Impurity scattering in highly anisotropic superconductors and inerband sign reversal of the order parameter.

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ABSTRACT

We discuss various mechanisms that can lead to interband sign reversal of the order parameter in a multiband superconductor. In particular, we generalize Abrikosov-Gor’kov solution of the problem of weakly coupled superconductor with magnetic and nonmagnetic impurities on the case of arbitrary order parameter anisotropy, including extreme cases as $d$-pairing or interband sign reversal of the order parameter, and show that interband scattering by magnetic impurities can stabilize an interband sign-reversal state. We discuss a possibility of such state in YBa$_2$Cu$_3$O$_7$ in the context of various experiments: Josephson tunneling, neutron scattering, isotope effect measurements.

Keywords superconductivity, anisotropy, YBa$_2$Cu$_3$O$_7$

1. In the last years new experiments with high-temperature superconductors revived the interest to the strongly anisotropic superconductivity. Under this name we mean superconductors where the order parameter $\Delta$ is strongly different on different parts of the Fermi surface. It includes the cases when strong anisotropy of $\Delta$ is implied by symmetry of the superconducting state, like $d$-wave pairing, one-band superconductors with a strong anisotropy, but the same symmetry as in the normal state (so-called extended $s$ pairing), or multiband superconductors with substantially different order parameters in different bands. It does not include superconductors with anisotropic electronic structure which does not result in anisotropic order parameter.

In this paper we shall concentrate on the case of multiband superconductors, more precisely on the case of extreme interband anisotropy when the sign of the order parameter is different in different bands. We shall discuss various conditions which can lead to such a state. First, we shall address the question whether a combination of phonon-induced attraction and homogeneous Coulomb repulsion may lead to interband sign reversal of the order parameter. Second, we shall derive a generalization of the Abrikosov-Gor’kov formula for superconductivity in dirty superconductors for arbitrary anisotropy of the order parameter, and will show that impurity scattering does not necessarily reduce the anisotropy of the superconducting state. Moreover, we shall point out cases when
impurity scattering can stabilize a solution with an interband sign reversal. Finally, we will describe a model when pairing appears due to electron-paramagnon scattering and the resulting state has an interband sign reversal of the order parameter. Finally, we will discuss various experiments in YBa$_2$Cu$_3$O$_7$, and show how they can be explained with the interband sign reversal concept.

2. The extension of the BCS theory for two or more superconducting bands was first worked out by Suhl, Matthias, and Walker, and independently by Moskalenko, and later elaborated on by many. It was realized that the fact that several bands cross the Fermi level is not sufficient to have considerable many-band effects in superconductivity. Only when the bands in question have a different physical origin, can a substantial effect appear. It has also been realized that the anisotropy, if any, appears already in the weak coupling (BCS) solution, while the proper strong coupling treatment adds no qualitative features.

Many high-$T_c$ cuprates not only have multsheet Fermi surfaces, but the actual bands at the Fermi level are qualitatively different. In particular, YBa$_2$Cu$_3$O$_7$ is known to have four sheets of the Fermi surface, all four having different physical origins. One is formed by the chains $p$-$d$-$\sigma$ states (seen by positron annihilation), another is an apical oxygen band (seen in de Haas-van Alphen experiments), and the last two are bonding and antibonding combinations of the two $p$-$d$ plane bands (seen by angular-resolved photoemission). Basing on the richness of the band structure of YBa$_2$Cu$_3$O$_7$, several groups pointed out that at least the two-band, or probably the whole four-band picture should be used to describe superconductivity in this system. Various experiments have been interpreted as indicating two or more different superconducting gaps.

We shall now remind the basic equations of the multiband BCS theory. The Hamiltonian has the following form:

$$H = \sum_{i,k,\alpha} \epsilon_{i,k} c_{i,k,\alpha}^\dagger c_{i,k,\alpha} + \sum_{i,k,k',\alpha,\beta} g_{ij} \epsilon_{i,k,\alpha}^* c_{i,k,\alpha}^\dagger c_{j,k',\beta}^\dagger c_{j,k',\beta}$$

where $\epsilon_{i,k}$ is the kinetic energy in the $i$-th band, $c_{i,k,\alpha}^\dagger$ and $c_{i,k,\alpha}$ are corresponding creation and annihilation operator, and $g_{ij}$ is the averaged pairing potential.

The order parameter $\Delta$ on the $i$-th sheet of the Fermi surface is given by the equation

$$\Delta_i = \sum_j \Lambda_{ij} \Delta_j \int_0^{\omega_D} dE \frac{\tanh(\frac{E^2 + \Delta_j^2}{2k_B T})}{\sqrt{E^2 + \Delta_j^2}}.$$

if the cut-off frequency $\omega_D$ is assumed to be the same for all sheets. $T_c$ is defined in the usual way by the effective coupling constant, $\log(2\gamma/\omega_D/\pi T_c) = 1/\lambda_{eff}$, $\gamma^* \approx 1.78$. The effective coupling constant $\lambda_{eff}$ in this case is simply the maximal eigenvalue $\lambda_{max}$ of the matrix $\Lambda_{ij} = g_{ij} N_j$, where $N_j$ is the density of states at the Fermi level (per spin) in the $j$-th band. $\Lambda_{ij}$ plays the role of the coupling constant $\lambda$ in the one-band BCS theory. Note that conventional (isotropic) $\lambda$ is also defined in terms of $\Lambda_{ij}$: $\lambda = \sum_i \Lambda_{ij} N_j/N = \sum_i \Lambda_{ij}/N$, where the mass renormalization for the $i$-th band, as measured, for instance, in de Haas-van Alphen experiments, is $\lambda_i = \sum_j \Lambda_{ij}$, and $N = \sum_i N_i$. Obviously $\lambda_{eff} \geq \lambda$, which means that due to larger variational freedom $T_c$ in the multiband theory is always larger than in the one-band theory. The two are equal in isotropic case, i.e. when $g_{ij}$ does not depend on $i,j$. An instructive example of the opposite case is the two-band model with $\Lambda_{11} = \Lambda_{22} = \lambda > 0$, $\Lambda_{12} = \Lambda_{21} = -\lambda$. Then $\lambda = 0$, while $\lambda_{eff} = 2\lambda$. Note that the last value is the same as when $\Lambda_{12} = \Lambda_{21} = \lambda$. The physical reason is that although there is no solution of Eq. 1 with $\Delta_j > 0$, $\Delta_2 > 0$, there is an obvious solution with $\Delta_1 = -\Delta_2 \neq 0$. Near $T_c$, the solution of Eq. 1 is $\Delta_2/\Delta_1 = (\lambda_{eff} - \lambda_{11})/\lambda_{12}$, demonstrating directly that the sign reversal of the order parameter, $\Delta_2/\Delta_1$, takes place when non-diagonal matrix elements $\Lambda_{12}$ and $\Lambda_{21}$ are negative. One can easily check that Eq. 1 may have a superconducting solution even for all $g_{ij} < 0$, i.e., when no attractive interaction is present in the system. The condition for that is $|g_{12}| > (|g_{1}|N_1^2 + |g_2|N_2^2)/2N_1N_2$. This is similar to the well-known fact that in a system with repulsion the superconductivity with higher angular momenta $(p,d)$ is possible, because of the sign reversal of the order parameter. The main difference is that in the example above the symmetry of the superconducting state is the same as of the normal state. Below we shall demonstrate that even a fully attractive interaction $g_{ij} \geq 0$ can lead to the sign reversal if (a) interband pairing interaction is weaker than Coulomb pseudopotential, or (b) there is strong interband scattering by magnetic impurities.
If $g$'s are electron-phonon pairing potentials, then Eq. 1 should be corrected for a Coulomb repulsion, which can be readily done by substituting $g_{ij} \rightarrow g_{ij} - U^*_{ij} \approx g_{ij} - U^*$, where the effective Coulomb repulsion $U^*$ is logarithmically renormalized in the same way as in one-band superconductivity theory ($U^*$ is assumed to be independent on $i,j$). A direct consequence of that is that if the interband electron-phonon coupling is weak, the situation with a negative gap, $g_{ij} - U^* < 0$, can easily be realized because of the interband repulsion. We illustrate that by numerical calculations presented in Fig. 1. In these calculations the following parameters had been used: $g_{12} = g_{22} = 0$, $N_1 = 4N_2$, and $g_{11} = N_1^{-1}$ so that to have $\lambda = 1$.

Several facts draw attention: First, in this model $T_c$ decreases with the increase of $\mu^* = U^* N$ substantially slower than in a one-band case when $T_c \rightarrow 0$ when $\mu^* \rightarrow \lambda$. Second, the order parameter induced in the second band (not superconducting by itself) is always negative; its absolute value reaches maximum when $|\Delta_1| = |\Delta_2|$, i.e., at $U^* = g_{pp} N_p / 2(N_p - N_c)$. Further below we shall discuss the relevance of this situation to $\text{YBa}_2\text{Cu}_3\text{O}_7$.

3. Now we shall analyze the influence of impurity scattering on the sign and anisotropy of the order parameter.
Following the standard way of including the impurity scattering in the BCS theory, one writes the equations for the renormalized frequency \( \tilde{\omega}_n \) and order parameter \( \Delta_n \) (\( n \) is the Matsubara index), which completely define the superconductive properties of the system:

\[
\hbar \tilde{\omega}_{jn} = \hbar \omega_n + \sum_{j'} \frac{\hbar^2 \tilde{\omega}_{j'n}}{2Q_{j'n}} (\gamma_{j,j'} + \gamma^{s}_{j,j'})
\]

\[
\tilde{\Delta}_{jn} = \Delta_j + \sum_{j'} \frac{\hbar^2 \tilde{\Delta}_{j'n}}{2Q_{j'n}} (\gamma_{j,j'} - \gamma^{s}_{j,j'})
\]

\[
\Delta_j = \pi T \sum_{j',n} A_{j,j'} \tilde{\Delta}_{j'n}/Q_{j'n}.
\]

The only difference from a specific case of multiband superconductivity (i.e., interband anisotropy), considered in a number of papers (see, e.g., Ref. 1), is that instead of the band indices in \( J, J' \), labeling Allen’s Fermi surface harmonics. Note that for a spherical (cylindrical) Fermi surface Allen’s harmonics reduce to spherical harmonics and \( J \) to \( \{L, m\} \) or just \( m \). For the interband anisotropy without intraband anisotropy \( J \) can be chosen to be the band index (so-called disjoint representation). Other notations in Eq. 2 have their usual meaning: \( \omega_n = (2n + 1)\pi T \), \( Q_{jn} = \sqrt{\tilde{\omega}_n^2 + \tilde{\Delta}_n^2} \), \( \gamma_{j,j'} \) is the scattering rate matrix by nonmagnetic impurities, and \( \gamma^{s}_{j,j'} \) is the same for magnetic impurities. Coupling matrix \( \Lambda \) is defined in the same way as Allen’s matrix \( \lambda_{J,J'} \) \( A_{J,J'} = V_{J,J'}^{\text{pairing}} N_{J,J'}(0) \).

Linearizing Eqs. 2 with respect to \( \Delta \) (an approximation valid near \( T_c \)), we get:

\[
\Delta_{jn}'(1 + G_{j}/2\omega_n) = \Delta_j + \sum_{j'} \Delta_{j'n}' \Gamma_{j,j'}/2\omega_n \Rightarrow \Delta_{jn}' = \sum_{j'} \Delta_{j'}(\delta_{j,j'} + g_{j,j'}/2\omega_n)^{-1},
\]

where we have defined

\[
Z_{jn} = 1 + G_{j}/2\omega_n; \quad \tilde{\Delta}_{jn} = \Delta_j + \sum_{j'} \frac{\tilde{\Delta}_{j'n}}{2\omega_n Z_{j'n}} \Gamma_{j,j'}
\]

\[
\Delta_{jn}' = \tilde{\Delta}_{jn}/Z_{jn}; \quad G_j = \sum_{j'} \gamma_{j,j'}; \quad Z_{jn} = \tilde{\omega}_{jn}/\omega_n
\]

\[
\Gamma_{j,j'} = \gamma_{j,j'} - \gamma^{s}_{j,j'}; \quad g_{j,j'} = \delta_{j,j'} G_j - \Gamma_{j,j'}.
\]

From this immediately follows that

\[
\Delta_j = 2\pi T \sum_{j',n} \frac{\omega_n < \omega_D}{\omega_n Z_{jn}} \sum_{k} \Delta_k (\delta_{k,j'} + g_{k,j'}/2\omega_n)^{-1},
\]

which after a usual trick with subtracting the clean limit, \( g = 0 \), and extending summation to infinity (a useful matrix formula is \((\hat{1} + \hat{\Lambda})^{-1} = \hat{1} - \hat{\Lambda}(\hat{1} + \hat{\Lambda})^{-1}\)), gives

\[
\Delta_j = \sum_{j',k} A_{j,j'} [L \delta_{j,k} - X_{j,k}] \Delta_k; \quad X_{j,j'} = \sum_{k} g_{k,j'}/2 \sum_{n} \omega_n^{-1}(\omega_n \delta_{j,k} + g_{j,k}/2)^{-1},
\]

where \( L = \ln(2\gamma^* \omega_D/\pi T_c) \). By introducing the eigensystem of \( g \), \( g_{j,j'} = \sum_{k} R_{j,k}^{-1} d_k R_{k,j'} \), we can express \( X \) in terms of the difference between the two incomplete gamma-functions:

\[
X_{j,j'} = \sum_{k} R_{j,k}^{-1} \sum_{n} \omega_n^{-1} d_k (\omega_n + d_k/2)^{-1} R_{k,j'} = \sum_{k} R_{j,k}^{-1} \chi(d_k) R_{k,j'},
\]

with \( \chi(x) = \psi(1/2 + x/2\pi) - \psi(1/2) \), which is the standard definition of the matrix function \( \hat{\chi} = \chi(\hat{g}) \).
in some cases it emerges quite naturally. A good example is a bilayer with spin fluctuations antiferromagnetically and strong in the interband channel. It looks on the first glance to be an artificial, unnatural condition. However, parameter of different sign in different bands. In this case the interaction must be small in the intraband channels where the order parameter has always the same sign. A possible solution is the well-known repulsive (for triplet pairing, it would be attractive). Thus it cannot lead to a superconducting solution.

\[
\Lambda = \sum_k (\Lambda^{-1}_{kk} + X_{jk})^{-1} L \Delta_k, \tag{4}
\]

which means that now \( T_c \) is defined by the maximal eigenvalue of the effective matrix \( \Lambda_{eff} = (\Lambda^{-1} + X)^{-1} \). As can be seen immediately from Eqs. 2 and 3 and the definition of \( g_{JJ'} \), diagonal non-magnetic scattering rates \( \gamma_{ii} \) have dropped out from Eq.(3). This is the manifestation of Anderson theorem for an anisotropic case: such scattering does not influence \( T_c \) (in a considered Born limit), as in isotropic superconductors. As will be discussed below, this argument works only for diagonal in \( J,J' \) non-magnetic scattering, while all other are, in principle, pair-breaking.

In the second order in \( \Lambda \) (assuming that \( AX \) is small),

\[
\Lambda_{eff} = \Lambda - \Lambda AX \Lambda. \tag{5}
\]

If we recall that \( \Delta \) forms the eigenvector of \( \Lambda \) for the maximal eigenvalue \( \lambda_{eff} \), we can immediately write the lowest-order correction to \( \lambda_{eff} \):

\[
\delta \lambda_{eff} = - \sum_{JJ'} \Delta_J X_{J,J'} \Delta_{J'}/L^2 \sum_J \Delta_J^2 \tag{6}
\]

It is illustrative to consider explicitly a two-band case for weak scattering \( (\gamma_{ij}, \gamma_{ij}' < T_c) \); the effective matrix is then given by:

\[
\hat{\Lambda}_{eff} = \hat{\Lambda} - \frac{\pi}{8Tc_0} \hat{\Lambda} \cdot \left( \begin{array}{cc} 2\gamma_{11}^2 & \gamma_{11}^2 + \gamma_{12} & \gamma_{12} - \gamma_{12} \\ \gamma_{21}^2 - \gamma_{21}^2 & 2\gamma_{22}^2 & \gamma_{22}^2 + \gamma_{21}^2 \end{array} \right) \cdot \hat{\Lambda}. \tag{7}
\]

When all \( \Lambda \)'s are equal (isotropic case), the standard Abrikosov-Gor'kov result is recovered: \( \delta \lambda \approx -\pi \lambda^2 (\gamma_{11}^2 + \gamma_{12}^2 + \gamma_{21}^2 + \gamma_{22}^2)/8Tc_0 \). The main point of the AG theory is that \( \gamma^* \) enters equations for \( \omega \) and \( \Delta \) with opposite signs. That is why the magnetic impurities appear to be pair-breakers, and the non-magnetic ones not. The above solution shows explicitly that in anisotropic case of Eqs. 2 only intraband non-magnetic scattering does not influence \( T_c \) (\( \gamma_{ii} \) drop out).

An interesting special case is \( \Lambda_{12}, \Lambda_{21} < \Lambda_{11}, \Lambda_{22} \). Then in the effective \( \Lambda \) matrix the nondiagonal element \( \Lambda_{eff}^{12} = \Lambda_{12} + \pi\Lambda_{11}\Lambda_{22}(\gamma_{12} - \gamma_{12}^2)/8Tc_0 \) becomes negative, if \( \gamma_{12} - \gamma_{12} > 8Tc_0\Lambda_{12}/\Lambda_{11}\Lambda_{22} \). As discussed above, this situation will lead to sign reversal of the order parameter. One can say that, when attractive interband coupling is relatively weak and the magnetic interband scattering is strong, the system chooses to have two gaps of the opposite signs, losing in pairing energy, but avoiding the pair-breaking due to interband scattering.

4. Maybe the most interesting example of an interband sign reversal of the order parameter is superconductivity due to electron-paramagnon interaction, in other words, superconductivity due to dynamic exchange of spin fluctuations. For simplicity, we consider the case of singlet pairing. In this case the pairing potential is \( V_{ij,kl} = \int d\mathbf{R} d\mathbf{R}' \sum_{\alpha\beta,\delta} \langle \alpha |J(r-\mathbf{R})\sigma_{\alpha\beta}|j\beta\rangle \chi(|\mathbf{R} - \mathbf{R}'|) \langle k\gamma|J(r-\mathbf{R})\sigma_{\gamma\delta}|l\delta\rangle \) where \( J \) is exchange interaction and \( \chi = \langle S(\mathbf{R}) S(\mathbf{R}') \rangle \) is spin-spin correlation function. Contrary to the electron-phonon interaction, this interaction is repulsive (for triplet pairing, it would be attractive). Thus it cannot lead to a superconducting solution where the order parameter has always the same sign. A possible solution is the well-known \( d \)-wave state, where the order parameter changes sign upon rotation by \( \pi/2 \) in the momentum space. Another possibility is to have order parameter of different sign in different bands. In this case the interaction must be small in the intraband channels and strong in the interband channel. It looks on the first glance to be an artificial, unnatural condition. However, in some cases it emerges quite naturally. A good example is a bilayer with spin fluctuations antiferromagnetically
correlated between the layers. This system models some superconducting cuprates, like YBa$_2$Cu$_3$O$_{7-\delta}$. If the one-electron tunneling between the layers is larger than superconducting gap, two bands are formed and well defined: bonding (symmetric) and antibonding (antisymmetric). By symmetry, only electron states of different parity can interact via exchange of antiferromagnetic (thus antisymmetric) spin fluctuations. Spin-fluctuation induces superconductivity in YBa$_2$Cu$_3$O$_{7-\delta}$ with sign reversal of the order parameter was quantitatively investigated in Ref.\textsuperscript{13}

5. Now we shall briefly discuss some experimental implications of the interband sign reversal of the order parameter, if it occurs in YBa$_2$Cu$_3$O$_7$. It is believed that four different bands cross the Fermi level in YBa$_2$Cu$_3$O$_7$: two plane bands, which are bonding (B) and antibonding (A) combinations of the individual planes’ states, the chain (C) band, and a small pocket formed mainly by apical oxygen states (which is not discussed here). We adopt here the point of view that the superconductivity originates in the plane bands, and is induced in the chain band by interband proximity effect. Thus we shall consider a superconducting state which is characterized by the order parameters of the opposite signs in the bonding and antibonding bands. For the following discussion we shall also mention that, according to calculations,\textsuperscript{14} the total density of states is small (∼ 15%), while its contribution in the plasma frequency $\omega_{py}^2 \propto N(0) \nu_{Fy}$, is considerable (∼ 50%). These finding are confirmed by the experiment: Maximal Fermi velocity was calculated\textsuperscript{15} to be $\sim 6 \times 10^7$ cm/s and corresponds to the point where the chain Fermi surface crosses the Γ–Y line. This value agrees well with the Raman experiments.\textsuperscript{16} Calculated plasma frequency anisotropy $\omega_{py}^2/\omega_{pz}^2 \simeq 1.75$, as discussed in Ref.\textsuperscript{17} is in agreement with the optical and transport measurements. Band A is, according to calculations, rather heavy, which also agrees with the experiment.\textsuperscript{18} Band B is light again. Both A and B bands are nearly tetragonal. Their relative contribution to the normal-state transport is defined by the partial plasma frequencies (Table I). Importantly, bands A and C at $q_z = 0$ can cross by symmetry, for instance they are degenerate with $\epsilon = E_F$ at $q_x = \approx 0.8\pi/a, \approx 0.2\pi/B, 0$). For all $q_z \neq 0$ these bands hybridize. This is the reason for YBCO being the most three-dimensional of all high-$T_c$ cuprates. An extremal orbit in $q_z = 0$ plane, which appears because of the $A − C$ hybridization, has been seen in de Haas-van Alphen experiments.\textsuperscript{19}\textsuperscript{20}

Which of the three possible mechanisms (if any) can be operative in YBa$_2$Cu$_3$O$_7$ and produce interband sign reversal? It appears that any of the three can. First, it is possible that the phonon-induced interaction is stronger in the intraband than in the interband channel. For instance, one of the key feature of the electronic structure of YBa$_2$Cu$_3$O$_7$ are extended saddle points at van Hove singularities. It was shown that extending of the saddle points is directly related to the geometry (warping) of the CuO$_2$ planes.\textsuperscript{21} Saddle points appear only in the antibonding band. Thus, one may expect the saddle points regions to interact particularly strongly with the phonons of the even parity (gerade), with correspondingly weaker intraband interaction. Second, the main type of magnetic defects in YBa$_2$Cu$_3$O$_7$ are localized magnetic moments on Cu, which appear near O vacancies and other imperfections. In view of very strong interplane antiferromagnetic correlations of the Cu spins in underdoped compounds, it seem plausible that each defect-related Cu moment in the fully doped system induced magnetic moment at the nearest Cu site in the next plane. Such pair of spins will have exactly that symmetry, that is needed to stabilize the superconductivity with the opposite signs of the order parameter. Third, the popular model of the spin-fluctuation superconductivity, which is usually associated with the $d$-pairing, in fact always has another solution, namely that with the angular $s$-symmetry and interband sign reversal.\textsuperscript{22} Which of the two solutions is more stable depends on whether or not the spin fluctuations are antiferromagnetically correlated between the planes. If they are, the second solution is always more stable. Correspondingly, if one accepts the results of the numerous neutron scattering experiments, which do show the interplane correlation, and assumes the spin-fluctuation induced superconductivity, the resulting state is one with the interband sign reversal. Note that this is true both for the overdamped magnetic excitation deduced by Pines and co-workers from the NMR data, and for the finite-energy paramagnon as deduced from the neutron scattering.

Now we shall briefly discuss some experiments, probing the symmetry and/or anisotropy of the superconducting state. It is instructive to single out those few experiments that deal with the symmetry of the order parameter. Those are Josephson tunneling experiments and neutron scattering experiments. Both were discussed in detail, in the context of the interband sign reversal, in Refs.\textsuperscript{23} and \textsuperscript{24} respectively. Numerous Josephson tunneling experiments, performed in the last two years, show that the phases of the tunneling current along crystallographic
a and b directions are shifted by $\pi$. A natural interpretation is in the form of the one-band $d_{x^2-y^2}$ state. As it was pointed out in Ref. 19 because of the strong difference in effective masses, the chain band in YBa$_2$Cu$_3$O$_7$ contribute substantially into the transport in b direction. Thus, the relative phase of the tunneling currents in the interband sign reversal model depends on whether the sign of the order parameter in the chain band is the same as in the bonding band (which dominates in-plane transport) or as in the antibonding band. In Ref. 19 it was argued that the latter is true, since the chain and the antibonding band anticross at the Fermi level. In this case reasonable numerical estimates lead to the conclusion that the total tunneling current will have the desired phase shift, unless the chain band gap is too small. The authors of Ref. 19 made an opposite assumption, namely that the signs of the order parameter in the chain and the antibonding band are different. This makes the gap disappear near the band crossing and thus fits with various experiments indicating vanishing minimal gap. However, it is more difficult to explain the Josephson experiments in this model (one would have to assume that both in the chain and in the bonding bands gaps are much smaller than in the antibonding band).

While the interband sign reversal model offers an alternative explanation of the Josephson experiments, it appears to be the only one able to explain the neutron scattering result in a plausible manner. Currently there is a consensus in the neutron scattering community that in fully doped YBa$_2$Cu$_3$O$_7$ a sharp peak in the imaginary part of magnetic susceptibility $\chi''$ appears below $T_c$ at the plane wave vector $\mathbf{q}=(\overline{\frac{\pi}{a}}, \frac{\pi}{b})$, which shows perfect antiferromagnetic correlations between the planes. The most obvious effect of the onset of superconductivity on magnetic susceptibility is the appearance of a peak in $\chi'$ at $h\omega = 2\Delta$, which however is allowed only if the correspondent wave vectors connects the parts of the Fermi surface with the opposite signs of the order parameter. Another effect is the appearance of a peak in $\chi''$ at $h\omega = \Delta + \xi_{vH}$, where $\xi_{vH}$ is the position of the van Hove singularity. In the RPA approximation the two peaks enhance each other, thus leading to a strong effect. However, the only way to ensure that the peak has no ferromagnetic (between the planes) component is to assume that there is no intraband sign reversal at $\mathbf{q}=(\overline{\frac{\pi}{a}}, \frac{\pi}{b})$, but only interband sign reversal.

It is worth noting that intraband sign reversal does not exclude any angular anisotropy of the order parameter. Moreover, the specific model considered in Ref. 19 appeared to have rather strong angular variation of the gap value, with the symmetry close to $(x^2-y^2)^2$. There are numerous experiments pointing to large variation of the absolute value of the gap in YBa$_2$Cu$_3$O$_7$. As long as these experiments do not distinguish between $d$- and strongly anisotropic $s$-wave states we do not discuss them here. A final note concerns isotope effect, which is known to be small at the optimal doping (the highest $T_c$), but increases rapidly when superconductivity is suppressed by oxygen reduction, Pr doping, or other means. Such a behavior is very typical for superconductivity of mixed phonon and non-phonon origin. However, both components should work constructively, and not destructively. For instance, in a regular electron-phonon isotropic superconductor spin fluctuations, if any, are pair-breaking, and not pairing. One possibility to rationalize the isotope affect is to say that the electronic component is non-magnetic (like acoustic plasmons), or that the major process is two-magnons exchange. Another possibility is to say that the phonon- and spin fluctuation-induced interaction are separated in the phase space. For instance, if (as it is usually assumed) spin fluctuations are operative near $\mathbf{q}=(\overline{\frac{\pi}{a}}, \frac{\pi}{b})$, but phonons only at small $\mathbf{q}$'s, both can be pairing in the contest of the $d_{x^2-y^2}$ superconductivity. Alternatively, if spin fluctuation are operative only in the interband channel, as it follows from the neutron scattering experiments, and phonons only in the intraband channel (in other words, only even $[gerade]$ phonons interact considerably with the electrons), the isotope effect trends follow naturally.

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