A model independent analysis of $B \rightarrow X_s \ell^+ \ell^-$ decays in supersymmetry

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Recently the semileptonic decays $B \rightarrow X_s e^+ e^-$, $B \rightarrow X_s \mu^+ \mu^-$ in generic supersymmetric extensions of the Standard Model have been studied in ref. [1]. In this talk I review the main points of this analysis. SUSY effects are parameterized using the mass insertion approximation formalism and differences with MSSM results are pointed out. Constraints on SUSY contributions coming from other processes (e.g. $b \rightarrow s \gamma$) are taken into account. Chargino and gluino contributions to photon and $Z$-mediated decays are computed and non-perturbative corrections are considered. We find that the integrated branching ratios and the asymmetries can be strongly modified. Moreover, the behavior of the differential Forward-Backward asymmetry remarkably changes with respect to the Standard Model expectation.

1. Introduction

One of the features of a general low energy supersymmetric (SUSY) extension of the Standard Model (SM) is the presence of a huge number of new parameters. FCNC and CP violating phenomena constrain strongly a big part of the new parameter space. However there is still room for significant departures from the SM expectations in this interesting class of physical processes.

Recently we have investigated the relevance of new physics effects in the semileptonic inclusive decay $B \rightarrow X_s \ell^+ \ell^-$. This decay is quite suppressed in the Standard Model; however, new $B$-factories should reach the precision requested by the SM prediction and an estimate of all possible new contributions to this process is compelling.

Semileptonic charmless $B$ decays have been deeply studied. The dominant perturbative SM contribution has been evaluated in ref. [3] and later two loop QCD corrections have been provided [4]. The contribution due to $c \bar{c}$ resonances to these results are included in the papers listed in ref. [3]. Long distance corrections can have a different origin according to the value of the dilepton invariant mass one considers. $O(1/m_b^2)$ corrections have been first calculated in ref. [5] and recently corrected in ref. [6]. Near the peaks, non-perturbative contributions generated by $c \bar{c}$ resonances by means of resonance-exchange models have been provided in ref. [7]. Far from the resonance region, instead, ref. [11] (see also ref. [12]) estimate $c \bar{c}$ long-distance effects using a heavy quark expansion in inverse powers of the charm-quark mass ($O(1/m_c^2)$ corrections).

An analysis of the SUSY contributions has been presented in refs. [13,14] where the authors estimate the contribution of the Minimal Supersymmetric Standard Model (MSSM). They consider first a universal soft supersymmetry breaking sector at the Grand Unification scale (Constrained MSSM) and then partly relax this universality condition. In the latter case they find that there can be a substantial difference between the SM and the SUSY results in the Branching Ratios and in the forward–backward asymmetries. One of the reasons of this enhancement is that the Wilson coefficient $C_7(M_W)$ can change sign with respect to the SM in some region of the parameter space while respecting constraints coming from $b \rightarrow s \gamma$. The recent measurements of $b \rightarrow s \gamma$ have narrowed the window of the possible values of $C_7(M_W)$ and in particular a sign change of this coefficient is no more allowed in the Constrained MSSM framework. Hence, on one hand it is worthwhile considering $B \rightarrow X_s \ell^+ \ell^-$ in a more general SUSY framework then just the Constrained MSSM, and, on the other hand, the
The above mentioned new results prompt us to a reconsideration of the process. In reference \[18\] the possibility of new-physics effects coming from gluino-mediated FCNC is studied.

We consider all possible contributions to charmless semileptonic $B$ decays coming from chargino-quark-squark and gluino-quark-squark interactions and we analyze both $Z$-boson and photon mediated decays. Contributions coming from penguin and box diagrams are taken into account; moreover, corrections to the MIA results due to a light quark are considered. A direct comparison between the SUSY and the SM contributions to the Wilson coefficients is performed. Once the constraints on mass insertions are established, we find that in generic SUSY models there is still enough room in order to see large deviations from the SM expectations for branching ratios and asymmetries. For our final computation of physical observables we consider NLO order QCD evolution of the coefficients and non-perturbative corrections ($O(1/m_t^2)$, $O(1/m_b^2)$,...), each in its proper range of the dilepton invariant mass.

Because of the presence of so many unknown parameters (in particular in the scalar mass matrices) which enter in a quite complicated way in the determination of the mass eigenstates and of the various mixing matrices it is very useful to adopt the so-called “Mass Insertion Approximation” (MIA) \[17\]. In this framework one chooses a basis for fermion and sfermion states in which all the couplings of these particles to neutral gauginos are flavor diagonal. Flavor changes in the squark sector are provided by the non-diagonality of the sfermion propagators. The pattern of flavor change is then given by the ratios

$$ (\delta^{ij}_{AB}) = \frac{(m_{ij}^f)^2}{M_{ij}^2}, \quad (1) $$

where $(m_{ij}^f)^2$ are the off-diagonal elements of the flavor mass squared matrix that mixes flavor $i, j$ for both left- and right-handed scalars ($A,B =$Left, Right) and $M_{ij}$ is the average squark mass (see e.g. \[13\]). The sfermion propagators are expanded in terms of the $\delta$s and the contribution of the first two terms of this expansion are considered. We show that the graphs with a double MI can be safely neglected in this process. The genuine SUSY contributions to the Wilson coefficients will be simply proportional to the various $\delta$s and a keen analysis of the different Feynman diagrams involved will allow us to isolate the few insertions really relevant for a given process. In this way we see that only a small number of the new parameters is involved and a general SUSY analysis is made possible. The hypothesis regarding the smallness of the $\delta$s and so the reliability of the approximation can then be checked a posteriori.

Many of these $\delta$s are strongly constrained by FCNC effects \[13\] or by vacuum stability arguments \[21\]. Nevertheless it may happen that such limits are not strong enough to prevent large contributions to some rare processes.

2. Operator basis and general framework

The effective Hamiltonian for the decay $B \rightarrow X_s \ell^+ \ell^-$ in general low-energy SUSY models is the same in the SM \[16\] and in the MSSM \[13\], it is known at next-to-leading order and we refer to the cited articles for its expression. We find that SUSY can also modify (with respect to the SM) the matching coefficients of the operators

$$ Q_7 = \frac{e}{8\pi^2} m_b s_R \sigma^{\mu\nu} b_L F_{\mu\nu}, $$

$$ Q_8 = (s_R \gamma_\mu b_R) \gamma_\nu l, $$

$$ Q_{10} = (s_R \gamma_\mu b_R) \gamma_\nu \gamma_5 l. \quad (2) $$

However we have checked that the contribution of these operators is negligible and so they are not considered in the final discussion of physical quantities. SUSY contributions to other operators are of higher perturbative order and can be neglected.

The observables we have in mind are the differential branching ratio and the forward-backward asymmetry,

$$ R(s) \equiv \frac{d \Gamma(B \rightarrow X_s \ell^+ \ell^-)/ds}{\Gamma(B \rightarrow X_s e^\nu)} \quad (3) $$

$$ A_{FB}(s) \equiv \frac{\Gamma(B \rightarrow X_s \ell^+ \ell^-) - \Gamma(B \rightarrow X_s \ell^- \ell^+)}{\Gamma(B \rightarrow X_s \ell^+ \ell^-) + \Gamma(B \rightarrow X_s \ell^- \ell^+)} $$
\[ \int_{-1}^{1} d\cos\theta \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d\cos\theta \, ds} \frac{\text{Sgn}(\cos\theta)}{s} \]
\[ \int_{-1}^{1} d\cos\theta \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d\cos\theta \, ds} \]

where \(s = (p_{l^+} + p_{l^-})^2/m_b^2\), \(\theta\) is the angle between the positively charged lepton and the B flight direction in the rest frame of the dilepton system.

It is worth underlying that integrating the differential asymmetry given in eq. (4) we do not obtain the global Foward–Backward asymmetry which is by definition:

\[ \frac{N(\ell^+_R) - N(\ell^+_L)}{N(\ell^+_R) + N(\ell^+_L)} \equiv \]
\[ \int_{-1}^{1} d\cos\theta \int ds \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d\cos\theta \, ds} \frac{\text{Sgn}(\cos\theta)}{s} \]
\[ \int_{-1}^{1} d\cos\theta \int ds \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d\cos\theta \, ds} \]

where \(\ell^+_R\) and \(\ell^+_L\) stand respectively for leptons scattered in the forward and backward direction. To this extent it is useful to introduce the following quantity

\[ A_{FB}(s) \equiv \]
\[ \int_{-1}^{1} d\cos\theta \int ds \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d\cos\theta \, ds} \frac{\text{Sgn}(\cos\theta)}{s} \]
\[ \int_{-1}^{1} d\cos\theta \int ds \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d\cos\theta \, ds} \]

whose integrated value is given by eq. (6).

Eqns. (4) and (5) have been corrected in order to include several non-perturbative effects. We refer to [1] and to references therein for all the definitions concerning this issue.

3. Light \(t_R\) effects

In the Mass Insertion Approximation framework we assume that all the diagonal entries of the scalar mass matrices are degenerate and that the off diagonal ones are sufficiently small. In this context we expect all the squark masses to lie in a small region around an average mass which we have chosen not smaller than 250 GeV. Actually there is the possibility for the \(t_R\) to be much lighter; in fact the lower bound on its mass is about 70 GeV. For this reason it is natural to wonder how good is the MIA when a \(\bar{t}_R\) explicitly runs in a loop.

The diagrams, among those we have computed, interested in this effect are the chargino penguins and box with the \((\delta_{23})_{LR}\) insertion. To compute the light–\(t_R\) contribution we adopt the approach presented in ref. [27]. There the authors consider an expansion valid for unequal diagonal entries which gives exactly the MIA in the limit of complete degeneration.

4. Constraints on mass insertions

In order to establish how large the SUSY contribution to \(B \rightarrow X_s l^+ l^-\) can be, one can compare, coefficient per coefficient, the MI results with the SM ones taking into account possible constraints on the \(\delta s\) coming from other processes, in particular from \(b \rightarrow s\gamma\). A discussion about this issue can be found in ref. [1]. The most relevant \(\delta s\) interested in the determination of the Wilson coefficients \(C_7, C_9\) and \(C_{10}\) are \((\delta_{23})_{LL}, (\delta_{23})_{LR}, (\delta_{23})_{RL}, (\delta_{23})_{LL}\) and \((\delta_{23})_{LR}\).

5. Results

While the gluino sector of the theory is essentially determined by the knowledge of the gluino mass (i.e. \(M_{gl}\)), the chargino one needs two more parameters (i.e. \(M_2, \mu\) and \(\tan\beta\)). Moreover it is a general feature of the models we are studying the decoupling of the SUSY contributions in the limit of high sparticle masses: we expect the biggest SUSY contributions to appear for such masses chosen at the lower bound of the experimentally allowed region. On the other hand this considerations suggest us to constrain the parameters of the chargino sector by the requirement of the lighter eigenstate not to have a mass lower than the experimental bound of about 70 GeV [28]. The remaining two dimensional parameter space has yet no constraint. For these reasons we scan the chargino parameter space by means of scatter plots [1].

Thus, with \(\mu \simeq -160, M_{gl} \simeq M_{sq} \simeq 250\) GeV,
\[ M_B \simeq 50 \text{ GeV}, \tan \beta \simeq 2 \] one gets
\[ C_9^{MI}(M_B) = -1.2(\delta_{23}^u)_{LL} + 0.69(\delta_{23}^u)_{LR} - 0.51(\delta_{23}^u)_{LL} \]
\[ C_{10}^{MI}(M_B) = 1.75(\delta_{23}^u)_{LL} - 8.25(\delta_{23}^u)_{LR} \].

In order to numerically compare these values with the respective SM results we note that the minimum value of \((C_9^{SM}(s))_{SM}(M_B)\) is about 4 while \(C_{10}^{SM} = -4.6\). Thus one deduces that SM expectations for the observables are enhanced by the contributions neglecting the resonances.

Looking table 1 we see that the differences between the CMSSM and more general SUSY models are large. As a consequence of this, the sign of asymmetries can be the opposite respect to the SM estimate. As a consequence of the method, the sign and the value of the coefficient \(C_7\) has a great importance. In fact the integral of the BR (see eq. (3)) is dominated by the \(|C_7|^2/s\) and \(C_7C_9\) term for low values of \(s\). In the SM the interference between \(O_7\) and \(O_9\) is destructive and this behavior can be easily modified in the general class of models we are dealing with. It is worthwhile to note that with the Constrained MSSM cannot drive a change in the the sign of \(C_7\) while this changes the coefficient \(C_7\).

The integrated BRs and asymmetries for the decays \(B \to X_s e^+ e^-\) and \(B \to X_s \mu^+ \mu^-\) in the SM case and in the SUSY one (with the above choices of the parameters) are summarized in tab.1. There we computed the total perturbative contributions neglecting the resonances.

The results of tab.1 must be compared with the experimental best limit which reads [1]

\[ BR_{exp} < 5.8 \times 10^{-5}. \] (8)

Looking table 1 we see that the differences between SM and SUSY predictions can be remarkable. Moreover a sufficiently precise measure of the \(A_{FB}B\) and \(A_{FB}s\) can either discriminate between the CMSSM and more general SUSY models or give new constraints on mass insertions. Both these kind of informations can be very useful for model building.

6. Conclusions

In this paper a discussion about SUSY contributions to semileptonic decays \(B \to X_s e^+ e^-\), \(B \to X_s \mu^+ \mu^-\) is provided.

Given the constraints coming from the recent measure of \(b \to s \gamma\) and estimating all possible SUSY effects in the MIA framework we see that SUSY has a chance to strongly enhance or depress semileptonic charmless B-decays. The expected direct measure will give very interesting informations about the SM and its possible extensions.

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Table 1
Integrated BR, $A_{FB}$ and $\overline{A}_{FB}$ in the SM and in a general SUSY extension of the SM for the decays $B \rightarrow X, e^+ e^-$ and $B \rightarrow X, \mu^+ \mu^-$. The second and third columns are the extremal values we obtain with a positive $C_7^{eff}$ while the fourth one is the $C_7^{eff} < 0$ case. The actual numerical inputs for the various coefficients can be found in the text. The BR is just the integral of $R(s)$ multiplied by the BR of the semileptonic dominant $B$ decay ($BR(B \rightarrow X, e\nu) = 0.105$).

| Observable | SM | SUSY maximal | SUSY minimal | SUSY $(C_7 < 0)$ |
|------------|----|--------------|--------------|-----------------|
| $BR(e)$    | 9.6 $10^{-6}$ | 4.3 $10^{-5}$ | 3.9 $10^{-6}$ | 3.9 $10^{-5}$   |
| $A_{FB}(e)$ | 0.23 | 0.33 | -0.18 | 0.31 |
| $\overline{A}_{FB}(e)$ | 0.071 | 0.24 | -0.19 | 0.11 |
| $BR(\mu)$  | 6.3 $10^{-6}$ | 4.0 $10^{-5}$ | 1.6 $10^{-6}$ | 3.4 $10^{-5}$   |
| $A_{FB}(\mu)$ | 0.23 | 0.33 | -0.18 | 0.31 |
| $\overline{A}_{FB}(\mu)$ | 0.11 | 0.27 | -0.27 | 0.15 |

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