Dynamics of large-diameter water pipes in hydroelectric power plants

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Abstract. An outline is made of physical behaviour of water-filled large pipes. The fluid-wall coupling, the key factor governing the pipe dynamics, is discussed in some detail. Different circumferential pipe modes and the associated cut-on frequencies are addressed from a theoretical as well as practical point of view. Major attention is paid to the breathing mode in view of its importance regarding main dynamic phenomena, such as water hammer. Selected measurement results done at EDF are presented to demonstrate how an external, non-intrusive sensor can detect pressure pulsations of the breathing mode in a pressure pipe. Differences in the pressure measurement using intrusive and non-intrusive sensors reveal the full complexity of large-diameter pipe dynamics.

1. Introduction
The effect of fluid-structure coupling (FSC) was known since more than a century ago¹. Studies on transient pipe dynamics have already started at the turn of 20th century². Various models have been established so far to simulate effects of non-rigid pipe walls. Simplified formulae of sound speed subjected to pipe external mechanical conditions were published in papers³ and books⁴,⁵. Further enhancement of pipe modelling was done by allowing the pipe to move via junction coupling which is not included in the classical pipe models⁶. A broad account of pipe dynamics under varying external boundary conditions aimed at diagnostics applications was recently presented in thesis work⁷.

Based on thin-wall assumptions a comprehensive FSC model of a free fluid-filled cylindrical shell has been produced by Lin and Morgan⁸. It has been shown that the sound speed is not constant but changes with frequency. Using a frequency-dependent multi-mode model the excitation by a point source located in a free elastic cylindrical shell has been worked out⁹. Similar modelling has been extended to the propagation of pulsation and vibration energy along fluid-filled cylindrical shells¹⁰.

This paper focuses at the dynamics of a large fluid-filled pipe which, free at the outer surface. The ultimate goal is to show how an external strain sensor can be employed to measure pressure pulsations using a pipe model which allows for pressure variation in axial, circumferential and radial directions.

2. Model of a free, thin-walled fluid-filled pipe
Fig. 1 shows the coordinate system used, shell geometry and wall displacements. The coupled fluid-wall motion can be expressed using the equations of motion of the fluid and the wall and the equation of continuity of radial motions at the fluid-wall interface, Eq. (1). The symbols stand for: t – time, p – dynamic pressure, E, ν, ρw – Young’s modulus, Poisson ratio and mass density of the wall, B, ρf – bulk modulus and mass density of the fluid, Ξ = 3×3 differential operator of shell elasticity. In what follows Flügge’s operator Ξ will be employed¹¹.
Assuming harmonic motion with angular frequency \( \omega = 2\pi f \) Eq. (1) can be resolved by factorization. This leads to a radial pressure distribution which is governed by Bessel functions \( J_n(kr) \), \( n \in \mathbb{Z} \). In the axial and circumferential sense both the fluid and the wall satisfy \( \exp(\pm ikx) \) and \( \cos(n\theta+\phi) \) laws. The symbols \( k_x \) and \( k_r \) denote the axial and radial components of the wavenumber \( k = \alpha/c_{\text{phys}} \), \( c_{\text{phys}} \) being the speed of sound in the unbounded fluid.

\[
B \Delta p - \rho_f \frac{\partial^2 p}{\partial t^2} = 0, \quad \left( \frac{4Eh}{d^2(1-\nu^2)} \right) \Delta - h \rho \frac{\partial^2 w}{\partial t^2} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = p_{r=d/2}^{(0)} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \alpha \frac{\partial p}{\partial r} + \rho_f \frac{\partial^2 w}{\partial t^2} = 0 \tag{1}
\]

To resolve the equations (1) a power series development is used. For any particular solution \( k_r/k_x \) the relationship between the radial displacement amplitude of the wall \( W \) and the fluid pressure amplitude \( P \) can then be obtained in the following way:

\[
\frac{P}{W} = \alpha^2 \frac{\rho_f d}{2} \frac{J_n(\kappa)}{J_n'(|\kappa|)} , \quad \kappa = \frac{k_r d}{2}, \quad J_n'(|\kappa|) = \frac{dJ_n(|\kappa|)}{d\kappa} \tag{2}
\]

The \( n = 0 \) shell order represents the axially symmetric case, called breathing motion. In this case the hoop (circumferential) strain due to breathing is simply \( \varepsilon_{\theta \theta} = 2\omega d/c_{\text{phys}} \), which allows direct use of Eq. (2) in providing the strain-pressure relationship. This relationship will be used as a basis of the method of non-intrusive pressure pulsation measurement, further discussed in §4.

3. Wave motion

Any wall deformation of a straight shell can be decomposed into circumferential orders \( n = 0, 1 \ldots \) Fig. 2. To each \( n \) is associated a number (theoretically infinite) of radial pressure distributions indexed by \( s = 1, 2, \ldots \) as defined in §2. Each pair \( n,s \) physically describes one particular wave entirely defined by wavenumber pairs: radial \( k_{r,n,s} \) and axial \( k_{k,n,s} \). A given wave can be either propagating or evanescent in dependence of whether its axial wavenumber \( k_{k,n,s} \) is purely real (propagating), or not (evanescent).

At low frequencies only a few of these waves can propagate. These are pulsation wave \((n = 0, s = 1)\), torsional wave \((n = 0, s = 2)\), extensional wave \((n = 0, s = 3)\), and flexural wave \((n = 1, s = 1)\). Evanescent waves may exist at low frequencies too; these waves do not propagate but instead decay exponentially. As the frequency increases, more and more evanescent waves turn into propagating ones. The frequency at which the conversion takes place is called cut-on frequency. Table 1 shows the first cut-on frequencies for waves of different orders relative to two water-filled steel pipes of 10.5 mm thickness used at EDF hydroelectric power plants. These frequencies are very low which means that in the frequency range of a few tens of Hz a large number of different waves may propagate.

| Table 1: First cut-on frequencies in Hz of water-filled 10.5-mm thick steel pipes | order n | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|--------------------------------|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| dia 2.4 m                     | 1.81   | 5.78| 12.2| 21.3| 33.2| 48.1| 66.0| 87.0| 111 | 139 | 169 |
| dia 5.5 m                     | 0.241  | 0.777| 1.66| 2.94| 4.63| 6.78| 9.40| 12.5| 16.1| 20.3| 25.0|

With the exception of torsional wave any given type of wave occurs simultaneously in the fluid and in the wall due to the fluid-wall coupling. Each fluid-wall wave pair has the same axial wavenumber, while the radial pressure distribution in the fluid is governed by the matching radial wavenumber.
The wave defined by $n = 0$, $s = 1$, called the pulsating wave, is of a particular importance: this wave typically generates the largest part of pressure pulsation within the pipe. However other $n = 0$ waves superpose to the pulsation wave, each of these having its own amplitude and its own speed. This is also true for any order other than $n = 0$, so the total wave motion will consist of a superposition of groups of waves of different orders with each group consisting of different wave types: some propagating, the others evanescent. If the flow in the fluid is non-homogeneous or of non-negligible speed the physical picture of wave motion becomes even more complex.

4. The non-intrusive measurement of pressure pulsations

A PVDF wire connected to a charge amplifier has been proposed as a sensor for the detection of pressure pulsations in a pipe\(^1\). It responds to the hoop surface strain relative to the line along which it is wound to the pipe. Where the breathing mode ($n=0$) is concerned the radial displacement and the integrated circumferential strain are proportional to each other, while for $n > 0$ modes the integrated circumferential strain is zero. The objective is thus to measure the integrated circumferential strain and to deduce from it the internal pressure originating from the pulsation waves using Eq. (2).

![Graphs showing pressure pulsation measurements](image)

Figure 3. Pressure pulsation in a 2.4m dia 10.5mm thick steel pressure pipe of a small hydroelectric power plant. Top: normal regime; bottom: regime of strong oscillations. Left: time history; right: RMS pressure spectrum. Symbols: w – external wire sensor, p – pressure sensor.

On Fig. 3 are shown pressure pulsations measured in a pressure pipe using three closely spaced PVDF sensors and a classical intrusive piezo sensor placed nearby. The measurements are shown for two regimes of operation in both time and frequency domains. Good overall matching can be seen in both cases. The mismatch in the spectra above 50 Hz is believed to originate from the difference in the character of the measured pressures: the intrusive sensor refers to a single point while the wire outputs the pressure averaged around the circumference. The difference between the two increases with frequency as a consequence of increase in number of propagating higher order waves.
The mounting of the external wire sensor can be simply done using a double-face adhesive tape. In some cases it is sufficient to install the wire directly on the pipe as shown on Fig. 4. The wire is connected to a charge amplifier. The signal from the wire is converted into frequency domain and the relationship (2) is applied to obtain the pressure spectrum. Alternatively, Eq.(2) can be brought in a form of a filter and the conversion of strain to pressure can then be done directly in time domain.

5. The wave speed in a pipe

The speed of sound in a free (unbound) fluid \(c_{f,\infty}\) is given by Newton-Laplace equation:

\[
c_{f,\infty} = \sqrt{\frac{B}{\rho_f}}
\]

The elasticity of pipe wall makes the apparent fluid bulk modulus, and thus the sound speed, drop. In the case of heavy liquids, i.e. water, and small wall thickness the decrease of sound speed can be rather substantial. The term “sound speed” is usually used for the speed of pulsation waves. Waves other than pulsation waves have own speed of propagation which varies with frequency in a rather complex fashion. For any type of wave the speed is obtained from the wavenumber by \(c = \alpha/k_s\).

The sound speed in function of frequency was computed for two EDF water-filled steel pipes using the solution for wavenumber \(k_s\) based on Eq. 1, Fig. 5 left. The nominal speed \(c_{f,\infty}\) was taken to be 1450 m/s. The drop of speed is seen to be significant and increases with frequency. Fig. 5, right, shows a particular cost function \(G\) used to obtain the actual speed in the 2.4m dia pipe from pulsation measurements. The function \(G\), calculated for a range of assumed speeds from the signals measured by 3 PVDF sensors, indicates the true speed by the position of its minimum.

6. Vibration of pipe wall

In order to monitor wall vibration six accelerometers were mounted around the circumference of the dia 2.4m pipe. The instantaneous radial displacement of the wall is shown in Fig. 6 at some selected...
frequency bands. The plots clearly show the complexity of pipe motion which is a consequence of the superposition of a multitude of waves having different circumferential orders. In view of Eq. 1 such complex vibration patterns indicate a complex distribution of internal pressure pulsations. It can be thus concluded that point measurement of pressure pulsations using classical intrusive sensors may not be representative where global pulsations averaged along the circumference are concerned.

Figure 6. Instantaneous displacement of the 2.4m pressure pipe.

7. Conclusions
It has been demonstrated that an external wire strain sensor wound around a large pipe can be used for the measurement of pressure pulsations within the pipe. This requires use of a mathematical model of fluid-wall coupling. The measurements done on a large pipe using the wire sensor and a classical intrusive sensor done in situ have shown satisfactory global matching. An array of 3 wire sensors was shown to be capable of measuring the true speed of sound in the pipe.

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