**J/ψ** production at the Tevatron and RHIC from s-channel cut

J.P. Lansberg* and H. Haberzettl†

*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 19, 69120 Heidelberg, Germany
†Center for Nuclear Studies, Department of Physics, The George Washington University, Washington, DC 20052, USA

E-mail: lansberg@tphys.uni-heidelberg.de, helmut@gwu.edu

**Abstract.**

We report on our recent evaluation of the s-channel cut contribution to J/ψ hadro-production. We show that it is likely significantly larger than the usual cut contribution of the colour-singlet model (CSM), which is known to underestimate the experimental measurements. Here the s-channel cut develops for configurations with off-shell quarks in the bound state. A correct treatment of its contribution requires the introduction of a four-point function, partially constrained by gauge invariance and limiting behaviours at small and large momenta. When the unconstrained degrees of freedom are fixed to reproduce the Tevatron data, we show that RHIC data are remarkably well reproduced down to very low transverse momenta $P_T$ without need of resummation of initial-state gluon effects. This unique feature might be typical of s-channel cut contribution.

**Keywords:** Quarkonia, Gauge invariance

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1. **INTRODUCTION**

More than ten years after the first measurements by the CDF Collaboration of the direct production of J/ψ and ψ′ at $\sqrt{s} = 1.8$ TeV [1, 2] we are still facing disagreements between theoretical predictions from the various available models and experimental studies of the cross section and the polarisation from the Tevatron and RHIC (for reviews see [3]).

CDF [4] recently confirmed their previous polarisation measurement [5] showing an unpolarised or slightly longitudinally polarised prompt J/ψ yield. This has reinforced the doubts cast on the dominance of the Colour Octet Mechanism (COM) coming from the application of NRQCD [6]. At the same time, many new results became available, e.g. the long-awaited NLO QCD corrections to the CSM [7] –showing significant enhancement of the cross section, an up-to-date proof [8] of NRQCD factorisation; an improved treatment of NRQCD factorisation in fragmentation regions where three heavy quarks have similar momenta [9] and last but not least a recent evaluation of the dominant $\alpha_s^5$ (NNLO) [10] correction to Υ(nS) production, the latter solving the longstanding conflict between the experimental measurements from the Tevatron at mid and large $P_T$ [11, 12] and the prediction from the CSM [13].

Considering that none of the existing theoretical approaches could reproduce all
available experimental data, we undertook in [14] a systematic study of the cut contributions due to off-shell and non-static quarks. In particular, we questioned the assumption of the CSM that takes the heavy quarks forming the quarkonium ($Q$) as being on-shell [13]. If they are not, the usual $s$-channel cut contributes to the imaginary part of the amplitude and need to be considered on the same footing as the CSM cut.

Current conservation for such off-shell configuration responsible for the $s$-channel cut imposes the introduction of an additional four-point function, or contact current [15], accounting for the interactions between the $c\bar{c}$ pair emitting the external gluon. In fact, this mechanism arises because of the possibility that the outgoing gluon is emitted by the particle interacting in the dressed $c - \bar{c} - J/\psi$ vertex (see Figure 1 (a)), as depicted in Figure 1 (b). The pair of on-shell quarks that makes the final $J/\psi$-gluon state is now in a colour-octet state which thus recovers the necessity for such configurations as a natural consequence of restoring gauge invariance.

Although current conservation imposes the introduction of such a 4-point vertex, it does not enable to relate it univocally to the 3-point one. Yet, there exist certain minimal requirements [15, 16] which such a 4-point function should satisfy. The 4-point function proposed in [14] provided a conserved current but was not entirely satisfactory since it contained poles (by construction similar to the basic direct and crossed contributions), and such poles for the contact current are unphysical and therefore should be avoided [15].

Another caveat to avoid was formerly identified by Drell and Lee [15]. Indeed, the minimal substitution prescription $\partial^\mu \rightarrow \partial^\mu + iQA^\mu$ ($Q$: charge, $A^\mu$: vector potential) in an effective Lagrangian corresponding to the dressed hadronic vertex is deficient in that it violates the high-energy scaling behaviour, because in avoiding poles for the 4-point function, it partially replaces the true momentum dependence of the vertices by constants.

Such an issue can be easily avoided in the approach which we shall follow and which was applied to pion photoproduction processes [17, 18, 19, 20]. As we shall show in the following, it is hence possible to build a 4-point vertex encompassing two limiting behaviours, when the final-state gluon is soft or hard [21]. In turn, we shall show that this enables to reproduce experimental data from the Tevatron up to mid $P_T$ by adjusting the unconstrained parameters of the 4-point vertex and hence to get a remarkable agreement with data from RHIC down to low $P_T$.

2. OUR APPROACH

2.1. The three-point function

We shall follow the approach developed in [14], where the transition $q\bar{q} \rightarrow Q$ is described by the 3-point function

$$\Gamma^{(3)}_\mu (p, P) = \Gamma (p, P) \gamma_\mu ,$$

(1)
where \( P \equiv p_1 - p_2 \) and \( p \equiv (p_1 + p_2)/2 \) are the total and relative momenta, respectively, of the two quarks bound as a quarkonium state, with \( p_1 \) and \( p_2 \) being their individual four-momenta. Ansatz \((1)\) amounts to representing the vector meson as a massive photon with a non-local coupling. The generic picture of the physical origin of the dressed vertex function \( \Gamma(p, P) \) is given in Figure 1(a). In the present work, we describe the relative-momentum distribution \( \Gamma(p, P) \) of the quarks phenomenologically as a Gaussian form, function of the square of the relative c.m. 3-momentum \( \vec{p}^2 \) of the quarks, which can be written in a Lorentz invariant form as

\[
\vec{p}^2 = -p^2 + \frac{(p \cdot P)^2}{M^2}.
\]

Explicitly, we have

\[
\Gamma(p, P) = Ne^{-\frac{\vec{p}^2}{\Lambda^2}},
\]

with a normalisation \( N \)—fixed by the leptonic-decay width \([14]\)— and a size parameter \( \Lambda \), which can be obtained from studies in relativistic quark models \([23]\).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Illustration of the mechanisms (a) contributing to the dressing of the 3-point function \( \Gamma^{(3)} \) and (b) responsible for the 4-point function \( \Gamma^{(4)} \): the external gluon is attached here within gluon loops of the dressed vertex, thus producing a diagram without poles and with a kinematic behaviour genuinely different from the initial 3-point vertex.}
\end{figure}

### 2.2. The four-point function: minimal substitution

Instead of directly employing the generalised Ward–Takahashi relations \([16]\) for the complete current, we will rather make use of an equivalent condition for the contact current similarly to what was done in \([17]\) in the pion-photoproduction case.

First let us write the 4-point function depicted in Figure 1(b) as

\[
\Gamma^{(4)} = -ig_s T^a_{ik} M^\mu_{\nu} \gamma^\mu,
\]

where \( g_s \) is the strong coupling constant, \( T^a_{ik} \) the colour matrix, and \( \mu \) and \( \nu \) are the Lorentz indices of the outgoing \( J/\psi \) and gluon, respectively. For simplicity, we have suppressed all indices on the left-hand side. The \( c - \bar{c} - J/\psi \) vertex function \( \Gamma^{(3)} \) with the kinematics of the direct graph is denoted here by \( \Gamma_1 \) and for the crossed graph by \( \Gamma_2 \), i.e., \( \Gamma_1 = \Gamma (c_1 - \frac{\vec{P}}{2}, P) \) and \( \Gamma_2 = \Gamma (c_2 + \frac{\vec{P}}{2}, P) \), as shown in Figures 3(a) and (b). The gauge-invariance condition for the contact current \( M^\nu_{\nu} \)
for the outgoing gluon with momentum $q$ reads now

$$q_{\nu}M'_{c} = \Gamma_{1} - \Gamma_{2}$$  \hspace{1cm} (4)

The contact current can be now constructed as usual $^{17,18,19,20}$ in terms of an auxiliary function $F = F(c_{1}, c_{2}, q)$ which contains the remaining unconstrained degrees of freedom of the problem. This gives

$$M'_{c} = \frac{(2c_{2} + q)\nu (\Gamma_{1} - F)}{(c_{2} + q)^{2} - m^{2}} + \frac{(2c_{1} - q)\nu (\Gamma_{2} - F)}{(c_{1} - q)^{2} - m^{2}},$$  \hspace{1cm} (5)

where we take $c_{1}^{2} = c_{2}^{2} = m^{2}$ and $P^{2} = M^{2}$ from the beginning, with $m$ and $M$ being the masses of the quark and the $J/\psi$, respectively. One easily verifies that this additional contact current satisfies the gauge-invariance condition (4).

$F(c_{1}, c_{2}, q)$ has now to be chosen so that the current (5) satisfies crossing symmetry (i.e., symmetry under the exchange $c_{1} \leftrightarrow -c_{2}$) and is free of singularities. Defining the constant $\Gamma_{0}$ as the (unphysical) value of the momentum distribution $\Gamma(p, P)$ when all three legs of the vertex are on their respective mass shells, we should then have $F(c_{1}, c_{2}, q) = \Gamma_{0}$ when $(c_{2} + q)^{2} = m^{2}$ or $(c_{1} - q)^{2} = m^{2}$.

A priori, $F = \Gamma_{0}$ everywhere should be satisfactory. However, this corresponds to the minimal substitution discussed by Drell and Lee $^{15}$ and later by Ohta $^{22}$. As we mentioned above, this does not provide the correct scaling properties at large energies, which means within the present context that $F = \Gamma_{0}$ would not lead to the expected $P_{T}$ scaling of the amplitude. Numerically, this choice overshoots the experimental data by more than one order of magnitude at $P_{T} = 20$ GeV as shown on Figure 2.

**FIGURE 2.** Comparison between the $J/\psi$ production cross section from $s$-channel cut obtained with the minimal substitution ($F = \Gamma_{0}$) for the 4-point function and the CDF data $^{2}$. See below for details on the cross-section evaluation.
2.3. The four-point function: our proposal

To avoid such abnormal scaling behaviour, we have to impose, in the large relative-momentum region, that the contact term and therefore the function \( F(c_1, c_2, q) \) exhibit a fall-off similar to the 3-point vertex functions.

The simplest crossing-symmetric choice satisfying this property is \[ F = \Gamma_1 + \Gamma_2 - \frac{\Gamma_1 \Gamma_2}{\Gamma_0}. \] \hfill (6)

The solution we propose here is to build \( F(c_1, c_2, q) \) from these two limiting cases:

\[ F = \Gamma_0 \] at low momentum,
\[ F = \Gamma_1 + \Gamma_2 - \frac{\Gamma_1 \Gamma_2}{\Gamma_0} \] at large momentum.

To this end, it is practical to choose the following simple Ansatz

\[ F(c_1, c_2, q) = \Gamma_0 - h(c_1 \cdot c_2)(\Gamma_0 - \Gamma_1)(\Gamma_0 - \Gamma_2), \] \hfill (7)

where the (crossing-symmetric) function \( h(c_1 \cdot c_2) \) rises to become unity for large relative momentum.

Note that \( F = \Gamma_0 \) at the poles is satisfied independently of \( h(c_1 \cdot c_2) \). Indeed, it is multiplied on the right by a factor which vanishes at the poles since either \( \Gamma_1 = \Gamma_0 \) or \( \Gamma_2 = \Gamma_0 \). A phenomenological choice for the interpolating function \( h(c_1 \cdot c_2) \) we can then propose is

\[ h(c_1 \cdot c_2) = 1 - a \frac{\kappa^2}{\kappa^2 - (c_1 \cdot c_2 + m^2)}, \] \hfill (8)

with two parameters, \( a \) and \( \kappa \). This choice is in no way unique: in a manner of speaking, this choice is simply a way of parameterising our ignorance by employing minimal properties of \( \Gamma^{(4)} \).

Other choices could be analysed but our conclusions, that \( s \)-channel cut contribution can be large and can indeed help to reproduce the experimental data, would not be affected.

3. RESULTS

For the Tevatron and RHIC kinematics, the direct \( J/\psi \) are produced by gluon fusion and a final-state gluon emission is required to conserve \( C \)-parity and provide the \( J/\psi \) with its \( P_T \). The relevant diagrams for the \( s \)-channel cut of the LO gluon fusion process are shown on Figure 3. We use the same normalisation of \( \Gamma^{(3)} \) as in \[14\], \( m_c = 1.87 \text{ GeV} \) and \( \Lambda = 1.8 \text{ GeV} \). As shown in \[14\], \( \Lambda \) can be varied between 1.2 and 2.2 GeV without affecting much the results. The same statement holds here.
FIGURE 3. (a) & (b) Leading-order (LO) s-channel cut diagrams contributing to $gg \rightarrow J/\psi g$ with direct and crossed box diagrams employing the $c - \bar{c} - J/\psi$ vertex. The crosses indicate that the quarks are on-shell. (c) Box diagram with the 4-point vertex $c - \bar{c} - J/\psi - g$. Diagrams with reversed quark lines are not shown.

The partonic differential cross section obtained from the amplitude calculated from our model (see [14] for details) is summed over the gluon polarisations, i.e.,

$$\frac{d\sigma_r}{dt} = \frac{1}{16\pi s^2} \sum_{p,q,s=T_1,T_2} |M^{pqrs}|^2, \quad r = L,T_1,T_2,$$

(9)

where $|M^{pqrs}|^2$ is the squared polarised partonic amplitude for $g_p(k_1)g_q(k_2) \rightarrow Q_r(P)g_s(q)$ averaged only over colour for polarised cross sections. Here, $p, q, r$ and $s$ are the helicities of the respective particles, and $\hat{s} = (k_1 + k_2)^2$, $\hat{t} = (k_2 - q)^2$ and $\hat{u} = (k_1 - q)^2$ are the Mandelstam variables for the partonic process. The relation to the double-differential polarised cross section in transverse momentum $P_T$ and rapidity $y$ then is given by

$$\frac{d\sigma_r}{dy dP_T} = \int_{x_1}^{1} dx_1 \frac{2sP_T g(x_1) g(x_2(x_1)) d\sigma_r}{\sqrt{s} (\sqrt{sx_1} - E_T e^y)} dt.$$

(10)

In the present calculations, we use the LO gluon distribution $g(x)$ of [24].

Figure 4 shows our results with parameter values $a = 4$ and $\kappa = 4.5$ GeV for $\sqrt{s} = 1.8$ TeV in the pseudorapidity range $|\eta| < 0.6$. The values of $a$ and $\kappa$ were chosen to reproduce the cross-section measurement of direct $J/\psi$ by CDF [2] up to about $P_T = 10$ GeV. At higher $P_T$, our curve falls below the data as expected from the genuine $1/P_T^2$ scaling of a LO box diagram. We expect higher-order corrections incorporating fragmentation-type topologies ($\sim 1/P_T^2$) [10, 25] and associated-production channels to fill the gap between data and theory at high $P_T$ [26].

Figures 5 show our results at $\sqrt{s} = 200$ GeV, still with $a = 4$ and $\kappa = 4.5$ GeV, compared with the PHENIX data [27] for the central rapidity region $|y| < 0.35$ (a) and the forward one $1.2 < |y| < 2.2$ (b).

$J/\psi$ polarisation measurements at the Tevatron exist only for the prompt yield, we have thus computed $\alpha$ from our direct-$J/\psi$ cross sections in two extreme cases,

\footnote{Note that the PHENIX analysis deals with the total $J/\psi$ yield, whereas our computation is for the direct yield. For the PHENIX kinematics, the $B$ feeddown can be safely neglected. To what concerns the feeddown from $\chi_c$, it is likely to affect the polarisation observable $\alpha$ (see later), but normally much less the $P_T$ dependence.}
one where the $J/\psi$'s from $\chi_c$ are 100% transverse and another where they are 100% longitudinal, the first scenario being the more likely one. Figure 6 shows the comparison between this computation and the recent results by CDF at $\sqrt{s} = 1.96$ TeV [4]. Figures 7 show the polarisation of the direct yield for the central and forward rapidity regions at RHIC. At very small $P_T$, the $J/\psi$ is found to be rather transversal, $\alpha$ being systematically larger for larger rapidity.
\[ \alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L} \]

CDF data for prompt J/\( \psi \) prompt with J/\( \psi \) from \( \chi_c \) 100\% Transverse

CDF data for prompt J/\( \psi \) prompt with J/\( \psi \) from \( \chi_c \) 100\% Longitudinal

FIGURE 6. Prompt J/\( \psi \) polarisation: theory vs. CDF data [1].

FIGURE 7. (Right) Direct J/\( \psi \) polarisation as a function of \( P_T \) for the RHIC kinematics in the rapidity intervals |\( y \)| < 0.35 and 1.2 < |\( y \)| < 2.2. (Left) Direct J/\( \psi \) polarisation integrated over \( P_T \) as a function of the rapidity for the RHIC kinematics.

4. CONCLUSION

In [14], we showed that there exist two singularities contributing to the imaginary part of the amplitude for \( gg \rightarrow J/\psi g \). The first can be identified to the CSM contribution when the static limit is taken (no relative momentum between quarks). The second can be referred to as an \( s \)-channel cut and was never considered before [14].

To deal with such configurations, we have to introduce a four-point function \( c - \bar{c} - J/\psi - g \), complementing the information given by the three-point function (or Bethe-Salpeter amplitude) \( c - \bar{c} - J/\psi \). Such a four-point function is a priori constrained by a low-energy limit (when the emitted gluon is soft) and a scaling limit (when the emitted gluon is hard). Given those two physical constraints, we constructed a four-point functions exhibiting a dependence only on two parameters, which we fixed to reproduce the Tevatron measurements up to mid \( P_T \). We then used the latter to compute the cross section for the RHIC kinematics, for which we obtained a striking agreement with PHENIX data. This agreement can be employed [28] to investigate on the kinematical effects attributable to the final-state-gluon emission in studies of shadowing effects on J/\( \psi \) production in pA,
$dA$ and $AA$ collisions, in the spirit of the study [29, 30]. Our prediction for the polarisation for the prompt $J/\psi$ yield at mid $P_T$ at the Tevatron is mostly longitudinal.

In the COM, colour-octet matrix elements account for transitions between a coloured heavy-quark pair into a quarkonium by soft unseen gluon emissions in the final state. In the present approach, the 4-point function accounts for gluon exchanges between the heavy quarks emitting the final-state gluon. As for the matrix elements of NRQCD, which are unknown and then fit, we fixed the unconstrained parameters of this function in order to reproduce the experimental data at $\sqrt{s} = 1.8$ TeV from the CDF collaboration at the Tevatron for $P_T \lesssim 10$ GeV.

Contrary to usual results obtained with LO calculations, our approach agrees with data down to very low values of the transverse momentum without need of resummation of initial-state gluon effects. This feature could be attributed to the threshold associated with the cut in the $s$-channel and should be analysed in more details in the future.

Before drawing further conclusions, several points have to be addressed: Firstly, the size of the real part of the amplitude has to be evaluated. When fixing the parameter of our four-point function to describe the Tevatron data we have implicitly assumed that such a real part was small; this has to verified. Secondly, the four-point function we proposed here has to be applied to other regimes of production: a similar enhancement by inclusion of the $s$-channel cut is expected in all production processes where the $J/\psi$ is associated with a gluon, e.g., photon-photon collision at LEP as well as in photo- and lepto-production at HERA. On the other hand, other observables insensitive to the COM or the $s$-channel cut – and thus to the ambiguity attached to the description of the four-point function – should be studied in the future, especially at the LHC. To conclude, let us mention two promising new observables, $J/\psi$ production in association with a $c\bar{c}$ pair [26] and the hadronic activity around the $J/\psi$ [31].

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