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Precise damping and stiffness extraction in acoustic driven cantilever in liquid

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In this paper, we first explain how to extract accurately the driving force acting on the acoustic driven atomic force microscope cantilever in liquid from the measured resonance curve. We present a model that includes the driving force to extract precisely the damping and the stiffness of the tip sample interaction. The model is validated by an experimental test based on two independent methods to measure the hydrodynamic drag coefficient of a sphere moving perpendicular to flat surface.

I. INTRODUCTION

The atomic force microscope (AFM) provides a sensitive sensor to investigate materials properties at nanometer scale. The surface topography and various properties of organic and inorganic surfaces have been obtained with high resolution in vacuum, in air and in liquid. The dynamic AFM is widely used to image soft material especially when the deflection oscillation amplitude of the cantilever. Thus, the whole motion of the tip is the sum of the base displacement and the stiffness of the cantilever force constant, and \( \gamma_{\text{bulk}} \) is the bulk damping coefficient far from the surface.

In the ideal situation, the driving force in liquid medium \( F_{\text{drive}} \) induced by the displacement of the cantilever base can be calculated using the Euler-Bernoulli beam theory. The analytical expression of the driving force is

\[
F_{\text{drive}} = (m^* \omega^2 - j \omega \gamma_{\text{bulk}}) \beta A_b e^{i \phi_{\text{out}}}, \tag{2}
\]

where \( m^* \) is the effective mass of the cantilever, \( k_c \) is the cantilever force constant, and \( \gamma_{\text{bulk}} \) is the bulk damping coefficient far from the surface.

Furthermore, the piezo actuator transmits vibration to the surrounding fluid, which then applies an additional force to the cantilever. This makes a challenge and complicates the extraction of the interaction properties: force, damping, and stiffness.

In this work, we first explain how to extract accurately the driving force acting on the cantilever in liquid from the measured resonance curve. Then, we present a model that includes the driving force to extract precisely the damping and the stiffness of the tip sample interaction.

II. THEORETICAL MODELING

Fig. 1 shows the model for a cantilever excited acoustically by a piezo actuator. For a cantilever excited at frequency \( \omega \), the deflection of the tip \( X(t) \) is described by the oscillator model equation:

\[
m^* \ddot{X} + \gamma_{\text{bulk}} \dot{X} + k_c X = F_{\text{drive}}, \tag{1}
\]

where \( m^* \) is the effective mass of the cantilever, \( k_c \) is the cantilever force constant, and \( \gamma_{\text{bulk}} \) is the bulk damping coefficient far from the surface.

By substituting expression (3) in (1) and using \( X(t) = A_{\text{free}} e^{(i \omega \tau + \phi_{\text{free}})} \), we get

\[
F_1 \quad k_c \quad A_{\text{free}} \quad \left[ 1 - \left( \frac{\omega}{\omega_0} \right)^2 \cos(\phi_{\text{free}}) - \left( \frac{\omega}{\omega_0 Q} \right) \sin(\phi_{\text{free}}) \right], \tag{4a}
\]

\[
F_2 \quad k_c \quad A_{\text{free}} \quad \left[ 1 - \left( \frac{\omega}{\omega_0} \right)^2 \sin(\phi_{\text{free}}) + \left( \frac{\omega}{\omega_0 Q} \right) \cos(\phi_{\text{free}}) \right]. \tag{4b}
\]
In practice, the knowledge of the driving forces and the slip for the liquid on both surfaces; this assumption is valid for a sphere. In the above equation, we have assumed no boundary condition at the surface and the sphere, and \( v \) is the velocity of the fluid. In our experiment because we have used a hydrophilic glass sphere and mica substrate. For a sphere approaching to the surface with a constant velocity (drainage experiment), the dynamic viscosity of the fluid, \( D \) is the distance between the surface and the sphere, and \( V \) is the velocity of the sphere. In the above equation, we have assumed no boundary slip for the liquid on both surfaces; this assumption is valid in our experiment because we have used a hydrophilic glass sphere and mica substrate. For a sphere approaching to the surface with a constant velocity (drainage experiment), the dynamic viscosity of the fluid, \( D \) is the distance between the surface and the sphere, and \( V \) is the velocity of the sphere. In the above equation, we have assumed no boundary slip for the liquid on both surfaces; this assumption is valid for a sphere. In the above equation, we have assumed no boundary condition at the surface and the sphere, and \( v \) is the velocity of the fluid. In our experiment because we have used a hydrophilic glass sphere and mica substrate. For a sphere approaching to the surface with a constant velocity (drainage experiment), the dynamic viscosity of the fluid, \( D \) is the distance between the surface and the sphere, and \( V \) is the velocity of the sphere. In the above equation, we have assumed no boundary slip for the liquid on both surfaces; this assumption is valid in our experiment because we have used a hydrophilic glass sphere and mica substrate. For a sphere approaching to the surface with a constant velocity (drainage experiment), the dynamic viscosity of the fluid, \( D \) is the distance between the surface and the sphere, and \( V \) is the velocity of the sphere. In the above equation, we have assumed no boundary slip for the liquid on both surfaces; this assumption is valid.
length = 200 μm thickness = 0.8 μm). We have used spherical borosilicate particle (GL0186B/45-53, MO-Sci Corporation) with a diameter of 102.8 μm. The sphere was glued to the end of the cantilever using epoxy (Araldite, Bostik, Coubert) (Fig. 2). The substrate is fixed on multi-axis piezo-stage (NanoT series, Mad City Labs) that allows a large displacement (Up to 50 μm for applied voltage of 10 v) with a high accuracy under closed loop control. The amplitude and phase of the tip were measured using a lock-in amplifier (Signal Recovery Model 7280) with an integration time of 100 ms.

First, we perform a drainage experiment in order to measure the hydrodynamic drag coefficient and to calibrate in situ the cantilever with the sphere attached to its end. This step was performed in contact mode (the acoustic excitation was switched off). The force was generated by the rapid approach (or withdraw) of the mica substrate to the sphere using the piezo-stage. To analyze the recorded data, in this part, we have followed the procedure given by Honig and Ducker and Zhu et al. The measured deflection for the approach and retract is shown in Fig. 3(a).

The deflection of the cantilever (def) is related to the force (F_h) by: $\frac{\text{def}}{V} = \frac{F_h}{kcV} = \frac{6\pi \eta g R^2}{kcD}$. The actual velocity of the approach (retract) was obtained from the time derivative of the distance. From the fit shown in Fig. 3(b), we obtain $\frac{6\pi \eta g R^2}{kc} = 0.261 \text{ nm/s}$, using the numerical values of the sphere radius $R = 51.4 \mu m$ and the water viscosity $\eta = 0.89 \text{ mPa s}$, we get the stiffness value $k_c = 0.17 \text{ N/m}$. Note that the curve plotted in Fig. 3(b) is no more than the hydrodynamic drag coefficient divided by the stiffness $\frac{F_h}{k_c}$.

Now, we perform experiment in dynamic mode to measure the hydrodynamic interaction damping $\gamma_{int}$. First, we need to measure the exact value of the driving force acting on the tip. This force is determined from the resonance curves of the cantilever in bulk free from any interaction. Fig. 4 shows the resonance curves of the cantilever with the sphere attached measured by thermal response and acoustic excitations. The fit of the thermal response curve (Fig. 4(a)) using the standard harmonic oscillator model allows to extract the resonance frequency $\omega_0$ and the quality factor $Q$ of the cantilever far from any interaction. Then, we use Eqs. (4a) and (4b) to calculate the driving force $F_1$ and $F_2$ shown in Fig. 4(c).

Then, we vibrate the cantilever at a given frequency and measure the amplitude and phase of the tip as the substrate approaches the sphere glued at the end of the cantilever with a very low rate. On Fig. 5(a) is shown the amplitude of the cantilever versus the distance for different frequencies of vibration. Close to the contact position, the damping is
infinite, and thus as expected from Eq. (5) \( A = A_b \) when \( \gamma_{\text{int}} \to \infty \), the measured cantilever deflection amplitude for all frequencies is equal to the base vibration amplitude. The amplitude of the base vibration was determined in this experiment for the vibrating frequencies 100, 200, and 1000 Hz to be 1.86, 2.04, and 2.06 nm, respectively.

After measuring the driving force, the quality factor \( Q \), the resonance frequency \( \omega_0 \), and the base amplitude \( A_b \), we can apply Eq. (7a) to extract the interaction damping versus the distance from the measured amplitude and phase.

Fig. 6 shows the damping \( \gamma_{\text{int}} \) versus the distance \( D \) for different frequencies extracted from the data of the amplitude and phase plotted in Fig. 5(a) and Fig. 5(b). Note here that damping extracted for different vibration frequencies coincide with each other. We have also reported on this figure, the hydrodynamic drag coefficient \( \gamma_h = \frac{F_h}{V} \) measured using the contact mode. As expected, the drag coefficient \( \gamma_h \) measured with a contact mode coincides with the damping coefficient \( \gamma_{\text{int}} \) measured with the dynamic mode.
IV. CONCLUSION

In summary, we have shown how to extract accurately the acoustic driving force acting on the cantilever in liquid from the measured resonance curve. We have presented a model that includes the driving force to extract the damping and the stiffness of the tip sample interaction. Finally, experimental measurements were performed to validate the model.

The three steps of our method are recalled below:

1. Measure the quality factor and resonance frequency from the thermal response of the cantilever (actually, this operation is available in most new AFMs).
2. Measure the acoustic driven resonance spectra of the cantilever and then extract the driven force using Eqs. (4a) and (4b).
3. Use Eqs. (7a) and (7b) to convert amplitude and phase data to damping and stiffness.

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APPENDIX: CONTRIBUTION OF THE ADDITIONAL FLUID ADDED MASS AND ADDED DAMPING CLOSE TO THE SURFACE

During the interaction to describe the cantilever motion, we have to add to the second term of Eq. (1), all the contributions due to the interaction.

The equation of motion can be written as

\[ m' \dddot{X} + \gamma_{\text{bulk}} \ddot{X} + k_c X = (F_1 + jF_2)e^{i\omega t} - k'_{\text{int}} Z - \gamma'_{\text{int}} \dot{Z} - \gamma'_{\text{add}} \dot{Z} - m'_{\text{add}} \ddot{Z}, \]  

(A1)

where \( k'_{\text{int}}, \gamma'_{\text{int}} \) are, respectively, the interaction stiffness and damping due to pure tip-sample interaction and \( m'_{\text{add}}, \gamma'_{\text{add}} \) are the additional fluid mass and damping due to the cantilever beam vibration close to the surface. Using the expressions of the tip deflection \( X(t) = A e^{i(\omega t + \phi)} \) and total tip displacement \( Z = X(t) + A_x e^{i\phi} \), we get

\[
-k'_{\text{int}} Z - \gamma'_{\text{int}} \dot{Z} = - (k'_{\text{int}} - m'_{\text{add}} \omega^2) Z - (\gamma'_{\text{int}} + \gamma'_{\text{add}}) \dot{Z} = -k_{\text{int}} Z - \gamma'_{\text{int}} \dot{Z}.
\]  

(A2)

With \( k_{\text{int}} = k'_{\text{int}} - m'_{\text{add}} \omega^2 \) and \( \gamma'_{\text{int}} = \gamma'_{\text{int}} + \gamma'_{\text{add}} \). By substituting Eqs. (A2) in (A1), we get Eq. (5) of the text

\[ m' \dddot{X} + (\gamma_{\text{bulk}} + \gamma_{\text{int}}) \ddot{X} + (k_c + k_{\text{int}}) X = (F_1 + jF_2)e^{i\omega t} - (k_{\text{int}} + j\omega \gamma_{\text{int}})A_x e^{i\phi}. \]