SELLING BY CLICKS OR LEASING BY BRICKS? A DYNAMIC GAME FOR PRICING DURABLE PRODUCTS IN A DUAL-CHANNEL SUPPLY CHAIN

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Abstract. In this paper, we discuss if and which pricing policies by a manufacturer who sells its products online motivate a retailer as an independent part to enter the market to provide selling and leasing options through a brick store. Moreover, the impact of online shopping preferences and brand image on end-user behavior is examined, and different consumption patterns are considered. For this purpose, a dynamic game is applied to model a supply chain consisting of one manufacturer and one retailer. The model aims to specify the optimal pricing policies in the second-hand market and according to physical utility associated with depreciation, brand image, and online shopping preferences for different end-users in an infinite time horizon. Markov perfect equilibria are considered as the solution concept to predict the behavior of end-users in the long term. The results revealed that enriching brand image always benefits the manufacturer and the retailer, while it does not mean there is the same optimal brand image level for both manufacturer and retailer. Besides, the improvement of physical utility makes more demand for leasing products and motivates the retailer to be active in the market. Notably, online shopping preferences play a prominent role in market segmentation and retailer decision as a result. Also, growing production costs have a significant reverse effect on the profitability of both manufacturer and retailer. Therefore, the manufacturer must focus on economic production.

1. Introduction. Since the expansion of internet infrastructure and information technology is remarkable, many manufacturers supply their products through an online store. Also, durable products have been affected by this trend. In the line of rising this trend, manufacturers of durable products such as Apple, Panasonic, HP, and Lenovo tend to supply their products through the dual-channel, namely online and physical stores.

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The benefits and disadvantages of each channel in a dual-channel supply chain significantly impact end-user preferences [43]. For instance, a purchase from a physical store costs the end-users a fortune since the end-users have to travel from their location to the store location. Regardless of the time wasted and the money spent on traveling this distance, the end-user may be at risk of being infected by the Coronavirus, an epidemic disease nowadays. However, some end-users would like to see products in person, touch them, and even try them before shopping.

On the other hand, online stores help the end-users save their time and provide them a vast range of choices in terms of color, size, or application. Although, these stores have some drawbacks such as lacking security for shopping, long waiting time for delivery, shipping cost, and unsuitable product delivery. In addition, this is the main reason why the end-users have different preferences for shopping online or through a brick store. Moreover, many attempts have been made to study the dual-channel supply. Nevertheless, a few studies examine durable products in a dual-channel supply chain [48]. Therefore, several questions in this regard remain to be addressed. For instance, it is essential to know which conditions encourage the retailer to enter the market in which the manufacturer directly sells the products through the online channel and the retailer supply them through a brick store with selling and leasing options. This paper is structured to address this question.

According to the durable products nature, leasing and selling are two popular ways of supplying durable products. Leasing is an attractive strategy for procuring a wide range of products [45]. That means the leasing market covers leasing demand and provides more demand for the manufacturer who has only selling options to the end-users. Leasing is a financial tool that ensures the modernization of enterprises [7]. There are considerable durable manufacturers such as Apple, Lenovo, Microsoft, etc., which are selling their products online. However, they take advantage of the retailers such as Best Buy [18], which provides various transaction ways, including selling and leasing through physical stores, to meet the need of end-users and encourage them to use the products more.

The past decades have seen many studies regarding leasing and selling policies [8, 47]. Overall, the studies conducted in this regard focused on investigating leasing policies versus selling and other related subjects, while the impact of the dual or multi-channel supply chain on end-users’ behavior has been mainly neglected. It is also noteworthy that the presented models describing leasing and selling policies are mostly static [3].

In this paper, a dynamic framework is considered to specify different end-users’ consumption patterns in the coming periods. A dynamic game follows game theory where players move sequentially or repeatedly [9]. In dynamic games, as the game progresses, players obtain information, complete or partial, on past actions of other players and apply this information in choosing their own actions [5]. In this paper, the manufacturer predicts the consumption patterns of end-users and then determines the selling price to the end-users and wholesale price to the retailer for new products. Then, by considering the selling prices and consumption pattern, the retailer determines the leasing price of new products. Finally, selling prices for the used products in the second-hand market will be determined. The model considers a dual-channel supply chain for durable products in different transaction ways (Leasing and selling) regarding the dynamic approach. It enables the manufacturer and the retailer to predict the consumption patterns for different customers in the long term and adopt optimal selling and leasing prices for new products. The
dynamic approach also considers depreciation and second-hand market with considerable detail. The model predicts the number of used products with different ages and accordingly different prices for them. Also, the dynamic approach indicates the interaction between leasing, selling, and used products better. For example, a change in the leasing price changes the selling and second-hand markets in the current and coming periods. However, the effect of these changes is not the same in all the periods. A dynamic model indicates the interaction between markets accurately. The proposed model examines optimal pricing policies regarding the second-hand market, physical utility concerning depreciation, brand image, and online shopping preferences for different end-users in an infinite time horizon. Markov perfect equilibria are considered to predict the behavior of end-users in the long term. Besides, a numerical example is provided to clarify the practicality of the proposed model in detail. The main aims of this paper are summarized as follows:

- Predicting the end-users’ consumption pattern for the future employing a dynamic game.
- Addressing the impact of online shopping preferences and brand image on consumption patterns.
- Determining the optimal prices of new and second-hand products.
- Discussing the pricing policies of a manufacturer who sells its products online encourages a retailer to enter the market to provide selling and leasing options for end-users.

The proposed model presents a dynamic dual-channel supply chain for durable products with different transaction ways (Leasing and selling). It enables the manufacturer and the retailer to accurately predict the consumption patterns for different customers in the long term and adopt optimal selling and leasing prices for new products. Also, depreciation is unavoidable, and the used products of various ages do not have the same utilities and prices. In other words, a one-period used product differs from a five-period used product in terms of utility. Thus, in the proposed dynamic model also depreciation and second-hand market with considerable detail are considered. The model predicts the number of used products with different ages and accordingly different prices for used products. Each time, the one-period used products come from the newly produced ones in the previous period, the two-period used products come from one-period used products in the previous period, and so on. Also, the presented model is the first to consider the brand image impact on a dual-channel supply chain. The end-users’ response to the brand image on the online store, physical store, and second-hand market is considered to be different.

The rest of the paper is organized as follows: Section two gives an overview of the previous studies conducted on this study and other areas of interest related to this paper. The third section presents a dynamic framework in which players, namely a manufacturer, a retailer, and the second-hand market, compete. Also, all the related equations and the solution concept are provided in this section. The numerical results are given in the fourth section. The managerial insights and practical implications are presented in the fifth section. Finally, section six delivers a summary associated with the conclusions obtained in this study and future research.

2. Literature review. Nowadays, a considerable number of manufacturers extend their traditional retail channels with an e-channel. This trend has a remarkable impact on the market of durable products [47]. Regarding many studies published on a variety of policies and models focused on deteriorating items [33, 34, 32, 35],
this paper focuses on durable products and considers different transaction ways for them.

There are considerable studies conducted on dual-channel supply chains up to now [19, 40, 46, 11, 51, 56]. Most of the authors that considered the dual-channel supply chain adopted the game theory approach in their studies. Yugang Yu et al. examined a producer’s dual-channel decision in a shopping complex according to consumer channel preference between the direct and indirect channels [52]. Soleimani investigated the pricing decisions of a dual-channel supply chain consisting of one retailer and one producer whose task is producing a product and selling it to the end-user directly or by the retailer [40]. Also, the optimum product price and trade-in discount was examined by Cao et al. in 2018 [10]. They discussed three-channel choices: only offline, online, and double channels to offer trade-in service. Wang et al. examined the pricing issue of complementary products based on a fuzzy dual-channel supply chain in which two producers and one retailer exist [44]. Yoo et al. combined a variety of channel structures and considered a monopoly producer and online and offline outlets to examine how the characteristic channel structure and various market conditions balance the effect of Internet channel entry on the channel members and consumers [50]. Nasiri et al. assumed an uncertain environment for location-allocation decisions and inventory control of distribution centers. According to this environment, this study extended a combined-integer non-linear programming model to specify the optimal pricing of products in various sales and distribution channels [29]. Lai et al. examined the optimal pricing and manufacture decisions in a channel supply chain based on the symmetric and asymmetric information cases and reached acceptable results [21]. According to the study conducted by Sun et al. [41], a dual-channel supply chain is investigated. Hence, a supplier sells a product through a direct channel and two various duopolistic retailers. Furthermore, the optimal pricing and service strategies for a dual-channel retailer were studied by Guo et al. [16]. In this research, the retailer sells the product using a traditional retail channel and an online channel.

It is noteworthy to mention that other factors affect supply chain performance in addition to applying a dual-channel supply chain [25, 26, 27, 53]. Until now, the market of durable products has been a striking subject in economics concerning the previous studies [20]. However, scant literature on the dual-channel supply chain addresses durable products and their effect on manufacturers and retailers [47, 48]. Leasing and selling are considered as two popular ways of durable product transactions, which motivated some studies to indicate the relation between their markets [3, 55, 42]. Most authors examined selling and leasing options irrespective of the dual-channel supply chain for durable products. Li et al. addressed a comparison between selling and leasing for new and remanufactured products in the electric vehicle industry [24]. Agrawal et al. indicated that disposal fees and promoting remanufacturing based on some conditions constitute a rise in the environmental effect. It was revealed that consumers' education effectively enhances the relative environmental performance of leasing [2]. The defaultable lease rate term structure with endogenous default was investigated by Agarwal et al. in 2011 [1]. The study defined the trade-off between debt and lease financing as a simultaneous equation problem to specify the lease rate. In addition, the main correlation between the leasing contracts and several factors affecting the value of these contracts were examined by BĂRBULESCU et al. in 2016 [36]. The previous studies assumed more generic assumptions, including static interactions.
In 2001, Huang et al. presented a dynamic monopoly model of leasing, selling, and secondary markets [20]. They examined only one manufacturer who supplies his products in cash and on leasing. In 2015, Andrikopoulos and Markellos examined the dynamic relationship between leasing price variations and selling price variations [3]. Concerning their study, both sellers and leasers bought their products from manufacturers. Therefore, their competition was merely horizontal, and the depreciation and secondary market were not considered in their model.

In 2012, Xiong et al. presented a dual-channel supply chain model for durable products for the first time [47]. They assumed that leasing and selling policies were possible for the retailer channel, and used products were considered as new ones. In 2016, Yan et al. applied virtually the same assumptions, but they deemed the used product’s price to be different from the new ones’ price [48]. In both studies of Xiong et al. and Yan et al., a static model was presented [47, 48]. However, to provide a more realistic assumption, this paper considers a dynamic approach to indicate the relationship between the leasing and selling market in more detail. Furthermore, the end-users are not indifferent to buy a new or second-hand product. Moreover, inflation is not avoidable in many cases [38, 4]; therefore, the impact of time on the value of profit is considered.

The brand image is considered a notable parameter in the end-users utility function [22]. In fact, the brand image includes products’ appeal, ease of use, functionality, fame, and overall value. In other words, brand image is the consumers' mental feedback when purchasing a product [30]. This paper follows [39], [31], and [22] to discuss the brand image effect on end-users’ behavior.

The presented model in this paper bears a resemblance to the one given by Huang et al. in 2001, with considerable alterations. In Huang et al.’s study, a dynamic relationship was considered between the leasing and selling market. It was assumed that the manufacturer offers both leasing and selling options. Nevertheless, in this paper, an independent retailer provides a selling or leasing option through a physical store, and the manufacturer only provides the selling option in her online store. A manufacturer-owned e-channel may refuse to adopt a leasing strategy due to some reasonable reasons, such as the lack of an effective supervisory mechanism in leasing that causes the renters to abuse this situation [47]. Moreover, the impact of brand image and online shopping preferences on the consumption pattern is examined here, which was not considered by Huang.

### TABLE 1: of dual-channel studies and transaction ways for durable products studies in the literature with the present study

| Previous studies | Transaction Ways | Dynamic nature | More than one channel | Game theory approach | Second hand Market | Durable Products | Depreciation | Brand Image |
|------------------|------------------|----------------|-----------------------|---------------------|------------------|-----------------|--------------|-------------|
| Cao et al., 2018 [10] | Leasing | * | * | * |
| Wang et al., 2018 [43] | Selling | * | * | * |
| Wang et al., 2019 [44] | Leasing | * | * | * |
| Lai et al., 2018 [21] | Selling | * | * | * |
| Soleimani, 2016 [40] | Leasing | * | * | * |
| Yu et al., 2020[52] | Selling | * | * | * |
Table 1 gives brief literature on dual-channel supply chains and various transaction ways to highlight the gaps in the previous research and illustrate this paper’s main contributions in more detail. Accordingly, the significant gaps that can be observed in the earlier studies are as follows:

| Authors | Year | * | * | * |
|---------|------|---|---|---|
| Zhu et al., 2020 | [57] | * | * | * |
| Sun et al., 2021 | [41] | * | * | * |
| Zhang and Xiao, 2013 | [54] | * | * | * |
| Yang et al., 2018 | [49] | * | * | * |
| Guo et al., 2021 | [16] | * | * | * |
| Hsieh et al., 2014 | [19] | * | * | * |
| Yoo and Lee, 2011 | [50] | * | * | * |
| Xiao and Shi, 2016 | [46] | * | * | * |
| Zhao et al., 2017 | [56] | * | * | * |
| Chen et al., 2017 | [11] | * | * | * |
| Nasiri et al., 2021 | [29] | * | * | * |
| Agarwal et al., 2011 | [3] | * | * | * |
| Rogers and Rodrigues, 2015 | [37] | * | * | * |
| Agrawal et al., 2012 | [2] | * | * | * |
| Andriopoulos and Markellos, 2015 | * | * | * | * |
| Li et al., 2021 | [24] | * | * | * | * |
| Harniulescu and Enache, 2016 | [36] | * | * | * |
| Yan et al., 2016 | [48] | * | * | * | * | * | * | * |
| Bamshi et al., 2016 | [17] | * | * | * | * | * | * |
| This paper | | * | * | * | * | * | * | * | * | * |
The scant literature considered dual-channel for durable products in which the leasing and selling issues were rarely investigated at the same time.

- The dynamic nature has been neglected.
- The brand image has not been investigated in a dual-channel supply chain.
- Only a few studies have been conducted regarding the depreciation and the second-hand market effects in pricing new durable products.
- The consumption patterns for different customers have been rarely examined.

The main contributions of this study can be summarized as follows:

1. In this paper, a dual-channel supply chain is specified for durable products, and a retailer is considered who independently offers different transaction ways at the same time. In fact, this retailer buys durable products at wholesale prices and offers leasing and selling options through the physical store. Whereas the manufacturer only provides selling options through the online channel.

2. The effect of brand image on a dual-channel supply chain for durable products is examined with respect to the different response levels for different channels.

3. A dynamic approach is applied to find consumption patterns in the coming periods.

The consumption pattern of different customers is predicted in the long term, while the impact of physical utility concerning depreciation, second-hand market, and the online shopping preferences on different consumption patterns are considered.

3. Problem statement. In this section, the proposed model is explained in detail. A manufacturer selling the products to the end-users directly through her online store is considered. Also, a retailer can hold this responsibility to offer the products through selling or leasing transaction ways [48]. The retailer buys the products wholesale from the manufacturer and provides different transaction ways for end-users through its physical stores. This paper assumes that transportation cost is included in the production costs. However, some authors have proposed various methods to reach an optimal type and cost of transportation in supply chain management [12, 14, 15, 13].

The products are leased for only one period, and leasing is considered for only new products. In the next period, these products are sold as second-hand products by the retailer. The total potential number of end-users is considered stable over time. The difference between end-users is represented by parameter $\theta$, which implies the end-user of type $\theta$. It is noteworthy that $\theta \in [0, 1]$ follows a uniform distribution. In fact, different end-users have different preferences, which are shown in more detail through the reservation price vector for end-users ($RP(\theta)$). Reservation price means the maximum value that an end-user pays to buy or lease a product.

The pricing decisions made by the manufacturer, the retailer, and end-users are modeled as a dynamic game. Since pricing decisions are made periodically, i.e., seasonally or annually, the manufacturer controls her profit each period. Therefore, she specifies the new products’ prices sold directly to the end-users and their selling price for the retailer. The retailer determines the leasing price of the new product. Based on the dynamic game considered in this paper, the players generally aim to maximize their net present value. The discount rate for all the players is considered the same, and there is no hidden information.
Figure 1 highlights the relationship between the manufacturer, online and physical stores, selling and leasing options, and other related parameters. Also, this figure clarifies the proposed model.

3.1. Symbols. The parameters considered in this model are as follows:

- \( C \)
  - The production cost ($)
- \( LS \)
  - The lifespan of each product
- \( d_1 \)
  - The deal cost of end-users ($)
- \( d_2 \)
  - The deal cost of the retailer ($)
- \( pop \)
  - Number of potential end-users
- \( \rho \)
  - Discount rate \((0 \leq \rho \leq 1)\)
- \( \varphi \)
  - The level of brand image \((0 \leq \varphi \leq 1)\)
- \( C''t \)
  - The cost of providing a specific level of brand image at period \( t \)
- \( EPV_r \)
  - The minimum expected net present value of the retailer
- \( \alpha \)
  - Online shopping preferences, the percentage of end-users preferring online shopping in comparison to the physical store
- \( \alpha_T \)
  - The threshold of retailers for online shopping preferences
- \( \lambda_1 \)
  - Response level of the end-user type \( \theta \) to the brand image for buying a new product.
- \( \lambda_2 \)
  - Response level of the end-user type \( \theta \) to the brand image for buying a new product.
- \( \lambda_3 \)
  - Response level of the end-user type \( \theta \) to the brand image for buying a new product.

The indices considered in this model are as follows:
The reservation price for the end-user type $\theta$ that aims to buy a new product for one period ($) when $k = 1, \cdots, LS - 1$

The reservation price for the end-user type $\theta$ that aims to buy a $k$-period used product for one period ($) when $k = 1, \cdots, LS - 1$

The reservation price for the end-user type $\theta$ that aims to lease a new product for one period ($) when $k = 1, \cdots, LS - 1$

The population of end-users who buy a new product at period $t$

The population of end-users who lease a new product at period $t$ for one period

The percentage of end-users type $\theta$ buying a new product at period $t$

The percentage of end-users type $\theta$ buying a $k$-period used product at period $t$ when $k = 1, \cdots, LS - 1$

The percentage of end-users type $\theta$ leasing a new product at period $t$

The percentage of end-users type $\theta$ that tend not to use any product at period $t$

The payoff function for end-user $\theta$ per period ($$

The profit function for the retailer per period ($$

The profit function for the manufacturer per period ($$

The action vector of an end-user $\theta$ at period $t$

It takes 1 when the end-user $\theta$ buys a new product at period $t$ and otherwise 0.

It takes 1 when the end-user $\theta$ buys a $k$-period used product at period $t$ and otherwise 0.

It takes 1 when the end-user $\theta$ leases a new product at period $t$ and otherwise 0.

It takes 1 when the end-user $\theta$ is with no action and does not use any product at period $t$ and otherwise 0.

The reaction function of the manufacturer at period $t$

The reaction function of the end-users at period $t$

The reaction function of the retailer at period $t$

The net present value of the end-user $\theta$

The net present value of the retailer

The net present value of the manufacturer

Index of a $k$-period used product when $k = 1, \cdots, LS - 1$

Index of period

The other symbols considered in this model are as follows:
3.2. Decision variables. The decision variables considered in this paper are as follows:

- $P^t$: The price vector at period $t$ ($\$\$)
- $p^t_k$: The selling price for a $k$-period used product at period $t$ ($\$\$)$ when $k = 1, \cdots, LS - 1$
- $p^t_0$: The selling price for a new product at period $t$
- $p^t_r$: The wholesale price of the products sold to the retailer at period $t$ ($\$\$)$
- $r^t$: The price of leasing a new product for one period at period $t$ ($\$\$)$

3.3. End-users. Concerning (1) end user’s action at period $t$ is represented by $\alpha^t(\theta)$. Their action only depends on their decision in the last period and prices in this period. In each period, such as $t$, end-users have some specific behaviors. In fact, they may decide to purchase a new product or a $k$-period used product, or they tend to lease a new product. Also, they may prefer not to take any action and not to use any product.

$$\alpha^t(\theta) = (n^t, s^t_k(\theta), l^t(\theta), w^t(\theta))$$

When

$$n^t + \sum_{k=1}^{LS-1} s^t_k(\theta) + l^t(\theta) + w^t(\theta) = 1$$

Concerning (1), in each period, only one of the binary vector ingredients takes 1.

The preferences of the end-users are shown through the reservation price vector (2). Reservation price means the maximum value that an end-user tends to pay for buying a new or a $k$-period used product or leasing a new one. Different end-users have different reservation prices. That means the maximum value that an end-user pays to buy or lease a new product or buy a used product differs from other end-users. Therefore, we define the end-user of type $\theta$ when it follows the unique distribution ($\theta \sim U[0,1]$). The reservation price is a linear function of $\theta$. Those end-users with higher amounts of $\theta$ tend to pay more as the reservation price, and those with the lower amount of $\theta$ tend to pay less as the reservation price.

$$Rep(\theta) = (Rep_0(\theta), \cdots, Rep_{LS-1}(\theta), Rep_L(\theta), 0)$$

However, the payoff of the end-users type $\theta$ at time $t$ depends on the action they have taken in the previous periods and the price vector in this period. The dynamic behavior of end-users is based on the Markov process as shown below:

$$F_\theta = F_\theta(\alpha^{t-1}(\theta), \alpha^t(\theta), P^t)$$

3.4. Manufacturer. The profit function of the manufacturer is estimated by (4). In accordance with (4), the incomes come from two sources, including selling products to end-users and selling products to the retailer in each period.

$$F_m = [N^t, L^t, P^t, \varphi^t] = (p^t_0 - C^t)N^t\alpha^t + (p^t_r - C^t)(L^t + N^t(1 - \alpha^t)) - C^t\varphi^t$$

When

$$P^t = (p^t_r, p^t_0, \cdots, p^t_{LS-1}, r^t).$$
3.5. **Retailer.** The profit function of a retailer in period $t$ consists of three components. The first one is about leasing new products for one period, which is denoted by $(r^t - p^t_r) L^t$. The second is related to the products leased in the previous period, and now they are collected and sold as the used products by the retailer. The second one is represented by $(p^t_1 - d_r) L^{t-1}$. Finally, the third one shows the number of products sold to the end-users who prefer shopping through physical stores. The third one is demonstrated by $(p^t_0 - p^t_r) N^t (1 - \alpha)$.

$$F_r = [L^t, L^{t-1}, N^t, P^t] = (r^t - p^t_r) L^t + (p^t_1 - d_r) L^{t-1} + (p^t_0 - p^t_r) N^t (1 - \alpha) \quad (6)$$

When $(r^t - p^t_r) L^t + (p^t_1 - d_r) L^{t-1}$ is negative, it means that the retailer will refuse to enter the leasing market. However, when (6) is positive, it does not necessarily mean the retailer will enter the market. In many cases, the retailer expects a minimum income to enter the market. This income directly depends on the end-user’s online shopping preference and leasing willingness. In the following, the threshold of associated with the retailer’s entrance to the market will be discussed in detail.

3.6. **Concept of the solution.** This paper presents a developed form of the framework proposed by Huang et al. in 2001. In fact, Huang et al. proposed a dynamic monopoly model of leasing, selling, and secondary markets, with a finite lifespan of products under an infinite time horizon and non-trivial transaction costs. In the new framework, a distinct Bellman equation is considered for the manufacturer, and also a new Bellman equation is assigned to the retailer while some further steps are provided to find an optimal solution. This optimal solution consists of the optimal selling price of new products to end-users and the wholesale price to the retailer, the leasing price of new products, and the selling prices of $k$-period used products under a specific level of brand image in each period.

It is assumed that the manufacturer specifies the selling prices for the retailer ($p^t_r$) and end-users ($p^t_0$). After that, the retailer determines the leasing price ($r^t$). Finally, prices of selling used products are determined implicitly based on selling and leasing decisions for new products. In order to find optimal selling prices, the manufacturer must predict the reaction of end-users when making pricing decisions. Thus, the Bellman equation of end-users is analyzed. After that, the Bellman equation of the manufacturer and her reaction to the end-user’s behavior are addressed. In the next step, the Bellman equation of the retailer and his reaction to the selling prices are examined. Finally, the clearance conditions are discussed to specify the used products’ prices based on new products’ selling and leasing prices.

3.6.1. **Bellman equation of end-users.** The Bellman equation for an end-user of type $\theta$ is obtained through backward induction. In fact, the process of dynamic programming assures that the solutions employed for these equations have the necessary subgame perfection. The equations indicate the net present value functions by the state vectors, which give the information needed for reasonable decision-making [20].

The net present values in these equations give the end-user of type $\theta$ the necessary information to decide reasonably in the $t$ period. The Bellman equation considers an end-user of type $\theta$ as the net present value function is written (7).

$$PV^T_\theta [\alpha^{t-1}(\theta), P^t] = \max_{\alpha^t(\theta)} \{ F_\theta [\alpha^{t-1}(\theta), \alpha^t(\theta), P^t] + \rho \cdot PV^{T+1}_\theta [\alpha^t(\theta), P^{t+1}] \} \quad (7)$$
The end-user of type $\theta$ takes action $\alpha^t(\theta)$ based on their previous action and the price vector in the period $t$ (8), and this action is made in line with maximizing the net present value function.

$$R^t_0[\alpha^{t-1}(\theta), p^t] = \alpha^t(\theta)$$

(8)

The price vector is basically comprised of the product selling prices to the end-users and the retailer (determined by the manufacturer), the leasing price (determined by the retailer), and the price of used products with different remaining lifespans (which is implicitly determined). The second-hand market is assumed to be competitive since the used products in each period are supplied by the retailer aiming to sell the products leased in the previous period and the end-users who were owners in the last period.

This competitive market implies that clearance conditions (17) will equalize the demand and supply of the used products at any period.

3.6.2. Bellman equation of the manufacturer. To obtain the manufacturer’s net present value function, $h^t(\theta)$ is defined as the consumption pattern in each period, which is written by (9):

$$h^t(\theta) = (h^t_0(\theta), \cdots, h^t_{LS-1}(\theta), h^t_{LS}(\theta), h^t_{LS+1}(\theta))$$

(9)

Where $h^t_k(\theta), 1 \leq k \leq LS$ indicates the percentage of end-users of type $\theta$ that own the $k$ period-used products in the $t$ period.

With respect to (9), $h^t_0(\theta)h^t_{LS}(\theta)$ and $h^t_{LS+1}(\theta)$ represent the percentage of end-users type $\theta$ who buy a new product, lease a new product, and who did not take any action during the period $t$ respectively. Therefore, $h^t_k(\theta), h^t_{LS}(\theta)$ and $h^t_{LS+1}(\theta)$ is not negative, and their sum is equal to one. The total number of end-users who purchased new products in the $t$ period is calculated by (10).

$$N^t(\theta) = \int_0^1 h^t_0(\theta)f(\theta)d\theta$$

(10)

Moreover, with respect to (11), $h^t(\theta)$ in the subsequent periods is evolved by the reaction functions for end-users;

$$h^{t+1}(\theta) = \sum_{k=0}^{LS+1} R^t_0[\alpha_{k}^{t-1} = 1, P^t]h^t_k(\theta)$$

(11)

In addition, the Bellman equation for the manufacturer is defined as follows:

$$PV^t[h^t(\theta)] = \max_{p^t_0, p^t_r} \{F_m[N^t, L^t, \varphi^t] + \rho \cdot PV^{t+1}[h^{t+1}(\theta)]\}$$

(12)

Where the profit function of the manufacturer is

$$F_m = [N^t, L^t, \varphi^t] = (p^t_0 - C^t)N^t\alpha^t + (p^t_r - C^t)(L^t + N^t(1 - \alpha^t) - C^t\varphi^t)$$

The manufacturer seeks to increase her net present value through her Bellman equation (12). For this purpose, she must specify the optimal selling prices of new products to the retailer and end-users. Concerning (10) and (11), the manufacturer predicts the number of end-users based on their specific choices (buying a new product, leasing a new product, buying a used product, or without any action) regarding new product prices. Then she adopts selling decisions (13) to maximize (11).

$$R^t_m[h^{t-1}(\theta)] = p^t_0k + p^t_r$$

(13)
3.6.3. Bellman equation of retailer. The total number of end-users tending to lease new products in the period \( t \) and \( t - 1 \) is defined by (14).

\[
(L^{t-1}, L^{t}) = \int_{0}^{1} (h_{LS}^{t-1} (\theta), h_{LS}^{t} (\theta)) f(\theta) d\theta
\]

The profit function per period for the retailer at \( t \) period is calculated by (15).

\[
F_r = [L^t, L^{t-1}, N^t, P^t] = (r^t - p^t) L^t + (p^t_0 - d_2) L^{t-1} + (p^t_r - p^t) N^t (1 - \alpha)
\]

The Bellman equation for the retailer is defined by (16):

\[
PV^t_r [h^{t-1} (\theta), P^t] = \max_{r^t} \{ F_r [L^{t-1}, L^t, N^t, P^t] + \rho \cdot PV^{t+1}_r [h^t (\theta)] \}
\]

The optimal leasing price of a new product depends on the selling prices them and the end-user’s consumption patterns in the last period. The leasing price is obtained by (17).

\[
R^t_r [h^{t-1} (\theta), p^t_0, p^t_r] = r^t
\]

The retailer reacts to the mentioned factors and adopts his optimal leasing price (17) based on the selling prices of this period and the consumption pattern of the end-users in line with maximizing the total net present value (his Bellman equation (16)).

3.6.4. Clearance conditions and second-hand market. The clearance condition mentions that the total number of \( k \)-period used products in period \( t \) is equal to the total number of \( k \) + 1- period used products in the period \( t + 1 \).

If the total number of new products in the period \( t \) is shown by \( NP^t_0 \), and \( NP^t_k \) demonstrates the total number of \( k \)-period used products. Therefore, the \( LS - 1 \) clearance conditions at the period \( t \) are represented by (18) in which \( NP^t_0 = L^t + N^t \) and \( NP^t_k = \int_{0}^{1} h^{t+1}_k (\theta) f(\theta) d(\theta) \), with, \( 0 < k < LS \);

\[
NP^t_k = NP^{t+1}_{k+1} \quad \& \quad 0 \leq k < LS - 1
\]

According to (18), \( LS - 1 \) clearance conditions are applied to determine the used product prices (\( p^t_k \)). In fact, these prices are specified implicitly after determining the selling price of new products by the manufacturer, and the leasing price by the retailer, and the percentage of end-users based on their previous choices.

\[
R^t_k [h^{t-1} (\theta), p^t_0, r^t] = p^t_k
\]

The overall steps of finding the Markov-perfect equilibrium solution of the model in each period are organized as follows: initially, the end-users are classified into multi categories with respect to the reaction of different end-users to a default price vector. Then, the manufacturer considers the number of end-users in each group and chooses the optimal selling price to end-users and wholesale price to the retailer in order to maximize her Bellman equation (11). In the next step, based on the manufacturer’s optimal selling prices and the reaction of different types of end-users under a defined brand image level, the retailer specifies the leasing price for one period (17) to maximize his net present value Bellman equation (16). The last decision is related to the pricing \( k \)-period used products under the clearance conditions (19).
3.6.5. Steady limit. A Markov-perfect equilibrium is considered a solution in the infinite time horizon of an independent system. In this equilibrium, all the explicit time dependence is ignored, and the prices and aggregate end-user behaviors are independent of time. Nevertheless, cyclic behaviors like the Edgeworth cycles are time dependence is ignored, and the prices and aggregate end-user behaviors are considered in several studies [28]. In accordance with this equilibrium, the game is considered in the strategy space. It converges to a fixed point during a limited number of periods, beginning from a specified initial state. When this point is more natural than any other steady limit in the game, it is considered the focal point. It is noteworthy that the Bellman equations in the steady limit are simplified because the explicit time dependences are neglected. The focal point is defined by a pair of vectors, namely \( h^*(\theta) \) and \( P^* \), and the following conditions are satisfied:

\[
h^*(\theta) = \sum_{k=0}^{LS+1} R_\theta(\alpha_{k}(\theta)) = 1,\ P^*[h^*_k(\theta)]
\]

\[
P^* = (R_m[h^*(\theta)]) = p_0^* \& p_r^*, \ R_1[h^*(\theta), p_0^*, r^*] = p_1^*, \cdots, \ R_r[h^*(\theta), p_0^*, p_r^*] = r^*
\]

Furthermore, \( h^*(\theta) \) and \( P^* \) meet the Bellman equations at the focal point.

\[
PV_\theta[\alpha(\theta), P^*] = \max_{\alpha(\theta)} \{ F_\theta[\alpha(\theta), R(\alpha(\theta), P^*), P^*] + \rho \cdot PV_\theta[R(\alpha(\theta), P^*), P^*] \}
\]

\[
PV_m[h^*(\theta)] = \max_{p_0^*, p_r^*} \{ F_m[N[h^*(\theta)], L_0[h^*(\theta)], P^*] + \rho \cdot PV_m[h^*(\theta)] \}
\]

\[
PV_r[h^*(\theta), p_0^*, p_r^*] = \max_{p_r^*} \{ F_r[L[h^*(\theta)], L[h^*(\theta)], p_0^*, p_r^*] + \rho \cdot PV_r[h^*(\theta), p_0^*, p_r^*] \}
\]

(23) and (24) indicate that the dynamic problem becomes a static problem under steady limit conditions for both the manufacturer and retailer.

3.7. Case study. The physical utility of the product is normalized for its entire life span to be 1. Therefore, the physical utility of the products in each period is a number between 0 and 1. For the sake of clarification, the physical utility of a new product for its first period, the second period, and the i period is considered equal to \( \Delta_1, \Delta_2, \Delta_i \). Then it is expected that \( \Delta_1 + \cdots + \Delta_i + \cdots + \Delta_N = 1 \& \Delta_i > \Delta_{i+1} \).

The simplest case is when a product’s lifespan is two periods \( (LS = 2) \). For instance, durable electrical devices such as printers, industrial refrigerators, or home appliances that follow the exponential distribution are regarded as the best examples to be assumed as products with a two-period lifespan. Since the exponential distribution has the property of memory lessness, they are considered new in the first period of their lifespan. While in the second period of their lifespan, they are called used products.

The reservation price of the end-user of type \( \theta \) means the maximum amount that an end-user tends to pay to buy or lease a product. It consists of two parts. The first part is related to the product’s physical utility. The second part is the effect of the brand image on end-user behavior. This paper follows [22] to reflect the brand image effect on end-user behavior. \( \theta \) follows the unique distribution \( (\theta \cup [0,1]) \). The reservation price is a linear function of \( \theta \). End users with higher amounts of \( \theta \) tend to pay higher reservation prices, and end-users with lower amounts of \( \theta \) tend to pay less.
The reservation price of the end-user of type $\theta$ aiming to buy a new product is shown by $\text{Rep}_0(\theta) = \Delta_\theta + \lambda_1 \varphi$. Also, $\text{Rep}_1(\theta) = \Delta_\theta + \lambda_2 \varphi$ represents the reservation price of the end-user of type $\theta$ tending to lease a new product. Furthermore, $\text{Rep}_2(\theta) = \Delta_\theta + \lambda_3 \varphi$ indicates the utility of the end-user of type $\theta$ tending to buy a used product.

The brand image level is denoted by $\varphi$ which belongs to the interval $[0,1]$. The higher level of the brand image increases the end-user’s tendency to choose the default product through one of the channels (direct selling by manufacturer, leasing channel by the retailer, and used product in the second-hand market).

The end-users brand image response is shown by $\lambda_i$, which is between zero and one ($0 \leq \lambda_i \leq 1$). That means the brand image has not had the same impact on different end-users. For example, the end-users eager to pay more money to buy new products care more about the band image than the end-users who lease a new product only for one period because they aim to use the durable products in the long term.

| Table 2. The payoff matrix for end-user $\theta$ with $N=2$. |
|----------------------------------------------------------|
| $N^1(\theta)$ | $S^1(\theta)$ | $L^1(\theta)$ | $W^1(\theta)$ |
| $\Delta_1 \theta + \lambda_1 \varphi^f - p_0$ | $\Delta_2 \theta + \lambda_3 \varphi^f$ | $\Delta_1 \theta + \lambda_2 \varphi^f - r^f + p_1^1 - d_2$ | 0 |
| $\Delta_1 \theta + \lambda_1 \varphi^f - d_2$ | $\Delta_2 \theta + \lambda_3 \varphi^f - p_1^1$ | $\Delta_1 \theta + \lambda_2 \varphi^f - r^f$ | 0 |
| $\Delta_1 \theta + \lambda_1 \varphi^f - p_0$ | $\Delta_2 \theta + \lambda_3 \varphi^f - p_1^1$ | $\Delta_1 \theta + \lambda_2 \varphi^f - r^f$ | 0 |
| $\Delta_1 \theta + \lambda_1 \varphi^f - p_0$ | $\Delta_2 \theta + \lambda_3 \varphi^f - p_1^1$ | $\Delta_1 \theta + \lambda_2 \varphi^f - r^f$ | 0 |

The payoff matrix highlights the difference between the reservation price and the amount of money that the end-users must pay.

3.7.1. End-users’ behavior in the steady limit. The variables under steady limit conditions are independent of the time at the focal point. $L, S, N$, and $W$ represent the leasing, purchasing second-hand products, purchasing new products, and without any action policies. Figure 2 indicates all possible consumption patterns under a default price vector.

There is a minimum amount of leasing price in which a retailer enters the leasing market and provides a leasing option. This amount is essential because leasing price has a considerable impact on end-users consumption patterns. To reach this minimum amount, the first and second part of the retailer’s profit function (6) should not be negative.

\[
(r - p_r)L + (p_1 - d_2)L > 0 \rightarrow (r - p_r + p_1 - d_2) > 0 \rightarrow r > p_r - p_1 + d_2 \quad (25)
\]

Moreover, the retailer is aware of the fact that when the leasing price exceeds a specific level, the end-users prefer to purchase a new product in every period. As a result, (26) is the constraint that must be considered.

\[
p_r - p_1 + d_2 < r < p_0 - p_1 + d_1 \quad (26)
\]

As long as (26) is met by $r, p_0, p_r$, and $p_1$ are satisfied, the end-users adopt the following policies when $1 \geq \theta_1 \geq \theta_2 \geq \theta_3 \geq 0$ (Appendix A).
Table 3. End-users policy in various classification

| Policy | Interval         |
|-------|------------------|
| LL    | $(\theta_1, 1)$  |
| NS    | $(\theta_2, \theta_1)$ |
| SS    | $(\theta_3, \theta_2)$ |
| WW    | $(0, \theta_3)$  |

Where $\theta_1 = \frac{r(1+\rho)-p_0+\varphi(\lambda_1+\rho\lambda_3-(1+\rho)\lambda_2)}{\rho(2\Delta_1-1)}$, $\theta_2 = \frac{p_0-(1+\rho)p_1-\varphi(\lambda_1-\lambda_3)}{2\Delta_1-1}$, and $\theta_3 = \frac{p_1-\varphi\lambda_3}{1-\Delta_1}$.

The clearance condition is solved in which the percentage of end-users who choose the pure leasing strategy is equal to the percentage of end-users tending to purchase used products (27).

$$1 - \theta_1 = \theta_2 - \theta_3$$ (27)

$$\rho_1^*(p_0, r) = \frac{(1-\Delta_1)((\rho-1)p_0+\varphi\lambda_1(1-\rho)-2\rho\Delta_1+\rho+r(1+\rho)-\varphi(1+\rho)\lambda_2)}{\rho(\rho-\Delta_1\rho+\Delta_1)}$$ (28)

Regarding (28), the price of purchasing the used products is obtained in terms of $p_0$ and $r$.

### 3.7.2. The manufacturer’s behavior in the steady limit.

Actually, it is up to the manufacturer to determine the selling prices to the end-users and wholesale price to the retailer, which is conducted in line with maximizing her total benefit. The wholesale price to the retailer is generally less than the selling price to the end-users.

The manufacturer decides to sell prices at a specific brand image-level based on end-user classification and the number of end-users in each category in the last period.

Considering Table 3, the percentage of end-users tending to buy a new product or lease a new one are calculated by (29) and (30), respectively.

$$N(p_0, p_1, r) = \frac{\theta_1 - \theta_2}{2} = \frac{(1+\rho)(r-p_0+\varphi(\lambda_1-\lambda_2)+\rho p_1)}{2\rho(2\Delta_1-1)}$$ (29)

$$L(p_0, p_1, r) = 1 - \theta_1 = \frac{\rho(2\Delta_1-1)-r(1+\rho)+p_0-\varphi(\lambda_1+\rho\lambda_3-(1+\rho)\lambda_2)}{\rho(2\Delta_1-1)}$$ (30)

The Bellman equation for the manufacturer at the focal point under a steady limit is rewritten (31).

$$PV_m[h^*(\theta)] = \max_{p_0} \{F_m[N[h^*(\theta)], L[h^*(\theta)], P^*] + \rho \cdot PV_m[h^*(\theta)]\} PV_m[h^*(\theta)]$$

$$= \frac{(p_0-C) \cdot N\alpha + (p_r-C) \cdot (L+N(1-\alpha))-C'\varphi}{1-\rho}$$ (31)

### 3.7.3. The retailer’s behavior at the focal point under the steady limit.

At the focal point under the steady limit and concerning the classification of end-users, optimal selling prices of new products, and the level of brand image, the retailer tries to determine the optimal leasing price for using a new product for one period to maximize his net present value. With respect to (14), the retailer’s profit function contains two parts. Thus, leasing new products and selling the leased products
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as used in the second-hand market constitutes the profit. Therefore, the Bellman
equation for the retailer at the focal point under a steady limit is rewritten (32).

\[ PV_r[h^*(\theta), p^*_0, p_r] = \max_r \{ F_r[N[h^*(\theta)], L[h^*(\theta)], p^*_0, p^*_r] + \rho \cdot PV_m[h^*(\theta), p^*_0, p^*_r] \} \]

\[ PV_r = \frac{(r - p_r)L + (p_1 - d_2)L + (p_0 - p_r)N(1 - \alpha)}{1 - \rho} \]

(32)

As mentioned before, the manufacturer specifies the selling prices to end-users \((p_0)\) and the wholesale price to the retailer \((p_r)\). Then, the retailer determines the leasing
price \((r)\). Therefore, at the focal point under a steady limit, a Stackelberg game
occurs. According to (32), the retailer could find the optimal amount of leasing price
\((r)\) that is a function of selling price to end-user \((p_0)\) and wholesale price to retailer
\((p_r)\). By replacing that function into (31) and optimizing (31) in terms of selling
prices to end-users \((p_0)\) and wholesale price to the retailer \((p_r)\) simultaneously, the
optimal selling prices subject to the constraints \(1 \geq \theta_1 \geq \theta_2 \geq \theta_3 \geq 0\) and (26) by
applying a standard optimization procedure such as the Frank-Wolfe method are
calculated.

Sometimes, a retailer may consider a threshold for \(\alpha\) to enter the market. Based
on his minimum expected profit \((EPV_r)\), the threshold \(\alpha\) is defined as \(\alpha_T\).

\[ EPV_r = \frac{(r - p_r)L + (p_1 - d_2)L + (p_0 - p_r)N(1 - \alpha)}{1 - \rho} \rightarrow \]

\[ \alpha_T = 1 + \frac{(r - p_r) + p_1 - d_2)L - (1 - \rho) \cdot EPV_r}{(p_0 - p_r)N} \]

The end-users tendency to buy online is denoted by \(\alpha\), which ranges from 0 to 1.
When \(\alpha\) is less than \(\alpha_T\), it means that the dealer enters the market. Besides, when
\(\alpha_T \geq 1\), it means that the leasing market is profitable enough for the dealer to enter
the market, and he does not care about his contribution from the selling market.
On the other side, when \(\alpha_T < 0\), it means that both the selling market and leasing
market are not profitable enough totally, and the dealer would refuse to enter the
market. This threshold of can vary for each retailer and depends on the minimum
profit expected for the retailer.

4. Numerical example. In this section, a numerical example is given to illuminate
the practicality of the proposed model. The proposed model could be considered
for any manufacturer who produces durable products and sells its products through
their online stores (such as HP, Apple, ...) and provides various transaction ways
by a retailer (such as Best Buy) in physical stores [18]. To cover various demands,
the manufacturer sells its products to the retailer at a wholesale price and provides
both selling and leasing options through the physical stores [48]. Collecting data
from different sources, such as the selling or leasing systems, enables us to estimate
the parameters by statistical methods. The parameter values combine the selling
and leasing options from local businesses and the related literature in our paper.

The parameters’ values and the optimal values of variables to solve the problem
are presented as follows:
According to the aforementioned explanations, a numerical example is provided here. This problem is an example of durable products with memoryless properties, such as printers. Also, a two-period model is considered in a market with one million potential end-users. The minimum expected net present value of the retailer is 250000. The parameters and optimal values of the variables are presented at the beginning of this section. The problem is finding if the retailer will enter the market and look for optimal prices in a balance point between players (the manufacturer, the retailer, and owners of used products).

The following steps describe the method employed here to solve the problem under a steady limit.

**Step 1.** According to (18)-(20) and Table 4, $p_1, N,$ and $L$ are calculated in terms of $r$ and $p_0.$

$$p_1 = -0.002p_0 + 0.302r + 0.035$$  \hspace{1cm} (33)

$$L = 1000 - 2.872r + 1.443p_0$$  \hspace{1cm} (34)

$$N = 1.865r - 1.438p_0 + 0.05$$  \hspace{1cm} (35)

**Step 2.** Specifying the optimal leasing price in terms of $p_0$ and $p_r$ so that the Bellman equation for the retailer (32) is maximized.

$$r^*(p_0, p_r) = 0.23 + 0.26p_0 + 0.375p_r$$  \hspace{1cm} (36)

**Step 3.** Replacing the $r^*$ obtained in the last step, in all constraints, including (26), $1 \geq \theta_1 \geq \theta_2 \geq \theta_3 \geq 0,$ and other variables such as $p_1, L, N,$ and $PV_m.$

**Step 4.** Maximizing the manufacturer’s profit function in each period in terms of $p_r$ and $p_0$ under the constraints (26) and $1 \geq \theta_1 \geq \theta_2 \geq \theta_3 \geq 0.$

**Step 5.** Determining the optimal values for all variables.

In the steady limit, the end-users coverage to some specific behavior. In fact, they are categorized into four classes, including always leasing a new product, buying a new product and using it up to its end of the lifespan, always buying a second-hand product, and finally, people with no use of products. For the end-users tending to use a new product in every period, adopting the leasing policy in two periods
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(LL) is more economical than adopting buying a new product each period (NN). Besides, more detailed information regarding this claim is presented in Appendix A.

The luxury end-users in the LL category are calculated as 5.7% of the potential population in this example. Furthermore, many end-users prefer adopting a new product and using it during the next period (NU) (17.8%*2=35.6%). On the other hand, the percentage of end-users tending to lease a new product in each period (L = 5.7% ) is equal to the percentage of end-users tending to buy a second-hand product (θ₃ − θ₂ = 5.7% ). However, pricing policies in this example make the most percentage of end-users (52.9%) take no action and refuse to use the product. The online shopping preference is equal to one, so that all end-users prefer online buying. But according to the results (αᵣ > α = 1), the leasing market is profitable enough for motivating the retailer to enter the market.

Regarding this example, some questions are raised: How do the brand image and the response level affect the end-user’s consumption pattern? How do channel preferences attract end-users? How do production cost changes optimal pricing decisions and end-users choices? How does the physical utility, depreciation, and second-hand market affect the end-user’s behavior? Furthermore, to what extent the dealing cost is essential for players? The sensitivity analyses are provided in the next section to answer these questions.

5. Managerial insights and practical implications. This section aims to provide comprehensive managerial insights by examining the impact of the most influential parameters, including brand image, physical utility, production cost, dealing cost, response levels of different end-users to the brand image on end-users behavior, and pricing policies. A dual-channel comprises an online store, and a physical store makes a profit from a wide range of end-users with different preferences. One of the most important reasons for a manufacturer to consider a dual-channel structure is market demand [23]. Applying a dual-channel supply chain versus a single-channel depends on the different factors [6], such as online shopping preferences among end-users, providing different transaction ways in different channels, the level of brand image, and channels’ response level, which affects the profit directly.

An appropriate level of the brand image associated with response levels of different end-users to the brand image plays a prominent role in controlling demand in all segments, pricing policies, and profits. More selling price with a higher brand image is expected that brings more demand. Enriching the brand image up to a specific level is beneficial to the manufacturer and retailer simultaneously. Lack of enough brand image makes the retailer less eager to enter the market and makes less profit for the manufacturer. Besides, higher levels of physical utility motivate the retailer to enter the market; therefore, the manufacturer should deliver a higher level of physical utility for new products by taking some actions such as providing more options, ergonomic design, or new services. The increasing physical utility makes much demand. Also, higher response levels in different channels, namely online and brick, mean more demand for those channels and simultaneously higher selling and leasing prices. Dealing with the cost is another critical parameter for the retailer to decide whether to enter the market. When the dealing cost increases, the manufacturer reduces the wholesale price to the retailer to support the retailer and prevent losing the lease market to some extent.
5.1. **Impact of brand image on optimal variables.** According to Figure 3, the end-users reservation price increases in line with the brand image. Higher brand image-level constitutes more demand compared to the past; as a result, higher prices \((p_0, p_r, r, \text{ and } p_1)\) are created. Consequently, Figure 2 indicates the number of people who prefer not to use any product for two periods in a row (WW policy) decreases dramatically. The number of end-users in the NS group escalates significantly while the number of end-users in LL&SS groups reduces simultaneously. That means more end-users tend to have a product for the long term or till the end of its lifespan.

According to Figure 4, the marginal profit of retailers significantly depends on online shopping preference \((\alpha)\). In the numerical example, \(\alpha\) is assumed to be close to 1. It means that everyone prefers online shopping, and the retailer offers the products through only leasing options. Otherwise, the retailer always takes advantage of the selling sector by increasing the level of brand image. With respect to Figure 4, a higher level of the brand image leads the demand for buying a new product to grow dramatically and triggers more profit for the manufacturer.
5.2. **Impact of physical utility on variables.** Overall, a higher physical utility for a new product comes from new technology or more ergonomic design, making end-users more interested in adopting the LL policy and taking advantage of a new product in each period. In this situation, a higher leasing price and lower selling price for used products are expected. Since the used product price decreases, the end-user group with the SS policy becomes more, and the number of people adopting the NS policy reduces. Therefore, the total demand increases, and the number of end-users adopting WW policy falls. Concerning 6-7, a rise in the physical utility for a new product constitutes an increase in the marginal profit per product for both the manufacturer and the retailer. As a result, the manufacturer and the retailer simultaneously take advantage of the idea of enriching the physical utility of new products.

5.3. **Impact of production cost on variables.** According to Figure 8, production cost plays a prominent role in pricing new products. More production cost leads to increasing the selling and leasing price of a new product. As a result, a reduction in the total demand will be observed, and more end-users adopt an active policy. In addition, end-users taking NS, LL, and SS policies drop dramatically (Figure 8).
Figure 6. The amounts of selling and leasing prices under various physical utilities.

Figure 7. The amounts of marginal profit for the manufacturer and the retailer under various physical utilities.

Figure 8. Consumption pattern of each classification under various production costs.
Figure 9. The amounts of selling and leasing prices under various production costs

Figure 10. The amounts of marginal profit for the manufacturer and the retailer under various production costs

Figure 10 indicates that increasing the production cost reduces the margin profit of the manufacturer and the retailer. Variations for the manufacturer are much tangible. However, when the production cost exceeded the specific threshold (in this example, \( C=0.7 \)), the retailer would not be interested in continuing its business since it would no longer be profitable for him.

5.4. Impact of dealing cost on variables. The retailer must incur the dealing cost \( (d_2) \) when he decides to sell the product leased in the previous period as a used product, and it is less than the dealing cost of individuals in the market \( (d_1) \) when they want to sell their second-hand ones. In accordance with Figure 12, higher amounts of the dealing cost lead the retailer to increase the second-hand products' price, and consequently, less demand in the second-hand market will be created. The selling price to the end-users is stable, but the wholesale price has an opposite relation with the dealing cost since the aim here is to prevent increasing the leasing price as much as possible and keep it down in the market. It is worth mentioning that more end-users with leasing policies bring more sales for the manufacturer. Increasing dealing costs make a complete loss for the retailer.

Moreover, for high dealing cost levels, even second-hand markets will not exist (Figure 12). When there is no second-hand market, the leasing policy is meaningless. Less demand means less profit for the manufacturer as well (Figure 13).
5.5. **Impact of response levels of end-users to the brand image.** According to Figures 14-16, when the response level of end-users to the brand image increases, more demand in all segments is expected. Increasing total demand means higher prices and more benefits for both the manufacturer and retailer. However, increasing
response levels have a more intensive impact on the manufacturer. This analysis indicates the considerable importance of choosing the target market correctly.

**Figure 14.** Consumption pattern of each classification under various response levels

**Figure 15.** The amounts of selling and leasing prices under various response levels

**Figure 16.** The amounts of marginal profit per product for the manufacture under various response levels
5.6. Impact of online shopping preference on variables. There are some ways to calculate channel preferences for both online and physical stores. For instance, the physical distance between the end-user location and physical store or delivery time for an online store can be considered as factors that directly impact the online shopping preference [50]. Other factors such as lead time, website availability, internet connection, or online payment security influence online shopping preferences. However, this paper does not consider them in detail and assumes online shopping preferences as a combination of those factors. Since we examine the end-user’s behavior based on online shopping preference to demonstrate the manufacturer pricing policies and specifies the time the retailer decides to enter the market or leave it, $\alpha$ is defined as the percentage of end-users preferring online shopping compared to the physical store.

![Figure 17. The amounts of marginal profit per product for the manufacturer and retailer based on various alpha](image)

When $\alpha = 1$, it means that all the end-users prefer shopping online. Therefore, the manufacturer sells new products exclusively, and the retailer has only the leasing market. When $\alpha = 0$, it means no one prefers online shopping, and all end-users tend to use the physical store; therefore, the retailer has the selling and leasing market completely. In practice, different end-users are with various preferences ($0 \leq \alpha \leq 1$) Figure 17 illustrates the impact of online shopping preference on the various variables.

Sometimes, a retailer may consider a threshold for $\alpha$ to enter the market. Based on the minimum profit the retailer expects, the threshold of $\alpha$ is defined as $\alpha_T$. In this example, $\alpha_T$ is equal to 1.17. Thus, the leasing market itself has enough attraction for the retailer to enter the market. However, retailer profit increases considerably when online shopping preferences decrease.

6. Conclusion. This paper attempted to determine the optimal dynamic pricing for durable products according to the customer consumption pattern for a dual-channel supply chain. In the presented model, the real assumptions are considered. First, in addition to selling, the retailer can lease the products. Second, depreciation and its effect on physical utility have been investigated. Third, the second-hand product market is observed as an undeniable problem in the model. Fourth, the
effect of the image brand and the level of customer response to it on the consumption pattern has been studied.

The main conclusions drawn in this paper are as follows:

- A threshold ($\alpha_T$) is defined in this study. If the number of online shopping preferences was higher than ($\alpha_T$), the retailer would refuse to enter the market. When $\alpha_T > 1$, it means that the leasing market is enough for the retailer, and he enters the market even if he has no sales and the manufacturer herself sells the products through the online store.
- Since the brand image has a striking impact on the end-users and motivates them to buy or lease the products, the manager who seeks to become a winner in the competitive market needs to increase the level of brand image as much as possible.
- Enriching the physical utility level for new products, such as incorporating more options, influences both the manufacturer and the retailer’s profit. In fact, bringing new technology or more ergonomic design is considered one of the manager’s priorities whose aim is to increase profitability. In this case, more end-users tend to adopt the leasing policy and benefit from a new product each period that motivates the retailer to enter the market or remain active.
- Growing production costs have a significant reverse effect on the profitability of both manufacturer and retailer. Therefore, the manufacturer must focus on economic production since a rise in production costs constitutes an increase in the selling prices of new and second-hand products and the leasing price, and ultimately reduces demand, the profit of the manufacturer and retailer.
- Besides, the dealing cost is a critical parameter for the retailer to stay or quit the market. When the dealing cost increases, the manufacturer supports the retailer by reducing the wholesale price to the retailer and preventing losing the leasing market.
- Striking a balance between the production costs and the selling and leasing prices is considered a challenging decision for the manager seeking to have more end-users. Thus, it is worth devoting attention to this decision and optimizing the new products’ demand rate.
- The primary purpose of a retailer is to make more profit so that he needs more end-users who adopt the leasing policy. In this condition, he must make an excellent managerial decision, decrease the leasing price, and support the improvement of physical utility in each period.

Future research should further develop and confirm these initial findings by expanding the framework of this study. The presented model works properly for durable products, and it covers leasing and cash selling. A monopoly market with a manufacturer and a retailer is considered in this paper, while more players in a more competitive condition could be examined. This model can be extended by providing leasing options for both new and second-hand products for more than one period. Also, other transaction ways such as finance can be discussed in future studies. Future suggestions are considering more factors in the model, such as lead time, transportation costs, online shopping promotions, and marketing activities.
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Appendix A. First, it will be proved that there are at most five different types of behavior for customers, including NN, LL, SS, WW, and NL. By considering (20) and (21), end-users converge to some repetitive behavior at the focal point. All the product’s repetitive behavioral patterns with a two-period life span in an infinite time horizon include NN, SS, LL, WW, LW, WL, SL, LS, SW, WS, SN, NS, LN, NL, W, and NW. The permutation in repetitive behavior patterns has the same meaning. For example, in the pattern ...LW LW LW..., there is no difference between the RW pattern and W R. After ignoring permutation behaviors, only ten behavior remain for review. At the focal point, customers adopt policies that satisfy (22). If we assume \( z = R[y, P] \), (22) will be rewritten as (37). s means the reaction to a and p is chosen to maximize (37).

\[
RP_\theta[y, z, P] = PV_\theta[y, z, P] = F_\theta[y, z, P] + \rho \cdot PV_\theta[z, P]
\]  

(37)

A necessary condition should be considered; end-user’s behavior in the focal point’s steady limit is repetitive. If \( R[z, P] = y \), then \( R[y, P] = z \) and vice versa. Obviously, for the algorithms, NN, LL, SS, WW; this condition is simply met. Now we examine LS, WL, NS, WS, WN, LN patterns. Concerning table 2, it is clear that end-users at time \( t - 1 \) with the actions, including leasing, purchasing a used product and no action, choose the same action in the time \( t \) because they have the same payoff functions. Therefore, the optimal choice is similar to them.

\[
R_\theta[L, P] = R_\theta[S, P] = R_\theta[W, P]
\]  

(38)

implies that LS, WL, and WS algorithms cannot exist at the focal point. For example, on the one hand, in terms of the necessary condition, if \( R_\theta[S, P] = L \), then \( R_\theta[L, P] = S \). On the other hand, according to (38), we have \( R_\theta[S, P] = R_\theta[L, P] \) that is quite contradictory.

Furthermore, WN and LN patterns cannot be at the focal point. We will prove the WN pattern; the proof of the LN pattern is similar. If WN can exist as an algorithm at the focal point under steady limit condition, the necessary condition, if \( R_\theta[W, P] = N \), then \( R_\theta[N, P] = W \), should be satisfied. However, at the following,
it will be proved that if $R_0[W, P] = N$, then $R_0[N, P] \neq W$.

IF $R[W, P] = N$ then $PV[W, P] \equiv RP_0[W, N, P]$

$$F_0[W, N, P] + \rho \cdot PV_0[W, P]$$

$RP_0[W, N, P] = \text{Max} \{RP_0[W, N, P], RP_0[W, L, P], RP_0[W, W, P], RP_0[W, U, P]\}$

$RP_0[W, N, P] \geq RP_0[W, W, P]$  

$\Delta_1 + \varphi \lambda_1 - p_0 + \rho \cdot PV_0[N, P] \geq \rho \cdot PV_0[W, P]$  \hspace{1cm} (39)

Now consider

$$RP_0[N, W, P] = p_1 - \alpha + \rho \cdot PV_0[w, P]$$

$$RP_0[N, N, P] = \Delta_1 + \varphi \lambda_1 - p_0 + p_1 - d_1 + \rho \cdot PV_0[N, P]$$

By concerning (39), the relation $RP_0[N, N, P] > RP_0[N, W, P]$ will be obtained. Therefore if $R[W, P] = N$ then $R[N, P] \neq W$ It also is proved that LL always dominates NN policy.

$$PV_0[N, P] = \Delta_1 + \varphi \lambda_1 - p_0 + p_1 - d_1 \quad PV_0[L, P] = \Delta_1 + \varphi \lambda_2 - r \quad \forall \theta$$

If $\lambda_1 > \lambda_2$ and (25) is met, the following relation is obtained.

$$PV_0[N, P] < PV_0[L, P]$$  \hspace{1cm} (40)

The next group with the most willing to pay $(\theta_2, \theta_1)$ belongs to those who prefer to buy a new item and use it in the next period. The third group $(\theta_3, \theta_2)$ includes people who always use second-hand items, and finally, people with the least willing to pay $(0, \theta_3)$ do not use any item. To find the amount of $\theta_1, \theta_2, \theta_3$ and $\theta_4$, the net present value functions at the focal point for the optimal consumption patterns are written as:

$$PV_0[L, P] = \frac{\Delta_1 + \varphi \lambda_2 - r}{1 - \rho}$$  \hspace{1cm} (41)

**Proof.**

$$PV_0 = F_0[L, L, P] + \rho PV_0[L, P] \rightarrow (1 - \rho) PV_0[L, P] = \delta_1 + \varphi \lambda_2 - r$$

$$\rightarrow PV_0[L, P] = \frac{\Delta_1 + \varphi \lambda_2 - r}{1 - \rho}$$

$$PV_0[N, P] = \frac{\delta_2 + \varphi \lambda_3 + \rho(\Delta_1 + \varphi \lambda_1 - q_0)}{1 - \rho^2} \quad \text{when } \theta \in (\theta_2, \theta_1), \theta \in NS$$  \hspace{1cm} (42)

$$PV_0[N, P] = \frac{\Delta_1 + \varphi \lambda_1 - p_0 + \rho(\delta_2 + \varphi \lambda_3)}{1 - \rho^2} \quad \text{when } \theta \in (\theta_2, \theta_1), \theta \in NS$$  \hspace{1cm} (43)

$$PV_0[S, P] = \frac{\Delta_2 + \varphi \lambda_3 - p_1}{1 - \rho} \quad \text{when } \theta \in (\theta_3, \theta_2), \theta \in UU$$  \hspace{1cm} (44)

$$PV_0[W, P] = 0 \quad \text{when } \theta \in (0, \theta_3), \theta \in WW$$  \hspace{1cm} (45)
To find amounts of $\theta_1, \theta_2$ and the following equations are solved:

$$\theta \in LL \quad PV_\theta[L, P] = PV_\theta[S, P] \quad \theta \in NS \quad (46)$$

$$\theta_1 = \frac{r(1 + \rho) - p_0 + \varphi(\lambda_1 + \rho \lambda_3 - (1 + \rho) \lambda_2)}{\rho(2\Delta_1 - 1)}$$

$$\theta \in NS \quad PV_\theta[S, P] = PV_\theta[U, P] \quad \theta \in UU \quad (47)$$

$$\theta_2 = \frac{\varphi(\lambda_3 - \lambda_1) - (1 + \rho)p_1 + p_0}{(2\Delta_1 - 1)}$$

$$\theta \in UU \quad PV_\theta[S, P] = PV_\theta[W, P] \quad \theta \in WW \quad (48)$$

$$\theta_3 = \frac{p_1 - \varphi \lambda_3}{1 - \Delta_1}$$

$$\theta \in WW \quad (49)$$