Synchronization of Delayed Neural Networks With Actuator Failure Based on Stochastic Sampled-Data Controller

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ABSTRACT This paper addresses the master-slave synchronization problems of delayed neural networks with actuator failure based on stochastic sampled-data controller. To simplify the analysis process, only two different sampling periods whose occurrence probabilities follow the Bernoulli distribution are considered. In addition, it can be further extended to cases with multiple random sampling periods. The sampling system with random parameters is transformed into a continuous system through applying the input delay method. The novelty of this article is to consider the problem of actuator failure which may exist in the real world. By constructing a new type of Lyapunov-Krasovskii function (LKF), a sampling controller for neural networks synchronization system is designed. Using Jensen’s inequality, Wirtinger’s inequality and convex optimization methods, the stability criterion of neural networks with low conservativeness is acquired. Meanwhile, the controller gain matrix can be obtained through solving the linear matrix inequalities (LMIs). One numerical example provides feasibility and advantages of theoretical results.

INDEX TERMS Stochastic sampling, actuator failure, linear matrix inequalities (LMIs), neural networks.

I. INTRODUCTION

Nowadays, there has been a large number of researches on neural networks because they are widely used in various fields such as signal processing, image processing, pattern recognition, optimization, and associative memory design and so on [1], [2]. For the control community, the attractiveness of neural networks is that they can fully approximate complex nonlinear mapping relationships, and they can learn and adapt to the dynamic characteristics of uncertain systems. In this form, the introduction of neural networks into control systems is an inevitable trend in the development of control disciplines. Based on the method of linear matrix inequalities (LMIs), many studies have devoted to the stability analysis of master-slave synchronization problems of neural networks. There are lots of achievements gained in recent years [3]–[7]. However, in reality, the time delays inevitably occur in neural networks, which may result instability, oscillation, poor performance of the system [8]. Using the input delay method, the study of synchronous control on neural networks with delays has become a focus topic [9]. Hence, the stability analysis of master-slave synchronization problems of delayed neural networks has received an increasing attention [10], [11].

Along with the researches on neural networks, the synchronous problem of neural networks system has gradually turned into an indispensable research area. The synchronous control of the neural networks is under the stability for the neural networks. A lot of significant methods for master-slave synchronization have been mentioned in recent years, such as pinning control [12], impulsive control [13], [14], event-triggered control [15] and sampled-data control [16], [17].

Benefit from advancement of computer technology, the sampled-data control systems have been attracted increasing interest in the past decades [18]. For synchronization,
the sampled-data only needs the information about the state of the system at the sampling instants [19]. The characteristic of this method is to reduce the transmission of information and improve the efficiency of control. There is an important issue to choose the sampling period when using sampled data control to achieve neural networks synchronization. A longer sampling interval will bring lower communication channel occupation, less signal transmission, and fewer drive controllers signal [20], [21]. Hence, how to obtain a larger upper bound of the sampling periods is the focus of the method [22]. In the past, many studies have been devoted to sampling all signals at a constant rate on a single-rate digital control system. However, continuous sampling is not applicable because of the human factors and uncertain interference of environmental [23]. Thus, it is especially significant to consider time-varying sampling for synchronous control [24]. It should be noted that in spite of sampled-data control technologies have been developed well in control theory, the synchronization problem of particular sampled-data on networks has so far attracted very little attention because of the random interference and mathematical complexity of the constrained system [25]. It is worth mentioning that, in [26], a new method about stochastic switched sampled-data control with time-varying has been proposed. In [27], the probabilistic sampling \( H_{\infty} \) problem of sampled-data systems with parameter uncertainties based on input delay method has been researched. To simplify the analysis process, only two sampling intervals have been considered. Stimulated by [27], Cao et al. have designed a random variable sampling controller for multiple intervals of delayed neural networks in [2].

In another research field, an inevitable disadvantage that we should not ignore is the sensor or actuator failures in various situations [28]. It is noted that, in [29]–[31], the actuator is assumed that there is not failed. But in actual systems, especially in networks conditions, actuator failures can inevitably occur. This will severely degrade the performance of systems. Even worse, the systems may become unstable. Therefore, in order to determine the reliability of the sampled data, it is meaningful to study the stability of the sampled data on the neural networks in the case of actuator failure.

Inspired by the above ideals, the main research contents and results of the paper are generalised as follows: (1) In a set of LMIs, the new stability conditions can be expressed by constructing a suitable Lyapunov functional and using Jensen’s inequality, Wirtinger-type inequality and reciprocally convex method; (2) It is assumed that the sampling time varies with time and can be arbitrarily switched between two different values. Moreover, the method can also be applied to multiple random sampling times; (3) Unlike previous studies, the article proposes a reliable control scheme for the delayed neural network without actuator failure via stochastic sampled-data control. Sufficient conditions are presented to guarantee the stability and the desired controller can be obtained.

II. PROBLEM STATEMENT AND PRELIMINARY

Consider the neural networks as follow:

\[
\dot{x}(t) = -Ax(t) + Bg(x(t)) + Cg(y(t - d(t))) + J(t),
\]

where \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \) represents the state vector. \( A = \text{diag}[a_1, a_2, \ldots, a_n] \) denotes the positive definite diagonal matrix with \( a_i > 0 \). \( B = (b_{ij})_{n \times n}, C = (c_{ij})_{n \times n} \) are the connection weight matrices. \( J(t) = [J_1(t), \ldots, J_n(t)]^T \) denotes a constant input vector. \( g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T \) represents the neuron activation function. The delay \( d(t) \) is continuous and satisfies the conditions: \( 0 \leq d(t) \leq d, \dot{d}(t) \leq \mu, \) where \( d, \mu \) are known scalars.

A assumption of neuron activation function can be described as follow.

**Assumption 1:** The neuron activation function \( g_i(\cdot) \) is continuous and bounded, if there exist constants \( \sigma_i^- \) and \( \sigma_i^+ \)

\[
\sigma_i^- \leq \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leq \sigma_i^+, \quad i = 1, 2, \ldots, n,
\]

where \( s_1, s_2 \in \mathbb{R}^n \), and \( s_1 \neq s_2 \).

When the system (1) is the master system in the paper, and a slave system is ascertained as:

\[
\dot{y}(t) = -Ay(t) + Bg(y(t)) + Cg(y(t - d(t))) + J(t) + u^F(t),
\]

where the structure is same as (1). \( u^F(t) \in \mathbb{R}^n \) represents the measurable control with failure described by

\[
u^F(t) = \mathcal{F}(t)u(t),
\]

where \( \mathcal{F}(t) \) is the time-varying failure level of actuators.

Combining (1), (3) and (4) with \( e(t) = y(t) - x(t) \), the synchronization error system (SES) can be formulated as:

\[
\dot{e}(t) = -Ae(t) + Bf(e(t)) + Cf(e(t - d(t))) + \mathcal{F}(t)u(t),
\]

where \( f(e(t)) = g(y(t)) - g(x(t)), f(e(t - d(t))) = g(y(t - d(t))) - g(x(t - d(t))) \). Now, the function \( f_i(s) \) meets the conditions as follow:

\[
\sigma_i^- \leq \frac{f_i(s)}{s} \leq \sigma_i^+, \quad i = 1, 2, \ldots, n,
\]

where \( s \in \mathbb{R} \) and \( s \neq 0 \).

In this article, the sampled-data feedback controller \( u(t) \) used in synchronizing neural networks is generated by the zero-order holder (ZOH), which is mathematically modeled and includes using conventional digital-to-analog converter to complete practical signal reconstruction, and converting discrete signals into continuous signals. This is to say, it is keep the sampling value unchanged during the sampling interval and continue until the next sampling interval, with a battery of holding time \( 0 = t_0 \leq t_1 \leq \cdots \leq t_k \leq \cdots \leq \lim_{k \to +\infty} t_k = +\infty, \) i.e., Only the measured discrete
sampled data is available for control purposes. Summarizing
the above, the following form is the representation for the
sampled-data controller:
\[ u(t) = Ke(t_k), \quad t_k \leq t < t_{k+1}, \quad (7) \]
where \( K \) denotes the gain matrix of sampled-data controller
to be resolved, \( t_{k+1} - t_k \equiv h \) is defined as the sampling period,
with any \( k \geq 0 \), and \( h > 0 \) denotes the upper limit of sampling
periods.

In addition, instead of the actuator outage, define the failure
level \( F(t) \equiv diag(F_1(t), F_2(t), \ldots, F_n(t)) \) with \( F_a(t)(a = 1, 2, \ldots, n) \)
satisfying
\[ F_a \leq F_a(t) \leq \bar{F}_a, \]
where \( F_a, \bar{F}_a \in (0, 1] \) are the lower and upper bound of
\( F_a(t) \). Specially, \( F_a = \bar{F}_a = 1 \) represents that no failures
happen on \( u(t) \). Here, \( F(t) \) is divided into following forms:
\[ F(t) = \tilde{F}_a + \Delta F(t), \]
where \( \tilde{F} = diag(\tilde{F}_1, \tilde{F}_2, \ldots, \tilde{F}_n) \), and the uncertainty
term \( \Delta F(t) = diag(\Delta F_1(t), \Delta F_2(t), \ldots, \Delta F_n(t)) \)
with \( \Delta F_a(t) \leq \tilde{F}_a - \frac{\tilde{F}_a - F_a}{2} \).

Let \( \Delta F(t) = \Gamma(t)\bar{F} \) with \( \tilde{F} = diag(\tilde{F}_1, \tilde{F}_2, \ldots, \tilde{F}_n, -\frac{\tilde{F}_a - F_a}{2}) \), then \( \Gamma(t)\bar{F} \leq 1 \). Combined with (7), we have
\[ uF(t) = F(t)Ke(t_k), \quad t_k \leq t < t_{k+1}. \quad (8) \]

So Eq.(5) can be expressed as
\[ \dot{\xi}(t) = -A\xi(t) + Bf(e(t)) + Cf(e(t) - d(t)) + F(t)Ke(t_k). \quad (9) \]

Given the time-varying delay \( \tau(t) = t - t_k \) which is
piecewisely linear, and \( \tau(t) \leq t_{k+1} - t_k \). There, we can
transform (9) into:
\[ \dot{\xi}(t) = -A\xi(t) + Bf(e(t)) + Cf(e(t) - d(t)) + F(t)Ke(t - \tau(t)). \quad (10) \]

Assuming that there are two sampling intervals, the values are \( c_1, c_2 \) and \( 0 < c_1 < c_2 \) whose happening probabilities are known:
\[ \Prob[h = c_1] = \gamma, \quad \Prob[h = c_2] = 1 - \gamma. \quad (11) \]

When the sampling interval is \( c_1 \), \( \tau(t) \in [0, c_1) \) with its probability:
\[ \Prob[0 \leq \tau(t) < c_1|h = c_1] = 1. \quad (12) \]

When the sampling interval is \( c_2 \), the probabilities of \( \tau(t) \in [0, c_1) \) and \( \tau(t) \in [c_1, c_2) \) are \( \frac{c_2 - c_1}{c_2} \) and \( \frac{c_2 - c_1}{c_2} \), respectively.

The corresponding probabilities are as follows:
\[ \Prob[0 \leq \tau(t) < c_1|h = c_2] = \frac{c_1}{c_2}, \quad (13) \]
\[ \Prob[c_1 \leq \tau(t) < c_2|h = c_2] = \frac{c_2 - c_1}{c_2}. \quad (14) \]

Therefore, the probability of \( \tau(t) \) can be calculated as:
\[ \Prob[0 \leq \tau(t) < c_1] = \Prob[0 \leq \tau(t) < c_1|h = c_1] \times \Prob[h = c_1] \]
\[ + \Prob[0 \leq \tau(t) < c_1|h = c_2] \times \Prob[h = c_2] \]
\[ = \gamma + \frac{c_1}{c_2}(1 - \gamma), \quad (15) \]
\[ \Prob[c_1 \leq \tau(t) < c_2] = \Prob[c_1 \leq \tau(t) < c_2|h = c_1] \times \Prob[h = c_1] \]
\[ + \Prob[c_1 \leq \tau(t) < c_2|h = c_2] \times \Prob[h = c_2] \]
\[ = \frac{c_2 - c_1}{c_2}(1 - \gamma). \quad (16) \]

The stochastic variable \( \beta(t) \) that satisfies a Bernoulli dis-
tribution is defined as:
\[ \beta(t) : \begin{cases} 1, & 0 \leq \tau(t) < c_1 \\ 0, & c_1 \leq \tau(t) < c_2. \end{cases} \quad (17) \]

Then we have:
\[ \Prob[\tau(t) = 1] = \Prob[0 \leq \tau(t) < c_1] = \gamma + \frac{c_1}{c_2}(1 - \gamma) \equiv \beta, \]
\[ \Prob[\tau(t) = 0] = \Prob[0 \leq \tau(t) < c_2] = \frac{c_2 - c_1}{c_2}(1 - \gamma) \equiv 1 - \beta. \quad (18) \]

We can easily get
\[ \E[\beta(t) - \beta] = 0, \quad \E[(\beta(t) - \beta)^2] = \beta(1 - \beta). \quad (19) \]

Comprehend the above conditions, SES (10) is converted into:
\[ \dot{\xi}(t) = -A\xi(t) + Bf(e(t)) + Cf(e(t) - d(t)) + \beta(t)F(t)Ke(t - \tau(t)) \]
\[ + (1 - \beta(t))F(t)Ke(t - \tau_2(t)), \quad (20) \]

where \( 0 \leq \tau_1(t) < c_1, c_1 \leq \tau_2(t) < c_2 \).

Next are the lemmas that will be applied in the proof.

Lemma 1 [19]: Let \( x(t) \) be a differentiable function:
\[ x(t_1, t_2) \rightarrow \mathbb{R}^n. \]
Given a vector \( \xi \in \mathbb{R}^m > 0 \), symmetric matrices \( R \in \mathbb{R}^{m \times m} \)
and \( N_1, N_2 \in \mathbb{R}^{n \times m} \), the inequality is established:
\[ -\int_{t_1}^{t_2} \xi^T(s)R\xi(s)ds \]
\[ \leq (t_2 - t_1)\xi^T[N_1^T R^{-1} N_1 + \frac{(t_2 - t_1)^2}{3} N_2^T R^{-1} N_2] \xi \]
\[ + 2\xi^T \left[ N_1^T x(t_2) - x(t_1) \right] - 2N_2^T \int_{t_1}^{t_2} \xi^T x(s)ds \]
\[ - 2(t_2 - t_1)\xi^T N_2 x(t_2) - x(t_1)). \]

Lemma 2 [33]: For given positive integers \( n, m \), a scalar \( \alpha \in (0, 1) \), a \( n \times n \)-matrix \( G > 0 \) and two matrices \( M_1 \) and \( M_2 \) in \( \mathbb{R}^{m \times m} \), for all vector \( \zeta \in \mathbb{R}^m \), the following function \( \Theta(\alpha, G) \) can be got by:
\[ \Theta(\alpha, G) = \frac{1}{\alpha} \xi^T M_1^T GM_1 \xi + \frac{1}{1 - \alpha} \xi^T M_2^T GM_2 \xi. \]
if there exist matrix $X \in \mathbb{R}^{n \times n}$ and $[G \ X] > 0$, the following inequality holds:

$$
\min_{\alpha \in (0,1)} \Theta(\alpha, R) \geq \begin{bmatrix} M_1 \zeta \varepsilon \ G & X \\ M_2 \zeta & G \end{bmatrix} \begin{bmatrix} M_1 \zeta \varepsilon \\ M_2 \zeta \end{bmatrix}.
$$

Lemma 3 [32]: For given $\Gamma^T = \Gamma_1, \Gamma_2$ and $\Gamma_3$ with appropriate dimensions, if $\Delta(t) \Delta(t) \leq I$, then

$$
\Gamma_1 + \Gamma_2 \Delta(t)/3 + \Gamma_3 \Delta(t) \Gamma_2 < 0.
$$

It is equivalent that there exists $\varepsilon > 0$ satisfying

$$
\Gamma_1 + \varepsilon \Gamma_2 \Gamma_2 + \frac{1}{\varepsilon} \Gamma_3 \Gamma_3 < 0.
$$

### III. MAIN RESULTS

To simplify the matrix expression, the block input matrix can be defined as $e_i = e_i = [0, \ldots, 0, \eta_i, 0, \ldots, 0, \eta_i, 0, \ldots, 0]$ $(i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)$, the rest of the symbols used as follows:

$$
W_i = \text{diag}(\sigma_i^+ \sigma_i^+ \sigma_2^+ \ldots \sigma_n^+ \sigma_n^+),
$$

$$
W_2 = \text{diag}(\sigma_1^+ \sigma_1^+ \sigma_2^+ \ldots \sigma_n^+ \sigma_n^+),
$$

$$
\eta^T(t) = [e^T(t), f^T(e(t))],
$$

$$
\sigma(t) = [e^T(t), e^T(t - d, e^T(t), f^T(e(t)), f^T(e(t - d)), e^T(t - \tau_1(t)), e^T(t - \tau_2(t)), e^T(t - c_2)].
$$

**Theorem 1:** For given scalars $\beta, \mu, d, \varepsilon, \eta_1, \eta_2, c_1, c_2$ and controller gain matrix $K$, the error system (20) is stochastically stable, if there exist positive matrices $P_1, P_2, P_3, P_4, R_1, R_2, Q_1, Q_2$ and positive diagnose matrices $V_1, V_2$, any appropriate dimensional matrices $N_1, N_2, X_1, X_2, G$ satisfying:

$$
\gamma_1 = \begin{bmatrix} S_2 & X_1 \\ * & S_2 \end{bmatrix} > 0,
$$

$$
\gamma_2 = \begin{bmatrix} R_2 & X_2 \\ * & R_2 \end{bmatrix} > 0,
$$

$$
\Xi_1 = \begin{bmatrix} \varepsilon & \Gamma^T_1 \\ \Gamma^T_2 & dN_i - d^2N_2 \\ * & - \varepsilon I \\ * & * & - \varepsilon I \\ * & * & * & - dZ_i \\ * & * & * & * & - 3dZ_1 \end{bmatrix} < 0,
$$

where

$$
\Xi_1 = 2e^T(\varepsilon P_1 + 2\Pi^T_1(Q_1 + Q_2)\Pi_1 - \Pi_1^T Q_1 \Pi_2 - (1 - \mu)\Pi^T_2 Q_2 P_1 + d^2 \varepsilon^T Z_1 d \varepsilon_2 + d^2 \varepsilon^T Z_2 e^T e_1 - e^T Z_2 e_1 + 2 \Pi^T_4 \varepsilon_1 \varepsilon_2 - 2 \Pi^T_4 \varepsilon_1 \varepsilon_2 - 2d \Pi^T_4 \varepsilon_1 \varepsilon_2 + \beta \varepsilon_1 S_1 \varepsilon_1 - \varepsilon_1^T L_1 e_1 - (1 - \beta) \varepsilon_1 R_1 e_1 + \varepsilon_1^T L_2 e_2 - \beta \Pi_5 \varepsilon_2 \Pi_5 - (1 - \beta) \Pi_6 \varepsilon_2 \Pi_6 + 2 \Gamma_1 \Gamma_2 + \Pi_1 \Gamma_3 \Gamma_3 + \Pi_3 \Gamma_4 \Gamma_4,
$$

$$
\Pi^T_2 = \begin{bmatrix} e_2^T, e_4^T \end{bmatrix}, \quad \Pi^T_2 = \begin{bmatrix} e_3^T, e_1^T \end{bmatrix}, \quad \Pi^T_3 = \begin{bmatrix} e_5^T, e_9^T \end{bmatrix}, \quad \Pi^T_4 = \begin{bmatrix} e_2^T, e_4^T, e_3^T, e_1^T, e_6^T, e_7^T, e_5^T, e_9^T, e_1^T, e_1^T, e_1^T, e_1^T, e_1^T, e_1^T, e_1^T, e_1^T, e_1^T, e_1^T, e_1^T \end{bmatrix},
$$

$$
L_1 = \beta S_1 + (1 - \beta) R_1, L_2 = \beta S_2 + (1 - \beta)(e_2 - c_1)^2 R_2
$$

$$
\Pi^T_5 = \begin{bmatrix} e_6^T - e_7^T, e_1^T - e_6^T \end{bmatrix}, \quad \Pi^T_6 = \begin{bmatrix} e_3^T - e_9^T, e_7^T - e_3^T \end{bmatrix},
$$

$$
\Gamma_1 = [\varepsilon_1 I, 0, \varepsilon_2 I, 0, 0, 0, 0, 0, 0, 0, 0, 0],
$$

$$
\Gamma_2 = [-G A, 0, -G B, 0, \beta G F K, 0, (1 - \beta) G F K, 0, 0, 0, 0, 0],
$$

$$
\Gamma_3 = [0, 0, 0, 0, 0, \beta G F K, 0, (1 - \beta) G F K, 0, 0, 0, 0],
$$

$$
\gamma_3 = [-W_1 V_1 W_2 V_1, * - W_1 V_1 W_2 V_2].
$$

**Proof:** Constructing the LKF candidate as follows:

$$
V(t) = \sum_{i=1}^{4} V_i(t), \quad t \in [t_k, t_{k+1}],
$$

where

$$
V_1(t) = e^T(t) P_1 e(t),
$$

$$
V_2(t) = \int_{t-d}^{t} \eta_i^T(s) Q_1 \eta_i(s) ds + \int_{t-d}^{t} \eta_1^T(s) Q_2 \eta_1(s) ds,
$$

$$
V_3(t) = \int_{d-t}^{t} \varepsilon (s) Z_1 \varepsilon (s) ds + d \int_{d-t}^{t} \varepsilon (s) Z_2 \varepsilon (s) ds,
$$

$$
V_4(t) = \beta \left[ \int_{t-c_1}^{t} \varepsilon (s) S_1 \varepsilon (s) ds + c_1 \int_{t-c_1}^{t} \varepsilon (s) S_2 \varepsilon (s) ds \right],
$$

The following equation represents the infinitesimal generator (defined as $\mathcal{L}$) of $V(t)$:

$$
\mathcal{L} V(t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left[ V(t + \Delta t) - V(t) \right].
$$

Calculate the time derivation of $V(t)$ on the trajectory of the error system (20), we can obtain that

$$
\mathcal{L} V_1(t) = 2 \varepsilon^T(t) P_1 e(t),
$$

$$
\mathcal{L} V_2(t) \leq \eta_i^T(s) Q_1 \eta_i(s) (t - d) Q_1 \eta_i(s) (t - d) + \eta_i^T(s) Q_2 \eta_i(s) (t - d) + \eta_1^T(s) Q_2 \eta_1(s) (t - d),
$$

$$
\mathcal{L} V_3(t) = d \varepsilon^T(t) Z_1 \varepsilon (t) - \int_{t-d}^{t} \varepsilon^T(s) Z_1 \varepsilon (s) ds + d^2 \varepsilon^T(t) Z_2 \varepsilon (t) - d \int_{t-d}^{t} \varepsilon^T(s) Z_2 \varepsilon (s) ds,
$$

$$
\mathcal{L} V_4(t) = \beta \varepsilon^T(t) S_1 \varepsilon (t) - \varepsilon^T(t - c_1) L_1 \varepsilon (t - c_1).
\[-(1 - \beta)e^T(t - c_2)R_1e(t - c_2) + \dot{e}^T(t)L_2\dot{e}(t) \]
\[-\beta c_1 \int_{t-c_1}^t \dot{e}^T(s)S_2\dot{e}(s)ds \]
\[\neg (1 - \beta)(c_2 - c_1) \int_{t-c_2}^{t-c_1} \dot{e}^T(s)R_2\dot{e}(s)ds. \quad (28)\]

By using Lemma 1, we obtain:
\[-\int_{t-d}^{t} \dot{e}^T(s)Z_1\dot{e}(s)ds \leq d\eta^T(t) \left[ N_1^TZ_1^{-1}N_1 + \frac{d^2}{3}N_2^TZ_1^{-1}N_2 \right] \eta(t) \]
\[+ 2\eta^T(t) \left[ N_1^T(e(t) - e(t - d)) - 2N_2^T \int_{t-d}^{t} \dot{e}^T(s)ds \right] \]
\[\neg 2d\eta^T(t)N_2^T \left[ e(t) + e(t - d) \right]. \quad (29)\]

For any compatible matrix \(X_1\) and \(Y_1 = \begin{bmatrix} S_2 & X_1 \\ \ast & S_2 \end{bmatrix} > 0\), when \(0 < \tau_1(t) < c_1\), the following inequality can be obtained by Jensen’s inequality and Lemma 2:
\[-\beta p_1 \int_{t-c_1}^t \dot{e}^T(s)S_2\dot{e}(s)ds \leq -\beta \eta^T(t)P_1S_2P_1^T \eta(t). \quad (30)\]

Similarly, for any compatible matrix \(X_2\) and \(Y_2 = \begin{bmatrix} R_2 & X_2 \\ \ast & R_2 \end{bmatrix} > 0\), when \(c_1 < \tau_2(t) < c_2\), we can obtain:
\[-(1 - \beta)p_2 \int_{t-c_2}^{t-c_1} \dot{e}^T(s)R_2\dot{e}(s)ds \leq -(1 - \beta)\eta^T(t)P_2S_2P_2^T \eta(t). \quad (31)\]

In addition, combining with (20), for any properly dimensional matrix \(G\) and scalar \(\epsilon_1\) and \(\epsilon_2\), we can get:
\[0 = E\left[ 2[\epsilon_1\dot{e}^T(t) + \epsilon_2\dot{e}^T(t)]G \times [-\dot{e}(t) - Ae(t) \]
\[\neg Bf(e(t)) + C^T f(e(t) - d(t)) - \beta(t)F(t)Ke(t - \tau_1(t)) \]
\[\neg (1 - \beta(t))F(t)Ke(t - \tau_2(t)) \right] \]
\[= 2\eta^T(t)[\Gamma_1^T\Gamma_2 + \Gamma_1^T F(t)\Gamma_3] \eta(t). \quad (32)\]

According to sector bound condition (6), for diagonal matrices \(V_1 > 0\) and \(V_2 > 0\), \(Y_3 = \begin{bmatrix} -W_1V_1 & W_2V_1 \\ \ast & -V_1 \end{bmatrix}\), \(Y_4 = \begin{bmatrix} -W_1V_2 & W_2V_2 \\ \ast & -V_2 \end{bmatrix}\), we can get:
\[0 \leq \eta^T(t)P_3Y_3P_3^T \eta(t), \quad (33)\]
\[0 \leq \eta^T(t)P_3Y_4P_3^T \eta(t). \quad (34)\]

Then, combining \(LV(t)\) with (32),(33), and (34), we get
\[LV(t) \leq \eta^T(t)\Xi \eta(t). \quad (35)\]

where
\[\Xi = \Xi_1 + \Gamma_3^T F(t) \Gamma_3 + dN_1^T S_2^{-1}N_1 + \frac{d^3}{3}N_2^T S_2^{-1}N_2.\]

By utilizing Lemma 3 and Schur complement, \(\Xi(t) < 0\) can be expressed as Eq.(23). Then, the synchronization error system is stochastically stable. Therefore, the system (1) and (3) are stochastically synchronous. The proof is accomplished. \(\square\)

**Remark 1:** Theorem 1 guarantees that the system (1) and (3) are stochastically synchronized based on sampled-data controller with actuator fault. However, in the most of literatures, such as in [25, 27, 2], the problem of actuator failures for neural networks is not taken into account. But in the practical application, the impact of actuator failures and parameters uncertainty cannot be ignored. Therefore, we consider the sensor or actuator failures for delayed neural networks.

**Remark 2:** In [2], an improved FMB integral inequality was introduced. The upper bound provided by the improved FMB is closer than the upper bound taken under Jensen’s inequality. Thence, the tighter bounding inequality in Lemma 1 is used to deal with the integral item \(- \int_{t-d}^{t} \dot{e}^T(s)Z_1\dot{e}(s)ds\), which may bring less conservative result.

**Remark 3:** The multiple sampling problem can be solved by introducing a random variable \(\beta(t)\) that satisfies the Bernoulli distribution. In order to deal with this problem, the sampling system is transformed into a continuous system with random parameters. In spite of Bernoulli distributions was used to random packet losses, uncertain observation. However, it seems that few attempts have been made to use it to solve problems associated with multiple sampling.

According to the conditions given in Theorem 1, the controller gain matrix \(K\) will be derived in the following theorem.

**Theorem 2:** For given scalars \(\beta, \mu, d, \epsilon, \epsilon_1, \epsilon_2, c_1, c_2\), the error system (20) is stochastically stable, if there exist positive matrices \(P, \tilde{Z}_1, \tilde{Z}_2, \tilde{S}_1, \tilde{S}_2, \tilde{R}_1, \tilde{R}_2, \tilde{Q}_1, \tilde{Q}_2, \tilde{\Sigma}\) and positive definite matrices \(\tilde{V}_1, \tilde{V}_2\), any appropriate dimensional matrices \(\tilde{N}_1, \tilde{N}_2, \tilde{\dot{X}}_1, \tilde{\dot{X}}_2, \tilde{G}, \tilde{K}\) satisfying:
\[\tilde{\dot{X}}_1 = \begin{bmatrix} \tilde{S}_2 & \tilde{X}_1 \\ \ast & \tilde{S}_2 \end{bmatrix} > 0, \quad (36)\]
\[\tilde{X}_2 = \begin{bmatrix} \tilde{R}_2 & \tilde{X}_2 \\ \ast & \tilde{R}_2 \end{bmatrix} > 0, \quad (37)\]
\[\begin{bmatrix} \tilde{\Xi}_1 & \epsilon \Gamma_3^T F(t) & \Gamma_3 \\ \ast & -\epsilon I & 0 \\ \ast & \ast & -\epsilon I \end{bmatrix} \begin{bmatrix} \tilde{d}N_1 & -d^2N_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0, \quad (38)\]

where
\[\tilde{\Xi}_1 = 2e_1^T Pe_1 + 2\Pi_1^T (\tilde{Q}_1 + \tilde{Q}_2) \Pi_1 - \Pi_1^T \Pi_2 \]
\[-(1 - \mu)\Pi_1^T \tilde{Q}_2 \Pi_3 + d_1^2 \tilde{Z}_1 e_{12} + d_1^2 \tilde{Z}_2 e_{12} - e_1^T \tilde{Z}_2 e_{12} \]
\[+ 2\Pi_3^T \left[ N_1^T (e_1 - e_2) - 2N_2^T e_{12} \right] - 2\Pi_4^T \tilde{N}_2 (e_1 + e_2) \]
\[+ \beta \epsilon_1^T \tilde{S}_1 e_1 - \epsilon_1^T \tilde{L}_1 e_1 - (1 - \beta) \epsilon_1^T \tilde{R}_1 e_9 + \epsilon_1^T \tilde{L}_2 e_{12} \]
\[- \beta \Pi_5 \tilde{X}_1 \Pi_2^T (1 - \beta) \Pi_6 \tilde{N}_2^T + 2\Gamma_3^T \Gamma_2 + \Pi_1 \tilde{Y}_3 \Pi_1^T \]
\[+ \Pi_3 \tilde{Y}_4 \Pi_3^T, \]
\[\Gamma_2 = [ -\bar{A} \tilde{G}, 0, -\bar{G}, \bar{B} \tilde{G}, 0, \beta \tilde{F} \tilde{K}, 0, (1 - \beta) \tilde{F} \tilde{K}, 0, 0, 0, 0, 0, \]
\[\Gamma_3 = [0, 0, 0, 0, 0, 0, \beta \tilde{F} \tilde{K}, 0, (1 - \beta) \tilde{F} \tilde{K}, 0, 0, 0, 0, \]
\[ \tilde{L}_1 = \beta \tilde{S}_1 + (1 - \beta) \tilde{R}_1, \tilde{L}_2 = \beta c_1^2 \tilde{S}_2 + (1 - \beta)(c_2 - c_1)^2 \tilde{R}_2, \]
\[ \tilde{Y}_3 = \begin{bmatrix} -W_1 \tilde{V}_1 & W_2 \tilde{V}_2 \\ * & -\tilde{V}_1 \end{bmatrix}, \tilde{Y}_4 = \begin{bmatrix} -W_1 \tilde{V}_2 & W_2 \tilde{V}_2 \\ * & -\tilde{V}_2 \end{bmatrix}. \]

The other notations are consistent with Theorem 1. Then, the controller gain matrix \( K \) can be derived as
\[ K = \tilde{K} \tilde{G}^{-T}. \] (39)

**Proof:** Define
\[ G = \tilde{G}^{-1}, \tilde{P} = \tilde{G} \tilde{P} \tilde{G}^T, \tilde{S}_1 = \tilde{G} S_1 \tilde{G}^T, \tilde{S}_2 = \tilde{G} S_2 \tilde{G}^T, \tilde{R}_1 = \tilde{G} R_1 \tilde{G}^T, \tilde{R}_2 = \tilde{G} R_2 \tilde{G}^T, \]
\[ W_1 = \tilde{G} W_1 \tilde{G}^T, W_2 = \tilde{G} W_2 \tilde{G}^T, \tilde{H}_1 = \tilde{H} \tilde{G}^T, \]
\[ \tilde{\gamma}_1 = \tilde{G} \gamma_1 \tilde{G}^T, \tilde{\gamma}_2 = \tilde{G} \gamma_2 \tilde{G}^T, \tilde{\tilde{\gamma}}_1 = \tilde{X}_1 \tilde{G}^T, \tilde{\tilde{\gamma}}_2 = \tilde{X}_2 \tilde{G}^T. \]

Pre and postmultiplying (23) with \((X, X, \ldots, X, I, I, I)\) and its transpose, respectively. We can get (38). The corresponding controller gain matrix \( K \) can be got by (39). The proof is accomplished.

Specifically, it is worth noting that the stability analysis with controller failures at two sampling intervals has been considered in the previous part of this paper. If we extend the sampling interval to three or more, we can get more general analysis results of the stability of random sampling control in systems (10). In general, we can extend the case that there are \( n \) sample intervals \( c_1, c_2, \ldots, c_n \) with \( 0 < c_1 < c_2 \cdots < c_n \). The probability of the occurrences is

\[ \text{Prob} \{ h = c_i \} = \tilde{\gamma}_i (i = 1, 2, \ldots, n), \sum_{i=1}^{n} \tilde{\gamma}_i = 1. \]

Similar to the subsection of the main method, the probability of \( \tau (t) \) is calculated as

\[ \text{Prob} \{ 0 \leq \tau (t) < c_1 \} = \tilde{\gamma}_1 + \frac{c_2 - c_1}{c_2} \tilde{\gamma}_2 + \cdots + \frac{c_n - c_{n-1}}{c_n} \tilde{\gamma}_n = \beta_1, \]
\[ \text{Prob} \{ c_1 \leq \tau (t) < c_2 \} = \frac{c_2 - c_1}{c_2} \tilde{\gamma}_2 + \frac{c_3 - c_2}{c_3} \tilde{\gamma}_3 + \cdots + \frac{c_n - c_{n-1}}{c_n} \tilde{\gamma}_n = \beta_2, \]
\[ \vdots \]
\[ \text{Prob} \{ c_{n-1} \leq \tau (t) < c_n \} = \frac{c_n - c_{n-1}}{c_n} \tilde{\gamma}_n = \beta_n. \]

Note that \( \sum_{i=1}^{n} \beta_i = 1 \). Then the indicator function is denoted as:
\[ \Lambda_i = \Lambda \{ \tau (t) \in [c_{i-1}, c_i] \}, \quad i = 1, 2, \cdots, n. \]

We can figure out
\[ E \{ \Lambda_i \} = \beta_i, \]
\[ E \{ (\Lambda_i - \beta_i)^2 \} = \beta_i (1 - \beta_i). \]

**TABLE 1.** The upper bounds of \( c_2 \) for different values of \( \beta (c_1 = 0.01) \).

| \( \beta \) | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|
| \( c_2 \) | 0.14 | 0.16 | 0.24 | 0.41 | 1.19 |

In this case, the error system (10) is expressed as follow:
\[ \dot{\varepsilon} (t) = -A \varepsilon (t) + Bf (\varepsilon (t)) + C \varepsilon (t - d (t)) + \sum_{i=1}^{n} \Lambda_i \tilde{\gamma}_i (t) K e (t - \tau (t)). \] (40)

where \( c_{i-1} < \tau (t) < c_i \). Then, similar to Theorem 1, for multiple sampling situations, we can get corresponding results.

Specifically, when \( \tilde{F}_\alpha = \tilde{F}_\alpha = 1 \), which means actuator works properly, in this case, the SES (20) is transformed into the following form:
\[ \dot{\varepsilon} (t) = -A \varepsilon (t) + Bf (\varepsilon (t)) + C \varepsilon (t - d (t)) + \beta (t) K e (t - \tau (t)) + (1 - \beta (t)) K e (t - \tau (t)). \] (41)

**Corollary 1:** For given scalars \( \beta, \mu, \) and \( \gamma \), this error system is stochastically stable, if there exist positive matrices \( P, Z_1, Z_2, S_1, S_2, R_1, R_2, Q_1, Q_2 \) positive diagnose matrices \( V_1, V_2, V_3, V_4 \) for appropriate dimensional matrices \( \tilde{N}_1, \tilde{N}_2, \tilde{X}_1, \tilde{X}_2, \tilde{G}, \tilde{L} \) such that the following LMIs holds:
\[ \begin{align*}
\gamma_1 & > 0, \\
\gamma_2 & > 0, \\
\gamma_3 & > 0.
\end{align*} \]

The notations are the same as in Theorem 1. Then, the master and slave neural networks realize synchronization under stochastic sampling without actuator failure. The gain matrix \( K \) can be obtained by \( K = \tilde{K} \tilde{G}^{-1} \).

**IV. NUMERICAL EXAMPLES**

In this section, a persuasive example is presented to verify the feasibility of the method.

**Example 1:** The systems (1) and (3) are considered with parameters as follows:
\[ A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -0.1 \\ -5 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{bmatrix}. \]

The neuron activation functions are
\[ f_i (x_i (t)) = \frac{e^{x_i} - e^{-x_i}}{e^{x_i} + e^{-x_i}}, \quad (i = 1, 2). \]

A straightforward calculation yields \( W_1 = 0.1I \) and \( W_2 = 0.3I \). Here, we assume that there are no failures happen to \( u_\alpha(t) \), that is \( \tilde{F}_\alpha = \tilde{F}_\alpha = 1 \). The time-varying delay is \( d (t) = \frac{d}{1 + t} \), and it can be obtained that \( d = 1, \mu = 0.25 \). Next, when choosing \( \epsilon_1 = 1 \) and \( \epsilon_2 = 0.1 \), we consider two cases and the results are given in Table 1-2.

The initial conditions are set \( x(0) = [0.4, 0.3]^T \) and \( y(0) = [0.2, 0.4]^T \). The response of \( x(t) \) and \( y(t) \) are depicted.
TABLE 2. The upper bounds of $c_2$ for different values of $c_1$ ($\beta = 0.8$).

| $c_1$  | 0.01 | 0.03 | 0.3  | 0.05 | 0.07 |
|--------|------|------|------|------|------|
| $c_2$  | 0.60 | 0.45 | 0.29 | 0.39 | 0.13 |

in Fig.1 and 2, respectively. If the condition of Theorem 1 is satisfied, we can verify the stability of the error synchronization system. Simultaneously, we readily receive the controller gain matrix $K$. By handling the LMIs (42)-(44), the value for the controller parameters $K$ could be gained correspondingly as follow:

$$K = \begin{bmatrix} -10.5773 & -0.15307 \\ -2.71456 & -11.2574 \end{bmatrix}.$$  

Using the obtained gain $K$, the correspondence of controller $e(t)$ is displayed in Fig.3, and the error state $u(t)$ is displayed in Fig.4. Then according to Theorem 1, it can be concluded that the synchronization between drive system (1) and response system (3) is achieved. It is shown that the trend of the state variable eventually goes to 0 in Fig.3, which also verifies the effectiveness and the feasibility of our method. Fig.5 shows the stochastic parameter $h$.

V. CONCLUSION

In this paper, a new way of stochastic sampled-data control for delayed neural networks with actuator failure via stochastic sampling has been researched. The sampling data controller with only two sampling intervals are considered and it can be extended to $n$ sampling intervals. A new LKF has been constructed to hold more information about the characteristics of the actual sampling mode. The conditions established in Theorem 1 reduce conservatism and guarantee the synchronization of master-slave systems. Moreover, we can synthesize the sampled-data controller gain matrix through solving the LMIs, under the allowable maximum
sampling period. The effectiveness of our approach has been verified via a illustrative examples given. In our future work, we will consider the effect of parameter variation for the delayed neural networks on the behavior of the stochastic sampled-data controller. Also, we will explore the robustness of the proposed approach.

REFERENCES

[1] P. Shi, F. Li, L. Wu, and C.-C. Lim, “Neural network-based passive filtering for delayed neutral-type semi-Markovian jump systems,” IEEE Trans. Neural Netw. Learn. Syst., vol. 28, no. 9, pp. 2101–2114, Sep. 2017.

[2] R. Rakkiyappan, N. Sakthivel, and J. Cao, “Stochastic sampled-data control for synchronization of complex dynamical networks with control packet loss and additive time-varying delays,” Neural Netw., vol. 66, pp. 46–63, Jun. 2015.

[3] Z.-G. Wu, P. Shi, H. Su, and J. Chu, “Stochastic synchronization of Markovian jump neural networks with time-varying delay using sampled data,” IEEE Trans. Cybern., vol. 43, no. 6, pp. 1796–1806, Dec. 2013.

[4] Y. Kan, J. Lu, J. Qiu, and J. Kurths, “Exponential synchronization of time-varying delayed complex-valued neural networks under hybrid impulsive controllers,” Neural Netw., vol. 114, pp. 157–163, Jun. 2019.

[5] Z.-G. Wu, J. Lam, H. Su, and J. Chu, “Stability and dissipativity analysis of static neural networks with time delay,” IEEE Trans. Neural Netw. Learn. Syst., vol. 23, no. 2, pp. 199–210, Feb. 2012.

[6] X. Le and J. Wang, “Robust pole assignment for synthesizing feedback control systems using recurrent neural networks,” IEEE Trans. Neural Netw. Learn. Syst., vol. 25, no. 2, pp. 383–393, Feb. 2014.

[7] Z. Guo, J. Wang, and Z. Yan, “Attractionality analysis of memristor-based cellular neural networks with time-varying delays,” IEEE Trans. Neural Netw. Learn. Syst., vol. 25, no. 4, pp. 704–717, Apr. 2014.

[8] D. Zeng, R. Zhang, Y. Liu, and S. Zhong, “Sampled-data synchronization of chaotic Lu’s systems via input-delay-dependent-free-matrix zero equality approach,” Appl. Math. Comput., vol. 315, no. 15, pp. 34–46, Dec. 2017.

[9] K. Shi, X. Liu, H. Zhu, S. Zhong, Y. Liu, and C. Yin, “Novel integral inequality approach on master–slave synchronization of chaotic delayed Lu’s systems with sampled-data feedback control,” Nonlinear Dyn., vol. 83, no. 3, pp. 1259–1274, Feb. 2016.

[10] A. Chandrasekar, R. Rakkiyappan, F. A. Rihan, and S. Lakshmanan, “Exponential synchronization of Markovian jumping neural networks with partly unknown transition probabilities via stochastic sampled-data control,” Neurocomputing, vol. 133, pp. 385–398, Jun. 2014.

[11] M. Syed Ali and N. Gunasekaran, “Sampled-data state estimation of Markovian jump static neural networks with interval time-varying delays,” J. Comput. Appl. Math., vol. 343, pp. 217–229, Dec. 2018.

[12] J.-L. Wang, H.-N. Wu, T. Huang, S.-Y. Ren, and J. Wu, “Pinning control for synchronization of coupled reaction-diffusion neural networks with directed topologies,” IEEE Trans. Syst., Man, Cybern., Syst., vol. 46, no. 8, pp. 1109–1120, Aug. 2016.

[13] J. Lu, D. W. C. Ho, J. Cao, and J. Kurths, “Single impulsive controller for globally exponential synchronization of dynamical networks,” Nonlinear Anal., Real World Appl., vol. 14, no. 1, pp. 581–593, Feb. 2013.

[14] H. Zhang, T. Ma, G.-B. Huang, and Z. Wang, “Robust global exponential synchronization of uncertain chaotic delayed neural networks via dual-stage impulsive control,” IEEE Trans. Syst., Man, Cybern., Syst., vol. 40, no. 3, pp. 831–844, Jun. 2010.

[15] H. Dai, J. Jia, L. Yan, F. Wang, and W. Chen, “Event-triggered exponential synchronization of complex dynamical networks with cooperatively directed spanning tree topology,” Neurocomputing, vol. 330, no. 22, pp. 355–368, Feb. 2019.

[16] C.-K. Zhang, Y. He, and M. Wu, “Exponential synchronization of neural networks with time-varying mixed delays and sampled-data,” Neurocomputing, vol. 74, nos. 1–3, pp. 265–273, Dec. 2010.

[17] Y. Liu, B.-Z. Guo, J. H. Park, and S.-M. Lee, “Nonfragile exponential synchronization of delayed complex dynamical networks with memory sampled-data control,” IEEE Trans. Neural Netw. Learn. Syst., vol. 29, no. 1, pp. 118–128, Jan. 2018.

[18] S. J. S. Theesaran, S. Banerjee, and P. Balasubramaniam, “Synchronization of chaotic systems under sampled-data control,” Nonlinear Dyn., vol. 70, no. 3, pp. 1977–1987, Sep. 2012.

[19] H. Zeng, K. Teo, Y. He, H. Xu, and W. Wang, “Sampled-data synchronization control for chaotic neural networks subject to actuator saturation,” Neurocomputing, vol. 185, no. 18, pp. 1656–1667, Oct. 2017.

[20] J.-G. Lu and D. J. Hill, “Global asymptotical synchronization of chaotic Lu’s systems using sampled data: A linear matrix inequality approach,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 55, no. 6, pp. 586–590, Jun. 2008.

[21] C.-K. Zhang, Y. He, and M. Wu, “Improved global asymptotical synchronization of chaotic Lu’s systems with sampled-data control,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 56, no. 4, pp. 320–324, Apr. 2009.

[22] X.-C. Shang-Guan, Y. He, W.-J. Lin, and M. Wu, “Improved synchronization of chaotic Lu’s systems with time delay using sampled-data control,” J. Franklin Inst., vol. 354, no. 3, pp. 1618–1636, Feb. 2017.

[23] T. H. Lee, J. H. Park, S. M. Lee, and O. M. Kwon, “Robust synchronization of chaotic systems with randomly occurring uncertainties via stochastic sampled-data control,” Int. J. Control, vol. 86, no. 1, pp. 107–119, Jan. 2013.

[24] B. Shen, Z. Wang, and X. Liu, “Sampled-data synchronization control of dynamical networks with stochastic sampling,” IEEE Trans. Autom. Control, vol. 57, no. 10, pp. 2644–2650, Oct. 2012.

[25] Y. Liu and S. Lee, “Synchronization of chaotic Lu’s systems using sampled-data PD control,” Nonlinear Dyn., vol. 85, no. 2, pp. 1–12, Mar. 2016.

[26] J. Wang, K. Shi, Q. Huang, S. Zhong, and D. Zhang, “Stochastic switched sampled-data control for synchronization of delayed chaotic neural networks with packet dropout,” Appl. Math. Comput., vol. 335, pp. 211–230, Oct. 2018.

[27] H. Gao, J. Wu, and P. Shi, “Robust sampled-data H∞ control with stochastic sampling,” Automatica, vol. 45, no. 7, pp. 1729–1736, Jan. 2009.

[28] S. H. Lee, M. J. Park, O. M. Kwon, and P. Selvaraj, “Improved synchronization criteria for chaotic neural networks with sampled-data control subject to actuator saturation,” Int. J. Control, Autom. Syst., vol. 17, no. 9, pp. 2430–2440, Sep. 2019.

[29] X. Yang, J. Cao, and J. Lu, “Synchronization of Markovian coupled neural networks with nonidentical node-delays and random coupling strengths,” IEEE Trans. Neural Netw. Learn. Syst., vol. 23, no. 1, pp. 60–71, Jan. 2012.

[30] R. Li, X. Gao, and J. Cao, “Exponential synchronization of stochastic memristive neural networks with time-varying delays,” Neural Process. Lett., vol. 50, no. 1, pp. 459–475, Feb. 2019.

[31] C. Ge, B. Wang, X. Wei, and Y. Liu, “Exponential synchronization of a class of neural networks with sampled-data control,” Appl. Math. Comput., vol. 315, pp. 150–161, Dec. 2017.

[32] J. Zhao, S. Xu, and J. H. Park, “Improved criteria for the stabilization of T-S fuzzy systems with actuator failures via a sampled-data fuzzy controller,” Fuzzy Sets Syst., vol. 392, pp. 154–169, Aug. 2020.

[33] P. Park, J. W. Ko, and C. Jeong, “Reciprocally convex approach to stability of systems with time-varying delays,” Automatica, vol. 47, no. 1, pp. 235–238, Jan. 2011.

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