Evaluation and Recalibration of Empirical Constant for Estimation of Reference Crop Evapotranspiration against the Modified Penman Method

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Abstract. The major demand in our country is irrigation demand. Looking to the low irrigation potential and small water resources, it is felt necessary to see that water must be used economically and efficiently. This may be achieved by using latest methods of determination of water requirements for crops and applying the proper water management practices. Evapotranspiration (ET) is a basis for calculation of water requirement for crops. The various popular empirical equations for reference crop evapotranspiration (ETr) belong to three categories namely, Temperature, Radiation based methods and Combined methods. The above methods are site specific; hence it is necessary to recalibrate the coefficients for applying them in India. In the present paper, the standard combined method namely FAO modified Penman method was used to recalibrate the constants in temperature based (TB) methods and it can also be used to determine the ET, for the selected station. Four TB evapotranspiration models namely Blaney-Criddle, Romanenko, Kharrufa, and, Thornthwaite methods are recalibrated and the constant in each method are redefined for the data from Lekkur station, Cuddalore district in India. The result shows that, large error existed when ET has been calculated with original constants. Hence regression equations were developed to minimise these variations in magnitude. It was found that out of four methods the Blaney-Criddle method suits better for the selected region.

Keywords: Reference crop evapotranspiration (ETr), FAO modified Penman method, temperature-based methods

1. Introduction
Evapotranspiration (ET) encompasses the total evaporation from plant body and surrounding areas, either potential or actual evapotranspiration may be of concern. Potential evapotranspiration or reference crop evapotranspiration (ETr) is the rate that would occur under some specified standard condition of vegetation with an unrestricted supply of water in the soil and without advection or heat storage effects. Even though ETr is considered as vital parameter for the terrestrial hydrologic system, measurement of ETr is difficult, due to its spatial and temporal variation. ETr is a major determinant variable in water cycles as well as it plays a major role for the redistribution of water on the earth’s surface. ET is the most significant component in the areas where the irrigation is a major component of agriculture due to low precipitation [1]. ET of other crops can be obtained by multiplying ETr with crop coefficients, which vary with the crop and its growth stage. Even though standard Penman model gives accurate result of ET value [2], it needs more numbers of data and extensively experienced
person to use that equation correctly [3]. Therefore, in order to select suitable method to use it in India, it is mandatory to analyse and evaluate the various forms of existing temperature-based (TB) models with the standard Penman model. During the past several decades, many literature showed that the evapotranspiration has being extensively studied in hydrology and water resources management [4-10].

2. Study Area
The meteorological data collected from Lekkur station has been used to calculate ET in the present study. This station is located in Thittakudi which is taluk headquarters of Cuddalore district in Tamil Nadu, India at a latitude of 11.416° N and longitude of 79.116°E. Several meteorological variables namely air temperature, wind speed, relative humidity, sunshine hours and wind speed for the period of 1975 to 2003 have been used for estimation of ET and development of regional equation. The monthly average main climatic parameters are shown in Table 1.

| Month | Humidity in % | Maximum Temperature in °C | Minimum Temperature in °C | Sunshine in hrs/day | Wind Velocity in km/h |
|-------|---------------|----------------------------|----------------------------|---------------------|----------------------|
| Jun   | 52.99         | 34.89                      | 26.47                      | 6.85                | 8.51                 |
| Jul   | 55.71         | 33.70                      | 25.47                      | 5.56                | 7.49                 |
| Aug   | 59.00         | 33.13                      | 24.75                      | 6.06                | 7.06                 |
| Sep   | 65.70         | 32.76                      | 24.05                      | 6.63                | 5.19                 |
| Oct   | 72.51         | 30.94                      | 23.37                      | 5.82                | 3.73                 |
| Nov   | 75.56         | 28.74                      | 22.22                      | 5.55                | 4.43                 |
| Dec   | 71.38         | 27.13                      | 20.83                      | 6.48                | 5.68                 |
| Jan   | 65.74         | 28.05                      | 20.70                      | 8.14                | 5.21                 |
| Feb   | 60.85         | 30.19                      | 21.33                      | 9.34                | 5.11                 |
| Mar   | 56.91         | 33.02                      | 22.98                      | 9.67                | 5.47                 |
| Apr   | 56.54         | 35.17                      | 26.23                      | 9.30                | 6.55                 |
| May   | 54.04         | 35.14                      | 26.74                      | 8.59                | 7.43                 |

3. Reference Crop Evapotranspiration using Temperature – Based methods
Two important methods namely temperature-based and radiation-based methods along with the FAO modified Penman method were studied by Jothiprakash et al [11], [12]. For calculating irrigation demand ETr is to be determined. In this paper following four TB models along with FAO modified Penman methods are used to calculate ETr.

3.1 Blaney – Criddle Method
Blaney-Criddle method is considered as the one of the simplest method because ET0 is calculated using measured temperature data alone. However this is not a very accurate method; it gives a rough estimate only. This method is inaccurate under "extreme" climatic conditions. This method gives underestimated ET0 value in windy, dry, sunny areas, and overestimated value in calm, humid, clouded areas. The mathematical expression of the Blaney-Criddle method for the estimation of ET0 has the form [13].

\[ \text{ET}_0 = a + b (0.46 T + 8.13) \]  

where,
ET\textsubscript{o} = Reference crop evapotranspiration in mm/day
a & b = Calibration factors
p = Mean daily percentage of total annual daytime hours.
T = Mean daily air temperature in °C
a = 0.0043 RH\textsubscript{min} - (n/N) – 1.41
b = ((0.82 – (0.0041RH\textsubscript{mean}) + (1.07 n/N) + (0.066U) – (0.006RH\textsubscript{mean} n/N) – (0.0006RH\textsubscript{mean} U)
\newline
n/N = Mean ratio of actual to possible sunshine hours.
RH\textsubscript{min} = Minimum daily relative humidity
U = Wind speed at a height of 2 m from ground surface.

3.2 Thornthwaite Method
This method was developed for the east-central U.S. It is simpler than Penman’s equation because this method requires less climatic data. Bautista et.al [14] concluded that this method worked very well during the rainy months. The equations is as follows
\[
PET = 1.6 L_d \left( \frac{10T}{I} \right)^a
\]  
(2)

where,
PET - Potential evapotranspiration in cm/month - to get a per day amount, we assume there are 30 days per month, so PET in mm/day = PET*10 (mm/cm)/30 days/month
L\textsubscript{d} - Daytime hours in units of 12, for example if there are 10 daytime hours, then L\textsubscript{d}=10/12
I - The annual heat index that is computed from the monthly heat indices
T - Average temperature for the day or month in °C

\[ I = \sum_{j=1}^{12} i_j \]  
(2.1)

where \( i_j \) is computed as
\[ i_j = \left( \frac{T_j}{5} \right)^{1.514} \]  
(2.2)

‘a’ is computed as either:
\[ a = 67.5 \times 10^{-8} I^3 - 77.1 \times 10^{-6} I^2 + 0.0179 I + 0.492 \]  
(2.3)

3.3. Romanenko method
An evapotranspiration equation was derived by Romanenko [15] based on the relationship using mean temperature and relative humidity is as follows.
\[ E = 0.0018(25 + T)^2(100 - rh) \]  
(3)

where
T= Mean air temperature in °C and
Rh = Mean monthly relative humidity in %.
The units of E are mm/month and based on a 31 day month - divide by 31 for mm/day.

3.4. Kharrufa method
Kharrufa [16] derived an equation through relationship of ET/p and T in the form of
\[ ET = 0.34pT_a^{1.3} \]  
(4)

where,
ET = Kharrufa Potential evapotranspiration (in mm/month) and T\textsubscript{a} and p have the same definitions earlier.
3.5 FAO modified Penman method

This method includes both climatological effect of aerodynamic and temperature variations. The equation used to estimate ETr is given by:

\[ ET_r = C_p \left[ WR_n + (1-W) \cdot f(u) \cdot (e_s - e_a) \right] \]  (5.1)

where,

- \( ET_r \): Reference crop evapotranspiration in mm/day
- \( W \): temperature and altitude dependent weighting factor
- \( R_n \): Net radiation in equivalent evaporation in mm/day
- \( R_{ns} = R_n - R_{nl} \)  (5.2)
- \( R_s = \text{Solar radiation in mm/day} = (a + b \cdot n/N)R_a \)  (5.4)
- \( n \): Actual sunshine hours
- \( N \): Maximum possible sunshine hours
- \( \alpha \): Reflection factor (0.25) (albedo)
- \( R_a = \text{Extra terrestrial radiation in mm/day} \)

\[ f(T) = \text{Effect of temperature on long wave radiation (} R_{nl} \text{)} \]
\[ f(e_a) = \text{Effect of actual vapour pressure on long wave radiation (} R_{nl} \text{)} \]
\[ f(n/N) = \text{Effect of ratio between actual and maximum bright sunshine hours on long wave radiation (} R_{nl} \text{)} \]
\[ f(u) = \text{Wind related function} = 0.27 \left[ 1 + U/100 \right] \]  (5.6)

If the wind speed data are available at height other than 2 meter, they need to be converted to wind speed corresponding to 2 meter height using 1/7th power law as recommended by F.A.O. [13].

\[ U_{2m} = U_x \left[ 2 / x \right]^{1/7} \]  (5.8)

where \( U_{2m} \) is the wind speed at 2 m height from ground surface and \( U_x \) is the wind speed measured at ‘x’ m height from ground surface. In the present study the \( U_{day} / U_{night} \) ratio is assumed as 1.5 [2]. The ‘a’ and ‘b’ coefficient in the radiation formula are assumed to be 0.25 and 0.51 respectively. In the FAO modified Penman formula, values of ‘W’ temperature and altitude dependent weighting factor, net long wave radiation in mm/day (\( R_{nl} \)), extra terrestrial radiation in mm/day (\( R_a \)), the mean daily duration of maximum possible sunshine hours (\( N \)) and the value adjustment factor \( C_p \) were taken from the FAO 24 [13].

4. Results and Discussion

The mean monthly values of ET, were estimated using the various methods discussed above with original constant values. Table 2 shows estimated ETr values with original constant using above discussed method as well as the percentage error for all method with reference to FAO modified Penman method. From the Table 2 it can be seen that the average annual error between FAO modified Penman method and other methods varies from −27.1% to 19.6%. The overestimated and underestimated values of ETr were represented by positive error and negative error respectively with
reference to the standard method. With the original constant values almost all the methods has
overestimated the ET\(_r\) values except Thronthwaite method. The highest error was noticed when
Thronthwaite method was used and Blaney – Criddle method yielded least error.

| Month | ET\(_r\)\(_\text{Pen}\) | ET\(_r\)\(_\text{Bla}\) | Error (%) | ET\(_r\)\(_\text{Thron}\) | Error (%) | ET\(_r\)\(_\text{Rom}\) | Error (%) | ET\(_r\)\(_\text{Kha}\) | Error (%) |
|-------|----------------|----------------|-----------|----------------|-----------|----------------|-----------|----------------|-----------|
| Jun   | 203.4          | 298.1          | 46.6      | 162.4          | -20.1     | 271.6          | 33.6      | 207.7          | 2.1       |
| Jul   | 181.2          | 187.8          | 3.6       | 113.9          | -37.2     | 247.1          | 36.4      | 162.0          | -10.6     |
| Aug   | 178.6          | 179.7          | 0.6       | 111.9          | -37.4     | 223.3          | 25.0      | 171.4          | -4.0      |
| Sep   | 158.7          | 122.4          | -22.9     | 112.9          | -28.9     | 183.3          | 15.5      | 183.4          | 15.6      |
| Oct   | 132.2          | 51.6           | -61.0     | 78.0           | -41.0     | 139.1          | 5.3       | 150.7          | 14.0      |
| Nov   | 115.3          | 48.9           | -57.6     | 54.6           | -52.6     | 115.5          | 0.1       | 131.9          | 14.4      |
| Dec   | 121.1          | 94.7           | -21.7     | 47.6           | -60.7     | 126.9          | 4.9       | 141.9          | 17.2      |
| Jan   | 138.0          | 143.1          | 3.7       | 66.3           | -52.0     | 155.4          | 12.5      | 183.5          | 32.9      |
| Feb   | 164.7          | 199.2          | 20.9      | 101.1          | -38.6     | 188.9          | 14.6      | 228.0          | 38.4      |
| Mar   | 200.5          | 256.5          | 27.9      | 153.7          | -23.4     | 226.6          | 13.0      | 262.5          | 30.9      |
| Apr   | 207.1          | 313.3          | 51.3      | 229.8          | 10.9      | 253.5          | 22.4      | 285.4          | 37.8      |
| May   | 210.9          | 342.2          | 62.3      | 235.2          | 11.5      | 274.9          | 30.4      | 271.3          | 28.7      |
| Annual| 2011.7         | 2237.5         | 11.2      | 1467.2         | -27.1     | 2406.1         | 19.6      | 2379.8         | 18.3      |

\(ETr_{\text{Pen}}\) = FAO modified Penman method, \(ETr_{\text{Bla}}\) = Blaney-criddle, \(ETr_{\text{Thron}}\) = Thronthwaite, \(ETr_{\text{Kha}}\) = Kharrufa method, \(ETr_{\text{Rom}}\) = Romanenko method.

4.1. Correlation of monthly evapotranspiration estimated with original constants

By using following linear regression equation, the monthly ET\(_r\) values estimated using above
mentioned methods were correlated with FAO modified Penman method.

\[Y = AX + B\]  \hspace{1cm} (6)

Where, \(Y\) - ET, estimated using FAO modified Penman method and \(X\) - ET, estimated from above
mentioned four TB methods. As far as the R\(^2\) values are concerned, the Romanenko method has the
highest R\(^2\) value of 0.95 with FAO modified Penman method which is given in Figure 1. The Blaney –
Criddle method gave least percentage error of mean monthly ETr estimation, has resulted with a R\(^2\) of
0.898.

Figure 1 shows the scattered plot of monthly ETr values obtained by using above TB
methods against FAO modified Penman method. Figure 1 shows a poor correlation within the TB
methods except Romanenko method.
4.2. Modification to the constants used in temperature-based ET, equations

The empirical equations, as used in this study, may be consistent in the areas and over the intervals for which they have been evolved, but massive inaccuracy can be predicted when they are extrapolated to different climatic regions without recalibrating the constants involved in the formulae [17]. Hence it is vital to recalibrate the constants involved in each equation. Therefore, constants involved in each equations were recalibrated against FAO modified Penman method using “Linear Regression equation” [18] to improve the performance of each method. The constant values of 0.46, 0.0018, 0.34, 16 used in Blaney – Criddle, Romanenko, Kharrufa, and Thronthwaite method respectively are recalibrated. The mean monthly ET, estimated using various TB methods with the recalibrated constant values are shown in Table 3. Table 3 shows that the average percentage of error varies from 0.06% to -0.06%. As far as recalibrated constant values are used, all the methods are gives under estimated ET values. But the results shows that there is a significant improvement with recalibrated constant value.

The R² value shows that there is no much variation with that of R² estimated using original constants. But there is a big trade in intercept and slope. The calculated ET, with recalibrated constant from the above discussed method are depicted as a scatter plot against FAO modified Penman method is shown in Figure 2. Figure 2 illustrates that there is no significant changes in the scattering pattern of ET, after calibration of constant and it shows only a change in magnitude of ET, with that of the FAO modified Penman method.

The original constant and recalibrated constant values against FAO modified Penman method is shown in Table 4. From the Table 4 it is evident that there is great variation in the recalibrated constant of Blaney – Criddle and Thronthwaite method compared to original constant values. The annual difference of ET, values among the above referred TB methods against FAO modified Penman method before and after recalibration are shown in Table 5. Table 5 shows that the
level of difference of annual ET\(_r\) is large for all the methods except Blaney-Criddle against FAO modified Penman method with original constant value and it can be seen from the same table that the difference of annual ET\(_r\) is significantly reduced for all the methods with help of recalibrated constant values.

Table 3. Mean monthly estimated ET\(_r\) in ‘mm’ using selected methods with recalibrated constant values

|     | ET\(_r\) Pen | ET\(_r\) Bl | Error (%) | ET\(_r\) Thon | Error (%) | ET\(_r\) Rom | Error (%) | ET\(_r\) Kha | Error (%) |
|-----|--------------|-------------|------------|---------------|------------|---------------|------------|---------------|------------|
| Jun | 203.4        | 204.3       | 0.44       | 187.8         | -7.64      | 209.7         | 3.10       | 172.6         | -15.12     |
| Jul | 181.2        | 168.0       | -7.29      | 163.2         | -9.92      | 195.1         | 7.69       | 147.7         | -18.47     |
| Aug | 178.6        | 165.3       | -7.44      | 162.2         | -9.17      | 181.0         | 1.36       | 152.9         | -14.40     |
| Sep | 158.7        | 146.5       | -7.73      | 162.7         | 2.51       | 157.3         | -0.89      | 159.4         | 0.42       |
| Oct | 132.2        | 123.2       | -6.79      | 145.1         | 9.77       | 131.1         | -0.81      | 141.6         | 7.14       |
| Nov | 115.3        | 122.3       | 6.05       | 133.2         | 15.51      | 117.1         | 1.52       | 131.4         | 13.90      |
| Dec | 121.1        | 137.4       | 13.48      | 129.6         | 7.09       | 123.9         | 2.33       | 136.8         | 13.01      |
| Jan | 138.0        | 153.3       | 11.06      | 139.1         | 0.79       | 140.7         | 1.95       | 159.4         | 15.51      |
| Feb | 164.7        | 171.7       | 4.25       | 156.8         | -4.84      | 160.6         | -2.51      | 183.6         | 11.47      |
| Mar | 200.5        | 190.6       | -4.95      | 183.4         | -8.52      | 183.0         | -8.73      | 202.4         | 0.96       |
| Apr | 207.1        | 209.3       | 1.03       | 222.0         | 7.17       | 199.0         | -3.95      | 214.9         | 3.74       |
| May | 210.9        | 218.8       | 3.75       | 224.7         | 6.57       | 211.6         | 0.35       | 207.2         | -1.74      |
| Annual | 2011.7      | 2010.5     | -0.06      | 2009.9      | -0.09     | 2010.0       | -0.08     | 2009.9       | -0.09     |

Table 4. Comparison of parameter values before and after calibration in temperature-based methods

| Method         | Original constant | Recalibrated constant |
|----------------|-------------------|-----------------------|
| Blaney-Criddle | 0.46              | -0.104                |
| Thornthwaite   | 16                | 25.689                |
| Romanenko      | 0.0018            | 0.002                 |
| Kharrufa       | 0.34              | 0.294                 |

Table 5. The annual difference in ET\(_r\) (mm/year) value between temperature-based method against FAO modified Penman method with original and Recalibrated constants

| Method         | Difference with original constants | Recalibrated constants difference |
|----------------|-----------------------------------|----------------------------------|
| Blaney-Criddle | -225.81                           | 1.15                             |
| Thornthwaite   | 544.45                            | 1.79                             |
| Romanenko      | -394.46                           | 1.64                             |
| Kharrufa       | -368.10                           | 1.76                             |
5. Conclusion

The reference crop evapotranspiration depends upon the meteorological variables. Hence different methods have been developed and applied depending upon the available data. In the present study four temperature-based methods developed in other countries has been evaluated and compared with FAO modified Penman method. After evaluation, the constants in each method were recalibrated, so that it suits better to the present study area Thittakudi, Cuddalore district. Based on the comparison of estimated ET, with original and recalibrated constants for temperature based methods, it was observed that there were large errors developed when original constants of 0.46, 16, 0.0018 and 0.34 of Blaney-Criddle, Thronthwaite, Romanenko and Kharrufa respectively used. In order to minimize the error, the constants were recalibrated and obtained respectively as 0.104, 25.68, 0.0015, 0.293. Thus based on the recalibrated constants and $R^2$ values it can be concluded that Romanenko as well as Blaney-Criddle method may be used for ET estimation in the study area.

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