Strong linear polarization of bremsstrahlung emissivity in photospheres of magnetic white dwarfs

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Abstract. We discuss the strong linear polarization and the appreciable decrease of the bremsstrahlung emissivity at frequencies below the electron cyclotron frequency (infrared wave band) in the photospheres of the isolated magnetic white dwarfs. In the photospheres of strongly magnetized white dwarfs \( B > 10^7 \) G, \( T \sim 10^4 \) K, the electron’s Larmor radius becomes smaller than the characteristic impact parameter of close Coulomb collisions in a non-magnetized plasma. Thus, the cyclotron period of the electron becomes smaller than the duration of all distant collisions and of most close collisions. The magnetic field effectively “freezes” the electron motion in the plane transverse to the magnetic field lines. The resulting motion is nearly one-dimensional and parallel to the magnetic field, inducing a strong linear polarization of the bremsstrahlung emission. Being attached to a magnetic field line, an electron cannot approach an ion as closely as it does in the case in which the magnetic field is absent. Thus, the bremsstrahlung emissivity appreciably decreases. We analytically compute an approximation to the spectrum of the strongly linear polarized bremsstrahlung emissivity at the frequencies below the electron cyclotron frequency.

1. Introduction
In the photospheres of the strongly magnetized white dwarfs, the electron’s Larmor radius

\[
 r_B = \frac{v_T}{\omega_B}
\]

becomes smaller than the characteristic impact parameter of close Coulomb collisions in a non-magnetized plasma

\[
 r_s = \frac{Ze^2}{(mev_T^2)}.
\]

Here, \( v_T = (k_BT/me)^{1/2} \) is the thermal velocity of electrons, \( \omega_B \) is the electron cyclotron frequency, \( m_e \) is the electron mass, \( e \) is the elementary charge, \( Z \) is the ion charge number, and \( k_B \) is the Boltzmann constant. The ratio

\[
 r_B \ll r_s
\]

is realized in magnetic fields \( B > 10^7 \) G at temperatures \( T \sim 10^4 \) K:

\[
 r_B/r_s = 1.3Z^{-1}(T/10^4 \text{ K})^{3/2}(B/10^7 \text{ G})^{-1} < 1.
\]

Under the condition of Eq. (3), the Coulomb collisions dramatically change in comparison with the collisions in a non-magnetized plasma (Zheleznyakov 1997; Zheleznyakov et al. 1999;
Koryagin 2000). The collision transport frequencies (which characterize the non-radiative relaxation processes) are determined by close collisions with impact parameters of the order or smaller than the distance

$$L_u = (Zm_e c^2 / B^2)^{1/3}. \quad (4)$$

In these collisions, an electron transits the region near an ion for a time of the order or smaller than the cyclotron period. The distance of Eq. (4) coincides with the radius of a highly excited non-magnetized atom in which an electron orbits a nucleus for a time equal to the given electron cyclotron period. The distance given by Eq. (4) is smaller than the parameter \( r_s \) and is larger than the Larmor radius given by Eq. (1). In the case of close collisions, the electron trajectory becomes quasi-trapped. The particle returns to the ion many times before it overcomes the attraction of the scattering center (Zheleznyakov et al. 1999; Schmidt 2000; Hu et al. 2002a; Hu et al. 2002b; Correa et al. 2005).

In the case of distant collisions with impact parameters \( p_h \gg L_u \), the ion Coulomb field accelerates an electron along a magnetic field line at the initial path to a scattering center and, then, decelerates it down to the initial velocity at the final part of the trajectory. The component transverse to the magnetic field of the ion Coulomb field only causes a slow drift of the electron in the crossed electric and magnetic fields. The resulting change of the electron pitch-angle is exponentially small.

The qualitative change of the electron-ion collisions must influence the bremsstrahlung. We consider the bremsstrahlung under the criterion given by Eq. (3) at the frequencies below the electron cyclotron frequency:

$$\omega \ll \omega_B, \quad (5)$$

which fall into the infrared or optical wavebands for the strongly magnetized white dwarfs:

$$\lambda \gg 2\pi c / \omega_B = 11 (B/10^7 \text{ G})^{-1} \mu\text{m}.$$  

2. Polarization

It is known that at the frequencies which satisfy Eq. (5), a magnetic field significantly suppresses the high frequency plasma conductivity in a plane transverse to the magnetic field. This suppression does not require the qualitative change of collisions. It stems from the limited displacement of an electron in a direction transverse to the magnetic field plane. This displacement is smaller than twice the Larmor radius. Then, in accordance with the Kirchoff law (Bekefi, 1966), the bremsstrahlung emissivity for the electromagnetic waves polarized in a plane transverse to the magnetic field is also suppressed. As a result, the bremsstrahlung emissivity becomes strongly linear polarized (Gnedin & Pavlov 1974). The plane of polarization is parallel to the magnetic field vector and the wave vector. The beam pattern of the linear polarized bremsstrahlung emissivity corresponds to the radiation of a dipole directed along the magnetic field:

$$\eta_\nu(\omega, \Theta) = \eta_\nu(0) \sin^2 \Theta, \quad (6)$$

where \( \Theta \) is the angle between the wave vector and the magnetic field, \( \eta_\nu(0) \) is the spectral emissivity per unit solid angle in a direction transverse to the magnetic field.

3. Spectrum

We divide the considered frequencies into several sub-bands. At frequencies lower than the inverse time of the electron’s transit of a region near the ion in a collision with the impact parameter given by Eq. (2):

$$\omega \ll \omega_s = mv^3_T / (Ze^2), \quad \lambda \gg 2\pi c / \omega_s = 8 (T/10^4 \text{ K})^{-3/2} \mu\text{m}, \quad (7)$$
the bremsstrahlung is determined by distant collisions with impact parameters \( p_h \gg r_s \). In these collisions, the ion weakly disturbs the electron motion. However, the spectral emissivity of an electron diverges with the decrease of its velocity \( v \parallel \) along the magnetic field proportionally to \( v \parallel^{-1} \). As a result, the bremsstrahlung emissivity integrated over the Maxwell velocity distribution contains the logarithmic factor which coincides with the Coulomb logarithm without the magnetic field:

\[
\eta^{(d)}_{\omega 0} = \frac{\sqrt{2/3}}{\pi^{3/2} m_i^2 c^6 v_T} \ln \left( \frac{m v_f^2}{\omega Z e^2} \right). \tag{8}
\]

This spectral emissivity for only one polarization in a direction transverse to the magnetic field is 2 times smaller than the analogous isotropic value in a non-magnetized plasma. The total emissivity integrated over all directions and summed over the two polarizations spectral power \( 8 \pi \eta_{\omega 0}/3 \) is 6 times smaller than the corresponding value in the absence of a magnetic field. The decrease of the bremsstrahlung emissivity in comparison with the case of a non-magnetized plasma is conditioned by the exponential suppression of the change of the electron pitch-angle in the distant collisions.

At the higher frequencies

\[
\omega \gg \omega_s = m v_f^2 / (Ze^2), \quad \lambda \ll 2 \pi c / \omega_s = 8 (T/10^4 \text{ K})^{-3/2} \mu \text{m}, \tag{9}
\]

the contribution from distant collisions decreases for increasing frequencies as \( \omega^{-2/3} \):

\[
\eta^{(d)}_{\omega 0} = \frac{0.57 n_i n_e Z^2 e^6}{\pi^{3/2} m_i^2 c^3 v_T} \left( \frac{\omega}{\omega_s} \right)^{-2/3}. \tag{10}
\]

In absence of magnetic field, the bremsstrahlung emissivity spectrum is flat at the frequencies given by Eq. (9). The flat spectrum is determined by hyperbolic trajectories with impact parameters \( p \ll r_s \) in which an electron approaches an ion much closer than the impact parameter. The strong magnetic field prevents the electron displacement in a direction transverse to the magnetic field plane, and the distance of the closest electron-ion approach equals the impact parameter. This effect explains the power-law decrease of the bremsstrahlung emissivity from the distant collisions.

The bremsstrahlung from close collisions with impact parameters \( p_h \sim L_i \) is determined by the parts of the quasi-bound electron trajectory for which the electron moves from the ion to a turning point and back to the ion for a time of the order or longer than the period of the emitted electromagnetic wave \( 2 \pi / \omega \). The number of such parts increases with the increase of the frequency \( \omega \). This determines the increase of the bremsstrahlung emissivity from the close collisions as \( \omega^{4/3} \):

\[
\eta^{(c)}_{\omega 0} = \frac{1.6 n_i n_e Z^2 e^6}{\pi^{3/2} m_i^2 c^3 v_T} \left( \frac{\omega}{\omega_B} \right)^{4/3} \ln^{4/3}(r_s/r_B). \tag{11}
\]

The minimum emissivity from the distant and close collisions \( \eta_{\omega 0} = \eta^{(d)}_{\omega 0} + \eta^{(c)}_{\omega 0} \) falls onto the frequency

\[
\omega_1 = 0.42 \omega_s^{1/3} \omega_B^{2/3} / \ln^{2/3}(r_s/r_B), \quad \lambda_1 = 2 \pi c / \omega_1 = 23 (B/10^7 \text{ G})^{-2/3} (T/10^4 \text{ K})^{-1/2} \ln^{2/3}(r_s/r_B) \mu \text{m}. \tag{12}
\]

The extrapolation of the emissivity of Eq. (11) up to the frequencies of the order of the cyclotron frequency — the upper limit of the considered band, see Eq. (5) — yields the value of the order of the bremsstrahlung emissivity without the magnetic field.

Thus, the bremsstrahlung spectrum is nearly flat at low frequencies — those which fulfill Eq. (7) — and under these conditions it is described by Eq. (8). At higher frequencies, \( \omega_s \ll \omega \ll \omega_1 \), the spectrum decreases as the frequency increases following the power law of Eq. (10). It achieves the minimum value at a frequency given by Eq. (12), whereas within the interval \( \omega_1 \ll \omega \ll \omega_B \), it increases following the power law of Eq. (11).
4. Discussion
In this contribution we have shown that the linear polarization of the bremsstrahlung emissivity coincides with the polarization of one of the normal electromagnetic waves in a magnetized plasma (ordinary wave). Thus, the propagation effects (the transverse Faraday effect) do not destroy the linear polarization. Applying the Kirchoff law (Bekefi 1966), we conclude that the extraordinary electromagnetic wave is less reabsorbed than the ordinary one. Thus, the outgoing waves of the extraordinary polarization come from the deeper and hotter layers of the photosphere. So, the mean polarization vector of the observed continuum radiation at the frequencies below the minimum electron cyclotron frequency at the star equator is expected to be transverse to the average projection of the white dwarf magnetic field onto the plane of the sky.

5. Conclusions
In the photospheres of the strongly magnetized white dwarfs, the bremsstrahlung emissivity at the frequencies below the electron cyclotron frequency becomes 1) strongly linear polarized, 2) anisotropic, and 3) decreases in magnitude compared with its value in a non-magnetized plasma.

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