Students’ common difficulties and approaches while solving conceptual problems with non-identical light bulbs in series and parallel

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Abstract
We discuss a study of student reasoning difficulties and approaches while solving problems about the brightness of non-identical light bulbs connected in series and parallel. The questions about the light bulbs can be solved quantitatively even though they were posed as conceptual problems. We compare the performance of introductory physics students with that of a set of physics PhD students and find that these problems related to non-identical light bulbs are difficult even for PhD students. We also conducted individual interviews with six introductory students to obtain an in-depth understanding of their approaches and rationale for solving the problems in a particular way. We discuss the conceptual difficulties displayed in the interviews and in the written responses in which introductory physics students were asked to explain their reasoning. In addition to confirming some misconceptions which have previously been observed in the context of equal wattage light bulbs, use of unequal wattage light bulbs in this research reveals misconceptions not documented previously.

Keywords: conceptual difficulties, expertise, problem solving, misconceptions

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Introduction

Several prior studies have focused on the difficulties of introductory physics students with electrostatics and electrical circuits and strategies that may help students learn these topics better [1–15]. For example, the investigation by Eylon and Daniel suggests that macro–micro relationships may be the missing link between electrostatics and electrodynamics in students’ reasoning [1]. McDermott and Shaffer investigated introductory physics students’ reasoning difficulties with concepts in electricity such as current, voltage and resistance and used research as a guide to develop a tutorial to help introductory physics students learn the concepts [2, 3]. Thacker et al investigated student understanding of transients in direct current electric circuits [4] and Itza-Ortiz et al studied students’ models of Newton’s second law in mechanics and electromagnetism and found that three different models were prevalent in student reasoning [5]. Singh and collaborators have investigated student reasoning difficulties with Gauss’s law, electrical circuit elements and with conductors and insulators and developed tutorials on Gauss’s law to help students [6–8]. Guruswamy et al studied student understanding of transfer of charge between conductors [9] and Guisasola et al have carried out investigations related to student understanding of capacitors [10, 11]. Several research-based assessment tools that focus on topics in current electricity or electrostatics have been developed [12–14].

Here we discuss an investigation that focused on introductory physics students’ common difficulties and approaches while solving conceptual problems involving the brightness of two non-identical light bulbs connected in series or parallel to each other and to a battery with no internal resistance in a circuit. Students were told that they can assume that the light bulbs were Ohmic and that the brightness of the light bulbs is proportional to the power dissipated. Students were either given the wattage (for the standard power supply of 120 V in the US) or the resistance of each non-identical light bulb connected in series or parallel in the circuit and asked which bulb will be brighter and why. We also conducted individual interviews with six introductory physics students to get a better understanding of their reasoning and origins of their conceptual difficulties and rationale for solving the conceptually posed problems using a particular approach. We discuss the conceptual difficulties and also compare their performance with those of the physics PhD students. In addition to confirming some misconceptions which have previously been observed in the context of equal wattage light bulbs, use of unequal wattage light bulbs reveals misconceptions not documented previously.

Goals and motivation

Our goals and motivation for conducting this study were as follows.

- How well do introductory physics students perform on conceptual circuit problems involving non-identical light bulbs in series and parallel and how different is their average performance in the wattage version of the problem compared to the resistance version of the problem?
- How does the performance of introductory physics students compare with that of the physics PhD students?
- What kinds of common reasoning difficulties do students have with non-identical light bulbs in series and parallel?
• Does use of unequal wattage light bulbs in this research reveal misconceptions not documented previously in addition to confirming some misconceptions which have previously been observed in the context of equal wattage light bulbs?
• Do introductory physics students use any quantitative reasoning to answer these questions which are posed as conceptual questions (not involving a number for the answer)?

Method of investigation

All introductory physics classes involved in this study were traditionally taught classes at a large research university in the US in which the instructor lectured during the regular class hours and the only recitation hour each week was focused on the TA solving homework problems on the board, fielding questions from students and then giving a quiz in the last 20 min of class. The introductory physics students in the calculus-based courses were asked to explain their reasoning after answering the free-response questions. The free-response format was useful for understanding students’ thought processes since they had to clarify their reasoning. We also developed a multiple-choice version of the questions and administered it to physics PhD students in their first semester in a mandatory, semester long, teaching assistant (TA) training course at the same university. Most of these PhD students came with an undergraduate degree and had not taken a course beyond undergraduate level in electricity and magnetism. These types of questions related to introductory physics course content were often used in the TA training course to investigate whether the PhD students knew the content they were teaching (multiple-choice format was used in this TA training course so that the analysis of data was less time consuming and common difficulties of TAs related to introductory course material could be promptly discussed with them in the following class). As can be seen from the free-response and multiple-choice versions below, they are very similar and the multiple-choice version does not include a reasoning for the choices that a student selects (e.g., it does not say that 100 W bulb will be brighter for a certain reason). In other words, questions in the multiple-choice format posed to the PhD students in this study do not provide hints for the correct answers that may lead to improved student performance compared to the corresponding free-response questions nor do they present distractor choices with common reasoning difficulties that can worsen student performance.

There were two versions of both the free-response and multiple-choice questions. In one version of the questions (which we call the wattage version), students were given the wattage of the light bulbs which was for the case when the light bulbs were connected to a standard power supply. In the other version of the questions (which we call the resistance version), they were given the resistance of each light bulb. Before attempting to answer the questions in both the free-response and multiple-choice formats, students were told to assume that the brightness of the light bulbs is proportional to the power dissipated. Here are the multiple-choice questions in the wattage version (correct answers are in italic).

1. Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in parallel to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be Ohmic?
   (a) 100 W light bulb will be brighter.
   (b) 25 W light bulb will be brighter.
   (c) The light bulbs will be equally bright.
(d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
(e) None of the above

(2) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in series to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be Ohmic?
(a) 100 W light bulb will be brighter.
(b) 25 W light bulb will be brighter.
(c) The light bulbs will be equally bright.
(d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
(e) None of the above

The following are the multiple-choice questions in the resistance version.

(1) Two light bulbs have 200 Ω and 500 Ω resistances. They are connected in parallel to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be Ohmic?
(a) 200 Ω light bulb will be brighter.
(b) 500 Ω light bulb will be brighter.
(c) The light bulbs will be equally bright.
(d) Initially, they will be equally bright but after a few minutes the 200 Ω bulb will be brighter.
(e) None of the above

(2) Two light bulbs have 200 Ω and 500 Ω resistances. They are connected in series to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be Ohmic?
(a) 200 Ω light bulb will be brighter.
(b) 500 Ω light bulb will be brighter.
(c) The light bulbs will be equally bright.
(d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
(e) None of the above

The following are the free-response questions in the wattage version.

(1) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in parallel to each other and to a 20 V ideal power supply. Which light bulb is brighter assuming their resistances to be Ohmic? You must explain your reasoning.

(2) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in series to each other and to a 20 V ideal power supply. Which light bulb is brighter assuming their resistances to be Ohmic? You must explain your reasoning.

Similar free response questions were also administered to some introductory physics students in the resistance version. All students involved in this study received traditional lecture-based instruction on circuits in their relevant courses before they answered the questions. The questions were administered either in the wattage or resistance version in pairs on written quizzes in the recitation classes after relevant instruction in traditional lecture format.
The resistance version of the questions was administered to 241 students from two calculus-based introductory physics classes. The results for these two classes are very similar (differences are not statistically significant). The wattage version of the questions was administered to 103 students from a different calculus-based introductory physics class. One class which was given the resistance version had the same instructor as the class in which the students were given the wattage version.

We note that the resistance version of the questions posed is easier than the wattage version because students have an additional step to work out in the wattage version, i.e., they have to figure out the resistance of the light bulbs from the wattage provided for the standard power supply. We hypothesised that even the students who can answer the resistance version correctly may not be able to answer the wattage version correctly due to the difficulty in the initial conversion from the wattage to the resistance of each non-identical light bulb. By comparing the student performance on these two versions of the questions, we expected to obtain a measure of the percentage of students who had difficulty converting from the wattage of a bulb to the resistance in the wattage version. The free-response questions were graded using rubrics which were developed by the two investigators together. A subset of the free-response questions was graded and categorised separately by the investigators. After comparing the grading and categorisation, the investigators discussed any disagreements and resolved them with a final agreement of better than 95%.

We also conducted in-depth individual interviews using a think aloud protocol to get a better insight into student reasoning. These hour long interviews were conducted with six students from a calculus-based course in which the written test was not administered. They were all volunteers whose first midterm exam scores were around the class mean. During the interviews, students were asked to work on both versions of the questions while thinking aloud and also articulate if there is any difference in the difficulty level of the two versions from their perspective. Interviewed students were first asked about the wattage version and then about the resistance version. The analysis of student responses to the interview questions yielded further insight into student difficulties with the brightness of the non-identical light bulbs.

Results

Before discussing the results, we reiterate that all students were specifically given that the brightness of the light bulbs is proportional to the power dissipated and that the bulbs have Ohmic resistances. For the wattage version of the questions, students have to make use of the fact that the standard wattage rating is for light bulbs connected in parallel to the standard power supply (and to the other appliances). They had to make use of the relation between the power dissipated, voltage and resistance to calculate the resistance of each non-identical light bulb. When the bulbs are connected in parallel as in the standard household connection, the higher wattage bulb has a higher power dissipated. This implies that the light bulb with a higher wattage rating has a lower resistance, making use of the fact that the power is potential difference squared over the resistance but the potential difference is fixed in a parallel circuit. Then, to determine which light bulb is brighter when connected in series, students must realise that the light bulbs have the same current through them in series. Therefore, the light bulb with a higher resistance (lower wattage rating for the standard power supply) will have a higher power dissipated in a series circuit.
Table 1 Introductory students’ responses to free-response questions about light bulbs connected in parallel or in series with known resistances. The bold numbers are the percentages of correct responses for the corresponding questions.

| Light bulbs connected in parallel (N = 241) | Light bulbs connected in series (N = 241) |
|-------------------------------------------|----------------------------------------|
| 200 Ω is brighter                         | 71%                                    |
| 500 Ω is brighter                         | 19%                                    |
| Same brightness                           | 18%                                    |
| Whichever bulb is first from battery in   | 17%                                    |
| the direction of current flow             |                                        |
| Other responses                           | 5%                                     |

Table 2. Introductory students’ responses to free-response questions about light bulbs connected in parallel and in series (in that order) with known resistances. The bold number is the percentage of correct responses for both questions.

Parallel, series Problems with known resistances (N = 241)

| 200 Ω, 500 Ω      | 46%          |
|-------------------|--------------|
| 200 Ω, same       | 15%          |
| 500 Ω, 200 Ω      | 10%          |
| 200 Ω, 200 Ω      | 7%           |
| 500 Ω, 500 Ω      | 5%           |
| Other responses   | 16%          |

Light bulbs connected in parallel or in series with resistance provided

Table 1 shows that when the question about the light bulbs connected in parallel was posed as a free response question, 71% of the introductory physics students correctly answered that the 200 Ω light bulb is brighter. About 18% of them thought that the bulb with a higher resistance will be brighter and 5% of them claimed that the two bulbs will have the same brightness. For the case in which the light bulbs are connected in series, 53% of them answered the question correctly and a higher percentage (17%) claimed that the brightness of both bulbs should be the same. Moreover, for the series connection of the bulbs, 5% of students claimed that whichever bulb is first from the battery in the direction of current flow will be brighter.

To better understand the answering patterns for these questions correctly, we analysed the distribution of introductory students’ responses for both (series and parallel) situations together. The results are shown in table 2. Table 2 shows that 46% of the introductory students answered both questions correctly and noted that for light bulbs connected in parallel, the one with lower resistance will have a higher current and thus will be brighter. Moreover, 15% of them answered the question about the light bulbs connected in parallel correctly but claimed that the light bulbs connected in series should have the same brightness. According to the reasoning used by some of these students, only the current through the light bulb determines the brightness. Some of these students correctly noted that for the light bulbs connected in series, the current is the same but they did not take into account the differences in resistance of the light bulbs and concluded that same current implies that the brightness should be the same for the two non-identical light bulbs. Also, 10% of the students claimed that the 500 Ω light bulb will be brighter when connected in parallel and the 200 Ω light bulb will be brighter in series.
The correct answer for these questions requires the ability to determine which light bulb has a larger resistance. Once the students find the resistance, the questions are the same as the ones discussed above in which resistance is provided.

Table 3 suggests that the question about the light bulb in series is much more difficult for introductory students than when they are connected in parallel. Table 3 also shows that 49% of the students answered the question about the light bulbs in parallel correctly but only 8% of them answered the question about the bulbs in series correctly. More than half of the students claimed that the 100 W bulb is brighter when the bulbs are connected in series. Written explanations suggest that many students expected the brightness to be the same as in parallel when the bulbs are connected in series.

Table 4 shows that only 6% of introductory students answered both the series and parallel questions correctly in the wattage version. Table 4 also shows that 24% of them thought that the 100 W light bulb is always brighter no matter how they are connected (parallel or series). Moreover, 22% of the students chose options for both questions that were incorrect. Interviews and written explanations suggest that at least some of these students thought that the 100 W light bulb has a higher resistance than the 25 W bulb. Table 4 also shows that 18% of the students claimed that the 100 W light bulb is brighter when it is connected in parallel and

| Light bulbs connected in parallel (N = 103) | Light bulbs connected in series (N = 103) |
|------------------------------------------|------------------------------------------|
| 100 W is brighter                        | 49%                                      |
| 25 W is brighter                         | 54%                                      |
| Same brightness                          | 29%                                      |
| 0%                                        | 8%                                       |
| Whichever bulb is first from battery in  |                                          |
| the direction of current flow             |                                          |
| 0%                                        | 7%                                       |
| Other responses                           | 10%                                      |
|                                          | 8%                                       |

| Problems with known power (N = 103)       |
|------------------------------------------|
| Parallel, Series                         |
| 100 W, 25 W                              | 6%                                       |
| 100 W, 100 W                             | 24%                                      |
| 25 W, 100 W                              | 22%                                      |
| 100 W, same                              | 18%                                      |
| 100 W/25 W, whichever bulb in series is  |
| first from battery in the direction of   |
| current                                    | 7%                                       |
| Other responses                           | 22%                                      |

**Light bulbs connected in parallel or series with wattage for each bulb provided for the standard power supply**

The correct answer for these questions requires the ability to determine which light bulb has a larger resistance. Once the students find the resistance, the questions are the same as the ones discussed above in which resistance is provided.

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both bulbs have the same brightness when they are connected in series. Interviews and written
explanations suggest that some of these students regarded the current as the only factor that
determines the brightness and argued that when the bulbs are in parallel, the 100 W bulb has
more current flowing through it and when the bulbs are in series, they have the same current
(and therefore they will be equally bright in series).

In summary, the data presented in tables 1–4 together show that students who worked on
the resistance version performed much better than those who worked on the wattage version.
As noted, the wattage version has an extra step because students must first equate power with
the square of the potential difference divided by the resistance to calculate the resistance of
the light bulb and the two versions were given to similar calculus-based introductory physics
classes (with the instructor being the same for two sections of the course in which the wattage
and resistance versions were administered). Thus, introductory students on average performed
worse on the wattage version and found this version more difficult. Introductory student
explanations in the written responses and individual interviews with students suggest that
many students had difficulty in finding the resistance of each light bulb from the wattage
provided.

Comparison of the performance of introductory students with physics PhD students

As noted earlier, we administered both versions of the questions in the multiple-choice format
to physics PhD students in their first semester in a mandatory semester long TA training
course. The performance of the PhD students can be helpful in benchmarking the level of
difficulty of the questions so that the performance of the introductory students can be viewed
in light of the PhD student performance. In the TA training courses, the wattage version of the
questions was administered to 42 PhD students in two consecutive years (26 students in one
class and 16 in another class). The 26 PhD students in one of the classes were also given the
resistance version of the questions on a separate day. As can be seen from table 5, PhD
students’ performance in most situations is better than that of the introductory students.
However, even the PhD students had difficulties in answering some of these questions.
Moreover, if we only consider the performance of the 26 PhD students who answered both
versions, 85% answered the question about light bulbs in parallel and 46% answered the
question about light bulbs in series correctly in the resistance version. But in the wattage
version, 71% of the PhD students answered the question for light bulbs in parallel correctly
and 43% answered the question for light bulbs in series correctly. Overall, the performance of
PhD students suggests that these questions related to the brightness of non-identical light bulb
are quite challenging even for those who have finished the entire undergraduate curriculum in
physics and are in the first semester of their physics PhD programme.

Discussion

Written explanations and individual interviews with introductory students suggest that they
had many common reasoning difficulties with the brightness of two non-identical light bulbs
in series and parallel. Here, we discuss some student difficulties while answering both ver-
sions of the circuit questions. Some of the difficulties discussed below have also been found
in situations in which many identical light bulbs were used in various circuit configurations
[2, 3] but others are revealed due to the fact that in the situation presented in this study, the
light bulbs are non-identical.

Current alone determines the brightness of a light bulb. This is one of the most common
misconceptions. Due to this misconception, 15%–18% of the students answered the question
**Table 5.** PhD students’ responses to multiple-choice questions about light bulbs connected in parallel or in series. The first two columns with numbers are for bulbs with known wattage rating for a traditional power supply and the last two columns with numbers are for bulbs with known resistances. The bold numbers are the percentages of correct responses for the corresponding questions.

| Light bulbs connected in parallel (PhD level N = 42) | Light bulbs connected in series (PhD level N = 42) | Light bulbs connected in parallel (PhD level N = 26) | Light bulbs connected in series (PhD level N = 26) |
|---------------------------------------------------|--------------------------------------------------|---------------------------------------------------|--------------------------------------------------|
| 100 W is brighter                                  | 71%                                              | 200 Ω is brighter                                  | 85%                                              |
| 25 W is brighter                                   | 12%                                              | 500 Ω is brighter                                  | 8%                                               |
| Same brightness                                   | 17%                                              | Same brightness                                   | 8%                                               |
| Other responses                                    | 0%                                               | Other responses                                   | 0%                                               |

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with light bulbs connected in parallel correctly but the one with light bulbs connected in series incorrectly. These students claimed that the light bulb with more current will always be brighter and the light bulbs with the same current through them will be equally bright. In the interviews, even those students who noted that they have to find the power dissipated in each light bulb to compare their brightness, sometimes only focused on current. For example, to answer the question about the two light bulbs connected in series correctly, students must know that the current is the same for both light bulbs and also know the resistance of each bulb (or the voltage across each bulb). Sometimes, the students ignored the information about the resistances of the bulbs (especially in the wattage version in which they had to calculate the resistance) and only focused on the current to determine the brightness. The following statements are from a student who answered both versions of the questions in the interview:

For the wattage version in series, the student ignored the resistance and incorrectly noted that ‘the bulb brightness will be equal, because current is equal in series. With the current equal, the brightness has to be the same.’ For the resistance version, the same student correctly noted that in series ‘the current is the same through both bulbs. Since the current is the same, the voltage drop across the 500 \( \Omega \) resistor will be greater. The 500 \( \Omega \) light bulb is brighter.’

Further probing suggests that this student did not know how to compute the resistance of the light bulbs for the wattage version of the questions so he simply chose to ignore it and assumed that the current alone determines the brightness in the wattage version. This type of inconsistency in responses to different versions of the questions and ignoring certain information that students do not know how to find suggests that these introductory students are still developing expertise [17–21].

Consideration of resistance of the light bulbs alone determines their brightness. The difficulties of the students who incorrectly claimed that the resistance alone determines the brightness of the light bulbs fall into several categories. One group of students claimed that the current is the same through both light bulbs without considering how the bulbs are connected in a circuit. Another group of students claimed that the light bulb with less resistance always has more current flowing through it even if it is connected in series to the other light bulb. Some students were confused about whether they should take into account the equivalent resistance of the entire circuit or consider the resistance of each light bulb separately especially when connected in series, to determine the brightness of each bulb. These students often asserted that when the light bulbs are connected in series, the resistance applies to the entire circuit and not to each bulb separately. Therefore, the brightness of both light bulbs in series should be the same. For example, one student claimed the following: ‘the brightness of the light bulbs is the same. In series, the resistance is evenly distributed.’

The light bulb with a larger wattage rating for a standard power supply will always be brighter. 24% of the students claimed that the 100 W light bulb is always brighter than the 25 W bulb no matter how they are connected to each other (in parallel or in series). In the interviews, some students explicitly mentioned that the 100 W light bulb is always brighter than the 25 W bulb because that is how they are manufactured and that fact cannot be changed by changing the circuit. This misconception has not been documented in previous studies. When interviewed students were asked explicitly about how the bulbs are connected to the standard power supply in our homes and offices, they often had difficulty articulating that they are connected in parallel to the power supply. Moreover, few students who provided written explanation for the free-response wattage questions mentioned that the light bulbs are connected in parallel to the standard power supply as part of their explanations.

Failure to recognise the relation between resistance of a light bulb and power dissipated. As mentioned earlier, the wattage version is much more difficult because students have to first figure out which of the two light bulbs has a higher resistance. During the interviews, students
were encouraged to articulate the connection between the two versions of problems. However, not a single interviewed student noted that there is an extra step involved in going from the wattage version to the resistance version. They could only state the literal difference between the two versions, namely, one provides the wattage of the bulbs for the standard power supply and the other provides the resistances. Even asking students explicitly to relate the wattage of the bulbs to their resistance and thus explore the connection between the two versions rarely helped. For the resistance version with light bulbs in parallel, some students correctly noted that since \( P = IV \) and the voltage \( V \) is the same for both bulbs, the light bulb with a higher wattage rating for the standard power supply will have more current through it. However, they often did not differentiate between bulbs in parallel and series and incorrectly claimed that this fact also applies to the bulbs connected in series.

**Wattage rating of a light bulb is proportional to the resistance.** Some students incorrectly claimed that the bulb with a higher wattage for a standard power supply will have a higher resistance. The reasons given by these students were sometimes simple and sometimes quite convoluted. Some of the students who claimed that the wattage rating of a bulb is proportional to the resistance claimed that a higher resistance of a tungsten filament in the bulb will make the bulbs glow brighter. They noted that higher resistance implies a higher power dissipated and hence a brighter bulb. Therefore, the higher wattage bulbs which glow brighter must have a larger resistance. To our knowledge, this misconception has not been documented earlier. The following student responses to the wattage version manifest the difficulty due to different reasons (sometimes via very convoluted reasoning): ‘In series, the 100 W bulb will be brighter. It has a higher resistance, which will use more voltage. The 25 W bulb has less resistance, but the same current. This will produce less heat/light than the 100 W bulb,’ and ‘The 25 W light bulb will be brighter. In series, more current will flow through the path of least resistance. Power is measured in watts. Power \( \equiv \overline{I}R. \) Assuming each has equal currents, the 100 W bulb will have a higher resistance. Therefore, the 25 W bulb will have more current when connected in parallel.’

**Failure to understand the basic features of circuits in parallel and series.** From the interviews, we find that when many introductory students see an electrical circuit, the first feature that comes to their minds is the relation between the equivalent resistance and individual resistances even though it may not be relevant for the given question. Even when students were asked explicitly about these issues, some interviewed students did not understand that the voltage is the same across resistances in parallel and current is the same for the resistances in series. Students who were explicitly asked sometimes admitted that they did not know these things while others incorrectly noted that the current is the same for resistances in parallel and the voltage is the same across resistances in series. Some students noted that when 200 \( \Omega \) and 500 \( \Omega \) light bulbs are connected in parallel, the 200 \( \Omega \) light bulb is brighter because it has more current flowing through it and hence has a higher voltage across it. They claimed that a larger current necessarily means a higher voltage and hence brighter light bulb without realising that the resistors connected in parallel have the same voltage across them (and it is the power dissipated that is different in different bulbs). These difficulties were much more pronounced than the previous studies because the light bulbs had different resistances.

**In series the light bulb that is closer to the power supply in the direction of current flow has a higher voltage across it:** For bulbs in series, some students incorrectly claimed that the voltage across the light bulb which is first after the power supply in the direction of current flow is more than the voltage across the second light bulb because the current flowing through the first light bulb will cause a ‘voltage drop’ so there is less available for the next light bulb. For example, one student noted that ‘It would depend on which one came first in the series
connection because the voltage would drop after going across the resistances of the bulbs. So
the first bulb would be the brightest because it got the most voltage.’ This finding is consistent
with earlier findings for identical resistance in a circuit [2, 3].

*Failure to realise that it is the power dissipated that determines the brightness.* Although
it was specifically written on top of the sheet on which students were given these problems
that the brightness of the light bulbs is proportional to the power dissipated, many students
did not consider the power explicitly when working on either version of the questions.

*Confusion between the concepts of resistance, current, voltage and power:* Some students
used the words resistance and power interchangeably. Others used current and power
interchangeably. Yet, others used voltage and power interchangeably. This type of confusion
is similar to students confusing the concepts of force, energy and momentum in introductory
mechanics.

*Do students solve these circuit problems using conceptual or quantitative approaches?*
The questions about the light bulbs in this investigation are such that either students are given
the wattage of the light bulbs for a standard power supply or the resistance of the light bulbs
and asked which bulb will be brighter. These questions can be answered quantitatively by
analysing the equation for the power dissipated in each light bulb for each case (parallel or
series). In the wattage version, students have to first figure out the resistance of each of the
light bulbs using the relation between power, resistance and voltage.

While a majority of the introductory students provided reasoning since they were
explicitly asked to do so, many students’ reasoning did not involve any equation. An analysis
of the introductory students’ work shows that, for the wattage version when the light bulbs are
in parallel, only 26% of the students provided reasoning involving equations (e.g., those
involving \( R = \frac{V^2}{P}, P = \frac{V^2}{R}, P = \frac{i^2R}{} \)) and when the light bulbs are in series, only 14% of
the students wrote any equation to reason about the question. For the resistance version,
approximately 40% of the students wrote any equation to answer the question for both series
and parallel cases. However, students who used any equations were more likely to answer the
question correctly than those who did not write any equations.

In a previous study, when Mazur gave a group of Harvard students quantitative problems
related to power dissipation in a circuit, students performed significantly better than when an
equivalent group was given conceptual questions about the relative brightness of the light
bulbs in similar circuits [22]. In solving the quantitative problems given by Mazur, students
applied Kirchhoff’s rules to write down a set of equations and then solved the equations
algebraically for the relevant variables, from which they calculated the power dissipated.
When the conceptual circuit question was given to students in similar classes, many students
appeared to guess the answer rather than reasoning about it systematically and performance
was worse [22]. For example, if students are given quantitative problems about the power
dissipated in each (identical) headlight of a car with resistance \( R \) when the two bulbs are
connected in parallel to a battery with an internal resistance \( r \) and then asked to repeat the
calculation for the case when one of the headlights is burned out, the procedural knowledge of
Kirchhoff’s rules can help students solve for the power dissipated in each headlight even if
they cannot conceptually reason about the current and voltage in different parts of the circuit.
To reason without resorting explicitly to mathematical tools (Kirchhoff’s rules) that the single
headlight in the car will be brighter when the other headlight is burned out, students have to
reason in the following manner. The equivalent resistance of the circuit is lower when both
headlights are working so that the current coming out of the battery is larger. Hence, more of
the battery voltage drops across the internal resistance \( r \) and less of the battery voltage drops
across each headlight and therefore each headlight will be less bright. If a student deviates
from this long chain of reasoning required in conceptual understanding, the student may not make a correct inference.

One relevant question is why more students did not invoke equations to answer the conceptually posed circuit questions in this study or in Mazur’s study even though it is likely to help them answer the question correctly. Discussions with students and instructors who teach introductory physics suggest that many students are hesitant to use equations to answer these questions because they were asked as conceptual questions (which light bulb is brighter as opposed to ‘what is the power dissipated in each bulb’). Several instructors claimed that many introductory students only think of using an equation when they are asked to calculate a physical quantity explicitly. In particular, in the questions discussed here, if the students were asked to calculate the power dissipated (instead of being asked which bulb is brighter), they would have resorted to a quantitative analysis and attempted to explicitly invoke relevant equations. Prior research in mechanics supports this claim and suggests that many students are reluctant to convert a problem posed conceptually into a quantitative problem even when explicitly asked to do so because the task of first converting a conceptual problem to a quantitative problem is cognitively demanding \[1\] (even though they were more likely to obtain the correct answer using a quantitative approach). The students often used their gut feelings rather than explicitly invoking relevant physics concepts or principles for the conceptual problems \[22, 23\].

Summary

We investigated student difficulties and approaches while solving conceptual problems about the brightness of non-identical light bulbs by administering written questions to students in introductory physics courses and interviewing six students. One version of the questions provided the resistance of each light bulb while the other version provided the wattage rating of each light bulb when connected to a standard power supply. We find that many introductory students had difficulty in determining the brightness of light bulbs under different conditions despite being told explicitly that the brightness of a light bulb is proportional to the power dissipated and that the bulbs have Ohmic resistances.

By comparing student performance on the two versions, we found that the wattage version of the questions was much more difficult than the resistance version for introductory students because it involved an additional step to solve the problem. In particular, to answer the wattage version correctly, students must first figure out the resistance of the light bulbs and then the power dissipated under different conditions. Although the non-identical light bulb questions in this study were posed as conceptual questions, students who attempted to use equations to reason about their brightness were more likely to answer them correctly although a majority of introductory physics students did not use equations explicitly in their reasoning. Moreover, we find that these questions are quite challenging for first year PhD students, although they performed better than introductory students.

While one may assume \textit{a priori} that students may have less difficulty in reasoning about the brightness of the non-identical light bulbs used in this research in simple series and parallel configurations compared to those used in the earlier studies (e.g., in which the battery with an internal resistance is connected to many identical light bulbs in more complex circuits), we find that students had many conceptual difficulties. The case of ‘identical’ light bulbs in previous research can hide some of the misconceptions revealed here in the case of non-identical light bulbs.
Students need explicit guidance in comprehending that the brightness of a light bulb can change depending on how it is connected in a circuit although how one connects it in circuit will not change the resistance of an Ohmic light bulb. Some students claimed that a higher current always implies a higher voltage across the bulb (even if the light bulbs are in parallel) or a higher resistance always implies a lower current through a light bulb (even if both bulbs are in series). Moreover, some students had a tendency to associate the brightness of a light bulb only with the current or the voltage as opposed to the power dissipated, which often led to incorrect inferences. Also, many students assumed that the light bulbs will have the same brightness regardless of how they are connected in a circuit. Some of these difficulties about non-identical light bulbs are similar to those revealed in prior studies in the context of identical light bulbs [2, 3] but use of non-identical light bulbs revealed misconceptions not previously documented. Curricula and pedagogies developed to improve students’ understanding should take into account these difficulties.

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