High Scale Study of Possible $B_d \to \phi K_S$ CP Physics

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Abstract

Some rare decay processes are particularly sensitive to physics beyond the Standard Model (SM) because they have no SM tree contributions. We focus on one of these, $B_d \to \phi K_s$. Our study is in terms of the high scale effective theory, and high scale models for the underlying theory, while previous studies have been focusing on the low scale effective Lagrangian. We examine phenomenologically the high scale parameter space with full calculations, but largely report the results in terms of mass insertion techniques since they are then easily pictured. We also determine the ranges of different mass insertions that could produce large non-SM CP effects. Then we exhibit classes of high scale models that can or cannot provide large non-SM CP effects, thus demonstrating that data on $B_d \to \phi K_s$ can probe both supersymmetry breaking and the underlying high scale theory and even make relatively direct contact with string-motivated models. We provide a novel and systematic technique to understand the relations between high and low scale parameters from RGE running. We include all constraints from other data, particularly $b \to s\gamma$ and EDMs.

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1 Introduction

CP violation is a fascinating subject in particle physics. Complex parameters in the Lagrangian could give rise to numerous interesting observables in low energy experiments. Many, especially those associated with the third generation of quarks, are either not well measured or controversial. They represent new opportunities to discover new physics and uncover new flavor physics in the near future. If possible FCNC parameters are small, some presently unknown mechanism or symmetry has to be found to explain why they are small. Sizable new CP violations, as well as the KM phase itself, probe fundamental flavor physics, which is closely related to supersymmetry breaking and string theory.

In the Standard Model, the KM phase $\delta_{KM}$ and the strong phase $\theta_{QCD}$ are the only sources for CP violation. The bounds on the electric dipole moment (EDM) of the neutron imply that $\theta_{QCD} < 10^{-9}$, so we will ignore it for b physics. Therefore, effectively the only source of CP violation in the SM is the non-zero $\delta_{KM}$. Many experiments have been analyzed to measure this phase and consistent results would increasingly establish the KM mechanism of CP violation. Before the measurement of the time dependent CP asymmetry in the process of $B_d \to \phi K_S$, all experiments indeed gave a consistent measure of $\delta_{KM} = 60^\circ \pm 14^\circ$ [1]. On the other hand, many proposals for new physics beyond the Standard Model allow new sources of CP violation. Some of them are constrained by the experimental bounds from electric dipole moments and measurements in Kaon physics. Others could give rise to potentially interesting deviations.

Low energy supersymmetry is the most compelling candidate for the new physics [2]. The most general soft supersymmetry breaking Lagrangian carries a significant number of phases which could be interesting new sources of the CP violation. Therefore, it is important to study the ways in which CP violation in the soft supersymmetry breaking Lagrangian could show up in the experiments.

Since processes involving the superpartners (hence the new parameters) necessarily occur at loop level, the new CP violating effects could only be significant in the processes where SM contributions are also suppressed. Important examples of such processes include $b \to s\gamma$ and $B_d \to \phi K_S$.

It is interesting to notice that recently there is a potential discrepancy between the CP asymmetry measured in the process $B_d \to \phi K_S$ (denoted by $S_{\phi K_S}$) and the SM prediction:

$$S_{\phi K} \simeq S_{\psi K} \simeq \sin 2\beta = 0.736$$ (1)

The latest results from BaBar [3, 5] is

$$S_{\phi K_S} = 0.47 \pm 0.34_{-0.06}^{+0.08},$$ (2)

and from BELLE [4, 5] is

$$S_{\phi K_S} = -0.96 \pm 0.50_{-0.11}^{+0.09}.$$ (3)

Because this process is one of a few that are unusually sensitive to CP physics beyond the SM, it is worthwhile studying it in detail however the experimental situation is finally resolved. And of course if a deviation from the SM prediction is confirmed it is exceptionally important.
Since $\delta_{KM}$ is the only source of CP violation in the SM, if a deviation from the SM prediction for $S_{\phi K}$ is confirmed, it will be a clear sign of new physics beyond the SM and the new physics must be relevant to the weak scale. The superpartners in the loops must be at a mass scale that guarantees their production at the Tevatron and LHC. In the Minimal Supersymmetric Standard Model (MSSM), there are 42 new CP violation phases in addition to $\delta_{KM}$ and $\theta_{QCD}$. Although many of these phases are already constrained by various experiments [6], there are still plenty of them which are not strongly constrained. It was shown in [7, 8, 9, 10, 11], that by tuning some of these not strongly constrained CP violation phases at the weak scale, BELLE’s result could be described without violating other experimental bounds.

The most stringent contraints on CP violation arise from several electric dipole moment measurements. Since we are mostly interested in CP violation associated with the last generations of quarks, the most relevant constraint will come from the mercury EDM which depends on the chromo-magnetic operator of the strange quark (see detailed discussion in Sec. 2). From the point of view of a general low energy MSSM, this should not give a direct constraint on CP violation observed in flavor changing B decays since the EDM is only sensitive to flavor diagonal phases. However, we have to bear in mind that the soft Lagrangian is actually defined at some high scale at which supersymmetry is broken. Generically, we expect soft parameters at that scale to carry large phases that can give large deviations from the Standard Model in CP violating observables. The renormalization group running mixes the flavor-diagonal and flavor off-diagonal parameters and the mixing is enhanced by the large Logs in the conventional picture of gauge coupling unification. One of the main results of this paper is the study of this effect and its implications for the high energy allowed parameter space.

With better understanding of the allowed high energy parameter space, we then study and constrain some models of flavor structures. Previous studies have focused on weak scale phenomenology. Here instead we emphasize the properties of the high scale theory and their relation to the data. We do both a phenomenological analysis of what high scale properties would be needed to explain a deviation from the SM, and an examination of what kinds of high scale underlying theories could or could not have the needed properties.

This paper is organized as follows. We describes the most relevant CP violation observables and constraints in Section 2. A model independent study of the input high scale parameter space is performed in Section 3, utilizing both semi-analytic and numerical methods to study the RGE effects. With the knowledge obtained from the model independent study, we examine several classes of models of high scale flavor structure in Section 4. We present our conclusions in Section 5 and a detailed semi-analytic study of MSSM RGE effects using a new approach in the appendix, which provides a clear picture of the interplays among the flavor parameters.

2 CP Violation Observables and Constraints

In this section, we review the main experimental contraints and prospects of detecting CP violation beyond the Standard Model. We discuss the Mercury EDM and $b \rightarrow s \gamma$ as they
usually provide the most useful constraints. We use $B_d \rightarrow \phi K_S$ as the main example of an observable potential deviation. Other constraints involving first or second families of quarks are less important for our purpose. We will not discuss them here, but still require our results to satisfy those experimental bounds. We emphasize that, although mass insertions generally provide a clearer picture of the flavor changing, we make use of the full squark mass matrix in our calculation. The necessity is evident in the discussions of the importance of various multiple-mass-insertions in the following.

2.1 Mercury Electric Dipole Moment

The EDM bound on Hg puts constraints on the CP violation phases of the soft terms. It requires the strange quark chromo EDM (CEDM) to satisfy

$$|e d_s^C| < 5.8 \times 10^{-25} \text{ecm},$$

(4)

assuming vanishing up and down quarks CEDM. However, there are significant theoretical hardronic uncertainties in extracting $d_s^C$ from Hg$^{199}$. In the MSSM, although chargino exchange diagrams can give sizeable contributions to the strange quark CEDM, the contribution is usually dominated by the gluino exchange diagrams

$$e d_s^C = c e \frac{\alpha_s m_3^2}{4 \pi m_\tilde{q}^2} \text{Im} (\Delta_{22}^{d,LR} \times L(x) + L \leftrightarrow R),$$

(5)

with $c = 0.91$, $L(x)$, where $x = m_\tilde{q}^2/m_\tilde{g}^2$, is a loop function (or its appropriate derivative). Notice it is different from $c = 3.3$ usually quoted in the literature. This difference comes from the different definition of the chromo-dipole operator. The chromo-dipole operator used in obtaining eq. (5) is defined as $\frac{i}{2} g s q_i t^a C_{\mu \nu}^a \sigma^{\mu \nu} \gamma_5 q_i$ which includes the strong coupling $g_s$. With this definition, which is also used in a recent study, there should be no large scaling of this operator.

$\Delta_{f_j}^{d,LR}$, $(f = u, d)$ is a generic mixing parameter between left-handed $i$-th generation and right-handed $j$-th generation of up- or down-type squarks. It could involve single or multiple mass insertion parameters which are directly defined from the soft parameters. Several examples are

$$\Delta_{22}^{d,LR} = (\delta_{LR}^d)_{22},$$

$$\Delta_{22}^{d,LR} = \frac{1}{2} (\delta_{LL}^d)_{23} (\delta_{LR}^d)_{32}, \quad \frac{1}{2i} (\delta_{LR}^d)_{23} (\delta_{RR}^d)_{23},$$

$$\Delta_{22}^{d,LR} = \frac{1}{3i} (\delta_{LR}^d)_{23} (\delta_{LR}^d)^*_{32}, \quad \frac{1}{3i} (\delta_{LL}^d)_{23} (\delta_{RR}^d)_{33}.$$ (6)

In the cases of double mass insertion, there is an extra factor of 1/2 coming from the Taylor expansion. Although flavor changing parameters do not contribute to the EDM at leading order,

\footnote{In our study, we therefore allow the theoretical calculation to have a factor of 3 uncertainty.}
they do enter at the next order through combinations with other flavor parameters. A particular
triple mass insertion is studied in Ref \[17\]. Using Eq. 5, we obtain for that combination
\[
ed^C_s = c \frac{\alpha_s}{4\pi} \frac{m_\tilde{g}}{m_\tilde{q}} \left( -\frac{11}{30} \right) \frac{1}{3!} \text{Im}((\delta^d_{LL})_{23}(\delta^d_{LR})_{33}(\delta^d_{RR})_{32}),
\]
where we have set gluino and average squark masses to be equal, \( \tilde{m}_g = \tilde{m}_q \), when evaluating
the loop function. Notice also the extra \( 1/6 \) coming from the Taylor expansion. Combining
this factor with the the scaling behavior, we found that the constraint of Ref. \[17\] should be
about a factor of 20 less restrictive than they report. \(^2\) Nevertheless, this combination will
still provide some constraint on the high energy CP violating parameters.

2.2 \( b \to s \gamma \)

\( b \to s \gamma \) is a process where the Standard Model tree level contribution is absent. Therefore,
the SUSY contribution, which enters at one-loop order, could be comparable to the Standard
Model processes. As a result, new flavor parameters in the soft Lagrangian will be significantly
constrained by this process. We use the following bound in our calculations:
\[
2.0 \times 10^{-4} < \text{BR}(b \to s \gamma) < 4.5 \times 10^{-4}
\]  
(8)

In the MSSM, the SUSY contribution to BR(\( b \to s \gamma \)) is usually assumed to be dominated
by the chargino loop contribution to the \( O_{7\gamma} \) (or \( O'_{7\gamma} \)) operator:
\[
C_{7\gamma}^{\text{New}} \propto g_2 \frac{m_b \tan \beta}{m_W} V_{12} \Delta_{23}^{u,LL} \times L_{\tilde{h}} \tilde{W}^c
+ g_2 \frac{\lambda^2 m_b \tan \beta}{m_W^2} \Delta_{33}^{u,LR} V_{22} \times L_{\tilde{h}}
+ g_2 \frac{m_b}{M_2} X_{23} V_{11} \times L_{\tilde{W}}.
\]  
(9)

where \( X_{23} \) is either \( \Delta_{23}^{u,LL} \) or \( \lambda^2 \). \( V_{ij} \) are chargino mixing matrix elements. The three different
lines correspond to Higgsino-Wino, Higgsino, and Wino loops, respectively. Typically, the
Higgsino-Wino loop gives the dominant contribution. The others could be important in some
circumstances as well. Eq. 4 is schematic and designed to show the dependence on various
flavor parameters. We have not shown explicitly the charged Higgs contribution as it does
not depend on flavor changing soft parameters. There is a similar expression for the chirality
flipped \( O'_{7\gamma} \) operator as well \[19\].

\( \Delta_{ij}^{u,AB} \) s should again be understood as compounded parameters. In this paper, we are mainly
interested in the flavor physics parameters associated with the down sector. However, as we
will argue in section 4, some of them are related to the up-sector flavor parameter via \( SU(2) \)

\(^2\)In a subsequent study \[18\], the same group of authors changed their result after private communication
from us. Their later results agreed with the results presented in this paper.
gauge symmetry and CKM mixings. Therefore, \( Br(b \to s\gamma) \) will provide important constraints in those cases. On the other hand, the current experiments do not impose strong constraints on the CP asymmetry of \( b \to s\gamma \). So we will not discuss it here.

### 2.3 \( B_d \to \phi K_S \)

The time-dependent CP asymmetry measured in the process of \( B_d \to \phi K_S \) or \( B_d \to J/\psi K_S \) is expressed in the following way:

\[
a_f(t) = \frac{\Gamma(B^0_d(t) \to f) - \Gamma(B^0_d(t) \to f) - \Gamma(B^0_d(t) \to f) + \Gamma(B^0_d(t) \to f)}{\Gamma(B^0_d(t) \to f) + \Gamma(B^0_d(t) \to f)} = C_f \cos \Delta M_{B_d} t + S_f \sin \Delta M_{B_d} t
\]

where \( f = \phi K_S \) or \( J/\psi K_S \) depending on which process we are studying. Defining as usual

\[
\lambda_f \equiv \left( \frac{q}{p} \right) \frac{\tilde{A}_f}{A_f},
\]

we have

\[
C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f = \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2}
\]

In the Standard Model, we have

\[
\lambda_{\phi K_S} = \left( \frac{q}{p} \right) \frac{\tilde{T}_{\phi K_S}}{A_{\phi K_S}} = -\frac{V_{td} V_{ts}^* V_{cs} V_{cd}^*}{V_{td}^* V_{ts}^* V_{cs} V_{cd}}
\]

This is invariant under redefinitions of phases.

The SUSY contributions to \( \lambda_f \) can also be written in a phase rotation invariant fashion. To simplify the formula, we assume SUSY contributions only modify the decay amplitude significantly, not the mixing part. We obtain

\[
\lambda_{\phi K_S} = -e^{-i2\beta} \frac{1 + r_{23}e^{i\theta_{23}} + r_{32}e^{i\theta_{32}}}{1 + r_{23}e^{-i\theta_{23}} + r_{32}e^{-i\theta_{32}}},
\]

where we have defined

\[
r_{23}e^{i\theta_{23}} = \frac{b_{23}}{a} \left( \frac{\Delta_{23}^{d,LR}}{V_{ts}^* V_{tb}} \right) \frac{M_3}{|M_3|}, \quad r_{32}e^{i\theta_{32}} = \frac{b_{32}}{a} \left( \frac{\Delta_{32}^{d,LR}}{V_{ts}^* V_{tb}} \right)^* \frac{M_3^*}{|M_3|},
\]

in order to distinguish the \( O_{8g} \) and \( O'_{8g} \) contributions. \( \frac{b_{23}}{a} \) is the ratio of the magnitudes of the SUSY and SM contributions. \( \theta_{23} \) and \( \theta_{32} \) are explicitly phase-rotation invariant. They are two of the 42 new CP violation phases in the MSSM\[20\]. In other words, a non-zero \( \theta_{23} \) or \( \theta_{32} \) characterizes CP violation beyond the KM mechanism. From eq.\[14\], it’s clear that when \( \theta_{23} \)
and $\theta_{32}$ are both zero, the time dependent CP asymmetry measured in the B factories should be

$$S_{\phi K} = \sin 2\beta = 0.736$$  \hspace{1cm} (16)

and thus confirm the standard model KM mechanism of CP violation. Any deviation from this result would imply a new CP violation source other than $\delta_{KM}$. In the framework of the MSSM, it would imply that some CP violation phase such as $\theta_{23}$ or $\theta_{32}$ is non-zero. Notice that although the SM CP violation phase $\beta$ and the new phase(s) $\theta_{23}$ (or $\theta_{32}$) are independent parameters from the low energy effective theory point of view, it’s still possible that these phases are correlated in fundamental flavor physics. Generically, the number of independent phases in the fundamental theory could be less than the number of phases carried by low energy parameters.

In the Wolfenstein parameterization, $V_{ts}$ and $V_{tb}$ are real. In the MSSM, the gluino phase can be rotated to zero by using the $U(1)_R$ symmetry. Then $\theta_{23}$ and $\theta_{32}$ are just the phases of $\Delta_{23}^{d,LR}$ and $\Delta_{32}^{d,LR}$, respectively. In the following, we will use this parameterization, but one should keep in mind that when we say there is a non-vanishing phase of these two MIs, what we really mean is the phase of the combined quantity in eq(15).

It was shown in [8] that if at the weak scale, \[ \Delta_{23}^{d,LR}|_W \text{ or } \Delta_{32}^{d,LR}|_W \sim O(10^{-2}) \times e^{i\phi}, \] where $\phi$ is a non-trivial phase, then $S_{\phi K}$ can deviate significantly from its SM value. We will follow Ref. [8] to calculate the hadronic matrix elements, using the BBNS method [21]. In this approach, the strong phases arise from four classes of diagrams, vertex corrections, penguins, hard scattering with spectator quarks and annihilation diagrams. It is important to properly account for the power corrections originated from the latter two classes of diagrams. These contributions should be subleading in the BBNS factorization, but they involve infrared divergent integrals. To regularize them, we follow BBNS and parameterize the integrals as $\Delta = (1 + \rho e^{i\phi}) \log(m_B/\lambda_b)$ with $\lambda_b = 500$ MeV. This parameterization introduces hadronic uncertainties into our calculation. However, as discussed in greater details in [8], the uncertainties affect the branching ratios more than the CP asymmetries, and we focus on the CP asymmetries in the present paper. The uncertainties also decrease quickly for heavier gluino masses if we assume the validity of BBNS factorization and choose moderate values of $\rho$. We henceforth set $\rho = 0$ since our gluino will be heavy. In the present paper we also mainly compare CP asymmetries for different models, and we expect relative results to be even less sensitive to hadronic physics.

In this paper, we would like to study high scale models which can give large non-SM CP violation in $B_d \to \phi K_S$. We will use these as examples of classes of models which could give rise to interesting low energy CP violation beyond the SM. One interesting result is that some

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\(^3\)Hereafter, when necessary we will use $(...)|_W$ and $(...)|_\Lambda$ to denote the weak scale and GUT scale values, respectively.
classes of high scale theories do not allow one to obtain results such as eq. (17) without violating other constraints. Thus the low scale data can rather directly probe high scale theories.

3 Model Independent Study of High Energy Parameter Space

3.1 RGE Running of High Energy FCNC Parameters

Supersymmetry is used to stabilize the large hierarchy between the GUT scale and weak scale. Therefore, the soft parameters at the input scale, which is assumed to be close to the GUT scale, are related to those in the low energy effective soft Lagrangian only after the inclusion of radiative corrections enhanced by large logarithms. For CP violation in FCNC processes it is crucial to take those RGE running effects into account. In this section, we briefly summarize the results which are very useful for qualitatively understanding the constraints on the high energy parameter space (see the Appendix for a detailed study).

As explained in the Appendix, a systematic and novel way to study the RGE running of the flavor parameters is to use what we call the High Energy SuperCKM (HES) basis. In the HES basis, we are dealing with approximate physical parameters and enjoying the presence of several small parameters, such as $\lambda \sim 0.22$. The qualitative result of that study is summarized in Table 1.

| Parameter | Universal Contribution | Feeds into |
|-----------|------------------------|------------|
| $(\delta^{d}_{LL})_{23}$ | $\sim \eta \lambda^2 y^2_2 \sim 0.01$ | - |
| $(\delta^{d}_{RR})_{23}$ | $\sim \eta^2 y^2_2 y^2_2 \lambda^2 < 10^{-4}$ | - |
| $(\delta^{d}_{LR})_{23}$ | $\sim \eta \lambda^2 y^2_2 < 10^{-4}$ | $(\delta^{d}_{LL})_{23} \sim 50(\delta^{d}_{LR})_{23}(\delta^{d}_{LR})_{33}$ |
| $(\delta^{d}_{LR})_{32}$ | $\sim \frac{m^2}{m^2} \eta^2 y^2_2 \lambda^2 < 10^{-5}$ | $(\delta^{d}_{RR})_{23} \sim 100(\delta^{d}_{LR})_{32}(\delta^{d}_{LR})_{33}$ |

Table 1: RGE Analysis of High Energy FCNC Parameters. $\eta \sim |t_{EW} - t_{GUT}|/16\pi^2 \sim 0.2$ is the loop integration parameter.

Notice that starting from a universal boundary condition at the high scale, the FCNC parameters still acquire non-zero values due to small CKM mixing effects enhanced by the RGE running. We call them universal contributions to the flavor parameters. Important effects of this type are summarized in the second column of Table 1. They should be regarded as important “model-independent” values of those parameters.

Of course, one could start with a set of non-zero FCNC parameters in such a way that their initial values exactly cancel the RGE generated contribution. This would require a conspiracy between the high energy fundamental flavor physics and the radiative corrections associated with lower energy scales. We do not consider such a possibility.
It is also important to notice that different FCNC parameters generically mix through the RGE running. In particular, starting with one FCNC parameter, others will be generated through the mixing. We document some of the most important mixing effects in the third column of Table 1. One of the obvious features is the “decoupling” behavior of LL and RR flavor parameters. Being dimension two parameters, they would not enter the running of the trilinears. As discussed in the appendix, the mixings between LL and RR flavor parameters are also suppressed either by second generation quark masses or by small CKM mixings.

Using the results of the RGE study, we now examine the connection of the high energy FCNC soft parameters and the low energy observables. We again remark that while we present results for simplicity in terms of mass insertions, we actually do full numerical analyses without using the mass insertion approximation.

### 3.2 High Scale Parameter Space

We focus on the three observables discussed in the previous section. In a generic flavor model, all of the flavor parameters could be non-zero at high scale. However, in order to illustrate the constraints effectively, we study the cases in which only one of those parameters is non-zero at a time. The scenario is then referred to by the sole non-vanishing input scale flavor parameter, e.g., \((\delta^d_{LL})_{23}|_{\Lambda_{GUT}}\) scenario which implies that \((\delta^d_{LL})_{23}\) is the only non-vanishing mass insertion at input scale \(\Lambda_{GUT}\). We will identify the input scale as the GUT scale, where the gaugino masses are

\[
M_1 = M_2 = M_3 = 300 \, \text{GeV},
\]

unless otherwise noted. The overall scale of the squark mass matrices \(m_0\) and trilinear terms are set to be 300 GeV as well. We will choose \(\tan \beta = 15\) at weak scale throughout this section.

#### 3.2.1 \((\delta^d_{LL})_{23}|_{\Lambda_{GUT}}\)

\((\delta^d_{LL})_{23}\) could contribute to \(B_d \to \phi K_S\) through the chromo-dipole operator \(O_{8g}\)

\[
O_{8g} = -\frac{g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b
\]

with \(C_{8g} \propto (\delta^d_{LL})_{23} (\delta^d_{LR})_{33}\). If it had a large CP violating phase it could give rise to a large deviation from the SM value of the CP asymmetry of \(B_d \to \phi K_S\).

Unlike many other cases, this CP violating parameter is not constrained by the EDM bound at the high scale. The reason is as follows. The lowest order contribution of \((\delta^d_{LL})_{23}\) to the mercury EDM is proportional to \((\delta^d_{LR})_{32}^* (\delta^d_{LL})_{23}\). Using the result of Table 1 we see that starting with only \((\delta^d_{LL})_{23}\) at the high scale would not generate \((\delta^d_{LR})_{32}\) through RGE running. Therefore, this type of contribution to the EDM is suppressed. The next order contribution is through the combination \((\delta^d_{RR})_{23} (\delta^d_{LR})_{32}^* (\delta^d_{LL})_{23}^* \propto (\delta^d_{RR})_{23} C_{8g}\). As indicated in Table 1 the universal contributions to \((\delta^d_{RR})_{23}\) due to RGE running is small. Adding the fact that LL and RR FCNC soft parameters do not mix significantly under the RGE running, this type of
Figure 1: The correlation of $S_{\phi K_S}$ (solid lines) and BR($b \to s\gamma$) (dashed lines) is shown in this plot. The $x$- and $y$- axes are the real and imaginary parts of $(\delta_{LL}^d)^{23}$ at the input scale, respectively. The CEDM of the $s$ quark receives relatively small contributions in this scenario and imposes no constraint. Here we set $M_2 = M_3$ at the input scale.

contribution is also suppressed. To summarize, the scenario in which $(\delta_{LL}^d)^{23}$ is the only flavor off-diagonal complex soft parameter at the high scale is almost unconstrained by low energy EDM measurements. Notice that the EDM constraint is usually the most stringent bound on the amount of the CP violation in the theory. Therefore, we consider $(\delta_{LL}^d)^{23}$ as a promising candidate for large CP violation which could be probed in rare B-decay processes, such as $B_d \to \phi K_S$.

$(\delta_{LL}^d)^{23}$ is constrained by $b \to s\gamma$ because it enters both the gluino and chargino diagrams.\(^5\) Our numerical studies of $(\delta_{LL}^d)^{23}$ are shown in Fig. 1 and Fig. 2.\(^6\) We see that it is possible to satisfy the $b \to s\gamma$ bound and generate an interesting CP violating effect in $B_d \to \phi K_S$ at the same time. More importantly, while giving interesting low energy CP violation this scenario is not constrained by the current EDM bound. For the entire parameter space shown in Fig. 1, $|e\tilde{d}^c_3| < 8.5 \times 10^{-26} e\text{cm}$.

\(^5\)In the SCKM base, The contribution from chargino and up-type squark diagram involves the factor $V_{CKM}^\dagger (\tilde{m}^2_u)^{LL} V_{CKM}$, which is precisely the left-handed down type squark mass matrix $(\tilde{m}^2_d)^{LL}$. Hence, $(\delta_{LL}^d)^{23}$ also contributes to $b \to s\gamma$ through chargino diagram.

\(^6\)We used leading order running of Wilson coefficients for all the calculations presented in this article.
Figure 2: The correlation of $S_{\phi K_S}$ (solid lines) and BR($b \to s\gamma$) (dashed lines) is shown in this plot. Here in contrast to Fig. 1 we scan $M_2$ (in GeV) and $|\langle \delta^{d}_{LL} \rangle_{23}|$ with $M_3 = 300\text{GeV}$ and $\text{Arg}(\langle \delta^{d}_{LL} \rangle_{23}) = \pi/2$ fixed at the input scale.

3.2.2 $(\delta^{d}_{RR})_{23}|_{\Lambda_{\text{GUT}}}$

$(\delta^{d}_{RR})_{23}$ could contribute to $S_{\phi K_S}$ through the $O'_{89}$ operator as $C'_{8g} \propto (\delta^{d}_{LR})^*_{33}(\delta^{d}_{RR})_{23}$. Since the right-handed rotations are not constrained by the Standard Model CKM matrix, $(\delta^{d}_{RR})_{23}$ is not in principle related to $(\delta^{u}_{RR})_{23}$. Therefore, $(\delta^{d}_{RR})_{23}$ is not strongly constrained by $b \to s\gamma$.

On the other hand, the mercury EDM strongly constrains the allowed CP violation carried by $(\delta^{d}_{RR})_{23}$. The leading order contribution is the combination $(\delta^{d}_{LR})_{23}(\delta^{d}_{RR})^*_{23}$. The universal RGE contribution to $(\delta^{d}_{LR})_{23}$ is suppressed. Combining this with the fact that RGE evolution would not mix $(\delta^{d}_{RR})_{23}$ with $(\delta^{d}_{LR})_{23}$, we conclude the that leading order contribution to the EDM would not strongly constrain $(\delta^{d}_{RR})_{23}$. The next order contribution is $(\delta^{d}_{LL})_{23}(\delta^{d}_{LR})^*_{33}(\delta^{d}_{RR})_{23} \propto (\delta^{d}_{LL})_{23}C'_{8g}$. As indicated in Table I, there is a universal RGE contribution to $(\delta^{d}_{LL})_{23} \sim 0.01$. Hence, a large non-SM CP violation in $S_{\phi K_S}$ from $C'_{8g}$ would almost generically imply a large contribution to the mercury EDM. Therefore, the prospect of getting a large CP violation effect from $(\delta^{d}_{RR})_{23}$, such as proposed in Ref. [9], is necessarily constrained by the mercury EDM bound, as indicated in Fig 3.
Figure 3: This figure shows the correlation between $S_{\phi K_s}$ (solid lines) and the $s$-quark CEDM (dashed lines in the unit of $10^{-25} e\text{cm}$). $(\delta_{RR}^d)_{23}$ nearly does not contribute to $b \to s\gamma$. In view of the uncertainties associated with the $s$-quark CEDM, we allow a relaxation of a factor of 3 of the current experimental bound.

3.2.3 $(\delta_{LR}^d)_{23}|_{\Lambda_{\text{GUT}}}$

$(\delta_{LR}^d)_{23}$ could contribute to $S_{\phi K_s}$ through the $O_{8g}$ operator as $C_{8g} \propto (\delta_{LR}^d)_{23}$. It could give rise to a large deviation from the Standard Model predictions.

The leading order contribution to the mercury EDM comes from combination $(\delta_{RR}^d)_{23}^* (\delta_{LR}^d)_{23} \propto (\delta_{RR}^d)_{23} C_{8g}$. It does not strongly constrain $(\delta_{LR}^d)_{23}$ since RGE running would not induce $(\delta_{RR}^d)_{23}$ either from $(\delta_{LR}^d)_{23}$ or from universal contributions. The next order contribution comes from the combination $(\delta_{LR}^d)_{23} (\delta_{LR}^d)_{33}^* (\delta_{LR}^d)_{32}$. The universal contribution to $(\delta_{LR}^d)_{32}$ is highly suppressed. Since $(\delta_{LR}^d)_{32}$ (approximately) does not mix with $(\delta_{LR}^d)_{23}$ in RGE running, the contribution to the EDM is again suppressed at this order. Therefore, although $(\delta_{LR}^d)_{23}$ does feed strongly into other soft parameters such as $(\delta_{LL}^d)_{23}$, CP violation in $(\delta_{LR}^d)_{23}$ would not be very strongly constrained by the mercury EDM.

$b \to s\gamma$ provides interesting constraints on $(\delta_{LR}^d)_{23}$. For a large class of models, $(\delta_{LR}^d)_{23} m_t / m_b \sim (\delta_{LR}^u)_{23}$. $(\delta_{LR}^d)_{23}$ contributes to $b \to s\gamma$ through the combinations $(\delta_{LR}^u)_{23}^* (\delta_{LR}^u)_{33}$ and $(\delta_{LR}^u)_{23} (\delta_{LL}^u)_{23}$.

Numerical study of the $(\delta_{LR}^d)_{23}$ scenario is shown in Fig. 5. We also include a study of the scenario where $(\delta_{LR}^d)_{23}$ is not related to $(\delta_{LR}^u)_{23}$, which is set to zero at the high scale. The result is shown in Fig. 4.
The interplay between $S_{\phi K_S}$ (solid lines) and $\text{BR}(b \to s\gamma)$ (dashed lines) is shown. We assume that $(\delta_{LR}^d)^{32} = 0$ is not related to $(\delta_{RR}^d)^{23}$ at the input scale. As a consequence, both $\text{BR}(b \to s\gamma)$ and the $s$-quark CEDM become less constraining as opposed to the more usual case in Fig.5 where $A_{i}^{U,L} = A_{i}^{D,L}$ at the input scale with $\tilde{A}_{ij}^{f} = Y_{ij}^{f} A_{j}^{f,L}$ being the trilinear term ($f = U, D$).

3.2.4 $(\delta_{LR}^d)^{32}|_{\Lambda_{GUT}}$

$(\delta_{LR}^d)^{32}$ will contribute to $S_{\phi K_S}$ through the $O'_{8g}$ operator as $C_{8g}^u \propto (\delta_{LR}^d)^{32}$. Since this is the first order in the mass insertion, generically, $(\delta_{LR}^d)^{32}$ could give rise to larger CP violation in $C_{8g}^u$ than $(\delta_{RR}^d)^{23}$. According to Table 1 an input scale $(\delta_{LR}^d)^{32}$ also feeds strongly into $(\delta_{RR}^d)^{23}$ through RGE running. Therefore, it could induce a larger contribution to $C_{8g}^u$ through $(\delta_{RR}^d)^{23}$.

Since $(\delta_{RR}^d)^{32}$ is not generically related to $(\delta_{RR}^d)^{32}$ at the high scale (see Section 4), $b \to s\gamma$ does not seriously constrain this scenario.

Since the CP violation entering $S_{\phi K_S}$ is from the operator $O'_{8g}$, it is again strongly constrained by the $(\delta_{LL}^d)^{23}C_{8g}^{u'}$ contribution to the EDM. The constraints are similar to those derived in the $(\delta_{RR}^d)^{23}$ scenario. The numerical result is presented in Fig. 5. The conclusion is that the prospect of CP violation in $(\delta_{LR}^d)^{32}$ is highly constrained by mercury EDM bound.
Figure 5: This figure shows the interplay among $S_{\phi K_S}$ (black solid lines), BR($b \to s\gamma$) (blue dash lines) and the $s$-quark chromo-EDM (red dash-dotted lines, in the unit of $10^{-25}e\text{cm}$). We assume $A_{U,L}^{ij} = A_{D,L}^{ij}$ at the input scale, where $\tilde{A}_{ij}^f = Y_{ij}^f A_{ij}^{f,L}$ is the trilinear term ($f = U, D$).

4 Models of High Energy Flavor Structure

In this section, we study several different models of high energy flavor structure. Our primary interest is in which models can have large low energy CP violation beyond the Standard Model and which ones cannot, for generic reasons. Given its considerable current interest, we will focus on the $B_d \to \phi K_S$ CP asymmetry. Our analysis could be generalized to other low energy CP violating observables.

4.1 General Estimates from Supergravity

In this section, we give general estimates of various mass insertion parameters using the general supergravity Lagrangian. To begin with, we discuss a parameterization of the trilinears with rather general assumptions about the supergravity induced soft terms.

In supergravity based models, the trilinear terms can be written as

$$\tilde{A}_{ij}^U = Y_{ij}^U A_{ij}^U \quad \tilde{A}_{ij}^D = Y_{ij}^D A_{ij}^D.$$  

Notice there is no matrix product in these two equations. It is in this parameterization that the absolute values of $A_{ij}^{U,D}$ are at the order of $O(m_{3/2})$. One can certainly write trilinears as $\tilde{A}^D = Y^D \cdot \tilde{A}^D$ where a matrix product is used. In this form, it’s not guaranteed that every
Figure 6: The correlation between $S_{K_S}$ (solid lines) and the $s$-quark CEDM (dashed lines, in the unit of $10^{-25} ecm$) is shown as a contour plot. $\text{Br}(B \to X_s \gamma)$ imposes no constraint here, while the $s$-quark CEDM is very constraining. We relax the CEDM bound by a factor of 3 to remind the reader of its large uncertainties.

An element of $\bar{A}$ is of order $O(m_{3/2})$ and the structure of $\bar{A}^D$ could be quite different from $A^D$ in eq. (20). Since we assume the MSSM Lagrangian arises from supergravity theory, we will use the parameterization in eq. (21).

To get a good estimate of $\delta_4^{\alpha LR}$, a reparameterization of eq. (20) will be useful. Take $A^D$ as an example. It can be written as

$$A^D_{ij} = A^D_0 + A^D_i + A^D_j + A^{D^L}_{ij} \delta_{i1} \delta_{j1} + A^{D^R}_{ij} \delta_{i2} \delta_{j2} + A^{D^L}_{ij} \delta_{i1} \delta_{j2} + A^{D^R}_{ij} \delta_{i2} \delta_{j1} + A^{D^L}_{ij} \delta_{i1} \delta_{j1} + A^{D^R}_{ij} \delta_{i2} \delta_{j2} \quad (21)$$

where

$$A^D_0 = A^D_{33} \quad (22)$$

$$A^D_i = A^D_i \quad A^D_j = A^D_j$$

$$A^{D^L}_{ij} = A^{D^L}_{13} - A^{D^L}_{33} \quad A^{D^L}_{23} - A^{D^L}_{33}$$

$$A^{D^R}_{ij} = A^{D^R}_{31} - A^{D^R}_{33} \quad A^{D^R}_{32} - A^{D^R}_{33}$$

$$A^{D^L}_{11} = A^{D^L}_{11} - A^{D^L}_{13} - A^{D^L}_{31} + A^{D^L}_{33}$$

$$A^{D^R}_{11} = A^{D^R}_{11} - A^{D^R}_{13} - A^{D^R}_{31} + A^{D^R}_{33}$$

$$A^{D^L}_{12} = A^{D^L}_{12} - A^{D^L}_{13} - A^{D^L}_{32} + A^{D^L}_{33}$$

$$A^{D^R}_{12} = A^{D^R}_{12} - A^{D^R}_{13} - A^{D^R}_{32} + A^{D^R}_{33}$$

If Yukawas are hierarchical, terms with a prime on the RHS of eq. (21) have to multiply small elements of the Yukawa matrix to give the corresponding elements in the trilinears $\bar{A}^D$. Thus
under the assumption that all $A_{ij}$ are of order the gravitino mass and the Yukawas are hierarchical, these terms are suppressed by at least $m_s/m_b \approx \lambda^2$ where $\lambda = 0.22$ is the Cabbibo angle. Hence we can safely neglect them. Keeping this in mind, one has

$$\tilde{A}_{ij}^U \approx Y_{ij}^U (A_{ij}^U + A_{ij}^{D,L} + A_{ij}^{U,R})$$

$$\tilde{A}_{ij}^D \approx Y_{ij}^D (A_{ij}^D + A_{ij}^{D,L} + A_{ij}^{D,R})$$

In most models, $A_{ij}^{U,L} = A_{ij}^{D,L} \equiv A_{ij}^Q$ since $u_i$ and $d_i$ are in the same $SU(2)$ doublet. Under this assumption, the above equations reduce to

$$\tilde{A}_{ij}^U = Y_{ij}^U (A_{ij}^U + A_{ij}^Q + A_{ij}^{U,R})$$

$$\tilde{A}_{ij}^D = Y_{ij}^D (A_{ij}^D + A_{ij}^Q + A_{ij}^{D,R})$$

The authors of [26] showed that trilinears of many SUSY breaking models follow this parameterization even without the assumption of hierarchical Yukawa couplings. Thus eq. (24) can be applied to a wide range of models.

To relate the trilinear terms to observables, we need to rotate $\tilde{A}$ to the SCKM base. To be precise, let’s define the down-type squark mass insertion (MI) as

$$\delta_{ij}^{d,LR} = \frac{(K_D^D \tilde{A}^D K_R^{D\dagger})_{ij} v_d}{(\tilde{m}^D_{LL})_{ii}(\tilde{m}^D_{RR})_{jj}}$$

where $v_d$ is the vev of the down type higgs and matrices $K^D$ are the rotation matrices that diagonalize Yukawa matrices. Among these $\delta_{ij}^{d,LR}$s, the most interesting ones are $\delta_{23}^{d,LR}$ and $\delta_{32}^{d,LR}$ since they are directly related to the $B \rightarrow \phi K_S$ signal. (Notice that $\delta_{ij}^{d,RL} = \delta_{ji}^{d,LR}$ so we only need to study $\delta_{ij}^{d,LR}$.) By using trilinears as written in eq. (24), we obtain

$$\delta_{23}^{d,LR} = \frac{1}{(\tilde{m}^D_{LL})_{22}(\tilde{m}^D_{RR})_{33}} \times \left( m_b (A_{31}^{D,L} - A_{21}^{D,L}) (K_L^D)_{23} (K_L^{D\ast})_{33} + m_b (A_{1}^{D,L} - A_{2}^{D,L}) (K_L^D)_{21} (K_L^{D\ast})_{31} + m_s (A_{3}^{D,R} - A_{2}^{D,R}) (K_R^D)_{23} (K_R^{D\ast})_{33} + m_s (A_{1}^{D,R} - A_{2}^{D,R}) (K_R^D)_{21} (K_R^{D\ast})_{31} \right)$$

Using the fact $m_s \ll m_b$, $(K_L^D)_{23} (K_L^{D\ast})_{33} \ll (K_L^D)_{21} (K_L^{D\ast})_{31}$ and the definitions in eq. (22) we simplify the above formula to

$$\delta_{23}^{d,LR} \approx \frac{m_b (-A_{2}^{D,L}) (K_L^D)_{23} (K_L^{D\ast})_{33}}{m_\tilde{\nu}^2} = \frac{m_b (A_{23}^{D,L} - A_{23}^{D,L}) (K_L^D)_{23} (K_L^{D\ast})_{33}}{m_\tilde{\nu}^2}$$

(27)

Using the same approximation, one also gets

$$\delta_{32}^{d,LR} \approx \frac{m_b (-A_{2}^{D,R}) (K_R^D)_{33} (K_R^{D\ast})_{23}}{m_\tilde{\nu}^2} = \frac{m_b (A_{32}^{D,R} - A_{32}^{D,R}) (K_R^D)_{33} (K_R^{D\ast})_{23}}{m_\tilde{\nu}^2}$$

(28)

Eq. (27) give a quite good estimate of $\delta_{23}^{d,LR}$ and $\delta_{32}^{d,LR}$ at the high scale. They suggest that by increasing the splitting of trilinears between the 2nd and 3rd family and/or increasing specific elements in the mixing($K^D$), these two MIs could be enhanced.

Next, we make numerical estimates of individual mass insertion parameters.
4.1.1 $\delta^{d,LR}_{23}$

The general form is given in eq. (29). The left-handed rotation $K^D_L$ is constrained not only by unitarity, but also by the CKM matrix:

$$V^{CKM}_{23} = (K^U_L)_{21}(K^D_L)_{31} + (K^U_L)_{22}(K^D_L)^*_{32} + (K^U_L)_{23}(K^D_L)^*_{33} = \lambda^2$$

where $\lambda = 0.22$ is the Cabibbo angle. If there are no large cancellations among the 3 terms in the middle part of the above formula, one expects that each of them is less than or around $\lambda^2 \approx 0.04$. This suggests $(K^D_L)_{23}$ is much less than 1 and implies a large suppression on $\delta^{d,LR}_{23}$. To avoid this suppression, one has to assume both of the last two terms in the above equation are large and they somehow cancel each other to get a small number $\lambda^2$. This could happen in models with democratic Yukawa couplings. If this cancellation indeed happens, then from the unitarity constraint:

$$|(K^D_L)_{13}|^2 + |(K^D_L)_{23}|^2 + |(K^D_L)_{33}|^2 = 1$$

the maximum mixing we can have is $|(K^D_L)_{23}(K^D_L)^*_{33}| \approx 0.5$. In gravity-mediated SUSY breaking models, it’s natural to assume that both $A^D$ and $m_q$ are of order $O(m_{3/2})$. Thus one gets $\delta^{d,LR}_{23} \approx 0.5 \times m_b/m_{3/2}$. Suppose $m_{3/2} \approx 200\text{GeV}$, then $\delta^{d,LR}_{23} \approx O(0.01)$.

This estimation is made at the high scale. To make contact with observables, one must take the RGE running effect into account. The main RGE effect is to enhance the diagonal terms in the squark mass matrix due to the gluino contribution. The off-diagonal terms don’t run much [27]. Thus in eq. (27) and eq. (28), only the denominators are significantly affected by the RGE running. The RGE running of the diagonal squark masses is approximately

$$m^2_q|_{W} \approx 6m_{1/2}^2 + m_0^2 = 6(\sqrt{3}m_{3/2})^2 + m_{3/2}^2 = 19m_{3/2}^2$$

On the second line we assumed a dilaton dominated SUSY breaking scenario: $m_{1/2} = \sqrt{3}m_{3/2}$ and $m_0 = m_{3/2}$. In the above formula, $m_{1/2}$ and $m_0$ on the RHS should take their high scale values. This formula shows the low scale value of $\delta^{d,LR}_{23}$ will get a factor of 19 suppression from its high scale value in the dilaton dominated SUSY breaking scenario, or a factor of 7 if one assumes $m_{1/2} \approx m_0 \approx m_{3/2}$. Thus the natural value of $\delta^{d,LR}_{23}$ is too small to give a large deviation from the SM for the $B_d \rightarrow \phi K_S$ process.

To compensate the RGE suppression, one needs to increase the splitting between $A^D_{23}$ and $A^D_{33}$. For example, if we take $A^D_{23} = -A^D_{33} = 4m_{3/2}$, there will be a factor of 8 enhancement in the numerator of eq. (27). This allows $\delta^{d,LR}|_{W}$ to be of order 0.01 which could give rise to large CP violation in $B_d \rightarrow \phi K_S$ if we assume the cancellation happens in eq. (30).

From the discussion above, we see that in order to use a large $\delta^{d,LR}_{23}$ generating CP asymmetry in the $B_d \rightarrow \phi K_S$ process, the following conditions should be satisfied:

- Large mixing in $K^D_L$: $|(K^D_L)_{23}(K^D_L)^*_{33}| \sim 0.5$
• Large splitting between $A_{23}^D$ and $A_{33}^D$.

As we discussed, to satisfy the first condition, a large fine-tuning may be required.

4.1.2 $\delta_{32}^{d,LR}$.

The basic formula for $\delta_{32}^{d,LR}$ is shown in eq. (28). The numerical estimate is similar to the $\delta_{23}^{d,LR}$ case and there are two conditions to be satisfied if one uses $\delta_{32}^{d,LR}\big|_\Lambda$ to generate a large non-SM CP violation:

• Large splitting between $A_{32}^D$ and $A_{33}^D$: $(A_{32}^D - A_{33}^D) \sim 8m_3/2$

• Large right-handed mixing $|((K_R^D)_{23}^*(K_R^D)_{33}| \sim 0.5$.

Unlike $(K_L^D)_{23}$, there is no CKM constraints on $(K_R^D)_{23}$.

4.1.3 Double Mass Insertions from $\delta_{23}^{d,LL}$ and $\delta_{23}^{d,RR}$.

At the weak scale, both single MI and double MI can give significant contribution to $B \to \phi K_S$ process. In the MI approximation, a double MI: $\delta_{32}^{d,LR} \times \delta_{32}^{d,RR}$ has an effect similar to a single $\delta_{32}^{d,LR}$ MI, and $\delta_{33}^{d,LR} \times \delta_{23}^{d,LL}$ has an effect similar to $\delta_{23}^{d,LR}$.

The size of $\delta_{32}^{d,RR}$ can be estimated as follows. Assuming the right-handed down-type squark mass matrix is diagonal in the gauge eigenstates, we have

$$
\delta_{32}^{d,RR} = \frac{(K_R^D m_2^2 K_R^{D\dagger})_{32}}{m_{3/2}^2} 
\approx \frac{(\tilde{m}_D^2 - \tilde{m}_D^2)(K_R^D)_{32}(K_R^{D*})_{33}}{m_{3/2}^2}
\approx (K_R^D)_{32}(K_R^{D*})_{33} \leq \frac{1}{2}
$$

Notice there is no fermion mass suppression and the only restriction on $K_R^D$ is unitarity. For $\delta_{23}^{d,LL}$, a similar estimate gives

$$
\delta_{23}^{d,LL} \approx (K_L^D)_{23}(K_L^{D*})_{33} \sim \begin{cases} 
\lambda^2 & \text{without cancellation} \\
0.5 & \text{with cancellation}
\end{cases}
$$

As in the single mass insertion case, the above estimates are at the high scale. The RGE will induce large suppressions at the weak scale.

4.2 Abelian Flavor Symmetry Models

The discussion in the previous section is quite generic, not relying on any flavor models. In this section, we study abelian flavor symmetry models and discuss whether they can give rise to a non-SM CP violation.
Abelian flavor symmetry models are interesting since they give a nice explanation for the observed hierarchical structure of fermion masses and the CKM matrix. In the supergravity framework, the soft SUSY breaking terms are related to the quantum numbers of the flavor symmetries. Therefore from a flavor symmetry model, we can calculate the parameters of the soft supersymmetry breaking Lagrangian and learn whether such models can give large deviations from low scale SM CP violation.

Suppose there is a $U(1)_X$ flavor gauge symmetry at the unification scale. A field $\phi$ is a SM singlet but carries $U(1)_X$ charge: $X_{\phi} = -1$. Gauge invariance requires that the superpotential take the form:

$$W = \sum_{ij} Y^D_{ij} \theta(q_i + d_j + h_d) \left( \frac{\phi}{M_P} \right)^{q_i + d_j + h_d} Q_i D_j H_d$$

$$+ \sum_{ij} Y^U_{ij} \theta(q_i + u_j + h_u) \left( \frac{\phi}{M_P} \right)^{q_i + u_j + h_u} Q_i U_j H_u$$

Here $M_P$ denotes the Planck scale. $q_i$, $d_j$ and $h_d$ are the $U(1)_X$ charges for $Q_i$, $D_j$ and $H_d$, respectively. $Y^D_{ij}$ are some $O(1)$ numbers. $\theta(x) = 1$ for $x \geq 0$ and 0 otherwise. To get a hierarchical Yukawa matrix, $\langle \phi \rangle / M_P$ should be a small number. We assume it is approximately $\lambda \approx 0.22$. By choosing the $U(1)_X$ charges correctly, one can generate hierarchical Yukawa matrices such as:

$$Y^U \propto \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} \quad Y^D \propto \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^1 & 1 & 1 \end{pmatrix}$$

This set of Yukawas could give correct fermion masses and a correct CKM matrix. Our discussion below doesn’t depend on the detailed structure of the Yukawas.

If the superpotential is coming from heterotic string theory, modular invariance conditions should be satisfied. To specify these conditions, we first write down the Kähler potential for the moduli fields:

$$K = -\sum_{\alpha} \log(T_{\alpha} + T_{\alpha}^*)$$

Here we denoted both T-type and U-type moduli fields collectively by $T_{\alpha}$. For each matter field $\Phi$, we denote the modular weights as $n^\alpha_{\Phi}$, corresponding to $T_{\alpha}$. In the superpotential, $Y^U_{ij}(Y^D_{ij})$ may also depends on $T_{\alpha}$ and have modular weights: $n^\alpha_{U,ij}(n^\alpha_{D,ij})$. To keep the theory modular transformation invariant, the following conditions should be satisfied:

$$(q_i + d_j + h_d)n^\alpha_{\phi} + n^\alpha_{Q_i} + n^\alpha_{D_j} + n^\alpha_{H_d} + n^\alpha_{D,ij} + 1 = 0$$

In the following, we would like to argue that in this model, it’s difficult to give a large beyond the SM contribution to the $B_d \rightarrow \phi K_S$ process. From the discussion of the previous section, we know that to get a large $\delta^{d,LR}_{32}$, $(K^D_{R})_{23}$ should be at $O(1)$. This requires 32 and 33
entries in $Y^D$ have similar magnitude, which implies that they have the same powers of $\lambda$ due to eq.\,[33], i.e.

$$q_3 + d_2 + h_d = q_3 + d_3 + h_d \quad \Rightarrow \quad d_2 = d_3. \quad (38)$$

In the flavor symmetry models, one usually assumes $Y_{ij}$ in eq.(35) are $O(1)$ and tries to explain the hierarchical structure by using different powers of $\phi/M_P$. Under this assumption, $Y_{ij}$ should be independent of moduli fields $T_\alpha$, or depend on $T_\alpha$ in the same form. Therefore, all $n_{D,ij}^\alpha$ are equal. Then using $d_2 = d_3$, we have

$$n_{D_2}^\alpha = n_{D_3}^\alpha \quad (39)$$

The two relations: $d_2 = d_3$ and $n_{D_2}^\alpha = n_{D_3}^\alpha$ have important implications for the soft terms. To calculate them we assume a diagonal Kähler metric for the observable sector fields but allow non-universality of the diagonal elements since different fields can have different modular weights. The scalar masses and trilinear terms take the form\,[28]:

$$m_{ij}^2 = m_{3/2}^2(1 + \phi_i + 3 \cos^2 \theta \sum_\alpha n_i^\alpha \Theta^2_\alpha) \delta_{ij} \quad (40)$$

$$\tilde{A}_{ij}^D = A_{ij}^D Y_{ij}^D = m_{3/2}(-\sqrt{3} \sin \theta + (q_i + d_j + h_d)) Y_{ij}^D \quad (41)$$

In the above equations, $\theta$ is the Goldstino angle and $\sum \Theta^2_\alpha = 1$. For the right-handed down type squarks, the second and third generations have the same $U(1)_F$ charge and same modular weights, so we have

$$\tilde{m}_{D_2}^2 = \tilde{m}_{D_3}^2. \quad (42)$$

Therefore, $\delta_{32}^{d,RR} = 0$. For the trilinears, notice that they satisfy the parameterization of eq.\,[24]. Thus eq.(28) should give a good estimate for $\delta_{32}^{d,LR}$ and we have

$$\delta_{32}^{d,LR} \approx \frac{m_d(A_2^D - A_3^D)(K_{R_3}^D)_{33}(K_{R_2}^{D*})_{23}}{m_q^2} \quad (43)$$

$$= \frac{m_d(d_2 - d_3)m_{3/2}(K_{R_3}^D)_{33}(K_{R_2}^{D*})_{23}}{m_q^2}$$

$$= 0$$

So in the large right-hand down type quark mixing case ($Y_{32}^D \approx Y_{33}^D$), we have $\delta_{32}^{d,RR} \approx \delta_{32}^{d,LR} \approx 0$. By the same argument, one can show that in the large left-hand down quark mixing case: $Y_{23}^D \approx Y_{33}^D$, we have $\delta_{23}^{d,RR} \approx \delta_{23}^{d,LR} \approx 0$. Therefore, in the large mixing cases, it’s difficult to generate large non-SM CP effects.

One can also try models with $O(\lambda)$ suppressed mixing. For example, $Y_{32}^D : Y_{33}^D = O(\lambda)$. Then $(K_{R_3}^D)_{23} \approx \lambda$ and according to the estimate in section 4.1.2 $|\delta_{32}^{LR}|$ will be much less than 0.01. In this case, using the formula in section 4.1.3, we get $|\delta_{23}^{RR}| \approx \lambda$ at the GUT scale. After running down to the weak scale and therefore including a factor of about 7 RGE suppression, we get $|\delta_{23}^{RR}| \approx \lambda/7 \approx 0.03$. This MI contributes to $B_d \to \phi K_S$ via a double mass insertion,
which involves $\delta_{33}^{LR}$. To estimate $\delta_{33}^{LR}$, let’s assume $m_\tilde{q} = \mu = 600$ GeV and $\tan \beta = 50$. Then we have

$$
\delta_{33}^{LR} \approx \frac{m_\tilde{b}\mu \tan \beta}{m_\tilde{q}^2} \approx 0.24 \quad (44)
$$

where $m_\tilde{b}$ is the running b-quark mass. Thus we can estimate the double mass insertion

$$
\delta_{23}^{RR} \times \delta_{33}^{LR} \approx 0.03 \times 0.24 \approx 0.007 \quad (45)
$$

Remember for the double mass insertion case, there is an extra $1/2$ suppression from the Taylor expansion. In addition, for this case, the 2nd derivative of the loop function is smaller than the 1st derivative for the region around $m_\tilde{g}^2/m_\tilde{q}^2 = 1$. Thus there is no large deviation from the SM for the CP asymmetry in the $B_d \to \phi K_S$ process. Similar arguments apply to the $Y_D^{ij}$ cases. Therefore, without fine-tuning some parameters, we conclude that in the $O(\lambda)$ suppressed mixing cases, it’s also difficult to obtain a large non-SM $B_d \to \phi K_S$ result.

In summary, for Abelian flavor symmetry models, under the assumptions:

- The coefficients $Y_D^{ij}$ and $Y_U^{ij}$ in eq.(34) are all $O(1)$ and T-moduli independent
- Kähler metrics are diagonal(and allowed to be non-universal) for the matter fields

it is unlikely to get a large non-SM contribution to the CP asymmetry of $B_d \to \phi K_S$ process.

### 4.3 Family Dependent Kähler Potential

In the previous section we saw that if a $U(1)_F$ flavor symmetry model is assumed, it’s difficult to give a large deviation from the SM for the $B_d \to \phi K_S$ process. Therefore, in this section, we won’t specify a particular flavor symmetry model. Instead, we take the Yukawas as given parameters. Hence there are no direct relations between the Yukawas and the soft terms. Then using a family dependent (but still diagonal) Kähler potential, we will show that it’s possible to get a large SUSY contribution to the CP asymmetry in $B_d \to \phi K_S$.

We first give a model which has large mixing between the 2nd and 3rd generation right-handed down type quarks. By splitting $A_D^2$ and $A_D^3$, we obtain a large $\delta_{32}^{d,LR}$ and therefore a large contribution to $B_d \to \phi K_S$. Then we give a model which has large mixing between the 2nd and 3rd generation left-handed quarks. By splitting $A_Q^2$ and $A_Q^3$, we have a large $\delta_{23}^{d,LR}$ and it also gives a large contribution to $B_d \to \phi K_S$. In the large $\delta_{32}^{d,LR}$ case, the EDM bound puts strong constraints on how big the SUSY contribution to $B_d \to \phi K_S$ can be, and in the large $\delta_{23}^{d,LR}$ case $b \to s\gamma$ constrains it.

It’s also worth pointing out that in both models, the source of the CP violation phases reside in the Yukawas and all the SUSY breaking $F$ terms($F_S$ and $F_T$) are real. If one takes the MSSM as an effective field theory, only the invariant phases, such as the SM CP violation phase $\delta_{KM}$ or SUSY CP violation phase in eq.(15), are physical phases so that it doesn’t matter whether we put the SUSY phases in the Yukawas or $F$ terms. But from the underlying theory
point of view, Yukawas and $F$ terms have different origins and represent different physics. Thus it’s quite interesting that in our models, the only source of CP violation is in $Y_u$ and $Y_d$ and this may have important impact on string model building.

4.3.1 Large $\delta_{32}^{d,LR}$ case.

In this model, we take the Yukawas as given parameters. They have the following form at the high scale:

\[
Y_u = a_u \times \text{diag}\{m_u, m_c, m_t\}
\]

\[
Y_d = a_d \times V_{CKM} \cdot \text{diag}\{m_d, m_s, m_b\} \cdot U
\]

In these equations, $a_u$ and $a_d$ are normalization factors depending on $\tan \beta$. $V_{CKM}$ and the quark masses should take their high scale values. $U$ is a matrix which generate a right handed down-type quark mixing:

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \omega & e^{i\phi} \sin \omega \\
0 & -e^{-i\phi} \sin \omega & \cos \omega
\end{pmatrix}
\]

Notice if we don’t have soft susy breaking terms, the phase $\phi$ in $U$ is not observable and can be rotated away by field redefinition. But with soft terms, certain combinations of $\phi$ and phases in soft terms are physical observables and can’t be rotated away. Also notice we assume the above Yukawas are for already canonically normalized fields.

To calculate the soft terms, we assume a mixed dilaton/moduli susy breaking scenario and assume we are in the weakly coupled heterotic orbifold vacuum. The soft terms take the form as [29]

\[
m_{1/2} = \sqrt{3}m_{3/2} \sin \theta e^{-\gamma_S}
\]

\[
m_i^2 = m_{3/2}^2(1 + 3 \cos^2 \theta \tilde{n}_i \cdot \tilde{\Theta}^2)
\]

\[
A_{ijk} = -\sqrt{3}m_{3/2}(\sin \theta e^{-\gamma_S} + \cos \theta \sum_{\alpha=1}^3 e^{-i\gamma_\alpha} \Theta_{\alpha})
\]

\[
\times \left[1 + n_i^\alpha + n_j^\alpha + n_k^\alpha + (T_\alpha + T_\alpha^*) \partial_\alpha \log(Y_{ijk})\right]
\]

In these formulas, $i, j, k$ denote different MSSM fields. $\theta$ is the goldstino angle and $\sum \Theta_\alpha^2 = 1$ where $\alpha = 1, 2, 3$ correspond to diagonal T-moduli fields associated with 3 compactified complex planes. For simplicity we assume only these moduli fields are relevant for soft term calculations. The $\gamma_S$ and $\gamma_\alpha$ are the phases for $F_S$ and $F_{\Theta_{\alpha}}$ and we set them to be zero, so all the CP violation sources are in the Yukawas. $n_i^\alpha$ are the modular weights of a field $i$ with respect to $\alpha$—moduli. They are negative fractional numbers. For fields $\phi$ in the untwisted sector, the modular weights are $n_\phi = (-1, 0, 0)$ or $(0, -1, 0)$ or $(0, 0, -1)$ depending on which complex plane field $\phi$ is on. Modular weights for twisted fields are a little bit more complicated and actually we don’t need to know them in this model.
Generally, the Yukawas $Y_{ijk}$ depend on moduli fields so that the last term in eq.(50) is nonzero. Since we take the Yukawas as input numbers, there derivatives respect to $T_\alpha$ can not be determined. Thus we have to assume their contributions to the trilinears are small and neglect them. Then the above formula for the trilinears satisfies the parameterization in eq. (24).

Using these equations we have

$$\Delta^A_{23} \equiv A^D_2 - A^D_3 = -\sqrt{3}m_{3/2}\cos \theta \sum_{\alpha=1}^{6} e^{-i\gamma_\alpha} \Theta_\alpha (n^\alpha_{D_2} - n^\alpha_{D_3})$$

$$= -\frac{m_{1/2}e^{i\gamma_S}}{\tan \theta} \sum_{\alpha=1}^{6} e^{-i\gamma_\alpha} \Theta_\alpha (n^\alpha_{D_2} - n^\alpha_{D_3})$$

From the discussions in section 4.1 we learned that to get large $\delta_{32}^{LLR}$, $\Delta^A_{23}$ should be large. Thus we need a small goldstino angle $\theta$ which means a moduli dominated susy breaking scenario is preferred.

In this model, we use the following parameters:

$$m_{3/2} = 420\text{GeV} \quad \sin \theta = \frac{1}{\sqrt{7}} \quad \omega = \frac{\pi}{4} \quad \tan \beta = 40.$$  \hspace{1cm} (51)

The relevant modular weights and $\Theta_\alpha$ are shown in Table 2. We assume the other matter fields have family independent modular weights. Therefore, all other fields have family independent trilinear and scalar masses. Notice $n_{D_2}$ and $n_{D_3}$ are different which means they have different Kähler potentials

$$K = \frac{|D_2|^2}{T_2 + \overline{T}_2} + \frac{|D_3|^2}{T_3 + \overline{T}_3} + \ldots$$  \hspace{1cm} (52)

This difference will only show up in the soft terms if $\cos \theta \neq 0$.

Taking these parameters as high scale input, we use RGEs to run the soft Lagrangian to the weak scale and calculate observables such as $S_{\phi K_{\phi}}$, the strange quark CEDM, $BR(b \to s\gamma)$, the higgs mass, etc. We scan the phase $\phi$ in eq.(47) from 0 to $2\pi$. As explained before, for the
large $\delta_{32}^{d,LR}$ case, it’s the EDM bound that is most difficult to satisfy. We show the correlation between predictions of $S_{\phi K_S}$ and $ed_s^C$ in figure 7. The upper bound on $|ed_s^C|$ is $5.8 \times 10^{-25} ec m$. The theoretical prediction shown in this figure has a large theoretical uncertainty. If we allow factor 3 theoretical uncertainty, from this figure we find the smallest $S_{\phi K_S}$ is about -0.3. We checked that for this model, other experimental constraints such as $BR(b \rightarrow s\gamma)$ and the higgs mass bound are satisfied.

![Large $\delta_{32}^{d,LR}$ scenario](image)

Figure 7: CEDM vs. $S_{\phi K_S}$ in the large $\delta_{32}^{d,LR}$ scenario. We scan the phase $\phi$ in eq. (47) from 0 to $2\pi$. Two horizontal lines are the experimental bounds on the s-quark CEDM. If we use the exact CEDM bound, the smallest $S_{\phi K_S}$ we can get in this model is around 0.14. If we allow a factor 3 theoretical uncertainty on the CEDM prediction, the smallest $S_{\phi K_S}$ in this model is around -0.33.

### 4.3.2 Large $\delta_{23}^{d,LR}$ case.

To make this MI large, we need a large mixing of the 2nd and 3rd generation left-handed quarks. Thus we use the following Yukawas:

$$Y_u = a_u \times U \cdot \text{diag}\{m_u, m_c, m_t\}$$

$$Y_d = a_d \times U \cdot V_{CKM} \cdot \text{diag}\{m_d, m_s, m_b\}$$

The parameters are:

$$m_{3/2} = 360\text{GeV} \quad \sin \theta = \frac{1}{\sqrt{7}} \quad \omega = \frac{\pi}{7} \quad \tan \beta = 24$$

(53)

The $\Theta_\alpha$ and modular weights are shown in Table 3. Due to the different modular weights for $Q_2$
Table 3: $\Theta_i$ and modular weights for some particles

|       | $T_1$ | $T_2$         | $T_3$         |
|-------|-------|---------------|---------------|
| $\Theta_i$ | 0     | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $n_{Q_1}$ | 0     | -1            | 0             |
| $n_{Q_2}$ | 0     | -1            | 0             |
| $n_{Q_3}$ | 0     | 0             | -1            |
| $n_{H_u}$ | 0     | -1            | 0             |
| $n_{H_d}$ | 0     | -1            | 0             |

and $Q_3$, the trilinears $A^Q_2$ and $A^Q_3$ are split. (Notice $A^Q = A^{U, L} = A^{U, R}$.) From eq.(27), we see that $\delta_{23}^{d,LR}$ will be large. We assume modular weights for other particles are family independent. We scan the phase $\phi$ in the $U$ matrix. As explained before, in the large $\delta_{23}^{d,LR}$ case, the SUSY contribution to $S_{\phi K_S}$ is mainly constrained by $BR(b \rightarrow s \gamma)$. We show the correlations between them in figure 8. From this figure, we see that the smallest $S_{\phi K_S}$ we can get without violating the $BR(b \rightarrow s \gamma)$ bound is about 0.13.

Figure 8: $BR(b \rightarrow s \gamma)$ vs. $S_{\phi K_S}$ in the large $\delta_{23}^{d,LR}$ scenario. Two horizontal lines are the bounds on $BR(b \rightarrow s \gamma)$. The smallest $S_{\phi K_S}$ in this model without violating the $b \rightarrow s \gamma$ constraint is around 0.13.
5 Conclusions

Fundamental flavor physics is expected to manifest itself in the flavor structure of the soft supersymmetry Lagrangian at the supersymmetry breaking scale. In principle, there could be large CP violating parameters in the soft Lagrangian. They could give rise to large deviations from Standard Model prediction of the CP violating observables. One example is the recent potential deviation in $S_{\phi K_S}$ of $B_d \rightarrow \phi K_S$. On the other hand, CP violation in the soft Lagrangian at high scale is constrained by low energy observables such as EDM and $b \rightarrow s\gamma$. In this paper, we give a model independent analysis of the constraints on the high energy parameter space. Several interesting scenarios are identified which both satisfy the experimental constraints and give rise to large CP violation beyond the Standard Model. With the help of these results, we further investigated classes of high energy flavor structure in the soft parameters. We found that large classes of such models could not produce large deviation from SM in processes such as $B_d \rightarrow \phi K_S$ and satisfy the constraints at the same time. Finally, we presented several scenarios where such a goal can be achieved. Thus such data could point toward some classes of high scale theories as favored. Further investigation along those directions are clearly very interesting and important especially if BELLE result on $S_{\phi K_S}$ of $B_d \rightarrow \phi K_S$ is confirmed.

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A RGE study of flavor parameters

In this section, we analyze the RGE running of FCNC soft parameters. The purpose of this study is to establish a systematic way to understand semi-analytically the running of flavor off-diagonal parameters in the MSSM. It provides an understanding complementary to that of the precise numerical study (which is presented in Section 3) of the connection between high and low scale flavor parameters. The techniques of this appendix will be generally useful in future analysis.

In general, the running of a specific parameter in the SUSY Lagrangian is basis-dependent since the RGEs depends on Yukawa couplings. The usual practice is to write the RGE equations in the gauge eigenstate basis (so that the gauge interactions are flavor diagonal). However, in this basis it is not possible to give a systematic and model-independent study of the running of the FCNC parameters due to the uncertainties of the Yukawa couplings. Therefore, it is more useful to study the running in the so called superCKM basis where there is a more direct connection of the low energy observables and high energy parameters.

It is neither convenient, nor necessary, to diagonalize the Yukawa couplings at each stage of the running. In order to get a systematic estimate of the effect of the running, it is enough
to run the RGE in the superCKM basis defined by high energy input Yukawa couplings. More specifically, we diagonalize the Yukawa matrices at the high energy input scale. This diagonalization defines a high energy superCKM (HES) basis. We then run the RGE equations in this basis. Now, in this basis, the RGE equations are different from those written for the gauge eigenstates. However, it can be shown that the only new parameters showing up the RGEs are the CKM matrix \((V_{CKM})\) obtained by diagonalizing the Yukawa matrices at the high scale. Due to the facts (which we will justify) that the CKM matrix has only small running \([30]\), and the diagonal Yukawa couplings in this basis are proportional to quark masses, it is thus possible to obtain reliable estimates of the running.

We begin with Yukawa couplings. In the gauge eigenstate basis, the RGEs of Yukawa couplings are

\[
\frac{dY_u}{dt} = \frac{1}{16\pi^2} \left[ 3Y_u Y_u^\dagger Y_u + Y_d Y_d^\dagger Y_u + 3Tr(Y_u^\dagger Y_u)Y_u - \left( \frac{16}{3}g_3^2 + 6g_2^2 + \frac{13}{15}g_1^2 \right) Y_u \right],
\]

\[
\frac{dY_d}{dt} = \frac{1}{16\pi^2} \left[ Y_u Y_u^\dagger Y_d + 3Y_d Y_d^\dagger Y_d + 3Tr(Y_d^\dagger Y_d)Y_d - \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2 \right) Y_d \right],
\]

(54)

where we have suppressed all leptonic Yukawa couplings. We then rotate to the HES basis by applying the transformation \(Y \rightarrow V_{L}^* Y V_{R}^T\) on both sides of Eq. 54. We then obtain, in the HES basis (we still denote the Yukawa coupling in this basis by \(Y\))

\[
\frac{dY_u}{dt} = \frac{1}{16\pi^2} \left[ 3Y_u Y_u^\dagger Y_u + V_{CKM}^2 Y_u Y_u^\dagger V_{CKM}^T Y_u + 3Tr(Y_u^\dagger Y_u)Y_u - \left( \frac{16}{3}g_3^2 + 6g_2^2 + \frac{13}{15}g_1^2 \right) Y_u \right],
\]

\[
\frac{dY_d}{dt} = \frac{1}{16\pi^2} \left[ V_{CKM}^T Y_u Y_u^\dagger V_{CKM}^2 Y_d + 3Y_d Y_d^\dagger Y_u + 3Tr(Y_d^\dagger Y_d)Y_d - \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2 \right) Y_d \right]
\]

(55)

where the high energy CKM matrix \(V_{CKM}\) is defined by \(V_{L}^T V_{d}^L\). In order to get an approximate solution to these RGE equations, we make the following assumptions

1. The Yukawa matrices stay approximately diagonal in the running. Therefore, approximately, individual diagonal entries run independently and are proportional to quark masses.

2. The off-diagonal running terms, which are proportional to \(V_{CKM}^2\), can be treated as a perturbation. The main effect of such a perturbation is to generate off diagonal Yukawa couplings.

3. The CKM matrix does not run very much (or the running effect is subleading). Therefore, we have a systematic expansion (in terms of \(\lambda\)) of the RGE effects.

From these assumptions, we can solve for the additional mixing generated by the running (the RGE running of the diagonal terms is well known and dominated by SU(3) gauge coupling

\[\text{We are using the convention of Ref. [2].}\]
and third generation Yukawas) by substituting diagonal Yukawa couplings in to Eq. 55 and integrating approximately.

As a result, we have the following approximate solutions

\[
Y_u|W \sim \begin{pmatrix} y_u & \eta y_b \rho \\ y_c & y_t \end{pmatrix} + \eta y_b \rho \begin{pmatrix} y_u \lambda^5 & y_c \lambda^5 & y_t \lambda^3 \\ y_u \lambda^3 & y_c \lambda^2 & y_t \lambda^2 \end{pmatrix},
\]

\[
Y_d|W \sim \begin{pmatrix} y_d & \eta y_t \rho \\ y_s & y_b \end{pmatrix} + \eta y_t \rho \begin{pmatrix} y_d \lambda^5 & y_s \lambda^5 & y_b \lambda^3 \\ y_d \lambda^3 & y_s \lambda^2 & -y_b \lambda^2 \end{pmatrix},
\]

where \( \eta \sim |t_{EW} - t_{GUT}|/16\pi^2 \sim 0.2 \). Since the diagonal Yukawa couplings are proportional to the quark masses, the most significant modification of the flavor mixing generated by RGE running are \( \delta V_{13} \sim 0.1 \lambda^3 \) and \( \delta V_{23}^L \sim 0.1 \lambda^2 \). We see that the 13 and 23 elements of the CKM matrix, or a hierarchical \( V^L \) proportional to the CKM matrix, could run about 10 percent. This is consistent with the assumption we made in solving the RGE equations.

Next, we look at the running of the trilinears. In the HES basis, the RGE equation for the trilinear \( \tilde{A}_d \) is

\[
\frac{d\tilde{A}_d}{dt} = \frac{1}{16\pi^2} \{4\tilde{A}_d Y_d^\dagger Y_d + 5Y_d^\dagger Y_d \tilde{A}_d + V_{CKM}^T Y_u Y_u^\dagger V_{CKM}^\dagger \tilde{A}_d + 2V_{CKM}^T \tilde{A}_d Y_u^\dagger V_{CKM}^\dagger Y_d
\]

\[
+ \tilde{A}_d (3Tr[Y_d^\dagger Y_d] - \frac{16}{3} g_3^2) + Y_d (\frac{16}{3} g_3^2 M_d + 6Tr[Y_d^\dagger \tilde{A}_d])\},
\]

where we suppressed subleading terms (such as terms proportional to electroweak gauge couplings). First, we observe that the running of the diagonal terms of the trilinears is almost always dominated by the term proportional to the gluino mass (with the possible exception of a large 3rd generation diagonal trilinear coupling). Intuitively, the running of the off-diagonal terms are almost proportional to themselves. Therefore, their running should not be very significant. To gain an approximate understanding of the running, we could expand the RGE equations, as we have done in the case of the Yukawa couplings, in terms of small (off-diagonal) parameters such as \( \lambda \sim 0.22 \). We write the RGE equation as

\[
16\pi^2 \frac{d\tilde{A}_d}{dt} = \mathcal{A}_d
\]

where the right-hand-side \( \mathcal{A}_d \) is a \( 3 \times 3 \) matrix. In terms of small parameters, the flavor off-diagonal entries of the last two generations are

\[
(\mathcal{A}_d)_{33} \sim -(y_d^2 + y_t y_b)\lambda^2 (\tilde{A}_d)_{33} + (y_t y_b + 4y_b^2) (\tilde{A}_d)_{23} + \lambda y_t y_b (\tilde{A}_d)_{13}
\]

\[
(\mathcal{A}_d)_{32} \sim 4\Delta y_b (\tilde{A}_d)_{33} + 5y_b^2 (\tilde{A}_d)_{32},
\]

where \( \Delta \sim \eta y_b y_t^2 \lambda^2 \), coming from term \( \tilde{A}_d Y_d Y_d^\dagger \) in the RGE. The result of RGE running of off diagonal entries of trilinear couplings is approximately \( \tilde{A}_d = \eta \mathcal{A}_d \). Of course, it is always
understood that the entries in matrix $\mathcal{M}$ should be taking some appropriate intermediate values. Useful estimates can be obtained from those expressions. For example, we can derive that even without the presence of off-diagonal terms in the trilinears, by RGE running, we will have

$$ (\delta_{LR}^d)_{23} \sim \frac{v_d \eta \lambda^2 y_t^2 (\tilde{A}_d^*)_{33}}{m_q} \sim 10^{-4}, $$

(61)

and

$$ (\delta_{LR}^d)_{32} \sim \frac{4 \eta \Delta y_b v_d (\tilde{A}_d^*)_{33}}{m_q} \sim 10^{-5} $$

(62)

Now we turn to consider the running of the soft masses. First, we consider the running of the right-handed down-type soft mass parameters. The RGEs, again in the HES basis, are

$$ 16\pi^2 \frac{d m_D^2}{dt} = 16\pi^2 [2 Y^T_d Y_d m_D^2 + 2 m_D^2 Y_d^T Y_d + 4 Y_d^T Y_d m_Q^2 Y_d + 4 m_D^2 Y_d^T Y_d + 4 \tilde{A}_d \tilde{A}_d - \frac{32}{3} g^2 |M_3|^2]. $$

(63)

We could write the RHS as a $3 \times 3$ matrix $\mathcal{M}_D$ and expand it in terms of small parameters. For the last two generations, we have approximately

$$ (\mathcal{M}_D)_{23} \sim 2 \Delta y_b [(m_{D_{33}})^2 + (m_{D_{22}})^2 + 2(m_{Q_{33}})^2] + 4(\tilde{A}_d)^*_{32}(\tilde{A}_d)_{33}. $$

(64)

Some important results can be derived from Eq. (64). First, if we begin at the input scale with a non-zero $(\tilde{A}_d)_{32}$, we would induce a right-handed mixing term through RGE running

$$ (\delta_{RR}^d)_{23} \sim \frac{4 (\tilde{A}_d)^*_{32}(\tilde{A}_d)_{33}}{m_q^2} \sim \frac{4 \eta (\delta_{LR}^d)_{32}}{v} \frac{\tan \beta}{\tan \beta} \sim 100 (\delta_{LR}^d)_{32}(\delta_{LR}^d)_{33}. $$

(65)

Notice that a $\tilde{A}_{23}$ entry, on the other hand, does not generate a large RR mixing. This fact will be important in the search for viable high scale models.

On the other hand, if there are only diagonal terms in the soft masses, the RGE running could generate an off-diagonal mixing

$$ (\delta_{RR}^d)_{23} \sim 2 \Delta y_b [(m_{D_{33}})^2 + (m_{D_{22}})^2 + 2(m_{Q_{33}})^2]/m_q^2 < 10^{-4} $$

(66)

which is quite suppressed (comparing with the $LL$ case studied below).

We also note that the mixing between $(\delta_{LL}^d)_{23}$ and $(\delta_{RR}^d)_{23}$ is highly suppressed by second generation quark masses and/or higher power of CKM mixing $\lambda$.

---

8 Notice that in our notation \cite{2}, LR part of the squark mass matrix corresponds to $\tilde{A}^*$.  
9 In our estimation, we typically take the trilinear part to be the dominant part in the LR sector of the squark mass matrix. This assumption could be violated for the diagonal elements of LR sector, especially the down-sector 33 element, in the very large $\tan \beta$ and $\mu$ regime of the parameter space. In that case, the estimates proportional $\tilde{A}_{33}^d$ (or mass insertions proportional to $(\delta_{LR}^d)_{33}$ would be enhanced.)
Finally, the RGE running of the soft masses of the left-handed squarks, in the HES basis, are

\[
\frac{dm^2_{\tilde{Q}_d}}{dt} = \frac{1}{16\pi^2} \left\{ V_{CKM}^T Y_u \tilde{Y}_u \tilde{V}_u^* m^2_{\tilde{Q}_d} + m^2_{\tilde{Q}_d} V_{CKM}^T Y_u \tilde{Y}_u \tilde{V}_u^* + 2V_{CKM}^T Y_u m^2_{\tilde{U}_u} \tilde{Y}_u \tilde{V}_u^* \right\} \\
+ 2m^2_{H_u} V_{CKM}^T Y_u \tilde{Y}_u \tilde{V}_u^* + 2V_{CKM}^T \tilde{A}_u \tilde{A}_u^* \tilde{V}_u^* \\
+ Y_d \tilde{Y}_d m^2_{\tilde{Q}_d} + m^2_{\tilde{Q}_d} \tilde{Y}_d \tilde{Y}_d^* + Y_d m^2_{\tilde{D}_d} \tilde{Y}_d^* + 2Y_d m^2_{\tilde{Y}_d} \tilde{Y}_d^* + 2\tilde{A}_d \tilde{A}_d^* - \frac{32}{3} g^2_3 |M_3|^2 \right\},
\]

(67)

which give

\[
(M_{\tilde{Q}})_{23} \sim -y_t^2 \lambda_2 [(m^2_{\tilde{Q}})_{33} + (m^2_{\tilde{Q}})_{22} + 2(m^2_{\tilde{U}_u})_{33} + 2m^2_{H_u}], \\
+ \Delta y_b [(m^2_{\tilde{Q}})_{33} + 2(m^2_{\tilde{Q}})_{23} + 2m^2_{H_d}] + y_b^2 (m^2_{\tilde{Q}})_{23} \\
+ 2(\tilde{A}_u)_{23} (\tilde{A}_u)_{33}^* + 2(\tilde{A}_d)_{23} (\tilde{A}_d)_{33}^* + \lambda (\tilde{A}_u)_{13} (\tilde{A}_u)_{33}. \]

(68)

Therefore, in the absence of any off-diagonal terms, the RGE running will generate a

\[
(\delta^d_{LL})_{23} \sim \eta y_t^2 [(m^2_{\tilde{Q}})_{33} + (m^2_{\tilde{Q}})_{22} + 2(m^2_{\tilde{U}_u})_{33} + 2m^2_{H_u}] / m^2_{sq} \sim 4\eta y_t^2 = 0.01.
\]

(69)

Notice also, although a large $\tilde{A}_{23}$ does not generate a large $RR$ mixing, it will generate a sizable $LL$ mixing via RGE running.

Similarly, the mixing between $(\delta^d_{LL})_{23}$ and $(\delta^d_{RR})_{23}$ is highly suppressed as well.

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