 3.3.

In order to obtain a concurrency-resistant algorithm, we can first observe that the algorithm of [14] also works if several nodes try to broadcast the same message. Hence, in order to use a similar idea as in the $\infty$-interval connected case, we have to make sure that in the case of multiple broadcast messages, some node detects that there are at least two messages. Using the same analysis as for a single broadcast message, one can see that after $O(n^2/\log n)$ rounds, every node knows at least one of the broadcast messages. Our next goal is to have at least one node which knows at least two different broadcast messages. As long as this is not the case, in every round, there is at least one edge connecting two nodes $u$ and $v$ which know different broadcast messages. For the next $O(n \log n)$ rounds, each node now transmits the broadcast message it knows with probability $1/n$ (assuming a node knows exactly one broadcast message). The probability that either $u$ or $v$ decide to broadcast
their message and no other node in the network decides to broadcast is at least $2/en$ and thus after $O(n \log n)$ rounds, w.h.p., there is at least one node which knows two broadcast messages.

Now, we can get a concurrency-resistant single-message broadcast algorithm in the same way as for $\infty$-interval connected graphs. We run the algorithm of [14] again such that each node which knows at least two different broadcast messages after the first phase, broadcasts $\perp$. The time complexity of the concurrency-resistant broadcast algorithm is $O(n^2/\log n)$ and therefore the first statement of Theorem 1.2 now directly follows from Lemma 3.4.

### 3.3.3 (Setting III) $T$-Interval Connected Dynamic Networks

For arbitrary $T$-interval connected graphs, we adapt the algorithm of [1]. We assume that the adversary is $\tau$-oblivious and we define $\psi := \min\{\tau, T\}$. We assume that $\psi = \Omega(\log^2 n)$, note that otherwise, Theorem 1.3 already follows from the statement of Theorem 1.2 for $T = 1$. We also assume that $\psi = O(n)$, as otherwise Theorem 1.3 follows from Theorem 1.1 for $T = \infty$. The algorithm of [1] runs in phases of length $\psi$. The progress in a single phase is analyzed in the proof of Theorem 1.1. A phase of length $\psi$ is called successful if at least an $\Omega(\psi/(n \log^2 n))$-fraction of all uninformed nodes are informed and it is shown, in the proof of Theorem 1 of [1], that a phase is successful with probability at least $\psi/(8\mu n)$.

Assume that we want to obtain a $k$-limited single-message broadcast algorithm. If $k \leq n/2$, as long as fewer than $k$ nodes are informed, the number of uninformed nodes is at least $n/2$. Hence, in this case, in a successful phase, at least $\Omega(\psi/\log^2 n)$ nodes are newly informed. Hence, to inform $k \leq n/2$ nodes, we need $O(k \log^2(n)/\psi)$ successful phases and we thus need $O(kn \log^2(n)/\psi^2)$ phases in total to inform $k$ nodes with probability at least $1/2$. The number of rounds to solve $k$-limited single-message broadcast is therefore $O(kn \log^2(n)/\psi) = O(kn \log^2(n)/\min\{\tau, T\})$. For $k > n/2$, we run the complete single-message broadcast algorithm of [1] and obtain a time complexity of $O(n^2 \log^3(n)/\min\{\tau, T\})$. The second statement of Theorem 1.3 now follows directly from Lemma 3.2.

To obtain a concurrency-resistant algorithm we use the same approach as in the 1-interval connected case. We first run the algorithm of [1]. From the description and analysis of the algorithm is not hard to see that the algorithm also works if several source nodes start the algorithm with the same broadcast message. In addition, the transmission behavior of a node does not depend on the content of the broadcast message and thus, if there are 2 or more broadcast messages, one can easily see that at the end of the algorithm, either there exists at least one node which knows at least two different broadcast messages or each node knows exactly one message. If every node knows exactly one broadcast message, we can create one node which knows at least two different broadcast messages in $O(n \log n)$ rounds in the same way as in the 1-interval connected case. The concurrency-resistant algorithm is then completed by running the algorithm of [1] once more, where every node which knows at least two different broadcast messages, uses the algorithm to broadcast $\perp$ to all nodes. The total time complexity of the algorithm is $O(n^2 \log^3(n)/\min\{\tau, T\} + n \log n)$ and therefore the first statement of Theorem 1.3 follows from Lemma 3.4.

### 4 Multi-message Broadcast Using Network Coding

To solve the multi-message broadcast problem in this setting we use randomized linear network coding (RLNC) to increase throughput while using an adapted version of the homogeneous randomized broadcast protocol by Clementi et al. [14] as the underlying broadcast protocol. The RLNC algorithm thus tells the nodes what to send and the broadcast protocol tells them when to send. We describe and analyze the RLNC algorithm based on the elegant work by Haeupler [23].

Let us assume that $M_1, M_2, \ldots, M_s$ are the $s$ broadcast messages initially given to the source nodes. We can represent these messages as vectors over a finite field $\mathbb{F}_q$, where $q$ is a prime or prime power. For all $1 \leq i \leq s$, let $\overrightarrow{m_i} \in \mathbb{F}_q^l$ be the vector representation for $M_i$, where $l$ is the maximum length of a message in base $q$ notation. In each round, each node that decides to transmit, sends a packet $(\overrightarrow{\mu}, \overrightarrow{m})$, where $\overrightarrow{\mu}$ is a
coefficient vector and \( \overrightarrow{m} \) is a message vector (i.e., a linear combination of the broadcast messages) such that

\[
\overrightarrow{m} = (\mu_1, \mu_2, \ldots, \mu_s) \in \mathbb{F}_q^s \quad \text{and} \quad \overrightarrow{m} = \sum_{i=1}^{s} \mu_i \overrightarrow{m}_i \in \mathbb{F}_q^l.
\]

When a node has received \( s \) packets with \( s \) linearly independent coefficient vectors, it can reconstruct all the \( s \) broadcast messages by applying Gaussian elimination. In other words, when the span of the received coefficient vectors by a node is the full space \( \mathbb{F}_q^s \), then it can decode all the broadcast messages. Here we explain the algorithm to solve the multi-message broadcast problem in this setting.

**Algorithm.**

Initially, each node has only one packet. A source node which has broadcast message \( \mathcal{M}_i \), has the packet \( (\overrightarrow{m}_i, \overrightarrow{m}) \), where \( \overrightarrow{m}_i \) is the \( i^{th} \) standard basis vector. A node which is not a source node has the packet \( (\overrightarrow{0}, \overrightarrow{0}) \).

During the execution, each node \( v \) keeps a complete span \( \psi(v) \) of all the packets it has received (i.e., \( \psi(v) \) is the set of all the linear combinations of the received packets). In each round, each node \( v \) chooses a packet, uniformly at random, from \( \psi(v) \) and transmits it with probability \( 1/n \).

**Analysis.**

Let \( \chi(v) \) denote the set of all linear combinations of the coefficient vectors in \( \psi(v) \) for node \( v \). During the algorithm execution, for any node \( v \), by receiving more messages the coefficient subspace \( \chi(v) \) grows monotonically to the full space \( \mathbb{F}_q^s \). We say that node \( v \) in round \( r \) knows about \( \overrightarrow{m} \in \mathbb{F}_q^s \) if \( \overrightarrow{m} \) is not orthogonal to \( \chi(v) \) in that round, that is, there exists some vector \( \overrightarrow{c} \in \chi(v) \) such that \( \langle \overrightarrow{c}, \overrightarrow{m} \rangle \neq 0 \).

**Lemma 4.1.** (Lemma 4.2 from [23]) If node \( u \) knows about vector \( \overrightarrow{m} \in \mathbb{F}_q^s \) and transmits a packet to node \( v \), then \( v \) knows about \( \overrightarrow{m} \) afterwards with probability at least \( 1 - 1/q \). Furthermore, if a node knows about all the vectors in \( \mathbb{F}_q^s \), it is able to decode all the \( s \) broadcast messages.

**Lemma 4.2.** In each round of the algorithm execution, for any vector \( \overrightarrow{m} \in \mathbb{F}_q^s \), if there exists some node that does not know about \( \overrightarrow{m} \), then there exists at least one new node knowing about \( \overrightarrow{m} \) afterwards with probability at least \( 1/2en \).

**Proof.** Let us fix an arbitrary vector \( \overrightarrow{m} \in \mathbb{F}_q^s \) and some positive integer \( r \) such that there exists some node in the network in round \( r \) that does not know about \( \overrightarrow{m} \). Since the network is connected, there exists some node \( v \) which does not know about \( \overrightarrow{m} \) and is connected to node \( u \) which knows about \( \overrightarrow{m} \), in round \( r \). Knowing the fact that all the informed nodes transmit with probability \( 1/n \), the probability that \( u \) is the only neighbor of \( v \) transmitting in round \( r \) is at least

\[
\frac{1}{n} \left( \frac{1}{n} \right)^{n-1} > \frac{1}{en}.
\]

Based on Lemma 4.1, if \( u \) is the only transmitting neighbor of \( v \), then \( v \) knows about \( \overrightarrow{m} \) afterwards with probability \( 1 - 1/q \geq 1/2 \). Therefore, in each such a round, one new node knows about \( \overrightarrow{m} \) afterwards with probability at least \( 1/2en \).

**Theorem 1.4 (repeated).** Using linear network coding, in 1-interval connected dynamic networks with communication capacity 1 and a 1-oblivious adversary, \( s \)-multi-message broadcast can be solved in time \( O(n^2 + ns) \), w.h.p.

**Proof.** Let us assume that the algorithm runs for \( cn(n+s) \) rounds for some constant \( c \). Then due to Lemma 4.2, for any arbitrary coefficient vector \( \overrightarrow{m} \in \mathbb{F}_q^s \), the number of nodes that know about it by the end of the algorithm execution is dominated by a binomial random variable \( X \sim Bin(n(n+s), 1/2en) \). Choosing a large enough constant as \( c \) and applying Chernoff bound, we can show that the probability of having one node not knowing about \( \overrightarrow{m} \) is less than \( 1/nq^s \). Applying union bound over all \( q^s \) coefficient vectors in \( \mathbb{F}_q^s \), all nodes know about all coefficient vectors in \( \mathbb{F}_q^s \) w.h.p., hence, they can decode all the broadcast messages.
5 Multi-message Broadcast Lower Bound

In this section we give a lower bound for solving the multi-message broadcast problem in an $\infty$-interval connected network controlled by a 0-oblivious adversary.

Recall that in an $\infty$-interval connected dynamic network, there is a static connected spanning subgraph which is present every round. In the following, we refer to this graph as the stable subgraph. We first prove a simple lower bound for the case where the stable subgraph has a non-constant diameter. With a more involved argument, we then extend the lower bound to the case where the stable subgraph has a constant diameter.

**Lemma 5.1.** Assume that $G$ is an $n$-node graph with maximum degree $\Delta = O(1)$. If $G$ is the stable subgraph of an $\infty$-interval connected dynamic radio network, with a 0-oblivious adversary, every $s$-multi-message broadcast algorithms requires at least $\Omega(ns/c)$ rounds, where $c \geq 1$ is the communication capacity of the network.

**Proof.** Recall that a 0-oblivious adversary can construct the communication graph of a given round $r$ after the random decisions of all nodes in round $r$. A 0-oblivious adversary therefore in particular knows which nodes are transmitting in round $r$ before determining the graph of round $r$. Given an $s$-multi-message broadcast algorithm $\mathcal{A}$, a 0-oblivious adversary constructs the sequence of communication graphs as follows. In every round in which 2 or more nodes decide to transmit, the communication graph is a complete graph. In all other rounds, the communication graph is only the stable graph $G$. Hence, in rounds with 2 or more nodes transmitting, all $n$ nodes will experience a collision and therefore the algorithm cannot make any progress. In rounds where 0 nodes are transmitting, there clearly also cannot be any progress. In rounds where exactly one node $v$ is transmitting, the message of $v$ only reaches its at most $\Delta = O(1)$ neighbors in $G$. Because we have $s$ broadcast messages of $B$ bits and because each broadcast message only has one source node, over the whole algorithm, the nodes in total need to learn $\Theta(nsB)$ bits of information. As every message sent by the algorithm can contain only $O(cB)$, in each round, the total number of bits learned by any node is also at most $O(cB)$. The lemma therefore follows. \qed

To prove the lower bound for constant-diameter stable subgraphs, we use the hitting game technique introduced by Newport in [34]. This is a general technique to prove lower bounds for solving various communication problems in radio networks. Using this technique, one first defines an appropriate combinatorial game with respect to the problem such that a lower bound for the game can be proved directly. It is shown that an efficient solution for the radio network problem helps a player to win the game efficiently. Consequently, the game’s lower bound can be leveraged to the problem’s lower bound.

For the sake of proving this lower bound, we define a combinatorial game called $(\alpha, \beta)$-hitting game. Assuming the existence of a distributed algorithm $\mathcal{A}$ which solves multi-message broadcast in the desired setting, we will show that a player can simulate the execution of $\mathcal{A}$ in an $\infty$-interval connected dynamic network called the target network. Then, the player uses the transmitting behavior of the nodes in the target network, while running $\mathcal{A}$, to win the game efficiently.

**$(\alpha, \beta)$-hitting Game.** The game is defined for two positive integers $\alpha$ and $\beta$, where $\beta < \alpha$. It proceeds in rounds, and a player, which is represented by a probabilistic automaton $\mathcal{P}$, plays the game against a referee. At the beginning of the game, the referee arbitrarily partitions the set $[\alpha + \beta]$ into two disjoint sets $A$ and $B$, such that $|A| = \alpha$ and $|B| = \beta$. This partitioning can be seen by the player and its sole purpose is the ease of target network construction that will be explained later. Accordingly, the referee selects $\beta$ elements from $A$ uniformly at random, and it generates a random permutation of these elements represented by $\langle a_1, a_2, \ldots, a_\beta \rangle$. The target set $R$ is defined based on this sequence, given by

$$R = \{ (a_1, 1), (a_2, 2), \ldots, (a_\beta, \beta) \}.$$  

\footnote{For two integers $a \leq b$, $[a, b]$ denotes the set of all integers between $a$ and $b$ (including $a$ and $b$). Further, for an integer $a \geq 1$, we use $[a]$ as a short form to denote $[a] := [1, a]$.}
is kept secret from the player. In each round, the player proposes a guess \((x,y)\), and the referee responses to the player whether the guess was in the target set or not. In case the guess belongs to \(R\), the referee removes the guess from \(R\) at the end of the round. The player wins the game after \(r\) rounds, if and only if at the end of round \(r\) the set \(R\) is empty. The following lemma shows a lower bound for this game, which is adapted from Lemma 3.2 in [20].

**Lemma 5.2.** (Adapted from [20]) There does not exist a player that can win the \((\alpha, \beta)\)-hitting game in \(o(\alpha \beta)\) rounds with high probability\(^4\).

**Lemma 5.3.** Let \(s\) and \(n\) be positive integers, where \(s < n\). If algorithm \(A\) solves multi-message broadcast problem with \(s\) source nodes in any \(\infty\)-interval connected dynamic \(n\)-node network against a 0-oblivious adversary in \(f(n, s)\) rounds in expectation, then it is possible to win the \((n - s, s)\)-hitting game in expected \(O(f(n, s))\) rounds.

**Proof.** For the following discussion in this proof, we fix an arbitrary instance \(I\) of the \((\alpha, \beta)\)-hitting game, where \(\alpha = n - s\) and \(\beta = s\). Based on \(I\), we first define a particular \(\infty\)-interval connected dynamic \(n\)-node network called the **target network**. To guarantee the \(\infty\)-interval connectivity of the target network we show that there exits a fixed connected \(n\)-node graph we call the **\(s\)-clique-star** which is a subgraph of the graph representations of the target network in all rounds of any execution. Since one cannot construct the target network based on an instance of the game with no knowledge about the secret information that the referee has, the player simulates the execution of \(A\) on the target network. However, we will see later that with the information revealed gradually by the referee during the game and the power of a 0-oblivious adversary, the simulation by the player is completely consistent with the execution of \(A\) on the target network. Then we show that how the simulation allows the player to win the game by the end of the broadcast. We start with the definition of the \(s\)-clique-star graph and the target network.

**\(s\)-Clique-Star Graph.** Consider an \(n\)-node static graph \(G\) which is defined for some positive integer \(s < n\). The nodes are partitioned into two disjoint sets of size \(s\) and \(n - s\), such that the \(n - s\) nodes form a clique, and each of the \(s\) nodes is connected to exactly one node in the clique. Each of these \(s\) nodes is called an **external node**, and any node in the clique which is connected to at least one external node is called a **bridge node**. Rest of the nodes are called **internal nodes**. You can see the graph in Figure 1.

![Figure 1: The \(s\)-clique-star graph. For some positive integer \(j \leq s\), \(b_1, b_2, \ldots, b_j\) are bridge nodes, and \(e_1, e_2, \ldots, e_s\) are external nodes.](image)

**The Target Network.** The target network is an \(\infty\)-interval connected dynamic \(n\)-node network \(G^t = (G_1, G_2, \ldots)\), such that for all \(r \geq 1\), \(G_r\) is either a complete graph or a complete graph lacking exactly one edge. Let us

\(^4\)We say that a probability event happens with high probability (w.h.p.) if it happens with probability at least \(1 - 1/n^c\), where \(n\) is the number of nodes and \(c > 0\) is a constant which can be chosen arbitrarily large by adjusting other constants.
assume that in the game instance $I$, the partitioned sets determined by the referee are $A$ and $B$, and the target set is

$$R = \{(a_1, 1), (a_2, 2), \ldots, (a_s, s)\},$$

such that among $a_1, \ldots, a_s$, there are $j$ different values, where $j \leq s$. Based on this instance of the game we first construct and instantiate a fixed $n$-node $s$-clique-star graph $G^{cs}$ with $j$ bridge nodes. Then for any $i \geq 1$, we construct $G_i$ by adding edges to $G^{cs}$.

We construct and instantiate $G^{cs}$ as follows. We have $n$ processes with IDs from 1 to $n$. The processes with IDs equal to the mentioned $j$ different values are assigned to the $j$ bridge nodes of $G^{cs}$. Additionally, the $s$ processes whose IDs belong to $B$ are assigned to the external nodes. We connect each external node $e_i$, where $1 \leq i \leq s$, to the bridge node with ID equal to $a_i$. Moreover, we randomly assign the broadcast messages $M_1 := 1, M_2 := 2, \ldots, M_s := s$ to processes assigned to the clique.

In any round $r$, for constructing the target network we consider the following three possibilities of transmitting behavior of the processes in $G^i$: (1) more than one node transmit, (2) only one node transmits and it is a bridge node transmitting $M_i$ such that node $e_i$ is its neighbor in $G^{cs}$, (3) otherwise. In case of possibility (1) or (2), $G_r$ is a complete graph over all $n$ nodes. In case of possibility (3), $G_r$ contains all edges among the nodes except the edge between the transmitter which is transmitting $M_i$ and the node $e_i$.

For a complete construction and instantiation of the target network based on an instance of the game, one needs to also have the secret information that the referee of the game has (since it sometimes needs to recognize the bridge nodes). However, we show that without knowing the secret information the player can simulate execution of $A$ on the target network with the gradual information that the referee reveals during the game.

The Simulation. The player simulates the execution of $A$ on the target network round by round. In each round, according to the transmitting behavior of the nodes it generates at most one guess for the game. Then playing the game for one round with the generated guess (if any), the player finishes the simulation of the current round by simulating the receive behavior of the nodes. It continues simulating next rounds and playing the game until it wins the game. In each round that only one node transmits, if it is not an external node, the player generates a guess consisting of its ID and the message it transmits. Otherwise, it does not generate any guess.

The receive behavior of the nodes in each round $r$ is simulated by the player based on the transmitting behavior of the nodes and the response of the referee to the generated guess. If more than one node transmits in a round, the player simulates all the nodes to receive collision. If an external node is the only transmitter in round $r$ and it is transmitting message $M_i$, then all the nodes except $e_i$ receive $M_i$ successfully. If a non-external node transmitting $M_i$ is the only transmitter in round $r$, then the player generates a guess and plays the game. If the guess is correct, then all the nodes receive $M_i$. Otherwise, all the nodes except $e_i$ receives $M_i$ successfully.

The receive behavior of the nodes in the simulation is completely consistent with that of the execution of $A$ on the target network. In both of them whenever more than one node transmits all the nodes receive collision. The only case in some round of the execution of $A$ the target network is not a complete graph is when that a bridge node which has neighbor $e_i$ in $G^{cs}$ and transmitting $M_i$. And this case can be recognized by the player when it plays the game with the corresponding generated guess. Therefore, we can conclude that the simulation by the player is completely consistent with the execution of $A$ on the target network.

By the end of the simulation, the player wins the game. Considering the receive behavior of the nodes in the simulation, the only way that an external node receives the broadcast message $M_i$ is to receive it directly from its corresponding bridge node in $G^{cs}$. And, it has to happen in a round that there exists only one transmitter since otherwise all nodes receive collision. Moreover, by the end of the simulation, for all $i$, the external node $e_i$ should have received the message $M_i$. Hence, for all $i$, there should exists a round in the simulation that a bridge node transmitting $M_i$ which is a neighbor of $e_i$ in $G^{cs}$ is the only transmitter. Therefore, based on the

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$^3$By instantiation of an $n$-node network, we mean assigning $n$ processes with unique IDs to the nodes of the network.
guess generation approach of the player, we can conclude that the player guesses all the elements of the target set by the end of the broadcasting algorithm simulation.

Finally since in each round of the simulation the player generated at most one guess, if $\mathcal{A}$ solves the multi-message broadcast problem with $s < n$ source nodes in any $\infty$-interval connected network in $f(n, s)$ rounds, a player can win any instance of $(n - s, s)$-hitting game in at most $f(n, s)$ rounds by simulating the execution of $\mathcal{A}$ on the target network which is constructed based on the given instance of the game.

**Theorem 1.5 (restated).** In $\infty$-interval connected dynamic networks with communication capacity $c \geq 1$ and a 0-oblivious adversary, any $s$-multi-message broadcast algorithm requires at least time $\Omega(ns/c)$, even when using network coding. Further, there is a constant-diameter $\infty$-interval connected network with communication capacity 1 such that any store-and-forward algorithm requires at least $\Omega((ns - s^2)/c)$ rounds to solve $s$-multi-message broadcast against a 0-oblivious adversary.

*Proof.* For the sake of contradiction let us assume that there exists an algorithm $\mathcal{A}$ that can solve the multi-message broadcast for $s$ source nodes in any $\infty$-interval connected dynamic $n$-node network in $f(n, s) = o(ns - s^2)$. Then based on Lemma 5.3, we can construct a player to win any instance $(\alpha, \beta)$-hitting game in $o(\alpha \beta)$ rounds. This contradicts Lemma 5.2.

6 Conclusions

In the paper, we studied multi-message broadcast in $T$-interval connected radio networks with different adaptive adversaries. In all considered cases, we shows that if $c$ broadcast messages can be packed into a single message (of the algorithm), $s$ broadcast messages can essentially be broadcast in $s/c$ times the time required to broadcast a single message. In one case ($\infty$-interval connected dynamic networks with a 0-oblivious adversary), we also showed that up to logarithmic factors, our algorithm is optimal. Note that using techniques from [20, 1], at the cost of one logarithmic factor, this lower bound can also be adapted to work in the presence of a 1-oblivious adversary.

A multi-message broadcast time which is roughly $s/c$ times as large as the time for broadcasting a single message seems not very spectacular. Such an algorithm essentially always runs just one single-message broadcast algorithm at each point in time (where for $c > 1$, the algorithm each time broadcasts a collection of messages). However, we believe that it will be interesting to see whether the time complexity can be significantly improved in any of the adversarial dynamic network settings considered in this paper. When using store-and-forward algorithms, such an improvement would imply that the algorithm can use some form of pipelining in an efficient manner. I might also be interesting to study somewhat weaker (adversarial) dynamic network models which allow some pipelining when broadcasting multiple messages.

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