TWO-STAGE MEAN-RISK STOCHASTIC MIXED INTEGER OPTIMIZATION MODEL FOR LOCATION-ALLOCATION PROBLEMS UNDER UNCERTAIN ENVIRONMENT

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Abstract. The problem of the optimal location-allocation of processing factory and distribution center for supply chain networks under uncertain transportation cost and customer demand are studied. We establish a two-stage mean-risk stochastic 0-1 mixed integer optimization model, by considering the uncertainty and the risk measure of the supply chain. Given the complexity of the model this paper proposes a modified hybrid binary particle swarm optimization algorithm (MHB-PSO) to solve the resulting model, yielding the optimal location and maximal expected return of the supply chain simultaneously. A case study of a bread supply chain in Shanghai is then presented to investigate the specific influence of uncertainties on the food factory and distribution center location. Moreover, we compare the MHB-PSO with hybrid particle swarm optimization algorithm and hybrid genetic algorithm, to validate the proposed algorithm based on the computational time and the convergence rate.

1. Introduction. The problem of location-allocation is one of the classic problems in operations research. The general use of it is in production, logistics, medical treatment, and even military affairs, such as the location of factories, warehouses, distribution centers, fire stations, and missile warehouses ([13, 52, 14, 34, 44, 9, 2010 Mathematics Subject Classification. Primary: 90C15; Secondary: 90C90.
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The importance of location-allocation lies in that once the location is selected and the distribution mode is determined. It will directly affect the following such as the service mode, service quality, service efficiency, service cost, thus affecting the profit and market competitiveness, and even determining the survival of enterprises. Once the site is improperly located, the consequences it brings are challenging to make up through other management follow-up measures. Therefore, the study of location-allocation is of great economic and social significance.

In real life, regarding market fluctuations, we cannot get accurate information about transportation costs, demands, and other parameters. In the problem of location-allocation, a practical approach is two-stage stochastic programming ([32]). Two-stage stochastic programming provides a natural and widely applicable modeling framework for optimization problems under uncertainty where decisions are made sequentially in two stages. Birge [7] and Shapiro et al. [42] highlight the rationality of establishing a two-stage stochastic programming model. In the first stage, decisions are made before the uncertain parameters are realized. In the second stage, (recourse) decisions are made in response to the pre-determined first-stage decisions and the realization of the uncertain parameters. And the two-stage stochastic programming problems have been widely studied ([23, 22, 8, 38, 54, 46, 18, 11, 10, 49, 6, 21, 24]). In the two-stage location-allocation model, the decision variables (the first-stage location variables) are determined. Then the allocation variables (the second-stage variables) are solved according to the realization of scenarios. Many scholars have studied it, such as Chau [8] developed a two-stage dynamic model to assist construction planners in developing the optimal strategy for building potential intermediate transit centers under the model. Ricoramirez et al. [38] studied the optimal location of booster disinfection stations in the water distribution network, established a two-stage stochastic mixed integer linear programming model, and proposed a simplified stochastic method to solve the problem. Yang [54] studied the airline network design problem under random demand, aiming to determine the hub location, flight routes, and flow allocation, and proposed a two-stage stochastic model to formulate the problem. Soleimani et al. [46] studied the location-allocation problem in a closed-loop supply chain with two extensions. They proposed a mean-risk two-stage stochastic programming model, taking into account the uncertainty of demand and price of new and returned products, as well as risk measurement. Table 1 lists the representative literature and research focus on location-allocation in the supply chain field.

From Table 1, we know that most two-stage stochastic programming problems are risk-neutral; that is, they take the expected cost (or expected profit) as an objective function and do not consider the risk. However, compared with the risk-neutral approach, the risk-averse approach considers the effects of the variability of random outcomes. Policy makers can evaluate strategies based on their risk appetite ([32]). Moreover, Klastorin et al. [39], Noyan et al. [33] and Sun et al. [48] indicated that conditional value-at-risk (CVaR) was a law invariant consistent risk measure of particular importance and the basis for other law invariant coherent risk measures. In the location-allocation problem, due to the existence of uncertain factors, the two-stage stochastic method, which only considers the expected cost (or profit) without viewing any risk measures, cannot meet the requirements of the actual situation. Therefore, to deal with the real market volatility in such problems,
Table 1. Gap analysis of various research focus of supply chain.

| Reference | Location type | Random parameters | Stochastic approach | Risk approach | Solving algorithm |
|-----------|---------------|-------------------|---------------------|---------------|-------------------|
| [27]      | Relief centers | Demand, supply    | Two-stage           | Risk-neutral  | Heuristic         |
| [16]      | Distribution center | //              | Two-stage           | Risk-neutral  | Matheuristic      |
| [32]      | Emergency facility | Demand           | Two-stage           | CVaR          | Bender decomposition |
| [6]       | Warehouse       | //                | Two-stage           | Risk-neutral  | Branch-and-bound  |
| [21]      | Factory         | //                | Two-stage           | Risk-neutral  | Heuristic         |
| [24]      | Factory, warehouse | //             | Two-stage           | Risk-neutral  | Heuristic         |
| [37]      | Facility        | Throughput costs  | One-stage           | Risk-neutral  | Heuristic         |
| [4]       | Factory         | Demand            | One-stage           | Risk-neutral  | L-shaped          |
| [5]       | Facility        | Demand            | Two-stage           | Risk-neutral  | Branch-and-bound  |
| [30]      | Distribution center | //             | One-stage           | Risk-neutral  | Heuristic         |
| [50]      | Facility        | Load time          | Two-stage           | CVaR          | Decomposition     |
| [19]      | Facility        | Demand            | One-stage           | Risk-neutral  | Combined simulated annealing |

this paper considers relevant risks and uses CVaR to conduct modeling, to get a more effective solution.

As can be seen from Table 1, in the traditional location-allocation problem, it is always supposed that the unit transportation cost from source to destination is a definite real number. However, under special circumstances, such as natural disasters, wars and traffic congestion in cities, the planned route network may be destroyed, which makes the unit transportation cost from source to destination uncertain, that is, an uncertain number. In most cases, the transportation cost is random ([3, 29]). Therefore, in this paper, we consider the location allocation problem with random transportation cost.

A supply chain system with optimal performance should maximize the value of internal activities while developing stable partnerships to maximize the value of external activities. Supply chain management realizes the integration and reconfiguration of nodal enterprise resources. In this paper, we consider the supply chain system, cancelled the original exists between manufacturers and customers of buffer inventories, between manufacturers and customers to establish a warehouse center as the distribution center, the dispersed custody of stock material together, improve the efficiency of the storage, loading, and unloading, etc., through rational planning distribution center of transportation planning and distribution plan, thereby reducing the transportation cost of the entire supply chain, speed up the logistics turnover.

In addition, as far as we know, there is a dearth of research on the location-allocation supply chain with multi-product, multi-supplier, multi-processing factory, multi-distribution center, and multi-customer under the random transportation cost and demand. Thus, to fill this gap, this paper considers the coordination mechanism of the location-allocation supply chain system composed of multi-supplier, multi-processing factory, multi-distribution center, and multi-customer. A two-stage mean-risk stochastic 0-1 mixed integer optimization model is established, keeping in view the location of the processing factory, distribution center, and the risk related to the total cost.

In the first stage, the location variable is the decision variable in the supply chain network. In the second stage, the decision variables because the way it is determined
in different realization scenarios makes the decision process as “here and now.” The quantity of transportation between each member in the supply chain network is the variable of the second stage, which is related to the different scenarios. Meanwhile, we will consider the risk measurement as the mean risk function in the objective function.

Due to the complexity of the model (see Section 3), we find that the traditional algorithm cannot solve the two-stage mean-risk stochastic 0-1 mixed integer optimization model. Therefore, developing the algorithms is another contribution to this study. In particular, we propose a MHB-PSO, which combines the synchronous dynamic learning factor and compression factor, and adds them to binary particle swarm optimization algorithm (B-PSO) to improve the convergence speed of the algorithm and ensure the global convergence of the algorithm. The main advantage of this proposed algorithm is its simple, fast searching speed and highly efficient. In the iterative process, the simplex algorithm is used to solve the second stage problem. Finally, MHB-PSO is compared with hybrid PSO and hybrid genetic algorithm (GA) to verify the effectiveness of the algorithm.

The main contributions of this paper are summarized as follows,

- Considering the uncertainty of supply chain, a two-stage mean-risk 0-1 mixed integer stochastic optimization model is established.
- This paper proposes a MHB-PSO to solve the two-stage mean-risk 0-1 mixed integer stochastic optimization model.
- We compare the MHB-PSO with hybrid PSO and hybrid GA, to validate the proposed algorithm based on the computational time and the convergence rate.

The arrangement of this paper is as follows. Section 2 illustrates the mathematical preliminaries for the CVaR. And Section 3 presents a formulation for the two-stage mean-risk stochastic 0-1 mixed integer optimization model. Section 4 then provides a solution algorithm for the model. Section 5 presents and discusses the computational results of a numerical example. The last Section 6 concludes with some future research directions.

2. Preliminaries. Risk measures are functionals that use a scalar value to represent the risk associated with a random variable, and they provide a means to compare the outcomes based on a decision maker’s risk preferences ([31]). The law-invariance and consistency of the risk measurement are axiomatized in [2]. CVaR is a law-invariant coherent risk measure of particular importance, which is a basis for other law-invariant coherent risk measures ([39, 33, 35]). Based on the nature of CVaR, we employ CVaR to measure the risk of the supply chain cost caused by the uncertainty of the demand and transportation cost. To facilitate a solution, we apply some properties of CVaR and express CVaR as linear programming of a discrete distribution. Next, we present some related definitions and relations.

**Definition 2.1.** ([40]) For a random loss variable $X$, the CVaR at a confidence level $\alpha \in (0, 1)$ is given by

$$
\text{CVaR}_\alpha(X) = \min_{\eta \in \mathbb{R}} \left\{ \eta + \frac{1}{1 - \alpha} \mathbb{E}(|X - \eta|+) \right\},
$$

(1)

where $[x]_+ = \max(x, 0)$.
The random variable \( X \) is defined on a probability space \((\Omega, \mathcal{F}, P)\), where \( \Omega \) denotes the sample space for \( X \), \( \mathcal{F} \) denoting a \( \sigma \)-algebra on \( \Omega \), and \( P \) is a known probability distribution, respectively.

A risk-averse decision-maker could choose a larger \( \alpha \) value e.g., \( \alpha = 0.95 \) for the confidence level. Besides this, the ability to specify the confidence level allows for greater flexibility to capture a wide range of risk preferences, including risk neutral \((\alpha = 0)\) and pessimistic worst-case \((\alpha \to 1)\) preferences.

Suppose \( X \) is a stochastic variable with realization \( \chi_1, \ldots, \chi_j, \ldots, \chi_n \) and corresponding probabilities \( p_1, \ldots, p_n \). Then, the CVaR problem can equivalently be formulated as the following linear program \((31)\),

\[
\min\{ \eta + \frac{1}{1-\alpha} \sum_{j \in [n]} p_j w_j : w_j \geq \chi_j - \eta \ \forall j \in [n], w \in \mathbb{R}^n_+, \eta \in \mathbb{R} \},
\]

where \([n] = \{1, \ldots, n\}\), \( w = [w_1, \ldots, w_n] \).

In modeling, the realization of the stochastic variable \( X \) would be the decision variables representing the realizations of the decision-based stochastic outcome of interest. As it is impossible to know the order of the decision-making related realizations in advance, the optimal representation of CVaR \((2)\) is very important for developing the mathematical program.

3. **Two-stage mean-risk 0-1 mixed integer stochastic optimization model.**

In this section, we establish a two-stage mean-risk stochastic 0-1 mixed integer optimization model under the uncertainty of customer demand and transportation cost. The first-stage decision is related to processing factory location and distribution center location. The second-stage decision is associated with the transportation cost, inventory cost, and demand, whose goal is to minimize the expected value of the cost function. Meanwhile, in the objective function, we also consider the CVaR of the whole cost function. The structure of the location-allocation supply chain network is shown in Figure 1. In particular, we considered \( S \) suppliers, \( I \) processing factories, \( J \) distribution centers, and \( K \) types of customers. A typical supplier, processing factory, distribution center, and customer are represented by \( s, i, j, \) and \( k \) respectively. The links in the supply chain network denote the transaction links. The supplier supplies \( V \) kinds of raw materials, the processing factory produces \( L \) kinds of products, and then transport them to the distribution centers, which meets the customer’s random demand as far as possible. The objective of the company is to maximize the expected profit by choosing the optimal number of processing factories and distribution centers in the market area on the premise of meeting random customer demand as far as possible.

For simplicity, we apply the following symbols for this model. All the vectors used in this paper are assumed to be column vectors.

3.1. **Notation.**

**Indices and set**

- \( s \) Suppliers index;
- \( i \) Processing factories index;
- \( j \) Distribution centers index;
- \( k \) Customers index;
- \( v \) Raw materials index;
- \( l \) Products index;
- \( S \) Set of the suppliers;
Figure 1. Network structure of location-allocation supply chain.

$I$ Set of the processing factories;
$J$ Set of the distribution centers;
$K$ Set of the customers;
$V$ Set of the raw materials;
$L$ Set of the products;

Parameters

$f_i$ Fixed cost of operating processing factory $i$;
$g_j$ Fixed cost of operating distribution center $j$;
$a_{vs}$ The ability of supplier $s$ to provide raw material $v$;
$r_{vs}$ The cost of raw materials $v$ provided by the supplier $s$;
$n_{vl}$ The quantity of raw material $v$ required for processing product $l$;
$b_{il}$ The ability of processing factory $i$ to produce product $l$;
$q_{il}$ The cost of producing product $l$ in the processing factory $i$;
$\tau_{lj}$ The ability of distribution center $j$ to store product $l$;
$w_{lj}$ The inventory cost of product $l$ stored in distribution center $j$;
$h_l$ Retail price of unit product $l$;
$\tilde{t}_{vsi}$ Random transportation cost of supplier $s$ transporting raw material $v$ to processing factory $i$;
$\tilde{m}_{ilt}$ Random transportation cost of processing factory $i$ transporting product $l$ to distribution center $j$;
$\tilde{u}_{ljk}$ Random distribution cost of distribution center $j$ transporting product $l$ to consumer $k$;
$\tilde{d}_{lk}$ Random demand of consumer $k$ for product $l$;
$\xi(\omega)$ Random demand and transportation cost vector $\xi(\omega) = (\tilde{t}_{vsi}(\omega), \tilde{m}_{ilt}(\omega), \tilde{u}_{ljk}(\omega), \tilde{d}_{lk}(\omega))$. 

\[ \tilde{u}_{ijk}(\omega), \tilde{u}_{ik}(\omega); \]
\[ \lambda \text{ Weight of CVaR; } \]
\[ E(\cdot) \text{ Expectation; } \]

**Variables**

- \( e_i \): Binary variables, open processing factory \( i \), value 1, otherwise 0, a first-stage decision variable;
- \( c_j \): Binary variables, open distribution center \( j \), value 1, otherwise 0, a first-stage decision variable;
- \( x_{esi} \): The quantity of raw materials \( v \) transported by the supplier \( s \) to the processing factory \( i \), a second-stage decision variable;
- \( y_{lij} \): The quantity of product \( l \) transported by the processing factory \( i \) to the distribution center \( j \), a second-stage decision variable;
- \( z_{ljk} \): The quantity of product \( l \) transported by the distribution center \( j \) to the consumer \( k \), a second-stage decision variable.

### 3.2. The model.

**Assumptions**

1. Customer’s demand cannot be overserved, but it is possible that they are not fully satisfied.
2. Random demand and transportation cost vector \( \xi(\omega) = (\tilde{t}_{esi}(\omega), \tilde{m}_{lij}(\omega), \tilde{u}_{ijk}(\omega), \tilde{d}_{lk}(\omega))) \) is defined on probability space \( (\Omega, \mathcal{F}, \mathcal{P}) \).
3. The probability space \( (\Omega, \mathcal{F}, \mathcal{P}) \) is finite.

Based on the above assumptions and description, we establish a two-stage mean-risk stochastic mixed integer optimization model. The proposed model considers the expected cost and its risk minimization of the supply chain, and obtained by a convex combination of risk measurement and expected cost function, that is, use convex combination to transform the multi-objective programming into a single-objective programming. As a function of the random parameter vector \( \xi(\omega) \), the total cost \( \sum_I f_i e_i + \sum_J g_j c_j + Q(e, c, \xi(\omega)) \) is obviously random variable. We can incorporate a risk measure CVaR on the total cost. Note that the mapping CVaR is translation equivariant, we have \( \text{CVaR}_\alpha(\sum_I f_i e_i + \sum_J g_j c_j + Q(e, c, \xi(\omega))) = \sum_I f_i e_i + \sum_J g_j c_j + \text{CVaR}_\alpha(Q(e, c, \xi(\omega))) \). We consider the expected cost and its risk minimization of the supply chain. That is, \((1 - \lambda)E[\sum_I f_i e_i + \sum_J g_j c_j + Q(e, c, \xi(\omega))] + \lambda \text{CVaR}_\alpha(\sum_I f_i e_i + \sum_J g_j c_j + Q(e, c, \xi(\omega)))\).

Next, we present a two-stage mean-risk stochastic 0-1 mixed integer optimization model as follows,

\[
\begin{align*}
\min_{e, c} \quad & \sum_I f_i e_i + \sum_J g_j c_j + (1 - \lambda)E[Q(e, c, \xi(\omega))] \\
& + \lambda \text{CVaR}_\alpha(Q(e, c, \xi(\omega))) \\
\text{s. t.} \quad & e_i \in \{0, 1\}, \forall i \in I, \\
& c_j \in \{0, 1\}, \forall j \in J,
\end{align*}
\]
where \( e = (e_1, \cdots, e_I), \ c = (c_1, \cdots, c_J), \ \alpha \in [0,1), \ \lambda \in [0,1], \ Q(e, c, \xi(\omega)) \) is the optimal value of the second-stage problem

\[
Q(e, c, \xi(\omega)) = \min_{x, y, z} \sum_V \sum_S \left( r_{vsi} \sum_I x_{vsi} \right) + \sum_I \sum_L \left( q_{il} \sum_J y_{lij} \right) \\
+ \sum_V \sum_S \sum_I \tilde{t}_{vsi}(\omega) x_{vsi} + \sum_L \sum_I \sum_J \tilde{m}_{ij}(\omega) y_{lij} + \sum_L \sum_J \sum_K \tilde{u}_{ijk}(\omega) z_{ijk} \\
+ \sum_L \sum_J \left( w_{lj} \sum_I y_{lij} - \sum_K \tilde{t}_{ljk}(\omega) \right) - \sum_K \left( h_l \sum_J \sum_K \tilde{t}_{ljk}(\omega) \right) \\
(4)
\]

\[
\text{s. t. } \sum_I x_{vsi} \leq a_{vs} \forall v, s, (5a) \\
\sum_L (n_{vi} \sum_J y_{lij}) \leq \sum_S x_{vsi} \forall v, i, (5b) \\
\sum_J y_{lij} \leq e_i b_{iJ} \forall i, l, (5c) \\
\sum_K z_{ijk} \leq \sum_J y_{lij} \forall l, j, (5d) \\
\sum_K z_{ijk} \leq c_{ij} \tau_{ij} \forall l, j, (5e) \\
\sum_J z_{ijk} \leq \tilde{d}_{lk}(\omega) \forall l, k, (5f) \\
x_{vsi}, y_{lij}, z_{ijk} \geq 0, \forall v, s, i, j, k, l. (5g)
\]

where, \( \tilde{t}_{vsi}(\omega), \tilde{m}_{ij}(\omega), \tilde{u}_{ijk}(\omega) \) and \( \tilde{d}_{lk}(\omega) \) are the realizations of the random transportation cost \( \tilde{t}_{vsi}, \tilde{m}_{ij}, \) random distribution cost \( \tilde{u}_{ijk}, \) and random demand \( \tilde{d}_{lk}, \) respectively, for any \( \omega \in \Omega. \)

In the two-stage mean-risk stochastic 0-1 mixed integer optimization model, the objective function (3) depicts the expected cost and its risk minimization of the supply chain. In particular, for a convex and nondecreasing mapping CVaR, the mean-risk function preserves the convexity, which is crucial in developing effective exact solution algorithms. In addition, when \( \lambda = 0, \) (3) is a risk-neutral two-stage stochastic 0-1 mixed integer optimization model. When \( \lambda = 1, \) (3) is a risk-averse two-stage stochastic 0-1 mixed integer optimization model. The second-stage problem (4) consists of seven parts. The initial one is the cost of raw materials than the production cost of the processing factory; transportation costs consist of third and fourth parts. The last three parts include the distribution cost, inventory cost, and the revenue obtained by selling products. Constraints (5a), (5c), and (5e) represent the raw material constraints of suppliers, the production capacity constraints of processing plants, and the distribution capacity constraints of distribution centers, respectively. Constraints (5b) and (5d) represent the balance conditions of raw materials and products, respectively. Constraint (5f) ensures that customer’s demand can not be overserved. The final constraint (5g) guarantees the non-negativity of decision variables.

In model (3)-(4), the two-stage process of the location-allocation problem is shown in Figure 2. The decision vector \( e \) and \( c \) are the first-stage decision that must be considered before the realizations of random demand and transportation.
cost vector $\xi(\omega)$ coming out. In the second-stage, the random demand and transportation cost $\xi(\omega)$, $e$, and $c$ are known. The decision variables are $x$, $y$, $z$. The goal is to minimize costs and risks in the supply chain.

![Figure 2](image_url)

**Figure 2.** Two-stage process of location-allocation problem.

For each $\xi(\omega)$, (5a) contains $|V||S|$ constraints, (5b) contains $|V||I|$ constraints, (5c) contains $|I||L|$ constraints, (5d) and (5e) contains $|L||J|$ constraints, respectively, (5f) contains $|V||S||I|$ constraints, (5g) contains $|V||S||I| + |L||I||J| + |L||J||K|$ constraints. So problem (4) has $|V||S||I| + |V||I||L| + 2|L||J| + |L||K| + |V||S||I| + |L||I||J| + |L||J||K|$ constraints for each $\xi(\omega)$. This makes model (3)-(4) very complicated and difficult to solve. Based on the complexity of model (3)-(4), in the following part, we propose a hybrid intelligent algorithm that can effectively solve it.

4. **Solution algorithm.** This section concentrates on the computation of the two-stage mean-risk stochastic 0-1 mixed integer optimization model (3)-(4). We know that the mean-risk function maintains convexity, which is crucial for the development of efficient solution algorithms. In what follows, we denote by $\xi(\omega_s')$ the $s'$-th scenario and the corresponding nominal probability is $p_{s'}$, $s' \in S'$, where $S'$ is the index set of scenarios.

The classical methods for solving mixed integer programming problems are Benders’ decomposition method ([43, 41]), cutting-plane method ([53, 1]), branch and bound method ([51, 47]), branch and cut method ([36]), etc. The main idea of these algorithms is to use dual theory to derive and generate the family of cutting planes in the iterative process, and add the optimal cutting to the constraint, to reduce the feasible region of solving the problem. If the above algorithm is used to solve model (3)-(4), similar to formula (37) in [31], the two-stage problem is first transformed into a single-stage problem, and then the single-stage problem is solved. However, there are $2|V||S| + |V||I| + |I||L| + 2|L||J| + |L||K| |S'| + 2|S'| + |V||S||I| + |L||I||J| + |L||J||K|$ constraints for single-stage problems. When $S'$ is very large, there are many constraints, and integer variables exist in the objective function and constraint, which makes the derivation and generation of cutting plane family very difficult. For example, in the location-allocation problem in the Section 5, $|V| = 1, |S| = 2, |I| = 4, |J| = 10, |K| = 4, |L| = 1$ and $|S'| = 1000$. If it is converted into a single-stage problem to be solved, there are 70088 constraints, and it
is complicated to solve. Therefore, it is complicated to solve model (3)-(4) with the above algorithm. Moreover, the traditional optimization algorithm cannot solve the two-stage mean-risk stochastic 0-1 mixed integer programming problem. Therefore, this paper designs a hybrid intelligent algorithm that can effectively solve model (3)-(4) without transforming the two-stage problem into a single-stage problem.

Medsker [26] introduced numerous ideas for designing hybrid intelligent systems, which integrate a variety of intelligent algorithms to produce more powerful and effective algorithms. It lays a foundation for solving mixed integer programming problems. Among several intelligent algorithms, PSO has the advantages of fast search rate, highly efficient, and manageable. So, based on the above discussion and the complexity of the problem considered in this paper, a MHB-PSO is proposed to solve the two-stage mean-risk stochastic 0-1 mixed integer optimization model (3)-(4).

In order to use MHB-PSO to solve model (3)-(4) more effectively, we make the following modifications to MHB-PSO:

(a) We combined the synchronous dynamic learning factor and the compression factor. We added them to the B-PSO, which improved the convergence speed of the algorithm and guaranteed the global convergence of the algorithm.

(b) We introduce boundary conditions in B-PSO to enhance the search efficiency of particles and avoid the expansion and dispersal of particle swarm.

(c) Embed the simplex algorithm into MHB-PSO to enhance the search efficiency of particles and avoid the expansion and dispersal of particle swarm.

Next, we introduce the detailed solving process of the algorithm.

Let \( \mathbf{X} = \{ e_1, \ldots, e_I, c_1, \ldots, c_J \} \), \( D = I + J \). Suppose that in the \( D \) dimensional objective search space, there are \( N \) particles forming a community, in which the \( i' \)th particle is represented as a \( D \)-dimensional vector \( \mathbf{x}_{i'} = (\chi_{i'1}, \ldots, \chi_{i'D}), \) \( i' = 1, 2, \ldots, N \). The flight velocity of the \( i' \)th particle is also a \( D \)-dimensional vector, which is written as \( \mathbf{v}_{i'} = (v_{i'1}, v_{i'2}, \ldots, v_{i'D}), \) \( i' = 1, 2, \ldots, N \). The personal best position and global best position of particle \( i' \) in the \( k' \)th iteration denoted as \( P^k_{bi'} \) and \( P^k_{gi'} \), respectively.

Denote \( \text{Fit}(\cdot) \) is the fitness function, and let the fitness of each particle be the minus of the first-stage value, i.e.,

\[
-\text{Fit}(\mathbf{X}) = \sum_I f_i e_i + \sum_J g_j c_j + (1 - \lambda)E[Q(\mathbf{X}, \xi(\omega))] + \lambda \text{CVaR}_\alpha(Q(\mathbf{X}, \xi(\omega))).
\]

Therefore, the particles of smaller the target value in the first-stage are evaluated with higher fitness. Among them, for each \( \xi(\omega) \), the second-stage function value \( Q(\mathbf{X}, \xi(\omega)) \) is calculated by simplex algorithm.

In the process of calculation, the updated formula of particle swarm velocity vector in the \( k' \)-th iteration is as follows:

\[
\mathbf{v}_{i'}^{k'+1} = \varrho \mathbf{v}_{i'}^{k'} + c_1 \cdot \text{rand} \cdot (P^k_{bi'} - \mathbf{x}_{i'}^{k'}) + c_2 \cdot \text{rand} \cdot (P^k_{gi'} - \mathbf{x}_{i'}^{k'}), \tag{6}
\]

where \( c_1, c_2 \in [c_{\text{min}}, c_{\text{max}}] \) are the dynamic learning factors, the learning factor at the \( k' \)-th iteration is

\[
c_1 = c_2 = c_{\text{min}} + \frac{(c_{\text{max}} - c_{\text{min}})k'}{T},
\]

and satisfies \( c_1 + c_2 \geq 4 \), \( T \) is the maximum number of iterations. \( \varrho \) is the compression factor which is set by the following expression ([12])

\[
\varrho = \frac{2}{|2 - \sqrt{\zeta - \sqrt{\zeta^2 - 4\beta}|},
\]

where \( \zeta = \frac{c_{\text{min}}}{\sqrt{(c_{\text{max}} - c_{\text{min}})/T}} \) and \( \beta = \frac{c_{\text{max}} - c_{\text{min}}}{T} \).
where $\varsigma = c_1 + c_2$. 

In (6), the first term depicts the velocity of the previous iteration of particles, which is used to ensure the global convergence of the algorithm. The second and third terms guarantee the local convergence of the algorithm. As the number of iterations increase, $c_1$ and $c_2$ linearly increase, and $\varrho$ decreases. In this way, it can ensure that at the beginning of the algorithm, each particle can detect a better region in the global range with a greater speed step. Later in the iteration, a smaller $\varrho$ ensures that the particle can do a fine search around the extreme point so that the algorithm has a greater probability to converge to the optimal global solution.

To improve the search efficiency of particles, avoid the expansion and dispersal of particle swarm, and bypass the blind search of particles in a wide range, we introduce the following boundary conditions:

$$
(\mathcal{V}_i^{k+1})_d = \begin{cases} 
\text{rand} \cdot (\mathcal{V}_{\max} - \mathcal{V}_{\min}) + \mathcal{V}_{\min}, & (\mathcal{V}_i^{k+1})_d > \mathcal{V}_{\max} \\
\text{rand} \cdot (\mathcal{V}_{\max} - \mathcal{V}_{\min}) + \mathcal{V}_{\min}, & (\mathcal{V}_i^{k+1})_d < \mathcal{V}_{\min} \\
(\mathcal{V}_i^{k+1})_d, & \text{otherwise}
\end{cases}
$$

(7)

In (7), $\mathcal{V}_{\max}$ is the maximum velocity limit and $\mathcal{V}_{\min}$ is the minimum velocity limit.

Since the value and change of the particle in the state space are limited to the two values of 0 and 1, and the $v_i^{k+1}$ of each dimension of velocity represents the possibility that the value of position $x_i^{k+1}$ is 1. The location update equation ([20]) is expressed as follows:

$$
x_i^{k+1} = \begin{cases} 
1, & \mathcal{J}(\mathcal{V}_i^{k+1}) > \text{rand} \\
0, & \text{otherwise}
\end{cases}
$$

(9)

In (8), $\mathcal{J}(\mathcal{V}_i^{k+1})$ is a sigmoid function. In (9), $X_{k+1}^{i'} = (x_{i_1}^{k+1}, \ldots, x_{i_d}^{k+1}, \ldots, x_{i_{d'}}^{k+1})$ denotes the position of the $i'$ particle after the $k$-th iteration.

Based on the above description, we give the detailed calculation process of the modified hybrid binary particle swarm optimization algorithm.

**Algorithm 1: Modified Hybrid Binary Particle Swarm Optimization Algorithm (MHB-PSO)**

**Step 0:** Initialize the particle swarm. The population size is $N$, maximum number of iterations $T$, $\mathcal{V}_{\max} > \mathcal{V}_{\min}$, $c_{\max} > c_{\min} > 0$, $P_0^0 = \text{ones}(N, 1)$, $P_g^0 = \text{eps}$, $X_{\text{best}} = \text{ones}(1, I + J)$, $k' = 0$, $i' = 1$.

**Step 1:** Initializes particle velocity and position. Randomly obtain the initial population of binary code $X^0 = \text{round}(\text{rand}(N, I + J))$, $P = X^0$, $v^0 = \text{rand}(I', I + J) \cdot (\mathcal{V}_{\max} - \mathcal{V}_{\min}) + \mathcal{V}_{\min}$.

**Step 2:** If $k' > T$, Stop. Return the particle $X_{\text{best}}$ as the optimal solution, and $-\text{Fit}(X_{\text{best}})$ as the optimal value.

**Step 3:** If $i' > N$, set $k' = k' + 1$, $i' = 1$, go to Step 2.

**Step 4:** Calculate the fitness $\text{Fit}(X_{i'}^{k'})$ through the simplex algorithm and (2).

**Step 5:** Update the individual’s optimal position and value. If $\text{Fit}(X_{i'}^{k'}) > P_{bi}', P_{bi}' = X_{i'}^{k'}$, else $P_{bi}' = P_{bi}$.

**Step 6:** Update the global optimal position and value. If $P_{bi}' > P_{g}', X_{\text{best}} = P_{bi}'$, $P_{g}' = P_{bi}'$, else $P_{g}' = P_{g}'$. 

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Step 7: Update velocity particles by (6).
Step 8: Boundary condition treatment by (7).
Step 9: Update location particle by (8)-(9).
Step 10: Set $i' = i' + 1$, go to Step 3.

Remark 1. In the iterative process, the update formula of particle swarm velocity vector in [12] differs from (6) in that $c_1, c_2 \in [c_{min}, c_{max}]$ are constants. We combine the synchronous dynamic learning factor and compression factor, and added them to the MHB-PSO, which improve the convergence speed of the algorithm.

Remark 2. In the iterative process, the boundary condition (7) is not considered in [12] and [20]. In MHB-PSO, we consider the boundary condition (7) to avoid the expansion and dispersal of particle swarm and improve the search efficiency of particles.

5. Numerical results. In this section, we apply the MHB-PSO to solve a practical case, and to provide a discussion of the results. All the program codes are written on MATLAB R2014a using Lenovo computers running on Intel(R) Core(TM) i7-8565U CPU @ 1.80 GHz, 8.00-GB memory. Throughout the computational experiments, the parameters in MHB-PSO are taken as: $N = 100$, $V_{max} = 10$, $V_{min} = -10$, $c_{min} = 2$, $c_{max} = 2$, $\lambda = 0.1$, $\alpha = 0.95$. The stopping criterion for the algorithm is $T = 1000$.

In the respective tables of the numerical results, “Pro” denotes the total supply chain’s profit, $\text{Pro} = -(\sum_{i} f_{ei} + \sum_{j} g_{cj} + E[Q(e, c, \xi(\omega))])$. “Val” denotes the optimal value for problem (3), $\text{Val} = -F_{\text{best}}$. $\mathbf{e}_{\text{best}}$ and $\mathbf{c}_{\text{best}}$ denotes the optimal solutions for problem (3). $x_{usi}^* = \sum_{s'} p_{si} x_{usi}^{s'}$ denotes the expected quantity of raw material $v$ transported from the supplier $s$ to the processing factory $i$, $y_{lij}^* = \sum_{s'} p_{s'i} y_{s'lij}^*$ denotes the expected quantity of product $l$ transported from the processing factory $i$ to the distribution center $j$, $z_{ljk}^* = \sum_{s'} p_{s'lj} z_{s'lj}^*$ denotes the expected quantity of product $l$ transported from the the distribution center $j$ to the consumer $k$, where, $(x_{usi}^{s'}, y_{s'lij}^{s'}, z_{s'lj}^{s'})$ represents the optimal solution of problem (4) in scenarios $s'$. “IT” denotes the computing time in seconds. In what follows, we provide a description of the test problem.

With the advancement of the economy, China’s new retail business model has developed rapidly. In 2018, “Tao xian da” appeared on the homepage of Taobao APP. Tao xian da is a combination of Taobao and Fresh Hema products, and a subsidiary of Ali’s Tmall supermarket group, which provides online and offline integrated solutions for traditional supermarkets and improves efficiency through a digital operation. In the same year, Rt-mart also joined the “Tao xian da strategy”. The Rt-mart consumers now can just click on the Taobao APP and buy all their goods inside Rt-mart, which undoubtedly considerably expanded the user scale of “Tao xian da”. Taking Shanghai bread supply chain network as an example, this paper studies the specific influence of uncertainty on location-allocation in the supply chain. In the supply chain network, two flour factories provide raw materials. The bread production in the chosen four food factories, along with its retail in ten Rt-mart supermarkets to meet the daily bread needs of users using “Tao xian da” in four demand areas of Shanghai. As shown in Figure 3, the specific location of the flour factory is marked by two grey-green houses, namely Fuxin third flour factory (supplier 1) and Fuxin flour factory (supplier 2). The specific location of the food factory is marked by four green triangles, namely the Yingyuan...
food factory, Changli food factory, Ziyan food factory, and Sunhong food factory. They are recorded as processing plants 1-4. The specific location of the Rt-mart supermarket is marked with ten yellow houses. They are Meilanhu Rt-mart, Anting Rt-mart, Nanxiang Rt-mart, Yangpu Rt-mart, Sijing Rt-mart, Chunshen Rt-mart, Kangqiao Rt-mart, Songjiang Rt-mart, Fengxian Rt-mart, and Nicheng Rt-mart. They are recorded as distribution centers 1-10. The consumer demand area is marked by four minions, which are defined as “Demand area 1 (green), 2 (gray), 3 (red), and 4 (blue)”. Demand area 1 consists of Jiading district, Baoshan district, Putuo district, Changning district, Xuhui district, Huangpu district, Jingan district, Hongkou district and Yangpu district. Demand area 2 consists of Qingpu district, Songjiang district and Minhang district. Demand area 3 consists of Jinshan district and Fengxian district. Demand area 4 consists of Pudong new district. As Chongming district is an independent island, the consumer demand in Chongming district is not considered in this calculation example.

![Location of flour factory, food factory, Rt-mart and bread demand area in Shanghai.](image)

For the convenience of calculation, this paper only considers a class of bread with the same price, and the unit raw materials (including dry yeast, butter, high-gluten powder, sugar, mixed dried fruit and salt, etc.) production unit bread, i.e., $V = 1, L = 1, n_{vl} = 1$. Also, assume that Rt-mart can successfully deliver bread to the demand area. The relevant parameters, random transportation costs and random demand settings are shown in the Appendix.

In order to solve the random location-allocation problem, for any feasible solution, we using Monte Carlo method generate 1000 random sampling points $\xi_{s'}(\omega), s' = 1, 2, \cdots, 1000$, for random simulation (such sample size is sufficient for simulation of randomly expected value). And the corresponding probability is $p_{s'} = \cdot \cdot \cdot$
1/1000. We put the data in Tables 8 and the generated sample points into two-stage mean-risk stochastic 0-1 mixed integer optimization model (3)-(4) and used MHB-PSO to solve it. The numerical results are given in Tables 2-7, and Figures 4, 5, 6, respectively.

**Table 2. Numerical optimal solution and value of the example**

| $e_{best}$ | $c_{best}$ | $x^*_1$ | $y^*_1$ | $z^*_1$ | $\text{Val}$ |
|------------|------------|---------|---------|---------|-------------|
| $(1,1,1,0)$ | $(1,0,0,0,1,0,1,0)$ | $x^*_11 = 495.0740$ | $y^*_125 = 424.7554$ | $z^*_114 = 456.9544$ | $-1.2799e+04$ |
|            |            | $x^*_12 = 935.0000$ | $y^*_127 = 466.7951$ | $z^*_125 = 443.1467$ |            |
|            |            | $x^*_13 = 69.9260$ | $y^*_129 = 43.4494$ | Others=0 |            |
|            |            | $x^*_123 = 382.7580$ | $y^*_131 = 52.9868$ |            |            |
|            |            | $y^*_111 = 495.0740$ | $y^*_133 = 399.6973$ |            |            |
|            |            | $y^*_112 = 935.0000$ | $y^*_135 = 52.9868$ |            |            |
|            |            | $y^*_113 = 69.9260$ | $y^*_139 = 399.6973$ |            |            |
|            |            | $y^*_125 = 424.7554$ | $y^*_131 = 52.9868$ |            |            |
|            |            | $y^*_127 = 466.7951$ | $y^*_133 = 399.6973$ |            |            |
|            |            | $y^*_129 = 43.4494$ | $y^*_135 = 52.9868$ |            |            |
|            |            | $y^*_12 = 935.0000$ | $y^*_139 = 399.6973$ |            |            |
|            |            | $y^*_13 = 69.9260$ | $y^*_1 = 1.2952e+04$ |            |            |
|            |            | $y^*_1 = 1.2952e+04$ | $y^*_1 = 1.2952e+04$ |            |            |
|            |            | $z^*_14 = 456.9544$ | $z^*_123 = 382.7580$ |            |            |
|            |            | $z^*_152 = 443.1467$ | $z^*_12 = 935.0000$ |            |            |
|            |            | $z^*_171 = 2.6723$ | $z^*_125 = 424.7554$ |            |            |
|            |            | $z^*_172 = 7.1685$ | $z^*_14 = 456.9544$ |            |            |
|            |            | $z^*_173 = 7.1685$ | $z^*_152 = 443.1467$ |            |            |

Figure 4 shows the structure diagram of the location-allocation supply chain network, and the specific results are given in Table 2. To meet customers' needs to the greatest extent, enterprises choose Yingyuan, Changli, and Ziyan food factories to produce bread, and pick Meilanhu Rt-mart, Sijing Rt-mart, Kangqiao Rt-mart, and Fengxian Rt-mart to deliver bread to customers. At this time, the maximum profit of the supply chain is $1.2952e+04$ CNY. The demand scope of Demand area 1 is $[485.5000, 510.5000]$, which is supplied by Meilanhu Rt-mart with 495.0740 and Kangqiao Rt-mart with 2.6723, respectively. The demand scope of Demand area 2 is $[475.5000, 500.5000]$, which is supplied by Sijing Rt-mart with 477.7422 and Kangqiao Rt-mart with 7.1685, respectively. The demand scope of Demand area 3 is $[430.5000, 455.5000]$, which is supplied by Fengxian Rt-mart with 443.1467. The demand scope of Demand area 4 is $[440.5000, 465.5000]$, which is supplied by Kangqiao Rt-mart with 456.9543. Yingyuan food factory purchased 495.0740 units of raw materials from Fuxin third flour factory and processed them into bread, and put them all in Meilanhu Rt-mart for distribution. Changli food factory purchased 935.0000 units of raw materials from Fuxin third flour factory. It processed them into bread, were placed in the Sijing Rt-mart, Kangqiao Rt-mart, and Fengxian...
Rt-mart for distribution. The quantity was 24,7554, 466,7951, and 43,4494, respectively. Ziyan food factory purchased 69,9260 units of raw materials from Fuxin third flour factory, and 382,7580 units of raw materials from Fuxin flour factory for processing into bread, were placed in the Sijing Rt-mart and Fengxian Rt-mart for distribution. The quantity was 52,9868 and 399,6973, respectively. At the same time, it can be seen from Figure 4 that the four Rt-mart selected are located in four demand areas, satisfying the nearby supply principle. And the chosen food factory location and the four Rt-mart location is relatively close.

In order to further evaluate the performance of our proposed MHB-PSO, we compare it with other discrete hybrid algorithms, such as binary PSO ([12]) and GA ([15, 45]). The numerical results are shown in Table 3 and Figure 5.

| Algorithm       | \(e_{best}\)       | \(c_{best}\)       | Pro Val     | Val       | TI         |
|-----------------|---------------------|---------------------|-------------|-----------|------------|
| MHB-PSO         | (1, 1, 1, 0), (1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1) | 0, 2952e+04 | -1.2799e+04 | 9849.4900 |
| Hybrid PSO      | (0, 1, 1, 1), (0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1) | 0, 2861e+04 | -1.2683e+04 | 11169.2300 |
| Hybrid GA       | (0, 1, 0, 1), (0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1) | 0, 2855e+04 | -1.2660e+04 | 12035.1647 |

![Figure 5. Comparisons of different algorithms](image)

In Table 3, we adopted three intelligent algorithms to solve the supply chain bi-level location-allocation problem, and obtained different optimal solutions and optimal values. Among them, when MHB-PSO is applied to solve the problem, the supply chain profit is the largest, and the algorithm has a shorter computing time. It can be seen from Figure 5 that MHB-PSO converges faster than Hybrid PSO and Hybrid GA, indicating that MHB-PSO is more suitable for two-stage mean-risk stochastic mixed integer programming.

To better prove the performance of MHB-PSO, we verified the stability of the MHB-PSO by setting different iteration numbers, population size and different parameters. The numerical results are shown in Table 4. The last column represents
the two-stage mean-risk stochastic mixed integer programming. It can effectively solve the two-stage mean-risk stochastic mixed integer optimization model. When the model does not consider CVaR, the relative error, which is defined by
\[
\text{Error} = \frac{\text{Optimal Val} - \text{Val}}{\text{Optimal Val}} \times 100\%.
\]

### Table 4. Results of MHB-PSO with different parameters

| System Parameters | Results | Val | Error(%) |
|-------------------|---------|-----|----------|
| T | N | \(c_{\min} \ c_{\max}\) | \(c_{\text{best}}\) | \(c_{\text{best}}\) |
| 50 | 10 | 2.0 2.1 | (1,1,1,0) | (1,0,0,1,0,1,1,0,1,0) | -1.2690e+04 | 0.99 |
| 100 | 10 | 2.0 2.1 | (0,1,1,1) | (0,0,1,0,1,1,0,1,0,1,0) | -1.2712e+04 | 0.76 |
| 100 | 20 | 2.0 2.1 | (1,1,1,0) | (0,0,0,1,0,1,1,0,1,0) | -1.2780e+04 | 0.23 |
| 2000 | 10 | 2.0 2.1 | (1,1,1,0) | (0,0,0,1,0,1,1,0,1,0) | -1.2780e+04 | 0.23 |
| 1000 | 100 | 2.0 2.1 | (1,1,1,0) | (1,0,0,0,1,0,1,1,0,1,0) | -1.2799e+04 | 0.08 |
| 1000 | 1000 | 2.0 2.1 | (1,1,1,0) | (1,0,0,0,1,0,1,1,0,1,0) | -1.2799e+04 | 0.08 |
| 1000 | 10 | 2.0 2.5 | (0,1,1,0) | (0,0,1,0,0,0,0,1,1) | -1.2801e+04 | 0.06 |
| 1000 | 10 | 2.0 3.0 | (1,1,0,1) | (0,0,0,1,0,1,1,0,1,0) | -1.2809e+04 | 0.00 |
| 1000 | 10 | 2.0 4.0 | (1,1,1,0) | (0,0,0,1,0,1,1,0,1,0) | -1.2780e+04 | 0.00 |
| 1000 | 10 | 2.5 3.0 | (1,1,0,1) | (0,0,0,1,1,0,1,0,1,0) | -1.2721e+04 | 0.09 |
| 1000 | 10 | 2.5 4.0 | (1,1,1,0) | (0,0,0,1,0,1,1,0,1,0) | -1.2760e+04 | 0.38 |

In Table 4, when we set different parameters, iteration numbers and population size in the MHB-PSO, the relative error is no more than 0.99%, which indicates that the MHB-PSO has strong robustness to parameters. It can effectively solve the two-stage mean-risk stochastic mixed integer programming.

### Table 5. Supply chain profit with random and expected transportation cost and demand

| \(\xi(\omega)\) | \(c_{\text{best}}\) | \(c_{\text{best}}\) | Pro |
|----------------|----------------|----------------|-----|
| Random | (1,1,1,0) | (1,0,0,0,1,0,1,0,1,0) | 1.2952e+04 |
| Expected | (0,1,0,1) | (0,0,0,1,0,0,1,1,0,1,0) | 1.2819e+04 |

Table 5 shows the supply chain profit under random and expected cost demand. Table 5 shows that in these two cases, the location decision of the supply chain is different. In the case of random cost demand, the supply chain profit is greater than expected, the supply chain profit increases by 1.04%. This indicates that the cost of ignoring the randomness of cost requirements in selection decisions is 133 CNY. Through analysis, it is shown that the stochastic programming model is more suitable for supply chain location modeling than the deterministic programming model.

### Table 6. Comparison of supply chain profit of model with \(\lambda = 0.1\) and \(\lambda = 0\)

| \(\lambda\) | \(c_{\text{best}}\) | \(c_{\text{best}}\) | Pro |
|-------------|----------------|----------------|-----|
| \(\lambda = 0\) | (1,1,1,0) | (0,0,1,1,0,1,1,0,1,0) | 1.3007e+04 |
| \(\lambda = 0.1\) | (1,1,1,0) | (1,0,0,0,1,0,1,0,1,0) | 1.2952e+04 |

In Table 6, we compare the supply chain of a typical two-stage stochastic mixed integer optimization model (\(\lambda = 0\)) with that of a two-stage mean-risk stochastic mixed integer optimization model. When the model does not consider CVaR, the
optimal location of supply chain are \( \mathbf{e}_{\text{best}} = (1, 1, 1, 0) \) and \( \mathbf{c}_{\text{best}} = (0, 0, 1, 0, 1, 0, 1, 0) \), and the profit of supply chain is 1.3007e + 04 respectively. When the model considers CVaR, the optimal location of supply chain are \( \mathbf{e}_{\text{best}} = (1, 1, 1, 0) \) and \( \mathbf{c}_{\text{best}} = (1, 0, 0, 0, 1, 0, 1, 0) \), and the corresponding profit of supply chain is 1.2952e + 04 respectively. The numerical results show that the supply chain profit is more conservative than those without CVaR. This conclusion is consistent with practice.

\[
\lambda = \begin{cases} 
0 & \text{if } Val = 0 \\
1 & \text{otherwise}
\end{cases}
\]

In model (3), different values of \( \lambda \) have different effects on decision-making. Decision makers can obtain the expected benefits by adjusting the weight of the CVaR. Figure 6 shows the effect of \( \lambda \) on the profit of supply chain and fitness function values, where the fitness function value is equal to the negative objective function value, i.e. \( \text{Fit} = -\text{Val} \). With \( \alpha \) fixed and \( \lambda \) changing from 0.1 to 0.9, the supply chain profit and fitness function values are reduced, respectively. That is, the greater the weight of the risk, the more conservative is the decision making. When \( \lambda \) is unchanged, a lower confidence level corresponds to a larger expected supply chain profit and fitness function value. That is, high risk corresponds to high expected returns.

![Figure 6. Values of fitness functions and supply chain profit with different confidence](image)

**Table 7.** Effect of raw material cost and retail price on supply chain profit

| \((r_{11}, r_{12})\) | \(h_1\) | \(\mathbf{e}_{\text{best}}\) | \(\mathbf{c}_{\text{best}}\) | \(\text{Pro}\) |
|---------------------|--------|----------------------|----------------------|---------|
| (4.6,4.8)           | 14.0   | (1, 1, 1, 0)         | (0, 0, 0, 1, 0, 1, 0, 1, 0) | 1.1989e + 04 |
| (4.6,4.8)           | 14.5   | (1, 1, 1, 0)         | (1, 0, 0, 0, 1, 0, 1, 0, 1, 0) | 1.2952e + 04 |
| (4.6,4.8)           | 15.0   | (1, 1, 1, 0)         | (1, 0, 0, 0, 1, 1, 0, 1, 0) | 1.3809e + 04 |
| (4.3,4.8)           | 14.5   | (0, 1, 1, 1)         | (0, 0, 0, 1, 0, 1, 0, 1, 0) | 1.3233e + 04 |
| (4.9,4.8)           | 14.5   | (0, 1, 0, 1)         | (0, 0, 0, 1, 0, 1, 0, 1, 1) | 1.2500e + 04 |
| (4.6,4.5)           | 14.5   | (0, 1, 0, 1)         | (0, 0, 0, 1, 0, 0, 0, 1, 1) | 1.3063e + 04 |
| (4.6,5.0)           | 14.5   | (1, 1, 1, 0)         | (0, 0, 0, 1, 1, 0, 1, 0, 1, 0) | 1.2914e + 04 |
| (4.3,4.5)           | 14.5   | (0, 1, 1, 1)         | (0, 0, 0, 1, 0, 1, 1, 1, 1) | 1.3424e + 04 |
| (4.9,5.0)           | 14.5   | (0, 1, 0, 1)         | (0, 0, 0, 1, 0, 0, 1, 0, 1) | 1.2361e + 04 |
With the advent of disruptive technology, effective cost reduction is an effective means to reap more benefits and higher profitability. Table 7 shows the effect of the raw materials costs on the optimal location and profitability of the supply chain, as can be seen from Table 7, by reducing the cost of the raw materials, the corresponding supply chain profit increases. This suggests that decision-makers can choose low-cost raw materials to obtain more benefits and profitability. Moreover, retailers can influence consumer demand by adjusting their prices in the face of consumer demand uncertainty. In Table 7, as retail prices increase, the corresponding consumer demand will decrease, but the supply chain profit will increase. This shows that enterprises can set reasonable retail prices to obtain consistent benefits.

6. **Conclusion.** In this paper, the optimal location-allocation of supply chain network under uncertain transportation cost and customer demand are studied. At the same time, the risk caused by uncertainty is considered. A two-stage mean-risk stochastic 0-1 mixed integer optimization model is established. Given the complexity of the model, this paper proposes a MHB-PSO to solve the problem studied. Taking Shanghai bread supply chain as an example, this paper studies the influence of uncertainty on the location of food factory and distribution center. By solving the problem, the optimal location and distribution of processing factory and distribution center are obtained, and then the maximum expected profit is achieved. Moreover, the effects of raw material cost, retail price, the weight of CVaR, and confidence level on the expected return are also analyzed. Our numerical analysis shows that decision-makers can make decisions based on their own risk preferences. Supply chain profits can be improved by reducing raw material costs or increasing retail prices. The computational results suggest that our MHB-PSO is better suited to a two-stage mean-risk stochastic 0-1 mixed integer optimization model.

Future research can consider the robustness and uncertainty of supplier supply and consumer demand in the supply chain network simultaneously, to investigate the interaction among the various forms of uncertainty, and their effects on the supply chain system.

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**Appendix.** The parameters related to flour factory, food factory and Rt-mart supermarket are shown in Table 8, where, $g_{i1}$ includes the daily cost of electricity, labor, and oven usage. $f_i$ is the daily rent. $w_{1j}$ includes the daily cost of refrigeration, the use of freezers, $g_j$ is the daily rent.

Table 9 shows the transportation cost from the flour factory to the food factory. Transportation costs include fuel costs, vehicle costs and driver wages. The formula for calculating the unit transportation cost is given below,

$$fuel\ costs = distance \ast \ fuel\ consumption \ast \ oil\ price,$$

$$Unit\ transportation\ cost = (fuel\ costs + \ vehicle\ costs + \ driver\ wages) /1000.$$

For example, the distance from Fuxin third flour factory to Yingyuan food factory is 14.5 km, the fuel consumption is 12 L/100 km, the oil price is 6.67 CNY / L, the vehicle cost is 80 CNY (Chinese Yuan), and the driver wage is 90 CNY. Each shipment of 1000 units of raw materials, hence, $(14.5 \ast 12/100 \ast 6.67 + 80 + 90)/1000 = 0.18$ CNY. Due to traffic congestion, traffic flow and other uncertain factors, the transportation cost is obtained randomly in $[0.18, 0.21]$. In Table 9, we
Table 8. Parameters for suppliers, processing factories, and distribution centers.

| Index | Suppliers | Processing plants | Distribution centers |
|-------|-----------|--------------------|----------------------|
| $s, i, j$ | $a_{is}$ | $r_{is}$ | $b_{i1}$ | $q_{i1}$ | $f_{i}$ | $w_{ij}$ | $g_{j}$ |
| 1 | 1500 | 4.6 | 850 | 0.80 | 185 | 500 | 0.25 | 135 |
| 2 | 1200 | 4.8 | 935 | 0.75 | 180 | 480 | 0.25 | 130 |
| 3 | / | / | 845 | 0.81 | 200 | 450 | 0.25 | 120 |
| 4 | / | / | 950 | 0.76 | 160 | 470 | 0.25 | 125 |
| 5 | / | / | / | / | / | 510 | 0.25 | 140 |
| 6 | / | / | / | / | / | 490 | 0.25 | 130 |
| 7 | / | / | / | / | / | 520 | 0.25 | 145 |
| 8 | / | / | / | / | / | 505 | 0.25 | 143 |
| 9 | / | / | / | / | / | 460 | 0.25 | 123 |
| 10 | / | / | / | / | / | 485 | 0.25 | 133 |

Use $\mathcal{U}(0.18, 0.21)$ to represent a random variable with uniform distributed on $[0.18, 0.21]$.

Similarly, Table 10 shows the random transportation cost of food factory transporting product to Rt-mart. Table 11 shows the random distribution cost of Rt-mart transporting product to consumer. According to the market survey statistics, the daily online order of 8000 is realized in Yangpu district Rt-mart, assuming that the bread order at the same price accounts for 1%, that is, the demand of Yangpu district is 80. After statistical calculation, the random daily demand of consumers is given in Table 12, where $h_1 = 14.5$ CNY.

Table 9. Random transportation cost from flour factory to food factory.

| Flour factory | Yingyuan food factory | Changli food factory | Ziyan food factory | Sunhong food factory |
|---------------|-----------------------|----------------------|-------------------|---------------------|
| Fuxin third   | $\mathcal{U}(0.18, 0.21)$ | $\mathcal{U}(0.12, 0.16)$ | $\mathcal{U}(0.25, 0.28)$ | $\mathcal{U}(0.40, 0.43)$ |
| Fuxin         | $\mathcal{U}(0.45, 0.49)$ | $\mathcal{U}(0.25, 0.28)$ | $\mathcal{U}(0.11, 0.15)$ | $\mathcal{U}(0.27, 0.30)$ |

Table 10. Random transportation cost of food factory transporting product to Rt-mart supermarket.

| Rt-mart         | Yingyuan food factory | Changli food factory | Ziyan food factory | Sunhong food factory |
|-----------------|-----------------------|----------------------|-------------------|---------------------|
| Meilanhu        | $\mathcal{U}(0.26, 0.30)$ | $\mathcal{U}(0.37, 0.40)$ | $\mathcal{U}(0.46, 0.49)$ | $\mathcal{U}(0.65, 0.68)$ |
| Anting          | $\mathcal{U}(0.46, 0.49)$ | $\mathcal{U}(0.41, 0.45)$ | $\mathcal{U}(0.40, 0.44)$ | $\mathcal{U}(0.68, 0.72)$ |
| Nanxiang        | $\mathcal{U}(0.31, 0.34)$ | $\mathcal{U}(0.27, 0.31)$ | $\mathcal{U}(0.33, 0.36)$ | $\mathcal{U}(0.56, 0.60)$ |
| Yangpu          | $\mathcal{U}(0.08, 0.11)$ | $\mathcal{U}(0.17, 0.20)$ | $\mathcal{U}(0.34, 0.37)$ | $\mathcal{U}(0.41, 0.44)$ |
| Sijing          | $\mathcal{U}(0.45, 0.49)$ | $\mathcal{U}(0.27, 0.31)$ | $\mathcal{U}(0.18, 0.21)$ | $\mathcal{U}(0.48, 0.52)$ |
| Chunshen        | $\mathcal{U}(0.36, 0.39)$ | $\mathcal{U}(0.12, 0.16)$ | $\mathcal{U}(0.06, 0.09)$ | $\mathcal{U}(0.33, 0.37)$ |
| Kangqiao        | $\mathcal{U}(0.26, 0.29)$ | $\mathcal{U}(0.11, 0.15)$ | $\mathcal{U}(0.24, 0.28)$ | $\mathcal{U}(0.19, 0.23)$ |
| Songjiang       | $\mathcal{U}(0.58, 0.62)$ | $\mathcal{U}(0.37, 0.41)$ | $\mathcal{U}(0.21, 0.25)$ | $\mathcal{U}(0.51, 0.55)$ |
| Fengxian        | $\mathcal{U}(0.58, 0.62)$ | $\mathcal{U}(0.36, 0.40)$ | $\mathcal{U}(0.24, 0.27)$ | $\mathcal{U}(0.30, 0.34)$ |
| Nicheng         | $\mathcal{U}(0.63, 0.67)$ | $\mathcal{U}(0.52, 0.55)$ | $\mathcal{U}(0.54, 0.57)$ | $\mathcal{U}(0.22, 0.25)$ |
Table 11. Random distribution cost of distribution center transporting product to consumer.

| Rt-mart | Demand area 1          | Demand area 2          | Demand area 3          | Demand area 4          |
|---------|------------------------|------------------------|------------------------|------------------------|
| Meilianlu | $\mathcal{W}(1.00, 1.50)$ | $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(2.00, 2.50)$ | $\mathcal{W}(1.70, 2.20)$ |
| Anting  | $\mathcal{W}(1.00, 1.50)$ | $\mathcal{W}(1.40, 1.90)$ | $\mathcal{W}(1.90, 2.40)$ | $\mathcal{W}(1.80, 2.30)$ |
| Nanxian | $\mathcal{W}(1.00, 1.50)$ | $\mathcal{W}(1.40, 1.90)$ | $\mathcal{W}(1.90, 2.40)$ | $\mathcal{W}(1.70, 2.20)$ |
| Yangpu  | $\mathcal{W}(1.10, 1.60)$ | $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(2.00, 2.50)$ | $\mathcal{W}(1.20, 1.70)$ |
| Sijing  | $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(1.00, 1.50)$ | $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(1.50, 2.00)$ |
| Chunshen| $\mathcal{W}(1.40, 1.90)$ | $\mathcal{W}(1.10, 1.60)$ | $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(1.40, 1.90)$ |
| Kangqiao| $\mathcal{W}(1.40, 1.90)$ | $\mathcal{W}(1.40, 1.90)$ | $\mathcal{W}(1.60, 2.10)$ | $\mathcal{W}(1.10, 1.60)$ |
| Songjiang| $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(1.00, 1.50)$ | $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(1.60, 2.10)$ |
| Fengxian| $\mathcal{W}(1.70, 2.20)$ | $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(1.00, 1.50)$ | $\mathcal{W}(1.60, 2.10)$ |
| Nicheng | $\mathcal{W}(1.70, 2.20)$ | $\mathcal{W}(1.70, 2.20)$ | $\mathcal{W}(1.50, 2.00)$ | $\mathcal{W}(1.00, 1.50)$ |

Table 12. Random demand of consumer.

| Demand area 1          | Demand area 2          | Demand area 3          | Demand area 4          |
|------------------------|------------------------|------------------------|------------------------|
| $\mathcal{W}(500, 525) - h_1$ | $\mathcal{W}(490, 515) - h_1$ | $\mathcal{W}(445, 470) - h_1$ | $\mathcal{W}(455, 480) - h_1$ |

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