Chiral Corrections to the Hyperon Vector Form Factors

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Abstract

We present the complete calculation of the SU(3)-breaking corrections to the hyperon vector form factors up to $O(p^4)$ in the Heavy Baryon Chiral Perturbation Theory. Because of the Ademollo–Gatto theorem, at this order the results do not depend on unknown low energy constants and allow to test the convergence of the chiral expansion. We complete and correct previous calculations and find that $O(p^3)$ and $O(1/M_0)$ corrections are important. We also study the inclusion of the decuplet degrees of freedom, showing that in this case the perturbative expansion is jeopardized. These results raise doubts on the reliability of the chiral expansion for hyperons.
1 Introduction

At present, the most precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{us}$ [1] is obtained from kaon semileptonic decays (for recent reviews see e.g. [2] and references therein). In this case, the uncertainties from both experiments and theory allow to reach the percent precision, which could be further improved in the near future. The experiments allow to extract with good accuracy the product $|V_{us} \cdot f_{+}(0)|^2$ where $f_{+}(0)$ is the vector form factor (vff) at zero momentum transferred. Because of the conservation of the vector current, the latter is known up to SU(3)-breaking corrections, which are suppressed because of the Ademollo-Gatto theorem [3]. The leading contribution to these corrections can be determined univocally in Chiral Perturbation Theory [4, 5], while the subleading corrections can been estimated via quark models [5], large-$N_c$ [6] or in a model independent way via Lattice QCD simulations [7, 8, 9].

Since $V_{us}$ together with $V_{ud}$ allow the most stringent test of CKM unitarity, it would be interesting to have other independent estimates for these quantities.

It has been recently pointed out [10] that $V_{us}$ can be extracted also from hyperon semileptonic decays in a similar way as from kaon decays. In particular, experimental data can be combined to extract the product $|V_{us} \cdot f_{1}(0)|^2$ where $f_{1}(0)$ is the hyperon vector form factor at zero momentum. As for the kaon case, Ademollo-Gatto theorem protects $f_{1}(0)$ from the leading SU(3)-breaking corrections and make these decays, at least in principle, good alternatives for the extraction of $V_{us}$. There exist several estimates for $f_{1}(0)$ using quark models [11], large-$N_c$ [12] and chiral expansions [13, 14, 15]. However, they disagree between each other, and a model independent estimate becomes mandatory (see also [16] for a recent discussion). In ref. [8, 17], a lattice study has been implemented demonstrating that the method of [7] for the extraction of SU(3)-breaking corrections from the lattice can be performed also in the hyperon case. However, because of the limitations of the actual numerical simulations it is necessary to estimate the leading chiral corrections to drive the extrapolation to low quark masses. We thus need an estimate for the leading chiral corrections to the hyperon vff. However, existing calculations [13, 14, 15] in (Heavy) Baryon Chiral Perturbation Theory (H)B$\chi$PT [18] for these amplitudes are not complete (and disagree between each other).

In this paper we perform the full $O(p^4)$ calculation in H$\chi$PT of hyperon decays. We find that the subleading $O(p^4)$ contributions (including $1/M_0$ relativistic corrections) are important and cannot be neglected. The results present cancellations between different contributions that make the estimates strongly dependent on the low energy parameters (masses and couplings). We also study the inclusion of decuplet degrees of freedom in the H$\chi$PT formalism developed in [19, 20], finding huge contributions that make the calculation unreliable. Because of these results, H$\chi$PT does not seem to be of help for the determination of $f_{1}(0)$ and lattice simulations closer to the chiral limit have to be performed to obtain a model independent estimate of these quantities.

The paper is organized as follows. In section 2 we review H$\chi$PT and fix our notations. In section 3 we present the results for the vff, in particular $O(p^2)$, $O(1/M_0)$ and $O(p^3)$ corrections are studied in subsections 3.1, 3.2 and 3.3 respectively and the final discussion on octet contributions is given in subsection 3.4. In section 4 we present our results for the decuplet contributions. The conclusions are given in section 5. Finally, in the appendices A and B we give the explicit expressions for the $O(p^3)$ octet and decuplet corrections respectively.
2 The effective field theory for (heavy) baryons

Chiral Perturbation Theory (χPT) is the effective field theory for the lightest mesons of QCD. Since these states are the quasi-Goldstone modes of the spontaneously-broken SU(3)\(_L\)×SU(3)\(_R\) chiral symmetry, a perturbative expansion can be performed as long as energies below the QCD scale (∼ \(m_\rho\)) are considered [21]. The symmetries of the underlying theory (QCD) fix completely the form of the interactions [22]. At each order of the expansion, the high energy dynamics is encoded in a finite number of effective couplings. These low energy constants (LECs) can be estimated using experiments or with non-perturbative approaches, such as Lattice QCD.

In principle, χPT can be extended to include fermionic degrees of freedom, (e.g. baryons), whose properties under chiral transformations fix their couplings to mesons. The lightest baryons, however, lie above the regime of validity of χPT and a naive inclusion of these states would break the power counting. A way to overcome this problem has been found in [18] by adapting the method for heavy quarks of ref. [23]. The Heavy Baryon Chiral Perturbation Theory (HBχPT) of ref. [18] is obtained by integrating out the heavy components of the fermionic fields that ruin the power counting, i.e. considering baryons in the non-relativistic limit. In this way, only light degrees of freedom are dynamical and the power counting is consistent at all orders of the chiral expansion. In HBχPT we thus have a double expansion: the chiral expansion in powers of \(p/\Lambda_{\chi}\) (\(\Lambda_{\chi} \sim 1 \text{ GeV}\)) and the heavy baryon expansion in powers of \(p/M_B\). Since \(M_B \sim \Lambda_{\chi}\) the two expansions can be joined together. As we will see, however, the degree of convergence of the perturbative series is rather poor and higher order corrections give important contributions.

In order to fix our convention we will report here the effective Lagrangian for baryons and mesons:

\[
\mathcal{L} = \mathcal{L}_M^{(2)} + \mathcal{L}_B^{(1)} + \mathcal{L}_B^{(2)} + \mathcal{L}_B^{(3)} + \ldots
\]  

The purely mesonic sector reads [24]:

\[
\mathcal{L}_M^{(2)} = \frac{f^2}{2} \langle A^\mu A_\mu + \rho \chi \rangle ,
\]  

where \(\langle \ldots \rangle\) indicates the trace on flavor indices and

\[
A_\mu = \frac{1}{2} i \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right) , \quad \chi = \frac{1}{2} \left( \xi^\dagger M \xi^\dagger + \xi M \xi \right) , \quad M = \text{diag} \left( m_u, m_d, m_s \right) ,
\]

\[
\xi = e^{\Phi / f} , \quad \Phi = \begin{pmatrix} \pi^0 \sqrt{2} + \eta \sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0 \sqrt{2} + \eta \sqrt{6} & K^0 \\ K^- & K^0 & -\frac{2}{\sqrt{6}} \eta \end{pmatrix}.
\]

The only parameters appearing in \(\mathcal{L}_M^{(2)}\) are the meson decay constant \(f\) (normalized so that \(f_\pi \simeq 132 \text{ MeV}\)) and the meson masses, which in the SU(2) limit \((m_\ell = m_u = m_d)\) read:

\[
m_\pi^2 = 2 \rho m_\ell , \
m_K^2 = \rho (m_\ell + m_s) , \
m_1^2 = \frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2 .
\]

The leading order \(O(p)\) HBχPT Lagrangian is [18]:

\[
\mathcal{L}_B^{(1)} = \langle \bar{B} i v^\mu D_\mu B \rangle + 2D \langle \bar{B} S^\mu \{ A_\mu , B \} \rangle + 2F \langle \bar{B} S^\mu [ A_\mu , B ] \rangle ,
\]
\[ D_\mu B = \partial_\mu B - i[V_\mu, B], \quad V_\mu = \frac{1}{2}i \left( \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right), \]  
\( B = \begin{pmatrix} \frac{\Sigma_0}{\sqrt{2}} + \frac{A}{\sqrt{6}} & \Sigma^+_0 p & \frac{n}{\sqrt{2}} + \frac{A}{\sqrt{6}} \vspace{12pt} \\
\Sigma^-_0 & \frac{n}{\sqrt{2}} & -\frac{2}{\sqrt{6}} \vspace{12pt} \\
\Xi^- & \Xi^0 & \Xi 
\end{pmatrix}, \]

\( v^\mu = p^\mu / M_0 \) is the four-velocity of the baryon and \( S^\mu = i\gamma_5 \sigma^{\mu\nu} v_\nu / 2 \) is the spin operator. \( F \) and \( D \) are the axial couplings, whose physical values are known with good accuracy [10]. From eq. (6) we can read the baryon propagator:

\[ \frac{i}{k \cdot v + i\varepsilon}, \]

where \( k \sim \mathcal{O}(p) \) is the off-shell momentum carried by the light field component \( B \) and, at this order, is related to the momentum \( p^\mu \) of the physical baryon via the relation

\[ p^\mu = M_0 v^\mu + k^\mu, \]

where \( M_0 \) is the baryon mass in the chiral limit and \( p^2 = M_0^2 \) on-shell.

At the next order we have \( \mathcal{L}^{(2)}_B \), which receives contributions both from genuine \( \mathcal{O}(p^2) \) chiral corrections and from \( 1/M_0 \) corrections:

\[ \mathcal{L}^{(2)}_B = \mathcal{L}^{(2)}_B (p^2) + \mathcal{L}^{(2)}_B (1/M_0), \]

\[ \mathcal{L}^{(2)}_B (p^2) = \sigma \langle \chi \rangle \langle B B \rangle + 2 b_D \langle \bar{B} \{ \chi, B \} \rangle + 2 b_F \langle \bar{B} [\chi, B] \rangle + \ldots \]

\[ \mathcal{L}^{(2)}_B (1/M_0) = \frac{1}{2M_0} \langle \bar{B} \left( (v^\mu \partial_\mu)^2 - \partial^\mu \partial_\mu \right) B \rangle \]

\[ + \frac{i v^\nu}{M_0} D \langle \partial_\mu \bar{B} S^\mu \{ A_\nu, B \} - \bar{B} S^\mu \{ A_\nu, \partial_\mu B \} \rangle \]

\[ + \frac{i v^\nu}{M_0} F \langle \partial_\mu \bar{B} S^\mu \{ A_\nu, B \} - \bar{B} S^\mu \{ A_\nu, \partial_\mu B \} \rangle + \ldots \]

where we wrote explicitly only those operators giving a non-vanishing contribution to the hyperon vff. The three terms in eq. (12) give the lowest order chiral corrections \( \delta M_B \sim \mathcal{O}(p^2) \) to the baryon mass \( M_0 \):

\[ \frac{i}{k \cdot v - \delta M_B + i\varepsilon} \approx \left[ \frac{i}{k \cdot v + i\varepsilon} + \frac{i}{k \cdot v + i\varepsilon} (-i\delta M_B) \right] \left[ \frac{i}{k \cdot v + i\varepsilon} \right], \]

and read:

\[ \delta M_N = 2 b_F (m_s - m_\ell) - 2 b_D (m_\ell + m_s) - \sigma (2 m_\ell + m_s), \]

\[ \delta M_\Sigma = -4 b_D m_\ell - \sigma (2 m_\ell + m_s), \]

\[ \delta M_\Lambda = -\frac{4 b_D}{3} (m_\ell + 2 m_s) - \sigma (2 m_\ell + m_s), \]

\[ \delta M_\Xi = -2 b_F (m_s - m_\ell) - 2 b_D (m_\ell + m_s) - \sigma (2 m_\ell + m_s). \]

The terms in the first line of eq. (13) correspond to the \( 1/M_0 \) corrections to the baryon propagator:

\[ \frac{i}{p - M_0 + i\varepsilon} \rightarrow \frac{i}{k \cdot v + i\varepsilon} + \frac{i}{k \cdot v + i\varepsilon} \left( \frac{k^2 - (k \cdot v)^2}{2M_0} \right) \frac{i}{k \cdot v + i\varepsilon}, \]
while the terms in the second and third line are the relativistic corrections to the leading order (LO) axial couplings of eq. (6). Note that in $\mathcal{L}_B^{(2)}(1/M_0)$ no new LECs appear since all the coefficients are fixed by Lorentz symmetry. On the other hand, the coefficients of the operators in $\mathcal{L}_B^{(2)}(p^2)$ are undetermined. However, since in the calculation of the vff at $\mathcal{O}(p^4)$ only the baryon mass-shift operators of eq. (12) will contribute, no unknown LEC appears at this order.

Since each meson loop corresponds to corrections of $\mathcal{O}(p^2)$, operators of order $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ of the HB$\chi$PT Lagrangian can only enter at tree level in the calculation of the vff at $\mathcal{O}(p^4)$. However, because of the Ademollo-Gatto theorem, the first tree level non-vanishing contribution starts at $\mathcal{O}(p^5)$ (since the LO is $\mathcal{O}(p)$), therefore only the operators in $\mathcal{L}_B^{(1)}$ and $\mathcal{L}_B^{(2)}$ will contribute. As a by-product, in the vff at $\mathcal{O}(p^4)$, loop contributions are finite and no unknown LEC appears.

### 3 Vector form factors for hyperons

The vector form factor $f_1(q^2)$ for baryons is defined via the matrix element of the SU(3) vector current as follows:

$$
\langle B_2| V^\mu | B_1 \rangle = \mathcal{B}_2(p_2) \left[ \gamma^\mu f_1(q^2) - i \sigma^{\mu\nu} q_\nu f_2(q^2) + \frac{q^\mu}{M_1 + M_2} f_3(q^2) \right] B_1(p_1).\tag{17}
$$

where $q^\mu = p_1^\mu - p_2^\mu$. In the SU(3) limit the vff at zero momentum $f_1(0)$ are fixed by the conservation of the SU(3)$_V$ charge. Out of the SU(3) limit Ademollo-Gatto theorem states that linear corrections in the breaking ($m_s - m_\ell$) vanish, i.e.

$$
f_1(0) = f_1^{SU(3)}(0) + \mathcal{O}(m_s - m_\ell)^2.\tag{18}
$$

The expression for the vector current in HB$\chi$PT can be extracted from eqs. (2), (6) and (11) by coupling to the system an external vector current and by varying the action with respect to it. The vff at zero momentum can thus be extracted by looking at the terms proportional to $v^\mu$ (see [25]) in the heavy baryon limit of the matrix element of eq. (17).

In the following, we will parametrize chiral corrections to the vff as follows:

$$
f_1(0) = \alpha^{(1)} \left[ 1 + \alpha^{(2)} + \left( \alpha^{(3)} + \alpha^{(1/M)} \right) + \ldots \right].\tag{19}
$$

$\alpha^{(1)} = f_1^{SU(3)}(0)$ is the tree-level $\mathcal{O}(p)$ amplitude. $\alpha^{(2)}$ is the one-loop correction ($\mathcal{O}(p^3)$) and it is $\mathcal{O}(p^2)$ with respect to $\alpha^{(1)}$. $\alpha^{(3)}$ and $\alpha^{(1/M)}$ are respectively the $\mathcal{O}(p^4)$ chiral contribution and the leading $1/M_0$ corrections, and both are $\mathcal{O}(p^3)$ with respect to $\alpha^{(1)}$.

The tree-level amplitudes give just the SU(3)$_V$ charges and read:

$$
\alpha^{(1)}_{\Sigma^+ - n} = -1, \quad \alpha^{(1)}_{\Lambda p} = -\sqrt{\frac{3}{2}}, \quad \alpha^{(1)}_{\Xi^- - \Lambda} = \sqrt{\frac{3}{2}}, \quad \alpha^{(1)}_{\Xi^- - \Sigma^0} = \frac{1}{\sqrt{2}}.\tag{20}
$$

#### 3.1 $\mathcal{O}(p^2)$ corrections

The leading SU(3)-breaking corrections occur at one-loop in the chiral expansion. The relevant diagrams, which give a non-vanishing contribution at this order, are those labeled as (z1), (ta), (tb), (s1a) and (s1b) in figure 1. These diagrams can be divided into two classes: tadpoles [(ta),(tb)] and sunsets [(z1),(s1a),(s1b)]. While the latter will receive $1/M_0$ corrections, in the former baryons do not
Figure 1: Feynman diagrams of octet contributions to the vff up to $O(p^4)$ in the chiral expansion. Solid (dashed) lines are baryons (mesons), crosses are vector current insertions, filled circles are $O(p^2)$ operators, empty boxes are $1/M_0$ corrections and crossed boxes are $1/M_0$ corrections to the vector current. The graphs with multiple insertions of $O(p^2)$ or $1/M_0$ operators are a short-hand notation and correspond to the sum of the corresponding graphs with single insertions.

The results we found for $O(p^2)$ corrections, according to the notation of eq. (19), read

$$
\alpha^{(2)}_{\Sigma^-\Lambda} = -\frac{3}{8}(H_{\eta K} + H_{\pi K}) - \frac{9}{8}(D - F)^2(H_{\eta K} + H_{\pi K}) + D^2H_{\pi K},
$$

$$
\alpha^{(2)}_{\Lambda\rho} = -\frac{3}{8}(H_{\eta K} + H_{\pi K}) - \frac{1}{8}(D + 3F)^2(H_{\eta K} + H_{\pi K}) - D^2H_{\pi K},
$$

$$
\alpha^{(2)}_{\Xi^-\Lambda} = -\frac{3}{8}(H_{\eta K} + H_{\pi K}) - \frac{1}{8}(D - 3F)^2(H_{\eta K} + H_{\pi K}) - D^2H_{\pi K},
$$

$$
\alpha^{(2)}_{\Xi^-\Sigma^0} = -\frac{3}{8}(H_{\eta K} + H_{\pi K}) - \frac{9}{8}(D + F)^2(H_{\eta K} + H_{\pi K}) + D^2H_{\pi K} .
$$

The $O(p^2)$ function $H_{pq}$ is defined as

$$
H_{pq} \equiv \frac{1}{(4\pi f)^2} \left[ m_p^2 + m_q^2 - 2m_p^2m_q^2 m_p^2 m_q^2 \log \left( \frac{m_q^2}{m_p^2} \right) \right],
$$

(22)
and satisfies the Ademollo-Gatto theorem.

In eq. (21) the contributions independent of $F$ and $D$ come from tadpoles while the rest come from sunset diagrams. It is interesting to notice that the tadpole contributions is universal (the same for all the channels) and is actually the same as in the $K^0 \rightarrow \pi^- v_{ff}$:

$$-\frac{3}{8} (H_{\eta K} + H_{\pi K}) = -0.023.$$  \hspace{1cm} (23)

The sunset contributions, on the other hand, are different in each channel, and can have either signs. Moreover, these contributions receive important $1/M_0$ and $\delta M_B$ corrections as shown in the next sections.

The results in eq. (21) agree with those of Krause [13] and Kaiser [15]. However they do not agree with those of Anderson and Luty [14], which seem to fail the overall sign for the tadpole contributions. The total $O(p^2)$ corrections, using physical values for masses and couplings (see table 1), are reported in table 2.

### 3.2 $O(1/M_0)$ relativistic corrections

The $O(1/M_0)$ corrections due to the non-relativistic expansion are produced by the terms in eq. (13). There are corrections to the propagators, the diagrams (z3a), (s3g), (s3h) in figure 1, to the strong vertices [(z3b),(z3c),(s3a),(s3b),(s3c),(s3d)] and to the vector current [(s3e),(s3f)]. All these contributions come from sunset diagrams, are $F$ and $D$ dependent and provide the relativistic corrections to the sunset contributions of eq. (21). They give the following contributions to the vff:

$$\alpha_{\Sigma^- n}^{(1/M)} = -\frac{9}{8}(D - F)^2(H_{\eta K}^\prime + H_{\pi K}^\prime) + D^2 H_{\pi K}^\prime,$$

$$\alpha_{\Lambda p}^{(1/M)} = -\frac{1}{8}(D + 3F)^2(H_{\eta K}^\prime + H_{\pi K}^\prime) - D^2 H_{\pi K}^\prime,$$

$$\alpha_{\Xi^- \Lambda}^{(1/M)} = -\frac{1}{8}(D - 3F)^2(H_{\eta K}^\prime + H_{\pi K}^\prime) - D^2 H_{\pi K}^\prime,$$

$$\alpha_{\Xi^- \Sigma^0}^{(1/M)} = -\frac{9}{8}(D + F)^2(H_{\eta K}^\prime + H_{\pi K}^\prime) + D^2 H_{\pi K}^\prime,$$  \hspace{1cm} (24)

where

$$H_{p,q}^\prime \equiv -\frac{2\pi}{3(4\pi)^2M_0} \frac{(m_p - m_q)^2}{m_p + m_q} \left( \frac{m_p^2 + 3m_p m_q + m_q^2}{m_p + m_q} \right),$$  \hspace{1cm} (25)

satisfies the Ademollo-Gatto theorem. The contributions of eqs. (24) were not considered in refs. [14, 15]. They agree with the $1/M_0$ expansion of the relativistic result of ref. [13]. This is due to the fact

| $m_\pi$ | 0.138 | $M_N$ | 0.939 | $M_{\Delta_0}$ | 1.232 |
| $m_K$ | 0.496 | $M_{\Sigma}$ | 1.193 | $M_{\Delta_1}$ | 1.384 |
| $m_\eta$ | 0.548 | $M_\Lambda$ | 1.116 | $M_{\Delta_2}$ | 1.533 |
| $f$ | 0.132 | $M_\Xi$ | 1.318 | $M_{\Delta_3}$ | 1.672 |
| $D$ | 0.804 | $M_0$ | 1.151 | $\Delta$ | 0.231 |
| $F$ | 0.463 | $C$ | 1.6 | | |

Table 1: Numerical values for masses and couplings used in the text (dimensionful quantities are in GeV).
that the relativistic corrections of eq. (24), at this order, are non-analytic in the quark masses, thus according to the discussion in [26], do not break the chiral expansion.

The physical values for the corrections in eq. (24) are reported in table 2. They are important, as already noticed in [13], and tend to cancel the $F$ and $D \mathcal{O}(p^2)$ contributions.

Moreover, these corrections contain the new parameter $M_0$, which is actually defined as the baryon mass in the chiral limit. At this order, however, it should be safe to choose any physical baryon mass, as long as the perturbative expansion holds, since the difference would be of $\mathcal{O}(p^5)$. By varying $M_0$ one can check this assumption and infer on the importance of higher order effects. The result is illustrated in figure 2, which shows that the dependence on $M_0$ is quite strong. In the plot is actually reported the dependence on $M_0$ of the full SU(3)-breaking corrections up to $\mathcal{O}(p^4)$. This fact was somewhat expected since, as observed also in the mesonic case, the chiral expansion with three flavors converges very slowly and contributions as large as the leading one can be expected from the resummation of higher order corrections.

### 3.3 $\mathcal{O}(p^3)$ corrections

$\mathcal{O}(p^3)$ corrections to the vff are produced by one-loop diagrams via the insertion of one $\mathcal{O}(p^2)$ operator. There are many $\mathcal{O}(p^2)$ operators with unknown LECs in the HBχPT. However, only those generating baryon mass shifts and reported explicitly in eq. (12) give non-vanishing contributions to the vff. This allows to study also $\mathcal{O}(p^3)$ corrections without the uncertainties coming from higher order LECs.

In order to calculate these contributions to the vff one can just add mass-shift effects to the baryon propagators in the $\mathcal{O}(p^2)$ diagrams. Note that tadpole diagrams do not contribute because baryons do not enter the loop (external lines are taken on-shell at the shifted mass value). Therefore, also the $\mathcal{O}(p^3)$ corrections, as the $1/M_0$ ones, will be proportional to the axial couplings $D$ and $F$. There is however a subtlety. At this order, indeed, the incoming and the outcoming baryons will no longer be degenerate in mass. In particular this means that, at $q^2 = 0$, $q^\mu \neq 0$ and a non vanishing momentum has to be injected into the loop. We can parametrize on-shell momenta as follows:

\[ p_1^\mu = M_1 v^\mu , \quad p_2^\mu = M_1 v^\mu - q^\mu , \quad M_{1(2)} = M_0 + \delta M_{1(2)} \]
\[ q^2 = 0, \quad p_2^2 = M_1^2 - 2M_1 v \cdot q = M_2^2, \]
\[ \Rightarrow \quad v \cdot q = \frac{M_1^2 - M_2^2}{2M_1} \approx (\delta M_1 - \delta M_2) + O(\delta M)^2. \]  

(26)

Since \( \delta M_{1(2)} \sim O(p^2) \) also \( v \cdot q \sim O(p^2) \) and it can be expanded as a mass insertion. The corresponding contribution is represented as a mass insertion on external lines in the graphs (2a), (s2a) and (s2b) in figure 1.

The explicit expression for the \( O(p^3) \) chiral corrections is rather lengthy and is reported in appendix A. Notice that, because of the Ademollo-Gatto theorem only the differences of baryon masses appear \( (\delta M_1 - \delta M_2 = M_1 - M_2) \), and the result does not depend on the choice of \( M_0 \).

The same contributions were calculated in ref. [14]. However we do not agree with that result. Ademollo-Gatto theorem can be checked explicitly from our eqs. (37)-(40). In [14] the authors declare to have made the same check too, unfortunately only the formulae for \( m_\pi = 0 \) are given in [14]. Numerically our results give smaller SU(3)-breaking corrections (see table 2). However they still remain important with respect to the leading corrections and cannot be neglected.

Note that these contributions do not receive \( 1/M_0 \) relativistic corrections at this order in the chiral expansion. These contributions, indeed, would be \( O(p^5) \), which we did not consider here.

### 3.4 Final results for octet contributions

In table 2 we showed the numerical estimates for the sum of all 1-loop contributions up to \( O(p^4) \) using the physical values for masses and couplings of table 1. The final results are slightly smaller than previously claimed\(^1\) in ref. [14]. However they are still large compared to what expected by CKM unitarity [10], and opposite in signs with respect to quark model estimates [11]. The explicit calculation of the vff shows that \( O(p^3) \) and \( 1/M_0 \) corrections are not small and the expansion converge very slowly. By varying \( M_0 \) (see figure 2) and the parameters \( D \) and \( F \) between their physical value (table 1) and what is expected to be their value in the chiral limit (see e.g. [18]), SU(3) corrections vary substantially. This is mainly due to the fact that the dependence on \( D \) and \( F \) is quadratic and the various corrections in table 2 tend to cancel each other. Going beyond the \( O(p^4) \) calculation would be challenging and useless since unknown LECs would appear. The impact of higher \( 1/M_0 \) corrections, on the other hand, could be studied by calculating the amplitude within a relativistic framework (see e.g. [26]). Note also that lattice QCD [17] and quark models calculations [11] suggest that local contributions (which would start at \( O(p^3) \) in the chiral expansion) give a negative contribution.

Another issue is represented by contributions from the decuplet states. In the calculation above decuplet states are taken to be integrated out, and their contribution reabsorbed into the parameters of the chiral Lagrangian. Decuplet degrees of freedom, however, are not so heavy to be safely considered frozen and non-analytic contributions can be important. We will study their effects in the next section. For all these reasons, the uncertainty in the SU(3)-breaking corrections of table 2 should be taken of order one.

As last remark we must say that the lack of convergence of the \( O(p^4) \) Lagrangian does not appear peculiar of the vff only, but of the 3-flavors HBχPT expansion itself. In view of these results, we think that it is dangerous to trust one-loop calculations, especially if \( 1/M_0 \) and \( O(p^3) \) corrections are not taken into account. For other quantities, indeed, the lack of convergence can hide itself behind the ignorance of the LECs.

\(^1\)Note that in [14] authors used chiral limit values for \( F \) and \( D \), which are sensibly smaller than the physical ones. We preferred to use the latter which are better known. The difference, assuming the chiral expansion holds, is an higher order effect.
\[
\Sigma^- \rightarrow n, \quad \Lambda \rightarrow p, \quad \Xi^- \rightarrow \Lambda, \quad \Xi^- \rightarrow \Sigma^0
\]

| Decay       | \(\alpha^{(1)}\) | \(\alpha^{(2)}(\times 10^2)\) | \(\alpha^{(3)}(\times 10^2)\) | \(\alpha^{(1/M)}(\times 10^2)\) | All \((\times 10^2)\) |
|-------------|------------------|-----------------------------|-----------------------------|-------------------------------|------------------|
| \(\Sigma^- \rightarrow n\) | -1 | +0.7 | +6.5 | -3.2 | +4.1 |
| \(\Lambda \rightarrow p\) | \(-\sqrt{3}/2\) | -9.5 | +4.3 | +8.0 | +2.7 |
| \(\Xi^- \rightarrow \Lambda\) | \(\sqrt{3}/2\) | -6.2 | +6.2 | +4.3 | +4.3 |
| \(\Xi^- \rightarrow \Sigma^0\) | \(1/\sqrt{2}\) | -9.2 | +2.4 | +7.7 | +0.9 |

Table 2: Numerical estimates of the chiral corrections \((\text{All} = \alpha^{(2)} + \alpha^{(3)} + \alpha^{1/M})\) to the vff. The physical values used for the parameters are reported in table 1.

4 Dynamical Decuplet

The calculations made so far rely on the assumptions that all other hadronic states can be safely integrated out. This is a good approximation if the relevant momenta are much smaller than the scale corresponding to higher hadronic states. In particular we need \(p \sim m_\pi, K \ll \Delta\), where \(\Delta\) is the mass shift between octet baryons and the lightest excitations. In QCD, these excitations are represented by the decuplet states, which unfortunately are rather light \(\Delta \sim 230\ \text{MeV}\). This means that these states can give dangerous non-analytic contributions that cannot be reabsorbed into local counterterms. Whether these contributions are actually important depends both on how the decuplet couples to the other fields and on the specific quantity under study. In order to make this statement more quantitative \((\text{HB}\chi\text{PT})\) has been extended also to these degrees of freedom \([19]\).

An issue that arises in embedding the decuplet within the framework of \(\text{HB}\chi\text{PT}\) is how to deal with the decuplet mass shift scale \(\Delta\). This scale, indeed, is neither large enough to be integrated out, nor is a chiral parameter, which could be tuned to be arbitrary small to make the expansion hold. In \([20]\) a phenomenological expansion was proposed where \(\Delta \sim m_\pi\) is treated as a small parameter of \(\mathcal{O}(p)\). Within this framework the decuplet contributions to the \(\text{HB}\chi\text{PT}\) Lagrangian can be organized as follows:

\[
\mathcal{L}_{10} = \mathcal{L}_{\Delta}^{(1)} + \mathcal{L}_{\Delta}^{(2)} + \mathcal{L}_{\Delta}^{(3)} + \ldots
\]

(27)

where the leading order terms read

\[
\mathcal{L}_{\Delta}^{(1)} = -iT^\mu T^\nu D_\nu \partial_\mu + \Delta \overline{T}^\mu T_\mu + C \left( \overline{T}^\mu A_\mu B + \overline{T} A_\mu T^\mu \right) + 2 \mathcal{H} \overline{T}^\mu S^\nu A_\nu T_\mu,
\]

(28)

\[
D_\mu T_\nu = \partial_\mu T_\nu - iV_\mu T_\nu.
\]

In eq. (28) flavor indices are not explicitly shown and \(T_\mu\) is the light component of the spin-3/2 Rarita-Schwinger field associated to the decuplet, the propagator of which (in \(d\)-dimensions) reads:

\[
\frac{i}{k^\nu - \Delta + i\varepsilon} \left( v_\mu v_\nu - g_\mu v_\nu - 4 \frac{d - 3}{d - 1} S_\mu S_\nu \right).
\]

(29)

In eq. (28), \(\mathcal{C}\) and \(\mathcal{H}\) are the effective couplings associated to the decuplet-octet-meson and the decuplet-decuplet-meson vertices, respectively. \(\mathcal{C}\) can be extracted from \(\Delta \rightarrow \pi N\) decays \([19]\). \(\mathcal{H}\), which is poorly known, does not contribute, however, to the hyperon vff at the order we are working.

Analogously to the octet case, we can parametrize the \(\text{SU}(3)\)-breaking corrections due to the decuplet as follows:

\[
\alpha^{(1)} \left[ \beta^{(2)} + \left( \beta^{(3)} + \beta^{(1/M)} \right) \right] + \ldots.
\]

(30)

The lowest order decuplet corrections \(\beta^{(2)}\) to the vff are \(\mathcal{O}(p^2)\) with respect to the tree-level amplitudes \(\alpha^{(1)}\) and are represented by the diagrams \([(d1a),(d1b),(zd1)]\) in figure 3. All the contributions come
Figure 3: Feynman diagrams for decuplet contributions to hyperon vff. Notations are the same as in figure 1 with double line and empty circles representing respectively decuplet propagators and SU(3)-breaking decuplet mass-shift operators (δMΔ in the text).

from sunset diagrams and, as in the octet case, they sum up to give finite results, according to the Ademollo-Gatto theorem. We find:

\[
\begin{align*}
\beta_{\Sigma-n}^{(2)} &= \frac{2}{3}C^2 \left( G_{\eta K} - \frac{3}{8}H_{\eta K} \right) + \frac{4}{3}C^2 \left( G_{\pi K} - \frac{3}{8}H_{\pi K} \right), \\
\beta_{\Lambda p}^{(2)} &= \frac{2}{3}C^2 \left( G_{\pi K} - \frac{3}{8}H_{\pi K} \right), \\
\beta_{\Xi-\Sigma}^{(2)} &= \frac{2}{3}C^2 \left( G_{\eta K} - \frac{3}{8}H_{\eta K} \right), \\
\beta_{\Xi-\Sigma^0}^{(2)} &= -\frac{4}{3}C^2 \left( G_{\eta K} - \frac{3}{8}H_{\eta K} \right) - \frac{2}{3}C^2 \left( G_{\pi K} - \frac{3}{8}H_{\pi K} \right),
\end{align*}
\]

where

\[
G_{pq} \equiv \frac{\Delta^2}{(4\pi f)^2} \left[ \frac{m_p^2 + 3m_q^2 - 4\Delta^2}{2(m_p^2 - m_q^2)} \log \left( \frac{m_p^2}{m_q^2} \right) \right] + \left( \begin{array}{c}
\frac{\Delta}{(4\pi f)^2} \left[ \frac{m_p^2 + 3m_q^2 - 4\Delta^2}{2(m_p^2 - m_q^2)} \sqrt{m_p^2 - \Delta^2} \arccos \left( \frac{\Delta}{m_p} \right) \right] + \\
+ \frac{m_p^2 + 3m_q^2 - 4\Delta^2}{2(m_p^2 - m_q^2)} \sqrt{m_q^2 - \Delta^2} \arccos \left( \frac{\Delta}{m_q} \right) \end{array} \right]
\]

satisfies the Ademollo-Gatto theorem.

As expected, in the limit \( \Delta \to \infty \) the decuplet decouples and its contribution goes to zero. In fact,

\[
\lim_{\Delta \to \infty} \left( G_{pq} - \frac{3}{8}H_{pq} \right) = O \left( \frac{p^4}{\Delta^2} \right),
\]

(32)
and the contributions of eq. (31) can be reabsorbed into local $O(p^5)$ counterterms. As an example, in figure 4 we show the plot of the $O(p^2)$ decuplet corrections to the $\Sigma^- \rightarrow n$ vff as a function of $\Delta$.

The resulting values for these decuplet contributions are given in table 3. They are sizable, however, in order to have a reliable estimate, we need also to include the subleading $O(p^3)$ corrections.

As for the octet case the $O(p^3)$ corrections come from the insertion of $O(p^2)$ mass-shift operators in one-loop diagrams. In this case, we have two contributions, coming from the insertion of decuplet and octet mass shifts (diagrams (d2c), (d2d), (zd3) and (d2a), (d2b), (zd2) in figure 3, respectively):

$$\beta^{(3)} = \beta^{(3)}(\delta M_\Delta) + \beta^{(3)}(\delta M_B).$$

The corrections due to the decuplet mass splitting read:

$$\begin{align*}
\beta^{(3)}_{\Sigma^- n}(\delta M_\Delta) &= C^2 \left[ \frac{2}{3} \delta M_{\Delta_1} G'_{\eta K} + \frac{4}{3} \left( \frac{4 \delta M_{\Delta_0} - \delta M_{\Delta_1}}{3} \right) G'_{\pi K} \right], \\
\beta^{(3)}_{\Lambda}(\delta M_\Delta) &= -C^2 \left[ \frac{2}{3} \delta M_{\Delta_1} G'_{\pi K} \right], \\
\beta^{(3)}_{\Xi}(\delta M_\Delta) &= C^2 \left[ \frac{2}{3} \delta M_{\Delta_1} G'_{\eta K} + \frac{2}{3} \left( \delta M_{\Delta_1} - \delta M_{\Delta_2} \right) G'_{\pi K} \right], \\
\beta^{(3)}_{\Xi^-}(\delta M_\Delta) &= -C^2 \left[ \frac{4}{3} \left( \frac{\delta M_{\Delta_1} + \delta M_{\Delta_2}}{2} \right) G'_{\eta K} + \frac{2}{3} \left( \frac{2 \delta M_{\Delta_1} + \delta M_{\Delta_2}}{3} \right) G'_{\pi K} \right],
\end{align*}$$

where

$$G'_{pq} = \frac{\partial G_{pq}}{\partial \Delta},$$

obviously satisfies the Ademollo-Gatto theorem. $\delta M_{\Delta_i}$ are the SU(3)-breaking mass shifts with respect to the scale $\Delta$, i.e.:

$$\delta M_{\Delta_i} = M_{\Delta_i} - (M_0 + \Delta),$$

and $\Delta_{0,1,2,3}$ correspond to the decuplet states $\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$ and $\Omega^-$ respectively.

The corrections in eq. (34) are numerically small (see table 3) and, as the $O(p^2)$ ones, vanish in the limit $\Delta \rightarrow \infty$. Note, however, that the contributions with small corrections in table 3 (i.e. those

Figure 4: $O(p^2)$ decuplet contributions as a function of $\Delta$. 
below $\sim 1\%$ result by accidental cancellations, so that their values strongly depend on the specific choice of masses and couplings.

Finally, we have the corrections from octet mass shifts. As shown in figure 3 the only contributions come from mass insertions on external legs, which shift the transferred momentum as discussed in section 3.3. The explicit expression for the corresponding $\beta(3) (\delta M_B)$ contributions are lengthy and reported in appendix B. The Ademollo-Gatto theorem has been checked as well as the decoupling limit.

At a first sight, both the $\beta(3) (\delta M_\Delta)$ and $\beta(3) (\delta M_B)$ contributions depend on the choice of $M_0$ (via the mass shifts $\delta M_\Delta$ and $\delta M_B$). However, it can be checked that this dependence cancel in the sum $\beta(3)$ of eq. (33), and the total $\mathcal{O}(p^3)$ corrections are independent of the explicit choice of $M_0$ as in the octet contribution $\alpha(3)$.

Evaluating the $\beta(3) (\delta M_B)$ contributions numerically (see table 3) we can notice that, in general, these corrections are huge. These results seem to signal a breaking of the perturbative expansion.

We would like to stress again that the estimates in tables 2 and 3 do not rely on unknown higher order LECs. They involve just the well known tree level parameters: masses, decay constants and 3-particle couplings (table 1). As long as the perturbative expansion holds, chiral corrections to these parameters must be considered of higher order, which justifies the use of the physical values.

### Table 3: Decuplet contributions to the chiral corrections ($\times 10^2$) to the vff.

| Decay       | $\beta(2)$ | $\beta(3) (\delta M_\Delta)$ | $\beta(3) (\delta M_B)$ |
|-------------|------------|-------------------------------|-------------------------|
| $\Sigma^- \to n$ | -3.1       | -1.8                          | +38.1                   |
| $\Lambda \to p$ | +1.5       | -0.01                         | +6.9                    |
| $\Xi^- \to \Lambda$ | -0.05      | -0.6                          | +18.4                   |
| $\Xi^- \to \Sigma^0$ | +1.6       | -0.2                          | -1.3                    |

Numerical values for the parameters are spelled out in table 1.

### 4.1 Discussion

The results obtained for the decuplet contributions deserve more discussion. Already in the pure octet case we observed a slow convergence of the chiral expansion. However, with the inclusion of decuplet states, it seems that the perturbative series breaks completely. Based on the fact that the decuplet couples to mesons stronger than the octet ($C_2^2/D_2^2 \sim 4$) we could have expected a large contribution from the former. This however cannot explain the large $\beta(3)$ contributions with respect to $\beta(2)$.

Notice also that $\mathcal{O}(1/M_0)$ corrections cannot cure the expansion since, at this order, they can only affect $\beta(2)$. Relativistic corrections to $\beta(3)$ are indeed of higher order in our power counting.

An alternative approach which could allow to resum all the $1/M_0$ corrections is the use of the relativistic formulations for Baryon Chiral Perturbation Theory [26]. These expansions are based on the use of modified regularizations, which allow to reabsorb into the LECs the terms that would break the power counting. In the particular case of the hyperon vff discussed here, this implementation should work in a peculiar way since all one loop contributions are finite, thus independent on the regularization scheme. Although these approaches would improve the calculation made here and probably help in reducing the anomalous decuplet contributions, is unlike that these could heal the problem completely, since extraordinary fine tuned relativistic corrections would be needed.

Moreover, there are clues indicating that the observed breaking of the chiral expansion is more related to the flavor power counting rather than to the relativistic corrections. In the standard power
counting, indeed, the quark masses are $O(p^2)$ so that meson masses ($m_{\pi,K,...}$) are $O(p)$ while baryon splittings ($\delta M_B$, $\delta M_\Delta$) are $O(p^2)$. Following [20] we treated the octet-decuplet mass shift $\Delta$ as a phenomenological parameter of $O(p)$ since numerically $\Delta \sim m_\pi$. However, while SU(2)-breaking baryon mass splittings are suppressed with respect to $\Delta$, the SU(3) ones are not. Note that if $\Delta$ is considered to be $O(p^2)$, the chiral corrections get even worse, while considering $\Delta \sim O(1)$ would correspond to integrating out the decuplet. It does not seem that the convergence problem can be cured in this way and doubts arise on the reliability of (H)B$\chi$PT with three flavors$^2$.

5 Conclusions

Recent progresses in the extraction of $V_{us}$ from hyperon semileptonic decays [10] made the estimation of the SU(3)-breaking effects for the corresponding vector form factors a central issue. Numerical studies [8, 17] demonstrate the possibility to estimate such corrections. However, current simulations are not yet able to catch the non-analytic contributions of the meson loops, which in these processes are the dominant ones because of the Ademollo-Gatto theorem. In this paper we estimated these contributions in the framework of Heavy Baryon Chiral Perturbation Theory. We performed a full $O(p^4)$ calculation including relativistic ($1/M_0$) corrections, extending and correcting previous analysis [13, 14, 15]. An important fact is that the Ademollo-Gatto theorem guarantees for these quantities the absence of local contributions at this order, therefore the final estimates are free from unknown parameters. We show that the would-be subleading $O(p^4)$ contributions are important and signal a poor convergence of the chiral expansion. This might not be so bad, anyway. We know that the three-flavor chiral expansion is slowly converging, and also in the case of kaon semileptonic decays [5, 7], important contributions come from the subleading local contributions, which can be estimated, for instance, using Lattice QCD.

The chiral expansion for baryons, however, presents also other complications. In particular the calculations made above rely on the fact that heavier hadronic states could be safely decoupled. This approximation may however be not so good for the decuplet states, which are only slightly heavier than the octet baryons. These states have been implemented into the (H)B$\chi$PT framework in [19, 20]. We thus evaluated the corresponding contributions to the vff to $O(p^4)$. We found that $O(p^3)$ decuplet contributions, in general, may be important, in agreement with analysis made for other quantities (see e.g. [19]). The $O(p^4)$ contributions, however, are anomalously huge, signaling a breaking of the perturbative expansion. These results arise doubts on the consistency of the chiral expansion with the decuplet, at least for the 3-flavors case.

Recently, several progress have been made towards a consistent relativistic formulation of the chiral expansion for baryons [26]. Such approaches could in principle remove part of the uncertainties connected to the $1/M_0$ expansion and its slow convergence. It is unlike, however, that they could solve the issues connected to the decuplet states, which seem more related to the structure of the power counting of the chiral flavor parameters.

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$^2$The problem of the convergence of the (H)B$\chi$PT with three flavors has already been discussed in other cases (see e.g. [27]). However, vff allow for a more reliable test of the perturbative expansion because, unlike other quantities, do not depend on unknown LECs at this order.
A Octet $\mathcal{O}(p^3)$ contributions

We present here the explicit expressions for the $\mathcal{O}(p^3)$ corrections $\alpha^{(3)}$ due to octet baryon contributions to hyperon vff:

$$\alpha^{(3)}_{\Sigma^*} =$$

$$\frac{1}{144 \pi f^2 (m_\eta + m_K)^3 (m_K + m_\pi)^2} \left\{ 3 F^2 \left[ 6 m_\eta^3 (m_K + m_\pi)^2 + m_\eta^2 (m_K^2 + 2 m_K m_\pi + m_K m_\pi^2 - 10 m_\pi^3) \right] + 2 m_\eta m_K \left[ m_K^2 + 2 m_K^2 m_\pi + m_K m_\pi^2 - 10 m_\pi^3 \right]^2 + m_K^2 \left[ m_K^2 + 2 m_K^2 m_\pi + m_K m_\pi^2 - 10 m_\pi^3 \right] \right\} (M_N - M_\Sigma)$$

$$-3 D F \left[ 4 m_\eta^3 (m_K + m_\pi)^2 (M_N - M_\Sigma) + m_\eta^2 (m_K^3 (3 M_\Lambda + 2 M_N - 5 M_\Sigma) - m_K^2 m_\pi (3 M_\Lambda - 4 M_N + M_\Sigma)) + m_K^2 \left[ m_K^2 (3 M_\Lambda + 2 M_N - 5 M_\Sigma) - m_K^2 m_\pi (3 M_\Lambda - 4 M_N + M_\Sigma) \right] \right.$$

$$+ m_\eta \left[ -3 M_\Lambda + 2 M_N + M_\Sigma \right] m_\pi \left[ -3 M_\Lambda + 2 M_N + M_\Sigma \right] + m_K m_\pi \left( -3 M_\Lambda + 2 M_N + M_\Sigma \right) + 3 m_\pi^3 \left( M_\Lambda - 4 M_\Sigma + M_\Sigma \right)$$

$$+ m_K \left[ m_K^2 (3 M_\Lambda + 2 M_N - 5 M_\Sigma) - m_K^2 m_\pi (3 M_\Lambda - 4 M_N + M_\Sigma) \right] + 3 m_\pi^2 \left( M_\Lambda - 4 M_N + 3 M_\Sigma \right) \right.$$ $$\left. - D^2 \left[ 6 m_\eta^3 (m_K + m_\pi)^2 (M_N - M_\Sigma) + m_\eta^2 (m_K^3 (3 M_\Lambda + M_N - 4 M_\Sigma) + m_K m_\pi \left( -3 M_\Lambda + 2 M_N + M_\Sigma \right) + m_K m_\pi \left( -3 M_\Lambda + 2 M_N + M_\Sigma \right) \right] \right.$$ $$+ m_K^2 \left[ m_K^2 (3 M_\Lambda + M_N - 4 M_\Sigma) + m_K m_\pi \left( -3 M_\Lambda + 2 M_N + M_\Sigma \right) \right] \right.$$ $$+ m_K m_\pi \left( -3 M_\Lambda + M_N + 2 M_\Sigma \right) + 3 m_\pi^3 \left( M_\Lambda - 10 M_N + 7 M_\Sigma \right) \right] \right\}, \quad (37)$$

$$\alpha^{(3)}_{\Lambda N} =$$

$$\frac{1}{48 \pi f^2 (m_\eta + m_K)^3 (m_K + m_\pi)^2} \left\{ -3 F^2 \left[ 2 m_\eta^3 (m_K + m_\pi)^2 - m_\eta^2 (m_K^2 + 2 m_K m_\pi + m_K m_\pi^2 - 2 m_\pi^3) \right] \right.$$ $$- 2 m_\eta m_K \left[ m_K^2 + 2 m_K^2 m_\pi + m_K m_\pi^2 - 2 m_\pi^3 \right] + m_K^2 \left[ m_K^2 + 2 m_K^2 m_\pi + m_K m_\pi^2 - 2 m_\pi^3 \right] \left( M_\Lambda - M_N \right)$$

$$+ D F \left[ 4 m_\eta^3 (M_\Lambda - M_N) (m_K + m_\pi)^2 + m_\eta^2 (m_K^3 (5 M_\Lambda - 2 M_N - 3 M_\Sigma) - m_K^2 m_\pi (3 M_\Lambda + 2 M_N - 3 M_\Sigma)) \right.$$ $$+ m_\eta \left[ -3 M_\Lambda + 4 M_N + M_\Sigma \right] m_\pi \left[ -3 M_\Lambda + 4 M_N + M_\Sigma \right] + m_K m_\pi \left( -3 M_\Lambda + 4 M_N + M_\Sigma \right) + 3 m_\pi^3 \left( 3 M_\Lambda - 2 M_N - 3 M_\Sigma \right)$$

$$- m_K^2 \left[ m_K^2 (5 M_\Lambda - 2 M_N - 3 M_\Sigma) - m_K^2 m_\pi (3 M_\Lambda - 4 M_N + M_\Sigma) \right] + m_K^2 \left( m_K^3 (5 M_\Lambda - 2 M_N - 3 M_\Sigma) \right) + D^2 \left[ 2 m_\eta^3 \left( M_\Lambda - M_N \right) (m_K^3 (m_K + m_\pi)^2 \right.$$ $$+ m_\eta^2 \left( m_K^2 (m_K + m_\pi)^2 + m_\eta^2 (m_K m_\pi^2 (2 M_\Lambda + M_N - 3 M_\Sigma) \right.$$ $$+ m_K^2 m_\pi \left( M_\Lambda + 2 M_N + M_\Sigma \right) m_K \left( -4 M_\Lambda + M_N + 3 M_\Sigma \right) \right.$$ $$+ m_\pi \left( -4 M_\Lambda + M_N + 3 M_\Sigma \right) \right.$$ $$+ m_K \left( m_K^2 (2 M_\Lambda + M_N - 3 M_\Sigma) \right) + m_K \left( m_K^2 (M_\Lambda + 2 M_N - 3 M_\Sigma) \right) \right.$$ $$\left. + m_K \left( -4 M_\Lambda + M_N + 3 M_\Sigma \right) \right.$$ $$\left. - m_\pi \left( -4 M_\Lambda + M_N + 3 M_\Sigma \right) \right]\right\}, \quad (38)$$

$$\alpha^{(3)}_{\Xi^* - \Lambda} =$$

$$\frac{1}{48 \pi f^2 (m_\eta + m_K)^3 (m_K + m_\pi)^2} \left\{ -3 F^2 \left[ 2 m_\eta^3 (m_K + m_\pi)^2 - m_\eta^2 (m_K^2 + 2 m_K m_\pi + m_K m_\pi^2 - 2 m_\pi^3) \right] \right.$$ $$- 2 m_\eta m_K \left[ m_K^2 + 2 m_K^2 m_\pi + m_K m_\pi^2 - 2 m_\pi^3 \right] - m_K^2 \left[ m_K^2 + 2 m_K^2 m_\pi + m_K m_\pi^2 - 2 m_\pi^3 \right] \left( M_\Lambda - M_\Xi \right)$$

$$+ D^2 \left[ 2 m_\eta^3 (m_K + m_\pi)^2 (M_\Lambda - M_\Xi) + m_\eta^2 \left( m_K^2 (m_K^2 + 2 m_K m_\pi + m_K m_\pi^2 - 2 m_\pi^3) \right) \right.$$ $$\left. + m_K \left( m_K^2 (2 M_\Lambda - 3 M_\Sigma + M_\Xi) \right) + m_K \left( m_K^2 \left( -4 M_\Lambda + 3 M_\Sigma + M_\Xi \right) \right]\right\}, \quad (39)$$
\[ + m^2_K m_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) - m^3_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) + 2 m_\eta m_K (m_K m_\pi^2 (2 M_\Lambda - 3 M_S + M_\Xi) \\
+ m^3_K (-4 M_A + 3 M_S + M_\Xi) + m^2_K m_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) - m_\eta (M_\Lambda - 3 M_S + 2 M_\Xi) \\
+ m^2_K (4 M_A + 3 M_S + M_\Xi) + m^2_K m_\pi (4 M_A + 3 M_S + M_\Xi) + m^2_K m_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) \\
- m^3_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) + D F \left[ -4 m^3_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) + m^2_\eta (3 m^2_\pi (3 M_A + M_\Xi - 4 M_\Xi) \\
- m^2_K m_\pi (M_\Lambda + 3 M_S - 4 M_\Xi) + m_K m^2_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) + m^3_\pi (-5 M_A + 3 M_S + 2 M_\Xi) \\
+ 2 m_\eta m_K (3 m^3_\pi (3 M_A + M_S - 4 M_\Xi) - m^2_\pi m_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) \\
+ m^3_\pi (-5 M_A + 3 M_S + 2 M_\Xi) + m^2_\pi (3 m^3_\pi (3 M_A + M_S - 4 M_\Xi) - m^2_\pi m_\pi (M_\Lambda + 3 M_S - 4 M_\Xi) \\
+ m_m m_\pi (M_\Lambda - 3 M_S + 2 M_\Xi) + m^3_\pi (-5 M_A + 3 M_S + 2 M_\Xi)) \right) \right] \right\} \tag{39}
\]

\[ \alpha_{(3)} = \frac{1}{144 \pi^2 (m_\eta + m_K)^2 (m_K + m_\pi)^2} \left\{ -3 F \left[ 6 m^3_\pi (m_K + m_\pi)^2 + m_\eta (m^3_\pi + 2 m^2_\pi m_\pi + m_K m_\pi^2 - 10 m^3_\pi) \right] \right. \\
+ 2 m_\eta m_K (m^3_\pi + 2 m^2_\pi m_\pi + m_K m_\pi^2 - 10 m_\pi) + m^2_K (m_K + 2 m^2_\pi m_\pi + m_K m_\pi^2 - 10 m_\pi) \left. \right] (M_\Sigma - M_S) \right) \right) \\
+ D F \left[ 6 m^3_\pi (m_K + m_\pi)^2 (M_\Sigma - M_S) - m_\eta (m^3_\pi (3 M_A + 7 M_S - 10 M_\Xi) + m^3_\pi (3 M_A - 4 M_S + M_\Xi) \\
+ m_K m_\pi^2 (-3 M_A + 2 M_S + M_\Xi) + m^2_K m_\pi (-3 M_A + 2 M_S + M_\Xi) + m^2_K m_\pi (-3 M_A + 2 M_S + M_\Xi) \\
- m_\eta (3 M_A + 7 M_S - 10 M_\Xi) + m^2_\pi (3 M_A - 4 M_S + M_\Xi) + m_K m_\pi^2 (-3 M_A + 2 M_S + M_\Xi) \\
+ m_\eta m_K (-3 M_A + 3 M_S + 2 M_\Xi) - 3 D F \left[ 4 m_\eta (m_K + m_\pi)^2 (M_\Sigma - M_S) - m_\eta (-m^2_\pi m_\pi (3 M_A + M_\Sigma - 4 M_\Xi) \right. \\
+ 3 m^3_\pi (M_\Lambda + 3 M_S - 4 M_\Xi) + m^2_K (3 M_\Lambda - 5 M_S + 2 M_\Xi) + m_K m_\pi^2 (-3 M_A + 2 M_S + M_\Xi) \\
- 2 m_\eta m_K (-m^2_\pi m_\pi (3 M_A + M_\Sigma - 4 M_\Xi) + m^3_\pi (M_\Lambda + 3 M_S - 4 M_\Xi) + m^3_\pi (3 M_A - 5 M_S + 2 M_\Xi) \\
+ m_K m_\pi^2 (-3 M_A + 3 M_S + 2 M_\Xi) - m_\eta (m_\pi^2 m_\pi (3 M_A + M_\Sigma - 4 M_\Xi) + 3 m^2_\pi (M_\Lambda + 3 M_S - 4 M_\Xi) \\
+ m_\eta m_K (3 M_A - 5 M_S + 2 M_\Xi) + m_K m_\pi^2 (-3 M_A + 3 M_S + 2 M_\Xi) \right) \right. \} \right\} \tag{40}
\]

### B Decuplet $O(p^3)$ contributions

In this appendix we present the explicit expressions for the $O(p^3)$ corrections $\beta_{(3)}(\delta M_B)$ due to the octet baryon mass corrections in decuplet contributions:

\[ \beta_{(3)}(\delta M_B) = \frac{C^2}{216 \pi^2 f^2} \left\{ \begin{array} {c}
3 \left[ m^6_\eta (-8 \delta M_N + 5 \delta M_S) - 18 m^4_K \delta M_S \Delta^2 + 4 m^2_K (13 \delta M_N - \delta M_S) \Delta^4 \\
+ 32 (-\delta M_N + \delta M_S) \Delta^6 + 2 m^4_\eta (8 \delta M_N - 11 \delta M_S) + (10 \delta M_N + 11 \delta M_S) \Delta^2 \right] \\
+ m^2_\eta (9 m^4_K \delta M_S + 4 m^2_K (-17 \delta M_N + 11 \delta M_S) \Delta^2 + 4 (5 \delta M_N - 17 \delta M_S) \Delta^4) \right\} \frac{\text{arccos} \left( \frac{\Delta}{m_\eta} \right)}{(m^2_\eta - m^2_K)^2 \sqrt{m^2_\eta - \Delta^2}} \\
+ \left\{ -12 m^2_K m^2_\eta \Delta^4 (17 \delta M_N m^2_\eta - 5 m^2_\pi \delta M_S + 16 \delta M_N \Delta^2 - 16 \delta M_S \Delta^2) \\
+ m^8_\eta (71 \delta M_N m^2_\pi - 80 m^2_\pi \delta M_S + 91 \delta M_N \Delta^2 + 98 \delta M_S \Delta^2) + 2 m^4_K \Delta^2 (\delta M_N (51 m^4_\pi + 416 m^2_\eta \Delta^2 - 112 \Delta^4) \\
- 2 m^2_\eta (21 m^4_\pi + 64 m^2_\eta \Delta^2 - 56 \Delta^4)) + m^2_K (4 \delta M_S (12 m^4_\eta + 92 m^2_\eta \Delta^2 - 131 \Delta^4) \\
+ \delta M_N (-3 m^2_\eta - 764 m^2_\eta \Delta^2 + 92 \Delta^4)) + 2 m^2_\eta (m^8_\eta (41 \delta M_N - 32 \delta M_S) - 6 m_\eta^2 (\delta M_N - 13 \delta M_S) \Delta^4 \\
- 2 m^2_K (53 \delta M_N m^2_\pi - 80 m^2_\pi \delta M_S + 25 \delta M_N \Delta^2 + 119 \delta M_S \Delta^2) \right. \right\} \end{array} \right. \]
The text seems to be a mathematical expression involving various variables and constants. It appears to be a complex algebraic or geometric formula, possibly related to a specific field of study such as physics or engineering. The expression includes terms like $m^2$, $\delta M$, $\Delta^2$, and other similar symbols, indicating a high level of mathematical complexity.
\[
\beta^{(3)}_{\xi - \Lambda} (\delta MB) = \\
\frac{\mathcal{C}^2}{72 \pi^2 f^2} \left\{ m^6_\eta \left[ -8 \delta M_\pi + 5 \delta M_\Xi \right] - 18 \eta^4 K \delta M_\Xi \Delta^2 + 4 \eta^2 K \left( 13 \delta M_\Lambda - \delta M_\Xi \right) \Delta^4 \\
+ 32 \left( -\delta M_\Lambda + \delta M_\Xi \right) \Delta^6 + 2 m^4_\eta \left( m^2_K + 8 \delta M_\Xi - 11 \delta M_\Xi \right) \Delta^2 \\
+ m^2_\eta \left( 9 m^2_K \delta M_\Xi + 4 \eta^2 K \left( -17 \delta M_\Lambda - 11 \delta M_\Xi \right) \Delta^2 + 4 \left( 5 \delta M_\Lambda - 17 \delta M_\Xi \right) \Delta^4 \right) \right\} \arccos \left( \frac{\Delta}{m_\eta} \right) \\
\times \frac{\frac{\Delta}{m_\eta}}{\sqrt{m^2_\eta - \Delta^2 (m^2_\eta - m^2_K)}} \\
+ \left[ m^6_\eta \left( -8 \delta M_\Lambda + 5 \delta M_\Xi \right) + 32 m^4_\eta \left( \delta M_\Lambda - \delta M_\Xi \right) \Delta^6 \\
+ 4 m^2_\eta \left( m^4_\Xi \delta M_\Xi \Delta^4 \left( -17 \delta M_\Lambda + 5 \delta M_\Xi \right) + 5 \delta M_\Lambda \Delta^2 + 16 \delta M_\Xi \Delta^2 \right) \\
+ m^4_\eta \left( 22 \delta M_\Xi \left( m^2_\Xi + \Delta^2 \right) + 4 \delta M_\Xi \left( 7 m^2_\Xi + 5 \Delta^2 \right) \right) + m^4_\Xi \left( \delta M_\Xi \left( m^4_\Xi + 72 m^2_\Xi \Delta^2 - 68 \Delta^4 \right) \\
- 4 \delta M_\Xi \left( m^4_\Xi + 39 m^2_\Xi \Delta^2 - 5 \Delta^4 \right) \right] + 2 m^4_\Xi \Delta^2 \left( 4 \delta M_\Xi \left( 5 m^4_\Xi + 24 m^2_\Xi \Delta^2 - 4 \Delta^4 \right) \\
+ m^2_\Xi \left( m^4_\Xi - 16 m^2_\Xi \Delta^2 - 16 \Delta^4 \right) \right] \\
+ m^4_\Xi \left( m^6_\Xi - 18 m^4_\Xi \Delta^2 + 56 m^2_\Xi \left( \delta M_\Lambda - \delta M_\Xi \right) \Delta^4 + 64 \left( -\delta M_\Lambda + \delta M_\Xi \right) \Delta^6 \\
+ m^6_\Xi \left( 9 m^4_\Xi \delta M_\Xi + 4 m^2_\Xi \left( -19 \delta M_\Xi + 28 \delta M_\Xi \right) \Delta^2 + 88 \left( \delta M_\Lambda - \delta M_\Xi \right) \Delta^4 \right) \\
+ 2 m^4_\Xi \left( 10 \delta M_\Xi \left( m^2_\Xi - \Delta^2 \right) + \delta M_\Xi \left( -19 m^2_\Xi + \Delta^2 \right) \right) + 2 m^2_\Xi \left( m^8_\Xi \left( 2 \delta M_\Xi - 5 \delta M_\Xi \right) \\
- 2 m^8_\Xi \left( 13 \delta M_\Xi \Delta^4 + m^2_\Xi \left( -16 \delta M_\Xi m^2_\Xi + 22 m^2_\Xi \delta M_\Xi + 24 \delta M_\Xi \Delta^2 - 36 \delta M_\Xi \Delta^2 \right) \\
+ 4 m^2_\Xi \Delta^2 \left( \delta M_\Xi \left( -14 m^4_\Xi + m^2_\Xi \Delta^2 - 16 \Delta^4 \right) + \delta M_\Xi \left( 5 m^4_\Xi - 13 m^2_\Xi \Delta^2 + 16 \Delta^4 \right) \right) \\
- m^4_\Xi \left( m^4_\Xi + 44 m^2_\Xi \Delta^2 - 114 \Delta^4 \right) + 2 \delta M_\Xi \left( m^4_\Xi - 34 m^2_\Xi \Delta^2 + 45 \Delta^4 \right) \right) \right] \\
\times \arccos \left( \frac{\Delta}{m_\eta} \right) \\
\times \sqrt{m^2_\eta - \Delta^2 \left( m^2_\eta - m^2_K \right)^2} \\
+ \left( \delta M_\Lambda - \delta M_\Xi \right) \left[ 13 m^6_\eta + 2 m^4_\eta \Delta^2 + 88 m^2_\eta \Delta^4 + 64 \Delta^6 + 9 m^4_\eta \left( m^2_\eta - 2 \Delta^2 \right) \\
- 2 m^6_\eta \left( 19 m^4_\eta + 56 m^2_\eta \Delta^2 + 28 \Delta^4 \right) \right] \arccos \left( \frac{\Delta}{m_\eta} \right) \\
\times \sqrt{m^2_\eta - \Delta^2 \left( m^2_\eta - m^2_K \right)^2} \\
+ \Delta \left[ 9 m^6_\eta \delta M_\Lambda - 9 m^4_\Xi \delta M_\Xi + 2 m^2_\Xi \left( 13 \delta M_\Lambda - \delta M_\Xi \right) \Delta^2 \right. \\
+ 16 \left( -\delta M_\Lambda + \delta M_\Xi \right) \Delta^4 + m^2_\Xi \left( -21 m^4_\Xi \left( -17 \delta M_\Lambda - 13 \delta M_\Xi \right) \Delta^2 \right) \left( \delta M_\Lambda - \delta M_\Xi \right) \Delta^2 \right. \\
+ \left. \frac{2 \Delta}{m^2_\Xi - m^2_K} \right] \left[ m^4_\Xi \left( \delta M_\Lambda + 5 \delta M_\Xi \right) + 4 m^2_\Xi \left( \delta M_\Lambda - \delta M_\Xi \right) \Delta^2 \\
+ m^2_\Xi \left( -9 \delta M_\Lambda m^2_\Xi + 3 m^2_\Xi \delta M_\Xi + 4 \delta M_\Lambda \Delta^2 - 4 \delta M_\Xi \Delta^2 \right) \\
+ m^2_\Xi \left( 5 \delta M_\Lambda m^2_\Xi + 3 m^2_\Xi \left( \delta M_\Lambda - 3 \delta M_\Xi \right) + m^2_\Xi \delta M_\Xi - 8 \delta M_\Lambda \Delta^2 + 8 \delta M_\Xi \Delta^2 \right) \right] \\
\times \Delta \left( \delta M_\Lambda - \delta M_\Xi \right) \left[ 9 m^4_\Xi + 9 m^4_\Xi + 28 m^2_\Xi \Delta^2 - 32 \Delta^4 + m^2_\Xi \left( -42 m^2_\Xi + 28 \Delta^2 \right) \right] \log \left( \frac{m^2_\Xi}{m^2_K} \right) \\
\left( \frac{m^2_\Xi}{m^2_K} \right) \right], \tag{43}
\end{align*}

\[
\beta^{(3)}_{\xi - \Sigma_0} (\delta MB) = \\
- \frac{\mathcal{C}^2}{216 \pi^2 f^2} \left\{ 9 \left( \delta M_\Sigma + \delta M_\Xi \right) \left[ -m^4_\eta - 3 m^2_\eta m^2_K + 14 m^2_\eta \Delta^2 + 6 m^2_K \Delta^2 - 16 \Delta^4 \right] \right\} \arccos \left( \frac{\Delta}{m_\eta} \right) \\
\times \frac{\frac{\Delta}{m_\eta}}{\sqrt{m^2_\eta - \Delta^2 \left( m^2_\eta - m^2_K \right)^2}} \\
- \left[ \left( m^2_K \left( 20 \delta M_\Sigma + 7 \delta M_\Xi \right) \right) - 144 m^4_\Xi \left( \delta M_\Sigma + \delta M_\Xi \right) \Delta^4 \\
+ 2 m^6_\Xi \left( 14 \delta M_\Sigma m^2_\Xi - 5 m^2_\Xi \delta M_\Xi + 94 \delta M_\Sigma \Delta^2 + 95 \delta M_\Xi \Delta^2 \right) \\
+ m^4_\Xi \left( 9 m^2_\Xi \delta M_\Xi - 8 m^2_\Xi \left( 43 \delta M_\Sigma + 29 \delta M_\Xi \right) \Delta^2 - 4 \left( 43 \delta M_\Sigma + 65 \delta M_\Xi \right) \Delta^4 \right) \right],
\end{align*}

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$$+ 2 m^2_\pi \Delta^2 \left( 2 \delta M_S \left( 27 m^4_\pi + 97 m^2_\pi \Delta^2 - 8 \Delta^4 \right) + 4 \delta M \left( 45 m^4_\pi + 166 m^2_\pi \Delta^2 + 16 \Delta^4 \right) \right)$$
$$- m^2_\eta \left( m^6_K \left( 16 \delta M_S + 29 \delta M \right) - 2 \Delta^2 \left( 9 m^2_\pi - 8 \Delta^2 \right) \right) \left( 4 \delta M_S m^2_\pi + 5 m^4_\pi \delta M - 2 \delta M_S \Delta^2 + 2 \delta M \Delta^2 \right)$$
$$+ m^4_K \left( -44 \delta M_S m^2_\pi - 82 m^4_\pi \delta M_S - 8 \delta M_S \Delta^2 + 10 \delta M \Delta^2 \right) + m^2_K \left( \delta M \left( 45 m^4_\pi + 128 m^2_\pi \Delta^2 - 116 \Delta^4 \right) \right)$$
$$+ 4 \delta M \left( 9 m^4_\pi + 4 m^2_\pi \Delta^2 - 7 \Delta^4 \right) \right) \right) \left[ \arccos \left( \frac{\Delta}{m_K} \right) \sqrt{m^2_\pi - \Delta^2 \left( m^2_K - m^2_\pi \right)} \right]$$
$$- \left[ - m^4_K \left( 2 \delta M_S + \delta M \right) \left( m^2_\pi - 2 \Delta^2 \right) - 2 \delta M \left( m^6_\pi + 32 m^4_\pi \Delta^2 - 58 m^2_\pi \Delta^4 + 16 \Delta^6 \right) \right] + \delta M \left( 11 m^6_\pi - 62 m^4_\pi \Delta^2 + 28 m^2_\pi \Delta^4 + 32 \Delta^6 \right) - 2 m^2_K \left( \delta M \left( -14 m^4_\pi + 10 m^2_\pi \Delta^2 + 22 \Delta^4 \right) \right)$$
$$+ \delta M \left( 5 m^4_\pi - 46 m^2_\pi \Delta^2 + 50 \Delta^4 \right) \right) \arccos \left( \frac{\Delta}{m_\pi} \right) \sqrt{m^2_\pi - \Delta^2 \left( m^2_K - m^2_\pi \right)} \right.$$
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