Dalitz plot analysis of $D^0 \rightarrow K^0_S \pi^+ \pi^-$ decays in a factorization approach

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Abstract. A quasi two-body QCD factorization is used to study the $D^0 \rightarrow K^0_S \pi^+ \pi^-$ decays. The presently available high-statistics Dalitz plot data of this process measured by the Belle and BABAR Collaborations are analyzed together with the $\tau^- \rightarrow K^0_S \pi^- \nu_\tau$ decay data. The total experimental branching fraction is also included in the fits which show a very good overall agreement with the experimental Dalitz plot density distributions. The branching fractions of the dominant channels compare well with those of the isobar Belle or BABAR models. We show that the branching fractions corresponding to the annihilation amplitudes are significant.

1 Introduction

Preliminary results of the Dalitz plot analysis of the $D^0 \rightarrow K^0_S \pi^+ \pi^-$ decays have been already shown during the MESON2012 Workshop [1]. One of our aims was the construction of D-decay amplitudes in which unitarity is preserved in two-body subchannels like in $K^0_S \pi$ S-wave and $\pi\pi$ S- and P-waves. Below we present some new results recently published in Ref. [2]. Starting from the weak effective Hamiltonian, 28 tree and annihilation (W-exchange) amplitudes build up the full $D^0 \rightarrow K^0_S \pi^+ \pi^-$ amplitude. The meson-meson final state interactions are described by the kaon-pion and pion scalar and vector form factors for the S and P waves and by Breit-Wigner formulae for the D waves. Unitarity, analyticity and chiral symmetry are used to constrain functional forms of the form factors which group several resonances in a given partial wave. This, together with charge symmetry, allows to reduce the 27 non-zero amplitudes into 10 effective amplitudes depending on 33 free parameters.

2 Meson-meson form factors and effective mass distributions

The final state strong interactions between mesons influence the functional dependence of the meson-meson form factors on the effective mass variables. An important role in the decay amplitude is played by the scalar form factors. In Fig. 1 the $K\pi$ scalar form factor is shown. On the left panel one can see two maxima lying below 1.5 GeV which correspond to the strange scalar resonances $K^*(800)$

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and $K^*(1430)$. In Fig. 2 the pion scalar form factor is plotted and three scalar resonances are jointly described. In the left panel the two maxima correspond to the $f_0(500)$ and $f_0(1400)$ resonances while the deep minimum at about 1 GeV is related to $f_0(980)$.

![Figure 1](image1.png)

**Figure 1.** The modulus (left panel) and the phase (right panel) of the $K\pi$ scalar form factor $F^{K\pi}_{0}$ as function of the $K\pi$ effective mass for two values of the $f_K/f_\pi$ ratio, where $f_K$ and $f_\pi$ are the kaon and pion decay constants.

![Figure 2](image2.png)

**Figure 2.** The modulus (left panel) and phase (right panel) of the pion scalar form factor $F^{\pi^+\pi^-}_{0}(m_0)$, resulting from the fit to the Belle data, are plotted as functions of the $\pi\pi$ effective mass. The dark bands represent their variations when the parameters vary within their errors. They are compared with the same form factor obtained in Ref. [3] with different parameters (dashed line) and with the form factor calculated from the Muskhelishvili-Omnès equations [4] (dotted-dashed line).

A joint $\chi^2$ fit to the Belle Dalitz plot density distribution [5], to the $\tau^\to K^0_{S}\pi^-\nu_\tau$ decay data [6] and to the total experimental branching fraction is carried out to fix the 33 free parameters. These are mainly related to the strengths of the scalar form factors and to unknown meson to meson transition form factors at a large momentum transfer squared equal to the $D^0$ mass squared. The present model Dalitz plot density distribution is shown in the left panel of Fig. 3. The Dalitz variables projections seen in the right panel of Fig. 3 and in Fig. 4 indicate that a good overall agreement to the Belle Dalitz
Table 1. Branching fractions (Br) for different quasi two-body channels calculated for the best fit to the Belle data \cite{5}. The branching fractions for the tree amplitudes (tree) and the lowest values for the annihilation amplitudes (Ann. low.) are also given. The errors are statistical. All numbers are in per cent.

| Channel | Br (%) | Br (tree) | Ann. low. |
|---------|--------|-----------|-----------|
| \([K^0_\pi^-]_{S}\pi^+\) | 25.0±3.6 | 8.2 ± 0.1 | 7.9±0.1 |
| \(K^0_S[\pi^-\pi^+]_S\) | 16.9±1.3 | 14.7 ± 0.2 | 2.9±0.1 |
| \([K^0_\pi^-]_{P}\pi^+\) | 62.7±4.5 | 24.7 ± 5.7 | 8.7±3.0 |
| \(K^0_S[\pi^-\pi^+]_P\) | 22.0±1.6 | 4.4 ± 0.1 | 6.7±0.04 |

**Figure 3.** Left panel: Dalitz plot distribution from the fit to the Belle data \cite{5}. Comparison of the \(K^0_\pi^-\) effective mass squared distributions for our model (solid curve) with the Belle data \cite{5} (points with error bars).

plot density distribution is achieved. The scalar and vector \(K\pi\) form factors have been constrained by a fit to the \(\tau^- \rightarrow K^0_\pi^-\nu\tau\) data. The red solid line in Fig. 5 shows the model distribution compared to the data of Ref. \cite{6}. A separate good quality fit to the BABAR isobar model distributions \cite{7} has been performed leading to a similar set of parameters as that obtained for the fit to the Belle data.

The four most important channel contributions to the branching fraction are given in Table 1. The \(S\)-wave channel branching fractions are sizable. The last column gives the lowest values of the annihilation parts. These values are significant in comparison with the tree branching fractions and indicate the importance of the W-exchange contributions to the full decay amplitude.

Our \(D^0 \rightarrow K^0_S\pi^+\pi^-\) decay amplitude could be a useful input for determinations of the \(D^0 - \bar{D}^0\) mixing parameters and of the Cabibbo-Kobayashi-Maskawa angle \(\gamma\) (or \(\phi_3\)). Upon request, we can provide numerical values for our amplitudes.

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Figure 4. Left panel: comparison of the $K^0_S\pi^+$ effective mass squared distributions for the best fit (solid curve) with the Belle data [5]. Right panel: as in left panel but for the $\pi^+\pi^-$ effective mass squared.

Figure 5. Comparison of the fit of the model (red solid line) with the $K^0_S\pi^-$ effective mass distribution of the Belle data on the $\tau^-	o K^0_S\pi^-\nu_\tau$ decays [6].

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