QCD analysis of $F_2^\gamma(x, Q^2)$: an unconventional view $^1$

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Abstract. Elements of the alternative approach to hard collisions of photons, proposed recently by the author, are reviewed, with particular attention to QCD analysis of $F_2^\gamma$. This approach is based on clear separation of genuine QCD effects from those of pure QED origin and does not rely on the assumption that parton distribution functions of the photon behave as $\alpha/\alpha_s$. It differs significantly from the conventional one, as illustrated on the example of charm contribution to $F_2^\gamma$, recently measured at LEP.

INTRODUCTION

In [1] I have proposed an alternative approach to QCD analysis of $F_2^\gamma(x, Q^2)$, and by implication to any hard process involving initial photon, which differs substantially from the conventional one. It builds in part on arguments advocated for a long time by the authors of [2] and agrees with the approach to calculations of direct photon production at HERA pursued in [3]. This alternative approach is based on two ingredients:

- Clear and systematic separation of genuine QCD effects from those of pure QED origin, which leads to unambiguous and universal definition of the concepts “leading” and “next-to-leading” order of QCD.
- Acknowledgement of the fact that parton distribution functions (PDF) of the photon are proportional to $\alpha$ and not, as assumed in the conventional approach, to $\alpha/\alpha_s$.

The general expression for $F_2^\gamma(x, Q^2)$ has the following structure

$$\frac{1}{x} F_2^\gamma(x, Q^2) = q_{NS}(M) \otimes C_q(Q/M) + \frac{\alpha}{2\pi} \delta_{NS} C_\gamma + \langle e^2 \rangle \Sigma(M) \otimes C_q(Q/M) + \frac{\alpha}{2\pi} \langle e^2 \rangle \delta \Sigma C_\gamma + \langle e^2 \rangle G(M) \otimes C_G(Q/M)$$

(1)

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where quark nonsinglet and singlet and gluon distribution functions satisfy the evolution equations

\[ \frac{d\Sigma(x, M)}{d\ln M^2} = \delta_\Sigma(k_q + P_{qq} \otimes \Sigma + P_{qG} \otimes G), \]
\[ \frac{dG(x, M)}{d\ln M^2} = k_G + P_{Gq} \otimes \Sigma + P_{GG} \otimes G, \]
\[ \frac{dq_{NS}(x, M)}{d\ln M^2} = \delta_{NS}(k_q + P_{NS} \otimes q_{NS}), \]

where \( \delta_{NS} = 6n_f (\langle e^4 \rangle - \langle e^2 \rangle^2) \), \( \delta_\Sigma = 6n_f \langle e^2 \rangle \). The splitting and coefficient functions \( k_i, C_j \) can be expanded in powers of \( \alpha_s(\alpha_s(M)/2\pi) \) as

\[ k_q(x, M) = \frac{\alpha}{2\pi} \left[ k_q^{(0)}(x) + a(M)k_q^{(1)}(x) + a^2(M)k_q^{(2)}(x) + \cdots \right], \]
\[ k_G(x, M) = \frac{\alpha}{2\pi} \left[ a(M)k_G^{(1)}(x) + a^2(M)k_G^{(2)}(x) + \cdots \right], \]
\[ P_{ij}(x, M) = a(M)P_{ij}^{(0)}(x) + a^2(M)P_{ij}^{(1)}(x) + \cdots, \]
\[ C_q(x, Q/M) = \delta(1 - x) + a(M)C_q^{(1)}(x, Q/M) + a^2(M)C_q^{(2)}(x, Q/M) + \cdots, \]
\[ C_G(x, Q/M) = a(M)C_G^{(1)}(x, Q/M) + a^2(M)C_G^{(2)}(x, Q/M) + \cdots, \]
\[ C_\gamma(x, Q/M) = C_\gamma^{(0)}(x, Q/M) + a(M)C_\gamma^{(1)}(x, Q/M) + a^2(M)C_\gamma^{(2)}(x, Q/M) + \cdots, \]

where the lowest order coefficient functions \( k_q^{(0)} \) and \( C_\gamma^{(0)} \)

\[ k_q^{(0)}(x) = (x^2 + (1 - x)^2), \]
\[ C_\gamma^{(0)}(x, Q/M) = \left( x^2 + (1 - x)^2 \right) \left[ \ln \frac{M^2}{Q^2} + \ln \frac{1 - x}{x} \right] + 8x(1 - x) - 1 \]

are unique and due entirely to QED coupling of the initial photon to the primary \( q\bar{q} \) pair. It is their presence what distinguishes hard collisions of photons from those of hadrons. In [1] I have discussed conceptual as well as numerical differences between the results obtained within the conventional and alternative approaches for the pointlike part of \( F_2^\gamma \) in the nonsinglet channel and under the simplifying assumption \( \beta_1 = 0 \). In this talk I will concentrate on proper definitions of the concepts “leading order” (LO) and “next–to–leading order” (NLO) of QCD and on the discussion of their implications for charm contribution \( F_2^{\gamma,c}(x, Q^2) \) to photon structure function, recently measured at LEP [4,5].

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2) For their definitions as well as other details of the alternative approach, see [1].
3) Throughout this paper we restrict ourselves to the case of real target photon, although the basic conclusions hold for the virtual photon as well. For the latter, however, the expression for \( C_\gamma^{(0)} \) differs slightly from (12).
DEFINING LEADING AND NEXT-TO-LEADING ORDERS OF QCD

Although the definition of the concepts “LO” and “NLO” in hard collisions of photons is a matter of convention, it is preferable to define them in a way that retains the same basic content these concepts have in collisions of leptons and hadrons. Let me recall, for instance, their meaning in the case of the familiar ratio

$$R_{e^+ e^-} (Q) \equiv \frac{\sigma(e^+ e^- \to \text{hadrons})}{\sigma(e^+ e^- \to \mu^+ \mu^-)} = \left(3 \sum_{i=1}^{n_f} e_i^2\right) \left(1 + r(Q)\right).$$  \hspace{1cm} (13)

The prefactor $R_{\text{QED}} \equiv 3 \sum_{i=1}^{n_f} e_i^2$ multiplied by unity in the brackets of (13) comes purely from QED, whereas genuine QCD effects are contained in $r(Q)$, which is given as expansion in powers of $\alpha_s$ as

$$r(Q) = \frac{\alpha_s(M)}{\pi} \left[1 + \frac{\alpha_s(M)}{2\pi} r_1(Q/M) + \cdots \right].$$ \hspace{1cm} (14)

For the quantity (13) it is a generally accepted practice to include $R_{\text{QED}}$ in theoretical expressions but disregard it when defining the “LO” and “NLO” of QCD. For instance, the NLO approximation of (13) implies retaing first two terms in (14), i.e. the first three terms in (13). Let me emphasize that only if at least first two nontrivial powers of $\alpha_s$ in (13) are taken into account can this truncated expansion be associated with a well-defined renormalization scheme. And it is this association what makes in my view the essential feature of the concept “NLO QCD approximation”. The same convention should be adopted for all physical quantities getting contributions from pure QED. I think it is preferable to use the terminology that avoids potential confusion which might arise from mixing orders of $\alpha_s$ and $\alpha$. Note that for both $R_{e^+ e^-}$ and $F_2^\gamma (x, Q^2)$, discussed in the next Section, the purely QED contributions $R_{\text{QED}}$ and $F_2^{\gamma, \text{QED}}$ are finite and unique and there is thus no reason why they should be treated differently as far as the definitions of the “LO” and “NLO” of QCD are concerned. The implications of the above considerations for $F_2^\gamma (x, Q^2)$ are the following:

- As shown in [1] the LO QCD expression for $F_2^\gamma$ contains in addition to terms included in the conventional LO analysis of $F_2^\gamma$ (i.e. those proportional to $k_q^{(0)}$ and $P_{ij}^{(0)}$), also terms involving $k_q^{(1)}$, $C_q^{(1)}$, $C_{\gamma}^{(0)}$ and $C_{\gamma}^{(1)}$. As all these functions are known, there is, however, no obstacle to performing such an analysis. As shown in [1] for the pointlike part of $F_2^\gamma (x, Q^2)$ in the nonsinglet channel, the results in these two approaches are numerically significantly different, the single most important contribution to this difference coming from $C_{\gamma}^{(1)}$. The coefficient function $C_q^{(1)}$ enters $F_2^\gamma$ already at the LO due to the fact that it does so in the convolution with purely QED part of quark distribution function of the photon, which has no analogue in hadronic collisions.

\[^4\text{Contrary to hadronic collisions, where it appears first at the NLO.}\]
The derivative (15) taken from the fit to LEP data (solid squares) compared to pure QED formula $a^{\text{QED}}(x) = 3 \sum_{i=1}^{n_f} e_i^4 x (1 + (1 - x)^2)$ for $n_f = 3, 4$. Both data and curves in units of $\alpha$.

- The NLO QCD analysis of $F_2^\gamma(x, Q^2)$ requires the knowledge of two quantities, $k_q^{(2)}$ and $C_\gamma^{(2)}$, that have not yet been calculated and thus is at the moment impossible to perform. Note, that in the conventional approach $C_\gamma^{(0)}$, which has nothing to do with QCD, appears only at the “NLO”, and $C_\gamma^{(1)}$, which involves evaluation of Feynman diagrams with a single QCD vertex is not used even there!

For clear and unambiguous definition of the terms “LO” (“NLO”) it is thus vital to agree on the basic criterion, namely that they refer to perturbative expansions of physical quantities retaining the first (first two) nontrivial powers of $\alpha_s$. The purely QED contribution to $F_2^\gamma(x, Q^2)$ is irrelevant from this point of view, but may be retained for comparison with experiment as it actually dominates scaling violations of $F_2^\gamma(x, Q^2)$ in most of accessible range of $x$. To identify genuine QCD effects one has to look for subtler effects than the dominant $\ln Q^2$ rise, like, for instance, the $x$-dependence of the slope

$$a(x) \equiv \frac{dF_2^\gamma(x, Q^2)}{d \ln Q^2}.$$  \hspace{1cm} (15)

Compared to $F_2^p(x, Q^2)$, for which scaling violations are due entirely to QCD effects, the nonzero slope (15) is by itself no sign of QCD effects, as these are given by the difference $\Delta(x) \equiv a(x) - a^{\text{QED}}(x)$. Fig. 1, based on numbers taken from [6], shows that for $x \gtrsim 0.5$ the precision of currently available data is insufficient to identify genuine QCD effects, although some indication of the turnover to negative $\Delta(x)$, expected theoretically, is visible there.
CHARM CONTRIBUTION TO $F_2^γ(X, Q^2)$

The QCD analysis of $F_2^γ(x, Q^2)$ in the region $x > 0.1$ provides possibly the simplest illustration of the differences between the conventional and alternative approaches to hard collisions of photons [7], since $F_2^γ$ is dominated in this region by the direct photon contribution, $F_2^{\text{dir}}$, which does not involve any PDF. It can be written as

$$F_2^{\text{dir}}(x, Q^2) = F_2^{\text{QED}}(x, Q^2) + \alpha_s(Q)F_2^{(1)}(x, Q^2) + \alpha_s^2(Q)F_2^{(2)}(x, Q^2) + \cdots,$$

where the coefficients $F_2^{(k)}$, $k \geq 1$ are calculable in perturbative QCD and the lowest order term $F_2^{(0)} \equiv F_2^{\text{QED}}$ comes from pure QED diagram in Fig. 2a. In the conventional approach the “NLO” approximation of the direct photon contribution is defined [8] by taking the first two terms in (16) including the purely QED contribution $F_2^{\text{QED}}$. However, as argued in the preceding Section, this definition does not have the basic attribute of genuine NLO QCD approximation. The inclusion of direct photon contributions of the order $\alpha^2\alpha_s^2$, coming from diagrams like those in Fig. 2e,g is vital not only for establishing the genuine NLO character of the
direct photon contribution itself, but also for ensuring [7] factorization scale invar-
ance of the full expression for $F_{2,c}^\gamma$. The latter involves adding the resolved photon
contributions up to the order $\alpha^2\alpha_s^2$, coming from diagrams like those in Fig. 2f,h,j.

SUMMARY AND CONCLUSIONS

The alternative approach to QCD analysis of $F_{2}^\gamma$, proposed recently by the au-
thor, differs substantially and in a number of aspects from the conventional one. It
satisfies factorization scale invariance in a way that does not rely on physically
untenable assumption that quark distribution functions of the photon behave as
$O(\alpha/\alpha_s)$. The simplest implications of this difference are illustrated on the case of
charm contribution to $F_{2}^\gamma(x, Q^2)$, recently measured at LEP.

To be useful phenomenologically the proposed approach needs to be further elab-
orated by extending it to the singlet sector and merging it with the hadronic con-
tributions. Work on this program is in progress. The NLO QCD analysis in the
proposed approach requires evaluation of several so far unknown quantities and is
thus currently impossible to perform. In view of the quality and number of experi-
mental data on $F_{2}^\gamma$, this is at the moment no serious drawback and a complete LO
QCD analysis seems sufficient for phenomenological purposes.

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