INFERRING INFORMATION ABOUT ROTATION FROM STELLAR OSCILLATIONS

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Abstract

The first part of this paper aims at illustrating the intense scientific activity in the field of stellar rotation although, for sake of shortness, we cannot be exhaustive nor give any details. The second part is devoted to the rotation as a perturbation effect upon oscillation frequencies. The discussion focuses on one specific example: the $p$-modes frequency small separation which provides information about properties of the stellar inner layers. It is shown that the small separation can be affected by rotation at the level of 0.1-0.2 $\mu$Hz for a 1.4 $M_\odot$ model rotating with an equatorial velocity of 20 km/s at the surface. This is of the same order of magnitude as the expected precision on frequencies with a 3 months observation and must therefore be taken into account. We show however that it is possible to recover the small separation free of these contaminating effects of rotation, provided enough high quality data are available as will be with space seismic missions such as Eddington.

Key words: Stars: rotation – Stars: oscillations

1. Introduction

The importance of the role of rotation in stellar evolution has long been recognized. Effects of rotation is nowadays taken into consideration in many stellar studies and is often included in the modelling of stars, their structure and evolution. Stars of different masses and ages have rotational velocities which span a wide range (Figure 1). Determining the dependence of the rotational properties of stars with mass and age is an active field. One important issue is to understand and get a global picture of the evolution of the rotation rate inside a star:

- with age from the PMS stage (Matthieu, 2003) up to the stage of compact objects (Woosley, Heger, 2003)
- with mass from early O-A-B type stars down to late F-G-K type stars

The global picture is essential for understanding observed properties of open clusters or, to give another example, for explaining the origin of of gamma ray bursts which progenitors must be massive rotating stars. Then the goal is to explore the consequences of the evolution of the angular momentum on the evolution of stars and galaxies. Progress in understanding evolution of angular momentum will result in improving the age determinations based on isochrone fitting, to understand the evolution of massive stars, the chemical evolution of galaxies. As a feedback, this will provide improved theories of transport and evolution of stellar angular momentum in stellar conditions.

Efforts to measure stellar rotation rates - at least projected rotational velocities $v\sin i$ - for many stars of different types have recently been intensified.

A homogeneous sample of $v\sin i$ measurements has been obtained for A-B type stars (Royer et al. 2002a,b). The resulting histograms in Figure 2 show that the bulk of stars is found in the range 20 - 100 km/s indicating that fast rotation is common among A-B stars; $v\sin i$ values of 200-300 km/s are not infrequent.

It is necessary to determine the projected rotational velocity of $\delta$ Scuti stars before any seismological study of these rapidly rotating variable stars be possible. This is particularly important for $\delta$ Scuti stars in open clusters as for cluster stars, other parameters, chemical composition, age - are known. Projected rotational velocities for a few $\delta$ Scuti stars have been measured in Praesepe or NGC 6134 cluster (Rasmussen et al., 2002) for instance.

In order to study connections between rotation, X ray-emission, lithium abundance and stellar activity (Cutisposto et al (2003)), Cutisposto et al (2002) have determined the $v\sin i$ of a large sample of F-G-K type objects, in the solar neighborhood, selected for their high $v\sin i$ (Figure 3). With the purpose of determining possible latitudinal differential rotation and its correlation with stellar activity in stars other than the Sun, Reiners and Schmidt (2003) have obtained projected rotational velocities for a sample of field F-G-K type PMS and MS stars. Of interest here is that, although these are typically much slower rotators than A-B type stars, $v\sin i$ values exceeding 15 km/s are not uncommon, as we may see in Figure 3.

Determination of projected rotational velocities in very young cluster stars is also carried out (Herbst et al., 2001; Terndrup et al., 2002) as well as measurements of rotation periods and latitudinal differential rotation (Petit et al., 2002; Cameron et al. 2002; Barnes, 2003) with the aim of providing constraints in the modelling of the evolution of angular momentum during the very early phases of star formation and PMS stages.
2. Rotation in stellar models

2.1. Stellar evolutionary models with rotation

Inclusion of rotation in calculations of 1D evolutionary models is now performed by several groups.

Meynet, Maeder (1997) (see also Meynet, Maeder 2000 and ref. therein) have developed an evolutionary code which is based on a modified version of Zahn’s prescription which treats rotation effects as diffusion and advection processes (Maeder, Zahn, 1998). The modelling works have stimulated new theoretical studies about the rotation rate profile in the µ-gradient zone (Talon, Charbonnel 2003; Palacios et al, 2003) and transport of angular momentum by gravity waves (Talon et al, 2002). The observational constraints are the abundances of chemical elements at the surface of stars which can give hints about the rotationally internal mixing processes. A number of calculations with the Geneva ‘rotating’ code has enabled investigations of several problems such as He overabundances in O stars (Maeder, Meynet, 2000a), more generally evolution of - and yields by - rotating massive stars (Maeder, Meynet, 2003; Meynet et al., 2003).

The Yale group studies rotating solar type stars with models built following the Endal and Sofia (1976) approach which includes a somewhat different prescription for the evolution of the angular momentum and rotationally induced mixing, (Sills et al. 2000). For instance, Barnes et al. (2001) investigate the possibility that disk locking can efficiently spin down young stars as observed in young clusters. Barnes (2003) compiles available observations of rotation periods of cluster stars, classifies the stars as belonging to different sequences and proposes an interpretation of these observations as related to the properties of stellar magnetic fields.

Using Endal and Sofia’s code, another group also studies the influence of rotation on the evolution of massive stars (Heger et al. 2000a,b). Observations seem to confirm the model prediction which states that main sequence OB stars are already mixed which causes boron to be depleted without nitrogen being affected (Venn et al. 2002).

All these models show that the evolutionary tracks are significantly modified by fast rotation and any conclusions drawn from the position of a star in the HR diagram are then affected, particularly ages, isochrones and masses, amount of overshooting (Maeder, Meynet 2000b; Heger, Langer, 2000a; Maeder, Meynet, 2003; Palacios et al, 2003)

Calculation of binary evolution including the rotation of both components, transport of angular momentum and tidal processes are also now being carried out. Rotation appears to be able to decrease significantly the efficiency of the accretion process in some cases; it has a strong effect on the evolution of massive binary systems, playing an important role in the formation of a black hole in a binary system for instance or in the evolution of progenitors of type I supernovae (Langer et al, 2003 and ref. therein)

2.2. 2D-3D modelling

In order to provide a prescription in 1D models as close to the reality as possible, one needs a better understanding
Figure 2. Histograms showing the relative number of stars found with a $v \sin i$ in a given range. Data are from Royer et al. 2000. The solid line represents A stars (164 objects), the dashed line corresponds to B stars (53 objects).

...of transport and mixing processes in the interior of stars; this can come from 2D and 3D calculations.

Fully 2D rotating models of stars show for instance that the shape of a convective core changes with increasing rotation rates from a spherical to an aligned with the rotation axis geometry (Deupree, 2001).

3D modelling of a rotating spherical shell which is a simplified representation of the convective core of a rotating A type star shows that indeed rotation strongly influences the convective motions; it provides clues about the properties of overshooting which appears to vary with latitude creating asymmetries in the extent of the penetration (Brun et al., 2003).

3D MHD computations of a rotating spherical shell representing the solar outer convective envelope are performed in order to study the relations between differential rotation, turbulent convection and stellar magnetic activity with the conclusion that rotation and convection can generate magnetic activity but for these to be cycles, the existence of a tachocline might be crucial (Brun, 2003).

3. Stellar rotation and oscillations

Stellar oscillation frequencies $\nu_{n,\ell,m}$ are labelled with 3 characteristic numbers: the radial order $n$, the degree $\ell$ and the azimuthal order $m \in [-\ell, \ell]$. In absence of rotation, the frequencies are $2l + 1$ degenerate $\nu_{n,\ell,m} = \nu_{n,\ell,0}$ for $m \in [-\ell, \ell]$. Rotation breaks the azimuthal symmetry and lifts the $m$-degeneracy.

If rotation is slow enough and its angular rate (hereafter denoted $\Omega$) is independent of latitude, then the first order perturbation method in $\Omega$ predicts an equidistant frequency splitting, $\delta_{n,\ell}$, between consecutive $m$-components within each $(n, \ell)$ multiplet. For uniform rotation, the values of $\delta_{n,\ell}$ are proportional to $\Omega$ and allow to measure the rotation rate, free of the $\sin i$ uncertainty. In the case of a shellular rotation, $\Omega = \Omega(r)$, the $\delta_{n,\ell}$ provide mode-dependent mean values of the interior rotation rate. If we define the symmetric frequency splitting as

$$\delta_{nl} = \frac{\nu_{n,\ell,m} - \nu_{n,\ell,-m}}{2m}$$

then the connection between such splitting and the interior rotation remains accurate up to $\Omega^2$ because the terms $\propto \Omega^2$ are even in $m$.

3.1. Seismic measurements of rotation rates

Seismic measurements of rotation deduced from frequency splittings, i.e. from $\delta_{nl}$ (Eq.1), are now available for a few individual stars. For instance, seismic measurements of the rotation rates of variable white dwarfs seem to be the easiest approach and possibility of determining a nonuniform rotation for these stars has been explored (Kawaler et al., 1999).

For other types of pulsating stars, the rotation rate deduced from seismic measurements is model dependent as it usually relies on a mode identification which is itself model dependent. Today $\beta$ Cephei stars are the best candidates to provide reliable mode identifications, hence the deduced rotation rates, which we can trust (Thoul et al. 2003; Handler et al., 2003). For $\delta$ Scuti stars, the process of mode identification remains difficult and this keeps one from determining their rotation rates with enough confidence, except perhaps for FG Vir for which line-profile and light variations are available (Mantegazza, Poretti 2002).

For solar-like oscillating stars, it is also proposed to measure directly the total angular momentum (Pijpers, 2003); this requires a large set of high quality frequencies as will be obtained by the forthcoming seismological space missions (MOST, COROT, Eddington).

3.2. Rotational effects on mode excitation

Rotation affects driving and damping of oscillation modes to different extent depending on the type of stars and of excited modes (Lee 1998; Ushomirsky, Bildsten, 1998; Saio et al., 2000; Lee, 2001). Correlations appear to exist between the amplitudes of the excited modes and the $v \sin i$ of $\delta$ Scuti stars in Praesepe cluster (Suarez et al., 2002); this is an important issue regarding possible selection effects which are responsible for the complicated patterns of oscillation frequency spectra of these stars and further work is necessary to confirm it. The Eddington space mission is well suited for such statistical studies from which we expect more complete oscillation spectra and the value of the rotational velocity $v$ instead of the projected one $v \sin i$ for a large sample of stars.
3.3. Rotational effects on frequencies

One must however be aware that rotation can also hinder some seismic studies (Dziembowski & Goode, 1992 (DG92)).

For stars oscillating with opacity driven modes such as δ Scuti and β Cephei variables which oscillate in the low frequency- low radial order range- and as these stars rotate fast, third order effects in Ω as measured by $\Omega^3/\nu_{n,\ell}^2$ range in the interval $\sim 0.01 - 0.5 \, \muHz$ and must therefore generally be taken into account in calculating the splittings (Goupil et al., 2001; Dziembowski et al., 1998; Goupil & Talon, 2002; Pamyatnykh, 2003)

For very fast rotating stars as for instance some δ Scuti stars known to have a $v \sin i \sim 250 \, \text{km/s}$, even a third order perturbation is no longer valid and a nonperturbative approach is the only appropriate one. Such investigations have been initiated with a special attention to low frequency gravity modes (Clement 1981-1998; Lee & Saio, 1987; Dintrans & Rieutord, 2000) which can apply to γ Dor stars. In order to consider acoustic modes, the effect of the centrifugal force on the equilibrium state of the star and on the wave motions must also be included. Based on the numerical approach of Rieutord & Valdettaro (1997) which allows to include a large number of spherical harmonics in the description of each individual oscillation mode without being numerically too time consuming, Lignières et al. (2002) and Lignières (2003, in prep) have calculated the oscillation frequencies of a rotating polytrope with an oblateness up to $\epsilon = 0.15$. Results show that the mode amplitude inside the star even at this relatively small oblateness may become a quite complicated function of $r$ and $\theta$, significantly departing from a single spherical harmonic description. These calculations also confirm that the small separation is affected particularly when it involves mixed modes (see also Sect. 3.4 below).

For solar-like stars, the small separation between $\ell = 0$ and a $\ell = 2$ modes as well as $\ell = 1$ and $\ell = 3$ can become so small that rotation, even at a modest rate of 15 km/s, leads to a significant departure from the single spherical harmonics representation of individual modes.

In the framework of a perturbative approach, this means that the radial mode $\ell = 0$ is no longer purely radial but is contaminated with a $\ell = 2$ contribution and conversely the $\ell = 2$ mode is contaminated with a $\ell = 0$ contribution. This has severe consequences already stressed in Soufi et al. (1998), Dziembowski (1997), Dziembowski & Goupil (1998)

3.4. Rotation as a perturbation

The small separation $d_{n,\ell}^{(0)}$ is defined as:

$$d_{n,\ell}^{(0)} = \nu_{n,\ell}^{(0)} - \nu_{n-1,\ell+2}^{(0)}$$

where $\nu_{n,\ell}^{(0)}$ is the eigenfrequency of mode with degree $\ell$ and radial order $n$ in absence of rotation. The small separation $d_{n,\ell}^{(0)}$ is known to provide information about the structural properties of the inner layers of the star (Gough 1998; Roxburgh, Vorontsov, 1998). However, rotation modifies the structure of the star and its oscillation frequencies. When rotation is fast enough, the frequencies can be modified to an extent that the small separation $d_{n,\ell}^{(0)}$ is significantly affected.

This is illustrated below. Only the symmetrical part of the centrifugal distortion is directly included in the stellar model as an effective gravity, no mixing due to rotation is taken into account. A uniform rotation is assumed.

Figure 3. Histograms as in Fig.2 but for F type stars (32 objects), G type stars (91 objects), K type stars (19 objects). Data are from Cutispoto et al. 2002.
and evolution of the stellar model is performed assuming conservation of total angular momentum. Other effects of rotation on the oscillation frequencies are taken into account by means of a perturbation method. For moderate rotation rates as are expected for most solar-like oscillators, cubic order terms, $O(\Omega^3/\nu^2)$, in the perturbation ($\sim 10^{-6} - 10^{-5}$ mHz) can be neglected and only second order effects in $\Omega$ are considered. In that case, one obtains, for each $m$ component of a given multiplet of modes $\ell, n$:

$$\nu_{n,\ell,m} = \nu_{0,n,\ell} + m C_{n,\ell} + D_{n,\ell,m}$$

where $\nu_{0,n,\ell}$ is the eigenfrequency of $(n, \ell)$ mode of a stellar model built with rotation through an effective gravity; $C_{n,\ell}$ represents the first order (Coriolis) correction due to rotation and reduces to $m\Omega(C_{n,\ell} - 1)$ for a uniform rotation in the observer frame with $C_{n,\ell}$ the Ledoux constant (DG92). The quantity $D_{n,\ell,m}$ is a $O(\Omega^2)$ contribution which takes into account the effect of the nonspherically symmetric centrifugal distortion of the stellar model and the second order correction due to Coriolis force.

For nondegenerate modes in a rotating star, the small separation is then given by

$$d_{n,\ell,m} = \nu_{0,n,\ell} - \nu_{0,n-1,\ell+2,m}$$

From Eq. (3), the small separation of non degenerate $m = 0$ modes can be written as:

$$d_{n,\ell,0} = \Delta_{n,\ell,0} + (D_{n,\ell,0} - D_{n-1,\ell+2,0})$$

where we have defined

$$\Delta_{n,\ell,m} = \nu_{0,n,\ell} - \nu_{0,n-1,\ell+2}$$

Note that in the general case of $m \neq 0$ components, we can define the small separations as

$$d_{n,\ell,m} = \frac{1}{2} (\nu_{n,\ell,m} + \nu_{n,m,-m} - \nu_{n-1,\ell+2,m} - \nu_{n-1,\ell+2,m})$$

which provide expressions similar to Eq. (5) for each $m$.

For high frequency $p$-modes such as those detected in the Sun and a few other stars, modes $(n, \ell)$ and $(n-1, \ell+2)$ are systematically near degenerate if the star is moderately rotating (Soufi et al 1998) i.e. their frequency differences are so small that a description with a single spherical harmonics for each mode is no longer valid (Chandrasekhar & Lebovitz, 1962). Each of the two near degenerate modes must be described as a combination of the spherical harmonics with degrees $\ell$ and $\ell + 2$. Their frequencies become:

$$\hat{\nu}_{n,\ell,m} = \frac{1}{2} (\nu_{n,\ell,m} + \nu_{n-1,\ell+2,m} - \sqrt{d^2_{n,\ell,m} + 4H^2_{n,\ell,m}})$$

and

$$\hat{\nu}_{n-1,\ell+2,m} = \frac{1}{2} (\nu_{n,\ell,m} + \nu_{n-1,\ell+2,m} + \sqrt{d^2_{n,\ell,m} + 4H^2_{n,\ell,m}})$$

where $H_{n,\ell,m}$ is a coupling term between both near degenerate modes; $\nu_{n,\ell,m}$ is given by Eq.3 and $d_{n,\ell,m}$ by Eq.4.

Hence for a rotating star, the observed small separation of near degenerate $m = 0$ modes is given by:

$$\hat{d}_{n,\ell} = \hat{\nu}_{n,\ell,0} - \hat{\nu}_{n-1,\ell+2,0}$$

where $\hat{d}_{n,\ell}$ differs from the small separation $d^{(0)}_{n,\ell}$ because the oscillation frequencies are modified by rotation according to Eq.(3-7-8). For later purpose, we note that Eq.7-8-9 imply

$$d^2_{n,\ell} = d^2_{n,\ell,0} + 4H^2_{n,\ell,0}$$

These changes due to rotation are quantified below for 1.40$M_\odot$ models with solar chemical composition and no overshooting. The stellar models have been computed, assuming either no rotation or an initial rotational velocity $v = 10, 15, 20, 30$ and $35$ km/s. Ages of the models have been chosen so that they have the same effective temperature ($\log T_{\text{eff}} = 3.810$) hence approximately fall at the same location in the HR diagram (changes in luminosity are negligible). The selected models are characterized by rotational velocities $v = 9.9, 14.8, 19.7, 29.6, 34.6$ km/s, rotation rates $\Omega = (0.82, 1.23, 1.64, 2.47, 2.88) 10^{-5}$ rad/s and ratios $\epsilon = \Omega/(GM/R^3)^{1/2} \sim (2.5, 3.8, 5.1, 8.9) 10^{-2}$ respectively.

Oscillation frequencies for these models have been computed for modes with degrees $\ell = 0 - 3$ in the radial order range $n = 6 - 25$. This corresponds to a frequency range of about 500-2000 mHz. Modes $\ell = 0, 2$ and modes $\ell = 1, 3$ are considered systematically near degenerate and their frequencies are computed according to Eq.7,8. For each model with initial velocity $v = 10, 15, 20, 30$ and $35$ km/s, the small separation $d_{13}$ for $m = 0$ modes is given by Eq.9 and is shown in Figure 4 (top panel) in function of the frequency $\nu_{n,\ell,0}$ for each model. The effect of rotation - which is mainly due to centrifugal distortion - can clearly be seen. The curves are seen to deviate from the $v = 0$ one, the departure increases with $\Omega$ as expected. At small enough $v$, only the highest frequency part is affected but for higher velocities $v > 15$ km/s the whole range of modes is affected.

As the effect of rotation is to increase the small separation, the point corresponding to the rotating model in the small separation versus large separation (CD) diagram (Christensen-Dalsgaard, 1993) would indicate a star which is younger than it is in reality. Hence a CD diagram, in addition to the chemical composition, must add the rotation as a parameter.

For a $v = 20$ km/s for instance, the small separation is increased by roughly 0.15 mHz. This departure is of the same order of magnitude as the the variation of the small separation with frequency which provides information about the inner layers and require a precision of $\sim 0.2$ mHz on individual frequencies i.e. which corresponds to $\sim 3$ months of observation. Hence rotation must be taken into account when the star rotates fast ($v > 15$ km/s for solar-like oscillating stars).

Fortunately, it is possible to remove the disturbing effects of the rotation on the small separation and to recover the
small separation $d_{n,\ell}^{(0)}$ (Eq(2)) free of rotational effects from the observed frequencies (Eq.7-8).

For solar-like oscillations i.e with frequencies following asymptotic relations, a good approximation when the rotation is moderate is

$$ d_{n,\ell}^{(0)} \sim \Delta_{n,\ell,0} $$

(11)

This means that the contribution of the spherically symmetric distortion of the equilibrium model is not removed but is quite negligible here.

As solar-like modes are predominantly acoustic, the coefficients in Eq.(3),(7), and (8) can be approximated by

$$ D_{nt,m} \approx Q_{\ell,m} V_{n,\ell} \quad H_{nt,m} = Q_{\ell,\ell+2,m} V_{n,\ell} $$

(12)

The coefficient $V_{n,\ell}$ is roughly the same for both near degenerate modes i.e. $V_{n,\ell} \sim V_{n-1,\ell+2}$. Geometrical factors are

$$ Q_{\ell,m} = \frac{\ell - 3 m^2}{4\ell - 3} \quad Q_{\ell,\ell+2,m} = \frac{3}{2} \beta_\ell \beta_{\ell+2} $$

(13)

with

$$ \Lambda = \ell (\ell + 1) \quad \beta_\ell = \left( \frac{\ell^2 - m^2}{4\ell^2 - 1} \right)^{1/2} $$

(14)

From Eq.(5), one derive

$$ \Delta_{n,\ell,0} = d_{n,\ell,0} - \left( Q_{\ell,0} - Q_{\ell+2,0} \right) V_{n,\ell} $$

(15)

where the difference $d_{n,\ell,0}$ using Eq.(10) is given by :

$$ d_{n,\ell,0} = \left( \frac{a_{n,\ell,0}^2 - 4 H_{n,\ell}^2}{V_{n,\ell}} \right)^{1/2} $$

(16)

i.e.

$$ d_{n,\ell,0} = \left( \frac{a_{n,\ell,0}^2 - 4 H_{n,\ell}^2}{V_{n,\ell}} \right)^{1/2} $$

(17)

With the help of Eq.(11-15), we can calculate the small separation $d_{n,\ell}^{(0)}$ for two near degenerate modes, $(n, \ell, m = 0)$ and $(n - 1, \ell + 2, m = 0)$, which is free of rotational effects and which depends only on the observed frequencies and the coefficient $V_{n,\ell}$. This coefficient can in turn be obtained from the observed frequencies. Let define:

$$ s_{n,\ell,m} = \frac{\hat{\nu}_{n,\ell,m} + \hat{\nu}_{n,\ell,-m}}{2} $$

(18)

For asymptotic acoustic modes, one then derive for $\ell = 0, 2$ near degenerate modes

$$ V_{n,0} = \frac{s_{n-1,2,1} - s_{n-1,2,2}}{Q_{2,1} - Q_{2,2}} $$

(19)

and for $\ell = 1, 3$ near degenerate modes

$$ V_{n,1} = \frac{s_{n-1,3,2} - s_{n-1,3,3}}{Q_{3,2} - Q_{3,3}} $$

(20)

Figure 4 (bottom panel) illustrates how efficiently the small separation $d_{n,\ell}^{(0)}$ (Eq.2) can be recovered from the observed frequencies $\hat{\nu}_{rot}$ (Eq.(4,5) for a rotating star using Eq.(15), (17) and (20). All \(\nu \neq 0\) curves (recovered) $d_{n,3/5}^{(0)}$ approximately coincide with the $\nu = 0$ one.

As a second example, we built a model equivalent to Roxburgh’s model in the Eddington assessment report (Favata et al., 2000) i.e a 1.5$M_\odot$ with no overshooting and an equivalent evolutionary main sequence stage. Figure 5 (top panel) shows the small separations $d_{0,2}^{(0)}$ and $d_{1,3}^{(0)}$ for a nonrotating model, which are quite comparable with the same quantities for Roxburgh’s model in Fig.2.7.

Figure 5 (bottom panel) shows the small separation $d_{1,3/5}^{(0)}$ for a 1.5$M_\odot$ model built with the same effective temperature but with a uniform rotation rate $\Omega = 3.94 \times 10^{-5}$ rad/s corresponding to a rotational velocity $v = 55.6$ km/s (and $\epsilon = 0.15$). The small separation calculated with the observed frequencies according to Eq.(7)-(9) significantly differs from the small separation in absence of rotation. The difference increases with the frequency and reaches 2.2 $\mu$Hz at $\sim 1500 \mu$Hz. However, combining the frequencies as described above i.e using Eq.(11), (15) and (17), it is found that one can remove most effects of rotation and recover a small separation free of these contaminations at the level $< 0.2 \mu$Hz as can be seen in Figure 5

3.6. IDEAL CASE FOR INVERSION OF ROTATION PROFILE

For stars with opacity driven modes, it is expected that one can obtain detailed information about the rotation profile inside the stars providing a few appropriate modes are excited, detected and identified (Goupil et al., 1996).

For stars with stochastically excited modes, it is known that the asymptotic p-modes can hardly give very localized information about rotation everywhere inside the star.
Figure 5. **Top:** Small separations $d_{0.2}/3$ (filled circles) and $d_{1.3}/5$ (dashed line with open circles) for a $M = 1.54 M_\odot$ ($\log T_{\text{eff}} = 3.820$) without rotation versus frequency both in $\mu$Hz. **Bottom:** $d_{1.3}/5$ with no rotation as top (dashed line with open circles); open squares : $\hat{d}_{1.3}/5$ including effects of near-degeneracy for an equivalent model but with a rotational velocity $v = 55.6$ km/s; open triangles: $d_{13}/5$ recovered after removing contamination by rotational effects. Note the change of vertical scale between the top and bottom panels.

(Gough, 1998, Goupil et al., 1996). However, this is true only if the excited modes are of purely acoustic nature. In more evolved stars, among the modes that may be stochastically excited, some may have a mixed character and thus exhibit some information about the structure and rotation in the inner layers.

Fig. 6 shows the expected distribution of the oscillation amplitudes (luminosity amplitude $\delta L/L$) of stochastically excited modes with frequency for a $1.60 M_\odot$ main sequence stellar model with a central hydrogen abundance $X_c = 0.2$.

The stellar model is built with Böhm-Vitense (1958)'s formulation of the mixing-length (MLT) and assumes the Eddington classical gray atmosphere. The eigenmodes and eigenfrequencies are calculated with the adiabatic ADIPLS pulsation code (Christensen-Dalsgaard, 1996). The calculations of the mode amplitudes make use of the adiabatic assumption for relating $\delta L/L$ to the rates $P$ and the rates $\eta$ at which the mode are excited and damped respectively. The excitation rates $P$ are computed according to the stochastic excitation model of Samadi & Goupil (2001). The calculations assume - as in Samadi et al. (2003) - a Lorenzian function for modelling the convective eddy time-correlations. Using the same physics (MLT) for a solar model leads to an underestimation of the excitation power $P$ compared with the observations. In order for the maximum in $P$ to match that of Chaplin et al. (1998)'s seismic observations, the solar excitation rates $P$ must be multiplied by a factor 13. Accordingly, we multiply by the same factor the computed excitation rates for the stellar model considered here. The damping rates $\eta$ are obtained from Houdek et al (1999)'s tables.

Figure 6 displays the amplitudes of luminosity fluctuations $\delta L/L$ for each mode in the frequency range expected to be sensitive to a stochastic excitation by turbulent convection for a $M = 1.60 M_\odot$ main sequence star. The Eddington detection threshold is also plotted. From the comparison, one expects many modes to be detected for such a star. Among these detected modes, a few are mixed modes which are seen to deviate from the general trend in Figure 6. These modes have amplitudes smaller than the other modes in the same frequency interval; this is due to their mixed nature, they have amplitudes in the inner layers which increase their inertia compared to that of the neighboring pure $p$-modes.

Assuming that rotational splitting of all mixed modes with amplitudes above the threshold is measured, Lochard et al. (2003 in prep) found that rotation rates in the $\mu$-gradient zone and in the chemically homogeneous envelope could be determined with some precision. This would provide a valuable constraint on theories of angular momentum transport in stellar interiors.
