The use of MRT-lattice Boltzmann method for the prediction of fluid solid flow

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Abstract. Computation of fluid solid flow requires an accurate scheme to capture the moving boundary problems for solid objects moving in a fluid. In this paper, a multi-relaxation time lattice Boltzmann method is coupled with the Newton’s second law to predict the flow characteristics around obstacle placed in a channel. The treatment of the curved boundary is based on the conventional bounce-back boundary scheme and interpolations. Good agreement with the previous studies by other methods indicates good applicability of the present scheme.

1. Introduction
Lattice Boltzmann method (LBM) is a powerful numerical technique for simulating fluid flows and modelling the physics in fluids [1-3]. This method is as an extension of the lattice gas automata (LGA) or as a special discretisation from the Boltzmann equation of the kinetic theory [4-8]. In lattice Boltzmann simulations, collision and propagation routines are performed to model the flow density and momentum which can be determined via distribution functions in phase space. The resulting information then enables the fundamentals quantities such as the drag or lift forces to be computed. Overall, the LBM with single-relaxation Bhatnagar-Gross-Krook (BGK) model provides a convenient and straightforward approach for solving continuum flow problems describes by the Navier-Stokes equation at low Reynolds numbers. Recently, the LBM has been improved to its stability and has been successfully extended for the application of flows at high Reynolds [2,8].

A major advantage of lattice Boltzmann method is its ease and accurate treatment at complicated boundary geometries. The conventional bounce-back (BB) scheme provides a particularly straightforward approach for modeling no-slips conditions on solid surfaces in macroscopic flows. In this scheme, the outgoing directions of the distribution function at the boundary sites are simply specified as the reverse of the incoming directions. In practice, there are two basic types of BB scheme, namely the “on-site” scheme [9] and the “mid-plane” scheme [10]. It is well-known that the on-site BB scheme is simpler, but has only first-order accuracy, whereas the mid-plane BB scheme provides second-order accuracy in both space and time [11].

Curved boundary treatments conventionally used in LB simulation when the uniform lattice is mapped in space and has been suggested as a means of improving the accuracy of the stair-shaped approximation. Several strategies have been proposed for dealing with complex geometry, curved boundaries in LBM. The first approach is to use a body-fitted (arbitrary) mesh and to execute the distribution functions throughout the entire computational domain [12-13]. The second strategy also applies an interpolation-based approach, but under a uniform Cartesian mesh, to track the position of...
boundary. Then, the on-site or off-site BB scheme is executed at the boundary surface depending on the location of the boundary relative to the lattice nodes [14-15]. The latest one is based upon the utilization of the immersed boundary treatments [15-17]. Due to their characteristics of a superior numerical accuracy, an intuitive approach and an inherent reliability, the interpolation type models are the most commonly employed technique for resolving curved boundary problems in LB simulations. Therefore, in the present study, MRT-LBM will be coupled with the interpolation type of boundary model to predict the flow around obstacles.

2. Mathematical Modelling of MRT-LBM

In MRT-LBM model, the physical space is discretized into uniform lattice nodes. Every node in the network is connected with its neighbors through a number of lattice velocities that are to be determined through the selected model.

Recently, Lallemand and Luo [17] suggested that the use of a MRT model could improve the numerical stability. The collision step in velocity space is difficult to perform; it is more convenient to perform the collision process in the momentum space [2].

The multi-relaxation-time lattice Boltzmann equation reads

$$f_i(x + c_i \Delta t, t + \Delta t) - f_i(x, t) = -M^{-1}S [m(x, t) - m_\infty(x, t)]$$  \hspace{1cm} (1)

where is the distribution function for particles with velocities $c_i$ at time $t$. Velocity $x$ is equal to rate of change of position $\Delta x dt$ per unit time $dt$. $m(x, t)$ and $m_\infty(x, t)$ are vectors of moments, $m = (m_0, m_1, m_2, ..., m_n)^T$. The relaxation matrix $S$ is a diagonal matrix.

The mapping between velocity and moment spaces can be performed by linear transformation as follows

$$m = Mf$$  \hspace{1cm} (2)

$$f = M^{-1}m$$  \hspace{1cm} (3)

The matrix $M$ for two-dimension nine-velocity model (D2Q9) is:

$$M = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 & 4 \\
-2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\
0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 1 & 0 & -1 & 1 & -1 & 1 & 1 \\
0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1 \\
0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
\end{bmatrix}$$

The moment vector $m$ is:

$$m = \{\rho, e, e, j_x, j_y, q_x, q_y, P_{xx}, P_{yy}\}^T$$  \hspace{1cm} (4)
where, $\rho$ is the fluid density, $\varepsilon$ is related to the square of the energy $e$, $j_x$ and $j_y$ are the mass flux in two directions, and $P_{xx}$ and $P_{xy}$ correspond to the diagonal and off-diagonal component of the viscous stress tensor.

The equilibrium of the moment $m^{eq}$ is:

$$m_0^{eq} = \rho$$
(5a)

$$m_1^{eq} = -2\rho + 3(j_x^2 + j_y^2)$$
(5b)

$$m_2^{eq} = \rho - 3(j_x^2 + j_y^2)$$
(5c)

$$m_3^{eq} = j_x$$
(5d)

$$m_4^{eq} = -j_x$$
(5e)

$$m_5^{eq} = j_y$$
(5f)

$$m_6^{eq} = -j_y$$
(5g)

$$m_7^{eq} = (j_x^2 - j_y^2)$$
(5h)

$$m_8^{eq} = j_xj_y$$
(5i)

where

$$j_x = \rho u_x \quad \text{and} \quad j_y = \rho u_y$$
(6)

The diagonal matrix $S$ is

$$S = \begin{bmatrix}
s_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & s_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & s_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & s_3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & s_4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & s_5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & s_6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & s_7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & s_8
\end{bmatrix}$$

In compact notation $S$ can be written as:
\[ S = \text{diag}(1.0, 1.4, 1.4, s_3, 1.2, s_5, 1.2, s_7, s_8) \]  

(7)

where \( s_7 = s_8 = 2/(1 + 6\nu) \), \( s_3 \) and \( s_5 \) are arbitrary, can be set to 1.0.

What should we mention here, it is possible to recover the SRT-LBM solution from MRT-LBM by setting

\[ s_1 = s_2 = s_4 = s_6 = s_7 = s_8 = 1/\tau \]  

(8)

Furthermore, we use D2Q9 model (figure 1) and the nine discrete velocities are given by

\[
c = \begin{bmatrix}
0 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 1
0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1
\end{bmatrix}
\]  

(9)

Figure 1. 9-velocity LBE model on the 2D square lattice.

The macroscopic density and velocities can be computed simply by moment integration as

\[
\rho = \sum_i f_i
\]  

(10)

\[
u = \frac{1}{\rho} \sum_i c_i f_i
\]  

(11)

3. Treatment on Fluid Solid Boundary

In this present work, we applied the interpolated bounce-back boundary-condition to model no slip boundary-condition with curved boundaries [17]. The method was initially proposed by Ladd [8] which is the modified version of classical BB method by adding an adjustment term to the bounce back. However, this method has some limitation which suffers low resolution for general surface geometry. In order to overcome limitation, Lallemand and Luo [17] proposed a curve boundary treatment as follow;
Lallemand’s model applied the combination of simple bounce-back and quadratic interpolation. Generally, this technique of quadratic interpolation model will give more accurate and more stable results. This model is an extension of Bouzidi et al. [18]. Figure 2 illustrates boundary lattice link, with two end node $X_s$ in a solid domain and $X_f$ in a fluid domain. $X_b$ is the intersection node between solid surface and fluid surface, with midpoint of lattice link is denoted by $X_m$.

The location of the solid boundary can be determined from the value of the fraction of boundary lattice link in the solid domain, $\Delta$ based on Lallemand’s model where

$$\Delta = \frac{|X_s - X_b|}{|X_s - X_f|}$$

(13)

For $\Delta \leq 1/2$, where the midpoint $X_m$ locates between $X_b$ and $X_f$, the midpoint velocity $V_m$ can be obtained as bellow

$$V_m = \frac{0.5V_b + (0.5 - \Delta)V_f}{1 - \Delta}$$

(14)

while for $\Delta \geq 1/2$,

$$V_m = \frac{3/2V_b + (\Delta - 0.5)V_f}{2 - \Delta}$$

(15)

The hydrodynamic force exerted on the solid particle at any boundary nodes can be obtained by using equation (16)

$$\vec{F}(x,t) = e_a \left[ f_a(x_s,t) + f_a(x_f,t + \Delta t) \right]$$

(16)

4. Results Validation and Discussion

The physical domain of the problem is shown in figure 3. The proposed model applied the MRT-LBM model to simulate the fluid flow past a circular static solid in a channel. Figure 3 shows the setup in the simulation with length, height and inlet velocity.

The value of Reynolds number $Re$ can be obtained by using the following equation:
\[ \text{Re} = \frac{U_{\text{max}} H}{v} \]

(17)

Figure 3. The problem geometry.

For the purpose of code validation, the computed results in terms of velocity distribution along the channel length were compared with Aidun et al. [19-20]. Excellent agreement was obtained showing the applicability of the numerical scheme.

In our next computation, the flow characteristics around a static cylinder were studied at Reynolds number of 90 by using MRT-LBM model couple with curve-boundary treatment. Figure 5a - 5d shows the plot of velocity contour at different computational times.

Figure 5 clearly demonstrates the flow development in terms of velocity contour until steady state condition. The flow velocity becomes higher near both upper and lower side of the blockage to satisfy the requirement of conservation of mass. Negative value of velocity downstream of the cylinder indicating the presence of vortices resulted from inverse pressure on the flow field.
5. Conclusion
Numerical computations of flow around cylindrical solid obstacle were out using MRT-LBM method coupled with curve-boundary model. It is observed that the proposed method was capable in predicting all the flow features for the presence case. Future efforts need to extend the current formulation for investigation at various types of solid fluid flow related to real engineering problems.

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