Magnetic charge superselection in the deconfined phase of Yang-Mills theory

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The vacuum expectation value of an operator carrying magnetic charge is studied numerically for temperatures above the deconfinement temperature in SU(2) and SU(3) gauge theory. By analyzing its finite size behaviour, this is found to be exactly zero in the thermodynamical limit for any $T > T_c$ whenever the magnetic charge of the operator is different from zero. These results show that magnetic charge is superselected in the hot phase of quenched QCD.

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I. INTRODUCTION

Color confinement is one of the most elusive phenomena in QCD. A natural working hypothesis is that confinement is the consequence of the breaking of some symmetry. A possible candidate is the symmetry related to Abelian magnetic charge conservation \cite{1}. The underlying idea is that a description of the theory exists in terms of dual fields, which have non-zero magnetic charge and are weakly coupled in the confined phase. This fits the idea of dual superconductivity of the vacuum as the mechanism of color confinement \cite{2, 3}.

With this scenario in mind, an operator $\mu$ carrying magnetic charge has been constructed in terms of the fundamental variables of the theory \cite{4, 5, 6, 7} for several Abelian projections and its vacuum expectation value $\langle \mu \rangle$ has been studied across the deconfinement phase transition in SU(2) \cite{8} and SU(3) \cite{9} gauge theories by lattice calculations. It has been shown that the magnetic symmetry is broken in the confined phase and implemented \`a la Wigner in the deconfined phase. For the quenched theory the confined and the deconfined phase are defined in terms of the string tension or in terms of the vacuum expectation value of the Polyakov line. In the critical region below the deconfinement temperature $T_c$, $\langle \mu \rangle$ scales with the critical indices of the transition. This shows the interconnection between confinement-deconfinement and breaking-restoring of the magnetic symmetry. $\langle \mu \rangle$ is indeed a disorder parameter for the deconfining phase transition. This analysis has been extended to full QCD with similar results \cite{10, 11} and an extension to SU($N$) is on the way \cite{12}. In the full QCD case the Polyakov line is not an order parameter and the transition is indicated by a drop of the chiral condensate, which is not either an order parameter at non-zero quark mass, so that $\langle \mu \rangle$ is the only available (dis)order parameter and could provide a genuine criterion for confinement.

It has been found numerically \cite{8, 9, 12} and then supported by analytical arguments \cite{14} that the behaviour of $\langle \mu \rangle$ is independent of the Abelian projection, implying that dual superconductivity is an intrinsic property of the confining vacuum.

While $\langle \mu \rangle$ has been studied in great detail in the confined (low temperature) region up to the critical temperature $T_c$, several aspects have still to be explored regarding the behaviour of $\langle \mu \rangle$ in the deconfined (high temperature) region. In this regime the magnetic symmetry is implemented \`a la Wigner, so that the Hilbert space of the theory is split into mutually orthogonal sectors labelled by the value of the magnetic charge (superselection). As a consequence, in the thermodynamic limit $\langle \mu \rangle = 0$ exactly for any $T > T_c$ if $\mu$ carries a net magnetic charge that is different from zero. While it has already been found \cite{8, 9, 12} that, in the case where $\mu$ creates a single magnetic monopole, $\langle \mu \rangle$ goes to zero exponentially as a function of the lattice size (i.e. in the thermodynamical limit) in the extreme weak coupling (infinite temperature) limit, a detailed finite size study of $\langle \mu \rangle$ in the whole deconfined region down to $T_c$ is still lacking. Such a study would reveal whether $\langle \mu \rangle$ is indeed strictly zero for any $T > T_c$. Moreover, a more extensive study performed on operators creating various monopole configurations with either zero or non-zero net magnetic charge would clarify if the observed phenomenon is really a manifestation of magnetic charge superselection in the deconfined phase of Yang-Mills theories.

It is the purpose of the present work to give an answer to those open questions. We do that both for SU(2) and SU(3) pure gauge theories.

The rest of the paper is organised as follows. In Sect. \ref{sec:technical} we give the technical details of our lattice simulations. In Sect. \ref{sec:results} we discuss our results for SU(3) and SU(2) gauge theory. Finally, Sect. \ref{sec:conclusions} summarizes our conclusions.
II. THE CALCULATION

On the lattice, the disorder parameter is defined as
\[ \langle \mu \rangle = \frac{\int (DU) e^{-\beta S_M}}{\int (DU) e^{-\beta S}} , \]
(1)

where \( S \) is the Wilson action and \( S_M \) is the “monopole” action, obtained from \( S \) by modifying the temporal plaquette at time zero by the procedure developed in Refs. [6, 8, 9]. By this procedure a single monopole can be created or a generic configuration of monopoles and antimonopoles.

Note that, because of the abelian projection independence of \( \langle \mu \rangle \), there is no need to perform explicitly an Abelian projection as done in Refs. [8, 9, 12]. To use the terminology introduced in [13], we will be studying \( \langle \mu \rangle \) in a “random projection”.

In order to study the dependence on the total magnetic charge, we have computed \( \langle \mu \rangle \) in the extreme weak coupling for monopoles of different charges and for dipole field configurations obtained by putting two monopoles with opposite magnetic charge at distance \( d \). We have also investigated “double monopole” configurations obtained by putting two monopoles of equal charge at distance \( d \).

If confinement is related to magnetic charge symmetry as discussed above, we expect that, in the deconfined region, \( \langle \mu \rangle = 0 \) in the infinite volume limit whenever the net magnetic charge is non-zero and that it can stay different from zero if the net magnetic charge is zero.

Instead of \( \langle \mu \rangle \), we computed numerically
\[ \rho = \frac{d}{d\beta} \log \langle \mu \rangle = \langle S \rangle_S - \langle S_M \rangle_{S_M} , \]
(2)

where the subscript indicates the action with respect to which the average has been taken. \( \rho \) carries the same physical information as \( \langle \mu \rangle \) and is much easier to determine in Monte Carlo simulations [5, 6, 7]. The value of \( \langle \mu \rangle \) is related to \( \rho \) by the relationship
\[ \langle \mu \rangle = \exp \left( \int_{\beta}^{\beta'} \rho(\beta')d\beta' \right) . \]
(3)

As it is clear from Eq. 3, \( \lim_{N_s \to \infty} \langle \mu \rangle = 0 \) in the whole deconfined region \( (T > T_c, \beta > \beta_c) \) if \( \lim_{N_s \to \infty} \rho = -\infty \) for every \( \beta > \beta_c \).

In our investigation, we used lattices of sizes \( N_s^3 \times 4 \) with \( N_s \) ranging from 16 to 48. Measurements have been performed sampling over 5000-10000 configurations. The statistical errors are shown in the figures. Other technical aspects of our simulations are similar to those described in [8, 9].

In the following section we shall discuss in details our numerical results for SU(3) and SU(2).

![Fig. 1](image1.png)

FIG. 1: \( \rho \) parameter in the weak coupling limit as a function of the spatial lattice size for different values of the whole magnetic charge in SU(3) gauge theory. Open circles refer to a single monopole of charge 2, filled circles refer to a dipole of zero net magnetic charge, filled triangles refer to a single monopole of charge 4, open triangles refer to two monopoles of charge 2 put at a distance of 2 lattice spacings apart from each other, filled squares refer to two monopoles of opposite charge but different kind (\( \lambda_3 \) and \( \lambda_3' \) generators) put at a distance of 2 lattice spacings apart from each other.

![Fig. 2](image2.png)

FIG. 2: \( \rho \) parameter in the weak coupling limit as a function of the spatial lattice size for different values of the whole magnetic charge in SU(2) gauge theory. Bullets refer to a single monopole of charge 2, squares refer to a dipole of zero net magnetic charge, diamonds refer to a single monopole of charge 4, triangles refer to two monopoles of charge 2 put at a distance of 2 lattice spacings apart from each other.
III. NUMERICAL RESULTS

Let us first discuss the results obtained in the extreme weak coupling limit for different operators $\mu$ creating different monopole configurations. In Fig. 1 and in Fig. 2 we show, respectively for SU(3) and SU(2) pure gauge theories, results obtained for $\rho$ as a function of $N_s$ in the limit $\beta \to \infty$ for different monopole configurations. As already explained in Refs. [8, 9], in order to perform this limit one has to find the absolute minima taken over the ensemble of gauge configurations for both $S$ and $S_M$: the minimum of $S$ is trivially obtained on the configuration with all the links equal to one, while for $S_M$ we have applied a simulated annealing procedure till a very high value of $\beta$ followed by straight cooling protracted till the value of the minimum stabilizes.

As it is clearly visible from the figures, $\rho(\beta = \infty)$ diverges linearly with $N_s$ whenever the net magnetic charge is different from zero and it reaches a constant value as a function of $N_s$ otherwise [15]. It is interesting to notice that the two cases of a single monopole of charge 4 and...
of 2 monopoles of charge 2 at distance $d$ from each other lead to the same $\rho$ apart from a $N_s$-independent correction which is most likely due to the interaction between the two monopoles.

We have also studied the finite size behaviour of $\rho$ in the whole deconfined region for the case of a single monopole of magnetic charge 2. Let us first discuss the case of SU(3). In Fig. 4 we show the behaviour of $\rho/N_s$ as a function of $\beta$ for different values of $N_s$, with $\beta_c \approx 5.69$ for $N_s = 4$. In particular we have verified that, very close to $\beta_c$, $\rho$ scales approximately as $N_s^2$, plus small corrections proportional to lower powers of $N_s$: this is apparent from Fig. 5, where the values of $\rho/N_s$ obtained for different values of $N_s$ fall exactly on top of each other when plotted as a function of $\beta$. This is consistent with the behaviour of $\rho$ on the other side of the phase transition, where it also diverges as $N_s^{1/\nu}$, with $\nu = 1/3$ (first order phase transition) for SU(3).

The finite size behaviour of $\rho$ for SU(2) is shown in Fig. 6. Again for $\beta > \beta_c \approx 2.29$ the scaling is well described by the ansatz $\rho \approx c N_s^{1/\nu}$ (see Fig. 7), where $\nu \approx 0.63$, i.e. the value for the 3d Ising model [8].

IV. CONCLUSIONS

We have investigated the finite size scaling behaviour of $\rho = \frac{d}{ds} \log \langle \mu \rangle$ in the deconfined region of both SU(2) and SU(3) pure gauge theories. We have shown that, whenever the magnetic charge created by $\mu$ is different from zero, $\rho$ diverges to $-\infty$ for $T > T_c$ as the spatial volume is increased, i.e. that $\langle \mu \rangle = 0$ in the thermodynamical limit. This supports the expectation that in the deconfined phase of Yang-Mills gauge theories magnetic charge is superselected. In the case where $\mu$ creates a magnetic monopole, we have shown that $\rho$ diverges linearly in the weak coupling limit, and more and more rapidly as $T \to T_c$ from above, where it diverges as $N_s^{1/\nu}$, thus proving that $\langle \mu \rangle$ is strictly zero for every temperature $T > T_c$. The results presented here complete the argument of [8, 9, 13] that $\langle \mu \rangle$ is a disorder parameter for the deconfinement phase transition.

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[15] One has to be careful in studying the infinite volume limit of $\rho$. The overall signal is obtained by integrating, over larger and larger volumes as $N_s$ increases, a signal density which fades out as the distance from the monopole (or dipole) center increases: so it happens that the finite precision used in the simulation can artificially wash out an otherwise linear behaviour with $N_s$, and one as to increase the precision to recover the correct result. This has been taken care of by performing the computation on large lattices with increased precision (single to double).