Localization of plastic flow in one-dimensional and two-dimensional problems

N A Kudryashov, P N Ryabov, R V Muratov
National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe shosse 31, Moscow, Russia
E-mail: nakudr@gmail.com, pnryabov31@gmail.com

Abstract. In this paper one-dimensional and two-dimensional models are used for investigation of the processes of adiabatic shear bands (ASB) formation. Significant characteristics of a localization process at different nominal strain-rates are studied in the series of the numerical simulations. The results obtained for one-dimensional and two-dimensional problems are compared.

1. Introduction
Localization of plastic flow in materials under high-speed loading is one of the mechanisms of material failure. Intensive material loads are often encountered in many processes in the military, aerospace and nuclear industries, where high reliability is required [1, 2, 3, 4, 5]. Therefore, in recent decades, a large number of studies have been devoted to the investigation of the phenomenon of plastic flow localization. One of the mechanisms of plastic flow localization in materials undergoing shear deformations is the formation of adiabatic shear bands (ASB). Traditionally the following phenomenon is studied using experimental and numerical approaches. Since the mathematical models describing the adiabatic shear bands formation processes included highly nonlinear terms analytical and numerical studies are often limited to only one dimensional case. So, in the present work, we investigate the possibility of generalizing the results obtained for one-dimensional models to the multidimensional case.

2. Mathematical model of the processes of plastic flow localization
In the present work we consider a large deformations of an elasto-plastic material. A motion of continuum substance is described by a classical continuity equations which have the form of mass conservation low (1), the Cauchy momentum equation (2) and conservation of energy (3). The above mentioned equations have the following form

\[
\frac{\partial \rho}{\partial t} + \text{div} (\rho \vec{v}) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t} (\rho \vec{v}) + \text{div} (\rho \vec{v} \times \vec{v} - \sigma) = 0. \tag{2}
\]

\[
\frac{\partial}{\partial t} (\rho E) + \text{div} (\rho E \vec{v} - \sigma \vec{v} - k \text{grad} T) = 0, \tag{3}
\]
Table 1. Thermophysical parameters of depleted uranium and aluminum alloy.

| Material | $C_v$, J/(kg°C) | $k$, W/(m°C) | $\rho_0$, g/cm$^3$ | $B$, GPa | $\mu$, GPa | $\sigma_0$, MPa | $a$, 1/°C | $\dot{\varepsilon}_y$, 1/s | $n$ |
|----------|----------------|-------------|-------------------|--------|---------|-------------|------|----------------|-----|
| DU       | 117            | 28          | 18.6              | 129    | 58      | 1079        | 0.0085 | 1              | 0.007 |
| Al       | 875            | 149         | 2.77              | 69.6   | 28      | 337         | 0.0017 | 1              | 0.01  |

where $\rho$ is density, $\vec{v}$ is velocity, $T$ is temperature, $E = e(T) + \vec{v}^2/2$ is the total energy and $e(T)$ is the specific internal energy, $\sigma$ is Cauchy stress tensor.

Next we suppose that the total deformation can be split in elastic and plastic parts according to relation $\varepsilon = \varepsilon^e + \varepsilon^p$. We use Hooke’s law for elastic part and associative flow rule for plastic part of deformation. In the result, we obtain a following equations

$$\dot{\sigma} = 2\mu(\dot{\varepsilon} - \dot{\varepsilon}^p), \quad \dot{\varepsilon}^p = \frac{3s}{2\sigma}, \quad \dot{\varepsilon} = \sigma^{-1}_y(\sigma, T),$$

(4)

where $s$ is a deviatoric stress tensor, $\dot{\varepsilon}$ is a deviatoric strain-rate tensor (index $p$ for plastic part), $\dot{\sigma}$ is an objective stress rate (we use Jaumann derivative), $\dot{\varepsilon}$ and $\sigma$ are effective strain-rate and effective stress respectively

$$(\dot{\varepsilon}^p)^2 = \frac{2}{3} \dot{\varepsilon}^p : \dot{\varepsilon}^p, \quad \sigma^2 = \frac{3}{2} s : s.$$

The function $\sigma_y^{-1}(\dot{\varepsilon}, T)$ is a plasticity flow law:

$$\sigma_y^{-1}(\sigma, T) = \dot{\varepsilon}_y \max \left[0, \left(\frac{\tau}{\sigma \cdot g(T)}\right)^{\frac{1}{m}} - 1\right],$$

(5)

where $g(T) = 1 - aT$ is temperature softening factor and $g(T)$ is equal to zero when a temperature reaches the melting point. Equation of state can be written in the form

$$e(T) = C_v T, \quad p(\rho) = B \left(\frac{\rho}{\rho_0} - 1\right).$$

(6)

So equations (1) - (6) with initial and boundary conditions is a closed system of equations which is described the processes of plastic flow localization at a large deformations.

In the present work we consider the process of plastic flow localization in two materials. The first one is depleted uranium DU-0.75Ti (DU) and the second one is aluminium alloy Al 7036 (Al). The thermo-physical parameters of materials studied are listed below in Table 1.

3. Numerical analysis of the problem

To study the process of plastic flow localization in Al and DU we use the numerical analysis. To integrated the above mentioned system of equation the finite volume method is used. The detail description of the numerical methodology is given in [6].

To perform numerical experiments we consider the pure shear of rectangular specimen with characteristic hight $H$. The size of the specimen considered is $10 \times 10$ mm, so $H = 10$ mm. Bottom boundary of the specimen is fixed, and the lower boundary is moving at a constant velocity $v_0 = \dot{\varepsilon}_0 H$. We use the periodic boundary conditions along $x$-axis. Initial temperature of the specimen is equal to zero, velocity is linearly distributed, deviatoric stress has random
initial distribution [6]. We perform the numerical experiments for each material studied at the following value of nominal strain rate \( \dot{\varepsilon}_0 = 2 \cdot 10^4 \) s\(^{-1} \); \( 4 \cdot 10^4 \); \( 6 \cdot 10^4 \); \( 8 \cdot 10^4 \); \( 10^5 \) s\(^{-1} \). Numerical experiments were performed in one dimensional and two dimensional cases. In one dimensional case we use \( 1 \times 2000 \) spatial cells and in two dimensional case \( 2000 \times 2000 \) spatial cells were used.

On Fig. 1, 2 we present the temperature and velocity distributions for DU and Al respectively. From Fig. 1, 2 we see that both temperature and velocity distributions are stepped. The dark lines on the figures correspond to the localization areas where temperature reaches its maximum value. For DU this value is equal to \( 1200 \) °C, and for Al the maximum temperature is equal to \( 800 \) °C.
610 °C. We also see that dark lines lies approximately on the same distance between each other. Numerical experiments show that the characteristic distance between shear bands significantly depends on the value of nominal strain rate. The following dependencies are illustrated on Fig. 3 (both for one dimensional and two dimensional case). From obtained results we see that the characteristic distance between ASB do not depend of the number of spatial variables. Also we obtain that, this is not true for all characteristics of the process of shear bands formation. In particular, the localization time increases when we considering a two-dimensional case.

4. Conclusion
In the present work we have formulated the mathematical model for describing the processes of plastic flow localization in one dimensional and two dimensional cases. We present the numerical algorithm to perform the numerical integration of the problem considered. The results of numerical experiments on deformation of aluminium and depleted uranium specimens are presented.

5. Acknowledgments
This research was supported by Russian Science Foundation Grant No 21-71-00102.

References
[1] D Rittel, 2005 Mater. Lett. 59 1845–48.
[2] D A Shockey, J W Simons, C S Brown and T Kobayashi 2007 Exp. Mech. 47 723–732.
[3] N A Kudryashov, R V Muratov and P N Ryabov 2017 Autom. Control. Comput. Sci. 51 614–620
[4] N A Kudryashov, R V Muratov and P N Ryabov 2018 Appl. Math. Comput. 338 164
[5] R C Batra and D Liu 1989 J. Appl. Mech. 56 527
[6] N A Kudryashov, R V Muratov and P N Ryabov 2021 Commun. Nonlinear Sci. Numer. Simul. 101 105858