Second-order Markov chain models of rainfall in Ibadan, Southwest Nigeria

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Abstract. Understanding the dynamics of rainfall is important for maximizing the agricultural potential, environmental planning, as well as policy development. In this study, a two-state second-order Markov chain model was developed for the annual and daily occurrence of different rainfall sequences in Ibadan, southwestern Nigeria. The annual variation was adapted to a sine series model. The model was able to account for more than 30% of the observed variation. Markov chain approach to modelling rainfall in tropical region is unique because it considers the “memory” – previous rainfall events, which is lacking in other methods. The daily occurrences showed patterns consistent with tropical regions. Results from this study is significant for economic, health and environmental planning in the location.

1. Introduction
Nigeria lies in the tropical zone and its climate can be regarded as a microcosm of the African climate. Four distinct climatic zones can be identified in the country, namely: coastal/tropical rainforest, guinea savanna, midland and Sahel/arid [1]. The two dominant seasons in Nigeria are the wet and dry seasons. The amount and frequency of rainfall, as well as the onset of wet season, is usually seen to reduce as one move away from the coastal States in the South [2]. The country experiences the “little dry season”, caused by the Inter Tropical Discontinuity (ITD), around July – August. The ITD is a consequence of two air masses (the tropical maritime air mass and tropical continental air mass) interacting.

Rainfall, the end product of complex atmospheric processes, is significant for the economy of tropical countries like Nigeria. Rainfall determines to a large extent the type of crops/animals, crop yield, length of growing season obtainable in the region. The importance of rainfall has necessitated various investigation into different methods of modelling and forecasting rainfall. Methods proposed include stochastic methods, probability distribution, wavelet decomposition, discrete autoregressive model, time fractional model and Markov chain model.

Markov chain models are relatively easy to derive, does not require understanding of underlying dynamics and it has modest computational requirements. It has the advantages of effective short-range forecasting, production of surrogates and ability to capture unique regional weather signatures. In Markov chain probability models, the current state of an event is assumed to be dependent on n numbers of previous states. The order n of the Markov chain is the number of preceding events considered. Modelling of rainfall has been considered using first order [3], second order [4] and
Higher orders [5]. A two-state Markov chain model had been used to study weekly rainfall in drought prone areas of India [6]. Markov chain modelling was also applied to weekly rainfall in Ethiopia for agricultural planning [7]. Similar approach was used to model monthly precipitation in Laos [8]. The aim of this work is to develop a Markov chain model for rainfall in Ibadan, southwestern Nigeria.

2. Methods
The study location, Ibadan (7.3775°N, 3.9470°E) is a coastal location in Southwestern Nigeria. Daily precipitation values for Ibadan was obtained from the Archives of the Nigerian Meteorological Services (NIMET) from 1972 – 2013. The organization carried out quality control of meteorological data. In this study, a rain event is assumed to occur when the mean rainfall is greater than 2 mm.

In this study a two-state, second order Markov model was considered. The possible sequences are rain, rain, rain (rrr), dry, rain, rain (drr), rain, dry, rain (rdr), and dry, dry, rain (ddr). Using the same approach as Stern and Coe [4], a fourth order oscillatory model was considered.

\[ f(x) = \sum_{k=1}^{n} a_k \sin(b_k x + c_k) \]

where \( n = 4 \) and the constants \( a, b, c \) are parameters to be determined using nonlinear least square fitting. \( f(x) \) is the number of occurrences of the sequences. The sine series model was considered due to the oscillatory pattern observed, however, any other convenient model can be used.

3. Results
A Markov chain model was developed for rainfall over Ibadan, southwestern Nigeria and the result shown in Fig. 1. The confidence interval for the model is also shown. The performance of the model was determined using the coefficient of determination and root mean square error. The values of 0.36, 0.37, 0.32 and 0.25 were obtained as the coefficient of determination for rrr, drr, rdr and ddr sequences respectively. In the same vein, the root mean square values for rrr, drr, rdr, and ddr were found to be 3.99, 3.62, 4.19 and 3.95 respectively. The probability of occurrences of any of the sequences on any given day was also investigated. The results are shown in Fig 2 with a 3-day running mean.

4. Discussion
Using a two-state, second order Markov chain model, the rainfall pattern in Ibadan was investigated. The proposed model was found to account for over 30% of the variation in yearly occurrences in all the sequences examined except ddr where only 25% of the variation could be explained. In Ibadan, the occurrence of rrr sequence varies between 2 and 18 events during the period under consideration. However, this was increased to between 30 and 45 events per year for the ddr sequence.

In Fig 2, the Markov chain was considered for daily occurrences of different sequences. Considerable probability of three consecutive days of rainfall (rrr) was observed during the raining season in the location. The drr sequence suggests that rainfall in the region increases consistently in the region till a peak is attained. The pattern of rdr depicts two rainfall peaks. These peaks become more prominent in the ddr sequence. The break, which occurs around July – August is attributed to ITD.
Fig. 1. A fourth order oscillatory model for annual occurrences of different rain sequences of rainfall for Ibadan.

Fig. 2. Second order Markov chain curves for probabilities of daily rainfall in Ibadan. The red lines denote a 3-day running mean.
5. Conclusions

This study investigates the probability of daily and annual occurrences of four different rainfall sequences in Ibadan using Markov chain model. The probability of annual rainfall was adapted to a sine series model due to its oscillatory nature. The probability of daily occurrences of rainfall in the region suggests a long raining season and a short one separated by a little break. This work has been able to determine the probability of precipitation based on previous day events. This is particularly important for agricultural and environmental planning. Furthermore, the intensity of rainfall is seen to have increasing intensity as the season progresses. Results from this study is significant for economic, health and environmental planning in the location. This study should be extended to other regions of the country and continent.

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