Failure Test Optimization Method of Silicon Pressure Sensor Based on Zero Failure Data

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Abstract. With the increasing reliability of the silicon pressure sensor, the phenomenon of zero-failure data often appears in the reliability test. This paper studies the optimal confidence limit method and distribution curve method to obtain the confidence coefficient-α and the constant-c (c>1) for the reliability analysis of zero-failure data of silicon pressure sensor, subsequently, the two methods were tested in different groups, by analyzing the influence of the number of test groups on the prediction analysis results, the optimal number of groups for the reliability prediction test of the silicon pressure sensor was obtained. The simulation results show that the best prediction results can be obtained when the confidence coefficient-α ∈ [0.3,0.6], c ∈ [4-6] and the number of groups is 10 to 13. This method improves the accuracy of failure rate estimation and reliability test efficiency with limited samples.

1. Introduction

With the development of technology, the requirement of component reliability is higher and higher. Nowadays, due to the high reliability of components, the traditional reliability evaluation method is no longer applicable to predict their reliability [1]. For the general equipment reliability analysis problem based on Weibull distribution, the least square method can be adopted, assuming that shape parameters remain unchanged and scale parameters change with the characteristics of equipment type, to provide two-parameter Weibull distribution parameter estimation [2].

The characteristic of Bayes method is to make full use of the prior information, but sometimes the prior information is not easy to obtain. For this kind of problem, the prior information can be expressed as the interval estimation of the reliability function [3-4]. In the absence of failure data, the confidence interval of equipment reliability is difficult to estimate. The confidence interval of equipment reliability can be obtained by combining bayes model and variance propagation technique [5]. Similarly, for system reliability, a dynamic Bayes estimation method can be used: take the predicted results as the prior knowledge of the reliability in the next stage, and use the maximum entropy method to determine the prior distribution of the system reliability [6]. In order to solve the challenge of fault modeling and prediction caused by zero-failure data, statistical prediction program can be developed in combination with Bayes model to provide the point estimation of the cumulative number of faults in the future period, and the impact of different assumptions on the failure time can also be evaluated [7-10].

Later, regarding the reliability prediction of zero-failure data, some scholars put forward innovative methods such as E-Bayes (Expected Bayesian Method) [11], for the aerobearing whose life obeys Weibull distribution, E-Bayes method is used to study the influence of different parameter values and number of different test groups on the prediction accuracy, and the optimal data for the reliability
analysis of aerobearing is finally determined, but the life obeying exponential distribution is not considered [12]. This paper focuses on the study of silicon pressure sensors whose life follows exponential distribution, discuss the parameter estimates of the optimal confidence limit method and the E-Bayes-based distribution curve method to select a suitable method for the analysis of silicon pressure sensors, the appropriate number of test groups can be obtained by testing the above two methods, and the suggestions for reliability evaluation of silicon pressure sensors were given.

2. Related Work

Failure rate is one of the main prediction targets in product reliability analysis. Product life is basically subject to index, normal, weibull distribution, etc. Here, products subject to exponential distribution are discussed and analyzed. About the exponential distribution, the distribution function \( F(t) \), the probability density function \( f(t) \), reliability function \( R(t) \), failure rate function \( \lambda(t) \) as shown in the following type

\[
F(t) = \begin{cases} 
0, & t \leq 0, \\
1 - e^{-\lambda t}, & t > 0. 
\end{cases} 
\]  

(1)

\[
f(t) = \lambda e^{-\lambda t} 
\]  

(2)

\[
R(t) = 1 - F(t) = e^{-\lambda t}, \ t > 0 
\]  

(3)

\[
\lambda(t) = \frac{f(t)}{R(t)} = \lambda 
\]  

(4)

The \( \lambda \) is the failure rate of the product, and \( \lambda > 0 \). Figure 1 shows that the probability density curve becomes steeper and steeper with the increase of \( \lambda \).

![Figure 1 Exponential probability density](image)

The timing truncation test with a total number of \( m \) is carried out for the product, where the truncation time in the ith (\( i=1, 2, \ldots, m \)) timing truncation test is \( T_i \) (\( T_1 < T_2 < \cdots < T_m \)), and the number of corresponding test samples in this group is \( n_i \). If no samples fail at the end of the test, then \( (n_i, T_i) \) is the zero-failure data [5].

The design life of silicon pressure sensors in the current market is about 50 years. Based on this, we select 6 groups of medium difference products with failure efficiency \( \lambda = 1.5 \times 10^{-6} \) to \( 2.5 \times 10^{-6} \) for simulation analysis, and obtain the optimal grouping method for life test of silicon pressure sensors within this range. First, Monte Carlo method was used to generate 400 random numbers with failure rates subject to exponential distribution, and the generated random numbers were arranged from small to large in an orderly manner. Every 5 data was a group, and the first 20 Numbers were selected. The smallest number in each group was reserved as a decimal as the timing truncation time \( T_i \), and it was considered that the product would never fail at the truncation time. The six groups of timing truncation time are shown in Table 1 below.
Table 1. Timing cut-off time under different failure rates.

| λ_1=1.5×10^{-6} | λ_1=1.7×10^{-6} | λ_1=1.9×10^{-6} | λ_1=2.1×10^{-6} | λ_1=2.3×10^{-6} | λ_1=2.5×10^{-6} |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 613.4           | 541.2           | 267.6           | 1163.2          | 1371.6          | 203.4           |
| 11659.1         | 10287.4         | 5868.6          | 5780.3          | 11096.2         | 4460.1          |
| 19898.9         | 17557.8         | 16764.5         | 13164.2         | 16152.8         | 12741.0         |
| 27567.2         | 24324.0         | 30541.6         | 20961.4         | 24497.6         | 23211.6         |
| 33052.3         | 29163.8         | 36582.7         | 29636.2         | 27569.9         | 27802.8         |
| 41958.8         | 37022.4         | 46183.4         | 32636.2         | 4460.1          | 42817.6         |
| 613.4           | 541.2           | 267.6           | 1163.2          | 1371.6          | 203.4           |
| 11659.1         | 10287.4         | 5868.6          | 5780.3          | 11096.2         | 4460.1          |
| 19898.9         | 17557.8         | 16764.5         | 13164.2         | 16152.8         | 12741.0         |
| 27567.2         | 24324.0         | 30541.6         | 20961.4         | 24497.6         | 23211.6         |
| 33052.3         | 29163.8         | 36582.7         | 29636.2         | 27569.9         | 27802.8         |

With the progress of the test, the probability of product failure gradually increases, so it is assumed that n decreases successively, and there are s products that fail at the time of T. The s = n + n + ... + n, assuming 210 products were tested. Group number and partial truncation time are shown in Table 2.

Table 2. Zero failure data grouping.

| Number of clusters | n_i  | s_i  | Number of clusters | n_i  | s_i  |
|-------------------|------|------|-------------------|------|------|
| 1                 | 20   | 210  | 1                 | 10   | 55   |
| 2                 | 19   | 190  | 2                 | 12   | 9    |
| 3                 | 18   | 171  | 3                 | 13   | 8    |
| 4                 | 17   | 153  | 4                 | 14   | 7    |
| 5                 | 16   | 136  | 5                 | 15   | 6    |
| 6                 | 15   | 120  | 6                 | 16   | 5    |
| 7                 | 14   | 105  | 7                 | 17   | 4    |
| 8                 | 13   | 91   | 8                 | 18   | 3    |
| 9                 | 12   | 78   | 9                 | 19   | 2    |
| 10                | 11   | 66   | 10                | 20   | 1    |

Confidence coefficient for the reliability analysis of device zero-failure data is obtained by calculating the classical confidence upper limit $\lambda_{CU}$ and MTBF at different confidence levels using the optimal confidence limit method, given by equation (5), where MTBF, the mean time between failures, is equal to the inverse of the failure rate when the product life obeying the exponential distribution, that is: $MTBF = 1/\lambda$. The formula for calculating the relative error of the failure rate obtained by the classical confidence upper limit $\lambda_{CU}$ is given by equation (6). Then, the optimal grouping number is discussed based on the optimal confidence coefficient $\delta_{CU}$.

$$\lambda_{CU} = -\ln \alpha / T, MTBF = 1 / \lambda_{CU}$$

$$\delta_{CU} = |(\lambda_{CU} - \lambda) / \lambda|$$

When using the distribution curve method, the optimal constant $c$ is obtained first, and then the optimal grouping of the test is discussed. For the product whose life is exponentially distributed, its failure rate $\lambda$ least squares estimate is given by equation (7).
\[ \hat{\lambda} = \frac{- \sum_{i=1}^{m} T_i \ln(1 - \hat{p}_i))}{\sum_{i=1}^{m} T_i^2} \]  

(7)

In equation (7), \( \hat{p}_i \) is the E-Bayes estimation of \( p_i \). Assuming that the failure probability of the test product at the time of \( T_i \) is \( p_i \). And the multi-layer prior density function \( \pi(p_i) \) is given by equation (8), in the case of square loss, the E-Bayes estimation of \( p_i \) is given by equation (9). Where 0 < \( p_i \) < 1, \( b \) is the super parameter, and 1 < \( b \) < \( c \) (c > 1, c is a constant), the prior distribution of \( b \) is the uniform distribution on (1, c). The value of \( c \) is 2 to 9 [12].

\[ \pi(p_i) = \frac{1}{c-1} \int_{c}^{b} (1 - p_i)^{b-1} db \]  

(8)

\[ \hat{p}_i = \frac{1}{c-1} \ln\left(\frac{s_i + c + 1}{s_i + 2}\right) \]  

(9)

3. Failure rate prediction analysis of different groups

Firstly, when using the optimal confidence limit method, the confidence coefficient \( \lambda \) is changed by using equation (5) and equation (6) under the condition that the number of groups ranges from 1 to 20. The relative error \( \alpha \) under different failure rates was calculated, and the results of partial grouping (groups 10, 13 and 15) were shown in the Figure 2.

![Figure 2](image)

(a) Relative error of the predicted failure rate with different confidence (\( X = 10 \)). (b) Relative error of the predicted failure rate with different confidence (\( X = 13 \)). (c) Relative error of the predicted failure rate with different confidence (\( X = 15 \)).

As can be seen from Figure 2, on the whole, the relative error of the predicted value of failure efficiency decreases first and then increases, and the value of optimal confidence coefficient \( \lambda \)
decreases with the increase of the number of test groups $X$. And for the same test group, the higher the failure rate is, the smaller the optimal confidence coefficient $\lambda_B$ is. By comparing Figure 2 (a) with Figure 2 (b), it can be seen that if the number of test groups decreases, the prediction error will increase. At the same time, it can also be seen from Figure 2 (c) that for the optimal confidence limit method, when the confidence coefficient value slightly deviates from the optimal value $\lambda_B$, the predicted value will show a significant deviation. Therefore, we believe that the test data should not be too much, or the phenomenon of "rush" will occur.

Subsequently, on the basis of the optimal confidence coefficient $\lambda_B$ for each failure efficiency obtained in the above test, the test group was changed from group 1 to group 20 according to equation (5) and equation (6), and the relative error of the failure efficiency $\delta_{CU}$ was determined (the relative error of each failure efficiency $\delta_i$ in Table 3 is $\delta_{CUi}$), and the optimal predicted value was obtained as shown in Table 3.

Table 3 The optimal value of each failure rate prediction under different test groups.

| $X$ | $\alpha_{B10}$ | $\alpha_{B11}$ | $\alpha_{B15}$ |
|-----|----------------|----------------|----------------|
| 10  | 0.0974151      | 0.0805728      | 0.0094032      |
| 13  | 0.0974135      | 0.0805718      | 0.0094037      |
| 15  | 0.1050646      | 0.0703409      | 0.0094032      |
| 18  | 0.0623902      | 0.0170986      | 0.0477619      |
| 20  | 0.0011957      | 0.0616408      | 0.0592041      |

According to Table 3, generally speaking, with the increase of test samples, the prediction accuracy is higher, with the increase of failure rate, the number of corresponding optimal test groups should also increase. At the same time, on the basis of considering the test cost, the test consumption should be minimized within the allowable engineering error. It can be seen from the above data that when the test groups exceed 10, the relative error is basically controlled within 1%, meeting the engineering requirements. Therefore, in the case of limited test samples, the number of experimental groups can be selected from 10 to 15. The parameter $\lambda_B$ in the optimal confidence limit method is from 0.3 to 0.6, and the larger the failure rate is, the smaller the value of $\lambda_B$ is correspondingly.

Finally, in combination with equation (7) and equation (9), the distribution curve method is used to verify the reliability prediction feasibility of products under the above grouping conditions. In engineering applications, the value of $c$ is determined according to specific products, $c$ here take from 2 to 9. The number of test groups were 10, 13 and 15, respectively. The failure efficiency was estimated by least square method in equation (7), and the results are shown in Figure 3.

As shown in Figure 3, with the increase of the number of test samples, the constant $c$ applicable to the least squares estimation method of the failure rate $\delta$ is decreasing; the constant $c$ suitable for the least squares estimation of the failure rate $\delta$ is decreasing with the increase of the failure rate. At the same time, it can be seen that the value range of $c$ that meets the engineering requirements is from 4 to 6. In
addition, when the number of experimental groups is 15, the least square estimation is not applicable, so it is suggested that the number of experimental groups should be between 10 and 13.

4. Conclusion
Two widely used methods, optimal confidence limit method and distribution curve method, are used to predict the life of the device. For these two methods, the following Suggestions are made:
1. The optimal number of groups for reliability life prediction test of this type of product can be set as 10 to 13;
2. When using the optimal confidence limit method, for a certain failure efficiency, the relative error presents a nonlinear change, with the increase of confidence, the relative error first decreases and then increases, and the predicted results meet the engineering requirements when is 0.3 to 0.6. At the same time, when the value of is less than the optimal confidence coefficient value, the prediction error is getting smaller and smaller with the increase of failure rate , while when the value of is more than the optimal confidence coefficient value, the prediction error shows a trend of getting larger and larger.
3. For the distribution curve method, the failure probability in this experiment is estimated by E-Bayes, where the optimal value range of the constant value is 4 to 6. Under the condition of constant failure efficiency, the optimal value of gradually decreases as the number of experimental groups increases, and notice that the larger the failure rate, the smaller the value of . The life of electronic products generally obeys the exponential or weibull distribution, and the next research plan plans to conduct the prediction analysis of the reliability life of electronic products with no failure data whose life obeys the weibull distribution, and study the optimization method of parameters and group number by considering the influence of environmental factors.

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