The Column Density Distribution of the Lyman-Alpha Forest: A Measure of Small Scale Power

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Absorption lines in the Lyα forest can be thought of as peaks in neutral hydrogen density along lines of sight. The column density distribution (the number density of absorption lines as a function of column density) is then a statistic of density peaks, which contains information about the underlying power spectrum. In particular, we show that the slope of the distribution provides a measure of power on scales smaller than those probed by studies of present-day large scale structure.

Two examples of power spectra are shown in Fig. 1a. Plotted are $\sigma_0(k_S)$ versus $k_S$, where $\sigma_0(k_S)$ can be regarded as the amount of power on scale $k_S$ for the power spectrum $P(k)$, linearly extrapolated to the redshift of interest. The CHDM model, because of neutrino free streaming, has less power than the CDM model on the scales shown. What kind of column density distribution would each predict?

Several steps are involved in the computation. We briefly mention a few important ones and refer the reader to Hui, Gnedin & Zhang¹ for details.

Let us denote the local gas overdensity by $\delta_b$. The neutral hydrogen density $n_{HI}$ is then proportional to $(1+\delta_b)^2T^{-0.7}J_{HI}^{-1}$ assuming ionization equilibrium, where $T$ is the temperature and $J_{HI}$ is the intensity of the radiation background at the hydrogen ionizing frequency. Using the approximate relation $T = T_0(1+\delta_b)^\alpha$ where $T_0$ is the temperature at mean gas density and $\alpha$ is some prescribed power law index (see Hui & Gnedin¹), one deduces that $n_{HI}$ is proportional to $\Omega_b^2T_0^{-0.7}J_{HI}^{-1}(1+\delta_b)^{2-0.7\alpha}$.

Now, consider a density peak along a line of sight, its neutral hydrogen column density is given by:

$$ N_{HI} = \int_{peak} n_{HI} dr \propto \frac{\Omega_b^2}{T_0^{0.7}J_{HI}}(1+\delta_b)^{2-0.7\alpha} \left[ -d^2\ln(1+\delta_b)/dr^2 \right]^{-0.5}, $$

where $r$ is the proper distance along the line of sight. We have Taylor expanded the density field around the peak (i.e. rewriting $1+\delta_b$ as $e^\xi$ and expanding $\xi$ to second order) so that the integrand $n_{HI}$ becomes a Gaussian.

Two important conclusions follow from this simple expression. First, for given $\delta_b$ and its second derivative, changing the combination of cosmological

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parameters $\Omega_\Omega^2 T_0^{-0.7} J_{HI}^{-1}$, which is uncertain by a factor of about 10 observationally, has the effect of rescaling $N_{HI}$. This means that for a column density distribution approximated by a power law (see Fig. 1b), the 'left-right' placement of the distribution is uncertain by an order of magnitude; in other words, at a fixed column density, this means a rescaling in the normalization of the distribution. Because of this it is hard to use the normalization of the column density distribution to learn something about the power spectrum.

Second, the column density depends on $1 + \delta_b$ and its second derivative. This means that if one knows the probability distribution of $1 + \delta_b$ and its first and second derivatives (with the first derivative vanishing) at any given point, one can calculate the column density distribution. To do so, one would need a way of evolving the density field $\delta_b$. We use the Zel’’dovich approximation as an efficient method which is sufficiently accurate in the mildly nonlinear regime $\delta_b < \sim 5$, corresponding to $N_{HI} < \sim 10^{14.5}$cm$^{-2}$ at $z = 3$ for typical cosmological parameters. An appropriate smoothing scale has to be chosen when using the approximation, to take into account orbit-crossing and also smoothing due to gas pressure on small scales. Without going into details, it turns out for most interesting cosmological models, the scale lies in the range $1 \text{Mpc}^{-1} < k < 10 \text{Mpc}^{-1}$ where $k$ is the comoving wave number. (Note this is where $\sigma_0 \sim 1$ in Fig. 1a). This is relatively small scale compared to scales familiar in studies of present-day large scale structure ($0.01 \text{Mpc}^{-1} < k < 1 \text{Mpc}^{-1}$).

Results of computations using the Zel’’dovich approximation are shown in Fig. 1b. A comparison is made with the results from a full hydrodynamic simulation and the agreement is encouraging. We note that the CHDM model produces a steeper distribution compared to the CDM model. This can be understood qualitatively by recalling that the CHDM model has less power than the CDM model on the relevant scales (Fig. 1a). This means that by $z = 3$, the CHDM density field is relatively less nonlinear compared with the CDM one i.e. there are proportionally fewer high density peaks compared to intermediate density ones, hence the steeper column density distribution of the CHDM model.

There are also other subtleties such as the effects of peculiar velocities, the definition of peaks, etc for which the reader is referred to Hui, et al. \footnote{There are also other subtleties such as the effects of peculiar velocities, the definition of peaks, etc for which the reader is referred to Hui, et al.}

Note also that, if one goes to sufficiently low density, because the CHDM has not developed very underdense regions, there would also be proportionally fewer very low density peaks compared to intermediate ones, but the column density range we show is not low enough to unambiguously see this effect. \footnote{Note also that, if one goes to sufficiently low density, because the CHDM has not developed very underdense regions, there would also be proportionally fewer very low density peaks compared to intermediate ones, but the column density range we show is not low enough to unambiguously see this effect.}

The above argument assumes that high column density also means high density $1 + \delta_b$ (or high peak height) and vice versa. From Eq. \footnote{The above argument assumes that high column density also means high density $1 + \delta_b$ (or high peak height) and vice versa. From Eq.} it is clear column density also depends on the peak width (the term involving second derivative of density). However, peak width and height are correlated and the correlation works out so that column density still increases with peak height.
Figure 1: 1a on the left shows $\sigma_0$ versus $k_S$ as defined in the text, for two power spectra at $z = 3$: solid line is a CDM (Cold Dark Matter) model with $\sigma_8 = 0.7$ and dashed line is a COBE-normalized CHDM (Cold+Hot Dark Matter) model with $\Omega_\nu = 0.2$, a tilt of $n = 0.9$ and no tensor modes. Both have $H_0 = 50$ kms$^{-1}$Mpc$^{-1}$ and $\Omega = 1$. 1b on the right shows the resulting column density distributions (number of absorption lines $N_{\text{line}}$ per unit column density $N_{\text{HI}}$ per unit redshift $z$ versus column density), open triangles for CDM and open squares for CHDM, computed using the Zel’doovich approximation. Solid triangles represent the same CDM model but computed using a full hydrodynamic simulation. Two different values of $\Omega_\nu^{2/3}T_0^{-0.7}J_{\text{HI}}^{-1}$ are chosen for the CDM and CHDM models respectively, which shifts the normalization of the points but not the slope, to clearly show the difference in their predicted slopes. See Hui, et al. 1 for details of the computations.

To conclude, while the normalization of the column density distribution provides only a weak constraint on the power spectrum due to large observational uncertainties in $\Omega_\nu^{2/3}T_0^{-0.7}J_{\text{HI}}^{-1}$, its slope can provide useful information on the amount of power on scales complimentary to those of other traditional cosmological studies.

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References

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