Active Wheelbase Design for Narrow Track Vehicle Stability

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ABSTRACT: In recent years, narrow track vehicles have gained considerable interest due to the compactness and its future potential. However, the vehicles are likely to have low stability against overturning especially during hard cornering. In this work, a new design concept named “Active Wheelbase” was proposed to improve the vehicle stability during cornering. A tricycle model was developed to analyze the effects of the active wheelbase design. Motion equations of the model were derived to investigate the dynamics characteristics of the vehicle. Multibody model was built in order to conduct simulations to verify the stability improvement by the active wheelbase design.

KEY WORDS: vehicle dynamics, chassis component, driving stability / active wheelbase, narrow track vehicle [B1]

1. Introduction

Narrow track vehicles have gained considerable amount of attention in recent years due to the increasing demand of compact mobility with flexible maneuverability especially in congested urban areas. The slim vehicle body size is highly desirable in heavy traffic environments and also reduces the requirement of large parking space. However, the narrow and light weight features of the vehicle are also causing low stability against rollover especially during hard cornering (¹). One of the most common solutions for this drawback is vehicle tilting (²), to improve the vehicle stability (³). But vehicle tilting has also its own limitation and side effect on stability control (⁴). Hence, this work proposes a novel approach named “Active Wheelbase” for narrow track vehicle stability.

2. Active Wheelbase for Vehicle Stability

2.1. The Concept of Active Wheelbase

Different from the vertical perspective in vehicle tilting, the active wheelbase approach varies the vehicle wheelbase in horizontal perspective. The vehicle wheelbase can be adjusted by changing the distance between the front and rear wheels. Taking a three wheeled tricycle model (Figure 1) as an example, in normal straight line movement, the vehicle wheelbase is the sum of the front wheel and rear wheels distances from the vehicle center of gravity ($l_f + l_r$). When the vehicle taking a left turn (Figure 1 bottom), the right (outer) rear wheel experiences greater wheel load as compared to the left (inner) rear wheel due to the lateral acceleration ($a$). By moving the right rear wheel (which has greater wheel load) nearer to the vehicle center of gravity, this active wheelbase approach is anticipated to improve the vehicle stability.

Fig. 1 Active Wheelbase Design in a Tricycle Model.
2.2. Active Wheelbase Design for Vehicle Stability

Based on the forces interaction during cornering in Figure 2, the vehicle rollover limit \((a_{lim})\) can be expressed as in Equation (1).

\[
a_{lim} = \frac{d_r}{2h_{cg}} \frac{l_f}{l_f + l_r} \tag{1}
\]

![Fig. 2 Forces Interaction During Cornering in a Tricycle Model.](image)

Substituting the change of the rear wheel distance \((\Delta l_r)\) into Equation (1), Equation (2) shows that positive change of the vehicle wheelbase \((\Delta l)\) increases will increase the vehicle rollover limit (improve stability).

\[
a_{lim} = \frac{d_r}{2h_{cg}} \frac{l_f}{l_f + l_r} \tag{2}
\]

Consider a tricycle with the vehicle specifications as shown in Figure 3, the relationship between vehicle rollover limit \((a_{lim})\) and the change of rear wheel distance \((\Delta l_r)\) can be plotted as shown in Figure 4, according to Equation (2). From Figure 4, assuming the maximum change of the rear wheel distance is 0.3 m (half of the diameter of the wheel), the active wheelbase design can increase the vehicle rollover limit by 18%.

![Fig. 3 Vehicle Specifications.](image)

3. Dynamics Behaviors of the Active Wheelbase Vehicle

3.1. Vehicle Motion Dynamics

Figure 5 illustrates the simple model of the active wheelbase vehicle. \(M\) is the vehicle mass, \(V\) is the vehicle speed, \(\gamma\) is the vehicle yaw rate, \(\delta\) is the steer angle, \(\beta\) is the slip angle at the vehicle center of gravity and \(\beta_r, \beta_l, \beta_a\) are the wheel slip angle respectively, \(C_r, C_i, C_c\) are the wheel cornering force respectively, and \(K_r, K_i, K_c\) are the wheel cornering power respectively. Based on the detailed derivation in \((6)\), the vehicle dynamics equation can be expressed in Equation (3).

\[
\begin{align*}
M \frac{d\beta}{dt} + (K_r + K_i + K_c) \beta + [MV + \frac{1}{2} (l_f K_i - l_r K_r - l_r K_r)] \gamma &= K_f \delta \\
\frac{dy}{dt} + (l_f K_f - l_r K_r - l_r K_r) \beta + \frac{1}{2} [l_f^2 K_f + l_r^2 K_r + l_r^2 K_r] \gamma &= l_f K_f \delta
\end{align*}
\tag{3}
\]

Under steady circular turning, \(\frac{d\beta}{dt} = 0\) and \(\frac{dy}{dt} = 0\). Substitute these conditions into Equation (3) to obtain the expression of vehicle slip angle and yaw rate as in Equation (4).

\[
\begin{align*}
\beta &= \frac{-MV^2 K_i l_r + l_r K_c (l_o - l_r) + l_o K_r (l_o - l_r)}{K_r [K_i (l_i + l_r) + K_r (l_r - l_r)] + K_c (l_o - l_o)^2 - MV^2 (l_f K_f - l_r K_r - l_r K_r)} \\
\gamma &= \frac{K_f [K_i (l_i + l_r) + K_r (l_r - l_r)]}{K_r [K_i (l_i + l_r) + K_r (l_r - l_r)] + K_c (l_o - l_o)^2 - MV^2 (l_f K_f - l_r K_r - l_r K_r)}
\end{align*}
\tag{4}
\]
3.2. The Effect of Active Wheelbase towards Wheel Load Distribution

On Figure 6, the equilibrium of vehicle horizontal forces can be expressed as in Equation (5). Similarly, the equilibrium of vehicle vertical forces in Figure 7 can be expressed as in Equation (6). Also, the equilibrium of moment at the center of gravity in Figure 8 can be expressed as in Equation (7). Finally, the equilibrium of moment at the center of gravity in Figure 9 can be expressed as in Equation (8).

\[ F_x + F_{rr} + F_{rl} = M_a \]  
\[ W_f + W_{rr} + W_{rl} = M_g \]  
\[ W_f l_f = W_{rr} (l_r - \Delta l_r) + W_{rl} (l_r + \Delta l_r) \]  
\[ W_f \Delta d_{cg} + W_{rl} \left( \frac{d_r}{2} + \Delta d_{cg} \right) + (C_f + C_{rr} + C_{rl}) h_{cg} = W_{rl} \left( \frac{d_r}{2} - \Delta d_{cg} \right) \]

Rearranging Equation (5) to Equation (8), the vehicle wheel load distribution can be expressed as in Equation (9). Based on the equation, the relationships between vehicle wheel load and lateral acceleration in normal (dotted lines) and active wheelbase (solid lines) vehicle are plotted in Figure 9. It is observed that the front wheel load remains constant in normal vehicle but decreasing in active wheelbase vehicle.

3.3. Vehicle Turning Radius

For narrow track vehicle, which total vehicle mass is below 500 kg, the vehicle wheel load and cornering power can be assumed to have linear relationship. The cornering powers of the vehicle can be expressed as in Equation (10), where \( K_f^* \), \( K_r^* \) are the cornering power of the wheels in normal vehicle (without active wheelbase).

\[ K_f = [1 - 2 \left( \frac{\Delta l_r}{l_r d_r} \left( \Delta d_{cg} + \frac{a}{g} h_{cg} \right) \right)] K_f^* \]
\[ K_r = [1 + 2 \left( \frac{\Delta l_r}{l_r d_r} \left( \Delta d_{cg} + \frac{a}{g} h_{cg} \right) \right)] K_r^* \]
\[ K_{rr} = [1 + 2 \left( \frac{\Delta l_r}{l_r d_r} \left( \Delta d_{cg} + \frac{a}{g} h_{cg} \right) \right)] K_r^* \]
Continue the derivation from Equation (4) and substituting the expressions in Equation (10), the expression of the vehicle turning radius can be obtained as in Equation (11), where \( R_c \) is the turning radius of a normal vehicle.

\[
\frac{V}{\gamma} = R_c \left( 1 - \frac{2l\Delta l \Delta d_{cg}}{l_f \Delta \delta} \right) + \sqrt{R_c \left( \frac{2l\Delta l \Delta d_{cg}}{l_f \Delta \delta} \right)^2 - \frac{8l\Delta l V^2}{l_f \Delta \delta g h_{cg}}} \]  

(11)

4. Simulation Analysis

4.1. Vehicle Model and Control System

In order to conduct simulation analysis, a simulation model of the tricycle vehicle was constructed in Multibody Dynamics (MBD) software ADAMS as shown in Figure 10. The specifications of the vehicle are summarized in Table 1.

Table 1: Vehicle Specifications of the Tricycle.

| Specification              | Value          |
|----------------------------|----------------|
| Wheelbase \((l)\)          | 1850 mm        |
| Track \((d_r)\)            | 500 mm         |
| Height of the Center of Gravity \((h_{cg})\) | 600 mm     |
| Length                     | 2500 mm        |
| Width                      | 600 mm         |
| Height                     | 1490 mm        |
| Total Mass \((M)\)         | 300 kg         |
| Tire                       | 125/80R13      |

An active wheelbase mechanism as shown in Figure 11 was designed to control the movement of the rear wheels. Both the rear wheels were connected together by a rectangular link structure that allows only forward and backward sliding movements between the wheels. The link structure was connected to the vehicle body by two pin joints at the center, where a motor was attached to control the rotation of the pin joints. The distance \((d)\) between the center joint to the wheel was 0.15 m.

![Fig. 11 Active Wheelbase Mechanism.](image)

The control system of the active wheelbase mechanism is illustrated in Figure 12. The vehicle speed, acceleration, steer angle and steer angular velocity are taken as inputs to calculate the appropriate turning angle \((\phi)\) for the active wheelbase mechanism motor to produce the desired rear wheel movement. The calculation is based on Equation (12), where \( C = 1 \) and the maximum wheel movement is limited to 0.3 m.

\[
\phi = \tan^{-1} \left( \frac{V l_f \delta + V l_f \delta + V^2}{C d} \right) 
\]  

(12)

4.2. Vehicle Cornering Simulation

A vehicle cornering simulation was conducted to investigate the dynamics behaviors and stability of the active wheelbase vehicle (blue line), with comparison to a normal vehicle (without active wheelbase) (red line). The vehicle velocity (10 m/s) and steer angle (2 deg) were controlled as in Figure 13 and 14 respectively.

![Fig. 12 Block Diagram of the Active Wheelbase Control.](image)

![Fig. 13 Vehicle Velocity (Simulation Condition).](image)
Figure 15 shows the rear wheel movement during the simulation. The active wheelbase control system is triggered by the steer angle input to move the rear wheel (0.27 m). As expected, no rear wheel movement occurs in normal vehicle.

Comparing the wheel load results between normal vehicle (Figure 16) and active wheelbase vehicle (Figure 16), the load of the front wheel remains the same in normal vehicle but reduces in active wheelbase vehicle. Part of the load is distributed to the rear wheel. The inner (left) rear wheel load is almost zero in both cases, indicates that the vehicle is running at rollover limit.

Figure 18 shows the turning radius of both vehicles. It is obvious that the turning radius of the active wheelbase vehicle is smaller than the normal vehicle in the same running condition. This has proven that the improved stability (higher rollover limit) in the active wheelbase vehicle.

Based on the simulation results, taking the turning radius of the normal vehicle as 50 m, the theoretical value of the turning radius can be calculated by using Equation (11) as follows:

\[
R = \frac{1}{2} \left[ R_C - \frac{2\Delta l_r \Delta d_{cg}}{l_f d_r \delta} \right] + \sqrt{\left( \frac{2\Delta l_r \Delta d_{cg}}{l_f d_r \delta} \right)^2 - \frac{8\Delta l_r}{l_f d_r \delta} \frac{V^2}{g h_{cg}}}
\]

\[
= \frac{1}{2} \left[ 50 + \sqrt{50^2 - \frac{8 \times 1.85 \times 0.27 \times 10^2}{1 \times 0.5 \times 0.035 \times g} \times 0.6} \right] \approx 41.6 m
\]

The turning radius calculation result of 41.6 m is very close to the simulation result in Figure 18. Taking this result, the vehicle wheel load also can be calculated by using Equation (9) as follows:

\[
W_f = \frac{l_r}{l} Mg \left[ 1 - 2 \frac{\Delta l_r}{l_r d_r} \left( \frac{\Delta d_{cg}}{g R} + \frac{V^2}{h_{cg}} \right) \right]
\]

\[
\cong 1350 \left( 1 - 2 \frac{0.27}{0.85 \times 0.5} \frac{0.035}{9.8 \times 41.6 \times 0.6} \right) \approx 1097 N
\]

Again, the front wheel load calculation result of 1097 N is closely agreed with the simulation result in Figure 17. The closely agreement between theoretical and simulation results has validated the theoretical derivation and simulation modeling of this work.
5. Conclusion

The objective of this work is to propose a novel approach named “Active Wheelbase” to improve narrow track vehicle stability especially during cornering. The development of this work can be summarized as follows:

- A theoretical tricycle model was developed to analyze the vehicle stability of the active wheelbase design.
- Dynamics behaviors on the vehicle motion dynamics, wheel load distribution, turning radius of the model were derived to investigate the dynamics characteristics of the proposed vehicle.
- Simulation model and control system were built in order to conduct simulations to verify the stability improvement by the active wheelbase design.
- From the simulation results, it is proven that the results are closely agreed with the theoretical calculations.

The closed agreement between simulation and theoretical results has validated the theoretical derivation and simulation modeling of this work in active wheelbase design for narrow track vehicle stability.

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