QCD corrections up to order $\alpha_s^2$ to polarized quark production in $e^+e^-$ -annihilation

V. Ravindran  
Mehta Research Institute of Mathematics and Mathematical Physics,  
Chhatnag Road, Jhusi, Allahabad-211019,  
India.

W.L. van Neerven  
Instituut-Lorentz,  
University of Leiden,  
PO Box 9506, 2300 RA Leiden,  
The Netherlands.

June 2000

Abstract

We present the calculation of the order $\alpha_s^2$ contributions to the cross section $e^+ + e^- \to \bar{q} + q$ where the incoming leptons as well as one of the outgoing (anti) quarks are longitudinally polarized. The computation is carried out for massless quarks so that it can be applied to light flavour production ($u,d,s$). Unfortunately the massless quark approach is not valid for heavy flavour production like $c,b,t$ even in the case when the centre of mass energy $Q$ is much larger than the quark mass $m$. This is in contrast to unpolarized scattering where this approach works rather well for $Q \gg m$. The reason for this can be attributed to the anomalous terms which are characteristic of polarized coefficient functions. Furthermore we also computed the order $\alpha_s$ corrections to the longitudinal, transverse and normal polarizations of heavy flavours with $m \neq 0$. The
latter have been presented earlier in the literature except for some contributions which are shown here for the first time. It turns out that the corrections to the longitudinal and transverse (in the plane) polarization are rather small. However the order $\alpha_s$ corrections to the normal (out of the plane) polarization are large so that second order contributions (for $m \neq 0$) are needed to get a better determination of this quantity.

PACS: 12.38.-t, 12.38.Bx, 13.65.+i, 13.88.+e
Keywords: electron-positron collisions, polarization, heavy flavour production, QCD corrections
1 Introduction

Quark production in electron positron annihilation provides us with additional information about the constants appearing in the standard model of the electroweak and strong interactions. This in particular holds for heavy flavour production. An example is the electroweak mixing angle $\theta_W$ which has been very accurately extracted from the forward-backward asymmetry measured for bottom quarks at LEP and SLC. Besides a more accurate determination of these constants heavy quark production can also reveal some signatures of new physics. They might be observable when linear colliders are built which are giving access to much larger energies than those available until now. These energies are large enough to observe top anti-top production which will be one of the issues under study. In particular of interest is the polarization of this quark because the spin can be easily measured from its decay products. This is possible because the mass of the top-quark is so big that it can decay electroweakly before it undergoes hadronization. The measurement of the polarization provides us with new tests of the standard model and it might even signal new interactions which are not predicted by this model. An example is given in [2] where the effect of anomalous chromoelectric couplings of the gluon to the top quark on the longitudinal polarization is studied. Another important quantity is the polarization which is perpendicular to the plane spanned by the momenta of the incoming electron and outgoing quark. This so called normal polarization is a time reversal odd observable which has implications for the observation of CP-violation in the neutral current sector. Polarized heavy quark production has been studied on the Born level in [3], [4], [5]. In most of these calculations the top quark spin is decomposed in the helicity basis. However one can also make other choices like the beamline basis and the off-diagonal basis. In [6] one has shown that choosing the off-diagonal basis the top and anti-top quarks are produced in one unique spin configuration only which depends on the helicities of the incoming leptons. QCD corrections to quark production have been computed in several papers [4], [7]-[12]. The previous references deal with the production of heavy quarks only. For the decay process, among which the spin density matrix, see [13], [14]. The corrections in the case of massive quarks are rather complicated even in first order. In the case of the helicity basis the most of them are done by one group only see [8]-[11] except for a few corrections which will be presented in this paper. The QCD corrections in the case of the off-diagonal basis have been calculated in [12] as far as the soft and virtual gluon corrections are concerned. However the contribution due to hard gluon bremsstrahlung has not been calculated in this basis. Therefore the authors in [12] choose an alternative by putting one of the quarks in a special spin configuration whereas the other (anti-) quark and the gluon were taken to be inclusive. Furthermore one has neglected the width of the $Z$-boson and the normal polarization was not considered. Because of the importance of polarized quark production one should have an independent check on the calculations in the literature. Therefore we want to repeat the computations done in [8]-[11] and include some order $\alpha_s$ corrections which were not considered before. Because of the complexity of the expressions for the first order
corrections we try to write the results as compact as possible. The procedure is akin to the one followed in [15] in which the forward-backward asymmetry and the shape parameter could be written in a compact form. The same was also achieved for the longitudinal and transverse cross section in [16]. The main problem is to reduce the number of Spence functions because they lead to unnecessarily long expressions. Exploiting several relations like the Hill-identity [17] one can reduce them to a minimum. Furthermore we will present the cross section in such a way that it holds for the longitudinal, transverse and normal polarization at the same time. Finally we concentrate on the second order corrections to the longitudinal polarization of massless quarks. They can be derived from the first moment of the timelike coefficient functions computed in [18–20]. In the case of unpolarized scattering [15], [16] the massless quark approach can be also applied to heavy quarks provided the centre of mass (CM) energy is much larger than the mass of the quark. These second order estimates are very useful [21] as long as the exact calculations are not available because the latter are very difficult to compute. Unfortunately the massless quark approach does not work for heavy flavour production in the case of polarized scattering. In this approach the coefficient functions are computed under the condition that the quark mass \( m \) is put to zero at the start of the calculation like in [15], [16] (for the first order see [23], [24], [25]). One can also compute these functions in the so called massive quark approach with \( m \neq 0 \) and taking the limit \( m \to 0 \) afterwards. In the calculation of the polarized coefficient functions both approaches lead to different results contrary to what we have seen for unpolarized scattering. In other words the zero mass limit does not commute with the integrations. This anomaly is due to chiral symmetry breaking when the quark becomes massive and it was discovered for the first Bjorken sum rule which is given by the first moment of the longitudinal spin structure function \( g_1(x, Q^2) \) in [26], [27]. For timelike processes like \( e^+e^- \) collisions it was discussed in [9], [11], [22]. Here we would like to emphasize that this anomaly does not affect the longitudinal polarization of the light flavours even if we regularize the collinear divergences by giving the light quark a fictitious mass. It turns out that this anomaly also appears in the quark operator matrix element so that it will be removed by mass factorization (see [24]). Hence we obtain the same result as for \( m = 0 \) where one can use n-dimensional regularization. However for heavy flavours, where the mass has a definite meaning, a subtraction via the quark operator matrix element is not justified so that these anomalous terms are retained in the QCD corrections. Therefore in the case of heavy quarks the polarized coefficient functions have to be calculated for non zero masses. These calculations are far from trivial because one cannot apply the tricks which were so successful for the calculation in [28] of the order \( \alpha_s^2 \) corrections to \( \sigma_{\text{tot}}(e^+e^- \to \text{hadrons}) \) with massive quarks. This is because the timelike coefficient functions cannot be written as the imaginary part of a forward scattering amplitude which is an essential ingredient for the computations in [28]. The paper will be organized as follows. In the section 2 we define the kinematics and give an outline of the calculation procedure. We also present the formulae for the spin dependent and spin independent parts of the cross section. In section 3 we show the results for the radiative corrections computed up to
first order in the strong coupling constant $\alpha_s$ for massive quarks. Here we also include the contributions which are proportional to the width of the Z-boson. The corrections will be extended for the longitudinal polarization of massless quarks up to order $\alpha_s^2$. In section 4 we discuss the effects of these corrections on the longitudinal, transverse and normal polarization of the detected quark. The long expressions for the order $\alpha_s$ corrected quark structure functions $W_i(x, Q^2, m^2)$ are presented in Appendix A. After integrating these functions over the Bjorken variable $x$ we could express them into a compact form in Appendix B.
2 Kinematics for polarized $e^+ e^-$-annihilation

Polarized heavy quark production in electron-positron annihilation is given by the following process

$$e^+(q_{e^+}, \lambda_{e^+}) + e^-(q_{e^-}, \lambda_{e^-}) \rightarrow V(q) \rightarrow H(p, s) + "X" ,$$

(2.1)

where H denotes the heavy quark and "X" represents any inclusive multi-partonic state containing the heavy anti-quark and the light (anti-) quarks and gluons. Further $\lambda_i, q_i$ denote the spin and momenta of the incoming leptons and $s, p$ stand for the spin and momentum of the heavy quark H detected in the final state. If we denote the momentum of the virtual vector boson $V$ ($V = \gamma, Z$) by $q$, the centre of mass energy $Q$ is defined by

$$q^2 = Q^2 = (q_{e^+} + q_{e^-})^2 .$$

(2.2)

The differential cross section corresponding to reaction (2.1) equals

$$d\sigma = \frac{1}{2Q^2} dPS \sum_{V_1V_2} L_{\mu\nu}^{(V_1V_2)} P^{(V_1V_2)} M_{\mu\nu,(V_1V_2)}^{(V_1V_2)}$$

$$\equiv \frac{1}{2Q^2} dPS \left[ L_{\mu\nu}^{(\gamma\gamma)} P^{(\gamma\gamma)} M_{\mu\nu,(\gamma\gamma)}^{\gamma\gamma} + L_{\mu\nu}^{(ZZ)} P^{(ZZ)} M_{\mu\nu,(ZZ)}^{ZZ} \right.$$  

$$+ L_{\mu\nu}^{(\gamma Z)} P^{(\gamma Z)} M_{\mu\nu,(\gamma Z)}^{\gamma Z} + L_{\mu\nu}^{(\gamma Z)}* P^{(\gamma Z)*} M_{\mu\nu,(\gamma Z)*}^{\gamma Z} \left] . \right.$$

(2.3)

Here $dPS$ denotes the multi-parton phase space including the heavy quark and $L_{\mu\nu}^{(V_1V_2)}$ and $M_{\mu\nu,(V_1V_2)}^{(V_1V_2)}$ are the leptonic and partonic matrix elements respectively. Further $P^{(V_1V_2)}$ represents the product of the propagators corresponding to the vector bosons $V_1$ and $V_2$. The leptonic tensors are given by

$$L_{\mu\nu}^{(\gamma\gamma)} = 4\pi\alpha Q_e^2 \left[ (1 - \lambda_{e^+} + \lambda_{e^-}) l_{\mu\nu} + (\lambda_{e^+} - \lambda_{e^-}) \bar{l}_{\mu\nu} \right] ,$$

$$L_{\mu\nu}^{(ZZ)} = \frac{4\pi\alpha}{c_w^2 s_w^2} \left[ \left( -2g_V^V g_e^A (\lambda_{e^+} - \lambda_{e^-}) + (g_e^V + g_e^A^2)(1 - \lambda_{e^+} + \lambda_{e^-}) \right) l_{\mu\nu} \right.$$

$$\left. + \left( -2g_V^V g_e^A (1 - \lambda_{e^+} + \lambda_{e^-}) + (g_e^V + g_e^A^2)(\lambda_{e^+} - \lambda_{e^-}) \right) \bar{l}_{\mu\nu} \right] ,$$

$$L_{\mu\nu}^{(\gamma Z)} = -\frac{4\pi\alpha}{c_w s_w^2} Q_e \left[ \left( -g_V^V (1 - \lambda_{e^+} + \lambda_{e^-}) + g_e^A (\lambda_{e^+} - \lambda_{e^-}) \right) l_{\mu\nu} \right.$$

$$\left. + \left( -g_V^V (\lambda_{e^+} - \lambda_{e^-}) + g_e^A (1 - \lambda_{e^+} + \lambda_{e^-}) \right) \bar{l}_{\mu\nu} \right] ,$$

(2.4)
with

\[ l^{\mu\nu} = q^\mu e^- q^\nu e^- + q^\nu e^+ q^\mu e^- - q^\mu e^+ q^\nu e^- - g^\mu\nu, \]
\[ \tilde{l}^{\mu\nu} = i\epsilon^{\mu\nu\alpha\beta} q_{\alpha} e^- q_{\beta}. \]  

(2.5)

Notice that the polarizations of the positron and the electron indicated by \( \lambda_{e^+} \) and \( \lambda_{e^-} \) respectively are defined by

\[ v_{\lambda_{e^+}}(q_{e^+}) \bar{v}_{\lambda_{e^+}}(q_{e^+}) = \frac{1}{2} \bar{q}_{e^+}(1 + \lambda_{e^+} \gamma_5), \]
\[ u_{\lambda_{e^-}}(q_{e^-}) \bar{u}_{\lambda_{e^-}}(q_{e^-}) = \frac{1}{2} \bar{q}_{e^-}(1 - \lambda_{e^-} \gamma_5), \]  

(2.6)

where the incoming leptons are taken to be massless. In the case \( \lambda_i = 1 \) the leptons are polarized along the direction of their momenta. When \( \lambda_i = -1 \) the leptons are polarized opposite to the direction of their momenta.

On the Born level the vertex for the coupling of the vector boson \( V \) to the fermions will be denoted by

\[ \Gamma_{a,\mu}^{V,(0)} = i (v_a^V + a_a^V \gamma_5) \gamma_\mu, \quad V = \gamma, Z, \]
\[ v_a^\gamma = -e Q_a, \quad a_a^\gamma = 0, \]
\[ v_a^Z = -\frac{e}{c_w s_w} g_a^V, \quad a_a^Z = \frac{e}{c_w s_w} g_a^A. \]  

(2.7)

The electroweak coupling constants are given by

\[ \alpha = e^2 / 4\pi, \quad c_w = \cos \theta_W, \quad s_w = \sin \theta_W, \]
\[ g_a^V = \frac{1}{2} T_a^3 - s_w^2 Q_a, \quad g_a^A = -\frac{1}{2} T_a^3. \]  

(2.8)

The electroweak charges for the leptons are equal to

\[ Q_a = 0, \quad T_a^3 = \frac{1}{2}, \quad a = \nu_l, \bar{\nu}_l, \quad l = e, \mu, \tau, \]
\[ Q_a = -1, \quad T_a^3 = -\frac{1}{2}, \quad a = e^-, \mu^-, \tau^-, \]  

(2.9)

and for the quarks we obtain

\[ Q_a = \frac{2}{3}, \quad T_a^3 = \frac{1}{2}, \quad a = u, c, t, \]
\[ Q_a = -\frac{1}{3}, \quad T_a^3 = -\frac{1}{2}, \quad a = d, s, b. \]  

(2.10)
The squared propagators $\mathcal{P}^{(V_1V_2)}$ are given by

\[
\mathcal{P}^{\gamma\gamma} = \frac{1}{Q^4}, \quad \mathcal{P}^{ZZ} = \frac{1}{(Q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2},
\]

\[
\mathcal{P}^{\gamma Z} = \frac{(Q^2 - M_Z^2) + i M_Z \Gamma_Z}{Q^2((Q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2)}, \tag{2.11}
\]

where we have introduced a finite width $\Gamma_Z$ for the Z-boson.

The differential cross section for the inclusive reaction Eq. (2.1) can be written as

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2Q^2} \int_1^{\sqrt{\rho}} dx \sum_{(V_1V_2)} \mathcal{L}^{(V_1V_2)}_{\mu\nu} \mathcal{P}^{(V_1V_2)} W^{\mu\nu,(V_1V_2)},
\]

\[
d\Omega = d\cos \theta \, d\phi, \quad x = \frac{2p \cdot q}{Q^2}, \quad \rho = \frac{4m^2}{Q^2}. \tag{2.12}
\]

Here $\theta$ is the polar angle of the outgoing quark with respect to the beam direction of the electron and $m$ denotes the mass of the heavy quark. The partonic tensor can be expressed into structure functions $W_i^{(V_1V_2)} (i = 1 - 11)$ in the following way

\[
W^{\mu\nu,(V_1V_2)} = \frac{N_c}{8\pi^2} Q^2 \left( g^{\mu\nu} W_1^{(V_1V_2)} + \frac{p^\mu p^\nu}{Q^2} W_2^{(V_1V_2)} + mg^{\mu\nu} s \cdot q \frac{1}{Q^2} W_3^{(V_1V_2)} \right)
\]

\[
+ mp^\mu p^\nu s \cdot q W_4^{(V_1V_2)} + mp^\mu s + sp^\nu W_5^{(V_1V_2)}
\]

\[
+ \frac{1}{Q^2} i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta W_6^{(V_1V_2)} + m\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta s \cdot q W_7^{(V_1V_2)}
\]

\[
+ \frac{m}{Q^2} i\epsilon^{\mu\nu\alpha\beta} p_\alpha s_\beta W_8^{(V_1V_2)} + \frac{m}{Q^2} i\epsilon^{\mu\nu\alpha\beta} q_\alpha s_\beta W_9^{(V_1V_2)}
\]

\[
+ mp^\mu s^\nu - sp^\mu W_{10}^{(V_1V_2)} + \frac{m}{Q^4} i(p^\mu \epsilon^{\nu\alpha\beta\gamma} p_\alpha q_\beta s_\gamma \cdot q W_{11}^{(V_1V_2)}), \tag{2.13}
\]

where $N_c$ denotes the number of colours (in QCD $N_c = 3$). Finally we have to specify the spin of the quark $H(p, s)$. In the rest frame i.e. $p = (m, 0)$ the spin vector is given by

\[
s = (0, \hat{W}), \quad \hat{W} = (\hat{W}^1, \hat{W}^2, \hat{W}^3), \quad \text{with} \quad \hat{W}^2 = 1, \tag{2.14}
\]

\[
8
\]
so that \( s^2 = -1 \) and \( s \cdot p = 0 \). In the CM frame of the electron positron pair we introduce the notations
\[
q_{e^-} = \frac{Q}{2}(1, 0, 0, 1), \quad q_{e^+} = \frac{Q}{2}(1, 0, 0, -1),
\]
\[
p = (E, |\vec{p}| \hat{n}), \quad n = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta),
\]
\[
E = \frac{1}{2}Qx, \quad |\vec{p}| = \frac{1}{2}Q\sqrt{x^2 - \rho}.
\]
In this frame the spin four-vector of the quark can be found after an appropriate Lorentz transformation so that it becomes equal to
\[
s = \left( |\vec{p}| \frac{\hat{n} \cdot \hat{W}}{m}, \hat{W} + \frac{|\vec{p}|^2(\hat{n} \cdot \hat{W})\hat{n}}{m(m + E)} \right), \quad \text{with} \quad \hat{n}^2 = 1.
\]
Note that the spin four-vector in the CM frame depends on the energy of the quark and hence depends on the integration variable \( x \) in Eq. (2.12). The computation of the cross section in Eq. (2.12) involves the contraction of the symmetric and antisymmetric parts of the leptonic tensor \( L^{(V_1 V_2)}_{\mu\nu} \) with \( W^{\mu\nu,(V_1 V_2)} \). The contraction with the symmetric part equals
\[
\int dx \ l_{\mu\nu} \ W^{\mu\nu,(V_1 V_2)} = \frac{N_c Q^4}{8\pi^2} \left[ \frac{1}{8} (1 + \cos^2 \theta) \left( v_q^1 v_q^2 T_1 + a_q^1 a_q^2 T_2 \right) \right.
\]
\[
+ \frac{1}{4} \sin^2 \theta \left( v_q^1 a_q^2 T_3 + a^1_q a_q^2 T_4 \right)
\]
\[
+ \frac{1}{2} \left( v_q^1 a_q^2 + v_q^2 a_q^1 \right) T_7 + \frac{1}{2} \left( v_q^1 a_q^2 - v_q^2 a_q^1 \right) T_8
\]
\[
+ v_q^1 v_q^2 T_9 \right],
\]
(2.17)
and for the antisymmetric parts we get
\[
\int dx \ \tilde{l}_{\mu\nu} \ W^{\mu\nu,(V_1 V_2)} = \frac{N_c Q^4}{8\pi^2} \left[ \frac{1}{4} \left( v_q^1 a_q^2 + v_q^2 a_q^1 \right) T_5 \cos \theta + \frac{1}{4} \left( v_q^1 a_q^2 \right. \right.
\]
\[
- v_q^2 a_q^1 \right) T_6 \cos \theta + v_q^1 v_q^2 T_10 + a_q^1 a_q^2 T_{11}
\]
\[
+ \frac{1}{2} \left( v_q^1 a_q^2 + v_q^2 a_q^1 \right) T_{12}
\]
\[
+ \frac{1}{2} \left( v_q^1 a_q^2 - v_q^2 a_q^1 \right) T_{13} \right],
\]
(2.18)
In the expressions above $T_i$ ($i = 1 - 6$) represent the unpolarized structure functions. The polarized structure functions can be decomposed as follows

\[
T_7 = T_{7,T} W_T \cos \theta + W_L \left[ T_{7,L_1} (1 + \cos^2 \theta) + T_{7,L_2} \sin^2 \theta \right],
\]

\[
T_8 = T_{8,T} W_T \cos \theta + T_{8,L} W_L (1 + \cos^2 \theta),
\]

\[
T_9 = T_{9,N} W_N \cos \theta \sin \theta,
\]

\[
T_{10} = T_{10,T} W_T + T_{10,L} W_L \cos \theta,
\]

\[
T_{11} = T_{11,T} W_T + T_{11,L} W_L \cos \theta,
\]

\[
T_{12} = T_{12,N} W_N \cos \theta,
\]

\[
T_{13} = T_{13,N} W_N \sin \theta.
\]

(2.19)

Notice that the polarized structure functions $T_i$ ($i = 7 - 13$) are expanded in terms of the longitudinal ($W_L$), the transverse $W_T$ and the normal polarization $W_N$ of the quark. They are defined by

\[
W_L = \hat{n} \cdot \hat{W}, \quad W_T = \hat{W}^3 - \hat{n} \cdot \hat{W} \cos \theta, \quad W_N = \hat{W}^2 \cos \phi - \hat{W}^1 \sin \phi.
\]

(2.20)

If we decompose the partonic structure functions as follows

\[
W_i^{(V_1V_2)} = \nu_1 V_1 \nu_2 V_2 W_i^{\nu_2} + a_1 V_1 a_2 V_2 W_i^{a_2} + (\nu_1 V_1 a_2 V_2 + a_1 V_1 \nu_2 V_2) W_i^{\nu_2,a_2}
\]

\[
+ (\nu_1 V_1 a_2 V_2 - a_1 V_1 \nu_2 V_2) W_i^{[\nu_2,a_2]},
\]

(2.21)

one can express the quantities $T_i$ into integrals over the partonic structure functions $W_i$. For the unpolarized structure functions we obtain

\[
T_1 = -4 \int_{\sqrt{\rho}}^1 dx W_1^{v_2}(x, Q^2, m^2),
\]

(2.22)

\[
T_2 = -4 \int_{\sqrt{\rho}}^1 dx W_1^{a_2}(x, Q^2, m^2),
\]

(2.23)

\[
T_3 = \int_{\sqrt{\rho}}^1 dx \left( \frac{\alpha_x^2}{2} W_2^{v_2}(x, Q^2, m^2) - 2W_1^{v_2}(x, Q^2, m^2) \right),
\]

(2.24)

\[
T_4 = \int_{\sqrt{\rho}}^1 dx \left( \frac{\alpha_x^2}{2} W_2^{a_2}(x, Q^2, m^2) - 2W_1^{a_2}(x, Q^2, m^2) \right),
\]

(2.25)

\[
T_5 = 2 \int_{\sqrt{\rho}}^1 dx \alpha_x W_6^{[\nu_2,a_2]}(x, Q^2, m^2),
\]

(2.26)
The results for the longitudinal polarized structure functions are given by

\[ T_{7,L} = \int_{\sqrt{\rho}}^{1} dx \left( -\frac{\alpha_x}{2} W^{(v_q,a_q)}_3(x,Q^2,m^2) \right), \quad (2.28) \]

\[ T_{8,L} = \int_{\sqrt{\rho}}^{1} dx \left( -\frac{\alpha_x}{2} W^{(v_q,a_q)}_3(x,Q^2,m^2) + \frac{\alpha_x^3}{8} W^{(v_q,a_q)}_4(x,Q^2,m^2) \right. \]

\[ \left. + \frac{\alpha_x}{2} W^{(v_q,a_q)}_5(x,Q^2,m^2) \right), \quad (2.29) \]

\[ T_{10,L} = \int_{\sqrt{\rho}}^{1} dx \left( \frac{\alpha_x^2}{4} W^{a_q^2}_7(x,Q^2,m^2) - \frac{\rho}{4} W^{a_q^2}_8(x,Q^2,m^2) \right) \]

\[ - \frac{x}{2} W^{a_q^2}_9(x,Q^2,m^2) \right), \quad (2.31) \]

Similar expressions are found for the transverse polarized structure functions

\[ T_{7,T} = \int_{\sqrt{\rho}}^{1} dx \left( -\frac{\sqrt{\rho}}{2} \alpha_x W^{(v_q,a_q)}_5(x,Q^2,m^2) \right), \quad (2.33) \]

\[ T_{8,T} = \int_{\sqrt{\rho}}^{1} dx \left( -\frac{\sqrt{\rho}}{2} \alpha_x W^{(v_q,a_q)}_5(x,Q^2,m^2) \right), \quad (2.34) \]

\[ T_{10,T} = \int_{\sqrt{\rho}}^{1} dx \left( -\frac{\sqrt{\rho}}{4} x W^{a_q^2}_8(x,Q^2,m^2) - \frac{\sqrt{\rho}}{2} W^{a_q^2}_9(x,Q^2,m^2) \right) \]

\[ - \frac{x}{2} W^{a_q^2}_9(x,Q^2,m^2) \right), \quad (2.35) \]

and the results for the normal polarized structure functions can be expressed into the form

\[ T_{9,N} = \int_{\sqrt{\rho}}^{1} dx \left( i\frac{\sqrt{\rho}}{8} \alpha_x^2 W^{a_q^2}_{11}(x,Q^2,m^2) \right), \quad (2.37) \]
\[ T_{12,N} = \int_{\sqrt{\rho}}^{1} dx \left( \frac{i\sqrt{\rho}}{2} \alpha_x W_{10}^{\{v,q\}}(x, Q^2, m^2) \right), \quad (2.38) \]

\[ T_{13,N} = \int_{\sqrt{\rho}}^{1} dx \left( \frac{i\sqrt{\rho}}{2} \alpha_x W_{10}^{\{v,q\}}(x, Q^2, m^2) \right), \quad (2.39) \]

with

\[ \alpha_x = \sqrt{x^2 - \rho}. \quad (2.40) \]

Analogous to deep inelastic lepton hadron scattering the leading contributions to the unpolarized spin structure functions \( W_i \) with \( i = 1, 2, 6 \) in Eqs. (2.22)-(2.27) are of type twist two. The same holds for the polarized structure function \( W_3 \). However \( W_4, W_5 \) and \( W_7, W_9, W_{10}, W_{11} \) contain twist two as well as twist three contributions. Notice that \( W_8 \) is due to the fact that the electroweak currents are not conserved. Furthermore the twist three part cancels in the combinations

\[ xW_4 + 4W_5, \quad -xW_7 + 2W_9, \quad (2.41) \]

which means that the leading contributions in \( m^2/Q^2 \) to the longitudinal structure functions \( T_{i,L} \) in Eqs. (2.28)-(2.32) are of twist two only. Notice that \( W_8 \) in the equations above is multiplied by \( \rho = 4m^2/Q^2 \) so that this term vanishes in the limit \( m \to 0 \). The transverse parts \( T_{i,T} \) in Eqs. (2.33)-(2.36) receive contributions from twist two as well as twist three. The same also applies to the normal parts \( T_{i,N} \) in Eqs. (2.37)-(2.39). A second feature of the above equations is that in the limit \( m \to 0 \) all transverse and normal parts vanish whereas the longitudinal parts \( T_{i,L} \) \( (i = 7, 10, 11) \) and the unpolarized quantities \( T_i \) \( (i = 1 - 5) \) tend to non zero values. After having carried out the integration over \( x \) in Eq. (2.12) the differential cross section can be written as

\[ \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-}, W) = \frac{d\sigma_U}{d\Omega} + W_L \frac{d\sigma_L}{d\Omega} + W_T \frac{d\sigma_T}{d\Omega} + W_N \frac{d\sigma_N}{d\Omega}, \quad (2.42) \]

where \( U \) represents the unpolarized cross section with respect to the outgoing quark. The four cross sections on the right hand side of Eq. (2.42) can be decomposed according to the vector bosons which appear in the intermediate state i.e.

\[ \frac{d\sigma_k}{d\Omega}(\lambda_{e^+}, \lambda_{e^-}) = N_c \alpha^2 \left[ \frac{d\sigma^{(\gamma\gamma)}_k}{d\Omega} + \frac{d\sigma^{(ZZ)}_k}{d\Omega} + \frac{d\sigma^{(\gamma Z)}_k}{d\Omega} \right], \quad k = U, T, L, N. \quad (2.43) \]

The results for the photon-photon interference term can be written as \(^1\)

\[ \frac{d\sigma^{(\gamma\gamma)}_U}{d\Omega} = Q^2 P^{\gamma\gamma} Q_e^2 Q_q \left[ (1 - \lambda_{e^+}\lambda_{e^-}) \left( \frac{1}{8} (1 + \cos^2 \theta) T_1 + \frac{1}{4} \sin^2 \theta T_3 \right) \right], \quad (2.44) \]

\(^1\)Very often one replaces \( \alpha \) by \( G_F M_Z^2 \sqrt{\frac{3}{2}} s w^2 c w^2 / \pi \) (see e.g. [4], [9]).
\[
\frac{d\sigma^{(\gamma\gamma)}_L}{d\Omega} = Q^2 \mathcal{P}^{\gamma\gamma} Q_e^2 Q_q^2 \left[ (\lambda_{e^+} - \lambda_{e^-}) \cos \theta T_{10,L} \right],
\]
\(\text{(2.45)}\)

\[
\frac{d\sigma^{(\gamma\gamma)}_T}{d\Omega} = Q^2 \mathcal{P}^{\gamma\gamma} Q_e^2 Q_q^2 \left[ (\lambda_{e^+} - \lambda_{e^-}) T_{10,T} \right],
\]
\(\text{(2.46)}\)

\[
\frac{d\sigma^{(\gamma\gamma)}_N}{d\Omega} = Q^2 \mathcal{P}^{\gamma\gamma} Q_e^2 Q_q^2 \left[ (1 - \lambda_{e^+} \lambda_{e^-}) \cos \theta \sin \theta T_{9,N} \right].
\]
\(\text{(2.47)}\)

The Z-Z interference term receives contributions from

\[
\frac{d\sigma^{(ZZ)}_t}{d\Omega} = Q^2 \mathcal{P}^{ZZ} \frac{c^4}{s^4} \left[ \right. - 2 g_e V g_e A (\lambda_{e^+} - \lambda_{e^-}) + (g_e V^2 + g_e A^2) (1 - \lambda_{e^+} \lambda_{e^-}) \left. \right]
\]
\[
\times \left\{ \frac{1}{8} (1 + \cos^2 \theta) (g_q V^2 T_1 + g_q A^2 T_2) + \frac{1}{4} \sin^2 \theta (g_q V^2 T_3 + g_q A^2 T_4) \right\}
\]
\[
+ \left\{ 2 g_e V g_e A (1 - \lambda_{e^+} \lambda_{e^-}) - (g_e V^2 + g_e A^2) (\lambda_{e^+} - \lambda_{e^-}) \right\}
\]
\[
\times \left\{ \frac{1}{2} g_q V g_q A \cos \theta T_5 \right\},
\]
\(\text{(2.48)}\)

\[
\frac{d\sigma^{(ZZ)}_t}{d\Omega} = Q^2 \mathcal{P}^{ZZ} \frac{c^4}{s^4} \left[ \right. 2 g_e V g_e A (\lambda_{e^+} - \lambda_{e^-}) - (g_e V^2 + g_e A^2) (1 - \lambda_{e^+} \lambda_{e^-}) \right]
\]
\[
\times \left\{ g_q V g_q A \left( (1 + \cos^2 \theta) T_{7,L1} + \sin^2 \theta T_{7,L2} \right) \right\}
\]
\[
+ \left\{ - 2 g_e V g_e A (1 - \lambda_{e^+} \lambda_{e^-}) + (g_e V^2 + g_e A^2) (\lambda_{e^+} - \lambda_{e^-}) \right\}
\]
\[
\times \left\{ \cos \theta \left( g_q V^2 T_{10,L} + g_q A^2 T_{11,L} \right) \right\},
\]
\(\text{(2.49)}\)

\[
\frac{d\sigma^{(ZZ)}_t}{d\Omega} = Q^2 \mathcal{P}^{ZZ} \frac{c^4}{s^4} \left[ \right. 2 g_e V g_e A (\lambda_{e^+} - \lambda_{e^-}) - (g_e V^2 + g_e A^2) (1 - \lambda_{e^+} \lambda_{e^-}) \right]
\]
\[
\times \left\{ g_q V g_q A \cos \theta T_{7,T} \right\}
\]
\[
+ \left\{ - 2 g_e V g_e A (1 - \lambda_{e^+} \lambda_{e^-}) + (g_e V^2 + g_e A^2) (\lambda_{e^+} - \lambda_{e^-}) \right\}
\]
\[
\frac{d\sigma_{N}^{(ZZ)}}{d\Omega} = Q^2 \mathcal{P}_{c_w^4, s_w^4}^{ZZ} \left[ -2 g_e V g_e A (\lambda_{e^+} - \lambda_{e^-}) + (g_e V^2 + g_e A^2)(1 - \lambda_{e^+} \lambda_{e^-}) \right] \\
\times \left\{ g_q V^2 \cos \theta \sin \theta T_{9,N} \right\} \\
+ \left\{ 2 g_e V g_e A (1 - \lambda_{e^+} \lambda_{e^-}) - (g_e V^2 + g_e A^2)(\lambda_{e^+} - \lambda_{e^-}) \right\} \\
\times \left\{ g_q g_q \sin \theta T_{12,N} \right\}.
\]

(2.50)

The photon-Z interference term consists of the following parts

\[
\frac{d\sigma_{L}^{(\gamma Z)}}{d\Omega} = 2Q^2 R_e \mathcal{P}_{c_w^2, s_w^2}^{\gamma Z} Q_e Q_q \left[ - g_e V (1 - \lambda_{e^+} \lambda_{e^-}) - g_e A (\lambda_{e^+} - \lambda_{e^-}) \right] \\
\times \left\{ \frac{1}{8} (1 + \cos^2 \theta) g_q V T_1 + \frac{1}{4} \sin^2 \theta g_q V T_3 \right\} \\
\times \left\{ - g_e V (\lambda_{e^+} - \lambda_{e^-}) + g_e A (1 - \lambda_{e^+} \lambda_{e^-}) \right\} \left[ \frac{1}{4} g_q^2 \cos \theta T_5 \right] \\
+ 2Q^2 \text{Im} \mathcal{P}_{c_w^2, s_w^2}^{\gamma Z} Q_e Q_q \left[ g_e V (\lambda_{e^+} - \lambda_{e^-}) - g_e A (1 - \lambda_{e^+} \lambda_{e^-}) \right] \\
\times \frac{1}{4} g_q^2 \cos \theta \text{Im} T_6,
\]

(2.51)

\[
\frac{d\sigma_{L}^{(\gamma Z)}}{d\Omega} = 2Q^2 R_e \mathcal{P}_{c_w^2, s_w^2}^{\gamma Z} Q_e Q_q \left[ - g_e V (1 - \lambda_{e^+} \lambda_{e^-}) - g_e A (\lambda_{e^+} - \lambda_{e^-}) \right] \\
\times \left\{ \frac{1}{2} g_q A \left( (1 + \cos^2 \theta) T_{7,L_1} + \sin^2 \theta T_{7,L_2} \right) \right\} \\
+ \left\{ g_e V (\lambda_{e^+} - \lambda_{e^-}) - g_e A (1 - \lambda_{e^+} \lambda_{e^-}) \right\} g_q V \cos \theta T_{10,L} \\
+ 2Q^2 \text{Im} \mathcal{P}_{c_w^2, s_w^2}^{\gamma Z} Q_e Q_q \left[ g_e V (1 - \lambda_{e^+} \lambda_{e^-}) - g_e A (\lambda_{e^+} - \lambda_{e^-}) \right]
\]

(2.52)
\[ \times \left\{ \frac{1}{2} g_q^A (1 + \cos^2 \theta) \text{Im} \mathcal{T}_{8,L} \right\} , \]  

\( \frac{d\sigma^{(\gamma Z)}}{d\Omega} = 2Q^2 \text{Re} \mathcal{P}^{\gamma Z} Q_e Q_q \left[ \left\{ - g_e^V (1 - \lambda_e^+ \lambda_e^-) + g_e^A (\lambda_e^+ - \lambda_e^-) \right\} \right] \\
\times \left\{ \frac{1}{2} g_q^A \cos \theta \mathcal{T}_{T,T} \right\} \\
+ \left\{ g_e^V (\lambda_e^+ - \lambda_e^-) - g_e^A (1 - \lambda_e^+ \lambda_e^-) \right\} g_q^T \mathcal{T}_{10,T} \right] \\
+ 2Q^2 \text{Im} \mathcal{P}^{\gamma Z} Q_e Q_q \left[ \left\{ g_e^V (1 - \lambda_e^+ \lambda_e^-) - g_e^A (\lambda_e^+ - \lambda_e^-) \right\} \right] \\
\times \left\{ \frac{1}{2} g_q^A \cos \theta \text{Im} \mathcal{T}_{8,T} \right\} , \]  

\( \frac{d\sigma^{(\gamma Z)}}{d\Omega} = 2Q^2 \text{Re} \mathcal{P}^{\gamma Z} Q_e Q_q \left[ \left\{ g_e^V (1 - \lambda_e^+ \lambda_e^-) - g_e^A (\lambda_e^+ - \lambda_e^-) \right\} \right] \\
\times \left\{ g_q^V \cos \theta \sin \theta \mathcal{T}_{9,N} \right\} \\
+ \left\{ - g_e^V (\lambda_e^+ - \lambda_e^-) + g_e^A (1 - \lambda_e^+ \lambda_e^-) \right\} \frac{1}{2} g_q^A \sin \theta \mathcal{T}_{12,N} \right] \\
+ 2Q^2 \text{Im} \mathcal{P}^{\gamma Z} Q_e Q_q \left[ \left\{ g_e^V (\lambda_e^+ - \lambda_e^-) - g_e^A (1 - \lambda_e^+ \lambda_e^-) \right\} \right] \\
\times \frac{1}{2} g_q^A \sin \theta \text{Im} \mathcal{T}_{13,N} \right] . \]  

The Born contributions (zeroth order in \( \alpha_s \)) to the unpolarized quantities denoted by \( \mathcal{T}_i^{(0)} \) (\( i = 1, 2, 3, 5 \)) can be found in \[3\], \[9\], \[15\]. The longitudinal \( \mathcal{T}_{7,L}^{(0)}, \mathcal{T}_{10,L}^{(0)} \) (\( i = 10, 11 \)) and the transverse polarized structure functions \( \mathcal{T}_{7,T}^{(0)} \) and \( \mathcal{T}_{10,T}^{(0)} \) were calculated in \[4\]. In the last reference one also finds the Born contribution to the normal polarized quantity given by \( \text{Im} \mathcal{T}_{13,N}^{(0)} \). Notice that the quantities \( \mathcal{T}_i \) not mentioned above all vanish in the Born approximation. The first order QCD contributions to \( \mathcal{T}_{9,N} \) and \( \mathcal{T}_{12,N} \) are also computed in \[4\] but the corrections to the other structure functions \( \mathcal{T}_i \) were neglected. The latter are computed in \[8\], \[9\], \[11\] (longitudinal) and \[10\] (transverse and normal). The exceptions are the order \( \alpha_s \) contributions to \( \text{Im} \mathcal{T}_6 \) and \( \text{Im} \mathcal{T}_{8,L} \) which will be presented in this paper for the first time. The second order QCD
corrections are not known yet but for light quarks like $u, d, s$ they can be computed for the longitudinal polarized structure functions $\mathcal{T}_{i,L} (i = 7, 10, 11)$ and the results are shown in the next section. Notice that the transverse and normal polarized structure functions $\mathcal{T}_{i,T}$ and $\mathcal{T}_{i,N}$ vanish for massless quarks.
3 Computation of the corrections up to order $\alpha_s^2$.

The partonic structure tensor defined in Eq. (2.13) is represented by the following perturbation series

$$ W_{\mu\nu}^{(V_1V_2)} = \sum_{k=0}^{\infty} \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^k W_{\mu\nu}^{(V_1V_2),(k)}, \quad (3.1) $$

where $\mu$ denotes the renormalization scale. The same expression for the perturbation series holds for the structure functions $W_i^{(V_1V_2)}$ and the functions $T_i$ in Eqs. (2.21)-(2.39). The structure tensor is determined by the following reaction

$$ V(q) \rightarrow H(p) + \bar{H}(p_1) + l(p_2) \cdots l(p_n), \quad (3.2) $$

where $l(p_i)$ ($i = 2, 3, \ldots, n$) represent the momenta of the light partons ((anti-)quarks and gluons) in the final state. If the matrix element of the process above is given by $M_{\mu\nu}^{(V_1V_2)}$ then the partonic tensor is obtained by integration over the multi-partonic phase space which also includes the heavy anti-quark i.e.

$$ W_{\mu\nu}^{(V_1V_2)} = \prod_{i=1}^{n} \int \frac{d^3p_i}{(2\pi)^32p_i^0} (2\pi)^4\delta^{(4)} \left( q - p - \sum_{j=1}^{n} p_j \right) M_{\mu\nu}^{(V_1V_2)}. \quad (3.3) $$

The computation of the phase space integrals is carried out in the rest frame of the vector boson $V$ and proceeds in the way as is given in [18] where the integrals are performed up to order $\alpha_s^2$ in the case of massless quarks. It can be easily extended for massive quarks up to order $\alpha_s$. However in second order the integrals for massive quarks become very tedious and we are only able to present the results for massless quarks.

In the Born approximation the zeroth order structure functions $W_i^{(V_1V_2),(0)}$ are determined by the following process

$$ V(q) \rightarrow H(p) + \bar{H}(p_1). \quad (3.4) $$

In this case the phase space integral is trivial and the partonic structure functions, presented in Eq. (A.1), are proportional to $\delta(1-x)$. From Eqs. (2.22)-(2.39) we obtain for the unpolarized structure functions

$$ T_1^{(0)} = \beta, \quad T_2^{(0)} = \beta^3, \quad T_3^{(0)} = \frac{1}{2}\beta^3, \quad T_4^{(0)} = 0, \quad T_5^{(0)} = \beta^2, \quad \text{Im} T_6^{(0)} = 0, \quad \beta = \sqrt{1-\rho}. \quad (3.5) $$

The longitudinal polarized quantities in the Born approximation are given by

$$ T_{7,L1}^{(0)} = -\frac{\beta^2}{4}, \quad T_{7,L2}^{(0)} = 0, \quad \text{Im} T_{8,L}^{(0)} = 0, \quad T_{10,L}^{(0)} = -\frac{\beta}{4}, \quad T_{11,L}^{(0)} = -\frac{1}{4}\beta^3. \quad (3.6) $$
The transverse polarized quantities equal
\[ T_{r,T}^{(0)} = -\frac{\beta^2}{4} \sqrt{\rho}, \quad \text{Im} \, T_{s,T}^{(0)} = 0, \quad T_{10,T}^{(0)} = -\frac{\beta}{4} \sqrt{\rho}, \quad T_{11,T}^{(0)} = 0. \quad (3.7) \]
For the normal polarized quantities we get
\[ T_{9,N}^{(0)} = 0, \quad T_{12,N}^{(0)} = 0, \quad \text{Im} \, T_{13,N}^{(0)} = \frac{\beta^2}{4} \sqrt{\rho}, \quad \beta = \sqrt{1 - \rho}. \quad (3.8) \]
The order \( \alpha_s \) corrections originate from the one-loop contributions to the Born reaction in Eq. (3.4) and the gluon bremsstrahlung process
\[ V(q) \rightarrow H(p) + \bar{H}(p_1) + g(p_2). \quad (3.9) \]
To facilitate the calculation we split the partonic tensor into a virtual (VIRT), a soft gluon (SOFT) and a hard (HARD) gluon part i.e.
\[ W_{\mu\nu}^{(V)}(V_1 V_2) = W_{\mu\nu}^{\text{VIRT}}(V_1 V_2) + W_{\mu\nu}^{\text{SOFT}}(V_1 V_2) + W_{\mu\nu}^{\text{HARD}}(V_1 V_2), \quad (3.10) \]
Starting with the virtual contribution the order \( \alpha_s \) corrected vector boson quark vertex reads
\[ \Gamma_{\alpha,\mu}^{V,(1)} = i \left[ \gamma_\mu (1 + C_1) \, v_\mu^V + \gamma_5 \gamma_\mu (1 + C_1 + 2C_2) \, a_\mu^V \right. \\
\left. + \frac{2p_\mu - q_\mu}{2m} C_2 \, v_\mu^V + \gamma_5 \frac{p_\mu}{2m} C_3 \, a_\mu^V \right], \quad V = \gamma, Z. \quad (3.11) \]
Here the functions \( C_i \) are given by
\[
\begin{align*}
\text{Re} \, C_1 &= \frac{\alpha_s}{4\pi} C_F \left[ - \frac{2 - \rho}{\beta} \ln(t) + 2 \right. \ln \left( \frac{\lambda^2}{m^2} \right) \left. - 4 - 3\beta \ln(t) \right. \\
&\quad \left. + \frac{2 - \rho}{\beta} \left\{ - \frac{1}{2} \ln^2(t) + 2 \ln(t) \ln(1 - t) + 2Li_2(t) + \frac{\pi^2}{3} \right\} \right], \\
\text{Re} \, C_2 &= -\frac{\alpha_s}{4\pi} C_F \left[ \frac{\rho}{\beta} \ln(t) \right], \\
\text{Re} \, C_3 &= \frac{\alpha_s}{4\pi} C_F \left[ (\rho - 3) \frac{\rho}{\beta} \ln(t) - 2\rho \right], \\
\text{Im} \, C_1 &= \frac{\alpha_s}{4\pi} C_F \left[ - \frac{2 - \rho}{\beta} \ln \left( \frac{\lambda^2}{m^2} \right) - 3\beta + \frac{2 - \rho}{\beta} \left\{ - \ln(t) \right\} \right], \\
\end{align*}
\]
\begin{align*}
\ln(1 - t)\} \\
\text{Im } C_2 &= \frac{\alpha_s}{4\pi} \pi C_F \left[ -\frac{\rho}{\beta} \right], \\
\text{Im } C_3 &= \frac{\alpha_s}{4\pi} \pi C_F \left[ (3 - \rho) \frac{\rho}{\beta} \right], \\
t &= \frac{1 - \beta}{1 + \beta},
\end{align*}

where \( C_F \) denotes the colour factor which is equal to \( C_F = (N_c^2 - 1)/2N_c \). In the equation above we have introduced a fictitious mass of the gluon \( \lambda \) which is needed to regularize the infrared divergence. The ultraviolet divergences, which cancel in the expressions above, can be regularized with the cut-off method \( (\int d^4k) \) which is known from old textbooks on QED. In this way one avoids the intricacies of the \( \gamma_5 \)-matrix prescription \( [23] \) which is characteristic of n-dimensional regularization. The mass renormalization is carried out in the pole mass scheme (on-shell renormalization). However one can also choose the \( \overline{\text{MS}} \)-scheme and carry out the analysis of the radiative corrections using the running mass (see e.g. \( [11] \)). The contributions from the Born reaction and the one-loop corrections are given by

\begin{align*}
W_{\mu\nu}^{(V_1V_2),(0)} + W_{\mu\nu}^{(V_1V_2),\text{VIRT}} &= \frac{N_c \beta}{32\pi^2} T R \left[ \frac{(1 + \gamma_5\gamma)}{2} (\hat{p} + m) \Gamma^{V_1,\mu}_{\gamma,\mu} (\hat{p} - m) \tilde{\Gamma}^{V_2,\nu}_{\gamma,\nu} \right] \\
&\quad \times \delta(1 - x), \\
W_{\mu\nu}^{(V_1V_2),\text{SOFT}} &= \frac{N_c \beta}{16\pi^2} S^{\text{SOFT}} T R \left[ \frac{(1 + \gamma_5\gamma)}{2} (\hat{p} + m) \Gamma^{V_1,(0)}_{\gamma,\mu} (\hat{p} - m) \Gamma^{V_2,(0)}_{\gamma,\nu} \right] \delta(1 - x),
\end{align*}

with \( \tilde{\Gamma}_\mu = \gamma_0 \Gamma^\gamma_\mu \gamma_0 \). The soft gluon part of the partonic tensor is given by

\begin{align*}
S^{\text{SOFT}} &= -\frac{\alpha_s}{8\pi^2} C_F \int_0^\omega d^3p_2 \left( \frac{m^2}{(p_1 \cdot p_2)^2} - \frac{m^2}{(p \cdot p_2)^2} - \frac{2p \cdot p_1}{(p_1 \cdot p_2)(p \cdot p_2)} \right).
\end{align*}

The above integral will be evaluated in the rest frame of the vector boson \( V \) where \( \omega \) is a cut off on the gluon energy \( E_2 \) which is taken to be much smaller than the quark mass \( m \) (i.e. \( \omega \ll m \)). This is the reason why like in the case of the contribution from the Born reaction and the virtual corrections the soft gluon part is proportional
to \( \delta(1 - x) \). The result is

\[
S_{\text{SOFT}} = \frac{\alpha_s}{4\pi} C_F \left\{ -2 \ln\left(\frac{4\omega^2}{\lambda^2}\right) + 2 - \frac{2 - \rho}{\beta} \left\{ \ln(t) - 2 \mathcal{L} i_2(t) - 2 \mathcal{L} i_2(-t) + \ln\left(\frac{4\omega^2}{\lambda^2}\right) \ln(t) - 2 \ln(t) \ln(1 - t) - 2 \ln(t) \ln(1 + t) + \ln^2(t) + \frac{\pi^2}{6} \right\} \right\}.
\]

Addition of the virtual \((V)\) and soft \((S)\) gluon parts leads to the expressions \(W^{(V_1V_2),V+S}_i\) in Eq. (A.3). Note that in the combination \(C_{V+S} = C_1 + S_{\text{SOFT}}\) the gluon regulator mass \(\lambda\) vanishes i.e.

\[
C_{V+S} = \frac{\alpha_s}{4\pi} C_F \left\{ - \left(2 + \frac{2 - \rho}{\beta} \ln(t)\right) \ln\left(\frac{4\omega^2}{m^2}\right) - \frac{2 - \rho}{\beta} \left\{ -4 \mathcal{L} i_2(t) - 2 \mathcal{L} i_2(-t) - 4 \ln(t) \ln(1 - t) - 2 \ln(t) \ln(1 + t) + \frac{3}{2} \ln^2(t) - \frac{\pi^2}{2} \right\} - 2 - \frac{5 - 4\rho}{\beta} \ln(t) \right\}.
\]

The hard gluon part of the partonic tensor can be calculated in a straightforward way and the results are given in Eq. (A.4). Notice that the upper bound on the integral over \(x\) in Eq. (2.12) for \(W^{(V_1V_2),HARD}_i\) is given by \(1 - 2m\omega/Q^2\) so that the energy cut off \(\omega\) is cancelled between \(W^{(V_1V_2),V+S}_i\) and \(W^{(V_1V_2),HARD}_i\). The results for \(T_i\) are given in Eqs. (B.1)-(B.18). We tried to shorten these expression as much as possible by minimizing the number of independent polylogarithms. Note that there is a difference in sign between \(T_{12,N}\) in Eq. (B.17) and the equivalent expression in Eq. 33 of [10]. However substitution of \(T_{6,N}\) (Eq. (B.16)) and \(T_{12,N}\) (Eq. (B.17)) in the cross section of Eq. (2.51) leads to the same result as presented in Eq. (16) in [4].

The computation of the order \(\alpha_s^2\) corrections for massive quarks is extremely tedious so that it has not been performed yet. The reason is that the partonic structure functions for timelike processes like \(e^+e^-\)-collisions cannot be written as the imaginary part of an amplitude. This is in contrast to deep inelastic scattering where \(q^2\) is spacelike or the total cross section \(\sigma_{tot}(e^+e^- \rightarrow \text{hadrons})\). In the latter case the cross section can be written as the imaginary part of the hadronic vacuum polarization function so that one is able to apply advanced methods to compute these type of quantities (see e.g. [28]). However for massless quarks we can compute the second order corrections to the partonic structure functions contributing to the longitudinal polarization. This is feasible using conventional techniques as is shown in [18]-[20]. In the case of massless quarks the computation is facilitated because one has the following relations

\[
W^{v_2}_{1} = W^{q_2}_{1} = -\frac{x}{2} W^{v_{a_2}}_{3},
\]
\[ W_{v^2} = W_2 a^2 = -\frac{x}{2} W_4 v a_q - 2 W_5 v a_q, \]
\[ W_6 v a_q = -\frac{x}{2} W_7 v^2 a^2 + W_9 v^2 a^2. \] (3.18)

which follow from Eq. (2.13) by putting \( s = p/m \). Since \( m = 0 \) the contributions coming from the vector currents \( v_q^2 \) lead to the same answer as those originating from the axial-vector currents given by \( a_q^2 \). In this case the matrix element only contains \( \gamma \)-matrices which anti-commute with the \( \gamma_5 \)-matrix. It also explains why the components proportional to \( v_q^2 \) and \( a_q^2 \) which show up in the unpolarized structure functions are the same as the contributions multiplying \( v a_q \) appearing in the polarized structure functions. The relations in Eq. (3.18) break down for \( m \neq 0 \) as will be discussed at the end of this section. The structure functions \( W_i \) in Eq. (2.13) can be related to the fragmentation functions \( F_i^h(x, Q^2) \) (unpolarized see [18], [19]) and \( g_i^h(x, Q^2) \) (polarized see [20]) defined for the process \( e^+ e^- \rightarrow h^+ X^- \). Here \( h \) represents a hadron in the final state which originates from a light (anti-) quark. Using the relations in Eqs. (2.22)-(2.32), the quantities \( T_i \) can be expressed into the first moments of the non-singlet quark coefficient functions as follows
\[ T_1 = T_2 = \int_0^1 dx \ C_{1,q}(x, \frac{Q^2}{\mu^2}), \] (3.19)
\[ T_3 = T_4 = \frac{1}{2} \int_0^1 dx \ \left( C_{1,q}(x, \frac{Q^2}{\mu^2}) - C_{2,q}(x, \frac{Q^2}{\mu^2}) \right), \] (3.20)
\[ T_5 = \int_0^1 dx \ C_{3,q}(x, \frac{Q^2}{\mu^2}), \] (3.21)
\[ T_{7,L_1} = -\frac{1}{4} \int_0^1 dx \ \Delta C_{5,q}(x, \frac{Q^2}{\mu^2}), \] (3.22)
\[ T_{7,L_2} = -\frac{1}{4} \int_0^1 dx \ \left( \Delta C_{5,q}(x, \frac{Q^2}{\mu^2}) - \Delta C_{4,q}(x, \frac{Q^2}{\mu^2}) \right), \] (3.23)
\[ T_{10,L} = T_{11,L} = -\frac{1}{4} \int_0^1 dx \ \Delta C_{1,q}(x, \frac{Q^2}{\mu^2}). \] (3.24)

Here \( \mu \) represents the mass factorization scale as well as the renormalization scale. However since all coefficient functions above are of the non-singlet type the dependence on the factorization scale vanishes while taking the first moments so that \( T_i \) only depends on the renormalization scale. Because of the relations in Eq. (3.18) the coefficient functions above are not mutually independent. In [20], it was demonstrated that one could derive the following relations between the polarized \( \Delta C_{i,q} \) (\( i = 1, 4, 5 \)) and the unpolarized \( C_{i,q} \) (\( i = 1, 2, 3 \)) coefficient functions. They are given by
\[ \Delta C_{1,q} = C_{3,q}, \ \Delta C_{4,q} = C_{2,q}, \ \Delta C_{5,q} = C_{1,q}, \] (3.25)
which means that in the case of massless quarks all quantities \( T_i \) are determined by three independent coefficient functions only. The order \( \alpha_s^2 \) contributions originate from the processes

\[
V \to H + H + g + g ,
\]

\[
V \to H + \bar{H} + H + \bar{H} ,
\]

\[
V \to H + \bar{H} + q + q ,
\]

which also includes the two-loop corrections to reaction (3.4) and the one-loop corrections to process (3.3). The corresponding coefficient functions have been computed in [18]-[20] for massless quarks and their first moments are presented in [15]. If we also include the contributions from the lower order processes in (3.4) and (3.9) (see also [23]-[25]) we obtain for massless quarks up to order \( \alpha_s^2 \)

\[
T_{m=0}^{m=0} = T_{m=0}^{n=0} = 1 + \frac{\alpha_s(\mu^2)}{4\pi} C_F [1] + \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \left[ C_F^2 \left( \frac{7}{2} \right) + C_A C_F \left( -\frac{11}{3} \ln \left( \frac{Q^2}{\mu^2} \right) \right) \right] + \frac{347}{18} - 44\zeta(3) + n_f C_F T_f \left( \frac{4}{3} \ln \left( \frac{Q^2}{\mu^2} \right) - \frac{62}{9} + 16\zeta(3) \right) ,
\]

\[
T_{m=0}^{n=0} = T_{m=0}^{n=0} = \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \left[ C_F^2 (-5) + C_A C_F \left( -\frac{22}{3} \ln \left( \frac{Q^2}{\mu^2} \right) \right) \right] + \frac{380}{9} + n_f C_F T_f \left( \frac{8}{3} \ln \left( \frac{Q^2}{\mu^2} \right) - \frac{136}{9} \right) ,
\]

\[
T_{m=0}^{m=0} = 1 + \left( \frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \left[ C_A C_F (-44\zeta(3)) + n_f C_F T_f (16\zeta(3)) \right] ,
\]

\[
T_{m=0}^{m=0} = -\frac{1}{4} T_{m=0}^{m=0} ,
\]

\[
T_{m=0}^{m=0} = -\frac{1}{2} T_{m=0}^{m=0} ,
\]

\[
T_{m=0}^{m=0} = T_{m=0}^{m=0} = -\frac{1}{4} T_{m=0}^{m=0} ,
\]

where we have chosen the \( \overline{MS} \)-scheme for the coupling constant renormalization. In the above equations the colour factors are given by \( C_A = N_c \) and \( T_f = 1/2 \) (for \( C_F \) see below Eq. (3.12)). Furthermore \( n_f \) is the number of light flavours and the scale \( \mu \) represents the renormalization scale. Notice that the sum \( T_1 + T_3 \) is equal to \( R_{e^+e^-} \) which is defined by \( \sigma_{tot}(e^+ + e^- \to \text{hadrons})/\sigma_{tot}(e^+ + e^- \to \mu^+ + \mu^-) \). Finally the
results obtained for the spin dependent quantities $\mathcal{T}_{7,L_1}$, $\mathcal{T}_{10,L}$, $\mathcal{T}_{11,L}$ depend on the way the limit $m \to 0$ is taken before or after the integration over the momenta. This is revealed by a comparison of the order $\alpha_s$ corrections in Eqs. (3.30)-(3.32) with those obtained for the same quantities in Eqs. (B.7), (B.10), (B.11). We find the following relation

$$\mathcal{T}^{(1),m\to 0}_{7,L_1} + 2\mathcal{T}^{(0),m\to 0}_{7,L_1} = \mathcal{T}^{(1),m=0}_{7,L_1}, \quad \mathcal{T}^{(1),m\to 0}_{i,L} + 2\mathcal{T}^{(0),m\to 0}_{i,L} = \mathcal{T}^{(1),m=0}_{i,L}, \quad i = 10, 11.$$  

\hspace{1cm} (3.33)

Therefore we expect that the same difference between the massless and massive quark approach will happen in higher order. This difference originates from the property that the $\gamma_5$-matrix commutes with the mass term in the trace (see e.g. Eq. (3.13)) contrary to the ordinary gamma-matrix with which it anti-commutes. Hence the relations between the polarized and the unpolarized partonic structure functions in Eq. (3.18) will break down for the subleading terms. The origin of this phenomenon is explained in \cite{11}. It does not show up in the relations between the $v^2_q$- and $a^2_q$-parts of the unpolarized structure functions. Therefore only the coefficient functions $\Delta C_i$ ($i = 1, 4, 5$) in Eqs. (3.22)-(3.24) will get an anomalous term while going from the massless to the massive quark approach. It also turns out that this term cancels in the combination $\Delta C_5 - \Delta C_4$ so that $\mathcal{T}_{7,L_2}$ in Eq. (3.23) will be unaffected at least up to order $\alpha_s$. It is unlikely that the latter will also hold in higher order of perturbation theory.

\footnote{A similar phenomenon has been observed in polarized deep inelastic lepton-hadron scattering for the structure function $\Delta g_1$ see \cite{26}.}
4 Results

In this section we will present the effect of the higher order QCD corrections to the polarization of the quark in $e^+ e^-$-collisions. When both the incoming leptons as well as the outgoing quark are unpolarized the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \sum_{\lambda_{e^+},\lambda_{e^-}} \left( \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-}, W) + \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-}, -W) \right) = \frac{1}{2} \sum_{\lambda_{e^+},\lambda_{e^-}} \frac{d\sigma_U}{d\Omega}.$$ \hspace{1em} (4.1)

When the incoming leptons are unpolarized but the quark is polarized the asymmetry is defined by

$$\frac{d\sigma_W}{d\Omega} = \frac{1}{4} \sum_{\lambda_{e^+},\lambda_{e^-}} \left( \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-}, W) - \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-}, -W) \right) = \frac{1}{2} \sum_{\lambda_{e^+},\lambda_{e^-}} \left( W_L \frac{d\sigma_L}{d\Omega} + W_T \frac{d\sigma_T}{d\Omega} + W_N \frac{d\sigma_N}{d\Omega} \right).$$ \hspace{1em} (4.2)

In the case the electron is polarized the asymmetry becomes equal to

$$\frac{d\sigma_W(\lambda_{e^-})}{d\Omega} = \frac{1}{2} \sum_{\lambda_{e^+}} \left( \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-}, W) - \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-}, -W) \right) = \sum_{\lambda_{e^+}} \left( W_L \frac{d\sigma_L}{d\Omega} + W_T \frac{d\sigma_T}{d\Omega} + W_N \frac{d\sigma_N}{d\Omega} \right).$$ \hspace{1em} (4.3)

When the positron is polarized and the electron is unpolarized we have the relation

$$\frac{d\sigma_W(\lambda_{e^+})}{d\Omega} = \frac{d\sigma_W(-\lambda_{e^-})}{d\Omega}.$$ \hspace{1em} (4.4)

The polarizations of the quark are defined by

$$P_k = W_k \sum_{\lambda_{e^+},\lambda_{e^-}} \frac{d\sigma_k}{d\Omega}, \quad P_k(\lambda_{e^-}) = W_k \sum_{\lambda_{e^+}} \frac{d\sigma_k}{d\Omega}, \quad k = L, T, N.$$ \hspace{1em} (4.5)

For the longitudinally polarized quark we choose the following spin vector

$$\hat{W} = \hat{n}, \quad \rightarrow \quad W_L = 1, \quad W_T = 0, \quad W_N = 0.$$ \hspace{1em} (4.6)

For the transversely polarized quark the spin vector is chosen to be in the plane spanned by the electron and the quark momenta

$$\hat{W} = \frac{\hat{n} \times (\hat{n} \times \hat{q}_{e^-})}{|\hat{n} \times (\hat{n} \times \hat{q}_{e^-})|}, \quad \rightarrow \quad W_L = 0, \quad W_T = -\sin \theta, \quad W_N = 0.$$ \hspace{1em} (4.7)
For the quark polarisation which is directed normal to the plan spanned by the electron and the quark momenta we make the choice

\[ \hat{W} = \frac{\hat{n} \times \hat{q}_e^-}{|\hat{n} \times \hat{q}_e^-|}, \quad \rightarrow \quad W_L = 0, \quad W_T = 0, \quad W_N = -1. \quad (4.8) \]

Notice that \( \hat{W} \) is chosen in the same way as in [10] but opposite the choice made in [4]. With the definitions above one can infer that on the Born level we have the following properties. In the case of longitudinally polarized quarks \((W_T = 0, W_N = 0)\) one obtains for \(W_L = \pm 1\)

\[
\sum_{\lambda_e^+} \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-} = W_L, W_L, \cos \theta = 1) = \sum_{\lambda_e^+} \frac{d\sigma_U}{d\Omega}(\cos \theta = 1)
\]

\[
\sum_{\lambda_e^+} \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-} = -W_L, W_L, \cos \theta = 1) = 0
\]

\[
\sum_{\lambda_e^+} \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-} = -W_L, W_L, \cos \theta = -1) = \sum_{\lambda_e^+} \frac{d\sigma_U}{d\Omega}(\cos \theta = -1)
\]

\[
\sum_{\lambda_e^+} \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-} = W_L, W_L, \cos \theta = -1) = 0
\]

(4.9)

The above relations hold for massive and massless quarks. From Eqs. (1.3) and (1.5) it follows

\[ P_L(\lambda_{e^-} = \pm 1, \cos \theta = \pm 1) = 1 \quad P_L(\lambda_{e^-} = \pm 1, \cos \theta = \mp 1) = -1 \quad (4.10) \]

For the transverse polarized cross section we also have a relation which only holds at threshold \(Q \rightarrow 2m\). It reads for \(\theta = \pi/2\) \((W_T = -1)\)

\[
\sum_{\lambda_e^+} \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-} = W_T, W_T, \cos \theta = 0) \rightarrow \sum_{\lambda_e^+} \frac{d\sigma_U}{d\Omega}(\cos \theta = 0)
\]

\[
\sum_{\lambda_e^+} \frac{d\sigma}{d\Omega}(\lambda_{e^+}, \lambda_{e^-} = -W_T, W_T, \cos \theta = 0) \rightarrow 0
\]

(4.11)

from which follows

\[ \lim_{Q \rightarrow 2m} P_T(\lambda_{e^-} = \pm 1, \cos \theta = 0) = \mp 1 \quad (4.12) \]

\[ ^3 \]In the case the anti-quark \( \bar{H} \) is detected in the final state we have \( P_k(\bar{H}) = -P_k(H) \) for \( k = L, T \) but \( P_N(H) = P_N(H) \).
When \( Q > 2m \) we get \(|P_T| < 1\). Far away from threshold the transverse polarization becomes very small and tends to zero. All relations above will be modified by QCD corrections as we will see below.

We will now discuss the effect of the higher order QCD corrections on the polarizations of the quarks where we neglect any higher order effect coming from the electro-weak sector. Our results are obtained by choosing the parameters given in Ref. [30]. The electro-weak constants are: \( M_Z = 91.187 \text{ GeV}/c^2, \Gamma_Z = 2.490 \text{ GeV}/c^2 \) and \( \sin^2 \theta_W = 0.23116 \). For the strong parameters we adopt \( \Lambda_{\text{MS}} = 237 \text{ MeV}/c \) at \( n_f = 5 \) so that the two-loop corrected running coupling constant equals \( \alpha_s(M^2_Z) = 0.119 \). Further we take the renormalization scale \( \mu = Q \). Furthermore we only study up-quark, bottom- and top quark production for which the following masses are chosen \( m_u = 0, m_b = 4.5 \text{ GeV}/c^2 \) and \( m_t = 173.8 \text{ GeV}/c^2 \). In our plots we will show the polarizations computed in different orders of perturbation theory. Hence we follow the notation in Eq. (3.1) and define \( P_k^i \) \((k = L, T, N)\) to be the order \( \alpha_s^i \) contribution to the polarization. Similarly we define \( P_{k,i} \) to be the order \( \alpha_s^i \) corrected polarization. Finally we only show figures where the electron beam is polarized. In the case the positron is polarized and the electron beam is unpolarized the figures for \( \lambda_{e^+} \) are the same as those for \( \lambda_{e^-} = \mp 1 \).

In Fig. 1 we have plotted the Born (zeroth order \( \alpha_s \)) and the higher order corrections to the longitudinal polarization of the up-quark at \( Q = M_Z \). The computation is done in the massless quark approach were the anomalous terms are absent. Since the QCD corrections for unpolarized beams are very small we could only show the order \( \alpha_s^2 \) corrected result \( P_{L,2} \). For this case we have plotted the ratios \( P_{L,1}/P_{L,0} \) and \( P_{L,2}/P_{L,1} \) in Fig. 2. From the latter figure we infer that the order \( \alpha_s \) corrections do not exceed the 6 pro-mille level. The order \( \alpha_s^2 \) corrections are at most 2 pro-mille. In the case of polarized beams they become larger. For \( \cos \theta = \pm 1 \) the order \( \alpha_s \) and order \( \alpha_s^2 \) corrections are 8.5% and 2.5% respectively. Similar features are shown in Fig. 3 by bottom production at the same CM energy which are computed in the massive quark approach so that the anomalous terms are implicitly present. Here the corrections for polarized beams are larger than those obtained for the up-quark and they amount to 25% at \( \cos \theta = \pm 1 \) but the corrections for unpolarized beams are so small that they could not be shown in the figure. In Fig. 4 we have studied the validity of the massless quark approach \((m = 0)\) and the massive quark approach \((m \to 0)\) for \( P_L \) in the case of bottom production at \( Q = M_Z \) with unpolarized lepton beams. To that order we have plotted the ratios of the various approaches with respect to the exact result computed for \( m = m_b = 4.5 \text{ GeV}/c^2 \). As expected the massless approach given by \( P_L^{(1)}(m = 0) \) does not work very well (see the dotted curve in Fig. 4) but also the massive approach \( P_L^{(1)}(m \to 0) \) is rather bad in particular near \( \cos \theta = \pm 1 \). It turns out that only for \( m_b < 0.2 \text{ GeV}/c^2 \) the difference between \( P_L^{(1)}(m \to 0) \) and \( P_L^{(1)}(m = 0.2) \) is less than 9%. For a comparison we have also shown the ratio of the unpolarized cross sections \( d^2 \sigma^{(1)}(m \to 0)/d^2 \sigma^{(1)}(m = 4.5) \) where no anomalous terms are present so that \( d^2 \sigma^{(1)}(m \to 0) = d^2 \sigma^{(1)}(m = 0) \). In this case the massless quark approach works rather well except near \( \cos \theta = \pm 1 \). This is the justification for neglecting the
bottom mass in the order $\alpha_s^2$ contributions to the forward backward asymmetry and the shape parameter in [13], [21]. In Fig. 5 we also plotted the longitudinal polarization of the top-quark which is produced at $Q = 500 \text{ GeV}$. Like in the two previous cases the order $\alpha_s$ corrections to $P_L$ are extremely small even when the incoming leptons are polarized. Notice that for up-quark and bottom-quark production at $Q = M_Z$ the $Z - Z$ interference term in Eq. (2.49) dominates the longitudinal polarization. However at $Q = 500 \text{ GeV}$ it turns out that the $\gamma - \gamma$ (Eq. (2.45)) and $\gamma - Z$ (Eq. (2.54)) interference terms become important too although they partially cancel each other because the latter is negative.

For an analysis of the transverse $P_T$ and normal $P_N$ polarization we have to limit ourselves to heavy quark production since these quantities vanish for massless quarks. In Fig. 6 we have shown $P_T$ at $Q = M_Z$ for bottom quarks. The QCD corrections are much larger than for $P_L$. This also holds for unpolarized beams. On the other hand the absolute values for $P_T$ are rather small which is mainly due to the fact that the bottom quark is produced far away from threshold $Q = M_Z \gg 2m_b$ so that $P_T \sim 2m_b/M_Z \ll 1$. For top-quark production at $Q = 500 \text{ GeV}$ (see Fig. 7) the situation is different. This quark is produced rather close to threshold and the ratio $2m_t/Q \sim 0.7$ is large enough so that for polarized beams the maximum value of $|P_T|$ is close to one. However due to the difference $Q - 2m_t$ there is a shift from $\theta = \pi/2$ to larger angles (here about $\theta = 2\pi/3$) where $P_T$ attains its maximum ($\lambda_e^- = -1$) or its minimum ($\lambda_e^- = 1$).

The normal polarization $P_N$ is presented for the bottom quark ($Q = M_Z$) in Fig. 8. From this figure we infer that $P_N$ is about a factor of five smaller than $P_T$ in Fig. 6. Furthermore the difference between the Born approximation $P_{N,0}$, which is wholly due to the quantity $\mathcal{T}_{13,N}^{(0)}$ in $d^2\sigma_N^{\gamma Z}$ (Eq. (2.55)), and the QCD corrected result $P_{N,1}$ is much larger than observed for $P_L$ and $P_T$. Notice that the QCD corrections to $P_N$ are not determined by the Z-peak in Eq. (2.51) only but they also receive contributions from the $\gamma - Z$ interference term in Eq. (2.55). Actually it is the structure function $\mathcal{T}_{13,N}^{(1)}$ (see Eq. (B.18)) which is mainly responsible for the difference in Fig. 8 between $P_{N,0}$ and $P_{N,1}$ around $\cos \theta = 0.8$ for $\lambda_e^- = 1$. When one is off-resonance the contributions in Eqs. (2.52)-(2.55) which are proportional to $\Im P^{\gamma Z} \sim M_Z \Gamma_Z$ are heavily suppressed. This is the reason why for top production at $Q = 500 \text{ GeV}$ (see Fig. 9) the Born approximation to the normal polarization given by $P_{N,0}$ is almost zero. Therefore in this case neither $\mathcal{T}_{13,N}^{(0)}$ nor $\mathcal{T}_{13,N}^{(1)}$, both appearing in Eq. (2.55), play any role anymore. Hence $P_N$ in Fig. 9, which is smaller than $P_T$ in Fig. 7 by a factor of forty, is completely dominated by the QCD contributions coming from the structure functions $\mathcal{T}_{9,N}^{(1)}$ (Eq. (B.10)) and $\mathcal{T}_{12,N}^{(1)}$ (Eq. (B.17)). This observation is important because besides of these corrections above, $P_N$ can also receive contributions coming from CP-violating terms in the neutral current sector which is a signal of new physics. Hence it is important to compute the QCD corrections beyond order $\alpha_s$ which is not an easy task as we have discussed below Eq. (3.4). Finally we want to emphasize that for the top-quark all interference terms are equally important for the determination of $P_T$ and $P_N$. This observation was already mentioned for $P_L$ above. This is because $t\bar{t}$-production occurs
far above the Z-boson resonance so that the dominance of the Z-propagator disappears.

As far as possible we have also made a comparison with some results obtained in the literature. First we agree with the results presented for the top-quark for polarized and unpolarized beams shown in Figs. 2, 9 in [4]. We also found agreement with the plots presented for $P_L$ and $P_T$ in [3]-[11] where the contributions $T_6^{(1)}$ (Eq. (B.6)) and $T_8^{(1)}$ (Eq. (B.9)) were not taken into account. However these contributions are negligible. On the other hand we disagree with the normal polarization $P_N$ plotted for the bottom quark (Fig. 2b) and the top-quark (Fig. 3b) in [10]. This is due to the difference in minus sign between $T_{12,N}^{(1)}$ and the equivalent expression in Eq. (33) of [10] which we have mentioned below Eq. (3.17). Note that the signs for $T_{12,N}^{(1)}$ (Eq. (B.16)) and $T_{12,N}^{(1)}$ (Eq. (B.17)) are the same as the contributions appearing in Eq. (16) of [4].

Before finishing this section we would like to comment on some results which are obtained in [12] and [3]. Here one has expressed $\hat{W}$ in Eq. (2.14) into the following orthonormal basis (see also [31])

$$\hat{W} = \sin \xi \cos \psi \hat{n}_1 + \sin \xi \sin \psi \hat{n}_2 + \cos \xi \hat{n}_3,$$

with

$$\hat{n}_1 = \frac{\hat{n} \times (\hat{n} \times e_3)}{|\hat{n} \times (\hat{n} \times e_3)|}, \quad \hat{n}_2 = \frac{\hat{n} \times e_3}{|\hat{n} \times e_3|}, \quad \hat{n}_3 = -\hat{n},$$

$$e_1 = (1, 0, 0), \quad e_2 = (0, 1, 0), \quad e_3 = (0, 0, 1).$$

(4.14)

Notice that $\hat{n}$ is the direction of the three-momentum of the observed quark in the CM frame of the electron-positron pair whereas $\hat{n}_3 = -\hat{n}$ points into the direction of the momentum $q = q_{e^+} + q_{e^-}$ in the rest frame of the observed quark. On the basis $e_i (i = 1, 2, 3)$ the vectors $\hat{n}_i$ take the following form:

$$\hat{n}_1 = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta),$$

$$\hat{n}_2 = (\sin \phi, -\cos \phi, 0),$$

$$\hat{n}_3 = -(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta).$$

(4.15)

Substituting Eq. (4.13) in Eq. (2.16) and using the parameterization for the quark momentum in (2.13) the spin four-vector $s$ in the CM frame of the incoming leptons has the following components

$$s^0 = -\frac{Q \alpha_x}{2m} \cos \xi,$$

$$s^1 = -\left(\frac{Q}{2m}\right) \cos \xi \sin \theta \cos \phi + \sin \xi \cos \psi \cos \theta \cos \phi + \sin \xi \sin \psi \sin \phi,$$

$$s^2 = -\left(\frac{Q}{2m}\right) \cos \xi \sin \theta \sin \phi + \sin \xi \cos \psi \cos \theta \sin \phi - \sin \xi \sin \psi \cos \phi,$$

(4.16)
\[ s^3 = -\left(\frac{Q_x}{2m}\right) \cos \xi \cos \theta - \sin \xi \cos \psi \sin \theta . \quad (4.16) \]

Similarly, using Eq.\((4.13)\) one finds that
\[
\begin{align*}
W_L &= \hat{n} \cdot \hat{W} = - \cos \xi , \\
W_T &= \hat{W}^2 - \hat{n} \hat{W} \cos \theta = - \sin \xi \cos \psi \sin \theta , \\
W_N &= \hat{W}^2 \cos \phi - \hat{W}^1 \sin \phi = - \sin \xi \sin \psi .
\end{align*}
\]

Comparing the equation above with our choices of the polarization vector in Eqs. \((4.6)\) - \((4.8)\) we obtain \(\xi = 0, \pi, \psi = \text{arbitrary} \) (longitudinal), \(\xi = \pm \pi/2, \psi = 0 \) (transverse), and \(\xi = \pm \pi/2, \psi = \pi/2 \) (normal) respectively. Notice that the same angles were also adopted in \([8] - [11]\). A different choice was made in \([12], [6]\) where the values of the angles are taken at \(\psi = 0\) and \(\xi = \text{arbitrary}\). In this case only the longitudinal and transverse polarization of the heavy quark can be studied since \(W_N = 0\). The reason for this choice is that the authors in \([12], [6]\) did not include the contributions coming from the imaginary parts of the virtual corrections and the Z-boson propagator so that the normal polarization vanishes. Furthermore in \([12]\) one has adopted three different bases for the spin vector which are called the helicity, the beamline and the off-diagonal bases. The helicity bases is defined by the condition
\[
\cos \xi = \pm 1 . \quad (4.18)
\]

which is equivalent to our choice for the longitudinal polarization of the quark. The beamline basis is given by
\[
\cos \xi = \frac{\cos \theta + \beta}{1 + \beta \cos \theta}, \quad \beta = \sqrt{1 - \rho} . \quad (4.19)
\]

This condition only applies to the Born and the soft plus virtual gluon contribution where there is no hard gluon emission. Here the spin of the quark is polarized along the positron momentum in the rest frame of the quark. The above equation is not valid when there is hard gluon emission. In the later case one obtains
\[
\cos \xi = \frac{x \cos \theta + \alpha_x}{x + \alpha_x \cos \theta}, \quad \alpha_x = \sqrt{x^2 - \rho} , \quad (4.20)
\]

which reproduces the Eq.\((4.19)\) in the limit \(x \to 1\) (soft gluon region) where \(\alpha_x \to \beta\). Since \(\cos \xi\) in Eq.\((4.20)\) is a function of the integration variable \(x\) in Eq. \((2.12)\) the choice above cannot be used in those expressions where the \(x\) integration is already done. Therefore in the case of hard gluon radiation the integral over \(x\) is very complicated since this variable appears in the numerator as well as denominator of the expression for \(\cos \xi\). This will lead to a non-trivial dependence of the cross section on \(\cos \theta\). Note that the choice in Eq.\((4.19)\) at the level of hard gluon emission is at variance with the interpretation that the spin of the top quark is polarized along the positron momentum direction in the rest frame of the quark. The third choice, called off-diagonal basis,
corresponds to the case when the like-spin configuration of the quark anti-quark pair vanishes identically. As for the beamline basis this choice leads to an \( x \)-dependence of \( \cos \xi \) which is non-trivial so that it can be only applied to the Born and soft plus virtual gluon contribution.

Summarizing the results obtained in this paper we have presented the complete first order QCD corrections to polarized (heavy) quark production. The most of these corrections were already calculated in \([8]-[11]\). We agree with these results except those obtained for the normal polarization in \([10]\). Further we were able to compress the expressions as far as possible by minimizing the number of polylogarithms. Moreover we also computed the second order QCD corrections to the production of light quarks when the latter are longitudinally polarized. Furthermore we discovered that the massless quark approach which was so successful to compute the higher order corrections to unpolarized quantities in the kinematical regime \( Q \gg m \) failed in the case of the longitudinal polarization. As we have seen for bottom-quark production this was not only due to the appearance of anomalous terms but also to the slow convergence to the zero mass limit. Our results reveal that the QCD corrections to the longitudinal polarization are rather small for light as well as heavy quarks except for very small or very large values of the scattering angle \( \theta \). These corrections become more important for the transverse polarization whereas in the case of the normal polarization they completely dominate the process. For this reason it is very important to compute the order \( \alpha_s^2 \) corrections to the normal polarization which is a difficult task.

ACKNOWLEDGMENTS

V. Ravindran would like to thank J. Blümlein, H.S. Mani and S.D. Rindani for discussions. The work of W.L. van Neerven was supported by the EC network ‘QCD and Particle Structure’ under contract No. FMRX–CT98–0194.
A Appendix A

Since the leptonic current is conserved for massless leptons, which implies $q_{\mu}L^{\mu
u} = q_{\nu}L^{\mu
u} = 0$, we present only those partonic tensors which contribute to the cross section (see Eqs. (2.12), (2.13)). In the Born approximation (3.4) we obtain

$$W(\gamma \gamma, V_1 V_2)_{(0)} = \left( v_1 V_1 v_2 V_2 \left[ -\frac{1}{4} \beta \right] + a_q V_1 a_q V_2 \left[ -\frac{1}{4} \beta^3 \right] \right) \delta(1 - x),$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = -\left( v_1 V_1 v_2 V_2 + a_q V_1 a_q V_2 \right) \beta \delta(1 - x),$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = \frac{1}{2} \left( v_1 V_1 a_q V_2 + a_q V_1 v_2 V_2 \right) \beta \delta(1 - x),$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = 0,$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = \frac{1}{2} \left( v_1 V_1 a_q V_2 + a_q V_1 v_2 V_2 \right) \beta \delta(1 - x),$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = \frac{1}{2} \left( v_1 V_1 a_q V_2 + a_q V_1 v_2 V_2 \right) \beta \delta(1 - x),$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = 0,$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = -\left( a_q V_1 a_q V_2 \right) \beta \delta(1 - x),$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = \frac{1}{2} \left( v_1 V_1 a_q V_2 + a_q V_1 v_2 V_2 \right) \beta \delta(1 - x),$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = \frac{1}{2} \left( v_1 V_1 a_q V_2 - a_q V_1 v_2 V_2 \right) \beta \delta(1 - x),$$

$$W(\gamma \gamma, V_1 V_2)_{(0)} = 0,$$

(A.1)

with $V_1, V_2 = \gamma, Z$. For the order $\alpha_s$ correction (3.6) it is convenient to decompose the partonic structure functions $W_i (i = 1 - 9)$ as follows

$$W_i^{(V_1 V_2), (1)} = W_i^{(V_1 V_2), V+S} + W_i^{(V_1 V_2), HARD},$$

with

$$W_i^{(V_1 V_2), V+S} = W_i^{(V_1 V_2), VIRT} + W_i^{(V_1 V_2), SOFT}.$$  (A.2)
The soft plus virtual gluon contributions are given by

\[ W_{1(V_1V_2)},V+S = -\frac{\beta}{2} \left[ v_q V_1 \, v_q V_2 \, C^{V+S} + a_q V_1 \, a_q V_2 \, \beta^2 \left( 2 \, \text{Re} C_2 + C^{V+S} \right) \right] \delta(1 - x), \]

\[ W_{2(V_1V_2)},V+S = -2\beta \left[ v_q V_1 \, v_q V_2 \left( \text{Re} C_2 + C^{V+S} \right) + a_q V_1 \, a_q V_2 \left( 2 \, \text{Re} C_2 + C^{V+S} \right) \right] \delta(1 - x), \]

\[ W_{3(V_1V_2)},V+S = \beta \left[ (v_q V_1 \, a_q V_2 + a_q V_1 \, v_q V_2) \left( \text{Re} C_2 + C^{V+S} \right) \right. \]

\[ \left. - (v_q V_1 \, a_q V_2 - a_q V_1 \, v_q V_2) \, i\text{Im} C_2 \right] \delta(1 - x), \]

\[ W_{4(V_1V_2)},V+S = \frac{2\beta}{\rho} \left[ (v_q V_1 \, a_q V_2 + a_q V_1 \, v_q V_2) \, \text{Re} C_2 \right. \]

\[ \left. + (v_q V_1 \, a_q V_2 - a_q V_1 \, v_q V_2) \, i\text{Im} C_2 \right] \delta(1 - x), \]

\[ W_{5(V_1V_2)},V+S = \frac{1}{2} \beta \left[ (v_q V_1 \, a_q V_2 + a_q V_1 \, v_q V_2) \left( -\frac{1}{\rho} \text{Re} C_2 + 3 \text{Re} C_2 + 2 \, C^{V+S} \right) \right. \]

\[ \left. + (v_q V_1 \, a_q V_2 - a_q V_1 \, v_q V_2) \left( -\frac{1}{\rho} \, i\text{Im} C_2 - \, i\text{Im} C_2 \right) \right] \delta(1 - x), \]

\[ W_{6(V_1V_2)},V+S = \beta \left[ (v_q V_1 \, a_q V_2 + a_q V_1 \, v_q V_2) \left( \text{Re} C_2 + C^{V+S} \right) \right. \]

\[ \left. - (v_q V_1 \, a_q V_2 - a_q V_1 \, v_q V_2) \, i\text{Im} C_2 \right] \delta(1 - x), \]

\[ W_{7(V_1V_2)},V+S = -\frac{\beta}{\rho} \left[ v_q V_1 \, v_q V_2 \, \text{Re} C_2 + a_q V_1 \, a_q V_2 \, \text{Re} C_3 \right] \delta(1 - x), \]

\[ W_{8(V_1V_2)},V+S = -\beta \left[ a_q V_1 \, a_q V_2 \left( 4 \, \text{Re} C_2 - \frac{1}{\rho} \, \text{Re} C_3 + 2 \, C^{V+S} \right) \right] \delta(1 - x), \]

\[ W_{9(V_1V_2)},V+S = -\frac{\beta}{2} \left[ v_q V_1 \, v_q V_2 \left( \frac{1}{\rho} \, \text{Re} C_2 - \text{Re} C_2 - 2 \, C^{V+S} \right) \right. \]

\[ \left. + a_q V_1 \, a_q V_2 \left( -4 \, \text{Re} C_2 + \frac{1}{\rho} \, \text{Re} C_3 - 2 \, C^{V+S} \right) \right] \delta(1 - x), \]

32
\[ W_{10}^{(V_1 V_2),V+S} = \frac{1}{2} \beta \left[ (v_q^{V_1} a_q^{V_2} - a_q^{V_1} v_q^{V_2}) \left( -\frac{1}{\rho} \text{Re} C_2 + 3 \text{Re} C_2 + 2 C^{V+S} \right) 
\right. \\
\left. + (v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}) \left( -\frac{1}{\rho} \text{Im} C_2 - i \text{Im} C_2 \right) \right] \delta(1-x), \]

\[ W_{11}^{(V_1 V_2),V+S} = -\frac{2\beta}{\rho} \left[ v_q^{V_1} v_q^{V_2} \text{Im} C_2 \right] \delta(1-x). \]

The hard gluon contributions are

\[ W_1^{(V_1 V_2),HARD} = C_F v_q^{V_1} v_q^{V_2} \left[ \frac{1}{\alpha_x(4-4x+\rho)^2(1-x)} \left( -20\rho - 11\rho^2 - \rho^3 \right) 
\right. \\
\left. +8x + 18\rho x - 2\rho^2 x - 2\rho^3 x + 8x^2 + 58\rho x^2 + 23\rho^2 x^2 
\right. \\
\left. +\rho^3 x^2 - 44x^3 - 74\rho x^3 - 8\rho^2 x^3 + 32x^4 + 18\rho x^4 - 4x^5 \right) \\
\left. + \frac{\ln(\xi)}{4\alpha_x^2(1-x)} \left( \rho - \rho^2 - 2\rho^3 x - 2x^2 + 7\rho x^2 + \rho^2 x^2 
\right. \\
\left. -2\rho x^3 - 2x^4 \right) \right], \]

\[ W_2^{(V_1 V_2),HARD} = C_F v_q^{V_1} v_q^{V_2} \left[ \frac{4}{\alpha_x^2(4-4x+\rho)^2(1-x)} \left( 4\rho + 35\rho^2 + 17\rho^3 + 2\rho^4 
\right. \\
\left. + 24x - 82\rho x - 110\rho^2 x - 22\rho^3 x - 40x^2 + 186\rho x^2 + 85\rho^2 x^2 
\right. \\
\left. +\rho^3 x^2 + 4x^3 - 126\rho x^3 - 8\rho^2 x^3 + 16x^4 + 18\rho x^4 - 4x^5 \right) \right]. \]
\[-\frac{\ln(\xi)}{\alpha_x^4(1-x)} \left( \rho - \rho^2 - 2\rho^3 + 10\rho^2x + 2x^2 - 13\rho x^2 - \rho^2x^2 + 2\rho x^3 + 2x^4 \right) \]

\[+ C_F a_q^{V_1} a_q^{V_2} \left[ \frac{4}{\alpha_x^3(4-4x+\rho)^2(1-x)} \left( 4\rho + 31\rho^2 + 18\rho^3 + 2\rho^4 
+ 24x - 106\rho x - 108\rho^2 x - 24\rho^3 x - 40x^2 + 274\rho x^2 
+ 95\rho^2 x^2 + 2\rho^3 x^2 + 4x^3 - 242\rho x^3 - 18\rho^2 x^3 + 16x^4 + 82\rho x^4 
+ 2\rho^2 x^4 - 4x^5 - 12\rho x^5 \right) \right] \]

\[-\frac{\ln(\xi)}{\alpha_x^4(1-x)} \left( \rho - 2\rho^2 - 2\rho^3 + 12\rho^2 x + 2x^2 - 15\rho x^2 - 2\rho^2 x^2 + 6\rho x^3 + 2x^4 - 2\rho x^4 \right) \]

\[W_{3(V_1V_2)\text{HARD}} = \frac{1}{2} C_F (v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}) \left[ -\frac{2}{\alpha_x^3(4-4x+\rho)^2(1-x)} \left( 68\rho^2 + 35\rho^3 
+ 4\rho^4 - 128\rho x - 208\rho^2 x - 45\rho^3 x + 48x^2 + 332\rho x^2 
+ 172\rho^2 x^2 + 5\rho^3 x^2 - 112x^3 - 304\rho x^3 - 52\rho^2 x^3 - 3\rho^3 x^3 
+ 104x^4 + 156\rho x^4 + 24\rho^2 x^4 - 64x^5 - 56\rho x^5 + 24x^6 \right) \right] \]

\[-\frac{\ln(\xi)}{2\alpha_x^4(1-x)} \left( 3\rho^2 + 4\rho^3 - 2\rho x - 21\rho^2 x + 18\rho x^2 + 5\rho^2 x^2 
- 4x^3 - 2\rho x^3 - 3\rho^2 x^3 + 6\rho x^4 - 4x^5 \right) \right], \]

\[W_{4(V_1V_2)\text{HARD}} = \frac{1}{2} C_F (v_q^{V_1} a_q^{V_2} + a_q^{V_1} v_q^{V_2}) \left[ \frac{8}{\alpha_x^7(4-4x+\rho)^2} \left( 64\rho^2 + 124\rho^3 
+ 45\rho^4 + 4\rho^5 - 528\rho^2 x - 404\rho^3 x - 63\rho^4 x + 112\rho x^2 \right) \right] \]
\[
+976 \rho^2 x^2 + 319 \rho^3 x^2 + 6 \rho^4 x^2 + 96 \rho x^3 - 432 \rho^2 x^3 - 18 \rho^3 x^3 \\
-176 x^4 - 764 \rho x^4 - 158 \rho^2 x^4 - 9 \rho^3 x^4 + 432 x^5 + 756 \rho x^5 \\
+75 \rho^2 x^5 - 336 x^6 - 206 \rho x^6 - \rho^2 x^6 + 80 x^7 + 6 \rho x^7 \\
\left( - \frac{2 \ln(\xi)}{\alpha_x^6} \left( 13 \rho^2 + 4 \rho^3 - 18 \rho x - 39 \rho^2 x + 58 \rho x^2 + 10 \rho^2 x^2 \\
-12 x^3 - 21 \rho x^3 + 4 x^4 + \rho x^4 \right) \right),
\]

\[
W_{5(V_1V_2),HARD} = \frac{1}{2} C_F \left( a_q V_i a_q V_j + a_q V_i V_j \right) \left[ - \frac{2}{\alpha_x^3(4 - 4x + \rho)^2(1 - x)} \left( 80 \rho + 96 \rho^2 \\
+37 \rho^3 + 4 \rho^4 - 96x - 440 \rho x - 272 \rho^2 x - 46 \rho^3 x + 400 x^2 \\
+736 \rho x^2 + 193 \rho^2 x^2 + \rho^3 x^2 - 568 x^3 - 446 \rho x^3 - 14 \rho^2 x^3 \\
+320 x^4 + 76 \rho x^4 + \rho^2 x^4 - 56 x^5 - 6 \rho x^5 \right) \\
- \frac{\ln(\xi)}{2\alpha_x^4(1 - x)} \left( 4 \rho + 5 \rho^2 + 4 \rho^3 - 26 \rho x - 22 \rho^2 x + 8 x^2 \\
+49 \rho x^2 + \rho^2 x^2 - 16 x^3 - 8 \rho x^3 + \rho x^4 \right) \right],
\]

\[
W_{6(V_1V_2),HARD} = \frac{1}{2} C_F \left( a_q V_i a_q V_j + a_q V_i V_j \right) \left[ \frac{8}{\alpha_x(4 - 4x + \rho)^2(1 - x)} \left( - 8 + 12 \rho \\
+8 \rho^2 + \rho^3 + 12x - 31 \rho x - 9 \rho^2 x - 2 x^2 + 18 \rho x^2 + \rho x^3 - 2 x^4 \right) \\
+ \frac{2 \ln(\xi)}{\alpha_x^2(1 - x)} \left( \rho^2 + x - 3 \rho x + x^3 \right) \right],
\]

\[
W_{7(V_1V_2),HARD} = C_F \left( a_q V_i V_j \right) \left[ \frac{2}{\alpha_x^3(4 - 4x + \rho)^2} \left( - 64 \rho - 36 \rho^2 - 4 \rho^3 + 96x \\
+172 \rho x + 39 \rho^2 x - 200 \rho x^2 - 102 \rho x^2 + \rho^2 x^2 + 104 \rho x^3 - 6 \rho x^3 \right) \right)
\]

35
\[\ln(\xi) \left( -4\rho - 4\rho^2 + 15\rho x - 8x^2 + \rho x^2 \right)\]

\[+ C_F a_q^V_1 a_q^V_2 \left[ \frac{2}{\alpha_x^2 (4 - 4x + \rho)^2} \left( -64\rho - 16\rho^2 - \rho^3 + 96x \right) \right.\]

\[+ 132\rho x + \rho^2 x - 3\rho^3 x - 200x^2 - 46\rho x^2 + 19\rho^2 x^2\]

\[+ 120x^3 - 22\rho x^3 - 16x^4 \right) \]

\[+ \ln(\xi) \left( -4\rho - \rho^2 + 15\rho x - 3\rho^2 x - 8x^2 + \rho x^2 \right)\],

\[W_8^{(V_1 V_2),HARD} = C_F a_q^V_1 a_q^V_2 \left[ -\frac{2}{\alpha_x (4 - 4x + \rho)(1 - x)} \left( \frac{52\rho + 31\rho^2 + 4\rho^3 - 56x}{-130\rho x - 34\rho^2 x + 112x^2 + 72\rho x^2 - \rho^2 x^2 - 56x^3 + 6x^3} \right) \right.\]

\[+ \left. \ln(\xi) \frac{\ln(\xi)}{2\alpha_x^2 (1 - x)} \left( \rho - 4\rho^2 + 10\rho x - 8x^2 - \rho x^2 \right) \right)\],

\[W_9^{(V_1 V_2),HARD} = C_F v_q^V_1 v_q^V_2 \left[ -\frac{1}{\alpha_x (4 - 4x + \rho)(1 - x)} \left( -8 - 17\rho - 4\rho^2 \right) \right.\]

\[+ 22x + 18\rho x - 4x^2 + 3\rho x^2 - 10x^3 \right)\]

\[+ \frac{\ln(\xi)}{4\alpha_x^2 (1 - x)} \left( \rho + 4\rho^2 - 4x - 10\rho x + 8x^2 - 3\rho x^2 + 4x^3 \right) \right)\]

\[+ C_F a_q^V_1 a_q^V_2 \left[ \frac{1}{\alpha_x (4 - 4x + \rho)^2(1 - x)} \left( 32 - 4\rho + \rho^2 - 120x \right) \right.\]

\[+ 62\rho x + 30\rho^2 x + 4\rho^3 x + 120x^2 - 144\rho x^2 - 35\rho^2 x^2\]

\[+ \ln(\xi) \frac{\ln(\xi)}{4\alpha_x^2 (1 - x)} \left( \rho - 4x - 2\rho x + 4\rho^2 x + 8x^2 \right) \]
\begin{align*}
W^{(V_1V_2)}_{10, \text{HARD}} &= \frac{1}{2} C_F (v_q^{V_1} a_q^{V_2} - a_q^{V_1} v_q^{V_2}) \left[ \frac{2}{\alpha_x (4 - 4x + \rho)^2 (1 - x)} \right] \left[ 76\rho + 33\rho^2 \\
&\quad + 4\rho^3 - 72x - 190\rho x - 38\rho^2 x + 144x^2 + 120\rho x^2 + \rho^2 x^2 \\
&\quad - 72x^3 - 6\rho x^3 \right] + \frac{\ln(\xi)}{2\alpha_x^2 (1 - x)} \left( \rho + 4\rho^2 - 14\rho x + 8x^2 + \rho x^2 \right), \\
W^{(V_1V_2)}_{11, \text{HARD}} &= 0.
\end{align*}

In the expressions above we have introduced the following notations

\begin{align*}
\alpha_x &= \sqrt{x^2 - \rho}, \quad \xi = \frac{\rho - 2x - 2\alpha_x}{\rho - 2x + 2\alpha_x}.
\end{align*}
B Appendix B

In this appendix we present the order $\alpha_s$ contributions to the structure functions $T_i$ Eqs. (2.21)-(2.32) computed in section 3. The unpolarized parts are given by

$$\mathcal{T}_1^{(1)} = C_F \left[ \frac{1}{2} \rho (1 + \rho) F_1 + \sqrt{\rho} (1 - 3 \rho) F_2 + 2 (8 - 5 \rho - \rho^2) F_3 + 2 \beta F_4 ight]$$

$$+ \frac{1}{2} \beta (2 + 13 \rho) + \frac{1}{2} (64 - 39 \rho - 7 \rho^2) L_{i_2}(t) + \frac{1}{4} (-48 + 36 \rho - 5 \rho^2) \ln(t)$$

$$+ 2 (4 - 3 \rho - \rho^2) \ln(t) \ln(1 + t) \right], \quad \text{(B.1)}$$

$$\mathcal{T}_2^{(1)} = C_F \left[ \frac{1}{2} \rho (1 + 2 \rho) F_1 + \sqrt{\rho} (1 - 4 \rho) F_2 + 2 (8 - 13 \rho + 2 \rho^2) F_3 + 2 \beta (1 - \rho) F_4 ight]$$

$$+ \frac{1}{2} \beta (2 + \rho) + \frac{1}{2} (64 - 103 \rho + 18 \rho^2) L_{i_2}(2) + \frac{1}{4} (-48 + 60 \rho - 9 \rho^2) \ln(t)$$

$$+ 2 (4 - 7 \rho) \ln(t) \ln(1 + t) \right],$$ \quad \text{(B.2)}

$$\mathcal{T}_3^{(1)} = C_F \left[ - \frac{1}{2} \rho (1 + \rho) F_1 + \sqrt{\rho} (-1 + 3 \rho) F_2 + 2 \rho (5 - \rho) F_3 + \beta \rho F_4 + 2 \beta (1 - \rho) ight]$$

$$+ \frac{1}{2} \rho (39 - 9 \rho) L_{i_2}(t) + \rho (-7 + 3 \rho) \ln(t) + 6 \rho \ln(t) \ln(1 + t) \right], \quad \text{(B.3)}$$

$$\mathcal{T}_4^{(1)} = C_F \left[ - \frac{1}{2} \rho (1 + 2 \rho) F_1 + \sqrt{\rho} (-1 + 4 \rho) F_2 + 2 \rho (1 + 2 \rho) F_3 ight]$$

$$+ \frac{1}{4} \beta (8 - 38 \rho + 3 \rho^2) + \frac{7}{2} \rho (1 + 2 \rho) L_{i_2}(t) + \frac{1}{8} \rho (-32 + 8 \rho - 3 \rho^2) \ln(t)$$

$$+ 2 \rho (1 + 2 \rho) \ln(t) \ln(1 + t) \right], \quad \text{(B.4)}$$

$$\mathcal{T}_5^{(1)} = C_F \left[ 4 \beta (-2 + \rho) G_1 + 2 (4 - 5 \rho) G_2 - 4 \sqrt{\rho} + 4 \rho + 8 (-1 + \rho) \ln(1 + t) ight]$$

$$+ 8 \ln(1 + t - \sqrt{t}) + 16 (-1 + \rho) \ln(1 - \sqrt{t}) \right]$$

38
\[ +2(2 - 4\rho + 3\beta\rho - 2\beta) \ln(t) \], \hspace{1cm} (B.5) \\

\[ \text{Im}\mathcal{T}_6^{(1)} = C_F \left[ 2\pi\rho\beta \right]. \hspace{1cm} (B.6) \]

The longitudinal polarized structure functions are equal to

\[ \mathcal{T}_{7, L_1}^{(1)} = C_F \left[ \beta(2 - \rho)G_1 - \frac{1}{4}(8 + 2\rho + 3\rho^2)G_2 - \frac{\sqrt{\rho}}{8}(8 + 29\rho) + \frac{1}{8}(2 + 35\rho) \\
+ 2(1 - \rho) \ln(1 + t) + \frac{1}{16}(-32 + 60\rho - 17\rho^2) \ln(1 + t - \sqrt{t}) \\
+ 4(1 - \rho) \ln(1 - \sqrt{t}) + \frac{1}{32}(-32 + 4\rho + 32\beta + 72\rho\beta + 17\rho^2) \ln(t) \right], \hspace{1cm} (B.7) \]

\[ \mathcal{T}_{7, L_2}^{(1)} = C_F \left[ \frac{\rho}{2}(10 + 3\rho)G_2 + \frac{\sqrt{\rho}}{2}(8 + 13\rho) - \frac{1}{2}(2 + 19\rho) \\
+ \frac{\rho}{4}(-24 + 7\rho) \ln(1 + t - \sqrt{t}) + \frac{\rho}{8}(24 - 52\beta - 7\rho) \ln(t) \right], \hspace{1cm} (B.8) \]

\[ \text{Im}\mathcal{T}_{8, L}^{(1)} = C_F \left[ -\frac{\pi}{2}\rho\beta \right], \hspace{1cm} (B.9) \]

\[ \mathcal{T}_{10, L}^{(1)} = C_F \left[ \frac{5\rho}{8}F_1 - \frac{\sqrt{\rho}}{4}(4 + \rho)F_2 - \frac{1}{2}(8 + \rho)F_3 - \frac{\beta}{2}F_4 + \frac{\beta}{2} \\
- \frac{1}{8}(64 + 3\rho)L_i2(t) + \frac{1}{4}(12 - 3\rho) \ln(t) - \frac{1}{2}(4 + 3\rho) \ln(t) \ln(1 + t) \right], \hspace{1cm} (B.10) \]

\[ \mathcal{T}_{11, L}^{(1)} = C_F \left[ \frac{\rho}{8}(5 - \rho)F_1 - \sqrt{\rho}F_2 + \frac{1}{2}(-8 + 7\rho - 3\rho^2)F_3 + \frac{\beta}{2}(-1 + \rho)F_4 \\
+ \frac{\beta}{2}(1 + 3\rho) + \frac{1}{8}(-64 + 61\rho - 25\rho^2)L_i2(t) + \frac{1}{4}(12 - 9\rho + \rho^2) \ln(t) \\
+ \frac{1}{2}(-4 + \rho - \rho^2) \ln(t) \ln(1 + t) \right]. \hspace{1cm} (B.11) \]
The transverse polarized structure functions are

\[ T_{1, T}^{(1)} = C_F \left[ \sqrt{\rho \beta}(2 - \rho)G_1 - \frac{\sqrt{\rho}}{4}(16 + 7\rho)G_2 + \frac{\sqrt{\rho}}{8}(48 + 17\rho) - \frac{\rho}{8}(62 + 3\rho) + 2\sqrt{\rho}(1 - \rho) \ln(1 + t) + \frac{\sqrt{\rho}}{16}(40 + 2\rho - 3\rho^2) \ln(1 + t - \sqrt{t}) + 4\sqrt{\rho}(1 - \rho) \ln(1 - \sqrt{t}) + \frac{\sqrt{\rho}}{32}(-104 + 62\rho + 168\beta + 16\rho\beta) + 3\rho^2) \ln(t) \right]. \]  

(B.12)

\[ \text{Im} T_{8, T}^{(1)} = C_F \left[ -\frac{\pi}{4}\sqrt{\rho \beta}(1 + \rho) \right], \]  

(B.13)

\[ T_{10, T}^{(1)} = C_F \left[ \frac{\sqrt{\rho}}{16}(4 + \rho)F_1 - \frac{5\rho}{8}F_2 + \frac{\sqrt{\rho}}{4}(-20 + 7\rho)F_3 - \frac{1}{2}\sqrt{\rho \beta}F_4 - \frac{3}{4}\sqrt{\rho \beta} + \frac{\sqrt{\rho}}{16}(-156 + 57\rho)L_i(t) + \frac{\sqrt{\rho}}{8}(26 - 13\rho) \ln(t) + \frac{\sqrt{\rho}}{4}(-12 + 3\rho) \ln(t) \ln(1 + t) \right], \]  

(B.14)

\[ T_{11, T}^{(1)} = C_F \left[ \frac{\sqrt{\rho}}{4}F_1 + \frac{\rho}{8}(-5 + \rho)F_2 - \sqrt{\rho}F_3 + \frac{1}{8}\sqrt{\rho \beta}(14 - 3\rho) - \frac{7}{4}\sqrt{\rho}L_i(t) + \frac{\sqrt{\rho}}{16}(20 - 12\rho + 3\rho^2) \ln(t) - \sqrt{\rho} \ln(t) \ln(1 + t) \right]. \]  

(B.15)

The normal polarized structure functions are

\[ T_{9, N}^{(1)} = C_F \left[ -\frac{\pi}{4}\sqrt{\rho}(1 - \rho) \right], \]  

(B.16)

\[ T_{12, N}^{(1)} = C_F \left[ -\frac{\pi}{4}\sqrt{\rho \beta}(1 + \rho) \right], \]  

(B.17)

\[ \text{Im} T_{13, N}^{(1)} = C_F \left[ \sqrt{\rho \beta}(\rho - 2)G_1 + \frac{\sqrt{\rho}}{4}(8 - 13\rho)G_2 + \frac{\sqrt{\rho}}{8}(20 + 9\rho) - \frac{\rho}{8}(3\rho + 26) - 2\sqrt{\rho}(1 - \rho) \ln(1 + t) + \frac{\sqrt{\rho}}{16}(24 - 2\rho - 3\rho^2) \ln(1 + t - \sqrt{t}) \right]. \]
$$-4\sqrt{\rho}(1 - \rho)\ln(1 - \sqrt{t}) + \frac{\sqrt{\rho}}{32}(40 - 62\rho - 8\beta + 48\rho\beta + 3\rho^2)\ln(t),$$  

(B.18)

where $\mathcal{L}i_2(x)$ is defined in [17]. Furthermore the functions $F_i$ ($i = 1 - 4$) and $G_i$ ($i = 1, 2$) appearing in the expressions above are given by

$$F_1 = \mathcal{L}i_2(t^3) + 4\zeta(2) + \frac{1}{2}\ln^2(t) + 3\ln(t)\ln(1 + t + t^2),$$

$$F_2 = \mathcal{L}i_2(-t^{3/2}) - \mathcal{L}i_2(t^{3/2}) + \mathcal{L}i_2(-t^{1/2}) - \mathcal{L}i_2(t^{1/2}) + 3\zeta(2)$$

$$+ 2\ln(t)\ln(1 + \sqrt{t}) - 2\ln(t)\ln(1 - \sqrt{t}) + \frac{3}{2}\ln(t)\ln(1 + t - \sqrt{t})$$

$$- \frac{3}{2}\ln(t)\ln(1 + t + \sqrt{t}),$$

$$F_3 = \mathcal{L}i_2(-t) + \ln(t)\ln(1 - t),$$

$$F_4 = 6\ln(t) - 8\ln(1 - t) - 4\ln(1 + t),$$

$$G_1 = \mathcal{L}i_2(-t^{3/2}) - 3\mathcal{L}i_2(-t^{1/2}) - 4\mathcal{L}i_2(t^{1/2}) - \mathcal{L}i_2(-t)$$

$$- \frac{1}{2}\zeta(2) - \frac{1}{8}\ln^2(t),$$

$$G_2 = \mathcal{L}i_2\left(\sqrt{\frac{t}{1 + t}}\right) - \frac{1}{8}\ln^2(t) - \frac{1}{2}\ln(t)\ln(1 + t) + \frac{1}{2}\ln^2(1 + t) - \frac{1}{2}\zeta(2),$$

(B.19)

where the variable $t$ is defined in (3.12). Further we are interested in the values taken by $T_i$ when $m \rightarrow 0$. In the unpolarized case they become

$$T_1^{(1), m \rightarrow 0} = T_2^{(1), m \rightarrow 0} = 1, \quad T_3^{(1), m \rightarrow 0} = T_4^{(1), m \rightarrow 0} = 2,$$

$$T_5^{(1), m \rightarrow 0} = 0i, \quad T_6^{(1), m \rightarrow 0} = 0,$$

(B.20)

whereas the longitudinal structure functions tend to the limits

$$T_{7L}^{(1), m \rightarrow 0} = \frac{1}{4}, \quad T_{8L}^{(1), m \rightarrow 0} = -1, \quad T_{8L}^{(1), m \rightarrow 0} = 0,$$

$$T_{10L}^{(1), m \rightarrow 0} = T_{11L}^{(1), m \rightarrow 0} = \frac{1}{2}.$$

(B.21)

All transverse and normal polarized quantities become zero in the limit $m \rightarrow 0$. 

41
References

[1] Proceedings: e+ e- Collisions at TeV Energies: The Physics Potential: Edited by P.M. Zerwas. Hamburg, Germany, DESY, 1996. 550p. (DESY 96-123D)

[2] S.D. Rindani, M.M. Tung, Phys. Lett. B424 (1998) 125.

[3] N.S. Craigie, Phys.Rept. 99 (1983) 69.

[4] J.H. Kühn, A. Reiter and P.M. Zerwas, Nucl. Phys. B272 (1986) 560.

[5] M. Anselmino, P. Kroll and B. Pire, Phys. Lett. B167 (1986) 113.

[6] S. Parke and Y. Shadmi, Phys. Lett. B387 (1996) 199.

[7] J.B. Stav and H. A. Olsen, Z. Phys. C57 (1993) 519, Phys. Rev. D50 (1994) 6775, ibid. D52 (1995) 1359, D54 (1996) 817, D56 (1997) 407.

[8] J.G. Körner, A. Pilaftsis and M.M. Tung, Z. Phys. C63 (1994) 509.

[9] S. Groote, J.G. Körner, M.M. Tung, Z. Phys. C70 (1996) 281.

[10] S. Groote, J.G. Körner, Z. Phys. C72 (1996) 255.

[11] S. Groote, J.G. Körner, M.M. Tung, Z. Phys. C74 (1997) 615.

[12] J. Kodaira, T. Nasuno and S. Parke, Phys. Rev. D59 (1999) 014023.

[13] C. Schmidt, Phys. Rev. D54 (1996) 3250.

[14] A. Brandenburg, M. Flesch, and P. Uwer, Phys. Rev. D59 (1999) 014001, Chech. J. of Phys. 50 (2000) Suppl. S1, 51.

[15] V. Ravindran and W.L. van Neerven, Phys.Lett. B445 (1998) 214.

[16] V. Ravindran and W.L. van Neerven, Phys.Lett. B445 (1998) 206.

[17] L. Lewin, ”Polylogarithms and Associated Functions”, North Holland, Amsterdam, 1983;
   R. Barbieri, J.A. Mignaco and E. Remiddi, Nuovo Cimento 11A (1972) 824;
   A. Devoto and D.W. Duke, Riv. Nuovo. Cimento Vol. 7, N. 6 (1984) 1.

[18] P.J. Rijken and W.L. van Neerven, Phys.Lett. B386 (1996) 422, ibid. B392 (1997) 207, Nucl. Phys. B487 (1998) 233.

[19] W.L. van Neerven, Acta Phys.Polon. B29 (1998) 2573.

[20] P.J. Rijken and W.L. van Neerven, Nucl. Phys. B523 (1998) 245.

[21] S. Catani and M.H. Seymour, JHEP 9907:023, (1999).
[22] B. Falk and L.M. Sehgal, Phys. Lett. B325 (1994) 509.

[23] D. de Florian and R. Sassot, Nucl. Phys. B488 (1997) 367.

[24] V. Ravindran, Phys. Lett. B398 (1997) 169, Nucl. Phys. B490 (1997) 272.

[25] M. Stratmann and W. Vogelsang, Nucl. Phys. B496 (1997) 41.

[26] R. Mertig and W.L. van Neerven, Z. Phys. C60 (1993) 489, Erratum ibid. C65 (1995) 360.

[27] O.V. Teryaev and O.L. Veretin, preprint hep-ph/9602362.

[28] K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, Phys. Lett. B371 (1996) 93, Nucl. Phys. B482 (1996) 213.

[29] W.L. van Neerven, Acta Phys.Polon. B29 (1998) 1175.

[30] C. Caso et al., Review of Particle Physics, Eur. Phys. J. C3 (1998) 19, 24, 87.

[31] R. Hagedorn, Relativistic Kinematics, W.A. Benjamin, Inc., New York and Amsterdam, 1964.
FIGURE CAPTIONS

FIG. 1 Longitudinal polarization $P_L$ of the up-quark up to second order in $\alpha_s$ at $Q = M_Z$ for polarized ($\lambda e^- = \pm 1$) and unpolarized (unpol) electrons. The positron beam is unpolarized. (a) Born $P_{L,0}$ (dashed line); (b) order $\alpha_s$ corrected $P_{L,1}$ (dashed dotted line); (c) order $\alpha_s^2$ corrected $P_{L,2}$ (solid line).

FIG. 2 Ratios $R_L$ of the higher order and lower order corrected longitudinal polarization of the up-quark at $Q = M_Z$ for unpolarized electrons and positrons. (a) Ratio of first order corrected and the Born contribution to the polarization $P_{L,1}/P_{L,0}$ (dashed line); (b) Ratio of the second order corrected and the first order corrected polarization $P_{L,2}/P_{L,0}$ (solid line).

FIG. 3 Longitudinal polarization $P_L$ of the bottom-quark up to first order in $\alpha_s$ at $Q = M_Z$ for polarized ($\lambda e^- = \pm 1$) and unpolarized (unpol) electrons. The positron beam is unpolarized. (a) Born $P_{L,0}$ (dashed line); (b) order $\alpha_s$ corrected $P_{L,1}$ (solid line).

FIG. 4 Ratios $R_L$ of order $\alpha_s$ contributions to various quantities for unpolarized electrons and positrons for bottom production at $Q = M_Z$. (a) Ratio of the longitudinal polarization in the massless quark approach with $m_b = 0$ and the exact longitudinal polarization $m_b = 4.5$ (dotted line); (b) Ratio of the unpolarized cross section $d^2\sigma/d\Omega$ for $m_b \to 0$ and $m_b = 4.5$ (dashed dotted line); (c) Ratio of the longitudinal polarization in the massive quark approach with $m_b \to 0$ and the exact longitudinal polarization for $m_b = 4.5$ (dashed line).

FIG. 5 Same as in Fig. 3 but now for top-quark production at $Q = 500$ GeV.

FIG. 6 Transverse polarization $P_T$ of the bottom-quark up to first order in $\alpha_s$ at $Q = M_Z$ for polarized ($\lambda e^- = \pm 1$) and unpolarized (unpol) electrons. The positron beam is unpolarized. (a) Born $P_{T,0}$ (dashed line); (b) order $\alpha_s$ corrected $P_{T,1}$ (solid line).

FIG. 7 The same as in Fig. 6 but now top-quark production at $Q = 500$ GeV.

FIG. 8 Normal polarization $P_N$ of the bottom-quark up to first order in $\alpha_s$ at $Q = M_Z$ for polarized ($\lambda e^- = \pm 1$) and unpolarized (unpol) electrons. The positron beam is unpolarized. (a) Born approximation ($\gamma - Z$-interference term) $P_{N,0}$ (dashed line); (b) order $\alpha_s$ (QCD) corrected results $P_{N,1}$ (solid line).

FIG. 9 Normal polarization $P_N$ of the top-quark up to first order in $\alpha_s$ at $Q = 500$ GeV for polarized ($\lambda e^- = \pm 1$) and unpolarized (unpol) electrons. The positron beam is unpolarized. (a) $\lambda e^- = -1$ (dashed line); (b) unpolarized electrons (solid line); (c) $\lambda e^- = 1$ (dotted line). The lower curves represent the Born approximation ($\gamma - Z$-interference term) $P_{N,0}$ whereas the upper curves stand for the order $\alpha_s$ (QCD) corrected results $P_{N,1}$. 

44
\[ P_L = \begin{cases} \lambda_e = -1 \\ \lambda_e = 1 \end{cases} \]

\[ P_{L,0} \quad P_{L,1} \quad P_{L,2} \]

\[ \cos \theta \]

**Fig. 1**
Fig. 2
\[ P_L = -\lambda_e \]

\[ \lambda_e = -1 \]

\[ \lambda_e = 1 \]

unpol

\[ \cos \theta \]

Fig. 3
Fig. 4
\[ \lambda_e = 1 \text{ unpol} \]
\[ \lambda_e = -1 \text{ unpol} \]

\[ P_L = P_{L,0}, P_{L,1} \]

Fig. 5
\[ \cos \theta = -1 \]

\[ \lambda_e = 1 \]

\[ \lambda_e = -1 \]

Fig. 6
\[ P_T,0 \]

\[ P_T,1 \]

\[ \lambda_e = -1 \]

\[ \lambda_e = 1 \]

Fig. 7
\[ P_{N,0} \quad P_{N,1} \]

\[ \lambda_e = -1 \quad \lambda_e = 1 \]

Fig. 8
\[ P_N = \begin{cases} 1 & \text{unpol} \\ \lambda_e = 1 \end{cases} \]

\[ \lambda_e = -1 \]

\[ \cos \theta \]

Fig. 9