How to evaluate the fatigue life of textile composite structures by the numerical analysis and the experimental data with scatter

Masaru Zako¹,²
¹ Graduate School of Engineering, Osaka University, Osaka, Japan
² Expert adviser of ITOCHU Techno-Solutions Corporation, Japan
zano@mit.eng.osaka-u.ac.jp

Abstract. The fatigue test data and Goodman diagram have employed in order to evaluate the safety for a long period of time. Since the discrepancies exist in the fatigue test data, a modified Goodman diagram with a failure probability and a confidence level has been proposed in this paper. The confidence level is a function of total number of test data. The proposed procedure has an advantage which can be applied to a small number of specimens. As a numerical example, a hydrogen gas tank for automobile made of CFRP has been picked up. FEM has been applied to obtain the stress under a pressure in the tank. The in and out of gas to tank is considered as a fatigue phenomenon. The safety of tank for repeated use has been evaluated by the comparison of maximum stress generated in the tank and the modified Goodman diagram with a probability of failure. It is recognized that the proposed procedure is a useful and reasonable method for safety evaluation.

1. Introduction
There are many kinds of fabrics, matrices and textiles, and lots of combinations of them. As they progress rapidly, many researchers and designers on them have the demand which needs to estimate their properties in a short time. It is not easily to prepare a large number of specimens and to conduct the experiments within a fixed period of time. The designers for textile and fibrous composites have the requirements which obtain not only the mechanical properties but also the characteristic on fatigue within a given period and with a small number of samples. However, it is difficult to obtain the characteristics on fatigue within a short time and to estimate the variation in data with a small number of specimens. On the other hand, ASME (Sec.III) [1] has proposed the code which shifts S-N curve to safety side. ASME Sec.III code consists of two processes. At first, S-N curve is got by the experimental data and the least squares method. A new S-N curve is obtained by shifting 1/2 to the stress amplitude or strain amplitude. And the other S-N curve is also obtained by shifting 1/20 to the number of cycle to failure. Using these two S-N curves, the conservative S-N curve is finally adopted as the design curve. This procedure is very simple and is widely used. It has a problem, however, that the shifting value does not change even if the variation in an experimental data is reduced by a modification or an improvement of production. The scatter of data can evaluate quantitatively with the theory of reliability. The probability of failure is equivalent to the safety factor, and the confidence level is concerned with the number of specimens. If it is possible to apply them to the treatment of fatigue test data, it will be a very useful method for designer to determine S-N curve with probability of failure and confidence level. The object of this study is to propose a procedure of determination of the shifting value based on reliability in order to satisfy designer’s requirements.
2. Probability with confidence level

2.1 Number of samples based on reliability

Figure 1 shows the relation between the number of specimens and the accumulated probability with a confidence level based on Median rank [2]. It is recognized that MIL specs requires 298 specimens and 28 specimens for A and B values, respectively. The confidence level has also a great influence on the number of specimens. Though it is difficult for a small number of specimens to obtain the sufficient probability, the probability may be revised by the employment of Figure 1. Equation (1) shows the relation between the number of specimens and the probability with a confidence level.

\[
n = \frac{\ln \gamma}{\ln(1 - F)}
\]

Where \(n\) is the total number of specimens, \(\gamma\) is the significance level and \(F\) is the probability of failure.

![Figure 1. Relation between the number of test specimens and the probability with a confidence level](image)

2.2 Equivalent stress (strain) amplitude

Figure 2 shows the typical example of S-N curves for S25C [3] and GFRP [4] obtained by bending fatigue test. GFRP consists of a glass roving cloth and polyester resin, and the volume fraction of fiber (V_f) is 35%. It is recognized that the test data on stress amplitude have scatter for number of cycles to failure, and that the distribution curves are not the same ones as shown in Figure 2(a). This tendency for S25C was made clear by Nishijima and he published as P-S-N diagram [3]. However, the distribution curves for the stress amplitude on a number of cycles to failure are same. GFRP has the same tendency as shown in Figure 2(b). This fact is very important to the arrangement of data.
Figure 2. Typical example of bending fatigue test data for S25C and GFRP

Figure 3 shows a procedure how to arrange the experimental data and how to get the distribution curve for the stress amplitude (or the strain amplitude) [5]. The oblique solid line is an analytical result which obtains from the least squares method. Figure 3(a) shows how to shift the stress amplitude at an arbitrary number of cycles ($N_c$). The stress amplitude ($S_1$) at a number of cycles ($N_1$) can convert into the stress amplitude ($S_c$) at an arbitrary number of cycles ($N_c$) by Equation (2).

\[ S_c = S_1 \left( \frac{N_c}{N_1} \right)^{m_0} \]  

(2)

Where $S_c$ is equivalent stress amplitude for the convenience and $m_0$ is the gradient of line. Figure 3(a) indicates a logical meaning that the specimen will be broken at $N_c$, if the stress amplitude is changed from $S_1$ to $S_c$. Figure 3(b) shows how to gather the experimental data in a number of cycles ($N_c$) in order to analyze the distribution curve of equivalent stress amplitude.

Figure 3. Procedure how to arrange the experimental data and how to get the distribution curve

2.3 S-N Curve for design with a probability of failure and a confidence level

The fatigue test data has been arranged for the analysis on probability of failure as shown in Figure 3. This procedure has an advantage which is able to get the probability of failure even if number of
specimens is few. The procedure has also the other advantage which can improve S-N curve in case of getting new test data. The estimated probability of failure $\hat{P}_f$ can be obtained by Equation (3).

$$\text{Pr}[\hat{P}_f < P_f] = 1 - \gamma$$

(3)

Where $P_f$ is the probability of failure and $\gamma$ is the significance level. Figure 4 shows the concept of determination of stress amplitude for design.

![Figure 4](image)

**Figure 4.** The concept of determination of stress amplitude for design based on estimated probability of failure and confidence level

The designer supposes some serious considerations and will determine an allowable stress amplitude ($S_d$). Of course, $S_d$ will be determined under the consideration which the estimated failure probability is lower than $P_f$ with confidence level $(1-\gamma)$. It means that the designer determines a probability of failure with confidence level instead of safety factor. The estimated probability of failure is calculated by one-side interval estimation as shown in Figure 4. The allowable stress amplitudes can be obtained by Equation (4). When the distribution of the equivalent stress amplitude follows the normal distribution, the allowable stress amplitude $S_d$ is calculated by Equation (4-a). In cases of log-normal and Weibull distributions, the allowable stress amplitudes ($S_d$) for each distribution are obtained by Equations (4-b) and (4-c), respectively.

$$S_d = \mu - \left\{ \Phi^{-1}(1-P_f) + \frac{\Phi^{-1}(1-\gamma)}{n} \sigma \right\}$$

(4-a)

$$S_d = \exp\left[ \mu - \left\{ \Phi^{-1}(1-P_f) + \frac{\Phi^{-1}(1-\gamma)}{n} \sigma \right\} \right]$$

(4-b)

$$S_d = \frac{\mu}{\Gamma(1+1/\alpha)} \left\{ \chi^2/(2n) \right\}^{-1/\alpha}$$

(4-c)

Where, $\mu$ is the mean value, $\sigma$ is the standard variation, $\alpha$ is the shape parameter of Weibull distribution, $\chi^2$ is chi-square, $\gamma$ is the significance level and $n$ is the total number of specimens, respectively. $S_d$ for the other distributions can get by the statistics procedure as same as Equation 4. The type of distribution is judged by the coefficient of correlation. Figure 5 shows the concept of S-N curve for design which is obtained from the above mentioned procedure.
Figure 5. S-N curve for design obtained by a probability of failure and a confidence level [6]

Figure 6 shows the fatigue test data under repeated tensile loading condition for unidirectional CFRP[7] and S-N curve obtained by the proposed procedure. The specimens are made of carbon fiber (T-700SH) and epoxy, and Vf is 60%. The computer system is useful for the calculation of probability of failure. The numerical analysis of probability of failure has been introduced in a computer system. S-N curves with each probability of failure as shown in Figure 6 are obtained by computer system. Using the system, S-N curve with probability of failure can be easily obtained. It is also recognized that the proposed procedure is useful for designer of structure because the statistical analysis reflects the total number of specimens as the confidence level.

Figure 6. An example of S-N curve for each probability of failure when the confidence level is 0.95 for CFRP (T700SH/Epoxy and 60% Vf)

3. A proposal of safety evaluation for CFRP hydrogen gas tank

3.1 Goodman diagram
Type 4 tank applies to a hydrogen gas tank to reduce the weight of automobile. Type 4 is the composites tank which is wound CFRP around a plastic liner tank. The putting in and taking out hydrogen gas looks like fatigue phenomenon. The high pressure gas safety institute of Japan (KHK) has established a standard for safety evaluation. As an example, the number of repeat cycles for automobile is required
30,000 cycles. Since it will take much time to conduct the fatigue test, the effective method for the evaluation is required. Goodman diagram[8] is applied to the judgement of occurrence of failure by fatigue. Figure 7 shows Goodman diagram with \( P_f = 0.5 \) for \( 10^5 \) cycles which obtains from the fatigue test of unidirectional CFRP shown in Figure 6. In Figure 7, the vertical axis is stress amplitude, the horizontal axis is mean stress and \( R \) is stress ratio, respectively. When \( R = 0 \), the loading condition means one side loading of tension when the minimum load is zero. The stress condition act on tank is one side repeated tension. For example, the maximum stress will be generated when the tank is filled with gas, and the minimum stress will be zero when the gas is completely released. If the maximum stress act on the tank is \( S \) and the minimum stress is zero, the stress amplitude \( S_a \) and the mean stress \( S_m \) will be \( S/2 \), respectively. The value has just plotted on the line of \( R = 0 \) in Figure 7. If the plotted point is lower than Goodman diagram of \( N_f = 10^5 \) cycles, it means that the tank does not break down up to \( 10^5 \) cycles.

![Goodman diagram for \( N_f = 10^5 \) cycles of an unidirectional CFRP](image)

Figure 7. Goodman diagram for \( N_f = 10^5 \) cycles of an unidirectional CFRP

3.2 Evaluation a hydrogen gas tank by Goodman diagram with probability of failure Figure 8 shows a hydrogen gas tank of type 4. The tank is 600 mm length with a plastics inner tank 240mm diameter, the capacity is 15 L and the pressure is 80Mpa. The tank is made up of the composite structure which is wound unidirectional CFRP (T700S/epoxy and 60%Vf) around a plastic liner tank. Figure 8 displays a quarter of the section of tank to make clear of structure. Figure 9 shows Goodman diagram for the probability of failure with confidence level 0.95 for 30,000 cycles which obtains from Figure 6. Three probability of failures, which are \( 10^{-3}, 10^{-4} \) and \( 10^{-5} \) have been picked up as the example.

![A quarter section of a hydrogen gas tank for an automobile and mesh size for analysis](image)

Figure 8. A quarter section of a hydrogen gas tank for an automobile and mesh size for analysis
Figure 9. Goodman diagram for 30,000 cycles with probability of failure ($P_f$) (confidence level is 0.95)

According to the regulation of KHK, the margin factor is set to 1.25. When 80MPa is assumed to tank pressure, 100MPa should be applied as the internal pressure. The numerical analysis has been carried out for full scale model and SACOM (FEM program code) has been employed. Since the outer layer has been wound unidirectional CFRP, the strength of unidirectional fiber reinforced plastics dominates the burst pressure of tank. Figure 10 shows the distributions of $S_L$, $S_T$ and $S_{LT}$ under 100MPa. From the numerical results, it is recognized that the maximum stress $S_L$ has been generated in hoop layer and the maximum stress is about 760MPa. When the gas is completely released, the stress ratio $R$ becomes zero. The stress amplitude equals to the mean stress and they are a half of the generated maximum stress. In this case, they are 380MPa and is plotted on Goodman diagram as point A in Figure 11(a).

It is shown that the hydrogen tank will be safe up to 30,000 cycle with the confidence level 0.95, because the plotted point has a distance from the line of $P_f=10^{-5}$. As the other example, we consider the situation which make repeated use of a half consumption. In this case, the maximum stress is 760MPa and the minimum stress is 380MPa. The mean stress is 570MPa and the stress amplitude is 190MPa, respectively. The point B in Figure 11(a) indicates the situation of a half consumption. Point B has a considerable distance from the line of $P_f=10^{-5}$ than point A. It is estimated that the operation B is a durable type than the operation A. Figure 11(b) shows the result of evaluation for $10^6$ cycles. In this case, the operation A contacts with the line of $P_f=10^{-5}$. It means that the tank will break at $10^6$ cycles with the probability of failure $P_f=10^{-5}$ and the operation B is durable even $10^6$ cycles. It is also recognized that the proposed procedure is simple and reasonable.
Figure 10. Numerical results of the distribution of $S_L, S_T$ and $S_{LT}$ under 100 MPa

Figure 11. Results of the safety evaluation by Goodman diagram with probanility of failure

4. Conclusion
An effective procedure to evaluate the safety for structures has been proposed. The proposed procedure consists of two parts. One is to decide the allowable stress with confidence level which is the function of the total number of specimens. The other is to evaluate the fatigue life by Goodman diagram with an estimated failure of probability. It is recognized that the proposed procedure is logical and simple. A hydrogen gas tank made of CFRP has been picked up as an example. The following points are clarified.

1) S-N curve with probability of failure can be regulated by the equivalent stress amplitude. Goodman diagram with the probability of failure is very useful for judgment of the safety of structure.
The confidence level is useful index for composites because it is applicable even if the number of specimens is small due to progress rapidly.

The probability of failure will be useful and reasonable index instead of a safety factor.

In the future, the verification of Goodman diagram for textile composites is necessary. The fracture in textile composites consists of some damages like matrix crack, debonding, delamination and fibre breaking etc. Damage mechanics can be applied to the judgment of occurrence and distinguish of complex damage modes. If the occurrence of these damages during fatigue make clear experimentally, the explicit estimation for life will be possible by the combination of damage mechanics and the data. In addition, the computer system is efficient, because the complicated formulation of statistics and FEM are included in the analyses. ITOCHU Techno-Solutions Corporation will publish the system as “Composites dream ver.4” in the near future.

Acknowledgments
A part of this research was sponsored by the project of NEDO in Japan. The author wishes to express their appreciations to Mr. M. Sugawara and Dr. M. Kawahara for the contribution of production of hydrogen gas tank. The author also wishes to extend my thanks to Dr. T. Kurashiki, Dr. K. Hanaki, Dr. X. Li and Dr. Mukouyama for their works on the analysis of fatigue data.

References
[1] ASME 1980 Boiler and Pressure Vessel code. Sec. III Div. I Appendix II
[2] Ichikawa M. et al 1987 Practical Reliability of Engineering for mechanical and Civil Engineer (Yokenndo) p 60
[3] Nishijima S. 1981 ASTM STP p75
[4] Tanimoto T. et al 1981 Proceeding of US-Japan conference on composite materials p145
[5] Zako M. Kurashiki T. and Hanaki S. 2001 A Statistical decision making procedure of S-N curve with fatigue limit JSTM Vol 50, No. 3 p279
[6] Zako M. Takano N. Ichikawa M. and Hanaki S. 1995 An evaluation System for Advanced materials based on Reliability Engineering JSME p1909
[7] NEDO report in Japan 2014 Fatigue test data of CFRP p52
[8] Goodman J. 1899 Mechanics Applied to Engineering Longman (London Green & Company)