The perturbative proton form factor reexamined

Bijoy Kundu, Hsiang-nan Li, Jim Samuelsson and Pankaj Jain

\textsuperscript{a}Department of Physics, IIT Kanpur, Kanpur-208 016, India

\textsuperscript{b}Department of Physics, National Cheng-Kung University
Tainan, Taiwan, Republic of China

\textsuperscript{c}Mehta Research Institute of Mathematics and Mathematical Physics
Jhusi, Allahabad, India

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Abstract

We recalculate the proton Dirac form factor based on the perturbative QCD factorization theorem which includes Sudakov suppression. The evolution scale of the proton wave functions and the infrared cutoffs for the Sudakov resummation are carefully chosen, such that the soft divergences from large coupling constants are diminished and perturbative QCD predictions are stabilized. We find that the King-Sachrajda model for the proton wave function leads to results which are in better agreement with experimental data compared to the Chernyak-Zhitnitsky wave function.
1. Introduction

Since the proposal of the improved perturbative QCD (PQCD) factorization formulas for exclusive processes, with the Sudakov resummation taken into account [1], there have been many applications in the literature, such as the pion form factor [2], photon annihilation into pions [3], the proton form factors [4, 5], pion Compton scattering [6], proton-anti-proton annihilation [7], and proton-proton Landshoff scattering [8]. These studies show that in the pion case the nonperturbative contributions from the end points of parton momentum fractions are moderated by Sudakov suppression, and perturbative predictions become relatively reliable. However, in the processes involving protons, because more partons share the external momentum, the infrared divergences associated with soft partons, which appear in hard scattering subamplitudes, are severer. It is then a concern whether Sudakov suppression of the end-point nonperturbative enhancements is strong enough to maintain the applicability of PQCD to the proton form factor at currently accessible energy scales.

The improved factorization formalism has been applied to the proton form factor [4]. However, the choice of the infrared cutoffs for the resummation was criticized [5]: The end-point enhancements are in fact not diminished completely by Sudakov suppression under the above choice of cutoffs, implying that PQCD predictions remain unreliable. A modified choice of the cutoffs has been proposed [5], and the soft enhancements were found to be suppressed. Unfortunately, it turned out that the PQCD contributions amount only to half of the data, and hence it was concluded that higher-order or higher-twist corrections may be important [5].

In this letter we shall recalculate the proton Dirac form factor based on the work of [4] by slightly modifying the infrared cutoffs for the resummation, and employing the more complete two-loop expression of the Sudakov factor. It will be shown that the end-point sensitivity is removed, and the PQCD predictions from one of the currently available models of the proton wave function match the experimental data well. We then confirm the applicability of the improved PQCD formalism for momentum transfer around few GeV. However, we emphasize that the uncertainty involved in our analysis is not negligibly small, and that the method in [6] based on the overlap integral of the proton wave functions may be regarded as a complementary approach to ours.
2. Factorization

According to the PQCD theory for exclusive processes \[10\], the proton Dirac form factor, can be factorized into two types of subprocesses: wave functions which contain the nonperturbative information of the initial- and final-state protons, and a hard subamplitude which describes the scattering of a valence quark of the proton off the energetic photon. The former can not be calculated perturbatively, and needs to be parametrized by a model or to be derived by nonperturbative methods such as QCD sum rules. The latter, characterized by a large momentum flow, is calculable in perturbation theory. We quote directly the factorization formula for the proton form factor derived in \[4\]:

\[
F_p^1(Q^2) = \int_0^1 (dx)(dx')(dk_T)(dk'_T)\bar{Y}_{\alpha'\beta'\gamma'}(k'_i, P', \mu) \times H_{\alpha'\beta'\gamma'\alpha\beta\gamma}(k_i, k'_i, Q, \mu)Y_{\alpha\beta\gamma}(k_i, P, \mu) ,
\]

with

\[
(dx) = dx_1dx_2dx_3\delta(\sum_{i=1}^3 x_i - 1) ,
\]

\[
(dk_T) = dk_{1T}dk_{2T}dk_{3T}\delta(\sum_{i=1}^3 k_{iT}) .
\]

\(P = (P^+, 0, 0)\) is the initial-state proton momentum, and \(x_i = k^+_i/P^+\) and \(k_{iT}\) are the longitudinal momentum fraction and transverse momenta of the parton \(i\), respectively. The primed variables \(P' = (0, P^-, 0)\), \(x'_i = k^-_i/P^-\) and \(k'_{iT}\) are associated with the final-state proton. \(Q^2 = 2P \cdot P'\) is the momentum transfer. In the Breit frame we have \(P^+ = P^- = Q/\sqrt{2}\). The scale \(\mu\) is the renormalization and factorization scale.

The initial distribution amplitude \(Y_{\alpha\beta\gamma}\), defined by the matrix element of three local operators in axial gauge \[11, 13\], is given by

\[
Y_{\alpha\beta\gamma} = \frac{1}{2\sqrt{2}N_c} \int \prod_{l=1}^3 \frac{dy_l^-dy_l^+}{(2\pi)^3} e^{ik_l \cdot y_l} \epsilon^{abc} \langle 0 | T[u^a_\alpha(y_1)u^b_\beta(y_2)d^c_\gamma(0)] | P \rangle
\]

\[= \frac{f_N(\mu)}{8\sqrt{2}N_c} [(\bar{\psi}C)_{\alpha\beta}(\gamma_5 N)_{\gamma} V(k_i, P, \mu) + (\bar{\psi}\gamma_5 C)_{\alpha\beta} N_{\gamma} A(k_i, P, \mu)
\]

\[\] - \((\sigma_{\mu\nu}P^{\nu} C)_{\alpha\beta}(\gamma^{\mu}\gamma_5 N)_{\gamma} T(k_i, P, \mu)\] ,

\(3\)
where $N_c = 3$ is the number of colors, $|P\rangle$ the initial proton state, $u$ and $d$ the quark fields, $a$, $b$ and $c$ the color indices, and $\alpha$, $\beta$ and $\gamma$ the spinor indices. In our notation, 1,2 label the two $u$-quarks and 3 labels the $d$-quark. The second form shows the explicit Dirac matrix structure [11], where $f_N$ is the normalization constant [12], $N$ the proton spinor, $C$ the charge conjugation matrix and $\sigma_{\mu\nu} \equiv \frac{1}{2} [\gamma_\mu, \gamma_\nu]$. The amplitude $\bar{Y}_{\alpha'\beta'\gamma'}(k'_1, P', \mu)$ for the final-state proton is defined similarly. By using the permutation symmetry [11] and the constraint that the total isospin of the three quarks is equal to $1/2$, it can be shown that the three functions $V$, $A$, and $T$ are not independent, and related to a single function $\psi$ by [11]

$$V(k_1, k_2, k_3, P, \mu) = \frac{1}{2} [\psi(k_2, k_1, k_3, P, \mu) + \psi(k_1, k_2, k_3, P, \mu)] ,$$

$$A(k_1, k_2, k_3, P, \mu) = \frac{1}{2} [\psi(k_2, k_1, k_3, P, \mu) - \psi(k_1, k_2, k_3, P, \mu)] ,$$

$$T(k_1, k_2, k_3, P, \mu) = \frac{1}{2} [\psi(k_1, k_3, k_2, P, \mu) + \psi(k_2, k_3, k_1, P, \mu)] .$$

The hard subamplitude $H_{\alpha'\beta'\gamma'\alpha\beta\gamma}$ is obtained from the photon-quark scattering diagrams, and the expressions for the integrands $\bar{Y}_{\alpha'\beta'\gamma'}H_{\alpha'\beta'\gamma'\alpha\beta\gamma}Y_{\alpha\beta\gamma}$ are referred to Table I in [4]. Employing a series of permutations of the parton kinematic variables, Eq. (1) in Fourier transform space reduces to

$$F_p^p(Q^2) = \sum_{j=1}^{8} \frac{8\pi^2}{27} \int_0^1 (dx)(dx')(db)[f_N(\mu)]^2 \times \tilde{H}_j(x_i, x'_i, b_i, Q, \mu) \Psi_j(x_i, x'_i, b_i, P, P', \mu) ,$$

(5)

with $b_i$ the conjugate variable to $k_{IT}$ and $(db) = db_1db_2/(2\pi)^4$. The explicit expressions of $\tilde{H}_j$ and of $\Psi_j$ in terms of $\psi$ will be below.

### 3. Sudakov Suppression

The Sudakov resummation of the large logarithms in $\psi$ leads to

$$\psi(x_i, b_i, P, \mu) = \exp \left[ -\sum_{l=1}^{3} s(x_l, w, Q) - 3 \int_w^\mu \frac{d\mu}{\mu} \gamma_q(\alpha_s(\mu)) \right] \times \phi(x_i, w) ,$$

(6)
where the quark anomalous dimension $\gamma_q(\alpha_s) = -\alpha_s/\pi$ in axial gauge governs the renormalization-group (RG) evolution of $\psi$. The function $\phi$, obtained by factoring the $Q$ dependence out of $\psi$, corresponds to the standard parton model. The exponent $s$ is written as

$$s(x, w, Q) = \int_w^{xQ/\sqrt{2}} \frac{dp}{p} \left[ \ln \left( \frac{xQ}{\sqrt{2}p} \right) A(\alpha_s(p)) + B(\alpha_s(p)) \right],$$

(7)

where the anomalous dimensions $A$ to two loops and $B$ to one loop are

$$A = \frac{C_F}{\pi} \alpha_s + \left[ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_0 \ln \left( \frac{e^{\gamma_E}}{2} \right) \right] \left( \frac{\alpha_s}{\pi} \right)^2,$$

$$B = \frac{2}{3} \frac{\alpha_s}{\pi} \ln \left( \frac{e^{2\gamma_E - 1}}{2} \right),$$

(8)

$n_f = 3$ being the number of flavors, and $\gamma_E$ the Euler constant. The two-loop running coupling constant,

$$\frac{\alpha_s(\mu)}{\pi} = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)} - \frac{\beta_1 \ln \ln(\mu^2/\Lambda^2)}{\beta_0^2 \ln^2(\mu^2/\Lambda^2)},$$

(9)

with the coefficients

$$\beta_0 = \frac{33 - 2n_f}{12}, \quad \beta_1 = \frac{153 - 19n_f}{24},$$

(10)

and the QCD scale $\Lambda \equiv \Lambda_{QCD}$, will be substituted into Eq. (7).

The infrared cutoff $w$ is chosen to be the inverse of a typical transverse distance among the three valence quarks. We try different definitions of this cutoff to determine its influence on the final result. One possible choice is $w = 1/b_{max}$, $b_{max} = \max(b_l)$, $l = 1, 2, 3$, adopted in [2], with $b_3 = |b_1 - b_2|$. As long as all of these mass scales are much larger than $\Lambda$, the Sudakov form factor should not give any suppression. As one of these scales gets close to $\Lambda$, the Sudakov form factor tends to zero and suppresses this region. We find that choosing the infrared cutoff in this fashion suppresses all the infrared divergences and leads to a self-consistent calculation of the form factor. However, this choice does not always correspond to a typical size of the three quark system. A more appropriate definition is obtained by considering it as a quark-diquark like configuration. The diquark constituents are taken to be those two quarks that are closest to each other in the transverse plane.
The typical transverse distance, $d_{typ}$: The transverse distance between the quarks A and B is the smallest among the three quarks. The diquark constituents are therefore considered to be the quarks A and B. The center of mass of the diquark, $\text{COM}_{dq}$, is taken to be the central point of the line that connects these two quarks. $d_{typ}$ is then defined as the distance between $\text{COM}_{dq}$ and the third quark C.

We now define the typical transverse distance, $d_{typ}$, as the distance between the center of mass of the diquark and the remaining third quark (Fig. 1). This is clearly a more reasonable measure of the distance in the three quark system that can be resolved by a gluon. We shall therefore take the infrared cutoff as $c w$, where $c$ is a parameter which is allowed to deviate slightly from unity. When we put $c = 1$, we recover the original choice of the cutoff. The introduction of this parameter $c$ is natural from the viewpoint of the resummation, since the scale $c w$, with $c$ of order unity, is as appropriate as $w$ [13]. We choose $c$ such that for a large number of randomly chosen triangles, of the type shown in Fig. 1, we get for the average $\langle d_{typ}/b_{\text{max}} \rangle = 1/c$. Defining $c$ in such a way, gives $c \approx 1.14$.

We find that both of these choices of the cutoff, lead to self-consistent calculations of the form factor in the sense that the form factor saturates at the large distance cutoff $b_c$. Remarkably, we find that with the small modification of $w$ into $c w$, which differs from what was used in [3], the results are in good agreement with experimental data. The dependence of the final results on the precise choice of scale $c w$ shows that large distance contributions cannot be completely dismissed, and give about 25-50% contribution at laboratory energies. Nevertheless, we find it encouraging that a physically motivated cutoff gives good agreement with experiments.

The choice of scales for the Sudakov resummation in Eq. (6) is compared
to that adopted in [4], where the different cutoffs $b_l$ are assigned to each exponent $s$ and to each integral involving $\gamma_q$:

$$
\psi(x_i, b_i, P, \mu) = \exp \left[ -\sum_{l=1}^{3} \left( s(x_l, b_l, Q) + \int_{b_l}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \right) \right] \times \phi(x_1, w) .
$$

(11)

The Sudakov factor in Eq. (11) does not suppress the soft divergences from $b_l \to 1/\Lambda$ completely. For example, the divergences from $b_1 \to 1/\Lambda$, which appear in $\phi(x_i, w)$ at $w \to \Lambda$, survive as $x_1 \to 0$, since $s(x_1, b_1, Q)$ vanishes and $s(x_2, b_2, Q)$ and $s(x_3, b_3, Q)$ remain finite. On the other hand, $w$ should play the role of the factorization scale, above which QCD corrections give the perturbative evolution of the wave function $\psi$ in Eq. (6), and below which QCD corrections are absorbed into the initial condition $\phi$. It is then not reasonable to choose the cutoffs $b_l$ for the Sudakov resummation different from $w$.

4. RG Evolution

The RG evolution of the hard scattering subamplitudes is written as

$$
\tilde{H}_j(x_i, x'_i, b_i, Q, \mu) = \exp \left[ -3 \sum_{l=1}^{2} \int_{\mu}^{t_{1l}} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \right] \times \tilde{H}_j(x_i, x'_i, b_i, Q, t_{j1}, t_{j2}) ,
$$

(12)

where the explicit expressions of $t$ are

$$
t_{11} = \max \left[ \sqrt{(1-x_1)(1-x'_1)}Q, 1/b_1 \right] ,
$$

$$
t_{21} = \max \left[ \sqrt{x_1x'_1}Q, 1/b_1 \right] ,
$$

$$
t_{12} = t_{22} = \max \left[ \sqrt{x_2x'_2}Q, 1/b_2 \right] .
$$

(13)

The first scales in the brackets are associated with the longitudinal momenta of the exchanged gluons and the second scales with the transverse momenta. The arguments $t_{j1}$ and $t_{j2}$ of $\tilde{H}_j$ denote that each $\alpha_s$ is evaluated at the largest mass scale of the corresponding gluon.
Inserting Eqs. (6) and (12) into Eq. (5), we obtain

\[
F^p_1(Q^2) = \sum_{j=1}^{2} \frac{4\pi}{27} \int_0^1 (dx)(dx') \int_0^\infty b_1 db_1 b_2 db_2 \int_0^{2\pi} \theta d\theta [f_N(cw)]^2 \\
\times \tilde{H}_j(x_i, x_i', b_i, Q, t_{j1}, t_{j2}) \Psi_j(x_i, x_i', cw) \\
\times \exp \left[ -S(x_i, x_i', cw, Q, t_{j1}, t_{j2}) \right],
\]

(14)

with

\[
\tilde{H}_1 = \frac{2}{3} \alpha_s(t_{11}) \alpha_s(t_{12}) K_0 \left( \sqrt{(1 - x_1)(1 - x_1')} Q b_1 \right) K_0 \left( \sqrt{x_2 x_2' Q b_2} \right),
\]

\[
\tilde{H}_2 = \frac{2}{3} \alpha_s(t_{21}) \alpha_s(t_{22}) K_0 \left( \sqrt{x_1 x_1' Q b_1} \right) K_0 \left( \sqrt{x_2 x_2' Q b_2} \right).
\]

(15)

The variable \( \theta \) is the angle between \( b_1 \) and \( b_2 \). \( K_0 \) is the modified Bessel function of order zero. The expressions for \( \Psi_j \) are

\[
\Psi_1 = \frac{2(\phi \phi')_{123} + 8(TT')_{123} + 2(\phi \phi')_{132} + 8(TT')_{132} - (\phi \phi')_{312} - (\phi \phi')_{231}}{(1 - x_1)(1 - x_1')},
\]

\[
\Psi_2 = \frac{2(\phi \phi')_{132} - 2(TT')_{123}}{(1 - x_2)(1 - x_1)} + \frac{(\phi \phi')_{123} - 8(TT')_{132} - 2(\phi \phi')_{321}}{(1 - x_3)(1 - x_1')},
\]

(16)

which group together the products of the initial and final wave functions in the notation

\[
(\phi \phi')_{123} \equiv \phi(x_1, x_2, x_3, cw) \phi(x_1', x_2', x_3', cw).
\]

(17)

\((TT')\) is defined similarly based on Eq. (4) but with \( \psi \) replaced by \( \phi \). The Sudakov exponent \( S \) is given by

\[
S(x_i, x_i', cw, Q, t_{j1}, t_{j2}) = \sum_{l=1}^{3} s(x_i, cw, Q) + 3 \int_{cw}^{t_{j1}} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \\
+ \sum_{l=1}^{3} s(x_i', cw, Q) + 3 \int_{cw}^{t_{j2}} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})).
\]

(18)

For the wave function \( \phi \), we will consider both the Chernyak-Zhitnitsky (CZ) model [11] and King-Sachrajda (KS) model [14]. They are decomposed
in terms of the first six Appel polynomials $A_j(x_i)$, which are eigensolutions
of the evolution equation for the nucleon wave function \cite{10, 15}
\[
\phi(x_i, w) = \phi_{as}(x_i) \sum_{j=0}^{5} N_j \left[ \frac{\alpha_s(w)}{\alpha_s(\mu_0)} \right]^{b_j/(4\beta_0)} a_j A_j(x_i),
\] (19)
with $\mu_0 \approx 1$ GeV. The constants $N_j$, $a_j$ and $b_j$ are given in Table I. $\phi_{as}(x_i) = 120x_1x_2x_3$ is the asymptotic form of $\phi$. The evolution of the dimensional constant $f_N$ is given by
\[
f_N(w) = f_N(\mu_0) \left[ \frac{\alpha_s(w)}{\alpha_s(\mu_0)} \right]^{1/(6\beta_0)},
\] (20)
with $f_N(\mu_0) = (5.2 \pm 0.3) \times 10^{-3}$ GeV$^2$ \cite{11}.

5. Numerical Results

In order to be able to calculate the seven-dimensional integral, Eq. (14), we use the VEGAS Monte Carlo routine \cite{16}. We set the Sudakov factor $\exp(-S)$ to unity in the small $b$ region where it displays a small enhancement, since in this region higher-order corrections should be absorbed into the hard scattering \cite{1}, instead of into the wave function, giving its evolution. Therefore we have also set the factor $\exp[-s(\xi, cw, Q)]$ to unity whenever $\xi Q/\sqrt{2} < cw$. As $cw$ approaches $\Lambda$, the Sudakov factor vanishes, implying that the whole integrand of Eq. (14) also vanishes.

First we choose the parameter value $c = 1$. The results of $Q^4 F^p_1$ for $\Lambda = 0.2$ GeV from the use of the KS wave function, along with the experimental data \cite{2, 4}, are shown in Fig. 4. The PQCD predictions amount only to about 60% of the data. It is then possible that higher-order or higher-
Fock-state contributions are necessary for the explanation of the data, which are certainly worth of further studies. However, before jumping to that conclusion, we investigate the effect from the freedom of varying the parameter $c$. The results with $c = 1.14$ are also displayed in Fig. 4. It is found that our predictions match the data well. Note that varying $c$ makes a difference in the resummation at the level of next-to-leading logarithms, which can be regarded as an uncertainty of our formalism. Therefore, we argue that the current data can be explained within the uncertainty of our approach.
Following Ref. [4], we should analyze how the contributions to $Q^4 F_1^p$ are distributed in the $b_1$-$b_2$ plane. The integration is done with both variables $b_1$ and $b_2$ cut off at a common value $b_c$. If the perturbative region dominates, most of the contributions will be quickly accumulated below a small $b_c$. The numerical outcomes (with $c = 1.14$) are shown in Fig. 3. All the curves, showing the dependence of $Q^4 F_1^p$ on $b_c$, increase from the origin and reach their full height at $b_c = 0.9/\Lambda$. The curves exhibit small humps at the high end of $b_c$, which imply that the evolution of the wave function gives a small negative contribution in the large $b$ region. A standard of self-consistency is that 50% of the whole amount of $Q^4 F_1^p$ is accumulated from the region with $\alpha_s/\pi$ smaller than 0.5. Based on this standard, the results with $Q^2 > 10$ GeV$^2$ are reliable. Therefore, the applicability of PQCD to the proton form factor at currently accessible energy scale $Q^2 \sim 35$ GeV$^2$ is justified.
Figure 3: Dependence of $Q^4 F_1^p$ on the cutoff $b_c$ with the KS wave function for $Q^2 = 12 \text{ GeV}^2$ (dotted line), $Q^2 = 16 \text{ GeV}^2$ (dashed line), $Q^2 = 25 \text{ GeV}^2$ (dense-dot line), and $Q^2 = 36 \text{ GeV}^2$ (solid line).

The CZ wave function is also employed, and the corresponding results are shown in Fig. 2. It is observed that the values are only about 2/3 and 3/4 of those derived from the KS model with $c=1.14$ and $c=1$ respectively, and are far below the data. Hence, we claim that the KS proton wave function is more phenomenologically appropriate.

6. Conclusion

In this work we have modified the choice of the infrared cutoffs for the resummation, and employed the more complete two-loop expression of the Sudakov factor compared to the previous analyses. With these modifications, we have been able to explain self-consistently the experimental data of the proton Dirac form factor for $Q^2 > 10 \text{ GeV}^2$, within the uncertainty of our formalism. We should emphasize that though the nonperturbative region denoted by $b \to 1/\Lambda$ does become less important in our analysis, the coupling constant $\alpha_s$ is not so small that we could consider the perturbative results as exact. Therefore, nonperturbative contributions may be comparable to the perturbative ones at the currently accessible energies. A complementary study based on nonperturbative approaches such as QCD sum rules and the determination of the transition of the proton form factor to PQCD, as
performed in [6], are then essential.

The analysis presented here is not conclusive even in the PQCD framework. The uncertainty at the level of next-to-leading logarithms indicates that higher-order corrections to the evolutions of the wave function and of the hard subamplitudes need to be evaluated. The contributions from higher Fock states should be investigated, which may be important in the intermediate energy range.

It is found that the KS wave function is more phenomenologically appropriate, which will be adopted in the future studies of QCD processes involving protons.

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References

[1] H-n. Li and G. Sterman, Nucl. Phys. B381 (1992) 129.

[2] T. Gousset and B. Pire, Phys. Rev. D 51 (1995) 15; R. Jakob and P. Kroll, Phys. Lett. B 315 (1993) 463.

[3] I.V. Musatov and A.V. Radyushkin, Preprint No. JLAB-Thy-97-07 [hep-ph/9702443].

[4] H-n. Li, Phys. Rev. D 48 (1993) 4243.

[5] R. Jakob, P. Kroll, M. Bergmann, and N.G. Stefanis, Z. Phys. C 66 (1995) 267.

[6] C. Corianò and H-n. Li, Phys. Lett. B 309 (1993) 409; Nucl. Phys. B434 (1995) 535.

[7] T. Hyer, Phys. Rev. D 47 (1993) 3875.

[8] M.G. Sotiropoulos and G. Sterman, Nucl. Phys. B 419 (1994) 77.

[9] J. Bolz and P. Kroll, Preprint No. WU B 95-35 [hep-ph/9603289].

[10] G.P. Lepage and S.J. Brodsky, in Quantum Chromodynamics (La Jolla Institute 1978), Proceedings of the Workshop, La Jolla, California, edited by W. Frazer and F. Henyey, AIP conf. Proc. No. 55 (AIP, New York, 1979); Report no. SLAC-PUB-2294, presented at the workshop on Current Topics in High Energy Physics, Pasadena, California, 1979 (unpublished); Phys. Rev. Lett. 43 (1979) 545; Phys. Rev. D22, (1980) 2157.

[11] V.L. Chernyak and A.R. Zhitnitsky, Yad. Fiz. 31 (1980) 1053 [Sov. J. Nucl. Phys. 31 (1980) 544]; Nucl. Phys. B216 (1983) 373; B246 (1984) 52; Phys. Rep. 112 (1984) 173.

[12] B.L. Ioffe, Nucl. Phys. B188 (1981) 317; B191 (1981) 591 (E).

[13] J. Botts and G. Sterman, Nucl. Phys. B325 (1989) 62.

[14] I.D. King and C.T. Sachrajda, Nucl. Phys. B279 (1987) 785.

[15] S.J. Brodsky and G.P. Lepage, Phys. Scr. 23 (1981) 945.
[16] Numerical Recipes in Fortran, second edition, W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery (Cambridge University Press, 1992).

[17] C.R. Ji, A.F. Sill and R.M. Lombard-Nelsen, Phys. Rev. D36 (1987) 165.

[18] G. Arnold et al., Phys. Rev. Lett. 57 (1986) 174.

Table I. Appel polynomial coefficients in Eq. (19) for the nucleon wave function \( \phi(x_i, w) \) of the CZ and KS models \([11, 14]\) with the scale \( \mu_0 \approx 1 \) GeV \([17]\).

| \( j \) | \( a_j \text{(CZ)} \) | \( a_j \text{(KS)} \) | \( N_j \) | \( b_j \) | \( A_j(x_i) \) |
|---|---|---|---|---|---|
| 0 | 1.00 | 1.00 | 1 | 0 | 1 |
| 1 | 0.410 | 0.310 | 21/2 | 20/9 | \( x_1 - x_3 \) |
| 2 | -0.550 | -0.370 | 7/2 | 24/9 | \( 2 - 3(x_1 + x_3) \) |
| 3 | 0.357 | 0.630 | 63/10 | 32/9 | \( 2 - 7(x_1 + x_3) + 8(x_1^2 + x_3^2) + 4x_1x_3 \) |
| 4 | -0.0122 | 0.003333 | 567/2 | 40/9 | \( x_1 - x_3 - (4/3)(x_1^2 - x_3^2) \) |
| 5 | 0.00106 | 0.0632 | 81/5 | 42/9 | \( 2 - 7(x_1 + x_3) + 14x_1x_3 + (14/3)(x_1^2 + x_3^2) \) |

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