On Gaugino Contributions to Soft Leptogenesis

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Abstract: We study the contributions to CP violation in right-handed sneutrino decays induced by soft supersymmetry-breaking gaugino masses including flavour effects and paying special attention to the role of thermal corrections. Using a field-theoretical as well as a quantum mechanical approach we compute the CP asymmetries and we conclude that for all the soft-supersymmetry breaking sources of CP violation considered, an exact cancellation between the asymmetries produced in the fermionic and bosonic channels occurs at $T = 0$ up to second order in soft supersymmetry-breaking parameters. Once thermal effects are included the new sources of CP violation induced by supersymmetry-breaking gaugino masses can be sizeable and they can produce the observed baryon asymmetry for conventional values of the $B$ parameter.

Keywords: Neutrino Physics, Beyond Standard Model.
1. Introduction

The discovery of neutrino oscillations makes leptogenesis a very attractive solution to the baryon asymmetry problem [1, 2]. In the standard type I seesaw framework [3], the singlet heavy neutrinos have lepton number violating Majorana masses and when decay out of equilibrium produce dynamically a lepton asymmetry which is partially converted into a baryon asymmetry due to fast sphaleron processes.

For a hierarchical spectrum of right-handed neutrinos, successful leptogenesis requires generically quite heavy singlet neutrino masses [4], of order $M > 2.4(0.4) \times 10^9$ GeV for vanishing (thermal) initial neutrino densities [4, 5] (although flavour effects [6–9] and/or extended scenarios [10, 11] may affect this limit). Low-energy supersymmetry can be invoked to naturally stabilize the hierarchy between this new scale and the electroweak one. This, however, introduces a certain conflict between the gravitino bound on the reheat temperature and the thermal production of right-handed neutrinos [12]. A way out of this conflict is provided by resonant leptogenesis [13]. In this scenario right-handed neutrinos are nearly degenerate in mass which makes the self energy contributions to the CP asymmetries resonantly enhanced and allowing leptogenesis to be possible at much lower temperatures.

Once supersymmetry has been introduced, leptogenesis is induced also in singlet sneutrino decays. If supersymmetry is not broken, the order of magnitude of the asymmetry and the basic mechanism are the same as in the non-supersymmetric case. However, as shown in Refs. [14, 15], supersymmetry-breaking terms can induce effects which are essentially different from the neutrino ones. In brief, soft supersymmetry-breaking terms involving the singlet sneutrinos remove the mass degeneracy between the two real sneutrino states of a single neutrino generation, and provide new sources of lepton number and CP violation. In this case, as for the case of resonant leptogenesis, it is the sneutrino self-energy contributions to the CP asymmetries which are resonantly enhanced. As a consequence, the mixing between the sneutrino states can generate a sizable CP asymmetry in their decays.
This scenario was termed “soft leptogenesis”. Altogether it was found that the asymmetry is large for a right-handed neutrino mass scale relatively low, in the range $10^5 - 10^8$ GeV, well below the reheat temperature limits, what solves the cosmological gravitino problem. However in order to generate enough asymmetry the lepton-violating soft bilinear coupling, $B$, responsible for the sneutrino mass splitting, has to be unconventionally small [14–17] *.

In soft leptogenesis induced by CP violation in mixing as discussed above an exact cancellation occurs between the asymmetry produced in the fermionic and bosonic channels at $T = 0$. Thermal effects, thus, play a fundamental role in this mechanism: final-state Fermi blocking and Bose stimulation as well as effective masses for the particle excitations in the plasma break supersymmetry and effectively remove this degeneracy.

In Ref. [20] the possibility of soft leptogenesis generated by CP violation in right-handed sneutrino decay and in the interference of mixing and decay was considered. These new sources of CP violation (the so called “new ways to soft leptogenesis”) are induced by vertex corrections due to gaugino soft supersymmetry-breaking masses. Some of these contributions, although suppressed by a loop factor and higher order in the supersymmetry-breaking parameters are relevant because they can be sizeable for natural values of the $B$ parameter. Furthermore it was found that, unlike for CP violation in mixing, these contributions did not require thermal effects as they did not vanish at $T = 0$.

In this work we revisit the role of thermal effects in soft leptogenesis due to CP violation in right-handed sneutrino decays induced by gaugino soft supersymmetry-breaking masses. In Sec.2 we describe the one-generation see-saw model in the presence of the soft supersymmetry-breaking terms and compute the relevant CP asymmetries in a field-theoretical approach. We find that for all soft supersymmetry-breaking sources of CP violation considered, at $T = 0$ the exact cancellation between the asymmetries produced in the fermionic and bosonic channels holds up to second order in soft supersymmetry-breaking parameters. In Sec.3 we recompute the asymmetries using a quantum mechanical approach, based on an effective (non hermitic) Hamiltonian. We find the same $T$ dependence of the resulting CP asymmetries. Finally in Sec. 4 we present our quantitative results and determine the region of parameters in which successful leptogenesis induced by the different contributions to the CP asymmetry is possible including the dominant thermal corrections as well as flavour-dependent effects associated with the charged lepton Yukawa couplings in this scenario.

2. The CP Asymmetry: Field Theoretical Approach

The supersymmetric see-saw model could be described by the superpotential:

$$ W = \frac{1}{2} M_{ij} N_i N_j + Y_{ij} \epsilon_{\alpha\beta} N_i L_{j}^{\alpha} H^{\beta}, \quad (2.1) $$

where $i, j = 1, 2, 3$ are flavour indices and $N_i, L_i, H$ are the chiral superfields for the right-handed (RH) neutrinos, the left-handed (LH) lepton doublets and the Higgs doublets with $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$ and $\epsilon_{12} = +1$.

*Extended scenarios [18, 19] may alleviate the unconventionally-small-$B$ problem.
The relevant soft breaking terms involving the RH sneutrinos \( \tilde{N}_i \) and SU(2) gauginos \( \tilde{\lambda}_2^a \) are given by

\[
\mathcal{L}_{soft} = - \left( A_{ij} Y_{ij} \epsilon_{\alpha \beta} \tilde{N}_i \tilde{\ell}_j^\alpha h^\beta + \frac{1}{2} B_{ij} M_{ij} \tilde{N}_i \tilde{N}_j + \frac{1}{2} m_2 \tilde{\lambda}_2^a P_L \tilde{\lambda}_2^a + \text{h.c.} \right). \tag{2.2}
\]

The Lagrangian for interaction terms involving RH sneutrinos \( \tilde{N}_i \), the RH neutrinos \( N_i \) and the \( \tilde{\lambda}_2^a \) with (s)leptons and higgsinos can be written as:

\[
\mathcal{L}_{int} = -Y_{ij} \epsilon_{\alpha \beta} \left( M_i \tilde{N}_i^\alpha \tilde{\ell}_j^\alpha h^\beta + \tilde{h}^\beta P_L \ell_j^\alpha \tilde{N}_i + \tilde{h} P_L N_i \tilde{\ell}_j^\alpha + \tilde{N}_i P_L \ell_j^\alpha h^\beta + A \tilde{N}_i \tilde{\ell}_j^\alpha h^\beta \right) - g_2 \left( \tilde{\lambda}_2^a P_L (\sigma_1)_{\alpha \beta} \ell_i^\alpha \tilde{\ell}_j^\beta - \frac{1}{\sqrt{2}} \tilde{\lambda}_2^a P_L (\sigma_3)_{\alpha \beta} \ell_i^\alpha \tilde{\ell}_j^\beta \right) + \text{h.c.} \tag{2.3}
\]

\( \ell_i^T = (\nu_i, \tilde{\nu}_i) \), \( \tilde{\ell}_i^T = (\tilde{\nu}_i, \tilde{\ell}_i) \) are the lepton and slepton doublets and, \( h^T = (h^+, h^0) \) and \( \tilde{h}^T = (\tilde{h}^-, \tilde{h}^0) \), are the Higgs and higgsino doublets. \( \tilde{\lambda}_2^\pm \) denotes \( \tilde{\lambda}_2^+ \) for \( \alpha \beta = 10 \) and \( \tilde{\lambda}_2^- \) for \( \alpha \beta = 10 \) with \( \sigma_{1,3} \) being the Pauli matrices, and \( P_{L,R} \) are the left or right projection operator.

The sneutrino and antisneutrino states mix with mass eigenvectors

\[
\tilde{N}_{i+} = \frac{1}{\sqrt{2}} (e^{i \Phi/2} \tilde{N}_i + e^{-i \Phi/2} \tilde{N}_i^*),
\]

\[
\tilde{N}_{i-} = \frac{-i}{\sqrt{2}} (e^{i \Phi/2} \tilde{N}_i - e^{-i \Phi/2} \tilde{N}_i^*), \tag{2.4}
\]

where \( \Phi \equiv \arg(BM) \) and with mass eigenvalues

\[
M_{ii\pm}^2 = M_i^2 \pm |B_{ii} M_{ii}|. \tag{2.5}
\]

From (2.3) and (2.4), we can write down the Lagrangian in the mass basis as

\[
\mathcal{L}_{int} = -Y_{ij} \epsilon_{\alpha \beta} \left\{ \frac{1}{\sqrt{2}} \tilde{N}_{i+} \left[ \tilde{h}^\beta P_L \ell_j^\alpha + (A_{ij} + M_i) \tilde{\ell}_j^\alpha h^\beta \right] + \frac{i}{\sqrt{2}} \tilde{N}_{i-} \left[ \tilde{h}^\beta P_L \ell_j^\alpha + (A_{ij} - M_i) \tilde{\ell}_j^\alpha h^\beta \right] + \tilde{h}^\beta P_L N_i \tilde{\ell}_j^\alpha + \tilde{N}_i P_L \ell_j^\alpha h^\beta \right\} - g_2 \left( \tilde{\lambda}_2^a P_L (\sigma_1)_{\alpha \beta} \ell_i^\alpha \tilde{\ell}_j^\beta - \frac{1}{\sqrt{2}} \tilde{\lambda}_2^a P_L (\sigma_3)_{\alpha \beta} \ell_i^\alpha \tilde{\ell}_j^\beta \right) + \text{h.c.} \tag{2.6}
\]

In what follows, we will consider a single generation of \( N \) and \( \tilde{N} \) which we label as 1. We also assume proportionality of soft trilinear terms and drop the flavour indices for the coefficients \( A \) and \( B \).

\(^{1}\)The effect of \( U(1) \) gauginos can be included in similar form.
As discussed in Refs. [14,15,20], in this case, after superfield rotations the Lagrangians (2.1) and (2.2) have two independent physical CP violating phases:

$$\phi_A = \text{arg}(AB^*),$$

$$\phi_g = \frac{1}{2}\text{arg}(Bm_2^*),$$

which we choose to assign to $A$ and to the gaugino coupling operators (the last two lines which are multiplied by $g_2$ in Eq.(2.6) ) respectively. So for the calculations below we will take $M$, $B$, $m_2$ and $Y_{1k}$ to be positive real and $A$ with phase $\phi_A$ and define a complex coupling $\tilde{g}_2 = g_2 \exp(i\phi_g)$ respectively.

As discussed in Ref. [15], when $\Gamma \gg \Delta M_{\pm} \equiv M_+ - M_-$, the two singlet sneutrino states are not well-separated particles. In this case, the result for the asymmetry depends on how the initial state is prepared. In what follows we will assume that the sneutrinos are in a thermal bath with a thermalization time $\Gamma^{-1}$ shorter than the typical oscillation times, $\Delta M_{\pm}^{-1}$, therefore coherence is lost and it is appropriate to compute the CP asymmetry in terms of the mass eigenstates Eq.(2.4).

We compute the relevant decay amplitudes following the effective field-theoretical approach described in [21], which takes into account the CP violation due to mixing and decay (as well as their interference) of nearly degenerate states by using resummed propagators for unstable mass eigenstate particles. The decay amplitude $\hat{A}^{a_k}_{\pm}$ of the unstable external state $\tilde{N}_{\pm}$ defined in Eq. (2.4) into a final state $a_k$ ($a_k \equiv s_k, f_k$ with $s_k = \tilde{c}^a_{k} h^\alpha$ and $f_k = \tilde{c}^a_{k} h^\beta$) is described by a superposition of amplitudes with stable final states:

$$\hat{A}^{a_k}_{\pm} = \left( A^{a_k}_{\pm} + i\nu_{\pm}^{a_k \text{abs}}(M^2_{\pm}) \right) - \left( A^{a_k}_{\mp} + i\nu_{\mp}^{a_k \text{abs}}(M^2_{\mp}) \right) \frac{i\Sigma_{\pm \pm}^{\text{abs}}}{M^2_{\pm} - M^2_{\mp} + i\Sigma_{\pm \mp}^{\text{abs}}},$$

$$\overline{A}^{a_k}_{\pm} = \left( A^{a_k*}_{\pm} + i\nu_{\pm}^{a_k \text{abs}}(M^2_{\pm}) \right) - \left( A^{a_k*}_{\mp} + i\nu_{\mp}^{a_k \text{abs}}(M^2_{\mp}) \right) \frac{i\Sigma_{\pm \mp}^{\text{abs}}}{M^2_{\pm} - M^2_{\mp} + i\Sigma_{\pm \pm}^{\text{abs}}}.$$  

(2.9)  

(2.10)

$A^{a_k}_{\pm}$ are the tree-level amplitudes:

$$A^{s_k}_{\pm} = \frac{Y_{1k}}{\sqrt{2}}(A^* + M)\epsilon_{\alpha \beta}, \quad A^{s_k}_{\mp} = -\frac{Y_{1k}}{\sqrt{2}}(A^* - M)\epsilon_{\alpha \beta},$$

$$A^{f_k}_{\pm} = \frac{Y_{1k}}{\sqrt{2}}[\tilde{u}(p_t)P_Rv(p_h)]\epsilon_{\alpha \beta}, \quad A^{f_k}_{\mp} = -\frac{Y_{1k}}{\sqrt{2}}[\tilde{u}(p_t)P_Rv(p_h)]\epsilon_{\alpha \beta}.$$  

(2.11)  

(2.12)

$\Sigma_{\alpha \beta}^{\text{abs}}$ are the absorptive parts of the $\tilde{N}_b \rightarrow \tilde{N}_a$ self-energies (see Fig. I):

$$\Sigma_{\mp \pm}^{(1)\text{abs}} = \Gamma M \left[ \frac{1}{2} + \frac{M^2_{\pm}}{2M^2} + \frac{|A|^2}{2M^2} + \frac{\text{Re}(A)}{M} \right],$$

$$\Sigma_{\pm \mp}^{(1)\text{abs}} = -\Gamma \text{Im}(A),$$

(2.13)  

(2.14)

with

$$\Gamma = \frac{\sum_k M|Y_{1k}|^2}{4\pi} = \frac{M(Y^\dagger Y)_{11}}{4\pi}.$$  

(2.15)
\begin{align*}
\mathcal{V}_{\pm k}^{a \text{abs}} & \text{ are the absorptive parts of the vertex corrections (see Fig. 2):} \\
\mathcal{V}_{+ k}^{a \text{abs}} (p^2) & = \frac{Y_{1 k}}{\sqrt{2} 32 \pi} \frac{3 m_2}{(g_2)^2} \ln \frac{m_2^2}{p^2 + m_2^2} \varepsilon_{\alpha \beta}, \quad (2.16) \\
\mathcal{V}_{- k}^{a \text{abs}} (p^2) & = -i \frac{Y_{1 k}}{\sqrt{2} 32 \pi} \frac{3 m_2}{(g_2)^2} \ln \frac{m_2^2}{p^2 + m_2^2} \varepsilon_{\alpha \beta}, \quad (2.17) \\
\mathcal{V}_{f k}^{a \text{abs}} (p^2) & = \frac{Y_{1 k}}{\sqrt{2} 32 \pi p^2} (A^* + M) (g_2^*)^2 \ln \frac{m_2^2}{p^2 + m_2^2} [\bar{u}(p_\ell) P_R v(p_\ell)] \varepsilon_{\alpha \beta}, \quad (2.18) \\
\mathcal{V}_{f k}^{a \text{abs}} (p^2) & = -i \frac{Y_{1 k}}{\sqrt{2} 32 \pi p^2} (A^* - M) (g_2^*)^2 \ln \frac{m_2^2}{p^2 + m_2^2} [\bar{u}(p_\ell) P_R v(p_\ell)] \varepsilon_{\alpha \beta}. \quad (2.19)
\end{align*}

\footnote{We notice that there is an irrelevant global $i$ factor in the tree level $A_{+k}$ and one-loop $\mathcal{V}_{+ k}^{a \text{abs}}$ amplitudes compared to $A_{+k}$ and $\mathcal{V}_{+ k}^{a \text{abs}}$ arising from the particular choice of global phase in the definition of $\tilde{N}_-$ in Eq.(2.4).}

### 2.1 The CP asymmetry

The CP asymmetry produced in the decay of the states $\tilde{N}_{i=\pm}$ which enters into the Boltz-
mann Equations (BE) is given by:

\[
\epsilon_k = \frac{\sum_{i=\pm} \gamma(\tilde{N}_i \to a_k) - \gamma(\tilde{N}_i \to \bar{a}_k)}{\sum_{i=\pm} \gamma(\tilde{N}_i \to a_k) + \gamma(\tilde{N}_i \to \bar{a}_k)},
\]

(2.20)

where we denote by \( \gamma \) the thermal averaged rates. In the rest frame of \( \tilde{N}_\pm \), (2.20) simplifies to:

\[
\epsilon_k = \epsilon^s_{+k} + \epsilon^s_{-k} + \epsilon^f_{+k} + \epsilon^f_{-k},
\]

(2.21)

where

\[
\epsilon^s_{\pm k} = \frac{\left(|\hat{A}^s_{\pm k}|^2 - |\hat{A}^{\bar{s}}_{\pm k}|^2\right) c^s_{\pm k}/M_{\pm}}{\sum_{i=\pm} \left(|\hat{A}_{\pm i}^s|^2 + |\hat{A}_{\pm i}^{\bar{s}}|^2\right) c^{a_i}/M_i},
\]

(2.22)

\[
\epsilon^f_{\pm k} = \frac{\left(|\hat{A}^f_{\pm k}|^2 - |\hat{A}^{\bar{f}}_{\pm k}|^2\right) c^f_{\pm k}/M_{\pm}}{\sum_{i=\pm} \left(|\hat{A}_{\pm i}^f|^2 + |\hat{A}_{\pm i}^{\bar{f}}|^2\right) c^{a_i}/M_i}.
\]

(2.23)

In Eqs. (2.22) and (2.23) \( c^s_i, c^f_i \) are the phase-space factors of the scalar and fermionic channels, respectively. As long as we neglect the zero temperature lepton and slepton masses and small Yukawa couplings, these phase-space factors are flavour independent and they are the same for \( i = \pm \). After including finite temperature effects in the approximation of decay at rest of the \( \tilde{N}_\pm \) they are given by:

\[
c^f_{\pm}(T) = c^f(T) = (1 - x_{\ell} - x_{\bar{h}}) \lambda(1, x_{\ell}, x_{\bar{h}}) \left[1 - f^eq_{\ell}\right] \left[1 - f^eq_{\bar{h}}\right],
\]

(2.24)

\[
c^s_{\pm}(T) = c^s(T) = \lambda(1, x_h, x_{\bar{\ell}}) \left[1 + f^eq_{h}\right] \left[1 + f^eq_{\bar{\ell}}\right],
\]

(2.25)

where

\[
f^eq_{h, \ell} = \frac{1}{\exp[E_{h, \ell}/T] - 1},
\]

(2.26)

\[
f^eq_{h, \ell} = \frac{1}{\exp[E_{h, \ell}/T] + 1},
\]

(2.27)

are the Boltzmann-Einstein and Fermi-Dirac equilibrium distributions, respectively, and

\[
E_{\ell, h} = \frac{M}{2}(1 + x_{\ell, h} - x_{\bar{h}, \ell}), \quad E_{h, \ell} = \frac{M}{2}(1 + x_{h, \ell} - x_{\bar{\ell}, h}),
\]

(2.28)

\[
\lambda(1, x, y) = \sqrt{(1 + x - y)^2 - 4x}, \quad x_a = \frac{m_a(T)^2}{M^2}.
\]

(2.29)

The thermal masses for the relevant supersymmetric degrees of freedom are [22]:

\[
m^2_{\ell}(T) = 2m^2_{h}(T) = \left(\frac{3}{8}g^2_2 + \frac{1}{8}g^2_Y + \frac{3}{4}\lambda^2\right) T^2,
\]

(2.30)

\[
m^2_{\ell}(T) = 2m^2_{\ell}(T) = \left(\frac{3}{8}g^2_2 + \frac{1}{8}g^2_Y\right) T^2.
\]

(2.31)
Here $g_2$ and $g_Y$ are gauge couplings and $\lambda_t$ is the top Yukawa, renormalized at the appropriate high-energy scale.

Substituting (2.9) and (2.10) into (2.22) and (2.23) we get in the numerators:

\[
|\tilde{A}_{s_\pm}^k|^2 - |\tilde{A}_{c_\pm}^k|^2 \simeq -4 \left\{ -\text{Im} \left[ A_{s_\pm}^k A_{c_\pm}^k \Sigma_{\pm\pm}^{\text{abs}} \right] \frac{M_{s_\pm}^2 - M_{c_\pm}^2}{(M_{s_\pm}^2 - M_{c_\pm}^2)^2 + |\Sigma_{\pm\pm}^{\text{abs}}|^2} \\
+ \text{Im} \left[ A_{s_\pm}^k A_{c_\pm}^k \Sigma_{\pm\pm}^{\text{abs}} \right] \right. \\
+ \text{Im} \left[ \Sigma_{\pm\pm}^{\text{abs}} \left( A_{s_\pm}^k A_{c_\pm}^k - A_{c_\pm}^k A_{s_\pm}^k \right) \right] \frac{\Sigma_{\pm\pm}^{\text{abs}}}{(M_{s_\pm}^2 - M_{c_\pm}^2)^2 + |\Sigma_{\pm\pm}^{\text{abs}}|^2} \right\},
\]

where we have used the relations $\Sigma_{\pm\pm}^{\text{abs}} = \Sigma_{\pm\pm}^{\text{abs}*}$ and $\Sigma_{\pm\pm}^{\text{abs}} = \Sigma_{\pm\pm}^{\text{abs}*}$. The $\simeq$ sign means that terms of order $\delta^3$ and higher are ignored with

\[
\delta_S \equiv \frac{|A|}{M} \cdot \frac{B}{M} \cdot \frac{m_2}{M}.
\]

The three lines in (2.32) correspond respectively to (i) CP violation in $\tilde{N}$ mixing from the off-diagonal one-loop self-energies (this corresponds to the effects originally considered in Refs. [14, 15]), (ii) CP violation due to the gaugino-mediated one-loop vertex corrections to the $\tilde{N}$ decay, and (iii) CP violation in the interference of vertex and self-energies.

In the denominator of (2.22) and (2.23) we consider only the tree-level amplitudes $|\tilde{A}_{s_\pm}^k|^2 + |\tilde{A}_{c_\pm}^k|^2 = 2|A_{s_\pm}^k|^2$, with $|A_{s_\pm}^k|^2 = Y_{ik}^2 \left[ |A|^2 + M^2 \pm 2M \text{Re}(\lambda_t) \right]$ and $|A_{c_\pm}^k|^2 = Y_{ik}^2 M_{s_\pm}^2$.

Using the explicit forms in Eqs.(2.14) and (2.19) we find that up to order $\delta^2$, the three contributions to the CP asymmetry from scalar and fermion decays verify:

\[
\begin{align*}
\epsilon_S^k &= \frac{e^S(T)}{e^S(T) + c^S(T)} \epsilon_S^{s_k}, \\
\epsilon_I^k &= -\frac{e^I(T)}{e^S(T) + c^S(T)} \epsilon_S^{s_k}, \\
\epsilon_V^k &= \frac{e^V(T)}{e^S(T) + c^S(T)} \epsilon_V^{s_k}, \\
\epsilon_I^k &= -\frac{e^I(T)}{e^S(T) + c^S(T)} \epsilon_I^{s_k}, \\
\epsilon_V^k &= \frac{e^V(T)}{e^S(T) + c^S(T)} \epsilon_I^{s_k}, \\
\epsilon_I^k &= -\frac{e^I(T)}{e^S(T) + c^S(T)} \epsilon_I^{s_k},
\end{align*}
\]

with

\[
\begin{align*}
\epsilon_S^{s_k} &= -K^0_k \frac{|A|}{M} \left( 1 \mp \frac{B}{2M} \right) \sin(\phi_A) \frac{2B\Gamma}{4B^2 + \Gamma^2}, \\
\epsilon_V^{s_k} &= -\frac{3K^0_k \alpha_2 m_2}{8} M \ln \frac{m_2^2 + M^2}{M^2} \left[ \frac{|A|}{M} \sin(\phi_A + 2\phi_g) - \frac{B}{M} \sin(2\phi_g) \pm \sin(2\phi_g) \right], \\
\epsilon_I^{s_k} &= \frac{3K^0_k \alpha_2 m_2}{4} M \frac{|A|}{M} \ln \frac{m_2^2 + M^2}{M^2} \sin(\phi_A) \cos(2\phi_g) \frac{\Gamma^2}{4B^2 + \Gamma^2},
\end{align*}
\]

where we have defined $\alpha_2 = \frac{g_Y^2}{4\pi}$ and the flavour projections $K^0_k$

\[
K^0_k = \frac{|Y_{1k}|^2}{\sum_k |Y_{1k}|^2}.
\]
Summing up the contribution from the decays of $\tilde{N}_+$ and $\tilde{N}_-$, one gets the three contributions to the CP asymmetry in Eq. (2.20)

$$\epsilon_k^S(T) = -K_0^0 \frac{|A|}{M} \sin (\phi_A) \frac{4B\Gamma}{4B^2 + \Gamma^2} \Delta_{BF}(T),$$
$$\equiv K_0^0 \Delta_{BF}(T) \epsilon^S,$$
$$\epsilon_k^V(T) = -\frac{3K_0^0 \alpha_2}{4} \frac{m_2}{M} \ln \frac{m_2^2}{m_2^2 + M^2} \left[ \frac{|A|}{M} \sin (\phi_A + 2\phi_g) - \frac{B}{M} \sin (2\phi_g) \right] \Delta_{BF}(T),$$
$$\equiv K_0^0 \Delta_{BF}(T) \epsilon^V,$$
$$\epsilon_k^I(T) = \frac{3K_0^0 \alpha_2}{2} \frac{|A|}{M} \ln \frac{m_2^2}{m_2^2 + M^2} \sin (\phi_A) \cos (2\phi_g) \frac{\Gamma^2}{4B^2 + \Gamma^2} \Delta_{BF}(T),$$
$$\equiv K_0^0 \Delta_{BF}(T) \epsilon^I,$$

where

$$\Delta_{BF}(T) = \frac{c^S(T) - c^I(T)}{c^S(T) + c^I(T)},$$

The asymmetry in Eq.(2.39) is the contribution to the lepton asymmetry due to CP violation in RH sneutrino mixing discussed in Refs. [14–16]. Eqs.(2.40) and (2.41) give the contribution to the lepton asymmetry related to CP violation in decay and in the interference of mixing and decay. They have similar parametric dependences as the ones derived in Ref. [20]. However as explicitly shown in the equations, the scalar and fermionic CP asymmetries, (2.34), cancel each other at zero temperature. Consequently we find that, up to second order in the soft supersymmetry-breaking parameters, all contributions to the lepton asymmetry in the soft supersymmetry scenario require thermal effects in order to be significant.

We finish by noticing that in this derivation we have neglected thermal corrections to the CP asymmetry from the loops, i.e., we have computed the imaginary part of the one-loop graphs using Cutkosky’s cutting rules at $T = 0$.

3. CP Asymmetry in Quantum Mechanics

In this section we recompute the asymmetry using a quantum mechanical (QM) approach, based on an effective (non hermitic) Hamiltonian [14,15,20]. In this language an analogy can be drawn between the $\tilde{N}$–$\tilde{N}^\dagger$ system and the system of neutral mesons such as $K^0$–$\bar{K}^0$ and its time evolution is determined in the non-relativistic limit by the Hamiltonian:

$$H = \left( \begin{array}{cc} M & B \frac{1}{2} \\ B \frac{1}{2} & M \end{array} \right) - \frac{i}{2} \left( \begin{array}{cc} \Gamma & E^* \frac{1}{M} \\ \frac{1}{M} E \Gamma \end{array} \right),$$

with $\Gamma$ given in Eq.(2.15).

In Refs. [14,15,20] the QM formalism was applied for weak initial states $\tilde{N}$ and $\tilde{N}^\dagger$. In practice it is possible to use the formalism to study the evolution of either initially
weak or mass eigenstates. So in order to study the dependence of the results on the choice of physical initial conditions we will compute the asymmetry in this formalism assuming either of the two possibilities for initial states. So we define the basis:

\[
\tilde{N}_1 = \left( a\tilde{N} + b\tilde{N}^\dagger \right), \\
\tilde{N}_2 = e^{i\beta} \left( b\tilde{N} - a\tilde{N}^\dagger \right). 
\]

The mass basis, Eq.(2.4) corresponds to \((a, b, \beta) = \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{\pi}{2} \right)\). Assuming that the physical initial states were pure \(\tilde{N}\) and \(\tilde{N}^\dagger\) corresponds to \((a, b, \beta) = (1, 0, \pi)\).

The decay amplitudes of \(\tilde{N}_1\) and \(\tilde{N}_2\) into fermions \(f_k = \ell_k^c \ell_k^d\) including the one-loop contribution from gaugino exchange are:

\[
A_{1k}^f = \left\{ Y_{1k}b - \frac{3Y_{1k}}{2M^2} (aM + bA^*) \left( \tilde{g}_2 \right) \frac{m_2}{16\pi} I_f \right\} \bar{\nu}(p_e) P_{RV} (p_h) \epsilon_{cd}, \\
\overline{A_{1k}}^f = \left\{ Y_{1k}a + \frac{3Y_{1k}}{2M^2} (bM + aA^*) \left( \tilde{g}_2 \right) \frac{m_2}{16\pi} I_f \right\} \bar{\nu}(p_h) P_{LV} (p_e) \epsilon_{cd}, \\
A_{2k}^f = -e^{-i\beta} \left\{ Y_{1k}a - \frac{3Y_{1k}}{2M^2} (bM + aA^*) \left( \tilde{g}_2 \right) \frac{m_2}{16\pi} I_f \right\} \bar{\nu}(p_e) P_{RV} (p_h) \epsilon_{cd}, \\
\overline{A_{2k}}^f = e^{-i\beta} \left\{ Y_{1k}b - \frac{3Y_{1k}}{2M^2} (aM - bA^*) \left( \tilde{g}_2 \right) \frac{m_2}{16\pi} I_f \right\} \bar{\nu}(p_h) P_{LV} (p_e) \epsilon_{cd},
\]

where the \(\overline{A}\) denotes the decay amplitudes into antifermions. The corresponding decay amplitudes into scalar \(s_k = \ell_k^c \ell_k^d\) and

\[
A_{1k}^{s_k} = \left\{ Y_{1k} (aM + bA^*) - \frac{3Y_{1k}}{2} (\tilde{g}_2) \frac{m_2}{16\pi} I_s \right\} \epsilon_{cd}, \\
\overline{A_{1k}}^{s_k} = \left\{ Y_{1k} (bM + aA) - \frac{3Y_{1k}}{2} (\tilde{g}_2) \frac{m_2}{16\pi} I_s \right\} \epsilon_{cd}, \\
A_{2k}^{s_k} = e^{-i\beta} \left\{ Y_{1k} (bM - aA^*) + \frac{3Y_{1k}}{2} (\tilde{g}_2) \frac{m_2}{16\pi} I_s \right\} \epsilon_{cd}, \\
\overline{A_{2k}}^{s_k} = e^{-i\beta} \left\{ Y_{1k} (aM - bA^*) - \frac{3Y_{1k}}{2} (\tilde{g}_2) \frac{m_2}{16\pi} I_s \right\} \epsilon_{cd},
\]

where

\[
\text{Re}(I_f) \equiv f_R = -\frac{1}{\pi} \left[ \frac{1}{2} \left( \ln \frac{m_2^2}{m_2^2 + M^2} \right)^2 + \text{Li}_2 \left( \frac{m_2^2}{m_2^2 + M^2} \right) - \zeta(2) \right], \\
\text{Re}(I_s) \equiv s_R = \frac{1}{\pi} \left[ \frac{1}{2} \left( \ln \frac{m_2^2}{m_2^2 + M^2} \right)^2 + \text{Li}_2 \left( \frac{m_2^2}{m_2^2 + M^2} \right) - \zeta(2) + B_0 \left( M^2, m_2, 0 \right) + B_0 \left( M^2, 0, m_2 \right) \right], \\
\text{Im}(I_f) \equiv f_I = \text{Im}(I_s) \equiv s_I = -\ln \frac{m_2^2}{m_2^2 + M^2}.
\]

The eigenvectors of the Hamiltonian in terms of the states \(\tilde{N}_1\) and \(\tilde{N}_2\) are:

\[
\left| \tilde{N}_L \right> = (ap + bq) \left| \tilde{N}_1 \right> + e^{-i\beta} (bp - aq) \left| \tilde{N}_2 \right>, \\
\left| \tilde{N}_H \right> = (ap - bq) \left| \tilde{N}_1 \right> + e^{-i\beta} (bp + aq) \left| \tilde{N}_2 \right>.
\]
where
\[
\frac{q}{p} = -1 - \frac{\Gamma|A|}{BM} \sin (\phi_A) - \frac{\Gamma^2|A|^2}{M^2B^2} \cos^2 (\phi_A) - \frac{i}{2} \frac{\Gamma^2|A|^2}{M^2B^2} \sin (2\phi_A).
\] (3.7)

At the time \( t \) the states \( \tilde{N}_1 \) and \( \tilde{N}_2 \) have evolved into
\[
\left\langle \tilde{N}_{1,2}(t) \right\rangle = \frac{1}{2} \left\{ \left[ e_L(t) + e_H(t) \pm C_0 (e_L(t) - e_H(t)) \right] \left| \tilde{N}_{1,1} \right\rangle \right.
\]
\[
+ e^{\mp i(\beta)} C_{1,2} (e_L(t) - e_H(t)) \left| \tilde{N}_{2,1} \right\rangle \right\},
\] (3.8)

where
\[
C_0 = ab \left( \frac{p}{q} + \frac{q}{p} \right), \quad C_1 = b^2 \frac{p}{q} - a^2 \frac{p}{q}, \quad C_2 = b^2 \frac{q}{p} - a^2 \frac{q}{p},
\] (3.9)

and
\[
e_{H,L}(t) \equiv e^{-i(M_{H,L} - \frac{i}{2} \sum_{i \neq j} G_{ii})t}.
\] (3.10)

The total time integrated lepton asymmetry is
\[
\epsilon_{Q_i}^{QM} = \sum_{i=1,2,a_k} \frac{\Gamma(\tilde{N}_i \rightarrow a_k) - \Gamma(\tilde{N}_i \rightarrow \bar{a}_k)}{\sum_{i=1,2,a_k} \Gamma(\tilde{N}_i \rightarrow a_k) + \Gamma(\tilde{N}_i \rightarrow \bar{a}_k)},
\] (3.11)

where \( \Gamma(\tilde{N}_i \rightarrow a_k) \) are the time integrated decay rates which from Eq.(3.8) are found to be
\[
\Gamma(\tilde{N}_i \rightarrow a_k) = \frac{1}{16\pi M} \left( |A_{ii}|^2 G_{ii} + \left| A_{j\neq i}^k \right|^2 G_{jj} \right.
\]
\[
+ 2 \left[ \Re \left( A_{ii}^{a_k} A_{j\neq i}^{a_k} \right) G_{ii}^R - \Im \left( A_{ii}^{a_k} A_{j\neq i}^{a_k} \right) G_{ii}^I \right] \). \] (3.12)

\( \Gamma(\tilde{N}_i \rightarrow \bar{a}_k) \) can be obtained from Eq.(3.12) with the replacement \( A_{ii}^{a_k} \rightarrow A_{ii}^{\bar{a}_k} \). We have defined the time integrated projections
\[
G_{1,2}^{11} = 2 \left( \frac{1}{1 - y^2} + \frac{1}{1 + x^2} \right) + 2 \left| C_0 \right|^2 \left( \frac{1}{1 - y^2} - \frac{1}{1 + x^2} \right)
\]
\[
\pm 8 \left[ \Re (C_0) \frac{y}{1 - y^2} - \Im (C_0) \frac{x}{1 + x^2} \right],
\] (3.13)
\[
G_{1,2}^{12} = 2 \left| C_{1,2} \right|^2 \left( \frac{1}{1 - y^2} - \frac{1}{1 + x^2} \right),
\]
\[
G_{11}^{R(2)} = 2 \left\{ \Re \left[ e^{\mp i\beta} C_{1(2)} \right] \frac{y}{1 - y^2} - \Im \left[ e^{\mp i\beta} C_{1(2)} \right] \frac{x}{1 + x^2} \right\}
\]
\[
\pm 2 \Re \left[ e^{\mp i\beta} C_{1(2)} \right] \left( \frac{1}{1 - y^2} - \frac{1}{1 + x^2} \right),
\] (3.14)
\[
G_{11}^{I(2)} = 2 \left\{ \Im \left[ e^{\mp i\beta} C_{1(2)} \right] \frac{y}{1 - y^2} + \Re \left[ e^{\mp i\beta} C_{1(2)} \right] \frac{x}{1 + x^2} \right\}
\]
\[
\pm 2 \Im \left[ e^{\mp i\beta} C_{1(2)} \right] \left( \frac{1}{1 - y^2} - \frac{1}{1 + x^2} \right),
\] (3.15)
in terms of masses and width differences coefficients:\footnote{We use the expression of $\Gamma_H \rightarrow \Gamma_L$ from Ref. [20]. Notice that with this definition $\Gamma_H \neq \Gamma_L$.}

\[
x = \frac{M_H - M_L}{\Gamma} = \frac{B}{\Gamma} - \frac{1}{2BM^2} \sin^2(\phi_A),
\]

\[
y = \frac{\Gamma_H - \Gamma_L}{2\Gamma} = \frac{|A|}{M} \cos(\phi_A) - \frac{B}{2M}.
\]

Substituting Eqs. (3.12)-(3.15) one can write the numerator in Eq. (3.11) as

\[
\sum_i \Gamma(\bar{N}_i \rightarrow a_k) - \Gamma(\bar{N}_i \rightarrow \bar{a}_k) \equiv \Delta \Gamma^{ak,R} + \Delta \Gamma^{ak, NR} + \Delta \Gamma^{ak,I},
\]

with

\[
\Delta \Gamma^{ak,R} = \frac{1}{2} \frac{c^{ak}}{16\pi M} \frac{x^2 + y^2}{(1 - y^2)(1 + x^2)} \left\{ |C_0|^2 \right\} \left[ |A_1^{ak}|^2 - |A_1^{\bar{ak}}|^2 + |A_2^{ak}|^2 - |A_2^{\bar{ak}}|^2 \right]
\]

\[
\sum_{\frac{1}{2}} \sum_{k} \left( |C_k|^2 - |C_{\bar{k}}|^2 \right) \left( |A_1^{ak}|^2 - |A_1^{\bar{ak}}|^2 + |A_2^{ak}|^2 - |A_2^{\bar{ak}}|^2 \right)
\]

\[
+ \left( |A_1^{ak}|^2 - |A_1^{\bar{ak}}|^2 + |A_2^{ak}|^2 - |A_2^{\bar{ak}}|^2 \right)
\]

\[
+ y \left[ \text{Re} \left( A_1^{ak} A_2^{\bar{ak}} - A_1^{\bar{ak}} A_2^{ak} \right) \text{Re} \left( e^{-i\beta} C_0 C_1 \right) + \text{Re} \left( A_2^{ak} A_1^{\bar{ak}} - A_2^{\bar{ak}} A_1^{ak} \right) \text{Re} \left( e^{i\beta} C_0 C_2 \right) \right]
\]

\[
- \left[ \text{Re} \left( A_1^{ak} A_2^{\bar{ak}} - A_1^{\bar{ak}} A_2^{ak} \right) \text{Re} \left( e^{-i\beta} C_1 \right) + \text{Re} \left( A_2^{ak} A_1^{\bar{ak}} - A_2^{\bar{ak}} A_1^{ak} \right) \text{Re} \left( e^{i\beta} C_2 \right) \right],
\]

\[
\Delta \Gamma^{ak, NR} = \frac{c^{ak}}{16\pi M} \frac{x}{1 - y^2} \left\{ \text{Re} \left( A_1^{ak} A_2^{\bar{ak}} - A_1^{\bar{ak}} A_2^{ak} \right) \text{Re} \left( e^{-i\beta} C_1 \right) + \text{Re} \left( A_2^{ak} A_1^{\bar{ak}} - A_2^{\bar{ak}} A_1^{ak} \right) \text{Re} \left( e^{i\beta} C_2 \right) \right\}
\]

\[
- \left[ \text{Re} \left( A_1^{ak} A_2^{\bar{ak}} - A_1^{\bar{ak}} A_2^{ak} \right) \text{Re} \left( e^{-i\beta} C_1 \right) + \text{Re} \left( A_2^{ak} A_1^{\bar{ak}} - A_2^{\bar{ak}} A_1^{ak} \right) \text{Re} \left( e^{i\beta} C_2 \right) \right],
\]

\[
\Delta \Gamma^{ak, I} = \frac{c^{ak}}{16\pi M} \frac{x}{1 + x^2} \left\{ \text{Re} \left( A_1^{ak} A_2^{\bar{ak}} - A_1^{\bar{ak}} A_2^{ak} \right) \text{Re} \left( e^{-i\beta} C_1 \right) + \text{Re} \left( A_2^{ak} A_1^{\bar{ak}} - A_2^{\bar{ak}} A_1^{ak} \right) \text{Re} \left( e^{i\beta} C_2 \right) \right\}.
\]

In writing the above equations we have classified the contributions as resonant, (non-resonant), $R$ ($NR$), depending on whether they present an overall factor $\frac{x^2 + y^2}{1 + x^2}$ (or no at all). We have labeled the remainder as interference term, $I$.

After substituting the explicit values for the amplitudes and the coefficients and neglecting all those terms which cancel in both basis we get that

\[
\Delta \Gamma^{f_k, R} = -c^f \Delta \Gamma^R_k,
\]

\[
\Delta \Gamma^{f_k, NR} = -c^f \Delta \Gamma^{NR}_k,
\]

\[
\Delta \Gamma^{f_k, I} = -c^f \Delta \Gamma^I_k.
\]

\footnote{We use the expression of $\Gamma_H \rightarrow \Gamma_L$ from Ref. [20]. Notice that with this definition $\Gamma_H \neq \Gamma_L$.}
with

\[ \Delta \Gamma_k^R = -\frac{1}{4\pi} Y^2_k \left[ (a^2 - b^2)^2 + (2ab)^2 \cos 2\beta \right] |A| \sin(\phi_A) \frac{x^2 + y^2}{x (1 - y^2)(1 + x^2)}, \quad (3.23) \]

\[ \Delta \Gamma_k^{NR} = \frac{3}{16\pi} Y^2_{1k} \alpha_2 \ln \frac{m_2}{m_2 + M^2} \left[ -|A| \sin(\phi_A + 2\phi_\theta) \\
+ yM \left( 2(2ab)^2 + (a^2 - b^2)^2 \cos(2\beta) \right) \sin(2\phi_\theta) \right], \quad (3.24) \]

\[ \Delta \Gamma_k^I = \frac{3}{16\pi} Y^2_{1k} \alpha_2 \ln \frac{m_2}{m_2 + M^2} \left[ \frac{2}{1 + x^2} |A| \sin(\phi_A) \cos(\beta) \cos(2\phi_\theta) \right]. \quad (3.25) \]

Eq. (3.22) explicitly displays the cancellation of the asymmetries at \( T = 0 \) also in this formalism for either initial mass or weak eigenstate right-handed sneutrinos. \footnote{We have traced the discrepancy with Ref. \[20\] to a missing \( \cos(\phi_f - \phi_s) \) factor in their expression for \( \sin \delta_\alpha \) in their Eq. (19). Once that factor is included, \( \epsilon^R \) in their Eq. (22) cancels against \( \epsilon^m_{\alpha i} \) in their Eq. (25), and \( \epsilon^i \) in their Eq. (23) cancels against \( \epsilon^i \) in their Eq. (24) so that the total asymmetry is zero at \( T = 0 \).}

Comparing Eqs. (3.29)–(3.31) with Eqs. (3.26)–(3.28) and Eqs. (2.39)–(2.41) we find that they show very similar parametric dependence though there are some differences in the numerical coefficients. In particular we find that \( \epsilon^{NR, QM}_k \) and the \( B \)-dependent (second term) in either the weak or mass basis \( \epsilon^{NR, QM}_k \) coincide with \( \epsilon^{S}_k \) and \( \epsilon^{I}_k \) and the \( B \)-dependent term in \( \epsilon^{I}_k \) derived in the previous section with the redefinition \( A \rightarrow 2A, B \rightarrow 2B \) and \( \sin(\phi_A) \rightarrow \pm \sin(\phi_A) \).
4. Results

Next we quantify the parameters for which successful leptogenesis induced by the different sources of CP violation discussed in the previous sections is possible by solving the corresponding set of Boltzmann Equations (BE).

The relevant classical BE describing the decay, inverse decay and scattering processes involving the sneutrino states in the framework of flavoured soft leptogenesis were derived in detail in Ref. [16] assuming that the physically relevant sneutrino states were the mass eigenstates (2.4). As we have seen in the previous section, the choice of the physical basis for the sneutrino states does not lead to important differences in the parametric form of the generated asymmetries, thus in what follows we will present our results assuming that the relevant sneutrino states are the mass eigenstates.

Including the $\tilde{N}_\pm$ and $N$ decay and inverse decay processes as well as all the $\Delta L = 1$ scattering processes induced by the top Yukawa coupling the final set of BE takes the form

\[
sHz \frac{dY_N}{dz} = -\left( \frac{Y_N}{Y_N^{eq}} - 1 \right) \left( \gamma_N + 4\gamma_l^{(0)} + 4\gamma_l^{(1)} + 4\gamma_l^{(2)} + 2\gamma_l^{(3)} + 4\gamma_l^{(4)} \right), \tag{4.1}
\]

\[
sHz \frac{dY_{\tilde{N}}{\text{tot}}}{dz} = -\left( \frac{Y_{\tilde{N}}{\text{tot}}}{Y_{\tilde{N}}^{eq}} - 2 \right) \left( \gamma_{\tilde{N}} + 3\gamma_{22} + 2\gamma_l^{(5)} + 2\gamma_l^{(6)} + 2\gamma_l^{(7)} + \gamma_l^{(8)} + 2\gamma_l^{(9)} \right) + \gamma_{\tilde{N}} \sum_k \epsilon_k(T) \frac{Y_{\Delta_k}}{2Y_{eq}^c}, \tag{4.2}
\]

\[
sHz \frac{dY_{\Delta_k}}{dz} = -\left\{ e^k(T) \left( \frac{Y_{\tilde{N}}{\text{tot}}}{Y_{\tilde{N}}^{eq}} - 2 \right) \gamma_{\tilde{N}} - \sum_j A_{kj} \frac{Y_{\Delta_j}}{2Y_{eq}^c} \gamma_N^{(k)} - \sum_j A_{kj} \frac{Y_{\Delta_j}}{2Y_{eq}^c} \left( \gamma_l^{(5)k} + 2\gamma_l^{(6)k} + 2\gamma_l^{(7)k} + Y_N \gamma_l^{(3)k} + 2\gamma_l^{(4)k} \right) \gamma_{\tilde{N}} + \gamma_{22}^{(2)k} + \frac{1}{2} \gamma_{\tilde{N}} \gamma_{22}^{(8)k} + 2\gamma_l^{(9)k} + 2\gamma_l^{(10)k} + 2\gamma_l^{(11)k} + 2\gamma_l^{(12)k} + 2\gamma_l^{(13)k} + 2\gamma_l^{(14)k} \right) \right\}. \tag{4.3}
\]

In the equations above $Y_{eq}^c \equiv \frac{15}{4\pi^2 g_s^*}$ and $Y_{eq}^{eq}(T \gg M) = 90\zeta(3)/(4\pi^4 g_s^*)$, where $g_s^*$ is the total number of entropic degrees of freedom, $g_s^* = 228.75$ in the MSSM.

Before proceeding let’s point out that, as recently discussed in Refs. [23], for resonant scenarios, the use of quantum BE [23, 24] (QBE) may be relevant. For standard resonant

\[\Delta L = 1\] scattering involving gauge bosons, $\Delta L = 2$ off-shell scattering processes involving the pole-subtracted s-channel and the u and t-channel, as well as the L conserving processes from N and $\tilde{N}$ pair creation and annihilation have not been included. The reaction rates for these processes are quartic in the Yukawa couplings and can be safely neglected.
leptogenesis they induce a $T$ dependence in the CP asymmetry which can enhance the produced baryon number. However, as shown in Ref. [17] for soft leptogenesis from CP violation in mixing, due to the thermal nature of the mechanism already at the classical level, the introduction of quantum effects does not lead to such enhancement and the required values of the parameters to achieve successful leptogenesis are not substantially modified. Thus in this work we study the impact of CP violation in sneutrino decay and the interference of mixing and decay on the values of the parameters required for successful leptogenesis in the context of the classical BE as described above.

In writing the BE relevant in the regime in which flavours have to be considered [6, 7, 11, 25], it is most appropriate to follow the evolution of $Y_{\Delta_k}$ where $\Delta_k = \frac{B_k}{3} - Y_{L_k} - Y_{L_{ks}} \equiv \frac{B_k}{3} - Y_{L_{tot}}^k$. This is so because $\Delta_k$ is conserved by sphalerons and by other MSSM interactions. In particular, notice that the MSSM processes enforce the equality of fermionic and scalar lepton asymmetries of the same flavour.

In Eqs.(4.1)–(4.3) we have accounted for the CP asymmetries in the $\tilde{N}_\pm$ two body decays to the order described in the previous sections. We have neglected higher order terms in supersymmetry-breaking parameters which could lead to differences in the distribution and thermal widths of $\tilde{N}_\pm$ and correspondingly we have written a unique BE for $Y_{\tilde{N}_{tot}} \equiv Y_{\tilde{N}_+} + Y_{\tilde{N}_-}$.

In Eq. (4.3) we have defined the flavoured thermal widths

$$\gamma_{\tilde{N}}^k = K_k^0 \gamma_{\tilde{N}}^k, \quad \gamma_{l}^{(l)} = K_k^0 \gamma_{l}^{(l)},$$

(4.4)

where the different $\gamma$’s are the thermal widths for the following processes (in all cases a sum over the CP conjugate final states is implicit):

$$\gamma_{\tilde{N}} = \gamma_{\tilde{N}}^f + \gamma_{\tilde{N}}^s = \gamma(\tilde{N}_+ \leftrightarrow \tilde{h}l) + \gamma(\tilde{N}_- \leftrightarrow h\tilde{l}),$$

$$\gamma_{\tilde{N}}^{(3)} = \gamma(\tilde{N}_- \leftrightarrow \tilde{e}^*\tilde{u}\tilde{q}),$$

$$\gamma_{22} = \gamma(\tilde{N}_- \leftrightarrow \tilde{e}^*\tilde{u}) = \gamma(\tilde{N}_- \leftrightarrow \tilde{e}^*\tilde{u}) = \gamma(\tilde{N}_- \leftrightarrow \tilde{e}^*\tilde{q}),$$

$$\gamma_{l}^{(5)} = \gamma(\tilde{N}_- \leftrightarrow \tilde{q}u) = \gamma(\tilde{N}_- \leftrightarrow \tilde{q}u),$$

$$\gamma_{l}^{(6)} = \gamma(\tilde{N}_- \leftrightarrow \tilde{q}u) = \gamma(\tilde{N}_- \leftrightarrow \tilde{q}u),$$

$$\gamma_{l}^{(7)} = \gamma(\tilde{N}_- \leftrightarrow \tilde{q}u) = \gamma(\tilde{N}_- \leftrightarrow \tilde{q}u),$$

$$\gamma_{l}^{(8)} = \gamma(\tilde{N}_- \leftrightarrow \tilde{q}u),$$

$$\gamma_{l}^{(9)} = \gamma(\tilde{N}_+ \leftrightarrow \tilde{u}l) = \gamma(\tilde{N}_+ \leftrightarrow \tilde{q}u),$$

$$\gamma_{l} = \gamma(N \leftrightarrow \ell h) + \gamma(N \leftrightarrow \tilde{e}^*\tilde{h}),$$

$$\gamma_{l}^{(0)} = \gamma(N \leftrightarrow \ell h) + \gamma(N \leftrightarrow \tilde{e}^*\tilde{h}),$$

$$\gamma_{l}^{(1)} = \gamma(N \leftrightarrow \ell h) + \gamma(N \leftrightarrow \tilde{e}^*\tilde{h}),$$

$$\gamma_{l}^{(2)} = \gamma(N \leftrightarrow \ell h) + \gamma(N \leftrightarrow \tilde{e}^*\tilde{h}),$$

$$\gamma_{l}^{(3)} = \gamma(N \leftrightarrow \ell h) + \gamma(N \leftrightarrow \tilde{e}^*\tilde{h}),$$

$$\gamma_{l}^{(4)} = \gamma(N \leftrightarrow \ell h) + \gamma(N \leftrightarrow \tilde{e}^*\tilde{h}).$$

(4.5)

The explicit expressions for the $\gamma$’s in Eq. (4.5) can be found, for example, in [27] for the case of Boltzmann-Maxwell distribution functions and neglecting Pauli-blocking and
stimulated emission as well as the relative motion of the particles with respect to the plasma.

The value of $A_{\alpha\beta}$ depends on which processes are in thermal equilibrium when leptogenesis takes place. As we will see below for any of the considered sources of CP violation, the relevant temperature window around $M \sim T$ corresponds to $T < (1 + \tan^2 \beta) \times 10^9$ GeV. In this regime the processes mediated by all the three charged lepton ($e, \mu, \tau$) Yukawa couplings are in equilibrium i.e. they are faster than the processes involving $\tilde{N}_\pm$ and one gets [26]

$$A = \begin{pmatrix} 0.93 & 6.25 & 6.25 \\ 3/31 & 1/30 & 1/30 \\ 3/31 & 1/30 & -19/30 \end{pmatrix}. \quad (4.6)$$

After conversion by the sphaleron transitions, the final baryon asymmetry is given by

$$Y_B = \frac{24 + 4n_H}{66 + 13n_H} Y_{B-L}(z \to \infty) = \frac{8}{23} \sum_k Y_{\Delta_k}(z \to \infty), \quad (4.7)$$

where $\sum_k Y_{\Delta_k}(z \to \infty)$ can be parametrized as

$$\sum_k Y_{\Delta_k}(z \to \infty) = -2(\bar{\epsilon}^S + \bar{\epsilon}^V + \bar{\epsilon}^I) \eta_{fla}, \quad (4.8)$$

with $\bar{\epsilon}^S$, $\bar{\epsilon}^V$, and $\bar{\epsilon}^I$ are given in Eqs.(2.39)–(2.41). $\eta_{fla}$ is the dilution factor which takes into account the possible inefficiency in the production of the singlet sneutrinos, the erasure of the generated asymmetry by $L$-violating scattering processes and the temperature and flavour dependence of the CP asymmetry. It is obtained by solving the array of BE above. Within our approximations for the thermal widths, $\eta_{fla}$ depends on the flavour projections $K^0_k$, on the Yukawa couplings $(YY^\dagger)_{11}$ and on the heavy mass $M$, with the dominant dependence on these last two arising in the combination

$$(YY^\dagger)_{11} v_u^2 \equiv m_{eff} M, \quad (4.9)$$

where $v_u$ is the vacuum expectation value of the up-type Higgs doublet, $v_u = v \sin \beta$ ($v=174$ GeV). There is a residual dependence on $M$ due to the running of the top Yukawa coupling as well as the thermal effects included in $\Delta_{BF}$ although it is very mild. Also, as long as $\tan \beta$ is not very close to one, the dominant dependence on $\tan \beta$ arises via $v_u$ as given in Eq. (4.9) and it is therefore very mild.

In Fig. III we plot $|\eta_{fla}|$ as a function of $m_{eff}$ for $M = 10^7$ GeV and for the the equally distributed flavour composition $K^0_1 = K^0_2 = K^0_3 = 1/3$. We consider two different initial conditions for the sneutrino abundance. In one case, one assumes that the $\tilde{N}$ population is created by their Yukawa interactions with the thermal plasma, and set $Y_{\tilde{N}}(z \to 0) = 0$. The other case corresponds to an initial $\tilde{N}$ abundance equal to the thermal one, $Y_{\tilde{N}}'(z \to 0) = Y_{\tilde{N}}^{eq}(z \to 0)$. As discussed in Ref. [16] for zero initial conditions, $\eta$ can take both signs depending on the value of $m_{eff}$ while for thermal initial conditions, on the contrary, $\eta > 0$. 

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Figure III: Efficiency factor $|\eta_{\text{fla}}|$ as a function of $m_{\text{eff}}$ for $M = 10^7$ GeV and $\tan \beta = 30$ and for $K_1^0 = K_2^0 = K_3^0 = 1/3$. The two curves correspond to vanishing initial $\tilde{N}$ abundance (solid black curve) and thermal initial $\tilde{N}$ abundance (dashed red curve).

Introducing the resulting $\eta_{\text{fla}}$ in Eqs.(4.8) and (4.7) we can easily quantify the allowed ranges of parameters for which enough asymmetry, $Y_B \geq 8.54 \times 10^{-11}$ [28], is generated. We plot in Fig. IV the resulting ranges for $B$ and $m_{\text{eff}}$ for the equally distributed flavour composition $K_1^0 = K_2^0 = K_3^0 = 1/3$ and for $|A| = m_2 = 1$ TeV and $\tan \beta = 30$. For different values of the CP phases and $M$ as explicitly given in the figure.

The upper panels in Fig. IV give the parameters regions for which the CP violation from pure mixing effects, $\epsilon^S$, can produce the observed asymmetry as previously described in [14–16]. Due to the resonant nature of this contribution, these effects are only large enough for $B \sim \mathcal{O}(\Gamma)$ which leads to the well-known condition of the unconventionally small values of $B$ and to the upper bound $M \lesssim 10^9$ GeV.

The central panels of Fig.IV give the corresponding regions for which CP violation from gaugino-induced vertex effects, $\epsilon^V$, can produce the observed baryon asymmetry. Despite being higher order in $\delta_S$ and including a loop suppression factor, $\alpha_2$, this contribution can be relevant because it is dominant for conventional values of the $B$ parameter. However, in order to overcome the loop and $\delta_S$ suppressions this contribution can only be sizeable for lighter values of the RH sneutrino masses $M \lesssim 10^6$ GeV (within the approximation used in this work: $\delta_S \ll 1$, $|A|, m_2 \sim \mathcal{O}(\text{TeV})$).

The parameters chosen in the figure are such that the second term in Eq.(2.40) dominates so that the allowed region depicts a lower bound on $B$. Conversely, when the first term in Eq.(2.40) dominates, $\epsilon^V$ becomes independent of $B$. In this case, for a given value of $M$ and $\delta_S$ the produced baryon asymmetry can be sizeable within the range of $m_{\text{eff}}$
values for which $\eta_{\text{fla}}$ is large enough. For example for $M = 10^5$ GeV, and $m_2 = |A| = 1$ TeV and $|\sin(\phi_A + 2\phi_g)| = 1$ with vanishing initial conditions

$$10^{-5} < \frac{m_{\text{eff}}}{\text{eV}} < 6.5 \times 10^{-4} \quad \text{or} \quad 8 \times 10^{-4} < \frac{m_{\text{eff}}}{\text{eV}} < 3 \times 10^{-2}, \quad (4.10)$$

where each range corresponds to a sign of the CP phase $\sin(\phi_A + 2\phi_g)$.

Finally we show in the lower panels of Fig.IV the values of $B$ and $m_{\text{eff}}$ for which enough baryon asymmetry can be generated from the interference of mixing and vertex corrections $\epsilon^I$, Eq.(2.41). Generically $\epsilon^S$, is subdominant to $\epsilon^S$ since both involve the same CP phase $\sin(\phi_A)$ while $\epsilon^I$ has additional $\delta_S$ and loop suppressions:

$$\frac{\epsilon^I}{\epsilon^S} = \frac{-3}{8} \frac{m_2}{M} \ln \frac{m_2^2}{M^2 + m_2^2} \frac{\cos(2\phi_g) \Gamma_B}{B}. \quad (4.11)$$

Consequently as seen in the above equation and illustrated in the figure, $\epsilon^I$ can only dominate for extremely low values of $B$ ($B \ll \Gamma$) for which it becomes independent of $B$.

Also we notice that for $M \lesssim 10^4$ GeV and $m_{\text{eff}} \gtrsim 10^{-2}$ eV the resulting baryon asymmetry generated by this contribution becomes independent of $m_{\text{eff}}$ since the $m_{\text{eff}}^2$ dependence from $\Gamma^2$ cancels the approximate $1/m_{\text{eff}}^2$ dependence of $\eta_{\text{fla}}$ in this strong washout regime.

In summary in this work we have quantified in detail the contributions to CP violation in right-handed sneutrino decays induced by soft supersymmetry-breaking gaugino masses paying special attention to the role of thermal effects. Using a field-theoretical as well as a quantum mechanical approach we conclude that for all the soft supersymmetry-breaking sources of CP violation considered, an exact cancellation between the asymmetries produced in the fermionic and bosonic channels occurs at $T = 0$ up to second order in soft supersymmetry-breaking parameters. However, once thermal effects are included the new sources of CP violation induced by soft supersymmetry-breaking gaugino masses can be sizeable and they can produce the observed baryon asymmetry for conventional values of the $B$ parameter.

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Figure IV: $B, m_{\text{eff}}$ regions in which successful soft leptogenesis can be achieved when flavour effects are included with $K_1 = K_2 = K_3 = 1/3$ and for different sources of CP violation. In all cases we take $A| = m_2 = 10^3$ GeV and $\tan \beta = 30$ and different values of $M$ and $\phi_A$ and $\phi_g$ as labeled in the figure (see text for details). The left (right) panels correspond to vanishing (thermal) initial $N$ abundance.