Supersymmetric Orientifolds in 6D
with D-Branes at Angles

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Abstract

We study a new class of N=1 supersymmetric orientifolds in six space-time dimensions. The world-sheet parity transformation is combined with a permutation of the internal complex coordinates. In contrast to ordinary orientifolds the twisted sectors contribute to the Klein bottle amplitude leading to new tadpoles to be cancelled by twisted open string sectors. They arise from open strings stretched between D7-branes intersecting at non-trivial angles. We study in detail the $\mathbb{Z}_3$, $\mathbb{Z}_4$ and $\mathbb{Z}_6$ permutational orientifolds obtaining in all cases anomaly free massless spectra.

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1. Introduction

In recent years we have seen the main activity in string model building shifting from the foremost promising class of heterotic Calabi-Yau compactifications to compact Type I and orientifold models. The latter class generically contains D-branes in the background supporting the gauge sector of the low energy theory, whereas gravity propagates in the ten-dimensional bulk. Therefore, such models became quite attractive in recent attempts to establish a unification scenario with the string scale as low as 1TeV [1].

In contrast to the millions of consistent models partly classified in the Calabi-Yau setting, so far we know comparatively few examples of orientifold models. This is mainly due to the fact that the only consistency condition we know of, namely tadpole cancellation, requires the complete knowledge of the one-loop partition function. Therefore we find ourselves restricted to toroidal orbifolds and Gepner models which were discussed in six flat dimensions in [2-8] and in four dimensions in [9-15].

In this paper we study a new class of orientifolds of Type IIB where the world-sheet parity transformation is combined with a $S_2$ permutation of the internal complex coordinates. In real coordinates the permutation is nothing else than a reflection of some of the internal coordinates, not preserving the complex structure. As a consequence, in these permutational orientifolds the loop channel twisted sector Klein bottle amplitudes are non-vanishing. It turns out that in tree channel only untwisted and $\mathbb{Z}_2$ twisted sectors propagate between the two cross-caps. In order to cancel tadpoles it is necessary to introduce D7-branes intersecting at non-trivial angles providing in open string loop channel sectors, which can be regarded as twisted open string sectors. As we will show, these new sectors automatically arise due to the geometric $\mathbb{Z}_N$ symmetry which simply rotates the D7-branes by an angle $\phi = 2\pi/N$. Inclusion of these twisted open string sectors indeed allows us to cancel the Klein bottle tadpoles and finally leads to massless closed and open string spectra satisfying the $N=1$ anomaly constraint.

In a different class of models [16] the request for twisted open string sectors was expressed, as well, however in the models discussed there also all twisted sectors appear in the tree channel Klein bottle amplitude. Thus, there are more tadpoles to cancel, which was argued to be impossible just by perturbative open string sectors. Let us emphasize that the models presented here and the models in [16] are not related by T-duality, as T-duality maps the left-right symmetric $\mathbb{Z}_N$ action to an asymmetric one.
Moreover, since in the permutational orientifolds all twisted sectors are left invariant, the non-supersymmetric Type 0B generalization \[17\] could lead to new tachyon-free orientifolds of the type discussed in \[18\].

This paper is organized as follows. In section two we will define our models and present the computation of the $\mathbb{Z}_3$ orientifold in very much detail. In section three we will discuss the $\mathbb{Z}_4$ and $\mathbb{Z}_6$ orientifolds, thereby focusing mostly on the new issues compared to the $\mathbb{Z}_3$ case. Finally, we will end with some conclusions.

2. The $\mathbb{Z}_3$ permutational orientifold

The six dimensional models studied in the course of this paper are defined as follows. We compactify Type IIB on the four torus $T^4$ with two complex coordinates $X = x_8 + ix_9$ and $Y = x_6 + ix_7$. Then we take the orientifold by the group $G + \Omega RG$, where the space-time symmetry group is $G = \mathbb{Z}_N$ with one of the four choices $N \in \{2, 3, 4, 6\}$ allowing for a crystallographic action. The supersymmetry preserving action of $\mathbb{Z}_N$ on the two complex coordinates is given by

$$\Theta : \begin{cases} X \to e^{2\pi i \frac{1}{N}} X \\ Y \to e^{-2\pi i \frac{1}{N}} Y \end{cases}. \tag{2.1}$$

In the orientifold the world-sheet parity transformation is always paired with the following $\mathbb{Z}_2$ operation

$$R : \begin{cases} X \to \bar{X} \\ Y \to \bar{Y} \end{cases}. \tag{2.2}$$

Thus, from the perspective of the complex coordinates it is nothing else than a permutation, whereas from the perspective of the real coordinates $x_i$ it is the reflection in the $x_7$ and $x_9$ direction. We also have to specify the action of $R$ on the Ramond sector ground states $R : |s_1 s_2 s_3 s_4 \rangle \to |s_1 s_2 - s_3 - s_4 \rangle$. Note, that the combination $\Omega R$ is related to $\Omega$ by T-duality in the $x_7$ and $x_9$ direction. However, under this T-duality the geometric $\mathbb{Z}_N$ operation $\Theta$ is mapped to $T\Theta T^{-1} = \hat{\Theta}$ acting in an asymmetric way on the left and right moving components of the fields $X$ and $Y$

$$\hat{\Theta} : \begin{cases} X_L \to e^{2\pi i \frac{1}{N}} X_L, & X_R \to e^{-2\pi i \frac{1}{N}} X_R \\ Y_L \to e^{-2\pi i \frac{1}{N}} Y_L, & Y_R \to e^{2\pi i \frac{1}{N}} Y_R \end{cases}. \tag{2.3}$$

Thus, the orientifolds we are going to discuss are not related via T-duality to the ordinary orientifolds.

To our knowledge permutational orientifolds were discussed for the first time in \[19\] in the context of non-supersymmetric string theory. The combination of $\Omega$ with $R$ has
dramatic consequences for the computation of the tadpole cancellation conditions as compared to usual orientifold models. First of all, we observe that $R$ does not commute with $\mathbb{Z}_N$, instead one encounters the relation

$$R\Theta^k = \Theta^{N-k}R,$$

(2.4)

where $\Theta$ denotes the generator of $\mathbb{Z}_N$. As a consequence, using the geometry of the Klein bottle as shown in figure 1 one obtains for the twist $g = (\Omega R\Theta^k)^2 = (\Omega R\Theta^l)^2 = 1$, so that in the tree channel only untwisted closed string states propagate along the tube. Therefore, we expect to find only untwisted tadpoles.

Moreover, in all orientifold models studied so far, the action of the world-sheet parity on the twisted sector ground states was such, that the sector twisted by $\Theta^k$ was mapped to the sector twisted by $\Theta^{N-k}$. Thus, only the untwisted and some $\mathbb{Z}_2$ twisted sectors gave a non-vanishing contribution to the Klein bottle amplitude, which were cancelled by annulus and Möbius contributions of D9- and D5-branes in most cases. Now, the permutation $R$ by itself also exchanges the $k$ twisted sector with the $N-k$ twisted sector, as can be seen directly from the definition of the twisted sector. In the $k$ twisted sector the field $X$ has the following monodromy

$$X(\sigma + 2\pi, \tau) = e^{2\pi i k/N}X(\sigma, \tau), \quad \overline{X}(\sigma + 2\pi, \tau) = e^{-2\pi i k/N}\overline{X}(\sigma, \tau),$$

(2.5)

which this mapped under the action of $R$ to

$$\overline{X}(\sigma + 2\pi, \tau) = e^{2\pi i k/N}\overline{X}(\sigma, \tau), \quad X(\sigma + 2\pi, \tau) = e^{-2\pi i k/N}X(\sigma, \tau).$$

(2.6)

Thus, after applying $R$ the field $X$ lies in the $N-k$ twisted sector. Therefore, the combined operation $\Omega R$ leaves the twisted sector invariant and one expects non-vanishing contributions to the Klein-bottle amplitude from all $N-1$ twisted sectors\footnote{This essential point was not realized in \cite{20} and renders the whole computation performed there fairly questionable.}. Note, that the $k$ twisted
sectors of the asymmetric $\mathbb{Z}_3$ orbifold are left invariant under $\Omega$ and are mapped to the $N - k$ twisted sectors under $\Omega R$. This confirms the T-duality relation of these models to ordinary orientifolds.

We are now facing the problem of how to introduce “twisted” open string sectors with the ability to cancel these tadpoles arising in the twisted Klein bottle amplitudes. In usual orientifolds open strings stretched between the same type of D-branes can be regarded as untwisted open string sectors and open strings stretched between a D9- and a D5-brane can be regarded as $\mathbb{Z}_2$ twisted open string sectors.

The problem of twisted open string sectors was also raised in a slightly different class of orientifolds in [16], where it was simply enforced by hand that $\Omega$ did not exchange the twisted sectors. The conclusion there was, that these “twisted” open string sectors are non-perturbative in nature and could be detected by using Type I - heterotic duality. We would like to make the following point clear. We are not claiming that these “non-perturbative” states found there do allow for a perturbative description. What we will show is, that in a different class of orientifolds twisted open string sectors can indeed be described in a perturbative way, namely by open strings stretched between D7-branes intersecting at non-trivial angles.

In the rest of this section we will construct the $\mathbb{Z}_3$ permutational orientifold in some detail revealing how nicely everything fits together. Before that let us remark, that the $\mathbb{Z}_2$ permutational orientifold is isomorphic to the ordinary one, as in that case the action of $\mathbb{Z}_2$ is the same on all four compact coordinates and it is a matter of taste which two are named X and Y. Thus, in that case one simply gets the T-dual of the Gimon-Polchinski model [3] containing instead of D9- and D5-branes two different sorts of D7-branes. The first non-trivial example is the $\mathbb{Z}_3$ orientifold.

2.1. Klein bottle amplitude

Before we can compute the Klein bottle amplitude we need to review some facts about the two dimensional $\mathbb{Z}_3$ lattice. In both the (67) and the (89) plane we choose the elementary cell of the $T^2$ torus as shown in figure 2.

The left-right moving momenta are given by

$$p_{L,R} = p^I \pm \frac{1}{2} L^I$$

(2.7)
with Kaluza-Klein (KK) momentum and winding given by

\[ P^I = \frac{\sqrt{2}}{r} (m_1 \vec{e}_1^* + m_2 \vec{e}_2^*) , \quad L^I = \frac{r}{\sqrt{2}} (n_1 \vec{e}_1 + n_2 \vec{e}_2) . \] (2.8)

Here we have chosen \( \alpha' = 2 \). The basis and the dual basis are

\[ \vec{e}_1 = (\sqrt{2}, 0), \quad \vec{e}_2 = (- \frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}) \]

\[ \vec{e}_1^* = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}), \quad \vec{e}_2^* = (0, \sqrt{\frac{2}{3}}) . \] (2.9)

In figure 2 we have also denoted the three fixed points of the \( \mathbb{Z}_3 \) action on each \( T^2 \). Note, that under the permutation \( R \) the origin is invariant whereas the other two fixed points get exchanged. Now we have to compute the Klein bottle amplitude

\[ \mathcal{K} = 8 c \int_0^\infty \frac{dt}{t^4} \text{Tr}_{U+T} \left[ \frac{\Omega R}{2} \left( \frac{1 + \Theta + \Theta^2}{3} \right) P_{GSO} e^{-2\pi t (L_0 + \bar{L}_0)} \right] , \] (2.10)

where the momentum integration in the non-compact directions has already been carried out and \( c = V_6/(8\pi^2\alpha')^3 \). Let us first discuss the untwisted sector.

a.) Untwisted sector

Unlike the purely \( \Omega \) orientifold where left-right combinations of states of the schematic form \( X\bar{X} \) and \( YY \) were contributing in the Klein bottle amplitude, in our case the states \( X\bar{X} \) and \( Y\bar{Y} \) are relevant. Since \( \Theta \) acts on these states as

\[ \Theta(X\bar{X}) = X\bar{X}, \quad \Theta(Y\bar{Y}) = Y\bar{Y} \] (2.11)

i.e. without any phase factor, each term in the sum \( (1 + \Theta + \Theta^2)/3 \) in (2.10) yields the same result in the oscillator part of the trace.
This is also true for the lattice part due to its $\mathbb{Z}_3$ symmetry. Note that $R$ is the reflection at the axes defined by $\vec{e}_1$, $R\Theta$ is the reflection at $\vec{e}_2$ and $R\Theta^2$ is the reflection at $\vec{e}_1 + \vec{e}_2$. Under the action of $\Omega R$ only KK momentum in the $x_6$ and $x_8$ direction and winding in the $x_7$ and $x_9$ direction survives. Using the expression in (2.8) and (2.9), one finds that $P^6 = (2m)/r$, $L^7 = \sqrt{3}nr$ and similarly for the second torus. It is now straightforward to compute the untwisted Klein bottle amplitude

$$K_U = c (1 - 1) \int_0^\infty \frac{dt}{t^4} \frac{\vartheta[1/2]}{\eta^{12}} \left( \sum_m e^{-\pi t m^2 \rho} \right)^2 \left( \sum_n e^{-\pi t n^2 \rho} \right)^2,$$

(2.12)

where the argument of the $\vartheta$-function is $\exp(-4\pi t)$ and as in [3] $\rho = r^2/\alpha'$.

b.) Twisted sectors

In the $\mathbb{Z}_3$ twisted sector the lattice part is trivial and the action of $\Omega R$ on the oscillator modes is analogous to (2.11). We have to sum over all nine fixed points in each twisted sector, however only one of them is invariant under the action of $\Omega R$. Thus, the $\Theta$ twisted sector contribution to the Klein bottle amplitude is

$$K_{\Theta} = c (1 - 1) \int_0^\infty \frac{dt}{t^4} \frac{\vartheta[1/2]}{\eta^{12}} \left( \sum_m e^{-\pi t m^2 \rho} \right)^2 \left( \sum_n e^{-\pi t n^2 \rho} \right)^2.$$  

(2.13)

The $\Theta^2$ twisted sector contribution to the Klein bottle amplitude can be obtained simply by exchanging $\vartheta[1/2]$ with $\vartheta[-1/2]$ in (2.13) and actually leads to the same partition function. Having everything expressed in terms of $\vartheta$-functions the transformation into tree-channel is straightforward and yields for the complete Klein bottle amplitude

$$\tilde{K} = c (1 - 1) \int_0^\infty \frac{dl}{3} \frac{\vartheta[1/2]}{\eta^{12}} \left( \sum_m e^{-\pi l m^2 \rho} \right)^2 \left( \sum_n e^{-\pi l n^2 \rho} \right)^2 +$$

$$\frac{\vartheta[1/2]}{\eta^{12}} \left( \sum_m e^{-\pi l m^2 \rho} \right)^2 \left( \sum_n e^{-\pi l n^2 \rho} \right)^2 +$$

$$\frac{\vartheta[1/2]}{\eta^{12}} \left( \sum_m e^{-\pi l m^2 \rho} \right)^2 \left( \sum_n e^{-\pi l n^2 \rho} \right)^2 +$$

(2.14)

with the argument exp($-4\pi l$). As expected we only find untwisted tadpoles and the numerical factors in front of the $\vartheta$-functions in (2.14) are exactly $4 \sin^2(\pi k/N)$. This guarantees that only $\mathbb{Z}_3$ invariant states from the untwisted closed string sector propagate along the tube of the tree channel Klein bottle. In other words, the three different terms
in (2.14) really constitute the complete projector $(1 + \Theta + \Theta^2)/3$ acting on the untwisted states in tree channel. Note, from our loop channel computation this matching appears to be some nice conspiracy between the numerical lattice factors from the untwisted sector and the number of invariant fixed points in the twisted sectors. In the open string sector we will promote this “completion of the projector in tree-channel” to our guiding principle in determining the relative normalizations of the different open string sectors.

2.2. Annulus amplitude

Since via T-duality in the $x_7$ and $x_9$ directions $\Omega R$ is related to $\Omega$, we expect that we need D7-branes to cancel the Klein-bottle tadpoles. Moreover, in the four internal coordinates these D7-branes should extend in the $x_6$ and $x_8$ direction and fill the complete six-dimensional uncompactified space-time. Let us assume first that we stuck together $M$ D7-branes on the $x_7 = x_9 = 0$ line fixed under $R$. The entire arrangement of D7-branes has to satisfy the geometric $\mathbb{Z}_3$ symmetry we would like to mod out. Therefore, we are forced to introduce rotated D7-branes, as well. More concretely, in both the (67) and the (89) plane we get the $\Phi_1 = 2\pi/3$ and $\Phi_2 = 4\pi/3$ rotated D7-branes as shown in figure 3.

Thus, we are automatically forced to introduce D7-branes intersecting at non-trivial angles which were discussed before in various papers dealing with supersymmetry preserving brane configurations [21]. Due to the analysis in [21] the rotated D7-branes depicted in figure 3 have eight unbroken supersymmetries, exactly what is needed for N=1 in six dimensions.
In order to derive the mode expansion for open strings connecting D-branes at angles in the \((67)\) and \((89)\) plane let us restrict first to the \((89)\) plane and assume for simplicity that one D-brane is stretching in the \(x_8\) direction. An open string ending on this D-brane satisfies the following boundary conditions

\[
\frac{\partial}{\partial \sigma} X^8(0, \tau) = 0, \quad X^9(0, \tau) = 0.
\] (2.15)

The other end of the open string ends on a different D-brane which is rotated by an angle \(\Phi = \pi \theta\) against the former D-brane. Therefore, the boundary conditions at \(\sigma = \pi\) have the following form

\[
\cos(\pi \theta) \frac{\partial}{\partial \sigma} X^8(\pi, \tau) + \sin(\pi \theta) \frac{\partial}{\partial \sigma} X^9(\pi, \tau) = 0
\]

\[
- \sin(\pi \theta) X^8(\pi, \tau) + \cos(\pi \theta) X^9(\pi, \tau) = 0.
\] (2.16)

The solution to these two equations is

\[
X_8 = \sum_{m \in \mathbb{Z} + \theta} \frac{1}{m} \alpha_m e^{-im\tau} \cos(m \sigma) + \sum_{n \in \mathbb{Z} - \theta} \frac{1}{n} \hat{\alpha}_n e^{-in\tau} \cos(n \sigma)
\]

\[
X_9 = \sum_{m \in \mathbb{Z} + \theta} \frac{1}{m} \alpha_m e^{-im\tau} \sin(m \sigma) - \sum_{n \in \mathbb{Z} - \theta} \frac{1}{n} \hat{\alpha}_n e^{-in\tau} \sin(n \sigma)
\] (2.17)

showing that one indeed obtains something deserving the name twisted open string sector.

The calculation for the world sheet fermions and for the \((67)\) plane is completely analogous. In the following we will denote the number of each type of D7-brane as \(M\). The annulus amplitude

\[
A = c \int_0^\infty \frac{dt}{t^4} \text{Tr}_{\text{open}} \left[ \frac{1}{2} \left( 1 + \Theta + \Theta^2 \right) P_{GSO} e^{-2\pi t L_0} \right]
\] (2.18)

has to be computed for all possible open strings connecting the three kinds of D7-branes.

a.) Untwisted sector

By untwisted open string sector we mean open strings stretched between two D-branes of the same kind. Since the \(\mathbb{Z}_3\) action rotates the D7-branes, it is clear that the \(\Theta\) and \(\Theta^2\) insertions in the trace in (2.18) vanish identically. The oscillator contribution to the trace is as usual and, due to the \(\mathbb{Z}_3\) symmetry, the lattice contribution is the same for each type of D7-brane. More concretely, the KK momentum in the \(x_6\) and \(x_8\) direction is quantized as \(P = m/r\) and for the winding in the \(x_7\) and \(x_9\) direction we find \(L = nr\sqrt{3}/2\). Thus, the complete untwisted annulus amplitude is

\[
\mathcal{A}_U = c (1 - 1) \int_0^\infty \frac{dt}{t^4} \frac{M^2}{4} \frac{\Theta[\frac{1}{2}]}{\eta^{12}} \left( \sum_m e^{-2\pi t m^2 \rho^2} \right)^2 \left( \sum_n e^{-2\pi t n^2 \rho^2} \right)^2
\] (2.19)
with the argument \( \exp(-2\pi t) \).

b.) Twisted sectors

Under twisted open string sectors we understand open string segments stretched between different types of D7-branes. Since the different D7-branes do not share any direction, the contribution from the lattice to the annulus amplitude is trivial and one easily gets

\[
A_{\Theta} = \sum_{i=1}^{3} A_{7_{i}7_{i+1}} = \kappa_1 c (1-1) \int_0^\infty \frac{dt}{t^4} \frac{M^2}{4} \frac{\vartheta[0/1]}{\eta^6} \frac{\vartheta[-1/2]}{\vartheta[1/2]} \frac{\vartheta[-1/3]}{\vartheta[1/2]},
\]

\[
A_{\Theta^2} = \sum_{i=1}^{3} A_{7_{i}7_{i-1}} = \kappa_2 c (1-1) \int_0^\infty \frac{dt}{t^4} \frac{M^2}{4} \frac{\vartheta[0/1]}{\eta^6} \frac{\vartheta[-1/2]}{\vartheta[1/2]} \frac{\vartheta[-1/3]}{\vartheta[1/2]},
\]

where in view of the more complicated \( \mathbb{Z}_4 \) and \( \mathbb{Z}_6 \) examples we have introduced normalization constants \( \kappa_1, \kappa_2 \) which will be determined by requiring that the tree channel amplitude contains the complete \( \mathbb{Z}_3 \) projector. Transformation to tree channel yields

\[
\tilde{A} = c (1-1) \int_0^\infty dl \frac{M^2}{6} \left[ \frac{\vartheta[1/2]}{\eta^{12}} \left( \sum_m e^{-\pi lm^2 \rho} \right)^2 \left( \sum_n e^{-\pi l m^2 \rho} \right)^2 + 3 \kappa_1 \frac{\vartheta[1/2]}{\eta^{12}} \vartheta[-1/3] \vartheta[1/3] \right] + 3 \kappa_2 \frac{\vartheta[1/2]}{\eta^{12}} \vartheta[-1/3] \vartheta[1/3] \right],
\]

(2.20)

with the argument \( \exp(-4\pi l) \). With the choice \( \kappa_1 = \kappa_2 = 1 \), up to a numerical factor this is the same as the Klein bottle amplitude (2.14) and in particular the \( \mathbb{Z}_3 \) projector is complete.

2.3. Möbius amplitude

The last one-loop amplitude to compute is the Möbius amplitude

\[
\mathcal{M} = c \int_0^\infty \frac{dt}{t^4} \text{Tr}_{\text{open}} \left[ \frac{\Omega R}{2} \left( \frac{1 + \Theta + \Theta^2}{3} \right) P_{GSO} e^{-2\pi t L_0} \right],
\]

(2.22)

where we also distinguish between untwisted and twisted contributions.

a.) Untwisted sector

Since the \( 7_1 \) branes are reflected onto themselves under \( R \), we expect to find a nonzero contribution for \( \mathcal{M}_{7_{1}7_{1}} \) with the \( \Omega R \) insertion. Similarly, we expect that \( \mathcal{M}_{7_{3}7_{3}} \) is only non-zero with the \( \Omega R \Theta^2 \) insertion and that \( \mathcal{M}_{7_{3}7_{3}} \) is only non-zero with the \( \Omega R \Theta \) insertion. The
trace over the oscillator part is as described in [3]. However, as in the annulus amplitude for the KK momenta in the \( x_6 \) and \( x_8 \) directions all \( P = m/r \) contribute but for the winding in the \( x_7 \) and \( x_9 \) direction only doubly wound modes \( L = nr\sqrt{3} \) are invariant. Thus, for the untwisted Möbius amplitude we obtain

\[
\mathcal{M}_U = -c(1 - 1) \int_0^\infty \frac{dt}{t^4} \frac{M^{0,1/2} M^{1/2}}{\eta^{12} \vartheta^{0,1/4}} \left( \sum_m e^{-2\pi t \frac{m^2}{\rho}} \right)^2 \left( \sum_n e^{-2\pi t n^2 \rho^3} \right)^2 (2.23)
\]

with the argument \( \exp(-4\pi t) \).

b.) Twisted sectors

Here one realizes that the \((7_1 \overline{7}_2)\) sector is invariant under \( \Omega R \Theta^2 \), the \((7_2 \overline{7}_3)\) sector is invariant under \( \Omega R \) and the \((7_3 \overline{7}_1)\) sector is invariant under \( \Omega R \Theta \). Thus, there are three contributions to the twisted Möbius amplitude. Using the general relation

\[
\frac{\vartheta^{a+1/2}_b}{\vartheta^{a+1/2}_{b+1/2}}(-q) = \frac{\vartheta^{(a+1)/2}_b}{\vartheta^{(a+1)/2}_{b+1/2}} \frac{\vartheta^{a/2}_b}{\vartheta^{a/2}_{b+1/2}} (q^2)
\]

we obtain for the \( \Theta \) twisted Möbius amplitude

\[
\mathcal{M}_\Theta = -\kappa_1 c(1 - 1) \int_0^\infty \frac{dt}{t^4} \frac{M^{0,1/2} M^{1/2}}{\eta^{6} \vartheta^{0,2} \vartheta^{-1/6} \vartheta^{1/3} \vartheta^{-1/3} \vartheta^{1/6} \vartheta^{1/6}} (2.25)
\]

and similarly for \( \mathcal{M}_{\Theta^2} \). Transformation to tree channel again reveals the appearance of the complete projector for \( \kappa_1 = \kappa_2 = 1 \).

\[
\tilde{\mathcal{M}} = -c(1 - 1) \int_0^\infty dl \frac{8M}{3} \left[ \vartheta^{1/2}_0 \vartheta^{0,1/2} \right]^2 \left( \sum_m e^{-\pi l m^2 \rho^4} \right)^2 \left( \sum_n e^{-\pi l n^2 \rho^3} \right)^2 + \\
3(\kappa_1 + \kappa_2) \left( \frac{\vartheta^{1/2}_0 \vartheta^{0,1/2} \vartheta^{1/2}_0 \vartheta^{0,1/3} \vartheta^{0,1/6} \vartheta^{0,1/6}}{\eta^{6} \vartheta^{0,2} \vartheta^{-1/6} \vartheta^{1/3} \vartheta^{1/3} \vartheta^{1/6}} \right) (2.26)
\]

with argument \( \exp(-8\pi l) \).

2.4. Tadpole cancellation and massless spectrum

Adding up all tree channel amplitudes from the Klein bottle, annulus and Möbius strip we extract only one single tadpole cancellation condition

\[
\frac{1}{6}(M^2 - 16M + 64) = \frac{1}{6}(M - 8)^2 = 0. (2.27)
\]
Thus, in contrast to the 32 D-branes usually appearing in pure $\Omega$ orientifolds, there are now only eight D7-branes of each kind. The next step is to compute the massless spectrum arising in both the closed and the open string sector and to show that it indeed satisfies anomaly cancellation.

The computation in the closed string sector differs from the analogous computation in the case of the usual $\Omega$ orientifolds. In the untwisted sector we find the $N=1$ supergravity multiplet and 3 hypermultiplets. In a twisted sector we have nine $\mathbb{Z}_3$ fixed points. The one fixed point invariant under $\Omega R$ contributes 1 hypermultiplet. The remaining eight fixed points form four pairs under the action of $\Omega R$ leading to 4 tensor multiplets and 4 hypermultiplets. Since there are two twisted sectors we end up with the massless closed string spectrum

$$1 \text{ SUGRA} + 8 \text{ tensor multiplets} + 13 \text{ hypermultiplets}.$$ 

In the open string sector we find in the untwisted sector one vector multiplet in the adjoint of $SO(8)$ and one hypermultiplet in the adjoint, as well. Moreover, there appears one further hypermultiplet in the adjoint from the twisted sector, so that we end up with

$$G=SO(8) \text{ with } 2 \text{ hypermultiplets in the adjoint}.$$ 

The closed and open string spectrum together satisfy the anomaly constraint

$$n_H - n_V + 29 n_T = 273. \quad (2.28)$$

It is not necessary to place all $7_1$ branes neither on top of each other nor at the fixed line of $R$. The position of the $7_2$ and $7_3$ branes are of course still related to the position of the $7_1$ branes due to the $\mathbb{Z}_3$ symmetry. Changing the position of the $7_1$ branes in the (68) plane is reflected in the effective theory as a Higgs branch on which the gauge symmetry is broken to some subgroups. For instance, placing four $7_1$ branes at a position $x_6 = x_8 = a$ with $a \neq 0$ and necessarily the remaining four $7_1$ branes at $x_6 = x_8 = -a$, we derive the following massless spectrum

$$G=U(4) \text{ with } 2 \times \text{Adj} + 2 \times 6.$$ 

This is related to the maximal spectrum via Higgsing and consistently satisfies the anomaly constraint, as well.

We presented a purely perturbative treatment of these new permutational orientifolds thereby describing twisted open string sectors by D7-branes intersecting at 60 and 120 degree angles. As we will see in the next sections some new aspects will arise when considering the other two possible $\mathbb{Z}_4$ and $\mathbb{Z}_6$ cases.
3. The $\mathbb{Z}_4$ permutational orientifold

In case of the $\mathbb{Z}_4$ orientifold some new issues arise which were absent in the $\mathbb{Z}_3$ case. A first attempt to choose the rectangular $\mathbb{Z}_4$ symmetric lattice with basis vectors in the (67) and (89) direction, respectively, failed, as in the tree channel Klein bottle amplitude the complete projector $(1 + \Theta + \Theta^2 + \Theta^3)/4$ did not show up. In particular in that case we found that the lattice part for the $\{1, \Theta^2\}$ insertions in the Klein bottle trace differed from the result for the $\{\Theta, \Theta^3\}$ insertions. The resolution to this puzzle is to start in the (89) plane with the lattice in figure 4.

![Figure 4]

and in (67) plane with the rotated lattice shown in figure 5.

![Figure 5]

We have depicted the $\mathbb{Z}_4$ fixed points by black circles and the additional $\mathbb{Z}_2$ fixed by white circles. From the figures it is evident that all four $\mathbb{Z}_4$ fixed points are also invariant under $R$ and that only eight of the sixteen $\mathbb{Z}_2$ fixed points are invariant. Thus, the $\Theta$ and $\Theta^3$ twisted sector contribution to the Klein bottle amplitude is weighted by a factor of four and the $\Theta^2$ twisted sector contribution is weighted by a factor of eight. The actual
computation is completely similar to the $\mathbb{Z}_3$ case and yields after all the following tree channel Klein bottle amplitude

\[
\tilde{K} = c (1 - 1) \int_0^\infty dl \, 2^6 \left[ \frac{d_l^{[1/2]}}{\eta^{1/2}} \right]^4 \left( \sum_m e^{-\pi l^{m^2} \rho} \right) \left( \sum_n e^{-\pi l^{2n^2} \rho} \right) \left( \sum_{\text{m}} e^{-\pi l^{2m^2} \rho} \right) \\
\left( \sum_{n} e^{-\pi l^{2n^2} \rho} \right) + 2 \frac{d_l^{[1/2]} \, d_l^{[1/2]} \, d_l^{[-1/4]} \, d_l^{[1/4]}}{\eta^{6} \, d_l^{[1/2]} \, d_l^{[-1/4]} \, d_l^{[1/4]}} + 2 \frac{d_l^{[1/2]} \, d_l^{[1/2]} \, d_l^{[1/2]} \, d_l^{[1/2]}}{\eta^{6} \, d_l^{[1/2]} \, d_l^{[1/2]} \, d_l^{[1/2]}}
\]

(3.1)

with the argument $\exp(-4\pi l)$. The last term in (3.1) arises in the $\Theta^2$ twisted sector and is actually zero. Nevertheless, its coefficient still demonstrates the appearance of the complete projector with numerical prefactors $4 \sin^2(\pi k/N)$ in front of the $d_l$-functions. The next step is to introduce appropriate D7-branes to cancel the tadpoles in (3.1). The right choice to do that is shown in figure 6.

Unlike the $\mathbb{Z}_3$ case, here we have two sets of D7-branes which are not related by $\mathbb{Z}_4$ rotations, namely the $\{7_1, 7_3\}$ and the $\{7_2, 7_4\}$ sets close under $\mathbb{Z}_4$ rotations. Thus, eventually we expect to get a product gauge group with two factors, one living on the first set of D7-branes and one living on the second set of D7-branes. Since there exists a $\mathbb{Z}_2$ subgroup leaving the different D7-branes invariant we get an action of the $\mathbb{Z}_2$ on the Chan-Paton factors of each D7-brane. As usual this action is described by a unitary matrix, denoted

Figure 6
as $\gamma_{\Theta^2}$. Due to the symmetry of the problem, we assume in the following that these four a priori different $\gamma$-matrices are all identical. The computation of the different terms in the annulus amplitude is now straightforward. However, one finds that in the loop channel the complete projector does not constitute itself, in fact the untwisted and $\Theta^{1,3}$ twisted sectors are fine, but the $\Theta^2$ sector is too small by a factor of two. Looking into the details reveals that in a $\mathbb{Z}_N$ orientifold the $\Theta^k$ twisted open string sector, $A_{\Theta^k}$, has to be weighted by an extra factor of

$$
\kappa_k = \frac{\sin^2 \left( \frac{\pi k}{N} \right)}{\sin^2 \left( \frac{\pi}{N} \right)}.
$$

(3.2)

This is similar to ordinary orientifolds with a discrete background NSNS two-form B-field [22,8], where also the sector of open strings stretched between D9- and D5-branes is weighted by an extra factor of $2^\text{g(B)/2}$. Analogously, also in our case it turns out that these prefactors are crucial for satisfying the anomaly constraint. Of course, it would be nice to have a deeper understanding of these extra factors from the loop channel point of view. Unfortunately, we cannot offer a satisfying explanation yet. The complete tree channel annulus amplitude is

$$
\tilde{A} = c (1 - 1) \int_0^\infty dl \frac{1}{4} \left[ M^2 \frac{\vartheta[1/2]^4}{\eta^{12}} \left( \sum_m e^{-\pi l m^2 \rho} \right) \left( \sum_n e^{-\pi l n^2 \rho} \right) \left( \sum_m e^{-\pi l 2m^2 \rho} \right) \right. 
$$

$$
\left. \left( \sum_n e^{-\pi l 2n^2 \rho} \right) + 8 \text{Tr}(\gamma_{\Theta^2})^2 \frac{\vartheta[1/2]^2 \vartheta[0]}{\eta^6 \vartheta[1/2]} \right] +
$$

$$
2 M^2 \frac{\vartheta[1/2]^2 \vartheta[1/2]}{\eta^6 \vartheta[1/2]} \vartheta[1/2] \vartheta[0] + 2 \text{Tr}(\gamma_{\Theta^2})^2 \frac{\vartheta[1/2]^2 \vartheta[0]}{\eta^6 \vartheta[1/2]} \vartheta[0] \vartheta[1/2] -
$$

$$
4 \text{Tr}(\gamma_{\Theta^2})^2 \frac{\vartheta[1/2]^2 \vartheta[0]}{\eta^6 \vartheta[0]} +
$$

$$
2 M^2 \frac{\vartheta[1/2]^2 \vartheta[1/2]}{\eta^6 \vartheta[1/2]} \vartheta[1/2] \vartheta[1/2] + 2 \text{Tr}(\gamma_{\Theta^2})^2 \frac{\vartheta[1/2]^2 \vartheta[1/2]}{\eta^6 \vartheta[1/2]} \vartheta[1/2] \vartheta[1/2] \vartheta[-1/4] \vartheta[1/4] \vartheta[0] \vartheta[-1/4] \vartheta[0] \vartheta[-1/4] \vartheta[1/4]
$$

(3.3)

with the argument $\exp(-4\pi l)$.

In the untwisted Möbius amplitude terms like

$$
\text{Tr}_{7173} \left( \frac{\Omega R}{8} P_{GSO} e^{-2\pi l L_0} \right)
$$

(3.4)
give a non-zero result, whereas due terms like
\[ \text{Tr}_{7_1,7_1} \left( \frac{\Omega R^2}{8} P_{GSO} e^{-2\pi t L_0} \right) \] (3.5)
are zero and do not contribute due to the action on the oscillator modes. In the twisted sector the traces
\[ \text{Tr}_{7_1,7_3} \left( \frac{\Omega R (\Theta + \Theta^3)}{4} P_{GSO} e^{-2\pi t L_0} \right) \quad \text{and} \quad \text{Tr}_{7_2,7_4} \left( \frac{\Omega R (1 + \Theta^4)}{4} P_{GSO} e^{-2\pi t L_0} \right) \] (3.6)
are non-vanishing. At first sight it might be confusing that the \( Z_2 \) twisted sectors contribute here, as in tree channel we expect to find some non-zero terms in the \( Z_4 \) twisted sectors. However, due to the different rules for the transformation from loop into tree channel in the Möbius amplitude and the relation (2.24), it finally comes out just right. Moreover, the sectors in (3.6) are exactly those weighted by this extra factor of two. This degeneration has to be taken into account in the Möbius amplitude, as well. As we shall find in the \( Z_6 \) example it can happen that the action of the \( Z_2 \) subgroup must be different in the degenerated sectors in order to guarantee completion of the projector in the tree channel Möbius amplitude. For the tree channel Möbius amplitude we find
\[ \tilde{M} = -c (1 - 1) \int_0^\infty dl \left[ 8 M \left( \sum_m e^{-\pi l \rho m^2} \right) \left( \sum_n e^{-\pi l \rho m^2} \right) + \left( 2 \kappa_2 \right) \frac{\eta^6 \vartheta_{[1/2]} \vartheta_{[0]} \vartheta_{[1/2]} \vartheta_{[1/4]} \vartheta_{[-1/4]} \vartheta_{[-1/4]}}{\eta^2 \vartheta_{[0]} \vartheta_{[1/2]} \vartheta_{[1/4]} \vartheta_{[-1/4]} \vartheta_{[-1/4]}} \right] \] (3.7)
with argument \( \exp(-8\pi l) \). Thus, also the Möbius amplitude confirms that the \( (7_1,7_3) \) as well as the \( (7_2,7_4) \) sector must be weighted by an extra factor \( \kappa_2 = 2 \). Note, that the second term in (3.7) is the sum of the \( \Theta \) and the \( \Theta^3 \) twisted open string sector.

Adding up all tree channel amplitudes we derive two tadpole cancellation conditions
\[ \frac{1}{4} (M - 16)^2 = 0, \quad \text{Tr}(\gamma \Theta^2) = 0, \] (3.8)
yielding \( M = 16 \) D7-branes of each type. The computation of the closed string massless spectrum gives one supergravity and 3 hypermultiplets from the untwisted sector. In the twisted sector one has to be very careful in treating the fixed points in the right way. The four \( Z_4 \) fixed points contribute eight further hypermultiplets. The eight \( Z_2 \) fixed points
invariant under $\Omega R$ give six hypermultiplets and the remaining eight fixed points yield three hypermultiplets and one tensormultiplet. Adding up all states we find

$$1 \text{ SUGRA} + 1 \text{ tensormultiplet} + 20 \text{ hypermultiplets}.$$  

For the open strings the untwisted sector gives a vectormultiplet in the adjoint of $U(8) \times U(8)$ and two hypermultiplets in the antisymmetric representation for each gauge factor. The $\Theta$ and $\Theta^3$ twisted sector contributes two hypermultiplets in the bifundamental representation and finally the $\Theta^2$ twisted open string sector yields another two hypermultiplets in the antisymmetric representation for each gauge factor. In the latter we have taken into account the extra degeneration in this sector. Summarizing we get

$$G = U(8) \times U(8) \text{ with } 4 \times (28, 1) + 4 \times (1, 28) + 2 \times (8, 8).$$

It is easily checked that the closed and open string spectrum above satisfies the anomaly constraint.

4. The $\mathbb{Z}_6$ permutational orientifold

In the $\mathbb{Z}_6$ orientifold there arises only one new issue as compared to the $\mathbb{Z}_3$ and $\mathbb{Z}_4$ examples to be discussed shortly. The rest of the computation is completely analogous and we will skip most of the details. Analogous to what we encountered in the $\mathbb{Z}_4$ case, we are free to choose in the (89) plane the lattice in figure 7.

![Figure 7](image)

In the (67) plane we are forced to take the rotated lattice as shown in figure 8.
The one $\mathbb{Z}_6$ fixed point is denoted by a white circle, the additional two $\mathbb{Z}_3$ fixed points by black circles and the additional three $\mathbb{Z}_2$ fixed points by grey circles. The behaviour of the fixed points under the reflection $R$ is obvious, namely three of the nine $\mathbb{Z}_3$ fixed points and four of the sixteen $\mathbb{Z}_2$ fixed points are invariant under $R$. Computing the Klein bottle
amplitude we realize that these numbers are exactly those needed to get the complete $\mathbb{Z}_6$ projector in tree-channel. The tadpoles appearing in the Klein bottle amplitude can be cancelled by introducing six different D7-branes rotated against each other by thirty degrees as shown in figure 9.

Under the action of $\mathbb{Z}_6$ the elements of two sets $\{7_1, 7_3, 7_5\}$ and $\{7_2, 7_4, 7_6\}$ are transformed among themselves. Open strings of type $(7_i \bar{7}_i)$ can be understood as the untwisted sector and open strings of type $(7_i \bar{7}_{i+3})$ as the $\Theta^3$ twisted sector. The remaining two $\Theta^{2,4}$ and $\Theta^{1,5}$ twisted sectors are given by open strings of type $(7_i \bar{7}_{i+2})$ and $(7_i \bar{7}_{i+1})$, respectively.
In the computation of the twisted sector annulus amplitude, we have to introduce those extra factors $[3.2]$. The resulting expressions are very similar to the $\mathbb{Z}_4$ annulus amplitude $[3.3]$.

The only sectors contributing to the loop channel Möbius amplitude are the untwisted one and the one arising from open strings in the $(7_i7_{i+2})$ sector. In the $(7_i7_3)$ sector, for instance, the two terms

$$
\text{Tr}_{7_i7_3} \left( \frac{\Omega R \Theta + \Theta^4}{2} P_{GSO} e^{-2\pi tL_0} \right)
$$

are non-vanishing. The first one gives rise to a contribution of the $\Theta$ twisted sector in the Möbius tree channel amplitude and the second one yields a contribution to the $\Theta^2$ twisted sector. The projector tells us that the relative normalization of these two factors in the tree channel must be $\tilde{M}_{\Theta^2} = 3\tilde{M}_\Theta$. However, from the annulus amplitude the overall normalization of the $(7_i7_{i+2})$ sector is fixed to be three. It is possible to repara that by requiring that in the three degenerated $(7_i7_{i+2})$ sectors the action of $\Omega R$ is not identical. One sector must be invariant and the other two must be exchanged under the action of $\Omega R$. Fortunately, our guiding principle again allows us to fix everything. Apparently, the different action of $\Omega R$ on the three degenerated sectors has dramatic consequences for the computation of the massless spectrum.

The two tadpole cancellation conditions in the $\mathbb{Z}_6$ case are

$$
\frac{1}{2} (M - 8)^2 = 0, \quad \text{Tr}(\gamma_{\Theta^3}) = 0.
$$

The computation of the closed string massless spectrum yields one supergravity and 3 hypermultiplets from the untwisted sector. Two further hypermultiplets come from the $\mathbb{Z}_6$ fixed point. The nine $\mathbb{Z}_3$ fixed points give rise to two tensormultiplets and eight hypermultiplets. Finally, the sixteen $\mathbb{Z}_2$ fixed points yield one tensormultiplet and five hypermultiplets. Adding up all these states we find

$$
1 \text{ SUGRA} + 3 \text{ tensormultiplets} + 18 \text{ hypermultiplets}.
$$

For the open strings the untwisted sector gives rise to a vectormultiplet in the adjoint of $U(4) \times U(4)$ and two hypermultiplets in the antisymmetric representation for each gauge factor. The $\Theta$ and $\Theta^5$ twisted sector contribute two hypermultiplets in the bifundamental representation. Taking into account the above mentioned subtlety in the $\Theta^{2,4}$ twisted open string sector we obtain four hypermultiplets in the antisymmetric representation and one hypermultiplet in the adjoint representation for each gauge factor. Finally, four further
hypermultiplets in the bifundamental representation arise in the $\Theta^3$ twisted open string sector. Summarizing we get

$$G=U(4) \times U(4) \text{ with } 6 \times (6, 1) + 6 \times (1, 6) + (\text{Adj, 1}) + (1, \text{Adj}) + 6 \times (4, 4),$$

which together with the closed string spectrum indeed satisfies the anomaly constraint.

5. Conclusion

We have studied a new class of orientifolds in which the world-sheet parity transformation is combined with a permutation of the internal coordinates. As the main new issue we encountered non-zero contributions to the twisted sector Klein bottle amplitude, which fortunately can be cancelled by introducing D7-branes intersecting at non-trivial angles into the background. We have studied all possible $N=1$ supersymmetric examples in six-dimensions, performed the complete tadpole cancellation computation and found after all reasonable anomaly-free massless spectra. We would like to emphasize again that the main tool or guiding principle used in the construction was the requirement of always getting the complete $\mathbb{Z}^N$ projector in the tree channel amplitude after modular transformation of the loop channel.

There exist a lot of open questions related to the permutational orientifolds we have presented in this paper. For instance, it is interesting to study models of this type in four space dimensions with $N=1$ supersymmetry \cite{23}. Moreover, these models are particularly suitable to yield non-tachyonic Type 0B generalizations \cite{17}. It is not evident what happens when one turns on some background two form flux and finally there might exist some dual F-theory descriptions.

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