The power of AQFT: the area law for entropy of localized quantum matter

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Abstract

The algebraic approach to QFT, which for several decades has enriched QFT with structural theorems, has recently shown its utility in various constructions of actual interest. In these lecture notes I explain how AQFT (in particular the modular theory of operator algebras) implies paradigmatic conceptual and mathematical changes while fully preserving the physical principles which underly QFT. As an illustration of actual interest I use holography on null-surfaces and the ensuing area law for entropy of localized matter in the vacuum state.

1 A post-standard model quantum field theory?

Looking at the present particle physics scene one may easily get the somewhat misleading impression that besides string theory and loop quantum gravity (and perhaps leaning back and just waiting for new results from the LHC machine in construction) there are no other worthwhile fundamental alternatives. Here I would like to point out the existence of a third way whose pursuit neither requires to wait for the moment when new experimental data resolve the Higgs issue or supersymmetry (which the impatiently expected LHC data may not even be able to deliver), nor does it involve the risks of a community getting lost in the blue yonder of the aforementioned highly speculative ideas.

This third way was always available but in the good times of the past, when ideas supported by existing computational methods led to a lot of progress, there was no strong motivation to think about it. It is based on the observation that QFT in contrast to QM has remained a conceptually unfinished project. Our knowledge has mainly been obtained by canonical or functional integral...
quantization of classical objects, i.e. by a parallelism to a world of classical field theory or in the words of the protagonist of field quantization Pascual Jordan, with the help of "classical crutches" [1].

We know about the existence of an autonomous world of QFT beyond classical analogs [2] from the result of some courageous explorers who, without much encouragement from people in the particle theory mainstream who were busy looking for new discoveries with the standard methods, have provided us with a wealth of structural insights into QFT. From very recent constructions of certain low-dimensional QFT models, which do not permit the attachment of a Lagrangian name [3][4], we also know that there are many more models than Lagrangian interactions; in fact the existence of a Lagrangian name is neither sufficient for insuring its existence nor necessary in order to know its physical content. With other words Lagrangian names do not make models more amenable for mathematical control, conceptual understanding of its physical content or nonperturbative computational access. The only thing which we can claim with considerable confidence is that the SM is most probably the best Lagrangian straight jacket (enriched with group symmetries and the gauge formalism) for the subatomic world which we could have hoped for.

In order to make progress beyond the SM one should therefore explore QFT beyond its narrow Lagrangian quantization setting using the very firm guidance of its underlying principles. In practical terms it means bringing the computational side of scientific production in particle theory into a better balance with conceptual developments. The number of open well-formulated theoretical problems is presently larger than at any other time in the history of particle theory (but one only becomes aware of this once one steps outside the lure of present fads). As a result of decades of uninterrupted success of particle physics in the last century there was little incentive to take up the unsolved conceptual problems which were left on the wayside; it is precisely this past success which makes a conceptual re-orientation at this late time difficult and time consuming. There is the widespread feeling that the relative ease with which the standard model was discovered is somehow related to the present difficulties to go beyond it; in more concrete terms one suspects that the conceptual mathematical setting which was essential for its formulation (i.e. Lagrangian quantization enriched with group representation theory) does not contain the right instruments needed for answering the remaining questions. This would amount to an admission that the success of renormalized perturbation theory and its finest product, the standard model, only temporarily sidelined the plea for a total autonomous framework of QFT and that the right time to return to this old issue with new vigor is now (even before LHC is running). The long lasting crisis in particle physics and the many failed attempts in trying to cure the symptoms with more of the same medicine forces us to take a new look at this old problem. In [5] I argued that self-interacting massive vector-mesons are only compatible with locality if they are accompanied by additional physical degrees of freedom whose

\[^1\text{Often invented for the only purpose of being able to grind it through the quantization mill.}\]
simplest realization is a scalar field.

Looking into the QFT books one may have the impression that after Wigner’s successful attempt to present the classification of particle spaces without reference to quantization (and its use in e.g. Weinberg’s text book only for additional support of the Lagrangian setting without investigating its potential for exploring Jordan’s dream of an autonomous QFT without classical crutches) not much has happened. Fortunately this first impression is not correct; there has been indeed steady progress in the pursuit of an autonomous presentation of QFT. Since in most textbooks the terminology “QFT” has been identified with the Lagrangian approach, it became customary practice to use the terminology LQP (local quantum physics, thus indicating that causal localization is its central concept) or AQFT (algebraic QFT, using operator algebra theory), when authors want to highlight that they are using a wider setting of QFT while maintaining its physical principles.

In a recent critical essay (last section in [5]) on the present situation I have argued that the new way of looking at particle physics sheds a new and different light on such important subjects as gauge theory and the Schwinger-Higgs screening mechanism \(^2\) for massive vector mesons. Some of these new perspectives will be presented in Mund’s contribution to this conference and published in the same proceedings \(^3\). There is also the rapidly changing (under the influence of AQFT) setting of QFT in CST and an emerging new perspective on perturbative gravity.

Here I will use the very interesting ideas about black hole thermal aspects as a motivating vehicle to present results about holography onto null-surfaces \(^6\) \(^7\) which draws heavily on operator algebra methods (in particular for the derivation of thermal aspects of localization and the area law for the entropy of localized quantum matter). The following small section about modular theory provides some remarks about the mathematical operator algebra formalism.

2 Modular theory and QFT in terms of positioning of monads

In the algebraic approach the starting point for structural investigations of Poincaré invariant QFT has been a spacetime-indexed net of operator algebras which is required to fulfill certain physically motivated and mathematically well-formulated properties related to causal locality and stability of relativistic quantum matter (energy positivity, KMS for thermal states). It is fairly easy to show that the individual local operator algebras (in contradistinction to the generated global algebra in a vacuum representation) do not contain operators which annihilate the vacuum (separability) but nevertheless generate a dense set of states from the vacuum (cyclicity) which changes together with their localization region.

\(^2\)This is often incorrectly called the Higgs “symmetry-breaking”.

\(^3\)The title of J. Mund’s contribution for the proceedings will be String-Localized Quantum Fields and Modular Localization.
This so-called Reeh-Schlieder property turns out to be the prerequisite for applying the Tomita-Takesaki theory \cite{2}; the latter is an extremely rich and profound mathematical theory within the setting of operator algebras. It associates with a pair of a weakly closed operator algebra (von Neumann algebra) acting cyclic and separating on a reference state vector \((A, \Omega)\) to two modular objects: a one-parametric automorphism group of \(A\) and a TCP-like anti-unitary reflection \(J\) (called the modular involution) whose adjoint action maps \(A\) into its commutant operator algebra \(A'\). This theory unfolds its conceptual power in a situation of several such algebras within a common Hilbert space. In fact placing a finite number of such operator algebras into certain relative positions\(^4\), the various individual modular automorphism groups generate noncompact spacetime symmetry groups and the action of the latter on the original finite number of operator algebras generates a net of operator algebras with the properties demanded by AQFT \cite{9}. The opposite way around, each AQFT permits a representation in terms of a modular positioning of a finite number of such operator algebras. In case of a Moebius-invariant chiral QFT the number is two, for 3-dimensional QFT one needs 3 algebras and 4-dimensional QFT can be generated from 6 appropriately positioned algebras: the number increases with increasing spacetime dimension. A closer look reveals that the so positioned algebras are necessarily isomorphic copies of one and the same operator algebra which according to Connes refinement of the Murray-von Neumann classification is the unique hyperfinite type III\(_1\) factor algebra \cite{11} often called monade because of its unique constructive role in QFT which follows precisely the idea of Leibniz about the basic properties which constitute physical reality.

This is not the place to give a mathematical account of the role of modular theory within the operator-algebraic formulation of QFT. But for conveying some aspects of its revolutionary new perspective a few remarks about its conceptual-philosophical content are in order.

If one identifies this distinguished unique hyperfinite type III\(_1\) von Neumann factor algebra as indicated with the role of monades in Leibniz’s construct of ideas about what constitutes reality, one finds a perfect match: the physical reality of relativistic local quantum matter in Minkowski spacetime originates from the (modular) positioning of a finite number of copies of one abstract monade; no sense of individuality can be attributed to the monade, apart from the fact that those hyperfinite factors allow inclusions and intersections (which are meaningless for points) it is as void of individual structure as a point in geometry (if you have seen one monade, you know them all). The rich content of a quantum field theoretical model including its physical interpretation (particles, spacetime and inner symmetries, scattering theory,...) is solely encoded in the relative positions of a finite number of its monades \cite{9}. In view of the conceptual simplicity

\(^4\)The theory is a vast extension of the notion of (uni)modularity of Haar measures in group algebras to the general setting of von Neumann algebras and their classification \cite{8}.

\(^5\)The requirement is that the positioning should be natural within the logic of modular operator theory (using the concept of modular inclusions and modular intersections). For higher than 3 spacetime dimensions the presently known descriptions still look somewhat concocted \cite{9}.
and the radical paradigmatic aspect of this totally autonomous and rigorous setting for characterizing particle physics, there is no way in which one could think of QFT as a closed theory. There is simply too much unexplored terrain beyond the Lagrangian quantization approach (which in view of its parallelism to classical physics cannot really be considered as an autonomous setting). The constructive use of this novel characterization of QFT is still terra incognita but the fact that it yields an alternative description of QFT is beyond any doubt.

The reader who is familiar with Vaughn Jones’s subfactor theory [10] will notice that the underlying philosophy of subfactor theory illustrates this monad picture, the only difference is that as a result of a different aim the subfactor theory is in many aspects simpler. During the last 3 decades subfactor theory has become a mathematical very mature and rich theory. The underlying motivation in this case was to generalize the theory of finite and compact groups by passing from Galois’s inclusion of commutative polynomial fields to noncommuting operator algebras. To achieve this it suffices to identify the monad with the simpler hyperfinite type $\text{II}_1$ factor on which a tracial state can be defined. This monad is too small for obtaining spacetime symmetries and localization-as well as thermal- properties; the implementation of these properties necessitate the use of previous field theoretic monades (as well as the replacement of Jones inclusions by modular inclusions). The theory of Jones inclusions is older and much more developed than the notion of modular inclusion (which is the important concept in the above monade presentation of QFT) and has served as a source of inspiration to AQFT. It was preceded by the DHR theory of localized endomorphism which in the Doplicher-Haag Roberts work was the crucial operator-algebraic concept to unravel the spacetime localization origin of statistics and global inner symmetries in QFT [2].

It is interesting to look at the way in which this modular theory based setting of QFT leads to spacetime symmetry, localization and causality in Minkowski spacetime i.e. to ask the question what is the algebraic germ for the rich geometrical aspects of QFT. It turns out that this is related to the previously mentioned positions of the dense subspaces which are cyclically generated by acting with the monade on the vacuum. The relation is made precise by modular theory in terms of the domain of the unbounded involutive Tomita operator $S$. This encoding of geometric aspects into abstract domain properties of unbounded operators is characteristic of Tomita operators; no other operator is capable to e.g. lead from the inclusion of domains of unbounded operators to inclusions of geometric localization regions in Minkowski spacetime. This way of getting from physical principles of local quantum physics to geometric properties is very different from say the better known Atiyah-Witten-Segal setting which uses the suggestive power of the classical geometric and topological aspects of euclidean functional integral representations for mathematical innovations. The mathematical results obtained in this way are independent of whether the motivating physical setting is metaphoric or autonomous (Euclidean functional integrals are benign metaphors i.e. the answers obtained with physical hindsight are cor-

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6For the derivation of the e.g. Jones polynomial it is of no importance whether the Euclidean Chern-Simons action fits into the Osterwalder-Schrader framework i.e. is asso-
rect even though they violate the functional integral representations from which they allegedly were derived.

AQFT is very different in its aims, it starts from mathematically rigorously formulated physical principle and derives consequences by operator algebraic methods. Using this approach in low dimensional QFT one arrives e.g. at braid group statistics \(^{12}\) in low-dimensional QFT in a way which is reminiscent of Jones subfactor theory. Whereas the quasiclassical approximation on functional integrals for Chern-Simons actions leads to Jones polynomials without revealing their physical interpretation in terms of particle statistics, the physical origin and interpretation in the algebraic approach remains clear throughout all computational steps.

One should perhaps mention that the discovery of modular operator theory was made independently at the same time by mathematicians (Tomita, with considerable enrichments by Takesaki) as well as by physicists (Haag-Hugenholtz-Winnink). Naturally physicists do not aim at the greatest mathematical generality but rather introduce mathematical concepts which are designed to solve a specific physical problem. The physical problem which brought H-H-W into the conceptual proximity of modular theory was the formulation of quantum statistical mechanics for open systems (i.e. directly in the thermodynamic limit) \(^{2}\). I do not know any other case in the history of mathematical physics where a fundamental mathematical theory was simultaneously (and independently) discovered by mathematicians and physicists. After both sides realized this the concepts and terminology quickly merged together. Even in the historical example of quantum mechanics, the Hilbert space theory was already fully available and it took the physicists some years to become aware of it. The relation of AQFT to the mathematical operator algebra theory is very different from the more fashionable but inherently metaphoric mathematics-particle physics relation.

3 Modular theory, classification and construction of models

The fact that the above presentation of Poincaré-invariant QFT in terms of positioning of monades is under rigorous mathematical control does not yet mean that it can be readily used for classifying and constructing models. To facilitate such a use it is advantageous to reformulate the monade assumption. One can show that the assumption of having a unitary representation of the Poincaré group and knowing its action on just one monade is equivalent to the previous relative placement of several copies. Some additional thinking reveals

ciated with a physical correlation function (it is not!) as long as its mathematical meaning is well-defined and its quasiclassical approximation is under mathematical control. In such metaphoric QFT-math connections it is also of no interest whether objects “derived” in this way really fulfill the starting relation or not.

7This means that one knows the position of the wedge algebra within the full algebra \(B(H)\) of all operators i.e. the inclusion \(A(W) \subset B(H)\).
that identifying this monade algebra with a wedge-localized operator algebra in the QFT to be constructed is a good starting point, because the modular objects of a wedge algebra have a well-known physical interpretation. Hence knowing a wedge algebra (i.e. its position in $\mathcal{B}(H)$), the action of the Poincaré group on it immediately leads to the knowledge of all wedge algebras: taking suitable intersection of these wedge algebras and unions of such intersections one obtains the full net of all spacetime indexed algebras i.e. the QFT associated with the original wedge-localized monade together with its covariant transformation property. If the intersections are the trivial algebra (i.e. complex multiples of the identity) then the wedge algebra has no associated QFT. From a practical point of view one does not compute directly with wedge algebras but rather with a generating system of wedge-localized operators which carry a known representation of the Poincaré group.

This rather abstract construction idea can be made to work under the quite strong restriction that there exist wedge-localized generators which, similar to free fields, upon their one-time application the vacuum create one particle states without the admixture of particle-antiparticle vacuum polarization clouds [13]. In case such vacuum-polarization-free-generators (PFG) exist for subwedge spacetime regions (i.e. for regions whose causal completion is smaller than a wedge) it can be shown (by a slight generalization of the Jost-Schroer theorem [14]) that the theory is free in the sense of being generated by free fields. In some sense which can be made precise, wedge-localized PFG’s are the best localized objects in interacting theories for which the field theoretic localization property and that of Wigner particles still coexist simultaneously; for subwedge localization the inexorable presence of interaction-caused vacuum polarization prevents the creation of only one particle states (without an particle/antiparticle polarization cloud) from the vacuum.

A detailed study of PFG wedge-localized generators with translation-invariant domains [15] reveals that they only exist in theories with a purely elastic S-matrix which is only possible in $d=1+1$ dimensions. It turns out that the resulting field theories are precisely those of the bootstrap-formfactor program [16] and the wedge-localized PFG’s are the Fourier transforms of the Zamolodchikov-Faddeev algebra generators (which in this algebraic setting receive a spacetime interpretation). Without the use of modular theory it was not possible to show that the calculated formfactors really belong to a well-defined QFT. This existence problem of QFT in this context of factorizing models has meanwhile been solved [17] by showing that the double-cone intersections of wedge-localized intersection are really nontrivial. Although these models have no real particle creation via scattering, their formfactors (matrix-elements of localized operators) exhibit a very rich vacuum polarization structure. Since they are analytic in a small region around zero coupling strength and since there are general structural arguments that their off-shell spacetime correlation functions admit no convergent power-series expansion for small couplings (they are at best only

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asymptotically convergent), these models suggest that on-shell quantities may have better perturbative properties than off-shell quantities. It would be very interesting to use these models in order to study this interesting problem in detail.

Without wedge-localized PFG’s, i.e. outside factorizing models there are some (presently still rather) vague ideas of how the fact that the S-matrix has the interpretation of a relative modular invariant may be used in model constructions. They are based on the suspicion that the old S-matrix bootstrap of the 60s failed (even in form of a perturbative construction) because this somewhat surprising and powerful connection of the scattering operator with wedge-localized algebras was not noticed; the hope is that its use (i.e. in a perturbative bootstrap-formfactor program) could have a chance to improve this situation. I expect that by combining some ideas of the old S-matrix approach with this recent framework of modular wedge localization one will obtain completely new insights into the nonperturbative structure of QFT.

It is my intense impression that the replacement of crossing and its substitution by Veneziano’s duality was the wrong turn at one of the most important cross roads of particle physics and that it is not possible to make progress in particle physics without a new problematization of the bootstrap-formfactor idea. In particular I expect that the geometric aspects of gauge theory which were important to arrive at the formulation of the standard model will not be useful in order to go beyond the already 30 year lasting stalemate of the SM.

4 Some concrete results about holography, localization entropy and black holes

AQFT and in particular its concept of modular localization is most important in situations in which the conventional quantization approach based on Lagrangians and functional integrals breaks down. Besides the previously mentioned factorizing models this is the case when one studies properties which have no natural association with individual fields. One such situation is the phenomenon of localization-thermality and in particular localization-entropy.

Using a rigorous algebraic formulation of holographic projection of localized quantum matter onto the causal horizon of the localization region, it is possible to compute the entropy generated by the vacuum polarization which occurs near the horizon. For technical details we refer to [6][7]; in the following we will a description of the main ideas. The perception that the surface of a localization region is the source of a very strong vacuum-polarization cloud was historically one of the first observations which showed the distinction between QM and QFT in the most dramatic way.

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9This means that its modular involution $J$ is related to that of a corresponding free theory $J_0$ through $J=J_0S_{scat}$ where $S_{scat}$ is the scattering matrix.

10Heisenberg became aware of (infinite) vacuum fluctuations when he tried to define a partial quantum charge by integrating the charge density over a finite volume. In the modern test function formulation of such problems the vacuum-fluctuation is kept finite by introducing a
The causal horizon of any (causally complete) region is a null-surface and the holographic projection keeps the global algebra, but radically changes its local net structure of the algebraic net of the bulk to that of the horizon. Although the change in the spacetime encoding leads to a different physical interpretation there is not much intrinsic meaning to say the theory "lives in the bulk region" or "lives on the horizon" because this is not a property which is intrinsic to the abstract algebraic substrate but rather depends on the way in which it is spatially organized.

The much advertised AdS-CFT holography\(^{11}\) is a very special kind of holographic projection onto a timelike boundary (meaning it contains the timelike direction, this is often called brane) at infinity; since it is an isomorphism (one-to-one holographic projection) under which the conformal spacetime covering group \(\widetilde{SO}(4,2)\) remains invariant. In that case the reprocessing of the (global) bulk spacetime net structure to that on the CFT boundary is rather straightforward and the name correspondence is more appropriate than holographic projection (which we will henceforth only use in case of null-surfaces). The AdS-CFT correspondence is one between QFT with the highest possible vacuum symmetries on geodetical complete globally hyperbolic manifolds of different spacetime dimensions; there are many models with lesser number of global symmetries which permit "partial" isomorphisms between algebras associated with subregions of different spacetimes with the same spacetime dimensions.

Unfortunately the idea of holography entered particle physics with a heavy metaphoric burden as a result of its alleged quantum gravity connection [19]. This prejudice has proven quite resistant and it is the root of a conjecture the famous Maldacena AdF-CFT conjecture [20] with led to thousands of papers in its bow wave [3]; it seems that no rational argument [21] is able to convince string theorists that the assumption on which it is based namely that holography in the above sense (of a change of spacetime encoding of a fixed algebraic quantum substrate) is capable of producing a QG theory on AdS (a contradiction in terms since QG is by definition background independent) is totally metaphoric [22] without any reasonable chance to ever be backed up by an autonomous physical argument. The conjecture which is backed up by a pathetically incomplete calculation outside any mathematical control is interpreted on the AdS side as a kind of dual QCD (strong QCD coupling limit where one expects confinement). The contradiction to the above rigorous AdS-CFT holography evaporates if one changes the terminology and interprets the Maldacena conjecture as one about an unknown weak coupling AdS theory corresponding to a strongly coupled QCD. If this would be confirmed by more trustworthy approximations it will be a sensational result (even if presently it has no conceptual basis at all) which is totally independent on ST or QG. The ST motivation could then be remembered

\(^{11}\)The idea that there exists a separate "gravitational holography" (different from the present field theoretic version which of course permits an extension to QFT in CST) is an metaphoric illusion. The Maldacena conjecture versus the Rehren theorem on AdS-CFT has been a subject of heated debates.
as a historical footnote and an insight into nonperturbative QCD could then be celebrated as the result of a collective effort of hundreds of researchers in thousands of papers (which would be a completely new sociological phenomenon in particle physics).

It seems that as a collateral effect of this kind of over-exposure of a fashionable subject the impressive progress about the validity of local covariance (leading to background independence) for quantum matter in CST which arose from a very nontrivial extension of the Haag-Kastler framework (including some convincing but not yet rigorous arguments that this continues to hold in a perturbative approach to the Einstein-Hilbert action) remained unnoticed [27][28].

By far the most interesting case is the holographic projection on null-surfaces (not a correspondence!) because it permits to focus on aspects which are pretty much out of reach within the spacetime organization which the bulk setting imposes on quantum matter. Among other things it allows to focus on thermal aspects of localization, in particular on those which underlie the behavior of quantum matter enclosed in black holes. Since QFT is not a theory of metaphoric miracles, there is a prize to pay in that other aspects, in particular particles and scattering theory, become blurred in the holographic projection.

Although holography has a priori nothing to do with gravity, it presents the only known case in which localization behind causal- and event- horizons may be a physical reality rather than a theoretical laboratory (Gedankenexperiment) to uncover interesting structural properties of QFT beyond Lagrangian quantization.

The setting of wedge-localized algebras offers the simplest illustration of the power of holography on null-surfaces. Since the linear extension of its upper (or lower) horizon is the lightfront, it is not surprising that the holography in this case may be considered as an autonomous formulation of the good old metaphoric "lightcone quantization". As its name indicates lightfront quantization (its more appropriate name) was invented as a different quantization and hence no attention was paid how it really links up with the original bulk QFT; not even in the case of free fields one finds clear statements about its relation to bulk matter and in the context of interactions its metaphoric aspect was rendering any credible use within the conceptual setting of QFT impossible. Lightfront holography not only explains why lightfront quantization failed to be applicable to interacting QFT, but also saves some of its physical motivations (simplifications in the description of certain properties). The reason for the failure of the l.c. quantization approach was that, although in case of free fields there is still a linear relation to a corresponding generating field of the lightfront algebra, the presence of interactions destroys any such linear relation. The only way of relating the bulk matter with its holographic projection consists in using the field-coordinatization-independent algebraic methods provided by AQFT. With other words holography cannot be done in the usual Lagrangian quantization setting and even Wightman’s more general formulation would be insufficient.

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Only in case of wedge-localized algebras in two-dimensional factorizing models one knows sufficient conditions which allow to uniquely invert the holographic map.
Although the rigorous mathematical description of lightfront holography (which is based on modular properties of operator algebras) is quite subtle, its intuitive physical content can be explained in terms of known classical causal propagation features for characteristic initial data. The starting observation is that with the exception of 2-dim. conformal relativistic field theories the characteristic data can only be specified on one lightfront (say on \(x = +t, x_\perp\) arbitrary). Specifying data on half the 3-dim. lightfront (null) plane determines the data within a wedge \((x > t, x_\perp\) arbitrary) whose upper causal horizon is the half-plane. By sliding the wedge into itself along the unique lightlike direction contained in its (say upper) horizon, one obtains a family of wedges whose upper causal horizons are sub-half-planes. But any (semi) compact region on the lightfront which is not semifinite in the \(x\)-directions and also two-sided transverse infinite does not cast any causal shadow at all i.e. such characteristic data do not have any associated ambient causal shadow region. The lightfront contains one unique lightray direction. The so called Wigner "little group" which leaves this lightray invariant is isomorphic to the 3-parametric subgroup of the Lorentz group, the Euclidean group \(E(2)\); its 2-parametric "translation" subgroup tilts the edge of the wedge within the lightfront\(^{13}\) (i.e. it changes the wedges, leaving their upper horizon in the same lightfront). Altogether the transformations which leave the lightfront invariant form a 7-parametric subgroup of the Poincaré group. The classical globally hyperbolic bulk theory has a symplectic inner product and the relative symplectic complement of the lightlike shifted classical wedge algebra inside the original wedge-localized classical subalgebra leads to a classical subalgebra which is localized in a "slab" of two-sided infinite transverse extension and compact lightlike extension. Finally the application of the Wigner translations "tilts" these slabs so that their intersections define a compact net structure on the lightfront. Modular operator theory elevates this construction into the algebraic formulation of interacting QFT.

The starting algebra is the wedge-localized algebra \(\mathcal{A}(W) \subset \mathcal{B}(H)\) as a subalgebra of all operators in the Hilbert space\(^{14}\). The slab-localized algebras are obtained as relative commutants of the lightlike translated wedge algebra \(\mathcal{A}(W_\alpha)\) within the original algebra \(\mathcal{A}(W_\alpha)' \cap \mathcal{A}(W) = \mathcal{A}\text{(slab)}\); this resolves the localization structure in the lightlike direction. Finally the intersection of these slab algebras with their tilted image under translations in the little group (by application of the Wigner little group translations) and their intersections define a local net structure. In this way both the longitudinal (lightray) as well as transverse localization structure is resolved. As was already apparent in the classical case of characteristic data the so-obtained local structure of the holographic lightfront projection is very different from that of the bulk; both structures are local in their own right, but nonlocal relative to each other.\(^{15}\)

\(^{13}\)The holographic projection of these Wigner "translations" act like \((x_\perp)_t - x_0\) Galilei velocity transformation in which space and time are interchanged.

\(^{14}\)Since the global lightfront algebra is equal to the global bulk algebra of full Minkowski spacetime, it cannot be used directly since it violates the "no vacuum annihilator" requirement of the Reeh-Schlieder theorem (the prerequisite for the application of modular theory).

\(^{15}\)As mentioned before, the AdS–CFT correspondence is a less radical type of holography.
Holography can be viewed as an extension of the algebraic isomorphism which in the new local covariance setting of QFT is the algebraic functorial image of the diffeomorphism covariance. This means that if one organizes the same abstract algebraic substrate on two different spacetimes, such that two globally hyperbolic submanifolds in the different spacetimes happen to be isometric, then the associated subalgebras are isomorphic i.e. physically indistinguishable (the quantum theory which would make them also mathematically identical is the still elusive QG). Null-surfaces have some common universal features and so do the extended chiral theories which are the algebraic targets of the holographic projections. In this case an (holographic) inversion is generally non-unique and one needs additional information to achieve uniqueness.

The (here suppressed) mathematical concept from modular operator theory are modular inclusions for resolving the longitudinal localization and modular intersections for achieving transverse localization.

The resulting local net structure on the lightfront is very interesting, because the inexorable vacuum fluctuations of QFT have all been compressed into the longitudinal lightray direction whereas there are none in the transverse direction. It is precisely this absence of transverse vacuum fluctuations which leads to an (transverse) area proportionality of those quantities (entropy, energy...) which in heat bath thermal setting used to be intensive quantities.

It is well known that the restriction of the vacuum state to localized regions (as e.g. the wedge) is a KMS thermal state at a fixed temperature whose value depends on how one normalizes the modular Hamiltonian; in the case of the wedge this is the boost generator (of the wedge-preserving boost). KMS thermal states are well-known to fulfill the second thermodynamic law in its most general formulation (no perpetuum mobile of second kind) even before a quantitative notion of entropy has been introduced.

The holographic projection greatly facilitates the computation of the details about the localization entropy which is associated with the wedge-restricted vacuum i.e. the proportionality coefficient in the area behavior. The localization-caused vacuum entropy of a sharply localized algebra is formally infinite for the same reasons that the entropy of the (translation-invariant) global bulk algebra in a heat-bath thermal KMS state diverges. Both algebras are what we previously called monades (hyperfinite type II$_1$ von Neumann factors), and in the standard treatment of heat-bath thermal behavior it is well known that one obtains the volume proportional entropy formula by approximating the translation-invariant monade with a sequence of finite box Gibbs states (the famous thermodynamic limit which defines thermodynamic equilibrium). The same idea works for the localization entropy, except that in this case the sequence consists of Gibbs states on fuzzy-localized approximands (type I factors) which converge to the

of a AdS bulk onto a CFT (timelike) brane for which the relative locality and the maximal spacetime symmetry is maintained even for the AdS bulk causal shadow originating from a compact CFT region.

An exception are the previously mentioned two-dim. factorizing models where one knows the ambient (bulk) Poincaré transformation properties of generators (± generators of the Zamolodchikov-Faddeev algebras) of the lightray algebra.
KMS state on the sharp localized algebra from the inside of the localization region. Since the argument involves conformal transformations, the metric difference between long and short distances become irrelevant.

Using the holographic representation of the sharp localized algebra, and introducing a lightlike interval of size $\varepsilon$ into which the vacuum polarization cloud of the approximands can spread$^{17}$, one obtains the following limiting logarithmic divergence behavior for the localization entropy

$$S_{\text{loc}} = \lim_{\varepsilon \to 0} A c |\ln \varepsilon|$$

where $A$ is the area$^{18}$ (of the edge of the wedge) and $c$ is a constant which measures the degrees of freedom of the holographically projected matter; in typical cases $c$ is equal to the Virasoro algebra constant. It turns out that the area behavior as well as the $|\ln \varepsilon|$ increase is a totally universal aspect (as was mentioned in the previous footnote) of localized quantum matter; its universality is linked to that of an auxiliary global heat bath thermal system on the lightfront (this isomorphism plays an important role in the derivation of the formula$[6][7]$). Changing the end points of the smaller interval on the lightray the entropy changes the ln-factor multiplicatively by the logarithm of the harmonic ratio of the 4 points.

On the other hand, as first observed by Bekenstein, the area behavior which one encounters in certain classical field theories of the Einstein-Hilbert kind is more special and has nothing to do with vacuum fluctuations. As already stated before, a computation of entropy based on quantum mechanical level counting is (unlike the present localization-entropy) inconsistent with the local covariance principle.

This conceptual insight is of relevance for the ongoing discussion about entropy of black holes. In the case of Schwartzschild black holes there are two ways in which the Hawking effect has been presented.

One way is to take as the relevant state the restriction of a certain static ground state on the extended matter algebra (which lives on the extended Schwartzschild spacetime). Upon restriction to the spacetime outside the black hole appears as a thermal KMS state. In this description the localization entropy is the unique entropy which the rules of quantum statistical mechanics relates with the thermal Hawking situation since the Hamiltonian associated with the Killing symmetry (which has continuous spectrum and therefore admits no Gibbs state) has to be approximated by a sequence of discrete spectrum Gibbs states Hamiltonians; this is precisely what the described sequence achieves. This QFT in CST setting of the Hawking effect is conceptually quite tight and does not offer direct support for speculative uses of black hole physics towards the

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$^{17}$This is not a standards short distance singularity in the sense of correlations between field-coordinatizations!

$^{18}$The product $A |\ln \varepsilon|$ is precisely the volume factor of a heat bath system on the lightfront whose Hamiltonian is the generator of the lightray translation. The isomorphism which maps this heat bath system (at $\beta = 2\pi$) to the vacuum-restricted localized one carries the thermodynamic limit into the inner approximand limit and maps the the longitudinal length factor $L$ in $V = AL$ (conformally) into the $|\ln \varepsilon|$. 

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still elusive quantum gravity. The Hawking radiation is matter radiation without any involvement of gravitons, it is fully understood in the setting of QFT in CST and the entropy is the localization entropy associated with this situation. To define entropy in any other way which is not related to the approximation of the modular Hamiltonian does not make much sense since it is only another aspect of the thermal manifestations of the Hawking effect. Whether it can be used to obtain hints about QG remains to be seen.

The second way of describing the Hawking effect is the more physical one. Instead of an equilibrium state one works with a non-equilibrium stationary state which models a collapsing star. Fortunately the Hawking radiation at future lightlike infinity does not depend on details, one only needs the to know the short-distance behavior of the matter two-point function (for free fields) at the point where the star radius passes through the Schwarzschild radius. This was Hawking’s intuitive idea which later was converted into piece of beautiful mathematical-conceptual physics in a paper by Fredenhagen and Haag. Although there has been significant progress in operator algebra theory on defining an entropy flux in stationary non-equilibrium states which replaces the equilibrium entropy, the application to the entropy issue of black holes still remains as an interesting open problem.

The reason for mentioning these ongoing investigation is obvious. The area of black hole entropy has been under intense investigation for its possible links to QG. Although I sympathize with taking big jumps into the conceptual blue yonder in certain situation, I strongly believe that this should not be done without securing a firm basis from where one could start such a leap (and to where one could return, if necessary). I do not think that ST is able to provide such a basis since it itself was founded on very muddy grounds. The energy and entropy concept used in ST is a global one in which quantum mechanical levels are filled and e.g. the entropy results from counting. As mentioned before this violates the local covariance principle of QFT in CST which underlies the more dynamical concept of localization entropy. On the other hand the LQG approach is far away from localizable particle physics and at least in its present form does not seem to animate particle physicists. As a particle physicist I am very surprised that the impressive progress about a new QFT framework which not only incorporates the new local covariance principle but also promises to shed new light on diffeomorphism invariance received so little attention.

In my mainly verbal presentation I glossed over two intermediate concepts which are indispensable for the derivation of the above formula. One is what Nicolov and Todorov in a recent paper very appropriately called the “covariant box” Gibbs state.

The other is a very deep analog of the so-called Nelson-Symanzik duality within the Euclidean Osterwalder-Schrader setting. Whereas in the N-S duality (for 2-dim. massive QFT in a periodic box place into a KMS thermal state)
state) the spatial periodicity is simply interchanged with the temperature "periodicity", the correlation functions of a rotational KMS state on a chiral algebra turn out to be selfdual under interchange of the temperature with its dual (and a temperature-dependent re-scaling of the angular coordinates). The case of the one-point function (the thermal expectation of the identity operator) coalesces with Verlinde’s duality for the partition function for which geometric arguments for its validity were proposed. It turns out that by viewing the partition function as an object of a complete thermal QFT one can use the powerful modular operator algebra theory to prove the temperature duality relation [23][24]. Since this duality relation is best known under its mathematical name SL(2,\mathbb{Z}) "modular identity", what looked like a coincidence of terminology has now a profound intrinsic connection.

5 Holographic symmetry and its relation to the classical BMS group

Another quite amazing consequence of null-surface holography is the emergence of a gigantic holographic symmetry group [7]. If we denote the generating point-like fields of the lightfront algebra by $A_i(x_\perp, x)$, with $x_\perp$ being the transverse and $x$ the longitudinal (lightray) coordinates, their commutation read for such generators read  

$$[A_i(x_\perp, x), A_j(x'_\perp, x')] = \delta(x_\perp - x'_\perp) \sum_{l(k) \geq 1} c_k \delta^{l(k)}(x - x') A_k(x_\perp, x)$$

The quantum mechanical transverse delta function comprises the absence of transverse vacuum fluctuations and appears in all composite field commutation relation and even in semiglobal objects (i.e. global in the longitudinal sense): its ubiquitous presence is the reason for the area behavior (by integration over $x_\perp$). The longitudinal scale dimension of the fields which contribute on the right hand side determines the degree $l(k)$ of the derivative of the longitudinal delta function according the well-known rules of dimensional matching. The automorphism group of this transverse extended chiral algebra is very big since it includes in addition to transverse Euclidean transformations the infinite group of $x_\perp$-dependent chiral diffeomorphisms 22 of the circle (in particular those which fix the point infinity). In case the null-surface is the upper horizon of a Minkowski spacetime double cone (i.e. the mantle of a frustum), its

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20This terminology refers to the properties of modular forms as studied in the first half of the 20th century by mathematicians as Hecke and others.
21Algebraic CFT, in particular algebraic chiral conformal theory always posseses pointlike covariant field generators [30]. There can be no reasonable doubt that this continues to hold for the transverse extended chiral theories which result from holography.
22The (extended) chiral theories which appear in null-surface holography, unlike chiral components of two-dimensional conformal models, do not come with an energy-momentum tensor and hence their diffeomorphism invariance (beyond Moebius-invariance) need a seperate discussion.
linear extension is the mantle of the (backward) lightcone and the transverse delta function in the above commutation relation is replaced by a delta function on the unit two-sphere (with the rotation group replacing the 3-parametric Euclidean group).

Parametrizing the mantle of the frustum in terms of spherical and lightray coordinates which run from \(-\infty\) (the apex) to \(+\infty\) we see that the holographic diffeomorphism group contains (unphysical) copies of the Lorentz group and the so-called \emph{supertranslations}. In the Penrose limit of an infinitely large double cone these become identical to the generators of the classical Bondi-Metzner-Sachs group which is a semi-direct product (or cross product) of the Lorentz group with the supertranslations. We remind the reader that the BMS group is defined in a geometric way without reference to QFT. Instead of looking for Killing isometries, one studies the much more general concept of transformations which fulfill the Killing equation in an appropriate asymptotic sense. The result is that semidirect product of angular dependent lightlike translations ("supertranslations") with the Lorentz group. The existence of a relation to the holographic group is not surprising in view of the similarity of Penrose’s picture for (future) lightlike infinity.

The surprising aspect is the fact that the conceptual relation with respect to properties of the bulk matter is much deeper in the quantum case. Since holography is a change of spacetime encoding in the same Hilbert space, the action of the holographic group is well-defined on the full algebra inasmuch as the global symmetry of the full algebra (Poincaré, conformal) has a well-defined action in terms of a geometric change of the null-surface. The only subgroup of the holomorphic group which acts as a diffeomorphism on the full algebra is the Poincaré group, all other holomorphic symmetries act in a "fuzzy" way which can only be described in terms of algebraic concepts (in the case of pointlike field coordinates in terms of a transformation on testfunction spaces). Naturally those quantum symmetries have no associated Noether theorem.

6 Conclusions

In these notes we set out to illustrate the power of the intrinsic algebraic formulation of QFT in which particle physics is not described in terms of individual field-coordinatizations but rather in terms of relations between spacetime-indexed operator algebras. The case of thermal manifestations of quantum matter behind causal/event horizons is most appropriate for this purpose because this phenomenon is outside the range of Lagrangian quantization. But as was indicated in the introduction, AQFT also offers a new perspective on gauge theories and the Higgs issue. In view of the expected observational progress expected from the LHC collider it should be very interesting to elaborate these ideas.
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