Possibility of robust detrended fluctuation analysis as a method for identifying fractal properties of geomagnetic time series

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Abstract. Geomagnetic data has been demonstrated to exhibit fractal properties, which are analysed using various fractal methods. These methods allow the characterization of geomagnetic activity during certain periods using the Hurst exponent. In this study, the geomagnetic activity during the quiet period of the month of December 2011 is analysed using the r-DFA method, of which viability to identify fractal properties of geomagnetic data has not been tested yet, and also using its established predecessor; the detrended fluctuation analysis (DFA). The results show that the r-DFA method is indeed viable to be used upon geomagnetic time series, with comparable if not better performance compared to its predecessor.

1. Introduction

The geomagnetic field of the earth is subject to the charged particles in the solar wind [1], which, if it caused a compression on the magnetosphere, a mayhem in the form of telecommunication system disruption, among many other things, would ensue not long after its occurrence [2]. Rest assured that this is not the customary case, as solar wind does not compress the magnetosphere violently all the time; an occurrence that is one of the traits attributed with a phenomenon called geomagnetic storm.

Earth’s geomagnetic activity varies as the time goes by. When there is no occurrence of geomagnetic storm, Earth’s geomagnetic activity is only affected by the daily variation caused by the solar and lunar activity. This period is called geomagnetic quiet period or quiet day. When geomagnetic storm is presently occurring, the force exerted by the solar wind towards the magnetosphere is greater than that of the solar and lunar activity; making the latter seem insignificant compared to the former. This period is called geomagnetic active period or active day, where geomagnetic activity is affected predominantly by geomagnetic storm.

Fractal properties is more often than not used as a trait to be analysed for understanding geomagnetic activity; it shows the self-similarity of an object under multiple levels of magnification [3]. The Hurst exponent, a parameter describing long term correlation of a time series is used to measure the fractal properties during certain geomagnetic period. The value of Hurst exponent varies from 0 to 1 (0<H<1), of which further details are explained in the following table.
Table 1. The possible ranges of Hurst exponent and its meaning.

| Ranges       | 0<H<0.5 | H=0.5 | 0.5<H<1 |
|--------------|---------|-------|---------|
| Category     | Anti-persistent | Fully Random | Persistent |
| Meaning      | Long-range Negative correlation | No discernible correlation (random) | Long-range Positive correlation |

One of several previous studies [4–7] which utilized Hurst exponent in analysing geomagnetic data and thus influenced this study was that of Hamid et al. [4], which uses several fractal methods in their study; the most notable being the detrended fluctuation analysis (DFA) method as it has been shown that it performed better than its contemporaries [4,8]. In 2017, Habib et al. devised an improved DFA method called robust detrended fluctuation analysis (r-DFA) which they claimed to have a better performance compared to its predecessor. Since its inception, it has been implemented by several studies in the field of hydrology [9–11] as well as other fields [12–14]. In this study, the relatively new robust detrended analysis (r-DFA) viability as a method to identify fractal properties of geomagnetic time series is tested by analysing the geomagnetic quiet day of December 16th, 2011. The accompanying results are compared with the result of its established predecessor.

2. Method and analysis

2.1. Data Selection

The data used in this study is courtesy of the Magnetic Data Acquisition Systems (MAGDAS), a division of the International Center for Space Weather Science and Education (ICSWSE), Kyushu University, Japan [15]. The details of the data are described in the following table.

Table 2. The details of Magnetic Data Acquisition Systems (MAGDAS) data used in this study.

| Station | Location (Geographic) | Period       | Data Resolution |
|---------|-----------------------|--------------|-----------------|
| DAV     | Davao, Philippines (07°N, 125.40°E) | December 16th, 2011 | 1 second (86400 values) |

2.2. The Method of Detrended Fluctuation Analysis (DFA) and Robust Detrended Fluctuation Analysis (r-DFA)

Detrended fluctuation analysis is one of the established fractal methods that is used frequently across all fields of science. It is reputable for its ability to generate a more accurate results compared to the other methods as it avoids the unwanted false-positive detection of scaling changes in a particular time series.

The analysis starts with the integration and division of the geomagnetic data, followed with a detrending process which generates the fluctuation of the time series, \( F(\tau) \). The details of the calculation can be found in the pioneering study by Peng et al.[16]. The relationship of \( F(\tau) \) against \( \tau \) is then plotted logarithmically;

\[ \langle F(\tau) \rangle_n \sim \tau^\alpha \] (1)
By obtaining the scaling exponent, $\alpha$, the Hurst exponent can be calculated according to the type of noise which the data belongs to, with detailed description in the following table.

| Types of noise                  | Fractional Gaussian noise | Non-stationary fractional Brownian noise |
|---------------------------------|---------------------------|-----------------------------------------|
| Hurst exponent                  | $H = \alpha$              | $H = \alpha - 1$                        |

After a particular geomagnetic data is computed using regular DFA method, the robust detrended fluctuation analysis (r-DFA) is commenced with the creation of surrogate datasets which will be analysed again using the regular DFA method. Piecewise linear regression will be used to optimise ‘crossovers’[17,18] which specifies the change in scaling of a data and also to minimise inevitable errors such as that caused by least-square errors. An ANCOVA (analysis of covariance) test will be performed if the data is confirmed to have crossovers, of which will be calculated to the exact amount the data supposedly has. Finally, the Hurst exponent can be calculated using the rules that apply to the regular DFA method. This method has been open sourced in the form of MATLAB program[19].

3. Results and Discussions

3.1. Detailed Description of Data

Figure 1 shows the time series of the global Dst index and the readings of geomagnetic northward component, H, at Davao station (DAV) on December 16th, 2011. It can be observed on the Dst index graph that on this day, the geomagnetic activity (illustrated with blue line) remains stable not too far from the adequate range of the zero baseline (illustrated with red line). Hence, this day can be classified as a quiet day.

Figure 1. Time series of global Dst index (top) and geomagnetic northward component, H (bottom), at Davao station (DAV) on December 16th, 2011

3.2. Detrended Fluctuation Analysis (DFA)

Figure 2 shows the result of detrended fluctuation analysis (DFA) on the geomagnetic data. A spectral limit of $2.7788 < \log \tau < 4.334$, or simply $\tau = 6$ hours until $\tau = 10$ minutes is to be imposed as the data beyond this range have been susceptible to the daily solar variation and the atmospheric tides, which introduce undesirable interference that may affect the data[4].
Figure 2. Detrended fluctuation analysis (DFA) of geomagnetic horizontal component, $H$, at Davao station (DAV) on December 16$^\text{th}$, 2011. $H$ inside the graph specifies the value of the Hurst exponent.

From the Figure 2 above, it is observed that on December 16$^\text{th}$, 2011, the geomagnetic time series of horizontal component, $H$, has the Hurst exponent value of 0.36, which renders the dataset to be anti-persistent ($H < 0.5$).

3.3. Robust Detrended Fluctuation Analysis (r-DFA)

Figure 3 shows the result of robust detrended fluctuation analysis (r-DFA) on the geomagnetic data. The open sourced algorithm provided by the originators of this method [19] is used extensively in this study, and no notable changes to the program are made, besides a few adjustment to a few parameters.

A few notable changes include the number of surrogate sets, the r-DFA order number, and the consideration of crossovers. The number of surrogate sets used is 5 sets, compared to over 100 sets used in previous study [20]. The simplicity of the choices made is for the sake of comparison with the other established methods, as the older methods does not have the same intricacy this method requires. This can be said as well towards the DFA order number used and the number of crossover considered. The r-DFA order number considered for comparison is restricted only for DFA of the 6$^\text{th}$ order, while the number of crossover considered is none, even though there is a possibility of a crossover to exist.

As for the limits of $\tau$ (or time) imposed to the other methods, in using this method, we decided not to restrict the timeframe of the analysis in the specified range already mentioned in the other methods. This may seem to be an imprudent decision and may render this whole ordeal as futile (perhaps it is), but, as the title of this study already suggests, this study is merely a test of r-DFA viability as a method to identify fractal properties of a geomagnetic time series. Besides that, we would like to see how the open sourced program stack up against the established methods on its own without any changes in the program.
Figure 3. Robust detrended fluctuation analysis (r-DFA) of geomagnetic horizontal component, $H$, at Davao station (DAV) on December 16th, 2011. $H$ inside the graph specifies the value of the Hurst exponent.

From the Figure 3 above, it is observed that on December 16th, 2011, the geomagnetic time series of horizontal component, $H$, has the Hurst exponent value of 0.11, which renders the dataset to be anti-persistent ($H < 0.5$). Even though the difference in result is quite sizable compared to its predecessor, it still reaches the same conclusion as its predecessor.

3.4. Results Summary

Table 4. The summary of the Hurst exponent results for each respective method. DFA for Detrended Fluctuation Analysis and r-DFA for Robust Detrended Fluctuation Analysis.

| Dataset               | DFA      | r-DFA    |
|-----------------------|----------|----------|
| December 16th, 2011   | 0.36±0.03| 0.11±0.01|

The Hurst exponent results for each method, namely DFA and r-DFA are unquestionably different from each other as the percentage difference between these values is over 106%. However, both of these results reached the same conclusion that the geomagnetic activity on the day of December 16th, 2011 showed anti-persistence tendencies ($H < 0.5$); a long-range negative correlation. These results are in accordance with several previous studies [4,21,22] which have stated that geomagnetic quiet day exhibits an anti-persistence behaviour. It is also worth noting that the r-DFA boasts a smaller amount of standard error compared to DFA. Therefore, without a doubt, r-DFA is viable to identify fractal properties of geomagnetic data. Still, there is an issue that needs to be addressed.

The most glaring problem r-DFA method has in this study, as we have mentioned earlier, is that there’s no spectral range limits imposed during our analysis. Nevertheless, if we base our understanding of the anti-persistence behaviour as a long-range negative correlation of a dataset, of which the quiet day certainly exhibits the characteristics of in this r-DFA analysis, regardless of the included external noises, we consider that the original program would suffice as a fractal method for geomagnetic time series on its own without any altercation whatsoever.

The next logical step would be testing this method against a longer dataset; longer than a day’s worth of data. Rabiu et al.[23] has analysed over five years of geomagnetic data for its periodicity variations, and yet the fractal properties have not been characterized. It is something that piqued our
interest to see whether this method holds up when used against longer datasets, and whether its capability to identify fractal properties can be relied upon.

4. Conclusion

In this work, we have tested the viability of the relatively new robust detrended analysis (r-DFA) as a method to identify fractal properties of geomagnetic time series. The results show that the method is indeed viable to be used upon geomagnetic time series and even had a performance that was comparable if not better than its predecessor by producing results that adheres to the long withstanding theory of anti-persistence behaviour in geomagnetic quiet day. On that note, we would consider r-DFA as our preferred method of choice in our upcoming studies, particularly studies of events involving phenomena of geomagnetic storm and solar flare, as well as identification of multiple effect on geomagnetic activity such as quasi-biennial oscillation (QBO) and equatorial plasma bubble (EPB).

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