Deflection of beams with sinusoidal perforation

A I Pritykin

Kaliningrad State Technical University, 1 Sovetskij ave., Kaliningrad, 236040, Russia
Immanuel Kant Baltic Federal University, 14 A. Nevskogo str., Kaliningrad, 236016, Russia

E-mail: prit_alex@mail.ru

Abstract. The presence of cutouts in the beams’ walls made of steel or from modern composite materials reduces their shear stiffness and increases deflections in comparison with beams of the same size with a solid wall. In this paper, we study the deflections of perforated beams with a sinusoidal shape of cutouts, which are widely used in construction recently. The cutouts’ effect on the perforated beams’ transverse bending was investigated analytically using the theory of composite bars and the finite element method. The aim of this work was to obtain an analytical dependence that allows to reliably evaluate the deformations of the beams with hexagonal cutouts with different angles of rounding. The study was conducted for the beams of different sizes, made by the non-waste technology from a rolling profile. An analysis of the calculating results for the beams made of different materials showed that with a decrease in the material elasticity modulus, the deflections of the beams increase, and a change in the yield strength of the material does not affect the deformation characteristics. The obtained dependence is applicable for the beams with a relative length $l_l/l_H \geq 10$ at a relative cutouts’ height $h/H = 0.667$. The discrepancies with FEM results do not exceed 1.5-3%.

Introduction

One of the main requirements for the beams made of composite materials, aluminum alloys or steels is to provide the necessary rigidity, i.e. the relative deflection limitation $f/l$. The presence of cutouts in the walls of perforated beams leads to a decrease in their shear stiffness and, as a result, to an increase in the deflections of the latter. For the pivotally supported beams, their growth can reach 30%, and for the rigidly clamped beams, the deflections can increase 1.5–2 times in comparison with the deflections of the same dimensions’ beams with a solid wall.

Recently, the number of publications devoted to determining the perforated beams’ deflections as well as the variety of methods used has increased. There are different approaches to the perforated beams’ deflections analytical calculation: according to the theory of composite bars; using the calculation scheme of Virendel; based on the minimum potential energy principle.

In the works of A.I. Pritykin [1-3], the analytical solution to the problem of bending a pivotally supported perforated beam with hexagonal cutouts of regular and rhomboid shape, made using the theory of composite bars, is in satisfactory agreement with the results of the FEM and experimental data. It was later included in Building Code 294 [4].

I. G. Raftoyannis and G. I. Ioannidis [5] proposed a dependence for calculating the deflections of a beam with hexagonal cutouts, based on the application of the Virendel principle, in which the total deflection of the beam is considered as the sum of the deflection from the bending moment and the shear...
strain of the spandrel beam from the transverse force. This introduces the concept of the effective moment of the perforated beam’s inertia and the effective shear area of the spandrel beam.

W. Yuan et al. [6] applied the principle of minimum potential energy when obtaining the analytical dependence for the deflections of perforated beams with cutouts of a hexagonal or round shape subject to the action of the uniformly distributed transverse load.

The same principle for the analytical calculation of the elastic deflections of perforated beams with hexagonal cutouts without rounding was proposed by S. Elaiwi et al. [7]; to increase the accuracy of the solution, the shear stiffness coefficient was determined using the ANSYS complex FEM calculations. It is indicated that for short beams within $10 \leq H \leq 12$ the calculation error does not exceed 6%.

The experiments held by Jamadar F.M. et al. [8] on the beams with round, hexagonal and rhomboid cutouts in order to choose their optimal shape showed good correspondence between the deflections and the FEM calculations in the ABAQUS system, but no analytical dependences were obtained.

In the works of S. Durif et al. [9, 10] the issues of strength and stability of beams with sinusoidal perforation by conducting experiments on full-scale beams were examined, the analysis of these results showed that two forms of their destruction are possible: the formation of 4 plastic joints in the cutouts’ angles or the loss of local stability in the cutouts’ sinusoidal part.

The work of L. Budi et al. [11] is devoted to optimizing the size of cutouts and the width of the lintels in the perforated beams with hexagonal cutouts, based on the stiffness and strength assessed by their stressed state.

However, the European building standards do not contain formulas for calculating the perforated beams’ deflections due to the difficulty in understanding the cutouts’ effect on shear strain. Only the Building Code of the Russian Federation [4] contains the recommendations for the analytical determination of the beams’ deflections with hexagonal cutouts of regular shape.

In connection with the recently appeared beams with sinusoidal perforation, developed by Arcelor Mittal, the question of assessing their deformation properties arises.

Although the numerical methods, among which the finite element method occupies a special place, make it possible to fairly accurately predict the deflections of perforated beams using various software systems, such as NASTRAN, ABAQUS, or ANSYS, they need to be used skillfully, since even a small error in the initial data can lead to the results’ serious misrepresentation. Therefore, the use of analytical dependencies, allowing to quickly evaluate the stiffness of the beams with a particular design, is preferable.

**Estimated Dependence**

In the beams with a sinusoidal cutouts’ shape that appeared in construction practice in order to reduce stress concentration, even with the basic shape of the cut in the form of a regular hexagon, the radius of the angles’ rounding can vary in a rather wide range. However, an increase in the radius leads to a decrease in the spandrel beam’s width, therefore, before performing the deflections’ calculations, we indicate the possible range of the angles’ rounding radius variation. The minimum spandrel beam’s width $c_{\text{min}}$ as a function of the fillet radius $r$ and the size of its horizontal side $a$ can be defined as:

$$c_{\text{min}} = a - 2r \tan 30^\circ.$$  \hspace{1cm} (1)

From the relation (1) the maximum rounding radius $r_{\text{max}}$ will take the form:

$$r_{\text{max}} = 0.866(a - c_{\text{min}}).$$  \hspace{1cm} (2)

In general, the change range of $c_{\text{min}}$ is quite wide, but it should be determined from the condition of the shear strength of the spandrel beam and ensuring its local stability. Even if we take the value $c_{\text{min}} = 0.13a$, then from (2) it follows that $r_{\text{max}} = 0.75a$. In this paper, we consider the options with the angles rounding relative radius value $\rho = r_{\text{max}} / a = 0.25$ (Fig. 1a) and $\rho = 0.5$ (Fig. 1b).
Figure 1. Type of beams with sinusoidal perforation: a) at $\rho = 0.25$; b) at $\rho = 0.5$

To indicate the beam’s dimensions, the following abbreviated form of recording is used in the work: 

$l - h_s - t_w - b_f - t_f - \beta = \rho$, in which the input quantities are interpreted as: $l$ - beam length, $h_s$ - beam wall height, $t_w$ - wall thickness, $b_f$ - flange width, $t_f$ - flange thickness, $\beta = h_i / h_s$ - relative height of cutouts, $\rho = r / a$ - the cutouts’ angles rounding relative radius (Figure 2a). The dimensions of the beam are indicated in centimeters.

When applying the theory of composite bars (TCB), the beam under consideration was represented in the form of two load-bearing horizontal tee bars located above and below the cutouts, and the elastic layer formed by the spandrel beams between the cutouts, ensuring the joint work of the bars (Figure 2b).

Figure 2. Beam with sinusoidal perforation: a) legend; b) design scheme for TCB

The differential equation of a composite beam’s bending has the form [3]

\[ w'' \frac{IK_x}{E Ai} w = \frac{MK}{E^2 Ai} \]

where $M$ - is a bending moment from the external load in an arbitrary beam section; $I$ - is the moment of the beam inertia over the cross section with a cutout; $A$ and $i$ - define the cross-sectional area and the T-belt inertia moment over the cutout; $E$ - is the Young’s modulus of the material; $K_x$ - is the stiffness coefficient of the elastic layer formed by the spandrel beams between the cutouts (dimension $K_x$ - N/mm²).

In [3], the equation (3) is solved by expanding the bending moment function in a Fourier series. The coefficient value $K_x$ is refined according to the FEM calculations. In this paper, the task was to obtain an analytical dependence for calculating the deflections of a pivotally supported perforated beam with sinusoidal perforation, with different relative lengths $l/H$ and different rounding radii of the cutouts’ angles.

It is important to know what perforation has a shear effect on the deflections of such beams. For a pivotally supported beam with a solid wall, the shear effect on the beam’s deflection can be estimated by the dependence [2]
\[
\max w_{\max} \approx w_{\max}^{TT} \left(1 + 12.5 \left(\frac{H}{l}\right)^2 \left(\frac{1}{6} \frac{\omega_f}{\omega_w}\right)\right),
\]

(4)

where \(H\) and \(l\) – define the beam height and length respectively; \(\omega_f\) - is the flange area; \(\omega_w\) - is the wall area; \(w_{\max}^{TT}\) - is the deflection of the beam from the distributed load, determined by the technical theory of bending. For a pivotally supported beam loaded with a uniformly distributed load \(q\)

\[
w_{\max}^{TT} = \frac{5ql^4}{384EI}.
\]

(5)

From the dependence (4) it follows that the greater is the shear influence, the greater the ratio \(\omega_f / \omega_w\), and since the wall perforation presence decreases \(\omega_w\), then the effect of the shift increases.

The increase in the relative length of the beam \(l / H\) reduces the shear role.

In [3], a dependence for estimating the deflections of the perforated beams with hexagonal cutouts of regular shape without rounding angles was obtained in the form:

\[
w_{\text{perf}} = w^{TT} \left(1 + 104 \beta H A / t_w^2\right),
\]

(6)

where the cross-sectional area of the T-belt \(A\) is calculated as:

\[A = t_f b_f + t_w (0.5(H - h) - t_f)\].

(7)

The given dependence (6), taking into account (5) and (7), gives satisfactory results if, as the moment of inertia included in (5), the value determined by the cross section passing through the cutout is used

\[I = I_{\text{sol}} - h^3 t_w / 12\],

(8)

where \(I_{\text{sol}}\) - is the inertia moment of a beam with a solid wall.

For the beams with sinusoidal cutouts, i.e., with cutouts having round radii, the dependence (6) needs to be adjusted, which can be done by introducing an additional function \(k(\rho)\). In this case, the dependence (6) takes the form

\[
w_{\text{perf}} = w^{TT} \left(1 + 104 k(\rho) \beta H A / t_w^2\right).
\]

(9)

To identify \(k(\rho)\) is only possible by calculating the FEM beams’ deflections, which, incidentally, showed that the dependence (9) can be extended to any type of the beam loading.

Based on the obtained dependence (9), the FEM calculations of the deflections of articulated perforated beams with a sinusoidal cutouts’ shape under the uniformly distributed load action were performed. The beams with hexagonal cutouts of the correct form with different radii of the angles’ rounding were considered (Figure 1). Numerical calculations were performed in the ANSYS software package using the finite elements Shell63.

The degree of rounding radius influence on the deflection can be determined by comparing the deflections from the shear deformation of the cantilever fragments of the beams with a length corresponding to the cutout pitch (Figure 3) under the action of the same transverse force. As it can be seen from Fig.3b with a relative fillet radius \(\rho = 0.25\), the deflection of the fragment decreases by 1.048 times. A similar calculation with a radius \(\rho = 0.5\) (Figure 3c) leads to an effect of 1.1 times. From the results obtained, it can be concluded that due to the presence of the radius, the additional shear coefficient can be approximated by the dependence:

\[k(\rho) = 1 - 0.18\rho\].

(10)

A slight increase in the beam’s rigidity in the angles’ rounding can be explained by the redistribution of material closer to the flanges of the beam while maintaining its volume.
Taking into account (10), the dependence (9) for the beam deflection with sinusoidal perforation, obtained according to the composite bars’ theory, can be represented as:

$$w_{\text{perf}}^{\text{TCB}} = w_{\text{perf}}^{\text{TT}} (1 + 104(1 - 0.18 \rho) \beta H A l \ell^3).$$

(11)

It should be emphasized that this dependence refers to the case of wall perforation with the regular hexagonal shape cutouts, having rounding in the range $0.25 \leq \rho \leq 0.75$.

### Results

Having taken the radii of angles’ rounding equal to $r = 0.75a$ for the regular hexagon shape cutout with $\vartheta = 60^\circ$ where $a$ – is the cutout side defined as $a = 0.5h / \sin \vartheta$, we obtain for a pivotally supported beam with the dimensions $11.8H - 75 - 0.9 - 19.9 - 1.4cm - 0.667 - 0.75$ at evenly distributed load $q = 10kN / m$ the deflection by FEM is equal to $w_{\text{perf}}^{\text{FEM}} = 4.92mm$ (Fig.4a), and according to the formula (11) - $w_{\text{perf}}^{\text{TCB}} = 4.919mm$. The results are almost the same. The comparison with beam deflection without rounding angles $w_{\text{perf}}^{\text{FEM}} = 5.0198mm$ (fig. 4b) shows that sinusoidal perforation increases the beam’s rigidity by only 2%.

It is possible to check the acceptability of dependence (11) by comparing the deflections $w_{\text{perf}}^{\text{TCB}}$ and the FEM calculation data $w_{\text{perf}}^{\text{FEM}}$ beam sizes $l - 90 - 1.2 - 19 - 1.78cm - 0.667 - 0.25$, shown in Fig. 5 for different lengths with a relative fillet radius $\rho = 0.25$. 

Figure 3. The beams’ fragments deformation: a) without rounding; b) if $\rho = 0.25$; c) if $\rho = 0.5$

Figure 4. Deflections of the beams with different perforations: a) at $\rho = 0.75$; b) at $\rho = 0$
Figure 5. The deflections $w_{\text{FEM \, perf}}$ and $w_{\text{TCB \, perf}}$ depending on relative length $l/H$

Discussion
If we compare the deflections of the correct form beams with hexagonal cutouts without rounding and with rounded cutouts, then we can note that the for the rounding with a relative radius $\rho = 0.5$ for the beams with relative length $l/H \approx 10$ the discrepancy does not exceed $2.5\%$. For the beams with $l/H \approx 20$ this discrepancy decreases to $\delta = 0.9\%$. From this it is possible to conclude that the rounding angles’ effect on the deflections of perforated beams with the regular shape hexagonal cutouts is insignificant. Perhaps, these design features will manifest themselves more significantly when assessing the beam’s bearing capacity. The obtained dependence (11) is quite universal and can even be used to calculate the perforated carbon-fiber traverse used in Airbus 380 airliners.

Summary
1. Based on the composite bars theory differential equation solution, the analytical dependence is obtained for the perforated beams’ deflections with sinusoidal cutouts of regular shape with their relative height $\beta = 0.667$, relative angles’ rounding radii in the range $0.25 \leq \rho \leq 0.75$ with the relative beam length $l/H \geq 9$.
2. The presence of rounding radii slightly increases the bending stiffness of the beam, since at the same wall area a part of it is redistributed closer to the flanges.
3. When calculating the beams’ deflections according to the dependence (11), the inertia moment should be taken over the section weakened by the cutout.
4. The deflections’ calculations according to the formula (11) are in satisfactory agreement with the FEM results.

References
[1] Pritykin A I 2015 Deflections of perforated beams with hexagonal cutouts: two forms of decision Industrial and civil engineering 5 111-118.
[2] Pritykin A I 2015 The castellated beams deflections calculated with theory of composed bars Mechanika 21(5) 367-371.
[3] Pritykin A I, Emelyanov K A 2018 Determination of deflections of beams with diamond-shaped wall perforation Bulletin of MGSU 13(7) 814-823.
[4] Building Code. Steel structures, Design rules. 294.1325800. (2017) 198.
[5] Raftoyiannis I G, Ioannidis G I 2006 Deflection of castellated beams under transverse loading Steel Structures 6 31-36.
[6] Yuan W-B, Yu N-T, Bao Z-S, Wu L-P 2016 Deflection of castellated beams subjected to uniformly distributed transverse loading International Journal of Steel Structures 16 (3) 813-821.
[7] Elaiwi S, Kim B, Li L 2019 Bending analysis of castellated beams Athens J. of Technology & Engineering 6 (1) 1-15.
[8] Jamadar F M, Kumbhar P D 2015 Parametric study of castellated beam with circular and diamond shaping openings International Research Journal of Engineering and Technology 2 (2) 715-722.
[9] Durif S, Bouchair A 2012 Behaviour of cellular beams with sinusoidal openings Steel Structures and Bridges 40 108-112.
[10] Durif S, Bouchair A, Vassart O 2013 Experimental tests and numerical modeling of cellular beams with sinusoidal openings Journal of Constructional Steel Research 82 (1) 72-87.
[11] Budi I, Sukamta W Partono 2017 Optimization analysis of size and distance of hexagonal hole in castellated steel beams Procedia Engineering 171 1092-1099.