BRST treatment of the Bohr collective hamiltonian at high spins

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Abstract: The BRST treatment of triaxial systems rotating at high spins is used to solve perturbatively the γ-independent Bohr collective hamiltonian.

1 Introduction

The standard model used to calculate nuclei at high spins is the cranking model. It is particularly successful in the prediction of energy levels, single-particle states, deformation parameters, etc. The problem of constructing the normal modes associated with the cranking model has been treated by several authors. All these treatments assume the quadrupole plus monopole pairing interaction. A different treatment of the cranking model at high spins is based on the application of the BRST symmetry to many-body problems. Since the collective coordinates are explicitly introduced, the treatment requires constraints and gauge conditions. An extension of the formalism to include transition probabilities has recently appeared.

In the BRST treatment there are no restrictions concerning the hamiltonian, other than those associated with the validity of the cranking model: small fluctuations in physical magnitudes relative to their expectation value (if this

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1 In particular, the velocity-dependent residual interactions may be treated on the same footing as the interactions which are directly responsible for the mean field. The importance of terms of this type insuring the Galilean invariance of the hamiltonian has been recently stressed in refs.
one does not vanish). In particular, this must be true for the collective angular momentum components $\vec{I}$ relative to the expectation value $\langle I_x \rangle = I$.

None of the previous formalisms has been carried out beyond the level of constructing normal modes and their corresponding transition matrix elements to the ground state. However (perturbative) calculations should also be performed within such basis. In the present letter we report for the first time on one of such calculations. It is performed by applying the formalism to a reasonably simple but non-trivial system, the Bohr collective hamiltonian \cite{Bohr35}. Our aim is: i) to verify that the problem of infrared divergencies has been properly solved and ii) to compare the perturbative results in the cranked basis with those in the laboratory system (which can be solved in this case). Emphasis is put in those diagrams associated with the modification of the rotational energies in excited states, since their systematic application may shed light on the still unsolved problem of identical bands.

The Bohr collective hamiltonian is constructed out of the five components of a quadrupole tensor $\alpha_\mu$ and their associated conjugate momenta $p_\mu$. We prefer to work with the “cartesian” components $q_\nu$

$$\alpha_o = q_\beta; \quad \alpha_{\pm 1} = \pm \sqrt{\frac{1}{2}}(q_y \pm iq_x); \quad \alpha_{\pm 2} = \sqrt{\frac{1}{2}}(q_\gamma \mp iq_z) \quad (1)$$

In terms of such coordinates and momenta, the kinetic energy and the two (independent) static invariants of the problem may be written

$$T = \frac{1}{2}(p_\beta^2 + p_\gamma^2 + p_x^2 + p_y^2 + p_z^2) \quad (2)$$

$$W_2 = q_\beta^2 + q_\gamma^2 + q_x^2 + q_y^2 + q_z^2 \equiv \beta^2 \quad (3)$$

$$W_3 = -2q_\beta^3 + 6q_\beta q_\gamma^2 + 3q_\beta(2q_\gamma^2 - q_x^2 - q_y^2) + 3\sqrt{3}q_\beta(q_x^2 - q_y^2)$$
$$-6\sqrt{3}q_x q_y q_z \equiv -2\beta^3 \cos 3\gamma \quad (4)$$

We work in a rotating frame of reference, and we formulate the problem within an overcomplete space consisting of the five original or intrinsic variables $q_\nu$ (and their conjugate momenta $p_\nu$) plus three collective coordinates, the Euler angles $\phi_\nu$ (and the three components of the collective angular momentum in the moving frame $I_\nu$), $[I_\mu, I_\nu] = -i\epsilon_{\mu\nu\rho}I_\rho$. The components of the intrinsic angular momentum are given in appendix \cite{Bohr35}.

Since the collective variables are raised to the level of true variables, some combination of the intrinsic variables belong to the spurious sector, which is

\footnote{These five original degrees of freedom of the Bohr hamiltonian are usually supposed to represent collective coordinates. Note that this is not the interpretation here, since we reserve the name collective (as opposed to intrinsic) to the Euler angles determining the position of the rotating frame.}
completed through the introduction of three Lagrange multipliers \( \Omega_\nu \) (and their conjugate momenta \( P_\nu \)) and six ghost (fermion) variables \( \eta_\nu, \bar{\eta}_\nu \) (and their conjugate partners \( \pi_\nu, \bar{\pi}_\nu \)).

The BRST hamiltonian for an ellipsoidal system reads

\[
H_{\text{BRST}} = H_B - \Omega_\nu (J_\nu - I_\nu) + \frac{1}{F_\nu} G_\nu P_\nu - \frac{A_\nu}{2F_\nu^2} P_\nu^2 \\
+ i\pi_\nu \bar{\pi}_\nu - \frac{1}{F_\nu} \bar{\eta}_\mu \eta_\nu [G_\mu, J_\nu] + i\epsilon_{\mu\nu\rho} \Omega_\rho \pi_\mu \bar{\pi}_\nu
\]

(5)

where \( H_B \) is the original Bohr hamiltonian, and \( G_\mu \) are gauge-fixing functions which we choose later. Three of the constants \( F_\nu, A_\nu \) disappear during the treatment, while the remaining three give rise to the spurious frequencies \( \omega_x, \omega_p \) (eqs. (13) and (18)).

The coordinates \( q_\beta, q_\gamma \) and the \( x \)-components of the intrinsic and collective angular momentum and of the Lagrange multipliers have the non-vanishing expectation values

\[
\langle q_\beta \rangle = \beta_o \cos \gamma_o ; \quad \langle q_\gamma \rangle = \beta_o \sin \gamma_o \\
\langle J_x \rangle = \langle I_x \rangle = 3\langle \Omega_x \rangle = I
\]

(6) (7)

The minimization of the hamiltonian \( H_{\text{BRST}} \) together with the condition (7) implies the minimization of the routhian \( \delta\langle (H - \Omega J_x) \rangle = 0 \) (\( \Omega \equiv \langle \Omega_x \rangle \)), and yields the value of the deformation parameters \( \beta_o(I), \gamma_o(I) \) and the moment of inertia \( \Im(I) \). Therefore the cranking solution is the starting point of a treatment of collective rotational motion which is exact in principle.

If the potential energy surface \( V = V(W_2, W_3) \) is independent of \( W_3 \) (i.e., \( \gamma \)-independent) the minimization condition yields

\[
\gamma_o = \pi/6 ; \quad \Im = 4\beta_o^2 ; \quad \langle p_x \rangle = 2\Omega \beta_o
\]

(8)

Since there is no restoring force in the \( \gamma \)-direction, the system acquires immediately the \( \gamma_o \) deformation that minimizes the kinetic energy.

Because \( W_2 = \beta^2 \), the value of \( \beta_o \) is determined from the eq. :

\[
V_1 - \frac{I^2}{8\beta_o^4} = 0
\]

(9)

3 This fact explains the difficulties in deciding empirically between the \( \gamma \)-unstable and the spheroidal rotor descriptions.
where $V_i \equiv \frac{d^2 V(\beta^2)}{d\beta^2} \big|_{\beta^2=\beta_0^2}$. The expectation value of the BRST hamiltonian is

$$\langle H_{\text{BRST}} \rangle = \frac{I^2}{2^3} + V(\beta_0^2) \quad (10)$$

In the next step we construct the normal modes. Each operator may consist of several terms which are labelled by the number of phonons through the supraindex ($i$) (for instance, $\langle a \rangle = a^{(0)}$). The normal modes are determined from the quadratic terms in the hamiltonian. Due to signature-invariance, these terms split into two parts which are labelled by the subindices $x$ and $\perp$, respectively. The expression for $H_{\text{ax}}^{(2)}$ is

$$H_{\text{ax}}^{(2)} = H_{\text{ax}}^{(2)} - \Omega J_{xx}^{(2)}$$

$$= \frac{i}{2}(p_\beta^{(1)2} + p_\gamma^{(1)2} + p_x^{(1)2}) + V_1(q_\beta^{(1)2} + q_\gamma^{(1)2} + q_x^{(1)2})$$

$$+ \frac{1}{2}V_2(\sqrt{3}q_\beta^{(1)} + q_\gamma^{(1)})^2 - \Omega J_{xx}^{(2)} \quad (11)$$

$$H_{\text{bx}}^{(2)} = -\Omega J_{xx}^{(1)} + \frac{1}{F_x}G_x P_x - \frac{A_x}{2F_x^2}P_x^2 + i\pi_x \bar{a}_x + i\eta_x \bar{b}_x \quad (12)$$

$$H_{\text{x}}^{(2)} = H_{\text{ax}}^{(2)} + H_{\text{bx}}^{(2)} = \omega_x(\Gamma_{1x}^\dagger \Gamma_{1x} + \frac{1}{2}) + \omega_\beta(\Gamma_{1x}^\dagger \Gamma_{1x} + \frac{1}{2})$$

$$+ \omega_\gamma(\Gamma_{1x}^\dagger \Gamma_{1x} - \Gamma_{0x}^\dagger \Gamma_{0x} + \bar{a}_x a_x + \bar{b}_x b_x) \quad (13)$$

The hamiltonian includes the two real vibrations with frequencies $\omega_\gamma = 2\Omega$ and $\omega_\beta = \sqrt{8V_1 + 3V_2}$ and a (supersymmetric) spurious sector arising from $(12)$ and from a term $(J_{xx}^{(1)})^2 / 2\Im_x$ obtained in $(11)$. The existence of this term is a consequence of the fact that the first two lines commute with the operator $J_{xx}^{(1)}$. The construction of the spurious sector is made as in [3,6,7]. The (arbitrary) frequency $\omega_x$ should disappear from any physical result. The (linear) gauge $G_x$ is the RPA angle which is obtained simultaneously with the dynamical moment of inertia from the RPA eqs.

$$[H_{\text{ax}}^{(2)}, G_x] = -\frac{i}{\Im_x} J_{xx}^{(1)} ; \quad [G_x, J_{xx}^{(1)}] = i \quad (14)$$

which yield

$$G_x = \frac{1}{4\beta_0} \left( 1 + \frac{3}{\Im_x} \right) q_x^{(1)} - \frac{4I\beta_0}{V_2^{3/2}}(\sqrt{3}p_\beta^{(1)} + p_\gamma^{(1)}); \quad \Im_x = \Im + \frac{16\Omega^2}{V_2} \quad (15)$$

Note the presence of the quadratic term $-\Omega J_{xx}^{(2)}$ (eq. (A.8)) in the hamiltonian with positive signature $(11)$, which has been overlooked in previous derivations of the normal modes in cranked systems.

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4 Note the relations $[\Gamma_{1s}, \Gamma_{1s}^\dagger] = -[\Gamma_{0s}, \Gamma_{0s}^\dagger] = [\bar{a}_s, a_s]_+ = [\bar{b}_s, b_s]_+ = 1$ for $s = x, p$. 

The quadratic Hamiltonian in the perpendicular direction is
\[ H_{a,\perp}^{(2)} = \frac{1}{2} \left( p_y \right)^2 + p_z^2 + V_1 (q_y \right)^2 + q_z^2 \right) - \Omega (J_{x,\perp}^{(2)} - I_{\perp}^{(2)}) \]
\[ H_{b,\perp}^{(2)} = -\Omega_y (J_y^{(1)} - I_y^{(1)}) + \frac{1}{F_y} G_y p_y - \frac{A_y}{2F_y} P_y^2 - \Omega_z (J_z^{(1)} - I_z^{(1)}) + \frac{1}{F_z} G_z \right) \]
(16)
\[ H_{\perp}^{(2)} = H_{a,\perp}^{(2)} + H_{b,\perp}^{(2)} \]
\[ = \frac{1}{2} \Omega + \omega_w (\Gamma_w \Gamma_w + \frac{1}{2}) + \omega_p (\Gamma_{1p} \Gamma_{1p} - \Gamma_{0p} \Gamma_{0p} + \bar{a}_p a_p + \bar{b}_p b_p) \]
(17)
where \( p = \pm \). The linear and quadratic terms of the collective angular momentum are obtained by means of the Holstein-Primakoff representation. The first (constant) term in (18) plus the rotational term in eq. (19) yield the quantal form \( I(I + 1) \). The real degree of freedom in (18) represents the wobbling motion with frequency \( \omega_w = 3\Omega \). There also appear two spurious sectors with (arbitrary) frequencies \( \omega_p \). The gauge functions and the moments of inertia are derived from a generalization of the RPA eqs. (14) (18)
\[ [H_{a,\perp}, G_y] = -\frac{i}{3y} J_y^{(1)} + iI \left( \frac{1}{3y} - \frac{1}{3} \right) G_z ; \quad [G_y, J_x^{(1)}] = i \]
\[ [H_{a,\perp}, G_z] = -\frac{i}{3z} J_z^{(1)} - iI \left( \frac{1}{3z} - \frac{1}{3} \right) G_y ; \quad [G_z, J_x^{(1)}] = i \]
(19) (20)
plus the condition \( [G_y, G_z] = 0 \). These eqs. yield \( G_y = -\beta_o^{-1} q_y^{(1)} \), \( G_z = -\beta_o^{-1} q_z^{(1)} \) and \( \Omega_y = \Omega_z = \beta_o^2 \).

The quadratic Hamiltonians (13) and (18) determine both the real and the spurious spectra associated with a given value of \( I \). The corresponding states span a space which is factorized into real, spurious and rotational subspaces
\[ |n_\beta, n_\gamma, n_w \rangle_I |n_{0\nu}, n_{1\nu}, n_{av}, n_{br} \rangle_I |IM = N (\Gamma_{1\beta}^I)^{n_\beta} (\Gamma_{1\gamma}^I)^{n_\gamma} (\Gamma_{1w}^I)^{n_w} |r \rangle_I \]
\[ \times (\Gamma_{0\nu}(\Gamma_{1\nu})^{n_{0\nu}} (\bar{a}_\nu)^{n_{av}} (\bar{b}_\nu)^{n_{br}} |sp \rangle_I |IM \]
(21)
where \( n_\beta, n_\gamma, n_w, n_{1\nu}, n_{0\nu} = 0, 1, 2, \ldots \) and \( n_{av}, n_{br} = 0, 1 \). The product of the vacua of the real and spurious excitations \( |r \rangle_I \) and \( |sp \rangle_I \), respectively) is also the yrast state for a given \( I \). The rotational states \( |IM \rangle \) are, in configuration space, the wave functions \( DS_{MK}^I \) of the rigid top. The rotational excitations \( DS_{MK}^I \) \( (K < I) \) are partly associated with the wobbling motion and partly with the spurious sector. Here \( K \) is the projection along the intrinsic \( x \)-axis.

We calculate the matrix elements of the quadrupole operator in the laboratory frame, since they are physical operators. In order to apply the formalism developed in (7) it is convenient to express the intrinsic operators with respect to the \( x \)-axis
\[ Q_{\mu}^{(lab)} = D_{\mu\nu}^2 (\phi) D_{\nu\rho}^2 (-\frac{1}{2}\pi, -\frac{1}{2}\pi, 0) \alpha_\rho \]
(22)
If acting within states having \( I_x \approx I \gg 1 \), the rotational matrices may be expressed in terms of the collective generators \( I_\nu \) times the operators \( E^2_+ \) increasing the angular momentum of the ground state from \( I \) to \( I + \iota \) [12]. In such a way we can label the components of (22) both by the number of phonons and by the change in \( I \). In particular, the zero-phonon component is

\[
(\alpha^{(\text{lab})}_2(0)) = -E^4_+ \beta_\alpha / \sqrt{2}
\]

(23)
corresponding to the transition matrix element between yrast states (which differ in two units of angular momentum).

The linear transition operators can be constructed from the expressions for the intrinsic variables in terms of the normal modes

\[
\begin{align*}
(\alpha^{(\text{lab})}_0)^{(1)} &= \frac{\beta_\alpha}{\iota \sqrt{I}} (\Gamma^1_\gamma + \Gamma_\gamma) ; \\
(\alpha^{(\text{lab})}_1)^{(1)} &= \frac{i \beta_\alpha}{\sqrt{I}} E^2_+ \Gamma_w \\
(\alpha^{(\text{lab})}_2)^{(1)} &= -\frac{1}{2 \omega_\beta} \sqrt{\frac{1}{\omega_\beta}} E^4_+ \left( \Gamma^b_\beta (\omega_\beta - 4 \Omega) + \Gamma_\beta (\omega_\beta + 4 \Omega) \right)
\end{align*}
\]

(24)
The classification of levels in terms of the quantum numbers \( n_\beta, n_\omega, n_\gamma \) allows for a new interpretation of the degeneracies associated with the \( \gamma \)-unstable spectrum: for each \( I \), there is vibrational spectrum with \( n_\gamma = 0, 1, 2, \ldots \). After \( n_\gamma = 2 \), they become degenerate with levels with one wobbling phonon more than the corresponding states with \( I \pm 1 \) (fig [1]).

The vanishing of the zero-phonon terms proportional to \( E^2_+ \) with \( \iota = 0 \) (diagonal matrix elements) and of the matrix elements increasing in one unit both the angular momentum and the number of wobbling bosons is due to the conservation of the \( \gamma \)-parity quantum number (see [14]).

Perturbative corrections can be calculated straightforwardly since \( H_{\text{BRST}} \) is free of zero frequency modes. The perturbative expansion yields the residual hamiltonian \( H_{\text{res}} = H^{(3)}_{\text{BRST}} + H^{(4)}_{\text{BRST}} + \cdots \), with

\[
H^{(3)}_{\text{BRST}} = \beta_\alpha V_2 (\sqrt{3} q^{(1)}_\beta + q^{(1)}_\gamma)(q^{(1)}_\beta - q^{(1)}_\gamma + q^{(1)}_x q^{(1)}_y q^{(1)}_z) + \beta^3_\alpha V_3 (\sqrt{3} q^{(1)}_\beta + q^{(1)}_\gamma)^3 - \Omega_x (J^{(2)}_x - I^{(2)}_x) - \Omega_y J^{(2)}_y - \Omega_z J^{(2)}_z - \frac{1}{F_\nu} \tilde{\eta}_\mu \eta_\nu [G_\mu, J^{(2)}_\nu] + \iota \epsilon_\mu \rho \Omega_\rho \pi_\mu \pi_\nu
\]

(25)

\[
H^{(4)}_{\text{BRST}} = \frac{1}{2} V_2 (q^{(1)}_\beta + q^{(1)}_\gamma)^2 (q^{(1)}_\beta - q^{(1)}_\gamma + q^{(1)}_x q^{(1)}_y q^{(1)}_z)^2 + \beta^2_\alpha V_3 (\sqrt{3} q^{(1)}_\beta + q^{(1)}_\gamma)^2 (q^{(1)}_\beta + q^{(1)}_x q^{(1)}_y q^{(1)}_z)^2 + \beta^4_\alpha \frac{1}{24} V_4 (\sqrt{3} q^{(1)}_\beta + q^{(1)}_\gamma)^4 + \Omega_y I^{(3)}_y + \Omega_z I^{(3)}_z
\]

(26)
The $J^{(2)}$ are given in appendix A and $I^{(i)}$ are obtained by means of the Holstein-Primakoff representation.

$H^{(3)}_{\text{BRST}}$ and $H^{(4)}_{\text{BRST}}$ give rise to the vertices shown in fig. 2. There are no vertices with an odd number of $\perp$-phonons due signature-invariance, and there are no vertices with an odd number of ghosts due to the fact that $H_{\text{BRST}}$ is an even-Grassman function.

The diagrammatic corrections of $o(\beta^2)$ to the energies of the yrast state and of the one $\beta$-phonon state are given in the second and third rows of fig. 2, respectively. Although individual diagrams depend on the spurious frequencies $\omega_x$ and $\omega_\perp$ this dependence cancels out from the final sums. The total corrections to the energies are

$$\Delta E(\text{yrast state}) = \frac{4}{3} + \frac{18I}{\omega_\beta \mathcal{I}^2} + \frac{3}{\omega_\beta^2} \left( -\frac{7}{3} V_1 + \frac{1}{8} V_2 + \frac{3}{8} V_3 + \frac{3^2}{96} V_4 \right)$$

$$+ \frac{18I}{\omega_\beta \mathcal{I}^2} \left( -8V_1 + 3V_2 + \frac{3^2}{6} V_3 \right)$$

$$- \frac{11}{8\omega_\beta^4 \mathcal{I}^2} \left( -8V_1 + 3V_2 + \frac{3^2}{6} V_3 \right)^2$$

(27)

$$\Delta E(\text{one } \beta\text{-phonon}) = \frac{36I}{\omega_\beta \mathcal{I}^2} + \frac{3}{\omega_\beta^2} \left( \frac{20}{3} V_1 + \frac{1}{2} V_2 + \frac{3}{2} V_3 + \frac{3^2}{24} V_4 \right)$$

$$+ \frac{36I}{\omega_\beta \mathcal{I}^2} \left( -8V_1 + 3V_2 + \frac{3^2}{6} V_3 \right)$$

$$- \frac{15}{2\omega_\beta^4 \mathcal{I}^2} \left( -8V_1 + 3V_2 + \frac{3^2}{6} V_3 \right)^2$$

(28)

The $\gamma$-independent Bohr collective hamiltonian may be analitically solved in the laboratory frame through the conventional method of separation of variables [14]. Within such a framework (which does not require the treatment of a spurious sector) corrections (27) and (28) can be reproduced analitically.

In the case of the harmonic motion [9], $V_1 = \text{constant}$ and $V_2 = 0$. Consequently, the “static” moment of inertia is $\mathcal{I} = 2I / \sqrt{2V_1}$, while the “dynamical” is $\mathcal{I}_x \to \infty$, as required to reproduce yrast energies increasing linearly with $I$.

A similar treatment can be applied to a $\gamma$-dependent potential, the main difference being that $\gamma_0 = \gamma_0(I)$ varies from the equilibrium value at $I = 0$ to the value (8) for large $I$. In this case the solution requires a numerical computation.

We have performed (to our knowledge, for the first time) a perturbative cor-
Fig. 1. The spectrum of $H_{\text{BRST}}^{(2)}$. The levels are classified by $(n_\beta, n_w, n_\gamma)$. Dark lines correspond to strong transitions between yrast states, lighter ones to transitions to vibrational states, and dotted lines to $\gamma$-parity inhibited transitions.

rection to zero order results appearing through the application of the cranking approximation. This has been done for the Bohr collective hamiltonian with a $\gamma$-independent potential energy surface. Although the empirical applicability of such a model is being increasingly recognized, we mostly consider the present contribution as a non-trivial exercise for future calculations with realistic systems. In particular, the replacement of the initial and final phonons by a fermion line and the corresponding modification of diagrams measures the difference between the moments of inertia associated with the even and odd systems (the problem of identical bands).

A Intrinsic angular momentum operators

The intrinsic angular momentum operators are defined by $J_\mu \equiv i\sqrt{10}(q,p)_\mu^1$. In cartesian components they are

\begin{align*}
J_x &= q_z p_y - q_y p_z + (\sqrt{3}q_\beta + q_\gamma)p_x - q_x(\sqrt{3}p_\beta + p_\gamma) \\
J_y &= q_x p_z - q_z p_x - (\sqrt{3}q_\beta - q_\gamma)p_y + q_y(\sqrt{3}p_\beta - p_\gamma) \\
J_z &= q_y p_x - q_x p_y - 2q_\gamma p_z + 2q_z p_\gamma
\end{align*}

(A.1) (A.2) (A.3)
Fig. 2. The vertices and diagrammatic contributions to the energies of the yrast and one \( \beta \)-phonon state. Here \( x(\perp) \) stands for \( \beta, \gamma, 1_x, 1_x(w, 1_p, 0_p) \) phonons and \( g \) for ghosts. The isolated crosses denote the expectation value of \( H_{BRST}^{(4)} \).

The expansion of the intrinsic coordinates and momenta gives

\[
\langle J_x \rangle = I; \quad \langle J_y \rangle = \langle J_z \rangle = 0 \quad (A.4)
\]
\[
J_x^{(1)} = 2\beta_o p_x^{(1)} + \frac{I}{2\beta_o} (\sqrt{3}q_{\beta}^{(1)} + q_{\gamma}^{(1)}) \quad (A.5)
\]
\[
J_y^{(1)} = -\beta_o p_y^{(1)} - \frac{I}{2\beta_o} q_z^{(1)}; \quad J_z^{(1)} = -\beta_o p_z^{(1)} + \frac{I}{2\beta_o} q_y^{(1)} \quad (A.6)
\]
\[
J_x^{(2)} = J_{xx}^{(2)} + J_{x\perp}^{(2)} \quad (A.7)
\]
\[
J_{xx}^{(2)} = (\sqrt{3}q_{\beta}^{(1)} + q_{\gamma}^{(1)})p_x^{(1)} - q_x(\sqrt{3}p_{\beta}^{(1)} + p_{\gamma}^{(1)}) \quad (A.8)
\]
\[
J_{x\perp}^{(2)} = q_x^{(1)} p_y^{(1)} - q_y^{(1)} p_z^{(1)} \quad (A.9)
\]
\[
J_y^{(2)} = q_x^{(1)} p_z^{(1)} - q_z^{(1)} p_x^{(1)} - (\sqrt{3}q_{\beta}^{(1)} - q_{\gamma}^{(1)}) p_y^{(1)} + q_y^{(1)} (\sqrt{3}p_{\beta}^{(1)} - p_{\gamma}^{(1)}) \quad (A.10)
\]
\[
J_z^{(2)} = q_y^{(1)} p_x^{(1)} - q_x^{(1)} p_y^{(1)} - 2q_{\gamma}^{(1)} p_z^{(1)} + 2q_z^{(1)} p_{\gamma}^{(1)} \quad (A.11)
\]
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