Noncommutative Dirac-Born-Infeld Action for D-brane

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Abstract

We derive the noncommutative Dirac-Born-Infeld action for the $D$-brane, which governs dynamics of $D$-brane with a NS-NS $B$-field in the low energy regime. Depending on some details of the path integral prescriptions, both ordinary Dirac-Born-Infeld action and noncommutative one can be obtained by evaluating the same Polyakov string path integral for the open string ending on the $D$-brane. Thus, it establishes the equivalence of the noncommutative Dirac-Born-Infeld action and the ordinary one.

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I. INTRODUCTION

The Dirichlet brane \([1]\), abbreviated to \(D\)-brane, is considered as one of the most important physical object to understand various aspects of string theories. It has been the key ingredient to the subjects of dualities \([2]\), black hole physics \([3]\), \(AdS/CFT\) correspondence \([4]\), and Matrix \(M\)-model in modern string theory \([5]\). Before the advent of the \(D\)-brane, Fradkin and Tseytlin \([6]\) computed the effective action for an open string coupled to \(U(1)\) gauge field and found that it is given by the Born-Infeld action at the tree level. Later Leigh \([7]\) studied the sigma model action for an open string in the \(D\)-brane background by requiring the conformal invariance and found that the effective action for the \(D\)-brane should be the Dirac-Born-Infeld (DBI) action in the low energy regime. Then the DBI action has been often adopted to discuss the diverse subjects in string theory in which the \(D\) brane plays an essential role. The \(Dp\)-brane is the \((p+1)\) dimensional hypersurfaces in space-time where the open strings can end and its dynamics is induced mostly by the open strings attached on it. The open string gives rise to the noncommutative geometry \([8]\) for the \(D\)-brane when a NS-NS \(B\)-field is present. The \(D\)-brane dynamics is then described by Yang-Mills gauge fields on noncommutative space-time \([9]\). Most recently Seiberg and Witten \([10]\) proposed an explicit relationship between the ordinary gauge fields and noncommutative gauge fields and in particular the equivalence of the ordinary Dirac-Born-Infeld action and the noncommutative one.

In the present paper we derive the noncommutative Dirac-Born-Infeld action \([11]\) for the \(D\)-brane, which governs dynamics of \(D\)-brane with a NS-NS \(B\)-field. We show that both ordinary DBI action and noncommutative one can be obtained by evaluating the same Polyakov string path integral for the open string ending on the \(D\)-brane. The difference in derivation of two DBI action only reside in some details of the path integral prescriptions. Thus, it establishes the equivalence of the noncommutative Dirac-Born-Infeld action and the ordinary one. The ordinary DBI action would be obtained \([8]\) if we employ the Neumann function as the Green function on the disk and treat the terms involving the NS-NS \(B\)-field.
and the $U(1)$ gauge field as interaction. When $B$-field is constant, we do not need to treat the term involving $B$-field as interaction. We may include this term in the kinetic part of the action, quadratic in string variables, and define the Green function with respect to it. In this case it is useful to employ our previous canonical analysis \cite{12}: The end points of the string, where the $U(1)$ gauge field is coupled to, obey noncommutativity and the classical action becomes equivalent to that of open string in the space-time with some effective metric, $G_E$. It suggests us that we may get the noncommutative DBI action where the space-time metric is replaced by the effective one $G_E$ and the ordinary $U(1)$ field strength by its noncommutative counterpart. We may include the term with $B$-field partly in the kinetic part and partly in the interaction term. Then, the derivation to be presented in this paper also suggests more general form of equivalence between the ordinary gauge fields and the noncommutative gauge fields, which is similar to one discussed in ref. \cite{10}.

**II. OPEN STRING ON D-BRANE AND DBI ACTION**

We begin with a brief review of the work of Fradkin and Tseytlin \cite{6} on DBI action. The bosonic part of the classical action for an open string ending on a $Dp$-brane with a $B$-field is given by

$$I = I_1 + I_2 = \frac{1}{4\pi\alpha'} \int_M d^2\xi \left[ G_{\mu\nu} \sqrt{h} h^{\alpha\beta}_{\mu} \frac{\partial X^\mu}{\partial \xi^\alpha} \frac{\partial X^\nu}{\partial \xi^\beta} - 2\pi i \alpha' B_{ij} \epsilon^{\alpha\beta} \frac{\partial X^i}{\partial \xi^\alpha} \frac{\partial X^j}{\partial \xi^\beta} \right]$$

where $\mu = 0, 1, \ldots, 9$ and $i = 0, 1, \ldots, p$ and $(\xi^0, \xi^1) = (\tau, \sigma)$. Here $G_{ij} = g_{ij} =$constant, $H = dB = 0$ and $h_{\alpha\beta} = \delta_{\alpha\beta}$. Since the longitudinal string variables $X^\mu$, $\mu = p + 1, \ldots, 9$ can be treated rather trivially, we will be concerned with the transverse variables $X^i$ only afterwards. At the tree level, the world surface of the open string is a disk on the $D$-brane. The interaction with $U(1)$ gauge field is introduced through a Wilson loop defined on $\partial M$, the boundary of the world surface,

$$W[A] = P \exp \left( -i \int_{\partial M} d\tau A_i(X) \dot{X}^i \right)$$
where \( \hat{\tau} \) is the parameter along \( \partial M \) and \( P \) denotes the path ordered product. To be explicit, we choose \( \hat{\tau} \) as
\[
\hat{\tau} = \begin{cases} 
\tau - 1 & : \hat{\tau} \in [-1, 0] \\
-\tau + 1 & : \hat{\tau} \in [0, 1].
\end{cases}
\] (3)

The effective action for the \( D \)-brane is given as the Polyakov string path integral on the disk
\[
\Gamma = \frac{1}{g_s} N \int D[X] \exp (-I) W[A]
\] (4)
where \( g_s \) is the string coupling constant and \( N \) is a normalization constant. Using the Stokes theorem we may write the Wilson loop operator in the string path integral as
\[
W[A] = \exp \left( -\frac{i}{2} \int_M d^2 \xi F_{ij} \epsilon^{\alpha\beta} \frac{\partial X^i}{\partial \xi^\alpha} \frac{\partial X^j}{\partial \xi^\beta} \right).
\] (5)
For a slow varying \( U(1) \) gauge field or a constant \( F_{ij} \)
\[
W[A] = \exp \left( -\frac{i}{2} \int_{\partial M} d\hat{\tau} F_{ij} X^i \frac{\partial X^j}{\partial \hat{\tau}} \right) = \exp(-I_3).
\] (6)
For constant \( B \)-field we also write
\[
I_2 = -\frac{i}{2} \int_M B_{ij} \epsilon^{\alpha\beta} \frac{\partial X^i}{\partial \xi^\alpha} \frac{\partial X^j}{\partial \xi^\beta} = -\frac{i}{2} \int_{\partial M} d\hat{\tau} B_{ij} X^i \frac{\partial X^j}{\partial \hat{\tau}}.
\] (7)
Therefore,
\[
I_2 + I_3 = -\frac{i}{2} \int_{\partial M} d\hat{\tau} (B + F)_{ij} X^i \frac{\partial X^j}{\partial \hat{\tau}}.
\] (8)
Note that \( (B + F) \) is invariant under the gauge transformation where
\[
A \rightarrow A + \Lambda, \quad B \rightarrow B - d\Lambda
\] (9)
for any one-form \( \Lambda \).

In order to evaluate the path integral it is convenient to diagonalize the space-time metric, introducing
\[
X^i_n = C^n_j Z^j_n, \quad (C^T g C)_{ij} = \delta_{ij}
\] (10)
\[ \int D[X] = \int \prod_{n \geq 0} dX^i_n = \int \prod_{n \geq 0} dZ^i_n \prod_{n \geq 0} \det C = \int D[Z](\det g)^{\frac{1}{4}}. \quad (11) \]

where we use \( \zeta(0) = \sum_n 1 = -1/2 \) and \( \det C = (\det g)^{-\frac{1}{4}}. \) Here we note that according to the canonical analysis \([12]\) the string normal modes, \( X^i_n \) are subject to the following constraints for the free open string

\[ X^i_n = X^i_{-n}, \quad n = 1, 2, \ldots \quad (12) \]

In evaluating the path integral we may treat \( I_2 \) and \( I_3 \) as interaction terms. It implies that the Green function on the disk is chosen as the Neumann function

\[ -\frac{\partial}{\partial \xi^\alpha} h^{\alpha \beta} \frac{\partial}{\partial \xi^\beta} N_G = \delta(\xi - \xi'). \quad (13) \]

Thus, the path integral may be written as

\[
\Gamma = \frac{1}{g_s} N(\det g)^{\frac{1}{4}} \int D[Z] \exp \left[ -I_1 - I_2 - I_3 \right] \\
= \frac{1}{g_s} N(\det g)^{\frac{1}{4}} \int d^{p+1}x \int [dz] \\
\quad \exp \left[ -\frac{1}{2} z G^{-1} z + i\pi \alpha' \left( C^T(B + F)C \right)_{ij} \int_{\partial M} d\hat{\tau} \frac{\partial z^i}{\partial \hat{\tau}} z^j \right]
\]

where \( z^i = Z^i|_{\partial M} \) and

\[ G(\hat{\tau}_1, \hat{\tau}_2) = N_G(z(\hat{\tau}_1), z(\hat{\tau}_2)), \quad G^{-1}G = \delta(\hat{\tau}_1 - \hat{\tau}_2). \quad (15) \]

Employing the result of ref. \([3]\), we have

\[
\Gamma = \frac{1}{g_s} N(\det g)^{\frac{1}{4}} \int d^{p+1}x \sqrt{\det (I + 2\pi \alpha' C^T(B + F)C)} \\
= \frac{1}{g_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}(\det g)^{\frac{1}{4}}} \int d^{p+1}x \sqrt{\det (g + 2\pi \alpha'(B + F))}
\]

where we choose \( N = 1/(2\pi)^p. \) Absorbing the factor \( (\det g)^{\frac{1}{4}} \) into the string coupling constant, we get the DBI Lagrangian

\[ L_{DBI} = \frac{1}{g_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}} \sqrt{\det (g + 2\pi \alpha'(B + F))}. \quad (17) \]
III. NONCOMMUTATIVE GEOMETRY

When we derive the DBI action, we treat the term with the $B$-field as interaction. However, since it is quadratic in string variables if $B$-field is constant, we may include it in the kinetic part of the action. The Green function is defined with respect to $I_1 + I_2$ instead of $I_1$. Our previous canonical analysis [12] shows that in this case the open string action is equivalent to that of free open string in the space time with metric given by $G_E$,

$$(G_E)_{ij} = \left(g - (2\pi\alpha')^2Bg^{-1}B\right)_{ij}. \quad (18)$$

The Hamiltonian and the string coordinate variable are written in the phase space $(Y_n^i, K_n^i)$ by

$$H = (2\pi\alpha')\frac{1}{2}p_i(G^{-1})^{ij}p_j + (2\pi\alpha')\sum_{n=1}^\infty \left\{ \frac{1}{2}K_{in}(G^{-1})^{ij}K_{jn} - \frac{1}{(2\pi\alpha')^2}n^2Y_n^i(G_E)_{ij}Y_n^j \right\}, \quad (19a)$$

$$X^i(\sigma) = x^i + i\theta^{ij}p_j \left(\sigma - \frac{\pi}{2}\right) + \sqrt{2}\sum_{n=1}^\infty \left(Y_n^i \cos n\sigma + \frac{i}{n}\theta^{ij}K_n^j \sin n\sigma\right) \quad (19b)$$

where $Y_n^i$ and $K_n^i$ satisfy the usual commutation relation

$$\{Y_n^i, Y_m^j\} = 0, \quad \{Y_n^i, K_m^j\} = \delta^i_j\delta_{nm}, \quad \{K_n^i, K_m^j\} = 0 \quad (20)$$

and

$$\theta^{ij} = -(2\pi\alpha')^2 \left(\frac{1}{g + 2\pi\alpha'B}\frac{1}{g - 2\pi\alpha'B}\right)^{ij} \quad (21a)$$

$$(G^{-1})^{ij} = \left(\frac{1}{g + 2\pi\alpha'B}\frac{1}{g - 2\pi\alpha'B}\right)^{ij}. \quad (21b)$$

In this representation, it is clear that the string variables are noncommutative. In particular, the ends points of the open string

$$z^i = X^i(0) = x^i - \frac{\pi}{2}i\theta^{ij}p_j + \sqrt{2}\sum_{n=1}^\infty Y_n^i, \quad (22a)$$

$$\bar{z}^i = X^i(\pi) = x^i + \frac{\pi}{2}i\theta^{ij}p_j + \sqrt{2}\sum_{n=1}^\infty (-1)^nY_n^i, \quad (22b)$$

satisfy
\[
[z^i, z^j] = i\pi \theta^{ij}, \quad [\bar{z}^i, \bar{z}^j] = -i\pi \theta^{ij}.
\]  

(23)

The vertex operators carrying momenta \(k\) and \(\bar{k}\) are associated with \(e^{ik^iz_i}\) and \(e^{i\bar{k}^i\bar{z}_i}\). Hence, their operator algebra are given as

\[
e^{ik^z}e^{iq^z} = e^{-i\frac{\pi}{2} k_i \theta^{ij} q_j} e^{i(k+q)\cdot z},
\]

\[
e^{i\bar{k}^z}e^{iq^z} = e^{i\frac{\pi}{2} \bar{k}_i \theta^{ij} q_j} e^{i(k+q)\cdot \bar{z}},
\]

\[
e^{ik^z}e^{iq^z} = e^{ik^z+iq^z}
\]

(24)

where we make use of the identity

\[
e^A e^B = e^{\frac{1}{2}[A,B]} e^{A+B}, \quad \text{if} \quad [[A, B], A] = [[A, B], B] = 0.
\]

(25)

The above noncommutative relations yield that the normal ordered product of two operators are given as the Moyal bracket \([13]\) as discussed in \([10]\). In general, a product of two functions of \(z\) is written as

\[
f(z)g(z) = \int \frac{dk}{2\pi} \int \frac{dq}{2\pi} e^{-i\frac{\pi}{2} k_i \theta^{ij} q_j} e^{i(k+q)\cdot z} \tilde{f}(k) \tilde{g}(q),
\]

(26)

where \(\tilde{f}\) and \(\tilde{g}\) are Fourier transformed functions of \(f\) and \(g\) respectively. It follows that normal ordered product of two operators satisfy

\[
: f(z) :: g(z) : = : f(z) * g(z) : 
\]

(27)

where

\[
f(z) * g(z) \equiv \exp \left( i\frac{\pi}{2} \theta^{ij} \frac{\partial}{\partial \xi^i} \frac{\partial}{\partial \zeta^j} \right) f(z + \xi)g(z + \zeta) \bigg|_{\xi = \zeta = 0}.
\]

(28)

The physical observables are often represented by Wilson loop operators. Consider a Wilson loop operator of \(U(1)\) gauge field on the \(D\)-brane, given as follows

\[
W_C[A] = P \exp \left[ \oint_C dX^i A_i(X) \right]
\]

(29)

where \(P\) denotes the path ordered product. Let us take that \(C\) is the boundary of the world surface of the open string on the \(D\)-brane, \(\partial M\) and is parameterize by \(\hat{\tau}\):
\[
X^i(\hat{\tau})|_{\partial M} = \begin{cases} 
  z^i : \quad \hat{\tau} \in [-1, 0] \\
  \bar{z}^i : \quad \hat{\tau} \in [0, 1] 
\end{cases}
\] (30)

and \(X^i(\hat{\tau} = -1) = X^i(\hat{\tau} = 1)\).

With the commutation relations Eq.(28), we write the expectation value of the Wilson loop operator as

\[
\langle W_C[A]\rangle = \int dz d\bar{z} \prod dX^i_n dP^i_n J(B) \exp \left[ \frac{i}{2\pi} \int d\tau \left\{ \left( \frac{dz^i}{d\tau} z^j - \frac{d\bar{z}^i}{d\tau} \bar{z}^j \right)(\theta^{-1})_{ij} \right. \right. \\
\left. \left. - H + \ldots \right\} \right] P \exp \left[ \oint_{\partial M} d\hat{\tau} dX^i(A(X)) \right] \\
\left. \times \exp \left[ \oint_{\partial M} d\hat{\tau} \frac{dX^i}{d\hat{\tau}} A_i(X) \right] \right] \]
\] (31)

where \(J(B)\) is a trivial Jacobian and \('...'\) denotes the kinetic terms for nonzero modes and constraint terms. Note the difference between the \(\tau\) ordered product and the path ordered product. If \(\hat{\tau}\)-ordering is employed, on \(\partial M\)

\[
e^{-i\pi/2 P^i_0} e^{i(P+Q)\cdot X} = e^{iP \cdot X} * e^{iQ \cdot X}. \] (32)

We may expand the Wilson loop operator as

\[
W_C[A] = I + \int_{\partial M} dX \cdot A + \frac{1}{2} \int_{\partial M} \int_{\partial M X_2 > X_1} dX_2 \cdot A(X_2) dX_1 \cdot A(X_1) + \ldots . \] (33)

Expanding \(A_i[X(\hat{\tau})]\) also and using Eq.(32) we get

\[
\langle W_C[A]\rangle = \left( I + \int_M d\sigma d\tau \left( \frac{\partial Y^i}{\partial \tau} \frac{\partial Y^j}{\partial \sigma} - \frac{\partial Y^i}{\partial \sigma} \frac{\partial Y^j}{\partial \tau} \right) \hat{F}_{ij} \right) + \ldots , \] (34a)

\[
\hat{F}_{ij} = \partial_i \hat{A}_j - \partial_j \hat{A}_i - \hat{A}_i * \hat{A}_j + \hat{A}_i * \hat{A}_j, \] (34b)

\[
\hat{A}_i = A_i - \frac{\pi}{4} \theta^{kl} \{ A_k, \partial_i A_l + F_{il} \} + O(\theta^2) \] (34c)

where

\[
Y^i(\sigma) = x^i + \sqrt{2} \sum_{n=1} Y_n^i e^{i n \sigma}. \] (35)
Note that
\[ dX \cdot A = d \left( x^i + \sqrt{2} \sum_{n=1}^{N} Y_n^i \right) A_i + \frac{\pi}{2} ip_i \theta^{ij} A_j. \]  

(36)

When we evaluate the expectation value of the Wilson loop operator, we may take the Wick contraction between \( p \) and \( A \). This procedure turns \( A \) into its noncommutative counterpart \( \hat{A} \). A similar argument can be found in ref. [14]. If we apply the (non-)Abelian Stokes theorem to the case of noncommutative algebra, we may find
\[
\langle W_C[A] \rangle = \left\langle \exp \left[ \int_M d\tau d\sigma \left( \frac{\partial Y_i}{\partial \tau} \frac{\partial Y_j}{\partial \sigma} - \frac{\partial Y_i}{\partial \sigma} \frac{\partial Y_j}{\partial \tau} \right) \hat{F}_{ij} \right] \right\rangle
\]

(37)

where
\[
\exp*(A) = \sum_n \frac{1}{n!} (A * A * \ldots * A).
\]

(38)

The noncommutative Stokes theorem needs a more rigorous proof.

**IV. NONCOMMUTATIVE DBI ACTION**

Being equipped with the canonical analysis [12] and the discussion on the noncommutative geometry given in the previous section, we evaluate the Polyakov string path integral. With the prescription given in the previous section the Polyakov string path integral representing the effective action is read as
\[
\Gamma = \frac{1}{g_s} N \int D[X] \exp (-I_1 - I_2) W[A] = \frac{1}{g_s} N \int D[Y, K] \exp \left[ \int d\tau \left( p_i \dot{x}^i + \sum_n K_{in} \dot{Y}^i_n - H \right) \right] W[A]
\]

where \( W[A] = P \exp \left( -i \int d\tilde{\tau} A_i(X) \dot{X}^i \right) \). As discussed if we include the \( B \)-field term in the kinetic part of the action, the Wilson loop operator in the Polyakov path integral may be rewritten as
\[
\langle W[A] \rangle = \left\langle \exp \left[ -\frac{i}{2} \int_M d^2 \xi \hat{F}_{ij} \epsilon^{\alpha \beta} \frac{\partial Y^i}{\partial \xi^\alpha} \frac{\partial Y^j}{\partial \xi^\beta} \right] \right\rangle.
\]

(40)
Following Seiberg and Witten \cite{10}, we substitute ordinary products for the $\ast$ products between $\hat{F}$ in Eq.\eqref{40}, since it makes difference only in terms with derivatives of $\hat{F}$. For a slow varying $U(1)$ gauge field, we may also write the Wilson loop operator as

\[ \langle W[A]\rangle = \left\langle \exp \left( -\frac{i}{2} \int_{\partial M} d\hat{\tau} \hat{F}_{ij} Y^i \partial Y^j \right) \right\rangle = \left\langle \exp \left( -\hat{I}_3 \right) \right\rangle. \] \hspace{1cm} (41)

Integrating out the momentum variables in Eq.\eqref{39},

\[ \Gamma = \frac{1}{g_s} \int D[Y] \exp \left( -I_E - \frac{i}{2} \int_M d^2\xi \hat{F}_{ij} \epsilon^{\alpha\beta} \frac{\partial Y^i}{\partial \xi^\alpha} \frac{\partial Y^j}{\partial \xi^\beta} \right), \] \hspace{1cm} (42a)

\[ I_E = \frac{1}{4\pi\alpha'} \int_M d^2\xi (G_E)_{ij} \sqrt{h} \epsilon^{\alpha\beta} \frac{\partial Y^i}{\partial \xi^\alpha} \frac{\partial Y^j}{\partial \xi^\beta}, \] \hspace{1cm} (42b)

we find that the string path integral Eq.\eqref{42a} coincides with Eq.\eqref{39}, if the effective metric $G_{ij}$ substitutes for the space-time metric $g_{ij}$ and the noncommutative field strength $\hat{F}$ for $(B + F)$. Then, the same procedure which leads to the ordinary DBI action, yields

\[ \Gamma = \frac{1}{g_s(2\pi)^p(\alpha')^\frac{p+1}{2} (\det G_E)^{\frac{1}{4}}} \int d^{p+1}x \sqrt{\det (G_E + 2\pi\alpha' \hat{F})} \] \hspace{1cm} (43)

Absorbing the factor $(\det g)^{-\frac{1}{4}}$ into the string coupling constant $g_s$ as before, we arrive at the noncommutative DBI Lagrangian

\[ \hat{L}_{DBI} = \frac{1}{G_s(2\pi)^p(\alpha')^\frac{p+1}{2}} \sqrt{\det (G_E + 2\pi\alpha' \hat{F})}, \] \hspace{1cm} (44a)

\[ G_s = g_s \left( \frac{\det G_E}{\det g} \right)^\frac{1}{4} = g_s \left( \frac{\det (g + 2\pi\alpha' B)}{\det g} \right)^\frac{1}{4}. \] \hspace{1cm} (44b)

V. CONCLUDING REMARKS

In ref. \cite{10} Seiberg and Witten discussed the equivalence between the noncommutative gauge theory and the ordinary one and the change of variables between them in an explicit form. The proposed equivalence was checked by comparing the noncommutative DBI with the ordinary one, both of which are supposed to describe the same $D$-brane with a NS-NS $B$-field. In the present paper we derive the noncommutative DBI action, evaluating the
Polyakov string path integral on a disk, which depicts the world surface of the open string ending on the $D$-brane. We get both noncommutative DBI and ordinary DBI from the same Polyakov string path integral. Thus, it is established that the noncommutative DBI action is equivalent to the ordinary one. Some details of the prescriptions for the path integral make the difference. If $B$-field is constant, the term involving the $B$-field can be treated as a part of interaction or as a part of kinetic term, since it is quadratic in string variables. In the former case we get the ordinary DBI action and in the latter case the noncommutative one. In ref. [10] two descriptions, one by the ordinary gauge theory and the other by the noncommutative one are shown to differ by the choice of regularization for the world-sheet theory; the Pauli-Villars regularization yields the ordinary commutative gauge symmetry while the point-splitting regularization yields the noncommutative one. The analysis of the string path integral in the present paper may be compared with theirs.

Since whether the term with $B$-field is put in the kinetic part or in the interaction part is optional, we may get a more general form of the noncommutative DBI action. We may split the term with the $B$-field into two and put one in the kinetic part and the rest in the interaction part. Then the string path integral will leads us to a more general form of the noncommutative DBI action. Thus, our description of the noncommutative DBI provides a useful tool to examine the interesting proposal made in [10] in some details. It is interesting to explore further its consequence [15]. The open strings attached to the multi-$D$-branes or to two different types of $D$-branes can be treated in similar ways. It is certainly interesting to understand the noncommutative non-Abelian DBI action in the framework presented here.

After having completed the work I found that equivalence between the ordinary DBI and the noncommutative DBI has been discussed also in refs. [16].

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