Erasure Correction for Noisy Radio Networks

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Abstract

The radio network model is a well-studied abstraction for modeling wireless multi-hop networks. However, radio networks make the strong assumption that messages are delivered deterministically. The recently introduced noisy radio network model relaxes this assumption by dropping messages independently at random.

In this work we quantify the relative computational power of noisy radio networks and classic radio networks. In particular, given a protocol for a fixed radio network we show how to reliably simulate this protocol if noise is introduced with a multiplicative cost of $\text{poly}(\log \Delta, \log \log n)$ rounds where $n$ is the number nodes in the network and $\Delta$ is the max degree. For this result we make the simplifying assumption that the simulated protocol is static. Moreover, we demonstrate that, even if the simulated protocol is not static, it can be simulated with a multiplicative $O(\Delta \log \Delta)$ cost in the number of rounds. Lastly, we argue that simulations with a multiplicative overhead of $o(\log \Delta)$ are unlikely to exist by proving that an $\Omega(\log \Delta)$ multiplicative round overhead is necessary under certain natural assumptions.

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1 Introduction

The study of distributed graph algorithms provides precise mathematical models to understand how to accomplish distributed tasks with minimal communication. A classic example of such a model is the radio network model of Chlamtac and Kutten [13]. Radio networks use synchronous rounds of communication and were designed to model the collisions that occur in multi-hop, wireless networks. However, radio networks make the strong assumption that, provided no collisions occur, messages are guaranteed to be delivered.

This assumption is overly optimistic for real environments in which noise may impede communication, and so we previously introduced the noisy radio network model where messages are randomly dropped with a constant probability [10]. We demonstrated that the runtime of existing state-of-the-art broadcast protocols deteriorates significantly in the face of random noise. Furthermore, we showed how to design efficient broadcasting protocols that are robust to noise.

However, it has remained unclear how much more computationally powerful radio networks are than noisy radio networks. In particular, it was not known how efficiently an arbitrary protocol from a radio network can be simulated by a protocol in a noisy radio network. A simulation with little overhead would demonstrate that radio networks and noisy radio networks are of similar computational power. However, if simulation were necessarily expensive then radio networks would be capable of completing more communication-intensive tasks than their noisy counterparts.

A trivial observation regarding the relative power of these two models is that any polynomial-length radio network protocol can be simulated at a multiplicative cost of $O(\log n)$ rounds where $n$ is the number of nodes in the network. In particular, we can simulate any protocol from the non-noisy setting by repeating every round $O(\log n)$ times. A Chernoff and union bound show that every message sent by the original protocol is successfully sent with high probability. However, $O(\log n)$ is a significant price to pay for noise-robustness for many radio network protocols. For example, the optimal known-topology message broadcast protocol of Gąsieniec et al. [25] uses only $O(D + \log^2 n)$ rounds, while the topology-oblivious Decay protocol of Bar-Yehuda et al. [6] takes $O(D \log n + \log^2 n)$ where $D$ is the diameter of the network. Moreover, a dependence on a global property of the network—the number of nodes in the graph—is surely overkill to correct for a local issue—faults.

1.1 Our Results

In this work we demonstrate that radio network protocols can be simulated by noisy radio networks with a multiplicative dependence on $\Delta$, the maximum degree of the network. Thus, we demonstrate that, for low-degree networks, noisy radio networks are essentially of the same computational power as radio networks. Moreover, we demonstrate that this dependence is more or less tight in two natural settings.

We give our results in three increasingly difficult settings which we sketch here, highlighting our insights and the technical challenges we overcome in each setting. In all of our simulations nodes store some notion of the round up to which they have successfully simulated and always help their neighbors that have made the least progress. In particular, nodes always simulate the round of their neighbor that has successfully simulated the fewest rounds. We also sketch our lower bound results here.

Local Progress Detection As a warmup we begin by providing a simulation with $O(\log \Delta)$ multiplicative round overhead in the setting where nodes have access to “local progress detection”.


Roughly, local progress detection enables nodes to know how many rounds of the original protocol each of their neighbors have successfully simulated. Our crucial insight in this setting is that allowing nearby nodes to simulate the same round of the original protocol while flexibly allowing distant nodes to simulate different rounds enables a multiplicative round overhead with a dependence on $\Delta$. We show this dependence by a novel “blaming chain” argument. (Section 4.1)

**Static Protocols**

Next we provide simulations for static protocols. Roughly, static protocols are protocols in which nodes know in which rounds they receive messages. In this setting, our simulations achieve a multiplicative $O(\log^3 \Delta \cdot \log \log n \cdot \log \log \log n) = \text{poly}(\log \Delta, \log \log n)$ round overhead without local progress detection. As we argue in Section 4.2, a number of well-known protocols (e.g. the optimal broadcast algorithm of Gąsieniec et al. [25]) are static. What makes this setting particularly difficult is that nodes have no way of knowing what sort of progress their neighbors have made. Simulating static protocols therefore isolates the challenge of efficiently sharing information locally in a noisy radio network. We overcome this challenge by using a distributed binary search which carefully silences nodes that search over divergent ranges. Moreover, we keep the range over which the binary search must search tractable by throttling nodes that make progress too quickly. (Section 4.2)

**General Protocols**

We also give simulations for arbitrary radio network protocols with a multiplicative $O(\Delta \log \Delta)$ overhead. The challenge of the general setting is the asymmetric nature of the radio network model: a node $v$ can communicate to its $\Delta$ neighbors in 1 round, but it takes $\Delta$ rounds until all of $v$’s neighbors can communicate with $v$. It is difficult, then, for a node that broadcasted to know with certainty that all of its neighbors received its message. For this reason, our simulation involves nodes exchanging messages with all neighbors in every round to preclude the possibility of a dropped message going unnoticed. We note that the $O(\Delta)$ difference between this result and our $O(\log \Delta)$ with local progress detection suggests that the primary challenge in simulation is not in recovering information lost to noise but rather in exchanging information regarding how much progress nodes have made. (Section 4.3)

**Lower Bounds**

Lastly, we show that two natural classes of simulations necessarily use $\Omega(\log \Delta)$ multiplicatively many more rounds than the protocols which they simulate. In particular, a simulation that either (1) does not use network coding, or (2) sends information in the same way as the original protocol requires $\Omega(\log \Delta)$ multiplicatively many more rounds than the original protocol. We also give a construction which we believe could prove an unconditional $\Omega(\log \Delta)$ simulation round overhead in the noisy setting. (Section 5)

## 2 Related work

### 2.1 Robust Communication

Several models have been studied to understand robust communication in models similar to the radio network model. El-Gamal [16] introduced the noisy broadcast model, which also assumes random errors, and focuses on studying the computation of functions of the inputs of nodes [18, 40, 26, 38]. The model of El-Gamal [16] differs from our model in that it assumes a complete communication network and single-bit transmissions. Rajagopalan and Schulman [42] introduced a similar model which again assumes single-bit transmissions and also does not have collisions as our model does. Several papers have been written on notions of noisy radio networks which assume the network admits geometric structure. Kranakis et al. [32] considers broadcasting in radio networks where
unknown nodes fail. In this work, unlike our own, nodes in the network admit some geometric structure; e.g. every node is at integer points on the line. In a similar vein, Kranakis et al. \cite{33} consider radio networks with possibly correlated faults at nodes. Like the previous work, this work assumes that there is an underlying geometric structure to the radio network; namely the network is a disc graph. There have also been several papers on noisy single-hop radio networks. See either Gilbert et al. \cite{24} for a study of an adversarial model or Efremenko et al. \cite{15} for a nice study of a random noise model. Another model which captures uncertainty is the dual graph model \cite{36,9,35,20,21}, in which an adversary chooses a set of unreliable edges in each round.

There has also been extensive work on two-party interactive communication in the presence of noise \cite{44,27,19}. In this setting, Alice and Bob have some conversation in mind they would like to execute over a noisy channel; by adding redundancy they hope to hold a slightly longer conversation from which they can recover the original conversation outcome even when a fraction of the coded conversation is corrupted. Multi-party generalizations of this problem have also been studied Braverman et al. \cite{8}. Our model can be seen as a radio network analogue of these interactive communication models in which erasures occur rather than corruptions.

Lastly, work on MAC layers has sought to provide abstractions for algorithms that hide low-level uncertainty in wireless communication \cite{22,34,43,29}. Radio networks differ from MAC layers as in radio networks it is not required that a sender receive an acknowledgement from a receiver.

2.2 Radio Networks

Since its introduction by Chlamtac and Kutten \cite{13}, the classic radio network model has attracted wide attention from researchers. The survey of \cite{41} is an excellent overview of this research area.

Here we focus the radio network literature that relates to our work. Much of previous work for the noisy radio network model focused on broadcast \cite{30}. For the classic model, Bar-Yehuda et al. \cite{6} gave a single-message broadcast algorithm for a known topology, which completes in $O(D \log n + \log^2 n)$ rounds. In Censor-Hillel et al. \cite{10} we showed that this protocol is robust to noise, completing in $O\left(\frac{\log n}{1-p}(D + \log n + \log \frac{1}{\delta})\right)$ rounds, with a probability of failure of at most $\delta$. In the classic model, single-message broadcast was then improved by Gasieniec et al. \cite{25} and Kowalski and Pelc \cite{31}, who showed that in the case of a known topology, $O(D + \log^2 n)$ rounds suffice. In Censor-Hillel et al. \cite{10} we showed that this protocol is not robust to noise, requiring in expectation $\Theta\left(\frac{p}{1-p}D \log n + \frac{1}{1-p}D\right)$ rounds, for broadcasting a message along a path of length $D$. We showed an alternative protocol, completing in $O(D + \log n \log \log n(\log n + \log \frac{1}{\delta}))$ rounds, with a probability of failure of at most $\delta$. For an unknown topology in the classic model, Czumaj and Rytter \cite{14} give a protocol completing in $O(D \log(n/D) + \log^2 n)$ rounds, which is optimal, due to the $\Omega(\log^2 n)$ and $\Omega(D \log(n/D))$ lower bounds of Alon et al. \cite{2} and Kushilevitz and Mansour \cite{37}, respectively. Ghaffari et al. \cite{23} give a $O(D + \text{poly} \log n)$-round protocol that uses collision detection.

2.3 Simulations

Lastly, simulations of models of distributed computation by other models of distributed computation is a foundational aspect of distributed computing, dating back to the 80’s–90’s with many simulations of various shared memory primitives, faults, and more (see a wide variety in, e.g., Attiya and Welch \cite{3}). It is also a focus for message-passing models with different features, being the motivation for synchronizers \cite{4,5}, and additional simulations \cite{11,12}. 
3 Model and Assumptions

In this section we formally define the classic radio network model, the noisy radio network model and discuss various assumptions we make throughout the paper.

Radio Networks A multi-hop radio network, as introduced in Chlamtac and Kutten [13], consists of an undirected graph \( G = (V,E) \) with \( n := |V| \) nodes. Communication occurs in synchronous rounds: in each round, each node either broadcasts a single message containing \( \Theta(\log n) \) bits to its neighbors or listens. A node receives a message in a round if and only if it is listening and exactly one of its neighbors is transmitting a message. If two or more neighbors of node \( v \) transmit in a single round, their transmissions are said to collide at \( v \) and \( v \) does not receive anything. We assume no collision detection, meaning a node cannot differentiate between when none of its neighbors transmit a message and when two or more of its neighbors transmit a message. Nodes are also typically assumed to have unbounded computation, though all of the protocols in our paper use polynomial computation.

We use the following notational conventions throughout this paper when referring to radio networks. Let \( \text{dist}(v,w) \) be the hop-distance between \( v \) and \( w \) in \( G \), let \( \Gamma(v) = \{ w \in V : \text{dist}(v,w) \leq 1 \} \) denote the 1-hop neighborhood of \( v \), and let \( \Gamma^{(k)}(v) = \{ w \in V : \text{dist}(v,w) \leq k \} \) denote the \( k \)-hop neighborhood of \( v \).

Noisy Radio Networks A multi-hop noisy radio network, as introduced in Censor-Hillel et al. [10], is a radio network with random erasures. In particular, it is a radio network where node \( v \) receives a message in a round if and only if it is listening, exactly one of its neighbors is broadcasting and a receiver fault does not occur at \( v \). Receiver faults occur at each node and in each round independently with constant probability \( p \in (0,1) \). As \( p \) will be treated as a constant it will be suppressed in our \( O \) and \( \Omega \) notation. We assume that in a given round a node cannot differentiate between a message being dropped because of a fault, i.e., a collision, and all of its neighbors remaining silent.

We consider this model because independent receiver faults model transient environmental interference such as the capture effect [39]. Moreover, we consider an erasure model—entire messages are dropped—rather than a corruption model—messages are corrupted at the bit level— because, in practice, wireless communication typically incorporates error correction and checksums that can guard against bit corruptions [17]. Our model can also be seen as modeling those cases when this error correction fails and the message cannot be reconstructed and is therefore effectively dropped.

Protocols A protocol governs the broadcast and listening behavior of nodes in a network. In particular, a protocol tells each node in each round to listen or what message to broadcast based on the node’s history. This history includes the messages the node received, when it received them and its initial private input. We assume that this private input includes the number of nodes, \( n \), the maximum degree, \( \Delta \), the receiver fault probability, \( p \), a private random string, and any other data nodes have as input in a protocol. Formally, a history for node \( v \) is an \( H \in \mathcal{H} \) where \( \mathcal{H} \) is all valid histories. \( \mathcal{H} = \{(i,R) : i \in I, R \subseteq M \times N\} \) where \( I \) is all valid private inputs, \( R \) gives what messages \( v \) has received in each round and \( M = \{0,1\}^{O(\log n)} \) is all \( O(\log n) \) bit messages a node could receive in a single round. Formally, a protocol \( P \) of length \( |P| := T \) is a function \( P : V \times [T] \times \mathcal{H} \rightarrow \{\text{listen}\} \cup M \) where \( P(v,t,H) = \text{listen} \) indicates that \( v \) listens in round \( t \) and \( P(v,t,H) = m \in M \) indicates that \( v \) broadcasts message \( m \in M \) in round \( t \).

\[ \text{listen}, \quad m \]

\[ \text{listen}, \quad m \]

1In contrast, a sender faults model in which an entire broadcast by a node is dropped might model hardware failures where independence of faults would be a poor modeling choice.
assume that $T$ is at most $\text{poly}(n)$.

**Simulating a Protocol in the Noisy Setting** We say that protocol $P'$ successfully simulates $P$ if, after executing $P'$ in the noisy setting, every node in the network can reconstruct the messages it would receive if $P$ were run in the faultless setting. In particular, for any set of private input $I \in I^V$—letting $H(v, I)$ be $v$’s history after running $P$ with private inputs $I$—it must hold that after running $P'$ with private inputs $I$, every $v$ can compute $H(v, I)$. Note that nodes running $P'$ can send messages not sent by $P$ or send messages in a different order than they do in $P$. We call $P$ the original protocol and $P'$ the simulation protocol. We measure the efficacy of $P'$ as the limiting ratio of $|P'|/|P|$ when $T$ is sufficiently large and $n$ goes to infinity, which we call the multiplicative overhead of a simulation.

## 4 Efficient Simulators for Noisy Radio Networks

We now provide our simulation results. We provide simulations in three settings of increasing difficulty. Throughout our results, we use the notion of a virtual round of node $v$, $t_v$, which measures how many rounds of $v$ has successfully simulated. All three simulations roughly work by having nodes first exchange their virtual rounds with their neighbors and then having nodes simulate the virtual round of their neighbor with the smallest virtual round. Our simulations differ in how each defines a virtual round and how each exchanges information regarding nodes’ virtual rounds.

As a warmup, we consider the setting where nodes have access to “local progress detection”. Local progress detection gives nodes oracle access to the virtual rounds of their neighbors. We show the following theorem.

**Theorem 1.** Let $P$ be a general protocol of length $T$ for the faultless radio network model. The described protocol $P'$ is of length $O(T \log \Delta + \log n + k)$ and successfully simulates $P$ in the noisy radio setting using local progress detection with probability at least $1 - \exp(-k)$, for any $k \geq 0$.

We next show how to efficiently spread virtual round information without using progress detection. In particular, we show the following theorem for protocols that are static.

**Theorem 2.** Let $P$ be a static protocol of length $T$ for the faultless radio network model. The simulation routine $\text{MainStatic}$ simulates $P$ in the noisy radio setting with high probability. The simulation takes $O((T + \log n) \Delta \log \Delta \log \log \log n)$ rounds.

The crucial property of static protocols that we leverage in our proof of this result is that nodes in a static protocol can efficiently learn their virtual round.

Lastly, we describe how to simulate any protocol with a $O(\Delta \log \Delta)$ multiplicative overhead.

**Theorem 3.** Let $P$ be a general protocol of length $T$ for the faultless radio network model. The simulation routine $\text{MainGeneral}$ simulates $P$ in the noisy radio setting with high probability. The simulation takes $O((T + \log n) \Delta \log \Delta)$ rounds.

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2When we write that an event occurs with high probability (w.h.p.), we mean that it occurs with probability $1 - \frac{1}{n^c}$, and that the constant $c = \Theta(1)$ can be made arbitrarily large by changing the constants in the routines.
4.1 Simulation with Local Progress Detection

We begin by giving our simulation with $O(\log \Delta)$ multiplicative overhead in the setting where nodes have access to local progress detection. Though, the assumption of local progress detection is strong, it highlights one of the main proof techniques we make use of when we provide simulations of static protocols, namely the “blaming chain”. Our main theorem of the section is as follows.

**Theorem 1.** Let $P$ be a general protocol of length $T$ for the faultless radio network model. The described protocol $P'$ is of length $O(T \log \Delta + \log n + k)$ and successfully simulates $P$ in the noisy radio setting using local progress detection with probability at least $1 - \exp(-k)$, for any $k \geq 0$.

The intuition behind $P'$ is simple: run $P$ but use local progress detection to force nearby nodes to stay locally synchronized. That is, nearby nodes are forced to make a similar amount of progress but far apart nodes are allowed to deviate in terms of the amount of progress they make.

To rigorously define local progress detection, we introduce our notion of how much simulation progress nodes have made, the virtual round. We say that a node $v$ successfully completed round $t$ if it has succeeded in taking the action in $P$ that it takes in $t$: $v$ successfully broadcasts its message $m$ of round $t$ if every node in $\Gamma(v)$ either has received $m$ or had a collision in the original protocol $P$ at round $t$; $v$ successfully completes its listening action of round $t$ if $v$ receives the message it receives in round $t$ of $P$ or a collision occurs at $v$ in round $t$ of $P$. We formally define the virtual round and local progress detection.

**Definition 1** (Virtual Round). The virtual round of node $v \in V$ in a given round of simulation $P'$ is the smallest $t_v \in \mathbb{Z}_{\geq 1}$ such that $v$ has not successfully completed round $t_v$ of $P$.

**Definition 2** (Local Progress Detection). In the noisy radio network model with local progress detection every node $v$ knows the virtual round of every node $w \in \Gamma(v)$ in every simulation round.

Having defined local progress detection, we are now equipped to describe our simulator. The protocol $P'$: Each node $v$ repeatedly does the following in each round of $P'$: Let $u$ be the node in $\Gamma(v)$ with minimal $t_u$. $v$ takes the action that it takes in round $t_u$ of $P$. That is, $v$ tries to “help” $u$ by simulating its virtual round.

We now prove the main theorem of this section.

**Proof of Theorem**. Let $v \in V$ and note that the virtual round $t_v$ never decreases. Hence for $x \in \mathbb{Z}_{\geq 1}$ we can define $D_{v,x}$ as the earliest round in $P'$ when $t_v \geq x$. Notice that $D_{v,T}$ just is the number of rounds that $v$ takes to simulate all rounds of the original protocol $P$. Thus, it will suffice for us to argue that $D_{v,T}$ is not too large for every $v$. We begin by finding a recurrence relation on $D_{v,x}$ saying that $v$ will advance soon after its 2-hop neighborhood $\Gamma^{(2)}(v)$ has virtual round at least $x$.

Consider when $t_v = x - 1$. If the action of $v$ at round $x - 1$ in $P$ is to broadcast, then once $v$‘s virtual round is minimal in $\Gamma^{(2)}(v)$, all of its neighbors will simulate its round. This takes $\max_{w \in \Gamma^{(2)}(v)} D_{w,x-1}$ rounds. Once all nodes in $\Gamma^{(2)}(v)$ have virtual round at least $x - 1$, every such node in $\Gamma(v)$ is simulating round $x - 1$. By definition of $P'$, $v$ then broadcasts until all of its neighbors that are not incident to a collision in round $x - 1$ of $P$ receive $v$‘s message without a fault occurring. Any neighbor of $v$ that is sent a message without collision in round $x - 1$ in $P$ will now take $G(p)$ rounds, a geometric random variable with constant expectation. Therefore, the number of rounds until $v$ has sent all its relevant neighbors its message from round $x - 1$ of $P$ is at
As such, to show an upper bound on that the length of a blaming chain in gives the value of \(Y_{v,x}\) as large as it is. In particular, a blaming chain is a sequence of nodes paired with simulated rounds that could explain why \(D_{v,x}\) is as large as it is. In particular, a blaming chain for \(D_{v,x}\) is a sequence of (node, round) tuples \((v_x, x), (v_{x-1}, x-1), \ldots, (v_1, 1)\) where \(v_i \in \Gamma^{(2)}(v_{i-1})\) and \(v_x = v\). We let \(\mathcal{C}(D_{v,x})\) stand for all such blaming chains of \(D_{v,x}\). We say \(|C| := \sum_{(v_i, i) \in C} Y_{v,i}\) is the length of blaming chain \(C\). Note that \(|\mathcal{C}(D_{v,x})|\) is at most \((\Delta^2)^{x-1}\). Also notice that \(D_{v,x}\) just is the length of the longest blaming chain in \(\mathcal{C}(D_{v,x})\). See Figure 1. Thus, we have that

\[
D_{v,x} = \max_{w \in \Gamma^{(2)}(v)} D_{w,x-1} + Y_{v,x}
\]

\[
= \max_{C \in \mathcal{C}(D_{v,x})} \sum_{v_i \in C} Y_{v,i}
\]

\[
= \max_{C \in \mathcal{C}(D_{v,x})} |C|
\]

As such, to show an upper bound on \(D_{v,x}\) it suffices to show that no blaming chain is too long. Notice that the length of a blaming chain in \(\mathcal{C}(D_{v,x})\) just is the sum of \(x\) random variables each of which

Figure 1: \(C \in \mathcal{C}(D_{v_3,2})\): path away from \(v_2, 2\). \(D_{v_3,2} = \max_{C \in \mathcal{C}(D_{v_3,2})} |C| = 6\). Blaming chain that gives the value of \(D_{v_3,2}\): red. Edge pointing to \(v_i, x\) labeled by the value of \(Y_{v_i,x}\).
is the max of at most $\Delta$ geometric random variables with constant expectation. In Lemma \[10\] of Appendix \[A\] we prove a Chernoff-style tail bound for such sums, showing that for any given $C \in C(D_v,x)$, we have that $\Pr[|C| \geq c(x \log \Delta + k)] \leq \exp(-k)$ for some constant $c > 0$. Letting $x = T$ and taking a union bound over all $C \in C(D_v,T)$ tells us that for a fixed $v$ we have $\Pr[\exists C \in C(D_v,T) \text{ s.t. } |C| \geq c(T \log \Delta + k)] \leq |C(D_v,T)| \cdot \exp(-k) \leq (\Delta^2)^{T-1} \cdot \exp(-k)$. Thus with probability at most $(\Delta^2)^{T-1} \cdot \exp(-k)$, a chain in $C(D_v,x)$ is of length more than $c(T \log \Delta + k)$ and so $D_{v,T}$ is of length more than $c(T \log \Delta + k)$ with probability at most $(\Delta^2)^{T-1} \cdot \exp(-k)$.

Taking a union bound over every vertex we have that there exists a vertex $v$ such that $D_{v,T}$ is more than $c(T \log \Delta + k)$ with probability at most $n(\Delta^2)^{T-1} \cdot \exp(-k)$. Setting $k \leftarrow \ln n + (2T - 2) \ln \Delta + k'$ we have that there exists a vertex $v$ such that $D_{v,x}$ is more than $O(T \log \Delta + \log n)$ is at most $\exp(k')$. Thus, we have that every vertex with probability at least $1 - \exp(k')$ has completed the protocol after $O(T \log \Delta + \log n)$ rounds of $P'$.

\[ \square \]

### 4.2 Simulation for Static Protocols

We now give our simulation results for static protocols with $\text{poly}(\log \Delta, \log \log n)$ multiplicative round overhead. Formally, a static protocol is as follows.

**Definition 3** (Static Protocol). A static protocol $P$ is a protocol in which every node can determine if it receives a message in each round of $P$ regardless of the private inputs of the nodes. Formally, each node $v$ is given a list, $M_v$, of the rounds in which it receives a message without collision in $P$.

We let $M_v$.getNextRound return the smallest round in $M_v$ such that for every $r \in M_v$ such that $r < M_v$.getNextRound, $v$ has received the message it receives in round $r$ of $P$. Since in a static protocol nodes know in which rounds they receive messages in $P$, nodes in a simulation can locally compute $M_v$.getNextRound. Also notice that even though static protocols assume nodes know a priori when they are supposed to receive messages, nodes are oblivious of the contents of these messages.

We focus on static protocols to understand how delays propagate in noisy radio networks and how to efficiently exchange information in noisy radio networks. This focus allows us to avoid some of the messier challenges of simulations in noisy radio networks (see Section \[6\] for further discussion).

However, we also note that a number of well-known protocols are static. For instance, assuming nodes know the network topology and use public randomness, the optimal broadcast algorithm of Gasieniec et al. \[25\] is static. In particular, under these assumptions nodes can compute the broadcast schedule of their neighbors for this protocol. The same is true of the Decay protocol of Bar-Yehuda et al. \[6\]. Since broadcast is the most studied problem in radio networks, the fact that the state-of-the-art broadcast algorithm is static gives strong evidence that studying static protocols is worthwhile. Moreover, though requiring that nodes know the network topology and use public randomness may seem restrictive, in the context of simulations this assumption is actually quite weak: we can dispense with both assumptions by simply running any primitive that informs nodes of the network topology or shares randomness before our simulation is run at a one-time additive round overhead. Any primitive to learn the network topology or share randomness from the classic radio network setting coupled with the $O(\log n)$ trivial simulation as described in Section \[1\] suffices here. This overhead is negligible if the simulated protocol is sufficiently long or we run many simulations on our network. Lastly, we note that it is often the case that nodes can even **efficiently**
compute the broadcast schedule of their neighbors—see Haeupler and Wajc [28]—and so this one-time cost is often quite small. Broadly speaking, then, any protocol in which knowing the topology and sharing randomness allows nodes to compute their neighbors’ broadcast schedule is static.

We now state the main theorem of this section and proceed to describe how we prove it.

**Theorem 2.** Let $P$ be a static protocol of length $T$ for the faultless radio network model. The simulation routine `MainStatic` simulates $P$ in the noisy radio setting with high probability. The simulation takes $O((T + \log n) \log^3 \Delta \log \log n \log \log \log n)$ rounds.

To prove our theorem we build upon the idea of the preceding section of using virtual rounds to locally synchronize nodes. However, in the current setting it is difficult for nodes to confirm when their broadcasts have succeeded, and so we must relax our definition of virtual rounds. In particular, for the remainder of this section we let the virtual round of each node be the minimum between the largest $t_v \in \mathbb{Z}_{\geq 1}$ such that $v$ receives a message without collision in $t_v$ and $v$ has successfully received all messages it receives up to round $t_v - 1$ in $P$ in our simulation and a throttling variable $L$. That is, the virtual round of $v$ is $\min(M_v.getNextRound, L)$. An important property of virtual rounds in this setting is that given $L$ a node can always compute its virtual round.

We use a throttling variable, $L$, for the following reason. Since we no longer assume nodes have access to local progress detection we must provide a means by which nodes can learn the virtual round of their neighbor. Each node $v$ learns the virtual round of its most delayed neighbor, $\min_{w \in \Gamma(v)} t_w$, via a distributed binary search coupled with a classic Decay algorithm [6]. An important technical challenge we overcome is how we keep the smallest and the largest virtual rounds in the network within a small additive value $Q$. If nodes were allowed to arbitrarily deviate in terms of simulation progress, our distributed binary search would have to search over a prohibitively large range. For instance, it is easy to see that given a matching between $n/2$ senders and $n/2$ receivers, after $T$ rounds of the senders broadcasting, the virtual rounds of the receivers would deviate by an additive $\Theta(\sqrt{T})$ with high probability. As such, we maintain $Q = O(\log n)$ by slowing down nodes that advance too quickly with our throttling variable $L$.

We now present the `MainStatic` simulation routine that simulates $P$ in the noisy setting (Algorithm 1). Its main subroutine, `LearnDelays` (Algorithm 2), informs each node of the virtual round of its most delayed neighbor. `LearnDelays` uses helper subroutines (i) `DistToActive` (Algorithm 3) which, given a set of “active” nodes, checks for each $v$ if there is a node within distance 2 of $v$ that is “active”, and (ii) `Broadcast` (Algorithm 4) which spreads a message to all neighbors of a node. We give each routine in pseudocode along with the lemma which gives its properties. Throughout our pseudocode we let $P(v, t)$ return the action taken by node $v$ in round $t$ of $P$. We end this section with the proofs of our lemmas and theorem.

**Algorithm 1 MainStatic for node $v$**

```
fv \leftarrow 1
for L = 1, 2, \ldots, T + O(\log n) do
    \text{for repeat } O(\log \Delta) \text{ times do}
        mv \leftarrow \text{LearnDelays}
        \text{do action } P(v, mv)
        \text{if } mv = M_v.getNextRound and v received a message then
            tv \leftarrow \min(L, M_v.getNextRound) \quad \triangleright \text{Throttle } v
```

We say that a node $v$ is most delayed in its 2-hop neighborhood when $t_v \leq \min_{w \in \Gamma^2(v)} t_w$. 


Lemma 1. Assume that $L - O(\log n) \leq t_v < L$ for all $v \in V$ and let $v$ be a most delayed node in its 2-hop neighborhood. After an innermost iteration of MainStatic, $t_v$ will increase by one with at least constant probability. Moreover, the running time of each innermost iteration is $O(\log^2 \Delta \log \log n \log \log \log n)$ rounds.

We now give our helper routine, LearnDelays, which informs nodes of their most delayed neighbors using a distributed binary search. The main challenge of performing a binary search in this setting is that nodes may deviate in terms of the ranges over which they are searching. This can cause nodes to interfere with the binary of their neighbors. The key idea we use to overcome this issue is to carefully mark nodes “silent” if they might interfere with another node's binary search. See Figure 2 for an illustration of LearnDelays.

Algorithm 2 LearnDelays for node $v$

Input: $L, t_v$

```plaintext
lo ← $L - O(\log n)$; hi ← $L$
while lo ≠ hi do ▷ repeats $O(\log \log n)$ rounds
  if v is marked as "active" iff $t_v \leq \lfloor \frac{lo+hi}{2} \rfloor$ and v is not marked “silent”
    dist ← DistToActive
    if dist is “=2” then mark v as "silent" until the end of LearnDelays
    if dist is “=0” or “=1” then
      hi ← $\lfloor \frac{lo+hi}{2} \rfloor$
    else
      lo ← $\lfloor \frac{lo+hi}{2} \rfloor + 1$
return lo
```

Lemma 2. Assume that $L - O(\log n) \leq t_v \leq L$ for all $v \in V$ and let $v$ be a most delayed node in its 2-hop neighborhood. After LearnDelays terminates, each 1-hop neighbor $w$ of $v$ will have $m_w = t_v$ with at least constant probability, i.e., every $w$ will try to help $v$ advance. The running time of LearnDelays is $O(\log^2 \Delta \log \log n \log \log \log n)$ rounds.

We now give DistToActive, the main helper routine for our distributed binary search routine.

Lemma 3. Let $A \subseteq V$ be any set of active nodes. After DistToActive terminates, a fixed node $v$ correctly learns its distance to the nearest active node as 1,2, or more than 2 with probability at
Algorithm 3 DistToActive for node $v$

Input: is $v$ active?

Output: “=0”, “=1”, “=2”, or “>2” as the smallest distance to an active node

- if $v$ is active then BROADCAST an arbitrary message X else stay silent
- if $v$ received X then BROADCAST an arbitrary message Y else stay silent
- if $v$ received X then return “=0”
- if $v$ received Y then return “=1”
- if $v$ received Y then return “=2”

return “>2”

least $1 - \frac{1}{\Delta^2 (\log \log n)^2}$. If $v$’s distance to the nearest active node is 0 then $v$ learns it as such with probability 1. The running time of DistToActive is $O(\log^2 \Delta \log \log \log n)$ rounds.

Lastly, we give Broadcast which spreads information among nodes.

Algorithm 4 Broadcast for node $v$

Input: a message to broadcast

- for $O(\log \Delta \log \log \log n)$ times do
  - for $i = 1, 2, \ldots, O(\log \Delta)$ do
    - $v$ broadcasts its message with probability $2^{-i}$

Lemma 4. Let $A \subseteq V$ be any set of nodes broadcasting the same message, while other nodes $V \setminus A$ are listening. After Broadcast terminates, a node $v$ with at least one broadcasting neighbor will receive the message with probability at least $1 - \frac{1}{\Delta^2 (\log \log n)^2}$. Nodes without neighbors will not receive any messages. The running time of Broadcast is $O(\log^2 \Delta \log \log \log n)$ rounds.

Proof of Lemma 4 (Broadcast). Call nodes in $A$ informed and let $m$ be the message known by nodes in $A$. Let the number of informed neighbors of $v$ be $[2^k, 2^{k+1})$ for $k \leq \log_2 \Delta$. An easy calculation shows that during the innermost iteration when $i = k$, $v$ will receive the message with probability $\Omega(1) \cdot p$. Since $p = \Omega(1)$, we have proven $v$ receives $m$ with constant probability in each outermost iteration. There are $O(\log \Delta \log \log \log n)$ outermost iterations. Thus, the probability that $v$ never receives a message is at most $\exp(-O(\log \Delta \log \log \log n)) \leq O\left(\frac{1}{\Delta^2 (\log \log n)^2}\right)$. Lastly, a running time of $O(\log^2 \Delta \log \log \log n)$ follows by the definition of Broadcast.

Proof of Lemma 3 (DistToActive). Let $u$ be $v$’s nearest active node. The cases when $\text{dist}(v, u) = 0$ and $\text{dist}(v, u) > 2$ are trivial by definition of DistToActive. Consider when $\text{dist}(v, u) = 2$. Let $w$ be a mutual neighbor of $v$ and $u$. The probability $v$ does not learn its distance as 2 is at most the probability that $w$ does not receive X or $v$ does not receive Y. By Lemma 4 and a union bound this probability is at most $O\left(\frac{1}{\Delta^2 (\log \log n)^2}\right)$. Lastly, the case when $\text{dist}(v, w) = 1$ holds for similar reasons. The running time of DistToActive follows from the running time of Broadcast.

Proof of Lemma 2 (LearnDelays). Let $v \in V$ be a most delayed node in its 2-hop neighborhood. The main idea of this proof is to show that nodes in $\Gamma(v)$ always update their binary search parameters in exactly the same manner as $v$. Furthermore, we show that, while nodes in $\Gamma^{(2)}(v) \setminus \Gamma(v)$ might
update their parameters differently, these nodes never interfere with the binary search performed by nodes in \( \Gamma(v) \).

By Lemma 3 and a union bound over \( v \)'s at most \( \Delta^2 \) 2-hop neighbors and the \( O(\log \log n) \) rounds of LEARNDELAYS, we have that every 2-hop neighbor of \( v \) correctly learns its distance to an active node with constant probability in every round of LEARNDELAYS. From this point on in the proof we condition on every 2-hop neighbor of \( v \) correctly learning its distance to an active node in every iteration of LEARNDELAYS. We say that nodes \( v \) and \( w \) deviate if at some point \( v \) and \( w \) have different values for \( lo \) or \( hi \). For any variable \( \alpha \) of LEARNDELAYS we let \( \alpha_i(v) \) stand for \( v \)'s value of \( \alpha \) in iteration \( i \). For example, \( hi_i(v) \) is \( v \)'s value of \( hi \) in iteration \( i \) of LEARNDELAYS.

We show that three properties hold in every iteration:

1. \( v \) is not marked silent.
2. \( v \) does not deviate from \( v \).
3. \( v \) correctly learns its distance to an active node in its 2-hop neighborhood.

Thus, we conclude that, in fact, in iteration \( i \) it holds that \( v \) is not marked silent.

(1) We would like to show that \( v \) is not marked silent in iteration \( i \). Assume for the sake of contradiction that \( v \) is marked silent. \( v \) being marked silent means that \( v \) is not active, meaning \( t_v > \frac{lo_i(v) + hi_i(v)}{2} \), but that there is a node in \( w \in \Gamma(v) \) that is active, meaning \( t_w \leq \frac{lo_i(w) + hi_i(w)}{2} \). By (3) of our inductive hypothesis we know that \( w \) has not deviated from \( v \), meaning \( \frac{lo_i(w) + hi_i(w)}{2} = \frac{lo_i(v) + hi_i(v)}{2} \) and so we then have

\[
\begin{align*}
t_w &\leq \frac{lo_i(v) + hi_i(v)}{2} \\
&= \frac{lo_i(v) + hi_i(v)}{2} \\
&< t_v
\end{align*}
\]

Thus, \( t_w < t_v \). However, since \( t_w \in \Gamma(v) \) this contradicts the fact that \( t_v \) is most delayed in its 2-hop neighborhood. Thus, we conclude that, in fact, in iteration \( i \) it holds that \( v \) is not marked silent.

(2) We would like to show that in iteration \( i \) it holds that \( u \in \Gamma(v) \) does not deviate from \( v \), i.e. \( hi_{i+1}(u) = hi_{i+1}(v) \) and \( lo_{i+1}(u) = lo_{i+1}(v) \). We case on whether or not in iteration \( i \) we have \( t_v \leq \frac{lo_i(v) + hi_i(v)}{2} \).

- Suppose that \( t_v \leq \frac{lo_i(v) + hi_i(v)}{2} \). By (1) of our induction hypothesis \( v \) has not been marked silent and so in this case \( v \) is active. If \( v \) is active then for any \( u \in \Gamma(v) \) (where potentially \( u = v \) since \( v \in \Gamma(v) \)) we have \( \text{dist}_i(u) \leq 1 \) and so \( hi_{i+1}(u) \leftarrow \frac{lo_i(u) + hi_i(u)}{2} \) and \( lo_{i+1}(u) \leftarrow lo_i(u) \). By (2) of our induction hypothesis we have that \( lo_i(u) = lo_i(v) \) and \( hi_i(v) = hi_i(u) \) for every \( u \in \Gamma(v) \). Thus, for \( u \in \Gamma(v) \) we have \( lo_{i+1}(u) \leftarrow lo_i(u) = lo_i(v) = lo_{i+1}(v) \) and
$hi_{i+1}(u) \leftarrow \left\lfloor \frac{lo_i(u) + hi_i(u)}{2} \right\rfloor = \left\lfloor \frac{lo_i(v) + hi_i(v)}{2} \right\rfloor = hi_{i+1}(v)$. Thus, in this case $v$ and $u$ do not deviate.

- Suppose that $t_v > \left\lfloor \frac{lo_i(v) + hi_i(v)}{2} \right\rfloor$ in iteration $i$. Combining the fact that $v$ is most delayed in its two-hop neighborhood, with (2) of our inductive hypothesis it follows that $dist_i(v) \geq 2$. Now assume for the sake of contradiction that $u$ deviates from $v$ in this iteration. That is, $dist_i(u) \leq 1$. Since $dist_i(v) \geq 2$ we know that $u$ must deviate from $v$ because there is an active node in $w \in \Gamma_i^{(2)}(v) \setminus \Gamma(v)$. However, by (3) of our inductive hypothesis any such node must have not deviated from $v$. Since $v$ is most delayed in its two-hop neighborhood it follows that $t_w \geq t_v \left\lfloor \frac{lo_i(v) + hi_i(v)}{2} \right\rfloor = \left\lfloor \frac{lo_i(w) + hi_i(w)}{2} \right\rfloor$ which contradicts the fact that $w$ is active. Thus, in this case $u$ must not deviate from $v$.

(3) We would like to show that if $w \in \Gamma_i^{(2)}(v) \setminus \Gamma(v)$ deviates from $v$ in iteration $i$ then $w$ is not active in iteration $j$ for any $j > i$. We demonstrate this by showing that any such $w$ is either marked silent in iteration $i$ or in every subsequent iteration $t_w$ is outside of the range over which $w$ is searching. Consider a $w$ that deviates from $v$. $w$ deviates if $dist_i(v) \leq 1$ but $dist_i(w) \geq 2$ or $dist_i(w) \leq 1$ but $dist_i(v) \geq 2$. Consider each case.

- Suppose $dist_i(v) \leq 1$ but $dist_i(w) \geq 2$. First notice that in this case $v$ is active since $v$ is minimal in its two-hop neighborhood and by (1) of our inductive hypothesis $v$ is not silent. Thus, since $v$ is active and $v$ is at most two hops from $w$ we have $dist_i(w) \leq 2$, meaning that $dist_i(w) = 2$. However, this means that $w$ is marked as silent and will therefore never be active in any iteration $j > i$.

- Suppose $dist_i(w) \leq 1$ but $dist_i(v) \geq 2$. By (3) of our inductive hypothesis we have that $lo_i(v) = lo_i(w)$ and $hi_i(v) = hi_i(w)$. Moreover, since $dist_i(v) \geq 2$ we know that in iteration $i$ it holds that $t_v > \left\lfloor \frac{lo_i(v) + hi_i(v)}{2} \right\rfloor$. Since $v$ is minimal in its two-hop neighborhood we have that $t_v \leq t_w$. Combining these facts we have that

$$t_w \geq t_v$$

$$\geq \left\lfloor \frac{lo_i(v) + hi_i(v)}{2} \right\rfloor$$

$$= \left\lfloor \frac{lo_i(w) + hi_i(w)}{2} \right\rfloor$$

Thus, $t_w > \left\lfloor \frac{lo_i(w) + hi_i(w)}{2} \right\rfloor$. Moreover, since $dist_i(w) \leq 2$ we know that $hi_{i+1}(w) = \left\lfloor \frac{lo_i(w) + hi_i(w)}{2} \right\rfloor$ and so $t_w > hi_{i+1}(w)$. Since $hi$ is non-increasing over iterations—i.e. $hi_{j+1}(w) \leq hi_j(w)$ for $j$ and any $w$—we will have that in every iteration $j > i$ it holds that $t_w > hi_{i+1}(w) \geq hi_j(w) \geq \left\lfloor \frac{lo_i(w) + hi_i(w)}{2} \right\rfloor$ meaning that $w$ cannot be active in iteration $j$ since $w$ is only active in iteration $j$ when $t_w \leq \left\lfloor \frac{lo_i(w) + hi_i(w)}{2} \right\rfloor$.

Having shown our induction we now argue that the value that $v$ will learn from its binary search is $t_v$, it’s virtual round. In particular, we argue that $v$ always updates its binary search parameters according to $t_v$. That is, in iteration $i$ we have $v$ reduces $hi$ when $t_v \leq \left\lfloor \frac{lo_i(v) + hi_i(v)}{2} \right\rfloor$ and $v$ increases $lo$ when $t_v > \left\lfloor \frac{lo_i(v) + hi_i(v)}{2} \right\rfloor$.
• If $v$ updates $hi$ in iteration $i$ then there must be some node $u \in \Gamma(v)$ which is active. That is, $t_u \leq \lfloor \frac{l_0(u)+hi(u)}{2} \rfloor$. By (2) we have $l_0(u) = l_0(v)$ and $hi(u) = hi(v)$ and by the fact that $v$ is most delayed in its two-hop neighborhood we know that $t_v \leq t_u$. Thus $t_v \leq t_u \leq \lfloor \frac{l_0(v)+hi(v)}{2} \rfloor = \lfloor \frac{l_0(v)+hi(v)}{2} \rfloor$.

• If $v$ updates $lo$ then no node in $\Gamma(v)$ is active and so certainly $v$ itself is not active. But since $v$ is never marked silent by (1) it must not be active because $t_v > \lfloor \frac{l_0(v)+hi(v)}{2} \rfloor$.

In either case we have that $v$ always updates its binary search parameters according to $t_v$ and so we have that $v$ will return $t_v$ from its binary search.

Thus, $v$ learns as its binary search value $t_v$ but since no node in $\Gamma(v)$ deviates from $v$ so too does every node in $\Gamma(v)$. Recalling that we conditioned on the fact that every node in $\Gamma(2)(v)$ always learns its distance to an active neighbor correctly which happens with constant probability, we conclude our claim. Lastly, our running time comes from straightforwardly summing the running time of subroutines and noting that by assumption every value we are binary searching over lies within a $O(\log n)$ range.

**Proof of Lemma 7 (MAINSTATIC).** Fix a $v$ that is most delayed in its 2-hop neighborhood such that $t_v < L$. Note that by definition of a virtual round and the fact that $t_v < L$ we have that $P(v, t_v)$ is a listening action where in round $t_v$ of $P$ it holds that $v$ is sent a message without collision. By Lemma 2 after LEARNDELAYS terminates with constant probability each 1-hop neighbor $w$ will have $m_w = t_v$. Thus, every node in the 1-hop neighborhood of $v$ will simulate round $t_v$, i.e. the one neighbor that has a message for $v$ will broadcast its message and the rest of $v$’s neighbors will be silent. This message will be successfully received with probability $p$ which is constant by assumption and so $t_v$ will be incremented with constant probability. The running time follows by summing subroutines.

**Proof of Theorem 3.** Let $Q := O(\log n)$. We will prove the following subclaim: Assume that for every vertex $v$ we have $L - Q \leq t_v \leq L$; letting $t'_v$ be $v$’s virtual round after $Q$ outermost iterations of MAINSTATIC we have $L \leq t'_v \leq L + Q$ w.h.p.

Note that this subclaim immediately proves the theorem. The length of $P$ is poly($n$) by assumption and so we can divide $P$ into poly($n$) chunks of rounds each of size $Q$. By induction and a union bound over chunks we have that after $KQ$ outermost iterations of MAINSTATIC it holds that for every node $v$ we have $QK \leq t_v \leq QK + 1$ w.h.p. Letting $K = \lceil \frac{|P|}{Q} \rceil$ yields that with high probability after $T + Q$ rounds every node $v$ is such that $|P| \leq t_v$, yielding our theorem.

We now prove the subclaim. Note that the assumption of our subclaim is that the preconditions of Lemma 1 are satisfied. For simplicity and without loss of generality we assume $L = 0$. Let $x \in \mathbb{Z}_{\geq 1}$ and define $D_{v,x}$ as the random variable giving the index of the earliest innermost iteration of MAINSTATIC when $t_v \geq x$. Using Lemma 1 we get the following following recurrence relation $D_{v,x} \leq \min_{w \in \Gamma(2)(v)} D_{w,x-1} + G_{v,x}(q)$, where $q$ is the constant probability of an inner iteration of MAINSTATIC advancing a node most delayed in its 2-hop neighborhood given by Lemma 1.

We now prove a tail bound for $D_{v,x}$ by union bounding over all “blaming chains.” A blaming chain for $D_{v,x}$ is a sequence of nodes paired with simulated rounds that could explain why $D_{v,x}$ took as long as it did. In particular, a blaming chain $C$ for $D_{v,x}$ is a sequence of (node, round) tuples $(v_0, x), (v_0, x - 1), \ldots, (v_1, 1)$ where $v_i \in \Gamma(2)(v_{i-1})$ and $v_x = v$. We let $C(D_{v,x})$ stand for such blaming chains. We say $|C| := \sum_{(v_i, i) \in C} G_{v_i,i}$ is the length of blaming chain $C$. Note that the
size of $\mathcal{C}(D_{v,x})$ is at most $(\Delta^2)^{x-1}$. Also notice that $D_{v,x}$ just is the length of the longest blaming chain in $\mathcal{C}(D_{v,x})$. Thus, we have that

$$D_{v,x} = \max_{w \in \Gamma^{(2)}(v)} D_{w,x-1} + G_{v,x}$$

$$= \max_{C \in \mathcal{C}(D_{v,x})} |C|$$

As such, to show an upper bound on $D_{v,x}$ it suffices to show that no blaming chain is too long. Notice that the length of a blaming chain in $\mathcal{C}(D_{v,x})$ just is the sum of $x$ geometric random variables with constant expectation. Lemma 10 of Appendix A gives a Chernoff-style tail bound for such sums and asserts that for any fixed blaming chain $C$ in $\mathcal{C}(D_{v,Q})$ we have

$$\Pr[|C| \geq c(Q \log \Delta + k)] \leq \exp(-k)$$

for some constant $c > 0$. A union bound over all blaming chains of all nodes gives us that

$$\Pr[\exists v \in V, D_{v,Q} \geq c(Q \log \Delta + k)] \leq n \exp(-k)$$

Setting $k \leftarrow 2Q \ln \Delta + O(\ln n)$, all $D_{v,Q} = O(Q \log \Delta + \log n) = O(\log n \log \Delta)$ w.h.p. Note that $D_{v,Q}$ counts innermost iterations, hence it takes $\frac{D_{v,Q}}{c \log \Delta}$ outermost iterations for the condition to be satisfied. Moreover, $c$ is the constant in the iteration range of the innermost for loop of MainStatic and $Q$ does not depend on it. Hence we can set $c > 0$ sufficiently large such that $\frac{D_{v,Q}}{c \log \Delta} = \frac{O(\log n)}{c} \leq Q$. Having shown that the number of outer iterations for every variable to arrive at virtual round $Q$ is at most $Q$, we conclude the subclaim. As we earlier argued, this gives our theorem. Note that our running time follows from summing the runtimes of our subroutines.

\[ \square \]

### 4.3 General Protocol Simulation

Here, we provide our results for arbitrary protocols. Our approach gives a $O(\Delta \log \Delta)$ multiplicative overhead.

**Theorem 3.** Let $P$ be a general protocol of length $T$ for the faultless radio network model. The simulation routine MainGeneral simulates $P$ in the noisy radio setting with high probability. The simulation takes $O((T + \log n)\Delta \log \Delta)$ rounds.

Again, we build on the notion of a virtual round for this setting. Our main challenge in this setting is that even if a node knows its neighbors are simulating its virtual round, the node cannot tell if the absence of a message indicates that it receives no message in this round in the original protocol or that a random fault occurred. As such, in order for node $v$ to advance its virtual round after hearing no messages from a neighbor, $v$ must confirm that every neighbor was silent in the simulated round. Let $P$ be the original protocol for the faultless setting. Define the token for a node $v$ in round $r$ of $P$ to be either the message that $v$ is sending in round $r$ of $P$ or an arbitrary message indicating “$v$ is not broadcasting” if $v$ is silent in round $r$ of $P$. Next, we (re-)define the virtual round of a node $v$ to be the largest $t_v \in \mathbb{Z}_{\geq 1}$ such that $v$ successfully received all tokens from all neighbors for rounds $1, 2, \ldots, t_v - 1$ (for a total of up to $(t_v - 1)\Delta$ tokens).

Our simulation algorithm works in two phases: every node first informs its neighbors of its virtual round; next, nodes help the neighbor with the smallest $t_v$ they saw by sharing the token for that round. We now present pseudocode for the SHAREKNOWLEDGE routine which shares messages from a node with all of its neighbors and MainGeneral which simulates $P$ in the noisy setting.
Algorithm 5 ShareKnowledge for node $v$

Input: a message, $B_v$, that $v$ wants to share

for $O(\Delta \log \Delta)$ rounds do
  $v$ broadcasts $B_v$ with probability $\frac{1}{\Delta}$, independently from other nodes

**Lemma 5.** After ShareKnowledge terminates, a fixed node $v$ successfully receives messages from all its neighbors with probability at least $3/4$ and successfully sends its message to all neighbors with probability at least $3/4$. The running time of ShareKnowledge is $O(\Delta \log \Delta)$ rounds.

**Proof.** Fix an arbitrary node $v$. Consider the event where $v$ receives a message from a fixed neighbor $w$ in a fixed iteration of ShareKnowledge. This event occurs iff $w$ broadcasts and all other neighbors of $v$, namely $\Gamma(v) \setminus \{w, v\}$, do not broadcast. This occurs with probability that is at least $\frac{1}{\Delta}(1 - \frac{1}{\Delta})^{\Gamma(v) \setminus \{w, v\}} \geq \frac{1}{\Delta}(1 - \frac{1}{\Delta})^{\Delta} \geq \Omega(\frac{1}{\Delta})$ by $(1 - \frac{1}{x})^x = \Omega(1)$. The probability that $v$ does not hear from $w$ after $O(\Delta \log \Delta)$ iterations is $(1 - \Omega(\frac{1}{\Delta}))^{\Delta \log \Delta} \leq \exp(-\Omega(\log \Delta)) \leq \frac{1}{4\Delta}$. Union bounding over all $|\Gamma(v)| \leq \Delta$ possibilities for $w$ we get that the probability of $v$ not sharing knowledge with all neighbors is at most $1/4$.

Algorithm 6 MainGeneral for node $v$

$t_v \leftarrow 1$

for $O(T \log \Delta + \log n)$ do
  $m_v \leftarrow$ ShareKnowledge($t_v$)
  ShareKnowledge(token for virtual round $m_v$)
  update $t_v$ if $v$ received all tokens for round $t_v$

**Lemma 6.** Let $v$ be a most delayed node in its 2-hop neighborhood. After one MainGeneral loop iteration, $t_v$ will increase by one with at least constant probability.

**Proof.** Fix a node $v$ that is most delayed in its 2-hop neighborhood. By Lemma 5 and a union bound over $v$’s at most $\Delta$ neighbors, the probability that the first call to ShareKnowledge successfully shares $v$’s message to all of its neighbors, and that all of $v$’s neighbors successfully share their tokens with $v$ is at least $\frac{1}{2}$. Since $v$ is minimal in its 2-hop neighborhood it follows that $m_w = t_v$ for $w \in \Gamma(v)$. Thus, $v$ will receive a token from every neighbor for its virtual round with probability at least $\frac{1}{2}$, meaning that $v$ increments its virtual round by one with constant probability.

**Proof of Theorem 3**. We use the same blaming chain proof technique as in Theorem 2 but use Lemma 6 instead of Lemma 1. The proof is completely analogous and hence omitted.

5 Lower bounds

In this section we argue that an $\Omega(\text{poly} \log \Delta)$ multiplicative overhead in simulation is necessary in two natural settings. In the first setting the simulation is not permitted to use network coding [1]. In the second setting the simulation must respect information flow in the sense that we show a lower bound when the graph is directed. It follows that any simulation with constant overhead either uses network coding or does not respect information flow. We also give a construction which we believe
could be used to show an $\Omega(\text{poly} \log \Delta)$ multiplicative overhead in simulation, even without any assumptions.

Additionally, we strengthen our lower bounds by proving them in the setting in which the simulation is granted “global control”. Informally, global control eliminates the need for control messages by providing a centralized scheduler that can synchronize nodes based on how faults occur. The scheduler, however, cannot read the actual contents of the messages.

**Definition 4 (Global Control).** We say that a noisy radio network has global control when (1) nodes know the network topology, (2) nodes learn which nodes broadcast in each round and at which nodes receiver faults occur in each round, and (3) all nodes have access to public randomness.

It is not difficult to see that access to global control is sufficient to achieve the local progress detection of Section 4.1. Moreover, notice that our simulation from Section 4.1 with $O(\log \Delta)$ multiplicative overhead is both non-coding and respects information flow. As such, it is optimal for both settings.

### 5.1 Non-Coding Simulations

We now define a non-coding protocol and prove that simulations that do not use network coding suffer a $\Omega(\log \Delta)$ multiplicative overhead.

**Definition 5 (Non-Coding).** We say that a simulation $P'$ in the noisy setting, which is simulating a protocol $P$ in the faultless setting, is non-coding if any message sent in $P'$ is also sent in $P$ (though possibly by different nodes).

We consider an isolated star with degree $\Delta$ where the center node wants to send $T$ messages to its neighbors. One can achieve a constant multiplicative overhead on this protocol by using an error correction code like Reed-Solomon [45]. However, the following lemma shows that if coding is not used no such overhead is possible.

**Lemma 7.** For any $T \geq 1$ and sufficiently large $\Delta$ there exists a faultless protocol of length $T$ on the star network with $\Delta + 1$ nodes such that any non-coding simulation of the protocol in the noisy setting with constant success probability requires $\Omega(T \log \Delta)$ many rounds even if the simulation has access to global control.

**Proof.** As noted, the network is a star with $\Delta$ leafs. Let $r$ be the central node of the star. In our faultless protocol, $r$ receives $T$ private inputs $M_1, M_2, \ldots, M_T$ each of $\Theta(\log n)$ bits. $r$ takes $T$ rounds to broadcast each input, broadcasting $M_i$ at round $i$.

Now consider a simulation of our protocol and assume for the sake of contradiction that it succeeds with constant probability. Let $C = C(p)$ be a constant such that $1 - p \geq \exp(-C)$ where $p = \Omega(1)$ is the fault probability of our noisy network. By the non-coding assumption, all messages sent by $P'$ must be in the set $\{M_1, \ldots, M_T\}$. Denote by $t_i$ the number of times $r$ broadcasts $M_i$. For the sake of contradiction, assume that $\sum_{i=1}^{T} t_i \leq \frac{1}{100C} T \log \Delta$. Hence $\min_{i \in [T]} t_i \leq \frac{1}{100C} \log \Delta$ by an averaging argument. Let $i^* = \arg \min_{i \in [T]} t_i$. Notice that the probability that a fixed node receives a message is independent of that of any other node since WLOG only $r$ ever broadcasts. Therefore, the probability that a fixed node does not receive $M_{i^*}$ is $(1 - p)^{t_{i^*}} \geq (1 - p)^{\frac{1}{100C} \log \Delta} \geq \exp(-C \frac{1}{100C} \log \Delta) \geq \Delta^{-1/100}$ by definition of $C$. Consequently, the probability that some node does not receive $i^*$ is at least $1 - (1 - \Delta^{-1/100})^\Delta \geq 1 - \exp(\Delta^{0.9/100})$ which tends to 1 as $\Delta \to \infty$. This contradicts our assumption that our simulation succeeds with constant probability. Therefore, no simulation protocol of length $\Omega(T \log \Delta)$ can deliver all messages with constant probability.  


5.2 Simulations that Respect Information Flow

In this section we show how any simulation with less than a $\Omega(\text{poly}(\log \Delta))$ multiplicative overhead must route information along different paths than those in the faultless setting. In particular, we show that an $\Omega(\text{poly}(\log \Delta))$ lower bound holds in a directed network where information in any simulation flows just as it does in the noiseless setting.

**Lemma 8.** Let $G = (L \cup R, E)$ be a complete directed bipartite graph with $|L| = |R| = \Delta$ that has an arc $(l, r)$ for all $l \in L, r \in R$. There exists a protocol $P$ of length $\Delta$ for the faultless setting on the directed network $G$ such that any protocol that works in the noisy setting with constant success probability requires $\Omega(\Delta \log \Delta)$ rounds. This bound holds even in the global control setting.

**Proof.** Let $L = \{l_1, l_2, \ldots, l_\Delta\}$ and $R = \{r_1, r_2, \ldots, r_\Delta\}$ be the set of nodes on both sides of the partition. Every node $l_i \in L$ gets private input $M_i$ and needs to broadcast it to all nodes in $R$. In the faultless protocol $P$, $l_i$ broadcasts $M_i$ in round $i \in [\Delta]$.

Now consider a noisy protocol $P'$. Since $G$ is directed, the only node from $L$ that has knowledge of $M_i$ is $l_i$. Moreover, we can assume without loss of generality that in any one round of $P'$ at most one node in $L$ broadcasts, since if this were not the case either no node in $R$ would be sent a message or a collision would occur at every node in $R$. Let $t_i$ be the number of rounds in $P'$ that $l_i$ broadcasts $M_i$. For the sake of contradiction, assume that the length of $P'$ is $c\Delta \log \Delta$ for a sufficiently small constant $c > 0$. Therefore, $\sum_{i=1}^{\Delta} t_i \leq c\Delta \log \Delta$ and there exists $i^* \in [\Delta]$ such that $t_{i^*} \leq c\log \Delta$. Since $M_{i^*}$ can only be broadcasted from $l_{i^*}$, $M_{i^*}$ is broadcasted at most $c\log \Delta$ times by an averaging argument. Thus, we have a star with central node $l_{i^*}$ and leaves given by $R$ with the assumption that $c\log \Delta$ rounds suffices to spread a message from $l_{i^*}$ to every node in $R$. The remainder of the proof is identical to the strategy given in Lemma 7 and hence is omitted. \qed

5.3 Unconditional Lower Bound Hypothesis

We do not believe there exists an $o(\text{poly}(\log \Delta))$ multiplicative overhead simulation, even for static protocols, and in this section we put forward a candidate hard example that might be used to prove this claim.

**Construction.** Let $G$ be a bipartite network with partition $(L, R)$, where $|L| = |R| = n$. Divide the nodes in $L$ into $\Delta$ groups of size $\frac{n}{\Delta}$, namely $L^1, \ldots, L^\Delta$. Let $l^1_i, \ldots, l^\Delta_i$ be the nodes in $L^i_i$. We repeat the following for $t = 1, 2, \ldots, \Delta$ iterations: pick a fresh independent permutation $\pi : [n] \to [R]$ and divide $R$ into $\Delta$ groups of size $n/\Delta$ according to $\pi$. Specifically, let $R^1 = \{\pi(1), \pi(2), \ldots, \pi(n/\Delta)\}$, $R^2 = \{\pi(n/\Delta + 1), \ldots, \pi(2n/\Delta)\}$, ..., $R^\Delta = \{\pi(n - n/\Delta + 1), \ldots, \pi(n)\}$. Note that the grouping $R$ changes between iterations unlike $L$ which remains fixed. Fully connect $l^1_i$ to all the nodes in $R^1$ for all $i \in [\Delta]$.

**Why we believe this protocol is hard.** Directly forwarding messages from a node $l \in L$ to each one of $l$’s neighbors requires a multiplicative $\Omega(\log \Delta)$ overhead before all of the neighbors receive the message. Thus, if we assume there is a $o(\log \Delta)$ overhead protocol, there must be a large fraction of messages that are delivered indirectly. That is, many nodes in $R$ receive many of the messages they need to simulate the original protocol from a different nodes than they do in the faultless protocol. However, indirectly delivering messages seems to require strictly more rounds

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*The previous version of this work mistakenly did not include the above random permutation in our candidate lower bound construction. Without this random permutation simulating the stated protocol is trivial. We have also added intuition as to why one would expect the above protocol is hard to simulate.*
Figure 3: A particular sample of our construction which we believe could help prove an unconditional lower bound. \( R_i \) labeled according to the first iteration of our construction. \( \Delta = 3, n = 9 \). Nodes (and incident edges) colored according to the round in which they broadcast in the faultless protocol.

than directly sending messages. Each \( r \in R \) roughly wants to receive private input from a random subset of \( L \). Therefore, if \( r \in R \) receives a message indirectly from a neighbor \( l \in L \), it is unlikely that the neighbors of \( l \) apart from \( r \) need this message to simulate the original protocol. Thus, while \( l \) delivers a message to \( r \), it blocks all other neighbors of \( l \) from receiving messages they need to simulate the protocol. Lastly, we note that this problem roughly corresponds to a random instance of index coding [7], for which the bounds are currently not fully understood.

6 Conclusion

In this paper we explored the computational power of noisy radio networks relative to classic radio networks. We demonstrated that any static radio network protocol can be simulated with a multiplicative cost of \( poly(\log \Delta, \log \log n) \) rounds. Moreover, we demonstrated that a general radio network protocol can be simulated with a multiplicative \( O(\Delta \log \Delta) \) round-overhead. We also showed lower bounds that suggest that any simulation of a radio network protocol by a noisy radio network requires \( \Omega(\log \Delta) \) additional rounds multiplicatively.

Though we give a simulation for general protocols, the main focus of technical content of this work has been static protocols. Static protocols capture a number of well-known protocols (e.g. the optimal broadcast algorithm of Gąsieniec et al. [25]) and this focus has enabled us to understand both how delays propagate (see our blaming chain arguments) and how to efficiently exchange information among nodes (see our distributed binary search) in noisy radio networks. Having given solutions to these challenges, we now state promising directions for future work in noisy radio networks.

First, it is an open question how to deal with neighbors with diverging notions of when one
another send messages in the simulated protocol. This is not an issue in static protocols and our general protocol solves this issue but only through a costly subroutine in which every node shares information with all of its neighbors. Thus, it remains unknown how such diverging perspectives might be dealt with and if a solution to this issue might expedite simulation time. “Backtracking” on faulty progress has been the subject of some interactive coding literature—see Haeupler [27]—and will likely prove insightful on this front.

Second, whether acknowledgements are necessary in simulations in noisy radio networks remains open. In our static protocol simulation we do not require that receivers send a message to senders acknowledging that they have successfully received the broadcast message. Though our general protocol simulation solves this issue with a $O(\Delta \log \Delta)$ round overhead, we leave as an open question whether some form of a poly log $\Delta$ solution for acknowledgements is possible[4] and whether such a solution is even necessary for efficient simulation. Lastly, we note that this question ties in nicely with past work on MAC layers [22; 34] where acknowledgments in a wireless setting are required.

As a final direction for future work we note that many of our techniques apply to simulations of faulty versions of other models of distributed computing. For instance, applying the techniques of our general protocol simulation to a receiver fault version of CONGEST almost immediately yields a simulation of CONGEST with receiver faults by CONGEST with a $O(\log \Delta)$ multiplicative round overhead. We expect further applications abound.

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4Such a solution could not simply have every neighbor receiver send a message to the sender as this would require $\Omega(\Delta)$ rounds.
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A Tail Bounds

In this section we prove the tail bounds that we use throughout this paper.

Lemma 9. Let $X_1, \ldots, X_n$ be independent random variables taking on integer values. Let $\mu_i = \mathbb{E}[X_i]$ and suppose that for every $i$ and every $t > 0$ we have $\Pr[X_i - \mu_i \geq t] \leq \exp(-ct)$ for some $c > 0$. Then $\Pr[\sum_{i \in [n]} X_i - \mu_i \geq t] \leq \exp(-ct)$ for $t \geq L \cdot n$, where $L > 0$ is a constant that depends only on $c$.

Proof. We use $V^+ = \max(V, 0)$ to denote the positive part of a real number; note that $\Pr[(X_i - \mu_i)^+ \geq t] \leq \exp(-ct)$ since probability is bounded above by 1. Fix $t \in \mathbb{Z}_{\geq 0}$; any event where $\sum_{i \in [n]} X_i - \mu_i \geq t$ gives a non-negative integer sequence $u_i = (X_i - \mu_i)^+$ where $\sum_{i \in [n]} u_i \geq t$. This, in turn, gives a non-negative integer sequence $u_i \leq (X_i - \mu_i)^+$ where $\sum_{i \in [n]} u_i = t$, for instance, by arbitrary decreasing positive $u_i$ until the sum is $t$. We will bound the probability of such a sequence existing.

Fix any non-negative integer sequence $u_i \in \mathbb{Z}_{\geq 0}$ where $\sum_{i \in [n]} u_i = t$ and $u_i \leq (X_i - \mu_i)^+$. It is well-known combinatorial fact that there are $\left(\binom{t+n-1}{n-1}\right) \leq \left(\frac{e(t+n-1)}{n-1}\right)^{n-1} \leq e^n(1 + \frac{t}{n})^n$ such sequences. Fixing one such sequence $u_i$, we have

\[
\Pr[\forall i \in [n], u_i \leq (X_i - \mu_i)^+] \\
\leq \prod_{i \in [n]} \Pr[(X_i - \mu)^+ \geq u_i] \\
\leq \exp \left( -c \sum_{i \in [n]} u_i \right) = \exp(-ct).
\]

Finally, union bounding over at most $e^n(1 + \frac{t}{n})^n$ sequences, we get $\Pr[\sum_{i \in [n]} X_i - \mu_i \geq t] \leq e^n(1 + \frac{t}{n})^n \exp(-ct)$. We can choose a sufficiently large $L$ such that both $1 + x \leq \exp(\frac{x}{2})$ for $x \geq L$, and $e^{1/L} \leq \exp(\frac{L}{2})$ hold. In this case $\Pr[\sum_{i \in [n]} X_i - \mu_i \geq t] \leq \exp(\frac{L}{2}) \exp(\frac{L}{2} t) \exp(-ct) \leq \exp(-c/3 \cdot t)$. \hfill \Box

Lemma 10. Let $Y_1, Y_2, \ldots, Y_T$ be independent maximums of at most $\Delta \geq 2$ independent geometric random variables with constant expectation. Then $\Pr[\sum_{i \in [T]} Y_i \geq C(T \log \Delta + t)] \leq \exp(-t)$ for all $t \geq 0$, where $C > 0$ is a constant.

Proof. Geometric random variable $G$ with constant expectation have a natural tail bound of the form $\Pr[G \geq t] \leq \exp(-ct)$. Since there are only finitely many geometric random variables in question, we can choose a sufficiently small constant $c > 0$ such that the bound holds for all of them. Consequently, the maximum of at most $\Delta$ such variables exhibits a tail bound. Let $G_1, G_2, \ldots, G_k$ where $k \leq \Delta$ be geometric random variables with constant expectation and set $Y := \max(G_1, \ldots, G_k)$. Then $\Pr[Y \geq t] \leq \sum_{i \in [k]} \Pr[G_i \geq t] \leq \Delta \exp(-ct)$. Setting $\mu := \frac{\ln \Delta}{c}$ and $t \leftarrow t' + \mu$, we get that $\Pr[Y - \mu \geq t'] \leq \exp(-ct')$.

Having obtained a tail bound for $Y$’s, we can apply Lemma 9, getting

\[
\Pr \left[ \left( \sum_{i \in [T]} Y_i \right) - T \frac{\log \Delta}{c} \geq t \right] \leq \exp \left( -\frac{c}{3} t \right) \quad \text{when } t \geq L \cdot T.
\]
By setting $t \leftarrow L \cdot T + t'$ and $c' := \frac{1}{\epsilon} + L$, we get that $\Pr \left[ \left( \sum_{i \in [T]} Y_i \right) \geq c' \cdot T \log \Delta + t' \right] \leq \exp(-\frac{c'}{3}t')$ for all $t' \geq 0$. Finally, setting $t' \leftarrow \frac{3}{\epsilon} t''$ and $C := \max(c'', \frac{3}{\epsilon})$ we get the final result $\Pr \left[ \left( \sum_{i \in [T]} Y_i \right) \geq C(T \log \Delta + t'') \right] \leq \exp(-t'')$ for all $t'' \geq 0$.\qed