A new study of energy levels of hydrogenic atoms and some molecules for new more general exponential screened coulomb potential

Abstract

In present research paper, the solutions of the modified Schrödinger (MSE) with new more general exponential screened coulomb (NMGES) potential, have been presented by means generalized Bopp’s shift method and standard perturbation theory, in the noncommutative three dimensional space phase (NC: 3D-RSP). The bound state energy eigenvalues, in terms of the generalized the hypergeometric function, the discreet atomic quantum numbers \( j = \lfloor \sqrt{s} \rfloor \ldots \lfloor \sqrt{s} \rfloor \), and \( m \) and \( n \), two infinitesimal parameters which are induced by position-position, in addition to, the dimensional parameters \((\Theta, f)\) of NMGES potential and the corresponding noncommutative Hamiltonian operator were obtained for hydrogenic atoms and the molecules \((CO, NO)\). We have also shown that, the total complete degeneracy of energy levels of NMGES potential equals the new values \( 2^n \). Furthermore, the global group symmetry (NC: 3D-RSP) corresponding NMGES potential reduce to the new subgroup (NC: 3D-RS) symmetries.

Keywords: schrödinger equation, hydrogenic atoms, more general exponential screened coulomb potential, noncommutative space and phase, star product and generalized Bopp’s shift method

Abbreviations: NMGES, new more general exponential screened coulomb potential; NC: 3D-RSP, noncommutativity three dimensional real space phase; CCRs, canonical commutations relations; NNCCRs, new noncommutative canonical commutations relations; SP, Schrödinger picture; HP, Heisenberg picture; MSE, modified Schrödinger equation

Introduction

The more general exponential screened coulomb (MGESC) potential is known to describe adequately the effective potential of a many–body system of a variety of fields such as the atomic, solid state, plasma and quantum field theory. In particularity, this potential used to calculate the bound state eigen values of molecules \((CO, NO)\). The noncommutativity of space–time, which known firstly by Heisenberg and was formalized by Snyder at 1947, suggest by the physical recent results in string theory. Very recently, several authors have attempted to obtain either the exact or approximate solutions of the non–relativistic Schrodinger equation or two relativistic (Klein–Gordon and Dirac) equations for different potentials in NC space. We want to extended, the study of Ita et al., to the case of extended quantum mechanics to the possibility of finding other applications and more profound interpretations in the sub-atoms scales based to the works and our previously works in this context. The no relativistic energy levels for hydrogenic atoms and molecules \((CO, NO)\), which interacted with NMGES potential in the context of NC space have not been obtained yet. The purpose of the present paper is to attempt study the MSE with NMGES potential (see below):

\[
V_{mg}(r) = -\frac{V_0}{r}(1 + (1+ar)^{1/2}a^{1/2}r^2)
\]

in (NC: 3D-RSP) symmetries using the generalized Bopp’s shift method which depend on the concepts that we present below in the third section. The new structure of extended quantum mechanics based to new NC canonical commutations relations (NNCCRs) in both Schrödinger and Heisenberg pictures (SP and HP), respectively, as follows (Throughout this paper, the natural units \( c = \hbar = 1 \) will be used):

\[
\begin{align*}
[x_i, p_j] &= [x_i(t), p_j(t)] = i\delta_{ij} \\
[x_i, x_j] &= [x_i(t), x_j(t)] = 0 \\
p_i, p_j &= [p_i(t), p_j(t)] = 0
\end{align*}
\]

However, the new operators \( \hat{\xi}(t) = [x_i(t) \mp p_j(t)] \) in (HP) are depending to the corresponding new operators \( \hat{\xi} = [x_i \mp p_j] \) in (SP) from the following projections relations:

\[
\hat{\xi}(t) = \exp(i\hat{H}_{mg}(t-t_0))*\xi*\exp(-i\hat{H}_{mg}(t-t_0))
\]

Here \( \xi = (x_i \mp p_j) \) and \( \xi(t) = (x_i(t) \mp p_j(t)) \), while the dynamics of new systems \( \frac{d\xi(t)}{dt} \) are described from the following motion equations in extended quantum mechanics:
quantum mechanics and it’s extension, while describe the dynamics of systems in (NC: 3D–RSP). The very small two parameters \( \Theta^n \) and \( \Theta^m \) (compared to the energy) are elements

\[
(f \ast g)(x, p) = \left( f \hat{g} \right)(x, p) = \left( f \ast g \right)(x, p) = \left( f - \frac{i}{2} \Theta^n \hat{p} f \hat{c} \hat{c} - g - \frac{i}{2} \Theta^m \hat{p} f \hat{c} \hat{c} \right)(x, p)
\]

where the notion \( \left( \hat{c}^n, \hat{c}^m \right)(x, p) \) denote to the \( \left( \frac{\partial}{\partial \beta^n}, \frac{\partial}{\partial \beta^m} \right) f(x, p) \). The effects of (space-space) and (phase-phase) noncommutativity properties, respectively induce the second and the third terms in the above equation. The organization scheme of the recently worked is given as follows: In next section, we briefly review the ordinary SE with MGESC potential on based ref.\(^3\). The Section 3 is devoted to studying the MSE by applying the generalized Bopp’s shift method for NMGESC potential. In the next subsection, by applying standard perturbation theory to find the quantum spectrum of \( n \)-th excited levels in for spin-orbital interaction in the framework of the global group (NC-3D: RSP) and then, we derive the magnetic spectrum for NMGESC potential. In the fourth section, we resume the global spectrum and corresponding NC Hamiltonian operator for NMGESC potential and corresponding energy levels of hydrogenic atoms and the molecules (CO, NO). Finally, the concluding remarks have been presented in the last section.

Overview of the eigenfunctions and the energy eigenvalues for MGESC potential for hydrogenic atoms and molecules (CO, NO):

In this section, we shall recall here the time independent SE for a MGESC potential \( V_{mg}(r) \), which studied by Ita et al.,\(^2\) and generalized to new form by Ita et al.,\(^3\) also in ref.:\(^4\)

\[
V_{mg}(r) = - \frac{e^2}{r} \left[ 1 + (1 + b) \exp(-2b) \right] V_{mg}(r) = - \frac{e^2}{r} \left[ 1 + (1 + ar) \exp(-2ar) \right]
\]

where \( a \to V_a \) and \( b \to \alpha \) are the strength coupling constant (the potential density of the MGESC potential) and the screened parameter (adjustable positive parameter), respectively. Part with exp. term of eq. \( 6 \) can be expanded in the power series of \( r \) up to the second term:

\[
\frac{1}{r^2} = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \left( 2\alpha r \right)^n = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \left( 2\alpha r \right)^n
\]

Inserting eq. (7) into eq. (6), explicit form of MGESC potential is obtained as:

\[
V(r) \equiv V_0 - \frac{2V_0}{r} - 2V_0 r
\]

If we insert this potential into the Schrödinger equation (3):

\[
\frac{d^2}{dx^2} + \frac{2}{r} \frac{d}{dr} \left[ \frac{l(l+1)}{r^2} \right] R_l(r) = 2l + 2 \left[ \frac{V_{mg}(r)}{r} \left( 1 + (1 + ar) \exp(-2ar) \right) \right] R_l(r) = 0
\]

Here \( \mu \) is the reduced mass of molecules (CO, NO) or the reduced mass of electron ant it’s nucleus for hydrogenic atoms. The electronic radial wave functions are shown as a function of the Laguerre polynomial in terms of some parameters:\(^5\)

\[
R_{\alpha}(\nu) = N_{\alpha}(2\beta)^{\frac{3-\alpha}{2}} \exp \left[ -\frac{\nu}{2} \right] \nu^{\alpha - \frac{1}{2}} \left( 2\beta \right)^{\alpha - \frac{3}{2}} (\nu)
\]

of two anti symmetric real matrices \( (\Theta^m, \Theta^n) = -(\Theta^n, \Theta^m) \) and \((\ast)\) denote to the new star product, which is generalized between two arbitrary functions \( f, g \) \((x, p) = \left( f \ast g \right)(x, p)\) to the new form \( \left( f \hat{g} \right)(x, p) \) in ordinary 3-dimensional space-phase:\(^{22,21}\)

\[
\Psi(r, \theta, \phi) = N_{mg}(2\beta)^{-1/2} \exp \left[ -\frac{\nu}{2} \right] \nu^{\alpha - 1/2} L_{\alpha}^{\nu}(\nu) \Psi(r, \theta, \phi)(11) \text{ and }
\]

\[
E_{mg} = V_0 e^{-\alpha} + 2\mu \left( \frac{V_0 + V_0 e^{-\alpha}}{n + l + 1} \right)^2
\]

With \( r_0 = 1.21282 \) and \( r_0 = 1.1508 \) for (CO and NO) molecules, for hydrogenic atoms, \( r_0 \) can be present the average dimension between the electron and the nucleus, \( N_{mg} \) is the normalization constant, \( \alpha = \frac{1}{2} \sqrt{4d(l+1) + 1} \), \( \beta^2 = \frac{2E_{mg}}{\alpha^3} \) and \( \nu_{mg} \) are the well-known spherical harmonic functions.

**Method and theoretical approach**

In this section, we shall give an overview or a brief preliminary for a NMGESC potential \( V_{mg}(r) \), in (NC: 3D-RSP) symmetries. To perform this task the physical form of modified Schrödinger equation (MSE), it is necessary to replace ordinary three-dimensional Hamiltonian operators \( \hat{H}(\rho, \xi) \), ordinary complex wave function \( \Psi(r) \) and ordinary energy \( E_{mg} \) by new three Hamiltonian operators \( \hat{H}_{mg}(\tilde{\rho}, \tilde{\xi}) \), new complex wave function \( \tilde{\Psi}(\tilde{r}) \) and new energies \( E_{mg} \), respectively. In addition to replace the ordinary old product by new star product \((\ast)\), which allow us to constructing the MSE in (NC: 3D-RSP) symmetries as:\(^{21-28}\)

\[
\tilde{\hat{H}}_{mg}(\tilde{\rho}, \tilde{\xi}) \tilde{\Psi}(\tilde{r}) = E_{mg} \tilde{\Psi}(\tilde{r}) = \tilde{H}(\tilde{\rho}, \tilde{\xi}) \ast \tilde{\Psi}(\tilde{r}) = E_{mg} \tilde{\Psi}(\tilde{r}) (13)
\]

The Bopp’s shift method employed in the solutions enables us to explore an effective way of obtaining the modified potential in extended quantum mechanics, it based on the following new commutators:\(^{35-34}\)

\[
\left[ \tilde{\hat{h}}, \tilde{\hat{p}} \right] = \left[ \tilde{\hat{h}}(t), \tilde{\hat{p}}(t) \right] = i\hbar \delta_t \text{ and } \left[ \tilde{\hat{p}}(t), \tilde{\hat{p}}(t) \right] = i\hbar \delta_t
\]

The new generalized positions and momentum coordinates \( \left( \tilde{\rho}, \tilde{\xi} \right) \) in (NC: 3D-RSP) are depended with corresponding usual generalized positions and momentum coordinates \( \left( \rho, \xi \right) \) in ordinary quantum mechanics by the following, respectively\(^{35-36}\)

\[
\left\{ \tilde{\rho}_1, \tilde{\rho}_2 \right\} = \left\{ \tilde{\rho}_1, \tilde{\rho}_2 \right\} = \left\{ \rho_1, \rho_2 \right\} = i\hbar \delta_t (15)
\]

The above equation allows us to obtain the two operators \( \tilde{\hat{\rho}} \) and \( \tilde{\hat{p}} \) in (NC: 3D-RSP)\(^{37-38}\)

\[
\left[ \tilde{\hat{\rho}}, \tilde{\hat{p}} \right] = \left[ \tilde{\hat{\rho}}^2, \tilde{\hat{p}}^2 \right] = \left[ \tilde{\hat{\rho}}^2 - \tilde{\hat{\rho}}^2, \tilde{\hat{p}}^2 + \tilde{\hat{p}}^2 \right] = i\hbar \delta_t \left( \tilde{\hat{\rho}} \tilde{\hat{p}} + \til\hat{p} \right)
\]

The two couplings \( L_\Theta \) and \( \tilde{\Theta} \) are \( \left( L_\Theta, L_\Theta, L_\Theta, L_\Theta \right) \) and \( \left( L_\Theta, L_\Theta, L_\Theta, L_\Theta \right) \) respectively and \( L_\Theta, L_\Theta, L_\Theta, L_\Theta \) are the

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three components of angular momentum operator \( \vec{L} \), while \( \Theta_0 = \theta_0 / 2 \). Thus, the reduced Schrödinger equation (without star product) can be written as:

\[
\hat{H}(\hat{\rho}_s, \hat{x}_s) \Psi(r) = E_s \Psi(r) = \hat{H}(\hat{\rho}_s, \hat{x}_s) \Psi(r) = E_s \Psi(r)
\]  

(17)

the new operator of Hamiltonian \( H_{\text{m-n}}(\rho_s, x_s) \) can be expressed as:

\[
H_{\text{m-n}}(\rho_s, x_s) \Rightarrow H_{\text{m-n}}(\rho_s, x_s) = \left[ \chi - \rho_0 \left( \frac{\rho_s}{2} \right), \rho_s \right] = \hat{\rho} + \frac{\rho_0}{2} x_s
\]  

(18)

Now, we want to find to the NMGESCT potential \( V_{\text{m-n}}(r) \):

\[
V_{\text{m-n}}(r) \Rightarrow V_{\text{m-n}}(r) = V_0 - \frac{2V_0}{r} - 2V_0 \alpha^2 r^2
\]  

(19)

After straightforward calculations, we can obtain the important term \( \left[ -\frac{V_0}{r} \right] \), which will be use to determine the NMGESCT potential in (NC: 3D-RSP) symmetries as:

\[
V_{\text{m-n}}(r) \Rightarrow \left[ -\frac{V_0}{r} \right] = \frac{V_0 \alpha}{r} - \frac{V_0 \alpha^2}{r} + \frac{V_0 \alpha^2}{r^2}
\]  

(20)

By making the substitution above equation into eq. (19), we find the global working new Hamiltonian operator \( H_{\text{m-n}}(r) \) satisfies the equation in (NC: 3D-RSP) symmetries:

\[
H_{\text{m-n}}(P_{t}, x_{t}) \Rightarrow H_{\text{m-n}}(P_{t}, x_{t}) = \left[ \left( \frac{2V_0 \alpha}{r} - \frac{V_0 \alpha^2}{r} \right) \theta + \frac{\rho_0 \theta_0}{2} \right] \hat{L} + \frac{\rho_0 \theta_0}{2}
\]  

(21)

where the operator \( H_{\text{m-n}}(P_{t}, x_{t}) \) is just the ordinary Hamiltonian operator with MGESC potential in commutative space:

\[
H_{\text{m-n}}(P_{t}, x_{t}) = \frac{P_3^2}{2\mu} + V_0 \alpha - \frac{V_0 \alpha^2}{r} + \frac{V_0 \alpha^2}{r^2}
\]  

(22)

while the rest two terms are proportional’s with two infinitesimals parameters \( H_{\text{m-n}}(r) \) and then we can considered as a perturbations terms \( H_{\text{m-n}}(r) \) in (NC: 3D-RSP) symmetries as:

\[
H_{\text{m-n}}(r) = \left( \frac{2V_0 \alpha^2}{r^2} \right) \theta \hat{L} + \frac{\rho_0 \theta_0}{2}
\]  

(23)

The exact modified spin-orbital spectrum for NMGESCT potential in global (NC: 3D- RSP) symmetries

\[
\frac{d^2}{dr^2} + \frac{2d}{r} \frac{d}{dr} \left( \frac{1}{r} \right) \hat{R}_{\text{m-n}}(r) + 2 \mu \left[ E_{\text{m-n}} + \frac{V_0}{r} (1 + \alpha + \alpha r \exp(-2\alpha r)) - \frac{2V_0 \alpha}{r} \right] \hat{L} \theta + \frac{\rho_0 \theta_0}{2}
\]  

\[
\hat{R}_{\text{m-n}}(r) = 0
\]  

(27)

The two terms which composed the expression of \( H_{\text{m-n}}(r) \) are proportional with two infinitesimals parameters \( (\theta \text{ and } \theta_0) \), thus, in what follows, we proceed to solve the modified radial part of the MSE that is, equation (27) by applying standard perturbation theory for their exact solutions at first order of two parameters \( \theta \text{ and } \theta_0 \).

The exact modified spin-orbital spectrum for NMGESCT potential in extended global (NC: 3D- RSP) symmetries

The purpose here is to give a complete prescription for determine the energy level of \( n^2 \) excited states, of hydrogenic atoms with NMGESCT potential, we first find the corrections \( E_{\text{m-n}} \) and \( E_{\text{d-m}} \), for hydrogenic atoms which have \( j = l+1/2 \) (spin up) and \( j = l-1/2 \) (spin down), respectively, at first order of two parameters \( \theta \) and \( \theta_0 \) obtained by applying the standard perturbation theory to find the following:

In this subsection, we apply the same strategy, which we have seen in our previously works, to reproduce both couplings to the new physical forms \( \gamma \Theta \vec{L} \vec{S} \) and \( \gamma \vec{L} \vec{S} \), respectively, to obtain the new forms of \( H_{\text{m-n}}(r, \Theta, \vec{B}) \) for 3D- NMGESCT potential as follows:

\[
H_{\text{m-n}}(r, \Theta, \vec{B}) = \gamma \left( \frac{2V_0 \alpha^2}{r^2} \right) \theta \hat{L} \vec{S} + \frac{\rho_0 \theta_0}{2}
\]  

(24)

Here \( \gamma = \frac{1}{137} \) is a new constant, which play the role of fine structure constant, we have chosen the two vectors \( \Theta \) and \( \vec{B} \) parallel to the spin \( \vec{S} \) of hydrogenic atoms. Furthermore, the above perturbative terms \( H_{\text{m-n}}(r) \) can be rewritten to the following new form:

\[
H_{\text{m-n}}(r, \Theta, \vec{B}) = \gamma \left( \frac{2V_0 \alpha^2}{r^2} \right) \theta \hat{L} \vec{S} + \frac{\rho_0 \theta_0}{2}
\]  

(25)

This operator traduces the coupling between spin \( \vec{S} \) and orbital moment \( \vec{L} \). The set \( (H_{\text{m-n}}(r, \Theta, \vec{B}), \vec{L}, \vec{S}^2, J, J_3) \) forms a complete of conserved physics quantities and for \( \vec{S} = 1/2 \), the eigen values of the spin orbital coupling operator are \( k_1 = \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \), \( k_2 = \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \) corresponding: \( j = l+1/2 \) (spin up) and \( j = l-1/2 \) (spin down), respectively then one can form a diagonal \((3 \times 3)\) matrix, with diagonal elements are \( \left( H_{\text{m-n}}(r) \right)_{11} \) and \( \left( H_{\text{m-n}}(r) \right)_{33} \) for NMGESCT potential in (NC: 3D-RSP) symmetries as:

\[
\left( H_{\text{m-n}}(r) \right)_{11} = \gamma k_1 \left( \frac{2V_0 \alpha^2}{r^2} \right) \theta \hat{L} \vec{S} + \frac{\rho_0 \theta_0}{2}
\]  

(26)

\[
\left( H_{\text{m-n}}(r) \right)_{33} = \gamma k_2 \left( \frac{2V_0 \alpha^2}{r^2} \right) \theta \hat{L} \vec{S} + \frac{\rho_0 \theta_0}{2}
\]  

(27)

After profound calculation, one can show that, the new radial function \( R_{\text{m-n}}(r) \) satisfying the following differential equation for NMGESCT potential:
where $T_2(-m,e,e,-n,e,\lambda+1)$ is obtained from the generalized hyper geometric function, $P_{\alpha}(\alpha_1,\ldots,\alpha_p,\beta_1,\ldots,\beta_q,\beta)$, for $p=3$ and $q=2$ while $\Gamma(x) = \int_0^{\infty} z^{x-1} e^{-z} dz$ denote to the usual Gamma function. After straightforward calculations, we can obtain the explicitly results:

$$T_1(n,a) = 2V_0^{1/2} T_1(n,a) = 2V_0^{1/2} \Gamma(n+2a+2) n! n! \int_0^{\infty} \left| L_{2a+1}^{n+1}(\nu) \right|^2 d\nu$$

(30)

$$T_2(n,a) = -V_0^{1/2} \Gamma(n+2a+2) n! n! \int_0^{\infty} \left| L_{2a+1}^{n+1}(\nu) \right|^2 d\nu$$

(31)

To evaluate the above factors $T_1(n,a), T_2(n,a)$ and $T_3(n,a)$, we apply the following special integration:

$$\int_0^{\infty} \exp(-\nu) L_{2a+1}^{n+1}(\nu) L_{2a+1}^{n+1}(\nu) d\nu = \frac{\omega E}{m! n! \Gamma(1+\beta) \Gamma(1+\lambda)} F_i(-m,e,e,-n,e,\lambda+1)$$

We have $\Lambda_{mod-\varepsilon} = \vec{B} \cdot \vec{J} - N_e$ denote to the modified Zeeman effect while $N_e = -\vec{S} \cdot \vec{B}$ is the ordinary Hamiltonian operator of Zeeman Effect. To obtain the exact noncommutative magnetic modifications of energy $E_{mod-\varepsilon}(n,m,\alpha)$, we just replace $k$ and $\Theta$ in the eq. (33) by the following parameters: $m$ and $\chi$, respectively:

$$E_{mod-\varepsilon}(n,m,\alpha) = -\frac{1}{2} \chi N_{\varepsilon_{mod-\varepsilon}}(2\beta^2) V_{k.f}(n,\alpha) B m$$

(37)

We have $-l \leq m \leq +l$, which allow us to fixing ($2l+1$) values for discreet number $m$.

**Results**

In the light of the results of the preceding sections, let us resume the modified eigenenergies $E_{mod-\varepsilon}(n,m,\alpha)$ of a hydrogenic atom with spin $S = 1/2$ for MSE with NMGESC potential obtained in this paper, the total energies corresponding $n^\text{th}$ excited states in NC: 3D-RSP symmetries are determined based to our original results presented on the Eqs. (33) and (37), in addition to the ordinary energy $E_{mod-\varepsilon}$ NMGESC potential, which presented in the eq. (13):

$$E_{mod-\varepsilon}(n,j,l,s,m,\alpha) = E_{mod-\varepsilon}(n,j,l,s,m,\alpha) - \frac{1}{2} \chi N_{\varepsilon_{mod-\varepsilon}}(2\beta^2) V_{k.f}(n,\alpha)(\Theta_k + B m \chi)$$

(38)

Thus, the extended global quantum group symmetry (NC: 3D-RSP) reduces to new quantum subgroup symmetry (NC: 3D-RS).

**The exact modified magnetic spectrum for NMGESC potential in extended global (NC: 3D-RSP) symmetries**

Further to the previously obtained results, now, we consider another physically meaningful phenomena produced by the effect of NMGESC potential related to the influence of an external uniform magnetic field $\vec{B}$, to avoid the repetition in the theoretical calculations, it’s sufficient to apply the following replacements:

$$\Theta \rightarrow \vec{B} \cdot \sigma \rightarrow \vec{B} \cdot \sigma$$

(35)

Here $\chi$ and $\sigma$ are two infinitesimal real proportional’s constants, and we choose the arbitrary external magnetic field $\vec{B}$ parallel to the (Oz) axis, which allow us to introduce the new modified magnetic Hamiltonian $H_{mod-\varepsilon}$ in (NC: 3D-RSP) symmetries as:

$$H_{mod-\varepsilon} = \left( 2V_0 \sigma^2 - \frac{V_0}{2\mu} \right) \chi \left( \frac{\sigma}{2\mu} \right) N_{mod-\varepsilon}$$

(36)

This is the main goal of this work. It’s clearly, that the obtained eigenvalues of energies are real’s and then the noncommutative diagonal Hamiltonian $H_{mod-\varepsilon}$ is Hermitian, furthermore it’s possible to writing the three elements as follows:

$$\left( H_{mod-\varepsilon} \right)_{ij} = -\frac{\Delta_{nc}}{2\mu} + H_{int-\varepsilon}$$

(39)

$$H_{int-\varepsilon} = \frac{\Delta_{nc}}{2\mu} - \frac{\Delta_{nc}}{2\mu} \vec{B} \cdot \vec{L} \cdot \sigma \cdot \vec{L}$$

(40)

Thus, the ordinary kinetic term for MGE potential $-\frac{\Delta_{nc}}{2\mu}$ and ordinary interaction $-\frac{V_0}{2\mu} (1+\alpha r^2) \exp(-2ar)$ are replaced by new modified form of kinetic term $-\frac{\Delta_{nc}}{2\mu}$ and new modified interactions.

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modified to the new form \((H_{\text{int-ang}} \text{ and } H_{\text{int-dng}})\). On the other hand, it is evident to consider the quantum number \(m\) takes \((2l+1)\) values and we have also two values for \(2(2l+1)\), thus every state in usually three dimensional space of energy for NMGESG potential will be \(2(2l+1)\) sub-states. To obtain the total complete degeneracy of energy level of the NMGESG potential in noncommutative three-dimension spaces-phases, we need to sum for all allowed values of \(l\). Total degeneracy is thus,

\[
2 \sum_{l=0}^{\infty} (2l+1) = 2n^2
\] (41)

\[
E_{\text{sc-ang}}(n,j,l,s,m,\alpha) = -V_0 e^{-\alpha} + 2\mu \left( \frac{V_0 + V_0 e^{-\alpha}}{n+l+1} \right)^2 - \frac{2}{2} \chi N_{\alpha} \left( 2p \right)^{2-2n} V_{\alpha f}(n,\alpha) \Theta(k(j,l,s) + Bn\chi) \] (42)

Paying attention to the behavior of the spectra (38) and (42) \(E_{\text{sc-ang}}(n,j,l,s,m,\alpha)\) and \(E_{\text{sc-dng}}(n,j,l,s,m,\alpha)\), it is possible to recover the results of commutative space (12) when we consider \((\Theta, \chi) \rightarrow (0,0)\). Finally, we can say that the results we have obtained in our recently research are more profound than the results listed in our reference.20

**Conclusion**

In this paper three-dimensional MSEs for NMGESG potential has been solved via Bopp’s shift method and standard perturbation theory in (NC: 3D-RSP) symmetries, we resume the main obtained results:

1. The exact energy spectrum \(E_{\text{sc-ang}}(n,j,l,s,m,\alpha)\) and \(E_{\text{sc-dng}}(n,j,l,s,m,\alpha)\) for \(n^N\) excited levels, for hydrogenic atoms,

2. Ordinary interaction \(\left( -\sum \left( 1+2+1 \alpha \right) \exp(-2\alpha r) \right)\) were replaced by new modified interactions \(3 \left( H_{\text{int-ang}} \text{ and } H_{\text{int-dng}} \right)\) for hydrogenic atoms.

3. The ordinary kinetic term \(-\Delta / 2\mu\) modified to the new form \(\Delta_{\text{sc}} / 2\mu \equiv \Delta - \Delta_{\text{L}} - \Delta_{\text{S}}\) for NMGESG potential,

4. Spectra of bounded diatomic molecules \((CO, NO)\) were studied analytically. NMGESG potential has been used to model the molecules \((CO, NO)\)

5. We have shown that, the group symmetry (NC: 3D-RSP) corresponding NMGESG potential reduce to the symmetry subgroup (NC: 3D-RS).

6. It has been shown that, the MSE presents useful rich spectrums for improved understanding of hydrogenic atoms and molecules \((CO, NO)\) influenced by the NMGESG potential and we have seen also that the modified of spin-orbital and modified Zeeman effect were appears du the presence of the two infinitesimal parameters \((\Theta, \chi)\) which are induced by position-position noncommutativity property of space.

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None.

**Conflicts of interest**

Author declares that there is no conflicts of interest.

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