Exponential Stability of Markovian Jumping Memristor-Based Neural Networks via Event-Triggered Impulsive Control Scheme

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\textbf{ABSTRACT} This paper studies the modeling and exponential stability problems for markovian jumping memristor-based neural networks (MJMNNs) via event-triggered impulsive control scheme (ETICS). The purpose is to design memristor-based neural networks (MNNs) which has markovian jumping parameters and hybrid time-variant delays to make the MNNs more general. Meanwhile, a state estimator is introduced to estimate system states through available output measurements. Furthermore, the proposed event-triggered scheme (ETS), which is also determined by markovian parameters, is used to determine whether there is an impulse and whether the system need to transmit the sampled state information. Then, by using Lyapunov-Krasovskii functional (LKF) and an improved inequality, exponential stable criterions are established. Finally, a numerical example is given to support the results.

\textbf{INDEX TERMS} Memristor-based neural networks, Markovian jumping, event-triggered impulsive control scheme, exponential stability.

\section{INTRODUCTION}
The memristor was postulated as the fourth basic circuit element in electrical circuits by Chua in 1971 [1]. And it was invented as a successful fabrication of a very compact nonvolatile nano scale memory by HP lab in 2008 [2]. The memristance is changed with charge and flux. It can be altered if there is a voltage or current applied to it [3]. Because there is a feature of memristor that is similar with the neurons in the human brain [4]. The memristor has a broad potential application [5]–[8]. For example, the memristor can be used in the neural networks (NNs). Recently, MNNs have provoked considerable attention [9]–[12]. The connection between two neurons in MNNs is realized by a memristor.

The model has strong computing power. It can be applied to brain simulation, combinatorial optimization, knowledge acquisition and pattern recognition [13].

In the traditional NNs, the information latching problem is ubiquitous. The problem can be solved by using the markovian chain [14]–[16]. Paper [17] studies the stability of delayed markovian generalized NNs where the transition rates of the modes are partly unknown. Paper [18] investigates the passivity of markovian jump systems with channel fading which uses event-triggered state feedback control. In order to solve the information latching problem, the markovian jumping will be considered in this paper.

The dynamic system can be classified into three categories which are continuous-time system, discrete-time system and dynamical system with impulses [19]. The third system combines the characteristics of the first and second system.
Impulsive effects exist in neural networks. For instance, switching phenomenon, frequency change, or sudden noise may cause instantaneous perturbations and abrupt changes of MNNs state. So, the impulse can be used to control the MNNs state. Compared with continuous control, impulsive control has a simple structure and can achieve the desired performance by small control input intermittently. Meanwhile, the impulsive control can reduce the communication bandwidth in MNNs. Therefore, the NNs with impulsive control has received much interest in recent years [20]–[22]. A class of heterogeneous delayed impulsive NNs with memristors and their collective evolution for multisynchronization has been studied in [23]. There are several results on global exponential stability of a fractional order cellular NNs with impulses and with time-varying and distributed delay has been presented in [24]. And the synchronization problem of impulsive NNs with mixed time-varying delays and linear fractional uncertainties has been examined in [25]. However, the research of MNNs with impulsive controller is inadequate. Hence, it is significant to analyze the dynamics of the impulsive control MNNs.

When signals transmit in a communication channel, the effectiveness of communication resources should be considered. In order to reduce the waste of communication resources, an event-triggered scheme has been proposed in the implementation of real-time system [26]–[30]. The paper [31] is concerned with the guaranteed cost control problem for a class of markov jump discrete-time NNs with event-triggered mechanism, asynchronous jumping, and fading channels. The paper [32] investigates the mixed H-infinity and passive filtering problem for a class of discrete-time networked singular markovian jump systems. The paper [33] investigates the event-triggered H∞ control problem for networked discrete-time markov jump systems subject to repeated scalar nonlinearities. In this paper, an event-triggered communication transmission scheme is adopted and an event generator is presented between the controller and sensor to reduce bandwidth waste.

Motivated by the above theoretical analyses, we study the modeling and exponential stability problems for MJMNNs via ETICS. The major contributions of this paper are as follows:

1. We study the exponential stability of general MNNs with markovian jumping and hybrid time-vary delays. The markovian jumping is used to solve the information latching problem. Meanwhile the discrete delay and distributed delay are both considered to make the model more general than previous results.

2. A new ETICS is applied to reduce the transmission of state information. The parameter of ETS is determined by markovian jumping. The ETS can not only determine whether there is an impulse generation, but also regulate the state estimator of the system.

3. Using the average values of the maximum and minimum of memristive synaptic weights can successfully convert the MNNs into traditional neural networks with uncertain parameters. And this method is much less conservative than using the maximum absolute values of memristive synaptic weights.

4. By constructing augmented Lyapunov-Krasovskii, which uses more information on the system and is effective for conservatism reducing, several new sufficient conditions that ensure MJMNNs with ETICS is exponential stable are derived. Then the gain matrices of proposed state estimator can be obtained by solving the proposed LMI conditions.

The rest of the paper is organized as follows. The model of error system is obtained and the preliminaries are given in section 2. The criteria of exponential stable of the error system are offered in section 3. In section 4, a numerical example is given to illustrate the effectiveness of our results. Finally, conclusions are drawn in section 5.

Through this paper, we have following notations. N denotes the set of positive integers. \( \mathbb{R}^n \) denotes the n-dimensional Euclidean space. \( \mathbb{R}^{n \times n} \) is the set of \( n \times n \) real matrices. * represents a term induced by symmetry. diag(·) denotes a block diagonal matrix. \( P^T \) is the transpose of matrix P. \( P \geq 0 \) (P < 0), where \( P \in \mathbb{R}^{n \times n} \), means that P is real positive semidefinite matrix (negative definite matrix). I and 0 are the identity and zero matrices with appropriate dimensions, respectively. \( \text{col}[E] \) is the closure of the convex hull of set E.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

In this paper, we discuss the markovian jumping MNNs of the following form:

\[
\dot{x}_i(t) = -a_i(\varphi(t))x_i(t) + \sum_{j=1}^{n} w_{ij}(x_i(t), \varphi(t))f_j(x_j(t)) + \sum_{j=1}^{n} w_{ij}(x_i(t), \varphi(t))f_j(x_j(t) - \tau_j(t)) + \sum_{j=1}^{n} w_{ij}(x_i(t), \varphi(t)) \int_{t-\rho(t)}^{t} f_j(x_j(s)) ds
\]

where \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n \) is the state vector, \( a_i(\varphi(t)) > 0 \) is the ith self-feedback connection weight, \( w_{0ij}(x_i(t), \varphi(t)), w_{1ij}(x_i(t), \varphi(t)), w_{2ij}(x_i(t), \varphi(t)) \) represent connection weights with the random jumping process \( \varphi(t) \). \( f_j(\cdot) \) is the activation functions. \( \tau_j(t) \) is the time-varying delay and \( \rho(t) \) is the distributed delay. They satisfy

\[
0 \leq \tau_j(t) \leq \tau_b, \quad \dot{\tau}_j(t) \leq \mu, \quad 0 < \rho(t) \leq \bar{\rho}
\]

where \( \tau_b, \mu \) and \( \bar{\rho} \) are constants. Let \( \varphi(t), t \geq 0 \) be a continuous-time markovian process with right continuous trajectories and taking values in a given finite set \( \mathbb{S} = \{1, 2, \ldots, N\} \), and the transfer rate is described as follows

\[
Pr\{\varphi(t + \Delta t) = q|\varphi(t) = p\} = \begin{cases} 
\pi_{pq} \Delta t + o(\Delta t), & \text{if } q \neq p \\
1 + \pi_{pq} \Delta t + o(\Delta t), & \text{if } q = p
\end{cases}
\]
where $\Delta t > 0$, $\lim_{\Delta t \to 0} \frac{\phi(\Delta t)}{\Delta t} = 0$. If $q \neq p$, then $\pi_{pq} \geq 0$ represents the transition rate from $p$ to $q$, otherwise $\pi_{pp} = -\sum_{q=1,q \neq p}^{N} \pi_{pq}$.

In (1), feedback weights $w_{0ij}(x_{i}(t), q(t))$, $w_{1ij}(x_{i}(t), q(t))$ and $w_{2ij}(x_{i}(t), q(t))$, should be relevant to the memristor. According to the bipolar switching scheme, a generalized definition of the memristive feedback weights is given below.

$$
\begin{align*}
\hat{w}_{0ij}(t) &= \begin{cases} 
\hat{w}_{0ij}(q(t)), & |x_{i}(t)| \leq T_{j}, \\
\hat{w}_{0ij}(q(t)), & |x_{i}(t)| > T_{j}, 
\end{cases} \\
\hat{w}_{1ij}(t) &= \begin{cases} 
\hat{w}_{1ij}(q(t)), & |x_{i}(t)| \leq T_{j}, \\
\hat{w}_{1ij}(q(t)), & |x_{i}(t)| > T_{j}, 
\end{cases} \\
\hat{w}_{2ij}(t) &= \begin{cases} 
\hat{w}_{2ij}(q(t)), & |x_{i}(t)| \leq T_{j}, \\
\hat{w}_{2ij}(q(t)), & |x_{i}(t)| > T_{j}, 
\end{cases}
\end{align*}
$$

for $i, j = 1, \ldots, n$, $T_{i} \geq 0$ represents the switching jumps, $\hat{w}_{0ij}(q(t))$, $\hat{w}_{0ij}(q(t))$, $\hat{w}_{1ij}(q(t))$, $\hat{w}_{1ij}(q(t))$, $\hat{w}_{2ij}(q(t))$, $\hat{w}_{2ij}(q(t))$ are known constant.

Let

$$
\hat{w}_{0ij}(q(t)) = \max\{\hat{w}_{0ij}(q(t)), \hat{w}_{0ij}(q(t))\},
$$

$$
\hat{w}_{0ij}(q(t)) = \min\{\hat{w}_{0ij}(q(t)), \hat{w}_{0ij}(q(t))\},
$$

$$
\hat{w}_{1ij}(q(t)) = \max\{\hat{w}_{1ij}(q(t)), \hat{w}_{1ij}(q(t))\},
$$

$$
\hat{w}_{1ij}(q(t)) = \min\{\hat{w}_{1ij}(q(t)), \hat{w}_{1ij}(q(t))\},
$$

$$
\hat{w}_{2ij}(q(t)) = \max\{\hat{w}_{2ij}(q(t)), \hat{w}_{2ij}(q(t))\},
$$

$$
\hat{w}_{2ij}(q(t)) = \min\{\hat{w}_{2ij}(q(t)), \hat{w}_{2ij}(q(t))\},
$$

$$
col\{\hat{w}_{0ij}(q(t)), \hat{w}_{0ij}(q(t))\} = \{\hat{w}_{0ij}(q(t)), \hat{w}_{0ij}(q(t))\},
$$

$$
col\{\hat{w}_{1ij}(q(t)), \hat{w}_{1ij}(q(t))\} = \{\hat{w}_{1ij}(q(t)), \hat{w}_{1ij}(q(t))\},
$$

$$
col\{\hat{w}_{2ij}(q(t)), \hat{w}_{2ij}(q(t))\} = \{\hat{w}_{2ij}(q(t)), \hat{w}_{2ij}(q(t))\}.
$$

Based on the theories of set-valued maps and differential inclusions, there exist $w_{0ij}(q(t)) \in [w_{0ij}(q(t)), w_{0ij}(q(t))]$, $w_{1ij}(q(t)) \in [w_{1ij}(q(t)), w_{1ij}(q(t))]$, $w_{2ij}(q(t)) \in [w_{2ij}(q(t)), w_{2ij}(q(t))]$, so that the system (1) can be written as follows.

$$
\begin{align*}
\dot{x}_{i}(t) &= -a_{i}(q(t))x_{i}(t) + \sum_{j=1}^{n} w_{0ij}(q(t))f_{j}(x_{j}(t)) \\
&\quad + \sum_{j=1}^{n} w_{1ij}(q(t))f_{j}(x_{j}(t) - \tau_{j}(t)) \\
&\quad + \sum_{j=1}^{n} w_{2ij}(q(t)) \int_{t-\rho_{j}(t)}^{t} f_{j}(x(s))ds.
\end{align*}
$$

(2)

To state conveniently, transform system (2) into the following vector form

$$
\begin{align*}
\dot{x}(t) &= -\hat{A}(q(t))x(t) + \hat{W}_{0}(q(t))f(x(t)) + \hat{W}_{1}(q(t)) \\
&\quad \times f(x(t - \tau(t)) + \hat{W}_{2}(q(t)) \int_{t-\rho(t)}^{t} f(x(s))ds.
\end{align*}
$$

(3)

where $x(t) = (x_{1}(t), \ldots, x_{n}(t))^{T}$, $f(x(t)) = (f_{1}(x_{1}(t)), \ldots, f_{n}(x_{n}(t)))^{T}$, $f(x(t - \tau(t))) = (f_{1}(x_{1}(t - \tau_{1}(t))), \ldots, f_{n}(x_{n}(t - \tau_{n}(t))))^{T}$, $\hat{A}(q(t)) = diag(a_{1}(q(t)), \ldots, a_{n}(q(t)))^{T}$, $\hat{W}_{0}(q(t)) = (w_{01}(q(t))^\top, \ldots, w_{0n}(q(t))^\top)_{n \times N}$, $\hat{W}_{1}(q(t)) = (w_{11}(q(t))^\top, \ldots, w_{1n}(q(t))^\top)_{n \times N}$, $\hat{W}_{2}(q(t)) = (w_{21}(q(t))^\top, \ldots, w_{2n}(q(t))^\top)_{n \times N}$.

The output measurement of system (3) can be assumed to satisfy

$$
y(t) = \hat{C}(q(t))x(t)
$$

(4)

where $y(t) \in \mathbb{R}^{m}$, $\hat{C}(q(t))$ represent connection weights with the random jumping process $q(t)$ and is known as a matrix with appropriate dimensions.

Let $\hat{W}_{0}(q(t)) = W_{0}(q(t)) + \Delta W_{0}(q(t))$, $\hat{W}_{1}(q(t)) = W_{1}(q(t)) + \Delta W_{1}(q(t))$, $\hat{W}_{2}(q(t)) = W_{2}(q(t)) + \Delta W_{2}(q(t))$, where

$$
\hat{W}_{0}(q(t)) = \frac{(w_{01}(q(t)) + \bar{w}_{01}(q(t)))_{n \times N}}{2},
$$

$$
\hat{W}_{1}(q(t)) = \frac{(w_{11}(q(t)) + \bar{w}_{11}(q(t)))_{n \times N}}{2},
$$

$$
\hat{W}_{2}(q(t)) = \frac{(w_{21}(q(t)) + \bar{w}_{21}(q(t)))_{n \times N}}{2},
$$

where $\Delta W_{0}(q(t))$, $\Delta W_{1}(q(t))$ and $\Delta W_{2}(q(t))$ are the time-varying parameter uncertainties, which assume that the following conditions are satisfied

$$
\begin{align*}
\Delta W_{0}(q(t)) &\leq M(q(t)) \Delta W_{1}(q(t)) \Delta W_{2}(q(t)) \\
&= M(q(t)) \Delta W_{1}(q(t)) \Delta W_{2}(q(t))
\end{align*}
$$

(5)

where $M(q(t))$, $E_{0}(q(t))$, $E_{1}(q(t))$ and $E_{2}(q(t))$ are known real constant matrices for all $q(t) \in S$, $F(q(t))$ is the unknown uncertain time-varying matrix satisfies the following condition

$$
F(q(t))^{T} F(q(t)) \leq I, \quad q(t) \in S.
$$

Remark 1: Different with the methods in [34], [35], which process the memristive weights by using the maximum absolute values, the approach in this paper divides the memristive weights into two parts. They are the average values of the maximum and minimum of memristive weights and uncertainties, respectively. Therefore, this method is effective to transform system (3) to traditional NNs with uncertainties and is less conservative.

Hence, the system with the impulsive controller can then be described by the following impulsive differential equation:

$$
\begin{align*}
\dot{x}(t) &= -\hat{A}(q(t))x(t) + \hat{W}_{0}(q(t))f(x(t)) \\
&\quad + \hat{W}_{1}(q(t))f(x(t) - \tau(t)) \\
&\quad + \hat{W}_{2}(q(t)) \int_{t-\rho(t)}^{t} f(x(s))ds,
\end{align*}
$$

(6)

where $x(t_{k}^{+})$ and $x(t_{k}^{-})$ denotes the right limit and left limit of function $x(t)$ at $t_{k}$, respectively. $t_{k}$ denotes the impulsive moment satisfying $0 \leq t_{0} < t_{1} < t_{2} < \cdots < t_{k} < \cdots$, $\lim_{k \to \infty} t_{k} = \infty$. Without loss of generality, it is assumed that $x(t)$ is right continuous at $t_{k}$, i.e., $x(t_{k}^{-}) = x(t_{k}^{+})$. $J_{k}$ is the impulsive control gain matrix.

Assumption 1: The activation function satisfies

$$
[f(x) - \beta_{1}x]^{T} [f(x) - \beta_{2}x] \leq 0, \quad \forall x \in \mathbb{R}^{n}
$$

(7)

where $\beta_{1}$, $\beta_{2}$ are real matrices with proper dimensions, and $\beta_{2} - \beta_{1} \geq 0$. 

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Remark 2: Considering of the limited network transmission resources, the system transmission should be reduced and the NNs bandwidth should be saved. So the ETS, an effective way to reduce the traffic consumption, is applied to this paper. Because the system in this paper is markovian jumping system, the parameter switching in ETS is also determined by markovian chain.

Then, the event generator will be introduced. Assume that the sensor is time-driven. The sensor measurements are sampled with period $h$, and the sampling instants are $kh(k = 0, 1, \ldots)$. Then, the event-triggered condition is given as follows:

$$
[x((k + j)h) - x(kh)]^T \Phi(q(t)) [x((k + j)h) - x(kh)] 
\leq \sigma x^T ((k + j)h) \Phi(q(t)) x((k + j)h) \quad (8)
$$

where $\Phi(q(t))$ is a positive symmetric definite matrix, $\sigma \in [0, 1)$, $x((k + j)h)$ is the current sampled sensor measurements. $j = 1, 2, \ldots, x(kh)$ is the latest transmitted sensor measurements. When it is not satisfied to the equal (8), the current sampled sensor measurements $x((k + j)h)$ can be transmitted by the event generator.

We consider the following two cases about the interval $[t_k, t_{k+1}].$

Case 1: When $t_{k+1} \leq t_k + h$, a function $d(t)$ is defined as

$$
d(t) = t - t_k \quad t \in [t_k, t_{k+1})
$$

Hence it can be derived that $d(t) \in (0, h].$

Case 2: When $t_{k+1} > t_k + h$, there exists a positive integer $n$, such that

$$
t_{k+1} = t_k + nh,
$$

It can be easily shown that

$$
[t_k, t_{k+1}) = \bigcup_{i=0}^{n-1} D_i
$$

where $D_0 = [t_k, t_k + h)$, $D_i = [t_k + ih, t_k + ih + h)$. Then, it can be defined that

$$
d(t) = \begin{cases} 
  t - t_k, & t \in D_0 \\
  t - t_k - ih, & t \in D_i, i = 1, 2, \ldots, n - 1. 
\end{cases}
$$

We can obtain that $d(t) \in (0, h].$ Meanwhile, $x(t_kh)$ and $t_kh + ih$ with $i = 1, 2, \ldots, n - 1$ satisfy equation (8).

In case 1, define $e_k(t) = 0$ for $t \in [t_k, t_{k+1}).$ In case 2, define

$$
e_k(t) = \begin{cases} 
  0, & t \in D_0 \\
  x(t_kh + ih) - x(t_kh), & t \in D_i, i = 1, 2, \ldots, n - 1. 
\end{cases}
$$

Then, it can be easily derived that for $t \in [t_k, t_{k+1})$

$$
e_k^T(t) \Phi(q(t)) e_k(t) \leq \sigma x^T (t - d(t)) \Phi(q(t)) x(t - d(t)). \quad (9)
$$

It is assumed that the time sequence $0 = t_0 < t_1 < \ldots < t_k < \ldots$ is the sampling time of sampled measurement $\hat{y}(t)$.

Hence,

$$
\hat{y}(t) = y(t_kh) = \tilde{C}(q(t)) x(t_kh), \quad t_k \leq t < t_{k+1}. \quad (10)
$$

Then the state estimator with actual input $\hat{y}(t) \in \mathbb{R}^m$ can be designed as follows:

$$
\begin{cases}
\dot{x}(t) = -\hat{A}(q(t)) \hat{x}(t) + K(q(t)) \hat{y}(t) - \tilde{C}(q(t)) \hat{x}(t), \\
\hat{x}(t_k) - \hat{x}(t_k^-) = J_k \hat{x}(t_k^-), \quad t_k < t. \quad (11)
\end{cases}
$$

where $\hat{x}(t)$ is the estimation of the neuron state, and $K(q(t))$ is the feedback gain matrix.

Note that when $q(t) = p$, where $p \in S$, $S = \{1, 2, \ldots, N\}$, then $\hat{A}(q(t)) = A_p$, $\hat{A}(q(t)) = \tilde{A}_p$, $\hat{W}qq(q(t) = p) = \hat{W}_{1xp}$, $\hat{W}_{1xx}(q(t) = p) = \hat{W}_{2xp}$, $\hat{C}(q(t) = p) = C_p$, $K(q(t) = p) = K_p$.

Denote $e(t) = x(t) - \hat{x}(t)$. Then from equation (6) and (11), the error NNs can be obtained as follows:

$$
e(t) = -[\hat{A}_p + K_p \tilde{C}_p] e(t) + (\hat{A}_p - \hat{A}_p + K_p \tilde{C}_p) x(t) - K_p \tilde{C}_p x(t - d(t)) + \tilde{W}_{1xp} f(x(s))ds \quad (12)
$$

By setting $\tilde{x}(t) = [x^T(t) \quad e^T(t)]^T$, it can be obtained from equation (6) and (12) as the following augmented system:

$$
\begin{cases}
\dot{\tilde{x}}(t) = -\tilde{A}_\tilde{p} \tilde{x}(t) + \tilde{B}_\tilde{p} \tilde{x}(t - d(t)) + \tilde{W}_{1xp} f(H \tilde{x}(t)) \\
+ \tilde{W}_{2xp} \int_{t-r(t)}^{t} f(H \tilde{x}(s))ds + \tilde{K}_p e_k(t) \\
\tilde{x}(t_k) - \tilde{x}(t_k^-) = J_k \tilde{x}(t_k^-). \quad (13)
\end{cases}
$$

where $\tilde{A}_\tilde{p} = \begin{bmatrix} \hat{A}_p & 0 \\ -\hat{A}_p + K_p \tilde{C}_p & \hat{A}_p + K_p \tilde{C}_p \end{bmatrix}$,

$$
\tilde{B}_\tilde{p} = \begin{bmatrix} 0 & 0 \\ -K_p \tilde{C}_p & 0 \end{bmatrix}, \quad \tilde{W}_{1xp} = \begin{bmatrix} \hat{W}_{1xp} & 0 \\ 0 & \hat{W}_{1xp} \end{bmatrix}, \quad \tilde{W}_{2xp} = \begin{bmatrix} \hat{W}_{2xp} & 0 \\ 0 & \hat{W}_{2xp} \end{bmatrix}, \quad \tilde{K}_p = \begin{bmatrix} 0 & J_k \\ 0 & 0 \end{bmatrix}.
$$

Remark 3: In the existing works [36]–[38], there are many continuous control methods, such as linear state feedback control. In order to save the limited network resources, sampled-data control [39] has been adopted. However, the sampled-data is essentially a kind of time-triggered control method and it may translate the unnecessary information. So this paper present the ETICS, which can reduce the wastage of communication bandwidth effectively.

Remark 4: Paper [40] proposed the event-triggered sampling control for stability and stabilization of MNNs with communication delays. The model proposed in this paper is more practical than paper [40], because they consider only MNNs. However, we consider not only markovian jumping but also impulsive disturbance for the MNNs. Meanwhile, the discrete delay and distributed delay are both considered to make the model more realistic.
Definition 1 [4]: Consider the MJMNIs via ETCS system (13). If there exist constants $M \geq 1$ and $\varepsilon > 0$ such that $\|\tilde{x}(t)\| \leq M \Psi e(-\varepsilon t)$, $\forall t \geq 0$, $\Psi = \sup_{-\tau_p < t < 0} \|\tilde{x}(t)\|$, then the origin of this system is exponential stable, where $\varepsilon$ is call the exponential convergence rate.

Lemma 1 (Peng-Park’s Integral Inequality) [41]: For any matrix $\begin{bmatrix} R & U \\ U^* & R \end{bmatrix} \geq 0$, positive scalars $h$ and $d(t)$ satisfying $0 < d(t) < h$, vector function $\tilde{x} : [-h, 0] \to \mathbb{R}^n$ such that the concerned integrations are well defined, then

$$-h \int_{-h}^{t} \chi^T(s) \bar{R} \tilde{x}^T(s) ds \leq \sigma^T \Omega \sigma,$$

where $\sigma = [x^T(t), x^T(t - d(t)), x^T(t - h)]^T$ and

$$\Omega = \begin{bmatrix} R & -U & U \\ -U & 2R + U + U^T & -U \\ U & * & -R \end{bmatrix}.$$

Lemma 2 [42]: For any positive definite matrix $M \in \mathbb{R}^{n \times n}$, scalars $h_2 > h_1 > 0$, vector function $\omega : [h_1, h_2] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:

$$\frac{(h_2^2 - h_1^2)}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \omega^T(s) M \omega(s) ds d\theta \leq - \left( \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \omega(s) ds d\theta \right)^T M \left( \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \omega(s) ds d\theta \right).$$

III. MAIN RESULTS

Theorem 1: For given constants $\tau_p, \mu, \tilde{\omega}, \sigma, h, M, \varepsilon$ and the estimator gain matrix $K_p$, under the ETS (9), the NNs (13) is exponential stable, if there exist symmetric positive definite matrices $P_p, Q, R, \xi$ and scalars $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3 > 0$, any matrices with appropriate dimensions $U$, such that the following LMI hold: $\forall t \in [0, \infty)$,

$$\prod_{i=1}^{k} \beta^p_i e^{-\tilde{\beta} t} \leq M^2, \quad \tilde{\beta} < \varepsilon,$$

$$\begin{bmatrix} R & U \\ * & R \end{bmatrix} \geq 0, \quad \Psi = \begin{bmatrix} \Theta & 0 \\ 0 & J \end{bmatrix} < 0,$$

where

$$\Theta = [\Theta_{ij}]_{8 \times 8}$$

(15)

$$\Theta_{11} = e^{\varepsilon t}(-P_p \tilde{A}_p - \tilde{A}_p^T P_p + \sum_{q=1}^{N} \pi_{pq} P_q + \varepsilon P_p + e^{\varepsilon h} Q - R + e^{\varepsilon h} L_1 - \tilde{1}_1 \tilde{p}_1), \quad \Theta_{12} = e^{\varepsilon t} U, \quad \Theta_{13} = e^{\varepsilon t} (P_p \tilde{B}_p + R - U),$$

$$\Theta_{15} = e^{\varepsilon t} (P_p \tilde{W}_{0xp} + e^{\varepsilon h} L_2 - \tilde{1}_2 \tilde{p}_2), \quad \Theta_{16} = e^{\varepsilon t} (P_p \tilde{W}_{1xp}), \quad \Theta_{17} = e^{\varepsilon t} (P_p \tilde{W}_{2xp}), \quad \Theta_{18} = e^{\varepsilon t} (P_p \tilde{K}_p), \quad \Theta_{22} = e^{\varepsilon t} (R - Q) - \tilde{\omega}, \quad \Theta_{23} = e^{\varepsilon t} (R - U)^T,$$

$$\Theta_{25} = e^{\varepsilon t} (-2R + U + U^T + \sigma^2 \Phi_p), \quad \Theta_{33} = e^{\varepsilon t} (-2R + U + U^T + \sigma^2 \Phi_p), \quad \Theta_{44} = e^{\varepsilon t} (-\mu L_1 - \tilde{1}_2 \tilde{p}_2), \quad \Theta_{46} = e^{\varepsilon t} (-\mu L_2 - \tilde{1}_2 \tilde{p}_2),$$

$$\Theta_{55} = e^{\varepsilon t} (\tilde{p}_3 L_3 + \tilde{p}_3 e^{\varepsilon t} \tilde{\omega} - \tilde{1}_1 \tilde{p}_1), \quad \Theta_{66} = e^{\varepsilon t} (-\mu L_3 - \tilde{1}_2 \tilde{p}_2), \quad \Theta_{77} = e^{\varepsilon t} (-\frac{\tau}{\mu} \tilde{p}_5), \quad \Theta_{88} =$$

$$e^{\varepsilon t}(-\Phi_p). \quad O = [-\tilde{A}_p, 0, \tilde{B}_p, 0, \tilde{W}_{0xp}, \tilde{W}_{1xp}, \tilde{W}_{2xp}, \tilde{K}_p]^T,$$
\[ \ell \mathcal{V}_d(\tilde{x}_t, t, \rho) \leq e^{\ell t}(e^{\rho t} \mathcal{F}^T (H \tilde{x}(t)) \hat{\xi}_f (H \tilde{x}(t))) - \frac{1}{\hat{\rho}} \left( \int_{t-\rho(t)}^t f(H \tilde{x}(s)) ds \right)^T \hat{\xi}(\int_{t-\rho(t)}^t f(H \tilde{x}(s)) ds). \]

From the inequality (9), we can get

\[ \sigma \tilde{T}(t - d(t)) \Phi_p \varepsilon(t - d(t)) - e^{\ell t(t)} \Phi_p e_k(t) \geq 0 \quad (18) \]

where \( \Phi_p = \begin{bmatrix} \Phi_0 & 0 \\ 0 & 0 \end{bmatrix} \).

From equation (7), one have

\[ \begin{bmatrix} \tilde{x}(t) \\ f(H \tilde{x}(t)) \end{bmatrix}^T \begin{bmatrix} \hat{\beta}_1 & \hat{\beta}_2 \\ \hat{\beta}_1 & I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ f(H \tilde{x}(t)) \end{bmatrix} \leq 0 \quad (19) \]

where \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \) are defined in equation (15), and \( H \) is shown in equation (13). Hence, for any scalars \( \tilde{\lambda}_1 > 0, \tilde{\lambda}_2 > 0 \), we can derive the following equations:

\[ \begin{align*}
- \tilde{\lambda}_1 \begin{bmatrix} \tilde{x}(t) \\ f(H \tilde{x}(t)) \end{bmatrix}^T \begin{bmatrix} \hat{\beta}_1 & \hat{\beta}_2 \\ \hat{\beta}_1 & I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ f(H \tilde{x}(t)) \end{bmatrix} & \geq 0 \\
- \tilde{\lambda}_2 \begin{bmatrix} \tilde{x}(t - \tau(t)) \\ f(H \tilde{x}(t - \tau(t))) \end{bmatrix}^T \begin{bmatrix} \hat{\beta}_1 & \hat{\beta}_2 \\ \hat{\beta}_1 & I \end{bmatrix} \begin{bmatrix} \tilde{x}(t - \tau(t)) \\ f(H \tilde{x}(t - \tau(t))) \end{bmatrix} & \geq 0.
\end{align*} \quad (20) \]

Combining equations (17) - (20) and using Schur complement Lemma, we get

\[ \mathcal{L} \mathcal{V}(\tilde{x}(t), t, p) \leq \chi^2(t)\Theta \chi(t) + \chi(t)OJ^T \Theta \chi(t) \]

where

\[ \chi^T(t) = (\tilde{x}^T(t - h) \quad \tilde{x}^T(t - d(t)) \quad \tilde{x}^T(t - \tau(t)) f^T(H \tilde{x}(t)) \]

\times f^T(H \tilde{x}(t - \tau(t))) \int_{t-\rho(t)}^t f^T(H \tilde{x}(s)) ds e^T(t). \]

From equation (21), it is clear that \( \mathcal{L} \mathcal{V}(\tilde{x}(t), t, p) \leq 0 \). Note that \( \mathcal{L} \mathcal{V}(\tilde{x}(t), t, p) = 0 \) if and only if \( \mathcal{V}(\tilde{x}(t), t, p) = 0 \). It is easy for us to get

\[ \mathcal{V}(\tilde{x}(0), r(0)) \leq \Lambda^2 \| \phi \|^2 \]

where

\[ \Lambda^2 = \lambda_{\text{max}}(P(r(0))) + h_0 \lambda_{\text{max}}(Q) + 2t_0 \lambda_{\text{max}}(L_2) + 2t_0 \lambda_{\text{max}}(L_4) + 2t_0 \lambda_{\text{max}}(L_4) + 2t_0 \lambda_{\text{max}}(\tilde{x}) \lambda_{\text{max}}(L_2) ^2 + \beta \lambda_{\text{max}}(\tilde{x}) \lambda_{\text{max}}(L_2) + 2t_0 \lambda_{\text{max}}(\tilde{x}) \lambda_{\text{max}}(L_2) + \lambda_{\text{max}}(\tilde{x}) \lambda_{\text{max}}(L_2) + \beta \lambda_{\text{max}}(\tilde{x}) \lambda_{\text{max}}(L_2) \]

When \( t = t_k \),

\[ \begin{align*}
\mathcal{V}(\tilde{x}(t_k), t_k, r(t_k)) &= e^{\ell t_k} \tilde{x}^T(t_k) \mathcal{P}_p \tilde{x}(t_k) + V_2(\tilde{x}(t_k), t_k, r(t_k)) \\
&+ V_3(\tilde{x}(t_k), t_k, r(t_k)) + V_4(\tilde{x}(t_k), t_k, r(t_k)) \\
&\leq e^{\ell t_k} \tilde{x}^T(t_k) \max(\lambda_{\text{min}}(P) \| I + J_k \| P(t) \| I + J_k \|) \tilde{x}(t_k) \\
&+ V_2(\tilde{x}(t_k), t_k, r(t_k)) + V_3(\tilde{x}(t_k), t_k, r(t_k)) + V_4(\tilde{x}(t_k), t_k, r(t_k)) \\
&\leq \max(1, \beta_1) V(\tilde{x}(t_k), t_k, r(t_k)) + V_2(\tilde{x}(t_k), t_k, r(t_k)) \\
&\leq \max(1, \beta_1) V(\tilde{x}(t_k), t_k, r(t_k)) + V_2(\tilde{x}(t_k), t_k, r(t_k)) \\
&= \beta_1^* V(\tilde{x}(t_k), t_k, r(t_k)) \forall t \in [t_k, t_{k+1}].
\end{align*} \]

\[ \begin{align*}
V(\tilde{x}(t), t, \phi(t)) &\leq V(\tilde{x}(t_k), t_k, r(t_k)) \leq \beta_1^* V(\tilde{x}(t_k), t_k, r(t_k)) \\
&\leq \beta_1^* \beta_1^* V(\tilde{x}(t_k), t_k, r(t_k)) \leq \ldots \\
&\leq \beta_1^* \beta_1^* \ldots \beta_1^* V(\tilde{x}(0), 0, 0) \leq \beta_1^* \beta_1^* \ldots \beta_1^* \Lambda^2 \| \phi \|^2.
\end{align*} \]

On the other hand,

\[ \begin{align*}
V(\tilde{x}(t), t, \phi(t)) &\geq e^{\ell t} \tilde{x}^T(t) \mathcal{P}_p \tilde{x}(t) \\
&\geq e^{\ell t} \lambda_{\text{min}}(P)\| \tilde{x}(t) \| ^2 = e^{\ell t} \| \tilde{x}(t) \| ^2
\end{align*} \]

where \( \lambda_{\text{min}}(P) \).

Therefore,

\[ \| \tilde{x}(t) \| \leq \Lambda \| \phi \| e^{\ell (e - \tilde{\beta}) t} \leq \frac{\Lambda M}{P^*} \| \phi \| e^{\ell (e - \tilde{\beta}) t}. \]

This completes the proof.

**Theorem 2:** For given constants \( \tau_p, \mu, \tilde{\rho}, \sigma, h, M, e, \) under the ETS (9), the NNs (13) is exponential stable, if there exist symmetric positive definite matrices \( P_1, P_2 \) such that the following LMI holds: \( \forall t \in [0, \infty) \),

\[ \sum_{i=1}^k \beta_i^* e^{-\beta_i t} \leq M^2, \beta < e \]

\[ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \]

\[ \begin{bmatrix} \tilde{\Theta} & \tilde{\Theta}_I \\ \tilde{\Theta}_I & \tilde{\Theta}_T \end{bmatrix} \leq 0 \quad (22) \]

\[ \tilde{\Theta} = [\tilde{\Theta}_{ij}]_{8 \times 8} < 0 \]

\[ \begin{bmatrix} \Omega_{11} & 0 \\ 0 & \Omega_{22} \end{bmatrix} \]

\[ \Omega_{11} = -P_1 \tilde{\Theta} - \tilde{\Theta}^T P_1 \sum_{q=1}^N \pi_{pq} P_{q1} \in P_1 \]

\[ + e^{\ell h} Q_1 - R_1 + e^{\ell h} L_1 - \frac{\tilde{\beta}_1^* \beta_2^* \beta_1^* \beta_1^*}{2}. \]
\[ \Omega_{21} = P_{p2} \tilde{A}_p - P_{p2} \tilde{A}_p + Y_p \tilde{C}_p, \]
\[ \Omega_{22} = -P_{p2} \tilde{A}_p - Y_p \tilde{C}_p - \tilde{A}_p^T P_{p2} - \tilde{C}_p^T Y_p^T + \sum_{q=1}^{N} \pi_{pq} P_{q2} + \varepsilon P_{p2} + e^{\varepsilon h} Q_1 - R_1 + e^{\varepsilon h} L_{11}. \]

**Theorem 2.**

In addition, the designed control gain matrices are given as \( K_p = P_{p2}^{-1} Y_p \).

**Proof.**

Pre-multiply and post-multiply \( \Psi \) in (14) by \( \text{diag}(I_{24 \times 24}, P_p) \) and \( \text{diag}(I_{24 \times 24}, P_p) \), one can have

\[
\tilde{\Psi} = \begin{bmatrix} \tilde{\Theta} & O \\ -2aP_p + a^2 \tilde{J} & \tilde{\Theta}_1^T F_p \tilde{\Theta}_2 + \tilde{\Theta}_2^T F_p \tilde{\Theta}_1 \end{bmatrix} (24)
\]

Then, (24) can be written as the following form

\[
\tilde{\Psi} \leq \begin{bmatrix} \tilde{\Theta} & O \\ -2aP_p + a^2 \tilde{J} + \alpha_1 \tilde{\Theta}_1^T \tilde{\Theta}_1 + \alpha_1^{-1} \tilde{\Theta}_2^T \tilde{\Theta}_2 \end{bmatrix} (25)
\]

It can be obtained that \( \tilde{\Psi} < 0 \), where \( \tilde{\Psi} \) is defined as (27). The rest of the proof follows directly from Theorem 1. \( \square \)

**Remark 5:**

In Theorem 1, a novel LKF (16) is constructed to deal with the delay terms. The integral term \( V_2(\tilde{x}_1, t, \sigma(t)) \) is used to deal with the sampled function \( d(t) \) of ETS. The integral term \( V_3(\tilde{x}_1, t, \sigma(t)) \) is used to deal with time-varying delay \( \tau(t) \). The integral term \( V_4(\tilde{x}_1, t, \sigma(t)) \) is used to deal with distributed delay \( \rho(t) \). Moreover, compared with Jensen's inequality [43], we use Peng-Park's integral inequality which is less conservative to process integral term \( -h \int_{t-h}^{t} \dot{\tilde{x}}^T(t) \dot{R} \dot{x}(t) ds \).

**Remark 6:**

If the impulsive control and Markovian jump are not considered, the system (13) can be degenerated to the following system:

\[
\ddot{x}(t) = -\hat{A}(t) \dot{x}(t) + \tilde{B}(t)(d(t)) + \tilde{W}_{0, sf}(H \tilde{x}(t)) + \tilde{W}_{1, sf}(H \tilde{x}(t) - \tau(t)) + \tilde{W}_{2, sf}(H \tilde{x}(t)) \int_{t-\rho(t)}^{t} f(H \tilde{x}(s)) ds + \tilde{K} e(t) \]  

where \( \tilde{A} = \begin{bmatrix} \hat{A} & 0 \\ -K \hat{C} & \hat{A} + K \hat{C} \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0 \\ -K \hat{C} \end{bmatrix}, \tilde{W}_{0, sf} = \begin{bmatrix} \tilde{W}_{0x} \\ \tilde{W}_{01} \end{bmatrix}, \tilde{W}_{1x} = \begin{bmatrix} \tilde{W}_{1x} \\ \tilde{W}_{11} \end{bmatrix}, \tilde{W}_{2x} = \begin{bmatrix} \tilde{W}_{2x} \\ \tilde{W}_{21} \end{bmatrix}, \tilde{K} = \begin{bmatrix} 0 \\ K \hat{C} \end{bmatrix}, H = [I \ 0]. \]

**Corollary 1:**

For given constants \( \tau_p, \tilde{p}, \beta, \sigma, h, M, \epsilon \), under the ETS (9), if there exist symmetric positive definite matrices \( P = \text{diag}((P_1, P_2), Q = \text{diag}(Q_1, Q_1), L_j = \text{diag}(L_{j1}, L_{j1})(j = 1, 2, 3), R = \text{diag}(R_1, R_1), \xi = \text{diag}(\xi_1, \xi_1), \), scalars \( I_1, I_2, \) and positive constant \( a \) and \( \alpha_1 \), any positive definite diagonal matrices \( U = \text{diag}(U_1, U_1) \) such that the following LMIs hold:

\[
\begin{bmatrix} R & U \\ \ast & R \end{bmatrix} \geq 0,
\]

\[
\tilde{\Psi} = \begin{bmatrix} \tilde{\Theta} & \tilde{O} & \tilde{\Theta}_1^T & \tilde{\Theta}_2^T \\ \ast & -2aP_p + a^2 \tilde{J} - \frac{1}{\alpha_1} I & 0 & 0 \\ \ast & \ast & -\frac{1}{\alpha_1} I & 0 \\ \ast & \ast & \ast & -\alpha_1 I \end{bmatrix} < 0 \]  

where \( \tilde{\Theta} \) and other parameters are same as the definition in the theorem 2.
IV. NUMERICAL EXAMPLES

In this section, a numerical example is presented to demonstrate the effectiveness of the results derived in this paper.

Example 1: Consider the MJMNMs (13) via ETICS with two modes. For modes 1 and 2, the dynamics of system with following parameters are described as

\[
\begin{align*}
    w_{011}(x_1(t), 1) &= \begin{cases} 
        0.8, & \text{for } |x_1(t)| \leq 0.5 \\
        1.0, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{012}(x_1(t), 1) &= \begin{cases} 
        0.7, & \text{for } |x_1(t)| \leq 0.5 \\
        0.9, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{021}(x_2(t), 1) &= \begin{cases} 
        -2.2, & \text{for } |x_2(t)| \leq 0.5 \\
        -2.4, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    w_{022}(x_2(t), 1) &= \begin{cases} 
        0.3, & \text{for } |x_2(t)| \leq 0.5 \\
        0.5, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    w_{111}(x_1(t), 1) &= \begin{cases} 
        0.2, & \text{for } |x_1(t)| \leq 0.5 \\
        0.4, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{112}(x_1(t), 1) &= \begin{cases} 
        0.4, & \text{for } |x_1(t)| \leq 0.5 \\
        0.6, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{121}(x_2(t), 1) &= \begin{cases} 
        0.3, & \text{for } |x_2(t)| \leq 0.5 \\
        0.5, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    w_{122}(x_2(t), 1) &= \begin{cases} 
        0.8, & \text{for } |x_2(t)| \leq 0.5 \\
        1.0, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    w_{211}(x_1(t), 1) &= \begin{cases} 
        0.4, & \text{for } |x_1(t)| \leq 0.5 \\
        0.6, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{212}(x_1(t), 1) &= \begin{cases} 
        0.2, & \text{for } |x_1(t)| \leq 0.5 \\
        0.4, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{221}(x_2(t), 1) &= \begin{cases} 
        0.5, & \text{for } |x_2(t)| \leq 0.5 \\
        0.7, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    w_{222}(x_2(t), 1) &= \begin{cases} 
        0.3, & \text{for } |x_2(t)| \leq 0.5 \\
        0.5, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    \tilde{A}(1) &= \begin{bmatrix} 
        4 & 0 \\
        0 & 7 
    \end{bmatrix}, \quad \tilde{C}(1) = \begin{bmatrix} 
        3 & 0 \\
        0 & 2 
    \end{bmatrix}, \quad \hat{A}(1) = \begin{bmatrix} 
        3 & 0 \\
        0 & 8 
    \end{bmatrix}.
\end{align*}
\]

\[
\begin{align*}
    w_{111}(x_1(t), 2) &= \begin{cases} 
        -0.5, & \text{for } |x_1(t)| \leq 0.5 \\
        -0.7, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{112}(x_1(t), 2) &= \begin{cases} 
        0.1, & \text{for } |x_1(t)| \leq 0.5 \\
        0.3, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{121}(x_2(t), 2) &= \begin{cases} 
        0.6, & \text{for } |x_2(t)| \leq 0.5 \\
        0.8, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    w_{122}(x_2(t), 2) &= \begin{cases} 
        0.4, & \text{for } |x_2(t)| \leq 0.5 \\
        0.6, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    w_{211}(x_1(t), 2) &= \begin{cases} 
        0.6, & \text{for } |x_1(t)| \leq 0.5 \\
        0.8, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{212}(x_1(t), 2) &= \begin{cases} 
        0.3, & \text{for } |x_1(t)| \leq 0.5 \\
        0.5, & \text{for } |x_1(t)| > 0.5 
    \end{cases} \\
    w_{221}(x_2(t), 2) &= \begin{cases} 
        -0.2, & \text{for } |x_2(t)| \leq 0.5 \\
        -0.4, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    w_{222}(x_2(t), 2) &= \begin{cases} 
        -0.05, & \text{for } |x_2(t)| \leq 0.5 \\
        -0.15, & \text{for } |x_2(t)| > 0.5 
    \end{cases} \\
    \tilde{A}(2) &= \begin{bmatrix} 
        5 & 0 \\
        0 & 4 
    \end{bmatrix}, \quad \tilde{C}(1) = \begin{bmatrix} 
        6 & 0 \\
        0 & 9 
    \end{bmatrix}, \quad \hat{A}(1) = \begin{bmatrix} 
        4 & 0 \\
        0 & 6 
    \end{bmatrix}, \\
    \beta_1 &= \begin{bmatrix} 
        0.3 & 0.2 \\
        0 & 2 
    \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} 
        0.5 & 0.2 \\
        0 & 0.95 
    \end{bmatrix}, \\
    J_k &= \begin{bmatrix} 
        0.3 & 0 \\
        0 & 0.3 
    \end{bmatrix}.
\end{align*}
\]

Let the nonlinear function be given by \( f(x_i) = (0.5x_{1i}(t) - \tanh(0.2x_{1i}(t))) + 0.2x_{2i}(t), 0.95x_{2i}(t) - \tanh(0.75x_{2i}(t)))^T \). The constant values are given as \( \tau_b = 0.4, \mu = 0.1, \varepsilon = 0.1, \theta = 0.1, \tilde{\vartheta} = 0.2, \sigma = 0.3 \).

The switching modes is transformed by the following markov transition probability matrix:

\[
\begin{bmatrix} 
    \pi_{11} & \pi_{12} \\
    \pi_{21} & \pi_{22} 
\end{bmatrix} = \begin{bmatrix} 
    -0.7 & 0.7 \\
    0.3 & -0.3 
\end{bmatrix}.
\]

Then, the control gain matrices and the event-triggered matrices can be calculated as follows:

\[
\begin{align*}
    K(1) &= 10^{-3} \times \begin{bmatrix} 
        0.5169 & -0.1918 \\
        0.8835 & 0.6896 
    \end{bmatrix}, \\
    K(2) &= 10^{-3} \times \begin{bmatrix} 
        -0.2145 & -0.2026 \\
        0.1202 & 0.3398 
    \end{bmatrix}, \\
    \Phi(1) &= \begin{bmatrix} 
        0.1090 & 0.0217 \\
        0.0217 & 0.0460 
    \end{bmatrix}, \\
    \Phi(2) &= \begin{bmatrix} 
        0.0865 & 0.0242 \\
        0.0242 & 0.0439 
    \end{bmatrix}.
\end{align*}
\]

The simulations are shown in Figures 1-4. Figures 1 shows the \( x(t) \) and its estimation. Figure 2 shows the estimation error \( e(t) \). Figure 3 draws one of the possible realisations of the markovian jumping mode. Figure 4 depicts the release instants and release intervals. Table 1 shows that the triggering times by using ETICS is less than other methods and average release period is longer in 10s. Meanwhile,
the ETICS drastically reduces the data transmission rate. All three indicators prove that ETICS is effectiveness to reduce the cost of bandwidth. From above simulation results, we can get that the system (13) with the conditions in Theorem 2 is exponential stable.

V. CONCLUSION

In this paper, the problem of MJMNNs has been investigated. In order to save the NNs bandwidth, ETICS is applied into this paper. Then, a state observer is designed to estimate the neuron states. By constructing augmented LKF, serval stability criteria and gain matrix were obtained. Finally, a numerical example has been offered to validate the effectiveness of our results. Because the hybrid delays and markovian jump are considered, our model is more practical. In addition, compared with other methods, ETICS’ triggering times is less and average release period is longer, which is more effective to reduce transmit rate. In the future work, the dynamics of semi-markovian jump Memristor-based Neural Networks will be studied. Meanwhile, the new event-triggering conditions can be researched.

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