Fragmentation of shells: an analogy with the crack formation in tree bark

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ABSTRACT

How does a shell explode into a series of fragments upon impact? The well-accepted explanation is Mott’s theory, which considers the fragmentation of shells as a random process controlled by defects. However, Mott’s theory is inadequate due to its assumption of energy conversion, and it is incapable of explaining the lack of change in saturation fragment length with the increase in expansion velocity. In this paper, we present a theory to explain the physical mechanism for fragmentation of shells and propose a highly efficient model for predicting the number of necks after fragmentation. We recognise that the fragmentation problem in shells is analogous to the cracking behaviour of tree bark, and closed-form solutions is obtained to describe the relationship between the expansion velocity and the number of necks with consideration of the strain-rate-dependent strength of the shell material. The theoretical results show excellent correlation with the experimental results.

1. Introduction

Fragmentation of shells is a classic and elementary problem in dynamic fracture, corresponding to the phenomenon where a shell body is subjected to the force of external pressure, causing the body to fracture into multiple pieces. A simplified one-dimensional ring is generally introduced to investigate the failure behaviour during fragmentation of the shell as shown in Figure 1. Over the past 50 years, ring fragmentation tests have mainly been conducted using an electromagnetic loading scheme [1–3], which was designed by Niordson [1]. Recently, a liquid-driving experimental technology based on the split Hokinson pressure bar was...
developed for conducting expanding ring tests by Zhang et al. [4]. Figure 1(a) shows the rings before and after the expansion test [3].

The well-accepted mechanism behind the fragmentation of shells was proposed by Sir Nevill Mott [5–7]. As shown in Figure 1(b), Mott postulated that fracture begins at one of a number of ‘weak points’ distributed throughout the material; when instantaneous fracture occurs at one weak point, stress waves (Mott’s waves) propagate away from the fracture, releasing the stress in the neighbourhood. Within the region of the specimen subjected to the action of the released wave, the material does not continue to stretch; thus failure is precluded. The size of the fragments produced is then determined by the distance travelled by the Mott’s wave. Since it is impossible to determine the travel time for a Mott’s wave, Mott made a simple assumption that the surface energy required to generate new fracture surfaces is provided by the local kinetic energy of the fragments (an inference known as Mott’s energy-conversion assumption). Accordingly, a formula to determine the mean length of a fragment was proposed based on the law of energy conservation

\[ l = \left( \frac{24G_c r^2}{\rho V^2} \right)^{1/3} \]

where \( G_c \), \( r \), \( \rho \) and \( V \) are the fracture energy, radius of the ring, density and velocity of the radial expansion of the ring, respectively. Grady et al. [8–10] further popularised Mott’s theory between the early 1980s and the late 2010s. A major defect of Mott’s theory is the assumption of energy conversion, as it is inconsistent with the law of conservation of energy, which suggests that the work done by an explosive load should equal the sum of the energy required to generate a new fracture surface, the kinetic energy of the fragments and other dissipated energy. Moreover, Mott’s theory does not consider the strain rate effect, which is another important factor in the fragmentation of shells.
Zhang and Ravi-Chandar [2,11] revealed that the onset of fragmentation is caused by the development of a part of the nucleated necks; thus the number of necks formed exceeds the number of fragments, and the variance of the fragment length decreases with the increase in the expansion velocity, suggesting that its value becomes zero when the expanding velocity is sufficiently high. More recently, both experimental results [3,4,12,13] and finite element simulations [14,15] have shown that defects have a negligible effect on the average neck spacing and the increasing rate for the number of necks decreases gradually with the increase of applied velocity, both circumstances which cannot be explained by the statistical fragmentation theory of Mott [14]. This has raised a question about whether or not the localisation process becomes deterministic at high expansion velocities. At the present time, the physical mechanism of the fragmentation of shells remains unclear. In this paper, we present a novel theory to explain the physical mechanism for the fragmentation of shells and propose an efficient theoretical model for predicting the number of necks after fragmentation.

2. The physical mechanism of fragmentation of shells

The principles governing the fragmentation process can be summarised in the following three criteria: (1) cracks initiate in a structure at locations where stress exceeds the strength of the material, (2) stress is released as a result of fracture so that stress will fall below the strength of the material at other locations in the structure and (3) energy consumed by the formation of cracks or necks is minimised (i.e. the principle of least action). The second criterion promotes more cracks being generated, while the third criterion suggests the opposite. The compromise between the two competing criteria determines the number and size of fragments/necks produced.

Before studying the process of fragmentation of shells, it is helpful to first introduce a natural phenomenon: the cracking behaviour of tree bark, as shown in Figure 2(a). The mechanism of crack formation in tree bark is illustrated schematically in Figure 2(b–d). Since the growth rate of the heartwood in a tree is higher than that of the bark, an expansion strain develops in the heartwood as the tree grows. This expansion strain is then transferred to the bark through interfacial shear stress, based on the shear lag principle [16,17]. For a single crack island generated during the tree growth period, from centre to edge, the interfacial shear stress increases from zero to a peak value, while the normal tensile stress decreases from a peak value to zero due to the free-edge effect [18], as shown in Figure 2(d). When tensile stress at the centre point of a bark segment is slightly lower than the tensile strength, the length of this segment will satisfy the requirements of both the second and third fragmentation criterion at the same time.
Similar fracture or necking mode is commonly seen in other layered materials or layer-like structures [19–22], such as fibre-reinforced composites [23], painted structures [24], craquelure in ceramics [25], dry bed-like fracture of lithium-ion batteries [26], periodic crack patterns in the thermally loaded Mo/Si multilayers [27], shrinkage cracks developed during cooling [28], the freezing of water droplets as they impact a cold surface [29]. For the failure happens in a brittle material, multiple cracks are formed [19–23]. If the failure happens in a plastic material, then multiple necks will form, such as the stretchability of thin metal films on elastomer substrates [30–32].

The fragmentation process of a shell is analogous to the process of crack formation in layer-like structures, such as tree bark. The shell is equivalent to the bark of the tree, and the high-pressure gas produced by the explosion is equivalent to the growth of heartwood in the tree’s trunk. The high-pressure gas and the shell stick tightly to each other before fragmentation, which is analogous to the adhesion between the bark and heartwood during the growth of the tree. The growth expansion strain in the outermost layer of the heartwood is considered to

![Figure 2](image-url)
be equivalent with the hoop expansion strain for the outermost layer of the high-pressure gas. The main difference between the fragmentation of shells and the cracking of tree bark is that the first process happens rapidly, while the latter happens over a long period of time. In this work, we consider the fragmentation of shell problem analogising to a rapidly growing tree, which can be solved using the approach reported by McGuigan et al. [17].

3. Theoretical model for fragmentation of shells

A one-dimensional shell is generally introduced to study the fragmentation of the shells [2–15]. A schematic diagram showing the fragmentation process for a shell is shown in Figure 3(a–c). The original radius of shell with an initial radius $r$ and thickness $h$ as shown in Figure 3(a); the radius increases to $kr$ during the plastic deformation stage as shown in Figure 3(b), and the elastic and plastic deformation is uniform before the occurrence of necking. However, at the instant where multi-necking/crack just occurs but gas leaking is not happened yet as shown in Figure 3(c), the elastic deformation in the two ends of segment is released due to the free-edge effect, resulting in the highest tension stress in the centre of the segment. The instantaneous elastic stress in the midpoint of each segment is equal to the tensile strength of shell material, corresponding to a segment length satisfying the requirements of both the second criterion and the third criterion for fragmentation. Since plastic deformation is not reversible, the stress in this segment depends mainly on the elastic deformation. Therefore, the fragmentation of a plastic shell with a radius of $r$ is approximately equal to the fracture of a brittle shell.

![Figure 3](image-url)

Figure 3. Fragmentation of shells. (a) Schematic diagram showing the shell before explosion, where the original radius of shell is equal to and an intermediate layer is assumed to be present between the gas and the shell. (b) Schematic for the instant of explosion, when the shell undergoes plastic deformation but necking/fracture has not yet occurred. The internal gas is significantly pressurised, resulting in hoop expansion, where the radius of the shell increases from $r$ to $kr$. (c) Schematic for the instant when necking/crack has just occurred but gas has not yet leaked. The shell and high-pressure gas are still in close contact, while the instantaneous tensile stress in the midpoint of each segment is equal to the tensile strength of shell material, and the tensile stress in the two ends of each segment is zero.
having a radius $kr$ into many fragments under the action of explosive loads, $k$ represents plastic deformation.

Let $p_{\text{max}}$ be the maximum impulse pressure acting on the inner side of the shell and $t_d$ be the duration of the explosion. During an explosion, the impulse pressure $p$ acting on the shells is very small in the initial stage (from 0 s to $t_e$), followed by an approximately linear increase (from $t_e$ to $t_d$), after which it drops suddenly to zero [3,12]. The explosion period can be approximated as $p \approx \left( \frac{p_{\text{max}}}{t_d - t_e} \right) (t - t_e)$. The impulse-momentum theorem suggests that $2\pi r \int_{t_e}^{t_d} p \text{dt} \approx 2\pi rhV$, which results in $p_{\text{max}} \approx \frac{2\rho Vh}{(t_d - t_e)}$. Hence, the maximum average hoop strain in the shell during the explosion is $\bar{\varepsilon} \approx \frac{2\rho Vh}{E(t_d - t_e)}$. Considering the tight contact between the high-pressure gas and the shell, the hoop expansion strain $\varepsilon$ in the outermost layer of the high-pressure gas (equivalent to the growth expansion strain in the outermost layer of the heartwood) is considered to be uniform along the hoop direction with a value calculated as follows:

$$\varepsilon \approx \frac{2\rho Vh}{E(t_d - t_e)}. \quad (2)$$

The effect of inertia plays a significant role in dynamic fracture of elastic bars [33]. However, as shown in Figure 1(b), the velocity along the hoop direction $V_h$ for all points on the shell is zero before the initiation of fragmentation, therefore, the inertia effect is assumed to be negligible in this work for investigating the fragmentation of shells. As shown in Figure 1(a), compare to the length of whole fragment, the length of flow localisation (necking) is smaller, thus we neglect its contribution to the fragment length. The high-pressure gas is usually much thicker than the shell, therefore the elastic stress in the thin shell is assumed to be uniform along the thickness direction. At the instant where multi-necking just occurs but gas leaking is not happened yet as shown in Figure 3(c), we neglect the effect of necking/crack on the strain distribution in the outermost layer of the high-pressure and adopt an explicit approximation that the outermost layer of the high-pressure gas with a uniform hoop expansion strain $\varepsilon$. For a fully bonded two-layer structure, once tension or compression strain develops in one of the two layers, stress will present in the other layer as well. For the crack formation in tree bark, these two layers are the bark and heartwood. For the fragmentation of shells, these two layers correspond to the shell and the high-pressure gas, assuming that the high-pressure gas stick tightly with the shell stick tightly during the loading process and before the fragmentation. Under this situation, when the gas is significantly pressurised, hoop expansion will generate in the outmost layer of the high-pressure gas, resulting tension stress in the shell. In order to establish a theoretical model, here we assume that the hoop expansion strain in high-pressure gas is transferred to the fragment of shell through interfacial shear stress.

For a fragmented shell segment as shown in Figure 3(c), the hoop tensile stress is the maximum at the centre of the segment while the shear stress is
zero along the radius direction. This suggests that the fragmentation of shell is caused mainly by the hoop tensile stress rather than shear stress. Under these assumptions, based on the free-body diagram shown in Figure 2(c), the interfacial shear stress \( \tau(s) \) and tensile stress \( \sigma(s) \) in the shell are related by the following equation \([17]\)

\[
h \frac{d\sigma(s)}{ds} = -\tau(s) \tag{3}
\]

The constitutive equation of the inter-layer is

\[
\tau(s) = \overline{G}(\varepsilon s - \phi(s))/\overline{h} \tag{4}
\]

where \( \phi(s) \) is the elastic displacement of the shell fragment, \( \overline{G} \) is the shear modulus of the inter-layer and \( \overline{h} \) is the thickness of the inter-layer. As shown in Figure 3(b,c), the plastic deformation in fragment is constant everywhere, does not affect the stress distribution of fragment, the tensile stress in the fragment, \( \sigma(s) \), is related to the elastic displacement of the shell through the following equation:

\[
\sigma(s) = E \frac{d\phi(s)}{ds} \tag{5}
\]

Combining Equations (3), (4) and (5) yields a differential equation

\[
\frac{d^2\phi(s)}{ds^2} - \frac{\overline{G}}{E\overline{h}h} \phi(s) + \frac{\overline{G}}{E\overline{h}h} \varepsilon s = 0 \tag{6}
\]

The general solution of Equation (6) is

\[
\phi(s) = C \sinh (\sqrt{\overline{G}/E\overline{h}hs}) + \varepsilon s \tag{7}
\]

where \( C \) is to be determined from the boundary conditions. Substituting Equation (7) into Equation (5), the tensile stress of the shell can be expressed as

\[
\sigma(s) = E[(C\sqrt{\overline{G}/E\overline{h}h}) \cosh (\sqrt{\overline{G}/E\overline{h}hs}) + \varepsilon] \tag{8}
\]

Considering the instant when necking has just occurred but gas has not yet leaked (which is shown in Figure 3c), the coefficient \( C \) in Equation (8) can be determined by the boundary condition \( \sigma(l/2) \approx 0 \)

\[
C \approx \frac{-\varepsilon}{\sqrt{\overline{G}/E\overline{h}h} \cosh (\sqrt{\overline{G}/E\overline{h}hl}/2)} \tag{9}
\]

The maximum tensile stress occurs in the midpoint of the segment as shown in Figure 2(d). Next, we look for a solution where this maximum stress is equal to the tensile strength, which corresponds to a segment length satisfying the requirements of both the second criterion and the third criterion for fragmentation. This solution can be written as \( \sigma_{\text{max}}(s) = \sigma(0) = \sigma^*(\dot{\varepsilon}) \), where \( \sigma^*(\dot{\varepsilon}) \) is the
strain-rate-dependent tensile strength and the strain rate is defined as $\dot{\varepsilon} = V/r$ [10]. For the case of crack formation in the tree bark, the tensile strength $\sigma^*(\dot{\varepsilon})$ will be equal to a constant value $\sigma^*$, without consideration of dynamic effect. This results in the following equation:

$$E \left[ -\frac{\varepsilon}{\cosh (\sqrt{G/Eh} l^*/2)} \cos h (0 + \varepsilon) \right] = \sigma^*(\dot{\varepsilon})$$ (10)

Solving Equation (10), the length of the neck segment can be determined as follows:

$$l^* \approx 2 \sqrt{\frac{Eh}{G}} \cosh^{-1} \left[ \frac{1}{1 - \sigma^*(\dot{\varepsilon})/E\varepsilon} \right]$$ (11)

Consequently, the number of necks after fragmentation can be calculated using Equation (12):

$$N \approx \frac{\pi r k}{\pi\varepsilon h} \sqrt{\frac{G}{Eh}} \left[ \cosh^{-1} \left( \frac{1}{1 - \sigma^*(\dot{\varepsilon})(t_d - t_e)/(2\pi r V)} \right) \right]^{-1}$$ (12)

### 4. Results and discussion

There are three important parameters in Equation (12) that need to be determined, the explosion period $t_d - t_e$, the rate-dependent strength $\sigma^*(\dot{\varepsilon})$ and $k^2G/\bar{h}$. Kahana et al. [3] and Zhang [12] conducted ring expansion tests of AZ31 magnesium alloy rings ($r = 16.3$ mm, $h = 1.7$ mm, $E = 45$ GPa, $\nu = 0.35, \sigma^* = 240$ MPa, $\rho = 1.8$ kg/m$^3$) and 1060 aluminium rings ($r = 16$ mm, $h = 1.5$ mm, $E = 72$ GPa, $\nu = 0.33, \sigma^* = 90$ MPa, $\rho = 2.68$ kg/m$^3$) respectively.

For the testing of commercially pure 1060 aluminium rings [3], the load increased from zero to a certain value very rapidly, and the ring finally fragmented into segments at about $20 \mu s$. For the test of 1060 pure aluminium rings [12], the pressure acting on the shells was very small at the initial stage, followed by a linear increase from $50 \mu s$ to $70 \mu s$, after which it dropped suddenly to zero. Therefore, for both experiments, we assume the effective impact time to be approximately $20 \mu s$.

Fragmentation of shells is a typical dynamic fracture problem, and the strain rate effect must be considered. Yu et al. [34] found that the tensile strengths of many metallic materials increase approximately linearly with the increase in strain rate over a range that is strain-rate-sensitive. This linear relationship is assumed to be applicable to the materials considered in this study. Based on the results of dynamic tests by Feng et al. [35], Xu et al. [36], and Khan and Huang [37], the tensile strength of AZ31 at strain rate of $10^4$ s$^{-1}$ is $\sim 1.7$ times of its quasi-static value [35,36], and the tensile strength of 1060 aluminium at
a strain rate of 6000 s\(^{-1}\) is \(~2\) times of its quasi-static value \([37]\). At this point, all parameters in Equation (12) have been determined, except for \(k^2G/h\), which can be determined by matching Equation (12) with one experimental data. The results suggest that \(k^2G/h = 27.78 \times 10^3\) GPa/m for AZ31 magnesium alloy rings and \(k^2G/h = 8.89 \times 10^3\) GPa/m for 1060 aluminium rings.

Figure 4 compares the predicted number of necks with the experimental observations by Kahana et al. \([3]\) and Zhang \([12]\). The predictions obtained by using Equation (12) show good agreement with the experimental results \([3,12]\). The number of necks increases dramatically at low expansion velocities, but the rate of increase slows gradually with a further increase in the expansion velocity. It is expected that the number of necks will reach a saturation value when the expansion velocity goes beyond a certain threshold. The flattening tendency of fragmentation can be explained in two ways. First, the expansion strain \(\varepsilon\) (based on the expansion velocity \(V\)) in Equation (8) has a nonlinear exponential relationship with the segment length for a constant value \(\sigma(s)\). Second, the tensile strength of the shell material increases with the increase in expansion velocity. As shown in Figure 4, the number of necks predicted when considering the strain rate effect is significantly smaller than that the number predicted without considering the strain rate effect.

5. Conclusion

In summary, this paper presents a new theoretical model to explain the physical mechanism of fragmentation in shells. At the instant of explosion, a two-layer structure is formed, where the outer layer is the shell and where the high-pressure gas generated by the explosion can be considered as the internal layer. Thus the fragmentation process for shells becomes similar to the
process of crack formation in tree bark. A theoretical model for predicting the number of necks after fragmentation is proposed with consideration of the strain-rate-dependent strength properties of the considered material. The model was calibrated for commercially pure 1060 aluminium alloy as well as for AZ31 magnesium alloy, and the predictions showed good agreement with the experimental results. Furthermore, it was shown that the strain rate effect strongly influences the fragmentation of the shells. We note that the presented model is only applicable to ring or cylindrical shell structure, but not suitable for shells of spherical shape. The shells with spherical shape will be further investigated in the future study.

The physical mechanism presented in this paper can be used to explain other types of fragmentation, such as bullets impacting on windows [38–41], thin layers of suspensions of non-Brownian particles that experience an impact [42], popping balloons [43] and so on. Due to the space–time effect, the hoop stress at locations near the impact region is significantly higher than the hoop stress at locations far from the impact region at the moment of impact, and a layer-like structure is formed. Consequently, the physical mechanism as shown in Figure 2 drives the rest of the process, and a star-shaped crack pattern is eventually formed.

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