Mathematical support of walking robots-tetrapods movement algorithms

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Abstract. This work presents a mathematical model that describes the movement of robots with four limbs (tetrapods). The relevance of the publication is justified by the popularity and increased demand for robotic devices used in production and everyday life. The main advantage of these machines is the ability to move over rough terrain, so are often used to perform transport and loading operations. The authors considered the most common movement scheme, taking into account the features of the kinematic scheme of this type devices. Tasks of transforming the coordinates of legs contact points of the supporting surface are formulated and solved to form tasks for controlling servo drives. The resulting mathematical expressions are universal for the considered robots and allow their statically stable movement in all directions and turning through a given angle. The obtained results can be used in development of algorithms of control programs for different microprocessor systems, in design of kinematics of these devices, in selection of optimal electromechanical equipment, as well as in creation of similar mathematical models for other types of walking robots. The materials of the article can be useful for engineers and scientists in the field of robotics and automation.

1. Introduction

The high level of automation sets new requirements for products at all levels of modern production, which has led to a significant development of microprocessor systems and electronics. So one of the most popular and common areas become robotics.

Modern robots are developed and implemented everywhere for a wide range of tasks: loading, sorting and transport works, diagnostics, assembly and repair of equipment and many others. Walking robots have gained worldwide fame: Atlas, PETMAN, LS3 and Cheetah (USA, Boston Dynamics:), Crabster (South Korea) and Spider-robot T8 (China, Robugtix), Halluc (Japan, fuRo), Quattroped (Taiwan, BioRoLA). The main advantage of such devices is the ability to move on uneven surfaces, cross barriers or climb and descend steps. However, their work is complicated by a number of problems: high energy costs, complexity of kinematics and control algorithms. At different stages of design of such devices it is necessary to take into account peculiarities of movement mechanism and algorithms of its operation [1-5].

The purpose of this work is to develop a mathematical support that allows to describe the principle of movement of the robot tetrapod, to implement coordinate transformations to determine the points of contact of the surface relative to the centre of the entire body and the kinematic diagrams of the legs [6].
2. The device of the walking robot
The considered robot consists of a body and four legs, driven by servo motors. Its simplified diagram is shown in Figure 1 (a).

Consider the coordinates of legs contact points of the surface, indicated in Figure 1 (b) and (c) according to the limb number, relative to the coordinate system (CS) associated with the original position of the body. The initial values of these parameters are represented as a 2x4 matrix: in the first line along the X axis, directed along the body, and in the second line - along the Y axis, directed across:

\[
R_{def,i,j} = \begin{bmatrix}
H/2 & H/2 & -H/2 & -H/2 \\
W/2 & W/2 & -W/2 & -W/2
\end{bmatrix}
\]  

(1)

where \( i = 1 \) is the X coordinate, \( i = 2 \) is the Y coordinate, \( j = 1..4 \) is the leg number of the tetrapod robot.

The algorithm of the robot body movement in space is implemented by successive permutation of limbs. Kinematics of the considered mechanism provides static stability, if at any moment at least three legs form a support triangle. Figure 1 shows with a dashed line the carrying limb at the moment when the robot does not rest on it, respectively (b) the unstable position and (c) stable.

3. Algorithm of tetrapod movement
It is necessary to develop a movement algorithm that provides static stability when moving the considered type of robots [7-10].

In the initial state, the centre of gravity B is in the zero of the absolute CS XY Figure 2 (a), the contact points of the legs are in the original positions. The XY coordinate system is not rigidly attached to the robot body and allows both CS and body movements.

Consider the time when leg 4 is transferred. It is necessary to move the body's centre of gravity to the geometric centre of the supporting triangle, formed by the limbs 1,2,3 (\( \Delta_{123} \)). In Figure 2 (b), point C indicates where the robot body should move. It is at the intersection of the median \( \Delta_{123} \).

Introduce the concept of a safe area of finding the centre of gravity of the robot body, in which it is possible to lift the carried leg without loss of stability. Its size depends on many factors, including the slope of the movement surface. In specific case at a horizontal surface the view of this area and its centre is similar to \( \Delta_{123} \). Let us mark this area in Figure 2 (b) as a safe triangle \( \delta_{123} \). The coordinates of the vertices are calculated by the method of proportion through the coefficient \( \lambda = 0..1 \), taking into account the fact that they are also located on the medians \( \Delta_{123} \).

When numerically solving the system of differential equations of motion of point B, the body moves in time. At achievement of the safe area \( \delta_{123} \) moves the leg 4 on \( \Delta X \) step distance, as shown in Figure 2 (c). The algorithm of further body movement can be different: to continue moving to point C, to stop...
body movement, or to move along the optimal trajectory to minimize the length of movement in the next step. In this work we will limit ourselves to the first variant (Figure 2 (d)). For the following transferable leg 1 coordinates of a new point C and triangles $\Delta_{234}, \delta_{234}$ are defined. In Figure 2 (d, e) the leg 1 moves to $\Delta X$ step forward.

![Diagram of robot walking](image)

**Figure 2.** Robot walking diagram «forward»

Since the absolute XY coordinates are not related to the body, their position must be adjusted so as not to accumulate the distance moved. In Figure 2 (f), the new origin of the $\Omega_1$ is at the centre of the quadrilateral formed by the contact points of all limbs. It also recalculates the centre of body B and the contact points.

For leg 3, the coordinates of point C and triangles $\Delta_{124}, \delta_{124}$ are calculated (Figure 2 (g)). At the same time the leg moves to bigger distance, than previous, but rather initial arrangement of $R_{def}$ on $\Delta X$. This allow to avoid the accumulating error of numerical calculation of differential equations, and also automatically forms a gait with the forward-interleaved step of the robot’s left and right limbs. Similarly for a leg 2 its new coordinates, a point C and triangles $\Delta_{134}, \delta_{134}$ are defined (Figure 2 (i, j)).

Then the absolute CS is corrected (Figure 2 (k, l)) and the algorithm is repeated from leg 4.

The movement algorithm sideways on $\Delta Y$ and on the diagonal ($\Delta X, \Delta Y$) is similar described above.

Consider the scheme of robot turning according to the algorithm described earlier (Figure 3).

Initially, the coordinates of point C, $\Delta_{123}, \delta_{123}$ are calculated for the moved leg 4 (Figure 3 (a)). The direction of its movement is calculated from a circle centered at the beginning of the absolute CS described around the rectangle of the initial limbs positions (b). Leg 1(c, d) moves sequentially. The body moves and turns on $\alpha\beta$ corner equal to a quarter of turn of one extremity. For two steps it will turn on a corner $\Delta \phi/2$ (d).
Figure 3 (e) shows the principle of absolute CS correction. They move to the centre of the figure formed by the contact points of all four legs, and turn around at the angle opposite to the current body rotation. The body will have an angle equal to zero (f).

Figure 3. Robot turning diagram

Legs 3 and 2 are sequentially rearranged Figure 3 (g, h) and Figure 2 (i, j). As with forward motion, the new coordinates of the moved limbs are calculated relative to the rectangle of the original positions $R_{def}$.

As a result, the following tetrapod movement algorithm is obtained:

1) the number of the leg to be moved is determined. In the simplest case, a vector with leg numbers is organized, providing a sequence $N_l = [4 1 3 2]$;

2) coordinates of the centre of the reference triangle, its vertices and vertices of the safe triangle and the new position of the leg to be moved, as well as the angle of the body turn are determined;

3) numerically solve differential equations of body motion;

4) when the body centre reaches the safe zone, movement of the moved leg begins. At that the body continues to move and turn;

5) steps 1-4 are executed twice, then the absolute CS is adjusted.

4. Mathematical model of movement

For a mathematical model, denote the 4th order unit matrix $E$, matrix $J_e$, representing as a subtraction of the matrix of units $J$ and identity matrix $E$, column-vector $e$ of length 4, filled with 1:
The rotation matrix for the angle $\varphi$ for the left-side CS and the positive direction of rotation counterclockwise:

$$M(\varphi) = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix}.$$  \hspace{1cm} (3)

Vector defining movement:

$$D_{xy} = \begin{pmatrix} \Delta X \\ \Delta Y \end{pmatrix}.$$  \hspace{1cm} (4)

Current coordinates of legs contact points in 2x4 matrix:

$$L = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{bmatrix}.$$  \hspace{1cm} (5)

At $n$-th step of program operation new coordinates of movable $j$-th leg are determined (the $j$-th column of the $L_n$ matrix is filled):

$$L_{*,j,n} = M(\Delta \varphi) \cdot R_{\text{def}} \cdot E_{*,j} + D_{xy},$$  \hspace{1cm} (6)

where $E_{*,j}$ - $j$-th column of a unit matrix, $\Delta \varphi$ - setting to the rotation angle of the body.

The coordinates of the contact points of the supporting legs are left unchanged. Setting on the angle of rotation of the body at the $n$-th step:

$$\alpha_{b,n} = \alpha_{b,n-1} + \Delta \varphi / 4,$$  \hspace{1cm} (7)

where $\alpha_{b,n-1}$ - previous value of body rotation angle.

The setting to the coordinates of the centre of the body $B_n$ corresponds to the geometric centre of the supporting triangle of legs $C_j$ for the moved $j$-th leg, defined as the arithmetic mean of each X and Y coordinate:

$$B_n = C_j = \frac{1}{3} \cdot L_{n-1} \cdot J_{e,j},$$  \hspace{1cm} (8)

where $J_{e,j}$ - $j$-th column of matrix $Je$.

Coordinates of a safe triangle corners are determined by coordinates of legs of a reference triangle for the moved $j$-th leg taking into account coefficient of scaling $\lambda$:

$$T_s = C_j \cdot (1 - \lambda) \cdot e^T + \lambda \cdot L.$$  \hspace{1cm} (9)

The column with the moving leg index in the $T_s$ matrix carries no information.

If the concrete speed of body movement $V_B$, leg $V_L$, angular speed $\omega_B$ are given, then the lengths of the body movement vector is $S_B = |B_n - B_{n-1}|$ and $j$-th leg is $S_{L,j} = |L_{*,j,n} - L_{*,j,n-1}|$. To simplify, we use Euler’s 1st order method of solving differential equations. Coordinate increments:
\[
\begin{align*}
\Delta X_{j,a} &= \frac{V_j \cdot \Delta t}{S_{ij}} (L_{a,j,a} - L_{a,j,a-1}); \\
\Delta \alpha_a &= \omega_a \cdot \Delta t; \\
\Delta B_n &= \frac{V_n \cdot \Delta t}{S_B} (B_n - B_{n-1});
\end{align*}
\]

where \( \Delta X_{j,a} \) – \( j \)-th column of the 2x4 increment matrix. The remaining columns are filled with zeros, \( \Delta t \) – fixed in time step of the calculation cycle.

In the process of numerical solution, the following actions are performed:

\[
\begin{align*}
L'_k &= L'_{k-1} + \Delta X_n; \\
B'_k &= B'_{k-1} + \Delta B_n; \\
\alpha'_k &= \alpha'_{k-1} + \Delta \alpha_n,
\end{align*}
\]

where \( k \) – intermediate steps for calculating differential equations of motion; \( L'_k, B'_k, \alpha'_k \) – current coordinates. At the beginning of the movement cycle, they are filled with the values from the previous step \( L_{n-1}, B_{n-1}, \alpha_{n-1} \).

In order to satisfy the condition where the center of the body at coordinates \( B \) enters the safe triangle \( T_s \) it is necessary to calculate the determinants of the following three matrices:

\[
\begin{align*}
\begin{bmatrix}
L_{1,m1} - B_1 & L_{1,m2} - L_{1,m1} \\
L_{2,m1} - B_2 & L_{2,m2} - L_{2,m1}
\end{bmatrix}, \\
\begin{bmatrix}
L_{1,m2} - B_1 & L_{1,m3} - L_{1,m2} \\
L_{2,m2} - B_2 & L_{2,m3} - L_{2,m2}
\end{bmatrix}, \\
\begin{bmatrix}
L_{1,m3} - B_1 & L_{1,m1} - L_{1,m3} \\
L_{2,m3} - B_2 & L_{2,m1} - L_{2,m3}
\end{bmatrix}
\end{align*}
\]

where the indices \( m_1, m_2, m_3 \) are selected from the matrix \( I_{sp} \), composed of the coordinate indices of the reference triangles for each \( j \)-th moving leg:

\[
\begin{bmatrix}
m_{1,j=1,4} \\
m_{2,j=1,4} \\
m_{3,j=1,4}
\end{bmatrix} = \begin{bmatrix}
2 & 1 & 1 & 1 \\
3 & 3 & 2 & 2 \\
4 & 4 & 4 & 3
\end{bmatrix}
\]

Geometrically, the determiners \( a_1, a_2, a_3 \) are the \( Z \) coordinate, perpendicular to the \( XY \) plane of the vector multiplication of the two vectors lying in it. One is the side of the triangle, the second is directed from point \( B \) to a particular vertex. If the determiners have the same sign when each side is bypassed sequentially, then point \( B \) lies inside the figure.

The new position of the coordinate system corresponds to the centre of the rectangle, formed by the contact points of all legs and is defined as the arithmetic mean of each coordinate:

\[
O_{1a} = \frac{1}{4} \cdot L_a \cdot e.
\]

The new coordinates of the body centre:

\[
B_{n+1} = M (-\alpha_{b,n}) \cdot (B_n - O_{1a})
\]

where \( \alpha_{b,n} \) – current solid rotation angle.
Since the axes must be rotated to the side opposite to the rotation of the body, the current rotation angle is taken with a negative sign. New contact points coordinates:

\[ L_{n+1} = M(-\alpha_{b,n}) \cdot (L_n - O_1) \cdot e^T. \]  

(16)

After that, the current rotation angle of the body is set to zero \( \alpha_{b,n+1} = 0 \).

In the process of solving differential motion equations, it is necessary to control the servo motors of the robot legs. They are related to the axes of rotation (shoulders) located at the edges of the body, and the coordinates of the contact points must be recalculated from the absolute CS XY \( L' \) in the Xl,Yl axis (Figure 4).

![Figure 4. Coordinate conversion for servo motors](image)

First \( L' \) is recalculated in CS, connected with the body Xb,Yb:

\[ Lb = M(-\alpha_{b,k}) \cdot (L'_k - B'_k) \cdot e^T. \]  

(17)

The results are then converted into a CS Xl,Yl, connected with the centres of the shoulders (starts) of the corresponding legs \( j \):

\[ L_j = L_b - C_j, \quad P_{\beta_{j-1,4}} = M(-\beta_j) \cdot L_{1,j}, \]  

(18)

where \( L_j \) – intermediate matrix 2x4, \( P \) – matrix of coordinates of contact points in axes, related to the beginning of robot legs, \( C_j \) – matrix of coordinates of shoulder(start) centre of robots legs, which depends on the geometric dimensions of the body. From Figure 4:

\[ C_j = \begin{bmatrix} H_b/2 & H_b/2 & -H_b/2 & -H_b/2 \\ W_b/2 & W_b/2 & -W_b/2 & -W_b/2 \\ 2 & 2 & 2 & 2 \end{bmatrix}, \]  

(19)

where \( H_b, W_b \) – Body size along and across, \( \beta \) – CS rotation angles of each legs, which depend on the robot device. According to Figure 4, the CSs are rotated byangles:

\[ \beta = \begin{bmatrix} 0 & \pi/2 & 3\pi/2 \end{bmatrix}. \]  

(20)
Depending on the kinematics of the connection of a particular limb servo motors, the angles of their deviation are calculated to achieve the required contact points.

5. Conclusion

In the work the problem of formation of the robot-tetrapod moving algorithm with synchronous movement along two coordinates and turning, providing static stability, is formulated and solved. Mathematical expressions of definition of legs contact points of reference surface and transformation of coordinates are obtained. Features of the algorithm includes: calculation of the safe zone of the position of the centre of gravity and of the coordinates of the moved leg relative to the initial position (elimination of accumulation of calculation errors) and periodical correction of absolute CS. The obtained results can be used in the development of algorithms of control programs for different microprocessor systems, in the design of kinematics of these devices, in the selection of optimal electromechanical equipment, as well as in the creation of similar mathematical models of other types of moving robots.

References

[1] Belter D and Skrzypczynsky P 2010 A biologically inspired approach to feasible gait learning for a hexapod robot International Journal of Applied Mathematics and Computer Science – Computational Intelligence in Modern Control Systems 20 pp 69-84
[2] Roy S, Singh A and Pratihar D 2011 Estimation of optimal feet forces and joint torques for online control of six-legged robot Robotics and Computer-Integrated Manufacturing 27 pp 910-7
[3] Vidoni R and Gasparetto 2011 A Efficient force distribution and leg posture for a bio-inspired spider robot Robotics and Autonomous Systems 59 pp 142-50
[4] Yang J. 2008 Omni directional walking of legged robots with a failed leg Mathematical and Computer Modeling 47 pp. 1372-88
[5] Gulyaev N, Kazantseva V, Shvetsov V, Sidorenko A and Sypin E 2016 Off-road autonomous walking robot South-Siberian scientific messenger 4 pp 185-9
[6] Ignatiev M, Vladimirov S, Sapozhnikov V, Sergeev M, Kuzmin D, Soloviev V, Ryzhov A and Lapinsky Ya 2016 Moving robots – problems and prospects Innovation and Expertise pp 128-137
[7] Briskin E, Zhoga V, Maloletov A and Chernyshov V 2009 Dynamics and motion control of walking machines with cyclic propulsion devices pp 189
[8] Won-Suk J and Baek-Kyu C 2018 Development of a walking algorithm on the uneven terrain for a hexapod robot Little Crabster200 Advances in Mechanical Engineering 10 pp 1-12
[9] Budanov V 2005 Movement planning algorithms of six-legged walking machine Fundamental and applied mathematics pp 197-206
[10] Pavlovsky V and Panchenko A 2012 Models and algorithm for controlling the movement of a small six-legged robot Mechatronics, automation, control 11 pp 23-8.