Hadronic Decays of $N$ and $\Delta$ Resonances in a Chiral Quark Model

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$\pi$ and $\eta$ decay modes of light baryon resonances are investigated within a chiral quark model whose hyperfine interaction is based on Goldstone-boson exchange. For the decay mechanism a modified version of the $^3P_0$ model is employed. Our primary aim is to provide a further test of the recently proposed Goldstone-boson-exchange constituent quark model. We compare the predictions for $\pi$ and $\eta$ decay widths with experiment and also with results from a traditional one-gluon-exchange constituent quark model. The differences between nonrelativistic and semirelativistic versions of the constituent quark models are outlined. We also discuss the sensitivity of the results on the parametrization of the meson wave function entering the $^3P_0$ model.

I. INTRODUCTION

The investigation of hadronic transitions of baryon resonances is currently of high interest [1]. On the experimental side, there are considerable efforts to measure these reactions in order to gain more and improved data on the resonance states. On the theoretical side, a quantitative description of the very details of the baryon ground and excited states represents a big challenge for all hadron models. Obviously the aim is to reach a comprehensive understanding of the low-energy hadron phenomenology on the basis of quantum chromodynamics (QCD).

A promising approach to low-energy hadrons consists in constituent quark models (CQMs). Starting from rudimentary attempts more than two decades ago, one has constantly improved the description and gained a lot of insight into the properties of hadrons at low and intermediate energies. Evidently, CQMs can at most be effective models of QCD in a domain where the fundamental theory is not (yet) accurately solvable. However, the concept of constituent quarks, in the beginning mainly motivated by symmetry considerations of hadron multiplets, nowadays gets more and more justified on the basis of QCD itself [2-4]. It appears that the spontaneous breaking of chiral symmetry (SB$\chi$S)
of QCD is responsible for the generation of constituent quarks as quasiparticles below a certain scale. Numerous evidences hint to a chirally broken phase (Nambu-Goldstone mode) of QCD.

Recently a chiral constituent quark model (CCQM) has been proposed that exploits the SBχS of QCD in deducing the hyperfine interaction of constituent quarks in light and strange baryons \[4\]. It relies on constituent-quark and Goldstone-boson fields as the relevant degrees of freedom in an effective interaction Lagrangian \[5\]. The so-called Goldstone-boson exchange (GBE) CQM introduces new symmetry properties into the hyperfine interaction of constituent quarks, which are rather different from traditional CQMs advocating one-gluon-exchange (OGE) dynamics \[6\]. The GBE CQM has turned out rather successful in producing an accurate description of the whole light and strange baryon phenomenology in a unified framework \[4\].

However, the reproduction of the baryon ground-state and resonance energies is just one item that has to be fulfilled by a successful hadron model. In addition, any CQM should also provide for a description of dynamical properties accessible through all types of reaction processes. Here we specifically study the performance of the GBE CQM in hadronic decays of \(N\) and \(\Delta\) resonances. Thereby we produce a further test of the reliability of the new kind of hyperfine interaction based on GBE.

We obtain three-quark wave functions for all the needed ground and excited baryon states by solving a differential Schrödinger-type equation with the stochastic variational method (SVM) \[7\]. These wave functions are then employed within a modified version of the \(^3P_0\) decay model \[8\] in order to calculate partial widths for \(\pi\) and \(\eta\) decays of \(N\) and \(\Delta\) resonance states up to \(\sim 1.8\) GeV. We compare the results to the experimental data and contrast them to an analogous study along a traditional version of the OGE CQM \[9\]. Our main aim is twofold: First we want to see how well the available data are reproduced by the GBE CQM, and second we wish to find possible differences between the two distinct types of CQMs.

In the following chapter we give a short description of the quark models used in the present study. We specify their parametrizations both in a nonrelativistic and a semirelativistic framework. In chapter 3 we explain the specific decay model we use here and give the pertinent formulae for the calculation of partial decay widths. The results are presented in chapter 4 along with a discussion of their sensitivity on different ingredients both in the CQMs and in the decay model. Our conclusions are given in chapter 5.

II. CONSTITUENT QUARK MODELS

Let us start by specifying the constituent quark models we use in the present study. The total three-quark Hamiltonian for baryons has the general form

\[
H = H_0 + V,
\]

where \(H_0\) is the kinetic-energy operator and \(V\) contains all the quark-quark interactions, i.e. confinement plus hyperfine potentials. For constituent quarks with effective masses of the order of a few hundred MeV the kinetic-energy operator
should be taken in relativistic form

\[ H_{0}^{SR} = \sum_{i=1}^{3} \sqrt{\vec{p}_i^2 + m_i^2}, \]  

(2)

with \( m_i \) the masses and \( \vec{p}_i \) the 3-momenta of the constituent quarks. A free Hamiltonian as in Eq. (2) leads to the so-called relativized or semirelativistic (SR) CQM [10]. It helps to avoid pathologies that usually appear in nonrelativistic constituent quark models [6]. We devote our attention primarily to the SR versions of the GBE and OGE CQMs as described below. Nevertheless, in order to provide a connection to previous studies of hadronic baryon decays, we consider also nonrelativistic (NR) versions of the two types of CQMs, which use the kinetic-energy operator in the form

\[ H_{0}^{NR} = \sum_{i=1}^{3} \left( m_i + \frac{\vec{p}_i^2}{2m_i} \right). \]  

(3)

A. GBE constituent quark model

For the CCQM relying on GBE dynamics we specifically adhere to the version published in Ref. [4]. It comes with a mutual quark-quark interaction

\[ V_{ij} = V_{\text{conf}} + V_{\chi}, \]  

(4)

with a confinement potential in linear form

\[ V_{\text{conf}}(r_{ij}) = V_0 + Cr_{ij} \]  

(5)

and the chiral interaction consisting of only the spin-spin part of the pseudoscalar meson exchange

\[ V_{\chi}(\vec{r}_{ij}) = \left[ \sum_{F=1}^{3} V_{\pi}(\vec{r}_{ij})\lambda_i^F \lambda_j^F + \sum_{F=4}^{7} V_{K}(\vec{r}_{ij})\lambda_i^F \lambda_j^F + V_{\eta}(\vec{r}_{ij})\lambda_i^8 \lambda_j^8 + \frac{2}{3} V_{\eta'}(\vec{r}_{ij}) \right] \vec{\sigma}_i \cdot \vec{\sigma}_j. \]  

(6)

Here \( \vec{\sigma}_i \) are the Pauli spin matrices and \( \lambda_i \) the Gell-Mann flavor matrices of the individual quarks. The meson-exchange potentials are parametrized in the form

\[ V_{\gamma}(\vec{r}_{ij}) = \frac{g_{\gamma}^2}{4\pi 12m_im_j} \left\{ \mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - \Lambda_\gamma^2 \frac{e^{-\Lambda_\gamma r_{ij}}}{r_{ij}} \right\}, \]  

(7)

with \( \mu_\gamma \) the meson masses, \( g_{\gamma} \) the meson-quark coupling constants, and \( \Lambda_\gamma \) the cut-off parameters resulting from the smearing of the \( \delta \)-functions (for details see Refs. [4] and also [1]). A single coupling constant \( g_0 \) is taken for all pseudoscalar octet mesons. In case of the SR GBE CQM it is set equal to the pion-quark coupling constant, whose value can be deduced from \( \pi N \) phenomenology via the Goldberger-Treiman relation. The coupling constant \( g_0 \) for the singlet \( \eta' \) is determined differently by a fit to the baryon spectra. The cut-offs \( \Lambda_\gamma \) scale with the phenomenological meson masses according to the rule
\[ \Lambda_\gamma = \Lambda_0 + \kappa \mu_\gamma. \] (8)

The strength and depth of the confinement potential (5) are determined by \( C \) and \( V_0 \), respectively. While these values have also been fitted to the baryon spectra, it is interesting to remark that for the SR GBE CQM the strength \( C \) comes out just in consistency with the QCD string tension. The parameter \( V_0 \) is needed merely to fix the ground-state level at the nucleon mass. All the parameter values are collected in Table I.

**TABLE I.** Parameters of the GBE CQM for the semirelativistic [4] and nonrelativistic [11] parametrizations.

| Parameters | SR    | NR    |
|------------|-------|-------|
| \( \frac{g^2}{4\pi} \) | 0.67  | 1.24  |
| \( (g_0/g_8)^2 \) | 1.34  | 2.23  |
| \( \Lambda_0 \) [fm\(^{-1}\)] | 2.87  | 5.82  |
| \( \kappa \) | 0.81  | 1.34  |
| \( C \) [fm\(^{-2}\)] | 2.33  | 0.77  |
| \( V_0 \) [MeV] | -416  | -112  |
| \( m_u = m_d \) [MeV] | 340   | 340   |
| \( \mu_\pi \) [MeV] | 139   | 139   |
| \( \mu_\eta \) [MeV] | 547   | 547   |
| \( \mu_\eta' \) [MeV] | 958   | 958   |

Table I also contains the parameters for a NR version of the GBE CQM [11], i.e. when the potential (5) is used together with the kinetic-energy operator (3). While the description of the \( N \) and \( \Delta \) is achieved with a similar quality (cf. Fig. III and Table IV), it is worthwhile to note the drastically different values of the fitted parameters (first 6 lines in Table I) in both the confinement and chiral interactions. In particular, the confinement potential becomes unrealistically weak, while the hyperfine potential gets much enhanced as compared to the SR case.

**B. OGE constituent quark model**

For the purpose of comparison to a different kind of quark-quark dynamics we employ a traditional OGE CQM. Specifically, it is the model following Bhaduri, Cohler, and Nogami (BCN) [9]. In this case the total potential has the form

\[ V_{ij} = V_0 + Cr_{ij} - \frac{2b}{3r_{ij}} + \frac{\alpha_s}{9m_i m_j} \Lambda^2 e^{-\Lambda r_{ij}} \vec{\sigma}_i \cdot \vec{\sigma}_j, \] (9)

i.e. it consists of a short-range Coulomb term, a linear confinement, and a flavor-independent spin-spin interaction. The parameter values for the original BCN potential were determined from a fit to the meson spectra, and they were
in baryon spectra, i.e. essentially confinement plus spin-spin hyperfine interactions. However, both the GBE and OGE CQMs also contain only the most important ingredients for the quark-quark interactions.

Before concluding this section a few remarks about the above versions of the GBE and OGE CQMs are in order. For both cases the model parameters used in a previous study [8] have been redetermined from a fit to the baryon spectra. Their values are summarized in Table II, from where it can be seen that they differ from the parameter set used in Ref. [9], specifically in the NR case.

TABLE II. Parameters of the OGE CQM after BCN [9] for the semirelativistic and nonrelativistic parametrizations.

| Parameters       | SR    | NR    |
|------------------|-------|-------|
| $b$              | 0.57  | 0.825 |
| $\alpha_s$      | 0.57  | 0.825 |
| $\Lambda$ [fm$^{-1}$] | 2.7   | 5     |
| $C$ [fm$^{-2}$]  | 3.12  | 2.26  |
| $V_0$ [MeV]      | -409  | -366  |
| $m_u = m_d$ [MeV] | 337   | 337   |

Again the spectra are produced in quite a similar manner by both the SR and NR versions (cf. Fig. 2 and Table II). Of course, the typical difficulties of OGE CQMs appear, e.g., with respect to the relative orderings of the lowest positive- and negative-parity excitations.

Before concluding this section a few remarks about the above versions of the GBE and OGE CQMs are in order. For both cases the models considered here contain only the most important ingredients for the quark-quark interactions in baryon spectra, i.e. essentially confinement plus spin-spin hyperfine interactions. However, both the GBE and OGE models bring about also further force components, such as central, tensor, and spin-orbit forces. While their
influence must be minor in the $N$ and $\Delta$ spectra (as demanded by their phenomenological structure) they could be of enhanced importance in dynamical observables such as hadronic widths, nucleon form factors, etc. In the study we present below we shall thus essentially explore the effects of the most prominent parts of the interquark forces. The GBE CQM has so far been published only with the spin-spin part of the quark-quark interaction \cite{7}, \cite{11}. For consistency in the comparison, also the OGE CQM is considered only with the spin-spin component.

TABLE III. Energies of baryon resonances predicted by the different CQMs considered in this work. For all models the nucleon mass is 939 MeV.

| $N^*$     | $J^\pi$ | GBE SR | GBE NR | OGE SR | OGE NR |
|-----------|---------|--------|--------|--------|--------|
| $N_{1440}$ | $1^+$   | 1459   | 1465   | 1578   | 1743   |
| $N_{1710}$ | $1^+$   | 1776   | 1712   | 1860   | 1925   |
| $\Delta_{1232}$ | $\frac{3}{2}^+$ | 1240   | 1232   | 1232   | 1232   |
| $\Delta_{1600}$ | $\frac{3}{2}^+$ | 1718   | 1585   | 1855   | 1967   |
| $N_{1520} - N_{1535}$ | $\frac{3}{2}^- - \frac{1}{2}^-$ | 1519   | 1529   | 1521   | 1531   |
| $N_{1650} - N_{1675} - N_{1700}$ | $\frac{1}{2}^- - \frac{5}{2}^- - \frac{7}{2}^-$ | 1647   | 1652   | 1691   | 1681   |
| $\Delta_{1620} - \Delta_{1700}$ | $\frac{3}{2}^- - \frac{5}{2}^-$ | 1642   | 1642   | 1621   | 1654   |
| $N_{1680} - N_{1720}$ | $\frac{5}{2}^+ - \frac{3}{2}^+$ | 1728   | 1679   | 1858   | 1883   |
III. THE $^3P_0$ MODEL FOR STRONG DECAYS

Investigations of hadronic decays have a long history with first attempts dating back to the early times of quark models. Still, the definite form of the decay operator is not yet known. Specific difficulties arising in strong interaction decays are connected with the extended sizes of both the baryons and the mesons involved in the decay process. Obviously one would require a reliable microscopic model that consistently accounts for the description of both the hadron states and the decay mechanism.

The simplest ansatz for the decay operator is furnished by the elementary emission model (EEM) \[13–15\]. Therein a pointlike meson is produced by a single constituent quark in the decaying baryon state. Evidently, this assumption leads to shortcomings, as found in a number of investigations with various CQMs (cf., for example, ref. \[16\]). A preliminary study of baryon decays for the relativistic GBE CQM along the EEM was performed in ref. \[17\].

An improved description of hadron decays is provided by the $^3P_0$ (or quark-pair creation) model. Here a $q\bar{q}$ pair is created from the vacuum and by a subsequent rearrangement the final meson and baryon states are produced. The $^3P_0$ model naturally allows to implement the extended structure of the emitted meson. By definition, the quark-antiquark pair must carry the quantum numbers of the vacuum, i.e. it is a color and flavor singlet, has positive $P$- and $C$-parity, total angular momentum $J = 0$ and carries total linear momentum zero. From $P = -(−1)^L$ and $C = (−1)^{L+S}$ one deduces as the simplest choice $L = S = 1$. The corresponding transition operator for the decay can thus be expressed as \[13\]

$$T = \gamma \sum_{i,j} \int d\vec{p}_q d\vec{p}_{\bar{q}} \delta(\vec{p}_q + \vec{p}_{\bar{q}}) \sum_m C_{1m1-m}^0 Y_1^m(\vec{p}_q - \vec{p}_{\bar{q}})(\chi_i^m(i,j)\phi_0(i,j)] b_i^\dagger(\vec{p}_q) d_j^\dagger(\vec{p}_{\bar{q}})$$

(10)

where, in evident notation, the momenta refer to the quark and antiquark states created by the operators $b_i^\dagger$ and $d_j^\dagger$, respectively. $Y_1^m(\vec{p}) = p^L Y_{1}^{m}(\hat{p})$ is a solid harmonics function, which gets coupled with the triplet spin wave function $\chi$ to give $J = 0$. $\phi_0$ is the flavor singlet wave function and the summation $\sum_{i,j}$ runs over spin and flavor indices. The pair-creation constant $\gamma$ is a dimensionless coefficient which is the only adjustable parameter of the model (apart from factors entering an eventual parametrization of the meson wave functions). Note that in Eq. (10) we have omitted a factor 3 in front of this constant which is frequently used to cancel a factor $1/3$ coming from the matrix element of color wave functions, which are not written out explicitly here.

The transition matrix element for the process $B \rightarrow B'M$ is then expressed as

$$\langle B'M \mid T \mid B \rangle \equiv \langle B'M \mid H \mid B \rangle = 3\gamma \sum_m C_{1m1-m}^0 \mathcal{I}_m = : \delta(\vec{P} - \vec{P}' - \vec{q}) A$$

(11)

Here, the factor 3 comes from the different possibilities of rearranging the quarks in the initial and final state, taking into account the symmetry of the wave functions. The momentum integral of Eq. (11) takes the form

$$\mathcal{I}_m = \int d\vec{p}_1 d\vec{p}_2 d\vec{p}_3 d\vec{p}_4 d\vec{p}_5 Y_1^m(\vec{p}_4 - \vec{p}_5) \delta(\vec{p}_4 + \vec{p}_5) \Phi_{pair}^{-m} \left[ \Psi_B(\vec{p}_1, \vec{p}_2, \vec{p}_4) \Phi_{B'} \right]^\dagger \left[ \Psi_M(\vec{p}_3, \vec{p}_5) \Phi_M \right]^\dagger \left[ \Psi_B(\vec{p}_1, \vec{p}_2, \vec{p}_3) \Phi_B \right].$$

(12)
Here, $\vec{p}_1, \vec{p}_2, \vec{p}_3$ are the individual quark momenta of the initial baryon $B$ which sum up to a total momentum $\vec{P} = \sum_{i=1}^{3} \vec{p}_i = 0$ in the rest frame of $B$. The meson carries away the momentum $\vec{q} = \vec{p}_3 + \vec{p}_5$, and the residual baryon $B'$ has momentum $\vec{P}' = \vec{p}_1 + \vec{p}_2 + \vec{p}_4 = -\vec{q}$, due to momentum conservation in the decay process. Finally, we denoted the combined spin-isospin wave functions involved in the decay process by $\Phi$.

In a next step, one separates the center-of-mass and relative motions in all hadron wave functions, what permits to carry out some of the integrations in Eq. (12):

$$I_m = \delta(\vec{P} - \vec{P}' - \vec{q}) \int d\vec{p}_x d\vec{p}_y Y_1^m(2\vec{q} + 2\vec{p}_y) \Phi^{\text{pair}} \left[ \Psi_B(\vec{p}_x, \vec{p}_y) \Phi_B^* \right] \left[ \Psi_M(-\frac{1}{2}\vec{q} - \vec{p}_y) \Phi_M^* \right] \left[ \Psi_M(-\vec{q} - \vec{p}_y) \Phi_M^* \right] \left[ \Psi_B(\vec{p}_x, \vec{p}_y) \Phi_B^* \right],$$

(13)

where $\vec{p}_x = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$ and $\vec{p}_y = \frac{1}{3}(2\vec{p}_3 - \vec{p}_1 - \vec{p}_2)$ are the momenta conjugate to the Jacobi coordinates $\vec{x}$ and $\vec{y}$.

In Ref. [8], it was observed that the $^3P_0$ model can be modified so as to reproduce the EEM in the limit of a pointlike meson. Taking also into account a relativistic boost effect, this requires the replacements

$$\gamma \longrightarrow \gamma \sqrt{\frac{\mu}{\omega}},$$

(14)

$$\mathcal{Y}_1^m(2\vec{q} + 2\vec{p}_y) \longrightarrow \mathcal{Y}_1^m \left( \frac{1}{2m} |\vec{q} + \frac{\omega}{m} \vec{p}_y | \right),$$

(15)

where $\mu$ is the mass of the emitted meson and $\omega = \sqrt{\mu^2 + q^2}$ its energy.

The partial decay width is then obtained by

$$\Gamma = \frac{1}{\pi} \frac{E \omega}{M_B} |\mathcal{A}|^2,$$

(16)

where $M_B$ is the mass of the decaying resonance, $E$ the energy of the final state baryon, and $\mathcal{A}$ is defined by Eq. (11). In Eq. (16) one still has to sum over final and to average over initial spin-isospin channels.

For the meson wave function in configuration space we first adopt a simple parametrization of the Gaussian type

$$\Psi_G(\vec{r}) = \frac{1}{(\pi R^2)^{3/4}} \exp \left( -\frac{r^2}{2R^2} \right),$$

(17)

where the parameter $R^2$ is related to the mean square radius of the meson by $\langle r^2 \rangle = \frac{3}{2} R^2$. While facilitating the calculations, this choice certainly cannot be regarded as a realistic representation of a meson wave function. We shall therefore investigate the influence of a different analytic form of meson wave functions on the baryon decay widths.

From the Fourier transform of the electromagnetic pion form factor, one can deduce a pion wave function that takes a Yukawa-like form:

$$\Psi_Y(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{m}{\sqrt{r}} \exp \left( -\frac{mr}{2} \right).$$

(18)

Here the parameter $m$ is related to the mean square radius of the meson by $\langle r^2 \rangle = 6/m^2$. Even if it is not physically meaningful, this expression may serve as a comparison to the Gaussian form.
A graphical representation of the meson wave functions is given in Fig. 3, where we compare the above forms to the wave function that follows from the original potential of Bhaduri et al. It can be seen that the exact wave function lies just between the extreme choices of a Yukawa and a Gaussian form. The parameters of Eqs. (17) and (18) have been fitted to give the same root mean square radius for the pion as the wave function from the potential of ref. [9], that is \( r_\pi = 0.565 \) fm. For simplicity we use the same parametrization for the wave function of the \( \eta \) meson.

**FIG. 3.** Meson wave functions in momentum (left) and configuration space (right). Gaussian and Yukawa forms are compared to the exact wave function following from the original quark-antiquark potential of Bhaduri et al. [9].

### IV. RESULTS FOR \( \pi \) AND \( \eta \) PARTIAL DECAY WIDTHS

In this section we shall present the results for the \( \pi \) and \( \eta \) decay modes of \( N \) and \( \Delta \) resonances, as predicted by the CQMs specified in Sec. 2. At the beginning we discuss some features of the baryon wave functions.

#### A. Three-quark wave functions

The solutions of the three-quark Hamiltonians have been obtained by solving the corresponding Schrödinger-type differential equations with the SVM [7]. The accuracy that is attained with respect to the eigenenergies in Table III is generally within a few percent even for the highest states considered. In this context the SVM was carefully counterchecked with complementary approaches, such as the Faddeev method [6,19]. Another measure for the accuracy of the solution of the three-quark problem is the mean square radius of the wave function. In the context of the present work this quantity is also useful for understanding some general characteristics of the results for decay widths, which are connected to the baryon sizes. In Table IV we therefore quote mean square radii of the \( N \) and \( \Delta \) ground state wave functions for the CQMs considered. The values refer to the case with pointlike constituent quarks. Therefore they are probably not realistic and must not be compared to experimental values. They are only useful to get insight into the relative extensions of the wave functions from each CQM.
TABLE IV. Mean square radii of $N$ and $\Delta$ from the various CQMs, assuming pointlike constituent quarks.

|               | GBE SR | GBE NR | OGE SR | OGE NR |
|---------------|--------|--------|--------|--------|
| $\langle r^2_N \rangle$ [fm$^2$] | 0.092  | 0.134  | 0.076  | 0.219  |
| $\langle r^2_\Delta \rangle$ [fm$^2$] | 0.152  | 0.172  | 0.115  | 0.288  |

Obviously, the values of the mean square radii are all rather small. Within each type of CQM, GBE or OGE, they are smaller in the semirelativistic cases, as it was already observed in ref. [10]. This may be viewed as a consequence of the stronger confinement generally needed in the semirelativistic CQMs. Inspection of the absolute magnitudes of the relevant quantities in Tables I, II, and IV shows, however, that confinement cannot be the only factor determining the mean square radii of the wave functions (note that the differences in confining strengths are much larger for the GBE parametrizations). A smaller extension of the baryon wave functions evidently implies even larger values for internal momenta (in the semirelativistic CQMs). This will help to explain certain results for decay widths involving high momenta in the next sections.

B. $\pi$ decays

The results for the partial widths of the $\pi$ decay modes of the $N$ and $\Delta$ resonances are shown in Table V. All values have been calculated with the Gaussian-type parametrization of the meson wave function of Eq. (17). For the baryons, the theoretical masses have been used as predicted by the different CQMs in Table III. In each case the strength parameter $\gamma$ introduced into the decay operator in Eq. (10) has been adjusted so as to reproduce the $\Delta_{1232} \rightarrow N\pi$ decay width. All the other decay widths can then be considered as genuine predictions of the CQMs along the modified $^3P_0$ model.

Table V also allows a comparison of the theoretical results to experimental data as compiled by the Particle Data Group (PDG) [12]. For the latter there arise two kinds of uncertainties: First, the total decay width of each resonance is given by a central value and a lower and upper bound. Second, the partial decay width has its own uncertainty. In Table V we quote the value for the $\pi$ decay widths deduced from the central value of the total width and first add the uncertainty from the partial decay width itself (numbers inside the parentheses in the last column). Then we indicate also the range of the total decay width by an upper and lower bound. We understand that the total uncertainty in a partial decay width must be estimated by combining both types of uncertainties (inherent separately in the total and partial widths).

Let us now examine the theoretical results in detail. For the $N_{1440}$ $\frac{1}{2}^+$ resonance the SR GBE prediction is obviously too large, whereas the pertinent NR result lies within the experimental error bars. The SR OGE result overshoots the experiment by far, its NR version is also much smaller than the SR one and lies just at the lower end of the
TABLE V. Decay widths of baryon resonances for the GBE and OGE constituent quark models both in nonrelativistic and semirelativistic parametrizations. A Gaussian-type meson wave function with $r_\pi = r_\eta = 0.565$ fm was used along with a modified $^3P_0$ decay model. Experimental data are from ref. [12]; for the quoted uncertainties refer to the text.

| $N^*$ | $J^P$ | $\Gamma(N^* \to N\pi)$ [MeV] | $\Gamma(N^* \to N\eta)$ [MeV] |
|-------|--------|-----------------------------|-----------------------------|
|       |        | Exp.                        | GBE SR | GBE NR | OGE SR | OGE NR | Exp.                        | GBE SR | GBE NR | OGE SR | OGE NR | Exp. |
| $N_{1440}$ | $^1_2^+$ | 517                         | 258   | 1064  | 161    |        | 207 ± 59        | 6      | 10   |
| $N_{1710}$ | $^1_2^+$ | 54                          | 14    | 202   | 8      |        | 15 ± 5          | 26     | 4    | 50   | 10   |
| $\Delta_{1232}$ | $^3_2^+$ | 120                         | 120   | 120   | 120    |        | 119 ± 1        |        |      |      |      |
| $\Delta_{1600}$ | $^3_2^+$ | 43                          | 34    | 174   | 14     |        | 61 ± 26        | 14     | 4    | 50   | 10   |
| $N_{1520}$ | $^3_2^-$ | 131                         | 161   | 108   | 168    |        | 66 ± 6        | 0      | 0    | 0    | 0    |
| $N_{1535}$ | $^3_2^-$ | 336                         | 75    | 462   | 109    |        | 67 ± 15       | 64     | 64   | 64   | 64   |
| $N_{1650}$ | $^1_2^-$ | 53                          | 5     | 87    | 8      |        | 109 ± 26      | 113    | 68   | 140  | 94   |
| $N_{1675}$ | $^3_2^-$ | 34                          | 35    | 40    | 52     |        | 68 ± 8        | 64     | 64   | 64   | 64   |
| $N_{1700}$ | $^3_2^-$ | 6                           | 6     | 7     | 9      |        | 10 ± 5        | 3      | 4    | 3    | 5    |
| $\Delta_{160}$ | $^3_2^-$ | 26                          | 3     | 41    | 5      |        | 38 ± 8        | 1      | 1    | 1    | 1    |
| $\Delta_{1700}$ | $^3_2^-$ | 28                          | 29    | 20    | 38     |        | 45 ± 15       | 4      | 2    | 6    | 6    |
| $N_{1680}$ | $^3_2^+$ | 85                          | 85    | 149   | 313    |        | 85 ± 7        | 0      | 1    | 2    | 6    |
| $N_{1720}$ | $^3_2^+$ | 377                         | 100   | 689   | 238    |        | 23 ± 8        | 15     | 11   | 30   | 25   |

For the next $\frac{1}{2}^+$ excitation of the nucleon, the $N_{1710}$, show a similar relative pattern as the ones for the Roper resonance, though all the values are smaller by about an order of magnitude. The fact that for each case, $N_{1440}$ and $N_{1710}$, the predictions of the SR parametrizations of both the OGE and GBE models exceed by far their NR counterparts can be readily understood observing the higher momentum components present in the SR parametrizations, as compared to the NR ones (cf. the discussion of the baryon wave functions in the previous Subsection). In case of the OGE SR this effect is enhanced by a phase space that is much too large (due to the bad prediction of the resonance energy).

For the $N_{1720} \frac{3}{2}^+$ resonance the results again have similar characteristics, with the SR cases drastically overshooting the experimental data. Here, however, none of the NR versions can come close to the rather small experimental width. This problem was already encountered in similar analyses [20,21] and may hint to a wrong symmetry assignment (or a strong mixing) of this state. Only for the $N_{1680} \frac{5}{2}^+$ resonance the GBE CQM produces correct results, both in its SR and NR versions. In this case the results from both variants of the OGE CQM are again too high.

For the negative-parity $N_{1535} \frac{1}{2}^-$ resonance the SR results are also much too high, whereas the predictions from experimental error bar. The results for the next $\frac{1}{2}^+$ excitation of the nucleon, the $N_{1710}$, show a similar relative pattern as the ones for the Roper resonance, though all the values are smaller by about an order of magnitude. The fact that for each case, $N_{1440}$ and $N_{1710}$, the predictions of the SR parametrizations of both the OGE and GBE models exceed by far their NR counterparts can be readily understood observing the higher momentum components present in the SR parametrizations, as compared to the NR ones (cf. the discussion of the baryon wave functions in the previous Subsection). In case of the OGE SR this effect is enhanced by a phase space that is much too large (due to the bad prediction of the resonance energy).

For the $N_{1720} \frac{3}{2}^+$ resonance the results again have similar characteristics, with the SR cases drastically overshooting the experimental data. Here, however, none of the NR versions can come close to the rather small experimental width. This problem was already encountered in similar analyses [20,21] and may hint to a wrong symmetry assignment (or a strong mixing) of this state. Only for the $N_{1680} \frac{5}{2}^+$ resonance the GBE CQM produces correct results, both in its SR and NR versions. In this case the results from both variants of the OGE CQM are again too high.

For the negative-parity $N_{1535} \frac{1}{2}^-$ resonance the SR results are also much too high, whereas the predictions from
the NR versions agree with experiment. For the $N_{1650} \frac{1}{2}^-$ the situation is just reversed. Most remarkably, in all instances the widths of the $N_{1535}$ resonance are larger than the ones of $N_{1650}$, contrary to experiment, where the $N_{1535}$ width appears to be smaller or is at most as large as the $N_{1650}$ width (taking into account the experimental uncertainties). Regarding the $L = 1, S = \frac{3}{2}$ multiplet $N_{1650} - N_{1675} - N_{1700}$, one notes that the SR parametrizations give approximately the correct ratios of these widths, as it is expected from the corresponding spin-isospin matrix elements. These features are not found for the NR parametrizations due to the exceedingly small value of the $N_{1650}$ width.

Concerning the negative-parity $N$ excitations, it is interesting to note that certain resonances are more sensible to the different parametrizations than others. Specifically, the S-wave resonances $N_{1535}$ and $N_{1650}$ (and likewise also $\Delta_{1620}$) appear to be 'structure-dependent', following the terminology of ref. [22]. This behaviour results in widths sometimes orders of magnitudes apart for different models. On the other hand, the D-wave resonances $N_{1520}$, $N_{1675}$, and $N_{1700}$ (and likewise also $\Delta_{1700}$) are found to be 'structure-independent'. Their decay widths are practically independent of the underlying spectroscopic model. These properties can be easily understood in the framework of the EEM (see ref. [18] for a thorough discussion), and evidently extend to the $^3P_0$ model, which is qualitatively very similar for orbital excitations.

The decay widths for the $\Delta$ resonances are practically all correct for the SR GBE CQM. In case of the other models the one or the other shortcoming appears.

C. $\eta$ decays

Table VI also gives the results for $\eta$ decays. Here we use the same spatial part for the meson wave function as for $\pi$ decays but the constant $\gamma$ is adjusted so as to reproduce the $\eta$ decay width of the $N_{1535}$ resonance. Note that this gives values for $\gamma$ about a factor 3 smaller than for the $\pi$ decays, in contrast to other works [20], where the same value was employed to describe both the $\pi$ and $\eta$ decays. This has several reasons, the most imminent one being the replacement according to Eq. (14). Furthermore, we use an unmixed flavor wave function for the $\eta$ meson, i.e. a pure flavor octet state. For non-strange decays as regarded in this work, a possible mixing would only influence the normalization of this wave function, which can effectively be absorbed into the coupling constant $\gamma$. Finally, an important contribution comes from our choice of phase space, as given by Eq. (16). We use a fully relativistic prescription and experimental values for the meson masses, in contrast to ref. [20], where a much higher, "effective" value for the pion mass was employed. A quick estimate of the magnitudes of these three effects shows indeed that we end up with about a factor of 3 difference in the constant $\gamma$ between $\pi$ and $\eta$ decays.

The $\eta$ widths of the Roper resonance $N_{1440}$ for the GBE parametrizations (NR as well as SR) are rigourously zero, since in both cases the theoretically predicted masses lie below the $\eta$ threshold, in accordance with experiment. For the OGE parametrizations, the decay $N_{1440} \rightarrow N\eta$ is possible, the corresponding widths remain rather small, however.
In total, there are four resonances predicted with considerable branching ratios in the $\eta$ decay channel. Only for the $N_{1535}$ and $N_{1650}$ resonances one can compare to experiment, since these are the only ones with an experimental width assigned by the PDG \cite{12}. The relative magnitudes of the experimental decay widths in both of these cases are missed by all theoretical models. This is again reminiscent of the EEM, where a similar effect is found. One may expect that the decays of these resonances are quite sensitive to spin-orbit and/or tensor forces in the quark-quark interaction. The inclusion of these force components would probably improve the description of both $N\pi$ and $N\eta$ decays for these resonances.

In addition to $N_{1535}$ and $N_{1650}$, also the widths of the $N_{1710}$ and the $N_{1720}$ resonances come out appreciably large. The PDG does not quote any experimental data for these states. This does not necessarily mean that their widths are vanishing or too small to be measured. It may simply be the case that experimental ambiguities do not (yet) allow for a reliable determination. In fact, there are single partial-wave analyses that assign an appreciable $\eta$ decay width, for example, also to the $N_{1710}$, see ref. \cite{23}.

D. Influences of the meson wave function

The modified $^3P_0$ decay model has two decisive ingredients: the pair-creation strength $\gamma$ and the parameter determining the extension of the meson wave function. While the former is merely a multiplicative constant, which may be suitably chosen to scale the overall strength of all decays, the latter is a nonlinear parameter, which may also alter the qualitative features of various predictions. In the following we consider certain different choices of the meson wave functions and examine their influences on the decay widths.

In Table VI we show results for decay widths when employing a Yukawa-like meson wave function, as given by Eq. (18), producing the same meson size as the Gaussian parametrization before. We have adjusted the parameter $\gamma$ again to fit the $\Delta$ and $N_{1535}$ widths for $N\pi$ and $\eta$ decays, respectively. However, as compared to Table V, the values change only little in this case.

By comparing the results in Tables V and VI it is immediately seen that the specific form of the meson wave function has only a minor influence on the predictions of the decay widths for the $\pi$ as well as $\eta$ decay modes. The qualitative features remain essentially unchanged. We have also performed calculations with the exact meson wave function produced by the potential of Bhaduri et al. (as shown in Fig. 3). They confirm the conclusion that the type of meson wave function is not decisive, provided its extension (meson radius) is kept the same.

We now focus the attention on the dependence of the results on the size of the meson. The meson wave functions employed in Tables V and VI both correspond to a radius of $r_\pi = 0.565$ fm. In the limit $r_\pi \to 0$ one expects to reproduce the results of the EEM. Thus it is interesting to look at an intermediate regime. Table VII gives the decay widths for the same case as in Table VI, but for a Gaussian-type wave function leading to a meson radius as small as 0.36 fm.
TABLE VI. Same as Table V but using a Yukawa-type meson wave function with \( r_s = r_\eta = 0.565 \) fm.

| \( N^* \) | \( J^\pi \) | \( \Gamma(N^* \to N\pi) \) [MeV] | \( \Gamma(N^* \to N\eta) \) [MeV] | \( \gamma \) |
|---|---|---|---|---|
| \( N_{1440} \) | \( \frac{1}{2}^+ \) | 528 | 363 | 1015 | 204 | (227 ± 18)\(^{+70}_{-59}\) | 6 | 15 |
| \( N_{1710} \) | \( \frac{1}{2}^+ \) | 59 | 10 | 179 | 7 | (15 ± 5)\(^{+5}_{-2}\) | 32 | 7 | 51 | 14 |
| \( \Delta_{1232} \) | \( \frac{3}{2}^+ \) | 120 | 120 | 120 | 120 | (119 ± 1)\(^{+5}_{-5}\) | 11 | (61 ± 26)\(^{+29}_{-10}\) |
| \( \Delta_{1660} \) | \( \frac{3}{2}^+ \) | 41 | 49 | 142 | 11 | (61 ± 26)\(^{+29}_{-10}\) |
| \( N_{1520} \) | \( \frac{3}{2}^- \) | 140 | 225 | 109 | 187 | (66 ± 6)\(^{+9}_{-5}\) | 0 | 1 | 0 | 1 |
| \( N_{1535} \) | \( \frac{1}{2}^- \) | 251 | 31 | 412 | 61 | (67 ± 15)\(^{+55}_{-17}\) | 64 | 64 | 64 | 64 |
| \( N_{1650} \) | \( \frac{1}{2}^- \) | 39 | 1 | 78 | 3 | (109 ± 26)\(^{+36}_{-3}\) | 110 | 59 | 138 | 86 |
| \( N_{1675} \) | \( \frac{1}{2}^- \) | 35 | 42 | 40 | 55 | (68 ± 8)\(^{+14}_{-4}\) | 3 | 6 | 3 | 7 |
| \( N_{1700} \) | \( \frac{1}{2}^- \) | 6 | 7 | 7 | 9 | (10 ± 5)\(^{+3}_{-3}\) | 1 | 1 | 1 | 1 |
| \( \Delta_{1620} \) | \( \frac{1}{2}^- \) | 20 | 1 | 39 | 2 | (38 ± 8)\(^{+8}_{-6}\) |
| \( \Delta_{1700} \) | \( \frac{3}{2}^- \) | 28 | 35 | 21 | 40 | (45 ± 15)\(^{+20}_{-10}\) |
| \( N_{1680} \) | \( \frac{3}{2}^+ \) | 98 | 144 | 158 | 379 | (85 ± 7)\(^{+6}_{-6}\) | 1 | 2 | 2 | 10 |
| \( N_{1720} \) | \( \frac{3}{2}^+ \) | 276 | 58 | 545 | 132 | (23 ± 8)\(^{+9}_{-5}\) | 14 | 11 | 25 | 22 |

First we note that the values for the constant \( \gamma \) obtained in this case are considerably larger than before. This is understandable, since in order to recover the results of the pointlike meson limit, one has to compensate for the effect of the \( \delta \) function, which then replaces the meson wave function. In particular, for the Gaussian form of Eq. (17) one has the relation

\[
(2\pi)^2 \delta(\vec{r}) = \lim_{R \to 0} \left( \frac{\pi}{R^2} \right) \frac{1}{4} \Psi_C(\vec{r}).
\] (19)

Most of the results for the decay widths are now rather different from before. They follow the general trend towards the predictions typical for the EEM. One of the characteristic results of the EEM is the extremely small decay width of the Roper resonance, as the first radial excitation of the nucleon; it is due to the orthogonality of the initial and final-state wave functions, which is strikingly felt in case of the EEM. The results of Table VII show the corresponding trend rather clearly: for all spectroscopic models the \( N_{1440} \) widths come out at least a factor of 2 smaller than before, while one is still rather far away from the pointlike limit.

Concerning the \( \eta \) decays one observes that the differences in the widths between the \( N_{1535} \) and \( N_{1650} \) resonances now increase in all cases. Again this follows the (unpleasant) trend towards the predictions typical for the EEM. As a result it appears favourable to use a decay model that permits the use of meson wave functions with finite extensions.
TABLE VII. Same as Table VI but using a Gaussian-type meson wave function with \( r_\pi = r_\eta = 0.36 \) fm.

| \( N^* \) | \( J^\pi \) | \( \Gamma(N^* \rightarrow N\pi) \) [MeV] | \( \Gamma(N^* \rightarrow N\eta) \) [MeV] | exp. |
|------|------|-----------------|-----------------|-----|
|      |      | GBE SR | GBE NR | OGE SR | OGE NR | GBE SR | GBE NR | OGE SR | OGE NR | exp. |
| \( N_{1440} \) | \( \frac{1}{2}^+ \) | 240 | 69 | 546 | 44 | (227 ± 18)\(^{+70}_{-59}\) | 2 | 4 |
| \( N_{1710} \) | \( \frac{1}{2}^+ \) | 6 | 13 | 63 | 26 | (15 ± 5)\(^{+30}_{-25}\) | 9 | 1 | 18 | 4 |
| \( \Delta_{1232} \) | \( \frac{3}{2}^+ \) | 120 | 120 | 120 | 120 | (119 ± 1)\(^{+5}_{-5}\) | | | | |
| \( \Delta_{1600} \) | \( \frac{3}{2}^+ \) | 0 | 2 | 24 | 63 | (61 ± 26)\(^{+20}_{-10}\) | | | | |
| \( N_{1520} \) | \( \frac{3}{2}^- \) | 89 | 88 | 81 | 137 | (66 ± 6)\(^{+9}_{-5}\) | 0 | 0 | 3 | 0 |
| \( N_{1535} \) | \( \frac{3}{2}^- \) | 584 | 106 | 953 | 195 | (67 ± 15)\(^{+55}_{-17}\) | 64 | 64 | 64 | 64 | (64 ± 19)\(^{+76}_{-15}\) |
| \( N_{1650} \) | \( \frac{1}{2}^- \) | 122 | 14 | 227 | 28 | (109 ± 26)\(^{+36}_{-33}\) | 128 | 80 | 156 | 109 | (10 ± 5)\(^{+4}_{-1}\) |
| \( N_{1675} \) | \( \frac{3}{2}^- \) | 26 | 22 | 32 | 46 | (68 ± 8)\(^{+14}_{-14}\) | 1 | 2 | 1 | 3 |
| \( N_{1700} \) | \( \frac{3}{2}^- \) | 4 | 4 | 5 | 8 | (10 ± 5)\(^{+3}_{-3}\) | 0 | 0 | 0 | 1 |
| \( \Delta_{1620} \) | \( \frac{1}{2}^- \) | 61 | 8 | 106 | 16 | (38 ± 8)\(^{+8}_{-6}\) | | | | |
| \( \Delta_{1700} \) | \( \frac{3}{2}^- \) | 21 | 18 | 17 | 34 | (45 ± 15)\(^{+20}_{-10}\) | | | | |
| \( N_{1680} \) | \( \frac{5}{2}^+ \) | 50 | 41 | 93 | 226 | (85 ± 7)\(^{+6}_{-6}\) | 0 | 0 | 1 | 3 |
| \( N_{1720} \) | \( \frac{3}{2}^+ \) | 489 | 85 | 1063 | 352 | (23 ± 8)\(^{+9}_{-5}\) | 12 | 8 | 24 | 23 |
| \( \gamma \) | | 20.575 | 20.695 | 22.699 | 17.997 | 6.844 | 10.060 | 6.430 | 6.619 |

V. SUMMARY AND CONCLUSION

In this work we investigated the theoretical description of \( \pi \) and \( \eta \) decays for \( N \) and \( \Delta \) resonances. In the first instance we were interested in the predictions of the specific chiral constituent-quark model whose hyperfine interaction is based on GBE dynamics \([4,11]\). A detailed comparison to the modern experimental data base \([12]\) is provided. We also studied the results relative to the predictions by a traditional CQM \([9]\) based on OGE but relying on the same type of force components as the GBE CQM. Furthermore we investigated the differences between a semirelativistic and a nonrelativistic description of the baryon states for both types of CQMs. For the decay mechanism a modified version of the \( ^3P_0 \) model \([8]\) was employed. We also examined the sensitivity of the results on the ingredients entering the decay operator, notably the analytical form and the extension of the meson wave functions.

From the present results it is still difficult to draw definite conclusions about the quality of the wave functions stemming from different CQMs. In fact, the various decay widths seem to be more determined by the choice of the SR or NR parametrizations rather than by the use of either type of dynamics, GBE or OGE. At this stage, we find a number of gross qualitative features that have been observed already before in similar studies along the classical \( ^3P_0 \) decay model.
It should be recalled that here we have not included spin-orbit or tensor forces into the quark-model Hamiltonians, especially because these force components are not yet provided by the published versions of the GBE CQM and we wanted to produce a consistent comparison with the other type of dynamics, namely the one resulting from OGE. Some decay widths are certainly sensitive to tensor and spin-orbit components in the wave functions. In this respect it may have been somewhat premature to make a comparison with experimental data at this stage.

In any case, our study reveals (and confirms previous such findings) that the description of strong decays of baryon resonances within present CQMs is not yet fully satisfactory. The reasons for the persisting difficulties may on the one hand reside in the baryon wave functions, which are probably not yet realistic enough. On the other hand one must realize that the $^3P_0$ decay model may also fall short as it is based on intuitive grounds and lacks a firm theoretical foundation. A consistent microscopic description of the strong-decay processes within the framework of CQMs thus remains a challenging task. One can think of a number of improvements to be done. For example, the proper inclusion of relativistic effects appears mandatory. The ultimate goal would, of course, be a unified description of the resonance spectra and the hadronic, as well as electromagnetic, transitions with the same dynamical scheme.

Acknowledgements The authors are indebted to Fl. Stancu, D. Rebreyend, and J.P. Bocquet for useful discussions. This work was supported by the Scientific-Technical Agreement 'Amadée’ between Austria and France under contract number II.9 and by the TMR contract ERB FMRX-CT96-0008.

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