Quantum magneto-oscillations in a two-dimensional Fermi liquid

Gregory W. Martin, Dmitrii L. Maslov, and Michael Yu. Reizer*

Department of Physics, University of Florida, P.O. Box 118440, Gainesville, FL 32611

(March 22, 2022)

Quantum magneto-oscillations provide a powerful tool for quantifying Fermi-liquid parameters of metals. In particular, the quasiparticle effective mass and spin susceptibility are extracted from the experiment using the Lifshitz-Kosevich formula, derived under the assumption that the properties of the system in a non-zero magnetic field are determined uniquely by the zero-field Fermi-liquid state. This assumption is valid in 3D but, generally speaking, erroneous in 2D where the Lifshitz-Kosevich formula may be applied only if the oscillations are strongly damped by thermal smearing and disorder. In this work, the effects of interactions and disorder on the amplitude of magneto-oscillations in 2D are studied. It is found that the effective mass diverges logarithmically with decreasing temperature signaling a deviation from the Fermi-liquid behavior. It is also shown that the quasiparticle lifetime due to inelastic interactions does not enter the oscillation amplitude, although these interactions do renormalize the effective mass. This result provides a generalization of the Fowler-Prange theorem formulated originally for the electron-phonon interaction.

Patterns of quantum magneto-oscillations in thermodynamic (de Haas-van Alphen effect) and transport (Shubnikov-de Haas effect) quantities encode three important parameters of a Fermi-liquid metal. The period of the oscillations gives the area of the extremal cross-section of the Fermi surface, the slope of the temperature dependence of the oscillations amplitude provides the quasiparticle effective mass, and the phase shift between oscillations of spin-up and-down electrons yields the (renormalized) spin susceptibility. Magneto-oscillations studies of the Fermi-liquid (FL) state in two-dimensions (2D) date back to the early 70s, when semiconductor heterostructures first became available [1]. Another surge of the activity in this field, which occurred in mid 90s, was stimulated by the discovery of the metallic state at $\nu = 1/2$ [2]. Recently, Shubnikov-de Haas oscillations have been used to determine the parameters of the “anomalous” metallic state in Si MOSFETs and other semiconductor heterostructures exhibiting an apparent metal-insulator transition in zero magnetic field [3–6]. Despite the long and successful history of quantifying FLs in 3D via magneto-oscillations, this method remains controversial in 2D. The primary goal of our paper is to resolve some of the open issues.

The first controversy is related to the applicability of the current theory of magneto-oscillations to the 2D case. The analysis of the experimental data in 2D is often based on the premise that the classic result for magneto-oscillations in a Fermi liquid for the 3D case, known as the “Lifshitz-Kosevich (LK) formula” [7–9], is transferrable to 2D upon a trivial change in the electron spectrum. The crucial features of the LK formula, i.e., its validity for arbitrarily strong interactions (without destroying the FL) and the fact that the FL parameters entering the formula are taken at zero magnetic field, survive on this premise. The deviations of the observed oscillation pattern in stronger fields from that predicted by the LK formula are ascribed to oscillations in the effective $g$-factor [1] and the effective mass [10]. On the other hand, there have been warnings that the 3D LK formula is non-transferrable to 2D [11,12] for any field strength. Hence the situation needs to be clarified. The second controversy—not specific to 2D—is related to the effect of quasiparticle damping. It is often mentioned in the literature that any scattering of quasiparticles, elastic and inelastic, contributes to the smearing of magneto-oscillations via the effective Dingle temperature (scattering rate) for a given process. Alternatively, Fowler and Prange [13] showed that the electron-phonon scattering rate does not appear in the oscillations amplitude due to the cancellation of two $T$–dependent parts of the Matsubara self-energy (cf. also [14]). To the best of our knowledge, this cancellation has never been discussed for other interactions, including the electron-electron one, which is of a primary importance for 2D electron systems. The last issue to be addressed in this paper (and not discussed previously in the literature) is the effect of interference between electron-impurity and electron-electron scattering on magneto-oscillations, neglected in the LK formula. The theory of interference effects in the ballistic regime [15], when $T \tau \gg 1$, where $\tau$ is the electron-impurity scattering time (we set $\hbar = k_B = 1$ throughout the paper), offers a plausible explanation of the metallic temperature dependence in the metallic phase of the 2D metal-insulator transition. Unusual (within the LK framework) temperature dependences of the oscillation amplitude are also commonly observed in Si MOSFETs [6,16], but the proper theory is currently lacking.

Our answers to these open questions are as follows. i) Although it is true that the LK formula does not work in 2D at $T = 0$ and in the absence of disorder, it is still applicable to the situation when finite temperature and/or disorder cause the oscillations to be exponentially small. ii) The cancellation of the scattering rate term in
the Matsubara self-energy is pertinent to any inelastic interaction, including the electron-electron one. Due to this cancellation, the scattering rate of inelastic processes does not enter the oscillation amplitude. iii) Interference between electron-impurity and electron-electron interactions gives a new, $T \ln T$, dependence of the amplitude’s argument, that can be interpreted equivalently either as a “$T$-dependent” effective mass or Dingle temperature. The functional form of this dependence is the same in the diffusive ($T\tau \ll 1$) and ballistic ($T\tau \gg 1$) regimes.

We limit our analysis to the de Haas-van Alphen effect and assume the chemical potential is fixed. The main features of the results for the de Haas-van Alphen effect, in particular, the $T$-dependence of the oscillation amplitude, are commonly expected to apply to the Shubnikov-de Haas effect as well, although a rigorous proof of that is currently lacking. Assuming a fixed chemical potential is not essential for the case of small oscillations, which is the focus of this paper (see below). We begin with a brief reminder of how the LK formula is derived in the Luttinger formalism [8,9]. The key issue here is whether the Luttinger formalism \[8,9\]. The key issue here is whether the ground states of an interacting system at $B = 0$ and in a finite field are not adiabatically connected in 2D. This fact has been emphasized by recent findings that the ground state of a 2D electron liquid is not a Fermi liquid even for $N \gg 1$, but rather a charge-ordered state \[17\]. Nevertheless, an absence of the full LK formula in 2D does not preclude a canonical analysis of magneto-oscillations, if under more restrictive conditions, as the FL-behavior is restored at higher energies.

The power-counting argument for (against) neglecting the oscillatory part of the self-energy in 3D (2D) is valid at $T = T_D = 0$. If the real and/or Dingle temperatures are sufficiently high, i.e.,

$$2\pi^2 \left( \frac{T}{\omega_c} + \frac{T_D}{\omega_c} \right) \gtrsim 1,$$

the amplitudes of all oscillatory quantities, including the self-energy, are exponentially small. Neglecting the oscillatory part of the self-energy, the amplitude of the first harmonic takes the form

$$A_1 = \left( 4\pi^2 T/\omega_c \right) \exp \left( -2\pi [\pi T + i\Sigma_0 (i\pi T, T)/\omega_c] \right),$$

where, due to condition (3), we limited the Matsubara sum by the first term $\Sigma_0 = \pi T$. The oscillatory part of $\Sigma$ results in a correction to $A_1$ which is itself of order $A_1$ (with exponential accuracy). The net contribution to $\Sigma$ is of order $\Sigma_0^2$, which is of the same order as $A_2$ for $\Sigma_{osc} = 0$. Thus, harmonics with $k \geq 2$ are affected by the oscillations in $\Sigma$ and a 2D analog of the LK formula, which includes the sum over all $k$, can only be derived in a perturbation theory for a specific interaction but not for a generic FL. However, the $k = 1$ harmonic does not include $\Sigma_{osc}$ and, as long as (3) is satisfied, the analysis can proceed as in the 3D case. In what follows, we assume that (3) is satisfied and the amplitude of the first (and only important) harmonic is given by (4).

Next we discuss whether the quasiparticle relaxation rate affects the amplitude of magneto-oscillations. We set $T_D = 0$ temporarily. Suppose that a quasiparticle relaxation rate is measured in a clean Fermi liquid, e.g., via electron heat conductivity, with a result that $1/\tau_e \propto T^2$. It seems natural to assume that the same rate contributes also to the Dingle temperature of magneto-oscillations. That this is not the case was shown for the electron-phonon interaction by Fowler and Prange \[13\]. Here we generalize their arguments for the electron-electron interaction in 3D and 2D, and then give a general theorem for an arbitrary interaction. For a generic FL in 3D, the
Matsubara self-energy, up to the quadratic in $\varepsilon_n$ and $T$ terms, can be written as

$$i\Sigma_0(\imath\varepsilon_n, T) = \alpha \varepsilon_n + i\beta v_F (p - p_F) + \gamma \left( (\pi T)^2 - \varepsilon_n^2 \right). \tag{5}$$

In addition to a direct calculation, the validity of the quadratic term in (5) is readily established by noticing that upon analytic continuation $\varepsilon_n \to \varepsilon + \imath 0$ this term gives the correct form for the imaginary part of the on-shell self-energy: $-\Im \Sigma^R_0 \propto (\pi T)^2 + \varepsilon^2$ [18]. The amplitude of the first harmonic (4) contains $i\Sigma_0(\imath \pi T, T)$, in which the quadratic term vanishes identically. The $T^2$ terms from higher Matsubara frequencies (legitimately considered within this scheme in 3D) increase the amplitude and cannot be interpreted as "damping."

In 2D, the integral over momentum transfers diverges logarithmically at the lower limit, changing the behavior of $\Im \Sigma^R_0$ to $E^2 \ln E$, where $E = \max\{\varepsilon, T\}$. This change does not alter the principal result. Consider the simplest case of a contact interaction. To the second order in this interaction, the quadratic term in (5) is replaced by

$$i\Sigma_0(\imath\varepsilon_n, T) = -\frac{U^2 m}{\pi^2 v_F^2} T \sum_{\omega_m = 0}^{\varepsilon_n - \pi T} \omega_m \ln (\varepsilon_F/\omega_m). \tag{6}$$

Although the sum in (6) does not have an analytic solution, it obviously vanishes for $\varepsilon_n = \pi T$.

To analyze a general case of a finite-range and dynamic interaction, including the screened Coulomb one, it is convenient to find the imaginary part of the retarded self-energy first and then continue back to Matsubara frequencies. On the mass-shell, $\Im \Sigma^R_0$ is given by

$$\Im \Sigma^R_0(\varepsilon) = -\frac{\pi}{2(2\pi)^D} \int d\omega F(\omega) \left[ \coth \frac{\omega}{2T} - \tanh \frac{\omega - \varepsilon}{2T} \right],$$

where $F(\omega) = \int d^D q \delta (\omega - v_F q \cdot \mathbf{q}) \Im V^R(\omega, q)$ and $V^R(\omega, q)$ is the retarded interaction potential. As a function of a complex variable $z$, $f(z) \equiv \Im \Sigma^R(z)$ has the following properties in the upper half-plane: i) all lines $\Im z = \pi (2n + 1) T$ are branch cuts on which $\Re f$ is continuous but $\Im f$ jumps up and down; ii) due to the fact that $\tanh(z - i\pi(n + 1/2)) = \coth z$, all points $z = i\pi (2n + 1) T$ are zeroes of $f(z)$. Thus, $f(z)$ is analytic in the band $0 \leq \Im z < \pi T$ that includes the Matsubara self-energy $\Sigma_0(\imath \varepsilon_0 = \pi T, T)$, which is equal to zero. Zeros of $f(z)$ at $z = i\pi (2n + 1) T$ with $n \geq 1$ do not lead to vanishing of $\Sigma_0(\imath \varepsilon_n, T)$ for $n \geq 1$ because those zeroes are separated from the real axis by branch cuts and thus are not accessible by analytic continuation.

The 2D case is special only in that the $q$-integration results in the $\ln \omega$ factor in $F(\omega)$ which does not change the reasoning given above. In particular, for a dynamically screened Coulomb interaction in 2D, $F(\omega) \propto \omega \ln |\omega|$ and still $\Sigma_0(\imath \pi T, T) = 0$. As this result does not depend on the particular form of the interaction, it can be viewed as a generalization of the Fowler-Prange theorem. Notice that in 3D the Fowler-Prange theorem is of limited applicability because nothing prevents one from considering $k > 1$ and lower values of $T + T_D$, when the effect of $\Sigma_0(\imath \varepsilon_n, T)$ needs to be taken into account. In 2D, one is bound to consider only $\Sigma_0(\imath \varepsilon_n, T)$ within the Luttinger approximation.

![FIG. 1. (a) The interference correction to the self-energy. (b) The vertex correction is assigned to either one of the vertices in (a) because the self-energy arises as insertions into the thermodynamic potential (closed loops). (c) Singlet/triplet-channel contributions to the effective potential.](image)

Finally, we discuss the effect of interference between electron-electron and electron-impurity scattering on magneto-oscillations, extending the analysis of the interference corrections to the self-energy in 2D from the diffusive ($T \tau \ll 1$) [19] to the ballistic ($T \tau \gg 1$) limit. The general form of the interference correction to the Matsubara self-energy is (see Fig. 1)

$$\Sigma^\text{int}_{0}(\imath \varepsilon_n, \mathbf{p}) = -2 T \sum_{\varepsilon_n(\omega_n - \varepsilon_n) > 0} \int \frac{d^2 q}{(2\pi)^2} V(i\omega_n, q) \times \Gamma(i\omega_n, q) G(i\varepsilon_n - i\omega_n, \mathbf{p} - \mathbf{q}),$$

where $G(i\varepsilon_n, \mathbf{p}) = (i\varepsilon_n - \xi_F + i\Im \varepsilon_n/2\tau)^{-1}$ and the effective interaction $V = V_s + 3V_t$ contains the contributions from both singlet and triplet channels [15]

$$V^{-1}_s (i\omega_n, q) = (2\pi e^2/q + F^0/\nu)^{-1} - \Pi (i\omega_n, q);$$

$$V^{-1}_\tau (i\omega_n, q) = \nu/F^0 - \Pi (i\omega_n, q),$$

where $\nu = m/\pi$. The factor of two in (7) accounts for two possibilities of including the vertex correction in Fig. 1a. The Fermi-liquid constants $F^0$ and $F^0_\tau$ [20] determine the renormalized charge and spin-suscetibilities, respectively. The vertex $\Gamma (i\omega_n, q) = \left( \sqrt{|\omega_n|^2 + (qv_F)^2} - 1 \right)^{-1}$ reduces to $\Gamma = (D q^2 + |\omega_n|^2)^{-1}$ and $\Gamma = \tau^{-1} (|\omega_n|^2 + (qv_F)^2)^{-1/2}$ in the diffusive and ballistic limits, respectively. The general form of the (small $q$) polarization operator

$$\Pi (i\omega_n, q) = -\nu [1 - |\omega_n|/\tau \Gamma (i\omega_n, q)] \tag{8}$$
reduces to \( \Pi (i\omega_n, q) = -\nu D q^2 / (D q^2 + |\omega_n|) \) in the diffusive limit, where \( D = v_F^2 T / 2 \), and to \( \Pi^0 (i\omega_n, q) = -\nu \left( 1 - |\omega_n| / \sqrt{(v_F q)^2 + \omega_m^2} \right) \) in the ballistic one. Omitting the details of lengthy but straightforward calculations, we give just the result for the self-energy valid to logarithmic accuracy

\[
i \Sigma_0^{\text{int}} (i\pi T, T) = -T \ln \left( \epsilon_F / T \right) Q \left( T \tau, F_0^0 \right) / 2 \epsilon_F T,
\]

where \( Q \left( T \tau, F_0^0 \right) = g_\rho (T \tau) + \left[ 3 F_0^0 / (1 + F_0^0) \right] g_\sigma (T \tau) \) and \( g_\rho/\sigma (x) \) are slowly varying functions which interpolate between the diffusive and ballistic regimes. The limiting values of \( g_\rho/|\sigma (x) \) are as follows: \( g_\rho (0) = 1 \), \( g_\rho (x \gg 1) = 3/2 \), \( g_\sigma (0) = 1 \), \( g_\sigma (x \gg 1) = 1/2 \). Apart from the numerical coefficients, the \( T \)-dependence of \( i \Sigma_0^{\text{int}} (\pi T, T) \) is the same in the diffusive and ballistic regimes. In that sense, the behavior of the self-energy is similar to that of the tunneling density of states [21]. Notice that the interference correction to the energy is similar to that of the tunneling density of states. In that sense, the behavior of the self-energy is similar to that of the tunneling density of states [21].

Following this interpretation, the interference effect leads to a replacement of the effective mass in the argument of the logarithmic temperature:

\[
\left( T \tau \right)^{-1} \ll 1 \quad \text{but not} \quad \left( T \tau \right)^{-1} \ll 1.
\]

However, due to the cancellation of the \( T^2 \ln T \) term in \( i \Sigma_0 (i\pi T, T) \) discussed earlier in this paper, the interference correction is the main nonlinear \( T \)-dependent term in the Matsubara self-energy, leading to a modification of the FL-formula. The \( T \ln T \)-dependence of the self-energy can be interpreted as a logarithmic \( T \)-dependence of the effective mass. Following this interpretation, the interference effect leads to a replacement of the effective mass in the argument of the exponential in \( A_1 \) by

\[
m^* (T) = m^* \left( 1 - \frac{m^* \ln \left( \epsilon_F / T \right) Q \left( T \tau, F_0^0 \right)}{2 \pi \epsilon_F T} \right). \tag{9}
\]

Notice that the effective mass is reduced by the singlet-channel interaction but enhanced by the ferromagnetic \( (F_0^0 < 0) \) interaction in the triplet channel. The \( \ln T \)-dependence of the effective mass is a characteristic feature of the “marginal Fermi liquid” model [22,23].

Equivalently, the nonlinear \( T \)-dependence of \( i \Sigma_0 (i\pi T, T) \) may be interpreted as a \( T \)-dependent Dingle temperature:

\[
T_D (T) = T_D \left( 1 - (T / \epsilon_F) \ln \left( \epsilon_F / T \right) Q \left( T \tau, F_0^0 \right) \right). \tag{10}
\]

One of the empirical procedures used in [6] to account for the observed deviations from the FL formula was to assume that the effective Dingle temperature has the same \( T \)-dependence as the zero-field resistivity. Comparing (10) with the result for the interference correction to the resistivity [15], we see that, although this recipe is not precise, it has some theoretical justification: the general structure of the \( T_D (T) \) and \( \rho (T) \) dependences is similar in the ballistic regime, except for a factor of \( \ln \left( \epsilon_F / T \right) \) present in \( T_D (T) \) but not in \( \rho (T) \).

This work was supported by NSF DMR-0077825. We are grateful to I. L. Aleiner, A. V. Chubukov, J. Klauder, and B. Z. Spivak for valuable comments and to M. E. Gershenson and V. M. Pudalov for extensive discussions of the experimental data.

* Permanent address: 5614 Naiche Rd, Columbus, OH 43213.

[1] See T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. 54, 437 (1982) and references therein.
[2] R. R. Du et al., Phys. Rev. Lett. 73, 3274 (1994); D. R. Leadlay at al., Phys. Rev. Lett. 73, 3274 (1994).
[3] S. J. Papadakis et al., Science 283, 2056 (1999).
[4] S. A. Vitkalov et al., Phys. Rev. Lett. 85, 2164 (2000).
[5] S. V. Kravchenko et al., Sol. St. Com. 116, 495 (2000); A. A. Shashkin et al., cond-mat/0301187.
[6] V. M. Pudalov et al., Phys. Rev. Lett. 88, 196404 (2002); V. M. Pudalov, M. E. Gershenson, and H. Kojima, cond-mat/0110160.
[7] I. M. Lifshitz and A. M. Kosevich, Zh. Eksp. Teor. Fiz. 9, 730 (1955) [Sov. Phys. JETP 2, 636 (1956)].
[8] J. M. Luttinger, Phys. Rev. 121, 1251 (1961).
[9] Yu. A. Bychkov and L. P. Gor’kov, Zh. Eksp. Teor. Fiz. 41, 1592 (1961) [Sov. Phys. JETP 14, 1132, (1962)].
[10] A. P. Smith, A. H. Macdonald, and G. Gunsins, Phys. Rev. B 45, 8829 (1992).
[11] K. Miyake and C. M. Varma, Sol. St. Com. 85, 335 (1993).
[12] S. Curnoe and P. C. E. Stamp, Phys. Rev. Lett. 80, 3312, (1998).
[13] M. Fowler and R. E. Prange, Physics 1, 315 (1965).
[14] S. Engelsberg and G. Simpson, Phys. Rev. B 2, 1657 (1970).
[15] G. Zala, B. N. Narozhny, I. L. Aleiner, Phys. Rev. B 64, 214204 (2001).
[16] V. M. Pudalov (private communication).
[17] A. A. Koulaikou, M. M. Fogler, and B. I. Skolkovski, Phys. Rev. Lett. 76, 499 (1996); M. P. Lilly et al., ibid. 82, 394 (1999); W. Pan et al., ibid 83, 820 (1999).
[18] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, Methods of quantum field theory in statistical physics, (Dover Publications, New York, 1963).
[19] See B. L. Altshuler and A. G. Aronov, in Electromagnet interactions in disordered conductors, edited by A. L. Efros and M. Pollak (Elsevier, 1985), p. 1., and references therein.
[20] As in [15], we make a model assumption of \( F_0^0 \) to have the same value in the diffusive and ballistic limits.
[21] A. M. Rudin, I. L. Aleiner, and L. I. Glazman, Phys. Rev. B 55, 9322 (1997).
[22] C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phys. Rev. Lett. 63, 1996 (1989).
[23] F. Pelzer, Phys. Rev. B 44, 293 (1991).