A Quark-Antiquark Condensate in Three-Dimensional QCD

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\abstract

Three-dimensional lattice QCD is studied by Monte Carlo simulations within the quenched approximation. At zero temperature a quark-antiquark condensate is observed in the limit of vanishing quark masses. The condensate vanishes continuously at the finite-temperature deconfinement phase transition of the theory. A natural interpretation of this phenomenon in the full theory with dynamical quarks is in terms of the spontaneous flavor symmetry breaking $U(N_f) \rightarrow U(N_f/2) \times U(N_f/2)$. In addition, the spectrum of low-lying Dirac operator eigenvalues is computed and found to be consistent with a flat distribution at zero temperature, in agreement with analytical predictions.
Although super-renormalizable, non-Abelian gauge theories in \((2+1)\) dimensions are notoriously difficult to analyze in perturbation theory due to the strong infrared divergences that appear already at leading orders. For this reason, a very appropriate framework for the study of such three-dimensional theories is that of lattice gauge theory \([1]\). In this letter we shall report on a series of Monte Carlo simulations aimed at clarifying the issue of flavor symmetry breaking and, at high temperature, flavor symmetry restoration in three-dimensional QCD.

One striking difference between massless \((2+1)\)-dimensional QCD and its \((3+1)\)-dimensional counterpart is the absence of chiral symmetry. The lowest representation of the Clifford algebra in three dimensions has dimension two. Spinors are thus naturally taken to be two-spinors, and as \(\gamma\)-matrices one can choose the three Pauli matrices \(\sigma_i\). On the surface there seems to be no room for an additional analogue of \(\gamma_5\), and hence no chiral symmetry. A fermionic mass term does, however, break parity \(P\) and time-reversal \(T\) symmetries \([3]\), symmetries that are both respected by the massless Lagrangian. By introducing an \(\textit{even}\) number of quark species \(N_f\), and combining the spinors in pairs into four-spinors \(\psi\), one can rewrite the fermionic part of the Lagrangian entirely in terms of three 4-dimensional \(\gamma\)-matrices \(\gamma_0, \gamma_1, \gamma_2\) and the four-spinors \(\psi\). A mass term of the kind \(m \bar{\psi}\psi\) can now be introduced \([2, 3]\). Such a mass term is \(P\) and \(T\) invariant. It does, however, break both of the “chiral” symmetries associated with \(\gamma_4\) and \(\gamma_5\). For this reason, a very appropriate framework for the study of such three-dimensional theories is that of lattice gauge theory \([1]\). In this letter we shall report on a series of Monte Carlo simulations aimed at clarifying the issue of flavor symmetry breaking and, at high temperature, flavor symmetry restoration in three-dimensional QCD.

It has been suggested that a plausible flavor symmetry breaking in the theory with an even number of flavors \(N_f\) should be that of \(U(N_f) \rightarrow U(N_f/2) \times U(N_f/2)\) \([1]\). The formation of a chiral condensate \(\langle \bar{\psi}\psi \rangle\) would be a signal for such spontaneous symmetry breaking. If correct, it would open up the possibility of comparing the detailed universality predictions \([4, 5]\) for the Dirac operator spectrum around eigenvalues near \(\lambda = 0\) with lattice Monte Carlo data, as has recently been done in the case of four-dimensional lattice QCD \([6]\). The first step in such a program is to establish the existence of the chiral condensate \(\langle \bar{\psi}\psi \rangle\).

The transcription of the formulation in terms of four-spinors and four-dimensional \(\gamma\)-matrices to a lattice theory is quite naturally done in terms of staggered fermions \([6]\). Interpreting three-dimensional staggered fermions in this manner has already been explored in several Monte Carlo analyses, see \(e.g.\) ref. \([7]\). The standard three-dimensional action for free staggered fermions, keeping the lattice spacing \(a\) explicitly,

\[
S_F + S_M = -\frac{1}{2} a^{d-1} \sum_{r,\mu} (-1)^{r_1+r_2+\ldots+r_{\mu-1}} [\chi(r) \chi(r+\mu) + \bar{\chi}(r+\mu) \chi(r)] + im a^d \sum_r \chi(r) \chi(r) \tag{1}
\]

can be transformed into a form with 2-component Dirac fields. In the limit \(a \rightarrow 0\) it will go to the continuum action for free Dirac fermions with two flavors and (if the kinetic terms is defined with respect to the same \(\gamma\) matrices; see below) masses \(m\) and \(-m\). In three dimensions there are two equivalence classes of irreducible representations of the Clifford algebra, \(\{\gamma_\mu\}\) and \(\{-\gamma_\mu\}\equiv \{\beta_\mu\}\).

Following ref. \([7]\) the new action can be defined on a lattice with twice the original lattice spacing:

\[
S_F + S_M = (2a^d) \sum_{r,\mu} \left\{ [\bar{\chi}(\gamma_\mu \otimes I) \partial_\mu u + \bar{d}(\beta_\mu \otimes I) \partial_\mu d + a[\bar{\chi}(I \otimes \gamma_\mu^T) \partial_\mu^2 d + \bar{d}(I \otimes \beta_\mu^T) \partial_\mu^2 u] + im[\bar{\chi}(I \otimes I) u + \bar{d}(I \otimes I) d] \right\} \tag{2}
\]
Here \( u \) and \( d \) are 2-component spinors and in the quark bilinears the first \( 2 \times 2 \) matrix acts on spinor indices while the second acts on flavor indices. One can identify a \( U(1) \times U(1) \) symmetry in eq. (2) as

\[
\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow e^{i \theta_1} \begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} \pi, \bar{\pi} \end{pmatrix} \rightarrow \begin{pmatrix} \pi, \bar{\pi} \end{pmatrix} e^{-i \theta_1},
\]

(3)

\[
\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} \cos \theta_2 & i \sin \theta_2 \\ i \sin \theta_2 & \cos \theta_2 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} \pi, \bar{\pi} \end{pmatrix} \rightarrow \begin{pmatrix} \pi, \bar{\pi} \end{pmatrix} \begin{pmatrix} \cos \theta_2 & i \sin \theta_2 \\ i \sin \theta_2 & \cos \theta_2 \end{pmatrix}.
\]

(4)

The symmetry in eq. (3) is a remnant of the three-dimensional “chiral symmetry” on the lattice, and it is broken by the inclusion of a mass term. The symmetry of eq. (3) is nothing but fermion number conservation.

We consider gauge group \( SU(3) \), and add to the fermionic part of the action the conventional Wilson gauge action (\( \beta \equiv 6/g^2 \)):

\[
S_{\text{gauge}} = \beta \sum_p (1 - \frac{1}{3} \text{Re} \; \text{Tr} U_p),
\]

(5)

where \( U_p \) is the product of links along a fundamental plaquette. The canonical ensemble of gauge field configurations was generated using a combination of microcanonical over-relaxation (MOR) \footnote{The analogous phenomenon for \( SU(2) \) (where it just amounts to a sign, and in fact can effectively flip the fermion boundary conditions from antiperiodic to periodic), was already pointed out in ref. \cite{15}.} and quasi-heat-bath (QHB) algorithms \cite{10}. An individual MOR or QHB step consisted of updating consecutively the three \( 2 \times 2 \) submatrices of the \( SU(3) \) link variable \cite{11}. In our implementation five MOR steps were followed by two QHB ones.

We now present the results of our Monte Carlo simulations. Instead of first considering symmetric \((2+1)\)-dimensional lattice volumes, we have immediately turned to lattices with the temporal direction much smaller than the two spatial directions. This has allowed us to study the finite-temperature behavior of the theory, while at the same time determining, in the low-temperature region, whether at all a quark-antiquark condensate is formed at zero temperature. As mentioned in the introduction, all simulations have been performed in the quenched approximation. This should have no bearing on the qualitative features as compared to the full theory with dynamical quarks, at least as long as the number of flavors is relatively small. We have monitored all relevant gauge-invariant correlation functions in both the matter and gauge sector, and in particular of course the crucial order parameters: the Polyakov line \( \langle W \rangle \) and the chiral condensate \( \langle \bar{\psi} \psi \rangle \). Since the gauge dynamics is unaffected by the fermions in this approximation, the theory should undergo a 2nd-order finite-temperature deconfining phase transition at an intermediate \( \beta \)-value \cite{12}. This particular detail may of course be modified by the presence of (light) dynamical quarks (just as its four-dimensional counterpart), but the qualitative feature of a switch from confining to deconfined characteristics should persist.

The finite-temperature phase transition in the gauge sector is associated to global \( Z(3) \) symmetry breaking. The Polyakov line aligns, in the deconfined phase, along the three complex roots of \( z^3 = 1 \) on the unit circle, \( i.e. \) along \( z \in \{1, \exp[2i\pi/3], \exp[4i\pi/3]\} \). In the theory with dynamical quarks, these act like a magnetic field that force the spontaneous symmetry breaking on the Polyakov line to occur on the physical real axis (\( i.e. \) along \( z=1 \)) \cite{13}, but in this quenched situation there is equal probability for the Polyakov line to fall into any of the three sectors. This is normally not a question of great concern, since in finite-volume simulations one in any case needs a modified working definition of the Polyakov line expectation value, which customarily is based on taking its absolute value. But as has recently been emphasized \cite{14}, the existence of complex phases in the Polyakov line condensate can have strong effects on the chiral condensate in quenched simulations.\footnote{The analogous phenomenon for \( SU(2) \) (where it just amounts to a sign, and in fact can effectively flip the fermion boundary conditions from antiperiodic to periodic), was already pointed out in ref. \cite{15}.}

We have seen this very clearly
in our simulations. In Figure 1 we display the (absolute value) of the Polyakov loop expectation value on a $40^2 \times 4$ lattice, as a function of the gauge coupling $\beta$. It shows the expected behavior of a smooth phase transition at an intermediate value of $\beta$, which we read off to be around $\beta \sim 15$. This is in good agreement with earlier Monte Carlo results \[16\] and mean field theory \[17\]. It is also in rough agreement with three-dimensional scaling, and earlier Monte Carlo results for a temporal extent of $N_T=2$ which gave a critical $\beta$-value of $\beta_c \approx 8.1$ \[12\]. If we re-instate the lattice spacing $a$, the definition of $\beta$ actually becomes $\beta \equiv 6/(g^2 a)$, that is, the coupling $g$ is dimensionful in three dimensions. With temperature given by $T = 1/(N_T a)$, perfect scaling in this theory would imply that for the physical critical temperature $T_c$ to stay fixed, the critical coupling $\beta_c$ should be $2 \times 8.1 = 16.2$ on our lattice with $N_T = 4$. Thus, although continuum scaling is not perfectly obeyed here, the deviation is nevertheless only around 5%.

While the behavior of the Polyakov line thus is completely as expected, the condensate $\langle \bar{\psi}\psi \rangle$ displayed at first a quite surprising behavior, which we eventually could attribute to the complex phases of the Polyakov line. Before presenting details on this particular issue, we first display, in Figure 2, a typical plot of our final results (here for a quark mass of $am_q = 0.1$). This figure, reminiscent of similar plots for SU(2) in 4 dimensions \[15\], suggests immediately that, in the limit of massless quarks, (1) a condensate is formed in the zero-temperature theory (corresponding to the phase with $\beta < \beta_c$), but not at high temperature – we shall confirm these observations with explicit extrapolations to the massless limit below – and (2) at the deconfinement transition this condensate approaches zero smoothly. Neither of these observations come as a surprise. First of all, the formation of a $\bar{\psi}\psi$-condensate at strong coupling is almost built in by the present lattice formulation. The strong-coupling, zero-temperature analysis of ref. \[18\] straightforwardly carries over to the present (2+1)-dimensional case, where it again predicts the formation of a chiral condensate.\footnote{The prediction is not as strong as in (3+1) dimensions, however, since one must view it in terms of a 1/d-expansion that quantitatively could fail for $d=3$.} Variational calculations in the Hamiltonian formalism \[19\] indicate the existence of the condensate also at finite couplings. Similarly, the extension to finite temperature \[20\] immediately predicts, at strong coupling, a symmetry-restoring phase transition at finite temperature.
Of course, details will depend on the quenched approximation. For instance, it is almost impossible to imagine that in the presence of a continuous deconfining phase transition the behavior of $\langle \bar{\psi}\psi \rangle$ could be anything but smooth in our case. To show that this is not what is found if one blindly measures $\langle \bar{\psi}\psi \rangle$ in this quenched simulation, we show in Figure 3 (top) a typical run history of that quantity.

For illustrative purposes we have here chosen $\beta = 14.6$ associated with $Z(3)$ breaking. Surprisingly, the chiral condensate displays what appears to be a clear-cut two-state signal, indicating the proximity of a discontinuous phase transition. The reason for this bizarre behavior becomes evident if we compare (see Figure 3 (middle)) with the corresponding time history of the (negative of the) real part of the Polyakov line, $\langle -\text{Re}W \rangle$. The apparent “two-state signal” appears to be reproduced in this variable, and in fact the data in the upper and middle part of Figure 3 are almost perfectly correlated. We recall that in the case of the Polyakov line, this behavior simply reflects the flipping around between the three different $Z(3)$ phases, and in no way implies a two-state signal in the properly defined Polyakov line. Furthermore, the anti-correlated behavior between $\langle \text{Re}W \rangle$ and $\langle \bar{\psi}\psi \rangle$ is in complete agreement with the observations of ref. [14]: When the Polyakov line lies along one of its two complex phases (and its real part hence is relatively small), the chiral condensate has an unphysical larger value than in the physical situation corresponding to a Polyakov line aligned along the real axis.

The problem described above can be solved in different ways. We have opted for the following simple method. The phase $\phi$ of $\langle W \rangle$ is measured. Then, if $\phi > \pi/3$, all the timelike link variables in the $t = 0$ time slice are multiplied by $\exp(-2\pi i/3)$. If $\phi < -\pi/3$, these variables are multiplied by $\exp(2\pi i/3)$. If we find $-\pi/3 \leq \phi \leq \pi/3$, the configuration is left unchanged. Thus, for the transformed configuration, the phase of $\langle W \rangle$ lies between $-\pi/3$ and $\pi/3$. The transformation as described is an exact symmetry of the quenched theory. Nevertheless, we prefer to discard the transformed configuration after performing measurements, and to continue the Monte Carlo process with the original configuration. In the bottom part of Figure 3 we show how the time history of $\langle \bar{\psi}\psi \rangle$ looks with this prescription. We have here used precisely the same starting configuration, and yet the result is radically different. There is now no trace of any metastability, and $\bar{\psi}\psi$ is nicely fluctuating around one particular, stable, value. We have applied this prescription wherever we detected a two-state signal in the $\langle \bar{\psi}\psi \rangle$ time history. This resulted in a
Figure 3: The Monte Carlo time history, averaged over 5 consecutive iterations, of the quark-antiquark condensate $\langle \bar{\psi}\psi \rangle$ for $am_q = 0.1$ measured on the original configurations (top), of the negative of the real part of $\langle W \rangle$ (middle) and of $\langle \bar{\psi}\psi \rangle$ measured on the transformed configurations (bottom) at $\beta = 14.6$ on a $40^2 \times 4$ lattice.
Figure 4: The quark-antiquark condensate $\langle \bar{\psi} \psi \rangle$ as a function of the quark mass $am_q$ at $\beta = 12.0$, corresponding to the low-temperature phase, for various volumes. Also shown is the value for the condensate at $am_q = 0$ in the infinite volume limit (square). 

While the data already shown indicate the presence of a non-vanishing quark-antiquark condensate at low temperature, we have performed a much more detailed analysis to really confirm this result. The problem of course is that for any finite volume $V$ there is no spontaneous symmetry breaking in the $am_q \to 0$ limit. Fortunately the behavior with increasing volume is, however, quite different in a phase of broken symmetry compared with a phase of restored symmetry. We can use this different behavior to test the hypothesis of spontaneous symmetry breaking in the confined phase of the theory. In Figure 4 we display data $\langle \bar{\psi} \psi \rangle$ for a variety of lattice volumes at $\beta = 12.0$ (corresponding to the confined phase). While the condensate eventually goes towards zero as $am_q \to 0$ for any of the lattices volumes, there is a clear trend towards increasing values, at fixed $am_q$, as the volume is increased. On the same figure we have also indicated the value of the condensate at $am_q = 0$ and extrapolated to $V = \infty$ as obtained from an entirely different analysis (see below). Considering the errors, this extrapolated value is completely consistent with the direct measurements at increasingly larger volumes, as shown in Figure 4. It finally remains to be tested if the behavior on the high-temperature side is consistent with a vanishing condensate in the limit of zero quark masses. In Figure 5 we show the convergence of $\langle \bar{\psi} \psi \rangle$ at $\beta = 17.0$. These data are clearly very different from those in the confined phase, with no discernible volume dependence. All points lie on a straight line that directly extrapolates to a vanishing condensate at $am_q = 0$.

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3The used prescription is, however, not entirely free of systematic error: to every transition between the physical and the unphysical values of $\bar{\psi} \psi$ in Figure 3 corresponds a spike in the refined value of $\bar{\psi} \psi$. As a result of these spikes, we expect the average $\langle \bar{\psi} \psi \rangle$ to be slightly biased upwards. This bias can be systematically reduced by increasing the lattice volume, thereby making the transitions between the phases of $\langle W \rangle$ increasingly rare.
We have shown that, at least in the quenched approximation and in our present latticized version, a chiral condensate is formed in (2+1)-dimensional QCD. This condensate disappears according to expectations at the finite temperature deconfinement phase transition at $T_c$. In the low-temperature phase it is thus possible to test the detailed predictions concerning the Dirac spectrum at small eigenvalues \[4, 5\]. In contrast to the situation in four-dimensional QCD, the prediction is here that the quenched microscopic spectral density should be completely flat.\[4\] To test this, we have performed a series of measurements of the lowest-lying Dirac operator eigenvalues, using the Ritz functional algorithm of ref. \[21\]. The density of low-lying eigenvalues at $\beta = 12.0$ is shown in Figure 6 for lattice sizes $8^3$ to $14^3$. Except for the largest lattice we computed the 10 lowest eigenvalues, on $14^3$ the 16 lowest ones. There is a clear trend towards a flat spectral distribution with increasing lattice volume. The dip very near $\lambda \sim 0$ is due to finite-size effects, and indeed this dip becomes more and more narrow as the volume is increased. While a small peak appears close to $\lambda = 0$, its significance is doubtful since it seems to be smallest on the largest lattice volume. In addition, it is not well correlated with the distribution of the smallest eigenvalue, as one would have expected from any genuine oscillatory behavior. From the flat plateaux we can extract an average value of the spectral density near the origin, i.e. $\rho(0)$. We find a slight volume dependence in the extracted values of $\rho(0)$. Extrapolating as $1/V$ to the infinite-volume limit we obtain $\rho(0) = 0.0315(19)$. Through a 3-dimensional analogue of the Banks-Casher formula this provides us with an independent measurement of the condensate: $\langle \bar{\psi} \psi \rangle = \pi \rho(0)$. Inserting the above value for $\rho(0)$, this gives $\langle \bar{\psi} \psi \rangle = 0.099(6)$, which is the value shown in Figure 4. It is consistent with our direct measurements. We have thus provided yet more independent confirmation of the formation of a condensate in the confined phase, while simultaneously testing the predictions about the microscopic spectral density of the Dirac operator \[4, 5\].

\[4\]This assumes that the quenched model with staggered quarks can be understood as the $N_f \to 0$ limit of the theory with an even number of quarks.

Figure 5: Same as Figure 4 but in the high-temperature phase, at $\beta = 17.0$. 

Figure 6: Density of low-lying eigenvalues at $\beta = 12.0$ for lattice sizes $8^3$ to $14^3$. Except for the largest lattice we computed the 10 lowest eigenvalues, on $14^3$ the 16 lowest ones. There is a clear trend towards a flat spectral distribution with increasing lattice volume. The dip very near $\lambda \sim 0$ is due to finite-size effects, and indeed this dip becomes more and more narrow as the volume is increased. While a small peak appears close to $\lambda = 0$, its significance is doubtful since it seems to be smallest on the largest lattice volume. In addition, it is not well correlated with the distribution of the smallest eigenvalue, as one would have expected from any genuine oscillatory behavior. From the flat plateaux we can extract an average value of the spectral density near the origin, i.e. $\rho(0)$. We find a slight volume dependence in the extracted values of $\rho(0)$. Extrapolating as $1/V$ to the infinite-volume limit we obtain $\rho(0) = 0.0315(19)$. Through a 3-dimensional analogue of the Banks-Casher formula this provides us with an independent measurement of the condensate: $\langle \bar{\psi} \psi \rangle = \pi \rho(0)$. Inserting the above value for $\rho(0)$, this gives $\langle \bar{\psi} \psi \rangle = 0.099(6)$, which is the value shown in Figure 4. It is consistent with our direct measurements. We have thus provided yet more independent confirmation of the formation of a condensate in the confined phase, while simultaneously testing the predictions about the microscopic spectral density of the Dirac operator \[4, 5\].

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Figure 6: The density of low lying eigenvalues for various volumes at $\beta = 12.0$. 
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