Possible Role of the WZ-Top-Quark Bags in Baryogenesis

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The heaviest members of the SM – the gauge bosons W, Z and the top quarks and antiquarks – may form collective bag-like excitations of the Higgs vacuum provided their number is large enough, both at zero and finite temperatures. Since Higgs vacuum expectation value (VEV) is significantly modified inside them, they are called “bags”. In this work we argue that creation of such objects can explain certain numerical studies of cosmological baryogenesis. Using as an example a hybrid model, combining inflationary preheating with cold electroweak transition, we identify “spots of unbroken phase” found in numerical studies of this scenario with such W – Z bags. We argue that the baryon number violation should happen predominantly inside these objects, and show that the rates calculated in numerical simulations can be analytically explained using finite-size pure gauge sphaleron solutions, developed previously in the QCD context by Carter, Ostrovsky and Shuryak (COS). Furthermore, we point out significant presence of the top quarks/antiquarks in these bag (which were not included in those numerical studies). Although the basic sphaleron exponent remains unchanged by the top’s presence, we find that tops help to stabilize them for a longer time. Another enhancement of the transition rate comes from the “recycling” of the tops in the topological transition. Inclusion of the fermions (tops) enhances the sphaleron rate by up to 2 orders of magnitude. We finally discuss the magnitude of the CP violation needed to explain the observed baryonic asymmetry of the Universe, and give arguments that the difference in the top-antitop population in the bag of the right magnitude can arise both from CP asymmetries in the top decays and in top propagation into the bags, due to Farrar-Shaposhnikov effect.

I. INTRODUCTION

A. The Baryogenesis

The question how the observed baryonic asymmetry of the Universe was produced is among the most difficult open questions of physics and cosmology. Sakharov [1] had formulated three famous necessary conditions: the baryon number and the CP violation, with obligatory deviations from thermal equilibrium. Although all of them are present in the Standard Model (SM) and standard Big Bang cosmology, the known mechanisms creating the baryon asymmetry can only produced effects many orders smaller than the observed amount, usually expressed as the ratio of the baryon density to that of the photons \( n_B/n_\gamma \sim 10^{-10} \). To find a scenario in which this puzzle can at least in principle be solved is one of the motivation of this paper.

It follows from general anomaly relation and, in particular, from known electroweak instanton and sphaleron solutions, that baryon number can be violated by certain processes providing a change in topology of the gauge field. In the broken phase of the SM we live in today these processes are basically frozen, by a very high barrier associated with Higgs VEV. On the other hand, in the symmetric phase – the electroweak plasma – the sphaleron rates are suppressed only by the powers of the coupling: those rates are high enough to wipe out any primordial asymmetry, if it has been generated cosmologically prior to the electroweak transition era. These consideration force us to search for resolution of the baryon asymmetry puzzle at narrow temperature interval at or right below the electroweak phase transition. For review of the field see e.g. [2, 3].

The bubbles associated with the expected 1-st order electroweak phase transition had attracted significant attention in 1980’s-1990’s. Moving walls of such bubbles may supply the necessary local deviations from thermal equilibrium. The rate of thermal sphaleron transitions, both in equilibrium and near the bubble walls, has been studied extensively, see [5, 6] and many more. However, as the experimental limits on the Higgs mass have over time evolved upward, to \( M_H > 116 \text{ GeV} \) today, it became clear that the first order transition is actually impossible in the SM. (Transition is a crossover for \( M_H > 80 \text{ GeV} \), see again [2] for details and the original references). After this has fact has been acknowledged, people either looked at various phenomena beyond the SM such as its supersymmetric extensions, which may still allow some window for the 1st order transitions.

An alternative is the modification of the standard cosmology. Instead of the usual Big Bang scenario, with its adiabatically slow crossing of the phase transition and tiny deviations from equilibrium, the so called “hybrid” scenario has been proposed [22, 23], see also [24, 25] and many subsequent works. It combines the end of the
inflation era with the establishment of the electroweak broken phase. In it the nonperturbative production of long-wavelength gauge and Higgs fields happens far from equilibrium, returning to equilibrium later, at a temperature well below the critical temperatures \( T_c \) of the electroweak transition; therefore it is sometimes called the “cold” electroweak scenario. While based on some fine tuning of the unknown physics of the inflation, it avoids many pitfalls of the standard cosmology, such as “error” of asymmetries generated before the electroweak scale.

In such a scenario there are coherent oscillations of the gauge/scalar fields [25–27]: those have been studied in detail in real-time lattice simulations [29–30]. Some of these authors focuses on the problem of generation of primordial magnetic fields, see e.g. [28] in the same scenario, which we will not discuss. The simulated models include two scalars - the inflaton and the Higgs boson – and the electroweak gauge fields of the SM, in the approximation that the Weinberg angle is zero (\( Z \) is degenerate with \( W \)). All fermions of the SM are ignored: the effect of the top quark in particular is the subject of the present paper. After inflation ends, all bosonic fields are engaged in damped oscillations for relatively short time, at the end of which the Higgs VEV and gauge fields stabilize to their equilibrium values, with the bulk temperature \( T_{\text{bulk}} \sim 50 \text{GeV} \), well below the critical (crossover) temperature.

In this work we will use the results of these simulations as a quantitative example, although some of our results should also be applicable for standard cosmology as well. We start with explaining some of the results of these simulations (the sphaleron size and rate) in simple analytical models, and then proceed to discuss what are changes induced by the top quarks.

Another ingredient of the baryogenesis problem, the CP violation, had also evolved during the last decades. The nature of observed CP-odd effects, first discovered in kaon decays and lately in decays of the mesons, is rather firmly established and its origins traced to the complex phase of the CKM matrix. It has been argued that it should enter as the so called Jarlskog determinant, with all mass differences explicitly present polynomially, leading to effect \( \sim 10^{-19} \), way too small for baryogenesis. The magnitude of the CP violation in the SM is however still hotly disputed: the effect must be proportional to the Jarlskog determinant only if all fermionic masses appear explicitly in the numerator. This would be the case if the relevant scale of all loops is much larger than all the masses. This is not the case e.g. in the famous CP violation in the kaon decays. It has been argued recently [35] that next-to-leading order dim-6 operator appears which contains \( 1/m^2 \) in the coefficient: if so, numerical study of the possible role of this operator [36] found CP effect even 4 orders of magnitude larger than needed! We will return to critical discussion of this issue at the end of the paper.

Vast literature exists on possible leptogenesis scenarios, generating the asymmetry in the leptonic sector, for overview see e.g. [3]. If this is the case however, leptonic asymmetry should still be converted into the baryonic asymmetry, again at or near the electroweak transition scale. We will not discuss this option in this paper, together with many other ideas beyond the SM.

\section{The \( W - Z - \text{top bags} \)}

Generic argument for multi-particle states due to Higgs attraction is that however weak the forces can be, if it is an universal attraction, a sufficiently large number \( N \) of particles it will become very strong. For example, an extremely weak gravity force binds the planets and stars. This generic argument however assumes that Higgs is light enough, so that sufficiently many particles can be collected inside the region of a size \( 1/M_H \), so that the universal Higgs-induced forces are now yet cut off by its mass. In the context of Higgs attraction it was discussed in particular for light quarks in [9], in which case the \( N_c \) is astronomically large and it worked only for very light Higgs bosons.

More recent round of ideas has focused, not surprisingly, on the heaviest member of the Standard Model (SM), the top quarks. The first question was whether a single top quark would significantly modify the Higgs vacuum in their vicinity to make a “bag” [10, 11]. This idea was soon refuted (see e.g. [12]) by quantum one-loop corrections. Similarly, the idea does not work for “realistic” Higgs mass and binary \( tt \) or \( tt \) systems.

Frogatt, Nielsen and collaborators [13] were the first to look at multi-top system. They provided simple hydrogen-atom-like estimates for a system of \( N = 12 \) top quarks (6 top and 6 anti-top for the lowest \( S_{1/2} \) orbital), which suggested that its binding energy can be large and nearly cancel the mass! Unfortunately, our more accurate mean field calculations [14] also refuted it. The binding is nonzero for massless Higgs, although it is much smaller than estimated in [13]. With the “realistic” Higgs mass we found that the 12 top-antitop system is unbound. Further discussion of the binding of the 12 top system can be found in [15], who refined the conclusion in the form of the maximal Higgs mass at which binding occurs:

\[
M_H < M_c(12) \approx 49 \text{GeV}
\]

Systematic study of the top-bags in relativistic regime has been started in the paper [14]. One-loop quantum corrections to the bags have been investigated in a separate paper [16], following basically the line of thought of Farhi et al. We have found that at Yukawa coupling corresponding to the top quark the quantum (one-loop) effect are still under control, for any \( N \), while it is not so for hypothetical fermions few times heavier than tops. Significant progress has been made in the companion paper [17], in which we have studied binding of various bags, filled with bosons \((W,Z)\) and fermions \((t, \bar{t})\). We have found that bosonic bags may exist even in vacuum,
for "realistic" Higgs masses \( \sim 100 \text{GeV} \), while purely top bags do not. Those bosonic bags should however have very large number of quanta, of the order of thousands, to get bound.

One reason to study the \( W - Z \)-top-bags is methodical: they are only the third (and highly relativistic) family of manybody bound objects, besides atoms and nuclei. Unfortunately their experimental production at accelerators seems to be impossible, mostly because it is highly improbable to produce many tops in a small vicinity from each other. In this paper we suggest another motivation for their study, the cosmological one. One of the main messages of this paper is that metastable (mechanically stabilized) multi-quanta bags replace the "metastable bubbles" considered 20 years ago, being the arena in which the baryon number violating sphaleron transitions (and possibly also CP violation) takes place.

C. Sphalerons with and without the Higgs

Semiclassical description of the tunneling through the barrier, separating topologically distinct gauge fields, is given by the famous electroweak instanton solutions, which however leads to extremely low tunneling rate per unit time and volume, normalized by the only scale of the theory being the electroweak theory are strongly renormalized near \( T_c \) has the energy \( E_{KM} \) (the height of the barrier) of about \( 14 \text{ TeV} \). So, at the temperatures at the electroweak transition scale \( T \sim T_c \sim 0.1 \text{ TeV} \), the corresponding Boltzmann factor is prohibitive

\[
\frac{\Gamma_{tunneling}}{T^4} \sim \exp(-4\pi/\alpha_w) \sim 10^{-170}
\]

At finite temperatures another option appears, the so called sphaleron transition, which is due to a thermal excitation onto the barrier. The original electroweak sphaleron solution found by Klinkhamer and Manton (KM) \cite{KM} has the energy \( E_{KM} \) (the height of the barrier) of about \( 14 \text{ TeV} \). So, at the temperatures at the electroweak transition scale \( T \sim T_c \sim 0.1 \text{ TeV} \), the corresponding Boltzmann factor is prohibitive

\[
\frac{\Gamma_{KM}}{T^4} \sim \exp(-E_{KM}/T) \ll 10^{-60}
\]

However, this estimate is too naive, as the parameters of the electroweak theory are strongly renormalized near \( T_c \). More accurate calculation of the equilibrium sphalerons rates at the electroweak cross region give much larger rates. For recent update see e.g. \cite{15} who estimated those in the range

\[
\frac{\Gamma}{T^4} \approx 10^{-20}
\]

including rather large preexponent calculated in Refs. \cite{6-8}. Such rates are however still too small for the solution of the baryogenesis puzzle.

The main reason of the increase of the rate, from the KM sphaleron to revised one is of course the reduction of the Higgs VEV from its vacuum value \( v = 246 \text{GeV} \) to near-zero. (Recall that there is no Higgs-related barrier in the symmetric phase, and thus no exponential suppression: but this is not a blessing since a transition rates faster than cosmological expansion leads to "wipe out".)

In the hybrid scenario one asks: what is the barrier height and the sphaleron transition rates for a no-Higgs-VEV bag of a specific size \( \rho \)? Although with a completely different (QCD) motivation, the analytic answer to this question was already known, given by the so called COS sphaleron \cite{19}. The main obstacle for finding it earlier was that, lacking a nonzero Higgs VEV, classical gauge theory is conformal and has no dimensional parameter. Thus the minimal sphaleron mass can only be defined if some additional condition is imposed, e.g. that the r.m.s. size of this object is fixed

\[
\rho^2 = \int r^2 B(r)^2 \, dr
\]

Adding another constraint, that the Chern-Simons number of the configuration \( N_{CS} \) is also fixed to some value, one can minimize the energy over all possible magnetic field configurations and find a "sphaleron path", the set of configurations which describe the barrier separating one topological valley from the next.

COS result for the energy as a function of the Chern-Simons number can be written in a parametric form, with the parameter \( k = -1/1.1 \)

\[
E_{COS} = \frac{3\pi^2}{g^2\rho}(1 - k^2)^2
\]

\[
N_{CS} = \frac{1}{4} \text{sign}(k)(2 + |k|)(1 - |k|)^2
\]

The \( k = 0, N_{CS} = 1/2 \) configuration is the top of the barrier -the COS sphaleron.

We will be arguing below that the topological objects found numerically in real-time are in fact close to COS sphalerons with the total energy

\[
E_{COS} \sim 2 \text{ TeV} \ll E_{KM} \approx 14 \text{ TeV}
\]

much smaller than in the broken phase. (Once again: it is only possible because they are located in the "no-Higgs spots" with depleted Higgs VEV.) Obviously the corresponding Boltzmann factor is dramatically reduced, falling into the range in which one can discuss baryogenesis.

D. The aims and the structure of this paper

In a sentence, we will show that the presence of the \( W - Z \)-top-bags should significantly enhance the rate of baryon number violating processes.

A bit more detailed summary is that there are at least three important effects we will consider:

(i) As we just discussed, the baryon number violation rates inside such bags are enhanced by many orders of
magnitude due to the absence of the Higgs VEV and related high barrier. In other words, instead of KM
sphalerons one should use the COS ones, reducing the barrier height from 14 to only about 2 TeV. We will be
able to analytically estimate the sphaleron rates, which compare well with numerical simulations.

(ii) Accounting for top “bags” in this setting we find that tops help mechanically stabilize the bags. In a
way, the metastable bags play a role similar to that of the bubbles (present if the electroweak transition be
of the 1st order), namely to enhance deviations from equilibrium for rather long time.

(iii) Last but not least, the “recycling” of top quarks, present in the bag, effectively lower the barrier further, by
about 600 GeV. CP odd effects may lead to top-antitop population difference in the bags, which will result in
asymmetric diffusion in the baryon number.

Now about the structure of the paper. We start in section [II] with the discussion of the the results of numerical
simulations [29] and then qualitatively explain them using the COS sphaleron. After that, we turn to top and
W bags in section [III]. We discuss their production and lifetime. Finally, in section [IV] we will study how the
sphaleron transition itself is changed, with the account for fermions.

II. HYBRID INFLATION SCENARIO: BOSONIC SIMULATIONS AND THEIR DISCUSSION

(i) One important finding of the simulations is that the initial coherent oscillations of scalars soon give way
to the usual broken phase. The most important feature is persistence of “no-Higgs spots” in which Higgs VEV is
very far from the equilibrium value $v$ and is instead close to zero. The gauge fields in them have however rather
high magnitude. Fig[1] (from [29]) show an example of a snapshot of the Higgs field modulus. Typically the
volume fraction occupied by such “no-Higgs spots” is of the order of several percents and is decreasing with time.

(ii) The second important findings is that of topologically nontrivial fluctuations of the gauge fields. As shown
in the lower Fig[1] those are well localized. According to the anomaly relation, changes in topology leads to viola-
tion of $B + L$ (baryon and lepton) numbers, via emission/absorption of fermions, 12 in the SM.

It was observed that topological fluctuations happen only inside the “no-Higgs spots” mentioned above. In-
deed, this becomes apparent from the distribution of the topological charge shown in the lower part of Fig[1]
for the same time configuration of the Higgs as shown in the upper part of the Fig[1]. Of course, it is only one snap-
shot, but the authors found from the simulations that it is true for the whole sample.

The fraction of no-Higgs spots which induced topological transitions is also in the range of a percent. More

FIG. 1: (Color online). The contour plots from [29] of the modulus of the Higgs field $|\phi(x)|^2$ (upper plot) and the topo-
logical charge density $Q(\hat{x}, t)$ at time $mt = 19$, for the model $A_1, N_s = 48$. Red (dark) areas in the upper plot correspond
to small VEV, while yellow (light) bulk corresponds to the broken phase. On the lower plot lumps of the topological
charge density appear as red regions (dark in black and white display). While most of the no-Higgs spots do not have the
topological transitions, all transitions seem to be inside the spots.

precise measure is the so called sphaleron rate is defined by the mean square deviation from zero of the Chern-
Simons number

$$\Gamma(t) = \frac{1}{m^4 V} \frac{d\Delta N_{CS}^2}{dt}$$

(here and below all quantities are defined via one characteristic mass parameter $m$: for the simulations its value is
about 264 GeV, close to $v$. For the definition see Appendix A.)

Since the process only exist for finite period of time, ref. [29] reports its time integrated value defined by some
time integral

$$I(mt) = \int_{t_i}^{t} d(mt) \Gamma(t)$$
during which it basically happen. (Too early there are no gauge fields and too late there are no no-Higgs spots.) By the end of the process \( mt < 45 \) it is in the range

\[
I \sim 10^{-4}
\]

(11)

for more details about different parameters sets see Table II of \[29\]. This quantity, as well as of course snapshots like those shown in Fig.4 directly give the spatial distance between the topological fluctuations \( R_{sph}m = 20..30 \).

![Fig. 2](image1)

**FIG. 2:** (Color online). From \[29\]. Two snapshots of the topological charge, at times \( mt = 18 \) and 19.

(iii) Space-time evolution of the topological charge \( Q \) is shown in Fig.2 as two snapshots, and in more detail in Fig.3 for the maximal \( B^2(t), E^2(t) \) and \( Q(t) \), for one particular fluctuation. One can see that the fluctuation at some time moment is very much concentrated in a small spherical cluster (the upper one in Fig.2 is followed by an expanding spherical shell (the lower one in Fig.2) which gets near-empty inside.

The decomposition into electric and magnetic components of the field is shown in Fig.3. One can see that the fluctuations starts as nearly (90%) magnetic object at \( mt \sim 17 \), with the electric field and thus the topological charge \( \sim E \hat{B} \) peaking some time later. Then there appears an expanding shell, followed by the magnetic field rebounding to its secondary maximum of smaller amplitude. We will return to discussion of all those features in the next chapter.

Some interesting findings of those studies we will not discuss but yet would like to mention. In Ref. \[31\] it has been shown that “hot spots” have some topology in terms of the Higgs winding number, and that their favorite values is about 1/2 (thus named half-knots). There is some correlation between this winding number and \( N_{CS} \) (and

![Fig. 3](image2)

**FIG. 3:** (Color online). From \[29\]. Upper: The spatial profile of the charge density peak for various times. Lower: The value of the electric, magnetic and charge density at the center of the peak as a function of time.
thus the sphaleron formation) which deserves to be studied further.

Before modelling these numerical results, let us summarize what is it exactly we would like to achieve:
(i) to model the structure of the “no-Higgs spots” by WZ-top bags
(ii) to model the topological fluctuations in them by the COS sphalerons, explaining their structure, size and ultimately the rate
(iii) to discuss specifically the modifications coming from the top quarks, as those were not included in numerical simulations.

A. “No-Higgs spots” as the WZ bags

In Ref.\[17\] we have made some first steps toward understanding of the states of the gauge bosons in certain spherically symmetric Higgs backgrounds. Our main finding was that while outside of the bag the gauge quanta $W,Z$ are massive and have 3 polarizations, in the bags the longitudinal component obtains additional strong repulsive potential. It also is mixed with electric polarization, which is indirectly affected by this potential. Only magnetic one remains unaffected by this repulsion: their properties were documented in \[17\].

In principle, at finite $T$ one needs a complete set of $W,Z$ states for the partition function of the bag. Furthermore, in order to model the “spots” appearing near reheating, one should also do it out of equilibrium, which is even more complicated. However, since such states are not yet known in full, we will start here with two grossly simplified limits, assuming that those spots contain so large number of $W$ that they can approximately be treated macroscopically.

Thermodynamically stable bubbles of the symmetric phase can only coexist with the bulk in the broken phase only at the transition temperature $T_c$, provided the phase transition is 1st order. As we already mentioned in the Introduction, such option has been excluded by the combination of experimental limits on Higgs mass and lattice studies of the transition. And we also consider a “cold model” in which the equilibrated bulk is at a temperature well below the critical one, $T < T_c$. Therefore, we discuss the objects which are not thermodynamically but only mechanically stable.

What we consider here are a very simple variant of the bag model. Let us assume that rescattering are rapid enough to make kinetic equilibrium inside the bag. There is no chemical equilibrium, as the number of $W$s produced are determined by early time dynamics: so we need to introduce both the internal temperature $T_{in}$ and the fugacity $\xi_W = \exp(\mu_W/T_{in})$. Rescatterings will force the internal momentum distribution to be thermal

\[N_W = V_{spot} g_W \int \frac{d^3k}{(2\pi)^3} \frac{1}{\xi_W^3 \exp(\epsilon(k)/T_{in}) - 1} (12)\]

even if there is no chemical equilibrium, say the number of gauge degrees of freedom can be different from the maximal number $g_W = 6$ (e.g. only magnetic modes excited, without electric or longitudinal ones, as we discussed in the companion paper) or $\xi_W$ different from 1.

Condition of mechanical stability we will write as

\[p_{in} = B + p_{bulk} (13)\]

with the bag constant $B$ created by the Higgs potential, assuming zero VEV inside

\[B = \lambda v^4/4 = m^2 v^2/4 (14)\]

(loop and T-dependent corrections can be easily included but are ignored. All other degrees of freedom of the SM such as quarks and leptons are ignored here because they have not yet been produced.) Using further Boltzmann approximation (ignoring 1 in denominator of (12)) and also ignoring $p_{bulk}$ for the estimate, one gets the mechanical stability condition in a form

\[B = g_W \xi_W \pi^2 T_{in}^4/45 (15)\]

or the internal temperature

\[T_{in} \xi_W^{1/4} (g_W)^{-1/4} = 0.66 m \approx 174 GeV (16)\]

So, if it is an equilibrium W gas (all factors are 1) the internal temperature inside the spot is indeed well above the electroweak critical temperature $T_c \sim 100 GeV$, making it a spot of symmetric phase.

Since the $W$ bag is only mechanically stabilized, it is relaxing by cooling and thus disappear relatively quickly. Indeed, this is what happens in simulations, giving the characteristic lifetime of the no-Higgs spots

\[\tau_{hot spots} \sim 20/m (17)\]

It is however important for the sphaleron rate that – in the bag approximation we use – the inside temperature $T_{in}$ is not changing while this bag shrinkage takes place, because it is related with the Higgs bag constant.

B. The sphalerons, their structure and size

Trying to understand the results of these numerical simulations, one can ask in particular the following questions:
(i) How so high-magnitude gauge field can be produced? Why does it happen only inside the no-Higgs spots?
(ii) What is the energy needed for topological fluctuations? What is the gauge field structure? Can one understand its subsequent evolution and decay products?
(iii) Can one estimate the rate of their production?

The fact that topological transitions happen only inside the no-Higgs spots is rather simple to understand: as we already mentioned in the introduction in the broken phase the height of the barrier (the mass of KM
sphaleron) is prohibitively large. In a spot the height
is not zero, however, as it has finite size \( \rho \). As we already
mentioned, it is given by the mass of the COS sphaleron
\( \sim 1/\rho \) (see \cite{21}).

In a scale-invariant classical YM theory the size \( \rho \)
is an arbitrary parameter. In a Big Bang however (as
well as in numerical models we discuss) this size is to
be determined by the optimal scenario maximizing the
transition rate.

Although the bag production is a result of a complicated
nonequilibrium process, it is reasonable to assume
that large-size bags are cut off exponentially in bag vol-
ume. Furthermore, we model it by a thermal fluctuation
with the inside temperature. In a simple bag model the
pressure is \( p = p_{in}(T) - B \) while the energy density is
\( \epsilon = \epsilon_{in}(T) + B \). Mechanical stability condition for the
total pressure of the bag \( p = 0 \) (more accurately, the out-
side pressure in the r.h.s., which is very close to zero),
or \( p_{in}(T) = B \). Furthermore, if the inside of the bag is
filled by massless gas of any kind, \( \epsilon_{in}(T) = 3p_{in}(T) = 3B \)
and thus the energy of the bag is \( 4BV \). Since our bags
are not asymptotically large, perhaps some coefficient (to
be later determined) can be used instead: so we finally
write the bag Boltzmann factor as \( \exp(-\beta BV) \)

So, the optimal size \( \rho_{o} \) is the extremum of the following expression

\[
\Gamma_{sph}(\rho) \sim \exp\left[-B\left(\frac{4\pi\rho^{3}}{3} - \frac{3\pi^{2}}{g^{2}\rho T_{in}}\right)\right] \quad (18)
\]

Here the first term schematically represents the proba-
bility to create large bags, while the second is the Bolt-
zzmann factor for the COS sphaleron, with the mass from
\cite{6}. The resulting optimal spot size is then

\[
\rho_{o} = \left(\frac{3\pi}{4\beta T_{in}g^{2}B}\right)^{1/4} \quad (19)
\]

independent on the degrees of freedom filling the bag.
Comparing it with the ‘numerically observed’ \( \rho_{\text{num}} \sim 3.9 \),
Figs. 2 and 3, at the moment of maximal \( B \), one may
extract the value of the parameter \( \beta \) which turns out a
bit smaller than \( 1/T_{in} \). We repeat that the model is crude
and we only need it to discuss later the influence of the
tops on the whole picture.

It is useful to discuss at this point how semiclassical are those sphalerons. Note that the parametrically large
quantity here is electroweak coupling \( 4\pi/g^{2} = 1/\alpha_{\text{ew}} \),
which determines that the sphalerons consists of many
gauge quanta and thus can be treated semiclassically. As
the radius contains its power \( 1/4 \) we get only factor \( 2 \)
from it. The number of gauge bosons involved can be estimated from the action/\( h \) which is about \( O(10) \). It is
perhaps still large enough for the semiclassical analysis
we use, but – as we will see later – not large enough to
ignore the back reaction from the 12 fermions associated
with the electroweak anomaly.

C. The shape and field structure of the sphaleron

Ideally the sphaleron transition may proceed with the
total energy of the gauge quanta exactly equal to to the
height of the barrier. In this case at the time the system
reaches the maximum – the “sphaleron moment” – the
kinetic (electric) energy is zero, after which the system
may fall downhill into a topologically different configura-
tion. We have already mentioned in the Introduction, it is 7 times
less than the KM sphaleron mass, and for the tempera-
tures we are dealing with \( T_{in} = 200 - 100 \text{GeV} \) it makes a
huge difference for the rate.

The sphaleron Boltzmann factor can now be estimated, as we know both the total energy and the internal tempera-
ture \( T_{in} \) from our bag model for the no-Higgs spot:

\[
\exp(-E_{\text{tot}}/T_{in}) \approx \exp(-2000 \text{GeV}/174 \text{GeV}) = 10^{-5} \quad (22)
\]

This is not yet the end of the estimate, since semiclassical sphaleron rate has also a significant preexponent. It has
not been calculated for COS sphaleron yet, so we use the
KM one

\[
\Gamma \sim \frac{\omega_{s}T^{3}}{2\pi m^{4}}N_{tr}N_{rot}(\frac{\alpha_{w}}{4\pi\alpha_{3}})^{3}\exp(-E_{\text{rot}}/T) \quad (23)
\]

which includes the unstable frequency \( \omega_{s} \approx 2M_{W} \), as well
as the numbers due to translational modes \( N_{tr} = 26 \) and
rotational modes \( N_{rot} \approx 5300 \). There are also factor ~
\( O(1) \) from the non-zero mode determinants. We used
here \( \alpha_{3} = g_{3}^{2}/4\pi = \sqrt{2}m_{W}g_{s}^{2}T \).
Combining all the factors we find that numerical value of preexponent and exponent nearly cancels out, leaving crudely

\[ \Gamma V m^4 \sim 10^{-1} \]  

with accuracy say an order of magnitude or so. With that accuracy it agrees with the results of the simulations which also finds that the number of sphaleron transitions per spot is indeed about several percents.

What should happen after the sphaleron moment? Sphaleron decay is classical rolling of the classical (high amplitude) gauge field downhill, from the (sphaleron) top into the next classical vacuum. This process was extensively studied numerically for KM sphaleron. Remarkably, an analytic solution of the time-dependent explosion of COS sphaleron has also been found, see the COS paper. We will not describe the details of that here, just give the expression for the late time \( t \gg \rho \) profile of the energy density of the expanding shell.

\[ 4\pi \epsilon(r,t) = \frac{8\pi}{g^2 \rho^2 r^2} \left( 1-k^2 \right)^2 \left( \frac{1}{1+(r-t)^2/\rho^2} \right)^3 \]

Comparing the explosion of COS sphaleron with numerical data one can see both the similarities and the differences between them. As seen from Fig. there is an empty shell formation at some time. However the inside of the shell does not remain empty: in fact the topology and magnetic field have a secondary peak (of smaller magnitude). Qualitatively it is easy to see why it happens. The COS sphaleron is a solution exploding in zero Higgs background, with massless gauge fields at infinity. In the numerical simulations we discuss such explosion happens inside the finite-size cavity. As the gauge bosons of the expanding shell hit the walls of the no-Higgs spot, they are massive outside. With some probability they get reflected by this wall back. This is the reason why a secondary splash (the second magnetic peak) is produced. It would be possible and interesting to study this problem separately, obtaining COS-like explosion in a spherically symmetric bag: we hope to do so elsewhere.

Let us make a small theoretical digression here. For both for KM and COS sphaleron explosions there was well known controversy about relation between the Chern-Simons number and the baryon charge for nonstatic solutions. At time going to infinity the Chern-Simons number during the sphaleron explosion has been found to be different from naively expected \( \Delta N_{CS} = 1/2 \). This happens because even a weak field at late time still preserves some topology. Nevertheless, it was somewhat troublesome before the issue of the fermion number violation has been resolved by Shuryak and Zahed in [32], who found the analytic solution of the Dirac eqn in a time-dependent exploding COS background. They found that any fermions starting from the sphaleron zero mode are indeed excited into physical (positive energy) states at late time. The outgoing fermions have simple momentum spectrum

\[ n_L(k) = \rho (2k \rho)^2 e^{-2k \rho}. \]

which is close to thermal with \( T_{eff} = 2/\rho \), and the average quark energy \( < E_q > = 3/\rho \). Thus the amount of baryon number violation by sphaleron transition is in fact as expected from the anomaly.

III. THE ROLE OF THE TOP QUARKS

A. Top quark production and concentration in the bags

Since the original numerical simulations have included the gauge fields but ignored fermions, we have to discuss first, at quite qualitative level, what their effects can be, relative to that of the gauge fields. Those tops would be added to the metastable bubbles of the symmetric phase, the no-Higgs spots, like the W discussed above. However, there are at least three important differences:

(i) their fugacity is expected to be larger, due to more rapid production rate of \( t \) relative to \( W \).

(ii) the degeneracy factor \( g_{t,t} = 12 \) is twice larger than for gauge bosons (6).

(iii) their binding to Higgs bags is several times larger than for \( W \), which makes it easier to satisfy the mechanical stabilization of the bag. (For details on that see [17].)

The first question is at what time and how the top quarks would be produced. Top quark has the largest coupling to Higgs: so it is produced first via \( HH \to \bar{t}t \) process. Let us crudely compare its rate relative to that of \( HH \to WW \)

\[ \frac{\Gamma(HH \to \bar{t}t)}{\Gamma(HH \to WW)} \sim \left( \frac{m_t}{m_W} \right)^4 \sim 20 \]

where in the first estimate follows from the fact that the (vacuum) masses are proportional to the corresponding coupling constants, which appears in the power 4 in the rate. This simple estimate suggests that top production may be by about an order of magnitude more rapid than that of \( W \), and thus they would in fact dominate in the formation of “spots”.

Obviously, specific simulations are needed to test top production rates. We are not currently able to evaluate top production, give out-of-equilibrium and from strongly fluctuating Higgs fields at the preheating stage. As a crude guess one may look at an equilibrium density in the bulk before the appearance of broken phase VEV

\[ n_{max}(\bar{t}t) = \frac{3g_{t,t}}{4\pi^2} T^3 \approx 1.1 T^3 \]

The next step is concentration of top quarks into the bags. The simple reason this is happening is that they are massive in the broken phase and massless in the symmetric one. As discussed in detail in [17], their binding can be as large as 100 GeV or so. Less trivial reason is that a “1d kink” of the Higgs field (a 2-d surface where Higgs VEV crosses zero) possesses fermion zero modes. In practice it means that top quarks can glue themselves
to the $\phi = 0$ surfaces and be transported along them, eventually into the remaining island of the symmetric phase. The binding energy in such case is nearly all the top mass, 172 GeV, several time the bulk temperature. It indicates that the effect can be rather robust: but only dedicated simulations can decide how effective this mechanism can actually be, and what fraction of the produced tops end up in the bags.

In order to see possible degree of such concentration, note that after tops are collected into the bags, they remain effectively massless, so their density inside is given by the same expression, with high $T_{in} \sim 200$ GeV needed to balance the bag pressure. The density in the bulk is given by the integral with the top mass and low $T_{bulk} \sim 50$ GeV

$$n_{bulk} = g_{tt} \int_{M}^{\infty} \frac{dE}{2\pi^2} \frac{\sqrt{E^2 - M^2}}{E} \exp(-E/T) \, (29)$$

which is smaller than the density $n_{max}$ inside the spots by about factor 300! It is thus clear, that after the broken phase is established in the bulk, most ($\sim 99\%$) tops should either be annihilated or collected into the bags.

Let us first estimate it from above, assuming that all heavy quarks can be collected into the remaining spots of the symmetric phase. If so, the number of tops per spot is just the ratio of their densities

$$N(\bar{t}t) = \frac{n(\bar{t}t)}{n_{spots}} \sim 10^2 \left(\frac{T}{m}\right)^3 \, (30)$$

where $T$ is a characteristic temperature of the Higgs field at early time, $T/m \sim 1$. Thus we expect $N_{t,\bar{t}}$ of the order of a hundreds of tops+antitops be collected into a no-Higgs spot.

Since we are not able to model the process quantitatively, in a highly fluctuating Higgs background we will introduce a parameter, normalizing this process to that of the W. So, we will denote the number of tops normalized to Ws which made into the “spots”:

$$\kappa = \frac{N_{\text{top}}}{N_{\text{spot}}} \, (31)$$

The population of the spots including W, tops and antitops would then be $(1 + 2\kappa)$ times larger than in bosonic simulations.

B. The W and the top quark lifetime in the bag

The lifetime of the W according to their weak decays is, in our units,

$$\tau_{W} m \sim \frac{m}{\Gamma_{W}} \approx 50 \, \, \, (32)$$

The lifetime for bound $W$’s is presumably reduced, according to their total energy, increasing by another factor 2 or so. This timescale is several times larger than the lifetime of the “bags” in the numerical simulations, which is of the order of $10^{-20}$ in the same units.

Thus the weak decays can be neglected (as they were in the numerical simulations) and the observed lifetime in the simulations should be attributed to evaporation of the W’s into the bulk, by thermal excitation. Indeed, the “hot spots” of the reduced Higgs VEV were considered there as mere fluctuations of the fields. We however look at them as metastable objects, which can come into mechanical equilibrium and decay more slowly due to heat transport as well as strong and weak interactions, ignored so far.

As discussed in [17] in detail, we found that tops tend to be on the surface of the bag, due to the influence of the “zero mode” phenomenon. For how long would $t, \bar{t}$ quarks remain in the bag, before their decay? Because of hierarchy of the couplings, one should first look at annihilation via strong interactions. One of such processes leads to production of other quarks via $tt \rightarrow \gamma \gamma$, another is the annihilation into gluons $tt \rightarrow \gamma \gamma$, which give the lifetime $\tau_{tt} \sim 200/T_{in}$, see Appendix B. This is about an order of magnitude longer time than the lifetime of the bosonic “no-Higgs spots” in [29].

(Another process $tt \rightarrow HH$ has a bit smaller coupling and less final states, especially in color, so it is clearly subleading.)

After strong annihilation of pairs, the remaining number of tops or antitops are due to random charge fluctuations during the formation stage

$$N_{t,\bar{t}} \sim \sqrt{N_{t,\bar{t}}} \, \, \, (33)$$

Assuming that this is the case, one may ask how the tops would decay further. In vacuum the physical top quark is much heavier than gauge quanta, so they decay $t \rightarrow b + W^{+}$ in the first order in weak interactions. The corresponding width is well known

$$\Gamma_{t} = \frac{a_{w}}{16} \left(\frac{m_{t}^{2}}{m_{W}^{2}}\right) \left(1 + 2 \frac{m_{W}^{2}}{m_{t}^{2}}\right) \left(1 - \frac{m_{W}^{2}}{m_{t}^{2}}\right)^{2} \, \, (34)$$

However, for both $t$ and $W$ bound in the bag it is by no means certain whether $m_{t} > m_{W}$ or not. As we had shown in [17], for large enough bags the lowest top levels are in fact lower than the lowest W ones. When the corresponding top energy levels are still above the W states, such weak reaction is possible. Crude estimate of the weak decay width can be obtained if in this formula we substitute masses by energies, writing (for $E_{t} > E_{W}$) the width as

$$\Gamma_{t} \sim 10^{-3} (1 - E_{W}/E_{t})^{2} \, \, (35)$$

In practice, this leads to a lifetime of the order of $\tau_{weak} \sim m_{t}/\Gamma \sim 10^{3}$. When the top quarks are heavier than $W$’s in the bag decays $t \rightarrow Wb$ become impossible. The weak decays would the proceed into the 3 body, say $t \rightarrow bl\nu_{t}$ like in
the usual beta decays. This produce an extra power of \( \alpha_w \) and much stronger dependence on the energy released because of 3 body phase space. This width, crudely estimated as

\[
\Gamma_i \sim \alpha_w^2 \frac{\Delta E_i}{E_i} \tag{36}
\]

where by \( \Delta E_i \) we mean the change in the total energy when one top disappears, is way too small and exceed all the scales in the problem (except the time of cosmological expansion).

In summary, \( m \tau_r \sim 200 \) is the lifetime due to to strong top annihilation, while their weak decays take no less than \( m t \sim 10^3 - 10^4 \) time, or even much more if the bags are large enough. This should be compared to \( \tau_W m \) of about 100 for the W’s, which would clearly be shorter and thus limit the lifetime of the bags. The lifetime \( \tau_m \) of about 20 for the bosonic spots in simulations is due to thermal evaporation: presumably a deeper binding and decrease of internal temperature due to tops will improve it. We conclude with a crude estimate of the bag lifetime as \( \tau_m \sim 100 \).

IV. THE SPHALERON TRANSITIONS IN THE TOP-STABILIZED BAGS

A. The exponential effects and the lifetime

As argued before, tops would be an important ingredient in the metastable bags with near-zero Higgs VEVs. Now we turn to the next question: how much the sphaleron rate can be affected by their presence, as compared to purely bosonic ones in the simulations [20]?

Since in this case there are more particles in the bags, with the tops added to gauge bosons, one may naively think that the bag energy would grow. But using a simple bag model we have already pointed out that this would not be the case: the mechanical stability require total pressure to be zero, which related the internal pressure to the bag constant \( B \). No matter which light degrees of freedom are inside, the bag energy would scale as \( BV \). The Boltzmann factors for sphalerons however would still depend on the number of degrees of freedom \( N_{DOF} \) in the bag because the temperature depends on it. This again follows from the mechanical stability condition

\[
N_{DOF} T_{in}^{3} + 1 \sim B \tag{37}
\]

Thus adding tops to the bag simply reduces the internal temperature. Returning to the exponential approximations for the sphaleron rate [18], we now put into it the optimal radius [19] and also rescale the temperature as in (37). One finds that the exponent of the sphaleron rate depends on the number of degrees of freedom as

\[
\sim (N_{DOF})^{0.25} \quad \text{changes by just few percents only.}
\]

We thus conclude that the cancellations due to mechanical stability condition (37) makes the effect of tops on the exponent small, within the level of uncertainties of the model used.

Now we turn to the discussion of the preexponent. Obviously it depends nontrivially on the presence of the tops in the bag, as fermions are related to the probability of topological fluctuations by the anomaly: we will turn to this effect in the next section. If the \( T^{4} \) in the rate comes from some thermal average, one may think that it will come multiplied by (at least) one power of \( DOF \) factor. If so, again the increase of the \( DOF \) in the bag due to tops would be partially canceled out by a reduced internal temperature, since the product is the bag pressure \( B \). The preexponent simply has to be recalculated, for COS sphalerons in the bag environment (in which many fields such as gluons are absent because of different cosmological scenario).

We have argued above that tops have longer lifetime in the bag, which may increase the overall bag lifetime. At the other hand, bag stability relies on \( W, Z \) presence and thus it cannot exist longer than the \( W \) lifetime, \( \tau_W m \sim 200 \). Thus the potential enhancement of the hot spot lifetime cannot be larger than factor 10.

B. Recycling the top quarks

The well known Adler-Bell-Jackiw anomaly require that a change in gauge field topology by \( \Delta Q \pm 1 \) must be accompanied by a corresponding change in baryon and lepton numbers, \( B \) and \( L \). More specifically, such topologically nontrivial fluctuation can thus be viewed as a “‘t’Hooft operator” with 12 fermionic legs. Particular fermions depend on orientation of the gauge fields in the electroweak \( SU(2) \): since we are interested in utilization of top quarks, we will assume it to be “up”. In such case the produced set contains \( t_r, b_r, g_r, c_r \), \( c_b, u_r, u_b, g_b \), \( r, b, g \), where \( r, b, g \) are quark colors. We will refer below to this process as the \( 0 \to 12 \) reaction. Of course, in matter with a nonzero fermion density many more reactions of the type \( n \to (12 - n) \) are allowed, with \( n \) (anti)fermions captured from the initial state.

Although in this work we try to follow thermal language (local kinetic equilibrium) as much as possible, let still discuss what should be done in a purely dynamical out-of-equilibrium setting. In principle, one should proceed quantum mechanically, starting and ending with certain number of ingoing and outgoing quark and gauge quanta in certain in and out states (e.g. fixed momenta) projecting those into the fermionic and gauge field configurations of the semiclassical theory of the sphaleron process. The sphaleron itself is just one – saddle – point at \( t = 0 \) on the path, which start from convergent waves and end with divergent ones.

The whole classical solution describing the expansion stage at \( t > 0 \) has been worked out for COS sphaleron explosion, and for the “compression stage” at \( t < 0 \) one can use the same solution with a time reversed. At very early time or very late times \( t \to \pm \infty \) the classical field become
weak and describe convergent/divergent spherical waves, which are nothing but certain number of colliding gauge bosons. Fermions of the theory should also be treated accordingly. Large semiclassical parameter – sphaleron energy over temperature – parametrically leads to the assumption that total bosonic energy is much larger than that of the fermions, so one usually ignores backreaction and consider Dirac eqn for fermions in a given gauge background. For KM sphaleron and effective $T$ we discuss, this parameter would be $\sim 70$, which is indeed large compared to 12 fermions. However in the case of COS sphaleron we are going to use the number is about $\sim 10$, comparable to the number of fermions produced. It implies that backreaction from fermions to bosons is very important.

To our knowledge, the only (analytic) solution to Dirac eqn of the “expansion stage” was obtained in [32], it describes motion from the sphaleron zero mode at $t = 0$ and all the way to large $t \to +\infty$, the outgoing physical state of fermions, with momentum distribution $|\tilde{\mathcal{P}}|^2$. A new element we are adding now is that its time-reflection can also describe the compression stage, from free fermions captured by a convergent spherical wave of gauge field at $t \to -\infty$ and ending at the sphaleron zero mode at $t = 0$.

How the corresponding fermionic levels move is schematically shown in Fig.4. In the Dirac sea notations, the level moves from some negative initial energy at early time to positive one at late time. For COS sphalerons the mean energy is $E_q = -3/\rho$ to $E_q = +3/\rho$ By symmetry, the level is crossing zero at $t = 0$: the corresponding fermionic zero energy state is known as the sphaleron zero mode. If the level was occupied (as most of levels below the surface of the Dirac sea are) it remains occupied and one gets new fermions produced. If there was antiquark (an unoccupied state or a hole, indicated by the open circle) it will remain unoccupied, thus having no meaning after the level moves to positive energy, where most levels are unoccupied. So, antiquarks – holes at negative energy sea – are actually decelerated by the radial electric field to zero energy modes, their energy being transmitted to the gauge field.

This conclusion is very important: it implies that energy of the initial fermions can be incorporated and used in the sphaleron transition. If initially fermions were in certain state (e.g. certain level of the Higgs bag $|\eta\rangle$), the fermion part of the sphaleron amplitude would include the amplitude of their capture into a “sphaleron doorway” state $|\text{doorway}\rangle$ by a convergent gauge field wave. As we will discuss in detail below, we will find that the optimum way to generate sphaleron transition is to use 3 initial top quarks, considering the $3 \to 9$ fermion process instead of the original $0 \to 12$ one. In order to satisfy Pauli principle and fit into the same sphaleron zero mode, colors of the 3 quarks of each flavor should all be different.

Let us show by simple estimates that under conditions we are discussing the $0 \to 12$ fermion production process is actually impossible. The final kinetic energy for fermion produced by COS sphaleron is $E_q = 3/\rho$ which is $\approx 200 \text{GeV}$ for the $m_\rho = 3.9$ example displayed as typical in Fig.3. Multiplied by 12 fermionic species produced it would require $2.4 \text{TeV}$ of energy which apparently exceeds the total available energy of the gauge fields in the topological fluctuations observed numerically.

(Note finally, that inclusion of the CP-violating processes – such as induced by CKM matrix – may lead to a small difference between recycling of the top and the antitop quark amplitudes. This asymmetry would then be a mechanism for the baryogenesis.)

The $3 \to 9$ fermion process saves a lot of energy, as in it the initial top quark energy can be completely transferred from the “sphaleron doorway state” to the gauge field during the compression stage. In the example we discuss, with $m_\rho = 3.9$, this mean energy of 3 top quarks is

$$\Delta E = 3 \times 3/\rho = 600 \text{GeV}$$

An estimate with exponential accuracy we find its enhancement by the factor of about

$$F_{\text{recycling}} \sim \exp(\Delta E/T) \sim 20$$

using the hot-spot temperature $T_{in}$. Note that the same amount of energy should also not be produced in the final state: thus in terms of energy balance this amount is doubled, saving $\sim 1200 \text{GeV}$ and making the transition possible. Yet this gain does not increase the rate any further, as the system falls downhill classically the probability of everything to happen being just 1, due to unitarity.

Can one continue to use the same mechanism for more initial-state fermions, for example considering the capture of 3 tops plus $n$ other quarks, gaining another factor $\sim e^{n \Delta E/T}$. In particular, should one focus on $6 \to 6$ transition, which is “energy neutral”? The answer to this question is negative: for quarks other than $t$ there is no reason to get trapped inside the no-Higgs spots, as Higgs effect on them (the mass) is too small compared to even low bulk $T_{out}$ and they occupy the bulk of the volume. Roughly their density is thus smaller than that of top quarks by the factor $(T_{out}/T_{in})^3$, which is few percents.

![FIG. 4: Schematic picture of fermionic level motion as a function of time. Open and closed circles indicate unoccupied and occupied states.](image-url)
The cube of this factor obviously cannot compensate the gain from the exponent.

The preexponent for the $3 \to 9$ fermion process should include projection of the initial states of fermions (say bound states of $t\bar{t}t$) in a bag onto the appropriate “fermionic doorway state” of the sphaleron process. As it was already mentioned, those have been found in [32] for COS sphalerons: they have simple exponential form. Lacking bound state wave functions in momentum representation, we can use the convolution with thermal state

$$P_t = \frac{1}{2n_t(T_{in})} \int_0^{\infty} dp \frac{(2\rho)^2 \exp(-2\rho)}{\exp(p - \mu_t/T_{in}) + 1}$$  \hspace{1cm} (40)

Factor 2 in denominator appears because we only integrate over momenta directed inward, and $n_t(T_{in})$ is the thermal quark density. Note that $\mu_0 \sim 4$ in numerical model we discuss, so the corresponding effective temperature of the doorway state is $1/(2\rho) \sim m_0/\rho \sim 30\text{GeV}$ is softer than the actual internal $T_{in} \sim 200\text{GeV}$. This mismatch would somehow reduce the preexponent. On the other hand, one can further optimize the rate, selecting optimal (higher than average) momenta of the top quarks, to affect even the exponent.

V. CP VIOLATION

Finally, let us discuss the last crucial component of the baryogenesis, the CP violation. As it is widely known, all so far observed CP violation in $K, B$ meson decays can be explained inside the SM, in which it comes from the phase of the CKM matrix.

(The only exception to this is a very recent result from D0 collaboration [37] on same-sign dilepton asymmetries which are about -0.01, a 3.2 standard deviation from much smaller SM prediction $(-2.3 \times 10^{-4})$. Its origin is believed to be the $B_d \leftrightarrow B_s$ oscillations: in a couple years we will know if it withstand further data samples from D0, CDF and LHCb.) Returning to SM and its CKM matrix, it would be desirable to establish a specific process and prediction, which will follow from it for the cosmological scenario under consideration. Since in this scenario two (of the three) Sakharov’s conditions – the deviation from equilibrium and the baryon number violation – are in a sense maximized, so the required magnitude of the CP violation in this scenario need to explain the observed asymmetry is minimized, to something of the order of $10^{-7}$ or so. Below we discuss whether such level of CP violation can actually be reached inside the SM.

A. Optimizing the one-loop CP-odd effective action

The magnitude of the CP violation can be derived by standard one-loop effective action which is $logdet D$, with the Dirac operator $D$ including Higgs and gauge fields. The well-known Jarlskog argument corresponds to the situation in which the field mass scale

$$F^2 \sim W_{\mu\nu} \gg m_q^2$$  \hspace{1cm} (41)

is larger than that of any quarks. In this case the coefficient of the effective operators must be proportional not only to the so called Jarlskog invariant $J$ but also to extremely small Jarlskog determinants

$$\delta L_{CP} \sim \frac{J}{F^2}(m_{q}^2 - m_{d}^2)(m_{u}^2 - m_{d}^2)(m_{b}^2 - m_{t}^2) \sim J m_{t}^4 m_{b}^4 m_{u}^4 m_{d}^4 / F^4$$  \hspace{1cm} (42)

because when each two same-charge quarks are degenerate the CP odd phase can be rotated away to zero, nullifying the CP-odd effect. Since the field inside the bags we discuss is of the scale $F \sim 100\text{GeV} \times \sqrt{N}$, this is indeed the case and thus we have to conclude that CP odd effect inside the bag is too small.

Let us now try to optimize the effect, by looking at all scales $F = m_t, m_b, m_c, m_s$, subsequently. The corresponding expressions and numerical values (for $J = 10^{-5}$ and actual quark masses) are

$$\delta L_{CP}(m_t) \sim \frac{J}{m_t^2}(m_{c}^2 - m_{d}^2)(m_{u}^2 - m_{d}^2)(m_{b}^2 - m_{t}^2)(m_{s}^2 - m_{d}^2) \sim 3 \times 10^{-13}\text{GeV}^4$$  \hspace{1cm} (43)

$$\delta L_{CP}(m_b) \sim J(m_{c}^2 - m_{u}^2)(m_{s}^2 - m_{d}^2) \approx 4 \times 10^{-7}\text{GeV}^4$$  \hspace{1cm} (44)

$$\delta L_{CP}(m_c) \sim J m_{c}^2 (m_{s}^2 - m_{d}^2) \approx 4 \times 10^{-7}\text{GeV}^4$$  \hspace{1cm} (45)

$$\delta L_{CP}(m_s) \sim m_{s}^4 J \approx 4 \times 10^{-9}\text{GeV}^4$$  \hspace{1cm} (46)

Thus, at this level of accuracy, the scales of $b$ and $c$ give comparable effects. There are however two more consideration which favor the $b$ quark scale as the optimal one.

One is related with “thermal masses” in the medium of the quark outside bags. Weak thermal (Klimov-Weldon) quark mass for left handed fermions due to scattering on thermal $W, Z$ is (see e.g. [38])

$$\delta M^2 = \frac{\pi T^2}{8}$$  \hspace{1cm} (47)

which for the outside temperature $T \sim 50\text{GeV}$ is about $(5\text{GeV})^2$. This independently points to the possibility to go down to the scale of the $b$ quark mass in all fermionic propagators.

The second consideration is specific to the hybrid scenario we discuss. In this case the light quarks are produced not from Higgs but from weak $W, Z, t$ decays, and gluons from quark annihilation: and at the initial period
of the electroweak phase under consideration both are not happened yet. For this reason we have not included gluonic mass in the previous paragraph. For the same reason we expect the mechanism due to a difference in s and d quark scattering on the “bags” be subleading to b scattering or b → t transitions.

In order to do the actual calculation one needs to calculate explicit CP violating Lagrangian and integrate it over the space-time volume. As a crude estimate, let us calculate explicit CP violating Lagrangian and integrate it over the space-time volume. As a crude estimate, let us calculate explicit CP violating Lagrangian and integrate it over the space-time volume. As a crude estimate, let us calculate explicit CP violating Lagrangian and integrate it over the space-time volume. As a crude estimate, let us calculate explicit CP violating Lagrangian and integrate it over the space-time volume. As a crude estimate, let us calculate explicit CP violating Lagrangian and integrate it over the space-time volume. As a crude estimate, let us calculate explicit CP violating Lagrangian and integrate it over the space-time volume.

The closest to it is the calculation in Ref.[35] for the scale below $m_c$, which produced the following operator of dimension-6

$$\delta L_{CP}(< m_c) \sim J m_c^2 e^{\mu \lambda \sigma}$$

$$[Z \mu W^\mu_\lambda W^\lambda_\sigma (W^\sigma_\nu W_\nu^\sigma + W_\sigma^\nu W_\nu^\sigma) + cc]$$

Some elements of its general structure, with 4-dim epsilon symbol is not yet calculated (to our knowledge).

Averaging this operator over fields found in numerical bosonic simulations has been performed in ref.[36], with the result four orders of magnitude larger than needed. It is however clear that one actually cannot apply this Lagrangian, since this operator was derived for a completely different field scale $F \ll m_c$, which is not the case in the numerical simulations in question. Not surprisingly, the obtained result is larger than the observed one: the dimension 6 of the field would produce very large contribution from the “hot spots” with very strong gauge fields. However, as follows from Jarlskog argument (repeated above) at larger field there is additional strong suppression due to mass differences, which will kill it as detailed above.

B. CP violation in top decays: the “penguins” and the final state interaction

It is well known that CP violating effects in particular meson decays are much larger than what one would get from the one-loop gradient expansion effective Lagrangians like the one discussed above. Instead of high order in weak interaction, one may find the needed four CKM matrices in the interference terms between lowest order weak interaction and strong radiative corrections, the so called “penguin” diagrams. Fig.5 shows two generic diagrams, which can interfere and produced the CP-violating asymmetries from the famous complex phase in the CKM matrix. As seen from the figure, there are six generic decays for all possible values of the final quarks:

$$t \to \bar{d}du, t \to \bar{s}sc, t \to \bar{d}dc, t \to \bar{b}bc; t \to \bar{s}su; t \to \bar{b}bu$$

The amplitude of the first one can be written as

$$A(t \to \bar{d}du) = V_{ud}^* V_{ub} F_{\text{penguin}}$$

in which we selected $D' = b$ and introduce the so called penguin factor, including in particular the propagator of an extra gluon and the strong coupling constants. The interference term between these two terms would provide the needed four CKM matrices, while the “time arrow” can be provided by a specific direction of the flow, e.g. leakage of bound tops into the unbound lighter quarks leaving the bag. (Note again, that in equilibrium there would be inverse processes which will cancel the asymmetry, but those are absent in the scenario we discuss: the light quarks are very much out of equilibrium and their density is negligible at the time we consider.)

The ratio of the two terms generates CP asymmetry, which for this decay is as large as

$$CP \sim \frac{V_{ub}^* F_{\text{penguin}}}{V_{ud}^*} \sim \eta F_{\text{penguin}}$$

Other modes have smaller asymmetry by powers of $\lambda$, but their decay rates are larger.

To get the average effect one has to evaluate the effective CP-odd Lagrangian or the imaginary part of the two terms, their interference. For all 6 decays mentioned above

$$\frac{\delta L_{CP}}{F^4} \sim \alpha_w^2 Im[V_{ud}^* V_{ub} F_{\text{penguin}} V_{td}^* V_{tb} F_{\text{penguin}}]$$

$$\sim \alpha_w^2 \lambda^6 F_{\text{penguin}} \approx 10^{-8}$$

where we have used the Wolfenstein parameterization (to $\lambda^4$ accuracy) of the CKM matrix, and include “penguin
suppression” factor $\sim 1/3$. The net effect of this Lagrangian would be a difference between the number of bound tops and antitops in the bag, and with “top recycling” leading to asymmetric sphaleron transitions and baryon asymmetry.

(Note that the quark mass differences entering the Jarlskog determinant argument of the previous subsection. It resists in the summation over the $D'$ quarks. If all three Penguin diagrams would be exactly the same, one would get the following combination

$$V_{tb}V_{ub}^* + V_{ts}V_{us}^* + V_{td}V_{ud}^*$$

which is zero because of the orthogonality condition of the two rows. Here however where different quark masses and “external conditions” (in form of the external thermal mass in the propagators of the $D'$ quarks) should help to produce different coefficients. Since the scale of such external effects are of the order of $F \sim$ few GeV, they clearly separating the $b$ penguin term from two others. As in the previous section, the remaining suppression of the order of $(m_s - m_d)/F$ may yet appear.)

Most CP in meson decays are due to mixing of the neutral meson-antimeson states: this but is not needed. Top/antitop decays are like those of the charged meson, which will have a CP effect due to the final state interaction, or “strong phases” as they are usually called. The first and second diagrams should be “dressed” by the gluon exchanges between the quarks (not shown in Fig.), it will result in some phases $\delta_{\text{direct}}, \delta_{\text{penguin}} \sim O(\alpha_s)$, which are the same for quarks and antiquarks. They are not the same for different final states, for example if the quark pair produced from a gluon in a penguin has small invariant mass, it corresponds to a pair produced at relatively large distances from the original quark, while the “direct” decay diagram is near-pointlike because of large $W$ mass. Standard arguments show that there is a difference in the decay rates of particle and antiparticle

$$|A(t \rightarrow f)|^2 - |A(\bar{t} \rightarrow \bar{f})|^2 \sim \sin(\delta_{\text{direct}} - \delta_{\text{penguin}})\sin(\delta_{CP})$$

(54)

Only recently the first such asymmetry has been observed in a difference between the width $\Gamma(B^+ \rightarrow \rho_0 K^+)$ decay and that of its CP conjugate. So, there is no doubt that partial widths of the $t$ and $\bar{t}$ decays can differ. While the CPT theorem forbids the difference in their total width, for time dependent bags well out of equilibrium one may still hope that it would be possible to go around strict CPT it and generate a difference in $t$ and $\bar{t}$ population, perhaps even of the magnitude comparable to the observed asymmetries in its decay modes.

C. Baryon charge separation during the bags formation

Another possible manifestation of the CP-odd phenomena is possible separation of the quarks vs antiquarks already during the process of bag formation, in the initial very off-equilibrium stage. If it happens, then the amount of tops and antitops in the bag would already be different, leading to an asymmetry in sphaleron transitions which “recycle” those tops, as detailed above.

Farrar and Shaposhnikov [38] were the first who discussed it in detail, in a specific case of strange quarks and near-equilibrium bubbles at $T = T_c$, assumed at the time to be the first order transition. Although we would use this phenomenon in a quite different setting, we would still call it “the Farrar-Shaposhnikov (FS) mechanism”. Its essence is the interference between the (CP-even) scattering/reflection on the bag wall and the (CP-odd) transition via the CKM matrix into a different flavor, reflection, and return to the original flavor. Let us remind the reader how it works. Let us start with flavor $f_1$: if it is reflected from the bag wall it gets the phase $\delta_1$. Alternatively it can turn into the flavor 2 and scatter with the phase $\delta_2$, picking on the way CP-odd phase from complex CKM matrix elements. The total baryonic current is a difference between those for quarks and antiquarks

$$|e^{i\delta_1} + Ce^{i\delta_2 + i\delta_{CP}}|^2 - |e^{i\delta_1} + Ce^{i\delta_2 - i\delta_{CP}}|^2 \sim \sin(\delta_1 - \delta_2)\sin(\delta_{CP})$$

(55)

The essential part of the argument is that two flavors under considerations should have different scattering on the bag, as the effect disappear at $\delta_1 = \delta_2$. This condition is especially difficult for light quarks, as they only weakly interact with the Higgs and thus the bag. FS has argued that the effect should be nonzero for $s$ relative to $d$ quarks, but only in a narrow strip of energies near the level crossings, where $s$ quarks are all reflected (large $\delta_1$) while $d$ are transmitted (negligible $\delta_2$). As a result, their estimate for the FS effect is proportional to a small parameter corresponding to the relative width of this region, $(m_s - m_d)/T \sim 10^{-3}$.

Another important part of their paper (which we would not remind here) is discussion of the unitarity and CPT constraints on possible effects, leading to a standard conclusion that in equilibrium (e.g. for a bubble wall at rest with the medium) the effect vanishes, as it indeed should from general considerations [1].

Let us now point out large differences which exist between the matter properties discussed by FS and that in the hybrid scenario under consideration. First of all, there are drastic differences in its chemical composition. Equilibrium matter at electroweak scale has about hundred effective degrees of freedom, including all quarks and leptons, as well as gluons. Therefore effective fermionic thermal masses are dominated by the gluonic part, with a scale $\sim T/2 \sim 50$ GeV. In the hybrid scenario there was not yet enough time to generate light quarks, leptons and gluons: the only quanta produced are weak gauge bosons $W$, $Z$, Higgs and the top quarks, all due to their large coupling to (strongly fluctuating) Higgs field. The spatial distribution of tops is very space and time-dependent, as they flee the regions with large Higgs VEV and are collected into the bags we discuss. While doing so, they can
also migrate, via the CKM matrix, into all other flavors, driving the FS mechanism and producing different top and antitop population in the bags.

For simplicity, let us imagine the bag to be very large and its boundary flat, separating the space into two halves, with symmetric ($\nu \approx 0$) and broken ($\nu \approx \nu_3$) phases. The flow, into symmetric phase, is driven by the difference in the chemical potential. As we have discussed throughout this paper, the temperature is also not constant over the divide, with $T \sim 50 GeV$ in the broken phase outside the bags. Tops in the bags (symmetric half) with energy less than the top mass outside $E < M_t (T_{out})$ cannot leave the bag and are reflected back if collide with the boundary: thus large $\delta_t$. If they however migrate into the $c$, $u$ quarks, those hardly see a bag at all, thus $\delta_c, \delta_u \approx 0$. Additional small parameter appearing in FS estimate is the part of the energies in which $c$, $u$ contributions do not cancel is $(m_c - m_u)/T \sim 1/50$, not so small. Together with Jarlskog $J$ from CKM, it will generate asymmetries of the order of $10^{-6}$ or so, which is right in the needed ballpark.

(A word of warning: The original FS estimate of the effect has been criticized in refs [39]. One of the points is that additional suppression appears from the widths of quarks coming from in media scattering, and we are not sure what is the final word in the polemics about it. Anyway, this criticism is much less relevant to the scenario discussed, as larger scale $m_c$ is substituting $m_s$ in FS, which is rather close to – much reduced relative to equilibrium matter – rescattering widths.)

VI. SUMMARY AND DISCUSSION

In summary, we have considered the role of the $W - Z$-top bags in baryogenesis. We have used, as a specific example, numerical simulations for the hybrid preheating scenario from Ref. [29].

(i) We first reproduced the main results of those simulation using the $W-Z$ bags and COS sphaleron, including qualitatively the sphaleron size and rate found numerically.

(ii) We then discussed the role of the top quarks and found them to be quite important. They are produced via $HH \rightarrow tt$ more effectively than $tW$. They are collected into the bags together with the gauge quanta, presumably producing bags of larger size, with improved lifetime. We however found that the mechanical stabilization condition (which makes bags with tops cooler than pure $W$-bags) basically makes the sphaleron exponent constant, independently of how many top quarks are in the rate.

(iii) In addition, there is an interesting effect of “top quark recycling”. This effect helps the transition, providing significant ($\sim 1/3$) fraction of the energy needed to reach the sphaleron mass. The recycling of 3 tops increases the rates by another factor 20.

(iv) We then provide arguments that optimal CP violation should happen either due to high order weak interaction in the area between the bags, at a scale of $m_t \sim 5 GeV$, or in top decays, or in top selection during bag formation due to Farrar-Shaposhnikov effect. Although we are not able to provide specific calculation of any of the three possibilities at this time, crude estimates show that any one of them can potentially provide a magnitude of $10^{-7}$ which, together with the estimated sphaleron rate, would generate the baryon asymmetry of the observed magnitude from known CKM-base mechanism alone. (If the D0 result on much larger CP violation in semileptonic $B_d$ decays would stand in time, one should of course also look at its origins as well.)

Let us finish this paper with a historical analogy. We do see stars on the sky which shine: yet basic nuclear physics was not enough to explain why they can do so, falling short by many orders of magnitude. It took decades of patient and specific work to uncovered the nontrivial ways in which the star cycles work. Those crucially depend on some catalysts, whose presence is very small but crucial. Perhaps the $W - Z$-top bags are also a kind of a catalyst, or a doorway state, helping to increase the probability of the baryon number violation and of the CP violation.

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Appendix A: The coupling constants and parameters

Let us remind the reader the hierarchy of the coupling constants, as it reflects the order in which various processes should be considered.

The largest coupling constant is still that of strong interaction,

$$\frac{g_s^2}{4\pi} = \alpha_s(100 GeV) \approx 0.11 \quad (A1)$$

but all the original bosonic fields of the electroweak theory $W, Z, H$ can only interact with strong interaction via quarks, so it has to play so to say the secondary role.

Quite close to it is the Yukawa coupling of the top quark to Higgs, if written in the same form it is

$$\frac{g_t^{\mu H}}{4\pi} = \alpha_t \approx 0.08 \quad (A2)$$

and this would be responsible for most of the effects we will discuss below. Other Yukawa couplings are much smaller and can be neglected: for example for the
next b quark such combination is smaller by the factor \((m_b/m_t)^2 \approx 7.4 \times 10^{-4}\).

Weak interaction coupling is naturally smaller

\[
\alpha^w \approx 0.03
\]  

(A3)

in the standard model: note however that in numerical simulation we discuss a much smaller value has been used, presumably for some methodical reasons. We will not include weak interactions except in top quark decays. Like in many other works, we will ignore electromagnetism (the U(1) gauge part), putting the Weinberg angle to zero: thus \(m_W = m_Z\).

It is well known, the Higgs mass (or self-coupling \(\lambda\)) remains the last unknown parameter of the SM. Our previous studies of the top-balls put the Higgs mass to a round number \(M_H = 100\, GeV\).

However, since we try to explain numerical bosonic simulations \[29\], let us mention that their choices for the Higgs mass and VEV are different. Details can be seen in their original work, for example the particular set of parameters, called A1 in \[29\] for which the pictures we used are based have rather heavy Higgs mass

\[
\lambda = g^2/2 = 0.00675, \quad m_H/m_W = 4.65
\]  

(A4)

and also a different the value of the electroweak coupling is \(g_W = 1/20\). The Higgs VEV is still \(v = 246\, GeV\) as in real world. The mass parameter of the Lagrangian related to physical Higgs mass by \(m = m_H/\sqrt{2}\) is chosen as a basic mass/energy/length/time unit, which we will also adopt in what follows. With realistic \(M_W\) it makes \(m = 264\, GeV\), not far from \(v\).

Appendix B: Strong annihilation

The differential cross sections for these two QCD reactions are well known, here we present them ignoring quark masses\[41\]:

\[
\frac{d\sigma_{tt\rightarrow \bar{q}q}}{dt} = \frac{4\pi\alpha^2}{s^2} \left[ 1 + u^2/s^2 \right]
\]  

(B1)

\[
\frac{d\sigma_{tt\rightarrow gg}}{dt} = \frac{4\pi\alpha^2}{s^2} \left( \frac{t^2 + u^2}{ut} \right) \left( \frac{8}{3} - \frac{6ut}{s^2} \right)
\]  

(B2)

Integrated it we get\[42\]

\[
\sigma_{tt\rightarrow \bar{q}q} \approx \sigma_{tt\rightarrow gg} \approx \frac{0.022}{s}
\]  

(B3)

Thus the contribution of those two processes to the decay rate of the top quark can thus be written as

\[
\Gamma_{tt} = 2 \times 0.022 \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} f(p_1)f(p_2) \int \frac{d^3p_3}{(2\pi)^3} f(p_3)
\]  

(B4)

where \(f(p)\) is the thermal distribution and \(\theta_{12}\) is the angle between momenta of two colliding tops. Assuming that logarithmic angular integral is regulated by \((1 - \cos(\theta_{min})) \approx 0.1\) we get the following simple answer for the strong annihilation time

\[
\tau_{tt} = 1/\Gamma_{tt} \approx 200/T
\]  

(B5)
A. D. Linde and I. Tkachev, Phys. Rev. Lett. 87, 011601 (2001) [arXiv:hep-ph/0012142].

[25] J. García-Bellido, M. Garcia Perez and A. Gonzalez-Arroyo, Phys. Rev. D 67, 103501 (2003) [arXiv:hep-ph/0208228].

[26] J. García-Bellido and D. Grigoriev, J. High Energy Phys. 01, 017 (2000).

[27] D. Grigoriev, “Electroweak Baryogenesis at and after Preheating: What's the difference?”, in the proceedings of the 35th Rencontres de Moriond: Electroweak Interactions and Unified Theories, E-print archive: hep-ph/0006115 (2000). J. García-Bellido, “Electroweak Baryogenesis from Preheating”, in the Proceedings of COSMO-99, hep-ph/0002256.

[28] J. García-Bellido, M. García-Perez and A. Gonzalez-Arroyo, Phys. Rev. D 69, 023504 (2004) [arXiv:hep-ph/0304285].

[29] J. García-Bellido, M. García-Perez and A. Gonzalez-Arroyo, Phys. Rev. D 69, 023504 (2004) [arXiv:hep-ph/0304285].

[30] A. Tranberg and J. Smit, JHEP 0311, 016 (2003) [arXiv:hep-ph/0310342]. Jon-Ivar Skullerud, Jan Smit, Anders Tranberg, JHEP 08(2003)045.

[31] Meindert van der Meulen, Denes Sexty, Jan Smit, Anders Tranberg, JHEP 02(2006)029. [hep-ph/0511080].

[32] E. Shuryak and I. Zahed, Phys. Rev. D 67, 014006 (2003) [arXiv:hep-ph/0206022].

[33] M. Kuchiev, [arXiv:0904.2899 [hep-th]].

[34] M. Kuchiev, Phys. Rev. D 79, 105004 (2009) [arXiv:0903.2079 [hep-th]].

[35] A. Hernandez, T. Konstandin and M. G. Schmidt, Nucl. Phys. B 812, 290 (2009) [arXiv:0810.4092 [hep-ph]].

[36] A. Tranberg, A. Hernandez, T. Konstandin and M. G. Schmidt, [arXiv:0909.4199 [hep-ph]].

[37] D0 collaboration. Evidence for an anomalous like-sign dimuon charge asymmetry, archive:1005.2757.

[38] G.R.Farrar and M.E.Shaposhnikov, hep-ph/9305274 Phys. Rev. Lett. 70, 2833 (1993); Phys. Rev. D 50, 774 (2004); reply to criticism: hep-ph/9406387

[39] M.B. Gavela, P. Hernandez, J. Orloff, O. Pene. arxiv: hep-ph/9312215 Mod. Phys. Lett. A9, 795 (1994). P. Huet, E. Sather. Phys. Rev. D 51, 379 (1995).

[40] G. Cvetic, Rev. Mod. Phys. 71, 513 (1999) arXiv:hep-ph/9702381.

[41] Those cannot be ignored entirely because this simplification leads to IR some divergences to be encountered below: yet those are logarithmic and be regulated by mass when needed.

[42] The second reaction, unlike the first, has logarithmic divergence which can be regulated by a quark mass: the result however depend weakly on the regulator and we keep the same coefficients for simplicity.