Newtonian gravity from Higgs condensates

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Abstract

We propose a description of Newtonian gravity as a long wavelength excitation of the scalar condensate inducing electroweak symmetry breaking. Indeed, one finds a $-\frac{G_F m_i m_j}{\eta r}$ long-range potential where $G_F$ is the Fermi constant and $\eta \equiv \frac{M^2}{2m^2}$ is determined by the ratio between the Higgs mass $M_h$ and the mass $m$ of the elementary quanta of the symmetric phase (‘phions’). The parameter $\eta$ would diverge in a true continuum theory so that its magnitude represents a measure of non-locality of the underlying field theory. By identifying $G \equiv \frac{G_F}{\eta}$ with the Newton constant and assuming the range of Higgs mass $M_h \sim 10^2 - 10^3$ GeV one obtains $m = 10^{-4} - 10^{-5}$ eV and predicts typical ‘fifth-force’ deviations below the centimeter scale. Relation to Einstein gravity and string theory is discussed. The crucial role of the first-order nature of the phase transition for the solution of the so-called ‘hierarchy problem’ is emphasized. The possible relevance of the picture for the self-similarity of the universe and for a new approach to the problem of dark matter is discussed.
1. Introduction

In this paper we shall present a simple mechanism to explain the physical origin of Newtonian gravity. However, the motivations for our proposal depend crucially on the description of spontaneous symmetry breaking (SSB) in scalar quantum field theories. For this reason, we shall address the main issue in sects. 2 and 3, after the preliminary discussion presented in this Introduction.

The ‘condensation’ of a scalar field, i.e. the transition from a symmetric phase where $\langle \Phi \rangle = 0$ to the physical vacuum where $\langle \Phi \rangle \neq 0$, has been traditionally described as an essentially classical phenomenon (with perturbative quantum corrections). In this picture, one uses a classical potential

$$V_{cl}(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4$$

where the phase transition, as one varies the $m^2$ parameter, is second order and occurs at $m^2 = 0$.

As discussed in ref. [1], the question of vacuum stability is more subtle in the quantum theory. Here, the starting point is the Hamiltonian operator

$$H = \int d^3 x \left[ \frac{1}{2} \left( \Pi^2 + (\nabla \Phi)^2 + m^2 \Phi^2 \right) + \frac{\lambda}{4!} \Phi^4 \right] :$$

after quantizing the scalar field $\Phi$ and the canonical momentum $\Pi$ in terms of annihilation and creation operators $a_k$, $a_k^\dagger$ of a reference vacuum state $|o\rangle$ ($a_k|o\rangle = (o|a_k^\dagger \rangle = 0$). These satisfy the commutation relations

$$[a_k, a_{k'}^\dagger] = \delta_{k,k'}.$$  \hspace{1cm} (1.3)$$

and, due to normal ordering, the quadratic part of the Hamiltonian has the usual form ($E_k = \sqrt{k^2 + m^2}$)

$$H_2 = \sum_k E_k a_k^\dagger a_k.$$ \hspace{1cm} (1.4)$$

for the elementary quanta of the symmetric phase (‘phions’).

Now the trivial vacuum $|o\rangle$ where $\langle \Phi \rangle = 0$ is clearly locally stable if phions have a physical mass $m^2 > 0$. However, is an $m^2 > 0$ symmetric vacuum necessarily globally stable? Could the phase transition actually be first order, occurring at some small but positive value of the physical mass squared $m^2 > 0$? The question is not entirely trivial just because [1] the static limit of the 2-body phion-phion interaction is not always repulsive. Besides the tree-level repulsive potential there is an induced attraction from higher-order
graphs. In this case, for sufficiently small values of \( m \), the trivial ‘empty’ state \(|\phi\rangle\) may not be the physical vacuum.

The answer to the question depends on the form of the effective potential \( V_{\text{eff}}(\phi) \) and it is not surprising that different approximations may lead to contradictory results on this crucial issue. The situation is similar to the Bose-Einstein condensation in condensed matter that is a first-order phase transition in an ideal gas. However, in interacting systems the issue is more delicate and often difficult to be settled experimentally. Theoretically is predicted to be a second-order transition in some approximations but it may appear as a weak first-order transition in other approximations \[2\].

We shall refer to \[1, 3\] for details on the structure and the meaning of various types of approximations to the effective potential and just report a few basic results:

i) the phase transition is indeed first order as in the case of the simple one-loop potential. This is easy to realize if one performs a variational procedure, within a simple class of trial states that includes \(|\phi\rangle\). In this case, one finds \[4\] that the \( m = 0 \) theory lies in the broken phase. Therefore the phase transition occurs earlier, for some value of the phion mass \( m = m_c \) that is still positive. This conclusion is confirmed by the results of ref.\[5\] that provides the most accurate non-perturbative calculation of the effective potential of \( \lambda \Phi^4 \) theories performed so far.

Understanding the magnitude of \( m_c \) requires additional comments. The normal ordering prescription in Eq.(1.2) eliminates all ultraviolet divergences of the free-field case at \( \lambda = 0 \) but for \( \lambda > 0 \) there are additional divergences. For this reason, one introduces an ultraviolet cutoff \( \Lambda \) and defines the continuum theory as a suitable limit \( \Lambda \to \infty \). In this case, however, one is faced with a dilemma since a meaningful description of SSB in quantum field theory must provide \( m_c = 0 \). Otherwise, from the existence of a non-vanishing mass gap controlling the exponential decay of the two-point function of the symmetric phase, and the basic axioms of quantum field theory \[3\] one would deduce the uniqueness of the vacuum (and, thus, no SSB). The resolution of this apparent conflict is that the continuum limit of the cutoff-regulated theory gives a vanishing ratio \( \epsilon = \frac{1}{\ln \frac{\Lambda}{M_h}} \)

\[
\frac{m_c^2}{M_h^2} \sim \epsilon
\]  

so that when the Higgs boson mass \( M_h \) is taken as the unit scale of mass, the possible values of the ‘phion’ mass \( 0 \leq m \leq m_c \) are naturally infinitesimal. In this sense, SSB is an \textit{infinitesimally weak} first-order phase transition where the magnitude of the ratio \( \frac{m}{M_h} \) represents a measure of the degree of non-locality of the cutoff-regulated theory.
ii) there is a deep difference between a ‘free-field’ theory and a ‘trivial’ theory where the interaction effects die out in the continuum limit. The former has a quadratic effective potential and a unique ground state. The latter, even for a vanishingly small strength $\lambda = O(\epsilon)$ of the elementary two-body processes can generate a finite gain in the energy density, and thus SSB, due to the macroscopic occupation of the same quantum state, namely to the phenomenon of Bose condensation. This leads to a large re-scaling of $\langle \Phi \rangle$. Indeed, one can introduce, in general, two distinct normalizations for the vacuum field $\phi$, say a ‘bare’ field $\phi = \phi_B$ and a ‘renormalized’ field $\phi = \phi_R$. They are defined through the quadratic shapes of the effective potential in the symmetric and broken phase respectively

$$
\frac{d^2V_{\text{eff}}}{d\phi_B^2}\bigg|_{\phi_B=0} \equiv m^2, \quad \frac{d^2V_{\text{eff}}}{d\phi_R^2}\bigg|_{\phi_R=\nu_R} \equiv M_h^2.
$$

Due to ‘triviality’, the theory is “nearly” a massless, free theory so that $V_{\text{eff}}$ is an extremely flat function of $\phi_B$. Therefore, due to (1.5), the re-scaling $Z_\phi$ relating $\phi_B$ and $\phi_R$ becomes very large. By defining $\phi_B^2 = Z_\phi \phi_R^2$, one finds $Z_\phi = O(1/\epsilon)$ or

$$v_R \sim v_B \sqrt{\epsilon} \quad (1.7)$$

Just for this reason, the rescaling of the ‘condensate’ $Z = Z_\phi$ is different from the more conventional quantity $Z = Z_{\text{prop}}$ defined from the residue of the shifted field propagator at $p^2 = M_h^2$. According to Källen-Lehmann decomposition and ‘triviality’ this has a continuum limit $Z_{\text{prop}} = 1 + O(\epsilon)$.

iii) the existence of two different continuum limits $Z_\phi \to \infty$ and $Z_{\text{prop}} \to 1$ reflects a fundamental discontinuity in the 2-point function at $p = 0$ ($p =$ Euclidean 4-vector). This effect is not totally unexpected and its origin should be searched in the infrared divergences of perturbation theory for 1PI vertices at zero external momenta. Of course, after the Coleman-Weinberg analysis, we know how to obtain infrared-finite expressions for 1PI vertices at zero external momenta. This involves summing up an infinite series of graphs of different perturbative order with different numbers of external legs, just as in the analysis of the effective potential that was taken as the starting point for our analysis. In this case, the second derivative of the effective potential gives $\Gamma^{(2)}(p = 0)$, the inverse susceptibility

$$
\chi^{-1} = \left. \frac{d^2V_{\text{eff}}}{d\phi_B^2} \right|_{\phi_B=\nu_B} = \frac{M_h^2}{Z_\phi}.
$$

Therefore, if $Z_\phi = O(1/\epsilon)$, one finds

$$
\frac{\Gamma^{(2)}(0)}{M_h^2} \sim \epsilon \quad (1.9)
$$

3
rather than \( \Gamma^{(2)}(0) = M_h^2 \) as expected for a free-field theory where

\[
\Gamma^{(2)}(p) = (p^2 + M_h^2)
\]  

(1.10)

Notice that the discrepancy found in the discrete-symmetry case implies the same effect for the zero-momentum susceptibility of the radial field in an \( O(N) \) continuous-symmetry theory. This conclusion, besides the general arguments of [8], is supported by the explicit calculations of Anishetty et al [10].

Notice that SSB requires the subtraction of disconnected pieces so that continuity at \( p = 0 \) does not hold, in general [11]. At the same time, a mismatch at \( p = 0 \) does not violate ‘triviality’ since no scattering experiment can be performed with exactly zero-momentum particles. On the other hand, for large but finite values of the ultraviolet cutoff \( \Lambda \), when ‘triviality’ is not complete, the discrepancy between \( \Gamma^{(2)}(0) \) and \( M_h^2 \) will likely ‘spill over’ into the low-momentum region \( p^2 \sim \epsilon M_h^2 \). In this region, we expect sizeable differences from the free-field form Eq.(1.10).

If really \( Z_\phi \neq Z_{\text{prop}} \) this result has to show up in sufficiently precise numerical simulations of the broken phase. To this end, the structure of the two-point function has been probed in refs. [12] by using the largest lattices considered so far. One finds substantial deviations from Eq.(1.10) in the low-\( p \) region and only for large enough \( p \), \( \Gamma^{(2)}(p) \) approaches the free field form (1.10). Also, the lattice data of refs.[12] support the prediction that the discrepancy between \( \Gamma^{(2)}(0) \) and the asymptotic value \( M_h^2 \) becomes larger when approaching the continuum limit. Notice that no such a discrepancy is present in the symmetric phase where \( \langle \Phi \rangle = 0 \).

In conclusion: theoretical arguments and numerical evidences suggest that in the limit \( k \to 0 \) the excitation spectrum of the broken phase can show substantial deviations from the free-field form \( E = \sqrt{k^2 + M_h^2} \). Due to the ‘triviality’ of the theory, the deviations from the free-field behaviour should, however, be confined to a range of \( k \) that becomes infinitesimal in units of \( M_h \) in the continuum limit \( \Lambda \to \infty \).

2. A gap-less mode in the broken phase

In this section we shall present a simple argument to illustrate the nature of the excitation spectrum of the broken phase in the limit \( k \to 0 \). Our analysis starts from the simple relation between the phion density \( n \) and the scalar field expectation value

\[
n = \frac{1}{2} m_\phi^2 \bar{\rho}_B \tag{2.1}
\]
underlying the ‘particle-gas’ picture of ref.[1].

Using Eq.(2.1) one can easily transform the energy density $\mathcal{E} = \mathcal{E}(n)$ into the effective potential $V_{\text{eff}} = V_{\text{eff}}(\phi_B)$. In this way, the $\phi_B = 0$ ‘mass-renormalization’ condition in Eq.(1.6) becomes

$$\frac{\partial \mathcal{E}}{\partial n} \bigg|_{n=0} = m. \quad (2.2)$$

Its physical meaning is transparent. If we consider the symmetric vacuum state ("empty box") and add a very small density $n$ of phions (each with vanishingly small 3-momentum) the energy density changes by $nm$ in the limit $n \to 0$. On the other hand, spontaneous symmetry breaking can be viewed as a phion-condensation process. This occurs at those values $\phi_B = \pm v_B$ where

$$\frac{dV_{\text{eff}}}{d\phi_B} \bigg|_{\phi_B=v_B} = 0 \quad (2.3)$$

By using Eq.(2.1) and defining the ground-state particle density

$$n_v = \frac{1}{2} m v_B^2, \quad (2.4)$$

we also obtain

$$\frac{\partial \mathcal{E}}{\partial n} \bigg|_{n=n_v} = 0 \quad (2.5)$$

Eq.(2.5) means that small changes of the phion density around its stationarity value do not produce any change in the energy of the system and one can add or remove an arbitrary number of phions at $k = 0$ without any energy cost. Just as in the non-relativistic limit of the theory. Therefore, the excitation spectrum in the limit $k \to 0$ has no gap and an expansion in powers of $k$ starts, in general, with a linear term $\tilde{E}(k) \sim k$. In this sense, the $k \to 0$ Fourier component of the scalar field, in the broken phase, behaves as a massless field. We now understand why the excitation spectrum $\tilde{E}$ cannot be $\sqrt{k^2 + M^2}$ at low $k$: this form does not reproduce $\tilde{E} = 0$ for $k = 0$.

Notice that this conclusion, although deduced within the framework of ref.[1], does not depend on the validity of Eq.(2.1). Indeed, Eq.(2.5) follows from Eq.(2.3) regardless of the precise functional relation between the phion density and the vacuum field. At the same time, this is only possible in a first-order phase transition where one can meaningfully investigate the broken phase in terms of physical elementary excitations of the symmetric phase. In this case in fact, however small the phion mass $m$ can be, there exists a non-relativistic limit $k \ll m$ where the scalar condensate will respond in a phase-coherent way. In this sense, the gap-less mode can be considered the Goldstone boson of a spontaneously broken continuous symmetry, the phase rotations of the condensate wave-function, that does not exist in the symmetric phase.
After this general discussion, let us now attempt a quantitative description of the energy spectrum of the broken phase. A first observation is that for large enough \( k \) we expect

\[
\tilde{E}(k) \sim \sqrt{k^2 + M_h^2}
\]

(2.6)

On the other hand, the region \( k \to 0 \) of low-density Bose systems can be analyzed in a universal way \(^2\) namely

\[
\tilde{E} \sim c_s k \quad \text{for } k \to 0
\]

(2.7)

where \( c_s \) is the sound velocity

\[
c_s = \frac{1}{m} \sqrt{\frac{4\pi n_v a}{m}}
\]

(2.8)

Here \( a \sim \frac{\lambda}{\pi^{7/6}} \) denotes the S-wave ‘phion-phion’ scattering length which enters the expression for the Higgs mass \(^3\)

\[
M_h^2 \equiv 8\pi n_v a
\]

(2.9)

Notice that Eq. (2.7) becomes a better and better approximation in the limit of very lowdensities \( n_v a^3 \to 0 \) where all condensed phions are found in the state at \( k = 0 \) and there is no population of the finite momentum modes (‘depletion’).

Let us analyze the situation in the case of spontaneous symmetry breaking in cutoff \( \lambda\Phi^4 \) theory. On one hand, ‘triviality’ requires a continuum limit with a vanishing strength \( \lambda = O(\epsilon) \) for the elementary 2-body processes. On the other hand, together with Eq.(1.5), this leads to \( aM_h \sim \sqrt{\epsilon} \). Therefore, since one takes \( M_h^2 \equiv 8\pi n_v a \) as a cutoff-independent quantity, we find

\[
n_v a^3 = O(\epsilon)
\]

(2.10)

When \( \epsilon \to 0 \), the phion-condensate becomes infinitely dilute so that the average spacing between two phions in the condensate, \( d = n_v^{-1/3} \) becomes enormously larger than their scattering length. In this limit, the energy spectrum (2.7) becomes exact ( in the limit \( k \to 0 \)).

Notice, however, that the phion density \( n_v = \frac{1}{2} m v_B^2 \), is very large, \( O(\epsilon^{-1/2}) \), in the physical units denoted by the correlation length \( \xi_h \equiv 1/M_h \). Indeed, \( \frac{\epsilon}{\xi_h} \sim \epsilon^{1/6} \). It is because there is such a high density of phions that their tiny 2-body interactions \( O(\epsilon) \) can produce a finite effect on the energy density.

In conclusion: spontaneous symmetry breaking in a cutoff \( \lambda\Phi^4 \) theory gives rise to an excitation spectrum that is not exactly Lorentz-covariant. The usual assumption \( \tilde{E}(k) \sim \sqrt{k^2 + M_h^2} \) is not valid in the limit \( k \to 0 \) where one actually finds a ‘sound-wave’ shape \( \tilde{E}(k) \sim c_s k \). This result reflects the physical presence of the scalar condensate.
3. A long-range potential in Higgs condensates

It is well known that condensed matter systems can support long-range forces even if the elementary constituents have only short-range 2-body interactions. Just for this reason, it is not surprising that the existence of a gap-less mode for \( k \to 0 \) in the broken phase can give rise to a long-range potential. For instance, when coupling fermions to a (real) scalar Higgs field with vacuum expectation value \( v \) through the Standard Model interaction term

\[
- m_i \bar{\psi}_i \psi_i (1 + \frac{h(x)}{v})
\]  

(3.1)

the static limit \( \omega \to 0 \) of the Higgs propagator

\[
D(k, \omega) = \frac{1}{E^2(k) - \omega^2 - i0^+}
\]  

(3.2)

gives rise to an attractive potential between any pair of masses \( m_i \) and \( m_j \)

\[
U(r) = -\frac{m_i m_j}{v^2} \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{E^2(k)}
\]  

(3.3)

By assuming Eqs.(2.6) and (2.7) for \( k \to \infty \) and \( k \to 0 \), and using the Riemann-Lebesgue theorem \(^{13}\) on Fourier transforms, the leading \( r \to \infty \) behaviour is universal. Any form of the spectrum that for \( k \sim m \) interpolates between the two asymptotic trends would produce the same result. At large distances \( r >> 1/m \) one finds (\( \eta \equiv \frac{c_s^2}{2m^2} = \mathcal{O}(\frac{1}{\epsilon}) \))

\[
U(r) = -\frac{G_F m_i m_j}{4\pi \eta} \frac{1}{r} [1 + \mathcal{O}(1/mr)]
\]  

(3.4)

where \( G_F \equiv 1/(v^2) \). In the physical case of the Standard Model one would identify \( G_F \sim 1.1664 \cdot 10^{-5} \text{ GeV}^{-2} \) with the Fermi constant. Notice that the coupling in Eq.(3.1) naturally defines the ‘Higgs charge’ of a given fermion as its physical mass. However, for nucleons, this originates from more elementary Higgs-quark and Higgs-gluon interactions. These effects can be resummed to all orders by replacing Eq.(3.1) with the alternative expression through the trace of the energy-momentum tensor \( \theta^\mu_\mu \), namely

\[
-\theta_\mu^\nu (1 + \frac{h(x)}{v})
\]  

(3.5)

This different normalization of the Higgs-fermion coupling reduces to the usual definition in the case of free quarks and yields exactly the nucleon mass when evaluating the matrix element between nucleon states (see for instance \(^{14}\))

\[
\langle N | \theta_\mu^\nu | N \rangle = m_N \bar{\psi}_N \psi_N
\]  

(3.6)
Notice that Eq.(3.5) is formally analogous to a Brans-Dicke theory \cite{15}. Here, however, the framework is very different since the $h$--field propagates in the presence of the phion condensate. Also, Eq.(3.5) represents the appropriate form to implement a ‘strong form’ of the Equivalence Principle \cite{16} by extending the notion of inertia to the electromagnetic field through scale non-invariant quantum effects \cite{17}.

We note that the strength of the long-range potential is proportional to the product of the masses and is naturally infinitesimal in units of $G_F$. It would vanish in a true continuum theory. Therefore, it is natural to relate this extremely weak interaction to the gravitational potential and to the Newton constant $G$ by identifying

$$\sqrt{\eta} = \sqrt{\frac{G_F}{G}} \sim 10^{17} \quad (3.7)$$

Notice, that the long-range $1/r$ potential is a direct consequence of the existence of the scalar condensate. Therefore, speaking of gravitational interactions makes sense only for particles that can induce variations of the phion density by exciting the gap-less mode of the Higgs field. In this sense, phions, although possessing an inertial mass, have no ‘gravitational mass’.

In the physical case of the Standard Model, and assuming the range of Higgs mass $M_h \sim 10^2 - 10^3$ GeV, this leads to a range of phion masses $m \sim 10^{-4} - 10^{-5}$ eV. The detailed knowledge of the spectrum $\tilde{E}(k)$ for $k \sim m$ would allow to compute the terms $\mathcal{O}(1/mr)$ in Eq.(3.4) and predict a characteristic pattern of ‘fifth force’ deviations below the centimeter scale.

4. **Summary and concluding remarks**

In this paper we have presented a simple physical picture where Newtonian gravity can be interpreted as a long-wavelength excitation of the scalar condensate inducing spontaneous symmetry breaking. We emphasize that our main result in Eq.(3.4) depends only on the Riemann-Lebesgue theorem on Fourier transforms \cite{13} and two very general properties of the excitation spectrum. Namely, the ‘diluteness’ condition Eq.(2.10) (that leads to the ‘sound-wave’ shape in Eqs.(2.7) and (2.8) for $k \to 0$) and the Lorentz-covariance for large $k$ (that leads to Eq.(2.6)). These general properties are expected to occur in any description of spontaneous symmetry breaking in terms of a weakly coupled Bose field. We emphasize, as in sect.2, that the assumption of a weakly first-order phase transition is essential. Only in this case, in fact, there is a non-relativistic regime $k \ll m$ where the scalar condensate
reacts with phase-coherence \[18\]. A more complete description of gravitational phenomena requires the detailed form of the spectrum \(\tilde{E}(k)\) and, in particular, the precise knowledge of the phion mass \(m\). Deviations from the Newton potential are expected at typical distances \(r \sim 1/m\) and could, eventually, be detected in the next generation of precise ‘fifth-force’ experiments \[20\].

We stress that the apparently ‘trivial’ nature of \(\lambda \Phi^4\) theories in four space-time dimensions should not induce to overlook the possibility that gravity can arise as a gap-less mode of the Higgs field. Indeed, our description is only possible if one assumes the existence of an ultimate ultraviolet cutoff so that the natural formulation of the theory is on the lattice and ‘triviality’ is never complete. In this case, however, the existence of a non-trivial infrared behaviour in the broken phase can be guessed from the equivalence of low-temperature Ising models with highly non-local membrane models on the dual lattice \[21\] whose continuous version is the Kalb-Ramond \[22\] model. Thus, in the end, a Higgs-like description of gravity would turn out to be equivalent, at some scale, to a Feynman-Wheeler theory of strings, as electromagnetism for point particles. This may be useful to establish a link between our description and alternative pictures of gravity, even with very different degrees of locality recalling, however, that the existence of the cutoff cannot be neglected (for instance when comparing with the action-at-distance theory of Hoyle and Narlikar \[23\]). At the same time, the basic idea that one deals with the same theory should allow to replace a description with its ‘dual’ picture when better suited to provide an intuitive physical insight.

Many readers, we realize, will be reluctant since the presently accepted point of view tends to regard Newtonian gravity as a well defined limit of a more fundamental theory, namely Einstein gravity. This is believed to lie outside of the Standard Model and to require fundamental and genuinely new interactions and particles (spin-2 gravitons). Apparently, this point of view is very natural since Einstein equations in the weak-field limit can be obtained \[24\] from flat space by requiring the Lorentz invariance of matrix elements for absorbing and emitting massless spin-2 quanta. However, this derivation assumes the validity of a fully Lorentz-covariant description of the elementary gravitational processes. In the presence of spontaneous symmetry breaking, this assumption is not necessarily true in the limit \(k \to 0\) if the departure from an exact Lorentz-covariant spectrum is the origin of gravitational interactions (and a consequence of the cutoff).

Quite independently of any application to gravity, the possible departure from a Lorentz-covariant excitation spectrum is a general feature in 4-dimensional interacting
Theories. The reason is that the usual normal-ordering procedure guarantees the local commutativity of Wick-ordered products of the field operator in the free theory but no such a procedure is known a priori for the interacting case. Thus the argument is circular since the proper normal-ordering procedure is only known after determining the vacuum and its excitation spectrum. In this respect, the usual approach, where one uses the normal-ordering definition of the free-field theory and defines the continuum theory as a suitable limit $\Lambda \to \infty$, is consistent. In fact, in this limit, the spectrum reduces simply to $\sqrt{k^2 + M^2}$ for all $k$ (except $k = 0$).

On the other hand, without considering these technical details, the very accurate equality between the inertial and gravitational mass of known particles should convince a skeptical reader that our result is not totally unexpected. Actually, to a closer inspection, a tight link between the physical origin of gravity and the physical origin of inertia is unavoidable. Only in this case, in fact, one can fully understand what, after Einstein, it is called Mach’s Principle, namely the consistent vanishing of inertia if gravity would be switched off. The point is that Einstein’s description of gravity is purely geometric and macroscopic. As such, it does not depend on any hypothesis about the physical origin of this interaction. Einstein’s construction would remain the same if the Newton potential would be experimentally known to behave as $\frac{1}{r} \exp(-\mu r)$ rather than as $1/r$. Just for this reason, general relativity, by itself, is unable to predict even the sign of the gravitational force (attraction rather than gravitational repulsion). Rather, Einstein had to start from the peculiar properties of Newtonian gravity to get the basic idea to transform the classical effects of this type of interaction into a metric structure. In this sense, it is not surprising that a few ‘crucial tests’ of general relativity in weak gravitational field merely verify two well established structures, namely Special Relativity (SR) and the Equivalence Principle (EP), and do not necessarily require a fundamental tensor theory. The reason is that the infinitesimal transformation to the rest frame of a freely falling elevator is of very general nature. For instance, it can also be obtained with a conformal transformation of length, time and mass. Just to illustrate this point, let us consider the relation among reference frames in the gravitational field of a large mass $M$ (e.g. the sun). Up to higher powers of the gravitational strengths, any bound observer $O(i)$ can be considered as performing circular orbits of radius $r(i)$. The ordering of the observers is such that $r(i) < r(i + 1)$ so that, assuming an overall weak-field condition $\frac{2GM}{c^2 r(1)} \ll 1$, the relation to the asymptotic reference frame $K(0)$ at spatial infinity can be approximated
as an infinitesimal Lorentz-transformation with the radial ‘escape’ velocity

\[ v^2(i) = \frac{2GM}{r(i)} \tag{4.1} \]

The set of metrics

\[ ds^2(i) = c^2 dt^2(i) - dr^2(i) - r^2(i) \left[ d\theta^2(i) + \sin^2 \theta(i) d\varphi^2(i) \right] \tag{4.2} \]

for the \( O(i) \) frames implies

\[ ds^2(0) = c^2 dt^2(i) \left[ 1 - \frac{2GM}{c^2 r(i)} \right] - \frac{dr^2(i)}{1 - \frac{2GM}{c^2 r(i)}} - r^2(i) \left[ d\theta^2(i) + \sin^2 \theta(i) d\varphi^2(i) \right] \tag{4.3} \]

for the \( K(0) \) frame that, indeed, is the Schwarzschild metric. The weak-field restriction means that Eq.(4.1) is valid up to higher order terms. For instance, one could replace the relativistic expression for the kinetic energy and obtain

\[ ds^2(0) = \frac{c^2 dt^2(i)}{\left[ 1 + \frac{GM}{c^2 r(i)} \right]^2} - dr^2(i) \left[ 1 + \frac{GM}{c^2 r(i)} \right]^2 - r^2(i) \left[ d\theta^2(i) + \sin^2 \theta(i) d\varphi^2(i) \right] \tag{4.4} \]

In this respect, Einstein’s definition of the Mach’s Principle provides the real physical meaning of his description of gravity. Namely, in a theory where the common origin of inertia and gravity is built in, general relativity can represent a very elegant and clever way to compute the weak-field corrections to measurements of length and time. After all, this interpretation of general relativity is consistent with its name that suggests a description of relative effects in gravitational fields rather than a truly dynamical explanation of the origin of the gravitational force. This interpretation is also consistent with alternative views \([29, 30, 31]\) that consider classical Einstein gravity as a weak-field effective theory generated by underlying quantum phenomena.

The new result is that the crucial ingredient for the Equivalence Principle, namely ‘inertial mass=gravitational mass’, turns out to be a deducible consequence of our present understanding of electroweak interactions where the non-trivial coexistence of ‘SR’ and ‘EP’ does not require to change the space-time but depends on the particular nature of the ground state. This also removes all self-contradictory consequences of accommodating gravity within exact Lorentz-covariance \([32]\) and represents a step towards a more comprehensive theory whose final form, of course, we are not able to predict.

A definite prediction, however, is that the gravitational force is naturally instantaneous. The velocity of light \( c \) has nothing to do with the long-wavelength excitations of the phion condensate that for \( k \to 0 \) propagate with the fantastically high speed \( c_s = \sqrt{\eta} c \sim \).
10^{17}$c. On the other hand, for $k \sim m$, i.e. at the joining of the two branches of the excitation spectrum, one recovers the expected result $dE/dk < c$. To a closer inspection, this apparently bizarre result appears less paradoxical than the generally accepted point of view that considers the inertial forces in an accelerated laboratory as the consequence of a gravitational wave generated by distant accelerated matter. Indeed, if the gravitational interaction propagates with the light velocity, distant matter must be accelerated before the inertial reaction is actually needed [33]. The same type of conclusions is suggested by the analysis of tidal forces [34].

Notice that, quite independently of our results and within the generally accepted framework of general relativity, the existence (or not) of faster than light signals is still an open possibility. For instance, regardless of the quantum phenomena that give rise to the ground state, it is known since a long time that constant energy-density solutions of Einstein equations contain, indeed, closed time-like curves [35]. More recently, the same type of questions have been raised by ‘inflation’ that, indeed, represents a superluminal mode of expansion introduced to reconcile the predictions of relativistic cosmology with the size of the observed universe. In the present approach [36], this is achieved through an extremely flat scalar potential that is very natural in a description where spontaneous symmetry breaking is an infinitesimally weak first-order phase transition. Actually, just the possibility to understand the extraordinary fine-tuning needed in inflationary models was one of the motivations behind the early attempt [1] to explain inertia and gravity from the same physical phenomenon. In fact, due to the large re-scaling $Z_\phi \sim \eta$ in Eqs.(1.6) and (1.7), one has actually a ‘2-parameter’ theory where the ‘renormalized’ value $v_R \sim G^{-1/2}$ is used for the W mass $M_{w}^{2} \sim \frac{g^{2}v_{B}^{2}}{4}$ and the ‘bare’ value $v_B \sim G^{-1/2}$ is used to generate Einstein’s lagrangian $\mathcal{L} \sim Rv_{B}^{2}$. After all, taking into account the crucial quantum phenomenon of the vacuum field rescaling, this is in the spirit of a spontaneously broken theory of classical Einstein gravity [29, 30, 31] where gravity, however, is induced by the Standard Model Higgs field [37, 38]. After some thought, however, this picture can hardly work. Indeed, besides introducing non-local effects that is difficult to accommodate within the traditional Einstein picture, the non-trivial presence of the phion condensate would likely require the introduction of a preferred metric structure. In fact, this unusual form of matter has an inertial mass and transmits gravity but does not generate any curvature: it is the quantum realization of the old-fashioned weightless aether whose long-range density oscillations are determined by the coupling of known matter to the gap-less mode of the Higgs field.

To better clarify this point, and in the spirit of a weak-field analogy, we note that
in classical Einstein theory, all forms of energy and matter contribute to the space-time curvature. Therefore, to take into account the peculiar medium that transmits gravity, one has to give up full general covariance. One possibility would be to require $\sqrt{g} = 1$ \[38, 40\] so that Einstein equations are replaced by their traceless counterpart. In this constrained formulation of gravity, there is no cosmological term from spontaneous symmetry breaking and, whatever the distribution of the known forms of matter, the trace of the energy-momentum tensor coincides with (minus) the scalar curvature $R$, up to an integration constant \[40\]. Therefore, to describe classical motions and up to higher-order $O(G^2)$ and $O(G_G F)$ terms, Eq.(3.5) can be used to generate Einstein lagrangian in a $\sqrt{g} = 1$ world. Connection with other descriptions of gravity (again for $\sqrt{g} = 1$) can be established if we separate out in Eq.(3.5) the long-wavelength part of the Higgs field $\tilde{h}(x)$ associated with the linear excitation spectrum of the scalar condensate. In this situation, and up to higher order terms, one can replace the lagrangian in Eq. (3.5) with $\left(\frac{\phi(x) \equiv \tilde{h}(x)}{v}\right)^2\frac{1}{16\pi G} \exp(\phi) R$ \[4.5\]

Eq.(4.5), formally, resembels string-inspired descriptions of Einstein gravity where, however, the ‘dilaton’ is identified with the gap-less, ‘tachionic’ mode of the Higgs field. In this respect, we find surprising that theoretical models of string cosmology seem to indicate, indeed, the ‘tachionic’ nature of the dilaton \[41\], at least in an early epoch of inflation where space-time is nearly flat. As anticipated, this may be a consequence of the non-trivial duality properties \[21\] of four-dimensional broken-symmetry Higgs phases.

To conclude, we want to mention two possible crucial implications of the scalar condensate at the cosmological level:

a) our picture, despite of its conceptual simplicity, provides a natural solution of the so-called ‘hierarchy-problem’. This depends on the infinitesimally weak first-order nature of the phase transition: in units of the Fermi scale, $M_h \sim G_F^{-1/2}$, the Planck scale $G^{-1/2}$ would diverge for a vanishing phion mass $m$. These three scales are hierarchically related through the large number $\sqrt{\eta} \sim 10^{17}$ that is the only manifestation of an ultimate ultraviolet cutoff. In this sense, spontaneous symmetry breaking in $(\lambda\Phi^4)_4$ theory represents an approximately scale-invariant phenomenon and it is conceivable that powers of the ‘replica-factor’ $10^{17}$ will further show up in a natural way. This speculation may not be too far from the actual physical situation in large-scale astronomy \[42\] where the strong experimental evidence for a hierarchical cosmology was clearly pointed out by de Vaucouleurs \[43\] long time ago. As a consequence, by accepting the common physical origin of both inertia and gravitation and the crucial role of the scalar condensate, it be-
comes natural to search the origin of the observed self-similarity of the universe \[44\] in the basic features of spontaneous symmetry breaking. In this way, one can hope to resolve the serious discrepancy \[44\] between the present models of galaxy formation and the present models of expansion of the universe.

b) as pointed out in sect.2, one expects tiny departures from the linear excitation spectrum at small \(k\) due to residual self-interaction effects within the condensate. The deviations from an exact ‘superfluid-regime’ can introduce a small friction that could become important in the typical astronomical large-scale and low-acceleration conditions. These effects, totally unobservable in laboratory tests, would show up as small deviations from the Newton force and could be interpreted as modifications of inertia and/or of gravity. It is not unconceivable that a closer look to the small residual self-interactions in the scalar condensate can explain the peculiar modification of inertia suggested by Milgrom \[45\] to resolve the observed mass discrepancy in many galactic system. Indeed, there is a very close analogy between the typical non-linear effects associated with the response of a non-trivial vacuum \[46\] and those actually needed to explain this particular effect \[47\]. Experimentally, this shows up when the acceleration of bodies becomes comparable to a universal acceleration field \(a_o \sim 10^{-8} \text{ cm sec}^{-2}\). Remarkably, this value is of the same order as (the unexplained part of) the anomalous acceleration toward the sun suggested by the Pioneer 10/11 \[48\] data. In our picture of gravity, it would be natural to interpret such effects as a tiny ‘braking’ of the scalar condensate.

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