Hadronic and New Physics Contributions to $B \to K^* \ell^+ \ell^-$

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ABSTRACT

The significance of the observed tensions in the angular observables in $B \to K^* \mu^+ \mu^-$ are dependent on the theory estimation of the hadronic contributions to these decays. Therefore, we discuss in detail the various available approaches for taking into account the long-distance hadronic effects and examine how the different estimations of these contributions result in distinct significance of the new physics interpretation of the observed anomalies. Furthermore, besides the various theory estimations of the non-factorisable contributions we consider a general parameterisation which is fully consistent with the analyticity structure of the amplitudes. We make a statistical comparison to find whether the most favoured explanation of the anomalies is new physics or underestimated hadronic effects within this general parametrisation. Moreover, assuming the source of the anomalies to be new physics, there is a priori no reason to believe that – in the effective field theory language – only one type of operator is responsible for the tensions. We thus perform a global fit where all the Wilson coefficients which can effectively receive new physics contributions are considered, allowing for lepton flavour universality breaking effects as well as contributions from chirality flipped and scalar and pseudoscalar operators.

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1 Introduction

Currently among the most significant particle physics measurements hinting to the observation of New Physics (NP) are the tensions between the Standard Model (SM) predictions and the corresponding experimental measurements in several $b \to s \ell^+\ell^-$ decays. The first tension was observed in the angular observable $P_0$ in the $B \to K^*\mu^+\mu^-$ decay with 1 fb$^{-1}$ data [1] at the LHCb with a significance of more than $3\sigma$ and later confirmed by the same experiment with 3 fb$^{-1}$ data [2]. $B \to K^*\mu^+\mu^-$ angular observables were also measured by the Belle [3], ATLAS [4] and CMS [5] experiments with larger theoretical uncertainties. Another measurement indicating larger than $3\sigma$ tension with the SM was observed by the LHCb [6] in the branching ratio $B_s \to \phi\mu^+\mu^-$. Several other tensions with a NP significance of $2.2-2.6\sigma$ have also been measured in the ratios $R_K$ and $R_{K^*}$ by the LHCb [7,8]. These tensions in the ratios if confirmed would establish the breaking of lepton flavour universality. Moreover, smaller tensions with the SM predictions (between 1 and $3\sigma$) are observed in the branching ratios of $B^0 \to K^0\mu^+\mu^-$, $B^+ \to K^+\mu^+\mu^-$, $B^+ \to K^{*+}\mu^+\mu^-$ [9] as well as in the baryonic decay of $\Lambda_b \to \Lambda\mu^+\mu^-\gamma$ [10]. All the observed tensions are in the decays with muons in the final state, at low dilepton invariant mass squared ($q^2$) and the measurements are below the SM predictions. All these tensions in the branching ratios, angular observables or $R$ ratios point to a coherent picture of deviations with the SM and they can all be explained with a common NP effect, namely about 25% reduction in the $C_9^{(\rho)}$ Wilson coefficient [11,13] (see also Refs. [14,19]).

While the tensions in the ratios are not very significant and below $3.5\sigma$ at the moment, in case they are confirmed by further experimental data, the only viable explanation would be NP since the theory predictions of these observables are very precise and robust [20,21] due to hadronic cancellations. On the other hand the observables $P_0^c(B \to K^*\mu^+\mu^-)$ and $\text{BR}(B_s \to \phi\mu^+\mu^-)$ both receive hadronic contributions which are difficult to estimate. Nevertheless, the confirmation of the $P_0^c(B \to K^*\mu^+\mu^-)$ anomaly by several measurements makes it unlikely that the tension in $P_0^c$ is due to statistical fluctuations and hence either underestimated hadronic effect or NP contributions are the more likely explanations [12,22–34]. The significance of the tension in $P_0^c$ depends on the precise treatment of the hadronic contributions [12,35,36].

The standard framework for the calculation of the hadronic corrections in the $B \to K^{*}\ell^+\ell^-$ decay [37–41], in the region where the dilepton invariant mass ($q^2$) is below the $J/\psi$ resonance, is the QCD factorisation (QCDf) method where an expansion of $\Lambda/m_b$ is employed [12,43]. Within this framework higher powers of $\Lambda/m_b$ remain unknown and are usually roughly estimated to be some fraction of the known leading order QCDf terms. However, there have been methods suggested for the estimation of the power corrections using light-cone sum rule (LCSR) techniques and employing dispersion relations [31] and the analyticity structure of the amplitudes [14] as well as an empirical model [45]. In the current paper we investigate how the different methods impact the $B \to K^{*}\ell^+\ell^-$ observables and in particular study the tension with $P_0^c$ within the several available implementations of the power corrections. We also examine how the significance of the preferred New Physics scenarios changes depending on the employed method for estimating the power corrections.

Alternatively, instead of making assumptions on the size of the power corrections they can be parameterised by a general function with a number of unknown free parameters [19,46–48] and then fitted to the data. In this case it is important to have the correct description of the general function not to disrupt the analyticity structure of the amplitude. Specifically, the ansatz should be in such a way as not to generate a pole in the longitudinal amplitude of the $B \to K^{*}\ell^+\ell^-$ especially if the data to $B \to K^*\gamma$ is to be considered since the longitudinal amplitude should vanish when the intermediate $\gamma$ becomes on-shell. We present a statistical comparison of both hadronic parameters and New Physics contributions to Wilson coefficients within this general parameterisation using the Wilks’ theorem [49].

Considering the $b \to s \ell^+\ell^-$ anomalies to be due to New Physics contributions there is a priori no reason to assume that such contributions would only appear in a single operator and in principle several operators could simultaneously affect $b \to s \ell^+\ell^-$ transitions. We discuss how the BR($B_s \to \mu^+\mu^-$) observable which is usually used to neglect potential contributions from scalar and pseudoscalar operators cannot be solely considered for such a conclusion, and that there are regions of parameter space that allow for large contributions to these operators and that in order to disregard the scalar and pseudoscalar contributions all $b \to s$ transitions should be globally considered. We perform NP fits in the most general case where all the relevant Wilson Coefficients including the scalar and pseudoscalar operators, can receive New Physics contributions and explore how well scenarios with extended NP contributions describe the $b \to s$ data and examine whether indeed simultaneous contributions to several operators are favoured or not.

The rest of the paper is organised as follows. In section 2 the general ansatz for the power corrections which respect the analyticity of the amplitude is given where we make statistical comparisons of the
hadronic and NP fits to $B \to K^*\mu^+\mu^-$ observables. In section 3, we discuss the various methods available for implementing the hadronic contributions relevant to $B \to K^*\ell^+\ell^-$ decay and examine the most favoured scenarios and the corresponding significance depending on the employed method. Finally, we discuss the global fit to all possible Wilson coefficients which impact the $b \to s\ell^+\ell^-$ transitions including scalar contributions in section 4, and give our conclusions in section 5.

2 Hadronic versus NP contributions in $B \to K^*\ell^+\ell^-$

The $b \to s\ell^+\ell^-$ transitions can be described via an effective Hamiltonian which can be separated into a hadronic and a semileptonic part:

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}^{\text{had}} + \mathcal{H}_{\text{eff}}^{\text{sl}},$$

where

$$\mathcal{H}_{\text{eff}}^{\text{had}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1,\ldots,6,8} C_i O_i,$$

$$\mathcal{H}_{\text{eff}}^{\text{sl}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=7,9,10,Q_1,Q_2,T} \left( C_i O_i + C'_i O'_i \right).$$

For the exclusive decays $B \to K^*\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$, the semileptonic part of the Hamiltonian which accounts for the dominant contribution, can be described by seven independent form factors $\hat{S}, \hat{V}_\lambda, \hat{T}_\lambda$, with helicities $\lambda = \pm 1, 0$. The exclusive $B \to V\ell\ell$ decay, where $V$ is a vector meson can be described by the following eight helicity amplitudes:

$$H_V(\lambda) = -i N' \left\{ C_{9\text{eff}}' \hat{V}_\lambda - C_{9}' \hat{V}_{-\lambda} + \frac{m_B^2}{q^2} \left[ \frac{2m_h}{m_B} (C_{7\text{eff}}' \hat{T}_\lambda - C_{7}' \hat{T}_{-\lambda}) - 16\pi^2 N_\lambda \right] \right\},$$

$$H_A(\lambda) = -i N' (C_{10} \hat{V}_\lambda - C_{10}' \hat{V}_{-\lambda}),$$

$$H_P = i N' \left\{ (C_{Q_2} - C_{Q_2}') + \frac{2m_h}{q^2} \left( 1 + \frac{m_s}{m_B} \right) (C_{10} - C_{10}') \right\} \hat{S},$$

$$H_S = i N' (C_{Q_1} - C_{Q_1}') \hat{S},$$

where the effective part of $C_{9\text{eff}}' (\equiv C_9 + \mathcal{Y}(q^2))$ as well as the non-factorisable contribution $N_\lambda(q^2)$ arise from the hadronic part of the Hamiltonian through the emission of a photon which itself turns into a lepton pair. Due to the vectorial coupling of the photon to the lepton pair, the contributions of $\mathcal{H}_{\text{eff}}^{\text{had}}$ appear in the vectorial helicity amplitude $H_V(\lambda)$. It is due to the similar effect from the short-distance $C_9$ (and $C_7$) of $\mathcal{H}_{\text{eff}}^{\text{sl}}$ and the long-distance contribution from $\mathcal{H}_{\text{eff}}^{\text{had}}$ that there is an ambiguity in separating NP effects of the type $C_{9\text{NP}}$ (and $C_{7\text{NP}}$) from non-factorisable hadronic contributions.

2.1 Most general ansatz for the nonfactorisable power corrections

The non-factorisable term $N_\lambda(q^2)$ contributing to $H_V(\lambda)$ is known at leading order in $\Lambda/m_b$ from QCDf calculations while higher powers can only be guesstimated within QCDf. These power corrections are usually assumed to be 10%, 20%, etc. of the leading order non-factorisable contribution. On the other hand, instead of making such a guesstimate on the size of the power corrections they can be parameterised by a polynomial with a number of free parameters which can be fitted to the experimental data.

In our previous work (Ref. [47]) we assumed a general $q^2$-polynomial ansatz for the unknown contributions

$$h_\lambda(q^2) = h_\lambda^{(0)} + \frac{q^2}{1 \text{GeV}^2} h_\lambda^{(1)} + \frac{q^4}{1 \text{GeV}^4} h_\lambda^{(2)}.$$

We used the measurements on $B \to K^*\mu^+\mu^-$ observables below the $J/\psi$ resonance to fit the free parameters $h_\lambda^{(0,1,2)}$. However, it turns out that this ansatz is not compatible with the general analyticity structure of the amplitude $H_V(\lambda)$ in the case of $\lambda = 0$, in particular there should be no physical pole in
the longitudinal amplitude for \( q^2 \rightarrow 0 \). In the current paper, we also consider the experimental result on \( BR(B \rightarrow K^*\gamma) \), thus compatibility with the analytical structure for \( q^2 \rightarrow 0 \) is mandatory.

We have now modified the \( h_\lambda(q^2) \) ansatz for \( \lambda = 0 \) and have kept the same ansatz for \( \lambda = \pm \) (see section 2.2)

\[
h_0(q^2) = \sqrt{q^2} \times \left( h_0^{(0)} + \frac{q^2}{1\text{GeV}^2} h_0^{(1)} + \frac{q^4}{1\text{GeV}^4} h_0^{(2)} \right).
\]

This modified definition for \( h_\lambda \) is the most general ansatz for the unknown hadronic contributions (up to higher order powers in \( q^2 \)) which is compatible with the analyticity structure assumed in Ref. [44].

The radiative decay \( B \rightarrow K^*\gamma \) can be described in terms of the helicity amplitudes \( H_V(\lambda = \pm) \)

\[
A_\lambda(B \rightarrow K^*\gamma) = \lim_{q^2 \rightarrow 0} \frac{q^2}{e} H_V(q^2 = 0; \lambda)
\]

\[
= \frac{i N m_B^2}{e} \left[ \frac{2\hat{m}_b}{m_B} (C_7 \hat{T}_\lambda(0) - C'_7 \hat{T}_{-\lambda}(0) - 16\pi^2 N_\lambda(q^2 = 0)) \right].
\]

with \( N_\lambda(q^2) \equiv \text{Leading order in QCDF} + h_\lambda(q^2 = 0) \) where the leading order contributions in QCDF includes the vertex corrections, spectator scattering and weak annihilation contributions and can be found in [42, 43, 50–52]. With the description in Eq. (8) for the power corrections, the \( B \rightarrow K^*\gamma \) decay can also be described correctly without developing a pole at \( q^2 \rightarrow 0 \).

### 2.2 The \( q^2 \)-dependence of \( H_V(\lambda) \) for \( \lambda = \pm \) and \( \lambda = 0 \)

We show in the following that the effect of NP contributions to observables from \( C_7 \) and \( C_9 \) can be embedded in the most general ansatz of the hadronic contributions. Thus it is possible to make a statistical comparison of a hadronic fit and a NP fit of \( C_9 \) embedded in the most general ansatz of the hadronic contributions. Thus it is possible to make a statistical comparison of a hadronic fit and a NP fit of \( C_9 \) (and \( C_7 \)) to the \( B \rightarrow K^*\mu^+\mu^- \) data. We note here that the form factors \( \hat{V}_\lambda, \hat{T}_\lambda \) appearing in \( H_V(\lambda) \) (Eq. [5]) have different \( q^2 \)-behaviours for \( \lambda = \pm \) and \( \lambda = 0 \).

#### 2.2.1 \( H_V(\lambda = \pm) \)

The helicity form factors \( \hat{V}_\pm = \hat{V}_{\pm}(q^2) \) and \( \hat{T}_\pm = \hat{T}_{\pm}(q^2) \) are written as

\[
\hat{V}_\pm(q^2) = \frac{1}{2} \left[ \left( 1 + \frac{m_{K^*}}{m_B} \right) A_1(q^2) \pm \frac{\lambda^{1/2}}{m_B(m_B + m_{K^*})} V(q^2) \right],
\]

\[
\hat{T}_\pm(q^2) = \frac{m_B^2 - m_{K^*}^2}{2m_B^2} T_2(q^2) \pm \frac{\lambda^{1/2}}{2m_B^2} T_1(q^2),
\]

with \( \lambda = m_B^2 + m_{K^*}^2 - 2(m_B^2 m_{K^*} + m_{K^*}^2 q^2 + m_B^2 q^2) \). Since \( V(q^2), A_1(q^2), T_1(q^2) \) and \( T_2(q^2) \) are all well-behaved functions of \( q^2 \) (e.g. see Fig. 2 in Ref. [53]), the helicity amplitudes \( \hat{V}_\pm \) and \( \hat{T}_\pm \) can be described in terms of polynomials in \( q^2 \) (see Fig. 1)

\[
\hat{V}_\pm = a_\pm \hat{V} + q^2 b_\pm \hat{V},
\]

\[
\hat{T}_\pm = a_\pm \hat{T} + q^2 b_\pm \hat{T},
\]

where \( a_\pm, b_\pm \) are determined by expanding the form factors \( \hat{V}_\pm \) and \( \hat{T}_\pm \).

With the above expansion for the helicity form factors in Eq. (10), the effect of \( \delta C_9^{\text{NP}} \) and \( \delta C_7^{\text{NP}} \) in \( H_V(\lambda = \pm) \) can be written as

\[
\delta H_V^{C_9,\pm}(\lambda = \pm) = -i N' \delta C_9^{\text{NP}} \left( a_\pm \hat{V} + q^2 b_\pm \hat{V} \right),
\]

\[
\delta H_V^{C_7,\pm}(\lambda = \pm) = -i N' 2\hat{m}_b m_B \delta C_7^{\text{NP}} \left( \frac{1}{q^2} a_\pm \hat{T} + b_\pm \hat{T} \right).
\]
Employing the polynomial ansatz of Eq. (7), the effect of the power corrections is

\[ \delta H^\text{PC}_V(\lambda = \pm) = i N' m_B^2 16\pi^2 \left( \frac{1}{q^2} h_\lambda^{(0)} + h_\lambda^{(1)} + q^2 h_\lambda^{(2)} \right), \]

which are compatible with the form factor terms in \( H_V(\lambda = \pm) \) and will not disrupt the analyticity structure of the amplitude.

Considering Eq. (11) and Eq. (12), New Physics effect can clearly be embedded in the more general case of hadronic contributions. Moreover, assuming \( b_{L, T}^V \) in the Taylor expansions of the form factors \( \tilde{T}_\pm, \tilde{V}_\pm \) to be zero, the \( \delta C_9 \) contributions correspond to \( h_\pm^{(1)} \) and the \( \delta C_7 \) contributions to \( h_\pm^{(0)} \) terms of the power corrections.

### 2.2.2 \( H_V(\lambda = 0) \)

The helicity form factors \( \tilde{V}_0(q^2) \) and \( \tilde{T}_0(q^2) \), are described in terms of the \( A_{12}(q^2) \) and \( T_{23}(q^2) \) with an extra term of \( 1/\sqrt{q^2} \) and \( \sqrt{q^2} \), respectively:

\[
\tilde{V}_0(q^2) = \frac{2mK^*}{\sqrt{q^2}} A_{12}(q^2) \quad \text{and} \quad \tilde{T}_0(q^2) = \frac{2\sqrt{q^2}mK^*}{m_B(m_B + m_{K^*})} T_{23}(q^2),
\]

where \( A_{12}(q^2) \) and \( T_{23}(q^2) \) are well-behaved functions of \( q^2 \) (see e.g. Fig. 2 in Ref. [53]). The helicity amplitudes \( \tilde{V}_0 \) and \( \tilde{T}_0 \) can then be described as a power expansion in \( q^2 \) in terms of

\[
\tilde{V}_0 = \frac{1}{\sqrt{q^2}} \left( a_0 + b_0 q^2 \right) \quad \text{and} \quad \tilde{T}_0 = \sqrt{q^2} \left( a_0 + b_0 q^2 \right),
\]

where \( a_0, b_0 \) are determined by expanding the form factors \( \tilde{V}_0 \) and \( \tilde{T}_0 \) (see Fig. 2).

Considering the expansion in Eq. (14) for the helicity form factors, the effect of \( \delta C_9^{\text{NP}} \) and \( \delta C_7^{\text{NP}} \) in \( H_V(\lambda = \pm) \) can be written as

\[
\delta H^{C_9^{\text{NP}}}_V(\lambda = 0) = -i N' \delta C_9^{\text{NP}} \left[ \frac{1}{\sqrt{q^2}} \left( a_0^T + q^2 b_0^T \right) \right],
\]

\[
\delta H^{C_7^{\text{NP}}}_V(\lambda = 0) = -i N' 2\tilde{m}_B m_B \delta C_7^{\text{NP}} \left[ \frac{1}{\sqrt{q^2}} \left( a_0^T + q^2 b_0^T \right) \right].
\]

Using the power expansion ansatz in Eq. (8), the effect of the power corrections is

\[
\delta H^{C}_V(\lambda = 0) = i N' m_B^2 16\pi^2 \left[ \frac{1}{\sqrt{q^2}} \left( h_0^{(0)} + q^2 h_0^{(1)} + q^2 h_0^{(2)} \right) \right],
\]

\[\text{Figure 1:} \] Form factors \( \tilde{V}_\pm \) and \( \tilde{T}_\pm \), where the solid lines correspond to the analytical expression and the dashed lines represent the expanded function. The helicity form factor error bands are calculated from the uncertainties and correlations of the “LCSR + Lattice” fit results for the traditional form factors \( V, A_{1,2} \) and \( T_{1,2,23} \) of Ref. [53].
which results in terms that are compatible with the form factor terms in $H_V(\lambda = 0)$ and will not disrupt the analyticity structure of the amplitude. And the embedding of the NP effects in the hadronic contributions remains valid. However, for $\lambda = 0$ when assuming $q_0^V, T$ in the Taylor expansions of the form factors $V_0, T_0$ to be zero, $C_5$ and $C_7$ both correspond to $h_0^{(0)}$.

Considering $H_V(\lambda = 0)$ it might seem that the longitudinal amplitude would have a pole at $q^2 \to 0$. However, it should be noted that for the longitudinal transversity amplitude one should consider $H$ and the dashed lines correspond to the expanded functions.

### 2.3 Hadronic fit vs NP fit to $\delta C_{7,9}$

In order to investigate whether the $B \to K^+\mu^+\mu^-$ data are better explained by assuming NP or underestimated hadronic contributions, we have done separate fits for each case where only the low $q^2$ data have been used (see also Ref. [47]). For the fits we have considered $\text{BR}(B \to K^+\gamma)$ [54], $\text{BR}(B^+ \to K^{*+}\mu^+\mu^-)_{q^2\in[1.1-6.0] \text{ GeV}^2}$ [49], and the CP averaged observables of the $B \to K^*\mu^+\mu^-$ decays [55] in the low $q^2$ bins up to 8 GeV$^2$. For the theory predictions SuperIso v4.0 [56,57] has been used.

The SM prediction of $B^{(+)} \to K^{(*)}\mu^+\mu^-$ observables can be found in Ref. [55] and using the “LCR+Lattice” result for the $T_1(0)$ form factor [53] (see e.g. Ref. [58] regarding the effect of the form factor choice) we have $\text{BR}(B \to K^+\gamma) = (4.29 \pm 0.85) \times 10^{-5}$.

For the hadronic fit, employing the parameterisation of section 2.1 we have varied the 18 free parameters describing the complex $h_{+,-,0}^{(0,1,2)}$. Most of the fitted parameters are consistent with zero (see Table 1) as they have large uncertainties, however, this can be changed with more precise experimental results and finer $q^2$ binning.

| Observables in the low $q^2$ bins up to 8 GeV$^2$ | $\chi^2_{SM} = 54.9$, $\chi^2_{min} = 14.7$ |
|-----------------------------------------------|-----------------------------------------------|
| $h_0^{(0)}$ | $(1.67 \pm 2.15) \times 10^{-4}$ | $(-1.17 \pm 1.84) \times 10^{-4}$ |
| $h_1^{(0)}$ | $(1.55 \pm 32.01) \times 10^{-5}$ | $(-1.65 \pm 2.35) \times 10^{-4}$ |
| $h_2^{(0)}$ | $(-1.65 \pm 72.01) \times 10^{-6}$ | $(4.36 \pm 3.73) \times 10^{-5}$ |
| $h_0^{(1)}$ | $(-2.13 \pm 1.77) \times 10^{-4}$ | $(4.79 \pm 3.24) \times 10^{-4}$ |
| $h_1^{(1)}$ | $(3.69 \pm 12.56) \times 10^{-5}$ | $(-5.31 \pm 3.71) \times 10^{-4}$ |
| $h_2^{(1)}$ | $(1.29 \pm 1.84) \times 10^{-5}$ | $(5.79 \pm 6.93) \times 10^{-5}$ |
| $h_0^{(2)}$ | $(-3.61 \pm 36.99) \times 10^{-5}$ | $(6.89 \pm 4.52) \times 10^{-4}$ |
| $h_1^{(2)}$ | $(3.63 \pm 2.98) \times 10^{-4}$ | $(-6.52 \pm 2.77) \times 10^{-4}$ |
| $h_2^{(2)}$ | $(-3.97 \pm 4.45) \times 10^{-5}$ | $(8.55 \pm 4.12) \times 10^{-5}$ |

Table 1: Hadronic power corrections fit to $\text{BR}(B \to K^+\gamma)$, $\text{BR}(B^+ \to K^{*+}\mu^+\mu^-)_{q^2\in[1.1-6.0] \text{ GeV}^2}$ and the $B \to K^*\mu^+\mu^-$ observables in the low $q^2$ bins up to 8 GeV$^2$.

We used the same set of observables to make one and two operator NP fit to $\delta C_9$ and $\delta C_{7,9}$ assuming the Wilson coefficients to be either real or complex in Table 2. Interestingly the real parts of the best fit.
point for $\delta C_9$ in all four cases are compatible within their 68% confidence level and all these NP scenarios have a larger than 4$\sigma$ significance better description of the data compared to the SM hypothesis. The fits suggest sizeable imaginary parts for the Wilson coefficients, with rather large uncertainties. In principle considering the CP asymmetric observables of the $B \to K^*\ell^+\ell^-$ decay should allow us to further constrain the imaginary parts of the Wilson coefficients but the current experimental data on the relevant T-odd CP asymmetries \cite{38,40} such as $A_{7,8,9}(B \to K^*\mu^+\mu^-)$ are not stringent enough to put any significant constraints on the imaginary parts.

As shown in section 2.2 the effect of New Physics contributions to observables from $C_7$ and $C_9$ can be embedded in the more general case of the hadronic contributions. Due to the embedding, any lepton flavour universal New Physics contribution to the Wilson coefficients, $C_7$ and $C_9$ can be simulated by some hadronic effect and it is not possible to rule out underestimated hadronic explanation in favour of the NP one by only considering CP-averaged $B \to K^*\mu^+\mu^-$ observables.\cite{40} However, there can be a statistical comparison between the NP fit versus the hadronic contribution fit. In Table 2 the significance of the improvement of the fit in the hypothesis with more free parameters has been compared to the that with less free parameters using the Wilks’ theorem. While the hadronic solution and the NP explanation both have a better description of the measured data with a significance of larger than $3\sigma$, there is always less than $1.5\sigma$ improvement when going from the NP fit to the hadronic one. Compared to the scenarios with real contributions to $C_9$ or $C_{7,9}$, the NP fit has $\sim 2\sigma$ improvement when the Wilson coefficients are considered complex.



\begin{table}
\begin{tabular}{|c|c|c|c|c|}
\hline
best fit value & $\chi^2_{\text{min}}$ & $\chi^2_{\text{min}}$ \\
\hline
$\delta C_9$ & $-1.15 \pm 0.22$ & 38.1 & $-1.03 \pm 0.25 + i(-2.04 \pm 0.58)$ & 33.9 \\
$\delta C_7$ & $0.04 \pm 0.03$ & 36.0 & $(0.03 \pm 0.03) + i(0.09 \pm 0.05)$ & 30.3 \\
$\delta C_9$ & $-1.47 \pm 0.31$ & & $(-1.30 \pm 0.35) + i(-2.40 \pm 0.73)$ & \\
\hline
\end{tabular}
\caption{One and two operator NP fits for real (complex) $\delta C_9$ and $\delta C_{7,9}$ on the left (right) considering the same observables as mentioned in the caption of Table 1.}
\end{table}


\begin{table}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
nr. of free parameters & 1 (Real $\delta C_9$) & 2 (Real $\delta C_7, \delta C_9$) & 2 (Complex $\delta C_9$) & 4 (Complex $\delta C_7, \delta C_9$) & 18 (Complex $h_{+,-,0}$) \\
\hline
0 (plain SM) & 41$\sigma$ & 40$\sigma$ & 42$\sigma$ & 41$\sigma$ & 31$\sigma$ \\
1 (Real $\delta C_9$) & & 1.5$\sigma$ & 2.1$\sigma$ & 2$\sigma$ & 1.5$\sigma$ \\
2 (Real $\delta C_7, \delta C_9$) & & & & 1.9$\sigma$ & 1.4$\sigma$ \\
2 (Complex $h_{+}$) & & & & & 1$\sigma$ \\
4 (Complex $\delta C_7, \delta C_9$) & & & & 0.95$\sigma$ & \\
\hline
\end{tabular}
\caption{Improvement of the hadronic fit and the scenarios with real and complex New Physics contributions to Wilson coefficients $C_7$ and $C_9$ compared to the SM hypothesis and compared to each other.}
\end{table}

The slight differences of Table 1\footnote{In principle, the embedding can be broken even with flavour universal New Physics contribution to the Wilson coefficients, $C_7$ and $C_9$ since the imaginary parts in Wilson coefficients correspond to CP-violating “weak” phases while the imaginary parts of the hadronic contributions correspond to CP-conserving “strong” phases \cite{38,40,59,60}. However, the current data on CP-asymmetric observables for $B \to K^*\ell^+\ell^-$ \cite{2,61} and $B \to K^*\gamma$ \cite{54,52,63} are not constraining enough to allow us to make this differentiation.} compared to the relevant similar results of Ref. 17 are due to the modified parameterisation of the hadronic contributions for $\lambda = 0$, and also due to the inclusion of two additional observables, $\text{BR}(B \to K^*\gamma)$ and $\text{BR}(B^+ \to K^{*+}\mu^+\mu^-)\int_{4.1-6.0}$ GeV$^2$. Nonetheless, the conclusion remains the same; adding 14-17 more parameters compared to the New Physics fit does not significantly improve the fit (although the improvement of the hadronic fit compared to NP one is now slightly larger).

Thus, at the moment the statistical comparison favours the NP explanation and more constraining data on CP-asymmetric observables would be needed to determine whether it should be real or complex.
However, the situation stays inconclusive. With the set of observables considered in this analysis, the NP fit can be embedded in the hadronic fit; in this case one cannot disprove the hadronic option in favour of the NP one as discussed above.

With the present results, there is no indication that higher powers of $q^2$ than what is attainable by NP contributions to $C_1$ and $C_2$ would be required to explain the $B \to K^* \mu^+ \mu^-$ data. However, this might be due to the size of the current $q^2$ bins which can potentially smear out a significant $q^2$ dependence and thus smaller binning can shed more light on this issue.

## 3 Various theory estimations of the hadronic contributions

The short-distance NP contributions due to $\delta C_9^{NP}$ (and/or $\delta C_7^{NP}$) can be mimicked by long-distance effects in $h_\lambda$. Therefore, a proper estimation of the size of the hadronic contributions is highly desirable and crucial in determining whether the observed anomalies in $B \to K^* \mu^+ \mu^-$ observables results in a significant NP interpretation. There are different approaches offered in the literature in order to estimate the hadronic contributions.

### 3.1 Various approaches

In the “standard” method the hadronic contributions are estimated using the QCD factorisation (QCDf) formalism where the factorisable as well as non-factorisable contributions from vertex corrections [50,66], weak annihilation and spectator scattering [42,43] are taken into account. However, higher powers of $O(1/m_b)$ remain unknown within the QCDf formalism. In the so-called “full form factor” method (see i.e. Ref. [35]), only the power corrections to the non-factorisable piece in the QCDf formula are not known and are usually guesstimated to be 10%, 20% or even higher percentages compared to the leading non-factorisable contributions.

Among the hadronic contributions, the most relevant contributions are due to the charm loops arising from the current-current operators $O_{1,2}$. The power corrections relevant to these charm loops, the soft gluon effects, have been estimated in Ref. [31] using the LCSR formalism in the $q^2 \lesssim 1$ GeV$^2$ region where $q^2 \ll 4m_b^2$ holds. The results are extrapolated up to the $J/\psi$ resonance by employing dispersion relations and using the experimental data from $B \to J/\psi K^*$ and $B \to \psi(2S)K^*$ decays. However, in the theoretical input of the dispersion relation the leading order non-factorisable effects (available from QCDf calculations [42,43]) which have an important contribution to the analyticity structure are not included. Moreover, the phases of the resonant amplitude relative to the short-distance contribution for each of the three amplitude structures (for both) resonances are just set to zero.

It is claimed in Ref. [32] that for the $B \to K^* \ell^+ \ell^-$ decay hadronic contributions from the $s$-quark (i.e. the $\phi$ meson pole) have a $1/q^2$ factor for the transverse polarisation and hence get enhanced at small $q^2$. Therefore, in order to have a precise estimation it would be preferable to have separate dispersion relations for the $c$-quark and the $s,b$-quarks. This has not been done since the theory calculations for one of the relevant contributions (e.g. Refs. [50,66]) are not available in a flavour separated way.

One way to compensate the missing leading order factorisable corrections in the Khodjamirian et al. method is to just add these missing contributions to the phenomenological model. However, the theoretical error which enters this procedure is unclear. This is done for example in Ref. [19], while the subleading hadronic contributions have been accounted for by considering the phenomenological description of Ref. [31] valid up to $q^2 \lesssim 9$ GeV$^2$ (referred to as PMD in Ref. [19]).

In Ref. [44], the most promising approach to the hadronic contributions is offered, which may lead to a clear separation of hadronic and NP effects. The authors consider the analyticity of the amplitude. Building upon the work of Refs. [31,32], both the leading and subleading hadronic contributions arising from the charm loop contributions of the current-current operators $O_{1,2}$ have been estimated. The calculations are performed at $q^2 < 0$ where the theory predictions for the leading terms in QCDf [42,43,50,66] as well as the subleading terms in LCSR [31,32] are reliable and in combination with the experimental information on the $B \to J/\psi K^*$ and $B \to \psi(2S)K^*$ decays, the hadronic contributions due to the charm loops are estimated in the physical region up to the $\psi(2S')$ resonance. They use the well-known $z$ parameterisation (see e.g. [67,60]).

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2The LHCb has provided a finer binning using the method of moments where the bins have a range of $\sim 1$ GeV$^2$, however, compared to the results with larger bins obtained with the maximum likelihood method, the experimental uncertainties are currently much larger.

3 Most recently (in Ref. [70]), the authors have analysed the convergence of the $z$ expansion in great detail. We still use their explicit results based on the expansion up to the $z^2$ terms.
The authors of this paper argue that due to nonperturbative OZI mechanism and the $\alpha_s$ suppression, the light hadron resonances can be neglected so the separation of the $s$-quark current becomes phenomenologically important only if one is interested in the local effect at $q^2 \approx m_\psi^2$ which can be resolved if an appropriate binning is considered \[71\].

Unfortunately the correlations among the theoretical uncertainties of the complex parameters describing the parameterisation of the hadronic contributions have not been provided. Nonetheless, the uncertainties of each of the parameters are available which used without the correlations leads to a very conservative theory estimation of the hadronic contributions.

Finally in Ref. \[45\], all the hadronic contributions, from the charm (and light quark) resonances are modeled as Breit-Wigner amplitudes. The effect of the $J/\psi$ and $\psi(2S)$ (and the rest of the) resonances on $B \to K^*\mu^+\mu^-$ observables are estimated (up to an overall global phase for each resonance) using measurements on the branching fractions and polarisation amplitudes of the resonances. The overall phase can be assessed from simultaneous fits to the short- and long-distance components in the $K^*\mu^+\mu^-$ final states. However, since this measurement is currently not available, in Ref. \[45\] all possible values for the overall phase of each resonant state have been assumed and therefore the results are rather unconstraining. The theory predictions of both Ref. \[31\] and Ref. \[44\] can be reproduced with appropriate choices for the unknown parameters entering the empirical model.

### 3.2 Comparison of the different approaches

To show how the various theory estimations differ in their predictions of $B \to K^*\mu^+\mu^-$ observables the SM results for $d\text{BR}/dq^2$ and $P_5'$, using the various implementations of the hadronic contributions are given in Fig. 3 and Fig. 4 respectively. In the “standard” method, the predictions are given for below $q^2 = 8$ GeV$^2$ where QCDf calculations are reliable while the phenomenological model of Khodjamirian et al. is considered up to $q^2 < 9$ GeV$^2$ and only the Bobeth et al. method has a prediction for also between the $J/\psi$ and $\psi(2S)$ resonances. Interestingly the central values of the latter two methods increase the tension with experimental measurement for both $d\text{BR}/dq^2$ and $P_5'$ and it seems that the contribution from the power corrections tend to further escalate the tension with the data. The theory error of these predictions however, are larger (for the Bobeth et al. method this is due to the lack of correlations among uncertainties, which are not given in Ref. \[44\]).

**Figure 3:** The SM predictions of $d\text{BR}(B \to K^*\mu^+\mu^-)/dq^2$ within various implementations of the hadronic contributions without (with) the theory uncertainties on the left (right). For the “QCDf” implementation the full form factor method has been considered, with a 10% error assumption for the power corrections. The theory error of the Khodjamirian et al. implementation is obtained by considering the relevant parameter uncertainties that goes into the phenomenological formula. For the theoretical uncertainty of the Bobeth et al. method the correlations of the parameters describing the hadronic contributions have not been used. The theoretical uncertainty of the method where the leading order non-factorisable contributions are added to the phenomenological model of Ref. \[31\] (Khodjamirian et al. + “missing QCDf”) are not shown.

The significance of the NP interpretation for the $B \to K^*\mu^+\mu^-$ anomalies clearly depends on the theory estimations of the hadronic contributions. In Table 4 the significance of different NP scenarios (for one operator fits to $\delta C_7$, $\delta C_9$ or $\delta C_{10}$) are given using the “standard” implementation (with 10% error assumption on the power corrections) and the Bobeth et al. implementation of the non-factorisable corrections. While in both implementations New Physics contribution to $C_9$ constitutes the favoured scenario, the significance and the best fit values are different.
Figure 4: The SM predictions of $P'_5(B \to K^* \mu^+ \mu^-)$ observables within various implementations of the hadronic contributions as described in the caption of Fig. 3.

Table 4: The $\chi^2$ of the one operator NP fit compared to the SM within the “standard” QCDf method (with a 10% error assumption on the power corrections) and the Bobeth et al. method. The observables considered in the fit include $\text{BR}(B \to K^* \gamma)$, $\text{BR}(B^+ \to K^{*+} \mu^+ \mu^-)$ in the [1.1-6.0] and [15-19] GeV$^2$ bins and all the $B \to K^* \mu^+ \mu^-$ observables in both the high and low $q^2$ bins.

4 Fit to NP including scalar & pseudoscalar operators

Assuming the observed tensions in $b \to s \ell^+ \ell^-$ data to be due to New Physics contributions there is in principle no reason that NP contributions should affect only one or two Wilson coefficients. In particular, a complete NP scenario incorporates many new particles and can have extended Higgs sectors, affecting the Wilson coefficients $C_7 \cdots C_{10}$ and requiring scalar and pseudoscalar contributions. It is often considered that the data on $\text{BR}(B_s \to \mu^+ \mu^-)$ remove the possibility to have large scalar and pseudoscalar Wilson coefficients $C_{Q1,2}$ (see e.g. Ref. [72] for the definition of the relevant operators). While this is rather true for $C_{Q1}$, there exists a degeneracy between $C_{10}$ and $C_{Q2}$ which makes it possible to have simultaneously large values for both Wilson coefficients. To demonstrate this, we perform a fit to $\text{BR}(B_s \to \mu^+ \mu^-)$ when the three Wilson coefficients $C_{10, Q1,2}$ are varied independently. The results can be seen in Fig. 5 where two dimensional projections of the constraints by $\text{BR}(B_s \to \mu^+ \mu^-)$ are shown on the $C_{10, Q1,2}$ Wilson coefficients. While $C_{Q1}$ is still limited between $\pm 0.2$, both $C_{10}$ and $C_{Q2}$ can have large values, due to the compensation in the $\text{BR}(B_s \to \mu^+ \mu^-)$ formula (see e.g. Ref. [72]).

Figure 5: Two-dimensional projection of the three operator fit to $C_{10}$, $C_{Q1}$, and $C_{Q2}$. The (light) red contours correspond to the (68) 95% C.L. regions.
We consider also the case where $C_Q \equiv C_{Q_1} = -C_{Q_2}$, and $C_{10}$ are varied separately. The results are shown in Fig. 6. In such a case the degeneracy between $C_{Q_2}$ and $C_{10}$ is broken, and the scalar and pseudoscalar contributions are limited between $\pm 0.2$. It is however remarkable that $C_{10}$ can take large values, whereas it is limited between $\pm 1$ when $C_{Q_1}$ and $C_{Q_2}$ are set to zero.

![Two operator fit to the BR($B_s \to \mu^+\mu^-$) with New Physics contributions in $C_{10}$ and $C_{Q_1} = -C_{Q_2}$. The (light) red contours correspond to the (68) 95% C.L. regions.](image)

**Figure 6:** Two operator fit to the BR($B_s \to \mu^+\mu^-$) with New Physics contributions in $C_{10}$ and $C_{Q_1} = -C_{Q_2}$. The (light) red contours correspond to the (68) 95% C.L. regions.

As a consequence, the branching ratio of $B_s \to \mu^+\mu^-$ cannot be used to set simultaneously strong constraints on $C_{10}$ and $C_{Q_{1,2}}$ in generic NP scenarios, but can only be used to justify why $C_Q$ or $C_{Q_2}$ can have very limited contributions. Conversely while the measurement of the branching ratio of $B_s \to \mu^+\mu^-$ is used to justify why the scalar and pseudoscalar contributions are set to zero in a specific fit, it cannot be used to set constraints on $C_{10}$ any more.

We have thus expanded our study to include NP in the global fit to $b \to s\ell^+\ell^-$ from other Wilson coefficients besides $C_7$ and $C_9$ to also include the chromomagnetic operator as well as the axial-vector, scalar and pseudoscalar operators with overall 10 independent Wilson coefficients $C_7, C_8, C_9, C_{10}, C_{Q_1}, C_{Q_2}$ (assuming lepton flavours to be $\ell = e, \mu$). Considering the operators where the chirality of the quark currents is flipped (primed Wilson coefficients) there will be 20 free parameters.

To perform our fits, the theoretical correlations and errors are computed using SuperIso v4.0, which incorporates an automatic multiprocessing calculation of the covariance matrix for each parameter point. We have considered 10% error assumption for the power corrections. The experimental correlations are also taken into account. The Minuit library [73] has been used to search for the global minima in high dimensional parameter spaces. For each fit we carefully searched for the local minima in order to find the global minima.

We first consider fits to one single Wilson coefficient. In Table 5 the one-dimensional fit results are given for the Wilson coefficients $C_7, C_8, C_{Q_1}, C_{Q_2}$ as well as for the $O_{XY}$ basis which is well motivated in several NP models, where $X$ indicates the chirality of the quark current and $i$ and $Y$ stand for the flavour index and chirality of the lepton current, respectively (see e.g. Ref. 1). New physics contributions to the primed Wilson coefficients, as well as $C_7, C_{Q_1}, C_{Q_2}$ are disfavoured in the fit, the same is also true for the axial-vector coefficient $C_{10}$ when lepton flavour universality is assumed. In all favoured scenarios whenever lepton flavour universality violation is allowed, the fit is improved which is due to the tensions in $R_{K^{\ast}}$ measurements. The most favoured scenario in the one-dimensional fit is when there is NP in $C_8^\mu$ with a significance of 5.8$\sigma$. The scenario with NP in $C_{LL}^\mu$ has also a large significance of 5.8$\sigma$.

We now turn to two dimensional fits. We performed 6 different fits, and their significance as well as the parameters of the best fit points are given in Table 6. A graphical representation showing the 68 and 95% C.L. contours is also provided in Fig. 7. The fit corresponding to $\delta C_{10}^\mu, C_9^\mu$ illustrates our discussion on the branching ratio of $B_s \to \mu^+\mu^-$, showing that the best fit point corresponds to $C_9^\mu \sim 0$, but $C_{10}^\mu$ can receive a rather large deviation from its SM value. Yet, the pull with the SM is only of 3.3$\sigma$. Scenarios with $C_{LL}$ and $C_{LR}$ improve the fits, leading to significances of more than 4$\sigma$. The most

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4The full list of observables considered in this study can be found in Ref. 55 where for dBR/d$\eta^2(B \to K^{\ast}\mu^+\mu^-)$ the binning of Ref. 55 with 3 fb$^{-1}$ data has been considered and the BR($B \to K^{\ast}\gamma$) and $R_K$ observables have also been added to the global fit.

5Since the experimental data on CP-asymmetric observables cannot put stringent constraints on the imaginary parts of the complex Wilson coefficients we have only considered real values for the Wilson coefficients.
The pull with the SM increases with the number of Wilson coefficients. The reason is that increasing the number of Wilson coefficients increases the number of degrees of freedom. In absence of real improvement in the fit, i.e. strong decrease in the best fit point values.

| b.f. value | $\chi^2_{\text{min}}$ | PullSM |
|------------|---------------------|--------|
| $\delta C_7$  | -0.01 ± 0.01 | 117.3 | 1.0σ |
| $\delta C_{Q_3}$ | -0.03 ± 0.08 | 118.2 | 0.2σ |
| $\delta C_{Q_4}$ | -0.91 ± 0.66 | 117.8 | 0.7σ |
| $\delta C_{Q_5}$ | 0.00 ± 0.01 | 118.2 | 0.2σ |
| $\delta C_{Q_9}$ | -0.77 ± 0.65 | 117.9 | 0.6σ |

Table 5: Best fit values and errors in the one operator fits to all the relevant data on $b \rightarrow s$ transitions, assuming 10% error for the power corrections.

favoured scenarios are for the case where there is NP in $C_9^u$ in combination with $\delta C_9^{\prime \prime}$, $\delta C_9$ or $\delta C_{10}^{\prime \prime}$, with significances of $\sim 5.5\sigma$.

| b.f. value | $\chi^2_{\text{min}}$ | PullSM |
|------------|---------------------|--------|
| $\delta C_{10}^{\prime}$ | 0.21 ± 0.25 | 117.5 | 0.9σ |
| $\delta C_{10}$ | 0.05 ± 0.19 | 118.2 | 0.3σ |
| $\delta C_{10}^{\prime \prime}$ | 0.67 ± 0.21 | 106.2 | 3.5σ |
| $\delta C_{10}^{\prime \prime}$ | -1.06 ± 0.28 | 102.6 | 3.9σ |
| $\delta C_{10}^{\prime \prime}$ | 0.04 ± 0.16 | 118.2 | 0.3σ |
| $\delta C_{2\prime}^{\prime}$ | -0.04 ± 0.29 | 118.2 | 0.1σ |

Table 6: Best fit values and errors in the two operator global fits, assuming 10% error for the power corrections.

We now expand the fits to the Wilson coefficients to 6, 10 and 20 dimensions. The results of the fits including $C_9^u$ are given in Table 7. The improvement column corresponds to the improvement in comparison to the previous set of Wilson coefficients, obtained using the Wilks’ theorem. Additional fit results can be found in the appendix, for $\{C_{10}, C_{Q_1}, C_{Q_2}\}$ (Table 9), $\{C_{10}^{\prime \prime}, C_{Q_1}^{\prime \prime}, C_{Q_2}^{\prime \prime}\}$ (Table 8), $\{C_{7}, C_{8}, C_{9}^{(c,\mu)}, C_{10}^{(c,\mu)}\}$ (Table 11) and $\{C_{7}, C_{8}, C_{9}^{(e,\mu)}, C_{10}^{(e,\mu)}, C_{Q_1}^{(e,\mu)}, C_{Q_2}^{(e,\mu)}\}$ (Table 12), including the best fit point values.

The pull with the SM increases with the number of Wilson coefficients. The reason is that increasing the number of Wilson coefficients increases the number of degrees of freedom. In absence of real improvement in the fit, i.e. strong decrease in the best fit point $\chi^2$, the increase of number of degrees of freedom will result in a reduced pull with the SM. This is confirmed by the improvement test, which reveals that adding Wilson coefficients to the “$C_9^u$ only” set does not bring any significant improvement.
Figure 7: Two operator global fits where the (light) red contour in the plots corresponds to the (68) 95% C.L. regions, assuming 10% error for the power corrections.

| Set of WC                          | Nr. parameters | $\chi^2_{\text{min}}$ | Pull_{SM} | Improvement |
|-----------------------------------|----------------|------------------------|------------|-------------|
| SM                                | 0              | 118.2                  | -          | -           |
| $C_9^{\mu}$                       | 1              | 84.6                   | 5.8$\sigma$| 5.8$\sigma$|
| $C_9^{(e,\mu)}$                   | 2              | 83.3                   | 5.6$\sigma$| 1.1$\sigma$|
| $C_7, C_8, C_9^{(e,\mu)}, C_{10}^{(e,\mu)}$ | 6          | 80.1                   | 4.9$\sigma$| 0.6$\sigma$|
| All non-primed WC                 | 10             | 78.2                   | 4.3$\sigma$| 0.3$\sigma$|
| All WC (incl. primed)             | 20             | 70.2                   | 3.5$\sigma$| 0.5$\sigma$|

Table 7: The $\chi^2_{\text{min}}$ values when varying different Wilson coefficients. In the last column the significance of the improvement of the fit compared to the scenario of the previous line is given, (assuming 10% error for the power corrections).

As a UV-complete NP model is likely to incorporate several new particles affecting all the Wilson coefficients, we give in Table 8 the best fit values when varying all the 20 Wilson coefficients. Many
of the Wilson coefficients have loose constraints which is due to the large number of free parameters compared to the numbers of observables and also lack of observables with sufficient sensitivity to those Wilson coefficients. The best fit values indicate potentially large contributions to the electron Wilson coefficients \(\delta C_9^{\mu}\) which is interesting as the few measurements on purely electron observables are much more SM-like than their muon counterparts and is mostly driven by the flavour violating observables \(R_{K^*}\). However, the large contributions are not statistically significant as there are many more muon than electron observables in the global fit. Specifically the favoured large contribution in the electron scalar coefficient is due to not having constraining experimental results on the \(B_s \rightarrow e^+e^-\) decay which would be sensitive enough to the scalar and pseudoscalar Wilson coefficients, the latter remaining currently completely unconstrained in the 20-dimensional fit. It can be noted that \(C_7\) and \(C_9^{\mu}\) are severely constrained with a very small error, revealing the compatibility between the constraints. \(C_8^{(\tau)}\) is much less constrained, as there are less observables sensitive to \(C_8\) in the fit. In addition, the muon scalar and pseudoscalar contributions can only have very small values. Finally, the best fit value of \(C_9^{\mu}\) is even smaller than for the one and two-dimensional fits, with 35% reduction of its SM value.

### Table 8: Best fit values for the 20 operator global fit to the \(b \rightarrow s\) data, assuming 10% error for the power corrections.

| \(\delta C_7\) | \(\delta C_8\) | \(\delta C_9^{\mu}\) | \(\delta C_{10}\) |
|----------------|----------------|----------------------|------------------|
| \(-0.01 \pm 0.03\) | \(0.90 \pm 0.54\) | \(-1.39 \pm 0.24\) | \(-4.10 \pm 0.55\) |
| \(0.01 \pm 0.03\) | \(-1.69 \pm 0.41\) | \(-0.08 \pm 0.27\) | \(1.31 \pm 1.13\) |
| \(0.23 \pm 0.60\) | \(-1.02 \pm 4.02\) | \(-0.16 \pm 3.32\) | \(2.70 \pm 1.13\) |

| \(C_{Q_1}^{\mu}\) | \(C_{Q_1}^{\tau}\) | \(C_{Q_2}^{\mu}\) | \(C_{Q_2}^{\tau}\) |
|----------------|----------------|----------------|----------------|
| \(-0.10 \pm 0.06\) | \(-0.83 \pm 3.60\) | \(-0.08 \pm 0.40\) | \(0.20 \pm 0.48\) |
| \(0.05 \pm 0.06\) | \(-0.93 \pm 3.42\) | \(-0.20 \pm 3.42\) | \(0.20 \pm 0.48\) |

### 5 Conclusions

Recent experimental measurements have shown tensions in some of the \(b \rightarrow s\) transitions. The most persistent tension which has been confirmed by several experiments is the anomaly in the angular observable \(P_5^\prime\) of the \(B \rightarrow K^* \mu^+ \mu^-\) decay. This decay however receives long-distance hadronic contributions that are difficult to calculate and consequently makes the SM predictions to be somewhat questionable. Hence the significance of the observed tension is quite dependent on how the non-factorisable contributions are estimated. In this paper we explored the various state-of-art methods for implementing the power corrections and demonstrated that while the various implementations of the unknown corrections offer different SM predictions and uncertainties, in all these cases, in the critical bin where the \(P_5^\prime\) anomaly is observed, the predictions roughly converge giving prominence to the observed tension.

Alternatively, instead of making assumptions on the size of the power corrections or using methods which intend to include these contributions (and hence introduce rather non-transparent systematic uncertainties and correlations) one can assume a general parameterisation for the power corrections and fit the unknown parameters of the ansatz to the \(B \rightarrow K^* \mu^+ \mu^-\) data. In this work, in addition to the \(B \rightarrow K^* \mu^+ \mu^-\) observables we have included data on \(BR(B \rightarrow K^* \gamma)\) which requires the ansatz for the power corrections to have the correct end-point behaviour as the virtual photon (which decays into the dimuon in \(B \rightarrow K^* \mu^+ \mu^-\)) becomes on-shell. The ansatz employed in this paper is the most general parameterisation (up to higher \(q^2\) terms) which respects the analyticity structure of the amplitudes and...
guarantees that the longitudinal amplitude disappears as $q^2 \to 0$.

Employing this model-independent ansatz we examined whether New Physics contribution to (real or complex) $C_9$ and $C_7$ Wilson coefficients (with 1-4 free parameters) is the favoured explanation for the anomalies or underestimated hadronic effects (modeled with 18 free parameters). A statistical comparison indicates that there is no significant preference in adding 14-17 parameters compared to the NP explanation. This is partly due to the experimental results not being constraining enough so that the 18 parameter fit of the power corrections are mostly consistent with zero and also since possible preference for a large $q^2$-dependence might be masked due to the $q^2$ smearing within the current ranges of the bins. Furthermore, when employing only CP-averaged flavour universal observables, due to the embedding of the NP contributions in the hadronic effects the latter cannot be ruled out in favour of the former while the opposite is possible. Therefore, whilst still the most favoured scenario is having real NP contributions in $C_9$, the picture remains inconclusive and more precise data with finer binning will be crucial in clarifying the situation, especially on CP-asymmetric observables which can differentiate the weak and strong phases emerging from NP and hadronic contributions, respectively.

Furthermore, we showed that while $\text{BR}(B_s \to \mu^+\mu^-)$ is very effective in constraining the scalar and pseudoscalar operators, the relevant Wilson coefficients cannot be neglected by only assuming this single observable and a global fit to all the $b \to s$ data is required where several Wilson coefficients can simultaneously receive NP contributions. Although, the various 1, 2, 6, 10 and 20 dimensional fits when varying different Wilson coefficient does not indicate any preference for NP beyond $C_9$, yet a large number of Wilson coefficients are very loosely bound or completely unconstrained in the case of electron scalar and pseudoscalar operators. This is interesting since especially with the indication of lepton flavour universality violation from the $R_K$ and $R_{K^*}$ ratios, there is motivation to investigate the electron and muon sectors separately for the scalar and pseudoscalar operators.

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A Additional fit results

All observables with $\chi^2_{SM} = 118.2$

$(\chi^2_{min} = 117.7; \text{Pull}_{SM} = 0.1\sigma)$

| $\delta C_{10}$ | $C_{Q_1}$ | $C_{Q_2}$ |
|-----------------|-----------|-----------|
| $0.28 \pm 0.29$ | $-0.16 \pm 0.04$ | $0.15 \pm 0.34$ |

Table 9: Best fit values for the three operator $\{\delta C_{10}, C_{Q_1}, C_{Q_2}\}$ global fit to the $b \to s$ data, assuming 10% error for the power corrections.

All observables with $\chi^2_{SM} = 118.2$

$(\chi^2_{min} = 101.4; \text{Pull}_{SM} = 2.6\sigma)$

| $\delta C_{10}^{\mu}$ | $\delta C_{10}^{e}$ | $\delta C_{10}^{\nu}$ | $C_{Q_1}^{\mu}$ | $C_{Q_2}^{\mu}$ | $C_{Q_1}^{e}$ | $C_{Q_2}^{e}$ |
|--------------------------|---------------------|-----------------------|----------------|----------------|----------------|----------------|
| $0.39 \pm 0.30$          | $-0.74 \pm 0.38$    | $-0.11 \pm 0.26$      | $0.03 \pm 0.22$ | $0.18 \pm 3.13$ | $0.00 \pm 6.62$ |

Table 10: Best fit values for the six operator $\{\delta C_{10}^{\mu}, C_{Q_1}^{\mu}, C_{Q_2}^{\mu}\}$ global fit to the $b \to s$ data, assuming 10% error for the power corrections.

All observables with $\chi^2_{SM} = 118.2$

$(\chi^2_{min} = 80.1; \text{Pull}_{SM} = 4.9\sigma)$

| $\delta C_7$ | $\delta C_8$ | $\delta C_9$ | $\delta C_{10}$ |
|---------------|---------------|---------------|----------------|
| $0.03 \pm 0.05$ | $-0.21 \pm 0.66$ | $-1.23 \pm 0.23$ | $-2.51 \pm 2.24$ |
| $\delta C_7^{\mu}$ | $\delta C_9^{\mu}$ | $\delta C_{10}^{e}$ | $\delta C_{10}^{\nu}$ |
| $-1.23 \pm 0.22$ | $-2.52 \pm 2.26$ | $0.03 \pm 0.25$ | $-1.35 \pm 0.50$ |

Table 11: Best fit values for the six operator $\{\delta C_7, \delta C_8, \delta C_9^{\mu}, \delta C_{10}^{e}\}$ global fit to the $b \to s$ data, assuming 10% error for the power corrections.

All observables with $\chi^2_{SM} = 118.2$

$(\chi^2_{min} = 79.9; \text{Pull}_{SM} = 4.2\sigma)$

| $\delta C_7$ | $\delta C_8$ | $\delta C_9^{\mu}$ | $\delta C_{10}^{e}$ |
|---------------|---------------|---------------------|--------------------|
| $0.03 \pm 0.05$ | $-0.20 \pm 0.65$ | $-1.23 \pm 0.22$ | $-2.52 \pm 2.26$ |
| $\delta C_9^{\mu}$ | $\delta C_{10}^{e}$ | $\delta C_{10}^{e}$ | $\delta C_{10}^{\nu}$ |
| $-1.23 \pm 0.22$ | $-2.52 \pm 2.26$ | $0.03 \pm 0.25$ | $-1.35 \pm 0.50$ |
| $C_{Q_1}^{\mu}$ | $C_{Q_1}^{e}$ | $C_{Q_2}^{\mu}$ | $C_{Q_2}^{e}$ |
| $-0.15 \pm 0.07$ | $-0.22 \pm 3.17$ | $0.12 \pm 0.30$ | $0.02 \pm 5.92$ |

Table 12: Best fit values for the ten operator $\{\delta C_7, \delta C_8, \delta C_9^{\mu}, \delta C_{10}^{e}, C_{Q_1}^{\mu}, C_{Q_1}^{e}, C_{Q_2}^{\mu}, C_{Q_2}^{e}\}$ global fit to the $b \to s$ data, assuming 10% error for the power corrections.
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