Method for calculating the garret window light transmittance coefficient at clear sky CIE

Kh M Guketlov *, H A Khachetlov, K A Kibishev

Kabardino-Balkarian State University named after H.M. Berbekov, 173, Chernyshevsky St., Nalchik, 360004, Russia

E-mail: 123guket@rambler.ru

Abstract. The method for calculating the anti-aircraft lamps’ transmittance, which makes it possible to take into account the brightness distribution dynamics of the clear sky CIE over time, depending on the Sun position in the sky has been developed. The proposed method will significantly increase the natural illumination calculating accuracy in the spaces with garret windows in the daily and annual cycles.

Introduction.
At present, garret windows are widely used for upper natural lighting of buildings under construction in the Russian Federation, both in the northern and southern regions. This is due to the fact that they have a number of advantages in comparison with other types of lamps, for example, such as high light activity, design simplicity, low metal consumption, low weight and good performance.

Based on the fact that in the current lighting calculations of garret windows only the cloudy sky CIE is taken into account [1], we set the task of determining the garret windows’ transmittance with CIE clear sky.

Unlike the cloudy sky CIE, the light flux incident on the input cavity of the light passage way consists of two components: the flux coming from the sky \( F^s \) and the stream coming from the Sun \( F^\Theta \). In addition, the light transmission coefficient is a variable, since the brightness distribution of the clear sky CIE changes over time with a change in the position of the Sun in the sky.

In accordance with [2], the light transmission coefficient of the garret window from the scattered component of the clear sky CIE can be represented in the form of two terms

\[
\tau^{c.s.} = u_c + \omega_c,
\]

where \( u_c \) – is the fraction of the light flux coming from the sky, which, having fallen on the passage way from the outside, will pass through it directly into the room; \( \omega_c \) – is the fraction of the light flux coming from the sky, which, having fallen on the passage way’s walls, after multiple reflections will enter through the exit passage way of the passage way into the room.
Term $u_c$ is determined by the ratio of the luminous flux coming from the sky, directly passing into the room $F_{\text{outp}}^s$, to the light flux coming from the sky at the entrance to the light passage way $F_{\text{inp}}^s$ (Figure 1)

$$ u_c = \frac{F_{\text{outp}}^s}{F_{\text{inp}}^s}, $$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The scheme of the light flux passage coming from the sky through the garret window}
\end{figure}

The luminous flux incident on the exit passage way of the light passage way is determined by integrating illumination over the area of the exit passage way:

$$ F_{\text{outp}}^s = \int_{a}^{b} \int_{0}^{a} E_A dx dy, $$

where $a, b$ – are the length and width of the light passage way; $E_A$ – is the point illumination $A$, lying in the plane of the outlet, from the sky area visible through the inlet of the passage way (Fig. 2), is determined by the formula

$$ E_A = \int_{\alpha_1}^{\alpha_2} \int_{0}^{Z_2(\theta_i, \alpha)} L(z, \alpha) \sin z \cos \alpha dz d\alpha + \int_{\alpha_1}^{\alpha_2} \int_{0}^{Z_2(\theta_i, \alpha)} L(z, \alpha) \sin z \cos \alpha dz d\alpha + \\
+ \int_{\alpha_3}^{\alpha_4} \int_{0}^{Z_2(\theta_i, \alpha)} L(z, \alpha) \sin z \cos \alpha dz d\alpha + \int_{\alpha_3}^{\alpha_4} \int_{0}^{Z_2(\theta_i, \alpha)} L(z, \alpha) \sin z \cos \alpha dz d\alpha, $$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ – are the integration limits, expressed by the geometric dimensions of the light passage way $\alpha_1 = \beta_1 = \arctg x/y; \ \alpha_2 = \alpha_1 + \beta_2 = \beta_1 + \arctg y/x + \arctg (a - y)/x$;

$\alpha_3 = \alpha_2 + \beta_3 = \arctg x/(a - y) + \arctg (b - x)/(a - y)$;

$\alpha_4 = \alpha_3 + \beta_4 = \alpha_3 + \arctg (a - y)/(b - x) + \arctg x/a$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{The scheme of the light flux passage coming from the sky through the garret window}
\end{figure}
\[ \alpha = \alpha_1 + \arctg \frac{y}{x}; \quad L(z, \alpha) \] is the brightness in a random part of the sky \( Z_2(\theta_1, \alpha), \ Z_2(\theta_2, \alpha) \) - are the equations of the lamp inlet contour visible from the calculated point; \( \theta_1, \ \theta_2, \ z \) - define the garret window distance to the inlet sides of the lamp and sky.

The equation (4) is solved in accordance with [3].

**Figure 2.** To the illumination calculation at the point A of the outlet passage way

The luminous flux incident on the light passage way inlet is determined by the formula

\[ F_{\text{inp}}^{\text{s}} = S_{\text{inp}} E_{\text{h}}^{\text{c.s.}} \]

where \( S_{\text{inp}} \) - is the light passage way area; \( E_{\text{h}}^{\text{c.s.}} \) - is the external horizontal illumination from the clear sky CIE is determined by the formula

\[ E_{\text{h}}^{\text{c.s.}} = 2\int_0^\pi \int_0^{\pi/2} L(z, \alpha) \sin z \cos \alpha dz d\alpha \]

Since the right-hand side of (3) cannot be solved analytically, the Simpson cubature formula [4] was applied to solve the integral. Let us break the passage area with a grid of rectangles with the side dimensions \( \Delta a = a/n; \Delta b = b/m \) (fig. 3). Then (2) takes the form

\[ u_c = \frac{\Delta a \Delta b \sum_{i=1}^{n} \sum_{j=1}^{m} A_{i,j} E_{i,j}}{9 S_{\text{inp}} E_{\text{h}}^{\text{c.s.}}} \]

where \( A_{i,j} \) - defines the constant coefficients; \( E_{i,j} \) - is the illumination in grid nodes, determined by (4).
Figure 3. The calculated points’ location in the light passage way outlet plane

A component that takes into account the reflected component of the light flux passing through the light passage way of the lamp from the sky of uniform brightness in accordance with [2] is determined by the equation:

\[ \omega = \frac{S_{inp} (1-u)^2 \rho}{S_s} \left( 1 - \rho + 2 \rho (1-u) \frac{S_{inp}}{S_s} \right) \]  \hspace{1cm} (8)

where \( S_s \) and \( \rho \) – show the area and reflection coefficient of the side faces of the light passage way.

Taking the problem complexity of determining the multiple reflections with uneven brightness of the clear sky CIE, we restrict ourselves to its approximate solution by introducing the factor in (8)

\[ \nu = \frac{L_{av}}{L_{av}^s} , \]  \hspace{1cm} (9)

where \( L_{av}^s \) – is the average brightness of the sky,

\[ L_{av}^s = \frac{E_{c,s}}{\sigma} , \]  \hspace{1cm} (10)

where \( L_{av} \) – is the average brightness of the sky visible through the passage way (Fig. 2),

\[ L_{av} = \left( \sum_{i=1}^{n} \frac{E_i}{\sigma} \right) / n , \]  \hspace{1cm} (11)

where \( E_i \) – is lighting in grid nodes; \( \sigma \) – is the solid angle projection of the sky section visible from the calculated point on the horizontal plane (Fig. 2); \( n \) – is the number of grid nodes.

Thus, the second term (1) is determined from the equation...
To find the light transmission coefficient of the anti-aircraft lamp from the straight line component of the Sun, it is possible to use the empirical formulas obtained experimentally [5]:

For $0.5 \leq i < 2$

$$\tau^\Theta = -0.14 + 0.759i - 1.172\rho + 0.112H^\Theta - 0.142i^2 + 1.5\rho^2 - 0.0031iH^\Theta - 0.003\rho H^\Theta,$$

(13)

For $2 \leq i \leq 6$

$$\tau^\Theta = -0.0329 + 0.1255i + 0.8182\rho + 0.00755H^\Theta - 0.00755i^2 - 0.04375\rho i - 0.00025liH^\Theta - 0.00607\rho H^\Theta,$$

(14)

where $\rho$ – is the reflection coefficient of the light passage way side faces; $H^\Theta$ – defines the Sun height, degrees; $i$ – is the lantern light index,

$$i = \frac{ab}{h(a+b)},$$

(15)

where $h$ – is the height of the garret window light passage way.

The total luminous flux from the sky and the Sun entering into the room through the garret windows is determined by the formula

$$F_{outp} = F^s\tau^{c,s} + F^\Theta\tau^\Theta$$

(16)

Fig. 4 shows the change in light transmission coefficient of the garret window for direct Sunlight, for the scattered light from a clear sky CIE $\tau^\Theta$ and with a uniform brightness of the sky. The curves are plotted for a lamp measuring 2.7 x 2.7 x 0.65 m with a reflection coefficient of the side faces $\rho = 0.6$.

Figure 4. 1 – $\tau^\Theta$; 2 – $\tau^{c,s}$; 3 – with a uniform sky brightness

Summary

The proposed method will significantly improve the calculating accuracy of natural illumination in the spaces with garret windows in the daily and annual cycles, which is important for differentiating the light climate when designing the natural and combined lighting systems.

References

[1] SNiP 23-05-95* Natural and artificial lighting 2004 (Moscow) 53.
[2] Kireev N N 1975 Method for calculating the transmittance of garret windows without filling *Lighting engineering* **8** 10-12.

[3] Kireev N N, Guketlov H M 1983 The computer calculation on natural lighting of the room from the clear sky CIE *Scientific works collection NIISF, Studies in building lighting engineering* 29-33.

[4] Demidovich B P, Maron I A 1963 Fundamentals of Computational Mathematics, Fizmatgiz, Moscow.

[5] Guketlov H M 2014 Determination of light transmission of garret windows from the direct component of the Sun *Problems of modern science and education* **5** (23) 91-95.