Measuring Ship Collision Risk in a Dense Traffic Environment

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ABSTRACT: Collision risk measurement is an essential topic for ship collision prevention. Many risk measures, i.e. DCPA/TCPA, etc., decouple the ship traffic into several pairs of ships and then evaluate the risk in each pair. This kind of measurement loses some information of the entire traffic and might include some biases in risk measurement, especially in multiple-ship scenarios. In this article, Imminent Collision Risk Assessment (ICRA) is extended, which formulates collision risk as a ratio of reachable maneuvers leading to a collision and all reachable maneuvers (velocities). Two groups of scenarios have been simulated to show the ICRA is suitable for assessing the collision risk in multiple-ship scenarios. Moreover, two improvements have been introduced: (1) a generalized velocity obstacle algorithm is introduced to collect the maneuvers leading to collisions, which considers ship dynamics; (2) the constraints of forces are considered in the formulation of reachable maneuvers. As a result, the proposed measurement helps one ship assess the risk of approaching obstacles which are difficult to avoid the collision in terms of own-ship’s dynamics and kinetic constraints.

1 INTRODUCTION

Risk metrics are important for collision prevention at sea. When one ship encounters with obstacles, the Officer On Watch (OOW) needs to appraise the dangerous levels of these obstacles for decision making, e.g. continue with current operations or take new actions. The importance of risk metrics is also stipulated in international regulations for prevention collision at sea (COLREGs), which requests the OOWs to “make a full appraisal of the situation and of the risk of collision” (Organization), 1972). Hence, various collision risk metrics have been developed and proposed in past decades and these metrics have become the core of various collision alert systems (Goerlandt, Montewka, Kuzmin, & Kujala, 2015) and automatic collision avoidance systems (Johansen, Perez, & Cristofaro, 2016).

Most of the collision risk metrics usually choose a pair of ships from traffic to evaluate the risk. In each pair, the ship under our control is usually called own ship (OS) and the other is target ship (TS). By choosing different TSs, different pairs of ships are obtained and the collision risk in each pair is evaluated. That means the ship out of the pair is temporarily ignored. Researchers use numerous indicators to calculate the collision risk which is also named as collision risk index (CRI). In these indicators, two frequently used ones are Distance to Closest Point of Approach (DCPA) and Time to Closest Point of Approach (TCPA). By this approach, the OOWs can calculate the CRI of each TS, find the TS in conflict with the OS (whose CRI over the threshold), and identify the most dangerous TS.

This group of methods, however, have difficulties in showing the entire risk level of the traffic for the OS
when the OS encounters with multiple ships. In
technique level, there are no agreements on
combining various CRIs into one number which
represents the risk of the entire traffic. There are some
alternatives, such as average, sum, and maximum,
while they more or less have some drawbacks. The
“average” underestimates the most dangerous TS; the
“maximum” ignores the influence from non-
conflicting TSs; the “sum” offers limited information
about collision event in each pair.

Furthermore, when we decouple the traffic in
several pairs of ships, we lose some information about
traffic and introduce some biases in collision risk
assessment. The biases of risk are caused by two
aspects: (1) the risk caused by a non-conflicting TS is
ignored. Although a non-conflicting TS does not
directly have a conflict with the OS, it might block
some operations of the OS which might result in an
inevitable encounter between the OS and another TS;
(2) the risk caused by traffic characteristics is ignored.
For example, the CRI value in each pair of ships are
the same, but well-organized traffic seems to be safer
than others (see Section 3.2 for details).

This paper offers a new perspective to evaluate the
collision risk for the OS, which considers all the target
ships together. The risk measurement presented in
literature (Y. Huang, Gelder, & Mendel, 2016) and (Y.
Huang & van Gelder, 2019) is applied to multiple-
ship cases, which is named as Immediate Collision
Risk Assessment (ICRA) in this paper. Moreover,
based on the original ICRA, the ship’s dynamics and
constraints on forces are considered. The structure of
this paper is as follows: the background and gaps of
existing collision risk assessment are presented in this
section; the details of ICRA and the improved ICRA
are shown in Section 2; Section 3 collects three groups
of scenarios which show the performance of ICRA
and improved ICRA; at the end, discussion and
conclusion are presented in Section 4 and Section 5,
respectively.

2 COLLISION RISK MEASUREMENT

2.1 Immediate Collision Risk Assessment Method

Immediate collision risk assessment (ICRA) (Y.
Huang et al., 2016) measures the collision risk with
the aid of “room-for-maneuver” (Degre & Lefevre,
1981). In (Y. Huang & van Gelder, 2019), this concept
is further developed. The construction of the ICRA
follows some steps: firstly, the encounter scenarios
have been projected from geography space into
velocity space (V-space) and the velocities leading to
collision are collected in Velocity Obstacle (VO) set;
secondly, a set of velocities that one ship can achieve
is denoted as reachable velocity (RV) set; lastly, the
overlap of VO set and RV set are the reachable
velocity leading to collision and the collision risk is
measured by the percentage of the overlap, i.e.:

\[
ICRA = \frac{S(VO \cap RV)}{S(RV)},
\]

where \( S(\bullet) \) represents an operation that calculates
the area of the inputted polygon, e.g. \( S(RV) \) is the
area of RV set; \( VO \cap RV \) represents the overlap of
VO set and RV set, as shown in Fig. 1.

The formulation of VO set is relying on velocity
obstacle algorithm. In some maritime studies, this
algorithm is also named as Collision Threat Parameter
Area (CTPA) (Lenart, 1983; Szlapczynski & Krata,
2018) or Collision Danger Sector (CDS) (Pedersen,
Inoue, & Tsugane, 2003). Readers who are interested
in this algorithm can read more in the literature
(Fiorini & Shiller, 1998) and its applications in
maritime studies can be found in (Y. M. Huang, van
Gelder, & Wen, 2018) and (P. Chen, Huang, Mou, &
van Gelder, 2018). The construction of RV set is
related with time to collision and maneuverability of
the ship. For the sake of simplification, some
researchers use constant maximal speed and
instantaneous heading changes to construct the RV set
(Westrenen & Ellerbroek, 2017).

![Figure 1 The illustration of ICRA measurement](image)

In this paper, we employed the VO algorithm
proposed in the literature (Fiorini & Shiller, 1998),
in which the target-ship is assumed to keep the constant
speed and course. The RV set is simplified as the half
of the whole V-space of the OS, i.e. velocity in surge
direction accepts \( v_s \in [0, v_{max}] \) and velocity in sway
direction is \( v_c \in [-v_{max}, v_{max}] \). One example is shown in
Fig. 3 (2).

2.2 Improved ICRA

In the previous section, the constructions of VO set
and RV set accept some simplifications. Specifically,
the ship’s dynamics is ignored and the RV set is
simply equal to the half of V-space. However, these
simplifications influence the performance of I-ICRA
measurement in close range. For example, the in close
range, the many collision-free solutions, i.e.
\( v \in VO \cap RV \), are not reachable regarding ship’s
dynamics.

To solve this problem, we use generalized velocity
obstacle (GVO) to collect the velocity set leading to
collisions, which was proposed in the literature
(Bareiss & van den Berg, 2015) and applied to ship
collision avoidance in the literature (Y. M. Huang,
Chen, & van Gelder, 2019).

2.2.1 The motion model of the ship using velocity as the
input

The ship dynamics model used in this paper is
from literature (Fossen, 2002). \( \mathbf{x} \) and \( \mathbf{u} \) denote
states of the ship and inputted forces. \( \mathbf{x} \) consists of
the position of the ship, heading, surge speed, sway
speed and yaw rate, i.e. \( [x, y, y', u, v, r]^T \). The inputted forces will modify the states of the ship, see Fig. 2 (1).

\[
\tau = K_p \left( u^* - Vx \right) - K_d \dot{V}x ,
\]

(2)

here, \( K_p \) and \( K_d \) are feedback matrices; \( u^* \) is desired velocity contains the desired surge speed, sway speed, and heading. In return, we have a new motion model:

\[
\begin{bmatrix}
I & 0 \\
0 & M
\end{bmatrix} + BK_d V \dot{x} = \begin{bmatrix}
R(\psi) \\
-C(\psi) & -D(\psi) & -V - K_d Vx
\end{bmatrix} + BK_d u^* ,
\]

(3)

here, \( v = [u, v, r]^T \) is velocity states; \( C(\psi), D(\psi), M \), and \( R(\psi) \) are Coriolis, damping, mass, and rotation matrices, respectively; \( I \) is a 3-by-3 identical matrix and \( B = \begin{bmatrix} 0^{3 \times 3}, I^{3 \times 3} \end{bmatrix} \). This equation can also be rewritten in a general form:

\[
\ddot{x} = f(x, u^*) = f_1(x, K_p, K_d) + f_2(x, K_p, K_d) u^* ,
\]

(4)

where \( f(\bullet) \) is nonlinear.

2.2.2 The desired velocity leading to a collision

Firstly, we define the collision event at time \( t \) as an event that one ship violates a minimum safety region of the other ship at time \( t \), which is formulated as:

\[
P_i(t) = P_i(t) \oplus \text{ConfP} ,
\]

(5)

\( P_i \) and \( P \) are the position of the OS and the TS; \( \text{ConfP} \) is the minimum safety region; \( P_i(t) \oplus \text{ConfP} \) is a set of safety region regarding the target ship.

Secondly, we formulate the relation between \( P_i \) and \( u^* \) in the help of a linearization of equation (4) around its initial state \( x^0 \) and initial input \( u^0 \):

\[
x_i(t) \approx \ddot{x}_i(t) + G(t) \left( u^* - u^0 \right) ,
\]

(6)

where \( G \) is a response matrix; \( \ddot{x}_i(t) \) is the trajectory of the OS given the initial state and inputs, which is calculated via Runge-Kutta Integration. Since we only need the position of the OS, we introduce a matrix \( C \), which:

\[
P_i(t) = CX_i(t) = CX_i(t) + CG(t) \left( u^* - u^0 \right)
\]

\[
= \ddot{P}_i(t) + CG(t) \left( u^* - u^0 \right)
\]

(7)

here, \( \ddot{P}_i(t) \) is the estimated trajectory of the OS with initial input \( u^0 \).

Thirdly, we substitute Equation (7) to Equation (5) and formulate the changes on inputs leading to collision:

\[
\left( u^* - u^0 \right) \in (CG)^{-1} \left( \left[ P_i(t) - \ddot{P}_i(t) \right] \oplus \text{ConfP} \right) = sUO(t) .
\]

(8)

This set only collects \( u^* \) resulting in a collision at time \( t \). Thus, if we sum the \( sUO(t) \) that \( \forall t \in (0, \infty) \), we obtain all \( u^* \) leading to a collision in the future, which is named as UO set.

Let say, the force in each direction is satisfying constraints:

\[
\tau_n \leq \tau \leq \tau_{ab} .
\]

(9)

Then, we can formulate the forces as a function of the states and the desired velocity according to Equation (3) and Equation (2), i.e.:

\[
\tau = \left( K_p - K_d V f_2 \right) u^* - \left( K_p V x - K_d V f_1 \right) .
\]

(10)

Combining Equation (9) and (10), we derive the constraints on the desired velocity \( u^* \):

\[
(K_p - K_d V f_2)^{-1} (\tau_n + K_p V x - K_d V f_1) \leq u^* \leq (K_p - K_d V f_2)^{-1} (\tau_n + K_p V x - K_d V f_1) .
\]

(11)

Equation (11) is the reachable velocity set satisfying the constraints on forces given a PD controller.

3 CASE STUDIES

Three groups of scenarios have been designed in this section. The performance of ICRA in multiple-ship encounters and different traffic modes are presented in Section 3.1 and Section 3.2, respectively. In Section 3.3, a demonstration of I-ICRA considering ship dynamics and constraints is shown.
3.1 Performance of ICRA in multiple-ship scenarios

Three encounter scenarios have been simulated to show the performance of the ICRA. In each scenario, the own-ship is placed at the origin heading to the North with speed at 10 knots, while the number of target ships is increasing from one to three. In the first scenario, the OS only encounters with one target-ship (TS1) whose DCPA is 0.5 [NM] and TCPA is 0.25 [h]. In the second scenario, one extra target-ship (TS2) is introduced and the DCPA and TCPA remain the same as that of the TS1. In the last scenario, the OS encounters with three target ships together, namely TS1, TS2, and TS3. The details of the settings are presented in Table 1.

Table 1. Settings of scenario

| Ship   | Position [NM] | Heading [deg] | Speed [knot] | DCPA [NM] | TCPA [h] |
|--------|---------------|---------------|--------------|-----------|----------|
| Own    | (0,0)         | 000           | 10.0         | -         | -        |
| TS1    | (0.65,1.44)   | 358           | 16.0         | 0.5       | 0.25     |
| TS2    | (-2.45,1.80)  | 081           | 8.5          | 0.5       | 0.25     |
| TS3    | (3.38,3.02)   | 268           | 14.7         | 0.5       | 0.25     |

In a two-ship scenario, the TS1 approaches the OS from its stern and the ship blocks the starboard-turn options of the OS, as shown in Fig. 3 (2). The blue area is the VO set which collects the velocity of the OS leading to a collision with the TS1. The rest of the area is collision-free for the OS, which also can be interpreted as the “room-for-maneuver”. According to Section 2.1, the percentage of the VO shows the danger level of the OS which is 0.749. That means, the ship still has 0.251 chance to avoid the collision.

3.2 Well-organized traffic versus chaotic traffic

In this section, the influence of ship traffic on the measurement of collision risk is shown. Three scenarios are simulated, in which three target ships are involved, namely TS1, TS2, and TS3. The same ship in different scenarios has the same DCPA, TCPA, and relative distance, while the position and velocity are slightly different. For example, the TS1 in each scenario has different positions and speeds, but the same settings of DCPA, TCPA, and relative distance.

In the first scenario, three ships are grouped as a vessel train (L. Y. Chen, Hopman, & Negenborn, 2018), specifically these vessels have the same velocity and keep the formation in purpose. In the second and the third scenarios, each target ship keeps its relative distance to the OS in the first scenario, but the bearings of each target ship are changed. In the second scenario, the bearing of the target ship is changed in a small angle, say an arbitrary angle smaller than 60 degrees (See Fig. 4); in the last scenario, the changing range is enlarged to 240 degrees. An illustration is shown in Fig. 4. The TS3 is located at Point A in the first scenario, while the TS3 is randomly located on the arc BC and DE in the second and third scenario respectively. The details of the settings are shown in Table 2.

When we introduce one more ship (TS2) whose DCPA and TCPA are the same as the that of TS1, the entire collision risk is undefined by traditional methods (CRI methods), especially the new ship has the same CRI with the TS1. As we can expect that more ships in the same area might increase the collision risk, but how do the new CRI influence the original CRI is unclear. ICRA offers a solution to this problem. One more ship blocks some extra “room-for-maneuver” which leads to less chance to avoid collision dangers, as shown in Fig. 3 (2). As a result, the encounter scenario would be more dangerous than the previous scenario. As we show, the ICRA, in this case, raises from 0.749 to 0.925, which means the number of solutions for the OS to avoid collision decrease and the OS is more dangerous than the previous case. When the OS encounters with three ships together, the area of “room-for-maneuver” is shrunk furthermore. When the whole velocity space is occupied by the VO sets, that means, the collision is inevitable in the future and the ICRA reaches 1.
Table 2. Settings of scenarios

| Case | Ship | Position [NM] | Heading [deg] | Speed [knot] | DCPA [NM] | TCPA [NM][h] |
|------|------|---------------|---------------|--------------|------------|--------------|
| Own Ship | (0.0) | 000 | 10.0 | - | - | |
| 1 | TS1 | (-0.29,4.99) | 174 | 10.0 | 0.05 | 0.25 |
| 2 | TS2 | (-0.19,4.00) | 174 | 10.0 | 0 | 0.20 |
| 3 | TS3 | (-0.10,3.00) | 174 | 10.0 | 0.05 | 0.15 |
| 2 | TS1 | (-1.23,4.84) | 153 | 10.0 | 0.05 | 0.25 |
| 3 | TS2 | (0.14,4.00) | 184 | 10.0 | 0 | 0.20 |
| 3 | TS3 | (-0.46,2.97) | 161 | 10.0 | 0.05 | 0.15 |

Fig. 5. The V-Space of the OS when it encounters with three ships in three cases, namely well-organized case, disorder case, and chaotic case. (the DCPA, TCPA and relative distance of one ship, e.g. TS1 in case 1, are as the same as the ship in other cases.)

Table 3 Settings of scenario

| Case | Ship | Position [NM] | Heading [deg] | Speed [knot] | DCPA [NM] | TCPA [NM][h] |
|------|------|---------------|---------------|--------------|------------|--------------|
| Own Ship | (0.0) | 000 | 10.0 | - | - | |
| TS1 | (4.24,4.24) | 248 | 18.4 | 0 | 0.25 |

Following the methods presented in Section 2, we generate VO set and UO set of the TS1 and present in Fig. 6 (2) and (3). In Fig. 6 (2), we ignore the dynamic model of the OS and assume the OS can change its velocity immediately. In return, we calculate the I-ICRA = 0.38, which means the ship has more than half chance to avoid a collision and the encounter scenario, which is not such urgent. However, if we consider the ship’s dynamics and constraints (e.g. the maximal forces and moment), the I-ICRA rises to 0.63. That is because most of the collision-free solutions in Fig. 6 (2) are not reachable given constraints and the PD controller. In the UO set, the shadow area represents the velocity is reachable for the OS but leading to collision; the white region is the reachable and collision-free solution to the ship.
4 DISCUSSION

4.1 Comparing between ICRA and other CRI methods (based on CPA)

Three difference between CRI methods and ICRA are identified in this paper. Firstly, ICRA defines the collision risk as the chance of avoiding a collision which considers the ability of the OS to avoid a collision, while most of CRI methods ignore this part. Secondly, ICRA maps all the obstacles into resolution space (i.e., V-space) together and then measures the collision risk, whereas the CRI methods decouple the traffic first and then assess the risk in each pair of ships. Thirdly, the construction of ICRA is relatively independent of the experts’ judgment.

4.1.1 ICRA considers the ability of the OS to avoid a collision

Fig. 7 shows the bow-tie model of the OS encounters with three target-ships (TS1, TS2, and TS3). The collision between TS1 and the OS means the path (say Path 1) between TS1 and the top event (or “Collision”) is connected.

The CRIs indicate the connectivity of these paths. If one TS’s risk index exceeds the threshold, the path between this TS and the OS is connected, which implies this TS is dangerous for the OS. Then, a collision alert is triggered. However, this approach ignores the “barriers” on the path which can block these paths from TSs to the top event. Here, the barriers can be interpreted as maneuvers. Before collisions happen, the OS is capable to take all kinds of maneuvers to block the paths, i.e. avoid the collision. In Section 3.2, we show that even the upcoming ship (e.g. TS3) has the same CRI, the room-for-maneuver of the OS to avoid collision is different. In the first scenario, the OS can take either port turns or starboard turns to avoid the collision, while in the last scenario, the OS can avoid the collision if and only if the ship chooses a hard port-turn. Thus, the last scenario should be more dangerous than in the first scenario. The CRI methods only consider the dangers level of “threats” (the approaching ships) but ignore the chance of the OS to avoid the “threats”. Therefore, CRI methods cannot distinguish the difficulty of the OS tackling these threats.

ICRA is designed to consider the ability of the OS’s maneuverability to prevent a collision. In returns, ICRA not only measures the danger levels of the threats (TSs) but also present the ability of the OS blocks the paths. When the ICRA exceeds the thresholds, it basically tells the officer on board that the coming threats (TSs) are not only dangerous but also difficult to find a solution to prevent collisions.

4.1.2 ICRA measures the collision risk of traffic as a whole

The ICRA measures the collision risk as a whole, which can prevent two drawbacks of CRI methods in multiple-ship cases.

The decoupling technique loses some information about the traffic, which results in biases in collision risk assessment. As Section 3.2 shown, a well-organized scenario is less dangerous than the traffic in chaos, even the risk index of each pair in these scenarios remains the same. The CRI cannot show these differences to the OOW, but the ICRA could.

Inconvenience in finding conflict resolutions. CRI only shows the risk in pairs and ignore the impacts of other ships. Thus, when we find one solution reducing the risk in one pair of ships, we cannot guarantee this solution can also reduce the risk in another pair. In some worse cases, this solution might create some new conflicts. Thus, the OOWs need to try and test the solutions in each pair of ships, until they can find the one which reduces all the conflict in each pair of ships. Conversely, the ICRA measures the collision risk as a whole and it can directly identify collision-free solutions to the OOWs. The target-ship who is temporarily not in conflict is also considered in the ICRA. As a result, the solutions identified by the ICRA method can solve all the conflicts and would not create a new conflict.

4.1.3 Independent from the experts’ judgment

The setting and meaning of ICRA are independent of the OOW and experts, while the construction of CRI methods strongly relies on expert’s knowledge. Moreover, there is a lack of general agreements on the settings of CRI (Goerlandt et al., 2015). That means the
same scenario might have different CRIs and different conclusions when the different experts are involved. On the other hand, the construction of the VO set relies on the obtained traffic data and the RV set depends on the maneuverability of the ship, which is relatively independent of experts. Additionally, the meaning of ICRA is also clear. When the value of ICRA reaches 1, then the OS is inevitable collide with obstacles, even the collision has not happened yet. When ICRA is 0.5, that means if the OS chooses the solutions randomly, the ship still has 50% to collide with other ships.

4.2 Potential applications

The ICRA offers a new perspective to measure collision risk, which can rich the tools for risk-informed decision making on board. In literature (Goerlandt et al., 2015), researchers proposed a framework for risk-informed collision alert, which helps share situational awareness between experts and OOWs. Some widely used indicators are listed, but few indicators reflect the ability of the ship to avoid a collision and consider the entire traffic. ICRA can be used as one indicator in this framework which offers some information about the difficulty of the OS ship avoiding collision with the entire traffic.

ICRA also can be used in risk-based decision making, e.g. collision avoidance. ICRA consists of VO set and RV set which can help the OOWs to eliminate the solutions leading to collisions and find the collision-free solutions to all the encountering ships.

5 CONCLUSION

In this paper, Immediate Collision Risk Assessment (ICRA) is proposed to measure collision risk in a dense traffic environment, i.e. multiple-ship scenarios. The collision risk is measured by the percentage of the maneuvers (velocities) leading to a collision. To tackle the dynamics of ship and constraints on forces, an improved-ICRA is proposed, where the generalized velocity obstacle (GVO) algorithm is applied.

Three groups of scenarios have presented. The first group of scenarios shows the performance of ICRA when the number of Target Ship (TS) is increasing; the second group of scenarios shows the proposed ICRA is enabled to measure the collision risk in different traffic modes, specifically well-organized traffic case and chaotic traffic case. These two cases show ICRA is suitable to use in multiple-ship scenarios. The last scenario demonstrates the improved-ICRA that considers ship dynamics and force constraints. It shows that the collision risk is underestimated when we ignore the ship dynamics and constraints on forces.

Three features of ICRA have been identified in this paper: (1) it measures the collision risk considering the ability of the Own Ship (OS) to avoid dangers; (2) it measures collision risk of the entire traffic instead of decoupling the traffic, which is more suitable in multiple-ship scenarios; (3) the measurement is independent from experts’ opinions. We believe that the proposed ICRA offers a new perspective in collision risk measurement, which not only enriches the choices in the developments of risk-informed collision alert systems but also can support the risk-based collision avoidance in multiple-ship scenarios.

Future research will consider the following directions. Firstly, the influence of regulations, e.g. COLREGs, will be included. If the OS complies with regulations, the size of RV set will be modified and then the measured risk is changed, e.g. (Y. Huang & van Gelder, 2019). Secondly, the environmental disturbance would be considered to support collision avoidance in different environmental conditions. Thirdly, the potentials of using ICRA on board ship and in vessel traffic service center in various scenarios need more studies.

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