A Monte Carlo simulation of solar neutron-decay protons is performed using a simple interplanetary magnetic field (slab model) in interplanetary space. The time profile of solar neutron-decay protons obtained by our simulation is compared with observations. It is shown that the simple model can predict coherent pulses at the start of events. Magnetic field parameters used in the comparison can suggest the mean free path of protons near the Earth. The values are discussed in relation to other published results. Also, the comparisons show that it is difficult to determine the angular distribution of solar neutrons emitted at a flare site as long as the presently available solar neutron-decay protons are used.

1. Introduction

Many investigations of the propagation of charged particle through an interplanetary medium have been made by observing solar energetic particles (see e.g. Palmer, 1982; Bieber et al., 1994). However, when direct solar-energetic particles are used to study such transports, it is difficult to distinguish propagation between solar corona and interplanetary medium.

Evenson et al. (1983a) reported on protons resulting from the decay of solar neutrons, and suggested that observing neutron-decay protons provides an independent way of determining the parameters for the propagation of charged particles in the solar system, because the protons may be mainly influenced by the interplanetary condition in the vicinity of the Earth. Thus, it is very interesting to analyze the time profile of solar neutron-decay protons in order to estimate the transport of charged particles in interplanetary magnetic fields. Evenson et al. (1983b) tried to understand the observed time profile of protons with a spherically symmetric model of isotropic diffusion. Ruffolo (1991) also calculated the time evolution by using the pitch-angle transport equation (PATE), including the spiral and focusing magnetic field (SF) derived by Parker (1958, 1967) and compared his calculation with his new analyzed data indicating coherent pulses at the start of events. Kurganov and Ostryakov (1992) made Monte Carlo simulation including SF to get the time profile and showed the sharp increases of intensity at the beginning of events. They supposed an isotropic pitch-angle scattering. Zhang (1993) computed time profile by solving PATE including the radial variation of magnetic power spectrum in addition to SF. Also, he pointed out that focusing effect is not important when the distance from the Sun becomes comparable to 1 AU, because $L$, the focusing length of the field, becomes larger than a mean free path. Ruffolo (1991) and Zhang (1993) used the pitch-angle scattering coefficient $D\mu_q$ derived from quasi-linear theory for their calculation. Here, $\mu$ denotes pitch-angle cosine. But, Ruffolo (1991) stated that he used $D\mu_q$ merely as a convenient and well-understood parametarization.

In this paper, the time profile of neutron-decay protons is discussed on the basis of a more fundamental scattering process by computing directly the particle's trajectories through the simulated magnetic field fluctuations near the Earth. Generally, solar neutron-decay protons are purely observed only during a few hours from the onset without any contamination of solar protons directly originating...
from the Sun. As the first step of our computation, we neglected the spiral and focusing Parker magnetic field lines. However, it will become soon obvious that even this small scale assumption can give good fit to the observed time profile even after 5 hours since flares. Further, magnetic field fluctuation is assumed to be approximately expressed by a slab model. That is, the fluctuating magnetic field is perpendicular to the average magnetic field strength of 5 nT near the Earth. When a neutron decays into a proton on a field line, the proton moves back and forth along this average field line because of the small fluctuating field, and comes accidentally to the observation point by a diffusion-like process. Here, in computing the time profile of neutron-decay protons, it is necessary to assume neutron emission at the Sun; both isotropic and anisotropic angular distribution proposed by Hua and Lingenfelter (1987) are taken into account. The time profiles of the protons obtained are compared to the data re-binned by Ruffolo (1991) for both cases of isotropic and anisotropic emission of neutrons at the Sun. It is shown that our simple model can produce the coherent pulse at the start of events pointed out by Ruffolo (1991), and Kurganov and Ostryakov (1992).

2. Model

In Fig. 1, a schematic drawing in the equatorial plane is shown. In the figure, E means the Earth (observation point) and denotes the origin of our coordinate system, and the z-axis coincides with the direction of average magnetic field \( \langle B \rangle \); x is perpendicular to the plane, and y is the direction at a right angle to \( \langle B \rangle \) following the right-handed coordinate system. Perturbation fields in x and y directions are denoted by \( \Delta B_x \) and \( \Delta B_y \), respectively, and superimposed on \( \langle B \rangle \) by the following observed magnetic field parameters. That is, \( \eta = (\Delta B^2)^{1/2}/\langle B \rangle \) which is defined as

\[
(\eta)^2 = 2 \int_0^\infty \rho(k)dk,
\]

![Fig. 1. Schematic drawing of the coordinates. The origin is at E supposed to be the Earth. The z-axis coincides with the direction of \( \langle B \rangle \), and y-axis is perpendicular to the x-z plane.](image-url)
where \( p(k) \) and \( k \) are the spatial power spectrum of the magnetic field fluctuation expressed as

\[
p(k) = \frac{A}{\left[1 + (2\pi kL_c)^2\right]^\gamma/2},
\]

and a wave number respectively. Here, \( A \) is a constant depending upon the magnitude of \( \eta \), correlation length \( L_c \), and index of power spectrum \( \gamma \). \( L_c \) and \( \gamma \) are taken as \( 6 \times 10^{11} \) cm and 1.5 (Hedgecock, 1975), respectively. Values of \( \eta \) are tentatively taken as 0.2, 0.3, and 0.4 with \( (B) = 5 \) nT so as to fit the time profile of proton data given by Ruffolo (1991). The method of producing the simulated magnetic field fluctuation was described in Kato and Sakai (1985). Both the spiral shape of the magnetic field line and the change of magnetic field strength with distance from the Sun are neglected for simplicity. This means that an average field line is straight and constant, and directed toward the \( z \)-axis. Even this assumption, it will be shown later that our model can give good fit to the observed time profile even after 5 hours since flares.

Fig. 2. The dependence of particles coming from \( \alpha^+ \) and \( \alpha^- \) directions upon the elapsed time after a flare. The energy is 40 MeV and the case of \( \eta = 0.3 \) is shown.
Neutrons emitted simultaneously from a flare site decay into protons with the decay constant of 896 sec (Particle Data Group, 1988) in the rest frame in some places of interplanetary space. The trajectories of protons are computed in the average field with fluctuations. The computation shows that protons are gyrating and move back and forth along the field lines like a diffusion process because of the fluctuating fields. At times, protons are quasi-trapped between stronger fields. If the field line is connected to an observation point, some of the protons can be observed. In our actual calculation, the procedure registering the coordinate of proton is inverse by referring Ogura and Kodama (1989). That is, neutrons are emitted as $\alpha = 0$, and decay into protons on the Sun-Earth line, where angle $\alpha$ is measured from the Sun-Earth line. The protons move back and forth along a straight magnetic field line. When the protons intersect the Earth's orbit, then the values of $\alpha$, time, $x$ and $y$ are recorded.

3. Results

In the next section, we will compare our results of simulation with the data given by Ruffolo (1991) which are plotted in a unit of distance travelled by particles. The distance is the measured velocity times the time since the flare, i.e., $v(t - t_{flare})$, where $v$, $t$ and $t_{flare}$ are proton velocity, time and time of flare, respectively. In this plotting, time evolution becomes independent of particle's energy, as pointed out by Ruffolo (1991). Considering this point, and referring Kurganov and Ostryanov (1992), we fixed here particle's energy at 40 MeV.

Figure 2 shows the dependence of 40 MeV neutron-decay protons coming from the direction of $\alpha^*$ and $\alpha^*$ (see Fig. 1) upon the elapsed time when neutrons are isotropically emitted at the Sun. From the figure, it is clear that protons in the range of $\alpha^*$ can more easily approach the Earth than protons in the region of $\alpha^*$ at the first stage. The reason is very simple and can be explained as follows. Protons decayed from neutrons move in the same direction as the neutrons. So the pitch-angle of protons in the region of $\alpha^*$ to the average magnetic field is directed away from the Earth and smaller than $\phi = 45^\circ$ which is the direction of nominal Parker field line measured from the Sun-Earth line. Then these protons have to return by a scattering process in order to be observed. On the other hand, protons in the range of $\alpha^*$ can easily

![Graph E55](image1)

**Fig. 3.** The time profile of solar neutron-decay protons of 40 MeV obtained by us is shown for E55, corresponding to the event of 24 April 1984 (55°E). Solid line denotes an isotropic injection case at the flare site, and broken line is an anisotropic case proposed by Hua and Lingenfelter (1987).

![Graph E72](image2)

**Fig. 4.** The time profile of solar neutron-decay protons of 40 MeV obtained by us is shown for E72, corresponding to the event of 3 June 1982 (72°E). Solid line denotes an isotropic injection case at the flare site, and broken line is an anisotropic case proposed by Hua and Lingenfelter (1987).
Propagation of Solar Neutron-Decay Protons near the Earth

Approach the Earth because the pitch-angle is directed toward the Earth. As time elapses, the distribution around $\alpha = 0$ approaches a gaussian one around $\alpha = 0$. Also, even after 300 min. from a flare, protons beyond $\alpha^* = 20^\circ$ are less than 20% of the total protons. On the other hand, protons can not move far beyond the distance of order of the gyro-radius in $x$- (off the ecliptic plane) and $y$-directions. This is due to the difficulty of crossing magnetic field lines.

In Figs. 3 and 4, the time profiles of solar neutron-decay protons of 40 MeV obtained by us are shown both for E55, corresponding to the event of 24 April 1984 (55°E), and for E72, corresponding to the event of 3 June 1982 (72°E), respectively. The value of $\eta$ used here is 0.3. The reason is described in Discussions. Solid line denotes an isotropic injection case at a flare site, and the broken line is an anisotropic case proposed by Hua and Lingenfelter (1987). In Fig. 5, angular distribution of neutron emission at a flare site reproduced from their figure 6 is shown, and an isotropic case is added as the straight line of $1/2\pi$. Here, $\theta$ is the zenith angle at the point of flare. Also, Bessel function means the differential ions spectrum in momentum/nucleon accelerated at flare (Ramaty, 1979). $B$ and $T$ denote the acceleration efficiency and the mean time for escaping from the acceleration regime, respectively. Horizontal $\delta$ ($89^\circ$) denotes the direction of downward beaming of accelerated ions and means $\delta$ function of $\theta = 89^\circ$.

![Fig. 5. Angular distribution of neutron emission at a flare site, reproduced from figure 6 of Hua and Lingenfelter (1987); an isotropic case is added as the straight line of $1/2\pi$. Here, $\theta$ is the zenith angle at the point of flare. Also, Bessel function means the differential ions spectrum in momentum/nucleon accelerated at flare (Ramaty, 1979). $B$ and $T$ denote the acceleration efficiency and the mean time for escaping from the acceleration regime, respectively. Horizontal $\delta$ ($89^\circ$) denotes the direction of downward beaming of accelerated ions and means $\delta$ function of $\theta = 89^\circ$.](image-url)
direction of downward beaming of accelerated ions and means $\delta$ function of $\theta = 89^\circ$. For E55, the isotropic case gives higher intensity than the anisotropic case by almost 10%. For E72, an anisotropic case has higher intensity than an isotropic case by $\sim50\%$. Recalling our results of simulation that only protons decaying near the Earth from neutrons can reach an observation point, and further that protons in the range of $\alpha$ can come more easily, for the 55°E (or 72°E) event, only neutrons around and above 55° (or 72°) in Fig. 5 contribute mainly to the number of protons observed in the simulation. Thus, the intensity differences in Figs. 3 and 4 can be reasonably understood by referring to Fig. 5. Here, it is noted that the general trend of the time profiles both for isotropic and anisotropic cases look similar to each other, as already shown in both Figs. 3 and 4.

4. Discussions

In the paper of Ruffolo (1991), a coherent pulse of protons is emphasized by plotting his data by linear scale in the ordinate. If one plots the data in a logarithmic scale, the coherent pulse is not so pronounced. Difference of trend of time profile between Evenson et al. (1985) and Ruffolo (1991) re-plotted in real time scale looks like small and both trend seem to be even plotted in error bars. Ruffolo (1991) suggested that his time profile data are independent of energy of particle in the unit of time multiplied by velocity of particle ($=s$). This is very convenient for us. Because our calculation uses a fixed energy. Also, numbers of data points are much larger than Evenson et al. (1985). So the data given by Ruffolo (1991) are used for comparison. However, the data are re-plotted in a logarithmic scale in the ordinate.

We made simulation for $\eta = 0.2, 0.3, \text{and} 0.4$. From the comparison of our results of simulations with the observation, we find that $\eta = 0.3$ gives the best fit to the data. So $\eta = 0.3$ is used for the following comparisons. The E55 case is shown in Figs. 6 and 7, corresponding to isotropic and anisotropic emission of neutrons at the flare site, respectively. Also, the E72 case is shown in Figs. 8 and 9, corresponding to an isotropic and anisotropic emission of neutrons at the flare site, respectively. From Figs. 6, 8 and 9, it is clear that there is no difference in goodness of fit for isotropic and anisotropic cases. Although the goodness of fit of Fig. 7 looks worse around the peak, the general trend of time profile seems well fitted. So it seems difficult to determine from the comparison with the presently available data which simulation

Fig. 6. Time profile of solar neutron-decay protons obtained by our simulation is shown together with the data (24 April 1984) cited from Ruffolo (1991). This case is an isotropic injection, and $\eta$ is 0.3.

Fig. 7. Time profile of solar neutron-decay protons obtained by our simulation is shown together with the data (24 April 1984) cited from Ruffolo (1991). This case is an anisotropic injection model (Hua and Lingenfelter, 1987), and $\eta$ is 0.3.
results (either isotropic or anisotropic case) give a better agreement with the observation. This is again due to the fact that only protons decayed from neutrons near an observation point are detectable, as mentioned previously. However, if the angular distribution of decay protons is simultaneously observed, useful informations regarding the angular distribution of neutron emission can be obtained as suggested by Ruffolo (1991).

Our model is very simple, and neglects the well-known Parker field pattern with focussed and spiral patterns. Nevertheless, good fitness to the data is obtained as already shown in the above. We discuss this fact in the below. Around the peak of intensity in time profile, ~99% of protons come from the region of α < 16°. Even after 300 min. (or ~10 AU in unit of s), the contribution of protons with α < 10° and <20° becomes ~60% and ~80%, respectively. That is, the observed protons are limited to protons decayed from neutrons near the Earth. So the focusing effect may be ineffective. This is one of reason why we can get a good fit to observed time profile.

Here, again, we would like to point out that the focusing effect is not so efficient in the rise time regime from different point of view. From the first rise of event to ~30 min., most of protons come from within
When $\alpha = 15^\circ$, the angle of the nominal Parker field line to radial direction is $-40^\circ$ which is the same direction with the pitch-angle of neutron-decay proton. If the proton moves toward the Earth along the Parker field line, the angle of $40^\circ$ becomes $31.8^\circ$ by the invariance of magnetic moment (focusing effect) and the path length through the field line becomes $-0.32$ AU. Thus, if a proton with 40 MeV travels along the path of 0.32 AU through Parker field, then the arrival time is obtained 11.6 min. by taking an average between $40^\circ$ and $31.8^\circ$ (case A). On the other hand, in the case of no focusing, the time will be simply calculated by $0.32$ AU/$v \cos 45^\circ$, where $v$ denotes the velocity of 40 MeV proton. This corresponds to $-13.3$ min. (case B). The difference between case A and B is 1.7 min. It seems too small to see this difference by observations. So we don’t think the focusing effect gives a great difference in rise time region. In addition, if we consider that scattering mean free path is around $-0.1$ AU, the focusing effect would be much smaller. Because a proton with $\alpha > 15^\circ$ will be at least more than one time scattered before it reaches an observation point and the pitch-angle may be changed more than or less than the expectation due to focusing effect. Further, if we consider that magnetic fields are fluctuating around an averaged magnetic field, the pitch-angle of proton fluctuates around $\pm 20^\circ$ for $\eta = 0.3$. This effect also obscures the focusing effect.

Scattering mean free path $\lambda_{nt}$ of 40 MeV protons is estimated in the following two ways for the fixed parameters of $\eta = 0.3$, $\gamma = 1.5$, and $L_c = 6 \times 10^{11}$ cm, which are used for the above comparisons. First, the numerical value of the pitch-angle scattering coefficient ($D_\mu^{11}$) is calculated by the expression of Goldstein (1977) based upon quasi-linear theory. Then, by using the relation among this $D_\mu^{11}$, the spatial diffusion coefficient ($\kappa_\mu$) (Jokipii, 1966; Hasselmann and Wibberenz, 1968), and also the well-known expression $\lambda_{nt} = 3 \kappa_\mu / v$, where $v$ is proton velocity, $\lambda_{nt} \sim 0.09$ AU is obtained.

Also, by following the proton’s trajectories (Sakai, 1988) for the same parameters as above, the computation of the value of $\kappa_\mu^{11\text{sim}} = \lim_{\Delta t \to \tau} \left( \Delta z^2 \right) / 2 \Delta t$ gives $\lambda_{nt} \sim 0.081$ AU, where $\Delta t$, $\tau$ and $z$ are time, correlation time of $L_c/v$ and displacement of the particle along the averaged magnetic field direction, respectively. This is a direct derivation of the spatial diffusion coefficient without using $D_\mu^{11}$ derived from quasi-linear theory. These values seem compatible with the results around 1 AU obtained by Zhang (1993). On the other hand, Evenson et al. (1983b), Ruffolo (1991), and Kurganov and Ostreyanov (1992) give nearly $\lambda \sim 0.3$ AU, which is obtained by fitting the time profile of protons observed by Evenson et al. (1985). This discrepancy of $\lambda_{nt}$ is considered to be a long-standing problem, which is summarized by Palmer (1982). Our trajectories’ computation does not explicitly have any difficulty near the pitch-angle of 90°. And, if $\eta$ becomes larger than 0.1, it shows trapping of particles among stronger fluctuating fields. This is a non-linear effect. In this sense, it seems to need attention that $D_\mu^{11}$ derived from quasi-linear theory is simply applicable to the computation of time profile of protons for $\eta > 0.1$. Here, it is worthwhile to note that if $\eta$ becomes smaller than 0.1, $\kappa_\mu^{11\text{sim}} = \lim_{\Delta t \to \tau} \left( \Delta z^2 \right) / 2 \Delta t$ approaches the value derived from $D_\mu^{11}$ of quasi-linear theory (Sakai, 1988). Here, if we calculate inversely the value of $\eta$ corresponding to $\lambda_{nt} = 0.3$ AU by using the relations ($D_\mu^{11}$, $\kappa_\mu$, and $\lambda_{nt}$) mentioned above for the same $L_c$, $\gamma$ and $\eta$ is estimated $\sim 0.2$, although it is not clear whether quasi-linear theory is still applicable.

For further discussion, observations of interplanetary field conditions at proton events, that is, $\phi$, $\langle B \rangle$, $\eta$, $\gamma$ and $L_c$ are necessary. In addition, the pitch-angle diffusion coefficient near $\mu = 0$ might be included, as pointed by Bieber et al. (1994), to compute the transport equation. Also, we feel that the slab model itself might be reconsidered for large fluctuation fields.

Finally, we should note that our calculation is based upon the neglecting the spiral and focusing Parker magnetic field line, and that checking this validity will be needed in future work.

5. Summary

The method of our simulation, starting from producing fluctuation of magnetic field in the interplanetary space, is a new approach to understand the propagation of neutron-decay protons near the Earth.
Can our simulation based upon a simple slab model fit the time profiles of 3 June 1982 and 24 April 1984 (Ruffolo, 1991)? We found that $\eta = 0.3$ with the parameters of interplanetary conditions of $L_c = 6 \times 10^{11}$ cm and $\gamma = 1.5$ can fit their data well. This fact implies a mechanism of propagation which governs near the Earth. Also, there is no distinct difference in fitting for isotropic and anisotropic injection of neutrons at the flare site, if we consider the error bars.

In addition, the above parameters suggest the mean free path of the protons. If we use $D_{\mu}^{nl}$, with $L_c = 6 \times 10^{11}$ cm, $\gamma = 1.5$, and $\eta = 0.3$, we get $\lambda_{nl} \sim 0.09$ AU. Further, a shorter mean free path is obtained by direct computation of $\lambda_{nl} = \lim_{\Delta \tau \rightarrow 0} \left( \frac{\Delta \xi^2}{2 \Delta t} \right)$, giving $\lambda_{nl} \sim 0.081$ AU. These values are smaller than the one obtained by Evenson et al. (1983b), Ruffolo (1991), and Kurganov and Ostryakov (1992). One discrepancy may be attributed to particle trapping among stronger fields in the simulation, and the use of $D_{\mu}^{nl}$ for $\eta > 0.1$ in computing PATE.

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