A brief note on the counter-intuitive region of a square plate

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Abstract This paper presents a numerical investigation of the counter-intuitive response in elastic, perfectly plastic square plates subjected to impulsive loading. A map of the counter-intuitive region is obtained based on non-dimensional numbers. Asymmetrical response of square plates in the counter-intuitive region is also reported.

Keywords: Counter-intuitive response, square plate, asymmetrical response

1 Introduction

Counter-intuitive behaviour of an elastic-plastic beam subjected to blast loading (i.e. the final mid-deflections of the beam rest on the opposite direction of the loading) was initially reported by Symonds and Yu [1]. This phenomenon, which is associated with the compressive instability and may become extremely sensitive to some material and structural parameters, has been observed in various structures. Most publications were reported for beams, which will not be reviewed in this brief note. An interested reader may find the latest progresses from [2-4]. Counter-intuitive phenomena were reported for other structures, i.e. spherical caps [5] and square plates [6], with little generalised analysis.

This brief note employs non-dimensional numbers and numerical modelling to obtain a map of counter-intuitive region for elastic, perfectly plastic square plates. Same procedure
can be used to study other two dimensional elements (e.g. circular and rectangular plates). Asymmetrical counter-intuitive response, which was observed in beams, is first reported in square plate.

2 Problem description and dimensional analysis

Consider a fully supported metal square plate subjected to blast loading with length $L$, thickness $h$, material properties $E$ (Young’s modulus), $E_t$ (tangent modulus), $\sigma_0$ (yield strength) and $\rho$ (density) and initial velocity $V_0$. Initial velocity of the plate is used to replace the impulsive pressure load represented by the total blast impulse ($I$) per unit area based on an initial uniform velocity distribution, i.e.

$$V_0 = \frac{I}{\rho h}.$$  (1)

A dimensional analysis implies that the maximum deflection $\delta_{\text{max}}$ of the plate can be expressed in a non-dimensional form,

$$\frac{\delta_{\text{max}}}{h} = f \left( \frac{h}{L}, \frac{E_t}{E}, \frac{\sigma_0}{E}, V_0 \sqrt{\frac{\rho}{E}} \right).$$  (2)

If material properties of the plate are fixed, $E_t/E$ and $\sigma_0/E$ become constants, then $\frac{\delta_{\text{max}}}{h}$ will depend only on the non-dimensional numbers $h/L$ (slenderness) and $V_0 \sqrt{\rho/E}$.

3 Finite element simulations and results

Finite element simulations are performed using the commercial code ABAQUS explicit [7]. Due to symmetry, only a quarter of the plate is modelled as a three-dimensional solid using 50×50 S4R shell elements. Following material parameters were used for all
simulations: $E=70$ GPa, $\nu=0.33$, $\rho=2700$ kg/m$^3$, $\sigma_0=220$ MPa. It has been shown in trail simulations that the selected FE model is sufficiently accurate for an elastic-plastic response.

In order to ensure that the blast problem can be represented by a simplified impulsive problem, a series of numerical simulations were run to compare the mid-point deflection under impulsive loading and pulse blast loading. A reference impulsive load of 267.3 Pa.s with a corresponding $V_0=16.5$ m/s was chosen. Pulse loads were generated with same impulse but different duration time $T_s$. Figure 1(a) shows the mid-point deflections corresponding to impulsive loading and pulse loadings with $T_s=1.9$ ms (which is the characteristic response time $T_c$ when the peak deflection is achieved) and $T_s=0.1$ ms. It can be seen that when the pulse loading duration is much smaller than $T_c$, the plate response is similar to the one obtained with impulsive loading. However, when the pulse loading duration is comparable with $T_c$, the pulse loading cannot be simplified into an impulsive loading. To find the critical pulse loading duration, below which the actual pulse loading problem can be simplified into an impulsive loading problem in the study of counter-intuitive phenomenon, mid-point deflections from pulse loadings with same impulse but different duration time $T_s$ were compared in Fig.1(b). It was found that when the loading duration is less than 0.105 ms (or 6% of $T_c$), the pulse loading problem can be represented by the impulsive loading problem.

![Figure 1 Comparison of mid-point deflections under a) impulsive loading and pulse loading; b) pulse loading with different duration](image-url)
Numerical simulations were performed using same non-dimensional numbers with different parameters (e.g., $h/L=0.006$ with $h=0.006$ and $L=1$, $h=0.0045$ with $L=0.75$ and $h=0.003$ and $L=0.5$, respectively). The counter-intuitive response was observed for the three above cases in the same range of $V_0\sqrt{\rho/E}$ showing that the counter-intuitive response is controlled only by non-dimensional numbers in Eq.(2).

In this study, the counter-intuitive region is searched numerically in the following range of non-dimensional numbers

$$0.004 < \frac{h}{L} < 0.0065, \quad 0.002 < V_0 \sqrt{\frac{\rho}{E}} < 0.004,$$

because they represent realistic problems, which may involve in counter-intuitive response.

A large quantity of numerical simulations were performed to define the counter-intuitive region (or map) of the square plate. A typical counter-intuitive response of the mid-point deflection of a square plate ($h=0.006$ m, $L=1$ m) subjected to an impulsive load ($V_0=16.5$ m/s) is shown in Fig.1(a). The deformation of the plate at different time is depicted in Fig.2. It can be seen that the final mid-point deflection of the plate comes to rest in the direction opposite to the applied load.

Figure 3 shows the map of the counter-intuitive response represented by the shaded area in the plane of $V_0\sqrt{\rho/E}$ and $h/L$. It can be observed that the map does not follow a preferential shape. However, the counter-intuitive response is always observed in $0.0029<V_0\sqrt{\rho/E}<0.0036$ for the range of non-dimensional numbers $h/L$ studied.

![Figure 2 Deformation of a square plate subjected to an impulse load $I$](image)
An interesting phenomenon, i.e. asymmetrical response of the square plate, was observed when a very small misalignment was introduced to the full model of the plate as shown in Fig.4. This phenomenon has been reported for beams both experimentally [8] and numerically [9]. The misalignment illustrated in Fig.4 is used to activate the asymmetrical response of the square plate [9]. Points A, B and C are coplanar. The initial misalignment results in a very small initial rotation angle of approximate 0.01° (Fig.4).

Figure 5 shows the displacement-time histories from numerical simulations of points A, B and C for two different cases: (I) \( h/L = 0.006 \) and \( V_0 \sqrt{\rho/E} = 0.007856 \); (II) \( h/L = 0.006 \) and \( V_0 \sqrt{\rho/E} = 0.003241 \). It can be seen that the plate vibrates symmetrically in case-I because points A and C have almost identical responses. For case-II, asymmetrical response is observed. It is also noticed that for case B the plate behaves counter-intuitively.
Figure 4 Initial misalignment of fully-clamped square plate

Figure 5 Deflection-time histories of points A, B and C using following non-dimensional parameters: a) $h/L = 0.006$ and $V_0 \sqrt{\rho/E} = 0.007855844$; b) $h/L = 0.006$ and $V_0 \sqrt{\rho/E} = 0.003240536$.

4 Concluding remarks

The numerical study conducted in this paper shows that the counter-intuitive response may occur in a square plate when subjected to an impulsive loading. A map of the counter-intuitive region defined by non-dimensional numbers is obtained for $0.0029 < V_0 \sqrt{\rho/E} < 0.0036$ and $0.004 < h/L < 0.0065$. The asymmetrical response of the square plate is also reported. Same procedure and method can be applied to other two-dimensional structural elements (e.g. circular plates) and consider the influence of the material properties.
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