Testing discrete symmetries with kaons: status and perspectives

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Abstract. The status of present experiments and future projects with kaons is reviewed, focusing on prospects for discrete symmetries tests.

1. Introduction
Since their discovery in 1947, K mesons, or kaons, turned out to be one of the most interesting and promising system to study discrete symmetries in particle physics. The discovery of the $\tau - \theta$ anomaly [1] and of CP violation in $K^0$ decays [2], and the subsequent theory development are two bright examples of the outcomes of these studies.

The present report focuses on the status of present experiments and future projects on this topic; for general and historical reviews the reader is referred to several excellent papers and books in the literature [3].

2. Kaon experiments at hadron machines
2.1. The discovery of direct CP violation
Tens of years after CP violation was established in the $K^0 - \bar{K}^0$ mixing, the $\epsilon'/\epsilon$ parameter was definitely measured to be different from zero, representing the first indication of direct CP violation in the decay amplitudes, and verifying the Cabibbo-Kobayashi-Maskawa (CKM) theory. This important result was accomplished by two experiments: KTeV at Fermilab and NA48 at CERN. They were third-generation experiments, both taking data at the end of the 90s, using high-energy proton beams to produce long- and short-lived neutral kaons on a fixed target.

In the KTeV experiment [4], two neutral kaon beams were formed from the secondary particles produced by 800 GeV/c protons colliding on a beryllium oxide target using a system of collimators, absorbers and sweeping magnets. The neutral kaon decays were detected in 110 - 158 m range from the production target. One of the beams passed through an active regenerator, made of scintillator, which produced a coherent mixture of $K_S$ and $K_L$ states. The regenerator alternated between the two neutral beams in order to reduce systematic differences between $K_L$ and $K_S$ decays.
The charged decay products were detected in a drift chamber spectrometer. The spectrometer was equipped with two chambers before and two after an analyzing magnet. Each chamber measured charged particle tracks in horizontal and vertical views. The neutral decay products were measured in a calorimeter composed by an array of 3100 Cesium Iodine (CsI) crystals, located after the spectrometer. A nearly hermetic photon veto system (up to 100 mrad) rejected background events for the $\pi^0\pi^0$ mode coming from interactions in the regenerator, semileptonic and $K_L \rightarrow \pi^0\pi^0\pi^0$ decays.

2.3. NA48 experiment at CERN

The main feature of the NA48 experiment [5] was the two simultaneous, almost collinear beams of neutral kaons derived from proton beams from the CERN SPS, and delivered to two fixed targets. The kaon beams had a common decay region and decays from both beams were recorded with the same detector. Both targets were made from beryllium and had a length of 400 mm and a diameter of 2 mm. The far target was located 126 m before the beginning of the decay region while the near target was only 6 m up-stream of the fiducial region and displaced by 7.2 cm in the vertical direction from the axis of the far-target beam. The two beam axes had an angle of 0.6 mrad with respect to each other and crossed at the longitudinal position of the electromagnetic calorimeter, 120 m down-stream of the near target. In both beams, charged particles were deflected by sweeping magnets. Particle decays from the far target were almost exclusively $K_L$ decays, while decays originating from the near target were mainly $K_S$ (and neutral hyperon) decays with, however, a small component of $K_L$ decays. The NA48 detector was designed to measure with high precision momenta of both charged and neutral particles. The charged particle reconstruction was provided by a magnetic spectrometer, with 4 drift chambers and a calorimeter. The reconstruction of photon energy, direction, time, and position was given by a LKr calorimeter.

2.4. The legacy of NA48 and KTeV experiments

The two collaborations published the final measurement of $\Re(\epsilon'/\epsilon)$ as $(19.2 \pm 2.1) \times 10^{-4}$ (KTeV) [6] and $(14.7 \pm 2.2) \times 10^{-4}$ (NA48) [7] where $\Re(\epsilon'/\epsilon)$ is obtained from the double ratio of decay rates:

$$\frac{\Gamma(K_L \rightarrow \pi^0\pi^0)/\Gamma(K_S \rightarrow \pi^0\pi^0)}{\Gamma(K_L \rightarrow \pi^+\pi^-)/\Gamma(K_S \rightarrow \pi^+\pi^-)} = \frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \simeq 1 - 6\Re(\epsilon'/\epsilon),$$

with

$$\eta_{00} = |\eta_{00}|e^{i\phi_{00}} = \frac{\langle \pi^0\pi^0 | T | K_L \rangle}{\langle \pi^0\pi^0 | T | K_S \rangle} = \epsilon - 2\epsilon'$$

$$\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}} = \frac{\langle \pi^+\pi^- | T | K_L \rangle}{\langle \pi^+\pi^- | T | K_S \rangle} = \epsilon + \epsilon'.$$

Combining all the recent measurements, the world average is $\Re(\epsilon'/\epsilon) = (16.8 \pm 2.0) \times 10^{-4}$ [8], clearly demonstrating the existence of the CP violation in decay. However, due to theoretical uncertainties in the hadronic matrix elements, to get information from $\Re(\epsilon'/\epsilon)$ on the Standard Model (SM) and New Physics (NP) beyond it, is difficult and remains a challenge to theoretical calculations. Recent progresses in lattice QCD calculations promise to solve this issue [9].

The KTeV collaboration measured also the $\Im(\epsilon'/\epsilon)$ parameter, exploiting the coherent regeneration phenomenon to modify the pure $K_L$ beam into a coherent superposition of $K_L$ and $K_S$. The fit of the measured $\pi^+\pi^-$ (and $\pi^0\pi^0$) decay intensity downstream the regenerator yielded the best CPT tests in two pion decays [6]: $\phi_{00} - \phi_{+-} \simeq -3\Im(\epsilon'/\epsilon) = (0.30 \pm 0.35)^\circ$ and
\(\phi_s - \phi_{SW} = (0.40 \pm 0.56)^\circ\), with \(\phi_s \simeq (\phi_{00} + 2\phi_{+-})/3\), \(\phi_{SW} = \arctan(2\Delta m/\Delta \Gamma)\) the so-called superweak phase, and consistent with no CPT violation.

The unitarity relation, originally derived by Bell and Steinberger [10]:

\[
\left( \frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{SW} \right) \left[ \frac{\Re \epsilon}{1 + |\epsilon|^2} - i \Im \epsilon \right] = \frac{1}{\Gamma_S - \Gamma_L} \sum_f A^*(K_S \to f) A(K_L \to f) \equiv \sum_f \alpha_f ,
\]

where the sum runs over all accessible final states \(f\) appearing in the decay amplitudes \(A(K_{SL} \to f)\), can be used to bound the CPT violating parameter \(\Im \epsilon\) (and other parameters), after having provided all the \(\alpha_f\) parameters, \(\Gamma_S, \Gamma_L\), and \(\phi_{SW}\) as inputs.

The analysis including the latest results by the KTeV collaboration [6] yields significant limits on \(\Im \epsilon\) and the neutral kaon mass difference (tests of the CPT symmetry): \(\Im \epsilon = (-1.5 \pm 1.6) \times 10^{-5}\) and \(|m_{\bar{K}_0} - m_{K_0}| < 4.8 \times 10^{-19}\) GeV at 95% CL, slightly improving the results obtained in a previous analysis [12].

2.5. NA48/2 experiment at CERN

The NA48/2 collaboration at CERN operated with simultaneous \(K^+\) and \(K^-\) beams of 60 GeV/c in 2003-2004. The charge asymmetry of the Dalitz plot linear slopes was measured for both \(K^\pm \to \pi^\pm \pi^+ \pi^-\) and \(K^\pm \to \pi^\pm \pi^0 \pi^0\) decays, being \(A_y \not= 0\) a manifestation of CP violation in decay. The charge asymmetries were measured to be \(A_y^g = (-1.5 \pm 2.2) \times 10^{-4}\) with \(3.11 \times 10^9\) \(K^\pm \to \pi^\pm \pi^+ \pi^-\) decays, and \(A_y^g = (1.8 \pm 1.8) \times 10^{-4}\) with \(9.13 \times 10^7\) \(K^\pm \to \pi^\pm \pi^0 \pi^0\) decays [13], still consistent with no CP violation.

2.6. Searches for rare kaon decays at LHCb

It’s worth mentioning that searches for rare kaon decays are also possible at LHC, at least for the cleanest decay channels with a closed kinematics and a controllable background. This holds for the \(K_S \to \mu^+ \mu^-\) decay, which is a Flavour Changing Neutral Current (FCNC) transition that has not yet been observed. This decay is suppressed in the SM, with an expected branching fraction [14] \(\text{BR}(K_S \to \mu^+ \mu^-) = (5.0 \pm 1.5) \times 10^{-12}\), while the current experimental upper limit is 3.2 \(\times 10^{-7}\) at 90% confidence level (C.L.) [15]. Although the dimuon decay of the \(K_L\) meson is known to be \(\text{BR}(K_L \to \mu^+ \mu^-) = (6.84 \pm 0.11) \times 10^{-9}\) [8], in agreement with the SM, effects of new particles can still be observed in \(K_S \to \mu^+ \mu^-\) decays.

The LHCb detector, described in detail in Ref.[16], is a single-arm forward spectrometer covering the pseudorapidity range \(2 < \eta < 5\). A search for the decay \(K_L \to \mu^+ \mu^-\) is performed, based on a data sample of 1.0 fb\(^{-1}\) of pp collisions at \(\sqrt{s} = 7\) TeV collected by the LHCb experiment at the Large Hadron Collider. The observed number of candidates is consistent with the background only hypothesis, yielding an upper limit of \(\text{BR}(K_S \to \mu^+ \mu^-) < 11(9) \times 10^{-9}\) at 95 (90)% confidence level. This limit is a factor of thirty below the previous measurement [17, 18].

3. Future kaon experiments at hadron machines

3.1. The intensity frontier for NP searches

The present collider searches at high transverse momenta exclude wide regions of the parameter space for NP thus increasing the importance of the study of rare processes sensitive to very high

\(^1\) The two physical states can be expressed as \(|K_0\rangle \propto |K_1\rangle + \epsilon_2 |K_2\rangle\) and \(|K_L\rangle \propto |K_1\rangle + \epsilon_L |K_0\rangle\), with \(\epsilon_2\) and \(\epsilon_L\) two small complex parameters describing the CP impurity in the physical states. One can equivalently define \(\bar{\epsilon} \equiv (\epsilon_2 + \epsilon_L)/2\), and \(\delta \equiv (\epsilon_2 - \epsilon_L)/2\); adopting a suitable phase convention (e.g. the Wu-Yang phase convention [11]) \(\bar{\epsilon} \not= 0\) implies T violation, \(\delta \not= 0\) implies CPT violation, while \(\delta \not= 0\) or \(\bar{\epsilon} \not= 0\) implies CP violation.
energy. This is true, in particular, for the $K \rightarrow \pi \nu \bar{\nu}$ decays, whose SM predictions are known with very high precision [19]. The branching ratio expectation of the CP-violating $K_L \rightarrow \pi^0 \nu \bar{\nu}$ channel is: $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (0.243 \pm 0.039 \pm 0.006) \times 10^{-10}$, while for the charged mode is: $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (0.781 \pm 0.075 \pm 0.029) \times 10^{-10}$. In both cases, the first error summarizes the parametric uncertainties, while the second one the remaining theoretical error. The extraction of the hadronic part of the amplitude is obtained from the very well measured $K_{E3}$ decay. The main uncertainties are due to the knowledge of the CKM matrix elements.

Processes mediated by FCNC are suppressed in the SM by the GIM mechanisms [20]. Further suppression can originate from hierarchies in CKM matrix or helicity. In some models of NP large enhancement with respect to SM rates are possible. In this sense the $K \rightarrow \pi \nu \bar{\nu}$ modes play a key role in seeking NP beyond the SM. This is clear from Fig.1, which reproduces the BR values predicted by the SM and by some models of NP [21]. Apart from establishing a direct signal of NP, the correlation of the BR of the two modes can be exploited to probe the flavour structure of NP theories and, therefore, to distinguish among different classes of NP scenarios. The exclusion regions given by the Grossman-Nir consistency conditions [22], limiting the BR ratio of the two decay modes, and by the only measurement of the charged mode [23] are also reported in Fig.1.

![Figure 1](image_url)

**Figure 1.** Correlation between the branching ratios of the charged and neutral $K \rightarrow \pi \nu \bar{\nu}$ decays showing the predictions of SM and some NP models [21]

The measurement $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.73 \pm 1.15) \times 10^{-10}$ is based on 7 events observed by E959 and E787 experiments at BNL [23], and is compatible with the SM prediction. The CP-violating neutral $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay has not been observed yet. An upper limit to the BR was set by the E391 collaboration at KEK [24]: $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 260 \times 10^{-10}$ (90% C.L.).
3.2. NA62 experiment at CERN

The main goal of the NA62 experiment at CERN [25] is to collect about 100 K$^+ \rightarrow \pi^+ \nu \bar{\nu}$ events with 10% of background in two years of data taking. This implies to collect more than 10$^{13}$ K decays with a background rejection factor of at least 10$^{12}$, assuming a signal acceptance of 10%.

The experimental method relies on exploiting a decay-in-flight technique with an intense charged kaon beam at high energy. This requires a beam of un-separated charged hadrons (with 6% K$^+$), a long decay region with an extended detector and the event-by-event measurement of kaon momenta. Positive charged kaons will be used in order to get, at the same proton energy and flux, higher kaon fluxes and lower pion background.

The incoming kaon is measured by the Gigatracker system in the beam. The charged decay particle is measured by the straw-chamber spectrometer and is identified by the Ring Imaging Cerenkov (RICH) detector and the muon-veto sampling calorimeter. The LKr calorimeter, originally built for NA48, is used as a veto for forward photons. Photons at large angles are intercepted by a series of 12 ring-shaped veto counters constructed using lead-glass blocks from the OPAL electromagnetic barrel calorimeter.

The NA62 analysis strategy is based on the accurate kinematic reconstruction of all the particles detected in the event to disentangle the signal from the huge amount of background processes, a precise timing to associate correctly the $\pi^+$ with the parent K$^+$, a system of efficient vetoes to reject events with $\gamma$ and $\mu$ in the final state, effective particle identification systems to identify K$^+$ among other particles of the intense hadron beam and to distinguish $\pi^+$ from $\mu^+$ and $e^+$ in the final states.

At present, the construction of the experimental apparatus is very advanced (see Ref. [26]). A commissioning run has taken place in the second half of 2012, with a subset of the detector and of the data acquisition and read-out systems. Other technical runs are foreseen during 2013, while completing the installation of detectors, which will continue in 2013 and 2014. The first physics run is expected at the end of 2014 (after CERN machines shutdown).

It’s worth mentioning that at an early stage of the NA62 project, in 2007-08, physics data have been collected using an optimized kaon beam line and the former NA48/2 detector to measure the ratio $R_K = \Gamma(K^\pm \rightarrow e\nu\bar{e})/\Gamma(K^\pm \rightarrow \mu\nu\bar{\mu})$. The result $R_K = (2.488 \pm 0.010) \times 10^{-5}$ is the most precise measurement to date and agrees with the SM predictions [27].

3.3. KOTO experiment at J-PARC

The KOTO experiment [28, 29] at the new high-intensity proton accelerator facility J-PARC (Japan Proton Accelerator Research Complex) [30] aims to observe for the first time the rare decay $K_L \rightarrow \pi^0 \nu\bar{\nu}$. The challenge consists in being a so-called nothing in - nothing out experiment, with both initial state and final decay products completely neutral.

A decay volume for $K_L$ is surrounded by particle detectors. The signature of a $K_L \rightarrow \pi^0 \nu\bar{\nu}$ decay is that there are two photons from a $\pi^0$ decay and no other visible particles in the final state. An electromagnetic calorimeter is placed downstream of the decay volume to detect the two photons. All the $K_L$ decay modes except $K_L \rightarrow \pi^0 \nu\bar{\nu}$ and $K_L \rightarrow \gamma\gamma$ have at least two charged particles, or two or more extra photons in the final state. These decays can be rejected by detecting additional particles with the surrounding detectors. The $K_L \rightarrow \gamma\gamma$ decays can be rejected by requiring a finite transverse momentum for the two photon system. In the case of the $K_L \rightarrow \pi^0 \nu\bar{\nu}$ decay, the two photon system has a finite transverse momentum, because the undetected two neutrinos take some momentum away. The decay vertex is calculated and the $K_L \rightarrow \pi^0 \nu\bar{\nu}$ decay reconstructed from two photons in the calorimeter, with the assumption that the two photons come from a $\pi^0$ decay on the line of flight.

The new neutral beam-line has been built at the Hadron Experimental Hall of J-PARC. The CsI crystal calorimeter (made of the crystals used in the past by the KTeV experiment), the charged-particle veto counters (CV) in front of it, and the main-barrel photon veto counters (MB)
which surround the decay volume in the vacuum vessel, have been built, while the construction of the front-end photon counters is being completed.

After the major Earthquake in Japan on 11 March 2011 (no damages to beam line and KOTO detector) J-PARC started re-commissioning on December 2011 and resumed operations on January 2012. Commissioning and engineering runs have been scheduled during beam times at the beginning of 2013. In the first physics run, expected for May - June 2013, the KOTO experimental sensitivity will cross the Grossman-Nir bound allowing new physics search to start. After the long shutdown of the J-PARC accelerators from August 2013 to January 2014 - with an upgrade of the linac - the KOTO experiment is expected to start a series of long physics runs, reaching the final sensitivity at the level of the SM prediction of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$, therefore extensively exploring the parameter space of NP models.

### 3.4. ORKA experiment at FNAL
ORKA is a proposed experiment [31] aiming to detect about 1000 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ events and to measure the corresponding branching ratio with 5% precision using the Fermilab Main Injector high intensity proton source. The detector design is based on the former BNL E787/E949 experiments, which detected seven candidate events [23]. Two orders of magnitude improvement in sensitivity relative to the BNL experiments comes from enhancements to the beam line and the detector acceptance.

The ORKA collaboration has the support of the FNAL PAC and the FNAL directorate. The collaboration expects to begin construction of ORKA in 2014 and begin collecting $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decays in 2017.

### 3.5. TREK experiment at J-PARC
In the $K^+ \rightarrow \pi^0 \mu^+ \nu$ decay, the transverse muon polarization $p_t$ (the perpendicular component of the muon spin vector relative to the decay plane determined by the momentum vectors of the muon and the pion in the $K^+$ rest frame) is a T -odd quantity. The SM prediction of $p_t$ is almost vanishing ($\sim 10^{-7}$) and a non-zero value for such quantity at a level above $10^{-4}$ would be an indication of time-reversal violation in NP, still allowed by the current limit ($p_t < 5 \times 10^{-3}$) set by the KEK E246 experiment [32].

A successor to E246 is another new kaon experiment at J-PARC, E06 TREK [33], aiming at a $p_t$ sensitivity of $10^{-4}$. This experiment is conducted in conjunction with the E36 one aiming to measure the $R_K$ ratio with a relative uncertainty of $\sim 2.5 \times 10^{-3}$, which is about half of the current world record. The physics run of E36 is expected in 2014-15.

### 3.6. Other projects
Project X is a proposed proton accelerator complex at Fermilab that would provide particle beams to multiple experiments searching for rare and hard-to-detect phenomena [34]. Construction for Project X could begin in 2016 and be completed in 2021. The plans include ultra-rare $K^\pm$ and $K_L$ decay measurements, and neutral kaon interferometry and CPT tests at a very high level of sensitivity.

The KLOD R&D program for a $K_L$ measurement at Protvino should also be mentioned, as well as the OKA project, in the same laboratory, using a separated charged kaon beam.
4. Kaon experiments at $e^+e^-$ collider

4.1. Neutral kaon interferometry at $\phi$-factory

At a $\phi$-factory neutral kaons are produced in entangled pairs with the $\phi$-meson quantum numbers $J^{PC} = 1^{--}$:

$$|i\rangle = \frac{1}{\sqrt{2}} \{ |K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \}$$

$$= \frac{\mathcal{N}}{\sqrt{2}} \{ |K_S\rangle |K_L\rangle - |K_L\rangle |K_S\rangle \},$$

(4)

where $\mathcal{N} = \sqrt{(1 + |\epsilon_S|^2)(1 + |\epsilon_L|^2)}/(1 - \epsilon_S \epsilon_L) \simeq 1$ is a normalization factor.

The observable quantity is the double differential decay rate of the state $|i\rangle$ into decay products $f_1$ and $f_2$ at proper times $t_1$ and $t_2$, respectively. After integration on $(t_1 + t_2)$ at fixed time difference $\Delta t = t_1 - t_2$, the decay intensity can be written as follows [35]:

$$I(f_1, f_2; \Delta t \geq 0) = C_{12} |\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_S \Delta t} - 2|\eta_1||\eta_2| e^{-\frac{\Gamma_L + \Gamma_S}{2} \Delta t} \cos[\Delta m \Delta t + \phi_2 - \phi_1] \} .$$

(5)

This expression is valid for $\Delta t \geq 0$, while for $\Delta t < 0$ the substitutions $\Delta t \rightarrow -\Delta t$ and $1 \leftrightarrow 2$ have to be applied, and with $\Delta m = m_L - m_S$,

$$\eta_i \equiv |\eta_i|^2 e^{i\phi_i} = \frac{\langle f_1 |T| K_L \rangle}{\langle f_1 |T| K_S \rangle} ,$$

(6)

$$C_{12} = \frac{|\mathcal{N}|^2}{2(\Gamma_L + \Gamma_S)} \frac{\langle f_1 |T| K_S \rangle \langle f_2 |T| K_S \rangle}{\langle f_1 |T| K_L \rangle \langle f_2 |T| K_L \rangle} .$$

(7)

Due to the huge difference in the lifetimes of the physical states ($\tau_L \gg \tau_S$), for $t_1 \gg t_2, \tau_S$ (or $t_2 \gg t_1, \tau_S$) the decay intensity in eq.(5) behaves like the initial state were an incoherent mixture of states $|K_S\rangle |K_L\rangle$ and $|K_L\rangle |K_S\rangle$. Hence the detection of a kaon at large times $\Delta t$ $\Rightarrow$ $\Delta t$ and $1 \leftrightarrow 2$ have to be applied, and with $\Delta m = m_L - m_S$.

4.2. KLOE experiment at DAΦNE

DAΦNE, the Frascati $\phi$-factory is an $e^+e^-$ collider working at a center of mass energy of $\sqrt{s} \sim 1020$ MeV, corresponding to the peak of the $\phi$ resonance. The $\phi$-meson production cross section is $\sim 3 \mu b$, and its decay into $K^0\bar{K}^0$ has a branching fraction of 34%, yielding $\sim 10^6$ $K^0\bar{K}^0$ pairs per pb$^{-1}$ of integrated luminosity.

The KLOE detector is a 4$\pi$ detector setup, which is able to measure both charged and neutral particles. It consists of a large volume drift chamber [36], which provides excellent momentum and vertex reconstruction for charged particles, and a barrel shaped electromagnetic calorimeter with two end-caps [37], made from lead and scintillating fibers, which surrounds the drift chamber. The energy deposits of charged and neutral particles in the calorimeter are measured with very good time resolution, which allows for the identification of charged particles based on their time of flight. Drift chamber and calorimeter are enclosed in a superconducting solenoid, providing an axial 0.52 T magnetic field.

The KLOE experiment at DAΦNE completed its first data taking campaign in March 2006 with a total integrated luminosity of $\sim 2.5$ fb$^{-1}$, corresponding to a production of $\sim 7.5 \times 10^9$ $\phi$-mesons and $\sim 2.5 \times 10^9$ $K^0\bar{K}^0$ pairs.

The quantum interference between the two kaon decays in the CP violating channel $\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^-\pi^+\pi^-$ has been observed for the first time by KLOE [38]. This decay mode is
very rich in physics; in fact several kinds of possible decoherence and CPT violation mechanisms, which in some cases might be justified in a quantum gravity framework, could show up as a deviation from the quantum mechanical prediction in eq.(5), especially in the region at $\Delta t \approx 0$, where $I(\pi^+\pi^-, \pi^+\pi^-; \Delta t = 0) = 0$. No deviation from the expectations of quantum mechanics and CPT symmetry is observed, while the precision on the measurements of the corresponding parameters is very high, and - in some cases - reaches the interesting Planck scale region. These results, together to other CPT symmetry tests in $K_S \to \pi\nu\nu$ decays, have been thoroughly reviewed in Refs. [35, 39, 40]. In the following very recent KLOE results on CP and CPT tests are reported, while a detailed discussion is referred elsewhere [41, 42, 43].

4.3. Search for CP violation in $K_S$ decay

The decay $K_S \to 3\pi^0$ violates CP invariance. The parameter $\eta_{000}$, defined as the ratio of $K_S$ to $K_L$ decay amplitudes, can be written as:

$$\eta_{000} = \frac{\langle 3\pi^0 | T | K_S \rangle}{\langle 3\pi^0 | T | K_L \rangle} = \epsilon_S + \epsilon'_{000},$$

where $\epsilon'_{000}$ is due to a direct CP-violating term. Since we expect $\epsilon'_{000} \ll \epsilon_S$ (at lowest order in Chiral Perturbation Theory one has [44, 45]: $\epsilon'_{000} = -2\epsilon'$, with $\epsilon'$ the direct CP violation parameter in $\pi\pi$ decays), it follows that $\eta_{000} \approx \epsilon_S$, and therefore in the SM one has (assuming CPT invariance, i.e. $\epsilon_S = \epsilon$) $\text{BR}(K_S \to 3\pi^0) \sim 1.9 \times 10^{-9}$ to an accuracy of a few $\%$, making the direct observation of this decay quite a challenge.

The best upper limit on $\text{BR}(K_S \to 3\pi^0)$ comes from the analysis of 450 pb$^{-1}$ data collected by the KLOE experiment in years 2001-2002 [46]: $\text{BR}(K_S \to 3\pi^0) < 1.2 \times 10^{-7}$ at 90\% C.L. In a new improved analysis, using 1.7 fb$^{-1}$ of data collected in years 2004-2005, $n_s = 0$ candidate events for the signal are found, with $n_{bkg} = 0$ background events expected from Monte Carlo with an effective statistics of two times that of the data. Hence the final KLOE upper limit is [41, 42]:

$$\text{BR}(K_S \to 3\pi^0) < 2.6 \times 10^{-8} \quad \text{at 90\% C.L.},$$

which is almost five times lower than the latest published result [46]. This limit can be directly translated into a limit on $|\eta_{000}|$:

$$|\eta_{000}| = \sqrt{\frac{\tau_L \text{BR}(K_S \to 3\pi^0)}{\tau_S \text{BR}(K_L \to 3\pi^0)}} < 0.0088 \quad \text{at 90\% C.L.}$$

4.4. CPT and Lorentz symmetry test using neutral kaon interferometry

CPT invariance holds for any realistic Lorentz-invariant quantum field theory. However a very general theoretical possibility for CPT violation is based on spontaneous breaking of Lorentz symmetry, as developed by Kostelecký [47, 48, 49], which appears to be compatible with the basic tenets of quantum field theory and retains the property of gauge invariance and renormalizability (Standard Model Extensions - SME). In SME for neutral kaons, CPT violation manifests to lowest order only in the mixing parameter $\delta$, (e.g. vanish at first order in the decay amplitudes), and exhibits a dependence on the 4-momentum of the kaon:

$$\delta \approx i \sin \phi_{SW} e^{i\phi_{SW}} \gamma_K (\Delta a_0 - \vec{\beta}_K \cdot \Delta \vec{a}) / \Delta m$$

where $\gamma_K$ and $\vec{\beta}_K$ are the kaon boost factor and velocity in the observer frame, and $\Delta a_\mu$ are four CPT- and Lorentz-violating coefficients for the two valence quarks in the kaon.
Following Ref. [48], the time dependence arising from the rotation of the Earth can be explicitly displayed in eq.(11) by choosing a three-dimensional basis $(\hat{X}, \hat{Y}, \hat{Z})$ in a non-rotating frame, with the $\hat{Z}$ axis along the Earth’s rotation axis, and a basis $(\hat{x}, \hat{y}, \hat{z})$ for the rotating (laboratory) frame (see Fig.2). The CPT violating parameter $\delta$ may then be expressed as:

\[
\delta(\vec{p}, t_{\text{sid}}) = i \sin \phi_{SW} e^{i \phi_{SW}} \frac{\Delta m}{\gamma_K} \{ \Delta a_0 + \beta_K \Delta a_Z \cos \theta \sin \chi \\
- \beta_K \Delta a_Z \sin \theta \cos \phi \sin \chi \\
- \beta_K \Delta a_X \sin \theta \sin \phi \sin \Omega_{t_{\text{sid}}} \\
+ \beta_K \Delta a_X \cos \theta \sin \chi \cos \Omega_{t_{\text{sid}}} \\
+ \beta_K \Delta a_Y \cos \theta \cos \phi \cos \chi \cos \Omega_{t_{\text{sid}}} \\
+ \beta_K \Delta a_Y \sin \theta \cos \phi \cos \chi \sin \Omega_{t_{\text{sid}}} \\
+ \beta_K \Delta a_Y \sin \theta \sin \phi \cos \Omega_{t_{\text{sid}}}
\}
\] (12)

where $\vec{p}$ is the kaon momentum, $t_{\text{sid}}$ is the sidereal time, $\Omega$ is the Earth’s sidereal frequency, $\cos \chi = \hat{z} \cdot \hat{Z}$; $\theta$ and $\phi$ are the conventional polar and azimuthal angles of the kaon momentum defined in the laboratory frame about the $\hat{z}$ axis. The sensitivity to the four $\Delta a_\mu$ parameters can be very different for fixed target and collider experiments, showing complementary features [48]. At a fixed target experiment usually the kaon momentum direction is fixed, while $|\vec{p}|$ might vary within a certain interval. On the contrary, at a $\phi$-factory kaons are emitted in all directions with the characteristic $p$-wave angular distribution $dN/d\Omega \propto \sin^2 \theta$, while $|\vec{p}|$ is almost fixed\(^2\).

At KLOE the analysis strategy to measure the four $\Delta a_\mu$ parameters is based on exploiting neutral kaon interferometry. In particular when $f_1 = f_2 = \pi^+ \pi^-$ the corresponding $\eta_i$ parameters can be slightly different for the two kaons due to the momentum dependence of the CPT violation effects as come from eq.(12):

\[
\eta_1 = \epsilon_L + \epsilon' = \epsilon - \delta(\vec{p}_1, t_{\text{sid}}) + \epsilon' \\
\eta_2 = \epsilon_L + \epsilon' = \epsilon - \delta(\vec{p}_2, t_{\text{sid}}) + \epsilon',
\] (13)

\(^2\) At DAΦNE $|\vec{p}|$ is not fixed because of a small $\phi$ meson momentum $\vec{p}_\phi$ in the laboratory frame ($|\vec{p}_\phi| \simeq 13$ MeV/c).
with $\vec{p}_2 = \vec{p}_\phi - \vec{p}_1$. The distribution $I(f_1, f_2; \Delta t)$ is sensitive to any deviation from unity of the ratio $\eta_1/\eta_2$ in the interference region (i.e. $\Delta t \approx 0$). Therefore a suitable analysis of the decays $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^-, \pi^+ \pi^-$ as a function of sidereal time and kaon momenta can provide a measurement of the four parameters $\Delta a_\mu$.

The two kaons are distinguished by their emission in the forward ($\cos \theta > 0$) or backward ($\cos \theta < 0$) hemispheres, as sketched in Fig.3. The data sample is divided in two subsets in which the kaons going in the forward direction ($\cos \theta > 0$) are emitted in a quadrant along ($\cos \phi > 0$) or opposite ($\cos \phi < 0$) to the $\phi$ momentum $\vec{p}_\phi$, thus having a higher (or lower) value of $\gamma_K$ than the companion kaons emitted in the backward direction (see Fig.3). Moreover the data are divided into four bins of sidereal time. In this way fitting simultaneously the corresponding eight $I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t)$ distributions one is able to observe possible modulation effects induced by the CPT-violating parameter $\delta$ in eq.(12) as a function of sidereal time and kaon momentum.

![Figure 3. Sketch of the quadrant subdivisions as seen in a top view of the KLOE detector; $p_\phi$ is directed along the $\hat{x}$ axis.](image-url)

It is worth noting that the presence of the small momentum $p_\phi$ makes $\gamma_{K,1} \neq \gamma_{K,2}$ on an event-by-event basis, which is a necessary condition in order to have the $I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t)$ distribution sensitive to the CPT violation effects induced by the $\Delta a_0$ parameter.

Using 1.7 fb$^{-1}$ of data collected in years 2004-2005, the preliminary results are [43]:

\[
\begin{align*}
\Delta a_0 &= (-6.2 \pm 8.2_{\text{stat}} \pm 3.3_{\text{syst}}) \times 10^{-18} \text{ GeV} \\
\Delta a_X &= (+3.3 \pm 1.6_{\text{stat}} \pm 1.5_{\text{syst}}) \times 10^{-18} \text{ GeV} \\
\Delta a_Y &= (-0.7 \pm 1.3_{\text{stat}} \pm 1.5_{\text{syst}}) \times 10^{-18} \text{ GeV} \\
\Delta a_Z &= (-0.7 \pm 1.0_{\text{stat}} \pm 0.3_{\text{syst}}) \times 10^{-18} \text{ GeV}
\end{align*}
\]

They can be compared to similar results obtained in the B meson system [50], where an accuracy on the $\Delta a_\mu^B$ parameters of $\mathcal{O}(10^{-13}\text{GeV})$ has been reached.

5. Future kaon experiments at $e^+e^-$ collider

5.1. The KLOE-2 experiment at DA$\Phi$NE improved in luminosity

After a successful experimental test [51], DA$\Phi$NE has been upgraded implementing an innovative collision scheme based on a crab-waist configuration, providing an improvement in the peak luminosity of a factor $\sim 3$.

The KLOE-2 experiment aims to continue and extend the physics program of its predecessor by collecting $\mathcal{O}(10 \text{ fb}^{-1})$ of data at the upgraded DA$\Phi$NE with an improved KLOE detector. The KLOE-2 physics program has been described in detail in Ref. [52] and among the main issues includes neutral kaon interferometry and tests of discrete symmetries and quantum mechanics. Improvements of about one order of magnitude in almost all present limits on CPT violation and decoherence parameters are expected [35, 39, 40, 52].
The upgrade of the KLOE detector would consist of the addition of (i) an inner tracker based on cylindrical GEM technology for the improvement of tracking and decay vertex resolution close to the interaction point (IP), (ii) a \( e^\pm \) tagging system for the \( \gamma \gamma \) physics, and (iii) two calorimeters in the final focusing region to improve acceptance and efficiency for photons coming from the IP and neutral kaon decays inside the detector volume.

The KLOE solenoid constitutes a strong perturbation on the machine optics, modifying the DA\( \Phi \)NE working conditions with respect to the test experiment described in Ref. [51]. The commissioning phase of the upgraded machine is in progress and is expected to be concluded by the year 2013. In the meanwhile the construction of the KLOE detector upgrades have been completed, their installation started, and is foreseen to be completed for mid 2013.

5.2. Direct test of Time reversal symmetry at a \( \phi \)-factory

In the context of a local quantum field theory with Lorentz invariance and Hermiticity, as in the Standard Model, the validity of the CPT theorem ensures an automatic theoretical connection between any source of CP violation and a corresponding T (time reversal) violation. Even though CPT invariance has been confirmed by all present experimental tests, particularly in the neutral kaon system with stringent limits to possible CPT violation effects [8, 38, 39, 40], the theoretical connection between CP and T symmetries does not imply an experimental identity between them. In fact a direct evidence of T violation should result from an experiment clearly showing the violation, independently from the results of CP violation and without any connection with them. For unstable systems, the associated irreversibility looks like it prevents a true test of T symmetry [53].

In case of a transition process, due to the antiunitarity of the operator implementing the symmetry transformation, T invariance requires that the rate for the reaction \( i \rightarrow f \) equals that of the reaction \( f_T \rightarrow i_T \), with \( i \) and \( f \) states exchanged and T \( \text{inverted} \) (for spinless particles this corresponds to the reaction \( f \rightarrow i \)).

In the past the measurement of a non-zero value of the Kabir asymmetry [54], comparing the rates of the process \( K^0 \rightarrow K^0 \) and its T conjugated one \( \bar{K}^0 \rightarrow K^0 \), has been presented as a proof for T violation [55, 56]. However, this process has the feature that \( K^0 \rightarrow K^0 \) is a CPT even transition, so that T and CP transformations are identical in this case, and the corresponding observables are not independent. Therefore it is impossible to separate T violation from CP violation in the Kabir asymmetry.

In order to overcome this difficulty it has been suggested to exploit the Einstein-Podolsky-Rosen (EPR) [57] entanglement of neutral mesons produced at a \( \phi \)-factory (or B-factory) [58, 59]. In fact in this case the neutral kaon pair can be rewritten in terms of any pair of orthogonal states \( K_+ \) and \( K_- \), e.g. the CP eigenstates:

\[
|i\rangle = \frac{1}{\sqrt{2}}\{(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle)\} = \frac{1}{\sqrt{2}}\{(|K_+\rangle|K_-\rangle - |K_-\rangle|K_+\rangle)\} . \tag{15}
\]

Thus, exploiting the perfect anticorrelation of the states implied by Eq. (15), it is possible to have a “flavor-tag” or a “CP-tag”, i.e. to infer the flavor (\( K^0 \) or \( \bar{K}^0 \)) or the CP (\( K_+ \) or \( K_- \)) state of the still alive kaon by observing a specific flavor decay (\( \pi^+ \ell^- \nu \) or \( \pi^- \ell^+ \bar{\nu} \)) or CP decay (\( \pi^+ \pi^- \) or \( \pi^0 \pi^0 \)) of the other (and first decaying) kaon in the pair, as explained in detail in Ref. [60]. In this way one can experimentally access other transitions than \( K^0 \rightarrow K^0 \) or \( \bar{K}^0 \rightarrow K^0 \). These new accessible processes can be divided into four categories of events, as summarized in Table 1.

For instance, one can consider \( K^0 \rightarrow K_+ \) as the reference process, where the initial state \( K^0 \) is identified at time \( t_1 \) with the flavor tag, and the final state \( K_+ \) is identified at a subsequent time \( t_2 \geq t_1 \) with a CP decay.
Table 1. Scheme of possible reference transitions and their associated T, CP or CPT conjugated processes accessible at a \( \phi \)-factory.

| Reference | T-conjug. | CP-conjug. | CPT-conjug. |
|-----------|-----------|------------|-------------|
| \( K^0 \to K_+ \) | \( K_+ \to K^0 \) | \( K^0 \to K_+ \) | \( K_+ \to K^0 \) |
| \( K^0 \to K_- \) | \( K_- \to K^0 \) | \( K^0 \to K_- \) | \( K_- \to K^0 \) |
| \( \bar{K}^0 \to K_+ \) | \( K_+ \to \bar{K}^0 \) | \( K^0 \to K_+ \) | \( K_+ \to K^0 \) |
| \( \bar{K}^0 \to K_- \) | \( K_- \to \bar{K}^0 \) | \( K^0 \to K_- \) | \( K_- \to K^0 \) |

I) The T transformed process is \( K_+ \to K^0 \); any asymmetry in the rate between \( K^0 \to K_+ \) and \( K_+ \to K^0 \) would be a genuine T violating effect.

II) The CP transformed process is \( \bar{K}^0 \to K_+ \); any asymmetry in the rate between \( K^0 \to K_+ \) and \( \bar{K}^0 \to K_+ \) would be a genuine CP violating effect.

III) The CPT transformed process is \( K_+ \to \bar{K}^0 \); any asymmetry in the rate between \( K^0 \to K_+ \) and \( K_+ \to \bar{K}^0 \) would be a genuine CPT violating effect.

One may check that the events used for the asymmetries in I), II), and III) are completely independent from each other.

Based on a similar strategy, the first direct observation of T violation, in the sense discussed above, has been accomplished in the neutral B meson system by the Babar collaboration [61, 62]. The same methodology, with some differences, can be implemented at a \( \phi \)-factory, and the KLOE-2 experiment at DAΦNE could make in the next future the first significant T symmetry test in the neutral K system, as discussed in detail in Ref. [60].

5.3. Bells inequality and discrete symmetries

Entanglement and its consequences - in particular the violation of Bell inequalities, which defies our concepts of realism and locality [63] - have been proven to play key roles in Nature by many experiments for various quantum systems.

A recent formulation of the Bell’s inequality for entangled neutral kaons produced at a \( \phi \)-factory [64] overcomes drawbacks in the past formulations, making the inequality experimentally testable also for the \( K^0 - \bar{K}^0 \) system. It’s worth mentioning that - surprisingly - CP violation in this case turns out to be a key ingredient to make the inequality violated by Quantum Mechanics. Feasibility studies to evaluate the experimental sensitivity of the test at KLOE-2 are in progress.

6. Conclusions

After direct CP violation has been firmly established in neutral kaon decays, in a couple of years, the next generation of kaon experiments at hadron machines (NA62 and KOTO) will be operational, definitely entering a new precision era in probing the SM, and in searching for New Physics in a way complementary to the LHC experimentation.

Other kaon experiments planned in the future will continue this program or address other specific issues, e.g. the transverse muon polarization in \( K^+ \) decays.

The neutral kaon system still constitutes an excellent laboratory for the study of discrete symmetries, and a \( \phi \)-factory represents a unique opportunity to push forward these studies. CPT symmetry and quantum mechanics tests will be one of the key issues at KLOE-2, and their precision will be further improved, while new ideas - e.g. the direct T-symmetry test - might also be implemented.
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