Interacting dark energy collapse with matter components separation

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Abstract. We use the spherical collapse model of structure formation to investigate the separation in the collapse of uncoupled matter (including dark matter and baryons) and coupled dark matter in an interacting dark energy scenario. Following the usual assumption of a single radius of collapse for all species, we show that we only need to evolve the uncoupled matter sector to obtain the evolution for all matter components. This gives us more information on the collapse with a simplified set of evolution equations compared with the usual approaches. We then apply these results to five quintessence potentials and show how we can discriminate between different quintessence models.

Keywords: dark matter simulations, dark energy theory, semi-analytic modelling

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1 Introduction

The field of cosmology, although receiving an accumulation of precision observations in the recent years and despite their consistence with the “concordance model”, ΛCDM, is yet to decide on the content of the universe. The wealth of data available ranges from large scale geometric assessments like the precise measurements of the Cosmic Microwave Background (CMB) [1], intermediate scale evaluations like Baryon Acoustic Oscillations (BAO) [2] and more local scales measures of the Hubble constant ($H_0$) via observations of standard candles like Cepheid variable stars [3] or Type Ia Supernovae (SNIa) [4]. Although the simplicity of the concordance model would place it in a prominent position, given its compatibility with observations, it suffers from theoretical shortcomings that question our understanding
of gravity and our knowledge of physics: the fine tuning problem and the coincidence problem (for a review, see [5]). Introduction of dynamical forms of dark energy (hereafter DE) was then considered to try and overcome those problems [5–7], and quintessence scalar fields [8–10] are the most obvious candidates (for review see [11]). However if they can overcome the first problem, the second remains in various forms. In this framework, DE and dark matter (DM) are the fundamental building blocks of the cosmological standard model based on general relativity, which leaves unknown and dark nearly 96% of the Universe’s energy content. Although coupling to baryonic matter is tightly constrained by local gravity tests [12], this undetermined aspect opens the door to all kinds of couplings within the dark sector, including non-linear DE self-gravitating clustering [13, 14] and non-minimal coupling between the two dark species, introduced either by ad hoc arguments [15] or because dark energy and dark matter are unified in the context of some framework [16–18]: the implications of interacting DE-DM on cosmological observations were examined in works such as [15, 19–22, and references therein], while analysis of CMB and BAO such as [23, 24], or of gamma-ray bursts as in [25], have constrained some coupled quintessence models. Studies have demonstrated that signatures of such coupled models can be found in the background expansion history of the universe and on cosmic structure formation, especially at larger, galaxy clusters, scales [26–30]. Another consequence of interacting dark sector could be the violation of the Equivalence Principle [31–35].

Dynamic DE impact on structure formation, in the form of a quintessence scalar field, was first restricted to linear or perturbation theory of structure evolution [36]. In the mildly non-linear regime, [37, 38] found the smallest scale of quintessence fluctuations are larger than the clusters scale, so this clustering should be negligible. Nevertheless, studies of the fully non-linear regime showed marked differences in DM structures between DE models that cluster and those that do not [13]. The clustering properties of DE models remain however an open question. Non-linear structure formation is traditionally studied with N-body simulations, as e.g. in [39, 40]. Insofar as investigating structure formation with DE, since no dynamical model seem preferred, the flexibility of semi-analytical methods have lead to numerous studies such as [13, 14, 38, 41–48]. The methods used, prevalent at the core of fully semi-analytic galaxy formation models, i.e in [49–55], revolve around similar mass function-determining schemes as the methodology developed by Press & Schechter [56, hereafter PS]. This method uses a spherically symmetric dynamical model to relate the collapse of massive structures to a density threshold in the linearly extrapolated density field. In this way, it is possible to apply Gaussian statistics to the initial density field in order to count the numbers of collapsed structures above a given mass threshold at a particular epoch. The spherical collapse model [57–69] or its variants are therefore at the root of such approach.

Non-gravitational interactions is expected to impact significantly on the free fall of DM, whether it links baryons with DM [70] or originates in interacting DE [71, 72]. This investigation aims at revealing that impact in the spherical collapse model, as suggested in [31, 32], in its dependence on the DE model involved and on the difference between the collapses of coupled and uncoupled matter species, as mark of a baryon/DM segregation. For that purpose, we singled out a range of quintessence potentials and a range of interaction parameters to gauge their influence on the violation of the Equivalence Principle between coupled and uncoupled matter manifested by this segregation: the double exponential potential [73], the original inverse power potential [9] and the infinite sum of inverse power potential [74], the supergravity motivated potential (SUGRA) [75], and the superstring theory motivated potential of [76]. For these models, we used the spherical collapse model to obtain separately
the uncoupled matter and DM overdensity parameters evolution and their critical values, arguably showing how the baryon/DM drift can manifest itself depending on the DE model. We introduced a novel approach, noticing that a single collapse radius kept for all species lead to algebraic relations between their overdensities. Thus we simplified the treatment of dynamics which entails following dust-like uncoupled matter in a DE environment. This simpler evolution manages to recover previous results such as [14, 48] and in the process allows to differentiate between coupled and uncoupled matter evolution.

The paper is organised as follows: in section 2, the evolution equations for both the baryons or uncoupled matter and for the coupled DM are detailed, followed by the forms of the potentials; section 3 presents our analysis of the segregated spherical collapse semianalytical model. Finally, in section 4 we discuss the results and present our conclusions.

2 The spherical collapse in interacting DM/DE quintessence models

The spherical collapse model (pioneered in [57–61] and summarised in [62]) is a powerful tool of semianalytical methods to study gravitational clustering, e.g. [63–69], in particular with many different models of interacting DE as in [13, 14, 38, 41–48].

The DE models considered here are interacting quintessence dynamical scalar fields. The governing equations of motion come from Einstein’s field equations (Friedmann and Raychaudhuri equations) and Bianchi identities (fluids energy density conservations), for a multicomponent fluid with a top hat density model. The top hat model assumes a Friedmann-Lemaître-Robertson-Walker (FLRW) background and a higher curved FLRW spherical collapsing patch. In this paper we will always assume the background to be flat (no curvature). The multicomponent fluid comprises baryons, uncoupled DM, coupled DM and DE. We treat the baryons and uncoupled DM in a single uncoupled matter fluid. The Bianchi identities for each species reflect the coupling of DE as well as its clustering properties in the collapsing patch with heat fluxes. The scalar field obeys a corresponding Klein-Gordon equation. The type of quintessence is entirely determined by its potential, however an important part of the model, as far as structure formation is concerned, is given in the interaction parameter as well as the clustering parameter.

2.1 Background evolution

we will use the following notations

- energy densities:
  - $\rho_\phi$ for DE energy density (quintessence from a scalar field $\phi$), $\rho_m$ for total matter, $\rho_u$ for uncoupled matter (including uncoupled DM and baryons) and $\rho_c$ for coupled DM. These energy densities are related by

  $$\rho_m = \rho_u + \rho_c. \tag{2.1}$$

  For the total energy density, we use

  $$\rho_{\text{tot}} = \rho_m + \rho_\phi. \tag{2.2}$$
• density parameters
  we define the density parameter for each species as
  \[ \Omega_i = \frac{\rho_i}{\rho_{\text{tot}}} \]  
  (2.3)

• FLRW Hubble parameter defined in terms of scale factor \( a \):
  \[ H = \frac{1}{a} \frac{da}{dt} = \frac{\dot{a}}{a} \]  
  (2.4)

2.1.1 Background Friedmann evolution
The Einstein field equations for the flat FLR W background are reduced to the Friedmann and Raychaudhuri equations for the background scale factor \( a \)

\[ H^2 = \frac{\kappa^2}{3} (\rho_m + \rho_\phi) \quad \dot{H} + H^2 = -\frac{\kappa^2}{6} (\rho_m + \rho_\phi + 3P_\phi), \]  
with \( P_\phi \) being the pressure of the dark energy.

2.1.2 Energy density conservation equations
Aside from the Einstein equations, the conservation of each cosmic fluid component is governed by the corresponding Bianchi identity for the homogeneous FLRW model, so we have for the uncoupled, coupled matter and DE

\[ \dot{\rho}_u + 3H\rho_u = 0, \quad \dot{\rho}_c + 3H\rho_c = \rho_c B' \dot{\phi}, \quad \dot{\rho}_\phi + 3H (\rho_\phi + P_\phi) = -\rho_c B' \dot{\phi}, \]  
(2.6)

where the DE-DM interaction manifests in the heat fluxes \( \Gamma_c = -\Gamma_\phi = \rho_c B' \dot{\phi}, \) with \( B' = \frac{dB(\phi)}{d\phi} \) the interaction rate. The first proposals for such a flux appeared in [15] with the choice of a constant \( B' \). Other forms can be chosen, as, e.g., in [77], however we will concentrate here on the form \( B' = -C\kappa \) with the free parameter \( C \) chosen appropriately (\( C = 0 \) representing of course absence of DE-DM coupling). Both matter conservation equations (2.6-a,b) can be integrated, yielding

\[ \rho_u = \rho_0 \Omega_u (\frac{a_0}{a})^3, \quad \rho_c = \rho_0 \Omega_c (\frac{a_0}{a})^3 e^{B - B_0}, \]  
(2.7)

where \( \rho_0 \) is the critical density at present, the subscript 0 denoting quantities at present, so \( B_0 = B(\phi_0) \). Note that \( e^B \) can be interpreted as the mass of the coupled DM.

2.2 Choice of potential
We chose a range of potentials \( V(\phi) \) likely to reflect a variety of DE behaviours. In this framework, the energy density and pressure of the scalar field are given by

\[ \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad P_\phi = \frac{\dot{\phi}^2}{2} - V(\phi). \]  
(2.8)

They come into play in eq. (2.6-c) which is, for a scalar field, the Klein-Gordon equation

\[ \ddot{\phi} + 3H \dot{\phi} + V' = -\rho_c B'. \]  
(2.9)
2.2.1 Double exponential
The double exponential potential was introduced by [73] to build a more realistic DE candidate out of the two regimes of the single exponential potential [8, 78]: one which tend to dominate the energy density, the other which mimics the time evolution of the background. The result is called double exponential potential (2EXP), which follows its name

\[ V(\phi) = \Lambda_\phi^4 \left( e^{-\lambda_1 \phi} + e^{-\lambda_2 \phi} \right). \]  

(2.10)

Throughout the rest of the paper all numerical evolutions for this potential are done with the parameter values \( \lambda_1 = 20.5 \) and \( \lambda_2 = 0.5 \). The value of \( \Lambda_\phi \) is fine-tuned to achieve a present value of \( \Omega_\phi = 0.7 \).

2.2.2 Ratra-Peebles

The first proposal of a late time scalar field quintessence was using an inverse power potential [9], designed to obtain the desired runaway potential evolution after initial kinetic starts. It is now commonly referred to as the Ratra-Peebles (RP) potential

\[ V(\phi) = \frac{\Lambda_\phi^4 + \alpha_\phi \phi^4}{\phi^{2 \alpha_\phi}}. \]  

(2.11)

We settle for our numerical evolutions on the parameter value \( \alpha_\phi = 6 \), whereas \( \Lambda_\phi \) is fine-tuned to a present value of \( \Omega_\phi = 0.7 \).

2.2.3 Steinhardt
In [74], Steinhardt propose this new form of potential by taking the infinite sum of inverse power potentials. It can be written as (SP)

\[ V(\phi) = \Lambda_\phi^4 e^{-\kappa \phi} \left( 1 - \frac{1}{\pi^2} \right). \]  

(2.12)

The Steinhardt potential has no extra free parameters and \( \Lambda_\phi \) is fine-tuned to achieve \( \Omega_\phi = 0.7 \) at present.

2.2.4 SUGRA

[75] motivated this potential from supergravity models and null vacuum superpotential expectation value. It takes the form (SG)

\[ V(\phi) = \frac{\Lambda_\phi^4 + \alpha_\phi \phi^4 \phi^{2 \kappa^2 / 2}. \]  

(2.13)

For our numerical results we used \( \alpha_\phi = 11 \), and fine-tuned \( \Lambda_\phi \) to achieve a value of \( \Omega_\phi = 0.7 \) at present.

2.2.5 Albrecht-Skordis

The combination of a single exponential with a polynomial, motivated by superstring theory, introduced by [76] creates a local minimum in which the field can oscillate. The parameters orders of magnitude are then more natural, however they still require fine tuning caused by the coincidence problem. It can take the form (AS)

\[ V(\phi) = \Lambda_\phi e^{-\kappa \phi} \left[ A + \kappa \phi^2 - B \right]^2. \]  

(2.14)

The values of the parameters we used for our numerical evolutions were \( A = 0.01 \), \( B = 33.96 \) and \( \lambda = 8 \). Again, \( \Lambda_\phi \) is fine-tuned to obtain \( \Omega_\phi = 0.7 \) at present.
2.3 Collapse evolution equations

The aim of this study is to extend the usual spherical collapse to follow separately the evolution of the coupled and uncoupled (including baryons) components of matter. In the usual scenario, a FLRW flat background contains a central spherical patch, modeled as a positively curved FLRW with scale factor \( r \). That scale factor is assimilated to the radius of the spherical patch [13, 14, 38, 41–48].

2.3.1 Uncoupled collapse

The collapsing uncoupled matter in this framework follows the conservation equation using the Hubble parameter \( \mathcal{H} = \frac{\dot{r}}{r} \) in the collapsed patch,

\[
\dot{\rho}_u + 3\mathcal{H}\rho_u = 0,
\quad (2.15)
\]

where henceforth we will denote with a \(*\) subscript all collapsing quantities. This integrates to

\[
\rho_u \equiv \rho_{u i} \left( \frac{r_i}{r} \right)^3,
\quad (2.16)
\]

where the \( i \) subscript denotes values at some arbitrary initial time of collapse. Therefore the ratio of the uncoupled collapsed matter to its background (2.7-a) is then related to the radius through

\[
\frac{\rho_u}{\rho_u} \propto \left( \frac{a}{r} \right)^3.
\quad (2.17)
\]

2.3.2 Coupled collapse

On the same token, the collapsing coupled DM follows the conservation equation

\[
\dot{\rho}_c + 3\mathcal{H}\rho_c = \rho_c \dot{\phi}_c
\quad (2.18)
\]

(with \( B_c' = B'(\phi) \)) which integrates to

\[
\rho_c \equiv \rho_{ci} \left( \frac{r_i}{r} \right)^3 e^{B_c - B_{ci}},
\quad (2.19)
\]

and therefore the ratio of the coupled collapsed matter to its background (2.7-b) is also related to the radius through

\[
\frac{\rho_c}{\rho_c} \propto \left( \frac{a}{r} \right)^3 e^{B_c - B}.
\quad (2.20)
\]

2.3.3 Simplified evolution

Here we point out that this parallel evolution of coupled and uncoupled collapse relies heavily on the implicit assumption that all species collapse within the same radius, \( r = r_c = r_u = r_\phi \). Introducing the definition of the energy density contrast,

\[
1 + \delta_m = \frac{\rho_{m*}}{\rho_m}, \quad 1 + \delta_u = \frac{\rho_{u*}}{\rho_u}, \quad 1 + \delta_c = \frac{\rho_{c*}}{\rho_c},
\quad (2.21)
\]
one can rewrite the radius of spherical collapse (2.17) into

\[ \left( \frac{a}{a_i} \right)^3 \left( 1 + \delta_i \right) = 1 + \delta_u. \]  

(2.22)

Without loss of generality, the parallel between eqs. (2.17) and (2.20), and the total matter, can be expressed in the relations

\[ 1 + \delta_c = (1 + \delta_u) \cdot e^{B_\star - B}, \quad \quad 1 + \delta_m = (1 + \delta_u) \cdot \frac{\rho_{c0} e^{B_\star - B_0} + \rho_{u0}}{\rho_{c0} e^{B - B_0} + \rho_{u0}}. \]  

(2.23)

However these relations assume the initial conditions for \( \delta_c \) and \( \delta_u \) to be the same. We relax in Appendix A this condition, showing that the final results are not significantly affected. For simplicity, we will keep assuming the equality of initial conditions for the rest of the paper, as is implicit in previous results [14, 46, 48].

We thus emphasize that the assumption, commonly made in the field of DE spherical collapse [13, 14, 38, 41–48], setting the same radius of collapse for all species \( (r = r_c = r_u = r_\phi) \), implies that all matter density contrasts are related to the uncoupled case and their dynamics can be obtained through that of the uncoupled matter. This simplifies greatly the treatment and governing equations previously used for coupled and collapsing DE spherical collapse, e.g. as in [14, 46, 48], since one only needs to evolve the uncoupled DM overdensity equations to get the coupled one via the above constraints. This means that we can concentrate on the evolution of the uncoupled matter, and recover the full evolution for all components.

### 2.3.4 Other assumptions

The central spherical patch is treated as a curvature varying Friedmann model with the Raychaudhuri equation while the background is a regular Friedmann model with lower density. The backreaction is neglected invoking the Birkhoff theorem, assuming some extension of it to cosmological backgrounds can be applied and the existence of a global separation shell [79–81] within the boundary between the two regions.

### 2.3.5 Non-linear evolution

We now will get the overdensity evolution equations for the uncoupled DM component since the algebraic relation with the coupled overdensity, from the unique radius constraint, renders the coupled evolution equations superfluous.

Using eq. (2.22), together with the Bianchi energy conservations (eqs. 2.6-a and 2.15) for the Hubble parameters of the background and collapsing regions yields the overdensity first derivative

\[ \dot{\delta}_u = 3 (1 + \delta_u) [H - \mathcal{H}]. \]  

(2.24)

The Einstein field equations in this frame are reduced to the Friedmann and Raychaudhuri equations for the background scale factor \( a \) (eqs. 2.5) and the collapsing radius \( r \):

\[ \dot{\mathcal{H}} + \mathcal{H}^2 = -\frac{k^2}{6} \sum (\rho + 3P)_\star, \]  

(2.25)

to which the Klein-Gordon equations for the background (2.9) and collapsing region are added in the form, in the case of totally collapsing quintessence,

\[ \ddot{\phi}_\star + 3H \dot{\phi}_\star + V'_\star = -\rho_{c\star} B'_\star. \]  

(2.26)
The case of the homogeneous quintessence obeys to the same equations except that in eq. (2.26), the collapsing Hubble parameter is replaced by that of the background. The overdensity evolution equation follows by differentiating eq. (2.24)

$$\frac{\delta_u}{1 + \delta_u} - \frac{\delta_u^2}{(1 + \delta_u)^2} = 3 \left[ \dot{H} - \dot{\Omega} \right].$$  \hspace{1cm} (2.27)

Eventually one gets, together with using eqs. (2.23-a, 2.25 and 2.5), (recall from it and from eq. 2.7-b that the coupled quantities are linked with the uncoupled ones)

$$\ddot{\delta}_u = -2 \frac{\dot{a}}{a} \delta_u + 4 \frac{\delta_u^2}{3} + \kappa^2 (1 + \delta_u) \left[ \delta_u \rho_u + \delta_c \rho_c \right] + \frac{\kappa^2}{2} \left[ (\rho_{\phi} + 3P_{\phi}) - (\rho_\phi + 3P_\phi) \right] (1 + \delta_u).$$ \hspace{1cm} (2.28)

If we have no coupled matter, that is if \( \rho_c = 0 \), this is the usual equation for the evolution the spherical collapse of uncoupled matter as seen in [14]. Following this evolution of \( \delta_u \) and using eq. (2.23-b) we can re-obtain the full matter evolution of [14].

### 2.3.6 Linear evolution

The linear evolution equations for the collapsing spherical patch starts from the quintessence field linearisation with \( \phi_* = \phi + \delta \phi \) of the Klein-Gordon eq. (2.26). We use the linearisation of eqs. (2.23)

$$\delta_{m,L} = \delta_{u,L} + \frac{\Omega_{\psi}}{\Omega_{\phi} + \Omega_{\psi} e^{B\phi} - B^2 \delta \phi}, \hspace{1cm} \delta_{c,L} = \delta_{u,L} + B' \delta \phi,$$  \hspace{1cm} (2.29)

so we are left with the linear part of eq. (2.26)

$$\delta \dot{\phi} + \left[ 3H + \rho_c B' \right] \delta \dot{\phi} + \left[ V'' + \rho_c \dot{\phi} B'' \right] \delta \phi + \delta_{u,L} \dot{\phi} + \delta_{c,L} B' \rho_c = 0.$$  \hspace{1cm} (2.30)

The linearisation of the field source term of eq. (2.28) yields

$$[(\rho_{\phi} + 3P_{\phi}) - (\rho_\phi + 3P_\phi)]_L = 2 \left( 2 \delta \dot{\phi} - V' \delta \dot{\phi} \right),$$  \hspace{1cm} (2.31)

so the density evolution linearises as (again recall that eq. 2.29-b relates coupled and uncoupled overdensities)

$$\ddot{\delta}_{u,L} = -2 \frac{\dot{a}}{a} \dot{\delta}_{u,L} + \kappa^2 \left[ 2 \delta \dot{\phi} - V' \delta \dot{\phi} \right] + \frac{\kappa^2}{2} \left[ \delta_{u,L} \rho_u + \delta_{c,L} \rho_c \right].$$  \hspace{1cm} (2.32)

Again, if we have no coupled matter, this is the usual linear equation for the spherical collapse evolution of uncoupled matter as seen in [14]. Following this evolution of \( \delta_{u,L} \) and using eq. (2.29-a) we can re-obtain the full matter linear evolution of [14].

### 3 Numerical implementation and results: critical density evolution

In order to follow the evolutions of the segregated uncoupled matter/DM spherical collapse for the five potentials we selected and inhomogeneous coupled DM to DE models, a matlab code was produced ad hoc. Here we present the results of our computations.
3.1 top hat and critical overdensity evolutions

3.1.1 collapse of uncoupled matter/coupled DM

To provide the reader with comparison points to previous works [13, 14, 38, 41–48], we present here the diagram of one example of time evolution for the linear and non-linear spherical top-hat collapse in a coupled inhomogeneous quintessence model. We make the arbitrary choice of using the Ratra-Peebles potential, fixing the uncoupled matter to baryons, $\Omega_u = 0.05$, for a range of coupling $C \in [-0.1; 0.1]$. We display this plot of $\delta_u$ vs $\log a$ in figure 1. It shows that the linear values of overdensities go up with coupling, and consequently that the corresponding collapse time decreases, as expected [14, 46, 48]. This appears mainly as a consequence of background evolution, as induced from eq. (2.7-b). The effect of inhomogeneous quintessence also speeds up collapse time, as seen in the last terms of eq. (2.28) or the second linear terms of eq. (2.32). This behaviour is similar for the other potentials and varying $\Omega_u$ with a fixed $C$ gives the same or opposite behaviour depending on the sign of the chosen $C$ (for more details, see section 3.1.3).

3.1.2 relative linear overdensities evolution

As we have followed separately the coupled (DM) and uncoupled (including baryons) matter components, as opposed to previous works [14, 46, 48], it is interesting to confront all the models used in their relative linear overdensity evolution with respect to the uncoupled component. In figure 2, we display the evolutions of the coupled and total overdensities relative to the uncoupled one, for the models involving the five potentials described in section 2.2 in the scaling regime, and for a range of coupling. Again, we fix the uncoupled matter to baryons, $\Omega_u = 0.05$, for a range of coupling $C \in [-0.1; 0.1]$ for each model.

In all cases, absence of coupling makes all species behave the same way (we see, as expected, an horizontal line). Because the total overdensity combines coupled and uncoupled ones, as from eqs. (2.21)

$$\frac{\delta_m}{\delta_u} = \frac{\delta_c}{\delta_u} + \frac{\rho_u}{\rho_m} \left(1 - \frac{\delta_c}{\delta_u}\right),$$

Figure 1. Spherical collapse evolution, both linear and non-linear, of the uncoupled matter, setting $\Omega_u = 0.05$, in inhomogeneous coupled Ratra-Peebles model with couplings $C = -0.1, -0.05, 0, 0.05$ and 0.1 from bottom to top.
we see that its departure from $\delta_u$ is always less than that of $\delta_c$. This departure is small when varying $C$, however in the range of chosen variations, it appears markedly smaller for $\delta_m$ than for $\delta_c$. In other words, for $C > 0$ we have $\delta_c < \delta_m < \delta_u$, while for $C < 0$ we have $\delta_c > \delta_m > \delta_u$. Negative couplings, with the form of eq. (2.6-b) and $B' = -C\kappa$, seem to induce stronger collapse in the coupled DM than in the uncoupled matter, whereas positive couplings have the opposite effect, although with a much stronger discrepancy than for the negative couplings in all models. This can also be interpreted from eq. (2.23-a) as $\phi - \phi > 0$. The qualitative behaviour for the double exponential, Ratra-Peebles, Steinhardt and SUGRA potentials is quite regular. However the Albrecht-Skordis potential displays quite strong oscillations in the linear overdensity near late times, reflecting its nature driving the field towards its local minimum around which it oscillates. This will be discussed further in section 3.2.
Figure 3. Critical overdensities of a Steinhardt model as a function of the non-linear collapse redshift. The upper panels are for a positive coupling ($C = 0.05$) and the lower panels for a negative coupling ($C = -0.05$). The left panels represent a homogeneous quintessence model with the total matter density contrast (solid lines) and as a reference the $\Lambda$CDM result (dotted line). The right panels represent an inhomogeneous quintessence model, with the density contrasts for uncoupled matter (dotted line), coupled matter (solid line) and total matter (dashed line). The curves lightness reflect the values of $\Omega_u \simeq 0.05, 0.11, 0.18, 0.24$ and $0.30$ (darker to lighter).

3.1.3 critical overdensities evolution

We now present the diagrams of critical overdensities as a function of their collapse redshift for the Steinhardt potential, used for illustration purposes. This time we fix the coupling respectively to $C = 0.05$ and $C = -0.05$, for a range of uncoupled matter parameter $\Omega_u \in [0.05; 0.3]$, the upper limit corresponding to no coupled matter (recall that we fix $\Omega_m = \Omega_u + \Omega_c = 0.3$). Representing that upper limit therefore implies that the curve for the coupled component is not defined and that we have identity of the uncoupled and total curves, as seen in figure 3. Note that the extremal curves for both the positive and negative couplings (upper and lower panels) represent the same result $\Omega_u = 0.3$. Moreover, with such parameterisation, growing $\Omega_u$ is indirectly equivalent to decreasing the amount of coupling $|C|$. Thus varying $\Omega_u$ can be mapped to varying $C$, accounting for the sign of the constant $C$ chosen. In figure 3, the leftmost panels gives the homogeneous quintessence models behaviour while the rightmost panels recall the total matter (dashed line) collapse of the inhomogeneous coupled model for comparison with previous works [13, 14, 46, 48]. Those panels also provide the uncoupled (dotted) and coupled (solid) evolutions. The scale and range of the diagrams is kept for each panel within a row for comparison purpose. Upper panels correspond to negative coupling, while lower panels show results for positive coupling.

As is now known [14, 46, 48], the variations induced by turning on coupling in homogeneous quintessence are markedly smaller than that induced in collapsing models (compare left and right, dashed lines, panels). Similarly as seen in figure 1, increasing the value of $C$ (or alternatively, varying $\Omega_u$ as described above) both increase the corresponding value of
Figure 4. Value of the uncoupled matter critical overdensity at a reference critical redshift as a function of the coupling (left panels) or $\Omega_u$ (middle and right panels). The upper panels are for a critical redshift of $z_{\text{crit}} = 0.5$ and the lower panels for $z_{\text{crit}} = 1.5$. In the left panels we use $\Omega_u = 0.05$; in the middle panels we use a negative coupling $C = -0.05$ and in the right panels a positive coupling $C = 0.05$. Each panel shows the results for the five potentials we study, from bottom to top: Ratra-Peebles, SUGRA, Steinhardt, double exponential and Albrecht-Skordis. The dotted line shows the critical overdensity value at the reference redshift in a $\Lambda$CDM model.

critical linear overdensity and the collapse redshift. Moreover, we see clearly from the spacing between corresponding curves in the right panels that the critical overdensity scales linearly with $\Omega_u$ and increases with decreasing $C$, the upper and lower sets joining at the maximum $\Omega_u$, as mentioned above. Thanks to the simplifying assumption that leads to eqs. (2.23), the dynamics needs only to be followed in $\delta_{u,\text{crit}}$, shown in the right panels. It displays, in the negative coupling range, a significantly lower collapse threshold $\delta_{u,\text{crit}}$ for uncoupled matter than for coupled DM, consistent with eq. (3.1), with a sharper decrease towards later times for uncoupled matter. The picture is inverted in the positive coupling range: the DM collapse threshold $\delta_{c,\text{crit}}$ is significantly lower than for uncoupled matter in that case and the decrease towards later times is sharper than for uncoupled matter. This vindicates the claim for baryon-DM segregation with coupled DE. Those results are qualitatively similar with the other potentials explored. However some features are specific to each potential. In the present case, the most negative couplings present a sharp change in the critical overdensity behaviour beyond a redshift that goes smaller with increased coupling. This appears to be a signature of the potential itself as we will show in the study of the next section.

3.2 critical overdensities dependence on coupling

3.2.1 general remarks

We synthesise our results for all the potentials in figure 4, representing their variations in critical overdensities as a function of coupling by changing either the value of $C$ or the value of $\Omega_u$. Recall from section 3.1.3 that increasing $\Omega_u$ decreases the amount of coupling $|C|$. We can see this as a mapping from central panels (fixed $C = -0.05$) into the the negative range of $C$ in the left panels (fixed $\Omega_u = 0.05$); and as an inverse mapping from the right panels
(fixed $C = 0.05$) into the positive $C$ range in the left panels. In particular, the curve features found for SP, AS, 2EXP in the respective $C > 0$ and $C < 0$ ranges of the left panels are found in the corresponding rightmost and central panels. Note also that the features of SP in figure 4 agree with those of figure 3. Those features seems to characterise each potential.

A striking first observation is that, at fixed time ($z_{\text{crit.}}$), the critical overdensities generally are decreasing with $C$, in agreement with section 3.1.3’s considerations. This can be understood as fixing $z_{\text{crit.}}$ requires to vary initial $\delta_i$ to maintain that collapse time so as to compensate the effect of the variations of $C$ in the exact opposite way. From figure 1, increased $C$ leads to decreased collapse time, so a smaller initial $\delta_i$ is required to compensate that effect, leading to a smaller final linear overdensity.

### 3.2.2 discriminating models

The RP and SG potentials display the least amount of features: in fact they behave linearly in $\Omega_u$. They are indeed both variations around the inverse power law that delivers regular background evolutions with marked tracker phases.

The 2EXP potential displays sections, mostly in the $C > 0$ part, where it also behaves linearly, with a larger slope than the RP and SG. This can be conjectured as reflecting the scaling solution behaviour. However it has an hybrid behaviour as its critical overdensity asymptotes to a constant (the $\Lambda$CDM critical overdensity) in the $C < 0$ region. That behaviour can be attributed to oscillations around its dominating solution.

The SP potential offers the most features: it matches the 2EXP linear $C > 0$ behaviour as well as its more or less constant $\Lambda$CDM asymptote $C < 0$ region at large $\vert C \vert$ but shows an intermediate region with similar slope to the RP and SG. One can conjecture a link with that potential being built of an infinite sum of inverse powers.

Finally, the AS potential brings the most untypical features: it doesn’t admit solutions for $C > 0.05$ and it decreases faster than any other model for $C > 0$. Whereas for $C < 0$ it oscillates and asymptotes to a constant $\Lambda$CDM value, like the 2EXP potential. This can be interpreted from the fact that the AS potential offers a local minimum for the field to be trapped in and oscillate around. From eqs. (2.6-b,c), we see that $C > 0$ feeds more energy in the DE sector (assuming DE slow roll so $\dot{\phi} > 0$ as all chosen potentials decay with field, in a first approximation), thus feeding the oscillations, whereas $C < 0$ has the opposite energy transfer and thus dampens the oscillations around the potential local minimum, where the model effectively behaves like a cosmological constant. Thus similarly as with 2EXP but in a stronger way, as the oscillation behaviour is stronger in AS than 2EXP, we have the dampened behaviour in the constant asymptote region and the excited behaviour in the negative slope region.

Of course this requires deeper investigations.

### 4 Discussion and conclusion

In this work we have presented results supporting the segregation of uncoupled matter and coupled DM in interacting DE models and have done so with the exposition of an innovative and simplifying approach to the dynamics of such system. The method employed relies on the assumption common in the field that all species collapse within the same top hat radius. The novel treatment comes from relying on the algebraic relations between species overdensities derived from that assumption to reduce the need to follow complex dynamics of the collapse to the simpler evolution of uncoupled matter. This also clarifies the reason why most of the
effects we found are mainly driven by background behaviours, principal components in the evolution equation outside of the uncoupled overdensity. Exploring the segregated spherical collapse, we made contact with previous work and established, in this model, the difference in behaviour between coupled matter and DM interacting with DE from (a) the differences in linear overdensity evolution between species, (b) the differences in critical overdensity functions between species, and (c) an original synthesis of critical overdensity as a function of coupling, directly or indirectly. On a smaller note, we also established the equivalence between variations of the coupling and variations of the amount of coupled DM. In the interaction model we chose, positive coupling corresponds to feeding the DE sector from the coupled DM density whereas negative coupling induces the opposite. This is a cornerstone to understand (a) that negatively coupled DM collapses more than uncoupled matter while the opposite is true of positively coupled DM; (b) that instabilities in the behaviour of some of the chosen potentials are enhanced by positively coupled DM collapse while the negatively coupled DM dampens them. This confirms the potential detectability of dark sector coupling in a similar fashion as, e.g. the tidal streams of stars in the Sagittarius dwarf galaxy due to DM, as simulated by [71], in this case with a distinct halo collapse threshold between baryons and coupled matter. Finally, it is possible to discriminate between potentials in their response to the amount of coupling. Those behaviours certainly are calling for further investigations.

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A Unequal initial overdensities

In section 2.3.3, we assumed that the initial values for the overdensities in coupled and uncoupled matter were equal, as in and most works in the field, like [13, 14, 38, 41–48]. It is however possible to keep independent the overdensity of each species. In particular, the coupled to uncoupled overdensity relation reads in general

$$1 + \delta_c = (1 + \delta_u) \frac{1 + \delta_{ci}}{1 + \delta_{ui}} \frac{e^{B_i - B}}{e^{B_i - B_i}} = (1 + \delta_u) \Delta e^{B_i - B}, \quad (A.1)$$

defining $\Delta = \frac{1 + \delta_{ci}}{1 + \delta_{ui}} e^{B_i - B_i}$, such that the assumptions of equal matter ($\delta_{ci} = \delta_{ui}$) and zero quintessence initial overdensities ($\phi_{si} = \phi_i$) reduce to $\Delta = 1$, and thus eq. (A.1) simplifies into relation (2.23-a) used in section 2.3.3. For the total matter relation (2.23-b), the freedom transcribes as

$$1 + \delta_m = (1 + \delta_u) \frac{\rho_u}{\rho_m} + (1 + \delta_c) \frac{\rho_c}{\rho_m} = \frac{1 + \delta_u}{\rho_m} \left( \rho_u + \rho_c \Delta e^{B_i - B} \right) = (1 + \delta_u) \cdot \frac{\Omega_c 0}{\Omega_m 0} \Delta e^{B_i - B_0} + \frac{\Omega_u 0}{\Omega_m 0}, \quad (A.2)$$
which again simplifies when $\Delta = 1$. The linear versions of eqs. (A.1) and (A.2) then read as

$$
\delta_{c,L} = \delta_{u,L} + B'\delta\phi + \delta_{ci} - \delta_{ui} - B'_i\delta\phi_i, \quad \delta_{m,L} = \delta_{u,L} + \left( \frac{B'\delta\phi + \delta_{ci} - \delta_{ui} - B'_i\delta\phi_i}{1 + \frac{\Omega_m}{\Omega_c} B_0}\right).
$$

(A.3)

The coupled overdensity equation can then be used, together with its non-linear counterparts, in the correspondingly modified scheme formed after eqs. (2.26), (2.32), (2.30) and (2.32) to follow the uncoupled evolution. Then, the coupled values can be transcribed algebraically from the uncoupled values. Since $\delta_{ci}$ and $\delta_{ui}$ are small, $\Delta \simeq 1$. As the differences in evolution are small and do not bring further light into the question of the segregated spherical collapse, we only present in the main paper the results for $\Delta = 1$.

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