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Collectively Enhanced Interactions in Solid-State Spin Qubits

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We propose and analyze a technique to collectively enhance interactions between solid-state quantum registers composed from random networks of spin qubits. In such systems, disordered dipolar interactions generically result in localization. Here, we demonstrate the emergence of a single collective delocalized eigenmode as one turns on a transverse magnetic field. The interaction strength between this symmetric collective mode and a remote spin qubit is enhanced by the square root of the number of spins participating in the delocalized mode. Mediated by such collective enhancement, long-range quantum logic between remote spin registers can occur at distances consistent with optical addressing. A specific implementation utilizing nitrogen-vacancy defects in diamond is discussed and the effects of decoherence are considered.

Harnessing collective phenomena by utilizing ensembles of identical particles is a powerful tool, which has been exploited in effects ranging from superradiance to scattering suppression [1]. The coherent dynamics resulting from interactions with individual constituents of an ensemble are often too weak to be observed directly; however, as evidenced by experiments in systems such as Rydberg atoms [2–4], cavity QED [5,6], atomic ensembles [7,8], and solid-state qubits [9], collective enhancement provides a natural route to overcoming this challenge. Here, we demonstrate that, for electronic spin quantum registers, such collective effects enable an extended coherent coupling over large distances—an essential prerequisite for quantum information processing.

Owing to favorable coherence properties, electronic spins associated with pointlike defects in solid-state systems have garnered significant recent interest as candidates for room-temperature quantum registers. Quantum control of such spins can be achieved using a combination of optical, magnetic, and electric fields. While our considerations apply to a variety of electronic spin qubits [10–12], here we focus on the nitrogen-vacancy (NV) center in diamond. The NV center harbors an electronic spin (S = 1), which can be optically initialized, coherently manipulated, and readout on subwavelength scales [13–15]. These results have sparked several recent proposals which utilize networks of NV registers as the platform for a scalable quantum information processor [16–19]. However, for any spin qubit candidate, two crucial challenges remain to be addressed: (1) the weakness of the magnetic dipolar interactions on distances compatible with individual optical addressing and (2) the disorder in spin positioning due to inherent imperfections during defect creation.

In this Letter, we present a novel approach to remote quantum logic which harnesses collectively enhanced interactions to overcome both of the above challenges.

The key idea underlying our proposal is to associate a single, robust qubit with a collective, generally disordered spin ensemble (Fig. 1). If the spins behave in an aggregate fashion, such a qubit can produce a large state-dependent magnetic field, leading to enhanced long-range coupling between ensembles; this is reminiscent of tailored light-matter interactions achieved via atomic ensembles [20]. However, we note that quenched disorder naturally leads to localization in solid-state spin systems [21] due to random flip-flop interactions. Similar to Anderson localization [22], this implies that each eigenmode of the ensemble is composed of only a few spins. Here, we demonstrate the use of a uniform transverse magnetic field to overcome this issue. The applied field causes the symmetric W state [23,24] to become an approximate eigenstate of the Hamiltonian, thereby enabling us to harness it as a collective qubit. Moreover, we show that this particular state is largely insensitive to the underlying spin distribution and hence robust to effects of disorder.

To be specific, we now describe our proposal in the context of NV diamond color centers. The largest energy scale in this system (Δ) is set by a combination of the zero-field splitting (2.87 GHz) and the projection of

![Diagram](https://example.com/diagram.png)

FIG. 1 (color online). High-density NV spin ensemble distributed randomly within a sphere of diameter r, with an average distance a. The NV centers have three internal spin states that are split by a zero-field splitting and a Zeeman field.
Thus, the number of selectively drive the transition between
tial overlap with the collective
that the external driving
without populating any other collective states, provided
all ensemble spins polarized into
is either 0 or 1. The effective Hamiltonian is
arises due to the perturbation created by the transverse
description is justified. The second-largest energy scale
is larger than the characteristic strength of
manifolds merely leadsto a renormalization of
m
has different energies, leading to a "blockade-
tions localizes all such eigenstates, even in three dimen-
contrast to the situation without a transverse field, where
the qubit splitting is tuned resonant with only the
state and the notation
refers
to the combined state with the NV qubit being in
one finds the existence of a single eigenstate with
collective enhancement factor, defined as
which for a symmetric eigenmode characterizes the num-er of participating ensemble spins. As expected, in the
absence of a transverse field, disorder localizes all eigen-
n states and, as depicted in Fig. 2 [dark gray (blue) circles],
N_{c} \ll N for all eigenstates. On the other hand, In the case
of a moderate transverse field \( \Omega = \mu B \), with \( B \approx 40 \) G,
one finds the existence of a single eigenstate with \( N_{c} \approx 70 \sim N \). While the specific details of this state depend
on the microscopic details (e.g., spin distribution within
the ensemble, higher-order couplings to the \( m_{s} = -1 \) state,
and magnitude of the applied transverse field), its collec-
tive nature is rather robust. In particular, as one varies the
strength of the transverse field \( B \), there exists a large
parameter regime where \( N_{c} > 50 \) (Fig. 2). This result

\[ V_{ij} = (1 - 3 \cos^{2} \theta_{ij}) \frac{\mu^{2}}{|r_{i} - r_{j}|^{3}} \times \left\{ \frac{1}{4} \left[ 1 + \sigma_{i}^{z^{(j)}} \right] \left[ 1 + \sigma_{j}^{z^{(j)}} \right] - \sigma_{i}^{z^{(j)}} \sigma_{j}^{z^{(j)}} - \sigma_{i}^{z^{(j)}} \sigma_{j}^{z^{(j)}} \right\} \].

where \( r_{i} \) denotes the position, \( \mu \) characterizes the magnetic
dipole moment, and \( \theta_{ij} \) is the angle between the NV axis
and the vector connecting sites \( r_{i} \) and \( r_{j} \). The total
Hamiltonian including both on-site and interaction terms
is then given by

\[ H = \Delta / 2 \sum_{i} \sigma_{i}^{x} + \Omega \sum_{i} \sigma_{i}^{z} + \sum_{i<j} V_{ij}. \]

Now, let us turn to the enhanced coupling between an
isolated NV defect (hereafter termed "qubit") and the
collective ensemble, separated by the distance \( R \). We envi-
sion the ensemble to be initialized into the \( |0\rangle \) state, while
the NV qubit is initialized to the \( m_{s} = 1 \) state. By ensuring
that the qubit splitting is tuned resonant with only the \( |W\rangle \)
state, one finds that the effective dynamics are restricted
to the single-excitation manifold of the combined qubit-
ensemble system; to lowest order, these dynamics are
governed by

\[ H_{\text{eff}} = \sqrt{N_{c} \frac{\mu^{2}}{R}} \left( \sum_{q} |0_{q} \rangle \langle 0_{q}| W \right) \text{ + H.c.}, \]

where \( N_{c} \) characterizes the approximate number of spins
participating in the \( |W\rangle \) state and the notation \( |0_{q} \rangle \) refers
to the combined state with the NV qubit being in \( m_{s} = 1 \)
and with the ensemble spins being in \( |0\rangle \). Consistent with
subwavelength techniques such as stimulated emission
depletion microscopy \( (R = 100 \) nm), we will assume that
the NV qubit can be manipulated and readout indepen-
dently of the ensemble \[29\].

To support the qualitative picture presented above, we
now perform exact diagonalization of the interacting spin
Hamiltonian. In the majority of the numerics, we restrict
ourselves to \( m \leq 2 \) excitations; however, we check the
validity of our results by including the \( m = 3 \) manifold
for slightly smaller system sizes. For each eigenstate \( |\phi\rangle \),
we calculate the collective enhancement factor, defined as

\[ N_{c} = \left( \sum_{i} \langle 0_{i} \ldots 1_{l} \ldots 0_{n} \rangle |\phi\rangle \right)^{2}, \]

which is for a symmetric eigenmode characterizes the num-er of participating ensemble spins. As expected, in the
absence of a transverse field, disorder localizes all eigen-
n states and, as depicted in Fig. 2 [dark gray (blue) circles],
N_{c} \ll N for all eigenstates. On the other hand, In the case
of a moderate transverse field \( \Omega = \mu B \), with \( B \approx 40 \) G,
one finds the existence of a single eigenstate with \( N_{c} \approx 70 \sim N \). While the specific details of this state depend
on the microscopic details (e.g., spin distribution within
the ensemble, higher-order couplings to the \( m_{s} = -1 \) state,
and magnitude of the applied transverse field), its collec-
tive nature is rather robust. In particular, as one varies the
strength of the transverse field \( B \), there exists a large
parameter regime where \( N_{c} > 50 \) (Fig. 2). This result
finding the qubit in the $m_x=1$ state, $p_q$, exhibits collectively enhanced Rabi oscillations. The frequency of these oscillations is enhanced by nearly an order of magnitude relative to that expected for bare dipolar interactions between two individual NV qubits at a similar distance. The numerics also allow us to obtain the time required for an interaction-induced $\pi$ pulse, $t_\pi$, and from this, one can derive the effective distance $R$ associated with $H_{\text{eff}}$. Surprisingly, in all cases, we observe that this distance corresponds not to $R-r$, but instead to the distance between the NV qubit and the center of the ensemble. We study the effects of putting the qubit closer to the ensemble by calculating the collectively enhanced coupling strength $V_c$ (as extracted from the numerically obtained $t_\pi$). As shown in Fig. 3, we find that only for distances very close to the ensemble does the collective enhancement deviate from the asymptotic $1/R^3$ scaling; e.g., the qubit is coupled to individual spins rather than to the entire ensemble.

**Experimental realization and decoherence.**—Thus far, our discussion has assumed that both the NV qubit and the ensemble spins are perfectly decoupled from the environment. In any experimentally realistic scenario, however, there are two natural error sources which will be present: spin decoherence and spin relaxation. We are particularly interested in the scaling of the error rates with $N$, as this may adversely affect the scaling fidelity of our proposed long-range gates [19]. As the error processes act locally on individual spins, we first calculate the error rate for a single spin and multiply the result by $N$ to obtain the rate for the collective state. For simplicity, we assume that the collective state is the previously described $|W\rangle$ state in which a single excitation is shared among all $N$ spins.

First, let us consider the effects of spin decoherence. The worst-case scenario for such decoherence is given by the leaking out into nonsymmetric states. Consequently, the error probability after a single $T_2$ decoherence event on spin $i$ is given by the probability to leave the $|W\rangle$ state,

$$p_{\tilde{W}} = p_{T_2} [1 - |\langle W|\sigma_i^{(i)}|W\rangle|^2] = \frac{4}{N} p_{T_2} \left(1 - \frac{1}{N}\right),$$  

(5)

where $p_{T_2}$ is the single spin decoherence rate. For large $N$, this result is essentially independent of $N$ (after weighing with the number of spins); therefore, the effect of $T_2$ processes on such a collective $|W\rangle$ state does not get enhanced by system size and, in fact, is only slightly worse than for a single spin; i.e., it can be expressed in terms of an effective coherence time $T_2^{\text{eff}}$.

Second, we consider the errors arising from phonon-induced spin relaxation processes ($T_1$). Here, we must distinguish between processes which flip an ensemble spin from $m_z=1$ to $m_z=0$, and the reverse. This asymmetry can easily be seen by noting that the $|W\rangle$ state has only one spin in $m_z=1$, while all other spins are in $m_z=0$. We denote the error probability associated with these two events as $p_{T_1}^{1\rightarrow0}$ and $p_{T_1}^{0\rightarrow1}$, respectively. For $p_{T_1}^{1\rightarrow0}$,
the state $|0\rangle$ with all ensemble spins in $m_s = 0$ is not affected at all, while the probability to flip from the $|W\rangle$ state into $|0\rangle$ is given by
\[ p_{W-0} = p_{T_1^{0-1}}|0\rangle|\sigma[e^{\frac{i\theta}{2}}]|W\rangle|^2 = \frac{p_{T_1^{0-1}}}{N}, \] (6)
which is again independent of the size of the ensemble after rescaling with $N$. However, this is not the case for $T_1^{0-1}$ processes. Both the $|0\rangle$ and the $|W\rangle$ state are strongly affected by such processes, since the existence of any additional spin in the $m_s = 1$ state corresponds to an effective magnetic impurity; this impurity modifies the energy of the collective state, thus tuning it out of resonance with the NV qubit. Additionally, this new state is also no longer an eigenstate of the Hamiltonian; numerical simulations demonstrate that this state dephases very quickly due to dipolar interactions within the ensemble. Thus, since any single spin $T_1^{0-1}$ error will immediately decohere the collective state, the effective error rate owing to $p_{T_1^{0-1}}$ is enhanced by $N$ and scales with the size of the ensemble.

While the system size scaling of $p_{T_1^{0-1}}$ errors might seem unfortunate, in solid-state spin systems, it is often the case that $T_1 \gg T_2$. Our proposed protocol is particularly useful in cases where $T_1/N$ remains longer than $T_2$, implying that the ensemble’s noise is dominated by decoherence as opposed to the enhanced relaxation. The specific example of NV centers highlights this crucial point. The decoherence of the NV originates from fluctuating magnetic fields as neighboring pairs of dipoles flip-flop $[30,31]$. Even at low temperatures it is impossible to freeze out such magnetic fluctuations and $T_2$ remains on the order of milliseconds $[30,32]$. On the other hand, the relaxation of the NV is thought to originate from an Orbach spin-phonon process; such a process has an \textit{exponential} dependence on temperature and implies that even moderate cooling can yield exceedingly long $T_1$ times ($\gg 1$ s at cryogenic temperatures) $[33-35]$. By liquid nitrogen temperatures, the errors introduced by the enhanced $T_1$ processes are already subpercent, enabling us to focus on the effects of decoherence. An alternate approach to combat the enhanced relaxation of the collective state is to utilize conventional dynamical decoupling techniques (e.g., Waugh-Huber-Haeberlen (WAHUHA)) $[36]$ to suppress dipolar interactions within the individual ensembles.

Collective quantum gates.—We now turn to a possible application where isolated NV qubits are interspersed with high-density NV ensembles, forming a regular structure, as depicted in Fig. 4(a). The qubits are used for initialization, single-qubit rotations, and readout. Two-qubit gates between remote spin qubits are mediated by the ensemble between them and thus benefit from collectively enhanced interactions. We would like to point out that such architectures put only modest requirements on the positioning of the NV centers; in particular, the positional disorder

within the NV ensembles is essentially irrelevant. The gate time $t_g$ is limited by the SWAP time $t_\pi$ required to transfer the information from one of the qubits to the ensemble (required four times per gate operation) $[37]$. The resulting error (assuming $T_1/N \gg T_2$) of the gate is given by $e = 1 - \exp\left(-4\pi/\gamma T_1^{\text{eff}}\right)$ in the presence of spin echo decoupling $[38]$. For an error of $e = 10^{-2}$, this translates to a required coherence time of $T_1^{\text{eff}} = 11$ ms, which can be readily realized in isotopically pure diamond samples $[30,32]$ or by using dynamical decoupling pulses $[31,39]$. The requirements on the coherence time can be further relaxed by increasing the number of spins in the ensemble or by reducing the qubit-ensemble separation.

An architecture featuring even better gate fidelities can be realized using a collective encoding scheme for the qubits $[see~Fig.~4(b)]$. There, the logical $|0\rangle$ state corresponds to all nuclear spins being polarized, while the logical $|1\rangle$ state is a collective nuclear spin $|W\rangle$ state. This state can be prepared by applying a microwave pulse to map the electronic $|W\rangle$ state onto a nuclear spin $|W\rangle$ state $[24]$. The time scale for such a single-qubit operation is limited to approximately 100 kHz by the hyperfine splitting of the NV centers in the $m_s = 1$ state ($\gamma = -2.14$ MHz for $^{14}$N) $[40]$. Operation at cryogenic temperatures allows for resonant readout of the $|W\rangle$ state via the zero phonon line without significant background fluorescence $[41]$. In this collective qubit architecture, two-qubit gates between ensembles are enhanced by a factor of $N$ instead of $\sqrt{N}$, thus leading to a SWAP time of $t_\pi = 70$ $\mu$s. Thus, we find that a gate error of $e = 10^{-2}$ requires a coherence time of $T_1^{\text{eff}} = 700$ $\mu$s, while for $e = 10^{-4}$, a coherence time of $T_1^{\text{eff}} = 3$ ms is needed $[30,32,42,43]$.

In summary, we have shown that collectively enhanced interactions can be realized between a NV qubit and a mesoscopic NV ensemble. Our proposed approach relies upon a transverse magnetic field to inhibit the localization.

FIG. 4 (color online). Scalable architectures with collectively enhanced interactions, corresponding to a lattice spacing of $R = 100$ nm, compatible with subwavelength optical addressing. (a) Individually addressable NV qubits [light gray (yellow)] are used for single-qubit operations (SQR), whereas the collectively enhanced interaction with an ensemble is used to mediate two-qubit gates via SWAP operations. (b) NV ensembles are used as collective qubits, where also single-qubit operations are performed using the collective $|W\rangle$ state.
of a symmetric $W$ eigenstate. Our work enables the realization of collectively enhanced quantum gates with high fidelity and provides an important step towards the realization of scalable quantum information architectures involving solid-state electronic spins.

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