PHOTONIC CRYSTAL WAVEGUIDES FOR PARAMETRIC DOWN-CONVERSION

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Photonic crystals create dramatic new possibilities for nonlinear optics. Line defects are shown to support modes suitable for the production of pairs of photons by the material’s second order nonlinearity even if the phase-matching conditions cannot be satisfied in the bulk. These structures offer the flexibility to achieve specific dispersion characteristics and potentially very high brightness. In this work, two phase matching schemes are identified and analyzed regarding their dispersive properties.

Keywords: Nonlinear Optics, Parametric processes, Photonic integrated circuits

1. INTRODUCTION

For many applications there is a need for new sources of entangled photon pairs that are brighter or show certain dispersion characteristics. In quantum communication for example, it is entanglement based quantum key distribution, quantum teleportation and, most importantly, quantum repeaters that all depend on a good supply of entangled photon pairs.

But photon pairs have more traditional applications as well. Phase measurements and lithography may one day employ entanglement to beat their classical limits. In these scenarios one would not only look for entangled pairs, but possibly even for entangled states of multiple photons. At present, though, the efforts to even create enough pairs to begin with, far outweighs the possible gains in precision. It is yet more difficult and costly in terms of resources to interferometrically engineer higher entangled states from pairs.

This would not necessarily be the case if only there were sources available that could be plugged into a wall outlet and had fiber outputs providing the entangled pairs, or even higher dimensional entangled photon states. To date the majority of experiments obtains their pairs from parametric down-conversion in bulk materials.\(^1\)

One standard way of achieving nonlinear interaction in materials that are not a priori suitable for this purpose is to use periodic poling. In periodic poling the sign of the nonlinear tensor component is reversed periodically. However, the linear dispersion characteristics all stay the same. Still, periodically poled crystals and
waveguides have been used as high-yield sources of entangled photon pairs\textsuperscript{2,3}.

Photonic crystals allow us to go beyond the properties of natural materials in many ways\textsuperscript{4}. They have been shown to exhibit super strong or very small dispersion, enhance nonlinear interactions\textsuperscript{5}, and to confine waves in guides and resonators. Therefore it only seems natural to consider photonic crystals for the production of entangled photon pairs\textsuperscript{6}. De Dood \textit{et al.}\textsuperscript{7} recently suggested to exploit form birefringence in multilayer stacks (one-dimensional photonic crystals) for achieving phase-matching in GaAs.

2. PARAMETRIC DOWN-CONVERSION IN PHOTONIC CRYSTAL DEFECT WAVEGUIDES

Spontaneous parametric down-conversion is difference-frequency generation where a pump beam at angular frequency $\omega_p$ and wavevector $k_p$ irradiates a material with some nonlinear tensor $\chi^{(2)}$. Then there is a probability to create pairs of photons at $(\omega_1, k_1)$ and $(\omega_2, k_2)$ so that $\omega_1 + \omega_2 = \omega_p$ (energy conservation) and $k_1 + k_2 = k_p$ (phase-matching). In some cases phase-matching can even be arranged such that the photon pairs are polarization entangled\textsuperscript{1}.

Obviously, $\chi^{(2)}$ should be large and one should choose the largest component of the tensor by selecting the polarizations and directions of the involved light fields to maximize the effective nonlinearity. In bulk materials, however, it is not always possible to work in the maximizing configuration because the natural dispersion of the material requires one to choose certain directions in order to satisfy the phase-matching condition.

Periodicity can come to the rescue. In a periodic structure we know from Bloch’s theorem that the solutions of Maxwell’s equation will be periodic functions multiplied by a plane wave. The periodic functions are labeled by a wavevector $k$ and we only need to consider wavevectors within the first Brillouin zone. This means that the phase-matching condition now reads $k_1 + k_2 = k_p + G$ where $G$ is any wavevector of the reciprocal lattice. In other words, any wavevector we consider can be mapped to one in the first Brillouin zone and phase-matching has to be satisfied within this zone.

Periodicity has been used in the form of periodic poling. In certain crystals such as LiNbO$_3$ and KTiOPO$_4$ it is possible to reverse the sign of the nonlinear optic coefficient. This will make it possible to achieve phase-matching in otherwise forbidden interaction schemes and therefore to utilize the maximum nonlinearity and a long interaction region. Other materials that are promising because of their large nonlinearity, such as GaAs, cannot easily be poled, even though some progress has been made\textsuperscript{8}.

Because the number of pairs created in parametric down-conversion is in the low excitation regime linear with the applied pump power there is usually no need to go to pulsed sources. However, for applications that need more than one photon pair at a time, such as entanglement swapping, one needs to down-convert from ultra-fast
pulses\textsuperscript{9}. In this case, in addition to the phase-matching condition one should ideally achieve group matching as well. Otherwise there will be some differential group delay between the pump and converted waves, or even between the two converted waves if they have different polarization. In the source of entangled photon pairs first described by Kwiat \textit{et al.}\textsuperscript{1} differential group delay severely limits the achievable entanglement and/or brightness, when it is used with a fast pulsed pump and not limited to a very short interaction region\textsuperscript{10}.

While periodic poling solves the phase-matching problem, it does not change the group velocities, because the effective refractive index is not changed. A natural consequence is therefore to not only modulate the nonlinearity but the dielectric as a whole, i.e. to look at photonic crystal structures. In contrast to earlier suggestions\textsuperscript{6,7}, here we will concentrate on waveguide structures that are formed by a line defect in a slab-type photonic crystal as shown in Fig. 1. The purpose of having a waveguide is a long interaction region and good mode overlap.

![Image of photonic crystal waveguide](image)

**Fig. 1.** (Left) A membrane of GaAs with a hexagonal array of holes except for one missing row. The missing row forms an increased-index waveguide. (Right) The shaded area represents the lowest lying gap in the even optical bands within the region outside the light cone for a slab of material with dielectric constant $\epsilon = 13$ patterned with an hexagonal array of holes with lattice constant $a$. One sees that the bottom of the gap increases considerably with the hole size. It is important to remember that confinement perpendicular to the plane of the slab is only due to total internal reflection. Therefore the bandgaps are not true gaps, but rather gaps for light propagating in the slab.

Waveguides in photonic crystal slabs (PCS) have first been studied theoretically by Johnson \textit{et al.}\textsuperscript{11}. There are many possibilities to create waveguides in photonic crystals. The most popular configuration is a line defect in a lattice of air-holes produced by reactive ion etching with subsequent undercut so as to create a free-standing membrane of materials such as GaAs, InP or Si. Spectacular structures up to 1 cm long have been demonstrated with losses as low as 0.76 dB/cm\textsuperscript{12}.

AlGaAs is a favorable material from many points of view. The fact that its
lattice constant is almost independent of the Aluminum fraction makes it possible to grow arbitrary heterostructures. Thus, the fundamental electronic band gap can be chosen freely. It has very high refractive index varying from about 3.6 (for 15% Aluminium) at 775 nm to about 3.3 at 1550 nm. The reported nonlinear susceptibilities scatter but in recent years a consensus seems to have been reached at values around 100 pm/V, which is about 5 times higher than for LiNbO$_3$ and almost 50 times that of $\beta$-BaB$_2$O$_4$. The maximum effective nonlinearity is achieved for three identical polarizations parallel to the [111] direction. Further, if any of the interacting waves is polarized along [011] the effective nonlinearity is independent of the other waves’ polarization. Growth of AlGaAs is usually done on [100] cut wafers. Light propagating in a membrane parallel to the surface could therefore propagate in any direction orthogonal to [100].

Fully three-dimensional band structure calculations using the MIT Photonic Bands package helped identify possible phase-matching schemes. In the literature there is very little data on band gaps for PCS as a function of either hole radius, or slab thickness or dielectric constant. Since for the problem at hand we need to be able to choose the location of the band gap with some care, I first calculated a series of bandstructures resulting in a map of the gap locations as a function of the hole radius. The shaded area in Fig. 1(right) shows the bandgap for even modes in an hexagonal array of holes patterned into a membrane as a function of the hole radius.

A photonic crystal defect waveguide in which the defect has increased refractive index with respect to the periodic structure around it can support two types of guided modes. There are index-guided modes lying below the lowest PCS bands and modes that lie within the band gap of the PCS. Obviously, the confinement will be better for the latter, but if there are no sharp bends, index guided modes should not suffer from much higher losses. Further, in a PCS that is symmetric about its central plane the bands split into even and odd ones, closely corresponding to their TE and TM polarized counterparts in two-dimensional photonic crystals.

The allowed regions for defect waveguide modes are determined by projecting the bands of the perfect PCS onto the guiding k-direction. This procedure masks all areas in the $(\omega, k_x)$ strip that are covered by states that are extended in the plane and can thus not support modes that would be localized to a defect. This is in addition to the restriction that all guided modes lie outside the lightcone of the surrounding medium. In the cases investigated here, the surrounding medium is air around the free-standing Al$_{0.15}$Ga$_{0.85}$As membrane. The material choice is mainly motivated by the desire to create photon pairs at telecommunications wavelengths, i.e. 1550 nm. The pump would then have to be at 775 nm for a symmetric source which requires a 10% Aluminium fraction or higher. At 15% one should be safely outside all exciton and impurity resonances and expect very little absorption for the pump.

From studying gap maps for hexagonal and square lattices of holes, we can conclude that either may be suitable for waveguiding, with the hexagonal ones yielding...
larger gaps and thus possibly more flexibility. The most basic defect waveguide is a single missing row of holes (see Fig. 1), in which waves then propagate in the \( \Gamma-K \) direction of the perfect PCS. It is well known\(^1\) that such a waveguide is not single-mode. Single-mode behavior can be brought about by decreasing the width of the defect\(^2\). It is clear that for a proper source, suitable for the generation of entangled pairs single mode behavior will be essential.

\[ \text{Frequency \( [c/a] \)} \]

![Dispersion diagram (frequency vs. \( k_x \)) for the \( z \)-even modes of a structure like in Fig. 1 with the projected bands of the PCS and the lightcone masked out. The hole radius is 0.38a, the dielectric constant is 13.0 for the upper section and 10.8 for the lower one. The close to horizontal modes in the upper section are gap-guided defect modes, whereas in the lower section we only find index-guided modes. The straight line indicates a potential phase-matching scheme at the points where it intersects with the waveguide modes. For easy reference a (vacuum) wavelength scale is plotted assuming a lattice constant of \( a = 232.5 \) nm.]

Figure 2 shows a scenario in which photons from a \( z \)-even gap-guided mode can down-convert into two photons of a \( z \)-even index-guided band.\(^a\) Figure 3 on the other hand shows a configuration in which pump photons in a \( z \)-odd index-guided band down-convert into photon pairs of a \( z \)-even index-guided band. The lattice is responsible for producing such a vastly different dispersion for the two different symmetries\(^4\).

In any case, the relative strength of the nonlinear interaction will depend on the overlap of the three fields involved. The amplitude of the one photon pair term in the output state will be proportional to (in first order perturbation theory)

\[
\int_V d^3 \mathbf{r} \chi_{ijk}^{(2)} E_i E_j E_k ,
\]

\(^a\)Unfortunately the frequency domain method used in the MIT Photonic Bands package cannot treat dispersive materials. Therefore the calculation was split into two, one for the low refractive index at long wavelengths, and one for the high refractive index and shorter wavelength.
Fig. 3. Similar to Fig. 2, except that the upper section now shows the z-odd bands instead of the z-even ones. A line again connects points that match \( k_p = k_1 + k_2 \) and \( \omega_p = \omega_1 + \omega_2 \).

where \( E_{i,j,k} \) are three components of the electrical field and we’ll have to take the sum over all indices. One of the field components would be the pump (high frequency) and the two other ones the down-conversion fields (low frequency). For the case where the low-frequency modes are degenerate the integrand reduces to \( E_i E_j^2 \). In the case of a periodic structure, we can take the integral over a unit cell. Obviously, the field distribution within the unit cell has to be similar between the three fields to achieve substantial overlap. Also, the fields will only contribute within the material, but not in the air space within the holes or above and below the slab. Preliminary calculations show that at least the phase-matching scheme shown in Fig. 2 achieves reasonable overlap for band number 9 (counted from 0 frequency) at \( k = 0.44 \) with band number 1 at \( k = 0.22 \). It is part of our ongoing work to calculate the effective nonlinearity in absolute units.

3. DISCUSSION

The goal for parametric down-conversion in photonic crystals is to exploit the high nonlinearity of certain semiconductors and possibly achieve group velocity matching. Group velocity matching is similar to achieving phase-matching over an extended bandwidth, if the band curvature is not too big.

It is obvious that the above schemes do not yet achieve group-matching, where the second one is closer to the goal but still too far away to be practically relevant for this purpose. Given that the gap-guided modes exhibit very small group velocities, it would be exciting if there was a way to phase-match between bands of different gaps. However, since the gaps in a PCS are typically shifted up in frequency by the vertical confinement it seems very difficult to find gaps at one frequency and twice the frequency simultaneously, where both are also below the light cone in the
waveguiding direction.

For photon pairs created by ultrafast pulses one has to compare the differential group delay (DGD) with the pulse duration. If the group velocities are \( u_{pump} \) and \( u_{dc} \) then the DGD per length is just \(|1/u_{pump} - 1/u_{dc}|\), which means that for small group velocities it will be more difficult to achieve a small DGD.

Yet, for some applications, such as interfacing to electromagnetically induced transparency (EIT) and stored light it is desirable to have very narrow-band down-conversion sources. For this purpose various groups have considered counter-propagating solutions, which are typically phase-matched only in a point, or equivalently, have an extremely high DGD, because one of the velocities is negative. For this purpose the first even-even-even phase-matching scheme (Fig. 2) appears to be a very good solution.

4. CONCLUSIONS

So far, we have only considered degenerate cases, i.e. both down-conversion photons have the same frequency. Asymmetric sources have applications in the preparation of single-photon states and in schemes where one photon needs to be detected with high efficiency, whereas the other needs to propagate with very low loss through an optical fiber. Clearly, there are many solutions for non-degenerate phase-matching to be found in the above diagrams. Whether a certain solution is interesting depends on the details of the intended application.

I have shown that phase matching is in principle possible in photonic crystal waveguides. The coupling of both the pump light and the down-converted one in and out of the waveguide respectively is a challenge. Cleaved edges can be used\(^{15}\), as well as fiber tapers\(^{16}\), where the latter have achieved the highest reported coupling efficiency to date.

The biggest interest in photon pairs is associated with entanglement. With a single waveguide mode and polarization there can be no entanglement a priori. However, the photons of a pair are still created simultaneously and therefore it should always be possible to construct a source of time-bin entangled photon pairs as is customary for photon pairs from waveguides\(^3\). With a solution that creates pairs in two different polarizations one can, using two identical waveguides, even construct a source of polarization entangled photon pairs.

In conclusion, we have seen that it is possible to exploit the high optical nonlinearity of AlGaAs and to achieve phase-matching in a slab-type photonic crystal defect waveguide. The next steps are to perform fully dispersive calculations and to tune the waveguide modes by changing the waveguide’s width and edge shape so as to give the involved modes the desired dispersion. While some details are still missing in this picture an ultra bright source of photon pairs at telecommunication wavelengths based on photonic crystals seems within reach.
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