Improved Geometrical Scaling at the LHC

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We show that geometrical scaling exhibited by the $p_T$ spectra measured by the CMS collaboration at the LHC is substantially improved if the exponent $\lambda$ of the saturation scale depends on $p_T$. This dependence is shown to be the same as the dependence of small $x$ exponent of $F_2$ structure function in deep inelastic scattering taken at the scale $p_T \simeq Q/2$.

Recently in Refs.\cite{1, 2} it has been shown that $p_T$ spectra measured by the CMS collaboration \cite{3} at the LHC exhibit geometrical scaling. Geometrical scaling was first introduced in the context of Golec-Biernat–Wüsthoff (GBW) model \cite{4} of deep inelastic scattering (DIS) in Ref.\cite{5}. There, a reduced $\gamma^*\text{W}$ model \cite{4} of deep inelastic scattering was first introduced in the context of Golec-Biernat–Wüsthoff (GBW) model \cite{4} of deep inelastic scattering (DIS) in Ref.\cite{5}. Geometrical scaling spectra measured by the CMS collaboration \cite{3} at the LHC exhibit geometrical scaling, i.e. they fall on one energy-independent universal curve $F(\tau)$

$$dN_{\text{ch}} / dy dp_T^2 = \frac{1}{Q_0^2} F(\tau)$$

if plotted in terms of the scaling variable

$$\tau = p_T^2 / Q_{\text{sat}}^2$$

with $\lambda \sim 0.27$. In essence, geometrical scaling for the $p_T$ spectra \cite{5} boils down to the prescription that allows to relate multiplicity distributions at two different energies $W$ and $W'$. If

$$dN_{\text{ch}} / dy dp_T^2 (p_T, W) = \frac{dN_{\text{ch}}}{dy dp_T^2} (p_T', W')$$

then the transverse momenta at which Eq.\cite{7} holds, satisfy

$$p_T' = p_T \left( \frac{W'}{W} \right)^{\lambda/(\lambda+2)}.$$\hspace{1cm}(8)

This formula is independent of $Q_0$ and of the overall energy scale of $W$ or $W'$. So the only relevant parameter of geometrical scaling is exponent $\lambda$.

Equations \cite{7} and \cite{8} allow to rescale $p_T$ of known spectrum at energy $W$ to another energy $W'$, and predict $p_T$ spectrum at this energy, provided we know the value of $\lambda$. In the following, transverse momentum spectra obtained that way will be referred to as rescaled spectra.

Alternatively, if we do not know $\lambda$ but we know spectrum at $W$, we can find $\lambda$ by changing its value until equality \cite{7} is satisfied, i.e. until the rescaled and true spectra coincide within errors.

In dipole models of DIS the quality of phenomenological fits is further increased provided one incorporates DGLAP $Q^2$ dependence \cite{10} of the saturation scale \cite{2}. Furthermore, since one "measures" $Q_{\text{sat}}$ with a $Q^2$ dependent probe (e.g. with virtual photon or a $p_T$ hadron), effective saturation scale $Q_{\text{sat, eff}}$ acquires some dependence on the virtuality of the probe. These two effects may be conveniently accounted for by replacing $\lambda \rightarrow \lambda(Q^2)$. Indeed, for large $Q^2$ DIS structure function $F_2$ that is directly related to the saturation scale \cite{4} behaves as:

$$F_2(x, Q^2) \sim \sigma_0 Q_{\text{sat, eff}}^2 \sim \frac{1}{x^{\lambda_{\text{sat}}(Q)}}.$$\hspace{1cm}(9)
Power $\lambda_{\text{eff}}(Q)$ has been extracted from the HERA data \cite{[1]} (see e.g. recent Ref.\cite{[12]} and references therein) as shown in Fig. 1. In the same figure we plot an eyeball fit to the experimental points given by a simple function

$$
\lambda_{\text{eff}}(Q) = 0.13 + 0.1 \left( \frac{Q^2}{10} \right)^{0.35}.
$$

An interesting question arises, whether exponent $\lambda$ that governs geometrical scaling in hadronic collisions exhibits any $p_T$ dependence and, if yes, whether it is similar to the one obtained in DIS. For $p_T$-dependent $\lambda$ formula \cite{[8]} takes the following form:

$$
p_T^2 \left( \frac{p_T}{W'} \right)^{\lambda(p_T)} = p'_T^2 \left( \frac{p_T}{W} \right)^{\lambda(p'_T)}.
$$

A simple way to calculate approximate $p_T$ dependence of $\lambda$ is to use Eq.\cite{[8]} bin by bin in $p_T$ instead of an exact equation \cite{[11]}. For slowly varying $\lambda(p_T)$ such a procedure should give a good first order approximation. To this end we choose to rescale transverse momenta of CMS multiplicity spectra at $W = 0.9$ and 7 TeV to the reference energy $W' = 2.36$ TeV for some initial value of $\lambda$. Next, we compare the rescaled spectra with the experimental data at $W'$ and repeat the whole procedure until the rescaled and true spectra coincide \cite{[7]}. In that way we obtain $\lambda(p_T)$. Since in general for $W'$ there is no data point at $p'_T$ obtained from \cite{[8]}, we have to interpolate the reference spectrum and its errors (in the following we neglect interpolation errors). The result of this interpolation is depicted in Fig. 3 by a grey band. In order to estimate the error of $\lambda(p_T)$ we add in quadrature errors of the $W$ spectrum and the interpolated error of the reference $W'$. Numerical care is needed before quantitative conclusions concerning small $p_T$ part can be drawn. One should also stress at this point that further analysis of low $p_T$ data become too noisy to draw definite conclusions.

Final conclusion that has to be drawn from Fig. 2 is that geometrical scaling with constant $\lambda$ is certainly a good first approximation, but a mild $p_T$ dependence of $\lambda$ improves substantially the quality of geometrical scaling.

![Fig. 1. Dependence of $\lambda_{\text{eff}}$ on $Q^2$ from HERA (HERA data points H1 after Ref.\cite{[12]}).](image1)

![Fig. 2. Dependence of $\lambda$ on $p_T$. Open circles correspond to $\lambda$ obtained by rescaling 0.9 TeV data to the reference energy of 2.36 TeV, whereas open triangles correspond to 7 TeV. Exponent $\lambda_{\text{eff}}$ extracted from HERA depicted by full pentagons (H1) and stars (Zeus) is plotted in function of $p_T = Q/2$ (see text). Solid line corresponds to Eq.\cite{[10]} taken at $Q = 2p_T$.](image2)
This is depicted in Fig. 3 where we plot the $p_T$ spectra in terms of the rescaled momentum $p'_T$ in the vicinity of 1 GeV where the difference between constant $\lambda = 0.27$ and "running" $\lambda$ of Eq. (10) is most pronounced. Black points and the shaded band correspond to the CMS spectrum (and its interpolation) at 2.36 TeV. Blue and red points (connected by dashed lines) correspond to 0.9 and 7 TeV spectra respectively, rescaled to the reference energy of 2.36 TeV for constant $\lambda$ and "running" $\lambda_{\text{eff}}(2p_T)$ of Eq. (10). An improvement for "running" $\lambda$ is evident. For other $p_T$ intervals where the difference between constant and "running" $\lambda$ is not large, the quality of geometrical scaling is – obviously – comparable.

Geometrical scaling in hadronic collisions is by far less obvious than in DIS. In DIS we have at our disposal simple theoretical (GBW) model [1] that allows to identify kinematical variables relevant for geometrical scaling. In hadronic collisions such models exist [13–17] but they rely on $k_T$ factorization which has not been proven for soft particle production in central rapidity. Nevertheless, if $k_T$ factorization is assumed, like in the recent studies of Refs. [16, 17], then the proportionality of multiplicity of produced gluons to the saturation momentum, and therefore geometrical scaling – assuming local parton-hadron duality – can be derived in a rather straightforward way (see e.g. [13]). Nevertheless, the exact form of the the scaling variable $\tau$, that in principle may depend also on rapidity, is to some extent a matter of educated guess. Luckily, for constant $\lambda$ some uncertainties cancel out in Eq. (5), showing that the only relevant parameter is exponent $\lambda$.

Another notable difference between DIS and hadronic collisions is that in DIS we deal with totally inclusive cross-section, whereas in pp both hadronization and final state interactions play essential role. Nevertheless the imprint of the saturation scale $Q_{\text{sat}}$ is visible in the spectra, which means that the information on the initial fireball survives until final hadrons are formed.

In this letter we have shown that the quality of geometrical scaling improves if the exponent $\lambda$ becomes $p_T$-dependent. We have computed this dependence by rescaling $p_T$ spectra at 0.9 and 7 TeV to the reference energy 2.36 TeV, however we have also checked that rescaling 0.9 and 2.36 TeV spectra to 7 TeV or 7 and 2.36 TeV spectra to 0.9 TeV gives qualitatively the same results. Not only $p_T$ spectra rescaled from different energies to the reference energy $W'$ agree (which is the essence of geometrical scaling), but the $p_T$ dependence of the exponent $\lambda$ agrees with the dependence obtained from DIS $\lambda_{\text{sat}}(Q)$, at the scale $Q \sim 2p_T$. We find this last result remarkable, since it provides a direct link between two different types of reactions.

Several points require further clarification. First of all new large $p_T$ data of good quality will be of importance to test the range of applicability of geometrical scaling and of the discussed similarity with DIS. Also low $p_T$ data, where hadronic $\lambda(p_T)$ deviates from the one from DIS, is required to see whether this deviation signals an onset of a new production mechanism common for different energies, or whether different energies require different $\lambda(p_T)$ violating geometrical scaling in this region. It will be interesting to verify if geometrical scaling works also in heavy ion collisions. If so, $p_T$ spectra in heavy ion collisions measured at different energies and at different centralities will allow find $A$ dependence and impact parameter dependence of $Q_{\text{sat}}$.

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