Auxiliary information based generally weighted moving coefficient of variation (AIB-GWMCV) control chart

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Abstract. Statistical Process Control (SPC) is a statistical technique to accurately determine process performance. It has been widely used in manufacturing industries and services. The most powerful instrument used in SPC is the control chart. It is designed to observe and detect timely assignable causes of the process. In general, the control chart is employed to detect shifts in process location and process dispersion. Monitoring the coefficient of variation (CV) is efficient in SPC when the standard deviation process changes with the mean. Also, the mean itself is expected to fluctuate with time but considered to be in-control. Recently, some studies investigate using auxiliary information to enhance the sensitivity of the CV control chart. This study investigates auxiliary information based CV-GWMA (AIB-GWMCV) control chart by using log-normal transformation to detect the small to large shifts in process CV. It turns out the proposed control chart performs better than the CV-GWMA control chart using log-normal transformation without auxiliary information. Also, a real case is presented to display the application of the AIB-GWMCV control chart. Results show that the proposed control chart is faster to detect shifts in the process CV than the control chart without auxiliary information.

1. Introduction
Control charts become the most powerful instrument used in SPC. It is designed to observe the behavior of the process and to detect timely assignable causes of the process [1]. The principle of SPC is that the process cannot be declared as in-control until both the process dispersion and the process location is stable. When standard deviation changes with the mean and mean itself timely fluctuate but considered to be in-control then the CV based control charts can be efficiently used to keep the stability of the process. Shewhart’s CV chart was first introduced by [2]. They conclude that it has an attractive performance than \( \bar{X} \) and \( S \) chart [3-4]. Adaptive Shewhart CV chart implementing VSS strategy was presented by [5]. Moreover, they implemented log-normal transformation to get the normal distribution of CV statistic. The proposed chart compared with Shewhart’s CV chart [2], synthetic CV chart [6], and VSI chart [7] and showed that it is more efficient to detect shifts of CV than existing charts. Further, EWMA-type CV [8] and DEWMA-type CV [9] were investigated to increase the efficiency in detecting process CV. In a recent study, the log-normal transformation was applied in an EWMA-type CV chart named EWMCV chart [10]. They conclude that EWMA-type CV with RSS performs better than EWMA-type CV with SRS. The GWMA based CV control chart was evolved by [11] and they concluded that it has better performance than the CV-EWMA chart [8] and CV-DEWMA chart [9].
In these control charts, the monitoring process dispersion is constructed by using a single quality characteristic. Recently, researchers have been proposed a method to increase the detecting ability by adding an auxiliary variable that correlated with the study variable. Haq [12] proposed two charts for efficient monitoring of process dispersion by using the regression type auxiliary estimator of the population variance called AIB-EWMA-I and using the cumulative distribution transformation called AIB-EWMA-II. The result showed that the AIB-EWMA-I chart performs better than the AIB-EWMA-II chart when detecting process dispersion. Abbasi [13] presented some varieties of auxiliary information based ratio form, regression form, and hybrid estimator to enhance the shift detection ability of the CV charts. The result showed that it is significantly efficient than competing charts without auxiliary information for moderate to high correlation levels. Further, a study about utilizing auxiliary information in the EWMCV chart is investigated by [14]. They conclude that the AIB-EWMCV chart outperforms the EWMCV chart developed by [10] in terms of quickly detect out-of-control signals.

This study investigates new auxiliary information based CV-GWMA (AIB-GWMCV). We proposed the AIB-GWMCV control chart utilizing the log-normal transformation suggested by [5]. The capability of control charts in detecting shifts can be evaluated with the Average Run Length (ARL) value. In the current study, we discuss the capability of control charts with different sample sizes and different combination parameters of $q$ and $\omega$. Also, we display the demonstration of the proposed control chart to monitor process CV in the production process of NPK fertilizer.

2. Methodology

2.1. Control chart structure and log-normal transformation for CV statistic

This subsection describes the CV-GWMA control chart proposed by [11], one of the developed CV charts using the GWMA structure. Further, the CV statistic will be transformed with log-normal transformation suggested by [5].

2.1.1. CV-GWMA control chart

Let $\{Y_{1i}, Y_{2i}, \ldots, Y_{ni}\}$ denote sample random variable $Y_{ij}$ which normally distributed such that $Y_{ij} \sim N(\mu, \sigma)$, where $\mu_0$ is in-control mean and $\sigma_0$ is in-control for $i$ time. Then, the CV of the population ($\gamma$) is explained as $\gamma = \frac{\sigma}{\mu}$. Let the mean, SD, and CV of a random sample for $i$ time is explained as

$$\bar{Y}_i = \frac{1}{n} \sum_{j=1}^{n} Y_{ij}, \quad S_i = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (Y_{ij} - \bar{Y}_i)^2}, \quad W_i = \frac{S_i}{\bar{Y}_i}, \quad -\infty < \gamma \leq \infty. \quad (1)$$

Let $M$ denotes count the number of samples until an event A first occurs since a previous of event A occurs, $P_i = P(M = i) = \overline{P}_{i-1} - \overline{P}_i$ denotes the probability that event A occurs in the first $i$ sample and $\overline{P}_i = P(M > i)$ denotes the probability that the event A does not occur in the first $i$ samples

$$\sum_{i=1}^{\infty} P(M = m) = P(M = 1) + P(M = 2) + \cdots + P(M = i) + P(M > i) = 1. \quad (3)$$
Where \( P(M = 1) \) denotes a weight of current observation, \( P(M = 2) \) denotes a weight of previous observation, \( P(M = i) \) denotes a weight of the oldest observation, and \( P(M > i) \) denotes a weight for target value. The GWMA-type CV statistic for \( i \) time is explained as,

\[
G_i = P(M = 1)W_i + P(M = 2)W_{i-1} + \ldots + P(M = i)W_1 + P(M > i)G_0
\]

\[
G_i = (\overline{P}_0 - \overline{P}_1)W_i + (\overline{P}_1 - \overline{P}_2)W_{i-1} + \ldots + (\overline{P}_{i-1} - \overline{P}_i)W_1 + \overline{P}G_0.
\]

(4)

For easy computation, \( q^i \) is modified into \( \overline{P}_i \). Consequently, equation (2) can be written as

\[
G_i = (q^{i^0} - q^{i^w})W_i + (q^{i^w} - q^{i^w})W_{i-1} + \ldots + (q^{i^{(i-1)^w}} - q^{i^w})W_1 + q^{i^w}\mu_w
\]

(5)

\( q \) is defined as a design parameter in the interval \( 0 < q < 1 \). The \( \omega \) is a parameter determined by the practitioner and the target value is determined by \( G_0 = \mu_w \). The expected value of \( G_i \) can be computed as

\[
E(G_i) = E\left((\overline{P}_0 - \overline{P}_1)W_i + (\overline{P}_1 - \overline{P}_2)W_{i-1} + \ldots + (\overline{P}_{i-1} - \overline{P}_i)W_1 + \overline{P}G_0\right)
\]

\[
E(G_i) = E\left((q^{i^0} - q^{i^w})W_i + (q^{i^w} - q^{i^w})W_{i-1} + \ldots + (q^{i^{(i-1)^w}} - q^{i^w})W_1 + q^{i^w}\mu_w\right)
\]

(6)

and variance of \( G_i \),

\[
Var(G_i) = \left((\overline{P}_0 - \overline{P}_1)^2 + (\overline{P}_1 - \overline{P}_2)^2 + \ldots + (\overline{P}_{i-1} - \overline{P}_i)^2\right)\sigma_w^2
\]

\[
Var(G_i) = \left((q^{i^0} - q^{i^w})^2 + (q^{i^w} - q^{i^w})^2 + \ldots + (q^{i^{(i-1)^w}} - q^{i^w})^2\right)\sigma_w^2
\]

\[
Var(G_i) = Q\sigma_w^2
\]

(7)

where, \( Q = \sum_{k=1}^{i} (q^{k^{(k-1)^w}} - q^{k^w})^2, i \geq 1 \). The lower control limit (LCL), central limit (CL), and upper control limit (UCL) of the CV-GWMA control chart is calculated as

\[
LCL_i = \mu_w - L\sqrt{Q_i}\sigma_w
\]

\[
CL_i = \mu_w
\]

\[
UCL_i = \mu_w + L\sqrt{Q_i}\sigma_w
\]

(8)

(9)

(10)

where \( L \) explains the width of control limits. The \( \mu_w \) and \( \sigma_w^2 \) [15] calculated as

\[
\mu_w \approx \gamma \left(1 - \frac{0.25 - \gamma^2}{n}\right)
\]

\[
\sigma_w^2 \approx \frac{\gamma^2}{n} \left(0.5 + \frac{0.4375}{n} + \gamma^2 \left(1 + \frac{1+9\gamma^2}{n}\right) - (\mu_w - \gamma)^2\right)
\]

(11)

(12)
2.1.2. Log-normal transformation for CV statistic

Log-normal transformation is used to transform the distribution of CV into a normal distribution. Castagliola et al [5] suggested this transformation form since the CV sample has a unimodal right-skewed distribution, so its distribution may become a preferred choice to transform CV statistic. The transformation form for \( i \) time is explained as,

\[
Q = a + b \ln \left( \gamma_i - c \right),
\]

where, \( \gamma_i = \frac{S_i}{Y_i} \), \( a, b > 0 \) and \( c \) can be obtained depend on \( n \) and \( \left( \gamma_0 \right) \) of the process, such that \( T_i \sim N(0,1) \).

\[
b = \frac{F_N^{-1}(r)}{\ln \left( \frac{x_{0.5} - x_r}{x_{1-r} - x_{0.5}} \right)},
\]

\[
a = -b \ln \left( \frac{x_{0.5} - x_r}{1 - \exp \left( \frac{F_N^{-1}(r)}{b} \right)} \right),
\]

\[
c = x_{0.5} - e^{-a/b}.
\]

\( F_N^{-1}(r) \) is the quantile function of \( N(0,1) \). \( x_r = F^{-1}(r \mid \eta_i, \gamma_0) \) is the quantile function noncentral-t distribution with the parameter of non-centrality \( \gamma_0 \), degrees of freedom \( \eta_i \), and probability \( r \). \( x_{0.5} = F^{-1}(0.5 \mid \eta_i, \gamma_0) \) and \( x_{1-r} = F^{-1}(1-r \mid \eta_i, \gamma_0) \) are the quantile order of 0.5 and \( 1-r \). The value of \( r \) is determined by the practitioner in range \( 0.01 \leq r \leq 0.1 \).

2.2. Proposed AIB-GWMCV control chart

Let \( \left( Y_j, X_j \right) \) denote random sample with \( j = 1,2,\ldots,n \) as samples of size \( n \) at time \( i = 1,2,\ldots,m \) bivariate normally distributed, i.e. \( (Y, X) \sim N_2(\mu_Y, \mu_X, \sigma_Y, \sigma_X, \rho_{XY}) \) where \( \mu_Y, \sigma_Y \) are the process mean and process standard deviation variable of interest \( Y \) and \( \mu_X, \sigma_X \) are the process mean and process standard deviation auxiliary variable \( X \). While \( \rho_{XY} \) is the correlation coefficient between the study variable \( Y \) and auxiliary variable \( X \). A suitable selection of the auxiliary information is very important when working with the AIB control chart for the monitoring process. The auxiliary information is a variable that should be stable and does not change when a shift occurs in the study variable. If it occurs, that variable should be included in the list of study variables, and a multivariate control chart is more suitable for this condition [16]. Following Haq and Khoo [17], AIB difference estimators for CV is defined by

\[
M_{Y,i} = Q_{Y,i} - \rho_{XY}^* Q_{X,i},
\]

where,
\[ Q_{Y,i} = a + b \ln(Y_{i}), \quad Q_{X,i} = a + b \ln(X_{i}), \]
\[ \rho_{XY}^2 \text{ denotes the correlation of } Q_{Y,i} \text{ and } Q_{X,i}, \]
which is estimated by using Monte Carlo simulation.

The expected value and variance \( M_{Y,i} \) are defined as follows,
\[ E(M_{Y,i}) = 0, \quad V(M_{Y,i}) = \left(1 - \rho_{XY}^2\right), \] (18)

Now, \( M_{Y,i} \) is transformed into standard normal transformation defined in equation (20),
\[ M'_{Y,i} = \frac{M_{Y,i} - E(M_{Y,i})}{\sqrt{1 - \rho_{XY}^2}}, \] (20)

As a result, \( M'_{Y,i} \) is normally distributed, such that \( M'_{Y,i} \sim \mathcal{N}(0,1) \). The statistic of AIB-GWMCV for \( i \) time is defined as follows,
\[ A_i = \left(q^{\sigma^2} - q^v\right)M'_{Y,i} + \left(q^{v} - q^{w}\right)M'_{Y,i-1} + \cdots + \left(q^{v(i-1)} - q^v\right)M'_{Y,i} + q^v \mu_{M_i}, \] (21)

\[ q = 1 - \lambda \quad \text{ is design parameter in the interval } 0 < q < 1. \]

The expected value and variance of \( A_i \) can be computed as follows,
\[ E(A_i) = E\left(\left(q^{\sigma^2} - q^v\right)M'_{Y,i} + \left(q^{v} - q^{w}\right)M'_{Y,i-1} + \cdots + \left(q^{v(i-1)} - q^v\right)M'_{Y,i} + q^v \mu_{M_i}\right) \]
\[ E(A_i) = \mu_{M_i}, \] (22)

\[ V(Q_i) = \left(\left(q^{\sigma^2} - q^v\right)^2 + \left(q^{v} - q^{w}\right)^2 + \cdots + \left(q^{v(i-1)} - q^v\right)^2\right)\sigma_{M_i}^2 \]
\[ V(Q_i) = Z_i \sigma_{M_i}^2, \] (23)

supposing \( Z_i = \sum_{k=1}^{i} \left(q^{v(k-1)} - q^{v}\right)^2 \), \( i \geq 1 \), control limits are defined as follows
\[ LCL_i = \mu_{M_i} - L \sigma_{M_i} \sqrt{Z_i}, \] (24)
\[ CL_i = \mu_{M_i}, \] (25)
\[ UCL_i = \mu_{M_i} + L \sigma_{M_i} \sqrt{Z_i}, \] (26)

supposing \( L \) is the width of control limits, a process is considered out-of-control when the \( A_i \) value falls outside the range of control limits.
2.3. AIB-GWMCV control chart charting procedures

These steps are given below for executing the AIB-GWMCV control chart:

1. Consider the process parameters of CV \( \gamma_0 \) for the study variable \( Y \) and auxiliary variable \( X \). It can be also estimated from previous data sets or compute the sample of CV from the data set \( \gamma_0 \).

2. Compute \( a, b, c \) based on \( \gamma_0, n, r \) and width of control limits \( L \). \( L \) is determined such that \( ARL_0 = 370 \).

3. Compute \( Q_{Y,i} \) and \( Q_{X,i} \) by utilizing equation (18) for each time.

4. Compute \( M_{Y,i} \) by utilizing equation (17) for each time.

5. Transform \( M_{Y,i} \) into standard normal distribution by utilizing equation (20).

6. Compute \( A_i \) by utilizing equation (21) for each time.

7. Compute the mean and variance of \( A_i \) that used to determine control limits.

8. Compute LCL, CL, and UCL by utilizing equation (24), (25), (26), respectively.

9. Construct AIB-GWMCV chart by comparing statistic \( A_i \) and control limits.

10. When \( A_i \) values fall outside of the control limit value, then investigate the source of out-of-control signals. Thus, eliminate it to get the in-control process if it is detected as an assignable cause.

3. Result and Discussions

3.1 Simulation procedures

The performance of the proposed chart can be evaluated with ARL. ARL defines as an average number of points plotted until out-of-control signals are indicated. There are several numerical methods in the literature to measure ARL. We utilize Monte Carlo simulation to measure ARL that is categorized as \( ARL_0 \) and \( ARL_s \). When the process under in-control, the \( ARL_0 \) value should be larger to keep away from false alarms. While \( ARL_s \) values should be small indicating shifts are rapidly detected when out-of-control condition.

The algorithms to determine the ARL of the proposed control chart are given below:

1. Consider the shift of the process \( \tau = \frac{\gamma_1}{\gamma_0} \). \( \gamma_0 \) denotes CV under the in-control condition and \( \gamma_1 \) denotes CV under out-of-control conditions. We use \( \tau = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 2.0, 3.0, 4.0 \). Note that \( \tau = 1 \) denotes no shift in the process.

2. Compute \( a, b, c \) based on \( \gamma_0, n, r \) and width of control limits \( L \). \( L \) is determined such that \( ARL_0 = 370 \).

3. Generate 1,000 subgroups of random samples that bivariate normally distributed from a shift process with samples of size \( n \) and in-control parameters are \( \mu_y = \mu_x = \mu_0 = 10 \), \( \sigma_y = \sigma_x = \sigma_0 = 1 \) and \( \gamma_y = \gamma_x = \gamma_0 = 0.1 \). The coefficient correlation between the study variable and the auxiliary variable is considered such that \( \rho_{xy} = 0.3, 0.5, 0.7, 0.9 \).

4. Compute mean, standard deviation, and CV for each subgroup/time of variable.

5. Compute statistic \( Q_{Y,i} \) and \( Q_{X,i} \) by utilizing equation (18) for each subgroup/time, and compute statistic \( M_{Y,i} \) by utilizing equation (17).

6. Transform \( M_{Y,i} \) into standard normal distribution by utilizing equation (20).
7. Compute \( A_i \) by utilizing equation (21) for each subgroup/time.
8. Compute the expected value and the variance of \( A_i \) that used for control limits.
9. Compute LCL, CL, and UCL defined in equation (24), (25), and (26).
10. Construct AIB-GWMCV chart by comparing statistic \( A_i \) and control limits. When the process is considered out-of-control, record numbers of subgroups as run length.
11. Repeat the whole process 1,000 times to get \( ARL_0 \) and \( ARL_s \).

3.2. Performance Evaluation of proposed control chart

We evaluate the performance of the control chart by measuring ARL. The smaller \( ARL \) values, the better performance of the control chart. Here, we consider sample sizes \( n = 5, 12 \) by using two combination parameters \( q \) and \( \omega \), i.e. \((0.75,0.8)\) and \((0.9,0.9)\) at fixed \( ARL_0 = 370 \). Moreover, we compute the ARL value for the AIB-GWMCV chart at some different correlation levels. We used different correlation levels such that \( \rho_{XY} = 0.0, 0.3, 0.5, 0.7, 0.9 \). Note that for correlation level \( \rho_{XY} = 0 \) represents the CV-GWMA control chart using log-normal transformation without auxiliary variable. Table 1 displayed the parametric values \( a, b, c \) for transformed statistics and the constant value of \( L \) for the AIB-GWMCV control chart based on \( \gamma_0 = 0.1 \) \( q, \omega \), and \( n \).

Table 2 and Table 3 report the average run length (ARL) comparison of the AIB-GWMCV chart for different correlation levels and sample sizes. Table 2 presents the performance of the proposed control chart and CV-GWMA control chart using log-normal transformation without auxiliary variable by using combination parameters \( q = 0.75 \) and \( \omega = 0.8 \). According to Table 2, it is concluded that for sample sizes \( n = 5 \), the proposed control chart is more efficient in detecting small to large shifts at low to high correlation levels than the CV-GWMA control chart using log-normal transformation without auxiliary variable. For example, at the shift \( \tau = 1.1 \), the CV-GWMA control chart using log-normal transformation without auxiliary variable has \( ARL_0 \) equal to 178.840. Meanwhile, the \( ARL_0 \) of proposed control charts with correlation levels \( \rho_{XY} = 0.3, 0.5, 0.7, 0.9 \) are 154.356, 150.154, 146.100, 76.472, respectively.

| \((q, \omega)\) | \(n = 5\)                  | \(n = 12\)                 |
|---------------|-----------------------------|-----------------------------|
|               | \(a = 9.8864, b = 6.8112, c = -0.1426\) | \(a = 15.412, b = 9.9859, c = -0.1167\) |
| \(\rho\)      | 0.0                         | 0.0                         |
|               | 0.3                         | 0.0                         |
|               | 0.5                         | 0.3                         |
|               | 0.7                         | 0.5                         |
|               | 0.9                         | 0.7                         |
| (0.75,0.8)    | 3.031                       | 3.021                       |
| (0.9,0.9)     | 2.89                        | 2.91                        |

For sample sizes \( n = 12 \), Also the proposed control chart demonstrates a better performance to detect small to large shifts at low to high correlation levels than the CV-GWMA control chart using log-normal transformation without auxiliary variable. For example, at the same shift, the \( ARL_0 \) for CV-GWMA control chart using transformation log-normal without auxiliary variable is 63.694. Meanwhile, the \( ARL_0 \) for the proposed control chart with correlation levels \( \rho_{XY} = 0.3, 0.5, 0.7, 0.9 \) are 62.070, 59.775, 51.164, 23.732, respectively. From the results, it indicates that an increase in sample sizes cause early detection of the proposed control chart at low to high correlation levels.
Table 3 provides the performance of the proposed control chart and CV-GWMA control chart using log-normal transformation without auxiliary variable with \( n = 5 \) and \( n = 12 \) for the same value of combination parameter \( q = 0.9 \) and \( \omega = 0.9 \) at correlation levels \( \rho_{XY} = 0.3, 0.5, 0.7, 0.9 \). From Table 3, it is implied that the increase in sample size will significantly increase the performance of the control chart in detecting shifts of process CV. For example, at shift 1.4 and sample of sizes \( n = 5 \), the \( ARL_s \) for CV-GWMA control chart using log-normal transformation without auxiliary variable is 13.680. Whereas, the \( ARL_s \) for the proposed control chart with correlation levels \( \rho_{XY} = 0.3, 0.5, 0.7, 0.9 \) are 12.592, 11.750, 9.922, 5.920, respectively. Also, the proposed chart with sample sizes \( n = 12 \) is more sensitive than the CV-GWMA control chart using log-normal transformation without auxiliary variable. It can be known at the same shift, the \( ARL_s \) for CV-GWMA control chart using transformation log-normal without auxiliary variable is 5,046. Whereas, the \( ARL_s \) for the proposed control chart with correlation levels \( \rho_{XY} = 0.3, 0.5, 0.7, 0.9 \) are 4.870, 4.764, 3.914, 2.276, respectively.

From these simulation studies, we can conclude that as \( \rho_{XY} \) increases from low to high correlation levels, the performance of the proposed control chart gets better and better, as compared to the CV-GWMA control chart using log-normal transformation without auxiliary variable. Furthermore, larger \( q \) enhances the sensitivity of the AIB-GWMCV control chart. A value of \( \lambda \) in the interval \( 0.05 \leq q \leq 0.25 \) is the practical approach [1]. It is equal to the value of \( q \) in the interval \( 0.75 \leq q \leq 0.95 \).

Table 2. ARL values of AIB-GWMCV for \( q = 0.75 \) and \( \omega = 0.8 \) at fixed \( ARL_0 = 370 \)

| Shift (\( \tau \)) | Sample Size (\( n \)) | \( \rho_{XY} \) | 0.0 | 0.3 | 0.5 | 0.7 | 0.9 |
|-------------------|------------------------|----------------|-----|-----|-----|-----|-----|
| 1.0               | 5                      | 375.668        | 382.996 | 376.568 | 369.450 | 372.588 |
|                   | 12                     | 371.783        | 366.648 | 372.566 | 370.480 | 370.250 |
| 1.1               | 5                      | 178.840        | 154.356 | 150.154 | 146.100 | 76.472 |
|                   | 12                     | 63.694         | 62.070 | 59.774 | 51.164 | 23.732 |
| 1.2               | 5                      | 51.960         | 51.416 | 48.592 | 40.456 | 19.822 |
|                   | 12                     | 17.240         | 16.852 | 15.566 | 13.516 | 6.486 |
| 1.3               | 5                      | 27.194         | 26.818 | 23.106 | 21.034 | 10.528 |
|                   | 12                     | 8.466          | 8.462 | 8.450 | 8.886 | 3.766 |
| 1.4               | 5                      | 18.098         | 15.906 | 14.564 | 12.242 | 6.810 |
|                   | 12                     | 5.680          | 5.666 | 5.438 | 4.540 | 2.504 |
| 1.5               | 5                      | 11.752         | 1.762 | 10.406 | 8.804 | 5.054 |
|                   | 12                     | 3.992          | 3.954 | 3.926 | 3.356 | 1.902 |
| 2.0               | 5                      | 5.118          | 4.784 | 4.474 | 3.872 | 2.598 |
|                   | 12                     | 1.918          | 1.906 | 1.900 | 1.868 | 1.160 |
| 3.0               | 5                      | 2.912          | 2.596 | 2.456 | 2.288 | 1.662 |
|                   | 12                     | 1.228          | 1.218 | 1.210 | 1.128 | 1.006 |
| 4.0               | 5                      | 2.210          | 2.132 | 2.032 | 1.894 | 1.512 |
|                   | 12                     | 1.070          | 1.070 | 1.048 | 1.036 | 1.002 |
Table 3. ARL values of AIB-GWMCV for $q = 0.9$ and $\omega = 0.9$ at fixed $ARL_0 = 370$

| Shift $\tau$ | Sample Size $n$ | $\rho_{XY}$ | 0.0  | 0.3  | 0.5  | 0.7  | 0.9  |
|--------------|-----------------|-------------|------|------|------|------|------|
| 1.0          | 5               | 367.260     | 369.038 | 374.432 | 368.504 | 372.624 |
|              | 12              | 369.397     | 369.622 | 379.702 | 367.613 | 370.052 |
| 1.1          | 5               | 120.326     | 111.484 | 104.164 | 88.530  | 46.786 |
|              | 12              | 45.874      | 39.022  | 38.004  | 31.676  | 14.968 |
| 1.2          | 5               | 35.698      | 34.222  | 32.800  | 27.862  | 15.224 |
|              | 12              | 13.690      | 12.962  | 11.704  | 11.146  | 5.384  |
| 1.3          | 5               | 19.392      | 18.338  | 17.004  | 15.254  | 8.652  |
|              | 12              | 7.298       | 7.060   | 6.610   | 6.034   | 3.152  |
| 1.4          | 5               | 13.680      | 12.592  | 11.750  | 9.922   | 5.920  |
|              | 12              | 5.046       | 4.870   | 4.764   | 3.914   | 2.276  |
| 1.5          | 5               | 9.528       | 9.494   | 11.782  | 7.438   | 4.664  |
|              | 12              | 3.976       | 3.522   | 3.432   | 3.014   | 1.792  |
| 2.0          | 5               | 4.410       | 4.156   | 4.101   | 3.656   | 2.418  |
|              | 12              | 1.888       | 1.894   | 1.820   | 1.576   | 1.146  |
| 3.0          | 5               | 2.494       | 2.392   | 2.226   | 2.090   | 1.600  |
|              | 12              | 1.208       | 1.200   | 1.152   | 1.094   | 1.006  |
| 4.0          | 5               | 2.040       | 1.968   | 1.906   | 1.794   | 1.412  |
|              | 12              | 1.080       | 1.056   | 1.044   | 1.018   | 1.002  |

3.3 An illustrative example

This section presents an illustrative example using real data set to show the application of the AIB-GWMCV control chart for monitoring the NPK fertilizer production process. We consider nitrogen as the study variable ($Y$) and phosphorus as an auxiliary variable ($X$). These variables consist of 27 subgroups with samples of size $n = 5$, respectively. Assuming $\gamma_0 = 0.1$ for an in-control process, the coefficient $a, b$ and $c$ are calculated such that $a = 9.8864$, $b = 6.8112$, and $c = -0.1426$. The main quality characteristic ($Y$) is normally distributed with parameters $\mu_Y = 14.157$, $\sigma_Y = 1.186$, $\gamma_Y = 0.08$ and rounded to $\gamma_0 = 0.1$. Also, the auxiliary variable ($X$) is normally distributed with parameters $\mu_X = 15.301$, $\sigma_X = 1.754$, $\gamma_X = 0.11$. It is known that $\rho_{XY} = 0.3$. We also consider combination parameters $q = 0.9$ and $\omega = 0.9$ for this application.

![Figure 1. CV-GWMA control chart using log-normal transformation without auxiliary variable](image-url)
According to Figure 1, it can be seen that the CV-GWMA control chart using log-normal transformation without auxiliary variable detects an out-of-control process at 25th observation. Meanwhile, based on figure 2, it is inferred that process is stated out-of-control at 7th observation when utilizing the auxiliary variable for monitoring process CV. According to these results, the proposed control chart can detect earlier out-of-control signals than the CV-GWMA control chart using log-normal transformation without auxiliary variable. Thus, based on Monte Carlo simulation and real application, the proposed control chart has better performance than the CV-GWMA control chart using log-normal transformation without auxiliary variable.

![Figure 2. AIB-GWMCV control chart](image)

4. Conclusion

We have proposed an AIB-GWMCV control chart to monitor the process CV. According to performance evaluation results, by utilizing the auxiliary variable, the detection ability of the control chart increases for each shift of process CV in all considered combination parameters $q$ and $\omega$. Furthermore, the detection ability of the proposed chart increases with an increase in levels of $\rho$. For small sample sizes, the proposed control chart quickly loses its ability to detect shifts, particularly at low correlation levels compared to large sample sizes. The application of the proposed control chart using a real data set supports the simulation results. Consequently, compared to the CV-GWMA control chart using log-normal transformation without auxiliary information, the AIB-GWMCV control chart has better performance in terms of faster detection of shifts process CV. Using more than one auxiliary variable in the proposed chart can be investigated in future research. Also, some auxiliary information estimators can be evaluated.

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