MAGNETIC ENERGY PRODUCTION BY TURBULENCE IN BINARY NEUTRON STAR MERGERS

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ABSTRACT

The simultaneous detection of electromagnetic and gravitational wave emission from merging neutron star binaries would greatly aid in their discovery and interpretation. By studying turbulent amplification of magnetic fields in local high-resolution simulations of neutron star merger conditions, we demonstrate that magnetar-level (\(\gtrsim 10^{16}\) G) fields are present throughout the merger duration. We find that the small-scale turbulent dynamo converts 60% of the randomized kinetic energy into magnetic fields on a merger timescale. Since turbulent magnetic energy dissipates through reconnection events that accelerate relativistic electrons, turbulence may facilitate the conversion of orbital kinetic energy into radiation. If \(10^{-3}\) of the \(\sim 10^{53}\) erg of orbital kinetic available gets processed through reconnection and creates radiation in the 15–150 keV band, then the fluence at 200 Mpc would be \(10^{-7}\) erg cm\(^{-2}\), potentially rendering most merging neutron stars in the advanced LIGO and Virgo detection volumes detectable by Swift BAT.

Key words: gamma-ray burst: general – gravitational waves – magnetohydrodynamics (MHD) – stars: neutron – turbulence – X-rays: general

Online-only material: color figures

1. INTRODUCTION

The inspiral and coalescence of binary neutron star (BNS) systems is a topic of increasingly intensive research in observational and theoretical astrophysics. It is anticipated that the first direct detections of gravitational wave (GW) will be from compact binary mergers. BNS mergers are also thought to produce short-hard gamma-ray bursts (SGRBs; Eichler et al. 1989; Narayan et al. 1992; Belczynski et al. 2006; Metzger et al. 2008). Simultaneous detections of a prompt GW signal with a spatially coincident electromagnetic (EM) counterpart dramatically increase the potential science return of the discovery. For this reason, there has been considerable interest as to which, if any, detectable EM signature may result from the merger (Metzger & Berger 2012; Piran et al. 2013). Other than SGRBs and their afterglows, including those viewed off-axis (Rhoads 1997; van Eerten & MacFadyen 2011), suggestions include optical afterglows associated with the radioactive decay of tidally expelled -process material (Li & Paczyński 1998; Metzger et al. 2010; Rezzolla et al. 2011; Giacomazzo et al. 2011). The largest eddies produced are on the order of km scale and rotate at \(\sim 0.1c\), setting the cascade time \(t_{\text{eddy}}\) and kinetic energy injection rate \(\epsilon_K \equiv E_K / t_{\text{eddy}}\), at 0.1 ms and \(5 \times 10^{50}\) erg s\(^{-1}\), respectively. When kinetic equipartition is reached, each turbulent eddy contains \(5 \times 10^{46}\) erg of magnetic energy, and a mean magnetic field strength

\[
B_{\text{RMS}} \gtrsim 10^{16} G \left( \frac{\rho}{10^{13} \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{v_{\text{eddy}}}{0.1c} \right). \tag{1}
\]

Whether such conditions are realized in merging neutron star systems depends upon the dynamo saturation time \(t_{\text{sat}}\) and equipartition level \(E_{\text{sat}} / E_K\). In particular, if \(t_{\text{sat}} \lesssim t_{\text{merge}}\) then turbulent volumes of neutron star material will contain magnetar-level fields throughout the early merger phase. Once saturation is reached, a substantial fraction of the injected kinetic energy, \(0.7 \epsilon_K\), is resistively dissipated (Haugen et al. 2004) at small scales. Magnetic energy dissipated by reconnection in optically thin surface layers will accelerate relativistic electrons (Blandford & Eichler 1987; Lyutikov & Uzdensky 2003), potentially yielding an observable EM counterpart, independently of whether the merger eventually forms a relativistic outflow capable of powering a short gamma-ray burst (GRB).

In this Letter, we demonstrate that the small-scale turbulent dynamo saturates quickly, on a time \(t_{\text{sat}} \lesssim t_{\text{merge}}\), and that \(10^{16}\) G magnetic fields are present throughout the early merger phase. This implies that the magnetic energy budget of merging BNSs is controlled by the rate with which hydrodynamical instabilities randomize the orbital kinetic energy. Our results are derived from simulations of the small-scale turbulent dynamo operating in the high-density, trans-relativistic, and highly conductive material present in merging neutron stars. We have carefully examined the approach to numerical convergence and report grid resolution criteria sufficient to resolve aspects of the small-scale dynamo. Our Letter is organized as follows.
The numerical setup is briefly described in Section 2. Section 3 reports the resolution criterion for numerical convergence of the dynamo completion time and the saturated field strength. In Section 4, we assess the possibility that magnetic reconnection events may convert a sufficiently large fraction of the magnetic energy into high-energy photons to yield a prompt EM counterpart detectable by high-energy observatories including Swift and Fermi.

2. NUMERICAL SCHEME AND PHYSICAL SETUP

The equations of ideal relativistic magnetohydrodynamics (MHD) have been solved on the periodic unit cube with resolutions between 163 and 10243:

\[ \nabla \cdot \mathbf{v} = 0 \quad (2a) \]
\[ \nabla \times \mathbf{v} = 0 \quad (2b) \]
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad (2c) \]

Here, \( \mathbf{T} \) is the stress tensor, \( \mathbf{v} \) is the velocity four-vector, and \( \mathbf{B} \) is the magnetic field four-vector, and \( \rho \) is the rest mass density. \( \nabla \cdot \mathbf{v} \) represents the divergence of the fluid motion, \( \nabla \times \mathbf{v} \) represents the curl of the fluid motion, and \( \frac{\partial \mathbf{B}}{\partial t} \) represents the change of the magnetic field with time.

We have employed a realistic micro-physical equation of state (EOS) appropriate for the conditions of merging neutron stars. It includes contributions from high-density nucleons according to a relativistic mean field model (Shen et al. 1998, 2010), a relativistic and degenerate electron–positron component, neutrino and anti-neutrino pairs in beta equilibrium with the nucleons, and radiation pressure. For our conditions, all the components make comparable contributions to the pressure. We have also employed a simpler gamma-law EOS and found close agreement for the conditions explored in this Letter, indicating that the MHD effects are insensitive to the EOS for transonic conditions. The models presented in our resolution study use the far more expensive gamma-law EOS.

All of the simulations presented in this study use the HLLD approximate Riemann solver (Mignone et al. 2009), which has been demonstrated as crucial in providing the correct spatial distribution of magnetic energy in MHD turbulence (Beckwith & Stone 2011). The solution is advanced with an unsplit, second-order MUSCL-Hancock scheme. Spatial reconstruction is accomplished with the piecewise linear method configured to yield the smallest possible degree of numerical dissipation. The divergence constraint on the magnetic field is maintained to machine precision at cell corners using the finite volume CT method of Tóth (2002). Full details of the numerical scheme may be found in Zrake & MacFadyen (2012, 2013).

3. RESULTS

3.1. Growth and Saturation of the Magnetic Energy

Magnetic fields are amplified in our simulations by the small-scale turbulent dynamo. Turbulent fluid motions stretch and fold the magnetic field lines, causing exponential growth of magnetic energy (e.g., Moffatt 1978). This growth is attributed to the advection and diffusion of the magnetic field (Equation (2c)). When the magnetic field is weak (\( E_M \ll E_K \)), it evolves passively and the turbulence is hydrodynamical. This limit is referred to as small-scale kinematic turbulent dynamo, and is well described by Kazantsev’s model (Kazantsev 1968). This model predicts that the power spectrum of magnetic energy peaks at the resistive scale \( \ell_R \) and obeys a power law \( k^{3/2} \) at longer wavelengths. The kinematic phase ends when the magnetic field acquires sufficient tension to modify the hydrodynamic motions, after which a dynamical balance between kinetic and magnetic energy is established.

Numerical simulations of MHD turbulence are typically limited to magnetic Prandtl numbers \( Pm \equiv v/\eta \approx 1 \). However, neutron star material is characterized by much smaller \( Pm \gg 1 \), with the viscous cutoff due to neutrino diffusion occurring around 10 cm, while the resistive scale is significantly smaller (Thompson & Duncan 1993). However, the disparity between true and simulated magnetic Prandtl number does not influence our conclusions, since dynamos are generically easier to establish in the high \( Pm \) regime than the small (Haugen et al. 2004; Ponty & Plunian 2011).

We use an initially uniform, pulsar-level (1011 G) seed magnetic field. This field is sub-dominant to the kinetic energy by 10 orders of magnitude, so that the initial field amplification is expected to be well described by Kazantsev’s model. Indeed, we find that during this phase the power spectrum of magnetic energy follows \( k^{3/2} \) (Figure 4), peaking at around 10 grid zones, which we identify as the effective scale of resistivity. The saturation process begins at even earlier times with increasing numerical resolution. This reflects the fact that during the kinematic phase, magnetic energy exponentiates on a timescale controlled by shearing at the smallest scales. In numerically converged runs, full saturation occurs with \( E_M \approx 0.6 E_K \) and is characterized by scale-by-scale super-equipartition, with \( E_M \approx 4 E_K \) at all but the largest scale.

3.2. Numerical Convergence

The same driven turbulence model was run through magnetic saturation at resolutions \( 16^3, 32^3, 64^3, 128^3, 256^3 \), and \( 512^3 \). Another model at \( 1024^3 \) was run through the end of the kinematic phase, but further evolution was computationally prohibitive. Figure 1 shows the development of \( B_{RMS}, E_M \), and \( E_K \) as a function of time at each resolution.

We find that sufficiently resolved runs (\( \gtrsim 512^3 \)) attain mean magnetic field strengths of \( 10^{16} \) G within two large-eddy rotations. All models with resolutions \( \gtrsim 32^3 \) eventually attain mean fields of \( \gtrsim 3 \times 10^{16} \) G. The saturated field strength increases until resolution \( 256^3 \). We find that the kinematic growth rate is higher at each higher resolution, while the timescale for the nonlinear saturation converges at \( 256^3 \) to roughly five large-eddy rotation times, or about 0.5 ms for the physical parameters of BNS mergers.

In order to quantitatively describe the time development of magnetic energy \( E_M(t) \), we describe it with an empirical model,

\[
E_M(t) = \begin{cases} 
E_M(0) e^{t/t_0} & 0 < t < t_1 \\
E_M(t_1) e^{(t-t_1)/t_2} & t_1 < t < t_{nl} \\
E_{nl} + (E_{sat} - E_{nl})(1 - e^{-(t-t_{nl})/t_3}) & t_{nl} < t < \infty 
\end{cases}
\]

where the six parameters (summarized in Table 1) are obtained by a least-squares optimization. Figure 2 shows the empirical
model given in Equation (3) applied to a representative run at 128^3. The first phase, t < t_1, is a startup transient and lasts until the hydrodynamic cascade is fully developed at \( t_{\text{edd}} \). The kinematic dynamo phase lasts between \( t_1 < t < t_{\text{edd}} \), during which the magnetic energy exponentiates on the timescale \( t_1 \). At \( t_{\text{edd}} \), the smallest scales reach kinetic equipartition and the growth rate slows. In the final stage, \( E_M \) asymptotically approaches \( E_{\text{sat}} \) on the timescale \( t_2 \). We define the dynamo completion time \( t_{\text{sat}} \) as \( t_{\text{edd}} + t_2 \). Figure 3 shows the best-fit \( E_{\text{sat}}, t_{\text{sat}} \), and \( t_1 \) as a function of the resolution.

The magnetic energy \( E_{\text{sat}} \) at dynamo completion shows signs of converging to a value of 0.6 \( E_K \) by resolution 512^3. The timescale \( t_2 \) on which the magnetic energy asymptotically approaches \( E_{\text{sat}} \) is consistently \( \approx 5t_{\text{edd}} \) at different resolutions. The dynamo completion time \( t_{\text{sat}} \) is numerically converged at \( \approx 5t_{\text{edd}} \) by 256^3. The best-fit kinematic growth time follows a power law in the resolution, \( t_1 \propto N^{-0.6} \). This is consistent with the value of \(-2/3\) expected if the dynamo time is controlled by shearing at the smallest scale, the cascade is Kolmogorov (i.e., \( u_t \propto \ell^{1/3} \)), and the viscous cutoff \( \ell_v \) occurs at a fixed number of grid zones. In that case, \( t_1 \sim t_v \propto \ell_v^{2/3} \sim N^{-2/3} \).

### 3.3. Power Spectrum of Kinetic and Magnetic Energy

The time development of kinetic and magnetic energy power spectra has been studied for a single run with resolution 512^3. We present three-dimensional, spherically integrated power spectra with the dimensions of erg cm^{-3} m defined as

\[
P_K(k_i) = \frac{1}{\Delta k_i} \sum_{k \in \Delta k_i} |f_k[v/\sqrt{\rho/2}]|^2 \tag{4a}
\]

\[
P_M(k_i) = \frac{1}{\Delta k_i} \sum_{k \in \Delta k_i} |f_k[B/\sqrt{8\pi}]|^2, \tag{4b}
\]

where the Newtonian versions of kinetic and magnetic energy are appropriate since the conditions are only mildly relativistic. The definitions in Equations (4) satisfy \( \int P(k)dk = \langle E \rangle \) for \( P_K \) and \( P_M \). Figure 4 shows the power spectrum of kinetic and magnetic energy at various times throughout the growth and saturation of magnetic field. During the kinematic phase, the kinetic energy has a power spectrum \( P_K(k) \propto k^{-5/3} \) consistent with the Kolmogorov theory for incompressible hydrodynamical turbulence, while \( P_M(k, t) \propto e^{t/(5k^{3/2})} \) consistent with Kazantsev’s model. \( P_M(k) \) maintains the same shape, but exponentiates in amplitude at the timescale \( t_1 \) which is controlled by shearing at the resistive scale. According to Kazantsev’s model, \( P_M(k) \) should peak at the resistive scale. This is consistent with the observed peak in the magnetic energy at roughly 10 grid zones, the same scale at which we observe the viscous cutoff. This is also consistent with \( P_M = 1 \) expected from the numerical scheme employed.

When the magnetic energy at the resistive scale surpasses the level of the kinetic energy at that scale, \( P_M(k) \) changes...
Figure 2. Time history of the magnetic energy for a representative run at 128³, together with the empirical model (Equation (3)) with best-fit parameters. The horizontal dashed line indicates the magnetic energy, $E_{\text{sat}}$, at the dynamo completion. From left to right, the vertical dashed lines mark the end of the startup, kinematic, and saturation phases.

(A color version of this figure is available in the online journal.)

Figure 3. Top: convergence study of the kinematic dynamo growth time $\tau_1$ (blue) and the dynamo completion time $t_{\text{sat}}$ (green) defined as $t_{\text{nl}} + \tau_2$. Bottom: convergence study of the best-fit model parameter $E_{\text{sat}}/E_K$ expressed as the ratio of magnetic to kinetic energy. The converged value of the volume-averaged $E_M/E_K \approx 0.6$. Nevertheless, at intermediate wavelengths $E_M(k)/E_K(k) \approx 4$. As shown in Figure 2, the largest scales remain kinetically dominated. This indicates the suppression of coherent magnetic structure formation near the integral scale of turbulence.

(A color version of this figure is available in the online journal.)

4. DISCUSSION

In this Letter we have determined the timescale and saturation level of the small-scale turbulent dynamo operating in the conditions of BNS mergers. We have presented numerically converged simulations showing that magnetic fields are amplified to the $\sim 10^{16}$ G level within a small fraction of the merger dynamical time ($t_{\text{sat}} \lesssim t_{\text{merge}}$), independently of the seed field strength. If hydrodynamical instabilities create fluctuating velocities on the order of 0.1c as indicated by global simulations, then each 1 km$^3$ turbulent volume dissipates $\gtrsim 10^{46}$ erg of magnetic energy per 0.1 ms. If $f_{\text{turb}}$ represents the fraction of the merger remnant that contains such turbulence, the magnetic energy dissipated during the merger is at least

$$E_{\text{diss}} \sim 10^{49} \text{ erg} \times \left( \frac{f_{\text{turb}}}{10^{-2}} \right) \left( \frac{t_{\text{merge}}}{1 \text{ ms}} \right).$$  

(5)

A fraction of that dissipation will occur through magnetic reconnection in optically thin surface layers, supplying relativistic electrons which synchrotron radiate in the merger remnant magnetosphere. If 5% of that magnetic energy dissipation creates radiation in the 15–150 keV band, then the fluence at 200 Mpc...
Figure 4. Time development of the spectrum of kinetic (left) and magnetic (right) energy, given in erg cm\(^{-3}\) m for resolution 512\(^3\). During the startup phase the hydrodynamic cascade is not yet fully developed; this phase lasts for \(t_{\text{eddy}}\). The kinematic phase refers to the time when the hydrodynamic cascade is fully developed, but the magnetic energy is still energetically sub-dominant. During the kinematic phase, the kinetic energy power spectrum is consistent with a Kolmogorov cascade, having a slope \(k^{-5/3}\) over intermediate wavelengths. Meanwhile, the magnetic energy spectrum is consistent with the Kazantsev description, having a positive slope \(k^{3/2}\) over the same intermediate wavelengths. During this phase the magnetic energy exponentiates rapidly, with an \(e\)-folding time controlled by shearing at the smallest available scales. After saturation, the magnetic energy spectrum conforms to the kinetic energy spectrum, such that \(E_M(k) \approx 4 E_K(k)\) over the intermediate wavelengths. The kinetic energy spectrum after the dynamo completion has a slightly shallower slope than during the kinematic phase, but is still consistent with a 5/3 spectral index.

would be \(10^{-7}\) erg cm\(^{-2}\), potentially rendering most merging neutron stars in the advanced LIGO and Virgo detection volumes detectable by Swift BAT. If so, then merging neutron stars are accompanied by a prompt EM counterpart, independently of whether a later merger phase yields a collimated outflow capable of powering a short GRB.

We suggest that merger flares may be present in the current sample of short GRBs and may be roughly isotropic on the sky since they are seen to distances where the cosmological matter distribution becomes homogeneous (e.g., Tanvir et al. 2005). Searches for merger flares should seek to identify short flares, not unlike soft-gamma repeaters, among the short burst population. If mergers also produce short GRBs, then the flaring activity could be detectable as a separate component of the emission, which may appear as a precursor (e.g., Troja et al. 2010) if there is a considerable delay between merger onset and the formation of short GRB engine (e.g., Baiotti et al. 2008). The possibility of prompt flares from BNS mergers has also been suggested to follow from a shock breakout scenario (Kytotoku et al. 2012) or tidal interaction in the late inspiral (Dall’Osso & Rossi 2013).

Finally, it has been shown that magnetic fields will directly influence the GW signature and remnant disk mass if they exist at the 10\(^{17}\) G level (Etienne et al. 2012). Such strong fields are unlikely on evolutionary grounds in merging BNSs, but our results suggest they may be revived up to the scale of turbulent cells during the merger. To exert influence, such turbulent cells would have to fill a considerable fraction of the merger volume. As we have shown in this Letter, the overall magnetic energy budget is controlled by the prevalence \(f_{\text{turb}}\) and vigor \(\epsilon_K\) of the turbulent volumes. This fact motivates the use of higher resolution global simulations aimed at measuring \(f_{\text{turb}}\) and \(\epsilon_K\).

The presence of strong magnetic fields may also aid in the ejection of neutron-rich material from surface layers of the merger remnant, possibly enhancing the enrichment of interstellar medium by \(r\)-process nuclei (Freiburghaus et al. 1999; Rosswog et al. 1999; Hotokezaka et al. 2013). Enhanced production of \(r\)-process nuclei also increases the likelihood of EM detection by radioactive decay powered afterglows, or “kilonovae” (Li & Paczyński 1998; Metzger et al. 2010).

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