Hertz’s viewpoint on quantum theory

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Abstract

In 19th century (in the process of transition from mechanics to the field theory) the German school of theoretical physics confronted problems similar to the basic problems in the foundations of quantum mechanics (QM). Hertz tried to resolve such problem through analysis of the notion of a scientific theory and interrelation of theory and experiment. This analysis led him to the Bild (image) conception of theory (which was latter essentially developed, but also modified by Boltzmann). In this paper we claim that to resolve the basic foundational problems of QM, one has to use the Bild conception and reject the observational viewpoint on physical theory. As an example of a Bild theory underlying QM (treated as an observational theory), we consider prequantum classical statistical field theory (PCSFT): theory of random subquantum fields.

keywords: Hertz Bild theory, descriptive and observational theories, hidden variables in electromagnetism vs quantum mechanics, quantum theory as observational theory, prequantum classical statistical field theory.

1 Introduction

During one hundred years quantum theory has been suffering of endless debates about its meaning and interpretation. I claim that this unacceptable
situation is the result of neglect by the fathers of quantum mechanics (QM) the extensive study of similar problems by the traditional German school in physics: ignoring the works of Helmholtz, Hertz, and Boltzmann, see, e.g., (Hertz, 1899; Boltzmann, 1905, 1974). Consciously or unconsciously Bohr, Heisenberg, Einstein, Pauli and other main contributors to foundations of quantum theory (but excluding Schrödinger, see, for example, (D’ Agostino, 1992)) ignored the historical lessons of the debate on the interrelation between theory and experiment which was initiated by transition from Newtonian mechanics to Maxwellian electromagnetism (Hertz, 1899). In particular, in this debate the problem of hidden variables was enlighten by Hertz - may the first time in history of science (Hertz, 1899). In the light of this debate the following debate between Bohr and Einstein can be characterized by lack of deep philosophic analysis (Einstein, Podolsky, & Rosen, 1935; Bohr, 1935). I am not afraid to call the latter debaters naive - by taking into account the lessons of the aforementioned debate about electromagnetism.

In this paper I shortly present the views of Hertz, see, e.g., (Hertz, 1899), see also (Boltzmann, 1905, 1974), on scientific theory - the Bild (image) conception, section 2.1. Here I follow the works (D’ Agostino, 1992; Miller, 1984). At the end of this section there are discussed various approaches to the notion of theory, the descriptive, Bild, and observational approaches. By speaking about a scientific theory one has to specify its type. Then I proceed to quantum physics. I consider the present situation in quantum foundations by appealing to the Herzian Bild conception, section 2.2. In section 2.3 there are formulated the rules of correspondence between two theories of different types (especially their mathematical structures). Finally, in section 3 there is presented a theory of micro-phenomena based on the Bild conception, pre-quantum classical statistical field theory (PCSFT), see (Khrennikov, 2007a, b, 2014, 2017c).

Since this issue is devoted to ontology of quantum theory, it is useful to stress the impact of the Bild conception to the quantum foundational debate about realism, including Einstein-Bohr debate about completeness of QM. From the Bild-viewpoint, realism in physics as well as any other area of scientific research is reduced to experimental facts. This is exactly Bohr’s po-

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1This paper is continuation of my previous works in which the Bild-conception was explored in quantum physics (Khrennikov, 2017a, b, c). It is also important to remark that similar approach to quantum theory was supported by Schrödinger, see (D’Agostino, 1992).
Thus the only realistic component of quantum physics are outcomes of measurements (Bohr’s ‘phenomena’). Any physical theory is only about human images of natural phenomena. At the same time these images are created on the basis of human’s interaction with nature.

In (Khrennikov, 2007c) I tried to establish relation between the Hertz-Boltzmann Bild viewpoint and the ontic-epistemic viewpoint (Atmanspacher & Primas, 2005) on the notion of scientific theory. However, this is a complex problem. Observational theories of the present paper can be definitely treated as epistemic theories. However, the Bild conception is not about reality as it is (as in an ontic theory), it is about human images of reality.

2 Method

2.1 Bild conception

Hertz’ discovery of radio-waves was connected with his deep analysis of the Maxwellian electromagnetism from the viewpoint of interrelation between theory and experiment. Electromagentism, in Hertz’s opinion, based on the action at a distance principle was only a “first approximation to the truth.” And he worked hardly to approach the final true theory. From the formal viewpoint, he tried to create a mechanical model of electromagnetic phenomena. However, these studies led him to understanding that it seems to be impossible to construct such a model without invention of hidden variables of the mass type, so called concealed masses. In turn, this led him to deep philosophical and methodological studies devoted to meaning of ‘theory’ in science.

However, Bohr would say that a Bild-type theory has nothing to do with physics and he would refer to it as a metaphysical theory. At the same time he was not so much interested in no-go theorems for descriptive or Bild-type theories. In principle, he could not exclude that such ‘beyond quantum theories’ might be constructed. But, they would not have any value for physics (Plotnitsky, 2006, 2009).

He was not able to complete his project on the mechanical theory of electromagnetism. (Ironically the same fate befell Einstein who in turn spent the last 20 years of his life by attempting to create the classical field theory of quantum phenomena, see, e.g., (Einstein & Infeld, 1961).) However, Hertz’s contribution was very valuable to the methodology of science. And it influenced strongly Boltzmann and Schrödinger and through Botzmann’s works Planck (and may be even Einstein), see (Miller, 1984).
The main impact of these studies was relative liberation of theory from experiment. One of Hertz’ fundamental statements is that “We become convinced that the manifold of the actual universe must be greater than the manifold of the universe which is directly revealed to our senses.” See (Hertz, 1899).

Hertz explored heavily Helmholtz’s principle about a parallelism between concepts and perceptions. However, Hertz rejected Helmholtz’s claim that this parallelism uniquely determines the theory consistent with experimental facts. Hertz questioned the later (so to say the strong version of the parallelism principle) and claimed that there exists a multiplicity of representations satisfying the requirement of Helmholtz’s parallelism: “The images [Bilder] which we may form of things are not determined without ambiguity by the requirement that the consequents of images must be images of consequents. Various images of the same objects are possible, and these images may differ in various aspects.” See (Hertz, 1899).

It is even more important for our present considerations that Hertz stated that Helmholtz’s parallelism of laws does not even work if a theory is limited to visible quantities. Only the introduction of hidden quantities allows creation of a consistent theory: “If we try to understand the motions of bodies around us, and refer to simple and clear rule, paying attention only to what can be directly observed, our attempts will in general fail. We soon become aware that the totality of things visible and tangible do not form a universe conformable to law, in which the same result always follow from the same conditions.” See (Hertz, 1899).

From Hertzian perspective, a theory is not a true description of nature (Botzmann’s “complete congruence with nature”) or at least a best approximation of it, but a theory is “mere a representation (Bild) of a nature ... which at the present allows one to give the most uniform and comprehensive account of totality of phenomena” See (Boltzmann, 1905).

Of course, this viewpoint on the conception of theory represents a failure from the perspective of the traditional descriptive conception of theories. However, this liberation of theory from experiment has its big advantage, since it liberates scientist’s mind from rigid constraints of the present experimental situation.

Treatment of theory as a consistent system of mental images leads to its causality. The latter is a consequence of causality of human reasoning. In his reasoning a human cannot do anything else than to proceed from cause to its consequence. At the same time causality should not be treated as a purely mental (logical) feature of a theory. We recall that Helmholtz’s parallelism between sensation and perception played the fundamental role in establishing
of the Bild conception. Therefore the causal structure of human reasoning is the result of evolutionary experiencing of humans observing causality in natural processes.

Since the Hertz(-Botzmann) viewpoint on the notion of theory has not been commonly accepted, it is useful to specify it by calling ‘Bild-theory’. It should be distinguished from ‘descriptive theory’ attempting to provide “complete congruence with nature.” Besides Bild and descriptive theories, we consider ‘observational theory’ operating only with outputs of observations. This sort of theory can also be called ‘sensational theory’, in contrast to Bild theory which can be called ‘perceptional theory’. The same experimental situation can be represented by various types of theories: descriptive, Bild, and observational.

2.2 Hertzian viewpoint on foundations of quantum mechanics

For our considerations, the most important is that Hertzian analysis of methodology of science implies:

1. Any attempt to create a consistent (causal) theory on the purely experimental basis would lead to a failure;

2. Any consistent theory of natural phenomena would contain hidden variables, quantities which are unapproachable for our perception (at least at the present time);

3. Generally in theory it is impossible to approach the one-to-one correspondence between theoretical concepts and experimental facts.

We state that these principles were totally ignored not only by fathers of QM (with a few exceptions such as Schrödinger), but even by practically all experts working in quantum foundations. The majority of them followed ‘the spirit of Copenhagen’ (Plotnitsky, 2016) and put tremendous efforts to proceed without taking into account Hertz 1, 2, i.e., to develop the formalism of observational (sensational) theory of micro-phenomena which is nowadays known as QM (cf. with Stapp’s analysis of the Copenhagen interpretation in (Stapp, 1972)). This approach led to the dead-end in the form of slogan: “Shut up and calculate!” (It is commonly assigned to Feynman). Following Hertz ideas, I claim that the basic problems of quantum foundations can be
resolved only by rejecting the spirit of Copenhagen and creation of a real quantum theory liberated from sensational paradigm.

At the same time it is important to understand that Einstein and his followers also suffered from ignoring the Bild conception about the meaning of scientific theory. They followed the old-fashioned descriptive understanding of theory and missed to explore the possibilities opened by Hertz 3 statement. Attempts to establish one-to-one correspondence between theory and experiment (as, e.g., in Bohmian mechanics) led either to invention of new concepts (such as, e.g., nonlocality in Bohmian mechanics) which do not match to ‘natural concepts’ generated by human experience or makes the project too complicated (as in the case of Einstein’s attempts to create the classical field model matching with micro-phenomena).

Of course, the main problem is the spirit of Copenhagen. The majority of the quantum community (especially the young generation) is oriented to the observational theory - QM. This theory is powerful and convenient, but it does not provide the consistent ‘Bild’ of micro-phenomena. The latter is disturbing. Surprisingly, it is disturbing not only for those who reject the Copenhagen interpretation (or at least understand its restrictive character), but even for its strongest and world’s famous supporters.

Measurement problem: It cannot be solved in the framework of the observational theory. One has to introduce hidden variables. (Of course, this viewpoint may be surprising: one should use unobservable variables to describe generation of outputs of measurement devices.) Bell understood the role of hidden variables in description of the process of quantum measurement very well. And he started the right project, but then he was disappointed by ‘nonlocality catastrophe’\(^4\). As was pointed out, the latter is resulted from ignoring the possibility provided by Hertz 3.

Acausality of QM. Von Neumann emphasized acausality of QM (von Neuman, 1955). He also pointed to specialty of quantum randomness, as ir-

\(^4\)During the 20 years of Växjö conferences on quantum foundations, I was lucky to meet in the private and relaxing atmosphere many leading experts in quantum theory and experiment, ‘big names’. Surprisingly, practically all of them dream for a new quantum theory which (soon or later) will replace the present quantum theory. Unfortunately, people do not like to speak openly about their dreams. (The later is understandable: typically dreams are too private and personal). Therefore young researchers live being sure that the present quantum theory is the final theory of micro-phenomena.

\(^5\)In spite of the common opinion that Bell ‘enjoyed’ nonlocality, in reality nonlocality came to him as unexpected consequence of his analysis of the EPR-Bohm correlations, see (Bell, 1964, 1987).
reducible randomness. (The latter claim is heavily explored in justification of specialty of randomness generated by quantum random generators.) Acausality of quantum theory is not surprising, generally acausality is a feature of observational theories. One cannot approach causality without transition to the Bild conception. Thus quantum acausality and specialty of quantum randomness are not the (mystical) physical features of micro-world, but the features of the use of Mach’s treatment of a physical theory.

**Perfect correlations.** The EPR correlations (Einstein, Podolsky, & Rosen, 1935) neither can be explained by the observational theory - without introducing hidden variables. Bell understood this well and his *original Bell inequality* (Bell, 1964) was derived to analyze this problem. However, at that time it was impossible to prepare singlet states with sufficiently high probability and to perform experiments to test the original Bell inequality. Therefore (to establish some relation to experiment) Bell was convinced to proceed with the CHSH-inequality. Later he had never mentioned the original Bell inequality and its the crucial difference from the CHSH-inequality. The latter has nothing to do with the perfect correlations and the EPR-argument (Khrennikov & Basieva, 2018). This paper also contains the analysis of the modern experimental situation and the novel possibilities to test the original Bell inequality as well as motivation to test it and not the CHSH-inequality.

**Quantum nonlocality.** It is considered as the most intriguing feature of quantum theory. The nonlocality prejudice is so strong, because it is supported by both camps in quantum foundations, those who use observational theory (QM) and those who use descriptive theories (such as Bohmian mechanics). In fact, typically two (totally different) nonlocalities generated by observational and descriptive theories are identified into aforementioned ‘quantum nonlocality’. Genuine quantum (observational) nonlocality is encoded in the tensor product structure and the projection postulate. The descriptive nonlocality is encoded in nonlocal equations of motions, such as in Bohmian mechanics, or in violation of Bell type inequalities (the latter issue is very delicate and we shall consider it in more detail below).

**Violation of Bell inequality.** By taking into account the big impact of the debates about the Bell type inequalities, see, for example, (Adenier, Fuchs, & Khrennikov, 2007; Adenier et al., 2008) and its impact to establishing the notion of quantum nonlocality we specially discuss Bell’s studies, from the viewpoint of the Bild conception. Bell suffered from the same problem as Einstein and Bohm. He took into account Hertz 1,2 statements, but ignored Hertz 3. He also tried to proceed in the old-fashioned descriptive framework
and to identify the experimental correlations with correlations based on hidden variables, see (Khrennikov, 2017 a, b, c; Khrennikov and Basieva, 2018). De Broglie understood well that such identification has no physical justification and that the Bell type inequalities cannot be derived for experimental correlations, see (Khrennikov, 2017 a, b).

Merging QM and general relativity. In this project the main efforts we set to ‘quantize gravity’. It seems that this activity is totally meaningless. One tries to transform the descriptive theory into the observational one. The situation is really paradoxical: one try to collect in one bottle all problems from resulting from ignoring Hertz 1, 2 and Herz 3, see (Khrennikov, 2017 d). It is not surprising that it does not work. Merging cannot be approached neither through quantization of gravity nor via naive descriptive ‘completion’ of quantum theory (in the spirit of Einstein or Bohm). Both QM and general relative have to be reconsidered from the viewpoint of the Bild conception.

2.3 Correspondence between mathematical formalisms of theories of different types

Since each theory is based on its mathematical formalism, it is useful to establish correspondences between mathematical formalisms of different types of theories representing the same experimental data. The basic elements of the mathematical formalism of a theory $\tau$ are its state space $S_\tau$ and the space of variables $V_\tau$, some space (may be very special) of real functions on $S_\tau$. For two theories $\tau_1$ and $\tau_2$, one can try to establish correspondence between their basic elements. This task is not straightforward. In particular, the notion of a state is different for different theories, e.g., for Bild and observational theories $\tau_B$ and $\tau_O$ (and we shall be interested in establishing correspondence between such two types of theories). A Bild-theory is causal and here the same initial condition implies the same consequence. Observational theories are often acausal. And let us consider such a case, i.e., $\tau_O$ is acausal. It would be naive to expect that it would be possible to establish straightforward correspondence between the state spaces of these theories. Causality is transformed into acausality through consideration of probability distributions. Therefore by establishing correspondence between $\tau_B$ and $\tau_O$ we have to consider some space (may be very special) of probability distributions $P_B$ on $S_B$ and map it onto the state space of $\tau_O$. (We assume that states of $\tau_O$ are interpreted statistically.) Then we have to construct two ‘physically natural
maps',

\[ J : P_B \rightarrow S_O, \quad J^* : V_B \rightarrow V_O. \quad (1) \]

Here ‘physically natural’ means consistent matching with the experimental facts. Both theories \( \tau_B \) and \( \tau_O \) have experimental justification through coupling to facts and the correspondence maps have to couple these experimental justifications. (We shall illustrate this statement by considering two theories of micro-phenomena, QM as \( \tau_O \) and PCSFT as \( \tau_B \)). Of course, theories \( \tau_B \) and \( \tau_O \) can differ by details of experimental justification. Therefore in correspondence provided by the maps \( J, J^* \) some of these details can be ignored.

Generally these maps are neither one-to-one nor onto. Let us consider this situation in more detail.

- A cluster of probability distributions on \( S_B \) can be mapped into a state from \( S_O \) (generally states of \( \tau_B \) and probability distributions of such states are unapproachable by \( \tau_O \)).

- A cluster of variables of \( \tau_B \) can be mapped into a variable of \( \tau_O \) (the observational description is often operational; it does not distinguish variables of a causal theory).

- Not all elements of \( S_O \) and \( V_O \) belong to the images \( J(P_B) \) and \( J^*(V_B) \). (Even observational theory \( \tau_O \) can contain its own ideal elements which need not be reflected in \( \tau_B \)).

In a Bild theory \( V_B \) is some space of functions on the state space \( S_B \), maps \( f : S_B \rightarrow \mathbb{R} \). Such theory is causal, the state \( \phi \) uniquely determines the values of all physical variables belonging \( V_B : \phi \rightarrow f(\phi) \).

### 3 Results: Correspondence between prequantum classical statistical field theory and quantum mechanics

In QM states are given by density operators acting in complex Hilbert space \( H \) (endowed with scalar product \( \langle \cdot | \cdot \rangle \)) and physical variables (observables) are represented by Hermitian operators in \( H \). Denote the space of density operators by \( S_{QM} \) and the space of Hermitian operators by \( V_{QM} \).
In PCSFT (Khrennikov, 2007a, b, 2014, 2017c) states are given by vectors of $H$ (in general non-normalized), i.e., $S_{PCSFT} = H$. Physical variables are represented by quadratic forms on $H$, i.e., maps of the form $f(\phi) = \langle \phi | A | \phi \rangle$, where $A \equiv A_f$ is a Hermitian operator. Denote the space of quadratic forms by the symbol $V_{PCSFT}$. Consider the space of probability distributions on $H$ with zero first momentum, i.e.,
\[
\int_H \langle \phi | a \rangle dp(\phi) = 0
\]
for any $a \in H$, and finite second momentum, i.e.,
\[
\mathcal{E}_p \equiv \int_H \|\phi\|^2 dp(\phi) < \infty.
\]
Denote this space of probability distributions by the symbol $P_{PCSFT}$. We remark that, instead of probability distributions, we can consider $H$-valued random vectors with zero mean value and finite second moment: $\xi = \xi(\omega)$, where $\omega$ is the chance parameter, such that $E[\xi] = 0$ and $E[\|\xi\|^2] < \infty$. Denote this space by the symbol $R_{PCSFT}$. We remark that if $H$ is finite-dimensional, these are usual complex vector-valued random variables; if $H$ is infinite-dimensional, then the elements of $R_{PCSFT}$ are random fields. For the latter, the basic example is given by the choice $H = L^2(\mathbb{R}^n)$. Here each B-field state $\phi$ is an $L^2$-function, $\phi : \mathbb{R}^n \mapsto \mathbb{C}$. Hence, each element of $R_{PCSFT}$ can be represented as a function of two variables, $\xi = \xi(x; \omega)$: chance parameter $\omega$ and space coordinates $x$. This is a random field [??]. We shall use the same terminology, ‘random fields’, even in the finite-dimensional case.

We remark that, for the state space $H = L^2(\mathbb{R}^n)$, the quantity $\mathcal{E}_p$ can be represented as $\mathcal{E}_p = \int_H \mathcal{E}(\phi) dp(\phi)$, where $\mathcal{E}(\phi) = \|\phi\|^2 = \int_{\mathbb{R}^n} |\phi(x)|^2 dx$ is field’s energy. Hence, $\mathcal{E}_p$ is the average of the field energy with respect to the probability distribution $p$ on the space of fields. We can also use the random field representation. Let $\xi = \xi(x; \omega)$ be a random field. Then its energy is the random variable $\mathcal{E}_\xi(\omega) = \int_{\mathbb{R}^n} |\xi(x; \omega)|^2 dx$ and $\mathcal{E}_p$ is the average of the latter (here $p$ is the probability distribution of the random field).

For any $p \in P_{PCSFT}$, its (complex) covariance operator $B_p$ is defined by its bilinear (Hermitian) form:
\[
\langle a | B_p | b \rangle = \int_H \langle a | \phi \rangle \langle \phi | b \rangle dp(\phi), \ a, b \in H,
\]
or, for a random vector $\xi$, we have: $\langle a|B\xi|b \rangle = E[\langle a|\xi\rangle\langle \xi|b \rangle]$. Generally a probability distribution (a random field) is not determined by its covariance operator (even under condition of zero average, see [2]). We remark that such complex covariance operator has the same mathematical properties as a density operator, besides normalization by the trace one; it is Hermitian, positively semidefinite, and trace class. (The latter property is important in the infinite-dimensional case, e.g., for the state space $H = L_2(\mathbb{R}^n)$).

PCSFT (the Bild-type theory) is connected with QM through the following formula. For $p \in P_{\text{PCSFT}}$ and $f \in V_{\text{PCSFT}}$, we have

$$\langle f \rangle_p = \int_\mathcal{H} f(\phi) dp(\phi) = \text{Tr}_p A_f,$$

where $A_f = \frac{1}{2} f^{(2)}(0)$, i.e., $f(\phi) = \langle \phi|A_f|\phi \rangle$. We remark that the covariance operator and the energy average are coupled through the simple formula:

$$\mathcal{E}_p = \int_\mathcal{H} ||\phi||^2 dp(\phi) = \text{Tr}_p;$$

in particular, by normalizing the covariance operator of a random field by average of field’s energy we obtain a density operator $\rho_p = B_p/\mathcal{E}_p$.

Let us consider the following maps $J$ and $J^*$, see (11), from PCSFT to QM,

$$J(p) = \rho_p, \quad J^*(f) = A_f.$$

This correspondence connects the averages given by the Bild and observational theories:

$$\frac{1}{\mathcal{E}_p} \langle f \rangle_p = \text{Tr}\rho_p A_f,$$

i.e., the QM and PCSFT averages are coupled with the scaling factor which is equal to the inverse of the average energy of the random field. Thus density operators are normalized (by average field energy) covariance operators of random fields and the Hermitian operators representing quantum observables correspond to quadratic forms of fields. We can also write the relation (8) in the form: $\langle \hat{f} \rangle_p = \text{Tr}\rho_p A_f$. If $\mathcal{E}_p << 1$, we can consider the quantity $g_p(\phi) \equiv \frac{f(\phi)}{\mathcal{E}_p}$ as amplification of the PCSFT physical variable $f$. Thus through coupling with PCSFT we can treat QM as an observational theory describing averages of amplified ‘subquantum’ physical variables.
In contrast to QM, PCSFT is causal: selection of a vector (‘field’) \( \phi \in H \) determines the values of all PCSFT-variables, quadratic forms of classical fields: \( \phi \rightarrow \langle \phi | A | \phi \rangle \).

For physical variables, the correspondence map \( J' \) is one-to-one, but the map \( J \) is not one-to-one. But it is a surjection, i.e., it is on-to map.

**Discussion**

The aim of this paper is to remind to the quantum foundational community studies of Hertz (and Boltzmann) on the Bild conception of physical theory; especially Hertz analysis of connection between theory and experiment. We emphasize the similarity of the problems discussed by Hertz in the process of transition from Newtonian mechanics to Maxwellian electromagnetism and the problems of interrelation between classical and quantum physical theories (including the problem of hidden variables). The Bild conception can be explored to resolve the basic problems of quantum foundations: measurement problem, acausality and irreducible quantum randomness, quantum nonlocality, merging QM and general relativity.

As an example of a Bild-type theory preceding QM (the latter is treated as an observational theory), we consider prequantum classical statistical field theory - PCSFT. In contrast to QM, PCSFT is not based solely on the observational data. It contains images which cannot be coupled straightforwardly to data. In particular, the EPR-Bohm correlations cannot be identified with the corresponding PCSFT-correlations, although numerically they coincide. There exists a natural correspondence between the mathematical entities of PCSFT and QM, the correspondence is not one-to-one. The same Bild theory can be coupled to a variety of observational theories and a variety of observational theories can represent the same experimental data. PCSFT can be coupled not only to QM, but to another observational theory based on threshold detection of random signals, see (Khrennikov, 2012).

Finally, we stress that the Bild conception can be used to develop a consistent theory of quantum(-like) cognition and interrelation between matter and mind, see (Khrennikov, 2010), cf. (Stapp, 2004).
Conflict of interest

I have no a financial or personal relationship with a third party whose interests could be positively or negatively influenced by the articles content.

References

Adenier, G., Fuchs, C. A. & Khrennikov, A. (Eds.), (2007). Foundations of Probability and Physics-4, Conf. Proc. 889, Melville, NY: AIP.

Adenier, G., Khrennikov, A. Yu., Lahti, P., Manko, V. I., & Nieuwenhuizen, Th.M. (Eds.), (2008). Quantum Theory: Reconsideration of Foundations-4, Conf. Proc. 962, Melville, NY: AIP.

Atmanspacher, H. & Primas, H. (2005). Epistemic and ontic quantum-realities. In: G. Adenier & A. Yu. Khrennikov (Eds.), Foundations of Probability and Physics-3, pp. 49-62, Conf. Proc. 750, Melville, NY: AIP.

Bell, J. (1964). On the Einstein-Podolsky-Rosen paradox. Physics, 1, 195-200.

Bell, J. (1987). Speakable and unspeakable in quantum mechanics. Cambridge: Cambridge Univ. Press.

Bohr, N. (1935). Can quantum-mechanical description of physical reality be considered complete? Phys. Rev., 48, 696-702.

Boltzmann, L. (1905). ber die Frage nach der objektiven Existenz der Vorgänge in der unbelebten Natur. In: J.A. Barth (Ed.), Leipzig: Populre Schriften.

Boltzmann, L. (1974) On the development of the methods of theoretical physics in recent times. In: B. McGuinness (Ed.), Theoretical Physics and Philosophical Problems. Vienna Circle Collection, vol 5. Dordrecht: Springer.

De Broglie, L. (1964). The current interpretation of wave mechanics: a critical study. Elsevier.

D’Agostino, S. (1992). Continuity and completeness in physical theory: Schrödinger’s return to the wave interpretation of quantum mechanics in the 1950’s. In: M. Bitbol & O. Darrigol (Eds.), E. Schrödinger: Philosophy and the Birth of Quantum Mechanics, pp. 339-360. Gif-sur-Yvette: Editions Frontieres.

Einstein, A., Podolsky, B. Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete?. Phys. Rev., 47 (10), 777-780.

Einstein, A. & Infeld, L. (1961). Evolution of physics: The growth of ideas from early concepts to relativity and quanta. New-York: Simon and Schuster.
Hertz, H. (1899). *The principles of mechanics: presented in a new form.* London: Macmillan.

Khrennikov, A. (2007a). Quantum mechanics as the quadratic Taylor approximation of classical mechanics: The finite-dimensional case. *Theor. Math. Phys.,* 152, 1111-1121.

Khrennikov, A. (2007b) To quantum averages through asymptotic expansion of classical averages on infinite-dimensional space. *J. Math. Phys.,* 48(1), Art. No. 013512.

Khrennikov, A. (2010). *Ubiquitous quantum structure: from psychology to finances,* Berlin-Heidelberg-New York: Springer.

Khrennikov, A. (2012). Quantum probabilities and violation of CHSH-inequality from classical random signals and threshold type detection scheme. *Prog. Theor. Phys.* 128, 31-58.

Khrennikov, A. (2014). *Beyond Quantum.* Singapore: Pan Stanford Publ..

Khrennikov, A. (2017a). After Bell. *Fortschritte der Physik (Progress in Physics),* 65, 1600014.

Khrennikov, A. (2017b) Bohr against Bell: complementarity versus non-locality. *Open Phys.,* 15, 734-738.

Khrennikov, A. (2017c) Quantum epistemology from subquantum ontology: Quantum mechanics from theory of classical random fields. *Ann. Phys.,* 377, 147-163.

Khrennikov, A. (2017d). The present situation in quantum theory and its merging with general relativity. *Found. Phys.* 47, 10771099.

Khrennikov, A. & Basieva, I. (2018). Towards experiments to test violation of the original Bell inequality. *Entropy*, 20(4), 280

Miller, A. I. (1984). *Imagery in scientific thought creating 20th-century physics.* Boston, MA: Birkhuser.

Plotnitsky, A. (2006). *Reading Bohr: Physics and philosophy.* Dordrecht: Springer.

Plotnitsky, A. (2009). *Epistemology and probability: Bohr, Heisenberg, Schrödinger, and the nature of quantum-theoretical thinking.* Springer: Heidelberg-Berlin-New York.

Plotnitsky, A. (2016). *The principles of quantum theory, from Planck’s quanta to the Higgs boson: The nature of quantum reality and the spirit of Copenhagen.* Springer: Heidelberg-Berlin-New York.

Stapp, H. P. (1972). The Copenhagen interpretation. *American J. Phys.* 40, 1098.
Stapp, H. P. (2004) *Mind, matter, and quantum mechanics*. Springer: Heidelberg-Berlin-New York.

Von Neuman, J. (1955). *Mathematical foundations of quantum mechanics*. Princeton: Princeton Univ. Press.