Dynamics modeling and analysis of the permanent-magnet synchronous motors bearing-rotor-magnetic field under rotor demagnetize

Zhou Bifeng¹,², Tang Guoning¹ and Luo Yiping³

Abstract
For the phenomenon of rotor demagnetization of Permanent Magnet Synchronous Motor (PMSM), studying the dynamic modeling and analysis of demagnetization rotor. Firstly, considering the unbalanced magnetic pulling (UMP) of the rotor under the demagnetization and the nonlinear Hertz contact generated by the rolling bearing, the double rectangular coordinate system of the stator and rotor is constructed, and the mathematical model of the unbalanced magnetic pull UMP is constructed with the rectangular coordinate as the variable. Then, the dynamics Jeffcott model of the bearing-rotor- magnetic fields coupling system in the condition of demagnetization is established. Based on this, the demagnetization status of permanent magnet in PMSM is described from three aspects: (1) rotor offset caused by demagnetization, (2) demagnetization angle, (3) decrease of air-gap MMF at the demagnetization angle. Through the analysis, it is found that there is a “saddle” point in the bifurcation plots of the rotor system with the change of demagnetization angle. The position of the “saddle” point in the bifurcation plots with the change of demagnetization angle is related to the speed. The research results of this paper provide a theoretical basis for demagnetization quantitative diagnosis and demagnetization vibration control of permanent magnet synchronous motor.

Keywords
Permanent magnet synchronous motor, demagnetization, unbalanced magnetic pulling, modeling, bearing-rotor-magnetic field

Introduction
With the wide application of permanent magnet synchronous motor (PMSM), the research on its specific fault (demagnetization) has become a hot topic. According to the demagnetization mechanism of PMSM studied by many scholars, it can be concluded that: Permanent magnet demagnetization of PMSM is inevitable.¹–³ For the demagnetization of PMSM, the existing research results mainly focus on the effective monitoring of demagnetization of PMSM and the improvement of PMSM performance after demagnetization, the flux linkage monitoring,⁴–⁶ modeling analysis of demagnetization⁷–¹⁰ and fault-tolerant control of demagnetization¹¹–¹⁴ of PMSM are mainly studied.

¹School of Mechanical Engineering, Hunan University of Science and Technology, Xiangtan, China
²Hunan Electrical College of Technology, Xiangtan, China
³College of Electrical Information, Hunan Institute of Engineering, Xiangtan, China

Corresponding author:
Zhou Bifeng, School of Mechanical Engineering, Hunan University of Science and Technology, No. 2 Taoyuan Road, Yuhu District, Xiangtan Hunan 411201, China.
Email: zhoubifeng99@163.com

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
Considering the stator flux observation of PMSM is affected by resistance variation, Wei et al. proposed a new stator flux observer of PMSM under the γ − δ rotating coordinates system. Qiu et al. proposed a design strategy of adaptive observer to monitor the state of permanent magnet (PM) in PMSM. De Bisschop et al. and Coenen et al. established an analysis model for the demagnetization detection of axial flux PMSM. Ruoho et al. and Ruoho and Arkkio studied the demagnetization model of PMSM by finite element analysis method, and analyzed the demagnetization performance of permanent magnet after overloading with surface mounted PMSM and the demagnetization behavior of permanent magnet under fault condition of PMSM. Hu et al. presented a cascaded robust fault-tolerant predictive control (CRFTPC) strategy with integral terminal sliding mode observer (IT-SMO) to achieve high performance speeds loop and current loop for permanent magnet synchronous motor (PMSM) drives. For the permanent magnet demagnetization in the PMSM can lead to the problem of load capacity reduction. Zhang et al. proposed a variety of fault-tolerant predictive control method based on the online detection of flux linkage.

However, PMSM often exists as a key component in the actual industrial system (such as permanent-magnet synchronous wind generators). The demagnetization of PMSMs not only affects the performance of the motor itself, the vibration of the motor due to demagnetization also considerably affects the system where the motor is located. So it is very important to study demagnetization vibration of PMSM.

Existing studies on the demagnetization vibration of PMSMs is relatively rare. Xiang et al. studied the influence of UMP on nonlinear dynamic behavior of motor system based on the Jeffcott rotor models for the demagnetization of electric vehicle of the PMSM. Liu et al. considered the effects of the UMP, investigated the nonlinear oscillations of a PMSM based on a Jeffcott rotor-bearing system. Zhang et al. established the dynamic model of bias rotor-bearing system based on the consideration of gyroscopic effect, nonlinear bearing force and UMP.

Some scholars have studied the demagnetization vibration of PMSMs and achieved promising results. However, the researchers investigated the variation of UMP in the constructed UMP model, which takes eccentricity and eccentricity angle as variables under the eccentricity state. Based on knowing the eccentricity and eccentricity angle of rotor, the UMP model can be built simply and directly. But: (1) If UMP model is based on polar coordinate variables (eccentricity and eccentricity angle), and the rotor dynamics model on the basis of rectangular coordinate variables. When there are variables that affect each other, the establishment of the two models will not be perfectly connected.

(2) In condition of the influence of demagnetization states of rotor permanent magnet which studied in a quantitative way on rotor dynamic characteristics, the eccentricity angle of rotor is a time-varying function related with rotational speed, which makes modeling more difficult.

Motivated by the above analysis, this study focuses on the dynamic modeling and analysis of the demagnetizing rotor of PMSMs. Firstly, considering the unbalanced magnetic pull (UMP) of the rotor under the demagnetization and the nonlinear Hertz contact generated by the rolling bearing, the double rectangular coordinate system of the stator and rotor is constructed, and the mathematical model of the unbalanced magnetic pull UMP is constructed with the rectangular coordinate as the variable, then, the dynamics Jeffcott model of the bearing-rotor- magnetic field coupling system in the condition of demagnetization rotor of the PMSM is established. Based on this, the demagnetization status of permanent magnet in PMSM is described from three aspects: demagnetization angle, demagnetization amount of the demagnetization angle and rotor offset caused by demagnetization.

In section 2, the UMP of the rotor and the nonlinear Hertz contact force of the rolling bearing under the demagnetization is studied, and the nonlinear dynamic model of rotor-bearing-magnetic field under demagnetization is construed. In section 3, based on the model, the operation characteristics of PMSM rotor under the quantitative change of permanent magnet demagnetization state are analyzed. Finally, section 4 gives relevant conclusions. This study provides a theoretical basis for the accurate demagnetization diagnosis and vibration control of PMSMs in the future.

Dynamic modeling

**Dynamic model of ball bearing-rotor-magnetic field coupling system**

Figure 1 shows the simplified schematic diagram of rotor section of PMSM. The rotor system consists of motor bearing, shaft, stator, rotor and air gap between stator and rotor.

The demagnetization of the permanent magnet in the rotor of the permanent magnet synchronous motor (PMSM) results in an uneven air-gap flux density in the motor, resulting in unbalanced magnetic pull (UMP) acting on the rotor, and considering the influence of rotor gravity. On this basis, the research on rotor system dynamics of PMSM can be transformed into the research on Jeffcott rotor system as shown in Figure 2 to simplify the research.

For Jeffcott rotor system, considering the gyroscopic effect caused by the offset of disc under UMP and the nonlinear force of rolling bearing, and the quality of
shaft is ignored. As shown in Figure 2, the shaft length is \( l \), the distance between the offset disc and the left bearing \( A \) is \( a \), the lumped mass of discs, polar moment of inertia and diameter moment of inertia are respectively as \( m_d, J_p, J_d \), and lumped mass of the rolling bearings \( A \) and \( B \) are respectively as \( m_A, m_B \).

If the permanent magnet of PMSM rotor is in the state of no demagnetization or axial uniform demagnetization, When the actual system is simplified to Jeffcott rotor system, the distance between the offset disk and the left bearing \( A \) is \( a = 1/2 \). If the permanent magnet of PMSM rotor is non-uniform demagnetization, the UMP produced by demagnetization is not acting on the center position of shaft, at this time, the actual rotor will be simplified as a Jeffcott rotor system in the disk offset state. That is to say, in the Jeffcott rotor system, the distance between the offset disc and the left bearing is related to the position and magnitude of the resultant force of UMP produced by demagnetization.

As shown in Figure 2, the coordinate system \( Ox'y'z' \) is established with the static equilibrium point of the support of the left rotor bearing as the origin. Let \( Ox \) as the rotor axis direction, \( Oy \) as the vertical direction and \( Ox \) as the horizontal direction. At any moment of the rotor motion, the center coordinates of the two bearings are \( (x_A, y_A) \) and \( (x_B, y_B) \) respectively, and the center coordinates of the disc are \( O(x, y) \). With the effect of UMP, the angle between the disc axis and the connecting lines of the bearings \( A \) and \( B \) is \( \psi \), the angles of the disc center around \( Ox, Oy \) axis are \( \theta_x \) and \( \theta_y \), and the rotation angular velocity of the disc is \( \omega \).

When the axial displacement and torsional deformation of the rotor are ignored, the generalized coordinates are taken as follows:

\[
q = (q_1, q_2, q_3, q_4) = [(x, \theta_y), [x_A, x_B], [y, \theta_x], [y_A, y_B]]
\] (1)

The nonlinear damped vortex-swing coupling dynamic equation of rolling bearing-rotor-magnetic field system is obtained by Lagrange equation as follows:

\[
M \ddot{q}^T + [Jq + C]q^T + Kq = F_M + F_s + G
\] (2)

Where \( M \) is the mass matrix, \( M = \text{diag}(M_A, M_B) \), \( M_a = \text{diag}(m_A, J_A, m_A, J_A) \).

\( J \) is the gyroscopic matrix, \( J = \begin{bmatrix} 0 & -J_1 \\ J_1 & 0 \end{bmatrix} \), \( J_1 = \text{diag}(0, J_p, 0, 0) \).

\( C \) is the damping matrix, \( C = \text{diag}(C_a, C_a) \), \( C_a \) is a partitioned matrix, satisfies: \( C_a = \begin{bmatrix} C_1 & C_2 \\ C_2^T & C_3 \end{bmatrix} \),

\[
C_1 = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \\
C_2 = \begin{bmatrix} -E & -E \\ E & -E \end{bmatrix},
\]

\[
C_3 = \begin{bmatrix} \frac{1}{l} (1 - \frac{a}{l})^2 c + \frac{c}{l^2} & \frac{1}{l} \left(1 - \frac{a}{l}\right) c + \frac{c}{l^2} \\ \frac{1}{l} \left(1 - \frac{a}{l}\right) c + \frac{c}{l^2} & \frac{1}{l} \left(1 - \frac{a}{l}\right) c + \frac{c}{l^2} \end{bmatrix}.
\]

\( K = \text{diag}(K_i, K_i) \) is the stiffness matrix.

\[
K_i = \begin{bmatrix} K_i & -K_i \Theta \\ -\Theta^T K_i & \Theta^T K_i \Theta \end{bmatrix}
\] is the stiffness matrix considering the rotor offset. Where \( \Theta \) is the coupling matrix of disc displacement and swing angle, it can be obtained from the geometric relationship that \( \Theta = \frac{1}{l} \begin{bmatrix} 1 - a & a \\ -1 & 1 \end{bmatrix} \), \( K_i \) is the stiffness matrix of the elastic shaft under rigid support regardless of the support deformation. The flexibility matrix of the shaft can be obtained by using the structure influence coefficient method as follows:

\[
\chi = \frac{1}{3EI} \begin{bmatrix} a^2(l - a)^2 & a(l - a)(l - 2a) \\ a(l - a)(l - 2a) & l^2 - 3la + 3a^2 \end{bmatrix}
\] (3)

Where, \( E \) is the elastic modulus of shaft, \( I \) is the moment of inertia of cross-section of shaft, then \( K_i = \chi^{-1} \).

\( F_M \) is the UMP of the rotor, \( F_s \) is nonlinear Hertz force of bearing and \( G \) is gravity of rotor.
Electromagnetic force of rotor under demagnetization (UMP)

In this section, the UMP $F_M$ of the rotor in the bearing-rotor-magnetic field coupling dynamic model established in the previous section will be analyzed. In view of the deficiency of traditional UMP model with rotor eccentricity and eccentricity angle as variables, the dual coordinate system of stator and rotor will be constructed, and the UMP model will be constructed with $x,y$ as the variable in this section.

The rotor system of PMSM is simplified as a Jeffcott rotor system, and the demagnetization state of PMSM is described equivalently with three parameters according to the vector sum of UMP caused by demagnetization of PMSM: (1) rotor offset $a$ caused by demagnetization, (2) demagnetization angle $2\beta$, (3) decrease of air-gap MMF at the demagnetization angle (air-gap MMF drop percentage) $\zeta$, as shown in Figure 3.

As shown in Figure 3, the rotating rectangular coordinate system $O'x'y'$ is established with the center of Jeffcott rotor disk as the origin. The Euler angles $\theta_1$, $\theta_2$ of the disc are ignored, and $O'x'y'$ is parallel to $Ox'y'$. Suppose that in the initial state, rotation coordinate system $Ox'y'$ is parallel to each corresponding coordinate axis in the coordinate system $Ox'y'$. $O''$ is the stator center corresponding to the Jeffcott rotor disc. In the three-dimensional coordinate system $Ox'yz$, $O''$ is a point in the $Oz$ axis, the coordinate axis $Ox$ and $Oy$ in the coordinate system $Ox'y'$ are translated to $O''$ along the coordinate axis $Oz$, and the stator coordinate system $O''x'y''$ is formed. Therefore, the values of $O''x$ and $O''y$ in coordinate system $O''x'y''$ are the same as $Ox$ and $Oy$ in coordinate system $Ox'y'$.

Let $\theta(t)$ be the angle between the connecting line of stator and rotor center $O'$, $O'$ and axis $x$, and when the rotor is running, it satisfies:

$$\theta(t) = \omega t + \theta_0$$  \hspace{1cm} (4)

Where $\theta_0$ is the angle between the resultant force of the rotor and the axis $x$ in the rotating coordinate system $O'x'y'$, $\omega$ is rotational angular velocity, $\beta$ is half of the demagnetization angle, $\psi$ is the space angle, $\sigma_0$ is the average air gap length; and $\epsilon(t)$ is the eccentricity scalar.

Let $\xi(t) = \frac{\partial \sigma}{\partial c}$ be the rotor eccentricity ratio. In the rotating coordinate system $x'O'y'$, the rotor eccentricity caused by the demagnetization of the permanent magnet is considered, and the air gap distribution $\delta(\psi,t)$ can be approximately expressed as follows:

$$\sigma(\psi,t) = \sigma_0 - \epsilon(t) \cos(\psi - \theta_0)$$
$$= \sigma_0 [1 - \xi(t) \cos(\psi - \theta_0)]$$
$$= \sigma_0 - \epsilon(t) \cos(\psi) - \epsilon'(t) \sin(\psi)$$  \hspace{1cm} (5)

Where $x'(t) = \epsilon(t) \cos(\theta_0)$ and $y'(t) = \epsilon(t) \sin(\theta_0)$ are the projection displacements of the eccentric vector in the $x'O'y'$ coordinate system. The permeability of the air gap is as follows:

$$\Lambda(\psi,t) = \frac{\mu_0}{\sigma(\psi,t)}$$
$$= \frac{\mu_0}{\sigma_0} (1 + \xi(t) \cos(\psi - \theta_0))$$
$$= \frac{\mu_0}{\sigma_0} (\sigma_0 + x'(t) \cos(\psi) + y'(t) \sin(\psi))$$  \hspace{1cm} (6)

Where $\mu_0$ is the permeability of vacuum.

Considering the demagnetization of the permanent magnets of PMSM, the fundamental wave of the air-gap MMF established by the current in the torque winding and the permanent magnet field in the rotor of PMSM is as follows:

$$F(\psi,t) = F_0 \cos(\omega t - p\psi - u)$$  \hspace{1cm} (7)

Where $p$ is the number of pole pairs of the torque winding, $F_0$ is the amplitude of the fundamental wave of the air-gap MMF, $u$ is the spatial initial phase angle of the fundamental wave of the air-gap MMF, and $w$ is the angular frequency of the winding current. The rotation angular velocity $\omega$ of the rotor and the angular frequency $w$ of the winding current satisfy $\omega = \frac{602\pi w}{p}$.

In the rotating coordinate system $x'O'y'$, and in consideration of the axial direction uniform demagnetization of the rotor of a PMSM, the change of the amplitude of the air-gap MMF caused by the demagnetization of the permanent magnet is equivalent mapped to demagnetization angle $\beta$ on both sides of the
coordinate \(-x'\) axis, and the decrease of air-gap MMF drops to \((1 - \zeta)^{\Phi}\) (at the demagnetization angle). At this time, the amplitude of the fundamental wave of the air-gap MMF \(F_\delta\) is:

\[
F_\delta = \left\{ \begin{array}{l}
\hat{F} \quad \psi \in (-\pi + \beta, \pi - \beta) \\
(1 - \zeta)\hat{F} \quad \psi \in (-\pi, -\pi + \beta) \cup (\pi - \beta, \pi)
\end{array} \right.
\]  

(8)

Where \(\beta \in (0, \pi)\), and \(\zeta = 1\) is the decrease of air-gap MMF at the demagnetization angle (air-gap MMF drop percentage).

Assuming that the permanent magnet is in normal condition and the air gap is uniform, the amplitude of flux density of air-gap for pair pole \(p\) is:

\[
\hat{B} = \frac{\mu_0}{\delta_0} \hat{F} = \frac{\mu_0}{\delta_0} \left( \frac{3}{4} N J \right) \left( \frac{3 \pi}{2 \pi 2p} \right)
\]  

(9)

Where \(N\) is the number of turns of each phase of the torque winding; and \(I\) is the amplitude of the excitation current of the torque winding.

Combine with equations (6)-(9), the Maxwell force in the \(x'\), \(y'\) direction can be obtained under the rotor eccentricity caused by the demagnetization of the permanent magnet of the PMSM:

\[
F_{Mx}(t) = \int_{0}^{2\pi} \frac{B_x^{2}(\psi,t)I_{o}}{2\mu_0} \cos \psi d\psi = \frac{I_{o} \mu_0}{2\delta_0} \int_{0}^{2\pi} (F_{o})^{2}(\sigma_0 + x'(t) \cos (\psi) + y'(t) \sin (\psi))^2 \cos \psi d\psi
\]  

(10)

\[
F_{My}(t) = \int_{0}^{2\pi} \frac{B_y^{2}(\psi,t)I_{o}}{2\mu_0} \sin \psi d\psi = \frac{I_{o} \mu_0}{2\delta_0} \int_{0}^{2\pi} (F_{o})^{2}(\sigma_0 + x'(t) \cos (\psi) + y'(t) \sin (\psi))^2 \sin \psi d\psi
\]  

(11)

Then, under the stator coordinate system \(O'x'y'\), the UMP in the axes direction of \(x'y'\) satisfies:

\[
F_{Mx}(t) = F_{My}(t) \cos (\omega t) - F_{My}(t) \sin (\omega t)
\]  

(12)

\[
F_{My}(t) = F_{Mx}(t) \sin (\omega t) + F_{My}(t) \cos (\omega t)
\]  

(13)

Through the integral solution of equations (10) and (11), and combined with the operation of equations (12) and (13), the electromagnetic force in \(x\) and \(y\) direction of the Jeffcott rotor system can be obtained, where \(F_{Mx}\) is the electromagnetic force in \(x\) direction, and \(F_{My}\) is the electromagnetic force in \(y\) direction. Namely

\[
F_M = [F_{Mx} \ 0 \ 0 \ F_{My} \ 0 \ 0 \ 0]^T
\]  

(14)

**Nonlinear Hertzian force model of rolling bearings**

In this section, the nonlinear Hertz force \(F_s\) which in the rolling bearing-rotor-magnetic field coupling dynamic model (2) will be analyzed.

The dynamic model of rolling bearing is shown in Figure 4, it is assumed that the outer ring and motor base are rigidly supported, regardless of the elastic deformation of motor base, the inner ring is fixed on the shaft rigidly, the rolling balls are arranged equidistant and pure rolling, the outer raceway \(r_o\), and the number of rolling balls is \(N_b\), because of the motion of the bearing and rotor system, if the angular velocity of the bearing inner ring is \(\omega\), then the angular velocity of the cages is \(\omega_b = \frac{\omega R_r}{R_o + R_r}\), and the position angle of the \(j-th\) ball at any time \(t\) is:

\[
\varphi_j = \frac{\omega R_r}{R_o + R_r} t + \frac{2\pi}{N_b} (j-1) + \varphi_0
\]  

(15)

Where \(j = 1,2,\ldots,N_b, \varphi_0\) is the initial position angle of the rolling balls.

According to the nonlinear Hertz theory of rolling bearing, the nonlinear Hertz force of bearing \(A\) is expressed as

\[
\left\{ \begin{array}{l}
F_{xs} = \sum_{j=1}^{N_b} k_b Y_j^2 A_j \cos \varphi_j \\
F_{sy} = \sum_{j=1}^{N_b} k_b Y_j^2 A_j \sin \varphi_j
\end{array} \right.
\]  

(16)

Where \(Y_j\) is the contact deformation of the rolling ball, satisfied: \(Y_j = x_j \cos \varphi_j + y_j \sin \varphi_j - y_0\). \(A_j\) is the Heavisiey function, satisfying:

\[
A_j = \left\{ \begin{array}{l}
0 \quad Y_j \leq 0 \\
1 \quad Y_j > 0
\end{array} \right.
\]
Similarly, the nonlinear Hertz contact force of bearing B can be obtained:

\[
F_s = \begin{bmatrix} 0 & 0 & F_{sA_x} & F_{sB_x} & 0 & 0 & F_{sA_y} & F_{sB_y} \end{bmatrix}^T
\]  

(17)

Thus, combined with equations (2), (14) and (17), the dynamic model of bearing-rotor-magnetic field coupling system under demagnetization is established.

Vibration characteristics analysis of bearing-rotor-magnetic field system

Parameter selection: \( J_p = 0.03481 kg \cdot m^2 \), \( J_d = 0.017405 kg \cdot m^2 \), \( c = 3.502 \times 10^3 N \cdot s/m \), \( c_a = 2.136 \times 10^3 N \cdot s/m \). Elastic modulus of shaft materia \( E = 2.09 \times 10^5 MPa \). Table 1 is the main parameters of PMSM, Table 2 is the main parameters of ball bearing of PMSM.

Table 1. The main parameters of PMSM.

| Variable                  | Unit | Value |
|---------------------------|------|-------|
| Average air gap length \( r_0 \) | mm   | 1     |
| Outer radius of the rotor \( r \) | mm   | 59    |
| Shaft diameter of the rotor \( d \) | mm   | 35    |
| Effective Core Length \( l_0 \) | mm   | 170   |
| Shaft length \( l \) | mm   | 410   |
| Number of pole pairs \( p \) | Pair | 2     |
| Phase number of motor \( M_p \) | Phase | 3    |
| Coil turn \( N \) | Turn | 11    |
| Rotor quality \( m_z \) | kg   | 20    |
| Stator current \( I_s \) | A    | 110   |
| Shaft material |              | 40Cr  |

Table 2. Main parameters of ball bearing of PMSM.

| Variable                  | Unit | Value |
|---------------------------|------|-------|
| Type |                      | JIS6306 |
| Quality of bearing \( m_A, m_B \) | kg   | 1     |
| Inner radius of outer ring \( R_o \) | mm   | 36    |
| Outer radius of inner ring \( R_r \) | mm   | 15    |
| Number of rolling balls \( N_b \) | 9    |
| Contact stiffness \( k_b \) | \( N/m^{3/2} \) | 13.34 + 10^9 |
| Bearing clearance \( \gamma_0 \) | \( \mu m \) | 10    |

Similarly, the nonlinear Hertz contact force of bearing B can be obtained:

\[
F_s = \begin{bmatrix} 0 & 0 & F_{sA_x} & F_{sB_x} & 0 & 0 & F_{sA_y} & F_{sB_y} \end{bmatrix}^T
\]  

(17)

Thus, combined with equations (2), (14) and (17), the dynamic model of bearing-rotor-magnetic field coupling system under demagnetization is established.

Vibration characteristics analysis of bearing-rotor-magnetic field system

Parameter selection: \( J_p = 0.03481 kg \cdot m^2 \), \( J_d = 0.017405 kg \cdot m^2 \), \( c = 3.502 \times 10^3 N \cdot s/m \), \( c_a = 2.136 \times 10^3 N \cdot s/m \). Elastic modulus of shaft materia \( E = 2.09 \times 10^5 MPa \). Table 1 is the main parameters of PMSM, Table 2 is the main parameters of ball bearing of PMSM.

In this paper, the influence of UMP caused by demagnetization on the dynamic characteristics of the rotor is mainly considered. The demagnetization of rotor permanent magnet is described in terms of rotor offset caused by demagnetization, demagnetization angle, decrease of air-gap MMF at the demagnetization angle. so this paper main research: (1) analysis of rotor vibration characteristics under different demagnetization angle; (2) analysis of rotor vibration characteristics under different decrease of air-gap MMF; (3) analysis of rotor vibration characteristics under different rotor offset.

Analysis of rotor vibration characteristics under different demagnetization angle

Assuming that the initial state is the ideal working state of the motor, ignoring the rotor eccentricity caused by motor assembly, that is, in the initial state of the PMSM, the rotor disc center coordinates \( O(x, y) \), the two support bearing center coordinates \( (x_A, y_A) \) and \( (x_B, y_B) \), and the rotation angle \( \theta_x \) and \( \theta_y \) of the disc center around \( x, y \) axis are zero.

The dynamic system is solved by using the ode45 solver in matlab. Considering that the system was affected by UMP, so the system is a time-varying differential equation system. In this paper, taking the rotor rotation period \( T = 300 \) as the step size, the corresponding ordinary differential equation is obtained, and then the periodic solution of the time-varying differential equation system is completed. In this paper, a total of 300 cycles are calculated, the unsteady data are discarded, and the steady-state solutions of the last 200 cycles are taken to discuss the dynamic characteristics of the rotor at speed \( \omega = 12.56 \) – 1256rad/s.

Figure 5 shows the bifurcation plots of rotor system under demagnetization angle and speed change. In the demagnetization state of PMSM, set the rotor offset (the distance between the rigid disk and the left bearing \( A \)) is \( a = l/2 \); the decrease of air-gap MMF \( \zeta = 0.3 \), and the demagnetization angle \( 2\beta \), and \( \beta \in \left[ 0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi \right] \).

According to Figure 5, it can be found that under different rotating speeds and different demagnetization angles, the PMSM runs in a complex operation state.
the PMSM runs in a complex operation state of periodic, quasi-periodic and chaotic alternations. At the same time, in Figure 5, under different demagnetization angles, the critical speed of the rotor system decreases with the increase of demagnetization angle firstly, when the demagnetization angle reaches $\frac{\pi}{2}$, the critical speed of the system decreases to the minimum value, and then increases with the increase of demagnetization angle. In the range of demagnetization angle $[0, \frac{\pi}{2}]$, the vibration characteristics of the rotor are positively correlated with the demagnetization angle. The larger of the demagnetization angle, the more obvious of the vibration characteristics; In the range of demagnetization angle $[\frac{\pi}{2}, \pi]$, the vibration characteristics of the rotor are negatively correlated with the demagnetization angle. With the increase of the demagnetization angle, the vibration characteristics weaken. Taking the demagnetization angle $\beta = \frac{\pi}{2}$ as the center, the vibration characteristics of the two demagnetization regions are similar.

In order to get a clearer conclusion, set the demagnetization interval $[0, \frac{\pi}{2}]$, rotor offset $a = 1/2$, the decrease of air-gap MMF $\zeta = 0.3$, and set the speed of PMSM are $\omega = 157.0759\text{rad/s}$ and $\omega = 471.2385\text{rad/s}$ respectively, analyzed rotor system bifurcation diagram with demagnetization angle as variable, as shown in Figure 6.

Figure 6(a) and (b) show the bifurcation plots of rotor system under different demagnetization angle. In Figure 6, the vibration characteristics of the rotor increase significantly with the increase of demagnetization amplitude angle firstly, then decrease, and increase sharply after passing the “saddle” point. That is, in the bifurcation plots of rotor system under different speed with demagnetization angle change, when the demagnetization angle is near a certain value, the vibration characteristics of the rotor system are obviously different from other regions, and the running track is obvious. In the same radial direction of the rotor axis trajectory, the difference between the outer diameter and the inner diameter is the smallest, and the rotor system runs periodically.

In Figure 6(a), there is a “saddle” point $\beta = \frac{11\pi}{40}$ in demagnetization interval $[0, \frac{\pi}{2}]$ at $\omega = 157.0759\text{rad/s}$, when $\beta = 0$, the rotor system operates in a single cycle; and if demagnetization occurs, $\beta \neq 0$, the rotor system runs periodically.

In Figure 6(a), there is a “saddle” point $\beta = \frac{11\pi}{40}$ in demagnetization interval $[0, \frac{\pi}{2}]$ at $\omega = 157.0759\text{rad/s}$, when $\beta = 0$, the rotor system operates in a single cycle; and if demagnetization occurs, $\beta \neq 0$, the rotor system runs periodically.

In Figure 6(a), there is a “saddle” point $\beta = \frac{11\pi}{40}$ in demagnetization interval $[0, \frac{\pi}{2}]$ at $\omega = 157.0759\text{rad/s}$, when $\beta = 0$, the rotor system operates in a single cycle; and if demagnetization occurs, $\beta \neq 0$, the rotor system runs periodically.
presents a bifurcation with a period of 11, and then with the demagnetization angle increases to the “saddle” point, the rotor system becomes a single cycle operation. When the demagnetization angle exceeds the “saddle” point, the rotor system presents a bifurcation with a period of 11 again. Meanwhile, in Figure 6(b), the “saddle” point $\beta = \frac{13\pi}{40}$ exists in the demagnetization interval $[0, \frac{\pi}{2}]$ at $\omega = 471.2385$rad/s. So compared with Figure 6(a) and (b), The position of the “saddle” point in the bifurcation plots of the rotor system with the change of demagnetization angle is related to the speed.

**Analysis of rotor vibration characteristics under different decrease of air-gap MMF**

From the above section, We can get that there is a “saddle points” in the bifurcation plots of rotor system under different speed with demagnetization angle change. Therefore, in order to analyze the vibration characteristics of rotor with different decrease of air-gap MMF at the demagnetization angle more clearly, the bifurcation diagram of the rotor system under different decrease of air-gap MMF and the change of speed was analyzed when the demagnetization angles $\beta = \frac{\pi}{15}$ and $\beta = \frac{11\pi}{40}$ were respectively taken. At this time, set the rotor offset (the distance between the rigid disk and the left bearing A) is $a = 1/2$; the decrease of air-gap MMF $\zeta \in [0, 0.2, 0.4, 0.6, 0.8]$.

According to Figure 7, it can be found that under different rotating speeds and different decrease of air-gap MMF at the demagnetization angles, the PMSM runs in a complex operation state. In Figure 7, the critical speed of the rotor system decreases with the increase of decrease of air-gap MMF, and the vibration characteristics of the rotor system increases obviously with the increase of decrease of air-gap MMF. Compared with Figure 7(a) and (b), the vibration characteristics of the rotor system are obviously different when the demagnetization angle of the rotor is at the “saddle point” position and far away from the “saddle” point.

In order to get a clearer conclusion, set the rotor offset is $a = 1/2$, demagnetization angles are $\beta = \frac{\pi}{15}$ and $\beta = \frac{11\pi}{40}$, and set the speed of PMSM are $\omega = 157.0759$rad/s and $\omega = 471.2385$rad/s respectively, analyzed rotor system bifurcation diagram with decrease of air-gap MMF as variable, as shown in Figure 8.

Figure 8(b) shows the bifurcation diagram of the rotor system under different decrease of air-gap MMF at the demagnetization angles when the speed $\omega = 157.0759$rad/s and the corresponding “saddle” point position is taken as the demagnetization angle. Corresponding to Figure 8(b), Figure 8(a) shows the bifurcation diagram of the rotor system under different decrease of air-gap MMF at “non-saddle point” positions. Comparing Figure 8(a) and (b), it can be found that when the demagnetization angle is near the “saddle” point, the decrease of air-gap MMF is not sensitive to the nonlinear dynamic characteristics of the rotor. As shown in Figure 8(b), the rotor system operates in a single cycle when the decrease of air-gap MMF increases. When the demagnetization angle is far from the “saddle” point in Figure 8(a), the decrease of air-gap MMF is sensitive to the nonlinear dynamic characteristics of the rotor. The system runs in multicycle, and vibration characteristics is obvious. Figure 8(c) shows the bifurcation plots of rotor system under different decrease of air-gap MMF when the speed is $\omega = 471.2385$rad/s and the demagnetization angle is
Compared with Figure 8(b), the current revolution speed and the demagnetization angle do not constitute a “saddle point” due to the change of speed. Therefore, the vibration characteristics of the rotor system are more obvious. However, the parameters in Figure 8(c) are closer to the “saddle” point than the parameters in Figure 8(a). Therefore, the vibration characteristics of the rotor in Figure 8(a) are more obvious when the decrease of air-gap MMF changes.

**Analysis of rotor vibration characteristics under different rotor offset**

Figure 9 shows the bifurcation plots of rotor system under different decrease of air-gap MMF at the demagnetization angles: (a) $\omega = 157.0759\text{rad/s}$, demagnetization angle $\beta = \frac{11\pi}{40}$; (b) $\omega = 157.0759\text{rad/s}$, demagnetization angle $\beta = \frac{11\pi}{40}$, and (c) $\omega = 471.2385\text{rad/s}$, demagnetization angle $\beta = \frac{11\pi}{40}$.

$\beta = \frac{11\pi}{40}$. Compared with Figure 8(b), the current revolution speed and the demagnetization angle do not constitute a “saddle point” due to the change of speed. Therefore, the vibration characteristics of the rotor system are more obvious. However, the parameters in Figure 8(c) are closer to the “saddle” point than the parameters in Figure 8(a). Therefore, the vibration characteristics of the rotor in Figure 8(a) are more obvious when the decrease of air-gap MMF changes.

Figure 10 shows the bifurcation plots of rotor system under rotor offset and speed change. In the demagnetization state of PMSM, set the decrease of air-gap MMF $\zeta = 1$, and the demagnetization angle $2\beta$, and $\beta = \pi/2$.

It can be seen from Figures 5, 7, 9, and 10, the displacement response value of the rotor increases suddenly near a certain speed. At this time, the speed corresponds to the critical speed of the rotor system, and the rotor amplitude corresponds to the resonance peak; Next, the dynamic characteristics of the rotor system will be analyzed with bifurcation diagram.
Figure 5 shows that, when the demagnetization angle is $\beta = 0$ (no demagnetization), and $\beta = \pi$ (uniform demagnetization in the circumferential direction), at this time, considering the UMP caused by demagnetization is zero. Therefore, the amplitude of the motor is small. If the rotor offset and the decrease of air-gap MMF is fixed, at the position $\beta = \frac{\pi}{2}$, the nonlinear dynamic characteristics of the rotor system are obviously stronger than other demagnetization angles, at this time, the critical speed of the system is the smallest and the resonance peak value is the largest.

Figure 7 shows that, If the rotor offset and the demagnetization angle is fixed, with the increase of the decrease of air-gap MMF at the demagnetization angle (air-gap MMF drop percentage), the nonlinear dynamic characteristics of the rotor system are obviously enhanced, the corresponding critical speed of the system decreases, and the resonance peak value increases.

Compare the bifurcation diagram($\beta = \pi/2$) in Figure 5(a = 1/2) with Figure 10, and compare the bifurcation diagram($\zeta = 1$) in Figure 7(a) ($a = 1/2$) with Figure 9. It can be found that the nonlinear characteristics of the rotor increases with the increase of the rotor offset ($a$ decreases), and the corresponding critical speed of the rotor system increases. As $a$ decreases, according to formula (3), the stiffness of the system increases, so the resonance peak value decreases.

**Conclusion**

For the phenomenon of rotor demagnetization of PMSM, considering the unbalanced magnetic pulling(UMP) of the rotor under the demagnetization and the nonlinear Hertz contact generated by the rolling bearing, the dynamics Jeffcott model of the bearing-rotor-magnetic field coupling system in the condition of demagnetization is established. Based on this, the demagnetization status of permanent magnet in PMSM is described from three aspects: (1) rotor offset caused by demagnetization, (2) demagnetization angle, (3) decrease of air-gap MMF at the demagnetization angle. And the influence of the dynamic characteristics is analyzed through above three aspects. The research results of this paper provide a theoretical basis for demagnetization fault quantitative diagnosis and demagnetization vibration control of permanent magnet synchronous motor.

For the dynamics Jeffcott model of the bearing-rotor-magnetic field coupling system in the condition of demagnetization, the influence of rotor permanent magnet demagnetization on rotor system operation was analyzed, and the conclusions are as follows:

- If the rotor offset and the decrease of air-gap MMF is fixed, there is a “saddle” point in the bifurcation plots of rotor system under different demagnetization angle.
- The position of the “saddle” point in the bifurcation plots of the rotor system with the change of demagnetization angle is related to the speed.
- If the rotor offset and the decrease of air-gap MMF is fixed, at the position $\beta = \frac{\pi}{2}$, the nonlinear dynamic characteristics of the rotor system are obviously stronger than other demagnetization angles, at this time, the critical
speed of the system is the smallest and the resonance peak value is the largest.

- If the rotor offset and the demagnetization angle is fixed, with the increase of the decrease of air-gap MMF at the demagnetization angle (air-gap MMF drop percentage), the nonlinear dynamic characteristics of the rotor system are obviously enhanced, the corresponding critical speed of the system decreases, and the resonance peak value increases.

- If the demagnetization angle and the decrease of air-gap MMF is fixed, the nonlinear characteristics of the rotor increases with the increase of the rotor offset \(a\) decreases, and the corresponding critical speed of the rotor system increases. The stiffness of the system increases, and the resonance peak value decreases.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work supported by National Natural Science Foundation of China (11972156).

**ORCID iDs**

Zhou Bifeng [https://orcid.org/0000-0003-2245-6722]

Tang Guoning [https://orcid.org/0000-0003-2708-0496]

Luo Yiping [https://orcid.org/0000-0002-2963-7270]

**References**

1. Li X and Wang S. Demagnetization analysis of the interior permanent magnet synchronous motor under different short circuit faults. *J China Coal Soc* 2017; 42: 626–632.

2. Tang X, Wang X, Li Y, et al. Demagnetization study for line-start permanent magnet synchronous motor during starting process. *Proc CSEE* 2015; 35: 961–970.

3. Choi G and Jahns TM. Investigation of key factors influencing the response of permanent magnet synchronous machines to three-phase symmetrical short-circuit faults. *IEEE Trans Energy Convers* 2016; 31: 1488–1497.

4. Wei H-F, Wei H-P, Zhang Y, et al. New stator flux observer of permanent magnet synchronous motor considering stator resistance perturbation under virtual – rotating coordinate system. *Control Decis* 2017; 32: 2301–2304.

5. Qiu T, Wen X, Zhao F, et al. Design strategy of permanent magnet flux linkage adaptive observer for permanent magnet synchronous motor. *Proc CSEE* 2015; 35: 2287–2294.

6. Liu K, Feng J, Guo S, et al. Identification of flux linkage map of permanent magnet synchronous machines under uncertain circuit resistance and inverter nonlinearity. *IEEE Trans Industr Inform* 2018; 14: 556–568.

7. De Bisschop J, Abdallh A, Sergeant P, et al. Identification of demagnetization faults in axial flux permanent magnet synchronous machines using an inverse problem coupled with an analytical model. *IEEE Trans Magn* 2014; 50: 1–4.

8. Coenen I, Van der Giet M and Hameyer K. Manufacturing tolerances: estimation and prediction of cogging torque influenced by magnetization faults. *IEEE Trans Magn* 2012; 48: 1932–1936.

9. Ruoho S, Dlala E and Arkkio A. Comparison of demagnetization models for finite-element analysis of permanent-magnet synchronous machines. *IEEE Trans Magn* 2007; 43: 3964–3968.
10. Ruoho S and Arkkio A. Partial demagnetization of permanent magnets in electrical machines caused by an inclined field. *IEEE Trans Magn* 2008; 44: 1773–1778.

11. Hu F, Luo D, Luo C, et al. Cascaded robust fault-tolerant predictive control for PMSM drives. *Energies* 2018; 11: 3087.

12. Zhang C, Wu G, Rong F, et al. Robust fault-tolerant predictive current control for permanent magnet synchronous motors considering demagnetization fault. *IEEE Trans Ind Electron* 2018; 65: 5324–5334.

13. Zhang C, Wu G-P, He J, et al. Sliding observer-based demagnetisation fault-tolerant control in permanent magnet synchronous motors. *J Eng* 2017; 2017: 175–183.

14. Zhang C, Wu G-P, He J, et al. Fault-tolerant predictive control for demagnetization faults in permanent magnet synchronous machine. *Trans China Electro Tech Soc* 2017; 32: 100–110.

15. Xiang C, Liu F, Liu H, et al. Nonlinear dynamic behaviors of permanent magnet synchronous motors in electric vehicles caused by unbalanced magnetic pull. *J Sound Vib* 2016; 371: 277–294.

16. Liu H, Wu Y, Wang X, et al. Nonlinear normal modes and primary resonance for permanent magnet synchronous motors with a nonlinear restoring force and an unbalanced magnetic pull. *Nonlinear Dyn* 2019; 97(2): 1197–1213.

17. Zhang A, Bai Y, Yang B, et al. Analysis of nonlinear vibration in permanent magnet synchronous motors under unbalanced magnetic pull. *Appl Sci* 2018; 8: 113.

18. Qiu JJ. *Nonlinear vibration of electromechanical coupled dynamic system*. Beijing: Science Press, 1996.