We describe the properties of complex nuclei, such as the Sn isotopes with mass numbers $A = 100 - 132$, in terms of the free nucleon–nucleon interaction as obtained from meson–exchange theory. This amounts to first calculating an effective interaction in which the free interaction is modified by the presence of the appropriate nuclear medium. The short–range correlations are included within the framework of Brueckner theory yielding the nuclear reaction matrix and the long–range correlations by using the reaction matrix in many–body perturbation theory to obtain an effective interaction. The resulting effective interaction is then employed in calculating the nuclear properties. Particular emphasis is placed on the ability of our calculation to describe systematic trends of the properties of these nuclei. Both successful achievements and problematic features are pointed out.

1 Introduction

One of the fundamental, yet unsolved problems of nuclear theory is to describe the properties of complex nuclei in terms of their constituent particles and the interaction among them. There are two major obstacles to the solution of this problem. Firstly, we are dealing with a quantal many–body problem which cannot be solved exactly. Secondly, the basic nucleon–nucleon (NN) interaction is not well known. Thus, it may be difficult to know whether an eventual failure to solve this problem should be ascribed to the many–body methods or the interaction model used. On the other hand, these two uncertainties are intimately connected. In any model chosen to approximate the original many–body problem one has to apply an interaction which is consistent with the particular degrees of freedom considered. This amounts to correcting the original interaction for the degrees of freedom not explicitly included in the many–body treatment, thus yielding a so–called effective interaction.

In principle, one should start from an NN interaction derived from the interaction between quarks. Although attempted, this program has not been quantitatively successful. Thus, one has to be content with using an NN interaction derived from meson–exchange models which reproduce the relevant
two–nucleon data. Such interactions are generally termed realistic interactions. Examples are the Paris, Bonn and Nijmegen potentials.

Once the basic NN interaction has been established, it should be employed in a quantal many–body approximation to the nuclear structure problem of interest. One such approximation is the spherical shell model, which has provided a successful microscopic approach for nuclei near closed shell. Away from closed shells the number of valence particles and single–particle orbits quickly becomes too large for the shell model to handle. On the other hand, there has been enormous progress in computer technology over the past years and this trend is likely to continue. It is therefore a challenge to use modern hardware computer technology coupled to efficient numerical methods developed in other fields of science and technology to handle complex nuclear structure problems.

This would indeed allow us to test the theory of realistic effective interactions in nuclei with many valence nucleons. Hitherto, realistic nuclear forces have mainly been applied to nuclei with two or a few valence particles beyond closed shells, such as the oxygen and calcium isotopes. Thus, by going to the tin isotopes, in which the major neutron shell between neutron numbers 50 and 82 is being filled beyond the $^{100}$Sn closed shell core, we have the opportunity of testing the potential of large–scale shell model calculations as well as the reliability of realistic effective interactions in systems with many valence particles. It should be noted that in many current shell model calculations the effective interaction is frequently either parametrized or adjusted in order to optimize the fit to the data. As a matter of principle we shall refrain from making any such adjustments and stick to the interaction obtained by a rigorous calculation consistent with the many–body scheme chosen. Only then will one be able to assess the quality and reliability of the interaction obtained and the possible needs for improvement. One limitation of the present work is that we only consider effective two–body forces. In systems with many valence particles one should in principle also include effective three– and many–body forces. This will however have to be deferred to future work.

In addition to being an exploratory calculation testing the shell model and realistic effective interactions over a wide range of Sn isotopes, the present work is also a calculation in its own right. Both the low–mass, very neutron–deficient and the heavy–mass, neutron rich Sn isotopes are very unstable and have only recently been identified and become accessible to spectroscopic studies. Thus, they represent a challenge to theoretical work as well.
The paper is organized as follows. In Sect. 2 we give a brief summary of the calculation of the effective interaction. Then, in Sect. 3 we discuss our shell model algorithm. The results are presented in Sect. 4 and concluding remarks in Sect. 5.

2 Calculation of the shellmodel effective interaction

The aim of microscopic nuclear structure calculations is to derive various properties of finite nuclei from the underlying Hamiltonian describing the interaction between nucleons. When dealing with nuclei, such as the tin isotopes with \( A \approx 100 \), the full dimensionality of the many–body Schrödinger equation for an \( A \)--nucleon system

\[
H \Psi_i(1, \ldots, A) = E_i \Psi_i(1, \ldots, A),
\]

becomes intractable and one has to seek viable approximations to Eq. (1). In Eq. (1), \( E_i \) and \( \Psi_i \) are the eigenvalues and eigenfunctions for a state \( i \) in the Hilbert space.

In nuclear structure calculations, one is normally only interested in solving Eq. (1) for certain low–lying states. It is then customary to divide the Hilbert space into a model space defined by the operator \( \mathcal{P} \) and an excluded space defined by the operator \( \mathcal{Q} \)

\[
P = \sum_{i=1}^{d} |\psi_i\rangle \langle \psi_i| \quad \mathcal{Q} = \sum_{i=d+1}^{\infty} |\psi_i\rangle \langle \psi_i|,
\]

with \( d \) being the size of the model space and such that \( \mathcal{P} \mathcal{Q} = 0 \). The assumption then is that the components of these low–lying states can be fairly well reproduced by configurations consisting of a few particles/holes occupying physically selected orbits. These selected orbitals define the model space.

Eq. (1) can then be rewritten as a secular equation

\[
PH_{\text{eff}} P \Psi_i = P(H_0 + V_{\text{eff}}) P \Psi_i = E_i P \Psi_i,
\]

where \( H_{\text{eff}} \) now is an effective Hamiltonian acting solely within the chosen model space. The term \( H_0 \) is the unperturbed Hamiltonian while the effective interaction is given by

\[
V_{\text{eff}} = \sum_{i=1}^{\infty} V_{\text{eff}}^{(i)},
\]

with \( V_{\text{eff}}^{(1)} , V_{\text{eff}}^{(2)} , V_{\text{eff}}^{(3)} , \ldots \) being effective one–body, two–body, three–body interactions etc. It is also customary in nuclear shell model calculations to add the
one–body effective interaction $V_{\text{eff}}^{(1)}$ to the unperturbed part of the Hamiltonian so that

$$H_{\text{eff}} = \tilde{H}_0 + V_{\text{eff}}^{(2)} + V_{\text{eff}}^{(3)} + \ldots,$$

where $\tilde{H}_0 = H_0 + V_{\text{eff}}^{(1)}$. This allows us, as in the shell model, to replace the eigenvalues of $\tilde{H}_0$ by the empirical single–particle energies for the nucleon orbitals of our model space, or valence space, e.g., $2s_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $0g_{7/2}$ and $0h_{11/2}$ for Sn isotopes under consideration. Thus, the remaining quantity to calculate is the two- or more–body effective interaction $\sum_{i=2}^{\infty} V_{\text{eff}}^{(i)}$. In this work we will restrict our attention to the derivation of an effective two–body interaction

$$V_{\text{eff}} = V_{\text{eff}}^{(2)},$$

using the many–body methods discussed in Ref. and reviewed below. The study of effective three–body forces will be deferred to a later work.

Our scheme to obtain an effective two–body interaction for the tin isotopes starts with a free nucleon–nucleon interaction $V$ which is appropriate for nuclear physics at low and intermediate energies. At present, there are several potentials available. The most recent versions of Machleidt and co–workers, the Nimjegen group, and the Argonne group have a $\chi^2$ per datum close to 1. In this work we will thus choose to work with the charge–dependent version of the Bonn potential models, see Ref. The potential model of Ref. is an extension of the one–boson–exchange models of the Bonn group, where mesons like $\pi$, $\rho$, $\eta$, $\delta$, $\omega$ and the fictitious $\sigma$ meson are included. In the charge–dependent version of Ref., the first five mesons have the same set of parameters for all partial waves, whereas the parameters of the $\sigma$ meson are allowed to vary.

The next step in our perturbative many–body scheme is to handle the fact that the repulsive core of the nucleon–nucleon potential $V$ is unsuitable for perturbative approaches. This problem is overcome by introducing the reaction matrix $G$ given by the solution of the Bethe–Goldstone equation

$$G = V + V \frac{Q}{\omega - \tilde{H}_0} G,$$

where $\omega$ is the unperturbed energy of the interacting nucleons, and $\tilde{H}_0$ is the unperturbed Hamiltonian. The operator $Q$, commonly referred to as the Pauli operator, is a projection operator which prevents the interacting nucleons from scattering into states occupied by other nucleons. In diagrammatic language the $G$–matrix is the sum over all ladder type of diagrams. This sum is meant to renormalize the repulsive short–range part of the interaction. The physical
interpretation is that the particles must interact with each other an infinite number of times in order to produce a finite interaction.

Since the perturbative interaction is $V - U$ rather than $V$, $U$ being an auxiliary one–body potential incorporated in $H_0$ along with the kinetic energy $T$, it is convenient to include insertions of $U$ to arbitrary order in the intermediate states of $G$. This can be done by redefining $G$ as

$$G = V + V \frac{Q}{\omega - QTQ} G.$$

It is further convenient to calculate $G$ using the double–partitioning scheme discussed in e.g., Ref. 1. A harmonic–oscillator basis was chosen for the single–particle wave functions, with an oscillator energy $\hbar \Omega$ given by $\hbar \Omega = 45A^{-1/3} - 25A^{-2/3} = 7.87$ MeV, $A$ being the mass number.

Finally, we briefly sketch how to calculate an effective two–body interaction for the chosen model space in terms of the $G$–matrix. Since the $G$–matrix represents just the summation to all orders of particle–particle ladder diagrams, there are obviously other terms which need to be included in an effective interaction. Long–range effects represented by core–polarization terms are also needed. The first step then is to define the so–called $\hat{Q}$–box given by

$$P\hat{Q}P = PGP + P \left( \frac{Q}{\omega - H_0} G + \frac{Q}{\omega - H_0} G \frac{Q}{\omega - H_0} G + \ldots \right) P. \quad (8)$$

The $\hat{Q}$–box is made up of non–folded diagrams which are irreducible and valence linked. A diagram is said to be irreducible if between each pair of vertices there is at least one hole state or a particle state outside the model space. In a valence–linked diagram the interactions are linked (via fermion lines) to at least one valence line. Note that a valence–linked diagram can be either connected (consisting of a single piece) or disconnected. In the final expansion including folded diagrams as well, the disconnected diagrams are found to cancel out. This corresponds to the cancellation of unlinked diagrams of the Goldstone expansion. We illustrate these definitions by the diagram shown in Fig. 1, where an arrow pointed upwards(downwards) is a particle(hole) state. Diagram (a) is irreducible, valence linked and connected, while (b) is reducible since the intermediate particle states belong to the model space (particle states outside the model space would be denoted by railed lines). Diagram (c) is reducible, valence linked and disconnected.
Figure 1: Different types of valence-linked diagrams. Diagram (a) is irreducible and connected, (b) is reducible, while (c) is irreducible and disconnected.

We can then obtain an effective interaction $H_{\text{eff}} = \tilde{H}_0 + V^{(2)}_{\text{eff}}$ in terms of the $\hat{Q}$-box, using the folded-diagram expansion

$$V^{(2)}_{\text{eff}}(n) = \hat{Q} + \sum_{m=1}^{\infty} \frac{1}{m!} \frac{d^m \hat{Q}}{d \omega^m} \left\{ V^{(2)}_{\text{eff}}(n-1) \right\}^m,$$

where $(n)$ and $(n-1)$ refer to the effective interaction after $n$ and $n-1$ iterations. The zeroth iteration is represented by just the $\hat{Q}$-box. Observe also that the effective interaction $V^{(2)}_{\text{eff}}(n)$ is evaluated at a given model space energy $\omega$, as is the case for the $G$-matrix as well. Here we choose $\omega = -20$ MeV. Less than 10 iterations were needed in order to obtain a numerically stable result. Note that all non-folded diagrams through third-order in the interaction $G$ are included in the $\hat{Q}$-box. For further details, see Ref. 1.

Another iterative scheme which has been much favored in the literature is one proposed by Lee and Suzuki 8. However, contrary to the folded-diagram expansion of Eq. (9), which gives those states having the largest overlap with the model space states, the Lee–Suzuki method converges to the lowest eigenstates regardless of their overlaps with the model space. Thus, Eq. (9) seems more appropriate for shell model calculations than the Lee–Suzuki scheme.

3 The shell model scheme

The effective two-particle interaction can in turn be used in shell model calculations. Both binding energies and excitation spectra are severe tests of the method. Furthermore, it is of importance to analyze nuclear systems with large number of degrees of freedom. Thus in the present work we have chosen the
Sn region with $Z = 50$ and $50 < N < 82$ and limit ourselves to effective two–particle matrix elements with $T = 1$. Two types of calculation are performed:

(I). Effective two–particle matrix elements are calculated based on a $Z = 50, N = 50$ symmetric core and with the active $P$–space based on the single–particle orbits $2s_{1/2}, 1d_{5/2}, 1d_{3/2}, 0g_{7/2}$ and $0h_{11/2}$. The corresponding single–particle energies are not known experimentally. At present we choose $\varepsilon(d_{5/2}^+)$ = 0.00 MeV, $\varepsilon(g_{7/2}^+)$ = 0.08 MeV, $\varepsilon(s_{1/2}^+)$ = 2.45 MeV, $\varepsilon(d_{3/2}^+)$ = 2.55 MeV and $\varepsilon(h_{11/2}^+)$ = 3.20 MeV. These data are in reasonable agreement with similar shell model calculations in this region, see for example Ref. 10. However, the single–particle energies of $s_{1/2}$ and $d_{3/2}$ have been adjusted in order to reproduce the lowest $1/2^+$ and $3/2^+$ states in $^{111}$Sn, see the discussion in Ref. 11. The shell model calculation then amounts to studying valence neutrons outside this core.

(II). Effective two–hole matrix elements are calculated based on a $Z = 50, N = 82$ asymmetric core and with the active $P$–space for holes based on the $2s_{1/2}, 1d_{5/2}, 1d_{3/2}, 0g_{7/2}$ and $0h_{11/2}$ hole orbits. The corresponding single–hole energies $\varepsilon(d_{5/2}^-)$ = 0.00 MeV, $\varepsilon(h_{11/2}^-)$ = 0.242 MeV, $\varepsilon(s_{1/2}^-)$ = 0.332 MeV, $\varepsilon(d_{3/2}^-)$ = 1.655 MeV and $\varepsilon(g_{7/2}^-)$ = 2.434 MeV are taken from Ref. 12 and the shell model calculation amounts to studying valence neutron holes outside this core.

The shell model problem requires the solution of a real symmetric $n \times n$ matrix eigenvalue equation

$$\tilde{H} |\Psi_k\rangle = E_k |\Psi_k\rangle,$$

where for the present cases the dimension of the $P$–space reaches $n \approx 2 \times 10^7$. At present our basic approach in finding solutions to Eq. (10) is the Lanczos algorithm; an iterative method which gives the solution of the lowest eigenstates. This method was already applied to nuclear physics problems by Whitehead et al. in 1977. The technique is described in detail in Ref. 13, see also Ref. 14.

4 Results and discussions

The results of the shell model calculation are presented in Tables 1–4. Our main intention is to gain insight about the effective interaction in nuclear systems and see to what extent our calculated two–particle matrix elements
Table 1: Excitation spectra for the light Sn isotopes.

| Jπ  | Exp. Jπ | Theory | Jπ  | Exp. Jπ | Theory |
|-----|---------|--------|-----|---------|--------|
| (2⁺) | 1.47 2⁺ | 1.73 | (2⁺) | 1.26 2⁺ | 1.42 |
| (4⁺) | 1.97 4⁺ | 2.10 | (4⁺) | 1.94 4⁺ | 1.99 |
| (6⁺) | 6⁺ | 1.96 | (6⁺) | 2.26 6⁺ | 2.37 |

| Jπ  | Exp. Jπ | Theory | Jπ  | Exp. Jπ | Theory |
|-----|---------|--------|-----|---------|--------|
| (2⁺) | 1.21 2⁺ | 1.36 | (2⁺) | 1.21 2⁺ | 1.44 |
| (4⁺) | 2.02 4⁺ | 2.15 | (4⁺) | 2.11 4⁺ | 2.37 |
| (6⁺) | 2.32 6⁺ | 2.36 | (6⁺) | 2.37 6⁺ | 2.47 |

can reproduce the general features of the experimental data in the Sn region. All experimental information in the present analysis is taken from the database of the National Nuclear Data Center at Brookhaven.

The Sn isotopes relevant for the calculation covers the range from ¹⁰²Sn to ¹³⁰Sn. Isotopes below ¹¹⁶Sn (light Sn) are treated based on the symmetric Z = N = 50 core whereas the isotopes above ¹¹⁶Sn (heavy Sn) are treated based on the asymmetric Z = 50, N = 82 core. This simplifies the shell model calculation, but in addition it is of interest to see how successful a hole–hole effective interaction calculated with respect to ¹³²Sn is.

The results in Table 1 show excitation spectra for the light Sn isotopes. Only some selected states are displayed. First of all, the well-known near constant 0⁺ – 2⁺ spacing is well reproduced. However the spacing is 0.1 – 0.2 MeV too large which indicates that our effective interaction produces a little too much pairing correlation compared to the experimental data. We believe this is related to the interaction between the two dominant low–lying d₅/₂ and g₇/₂ orbits and the intruder orbit h¹₁/₂. This orbit is essential for the constant 0⁺ – 2⁺ spacing throughout the Sn isotopes due to its large degeneracy. Such intruder orbits are difficult to handle by our effective interaction methods and the results indicate that further investigation is necessary on this point. A more complete analysis of the excitation spectra all the way up to ¹¹⁶Sn is under preparation. Preliminary analysis shows similar good agreements as in Table 1.

The resulting excitation spectra for the heavy Sn isotopes are shown in Table 2. Again the near constant 0⁺ – 2⁺ spacing is well reproduced all the way down to ¹¹⁶Sn, even better than for the light Sn isotopes. Also the addi-
Table 2: Excitation spectra for the heavy Sn isotopes.

|       | \(^{130}\text{Sn}\) |       | \(^{128}\text{Sn}\) |
|-------|-------------------|-------|-------------------|
| \(J^\pi\) | Exp. | \(J^\pi\) | Theory | Exp. | \(J^\pi\) | Theory |
| \((2^+)^\) | 1.22 | 2\(^+\) | 1.46 | \((2^+)^\) | 1.17 | 2\(^+\) | 1.28 |
| \((4^+)^\) | 2.00 | 4\(^+\) | 2.39 | \((4^+)^\) | 2.00 | 4\(^+\) | 2.18 |
| \((6^+)^\) | 2.26 | 6\(^+\) | 2.64 | \((6^+)^\) | 2.38 | 6\(^+\) | 2.53 |

|       | \(^{126}\text{Sn}\) |       | \(^{124}\text{Sn}\) |
|-------|-------------------|-------|-------------------|
| \(J^\pi\) | Exp. | \(J^\pi\) | Theory | Exp. | \(J^\pi\) | Theory |
| \(2^+\) | 1.14 | 2\(^+\) | 1.21 | \(2^+\) | 1.13 | 2\(^+\) | 1.17 |
| \(4^+\) | 2.05 | 4\(^+\) | 2.21 | \(4^+\) | 2.10 | 4\(^+\) | 2.26 |
| \(6^+\) | 2.61 | 6\(^+\) | 2.64 | \(6^+\) | 2.70 | 6\(^+\) | 2.70 |

|       | \(^{122}\text{Sn}\) |       | \(^{120}\text{Sn}\) |
|-------|-------------------|-------|-------------------|
| \(J^\pi\) | Exp. | \(J^\pi\) | Theory | Exp. | \(J^\pi\) | Theory |
| \(2^+\) | 1.14 | 2\(^+\) | 1.15 | \(2^+\) | 1.17 | 2\(^+\) | 1.14 |
| \(4^+\) | 2.14 | 4\(^+\) | 2.30 | \(4^+\) | 2.19 | 4\(^+\) | 2.30 |
| \(6^+\) | 2.56 | 6\(^+\) | 2.78 | \(6^+\) | 2.86 | 6\(^+\) | 2.86 |

|       | \(^{118}\text{Sn}\) |       | \(^{116}\text{Sn}\) |
|-------|-------------------|-------|-------------------|
| \(J^\pi\) | Exp. | \(J^\pi\) | Theory | Exp. | \(J^\pi\) | Theory |
| \(2^+\) | 1.22 | 2\(^+\) | 1.15 | \(2^+\) | 1.30 | 2\(^+\) | 1.17 |

Additional calculated states are in very good agreement with experiment. However, more detailed analysis of the results close to \(^{116}\text{Sn}\) indicates that our effective two–particle interaction has difficulties in reproducing the shell closure which is believed to occur in this region. The increase of the \(0^+ – 2^+\) splitting is not as sharp as found experimentally, even if the phenomenon is rather weak in the case of Sn. We have observed a similar feature around \(^{48}\text{Ca}\) which is generally agreed to be a good closed shell nucleus. Here the deviation between theory and experiment is severe. Preliminary analysis indicates that our effective interaction may be slightly too attractive when the two particles occupy different single–particle orbits. This may be related to the radial wave functions which in our calculation are chosen to be harmonic oscillator functions.

The next set of data we have analysed is the relative binding energies. Table 3 shows the results for the light Sn isotopes. In this case data for \(^{100}\text{Sn}\) and \(^{101}\text{Sn}\) are not known experimentally so we have calculated binding energies relative to \(^{102}\text{Sn}\) by the formula...
Table 3: Binding energies for the light Sn isotopes relative to $^{102}$Sn. For the definition, see Eq.11.

|       | $^{104}$Sn | $^{106}$Sn | $^{108}$Sn | $^{110}$Sn | $^{112}$Sn | $^{114}$Sn |
|-------|------------|------------|------------|------------|------------|------------|
| Experiment | -2.45     | -4.19      | -4.55      | -4.16      | -2.77      | -0.45      |
| Shell Model | -2.17     | -3.99      | -6.27      | -6.53      | -6.28      |

$$BE_r^{[102+n Sn]} = BE^{[102+n Sn]} - BE^{[102 Sn]} - n \left( BE^{[103 Sn]} - BE^{[102 Sn]} \right).$$  \hspace{1cm} (11)

In case of the heavy Sn isotopes the necessary data are known and the values in Table 4 are calculated by the formula

$$BE_r^{[132-n Sn]} = BE^{[132-n Sn]} - BE^{[132 Sn]} - n \left( BE^{[131 Sn]} - BE^{[132 Sn]} \right).$$  \hspace{1cm} (12)

Table 4: Binding energies for the heavy Sn isotopes. For the definition, see Eq.12.

|       | $^{130}$Sn | $^{128}$Sn | $^{126}$Sn | $^{124}$Sn | $^{122}$Sn | $^{120}$Sn | $^{118}$Sn | $^{116}$Sn |
|-------|------------|------------|------------|------------|------------|------------|------------|------------|
| Experiment | -2.09     | -3.64      | -4.79      | -5.47      | -5.64      | -5.26      | -4.28      | -2.61      |
| Shell Model | -2.24     | -4.60      | -6.99      | -9.39      | -11.77     | -14.11     | -16.39     | -18.58     |
| Mod. Shell Model | -2.09     | -3.72      | -4.81      | -5.32      | -5.22      | -4.51      | -3.15      | -1.12      |

For the light Sn isotopes experimental relative binding energies show a parabola structure with a minimum around $^{108}$Sn. This is an effect of the Pauli principle and the limited number of the degrees of freedom in the $P$–space for the valence particles. Here the dominant orbits are $g_{7/2}$ and $d_{5/2}$ which should give a minimum around $^{108}$Sn and a shell closure around $^{116}$Sn. Theory produces more binding with a minimum around $^{110}$Sn, again indicating too much influence of the $h_{11/2}$ orbit.

A similar and even more dramatic result is seen in the calculation of the relative binding energies for the heavy Sn isotopes. Experiment indicates a minimum around $^{124}$Sn–$^{122}$Sn and consequently a shell closure around $^{116}$Sn.
whereas theoretical binding energies increases linearly all the way down to $^{116}\text{Sn}$. Thus the shell model calculation uses all $P$–space degrees of freedom to produce ground state binding energies in clear contradiction to experiment.

This phenomenon of overbinding of nuclear systems when effective interactions from meson theory are used have been much discussed in the literature, see for example Ref. [16]. The arguments are that such matrix elements must be modified in order to reproduce the binding energies correctly. The so–called centroid matrix elements should be modified in order to reproduce experiment. However, no well recipe for doing this is available.

In our case we have investigated the heavy Sn isotopes and defined a global centroid by

$$W = \frac{1}{\dim} \sum_{j_1 \geq j_2} \frac{\sum_J (2J + 1) < j_1, j_2 \mid J \mid V \mid j_1, j_2 \rangle >}{\sum_J (2J + 1)}$$  

(13)

where $\dim = 160$, the total number of matrix elements in the present calculation. In the calculation of $^{130}\text{Sn}$ our theoretical binding energy gave $-2.24 \text{ MeV}$ whereas experiment gives $-2.09 \text{ MeV}$. Thus we made a global monopolar correction $W \cdot (n(n-1))/2$ to all matrix elements and adjust $W$ to reproduce the correct binding energy of $-2.09 \text{ MeV}$. This gave $W = +0.15 \text{ MeV}$. The modified binding energies are displayed in Table 4, now in very good agreement with experiment. Such a modification of the matrix elements has no effect on the excitation spectra and preserves the good agreement in this part of the calculation. A similar modification is not possible in the light Sn isotopes since essential data for $^{100}–^{102}\text{Sn}$ is not available.

5 Conclusion

We have presented the basic elements for a calculation of a realistic microscopic effective interaction. The interaction is derived from a modern meson–exchange NN potential using many–body perturbation theory. This is applied to the Sn isotopes ranging from $A = 102$ to $A = 132$ where both excitation spectra and relative binding energies are calculated. Excitation spectra are in good agreement with experiment without any adjustment of parameters related to either single–particle energies or matrix elements. Relative binding energies are calculated and show clear deviations from experiments. This indicates problems related to the methods used in calculating effective interactions from meson theory. The BONN CD potential[15] which is used in the present calculation produces more binding than previous versions and may be part of the reason for overbinding in the calculation. Another possible explanation is
related to the radial wave functions which are taken to be harmonic oscillators. This question will be investigated further.

We have shown that a global monopole term added to all matrix elements can cure the difficulties with the binding energies, at least in the heavy Sn isotopes. Only a change of binding energy for two holes from the calculated $-2.24$ Mev to $-2.09$ is needed for good agreement with experiment. Thus, in spite of the drastic difference shown in Table 4 we believe that only minor improvements are necessary to give good effective interactions to be used in shell model calculations.

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