Impedance Modeling and Stability Analysis of MMC Flexible DC System at the DC Side

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Abstract. Aiming at the stability of modular multi-level converter (MMC) on the dc side, this paper adopts harmonic linearization method to establish impedance model of MMC dc side. This method ignores the higher-order components of voltage and current, and considers the influence of the control loops to guarantee high accuracy at the same time. The high-frequency band of impedance presents inductive characteristics, and the mid- and the low-frequency band is related to the control loop. The parameters of the current loop have a greater influence on the impedance characteristics of the mid-frequency band while the voltage loop parameters have a greater influence on the low-frequency band. For the back-to-back transmission system based on MMC, the voltage at the dc side has obvious oscillation when the impedance mismatch or the stability margin is small. The risk of dc side voltage instability can be eliminated by optimizing the controller parameters and phase angle margin can be improved.

1. Introduction
With the rapid development of power electronics technology, more and more renewable energy and dc equipment are interfaced with the power grid. The concept of the dc grid has increasingly become the preferred solution for power distribution and transmission. As the normal interconnection between ac grid and dc grid, modular multi-level converter (MMC) has the advantages of high power quality, easy expansion, and capability of fault ride-through, which can effectively increase the reliability and flexibility of the power grid[1].

Nowadays, with higher proportion of renewable energy in the power system, the connection between the renewable energy and ac grid may lead to the stability problem of oscillation such as the resonance problems of MMC-based high voltage dc transmission in German and China[2-3]. For this problem, the impedance-based method has been an effective way to assess the stability of the system and the impedance of MMC must be obtained first.

As the new generation of voltage source converter, the impedance modeling and stability analysis for modular multilevel converter (MMC) at the ac side is quite fruitful[4-5]. However, with more MMC-based projects including multi-terminal dc grid, DC distribution networks put into operation[6]. The dc-side grid connection of MMC also faces stability problems. Reference [7] built both AC and DC impedance models of MMC by analyzing its operation principle, but it did not consider closed-loop control. Reference [8] gave the small-signal dc impedance model of MMC with the consideration of
circulating current suppression control. But the impedance model with the full control loop is still needed. The MMC control strategy in reference [9] is different from the classical control strategy in DQ coordinate system and has no generalization. Reference [10] built the dc model of MMC also based on harmonic linearization method, but the stability of MMC interconnection needs further analysis. There are steady-state harmonics of multiple fundamental frequency in the bridge arm current, capacitor voltage during MMC operation, which not only couple with each other, but also couple with the perturbation, and the dynamic characteristics are quite complicated. The harmonic linearization method can accurately reflect the relationship between the harmonics, and the obtained frequency-domain variables can be easily converted between the dq rotating coordinate system and the abc three-phase stationary coordinate system[11].

For the stability analysis of MMC at the dc side, the dc impedance model is built in this paper by using harmonic linearization method. The model considers the harmonic coupling characteristics of electric quantities in MMC and effects of different control methods. The Time-domain and frequency-domain simulation results show that the dc impedance model is of great accuracy. For the unstable working condition of back-to-back MMC power transmission, the controller parameters are optimized based on the proposed model to eliminate the phenomenon of dc-side oscillation.

2. Steady-state harmonics analysis
The basic topology of MMC is shown as Fig.1, $v_{ga}$, $v_{gb}$, $v_{gc}$ are the AC voltage of MMC. $v_{dc}$ is the DC voltage of MMC. Every arm of MMC contains N series submodules(SM) and an arm inductor. The arm inductor is used for the restriction of the circulating current of the arm. The forward direction of current is shown in Fig.1. $i_{au}$ stands for the current of the upper arm of phase a and $i_{al}$ stands for the current of the lower arm of phase a.

Since the structure of MMC is of high symmetries among 6 arms in three phases. Analyzing the relationship of the electrical quantity of the upper bridge arm of phase a can deduce the electrical quantity equation of the remaining bridge arms.

As it is shown in Fig.1, the average model of the upper arm in phase a of MMC can be derived as:

$$L \frac{di_{au}}{dt} = \frac{v_{dc}}{2} - v_{ga} - m_{au}v_{au}$$  \hspace{1cm} (1)
\[ C \frac{dv_{ua}}{dt} = m_{ua} i_{ua} \]  

(2)

\( v_{ua} \) is the sum of capacitor voltages of all SMs of the upper arm in phase a. \( m_{ua} \) is the insertion index of the upper arm in phase a. L is the arm inductor and C is the equivalent capacitor of all SMs of the upper arm.

By using Fourier expansion of state variables in (1) and (2), the steady-state harmonics at the frequency of \(-kf_1\), \(-f_1\), 0, \(f_1\), \(k f_1\) can be obtained. \( f_1 \) is the basic frequency and \( k \) is a positive integer. Considering that the high-frequency harmonics proportion is really small, the harmonics above 3 basic frequency is ignored in this paper[4]. For simplicity, the subscript of phase a is omitted in the following equations. The frequency-domain average model of the upper arm in phase a can be derived as:

\[ Z_l i_u = v_{dc} / 2 - v_0 - v_u \otimes v_0 \]  

(3)

\[ Y_c v_u = m_u \otimes i_u \]  

(4)

The vectors in (3) and (4) are shown as (5):

\[ i = \begin{bmatrix} I_1 \\ I_2 \\ I_1 \\ I_2 \\ I_1 \end{bmatrix}, \quad v = \begin{bmatrix} V_1 \\ V_2 \\ V_1 \\ V_2 \\ V_1 \end{bmatrix}, \quad m = \begin{bmatrix} M_1 \\ M_2 \\ M_1 \\ M_2 \\ M_1 \end{bmatrix}, \quad v_{dc} / 2 = \begin{bmatrix} V_{dc} / 2 \\ V_{dc} / 2 \\ V_{dc} / 2 \\ V_{dc} / 2 \\ V_{dc} / 2 \end{bmatrix}, \quad v_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

(5)

The matrices of impedance and admittance in (3) and (4) can be derived as:

\[ Z_l = 2\pi L \cdot \text{diag} \left[ -3f_1, -2f_1, -f_1, f_1, 2f_1, 3f_1 \right] \]  

(6)

\[ Y_c = 2\pi C \cdot \text{diag} \left[ -3f_1, -2f_1, -f_1, f_1, 2f_1, 3f_1 \right] \]  

(7)

It can be seen that equations (3) and (4) contain vector convolution terms. To simplify the equations, the convolution of vectors can be transformed into the multiplication of matrices. Then equations (3) and (4) can be rewritten as the followings.

\[ Z_l i_u = v_{dc} / 2 - v_0 - V_u m_u \]  

(8)

\[ Y_c v_u = I_u m_u \]  

(9)

In (8) and (9), the voltage and current matrices of the upper arm can be derived as:

\[ V_u = \begin{bmatrix} V_0 & V_1 & V_2 & V_1 & V_0 & V_1 & V_2 & V_1 & V_0 & V_1 & V_2 & V_1 & V_0 \end{bmatrix} \]

(10)
At the steady working point of MMC, the dc and basic-frequency component of $i_u$, and dc component of $v_u$ are known. According to the characteristics of the Fourier transform, the positive and negative frequency corresponding quantities are conjugated. Removing the variables at frequency $3f_1$, $-2f_1$, $-f_1$, 0, $f_1$, the equation (8) can be simplified as:

$$\begin{bmatrix}
I_1 \\
I_3
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
\frac{j2\pi L \cdot 2f_1}{2} & 0
\end{bmatrix}
\begin{bmatrix}
\bar{M}_1 \\
\bar{M}_3
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & V_3 & V_2 & V_1 & v_0 & \bar{v}_1 \\
0 & 0 & 0 & V_3 & V_2 & V_1 & v_0
\end{bmatrix}
\begin{bmatrix}
M_0 \\
M_1 \\
M_2 \\
M_3
\end{bmatrix}$$

In the same way, the equation (9) can be simplified by removing the variables at frequency $3f_1$, $-2f_1$, $-f_1$, 0. Combining simplified (9) and (12), the following equation can be derived:

$$\begin{bmatrix}
I_2 \\
I_4 \\
V_i \\
V_i
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{j2\pi L \cdot 2f_1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\bar{M}_1 \\
\bar{M}_2 \\
\bar{M}_3
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & V_3 & V_2 & V_1 & v_0 & \bar{v}_1 \\
0 & 0 & 0 & V_3 & V_2 & V_1 & v_0
\end{bmatrix}
\begin{bmatrix}
M_0 \\
M_1 \\
M_2 \\
M_3
\end{bmatrix}$$

The steady-state arm current and capacitor voltage at different frequencies can be obtained by solving (13).

3. DC Impedance modeling

Assuming that there is a small-signal perturbation at the dc side of MMC, and its frequency is $f_p$. The interaction between perturbation and steady-state harmonics will produce the small-signal perturbation at frequency $f_p$-$kf_1$, $f_p$+$kf_1$, $f_p$+$f_1$, $f_p$+$2f_1$, $f_p$+$3f_1$. Considering the symmetrical structure of MMC, the response to dc perturbation between the three-phase is the same. Analyzed with small-signal model, equation (8) and (9) can be rewritten as:

$$Z_{\hat{i}_a} = \hat{v}_{dc} / 2 - \hat{v}_u \hat{m}_u - M_u \hat{v}_u$$

(14)
Then the controlled insertion $\hat{m}_u$ is analyzed in small-signal model. For the typical control strategy of dc voltage control, power control, and current control, the corresponding control diagrams are shown in Fig.2. The power control means controlling the current at the ac side, which is equivalent to ac current closed-loop control.

\[ Y \hat{v}_u = I_u \hat{m}_u + M_i \hat{u}_u \]  \hspace{1cm} (15)

The small-signal of insertion index $\hat{m}_u$ is affected by power control, dc voltage control, and circulating current control, which can be derived as:

\[ \hat{m}_u = Q \cdot \hat{i}_u + Q_c \cdot \hat{u}_u + P \cdot \hat{v}_{dc} \]  \hspace{1cm} (16)

In (15), $Q$ is the transfer function matrix of current-loop perturbation. $Q_c$ is the function matrix of circulating current control loop perturbation. $P$ is the function matrix of voltage-loop perturbation. When MMC works under power control mode, $P$ is a null matrix.

From Reference [4], it can be known that the steady-state harmonics satisfy the following rules:

\[ f = \begin{cases} 3k f_i & \text{zero sequence} \\ (3k + 1) f_i & \text{positive sequence} \\ (3k + 1) f_i & \text{negative sequence} \end{cases} \]  \hspace{1cm} (17)

\[ f = \begin{cases} 2k f_i & \text{common- mode(CM) harmonic} \\ (2k + 1) f_i & \text{differential- mode(DM) harmonic} \end{cases} \]  \hspace{1cm} (18)

Since the perturbation response is the same among upper arms and lower arms in three-phase, so the perturbation satisfies the same rules as steady-state harmonics, as it is shown in (19) and (20).

\[ f = \begin{cases} f_p + 3k f_i & \text{zero sequence} \\ f_p + (3k + 1) f_i & \text{positive sequence} \\ f_p + (3k + 1) f_i & \text{negative sequence} \end{cases} \]  \hspace{1cm} (19)

\[ f = \begin{cases} f_p + 2k f_i & \text{CM} \\ f_p + (2k + 1) f_i & \text{DM} \end{cases} \]  \hspace{1cm} (20)

As it is shown in Fig.2, the inner current loop controls the phase current of MMC at the ac side. The phase current of MMC is the difference between the upper and lower arm current. This means only DM components are needed consideration for small-signal perturbation analysis of current control. So the $Q$ can be derived as:
\[ Q = \text{diag}[0 \ 0 \ a \ 0 \ b \ 0 \ 0] \]
\[
\begin{align*}
a &= 2 \left( H_i \left( j 2 \pi f_p \right) + j K_i \right) / V_{dc} \\
b &= 2 \left( H_i \left( j 2 \pi f_p \right) - j K_i \right) / V_{dc}
\end{align*}
\]

(21)

The output voltage loop controls the dc voltage of MMC. The amplitude of \( f_p \) will reduce by half after \( dq \) inverse transformation, and the frequency will change to \( f_p - f_1 \) and \( f_p + f_1 \). The perturbation at frequency \( f_p - f_1 \) is negative sequence and the perturbation at frequency \( f_p + f_1 \) is positive sequence. So the matrix \( Q \) can be derived as:

\[ P(:,4) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]
\[
\begin{align*}
c &= H_i \left( j 2 \pi f_p \right) H_r \left( j 2 \pi f_p \right) / V_{dc} \\
d &= H_i \left( j 2 \pi f_p \right) H_r \left( j 2 \pi f_p \right) / V_{dc}
\end{align*}
\]

(22)

As it is shown in Fig.2, the circulating current loop controls the double frequency circulating current in MMC arms. So only the common-mode perturbation of arm current will contribute to the insertion index. Can be derived as:

\[ Q = \text{diag}[0 \ e \ 0 \ 0 \ f \ 0] \]
\[
\begin{align*}
e &= 2 \left( H_i \left( j 2 \pi f_p \right) + j K_i \right) / V_{dc} \\
f &= 2 \left( H_i \left( j 2 \pi f_p \right) - j K_i \right) / V_{dc}
\end{align*}
\]

(23)

Combining (14),(15),and (16). The dc impedance of power-controlled MMC can be derived as:

\[ Z_{dc} = \frac{\hat{v}_{dc}}{i_{dc}} = \frac{\hat{v}_{dc}}{3i_{dc}} \]
\[
= \frac{2 \left( Z_L + M_u Z_m M_u + (M_u Z_m I + V_u)(Q + Q_u) \right)}{3} \]

(24)

In the same way, the dc impedance of voltage-controlled MMC can be derived as:

\[ Z_{dc} = \frac{Z_L + M_u Z_m M_u + (M_u Z_m I + V_u)(Q + Q_u)}{1.5U - 3M_u Z_m I_u P - 3W_u P} \]

(25)

4. Simulation verification of dc impedance model

To validate the accuracy of the proposed dc impedance model, an average simulation model of MMC has been built in PLCS. The simulation parameters are shown in Table.1. A voltage source perturbation is set at the dc side to obtain the frequency-domain impedance responses.

**Table 1. MMC simulation parameters**

| Parameter       | Value  |
|-----------------|--------|
| DC Voltage      | 20 kV  |
| AC Voltage      | 10 kV  |
| Active Power    | 10 MW  |
| Arm Inductor    | 8 mH   |
| Equivalent Capacitance | 0.4 mF |

The theoretical and simulation DC impedance of MMC under power control and voltage control are shown in Fig.3(a) and Fig.3(b). Responses predicted by the small-signal models are plotted as continuous lines, while responses obtained from circuit simulation are presented by circles at discrete frequency points. As it is shown in Fig.3, the theoretical impedance is in great agreement with the simulation results. At frequency above 200Hz, the dc impedance of MMC presents inductor
At frequency below 20Hz, the impedance is mainly affected by the outer voltage-loop. Due to the complex impedance characteristics, it is necessary to explore the influence of different control parameters on the DC impedance of MMC and the stability of multi-MMC interconnection.

5. Effects of controller parameters and stability analysis of cascaded system

For the MMC put into use in engineering projects, the electrical parameters and topology structure are relatively fixed, so the effects of the controller parameters and control methods are discussed in this paper. According to the dc impedance model in (22) and (23), the impedance change trend can be obtained by changing the controller parameters, which are shown in Fig.4 and Fig.5. From Fig.4(a) and Fig.5(a), it can be known that changing Kp of the controller has little effect on impedance characteristics, and impedance is only changed in the middle-frequency band. From Fig.4(b) and Fig.5(b), it can be known that changing Ki of the controller has a great influence on the impedance in low and middle frequency, which is an important way of impedance optimization.

Figure 3. DC impedance of MMC under different control methods

Figure 4. DC impedance of power-controlled MMC with different current controller parameters
The stability of cascaded converters in dc system can also be assessed by the impedance-based approach. Fig.6 shows the small-signal analysis diagram of a back-to-back MMC power transmission system. The voltage-controlled MMC can be equivalent to the series connection of voltage source $V_v$ and output dc impedance $Z_v$. The power-controlled MMC can be equivalent to its DC input impedance $Z_p$. $V_l$ is the voltage of the dc line.

**Figure 5.** DC impedance of dc voltage-controlled MMC with different voltage controller parameters

**Figure 6.** Small-signal diagram of back-to-back MMC power transmission system

From Fig.6, the dc voltage $V_l$ is:

$$V_l(s) = V_v(s) \cdot \frac{Z_v(s)}{Z_v(s) + Z_I(s)}$$

$$= V_v(s) \cdot \frac{1}{1 + \frac{Z_v(s)}{Z_I(s)}} = V_v(s) \cdot \frac{1}{1 + T_m(s)}$$

By using the Middlebrook Criterion\cite{12-13} for the minor loop gain $T_m(s)$, the phase difference at the frequency of impedance overlapping should be less than 180 degrees. The greater the phase margin is, the more stable the system will be. Fig.7(a) shows the dc impedance of 2 connected MMC. The current controller parameter of the power-controlled MMC is $(2 + 4000/s)$, and the voltage controller parameter of the voltage-controlled MMC is $(0.5 + 300/s)$. It can be seen that the impedance gain of two MMC overlaps at 53Hz, and the phase difference is 191 degrees, which doesn’t satisfy the requirements of the Nyquist criterion. To improve the phase margin, the current controller and voltage controller parameters are optimized to $(3 + 4000/s)$ and $(0.5 + 30/s)$ respectively, and the corresponding dc impedance of the two MMC is shown in Fig.7(b). To validate the stability analysis above, the time-domain simulation has been made and shown in Fig.9. The dc voltage waveform is shown in Fig.8(a), the controller parameters were changed for improvement at 2s, and the oscillation on the dc line
diminished. The Fourier analysis of dc voltage before and after parameters improvement is shown in Fig.8(b) and Fig.8(c) respectively. It can be seen that before improving the controller parameters, the dc voltage oscillates at 53Hz. After the improvement, the oscillation is cleared. The simulation results in Fig.8 are in great agreement with the impedance analysis in Fig.7.

![Figure 7. DC impedance of connected MMC](image)

![Figure 8. Time-domain simulations of back-to-back MMC power transmission system](image)

6. Conclusion
To analyze the dc-side stability of MMC power transmission system, a dc impedance small-signal model of MMC is proposed. The impedance characteristics and stability analysis are carried out under back-to-back power condition. The conclusions are as follows:

Considering the steady-state harmonics of MMC and different control methods, the dc impedance of MMC is established. A voltage source perturbation is added at the dc side of MMC to validate the correctness of the impedance model.

DC impedance of MMC has different characteristics at different frequency bands. It shows inductor characteristics at the high-frequency band and its impedance characteristics at low frequency is affected greatly by the control methods and controller parameters. The current controller has a great influence on the impedance at middle frequency and the voltage controller has a great influence on the impedance at low frequency.

Under back-to-back power transmission, the dc voltage of MMC may have the risk of instability due to the mismatch of two MMC. Through the improvement of controller parameters, the phase margin can be raised and the stability can be enhanced.

7. Reference
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