ADAPTIVE REDUCED-RANK PROCESSING USING A PROJECTION OPERATOR BASED ON JOINT ITERATIVE OPTIMIZATION OF ADAPTIVE FILTERS FOR CDMA INTERFERENCE SUPPRESSION

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ABSTRACT

This paper proposes a novel adaptive reduced-rank filtering scheme based on the joint iterative optimization of adaptive filters. The proposed scheme consists of a joint iterative optimization of a bank of full-rank adaptive filters that constitutes the projection matrix and an adaptive reduced-rank filter that operates at the output of the bank of filters. We describe minimum mean-squared error (MMSE) expressions for the design of the projection matrix and the reduced-rank filter and simple least-mean squares (LMS) adaptive algorithms for its computationally efficient implementation. Simulation results for a CDMA interference suppression application reveal that the proposed scheme significantly outperforms the state-of-the-art reduced-rank schemes, while requiring a significantly lower computational complexity.

Index Terms—Adaptive filters, iterative methods.

1. INTRODUCTION

In the literature of adaptive filtering [1], the designer can find a huge number of algorithms with different trade-offs between performance and complexity. They range from the simple and low-complexity least-mean squares (LMS) algorithms to the fast converging though complex recursive least-squares (RLS) techniques. In the last decades, several attempts to provide cost-effective adaptive filters with fast convergence performance have been made through variable step-size algorithms, data-reusing, sub-band and frequency-domain adaptive filters and RLS type algorithms with linear complexity. A challenging problem that remains unsolved by conventional techniques is that when the number of elements in the filter is large, the algorithm requires a large number of samples (or data record) to reach its steady-state behavior. In these situations, even RLS algorithms require an amount of data proportional to \(2M\) [1], where \(M\) is the number of elements of the filter, in order to converge and this may lead to unacceptable convergence and tracking performance. Furthermore, in highly dynamic systems such as those found in wireless communications, large filters usually fail or provide poor performance in tracking signals embedded in interference and noise.

An alternative and effective technique in low sample support situations and in problems with large filters is reduced-rank parameter estimation [2]-[8]. The advantages of reduced-rank adaptive filters are their faster convergence speed and better tracking performance than existing full-rank techniques when dealing with large number of weights. Several reduced-rank methods and systems are based on principal components analysis, in which a computationally expensive eigen-decomposition is required [3]-[4] to extract the signal subspace. Other recent techniques such as the multistage Wiener filter (MWF) of Goldstein et al. in [5] perform orthogonal decompositions in order to compute its parameters, leading to very good performance at the cost of a relatively high complexity and the existence of numerical problems for implementation.

In this work we propose an adaptive reduced-rank filtering scheme that employs a projection matrix based on combinations of adaptive filters. The proposed scheme consists of a joint iterative optimization of a bank of full-rank adaptive filters that constitutes the projection matrix and an adaptive reduced-rank filter that operates at the output of the bank of full-rank filters. We describe MMSE expressions for the design of the projection matrix and the reduced-rank filter along with simple LMS adaptive algorithms for its computationally efficient implementation. We assess the performance of the proposed scheme via simulations for CDMA interference suppression.

The rest of this paper is organized as follows. Section 2 states the basic reduced-rank filtering problem. Section 3 presents the novel reduced-rank scheme, the joint iterative optimization approach and the MMSE design of the filters. Section 4 introduces LMS algorithms for implementing the new scheme. Section 5 presents and discusses the numerical simulation results, while Section 6 gives the conclusions.

2. REDUCED-RANK MMSE PARAMETER ESTIMATION AND PROBLEM STATEMENT

The MMSE filter is the parameter vector \(\mathbf{w} = [w_1 \ w_2 \ldots \ w_M]^T\), which is designed to minimize the MSE cost function

\[
J = E[|d(i) - \mathbf{w}^H \mathbf{r}(i)|^2],
\]

where \(d(i)\) is the desired signal, \(\mathbf{r}(i) = [r_0^{(i)} \ldots r_{M-1}^{(i)}]^T\) is the received data, \((\cdot)^T\) and \((\cdot)^H\) denote transpose and Hermitian transpose, respectively, and \(E[\cdot]\) stands for expected value. The set of parameters \(\mathbf{w}\) can be estimated via standard stochastic gradient or least-squares estimation techniques [1]. However, the laws that govern the convergence behavior of these estimation techniques imply that the convergence speed of these algorithms is proportional to \(M\), the number of elements in the estimator. Thus, large \(M\) implies slow convergence. A reduced-rank algorithm attempts to circumvent this limitation in terms of speed of convergence and tracking capabilities by reducing the number of adaptive coefficients and extracting the most important features of the processed data. This dimensionality reduction is accomplished by projecting the received vectors onto a lower dimensional subspace. Specifically, let us introduce an \(M \times D\) projection matrix \(\mathbf{S}_D\) that carries out a dimensionality reduction on the received data as given by

\[
\mathbf{f}(i) = \mathbf{S}_D^H \mathbf{r}(i),
\]
where, in what follows, all $D$-dimensional quantities are denoted with a "bar". The resulting projected received vector $\bar{r}(i)$ is the input to a tapped-delay line filter represented by the $D$ vector $\bar{w} = [\bar{w}_1 \bar{w}_2 \ldots \bar{w}_D]^T$ for time interval $i$. The filter output corresponding to the $i$th time instant is

$$x(i) = \bar{w}^H \bar{r}(i).$$

(3)

If we consider the MMSE design in (3) with the reduced-rank parameters we obtain

$$\bar{w} = \bar{R}^{-1} \bar{p},$$

(4)

where $\bar{R} = E[\bar{r}(i)\bar{r}^H(i)] = \bar{S}_D^H \bar{R} \bar{S}_D$ is the reduced-rank covariance matrix, $\bar{R} = E[\bar{r}(i)r^H(i)]$ is the full-rank covariance matrix, $\bar{p} = E[d^j(i)r(i)] = \bar{S}_D^H \bar{p}$ and $\bar{p} = E[d^j(i)r(i)]$. The associated MMSE for a rank $D$ estimator is expressed by

$$J = \sigma_d^2 - \bar{p}^H \bar{R}^{-1} \bar{p},$$

(5)

where $\sigma_d^2$ is the variance of $d(i)$. Based upon the problem statement above, the rationale for reduced-rank schemes can be simply put as MMSE for a rank $r$ vector. A scheme using a projection operator based on adaptive filters. The matrix formed by a bank of full-rank adaptive filters and a reduced-rank novel approach based on the joint optimization of a projection matrix formed by a bank of full-rank adaptive filters and a reduced-rank adaptive filter.

### 3. PROPOSED REDUCED-RANK SCHEME

In this section we detail the principles of the proposed reduced-rank scheme using a projection operator based on adaptive filters. The new scheme, depicted in Fig. 1, employs a projection matrix $\bar{S}_D(i)$ with dimension $M \times D$ to process a data vector with dimension $M \times 1$, that is responsible for the dimensionality reduction, and a reduced-rank filter $\bar{w}(i)$ with dimension $D \times 1$, which accomplishes the second stage of the estimation process over a reduced-rank data vector $\bar{r}(i)$ to produce a scalar estimate $x(i)$. The projection matrix $\bar{S}_D(i)$ and the reduced-rank filter $\bar{w}(i)$ are jointly optimized in the proposed scheme according to the MMSE criterion.

![Fig. 1. Proposed Reduced-Rank Scheme.](image)

Specifically, the projection matrix is structured as a bank of $D$ full-rank filters $\bar{s}_d(i) = [s_1(i) \, s_2(i) \, \ldots \, s_{M,d}(i)]^T$ ($d = 1, \ldots, D$) with dimension $M \times 1$ as given by $\bar{S}_D(i) = [\bar{s}_1(i) \, \bar{s}_2(i) \, \ldots \, \bar{s}_{D}(i)]^T$. Let us now mathematically express the output estimate $\hat{x}(i)$ of the reduced-rank scheme as a function of the received data $r(i)$, the projection matrix $\bar{S}_D(i)$ and the reduced-rank filter $\bar{w}(i)$ (we will drop index $(i)$ of the components for ease of presentation):

$$x(i) = (\bar{w}_1^\ast s_{1,1}^\ast r_1 + \bar{w}_1^\ast s_{2,1}^\ast r_2 + \ldots + \bar{w}_1^\ast s_{M,1}^\ast r_M) +$$

$$+ (\bar{w}_2^\ast s_{1,2}^\ast r_1 + \bar{w}_2^\ast s_{2,2}^\ast r_2 + \ldots + \bar{w}_2^\ast s_{M,2}^\ast r_M) +$$

$$\vdots

+ (\bar{w}_D^\ast s_{1,D}^\ast r_1 + \bar{w}_D^\ast s_{2,D}^\ast r_2 + \ldots + \bar{w}_D^\ast s_{M,D}^\ast r_M)$$

$$= \bar{w}^H(i)\bar{S}_D^H(i)r(i) = \bar{w}^H(i)\bar{r}(i).$$

(6)

The MMSE expressions for the filters $\bar{S}_D(i)$ and $\bar{w}(i)$ can be computed through the following optimization problem:

$$J = E[(d(i) - \bar{w}^H(i)\bar{S}_D^H(i)r(i))^2]$$

$$= E[(d(i) - \bar{w}^H(i)\bar{r}(i))^2].$$

(7)

By fixing the projection $\bar{S}_D(i)$ and minimizing (7) with respect to $\bar{w}(i)$, the reduced-rank filter weight vector becomes

$$\bar{w}(i) = \bar{R}^{-1}(i)p(i),$$

(8)

where $\bar{R}(i) = E[\bar{S}_D^H(i)r(i)\bar{r}(i)^\ast]$, $\bar{S}_D(i) = E[\bar{r}(i)r^H(i)]$, $\bar{p}(i) = E[d^j(i)r(i)^\ast]$. We proceed with the proposed joint optimization by fixing $\bar{w}(i)$ and minimizing (7) with respect to $\bar{S}_D(i)$. We then arrive at the following expression for the projection operator

$$\bar{S}_D(i) = \bar{R}^{-1}(i)p_D(i)\bar{R}_w(i),$$

(9)

where $\bar{R}(i) = E[\bar{r}(i)r^H(i)]$, $\bar{P}_D(i) = E[d^j(i)r(i)\bar{w}^H(i)]$ and $\bar{R}_w(i) = E[\bar{w}(i)\bar{w}^H(i)]$. The associated MMSE is

$$J_{\text{MMSE}} = \sigma_d^2 - \bar{p}^H(i)\bar{R}^{-1}(i)\bar{p}(i),$$

(10)

where $\sigma_d^2 = E[(d(i))^2]$. Note that the filter expressions in (8) and $\bar{S}_D(i)$ is not a closed-form solution for $\bar{w}(i)$ and $\bar{S}_D(i)$ since (8) is a function of $\bar{S}_D(i)$ and (9) depends on $\bar{w}(i)$ and thus it is necessary to iterate (8) and (9) with an initial guess to obtain a solution. The MWF [6] employs the operator $\bar{S}_D = [\bar{p} \, \bar{R}_p \ldots \, \bar{R}_D \bar{p}]$ that projects the data onto the Krylov subspace. Unlike the MWF approach, the new scheme provides an iterative exchange of information between the reduced-rank filter and the projection matrix and leads to a much simpler adaptive implementation than the MWF. The projection matrix reduces the dimension of the input data, whereas the reduced-rank filter attempts to estimate the desired signal. The key strategy lies in the joint optimization of the filters. The rank $D$ must be set by the designer in order to ensure appropriate performance. In the next section, we seek iterative solutions via adaptive LMS algorithms.

### 4. ADAPTIVE LMS IMPLEMENTATION OF THE PROPOSED REDUCED-RANK SCHEME

In this section we describe an adaptive implementation and detail the computational complexity in terms of arithmetic operations of the proposed reduced-rank scheme.

#### 4.1. Adaptive Algorithms

Let us consider the MSE cost function

$$J = E[(d(i) - \hat{x}(i))^2] = E[(d(i) - \bar{w}^H(i)\bar{S}_D^H(i)r(i))^2].$$

(11)

By computing the gradient terms of (11) with respect to $\bar{w}(i)$ and $\bar{S}_D(i)$, and using the instantaneous values of these gradients, one can devise jointly optimized LMS algorithms for parameter estimation.
Let us first describe the computation of the gradients of (11) with respect to $\mathbf{w}(i)$ and $\mathbf{S}_D(i)$:

$$
\nabla_{\mathbf{w}(i)} J = \frac{\partial J}{\partial \mathbf{w}^* (i)} = - (d(i) - \mathbf{w}^H(i) \mathbf{S}_D^H(i) \mathbf{r}(i))^* \mathbf{S}_D^H(i) \mathbf{r}(i) = - e^*(i) \mathbf{S}_D^H(i) \mathbf{r}(i) = - e^*(i) \mathbf{r}(i),
$$

(12)

$$
\nabla_{\mathbf{S}_D(i)} J = \frac{\partial J}{\partial \mathbf{S}_D(i)} = - (d(i) - \mathbf{w}^H(i) \mathbf{S}_D^H(i) \mathbf{r}(i))^* \mathbf{r}(i) \mathbf{w}^H(i) = - e^*(i) \mathbf{r}(i) \mathbf{w}^H(i).
$$

(13)

By using the gradient rules $\hat{\mathbf{w}}(i+1) = \mathbf{w}(i) - \mu \nabla_{\mathbf{w}(i)} J$ and $\hat{\mathbf{S}}_D(i+1) = \mathbf{S}_D(i) - \eta \nabla_{\mathbf{S}_D(i)} J$, where $\mu$ and $\eta$ are the step sizes, the proposed jointly optimized and iterative LMS algorithms for reduced-rank parameter estimation are

$$
\hat{\mathbf{w}}(i+1) = \mathbf{w}(i) + \mu e^*(i) \mathbf{r}(i),
$$

(14)

$$
\hat{\mathbf{S}}_D(i+1) = \mathbf{S}_D(i) + \eta e^*(i) \mathbf{r}(i) \mathbf{w}^H(i).
$$

(15)

The LMS algorithms described in (14)-(15) have a complexity $O(DM)$. In our studies, we verified a performance significantly superior to full-rank estimation algorithms and that there is no local minima in the optimization procedure. The proposed scheme and algorithms trade-off a full-rank LMS adaptive filter against $D$ full-rank adaptive filters as the projection matrix $\mathbf{S}_D(i)$ and one reduced-rank adaptive filter $\hat{\mathbf{w}}(i)$ operating simultaneously and exchanging information.

### 4.2. Computational Complexity

Here, we provide the computational complexity in terms of additions and multiplications of the proposed schemes with LMS algorithms and other existing algorithms, namely the Full-rank LMS and the LMS version of the MWF, as shown in Table 1. The MWF has a complexity $O(DM^2)$, where the variable dimension of the vectors $M = M - d$ varies according to the the rank $d = 1, \ldots, D$. The proposed scheme is much simpler than the MWF and slightly more complex than the Full-rank (for $D << M$, as will be explained later).

| Algorithm | Number of operations per symbol |
|-----------|---------------------------------|
| Full-rank | $2M$                           |
| Proposed  | $2DM + D$                      |
| MWF       | $D(2M^2 - 3M + 1)$             |

Table 1. Computational complexity of LMS algorithms.

5. SIMULATIONS

In this section we analyze the proposed reduced-rank scheme and algorithms in a linear CDMA interference suppression application. Note that non-linear techniques like clipping, quantization, and equalization are also possible. We consider the uplink of a symbol synchronous BPSK DS-CDMA system with $K$ users, $N$ chips per symbol and $L$ propagation paths. Assuming that the channel is constant during each symbol interval and the randomly generated spreading codes are repeated from symbol to symbol, the received signal after filtering by a chip-pulse matched filter and sampled at chip rate yields the $M$-dimensional received vector

$$
\mathbf{r}(i) = \sum_{k=1}^{K} \mathbf{H}_k(i) \mathbf{A}_k \mathbf{C}_k \mathbf{b}_k(i) + \mathbf{n}(i),
$$

(16)

where $M = N + L - 1$, $\mathbf{n}(i) = [n_1(i) \ldots n_M(i)]^T$ is the complex Gaussian noise vector with $E[\mathbf{n}(i) \mathbf{n}^T(i)] = \sigma^2 \mathbf{I}$, the symbol vector is $\mathbf{b}_k(i) = [b_{k,i+L_s-1} \ldots b_{k,i} \ldots b_{k,i-L_s+1}]^T$, the amplitude of user $k$ is $A_k$, $L_s$ is the intersymbol interference span, the $((2L_s-1).N) \times (2L_s-1)$ block diagonal matrix $\mathbf{C}_k$ is formed with $N$-chips shifted versions of the signature $\mathbf{s}_k = [s_k(1) \ldots s_k(N)]^T$ of user $k$ and the $M \times (2 \cdot L_s - 1) \cdot N$ convolution matrix $\mathbf{H}_k(i)$ is constructed with shifted versions of the $L \times 1$ channel vector $\mathbf{h}_k(i) = [h_{k,0}(i) \ldots h_{k,L_s-1}(i)]^T$ on each column and zeros elsewhere. For all simulations, we assume $L_s = 8$ as an upper bound, use 3-path channels with relative powers given by 0, $-3$ and $-6$ dB, where in each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips and average the experiments over 100 runs. The system has a power distribution amongst the users for each run that follows a log-normal distribution with associated standard deviation 1.5 dB.

We compare the proposed reduced-rank scheme with the full-rank and the MWF implementation for the design of linear receivers, where the reduced-rank filter $\mathbf{w}(i)$ with $D$ coefficients provides an estimate of the desired symbol for the desired used (user 1 in all experiments) as given by

$$
\hat{b}(i) = \text{sgn} \left( \Re(\mathbf{w}^H(i) \mathbf{r}(i)) \right) = \text{sgn} \left( \Re(\mathbf{x}(i)) \right)
$$

(17)

where $\Re(\cdot)$ selects the real part, $\text{sgn}(\cdot)$ is the signum function.

We first consider the tuning of the rank $D$ with optimized step sizes for all schemes, as shown in Fig. 2. The results indicate that the best rank for the proposed scheme is $D = 3$ (which will be used in the remaining experiments) and it is very close to the MMSE. Our studies with systems with different processing gains show that $D$ is invariant to the system size, which brings considerable computational savings.

![Fig. 2. MSE performance versus rank (D).](image-url)
with 500 symbols and then switch to decision-directed mode. The results show that the proposed scheme has a significantly better performance than the existing approaches and is able to adequately track the desired signal.

6. CONCLUSIONS

We proposed a novel MMSE reduced-rank scheme based on the joint and iterative optimization of a projection matrix and a reduced-rank filter and a low complexity adaptive implementation using LMS algorithms. The results for CDMA interference suppression show a performance significantly better than existing schemes and close to the optimum MMSE.

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