A displacement-based analytical stress solution for various adhesive joint configurations

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The present work provides a simple and precise thermo-mechanical model for the prediction of stress distributions in adhesive joints. A displacement-based derivation is applied, using a general sandwich-type model for the overlap region. For the adhesive layer a linear displacement approach is proposed, which captures the variation of the shear stresses in the through-thickness direction. The model covers various joint configurations with linear-elastic material behavior under different load cases.

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1 Introduction

Bonded joints gain increasing significance with regard to lightweight constructions. Especially in the case of multi-material structures or fiber-reinforced composites, adhesive joints can be employed very advantageously. There are different approaches on how to predict adhesive stress distributions adequately, which enable a deep insight into the mechanical behavior and design of adhesive joints. Early analytical approaches were delivered by Volkersen [1], followed by Goland and Reissner [2], modeling the adherends as Euler-Bernoulli beams and the adhesive as a continuum of smeared linear-elastic springs, also referred to as a "weak-interface". However, these models were restricted to simple load cases of single lap joints (SLJ). Later studies by Bigwood and Crocombe [3] extended the approach by introducing a general sandwich-type model, considering the overlap region only and thus enabling the evaluation of varying joint configurations. Ojalvo and Eidinoff [4] were the first to investigate the impact of the adhesive’s thickness on the stress results by proposing a linear displacement field for the adhesive.

2 Governing equations of the general sandwich-type model

In Fig. 1 the underlying general sandwich-type model is presented. In order to derive the stress solutions in the adhesive layer, we first formulate equilibrium equations at an infinitesimal area segment of the sandwich-type structure with length \( dx \). A free body cut along the \( x \)-axis through the adhesive layer in Fig. 2 yields three equilibrium conditions

\[
N^{(i)}(z) \pm \tau^{(a)}_{zz} = 0, \quad V^{(i)}(z) \pm \sigma^{(a)}_{zz} = 0, \quad M^{(i)}(z) - V^{(i)} + \frac{h_i + t}{2} \tau^{(a)}_{zz} = 0, \quad i = 1, 2, \tag{1}
\]

for the upper \((i=1)\) and three for the lower \((i=2)\) part respectively, delivering six relations between the adhesive’s stresses \( \tau^{(a)}_{zz} \) and \( \sigma^{(a)}_{zz} \) and the adherends’ section forces and moments \( N^{(i)}, V^{(i)} \) and \( M^{(i)} \). A plane strain behavior is assumed for the entire model, neglecting the extensions in \( y \)-direction in the adherends and adhesive. The adherends are considered as beams with the displacements

\[
u^{(i)}(x, z) = u^{(i)}_0 + z \psi^{(i)}(x), \quad w^{(i)}(x, z) = w^{(i)}_0,
\]

where first order shear deformation theory is applied. The values \( u^{(i)}_0 \) and \( w^{(i)}_0 \) denote the midplane displacements of the beams and \( \psi^{(i)} \) their rotation about the \( y \)-axis. Inserting the displacement fields into a linear elastic plane strain material law yields

\[
\begin{pmatrix} N^{(i)} \\ M^{(i)} \end{pmatrix} = \begin{pmatrix} A^{(i)}_{11} & B^{(i)}_{11} \\ B^{(i)}_{11} & D^{(i)}_{11} \end{pmatrix} \begin{pmatrix} u^{(i)}_0 \\ \psi^{(i)}_0 \end{pmatrix} - \begin{pmatrix} N^{T(i)} \\ M^{T(i)} \end{pmatrix}, \quad V^{(i)} = k A^{(i)}_{55} \left( u^{(i)}_0 + \psi^{(i)} \right). \tag{3}
\]

Thereby thermal expansion of the adherends is considered by \( N^{T(i)} \) and \( M^{T(i)} \), which denote thermal section forces and moments. In the adhesive layer an in thickness direction linearly varying displacement approach is applied, which was first introduced by Ojalvo and Eidinoff. Following the assumption of a simplified continuum, the adhesive displacement field is expressed through the displacements of the adherends evaluated at their interface to the adhesive layer \( \hat{u}^{(i)} \) and \( \hat{w}^{(i)} \):

\[
u^{(a)}(x, z) = \hat{u}^{(1)}(1) + \frac{z}{t} \left( \hat{w}^{(2)}(2) - \hat{w}^{(1)}(1) \right), \quad w^{(a)}(x, z) = \frac{\hat{w}^{(1)}(1) + \hat{w}^{(2)}(2)}{2} + \frac{z}{t} \left( \hat{w}^{(2)}(2) - \hat{w}^{(1)}(1) \right). \tag{4}
\]

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We derive the constitutive equations of the adhesive layer from Hooke’s law under the assumption of plane strain behavior and with neglect of the adhesive normal stress $\sigma_{zz}^{(a)}$ as:

$$
\tau_{xz}^{(a)} = G_{xz}^{(a)} \varepsilon_{xz}^{(a)}, \quad \sigma_{zz}^{(a)} = E_{zz}^{(a)} \varepsilon_{zz}^{(a)}, \quad \text{with} \quad E_{zz}^{(a)} = \frac{E^{(a)}}{1 - \nu^{(a)^2}}.
$$

By inserting the above-mentioned kinematics and constitutive relations (2)-(5) into the equilibrium conditions (1), a displacement-based system of ordinary differential equations with constant coefficients is obtained. For simplified numerical evaluation this second-order system of six differential equations can be transformed into a first-order system of twelve differential equations

$$
\Psi' = A\Psi, \quad \text{with} \quad \Psi = \begin{pmatrix}
    u_0^{(1)}, u_0^{(2)}, w_0^{(1)}, w_0^{(2)}, u_0'^{(1)}, u_0'^{(2)}, w_0'^{(1)}, w_0'^{(2)}, \psi_1^{(1)}, \psi_1^{(2)}, \psi_2^{(1)}, \psi_2^{(2)}
\end{pmatrix}^T,
$$

where the vector $\Psi$ contains the structure’s unknown displacement variables and their derivatives. Solving the system requires an eigenvalue analysis with eigenvectors of higher order. The unknown constants are determined from underlying boundary conditions, resulting from the external section forces and moments at the left and right ends of the upper and lower adherend, depending on the specific load situation.

3 Results and discussion

The present model has been applied on a single-lap joint and a reinforcing patch under different types of loading. The distributions of adhesive shear and peel stresses occurring in the midplane of the overlap region are presented along the overlap length. The obtained results were verified by finite element analyses (FEA). In Fig. 3 mechanical tension is applied on a single lap joint with a steel-epoxy-steel layup. The stress predictions show symmetric distributions with stress peaks at the edges of the overlap where the external forces are applied. This behavior is confirmed in very good agreement by FEA. In Fig. 4 the stress results of a patch deforming due to a change in temperature of -100K are presented. Since the adhesive layer is relatively soft, the stresses arise from the different thermal expansion coefficients $\alpha_T$ of the aluminum and the steel adherend, which therefore deform to a different extent. This imbalance of deformation increases further towards the edge of the overlap region leading to local stress peaks. The FEA shows similar findings with minor deviations near the overlap edges.

4 Conclusion

The present model allows for a simple and efficient prediction of stresses in adhesive joints for various joint configurations. The stresses obtained from mechanical and thermal loads are captured well. A linear displacement approach allows for decent results by taking into account the often neglected thickness of the adhesive layer.

Acknowledgements Open access funding enabled and organized by Projekt DEAL.
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