Radiation induced acceleration of ions in a laser irradiated transparent foil

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Abstract
Radiation friction can have a substantial impact on electron dynamics in a transparent target exposed to a strong laser pulse. In particular, by modifying the quiver electron motion, it can strongly enhance the longitudinal charge separation field, thus inducing ion acceleration. We present a model and simulation results for such a radiation induced ion acceleration regime and study the scalings of the maximal attainable and average ion energies with respect to the laser and target parameters. We also compare the performance of this mechanism to the conventional ones.

1. Introduction
Several multi-petawatt laser facilities are under construction in Europe [1–3], and even more powerful 100–200 PW projects have been announced [4, 5]. A promising application of such powerful lasers is ion acceleration in plasma. In contrast to laser driven electron acceleration, for which a great progress has been achieved even with the existing lasers and reaching an 8 GeV energy level has been recently reported [6], energies of laser accelerated ions in experiments have not yet attained the level of 100 MeV per nucleon [7–9]. Though it is sufficient for such applications as proton radiography [10, 11] or materials stress testing [12], some others, including hadron therapy for cancer treatment [13, 14] or fast ignition of nuclear fusion [15], require higher ion energy [11, 16, 17].

Among various known mechanisms of laser driven ion acceleration the radiation pressure acceleration (RPA) [15, 18–29] stands out by a strong (linear) scaling of the ion energy with laser intensity. The details are peculiar to the cases of thin [19–21, 23–27] and thick [15, 18, 28, 29] targets (target thickness should be compared to the laser wavelength [25]). These cases are called the light sail (LS) and hole boring (HB) regimes, respectively. Independently of these distinctions the main idea underlying RPA is that a strong laser pulse, being reflected off a dense plasma target, pushes the plasma electrons forward, thus creating a longitudinal charge separation electric field, which pulls the ions of the target.

Furthermore, the prospective high power facilities mentioned at the beginning are designed to generate laser pulses in the intensity range $10^{23} – 10^{24}$ W cm$^{-2}$. In such a strong field electrons radiate so violently that their motion is strongly influenced by the recoil [30–32]. This effect, called radiation friction (RF), was recently studied in the dedicated experiments employing head-on collision of intense laser pulses with energetic electrons [33, 34] and ultrarelativistic positrons propagating in silicon [35]. In particular, it was clearly demonstrated [33, 34] that the recoil changed both the electron energy distribution and the Compton radiation spectrum, hence it should be properly taken into account in an accurate theoretical interpretation of the experimental data.

In spite of a notable activity, current understanding of the impact of RF on ion acceleration remains rudimentary. On the one hand, it was demonstrated that RF has a limited impact on ion acceleration in the LS regime [36–38]. This is natural, as in an opaque target only a minority of electrons can penetrate into the strong field region and probe the RF force. In contrast, it looks as RF can noticeably reduce ions energy...
in the HB case [39]. Furthermore, it can substantially modify the laser-plasma dynamics in a transparent target [40, 41]. In particular, it was established analytically and numerically [42, 43] that RF can substantially enhance charge separation in a laser driven transparent plasma. Though it was also demonstrated by particle-in-cell (PIC) simulations that under such conditions RF can enhance ion acceleration in a thin foil [39, 44], no convincing explanation has been proposed so far.

Classically, radiation recoil is described by introducing an RF force $F_{RF}$ into the electron equation of motion

$$\frac{dp}{dt} = -e\left( \frac{1}{c} [v \times B] \right) + F_{RF}. \tag{1}$$

The full-length expression for the force $F_{RF}$ is discussed in [45]. However (see appendix A), under the conditions of our interest it is enough to retain only the leading term

$$F_{RF} \approx \frac{2 \nu}{3} \alpha e E_{cr} \chi^2, \tag{2}$$

directed opposite to the electron velocity. Here $\nu > 0$ and $m$ are electron charge magnitude and mass, $\alpha = e^2/\hbar c$ is the fine structure constant, $E_{cr} = m^2 c^3 / e \hbar$ is the critical field of quantum electrodynamics, and

$$\chi = \left( \frac{e \hbar}{m^2 c^2} \right)^{1/2} \left( \frac{E}{1 + e \frac{1}{c} [v \times B]} \right)^{1/2} - \frac{1}{e^2} (E \nu)^2, \quad \gamma = \sqrt{1 + (p/mc)^2}$$

is called the dynamical quantum parameter [46].

The value of $\chi$ discriminates which description, the classical one ($\chi \ll 1$) or the quantum one, is valid for RF. Quantum corrections to the classical force (2) can be introduced by a replacement $F_{RF} \to g(\chi)F_{RF}$ [34, 47–49]. We approximate the correction factor $g(\chi)$ by [48]

$$g(\chi) \approx 1 - \frac{55\sqrt{3}}{16} \chi$$

with the accuracy 5% for $\chi < 0.03$.

Now consider a strong plane electromagnetic (laser) wave propagating in a plasma. As shown in [42], the longitudinal component of the equation of motion for a plasma electron in such a field can be cast into the form

$$\frac{du}{dt} = \frac{1}{2\gamma} \frac{da^2(\varphi)}{d\varphi} + \mu \frac{a^1 - v_x}{1 + v_x} - \sigma, \quad \mu = \frac{2 \nu}{3} \frac{\hbar \omega}{mc^2} \approx 1.18 \times 10^{-8} \frac{1}{\lambda(\mu m)}, \tag{5}$$

Here $u = p/mc$ is the dimensionless momentum of the electron, time $t$ is measured in units of $\omega^{-1}$, where $\omega = 2\pi c/\lambda$ and $\lambda$ are the laser frequency and wavelength, electron velocity $v$ is measured in the units of speed of light $c$, $\sigma$ is the dimensionless (measured in units $mc^2/\hbar$) longitudinal charge separation field. Besides, as discussed above, we introduce the factor (4) to account for small quantum corrections. Unless stated otherwise, we assume that the laser field is circularly polarized (CP), so that its dimensionless amplitude can be expressed as $a_\perp \equiv E_0/mc\omega = \{0, a(\varphi) \cos \varphi, a(\varphi) \sin \varphi\}$, where $\varphi = \omega(t - x/c)$ is the phase and $a(\varphi)$ is the carrier envelope. According to [42], in the specified setup the validity of equation (5) for a cold transparent plasma is restricted only by the assumptions that the propagating wave is of ultrarelativistic intensity ($a(0) \gg 1$) and the transverse component of the RF force can be considered a small perturbation to the transverse Lorentz force. The latter condition enables us to use the approximation $u_\perp \approx a(\varphi)$ (however, we take into account the change of direction of $u_\perp$ due to RF).

The first term on the right-hand side of equation (5), proportional to the slope of the wave envelope, is the relativistic ponderomotive force [50]. The second term arises indirectly due to RF as explained below [42, 43, 51–53]. Notably it is positive, meaning that in presence of RF the electrons are pushed forward harder. As a consequence, the longitudinal charge separation field in the plasma is enhanced. To understand such an apparently counter-intuitive behavior, assume that the carrier envelope is flat ($a(\varphi) = \text{const.}$), so that the ponderomotive force vanishes. If, furthermore, RF is neglected, then in a reference frame where the electron is at rest, on average its trajectory is a circle perpendicular to the laser propagation direction, see figure 1(a). In this case the velocity is parallel to the magnetic field, hence the Lorentz force $-e(v \times B)/c$ in equation (1) equals zero and there is no longitudinal acceleration. However, when RF is taken into account, it modifies the transverse electron motion. In particular, it turns the electron velocity to form an angle with the magnetic field as shown in figure 1(b). The resulting longitudinal component of the Lorentz force accelerates the electron forward (out of the page of the drawing). It is worth to emphasize that the longitudinal component of RF force is directed opposite to $v_x$, i.e. backwards (as expected for a friction force). Nevertheless, the resultant of the longitudinal Lorenz and the RF forces turns out to be positive.
Consequently, the energy gain from the electromagnetic wave is enhanced due to RF and overcomes the energy loss caused by radiation.

We have already studied [42, 43] how this effect enhances the generation of longitudinal waves in an extended plasma slab, assuming the ions were so heavy that their motion could be neglected (see [43] for details). On the contrary, here we show how it can be applied for ion acceleration. We consider an intense laser pulse, incident normally on a thin transparent hydrogen foil, with RF taken into account, and develop a 1D model capable for estimating the energy of accelerated ions for given laser and target parameters. We justify our model by PIC simulations and demonstrate that such a radiation induced acceleration (RIA) mechanism can produce ions with considerably higher energy than RPA.

In the present paper we consider a laser pulse that is strong enough to separate charges in the target completely. This regime is quite similar to a directed Coloumb explosion (DCE) considered as a valuable alternative mechanism for ion acceleration [13, 54–58]. However, in our case the ions are accelerated via attraction to the electrons dragged by the laser pulse, whereas in DCE the lighter ions are accelerated via repulsion of heavier ones in a structured target with laser pulse serving only to remove most of the electrons from the target.

2. Analytical model

On hitting the target, a laser pulse of ultrarelativistic intensity pushes the electrons forward, instantly accelerating them almost to the speed of light. Snapshots from a typical 1D PIC simulation of a pulse incidence on an underdense plasma foil, with and without RF taken into account, are presented in figure 2 (the details of the numerical approach are given in appendix B). One can see that with RF taken into account much more electrons are captured accelerating inside the laser pulse. As a consequence, the longitudinal charge separation field is also substantially stronger than in figure 2(b).

In order to study the ion motion, let us assume the target is initially so thin, that for a time being the majority of electrons is moving inside the laser pulse, remaining completely separated from the ions accelerating behind the pulse (see figure 2) and tweak the model (5) accordingly. First, since spatial distribution of the electrons is essential for ion acceleration, it is more convenient to rewrite equation (5) in the Eulerian form by implying that \( u = u(t, x) \) and \( d/dt = \partial/\partial t + v \partial/\partial x \), where the spatial variable \( x \) is normalized by \( c/\omega \). Second, due to initial neutrality of the target, inside the pulse the longitudinal charge
separation field $\sigma(t, x)$ in 1D coincides with the areal density of the charge located to the right of $x$,

$$\sigma(t, x) = \int_{x}^{\infty} \bar{n}(t, x') dx', \quad (6)$$

where $\bar{n} = n/n_c$ and $n_c = m_e \omega^2 / 4\pi e^2$ is the plasma critical density.

We assume that laser and plasma parameters are such that the ions are accelerated up to ultrarelativistic energies. This implies that, except for a short initial stage of interaction which we ignore, the longitudinal motion of electrons is also ultrarelativistic, i.e. $\gamma \approx u_c \gg u_\perp \approx a_1 + v_x \approx 2$ and

$$1 - v_x \approx \frac{a_1}{2u_c^2}. \quad (7)$$

Then, transforming from $(t, x)$ to the more convenient variables $(t, \varphi)$, we obtain

$$\frac{\partial u_c}{\partial t} + \frac{\partial}{\partial \varphi} \left( \frac{a_1^2}{2u_c^2} \right) g_{\varphi} - \frac{\mu_0 a_1^6}{4u_c^2} + \sigma = 0. \quad (8)$$

With the same notations and assumptions the continuity equation takes the form

$$\frac{\partial \sigma}{\partial t} + \frac{\partial \sigma}{\partial \varphi} \frac{a_1^2}{2u_c^2} = 0. \quad (9)$$

Under the above assumptions ion acceleration is governed by the total electron areal density inside the laser pulse

$$\sigma_p(t) = \sigma(t, \varphi = T) = \int_{-\infty}^{T} \bar{n}(t, \varphi') d\varphi', \quad (10)$$

where $T = \omega \tau_{\text{pulse}}$ is the dimensionless pulse duration. Indeed, in our model, after leaving the laser pulse the electrons mix up with the ions, hence cease to exert further influence on ion acceleration (for the RIA case this assumption is revised in the next section). Then the distribution of the dimensionless charge separation field behind the pulse at the moment $t$ can be written as

$$a_c(t, x) \sim \sigma_p(t) - \int_{x}^{x_R} \bar{n}(t, x') dx', \quad (11)$$

where $x_R$ is the position of the rightmost ion. It attains a maximum $\sigma_p(t)$ at $x_R$ and vanishes inside the ion cloud, see figure 2. In particular, the maximal longitudinal momentum and energy of the ions (i.e. at $x_R$) are given by

$$u_{i,\text{max}}(t) = \frac{m_e}{m_i} \int_{0}^{t} \sigma_p(t') dt', \quad E_{i,\text{max}} = m_e c^2 \sqrt{1 + u_{i,\text{max}}^2}, \quad (12)$$

where $m_i$ is the ion mass.

To find the accelerating field $\sigma_p(t)$, we consider two concurring mechanisms of charge separation [42, 43] in a transparent thin foil. Namely RIA, when the charge separation is governed by the third (RF induced) term in (8), and ponderomotive acceleration (PA), when it is governed by the second (ponderomotive) term.

In the PA case we neglect the third term in (8) and assume that $a(\varphi) = a_0 \alpha(\varphi/T)$, where the function $\alpha(\zeta)$ is such that (i) it rapidly vanishes for $|\zeta| \gtrsim 1$, and (ii) $\alpha(0) = 1$. By seeking a solution of the resulting equations in the form $u_c = a_0 \sqrt{T/T_0} \alpha(\varphi/T)/g(\varphi/T)$, $\sigma = a_0 \alpha(\varphi/T)/(2\sqrt{T_0})$, they transform to

$$\nu = \frac{g'}{g}, \quad \nu' g^2 - \alpha^2 \nu = 0. \quad (13)$$

Since they no longer contain large or small parameters, even without solving them we can claim that for an arbitrary pulse shape the charge captured inside the pulse can be estimated up to a numerical factor as

$$\sigma_p^{(\text{PA})}(t) = \sigma(t, \varphi = T) \sim \frac{a_0}{2\sqrt{T_0}}, \quad t > t_0 = \frac{a_0^2}{4T_0\sigma_0^2}. \quad (14)$$
Figure 3. The total charge of electrons, captured inside the laser pulse, vs time. Solid lines-1D PIC simulations, dashed lines-estimates (20) and (14). Laser and target parameters are the same as in figure 2.

The moment $t_0$ from which the estimation (14) becomes valid, is determined by the condition $\sigma_p^{(PA)}(t_0) = \sigma_0$, where

$$\sigma_0 = \frac{\omega d n_0}{c n_c}$$

is the initial areal density of the target, $n_0$ and $d$ are its initial density and thickness.

Figure 3 shows that, with RF taken into account, the acceleration process splits into two stages. On the initial stage the charge captured inside the pulse is conserved, meaning that all the electrons from the target are moving inside the laser pulse. However, since they are slower than the pulse, they gradually drift from its front to the back. Eventually, at some moment $t_{bd}$ when the electrons reach the region of the laser field not strong enough to balance the electrostatic field, a breakdown occurs [42, 43]. On the second stage the electron population is leaking away from the back of the pulse and the ion acceleration saturates.

In the case of RIA, we neglect the second term in (8) and estimate the longitudinal four-velocity of electrons inside the pulse by balancing the terms corresponding to the electrostatic force and the RF induced longitudinal Lorentz force:

$$u_x \sim a_3^2 \sqrt{\frac{\mu g}{\sigma}}.$$  \hspace{1cm} (16)

For $t < t_{bd}$ the value of the longitudinal field at the position of the leftmost electron inside the pulse is $\sigma_0$. Until the breakdown the leftmost electron propagates through the pulse and acquires the phase $\varphi = T$ [42]. Therefore the breakdown time can be obtained from the condition

$$\varphi(t_{bd}) = \int_0^{t_{bd}} (1 - v_x) dt \sim T.$$  \hspace{1cm} (17)

In the ultrarelativistic case under consideration (for the analysis of assumptions made in the derivation see appendix A) the integral can be evaluated by combining ((7) and (16)) and the estimate $a \sim a_0 \equiv a(0)$ for the laser field amplitude inside the pulse (the latter assuming the field is still strong enough to counteract the charge separation and RF induced longitudinal forces). Finally for the breakdown time we obtain

$$t_{bd} \approx \frac{\mu g a_0^4 T}{2 \sigma_0}.$$  \hspace{1cm} (18)

The meaning of this condition is precisely that the breakdown occurs when the leftmost electrons get to the back of the pulse.

For $t > t_{bd}$ the charge inside the pulse can be calculated by incorporating (16) into the continuity equation (9). After separating the variables assuming $\sigma(t, \varphi) = f(t) h(\varphi)$, we obtain a solution

$$\sigma \approx \frac{\mu g}{2t} \int a^4 d\varphi.$$  \hspace{1cm} (19)

By estimating $\int_{-\infty}^{T} a^4 d\varphi \sim a_0^4 T$, we arrive at the expression for a charge captured inside the laser pulse

$$\sigma_p^{(RIA)}(t) \sim \begin{cases} \sigma_0, & t < t_{bd} \\ \mu g a_0^4 T / 2t, & t > t_{bd}. \end{cases}$$  \hspace{1cm} (20)
Let us estimate the value of the quantum parameter \( (3) \). In the ultrarelativistic limit

\[
\chi \sim \frac{\mu g}{\alpha} \gamma a_0 (1 - v_x).
\]

Employing \((7)\) and \((16)\) and setting \( g = 1 \) as a zeroth order approximation we get

\[
\chi \sim \frac{\sqrt{\sigma_0 \mu}}{\alpha} = \sqrt{\frac{2 \hbar \omega \sigma_0}{3 m c^2 \alpha}}.
\]

It is worth noting that \( \chi \) is determined solely by the initial areal density \( \sigma_0 \) of the foil and does not depend on the laser strength (as long as the ultrarelativistic approximation is valid, see appendix A). This is in contrast to another example of RF driven laser-plasma phenomenon, the inverse Faraday effect [59], where quantum corrections grow with laser intensity tending to suppress the effect [60].

For \( n = 0.5n_c \) \( (\sigma_0 = \pi) \) used for 1D numerical simulations in this paper \( \chi \approx 0.026 \) and \( g(\chi) \approx 0.84 \). Estimates \((14)\) and \((20)\) for the time dependencies of the foil areal density in PA and RIA cases are in a good agreement with simulations results, see figure 3.

To justify the consistency of our analysis let us inspect the parameter range for which either one of the second or the third terms in the right-hand side of \((8)\) can be neglected. Using the same estimate \( \frac{d \sigma^2}{d \varphi} \sim \frac{a_0^2}{T} \) as above, we can see that the RF induced term dominates over the ponderomotive term if

\[
\mu g \sigma_0 a_0^2 T^2 > 1.
\]

The assumptions underlying our models are collected and discussed in appendix A. The resulting conditions \((A.1)\), \((A.4)\)–\((A.6)\) are fulfilled for the parameters used in the paper.

3. Results

According to the developed model, the time dependence of the maximal ion energy for RIA can be estimated by substituting the value of the electron areal density inside the laser pulse \((20)\) into \((12)\)

\[
\mathcal{E}_{\text{max}}^{(\text{RIA})}(t) \approx m_e c^2 \begin{cases} \sigma_0 t, & t < t_{bd}, \\ \mu g a_0^2 T \left(1 + \ln \frac{t}{t_{bd}}\right), & t > t_{bd}, \end{cases}
\]

where we assume for simplicity that ions are ultrarelativistic, \( \mathcal{E}_{\text{max}} \approx u_{\text{max}} \). In the PA case, using \((14)\) we obtain

\[
\mathcal{E}_{\text{max}}^{(\text{PA})}(t) \approx m_e c^2 a_0 \sqrt{\frac{t}{T}}.
\]

Estimates \((24)\) and \((25)\) are in a good agreement with 1D PIC simulations, see figure 4(a).

Let us also estimate an average ion energy \( \langle \mathcal{E}_i(t) \rangle \), which might be considered a more meaningful indicator of the acceleration process. Due to charge neutrality of the target, the charge of accelerating ions coincides with the charge of electrons captured inside the laser pulse, \( \sigma_i(t) \sim \sigma_p(t) \). Hence the energy increment of all ions over the duration \( \Delta t \) per cross section \( S \) can be written as \( d\mathcal{E}_i(t)/S = \frac{\sigma_p(t)}{2} \mathcal{E}_i(t) dt \).
where the accelerating field \( \alpha_0(t, x) \) averaged over \( x \) is estimated to be \( \sigma_p(t)/2 \). Integrating and dividing by the total number of ions \( \sigma_0 S \), we estimate the average ion energy as

\[
\langle \mathcal{E}_i(t) \rangle \sim \frac{\int_0^t \sigma_0^2(t') \, dt'}{2\sigma_0}.
\]

Substituting (20) and (14) into (26), we get

\[
\langle \mathcal{E}^{(\text{RIA})}_i(t) \rangle \sim \begin{cases} 
 m_e c^2 \frac{\sigma_0 t}{2}, & t < \tau_{bd}, \\
 m_e c^2 \frac{\mu g_0 d_0 T}{4} \left( 2 - \frac{\tau_{bd}}{t} \right), & t > \tau_{bd}
\end{cases}
\]

and

\[
\langle \mathcal{E}^{(\text{PA})}_i(t) \rangle \sim m_e c^2 \frac{a_0^2}{8\sigma_0 T} \ln \frac{t}{t_0}.
\]

In the RIA case, according to (27), for \( t < \tau_{bd} \) we have \( \langle \mathcal{E}^{(\text{RIA})}_i(t) \rangle \sim \mathcal{E}^{(\text{RIA})}_{i, \text{max}}(t)/2 \). This is natural, since all the particles participate in acceleration and the accelerating field varies from its maximum value \( \sigma_0 \) to zero. However, for longer \( t \) the estimate (27) is not accurate enough, because after the breakdown the electrons are accelerating leftwards creating a negative longitudinal field, which decelerates the ions, see figure 4(b). The estimate can be refined by taking into account that acceleration stops at \( t = 2\tau_{bd} \), when a half of the electrons have left the laser pulse, passed through the ion cloud and balanced the quasistatic accelerating field of the electrons remaining in the pulse. This way the corrected final averaged ion energy is estimated as

\[
\langle \mathcal{E}^{(\text{RIA})}_i(t) \rangle_{\text{lim}} \sim m_e c^2 \frac{3\mu g_0 d_0 T}{8}, \quad t > 2\tau_{bd}.
\]

As shown in figure 5, the resulting estimates (27)–(29) are in a good agreement with 1D PIC simulations. The results for linear polarization can be obtained by a substitution [43] \( a_0 \rightarrow a_0/\sqrt{2} \). One can also see that for strong laser pulses RIA is much more efficient than PA due to a stronger scaling with \( a_0 \).

Comparing (28) and (29), we get again the condition (23) for the dominance of RIA. Using this condition, we find that RIA should dominate over PA if \( I \gtrsim 10^{23} \text{ W cm}^{-2} \) for \( T = 35 \) (FWHM 30 fs) and \( \sigma = \pi \). This is in agreement with the simulation results in figure 5(b). For lower intensities, the ponderomotive force is stronger than the RF induced longitudinal force, in accordance with the results of simulations including RF which follow the estimate (28) for PA, see figure 5(b).

Furthermore, RIA can outperform in terms of the average ion energy the LS acceleration mechanism, which is commonly accepted as favorable for intense lasers and almost unaffected by RF [36], see figure 5(a) corresponding to the rightmost dots in figure 5(b). Let us discuss this point in more detail. According to a conventional 1D model for LS acceleration [19], the energy of the laser pulse is almost entirely converted into the energy of the ions. By taking into account that the optimal foil area density for LS is \( \sigma_0 \sim 2a_0 \) [20], this is equivalent to the scaling

\[
\langle \mathcal{E}^{(\text{LS})}_i \rangle \sim m_e c^2 \frac{\sigma_0 T}{2} \sim m_e c^2 \frac{a_0 T}{2}.
\]
Figure 6. 2D PIC simulation results for ion acceleration in a transparent target: charge density (color scale) and laser field (grey scale) distributions at \( t = 127 \) fs with (a) and without (b) RF; insets: sectional view along \( y = 0 \). Laser pulse parameters: \( I = 10^{14} \) W cm\(^{-2}\), FWHM pulse duration 30 fs, waist radius \( w = 6\lambda \), circular polarization; target density \( n_0 = 16n_c \), thickness \( d = \lambda \), initial position \( x_0 = 30\lambda \).

Figure 7. Ion spectra for RIA and PA at \( t = 1440 \) fs; inset: average ion energy for RIA and PA vs time. Laser and target parameters are similar to figure 6.

Then, by comparing (30) to (29), we conclude that RIA can outperform LS for

\[
\mu g a_0^2 \gtrsim \frac{4}{3},
\]

i.e. for \( a_0 \gtrsim 500 \) or \( I \gtrsim 7 \times 10^{23} \) W cm\(^{-2}\) assuming \( \lambda = 1 \) \( \mu \)m. This agrees very well with the numerical simulations, see figure 5(b).

It is worth noting that our comparison with LS is based on a 1D LS model developed in [19]. Employing multidimensional effects [23, 38], multilayer targets [22, 26, 27] or other ingenious methods to further boost the ion energy are not considered here. They are outside the scope of the present paper.

Let us discuss the difference between LS and RIA mechanisms. First, the LS spectrum is close to monoenergetic, while RIA spectrum is flat, see the inset in figure 5(a). The shape of the spectrum is important for the applications including the medical ones [13, 14, 27]. Second, the areal density of accelerated ions is of the order of \( \sigma_0 \) in both cases. However, this value and therefore the amount of accelerated ions is higher for LS. Third, in contrast to LS, in the RIA case only a small fraction of laser energy is transferred to particles (as is seen, e.g. from that the Gaussian laser profile in figure 2(a) remains undistorted, see also appendix A). Nevertheless, being distributed among much fewer particles (as compared to LS), this can still provide higher average ion energy.
In order to test RIA under more realistic conditions, we also performed 2D simulations with a focused laser pulse (waist radius \( w = 6\lambda \)), see figure 6. Here, the ponderomotive force expels the electrons in transverse direction, thus reducing the charge separation field \([42]\), see Figures 6(a) and (b). Besides, the laser field amplitude falls with time due to diffraction of the pulse. Finally, if the separation distance between the electron and ion layers becomes larger than their transverse width, then in 2D the acceleration force decreases. These unfavorable effects for RIA are partially compensated by choosing the value of the target charge areal density \( \sigma_0 \) higher than in 1D case. This reduces the breakdown time \( \tau_{bd} \) with respect to the transverse expansion and laser diffraction times and also reduces the distance between the electron and ion layers. As a result, RF still allows more electrons to propagate inside the pulse (compare figures 6(a) and (b)), thus increasing both the ions energy and the total number of accelerated ions, see figure 7. However, the estimates (24), (25) and (27)–(29) are no longer accurate quantitatively but should be refined by the correct consideration of all the 2D effects.

4. Conclusions

We examined the impact of RF on ion acceleration by strong laser pulses in a thin transparent foil. For each of the alternative acceleration mechanisms (RIA and PA, based on pushing the electrons forward either by the RF induced longitudinal Lorentz force, or by the ponderomotive force, respectively), we elaborated a proper 1D analytical model. As shown in figures 4(a) and 5, each model is in excellent agreement with 1D PIC simulations. By using either (27)–(29) or 1D PIC simulations, we identified the parameter regions where RF essentially enhances ion acceleration and RF induced acceleration can be favorable among the alternative acceleration mechanisms. The effect remains pronounced in multi dimensions, though in this case its accurate quantitative description is a complex separate problem still to be solved.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Appendix A. Limits of applicability of the RIA model

Let us briefly review the assumptions, which we made in order to consider electrons dynamics in the RIA case.

- First, we used the quantum corrected classical approximation for RF, which implies that the quantum parameter \( \chi \ll 1 \) \([31, 46]\). It leads to a restriction for the initial areal density \( \sigma_0 \) of the target

\[
\sigma_0 < \frac{\alpha mc^2}{\hbar \omega}.
\]  

(A.1)

To verify the consistency of the approximation, we made a quantum Monte-Carlo (MC) simulation \([61]\) with the same parameters as in figures 2–4. Figure A1 demonstrates that the quantum corrected Landau–Lifshitz (LL) and MC approaches provide very similar results, as expected. We also made a simulation using RF force without a quantum correction. The result for the maximum ion energy is approximately 15% higher as compared to the quantum corrected simulation. This very well corresponds to the value \( g(\chi) \approx 0.84 \).
Appendix B. Numerical approach

For numerical simulations we used the PIC code SMILEI \[61\], which allows to make calculations with RF modeled by LL force \(F_{RF}\) (2), quantum corrected force \(\dot{g}(\chi)F_{RF}\), fully quantum MC algorithm \[49\] and also

### Figure A1

Ion spectra with RF taken into account calculated by classical LL approach, quantum corrected approach (cLL) and fully quantum MC approach at \( t = 2h_{bd} \approx 9\) ps, \( n_0 = 600 (I \approx 10^{18} W cm^{-2})\); CP, FWHM 30 fs, target density \( n = 0.5n_c\), thickness \( d = \lambda = 1\) pm.

- Second, we assumed that the first and second terms in LL expression \[45\] for a classical RF force

\[
F_{RF} = \frac{2e^2}{3mc^2} \gamma \left\{ \left( \frac{\partial}{\partial t} + (v \cdot \nabla) \right) E + \frac{1}{c} \left[ v \times \left( \frac{\partial}{\partial t} + (v \cdot \nabla) \right) B \right] \right\} \\
- \frac{2e^2}{3mc^2} \gamma v \left\{ \left[ E \times B \right] + \frac{1}{c} [B \times (B \times v)] + \frac{1}{c^2} E(vE) \right\} \\
- \frac{2e^2}{3mc^2} \gamma^2 v \left\{ \left[ E + \frac{1}{c} (v \times B) \right]^2 - \frac{1}{c^2} (E^2) \right\}
\]

are much smaller than the third term, which we solely left in equation (1). Indeed, contributions from the first and second terms in the dimensionless form can be written as

\[
f_{i+2}^{RF} = \mu (1 - v_x) \left[ \gamma \alpha (1 - v_x) + a^2 \right],
\]

which is \( a^2 \gg 1 \) times smaller than the RF-induced term in equation (5).

- Third, we assumed that electron longitudinal motion is ultrarelativistic, i.e. \( u_x \gg a \). Using (16) with \( \sigma \sim \sigma_0 \) and \( a \sim a_0 \), this is equivalent to

\[
\mu g a_0^2 \gg \sigma_0.
\]

- Fourth, we assumed that \( u_z \approx a_0 \), i.e. that the transverse component of the RF force is much smaller than the Lorentz force \[42, 43\]. This means that \( \mu g a_0^2 \gamma^2 (1 - v_z) v_z \ll a_0 \), or, using (7) and (16) with \( \sigma \sim \sigma_0 \),

\[
\mu g a_0^2 \sigma_0 \ll 1.
\]

Note that this condition at the same time ensures that the total energy acquired by ions

\[
\frac{\epsilon_{tot}^R}{\epsilon_{tot}^R} \sim \sigma_0 S \langle \epsilon_{i}^{RIA} \rangle_{lim} \sim mc^2 \mu g a_0^2 T \sigma_0 S
\]

is much smaller than the energy of the laser pulse \( \epsilon_{las} \sim mc^2 a_0^2 TS \), i.e. that the approximation of undistorted external field is valid.

- Finally, we neglected the initial stage of acceleration, when the electron longitudinal velocity is not yet close to the speed of light. The duration of this stage \( t_{e,acc} \) can be estimated using (5) by neglecting all the terms in the rhs except for the RF induced term and assuming \( v_x \ll 1 \). From \( du_x/dt \sim \mu g a_0^2 \) we obtain \( u_x(t_{e,acc}) \sim \mu g a_0^2 t_{e,acc} \) and, by equating it to \( a_0 \), get \( t_{e,acc} \sim 1/\mu g a_0^2 \). This acceleration time should be much smaller than the breakdown time, \( t_{b,d} \ll b_{bd} \), which is equivalent to

\[
\frac{\mu g a_0^2 T}{\sigma_0} \gg 1.
\]
without RF. Our results are obtained with quantum corrected RF force. The interpolation formula for \( g(\chi) \), implemented in SMILEI (see (3) in [34]) provides the accuracy of 2\% for arbitrary \( \chi \).

In all the simulations we used hydrogen targets and laser pulses with the wavelength \( \lambda = 1 \mu m \) and a Gaussian temporal profile \( \phi(t) = a_0 \exp[-(t - 4T)^2/T^2] \). The dimensionless pulse duration \( T \) is related to FWHM duration by \( T \approx 0.57\sqrt{\omega_{FWHM}/c} \). For 2D simulations we used laser pulses with a supergaussian spatial profile, see [62] for the details of implementation of arbitrary pulse shapes in SMILEI. The intensity distribution in a focal plane was of the form \( I \sim \exp[-(y/w)^2] \), where the laser pulse waist \( w = 6a \) was taken.

For 1D simulations we used 1000 cells per wavelength to resolve the limit \( \Delta x < \lambda/a_0 \), established in [63]. Note that [63] did not take RF into account. However, we checked that the results are similar for \( \Delta x = \lambda/1000 \) and \( \Delta x = \lambda/500 \). We took 50 particles of each type per cell for low density foils and \( n\pi c \) particles per cell for high density ones; time step was equal to a space step divided by the factor \( c/0.95 \). 2D simulations were done inside a 500.\( a \times 1283 \) box with the cell side length \( dx = dy = \lambda/100 \) in both directions. Time step was chosen as \( dt = 0.95 dx/c\sqrt{\lambda} \), and the number of particles of each type per cell was equal to 16. Target density for LS simulations satisfied the condition [20] \( l/\lambda = a_0/\pi n \). Boundary conditions for particles and fields were absorbing. The target thickness was \( d = \lambda \), and the laser was focused on its left boundary located at \( x = 30\lambda \).

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