SAR image noise suppression of BEMD by the kernel principle component analysis

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Abstract
In the process of synthetic aperture radar image noise suppression by the bi-dimensional empirical mode decomposition (BEMD) algorithm, the edge effect is a key problem in the BEMD operation. To weaken this effect, an improved BEMD-kernel principal component analysis (BEMD-KPCA) method of image denoising is proposed in this study. Experimental results show that the BEMD-KPCA algorithm has a good capability of improving edge effects in the BEMD decomposition process and satisfying the requirement of the reliable decomposition results. Compared with the traditional BEMD method, the proposed approach has a good effect in suppressing speckle noise. Additionally, the denoised image from the decomposed components of the IMFs processed by the BEMD-KPCA method sufficiently preserves the edge and detail information, confirming its high coherency with the original image.

1 | INTRODUCTION

Synthetic aperture radar (SAR) image has been widely used in the remote sensing field. Some noises will be inevitably introduced in the SAR image acquisition, conversion and transfer, which will seriously affect the quality of the SAR image. In order to obtain real information for SAR image, denoising and suppressing the noises are an essential step to improve the image [1, 2]. In the processing of image denoising, it is necessary to remove the noise pollution for the given image as much as possible, and maintain all the important details information of the image. In recent years, some noise suppression theoretical methods for SAR image have been proposed and developed [3–6].

With the development of artificial intelligence algorithms, some new attempts have appeared to denoise SAR image. From a new perspective, the application of the empirical mode decomposition (EMD) fully shows advantages of the non-stationary time-frequency, nonlinear and adaptive capacity in the image noise removal processing [7]. It decomposes the input signal into multiple scales, which represent different time-frequency components of the original signal [34]. Compared with the other time-frequency signal processing methods such as short-time Fourier transform and wavelet transform, the EMD method does not rely on any priori basic functions and leads to an adaptive decomposition process by its own characteristics of the data, which should be better at revealing signal features involved in the time-frequency localization behaviour [8–11, 20]. As an adaptive signal denoise and analysis technology, the EMD has been applied in the study of earthquake, mechanical fault diagnosis, oceanography and other fields.

Based on the image decomposition in different directions for EMD decomposition, the decomposition process of the EMD approach is relatively inefficient. Nunes et al. [12, 13] extended the EMD of 1D case to the 2D, and introduced an algorithm
of bi-dimensional empirical mode decomposition (BEMD) into image processing. Due to the noise characteristics of practical SAR images, it is generally likely that the highest noise content of the image is extracted into the first intrinsic mode function (IMF) component for the natural property of the BEMD process, and the remaining fewer noise contents sneak into the subsequent ones [14, 15]. In the process of image decomposition, the BEMD algorithm has a close relationship with the correct interpolation operation. In the edge region, incorrect interpolation may occur due to the lack of extreme point constraints, which is the so-called edge effect in BEMD decomposition [15]. However, the edge effect in BEMD decomposition has not been well solved, and further research work is urgently needed in the application of BEMD.

To solve edge effects problem, some solutions and improvement methods such as adding additional extrema points in the image [16], setting specific screening frequency [17], non-parametric sampling for texture synthesis method [18], envelope estimation method [19] and application of support-vector-regression (SVR) extrapolation [20] are proposed, which reduce or suppress the edge pollution problem in the BEMD decomposition. Although these above methods can weaken and suppress the edge effects problem in image decomposition to a certain extent, however, the iterative calculation is complex, and the filtering stopping criterion needs to be constantly adjusted, which will lead to unreasonable and unreliable decomposition results.

To this end, an improved approach to weaken the edge effect of the BEMD and suppress the performance of image denoising is proposed. By using this algorithm, the image with noises firstly is decomposed to several intrinsic mode functions (IMFs) and the residual component; secondly, the extrema of the extrapolation IMFs neighbouring the edges are taken as positions of the mirror to new IMFs; thirdly, the number of the new IMF's component with noises are determined using the kernel principal component analysis (KPCA) method [21–23], next, the IMFs component with noises are processed by adaptive filter; and then the closed end mirroring expansion is introduced to further remove the edge effects of the BEMD; finally, the IMFs components done by adaptive filter, residual IMFs components and the residual component are reconstructed to obtain the image without noises, which is called the BEMD-KPCA method. The experimental results of removing noises from noisy images illustrate that the proposed approach to improve the edge effect of the BEMD gives a better result than the others in suppressing edge effects during processing the actual image, and the denoise image can maintain the edges and original details information of the image.

2.1 FUNDAMENTALS OF BEMD ALGORITHM

The BEMD algorithm utilizes extreme values, which are detected in the original image with noises or structured by the first derivative and higher-order derivative from the original image, to decompose the image signal [20]. The distance between extreme values can provide information to represent the image on the intrinsic length scale.

For a \( m \times n \) bi-dimensional image, the basic procedure of BEMD can be summarized as follows [6, 11, 12]:

1. Initialize the image with noise, to set \( n_0(x, y) = f(x, y) \) as the input image, and \( k = 1 \).
2. Initialize the parameters, set \( b_{k,0}(x, y) = r_{k-1}(x, y) \), and extract extrema points involved in the \( b_{k,1}(x, y) \).
3. By cubic spline, interpolate between local maxima and minima respectively, to obtain two envelope surfaces \( e_{max}(x, y) \) and \( e_{min}(x, y) \).
4. Calculate the mean envelope surface in terms of these two envelope surfaces, given by
   \[
   e_{mean}(x, y) = \frac{(e_{max}(x, y) + e_{min}(x, y))}{2}
   \]
5. Extract the residual \( b_{k,j}(m, n) \), which is equal to image \( b_{k,j-1}(x, y) \) subtracting mean envelope \( e_{mean}(x, y) \), given by
   \[
   b_{k,j}(x, y) = b_{k,j-1}(x, y) - e_{mean}(x, y)
   \]
6. Calculate the standard deviation SD, given by
   \[
   SD = \frac{\sum_{x=1}^{M} \sum_{y=1}^{N} \left| b_{k,j-1}(x, y) - b_{k,j}(x, y) \right|^2}{b_{k,j-1}^2(x, y)}
   \]
7. Repeat steps (2) to (6) until the calculated standard deviation is less than a predetermined criterion SD (the threshold of SD is generally taken as 0.2–0.3) and stop the iteration. Once this process is achieved, and the resulting signal will be considered as a proper mode, which is regarded that \( b_{k,j}(x, y) \) represents an intrinsic mode function (IMF), denoted as \( imf_k(x, y) = b_{k,j}(x, y) \).
8. Calculate the residual by the following relation
   \[
   r_k(x, y) = r_{k-1}(x, y) - imf_k(x, y)
   \]
9. Repeat steps (1) to (8), up to the \( K \)th times, \( K = (k + 1) \), when the residual \( r_k(x, y) \) is a monotonic signal, stop the process of BEMD decomposition and finally obtain all IMF components. Once the decomposition process has finished, the original image can be expressed as
   \[
   f(x, y) = \sum_{k=1}^{K} imf_k(x, y) + r_k(x, y)
   \]

The end of the decomposition process and the final residual component in BEMD is directly determined by the stopping criterion. Therefore, the choice of SD value is closely related to the number of IMFs obtained by BEMD decomposition. The SD value should be correctly selected according to the actual situation. To depict the details of the image, it should be the best
choice for a lower SD value [20]. The SD value in this paper is chosen to be 0.25.

In the result of BEMD decomposition, the residual error obtained by BEMD decomposition of 2D signals often contains some information about the original image features and characteristics [20]. In the following decomposition process, the factors discussed above cannot be ignored and must be considered for some influences and contributions to the original image.

3 IMAGE DENOISING BY USING THE BEMD-KPCA

3.1 Image denoising

Some noise may be mixed up with pure image information during image acquisition. Therefore, the real signal of the image can generally be regarded as a composite of noise and pure image information, which can be expressed as

\[ g(x, y) = f(x, y) + q(x, y) \]  

where \( g(x, y) \) is the real image with noises, \( f(x, y) \) is the original image without noises and \( q(x, y) \) is the noise.

After the real image with noises \( g(x, y) \) is decomposed by BEMD, assuming the number of IMF decomposition is \( k \), given by

\[ g(x, y) = \sum_{i=1}^{k} f_{\text{imf}_i}(x, y) + r_f(x, y) \]  

where \( f_{\text{imf}_i}(x, y) \) is the IMF components of \( g(x, y) \), and \( r_f(x, y) \) is their residual component. The removal noise in the image means that the image noise content \( q(x, y) \) is removed from \( g(x, y) \). The residual content \( r_f(x, y) \) decomposed by BEMD from \( g(x, y) \) will be neglected for small noise contribution.

The image with noise can be decomposed into some IMFs and residual trend component by BEMD algorithm. These IMFs not only include some high-frequency contents in the original real image, but also contain the contribution of the IMFs from the noise image [20, 24]. The noises in the image mainly exist in the high-frequency component. Therefore, it is a key problem for the image denoising to eliminate the high-frequency components with noise from image decomposition results. The noise in the image has an important contribution to some IMFs, but little to other low-frequency IMFs. Therefore, only by removing high-frequency IMFs which are full of noise components, the real image can be reconstructed by synthesizing the residual IMFs and the remaining trend component from the original noisy image [25].

3.2 KPCA

Assume \( X = [x_1, x_2, ..., x_m]^T \) to be an \( m \times n \) sample matrix of \( x_i \), the sample vector of \( X \). The mean value of \( x_i \) is estimated as

\[ \mu_i = \frac{1}{n} \sum_{j=1}^{n} X_i(j) \]

and then the sample vector \( X_i \) is centralized as

\[ \bar{X}_i = X_i - \mu_i = [\bar{x}_i^T, \bar{z}^T_i, ..., \bar{z}_m^T]^T \]

where \( \bar{x}_i = x_i - \mu_i \). Accordingly, the centralized matrix of \( X \) is

\[ \bar{X} = [\bar{x}_1^T, \bar{z}^T_1, ..., \bar{z}_m^T]^T \]

Finally, the co-variance matrix of the centralized dataset is calculated as

\[ \Omega = \frac{1}{n} \bar{X} \bar{X}^T \]

In PCA transformation, an orthonormal transformation matrix \( P \) is calculated to decorrelate \( \bar{X} \), that is \( \bar{Y} = P \bar{X} \) such that the covariance matrix of \( \bar{Y} \) is diagonal. The co-variance matrix \( \Omega \) of \( \bar{X} \) is symmetrical, so it can be written as

\[ \Omega = \Phi \Lambda \Phi^T \]

where, \( \Phi = [\Phi_1, \Phi_2, ..., \Phi_m] \) is the \( m \times m \) orthonormal eigenvector matrix and \( \Lambda = \text{diag}\{\lambda_1, \lambda_2, ..., \lambda_m\} \) is the diagonal eigenvalue matrix with \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m \). The terms \( \Phi_1, \Phi_2, ..., \Phi_m \) and \( \lambda_1, \lambda_2, ..., \lambda_m \) are the eigenvectors and eigenvalues of \( \Omega \) respectively. By putting

\[ P = \Phi^T \]

\( \bar{X} \) can be decorrelated, that is \( \bar{Y} = P \bar{X} \) and \( \Lambda = (1/n) \bar{Y} \bar{Y}^T \)

In PCA, the energy of a signal will concentrate on a small subset of the PCA transformed dataset, while the energy of noise will evenly spread over the whole dataset, that is, it fully de-correlates the original dataset, separating signal from noise [36].
However, PCA is a linear data processing method, which cannot
detect all the structures in a given data, especially some
nonlinear structures. In KPCA a set of multidimensional signals
$X_k$ ($k = 1, 2, \ldots, K$), is envisaged to be mapped through a non-
linear function $\Phi(X_k)$. In feature space then a linear PCA is per-
formed estimating the eigenvectors and eigenvalues of a matrix
of outer products, called a scatter matrix which for zero mean
data is given by $\Omega = \Phi \Phi^T$. These eigenvectors and eigenvalues
are related to the eigenvectors and eigenvalues of inner product
matrix (called kernel matrix $K = \Phi^T \Phi$). The central core matrix
can be expressed as [23],

$$
K_{ij} = \left( I - \frac{1}{K} \sum_{k=1}^{K} k_i k_j^T \right) \Phi^T \Phi \left( I - \frac{1}{K} \sum_{k=1}^{K} k_i k_j^T \right)
$$

(15)

where $k = [1, \ldots, 1]^T$ is a vector with dimension $K \times 1$, and
$I$ is a $K \times K$ identity matrix. Each element $k(i,j)$ of the kernel
matrix depends on the inner product $\Phi^T \Phi$, which can be
computed using only the data $X_k$ in input space.

The eigenvalues of the scatter matrix $\Omega$ coincide with the
eigenvalues of the kernel matrix $K$, the intrinsic composition
of $K_{ij}$ provides the necessary information to compute the pro-
tection of a vector of the input space $y_j$ in the feature space.
Considering the matrix $V$, the columns of which represent the $L$
eigenvectors of the kernel matrix, and $D$, a diagonal matrix
with the corresponding $L \leq K$ eigenvalues of both matrices, the
image $\Phi(y_j)$ of a point in input space, can be projected onto the $L$
directions spanned by the eigenvectors of the scatter matrix via,

$$
\varphi = D^{-1/2} \Phi^T \left( I - \frac{1}{K} \sum_{k=1}^{K} k_i k_j^T \right) \Phi^T \Phi y_j
$$

(16)

where $\Phi^T \Phi y_j$ represents a vector the components of which can be
computed using the kernel trick by,

$$
k_{ij} = \left[ k(X_{1},y_j), k(X_{2},y_j), \ldots, k(X_{n},y_j) \right]^T
$$

(17)

In the application of denoising, it is necessary to recon-
struct any data point in the feature space from the noise source
by using L principal component [23]. Interested in the position
of the corresponding points in the input space, the pre-
image of the denoised data sample in feature space needs to be
estimated.

### 3.3 The number of IMFs with noises

Speckle noise filtering method based on BEMD, its decompo-
sition after the former one or two IMFs components directly
remove all constituted by the noise, and then add up the recon-
struction of the rest in the IMF to achieve the purpose of noise.
However, top IMFs still contain more detailed information after
the noised image is decomposed by BEMD. SAR image denois-
ing method based on direct reconstruction of residual IMFs that
will lose a lot of details in the image. It is a very crude denois-
ing method, which will lead to the serious distortion in the
image.

From the above discussion, it is unreasonable to remove the
IMFs with noises directly. On the one hand, the number of IMFs with noises is not effectively determined. On the other
hand, the removed IMFs still contain more detailed information.
If the above operations are done, which will cause serious
images distortion for the SAR image. Based on the above prob-
lems, the kernel principle component analysis (KPCA) method
is introduced into SAR image processing [21, 26, 27]. Firstly,
the appropriate numbers of the IMFs with noises are selected
by the KPCA algorithm, then those selected noisy IMFs are fil-
tered by the wavelet to remove the noise from the image $g(x,y)$,
finally the filtered IMFs, the IMFs without noises and the resid-
uals component are reconstructed to generate the pure SAR
image.

Assume that $g(x,y)$ is a SAR image with noises, after the
image with noise is decomposed by BEMD, the IMF of the $K$
layer is $imf_k$. For the convenience of discussion, let $imf_k = f_k$,
then there is [37]:

$$
f_k = X = \begin{bmatrix}
x_1^1 & x_2^1 & \cdots & x_m^1 \\
x_1^2 & x_2^2 & \cdots & x_m^2 \\
\vdots & \vdots & \ddots & \vdots \\
x_1^n & x_2^n & \cdots & x_m^n
\end{bmatrix}
$$

(18)

The energy of the $K$th layer $imf_k$ is $\varepsilon(f_k)$. $\varepsilon(f_k)$ is defined as

$$
\varepsilon(f_k) = f_k f_k^T
$$

(19)

Assume

$$
f_k = g_k + W_k
$$

(20)

where, $g_k$ is the image component. $W_k$ is noise components in
the $K$th layer IMF.

$$
f_k - \bar{E}(f_k) = g_k + W_k - \bar{E}(g_k) + \bar{E}(W_k) = \bar{g}_k - \bar{E}(g_k) + \bar{W}_k - \bar{E}(W_k)
$$

(21)

where $\bar{E}$ is the mathematical expectation.

Assume that $\tilde{f}_k = f_k - \bar{E}(f_k)$, $\tilde{g}_k = g_k - \bar{E}(g_k)$, the Equation (20) computing time can be expressed as

$$
\tilde{f}_k = \tilde{g}_k + \bar{W}_k
$$

(22)

According to literature [37], the relationship between $\varepsilon(W_k)$
and $\varepsilon(W_k')$ is close seemingly satisfactory:

$$
\varepsilon(W_k) = \frac{\varepsilon(W_k')}{\beta} \rho^k, k \geq 2
$$

(23)

where, $\beta \approx 0.719, \rho \approx 2.01$. $\varepsilon(W_k')$ is the energy of noise con-
tained in the first layer of the IMF component.
Based on the Equation (15), assume
\[ C_{X} = E - (X - m_X)(X - m_X)^T \]
where, \( C_{X} \) is the covariance matrix of \( f_k \). Let \( \lambda_1 \geq \lambda_2 \geq ... \lambda_N \) is the eigenvalue of \( C_{X} \), and \( \varphi_1, \varphi_2, ..., \varphi_N \) is the corresponding feature vector. Let \( \Phi = [\varphi_1, \varphi_2, ..., \varphi_N]^T \), where \( \Phi \) is an orthonormal matrix vector.

Define \( Y = [Y_1, Y_2, ..., Y_N]^T = \Phi(X - m_X) \), from the PCA characteristics, the noise in \( g(x, y) \) is distributed in all components \( Y_i \), while noises are mainly concentrated in the components of the previous layers.

Then the \( \Phi \)-image of \( X \) can be reconstructed from its projections onto the first \( H \leq L \) principal components in the IMF by using a projection operator \( P_H \) as
\[ P_H \Phi(x) = \sum_{k=1}^{H} \lambda_k V^k \]

The procedure of kernel PCA is equivalent to that of standard PCA on the mapped data [37]. When \( H = L \), the \( \Phi \)-image is completely reconstructed,
\[ \Phi(x) = P_L \Phi(x) = \sum_{k=1}^{L} \lambda_k V^k \]

When \( H < L \), the previous \( H \) suitable principal components (IMF) are selected for reconstruction, then the approximate reconstruction value \( \Phi(x) \) is,
\[ \Phi(x) = \sum_{k=1}^{H} \lambda_k V^k \]

According to the decomposition characteristics of the KPCA, the image information part in \( \Phi(x) \) is mainly concentrated in the principal components of the previous layers,
\[ \varepsilon(\Phi(x)) = (\sum_{k=1}^{L} \lambda_k V^k)(\sum_{k=1}^{L} \lambda_k V^k)^T = \sum_{k=1}^{L} \lambda_k^2 \]

According to the Equations (25) and (28),
\[ r = \sum_{k=H+1}^{L} \lambda_k^2 / \sum_{k=1}^{L} \lambda_k^2 = \left[ \frac{\varepsilon(W_k)}{\varepsilon(f_k)} \right]^d \]

where, \( L \) is the number of IMF decompositions, the number of retained components is determined by the size of \( r \), and \( H \) is the number of the stopping filter layer.

In order to make the deleted noise energy equal to the noise energy contained in \( \Phi(f_k) \), a suitable \( H \) should be selected to make the Equation (29) hold. However, the noise is distributed in all the principal components, \( H \) value according to the Equation (29) will inevitably cause the loss of detail information.

In fact, \( H \) is difficult to choose. The parameter \( \theta \) \((0 < \theta < 1)\) is introduced in this paper, and the energy of noise component is satisfied [37],
\[ \sum_{k=H+1}^{N} \lambda_k^2 / \sum_{k=1}^{N} \lambda_k^2 \leq \theta \cdot \left[ \frac{\varepsilon(W_k)}{\varepsilon(f_k)} \right]^d \]

The choice of \( H \) in this paper is based on the following values, that is, if there is \( \eta \), let the following Equation (30) hold, then take \( H = \eta \).
\[ \sum_{k=H+1}^{N} \lambda_k^2 / \sum_{k=1}^{N} \lambda_k^2 \leq \theta \cdot \left[ \frac{\varepsilon(W_k)}{\varepsilon(f_k)} \right]^d \leq \sum_{k=H_1}^{N} \lambda_k^2 / \sum_{k=1}^{N} \lambda_k^2 \]

Based on the analysis of the PCA method, the number of layers decomposed by BEMD and the number \( H \) of filtered IMFs is finally determined.

### 3.4 Treatment of the edge effect

In the process of the BEMD, the Delaunay triangulation plays an important role. The effect of Delaunay triangulation is more effective when the point set is a convex set, but the split effect is affected when there are concave or multi-connected regions in the set. In addition, triangulation interpolation cannot be extrapolated due to edge effect. At the same time, in the process of modal decomposition, the edge of the data set will be divergent and distorted, which will pollute the data set and destroy the decomposition results. As this process continues, the edge effect will become more serious, which will eventually lead to the distortion of the IMFs for the decomposed image. To reduce or eliminate the edge effect in the process of the BEMD operation, a new approach to deal with the edge effect is proposed in this paper. The steps are as follows (Figure 1).

1. Extend the edge of the image to extract extreme points: $f_t(x, y) \in [f(x, y_1), ..., f(x, y_n), f(x_1, y_1), ..., f(x_n, y_n)]$ with a period length of $2T$. $f_{t+1}(x, y) \in [f(x, y_1), ..., f(x, y_n), f(x_1, y_1), ..., f(x_n, y_n)]$. 

![Flowchart of the BEMD-KPCA algorithm denoising scheme](attachment:flowchart.png)
where, $t_l \in [t_1, \ldots, t_9, t_1, \ldots, t_9]$. 

II. Symmetrical expansion processing is performed at the right edge of $f_1(x, y)$, that is, the original signal is mirrored on the right edge of $f_1(x, y)$, and is connected with $f_1(x, y)$ to form an even image $f_{2}(x, y)$ with a period length of $3T$.

\[
\begin{align*}
\tilde{f}_2(x,y) \in & \{f(x_1,y_1), \ldots, f(x_2,y_2), f((x_1,y_1))', \ldots, \}
\end{align*}
\]

where, $\tilde{f}_2 \in \{t_1, \ldots, t_9, t_1, \ldots, t_9 \}$.

III. Similarly, the periodic expansion is performed on the upper and lower sides of left and right extension image $f_{2}(x, y)$, respectively, to obtain images $f_{3}(x, y)$ and $f_{4}(x, y)$.

Upper extension image:

\[
\begin{align*}
\tilde{f}_3(x,y) \in & \{f(x_1,y_1), \ldots, f(x_2,y_2), f((x_1,y_1))', \ldots, \}
\end{align*}
\]

Lower extension image:

\[
\begin{align*}
\tilde{f}_4(x,y) \in & \{f(x_1,y_1), \ldots, f(x_2,y_2), f((x_1,y_1))', \ldots, \}
\end{align*}
\]

where, $\tilde{f}_2 \in \{t_1^1, \ldots, t_9^1, t_1, \ldots, t_9^1 \}$.

\[
\begin{align*}
\tilde{f}_3 \in & \{t_1^2, \ldots, t_9^2, t_1, \ldots, t_9^2, \}
\end{align*}
\]

V. The periodic extended image $A_2(x,y)$ is decomposed by BEMD, and the extension part of the IMF component is discarded to get the first IMF component after decomposition.

1. According to the termination condition SD of IMF screening, if the decomposition condition of the next layer of IMF is satisfied, the residual after the subtraction of the mean value will continue to be the edge period extended;
2. Repeat the above 1–3 steps until the end of the screening. Then, the values in the length $T$ of the first and last time periods of each component are emptied, and the component IMF$_{2}(x, y)$ and the final scale trend item $r_9(x,y)$ of the edge periodic extension are obtained.
3. According to Equations (8) and (9), the IMFs with the noise of the former H term is processed by wavelet filter, and then the filtered IMF components and the unprocessed low-frequency IMF components and the remainder are reconstructed to get the denoised image $f'(x, y)$.

### 4 ESTIMATION METHOD OF IMAGE

In the process of image denoising, some image quality indexes such as the Mean Squared Error (MSE), Standard Deviation (SD), Peak Signal Noise Ratio (PSNR), Equivalent Number of Looks (ENL) and Edge Preservation Index (EPI) are used to compare the filtering effect. MSE reflects the average gray level of the image, that is the average backscatter coefficient of the contained object [28]. The magnitude of the SD indicates the amount of information in the image [29]; ENL is an index that measures the relative intensity of coherent speckle noise in an image and describes the degree of averaging applied to the measurements during data formation and post processing [30, 31]. The larger the ENL number is, the weaker the speckle on the image is, and the better its interpretation is, which is defined as follows:

\[
\text{ENL} = I^2 / \sigma^2
\]

where, $I$ is the mean of all pixels in the image, and $\sigma$ is the standard deviation.

The edge preservation index (EPI) is a quantitative measure of edge preservation. In image noise suppress it is interested in despeckling while preserving the edges. For perfect edge preservation, EPI is required to be unity. Mathematically EPI is defined as [32]

\[
\text{EPI} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} |\hat{x}(i,j+1) - \hat{x}(i,j)|}{\sum_{i=1}^{n} \sum_{j=1}^{m} |x(i,j+1) - x(i,j)|}
\]

where $\hat{x}$ is de-speckled SAR image, $x$ is original SAR image, $n$ is the number of columns in SAR image and $m$ is the number of rows in SAR image. The higher the value of edge retention index, the better the preservation ability of the edges of the image. A better denoised SAR image is generally having a reasonable EPI value.

The PSNR is the ratio between the maximum possible power of a signal and the power of corrupting noise, which is the factor to measure the effect of image enhancement algorithm on image quality [15, 33]. The value of PSNR must be maximum as much as possible, and the higher the value means, the higher the image denoising quality will be. It is usually expressed and defined as [15, 32, 33].

\[
\text{PSNR} = 10 \log \frac{255^2}{\text{MSE}}
\]

where MSE is the mean squared error, given by

\[
\text{MSE} = \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} [f(x,y) - f'(x,y)]^2
\]
where \( f(x, y) \) and \( f'(x, y) \) are the original image without noise and denoised SAR image respectively. \( M \) is the number of columns in SAR image and \( N \) is the number of rows in SAR image. If MSE value is small, the PSNR value will be high and vice versa. The MSE value must be minimum as much as possible.

5 EXPERIMENT AND ANALYSIS

5.1 The decomposition of SAR image

To show the efficiency of the proposed approach and the performance, the examples are performed in the following. Consider a gray Lotus image (500 × 500 pixel) white noise given in Figure 2(a). Under the environment of matlab software, A Gaussian white noise is added into it to form a contaminated image by the white noise, where the mean square deviation of Gaussian white noise is 0.03. Traditional Lee filtering algorithm, BEMD algorithm and BEMD-KPCA algorithm are used to denoise the image in this paper. In the experiment, \( 3 \times 3 \) formwork is used in Lee filtering method, while the decomposition layers of the IMF in BEMD are four. This noisy image will be used in traditional Lee filtering algorithm, BEMD algorithm and BEMD-KPCA algorithm to denoise noises in the Lotus image, and evaluate the effectiveness of the proposed method in image denoising.

The Lotus image with Gaussian white noise will be decomposed by the BEMD coupled with the proposed approach to improve the edge effect into a few IMFs. Considering the high frequency characteristics of the noise, it can be found that the noise mainly involves the high frequency components of BEMD decomposition. In the first experiment of image denoising using BEMD algorithm, the first decomposition of the noisy image is done for only one time to extract an IMF, and then this IMF, as a new image, completes another decomposition, obtaining another IMF. Repeat the above steps, the top four IMFs resulting from the image may be approximately regarded as the noises. Then subtracting them from the original noisy image produces a new image, which is the result of image denoising (Figure 2).

From the Figure 2(b–e), it clearly shows that IMFs edges decomposed by the traditional BEMD method appear edge pollution phenomenon in the process of the IMF's decomposition since the triangle subdivision interpolation cannot be extrapolated. The top four-layer decomposition results of IMFs using Delaunay triangulation subdivision are given in Figure 2(d,e). This noisy image will be used as the first example to illustrate the effectiveness of the BEMD-KPCA method in image denoising.

The experimental results show that the edge of the IMFs component obtained from the second IMF2 decomposition by the traditional BEMD is divergent (black pollution), although the black pollution is not serious at this time, but it still has clearly visible edge distortion; from the third component of IMFs, its edge pollution (indicated by the dotted oval in Figure 2(d,e)) has become more serious than the second IMF component. The bad black pollution at each corner is clearly visible and a great spreading trend from four corners to the center. As the BEMD decomposition continued, edge effects of IMFs component decomposed by the traditional BEMD will gradually increase, which will affect the integrity of the IMFs components. Similarly, the first example of the denoising of the image with Gaussian white noise, the top four layers of IMF components decomposed by the BEMD-KPCA method is given in Figure 3. Obviously, the edge effect has been effectively suppressed, when the edge effect is treated by the mirror expansion approach in the decomposition of the improved BEMD. This result is better than that from the traditional BEMD algorithm without consideration of the edge effect, which shows that it is very important for image denoising by the BEMD method to improve the edge effect, and illustrates that the proposed approach is feasible in the treatment of the edge effect.

To evaluate the effectiveness of the BEMD-KPCA method in image denoising, Figure 4 shows the denoising results from Lee filter, the traditional BEMD algorithm, and the proposed approach (BEMD-KPCA), respectively. The proposed
approach gives a more satisfactory result than the others. Any spots contaminated by the noise almost cannot be seen from the denoised image. Compared with the proposed approach, the images done by the traditional BEMD and Lee filter are still some noises.

To interpret the effectiveness of the proposed denoising method, the PSNR is used as the measurement index of denoising. From the histogram of PSNR with different denoising methods (Figure 5), it can be seen that the values after denoising increase to some extent for any one of the above three denoising methods. The BEMD-KPCA algorithm proposed in this paper gets a first in increasing the PSNR, and is better than that of traditional BEMD and Lee filtering method. The proposed approach reduces the noise of the image and retains the details of the image as many as possible. The traditional BEMD results in higher values of PSNR than the Lee filter, but it loses some high frequency detail compositions in image resulting in image blurring.

5.2 Estimation of SAR image denoising effect

To further analyze the availability of our proposed method, we also apply it on the spaceborne SAR image (500-pixel × 500-pixel, L-band) with speckle noises (see Figure 6(a)). It can be clearly seen that the image has an obvious terrain features area. The matlab R2015b software is used to implement the proposed method. The traditional BEMD method, the Lee filter and the BEMD-KPCA method are adapted to remove noises for comparative analysis in the experiment. In the experiment, 3 × 3 formwork is used in the Lee filter method. The IMFs with the traditional BEMD belongs to the four-layer decomposition. Figure 6 shows the qualitative comparison of the proposed despeckling method with various existing methods.
As can be seen from the Figure 6, speckle noise in SAR images is removed to a certain extent after different filtering methods. Both Lee filter and the BEMD method can reach an acceptable result of the denoising, as shown in Figure 6b,c. However, compared with the denoised image by the proposed approach, those from both still maintain a definite distance. Some noise from the original image is removed by the Lee filter, but some edge details in the SAR image are not retained after filtering. Compared with the Lee filter and the BEMD-KPCA, the filtered SAR image processed by the BEMD has less noise, but the loss of edge detail is more serious. Obviously, the edge effect has a serious influence on the result of the denoising, which shows that some contents of the noise enter the other IMFs except for the first IMF.

From Figure 6(d), it is seen that the BEMD-KPCA approach gives the best result of removing the speckle noise. When the edge effect is treated by the KPCA in the decomposition of the BEMD, this result is better than that from the above two methods. The above results illustrate that the schedule combining the extrapolation expansion with the KPCA algorithm as BEMD-KPCA can effectively improve the edge effect of the BEMD operation and get the reliable decomposed results, thus enhancing the image denoising performance of the BEMD by a big margin.

To quantitatively compare and analyze the denoising effect of the proposed method, three regions (30-pixel × 30-pixel, 25-pixel × 25-pixel and 50-pixel × 50-pixel) are selected in the image, as shown in Figure 6(a). According to the definition of the ENL, the values for each region before and after denoising are calculated. All results are listed, as shown in Table 1. Compared with the ENLs before denoising, the values after denoising increase to some extent for any one of the above three denoising methods. The proposed approach gets a first in increasing the ENL, and the next one is the traditional BEMD.

| Table 1 | Comparison of ENL on the regions |
|---------|----------------------------------|
| Denoising method | Region 1 | Region 2 | Region 3 |
| Original SAR image | 4.3641 | 3.532 | 4.354 |
| Lee filter | 65.793 | 86.482 | 79.372 |
| BEMD | 65.465 | 87.072 | 80.763 |
| BEMD-KPCA | 86.376 | 99.437 | 97.347 |

Relatively, the Lee filter plays a worst role in improving the ENL after denoising. For the denoising effect of the three regions, the ENL of the BEMD-KPCA is 86.376, 99.437 and 97.347, respectively. Compared with the denoising effect of the Lee filter and BEMD method, the BEMD-KPCA method also has a significant advantage.

The index values of the whole image (Figure 6(a)) before and after denoising are shown in Table 2. It can be seen that MSE and SD of the image after denoised by the BEMD-KPCA algorithm are the smallest. Compared with the Lee filter and the BEMD method, the SD of the image denoised by the BEMD-KPCA algorithm reduced by 1.651 and 1.021 respectively. In terms of the PSNR of the image, the value of the image denoised by BEMD-KPCA algorithm is the highest, compared with the Lee filter and the BEMD method, and that of the image denoised by BEMD-KPCA method is increased by 1.4 and 0.892 respectively, which shows that the BEMD-KPCA algorithm can effectively separate the signal and noise in different frequency domains. In the aspect of EPI, the BEMD-KPCA algorithm is better than other filtering methods, which shows that the proposed approach has obvious advantages in image edge detail maintenance. According to the above analysis, the BEMD-KPCA algorithm has the best visual effects, denoising better and preserving image edges.

Using the Envisat SAR image (size of 512-pixel × 512-pixel) of the eastern region in Las Vegas with noise, the third experiment is introduced to further test the denoising effect of the proposed approach. Figure 7 presents comparison results of the denoised and edge retained effects for the Lee filter, BEMD, and BEMD-PCA, respectively. In this experiment, the size of formwork in Lee filter is 3 × 3, and decomposition layers of the BEMD method are four.
FIGURE 7 Comparison of denoised results for different filtering algorithms: (a) original SAR image, (b) Lee filter, (c) Wavelet filter and (d) BEMD-PCA

TABLE 3 Comparison of the de-noised effect and the edge retained effect

| Denoising method | MSE     | SD      | PSNR   | EPI   |
|------------------|---------|---------|--------|-------|
| Original image   | 132.762 | 11.522  | 26.900 | 1.000 |
| Lee filter       | 107.337 | 10.360  | 27.823 | 0.612 |
| BEMD             | 98.895  | 9.945   | 28.179 | 0.699 |
| BEMD-KPCA        | 84.135  | 9.173   | 28.881 | 0.768 |

As can be seen from the Figure 7, the BEMD-KPCA algorithm has the most obvious denoising effect compared with Lee filter and the traditional BEMD algorithm. The image denoised by the BEMD-KPCA algorithm is smoother, and image outline is clearer. The BEMD method loses many high frequency detail compositions, so the image is blurring. The results show that the BEMD-KPCA algorithm can effectively remove speckle noise from the SAR image, and retain the details of the image as many as possible.

The evaluation indexes of the denoising effect are shown in Table 3. As for the three algorithms, denoised images have larger PSNR and ENL than these of the original image. The PSNR and ENL in denoised image by the BEMD-KPCA algorithm is the largest, showing the best denoising and edge preserving ability. The MSE and SD of the three algorithms become less, and the MSE and SD of the BEMD-KPCA algorithm are the least, showing the best denoising ability. According to the definition of MSE, SD, PSNR, and EPI for the image before and after denoising, the data in Table 3 are corresponding to the visual effects in Figure 7. The above results indicate that the proposed approach really provides an effective approach for improving the edge effect of the BEMD operation to get reliable decomposed results and then enhance the image denoising performance.

6 CONCLUSIONS

The BEMD approach is widely used in the SAR image denoising through the decomposed results on different intrinsic scales, however, the edge effect is a noticeable problem. To reduce the influence of edge effects, the BEMD-KPCA algorithm is proposed in this study to improve the edge effects to enhance the denoising performance in the BEMD decomposition. In the edge treatment, the first one is an image mirror expansion operation through the extrapolates data toward the outside regions on the left and right sides of the image respectively, and the second is an expansion by the upper and lower mirror expansion with respect to the extrema nearest to the edge of the data resulted from the first step operation.

With consideration of the fact that the highest noise content of a SAR image is extracted into the front several layers of IMFs, the KPCA method is used to determine the number of IMFs containing the noise. This work introduces a procedure of the IMFs with noise filtered by adaptive filter approach. The IMFs with noise after filtering, the remaining IMFs and residual components are synthesized to obtain the reconstructed image, which is the denoised result.

After the analysis of SAR images with speckle noise denoised by the Lee filter, the BEMD method and the proposed approach, it is found that the BEMD-KPCA algorithm can enhance the performance of SAR image denoising. The BEMD-KPCA algorithm in this paper not only overcomes the shortcoming of the BEMD without edge treatment but also improves the edge effect of the BEMD processing. Compared with the image quality indexes such as MSE, SD, PSNR, ENL and EPI, the SAR image filtered by the BMED-KPCA algorithm has the best performance in the denoising effect.

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REFERENCES

1. Sharma, R., Panigrahi, R. K.: Improved patch-based NLM polsar speckle filter based on iteratively re-weighted least squares method. Iet Radar Sonar Nav. 12(1), 30–36 (2018)

2. Alparone, L., et al.: An adaptive order-statistics filter for sar images. Int. J. Remote Sens. 17(7), 1357–1365 (1996)

3. Abergel, R., et al.: Subpixelic methods for sidelobes suppression and strong targets extraction in single look complex SAR images. IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing 11(3), 775–786 (2018)

4. Sumantyo, J. T. S., Amini, J.: A model for removal of speckle noise in SAR images (Alos Palsar). Can. J. Remote Sens. 36(6), 503–515 (2008)

5. Antropov, O., et al.: Volume scattering modeling in polarim sar decompositions: study of alos palsa data over boreal forest. IEEE Trans. Geosci. Remote Sens. 49(10), 3838–3848 (2011)

6. Wakabayashi, H., Arai, K.: A method of speckle noise reduction for SAR data. Int. J. Remote Sens. 17(10), 1837–1849 (2010)

7. Huang, N. E., et al.: The empirical mode decomposition and the hilbert spectrum for nonlinear and non-stationary time series analysis. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences 454, 903–995 (1998)

8. Garcia, P. A., et al.: Fused empirical mode decomposition and wavelets for locating combined damage in a truss-type structure through vibration analysis. J. Zhejiang Univ. Sci. 14, 615–630 (2013)

9. Qiao, L., et al.: Perfect reconstruction image modulation based on bemd and quaternionic analytic signals. Sci. China Inf. Sci. 54(12), 2602–2614 (2011)

10. Zha, K., et al.: Incipient fault diagnosis of roller bearings using empirical mode decomposition and correlation coefficient. J. Vibration Engineering 15, 597–603 (2013)

11. Song, H., et al.: Analysis of ocean internal waves imaged by multichannel reflection seismics, using ensemble empirical mode decomposition. J. Geophys. Eng. 9, 302–311 (2012)

12. Nunes, J. C., et al.: Image analysis by bidimensional empirical mode decomposition. Image Vision Comput. 21(12), 1019–1026 (2003)

13. Nunes, J. C., et al.: Texture analysis based on local analysis of the bidimensional empirical mode decomposition. Mach. Vision Appl. 16(3), 177–188 (2005)

14. Bhiyan, S. M., et al.: Fast and adaptive bidimensional empirical mode decomposition using order-statistics filter based envelope estimation. EURASIP J. Adv. Signal Process 2008(1), 1–18 (2008)

15. An, F. P., et al.: Enhancing image denoising performance of bidimensional empirical mode decomposition by improving the edge effect. International Journal of Antennas and Propagation 2015, 1–12 (2015)

16. Biswas, M., Dey, D.: Bi-dimensional statistical empirical mode decomposition-based video analysis for detecting colon polyps using composite similarity measure. Adv. Intell. Syst. Comput. 309(51), 297–308 (2015)

17. Damerval, C., et al.: A fast algorithm for bidimensional emd. IEEE Signal Process Lett. 12(10), 701–704 (2005)

18. Liu, Z., Peng, S.: Boundary processing of bidimensional emd using texture synthesis. IEEE Signal Process Lett. 12(1), 33–36 (2004)

19. Mahraz, M. A., et al.: Motion estimation using the fast and adaptive bidimensional empirical mode decomposition. J. Real-Time Image Pr. 9(3), 491–501 (2014)

20. An, F. P., et al.: Edge effects of bemd improved by expansion of support-vector-regression extrapolation and mirror-image signals. Optik 126(21), 2985–2993 (2015)

21. Int, J. K., et al.: Tangent hyperplane kernel principal component analysis for denoising. IEEE Trans. Neural. Netw. Learn. Syst. 23(4), 644–656 (2012)

22. Galinsky, K. J., et al.: Fast principal-component analysis reveals convergent evolution of adh1b in Europe and East Asia. Am. J. Hum. Genet. 98(3), 456–472 (2016)

23. Teixeira, A. R., et al.: Kpca denoising and the pre-image problem revisited. Digit. Signal Process. 18(4), 568–580 (2008)

24. Zheng, Y., Qin, Z.: Region-based image fusion method using bidimensional empirical mode decomposition. J. Electron. Imaging 18(1), 013008 (2009)

25. Guo, S., et al.: Self-adaptive image denoising based on bidimensional empirical mode decomposition (bemd). Biomed. Mater. Eng. 24(6), 3215–3222 (2014)

26. Abdi, H., Williams, L. J.: Principal component analysis. Wiley Interdiscip. Rev. Comput. Stat. 2(4), 433–459 (2010)

27. Rahman, M., Atta, G. K.: Coherence pursuit: fast, simple, and robust principal component analysis. IEEE Trans. Signal Process. 65(23), 6260–6275 (2017)

28. Linderholm, A.: 2d empirical mode decompositions in the spirit of image compression. In: AeroSense, (International Society for Optics and Photonics), SPIE, Bellingham, WA (2002)

29. April, G.,Arsenault, H. H.: Properties of speckle integrated with a finite aperture and logarithmically transformed. J. Opt. Soc. Am., 66, 1160–1163 (1976)

30. Cui, Y., et al.: Unsupervised estimation of the equivalent number of looks in sar images. IEEE Geosci. Remote. Sens. Lett. 8(4), 710–714 (2011)

31. Anfinsen, S. N., et al.: Estimation of the equivalent number of looks in polarimetric synthetic aperture radar imagery. IEEE Geosci. Remote. Sens. Lett. 47(11), 3795–3809 (2009)

32. Ishib, M., et al.: Sar image despeckling by selective 3d filtering of multiple compressive reconstructed images. Prog. Electromagn. Res. 134, 209–226 (2013)

33. Huynh-Thu, Q., Ghanbari, M.: Scope of validity of psnr in image/video quality assessment. Electron. Lett. 44(13), 800–801 (2008)

34. Zhang, L., et al.: Two-stage image denoising by principal component analysis with local pixel grouping. Pattern Recognit. 43(4), 1531–1549 (2010)

35. Sahu, B. K., Swarni, P. D.: Image denoising using principal component analysis in wavelet domain and total variation regularization in spatial domain. Int. J. Comput. Appl. 71(12), 40–46 (2013)

36. Selkhat, B. V. D. S., et al.: Principal component analysis based image denoising implemented using LPG and compared to wavelet transform techniques. IJESRT 5(6), 673–680 (2016)

37. Li, C. C., Wang, W. W: SAR image despeckling based on empirical mode decomposition and kernel principle component analysis. Journal of Wuhan University of Science and Technology 39(3), 224–230 (2016)