Radiation from SU(3) monopole scattering.

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The energy radiated during the scattering of SU(3) monopoles is estimated as a function of their asymptotic velocity $v$. In a typical scattering process the total energy radiated is of order $v^3$ as opposed to $v^5$ for SU(2) monopoles. For charge $(1,1)$ monopoles the dipole radiation produced is estimated for all geodesics on the moduli space. For charge $(2,1)$ monopoles the dipole radiation is estimated for the axially symmetric geodesic. The power radiated appears to diverge in the massless limit. The implications of this for the case of non-Abelian unbroken symmetry are discussed.

1 Introduction

The moduli space approximation has proved to be a very useful tool in the study of magnetic monopoles. It assumes that at low velocities the dynamics of BPS monopoles is determined by the geodesic motion in the manifold of static BPS configurations. This ignores the effect of Lorentz contraction on the monopole and a similar approximation is made regarding the total electric charge of the monopole. For a Lorentz boosted BPS monopole with rest mass $M$ and velocity $v$, the true kinetic energy is $\frac{1}{2}M\sqrt{1-v^2}$ whereas the moduli space approximation gives $\frac{1}{2}Mv^2$. The approximation is thus accurate to order $v^4$, a similar statement is true regarding the total electric charge $q$ of the monopole.

However when more than one monopole is present another question needs to be considered. As the monopoles approach each other they will accelerate, and because these are charged objects, radiation will be produced. The amount of radiation measures the deviation from the moduli space approximation. In [1], Manton and Samols estimate the radiation from the scattering of two SU(2) monopoles. To a first approximation the authors assume that the heavy inner cores of the monopoles evolve according to the moduli space approximation. The core region is where the monopole fields are non-Abelian and it

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is where the energy density is concentrated. Outside this core region the fields are essentially Abelian. The radiation of massless fields that such a motion of the monopoles cores would produce was then estimated. Corrections to this will arise because of deviations of the monopole core fields to the geodesic approximation. These corrections were shown to be of sub-leading order and can be ignored. The authors assume, as will be done here, that the dominant energy loss is to long range massless fields, as opposed to the short range massive fields. SU(2) monopoles have only one massless field; outside their core they can be treated using the usual electrodynamics. To calculate the radiation one needs the multipole moments of the scalar, magnetic and electric fields which depend on the time dependent moduli space parameters. These give a field expansion of the monopole just outside the core region. The radiation produced from this region to infinity can then be calculated using the usual formulae for multipole radiation from electrodynamics (see [1] for a more detailed description). For two SU(2) BPS monopoles their scalar and magnetic dipole moments always vanish. The leading order scalar and magnetic radiation is given by the quadrupole radiation. The quadrupole radiation in the head-on collision of two pure monopoles (each of mass $4\pi$) was found to be approximately $17v^5$, where the asymptotic monopole velocity is $v$. This is likely to be the maximally radiating scattering geodesic due to the head-on nature of the collision. For dyons, the electric dipole moment can be nonzero, but with the restriction that the relative electric charge $q$ satisfies $q \approx v \ll 1$, the electric dipole radiation is also order $v^5$ [1].

More generally, the total time where radiation is produced in a scattering event is of order $R/v$, $v$ is the incoming velocity and $R$ is the separation where inter-monopole forces become appreciable, which is roughly their core radius. The monopole core radius is of order $1/M$ where $M$ is the mass, so $R \approx 1/M$. Denoting the dipole moment by $d_i$ and the quadrupole moment by $Q_{ij}$, then an order of magnitude calculation gives

$$\frac{d^2}{dt^2}d_i \approx Mv^2, \quad \frac{d^3}{dt^3}Q_{ij} \approx Mv^3. \quad (1.1)$$

The factors of $M$ are determined on dimensional grounds. Inserting this into the formulae for dipole and quadrupole radiation and multiplying by the time, $1/(Mv)$, gives an order of magnitude estimate of the dipole radiation in terms of the velocity as being of order $Mv^3$ and the quadrupole radiation of order $Mv^5$. The energy radiated from magnetic or scalar quadrupoles will generally be of order $Mv^5$, and the result of Manton and Samols is very likely to hold for higher charge SU(2) monopoles. Magnetic or scalar dipoles will radiate energy of order $v^3$ during a scattering process, but in the center of mass frame the scalar and magnetic dipole moments of all SU(2) monopoles vanish. It can similarly be shown that higher order multipole radiation is of higher order in $v$.

For the case of SU(3) monopoles with unbroken gauge group $U(1) \times U(1)$ one might expect the same arguments used for the SU(2) case will also hold here since outside the monopole cores the theory reduces to that of $U(1) \times U(1)$ gauge theory. However a difference arises in the long range behavior of the monopole fields, which in general have non-vanishing dipole moments. One can easily re-derive the formula for the energy radiated by a time dependent dipole in the $U(1) \times U(1)$ case. The formula for the radiated energy is just a
sum over the two U(1) factors and we find the result that the energy radiated in a typical
scattering process is order $v^3$. As such, this does not pose any problems as regards the
validity of the moduli space approximation, the kinetic energy is order $v^2$, so $E_{\text{rad}}/E_{\text{kin}} \approx v$,
for small monopole velocities the radiation produced is small, providing the coefficient of
$v^3$ in $E_{\text{rad}}$ is finite.

First we consider the case of charge (1, 1) monopoles. Their dipole moments are easily
determined since the point description of these monopoles is valid. Assuming the motion
of the monopoles inner core is that of a geodesic on the moduli space the total dipole
radiation can be calculated to be $v^3$ times a function of two geometric parameters (which
depend on the monopole geodesic) and the monopole masses. The radiation produced
remains small if one of the monopoles masses approaches zero.

For (2, 1) monopoles, due to the complexity of the metric we only calculate the dipole
radiation for a special axially symmetric geodesic. Again the dipole radiation produced
is a function times $v^3$. The case we are really interested in is the limit of non-Abelian
unbroken symmetry which occurs when the mass of the (0, 1) monopole is taken to zero.
Taking the naive limit, the function multiplying $v^3$ diverges implying that infinite radiation
is produced and thus invalidating the moduli space approximation. The argument given
around Eq. (1.1) for SU(2) monopoles does not work here as now there are two mass
parameters, it is a function of their ratio that diverges.

However since the monopoles are finite energy objects, it is not possible for them to
produce infinite radiation, therefore there is a problem with our naive reasoning. This can
be seen by examining the core size of the monopole in the massless limit. For a monopole of
mass $m$ its core size is of order $1/m$, this diverges in the massless limit. As the mass $m$ of the
(0, 1) monopole approaches zero, the size of its core region expands; for $m$ small enough the
core of the (0, 1) monopole will generically surround the cores of the two massive monopoles,
[2]. Inside this overall core region the fields are non-Abelian. However the energy density
is concentrated in a small region around the (2, 0) monopole. Just outside the overall
core region, where we measure the multipole moments, the multipole expansion will differ
significantly from that of a static configuration. This is because the fields in the core region
become highly relativistic in the massless limit. The assumption that the monopole fields
inside the core region are well approximated by the geodesic approximation breaks down
in the massless limit. Our results are therefore somewhat inconclusive, nonetheless in the
massless limit it appears that the time dependent monopole fields do have large deviations
from that predicted by the moduli space approximation.

In [1], and in the present work, the amount of radiation produced is used as a estimate
of the validity of the moduli space approximation. In [3], Stuart proves the validity of
the geodesic approximation at low velocities for SU(2) monopoles using rigorous analytical
methods. It would be interesting if the methods in [3] could be extended to higher gauge
groups, indeed it seems that such an approach is necessary as the calculations presented
here do not have sufficient rigor to confirm or reject the validity of the moduli space
approximation in the massless limit.
2 The radiation formula for SU(3) monopoles

We assume that the Higgs field is in the adjoint representation and we are in the BPS limit in which the scalar potential is zero but a nonzero Higgs expectation value is imposed at spatial infinity as a boundary condition. An SU(3) gauge theory can be broken by an adjoint Higgs mechanism to either U(1)×U(1) or U(2). The generators of SU(3) may be chosen to be two commuting operators $H_i$, $i = 1, 2$, with $\text{Tr} \ H_i H_j = \delta_{ij}$, together with ladder operators associated with the roots $\pm \alpha, \pm \beta, \pm (\alpha + \beta)$ that obey

$$[H_i, E_\gamma] = \gamma^i E_\gamma, \quad [E_\gamma, E_{-\gamma}] = \gamma \cdot H = H_\gamma,$$

for $\gamma$ any root. Define $\alpha = (-1/2, \sqrt{3}/2)$ and $\beta = (1, 0)$. We choose the singular gauge where the Higgs field $\phi$ is constant at spatial infinity, equal to $\phi_\infty$. Choosing this constant value to lie in the Cartan sub-algebra defines a vector $h$ by $\phi_\infty = h \cdot H$. If SU(3) is broken to U(1)×U(1), all roots have nonzero inner product with $h$ and there is a unique set of simple roots with positive inner product with $h$. If SU(3) is broken to U(2) then one of the roots, $\beta$ say, is perpendicular to $h$.

For any finite energy solution, asymptotically

$$B_i = \frac{x_i}{4\pi |x|^3} G,$$  

$G$ is a constant element of the Lie algebra of SU(3). The Cartan sub-algebra may be chosen so that $G = g \cdot H$. The quantization of magnetic charge determines $g$ as

$$g = \frac{4\pi}{e} \{k_\alpha \alpha + k_\beta \beta\},$$

where $e$ is the gauge coupling, and $k_\alpha, k_\beta$ are non-negative integers. Such a solution is called a $(k_\alpha, k_\beta)$ monopole and has mass $g \cdot h$.

When SU(3) is broken to U(1)×U(1) the topological charges of the monopoles are determined by the integers $k_\alpha, k_\beta$. All BPS monopoles may be thought of as superpositions of two fundamental monopoles given by embeddings of the charge one SU(2) monopole [4]. Associated with each root is an SU(2) sub-algebra. The two fundamental monopoles are obtained by embedding the charge one SU(2) monopole along the SU(2) sub-algebra associated to the simple roots, $\alpha$ and $\beta$ [4]. Each fundamental monopole has four zero modes, corresponding to its position and a U(1) phase. Embedding along the root $\alpha$ gives the $(1, 0)$ (or $\alpha$) monopole charged with respect to one of the unbroken U(1)’s. Similarly, one can embed along the root $\beta$ to give the $(0, 1)$ (or $\beta$) monopole charged with respect to the other unbroken U(1) group. A $(k_\alpha, 0)$ monopole or a $(0, k_\beta)$ monopole is made up of only one type of monopole and behaves like the corresponding SU(2) monopole. A $(k_\alpha, k_\beta)$ monopole has mass $(k_\alpha M + k_\beta m)$, with $M$ is the mass of a $(1, 0)$ monopole and $m$ the mass of a $(0, 1)$ monopole. BPS monopoles interact in different ways depending on whether they are of the same type or of different types. The metric on the moduli space of two SU(3)
monopoles of different type, the (1, 1) monopole is the Taub-NUT metric. The metric for a (2, 0) monopole is the Atiyah-Hitchin metric. The metric on the moduli space of (2, 1) monopoles mixes these interaction types.

We now consider the multipole radiation. For SU(2) monopoles a core region exists where the fields are non-Abelian. Outside this core region the fields can be gauge transformed to be exponentially close to Abelian fields, i.e. proportional to the same SU(2) generator. The core region is where the energy density of a monopole is concentrated. Outside this region the fields satisfy the source-less Maxwell equations. As regards the asymptotic region, the fields in the core region can be viewed as providing effective U(1) sources for the asymptotic fields. Just outside the core region the monopole fields are Abelian, and assuming no appreciable radiation has been emitted by the slow moving core, the fields are determined from the moduli space approximation. For example, the scalar field can be expanded as

\[ \phi = \tau_3 \left\{ v - \frac{g}{4\pi|x|} + \frac{d_s(t) \cdot \hat{x}}{4\pi|x|^2} + \ldots \right\} , \]  

where \( d_s(t) \) is the time dependent dipole moment, its time dependence is determined from the moduli space approximation, and \( \tau_3 \) is a fixed element of the SU(2) Lie algebra. Because outside of the core the monopole behavior is exactly that of U(1) dyons, the multipole radiation formulae is unchanged from the U(1) theory.

Turning now to the case of SU(3) broken to U(1)×U(1), almost everything said for SU(2) monopoles carries over here. Again there exists a core region outside of which the fields can be gauge transformed to lie in the Cartan sub-algebra, the fields satisfy the source-less U(1)×U(1) Maxwell wave equation outside the core. The core region again provides effective U(1)×U(1) sources for the asymptotic region. Using the orthogonal basis \( H_1, H_2 \), the field equations decompose into two linearly independent parts and the radiation formula can be derived as before. Each multipole moment has a \( H_1 \) and a \( H_2 \) component. Just outside the core region the scalar field may be expanded as

\[ \phi = h \cdot H - \frac{g \cdot H}{4\pi|x|} + \frac{d_s(t) \cdot \hat{x}}{4\pi|x|^2} + \ldots , \]  

where \( d_s(t) = d_s^1(t) H_1 + d_s^2(t) H_2 \), is the time dependent scalar dipole moment. The radiation formula is just a sum of the radiation produced from each component. Scalar, magnetic and electric dipole radiation will be produced. The general dipole radiation formula for SU(3) monopoles with maximal symmetry breaking is

\[ P(t) = \frac{1}{6\pi} \sum_{i=1,2} \left[ \ddot{d}_s^i \cdot \ddot{d}_s^i + \ddot{d}_s^b \cdot \ddot{d}_s^b + \ddot{d}_s^e \cdot \ddot{d}_s^e \right] , \]

where the subscripts \( s, b, e \) denote the scalar, magnetic and electric dipole moments respectively. The total power radiated, \( P \), is just the time integral of \( P(t) \), \( P = \int_{-\infty}^{\infty} dt \). We now consider in turn the case of (1, 1) and (2, 1) monopoles. The procedure is to find
the dipole moments as a function of the moduli space parameters. The time dependence of the moduli space parameters is determined from the geodesic equation. The radiated power from Eq. (2.6) is then integrated over the corresponding geodesic in the moduli space.

3 Charge (1, 1) monopoles

Charge (1, 1) monopoles are relatively easy to describe due to their simple point-like interactions. The monopole consists of two embedded charge one SU(2) monopoles, embedded along distinct roots, α and β. We want an expression for the dipole moments of the monopole fields, it turns out that these are obtained from a simple addition of the dipole moments of each embedded monopole. A charge one SU(2) monopole with electric charge $q$, magnetic charge $g$, positioned at $X \in \mathbb{R}^3$, may be written in the singular gauge as

$$
\phi(x) = \frac{(q^2 + g^2)^{1/2}}{4\pi|x - X|} H_3, \quad B_i(x) = \frac{g(x - X)}{4\pi|x - X|^3} H_i, \quad E_i(x) = \frac{q(x - X)}{4\pi|x - X|^3} H_i.
$$

This expression for the fields is valid outside the monopole core with exponentially small corrections, and is just an embedding of a U(1) dyon into SU(2). For charge (1, 1) SU(3) monopoles a similar expression can be written down for the fields. In [3], the Nahm transform was inverted for a variety of monopole charges to give explicit expressions for the monopole fields. For (1, 1) SU(3) monopoles, the long range fields are just a naive sum of the embedded SU(2) fields. Inside the monopoles cores and near the line joining the monopoles the expressions for the fields are more complicated. The fields considered in [3] were time independent, i.e. the moduli space parameters were time independent. Electric fields are absent in this case. Introducing time dependence of the moduli parameters generally gives rise to electric fields. The results in [3] show that when the electric fields are zero the multipole moments are just a naive sum of those from the individual monopoles. We assume that this is also true for non-zero electric fields. We can then write down the long range (1, 1) monopole fields. They are just what one obtains from an ansatz of point electric, magnetic and scalar charges. Using the root basis described earlier, a (1, 1) monopole with composed of a (1, 0) monopole with electric charge $q_1$ and a (0, 1) monopole with electric charge $q_2$ can be asymptotically written as

$$
\phi(x) = h \cdot H + \frac{(q_1^2 + g^2)^{1/2}}{4\pi|x - X_1|} H_\alpha + \frac{(q_2^2 + g^2)^{1/2}}{4\pi|x - X_2|} H_\beta, \quad (3.2)
$$

$$
B_i(x) = \frac{g(x - X_1)}{4\pi|x - X_1|^3} H_\alpha + \frac{g(x - X_2)}{4\pi|x - X_2|^3} H_\beta,
$$

$$
E_i(x) = \frac{q_1(x - X_1)}{4\pi|x - X_1|^3} H_\alpha + \frac{q_2(x - X_2)}{4\pi|x - X_2|^3} H_\beta.
$$
Here $X_1$ and $X_2$ are the positions of the two monopoles in space. We can read off the dipole moments which are given by

$$d_s = (q_1^2 + q_2^2)^{1/2} X_1 H_\alpha + (q_2^2 + g^2)^{1/2} X_2 H_\beta \quad (3.3)$$

$$d_b = -(g X_1 H_\alpha + g X_2 H_\beta) \quad (3.4)$$

$$d_e = -(q_1 X_1 H_\alpha + q_2 X_2 H_\beta). \quad (3.5)$$

To use the formula in Eq. (2.3) for the power radiated by scalar, electric and magnetic dipoles we have to decompose the dipole moments into orthogonal parts of the Lie algebra. Using our previous definitions for the roots $\alpha$, and $\beta$ we have $H_\alpha = \frac{1}{2}(-H_1 + \sqrt{3}H_2)$ and $H_\beta = H_1$. We then decompose the dipole moments into their $H_1$ and $H_2$ components, $d = d_1 H_1 + d_2 H_2$ and using Eq. (2.4) we get

$$P(t) = \frac{1}{6\pi}(2(g^2 + q_1^2)\dot{X}_1 \cdot \ddot{X}_1 + 2(g^2 + q_2^2)\dot{X}_2 \cdot \ddot{X}_2) \quad (3.6)$$

$$- [g^2 + q_1 q_2 + (g^2 + q_1^2)^{1/2}(g^2 + q_2^2)^{1/2}]\dot{X}_1 \cdot \dot{X}_2 \}.$$

Our assumption that the monopole cores evolve according to the moduli space approximation implies that the monopoles centers $X_1, X_2$ are determined by a geodesic on the moduli space. The overall center of mass, $R = (MX_1 + mX_2)/(M + m)$, can be ignored as it evolves with constant velocity. The relative moduli space has the Taub-NUT metric with positive mass [3, 4]. This is a four dimensional manifold whose parameters describe the relative position, $r$, and phase, $\chi$, of the monopole. The geodesics on the moduli space manifold are equivalent to the equations of motion derived from a Lagrangian. This Lagrangian can be derived from the point-particle interactions of the monopoles. The Lagrangian is given by

$$L = \frac{1}{2}(\mu + \frac{g^2}{8\pi r})\dot{r} \cdot \dot{r} + \frac{1}{2}(\frac{g^2}{8\pi r})^2(\mu + \frac{g^2}{8\pi r})^{-1}\dot{\chi} + w(r) \cdot \dot{r})^2, \quad (3.7)$$

with $\mu$ equal to the reduced mass, $\mu = Mm/(M + m)$. The relative monopole position, $r = X_1 - X_2$, is written in spherical polar coordinates $r$, $\theta$, $\phi$, and $w(r)$ is the Dirac monopole potential satisfying, $\nabla \times w(r) = r/r^3$. The conserved relative electric charge of the monopole, $q$, is conjugate to $\chi$, i.e. $q = (g^2/8\pi)(\mu + g^2/8\pi r)^{-1}(\dot{\chi} + w(r) \cdot \dot{r})$. In terms of the monopoles individual charges $q_1$ and $q_2$, the relative electric charge $q$ is given by $q = (q_1 - q_2)/2$. There also exists the conserved total electric charge $Q$, with

$$Q = \frac{Mq_1 + mq_2}{M + m}. \quad (3.8)$$

We henceforth set $Q = 0$ to restrict to motion on the relative moduli space. This determines $q_1$ and $q_2$ as functions of the relative electric charge $q$,

$$q_1 = \frac{2mq}{M + m}, \quad q_2 = -\frac{2Mq}{M + m}. \quad (3.9)$$
\( X_1, X_2 \) are given in terms of \( r \) and \( R \) by

\[
X_1 = R + \frac{m}{M + m} r, \quad X_2 = R - \frac{M}{M + m} r.
\]  

(3.8)

Using this in the above formula for the power radiated we have

\[
P(t) = \lambda \ddot{r} \cdot \dot{r},
\]

(3.9)

\[
\lambda = \frac{1}{3\pi} \left\{ \frac{m^2 (g^2 + q_1^2) + (g^2 + q_2^2) M^2}{(M + m)^2} \right\} + \left\{ \frac{mM}{6\pi (M + m)^2} (g^2 + q_1 q_2 + (g^2 + q_1^2)^{1/2}(g^2 + q_2^2)^{1/2}) \right\}.
\]

(3.10)

This formula contains terms of different order in the velocity \( v \), which is the order parameter in the problem. The electric charges \( q_1 \) and \( q_2 \) are really small parameters of the same order as the velocity in the moduli space approximation. So the terms involving the \( q_1, q_2 \) have the same order of magnitude as the quadrupole terms arising from the scalar and magnetic radiation. We retain the \( q_1, q_2 \) terms just to give a complete formula for the dipole radiation. It now remains to calculate \( \int \ddot{r} \cdot \dot{r} \) for a given geodesic on the moduli space. We fix the coupling constant by setting \( g = 4\pi \). The equation of motion for \( r \) resulting from the Lagrangian, Eq. (3.5), is

\[
(\mu + \frac{2\pi}{r})\ddot{r} = -\frac{2\pi}{r^3} \left\{ \frac{1}{2}(\dot{r} \cdot \dot{r}) r - (r \ddot{r}) \dot{r} \right\} + \frac{\dot{r} \times r}{r^3} + \frac{q^2}{4\pi r^3} r.
\]

(3.11)

From this it is not too hard to show that

\[
\ddot{r} \cdot \dot{r} = \frac{4\pi^2 E}{(\mu r^4 + 2\pi)^4},
\]

(3.12)

with \( E \) is the conserved energy given by

\[
E = \frac{1}{2} (\mu + \frac{2\pi}{r}) \{ \dot{r} \cdot \dot{r} + (\frac{q}{2\pi})^2 \}.
\]

(3.13)

Equation (3.12) simplifies considerably the task of calculating the power radiated as now all that is needed is to determine the separation parameter \( r \) as a function of time. So the total power radiated is

\[
P = 4\pi^2 E^2 \lambda \int_{-\infty}^{\infty} dt \frac{1}{(\mu r + 2\pi)^4}.
\]

(3.14)

We now change the integration variable from \( t \) to \( r \) to yield

\[
P = 8\pi^2 E^2 \lambda \int_{r_{\text{min}}}^{\infty} dr \frac{1}{r(\mu r + 2\pi)^4}.
\]

(3.15)
This is valid since geodesics on the Taub-NUT space are hyperbolae \[^6\], so in a scattering process \( r \) asymptotically approaches infinity and there is only one turning point. We need to determine \( \dot{r} \) as a function of \( r \), in order to do this we examine the equations of motion and use the conserved quantities. In addition to the energy there are two conserved vector quantities, the angular momentum \( \mathbf{J} \), and the vector \( \mathbf{K} \). The existence of the conserved vector quantity \( \mathbf{K} \) is a special feature of the Taub-NUT manifold, \[^8\], owing its existence to the self dual nature of the metric. Defining \( \mathbf{p} \) as

\[
\mathbf{p} = (\mu + \frac{2\pi}{r})\hat{\mathbf{r}} ,
\]

(3.16)

\( \mathbf{J} \) and \( \mathbf{K} \) are given by

\[
\mathbf{J} = \mathbf{r} \times \mathbf{p} - q\hat{\mathbf{r}} ,
\]

(3.17)

\[
\mathbf{K} = \mathbf{p} \times \mathbf{J} - (2\pi E - \frac{\mu q^2}{2\pi})\hat{\mathbf{r}} .
\]

It is not difficult to check from Eq. (3.5) that these vectors are indeed conserved. The angular momentum is a sum of the orbital angular momentum and the Poincaré contribution. The magnitude of the orbital angular momentum, \( l = |\mathbf{r} \times \mathbf{p}| \), is also conserved and satisfies

\[
J^2 = l^2 + q^2 ,
\]

(3.18)

with \( J^2 = \mathbf{J} \cdot \mathbf{J} \). We now choose a coordinate frame so that the \( \hat{z} \) component of \( \mathbf{r} \) is constant. This is achieved by taking the vector \( \mathbf{K} - \kappa \mathbf{J} \) to point in the \( \hat{z} \) direction (\( \kappa \) is a constant defined below). Then writing

\[
\mathbf{K} - \kappa \mathbf{J} = \frac{J^2 - q^2}{z_0}\hat{z} ,
\]

(3.19)

\[
\kappa = \frac{1}{q}(2\pi E - \frac{\mu q^2}{2\pi}) ,
\]

implies the \( \hat{z} \) component of \( \mathbf{r} \) is constant, equal to \( z_0 \). This can be checked using Eq. (3.17). The separation vector is then written in cylindrical coordinates \((\rho, \psi, z)\) as

\[
\mathbf{r} = \rho \hat{\rho} + z_0 \hat{z} ,
\]

(3.20)

\[
\hat{\mathbf{r}} = \hat{\rho} \hat{\rho} + \rho \hat{\psi} \hat{\psi} .
\]

The relative monopole motion is thus in a fixed plane. The angular momentum \( \mathbf{J} \) may be calculated in this basis, \( \mathbf{J} = J_\rho \hat{\rho} + J_\psi \hat{\psi} + J_z \hat{z} \), with

\[
J_\rho = -\frac{\rho q}{r} - (\mu + \frac{2\pi}{r})\rho \dot{\psi} z_0 , \quad J_\psi = (\mu + \frac{2\pi}{r})\dot{\rho} z_0 , \quad J_z = (\mu + \frac{2\pi}{r})\rho^2 \dot{\psi} - \frac{q z_0}{r} ,
\]

(3.21)

and \( r = (\rho^2 + z_0^2)^{1/2} \). From the equation for \( J_z \) in (3.21) we can express \( \dot{\psi} \) in terms of \( \rho \). We then use the energy equation (3.13) to determine \( \dot{\rho} \). Using \( \dot{\mathbf{r}} \cdot \mathbf{r} = \dot{\rho}^2 + \rho^2 \dot{\psi}^2 \), Eq. (3.13)
may be converted into an equation for $\dot{\rho}$ and since $r\dot{r} = \rho\dot{\rho}$, this can then be transformed into an equation for $\dot{r}$ in terms of $r$. Finally we need to determine the constants $J_z$ and $z_0$. The constant $J_z$ can be seen to be $J_z = -\kappa z_0$ using

$$J_z = \mathbf{J} \cdot \frac{(\mathbf{K} - \kappa \mathbf{J})}{|\mathbf{K} - \kappa \mathbf{J}|}.$$  

(3.22)

Then using the above expressions for $\mathbf{J}$ and $p$ in cylindrical coordinates we evaluate $(\mathbf{p} \times \mathbf{J})_3$ and insert into Eq. (3.19) using the definition of $\mathbf{K}$. Comparing both sides of the first equation in (3.19) we get

$$z_0 = \frac{q\{J^2 - q^2\}^{1/2}}{2\pi E}.$$  

(3.23)

If $z_0 = 0$, both monopoles move in the same plane. This occurs if $q = 0$ or $J^2 = q^2$ ($l = 0$), in the latter case the scattering is along a straight line. It can be checked that Eq. (3.19) has a sensible limit as $z_0 \to 0$. Using these values for $J_z$ and $z_0$ we get the following equation for $\dot{r}$,

$$\dot{r} = \frac{1}{(\mu r + 2\pi)}\{2\mu r^2(E - \frac{\mu q^2}{8\pi^2}) + 4\pi r(E - \frac{\mu q^2}{4\pi^2}) - J^2\}^{1/2}.$$  

(3.24)

Recalling Eq. (3.13), the limits of the integral are $r_{\min}$ and $\infty$, $r_{\min}$ is found by solving $\dot{r} = 0$. The energy $E$ can be expressed in terms of the asymptotic velocity, $v$, using Eq. (3.13) the asymptotic velocity is determined from

$$E = \frac{\mu}{2}\{v^2 + (\frac{q}{2\pi})^2\}.$$  

(3.25)

The magnitude of the conserved orbital angular momentum $l$ ($l^2 = J^2 - q^2$) can be written as $l = \mu vr_0$. Here $r_0$ is the asymptotic impact parameter. $r_0$ satisfies $r_0 \geq z_0$ because the monopoles separation is constant in the $\hat{z}$ direction, their relative velocity is in the $x$-$y$ plane. $r_0^2 = z_0^2 + r_\perp^2$ where $r_\perp$ is the planar impact parameter. The resulting integral for the radiated power can be done without too much difficulty with the result

$$P = \frac{\lambda v^3}{r_0} g(y),$$  

(3.26)

where

$$g(y) = \frac{1}{y^2}\{[1 + \frac{3}{y^2}][\frac{\pi}{2} - \sin^{-1}(1 + y^2)^{-1/2}] - \frac{3}{y}\}, \quad y = \frac{\mu r_0}{\pi(1 + q^2/4\pi^2 v^2)}.$$  

(3.27)

This is the result for the dipole radiation produced during a scattering of $(1, 1)$ monopoles. The result depends on the incoming velocity $v$, the relative electric charge $q$, the impact parameter $r_0$ and the monopole masses $M, m$. Recall from Eq. (3.10) and Eq. (3.7) that $\lambda$ is dependent on $q, M$ and $m$. The function $g(y)$ satisfies

$$\lim_{y \to 0} g(y) = \frac{4y}{15}, \quad \lim_{y \to \infty} g(y) = \frac{\pi}{2y^2}.$$  

(3.28)
For positive \( y \), \( g(y) \) has one turning point, its maximum, at \( y \approx 1.1 \), where \( g(y) \approx 0.15 \). The power radiated is maximal for the head on collision of two pure monopoles as might be expected, the case of \( r_0 = q = 0 \) with \( P = 4 \lambda \mu v^3/15 \pi \). In terms of \( v \), the relative velocity, the power radiated has leading order \( v^3 \). If one of the monopole masses approaches zero \( (\mu \to 0) \), the total radiation, \( P \), correspondingly decreases to zero. For large values of the impact impact parameter \( r_0, P \) falls off as roughly as \( v^3/r_0^3 \).

4 Charge \((2, 1)\) monopoles

For the previous case of charge \((1, 1)\) monopoles we were able to find a explicit formula for the dipole radiation produced from any geodesic motion. For charge \((2, 1)\) monopoles we can also compute the dipole moments and the moduli space metric is known. But the calculations are much more involved given the complexity of the \((2, 1)\) metric. We will restrict ourselves to computing the radiation produced from a single axially symmetric geodesic where the calculations simplify considerably. This scattering event was described in [3], it involves a head-on collision of the monopoles, we are especially interested in the massless limit. It is very likely that the radiation produced in this scattering event will be the maximal of all scattering events, again due to the head-on nature of the collision. The procedure of finding the radiation produced is exactly the same as the previous case; first find the dipole moments as a time dependent function of the moduli space parameter, the explicit time dependence is determined from the equation for geodesic motion. This is inserted into the dipole radiation formula and integrated over time to give the total power radiated.

The dipole moment of a \((2, 1)\) monopole configuration is not hard to evaluate. A \((2, 1)\) monopole can generally be thought of as a \((2, 0)\) monopole (an embedded charge two \(\text{SU}(2)\) monopole of mass \(2M\)) combined with a \((0, 1)\) monopole (an embedded charge one \(\text{SU}(2)\) monopole of mass \(m\)), in much the same way as a \((1, 1)\) monopole is a combination of a \((1, 0)\) monopole with a \((0, 1)\) monopole. We choose the coordinate system where the \((2, 0)\) monopole center of mass is at the origin. When the \((2, 0)\) monopole is centered it has no dipole moment. The total dipole moment is deduced from that of the \((0, 1)\) monopole.

The \((0, 1)\) monopole has a well defined position and the point approximation can be used as before to calculate its dipole moment. The above statements can be proved in the case of the axially symmetric geodesic. To show this, first notice that the dipole moment must point along the axis of symmetry. In the singular gauge introduced earlier we can write the Higgs field as

\[
\phi = v \mathbf{h} \cdot \mathbf{H} - \frac{(2\alpha + \beta)}{4\pi|x|} \mathbf{H} + \frac{(d^\alpha H^\alpha + d^\beta H^\beta)}{4\pi|x|^2} \hat{x} + \ldots. \tag{4.1}
\]

The fields are invariant up to gauge transform under rotations about the \(\hat{z}\) axis. To leave invariant \(v \mathbf{h} \cdot \mathbf{H}\) and \((2\alpha + \beta) \cdot \mathbf{H}/|x|\), the compensating gauge transform must have its constant and \(1/|x|^2\) terms in the Cartan sub-algebra \(\mathbf{H}\). Therefore the \(1/|x|^2\) component
of $\phi$ is also unchanged by the gauge transform. This implies that the $1/|x|^2$ component of $\phi$ is invariant under a rotation about the $\hat{z}$ axis without a corresponding gauge transform, which in turn means that $d^\alpha_x = d^\beta_x = d^\alpha_y = d^\beta_y = 0$. So the dipole moment points in the $\hat{z}$ direction and can be calculated knowing just the fields along the axis of symmetry which we compute below from the Nahm transform. This argument does not hold in the massless limit where the unbroken gauge symmetry is enhanced. Indeed the dipole moments of the fields have been analyzed for the minimal symmetry breaking case, $[10]$, and such a point interpretation is not possible.

The Nahm data is known explicitly for the axially symmetric configurations. Using the Nahm transform we can invert this to derive the monopole fields. To derive the monopole fields at a given point in space it is necessary to solve a linear equation with the Nahm data and the aforementioned point as input, from the solution to this equation one can find the monopole fields. It turns out that this equation is difficult to solve for points off the axis of symmetry, but the solution can be found for points on the axis of symmetry. Similar calculations have been done previously $[11, 5]$, in the present case the calculations are long and not very illuminating, we will just state here the results. The electric fields are zero and the scalar dipole moment $d_s$ is equal to the magnetic dipole moment $d_b$. In the frame where the $(2, 0)$ monopole is centered, the dipole moments are unchanged from those found in $[11]$ for $m = 0$ (the higher multipole moments do depend on $m$). As mentioned earlier the dipole moments for $m = 0$ are non-zero off the axis of symmetry, this is not the case here, where $m > 0$. The Nahm transform can be used thus to determine the dipole moments on the axis of symmetry. Denoting $r_\beta$ the position of the $(0, 1)$ monopole, the dipole moments are given by

$$d_s = d_b = \{\beta \cdot H\} r_\beta.$$  \hspace{1cm} (4.2)

This is exactly what one derives assuming that the total dipole moment is determined from the point-like $(0, 1)$ monopole. Inserting this in Eq. (2.6) we get the power radiated as

$$P = \frac{1}{3\pi} \int_{-\infty}^{\infty} dt \ddot{r}_\beta \cdot \dddot{r}_\beta.$$  \hspace{1cm} (4.3)

In fact it is not justified to restrict to the frame where the $(2, 0)$ monopole is centered as it is not a geodesic sub-manifold of the full moduli space. We really should work in the overall center of mass frame. Returning to the overall center of mass frame has the effect of changing the mass parameter $m$ of the $(0, 1)$ monopole to the reduced mass in the metric, $[9]$, in addition the formula for the power, (4.3), is multiplied by a function of $m/M$. We will continue to use the $(2, 0)$ monopole centered frame for simplicity as the formulas are more transparent.

In the notation introduced earlier we have two $\alpha$ monopoles, the $(2, 0)$ monopole, and one $\beta$ monopole, the $(0, 1)$ monopole. The axially symmetric geodesic that we consider has at one asymptote the spherically symmetric $\alpha + \beta$ monopole approaching the second $\alpha$ monopole. The monopoles collide at the origin, the $\beta$ monopole then scatters to spatial infinity, the two $\alpha$ monopoles remain coincident. The configuration asymptotically resembles the $\beta$ monopole separating from the charge two donut $\alpha$ monopole. Axial symmetry is preserved at all times, see $[\overline{13}]$ for more details.
The moduli space metric for charge \((2,1)\) monopoles is derived in [9]. This metric is equivalent to a Lagrangian describing the monopole dynamics. We consider the axially symmetric monopoles which form a one dimensional geodesic sub-manifold of the moduli space. The sub-manifold can be thought of as a union of two different regions. The Lagrangian in Region 1 is given by
\[
L = \frac{1}{2} a_1(D) \dot{D}^2 \quad \text{or,}
\]
\[
L = \frac{1}{2} \left\{ (\sinh DM \cosh DM - DM)(DM - \tanh DM) \frac{\cosh DM}{2D \sinh^3 DM} \right\} \dot{D}^2 .
\]
This region describes the \(\alpha\) monopole approaching and colliding with the spherically symmetric \(\alpha + \beta\) monopole. The position of the \(\beta\) monopole is well defined and is given by
\[
r_\beta = (0, 0, -\frac{D}{2} \coth DM) .
\]
In this region \(D\) satisfies \(0 \leq D < \infty\). The separation of the \(\alpha\) monopole from the \(\alpha + \beta\) monopole is approximately \(D\) for large \(D\). The collision occurs at \(D = 0\). After the collision the two \(\alpha\) monopoles coalesce to form an axially symmetric configuration and the \(\beta\) monopole scatters to spatial infinity, this is Region 2. The Lagrangian is given in Region 2 by
\[
L = \frac{1}{2} a_2(D) \dot{D}^2 \quad \text{or,}
\]
\[
L = \frac{1}{2} \left\{ (\sin DM \cos DM - DM)(DM - \tan DM) \frac{\cos DM}{2D \sin^3 DM} \right\} \dot{D}^2 .
\]
This region describes the \(\beta\) monopole separating from the donut configuration of the two \(\alpha\) monopoles. The position of the \(\beta\) monopole is
\[
r_\beta = (0, 0, -\frac{D}{2} \cot DM) .
\]
The donut configuration is at the origin. \(D\) is constrained to satisfy \(0 \leq D < \pi/M\). As \(D\) approaches \(\pi/M\), the \(\beta\) monopole approaches spatial infinity. The two regions fit together smoothly at \(D = 0\) and together form a geodesic sub-manifold of the full moduli space. In fact there is a two dimensional family of axially symmetric geodesics, one can act on the above family with a U(1) factor conserving the axial symmetry. For simplicity we restrict here to the one dimensional case.

To determine the total power radiated we need to find \(\ddot{r}_\beta \cdot \ddot{r}_\beta\), from Eq. (4.5) and Eq. (4.7) this can be expressed in terms of \(D\) and its times derivatives. \(D\) is then determined as a function of time from the Lagrangian, Eq. (4.4) and Eq. (4.6). We can express the total power radiated as that of a sum coming from Regions 1 and 2. Define the \(\hat{z}\) component of \(r_\beta\) in region \(i\) \((i = 1, 2)\) as \(b_i(D)\),
\[
b_1(D) = -\frac{D}{2} \coth DM , \quad b_2(D) = -\frac{D}{2} \cot DM .
\]
Now using Eq. (4.3) we have an expression for the radiated power as

\[
P = \frac{(2E)^{3/2}}{3\pi} \left\{ \int_0^\infty dD \frac{(2a_1 b''_1 - a'_1 b'_1)^2}{4a_1^{7/2}} + \int_{\pi/M}^\infty dD \frac{(2a_2 b''_2 - a'_2 b'_2)^2}{4a_2^{7/2}} \right\}, \tag{4.9}
\]

again \(a_1(D), a_2(D)\) are the metric coefficients in each region, \(E\) is the conserved energy, and \(a'_1\) denotes the derivative of \(a_1\) with respect to \(D\) etc. The two integrals correspond respectively to the radiation produced from the two sections of the geodesic. Both integrals can be computed numerically, their sum has the approximate behavior

\[
P \approx \frac{E^{3/2}}{12\pi(2M)^{1/2}} \left\{ \frac{0.01}{(0.3 + m/M)^{7/2}} + 4.3 \sqrt{\frac{M}{m}} \right\}. \tag{4.10}
\]

Holding \(M\) fixed we clearly see that the second term above diverges as \(m \to 0\). The divergence comes from Region 2, where the \(\beta\) monopole separates from the donut configuration of two \(\alpha\) monopoles. The divergence arises as \(D\) approaches \(\pi/M\), i.e. as the \(\beta\) monopole approaches infinity. Although we have only calculated the radiation from a single geodesic we expect to encounter a similar behavior for any geodesic in which the \(\beta\) monopole is asymptotically well separated from the two \(\alpha\) monopoles. As mentioned earlier we should really work in the overall center of mass frame. This has the effect of changing \(m\) to the reduced mass, and multiplying the overall result by a function of \(m/M\). We have omitted this correction term for simplicity, as \(m \to 0\) the correction becomes negligible.

To gain some insight as to the source of the divergence in Eq. (4.10) we examine more closely the monopole dynamics during the above scattering event. We restrict our attention to Region 2, where the divergence arises. The metric in Region 2, Eq. (4.6), has two terms, a \(m\) dependent term and a term independent of \(m\). The \(m\) dependent term [the second line in Eq. (4.6)] can be written as \(\frac{1}{2} m \dot{r}_\beta^2\), with \(r_\beta\) given by (4.7), it describes the kinetic energy of the \(\beta\) monopole positioned at \(r_\beta\). We call this the mass term, since it describes the extended particle-like behavior of a massive soliton. When \(m\) is taken to zero the static energy density is not concentrated around the position of the \(\beta\) monopole, all that remains of the \(\beta\) monopole is a cloud surrounding the massive \(\alpha\) monopoles. The term independent of \(m\) describes the cloud dynamics in the massless case, we denote this term as the cloud term.

Holding \(M\) fixed, if \(m > M\), the mass term is greater than the cloud term for all values of \(D\) (or \(|r_\beta|\)). We interpret this as meaning that the configuration is composed of the donut \(\alpha\) monopole and a well defined \(\beta\) monopole (its energy density localized around its position). The cloud term in the Lagrangian describes the interaction of the \(\beta\) monopole with the donut \(\alpha\) monopole. If \(m < M\), the cloud term is greater than the mass term for \(D\) less than some \(m\) dependent value, \(D_m\), which is determined from Eq. (4.4). Since from Eq. (4.7), \(|r_\beta|\) is an increasing function of \(D\), then the cloud term is greater than the mass term for \(|r_\beta|\) less than some value \(r_m\), roughly given as \(r_m \approx 1/m\). This is so because for large \(|r_\beta|\), Eq. (4.6) is approximately given by

\[
L \approx \frac{1}{2} \left( m + \frac{1}{|r_\beta|} \right) \dot{r}_\beta^2. \tag{4.11}
\]
For $|r_\beta| > r_m$ the mass term is greater than the cloud term, as $|r_\beta| \to \infty$ the mass term dominates. For $|r_\beta| \gg r_m$ the configuration should regain its interpretation as a distinct $\beta$ and donut $\alpha$ monopole. As $m \to 0$ with $|r_\beta| \ll r_m$ the cloud term dominates the kinetic energy of the configuration. Here, from analogy to the $m = 0$ case, we expect the configuration to look like the donut $\alpha$ monopole surrounded by some form of cloud configuration. Previously, the term cloud has been used only in the massless limit, there it denotes the region in space inside which the monopole fields do not commute with all the generators of the non-Abelian unbroken gauge group. We will use the same term here as meaning the overall core size of the monopole. The cloud we discuss here becomes identical to the more familiar cloud in the massless limit. Inside the cloud the fields are non-Abelian and the overall core of the monopole is defined by the cloud. We do not have explicit field information off the axis of symmetry so we cannot determine the nature of the cloud configuration and how it differs to that of $m = 0$ case, [12]. We expect the $\beta$ monopole to appear as a distinct soliton only when $|r_\beta| \gg r_m (\approx 1/m)$. The value of $r_m$ increases to infinity as $m \to 0$.

The apparent divergence in the radiation can be understood as follows. When the monopole cores overlap the monopoles lose their individuality. In Region 2, when the cores of the donut $\alpha$ monopole and the $\beta$ monopole overlap, the configuration resembles a cloud surrounding the donut $\alpha$ monopole. The cloud radius is given by $|r_\beta|$. Once $r_\beta$ is far enough from the donut $\alpha$ monopole, i.e. $r_\beta \gg r_m$, the $\beta$ monopole regains its individuality. This is as expected since the core size of a single $\beta$ monopole is of order $1/m$, or $r_m$. By examining the radiation formula, Eq. (4.3), it can be seen that the radiation produced is significant for all $|r_\beta| < r_m$. When $|r_\beta| \gg r_m$ the monopoles are almost non-interacting (the metric is flat in terms of $|r_\beta|$) and little radiation is produced. It is the cloud itself which is responsible for the diverging contribution to the radiation (the cloud is dynamical and moves with velocity $\dot{r}_\beta$). The donut $\alpha$ monopole is almost static and is not responsible for large amounts of radiation. As $m$ decreases towards zero, radiation is produced over a larger and larger time period and this is what causes the eventual divergence.

With $M$ fixed and $m \ll M$ the above calculations imply that the geodesic approximation breaks down as the total radiation produced becomes of the same order as the kinetic energy, or $P \approx E \approx \frac{1}{4} M v^2$ where $v$ is the asymptotic relative velocity of the incoming $\alpha$ and $\alpha + \beta$ monopoles. This occurs for incoming velocities $v$ of the order, $v^2 \approx m/M$; the velocity of the outgoing $\beta$ monopole, $\dot{r}_\beta$, is of the order 1. In fact, holding the incoming velocity $v$ fixed and reducing $m$, it is easy to see that the outgoing velocity of the $\beta$ monopole (considered as a function of $m$) increases without limit as $m \to 0$. So even if the incoming monopole velocities are small, the velocity of the outgoing $\beta$ monopole becomes relativistic once its mass is small enough and the radiation produced correspondingly increases.

In the previous case of charge $(1, 1)$ monopoles, as one of the masses is taken towards zero, the radiation produced remains finite. Both cases share the property that as one of the monopoles masses approaches zero the monopole core will increase to arbitrary large size. The difference lies in the fact that the monopole velocities remain small at all times for charge $(1, 1)$ monopoles. For charge $(2, 1)$ monopoles the velocity of the $(0, 1)$ monopole becomes relativistic at small enough masses. This is what causes the large radiation above.
The results above appear to indicate that the validity of the moduli space approximation will break down for charge \((2, 1)\) monopoles as the mass of the charge \((0, 1)\) monopole approaches zero. However we must first analyze more closely the assumptions made. The main assumption was that the monopole cores evolve according to the moduli space approximation. Using this, the dipole radiation of massless fields from the monopole core to spatial infinity was computed. Considering Region 2 of the above geodesic the monopole fields are non-Abelian inside the cloud (the overall core region). It is from the cloud to infinity that we calculated the radiation produced, in particular we determined the time dependent dipole moments outside the cloud where the fields are Abelian.

We have also seen above that as the \(\beta\) monopole regains its individuality its asymptotic velocity increases without limit as a function of \(m\) as \(m \to 0\). The fields in the cloud region will also have very large time derivatives. Thus the moduli space approximation implies that the fields will have large time derivatives even if the incoming monopole velocities in Region 1 are small. Because the fields in the cloud region are highly relativistic it is not justified to assume the the time dependent dipole moments outside the cloud are given by the geodesic approximation. To determine the time dependent dipole moments outside the cloud a proper consideration of the time dependent field equations is necessary. However this is a very difficult task. A proper inclusion of these effects will alter significantly the results found here for the dipole radiation produced in a scattering event. We know that the finite energy monopoles cannot actually radiate infinite energy, the question is whether or not their radiation is comparable to their kinetic energy as given by the geodesic approximation. The true time dependent dipole moments will be very different to that predicted by the moduli space approximation, and using these correct dipole moments in Eq. (2.6) may imply that the radiation produced is not too large. The only concrete conclusion that we can draw is that the true time dependent monopole fields in the cloud region will differ significantly from that predicted by the geodesic approximation.

We conclude by mentioning some possibilities for future work. It may be possible to make better progress by considering the SU(4) monopoles discussed in \([5]\), where explicit field information is known. As mentioned earlier the dipole moments change discontinuously when the unbroken symmetry group becomes non-Abelian. It would be useful to see how this occurs explicitly. It is likely that the divergence in the radiation found above is generic whenever there is a cloud configuration, again this can be tested by considering the monopoles charges in \([5]\).

It would also be helpful to repeat the calculation of Section 4 directly in the minimally broken theory. One must re-derive the radiation formula in terms of the multipole moments which are now non-Abelian. This appears to be difficult because of the non-Abelian nature of the field equations. There are two massive monopoles with well defined heavy cores, parametrized by coordinates which appear in the moduli space. There are further moduli representing the cloud radius and its SU(2) orientation. The same difficulty remains that for large cloud moduli there exist large regions in space where the fields are non-Abelian but the potential energy is small. The kinetic energy carried by the cloud parameters is however of the same order as that of the kinetic energy of the massive monopoles. This is what naively causes the divergence in the radiation produced. Finally, it appears to us
that a proper resolution of the questions raised here will require an analysis similar to that of [3] for higher gauge groups.

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