Dynamic Analysis of a New Type of Asymmetrical Parallel Mechanism Based on Lagrange Method

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Abstract. In this article, a novel over constrained 2-RRU&RSR parallel manipulator is proposed and analyzed. This mechanism is composed of a moving platform and a base, which are connected by two same configuration RRU legs and one RSR leg. The article uses the 2RRU&RSR parallel mechanism that can achieve 2R1T (two rotations and one translational motion) as the object. Based on the known kinematics forward solutions, the dynamics study was carried out. The dynamic model of the parallel mechanism was established using the Lagrange method and then it can be obtained that the dynamic inverse solution. Then ADAMS software was used to build a virtual prototype of this mechanism and the dynamics simulation was performed by using Lagrange method. Under the trend of a certain trajectory, the law of driving torque changes with time, paving the way for subsequent trajectory planning and control.

1. Introduction

The rapid development of the machinery industry and the wide application of less-degree-of-freedom agencies have prompted the optimization and upgrading of parallel institutions. One type of 2R1T three-degree-of-freedom parallel mechanism has been studied by more and more people.

The parallel robot has high rigidity and symmetry, low cost, small occupied space, and easy to realize the characteristics of its spatial motion [1,2,3]. Up to now, a great deal of related research works in this area has been described in literature. Xie F. G introduced a systematic type synthesis method for 2R1T LMPM based on the Grossmann Line Geometry, and discussed the typical application of this type parallel manipulators in five-axis machine tools in [4]. 3-RPS parallel manipulator is a typical 2R1T LMPM, which was firstly introduced by [5], the kinematic characteristics analysis, instantaneous motions analysis, kinematic synthesis, dimensional synthesis and singularity analysis, etc., of the parallel manipulator were presented in [6,7,8]. Fu Tie studied the configuration of the 2R1T parallel mechanism using the Lie group theory systematically [9]. Among them, the non-fully symmetrical parallel mechanism has obvious advantages over other symmetrical mechanisms in some special applications and can play a decisive role [10]. The advantages of the parallel mechanism are prominent. However, due to the different closed-loop structures, there is a nonlinear coupling of spatial mechanisms with multiple degrees of freedom, resulting in complex forward and inverse solution of the mechanism, rich singularity, small working space, and difficult control. There are several methods for the dynamic modelling of the parallel mechanism [11,12]: Newton Euler's method, Lagrange method, and D'Alembert's principle. The method used in this paper is the Lagrange method to study the dynamics.
of 2RRU&RSR. This method is easy to model and easier to read than the equation using the vector force relationship.

In this paper, a novel over constrained 2R1T 2RRU&RSR parallel manipulator is proposed. This mechanism is composed of a moving platform and a base, connected by two same structure RRU legs and one RSR leg. Three revolute joints fixed on the base are set to be the driven joints. The arrangement of this text is planned as follows. In the section 2, the form of the institution is described in detail. In the section 3, kinematics problems, including the analysis of the reverse position of the manipulator are solved by calculations. The fourth section is about the dynamic modelling analysis of the organization. In Section 5, the virtual dynamic modelling and simulation of 2RRU&RSR is described. In section 6, the conclusions of the 2RRU&RSR research are described.

2. Structure characteristics of the mechanism

In Figure 1, the 2-RRU&RSR parallel mechanism consists of a movable platform and a fixed platform, connected by three limbs: two RRU limbs and one RSR limb. The mechanism is symmetric with respect to RSR limb but not fully symmetrical. It is important to note that the two RRU limbs share a universal joint at the moving platform, which is named as a Hook joint “U”. The parallel mechanism is driven by three rotating joints “R” connected to the fixed platform as driving joints.

![Figure 1 3D model of 2-RRU&RSR](image1)

![Figure 2 The structural diagrams of 2-RRU&RSR](image2)

In Figure 2, A, B, and C (i = 1,2,3) represent the center of each joint of motion pairs on each limb respectively. The basic coordinate frame O - xyz is established on the fixed platform. The X axis is along the OA direction and the Y axis along the OA direction and Z axis is vertically upward. The dynamic coordinate system P - uvw is established on the moving platform. The V axis is located in the moving platform along the C1C2 direction. U axis located on moving platform and perpendicular to V axis. The W axis is perpendicular to the moving platform.

The RPY Euler angle (ψ θ ϕ) representing the attitude of the moving platform represents the angle of rotation of the moving platform around the fixed coordinate system x, y, z.

\[
R = \begin{bmatrix}
c\theta c\phi & c\psi s\theta - c\psi c\theta s\phi & c\psi s\theta + c\psi c\theta s\phi \\
c\phi & c\psi c\theta & -s\psi \\
-s\theta c\phi & c\psi s\theta + c\psi c\theta s\phi & c\psi c\theta - s\psi c\theta s\phi
\end{bmatrix} = \begin{bmatrix}
u_x & v_x & 0 \\
u_y & 0 & w_y \\
u_z & w_z & 0
\end{bmatrix} = \begin{bmatrix}u
v
w\end{bmatrix}
\]

(1)
Here “s” and “c” refers to sin and cos.

3. Kinematic analysis of the mechanism

As shown in the figure 2, under the base coordinate system $O-xyz$, the point $C_i$ can be expressed by vector $r = (x \ y \ z)^T$.

$$r = OA_i + AB_i + BC_i \quad i = 1, 2; \quad r = OA_i + AB_i + BC_i - C_i C_i \quad i = 3$$

(2)

Since $AB_i$ and $BC_i$ are perpendicular to one of the axis of the U joint at the same time, the two limbs and the intersection point $C_i$ of the branch chain are confined to the same plane and apart of this mechanism becomes a planar five-bar mechanism. Therefore, according to structural features, the following equations are obtained:

$$r^T c_i = 0, \quad i = 1, 2$$

(3)

Here, $c_i = c_j = (0 \ 1 \ 0)^T$

Similarly, for the RSR limb, the constraint imposed by the revolute joint restricts both $AB_i$ and $BC_i$ to be normal to the unit vector $c_i$ of the revolute joint axis. Thus, taking the dot product with $c_i$ on both sides of equation (3), leads to:

$$(r + C_i C_i)^T c_i = 0, \quad i = 3$$

(4)

Here, $c_i = R c_i, c_3 = (1 \ 0 \ 0)^T$.

It should be noted that since two RRU limbs can only move in a $Oxz$ plane, thus $u_y = 0$.

Equations (3) and (4) lead to

$$x = -a \sin(\psi) \sin(\theta), \quad y = 0, \quad z = z$$

(5)

$$\phi = 0$$

(6)

The forward position analysis of the mechanism is relative to the determination of the mobile platform pose given the position of $C_i$.

In accordance with the geometric constraint of the mechanism, since two RRU legs can only move in the plane $Oxz$, the $u$ axis of the moving frame $P-uvw$ can only moves in the same plane. So the constraint equation can be deduced as

$$A_0 \cos \phi + B_0 \sin \phi - C_0 = 0$$

(7)

Where

$$A_0 = 2 q_{12} (x_{b2} - x_{b1}), \quad B_0 = 2 q_{12} (z_{b2} - z_{b1}), \quad C_0 = q_{12}^2 + L_{b1b2}^2 - q_{22}^2, \quad L_{b1b2} = \sqrt{(x_{b1} - x_{b1})^2 + (z_{b1} - z_{b1})^2},$$

$$x_{b1} = -b - q_{11} \cos(\pi - \alpha_1), \quad z_{b1} = q_{11} \sin(\pi - \alpha_1),$$

$$x_{b2} = b + q_{21} \cos(\pi - \alpha_2), \quad z_{b2} = q_{21} \sin(\pi - \alpha_2)$$

Where $q_{ij}$ are the distance of $\| A_i B_i \|, \| B_i C_i \|$ and $\| A_i C_i \|$. Expanding equation (7), yields

$$\phi = 2 \arctan \left( \frac{B_0 \pm \sqrt{A_0^2 + B_0^2 - C_0^2}}{A_0 + C_0} \right)$$

(8)

Through equation (8), yields

$$x_{c1} = x_{b1} + q_{12} \cos(\phi), \quad z_{c1} = z_{b1} + q_{12} \sin(\phi)$$

(9)

In the same time, using the cosine theorem of the triangle $C_i C_3 B_i$ and the geometric relationship of the mechanism, yields
Implementing necessary addition and subtraction, yields

\[
A \cos^2(\psi) + B \cos(\psi) + C = 0
\]  

(11)

Where \( A = E^2 + D^2, B = -2KE, C = K^2 - D^2 + \frac{D^2x_{c1}^2}{a^2}, E = -a[b + q_{31} \cos(\pi - \alpha_3)] \),

\[
E = -a[b + q_{31} \cos(\pi - \alpha_3)], D = a[q_{11} \sin(\pi - \alpha_3) - z_{c1}], K = -\frac{L^2 + a^2 - q_{31}^2}{2} - x_{c1}^2, L = B_3C_1
\]

Expanding equation (11), yields

\[
\psi = \arccos\left(-\frac{B \pm \sqrt{B^2 - 4AC}}{2A}\right)
\]  

(12)

Next, substituting equation (12) into equation (5) yields

\[
\theta = \arcsin\left(-\frac{x_{c1}}{a \sin(\psi)}\right)
\]  

(13)

Lastly, given a set of \((\alpha_1, \alpha_2, \alpha_3)\), \(x, y, z, \psi \) and \(\theta\) can be solved using equations (9), (12) and (13).

Above all, the coordinate of the center of the moving platform can be written as the following formula:

\[
^o P = R^\psi P + ^o P
\]  

(14)

By importing parameters including Euler angles and limb lengths into the MATLAB software, the coordinates of the moving platform can be calculated immediately. Because the result of MATLAB is too long, it is very difficult to express integrity in the text.

4. Dynamic Modelling and Analysis of 2RRU&RSR

This article only considers the dynamic and potential energy of the organization's dynamic platform and its various branches, ignoring friction. The mechanism is divided into two systems to calculate: the dynamic platform and three limbs, and finally all the kinetic energy and potential energy are added together to obtain the total kinetic energy and potential energy of the system. Since this mechanism has three degrees of freedom: translation along the Z axis and rotation around the X axis and the Y axis, the pose parameters of the moving platform are:

\[
p = (Z_{\text{RI}}, \psi, \theta)
\]  

(15)

Here \( Z_{\text{RI}} \) is the coordinate of the moving platform in the fixed platform coordinate system, \( \psi \) and \( \theta \) are the angles that the movable platform rotates around the X axis of the fixed platform and the angles that the rotating platform rotates about the \( Y \) axis.

First, consider the kinetic energy and potential energy generated by the moving platform, \( \omega \) is the angular velocity of the moving platform. Then the kinetic energy of the moving platform is:

\[
E_k = \frac{1}{2} m v_{\text{RI}}^2 + \frac{1}{2} \omega^T I_\omega \omega = \frac{1}{2} (m \dot{Z}_{\text{RI}}^2 + \omega^T I_{\omega} \omega)
\]  

(16)

Where \( m \) represents the quality of the moving platform; \( v_{\text{RI}} \) is the linear velocity vector of the center of the platform and \( I_{\omega} \) is the inertial matrix of the motion platform relative to the inertia coordinate system, and \( I_{\omega} = R I_{\text{f}} R^T \), \( R \) is the transformation matrix of the primary axis of the branch relative to the fixed reference frame, \( I_{\text{f}} \) is the moment of inertia of the coordinate system of the orbiting platform.

Then, the kinetic energy and potential energy of the three limbs of the parallel mechanism are calculated respectively. The mass of the connecting rod can be assumed uniform, and that the center of mass is unchanged during the movement.
The second step is the calculation of the kinetic energy and potential energy of two RRU limbs and one RSR limb. Each limb has two rods. The mass of the links is $m_i$, where $i$ represents the number of branches and $j$ represents the lower rod and the upper rod. In order to simplify the process, this article makes the following assumptions: The rod connected to the fixed platform is called the lower rod, and the length is $l_1$; the rod connected to the moving platform is called the upper rod, and the length is $l_2$.

According to the spatial geometric relationship, the coordinates of the second R joints $B_1$, $B_2$ and $B_3$ can be obtained as follows:

$$
\begin{align*}
\begin{bmatrix}
x_{B_1} \\
y_{B_1} \\
z_{B_1}
\end{bmatrix} &= \begin{bmatrix} -b - l_i \cos(\pi - \alpha_i) \\ 0 \\ l_i \sin(\pi - \alpha_i) \end{bmatrix}, & \begin{bmatrix}
x_{B_2} \\
y_{B_2} \\
z_{B_2}
\end{bmatrix} &= \begin{bmatrix} b + l_i \cos(\pi - \alpha_i) \\ 0 \\ l_i \sin(\pi - \alpha_i) \end{bmatrix}, & \begin{bmatrix}
x_{B_3} \\
y_{B_3} \\
z_{B_3}
\end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ l_i \sin(\pi - \alpha_i) \end{bmatrix}
\end{align*}
$$

So the potential energy of lower rods and the upper rods can be written as follows:

$$P_{ij} = \frac{1}{2} m_i g Z R_i , P_{ij} = \frac{1}{2} m_i g (Z R_i + Z C_i) \quad (i = 1, 2, 3)$$

Then the kinetic energy of RRUs and RSR branches are solved by using the following formula:

$$E_{\omega j} = \frac{1}{2} (v_{\omega j}^T m v_{\omega j} + a_{\omega j}^T I_{\omega j} a_{\omega j}) \quad (i = 1, 2, 3; j = 1, 2)$$

The linear velocity of the connecting rod can be obtained by finding the vector diameter of the connecting rod center of mass, $v = \dot{r}$. $R_i$ is the center of the connecting rod. Therefore, the linear velocity component of the connecting rod along the x axis, y axis, and z axis can be calculated by deriving the coordinates of the center of mass of the connecting rod respectively.

The vectors of the upper and lower links of three branches’ centers are:

$$
\begin{align*}
\begin{bmatrix}
x_{B_{1i}} \\
y_{B_{1i}} \\
z_{B_{1i}}
\end{bmatrix} &= \frac{1}{2} \begin{bmatrix} x_{B_1} - b \\ 0 \\ z_{B_1} \end{bmatrix}, & \begin{bmatrix}
x_{B_{2i}} \\
y_{B_{2i}} \\
z_{B_{2i}}
\end{bmatrix} &= \frac{1}{2} \begin{bmatrix} x_{B_2} + x_{C_3} \\ 0 \\ z_{B_2} \end{bmatrix}, & \begin{bmatrix}
x_{B_{3i}} \\
y_{B_{3i}} \\
z_{B_{3i}}
\end{bmatrix} &= \frac{1}{2} \begin{bmatrix} x_{B_3} + x_{C_3} \\ 0 \\ z_{B_3} \end{bmatrix};
\end{align*}
\begin{align*}
\begin{bmatrix}
x_{B_{1i}} \\
y_{B_{1i}} \\
z_{B_{1i}}
\end{bmatrix} &= \frac{1}{2} \begin{bmatrix} 0 \\ y_{B_1} + b \\ z_{B_1} \end{bmatrix}, & \begin{bmatrix}
x_{B_{2i}} \\
y_{B_{2i}} \\
z_{B_{2i}}
\end{bmatrix} &= \frac{1}{2} \begin{bmatrix} 0 \\ y_{B_2} + y_{C_3} \\ z_{B_2} \end{bmatrix}, & \begin{bmatrix}
x_{B_{3i}} \\
y_{B_{3i}} \\
z_{B_{3i}}
\end{bmatrix} &= \frac{1}{2} \begin{bmatrix} 0 \\ y_{B_3} + y_{C_3} \\ z_{B_3} \end{bmatrix}
\end{align*}
$$

$R_{ij}$ is a coordinate transformation matrix between a link system and a fixed coordinate system, and $I_{ij}$ is a moment of inertia around a link coordinate system.

To establish the coordinate system of the rod, the z axis is perpendicular to the platform, the y axis coincides with the axis of rotation, and the x axis is determined by the right-handed helix rule. According to the spatial geometric relations, the angular velocity components of the connecting rod around the x, y and z axis can be obtained by deriving the time around the x, y and z axis:

Angular speed of lower robs and upper robs are:
Substituting the parameters into equation (19) yields the kinetic energy of the three branches.

The total potential energy and the total kinetic energy of the system are:

\[ P = P_1 + P_2 + P_3 + P_2 + P_3 + P_3 + P_3; \quad E = E_1 + E_1 + E_2 + E_2 + E_3 + E_3 + E_3 \]  \hspace{1cm} (22)

5. Dynamic Simulation of the Mechanism

Establish the three-dimensional model of the mechanism, introduce the established mechanism model into ADAMS, and simulate and analyze the dynamics of the organization. The modules in the ADAMS software use the Lagrange equation in the theory of multi-body system dynamics to establish a system dynamics model that can perform static, kinematic, and dynamic analysis on virtual mechanical systems, output displacement, velocity, acceleration, and force curves and so on.

Simulate the forces that drive the joints of each branch of the mechanism. The structural parameters of the parallel robot are in table 1:

| Table 1. Structural parameters of the mechanism. |
|----------------------------------|
| **Institutional parameters**    | **Length (mm)** |
| \( l_1 \)                      | 85             |
| \( l_2 \)                      | 120            |
| \( a \)                       | 70             |
| \( b \)                       | 64             |

Assuming that the initial position of the moving platform is where the angle \( \theta \) is always 0. Its trajectory is a circular motion around its own center with a radius of 20mm. The motion trajectory of the moving platform center is: \( x = 20 \cos \left( \frac{\pi}{2} t \right), y = 20 \sin \left( \frac{\pi}{2} t \right) \).

Where \( t \) is the time parameter. According to the given structural parameters, ADAMS is used for simulation analysis of driving torques of this mechanism.

Figure 3. The curve of position change over time.
Firstly, it can be seen from figure 3 that the curves of the position change of the moving platform. Secondly, the three driving angles changing in 8s and the driven rods move according to the sine curve in the figure 4. Thirdly, in figure 5, the driving torque of one of one RRU branch is always greater than that of the other RRU branch, which is equivalent to the larger torque of the two branches acting as a driven branch. It can be inferred that when the direction of the trajectory in the plane is opposite, the torque of the smaller branch will become the driven branch. From this simulation, when the sign of the angle changes, the force torque in the RSR branch also suddenly changes.

6. Conclusion
In this paper, (1) According to the characteristics of the 2RRU and RSR parallel robots, kinematics analysis is performed to obtain the positive solution of the motion platform, which lays a foundation for dynamic analysis. (2) The rigid body dynamic model is established by the Lagrange function balance method, and the corresponding displacement and force curves of the system are obtained through ADAMS virtual simulation. (3) Reasonably predict the behaviour of the organization and prepare for the next study.

The above analysis has very important significance for the optimization design and engineering application of this mechanism. It can also be used for dynamic analysis of similar parallel mechanisms.

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