Optimization of composite cylinder shell via a data-driven intelligent optimization algorithm

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Abstract. While composite material provides huge flexibility for design, the design optimization of composite structure is time consuming with low efficiency. This work combines finite element analysis for composite cylinder shell with a data-driven intelligent optimization algorithm (Bayesian optimization algorithm) and is aimed at maximizing eigenvalue buckling load. Through minimizing number of iterations as a derivative-free global optimization algorithm, Bayesian optimization is versatile and can be further applied to design advanced composite structure with more complicated scenarios, such as complex geometries and load conditions.

1. Introduction
Fiber-reinforced composite material possesses excellent mechanical performance such as high strength, stiffness and fatigue resistance, and are wildly used in aerospace, aircraft, civil engineering and automotive, among others[1-4]. Fiber-reinforced composite material is flexible for design by adopting different design strategies for instance fiber-matrix combinations[5-6]. Besides, fiber-reinforced composite material is typical anisotropic and can be tailored and optimized to meet specific needs for practical applications with comparison to traditional metal alloys[7-9]. However, composite material is designable, bringing in increasing complicacy to conduct design optimization[10-11]. In order to deal with such complicacy, fiber orientation angle such as 0, 90, and ±45 are generally employed in manufacturing[12]. However, the outstanding performances of composite structures are degraded by doing this.

Cylinder shell is a vital component of underwater vehicle which is a crucial equipment deployed in deep water and wildly used for multifunctionality[13-15]. Underwater vehicles are often made of metal alloys with thick hull to withstand huge external hydrostatic pressure. By employing composite material for pressure hull of underwater vessels, underwater vehicles can be more light-weighted, thus possessing greater load capacity and endurance with comparison to the metal counterparts[16-18]. Due to the flexibility of design for composite material, the performance of underwater vehicle made of composite material can be further optimized and enhanced to satisfy practical requirements.

Bayesian optimization algorithm is a global optimization algorithm, needing not to compute derivative. What’s more, when compared with other methods, it calls for fewer evaluations of objective function[19-23]. Hence Bayesian optimization algorithm is adopted to optimize. In practical engineering optimization, the objective function is often implicit or has a form of black box, therefore optimization method which doesn’t need derivative possesses big advantage[24-25]. Since the objective function is often non-convex and multimodal, a method is supposed to get global optimum. Other methods for instance evolution algorithms can meet this need[26]. Unfortunately, evolution algorithms call for a
large number of iterations of objective function. While Bayesian optimization algorithm is capable of obtaining the optimum with fewer evaluations[27]. If a numerical simulation is computationally expensive, the optimization would be computationally prohibited. Therefore Bayesian optimization algorithm is employed considering these problems.

A brief review of Bayesian optimization is presented in Section 2. Detailed procedure for Bayesian optimization for composite cylinder shell is then given. Results and discussion and the conclusions follow.

2. Bayesian optimization

Bayesian optimization algorithm consists of two components. The first component is a prior model over functions to model the objective function. Thanks to the flexibility in modelling functions, Gaussian process (GP) prior model is adopted[28]. The other is an acquisition function, aiming at deciding next point to search. Bayesian optimization algorithm is shown as below:

Table 1. Procedure of Bayesian optimization algorithm.

| Bayesian optimization algorithm |
|--------------------------------|
| For $g(x)$ subject to Gaussian process. |
| Evaluate $g(x)$ for $m_0$ points by conducting design of experiment, then construct Gaussian process model. Assign $m = m_0$. |
| while $m \leq N$ do: |
| Posterior probability distribution on $g(x)$ is updated based on past evaluation. |
| Based on current posterior distribution, acquisition function is optimized to get $x_m$. |
| Evaluate $y_m = g(x_m)$. |
| Increment $m$. |
| end. |

2.1 Gaussian process

Featured by prior mean function $\mu_0 : \mathcal{X} \rightarrow \mathbb{R}$ and covariance function $\kappa_0 : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$, Gaussian process $GP(\mu_0, \kappa_0)$ is a non-parametric Bayesian method. For $m$ sample points $x_{1:m}$, $g_i = g(x_i)$ are the unknown function values and $y_{1:m}$ are noisy observations. As for Gaussian process, $g_i = g(x_i)$ is assumed jointly Gaussian and the observations $y_i = y_{i:m}$ are subject to Gaussian distribution, thus bringing in the following model: $g | X$ is subject to Gaussian distribution. $M = \mu_0(x_i)$ stands for mean. $K = k(x_i, x_j)$ denotes covariance matrix. $y | g, \sigma^2 I$ is subject to Gaussian distribution $N(g, \sigma^2 I)$. 

Observations are represented by $D_m := \{x_i, y_i\}_{i=1}^m$ and $x$ represents the test point. Based on observations $D_m$, the random variable $g(x)$ is also normally distributed, possessing posterior mean and variance. The posterior mean and variance represent the prediction and uncertainty of the model, and are employed to select the next point $x_{1:m}$ to evaluate.

2.2 Acquisition functions

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\{x_m, y_m\}_{m=1}^N$ and $y_m \sim N(g(x_m), \eta)$ stand for observations. $\eta$ stands for variance of noise for function observations. Posterior is induced on basis of prior and observations. $a : \mathcal{X} \rightarrow \mathbb{R}^+$, which is acquisition function, makes decision about which point to evaluate next. Acquisition function depends on GP hyperparameters and the previous observations and is represented as $a(x; \{x_m, y_m\}, \theta)$. There are several extensively utilized acquisition functions[29].
Lower (upper) confidence bound is done exploiting lower confidence bounds (upper confidence bound for maximization) to construct acquisition function which minimizes regret during optimization. Acquisition function is obtained through

\[-\kappa \sigma(x; \{ x_m, y_m \}, \theta) + \mu(x; \{ x_m, y_m \}, \theta).\]

### 3. Numerical model

As for Bayesian optimization algorithm for composite cylinder shell, buckling load is the main factor to take into account\[30\]. The composite cylinder shell has a length of 1000mm, diameter 324mm and thickness 6mm respectively. The stacking strategy is symmetrically stacking, totally 20 plies for the composite cylinder shell. The finite element modelling and mesh algorithm for composite cylinder shell is presented as below in Figure 1 and in Figure 2 respectively. S4R element is used to model composite cylinder shell as for element selection. It is rigidly fixed for the left end. While for the right end, it is fixed except displacement along axial direction. The purple arrows stand for external uniform pressure. Carbon fiber-epoxy’s mechanical properties are presented as listed in table 2.

![Figure 1](image1.png)

Figure 1. Modelling composite cylinder shell in Abaqus.

![Figure 2](image2.png)

Figure 2. Mesh algorithm for composite cylinder shell.

| Mechanical properties for carbon fiber-epoxy composite material | Symbol and value |
|---------------------------------------------------------------|------------------|
| Three elastic modulus                                        | \( E_{11} = 121 \times 10^3 \text{ MPa}, E_{33} = 8.6 \times 10^3 \text{ MPa}, E_{22} = 8.6 \times 10^3 \text{ MPa} \) |
| Three shear modulus                                          | \( G_{12} = 3.35 \times 10^3 \text{ MPa}, G_{23} = 2.68 \times 10^3 \text{ MPa}, G_{13} = 3.35 \times 10^3 \text{ MPa} \) |
| Three Poisson’s ratio                                        | \( \nu_{12} = 253 \times 10^{-3}, \nu_{23} = 421 \times 10^{-3}, \nu_{13} = 253 \times 10^{-3} \) |
When it comes to design optimization of composite cylinder shell via Bayesian optimization algorithm, eigenvalue buckling load is evaluated when conducting experimental design, and then Gaussian process model is established and set up. By maximizing acquisition function, the next point is selected and eigenvalue buckling load is evaluated afterwards. With new data obtained, the Gaussian process model is updated and renewed. Depending on the updated Gaussian process model, following point is selected through optimizing acquisition function. After relatively few evaluations, the maximum eigenvalue buckling load is found.

The optimization is aimed at maximizing eigenvalue buckling load \( P(\theta^i) \).

Objective function: \( P(\theta^i) \).

Design domain: \( \theta^i \in [-89, 90], i = 1, 2, ..., 10 \).

The flowchart of Bayesian optimization algorithm for composite cylinder shell is shown as follows:

![Flowchart](image)

Figure 3. The flowchart of Bayesian optimization algorithm for composite cylinder shell.

### 4. Results and discussions

Thanks to power of covariance function, Gaussian process model is capable of representing many distributions. Generally, squared exponential kernel is adopted by default as for the covariance function for Gaussian process model. However, utilizing this covariance function makes sample functions
unrealistically smooth. Matern5/2 kernel is used instead, inducing twice-differentiable functions. LCB is adopted as acquisition function for the optimization.

The composite cylindrical shell has a symmetrical stacking sequence of 20 plies totally, we conduct optimization for 10 orientation angles. For other methods for instance genetic algorithm (GA), this 10 dimensional engineering problem will be computationally expensive and time consuming. As depicted in Figure 4, after 57 iterations of evaluation, Bayesian optimization algorithm finds maximum eigenvalue buckling load 4.1152MPa, with the fiber orientation angle for a symmetrically stacking sequence [90/90/90/90/76/-83/-50/-36/1/10]s. For this engineering problem, Bayesian optimization algorithm is able to deal with non-convex problem as an efficient derivative-free global optimization algorithm.

Bayesian optimization algorithm differs from other methods since a probabilistic model is constructed. This model is utilized to decide next point for evaluation. Bayesian optimization algorithm makes the most use of past evaluations of model without using gradient information. By doing this, Bayesian optimization algorithm benefits a lot with the ability to reach the optimum by performing relatively few computations for non-convex functions.

5. Conclusion
Bayesian optimization algorithm is utilized, maximizing eigenvalue buckling load for optimum layup of composite cylinder shell. Thanks to statistical model and flexible acquisition function, Bayesian optimization algorithm is capable of reaching optimum with efficiency, needing not prior knowledge of objective function. The powerful Bayesian optimization algorithm has potential to be further utilized under condition of progressive damage and fatigue.

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