Mark Wardle  
*Macquarie University, Sydney NSW 2109, Australia*

**Abstract.** A variety of processes play a role in the evolution of protostellar disks. Here I focus on the uncertain issue of magnetic field-disk coupling and its implications for magnetically-driven turbulence and disk-driven winds. At present it is clear that the magnetic field plays a crucial role in disk evolution, but detailed conclusions cannot be drawn because the complicated interplay between dynamics and the evolution of the grain population remains to be explored.

1. Introduction

Disks are the natural byproduct of the formation of stars: the resting place of material with too high an angular momentum to participate fully in the initial gravitational collapse that forms the central protostar. Accretion of an appreciable fraction of the star’s mass may (or may not) occur through the disk, and disks appear to be an essential component of the engine that powers protostellar jets. A somewhat bewildering array of physical processes come into play to determine disk structure and must be addressed if we wish to understand disk evolution, forming planetary systems, and especially to interpret essentially unresolved observations of disks.

To zeroth order the material in a disk is in orbit about the central object: in other words, the radial component of the gravitational field from the protostar is balanced primarily by rotation. Even moderately flattened structures supported by other means tend to be unstable to interchange instabilities - whether the support is by gas pressure, radiation pressure or magnetic stresses (e.g. Stehle & Spruit 2001), and must therefore be evolving rapidly. This implies that the internal signal speed in a disk that survives for many orbits is much less than the Keplerian speed. A slowly-evolving disk is thin because its scale height is determined by the balance between vertical pressure gradient and the tidal field of the central object.

One consequence is that the local dynamical time scale (i.e. the orbital time scale) is much shorter than the time scale on which the disk evolves. This makes physical sense, as a well-defined disk cannot otherwise be expected to exist for more than a few orbits. There is however, a flip side to this: the range of orbital time scales across the radial extent of the disk makes globally steady evolution most unlikely. It is unreasonable to expect that the radial variation of the processes that determine disk structure conspires to force the local behaviour to be just that needed to guarantee that $\dot{M}$ is constant all radii. Disk models that rely on this assumption to tie together the behaviour at different radii should be
regarded with caution, though they might represent the disk in a time-averaged sense.

A variety of processes play a role in the evolution of protoplanetary disks. Some factors are well-defined and operate on a scale large enough to be amenable to observation, such as tidal fields and irradiation by UV from companion stars or nearby stars, the X-ray irradiation from the central star, and self gravity if the disk mass \( \gtrsim 30\% \) of the stellar mass. Gravitational disturbance by planets forming within the disk is, of course, critical in the later stages of disk evolution.

The factors that operate on small-scales are not directly observable, requiring a resolution equivalent to the disk thickness or less. They do however control transport processes, the internal dynamics of the disk, and chemical evolution of the disk material. (This latter point may yield a way of teasing apart the processes that occur within disks.) Although much of the theoretical work on the physics operating on these scales is directed towards the identification of mechanisms responsible for angular momentum transport, note that protoplanetary disks need not necessarily be accretion disks. Although magnetospheric accretion and emission from boundary layers are observed in some systems, the total mass accreted onto the star may be quite small compared to the total disk mass, and so accretion may not be the defining characteristic of the disk structure.

Convection was an early proposed mechanism for the origin of angular momentum transport in protostellar disks (e.g. Lin & Papaloizou 1980). The idea was that convective cells couple different disk radii, thereby acting as an effective viscosity. The convection would be driven by internal heating due to the dissipation associated with the energy lost by the accreting material. However, simulations show that angular momentum is transported inwards rather than outwards (e.g. Cabot et al. 1987a,b), and in any case the convective cells are thin in the radial direction (ironically because of angular momentum conservation) and would be relatively inefficient as a mechanism for driving accretion.

The leading contender is now the magnetic field, which can transport angular momentum through the generation of turbulence via the magnetorotational instability (Balbus & Hawley 1991,2000; Balbus 2003), or through the centrifugal acceleration of material from the disk surfaces and the formation of a jet (e.g. Blandford & Payne 1982; Königl & Pudritz 2000). At present, it is not known what the initial radial distribution of mass and magnetic flux is. However, the initial poloidal field will have a strength of at least several milligauss, even if the ambient molecular cloud field is not particularly dragged in during the collapse that forms the disk. Even this weak field is sufficient to act as a seed for the magnetorotational instability at 1 AU (Salmeron & Wardle, in preparation).

Protostellar disks are weakly ionized, so the role of the magnetic field is complicated by its dependence on the trace charged species in the gas. Thus the evolution of protostellar disks is linked to chemistry, which determines the thermal and ionisation structure of the disk, and also to the population of dust grains which evolves through the accumulation of ices, sticking or shattering as a result of grain collisions, advection by turbulence, grain drag effects, and settling to the midplane (e.g. Weidenschilling & Cuzzi 1993).
2. Magnetic Coupling

The ability of the magnetic field to drive turbulence via the MRI or to produce a disk-driven wind is determined by its coupling to the weakly-ionised fluid. This is achieved by the collisions of charged species drifting in response to the electric and magnetic fields. The coupling is sensitive to ionisation rate, density and the very different mobilities of electrons, ions and grains.

The magnetic field in a weakly-ionised medium evolves according to the induction equation

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - c \nabla \times \mathbf{E}' \tag{1}
\]

where \(\mathbf{E}'\) is the electric field in the rest frame of the fluid, which is related to the current density \(\mathbf{J} = c/4\pi \nabla \times \mathbf{B}\) by

\[
\mathbf{E}' = \frac{\mathbf{J}}{\sigma_{||}} + \frac{\sigma_{H}}{\sigma_{H}^2 + \sigma_{P}^2} \frac{\mathbf{J} \times \mathbf{B}}{\mathbf{B}} - \left( \frac{\sigma_{P}}{\sigma_{H}^2 + \sigma_{P}^2} - \frac{1}{\sigma_{||}} \right) \frac{(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}}{\mathbf{B}^2}. \tag{2}
\]

Here \(\sigma_{||}\), \(\sigma_{H}\) and \(\sigma_{P}\) are the field-parallel, Hall and Pedersen conductivities and the three terms involving \(\mathbf{J}\), \(\mathbf{J} \times \mathbf{B}\) and \((\mathbf{J} \times \mathbf{B}) \times \mathbf{B}\) correspond to Ohmic, Hall, and ambipolar diffusion respectively.

Generally there is a trend from ambipolar to Hall and then Ohmic diffusion with increasing density and decreasing ionisation rate, which is why calculations of core collapse and disk formation have generally assumed that ambipolar diffusion dominates whereas solar nebula studies traditionally employ a resistive approximation. However, the Hall regime spans a surprisingly wide range of intermediate conditions and cannot be neglected. Furthermore, Hall diffusion qualitatively changes the vector evolution of the magnetic field, exhibiting a dependence on the strength of the magnetic field and a sensitivity to handedness that breaks many simple planar geometries that could otherwise be maintained in the presence of ambipolar or Ohmic diffusion.

The broad range of ionisation rates and densities obtaining over the radial and vertical extent of protostellar disks means that each form of diffusion will dominate in different regions. The ionisation rate is dominated by x-rays from the central low-mass star in the upper few \(\text{g cm}^{-2}\) of the disk (Glassgold, Najita & Igea 1997) and then by cosmic rays to depths of a few hundred \(\text{g cm}^{-2}\) (Hayashi 1981). The increase in ionisation rate and decrease in density with increasing height produces a trend from Ohmic through Hall to ambipolar diffusion with increasing height. If sufficient electrons are present Ohmic diffusion may never become completely dominant even at the midplane. The biggest uncertainty in estimates of the degree of coupling of the magnetic field to the disk is the grain population, as ions and electrons stick to grain surfaces and the mobility of the typical charge carriers is much reduced. If, on the other hand, grains have settled to the disk midplane, Hall diffusion dominates at the midplane over disk radii from 0.1 to 100 AU (Sano & Stone 2002a).

By way of example, the resulting vertical profile of the conductivity tensor in a minimum solar nebular model at 1AU is plotted in the presence and absence of 0.1 \(\mu\)m grains in the top panel of Fig. 1. The much larger conductivities in the no-grain model within three scale heights of the midplane reflects the lack
Figure 1. Upper panel: conductivity tensor components as a function of height above the midplane at 1AU in a minimum solar nebula for a dusty disk with 0.1\(\mu\)m grains and a model in which grains have settled to the midplane. The magnetic field strength is assumed to be 100 mG. Lower panel: The vertical profile of a coupling parameter \(\chi\) in the two models (solid lines), and the ratio of the Alfven speed to the sound speed and the square of that ratio (dotted lines). The magnetic field effectively couples to the disk material when \(\chi \sim v_A/c_s\); this criterion becomes \(\chi \sim v_A^2/c_s^2\) in the presence of significant Hall diffusion.

of grains – the charge is carried by more mobile free ions and electrons. The decoupling of ions from the magnetic field for densities \(\gtrsim 10^9\) cm\(^{-3}\) means that Hall component is larger than the Pedersen component between 1.5 and 4 scale heights, and is 80% of \(\sigma_p\) even at the midplane. In the 0.1 \(\mu\)m grain model, the Hall conductivity dominates the Pedersen conductivity within 4 scale heights of the midplane.

The variation of magnetic coupling with height above the midplane determines how deeply magnetic activity penetrates the disk and therefore the thickness of the “dead zone” (Gammie 1996) which is not affected by magnetic activity, and does not participate in accretion.
3. Magnetorotational Instability

One measure of whether the conductivity is sufficient for the magnetic field to interact with the disk material is whether the field is unstable to the magnetorotational instability. This is determined by comparing the coupling parameter \( \chi = \frac{\omega_c}{\Omega} \) to the ratio of Alfvén speed to sound speed, \( v_A/c_s \). Here \( \omega_c \) is the frequency above which ideal MHD breaks down and \( \Omega \) is the Keplerian frequency. If Hall diffusion is unimportant the criterion is \( \chi \sim > v_A/c_s \), if it is dominant (and the magnetic field has the correct orientation), then \( \chi \sim > (v_A/c_s)^2 \) is necessary.

The coupling parameter is plotted as a function of height in the lower panel of Fig. 1. The entire thickness of the disk is magnetically active in the absence of grains, whereas in the single-size grain model the layers above 2.5 scale heights are active. In both cases, Hall diffusion is important throughout the active regions.

If grains have settled to the disk midplane, Hall diffusion dominates at the midplane over disk radii from 0.1 to 100 AU (Sano & Stone 2002a). The implications for the magnetorotational instability are twofold. First, in the presence of Hall diffusion the instability operates under much weaker coupling than in the resistive or ambipolar diffusion cases, and therefore the potentially magnetically turbulent region is much more extensive than previously thought (Wardle 1999): the magnetic “dead zone” in which the field does not couple to the disk material is more restricted. This is illustrated in Fig. 2, which compares the linear development of the instability in models with and without Hall diffusion (Salmeron & Wardle 2003). If Hall diffusion is neglected, the instability grows in the upper layers of the disk and there is a substantial dead zone \( (z/h < 2) \). With Hall diffusion, the growth rate of the most unstable mode is increased somewhat and the mode penetrates deeper into the disk, to within a scale height of the midplane.

Second, in the nonlinear regime the turbulence grows and decays cyclically: as the magnetic field is amplified through the nonlinear growth of the instability, the charged particles become more tightly coupled to the magnetic field, Hall diffusion becomes less important and growth shuts off. The field then decays until Hall diffusion becomes important again, at which point the instability restarts and drives turbulence again (Sano & Stone 2002b).

4. Magnetically-driven Jets

So far, I have focussed on the dynamics of an initially weak field lying within the disk. However, coupling of a large-scale poloidal magnetic field to the rotation of the disk may produce a centrifugally-driven outflow. This mechanism has been suggested as the origin of protostellar jets, though it is unclear whether the outflow occurs just at the inner edge of the disk or extends over a significant fraction of the disk (Königl & Pudritz 2000). The poloidal field could plausibly be the remains of the partial dragging in of the original field threading the parent molecular cloud during the core collapse and disk formation.

The centrifugal acceleration mechanism relies on coupling the magnetic field to the disk and to the accelerated material, the field transferring angular momentum from the former to the latter. Clearly, as with the magnetorotational
instability, there must be sufficient coupling for this mechanism to work. In addition, there must be some slippage so that the magnetic field is not dragged inwards by the accreting material. Thus there are constraints on the magnitude of the diffusivity of the magnetic field. What appears to be unappreciated is that the nature of the coupling (i.e. whether ambipolar, Hall or Ohmic diffusion is dominant) determines the magnetic field direction as the lines emerge from the disk surface. This in turn, controls the tendency of material to slide along the field lines by the rotation (e.g. if the magnetic field lines are swept back, the acceleration is less efficient).

Simulations are currently unable to solve the complete disk+protostellar jet problem because of the large dynamic range in density between the disk and wind. Therefore disk-wind simulations are generally restricted to the wind region, with the magnetic field, density and fluid velocity specified at the base of the jet (roughly equivalent to the disk surface). Physically, these quantities are not independent but are related through the diffusion of the field within the disk, which simultaneously controls the bending of the field lines and how
matter is loaded onto them and lifted away from the disk (Wardle & Königl 1993; Wardle 1997). This is still very much an open problem.

5. Summary

The examples discussed here – magnetorotational instability and disk winds – show how the coupling between a magnetic field and the disk material likely plays a crucial role in the structure and evolution of protostellar disks. It should be borne in mind that these two examples need not be mutually-exclusive: within the disk the large-scale poloidal field may be the mean of a field which is tangled (cf. Fig. 3 of Blandford & Payne 1982) by turbulence driven by the magnetorotational instability.

The magnetic activity in protostellar disks is stratified because of the strong dependence of diffusion on the level of ionisation. This implies that when grains are present, the bulk of the disk is magnetically inactive within a few AU of the centre, where it is self-shielded from ionisation sources. Much of the magnetic activity, and hence accretion, occurs in the surface layers of the disk (Gammie 1986). When grains have settled to the midplane, Hall diffusion of the magnetic field dominates over much of the disk (Balbus & Terquem 2000, Sano & Stone 2002b). This tends to increase the extent of the magnetically-coupled region within the disk, and reduces the extent of the dead zone.

The separation of the disk into turbulent and inactive layers affects planet-building, which is seeded by the coagulation and settling of solid material to the midplane (e.g. Weidenschilling & Cuzzi 1993). This is complicated by the intriguing coupling between the grain population and the magnetic field. The grains determine the field diffusivity and therefore affects the fluid dynamics, the fluid motions in turn feed back on the grain population through the competing processes of coagulation, shattering, mantle accumulation or evaporation and settling mediated through the stirring and energy dissipation by magnetically-driven turbulence in the disk.

The interdependence of the grains and fluid dynamics is, at present, unexplored territory. There is some hope for constraining theoretical models using observational signatures of disk chemistry, which also be dependent on the stratification of heating and the evolution of the grain population.

References

Balbus, S. A. 2003, ARAA, 41, 555
Balbus, S. A. & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S. A. & Hawley, J. F. 2000, Sp Sci Rev, 92, 39
Balbus, S. A. & Terquem, C. 2001, ApJ, 552, 235
Blandford, R. D. & Payne, D. G. 1982, MNRAS, 199, 883
Cabot, W., Camuto, V. M., Hubickyj, O. & Pollack, J. B. 1987a, Icarus, 69, 387
Cabot, W., Camuto, V. M., Hubickyj, O. & Pollack, J. B. 1987b, Icarus, 69, 423
Gammie, C. F. 1996, ApJ, 457, 355
Glassgold, A. E., Najita, J. & Igea, J. 1997, ApJ, 480, 344
Hayashi, C. 1981, Prog Theor Phys Supp, 70, 35
Konigl, A. & Pudritz, R. E. 2000, in Protostars and Planets IV, ed. V. Mannings, A.P. Boss, & S. S. Russell (Tucson: U. Arizona), 759
Lin, D. N. C. & Papaloizou, J. 1980, MNRAS, 191, 37
Salmeron, R. & Wardle, M. 2003, MNRAS, 345, 992
Sano, T. & Stone, J. M. 2002a, ApJ, 570, 314
Sano, T. & Stone, J. M. 2002b, ApJ, 577, 534
Stehle, R. & Spruit, H. C. 2001, MNRAS, 323, 587
Wardle, M. 1997, in Proc. IAU Colloq. 163, Accretion Phenomena and Related Outflows, ed. D. Wickramasinghe, L. Ferrario & G. Bicknell (San Francisco: ASP), 561
Wardle, M. 1999, MNRAS, 307, 849
Wardle, M. & Königl, A. 1993, ApJ, 410, 218
Weidenschilling, S. J. & Cuzzi, J. N. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: University of Arizona Press), 1031