In this talk I review some recent developments which shed light on the main connections between structural glasses and mean-field spin glass models with a discontinuous transition. I also discuss the role of quantum fluctuations on the dynamical instability found in mean-field spin glasses with a discontinuous transition. In mean-field models with pairwise interactions in a transverse field it is shown, in the framework of the static approximation, that such instability is suppressed at zero temperature.

I. INTRODUCTION

There is much current interest in the study of disordered systems. These are characterized by the presence of quenched disorder, i.e. disorder which is frozen at a timescale much larger than the typical observation time. In particular, a large amount of experimental and theoretical work has been devoted to the study of spin glasses [1]. These are alloys where magnetic impurities are introduced in the system. The magnetic impurities are frozen inside the host material and the interaction between them is due to the conduction electrons. This is the RKKY interaction which can be ferromagnetic or antiferromagnetic depending on the distance between the frozen impurities. Hence, the site disorder in the system leads to frustrated exchange interactions.

But there are also intrinsically non disordered systems which display glassy behavior as soon as they are off-equilibrium. The most well known examples are structural glasses [2], for instance dioxide of silica $\text{SiO}_2$. Structural glasses are characterized by the existence of a thermodynamic crystalline phase below the melting transition. Under fast cooling the glass does not crystallize at the melting transition temperature $T_M$ and it stays in the supercooled state in local equilibrium. Instead if the temperature is slowly decreased this local equilibrium property is lost when the cooling rate is of the same order than the inverse of the relaxation time.

There are two main differences between glasses and spin glasses. The first difference has been already pointed out. It is that spin glasses are disordered systems while glasses are intrinsically clean. The second difference emerges from experimental measurements which show that in spin glasses there is a static quantity, the non linear susceptibility, which diverges at a critical temperature. This implies the existence of characteristic length scale or correlation length which diverges at the critical point. On the contrary in real glasses a divergence of a static susceptibility has not been observed. Hence, while in spin glasses there is common agreement that there is a true thermodynamic transition the situation for structural glasses is much less clear and such a thermodynamic transition (the so called ideal glass transition) is still only a theoretical concept.

The renewed interest in the connection between glasses and spin glasses comes from the observation that a certain class of mean-field microscopic spin-glass models seem to capture the essential features which are present in structural glasses. Furthermore, while spin glass mean-field models explicitly contain disorder it has been recently shown that this is not an essential ingredient and spin glass behavior is found even in microscopic models where disorder is absent. The connection appears then fully justified.

This connection between structural glasses and spin glasses at the classical level has been already emphasized in other talks in this conference (see the talks by S. Franz and G. Parisi). After reviewing recent developments in this direction I will address a different aspect of glassy behavior, this is the study of glassiness in the presence of quantum fluctuations. There are several reasons why quantum fluctuations are interesting [3]. The most compelling reason is that the low temperature behavior of a large variety of systems in condensed matter physics strongly depends on the presence of disorder, for instance the quantum Hall effect and the metallic insulator transition (see the talk by T. Kirkpatrick in these proceedings). Another reason is more theoretical and relies on the need for a better understanding of the role of randomness in quantum phase transitions in the regime where there is...
The talk is organized as follows. First I will review some recent work on the connections between glasses and spin glasses at the classical level, putting special emphasis in the difference between spin-glass models with continuous and discontinuous transitions. In section 4 I will show that a large class of exactly solvable models with discontinuous phase transition at finite temperature have a continuous phase transition at zero temperature. Particular results will be presented for the random orthogonal model. Finally I will discuss the implications of this result and present the conclusions.

II. GLASSES V.S. SPIN GLASSES

It has been realized quite recently that systems without disorder can have a dynamical behavior reminiscent of spin glasses. While this suggestive idea has to be traced back to Kirkpatrick and Thirumalai [4] only recently has it been shown how this idea works in some microscopic models. In particular, Mezard and Bouchaud [5] studied the Bernasconi model [6] by mapping it onto a disordered model (the $p$-spin Ising spin glass with $p = 4$) and thus finding evidence for glassy behavior. It has also been shown in [6] how it is possible to map the Bernasconi model (with periodic boundary conditions) into a disordered model and solve it exactly by means of the replica method. Within this approach it is possible to show that both, the ordered and the disordered model, have the same high temperature expansion. While the thermodynamics of the models is different at low temperatures (the disordered model does not have a crystalline state) the ordered model reproduces all the features of the metastable glassy phase found in the disordered model including the existence of a dynamical singularity.

There have been several microscopic models such as the sine or cosine model [8], fully frustrated lattices [9], matrix models [10], the Amit-Roginsky model [11] and mean-field Josephson junction arrays in a magnetic field [12,13] (see the talk by P. Chandra in this conference) where this approach has been successfully applied. The main conclusion which emerges from these studies is that quenched disorder is not necessary to have spin glass behavior but it can be self-generated by the dynamics. Physically this means the following: the relaxation of the system becomes slower as the temperature is decreased and the local fields, acting on the microscopic variables of the system, can be considered as effectively frozen. It seems also that all mean-field models where this mapping is possible are those which show the existence of a dynamical singularity above the static transition temperature. In fact, all models where this equivalence has been built up are characterized by a discontinuous transition. In the spin-glass language this corresponds to models with one-step replica symmetry breaking transition.

In what follows I will discuss the main results concerning mean-field spin glass models contrasting those with a continuous and discontinuous transition. The phase transition in spin glasses is described by an order parameter which is the Edwards-Anderson parameter (hereafter referred as $EA$ order parameter). Because spin glasses are intrinsically disordered systems the magnetization is not a good order parameter since long range order is absent. In fact, below the spin-glass transition the spins tend to freeze in certain directions which randomly change from site to site. While spatial fluctuations of the local magnetization are large the temporal fluctuations are quite small and the parameter which measures the local spin glass ordering is given by $\langle \sigma_i \rangle^2$. The $EA$ order parameter $EA=1/\sum_{i=1}^{N} <\sigma_i>^2$. The $EA$ parameter varies from zero to 1. If it is very close to 1 this means that the system is strongly frozen and thermal fluctuations are small. In spin-glass models with a continuous transition $q_{EA}$ is zero above the spin glass transition $T_g$ (in the paramagnetic phase) and continuously increases as the temperature is lowered below $T_g$ (i.e. within the spin glass phase). In these models one finds $q_{EA} \approx (T_g - T)$ which means that the critical exponent $\beta$ is equal to 1, a typical value found in mean-field disordered systems. The simplest example of this class of models is the Sherrington-Kirkpatrick model [12] defined by

$$H = - \sum_{i<j} J_{ij} \sigma_i \sigma_j$$

where the $J_{ij}$ are random Gaussian variables with zero mean and variance $1/N$.

In models with a discontinuous transition $q_{EA}$ is zero above $T_g$ but discontinuously jumps to a finite value below $T_g$. Examples of models with a transition of this type are $q$-states Potts glass models with $q \geq 4$ [14] and $p$-spin glass models with $p \geq 3$ [15]. In the limit $p, q \to \infty$ both class of models converge to the random energy model of Derrida [16] which is characterized by an order parameter which is 1 just below $T_g$. Because this model is a particular limit where the energies of the configurations are randomly distributed and also because it can be fully solved without the use of replicas it is usually referred to as the simplest spin glass [17]. The two types of transitions are shown in figure 1.

Another example of a model with a discontinuous transition has been recently introduced [8]. This is the random orthogonal model (we will use the initials ROM in the rest of the paper to refer to this model) defined by eq. (3) where now the $J_{ij}$ are matrix elements of a random...
orthogonal ensemble of matrices, i.e. $J_{ij}J_{jk} = \delta_{ik}$. In the ROM model $q_{EA}$ jumps discontinuously to 0.9998 at the transition point $T_g$. This model can be considered as a very faithful microscopic realization of the random energy model of Derrida. It is important to note that both discontinuous and continuous spin glass transitions are continuous from the thermodynamic point of view. This is related to one of the main subtleties of spin glasses where order parameters are functions $q(x)$ in the interval $[0:1]$ and thermodynamic quantities are integrals of moments of these functions $\int_0^1 q(x)dx$. The functional nature of the order parameter $q(x)$ is related to the existence of an infinite number of pure states in the spin-glass phase as shown by G. Parisi. In discontinuous transitions the $q(x)$ has a finite jump for $x \to 1$ and all the moments $\int_0^1 q^p(x)dx$ remain continuous at the transition, hence there is no latent heat.

FIG. 1. EA order parameter as a function of $T$ for a continuous transition (continuous line) and a discontinuous transition (long dashed line).

The body of these results apply to mean-field models with long ranged interactions. Quite surprisingly it appeared that these mean-field models with a discontinuous transition are a nice realization of the entropy crisis theory proposed for glasses long ago by Gibbs and Di Marzio and later on by Adam and Gibbs in two seminal papers. This is a heuristic theory which is based on the Kauzmann paradox and proposes the collapse of the configurational paradox as the mechanism for a thermodynamic glass transition. The simplest example where this transition occurs is the random energy model of Derrida where the phase transition coincides with the point at which the entropy collapses to zero. This transition corresponds to what theorists refer to as the ideal glass transition and lies below the laboratory glass transition (see the talk by Angell in this proceedings) which is defined as the temperature at which the relaxation time is of order of a quite few minutes (more concretely the viscosity is $10^{13}$ Poisse).

It is important to note that the ideal glass transition is a thermodynamic transition where the configurational entropy collapses to zero but still the total entropy can be finite since fluctuations inside a configurational state can still be present. In the random energy model, fluctuations in the low temperature phase are absent and the full entropy (which gets contributions from the configurational entropy and the entropy coming from local fluctuations inside one state) vanishes at the glass transition $T_g$ (in what follows we will denote by $T_g$ the ideal glass thermodynamic transition temperature). In the ROM the entropy at $T_g$ is of order $10^{-3}$ and can be considered quite small (the entropy per free spin is $\log(2)$).

The connection between structural glasses and spin glasses with a discontinuous transition would not be fully accomplished if dynamics is not taken into account. This was realized by Kirkpatrick, Thirumalai and Wolynes who noted the existence of a dynamical transition $T_d$ above the glass transition $T_g$ in connection with an instability found in the mode coupling theory of glasses (MCT). S. Franz and J. Hertz have shown that the dynamical equations of the Amit-Roginsky model (a model with pseudo-random interactions which, nevertheless, does not contain explicit disorder) can be mapped onto the dynamical equations of the $p$-spin spherical spin glass model with $p = 3$. At the dynamical transition a large number of metastable states (which grows exponentially with the size of the system) determines an instability in the relaxational dynamics of the system but does not induce a true thermodynamic transition. In the region $T_g < T < T_d$ the free energy of the system is given by the paramagnetic free energy $f_P$ but the dynamical response is fully determined by the presence of a very large number of metastable configurations $exp(NC^*)$ where $C^*$ is the so called configurational entropy or complexity. Note that the free energy of these metastable states is higher than the free energy of the paramagnetic state and there is no thermodynamic transition at $T_d$. As the temperature is decreased the number of metastable configurations decreases and so does their free energy. When the free energy of the metastable states equals the paramagnetic free energy there is a phase transition. Because the number of metastable solutions with equilibrium free energy are not exponentially large with the size of the system the configurational entropy $C^*$ also vanishes at this point.

*The local order parameter inside one state is given by the EA order parameter which is given by the relation $q_{EA} = \max_x q(x)$.  

\[ \int_0^1 \delta_{ik} \]
The suspicious reader will find it extremely unclear how all these quantities \( (T_d, T_g, q_{EA}, C^*) \) can be analytically computed. Fortunately it is not necessary to fully solve the dynamics in order to find these quantities and there are powerful techniques to compute them. One of the simplest procedures [24] works for discontinuous transitions of the type described here and consists in expanding the free energy around \( m = 1 \) \( (m \) parametrizes the one step replica symmetry breaking and corresponds to the size of the diagonal blocks with finite order parameter \( q \) in the Parisi ansatz [3]). The free energy is expanded in the following way,

\[
\beta f(q) = \beta f_P + (m - 1)\mathcal{V}(q)
\]  

(2)

where \( f_P \) is the paramagnetic free energy (independent of \( q \)) and \( \mathcal{V}(q) \) is a function called the potential [26]. Note in eq. (2) that \( \mathcal{V}(q) \) plays the role of an entropy contribution to the paramagnetic free energy except for the factor \( (m - 1) \). The dynamical transition corresponds to an instability in the dynamics and is obtained by solving the equations,

\[
\frac{\partial \mathcal{V}}{\partial q} = 0 ; \quad \frac{\partial^2 \mathcal{V}}{\partial q^2} = 0
\]  

(3)

which yield the transition \( T_d \) and the jump of the EA order parameter \( q_{EA}^d \) at the dynamical transition temperature. On the other hand, the static transition is obtained by solving the equations

\[
\frac{\partial \mathcal{V}}{\partial q} = 0 ; \quad \mathcal{V}(q) = 0
\]  

(4)

and yield the transition \( T_g \) and the jump of the EA order parameter \( q_{EA}^g \) at the glass transition temperature. In the range of temperatures \( T_g < T < T_d \) the complexity or configurational entropy \( C^* \) is given by the value of the potential \( \mathcal{V}(q) \) in the secondary minimum (see figure 2) in the region \( q_{EA}^d < q < q_{EA}^g \). For continuous transitions (such as that found in the Sherrington-Kirkpatrick model [24]) both temperatures \( (T_d \) and \( T_g \) coincide and there is no discontinuous jump of \( q_{EA} \) at the transition temperature. The behavior of \( \mathcal{V} \) as a function of \( q \) for different temperatures is shown in figure 2 for a discontinuous transition. Note that the behavior shown in figure 2 is quite reminiscent of a spinodal instability in first-order phase transitions. The behavior of the potential \( \mathcal{V}(q) \) determines the phase transition and in particular the existence of an instability at \( T_d \). The reason why a zero mode at \( T_d \) yields a divergent relaxation time in the dynamics is related to the one of the most prominent features of glassy systems: the dominance of an exponentially large of metastable states \( \exp(NC^*) \) at that temperature [23,25]. Note that in mean-field theory metastable states have an infinite lifetime, the time to jump from the metastable glassy phase to the paramagnetic state being equal to \( \exp(NB^*) \) where \( B^* = \max_{0 < q < q_{EA}^g} \mathcal{V}(q) \) is the height of the free energy barrier which separates the metastable and the paramagnetic phase. The extension of this approach to the computation of thermodynamic quantities below \( T_d \) in the metastable glassy phase has been considered in [26].

Summarizing, mean field spin-glass models with a discontinuous transition are good models to describe real glasses. The role of disorder is not essential and can be self-generated by the dynamics. These models show a thermodynamic transition (the ideal glass transition \( T_g \)) where the configurational entropy \( C^* \) collapses to zero and the EA order parameter jumps to a finite value. Concerning dynamics, these models are described by the mode coupling equations which are a good description of relaxational processes in real glasses at temperatures above but not too close to \( T_d \). The instability found at \( T_d \) is a mean-field artifact which should be wiped out by including activated processes over finite energy barriers. In this sense, mode coupling equations are genuine mean-field dynamical equations. The interested reader can find more details in [27].

![FIG. 2. Potential \( \mathcal{V}(q) \) for a spin glass model with a discontinuous transition. The different regimes are: \( T > T_d \) (continuous line), \( T = T_d \) (dotted line), \( T_g < T < T_d \) with complexity \( C^* = \min_{q > 0} \mathcal{V}(q) \) (dashed line), \( T = T_g \) with \( C^* = \mathcal{V}(q_{EA}^g) = 0 \) (dot-dashed line)

III. ISING SPIN GLASSES IN A TRANSVERSE FIELD

There is much recent interest in the study of quantum phase transitions (see the talks by T. Kirkpatrick, R. Oppermann and H. Rieger in this proceedings). These transitions appear at zero temperature when an external
parameter is varied. For a certain critical value of this parameter the system enters into the disordered phase. The general problem can be put in the following way. Let us consider the following Hamiltonian,
\[ H = \mathcal{H}_0 + \epsilon \mathcal{P} \]  \hspace{1cm} (5)
where \( \mathcal{H}_0 \) stands for the unperturbed Hamiltonian and \( \mathcal{P} \) is a perturbation which does not commute with \( \mathcal{H}_0 \), i.e. \( [\mathcal{H}_0, \mathcal{P}] \neq 0 \) and \( \epsilon \) denotes the strength of the perturbation. Let us suppose that the system for \( \epsilon = 0 \) is in an ordered phase at \( T = 0 \). As the control parameter \( \epsilon \) is varied and the strength of the perturbation \( \mathcal{P} \) increases, the new ground state of \( H \) becomes a mixture of all the eigenstates of \( \mathcal{H}_0 \) and this tends to disorder the system. For a certain value of the control parameter \( \epsilon \) the systems fully disorders. The effect of the control parameter \( \epsilon \) in quantum phase transitions is rather similar to the effect of temperature in classical systems. The difference is that now quantum effects are non dissipative while thermal effects are.

In the realm of disordered systems it is essential to understand the role of disorder in quantum phase transitions. The rest of this talk will be devoted to discuss this problem in the framework of disordered mean-field spin glass models as presented in the previous section. Let us discuss how the classical glassy scenario is modified in the presence of quantum fluctuations. This problem has been already addressed in the literature, in particular the question whether replica symmetry breaking survives to the effect of quantum fluctuations. Large amount of work have been devoted to the study of the Sherrington-Kirkpatrick Ising spin glass in a transverse field \(^{[28–32]}\). The model is defined by,
\[ H = \mathcal{H}_0 + \Gamma \mathcal{P} = - \sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^z \]  \hspace{1cm} (6)
where \( \sigma_i^z, \sigma_i^x \) are the Pauli spin matrices and \( \Gamma \) is the transverse field which plays the role of a perturbation. The indices \( i, j \) run from 1 to \( N \) where \( N \) is the number of sites. The \( J_{ij} \) are random variables Gaussian distributed with zero mean and 1/\( N \) variance. Note that for \( \Gamma = 0 \) this model reduces to the classical Sherrington-Kirkpatrick model which has a continuous transition to a replica broken phase with local spin-glass ordering in the \( z \) direction. The effect of the transverse field is to mix configurations and to suppress the order in the \( z \) direction. For a critical value of \( \Gamma \) the ordering in the \( z \) direction is completely suppressed at the expense of ordering in the \( x \) direction. Note that the effect of a perturbation in the \( z \) direction in the form of a longitudinal magnetic field \( P = - \sum_i \sigma_i^z \) has a quite different effect on the phase transition. The reason is that it commutes with the unperturbed Hamiltonian, hence it does not mix configurations.

The phase diagram of the model eq.(6) is reproduced in figure 3. There are two phases, the quantum paramagnetic (QP) and the quantum glass phase (QG). The transition is continuous all along the phase boundary and the QG phase resembles the classical one where replica symmetry is broken and a large number of pure states, with essentially the same free energy, contribute to the thermodynamics \(^{[1]}\). A large body of information on the SK model with a transverse field has been obtained from spin summation techniques \(^{[31]}\) and perturbative expansions \(^{[34]}\). But only very recently has a full understanding of this model been achieved through seminal work by Miller and Huse \(^{[33]}\) and in an independent way by Ye, Read and Sachdev \(^{[34]}\). They have been able to obtain the frequency response of the system as well as the crossover lines which separate regimes where thermal or quantum fluctuations are dominant. The values of the critical exponents as well as the nature of finite size corrections in the quantum critical point have been also numerically checked in \(^{[35]}\).

FIG. 3. Phase diagram for the SK model in a transverse field. The critical boundary separates the quantum paramagnetic phase (QP) from the quantum glass phase (QG).

In this framework one would like to understand the role of quantum fluctuations in systems with a discontinuous transition. In particular it is relevant to understand how quantum fluctuations could modify the dynamical instability \( T_d \) obtained in classical systems within the mode coupling approach. Note that at zero temperature the entropy must vanish and the idea of an entropy crisis as the transverse field \( \Gamma \) is varied is nonsense. On the other hand, since the statics and dynamics are inextrica-

\(^{†}\)The extensive free energies of the different solutions only differ by finite - i.e. non extensive- quantities.
bly linked in quantum phase transitions it is interesting to ask to what extent a dynamical instability (which is not related to any static singularity) can survive at zero temperature.

In this proceedings I want to discuss some recent results in a general family of solvable models which strongly suggest that the dynamical instability predicted in the mode coupling approach is completely suppressed at zero temperature when quantum fluctuations are taken dominant [30].

In particular we will show, always within this class of models, that the discontinuous transition becomes continuous at zero temperature. Particular results will be shown for the ROM (random orhogonal model). The implications of this and other results will be also discussed.

A. Mean-field models with pairwise interactions

The family of exactly solvable models we are interested in are quantum Ising spin glasses with pairwise interactions in the presence of a transverse field. These are described by the Hamiltonian,

$$\mathcal{H} = -\sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x - \Gamma \sum_i \sigma_i^z$$  \hspace{1cm} (7)

where $\sigma_i^x, \sigma_i^z$ are the Pauli spin matrices and $\Gamma$ is the the transverse field. The indices $i, j$ run from 1 to $N$ where $N$ is the number of sites. $J_{ij}$ are the couplings taken from an ensemble of random symmetric matrices.

Details about how to analytically solve the quantum model [3] can be found in [34]. This are based on matrix theory techniques introduced in [7] to solve glassy models without disorder. Here we want we present some of the main results of the analysis of [36]. In practice the simplest way to solve the model eq.(7) is by means of the replica trick where we compute the average over the disorder of the $n$-th power of the partition function making the analytical continuation $n \to 0$ at the end,

$$\beta f = \lim_{n \to 0} \frac{\mathbb{Z}^n_f - 1}{n}$$  \hspace{1cm} (8)

where

$$\mathbb{Z}^n_f = \int [dJ] \text{Tr} e^{\frac{n}{2} \mathcal{H}^n}$$  \hspace{1cm} (9)

and $\int [dJ]$ means integration over the random ensemble of matrices. This integral can be done using known methods in matrix theory [33,36]. The final result of eq.(8) can be written in terms of a generating function $G(x)$ which depends on the particular ensemble of $J_{ij}$ couplings via its spectrum of eigenvalues. For the two examples we will consider here we have $G_{SK}(x) = \frac{x^2}{2}$ (SK model) and $G_{ROM}(x) = \frac{1}{2} \log(\frac{\sqrt{1+4x^2} - 1}{2x}) + \frac{1}{2} \sqrt{1 + 4x^2} - \frac{1}{2}$ (ROM model).

By going to imaginary time and using the Trotter-Suzuki breakup we end up with a closed expression for the free energy. The final result is,

$$\mathbb{Z}^n_f = \int dQ \text{d} \Lambda \exp(-NF(Q, \Lambda))$$  \hspace{1cm} (10)

where

$$F(Q, \Lambda) = -\frac{nC}{N} + \frac{1}{M^2} \text{Tr}(Q\Lambda) - \frac{1}{2} \text{Tr}G(AQ) - \log(H(\Lambda))$$  \hspace{1cm} (11)

and the free energy is obtained by making the analytic continuation $\beta f = \lim_{n \to 0} \frac{F(Q^*, \Lambda^*)}{n}$ where $Q^*, \Lambda^*$ are solutions of the saddle point equations, $\Lambda^{tt'}_{ab} = \frac{\Lambda M^2}{2} \langle G'(AQ) \rangle^{tt'}_{ab}$ and $Q_{ab}^{tt'} = \langle \sigma_a^t \sigma_b^{t'} \rangle$. The average $\langle \cdot \rangle$ is done over the effective Hamiltonian in [12]. For $a = b$ we have translational time invariance and the order parameter becomes independent of the replica index, i.e $Q_{aa}^{tt'} = R(|t - t'|)$.

Once we have written a closed expression for the free energy eq.(11), one can obtain the static and dynamical transition temperatures according to eq.(12). Such a solution always exists for models with a single quantum paramagnetic phase. In particular, using eq.(3), a closed expression for the dynamical instability can be obtained (see [30]). For a continuous transition this equation can be written in the simple form

$$\chi_0^2 G''(\chi_0) = 1$$  \hspace{1cm} (13)

where $\chi_0 = \beta R_0$ is the longitudinal magnetic susceptibility and $R_0 = M^{-1} \sum_{t=0}^{M-1} e^{-\omega t} R_t$ is the Fourier transformed order parameter $R(t)$ in terms of the Matsubara frequencies $\omega_p = \frac{2\pi}{t'}$. Our main aim is to compute the order of the transition. We already know that some models within the family eq.(3) (for instance, the ROM) have a classical discontinuous transition with a dynamical instability above the
static transition. Is the discontinuous nature of this transition changed in the presence of quantum fluctuations?

To answer this question we consider the static approximation introduced by Bray and Moore in the context of quantum spin glasses [28]. This approximation considers $R(t - t')$ to be constant which amounts to take into account only the zero frequency behavior $p = 0$ (small energy fluctuations) in the set of order parameters $R_p$. We will later comment on the validity of this approximation.

Using this approximation one can write closed expressions for the paramagnetic free energy $f_p$ and the complexity $C$. It is found [34] that at $T = 0$ the dynamical transition $T_d$ and the static transition $T_s$ coincide and the complexity vanishes. The transition then becomes continuous. The value of the critical field and all the thermodynamic observables at the critical point at zero temperature can be expressed in terms of the longitudinal susceptibility $\chi_0$ which satisfies the following simple set of equations,

$$\chi_0^2 G''(\chi_0) = 1 ; \quad \Gamma - \frac{1}{\chi_0} = G'(\chi_0);$$  \hspace{1cm} (14)

Note that the first of eq. (14) can be obtained deriving the second of eq. (14) with respect to $\chi_0$. At the critical point the internal energy is given by $U = -\Gamma_c$ and the entropy $S = \frac{1}{T_c} G'(\chi_0) + 1 - \chi_0 + log(\chi_0)$). Above the critical point, always at zero temperature, the second of eq. (14) is still valid and yields the susceptibility as a function of $\Gamma$.

To go beyond the static approximation we should consider all the Matsubara modes $R_p$ in the saddle point equations. The difficulty of this problem is similar to that found in strongly correlated systems where an infinite set of parameters has to be computed in a self-consistent way [38]. Nevertheless we expect the order of the transition to be correctly predicted. The essential idea is that for a continuous transition at zero temperature the gap vanishes. It would be quite surprising that higher frequency modes could drastically modify the low frequency behavior. The order of the transition should not be determined by the decay of the correlation $R_t$ in imaginary time but for its infinite time limit which is the EA parameter at the transition point [2].

In the next subsection we analyze our results for the particular case of the ROM model and compare them with those obtained in case of the SK model.

**B. Results for the ROM**

As has been already said, the ROM has a classical discontinuous transition at zero transverse field where $q_{EA} = 0.962$ at the dynamical transition $T_d \simeq 0.134$ and $q_{EA} = 0.9998$ at the static transition $T_s \simeq 0.065$. At the static transition the configurational entropy or complexity $C^*$ vanishes and the total entropy is of order of $10^{-4}$. The locations of the static and dynamical transitions can be evaluated within the static approximation to obtain the results shown in figure 4. Both transition temperatures decrease as a function of the transverse field merging into the same point at zero temperature as one would expect for a continuous transition. In figure 5 we show the EA order parameter $q = <\sigma^2>$ as a function of $\Gamma$ as we move along the static ($q_{EA}^d$) and dynamical ($q_{EA}^s$) phase boundaries. Note that both EA order parameters $q_{EA}^d$ and $q_{EA}^s$ vanish at zero temperature like $T^2$, hence the jump in the order parameter dissapears at zero temperature.

![Figure 4: Phase boundaries $T_d(\Gamma)$ (lower line) and $T_s(\Gamma)$ (upper line) in the ROM in the static approximation. At zero transverse field $T_d \simeq 0.0646, T_s \simeq 0.1336.$](image-url)

By substituting the particular function $G(x)$ for the ROM model in the second of eq. (14) the susceptibility at zero temperature in the QP phase can be analytically obtained in the static approximation. One finds $\chi_0^{ROM} = \frac{1}{\Gamma_c - 1} \Gamma_c$ which diverges at the critical field $\Gamma_c = 1$. This is quite different to what is found in the SK model where $\chi_0^{SK} = \frac{\Gamma_c - \Gamma_c^2 - 1}{2}$ and is finite at the critical point $\Gamma_c = 2$. Note that in the static approximation the critical field is given by the maximum eigenvalue of the coupling matrix $J_{ij}$.
of this result for continuous quantum phase transitions
the critical field. To see clearly the general implications
of the instability at $T_d$, even in mean-field theory, could
come continuous at $T=0$ and there is no room for a
metastable glassy phase. We have argued in favour of this
result even beyond the static approximation. Particular
results have been presented for the ROM model where
it has been shown that some critical exponents at the
quantum phase transition should differ from the mean-
field exponents derived in the case of the SK model. It
is still too soon to understand the implications of this
result which deserves further investigation. The removal
of the instability at $T_d$, even in mean-field theory, could
be a general consequence of the non dissipative nature of
quantum processes. How general this result is for other
type of models remains an interesting open problem. In
this direction it would be very instructive to address the
problem presented here within the approach developed in
for the SK model as well as taking this research

FIG. 5. EA order parameter $q_{EA}^g$ (upper line) and $q_{EA}^d$
(lower line) in the ROM on the static and dynamical phase
boundaries boundaries as a function of the transverse field.
At zero transverse field $q_{EA}^g \approx 0.99983, q_{EA}^d \approx 0.961$. Both
$q_{EA}^g$ and $q_{EA}^d$ vanish linearly with $T^{\frac{\nu}{\omega}}$ at zero temperature.

Unfortunately, as we have said before, the static approx-
imation gives incorrect results for the thermody-
namic quantities. In particular, the entropy is finite at
zero temperature in the SK model $^{[12]}$ and infinite in the
ROM case. It is important to note that despite these
failures of the static approximation some exact results
can still be derived, in particular from $^{[12]}$, eq. ($^{[13]}$). When
the transition is continuous equation ($^{[13]}$) is exact. One
finds for the ROM model that the longitudinal suscep-
tibility $\chi_0$ diverges at the critical point. This is differ-
tent to what happens in the SK model where $\chi_0 = 1$ at
the critical field. To see clearly the general implications
of this result for continuous quantum phase transitions
we observe that $\chi_0$ is given by the decay in imaginary
time of the correlation function $R(t)$ via the relation,
$\chi_0 = \int_0^\beta R(t)dt$ (now the time $t$ has become a con-
tinuous variable in the $M \rightarrow \infty$ limit). In the SK model
the large time behavior of the $R(t)$ has been obtained
$^{[23],[24]}$. It is found that $R(t) \approx t^{-2}$ which at zero tem-
perature yields a finite value of the susceptibility. This
decay is necessarily slower in the case of the ROM model
where $\chi_0 = \infty$. Because the decay of $R(t)$ in imaginary
time is related to the quantum critical exponents via the
relation $R(t) \approx t^{-\frac{z}{d}}$ the $z$ exponent is probably not a
universal quantity and different mean-field models with
continuous transitions may have different exponents. For
the SK model $z = 2, \beta = 1, \nu = \frac{1}{4}$ yield the correct decay.

These results suggest that the value of the exponent $z$ is
larger than 2 in the ROM.

The exact computation of the critical exponents and
the full analysis of the problem beyond the static approx-
imation in the ROM remains an interesting open problem.

IV. CONCLUSIONS

In this talk I have reviewed some recent developments
in the theory of spin glasses which allow for a comparison
between glasses and spin glass models with a discontinu-
ous transition. I have stressed that the two most common
approaches to the glass transition, the thermodynamic
approach based on the Adam-Gibbs theory and the dy-
namical approach based on mode coupling theory, appear
quite naturally in the framework of mean-field spin
glass models with a discontinuous transition. I have also
stressed that disorder is not necessary to find spin-glass
behavior and a large family of non disordered mean-field
models indeed show spin-glass behavior. The connection
between disordered spin glasses and structural glasses, at
least at the mean-field level, appears to be fully justified.
Going beyond the mean-field level is a major open prob-
lem. It is well accepted that the dynamical instability at
$T_d$ is an artifact of the mean-field theory but it is unclear
if the entropy crisis survives in finite dimensions (see the
talk of S. Franz in these proceedings).

We have also discussed the role of quantum fluctua-
tions in glassy systems by studying the Ising spin glass in
a transverse field. In quantum phase transitions statics
and dynamics are inextricably linked. Then it is of rel-
ance to understand the role of complexity in non relax-
tional quantum dynamics. We have shown that the clas-
ical glassy scenario with a dynamical transition above
the thermodynamic transition is modified in the presence
of quantum fluctuations. This result has been obtained
in the framework of models with two spin interactions in
the presence of a transverse field. In models with a dis-
continuous finite temperature transition we have shown,
using the static approximation, that the transition be-
comes continuous at $T = 0$ and there is no room for a
metastable glassy phase. We have argued in favour of this
result even beyond the static approximation. Particular
results have been presented for the ROM model where
it has been shown that some critical exponents at the
quantum phase transition should differ from the mean-
field exponents derived in the case of the SK model. It
is still too soon to understand the implications of this
result which deserves further investigation. The removal
of the instability at $T_d$, even in mean-field theory, could
be a general consequence of the non dissipative nature of
quantum processes. How general this result is for other
type of models remains an interesting open problem. In
this direction it would be very instructive to address the
problem presented here within the approach developed in
for the SK model as well as taking this research
further by studying the zero temperature dynamical transition in quantum $p$-spin glass models \[39\] and Potts glass models \[40\].

Acknowledgements. I am very grateful to the following colleagues for fruitful collaborations in this and related subjects: J. V. Alvarez, S. Franz, D. Lancaster, E. Marinari, Th. M. Nieuwenhuizen, F. G. Padilla and G. Parisi. I acknowledge to the Foundation for Fundamental Research of Matter (FOM) in The Netherlands for financial support through contract number FOM-67596.

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