Emergence of cooperation induced by preferential learning

Jie Ren, Wen-Xu Wang, Gang Yan, and Bing-Hong Wang

1Department of Physics,
2Department of Modern Physics,
3Department of Electronic Science and Technology,
University of Science and Technology of China,
Hefei, 230026, PR China
(Dated: February 2, 2022)

The evolutionary Prisoner’s Dilemma Game (PDG) and the Snowdrift Game (SG) with preferential learning mechanism are studied in the Barabási-Albert network. Simulation results demonstrate that the preferential learning of individuals remarkably promotes the cooperative behavior for both two games over a wide range of payoffs. To understand the effect of preferential learning on the evolution of the systems, we investigate the time series of the cooperator density for different preferential strength and payoffs. It is found that in some specific cases two games both show the 1/f-scaling behaviors, which indicate the existence of long range correlation. We also figure out that when the large degree nodes have high probability to be selected, the PDG displays a punctuated equilibrium-type behavior. On the contrary, the SG exhibits a sudden increase feature. These temporary instable behaviors are ascribed to the strategy shift of the large degree nodes.

PACS numbers: 87.23.Kg, 02.50.Le, 87.23.Ge, 89.75.CC

I. INTRODUCTION

Cooperation is ubiquitous in real world, ranging from biological systems to economic and social systems [1]. However, the unselfish, altruistic actions apparently contradict Darwinian selection. Thus, understanding the conditions for the emergence and maintenance of cooperative behavior among selfish individuals is a central problem. Game theory together with its extensions [2, 3, 4, 5, 6, 7, 8], considered to be an important approach, provides a useful framework for investigating this problem. Two simple games, Prisoners’ Dilemma Game (PDG) [9] and Snowdrift Game (SG) [10], as metaphors for characterizing the evolution of cooperative behavior have drawn much attention from not only social but also biological and physical scientists [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. Interestingly, it is found that comparing with the square lattices, Scale-free networks provide a unifying framework for the emergency of cooperation. Some previous works have suggested that the introduction of “tit-for-tat” [6, 14] strategy can remarkably enhance the cooperative behavior. More recently, Nowak and May [13] found that the PDG with simple spatial structure can induce the emergence of cooperation, and in particular, spatial chaos is observed. In contrast, the work of Hauert and Doebley [26] demonstrates the spatial structure often inhibits the evolution of cooperation in the SG. Inspired by the idea of spatial game, much attention has been given to the interplay between evolutionary cooperative behavior and the underlying structure [17, 18, 19, 22, 23]. Since the surprising discovery of “small world” [26] and “scale-free” [27] structural properties in real networked systems, evolutionary games are naturally considered on the networks with these two kinds of structural features [16, 21, 22, 23]. Interestingly, it is found that comparing with the square lattices, Scale-free networks provide a unifying framework for the emergency of cooperation [23].

In the two games with network structure, such as square lattices (spatial structure), small world and scale-free structure, players interact only with their immediate neighbors. In each round, the score of each individual is the sum of the payoffs in the encounters with its neighbors. At the next generation, all the individuals could update their strategies (cooperate or defect) synchronously according to either the deterministic rule introduced by Nowak and May [13] or the stochastic evolutionary rule by Szabó and Tőke [17].

In this paper, we focus on the PDG and SG on scale-free networks mainly according to the stochastic update rules. However, we argue that such as in the social system, individual may not completely randomly choose a neighbor to learn from. “Rich gets richer” is a common feature in social and natural system, which reveals the ex-

*Electronic address: bbhwang@ustc.edu.cn
istence of preferential mechanism. It is indeed the preferential attachment mechanism of Barabási and Albert model (BA for short) 27 leads to the scale-free structural property in good accord with the empirical observations. Thus, in the present work, we present a preferential learning rule, the probability of which is governed by a single parameter, for better mimicking the evolution of real world system. The probability of choosing a neighbor for each individual depends on the degree of that neighbor. This assumption takes into account that the status of individuals can be reflected by the degree of them in various communities in nature and society, e.g. the leader usually interacts with large quantities of individuals. Interestingly, we find that the preferential learning mechanism promotes the cooperative behavior of both the PDG and SG. Several attractive properties for some specific parameter values are observed, such as the $1/f$-like noise of evolutionary cooperator density for both two games, which indicates the long range correlation of cooperation. In the SG, for some specific cases, the degree of cooperation displays a punctuated equilibrium-type behavior instead of steady state. In contrast, the PDG exhibits an absolutely different property of sudden jumps of cooperation. These two distinct behaviors are both attributed to the effect of leaders, i.e. the individuals with large connectivity.

The paper is arranged as follows. In the following section we describe the model in detail, in Sec. III simulations and analysis are provided for both the PDG and SG , and in Sec. IV the work is concluded.

II. THE MODEL

We first construct the scale-free networks using the BA model which is considered to be the most simple and general one. Starting from $m_0$ fully connected nodes, one node with $m$ links is attached at each time step in such a way that the probability $\Pi_i$ of being connected to the existing node $i$ is proportional to the degree $k_i$ of that node, i.e., $\Pi_i = k_i / \sum_j k_j$ with summation over all the existing nodes. Here, we set $m = m_0 = 2$ and network size $N = 5000$ for all simulations. The degree distribution of BA networks follows a power law $P(k) \sim k^{-3}$.

We consider the evolutionary PDG and SG on the networks. Without losing generality, we investigate the simplified games with a single payoff parameter following previous works 13, 17, 23. Figure (1) illustrates the encounter payoffs of both the PDG and SG. Each individual is placed on a node of the network and plays the games only with their immediate neighbors simultaneously. The total payoff of each player is the sum over all its encounters.

During the evolutionary process, each player is allowed to learn from one of its neighbors and update its strategy in each round. As mentioned early, each player chooses a neighbor according to the preferential learning rule, i.e., the probability $P_{i \rightarrow j}$ of $i$ selecting a neighbor $j$ is

$$P_{i \rightarrow j} = \frac{k_i^\alpha}{\sum_j k_j^\alpha},$$

where $\alpha$ is a tunable parameter and the sum runs over the neighbors of $i$. One can see when $\alpha$ equals zero, the neighbor is randomly selected so that the game is reduced to the original one. While in the case of $\alpha > 0$, the individuals with large degree have advantages to be selected; Otherwise, the small degree individuals have larger probability to be selected. In social and natural systems, some individuals with high status and reputation may have much stronger influence than others and the status of individuals can be reflected by the degree of them. Thus, the introduction of the preferential learning intends to characterize the effect of influential individuals on the evolution of cooperation. In parallel, we also investigate the performance of the systems with tendency of learning from the individuals with small degree.

After choosing a neighbor $y$, the player $x$ will adopt the coplayer’s strategy with a probability depending on the normalized total payoff difference presented in Ref. 21 as

$$W = \frac{1}{1 + \exp[(M_x/k_x - M_y/k_y)/T]},$$

where $M_x$ and $M_y$ are the total incomes of player $x$ and $y$, and $T$ characterizes the noise effects, including fluctuations in payoffs, errors in decision, individual trials, etc. This choice of $W$ takes into account the fact of bounded rationality of individuals in sociology and reflects nat-
ural selection based on relative fitness in terms of evolutionism. The ratio of total income of individual and its degree, i.e., $M_x/k_x$ denotes the normalized total pay-off. This normalization avoids an additional bias from the different degree of nodes. In the next section, we perform the simulations of the PDG and SG respectively and our goal is to find how the preferential learning affect the evolutionary cooperative behaviors of both PDG and SG.

### III. SIMULATION RESULTS

The key quantity for characterizing the cooperative behavior of the system is the density of cooperators $\rho_c$. Hence, we first investigate $\rho_c$ as a function of the tunable parameter $\alpha$ for different payoff parameter $b$ in the PDG, as shown in Fig. 2. The simulation results were obtained by averaging over last 10000 time steps of entire 20,000 time steps. Each data point results from an average over 20 simulations for the same type of network structure. In the initial state, the strategies of $C$ and $D$ are uniformly distributed among all the players. We figure out that, comparing with the case of no preferential learning, i.e., $\alpha = 0$, the cooperation is remarkably promoted not only for positive value of $\alpha$, but also for negative $\alpha$ in a wide range of $b$. For negative $\alpha$, the $\rho_c$ monotonously increases with the decrease of $\alpha$ and finally $\rho_c$ reaches a upper limit for very small $\alpha$. In contrast, in the case of positive $\alpha$, we find that $\rho_c$ increases dramatically and there exists a maximal value of $\rho_c$, which indicates that although the leaders with large degree play a key role in the cooperation, very strong influence of leaders will do harm to the persistence of cooperation and make individuals to be selfish. One can also find that the larger the value of $b$, the larger the value of $\alpha$ corresponding to the maximal $\rho_c$. Moreover, an interesting phenomenon is observed in Fig. 2, that is when $b$ is small, positive $\alpha$ leads to better cooperative behavior than the negative one; However, for large $b$, the system performs better when choosing negative $\alpha$. These results imply that if the income of defectors is only little more than that of cooperators, the leader’s effect will considerably enhances the cooperation; While if the selfish behavior is encouraged in the system (large $b$), the influential individuals will leads to the imitation of selfish behavior and reduce the cooperators density in a certain extent. On the contrary, restriction of leader’s influence (negative $\alpha$ decreases the selected probability of large degree individuals by their neighbors) results in better cooperation.

In parallel, we investigate the effect of preferential learning upon the SG. The simulation results are demonstrated in Fig. 3. Similar to the PDG, $\rho_c$ is improved by the introduction of preferential learning for nearly the entire range of $r$ from 0 to 1. In both sides of $\alpha = 0$, $\rho_c$ reaches an upper limit, which means that in the case of strong leader’s influence or without leaders, cooperation can be promoted to the highest level for the wide middle range of $b$. Contrary to the PDG, for very large $r$, the system still performs cooperative behavior, which is attributed to the fact that the rule of SG favors the cooperators, that is the cooperators yet gain payoff $1 - r$ when meeting defectors. Combining the above simulation results of both the PDG and SG, we can conclude that the preferential learning mechanism indeed plays an important role in the emergence of cooperation.

In the following, we analyze the time series of the cooperars density to give detailed description of the systems’ evolutionary behavior. We first study the PDG for negative value of parameter $\alpha$. Surprisingly, for some specific values of $b$ and $\alpha$, $1/f$-like noise is found. A prototypical example is exhibited in Fig. 4. The $1/f$-like noise
is observed frequently in real-world systems, including healthy physiologic systems [28, 29, 31], economical systems [31, 32], as well as traffic systems [33]. However, as far as we know, $1/f$ pattern hasn’t been reported in the study of evolutionary games. The $1/f$ noise denotes that the power spectrum of time series varies as a power-law $S(f) \sim f^{-\phi}$ with the slope $\phi = 1$. The spectrum exponent $\phi$ characterizes the nature of persistence or the correlation of the time series. $\phi = 2$ indicates zero correlation associated with Brownian motion, where as $\phi = 0$ corresponds to a completely uncorrelated white noise. $\phi > 2$ indicates positive correlation and persistence i.e., if the process was moving upward (downward) at time $t$, it will tend to continue to move upward (downward) at future times $t'$; $\phi < 2$ represents negative correlation and anti-persistence. The intermediate case, $S(f) \sim f^{-\phi}$, is a “compromise” between the small-time-scale smoothness and large-time-scale roughness of Brownian noise. Figure 4 (a) shows the time evolution for $b = 1.0$ and $\alpha = -1$, i.e. the case of restriction of leader’s influence. In this case, the density of cooperators remains stable with frequently fluctuations around the average value. Figure 4 (b) is the power spectrum analysis of the time series of cooperator density. A prototypical $1/f$-like noise is found with the fitted slope $\phi = 1.06$. This result indicates when the small degree individuals have large probability to be followed, i.e., suppress the influential leader’s effect, the nontrivial long range correlation of evolutionary cooperative behavior emerges. The similar phenomenon is also observed in the SG for the case of negative $\alpha$, as shown in Fig. 5. The emergence of the $1/f$ scaling is associated with the parameter values $\alpha = -1$ and $r = 0.5$. The discovered $1/f$ noise for both two games is partly ascribed to the lack of influence of leaders. Suppose that if the individuals with large connectivity are chosen with large probability, their strategy will be easily followed by a large number of persons, because those leaders usually gain very high income. Since the influential ones only take the minority, the evolutionary cooperative behavior will mainly determined by the minority. Besides, the strategies of those leaders are usually fixed due to their very high score, the long range correlation of the fluctuation of cooperator density is broken.

Then we investigate the evolutionary behavior of both the SG and PDG in the case of positive $\alpha$. For the SG, when the parameter $\alpha$ is close to zero, for arbitrary $b$, the level of cooperation remains stable with relatively small fluctuations around the average value. This property is remarkably changed for large value of $\alpha$, which means the influence of leaders becomes strong. As shown in Fig. 6, for $\alpha = 5$ and $r = 0.5$, the equilibrium is punctuated by sudden drops of cooperator density. After a sudden drop, the cooperation level $\rho_C$ will gradually increase until $\rho_C$ reaches the average value. The occurrence of these punctuated equilibrium-type behavior is ascribed to the strong influence of a small amount of leaders. As we have mentioned, the leader nodes usually get large payoffs, thus they tend to hold their own strategies and are not easily affected by their neighbors. However, those influential individuals still have small probability to follow their neighbors’ strategies. If an event that a leader shift his strategy to defector occasionally happens, the successful defector strategy will rapidly spread from the leader to his vicinities. Due to the connection heterogeneity of the scale-free networks, i.e., the leaders have large amount of neighbors, the imitation of a successful selfish behavior of the leader triggers the rapidly decrease of cooperator density. After the occurrence of a sudden drop, defectors become the majority and the selfish

![FIG. 4](image1.png)

(a) Time series of cooperator density $\rho_{oc}$ of the PDG for $b = 1.0$ and $\alpha = -1$. (b) Power spectrum analysis of (a).

![FIG. 5](image2.png)

(a) Time series of cooperator density $\rho_{oc}$ of the SG for $r = 0.5$ and $\alpha = -5$. (b) Power spectrum analysis of (a).
FIG. 6: The time evolution of cooperator density of the SG with $r = 0.5$, $\alpha = 5$ exhibits the punctuated equilibrium-type behavior.

FIG. 7: The sudden increase of cooperator density of the PDG with $b = 1.5$, $\alpha = 1$.

leader nearly gain nothing. Then under the influence of

the other leaders with cooperate strategies, the cooperator density will slowly recover to the steady state.

The evolutionary behavior of the PDG for the positive $\alpha$ also exhibits nontrivial feature as shown in Fig. 7. Contrast to the SG, the cooperation level shows some sudden increases. The mechanism that induces the temporary instability of cooperator density is the same as that of the sudden drops of the SG. The strategy shift of influential nodes plays the main role in the occurrence of the sudden increase. Opposite to the SG, the payoff matrix of the PDG favors the defect behavior, thus the cooperation level is quite low. An occasional strategy shift from defect to cooperate of a leader will lead to the imitation of its neighbors and a sudden increase occurs. However, the high cooperator density is instable in the PDG for large $b$, hence the sudden increase will rapidly decrease to the average value.

IV. CONCLUSION AND DISCUSSION

We have investigated the cooperative behavior of the evolutionary games resulting from the preferential mechanism. Comparing with the cases of random selection, i.e., $\alpha = 0$, preferentially selecting large degree nodes or small degree ones can promote the cooperation density of both the PDG and the SG over a wide range of payoffs. For the cases of negative value of $\alpha$, the systems perform the behavior of long range correlation, which is quantified by the $1/f$ scaling of power spectrum. Interestingly, in the case of positive value of $\alpha$, i.e., the large degree nodes have high probability to be selected for imitation, the SG exhibits a punctuated equilibrium-type behavior which is qualified by the occasional occurrence of sudden drops. In contrast to the SG, the PDG shows temporary instable behavior with the existence of sudden increase. The mechanism that leads to the instabilities of cooperation for both games are the strategy shift of influential nodes and the imitation of their neighbors. The instable behavior indicates that the strong influence of leader individuals will do harm to the evolutionary cooperative behavior of the systems. The present work implies that the existence of the preferential learning mechanism plays an important role in the emergence of cooperation in the heterogeneous networked systems.

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