Strange quark effects in electron and neutrino-nucleus quasi-elastic scattering

Andrea Meucci, Carlotta Giusti, and Franco Davide Pacati

Dipartimento di Fisica Nucleare e Teorica, Università di Pavia and
Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy

The role of the sea quarks to ground state nucleon properties with electroweak probes is discussed. A relativistic Green’s function approach to parity violating electron scattering and a distorted-wave impulse-approximation applied to charged- and neutral-current neutrino-nucleus quasi-elastic scattering are presented in view of the possible determination of the strangeness content of the nucleon.

I. INTRODUCTION

The nucleon is a bound state of three valence quarks. However, a sea of virtual $q\bar{q}$ pairs and gluons surrounds each valence quark and play an important role at distance scales of the bound state, where the QCD coupling constant is large and the effects of the color field cannot be calculated accurately. One simple way to probe the effects of the sea is to investigate whether strange quarks contribute to the static properties of the nucleon. The first evidence that the strange axial form factor $g_s A = G_s^A(Q^2 = 0)$ is different from zero and large was found at CERN by the EMC collaboration [1] in a measurement of deep inelastic scattering of polarized muons on polarized protons. In order to study the role of the strange quark to the spin structure of the nucleon various reactions have been proposed. Here we are interested in parity-violating (PV) electron scattering and neutrino-nucleus scattering. These two kinds of reactions can give us complementary information about the contributions of the sea quarks to the properties of the nucleon. While PV electron scattering is essentially sensitive to the electric and magnetic strangeness of the nucleon, neutrino-induced reactions are primarily sensitive to the axial-vector form factor of the nucleon.

A number of PV electron scattering measurements have been carried out in recent years. They are sensitive to the strangeness contribution by measuring the helicity-dependent PV asymmetry

$$A_{PV} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-},$$

where $d\sigma_+(-)$ is the cross section for incident right(left)-handed electrons from unpolarized targets (usually protons). $A_{PV}$ arises from the interference between electromagnetic and weak processes and depends on the electric (magnetic) form factors $G_E^M$, their weak counterparts $G_Z^E(M)$, and the axial form factor as seen in electron scattering $G_A^A$. It has been noted that the contribution of radiative correction must be calculated in order to allow a precise extraction of the strange axial form factor $G_A^A$ from a PV measurement of $G_A^A$. This usually prevents from a final determination of $G_A^A$ from this data. The SAMPLE [2] results at backward angle on proton and deuteron targets reported results for $G_M^A$ and $G_E^A$ at $Q^2 \approx 0.1$ (GeV/c)$^2$. The HAPPEX [3], A4 [4], and G0 [5] results at forward angles provide a linear combination of $G_E^A$ and $G_M^A$ over the range $0.1 \leq Q^2 \leq 1$ (GeV/c)$^2$, where the contribution from the axial term is usually suppressed by kinematical conditions. Three independent measurements are needed to extract $G_E^A$, $G_M^A$, and $G_A^A$ separately. The PV asymmetry from a spinless, isoscalar target, such as $^4$He, depends only on the electric form factors [6] and represents an interesting way via to avoid the problem of the axial term.

Neutrino reactions are a well-established alternative to PV electron scattering and give us complementary information about the contributions of the sea quarks to the properties of the nucleon. A measurement of $\nu(\bar{\nu})$-proton elastic scattering at Brookhaven National Laboratory (BNL) [7] suggested a non-zero value for the strange axial-vector form factor of the nucleon.
However, it has been shown in Ref. [8] that the BNL data cannot provide us decisive conclusions about the strange form factors when also strange vector form factors are taken into account. The FINeSSE [9] experiment at Fermi National Laboratory aims at performing a detailed investigation of the strangeness contribution to the proton spin via measurements of the ratio of neutral-current to the charged-current $\nu(\bar{\nu})N$ processes. When combined with the existing data on PV scattering, a determination of the strange form factors in the range $0.25 \leq Q^2 \leq 0.75$ would have to be possible with an uncertainty at each point of $\simeq \pm 0.02$ [10]. Since a significant part of the event will be from scattering on $^{12}\text{C}$, nuclear structure effects have to be clearly understood in order to give a reliable interpretation of the data.

II. PV ASYMMETRY IN INCLUSIVE ELECTRON SCATTERING ON NUCLEI

The helicity asymmetry for the scattering of a polarized electron on a target nucleus through an angle $\vartheta$ can be written from Eq. (1) as the ratio between the PV and the parity-conserving (PC) cross section, i.e.,

$$A = A_0 \frac{v_L R^\text{AV} + v_T R^\text{TA} + v'_T R^\text{VA}}{v_L R_L + v_T R_T} , \quad (2)$$

where $A_0 \simeq 1.799 \times 10^{-4} \frac{Q^2}{(\text{GeV}/c)^2}$ is a scale factor. The coefficients $v$ are derived from the lepton tensor components and are taken from Ref. [11]. The response functions $R$ are given in terms of the components of the hadron tensor and contain the interference between the electromagnetic and the weak neutral part of the current operator [11]. The single-particle electromagnetic part of the current is

$$j^\mu = F_1 \gamma^\mu + i \frac{\kappa}{2M} F_2 \sigma^{\mu\nu} q_\nu . \quad (3)$$

The single-particle current operator related to the weak neutral current is

$$j^\mu = F^\text{V}_1 \gamma^\mu + i \frac{\kappa}{2M} F^\text{V}_2 \sigma^{\mu\nu} q_\nu - G_A \gamma^\mu \gamma^5 . \quad (4)$$

The vector form factors $F^\text{V}_i$ can be expressed in terms of the corresponding electromagnetic form factors for protons ($F^\text{p}_i$) and neutrons ($F^\text{n}_i$), plus a possible isoscalar strange-quark contribution ($F^s_i$), i.e.,

$$F^\text{V}_i; p(n) = \pm \left\{ F^\text{p}_i - F^\text{n}_i \right\} /2 - 2 \sin^2 \theta_W F^{p(n)}_i - F^s_i/2 , \quad (5)$$

where $+(-)$ stands for proton (neutron) knockout and $\theta_W$ is the Weinberg angle ($\sin^2 \theta_W \simeq 0.2313$). The strange vector form factors are taken as

$$F^s_i(Q^2) = \frac{(\rho^s + \mu^s)\tau}{(1 + \tau)(1 + Q^2/M^2_V)^2} , \quad F^s_2(Q^2) = \frac{(\mu^s - \tau \rho^s)}{(1 + \tau)(1 + Q^2/M^2_V)^2} , \quad (6)$$

where $\tau = Q^2/(4M^2_p)$ and $M_V = 0.843 \text{ GeV}$. The quantities $\mu^s$ and $\rho^s$ are related to the strange magnetic moment and radius of the nucleus. The axial form factor is expressed as

$$G_A(Q^2) = \frac{1}{2} (\pm g_A - g_A^\Lambda) G , \quad (7)$$

where $g_A \simeq 1.26$, $g_A^\Lambda$ describes possible strange-quark contributions, and

$$G = (1 + Q^2/M^2_A)^{-2} . \quad (8)$$
The axial mass has been taken from Ref. [12] as $M_A = (1.026 \pm 0.021)$ GeV.

The inclusive PV electron scattering may be treated using the same relativistic approach which was already applied to the inclusive PC electron scattering [13] and to the inclusive quasi-elastic $\nu(\bar{\nu})$-nucleus scattering [14]. The components of the nuclear response are written in terms of the single-particle optical-model Green’s function, that is based on a bi-orthogonal expansion in terms of the eigenfunctions of the non-Hermitian optical potential and of its Hermitian conjugate. As it is discussed in Refs. [11, 13, 14], the flux is preserved and final state interactions (FSI) are treated in the inclusive reaction consistently with the exclusive one.

In order to evaluate the uncertainties of the model, we compare in Fig. 1 the asymmetries for $^{12}$C at $q = 400$ MeV/c and $^{16}$O at $\varepsilon_i = 1200$ MeV and $\vartheta = 32^\circ$ with different bound states and optical potentials as explained in the text. The results are rescaled by the factor $10^5$.

![FIG. 1: PV asymmetry for $^{12}$C at $q = 400$ MeV/c and $^{16}$O at $\varepsilon_i = 1200$ MeV and $\vartheta = 32^\circ$ with different bound states and optical potentials as explained in the text. The results are rescaled by the factor $10^5$.](image)

The sensitivity of PV electron scattering to the effect of strange-quark contribution to the vector and axial-vector form factors, is shown in Fig. 2 for $^{12}$C at $q = 500$ MeV/c, $\omega = 120$ MeV, and $\vartheta = 30^\circ$ as a function of the strangeness parameters, $\rho^s$, $\mu^s$, and $g_A^s$. The range of their values is chosen according to Refs. [3, 18]. The asymmetry reduces in absolute value up to $\approx 40\%$ as $\rho^s$ varies in the range $-3 \leq \rho^s \leq +3$, whereas it changes up to $\approx 15\%$ for $-1 \leq \mu^s \leq +1$. We note that, according to HAPPEX [3] results, $\rho^s$ and $\mu^s$ might have opposite sign, thus leading to a partial cancellation of the effects. The sensitivity to $g_A^s$ is very weak, as can be seen in the lower panel of Fig. 2.
FIG. 2: PV asymmetry for $^{12}$C at $q = 500$ MeV/c, $\omega = 120$ MeV, and $\vartheta = 30^\circ$ as a function of $\rho^s$ and $\mu^s$ (upper panel) and as a function of $g_A^s$ and $\mu^s$ (lower panel).

III. THE QUASI-ELASTIC NEUTRINO-NUCLEUS SCATTERING

The $\nu(\bar{\nu})$-nucleus cross section for the semi-inclusive process may be written as a contraction between the lepton and the hadron tensor. The lepton tensor is defined in a similar way as in electromagnetic knockout and it separates into a symmetrical and an anti-symmetrical component which are written as in Refs. [14, 19, 20]. The hadron tensor is given in its general form by suitable bilinear products of the transition matrix elements of the nuclear weak-current operator. Assuming that the final states are given by the product of a discrete (or continuum) state of the residual nucleus and a scattering state of the emitted nucleon and using the impulse approximation, the transition amplitude reduces to the sum of terms similar to those appearing in the electron scattering case [19, 21]. The single-particle current operator related to the weak current is

$$j^\mu = \left[ F_V^1 \gamma^\mu + i \frac{F_V^2}{2M} \sigma^\mu^\nu q^\nu - G_A \gamma^\mu \gamma^5 + F_\rho q^\mu \gamma^5 \right] O_\tau ,$$

(9)

where $O_\tau = \tau^\pm$ are the isospin operators for charged-current (CC) reactions, while $O_\tau = 1$ for neutral-current (NC) scattering. The induced pseudoscalar form factor $F_\rho$ contributes only to CC scattering but its effects are almost negligible. For NC reactions, the weak isovector form factor, $F_V^1$ and $F_V^2$, and the axial form factor are expressed as in Eqs. (5) and (7), whereas for CC scattering they are

$$F_V^i = F_P^i - F_N^i , \quad G_A = g_A G ,$$

(10)

where $g_A \simeq 1.26$ and $G$ is defined in Eq. (8). The single differential cross section for the quasi-elastic $\nu(\bar{\nu})$-nucleus scattering with respect to the outgoing nucleon kinetic energy $T_N$ is obtained after performing an integration over the solid angle of the final nucleon and over the energy and angle of the final lepton. We use in our calculations a relativistic optical potential with a real and
an imaginary part which produces an absorption of flux. This is correct for an exclusive reaction, but would be incorrect for an inclusive one. Here we consider situations where an emitted nucleon is detected and treat the quasi-elastic neutrino scattering as a process where the cross section is obtained from the sum of all the integrated exclusive one-nucleon knockout channels. Some of the reaction channels which are responsible for the imaginary part of the optical potential, like multi-step processes, fragmentation of the nucleus, absorption, etc. are not included in the experimental cross section as an emitted proton is always detected. The outgoing proton can be re-emitted after re-scattering in a detected channel, thus simulating the kinematics of a quasi-elastic reaction, only in few cases. The relevance of this contributions depends on kinematics and should not be too large in the situations considered here.

![Diagram](image)

**FIG. 3**: Upper panel: ratio of proton-to-neutron NC cross sections of the $\nu$ scattering on $^{12}$C. Lower panel: ratio of neutral-to-charged current cross sections of the $\nu$ scattering on $^{12}$C. Dashed lines are the results with no strangeness contribution, solid lines with $g_A^s = -0.10$, dot-dashed lines with $g_A^s = -0.10$ and $\rho^s = -0.50$, dotted lines with $g_A^s = -0.10$ and $\rho^s = +2$. Long dashed lines are the RPWIA results without strangeness contribution.

Since an absolute cross section measurement is a very hard experimental task due to difficulties in the determination of the neutrino flux in Ref. [22] was suggested to measure the ratio of proton to neutron (p/n) yield as an alternative way to separate the effects of the strange-quark contribution. This ratio is very sensitive to the strange-quark contribution as the axial-vector strangeness $g_A^s$ interferes with the isovector contribution $g_A$ with one sign in the numerator and with the opposite sign in the denominator (see Eq. (7)) and it is expected to be less sensitive to distortion effects than the cross sections themselves. In the upper panel of Fig. 3, the p/n ratio of the quasi-elastic $\nu$ scattering on $^{12}$C is displayed as a function of $T_N$. The RPWIA results are shown in the figure and they are almost coincident with the RDWIA ones. The p/n ratio for an incident neutrino is enhanced by a factor $\simeq 20$-$30\%$ when $g_A^s$ is included and by $\simeq 50\%$ when both $g_A^s$ and $\mu^s$ are included. A minor effect is produced by $\rho^s$, which gives only a slight reduction of the p/n ratio. Precise measurements of the p/n ratio appear however problematic due to the difficulties associated with neutron detection. This is the reason why the most attractive quantity to extract experimental
information about the strangeness content seems the ratio of the neutral-to-charged (NC/CC) cross sections. In fact, although sensitive to the strange-quark effects only in the numerator, the NC/CC ratio is simply related to the number of events with an outgoing proton and a missing mass with respect to the events with an outgoing proton in coincidence with a muon. Our RDWIA results for the NC/CC ratio of the quasi-elastic $\nu$ scattering on $^{12}$C are presented in the lower panel of Fig. as a function of the kinetic energy of the outgoing proton. The fact that the CC cross section goes to zero more rapidly than the corresponding NC one (because of the muon mass) causes the enhancement of the ratio at large values of $T_p$. The simultaneous inclusion of $g_A^s$ and $\mu^s$ gives an enhancement that is about a factor of 2 larger than the one corresponding to the case with only $g_A^s$ included. The effect of $\rho^s$ is very small.

[1] J. Ashman, et al., [European Muon Collaboration], Nucl. Phys. B 328, 1 (1989).
[2] D.T. Spayde, et al., Phys. Lett. B 583, 79 (2004).
[3] K. Aniol, et al., [HAPPEX Collaboration], Phys. Rev. Lett. 82, 1096 (1999); Phys. Rev. C 69, 065501 (2004); Phys. Lett. B 635, 275 (2006).
[4] F.E. Maas, et al., [A4 Collaboration], Phys. Rev. Lett. 94, 152001 (2005).
[5] D.S. Armstrong, et al., [G0 Collaboration], Phys. Rev. Lett. 95, 092001 (2005).
[6] A. Acha, et al., [HAPPEX Collaboration], nucl-ex/0609002.
[7] L.A. Ahrens, et al., Phys. Rev. D 35, 785 (1987).
[8] G.T. Garvey, et al., Phys. Rev. C 48, 761 (1993).
[9] S. Brice, et al., hep-ex/0402007. See also http://www-finesse.fnal.gov/index.html and http://www-boone.fnal.gov/.
[10] S.F. Pate, Phys. Rev. Lett. 92, 082002 (2004); S.F. Pate, G. MaLachlan, D. McKee, and V. Papavasiliou, hep-ex/0512032.
[11] A. Meucci, C. Giusti, and F.D. Pacati, Nucl. Phys. A 756, 359 (2006).
[12] V. Bernard, L. Elouadrhiri, and Ulf-G. Meissner, J. Phys. G 28, R1 (2002).
[13] A. Meucci, F. Capuzzi, C. Giusti, and F.D. Pacati, Phys. Rev. C 67, 054601 (2003).
[14] A. Meucci, C. Giusti, and F.D. Pacati, Nucl. Phys. A 739, 277 (2004).
[15] M.M. Sharma, M.A. Nagarajan, and P. Ring, Phys. Lett. B 312, 377 (1993).
[16] E.D. Cooper, et al., Phys. Rev. C 47, 297 (1993).
[17] G.A. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55, 540 (1997).
[18] D.H. Beck and R.D. McKeown, Ann. Rev. Nucl. Part. Sci. 51, 189 (2001).
[19] S. Boffi, et al., Electromagnetic Response of Atomic Nuclei, Oxford Studies in Nuclear Physics, Vol. 20 (Clarendon, Oxford, 1996); S. Boffi, et al., Phys. Rep. 226, 1 (1993).
[20] A. Meucci, C. Giusti, and F.D. Pacati, Nucl. Phys. A 773, 250 (2006).
[21] A. Meucci, C. Giusti, and F.D. Pacati, Phys. Rev. C 64, 014604 (2001); Phys. Rev. C 64, 064615 (2001).
[22] G.T. Garvey, et al., Phys. Lett. B 289, 249 (1992); G.T. Garvey, et al., Phys. Rev. C 48, 1919 (1993); C.J. Horowitz, et al., Phys. Rev. C 48, 3078 (1993); W.M. Alberico, et al., Nucl. Phys. A 623, 471 (1997); W.M. Alberico, et al., Phys. Lett. B 438, 9 (1998).