Leptogenesis and Low energy CP violation, a link

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Abstract. How is CP violation of low energy related to CP violation required from baryon number asymmetry? We give an example which shows a direct link between CP violation of neutrino oscillation and baryogenesis through leptogenesis.

When the sphaleron process is active, the sum of baryon number and lepton number is not a conserved quantity; \( \frac{d (B+L)}{dt} = \text{Anomaly} \neq 0 \). Therefore, the evolution equation of baryon number and lepton number becomes a coupled equation. The present baryon number can be written in terms of the "initial" lepton number and baryon number as:

\[
B_{\text{now}} = \frac{1}{2} (B - L)_{\text{ini}} + \frac{1}{2} (B + L)_{\text{ini}} \exp \left[ -\frac{\Delta t}{\tau} \right] - \frac{1}{2} (B - L)_{\text{ini}}.
\]

Fukugita and Yanagida proposed "baryogenesis without Grand unification" [1]. Then, \( B_{\text{ini}} = 0 \) while the lepton number production is possible because their model includes the heavy Majorana neutrinos and their decays lead to the lepton number asymmetry:

\[
L_{\text{ini}} \sim \Gamma[N \rightarrow l^- \phi^+] - \Gamma[N \rightarrow l^+ \phi^-].
\]

The purpose of my talk is give a specific example which shows "a direct link" between the size and sign of baryon number and CP violation in neutrino oscillation:

\[
P(\nu_{\mu} \rightarrow \nu_e) - P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e) \sim J = \text{Im} (K_{e1}K_{e2}^*) (K_{\mu1}K_{\mu2}^*)^*,
\]

where \( K \) is MNS matrix.

\[
J^{cc}_{\mu} = \bar{l}_L\gamma_\mu K_{\nu L}.
\]

\( J \sim P - \bar{P} = \Delta P \) is related to MNS matrix. By taking the basis in which the mass matrix for the heavy Majorana neutrinos and charged leptons are real diagonal, the MNS matrix can be obtained through the diagonalization of \( m_{\text{eff}} = -m_D \frac{1}{\tau} m_D^T \).

On the other hand the lepton number asymmetry in the same basis is given as:

\[
\epsilon_1 = \frac{\Gamma[N_1 \rightarrow l^- \phi^+] - \Gamma[N_1 \rightarrow l^+ \phi^-]}{\Gamma[N_1 \rightarrow l^- \phi^+] + \Gamma[N_1 \rightarrow l^+ \phi^-]} = -\frac{3M_1 \text{Im}(m_1^c m_D^*_{12})}{2M_2 \text{V}^2(m_D^* m_D)_{11}},
\]

where \( V \sim 1 \) (TeV) and we take \( M_2 \gg M_1 \). We consider "the minimal seesaw" which generates \( L \neq 0 \) and \( \Delta P \neq 0 \) simultaneously. The minimal model is (3, 2) model with 3 light neutrinos \( \nu_1, \nu_2, \nu_3 \) and 2 heavy Majorana neutrinos \( N_1 \) and \( N_2 \).

\[
\mathcal{L} = \bar{l} m_l l + \bar{\nu} m_D \nu_R + \frac{1}{2} (\bar{N}_R) c M J N_R,
\]

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In Fig. 1 and Fig. 2, we show the correlation (sin 2γ)

\[ V \] 

writing orthogonal matrix \( U \) as we have;

\[ m_D = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{pmatrix} = U_L \begin{pmatrix} 0 & 0 \\ m_2 & 0 \\ 0 & m_3 \end{pmatrix} V_R, \]

(7)

where \( U_L (V_R) \) is a 3 × 3 (2 × 2) unitary matrix. The important properties of the model are one light neutrino is exactly massless: \( \text{det}[m_D + m_D^T] = 0 \) and there are three CP violating phases since there are 3 = 6 − 3 imaginary elements in \( m_D \). By writing \( V_R \) as follows;

\[ V_R = \begin{pmatrix} c_R & s_R \\ -s_R & c_R \end{pmatrix} \text{exp}(i \frac{2 \gamma}{3}) \] 

we can show the leptogenesis is determined by a CP phase γ\(_R\),

\[ \epsilon_1 \sim -Im[(\bar{m}_D^2m_D^2)] \]

\[ \sim -(m_3^2 - m_2^2)^2 s_R^2 c_R^2 \sin 2 \gamma_R. \]

On the other hand, CP violation in neutrino oscillation, \( J \) in Eq.(3), depends on all three CP violating phases because \( K \) is determined by the diagonalization as \( -K^+m_D + m_D K^+ \) and it is sensitive to all CP phases in \( m_D \). We give an example for the model in which \( J \) is determined by leptogenesis phase γ\(_R\). Suppose \( U_L \) is a real orthogonal matrix [2] as \( U_L = O_{23}O_{12} \). MNS matrix \( K \) has the following form;

\[ K = O_{23}O_{12} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \exp(-i\phi) \\ 0 & \sin \theta \exp[i\phi] & \cos \theta \end{pmatrix} P, \]

(10)

where \( P \) is a diagonal phase matrix which is not relevant for \( J \), \( \theta \) and \( \phi \) are determined through the diagonalization of \( -U_L^T m_D V_R^T V_R m_D^T U_R^T \). Therefore \( \theta \) and \( \phi \) do not depend on \( U_L \) at all. It depends on \( m_2, m_3, \theta_R, \gamma_R \) besides \( M_1 \) and \( M_2 \). Those four quantities can be determined from the heavy Majorana decay width, \( \Gamma_1 \) and \( \Gamma_2 \) and two light neutrino masses scales \( \Delta m_{\text{atm}} \) and \( \Delta m_{\text{sol}} \). Taking \( \theta_{23} = \pi/4, \theta_{12} = \pi/4 \), we have;

\[ K = \begin{pmatrix} 1 & \frac{\cos \theta}{\sqrt{2}} & \frac{\sin \theta \exp(-i\phi)}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\cos \theta - \sqrt{2} \sin \theta \exp[i\phi]}{2} & \frac{\sin \theta \exp[i\phi]}{2} \\ \frac{1}{2} & \frac{\cos \theta + \sqrt{2} \sin \theta \exp[i\phi]}{2} & -\frac{\sin \theta \exp[i\phi]}{2} \end{pmatrix} P. \]

(11)

It is easy to see \( J \sim \sin \phi \). We can show there is a correlation between \( \phi \) and \( \gamma_R \). If \( \gamma_R \) vanishes, \( m_{\ell J} \) becomes a real symmetric matrix. Then, \( \phi \) must vanish in the limit. On the other hand, if \( \gamma_R \) does not vanish, the sign of \( \gamma_R \) determines the sign of lepton number asymmetry which in turn determines the excess of matter (anti-matter). In our model, the sign of \( J \) reflects the sign of \( \gamma_R \). We found the following correlation holds.

\[ B \rightarrow -B \Leftrightarrow L \rightarrow -L \Leftrightarrow \gamma_R \rightarrow -\gamma_R \Leftrightarrow \phi \rightarrow -\phi \Leftrightarrow J \rightarrow -J. \]

(12)

In Fig. 1 and Fig. 2, we show the correlation (sin 2γ\(_R\), sin \( \phi \), \( x + y \)), where \( x = \frac{\Gamma_1 V^2}{m_1^2} \) and \( y = \frac{\Gamma_2 V^2}{m_2^2} \), by identifying the two light neutrino masses as \( \Delta m_{\text{atm}}^2 \approx 5.5 \times 10^{-2} \) eV and \( \sqrt{\Delta m_{\text{sol}}^2} = (4 \sim 5) \times 10^{-3} \) (LMA). We take \( \frac{M_1}{M_2} = 0.1 \). The figures are obtained for fixed \((x + y) \times 10^{-2}\) (eV) and varying \( u = x - y \).
Summary

- leptogenesis phase ($\gamma_R$) certainly affects the neutrino oscillation CP violation through ($\phi$). However, if we measure $J$ only, we can not distinguish the phase for the leptogenesis ($\phi$) from the other phases in $U_L$ because only a certain combination of them appear. The isolation must be done using some other quantities, double $\beta$ decay etc [3].
- We show the correlation between CP violating phase for leptogenesis and CP violating phase for neutrino oscillation for a specific choice of $U_L$.

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