Analysis and Optimization of a Double-IRS Cooperatively Assisted System With a Quasi-Static Phase Shift Design

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Abstract—The analysis and optimization of single intelligent reflecting surface (IRS)-assisted systems have been extensively studied, whereas little is known regarding multiple-IRS-assisted systems. This paper investigates the analysis and optimization of a double-IRS cooperatively assisted downlink system (D-IRS-C), where a multi-antenna base station (BS) serves a single-antenna user with the help of two multi-element IRSs, connected by an inter-IRS channel. The channel between any two nodes is modeled with Rician fading. The BS adopts the instantaneous CSI-adaptive maximum-ratio transmission (MRT) beamformer, and the two IRSs adopt a cooperative quasi-static phase shift design. The goal is to maximize the average achievable rate, which can be reflected by the average channel power of the equivalent channel between the BS and user at low channel estimation and phase adjustment costs and computational complexity. First, we obtain tractable expressions of the average channel power of the equivalent channel in the general (Rician factor), pure line of sight (LoS), and pure non-line of sight (NLoS) regimes, respectively. Then, we jointly optimize the phase shifts of the two IRSs to maximize the average channel power of the equivalent channel in these regimes. The optimization problems are challenging non-convex problems. We obtain globally optimal closed-form solutions for some cases and propose computationally efficient iterative algorithms to obtain stationary points for the other cases. Next, we compare the computational complexity for optimizing the phase shifts and the optimal average channel power of D-IRS-C with those of a counterpart double-IRS non-cooperatively assisted system (D-IRS-NC) and a counterpart single-IRS-assisted system (S-IRS) at a large number of reflecting elements in the three regimes. Finally, we numerically demonstrate notable gains of the proposed solutions over the existing solutions at different system parameters. To our knowledge, this is the first work that optimizes the quasi-static phase shift design of D-IRS-C and characterizes its advantages over the optimal quasi-static phase shift design of the counterpart D-IRS-NC and S-IRS.

Index Terms—Intelligent reflecting surface (IRS), double IRSs, cooperation, quasi-static phase shift design, coordinate descent, optimization.

I. INTRODUCTION

STRINGENT requirements for future wireless networks, such as ultra-high data rate and energy efficiency, cannot be fully achieved with the existing wireless communication technologies. Intelligent reflecting surface (IRS), which consists of nearly passive, low-cost, reflecting elements with reconfigurable parameters, has been recently recognized as a promising solution for improving spectrum and energy efficiency [2], [3]. There have been extensive studies on IRS. In what follows, we restrict our attention to the optimization of base station (BS) beamforming and IRS phase shifts for an IRS-assisted single-cell network. In the existing works on optimal designs for IRS-assisted systems, BS beamforming designs are usually adaptive to instantaneous channel state information (CSI), whereas the phase shift designs can be classified into instantaneous CSI-adaptive phase shift designs (which adapt to instantaneous CSI) [4], [5], [6], [7], [8], [9], [10] and quasi-static phase shift designs (which adapt to CSI statistics and do not change over time slots during a certain period) [11], [12], [13], [14], [15], [16], [17]. Notice that a quasi-static (also termed statistical) phase shift design yields a low phase adjustment cost at some performance sacrifice compared with an instantaneous CSI-adaptive phase shift design. Considering the practical implementation issue, a quasi-static phase shift design may be more valuable [11], [12], [13], [14], [15], [16], [17].

Early research on IRS investigates the optimal design for an IRS-assisted system with a single IRS. For example, in [4], [5], [6], [7], [11], [12], [13], [14], and [15], the authors optimize the BS beamformer and IRS phase shifts to minimize the transmit power [4], outage probability [14], and average transmit power [15] and maximize the weighted sum rate [5], secrecy rate [6], energy efficiency [7], and ergodic rate [11], [12], [13]. Specifically, [4], [5], [6], [7] adopt instantaneous CSI-adaptive phase shift designs, whereas [11], [12], [13], [14], [15] consider quasi-static phase shift designs. Later study regarding IRS concentrates on the optimal design for an IRS-assisted system with more than one IRS. For instance, in [8], [9], [16], and [17], the authors optimize
the BS beamformer and IRS phase shifts to minimize the outage probability [16] and maximize the sum secrecy rate [8], received signal power [9], and ergodic achievable rate [17]. In particular, [8], [9] adopt instantaneous CSI-adaptive phase shift designs, whereas [16], [17] consider quasi-static phase shift designs. Notice that the abovementioned works on multi-IRS-assisted systems [8], [9], [16], [17] ignore channels between any two IRSs, referred to as inter-IRS channels, and hence cannot capture their interaction. To achieve the full potential of multi-IRS-assisted systems, [18], [19], [20], [21], [22], [23], [24], [25] explicitly model inter-IRS channels in double-IRS-assisted systems [18], [19], [20], [21], [22], [23], [24] and multi-IRS-assisted systems [25] and consider IRS cooperation. Specifically, in [18] and [19], the authors propose cascaded channel estimation methods for double-IRS cooperatively assisted systems. In [20] and [21], the authors show that the power gain with respect to (w.r.t.) the total number of reflecting elements of a double-IRS-assisted system is higher in order than that of the counterpart single-IRS-assisted system. In [22], [23], [24], and [25], the authors optimize the BS beamformer and IRS phase shifts to maximize the signal-to-interference-plus-noise ratio (SINR) [22], capacity [23], secrecy rate [24], and received signal power [25].

The existing works on double-IRS cooperatively assisted systems [18], [19], [20], [21], [22], [23], [24], [25] have two main limitations. Firstly, [18], [19], [20], [21], [22], [23], [24], [25] simply assume that the direct channel between the BS and user is entirely blocked for tractability. Secondly, [18], [19], [20], [21], [22], [23], [24], [25] solely consider instantaneous CSI-adaptive phase shift designs, which incur higher channel estimation and phase adjustment costs and computational complexities. Therefore, the analysis and optimization of the double-IRS cooperatively assisted systems with a more general channel model (e.g., capturing the direct channel) and a more cost-effective phase shift design remain open.

In this paper, we shall shed some light on the above issues. Specifically, we consider a double-IRS cooperatively assisted system (D-IRS-C), where a multi-antenna BS serves a single-antenna user with the help of two multi-element IRSs connected by an inter-IRS channel. The antennas at the BS and the reflecting elements at the IRSs are arranged in uniform rectangular arrays (URAs). The channel between any two nodes is modeled with Rician fading. In particular, the line of sight (LoS) components do not change during the considered time, and the non-line-of-sight (NLoS) components vary from time slot to time slot. The BS adopts the instantaneous CSI-adaptive maximum-ratio transmission (MRT) beamformer. The two IRSs adopt a cooperative quasi-static phase shift design to improve the average achievable rate at low channel estimation and phase adjustment costs and computational complexity. As the average achievable rate can be approximately reflected by the average channel power of the equivalent channel between the BS and user with a negligible approximation error, this paper focuses on the analysis and optimization of the average channel power¹ rather than the average achievable rate. The main contributions of this paper are summarized as follows.

- **Analysis of Average Channel Power:** First, we characterize the influences of the phase shifts of the two IRSs on the average channel power and divide the channel conditions into four cases accordingly. Then, we obtain the tractable expressions of the average channel power in the general (Rician factor), pure LoS, and pure NLoS regimes, respectively.

- **Optimization of Average Channel Power:** First, we jointly optimize the phase shifts of the two IRSs to maximize the average channel power in the general and pure LoS regimes, respectively. The corresponding optimization problems are challenging non-convex problems. We obtain globally optimal closed-form solutions for some cases and propose computationally efficient iterative algorithms to obtain stationary points for the other cases based on the coordinate descent (CD) and block coordinate descent (BCD) methods. Then, in each regime, we characterize the optimal average channel power when the total number of reflecting elements is large.

- **Comparison with Counterpart IRS-Assisted Systems:** First, we analyze and optimize the average channel powers of a counterpart double-IRS non-cooperatively assisted system (D-IRS-NC) and a counterpart single-IRS-assisted system (S-IRS). Then, we compare the computational complexities for calculating the quasi-static phase shift designs and optimal average channel powers of D-IRS-C, D-IRS-NC, and S-IRS. Specifically, we show that D-IRS-C can achieve a better performance and computational complexity tradeoff than D-IRS-NC and S-IRS when the total number of reflecting elements is sufficiently large.

- **Numerical Results:** We numerically demonstrate notable gains of the proposed solutions over the existing quasi-static phase shift designs. Furthermore, we numerically show that for D-IRS-C, the proposed quasi-static phase shift design is more desirable than the respective instantaneous CSI-adaptive phase shift design as long as the LoS components are sufficiently large.

**Notation:** Boldface lower-case letters (e.g., \(x\)), boldface upper-case letters (e.g., \(X\)), non-boldface letters (e.g., \(x\)), and calligraphic upper-case letters (e.g., \(\mathcal{X}\)) denote vectors, matrices, scalars, and sets, respectively. \(X^H\), \(X^T\), \(\text{tr}(X)\), and \(\|X\|_F\) denote the conjugate transpose, transpose, trace, and Frobenius norm of a matrix, respectively. \(\text{vec}(X)\) denotes the vectorization of a matrix. \(\text{diag}(x)\) denotes a square diagonal matrix with the elements in \(x\) on its main diagonal. \(\text{Diag}(\mathbf{X})\) denotes the leading diagonal of matrix \(X\). \(\|x\|_p\) denotes the Euclidean norm of a vector. \(\mathbb{P}\{x\}\) and \(\mathbb{E}\{x\}\) denote the real part and modulus of a complex number, respectively. \(\mathbb{C}\) denotes the expectation w.r.t. all random variables in the brackets. \(\otimes\) denotes the kronecker product. \(\mathbb{C}^{a \times b}\) and \(\mathbb{R}^{a \times b}\) denote the space of \(a \times b\) complex-valued and real-valued matrices, respectively. \(\mathbb{C}N(\mu, \sigma^2)\) denotes a circularly symmetric complex Gaussian random variable with mean \(\mu\) and variance \(\sigma^2\). \(1_T\) and \(0_T\) denote the \(T\) dimensional vectors with all

¹If not specified otherwise, the average channel power means the average channel power of the equivalent channel between the BS and user.
and also ignore the direct channel between the two IRSs and the IRSs’ locations, which can be approximately channel such as time difference of arrival (TDOA) techniques [26]. We assume that IRS 1 and IRS 2 are placed closer to the BS and user, respectively. We consider four channels, i.e., the direct channel $SU$, cascaded channels $(S1, 1U)$, $(S2, 2U)$, and $(S1, 12, 2U)$. The locations of the BS and IRSs are fixed and known, and the user’s location is static during a certain period and known. The BS is equipped with a URA of $M_S \times N_S$ antennas. Each IRS $l \in L$ is equipped with a URA of $M_l \times N_l$ reflecting elements, where $L \triangleq \{1, 2\}$ denotes the set of IRS indices. Let $T_i \triangleq M_i N_i$, $i = 1, 2$, denote the numbers of the BS’s antennas and IRSs’ reflecting elements. Let $T \triangleq T_1 + T_2$ denote the total number of reflecting elements in the system. Let $M_l \triangleq \{1, 2, \ldots, M_l\}$, $N_l \triangleq \{1, 2, \ldots, N_l\}$, and $T_l \triangleq \{1, 2, \ldots, T_l\}$ denote the corresponding sets of indices. The phase shifts of each IRS’s reflecting elements can be determined by a smart controller attached to it. The BS communicates to the two IRS controllers to configure the two IRSs’ phase shifts via separate reliable wireless links so that both IRSs jointly assist the downlink transmission from the BS to the user.

We consider a narrow-band system and adopt the block-fading model for small-scale fading. Let $H_{SU} \in C^{T_1 \times T_S}$, $H_{12} \in C^{T_2 \times T_1}$, $H_{SU}^{H} \in C^{1 \times T_S}$, and $H_{12}^{H} \in C^{1 \times T_2}$ represent the channel matrix between the BS and IRS $l \in L$, the channel matrix between IRS 1 and IRS 2, the channel vector between the BS and user, and the channel vector between IRS $l \in L$ and the user, respectively. Notice that in contrast with the existing works on double-IRS cooperatively assisted systems [18], [19], [20], [21], [22], [23], [24], we consider the direct channel between the BS and user. As the IRSs are usually far above the ground where scattering is relatively weak, we adopt the Rician fading model for all small-scale fading channels [11], [12], [13], [14], [15], [16], [17]. Specifically,

$$H_{ab} = \sqrt{\alpha_{ab}} \left( \sqrt{\frac{K_{ab}}{K_{ab} + 1}} H_{ab} + \sqrt{\frac{1}{K_{ab} + 1}} \bar{H}_{ab} \right),$$

$$h_{ab}^H = \sqrt{\alpha_{ab}} \left( \sqrt{\frac{K_{ab}}{K_{ab} + 1}} h_{ab}^H + \sqrt{\frac{1}{K_{ab} + 1}} \bar{h}_{ab}^H \right),$$

where $\alpha_{ab} > 0$ represents the large-scale fading power; $K_{ab} \geq 0$ represents the Rician factor; $H_{ab} \in C^{T_2 \times T_1}$ and $h_{ab}^H \in C^{1 \times T_1}$ represent the random normalized NLoS components in a slot with elements independently and identically distributed (i.i.d) according to $CN(0, 1)$; $\bar{H}_{ab} \in C^{T_2 \times T_1}$ and $\bar{h}_{ab}^H \in C^{1 \times T_1}$ represent the deterministic normalized LoS components with unit-modulus elements. Note that $H_{ab}$ and $h_{ab}^H$ do not change during the considered period, as the locations of the BS, IRSs, and user are assumed to be invariant [11], [12], [13], [14], [16].

Let $\lambda$ and $d (\leq \frac{\lambda}{2})$ denote the wavelength of transmission signals and the distance between two adjacent reflecting elements or antennas in each row and column of the URAs. Define $f(x^{(h)}, x^{(v)}, m, n) \triangleq 2\pi \lambda \sin x^{(h)}((m - 1) \cos x^{(h)} + (n - 1) \sin x^{(h)})$, $A^{(h)}(x^{(h)}, x^{(v)}, M, N) \triangleq e^{i f(x^{(h)}, x^{(v)}, m, n)}$, and $a^{(h)}(x^{(h)}, x^{(v)}, M, N) \triangleq \text{vec}(A^{(h)}(x^{(h)}, x^{(v)}, M, N)) \in \mathbb{C}^{MN}$. Then, $H_{ab}$ and $h_{ab}^H$ are modeled as [11], [12], [13], and [14]:

$$H_{ab} = a_{A,ab} a_{D,ab}^H ab \in \{S1, S2, 12\},$$

$$h_{ab} = a_{D,ab} ab \in \{1U, 2U, SU\},$$

where $a_{A,ab} \triangleq a_{\delta ab}^{(h)}(\delta_{ab}^{(h)}, \delta_{ab}^{(v)}, M_b, N_b)$ and $a_{D,ab} \triangleq a_{\varphi ab}^{(h)}(\varphi_{ab}^{(h)}, \varphi_{ab}^{(v)}, M_a, N_a)$. Here, $f(\theta^{(h)}, \theta^{(v)}, m, n)$ represents the difference of the corresponding phase changes over the LoS component; $\delta_{ab}^{(h)}(\delta_{ab}^{(v)})$ represents the azimuth (elevation) angle of arrival (AoA) of a signal from node $a$ to the URA at node $b$; $\varphi_{ab}^{(h)}(\varphi_{ab}^{(v)})$ represents the azimuth (elevation) angle of departure (AoD) of a signal from the URA at node $a$ to node $b$.

To reduce the channel estimation and phase adjustment costs, we consider a quasi-static phase shift design [11], [12], [13], [14], [15], [16], [17]. To be specific, the phase shifts of the two IRSs do not change with the NLoS components from slot to slot and remain constant during the considered period. Let $\theta_l \triangleq (\theta_{l,m,n})_{m \in M_l, n \in N_l} \in \mathbb{R}^{M_l \times N_l}$ denote the constant phase shifts.

The proposed framework can be extended to an IRS cooperatively assisted system with multiple IRSs and users. In this paper, we consider two IRSs and one user to obtain first-order design insights. As in [22] and [23], we use the shifted channel ($S2, 21, U$) and the cascaded channels passing each IRS more than once, e.g., the cascaded channel ($S1, 12, 21, U$), as they have much larger path loss. Note that [22] and [23] also ignore the direct channel $SU$ for simplicity.

As in [19], [20], and [22], the optimization of the element allocation between the two IRSs and the IRSs’ locations, which can be approximately tackled using exhaustive search or heuristic methods [27], are out of the scope of this paper.
phase shifts of IRS $l$, where the phase shift of its $(m,n)$-th element satisfies $\theta_{i,m,n} \in [0,2\pi)$. For notational convenience, we introduce $\phi_i \triangleq (\phi_{i,t})_{t \in T_i} = \text{vec}(\theta_i) \in \mathbb{R}^{T_i}$ to represent the phase shifts of IRS $l$, where its $t$-th element satisfies:

$$\phi_{i,t} \in [0,2\pi), t \in T_i, l \in L.$$  

The equivalent channel between the BS and user, denoted by $h_i^H(\phi_1, \phi_2) \in \mathbb{C}^{1 \times T_S}$, can be expressed as:

$$h_i^H(\phi_1, \phi_2) = h_{SU}^H + \sum_{l \in L} h_{uI}^H \text{diag}(v_i^T) H_{St} + h_{u2}^H(\phi_1) H_{12}^H \text{diag}(v_i^T) H_{St},$$

where $v_i \triangleq (e^{-j\phi_{i,t}})_{t \in T_i} \in \mathbb{C}^{T_i}$. Note that $h_{uI}^H \text{diag}(v_i^T) H_{St}^H$ represents the cascaded channel $(S1, IU)$, and $h_{u2}^H(\phi_1) H_{12}^H \text{diag}(v_i^T) H_{St}$ represents the cascaded channel $(S1, 12, 2U)$. We adopt the following assumptions on CSI: $\alpha_{ab}$, $K_{ab}$, $ab \in \{S1, S2, SU, IU, 12, 2U, 12\}$ are known, $\delta_{ab}^{(h)}(\delta_{ab}^{(v)})$, $\varphi_{ab}^{(h)}(\varphi_{ab}^{(v)})$, $ab \in \{S1, S2, SU, IU, 12\}$ are known, implying that $\mathbf{H}_{ab}, ab \in \{S1, S2, SU\}$ and $\mathbf{h}_{ab} \in \{1U, 2U, SU\}$ (i.e., CSI statistics) are known: $h_{U}(\phi_1, \phi_2)$ (i.e., instantaneous CSI) is known to the BS and user at each slot for given $\phi_1$ and $\phi_2$.

Remark 1 (CSI Estimation): Note that $\alpha_{ab}$ and $K_{ab}$ can be obtained by standard offline channel measurement [29]. Given the BS, IRSs, and user’s locations, $\delta_{ab}^{(h)}(\delta_{ab}^{(v)})$ and $\varphi_{ab}^{(h)}(\varphi_{ab}^{(v)})$ can be easily obtained using angle estimation techniques [30] at the beginning of the considered communication period. Thus, the cost for estimating CSI statistics for quasi-static phase shift design is negligible. With time division duplexing (TDD), at each slot, $h_e(\phi_1, \phi_2)$ can be obtained by setting the phase shifts to the optimized values, sending one pilot symbol from the user ($T_S$ pilot symbols from the BS sequentially), and estimating the received signal at the BS (user), using standard channel estimation methods [31]. Thus, the cost for estimating instantaneous CSI for quasi-static phase shift design is much lower than that for instantaneous CSI-adaptive phase shift design [19], [22], [23].

The BS serves the user via linear beamforming. Let $w \in \mathbb{C}^{T_S}$ with $\|w\|^2 = 1$ represent the normalized linear beamforming vector at the BS. Then, the received signal at the user can be expressed as:

$$Y = h_i^H(\phi_1, \phi_2) w \sqrt{P_S} X_S + Z,$$

where $P_S$ represents the transmit power of the BS, $X_S \in \mathbb{C}$ with $E[|X_S|^2] = 1$ represents an information symbol for the user, and $Z \sim \mathcal{CN}(0, \sigma^2)$ denotes the additive white Gaussian noise (AWGN).

We consider coding within each slot. To maximize the achievable rate (signal-to-noise ratio, SNR) at each slot, we adopt the instantaneous CSI-adaptive MRT beamformer at the BS [11], [12], [13], [14], [15], [17].

$$w = \frac{h_i(\phi_1, \phi_2)}{\|h_i(\phi_1, \phi_2)\|^2}.$$  

Thus, for given $\phi_1$ and $\phi_2$, the average achievable rate of D-IRS-C over slots, $C(\phi_1, \phi_2)$ (bit/s/Hz), is given by:

$$C(\phi_1, \phi_2) \triangleq E \left[ \log_2 \left( 1 + \frac{P_S}{\sigma^2} \right) \right].$$

Therefore, we would like to analyze and maximize $C(\phi_1, \phi_2)$. For tractability, we approximate $C(\phi_1, \phi_2)$ with its upper bound, $\log_2 \left( 1 + \frac{P_S}{\sigma^2} \gamma(\phi_1, \phi_2) \right)$, which is an increasing function of $\gamma(\phi_1, \phi_2)$. Note that in what follows, we let $ab$, $\bar{ab}$, and $b$ represent the Rician channel, LoS channel, and NLoS channel between node $a$ and node $b$, respectively. The Rician channel $ab$ can be viewed as the composition of the LoS channel $\bar{ab}$ and NLoS channel $\bar{ab}$.

III. Analysis

In this section, we analyze the average channel power of D-IRS-C in the general and special regimes, respectively.

A. Analysis in General Regime

In this part, we consider the analysis of the average channel power of D-IRS-C in the general regime. First, we characterize the influences of $\phi_1$ and $\phi_2$ on $\gamma(\phi_1, \phi_2)$.

Lemma 1 (Influences of Phase Shifts of D-IRS-C): (i) If $K_{S1}(K_{1U} + K_{12}) = 0$ (i.e., $K_{S1} = 0$ or $K_{1U} = K_{12} = 0$), then $\gamma(\phi_1, \phi_2)$ does not change with $\phi_1$ and $\phi_2$. (ii) If $K_{SU}(K_{1U} + K_{2U}) = 0$ (i.e., $K_{SU} = 0$ or $K_{1U} = K_{2U} = 0$), then $\gamma(\phi_1, \phi_2)$ does not change with $\phi_2$.

Proof: Please refer to Appendix A.

Lemma 1 indicates that IRS $l$ takes effect if and only if there exist two LoS channels through which signals can arrive at IRS $l$ from a neighbor node and depart from IRS $l$ to a neighbor node. Based on Lemma 1, we can divide the channel conditions into four cases according to the values of Rician factors, as shown in Fig. 2.

- **Case 0** ($K_{S1} = 0$ or $K_{1U} = K_{12} = 0$ and $K_{12} = K_{S2} = 0$ or $K_{2U} = 0$): $\gamma(\phi_1, \phi_2)$ does not change with $\phi_1$ or $\phi_2$ and hence is rewritten as $\gamma^0$.
- **Case 1** ($K_{S1} > 0$, $K_{1U} + K_{12} > 0$, and $K_{12} = K_{S2} = 0$ or $K_{2U} = 0$): $\gamma(\phi_1, \phi_2)$ changes only with $\phi_1$ and hence is rewritten as $\gamma^{(1)}(\phi_1)$.
- **Case 2** ($K_{S1} = 0$ or $K_{1U} = K_{12} = 0$, $K_{12} + K_{S2} > 0$, and $K_{2U} > 0$): $\gamma(\phi_1, \phi_2)$ changes only with $\phi_2$ and hence is rewritten as $\gamma^{(2)}(\phi_2)$.

It is obvious that $w$ in (8) is optimal for the maximization of the instantaneous SNR w.r.t. $w$ under $\|w\|^2 = 1$ at each slot and hence is optimal for the average rate maximization.

The special cases listed in this paper are those which not only correspond to special channel setups but also have closed-form solutions or iterative algorithms with closed-form updates in each iteration.
Fig. 2. The channel setups of D-IRS-C in the four cases.

- **Case 3** \((K_{S1} > 0, K_{IU} + K_{12} > 0, K_{12} + K_{S2} > 0, \text{and } K_{2U} > 0): \gamma(\phi_1, \phi_2) \text{ changes with both } \phi_1 \text{ and } \phi_2 \text{ and is also written as } \gamma^{(3)}(\phi_1, \phi_2).

Then, we characterize the average channel power of D-IRS-C in the four cases of channel conditions. For ease of exposition, we introduce several new notations. Firstly, for \(ab \in \{S1, S2, 1U, 2U, SU\}\), we define \(L_{ab} \triangleq \frac{K_{S1} K_{IU}}{K_{ab}}\) and \(L_{ab} \triangleq \frac{K_{S1} K_{IU}}{K_{ab}}\), which can be interpreted as the large-scale fading powers of the LoS channel \(\overrightarrow{ab}\) and NLoS channel \(\overrightarrow{ab}\), respectively. Obviously, \(L_{ab}\) and \(L_{ab}\) both increase with \(K_{ab}\), \(L_{ab}\) increases with \(K_{ab}\), and \(L_{ab}\) decreases with \(K_{ab}\). Secondly, we define \(L_{a_1 b_1 a_2 b_2} \triangleq \sum_{i=1}^{2} L_{a_i b_i, (a_1 b_1, a_2 b_2)} \in \{S1, S1\} \times \{\overrightarrow{12}, \overrightarrow{12}\} \times \{SU, 2U\}\), which can be interpreted as the large-scale fading powers of the cascaded channels \(\overrightarrow{a_1 b_1 a_2 b_2}\) and \(\overrightarrow{a_1 b_1 a_2 b_2 a_3 b_3}\), respectively. Thirdly, for \(a \in \{S, 1\}, l \in L, b \in \{2, U\}\), we define:

\[\Delta_{a_1 b_1 a_2 b_2} \triangleq \angle \left( \text{diag}(a_{D,a_2 b_2}^{H})a_{a_1 a_1} \right) \in \mathbb{R}^T,\]

\[\Delta_{a_1 b_1 a_2 b_2} \triangleq \text{diag}(a_{D,a_2 b_2}^{H})a_{a_1 a_1} \in \mathbb{C},\]

\[\Delta_{a_1 b_1 a_2 b_2} \triangleq \text{diag}(a_{D,a_2 b_2}^{H})a_{a_1 a_1} \in \mathbb{C}.\]

Note that \(\Delta_{a_1 b_1 a_2 b_2}\) represents the sum of the phase changes over the LoS channel \(\overrightarrow{ab}\) and LoS channel \(\overrightarrow{ab}\). \(\Delta_{a_1 b_1 a_2 b_2}\) depends on \(\varphi_{a_1 b_1 a_2 b_2}\), \(\varphi_{a_1 b_1 a_2 b_2}\), and \(\varphi_{a_1 b_1 a_2 b_2}\), which are determined only by the placement of the URA at node \(a\) and the locations of IRS \(l\) and node \(b\); \(\Delta_{a_1 b_1 a_2 b_2}\) depends on \(\delta_{a_1 b_1 a_2 b_2}\), \(\delta_{a_1 b_1 a_2 b_2}\), and \(\delta_{a_1 b_1 a_2 b_2}\), which are determined only by the placement of the URA at IRS 2 and the locations of the BS and IRS 1. Finally, we define:

\[A_{11} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{S1}^{H}\text{diag}(\overrightarrow{h_{11}}) \in \mathbb{C}^T \times T_I,\]

\[A_{12} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{S1}^{H}\text{diag}(\overrightarrow{h_{11}}) \in \mathbb{C}^T \times T_I,\]

\[A_{21} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{S2}^{H}\text{diag}(\overrightarrow{h_{11}}) \in \mathbb{C}^T \times T_2,\]

\[A_{22} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{12}^{H}\text{diag}(\overrightarrow{h_{11}}) \in \mathbb{C}^T \times T_3,\]

\[A_{3} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{12}^{H}\text{diag}(\overrightarrow{h_{11}}) \in \mathbb{C}^T \times T_1,\]

\[b_{11} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{S1}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T,\]

\[b_{12} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{S1}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T,\]

\[b_{21} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{S2}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T,\]

\[b_{22} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{12}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T,\]

\[b_{1} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{12}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T \times T_1,\]

\[b_{2} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{12}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T \times T_2,\]

\[b_{3} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{12}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T \times T_3,\]

\[b_{4} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{12}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T \times T_4,\]

\[b_{5} \triangleq \text{diag}(\overrightarrow{h_{11}}^{H})\overrightarrow{H}_{12}^{H}\overrightarrow{H}_{SU} \in \mathbb{C}^T \times T_5.\]

Based on the above notations, we characterize the average channel power of D-IRS-C in the general regime as follows.

**Theorem 1 (Average Channel Power of D-IRS-C in General Regime):** The average channel power of D-IRS-C in the four cases of general regime are given by (13)-(16), shown at the bottom of the page.

**Proof:** Please refer to Appendix B.

Theorem 1 indicates that the average channel power of D-IRS-C in Case 0 of the general regime does not depend on \(\phi_1\) or \(\phi_2\), and hence a quasi-static phase shift design is void in Case 0.

**B. Analysis in Pure LoS Regime**

In this part, we analyze the average channel power when the Rician factors go to infinity, corresponding to the regime
of pure LoS channels. Apparently, Cases 0, 1, and 2 are void in the pure LoS regime, and the pure LoS regime can be regarded as a special case of Case 3 of the general regime. Based on Theorem 1, we have the following result.

**Corollary 1 (Average Channel Power of D-IRS-C in Pure LoS Regime):** As $K_{SU} = K_{2U} = K_{12} = K_{SU} = K_{12} = K_{2U} \to \infty$, $\gamma(3)(\phi_1, \phi_2) \to \tilde{\gamma}(3)(\phi_1, \phi_2)$, where $\tilde{\gamma}(3)(\phi_1, \phi_2)$ is given by (17), shown at the bottom of the page.

As the Rician factors go to infinity, the large-scale fading power of each channel or cascaded channel consisting of at least one NLoS channel goes zero. Thus, Theorem 1 readily implies Corollary 1.

### C. Analysis in Pure NLoS Regime

In this part, we analyze the average channel power of D-IRS-C when the Rician factors are zero, corresponding to the regime of pure NLoS channels. Obviously, Cases 1, 2, and 3 are void in the pure NLoS regime, and the pure NLoS regime can be regarded as a special case of Case 0 in the general regime. From Theorem 1, we have the following result.

**Corollary 2 (Average Channel Power of D-IRS-C in Pure NLoS Regime):** If $K_{S1} = K_{S2} = K_{12} = K_{SU} = K_{1U} = K_{2U} = 0$, then $\gamma(0) = \tilde{\gamma}(0)$, where

$$\tilde{\gamma}(0) \triangleq \alpha_{SU}T_S + \alpha_{S1}\alpha_{1U}T_ST_1 + \alpha_{S2}\alpha_{2U}T_ST_2 + \alpha_{S1}\alpha_{12}\alpha_{2U}T_ST_1T_2.$$  

(18)

When the Rician factors become zero, the large-scale fading power of each channel or cascaded channel consisting of at least one LoS channel becomes zero. As a result, Theorem 1 readily implies Corollary 2. Analogously, a quasi-static phase shift design is ineffective for D-IRS-C in the pure NLoS regime.

### IV. OPTIMIZATION

In this section, we maximize the average channel power of D-IRS-C w.r.t. the phase shifts in the general and special regimes, respectively.

#### A. Optimization in General Regime

In this part, we maximize the average channel power of D-IRS-C w.r.t. the phase shifts in the general regime. As shown in Theorem 1, $\gamma(0)$ is irrelevant to $\phi_1$ and $\phi_2$ in the general regime. Thus, we optimize the phase shifts only for Cases 1, 2, and 3 in the general regime in the sequel.

10Pure LoS channels usually exist in the indoor scenario or far above the ground where scattering is relatively weak [32].

---

$$\tilde{\gamma}(3)(\phi_1, \phi_2) \triangleq \alpha_{SU}T_S + \alpha_{S1}\alpha_{1U}v_1^HA_{11}v_1 + \alpha_{S2}\alpha_{2U}v_2^HA_{21}v_2 + \alpha_{S1}\alpha_{12}\alpha_{2U}T_ST_Sv_2^H\alpha_{12}v_1^TA_{12}v_2
+ 2\Re\left\{\sqrt{\alpha_{SU}\alpha_{S1}\alpha_{1U}}b_1 + \sqrt{\alpha_{SU}\alpha_{S2}\alpha_{2U}}b_2 + \sqrt{\alpha_{SU}\alpha_{S1}\alpha_{12}\alpha_{2U}v_1^H}\text{diag}(v_2^HB_1)B_2v_1 + \sqrt{\alpha_{SU}\alpha_{S1}\alpha_{12}\alpha_{2U}}v_1^H\text{diag}(v_2^HB_1)B_3v_2 + v_2^H\left(\sqrt{\alpha_{SU}\alpha_{S1}\alpha_{12}\alpha_{2U}B_1^r}v_1^r + \sqrt{\alpha_{SU}\alpha_{S1}\alpha_{12}\alpha_{2U}B_5v_1}\right)\right\}$$

(17)
descent method. The idea is that at each step of one iteration, \( \phi_{t,t} \in T_t \) are sequentially updated by analytically solving the coordinate optimizations, each with a single variable. Specifically, in each coordinate optimization, we maximize \( \gamma^{(l)}(\phi_t) \) w.r.t. \( \phi_{l,t} \) for some \( t \in T_l \) with the other phase shifts being fixed. For ease of exposition, we rewrite \( \phi_t \) as \( (\phi_{l,t}, \phi_{l,-t}) \), where \( \phi_{l,-t} \) represents the elements of \( \phi_t \) except for \( \phi_{l,t} \). Then, given \( \phi_{l,-t} \) obtained in the previous step, the coordinate optimizations w.r.t. \( \phi_{l,t} \) for Case 1 with \( K_{12}, K_{1U} > 0 \) and Case 2 with \( K_{S2}, K_{12} > 0 \) are formulated as follows.

Problem 2 (Coordinate Optimization w.r.t. \( \phi_{l,t} \) for D-IRS-C in Case 1 = 1, 2 of General Regime): For Case 1 = 1, 2 and for all \( t \in T_l \),

\[
\phi_{l,t} \triangleq \arg\max_{\phi_{l,t} \in [0, 2\pi]} \gamma^{(l)}(\phi_{l,t}, \phi_{l,-t}).
\]

Problem 2 is a single variable optimization problem and hence is much simpler than Problem 1. Define:

\[
\begin{align*}
A_1 & \triangleq L_{ST,1T}A_{11} + L_{ST,12,2T}T_2A_{12} \in \mathbb{C}^{T_1 \times T_1}, \\
A_2 & \triangleq L_{ST,22,2T}A_{21} + L_{ST,12,2T}TSA_{22} \in \mathbb{C}^{T_2 \times T_2}, \\
b_1 & \triangleq \sqrt{L_{ST,11T}L_{ST,11,2T}}b_{11} + \sqrt{L_{ST,22,2T}L_{ST,12,2T}}b_{12} \in \mathbb{C}^{T_1}, \\
b_2 & \triangleq \sqrt{L_{ST,11T}L_{ST,22,2T}}b_{21} + \sqrt{L_{ST,11T}L_{ST,12,2T}}T_Sb_{22} \in \mathbb{C}^{T_2}.
\end{align*}
\]

After some basic algebraic manipulations, we can derive a closed-form optimal solution of Problem 2.

Lemma 2 (Optimal Solution of Problem 2): For Case 1 = 1, 2 and for all \( t \in T_l \), the unique optimal solution of Problem 2 is given by:

\[
\phi_{l,t}^1 = \Lambda \left( -\Lambda \left( \sum_{k \in T_1, k \neq t} A_{l,t,k} e^{-j\phi_{l,k}} + b_{l,t} \right) \right),
\]

where \( A_{l,t,k} \) and \( b_{l,t} \) represent the \((t, k)\)-th element of \( A_l \) given by (22) and the \(t\)-th element of \( b_l \) given by (23), respectively.

Proof: Please refer to Appendix D.

The details of the coordinate descent-based algorithm for Case 1 with \( K_{12}, K_{1U} > 0 \) and Case 2 with \( K_{S2}, K_{12} > 0 \) are summarized in Algorithm 1. Since Problem 2 can be solved analytically, the computation time of Algorithm 1 is relatively short. Specifically, the computational complexity of Step 4 of Algorithm 1 is \( O(T_l) \). Hence the overall computational complexity of each iteration of Algorithm 1 is \( O(T^2_l) \). Furthermore, we know that every limit point generated by Algorithm 1 is a stationary point of Problem 2, as Problem 2 has a unique optimal solution [33, Proposition 2.7.1].

2) Optimization for Case 3 of General Regime: As \( \gamma^{(3)}(\phi_1, \phi_2) \) depends on \( \phi_1 \) and \( \phi_2 \), we jointly optimize \( \phi_1 \) and \( \phi_2 \) in Case 3.

Problem 3 (Optimization for D-IRS-C in Case 3 of General Regime): For Case 3,

\[
\gamma^{(3)*} \triangleq \max_{\phi_1, \phi_2} \gamma^{(3)}(\phi_1, \phi_2) \\
\text{s.t. } (5),
\]

Algorithm 1 Coordinate Descent Algorithm for Obtaining a Stationary Point of Problem 1

1: Initialization: Select \( \phi_t \) satisfying the constraints in (5) as the initial point.
2: repeat
3: for \( t = 1, \ldots, T_l \) do
4: Calculate \( \phi_{l,t}^1 \) according to (24), and set \( \phi_{l,t} = \phi_{l,t}^1 \).
5: end for
6: until some convergence criterion is met.

where \( \gamma^{(3)}(\phi_1, \phi_2) \) is given by (16). Let \( (\phi_1^*, \phi_2^*) \) denote an optimal solution.

Apparently, Problem 3 is a more challenging non-convex problem than Problem 1, as it has a more complex objective function and more optimization variables. However, by the triangle inequality and Cauchy-Schwartz inequality, we can still obtain a globally optimal closed-form solution of Problem 3 for Case 3 with \( K_{S2} = K_{1U} = 0 \) or \( K_{12} = K_{SU} = 0 \).

Theorem 3 (Optimal Solution of Problem 3): (i) For Case 3 with \( K_{S2} = K_{1U} = 0 \), any \( (\phi_1^*, \phi_2^*) \) with

\[
\begin{align*}
\phi_1^* & = \Lambda \left( -\Lambda_{ST,1T} + \psi_1T_1 - \frac{1}{2} \left( \Lambda_{ST,1T} \right)^T 1_{T_1} \right), \\
\phi_2^* & = \Lambda \left( -\Lambda_{ST,22,2T} - \psi_1T_2 - \frac{1}{2} \left( \Lambda_{ST,1T} \right)^T 1_{T_2} \right),
\end{align*}
\]

for all \( \psi \in \mathbb{R} \), is an optimal solution of Problem 3. (ii) For Case 3 with \( K_{12} = K_{SU} = 0 \), any \( (\phi_1^*, \phi_2^*) \) with

\[
\begin{align*}
\phi_1^* & = \Lambda \left( -\Lambda_{ST,1T} + \psi_1T_1 \right), \\
\phi_2^* & = \Lambda \left( -\Lambda_{ST,22,2T} + \psi_1T_2 + \left( \Lambda_{ST,1T} \right)^T 1_{T_2} \right),
\end{align*}
\]

for all \( \psi \in \mathbb{R} \), is an optimal solution of Problem 3.

Proof: Please refer to Appendix E.

Theorem 3 indicates two facts for D-IRS-C in the general regime. Firstly, in Case 3 with \( K_{S2} = K_{1U} = 0 \), the optimal phase changes over the cascaded LoS channel \((ST,1T,2T)\) are the same as the phase changes over the LoS channel \(SU\). Secondly, in Case 3 with \( K_{12} = K_{SU} = 0 \), the optimal phase changes over the cascaded LoS channels \((ST,1T)\) and \((S,T,2T)\) are identical.

However, for Case 3 with \( K_{S2} > 0, K_{1U} > 0, \) and \( K_{12} + K_{SU} > 0 \), we cannot derive a globally optimal solution with the same method. Instead, we propose a computationally efficient iterative algorithm based on the coordinate descent method. The idea is that at each iteration, \( \phi_{l,t}, t \in T_t, l \in L \) are sequentially updated by analytically solving the coordinate optimizations. Specifically, in each coordinate optimization, we maximize \( \gamma(\phi_1, \phi_2) \) w.r.t. \( \phi_{l,t} \) for some \( t \in T_l, l \in L \) with the other phase shifts being fixed. For ease of exposition, we rewrite \( \phi_t \) as \( (\phi_{l,t}, \phi_{l,-t}, \phi_{-l}) \), where \( \phi_{l,-t} \) represents the elements of \( \phi_t \) except for \( \phi_{l,t} \), and \( -l \) represents the element in \( L \backslash \{1\} \). Then, given \( \phi_{l,-t} \) and \( \phi_{-l} \) obtained in the previous step, the coordinate optimization w.r.t. \( \phi_{l,t} \) is formulated as follows.

Problem 4 (Coordinate Optimization w.r.t. \( \phi_{l,t} \) for D-IRS-C in Case 3 of General Regime): For Case 3 and for all \( t \in T_l, l \in L \),

\[
\phi_{l,t}^1 \triangleq \arg\max_{\phi_{l,t} \in [0, 2\pi]} \gamma(\phi_{l,t}, \phi_{l,-t}, \phi_{-l}).
\]
Problem 4 is a single variable non-convex optimization problem and hence is much simpler than Problem 3. Define:

\[
C_1 \triangleq A_1 + L_{S1,12,2U}TSA_1^Tv_1^*v^T_1A_3^*
+ 2 \sqrt{L_{S1,12,2U}L_{S1,12,2U} T L_{S1,12,2U}} \mathbb{R}\{\text{diag}(v_1^H B_1)B_2}\) ∈ \mathbb{C}^{T_1 \times T_1},
\]

\[
C_2 \triangleq A_2 + L_{S1,12,2U}TSA_1^Tv_1^*A_3^H
+ 2 \sqrt{L_{S1,12,2U}L_{S1,12,2U} T L_{S1,12,2U}} \mathbb{R}\{B_1\text{diag}(v_1^H B_3)\} ∈ \mathbb{C}^{T_2 \times T_2},
\]

\[
d_1 \triangleq b_1 + \sqrt{L_{S1,12,2U}L_{S1,12,2U}} \text{diag}(v_1^H B_1)B_3 v_2
+ \sqrt{L_{S1,12,2U}L_{S1,12,2U}} B_1^*v_2^*v_1^*B_4v_1^*
+ \sqrt{L_{S1,12,2U}L_{S1,12,2U}}B_3^*v_2^*v_1^*B_5v_1^* ∈ \mathbb{C}^{T_2}.
\]

After some basic algebraic manipulations, we can derive a closed-form optimal solution of Problem 4.

**Lemma 3 (Optimal Solution of Problem 4):** For Case 3 and all \( t \in \mathbb{T}, l \in \mathbb{L} \), the unique optimal solution of Problem 4 is given by:

\[
\phi_{l,t}^* = \mathcal{A} \left( - \sum_{k \in \mathbb{T}, k \neq t} C_{l,t,k}e^{-j\phi_{l,k}} + d_{l,t} \right),
\]

where \( C_{l,t,k} \) represents the \((l, k)\)-th element of \( C_l \) given by (25), and \( d_{l,t} \) represents the \(t\)-th element of \( d_l \) given by (26).

**Proof:** Please refer to Appendix F.

The details of the coordinate descent-based algorithm are summarized in Algorithm 2. The computational complexities of Step 4 and Step 6 of Algorithm 2 are \( \mathcal{O}(T_1T_2T_3) \) and \( \mathcal{O}(T_1) \), \( l \in \mathbb{L} \). Hence, the computational complexity of each iteration of Algorithm 2 is \( \mathcal{O}(T_1T_2T_3 + T_3^2T_1) \). If \( T_1 = cT \) and \( T_2 = (1 - c)T \) for some \( c \in (0, 1) \), then the computational complexity of each iteration of Algorithm 2 becomes \( \mathcal{O}(T^3) \).

As Problem 4 can be solved analytically, the computation efficiency of Algorithm 2 is relatively high. Furthermore, we know that every limit point generated by Algorithm 2 is a stationary point of Problem 3, as Problem 4 has a unique optimal solution [33, Proposition 2.7.1].

3) **Optimal Average Channel Power in General Regime at Large Number of Reflecting Elements:** We characterize the average channel power of D-IRS-C in Case 0 and the optimal average channel power of D-IRS-C in Cases 1, 2, and 3 of the general regime at Large \( T_1, T_2, \) and \( T \).

**Theorem 4 (Optimal Average Channel Power of D-IRS-C in General Regime):**

(i) \( \gamma(0) \overset{T_1,T_2 \to \infty}{\sim} L_{S1,12,2U}L_{S1,12,2U} + L_{S1,12,2U}L_{S1,12,2U} \gamma(1)^* \overset{T_1 \to \infty}{\sim} L_{S1,12,2U}L_{S1,12,2U}T_2 \),

(ii) \( \gamma(2)^* \overset{T_1 \to \infty}{\sim} L_{S1,12,2U}L_{S1,12,2U}T_2 \),

(iii) \( \gamma(3)^* \overset{T_1 \to \infty}{\sim} L_{S1,12,2U}L_{S1,12,2U}T_2 \).

\[
\gamma(0) \overset{T_1,T_2 \to \infty}{\sim} L_{S1,12,2U}L_{S1,12,2U} + L_{S1,12,2U}L_{S1,12,2U} \gamma(1)^* \overset{T_1 \to \infty}{\sim} L_{S1,12,2U}L_{S1,12,2U}T_2 \),
\]

\[
\gamma(2)^* \overset{T_1 \to \infty}{\sim} L_{S1,12,2U}L_{S1,12,2U}T_2 \),
\]

\[
\gamma(3)^* \overset{T_1 \to \infty}{\sim} L_{S1,12,2U}L_{S1,12,2U}T_2 \).
\]

**Algorithm 2 Coordinate Descent Algorithm for Obtaining a Stationary Point of Problem 3**

1. **Initialization:** Select \( \phi_1 \) and \( \phi_2 \) satisfying the constraints in (5) as the initial point.

2. **repeat**

3. **for** \( l = 1, 2 \) **do**

4. Calculate \( C_l \) and \( d_l \) according to (25) and (26), respectively.

5. **for** \( t = 1, ..., T_1 \) **do**

6. Calculate \( d_{l,t} \) according to (27), and set \( \phi_{l,t} = \phi_{l,t}^* \).

7. **end for**

8. **end for**

9. **until** some convergence criterion is met.

**Proof:** Please refer to Appendix G.

By Theorem 4 and [23, Theorem 1], for D-IRS-C in the general regime, the optimal quasi-static phase shift design for Case 3 achieves the same average power gain in order w.r.t. the total number of reflecting elements \( T \) (i.e., \( \Theta(T^4) \)) as the optimal instantaneous CSI-adaptive phase shift design but with lower channel estimation and phase adjustment costs. Besides, for D-IRS-C in the general regime, the optimal average channel powers in Case 1 and Case 2 (i.e., \( \Theta(T^3) \)) are equivalent in order and are higher in order than the average channel power in Case 0 (i.e., \( \Theta(T^2) \)) and lower in order than the average channel power in Case 3 (i.e., \( \Theta(T^4) \)).

**B. Optimization in Pure LoS Regime**

In this part, we maximize \( \gamma(3)(\phi_1, \phi_2) \) w.r.t. \( \phi_1 \) and \( \phi_2 \) in the pure LoS regime.

**Problem 5 (Optimization for D-IRS-C in Pure LoS Regime):**

\[
\gamma(3)^* \triangleq \max_{\phi_1, \phi_2} \gamma(3)(\phi_1, \phi_2)
\]

s.t. (5),

where \( \gamma(3)(\phi_1, \phi_2) \) is given by (17). Let \( (\tilde{\phi}_1^*, \tilde{\phi}_2^*) \) denote an optimal solution.

As \( \gamma(3)(\phi_1, \phi_2) \) has a simpler form than \( \gamma(3)(\phi_1, \phi_2) \), Problem 5 is more tractable than Problem 3. We propose a computationally efficient iterative algorithm to obtain a stationary point of Problem 5 using the block coordinate descent method. Specifically, we divide the optimization variables into \( T_2 + 1 \) disjoint blocks, i.e., \( \phi_1, \phi_{2,t}, t \in \mathbb{T}_2 \), with the first block consisting of \( T_3 \) coordinates and each of the remaining blocks consisting of only one coordinate, and sequentially
update each block by analytically solving the corresponding block coordinate and coordinate optimization problems. Here, we treat $\phi$ as a single block, as the optimization problem w.r.t. $\phi$ can be analytically solved, with a lower computational complexity than the separate optimization problems w.r.t. $\phi_{1,t}, t \in T_1$, which will be seen shortly. The block coordinate optimization w.r.t. $\phi_1$ and the coordinate optimizations w.r.t. $\phi_{2,t}, t \in T_2$ are formulated as follows.

**Problem 6 (Block Coordinate Optimization w.r.t. $\phi_1$ for D-IRS-C in Pure LoS Regime):** In the pure LoS regime,

$$\hat{\phi}^1_1 \triangleq \arg\max_{\phi_1} \gamma^{(3)}(\phi_1, \phi_2),$$

s.t. $\phi_{1,t} \in [0, 2\pi), t \in T_1$.

**Problem 7 (Coordinate Optimization w.r.t. $\phi_{2,t}$ for D-IRS-C in Pure LoS Regime):** In the pure LoS regime, for all $t \in T_2$,

$$\bar{\phi}^t_{2,t} \triangleq \arg\max_{\phi_{2,t} \in [0, 2\pi)} \gamma^{(3)}(\phi_{2,t}, \phi_{2,-t}, \phi_1).$$

Define:

$$\bar{C}_2 \triangleq \alpha_{S2}a_{2U}A_{21} + \alpha_{S1}a_{1U}a_{2U}T_SA_3v_1^Tv_1T_A^H + 2\sqrt{\alpha_{S2}a_{2U}a_{S1}a_{1U}2\Re\{B_1\text{diag}(v_1^H)B_3\}} \in \mathbb{C}^{T_2 \times T_2},$$

$$d_2 \triangleq \sqrt{\alpha_{SU}a_{SU}a_{2U}b_{21} + \sqrt{\alpha_{SU}a_{SU}a_{S1}a_{1U}B_1v_1^\dagger}} + \sqrt{\alpha_{S1}a_{1U}a_{S1}a_{1SU}B_2\text{diag}(v_2^H)B_1v_1} + \sqrt{\alpha_{S1}a_{1U}a_{S1}a_{1SU}B_1v_1^\dagger},$$

(28)

(29)

After some basic algebraic manipulations, we can derive closed-form optimal solutions of Problem 6 and Problem 7 using the triangle inequality and Cauchy-Schwarz inequality.

**Lemma 4 (Optimal Solutions of Problem 6 and Problem 7):** (i) The unique optimal solution of Problem 6 is given by (30), shown at the bottom of the page. (ii) For all $t \in T_2$, the unique optimal solution of Problem 7 is given by:

$$\bar{\phi}^t_{2,t} = \Lambda\left(-\sum_{k \neq t} \bar{C}_{2,t,k}e^{-j\bar{\phi}_{2,k}} + \bar{d}_2\right),$$

(31)

where $\bar{C}_{2,t,k}$ represents the $(t, k)$-th element of $\bar{C}_2$ given by (28), and $\bar{d}_{2,t}$ represents the $t$-th element of $\bar{d}_2$ given by (29).

**Proof:** Please refer to Appendix H.

The details of the block coordinate descent algorithm are summarized in Algorithm 3. The computational complexities of Step 3, Step 4, and Step 6 are $O(T_1T_2)$, $O(T_1T_2^2)$, and $O(T_2)$, respectively. Hence the computational complexity of each iteration of Algorithm 3 is $O(T_1T_2^2)$. If $T_1 = cT$ and $T_2 = (1 - c)T$ for some $c \in (0, 1)$, then the computational complexity of each iteration of Algorithm 3 becomes $O(T^3)$. Since Problem 6 and Problem 7 have unique optimal solutions, respectively, every limit point generated by Algorithm 3 is a stationary point of Problem 5 [33, Proposition 2.7.1].

**Algorithm 3 Block Coordinate Descent Algorithm for Obtaining a Stationary Point of Problem 5**

1. **Initialization:** Select $\bar{\phi}_1$ and $\bar{\phi}_2$ satisfying the constraints in (5) as the initial point.
2. **repeat**
3. Calculate $\bar{\phi}_1^1$ according to (30), and set $\bar{\phi}_1 = \bar{\phi}_1^1$.
4. Calculate $\bar{C}_2$ and $\bar{d}_2$ according to (28) and (29), respectively.
5. for $t = 1, \ldots, T_2$
6. Calculate $\bar{\phi}_{2,t}^1$ according to (31), and set $\bar{\phi}_{2,t} = \bar{\phi}_{2,t}^1$.
7. **end for**
8. until some convergence criterion is met.

The following lemma characterizes how the optimal average channel power of the pure LoS regime scales $T_1$, $T_2$, and $T$, when they are large.

**Lemma 5 (Optimal Average Channel Power of D-IRS-C in Pure LoS Regime):** (i) $\gamma^{(3)}_1 \sim T_1T_2^{-\infty} a_{S1}a_{1U}a_{2U}T_ST_1^2T_2^2$.

(ii) $T_1 = cT$ and $T_2 = (1 - c)T$ for some $c \in (0, 1)$, then $\gamma^{(3)}_1 \sim T_1T_2^{-\infty} a_{S1}a_{1U}a_{2U}c^2(1 - c)^2T_ST_1^2T_2^2$.

**Proof:** Following the proof of Theorem 4, we can show Lemma 5.

**C. Optimization in Pure NLoS Regime**

As shown in Corollary 2, the average channel power in the pure NLoS regime, i.e., $\gamma^{(0)}$, does not change with $\phi_1$ or $\phi_2$. Thus, there is no need to further optimize the phase shifts in this regime. However, the deployment of reflecting elements still influences $\gamma^{(0)}$. The following lemma characterizes how $\gamma^{(0)}$ scales $T_1$, $T_2$, and $T$, when they are large.

**Lemma 6 (Optimal Average Channel Power of D-IRS-C in Pure NLoS Regime):** (i) $\gamma^{(0)}_1 \sim T_1T_2^{-\infty} a_{S1}a_{1U}a_{2U}T_ST_1T_2$.

(ii) $T_1 = cT$ and $T_2 = (1 - c)T$ for some $c \in (0, 1)$, then $\gamma^{(0)}_1 \sim T_1T_2^{-\infty} a_{S1}a_{1U}a_{2U}c(1 - c)T_ST_1T_2$.

**Proof:** By showing that $\lim_{T_1T_2 \to \infty} \frac{\gamma^{(0)}_1}{\alpha_{S1}a_{1U}a_{2U}T_ST_1T_2} = 1$ and $\lim_{T_1T_2 \to \infty} \frac{\gamma^{(0)}_1}{\alpha_{S1}a_{1U}a_{2U}c(1 - c)T_ST_1T_2} = 1$, we can show $\gamma^{(0)}_1 \sim T_1T_2^{-\infty} a_{S1}a_{1U}a_{2U}T_ST_1T_2$ and $\gamma^{(0)}_1 \sim T_1T_2^{-\infty} a_{S1}a_{1U}a_{2U}c(1 - c)T_ST_1T_2$, respectively.

**V. COMPARISON**

In this section, we consider two counterpart IRS-assisted systems, namely double-IRS non-cooperatively assisted system (D-IRS-NC) and single-IRS-assisted system (S-IRS), as shown in Fig. 3. The assumptions adopted for the two systems are similar to those for D-IRS-C in Section II. First, we analyze and optimize the average channel powers of their quasi-static phase shift designs. Then, we compare their optimal quasi-static phase shift designs with the optimal quasi-static phase shift design of D-IRS-C in computational complexity and average channel power.

$$\phi^*_1 = \Lambda\left(-\left(\sum_{k \neq t} \bar{C}_{2,t,k}e^{-j\bar{\phi}_{2,k}} + \bar{d}_2\right)\right),$$

(30)
A. Double-IRS Non-cooperatively Assisted System (D-IRS-NC)

In this part, we analyze and optimize the average channel power of the quasi-static phase shift design of D-IRS-NC, which is almost the same as D-IRS-C but has no inter-IRS channel. More specifically, there exist three channels, i.e., the direct channel $SU$ and cascaded channels $(S_1,U)$, $(S_2,U)$, represented by $h_{SU}^H$, $h_{SU}^H\text{diag}(v_1^H)H_{S1}$, and $h_{SU}^H\text{diag}(v_2^H)H_{S2}$, respectively, as shown in Fig. 3 (a). Thus, for D-IRS-NC, the equivalent channel between the BS and user, denoted by $h_{DNC,e}^H(\phi_1,\phi_2) \in \mathbb{C}^{1 \times T_s}$, is given by:

$$h_{DNC,e}^H(\phi_1,\phi_2) = h_{SU}^H + \sum_{l \in L} h_{SU}^H\text{diag}(v_l^H)H_{S1}, \quad (32)$$

where $H_{S1}$ and $h_{SU}^H$, $h_{SU}^H$ are given by (1) and (2), respectively. Note that $h_{DNC,e}^H(\phi_1,\phi_2)$ in (32) is identical to $h_{SU}^H(\phi_1,\phi_2)$ in (6) with $\alpha_1 = 0$. Let $\gamma_{DNC}(\phi_1,\phi_2) \triangleq \mathbb{E} \left[ \left\| h_{DNC,e}^H(\phi_1,\phi_2) \right\|_2^2 \right]$ represent the average channel power of the equivalent channel of D-IRS-NC.

First, we characterize the influences of $\phi_1$ and $\phi_2$ on $\gamma_{DNC}(\phi_1,\phi_2)$.

**Lemma 7 (Influences of Phase Shifts of D-IRS-NC):** For $l \in L$, if $K_{SU}K_{lU} = 0$ (i.e., $K_{SI} = 0$ or $K_{lU} = 0$), then $\gamma_{DNC}(\phi_1,\phi_2)$ does not change with $\phi_1$.

**Proof:** Following the proof of Lemma 1, we can show Lemma 7.

Based on Lemma 7, we can also divide the channel conditions of D-IRS-NC into four cases:

- **Case 0** ($K_{SI} = 0$ or $K_{lU} = 0$ and $K_{SU} = 0$ or $K_{lU} = 0$): $\gamma_{DNC}(\phi_1,\phi_2)$ does not change with $\phi_1$ or $\phi_2$ and hence is rewritten as $\gamma_{DNC}^{(0)}$.

- **Case 1** ($K_{SI} > 0$ and $K_{SU} = 0$ or $K_{SU} > 0$): $\gamma_{DNC}(\phi_1,\phi_2)$ changes only with $\phi_1$ and hence is rewritten as $\gamma_{DNC}^{(1)}(\phi_1)$.

- **Case 2** ($K_{SI} = 0$ or $K_{lU} = 0$ and $K_{SU} = 0$ or $K_{lU} > 0$): $\gamma_{DNC}(\phi_1,\phi_2)$ changes only with $\phi_2$ and hence is rewritten as $\gamma_{DNC}^{(2)}(\phi_2)$.

- **Case 3** ($K_{SI} > 0$, $K_{SU} > 0$, $K_{SU} > 0$): $\gamma_{DNC}(\phi_1,\phi_2)$ changes with both $\phi_1$ and $\phi_2$ and is also written as $\gamma_{DNC}^{(3)}(\phi_1,\phi_2)$.

Next, we characterize the average channel power of D-IRS-NC in the general, pure LoS, and pure NLoS regimes.
Case \( l = 1, 2 \). The optimal solution of the problem in (39) is given below.

**Lemma 8 (Optimal Solution of Problem in (39)):** The unique optimal solution of Problem in (39) for \( \gamma_1 \) is given by:

\[
\phi_{\text{DNC}, 1}^{(3)} = \Lambda(-\Delta_{STU} - \angle(r_{STU})1)_{T_1}. \tag{40}
\]

**Proof:** Following the proof of Theorem 2, we can show Lemma 8. \( \blacksquare \)

Lemma 8 can be viewed as a special case of Theorem 2. Lemma 8 indicates that for Case \( l = 1, 2 \) of D-IRS-NC in the general regime, the optimal phase changes over the cascaded LoS channel \((ST, TU)\) and \((SU, U)\) are identical to the phase changes over the LoS channel \(SU\). Furthermore, the computational complexity for calculating \( \phi_{\text{DNC}, 1}^{(3)} \) based on Lemma 8 is \( \mathcal{O}(T) \).

For D-IRS-NC, the maximization of \( \gamma_{\text{DNC}}^{(3)}(\phi_1, \phi_2) \) w.r.t. \( \phi_1 \) and \( \phi_2 \) in Case 3 of the general regime is formulated as:

\[
\gamma_{\text{DNC}}^{(3)} = \max_{\phi_1, \phi_2} \gamma_{\text{DNC}}^{(3)}(\phi_1, \phi_2) \tag{41}
\]

where \( \gamma_{\text{DNC}}^{(3)}(\phi_1, \phi_2) \) is given by (36). Let \( (\phi_{\text{DNC}, 1}^{(3)}, \phi_{\text{DNC}, 2}^{(3)}) \) denote an optimal solution. Besides, for D-IRS-NC, the maximization of \( \gamma_{\text{DNC}}^{(3)}(\phi_1, \phi_2) \) w.r.t. \( \phi_1 \) and \( \phi_2 \) in Case 3 of the pure LoS regime is formulated as:

\[
\gamma_{\text{DNC}}^{(3)} = \max_{\phi_1, \phi_2} \gamma_{\text{DNC}}^{(3)}(\phi_1, \phi_2) \tag{42}
\]

where \( \gamma_{\text{DNC}}^{(3)}(\phi_1, \phi_2) \) is given by (37). Let \( (\phi_{\text{DNC}, 1}^{(3)}, \phi_{\text{DNC}, 2}^{(3)}) \) denote an optimal solution.

The optimal solutions of the problem in (41) and the problem in (42) are given below.

**Lemma 9 (Optimal Solutions of Problem in (41) and Problem in (42)):** The unique optimal solutions of the problem in (41) and the problem in (42) are given by:

\[
\phi_{\text{DNC}, 1}^{(1)} = \phi_{\text{DNC}, 1}^{(3)} = \Lambda(-\Delta_{STU} - \angle(r_{STU})1), \tag{43}
\]

\[
\phi_{\text{DNC}, 2}^{(2)} = \phi_{\text{DNC}, 2}^{(3)} = \Lambda(-\Delta_{SU} - \angle(r_{SU})1)_{T_2}. \tag{44}
\]

**Proof:** Following the proof of Theorem 2, we can show Lemma 9. \( \blacksquare \)

Lemma 9 indicates that for D-IRS-NC in Case 3 of the general and pure LoS regimes, the optimal phase changes over the cascaded LoS channels \((ST, TU)\) and \((SU, U)\) and the LoS channel \(SU\) are identical. Furthermore, the computational complexity for calculating \( \phi_{\text{DNC}, 1}^{(3)}(\phi_{\text{DNC}, 1}^{(3)}) \) and \( \phi_{\text{DNC}, 2}^{(3)}(\phi_{\text{DNC}, 2}^{(3)}) \) based on Lemma 9 are \( \mathcal{O}(T) \).

Finally, we characterize the optimal average channel power of D-IRS-NC in the general, pure LoS, and pure NLoS regimes at large \( T_1, T_2, \) and \( T \).

**Theorem 1 (Optimal Average Channel Power of D-IRS-NC):** (i) For D-IRS-NC in the general regime, the optimal average channel power in Cases 1, 2, and 3 (i.e., \( \Theta(T^2) \)) is given by:

\[
\gamma_{\text{DNC}}^{(i)}(\phi_1, \phi_2) = \text{max}_{\phi_1, \phi_2} \gamma_{\text{DNC}}^{(i)}(\phi_1, \phi_2) \tag{45}
\]

where \( \gamma_{\text{DNC}}^{(i)}(\phi_1, \phi_2) \) is given by (36). Let \( (\phi_{\text{DNC}, 1}^{(i)}, \phi_{\text{DNC}, 2}^{(i)}) \) denote an optimal solution. Besides, for D-IRS-NC, the maximization of \( \gamma_{\text{DNC}}^{(i)}(\phi_1, \phi_2) \) w.r.t. \( \phi_1 \) and \( \phi_2 \) in Case 3 of the pure LoS regime is formulated as:

\[
\gamma_{\text{DNC}}^{(i)} = \max_{\phi_1, \phi_2} \gamma_{\text{DNC}}^{(i)}(\phi_1, \phi_2) \tag{46}
\]

where \( \gamma_{\text{DNC}}^{(i)}(\phi_1, \phi_2) \) is given by (37). Let \( (\phi_{\text{DNC}, 1}^{(i)}, \phi_{\text{DNC}, 2}^{(i)}) \) denote an optimal solution.

**Remark 1 (Quasi-static Phase Shift Design for D-IRS-NC):** The results in Section V.A extend the analysis and optimization results on quasi-static phase shift design for a multi-IRS non-cooperatively assisted system in [16], where the BS is equipped with a single antenna. Moreover, note that the optimal average channel power of D-IRS-NC with a quasi-static phase shift design and a large number of reflecting elements has not been characterized in the existing literature.

**B. Single-IRS-Assisted System (S-IRS)**

In this part, we analyze and optimize the average channel power of the quasi-static phase shift design of S-IRS where the BS serves the user with the help of one IRS indexed by 0 and equipped with a URA of \( M_0 \times N_0 \) elements, as shown in Fig. 3 (b). Define \( M_0 \leq \{1, ..., M\}, N_0 \leq \{1, ..., N\}, \) and \( T_0 \leq \{1, ..., M_0N_0\} \). Let \( \phi_0 \leq \{\phi_{0,t}\}_{t \in T_0} \in \mathbb{R}^{T_0} \) denote the constant phase shifts of IRS 0, where its \( t \)-th element satisfies:

\[
\phi_{0,t} \in [0, 2\pi), \quad t \in T_0. \tag{45}
\]

Accordingly, denote \( v_0 \leq \{\exp(j\phi_{0,t})\}_{t \in T_0} \in \mathbb{C}^T \). For a fair comparison, we let \( M_0N_0 = T_0 \) and place IRS 0 at IRS 1’s location in D-IRS-C. The other setups of S-IRS remain the same as those of D-IRS-C. Let \( \mathbf{H}_0 \in \mathbb{C}^{T \times T_0} \) and \( \mathbf{H}_0^H \in \mathbb{C}^{T_0 \times T} \) represent the Rician channel between the BS and IRS 0 and the Rician channel between IRS 0 and the user, respectively. Specifically, we have:

\[
\mathbf{H}_0 = \sqrt{\alpha_0}\left(\begin{array}{c}
K_0 \mathbf{S}_0 \mathbf{H}_0 + \mathbf{S}_0 \\
K_0 + 1
\end{array}\right), \quad \mathbf{H}_0^H = \sqrt{\alpha_0}v_0\left(\begin{array}{c}
K_0 \mathbf{S}_0^H \mathbf{H}_0^H + \mathbf{S}_0^H \mathbf{H}_0 \\
K_0 + 1
\end{array}\right). \tag{46}
\]
where $\alpha_{S0}, \alpha_{0U} > 0$ represent the large-scale fading powers; $K_{S0}, K_{0U} \geq 0$ represent the Rician factors; $H_{S0}, H_{0U} \in \mathbb{C}^{T \times Ts}$ and $h_{S0}, h_{0U} \in \mathbb{C}^{1 \times Ts}$ represent the random normalized NLoS components in a slot with elements i.i.d. according to $\mathcal{CN}(0,1)$; $H_{S0} = a_sH_{D,S0} \in \mathbb{C}^{T \times Ts}$ and $h_{SU} = a_{SU}H_{S0} \in \mathbb{C}^{1 \times Ts}$ represent the deterministic normalized LoS components with unit-modulus elements. Then, for S-IRS, the equivalent channel between the BS and user, denoted by $h_{SGL,e}(\phi_0) \in \mathbb{C}^{1 \times Ts}$, is given by:

$$h_{SGL,e}(\phi_0) = h_{SU} + h_{S0}^{H}\text{diag}(h_{0U}^{H})H_{S0}.$$ 

Let $\gamma_{SGL}(\phi_0) = \mathbb{E}[\|h_{SGL,e}(\phi_0)\|^2]$ represent the average channel power of the equivalent channel of S-IRS.

For ease of illustration, denote $A_0 \triangleq \text{diag}(h_{0U}^{H})H_{S0}h_{SU}^{H} \in \mathbb{C}^{T \times T}$ and $b_0 \triangleq \text{diag}(h_{SU}^{H})H_{S0}h_{SU} \in \mathbb{C}^{T \times T}$. Now, we characterize the average channel power of S-IRS in the general, pure LoS, and pure NLoS regimes.

Lemma 10 (Average Channel Power of S-IRS): (i) For any $K_{S0}, K_{0U} \geq 0$, $\gamma_{SGL}(\phi_0) = L_{S0,0U}^{(0)}v_0^{H}A_0v_0 + 2\Re \left\{ \sqrt{L_{S0,0U}^{(0)}2\alpha_{SU}T_S + (L_{S0,0U}^{(0)} + L_{S0,0U}^{(0)}T_S)T} \right\}$.

(ii) As $K_{S0}, K_{0U} \rightarrow \infty$, $\gamma_{SGL}(\phi_0) \rightarrow \gamma_{SGL}(\phi_0) \triangleq \alpha_{SU}T_S + \alpha_{SU}T_ST$.

(iii) If $K_{S0} = K_{0U} = 0$, then $\gamma_{SGL}(\phi_0) = \gamma_{SGL}0$, where $\gamma_{SGL}0 \triangleq \alpha_{SU}T_S + \alpha_{SU}T_ST$.

Proof: Following the proof of Theorem 1, we can show Lemma 10.

By Corollary 4 and Lemma 10, if $\alpha_{S0,U} = \alpha_{0SU} = \alpha_{SU}$, then $\gamma_{SGL} = \gamma_{SGL}0$. Noting that $\gamma_{SGL}$ does not change with $\phi_0$, we optimize the phase shifts only in the general and pure LoS regimes. Specifically, in the general regime, the maximization of $\gamma_{SGL}(\phi_0)$ w.r.t. $\phi_0$ is formulated as:

$$\gamma_{SGL} \triangleq \max_{\phi_0} \gamma_{SGL}(\phi_0) \ \text{s.t.} \ (45).$$

Let $\bar{\phi}_0^*$ denote an optimal solution of the problem in (46). In the pure LoS regime, the maximization of $\gamma_{SGL}(\phi_0)$ w.r.t. $\phi_0$ is formulated as:

$$\bar{\gamma}_{SGL} \triangleq \max_{\phi_0} \gamma_{SGL}(\phi_0) \ \text{s.t.} \ (45).$$

Let $\tilde{\phi}_0^*$ denote an optimal solution of the problem in (47).

Define $\Delta_{S0,0U} \triangleq -\frac{1}{2} \left\{ \text{diag}(a_{S0,0U}^{H})a_{S0,0U} \right\} \in \mathbb{R}^{T \times T}$ and $\gamma_{S0,0U} \triangleq a_{S0,0U}^{H}a_{S0,0U} \in \mathbb{C}$. The optimal solutions of the problem in (46) and problem in (47) are given below.

Lemma 11 (Optimal Solutions of Problem in (46) and Problem in (47)): The unique optimal solutions of the problem in (46) and problem in (47) are given by:

$$\phi_0^* = \tilde{\phi}_0^* = \Lambda \left( -\Delta_{S0,0U} - \frac{1}{2} \left( \gamma_{S0,0U}^{1/2} \right) \right).$$

Proof: Following the proof of Theorem 2, we can show Lemma 11.

Lemma 11 indicates that for S-IRS in the general and pure LoS regimes, the optimal phase changes of over the cascaded LoS channel $(\overrightarrow{S0}, \overrightarrow{SU})$ are identical to the phase changes of over the LoS channel $(\overrightarrow{SU})$. Furthermore, the computational complexities for calculating $\phi_0^*$ and $\bar{\phi}_0^*$ based on Lemma 11 are $O(T)$.

Finally, for S-IRS, we characterize the optimal average channel power in the general and pure LoS regimes and the average channel power in the pure NLoS regime at large $T$.

Lemma 12 (Optimal Average Channel Power of S-IRS): $\gamma_{SGL}^* \sim L_{S0,0U}^{(0)}T_ST^2$, $\tilde{\gamma}_{SGL}^* \sim \alpha_{SU}T_ST^2$, and $\bar{\gamma}_{SGL}^* \sim \alpha_{SU}T_ST^2$.

Proof: Following the proof of Theorem 4, we can show Lemma 12.

By Lemma 12 and [4, Proposition 2], for S-IRS, the optimal quasi-static phase shift design achieves the same average power gain in order w.r.t. $T$ (i.e., $\Theta(T^2)$) as the optimal instantaneous CSI-adaptive phase shift design in [4]. Besides, for S-IRS, the optimal average channel powers in the general and pure LoS regimes (i.e., $\Theta(T^2)$) are equivalent in order, and they are higher in order than the average channel power in the pure NLoS regime (i.e., $\Theta(T)$).

Remark 3 (Quasi-static Phase Shift Design for S-IRS): The results in Section V.B extend the analysis and optimization results on quasi-static phase shift design for a less general S-IRS in [11] where the direct channel is modeled as Rayleigh fading. Moreover, note that the optimal average channel power of S-IRS with a quasi-static phase shift design and a large number of reflecting elements has not been characterized in the existing literature.

C. Comparison

In this part, we compare the optimal quasi-static phase shift design of D-IRS-C with those of D-IRS-NC and S-IRS in computational complexity and average channel power. Specifically, based on the optimization and analytical results in Section IV, Section V.A, and Section V.B, we summarize the computational complexities and average channel powers of the optimal quasi-static phase shift designs of D-IRS-C, D-IRS-NC, and S-IRS in the general, pure LoS, and pure NLoS regimes in Table I. Here, we are interested in the growth rates of the computational complexity and average channel power w.r.t. the total number of reflecting elements.$^{13}$

1) Computational Complexity: In each case where the phase shift optimization is necessary, the closed-form optimal quasi-static phase shift design of D-IRS-C (if it exists) has the same computational complexity in order as the closed-form optimal quasi-static phase shift designs of D-IRS-NC and S-IRS, whereas the numerical quasi-static phase shift design of D-IRS-C has higher computational complexity in order than the closed-form optimal quasi-static phase shift designs of D-IRS-NC and S-IRS.

2) Optimal Average Channel Power: (i) In the general regime, the optimal average channel power of D-IRS-NC in Cases 1, 2, and 3 and that of S-IRS are identical in order.

$^{13}$The computational complexity results are derived from those in Section IV, Section V.A, and Section V.B by letting $T_1 = cT$ and $T_2 = (1 - c)T$ for some $c \in (0, 1)$ and $T \rightarrow \infty$. In some cases, there is no need to optimize the phase shifts, and hence we use $\mathcal{O}(1)$ for the computational complexity and consider the average channel power.
Besides, the optimal average channel powers of D-IRS-NC and S-IRS are identical in order in the pure LoS and pure NLoS regimes, respectively. This is because the underlying difference between D-IRS-NC and S-IRS lies in path loss which does not influence the order of growth of the optimal average channel power w.r.t. $T$. (ii) In the general regime, the optimal average channel power of D-IRS-C in each case is higher in order than the optimal average channel power of D-IRS-NC in the same case and the optimal average channel power of S-IRS. In the pure LoS and pure NLoS regimes, respectively, the optimal average channel power of D-IRS-C is higher in order than the optimal average channel powers of D-IRS-NC and S-IRS. This is because the additional inter-IRS channel in D-IRS-C can convey signals from the BS to the user.

3) Tradeoff: D-IRS-C achieves a better performance and computational complexity tradeoff than D-IRS-NC and S-IRS in the cases with closed-form optimal quasi-static phase shift designs and the cases that do not require optimizing phase shifts and achieves a different performance and computational complexity tradeoff in the other cases. For quasi-static phase shift designs, as phase shifts remain constant during a certain period, the computational complexity for optimizing phase shifts is negligible. Therefore, D-IRS-C is more desirable than D-IRS-NC and S-IRS in practice.

### VI. Numerical Results

In this section, we numerically evaluate the average rates of the phase shift designs for D-IRS-C, D-IRS-NC, S-IRS, and the counterpart system without any IRS (W/O-IRS) [34]. As shown in Fig. 4, for all systems, the BS and user are located at $(0, -25, 1.2)$ and $(0, 25, 1)$ (in m), respectively; for D-IRS-C and D-IRS-NC, IRS 1 and IRS 2 are located at $(-x, -y, 5)$ and $(-x, y, 5)$, respectively; for S-IRS, IRS 0 can be located at the locations of IRS 1, IRS 2, and the midpoint between IRS 1 and IRS 2, and the corresponding systems are termed $S$-IRS-Pos-1, $S$-IRS-Pos-2, and $S$-IRS-Pos-Mid, respectively.

| Regime          | System                  | Computational Complexity | Optimal Average Channel Power |
|-----------------|-------------------------|--------------------------|-------------------------------|
| General         | D-IRS-C                 | $O(1)$                   | $\Theta(T^2)$                 |
|                 | C-0                     | $O(T^3)$ (num.)          | $\Theta(T^3)$                 |
|                 | C-1                     | $O(T^3)$ (ana.)          | $\Theta(T^3)$                 |
|                 | C-2                     | $O(T^2)$ (ana.)          | $\Theta(T^2)$                 |
|                 | C-3                     | $O(T^2)$ (ana.)          | $\Theta(T^2)$                 |
|                 | S-IRS                   | $O(T)$ (ana.)            | $\Theta(T^2)$                 |
| Pure LoS        | D-IRS-C (C-3)           | $O(T^4)$ (num.)          | $\Theta(T^4)$                 |
|                 | D-IRS-NC (C-3)          | $O(T)$ (ana.)            | $\Theta(T^4)$                 |
|                 | S-IRS                   | $O(T)$ (ana.)            | $\Theta(T^4)$                 |
| Pure NLoS       | D-IRS-C (C-0)           | $O(1)$                   | $\Theta(T^4)$                 |
|                 | D-IRS-NC (C-0)          | $O(1)$                   | $\Theta(T^4)$                 |
|                 | S-IRS                   | $O(1)$                   | $\Theta(T^4)$                 |

In the simulation, we set $d = 1/4$, $x = 5$, $y = 20$, $M_S = N_S = 2$, $M_U = N_U = 2$, $N_S = 10$, $P_S = 5$ dBm, $\sigma^2 = -104$ dBm, $\varphi_{S1}^{(h)} = \varphi_{U1}^{(h)} = \pi/6$, $\varphi_{S2}^{(h)} = \varphi_{S2}^{(v)} = \pi/4$, $\varphi_{U2}^{(h)} = \varphi_{U2}^{(v)} = \pi/4$, $\varphi_{SU}^{(h)} = \varphi_{SU}^{(v)} = \pi/9$, $d_{1S1} = d_{1S2} = \delta_{S1} = \delta_{S2} = \delta_{S3} = \pi/5$, $\delta_{U1} = \delta_{U2} = \pi/4$, $K_{S1} = K_{S2} = K_{U1} = K_{U2} = K_{SU} = K_{SU} = K = 10$ dB, if not specified otherwise. We set $\alpha_{ab} = 1/(1000d_{ab}^{2\alpha})$ (i.e., $-30 + 10\alpha_{ab}\log_{10}(d_{ab})$ dB), $ab = S1, S2, 1U, 2U, SU$, where $\alpha_{ab}$ represents the corresponding path loss exponent. We choose $\alpha_{12} = 2.2$, $\alpha_{S1}, \alpha_{2U}, \alpha_{S2}, \alpha_{SU} = 2.3$, and $\alpha_{SU} = 3.7$ [4], [12], [22].

14 IRSs are usually placed far above the ground, and their locations are carefully selected. Thus, the inter-IRS channel experiences the fewest obstacles and weakest scattering, and the channels $S1, S2, 1U$, and $2U$ experience fewer obstacles and weaker scattering. Besides, the BS and user are usually located on the ground. Thus, the direct channel $SU$ experiences the most obstacles and strongest scattering [4], [12], [22].
that the upper bound, \( \log \) simplicity. Besides, for D-IRS-C, D-IRS-C-Random chooses the phase shifts uniformly at random, and D-IRS-C-ICSI adopts the optimal instantaneous CSI-adaptive phase shift as in [22].

Fig. 5, Fig. 6, Fig. 7, Fig. 8, Fig. 9, and Fig. 10 illustrate the average rate versus the transmit power \( P \), number of antennas at the BS \( T \), number of IRS 1’s reflecting elements \( T_1 \), IRS locations, total number of reflecting elements in the system \( T \), and Rician factor \( K \), respectively, under Rician fading. In these figures, D-IRS-C-MC represents the numerical expectation of the rate of D-IRS-C, and each other curve represents the analytical upper bound of the average rate of a scheme based on Jensen’s inequality. From Fig. 5, Fig. 6, Fig. 7, Fig. 9, and Fig. 10, we can see that D-IRS-C-MC and D-IRS-C are very close to each other, indicating that the upper bound, \( \log \left( 1 + \frac{1}{2} \gamma (\phi_1, \phi_2) \right) \), is a good approximation of \( C(\phi_1, \phi_2) \). In Fig. 9, D-IRS-C-T, D-IRS-NC-T, and S-IRS-Pos-1-T represent the asymptotic results for D-IRS-C, D-IRS-NC, and S-IRS-Pos-1 at large \( T \), respectively. For each of them, the gap between the general result at any \( T \) and asymptotic result at large \( T \) decreases with \( T \), which is in accordance with Theorem 4, Lemma 5, Lemma 6, Theorem 6, and Lemma 12. The gap between D-IRC-C-T and D-IRS-C is constant when \( T > 800 \), indicating that the asymptotic average channel power can well approximate the exact average channel power when \( T > 800 \) (cf. footnote 12). In Fig. 10(b), the gap between D-IRS-C-Gen and D-IRS-C-LoS (D-IRS-C-NLoS) decreases (increases) with \( K \), in accordance with Corollary 1 (Corollary 2); the gap between D-IRS-NC-Gen and D-IRS-NC-LoS (D-IRS-NC-NLoS) decreases (increases) with \( K \), in accordance with Corollary 3 (Corollary 4); and the gap between S-IRS-Pos-1-Gen and S-IRS-Pos-1-LoS (S-IRS-Pos-1-NLoS) decreases (increases) with \( K \), in accordance with Lemma 10.

Fig. 10. Average rate versus \( K \) under Rician fading.

From Fig. 5 and Fig. 6, we see that the average rate of each scheme increases with \( P \) and \( T \), respectively. From Fig. 7, we see that the average rate of the proposed solution is maximized when the two IRSs have the same number of elements, mainly due to the symmetric channel setup. From Fig. 8, we see that the average rates of D-IRS-C, D-IRS-NC, and S-IRS-Pos-1 are maximized when IRS 1 and IRS 2 are placed closest to the BS and user, respectively. From Fig. 9, we observe that the average rate of each scheme for IRS-assisted systems increases with \( T \), mainly due to the increment of reflecting signal power. Fig. 10 shows that the average rate of each quasi-static phase shift design increases with \( K \), mainly due to the increment of the channel power of each LoS component.

From Fig. 5, Fig. 6, Fig. 7, Fig. 9, and Fig. 10, we can see that D-IRS-C outperforms D-IRS-C-Random, D-IRS-Pos-Mid, and D-IRS-NC at any total number of reflecting elements and outperforms D-IRS-NC, S-IRS-Pos-1, and S-IRS-Pos-2 as long as the total number of reflecting elements is large enough. Specifically, the gain of D-IRS-C over D-IRS-C-Random comes from the optimization of the quasi-static phase shifts, and the gain of D-IRS-C over D-IRS-NC derives from the effective utilization of the inter-IRS channel that can convey signals from the BS to the user. The performance gains of D-IRS-C over D-IRS-NC, S-IRS-Pos-1, and S-IRS-Pos-2 depend on the average channel power of the cascaded channel \((S1, 12, 2U)\), which increases with \( T_1 \),
T2 (as shown in Fig. 9) and K_{S1}, K_{12}, K_{2U} (as shown in Fig. 10). Furthermore, D-IRS-C-ICSI outperforms D-IRS-C at the sacrifice of increased channel estimation and phase adjustment costs and overall computational complexity. Thus, quasi-static phase shift design has practical sense as long as the LoS components are sufficiently large.

Fig. 11 (a) and Fig. 11 (b) show the average rate versus the total number of reflecting elements T and the Rician factor K, respectively, in the general regime under the CDL-D model [28]. From Fig. 11 (a) and Fig. 11 (b), we can see that D-IRS-C still outperforms D-IRS-NC, D-IRS-Random, S-IRS-Pos-1, S-IRS-Pos-2, S-IRS-Pos-Mid, and W/O-IRS, as in Fig. 9 and Fig. 10 for Rician fading. This indicates that the proposed solution framework can be applied to a CDL model.15

VII. CONCLUSION

This paper investigated the analysis and optimization of the quasi-static phase shift design for D-IRS-C. Furthermore, this paper compared the optimal quasi-static phase shift design of D-IRS-C with those of D-IRS-NC and S-IRS. Both analytical and numerical results demonstrate notable gains of the proposed solutions over the existing solutions and reveal insights into designing practical IRS-assisted systems. This work can be extended to IRS-cooperatively-assisted systems with multiple IRSs and users.

APPENDIX A

PROOF OF LEMMA 1

By [35, Theorem 7.16] and [36, (A.26)], we can show that \( H_{S1} \overset{d}{\sim} \text{diag}(v_H^H)H_{S1} \) if \( K_{S1} = 0 \), \( H_{12} \overset{d}{\sim} \text{diag}(v_H^H)H_{12} \) if \( K_{12} = 0 \), and \( h_{U1}^H \overset{d}{\sim} \text{diag}(v_H^H)H_{U1} \) if \( K_{U1} = 0 \). Thus, we can show that \( h_{2U}^H \overset{d}{\sim} \text{diag}(v_H^H)H_{2U} \) if \( K_{2U} = 0 \). Therefore, we can readily obtain \( \gamma^{(0)}(\phi_1, \phi_2), \gamma^{(1)}(\phi_1, \phi_2), \) and \( \gamma^{(2)}(\phi_1, \phi_2) \).

APPENDIX B

PROOF OF THEOREM 1

After some basic algebraic manipulations, we can show that for all \( K_{S1}, K_{S2}, K_{12}, K_{1U}, K_{2U}, K_{SU} \geq 0 \),

\[
\gamma(\phi_1, \phi_2) = \sum_{i=0}^{3} \mathbb{E}[\|x_i^H\|^2_2] + 2 \sum_{j=0}^{3} \sum_{l=1}^{3} \Re \{\mathbb{E}[x_j^H x_l]\},
\]

where \( \mathbb{E}[x_j^H x_l] \) is given by (49)-(53) given below and (54)-(56) shown at the bottom of the next page.

\[
\begin{align*}
\mathbb{E}[\|x_0^H\|^2] &= \alpha_{SU}T_S, \\
\mathbb{E}[\|x_1^H\|^2] &= L_{SU}v_H^H \text{diag}(h_{U1}^H)HH_{S1}^H H_{U1}^H v_H + (L_{SU} + L_{S1U} + L_{SU}^2)T_S T_i, \\
\mathbb{E}[x_2^H x_0] &= \sqrt{L_{SU}L_{S1U}S_{12U}}v_H^2 \text{diag}(h_{U1}^H)HH_{S1}^H H_{SU}, \\
\mathbb{E}[x_3^H x_1] &= \sqrt{L_{SU}L_{S1U}S_{12U}}v_H^2 \text{diag}(h_{2U}^H)HH_{S2}^H H_{SU}, \\
\mathbb{E}[x_2^H x_1] &= \sqrt{L_{SU}L_{S1U}S_{12U}}v_H^2 \text{diag}(h_{2U}^H)HH_{S2}^H H_{SU}.
\end{align*}
\]

Therefore, we can readily obtain \( \gamma^{(0)}(\phi_1, \phi_2), \gamma^{(1)}(\phi_1, \phi_2), \) and \( \gamma^{(2)}(\phi_1, \phi_2) \).

APPENDIX C

PROOF OF THEOREM 2

By the triangle inequality and \( \Re\{x\} \leq |x|, x \in \mathbb{C} \), we can show the following lemma.

**Lemma 13:** Suppose \( A = xy^H \in \mathbb{C}^{M \times N} \) with \( x \in \mathbb{C}^M \) and \( y \in \mathbb{C}^N \), \( v \triangleq \left(e^{-j\phi}\right)_{n \in N} \in \mathbb{C}^N \) with \( N \triangleq \{n = 1, \ldots, N\} \), \( b \in \mathbb{C}^M \), and \( c_1, c_2 \in \mathbb{R} \). The unique optimal solution of the problem

\[
\max_{\phi} c_1 v^H A^H v + c_2 \Re\{v^H A b\}
\]

s.t. \( \phi_n \in [0, 2\pi], \ n \in N \),

is \( \phi^* = \Lambda (\angle(y) - \angle(x^H b)1_N) \).

Based on Lemma 13, we prove Theorem 2 in the following.

When \( K_{12} = 0 \), by (14), we have

\[
\gamma^{(1)}(\phi_1) = L_{SU}v_H^H T_{S1}G_{11}^H G_{11} v_1 + 2 \sqrt{L_{SU}L_{S1U}} \Re\{v_1^H G_{11}^H g_{11}\} + \gamma^{(0)},
\]

where \( G_{11} \triangleq H_{SU}^H \text{diag}(h_{SU}) \) and \( g_{11} \triangleq h_{SU}^H \text{diag}(h_{SU}) \). When \( K_{1U} = 0 \), by (14), we have

\[
\gamma^{(1)}(\phi_1) = L_{SU}v_H^H T_{S1}G_{12}^H G_{12} v_1 + 2 \sqrt{L_{SU}L_{S1U}} \Re\{v_1^H G_{12}^H g_{12}\} + \gamma^{(0)},
\]

where \( G_{12} \triangleq H_{SU}^H \text{diag}(a_{D,12}) \) and \( g_{12} \triangleq H_{SU}^H a_{A,12} \). When \( K_{12} = 0 \), by (15), we have

\[
\gamma^{(2)}(\phi_2) = L_{SU}v_H^H T_{S2}G_{21}^H G_{21} v_2 + 2 \sqrt{L_{SU}L_{S2U}} \Re\{v_2^H G_{21}^H g_{21}\} + \gamma^{(0)},
\]

15 As illustrated in footnote 6, the quasi-static phase shift designs for D-IRS-C, D-IRS-NC, S-IRS-Pos-1, S-IRS-Pos-2, and S-IRS-Pos-Mid are obtained by applying the the proposed methods to the approximate Rician fading model for the CDL-D model. The average rates of the proposed solutions and baseline schemes are evaluated under the actual CDL-D model (i.e., averaged over samples generated according to the CDL-D model).

16 We omit some details due to page limitation. A more detailed version of the proofs can be found in [37].
where $G_{22} \triangleq H_{SU}^T \text{diag}(h_{2U})$ and $g_{21} \triangleq h_{1U}$. When $K_{SU} = 0$, by (15), we have
\[
\gamma(2)(\phi_2) = L_{S1,T2}^2 T_2 v_2^H G_{22}^H g_{22} v_2 + 2 \sqrt{L_{S1,T2}^2 L_{S1,T2}^2 T_2^2} 3 \text{Re} \{v_2^H G_{22}^H g_{22} + \gamma(0)\},
\]
where $G_{22} \triangleq H_{12}^T \text{diag}(a_{12})$ and $g_{12} \triangleq h_{1U}$. Therefore, by (10), (11), (12), and Lemma 13, we can show (20) and (21).

**APPENDIX D**

**PROOF OF LEMMA 2**

By (14), (15), (22), and (23), we have:
\[
\gamma(1)(\phi_{l,t}, \phi_{l,-t}) = 2 \text{Re} \left\{ e^{j \phi_{l,t}} \left( \sum_{k \in T_l, k \neq t} A_{l,t,k} e^{-j \phi_{l,k}} + b_{l,t} \right) \right\}
+ \sum_{k \in T_l, k \neq t} e^{j \phi_{l,k}} b_{l,k}
+ \sum_{k \in T_l, k \neq t} A_{l,t,k} e^{j (\phi_{l,k} - \phi_{l,t})} + A_{l,t,t} + \gamma(0).
\]
Besides, by $\Re \{x\} \leq |x|$, $x \in \mathbb{C}$, we have (57), shown at the top of the next page. Where the equality holds if and only if $\phi_{l,t} = -\angle \left( \sum_{k \in T_l, k \neq t} A_{l,t,k} e^{-j \phi_{l,k}} + b_{l,t} \right)$ By noting that $\phi_{l,t} \in [0, 2\pi)$, we can show (24).

**APPENDIX E**

**PROOF OF THEOREM 3**

First, consider $K_{SU} = K_{1U} = 0$. By the triangle inequality and $\Re \{x\} \leq |x|$, $x \in \mathbb{C}$, we can show:
\[
\gamma(3)(\phi_1, \phi_2) \leq L_{ST,22}^2 T_2 T_2^2 + L_{S1,T2}^2 T_2 T_1 T_2^2 + \gamma(0)
+ L_{S1,T2}^2 T_2^2 T_2^2 + 2 \sqrt{L_{S1,T2}^2 L_{S1,T2}^2 T_1 T_2} |\gamma_{ST,ST'}|,
\]
where the equality holds if and only if $\phi_1$ and $\phi_2$ satisfy $\phi_1 = -\Delta_{ST,ST'}, \phi_1, \psi_1 \in \mathbb{R}, \phi_2 = -\Delta_{ST,ST'}, \psi_2 \in \mathbb{R}$ and $\angle (v_1^H \text{diag}(h_{12}^H a_{12})) + \angle (v_2^H \text{diag}(h_{12}^H a_{12})) = 0$. By noting that $\phi_{l,t} \in [0, 2\pi)$, $t \in T_l, l \in L$ and by choosing $\psi_1 = \frac{1}{2} \angle (\gamma_{ST,ST'}) + \psi$ and $\psi_2 = -\frac{1}{2} \angle (\gamma_{ST,ST'}) - \psi$ for all $\psi \in \mathbb{R}$, we can show that Statement (i). Next, consider $K_{SU} = K_{12} = 0$. By the triangle inequality and $\Re \{x\} \leq |x|$, $x \in \mathbb{C}$, we can show:
\[
\gamma(3)(\phi_1, \phi_2) \leq \sum_{l=1}^2 L_{S1,T2}^2 T_2^2 + \gamma(0)
+ 2 \sqrt{L_{S1,T2}^2 L_{S1,T2}^2 T_1 T_2} |\gamma_{ST,ST'}|,
\]
where the equality holds if and only if $\phi_1$ and $\phi_2$ satisfy $\phi_1 = \Lambda \left( -\Delta_{ST,ST'} + \psi_1 \right), \psi_1 \in \mathbb{R}, l \in L$ and $\angle (v_1^H \text{diag}(h_{12}^H a_{12})) + \angle (\gamma(0)) = 0$. By noting that $\phi_{l,t} \in [0, 2\pi)$, $t \in T_l, l \in L$ and by choosing $\psi_1 = \psi$ and $\psi_2 = \psi + \angle (\gamma(0))$ for all $\psi \in \mathbb{R}$, we can show Statement (ii).

**APPENDIX F**

**PROOF OF LEMMA 3**

By (16), (25), and (26), we have (60), show at the top of the next page. By $\Re \{x\} \leq |x|$, $x \in \mathbb{C}$, we can show (27).

**APPENDIX G**

**PROOF OF THEOREM 4**

First, consider $\gamma(0)$. Case 0. As $\lim_{t_1, t_2 \to \infty} L_{S1,T2}^2 T_2 T_1 T_2 \to 1$, we can show $\gamma(0) \Rightarrow t_1, t_2 \to \infty \Rightarrow L_{ST,22}^2 T_2 T_1 T_2 + 2 \sqrt{L_{ST,22}^2 L_{ST,22}^2 T_1 T_2} |\gamma_{ST,ST'}|$, $\gamma(1) \Rightarrow L_{ST,22}^2 T_2 T_1 T_2 - 2 \sqrt{L_{ST,22}^2 L_{ST,22}^2 T_1 T_2} |\gamma_{ST,ST'}|$. As $\lim_{t_1, t_2 \to \infty} L_{S1,T2}^2 T_2 T_1 T_2 = 1$ and $\lim_{t_1, t_2 \to \infty} L_{S1,T2}^2 T_2 T_1 T_2 = 1$, by the squeeze theorem, we have $\lim_{t_1, t_2 \to \infty} L_{S1,T2}^2 T_2 T_1 T_2 = 1$. Thus, we can show $\gamma(1) \Rightarrow L_{S1,T2}^2 T_2 T_1 T_2$. Then, consider

\[
E[\|x_1^H\|_2^2] = L_{S1,T2}^2 L_{S1,T2}^2 v_2^H \text{diag}(h_{12}^H a_{12}) v_1^H v_1^T \text{diag}(h_{12}^H a_{12}) v_2^H v_2 + L_{S1,T2}^2 L_{S1,T2}^2 v_1^H \text{diag}(a_{12}^H) H_{S1} H_{S1}^H v_1 + L_{S1,T2}^2 L_{S1,T2}^2 v_2^H \text{diag}(h_{12}^H a_{12}) v_2 + L_{S1,T2}^2 L_{S1,T2}^2 + L_{S1,T2}^2 L_{S1,T2}^2 T_2 T_1 T_2 (54)
\]
\[
E[x_1^H x_1] = L_{S1,T2}^2 L_{S1,T2}^2 v_1^H \text{diag}(v_1^H) \text{diag}(h_{12}^H a_{12}) v_1 v_1^H \text{diag}(h_{12}^H a_{12}) v_2^H v_2 + L_{S1,T2}^2 L_{S1,T2}^2 T_2 v_1^H \text{diag}(h_{12}^H a_{12}) v_1 v_1^H \text{diag}(h_{12}^H a_{12}) v_2 + L_{S1,T2}^2 L_{S1,T2}^2 v_2^H \text{diag}(h_{12}^H a_{12}) v_2 v_2^H \text{diag}(h_{12}^H a_{12}) v_1 (55)
\]
\[
E[x_1^H x_2] = L_{S1,T2}^2 L_{S1,T2}^2 v_1^H \text{diag}(v_2^H) \text{diag}(h_{12}^H a_{12}) v_2^H v_2 + L_{S1,T2}^2 L_{S1,T2}^2 v_2^H \text{diag}(a_{12}^H) H_{S1} H_{S1}^H v_2 (56)
\]
Case 2. By Cauchy-Schwarz inequality and the triangle inequality, we can show $\gamma_{(2)*}^{l} \leq \gamma_{(2)*} \leq \gamma_{(2)*}^{u}$, where

\[
\gamma_{(2)*}^{u} \triangleq L_{S_{2}T_{2}}^{l} \sum_{l \neq t} T_{2}^{l} T_{2}^{l} + 2 \sqrt{L_{S_{2}T_{2}}^{l} L_{S_{2}T_{2}}^{l}} T_{2}^{l} T_{2}^{l} + 2 \sqrt{L_{S_{1}T_{1}}^{l} L_{S_{1}T_{1}}^{l}} T_{1} T_{1} T_{1} + \gamma(0),
\]

\[
\gamma_{(2)*} \triangleq L_{S_{1}T_{1}}^{l} T_{1} T_{1} T_{1} + 2 \sqrt{L_{S_{1}T_{1}}^{l} L_{S_{1}T_{1}}^{l}} T_{1} T_{1} T_{1} + \gamma(0).
\]

As $\lim_{T_{1},T_{2} \to \infty} \gamma_{(2)*}^{u} / L_{S_{2}T_{2}}^{l} T_{2}^{l} T_{2}^{l} = 1$ and $\lim_{T_{1},T_{2} \to \infty} \gamma_{(2)*} / L_{S_{2}T_{2}}^{l} T_{2}^{l} T_{2}^{l} = 1$, by the squeeze theorem, we have $\lim_{T_{1},T_{2} \to \infty} \gamma_{(2)*} = 1$. Thus, we can show $\gamma_{(2)*} \leq \gamma_{(2)*}^{l} \leq \gamma_{(2)*}^{u}$, where

\[
\gamma_{(2)*}^{l} \triangleq L_{S_{1}T_{1}}^{l} T_{1} T_{1} T_{1} + 2 \sqrt{L_{S_{1}T_{1}}^{l} L_{S_{1}T_{1}}^{l}} T_{1} T_{1} T_{1} + \gamma(0),
\]

\[
\gamma_{(2)*} \triangleq L_{S_{1}T_{1}}^{l} T_{1} T_{1} T_{1} + 2 \sqrt{L_{S_{1}T_{1}}^{l} L_{S_{1}T_{1}}^{l}} T_{1} T_{1} T_{1} + \gamma(0).
\]

Case 3. By (61)-(64), Cauchy-Schwarz inequality, and the triangle inequality, we can show $\gamma_{(3)*}^{l} \leq \gamma_{(3)*} \leq \gamma_{(3)*}^{u}$, where

\[
\gamma_{(3)*}^{u} \triangleq \gamma_{(3)*}^{u} + \gamma_{(3)*}^{u} - \gamma(0) + L_{S_{1}L_{2}T_{2}}^{l} T_{1} T_{1} T_{2}^{l} + 2 \sqrt{L_{S_{1}L_{2}T_{2}}^{l} L_{S_{1}L_{2}T_{2}}^{l}} T_{2}^{l} T_{1} T_{2}^{l} + 2 \sqrt{L_{S_{1}L_{2}T_{2}}^{l} L_{S_{1}L_{2}T_{2}}^{l}} T_{1} T_{1} T_{2}^{l} + \gamma(0),
\]

\[
\gamma_{(3)*} \triangleq \gamma_{(3)*} + \gamma_{(3)*} - \gamma(0) + L_{S_{1}L_{2}T_{2}}^{l} T_{1} T_{1} T_{2}^{l} + 2 \sqrt{L_{S_{1}L_{2}T_{2}}^{l} L_{S_{1}L_{2}T_{2}}^{l}} T_{2}^{l} T_{1} T_{2}^{l} + 2 \sqrt{L_{S_{1}L_{2}T_{2}}^{l} L_{S_{1}L_{2}T_{2}}^{l}} T_{1} T_{1} T_{2}^{l} + \gamma(0).
\]

As $\lim_{T_{1},T_{2} \to \infty} \gamma_{(3)*}^{u} / L_{S_{1}L_{2}T_{2}}^{l} T_{2}^{l} T_{2}^{l} = 1$ and $\lim_{T_{1},T_{2} \to \infty} \gamma_{(3)*} / L_{S_{1}L_{2}T_{2}}^{l} T_{2}^{l} T_{2}^{l} = 1$, by the squeeze theorem, we have $\lim_{T_{1},T_{2} \to \infty} \gamma_{(3)*} = 1$. Thus, we can show $\gamma_{(3)*} \leq \gamma_{(3)*}^{l} \leq \gamma_{(3)*}^{u}$, where

\[
\gamma_{(3)*}^{l} \triangleq \gamma_{(3)*} + \gamma_{(3)*} - \gamma(0) + L_{S_{1}L_{2}T_{2}}^{l} T_{1} T_{1} T_{2}^{l} + 2 \sqrt{L_{S_{1}L_{2}T_{2}}^{l} L_{S_{1}L_{2}T_{2}}^{l}} T_{2}^{l} T_{1} T_{2}^{l} + 2 \sqrt{L_{S_{1}L_{2}T_{2}}^{l} L_{S_{1}L_{2}T_{2}}^{l}} T_{1} T_{1} T_{2}^{l} + \gamma(0),
\]

\[
\gamma_{(3)*} \triangleq \gamma_{(3)*} + \gamma_{(3)*} - \gamma(0) + L_{S_{1}L_{2}T_{2}}^{l} T_{1} T_{1} T_{2}^{l} + 2 \sqrt{L_{S_{1}L_{2}T_{2}}^{l} L_{S_{1}L_{2}T_{2}}^{l}} T_{2}^{l} T_{1} T_{2}^{l} + 2 \sqrt{L_{S_{1}L_{2}T_{2}}^{l} L_{S_{1}L_{2}T_{2}}^{l}} T_{1} T_{1} T_{2}^{l} + \gamma(0).
\]

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