R Symmetry Breaking Versus Supersymmetry Breaking

Ann E. Nelson  
Department of Physics 0319  
University of California, San Diego  
9500 Gilman Drive  
La Jolla, CA 92093-0319

and

Nathan Seiberg  
Department of Physics and Astronomy  
Rutgers University  
Piscataway, NJ 08855-0849

Abstract

We point out a connection between R symmetry and supersymmetry breaking. We show that the existence of an R symmetry is a necessary condition for supersymmetry breaking and a spontaneously broken R symmetry is a sufficient condition provided two conditions are satisfied. These conditions are: genericity, i.e. the effective Lagrangian is a generic Lagrangian consistent with the symmetries of the theory (no fine tuning), and calculability, i.e. the low energy theory can be described by a supersymmetric Wess-Zumino effective Lagrangian without gauge fields. All known models of dynamical supersymmetry breaking possess such a spontaneously broken R symmetry and therefore contain a potentially troublesome axion. However, we use the fact that genericity is not a feature of supersymmetric theories, even when nonperturbative renormalization is included, to show that the R symmetry can in many cases be explicitly broken without restoring supersymmetry and so the axion can be given an acceptably large mass.

UCSD/PTH 93-27  
RU-93-42  
hep-ph@xxx/9309299  

September 1993
1. Introduction

Dynamical Supersymmetry Breaking (DSB) provides an attractive way to achieve a large hierarchy between the Planck scale $M_P$ and the scale of supersymmetry breaking, and, in models where the weak scale is related to the supersymmetry breaking scale, can solve the gauge hierarchy problem \[1\]. In DSB models, all mass scales, including the scale of supersymmetry breaking, arise through dimensional transmutation, and thus are proportional to $M_P e^{-a/g^2}$, where $a$ is a constant of order $4\pi^2$ and $g$ is some effective coupling constant at the Planck scale. There are several examples of models which exhibit DSB \[4\, 5\]. In reference \[4\] the following guidelines for finding dynamical supersymmetry breaking models were suggested:

1. The classical potential should not have any noncompact flat directions, \textit{i.e.} it should be nonzero for large field strengths in all directions in field space.

2. The theory should contain a nonanomalous continuous global symmetry, and the complexity or absence of solutions to ’t Hooft’s anomaly matching conditions \[4\] should imply that this symmetry is spontaneously broken.

These conditions are generally sufficient for spontaneous supersymmetry breaking because if supersymmetry is unbroken, then the Goldstone boson from the spontaneously broken global symmetry should reside in a supermultiplet which also contains a massless fermion and another massless scalar. In a few cases, this second massless scalar could itself be a Goldstone boson of another spontaneously broken symmetry, or even if it is not a Goldstone boson, its target space can be compact. Otherwise the existence of another massless scalar implies an exact, non-compact vacuum degeneracy (flat directions). However the first condition is that classically there is no such degeneracy. For a nonperturbatively generated term to restore the degeneracy which is absent classically is implausible and usually not possible since in asymptotically free theories the dynamically generated terms can be shown to be less important for large field strengths than the classical terms.

No argument has been given that these criteria are \textit{necessary} for DSB, yet all known examples satisfy them by having no flat directions and a spontaneously broken $U(1)$ R symmetry. An argument based on the anomaly \[5\]

$$\{\overline{Q}_{\dot{\alpha}}, \psi^i_{\dot{\alpha}} \phi_i \} = \partial_i W \phi^i + C \frac{g^2}{32\pi^2} \lambda \lambda$$

(1.1)

2
(\(W\) is the superpotential, \(\lambda\) is a gluino and \(C\) is a constant depending on the gauge group) was given in reference \[2\] suggesting that in many gauge theories an exact, spontaneously broken \(R\) symmetry is a sufficient condition for supersymmetry breaking. In section 2 of this paper we argue that such an \(R\) symmetry is a feature of any DSB model where the low energy effective theory can be studied by integrating out the gauge dynamics, and in which the low energy effective Lagrangian is a generic Lagrangian consistent with the symmetries (no fine tuning). The existence of a spontaneously broken \(U(1)\) \(R\) symmetry is therefore a useful guideline for model builders attempting to break supersymmetry.

This line of reasoning leads to a serious problem. Spontaneously broken \(R\) symmetry implies the existence of an exactly massless Goldstone boson, the \(R\)-axion\(^1\). General theoretical arguments based on quantum gravitational effects \[3\], and on string theory \[\text{[7]}\], lead us to expect that any apparent global symmetries in an effective Lagrangian are accidental consequences of gauge symmetries and renormalizability, and are only approximate. (\(R\) symmetry is difficult to gauge because of the necessity of anomaly cancellation.) \(R\) symmetry breaking above the supersymmetry breaking scale is also necessary in order to be able to tune the cosmological constant to zero. Furthermore, there are strong phenomenological constraints on an exactly massless Goldstone boson. We conclude that to describe our world, any continuous \(R\) symmetry is likely to be explicitly broken. However, if an \(R\) symmetry is a necessary condition for supersymmetry breaking, and Nature does not have such an \(R\) symmetry, supersymmetry cannot be spontaneously broken!

In section 3 we examine the effects of explicit \(R\) symmetry breaking in the effective Lagrangian, in order to see whether supersymmetry can be broken without an exact \(R\) symmetry. We find that there are many cases where one can show that the effective Lagrangian is not the most general consistent with the symmetries, even when nonperturbative effects are considered, and that supersymmetry is spontaneously broken without an exact \(R\) symmetry.

\section{Criteria for supersymmetry breaking}

In flat space, assuming there are no Fayet Iliopoulos terms in the Lagrangian\(^2\), supersymmetry is unbroken at tree level if and only if there exist some values of the scalar

---

\(^1\) We refer to this Goldstone boson as an axion even though it is exactly massless.

\(^2\) since we are only interested in models where all mass scales arise dynamically
components of the chiral superfields $z_i$ for which

$$\frac{\partial}{\partial z_j} W = 0 \quad ("F terms")$$

and

$$z_j^* T^A_{jk} z_k = 0 \quad ("D terms") ,$$

where $W$ is the superpotential, the Kahler potential $K$ is $\sum_j z_j^* z_j$, and the $T^A$'s are the gauge generators. Since in general these are $m$ complex conditions and $r$ real conditions for $m$ complex unknowns, where $m$ is the number of chiral superfield components and $r$ is the number of gauge generators, one might naively suppose that supersymmetry breaking would be easy. In fact spontaneous supersymmetry breaking is surprisingly difficult to achieve [8].

Many interesting examples of gauge theories are known where effects of the gauge dynamics may be systematically computed. In such theories there is a limit where a superpotential coupling can be taken to zero, making the vacuum energy, and thus the supersymmetry breaking scale, arbitrarily small. Thus the scale of supersymmetry breaking is lower than the scale at which the gauge dynamics becomes strong. Then the gauge dynamics may be integrated out and an effective supersymmetric theory which contains only the light chiral superfields can be constructed. The following argument shows that in many such theories a supersymmetric vacuum is absent if there is a spontaneously broken continuous R symmetry.

We first construct an effective Lagrangian for the light degrees of freedom, in the limit where the couplings in the classical superpotential are weak. In general there are solutions to the D-term equations in eq. (2.1), which depend on a finite number of continuous parameters ("D-flat directions"). These represent the massless scalar degrees of freedom in the classical limit with no superpotential. For some theories one can find D-flat directions where the gauge symmetry is completely broken and the low energy effective theory has no gauge fields [2]. In other theories there are D-flat directions where the gauge symmetry is not completely broken, but all chiral superfields which transform nontrivially under the unbroken subgroup get large masses, and the remaining low energy effective theory consists of light gauge singlet fields and a supersymmetric gauge theory, but with no light chiral superfields carrying the unbroken gauge symmetries [4]. In both cases the effects of the nonperturbative gauge dynamics are understood using various methods [2, 8, 9, 10] and the scale of supersymmetry breaking can be made arbitrarily small compared to the scale at which the gauge dynamics become strong by tuning the superpotential parameters.
We will refer to either of these cases as calculable theories. The light chiral superfields $\phi_i$ may be constructed as gauge invariant functions of the parameters describing the D-flat directions. The effective supersymmetric Lagrangian has no gauge terms, but typically has a very complicated Kahler potential $K(\phi_i, \phi_i^\dagger)$.

We now write the superpotential $W(\phi_i)$ and make the crucial assumption of genericity, i.e. that the gauge dynamics will generate nonperturbatively additional terms in $W_{\text{eff}}$ which are generic (locally) holomorphic functions of the fields $\phi_i$ consistent with the global symmetries of the theory. In other words, the low energy effective Lagrangian has no fine tuned coefficients. This assumption of genericity is known to be satisfied in standard field theories. It has been shown explicitly to be true in supersymmetric theories for a number of examples (see, e.g. ref. [2, 10]), but there are also cases where it is not true [11].

If we restrict ourselves to values of $\phi$ for which $K_{ij}^*$ is finite and nonsingular (which typically means $\phi_i \neq 0, \infty$), the criterion for unbroken supersymmetry in the effective theory is

$$\frac{\partial W_{\text{eff}}}{\partial \phi_i} = 0.$$ (2.2)

We now examine whether or not there is a supersymmetric minimum for various possibilities for the global symmetries:

1. \textit{No symmetries.} If there are no global symmetries the conditions (2.2) represent $n$ equations for $n$ unknowns, which are soluble for generic $W_{\text{eff}}$, hence supersymmetry should be unbroken. It is possible to find special superpotentials with no global symmetries for which the conditions (2.2) are not soluble, however these do not satisfy the genericity assumption. They are unstable under small variations in the couplings.

2. \textit{Continuous global symmetry which commutes with supersymmetry.} Constraining the theory by means of a non-R global symmetry does not allow one to evade the conclusion that supersymmetry is unbroken. Because $W_{\text{eff}}$ is holomorphic, if there are $l$ generators of global symmetries, $W_{\text{eff}}$ may be written as a function of $n - l$ variables. For example, in the case of a $U(1)$ global charge where the fields $\phi_i$ have charges $q_i$, $W_{\text{eff}}$ is a function of the $n - 1$ variables $X_i = \phi_i/q_i^n$, where $i = 1, \ldots, n - 1$, and $q_n \neq 0$. Thus since $W_{\text{eff}}$ is independent of $l$ complex variables, $l$ of the equations (2.2) are automatically satisfied. The other equations give $n - l$ constraints for $n - l$ unknowns, which typically have solutions. The counting of equations and unknowns in the case of a global discrete symmetry is as in case 1 above (no symmetries).
3. Spontaneously broken \( U(1) \) \( R \) symmetry. The superpotential carries charge 2 under an \( R \) symmetry, and so if \( \phi_n \) receives an expectation value and carries \( R \) charge \( q_n \), \( W_{\text{eff}} \) may be written

\[
W_{\text{eff}} = \phi_n^{2/q_n} f(X_i) \quad ; \quad X_i = \frac{\phi_i}{\phi_n^{q_i/q_n}},
\]

where \( i < n \) and \( f \) is a generic function of the \( n-1 \) variables \( X_i \). Now for supersymmetry to be unbroken, the \( n \) equations

\[
\frac{\partial f}{\partial X_i} = 0 \quad (2.4)
\]

and

\[
f = 0 \quad (2.5)
\]

must be satisfied. (In deriving (2.4) and (2.5) we used the fact that \( \phi_n \) is finite and nonzero.) These are \( n \) equations with \( n-1 \) unknowns which cannot be satisfied for generic \( f \). Therefore, generically there is no supersymmetric solution. (It is however possible to find special superpotentials which will spontaneously break \( R \) but not supersymmetry; it is implausible that such a superpotential could be generated dynamically.) One may worry that there could be a supersymmetric minimum in the limit where the \( R \) symmetry is restored. Because the change of variables in (2.3) is singular in this limit, it is necessary to examine the behavior of \( K_{ij}^{-1} \) to decide whether or not supersymmetry is restored as \( \phi_n \to 0 \). One must also consider what happens as \( \phi_n \to \infty \). In the case where \( q_n < 0 \), the theory can have a stable ground state (with broken supersymmetry) only if \( K_{ij}^{-1} \) grows sufficiently fast. We expect this to be the case if the original theory has no classical flat directions. Otherwise the theory has no ground state.

We believe these arguments explain why previously discovered calculable models which are known to dynamically break supersymmetry also have an \( R \) symmetry. Our arguments that an \( R \) symmetry is necessary for DSB do not apply to noncalculable models, for which there is no separation between the supersymmetry breaking scale and the scale of the strong gauge dynamics. For instance there are several models which are believed to dynamically break supersymmetry in a regime where the theory is a strongly coupled chiral gauge theory. Examples include an \( SU(2n+1) \) gauge theory, with \( n > 1 \), coupled to chiral superfields in one antisymmetric tensor and \( 2n-3 \) antifundamental representations,
with the most general renormalizable gauge invariant superpotential consistent with the symmetries. Another example is an $SO(10)$ gauge theory with a single 16. However in all of these theories there is an accidental R symmetry, and the criteria of reference [2] are sufficient to show that supersymmetry is broken.

Our arguments also do not apply to nongeneric models. Below we give various examples of generic superpotentials to which our arguments do apply, and of nongeneric superpotentials which are counterexamples. All of these should be assumed to have the conventional $\sum_i z_i z_i^*$ form for the Kahler potential.

1. **Generic superpotential, exact unbroken R symmetry, spontaneously broken non-R symmetry, unbroken supersymmetry.** The superpotential is

$$\lambda S X^2 ,$$

which is the most general consistent with the R symmetry under which $S$ has charge zero and $X$ has charge 1, and with a non-R $U(1)$ symmetry under which $S$ has charge 2 and $X$ has charge $-1$. Supersymmetry is unbroken along the flat direction labeled by $a$

$$X = 0, \quad S = a ,$$

which leaves the R symmetry unbroken but does break the other $U(1)$ symmetry (we could have considered a broken R symmetry which is a linear combination of the two $U(1)$s). Note that the Goldstone boson from the spontaneously broken $U(1)$ has a massless scalar partner as required by supersymmetry, and there is a noncompact flat direction.

2. **Nongeneric superpotential, exact spontaneously broken R symmetry, unbroken supersymmetry.** This model has superpotential

$$S X^2 + S X Y + X^2 \bar{Y} + m Y \bar{Y} ,$$

where $S, \bar{Y}$ are superfields with R charge 2, and $X, Y$ have R charge 0. The supersymmetric vacua are

$$X = Y = \bar{Y} = 0, \quad S = a$$

and

$$X = m, \quad S = \bar{Y} = 0, \quad Y = -m ,$$
where $a$ is an arbitrary parameter characterizing a flat direction associated with the broken R symmetry. Note that this flat direction is required for supersymmetry, so that the Goldstone boson from the spontaneously broken R symmetry can have a massless scalar partner, and that the existence of such a flat direction requires a superpotential such as (2.8) which is carefully chosen so that equations (2.4) and (2.5) can be satisfied simultaneously. If other invariant terms such as $S$ are added, there is only an R preserving supersymmetric ground state with $S = 0$.

3. *Generic superpotential, exact R symmetry, broken supersymmetry.* This is the O’Raifeartaigh model with the superpotential

$$\mu^2 S + SQ^2 + mPQ, \quad (2.11)$$

which is the most general renormalizable superpotential consistent with an R symmetry and a $Z_2$ symmetry. All ground states break supersymmetry. The ground state with $S = 0$ does not spontaneously break the R symmetry. The R symmetry guarantees that the addition of generic nonrenormalizable terms consistent with the symmetries will not restore supersymmetry.

4. *Nongeneric superpotential, no R symmetry, broken supersymmetry.* The superpotential

$$\lambda_x XQ^2 + \lambda_y Y(Q^2 - \mu^2) + \lambda Q^3 + m_1 Q^2 + m_2 Q, \quad (2.12)$$

has no R symmetry, but no supersymmetric vacua. The addition of other terms nonlinear in $X$ or $Y$, which are allowed by all symmetries, would restore supersymmetry.

We have argued that in flat space any generic calculable model has DSB if it has a spontaneously broken R symmetry (an exact R symmetry being also a necessary condition). However, when the effects of *local* supersymmetry are included, the F-term conditions are replaced by

$$D_j W = W_j + \frac{1}{M_P^2} K_j W = 0 \quad \text{(supergravity)} \ . \quad (2.13)$$

In order to ensure that the effective cosmological constant is zero after supersymmetry is broken, $W$ must be of order $M_s^2 M_P$, where $M_s$ is the supersymmetry breaking scale. For a theory which exhibits supersymmetry breaking in flat space, since $W_j \sim M_s^2$ and $K_j = O(\langle z \rangle)$, the additional terms in (2.13) are negligible compared to $W_j$ unless some field strengths are of order $M_P$, in which case any effective field theory description breaks
down and we cannot answer the question of whether or not there is a supersymmetric minimum without a detailed understanding of Planck scale physics. We thus believe that our analysis is the most general one possible within the regime of validity of effective field theories.

This discussion has an immediate application to some popular models of supersymmetry breaking in string theory. The original version of these models [12] was based on gluino condensation in some gauge group. A dimension five operator

$$\int d^2 \theta S W^2_\alpha$$

(2.14)

couples the gauge superfields $W_\alpha$ to a dilaton field $S$ and induces upon integrating out the gauge dynamics an effective superpotential for $S$

$$W_{\text{eff}} = e^{-CS}$$

(2.15)

with some constant $C$ depending on the gauge group. Both the original theory with the coupling (2.14) and the effective theory with the superpotential (2.15) exhibit an anomaly free R symmetry under which $S$ is shifted by an imaginary amount. As follows from our general analysis supersymmetry is broken for any finite $S$ where the R symmetry is broken. The problem with this model is that the potential for $S$ slopes to zero at infinity and the theory does not have a ground state.

In order to avoid this problem, the authors of reference [13] proposed to add another gauge group and to couple it to $S$ as in (2.14). These models are referred to as “race track models.” The effective superpotential obtained after integrating out the two gauge sectors is

$$W_{\text{eff}} = e^{-C_1S} + Ae^{-C_2S}$$

(2.16)

where the constants $C_i$ depend on the two gauge groups. Unlike the previous situation, now there is no exact R symmetry. Correspondingly, the superpotential (2.16) leads to a supersymmetric ground state at finite $S$. More sophisticated versions of this proposal [14] involve also some other fields (moduli). Since more than one gauge group is used, the models do not have an R symmetry and therefore, as follows from our general analysis, they all exhibit supersymmetric ground states at finite field strength (as well as some nonsupersymmetric states).
3. Explicit R symmetry breaking and the R-axion

The necessity of a spontaneously broken R symmetry for supersymmetry breaking is troublesome, since it would imply the existence of a Goldstone boson, the R-axion, which could be in conflict with phenomenological and astrophysical observations. Also we do not expect on theoretical grounds that such a global symmetry can be exact [6, 7]. Fortunately, we have found that several ways of explicitly breaking the R symmetry can be shown not to restore supersymmetry. This is a consequence of the lack of genericity of many supersymmetric models and of a nonperturbative nonrenormalization theorem [11].

3.1. Higher dimension operators

There are many examples of models with an accidental R symmetry of the gauge invariant renormalizable terms, but in which this symmetry could be explicitly broken by higher dimension operators in either the superpotential or the Kahler potential. The following argument shows that for a general class of theories such symmetry breaking can only lead to supersymmetry restoration for field strengths which are at least as large as the dimensionful scale which suppresses the effects of the higher dimension terms.

Consider a theory of DSB with effective superpotential \( W_{eff} = W_{ren} + W_{dyn} \), with \( W_{ren} \) a renormalizable superpotential only containing terms of dimension four, and \( W_{dyn} \) a term generated by the nonperturbative dynamics of an asymptotically free gauge theory. This gauge theory generates a scale \( \Lambda_S \) through dimensional transmutation. On general grounds \( W_{dyn} \) must be proportional to a positive power of \( \Lambda_S \). (The restriction that \( W_{dyn} \) can only contain soft terms is an example of the lack of genericity of supersymmetric effective superpotentials.) We will work in a canonical basis for the fields, where the Kahler potential is \( \sum i |z_i z_i^*|^2 + \text{higher dimension terms} \). Now we also assume

1. \( W_{eff} = W_{ren} + W_{dyn} \) has an exact continuous \( U(1) \) R symmetry.
2. The only dimensionful terms in \( W_{eff} \) are characterized by the scale \( \Lambda_S \).
3. This theory spontaneously breaks supersymmetry (and R symmetry) at the scale \( \Lambda_S \).

Therefore, for all values of the fields

\[
\sum_i \left| \frac{\partial W_{eff}}{\partial z_i} \right|^2 + \sum_A \left( z_j^* T^A_{jk} z_k \right)^2 \geq \mathcal{O}(\Lambda_S^4).
\] (3.1)

4. The theory without \( W_{dyn} \) has no flat directions, and the tree level theory is scale invariant. Thus since \( W_{dyn} \) is generated by an asymptotically free interaction, it must
be proportional to a positive power of $\Lambda_S$ and hence must grow more slowly than $z^3$ for large field strength. Therefore, for any field strength $z$ which satisfied the D flatness condition and which is much larger than $\Lambda_S$,

$$\sum_i \left| \frac{\partial W_{\text{eff}}}{\partial z_i} \right|^2 \sim z^4.$$  \hspace{1cm} (3.2)

Now we wish to study whether higher dimension operators can restore supersymmetry. We assume these operators are characterized by at least one inverse power of a scale $m$. Higher dimension terms in the Kahler potential can help restore supersymmetry only if $K_{ij}^{-1}$ has zero eigenvectors, which can happen only for field strengths of order $m$. We can also add higher dimension terms to $W_{\text{eff}}$. Call the higher dimension terms $W_{\text{nonren}}$, which can be written

$$W_{\text{nonren}} = m^3 f(z_i/m), \quad f(x) \leq x^4 \text{ when } x < 1.$$  \hspace{1cm} (3.3)

These terms can only restore supersymmetry for

$$\frac{\partial W_{\text{nonren}}}{\partial z_i} \geq \Lambda_S^2,$$  \hspace{1cm} (3.4)

which can only happen for field strengths

$$z \geq (m\Lambda_S^2)^{1/3},$$  \hspace{1cm} (3.5)

which is greater than $\Lambda_S$. However for such large field strengths, because of our assumption 4,

$$\frac{\partial W_{\text{eff}}}{\partial z_i} \sim z^2.$$  \hspace{1cm} (3.6)

These F terms can only be canceled out by contributions from $W_{\text{nonren}}$ when the light field strengths are of order $m$.

We therefore conclude that if we have a renormalizable theory with DSB and no classical flat directions, higher dimension operators can only restore supersymmetry for field strengths of order the scale which characterizes them – typically the Planck mass. Understanding whether or not this actually happens requires a complete understanding of Planck scale physics.

If the higher dimension operators arise from integrating out heavy particles with a mass less than the Planck mass, then we can study the full theory to understand whether
or not supersymmetry is actually restored at large field strengths. In many cases there will not be any supersymmetric minimum, as shown by the following argument.

Consider a renormalizable theory which, in the limit where some massive parameter \( m \) goes to infinity, has a renormalizable low energy effective theory with an unbroken asymptotically free gauge theory, a continuous R symmetry, no flat directions, spontaneously broken supersymmetry, and no dimensionful parameters other than \( \Lambda_S \), the supersymmetry breaking scale in the effective theory. \( \Lambda_S \) will generally have some \( m \) dependence but we assume that there is a gauge parameter \( g \) which we can tune to make \( \Lambda_S(g, m) \ll m \). We want to understand whether the theory has a supersymmetric minimum. By our previous arguments, if such a minimum does exist, then when \( m \) is larger than \( \Lambda_S \), at least some of the light fields must have vevs of order \( m \) or larger at the minimum. Now we also assume that the full theory has no classical flat directions, and that classically when any of the fields have vevs of order \( m \) that there is at least one F or D term which is as large as \( \mathcal{O}(m^2) \).

Let us only consider D flat directions. The full superpotential is \( W_{\text{eff}} = W_{\text{tree}} + W_{\text{dyn}} \), where \( W_{\text{dyn}} \) is characterized by a scale \( \Lambda_S \) which can be made arbitrarily small compared with \( m \). A supersymmetric minimum requires that for all superfields

\[
\frac{\partial W_{\text{tree}}}{\partial z_i} + \frac{\partial W_{\text{dyn}}}{\partial z_i} = 0 .
\] (3.7)

If there is a supersymmetric minimum we know that at least one of the light fields in the low energy effective theory has vev larger than \( \mathcal{O}(m) \) and therefore there is at least one field for which

\[
\left| \frac{\partial W_{\text{tree}}}{\partial z_i} \right| \geq \mathcal{O}(m^2) .
\] (3.8)

So for this field we must have

\[
\left| \frac{\partial W_{\text{dyn}}}{\partial z_i} \right| = \left| \frac{\partial W_{\text{tree}}}{\partial z_i} \right| \geq \mathcal{O}(m^2) .
\] (3.9)

Thus a supersymmetric minimum could only exist for field strengths larger than \( m \) and then only if there is a contribution to \( W_{\text{dyn}} \), which for large field strength grows in some direction in the light field space faster than any of the tree level terms. Such a situation is possible when there are directions involving the light fields for which the classical potential grows more slowly than quartically. However there are many models, such as the example we will present in the next section, for which the potential grows quartically for any direction in the light field space, and which therefore cannot possess a supersymmetric minimum at any field strength.
This argument has assumed that $m \gg \Lambda_S$. However in a supersymmetric theory it
is not possible to have supersymmetry unbroken for a range of parameters but broken for
other values [8]. The only exception to this rule is when the behavior of the theory at large
field strengths changes [8]. We conclude that in such a model there is no finite range of $m$
for which supersymmetry is unbroken.

If one can find a calculable model where supersymmetry is dynamically broken, even
though there is no R symmetry, then $W_{\text{eff}}$ cannot be generic. In the next section we study
an example of such a model, and show how the lack of genericity can be exploited to give
DSB.

3.2. A model of dynamical supersymmetry breaking without an R symmetry

The simplest known calculable model of DSB, which was analyzed in detail in reference
[2], has gauge group $SU(3) \times SU(2)$, and superfields

$$Q \sim (3, 2), \quad \overline{U} \sim (\overline{3}, 1), \quad \overline{D} \sim (\overline{3}, 1), \quad \text{and} \quad L \sim (1, 2).$$

The classical superpotential is

$$W = \lambda Q \overline{D}L.$$  

This theory has no flat directions and an exact spontaneously broken R symmetry. The
$SU(3)$ gauge theory gets strong at a scale $\Lambda_S$ which is higher than the $SU(2)$ scale and
spontaneously breaks supersymmetry at a scale $\lambda^{5/14} \Lambda_S$. There is also an exact global
$U(1)$ symmetry, called hypercharge, which is not spontaneously broken.

In order to study the effects of explicit R symmetry breaking, we add to this theory
two chiral superfields $S$ and $\overline{S}$ which are $SU(2)$ singlets and transform like a triplet and
an anti-triplet under $SU(3)$. The hypercharge assignments of these fields are

$$Q \sim \frac{1}{6}, \quad \overline{U} \sim -\frac{2}{3}, \quad \overline{D} \sim \frac{1}{3}, \quad L \sim -\frac{1}{2}, \quad S \sim -\frac{1}{3}, \quad \overline{S} \sim \frac{1}{3}.$$  

The most general renormalizable tree level superpotential invariant under hypercharge is

$$W_t = \lambda LQ\overline{D} + \lambda_s LQS + \lambda' Q^2 S + \overline{\lambda} \overline{D}\overline{U}S + m\overline{S}S.$$  

When $m$ is large, we can integrate out $\overline{S}$ and $S$ and find the effective tree level superpo-
tential for the light fields

$$W_N = \lambda LQ\overline{D} + \frac{\lambda' \overline{\lambda}}{m} Q^2 \overline{D}U.$$  

13
For $\lambda'$ or $\overline{\lambda} = 0$ the model has an R symmetry and no flat directions and therefore satisfies the conditions of reference [2] for DSB. When $\lambda', \overline{\lambda} \neq 0$ but $m = 0$ there is no R symmetry and there is a classical flat direction with nonzero $S$ and $\overline{S}$.

We wish to know whether supersymmetry is broken in this theory when $m$ and all the $\lambda$'s are nonzero. Naively, the answer is yes, since when $m$ is large, $S$ and $\overline{S}$ decouple and we are left with a theory which is known to break supersymmetry. Then by the arguments of reference [8], supersymmetry should be broken for any value of $m$. This model is a specific example of the general class of theories described in the previous subsection, which we argued would have to break supersymmetry for large $m$ unless there is a direction in the light field space for which dynamically generated terms grow faster than the tree level potential. The only possible such direction is large $S$ and $\overline{S}$, since in this direction the potential only grows quadratically. However these fields decouple in the large $m$ limit and the supersymmetric minimum must involve expectation values larger than $m$ for at least one of the light fields $Q$, $\overline{U}$, $\overline{D}$, and $L$, and in all such directions the classical potential grows quartically. Thus this model cannot have a supersymmetric minimum, even though it has no R symmetry, due to the lack of genericity of the effective superpotential.

Let us analyze the low energy effective theory given by (3.14) in more detail. The scale at which the low energy $SU(3)$ theory becomes strong is $\Lambda_S = m^{1/7} \Lambda_3^{6/7}$, where $\Lambda_3$ is the $SU(3)$ scale when $m = 0$. If we ignore the small nonrenormalizable term in (3.14), and the term proportional to $\lambda$, then the effective theory is massless QCD with two flavors, for which one can show that instantons generate the effective superpotential

$$W_{\text{instanton}} = \frac{\Lambda_3^7}{Q^2 \overline{U} \overline{D}} ,$$

(3.15)

(the $SU(3) \times SU(2)$ gauge indices in $Q^2 \overline{U} \overline{D}$ are contracted in a gauge invariant way) which is the only term allowed by the symmetries. Adding the tree level terms we find

$$W_{\text{eff}} = \lambda L Q \overline{D} + \frac{\lambda \overline{\lambda}}{m} Q^2 \overline{U} \overline{D} + \frac{m \Lambda_3^6}{Q^2 \overline{U} \overline{D}} .$$

(3.16)

The R invariance of $W_{\text{eff}}$ is broken by the nonrenormalizable term. However this effective superpotential does not lead to a supersymmetric minimum. To see this, note that if the equation $\partial_{L_i} W_{\text{eff}} = \lambda Q_i \overline{D} = 0$ is satisfied (here $i$ is the $SU(2)$ index), and the D-term equations for the $SU(3)$ and $SU(2)$ are also satisfied, then $Q^2 \overline{U} \overline{D} = 0$ and the superpotential is singular.
To compare with our analysis in section 2, the superpotential (3.16) is not generic. The second term breaks the R symmetry but does not restore supersymmetry. However, we have so far neglected the possibility that nonzero $\lambda$, $\lambda'$ and $\bar{\lambda}$ could generate additional nonperturbative terms, such as $\Lambda_S^{7/2} \left( \frac{(LQ\bar{D})}{(Q^2U\bar{D})} \right)^{1/2}$ which would restore supersymmetry. However, the nonperturbative nonrenormalization theorem of reference [1] shows that this does not happen, the effective superpotential (3.16) is exact and does not receive any corrections. In other words, the superpotential is not generic and supersymmetry is broken.

3.3. Effects of higher dimension operators on the R axion mass

Despite the example in the previous section, it remains true that in all known models of DSB which have no mass scales in the classical Lagrangian, there is a massless R axion. In realistic models, the R axion gains a small mass from the QCD anomaly. This axion could be in conflict with astrophysical and phenomenological observations if the scale of R symmetry breaking is lower than $\sim 10^{10}$ GeV and with cosmology if the scale is higher than $\sim 10^{12}$ GeV [15, 16]. However, we now know that the R symmetry could be explicitly broken by nonrenormalizable terms without restoring supersymmetry. Such terms could solve the astrophysical problems associated with the R axion occurring in models of DSB at low energy [17], by giving the axion a mass which is too large to be produced in stars.

If the R symmetry is explicitly broken by operators of dimension five proportional to $1/m_P$, then the effects on the R axion mass are quite interesting. Assuming that the only dimensionful scale in the low energy effective theory is the supersymmetry breaking scale $\Lambda_S$, and that this is also the scale of spontaneous R symmetry breaking, the R axion mass squared is approximately

$$m_{\text{axion}}^2 \sim \frac{\Lambda_S^3}{m_P}.$$  \hspace{1cm} \text{(3.17)}

For $\Lambda_S$ of order $10^5$ GeV, this gives $m_{\text{axion}} \sim 10$ MeV, which is just barely light enough to be produced in supernovae. We conclude that the R axion is not an astrophysical problem for models of DSB in the visible sector, provided that the R symmetry can be broken by terms of dimension 5 and $\Lambda_S > 10^5$ GeV.

There can also be cosmological problems associated with overproduction of the R axion in the early universe, if the R symmetry breaking scale is higher than $\sim 10^{12}$ GeV [18]. A general analysis of the cosmological bounds on the axion coupling and mass [19] shows that this problem can also be solved if explicit symmetry breaking gives the axion a
large enough mass. For instance an axion with a mass given by eq. (3.17) does not cause any cosmological problems when the scales of R symmetry breaking and supersymmetry breaking are the same. However in models where dynamical supersymmetry breaking is driven by nonrenormalizable operators suppressed by the Planck mass, the R axion mass is typically not large enough to avoid cosmological overproduction [16].

3.4. R color

Another method for solving the R axion problem [2], is to introduce a new strong gauge group called R color, under which the R symmetry has an anomaly, in addition to the group whose dynamics are responsible for DSB. Nonperturbative R color effects can give the R axion a mass, but one must then worry about whether they can also induce new terms in the effective superpotential which could restore supersymmetry. The following argument shows that in many cases R color does not restore supersymmetry, due to the lack of genericity of the nonperturbatively generated terms.

Consider a theory whose classical lagrangian only has dimension four terms, with no classical flat directions, a spontaneously broken R symmetry, and supersymmetry breaking scale $\Lambda_S$. Now introduce R color by gauging a global symmetry of this theory, which becomes strong at a scale $\Lambda_R \ll \Lambda_S$. R color can nonperturbatively induce operators in the effective superpotential which always grow more slowly for large field strength than $O(z^3)$, and which break the R symmetry.

We write the effective superpotential as

$$W_{\text{eff}} = W_S + W_R ,$$

(3.18)

where $W_S$ is the effective superpotential of the theory in the limit where R color is turned off, and the remaining terms in $W_R$ are generated by nonperturbative R color effects. For all D-flat directions there is at least one superfield for which

$$\left| \frac{\partial W_S}{\partial z_i} \right| \geq \Lambda_S^2 .$$

(3.19)

Supersymmetry restoration requires that

$$\frac{\partial W_S}{\partial z_i} + \frac{\partial W_R}{\partial z_i} = 0 ,$$

(3.20)
thus to restore supersymmetry it is necessary that
\[ \left| \frac{\partial W_R}{\partial z_i} \right| \geq \Lambda_S^2. \] (3.21)

R color can only induce terms proportional to positive powers of \( \Lambda_R \), and for field strengths larger than \( \mathcal{O}(\Lambda_S) \)
\[ \left| \frac{\partial W_R}{\partial z_i} \right| < \mathcal{O}(z^2). \] (3.22)

Because of the lack of classical flat directions, for such field strengths
\[ \left| \frac{\partial W_S}{\partial z_i} \right| \sim \mathcal{O}(z^2), \] (3.23)
and so it is not possible to satisfy eq. (3.20) for field strengths larger than \( \mathcal{O}(\Lambda_S) \). So R color could only restore supersymmetry in the region where all field strengths are smaller than \( \Lambda_S \) and the supersymmetry breaking group is strongly coupled. In this region the contribution of the supersymmetry breaking group to the effective potential is larger than \( \mathcal{O}(\Lambda_S^4) \), and R color would have to make an equally important canceling contribution in order to restore supersymmetry. This is not possible since we are assuming that \( \Lambda_R \ll \Lambda_S \), and the R color contribution to the effective potential will be smaller than \( \max(\mathcal{O}(\Lambda_R^4), \mathcal{O}(z^4)) \).

We conclude that when R color is added to a DSB model with no classical flat directions or mass scales, then if R color is sufficiently weak nonperturbative R color effects cannot restore supersymmetry. Furthermore, we have previously noted that it is not possible to have a theory in which supersymmetry is unbroken for a range of parameters but broken for other values, so supersymmetry breaking should persist even when R color is as strong or stronger than the supersymmetry breaking gauge group.

4. Conclusions

We have shown that a continuous R symmetry is a necessary condition for spontaneous supersymmetry breaking and a spontaneously broken R symmetry is a sufficient condition, in models where the gauge dynamics can be integrated out and in which the effective superpotential is a generic function consistent with the symmetries of the theory. Therefore the existence of a nonanomalous R symmetry is a useful guideline for finding a model with dynamical supersymmetry breaking. This implies that a troublesome Goldstone boson, the R axion, is a typical feature of models which break supersymmetry dynamically.
We have also argued that because supersymmetry severely limits the terms which can be generated nonperturbatively in the effective superpotential, in some cases the effective superpotential is not generic and it is possible to dynamically break supersymmetry without an exact R symmetry. The R axion can be given an acceptably large mass, either by introducing higher dimension operators which explicitly break the R symmetry or by introducing a new gauge group under which the R symmetry is anomalous. We conclude by speculating on the attractive possibility that because of the lack of genericity of effective superpotentials, there may exist a model with a single gauge group, in which all mass scales arise through dimensional transmutation, supersymmetry is spontaneously broken, and there is no R symmetry or R axion at all. Such a model would be an excellent candidate for dynamical supersymmetry breaking near the weak scale.

Acknowledgments

It is a pleasure to thank T. Banks, D. Kaplan, S. Shenker and E. Witten for several useful discussions. A.N. would like to thank Rutgers University and the CERN theory group for their hospitality. This work was supported in part by DOE grants #DE-FG05-90ER40559 and #DOE-FG03-90ER40546, and by the Texas National Laboratory Research Commission under grant #RGFY93-206. A.N. was supported in part by an SSC fellowship and a Sloan fellowship.
References

[1] E. Witten, Nucl. Phys. B188 (1981) 513
[2] I. Affleck, M. Dine and N. Seiberg, Phys. Lett. 137B (1984) 187; Phys. Rev. Lett. 52 (1984) 493; Phys. Lett. 140B (1984) 59; Nucl. Phys. B256 (1985) 557
[3] G.C. Rossi and G. Veneziano, Phys. Lett. 138B (1984) 195; Y. Meurice and G. Veneziano, Phys. Lett. 141B (1984) 69; D. Amati, G.C. Rossi and G. Veneziano, Phys. Lett. 138B (1984) 195; D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, Phys. Rep. 162 (1988) 169
[4] G. ’t Hooft, in Recent Developments in Gauge Theories, ed. G. ’t Hooft et al., (Plenum, New York, 1980)
[5] K. Konishi, Phys. Lett. 135B (1984) 439
[6] S. Hawking, Comm. Math. Phys. 43 (1975) 199; Phys. Lett. 195B (1987) 337, G.V. Lavrelashvili, V. Rubakov, and P. Tinyakov, JETP Lett. 46 (1987)167; S. Giddings and A. Strominger, Nucl. Phys. B307 (1988) 854; S. Coleman, Nucl. Phys. B310 (1988) 643; T. Banks, Physicalia, 12 (1990) 19
[7] T. Banks and L. J. Dixon, Nucl. Phys. B307 (1988) 93
[8] E. Witten, Nucl. Phys. B202 (1982) 253
[9] G. Veneziano and S. Yankielowicz, Phys. Lett. 113B (1982) 321; T.R. Taylor, G. Veneziano, and S. Yankielowicz, Nucl. Phys. B218 (1983) 493; V.A. Novikov, M.A. Shifman, A. I. Vainshtain and V. I. Zakharov, Nucl. Phys. B223 (1983) 445; M. Peskin, in Problems in Unification and Supergravity, ed. G. Farrar and F. Henyey (AIP, New York, 1984)
[10] I. Affleck, M. Dine, and N. Seiberg, Phys. Rev. Lett. 51 (1983) 1026; Nucl. Phys. B241 (1984) 493
[11] N. Seiberg, to appear
[12] J.P. Derendinger, L.E. Ibanez and H.P. Nilles, Phys. Lett. 155B (1985) 65; M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. 156B (1985) 55
[13] N.V. Krasnikov, Phys. Lett. 193B (1987) 37; L. Dixon, V. Kaplunovsky, J. Louis and M. Peskin, unpublished; L. Dixon, talk presented at the APS, DPF Meeting at Houston (1990); V. Kaplunovsky, talk presented at the “String 90” workshop at College Station (1990)
[14] See e.g.: S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, Phys. Lett. 245B (1990) 409; J.A. Casas, Z. Lalak, C. Munoz and G.G. Ross, Nucl. Phys. B347 (1990) 243; B. deCarlos, J.A. Casas and C. Munoz, Phys. Lett. 263B (1991) 248
[15] Relevant reviews include J.E. Kim, Phys. Rep. 149 (1987) 1; R. Peccei, published in CP Violation, ed. C. Jarlskog (1988)
[16] T. Banks, D. B. Kaplan, and A. E. Nelson, preprint UCSD/PTH 93-26, RU-37 (1993)
[17] M. Dine and A.E. Nelson, Phys. Rev. D48 (1993) 1277
[18] J. Preskill, M.B. Wise, and F. Wilczek, Phys. Lett. 120B (1983) 127; L. P. Abbott and P. Sikivie, Phys. Lett. 120B (1983) 133; M. Dine and W. Fischler, Phys. Lett. 120B (1983) 137

[19] J. Ellis, D.V. Nanopoulos and M. Quiros, Phys. Lett. 174B (1986) 176