On the origin of the polytropic behavior in space plasmas

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Abstract. The paper addresses the connection between the polytropic behavior – the specific power-law relationship among the thermal plasma moments – and the functional form of the distribution of particle velocities and energies. Surprisingly, the polytropic behavior requires the statistical mechanics of the plasma particles to turn to the framework of kappa distributions. While it was already known that kappa distributions can lead to the polytropic relationship, the new result shows that the reverse derivation is also true; thus, the polytropic behavior has the role of a mechanism generating kappa distributions. Therefore, when observations confirm the existence of a polytropic behavior in plasma particles, then the framework of kappa distributions for describing particle velocities and energies can be indirectly confirmed.

1. Introduction

Space plasmas are collisionless and weakly coupled particle systems, characterized by a collective behavior that induces correlations among their particles. The distributions of particle velocities and energies are frequently described by a specific type of non-Maxwellian distributions, the so-called, kappa distributions (e.g., see the book of kappa distributions [1]). These distributions generalize the Maxwell-Boltzmann distribution that applies in ideal systems characterized by no correlations among their particles.

Kappa distributions were first introduced to describe magnetospheric electron data, by Olbert and its Ph.D. students and colleagues [2-4]; Binsack (1966) [2] was the first to publish the usage of kappa distributions, but he acknowledged that the kappa function was actually “introduced by Prof. Olbert of MIT in his studies of IMP-1.” Since then, single types of kappa distributions are frequently used to model the space plasma populations (e.g., [5-7]); however, more complicated models of kappa distributions have been employed to describe rare features (e.g., anisotropy, superposition) (e.g., [8-10]). A kappa distribution is primarily formulated to describe a Hamiltonian, i.e., the particle kinetic and potential energy ([11,12]; [1], Chapter 3).

Various mechanisms responsible to generate kappa distributions in space plasmas have been examined and verified. Some examples are: macroscopic extensivity of entropy [13], thermodynamics [14], superstatistics [15-18], effect of shock waves [19], weak turbulence [20], turbulence with a diffusion coefficient inversely proportional to velocity [21], effect of pickup ions [22], pump acceleration mechanism [23], polytropic behavior [24-26]; (see also: [1], Chapters 5,6,8,10,15,16). Also, common processes, characteristic of space plasmas, such as the Debye shielding and magnetic coupling, have an important role in the generation of kappa distributions in plasmas [27]. In general, long-range interactions or other causes of local particle correlations implicate the particle system in the statistical framework of kappa distributions [28]. Such an example is the state of a charged test particle in a constant temperature heat bath of a second species of charged particles, modeled by [29]. The time dependence of the distribution function of the test particle is given by a Fokker-Planck
equation for Coulomb collisions and wave-particle interactions; the stationary state of this equation can be described by kappa distributions (for certain choices of the involved parameters). (For more examples, see: [30-37].)

The kappa distributions have become increasingly widespread across the physics of space plasma processes, describing particles in the heliosphere, from the solar wind and planetary magnetospheres to the heliosheath and beyond, the interstellar and intergalactic plasmas: *inner heliosphere*, including solar wind (e.g., [13,20,26,38-48]), solar spectra (e.g., [49,50]), solar corona (e.g., [51-54]), solar energetic particles (e.g., [55,56]), corotating interaction regions (e.g., [57]), and solar flares related (e.g., [58,33,21,59]); *planetary magnetospheres*, including magnetosheath (e.g., [60,61]), magnetopause (e.g., [62]), magnetotail (e.g., [63]), ring current (e.g., [64]), plasma sheet (e.g., [65-67]), magnetospheric substorms (e.g., [68]), Aurora (e.g., [69]), magnetospheres of giant planets, such as Jovian (e.g., [70-73]), Saturnian (e.g., [74-76]), Uranian (e.g., [77]), Neptunian (e.g., [78]), magnetospheres of planetary moons, such as Io (e.g., [79]) and Enceladus (e.g., [80]), cometary magnetospheres (e.g., [81,82]); *outer heliosphere and the inner heliosheath* (e.g., [83-91,22,92-97,13]); (iv) *beyond the heliosphere*, including HII regions (e.g., [98]), planetary nebula (e.g., [99,100]), and supernova magnetospheres (e.g., [101]); and in cosmological scales (e.g., [102]). The kappa distributions have also been applied in general plasma-related analyses (e.g., [103-115,92,116-121]).

A breakthrough in the field came with the connection of kappa distributions with statistical mechanics and thermodynamics, accomplished by the following findings: (1) kappa distributions maximize entropy under the constraints of canonical ensemble [122]; (2) particle systems exchanging heat and reaching thermodynamic equilibrium are stabilized always into a kappa distribution [14]; and (3) polytropes - Systems characterized by a polytropic behavior - are uniquely consistent to kappa distributions and their statistical formalism [24]. The latter is the topic presented in this paper, that is, the equivalence between the polytropic behavior and the formalism of kappa distributions.

2. General profiles of thermal observables

The phase-space distribution function of plasma particles is constructed by considering the distribution of particle energy, \( p(E) \), and substituting the Hamiltonian function instead of the energy, \( f(\vec{r}, \vec{u}) = p[H(\vec{r}, \vec{u})] \). We consider the 1-particle Hamiltonian function, \( H(\vec{r}, \vec{u}) = \varepsilon_k(\vec{u}) + \Phi(\vec{r}) \), for an ideal-gas type of plasma, where the inter-particle dynamical terms are negligible and the potential energy depends only on the position vector. The kinetic energy is expressed in the reference frame of the co-moving system, i.e., \( \varepsilon_k(\vec{u}) = \frac{1}{2} m \cdot (\vec{u} - \vec{u}_0)^2 \), with \( \vec{u}_0 \) noting the bulk plasma velocity.

The integration of \( f(\vec{r}, \vec{u}) \) over both the positions and velocities gives the normalization of the probability distribution,

\[
1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\vec{r}, \vec{u}) d\vec{r} d\vec{u}.
\]  

(1)

The velocity distribution comes from the integration of \( f(\vec{r}, \vec{u}) \) over the positions \( \vec{r} \), while the positional distribution is derived from the integration over the velocities \( \vec{u} \) (Figure 1).
The statistical moments of velocity space are defined in the co-moving system. The global moments are given by:

\[ \mu_m = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\vec{r}, \vec{u}) \, d\vec{u} \, \left[ \vec{u} - \vec{u}_m \right]^m \, d\vec{r}, \quad (2) \]

while the unnormalized and normalized local moments are defined by:

\[ \bar{\mu}_m(\vec{r}) = \int_{-\infty}^{\infty} f(\vec{r}, \vec{u}) \left[ \vec{u} - \vec{u}_m \right]^m \, d\vec{u}, \quad (3) \]

and

\[ \mu_m(\vec{r}) = \frac{\int_{-\infty}^{\infty} f(\vec{r}, \vec{u}) \left[ \vec{u} - \vec{u}_m \right]^m \, d\vec{u}}{\int_{-\infty}^{\infty} f(\vec{r}, \vec{u}) \, d\vec{u}}. \quad (4) \]

The density profile is proportional to the positional distribution, that is, the unnormalized local distribution for \( m=0 \), \( f(\vec{r}) = \bar{\rho}_0(\vec{r}) \). The proportionality constant, \( n_\infty \), provides the density at a position where the effect of the potential energy is negligible; here, we consider that \( \vec{r}_\infty \to 0 \). Hence,

\[ n(\vec{r}) = n_\infty \cdot f(\vec{r}) = n_\infty \cdot \int_{-\infty}^{\infty} f(\vec{r}, \vec{u}) \, d\vec{u}. \quad (5) \]

The global temperature \( T \) is defined by the global 2nd moment, \( \mu_2 \),

\[ \frac{1}{2} d k u T \equiv \left\langle \varepsilon_k \right\rangle_{(\vec{r}, \vec{u})} = \frac{1}{2} m \mu_2 = \frac{1}{2} m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\vec{r}, \vec{u}) \, d\vec{u} \left[ \vec{u} - \vec{u}_m \right]^2 \, d\vec{r}, \quad (6) \]

(Note: The notions \( \left\langle X \right\rangle_{(\vec{r}, \vec{u})} \), \( \left\langle X \right\rangle_{\vec{r}} \), and \( \left\langle X \right\rangle_{\vec{u}} \) mean averaging the argument \( X(\vec{r}, \vec{u}) \) over the whole phase space, the positional space, and the velocity space, respectively.)

The temperature profile \( T(\vec{r}) \) is given by the normalized local 2nd moment, \( \mu_2(\vec{r}) \), i.e.,

\[ \frac{1}{2} d k u T(\vec{r}) \equiv \left\langle \varepsilon_k \right\rangle_{\vec{r}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\vec{r}, \vec{u}) \varepsilon_k(\vec{u}) \, d\vec{u} \, d\vec{r} = \int_{-\infty}^{\infty} n(\vec{r}) f(\vec{r}, \vec{u}) \varepsilon_k(\vec{u}) \, d\vec{u} \, d\vec{r}. \quad (7) \]

Therefore, the thermal pressure is given by

\[ p(\vec{r}) = n(\vec{r}) k u T(\vec{r}) = \frac{2}{d_k} \int_{-\infty}^{\infty} n(\vec{r}) f(\vec{r}, \vec{u}) \varepsilon_k(\vec{u}) \, d\vec{u}. \quad (8) \]

It is worth noting that the average temperature profile coincides with the global temperature.
3. Profiles of thermal observables in the case of kappa distributions

The specific relationship, \( p = g(n) \), constructed by the graph curve between the pressure \( p(\vec{r}) \) and density \( n(\vec{r}) \) for all the positions \( \vec{r} \), defines the generalized polytropic behavior of the space plasma.

Next, we will examine how this relationship is reduced to a power-law in the case of kappa distributions.

The kappa phase-space distribution function \([1,11,12,123,124]\) gives the probability distribution of a particle having its position and velocity in the infinitesimal intervals \([\vec{r}, \vec{r} + d\vec{r}]\) and \([\vec{u}, \vec{u} + d\vec{u}]\), respectively,

\[
\begin{align*}
    f(\vec{r}, \vec{u} ; \kappa, T) & \propto \left[ 1 + \frac{1}{\kappa} \cdot \frac{H(\vec{r}, \vec{u}) - \langle H \rangle_{(\vec{r}, \vec{u})}}{k_B T} \right]^{-\kappa-1} \\
    & = \left[ 1 + \frac{1}{\kappa} \cdot \frac{\varepsilon_k(\vec{u}) + \Phi(\vec{r}) - \langle \varepsilon_k \rangle_{(\vec{r}, \vec{u})} - \langle \Phi \rangle_{(\vec{r}, \vec{u})}}{k_B T} \right]^{-\kappa-1},
\end{align*}
\]

which can be rewritten as:

\[
\begin{align*}
    f(\vec{r}, \vec{u}) & \propto \left[ 1 + \frac{1}{\kappa} \cdot \frac{\langle H \rangle_{(\vec{r}, \vec{u})} + \frac{1}{\kappa} \cdot \frac{H(\vec{r}, \vec{u})}{k_B T}}{k_B T} \right]^{-\kappa-1} \\
    & \propto \left[ 1 + \frac{1}{\kappa - \frac{1}{k_B T}} \cdot \frac{H(\vec{r}, \vec{u})}{k_B T} \right]^{-\kappa-1} \approx \left[ 1 + \frac{1}{\kappa - \frac{1}{k_B T}} \cdot \frac{H(\vec{r}, \vec{u})}{k_B T} \right]^{-\kappa-1}.
\end{align*}
\]

The mean Hamiltonian defines the total degrees of freedom or dimensionality, \( d \), summing the kinetic and potential degrees of freedom,

\[
\frac{1}{\kappa} d = \frac{\langle H \rangle_{(\vec{r}, \vec{u})}}{k_B T} = \frac{1}{\kappa} d_k + \frac{1}{\kappa} d_\phi,
\]

where the potential degrees of freedom are defined similar to the kinetic ones,

\[
\frac{1}{\kappa} d_k = \frac{\langle \varepsilon_k \rangle_{(\vec{r}, \vec{u})}}{k_B T}, \quad \frac{1}{\kappa} d_\phi = \frac{\langle \Phi \rangle_{(\vec{r}, \vec{u})}}{k_B T}.
\]

(Note that \( d_\phi \) can be either positive or negative.)

The kappa index depends on the dimensionality as \( \kappa = \kappa(d) = \text{constant} + \frac{1}{\kappa} d \), so that the difference \( \kappa(d) - \frac{1}{\kappa} d \) remains invariant under changes of the dimensionality \( d \). Hence, the invariant kappa index \( \kappa_0 \) is defined by \( \kappa_0 = \kappa - \frac{1}{\kappa} d \); hence, \( \kappa(d) = \kappa_0 + \frac{1}{\kappa} d \). The physical meaning of the thermodynamic parameter kappa is better carried by its invariant value \( \kappa_0 \), because this is independent of the degrees of freedom \([1,10,11,12,28]\).

Throughout this analysis, we use the notion of the invariant kappa index \( \kappa_0 \), but the typical 3-dimensional index can be easily retrieved, \( \kappa_3 = \kappa_0 + \frac{1}{\kappa} d \). Then, the phase-space distribution \((11)\) is rewritten as,
The integration of the phase space kappa distribution over the positions leads to the standard kappa distribution of velocities. This is true for quite a large set of potential energy forms, which they can be analytically expressed and expanded (at a position \( r_0 \)), so that \( \Phi(\vec{r}) \approx \Phi(\vec{r}_0) + \frac{1}{2} k \cdot |\vec{r} - \vec{r}_0|^2 \) (e.g., [123]).

If we integrate the phase space kappa distribution over the velocity space, then we derive the positional kappa distribution:

\[
f(\vec{r}; \kappa_0, T) \propto \left[ 1 + \frac{1}{\kappa_0} \frac{e_k(\vec{u}) + \Phi(\vec{r})}{k_B T} \right]^{-\kappa_0^{-1} - \frac{1}{2} d_0},
\]  

where the density profile is proportional to the positional probability distribution, i.e.,

\[
n(\vec{r}) = n_\infty \left[ 1 + \frac{1}{\kappa_0} \frac{\Phi(\vec{r})}{k_BT} \right]^{-\kappa_0^{-1} - \frac{1}{2} d_0}.
\]  

Again, the density \( n_\infty \) refers to the position where the potential energy becomes zero or practically negligible.

The temperature profile is determined by the local mean kinetic energy,

\[
\frac{1}{2} d_k k_B T(\vec{r}) \equiv \left\langle \epsilon_k(\vec{u}) \right\rangle_u = \frac{\int_{-\infty}^{\infty} f(\vec{r}, \vec{u}; \kappa_0, T) e_k(\vec{u}) d\vec{u}}{\int_{-\infty}^{\infty} f(\vec{r}, \vec{u}; \kappa_0, T) d\vec{u}},
\]

hence,

\[
T(\vec{r}) = T_\infty \cdot \left[ 1 + \frac{1}{\kappa_0} \frac{\Phi(\vec{r})}{k_BT} \right], \quad \text{with} \ T_\infty = T \cdot \frac{\kappa_0}{\kappa_0 + \frac{1}{2} d_0}.
\]

Then, the thermal pressure, \( p(\vec{r}) = n(\vec{r}) k_B T(\vec{r}) \), becomes

\[
p(\vec{r}) = p_\infty \cdot \left[ 1 + \frac{1}{\kappa_0} \frac{\Phi(\vec{r})}{k_BT} \right]^{-\kappa_0^{-1} - \frac{1}{2} d_0}, \quad \text{with} \ p_\infty = n_\infty k_B T_\infty.
\]

The polytropic behavior is defined by the power-law relationship between thermal observables:

\[
p(\vec{r}) \propto n(\vec{r})^{\gamma} \Leftrightarrow n(\vec{r}) \propto T(\vec{r})^{\gamma - 1},
\]

where the exponent \( \gamma \) denotes the polytropic index (e.g., [24-26,123,125-128]):

\[
\gamma = 1 + \frac{1}{\nu} \Leftrightarrow \nu = 1 / (\gamma - 1).
\]

(Note that both the exponents \( \gamma \) and \( \nu \) have been called as the polytropic index.)

In the classical case of Maxwell-Boltzmann distributions, the integration of the exponential of Hamiltonian over the velocity space gives the exponential distribution of the potential energy,

\[
f(\vec{r}; \kappa_0 \to \infty, T) = \int_{-\infty}^{\infty} f(\vec{r}, \vec{u}; \kappa_0 \to \infty, T) d\vec{u} \propto e^{-\frac{e_k(\vec{u}) + \Phi(\vec{r})}{k_B T}},
\]

and

\[
\frac{1}{2} d_k k_B T(\vec{r}) = \int_{-\infty}^{\infty} e^{-\frac{e_k(\vec{u}) + \Phi(\vec{r})}{k_BT}} \epsilon_k(\vec{u}) d\vec{u} = \int_{-\infty}^{\infty} e^{-\frac{e_k(\vec{u})}{k_BT}} \epsilon_k(\vec{u}) d\vec{u} = \frac{1}{2} d_k T,
\]

thus, we obtain

\[
n(\vec{r}) = n_\infty \cdot e^{-\frac{\Phi(\vec{r})}{k_BT}}, \quad T(\vec{r}) = T, \quad p(\vec{r}) = n_\infty k_B T \cdot e^{-\frac{\Phi(\vec{r})}{k_BT}}.
\]
Therefore, in the case of Maxwell-Boltzmann distributions, we obtain \( p(\vec{r}) \propto n(\vec{r}) \), hence \( \gamma = 1 \) (or \( \nu \rightarrow \pm \infty \)), corresponding to a single thermodynamic process, the isothermal one.

In the case of kappa distributions, we obtain

\[
p(\vec{r}) \propto n(\vec{r})^{(k_0 + \frac{d}{e} + 1)},
\]

leading to the relationship between kappa and polytropic indices:

\[
\gamma = \frac{k_0 + \frac{1}{2} d}{k_0 + \frac{1}{2} d + 1}
\]

from which we can verify that the classical case of Maxwell-Boltzmann that corresponds to kappa index \( k_0 \rightarrow \infty \), leads to the isothermal case, i.e., \( \gamma = 1 \) or \( \nu = \pm \infty \).

4. Connection between kappa distributions and polytropic behavior

In the previous section, we have seen how the kappa distributions can lead to the polytropic behavior. Here, we will show that the opposite statement is also true. Namely, stationary velocity and/or energy distributions associated with polytropic behavior are always connected with the framework of kappa distributions. This recently shown result [24,123,127] is briefly presented below.

The Navier-Stokes momentum equation in a conservative external field \( F \) is

\[
m \cdot n \left[ \frac{\partial \vec{u}}{\partial \vec{r}} + (\vec{u} \cdot \nabla) \vec{u} \right] = -\nabla p + n \cdot \vec{F} + \nabla \cdot \nabla \Phi .
\]

Furthermore, considering: (i) the viscosity tensor \( \mathbf{R} \) can be neglected; (ii) the velocity vector field is independent of the position vector, i.e., the convective acceleration term vanishes, \( (\vec{u} \cdot \nabla) \vec{u} = 0 \); and (iii) the velocity field is stationary, \( \frac{\partial \vec{u}}{\partial t} = 0 \), then, the Navier-Stokes momentum equation above is reduced to the Euler’s equation:

\[
\nabla p(\vec{r}) = n(\vec{r}) \cdot \vec{F} = -n(\vec{r}) \cdot \nabla \Phi(\vec{r}) .
\]

It is trivial to show that both Maxwell-Boltzmann and kappa distributions are consistent with the polytropic relationship and the Euler’s equation. Then, we may ask: \textit{What is the most general distribution function of particle velocities/energies, consistent with an inviscid and polytropic particle flow under a conservative field?}

In order to answer this question, we use the polytropic relationship, \( n \propto T^\nu \), and express the density, temperature, and pressure, as arbitrary functions \( f, f^\nu, \) and \( f^{\nu+1} \), respectively, of the ratio \( \Phi(\vec{r})/(k_u T) \):

\[
n(\vec{r}) = n_\infty \cdot f \left[ \frac{\Phi(\vec{r})}{k_u T} \right]^\nu , \quad T(\vec{r}) = T_\infty \cdot f \left[ \frac{\Phi(\vec{r})}{k_u T} \right] , \quad p(\vec{r}) = p_\infty \cdot f ^{\nu+1} \left[ \frac{\Phi(\vec{r})}{k_u T} \right] ^{\nu+1} ,
\]

and setting that \( f(0)=1 \). Then, we substitute \( n(\vec{r}) \) and \( p(\vec{r}) \) in Euler’s equation, Eq.(28), and we find:

\[
\left[ 1 + (\nu + 1) \frac{T_\infty}{T} \cdot f ^{\nu} \left[ \frac{\Phi(\vec{r})}{k_u T} \right] \right] \cdot n(\vec{r}) \cdot \nabla \Phi(\vec{r}) = 0 ,
\]

leading to

\[
f ^{\nu} \left[ \frac{\Phi(\vec{r})}{k_u T} \right] = -\frac{1}{(\nu + 1) \frac{T}{T_\infty}} = \text{constant} \equiv 1/C ,
\]

and given that \( f(0)=1 \), Eq.(31) is solved to

\[
f \left[ \frac{\Phi(\vec{r})}{k_u T} \right] = 1 + \frac{1}{-(\nu + 1) \frac{T}{T_\infty}} \left[ \frac{\Phi(\vec{r})}{k_u T} \right] , \quad \text{or}
\]

\[
f ^{\nu} \left[ \frac{\Phi(\vec{r})}{k_u T} \right] = 1 + \frac{1}{-(\nu + 1) \frac{T}{T_\infty}} \left[ \frac{\Phi(\vec{r})}{k_u T} \right] ,
\]
\[
\Phi(\vec{r}) = \frac{k_B T}{\rho} \left( 1 + C \frac{\Phi(\vec{r})}{k_B T} \right), \quad \text{with} \quad C = -(v + 1) \frac{T}{T_c}.
\]

Hence, we find:
\[
n(\vec{r}) = n_c \left[ 1 + C \frac{\Phi(\vec{r})}{k_B T} \right]^v, \quad T(\vec{r}) = T_c \left[ 1 + C \frac{\Phi(\vec{r})}{k_B T} \right], \quad p(\vec{r}) = p_c \left[ 1 + C \frac{\Phi(\vec{r})}{k_B T} \right]^{v+1}.
\]

Now let’s find the constant \( C \). Averaging the temperature profile over the whole positional space is the global temperature, \( \left\langle T(\vec{r}) \right\rangle = T \), and using the definition of the potential degrees of freedom, \( \frac{1}{2} d_{\Phi} = \left( \Phi(\vec{r}) \right)_c / (k_B T) \), we find:
\[
C = -(v + 1) \frac{T}{T_c} = -(v + 1 + \frac{1}{2} d_{\Phi}).
\]

Finally, we observe that the constant \( C \) coincides with the kappa index \( \kappa_0 \) and the formulation of the thermal observables with that of kappa distributions (Figure 2). (For more details, see: [24].)

### Figure 2.
Equivalence between the thermal observables derived from (left) polytropic behavior and (right) kappa distributions. (For more details, see: [24].)

#### 5. Application: The adiabatic process

As it was shown in Section 5, the classical Maxwell-Boltzmann statistical framework does not describe the adiabatic or any other polytropic thermodynamic process besides the isothermal one. On the other hand, the adiabatic process is consistent with the framework of kappa distributions specifically for the kappa index given by:
\[
\kappa_0 = -\frac{1}{2} d_{K} - \frac{1}{2} d_{\Phi} - 1 \iff \text{adiabatic process}.
\]

This relationship can be fulfilled in the cases of (i) positive potential energies and negative kappa indices (e.g., see: negative kappa distributions in [11]), and (ii) negative potential energies (e.g., \( \Phi(\vec{r}) = \pm \frac{k}{b} |\vec{r}|^b \) with negative exponent \(-b<0\) and either positive or negative kappa indices. In Table, we present several examples of power-law potential energies and the corresponding kappa indices that may lead to the adiabatic process.
6. Conclusions
Classical collisional particle systems residing in thermal equilibrium have their particle velocity/energy distribution function stabilized into a Maxwell-Boltzmann distribution. On the contrary, space and astrophysical plasmas are exotic collisionless particle systems residing in stationary states characterized by the so-called kappa distribution function. Kappa distributions have become increasingly widespread across the physics of space plasma processes, describing particles in the heliosphere, from the solar wind and planetary magnetospheres to the heliosheath and beyond, the interstellar and intergalactic plasmas. A breakthrough in the field came with the connection of kappa distributions with statistical mechanics and thermodynamics: kappa distributions (i) maximize the entropy of nonextensive statistical mechanics under the constraints of canonical ensemble, (ii) characterize particle systems exchanging heat with each other eventually stabilized at thermal equilibrium, and (iii) constitute the unique description of particle energies consistent with polytropic behavior; while the classical Maxwell-Boltzmann statistical framework cannot describe the adiabatic or any other polytropic thermodynamic process besides the isothermal, the kappa distributions can describe the adiabatic and any other thermodynamic process for specific kappa indices.

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Acknowledgments
The work was supported in part by the project NNX17AB74G of NASA’s HGI Program.