Abstract: “Number” is an important learning dimension in primary mathematics education. It covers a large proportion of mathematical topics in the primary mathematics curriculum, and teachers use most of their class time to teach fundamental number concepts and basic arithmetic operations. This paper focuses on the nature of mathematics pedagogical content knowledge (MPCK) concerning arithmetic word problems. The aim of this qualitative research was to investigate how well the future primary school teachers in Hong Kong had been prepared to teach mathematical application problems for third and sixth graders. Nineteen pre-service teachers who majored in both mathematics and primary education were interviewed using two sets of scenario-based questions. The results revealed that innovative approaches were suggested for teaching third graders while the strategies suggested for teaching sixth graders were mostly based on a profound understanding of mathematical content knowledge. Many participants demonstrated sound knowledge about the sixth grader’s mathematical misconception, but most of them were unable to precisely indicate the third grader’s error in presenting a complete solution for a typical mathematics word problem. A deep understanding of elementary number theory seems to be a precondition for developing pre-service teachers’ MPCK in teaching arithmetic word problems.

Keywords: arithmetic word problems; elementary number theory; mathematics pedagogical content knowledge; primary mathematics; teacher education

1. Introduction

The core of primary mathematics is the understanding of number concepts and operations. Problem solving and daily life applications as part of mathematics teaching and learning in primary schools are highlighted in reform mathematics curricula across countries [1]. Apart from in-service professional development, pre-service teacher education is also a key concern to ensure the success of an innovative mathematics curriculum [2].

In reviewing the research literature on initial teacher preparation, there has been a longstanding interest in pre-service primary teachers’ knowledge of elementary number theory [3,4]. Nevertheless, far too little attention has been paid to their pedagogical knowledge for teaching arithmetic word problems although primary students always demonstrated difficulty in solving multi-step word problems involving whole numbers in international assessments, for example, the Trends in International Mathematics and Science Study (TIMSS) [5]. Primary students in Hong Kong can struggle with mathematics word problems even if they maintain an outstanding performance in TIMSS [5,6]. The purpose of this research was therefore to ascertain the nature of Hong Kong pre-service primary teachers’ mathematics pedagogical content knowledge (MPCK) on teaching lower- and upper-grade students to solve arithmetic word problems. The contributions of this paper would be of interest to mathematics educators, researchers, and curriculum developers.
1.1. Elementary Number Theory and Word Problems in Primary Mathematics Education

Elementary number theory mainly refers to the properties of integers such as parity, primality, additivity, and divisibility [7]. Four basic operations on whole numbers including addition, subtraction, multiplication, and division constitute a vital part of any primary mathematics curriculum [8]. Students are expected to develop an extended number sense and apply mathematical operations to solve different types of real-world science, technology, engineering and mathematics (STEM) problems during the primary education phase [9].

Word problems encompass various cognitive processes including mathematical and linguistic development [10]. They play a prominent role in primary mathematics education because of their application function. Verschaffel and his research team claimed that arithmetic word problems can “offer practice for everyday situations of applied problem solving and mathematical modeling in which learners will need what they have learnt in school” [11] (p. 2).

However, there are some difficulties often encountered by students in solving arithmetic word problems. According to Salemeh and Etchells [12], the sources of difficulties in solving word problems include inappropriate analysis, lack of reading comprehension skills, use of the wrong operation, and carelessness of students. As pointed out by Rich and Yadav [13], there is a tendency for primary students to solve mathematics word problems without consideration of context. It should be noted that this may be caused by their weak mental representation skills and poor reading comprehension skills [14]. Thus, helping students become confident word problem solvers is especially challenging for pre-service teachers due to their limited practical teaching experience.

There has been little research into pre-service teachers’ MPCK concerning the teaching of word problems. One recent study has been conducted by Csíkos and Szitányi [15] where a standardized interview protocol was used to determine 30 Hungarian pre-service and in-service primary and lower-secondary teachers’ MPCK in teaching word problem-solving strategies. Their inquiry approach involved five sets of interview questions but those regarding teaching methods for word problems were only asked of the in-service teachers. The findings showed that some pre-service teachers did not anticipate a textual answer from students even though answering in complete sentences is an important tradition in Hungary. Out of the six pre-service teachers, four considered using a narrative-oriented approach to teach the process of word problem solving whilst two preferred adopting a paradigmatic-oriented approach.

In Hong Kong, “number” is a significant learning dimension in primary mathematics education. About half of mathematics periods (44%) in primary schools are allocated to teaching relevant topics under this content area [16]. The basic arithmetic operations are broadly covered in the reform primary mathematics curriculum [17]. Unlike many Western countries, the ability of primary students to provide clear and unambiguous representation with a descriptive or conclusive statement in solving arithmetic word problems is emphasized in Hong Kong. Students do not achieve full marks for word problems on mathematics tests if they only present the correct answer and do not fully justify their thinking. Accordingly, teachers have to recognize individual differences in their students and hence support them in the process of mathematical word problem solving.

1.2. The Concept of MPCK

Mathematics pedagogical content knowledge (MPCK) refers to pedagogical content knowledge (PCK) in mathematics education. The fundamental notion of PCK is attributed to Shulman’s [18] theory of teacher knowledge. This particular type of knowledge is defined as the “special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding” [19] (p. 8). It is widely accepted that PCK is a domain of subject-specific knowledge for teaching.

Based on the work of Shulman [19], various models and theories regarding knowledge of mathematics teachers have been produced. For example, Ernest [20] proposed a model of cognitive structures connecting mathematics teachers’ knowledge, beliefs, and attitudes. He claimed that
both pedagogical and curricular knowledge were regarded as practical knowledge of mathematics teaching. Ball and her colleagues developed a “practice-based theory of mathematical knowledge for teaching” [21] (p. 389). In their theory, PCK includes knowledge of content and student, knowledge of content and teaching, and knowledge of content and curriculum.

In 2008, the term MPCK initially appeared in the Teacher Education and Development Study in Mathematics (TEDS-M) project [22]. TEDS-M was the first large cross-national study into teacher education systems but Hong Kong did not participate. It used questionnaires to collect large-scale data from 17 participating countries to measure pre-service primary and lower-secondary teachers’ mathematical competencies. A total of 32 question items were designed to investigate MPCK in the TEDS-M study. Pre-service teachers’ abilities of generating fruitful questions, explaining or representing mathematical concepts or procedures, analyzing and diagnosing students’ responses, providing appropriate feedback, and responding to unexpected mathematical issues were assessed [22]. Since there are different perceptions of MPCK, it is necessary to clarify the conception of MPCK by specifying its components for the present study.

An extensive search of literature indicated that MPCK is usually made up of two principle components. The first component is knowledge of instructional strategies for mathematics, such as the use of “analogies, illustrations, examples, explanations, and demonstrations” [19] (p. 9) in mathematics teaching. The second component is knowledge of students’ conceptions about mathematics [21,23]. Researchers are especially interested in how teachers deal with students’ mathematical errors and misconceptions [24]. For instance, the TEDS-M researchers found that pre-service teachers had a difficulty in assisting students to overcome or reduce their misconceptions in primary mathematics [25,26]. In regard to enactment of mathematics teaching, originally addressed by the TEDS-M study [22], it is believed that researching both components can foster a comprehensive understanding of MPCK among pre-service teachers.

1.3. Frameworks for Analyzing MPCK

Many studies have utilized qualitative research methods to uncover the successes and challenges that pre-service and in-service teachers are likely to experience in their teaching practice [27,28]. In terms of qualitative measurement, various frameworks have been developed to unpack the nature of MPCK. All such frameworks can be construed as progressions of the concept of PCK devised by Shulman [19,29].

Rowland and his colleagues in the United Kingdom collected data from classroom observations and proposed a “knowledge quartet” framework for the analysis of mathematics teaching [30–32]. Their framework consisted of four categories: foundation, transformation, connection, and contingency. Noting that the second category of the quartet “transformation” echoes Shulman’s [19] PCK. Undoubtedly, the observation of actual classroom practice is an ideal data collection method to identify teachers’ MPCK. However, this may not be practicable for cross-sectional studies of pre-service teachers with a large sample size.

Chick and her colleagues at the University of Melbourne developed a framework for analyzing Australian primary teachers’ MPCK in teaching decimals and subtraction algorithm, respectively [33,34]. They intended to apply their framework to not only data obtained from classroom observations but also information gathered from questionnaires and interviews. Their framework encompassed three categories and 20 sub-categories [33]. The evidence that identifies each sub-category of the framework is provided in Appendix A.

In Chick, Baker et al. [33] three-category MPCK framework (see Appendix A), the first category is “clearly PCK”, in which pedagogy and content are thoroughly intertwined. The contents of its 11 sub-categories can be summarized as follows: “knowledge of teaching strategies for mathematics, knowledge of students’ conceptions and misconceptions, knowledge of representations in mathematical concepts, and knowledge of resources and curriculum” [35] (pp. 68–69). The second category, namely “content knowledge in a pedagogical context”, involves five sub-categories. It covers the
profound understanding of fundamental mathematics, ability to deconstruct content knowledge to its key components, awareness of mathematical structure and connections, procedural knowledge, and methods of solutions [33]. The third category of the framework is “pedagogical knowledge in a content context”. It consists of four sub-categories referring to “knowledge of learning goals, knowledge of strategies for getting and maintaining student focus, and knowledge of classroom techniques in a specific content area” [35] (p. 69).

The analytical framework proposed by Chick, Baker et al. [33] has been applied by other researchers to investigate pre-service primary teachers’ MPCK in teaching various mathematical topics [36,37]. Its explicit descriptions of the components allow researchers to examine qualitative data in detail. Therefore, the current study utilized Chick, Baker et al. [33] framework (see Appendix A) to identify strengths and limitations in the MPCK held by the sampled pre-service primary teachers in Hong Kong.

There were two research questions guiding this study:

1. What is the nature of Hong Kong pre-service teachers’ MPCK in teaching arithmetic word problems at the third- and sixth-grade levels?
2. To what extent do Hong Kong pre-service teachers recognize and properly comment on primary students’ specific mathematical errors?

2. Materials and Methods

This cross-sectional study was designed to explore pre-service primary teachers’ MPCK across the four academic years of a teacher education program in Hong Kong. It aimed to determine how well the participating pre-service teachers had been prepared to teach arithmetic word problems for third and sixth graders. A qualitative research method was employed in the study. A series of two-phase scenario-based interviews was conducted in order to explore the nature of participants’ MPCK and knowledge of student errors. The following four sub-sections elaborate in more detail on the methods of data collection.

2.1. Participants

Nineteen individual interviews were carried out (with four each from Year 1 to Year 3, and seven from Year 4). Of the 19 interviewees, 14 were female and five were male. These participants had selected mathematics as their major in the undergraduate teacher education program with a concentration in primary education. As the sampled participants were mathematics major students, they had to do 20 mathematics modules throughout the four academic years. Apart from subject matter knowledge, the pedagogy of primary mathematics was also included in their studies. Under the program structure, the pre-service teachers were required to complete particular field experiences and placements: classroom observations during the first year, a 10-day attachment in a local primary school during the second year, as well as six- and eight-week practicums during the third and final years, respectively. It was anticipated that recruiting participants from different year groups would provide rich insights into the research questions.

2.2. Interview Protocol

The interviews comprised two sets of mathematics classroom scenarios. Each scenario consisted of two core questions concerning pre-service teachers’ MPCK in teaching arithmetic word problems (see Table 1). The interview questions aimed to investigate the types of teaching strategies the pre-service teachers preferred and their knowledge of students’ mathematical errors, especially regarding word problems for third and sixth graders.
These arithmetic word problems were selected from the mathematics papers of a large-scale assessment compulsory for all primary and secondary schools in Hong Kong, namely Territory-wide System Assessment (TSA) [38,39]. This was because the errors made by the primary students for these two multi-step word problems were typical. The errors such as “no descriptive statement, explanations, or conclusions” and “inversion of dividend and divisor” have been particularly discussed in the report of students’ mathematics performances on TSA [40], which was published by Hong Kong Examinations and Assessment Authority (HKEAA), although the answers are correct in both cases. Such shortcomings in responses to mathematics word problems are culturally referred to as “student errors” in the Hong Kong setting.

Two word problems and the answers given by the primary students displayed in Table 1 were shown to the interviewees. Each interviewee was required to describe methods to teach the third and sixth graders how to solve these mathematical application problems, and to suggest approaches that could be used to correct students’ mistakes.

To determine the usefulness of the interview protocol, the draft interview questions were trialed with one pre-service teacher from another program. After the pilot interview, a follow-up question was added after Question 1b—“If a third grader gives this following answer, what will you do?”, and Question 2b—“If a sixth grader gives this following answer, what will you do?” The additional question was considered necessary because the interviewee in the pilot study was able to indicate the issues with the students’ answers but he could not clearly express whether the answers were correct or not. Thus, an additional question—“Do you think the answer is correct?” was added in order to clarify interviewees’ thinking. The interview was semi-structured in design, allowing the interviewer to add follow-up questions if the need arose.

### 2.3. Procedures and Ethical Considerations

Three documents including information sheets, consent forms, and interview materials were given to the pre-service teachers who agreed to participate in the study and attend an individual interview. All 19 participants were identified through the program director in order to protect their rights to privacy. Each of the semi-structured interviews took about 20 min. The interviews were conducted in Cantonese (a mainstream dialect of Hong Kong people) and audio recorded. The interviewees remained anonymous in the recordings. Their personal information only including gender and year of study at the time of the data collection were reported in the research.
2.4. Data Analysis

The interview data were thematically analyzed with the assistance of NVivo software package [41]. A content analysis approach was adopted to generate categories in an inductive manner [42]. Before analyzing the qualitative data, the 19 audio-recorded interviews were transcribed and translated from Chinese to English and entered into NVivo.

In the data analysis, the interview transcripts were coded as a set of nodes under each core interview question. These nodes could exist in different forms such as a word, a phrase, or a sentence representing interviewees’ ideas. Hence, a list of sub-categories was generated by grouping nodes which were similar in meaning. Each sub-category was associated with an overarching category for further analysis. Moreover, the analysis of interviewees’ MPCK concerning word problems followed the analytical framework developed by Chick, Baker et al. [33]. Therefore, specific categories and sub-categories were used for classifying the nature of their MPCK (see Appendix A).

3. Results

3.1. An Arithmetic Word Problem for Third Graders

Question 1a shown in Table 1 aimed to determine the extent of pre-service teachers’ MPCK in teaching a class of third graders. All responses to Question 1a were classified into 10 types of teaching methods. Among the 19 interviewees, 17 provided more than one teaching method. On the basis of the MPCK categories and sub-categories proposed by Chick, Baker et al. [33], the teaching methods were classified into eight sub-categories under three categories. One example for each teaching method is presented in Table 2.

| Category 1 | Sub-Category 2 | Teaching Method | Selected Response of Interviewees |
|------------|----------------|-----------------|-----------------------------------|
| Clearly PCK | Teaching strategies | Role-playing | I will conduct a role-playing game to illustrate the problem. (Year 3, male) |
| | Student thinking | Identifying a particular student’s specific ways of thinking | I would like to invite four students to show their working on the blackboard. (Year 3, female) |
| | | Teaching with an aid of materials (real objects or worksheets) | For me, I prefer showing the tea set in the class, and then put a price sticker on each item. (Year 4, female) |
| | Appropriate and detailed representations of concepts | Teaching with an aid of pictures (showing or drawing pictures) | I will draw one teapot and four cups on the blackboard, and then write down the selling price for each item. (Year 1, female) |
| | Purpose of content knowledge | Addressing the content of elementary number theory in a third-grade curriculum | As students can do addition and multiplication at their third-grade level, I can further develop their combined addition and multiplication abilities. (Year 4, female) |
| Content knowledge in a pedagogical context | Deconstructing content to key components | Decomposing the problem into few sub-problems | Before working out its solution, I will request students to think about: How much does a teapot cost? How much do four cups cost? (Year 2, female) |
| | Procedural knowledge | Displaying every step for solving the problem | Write the following sequence on the blackboard: a descriptive statement “The price of the tea set is”, a solution “88+10×4”, and an answer “$128”. (Year 1, female) |
| | Methods of solutions | Demonstrating two methods for solving the problem | Compare the advantages between two solutions: “88+10×4” and “88 + 10 × 4”. (Year 4, female) |
| Pedagogical knowledge in a content context | Getting and maintaining student focus | Teaching by questioning | Ask the students to provide some methods for solving the problem. (Year 4, female) |
| | | Group discussions | Let students do oral presentations after group discussions. (Year 2, male) |

1, 2 These classifications are based on Chick, Baker et al. [33] MPCK framework.
In this particular analysis task, Chick, Baker et al. [33] framework for investigating MPCK of Australian primary mathematics teachers fits the teaching strategies suggested by the pre-service primary teachers in Hong Kong. A total of 45 methods for teaching a mathematics word problem in a third-grade classroom were coded. Seventeen teaching methods suggested by 13 pre-service teachers were placed in three sub-categories under the first category “clearly PCK”. These methods were referred to as reform-oriented approaches such as role-playing, addressing student thinking, and using appropriate representations of the concept of addition. The content being included in the third-grade curriculum was also mentioned. Twenty-one teaching methods provided by 15 pre-service teachers were classified into three sub-categories under the category of “content knowledge in pedagogical context”. The interviewees demonstrated sound procedural knowledge. Other methods covered by the second category included deconstructing content to key components, procedural knowledge, and methods of solutions. In the category of “pedagogical knowledge in content context”, seven teaching methods given by seven pre-service teachers were grouped into this sub-category. These methods included questioning and group discussion to maintain students’ focus.

The second question of the first classroom scenario required interviewees to make comments on the work of a third grader in solving the word problem (see Table 1, Question 1b). Six sub-categories emerged from an analysis of the interview data. Responses from each interviewee were grouped into more than one sub-category. These sub-categories were further classified into two categories. The categories were labeled as “comments on the student’s presentation” and “comments on the student’s working steps”. Table 3 presents the categories and shows a sample response for each sub-category.

Table 3. Examples for interviewees’ comments on a third grader’s work.

| Category                  | Sub-Category                          | Selected Response of Interviewees                                                                 |
|---------------------------|---------------------------------------|---------------------------------------------------------------------------------------------------|
| Comments on the student’s presentation | No descriptive statements (n = 9)     | This student did not write a description to explain his or her solutions. (Year 3, female)         |
|                           | Missing a unit of account (n = 9)     | A dollar sign should be included in the final answer. (Year 3, female)                             |
|                           | Unclear presentation (n = 3)          | Messy working. (Year 1, female)                                                                  |
| Comments on the student’s working steps | Incorrect use of brackets (n = 8)    | The bracket is unnecessary in the solution. (Year 2, female)                                      |
|                           | Correct solutions/answer (n = 7)      | Overall student’s solutions are correct. (Year 3, male)                                           |
|                           | Slow methods for getting the answer (n = 5) | The student should use “88 + (10 × 4)” for solving the problem so that he or she can get the answer quickly. (Year 4, male) |

1 Numbers in brackets indicate interviewees whose responses were included in this sub-category.

As shown in Table 3, the first category “comments on student’s presentation” involved three sub-categories. According to the HKEAA [40], the solution written by third graders was assessed to be “no descriptive statements, explanations, or conclusions” (p. 249). Of the 19 pre-service teachers, nine made a similar comment such as “no descriptions” and “missing a descriptive statement”. Their comments were identified as belonging to the first sub-category—no descriptive statements. Other comments indicating the existence of problems with an aspect of presentation but not specifying a lack of descriptive statements, such as “missing a dollar sign” (Year 1, female) and “an unclear solution” (Year 4, female) were classified as belonging to the second and the third sub-categories—missing a unit of account and unclear presentation.

The second category “comments on the student’s working steps” covered three sub-categories: incorrect use of brackets, correct solutions/answer, and slow methods for getting the answer. There were 20 responses in this category while some of them were problematic in terms of mathematics teaching. Examples of problematic comments are as follows:

The solution seems to be too long. (Year 2, female)
The bracket should be omitted. (Year 4, female)

Even though these comments were unclear as to the intention of the approach, others were reasonable as long as a logical explanation was provided. For instance, the following comment was
identified as belonging to the sub-category—incorrect use of brackets, but the interviewee provided a rationale:

When the student used a bracket in the solution, the calculation inside the bracket should be done first but the student did not. The working steps on the right- and left-hand side did not match each other, so I can see this student could not correctly use the bracket. (Year 4, female)

In short, fewer than half of the interviewees could correct the third grader’s error. Most of the pre-service teachers paid attention to how the student worked out the solutions but were not concerned if the student could use appropriate mathematical language to illustrate the solution. The significance of this observation is that the pre-service teachers might embrace more conventional beliefs about mathematics learning with the focus on getting right answers. They were unaware of missing a description statement in the solution because they accepted the student performing procedures and finding answers without understanding what they meant.

3.2. An Arithmetic Word Problem for Sixth Graders

The responses to Question 2a were identified as illustrating eight types of teaching strategies. Among the 19 interviewees, 15 suggested more than one teaching method. According to the Chick, Baker et al. [33] MPCK categories framework, these teaching strategies were classified into six sub-categories under three categories. An example for each teaching strategy is provided in Table 4.

| Category ¹ | Sub-Category ² | Teaching Method | Selected Response of Interviewees |
|-----------|----------------|-----------------|----------------------------------|
| Clearly PCK | Teaching strategies | Teaching with a similar problem as an example | Working out the solution of a similar mathematical application problem as a demonstration in the class. (Year 2, male) |
| | Student thinking | Identifying a particular student’s specific ways of thinking | Invite some students to show their working on the blackboard. (Year 4, female) |
| | Appropriate and detailed representations of concepts | Drawing pictures | Illustrate the application problem by drawing pictures on the blackboard. (Year 4, female) |

| Content knowledge in a pedagogical context | Profound understanding of fundamental mathematics | Teaching the concept of average | Using this application problem as an example to teach students the meaning of average. (Year 3, female) |
| | Methods of solutions | Demonstrating the best method for solving the problem | Writing an ideal solution “48×6÷8” on the blackboard. (Year 1, female) |

| Pedagogical knowledge in a content context | Getting and maintaining student focus | Teaching by questioning | At the beginning, I would like to ask questions to recall students’ shopping experiences. (Year 3, male) |
| | | Group discussions | Conduct a group discussion. (Year 2, male) |

¹, ² These classifications are based on Chick, Baker et al. [33] MPCK framework.

The teaching methods suggested by the pre-service teachers match Chick, Baker et al. [33] PCK framework. A total of 36 methods for teaching an arithmetic word problem in a sixth-grade classroom were coded. Five teaching methods provided by four pre-service teachers were coded into three sub-categories under the first category “clearly PCK”. These methods included teaching with examples, preparing worksheets, addressing student thinking, and using appropriate representations of the concept of average. Nineteen teaching methods suggested by 16 pre-service teachers were classified as three sub-categories under the second category “content knowledge in a pedagogical context”. These methods were referred to as pre-service teachers’ profound understanding of fundamental mathematics and their knowledge of methods of solution. In the category of “pedagogical knowledge in a content context”, interview data showed that 12 pre-service teachers suggested teaching by questioning and arranging group discussions in order to engage sixth graders in mathematics learning.

The second question of the second classroom scenario required interviewees to make comments on the work of a sixth grader for solving the word problem (see Table 1, Question 2b). Six sub-categories emerged from an analysis of the interview data. The responses from 10 interviewees were grouped into more than one sub-category. These sub-categories were classified into two categories. The categories
were labeled “comments on the student’s mathematical concepts” and “comments on the student’s working steps”. Table 5 presents the categories and shows a sample response for each sub-category.

Table 5. Examples for interviewees’ comments on a sixth grader’s work.

| Category                              | Sub-Category                          | Selected Response of Interviewees                                      |
|---------------------------------------|---------------------------------------|------------------------------------------------------------------------|
| Comments on the student’s             | Inversion of dividend and divisor (n=3)| This student transposed dividend and divisor.                          |
| mathematical concepts                 | Problematic student thinking (n=2)     | (Year 3, female)                                                       |
|                                       | Misconceptions of division/average (n=12)| The student has the wrong concept of division.                         |
|                                       |                                       | (Year 3, male)                                                         |
| Comments on the student’s working    | Incorrect methods of solution (n=10)   | Wrong solutions. He or she should write “(48×6)+8” instead of “8÷(6×48)”. |
| steps                                 | Correct answer (n=3)                   | The final answer is correct.                                           |
|                                       | Incorrect use of brackets (n=1)        | The student should omit the bracket. It is not necessary.              |
|                                       |                                       | (Year 3, male)                                                         |
|                                       |                                       | (Year 4, female)                                                       |

1 Numbers in brackets indicate interviewees whose responses were included in this sub-category.

In Table 5, the first category “comments on the student’s mathematical concepts” involved three sub-categories. According to the HKEAA [40], the solution written by the sixth grader was evaluated as “inversion of dividend and divisor” (p. 269). Among the 19 pre-service teachers, three were able to indicate this specific mathematical error. Their comments were identified as the first sub-category—inversion of dividend and divisor. Two pre-service teachers’ comments referring to the student’s general thinking ability were identified as the second sub-category—problematic student thinking.

The comments relevant to general mathematical concepts were grouped into the third sub-category—misconceptions of division/average. Twelve pre-service teachers commented on the student’s misunderstanding of concepts in division or average. However, half of them were not able to use appropriate mathematical language such as “dividend” and “divisor” to illustrate the student’s error. For instance,

Two sets of numbers “48×6” and “8” were inversed. (Year 4, female)

Eight people cannot be divided by “48×6”. (Year 4, female)

Apart from the conceptual errors in numbers, many interviewees were concerned with the student’s correct and incorrect steps for solving the word problem. Their comments were classified as the second category “comments on the student’s working steps”. It covered three sub-categories including incorrect methods of solution, correct answer, and incorrect use of brackets. It should be noted that nine pre-service teachers’ responses were classified as both first and second categories. For example, the following response was classified as belonging to four sub-categories under two different categories:

The student had a misunderstanding of concept of average. The numbers were inversed.
The error in solution tells me his way of thinking was problematic. [ . . . ] He could find out a right answer but his method of solution was absolutely wrong. (Year 2, male)

This interviewee made comments on the student’s mathematical concepts and presentation of the solution. He could only point out that the “numbers” were inverted but did not clearly mention inversion of the dividend and divisor so that might confuse the student. Apparently, the interviewee was not able to address this student’s error by using appropriate mathematical terms.

These findings showed that the pre-service teachers knew about the sixth grader’s mathematical misconceptions. However, they seemed to have difficulties applying the correct mathematical terminology. Due to a lack of mathematical language for division, most of the pre-service teachers were not able to accurately specify the student’s errors.

4. Discussion and Conclusions

These findings contributed to answering the research questions regarding the nature of pre-service teachers’ MPCK and their knowledge of student errors. The results indicated that there was a significant
difference of pre-service teachers’ MPCK between third- and sixth-grade mathematics, although it could be due to differences in the items as much as the grades. For teaching lower-grade students in solving arithmetic problems, a variety of reform-oriented approaches was suggested. Different kinds of learning activities which aimed to motivate lower grade students to learn were involved in their teaching suggestions which are in accordance with the reform curriculum aimed at promoting student-centered approaches.

On the other hand, the strategies suggested for teaching students in upper grades were mostly categorized as “content knowledge in a pedagogical context”. The majority of interviewees focused on demonstrating the fundamental mathematical concept, as well as the strategies for finding the correct answer. The findings also indicated that most of the pre-service teachers were not able to precisely address the lower grade student’s error in presenting a complete solution for a typical mathematics word problem. However, many interviewees demonstrated concrete knowledge about the upper grade student’s mathematical misconceptions even though only a few could specify the student’s errors. Some interviewees were also able to discuss what caused the sixth grader’s misunderstanding.

Furthermore, interview data also indicated that few pre-service teachers demonstrated sound knowledge of student errors. Of the 19 interviewees, 10 were not able to identify the third grader’s error in setting out. They did not identify a missing descriptive statement in the student’s work. Only three interviewees could specifically indicate a sixth grader’s mathematical error. In fact, pre-service primary teachers have been criticized for a lack of conceptual knowledge in previous studies [43–45]. Prior research has documented that pre-service primary teachers use procedural knowledge rather than conceptual knowledge and are weak at dealing with student misconceptions in primary mathematics [25,46,47]. These findings are consistent with the interview results in the current study, showing that some interviewees were not able to use their conceptual knowledge to comment on a student’s misconceptions of average.

In conclusion, a profound understanding of elementary number theory should be addressed in teacher education. It could be one of preconditions for the development of pre-service teachers’ MPCK in teaching arithmetic word problems at the primary school level. In order to further develop their MPCK, more reflection activities should be provided for pre-service teachers before and after the teaching practicum [48] because reflection can improve their teaching skills and pedagogical understanding [49].

**Funding:** This research was supported by Woosong Academic Research Funding 2020.

**Acknowledgments:** The author would like to acknowledge the pre-service teachers who voluntarily participated in the study.

**Conflicts of Interest:** The author declares no conflict of interest.
## Appendix A

### Table A1. Chick, Baker et al. Framework for Analyzing MPCK [33] (p. 299)

| Category | Sub-Category | Evident When the Teacher ... |
|----------|--------------|------------------------------|
| Clearly PCK | Teaching strategies—general (explicitly maths related) | Discusses or uses general strategies or approaches for teaching a mathematical concept |
| | Teaching strategies—specific (explicitly maths related) | Discusses or uses specific strategies or approaches for teaching a particular mathematical concept or skill |
| | Student thinking—general (excludes misconceptions) | Discusses or responds to possible student ways of thinking about a concept, or recognizes typical levels of understanding |
| | Student thinking—specific (excludes misconceptions) | Identifies a particular student’s specific level of understanding or ways of thinking about a concept |
| | Student thinking—misconceptions—general | Discusses or addresses typical/likely student misconceptions about a concept |
| | Student thinking—misconceptions—specific | Identifies a particular student as having a specific misconception about a concept |
| | Cognitive demands of task | Identifies aspects of the task that affect its complexity |
| | Appropriate and detailed representations of concepts | Describes or demonstrates ways to model or illustrate a concept (can include materials or diagrams) |
| | Knowledge of resources | Discusses/uses resources available to support teaching |
| | Curriculum knowledge | Discusses how topics fit into the curriculum |
| | Purpose of content knowledge | Discusses reasons for content being included in the curriculum or how it might be used |
| Content Knowledge in a Pedagogical Context | Profound understanding of fundamental mathematics | Exhibits deep and thorough conceptual understanding of identified aspects of mathematics |
| | Deconstructing content to key components | Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept |
| | Mathematical structure and connections | Makes connections between concepts and topics, including interdependence of concepts |
| | Procedural knowledge | Displays skills for solving mathematical problems (conceptual understanding need not be evident) |
| | Methods of solution | Demonstrates a method for solving a mathematical problem |
| Pedagogical Knowledge in a Content Context | Goals for learning—mathematics specific | Describes a goal for students’ learning directly related to specific mathematics content |
| | Goals for learning—general | Describes a goal for students’ learning not directly related to content |
| | Getting and maintaining student focus | Discusses strategies for engaging students |
| | Classroom techniques | Discusses generic classroom practices |

### References

1. Mullis, I.V.S.; Martin, M.O.; Foy, P. TIMSS 2007 International Mathematics Report: Findings from IEA’s Trends in International Mathematics and Science Study at the Fourth and Eighth Grades; Boston College: Chestnut Hill, MA, USA, 2008.
2. Hart, L.; Oesterle, S.; Swars, S.; Kajander, A. The Mathematics Education of Elementary Teachers: Issues and Strategies for Content Courses; Information Age Publishing: Charlotte, NC, USA, 2016.
3. Liljedahl, P.; Sinclair, N.; Zazkis, R. Number concepts with number worlds: Thickening understandings. *Int. J. Math. Educ. Sci. Technol.* **2006**, *37*, 253–275. [CrossRef]
4. Zazkis, R. *Relearning Mathematics: A Challenge for Prospective Elementary School Teachers*; Information Age Publishing: Charlotte, NC, USA, 2011.
5. Mullis, I.V.S.; Martin, M.O.; Foy, P.; Hooper, M. TIMSS 2015 International Results in Mathematics; TIMSS & PIRLS International Study Center: Boston College: Chestnut Hill, MA, USA, 2016.
6. Mullis, I.V.S.; Martin, M.O.; Foy, P.; Arora, A. TIMSS 2011 International Results in Mathematics; Boston College: Chestnut Hill, MA, USA, 2012.
7. Yan, S.Y. *Number Theory for Computing*, 2nd ed.; Springer: Berlin/Heidelberg, Germany, 2002.
8. Zazkis, R.; Mamolo, A. On numbers: Concepts, operations, and structure. In *The Second Handbook of Research on the Psychology of Mathematics Education*; Gutiérrez, A., Leder, C.G., Boero, P., Eds.; Sense Publishers: Rotterdam, The Netherlands, 2016; pp. 39–71.
9. Fernández-Cézar, R.; Garrido, D.; Solano-Pinto, N. Do Science, Technology, Engineering and Mathematics (STEM) experimentation outreach programs affect attitudes towards mathematics and science? A quasi-experiment in primary education. *Mathematics* 2020, 8, 1490. [CrossRef]

10. Kurz, T.L.; Gómez, C.; Jiménez-Silva, M. Guiding preservice teachers to adapt mathematics word problems through interactions with ELLs. *IJIME 2017*, 10, 32–51.

11. Verschaffel, L.; Schukajlow, S.; Star, J.; Van Dooren, W. Word problems in mathematics education: A survey. *ZDM 2020*, 52, 1–16. [CrossRef]

12. Salemeh, Z.; Etchells, M.J. A case study: Sources of difficulties in solving word problems in an international private school. *ElJEA 2016*, 2, 149–163.

13. Rich, K.M.; Yadav, A. Applying levels of abstraction to mathematics word problems. *TechTrends 2020*, 64, 395–403. [CrossRef]

14. Boonen, A.J.H.; De Koning, B.B.; Jolles, J.; Van Der Schoot, M. Word problem solving in contemporary math education: A plea for reading comprehension skills training. *Front. Psychol.* 2016, 7. [CrossRef]

15. Csíkos, C.; Szitányi, J. Teachers’ pedagogical content knowledge in teaching word problem solving strategies. *ZDM 2019*, 52, 165–178. [CrossRef]

16. Lam, L. Mathematics education reform in Hong Kong. In Proceedings of the 4th International Conference on Mathematics Education into the 21st Century: The Humanistic Renaissance in Mathematics Education, Palermo, Italy, 20–25 September 2002; Rogerson, A., Ed.; MEC21: Sicily, Italy, 2002; pp. 204–208.

17. Curriculum Development Council. *Supplement to Mathematics Education Key Learning Area Curriculum Guide: Learning Content of Primary Mathematics*; Education Department: Hong Kong, China, 2017.

18. Shulman, L.S. Paradigms and research programs in the study of teaching: A contemporary perspective. In *Handbook of Research on Teaching*, 3rd ed.; Wittrock, M.C., Ed.; Macmillan: New York, NY, USA, 1985; pp. 3–36.

19. Shulman, L. Knowledge and teaching: Foundations of the new reform. *Harv. Educ. Rev.* 1987, 57, 1–23. [CrossRef]

20. Ernest, P. The knowledge, beliefs and attitudes of the mathematics teacher: A model. *J. Educ. Teach.* 1989, 15, 13–33. [CrossRef]

21. Ball, D.L.; Thames, M.H.; Phelps, G. Content knowledge for teaching. *J. Teach. Educ.* 2008, 59, 389–407. [CrossRef]

22. Tatro, M.T.; Schwille, J.; Senk, S.; Ingvarson, L.; Peck, R.; Rowley, G. *Teacher Education and Development Study in Mathematics (TEDS-M): Conceptual Framework*; Teacher Education and Development International Study Center, College of Education, Michigan State University: East Lansing, MI, USA, 2008.

23. Johnson, J. Comparing the major definitions of mathematics pedagogical content knowledge. *JMETC 2017*, 8, 19–28.

24. Depaepe, F.; Verschaffel, L.; Kelchtermans, G. Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teach. Teach. Educ.* 2013, 34, 12–25. [CrossRef]

25. Senk, S.L.; Tatro, M.T.; Reckase, M.; Rowley, G.; Peck, R.; Bankov, K. Knowledge of future primary teachers for teaching mathematics: An international comparative study. *ZDM 2012*, 44, 307–324. [CrossRef]

26. Hsieh, F.J.; Lin, P.-J.; Wang, T.-Y. Mathematics-related teaching competence of Taiwanese primary future teachers: Evidence from TEDS-M. *ZDM 2011*, 44, 277–292. [CrossRef]

27. Crespo, S.; Oslund, J.A.; Parks, A.N. Imagining mathematics teaching practice: Prospective teachers generate representations of a class discussion. *ZDM 2010*, 43, 119–131. [CrossRef]

28. Mishra, L. Conception and misconception in teaching arithmetic at primary level. *J. Crit. Rev.* 2020, 7, 1047–1050. [CrossRef]

29. Shulman, L.S. Those who understand: Knowledge growth in teaching. *Educ. Res.* 1986, 15, 4–14. [CrossRef]

30. Rowland, T.; Huckstep, P.; Thwaites, A. Elementary teachers’ mathematics subject knowledge: The knowledge quartet and the case of Naomi. *J. Math. Teach. Educ.* 2005, 8, 255–281. [CrossRef]

31. Rowland, T.; Turner, F. A framework for the observation and review of mathematics teaching. *Math. Educ. Rev.* 2006, 18, 3–17.

32. Rowland, T.; Turner, F.; Thwaites, A.; Huckstep, P. *Developing Primary Mathematics Teaching: Reflecting on Practice with the Knowledge Quartet*; SAGE Publications: London, UK, 2009.
33. Chick, H.; Baker, M.; Pham, T.; Cheng, H. Aspects of teachers’ pedagogical content knowledge for decimals. In Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education, Prague, Czech Republic, 16–21 July 2006; Novotná, J., Moraová, H., Krátká, M., Stehliková, N., Eds.; PME: Cape Town, South Africa, 2006; pp. 297–304.

34. Chick, H.; Pham, T.; Baker, M. Probing teachers’ pedagogical content knowledge: Lessons from the case of the subtraction algorithm. In Proceedings of the 29th Annual Conference of the Mathematics Education Research Group of Australia Identities: Cultures and Learning Spaces, Canberra, Australia, 16–21 July 2006; Grootenboer, P., Zevenbergen, R., Chinnappan, M., Eds.; MERGA: Adelaide, Australia, 2006; pp. 139–146.

35. Lo, W.Y. Rethinking Teacher Preparation: Towards a Conceptual Framework for the Study of Mathematical Knowledge and Beliefs; Woosong University: Daejeon, Korea, 2016.

36. Livy, S.; Vale, C. First year pre-service teachers’ mathematical content knowledge: Methods of solution for a ratio question. MTED 2011, 13, 22–43.

37. Beswick, K.; Goos, M. Measuring pre-service primary teachers’ knowledge for teaching mathematics. MTED 2012, 14, 70–90.

38. HKEAA. Education and Manpower Bureau Territory-Wide System Assessment 2007, Primary 3 Mathematics, 3ME1; EMB: Hong Kong, China, 2007.

39. HKEAA. Education and Manpower Bureau Territory-Wide System Assessment 2007, Primary 6 Mathematics, 6ME2; EMB: Hong Kong, China, 2007.

40. HKEAA. Territory-Wide System Assessment 2007: Report on the Basic Competencies of Students in Chinese Language, English Language and Mathematics, Key Stages 1–3.; EMB: Hong Kong, China, 2007.

41. Paulus, T.M.; Lester, J.N. Using software to support qualitative data analysis. In Handbook of Qualitative Research in Education, 2nd ed.; Ward, M.R.M., Delamont, S., Eds.; Edward Elgar Publishing Limited: Cheltenham, Glos, UK, 2020; pp. 420–429.

42. Guest, G.; MacQueen, K.; Namey, E. Applied Thematic Analysis; SAGE: Thousand Oaks, CA, USA, 2012.

43. Bartell, T.G.; Webel, C.; Bowen, B.; Dyson, N. Prospective teacher learning: Recognizing evidence of conceptual understanding. J. Math. Teach. Educ. 2012, 16, 57–79. [CrossRef]

44. Kajander, A. Unpacking mathematics for teaching: A Study of preservice elementary teachers’ evolving mathematical understandings and beliefs. J. Teach. Learn. 2007, 5, 33–54. [CrossRef]

45. Karp, A. Analyzing and attempting to overcome prospective teachers’ difficulties during problem-solving instruction. J. Math. Teach. Educ. 2009, 13, 121–139. [CrossRef]

46. Tatro, M.T.; Schwille, J.; Senk, S.; Ingvarson, L.; Rowley, G.; Peck, R.; Bankov, K.; Rodriguez, M.; Reckase, M. Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M); International Association for the Evaluation of Educational Achievement (IEA): Amsterdam, The Netherlands, 2012.

47. Forrester, P.A.; Chinnappan, M. The predominance of procedural knowledge in fractions. In Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia, Fremantle, Australia, 3–7 July 2010; Sparrow, L., Kissane, B., Hurst, C., Eds.; MERGA: Adelaide, Australia, 2010; pp. 185–192.

48. Aguirre, J.; Zavala, M.; Katanyoutant, T. Developing robust forms of pre-service teachers’ pedagogical content knowledge through culturally responsive mathematics teaching analysis. MTED 2012, 14, 113–136.

49. Cheng, M.M.; Tang, S.Y.; Cheng, A.Y. Differences in pedagogical understanding among student–teachers in a four-year initial teacher education programme. Teach. Teach. 2013, 20, 152–169. [CrossRef]

© 2020 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).