Physics of Narrow Superconducting and Superfluid Channels: Critical Temperature and Associated Single Vortices Nucleation

M B Sobnack and F V Kusmartsev
Department of Physics, Loughborough University, Loughborough LE11 3TU, United Kingdom
E-mail: m.b.sobnack@lboro.ac.uk

Abstract. It is well known that in lower dimensions at any finite temperature one can thermally create single vortex excitations. This is because the boundary cut-off eliminates the logarithmic divergence of the vortex energy, thus eliminating the Berezinskii-Kosterlitz-Thouless transition which typically exists in 2D films. However, single vortices created in a channel are strongly attracted to the boundaries; as a result, their life-time is very short. Here we demonstrate that with rising temperature the density of spontaneously generated vortices and anti-vortices increases significantly faster than one would expect from conventional activation law. The two boundaries of the channels act as mirrors that multiply the effective number of vortices and anti-vortices spontaneously created and makes it practically equal to infinity. All these vortices, a few real bare and infinitely many imaginary ones, screen the vortex-vortex and vortex-boundary interactions. As a result of the screening, the life-time and density of vortices increase. This process constitutes a new mechanism for the destruction of phase coherence, superconductivity or superfluidity in narrow channels. We describe this effect using real space renormalization group techniques and derive an expression for the scale-dependent critical temperature at which the vortices destroy superconductivity/superfluidity in a cascade manner.

1. Introduction
High-temperature superconductors (HTS) are most promising materials for numerous practical applications. These include HTS magnets, and SQUIDs. Indeed, over the past years, there has been an enormous interest in the superconducting properties of narrow channels and quasi-one-dimensional systems [1, 2, 3, 4, 5, 6, 7, 8]. In particular, there is renewed interest in the breakdown of superconductivity in systems with confined geometries [8]. It has been shown that the geometry and the size determine the dynamics of the vortices in the presence of the transport current subject to a magnetic field orthogonal to the axis [9]. Moreover, it was shown that the character of the superconducting phase slips depends on the transverse size of the superconducting nanowires. For ultra-narrow nanowires, the phase slips ascribe a quantum character that change the character of the superconducting state and can even completely suppress it [8]. In this manuscript, we study the suppression of superconductivity in narrow superconducting thin tapes and nanowires and, in general, in narrow superfluid channels. However, in contrast with the above studies [8, 9] we do not study ultra narrow superconducting wires where quantum phase slips may occur, but we study superconducting/superfluid channels.
with widths much larger than the superconducting coherence length. We show that there is another mechanism of the suppression of the superconducting state which arises at high temperatures and is associated with the spontaneous cascade creation of vortices from the boundary.

The essence of the new mechanism is in the influence of the boundary on the vortex nucleation. For the narrow superconducting wires, the low energy excitations of the systems are the vortices created near the boundary. The point is that the energy required to create a quantised vortex in a system of characteristic size $L$ depends logarithmically on the size of the system, i.e., it requires an enormous amount of energy to create the stationary vortex. This is the main reason why these topological defects did not receive much attention when they were proposed by Onsager [10] in the 40’s and by Feynman [11] in the 50’s, and observed by Vinen [12] in the 70’s. It was not until Berezinsk˘i [13] showed that it costs much less energy to create a vortex- anti-vortex ($V$–$A$) pair (with the energy depending logarithmically on the pair separation) that their importance was realised. Berezinski recognised that these defects may play a significant role in phase transitions and qualitatively described the phase transitions stimulated by spontaneous vortex- anti-vortex pair generation. Kosterlitz and Thouless [14, 15] described the scaling relations of the transition. Since then, phase transitions in two dimensions (2D) have been the subject of a lot of interest given that the Berezinski -Kosterlitz-Thouless (BKT) transition [13-15] must arise in 2D systems such as superfluid films and quasi-2D superconductors.

Qualitatively, the BKT transition involves the nucleation of $V$–$A$ pairs, which reduces the superfluid density. This, in turn, diminishes the binding energy of $V$–$A$ pairs so that it becomes easier for more pairs to be created, reducing the superfluid density further. This causes a chain (renormalization) process that continues until the screening effect is large enough for members of $V$–$A$ pairs to become completely unbound. Vortices and anti-vortices then nucleate freely and spontaneously. Since single vortices destroy the phase coherence needed for 2D superfluidity, the superfluidity is destroyed and there is a jump discontinuity in the superfluid density at the transition.

The situation may be rather different in the case of “small” superfluid samples or in a near-edge layer (or a stripe) of macroscopic films. Due to the confinement, there is a cut-off in the interaction energy for individual vortices — not only for $V$–$A$ pairs — while the condensation energy associated with the vortex cores is twice smaller for the case of single vortex excitations. This can make nucleation of single vortices in confined geometries energetically more favorable than nucleation of $V$–$A$ pairs. In this study, we show that in this case, a single–vortex plasma arising due to thermal fluctuations can destroy superfluidity, playing the same role as the $V$–$A$ plasma does in the BKT transition existing in two-dimensional superfluid and superconducting films. Note that, for narrow superconducting films, the interaction between vortices and anti-vortices is screened due to the existence of the hard boundary. In such a strip, each hard boundary plays the role of a mirror and therefore each vortex or anti-vortex created in the strip is attracted to an infinite number of image vortices and anti-vortices created by the mirror. As a result, the vortex charge is spread over the line of its images and the images of the images, therewith leading to a complete screening of the vortex-vortex interaction. If such a screening would arise in a superconducting film, the superconducting state would not exist. The existence of the superconducting state in the narrow strip is due to the boundary which attracts the vortices or anti-vortices and therewith removing them from the strip, cleaning the coherent state. However, the attraction of vortices to the boundaries can be also screened and the effect depends on the width of the superconducting strip and increases when the temperature rises.

Thus, in our approach, we describe the interaction of a single vortex with two sample edges, which can effectively be described as its attraction to infinitely many imaginary vortices (IV) and anti-vortex (IA), located beyond the sample (vortices, just like 2D electric charges, are attracted to their mirror anti-charges). A vortex can penetrate into the sample only by overcoming
Figure 1. This figure shows schematically the channel (of width $2H = 2H_0$) studied in the paper. A vortex (or anti-vortex) is strongly attracted to the boundary, and the energy required to create such a vortex (anti-vortex) is calculated, in analogy with 2D electrostatics, by calculating the energy of interaction of the vortex (anti-vortex) with an infinite number of image vortices and an infinite number of image anti-vortices situated beyond the channel, giving a logarithmic energy. Even one created vortex has an infinitely many vortex and antivortex images. There even a few created vortices act and move as in a plasma of vortices. As the temperature increases, there will be further vortex (anti-vortex) excitations in the system, including some in the space between the vortex (anti-vortex) and the boundary. This vortex (anti-vortex) plasma edge renormalizes the attraction of the vortex (anti-vortex) to the boundary. The width of this plasma edge increases when the temperature rises and becomes equal to the width of the channel at the critical temperature. The problem is studied here using renormalization group techniques.

this vortex attraction to their multiple images of vortices and anti-vortices (V–IA and V–IV) interaction, which has a logarithmic character. The same is true for an anti-vortex, which can penetrate into the sample only by overcoming a similar anti-vortex to multiple images of vortices (A–IV) and anti-vortices (A–IA) interaction. For bulk superconductors, this attraction to the boundary is known as a surface or Bean-Livingston barrier. Let us assume that several thermally excited vortices have been created near the boundary. These non-stationary vortices induce infinitely many images which renormalize (decrease) the vortex attraction to the boundary. In a fashion similar to the screening of the V–A interaction in the BKT mechanism, now the imaginary plasma of V–IA , V–IV screens the “Coulomb” attraction. Eventually this chain process leads to the nucleation of unbound fluctuational vortices and anti-vortices in a relatively wide layer near the boundary and the order parameter associated with superfluidity is destroyed. The temperature at which this crossover happens can be defined as a new size–dependent critical temperature which is lower than the critical temperature $T_c$ of the bulk material. Because this crossover occurs very fast, the described phenomenon presents the same kind of topological phase transition as the BKT [13-15] [13, 14, 15] transition, the destruction of superconductivity in a superconducting disk [16] or the Williams-Shenoy model for the $\lambda$–phase transition in superfluid Helium [17, 18]. Note that the screening process described here in which one vortex generates infinitely many image vortices and antivortices has never been discussed before. This mechanism
is completely different from the one which we described in our previous PRL paper [16] where each vortex has only one (not infinitely many, as in the present case) image anti-vortex taking part in the vortex dynamics.

2. Formulation

2.1. Vortices in superconducting tapes and analogy with flows in classical hydrodynamics
The model we study is a quasi-1D channel of superfluid film (density $\rho_s$) of width $2H$, lying between $x = -H$ and $x = H$ and unbounded in the $y$-direction. Consider a vortex of vorticity $\kappa = h/m$ at an arbitrary position $r_0 = (x_0, y_0)$, $0 < x_0 < H$. The correct boundary condition on $x = \pm H$ is that any flow at the boundaries should be tangential, i.e., the normal component of the flow at the boundaries should be zero. This is achieved, in analogy with 2D electrostatics, by having an infinite array of image vortices (each of vorticity $\kappa$) beyond the channel at $r_{m} = (x_m, y_0)$ and an infinite array of image anti-vortices (of vorticity $-\kappa$) at $r_{m} = (x'_m, y_0)$, where

$$x_m = x_0 + 4Hm, \ m \in \mathbb{Z}, m \neq 0$$

and

$$x'_m = 2H - x_0 + 4Hm, \ m \in \mathbb{Z}$$

The energy required to create the vortex at $r_0$ is the energy with which it is attracted to the boundaries. This can be evaluated by calculating the energy of interaction of the vortex with all the image vortices and anti-vortices. We do this in analogy with 2D hydrodynamics (see, for example, Newton [19] and references therein) and after a lengthy calculation we arrive at

$$U_0(x_0) = 2q^2\ln \frac{2\cos kx_0}{kr_c} + E_c$$

(1)

for the energy required to create a vortex in the channel at $r_0 = (x_0, y_0)$. $r_c$ is the radius of the core of the vortex, $E_c$ is the potential energy associated with the vortex core and, where in analogy with the 2D Coulomb gas,

$$q = \frac{1}{2} \kappa \left( \frac{\rho_s}{\pi} \right)^{1/2} = \frac{h}{m} \sqrt{\pi \rho_s}$$

is the effective vortex charge. The core energy $E_c$ is sometimes written as $-\mu$ and is called the chemical potential of the vortex. $U_0(x_0)$ is the energy with which the vortex charge at $r_0$ is attracted to the boundary (surface) of the superfluid material. From now on, we shall drop the subscript “0” from $x_0$.

2.2. Screening of the vortex attraction to the boundaries and the renormalization group equations
At low temperatures, it is likely that only a few vortices will be present. These cannot nucleate and are attracted to the boundary. This will also happen with virtual vortices and anti-vortices originated from a vacuum state as well. At higher temperatures, however, there are likely to be many more vortex excitations, including some located in the space between $x$ and $H$. These have an attenuating effect on, and screen, the interaction $U_0(x)$. To take into account this screening effect, we follow Kosterlitz and Thouless [14, 15], Sobnack [16], Williams [17] and Shenoy [18] by introducing a scale-dependent dielectric constant

$$\varepsilon = 1 + 4\pi \chi.$$  

(2)
Figure 2. The logarithmic interaction of the vortex to the boundary far outweighs the vortex-vortex or vortex-anti-vortex interaction in the channel - the latter arises only on very short distances, smaller than the width of the channel. On larger distances, it weakens very fast as it is effectively the interaction between two charges, each distributed over a line of infinite length. With increasing temperature, the effective width of the channel decreases to $2H_l$ where $l$ is the scaling parameter (see text) and so does the range of the attraction between a vortex and an anti-vortex.

The effective susceptibility $\chi = \int_{r_c}^{H-r_c} \alpha(x)dn(x)$, where $\alpha(x) = q^2(H - x)^2/2k_BT$ is the polarizability and $n(x)$ is the number density of vortices. It is straightforward to show that

$$dn(x) = dx \exp(-U(x)/k_BT)/(2\pi r_c^2H),$$

where $U(x)$ is the screened interaction. In terms of these, Eq. (5) reads

$$\varepsilon = 1 + \frac{q^2}{k_BT} \frac{1}{r_c^2 H} \int_{r_c}^{H-r_c} dx (H - x)^2 \exp(-U(x)/k_BT).$$

Introducing the dimensionless superfluid density $K = q^2/(\pi k_BT)$ and the renormalized density $K_r = K/\varepsilon(r)$, the above equation becomes an equation for $K_r$,

$$K^{-1}_r = K^{-1} + \frac{\pi \bar{g}_0}{r_c^2 H} \int_{r_c}^{H-r_c} dx (H - x)^2 \exp \left[-2\pi K \ln \frac{2\cos kx}{k r_c} \right] ,$$

where $\bar{g}_0 = \exp(-E_c/k_BT)$. In deriving this equation we have implicitly assumed a rather low density of vortices, evident, for example, in the fact that we have used the unrenormalized charge $q$ instead of $q_r = q/\varepsilon(r)$ to determine the polarizability. We have also neglected the correction term in the interaction energy and used the bare energy $U_0(x)$. These approximations do not change the results and are necessary to prevent the equations from becoming intractable. Strictly, one needs to replace $K$ in the exponential by the renormalized density $K_r$ to make Eq. (3) self-consistent. However, at low temperatures, the integral is small, and Eq. (3) is the first two terms in the expansion of $K_r^{-1}$.

For temperatures in the neighbourhood of the cross-over where the superfluid density tends to zero, the perturbative expansion is not valid and here we follow José et al. [20] and use their
vortex-core rescaling technique. Eventually, one arrives at the differential renormalization group equations (see, for example, [16])

\[
\frac{dK_i}{dl} = K_i - \pi \frac{(H - r_c)^2}{r_c H} K_i^2 y_i
\]

\[
\frac{dy_i}{dl} = (1 - 2\pi K_i) y_i
\]

(4)

for the effective couplings \(K_i\) and \(y_i\), together with \(dH_i/dl = -H_i\). Here \(dl = \ln b\) and \(K_i\), \(y_i\) and \(2H_i\) are the dimensionless density, the fugacity and the width of the channel at scale \(l\). Note that in equations (4) \(H\) and \(r_c\) have their original bare values. For convenience, in numerical simulations presented we measure \(H\) in units of \(r_c\); then the equations becomes dimensionless.

For illustration (Fig. 3) we present the solutions to these equations for a superconducting strip of width \(H = 20r_c\). We find here that there are two different flows: one flow is going to decreasing vortex fugacity and increasing superconducting density. Therewith this flow recovers the superconducting state. The other flow is going on to increasing vortex fugacity and decreasing superconducting density and therewith recovers the normal state.

The critical (or fixed) point of the scaling equations is given by \(dK_i/dl = 0\) and \(dy_i/dl = 0\). These two equations have the nontrivial solution

\[
K^* = \frac{1}{2\pi} \quad \text{and} \quad y^* = \frac{2r_c H}{(H - r_c)^2},
\]

(5)

describing the critical point which is a saddle point (see the renormalisation group flows presented in Fig. 3). Thus, this critical point separates the superconducting and normal states in the narrow channel. Interesting enough is the fact that the critical superconducting density does not depend on the channel width, whereas the critical value for the vortex fugacity does depend on it. For example, for a wide strip the value for critical fugacity decreases as \(y^* \sim 2r_c/H\), while it increases rapidly when the channel width approaches the coherence length.

3. Vanishing of the superconducting state and the scaling behaviour in the vicinity of the critical point: results

Now we are in the position to investigate how the superconducting state vanishes. For this purpose we have to analyse the scaling in the vicinity of the critical point and connect this scaling with the thermodynamic behaviour of the system. To investigate the behaviour of the scaling, we write the scaling equations (4) in terms of scaled deviations from the fixed points by expanding \(K_i\) and \(y_i\) around \((K^*, y^*)\) as \(K_i = K^* (1 + k')\) and \(y_i = y^* (1 + y')\). To first order in \(k'\) and \(y'\), the scaling equations then read (we now drop the primes)

\[
\begin{pmatrix}
\hat{k} \\
\hat{y}
\end{pmatrix} =
\begin{pmatrix}
-1 & -1 \\
-1 & 0
\end{pmatrix}
\begin{pmatrix}
k \\
y
\end{pmatrix},
\]

(6)

where \(\hat{k} = dk/dl\) and similarly for \(\hat{y}\). Expanding \(k\) and \(y\) in eigenstates \(A_\pm(l) = A_\pm e^{\lambda_\pm l}\) of the fixed–point stability matrix above, the eigenvalues are \(\lambda_\pm = \pm (\sqrt{5} \mp 1)/2\). These define the relevant and irrelevant axes in the \(K_i - y_i\) plane. We assume, following existing procedure, that the relevant scaling field \(A_+\) is the temperature axis, \(A_+ \approx A|\varepsilon|\), where \(\varepsilon = (1 - T/T_c)\) is the deviation of the temperature \(T\) from the bulk transition temperature \(T_c\), and \(A\) is a constant.

The rescaling law for the free energy \(F\) per unit area implies

\[
Z(K_0, y_0, H) = e^{ -(F_1 - F_0)L^2} Z(K_1, y_1, H_1)
\]

\[
= e^{ -(F_1 - F_0)L^2} Z \left( A|\varepsilon| e^{\lambda_+ l}, A_- e^{\lambda_- l}, H e^{-l} \right),
\]

(7)
**Figure 3.** The scaling flows of the superconducting density $K$ and the vortex fugacity $y$. All flows are split into two classes associated with the superconducting and the normal states. They are separated by the fixed point represented by the red bold dot. The flows into superconducting state are characterised by vanishing fugacity $y$ and growing superconducting density $K$, while flows into the normal state are characterised by vanishing superconducting density and growing vortex fugacity.

where $Z$ is the partition function and $K_0$ and $y_0$ are the initial values of $K_l$ and $y_l$ at scale size $r_c$. The scaling stops when $H_l = H e^{-l}$ reaches a critical width $H_c = \xi = r_c e^{l}$. This happens when

$$l = l_c = \frac{1}{2} \ln \frac{H_c}{r_c}.$$  \hspace{1cm} (8)

Setting $l = l_c$ in Eq. (7), the partition function is well defined only if

$$\xi(= r_c e^{l_c}) = r_c |\epsilon|^{-1/\lambda^+}.$$  \hspace{1cm} (9)

Combining Eqs. (8) and (9) gives $|\epsilon|^2 = (r_c/H)^{\lambda^+}$, whence we obtain a new critical temperature

$$T_R^{R} = T_c \left[ 1 - \left( \frac{r_c}{H} \right)^{\lambda^+/2} \right], \quad \left( \frac{\lambda^+}{2} \approx 0.31 \right).$$ \hspace{1cm} (10)

This result implies that the superfluidity breaks down at a temperature which is lower than the bulk critical temperature $T_c$. The mechanism of vortex nucleation we have described leads to a depression of $T_c$ by an amount which varies inversely with the width of the superfluid channel. Previously we had done a very rough calculation on the same system (a quasi-1D superfluid channel) where we only considered two image anti-vortices (one beyond the boundary at $x = -H$ and the other beyond that at $x = +H$) and we obtained

$$T_R^{R} = T_c \left[ 1 - \left( \frac{r_c}{H} \right)^{1/2} \right]$$

for the temperature at which the superfluidity is destroyed [21]. Two image anti-vortices are blatantly not enough to satisfy the boundary condition of no normal flow at the boundaries of the system.
4. Conclusions/Discussion

We have described a mechanism of vortex nucleation in superfluid systems of size comparable to, or larger than, the characteristic size (coherence length) of the quasiparticles of the systems. A vortex created in a small superfluid system is strongly attracted to the boundary of the system and hence cannot nucleate. The attraction is due to the Coulomb attraction of the “vortex charge” to the “vortex charge” of the image anti-vortex. However, a fluctuating creation of several of these topological defects in the bulk of the superfluid, in the space between the vortex and the boundary, screens the V-IA attraction by renormalizing the superfluid density. This further improves the condition for the creation of more of such fluctuations. The screening effect of the plasma of vortex fluctuations continues until vortices nucleate freely. This leads to a cross-over at which the order parameter associated with the superfluidity of the system is destroyed.

We have used a real–space renormalization group method to derive the scaling laws in the vicinity of this cross-over, and shown that the cross-over occurs at a temperature which is lower than the bulk transition temperature \( T_c \). The amount by which \( T_c \) is lowered is equal to \( \Delta T_c \sim T_c (r_c/H)^a \) (with \( a \approx 1/3 \)), showing that the depression in the transition temperature increases with decreasing channel width \( H \).

Note that we have considered channels with widths larger than the coherence length, \( H > \xi \). In this regime the critical temperature decreases as the width decreases. It reaches a minimum when the width is of the order of the coherence length. When the width becomes smaller than the coherence length, the vortices cannot nucleate any more in the channel; therefore with the next decrease of the width the critical temperature may increase. However, at widths much smaller than the coherence length there may arise another phenomenon – quantum phase slip, corresponding to the vortex tunnelling from one edge of the channel to the other – an activity which is now seeing explosive interest [7, 22, 23, 24, 25, 26, 27].

The phenomenon described above is very general. Obviously it may arise in any small or porous system, like superfluid droplets, superfluid in pores of aerogels, superconducting quantum dots and so on. It may arise not only in superfluid or superconducting systems, but also in other condensed states, such as the magnetic state. In any case, topological vortex-like defects originating from the surface/boundary (like vortices in superfluids and superconductors) have the potential to destroy a condensed state provided that the system has a small size.

In this paper, we have presented a new effect that suppresses superconductivity/superfluidity in confined systems and driven by the boundary of the systems. More precisely, this suppression, although similar to the BKT transition, is driven by the creation of a plasma between the vortex and the boundary which screen the Coulomb attraction of the vortex to the boundary. In this respect, the driving mechanism is analogous to the mechanism proposed by Kusmartsev [28] for the nucleation of vortices in a flow of rotating superfluid \(^4\)He, where, instead of imaginary vortices and anti-vortices, imaginary half-vortex loops play a crucial role in the destruction of the superfluid state.

Here we have discussed the thermodynamics aspect of the new low temperature physics in narrow channels where each vortex is attached to an infinitely many image vortices and image anti vortices created due to the two channel boundaries. We expect that the phenomenon we have described is more general and also exists in both equilibrium and non-equilibrium physics and dynamics of vortices. It will have a strong influence on the phase slip phenomena as well as on the electrical current propagation through superconducting wires and in superfluid channels. We intend to investigate these new phenomena in the future studies.

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