Current Flow in a Cylindrical Nanopore with an Object–Implications for Virus Sensing

Mohammad Tajparast1 · Mladen Glavinovic2

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Abstract
Interest is growing in nanopores as real-time, low-cost, label-free virus size sensors. To optimize their performance, we evaluate how external electric field and ion concentrations and pore wall charges influence currents and object (disk) radius-current relationship using simulations. The physics was described using the Poisson-Nernst-Planck and Navier–Stokes equations. In a charged cylindrical nanopore with a charged disk, elevated external electric field produces higher (and polarity independent) ion concentrations and greater ion current (largely migratory). Elevated external ion concentrations also lead to higher concentrations (mainly away from the pore wall), greater axial electric field especially in the disk-pore wall space, and finally larger current. At low concentrations, current is disk radius independent. The current rises as concentrations increase. Interestingly, the rise is greater for larger disks (except when the pore is blocked mechanically). Smaller cross-sectional area for current flow or volume exclusion of electrolyte by object thus cannot be universally accepted as explanations of current blockage. Ion current rises when pore wall charge density increases, but its direction is independent of charge sign. Current-disk radius relationship is also independent of pore wall charge sign. If the pore wall and disk charges have the same sign, larger current with bigger disk is due to higher counter-ion accumulation in the object-pore wall space. However, if their signs are opposite, it is largely due to elevated axial electric field in the object-pore wall space. Finally in uncharged nanopores, current diminishes when disk radius increases making them better sensors of virus size.

Keywords Virus detection · Ion current · Pathogens · Cylindrical nanopore · Poisson-Nernst-Planck · Navier–Stokes

1 Introduction
Micropores have been used for a long time for rapid estimation of cell numbers, size, and “dwell time” (Coulter counter; [1, 2]). Interest in using pores as low-cost, rapid, and label-free estimators of properties of pathogens (such as viruses) is large and growing. However, viruses are smaller than cells and vary in size greatly (from several to several hundred nanometers [3]). Estimating their size—an important descriptor in their classification [4]—thus often requires nanopores. We classify the pore size on a logarithmic scale (with base 10; [5]). If the pore size ranges from 0.1 to 100 nm, it is considered a nanopore, and if it ranges from 0.1 to 100 µm, it is considered a micropore [5]. Nanopores are already used as sensors of single molecules—DNA [6–9], proteins [10–12], peptides [13, 14], and amino acids [15, 16], and solid-state nanopores as diagnostic tools for infectious diseases [17].

In all cases, the “object” size is estimated from a transient and discrete ionic current blockage during its translocation through the pore—the larger the object (and the smaller the cross-sectional area for current flow) the larger the current blockage. This “resistive-pulse method” [18] based on the size-exclusion principle [2] can also yield the shape of objects if pores with irregularities (a pore is considered irregular if its radius changes axially) are used [19, 20]. When an object passes through a pore, it often rotates. If the object is non-spherical, the cross-sectional area for the current flow changes and this leads to current fluctuations that contain information about how non-spherical the object is [21]. When a spherical object rotates, the cross-sectional area for the current flow does not change. Although this method is...
now used for size and shape characterization of larger viruses [4, 20, 22, 23], it is unclear whether it would be suitable for characterization of smaller viruses using nanopores. However, shape is a powerful descriptor of viruses that vary in their shape greatly and can be globular (coronaviruses), oval shaped (poxviruses), rod-like (as many plant viruses), or head and tail like (bacteriophages) [24].

What is critically important is to understand the relationship between the cross-sectional area for current flow and current. Presently, the “size-exclusion principle” is the most widely held view. Expanding on this notion, it has been argued that the volume exclusion of electrolyte solution by the object also contributes to the current decrease during its translocation [25, 26]. In this study, we challenge these notions as a general explanation of current blockage because ions are as a rule not uniformly distributed within charged pores. Screening charges formed to counterbalance surface charges are concentrated near the pore wall [27–29]. Given such charges’ distribution, the current-object size dependence cannot be one of progressive linear decrease, but it is unclear what the relationship is, and what influences it.

The object if charged also induces screening charges within nanopore. It has been suggested that this leads to larger current, partially counterbalancing the current decrease caused by smaller cross-sectional area for current flow and volume exclusion of electrolyte solution by the object, finally producing multi-level current changes [26]. This is an interesting, but a simplistic proposition because charged disk surfaces that may induce such currents will also largely block them mechanically. However, a charged object will affect the electric field (both its radial and axial components) within the nanopore (especially if the disk and the pore wall are charged differently). The radial electric field controls the radial migratory current around the disk and may change the overall axial migratory current in the disk-pore wall space, but it is unclear how and how much. Finally, even an uncharged object changes the electrostatics (and thus electro-kinetics) within the nanopore because it is a low permittivity material ($\varepsilon_{r,d} = 2$; [30]) within a highly polarizable core of the nanopore ($\varepsilon_{r,w} = 80$; [30]).

The presence of two charged surfaces in close proximity can change ion currents and conductivity greatly. Even a “simple” charge non-uniformity at the pore walls can lead to pronounced changes of ion fluxes [31], pore conductance [32], and current rectification [27, 32–35]. In engineering, such nanopores are used as ionic diodes and transistors [36–39] or serve as ion filters [29]. A systematic evaluation of how external factors and pore parameters influence the current in a charged cylindrical nanopore with a charged disk is clearly needed.

We evaluate how external electric field and ion concentrations and pore wall charges influence ion currents and disk radius-current relationship. The transport of $K^+$ and $Cl^-$ was simulated using the Poisson–Nernst–Planck (PNP) equations [40] and was coupled to the transport of water using the Navier–Stokes (NS) equation [41, 42]. The electrostatic-electro-kinetic-hydrodynamic variables (potential, mobile charge density and ion currents) were subsequently evaluated.

## 2 Methods

### 2.1 Geometry of Simulation Domain, Parameters, Constants, and Boundary Conditions

Figure 1 gives the simulation space which consists of a cylindrical nanopore, two compartments flanking the nanopore, and a piece of membrane separating the compartments. All subdomains and boundaries are enumerated, and their dimensions are given (Fig. 1A). Figure 1B depicts meshing of the simulation space and Fig. 1C shows its 3D representation. Table 1 lists the electrostatics, electro-kinetics, and fluidics simulation space and Fig. 1C shows its 3D representation. Table 1 lists the electrostatics, electro-kinetics, and fluidics constants (but also the dielectric constant of water and its viscosity) are homogeneous, isotropic, and the same as in the aqueous solution.

### 2.2 General Formulation of Mathematical Simulations

We used the Nernst-Planck equation to calculate ionic fluxes within a nanopore for both $K^+$ and $Cl^-$ ions. The Poisson equation was used to compute the electric potential ($\psi$) due to mobile charges (inside the nanopore and in two compartments flanking it) and fixed charges (on the pore and disk walls; Eq. (1)), while the Laplace’s equation (Eq. (2)) calculates the electric potential ($\phi$) due to applied external electric field (i.e., the potential difference between the upper and lower controlling edges) as follows:

\[-\nabla \cdot \varepsilon_0 \varepsilon_r \nabla \psi = \rho_e\]  

(1)
\[ \nabla^2 \phi = 0 \]  \hfill (2)

where \( \varepsilon_0 \) is the permittivity of vacuum, \( \varepsilon_r \) is the relative dielectric constant of a chosen subdomain (as shown in Fig. 1) there are three subdomains in the model; their \( \varepsilon_r \)'s are given in Table 2, and \( \nabla \) is the gradient operator. The charge density \( \rho_e \) (mobile) was calculated as follows:
Table 2 Model parameters and physical constants

| Params | Values | Unit | Description | Refs |
|--------|--------|------|-------------|------|
| T      | 300.0  | K    | Temperature | [30] |
| R      | 8.314  | J/(mol·K) | Universal gas constant | [30] |
| e      | 1.602 × 10⁻¹⁹ | C | Elementary charge | [30] |
| D_K    | 1.960 × 10⁻⁹ | m²/s | Diffusion coefficient of K⁺ ions | [56] |
| D_Cl   | 2.030 × 10⁻⁹ | m²/s | Diffusion coefficient of Cl⁻ ions | [56] |
| ρ      | 1.0 × 10³ | kg/m³ | Fluid density | [30] |
| μ      | 1.0 × 10⁻¹³ | Pa s | Fluid viscosity | [30] |
| ε₀     | 8.854 × 10⁻¹² | F/m | Permittivity of vacuum | [30] |
| ε_r,m  | 80.0 | Dimensionless | Relative permittivity of membrane | [30] |
| ε_r,d  | 2.0 | Dimensionless | Relative permittivity of object (disk) | [30] |

\[
\rho_e = F \sum z_a c_a = \epsilon \sum z_a n_a
\]

(3)

Note that \(c_a\) is the molal concentration of each ion in [mol/m³], \(z_a\) is the valence of ion \(a\), \(n_a\) is the number density of ion \(a\), and \(F\) is the Faraday constant (9.648 × 10⁴ C/mol). The contribution of fixed charges (\(σ_w\)-pore wall charge density and \(σ_o\)-disk charge density) is included through electrostatic boundary conditions (Table 1). The International System of Units (SI) is used in all simulations, and mol/m³ thus equals to mmol/liter (or simply mM).

The movement (convection–diffusion-migration) of ions in the electrolytic fluid was defined by the Nernst-Planck equation as follows:

\[
J_a = u_a - D_a \nabla c_a - m_a z_a F e \nabla \Phi
\]

(4)

where \(J_a\) is the molar flux in mol/(m²·s), whereas \(u\) denotes the fluid velocity calculated by the NS equation, \(D_a\) and \(m_a\) represent diffusivity and mobility of ion \(a\), which are related by \(m_a = D_a/(RT)\). \(R\) and \(T\) account for the universal gas constant \((R = 8.314 \text{ J/(mol·K)})\) and absolute temperature in Kelvin, respectively. Finally, \(Φ\) denotes the total electric potential, which is the sum of \(ϕ\) and \(ψ\).

Equation (5) gives the conservation of ionic mass in a steady-state situation.

\[
\nabla \cdot J_a = 0
\]

(5)

The Navier–Stokes equation (at steady-state condition), in the presence of external forces, was applied to model fluid velocity as follows:

\[
\rho (u \cdot \nabla u) = -\nabla p + \nabla \cdot [\mu (\nabla u + (\nabla u)ᵀ)] + F_e
\]

(6)

\[
\nabla \cdot u = 0
\]

(7)

Equation (6) describes the conservation of momentum, whereas Eq. (7) accounts for the conservation of mass. The \(ρ\), \(μ\), and \(p\) stand for fluid density, viscosity, and pressure (defined as the isotropic part of the fluidic stress tensor), respectively. The vector \(u\) denotes the fluid velocity. Finally, \(F_e\) accounts for the electric force per unit volume, calculated as \(F_e = -ρ_e \nabla \Phi\).

At nanoscales, the Reynolds number is expected to be very low and fluid flow laminar with negligible inertial effects [43]. Indeed, we find that the Reynolds number is < 0.01, even for the highest water velocity within the nanopore (i.e., very low ensuring that the flow is laminar). Moreover, the nanopore size and the Debye length [44] are comparable. We thus used no-slip condition in all simulations, but a slip condition may be a better choice when the surfaces are strongly hydrophobic. Finally, our choice of a no-slip boundary condition enables us to resolve the flow inside the electric double layer.

2.3 Mathematical Model in the Cylindrical Coordinate System

This study uses the cylindrical coordinate system (the 2D \(r-z\) plane). The simplified Navier–Stokes equation (note that the velocity and its gradient in the \(θ\)-direction are ignored, and there is an axial symmetry in the \(z\)-direction) in terms of components of the stress tensor \(τ\) are thus as follows:

\[
\tau_r \text{- component : } ρ \left( \mu \frac{∂u}{∂r} + v \frac{∂v}{∂z} \right) = -\frac{∂p}{∂r} - \left[ \frac{1}{r} \frac{∂}{∂r} \left( r τ_r \right) + \frac{∂τ_z}{∂z} \right] + F_{e,r}
\]

(8)

\[
\tau_z \text{- component : } ρ \left( \mu \frac{∂v}{∂r} + v \frac{∂v}{∂z} \right) = -\frac{∂p}{∂z} - \left[ \frac{1}{r} \frac{∂}{∂r} \left( r τ_r \right) + \frac{∂τ_z}{∂z} \right] + F_{e,z}
\]

(9)

The \(u\) and \(v\) are the \(r\)- and \(z\)-components of the fluid velocity, respectively. The \(τ_i\) are the \(ij\)-th components of the viscous stress tensor \(τ\), which for the Newtonian fluids in 2D \(r-z\) plane (in the cylindrical coordinate system) are defined as:

\[
τ_{rr} = -\mu \left[ 2 \frac{∂u}{∂r} - \frac{2}{3} (\nabla \cdot u) \right]
\]

(10)
\[ \tau_{zz} = -\mu \left[ 2 \frac{\partial v}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{u}) \right] \]  
\[ \tau_{rr} = -\mu \left[ \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z} \right] \]  
\[ \nabla \cdot \mathbf{u} = 1 \frac{\partial}{r \partial r} (ru) + \frac{\partial v}{\partial z} \]  

where \( \nabla \cdot \mathbf{u} \) is defined as: \( \tau_{zz} \) is given by:

The electrostatic force is defined as \( \mathbf{F}_e = -\rho_e \nabla \Phi \), whereas the gradient operator in the cylindrical coordinate system is given by:

\[ [\nabla \Phi]_r = \frac{\partial \Phi}{\partial r}, \quad [\nabla \Phi]_z = \frac{\partial \Phi}{\partial z} \]

When the components of the stress tensor (given in Eqs. (10)–(14)) are substituted into Eqs. (8) and (9) with constants \( \rho \) and \( \mu \), we obtain Eqs. (15) and (16) describing the Navier–Stokes equation in terms of velocity gradients.

\[ r \text{- component: } \rho \left( u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left( \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial^2 u}{\partial z^2} \right) - \rho_c \frac{\partial \Phi}{\partial r} \]  
\[ z \text{- component: } \rho \left( u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left( \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial^2 v}{\partial z^2} \right) - \rho_c \frac{\partial \Phi}{\partial z} \]  

Finally, note that the divergence operator in Eqs. (1) and (5) is defined as:

\[ \nabla \cdot \mathbf{J} = \frac{1}{r} \frac{\partial}{\partial r} (rJ_r) + \frac{\partial J_z}{\partial z} \]  

The \( J_r \) and \( J_z \) are the \( r \)- and \( z \)- components of vector \( \mathbf{J} \).

The system of coupled PNP and NS equations was solved using a finite element method based on a commercial software package COMSOL Multiphysics 4.3 (COMSOL, Burlington, MA, USA), whereas the postprocessing was performed using a MATLAB software package for scientific and engineering computation (MathWorks, Natick, MA, USA).

## 3 Results

### 3.1 Effect of External Electric Field on Ion Currents in a Charged Nanopore with a Charged Disk

The color-coded 2D distributions shown on top depict the effect of external axial electric field (\( E_{ex} \)) on the axial electric field (\( E_{ax} \)), \( K^+ \) and \( Cl^- \) concentrations (\( coK \) and \( coCl \)) within the nanopore (Fig. 2). To test whether the mesh sizes are adequately small (note that the mesh size differs depending on the location and is smaller near the disk and close to the edges; Fig. 1), we made some simulations where size was 100% bigger. In each set of 2D distributions, the results with double size mesh are shown as third panels (their \( V_u \) was +1 V). This change did not affect the 2D distributions of the \( E_{ax}, coK, \) and \( coCl \) (Fig. 2). Mesh size is thus adequately fine.

Radial profiles of the \( E_{ax}, coK, \) and \( coCl \) above the object-disk (at \( z = 4 \) nm which is indicated by the upper horizontal line; \( E_{ax,u}, coK,u, \) and \( coCl,u \)) and in the disk-pore wall space (at the disk’s axial mid-point; i.e., at \( z = 1.5 \) nm; \( E_{ax,o}, coK,o, \) and \( coCl,o \)), which are critical for understanding how current is generated and controlled, are also shown. Lower horizontal line depicts the axial midpoint (i.e., \( z = 0 \) nm). The \( coK,u \) is high in a very narrow space near the (negatively) charged pore wall. In the pore center it is low, but significantly above the value at the controlling edges (upper or lower). The \( coCl,o \) is similar in value near the pore wall, but is near zero close to the (positively charged) disk. Given that in between the pore and disk walls it differs significantly from the \( coK,u \), there is a significant \( coK \) axial gradient in small space near the disk tip. The \( coCl,o \) is low near the pore wall and rises towards its center, whereas the \( coCl,o \) (also low near the wall) rises to high values near the (positively) charged disk.

Neither the \( coK,u \) (and \( coK,o \)) nor \( coCl,u \) (and \( coCl,o \)) radial profiles change significantly, if the \( E_{ex} \) is reversed (Fig. 2A, B). However, as can be seen from 2D distributions the panels of \( coK \) (and \( coCl \)) are not identical if \( V_u \) is reversed. The \( coK,u \) (and \( coK,o \)) and \( coCl,u \) (and \( coCl,o \)) radial profiles should reveal some differences if different axial positions are chosen. The \( E_{ax,u} \) and \( E_{ax,o} \) radial profiles (which are quite uniform) change with the \( E_{ex} \) reversal. The \( E_{ax,o} \) profiles change too but more so (Fig. 2C). Complete \( V_u \) dependence reveals that the \( coK \) and \( coCl \) (near the pore wall and in the pore center) are both qualitatively similar–high at very positive and negative \( V_u \), and low at zero \( V_u \) (Fig. 2D, E). The \( E_{ax,u} \) however depend on the \( V_u \) linearly, but the \( E_{ax,o} \) are higher and more \( V_u \)-dependent than the \( E_{ax,u} \) (Fig. 2F).

All axial current densities–migratory (\( \sigma_{mig} \)), diffusive (\( \sigma_{diff} \)) and convective (\( \sigma_{conv} \))–are affected by the \( E_{ex} \) (Fig. 3). Figure 3A-D show their radial profiles and those of the total current density (\( \sigma_{tot} \)) for two extreme \( V_u \). The \( \sigma_{conv,u} \) is confined largely to the spaces near the pore wall (being zero near pore center)–a consequence of the presence of \( E_{ax} \) and elevated ion (counter-ion) concentration there that drive the water flow. When water flow changes direction with the \( E_{ax} \) reversal the \( \sigma_{conv,v} \) changes direction too. The \( \sigma_{diff,u} \) is also prominent near the pore wall and this is due to large \( coK \) and \( coCl \) axial gradients there (see the difference of \( coK,u \) and \( coK,o \) (and \( coCl,u \) and \( coCl,o \)) radial profiles (Fig. 2A, B)). The \( \sigma_{diff,u} \) profiles do not change significantly if the \( E_{ex} \) is reversed, and this is not surprising because the \( coK \) and \( coCl \) radial profiles do not change either. The \( \sigma_{mig,u} \) (the largest of all) is however non-zero at any radial distance, and its radial profiles are reversed with \( E_{ax} \) reversal. Two factors explain \( \sigma_{mig,u} \) profiles–non-zero and constant \( E_{ax} \) at any
radial distance, whose values reverse with $E_{ax}$ reversal, and $c_{0K}$ and $c_{0Cl}$ radial profiles that are essentially insensitive to the $E_{ex}$ reversal (Fig. 2A, B).

The $V_u$-dependence plots of the $I$s summarize these findings. The migratory currents ($I_{mig,u}$ and $I_{mig,o}$), which are the greatest at all $V_u$s, depend linearly on the $V_u$ (Fig. 3E-H). Given that neither the $c_{0K}$s nor $c_{0Cl}$s change greatly with the $E_{ex}$, linear dependence of $I_{mig}$s is as expected. The $K^+$ contributes the most to the $I_{mig}$ and this is also as expected--$K^+$ is a counter-ion to the negatively charged pore wall. Note that the $I_{conv,u}$ and $I_{conv,o}$ also depend linearly on $V_u$, unlike the $I_{diff,u}$ and $I_{diff,o}$ that do not. Finally, note that if there is no disk the $I_{conv,u}$ becomes more $V_u$-dependent whereas the $I_{mig,u}$ and $I_{tot,u}$ become less so. Small and largely $V_u$-independent $I_{diff,u}$ becomes even smaller and less $V_u$-dependent.

Fig. 2 Effect of external electric field on electric field and ion concentrations within a nanopore. External axial electric field ($E_{ax}$) alters axial electric field ($E_{ax}$) within a cylindrical nanopore with charged walls and an object (a disk), but also $K^+$ and $Cl^-$ concentrations ($c_{0K}$ and $c_{0Cl}$). Disk radius ($r_o$) was 3 nm, charge density at the pore wall ($\sigma_w$) was $-160 \text{ mC/m}^2$ (this is equivalent to $1 \text{ e/nm}^2$), and at the disk ($\sigma_o$), it was $+160 \text{ mC/m}^2$. Ion concentrations ($K^+$ and $Cl^-$) at the controlling edges of the upper ($c_{0K,u}$ and $c_{0Cl,u}$) and lower ($c_{0K,o}$ and $c_{0Cl,o}$) compartment were 400 mM. The color coded 2D distributions of $E_{ax}$, $c_{0K}$ and $c_{0Cl}$ are given on top for the voltage at upper controlling edges ($V_u$) with $V_u = -1 \text{ V}$ (left panels), $+1 \text{ V}$ (middle panels) and $+1 \text{ V}$ but with mesh doubled in size (right panels). In this case and throughout the text the potential ($V_d$) at the lower controlling edges was $0 \text{ V}$, whereas the pressure at both upper and lower controlling edges was $0 \text{ Pa}$ (see Table 1). The calibration bars are as indicated. A, B Radial profiles of $K^+$ concentrations at $z = 4 \text{ nm}$ (i.e., $2.25 \text{ nm}$ above the disk upper surface and from pore center to the wall; $c_{0K,u}$), or at $z = 1.5 \text{ nm}$ (i.e., from the axial center of the disk to the pore wall; $c_{0K,o}$) and corresponding $Cl^-$ concentrations ($c_{0Cl,u}$ and $c_{0Cl,o}$) near the pore wall and in the pore center. C Radial profiles of $E_{ax,u}$ and $E_{ax,o}$. D, E $V_u$ dependence of $c_{0K,u}$ (and $c_{0K,o}$) and $c_{0Cl,u}$ (and $c_{0Cl,o}$) near the pore wall and in the pore center. F $V_u$ dependence of $E_{ax,u}$ and $E_{ax,o}$. 

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3.2 Effect of External Ion Concentrations on Currents

If the external K⁺ and Cl⁻ concentrations (they are identical at both sides of the nanopore throughout this study) change, the coK and coCl will also change. How large the change is and where, is important as it influences the $E_{ax}$ and ultimately the current—its all three components. In the presence of an object the change may be difficult to predict. DNA translocation reduces the current compared to the baseline current at high concentration, but at low concentration it increases it [45]. As Fig. 4 shows the effect on the coKs and coCl{s is qualitatively as expected. The coK{u near (negatively charged) pore wall is high, but changes only moderately. In the pore center it is low but depends more on the external concentrations. Interestingly, it is above the external value (Fig. 4A, D). The coK{o is also low (but not as low as the coK{u) near the pore wall (the disk is positively charged), but rises more as external concentrations increase.

The coCl{u and coCl{o are also as expected near zero close to the pore wall, especially when the external concentrations are low. In the pore center, the coCl{u is above the external values (the disk is positively charged) and higher at high external concentrations. Finally, the coCl{o is quite high
near the disk and very dependent on external concentration (Fig. 4B, E). In general thus the $c_{\text{Cl}}$ also rise as the external concentrations increase but those that are the largest (the $c_{\text{Cl}_{\text{tot}}}$ near the disk and the $c_{\text{Cl}}$ near the pore center) rise the most (Fig. 4E). It is intuitively not very obvious how the $E_{\text{ax}_{\text{u}}}$ or $E_{\text{ax}_{\text{o}}}$ radial profiles may be influenced by the external $c_{\text{K}}$ and $c_{\text{Cl}}$. As these simulations show they both become more negative at high external K$^+$ and Cl$^-$ concentrations especially in the pore wall-disk space (Fig. 4C, F).

What should $\sigma_I$ profiles be given such $c_{\text{K}}$, $c_{\text{Cl}}$, and $E_{\text{ax}}$ radial profiles? The $\sigma_{\text{conv}_{\text{u}}}$ which is small and negative near the pore center (but zero at the disk walls; see Sect. 2) has a negative peak near the pore wall. The $\sigma_{\text{conv}_{\text{o}}}$ peak is also negative but is larger. They both become more negative at higher external concentrations (Fig. 5A). Elevated K$^+$ concentration near positively charged pore wall and positive $V_u$ lead to negative $I_{\text{tot}}$ that drives negative water flow (not shown) and thus negative $I_{\text{conv}}$ which becomes more negative at high external concentrations, and this remains the case regardless of axial position. The $\sigma_{\text{diff}_{\text{u}}}$ is positive and elevated near the pore wall but very insensitive to the external concentration change, whereas the $\sigma_{\text{diff}_{\text{o}}}$ is negative and small but depends on external concentration (Fig. 5B). Positive $\sigma_{\text{diff}_{\text{u}}}$ is not surprising due to an axial K$^+$ concentration gradient induced by the positive $V_u$ but it is more difficult to predict what the $\sigma_{\text{diff}_{\text{o}}}$ would be because of complex $c_{\text{K}}$ and $c_{\text{Cl}}$ distributions in disk-pore wall space. Finally, the $\sigma_{\text{mig}_{\text{u}}}$ is near zero in the pore center and negative near the pore wall at low concentrations. It becomes negative even in the pore center but especially near the pore wall at high concentrations. This is due to the significantly greater $c_{\text{Cl}_{\text{u}}}$ throughout nanopore and negative and larger $E_{\text{ax}}$ (Fig. 4). The $\sigma_{\text{mig}_{\text{o}}}$ is negative in the disk-pore wall space at low or high concentrations. This becomes much more so at elevated external concentrations near both the disk and pore wall. Given that the $\sigma_{\text{mig}}$ is the largest it determines the $\sigma_{\text{tot}}$ profile (Fig. 5C, D). Finally, note that with no disk the $I_{\text{conv}}$ becomes more concentration dependent whereas the $I_{\text{mig}_{\text{u}}}$ and $I_{\text{tot}_{\text{u}}}$ become less so. Small $I_{\text{diff}_{\text{u}}}$ becomes smaller and essentially concentration independent.

How $I$s depend on external concentrations is sometimes easy to predict, but not always. The total $I_{\text{conv}_{\text{u}}}$ and $I_{\text{conv}_{\text{o}}}$ are small but increasing as the external concentrations rise. They are carried largely by K$^+$ ions concentrated near the negatively charged pore wall (Fig. 5E). The total $I_{\text{diff}_{\text{u}}}$ is small, positive (carried by K$^+$ ions and increasing modestly as the external concentration rises). Interestingly, the total $I_{\text{diff}_{\text{o}}}$ which is even smaller is negative and carried by the Cl$^-$ ions (Fig. 5F). The $I_{\text{mig}_{\text{u}}}$ and $I_{\text{mig}_{\text{o}}}$ (the largest at any concentration and which determine the $I_{\text{tot}_{\text{u}}}$ and $I_{\text{tot}_{\text{o}}}$), which are negative and almost identical, increase as external concentrations rise (Fig. 5G, H). Finally, we show the 2D distributions with relative error needed to terminate simulations reduced by ten times (third panel in each case). This did not lead to any visible change demonstrating that the chosen relative error is adequately small.

### 3.3 Effect of Pore Wall Charges on Ion Currents

How much do the pore wall charges affect $c_{\text{K}}$ and $c_{\text{Cl}}$ within the nanopore with a charged disk, and what are the consequences for ion currents passing through? If the $\sigma_w$ changes from $-160$ to $0$ mC/m$^2$ to $+160$ mC/m$^2$, the $c_{\text{K}}$ and $c_{\text{Cl}}$ change greatly throughout the nanopore. The changes near the pore wall are as expected. Away from the pore wall, the $c_{\text{K}}$ changes little above the disk when $\sigma_w$ changes from $-160$ to $0$ mC/m$^2$ but increases significantly when the $\sigma_w$ rises to $+160$ mC/m$^2$. Below the disk the $c_{\text{K}}$ decreases to very low levels before rising, but very modestly. Interestingly, the $c_{\text{Cl}}$ changes similarly both above and below the disk. The $\sigma_w$ influences the $E_{\text{ax}_{\text{o}}}$ too but the effect is confined to or near the disk (Fig. 6).

The radial profiles confirm visual impressions. Near the pore wall, the $c_{\text{K}_{\text{u}}}$ and $c_{\text{Cl}_{\text{o}}}$ are either high (negative $\sigma_w$), the same as the $c_{\text{K}_{\text{o}}}$ in the pore center (zero $\sigma_w$) or low (positive $\sigma_w$). In the pore center, the $c_{\text{K}_{\text{u}}}$ is in all cases above the external value but is especially elevated for positive $\sigma_w$—probably an effect of positive $V_u$ (Fig. 6A). The $c_{\text{Cl}_{\text{u}}}$ near the pore wall is also easy to predict—high near positive wall, zero if it is negative and the same as in the pore center if the $\sigma_w$ is zero. In the pore center, it is always above its external value, but higher for positive $\sigma_w$. It is high near the (positively charged) disk but is still influenced by the $\sigma_w$ (Fig. 6B). The effect of $\sigma_w$ on the $E_{\text{ax}_{\text{o}}}$ is interesting. Whereas the $E_{\text{ax}_{\text{u}}}$ is always negative (with flat radial profile), it is more negative for negative $\sigma_w$. Finally, the $E_{\text{rad}_{\text{u}}}$ is both much more negative, and much more $\sigma_w$ dependent (Fig. 6C). We also evaluated the $E_{\text{rad}_{\text{o}}}$. If $\sigma_w$ changes the $E_{\text{rad}_{\text{o}}}$ should also change, and it does from being very near zero at radial distances below 2 nm to very significantly positive beyond (if the $\sigma_w$ is negative) or similarly significantly negative (positive $\sigma_w$). If $\sigma_w$ is zero, the $E_{\text{rad}_{\text{o}}}$ rises to positive but small values (Fig. 6C).

The $\sigma_w$ dependence of $c_{\text{K}}$, $c_{\text{Cl}}$, and $E_{\text{ax}}$ provides some additional insights. As the $\sigma_w$ changes from negative to positive, the $c_{\text{K}_{\text{o}}}$ and $c_{\text{K}_{\text{o}}}$ decrease from very high to low values near the pore wall, whereas in the pore center the $c_{\text{K}_{\text{u}}}$ rises from low values, and near the disk the $c_{\text{K}_{\text{o}}}$ remains low (Fig. 6D). In contrast, the $c_{\text{Cl}_{\text{u}}}$ rises greatly near the disk and pore walls, whereas the $c_{\text{Cl}_{\text{o}}}$ rises similarly near the pore wall but its rise in the pore center is less pronounced (Fig. 6E). Finally, the $E_{\text{ax}_{\text{u}}}$ is negative and not very $\sigma_w$ dependent, whereas the $E_{\text{ax}_{\text{o}}}$ is much more negative and its $\sigma_w$ dependence is complex (Fig. 6F).

The $\sigma_w$ effect on $\sigma_{\text{conv}_{\text{u}}}$ is pronounced but some changes are surprising (Fig. 7). The $\sigma_{\text{conv}_{\text{o}}}$ (zero in the pore center) is
**Fig. 4** Effect of external ion concentrations on concentrations and electric field within nanopore. Color-coded 2D distributions depict the effect of external ion concentrations on the c_{oK}, c_{oCl} and E_{ex} within nanopore. The V_{g} was 1 V, the \( \sigma_{w} \) was \(-160 \text{ mC/m}^2\), and the \( \sigma_{o} \) was \(+160 \text{ mC/m}^2\). Radial profiles of c_{oK,u} and c_{oK,o} (A), c_{oCl,u} and c_{oCl,o} (B), and E_{ax} (C) at external c_{K} and c_{Cl} as indicated. Concentration dependence of c_{oK,u} and c_{oK,o} (D) c_{oCl,u} and c_{oCl,o} (E) and E_{ax,u} and E_{ax,o} (F) near the pore wall, and in the pore center (or near the disk wall).
negative near the pore wall regardless of $\sigma_w$ sign, but more so if the $\sigma_w$ is negative. The $\sigma_{I,\text{conv},o}$ is more negative than $\sigma_{I,\text{conv},u}$ for similarly charged pore wall and disk. However, it is positive near the disk and negative near the wall if the $\sigma_o$ and $\sigma_w$ differ in sign (Fig. 7A). This is not surprising because water movement drives positive and negative ions in the same direction. What the $\sigma_{I,\text{diff},u}$ radial profiles should be is intuitively less obvious, owing to the spatial complexity of $c_{oK}$ and $c_{oCl}$ near the disk and pore walls. The $\sigma_{I,\text{diff},u}$ is zero in the pore center, negative near positively charged pore wall, but positive if it is negative. As already discussed this is likely due to the $V_o$ that affects $K^+$ and $Cl^-$ differently. The $\sigma_{I,\text{diff},u}$ is positive near positively charged pore wall, but negative and small if it is negatively charged (in both cases the disk was positively charged; Fig. 7B). The $\sigma_{I,\text{diff},o}$ is negative at any radial distance but more so near the pore wall (especially if the $\sigma_w$ is negative), as is the $\sigma_{I,\text{diff},o}$ which is larger and negative near both the disk and the pore walls.
regardless of the sign of their charges (Fig. 7C). Negative \( \sigma_{\text{I,mig}} \) s are not surprising given the positive \( V_u \). As expected the \( \sigma_{\text{I,tot}} \) is similar to that of \( \sigma_{\text{I,mig}} \) (the \( \sigma_{\text{I,conv}} \) and \( \sigma_{\text{I,diff}} \) are much smaller; Fig. 7D). 

The radial current densities (\( \sigma_{\text{I,rads}} \)) just above the upper disk surface (0.25 nm above) were also estimated. If the radial currents are significant, they may contribute to the axial currents in the disk-pore wall space. The \( \sigma_{\text{I,rad,convs}} \) are small (not shown). The \( \sigma_{\text{I,rad,diffs}} \) are indeed comparable to \( \sigma_{\text{I,diff}} \). If the \( \sigma_w \) is negative, the \( \sigma_{\text{I,rad,diff}} \) is present even at short radial distances and gradually increases near the disk’s tip. If the \( \sigma_w \) is positive, the \( \sigma_{\text{I,rad,diff}} \) is smaller and negative up to the disk’s tip, but then it rises to significant and positive values. Interestingly, if the \( \sigma_w \) is zero, the \( \sigma_{\text{I,rad,diff}} \) remains significant and negative near the tip (Fig. 7 B1). The \( \sigma_{\text{I,rad,mig}} \) s are also elevated and high near the tip, but they oppose the \( \sigma_{\text{I,rad,diff}} \) (Fig. 7 C1). Finally, the \( \sigma_{\text{I,rad,tots}} \) are moderate, present at any radial distance, rise near the tip, and are insensitive of the \( \sigma_w \) sign (Fig. 7 D1).

The \( I_{\text{mig,u}} \) and \( I_{\text{mig,o}} \) are the largest of all Is and very similar–negative (especially for high \( \sigma_w \)) and essentially independent of \( \sigma_w \) sign (Fig. 7G). Both the \( I_{\text{conv,u}} \) and \( I_{\text{conv,o}} \) are
small and depend differently on $\sigma_w$ (Fig. 7E), and the same is true for $I_{\text{diff},u}$ and $I_{\text{diff},o}$ (Fig. 7F). The $I_{\text{tot},u}$ and $I_{\text{tot},o}$ overlap completely (Fig. 7H). If no disk is present within nanopore, the $I_{\text{conv},u}$ becomes more $\sigma_w$-dependent. The $I_{\text{mig},u}$ and $I_{\text{mig},o}$ become not only less $\sigma_w$-dependent, but also smaller in value at higher $\sigma_w$ regardless of its sign. Finally, the $I_{\text{diff},u}$ becomes near zero at any $\sigma_w$.

### 3.4 In an Uncharged Nanopore, the Current Magnitude Diminishes as the Disk Size Increases

As already stated, the $I_{\text{tot}}$ is the smallest if the $\sigma_w$ is near zero. Interestingly in such a case, the $r_o$ dependence is qualitatively different. The 2D distributions shown on top compare the $\sigma_{\text{tot}}$, $E_{\text{ax}}$, and $\sigma_{\text{C}}$ for a short and long disk when the $\sigma_w$ is zero or positive and significant (i.e., 0 or $+160$ mC/m$^2$; Fig. 8). It is not evident based on visual observations whether or how $I_{\text{diff}}$ will differ, when the $\sigma_w$ is $+160$ mC/m$^2$ instead of 0 mC/m$^2$ regardless of disk radius. Both the $I_{\text{conv}}$ and $I_{\text{mig}}$ are clearly more negative when the $\sigma_w$ is $+160$ mC/m$^2$ instead of 0 mC/m$^2$ especially near the pore wall. The $\sigma_{\text{C}}$ and $E_{\text{ax}}$ distributions are also shown as they illustrate complex $\sigma_{\text{C}}$ (especially near the pore wall) and $E_{\text{ax}}$ (in the pore interior) changes that influence different currents. The $\sigma_+^-$ is depicted because it is a dominant ion within the nanopore (either both the $\sigma_w$
and $\sigma_o$ are positive or the $\sigma_o$ is positive and the $\sigma_w$ is zero; Fig. 8).

Figure 8A and B give the $E_{ax}$ and $c_{oCl}$ axial profiles at the radial mid-point of the disk-pore wall space (i.e., with the $r_o$ of 2 nm or 3.25 nm and from $z = 1.25$ nm to $z = 1.75$ nm). If the disk is small ($r_o = 0.5$ nm) the axial profiles of both $E_{ax}$ and $c_{oCl}$ are constant. Interestingly even in this case their values depend on the $\sigma_w$. If the $\sigma_w$ is 0 mC/m$^2$, the $E_{ax}$ is clearly more negative and the $c_{oCl}$ is lower. With large disk ($r_o = 3.0$ nm), the $E_{ax}$ is more negative at the lower end especially when the $\sigma_w$ is 0 mC/m$^2$ but is almost zero at the higher end regardless of the $\sigma_w$. The $c_{oCl}$ also changes. It is lower when the $\sigma_w$ is 0 mC/m$^2$ especially at the lower end. The $E_{ax}$ and $c_{oCl}$ are thus both affected by the $\sigma_w$ changes even when the disk is small, but if it is large, they also change significantly axially.

The currents depend also on $\sigma_w$ and at all disk radii, but there are important differences. If the $\sigma_w$ is zero, the $I_{conv}$ is...
also near zero and is largely \( r_o \)-independent. If the \( \sigma_w \) is positive, it is negative (having the most negative value for \( r_o \) of 2 nm). There is very little ion accumulation near uncharged pore wall, but if the wall is positively charged Cl\(^-\) accumulates. Given that the \( V_u \) is positive, water will move upwards (not shown) producing a negative \( I_{\text{conv}} \). The \( I_{\text{conv}} \) decreases eventually when very large disk blocks almost completely mechanically the water flow and \( I_{\text{conv}} \) (Fig. 8C). The \( I_{\text{diff}} \) is near zero for small \( r_o \) regardless of the \( \sigma_w \). It rises as the \( r_o \) increases but more when the \( \sigma_w \) is 0 mC/m\(^2\). Although the \( \text{coCl} \) is higher when the \( \sigma_w \) is large and positive, it changes more axially when the \( \sigma_w \) is 0 mC/m\(^2\). When the \( r_o \) is small, the \( \text{coCl} \) is not only low but changes very little axially (Fig. 8B). The \( I_{\text{mig}} \) is negative (and the largest of all \( I_s \)) if the \( \sigma_w \) is 0 mC/m\(^2\) and diminishes in amplitude (as does the \( I_{\text{tot}} \)) as \( r_o \) increases (Fig. 8E). It is difficult to give a simple explanation for this change–the \( E_{\text{ax}} \) does not change overall but becomes highly non-uniform axially and the \( \text{coCl} \) in fact increases. When the \( \sigma_w \) is positive (\(+\) 160 mC/m\(^2\)), the \( I_{\text{mig}} \) and \( I_{\text{tot}} \) become more negative with greater \( r_o \) (except when the disk comes very near the wall; Fig. 8E, F). The most likely explanation is the rise of Cl\(^-\) concentration in the disk-pore wall space (Fig. 8B).

3.5 At High External Ion Concentrations, Ion Current is More \( r_o \)-Dependent

As external concentrations increase the \( I_{\text{tot}} \) rises (Fig. 5H), but it is unclear whether, and if so how, its \( r_o \)-dependence may change. The \( V_u \) remained +1 V but the \( \sigma_o (+\) 160 mC/m\(^2\)) and \( \sigma_w (\sim 160 \text{ mC/m}^2) \) differed in sign (unlike in Fig. 8). At two comparatively low but different external concentrations (20 and 100 mM), the \( I_{\text{conv}} \) diminishes as \( r_o \) increases regardless of external concentrations, but its \( r_o \) dependence is greater at high concentrations (Fig. 9A). In contrast, the \( I_{\text{diff}} \) (also small) rises as \( r_o \) increases but is concentration independent (Fig. 9B). This is most likely because the axial concentration gradients in the disk-pore wall space–the K\(^+\) (near the pore wall) and Cl\(^-\) (near the disk)–increased. Finally, the \( I_{\text{mig}} \) (the largest current at any \( r_o \); Fig. 9D) is not \( r_o \)-dependent at low concentration but at high concentration it is not only larger, but becomes moderately \( r_o \)-dependent, rising as \( r_o \) increases except when the disk gets very near the pore wall (Fig. 9C).

4 Discussion

4.1 Background

The size of cells and generally micro-particles traversing through a micro-pore is often evaluated by measuring how much their transit reduces the current [2, 46, 47]. A basic idea is that a bigger cell (which leaves smaller cross-sectional area for the passage of ions) leads to smaller current. The cell shape can also be assessed (with irregular micro-pores) from current fluctuations produced by cell rotation [21]. However, it is still not clear why axial irregularity may be critical in inducing object rotation, whether the object would rotate in nanopores, and whether its rotation would be detectable [19–21]. Given that many viruses are at the nanoscale and that thus nanopores will have to be used for their detection a systematic evaluation of what controls the current and current-object size relationship in nanopores is clearly needed.

In this study, we determine using simulations how external factors (electric field and ion concentrations), and pore properties (pore wall charge density) affect the current and object (disk) radius-current relationship in cylindrical nanopores. Three components of axial ion current are dissected above the disk and in the disk-pore wall space–diffusive (due to concentration gradient; \( I_{\text{diff}} \)), convective (due to water movement; \( I_{\text{conv}} \)), and migratory (due to electric field; \( I_{\text{mig}} \)), as each component is controlled differently. Finally, we dissected the corresponding radial currents and estimate them just above the disk. Note that we do not consider osmotic gradient term in the interaction between ions and the fluid. This will be a part of our future study.

4.2 Effect of External Electric Field on the Current Flow

The effect of external electric field (\( E_{\text{ex}} \)) on ion currents was tested in a negatively charged nanopore with a positively charged large disk. The voltages applied at the controlling edges resemble those considered previously (ranging from –1 to +1 V; [48]). Greater \( E_{\text{ex}} \) leads to a proportionally greater \( I_{\text{mig}} \), but much smaller \( I_{\text{conv}} \) and \( I_{\text{diff}} \) also depend on the \( E_{\text{ex}} \). Note that not only at high voltages, but over a wide range of voltages (and thus over a wide range of \( E_{\text{ex}} \) values), the \( I_{\text{mig}} \) is the largest current and determines the \( I_{\text{tot}} \). At any point within the nanopore, the \( I_{\text{mig}} \) is determined by two factors: (a) \( E_{\text{ex}} \) and (b) ion concentrations. We tested the \( E_{\text{ex}} \) effect above the disk and in the disk-pore wall space. The \( E_{\text{ex}} \) radial profiles are uniform. The \( E_{\text{ex}} \) depends (linearly) on the \( E_{\text{ex}} \), but its dependence is greater in the pore wall-disk space. As in conically shaped nanopores, the \( E_{\text{ex}} \) influences the \( \text{coK} \) and \( \text{coCl} \) [25, 49, 50], which rise with greater \( E_{\text{ex}} \) regardless of tested location, and independently of \( E_{\text{ex}} \) polarity. Finally note that the disk presence within nanopore leads to greater \( V_{\text{u}} \)-dependence of the \( I_{\text{tot}} \) and \( I_{\text{mig}} \), but that of the \( I_{\text{conv}} \) diminishes.

4.3 Effect of External Ion Concentrations on Ion Currents and Their \( r_o \) Dependence

External K\(^+\) and Cl\(^-\) concentrations influence those within nanopore, but unequally. The concentrations varied over a
significant range (from 10 to 400 mM)–i.e., beyond values used experimentally (100–300 mM; [20]), and if they rise, the nanopore concentrations increase. Given that in these simulations the pore wall is negatively charged, and disk is positively charged K+ ions are counter-ions for the pore wall and co-ions for the disk surface. Opposite is the case for Cl− ions. As expected near the pore wall, the coK is high (but more so above the disk than in the disk-pore wall space), whereas in the pore center it is low but even lower near the disk in the disk-pore wall space. The coCl is opposite–low near the pore wall above the disk and in the disk-pore wall space. In the pore center, it is high (moderately) but higher near the disk in the disk-pore wall space. As the external concentrations rise, both coK and coCl rise regardless of location tested. The coK increases little near the pore wall above the disk, but more so in the disk-pore wall space. The coCl remains low near the pore wall, but in the pore center and especially near the disk in the disk-pore wall space, it rises significantly.

The Eax was negative (Vg was +1 V and Vd 0 V) except at low external concentrations in the center of the nanopore. It becomes progressively more negative as external concentrations increased, especially in the pore wall-disk space (near the pore wall or the disk). This is not entirely surprising because in that space the ion concentrations change greatly both radially and axially, especially when the disk almost reaches the pore wall. All current components increase—the Imig (the greatest), the Iconv (that adds to the Imig), and the Idiff (that opposes it) owing to elevate charges near the pore wall and greater Eax. The Itot, which is r independent at low concentration, becomes r dependent at high concentration, but only modestly. It is also worth pointing that if the disk is present within the nanopore the concentration dependence of the Itot,u and Imig,u rises but that of the Iconv,u diminishes.

4.4 Effect of Pore Wall Charges on Ion Concentrations and Currents and Current r Dependence

In this study, the σw changed from −160 to +160 mC/m². These values are equivalent to 1 e/nm²—thus equal or beyond those observed biologically or used experimentally. At the surface of biological membranes, the σw has been estimated at 40–160 mC/m² or 0.25 to 1 e/nm² [51, 52], and at the surface layer of SiO2 membrane at 26 mC/m² [53]. As expected the σw influences the ion concentrations greatly near the charged pore wall or disk surfaces. We find it surprising how large the σw influence is throughout the nanopore including in the pore center. It is also notable how
different the counter-ion and co-ion concentrations could be above and below the disk. The effect on the $E_{ax}$ is also difficult to predict—pronounced and complex in the disk-pore wall space, but also present though limited above the disk.

How $\sigma_w$ radial profiles and corresponding currents should depend on the $\sigma_w$ is sometimes possible to assess but not always. As expected the $\sigma_{\text{diff}}$ and $\sigma_{\text{lconv}}$ are confined near the pore wall regardless of the $\sigma_w$ value or its sign. However, the $\sigma_{\text{mig}}$, though greater near the pore wall, is not insignificant near the pore center. The $\sigma_{\text{mig}}$ rises greatly as $\sigma_w$ increases but its direction is independent of the $\sigma_w$ sign. The $I_{\text{mig}}-\sigma_w$ relationship is thus almost parabolic. Greater concentration of counter-ions near the pore wall contributes to greater $I_{\text{mig}}$ at higher $\sigma_w$. It however does not affect its direction ($K^+$ movement in one direction is the same as $Cl^−$ movement in the opposite direction). The $E_{ax}$ also changes (especially in the pore wall-disk space) influencing the $I_{\text{mig}}$, but the $E_{ax}-\sigma_w$ relationship is complex.

Given that the nanopore does not have current sources or sinks, the $I_{\text{tot}}$ has to be independent of where axially it is estimated and it is. The $I_{\text{mig}}$ which clearly makes the largest contribution to the $I_{\text{tot}}$ should thus depend very little on where they are axially estimated, and that is also the case. On the other hand much smaller $I_{\text{diff}}$ and $I_{\text{lconv}}$ may differ significantly depending on where axially they are estimated, and they clearly do. It has been suggested that the (a) presence of an object within nanopore leads to volume exclusion of electrolyte solution thus reducing the axial current flow and (b) charged object induces screening charges leading to larger axial currents, finally producing multi-level current changes [26]. However, this study clearly demonstrates that the $I_{\text{tot},u}$ (and $I_{\text{mig},u}$) become larger and not smaller (regardless of $\sigma_w$ sign) at high $\sigma_w$ if disk is present within the nanopore. However, if the $\sigma_w$ is low (or zero), the $I_{\text{tot},u}$ (and $I_{\text{mig},u}$) become smaller.

To show how the $\sigma_w$ influences the $I_r$ relationship we chose two $\sigma_w$s ($0$ and $+160 \text{ mC/m}^2$). In an uncharged nanopore, the $I_{\text{mig}}$ decreases in value (rather than increasing) with greater $r_o$ but remains the largest current contribution. The $I_{\text{lconv}}$ (negative and small for positive $\sigma_w$) becomes almost zero for all $r_o$s. The $I_{\text{diff}}$ is positive and near zero for small $r_o$ regardless of $\sigma_w$. It rises in value with greater $r_o$ but more so in an uncharged nanopore. Should the $I_{\text{diff}}$ be positive or negative and how should it change in value as the $r_o$ rises? Consider the $Cl^−$ (counter-ion) concentration near the disk in an uncharged pore. It is elevated near the disk but diminishes upwards. It thus produces a positive $I_{\text{diff}}$. Greater $I_{\text{diff}}$ at larger $r_o$ is also not unexpected. Larger $r_o$ leads to more fixed (and thus also more mobile) charges. Whereas a positively charged pore would lead to more charges ($Cl^−$ ions) the axial gradient is likely smaller. Overall, in an uncharged nanopore, the $I_{\text{tot}}$ decreases significantly in value with greater $r_o$ instead of increasing modestly as observed when $\sigma_w$ is positive and high. A comparison of $I_r$-relationships with $\sigma_w$s that are either negative or positive but high (the $\sigma_w$ was $+160 \text{ mC/m}^2$ in both cases) is also interesting. The $I_{\text{tot}}-r_o$ relationship depends on the $\sigma_w$ value but is largely independent of its sign (i.e., whether the $\sigma_w$ and $\sigma_o$ have the same or opposite signs). However, if the $\sigma_w$ and $\sigma_o$ have the same sign, larger $I_{\text{tot}}$ at greater $r_o$ is due to higher counter-ion accumulation in the object-pore wall space as suggested before [26]. In contrast, if the $\sigma_w$ and $\sigma_o$ are of opposite sign, similarly greater $I_{\text{tot}}$ is largely due to an elevated $E_{ax}$ in that space.

It is clear from the above that the uncharged nanopores would make better sensors of size and shape of nano-size objects (such as viruses). Note also, that in a charged nanopore (and to a lesser extent in an uncharged nanopore too), the $I_{\text{tot}}$ is almost $r_o$-independent for $r_o$s ranging from $0$ to $2–2.5 \text{ nm}$ in a $3.5 \text{ nm}$ radius nanopore. Suggestions that the $I_{\text{tot}}$ block is due to the volume exclusion of electrolyte solution by the object thus cannot be universally accepted. If the object does not get very close ($<1 \text{ nm}$) to the pore wall, it will not reduce the $I_{\text{tot}}$ significantly, because the ion current flows largely in $1 \text{ nm}$ space near the pore wall.

Our simulations are three-dimensional based on axial symmetry. True three-dimensional simulations can evaluate radially asymmetrical situations whereby one side of the disk is closer to the pore wall than the opposite side. Given that the $I_{\text{tot}}$ depends relatively little on the $r_o$ the $I_{\text{tot}}$ should not depend greatly on how asymmetrical the radial position of the disk is. The cross-sectional area for the current flow remains constant. However, if the disk is positioned non-centrally (i.e., if it is axially asymmetrical), the $I_{\text{tot}}$ changes. It diminishes linearly if the disk is positioned more upwards (15 nm change in axial position leads to $I_{\text{tot}}$ reduction of 39%; not shown).

#### 4.5 Comment on Continuum Poisson-Nernst-Planck and Navier–Stokes Simulations

Continuum PNP-NS simulations do not capture all features of the nanopore ion and water system. As shown by the molecular dynamics simulations water density and viscosity are greater near the pore wall due to non-electrostatic (described by Lennard–Jones potentials) and electrostatic interactions [54], whereas the Navier–Stokes equations assume that the water is incompressible. The ions are also layered near the pore wall and show a peak near, but not at its surface, because the ions cannot get closer to the surface than an ionic radius [54]. This is also not predicted by the continuum simulations. Nevertheless, most properties of the nanopore ion and water system are well described by the continuum PNP-NS equations for water and ion transport [55].
5 Conclusions

In a charged cylindrical nanopore with a charged disk elevated $E_{ax}$ leads to higher $E_{ex}$, $coK$, $coCl$, and $I_{tot}$ (which is largely determined by the $I_{mig}$ regardless of the $E_{ex}$ value). The $coK$ and $coCl$ rise (estimated above the disk or in the disk-pore wall space) is largely $E_{ex}$ polarity independent. The $I_{tot}$ also rises with greater $\sigma_w$ and interestingly it is $\sigma_w$ sign independent. If the $\sigma_w$ and $\sigma_o$ have the same sign, larger $I_{tot}$ is due to higher counter-ion accumulation in the object-pore wall space, but if their signs are opposite similarly larger $I_{tot}$ is mainly due to greater $E_{ax}$ in the object-pore wall space. Elevated external ion concentrations lead to greater $coK$ and $coCl$ (mainly away from the pore wall), raise the $E_{ax}$ (especially in the disk-pore wall space), and lead to larger $I_{tot}$. The $I_{tot}$ is not $r_o$-dependent at low concentrations, but at high concentrations it is, though only modestly. Surprisingly, at high concentrations the $I_{tot}$ rises modestly when disk becomes bigger. Regardless of external concentrations the $I_{tot}$ depends very little on the $r_o$ if it is < 2 nm. The $r_o$-independence of $I_{tot}$ for small disks is not surprising because the current flow is largely confined to the space near the pore wall. Smaller cross-sectional area for current flow (size-exclusion principle) or volume exclusion of electrolyte solution by the object thus cannot be universally accepted as explanations of current blockage. However, the radial currents near the charged disk surface due to screening charges and $E_{rad}$ can contribute to axial currents. Finally, if the pore wall is uncharged, the $I_{tot}$ diminishes significantly as $r_o$ increases due to progressively smaller $I_{mig}$ and larger $I_{diff}$, which opposes it. Though smaller than the $I_{mig}$, the $I_{diff}$ is not insignificant.

Author Contribution MIG: conceptualization, simulations, analysis, and writing. MT: model development and writing.

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Declarations

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