Double-electron ionization driven by inhomogeneous fields

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Abstract  Electron-electron correlation effects play an instrumental role in our understanding of sequential (SDI) and non-sequential double ionization (NSDI) mechanisms. Here, we present a theoretical study of NSDI driven by plasmonic-enhanced spatial inhomogeneous fields. By numerically solving the time-dependent Schrödinger equation for a linear reduced model of He and a double-electron time-evolution probability analysis, we provide evidence for the enhancement effects in NSDI showing that the double ionization yield at lower laser peak intensities is increased due to the inhomogeneity of the laser field. Furthermore, our quantum mechanical model, as well as classical trajectory Monte Carlo simulations, show that inhomogeneous fields are a useful tool for splitting the binary and recoil processes in the rescattering scenario.

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1 Introduction

Since 1982, when L’Huillier presented the first experimental observation of a large enhancement in the double-charge ionization yield of Xe driven by an intense infrared (IR) laser-field, a number of questions about electron-electron (e-e) correlation effects and their mechanisms have arisen \[1,2\]. The fact that those results could not be explained in the framework of SDI, where e-e correlation effects are assumed negligible, opened the path of considering the importance of such correlation effects in the ionization processes \[1,2,3,4,5\]. It was then that the concept of Non-Sequential Double Ionization (NSDI) arose as an explanation of the 1982 experiment \[6,7,8,9\]. However, in the NSDI mechanism there are several processes such as the shake-off, laser-field-assisted rescattering ionization, Rescattering Impact Double Ionization (RIDI) \[10,11,12,13\] and Rescattering Excitation with Subsequent Ionization (RESI), which might take place during the DI of atoms. The question of how to disentangle RIDI and RESI (and within RIDI the binary and recoil processes) are therefore still under investigation in the attosecond science community \[3\].

Prior studies addressing e-e correlation effects in laser-driven multiple ionization processes were done considering only spatially homogeneous fields, i.e. fields that do not present spatial variations in the region where the electron dynamics takes place. This is a legitimate assumption considering that in conventional laser-matter experiments the laser electric field changes in a region on the orders of micrometers, whereas the electron dynamics develops on a nanometric scale (see e.g. \[14\] for more details). However, since recent studies of post-ionization dynamics in spatially inhomogeneous fields \[15\] provides new physical effects and insights, a question arises as to the influence of spatial variation on the DI process. The aim of this work is to present a complete study of DI driven by plasmonic-enhanced spatially inhomogeneous fields with an investigation of NSDI in general, and the RESI and RIDI mechanisms in particular. To this end, we will employ both quantum mechanical approaches, based on the numerical solution of the time-dependent of the Schrödinger equation (TDSE) for two electrons in reduced dimensions, and classical schemes employing classical trajectories Monte Carlo (CTMC) simulations. Within the quantum framework, we employ a linear model for the helium atom, where the motion of both electrons is restricted to the direction of the laser polarization. Experience has shown that 1D models qualitatively reproduce strong-field phenomena such as the double-ionization knee structure \[16,17\] or above-threshold ionization \[18\] and intense-field double ionization mechanisms \[19\].
The rest of this paper is organized as follows. In the next section we present our theoretical tools, namely the TDSE and CTMC for two electrons in reduced dimensions. Then, in Section 3, we show a comparative study between DI driven by conventional laser pulses and DI governed by plasmonic-enhanced spatially inhomogeneous fields. We put particular emphasis on the two-electron momentum distribution, considering it represents one of the most detailed observables and is fully experimentally accessible. We end up with concluding remarks and a brief outlook.

2 Numerical model

We study the two-electron dynamics driven by plasmonic-enhanced fields via a fully quantum mechanical linear model of the helium atom and the integration of Newton’s equations in the framework of a classical trajectory Monte Carlo (CTMC) method. The ab-initio quantum mechanical calculations allow us to address the whole electron-electron (e-e) correlated dynamics by means of the numerical solution of the Time-Dependent Schrödinger Equation (TDSE) similarly to those used by Lein [19] and Watson [23][18]. The Hamiltonian of our 2e system reads (atomic units are used throughout the paper unless specified otherwise)

\[ H = \sum_{j=1}^{2} \left( \frac{p_j^2}{2} + V(z_j) + V_{\text{int}}(z_j, t) \right) + V(z_1, z_2), \]

where \( p_j = -i \frac{\partial}{\partial z_j} \) is the momentum operator corresponding to the \( j \)-th electron (\( j \)-th-e), \( j = 1, 2 \). \( V(z_j) = -\frac{Z}{\sqrt{z_j^2 + a}} \) and \( V(z_1, z_2) = \frac{1}{\sqrt{(z_1 - z_2)^2 + b}} \) are the attractive potential of the interaction of the \( j \)-th electron with the nucleus of charge \( Z \) and the repulsive e-e potential, respectively. The potential describing the interaction of the \( j \)-th electron with the spatially dependent laser electric field in length gauge is [24]

\[ V_{\text{int}}(z_j, t) = (z_j + \frac{\beta}{2} z_j^2) E_h(t), \]

where \( \beta \) denotes the inhomogeneity strength (see e.g. [24][25] for more details) of the plasmonic field and \( E_h(t) = E_0 \sin^2(\omega_0 t/2N) \sin(\omega_0 t + \varphi_0) \) is

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1 This contribution is dedicated to Ted Hänsch on the occasion of his 75th birthday. Although Prof. Hänsch is mostly regarded for “contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique”, and his contributions to laser cooling and physics of ultracold atoms, his influence on attosecond physics is hard to underestimate. For instance, he predicted at very early stages the possibility of generating attosecond pulse trains from phase locked harmonics [20][21] and pioneered and contributed to the initial studies of the high-order harmonics coherence [21][22]. His group has also developed decisive steps extending the frequency combs toward high frequencies regimes.
the spatially homogenous or conventional laser electric field. Here, $E_0$, $\omega_0$, $N$ and $\varphi_0$ are the laser electric peak amplitude, laser frequency, total number of cycles and carrier envelope phase (CEP), respectively.

The numerical algorithm used to solve the TDSE for our linear 1Dx1D He model is the Split Operator method described in Ref. [20,27]. This algorithm takes advantage of the Fast Fourier Transform (FFT) paradigm to evaluate the kinetic energy operators of Eq. (1) in the Fourier space. To speed up our calculations and redistribute the whole 2e wavefunction in position space –with a total number of points $N_T = N_1 \times N_2 \approx 4 \times 10^4 \times 4 \times 10^4 \approx 1.6 \times 10^9$ – on different computational nodes, $N_p$, we employ the message passing interface MPI parallelized version of the FFTW [28]. This implementation allows us to reach large electron excursions $z_j \gg \frac{\varphi_0}{\omega_0}$, which is typical for electrons driven by spatially inhomogeneous fields [24,25]. Each 1Dx1D TDSE calculation took about 11735 CPU-hours on $N_p = 1024$ cores in the Barcelona Supercomputer Center.

For the He linear model, we have fixed the soft-core parameters and the nucleus-charge to $a = b = 1$ a.u., and $Z = 2$, respectively. With these values, we obtain a 2e ground state energy of $E_{1,2} = -2.238$ a.u. Although the matching with the experimental data is not perfect, it is sufficient to qualitatively reproduce the 2e-dynamics driven by a linearly polarized laser-field [10,23,15,29]. The 2e wavefunction $\Psi_0(z_1, z_2)$ of the ground state is calculated via imaginary-time propagation switching off the potential of the laser electric field in Eq. (2), i.e. $V_{\text{ion}}(z_j, t) = 0$. The inner electron and the outer electron ionization potentials are then $I_p^{(i)} = 1.487$ and $I_p^{(o)} = 0.751$ a.u., respectively.

In order to follow the 2e dynamics driven by the plasmonic-enhanced spatially inhomogeneous fields, encoded in the the time-dependent wavefunction $\Psi(z_1, z_2, t)$, the wavefunction $\Psi_0(z_1, z_2)$ of the ground state is propagated in real time via TDSE with the Hamiltonian defined in Eq. (1). In addition, we compute the single-electron ionization, $P_{1e}(t)$, and 2e-ionization, $P_{2e}(t)$, as a function of time $t$. The 2e position distribution $|\Psi(z_1, z_2, t)|^2$ is split into three parts: (i) \{|$z_1|, |z_2| < z_a$\}, (ii) \{|$z_1| < z_a, |z_2| \geq z_a$\} or \{|$z_1| \geq z_a, |z_2| < z_a$\}, and (iii) \{|$z_1|, |z_2| \geq z_a$\} with $z_a = 90$ a.u.. The first region (i) describes the 2e bound wavefunction, $\Psi_0(z_1, z_2, t)$, part of $\Psi(z_1, z_2, t)$. The second one (ii) defines the single-electron ionization $\Psi_{1e}(z_1, z_2, t)$, which is the time dependent He$^+$ yield. And the third region (iii) includes the double-electron ionization $\Psi_{2e}(z_1, z_2, t)$ part, which represents the He$^{2+}$ time dependent yield production. Then, by integrating over regions (ii) and (iii) the single- and double-electron ionization $P_{1e}(t)$ and $P_{2e}(t)$ rates (He$^+$ and He$^{2+}$ production yields) are computed, respectively.

The final two-electron momentum distribution $S_{2e}(p_1, p_2) = |\Psi_{2eM}(p_1, p_2, t_F)|^2$ is evaluated half a laser cycle after the end of the IR laser field as the absolute square of the projection of the final 2e wavepacket $\Psi_{2e}(z_1, z_2, t_F)$ on the double-electron plane waves $\Phi_{p_1, p_2}(z_1, z_2) = \frac{1}{\sqrt{2\pi}} \exp[i (z_1 p_1 + z_2 p_2)]$ [19,11]. Furthermore, the correlated ion $S_{\text{He}^{2+}}(p_{\text{ion}})$ momentum distribution is
calculated by projecting $\hat{S}_{2e}(p_1, p_2)$ on the diagonal $p_1 = p_2$, which corresponds to the total 2e momentum $p = p_1 + p_2$. Thereby, via momentum conservation of the system, the ion momentum reads $p_{\text{ion}} = -(p_1 + p_2)$.

In order to supplement the quantum mechanical calculations and understand the physical origin of the effects of the plasmonic-enhanced field better, we implement CTMC simulations to investigate electron trajectories after ionization of helium under the so called RIDI mechanisms. The simulations are restricted to one dimension, namely the direction of field polarization, in which also the field inhomogeneity develops. The trajectories are launched at a starting time $t_0$, which is distributed probabilistically following the Ammosov-Delone-Krainov (ADK) formula \cite{30,31}, typically used to model strong field ionization \cite{32,33,34,35}

$$P(t_0, v_\perp) = \exp\left(-\frac{2(2I_p(t_0))^{3/2}}{3E_h(t_0)}\right)$$

(3)

corresponding to an atom centered at the origin. $I_p$ denotes the Stark shifted ionization potential \cite{36}

$$I_p(t_0) = I_p^{(0)} + \frac{1}{2}(\alpha_N - \alpha_I)E_h(t_0)^2$$

(4)

with $\alpha_N$ and $\alpha_I$ representing the polarizability of the atom and ion, respectively. The tunnel exit radius is assumed to be zero following the simple man’s model \cite{10}. The dynamics of each electronic trajectory after ionization is solved numerically by integrating the Newton’s equations of motion, which takes into account the laser field, but not the Coulomb potential following the model in \cite{11}.

If the electron returns to the ion ($z = 0$) at time $t_r$ with kinetic energy $E_{k1}(t_r)$ larger than the ionization potential $I_p^{(i)}$ of the second electron \cite{18}, this second, inner electron can be ionized as well. In this ionization process the kinetic energy of the first electron is reduced by $I_p^{(i)}$ and the second electron is born in the continuum with zero velocity

$$p_1(t_r) = \pm \sqrt{2\left(E_{k1}(t_r) - I_p^{(i)}\right)}$$

$$p_2(t_r) = 0.$$  

(5)

Here, the two different signs in $p_1$ describe the possibility of scattering the first electron into forward, binary, or backward, recoil, direction with respect to its momentum directly before the ionization of the inner electron. For each double ionization event, we calculated both options. The dynamics after the second ionization is again determined by the propagation in the laser field, where the Coulomb force is completely neglected \cite{11}.
3 Double-electron ionization

To study the e-e correlation effects we firstly compute the final single- and double-electron ionization yields as a function of the peak laser field intensity for a few-cycle IR pulse. This allows us to identify the intensity regions where the spatially inhomogeneous field substantially modifies the double-electron ionization process. Secondly, we compute the two-electron momentum distribution as a function of the inhomogeneity degree at a fixed intensity. This scan on $\beta$ provides enough evidence about the role of the inhomogeneous field in the 2e ionization process. Furthermore, we scrutinize if any e-e correlation effects can be found in this process.

3.1 He$^+$ and He$^{2+}$ ion yields

We numerically compute the final 2e-ionization yield by the procedure described in Section 2. The grid parameters used in those calculations are $N_1 = N_2 = 40960$ points and $\delta z_1 = \delta z_2 = 0.25$ a.u. The integration time step was chosen $\delta t = 0.025$ a.u. The results of the single- and double-electron ionization yield as a function of the peak laser field intensity for the homogeneous ($\beta = 0$) and inhomogeneous fields with $\beta = 0.005$ a.u., are depicted in Fig. 1.

A sizeable enhancement of the final 2e ionization $P_{2e}(t_F, I_0)$ is observed for the inhomogeneous field case when compared to the conventional one. Similar effects are also obtained in the comparison with the single-electron ionization yields. This enhancement clearly shows that the spatially inhomogeneous fields play an instrumental role in the NSDI of helium.

Naturally, the question about the origin of this enhancement arises. In order to answer it, we compute the single- $P_{1e}(t)$ and double-electron $P_{e2}(t)$ ionization yield as a function of time at a fixed peak intensity of $I_0 = 2 \times 10^{14}$ W/cm$^2$. Here we focus our attention on the intensity region where the double ion yield, He$^{2+}$, is enhanced by the inhomogeneous field. According to Fig. 1 one such region is $I_0 = 1 - 5 \times 10^{14}$ W/cm$^2$. The results of the time-evolved probabilities are depicted in Fig. 2.

For the single-electron ionization $P_{1e}(t)$ shown in Fig. 2(a), the “inhomogeneous” ionization yield is larger than the conventional one, in particular, at about 2.5 cycles of the IR laser. We could trace out the origin of this observation in a much stronger distortion of the laser-atomic potential barrier, which raises the probability of the first bound electron to ‘escape’ from the atom.

Fig. 2(b) shows a comparison of $P_{2e}(t)$ for conventional and inhomogeneous fields. About 3.4 cycles of the IR laser oscillations, the 2e ionization yield largely increases for the inhomogeneous field case with respect to the conventional one by more than 5-times. At this very low inhomogeneity degree of $\beta = 0.005$ a.u., and low IR peak intensity, this enhancement of the 2e-ionization rate is a very surprising result. Similar behaviour was previously observed in [15], where the double-electron ionization reaches higher
**Double-electron ionization driven by inhomogeneous fields**

**Fig. 1** Numerical TDSE calculations of the single- (red) and double-electron (blue) ionization yields of our linear 1Dx1D He 2e-model driven by homogeneous (line with circles) and inhomogeneous (dashed line) fields as a function of the laser peak intensity. The mean frequency of the IR laser field is $\omega_0 = 0.057$ a.u. (1.55 eV), the CEP is $\varphi_{\text{CEP}} = 0^\circ$ and the total number of cycles is $N = 4$ under a $\sin^2$ envelope.

**Fig. 2** Single- (a) and double-electron (b) ionization yields of our linear 1Dx1D He 2e-model driven by conventional and spatially inhomogeneous fields as a function time (see left axis). The IR laser field oscillations are depicted in red solid line. The peak intensity used to follow the two-electron dynamics is $I_0 = 2 \times 10^{12}$ W/cm$^2$. The other laser parameters are the same than those used in Fig. 1.

Yields leading to an enhancement in the intensity of the HHG signal. However, in that latter case a larger inhomogeneity degree of $\beta = 0.02$ a.u was used.

An hypothesis that might explain that result is based on the three step Corkum’s model [10,9,6] where (1) the first electron ionizes via tunnelling, (2) then this electron propagates in the continuum gaining energy from the laser field - in our case a spatially inhomogeneous field - and then when the field changes its sign the electron has a probability to re-collide with the ion core He$^+$. As a third step, this colliding electron can kick out the second inner electron if and only if the first electron kinetic energy is larger than
**Fig. 3** Numerical two-electron momentum distribution for various inhomogeneity degrees: $\beta = 0.000$ (a), 0.005 (b), 0.010 (c) and 0.015 a.u. (d). The color scale is $\log_{10}[S_{2e}(p_1, p_2)]$. Vertical and horizontal blue dashed lines denote the 2e momentum axes which help us to distinguish between correlated, (i) and (iii) quadrants, and anticorrelated, (ii) and (iv) quadrants, regions. The diagonal red dashed line $p_1 = p_2$ represents the max e-e correlation momentum points or the total 2e momentum $p = p_1 + p_2$. CTMC for the (e, 2e) mechanisms are superimposed in green-circles (recoil process) and in red-squares (binary process) in (c) and (d) panels. The laser-peak intensity used for these numerical calculations is $I_0 = 2 \times 10^{12}$ W/cm². The other laser parameters are the same as those used in Fig. 1.

$I_{(i)}^{(i)}$, the ionization potential of the second inner electron. This process is called double-ionization by re-scattering impact direct ionization (RIDI) or (e, 2e).

From the behaviour of electrons driven by spatially inhomogeneous fields (see e.g. [14]), it is very likely that the first-ionized electron gains a much larger energy compared to the conventional case. Thus, at the instant of recollision, the second electron would have a higher chance to be ionized in a spatially inhomogeneous field, which corresponds to an enhancement of the double electron ionization probability.
Another interesting observable, which contains information about the e-e correlation, is the 2e-momentum distribution. This observable has allowed to disentangle the common sequential and non-sequential double RESI, rescattering impact ionization and laser-field assisted rescattering ionization mechanisms [12,13,37]. Fig. 3 depicts $S_{2e}(p_1,p_2)$ for different $\beta$ parameters at the same fixed laser peak intensity of $I_0 = 2 \times 10^{14}$ W/cm$^2$. The double-electron map in Fig. 3(a) exhibits two large probability peaks on the first quadrant of the correlation region - in almost perfectly concordance with the data published in ref. [19]. This probability distribution indicates that both electrons prefer to leave on the same (positive) direction. It is understood that the repulsive e-e Coulomb potential plays an important role at those relative low peak intensity for the He model [19].

Note that a classical rescattering electron scenario (e, 2e) is not good enough for describing this NSDI mechanism of our He model at this peak intensity. From a classical viewpoint, the rescattering energy $E_{k,\text{max}} = 3.17U_p = 1.4$ a.u., is lower than the ionization potential of the inner electron $I_{p} \sim 1.5$ a.u. Instead, this double-electron ionization map could be understood as a laser-field-assisted rescattering process for which such a constraint does not apply [11,12,13,19,37]. As pointed out in [9,19], the driving laser field provides the rest of the required energy to remove the second electron at the instant of recollision.

For further interpretations of Fig. 3 we recall that finding double ionization in quadrants I and III corresponds to both electron momenta pointing in the same direction. In contrast, quadrants II and IV contain the cases of the electrons’ momenta pointing into opposite directions. When both electrons leave the atom in the same direction, we say they are correlated. Comparing the 2e-momentum distributions in Figs. 3(a) and (b), we find that the two electrons prefer to detach in opposite directions when driven by plasmonic-enhanced spatially inhomogeneous fields. This effect is even larger for an inhomogeneity degree of $\beta = 0.01$ and 0.015 a.u., as can be seen in Figs. 3(c)-(d). We note, however, the appearance of a small 2e-probability also in the correlated regions.

Naturally, questions about the physical mechanisms behind those effects in the 2e maps emerge. In order to address those questions, we superimposed our CTMC calculations on the TDSE results in Figs. 3(c) and (d) for the cases of binary (red-squares) and recoil (green-circles) processes. For those results, an excellent agreement between the TDSE and the CTMC calculations is found, in particular for the case of an inhomogeneity degree $\beta = 0.015$ a.u.. This clearly corroborates that the forward rescattering process with respect to the first incident electron direction, binary, is highly probable within that so-called (e, 2e) mechanisms if spatially inhomogeneous fields drive the two-electron system. Note that this agreement of TDSE and CTMC supports our previous observation that the 2e-particles are likely to prefer to leave the atom in opposed directions.
According to Weber et al. [8] the momentum distribution corresponding to the coordinates \( p = p_1 + p_2 \) (diagonal along \( p_1 = p_2 \)) and \( p^- = p_1 - p_2 \) (diagonal \( p_1 = -p_2 \)), are helpful for describing the importance of two effects: e-e repulsion and acceleration of the particles by the optical field. On the one hand, e-e repulsion does not change \( p \) but contributes to \( p^- \). On the other hand, the momentum transfer received from the field is identical. So, this part of acceleration does not change \( p^- \) but contributes to \( p \). Note, however, that this statement is only valid if the electric field does not depend on the position. Thereby, for the inhomogeneous field cases we cannot conclude that the acceleration part does not contribute to \( p^- \). This is in absolute concordance with what we observe in the 2e momentum maps for \( \beta = 0.010 \) and 0.015 a.u..

Additionally, in Fig. 4 we show the correlated ion He\(^{2+}\) momentum distributions corresponding to the \( S_{2e}(p_1,p_2) \) panels of Fig. 3. A first observation is that a sizeable momentum-shift is found for the recoiling ion as the inhomogeneity degree \( \beta \) increases. For the conventional field case depicted in Fig. 4(a), the full momentum width of the distribution is about \( \pm 2A_0 = \pm 2.6 \) a.u., where \( A_0 = E_0/\omega_0 \) is the maximum vector potential for the conventional field [7,19,38]. An asymmetry in the amplitude of the ion distribution \( S_{He^{2+}}(p_{ion}) \) is observed at \( \beta = 0 \) a.u. This is due to the employed laser-field being within the few-cycle regime, \( N = 4 \), see e.g. Ref. [8] about
the CEP effects. However, three peaks at about $p_{\text{ion}}^{(\text{max})} = \{-A_0, 0, +A_0\}$ are found. These might suggest that the laser-field assisted rescattering double ionization mechanism and the RESI mechanism take place simultaneously in such a complex correlated momentum map.

In case of inhomogeneous fields, the ion distribution shape strongly depends on the parameter $\beta$. While the inhomogeneity increases, the expectation value of the ion momentum $\langle p_{\text{He}^{2+}} \rangle$ is shifted from negative to positive momentum, indicating that the ion recoils in the completely opposite direction compared to the conventional field case. Note that these several peaks that appear in the ion-distributions, suggest the possibility of different interference paths in the processes of DI driven by that spatially inhomogeneous field.

So far, we have studied double ionization in He via scanning the 2e-momentum distribution over the inhomogeneity parameter $\beta$ at a fixed laser intensity. In order to obtain an insight about the 2e ionization when the laser-peak intensity increases, we compute and compare the momentum-momentum distributions for the conventional $\beta = 0$ and inhomogeneous $\beta = 0.005$ a.u fields. Additionally, our ab-initio TDSE calculations are compared with the CTMC simulations. The results are depicted in Fig. 5. While the peak intensity increases from 3 to $7 \times 10^{14}$ W/cm$^2$ for conventional fields, some pronounced lobes in the correlation regions are observed. Those density lobes are larger in the anticorrelated region. This is a signal that the e-e Coulomb repulsion force is losing its importance while the laser-field peak intensity increases. In particular, that effect is larger for the highest intensity. In addition, note that a better agreement between TDSE and CTMC is found in the cases of Figs. 5(c) and (e) as it is expected. This indicates that the (e, 2e) processes are the main mechanisms behind those calculations. However, in Fig. 5(a), a laser-field assisted rescattering DI process still dominates over the RIDI mechanisms. This we conclude from the poor agreement between the quantum mechanical and the classical calculations for the binary and recoil processes.

On the other hand, for inhomogeneous fields, Fig. 5(b), (d) and (f), the probability of 2e ionization with opposite momenta increases. This clearly indicates that the propagation of electrons under the influence of plasmonic field changes completely the 2e-dynamics. Note, that a signal in quadrant III of the correlation region is also observed, which is an indication that both electrons, independently of the incident direction of the first colliding particle, prefer to leave with negative momenta directions. Furthermore, while the laser peak intensity increases, the V-like shape in the quadrant III tends to be much closed, and also a strong signal along the diagonal $p_1 = p_2$ for $p_1 < 0$ is clearly observed. These facts are the signature of e-e correlation effects rapidly losing importance while the particles are propagating in the plasmonic field. Note, however, that e-e repulsion somehow is still present because of the large momentum density width along the diagonal $p_1 = -p_2$.

Finally, it is interesting to point out that up to $7 \times 10^{14}$ W/cm$^2$ the NSDI by inhomogeneous fields is still within the rescattering (e, 2e) scenario. This
Fig. 5 Numerical two-electron momentum distributions driven by homogeneous \( \beta = 0 \) (a), (c), (e), and inhomogeneous fields (b), (d), (f) with \( \beta = 0.005 \) a.u., for three different laser-peak intensities: \( I_0 = 3, 5, 7 \times 10^{14} \text{ W/cm}^2 \). The CTMC calculations for binary (red-squares) and recoil (green-circles) processes are superimposed on the momentum-momentum maps. Other laser parameters are the same as in Fig. 1.

Statement is supported by the CTMC simulations that agree very well with the TDSE calculations for all studied cases. This demonstrates that the isolation of binary and recoil processes is very sensitive to the laser peak intensity. We should note, however, that between 2 and \( 5 \times 10^{14} \) W/cm\(^2\), we ensure that those backward and forward rescattering processes could be separated, just by observing the anticorrelating and correlating regions of the momentum-momentum distribution.
4 Conclusions

Non-sequential double ionization of helium atoms driven by a plasmonic-enhanced spatially inhomogeneous fields has been theoretically investigated. By means of the fully numerical solution of the time-dependent Schrödinger equation, we observed that ion yield of He$^{2+}$ substantially increases while the inhomogeneous field drives the system. An analysis of the single- and double-electron time-evolution probabilities and the two-electron momentum distribution simulations of the binary an recoil mechanisms support that the main reason for this enhancement corresponds to a high accumulated energy of the first re-colliding electron when it is moving in the spatially inhomogeneous field.

An unexpected (e, 2e) mechanism at very low intensity, i.e. $I_0 = 2 \times 10^{14}$ W/cm$^2$ is observed with increasing inhomogeneity. This means that by engineering the inhomogeneous field the two different mechanisms, namely the laser-field-assisted re-scattering and the RIDI process can be isolated. Furthermore, our interpretation of the fully ab-initio TDSE for the two-electron momentum distributions by comparing to CTMC simulations, allowed us to distinguish between binary and recoil processes if and only if the spatially inhomogeneous field drives the system.

Still there are open questions, e.g. concerning the role of the e-e Coulomb potential while both identical particles are propagating within the spatially inhomogeneous field and how this effect is related to the 2e momentum distribution maps for larger laser peak intensities here used. We plan to address these questions in subsequent work.

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