Evidence for granularity, anisotropy and lattice distortions in cuprate superconductors and their implications

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Granularity, anisotropy, local lattice distortions and their dependence on dopant concentration appear to be present in all cuprate superconductors, interwoven with the microscopic mechanisms responsible for superconductivity. Here we review anisotropy and penetration depth measurements to reassess the evidence for granularity, as revealed by the notorious rounded phase transition, the evidence for the three dimensional nature of superconductivity, uncovered by the doping dependence of transition temperature and anisotropy, and to reassess the relevance of the electron-lattice coupling, emerging from the oxygen isotope effects.

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Establishing and understanding the phase diagram of cuprate superconductors in the temperature - dopant concentration plane is one of the major challenges in condensed matter physics. Superconductivity is derived from the insulating and antiferromagnetic parent compounds by partial substitution of ions or by adding or removing oxygen. For instance La2CuO4 can be doped either by alkaline earth ions or oxygen to exhibit superconductivity. The empirical phase diagram of La2−xSrxCuO4 depicted in Fig.1 shows that after passing the so called underdoped limit ($x_u \approx 0.05$), $T_c$ reaches its maximum value $T_c(x_m)$ at $x_m \approx 0.16$. With further increase of $x$, $T_c$ decreases and finally vanishes in the overdoped limit $x_o \approx 0.27$. This phase transition line is thought to be a generic property of cuprate superconductors [10] and is well described by the empirical relation

$$T_c(x) = T_c(x_m) \left(1 - 2 \left(\frac{x}{x_m} - 1\right)^2\right) = \frac{2T_c(x_m)}{x_m^2} (x - x_u)(x_o - x),$$

proposed by Presland et al.[11]. Approaching the endpoints along the $x$-axis, La2−xSrxCuO4 undergoes at zero temperature doping tuned quantum phase transitions. As their nature is concerned, resistivity measurements reveal a quantum superconductor to insulator (QSI) transition in the underdoped limit [12, 13, 14, 15, 16, 17, 18] and in the overdoped limit a quantum superconductor to normal state (QSN) transition [12].

Another essential experimental fact is the doping dependence of the anisotropy. In tetragonal cuprates it is defined as the ratio $\gamma = \xi_{ab}/\xi_c$ of the correlation lengths parallel ($\xi_{ab}$) and perpendicular ($\xi_c$) to CuO2 layers ($ab$-planes). In
the superconducting state it can also be expressed as the ratio $\gamma = \lambda_c/\lambda_{ab}$ of the London penetration depths due to supercurrents flowing perpendicular ($\lambda_c$) and parallel ($\lambda_{ab}$) to the ab-planes. Approaching a nonsuperconductor to superconductor transition $\xi$ diverges, while in a superconductor to nonsuperconductor transition $\lambda$ tends to infinity. In both cases, however, $\gamma$ remains finite as long as the system exhibits anisotropic but genuine 3D behavior. There are two limiting cases: $\gamma = 1$ characterizes isotropic 3D- and $\gamma = \infty$ 2D-critical behavior. An instructive model where $\gamma$ can be varied continuously is the anisotropic 2D Ising model. When the coupling in the $y$ direction goes to zero, $\gamma = \xi_x/\xi_y$ becomes infinite, the model reduces to the 1D case, and $T_c$ vanishes. In the Ginzburg-Landau description of layered superconductors the anisotropy is related to the interlayer coupling. The weaker this coupling is, the larger $\gamma$ is. The limit $\gamma = \infty$ is attained when the bulk superconductor corresponds to a stack of independent slabs of thickness $d$. With respect to experimental work, a considerable amount of data is available on the chemical composition dependence of $\gamma$. At $T_c$ it can be inferred from resistivity ($\gamma = \xi_{ab}/\xi_c = \sqrt{\rho_{ab}/\rho_c}$) and magnetic torque measurements, while in the superconducting state it follows from magnetic torque and penetration depth ($\gamma = \lambda_c/\lambda_{ab}$) data.

In Fig. 1b we displayed the doping dependence of $1/\gamma$ evaluated at $T_c$ ($\gamma_T$) and $T = 0$ ($\gamma_T=0$). As the dopant concentration is reduced, $\gamma_T$ and $\gamma_T=0$ increase systematically, and tend to diverge in the underdoped limit. Thus the temperature range where superconductivity occurs shrinks in the underdoped regime with increasing anisotropy. This competition between anisotropy and superconductivity raises serious doubts whether 2D mechanisms and models, corresponding to the limit $\gamma_T = \infty$, can explain the essential observations of superconductivity in the cuprates. From Fig. 1c, it is also seen that $\gamma_T(x)$ is well described by

$$\gamma_T(x) = \frac{\gamma_{T,0}}{x-x_u},$$

where $\gamma_{T,0}$ is the quantum critical amplitude. Having also other cuprate families in mind, it is convenient to express the dopant concentration in terms of $T_c$. From Eqs. 1 and 2 we obtain the correlation between $T_c$ and $\gamma_T$:

$$\frac{T_c(x)}{T_c(x_m)} = 1 - \left( \frac{\gamma_T(x_m)}{\gamma_T(x)} - 1 \right)^2, \quad \gamma_T(x_m) = \frac{\gamma_{T,0}}{x_m-x_u}$$

Provided that this empirical correlation is not merely an artefact of La$_{2-x}$Sr$_x$CuO$_4$, it gives a universal perspective on the interplay of anisotropy and superconductivity, among the families of cuprates, characterized by $T_c(x_m)$ and $\gamma_T(x_m)$. For this reason it is essential to explore its generic validity. In practice, however, there are only a few additional compounds, including HgBa$_2$CuO$_{4+\delta}$, for which the dopant concentration can be varied continuously throughout the entire doping range. It is well established, however, that the substitution of magnetic and nonmagnetic impurities depresses $T_c$ of cuprate superconductors very effectively. To compare the doping and substitution
driven variations of the anisotropy, we depicted in Fig. 2 the plot $T_c(x)/T_c(x_m)$ versus $\gamma_T(x_m)/\gamma_T(x)$ for a variety of cuprate families. The collapse of the data on the parabola, which is the empirical relation (3), reveals that this scaling form appears to be universal. Thus, given a family of cuprate superconductors, characterized by $T_c$, and $\gamma_T(x_m)$, it gives a universal perspective on the interplay between anisotropy and superconductivity.

Close to 2D-QSI criticality various properties are not independent but related by
\[
T_c \propto \Phi_0^2 R_2 \frac{d_s}{16\pi^3 k_B \lambda_{ab}^2(0)} \propto \gamma_T^{-\nu} \propto \delta^{z\nu},
\]
where $k_B$ is the Boltzmann constant, and $\Phi_0$ the elementary flux quantum. $\lambda_{ab}(0)$ is the zero temperature in-plane penetration depth, $d_s$ the thickness of the sheets, and $\nu$ the correlation length critical exponent of the 2D-QSI transitions. $\delta$ measures the distance from the critical point along the $x$ axis (see Fig. 4), and $R_2$ is a universal number. Since $T_c \propto d_s/\lambda_{ab}^2(0) \propto n_s^{\nu}$, where $n_s^{\nu}$ is the thickness of the sheets, becoming independent in the 2D limit $\nu \approx 1$. The relevance of $d_s$ was also confirmed in terms of the relationship between the iso-phase effect on $T_c$ and $1/\lambda_{ab}^2(0)$ [16, 31]. Moreover, together with the scaling form (4), the empirical relation (4) implies 2D-QSI and 3D-QSN transitions with $z = 1$, while the empirical relation for the anisotropy (Eqs. (2) and (3)), require $\nu = 1$ at the 2D-QSI critical point. Thus, the empirical correlations point to a 2D-QSI transition with $z = 1$ and $\nu = 1$. These estimates coincide with the theoretical prediction for a 2D disordered bosonic system with long-range Coulomb interactions, where $z = 1$ and $\nu \approx 1$. Here the loss of superfluidity is due to the localization of the pairs, which ultimately drives the transition. From the scaling relation $\lambda_{ab}$ it is seen that measurements of the out of plane penetration depth of sufficiently underdoped systems allow to estimate the dynamic critical exponent $\delta$ directly, in terms of $T_c \propto (1/\lambda_{ab}^2(0))^{z/(z+2)}$, which follows from Eq. (4) with $\delta_T = \lambda_c(0)/\lambda_{ab}(0)$. In Fig. 3 we displayed the data of Hosseini [25] for heavily underdoped YBa$_2$Cu$_3$O$_7$ single crystals. The solid line is $T_c = 170 (1/\lambda_{ab}^2(T = 0))^{1/3}$ and uncovers the consistency with the 2D-QSI scaling relation $T_c \propto (1/\lambda_{ab}^2(0))^{z/(z+2)}$ with $z = 1$.

We have seen that the doping tuned flow to the 2D-QSI critical point is associated with a depression of $T_c$ and an enhancement of $\gamma_T$. It implies that whenever a QSI transition is approached, a non vanishing $T_c$ is inevitably associated with an anisotropic but 3D condensation mechanism, because $\gamma_T$ is finite for $T_c > 0$ (see Figs. 4 and 2). This represents a serious problem for 2D models [27] as candidates to explain superconductivity in the cuprates, and serves as a constraint on future work toward a complete understanding. Note that the vast majority of theoretical models focus on a single Cu-O plane, i.e., on the limit of zero intracell and intercell $c$-axis coupling.

Since Eq. (4) is universal, it also implies that the changes $\Delta T_c$, $\Delta d_s$ and $\Delta (1/\lambda_{ab}^2(T = 0))$, induced by pressure or...
Indeed, the relative shift, Δ, which do not modify the lattice parameters, and are coupled to the superfluid. Evidence for this coupling merges from the oxygen isotope effect on using various experimental techniques on powders, thin films and single crystals. Further evidence for this coupling emerges from the isotope effect on the granularity of the cuprates and the failure of the Migdal-Eliashberg (ME) treatment of the electron-phonon interaction, predicting, 1

isotope exchange are not independent, but related by

\[
\frac{\Delta T_c}{T_c} = \frac{\Delta d_s}{d_s} + \frac{\Delta \left(1/\lambda_{ab}^2(0)\right)}{(1/\lambda_{ab}^2(0))} = \frac{\Delta d_s}{d_s} - 2\frac{\Delta (\lambda_{ab}(0))}{\lambda_{ab}(0)}. \tag{5}
\]

In particular, for the oxygen isotope effect \(^{16}\text{O} \text{vs.} \; ^{18}\text{O}\) of a physical quantity \(X\) the relative isotope shift is defined as \(\Delta X/X = (^{18}X - ^{16}X)/^{18}X\). In Fig.4 we show the data for the oxygen isotope effect in La\(_{2-x}\)Sr\(_x\)CuO\(_4\) [41, 42, 43], Y\(_{1-x}\)Pr\(_x\)Ba\(_2\)Cu\(_3\)O\(_{7-\delta}\) (∗: \(x=0, 0.2, 0.3, 0.4\)) [39, 41] and YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) (∇ [41]) in terms of \(\Delta (\lambda_{ab}(0))/\lambda_{ab}(0)\) versus \(-\Delta T_c/T_c\). The solid line indicates the flow to 2D-QSI criticality and provides with Eq.(5) an estimate for the oxygen isotope effect on \(d_s\), namely \(\Delta d_s/d_s = 3.3(4)\%\).

FIG. 4: Data for the oxygen isotope effect in underdoped La\(_{2-x}\)Sr\(_x\)CuO\(_4\): \(x=0.15, 0.08\), 0.086 [37], Y\(_{1-x}\)Pr\(_x\)Ba\(_2\)Cu\(_3\)O\(_{7-\delta}\) (∗: \(x=0, 0.2, 0.3, 0.4\)) [39, 41] and YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) (∇ [38], ▼ [11]) in terms of \(\Delta (\lambda_{ab}(0))/\lambda_{ab}(0)\) versus \(-\Delta T_c/T_c\). The solid line indicates the flow to 2D-QSI criticality and provides with Eq.(5) an estimate for the oxygen isotope effect on \(d_s\), namely \(\Delta d_s/d_s = 3.3(4)\%\).

Since upon oxygen isotope exchange the lattice parameters remain essentially unaffected [42, 43], the substantial isotope effect on the in-plane penetration depth uncovers the coupling between local lattice distortions and superfluidity and the failure of the Migdal-Eliashberg (ME) treatment of the electron-phonon interaction, predicting, 1/\(\lambda^2(0)\), to be independent of the ionic masses [41]. Evidence for this coupling emerges from the oxygen isotope effect on \(d_s\), the thickness of the superconducting sheets, upon isotope exchange, while the lattice parameters remain unaffected. Indeed, the relative shift, \(\Delta d_s/d_s \approx 3.3(4)\%\), apparent in Fig.3 implies local distortions of oxygen degrees of freedom, which do not modify the lattice parameters, and are coupled to the superfluid.

Further evidence for this coupling emerges from the isotope effect on the granularity of the cuprates. [45, 46]. Recently, it has been shown that the notorious rounding of the superconductor to normal state transition is fully consistent with a finite size effect, revealing that bulk cuprate superconductors break into nearly homogeneous superconducting grains of rather unique extent [42, 43, 46, 47, 48]. Even evidence for their surface and edge contributions to specific heat and penetration depth has been established [49]. A characteristic feature of a finite size effect in the temperature dependence of the in-plane penetration depth \(\lambda_{ab}\) is the occurrence of an inflection point giving rise to an extremum in \(d (\lambda_{ab}^2(T = 0)/\lambda_{ab}^2(T)) /dT\) at \(T_p\). Here \(\lambda_{ab}^2(T_p)\), \(T_p\) and the length \(L_c\) of the grains along the c-axis are related by [45, 46, 47, 48, 49]

\[
\frac{1}{\lambda_{ab}^2(T_p)} = \frac{16\pi^3 k_B T_p}{\Phi_0^2 L_c}. \tag{6}
\]

Recently we explored the effect of oxygen isotope exchange in Y\(_{1-x}\)Pr\(_x\)Ba\(_2\)Cu\(_3\)O\(_{7-\delta}\) on \(L_c\) by means of in-plane
FIG. 5: Magnetic field penetration profiles $B(z)$ for a $^{16}$O substituted (closed symbols) and a $^{18}$O substituted (open symbols) YBa$_2$Cu$_3$O$_{7-x}$-film measured in the Meissner state at 4 K and an external field of 9.2 mT, applied parallel to the surface of the film. The data are shown for implantation energies 3, 6, 10, 16, 22, and 29 keV starting from the surface of the sample. Solid curves are best fits to $B(z) = B_0 \cosh[(t-z)/\lambda_{ab}^c] \cosh(t/\lambda_{ab}^c)$. This is the form of the classical exponential field decay in the Meissner state $B(z) = B_0 \exp(-z/\lambda_{ab}^c)$, modified for a film with thickness $2t$ with flux penetrating from both sides. Taken from Khasanov et al.

penetration depth measurements\textsuperscript{46}. Note that the shifts are not independent but according to Eq.\textsuperscript{6} related by

$$\frac{\Delta L_c}{L_c} = \frac{\Delta T_{pc}}{T_{pc}} + \frac{\Delta \lambda_{ab}^2(T_{pc})}{\lambda_{ab}^2(T_{pc})}.$$ \hfill (7)

From the resulting estimates, listed in Table I, several observations emerge. First, $L_c$ increases systematically with reduced $T_{pc}$. Second, $L_c$ grows with increasing $x$ and upon isotope exchange ($^{16}$O, $^{18}$O). Third, the relative shift of $T_{pc}$ is very small. This reflects the fact that the change of $L_c$ is essentially due to the superfluid, probed in terms of $\lambda_{ab}^2$. Accordingly, $\Delta L_c/L_c \approx \Delta \lambda_{ab}^2/\lambda_{ab}^2$ for $x = 0, 0.2$ and $0.3$.

| $x$ | 0    | 0.2  | 0.3  |
|-----|------|------|------|
| $\Delta L_c/L_c$ | 0.12(5) | 0.13(6) | 0.16(5) |
| $\Delta T_{pc}/T_{pc}$ | 0.000(2) | -0.015(3) | -0.021(5) |
| $\Delta \lambda_{ab}^2(T_{pc})/\lambda_{ab}^2(T_{pc})$ | 0.11(5) | 0.15(6) | 0.15(5) |
| $^{16}T_{pc}$ (K) | 89.0(1) | 67.0(1) | 52.1(1) |
| $^{18}T_{pc}$ (K) | 9.7(4) | 14.2(7) | 19.5(8) |
| $^{16}L_c$(A) | 10.9(4) | 16.0(7) | 22.6(9) |

Table I: Finite size estimates for the relative changes of $L_c$, $T_{pc}$ and $\lambda_{ab}^2(T_{pc})$ upon oxygen isotope exchange for Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_{7-x}$\textsuperscript{46}.

To appreciate the implications of these estimates, we note again that for fixed Pr concentration the lattice parameters remain essentially unaffected\textsuperscript{42, 43}. Accordingly, an electronic mechanism, without coupling to local lattice distortions implies $\Delta L_c = 0$. On the contrary, a significant change of $L_c$ upon oxygen exchange requires local lattice distortions involving the oxygen lattice degrees of freedom and implies with Eq.\textsuperscript{7} a coupling between these distortions and the superfluid. A glance to Table I shows that the relative change of the grains along the $c$-axis upon oxygen isotope exchange is significant, ranging from 12 to 16%, while the relative change of the inflection point at $T_{pc}$, or the transition temperature, is an order of magnitude smaller. For this reason the significant relative change of $L_c$ at fixed Pr concentration is accompanied by essentially the same relative change of $\lambda_{ab}^2$, which probes the superfluid. This uncovers unambiguously the existence and relevance of the coupling between the superfluid and lattice distortions, involving the oxygen lattice degrees of freedom. Furthermore the substantial isotope effect on the in-plane penetration depth at $T = T_{pc}$ extends the evidence for the failure of the Migdal-Eliashberg (ME) theory of the electron-phonon
interaction, predicting $1/\lambda^2$ to be independent of the ionic masses $^{[44]}$, to finite temperature. Although the majority opinion on the mechanism of superconductivity in the cuprates is that it occurs via a purely electronic mechanism involving spin excitations, and lattice degrees of freedom are supposed to be irrelevant, the relative isotope shifts $\Delta L_c/L_c = \Delta \lambda_{ab}^2/\lambda_{ab}^2 \approx 14\%$ and $\Delta d_s/d_s \approx 3\%$ uncover clearly the existence and relevance of the coupling between the superfluid and local lattice distortions. Recent site-selective oxygen isotope ($^{16}$O/$^{18}$O) effect measurements of the in-plane penetration depth in $Y_{0.6}Pr_{0.4}Ba_2Cu_3O_{7-\delta}$, using the muon-spin rotation ($\mu$SR) technique show that the distortions arise from the oxygen sites within the CuO$_2$ planes (100 % within error bar) $^{[41]}$. Potential candidates are then the Cu-O bond-stretching-type modes showing a temperature dependence, which parallels that of the superconductive order parameter $^{[50]}$.

To summarize we observed remarkable consistency between the scaling properties of the experimental data for a variety of cuprates and those characterizing 2D-QSI transitions. The important implication there is that in cuprates a non vanishing transition temperature and superfluid density in the ground state are unalterably linked to a finite anisotropy. Furthermore, the oxygen isotope effect on the in-plane penetration depth and the spatial extent of the superconducting grains revealed the coupling between local lattice distortions and superfluidity, while the lattice parameters remain essentially unaffected. These findings raise serious doubts that 2D models $^{[36]}$, neglecting granularity and local lattice distortions are potential candidates to explain superconductivity in cuprates.

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