Dual Wavelength Chirped Pulse Regenerative Amplifier Based on Yb$^{3+}$:LuAlO$_3$ Crystal for Terahertz Applications

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Abstract

Compact diode-pumped chirped pulse regenerative amplifier systems with pulse repetition rate of hundreds kilohertz based on Yb$^{3+}$-doped crystals are of practical importance for wide range of applications such as materials processing, medicine, scientific research, etc. The aim of this work was to study the Yb$^{3+}$:LuAlO$_3$ crystal based dual wavelength chirped pulse regenerative amplifier.

Perovskite-like aluminate crystals have unique spectroscopic properties that allowed to use amplifier active element gain spectrum as an amplitude filter for amplified pulse spectrum and even obtained dual wavelength amplification without any additional components.

In our work a simple way to obtain dual-wavelength operation of chirped pulse regenerative amplifier by using the active medium gain spectrum as an amplitude filter for the formation of the amplified pulses spectrum demonstrated for the first time to our knowledge. Maximum output power of 5.4 W of chirped pulses (3.8 W after compression) and optical-to-optical efficiency of 22.5 % have been obtained for Yb:LuAP E//b-polarization at 200 kHz repetition rate. Compressed amplified pulse duration was about 708 fs while separate spectral components durations were 643 fs and 536 fs at 1018.3 nm and 1041.1 nm central wavelengths, respectively. Performed investigations show high potential of Yb$^{3+}$:LuAP crystals as active elements of compact diode pumped chirped pulse regenerative amplifiers.

Keywords: chirped pulse regenerative amplifier, ytterbium ions, diode pumping, lutetium aluminate crystals, gain narrowing effect.

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Регенеративный усилитель чирпированных импульсов на кристалле Yb$^{3+}$:LuAlO$_3$ с усилением отдельных спектральных компонент для применений в терагерцовой области спектра

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Компактные регенеративные усилители чирпированных импульсов с диодной накачкой, обеспечивающие частоту повторения усиленных импульсов в сотни килогерц, построенные на материалах, легированных ионами Yb$^{3+}$, представляют практический интерес для широкого ряда научных, промышленных и биомедицинских применений. Целью данной работы являлось исследование регенеративного усилителя на кристалле Yb$^{3+}$:LuAlO$_3$ с усилением отдельных спектральных компонент импульсов задающего лазера.

Кристаллы алюминатов со структурой перовскита обладают уникальными спектроскопическими свойствами, что позволяет использовать спектр усиления активной среды регенеративного усилителя в качестве амплитудного фильтра и усиливать отдельные участки спектра импульсов задающего лазера без каких-либо дополнительных оптических компонентов.

В данной работе впервые исследован простой подход, позволяющий использовать спектр усиления активной среды регенеративного усилителя как амплитудный фильтр для формирования спектра усиленного импульса в виде спектра состоящего из отдельных полос. Максимальная средняя выходная мощность 5,4 Вт (3,8 Вт после компрессора) с оптической эффективностью 22,5 % получена для поляризации излучения параллельной оси $b$ кристалла Yb:LuAP при частоте следования импульсов 200 кГц. Длительность сжатых импульсов составила 708 пс при учете влияния всех спектральных компонент, и 643 пс и 536 пс для спектральных компонент с центральными длинами волн 1018,3 нм и 1041,1 нм. Проведенные исследования показывают высокий потенциал использования кристалла Yb$^{3+}$:LuAP в качестве активного элемента компактных регенеративных усилителей чирпированных импульсов с диодной накачкой.

Ключевые слова: регенеративный усилитель чирпированных импульсов, ионы иттербия, диодная накачка, кристалл лютециевого алюмината, эффект сужения спектра усиления.

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Introduction

Compact diode-pumped chirped pulse regenerative amplifier (CPRA) systems with pulse repetition rate of hundreds kilohertz based on Yb$^{3+}$-doped crystals are of practical importance for wide range of applications such as materials processing, medicine, scientific research, etc.

One of the rapidly developing areas of research is the generation of THz radiation. Radiation of this spectral range is used in imaging and airport security inspection, cosmology and radio astronomy, atmospheric remote sensing, material measurements, medical sensing, etc.

Due to sub-picosecond pulse durations, μJ-level pulse energies and hence tens of MW pulse peak power levels CPRA systems can be successfully used for THz radiation generation by means of optical rectification [1–3] or difference frequency generation (DFG) techniques [4, 5]. In the case of DFG technique it is necessary to use laser source that emits at least two spectral components. For this purpose can be used Q-switched lasers [6, 7] which provide compact THz sources, optical parametric oscillators [8, 9] with complex cavities, two color semiconductor lasers [10] providing cost effective THz sources with relatively low power levels, etc. It is evident that application of ultrafast chirped pulse regenerative amplifier systems allowed power scaling of the generated THz radiation.

Unique spectroscopic properties of Perovskite-like aluminate crystals are well studied in our previous works, such as spectroscopy and high power CW [11], actively Q-switched [12] lasers, broad-band femtosecond chirped pulse regenerative amplifier with amplified pulse spectral broadening [13].

In this paper we study chirped pulse regenerative amplification of broad-band pulses in Yb$^{3+}$:LuAlO$_3$ (Yb:LuAP) crystal based system in presence of strong gain narrowing effect which led to dual wavelength shaping of the amplified pulse spectrum for the first time to the best of our knowledge.

Crystal growth

LuAP is a biaxial crystal of the “distorted perovskite” type (space group D$_{2h}^{16}$-Pbnm). Unlike the stable aluminates based on large-size rare-earths (RAIO$_3$, R = Gd-Er), the end-member orthorhombic aluminates of smaller-size rare-earths (RAIO$_3$, R = Tm, Yb, Lu) are considered as metastable compounds, i. e., they have no stability regions in the subsolidus and cannot be sintered employing traditional solid state reaction techniques. In the flux growth, TmAlO$_3$ and YbAlO$_3$ phases have been observed together with the corresponding garnet phases, while LuAlO$_3$ could not be obtained; instead of this, Lu$_5$Al$_2$O$_{12}$ and Lu$_3$O$_3$ phases have been recognized. The first reported LuAlO$_3$ single crystals were grown from the melt by Czochralski method. Phase equilibria studies in the Lu$_5$O$_3$-Al$_2$O$_3$ system [14, 15] have shown that the range of stability of the perovskite phase is quite narrow and its nucleation may occur only on cooling from the molten state, from temperatures well above the liquidus temperature. Based on solidification behavior of the LuAP melts, the schemes were developed for single crystal growth, the details of which can be found in [16]. LuAP:Yb single crystals for the present studies were grown by the vertical Bridgman method (or vertical directional crystallization) [16, 17] under Ar/H$_2$ atmosphere (5 vol% of H$_2$) using molybdenum containers 14 mm in diameter. High purity Lu$_2$O$_3$, Yb$_2$O$_3$ and crystalline Al$_2$O$_3$ were used as starting components; the selected concentrations of Yb were 2, 5 and 10 at.%. Due to a very small size mismatch between Yb$^{3+}$ ($\varepsilon_{VIII} = 0.985$ Å) and Lu$^{3+}$ ($\varepsilon_{VIII} = 0.977$ Å) ions [18] (around 0.8 % with respect to Lu$^{3+}$), the distribution coefficient of Yb$^{3+}$ ions in LuAlO$_3$ is close to unit and practically all Yb ions amount added to the melts is being incorporated into the lattice.

Spectroscopy

Polarized absorption spectra of Yb$^{3+}$(2 at.%):LuAP (corresponding ytterbium concentration was 4.02×10$_{20}$ cm$^{-3}$) at room temperature were registered by a Varian CARY-5000 spectrophotometer. Absorption cross-section spectra for three light polarizations parallel to the a, b and c crystallographic axes are shown in Figure 1.

Strong absorption is found for E/c light polarization with the peak absorption cross-section at 978.5 nm of about 6.6×10$_{20}$ cm$^2$ and spectral bandwidth FWHM of 4 nm.

It is well known that radiation trapping strongly affects the measured lifetime of Yb-doped materials because of significant overlap of the absorption and emission bands [16, 19]. The comparatively high index of refraction of LuAP ($n_0 = 1.923$) also increases the probability of reabsorption even in optically thin samples because of the total internal reflection. Thus the special methods discussed in the
literature [16, 19] should be used to determine the luminescence lifetime accurately. In our experiments we used a fine powder of Yb:LuAP crystal immersed in glycerin. The diameter of the powder particles was measured to be approximately 30–40 µm, several times lower than absorption length of the most heavily doped Yb\(^{3+}\)(10 at.\%):LuAP crystal (75 µm at 978.5 nm). The Yb ions contents in the samples were 2, 5 and 10 at.%. The samples were excited by 20 ns pulses at 978.5 nm and luminescence kinetics was registered with the use of a 0.3-m monochromator, fast Ge-photodiode with a rise time of < 20 ns and a 500 MHz digital storage oscilloscope. All the samples exhibited single exponential decays (see Figure 2a). Starting from certain powder content, the lifetime remained constant despite further dilution (Figure 2b), thus indicating that reabsorption effects became negligible. Emission lifetime for 10, 5, and 2 at.% Yb-doped crystals was measured to be 310 ± 10 µs, 380 ± 10 µs and 475 ± 10 µs, respectively (see Figure 2c). Taking into account that similar concentration quenching was observed for Yb:YAP starting from about 4 at.% of Yb doping concentration [20], we believe that the measured value of (475 ± 5) µs corresponds to the radiative lifetime of Yb\(^{3+}\)-ions in LuAP.

The stimulated-emission cross sections were calculated by use of the modified reciprocity method in which it is not necessary to know the Stark level structure of the Yb\(^{3+}\) manifolds (\(^{2}\)F\(_{5/2}\) and \(^{2}\)F\(_{7/2}\)) [21]:

\[
\sigma_{SE}(\lambda) = \frac{3}{8\pi^2} \frac{\exp(-hc/(kT\lambda))}{\tau_{rad}} \sum_{\beta} \frac{1}{\beta} \sigma_{abs}(\lambda) \exp(-hc/(kT\lambda)) d\lambda \tag{1}
\]
where $\tau_{rad}$ is the radiation lifetime of an active center; $c$ is the light velocity; $h$ and $k$ are Planck and Boltzmann constants, respectively; $T$ is the crystal temperature; $n$ is the refractive index of a crystal; $\alpha$ and $\beta$ denote the polarization state; and $\sigma_{abs}$ is the ground-state absorption cross section.

The stimulated-emission cross section spectra calculated with this method are presented in Figure 1.

The most intensive stimulated emission (SE) cross-section band at 999.6 nm has peak value of about $3.74 \times 10^{-20}$ cm$^2$ for $E//c$-polarization.

**Continuous wave laser experiment**

For laser operation the most interesting polarization states in the crystal are $E//c$ and $E//b$ ($a$ and $b$ are crystallographic axes) due to high stimulated-emission cross sections values. In comparison with Yb-doped YAP, the crystal of Yb:LuAP exhibits slightly higher stimulated-emission cross section, a close radiative lifetime and a comparable stimulated emission bandwidth [22].

For a CW laser experiments a set up with X-folded cavity design was used (see Figure 3). It consisted of two curved mirrors M1 and M2 and two plane mirrors: OC and HR.

![Figure 3](image)

**Figure 3** – Experimental setup of continuous wave diode-pumped Yb:LuAP laser: HR—highly reflective mirror; M1, M2-concave mirrors; OC—output coupler; LD—laser diode

The calculated TEM$_{00}$ mode diameter in the crystal was about 180 μm. As a pump source, a multiple single emitter InGaAs fiber-coupled laser diode (Ø105 μm, NA = 0.15) with a maximum output power of about 25 W was used. An “off-axis” pump layout was used for longitudinal pumping of the active element (see Figure 3). This pump arrangement was successfully tested in our previous work [11–13] and the main advantage of such a pump scheme is that all the cavity mirrors have highly reflecting coating at 900–1100 nm. The pump light was formed by a set of lenses into the spot with a diameter of about 180 μm (1/e$^2$).

A 2 mm long Yb(2 at.%):LuAlO$_3$ crystal was used as a gain medium. The crystal was a-cut to provide $E//b$ and $E//c$ polarized laser output. It was a slab with dimensions $2(a) \times 5(b) \times 1.5(c)$ mm$^3$; both $5 \times 2$ mm$^2$ lateral faces were maintained at 15 °C by means of copper plates (indium foil was used to improve thermal contact) and thermo-electrical cooling elements with water-cooled heat sink, while $1.5 \times 5$ mm$^2$ working faces were antireflection coated for pump and laser radiation.

The dependencies of the laser output power on the absorbed pump power for $E//b$– and $E//c$–polarized outputs and different OCs are shown in Figure 4. Absorbed pump power was real-time measured during the laser action.

![Figure 4](image)

**Figure 4** – CW laser performance of Yb:LuAP crystal for different polarizations and output coupler transmittances

The maximum CW output power of 9.6 W at absorbed pump power of 15.2 W with slope efficiency of 73.5 % was demonstrated for $E//b$ polarization with 5 % OC transmittance. With output coupler transmission of 10 % and 20 % the laser output power slightly decreased to 8.6 W and 6.6 W, respectively, while the corresponding slope efficiencies increased to 76.4 % and 84.5 %. Similar output powers were demonstrated for $E//c$ laser output. With 10 % output coupler transmittance 9.1 W of output power was
obtained at 14.5 W of absorbed pump power with 77.9 % slope efficiency. Output powers of 8.3 W and 7.2 W with slope efficiencies of 62.5 % and 73.6 % were obtained for 5 % and 20 % OCs, respectively.

**Dual wavelength chirped pulse regenerative amplifier experiment**

Experimental setup of chirped pulse regenerative amplifier is shown in Figure 5. As a seed source laser diode-pumped Yb:KYW oscillator was used which provided 100 fs pulse train with 70 MHz PRF and 25 nJ single pulse energy. The seed pulse spectrum was about 12.5 nm wide (FWHM) and centered at 1038 nm. A 10 m-long single mode Ø9/125 µm telecom fibre was used for pulse spectral broadening and temporal stretching ($t_{\text{pulse}} \approx 7.5$ ps). After passing through double-$\text{BaB}_2\text{O}_4$ Pockels cell based pulse picker and Faraday isolator, the seed pulse was injected into the RA. The isolator was employed to protect the seeder from high-intensity back reflections and, at the same time, for separating the amplified output pulse from the seed oscillator. The RA setup chosen for this experiment is quite common, employing a 40 mm-long double-$\text{BaB}_2\text{O}_4$ Pockels cell for pulse injection and ejection. Pulse repetition frequency (PRF) was chosen to be 200 kHz to prevent damage of the optical elements. For chirped pulse regenerative amplifier experiments was used the same active element as for CW experiments. The last unit of the amplifier system is compressor based on transmission diffraction grating with 1000 grooves per millimetre.

As one can see from the stimulated emission cross-section spectrum (Figure 1) for E//b polarization separate peaks around 1020 and 1040 nm are observed. Using broad-band seed source and taking into account strong gain narrowing effect we can estimate spectral shaping of the amplified pulses in such manner that the output pulse spectrum will consist of separate parts corresponding to SE cross-section spectrum of the amplifier gain medium. In the following experiment we studied of this idea. Seed and amplified pulse spectra obtained during the experiment are shown in Figure 6.
The maximum uncompressed average output power of 5.4 W was obtained for E//b- polarized light after 112 roundtrips (RT) of the pulse through the amplifier cavity, while the pulse spectral width (FWHM) was 2.9 nm at 1018.3 nm (short wavelength part) and 5.2 nm at 1041.1 nm (long wavelength part). When the incident pump power was 24 W, the optical-to-optical efficiency was as high as 22.5 %.

Then we adjust compressor unit in the following order: adjust compressor for best compression of short wavelength part of the amplified pulse spectrum separately, made sure that the long-wavelength part of the spectrum is also satisfactorily compressed separately and then adjust relative offset of the pulses relative to each other by moving the rear mirror of the compressor. Measured autocorrelation traces of short and long wavelength parts separately are shown in Figure 7.

![Figure 7](image_url)

**Figure 7** – Measured autocorrelation traces of short (a) and long (b) wavelength parts

After the adjusting of the compressor unit we measured autocorrelation trace of both spectral parts (Figure 8).

![Figure 8](image_url)

**Figure 8** – Measured autocorrelation traces of both spectral parts

During the experiments compressed pulse durations of 643 fs for short wavelength part and 536 fs for long wavelength part were achieved. Pulse duration of about 708 fs was obtained from autocorrelation trace of both spectral components demonstrating good overlap between components.

After simple calculations, one can expect that as a result of the difference frequency generation process, a radiation of about 6.5 THz spectral region can be obtained.

**Conclusion**

In conclusion, a simple way to obtain dual-wavelength operation of chirped pulse regenerative amplifier by using the active medium gain spectrum as an amplitude filter for the formation of the amplified pulses spectrum demonstrated for the first time to our knowledge. Maximum output power of 5.4 W of chirped pulses (3.8 W after compression) and optical-to-optical efficiency of 22.5 % have been obtained for Yb:LuAP E//b-polarization at 200 kHz repetition rate. Compressed amplified pulse duration was about 708 fs while separate spectral components durations were 643 fs and 536 fs at 1018.3 nm and 1041.1 nm central wavelengths, respectively.

Performed investigations show high potential of Yb\(^{3+}\):LuAP crystals as active elements of compact diode pumped chirped pulse regenerative amplifiers.

**References**

1. Schneider W., Ryabov A., Lombosi Cs., Metzger T., Major Zs., Fülöp J.A., Baum P. 800-fs, 330-μJ pulses from a 100-W regenerative Yb:YAG thin-disk amplifier at 300 kHz and THz generation in LiNbO\(_3\). *Opt. Lett.*, 2014, vol. 39, iss. 23, pp. 6604–6607. DOI: 10.1364/OL.39.006604
2. Koustuban Ravi, Wenqian Ronny Huang, Sergio Carbayo, Emilio A. Nanni, Damian N. Schimpf, Erich P. Ippen, and Franz. X. Kärtner. Theory of terahertz generation by optical rectification using tilted-pulsefronts. *Opt. Express*, 2015, vol. 23, iss. 4, pp. 5253–5276. 

DOI: 10.1364/OE.23.005253

3. Meyer F., Hekmat N., Mansourzadeh S., Fobbe F., Aslani F., Hoffmann M., Saraceno C.J. Optical rectification of a 100 W average power mode-locked thindisk oscillator. *Opt. Lett.*, 2018, vol. 43, pp. 5909–5912. 

DOI: 10.1364/OL.43.005909

4. Ding J.Y. Generation of quasi-single-cycle THz pulses based on broadband-phase-matched difference-frequency generation: high conversion efficiencies and output powers. *Conference on Lasers and Electro-Optics*, 2005, Baltimore, MD, vol. 2, pp. 1453–1455. 

DOI: 10.1109/CLEO.2005.202156

5. Saito K., Tanabe T., Oyama Y. THz-Wave Generation from GaP THz Photonic Crystal Waveguides under Difference-Frequency Mixing. *Optics and Photonics Journal*, 2012, vol. 2, no. 3, pp. 201–205. 

DOI: 10.4236/opj.2012.2.20300

6. Zhao Pu, Ramag Srinivasa, Yujie J. Ding and Ioulia B. Zotova. Investigation of terahertz generation from passively Q-switched dual-frequency laser pulses. *Optics Letters*, 2011, vol. 36, iss. 24, pp. 4818–4820. 

DOI: 10.1364/OL.36.004818

7. Bezotosnyi V.V., Cheshnev E.A., Gorbunkov M.V., Koromyslov A.L., Krokhin O.N., Mityagin Yu.A., Popov Yu.M., Savinov S.A., Tunkin V.G. Coherent THz Repetitive Pulse Generation in a GaSe Crystal by Dual-wavelength Nd:YLF Laser. *Physics Procedia*, 2015, vol. 72, pp. 405–410. DOI: 10.1016/j.phpro.2015.09.075

8. Schara J.E., Vodopyanov K.I., Fejer M.M. Intracavity terahertz-wave generation in a synchronously pumped optical parametric oscillator using quasi-phase-matched GaAs. *Opt. Lett.*, 2007, vol. 32, pp. 1284–1286. DOI: 10.1364/OL.32.001284

9. Walsh David A., Browne Peter G., Dunn Malcolm H., Rae Cameron F. Intracavity parametric generation of nanosecond terahertz radiation using quasi-phase-matching. *Opt. Express*, 2010, vol. 18, pp. 13951–13963. DOI: 10.1364/OE.18.013951

10. Hoffmann S., Hofmann M.R. Generation of Terahertz radiation with two color semiconductor lasers. *Laser & Photon. Rev.*, 2007, vol. 1, iss. 1, pp. 44–56. DOI: 10.1002/lpor.200710004

11. Alexander Rudenkov, Viktor Kisel, Anatol Yasukevich, Karine Hovhannesyan, Ashot Petrosyan, Nikolay Kuleshov. Spectroscopy and continuous wave laser performance of Yb$^{3+}$:LuAlO$_3$ crystal. *Opt. Lett.*, 2016, vol. 41, pp. 5805–5808. DOI: 10.1364/OL.41.005805

12. Alexander Rudenkov, Viktor Kisel, Anatol Yasukevich, Karine Hovhannesyan, Ashot Petrosyan, Nikolai Kuleshov. Compact 999.6 nm Actively Q-Switched Yb$^{3+}$:LuAlO$_3$ Laserfor Laser-Induced Breakdown Spectroscopy. *Devices and Methods of Measurements*, 2019, vol. 10, no. 2, pp. 119–127. DOI: 10.21122/2220-9506-2019-10-2-119-127

13. Alexander Rudenkov, Viktor Kisel, Anatol Yasukevich, Karine Hovhannesyan, Ashot Petrosyan, and Nikolay Kuleshov. Yb$^{3+}$:LuAlO$_3$ crystal as a gain medium for efficient broadband chirped pulse regenerative amplification. *Opt. Lett.*, 2017, vol. 42, pp. 2415–2418. DOI: 10.1364/OL.42.002415

14. Petrosyan A.G., Popova V.F., Gusrarov V.V., Shirinyan G.O., Pedrini C., Lecooq P. The Lu$_2$O$_5$–Al$_2$O$_3$ system: Relationships for equilibrium-phase and supercooled states. *Journal of Crystal Growth*, 2006, vol. 293, pp. 74–77. DOI: 10.1016/j.jcrysgro.2006.05.017

15. Petrosyan A.G., Popova V., Ugolkov V.L., Romanov D.P., Ovanesyan K.L. A phase stability study in the Lu$_2$O$_5$–Al$_2$O$_3$ system. *Journal of Crystal Growth*, 2013, vol. 377, pp. 178–183. DOI: 10.1016/j.jcrysgro.2013.04.054

16. Kühn Henning, Fredrich-Thornton Susanne T., Kränkel Christian, Peter Rigo, Petermann Klaus. Model for the calculation of radiation trapping and description of the pinhole method. *Opt. Lett.*, 2007, vol. 32, pp. 1908–1910. DOI: 10.1364/OL.32.001908

17. Petrosyan A.G. Crystal growth of laser oxides in the vertical Bridgman configuration. *Journal of Crystal Growth*, 1994, vol. 139, pp. 372–392. DOI: 10.1016/0022-0248(94)90190-2

18. Shannon R.D. Revised effective ionic radii and systematic studies of interatomic distances in halides and chalcogenides. *ActaCrystallogr A*, 1976, vol. 32, pp. 751–767. DOI: 10.1107/S0567739476001551

19. Sumida D.S., Fan T.Y. Effect of radiation trapping on fluorescence lifetime and emission cross section measurements in solid-state laser media. *Opt. Lett.*, 1994, vol. 19, pp. 1343–1345. DOI: 10.1364/OL.19.001343

20. Boulon G., Guyot Y., Canibano H., Hraisch E., Yoshikawa A. Characterization and comparison of Yb$^{3+}$ doped YAlO$_3$ perovskite crystals (Yb:YAP) with Yb$^{3+}$ doped Y$_5$Al$_2$O$_{12}$ garnet crystals (Yb:YAG) for laser application. *J. Opt. Soc. Am. B*, 2008, vol. 25, pp. 884–896. DOI: 10.1364/JOSAB.25.000884

21. Yasukevich A.S., Sheherbitskii V.G., Kisiel V.E., Mandrik A.V., Kuleshov N.V. Integral method of reciprocity in the spectroscopy of laser crystals with impurity centers. *Journal of Applied Spectroscopy*, 2004, vol. 71, no. 2, pp. 202–208. DOI: 10.1023/B:JAPS.0000032875.04400.a0

22. Viktor E. Kisel, Sergey V. Kurilchik, Anatol S. Yasukevich, Sergey V. Grigoriev, Sofya A. Smirnova, and Nikolay V. Kuleshov. Spectroscopy and femtosecond laser performance of Yb$^{3+}$:YAlO$_3$ crystal. *Opt. Lett.*, 2008, vol. 33, pp. 2194–2196. DOI: 10.1364/OL.33.002194