Mapping the Conditions for Hydrodynamic Instability on Steady-State Accretion Models of Protoplanetary Disks

Thomas Pfeil and Hubert Klahr
Max-Planck-Institut für Astronomie, Königstuhl 17, D-69117 Heidelberg, Germany; pfeil@mpia.de, klahr@mpia.de
Received 2018 June 19; revised 2018 December 16; accepted 2018 December 16, published 2019 January 29

Abstract

Hydrodynamic instabilities in disks around young stars depend on the thermodynamic stratification of the disk and on the local rate of thermal relaxation. Here, we map the spatial extent of unstable regions for the Vertical Shear Instability (VSI), the Convective Overstability (COS), and the amplification of vortices via the Subcritical Baroclinic Instability (SBI). We use steady-state accretion disk models, including stellar irradiation, accretion heating, and radiative transfer. We determine the local radial and vertical stratification and thermal relaxation rate in the disk, which depends on the stellar mass, disk mass, and mass accretion rate. We find that passive regions of disks—that is, the midplane temperature dominated by irradiation—are COS unstable about one pressure scale height above the midplane and VSI unstable at radii >10 au. Vortex amplification via SBI should operate in most parts of active and passive disks. For active parts of disks (midplane temperature determined by accretion power), COS can become active down to the midplane. The same is true for the VSI because of the vertically adiabatic stratification of an internally heated disk. If hydrodynamic instabilities or other nonideal MHD processes are able to create α-stresses (>10⁻⁵) and released accretion energy leads to internal heating of the disk, hydrodynamic instabilities are likely to operate in significant parts of the planet-forming zones in disks around young stars, driving gas accretion and flow structure formation. Thus, hydrodynamic instabilities are viable candidates to explain the rings and vortices observed with the Atacama Large Millimeter/submillimeter Array and Very Large Telescope.

Key words: accretion, accretion disks – hydrodynamics – instabilities – methods: numerical – protoplanetary disks

1. Introduction

Angular momentum transport and the associated accretion process in protoplanetary disks are driven by either winds (Wardle 1997; Pudritz et al. 2007; Königl et al. 2010; Bai & Stone 2013) or magnetic and hydrodynamic turbulence (Luest 1952; Shakura & Sunyaev 1973; Balbus & Hawley 1991). One of the considerable physical processes causing outward transport of angular momentum is the Magnetorotational Instability (MRI), which requires a sufficiently ionized shear flow in addition to weak magnetic fields. This linear instability works well in accretion disks of high temperature around black holes or neutron stars. Large parts of protoplanetary disks, however, have low ionization rates and gas densities outside ~0.3 au; thus, nonideal MHD effects, namely resistivity and ambipolar diffusion (Lesur et al. 2014; Gressel et al. 2015), largely hamper the MRI and thereby open a venue for hydrodynamic instabilities, as explored by Lyra & Klahr (2011).

These hydrodynamic mechanisms include the Subcritical Baroclinic Instability (SBI) (Klahr & Bodenheimer 2003; Petersen et al. 2007a, 2007b; Lesur & Papaloizou 2010), the Convective Overstability (COS) (Klahr & Hubbard 2014), which can be interpreted to be the linear phase of the SBI mechanism (Lyra 2014), and the Vertical Shear Instability (VSI) (Urpin & Brandenburg 1998; Urpin 2003; Nelson et al. 2012; Lin & Youdin 2015), which is the protoplanetary disk equivalent of the Goldreich–Schubert–Fricke Instability (Goldreich & Schubert 1967; Fricke 1968) in stars.

The radial and vertical stratification of the disk in temperature and density and the thermal relaxation timescale determine whether these instabilities can exist. For infinite cooling times, the stability constraints are given by the standard Solberg–Høiland criteria (Rüdiger et al. 2002). Malygin et al. (2017) have investigated detailed models of the radiative properties of a simple, nonaccreting power-law disk profile, identifying necessary conditions for the onset of instability by mapping where the infinite cooling time condition is sufficiently violated.

In this article, we replace the power-law disk models with a self-consistent 1+1D accretion disk model that allows us to determine the nontrivial temperature and density stratification of the gas as a result of gas accretion (Meyer & Meyer-Hofmeister 1982; Bell et al. 1997) and stellar irradiation (D’Alessio et al. 1998). Although the surface temperature of a disk can usually be nicely approximated by a power law, the midplane temperature can have a more complicated structure with varying gradients, reflecting the local optical depth and the rate of viscous heating (Bell et al. 1997; D’Alessio et al. 1998).

Gas accretion and the associated heating are assumed to be the result of an unspecified angular momentum transport mechanism, which could be turbulent viscosity or winds, described by the α-parameter (Shakura & Sunyaev 1973), which for our model determines the amount of thermal energy that is released inside of the disk.

A passive disk as defined here has a temperature structure in the planet-forming regions dominated by stellar irradiation (which is in our models the case for α ≲ 10⁻⁵), and an active disk is dominated by accretion heating in the planet-forming zone (in our models for α > 10⁻⁵).

The goal of this article is to learn, first, whether a passive disk is able to develop hydrodynamic instabilities and, second, whether the energy release from resulting mass accretion triggered by the instabilities will support or suppress the instabilities.

On the other hand, the mass accretion rate can also be thought to be the result of either another instability associated...
with nonideal MHD effects, like a strongly suppressed MRI (which would otherwise quench the instabilities we are aiming to investigate), or can be produced by magnetically driven disk winds or Hall MHD, also leading to some heating of the disk. Also, for that case, it would be interesting to know whether conditions for additional hydrodynamic instabilities could be given.

A prediction of the physical conditions inside of a protoplanetary disk makes it possible to determine where the necessary criteria for an instability are met and how fast the corresponding linear perturbations should grow with time. This is crucial to study the nature of angular momentum transport in circumstellar disks and to set up simulations of hydrodynamic instabilities under realistically modeled physical conditions (i.e., radial and vertical stratification).

The weak hydrodynamic instabilities are also of special interest for planet formation theory, because even if their contribution to angular momentum transport might be small, they drive the formation of nonlaminar flow features, such as zonal flows and vortices. Such pressure maxima are able to accumulate the inward-drifting dust particles and could, therefore, be the birthplaces of planetesimals and planets (Barge & Sommeria 1995; Klahr & Bodenheimer 2006). Such structures are observed lately in circumstellar disks by the Atacama Large Millimeter/submillimeter Array (ALMA) and the Very Large Telescope (VLT; van der Marel et al. 2013; Carrasco-González et al. 2016). For a recent review on the role of nonlaminar flow features on planetesimal formation, we refer to Klahr et al. 2018. Hartmann & Bae (2018) revisited the physics of the accretion process itself and found that even a relatively low \( \alpha \gtrsim 10^{-2} \) is enough to explain the observed accretion rates onto T-Tauri stars. They conclude that the hydrodynamic contribution to angular momentum transport might have been underestimated in the past and that, in some cases, hydrodynamic turbulence might even be more important to the accretion than magnetic contributions originating from the MRI.

In the following section, we give an overview of the basic physics of the investigated instabilities, their analytical growth rates, and the concepts of thermal relaxation used in the scope of this work. The 1+1D disk model and the used opacity model are described in Section 3. The general influence of the disk structure on stability, as well as the spatial distributions and growth rates of the introduced mechanisms, are presented in Section 4. Our stability maps, shown in Section 4.6, sum up the gained knowledge of the distribution of the susceptible regions for the investigated instabilities and parameter sets. One example of such a map is shown in Figure 1, for a disk model with input parameters \( M_{\text{disk}} = 0.1 M_\odot \), a moderate local viscous heating of \( \alpha = 10^{-3} \), and a central star of \( M_\star = 1 M_\odot \). We finally summarize and conclude in Section 5.

### 2. Instabilities

The hydrodynamic instabilities discussed here arise either from vertical shear or from the radial buoyancy in the disk. Both are linked to the radial gradient of temperature.

The VSI is a special case for the violation of the Rayleigh criterion—that is, by moving upward, gas can move also outward under conservation of the specific angular momentum, which leads to a release of kinetic energy. The COS and its weakly nonlinear extension the SBI are both special cases of thermal convection in the radial direction of the disk (i.e., of a radial superadiabatic stratification).

Another hydrodynamic effect creating vortices in disks is the so-called Zombie Vortex Instability (ZVI; Marcus et al. 2015; Umurhan et al. 2016b), but we will not include it in this investigation, because it is a nonlinear instability, making predictions on its occurrence more complicated than those for the linear instabilities we study here. A study by Lesur & Latter (2016) suggests that the instability may occur in the inner optically thick regions of disks (\( R < 0.3 \) au), where thermal relaxation takes long enough to allow for the necessary vertical internal gravity waves for the ZVI to operate. In contrast, Barranco et al. (2018) argue that, at larger distances from the star (\( R > 2 \) au or \( R > 16 \) au, depending on the assumptions of dust growth and settling) at sufficient height above the midplane (\( z > 1.5 \) \( R \)), dust might thermally decouple from the gas, allowing the ZVI to operate, provided gas opacities are small (see the discussion in Malygin et al. 2017). Barranco et al. (2018) also find in numerical experiments that even in cases where ZVI operates only above \( z > 1.5 \) \( R \), some turbulence still reaches the midplane. In summary, the conditions for ZVI are currently more a problem of the proper dust size and gas opacity model, rather than the influence of accretion onto the temperature gradients in the disk, which is the actual topic of our article. We therefore abstain from mapping out likely ZVI regions, while noting that the dust size distributions for protoplanetary disks constrained by observations combined with numerical models of the growth processes (e.g., Birnstiel et al. 2012; Estrada et al. 2016) will hopefully bring light to these outstanding issues.

---

**Figure 1.** Stability map for a solar nebula-like disk around a solar mass star with \( M_{\text{disk}} = 0.1 M_\odot \) and a moderate local viscous heating of \( \alpha = 10^{-3} \). The black lines indicate one, two, and three pressure scale heights, respectively. Vertical Shear Instability (VSI—green lines slanted to the right) can occur at larger radii, where cooling is efficient, or alternatively in the inner more optical thick parts of the nebula, which is vertically adiabatic, indicated by the possible occurrence of Vertical Convective Instability (VCI—vertical red lines). Convective Overstability (COS—horizontal blue lines) will occur in the irradiation-dominated outer parts only at a height of one pressure scale height and above, because the entropy gradient flips sign only some height above the midplane. In the inner parts of the nebula, where viscous heating is important and especially in the opacity transition zones (ice line 3 au–6 au and silicates evaporation zones 0.4 au–1.5 au), the entropy gradient is negative down to the midplane, forming a sweet spot for COS. The Convective Amplification of large-scale vortices (SBI—Gray Shaded region) can occur throughout most of the disk (0.3 au–50 au), because the surface density structure is shallower than the midplane density; thus, radial buoyancy (negative gradient of vertical integrated entropy) can much more easily be established.

---

Pfeil & Klahr

The Astrophysical Journal, 871:150 (20pp), 2019 February 1
The following sections briefly review the mechanisms for which we have predictions of linear growth rates. We want to point out that the growth rates presented in the following sections are derived for disk models with either vertical or adiabatic temperature stratification, whereas radiation transport will lead to stratifications that are somewhere between those ideal cases. The influences of disk structure and viscosity on the instabilities are further discussed in the following sections, whereas the treatment of radial flows was not incorporated in this study.

Our models do not include the effects of magnetic fields, and we are thus not able to treat their impacts on the evolution of the hydrodynamic instabilities beyond providing some heating of the disk as incorporated by $\alpha$. We explicitly assume that the nonideal MHD terms do allow for sufficient diffusion of magnetic fields; thus, hydrodynamic instabilities will not be suppressed (Lyra & Klahr 2011; Latter & Papaloizou 2018).

2.1. COS

Klahr & Hubbard (2014) considered finite thermal relaxation times ($\tau_{\text{relax}}$) in their linear, inelastic stability analysis of protoplanetary disks and found a new, thermally driven instability that can be described as radial convection on epicycles. The mechanisms can be explained as follows. An outward perturbation of a fluid element brings it into contact with a cooler surrounding because of the radial temperature gradient of the disk. While the gas parcel undergoes half an epicycle, it changes its temperature owing to heat exchange with its surrounding on a timescale $\tau_{\text{relax}}$. When it arrives at the initial radial distance to the star, its entropy is lower than the initial value, which leads to an inward acceleration due to buoyancy. On its extended half epicycle through the inner and hotter region of the disk, it undergoes an increase of temperature and entropy. When it finally arrives at the initial radius, it experiences an outward buoyancy force. This positive feedback caused by a radially buoyant stratification is called COS. The linear phase of the COS drives motions in the disk’s $R$–$z$ plane with a growth rate of

$$\Gamma_{\text{COS}} = \frac{-\gamma \tau_{\text{relax}} N_R^2}{2 \left( 1 + \gamma^2 \tau_{\text{relax}}^2 \left( \kappa_R^2 + N_R^2 \right) \right)}$$  

(Klahr & Hubbard 2014), where $\gamma$ is the heat capacity ratio of the gas, $\kappa_R$ is the radial epicyclic frequency, and $N_R^2$ refers to the square of the radial Brunt–Väisälä-frequency, which is indicating stability when positive and buoyancy-driven instability when negative. The necessary condition for COS to develop is, thus, given by a radially buoyant stratification ($N_R^2 < 0$), which is expected to be present in the optically thick regions of the disk. The growth rate’s dependency on perturbation wavenumber, therefore, enters Equation (1) via the thermal relaxation time $\tau_{\text{relax}} = \tau_{\text{diff}} \propto k^{-2}$ (see Section 2.5). The assumption of $\tau_{\text{relax}} = \tau_{\text{diff}}$ is justified in this case, because the necessary condition $N_R^2 < 0$ requires the disk to be optically thick.

The wavenumbers leading to significant growth of perturbations have to fulfill $k \gg k_R$, which implies radially elongated and vertically unextended motions ($k \approx k_c$) (Klahr & Hubbard 2014; Lyra 2014), and, thus, we estimate the influence of vertical stratification onto the growth rates as unimportant. Therefore, it is justified to use the Klahr & Hubbard (2014) growth rate, which was derived in a radially but not vertically stratified disk setup. The sign of $N_R^2$ directly depends on the radial (cylindrical) density and temperature structure of the disks in terms of

$$\beta_T = \frac{d \log (T)}{d \log (R)}$$  

$$\beta_p = \frac{d \log (\rho)}{d \log (R)},$$  

from which we derive the radial slope in pressure $\beta_p = \beta_T + \beta_T$ and specific entropy $\beta_s = \beta_T + (1 - \gamma)\beta_p$, resulting in the expression

$$N_R^2 = -\frac{1}{\gamma} \left( \frac{H}{R} \right)^2 \beta_k \beta_p \Omega^2,$$

(Raettig et al. 2013) where $H/R$ represents the disk’s aspect ratio with respect to the local pressure scale height $H$ (i.e., an expression for the temperature of the disk and $\Omega$ is the local Keplerian angular frequency). Because this overstability relies on entropy differences between perturbed fluid parcels and their surroundings, relaxation times have to be neither too small nor too large. In the first case, a fluid element would always adopt the temperature of its surroundings. Its movement would be isothermal and no buoyant force would act on it, which means that Rayleigh’s stability criterion applies to it. The latter case describes an adiabatic perturbation, where the fluid’s entropy stays constant during its epicyclic motion. This means that it follows a stable, buoyancy-adjusted epicycle (Latter 2016). The relaxation time for maximum growth of the linear phase was also calculated by Klahr & Hubbard (2014) to be

$$\tau_{\max,\text{COS}} = \frac{1}{\gamma \Omega}$$

This condition can be used to derive the instability’s maximum growing wavenumber $k_{\max,\text{COS}} = 1/\sqrt{D_E \tau_{\max,\text{COS}}}$ (see Section 2.5 Equation (23) for more detail). In this study, we are interested in the modes that fulfill this condition and thus grow fastest with a rate derived by Klahr & Hubbard (2014)

$$\Gamma_{\max,\text{COS}} = -\frac{N_R^2}{4\Omega}.$$  

Viscosity was shown to hinder the growth of COS, especially for small-scale perturbations, by Klahr & Hubbard (2014) and Latter (2016), yet realistic molecular viscosity is too low to provide an obstacle to the COS; thus, these considerations are not incorporated into our studies. The finite amplitude perturbations created by the COS can trigger its nonlinear phase, the SBI (Klahr & Bodenheimer 2003; Petersen et al. 2007a, 2007b; Lyra 2014), which amplifies existing vortices in the disk’s $R$–$\phi$ plane (see Section 2.2). These vortices are of interest for the growth of dust to planetesimals, because they are able to accumulate dust particles (Barge & Sommeria 1995).
Large anticyclonic vortices are quasi-2D structures in the $R$–$\phi$ plane of the disk. They have vertically little variation over more than a pressure scale height above the midplane (Meheut et al. 2012; Manger & Klahr 2018). Thus, to study their possible amplification in the SBI mechanism, which relies on the radial buoyancy, one has to consider their vertically integrated entropy and pressure structure. We, therefore, use the definition of a vertically integrated density $\Sigma$ and entropy $\tilde{S}$, also used in Klahr (2004) and Klahr et al. (2013)

$$\Sigma = \int_0^{\xi_{\text{max}}} \rho(R, z) dz$$

(7)

$$\tilde{S} = C_v \log(\tilde{K})$$

(8)

where we assume a polytropic equation of state for a 2D pressure $\tilde{P} = \tilde{K}\Sigma^{\gamma}$ with an entropylike potential temperature $\tilde{K}$. The heat capacity ratio has to be adjusted to a fi

$$\Sigma = \int_0^{\xi_{\text{max}}} \rho(R, z) dz$$

(7)

$$\tilde{S} = C_v \log(\tilde{K})$$

(8)

where we assume a polytropic equation of state for a 2D pressure $\tilde{P} = \tilde{K}\Sigma^{\gamma}$ with an entropylike potential temperature $\tilde{K}$. The heat capacity ratio has to be adjusted to a vertically integrated value $\gamma$, defined by Goldreich et al. (1986) as $\gamma = (3\gamma - 1)/(\gamma + 1) = 1.354$ for a typical mixture of hydrogen and helium gas ($\gamma = 1.43$). The investigation of the vertically integrated disk structure has also proven to be a useful tool in the study of planet disk interactions, where the radial entropy structure impacts the so-called horseshoe drag exerted on a planet (Baruteau et al. 2014). This work already showed that the radial entropy structure of the disk can be a nonmonotonic function with varying and sign changing gradients. We now adopt the definitions of the logarithmic gradients in column density $\beta_{\Sigma}$, vertically integrated pressure $\beta_p$, and entropy $\beta_{\tilde{S}}$ from Klahr (2004), given by

$$\beta_{\Sigma} = \frac{d \log(\Sigma)}{d \log(R)}$$

(9)

$$\beta_p = \frac{d \log(\tilde{P}\Sigma^{\gamma})}{d \log(R)}$$

(10)

$$\beta_{\tilde{S}} = \frac{d \log(\tilde{P}\Sigma^{\gamma})}{d \log(R)} = \beta_p - \gamma \beta_{\Sigma}.$$  

(11)

Using this, we find a vertically integrated, radial Brunt–Väisälä-frequency

$$\tilde{N}_R^2 = -\frac{1}{\tilde{\gamma}} \left( \frac{H}{R} \right)^2 \beta_{\tilde{S}} \beta_p \Omega^2,$$

which is formally similar to Equation (4), but can obtain significantly different values for the same radial stratification.

The growth rate for the SBI initially suggested by Lesur & Papaloizou (2010) and modified by Beutel (2012) using a COS-like stability analysis but Goodman et al. (1987) (GNG) like vortices instead of plane waves is approximately

$$\Gamma_{\text{SBI}} \approx -\frac{4\tilde{N}_R^2}{\omega(1 + \chi^2)} \left( \frac{\tilde{\gamma}}{1 + (\tilde{\gamma}/\omega_{\text{relax}})^2} \right)\frac{\omega_{\text{relax}}}{\omega(1 + \chi^2)}.$$  

(13)

To get an estimate for a vortex’ maximum growth rate, we set $\omega_{\text{relax}} = 1$ and determine the internal vortex angular frequency $\omega$ with the relation by GNG

$$\omega = \Omega \sqrt{\frac{3}{\chi^2 - 1}}.$$  

(14)

We choose the aspect ratio of the vortex (azimuthal versus radial extent) to be $\chi = 4$, which is a reasonable value for vortices that are not too affected by epicyclic instability (Lesur & Papaloizou 2009) and is typical for large-scale 3D $R$–$\phi$ vortices found in numerical simulations (Manger & Klahr 2018). Combining these assumptions with Equation (13), leads to an approximate growth rate of

$$\Gamma_{\text{SBI}} \approx -\frac{\tilde{N}_R^2}{4\Omega^2},$$

which is formally the same maximum growth rate as for the COS Equation (6), but note that the vertically “integrated” $\tilde{N}_R^2$ can significantly differ from the height-dependent $N_R^2$. This means that the local appearance of COS in a disk, which depends on $N_R^2 < 0$, is less widespread than SBI. However, if COS occurs, it can trigger the SBI when $\tilde{N}_R^2 < 0$ is additionally fulfilled. Raettig et al. (2013) confirmed the $\Gamma_{\text{SBI}} \approx -\tilde{N}_R^2$ dependency in numerical experiments.

Andrews et al. (2010) find a mean value for the radial slope of surface density in disks around young stars of $\beta_\Sigma = -0.9$. At the same time, they estimate a temperature profile of $\beta_T = -0.6$, which leads to a radially buoyant (unstable) situation with $\tilde{S} \propto R^{-0.25}$, whereas for the same radial stratification, the midplane density slope is $\beta_\Sigma = -2.1$, leading to a radial entropy slope of $S(z = 0) \propto R^{-0.3}$, which is stable. However, note that the radial density profile $\beta_\Sigma$ is a function of height $\beta_\Sigma(z) = \beta_\Sigma(z = 0) + (3 + \beta_\Sigma(z)) z^2$; thus, in the given example, starting from a height of $z = H$ and assuming a vertically constant radial temperature gradient (which is typical for the regions at large distance to the central star), one finds $\beta_\Sigma(z = H) = -0.9$. This leads to a radially declining entropy profile at one pressure scale height above the midplane with $S(z = H) \propto R^{-0.21}$ which is again unstable, a behavior that we will discuss in a later section, when we come to our models (see also the discussion in Lyra & Umurhan (2018)).

2.3. VSI

The VSI is the analog of the well studied Goldreich–Schubert–Fricke Instability (Goldreich & Schubert 1967; Fricke 1968) for protoplanetary disks. The VSI can develop if the disk has a vertical gradient in angular frequency, as well as the ability to cool sufficiently fast, to allow for vertical perturbations to develop, albeit counteracting buoyancy. The vertical shear is thus a necessary condition for the onset of VSI. It can be derived from radial hydrostatic equilibrium and depends on the vertical and radial stratification of the disk

$$\Omega(R, z) = \frac{GM_s}{(R^2 + z^2)^{3/2}} + \frac{1}{R\rho} \frac{\partial P(R, z)}{\partial R}$$

(16)

$$\frac{\partial \Omega(R)}{\partial z} = \frac{\partial \nu_\phi}{\partial z} = \frac{1}{2\Omega\rho^2} \left( \frac{\partial P}{\partial \rho} \frac{\partial \rho}{\partial z} - \frac{\partial P}{\partial \rho} \frac{\partial \rho}{\partial z} \right),$$

(17)

where $\nu_\phi$ is the azimuthal flow velocity of the gas, $P$ is the pressure and, $\rho$ gives the density. Vertically perturbed fluid parcels, which move along the curved isosurfaces of angular momentum, gain kinetic energy and circumvent Rayleigh’s stability criterion (Urpin & Brandenburg 1998). This instability therefore drives modes that are vertically elongated ($k_R/k_z \gg 1$).
(Arlt & Urpin 2004). Nelson et al. (2012), who were the first to show that the VSI can operate in protoplanetary disks, calculated the corresponding growth rate for a locally isothermal, compressible gas as under the shearing sheet approximation

$$\Gamma_{\text{VSI}}^{2} = \frac{-\kappa_{\rho}^{2}(\epsilon_{z} k_{z}^{2} + N_{z}^{2}) + 2\Omega\kappa_{R} k_{R} \frac{\partial \rho}{\partial z}}{c_{s}^{2}(k_{z}^{2} + k_{R}^{2}) + \kappa_{\rho}^{2} + N_{z}^{2}},$$  \hspace{1cm} (18)

where $N_{z}$ is the vertical buoyancy frequency, $c_{s}$ is the local sound speed, and $\Omega$ is the Keplerian angular frequency. They performed numerical simulation of the instability and found narrow, almost vertical motions, which caused $\alpha \sim 10^{-3}$. Other authors, such as Stoll & Kley (2014) and Manger & Klahr (2018), find turbulence associated with relatively low $\alpha$ values in the order of $10^{-4}$.

In this work, we are interested in the maximal growing perturbation of a certain radial wavenumber. Therefore, we use the following condition by Arlt & Urpin (2004) to get the corresponding fastest growing vertical wavenumber (see also Umurhan et al. 2016a)

$$k_{z} = \frac{k_{R} \partial_{z} j^{2}(R, z)}{2 \partial_{R} j^{2}(R, z)},$$ \hspace{1cm} (19)

where $j(R, z)$ represents the specific angular momentum. The vertically perturbed fluid parcels, which are prone to be unstable to the VSI, experience buoyant forces in the case of a stable stratification, which impede the instability’s growth. Fast thermal relaxation can overcome this obstacle, because it adjusts the fluid parcel’s temperature to the background temperature and therefore diminishes buoyancy-generating entropy differences. Lin & Youdin (2015) considered this effect and derived a critically slow relaxation timescale of $\tau_{\text{crit}} = |\partial_{z} v_{\phi}| / N_{z}^{2}$ for which the VSI can grow in a convectively stable disk. In contrast, a convectively unstable or neutral vertical stratification does not impede vertical perturbations and, thus, allows for VSI in the presence of sufficiently fast cooling. The behavior of VSI in such a polytropically stratified disk was also studied in numerical experiments by Nelson et al. (2012), who found a critical relaxation time of $\tau_{\text{relax}} \Omega \sim 10$. This leads to a necessary criterion for the onset of VSI in the convectively stable ($N_{z}^{2} > 0$) and unstable/neutral ($N_{z}^{2} < 0$) case.

$$\tau_{\text{crit}} = \begin{cases} \frac{|\partial_{z} v_{\phi}|}{N_{z}^{2}} & \text{for } N_{z}^{2} > 0 \\ 10 / \Omega & \text{for } N_{z}^{2} \leq 0 \end{cases}$$ \hspace{1cm} (20)

It is, therefore, necessary to investigate whether the vertical stratification is buoyantly neutral or unstable to convection, because the resulting change in the vertical disk structure might allow for VSI, even if cooling becomes less efficient.

It has to be noted that Equation (18) does not include the effects of viscosity, which cannot be treated extensively in this work. Lin & Youdin (2015) found that significant growth in setups including viscosity only occurs for modes with $k_{R} H \sim O(10)$. We are therefore limiting our study to a radial wavenumber in this order of magnitude to minimize potential shortcomings of the linear theory due to viscous damping in our disk models. For a full treatment of viscosity in a linear analysis, we refer to Latter & Papaloizou (2018).

### 2.4. Vertical Convective Instability (VCI)

Vertical convection is not a primary instability mechanism we study for this article. However, our 1+1D models do contain internal heating from viscosity, which in combination with the radiation transport, can generate vertical temperature stratification that becomes superadiabatic for sufficient optical depth.

Vertical buoyancy (VCI) is the consequence of such a temperature profile. Cameron (1978) first considered this to be a possible source of turbulent viscosity in protoplanetary accretion disks. Further study revealed that convection does not significantly influence the angular momentum transport but might be able to establish a vertically adiabatic stratification (Cabot et al. 1987; Klahr 2007). This makes vertical buoyancy an important aspect of the disk stability, even if it might not be able to drive turbulence in the disk, because it allows for the growth of VSI even if relaxation times exceed the value for the vertically isothermal stratification (Equation (20)).

More recent studies, such as that by Lesur & Ogilvie (2010), have revisited the possibility of vertical convection as a source of angular momentum transport in disks. They indeed found positive $\alpha$ stresses but pointed out that a continuous level of turbulence requires a steady maintenance of the unstable vertical temperature gradient, which in their work was superimposed by the input disk structure. Held & Latter (2018) point out that a secondary mechanism, such as spiral density waves due to an orbiting planet (Lyra et al. 2016; Boley & Durisen 2018) or dissipation of strong magnetic fields created by Hall MHD (Lesur et al. 2014), might be able to render the vertical entropy gradients unstable. For such a sustained unstable stratification, they also find substructures such as vortices and zonal flows emerging from convective instability. Whether the supporting mechanisms are able to provide the necessary thermal energy at the right places inside the disk is nonetheless uncertain. The main problem of vertical convection as a source of angular momentum transport, thus, remains to be the missing ability of the convective instability to self-consistently render the temperature gradient steeper than adiabatic.

The corresponding condition on the vertical stratification is given by the Schwarzschild criterion and can also be phrased in terms of a negative vertical entropy gradient (e.g., Pringle & King 2007)

$$\frac{\partial}{\partial z} C_{V} \log \left( \frac{P}{\rho^{\gamma}} \right) = \frac{\partial S}{\partial z} < 0 \quad \Rightarrow \quad \text{Instability},$$ \hspace{1cm} (21)

where $C_{V}$ represents the heat capacity at constant volume, $\gamma$ is the heat capacity ratio, and $S$ gives the specific entropy. The growth rate of the VCI can be determined via the vertical Brunt–Väisälä-frequency $N_{z}$ (Rüdiger et al. 2002)

$$\Gamma_{\text{VCI}} = \sqrt{-N_{z}^{2}} = \frac{1}{\sqrt{\gamma \rho}} \frac{\partial P}{\partial z} \log \left( \frac{P}{\rho^{\gamma}} \right).$$ \hspace{1cm} (22)

The violation of the criterion may lead to vertical convective motion, which could be treated in a mixing length model (Bell et al. 1997), or simply used to limit the the vertical temperature gradient to be adiabatic. We do not apply any of these
assumptions here for our disk modeling in the following section, because in the interplay with the other instabilities, it is unclear how vertical entropy transport will actually be established as long as the relevant numerical simulations have not been performed.

2.5. Thermal Relaxation

The thermal relaxation times of the disks’ material determine how fast a temperature perturbation decays or, in other words, how fast a spatially perturbed fluid parcel adopts the temperature of its new surroundings. This makes \( \tau_{\text{rel}} \) an important parameter for the COS (see Equation (1)), the SBI (see Equation (13)), and the VSI (see Equation (20)), because the growth of these instabilities relies on temperature differences between the perturbed flow and the background. Malygin et al. (2017) derived a detailed formalism for the calculation of the thermal relaxation times for the optically thin and for the optically thick regime in protoplanetary disks. Our model provides the necessary information to compute these timescales and, therefore, allows us to make statements about the linear growth phase of the instabilities considered here. The radiative transfer in the optically thick regime is limited by the diffusion of photons, which happens on a timescale

\[
\tau_{\text{diff}} = \frac{1}{\tilde{D}_E k^2},
\]

where \( k^2 = k_r^2 + k_z^2 \) is the wavenumber of the perturbation and

\[
\tilde{D}_E = D_E \cdot f = \frac{\zeta c}{\kappa_{\text{op}} \rho} \frac{4\eta}{1 + 3\eta}
\]

represents the effective energy diffusion coefficient, where \( \kappa_{\text{op}} \) refers to the opacity, \( \zeta \) represents the flux limiter to treat the transition from optical thick to thin properly (Levermore & Pomraning 1981), \( c \) is the speed of light, and \( \eta = E_R / (E_R + E_{\text{int}}) \) is the ratio between radiation energy density \( E_R = a T^4 \) and combined radiation and internal energy density \( E_{\text{int}} = \rho C_V T \), with the radiation constant \( a \) (see Malygin et al. 2017, Appendix A, for more detail).

For the optically thin case, most photons are able to directly leave the material (i.e., optical thin cooling). In that case, not only the radiation emitted by the dust grains but also the coupling between dust and gas will determine the loss rate of thermal energy.

The majority of thermal energy is stored in hydrogen molecules and helium atoms, which are the most abundant species in the disk. Those particles have no electric dipole moment, which makes them extremely inefficient coolants. They, therefore, have to transfer their energy to the emitting species, which are most importantly dust and ice particles. Their inner structure allows for the direct emission of infrared radiation via crystal lattice vibrations. The second timescale is therefore set by the collisional timescale for gas molecules to collide with dusty and icy grains

\[
\tau_{\text{coll}} = \frac{1}{n \sigma_c v_{\text{coll}}},
\]

where \( n \) refers to the number density of the grains, with the collisional cross section \( \sigma_c \). For this work, we adopted the estimates from Malygin et al. (2017), who estimated \( \sigma_c \approx 1.5 \times 10^{-9} \text{cm}^2 \). The thermal energy of gas particles, defined via kinetic gas theory, determines the most likely velocity for a molecule (or atom) at gas temperature \( T \)

\[
v_{\text{coll}} = v_g = \sqrt{\frac{3 k_B T}{\mu m_p}},
\]

with the Boltzmann constant \( k_B \) and mean molecular mass \( \mu m_p \). The target number density \( n \) is calculated for a constant dust-to-gas ratio of \( \epsilon_{d/g} = 0.02 \). The considered dust grains are micrometer sized, implying efficient coupling between dust and gas, which justifies the assumption of a constant \( \epsilon_{d/g} \) due to efficient mixing.

If the dust receives the kinetic energy of the gas via these collisions, its emission timescale is mainly defined by the blackbody emission rate

\[
\tau_{\text{emit}} = \frac{C_V}{16 \kappa_{\text{op}} \sigma T^3},
\]

where \( \sigma \) gives the Stefan–Boltzmann constant. Finally, the total thermal relaxation timescale of the disk’s material is determined by the slowest channel of energy transfer

\[
\tau_{\text{rel}} = \max(\tau_{\text{coll}}, \tau_{\text{diff}}, \tau_{\text{emit}}).
\]

This means that the thermal relaxation in the dense inner regions is dominated by the diffusion timescale and the regions farther away from the central star cool mostly via direct irradiation. Because temperatures decline with distance, optically thin cooling becomes less efficient as well, because the blackbody emission rate scales with \( T^{-3} \). Far above the midplane, where densities nearly reach those of typical molecular clouds, collisions become so unlikely that the energy transfer between hydrogen molecules and the emitting species becomes limited by the collisional timescale, which is completely in line with the arguments in Barranco et al. (2018), where this effect is even enhanced by dust growth and sedimentation.

3. Model

3.1. Structure Model

To calculate the physical conditions in the \( R-z \) plane of a protoplanetary disk, a 1+1D steady-state accretion disk model was used (Meyer & Meyer-Hofmeister 1982; D’Alessio et al. 1998). Comparable setups were also implemented by Bell & Lin (1994) and Bell et al. (1997) in their studies of the structure of equilibrium disks and the evolution and origins of FU-Orionis events. The model used consists of a radial series of vertical integrations, executed via finite differences in a cylindrical grid. Input parameters are the disk mass \( M_{\text{disk}} \), the \( \alpha \)-parameter, and the stellar mass \( M_\star \).

Classically, these models used the accretion rate \( \dot{M} \) as the input parameter, because this is the observable that was to be modeled, but in planet formation, we are more interested in the disk mass; thus, in a work-around, we define the desired disk mass and iteratively search for an accretion rate that is consistent with this disk mass for the given \( \alpha \) and \( M_\star \), similar to the method undertaken in Andrews et al. (2010), with the difference that they applied a 1D radial model, whereas we reconstruct the vertical structure as well. To determine the mass
of our disk model, we use an inner cutoff at 0.1 au and create an exponential truncation radius of \( R_e = 100 \text{ au} \) for all disk models, for the sake of a finite disk mass, by setting \( M(R) = e^{-R/100} \text{M}_\odot \) for the integration of the structure models, which leads to a truncation in \( \Sigma \) as a result. This means the accretion rate decreases by \( \sim 50\% \) at 60 au, thus mimicking the results of Lynden-Bell & Pringle (1974) for viscous ring spreading, implying a radial decline of accretion rates toward the truncation radius and negative accretion rates at larger radii and, thus, outside the regions we are investigating. The outward movement of material at large radii is a necessary consequence of angular momentum conservation in accretion disks and does not imply an unstable disk (where an accretion disk is always just quasi-steady by nature, which means that the viscous timescale is much larger than the dynamical timescale). Here we ignore for the time being that the disk truncation radius might also vary as function of stellar and disk masses. Thus, we define

\[
M_{\text{disk}} = 2\pi \int_{0.1 \text{ au}}^{100 \text{ au}} \Sigma(R)R \, dR,
\]

using the \( \Sigma(R) \) values as determined in our model.

When a star is on the main sequence, its stellar mass also defines its radius \( (R_\ast) \), effective temperature \( (T_{\text{eff}}) \), and luminosity \( (L_\ast) \) via standard mass–radius and mass–luminosity relations (Duric 2003; Salaris & Cassisi 2005; Weigert et al. 2009) (see Table 1). For pre-main-sequence stars that are the ones around which we find disks, the luminosity can be up to 50% larger (Baraffe et al. 1998), an effect that we neglect for the present article because it will only slightly increase the temperature of the disk, thus further increasing the likelihood of thermal-driven instabilities.

For each vertical disk column, we solve locally for the condition of vertical hydrostatic equilibrium

\[
\frac{dP}{dz} = \rho g_z = \frac{\rho G M_a z}{(R^2 + z^2)^{3/2}},\]

where \( G \) gives the gravitational constant. The dissipation of kinetic energy, caused by the \( \alpha \)-viscosity (Shakura & Sunyaev 1973), in a slightly modified way to include the vertical stratification of the disk is given by

\[
\nu = \alpha c_s \min(H, l_P),
\]

where \( H = c_s/\Omega \) is the standard pressure scale height in the midplane and \( l_P = P/(\partial_\nu P) \) is the local pressure scale height, to limit dissipation far above the midplane. Viscous heating is then equated with the gradient of radiative flux

\[
\frac{dF}{dz} = Q^+ = \frac{9}{4} \nu \Omega^2,
\]

in order to obtain vertical thermal balance. The resulting energy flux is furthermore associated with a temperature gradient via the flux-limited diffusion equation by Levermore & Pomraning (1981), which is used to obtain the disk’s temperature profile

\[
\frac{dT}{dz} = \frac{F}{D_T} = \frac{\rho \nu \Sigma}{4 c_s \xi T^3} F.
\]

The gas density is calculated with the ideal equation of state \( P = \rho T/\mu \), with gas constant \( R \) and mean molecular weight \( \mu = 2.33 \). Equations (30), (32), and (33) are integrated vertically top-down toward the midplane by use of finite differences (Bell et al. 1997). This kind of model does not allow for the modeling of radial energy transport, which is an appropriate approximation as long as the disk remains geometrically thin (Pringle 1981). In contrast to the model by Bell & Lin (1994), our calculations start at the highest grid-cell of the simulation domain, arbitrarily chosen, and not at a cell that fulfills \( \tau = \frac{2}{3} \). Therefore, the initial temperature is assumed to be equal to the temperature caused by stellar irradiation alone

\[
T_{\text{init}} = T_{\text{eff}} \frac{R_e}{R} \sin(\theta)\frac{1}{\tau^2} \]

where \( \theta \) corresponds to the approximate angle between the disk’s surface and the line of sight. In Bell et al. (1997), we still had to do two independent integration to cover the optical thick and thin parts of the disk: one going down from the \( \tau = \frac{2}{3} \) photosphere and one going up. This is now combined in one integration sweep.

The initial vertical energy flux is defined via the equilibrium condition for actively accreting disks (Pringle 1981)

\[
F_{\text{init}} = \frac{3 G M_a M}{8 \pi R^3} \left( 1 - \frac{R_e}{R} \right).
\]

This equation links the initial guess of the disk’s mass accretion rate \( M \) with the surface energy flux of a disk annulus. A vertical integration series is finished when the initial density guess \( \rho_{\text{init}} \) leads to a vanishing flux in the disk midplane. Otherwise, \( \rho_{\text{init}} \) is varied and the vertical integration is rerun. This procedure is repeated at every radial position, to obtain the whole \( R-z \) plane structure. The radial series is complete when the resulting disk mass fulfills \( |M_{\text{disk}} - M_{\text{d,init}}|/M_{\text{disk}} < 10^{-4} \), where \( M_{\text{disk}} \) is the input parameter of the model and \( M_{\text{d,init}} \) is the resulting disk mass of a radial integration series.

If this condition is not fulfilled, \( M \) is varied and the whole radial process is repeated until the disk mass fits the demanded value. The resulting disk structure is then used to calculate the analytical growth rates and instability criteria introduced in Section 2 for the whole \( R-z \) plane.

### 3.2. Opacity Model

The assumption of vertical thermal balance requires radiative transfer in the \( z \)-direction, which is realized via the flux-limited diffusion approach by Levermore & Pomraning (1981). The underlying temperature diffusion coefficient depends on the

### Table 1

| Stellar Parameters | Mass, Luminosity, Effective Temperature, and Radius Used for the Models Presented in This Work |
|--------------------|------------------------------------------------------------------------------------------|
| \( M_\ast (M_\odot) \) | \( L_\ast (L_\odot) \) | \( T_{\text{eff}} (T_\odot) \) | \( R_\ast (R_\odot) \) |
| 0.4                | 0.028                                      | 0.647                                      | 0.4                  |
| 0.6                | 0.13                                       | 0.775                                      | 0.6                  |
| 1.0                | 1.0                                        | 1.0                                        | 1.0                  |
| 1.5                | 5.063                                       | 1.328                                      | 1.275                |
opacity structure $\kappa_{\text{op}}(R, z)$ via

$$D_T = \frac{4c\varepsilon\alpha T^3}{\rho \kappa_{\text{op}}}. \quad (36)$$

The applied opacity model is, therefore, of great importance for the disk structure and its hydrodynamic stability, because it determines the magnitude of temperature gradients. For the calculations done in the scope of this work, the opacity model by Bell & Lin (1994) was used, which relies on the analytical expressions by Lin & Papaloizou (1980), who assumed grain sizes in the micrometer range. It provides frequency-independent mean opacities for eight different regions of protoplanetary disks, defined and ordered by their temperature. In each region, opacities are calculated by a specific power law in temperature and density

$$\kappa_{\text{op},i} = \kappa_{\text{op},i}(\rho)^{\alpha_i} T^{b_i}. \quad (37)$$

Region 1 contains ice grains and metal grains, and the opacity in region 2 is determined by the evaporation of the ice grains and the still-existing metal grains. Regions 3 and 4 are defined by the abundance of metal grains and their evaporation. At higher temperatures, molecular hydrogen dominates region 5 until hydrogen scattering determines opacities in region 6. When the gas is ionized, electron scattering and Kramer’s law take over in regions 7 and 8. In our model, temperatures do not exceed $\sim$1800 K, which means that only the opacities of region 1–5 matter for our considerations. Metal and water ice grains, therefore, determine the opacity in our model, which leads to two major drops in Figure 2 at the typical evaporation temperatures (water ice at $\sim$160 K; metal/silicate grains at $\sim$1000 K). We will see that the transition zones of opacity are the prime locations for radial buoyancy-driven instabilities, because the temperature-dependent opacities generate major fluctuations in the radial midplane temperature profile (see Figure 3), while leaving the average profile unchanged.

### 3.3. Radial Transport of Heat

Our 1+1D model consists of independent vertical slices of the disk. Thus, radial transport of heat via radiation transport is neglected, as is radial transport of entropy via local mass advection. Both could have an influence on the radial temperature structure and, thus, on the entropy profile, which is so important for the onset of the convective instabilities. Bitsch et al. (2015) created a model for an evolving disk around a solar mass star for an $\alpha = 5.4 \times 10^{-3}$, which used a full 3D radiation hydrodynamic simulation in axis symmetry. Despite their proper treatment of radial diffusion of radiation, they find the same peak values of the temperature gradient in the evaporation zone of the ice particles, which they fit with $\beta_T = -\frac{8}{7}$, which is the same value that we find in our much simpler simulations (see Figure 3(b)).

### 4. Results

To probe the parameter space of star-protoplanetary disk systems, we calculated a series of structure models for different values of each parameter ($M_{\text{disk}}, M_*, \alpha$). See Table 2 for an overview of our simulation parameters, including the mass accretion rate for each model. We do not change the assumed metallicity from the solar value assumed in Bell & Lin (1994), which would reflect in higher or lower opacities. In that sense, choosing a lower disk mass (compensated by slightly larger $\alpha$ to achieve the same accretion rate) would have a similar effect on the disk structure as decreasing the opacity. Nevertheless, investigations of the effect of metallicity should eventually be done in the context of better dust opacities, including the evolution of the dust population as in Birnstiel et al. (2012). Estrada et al. (2016) have put forward a model in which disk and dust are evolving and opacities are calculated from the local dust properties, yet their model does not determine the detailed vertical structure of disks and, thus, does not derive the local radial and vertical stratification of their disk as functions of $R$ and $z$.

#### 4.1. Disk Structure and Stability

Far away from the central object ($R \gtrsim 10$ au), densities are low and stellar irradiation dominates the thermal structure, which leads to a radial trend in temperature that scales with $R^{-0.5}$ (horizontal line in Figure 3(b)). The temperature profile closer to the star, which is strongly influenced by the disk’s

| $M_*$ ($M_\odot$) | $M_{\text{disk}}$ ($M_\odot$) | $\alpha$ | $M$ ($M_\odot$ yr$^{-1}$) |
|-----------------|-----------------|--------|-----------------|
| 0.4             | 0.1             | 0.001  | 7.7282 \times 10^{-9} |
| 0.6             | 0.1             | 0.001  | 8.4444 \times 10^{-9} |
| 1.0             | 0.1             | 0.001  | 1.0513 \times 10^{-8} |
| 1.5             | 0.1             | 0.001  | 1.27555 \times 10^{-8} |

Note. Stellar mass, disk mass, and $\alpha$ are the input parameters of the model. The mass accretion rate is determined iteratively in order to fit the input parameters.
varying optical depth structure, has a major impact on thermally driven instabilities. It can be seen in Figure 3(c) that entropy gradients drop below zero for the most opaque zones, which renders the disk radially buoyant (the pressure gradient is negative here) in the sense of the classical Schwarzschild criterion ($\partial_P \rho \partial_T > 0$) (e.g., Pringle & King 2007), which is the necessary condition for the onset of COS. Two-dimensional density and temperature structures are shown in Figure 4. As mentioned before, variations in the opacity (mostly with temperature) lead to complex behavior of the density and temperature gradients. Notably, one finds the steepest temperature gradient and the most shallow density gradient at the grain and ice evaporation lines (indicated by vertical lines) at which opacity and, thus, optical depth reaches a local extremum. The anticorrelation of temperature and density gradients is typical for viscous accretion disks, because they have a roughly constant pressure profile of about $R^{-2}$, which is a result of the $\alpha$-model for viscosity and the assumption of a constant accretion rate. Temperature is then also relatively high at these locations, because thermal energy transport needs stronger gradients with increasing optical depth. Therefore, the disk appears to be puffed up in the regions of maximal opacity, which can be seen as the kinks in the scale height profiles in Figure 4. The 2D temperature profile also shows strong vertical gradients in the viscously heated parts of the disk and a vertically isothermal structure in the regions dominated by stellar irradiation. The vertical gradients are of interest for the investigation of VSI.

The outer, vertically isothermal regions are prone to be unstable as a result of the VSI, because cooling times are low enough to allow for the growth of vertical perturbations. Figure 4(c) displays the thermal relaxation times for a fixed perturbation wavenumber of $k = 50/H$ (which was used as the radial wavenumber for the study of the VSI). It can be seen that relaxation times are low at distances $\gtrsim 10\,a_u$, which is a necessary condition for VSI growth. Additionally, the criterion $\tau_{\text{relax}} \Omega < 10$ (Nelson et al. 2012) for a polytropic stratification is fulfilled in those regions that are prone to be convective. This means that the VSI can also exist in the dense interior regions if the disk becomes buoyant there and a polytropic structure is established.

The disk’s vertical gradient in angular velocity is shown in Figure 4(d), where it can be seen that the vertical shear increases with height above the midplane. Its origin is the vertical variation of the radial pressure gradient. Radial hydrostatic equilibrium (i.e., the balance of gravitational force versus radial pressure gradient and centrifugal force) thus requires the rotation velocity to change, too. This vertical shear is the necessary condition for the VSI.

The disks’ stability in the context of self-gravity was checked by use of the Toomre criterion $Q = \kappa \rho C_s / (\pi G \Sigma) < 1$ (Toomre 1964), where $C_s = \partial P/\partial \Sigma$ represents a 2D speed of sound (Pringle & King 2007). Gravitational stability holds for...
The COS requires negative radial entropy gradients. Sufficient optical depth is needed for a disk to develop superadiabatic radial temperature gradients. This means that two COS-active regions are located in the regions of maximal opacity. An additional unstable zone arises at a certain height above the midplane ($z \sim 1 H$) and covers the whole radial extent of the disk. The reason for this is the change of sign in the radial entropy and pressure gradients in the disk, because there is always a region where the gradients are parallel. This unstable branch, therefore, also exists in disks without internal heating, as long as radial gradients in temperature exist. As was pointed out in Section 2.1, growth rates reach their maximum for relaxation times that fulfil Equation (5). This also defines the wavenumbers, which are growing with a maximal rate because of the dependency of cooling time to wavenumber and local opacity (Equation (23)). In regions where the optical depth is high, only spatially small perturbations (large $k$) can relax on a timescale suitably short for COS. Where the optical depth is small, only larger perturbations, which cool slower than small perturbations, are relaxing on a timescale that is ideal for COS. This means that the maximum growing perturbations need to be spatially small (large $k$) close to the midplane and become larger (smaller $k$) in the optically thinner upper and outer parts of the disk (see Figure 12 in the Appendix). For the maximal growing perturbations, we find growth rates of $\sim 10^{-5} \Omega$–$10^{-6} \Omega$.

Viscosity and its impact on small perturbations, as discussed in Klahr & Hubbard (2014) and Latter (2016), was not taken into account for our study.

As can be seen in Figure 5(a), a higher stellar mass (and luminosity) leads to a thermal structure in which the influence of accretion heating becomes less dominant. Therefore, entropy gradients are negative in large regions for low-mass stars with massive disks and smaller for high-mass stars with a disk of low mass compared to the stellar mass. Another important effect of an increased solar luminosity is the overall temperature increase of the disk. Ice sublimes at $\sim 160$ K, and, therefore, the second COS-active zone vanishes for high-mass stars (lower right panel in Figure 5(a)), because temperatures are too high to allow for the existence of opacity enlarging ice grains.

The opposite effect is visible for a larger disk mass (Figure 5(b)). Because of the increase of accretion heating and optical depth with this parameter, temperature and entropy gradients are both stronger. This also leads to the outward shift of the susceptible regions with increasing disk mass, which is the direct consequence of the movement of the dust and ice sublimation lines. For a disk of $0.01 M_\odot$, the ice sublimation line is so close to the star that no second unstable zone exists (upper left panel in Figure 5(b)). Because of the increasing densities, wavenumbers need to be small to allow for more-efficient cooling. (see Figure 12(b) in the Appendix)

An increase of the disk’s $\alpha$-parameter leads to enhanced viscous heating of the dense interior parts. Therefore, radial temperature gradients are also building up and growth rates for the COS become larger when viscous heating dominates the interior temperature profile.

Because the density profile is only slightly altered by the $\alpha$-parameter, thermal relaxation stays relatively constant in the outer regions, where the opacity is provided by ice and metal grains. In the zones closer to the star, temperatures increase...
because of the increased accretion heating and sublimation lines move outward with increasing $\alpha$. For an extremely low $\alpha = 10^{-5}$, we notice that midplane temperatures are low enough to allow for the existence of metal grains even at radii of $\sim 0.1$ au. Densities in these regions are high, thus leading to high opacities and, therefore, to an optically thick structure. At these locations, perturbation wavenumbers need to be very high to allow for efficient enough cooling (see Figure 12 in the Appendix).

4.3. SBI

Figure 6 displays the quantities introduced in Section 2.2. It can be seen that the vertically integrated pressure profile (b) scales with $R^{-2}$. The reason for this is the spatially constant mass accretion rate, which means that $\dot{M} \propto \nu \Sigma = \alpha c_{s}^{2} \Sigma \Omega^{-1} = \alpha \dot{P} \Omega^{-1} = \text{const.}$ (Lynden-Bell & Pringle 1974). Figure 6(c) shows a single large buoyancy unstable region in contrast to the two smaller zones found in the 3D profiles in Section 4.2. This means that vertically extended $R-\phi$ vortices, which can be treated as 2D structures, might survive in even larger regions than the linear $R-z$ modes of the COS. Migrating vortices could, therefore, be formed in the COS-active regions and move to the regions unaccessible for small-scale COS turbulence.

A parameter study reveals that the SBI’s growth rate has very similar dependency on the disk and stellar parameters as the COS. Figure 7 provides an overview of the SBI’s behavior for different parameters. The growth rates shown in Figure 7(a) shrink with increasing stellar mass. The reason for this lies in the decline of radial temperature and entropy gradients in the regions of high opacity as a result of outward movement of dust evaporation lines, which shifts the optically thick regions into zones of lower density and pressure. The opposite effect results from an increase of disk mass (Figure 7(b)) and $\alpha$-parameter (Figure 7(c)), which lead to an increase in accretion heating and, hence, enlarge the radial temperature gradients and vortex growth rates. As mentioned before, SBI growth occurs over almost the whole radial extent of the disk, but the highest growth rates are reached at the location of the ice line, where opacities are maximal. At this location, $\Gamma_{\text{SBI}}$ is in the order of $\sim 10^{-4}\Omega-10^{-3}\Omega$, which corresponds to a growth timescale of $\sim 10^{2}T_{\text{orb}}-10^{3}T_{\text{orb}}$ (local orbital timescales).
Figure 8. Squared vertical Brunt–Väisälä frequencies in units of $\Omega^2$. Blue color indicates a convectively unstable stratification. One parameter is varied per panel, and the other two are held constant at $M_*=1.0 M_\odot$, $\alpha=10^{-3}$, or $M_{\text{disk}}=0.1 M_\odot$.

Figure 9. Growth rates of the VSI in units of the local dynamical timescale $(\Gamma \Omega)^{-1}$. One parameter is varied per panel, and the other two are held constant at $M_*=1.0 M_\odot$, $\alpha=10^{-3}$, or $M_{\text{disk}}=0.1 M_\odot$. No growth occurs in white areas and in the midplane.
4.4. VCI

The onset of VCI in a protoplanetary disk requires the existence of sufficiently steep vertical temperature gradients. As soon as these gradients are at least adiabatic, the VSI can operate despite long cooling times, which drives our interest in the onset of VCI.

To steepen the vertical temperature gradients, radiative transfer needs to be inefficient enough to force the temperature to increase close to the midplane. In other words, densities and opacities need to be large and the accretion rate needs to be strong to ensure strong viscous heat production and corresponding temperature gradients. A temperature profile dominated by stellar irradiation is vertically isothermal and, thus, contradictory to the conditions for convection. Higher temperatures due to stellar irradiation lead to the evaporation of dust grains at larger radii and to an outward shift of convection zones. These effects are strongly visible in Figure 8(a). For these models, disk mass and $\alpha$-parameter are set to constant values of $M_{\text{disk}} = 0.1 M_\odot$ and $\alpha = 10^{-3}$. Two separate convection zones appear for stellar masses $\lesssim 2 M_\odot$ because of the decrease in opacity at $T \sim 160$ K. When the convection zones are shifted to larger radii, densities and rotation frequencies are no longer high enough to sustain the existence of the outer convection zone. An increase of stellar luminosity, therefore, leads to a disk that is less susceptible to convective instability. In the case of a solar mass star with a disk of $M_{\text{disk}} = 0.1 M_\odot$ and $\alpha = 10^{-3}$, convection zones span from $\sim 0.3$ to $1.1$ au and from $\sim 3$ to $3.2$ au close to the midplane.

An increase of the disk mass has the exact opposite effect on convectively unstable regions, as can be seen in Figure 8(b). As the mass increases, densities and, therefore, opacities increase, which heats up the disk at small radii. Again, the combination of stellar and accretion heating shifts the unstable zones to larger radii for larger disk masses, but because of the increase of densities at all radii, convection zones grow nonetheless. We conclude that disks with higher total mass are more prone to be unstable because of convection.

As can be seen in Figure 8(c), our disk model behaves quite similarly when the disk’s $\alpha$-parameter is increased. In that case, the efficiency of accretion heating becomes higher. Hence, temperature gradients increase and convection zones grow. As $\alpha$ becomes larger, the innermost parts of the disk become hotter and the sublimation lines move outward. Convection zones grow as the disk is heated and temperature gradients become larger. For an intermediate $\alpha$-value of $10^{-3}$, two spatially separated convection zones exist, which span from $\sim 0.35$ to $1.1$ au and from $\sim 3$ to $4$ au close to the midplane. It can be said that the more effective viscous heating becomes, the more susceptible a disk is to convective instability and, thus, indirectly to VSI. The growth rates of the instability have been calculated by use of Equation (22) and increase with height. They are generally in the order of $\sim 10^{-4} \Omega$.

4.5. VSI

For a disk to be unstable as a result of VSI, a vertical gradient in angular velocity is required. The stratification has to be neutral in the sense of buoyancy, or at least allow for sufficiently fast thermal relaxation to overcome restoring buoyancy forces. A vertically buoyant structure also allows for the growth of VSI if $\tau_{\text{relax}} \lesssim 10$ is fulfilled (Nelson et al. 2012). To calculate the
growth rates for the VSI with Equation (18), a certain radial wavenumber was chosen. We then used Equation (19) to determine which corresponding vertical wavenumber leads to maximum growth of a perturbation. The results of Lin & Youdin (2015) suggest that modes with large wavenumber decay in viscous disks because their growth time becomes similar to that of the viscous timescale. Therefore, we draw the conclusion that only perturbations with wavenumbers in the order of $k_{\text{g}}H \sim O(10)$ will grow significantly, where $H$ refers to the disk’s local pressure scale height. We chose $k_{\text{g}}H = 50$ for our investigation, because this value is $\sim O(10)$ but also sufficiently large to allow for efficient cooling at larger radii (see Equation (23)). Higher wavenumbers allow, in principle, for VSI in even larger areas beyond the adiabatic regions. A determination of the fastest growing wavenumbers, including the effects of realistic thermal relaxation models, requires a numerical study, as was was done by Lin & Youdin (2015), and goes beyond the scope of this work. Our investigation of the VSI, therefore, relies on the arbitrary choice of a $k_{\text{g}}$ in the order of magnitude that was suggested by Lin & Youdin (2015).

At larger distances to the star, the disk becomes optically thinner and stable to buoyancy, which renders it unstable to the VSI. As can be seen in Figure 9, growth rates increase with vertical distance to the midplane. Close to the midplane, they reach values of $\sim 10^{-4} \Omega - 10^{-3} \Omega$ and growth rates of up to $\sim 10^{-2} \Omega - 10^{-3} \Omega$ at heights larger than $z/R \sim 0.05$. For very faint, low-mass stars, temperatures far away from the star are quite low ($T \lesssim 20$ K). It can be seen in Equation (27) that optically thin relaxation times are strongly temperature dependent ($\tau_{\text{relax}} \propto T^{-3}$). Therefore, optically thin cooling dominates the outer regions of the disk for low-mass stars. This timescale can become so large that criterion (20) for VSI growth is no longer obeyed and no outer VSI-susceptible zone exists. Temperatures increase when the solar mass is increased and the outer regions become susceptible for VSI, as can be seen in Figure 9(a). Therefore, we find that the mass of the central star has different effects on the susceptibility to the VSI in the inner and in the outer parts of the disk. A low stellar mass (compared to the disk mass) leads to large convectively unstable zones, which are also VSI susceptible, but to slow thermal relaxation in the outer regions, which inhibits the growth there (Figure 9(a) upper panels). Larger stellar masses lead to smaller convection zones but to fast thermal relaxation far away from the star, which has the opposite effect on the VSI. A generally hotter disk due to a more massive star, therefore, becomes susceptible to VSI at larger radii but less VSI active at smaller radii.

An increase of the disk’s mass has the opposite effect on the inner VSI active regions. As described in the previous sections, low disk masses lead to a disk that is stable against buoyancy. This means that the inner VSI-susceptible regions are small for small disk masses and larger for large disk masses (see Figure 9(b)). The outer susceptible region shows a different behavior. When the disk mass is very small ($M_{\text{disk}} \sim 0.01 M_*$), densities also become small at larger radii. Consequently, collisions between molecules and dust particles become rare, and thermal relaxation slows down. This is why the outer VSI-susceptible region is smaller for smaller disk masses.

The $\alpha$-parameter defines the efficiency of viscous heating in the disk. Increasing this parameter, therefore, leads to enlarged convectively unstable zones, with the same effect on VSI active zones (see Figure 9(c)). The shape and size of the outer susceptible region depends mostly on thermal relaxation and, therefore, remains mostly unaffected by a variation of $\alpha$.

All in all, VSI active regions close to the star coincide with vertical convection zones and have the same dependency on the system parameters as convection. The extent of the outer susceptible zone depends on cooling times and is, therefore, favored in systems with high stellar mass, disk mass, and $\alpha$-parameter.

### 4.6. Stability Maps

The spatial distributions of the four discussed instabilities and the investigated parameter sets are summarized in Figure 10. It can be seen that the COS and the VCI share large parts of their susceptible regions, because both of them rely on the opacity structure of the disk in similar ways. The VSI operates also in the convectively unstable region, because vertical perturbations are enhanced there, as well as at radii larger than $\sim 10$ au where the disk becomes optically thinner. All hydrodynamic instabilities are favored in setups with small $M_*$ and large $\alpha$ and $M_{\text{disk}}$. These are the disks in which the temperature gradients are dominated by viscous heating instead of stellar irradiation. A disk that is viscously heated, thus, becomes susceptible to hydrodynamic turbulence in large parts. Disks with very low $\alpha \lesssim 10^{-4}$ or $M_{\text{disk}}$ are nonetheless unstable to VSI at larger radii and can develop COS in a thin region at $z \gtrsim 1 H$. If these instabilities are able to produce finite amplitude perturbations, they might be able to develop into large-scale SBI vortices, which are amplified in large parts of the disk.

### 5. Summary and Conclusions

In this article, we investigate the stability of active protoplanetary disks (where the temperature profile is set by accretion) and passive disks (in which the temperature profile is dominated by irradiation) by use of 1+1D steady-state accretion disk models including stellar irradiation. This allows for the treatment of flux-limited radiative transfer caused by viscous heating and makes it possible to apply detailed models of the local rate of thermal relaxation, thus making it possible for the first time to the authors’ knowledge to spatially map the growth rates of the COS, the SBI, and the VSI to the radial-vertical plane of a realistically stratified circumstellar disk.

We found that we can reproduce the radial temperature profile in full time-dependent 3D axis symmetric radiation hydrodynamic simulations as performed by Bitsch et al. (2015) for their model parameters, but can cover a much wider parameter space.

It is shown that even an almost quiescent disk, with an extremely low viscosity parameter of $\alpha = 10^{-5}$, becomes unstable to COS and VSI at radii $\gtrsim 10$ au and for the COS even closer to the star at heights of $\sim 1 H$ above the midplane. The turbulent structures resulting from such instabilities were shown to grow to large-scale vortices by various authors (Meheut et al. 2012; Lyra 2014; Manger & Klahr 2018).

A closer look was, therefore, also taken upon the growth rates and susceptible regions of these structures by consideration of the SBI mechanism. Our results show that the vertically integrated, radial stratification of the disk allows for positive growth rates over almost the whole radial extent of the disk,
between ~0.3 and 50 au at timescales of ~1000 \( T_\text{orb} \), even if \( \alpha \) has low values of \( 10^{-2} - 10^{-4} \). Vortices that evolve at large radii because of the perturbations caused by COS and VSI might thus be able to migrate toward the central star while being constantly forced by the SBI contributing to their longevity (Lesur & Papaloizou 2010; Paardekooper et al. 2010). These findings also indicate that the SBI mechanism could be a controlling mechanism for the frequently observed vortices in protoplanetary disks (van der Marel et al. 2013; Carrasco-González et al. 2016).

Inner regions of disks, which reach \( \alpha \)-values of \( 10^{-4} \), are already subject to strong enough viscous heating and evolve into a radially and vertically buoyant structure, as was shown in Sections 2.1 and 2.4. We have shown that these buoyantly unstable zones arise wherever the disk becomes optically thick enough to allow for the existence of strong vertical temperature gradients, which means that they are larger in disks with a high mass (\( \gtrsim 0.01 M_\odot \) for a 1 \( M_\odot \) star).

The resulting radial buoyancy, in combination with thermal relaxation, is able to drive COS at maximal growth rates of \( \sim 10^{-2} \Omega \). COS is generally favored by massive disks around low-mass stars and high \( \alpha \)-viscosity. The opacity structure and, thus, the relaxation time criterion determine the fastest growing modes, which means that short-wavelength perturbations (large \( k \)) grow best in optically thick regions and long-wavelength perturbations grow better in optically thinner regions.

VSI depends even more on radiative cooling and, thus, requires the optically thick parts of the disk to be at least buoyantly neutral, which impedes repelling forces on vertical perturbations. The internal heat production, caused by \( \alpha \)-values of \( 10^{-4} \), is sufficient to provide such a disk structure in the denser interior zones and, therefore, makes it possible for the VSI to arise, even if the necessary criterion for cooling by Lin & Youdin (2015) is not fulfilled. The finding that VSI can produce an \( \alpha = 10^{-4} - 10^{-3} \) (Nelson et al. 2012; Stoll & Kley 2014; Manger & Klahr 2018) enables the possibility that the VSI can maintain the thermal structure of the disk that it needs to operate. What needs to be shown is at what height in the disk the thermal energy will actually be released.

Two distinct regions of VSI growth exist at small radii between ~0.3–1.1 au and ~3–3.2 au (for model parameters of \( M_\text{gas} = 1 M_\odot \), \( M_\text{disk} = 0.1 M_\odot \), and \( \alpha = 10^{-3} \))

determined by the vertically polytropic structure and, therefore, is favored by small \( M_\text{gas} \), large \( \alpha \), and \( M_\text{disk} \). The growth rate of the instability increases vertically from \( \sim 10^{-5} \Omega \)–\( 10^{-2} \Omega \).

Future work should deal with more recent opacity models, including the evolution of the dust component of the disk and a variable chemical composition of the disk having influence on the adiabatic coefficient \( \gamma \). The latter in particular probably has an important influence on the strength of buoyancy-driven instabilities. Then it also should be possible to map out possible Zombie Vortex regions, as discussed by Barranco et al. (2018).

Passive disks with an assumed surface density profile inspired by the so-called minimum mass solar nebula (Chiang & Goldreich 1997) (i.e., \( \beta_\Sigma = -1.5 \) and radial temperature gradient determined only by irradiation) will have a radial increasing entropy structure and, therefore, will not be the subject of SBI. However, note that this steep density profile was extrapolated from the “solid” mass distribution in our solar system and not from the gas distribution around the young Sun, which should be completely different in the course of dust growth and pebble drift (Birnstiel et al. 2012). Modeling of an actively accreting disk, whether it is as simple as that in our model or a more elaborate 1+1D irradiation model as in D’Alessio et al. (2006) or even in a full 2D hydrodynamic model, gives a much shallower surface density profile than does \( \beta_\Sigma = -1.5 \), in range of the values derived from observations (Andrews et al. 2010) of \( \beta_\Sigma = -1.1 \) to –0.4. These values predict a wide region in a disk to have a negative, buoyantly unstable gradient in vertically integrated entropy (i.e., the condition for convective amplification of vortices (SBI)).

Our model is in general agreement with the more complicated models of disk structure (D’Alessio et al. 2006; Bitsch et al. 2015) if one considers the first two pressure scale heights above the midplane. Our predictions for stability/instability above that region are less reliable. Here we suggest additional work to investigate cooling rates and entropy structures in these dilute regions, but remember that those regions might also be influenced by magnetic fields because of their sufficient ionization state (Dzyurkevich et al. 2013) and yet be hampered by ambipolar diffusion. Typically, those would be the regions where a wind is being launched from the disk in an interplay of photoevaporation and magnetic fields (Pudritz et al. 2007; Königl et al. 2010).

We conclude that hydrodynamic instabilities can exist in large portions of protoplanetary disks and that they benefit from the release of accretion power if at least a fraction of the released heat gets deposited within one or two pressure scale heights above the midplane. However, note that even disks with very low accretion rates have a radial temperature stratification, which renders them unstable to SBI and partially unstable to VSI and COS, especially at radii \( \gtrsim 10 \) au. Observed disk profiles by Andrews et al. (2010) are indeed unstable to the SBI.

If the resulting \( \alpha \)-values from either these hydrodynamic instabilities or nonideal MHD effects are low to moderate (\( \alpha = 10^{-4} - 10^{-3} \)), as suggested by Raettig et al. (2013), and the heat is deposited around the midplane, then regions closer to the star also become vertically and radially buoyant and, therefore, susceptible to COS and VSI.

Thus, the largest caveat in our work is that, even if sufficient \( \alpha \)-values are measured in simulations of SBI and
it is currently not known where the kinetic energy resulting from the release of potential energy in the accretion disk is deposited. If at least a part of this energy is deposited close to the midplane of the disk, then hydrodynamic turbulence has a good chance to operate in large parts of the planet-forming regions of protoplanetary disks, even in the midplane, where nonideal MHD effects dampen otherwise dominant magnetic effects sufficiently (Lyra & Klahr 2011). With its well-known properties of forming vortices (Raettig et al. 2013; Manger & Klahr 2018) and zonal flows, hydrodynamic instability can be a major agent in forming planetesimals and, thus, determine the properties of planetary systems (Klahr et al. 2018).

We would like to thank the whole planet and star formation theory group of the Max-Planck-Institute for Astronomy in Heidelberg for many fruitful discussions of the topic and their help and advice. We especially thank Natascha Manger, Martin Schlecker, Bertram Bitsch, and Henrik Latter for useful suggestions, discussions, and advice. This research was supported by the Munich Institute for Astro- and Particle Physics (MIAPP) of the DFG cluster of excellence “Origin and Structure of the Universe” and in part at KITP Santa Barbara by the National Science Foundation under grant No. NSF PHY17-48958. The authors gratefully acknowledge the Gauss Centre for Supercomputing (GCS) for providing computing time for a GCS Large-Scale Project (additional time through the John von Neumann Institute for Computing (NIC) on the GCS share of the supercomputer JUQUEEN (Stephan & Docter 2015) and now JEWELS at Jülich Supercomputing Centre (JSC). GCS is the alliance of the three national supercomputing centers HLRS (Universität Stuttgart), JSC (Forschungszentrum Jülich), and LRZ (Bayerische Akademie der Wissenschaften), funded by the German Federal Ministry of Education and Research (BMBF) and the German State Ministries for Research of Baden-Württemberg (MWK), Bayern (StMWFK) and Nordrhein-Westfalen (MIWF). Additional simulations for the project of hydrodynamic instabilities in protoplanetary disks were performed on the THEO cluster owned by the MPIA and the HYDRA and DRACO clusters of the Max-Planck-Society, both hosted at the Max-Planck Computing and Data Facility in Garching (Germany). Part of this work was performed at the Aspen Center for Physics, which is supported by National Science Foundation grant No. PHY-1607761.

Appendix

Here, we provide information, which we deem valuable for numerical experiments on the presented instabilities.

In Figure 11, we present the radial surface density and temperature profiles for our chosen stellar and disk parameters, including $\alpha$. A numerical simulation, using these values in combination with the Bell & Lin opacity table (provided no $\alpha$ is used in the dynamic simulation) should then display a linear stability/instability as predicted in this article. Figure 12 gives the expected COS wavenumbers for maximum growth rates, which indicates the resolution in grid cells per pressure scale height needed in the numerical simulation to resolve them properly. Finally, Figure 13 displays the local radial stratification for three different heights ($z = 0, H, 2H$) in density, temperature, entropy, and relaxation time to be used in local linear analysis as well as local multidimensional nonlinear simulations. For completeness we also added local pressures scale heights, Richardson Number, maximum growth wavelength of the COS, and VSI growth rates. This should be useful for checking numerically our predictions as well as for an a posteriori investigation of whether the instabilities can provide their own viscous stresses and dissipation of energy, as we assumed a priori in constructing the stratification.

16
Figure 11. Toomre $Q$ (top row), column density $\Sigma$ (middle row), midplane temperature $T_{\text{mid}}$ and surface temperature $T_{\text{surf}}$ (bottom row) as a function of radius for the presented disk models for the different stellar parameters in column (a), disk masses in column (b), and $\alpha$-parameters in column (c). It can be seen that every model is stable in the sense of $Q > 1$. Increasing disk temperatures lead to higher $Q$ values for higher stellar mass or $\alpha$-parameter. Larger disk masses and, thus, column densities reduce $Q$. We also added the irradiation temperature $T$ to the bottom plots. This enables us to identify the transition radius from irradiation-dominated disks (passive) to accretion heating dominated disks (active), which is a function of disk mass, accretion rate ($\dot{M} \propto \alpha M_{\text{disk}}$), and opacities.
Figure 12. Maximally growing wavenumbers of the linear COS, corresponding to Figure 5. One parameter is varied per panel, and the other two are held constant at $M_*=1\ M_\odot$, $\alpha = 10^{-5}$, or $M_{\text{disk}} = 0.1\ M_\odot$. No growth occurs in the white areas.
Figure 13. Several disk properties of a model calculated with a disk mass of \(M_{\text{disk}} = 0.1 \, M_\odot\), stellar mass of \(M_\star = 1 \, M_\odot\), and \(\alpha = 10^{-3}\) for the midplane and at \(z = H\). Panel (a) shows the logarithmic radial density gradient, panel (b) displays the radial logarithmic temperature gradient, and panel (c) shows the radial entropy gradient. This means that entropy gradients are completely negative at \(z = 2H\) above the midplane. However, pressure gradients become radially positive, which means that the disk is radially stable against convection at \(z = 2H\), in the sense of the Schwarzschild criterion. At a height of \(z \approx 1H\) and in the midplane, pressure and entropy gradients are partially parallel (shown as the dashed section in the green line). This means that these regions are unstable in the sense of the Schwarzschild criterion, which can give rise to COS. Panel (d) shows the local scale height profile. In panel (e), cooling times for \(k = 1/H\) are shown. It can be seen that thermal relaxation becomes faster with height above the midplane, because densities decrease. The regime of collisionally limited relaxation due to extremely low densities is not covered at these heights and becomes dominant at \(z \sim 3H\). Panel (f) shows the radial Richardson number of the disk. The convectively unstable zone at \(z = 1H\) is visible, because negative Richardson numbers indicate instability in the sense of the classical Schwarzschild criterion without rotation taken into account. A Richardson number larger than \(-\frac{3}{2}\) indicates radial stability in the sense of the standard Solberg–Holland criteria. Panel (g) shows the perturbation wavelength for which the COS' growth rate becomes maximal: \(\gamma_{\text{relax}} \Omega = 1 \Rightarrow \lambda_{\text{max, COS}}/H = 4\pi^2\Omega^2/\gamma(\Omega H^2)\) with \(\tau_{\text{relax}} = 1/(k^2\Omega^2)\). In panel (h), VSI growth rates are displayed. The dotted parts indicate that growth is inhibited by cooling times \(>\tau_{\text{relax}}\) (Lin & Youdin 2015).

References

Andrews, S. M., Wilner, D. J., Hughes, A. M., Qi, C., & Dulmond, C. P. 2010, ApJ, 723, 1241
Arlt, R., & Urpin, V. 2004, A&A, 426, 755
Bai, X.-N., & Stone, J. M. 2013, ApJ, 769, 76
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Baraffe, I., Chabrier, G., Allard, F., & Hauschildt, P. H. 1998, A&A, 337, 403
Barge, P., & Sommeria, J. 1995, A&A, 295, L3
Barranco, J., Pei, S., & Marcus, P. 2018, ApJ, 869, 127
Baruteau, C., Crida, A., Paardekooper, S.-J., et al. 2014, in Protoplanets and Planets VI, ed. H. Beuther et al. (Tucson, AZ: Univ. Arizona Press), 667
Bell, K. R., Cassen, P. M., Klahr, H. H., & Henning, T. 1997, ApJ, 486, 372
Bell, K. R., & Lin, D. N. C. 1994, ApJ, 427, 987
Beutel, M. 2012, Bachelor’s thesis, Heidelberg Univ. Max Planck Institute for Astronomy
Birnstiel, T., Klahr, H., & Ercolano, B. 2012, A&A, 539, A148
Bitsch, B., Johansen, A., Lambrechts, M., & Morbidelli, A. 2015, A&A, 575, A28
Boley, A. C., & Durisen, R. H. 2018, ApJ, 861, 534
Cabot, W., Canuto, V. M., Hubickyj, O., & Pollack, J. B. 1987, Icar, 69, 387
Cameron, A. 1978, in The Moon and the Planets, Vol. 18 (Dordrecht: Kluwer Academic), 5
Carrasco-González, C., Henning, T., Chandler, C. J., et al. 2016, ApJL, 821, L16
Chiang, E. I., & Goldreich, P. 1997, ApJ, 490, 368
D’Alessio, P., Calvet, N., Hartmann, L., Franco-Hernández, R., & Servín, H. 2006, ApJ, 638, 314
D’Alessio, P., Cantó, J., Calvet, N., & Lizano, S. 1998, ApJ, 500, 411
Duric, N. 2003, Advanced Astrophysics (Cambridge: Cambridge Univ. Press)
Dzurkevich, N., Turner, N. J., Henning, T., & Kley, W. 2013, ApJ, 765, 114
Estrada, P. R., Cuzzi, J. N., & Morgan, D. A. 2016, ApJ, 818, 200
Fricke, K. 1968, ZAp, 68, 317
Goldreich, P., Goodman, J., & Narayan, R. 1986, MNRAS, 221, 339
Goldreich, P., & Schubert, G. 1967, ApJ, 150, 571
Goodman, J., Narayan, R., & Goldreich, P. 1987, MNRAS, 225, 695
Gressel, O., Turner, N. J., Nelson, R. P., & McNally, C. P. 2015, ApJ, 801, 84
Hartmann, L., & Bae, J. 2018, MNRAS, 474, 88
Held, L. E., & Latter, H. N. 2018, MNRAS, 480, 4797
Klahr, H. 2004, ApJ, 606, 1070
Klahr, H., 2007, in IAU Symp. 239, Convection in Astrophysics, ed. F. Kupka, I. Roxburgh, & K. L. Chan (Cambridge: Cambridge Univ. Press), 405
Klahr, H., & Bodenheimer, P. 2006, ApJ, 639, 432
Klahr, H., & Hubbard, A. 2014, ApJ, 788, 21
Klahr, H., Pfeil, T., & Schreiber, A. 2018, in Handbook of Exoplanets, ed. H. Deeg & J. Belmonte (Cham: Springer), I
Klahr, H., Raettig, N., & Lyra, W. 2013, in EPJ Web of Conf., Instabilities and Structures in Proto-Planetary Disks, ed. P. Barge & L. Jorda (Les Ulis: EDP Sciences), 04001
Klahr, H. H., & Bodenheimer, P. 2003, ApJ, 582, 869
Königl, A., Salmeron, R., & Wardle, M. 2010, MNRAS, 401, 479
Latter, H. 2016, MNRAS, 455, 2068

ORCID iDs

Thomas Pfeil @ https://orcid.org/0000-0002-4171-7302
Hubert Klahr @ https://orcid.org/0000-0002-8227-5467
