Shape control of the crystal-melt interface in the Bridgman method

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Abstract. The processes of silicon crystallization in flat-bottomed cylindrical graphite crucibles in the modes of conjugate convective heat transfer with allowance for latent heat are numerically investigated. The influence of hydrodynamics and heat transfer on the local shape of the crystal-melt interface has been studied. The hydrodynamics of a silicon melt is investigated in the modes of thermogravitational and thermal gravitational-centrifugal convection. First, the effect of the velocity of lowering the crucible into the cold zone was studied. Then, at the set crucible lowering velocity, the influence of the crucible angular velocity was studied. The problems were solved in a nonstationary formulation by the finite element method.

1. Introduction

Crystallization from melts occurs in systems that are inevitably non-isothermal and in a gravity field. Therefore, thermogravitational or thermal gravitational-capillary convection of various intensities develops in melts [1-3]. The spatial shape and intensity of the flow depend on the characteristic temperature difference, geometry, and absolute dimensions of the test section of the growth installation. As a rule, in all works related to the analysis of processes in many options of the single crystals growing technology using directional crystallization methods, it is stated that free convection is poorly controlled. In the Czochralski method, crystal rotation or crystal and crucible rotation are used to eliminate imperfections in the working installation and to control the hydrodynamics of melts. When the crystal and crucible rotate, the hydrodynamics of the melt is determined by the combined action of several mass and surface forces [1]. The physical nature of convective motion is gravitational centrifugal, and in the presence of free non-isothermal surface of the melt, the thermocapillary effect is also added. When analyzing the nature of melt flows in rotating crucibles in a rotating coordinate system, except for the buoyancy and centrifugal forces, Coriolis force should be taken into account [4, 5]. In the simplest geometry cases, the uniform rotation of a nonisothermal system in some range of rotation velocity similarly to a homogeneous magnetic field increases the stability threshold of mechanical equilibrium or convective flow, reducing its velocity [2, 4-6]. Therefore, to reduce the intensity of free convection, uniform crucible rotation can be applied in the Bridgman method [2]. In Bridgman's method, because of the radial temperature gradients caused by the difference in thermal conductivity of the melt and crucible walls, convective flows inevitably appear in the melt [2, 3]. Despite a large number of technological and numerical studies of crystallization processes in the classical Bridgman method, there are no answers to many questions due to the nonlinearity of the interaction of various mechanisms of flow generation. In the monograph [5] it was fairly noted that just a few papers study heat transfer processes for cases where buoyancy forces and rotation effects are involved. As Bridgman method
produces single crystals of different source materials for the creation of microelectronics products, optoelectronics, and nonlinear optics, researches aimed at searching for optimal modes of heat exchange and methods of melt hydrodynamics control that is quite actual. The method is also used for the crystallization of solar-grade silicon in flat-bottomed cylindrical and rectangular crucibles [2, 6]. Technological practice shows that in all methods of directional crystallization, the growth of crystals with a plane crystal-melt interface provides the most uniform radial-azimuthal distribution of electrophysical properties in the plane normal to the growth direction. By choosing the mode of convective heat transfer, one can correct the shape of the crystal-melt interface. The development of technologies for growing high-quality single crystals requires an understanding of the features of conjugate convective heat transfer with various combinations of thermal conductivity of melts, crystals, and crucible materials. This work is aimed at investigating these effects. Direct experimental studies of high-temperature technological processes are expensive and time-consuming. It is practically impossible to measure the characteristics of non-stationary temperature fields in the composite region of the crucible-melt-crystal of the growth apparatus. Therefore, it is advisable to numerically investigate the effect of conjugate convective heat transfer on the crystallization process and the shapes of crystal-melt interfaces. For the silicon-graphite system, these studies are the further development of works [2, 3].

2. Statement of the problem

When growing silicon crystals by the Bridgman method under technological conditions, the crucible with silicon melt is first kept in the upper part of the furnace (the stage of melting the charge and homogenization of the melt), which has a cylindrical shape, and then slowly descends from the zone with a constant temperature of the furnace walls to the zone with decreasing linearly wall temperature [2, 3]. In this case, the crucible bottom begins to cool down, and when the temperature at the crucible bottom reaches a little lower than the silicon crystallization temperature, crystal nucleation begins. The problems were solved in an axisymmetric formulation in a composite computational domain, whose diagram is shown in Figure 1, which represents only the right side of the axisymmetric computational domain. At the initial moment of time \( t = 0 \), the crucible is filled with the melt \( \Omega_1 \), overheated relative to the crystallization temperature. On the outside of the walls of the crucible \( S_7 \), a temperature profile is set, consisting of an upper section with a constant temperature and a lower section with a linear temperature distribution. The lowering of the crucible from the hot zone to the cold one is simulated by the velocity of movement of the temperature breakpoint on the outer edge of the \( S_7 \) crucible. The constant temperature in the upper part of the crucible corresponds to the initial temperature of the superheated melt. The processes of cooling the outer surface of \( S_7 \) and crystal growth continue until the region of solidified matter \( \Omega_2 \) occupies the entire inner space of the crucible. The upper free boundary of the melt \( S_3 \) is set flat and non-deformable. The total volume of the crystallized substance and melt is assumed to be constant. Convective heat transfer in a melt is described by the system of nonstationary thermogravitational convection equations in the Boussinesq approximation, which are represented in dimensionless variables of temperature, stream function, and velocity vortex, written in the cylindrical coordinate system \((r, z)\):
\[
\begin{aligned}
\left\{ \frac{\rho_f(T)c_f(T)}{\rho_f c_f} \frac{dT}{dt} + U \frac{\partial T}{\partial r} + V \frac{\partial T}{\partial z} = \lambda_f(T) \nabla^2 T + Q, \\
\frac{\partial \omega}{\partial t} + U \frac{\partial \omega}{\partial r} + V \frac{\partial \omega}{\partial z} - \frac{1}{r} \frac{\partial W^2}{\partial r} = \nabla^2 \omega - \frac{\omega}{r^2} - \frac{g : \beta_f(T) \cdot R_f^3 \Delta T}{\nu_f^2} \frac{\partial T}{\partial r}, \\
\frac{\partial W}{\partial t} + U \frac{\partial W}{\partial r} + V \frac{\partial W}{\partial z} + U \frac{W}{r} = \nabla^2 W - \frac{W}{r^2}, \\
\nabla^2 \psi - \frac{2}{r} \frac{\partial \psi}{\partial r} = r \cdot \omega, \\
U = - \frac{1}{r} \frac{\partial \psi}{\partial r}, \\
V = - \frac{1}{r} \frac{\partial \psi}{\partial r}, 
\end{aligned}
\] (1)

where \( U, V, W \) are the components of the velocity vector, \( T \) is the temperature, \( \omega \) velocity vortex, \( \psi \) is the stream function, \( \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \) is the Laplace operator, \( \Delta T \) is the characteristic temperature difference.

The dimensionless equations of heat conduction in the crystal and crucible have the following form, respectively:

\[
\frac{c_c \rho_s \frac{dT}{dt}}{c_f \rho_f} = \frac{\lambda_s}{\nu_f} \nabla^2 T + Q, \\
\frac{c_c \rho_c \frac{dT}{dt}}{c_f \rho_f} = \frac{\lambda_c}{\nu_f} \nabla^2 T. 
\] (2)

In the equations being solved \( c_c, c_s, c_f \) are the specific heat coefficients of the melt, crystal, and crucible, respectively; \( \lambda_s, \lambda_s, \lambda_c \) are the thermal conductivity coefficients; \( \rho_f, \rho_s, \rho_c \) are the densities; \( \nu_f \) is the kinematic viscosity of the melt; \( \beta_f \) is the volumetric thermal expansion coefficient of the melt; \( g \) is the acceleration due to gravity, \( R_f \) is the scale of geometric dimensions.

The latent heat is accounted for through an internal heat source. Let for a time instant equal to \( c \) the crystal-melt interface changes its position from \( B^0(t) \) to \( B^1(t) \) (Figure 2), respectively. The source power per unit volume in the domain \( \Omega^0(t) \), bounded by the surfaces \( B^0(t) \) and \( B^1(t) \), is equal to \( Q = \frac{\rho_f L}{\Theta} \). Thus, in equations (1) and (2) the value of \( Q \) is defined as:

\[
Q = \begin{cases}
\frac{\rho_f L}{\Theta}, & \text{in the area of } \Omega^0(t); \\
0, & \text{in the areas of } \Omega^1(t) \text{ or } \Omega^2(t).
\end{cases}
\]

It is clear that the position of the crystal-melt interface \( B^1(t) \) is not known in advance. It is determined by solving discrete analogs of equations, by their multiple solutions within one time step with a relaxation coefficient, while the position of the boundary \( B^2(t) \), corresponding to the isothermal surface with a temperature value equal to the crystallization temperature is redefined, and the position of the boundary \( B^1(t) \) remains unchanged.

At the boundaries between the crucible-melt, crucible-crystal, and melt-crystal, the conditions of ideal thermal contact are fulfilled, i.e., at the indicated boundary, the temperature and heat flux are inseparable:

\[
\lambda_f \left. \frac{dT}{dn} \right|_{S_{y^+}} = \lambda_s \left. \frac{dT}{dn} \right|_{S_{y^+}}, \quad \lambda_f \left. \frac{dT}{dn} \right|_{S_{1^+,2^+}} = \lambda_c \left. \frac{dT}{dn} \right|_{S_{1^-,2^+}}, \quad \text{or} \quad \lambda_s \left. \frac{dT}{dn} \right|_{S_{1^+,2^+}} = \lambda_c \left. \frac{dT}{dn} \right|_{S_{1^-,2^+}}.
\]
\[ T_{S_i} = T_{S_i}, \quad T_{S_{2i}} = T_{S_{2i}}, \quad T_{S_{3i}} = T_{S_{3i}}. \]

At the non-deformable boundary of the melt region and the melt-crystal interface, the no-flow conditions are set:

\[ \psi_{|S_i} = 0, \quad i = 1, 2, 3, 9. \]

On the inner rigid surfaces of the crucible walls and the melt-crystal interface, the adhesion conditions are satisfied; as a result, the condition for the velocity vortex is presented in the form:

\[ \omega_{|S_i} = \frac{\partial U}{\partial z} - \frac{\partial V}{\partial r}, \quad i = 1, 2, 9. \]

At the upper free boundary of the melt, the slip condition is applied:

\[ \omega_{|S_5} = 0. \]

The following conditions are set on the axis of symmetry:

\[ \psi_{|r=0} = 0, \quad \omega_{|r=0} = 0, \quad W_{|r=0} = 0, \quad \frac{\partial V}{\partial r}_{|r=0} = 0, \quad \frac{\partial T}{\partial r}_{|r=0} = 0. \]

The upper horizontal boundary of the computational domain is adiabatic:

\[ \left. \frac{\partial T}{\partial z} \right|_{S_{5,8}} = 0. \]

At the initial moment, the temperature in the entire system is constant and equal to the temperature in the hot zone of the furnace, there are no convective flows:

\[ T_{|t=0} = T^*, \quad \psi_{|t=0} = 0, \quad \omega_{|t=0} = 0, \quad W_{|t=0} = 0. \]

The temperature at the outer boundaries of the crucible \((S_6, S_7)\) changes over time:

\[ T(z)_{|S_{6,7}} = T^* + \frac{dT}{dz} \left( z - v^* t \right), \]

if \( T(z)_{|S_{6,7}} < T_{\text{min}} \), then \( T(z)_{|S_{6,7}} = T_{\text{min}} \), if \( T(z)_{|S_{6,7}} > T_{\text{max}} \), then \( T(z)_{|S_{6,7}} = T_{\text{max}} \), where \( T^* \) is the temperature in the hot zone of the furnace, \( T_{\text{min}} \) is fixed minimum cooling temperature, \( v^* \) is crucible lowering velocity, \( dT/dz \) is given temperature gradient.

The linear velocity of the inner boundaries of the crucible \((S_1, S_2)\) is a function of the radius:

\[ W_{|S_{1,2}} = r W^*, \]

where \( W^* \) is the dimensionless azimuthal velocity at \( r = 1 \).

The problems were solved by the finite element method. Linear functions on triangles were used to approximate the solution. Grid generation was based on the direct method of constructing the Delaunay triangulation over the largest angle with cellular acceleration. The grid was constructed for each time step, to track the shape of the crystal-melt interface at the current step and the next step with automatic determination of the volume of the crystallized substance. Besides, the grids were condensed towards the crystal-melt interface on both sides and towards the boundaries of the computational domain. The calculations were carried out at the following thermophysical parameters characteristic of silicon crystallization in graphite crucibles \([8, 9]\): thermal conductivity coefficients, \( \lambda \): for silicon melt 67 \( J/(m\cdot s\cdot K) \), crystal 22 \( J/(m\cdot s\cdot K) \); and graphite 50 \( J/(m\cdot s\cdot K) \); density, \( \rho \): for silicon melt and crystal 2530 \( kg/m^3 \); and graphite 1800 \( kg/m^3 \); specific heat, \( c \): for graphite 2100 \( J/(kg\cdot K) \); silicon melt and crystal...
1000 J/(kg·K); kinematic viscosity of silicon melt \( \nu = 3.4 \cdot 10^{-7} \text{ m}^2/\text{s} \); volumetric thermal expansion coefficient of silicon melt \( \beta_r = 1.4 \cdot 10^{-4} \text{ K}^{-1} \); latent heat of silicon melt \( L = 1.8 \cdot 10^6 \text{ J/kg} \); and Prandtl number of silicon melt \( Pr = 0.0128 \). Crucible is the cylindrical container made of graphite grade MPG-6 with the same wall and bottom thickness of 0.5 cm. The inner radius of the crucible was 5 cm.

3. Results and discussion
First, the effect of the crucible lowering rate on the hydrodynamics of the melt, heat transfer, and the shape of the crystal-melt interface was studied in the modes of nonstationary thermogravitational convection. Below are the results obtained with a discrete set of crucible lowering velocity \( V_c \) from 1 to 10 cm/h. The initial overheating of the melt relative to the crystallization temperature was constant and equal to 40 K. The temperature gradient on the outer side of the crucible walls was \( dT/dz = 35 \text{ K/cm} \).

![Figure 3](image-url)

**Figure 3.** Fields of isotherms and isolines of the stream function depending on the crucible lowering velocity \( V_c \) (cm/h), calculated taking into account the latent heat (1,2,3) and without taking into account (4) at times \( t \) (s): 1 – \( t = 7880 \); \( V_c = 2.81 \); 2 – 4420; \( V_c = 5 \); 3 – 2210; \( V_c = 10 \); 4 – 2210; 10.

Figure 3 shows the combined fields of isotherms and isolines of the stream function at times \( t \) (s), corresponding to the position of the point with \( T = 1410 \text{ K} \) in the temperature distribution on the outer side of the crucible walls at the level \( z = 5 \text{ cm} \) at different velocities of lowering the crucible into the cold zone \( V_c \). Here are the results of calculations taking into account (1, 2, 3) and without taking into account (4) the latent heat. As can be seen from the shape of the crystallization front, with an increase in the crucible lowering velocity, the distributions of axial and radial local heat fluxes change, influencing the local curvature of the crystal-melt interface. With an increase in the crucible lowering velocity, radial heat removal is increasingly manifested and the crystallization rate near the sidewall of the crucible is significantly higher than in the axial zone. With an increasingly concave crystal-melt interface and lateral cooling in the melt, the flow in the secondary vortex above the crystallization front becomes more and more intense. Here, the circulation of the melt occurs clockwise. In a more developed upper vortex, which is formed due to lateral heating above the breaking point in the temperature distribution on the outer side of the crucible wall, the circulation of the melt occurs counterclockwise. The directions and velocities of the flow in these vortices in the sections along the height of the melt layer above the crystal-melt interface are shown in Figure 4. Here data obtained at \( V_c = 1 \text{ cm/h} \) are added. Sections were selected that pass through the center of rotation of the secondary vortices above the crystal-melt interface approximately at the height of the interface between the main upper vortex and the lower secondary. According to the data in Figure 3 (4), the relative role of the phase transition heat released in front of the crystal-melt interface is clear at a qualitative level. In addition to a decrease in the crystallization rate, this factor affects the spatial shape and intensity of the melt flow over the crystal-
melt interface, maintaining an upward flow in the secondary vortex above the crystal-melt interface in the axial region.

**Figure 4.** Profiles of the axial velocity component in sections along $z \,(cm)$ at times $t \,(s)$ depending on the crucible lowering velocities $V_c \,(cm/h)$, a: $1 - V_c = 1; \ z = 4; \ t = 18489.8; \ 2 - 2.81; \ 5; \ 7880; \ 3 - 5; \ 4.8; \ 4420; \ 4 - 10; \ 4.5; \ 2210; \ b: \ 1 - V_c = 1; \ z = 4.2; \ t = 18489.8; \ 2 - 2.81; \ 5.5; \ 7880; \ 3 - 5; \ 5.7; \ 4420; \ 4 - 10; \ 5.6; \ 2210.$

Figure 5 shows the temperature distributions over the height of the melt layer in the section passing through the center of rotation of the main vortex $r = 3.7 \, cm$, at times $t \,(s)$ corresponding to the position of the temperature point with $T = 1410 \, K$ (and isotherms corresponding to the crystal-melt interface) in the temperature distribution on the outer side of the crucible walls at the level $z = 5 \, cm$ at different crucible lowering velocity $V_c$. Figure 5b shows on a larger scale a fragment of the temperature distribution in the cross-section $r = 3.7 \, cm$ at the same time moments $t \,(s)$ in the melt and near the crystal-melt interface. As can be seen, with the high thermal conductivity of a silicon melt, which is characteristic of liquid metals, the presence of two vortex flow structures does not distort the monotonic temperature distribution over the crystal-melt interface.

**Figure 5.** Temperature distribution in the section $r = 3.7 \, (cm)$ at times $t \,(s)$ at different crucible lowering velocities $V_c \,(cm/h)$: $1 - t = 7880; \ V_c = 2.81; \ 2 - 4420; \ 5; \ 3 - 2210; \ 10.$
However, a comparison of the profiles of the axial velocity component in Figure 4 and the shapes of the crystal-melt interface in Figure 3 shows that a change in the direction and velocity of the flow in the secondary vortex above the crystal-melt interface significantly affects the convective heat flux and the shape of the crystal-melt interface. Thus, the above calculation results show that the velocities of lowering the crucible into the cold zone significantly affect the shape of the crystal-melt interface. To study the effect of crucible rotation on the crystal-melt interface shape, as the initial state (without crucible rotation), a mode with a crucible lowering velocity was selected, which was closer to the optimal one, since the crystal-melt interface, in this case, was closer to flat (Figure 6 (1)). This is due to both changes in the ratios of axial and radial heat fluxes in the crucible walls in the crystal in the modes of unsteady thermal conductivity, and conjugate convective heat transfer. The studies were carried out with a discrete set of crucible rotation velocity in the range from 0 to 10 rpm. Figure 6 shows the evolution of the fields of stream function isolines and isotherms with an increase in the angular velocity of crucible rotation at the same time instant, counted from the start of the crucible movement into the cold zone of the furnace. Silicon-graphite system, crucible lowering rate 1 cm/h, temperature gradient on the outer side of the vertical crucible wall \( \frac{dT}{dz} = 35 \text{ K/cm} \).

**Figure 6.** Combined fields of isotherms and isolines of the stream function at time \( t = 18489.8 \text{ s} \), in convective mode at angular velocities: 1 – \( \omega_T \) (rpm) = 0; 2 – 0.5; 3 – 1; 4 – 2; 5 – 5; 6 – 7.5; 7 – 10 and in the thermal conductivity mode (8).
The initial temperature of the system was 40 K higher than the crystallization temperature. As the angular velocity of rotation of the crucible increases, the radial dimensions of the main convective vortex located above the crystal-melt interface, as can be seen, decrease monotonically. That is, the circulation of the liquid along the contour is shifted to the sidewall of the crucible. First, the flow rate to the crystal-melt interface in the paraxial region decreases, which is shown in Figure 7. In Figure 7, the profiles of the axial velocity component are shown in sections passing through the center of rotation of the main (upper) vortex. From the standpoint of technology, an important parameter is the amplitude of the flow rate of the hot melt on the solidification front. The local curvature of the crystal-melt interface depends on the velocity of the incident flow and the position of the amplitude of the descending melt flow. Figure 7 shows that as the angular velocity increases, the downward flow gradually disappears in the central part of the melt bulk. As can be seen in Figure 8, already at $\omega_T = 1 \text{ rpm}$, the center of the descending flow is displaced towards the crucible wall, but the amplitude of the descending flow is higher than that in the mode without crucible rotation. At the same time, there is practically no downward flow in the axial region. With an increase in the angular velocity, this tendency becomes more and more evident. At an angular velocity of 5 rpm, the circulation rate has increased quite sharply.

![Figure 7. Profiles of the vertical velocity component in sections along $z$ (cm) at the time $t = 18489.8$ s at different angular velocities $\omega_T$ (ob/min): 1 – $z = 8.4$; $\omega_T = 0$; 2 – 8.2; 1; 3 – 7.5; 5; 4 – 6.5; 10.](image1)

![Figure 8. Profiles of the radial velocity component in sections along $r$ (cm) at the time $t = 18489.8$ s at different angular velocities $\omega_T$ (rpm): 1 – $z = 4$; $\omega_T = 0$; 2 – 4; 1; 3 – 4.2; 5; 4 – 4.6; 10.](image2)

However, since the center of rotation remains at approximately the same distance from the crystal-melt interface and the vortex configuration changes, the increase in the velocity did not affect the local curvature of the crystal-melt interface at angular velocities $\omega_T \leq 1 \text{ rpm}$. In the range of angular velocities from 2 to 10 rpm, as can be seen in Figure 7, the shape of the crystal-melt interface approaches that characteristic of the thermal conductivity regime. This is because the convective heat flux to the crystal-melt interface in the axial region decreases. Figure 8 shows the profiles of the radial velocity component also in sections passing through the center of the main vortex. The data presented here provide additional information on the circulation rate in the vortices located above the crystal-melt interface, as well as show the presence of a low-intensity secondary vortex above the crystallization front with the circulation direction opposite to the circulation in the main vortex (which is observed in the initial regime). An important stage in the technological process is the appearance of a nucleus crystal. Therefore, the key point from the technological viewpoint is the initial stage of the isotherm with the temperature of crystallization at the boundary of the melt of the bottom crucible. To form a crystallographic structure of an ingot close to a single crystal, it is desirable to cool the crucible bottom near the axis of symmetry with a minimum area of the cooled region. In this case, the probability of the appearance of a single
crystal seed is quite high. The smaller its radial size on the flat bottom of the crucible, the higher is the probability of the appearance of a single crystal. In Figure 9, a bold line indicates the isotherm corresponding to the crystallization temperature. Shown here is the dependence of the shape of this isotherm on the angular velocity of rotation of the crucible at times close to the time it reaches the upper surface of the crucible bottom. Figure 9 (1) shows that in the convective mode without rotation, at the initial stage, a layer of solid matter is formed practically over the entire bottom area.

Figure 9. Fields of isotherms and isolines of the stream function at times $t$ (s) at different angular velocities $\omega_T$ (rpm): 1 – $t = 5788.6$; $\omega_T = 0$; 2 – $5451.4$; 1; 3 – $5282.8$; 5; 4 – $5170.4$; 10.

As the angular velocity of rotation increases, the area of the cooled zone in the paraxial region gradually decreases, and at $\omega_T = 10$ rpm the area of the cold spot becomes the same as in the heat conduction mode. The dependences of the radial distributions of the azimuthal component of the melt flow velocity on the crucible rotation velocity in Figure 10 show that at 10 rpm, a mode close to the solid-state rotation of the melt is realized.

Figure 10. Profiles of the azimuthal velocity component in sections along $z$ (cm): 1 – $z = 1$; 2 – 3; 3 – 5; 4 – 7; 5 – 9; a: at time $t = 5451.4$ s an angular velocity $\omega_T = 1$ rpm; b: at time $t = 5170.4$ s an angular velocity $\omega_T = 10$ rpm.
Here are the distributions of the azimuthal velocity component at different levels along the height of the melt layer. Thus, it is shown that without the rotation of the crucible, the thermal conductivity mode is not realizable in real technology, and at an angular velocity \( \Omega_T = 10 \text{ rpm} \), the convective flow is practically suppressed and heat transfer occurs in a mode close to the thermal conductivity mode.

Conclusions
Numerical simulation of the growth processes of silicon ingots by the Bridgman method in unsteady modes of thermal conductivity, thermogravitational and gravitational-centrifugal convection with a discrete set of angular velocities of crucible rotation varied from 0 to 10 rpm was carried out. The study was carried out with the initial overheating of the melt relative to the crystallization temperature of 40 K. It is revealed that the section with a temperature gradient equal to 35 K/cm at a given velocity moves uniformly upward from the lower edge of the crucible sidewall, simulating immersion in the cold zone.

A crucible made of graphite grade MPG-6 with the same wall and bottom thickness of 0.5 cm had the shape of a cylinder with an inner radius of 5 cm. The initial height of the silicon melt layer was 9.5 cm. It is shown that in nonstationary regimes of thermogravitational convection, the shape of the crystal-melt interface becomes more and more concave with an increase in the crucible lowering velocity from 1 cm/h to 10 cm/h. In non-stationary modes of thermal gravitational-centrifugal convection at a crucible lowering velocity of 1 cm/h, it is shown that when the crucible rotation velocity is increased to 10 rpm, crystallization is possible at the initial stage practically in the thermal conductivity mode. Thus, at the initial stage of crystal growth, the gravitational-centrifugal convection mode is preferable, because the axial region with a small radius is cooled below the crystallization temperature. This creates the possibility of nucleation of a single crystal. At subsequent stages of the process, thermogravitational or gravitational-centrifugal convection regimes with a low rotation rate are preferable, which will allow maintaining a flat crystal-melt interface.

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References
[1] Berdnikov V S, Vinokurov V A, Vinokurov V V 2017 Bulletin of the Russian Academy of Sciences: Physics 81 10) 1257–62
[2] Nepomnyashchikh A I, Presnyakov R V, Antonov P V, Berdnikov V S 2014 Inorganic Materials 50(12) 1185–90
[3] Mitin K A, Berdnikov V S, Kislitsin S A 2019 Computational Mechanics of Continua 12(1) 106–16
[4] Chandrasekhar S 1961 Hydrodynamic and hydromagnetic stability (Oxford: Clarendon Press) 652
[5] Gebhart B, Jaluria Y, Mahajan R L, Sammakia B 1988 Buoyency-induced flows and transport (Washington: Hemisphere Publishing Corporation)
[6] Meier D, Lukin G, Thieme N, et al. 2017 Journal of Crystal Growth 461 30–7
[7] Mokhtar Ben Sassi, Slim Kaddeche, Marcello Lappa, et al. 2017 Journal of Crystal Growth 458 154–65
[8] Stankus S V, Khairulin R A, Tyagel'skii P V 1999 High Temp. 37 529–34
[9] Stankus S V, Savchenko I V, Agadzhanov A S, Yatsuk O S, Zhmurikov E I 2013 High Temp. 51 179–82