Neutrino Data, CP violation and Cosmological Implications

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Abstract. Recent experimental data provides evidence for neutrino masses and leads to the possibility of leptonic mixing and CP violation. In this work special attention is dedicated to CP violation in the leptonic sector both at low and at high energies in the framework of seesaw with only three righthanded neutrinos added to the Lagrangean of the Standard Model. It is shown that leptogenesis is a possible and likely explanation for the observed baryon asymmetry of the universe. In this case neutrino masses are constrained yet CP violating phases at low energies can only be related to CP violation at high energies in the context of specific models.

1 Introduction

Recent evidence for neutrino masses available from experiments with solar [1], [2] and atmospheric neutrinos [3], reactor experiments [4], [5] neutrinoless double beta decay searches [6] astrophysics and cosmology [7] [8] entails the possibility of leptonic mixing and CP violation both at low energies and at high energies. CP violation in the leptonic sector can have important cosmological implications playing a rôle in the generation of the observed baryon number asymmetry of the universe (BAU) through leptogenesis [9].

There are several possible ways of generating neutrino masses in the context of minimal extensions of the Standard Model. The most straightforward one is the simple extension of the Standard Model (SM) by including one righthanded neutrino per generation. In this case the number of fermionic degrees of freedom for neutrinos equals those of all other fermions in the theory, this fact may be viewed as adding elegance to the theory. This is the framework on which the work presented in this talk is based. Special emphasis will be given to general results obtained in [10] and [11] and at the same time an attempt is made to present some recent important results obtained by other authors. It should be pointed out that it is also possible to generate neutrino masses in such a framework without requiring the number of righthanded and lefthanded neutrino fields to be equal.

It is well known that such an extension of the SM allows for the seesaw mechanism [12] to operate giving rise to three light and three heavy neutrinos of Majorana character as well as leptonic mixing and the possibility of CP violation in the couplings of these neutrinos to the charged leptons. The seesaw mechanism also provides a natural explanation for the smallness of neutrino masses. In this framework one of the most plausible scenarios for the generation of BAU is
the leptogenesis mechanism where a CP asymmetry generated through the out-of-equilibrium L-violating decays of the heavy Majorana neutrinos leads to a lepton asymmetry which is subsequently transformed into a baryon asymmetry by (B+L)-violating sphaleron processes [13].

The possibility of CP violation in the leptonic sector both at low and high energies, i.e., in the charged current couplings of heavy neutrinos – with implications for leptogenesis – and the charged current couplings of light neutrinos – with implications for low energy phenomenology that may possibly be observed in future experiments – raises the important question of whether it is possible to establish a direct connection between these two phenomena. This question is of special relevance and is related to another important one: If indeed BAU is generated through leptogenesis how can it be proved? In the discussion that follows it will become clear that this connection cannot be established in general it can only be established in special frameworks. This was shown in [10], by making use of a special parametrization which made explicit the fact that it is possible to have low energy CP violation without CP violation at high energies (meaning no leptogenesis). In addition it was shown in [11] that viable leptogenesis is possible without low energy CP violation, i.e., no CP violation at low energies resulting either from Dirac or from Majorana phases. Several authors have addressed the same question in the context of specific models [14].

In the next section we introduce the general framework and give the number of independent CP violating phases present in the Lagrangean. We show how these phases can be parametrized, for three generations, still in a weak basis and we also indicate how they appear in the physical basis, through the seesaw mechanism, where three phases are relevant for CP violation at low energies. In section three we briefly present the conditions for viable leptogenesis with special emphasis in the case of hierarchical heavy neutrinos. We show that for three generations there are three CP violating phases on which leptogenesis depends. In section four we comment on the connection between CP violation at high energies and at low energies. Section five shows how to build weak basis invariant conditions allowing to determine whether a particular Lagrangean does violate CP without the need to go to the physical basis. Section six contains the conclusions.

2 General Framework and Seesaw

After spontaneous symmetry breaking, the leptonic mass term for the minimal extension of the SM, which consists of adding to the standard spectrum one right-handed neutrino per generation, can be written as:

\[ \mathcal{L}_m = -\left[ \nu_R^T \nu_R + \frac{1}{2} \nu_R^T C M \nu_R + i \nu_R^T m_\ell l_R \right] + \text{h.c.} = -\left[ \frac{1}{2} n_L^T C M^* n_L + i \bar{n}_L m_\ell l_R \right] + \text{h.c.} \]  

(1)
where \( m, M \) and \( m_l \) denote the neutrino Dirac mass matrix, the right-handed neutrino Majorana mass matrix and the charged lepton mass matrix, respectively, and \( n_L = (\nu^0_L, (\nu^0_R)^c) \) (should be interpreted as a column matrix). In this minimal extension of the SM a term of the form \( \frac{1}{2} \nu^0_L C m_L \nu^0_L \) does not appear in the Lagrangean and the matrix \( \mathcal{M} \) is given by:

\[
\mathcal{M} = \begin{pmatrix}
0 & m \\
m^T & M
\end{pmatrix}
\]

with a zero entry on the (11) block. The right-handed Majorana mass term is \( SU(2) \times U(1) \) invariant, consequently it can have a value much above the scale \( v \) of the electroweak symmetry breaking, thus leading to the seesaw mechanism.

The number of independent CP violating phases was identified for this case [15] as being equal to \( n(n - 1) \) with \( n \) the number of generations. For three generations this number equals six. In the most general case, without imposing \( m_L \) equal to zero and with \( n \) lefthanded neutrino fields, \( n' \) righthanded neutrino fields this number is given [16] by \( nn' + \frac{n(n-1)}{2} \).

CP violation may be analysed either in a weak basis (WB) or in the physical basis. It is always possible to choose a WB where the matrices \( M \) and \( m_l \) are simultaneously diagonalized, in this WB all CP violating phases appear in the matrix \( m \). The matrix \( m \) can be written without loss of generality as the product of a unitary times a Hermitian matrix (polar decomposition) in this case it is clear how these six independent CP violating phases may appear in \( m \):

\[
m = U H = P_\gamma \bar{U}_\delta P_\tau \bar{H}_\sigma P_\beta \cdot
\]

The matrices \( P \) are diagonal unitary, in general \( P_\gamma \) can have three phases in the diagonal which can be rotated away through a redefinition of the lefthanded leptons, \( P_\tau \) and \( P_\beta \) only have two phases each, \( \bar{U}_\delta \) is a general unitary matrix with only one phase left, \( \bar{H}_\sigma \) is a Hermitian matrix with two of its phases factored out. After rotating away \( P_\gamma \) we are left with

\[
m = \bar{U}_\delta P_\alpha \bar{H}_\sigma P_\beta
\]

with six phases \( \rho, \alpha_1, \alpha_2, \sigma, \beta_1 \) and \( \beta_2 \) which cannot be eliminated.

In order to go to the physical basis let us start from the WB where \( m_l \) is already diagonal. The neutrino mass matrix \( \mathcal{M} \) is diagonalized by the transformation:

\[
V^T \mathcal{M} V = \mathcal{D},
\]

where \( \mathcal{D} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_{\nu_1}, M_{\nu_2}, M_{\nu_3}) \), with \( m_{\nu_i} \) and \( M_{\nu_i} \) denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write \( V \) and \( \mathcal{D} \) in the following block form:

\[
V = \begin{pmatrix}
K & R \\
S & T
\end{pmatrix}; \quad \mathcal{D} = \begin{pmatrix}
d & 0 \\
0 & D
\end{pmatrix}.
\]
From (5) and assuming the scale of $M$ much higher than that of $v$, one obtains, to an excellent approximation:

$$-K^\dagger m^{-1}M^T K^* = d,$$

(7)

together with the following exact relation:

$$R = mT^*D^{-1}.$$  

(8)

In the WB where the right-handed Majorana neutrino mass is also diagonal, it then follows, to an excellent approximation, that:

$$R = mD^{-1}.$$  

(9)

Equation (7) is the usual seesaw formula with $K$ a unitary matrix. The neutrino weak-eigenstates are related to the mass eigenstates by:

$$\nu^0_L = V_{i\alpha} \nu_{\alpha L} = (K, R) \begin{pmatrix} \nu_{1L} \\ N_{1L} \end{pmatrix} \begin{pmatrix} \alpha = 1, \ldots, 6 \end{pmatrix},$$

(10)

and thus the leptonic charged-current interactions are given by:

$$-\frac{g}{\sqrt{2}} (\overline{\nu_{iL}} \gamma_{\mu} K_{ij} \nu_{jL} + \overline{\nu_{iL}} \gamma_{\mu} R_{ij} N_{jL}) W^\mu + \text{h.c.}$$

(11)

From (10), (11) we see that $K$ and $R$ give the charged-current couplings of charged leptons to the light neutrinos $\nu_j$ and to the heavy neutrinos $N_j$, respectively. The unitary matrix $K$, which contains all the information about CP violation at low energies, can be parametrized as:

$$K = P_\xi \hat{U}_\delta P_\theta \rightarrow \hat{U}_\delta P_\theta$$

(12)

with $P_k = \text{diag}(\exp(i\xi_1), \exp(i\xi_2), \exp(i\xi_3))$, and $P_\theta = \text{diag}(1, \exp(i\theta_1), \exp(i\theta_2))$ leaving $\hat{U}_\delta$ with only one phase as in the case of the Cabibbo, Kobayashi and Maskawa matrix. Since $P_k$ can still be rotated away by a redefinition of the charged leptonic fields, $K$ is left with three CP-violating phases, one of Dirac type $\varrho$ and two of Majorana character $\theta_1$ and $\theta_2$. The matrix $K$ is the Maki, Nakagawa and Sakata mixing matrix [17]. The matrix $R$ is of relevance for leptogenesis even though the out of equilibrium decay of the heavy Majorana neutrinos responsible for the generation of $L \neq 0$ occurs in the symmetric phase, since it can be related to the initial Yukawa couplings and the masses of the heavy neutrinos through (8). It is clear from (11) that its entries are suppressed by $v/M$.

### 3 Comment on General Conditions for Leptogenesis

In this section, we identify the CP violating phases relevant for leptogenesis, obtained through the out of equilibrium decay of heavy Majorana neutrinos. We
have seen that there are six independent CP violating phases in the Lagrangian. Next we show which of these six independent CP violating phases contribute to lepton number asymmetry. Leptogenesis strongly depends on the masses of the heavy neutrinos and requires different conditions depending on whether these masses are hierarchical \[13\] or close to degenerate \[19\]. Thermal leptogenesis in the case of hierarchical heavy neutrinos only depends on four parameters \[20\]: the mass \(M_1\) of the lightest heavy neutrino together with the corresponding CP asymmetry \(\varepsilon_{N_1}\) in their decays, as well as the effective neutrino mass \(\tilde{m}_1\) defined as

\[
\tilde{m}_1 = (m^\dagger m)_{11}/M_1
\]

in the weak basis where \(M\) is diagonal real and positive, and finally, the sum of all light neutrino mass squared, \(m^2 = m^2_1 + m^2_2 + m^2_3\), which controls an important class of washout processes.

The computation of the lepton-number asymmetry, in this extension of the SM, resulting from the decay of a heavy Majorana neutrino \(N^j\) into charged leptons \(l^\pm_i\) \((i = e, \mu, \tau)\) leads to \[21\] :

\[
\varepsilon_{N_j} = \frac{g^2}{M_W} \sum_{k \neq j} \left[ \text{Im} \left( (m^\dagger m)_{jk}(m^\dagger m)_{jk} \right) \frac{1}{16\pi} \left( I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \frac{1}{(m^\dagger m)_{jj}}
\]

\[
= \frac{g^2}{M_W} \sum_{k \neq j} \left( M_k \right)^2 \text{Im} \left( (R^\dagger R)_{jk}(R^\dagger R)_{jk} \right) \frac{1}{16\pi} \left( I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \frac{1}{(R^\dagger R)_{jj}}
\]

(14)

with the lepton-number asymmetry from the \(j\) heavy Majorana particle, \(\varepsilon_{N_j}\), defined in terms of the family number asymmetry \(\Delta A^j_i = N^j_i - \overline{N}^j_i\) by :

\[
\varepsilon_{N_j} = \frac{\sum_i \Delta A^j_i}{\sum_i \left( N^j_i + \overline{N}^j_i \right)}
\]

(15)

\(M_k\) are the heavy neutrino masses, the variable \(x_k\) is defined as \(x_k = \frac{M_k^2}{M^2}\) and \(I(x_k) = \sqrt{x_k} \left( 1 + (1 + x_k) \log\left( \frac{x_k}{1 + x_k} \right) \right)\), the sum in \(i\) runs over the three flavours \(i = e, \mu, \tau\). From \[14\] it can be seen that the lepton-number asymmetry is only sensitive to the CP-violating phases appearing in \(m^\dagger m\) in the WB, where \(M\) and \(m_l\) are diagonal (or equivalently in \(R^\dagger R\)). Making use of the parametrization given by \[14\] it becomes clear that leptogenesis is only sensitive to the phases \(\beta_1, \beta_2\) and \(\sigma\). If these phases are zero \(m^\dagger m\) is real and no lepton number asymmetry is generated through the decay of heavy Majorana neutrinos.

Successful leptogenesis would require \(\varepsilon_{N_1}\) of order \(10^{-8}\), if washout processes could be neglected, in order reproduce the observed ratio of baryons to photons which is given by \[12\] :

\[
\frac{n_B}{n_\gamma} = (6.1^{+0.3}_{-0.2}) \times 10^{-10}.
\]

(16)
The computation of the effect of washout processes requires the integration of the full set of Boltzmann equations. Leptogenesis is a nonequilibrium process which takes place at temperatures $T \sim M_1$. This imposes an upper bound on the effective neutrino mass $\tilde{m}_1$ given by the “equilibrium neutrino mass” $\tilde{m}_1$: 

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g^\star \frac{v^2}{M_{\text{pl}}} \approx 10^{-3}\text{eV}$$

(17)

where $M_{\text{pl}}$ is the Planck mass ($M_{\text{pl}} = 1.2 \times 10^{19}$ Gev). The sum of all neutrino mass squared $\sum m^2$ is constrained, to be below 0.21 eV [25]. Which implies an upper bound on all light neutrinos masses of 0.12 eV. Furthermore relaxing this upper bound to 0.4 eV already requires strong degeneracy of the heavy neutrinos [25]. It is interesting to note that these bounds are compatible [26] with the present constraints on $|<m>|$ defined by:

$$|<m>| \equiv |m_1 K_{11}^2 + m_2 K_{12}^2 + m_3 K_{13}^2|$$

(18)

obtained from neutrinoless double beta decay, for which the Heidelberg-Moscow Collaboration gives [6]:

$$|<m>| = (0.05 - 0.84)\text{eV}.$$  

(19)

The leptogenesis scenario is an interesting explanation for BAU in agreement with all experimental data available at present.

4 On the Connection between CP Violation at Low and High Energies

The prospects of finding CP-violating effects at low energies, for instance in future neutrino factories, are extremely exciting. Yet it is important to notice that leptogenesis remains in principle a viable scenario even if there is no CP violation at low energies [11], conversely the observation of CP violation at low energies does not necessarily imply CP violation at high energies [10].

In the previous section it was shown that there is no leptogenesis for $\beta_1$, $\beta_2$ and $\sigma$ equal to zero. However the matrix $m_{\text{eff}} = -m_D^{-1} m^T$ which is diagonalized by the $V_{\text{MNS}}$ matrix, can still be complex in this case, due to the fact that there are three additional phases $\varphi, \alpha_1$ and $\alpha_2$ in the parametrization of $m$ and these do not cancel out in $m_{\text{eff}}$. Another simple way of reaching the same conclusion is by noting that any matrix can be diagonalized through a biunitary transformation and thus writing the matrix $m$ in the form:

$$m = U_1^\dagger D U_2$$

(20)

with the $U_i$ unitary matrices and $D_D$ a diagonal real matrix. If $U_2$ is real $m^\dagger m$ is also real and there is no leptogenesis. Yet $m_{\text{eff}}$ which is given by

$$m_{\text{eff}} = -m_D^{-1} m^T = -U_1^\dagger D U_2 D^{-1} U_2^T D_D U_1^*$$

(21)
can be a complex matrix, even in the limit of $U_2$ real, requiring $V_{MNS}$ also complex.

On the other hand from (7) we can write

$$\left( m \frac{1}{\sqrt{D}} \right) \left( m \frac{1}{\sqrt{D}} \right)^T = (iK\sqrt{d})(iK\sqrt{d})^T$$

with $\sqrt{d}$ and $\sqrt{D}$ diagonal real matrices such that $\sqrt{d}\sqrt{d} = d$, $\sqrt{D}\sqrt{D} = D$

it is clear that it is possible to choose the matrix $m$ after replacing $K$ by the expression given in [22] as [27]:

$$m = i\hat{U}_e P_\theta \sqrt{d} O c \sqrt{D},$$

$O^c$ is an orthogonal complex matrix, i.e. $O^c O^{c^T} = 1$ but $O^c O^{c\dag} \neq 1$. Particularizing for $\theta_1 = \theta_2 = 0$ together with $\varphi = 0$, there is no CP violation at low energies. Yet leptogenesis is sensitive to the combination $m^\dagger m$, which is given by:

$$m^\dagger m = \sqrt{D} O c^\dag d O^c \sqrt{D};$$

consequently, provided that the combination $O^c^\dag d O^c$ is CP-violating, we may have leptogenesis even without CP violation at low energies either of Dirac or Majorana type.

Equation (20) is also useful to reach the same conclusion in a simple way. In this notation

$$m^\dagger m = U_2^\dagger d^2 U_2$$

and viable leptogenesis requires a complex $U_2$ matrix. In this case from (21) it is clear that, unless one chooses a special form for the $U_1$ matrix, $m_{eff}$ is in general complex and there is also CP violation at low energies. However it is obvious that it is always possible to choose $U_1$ such that $V_{MNS}$ is real – this conclusion follows from the fact that both $U_1$ and $V_{MNS}$ are unitary matrices and appear in adjacent positions in the diagonalization of $m_{eff}$ so that $U_1$ can be redefined to absorb the phases of $V_{MNS}$. Furthermore this reasoning shows that, given a model with an arbitrary complex matrix $m$, in general one should expect manifestations of CP violation both at low and at high energies. The connection between these manifestations is model dependent and has been studied by many authors [14]. It is possible that all CP violating phenomena in nature have a common origin through a single phase in the vacuum expectation value of a complex scalar field [28].

5 Weak Basis Invariants and CP Violation

Given a Lagrangean still written in a weak basis it is useful to be able to analyse whether or not there is CP violation without necessarily having to go to the physical basis. For that purpose one must write WB invariant conditions which have to be verified in the case of CP conservation.
Let us write the most general CP transformation which leaves the Lagrangian invariant \[16\]:

\[
\begin{align*}
\text{CP}_L(CP)^\dagger &= U_{\gamma^0 C}^T L \quad \text{CP}_R(CP)^\dagger = V_{\gamma^0 C}^T R \\
\text{CP}_{\nu L}(CP)^\dagger &= U_{\gamma^0 C}^T \nu_L \quad \text{CP}_{\nu R}(CP)^\dagger = V_{\gamma^0 C}^T \nu_R
\end{align*}
\tag{26}
\]

where U, V, W are unitary matrices acting in flavour space and where for notation simplicity we have dropped here the superscript 0 in the fermion fields. Invariance of the mass terms under the above CP transformation, requires that the following relations have to be satisfied:

\[
\begin{align*}
W^TMW &= -M^* \\
U^\dagger mW &= m^* \\
U^\dagger m_lV &= m_l^*
\end{align*}
\tag{27-29}
\]

From these equations one obtains:

\[
\begin{align*}
W^\dagger hW &= h^* \\
W^\dagger HW &= H^* \\
U^\dagger h_iU &= h_i^*
\end{align*}
\tag{30}
\]

where \( h = m^\dagger m \), \( H = M^\dagger M \) and \( h_i = m_l m_l^\dagger \). It can be then readily derived, from (27), (30), through multiplications or commutators and applying traces (and determinants) that CP invariance requires, for instance:

\[
\begin{align*}
I_1 \equiv \text{ImTr}[hHM^* h^* M] &= 0 \\
I_2 \equiv \text{ImTr}[hH^2M^* h^* M] &= 0 \\
I_3 \equiv \text{ImTr}[hH^2M^* h^* MH] &= 0
\end{align*}
\tag{31}
\]

Since these \( I_i \) are WB invariant, they may be evaluated in any convenient WB. These conditions are sensitive to the phases \( \beta_1, \beta_1 \) and \( \sigma \) which are relevant for leptogenesis. Three additional interesting conditions can be obtained through the substitution of \( h \) by \( \bar{h} = m_l h_i m \). The strength of CP violation at low energies, observable for example through neutrino oscillations, can be obtained from the following low-energy WB invariant:

\[
Tr[h_{ef}, h_i]^3 = 6i \Delta_{21} \Delta_{32} \Delta_{31} \text{Im}\{(h_{ef})_{12}(h_{ef})_{23}(h_{ef})_{31}\}
\tag{32}
\]

where \( h_{ef} = m_{ef} m_{ef}^\dagger \) and \( \Delta_{21} = (m_\mu^2 - m_e^2) \) with analogous expressions for \( \Delta_{31}, \Delta_{32} \). This invariant is analogous to the one written for the quark sector in the context of the standard model in [29] where this technique was first applied. Several different WB invariant conditions, useful in the leptonic sector, and for specific models have been built using the same technique [16] [30].

6 Conclusions

Neutrino physics is a very lively subject both theoretically and experimentally. Several neutrino experiments are under study for the near future. From the
theoretical point of view a lot of work is being done on fundamental questions such as the origin of leptonic masses and mixing \(^{31}\). Neutrino properties have important cosmological implications, for example, the possibility of leptogenesis. If leptogenesis is the origin of the observed baryon asymmetry of the universe this implies constraints on neutrino masses both of light and of heavy neutrinos. Leptogenesis is one of the most promising scenarios, in part due to the fact that several other alternative proposals are on the verge of being ruled out. However it is likely that this will remain an open question still for some time.

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**References**

1. S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Lett. B 539, 179 (2002) [arXiv:hep-ex/0205075].
2. Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. 89, 011302 (2002) [arXiv:nucl-ex/0204009].
3. T. Nakaya [SUPER-KAMIOKANDE Collaboration], eConf C020620, SAAT01 (2002) [arXiv:hep-ex/0209036].
4. M. Apollonio *et al.* [CHOOZ Collaboration], Phys. Lett. B 466, 415 (1999) [arXiv:hep-ex/9907037].
5. K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. 90, 021802 (2003) [arXiv:hep-ex/0212021].
6. H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney, I. V. Krivosheina, Mod. Phys. Lett. A 16, 2409 (2001) [arXiv:hep-ph/0201231]; H. V. Klapdor-Kleingrothaus [Heidelberg-Moscow Collaboration], arXiv:hep-ph/0307330. To appear in the proceedings of 4th Workshop on Neutrino Oscillations and their Origin (NOON2003), Kanazawa, Japan, 10-14 Feb 2003.
7. C. L. Bennett *et al.*, Astrophys. J. Suppl. 148, 1 (2003) [arXiv:astro-ph/0302207].
8. D. N. Spergel *et al.*, Astrophys. J. Suppl. 148, 175 (2003) [arXiv:astro-ph/0302209].
9. M. Fukugita, T. Yanagida, Phys. Lett. B 174, 45 (1986).
10. G. C. Branco, T. Morozumi, B. M. Nobre, M. N. Rebelo, Nucl. Phys. B 617, 475 (2001) [arXiv:hep-ph/0101161].
11. M. N. Rebelo, Phys. Rev. D 67, 013008 (2003) [arXiv:hep-ph/0207236].
12. T. Yanagida, in Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe, ed. by O. Sawada and A. Sugamoto (KEK report 79-18, 1979), p.95, Tsukuba, Japan; M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, ed. by P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979), p.315; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44, 912 (1980).
13. V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155, 36 (1985).
14. R. N. Mohapatra, X. Zhang, Phys. Rev. D 46, 5331 (1992); H. B. Nielsen, Y. Takanishi, Phys. Lett. B 507, 241 (2001); E. Nezri, J. Orloff, JHEP 0304, 020 (2003); T. Endoh, T. Morozumi, T. Onogi, A. Purwanto, Phys. Rev. D 64, 013006 (2001); A. S. Joshipura, E. A. Paschos, W. Rodejohann, JHEP 0108, 029 (2001); F. Buccella, D. Falcone, F. Tramontano, Phys. Lett. B 524, 241 (2002); K. R. S. Balaji, W. Rodejohann, Phys. Rev. D 65, 093009 (2002); G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim, M. N. Rebelo, Nucl. Phys. B 640, 202 (2002); J. R. Ellis, M. Raidal, Nucl. Phys. B 643, 229 (2002); Z. Z. Xing, Phys. Lett. B 545, 352 (2002); J. R. Ellis, M. Raidal, T. Yanagida, Phys. Lett. B 546, 228 (2002); S. Davidson, A. Ibarra, Nucl. Phys. B 648, 345 (2003); P. H. Frampton, S. L. Glashow, T. Yanagida, Phys. Lett. B 548, 119 (2002); T. Endoh, S. Kaneko, S. K. Kang, T. Morozumi, M. Tanimoto, Phys. Rev. Lett. 89, 231601 (2002); T. Hambye, arXiv:hep-ph/0210048; G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim, I. Masina, M. N. Rebelo, C. A. Savoy, Phys. Rev. D 67, 073025 (2003); S. F. King, arXiv:hep-ph/0211228; S. Pascoli, S. T. Petcov, W. Rodejohann, arXiv:hep-ph/0302054; E. K. Akhmedov, M. Frigerio, A. Y. Smirnov, arXiv:hep-ph/0305322; A. Broncano, M. B. Gavela, E. Jenkins, arXiv:hep-ph/0307058; L. Velasco-Sevilla, arXiv:hep-ph/0307071; B. Dutta, R. N. Mohapatra, arXiv:hep-ph/0307163; V. Barger, D. A.Dicus, H. J. He, T. Li, arXiv:hep-ph/0310278; W. I. Guo, Z. Z. Xing, arXiv:hep-ph/0310326; W. Rodejohann, arXiv:hep-ph/0311142.

15. T. Endoh, T. Morozumi, T. Onogi, A. Purwanto, in [14].

16. G. C. Branco, L. Lavoura, M. N. Rebelo, Phys. Lett. B 180, 264 (1986).

17. Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

18. S. Davidson, JHEP 0303, 037 (2003) arXiv:hep-ph/0302075; W. Buchmuller, P. Di Bari, M. Plumacher, Nucl. Phys. B 665, 445 (2003) arXiv:hep-ph/0302092; G. F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia, arXiv:hep-ph/0310123.

19. A. Pilaftsis, T. E. J. Underwood, arXiv:hep-ph/0309342.

20. W. Buchmuller, P. Di Bari, M. Plumacher, Nucl. Phys. B 643, 367 (2002) arXiv:hep-ph/0205349; W. Buchmuller, P. Di Bari, M. Plumacher, Phys. Lett. B 547, 128 (2002) arXiv:hep-ph/0209301.

21. L. Covi, E. Roulet, F. Vissani, Phys. Lett. B 384, 169 (1996); M. Flanz, E. A. Paschos, U. Sarkar, Phys. Lett. B 345, 248 (1995) [Erratum, ibid. B 382, 447 (1995)]; M. Plumacher, Z. Phys. C 74, 549 (1997).

22. E. W. Kolb, M. S. Turner, “The Early Universe,” Addison-Wesley (1990) 547 p. (Frontiers in physics, 69).

23. W. Fischler, G. F. Giudice, R. G. Leigh, S. Paban, Phys. Lett. B 258, 45 (1991).

24. W. Buchmuller, T. Yanagida, Phys. Lett. B 302, 240 (1993).

25. W. Buchmuller, P. Di Bari, M. Plumacher (2003) in Ref. [18].

26. H. V. Klapdor-Kleingrothaus, U. Sarkar, Mod. Phys. Lett. A 18, 2243 (2003) arXiv:hep-ph/0304032.

27. This parametrization was first used by: J. A. Casas, A. Ibarra, Nucl. Phys. B 618, 171 (2001) arXiv:hep-ph/0103065; G. C. Branco, P. A. Parada and M. N. Rebelo, arXiv:hep-ph/0307119; J. Bernabeu, G. C. Branco, M. Gronau, Phys. Lett. B 169, 243 (1986).

28. A. Pilaftsis, Phys. Rev. D 56, 5431 (1997) arXiv:hep-ph/9707235; G. C. Branco, M. N. Rebelo, J. I. Silva-Marcos, Phys. Rev. Lett. 82, 683 (1999) arXiv:hep-ph/9810328.

29. E. Ma, arXiv:hep-ph/0307016 Published in these proceedings.