Bosonic interactions in a nonlocal theory in (2+1) dimensions

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Pseudo-Quantum Electrodynamics (PQED) provides an excellent description of the interaction between charged particles confined to a plane. When we couple pseudo-gauge field with a bosonic matter field, we obtain the so-called Scalar Pseudo-Quantum Electrodynamics (SPQED). In this work, we make a perturbative analysis of SPQED via Feynman diagrams. We compute the one loop Green functions: bosonic field self-energy, electromagnetic field self-energy and vertex corrections. Finally we consider the non relativistic interaction potential between two bosonic particles. We compute the radiative corrections to the usual Coulomb potential and comment on the analogies and the differences with the fermionic case.

I. INTRODUCTION

The description of electronic interactions in two-dimensional materials, such as graphene, sparked a renewed interest into nonlocal relativistic quantum field theories in reduced dimensionalities at low energies. The nonlocality in the space-time is generated by a dimensional reduction [3, 4] that has been largely discussed in literature [5, 7]. A well known example is the so-called Pseudo-quantum electrodynamics (PQED) [4], sometimes reduced QED is also used in the literature [8], which applies to the description of the electron-electron interactions in graphene and transition metal dichalcogenides (TMD’s). In the case of graphene, quantum corrections for the longitudinal conductivity and a driven quantum valley Hall effect are expected to emerge at very low temperatures and large coupling constant [9]. Furthermore, it has been shown that the correction to the electron g-factor in graphene also may be calculated from a rescaling of an effective length scale which is due to the appearance of a new diagram in the bosonic case.

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II. THE MODEL

We define the Lagrangian of SPQED as follows

\[ \mathcal{L} = -\frac{1}{4}F_{\mu\nu} \left( \frac{2}{\sqrt{\Box}} \right) F^{\mu\nu} + \partial_\mu \phi \partial^\mu \phi^* - m^2 \phi \phi^* + i\epsilon A_\mu \left( \frac{\partial^\mu}{\sqrt{\Box}} \phi^* \right) + e^2 A_\mu A^\mu \phi \phi^* + \frac{\xi}{2} A_\mu \frac{\partial^\mu}{\sqrt{\Box}} A^\nu, \]  (1)
where $F_{\mu\nu}$ is the usual field-intensity tensor of the $U(1)$ gauge field $A_\mu$, which mediates the electromagnetic interaction in 2D (pseudo electromagnetic field), $\Box$ is the d’Alembertian operator, $\phi$ is the massive charged Klein-Gordon field, $\phi \bar{\phi} \phi^* = \phi (\Box^\alpha \phi^\alpha) - (\Box \phi) \phi^*$, $\epsilon$ is coupling constant and $\xi$ is a gauge fixing parameter.

The Feynman rules for this theory in Minkowski space are as follows. The bare propagators are

$$\begin{align*}
\frac{p}{p} &= S_\phi^{(0)} (p) = \frac{i}{p^2 - m^2}, \\
\frac{\mu}{\nu} &= \Delta^{(0)}_{\mu\nu} (p) = \frac{-i}{2\sqrt{p^2}} \left[ g_{\mu\nu} - \left( 1 - \frac{1}{\xi} \right) \frac{p_\mu p_\nu}{p^2} \right]
\end{align*}$$

and the vertices,

$$\begin{align*}
\Gamma^{(1,2)}_0 \stackrel{\mu}{\mu} \stackrel{\nu}{\nu} &= \left( \Gamma^{(1,2)}_0 \right)^\mu = i \epsilon (p_1 + p_2)^\mu, \\
\Gamma^{(2,2)}_0 \stackrel{\mu}{\mu} \stackrel{\nu}{\nu} &= \left( \Gamma^{(2,2)}_0 \right)^{\mu\nu} = i \epsilon^2 g^{\mu\nu},
\end{align*}$$

where we are using the notation $\Gamma^{(N_A,N_B)}$ for the vertices where $N_A$ and $N_B$ is the number of gauge field $A_\mu$ and bosonic scalar fields $\phi$ interacting at the same vertex, respectively.

The non-local term present in the Maxwell Lagrangian in Eq. (1) renders the canonical dimension of the gauge field equal to one, in units of mass, while the scalar field has dimension 1/2. Therefore, the coupling constant $\epsilon$ is dimensionless in the $2 + 1$ space-time, and the theory is renormalizable, analogously to scalar QED$_4$.

The renormalization procedure in quantum-field theory leads us to the study of the primitively divergent Feynman diagrams. This information can be obtained from the degree of superficial divergence of a generic graph $\gamma$, given by

$$d(\gamma) = 3 - N_A - \frac{1}{2} N_B.$$  

Therefore, the quadratically divergent diagrams are those with $N_B = 2$ and $N_A = 0$ (scalar boson self-energy). For $N_B = 0$ and $N_A = 2$ the diagrams are linearly divergent (photon self-energy), whereas for $N_A = 2$ and $N_B = 2$ and for $N_A = 1$ and $N_B = 2$ the diagrams have logarithmic and linear divergence, respectively (vertex correction). In what follows, we will calculate these diagrams using the dimensional regularization procedure [17–19] as a way to obtain finite Feynman amplitudes and we will adopt Feynman’s gauge $\xi = 1$. Accordingly, the dimensionless coupling constant can be written as $\epsilon \rightarrow \epsilon \mu^\beta$, where $\mu$ is an arbitrary massive parameter and $\beta = 3 - D$.

### III. Radiative Corrections

In this section we discuss the results of the Feynman diagrams up at one loop for bosonic field self energy (subsection A) as well as for the gauge field (subsection B).

#### A. One loop bosonic field self energy

The one-loop corrections of scalar field propagator are shown in Fig. 1.

![Fig. 1: The complex scalar field propagator at one-loop corrections, which are quadratically divergent.](image)

We first evaluate the diagram shown in Fig. 2. We present in some detail this calculation and use the important relations in other diagrams throughout the paper.

![Fig. 2: One loop correction to scalar propagator which we will call by $-i\Sigma_1 (p^2, m^2)$](image)

Using the Feynman rules (2)-(4), we have

$$-i \Sigma_1 (p^2, m^2) = \int \frac{d^D k}{(2\pi)^D} i \epsilon^\mu \hat{\mu} (2p - k)^\mu \Delta^{(0)}_{\mu\nu} (k) \times$$

$$\times i \epsilon^\nu \hat{\nu} (2p - k)^\nu S_\phi^{(0)} (p - k).$$

We rewrite this relation as

$$-i \Sigma_1 (p^2, m^2) = -\frac{\epsilon^2 \mu^\beta}{2} \int \frac{d^D k}{(2\pi)^D} \frac{N}{[(p - k)^2 - m^2] \sqrt{k^2}},$$

where $N = (2p - k)^\mu (2p - k)_\mu$. To solve this integral we use the Eq. (A1) in Appendix A with $\alpha = 1$ and $\beta = 1/2$, make a change of variable $k \rightarrow k + px$ and use the dimensional regularization, with help of Eq. (A3) and Eq. (A4), to obtain

$$-i \Sigma_1 (p^2, m^2) = \Sigma_1^{\text{div}} (p^2, m^2) + \Sigma_1^{\text{finite}} (p^2, m^2),$$

where $\Sigma_1^{\text{div}} (p^2, m^2)$ represents the divergent term while
$\Sigma_1^{\text{finite}}(p^2, m^2)$ is the finite part of the diagram. Explicitly

$$\Sigma_1^{\text{div}}(p^2, m^2) = -\frac{e^2}{32\pi^2} \frac{1}{\varepsilon} \left(\frac{40p^2}{3} + 8m^2\right),$$  \hspace{1cm} (10)

$$\Sigma_1^{\text{finite}}(p^2, m^2) = -\frac{e^2}{32\pi^2} A(p^2, m^2, \mu),$$  \hspace{1cm} (11)

with

$$A(p^2, m^2, \mu) = \int_0^1 \frac{dx}{\sqrt{1-x}} \left[ 3\Delta_1 \ln \left(\frac{4\pi \mu^2 e^{\frac{x}{2}}}{\Delta_1 e^{\gamma}}\right) + 2p^2 (4 - 4x + x^2) \ln \left(\frac{4\pi \mu^2}{\Delta_1 e^{\gamma}}\right) \right],$$  \hspace{1cm} (12)

where $\Delta_1 = p^2(x^2 - x) + m^2x$ and $\gamma = 0.5772...$ is the Euler-Mascheroni constant.

The second diagram corrected the scalar field propagator is shown in Fig. 3. Following the Feynman rules, we can write

*FIG. 3: One loop correction to scalar propagator which we will cal by $-i\Sigma_2(p^2)$.***

$$-i\Sigma_2(p^2) = \int \frac{d^Dk}{(2\pi)^D} \frac{(ie\mu^2 g^{\mu\nu})}{2\sqrt{k^2}}.$$  \hspace{1cm} (13)

We observe that we have an infrared divergence. To avoid this problem, let’s introduce a mass term, $\bar{M}$, and after the calculations we take the limit $\bar{M} \to 0$. Thus

$$-i\Sigma_2(p^2) = \frac{3e^2\mu^2}{2} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{\sqrt{k^2 - \bar{M}}}.$$  \hspace{1cm} (14)

Using the Eq. (A4) with Eq. (A7) and taking the limits $\varepsilon \to 0$ and $\bar{M} \to 0$ the self-energy results into [20]

$$-i\Sigma_2(p^2) = 0.$$  \hspace{1cm} (15)

Therefore, this diagram does not contribute to the perturbative series.

The one loop corrections to complex scalar field is given by the contributions of Eq. [9] and Eq. [15], thus

$$-i\Sigma(p^2, m^2) = -i\Sigma_1 - i\Sigma_2 = -\frac{e^2}{32\pi^2} \frac{1}{\varepsilon} \left(\frac{40p^2}{3} + 8m^2\right) - \frac{e^2}{32\pi^2} A(p^2, m^2, \mu).$$  \hspace{1cm} (16)

After using the minimal subtraction scheme, where the divergent term of the self-energy is neglected, we find the renormalized amplitude given by

$$-i\Sigma_R(p^2, m^2) = -\frac{e^2}{32\pi} A(p^2, \mu^2 = m^2),$$  \hspace{1cm} (17)

where we have assumed $\mu^2 = m^2$ for the sake of simplicity. The pole of the full propagator, after we include the boson self-energy, yields the renormalized mass $m_R$ for the bosonic field, namely

$$m_R^2 = m^2 + \frac{e^2}{32\pi} A(p^2 = m^2, \mu^2 = m^2).$$  \hspace{1cm} (18)

Using Eq. (12), we have

$$m_R \approx \pm |m| \sqrt{1 + \frac{75\alpha}{8}}$$  \hspace{1cm} (19)

with $e^2 = 4\pi\alpha$ for comparison with QED. Eq. (19) shows that the main effect of repulsive interactions is to slightly increase the energy gap, as expected.

B. One loop gauge field self energy

One loop corrections to gauge field propagator are shown in Fig. 4 and Fig. 5, and are given by

$$-i\Pi^{\mu\nu}_1(p^2) = \int \frac{d^Dk}{(2\pi)^D} \frac{ie\mu^2 (2k + p)^\mu S^{(0)}_\phi (p + k) \times \epsilon_\mu e^{\nu \tilde{z}} (2k + p)^\nu S^{(0)}_\phi (k)}{2\sqrt{k^2 - \bar{M}}}.$$  \hspace{1cm} (20)

and

$$-i\Pi^{\mu\nu}_2(p^2) = \int \frac{d^Dk}{(2\pi)^D} \frac{ie\mu^2 \hat{g}^{\mu\nu} S^{(0)}_\phi (k)}{2\sqrt{k^2 - \bar{M}}}.$$  \hspace{1cm} (21)

*FIG. 4: d(G) = 1, because $N_A = 2, N_B = 0$.***

Up to that order these diagrams are the same as SQED$_3$ and using dimensional regularization we obtain...
with $\Delta$ as approximation to the photon propagator can be visualized in the ultraviolet regime. The Random Phase Approximation reflects the conservation of current $p$ and therefore $\Pi(p^2) = 1$ as $\mu \nu$.

FIG. 5: One loop correction to gauge field. For this diagram $d(G) = 1$ as $N_A = 2$, $N_B = 0$.

$$-i\Pi_1^{\mu\nu}(p^2) = \frac{ie^2}{8\pi} \{g^{\mu\nu} \left( \frac{1}{4m^2} - \frac{(p^2 - 4m^2)}{2} I(m, p) \right) + \frac{p^\mu p^\nu}{p^2} \left[ \frac{1}{4m^2} + \frac{(p^2 - 4m^2)}{2} I(m, p) \right] \},$$

(22)

and therefore

$$-i\Pi_2^{\mu\nu}(p^2) = -2i \epsilon^2 g^{\mu\nu} \sqrt{4m^2},$$

(23)

where

$$\pi(p^2) = \frac{-e^2}{8\pi} \left( \frac{1}{4m^2} + \frac{(p^2 - 4m^2)}{2} I(m, p) \right),$$

(25)

and

$$I(m, p) = \int_0^1 dx \frac{1}{\sqrt{m^2 + p^2(x^2 - x)}},$$

(26)

which reflects the conservation of current $p_\mu \Pi^{\mu\nu} = 0$.

Note that the use of dimensional regularization already renormalizes the diagram which has a linear divergence in the ultraviolet regime. The Random Phase Approximation to the photon propagator can be visualized as

$$\Delta_{\mu\nu} = \Delta^{(0)}_{\mu\nu} + \Delta^{(0)}_{\mu\alpha} (-i) \Pi^{\alpha\beta} \Delta^{(0)}_{\beta\nu} + \Delta^{(0)}_{\mu\alpha} (-i) \Pi^{\alpha\beta} \Delta^{(0)}_{\beta\nu} (-i) \Pi^{\lambda\nu} \Delta^{(0)}_{\mu\nu} + \ldots,$$

(27)

with $\Delta^{(0)}_{\mu\nu} = -\frac{ip_{\mu\nu}}{2\sqrt{p^2}}$, where $P_{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$. We can rewrite Eq. (27) so that we get

$$\Delta_{\mu\nu}(p) = \frac{-iP_{\mu\nu}}{2\sqrt{k^2 - \pi(p^2)}}.$$  

(28)

C. VERTEX CORRECTIONS

The first vertex to be analyzed is the interaction vertex that represents the interaction of the bosonic fields, $\phi^*$ and $\phi$, with the gauge field, $A_\mu$. The perturbative series is shown in Fig. 6.

![Diagram](image)

FIG. 6: One-loop corrections to the vertex $ieA_\mu \left( \phi^* \partial^\mu \phi \right)$.

This vertex correction, $\left( \Gamma_1^{(1,2)} \right)^\nu$, is linearly divergent.

With the use of Feynman rules we write

$$\left( \Gamma_1^{(1,2)} \right)^\nu = \frac{e^2}{3!} \left( ie\mu \xi \right)^3 \int \frac{d^Dk}{(2\pi)^D} (2k + p + q)^\nu \times S_\phi^{(0)}(k + p) \Delta^{(0)}_{\sigma\eta}(k)(k + 2p)^\eta \Delta^{(0)}_{\mu\nu}(k) S_\phi(k + q).$$

(29)

To compute the integral we introduce the Feynman parameters through Eq. $[A2]$ with $\alpha = \beta = 1$ and $\gamma = 1/2$, make a change of variables $k \rightarrow k - qx - pq$, then evaluate the integrals in $k$ variable with Eq. $[A5]$, Eq. $[A3]$, Eq. $[A4]$, using the expansions given by Eq. $[A6]$ and Eq. $[A7]$, and obtain

$$\left( \Gamma_1^{(1,2)} \right)^\nu = \left( \Gamma_1^{(1,2)} \right)^\nu + \left( \Gamma_1^{(1,2)} \right)^\nu,$$

(30)

with

$$\left( \Gamma_1^{(1,2)} \right)^\nu = \frac{e^3}{12\pi^2} \frac{1}{\epsilon} \left( p^\nu + q^\nu \right),$$

(31)

$$\left( \Gamma_1^{(1,2)} \right)^\nu = \frac{e^3}{\pi^2} B^\nu(p, q, m, \mu).$$

(32)
and using the expansions Eq. (A6) and (A7), we rewrite

\[
B^\nu (p, q, m, \mu) = -\frac{1}{96} \int_0^1 dx \int_0^{\frac{1-x}{y}} \frac{dy}{\sqrt{1-x-y}} \times \\
\times \left\{ (U_1)^\nu \ln \left( \frac{4\pi \mu^2}{\Delta_3 e^\gamma} \right) + (U_2)^\nu \ln \left( \frac{4\pi \mu^2}{\Delta_3 e^{2+\gamma}} \right) \\
- \frac{2}{\Delta_3} (U_3)^\nu \right\},
\]

where \( \Delta_3 = (px + qy)^2 - p^2x - q^2y + m^2(x + y) \) and \((U_1)^\nu, (U_2)^\nu \text{ and } (U_3)^\nu\) are given by

\[
U_1^\mu = q^\mu (4 - 4y) + p^\mu (4 - 4x), \tag{34}
\]
\[
U_2^\mu = p^\mu (1 - 2x) + q^\mu (1 - 2y), \tag{35}
\]
\[
U_3^\mu = p^\beta q_\beta p^\nu [(1 - 2x)(2 - x)2 - y + (x - 2x^2)] + p^\beta q_\beta q^\mu [(1 - 2y)(2 - x)2 - y + (x - 2x^2)] \\
\times (2 - y) + yx - 2y^2x - p^2(2x - x^2) \\
\times [p^\mu (1 - 2x) + q^\mu (1 - 2y)] - q^2 \\
\times [q^\nu (1 - 2y) (2y - y^2) + \\
+ p^\mu (1 - 2x) (2y - y^2)]. \tag{36}
\]

Next, we calculate the diagram in Fig. 8.

![FIG. 8: One-loop correction to the vertex \( ieA_\mu (\phi \partial^\mu \phi^*). \)
This vertex correction, \( (\Gamma_2^{(1,2)})^\alpha \), is linearly divergent.

This vertex diagram is given by

\[
(\Gamma_2^{(1,2)})^\alpha = 2 \times \frac{4}{2!} \int \frac{d^Dk}{(2\pi)^D} e^{i\phi^\mu g^\alpha\nu S_0^\phi (p - k)} \\
\times ie\mu^2 (2p - k)^\mu \Delta_\mu^{(0)} (k), \tag{37}
\]

where the first factor 2 is due to the multiplicity of the diagram. Introducing the Feynman parameters, Eq. (A1) with \( \alpha = 1 \) and \( \beta = \frac{1}{2} \), and making the change of variables \( k \rightarrow k + xp \), we have

\[
\Gamma_2^{(1,2)} = -e\mu^2 3^\nu \int_0^1 dx \frac{dx}{\sqrt{1-x}} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{|k^2 - \Delta_4|^2}, \tag{38}
\]

with \( \Delta_4 = p^2 (x^2 - x) + m^2 x. \) With the help of Eq. (A4) and using the expansions Eq. (A6) and (A7), we rewrite Eq. (38) as

\[
(\Gamma_2^{(1,2)})^\alpha = \left( \Gamma_2^{(1,2)} \right)^\alpha_{(\text{div})} + \left( \Gamma_2^{(1,2)} \right)^\alpha_{(\text{finite})}, \tag{39}
\]

where

\[
\left( \Gamma_2^{(1,2)} \right)^\alpha_{(\text{div})} = -\frac{e^3}{3\pi^2} p^\nu \left( \frac{1}{\varepsilon} \right), \tag{40}
\]

and

\[
\left( \Gamma_2^{(1,2)} \right)^\alpha_{(\text{finite})} = -\frac{e^3}{3\pi^2} p^\nu C (p, m, \mu), \tag{41}
\]

with

\[
C (p, m, \mu) = 1 \int_0^1 dx \frac{dx}{\sqrt{1-x}} \ln \left( \frac{4\pi \mu^2}{\Delta_4 e^\gamma} \right). \tag{42}
\]

Combining the results given by Eq. (30) and Eq. (39),

\[
\left( \Gamma_2^{(1,2)} \right)^\mu = \left( \Gamma_1^{(1,2)} \right)^\mu + \left( \Gamma_2^{(1,2)} \right)^\mu, \tag{43}
\]

we obtain the one-loop correction to this vertex

\[
\left( \Gamma^{(1,2)} \right)^\alpha = -\frac{e^3}{\pi^2} \left[ \left( \frac{5}{4}\alpha - \frac{1}{12}\gamma^\mu \right) \frac{1}{\varepsilon} + B^\nu (p, q, m, \mu) + p^\rho C (p, m, \mu) \right], \tag{44}
\]

with \( B^\mu (p, q, m, \mu) \) given by Eq. (33) and \( C (p, m, \mu) \) given by Eq. (42).

Finally we analyze the vertex that represents the interaction between the scalar fields \( \phi^* \) and \( \phi \) with two gauge fields \( A_\mu \). In Fig. 9 we display the one-loop corrections for this vertex.

![FIG. 9: One-loop corrections to the vertex \( e^2 A_\mu A^\nu \phi^* \phi. \)
First, we evaluate the diagram shown in Fig. 10.

![FIG. 10: One-loop correction to the vertex \( e^2 A_\mu A^\nu \phi^* \phi. \) This vertex correction, \( (\Gamma_3^{(1,2)})^\mu \), is logarithmically divergent.
This vertex diagram is given by
\[
\left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} = \frac{4}{3!} \int \frac{d^{D}k}{(2\pi)^{D}} ie^{2\mu\nu} \epsilon^{p} S_{\phi}^{(0)} (k + q) \times \\
\times i e \mu \frac{2}{(k + 2q)^{\alpha} \Delta_{\alpha\beta}^{(0)} (k)} i e \mu \frac{2}{(k + 2p)^{\beta} \times S_{\phi}^{(0)} (k + p)}. \quad (45)
\]

Using Eq. (A2) we obtain
\[
\left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} = g^{\mu\nu} \epsilon^{4\mu\nu} \int_{0}^{1} dx \int_{0}^{1-x} \frac{dy}{\sqrt{1-x-y}} \times \\
\times \int \frac{d^{D}k}{(2\pi)^{D}} \frac{N}{\left\{ k + (qx + py) \right\}^{2} - \Delta_{5}^{2}} \Gamma_{1}^{(2,2)} (2), \quad (46)
\]
where \( N = (k + 2q)^{\alpha} (k + 2p)^{\alpha} \) and \( \Delta_{5} = (x + py)^{2} - q^{2}x - p^{2}y + m^{2}(x + y) \).

Making the change of variables, using the Eq. (A3), Eq. (A4) and the expansions given by Eq. (A6) and Eq. (A7), we obtain
\[
\left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} = \left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} + \left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu}, \quad (47)
\]
where
\[
\left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} = g^{\mu\nu} \epsilon^{4\mu\nu} \frac{1}{6\pi^{2} \epsilon^{2}}, \quad (48)
\]
\[
\left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} = -\frac{e^{4} g^{\mu\nu}}{\pi^{2}} D (p, q, m, \mu), \quad (49)
\]
and
\[
D (p, q, m, \mu) = -\frac{1}{48} \int_{0}^{1} dx \int_{0}^{1-x} \frac{dy}{\sqrt{1-x-y}} \times \\
\times \left[ 3 \ln \left( \frac{4\pi^{2} \epsilon^{2} m^{2}}{(\Delta_{5} e^{\gamma} + \frac{1}{2})} \right) - 2f_{1} (p, q, x, y) \right], \quad (50)
\]
with
\[
f_{1} (p, q, x, y) = q^{2} (x^{2} - 2x) + p^{2} (y^{2} - 2y) + q^{2} p_{a} (2xy - 2x - 2y + 4). \quad (51)
\]

Finally we evaluate the last diagram of the vertex correction shown in Fig. [11]

**FIG. 11:** One-loop correction to the vertex \( e^{2} A_{\mu} A^{\nu} \phi^{*} \phi \), \( \left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu} \), which is logarithmically divergent.

This diagram has the following analytical structure, with \( p = p_{1} + p_{2} \),
\[
\left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu} = \frac{16}{21} \int \frac{d^{D}k}{(2\pi)^{D}} \frac{(ie^{2} \mu \epsilon^{\alpha}}{g^{\alpha\beta}} \Delta_{\alpha\beta}^{(0)} (k) \times \\
\times (ie^{2} \mu \epsilon^{\beta}) S_{\phi}^{(0)} (k - p). \quad (52)
\]

Using the Feynman parameterization of Eq. (A1), with \( \alpha = 1 \) and \( \beta = \frac{1}{2} \), and further making the change of variables \( k \rightarrow k + px \), we obtain
\[
\left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu} = -2e^{4} g^{\mu\nu} \epsilon^{2} \int_{0}^{1} dx \int \frac{d^{D}k}{(2\pi)^{D}} \frac{1}{\left\{ k^{2} - \Delta_{6}^{2} \right\}^{2}}, \quad (53)
\]
with \( \Delta_{6} = p^{2} (x^{2} - x) + m^{2}x \).

Solving the integral in \( k \) with Eq. (A4), Eq. (A6) and Eq. (A7) we obtain
\[
\left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu} = \left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu} + \left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu}, \quad (54)
\]
where
\[
\left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu} = \frac{-2e^{4} g^{\mu\nu}}{\pi^{2}} \epsilon^{2}, \quad (55)
\]
\[
\left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu} = \frac{-e^{4} g^{\mu\nu}}{\pi^{2}} E (p, m, \mu), \quad (56)
\]
and
\[
E (p, m, \mu) = \frac{1}{2} \int_{0}^{1} dx \frac{\ln \left( \frac{4\pi^{2} \epsilon^{2} m^{2}}{\Delta_{6} e^{\gamma}} \right)}{\sqrt{1-x}}. \quad (57)
\]

Finally, combining the results of Eq. (47) and Eq. (54), and defining
\[
\left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} = \left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} + \left( \Gamma_{2}^{(2,2)} \right)^{\mu\nu}, \quad (58)
\]
we obtain the result
\[
\left( \Gamma_{1}^{(2,2)} \right)^{\mu\nu} = \frac{-e^{4} g^{\mu\nu}}{\pi^{2}} \left\{ \frac{11}{6\epsilon} + D (p, q, m, \mu) + E (p, m, \mu) \right\}. \quad (59)
\]

**IV. DISCUSSION**

To understand the effects of interactions, it is instructive to analyze the behavior of the interaction potential of SPQED in the static limit. This potential describes how bosonic fields interact between each other. In the static limit, Eq. (26) can be written as
\[
I (m, -|k|^{2}) = \int_{0}^{1} dx \frac{1}{\sqrt{m^{2} - |k|^{2} (x^{2} - x)}}. \quad (60)
\]

The roots of the argument are \( x_{\pm} = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 + \frac{4m^2}{|k|^2}}. \) Note that \( x_- < 0 \) and \( x_+ > 1 \), therefore they lie outside the integration interval. The integral \((60)\) gives
\[
I(m, -|k|^2) = \frac{2}{|k|} \arccot \left( \frac{2m}{|k|} \right). \tag{61}
\]
Therefore, Eq. \((25)\) can be written as
\[
\pi(|k|^2) = -\frac{\alpha}{2} \left[ \sqrt{4m^2 - \left( \frac{|k|^2 + 4m^2}{|k|} \right)^2} \arccot \left( \frac{2m}{|k|} \right) \right], \tag{62}
\]
where \( \alpha = e^2/4\pi \) is fine-structure constant.

The static interaction potential, including the radiative corrections, is given by
\[
V(r) = \frac{\alpha}{\pi} \int_{0}^{\infty} \int_{0}^{2\pi} d|k| d\theta \left\{ \frac{|k| e^{-i|k|r\cos\theta}}{2\sqrt{|k|^2 + \pi(|k|^2)}} \right\}, \tag{63}
\]
with \( r = |x - y| \) and \( \pi(|k|^2) \) is given by Eq. \((62)\). Integrating over the angular variable, and defining \( |k| = my \), we have
\[
V_{\text{exact}}(r) = \alpha m \int_{0}^{\infty} dy \left\{ \frac{yJ_0(myr)}{y + \frac{\alpha}{2} \left[ -2 + \frac{(y^2 + 4)}{y^2} \arccot \left( \frac{2}{y} \right) \right]} \right\}. \tag{64}
\]
Eq. \((64)\) can be solved numerically. The result of the integration is shown by the black continuous line in Fig.\((12)\).

We provide an analytical estimate of the interaction potential at small and large distances by performing controlled approximations of Eq. \((64)\). We first discuss the result at small distances. Considering the first order approximation of the Dyson series of the denominator
\[
V_{\text{first}}(r) = \frac{\alpha}{r} - \frac{\alpha^2 m}{4} \int_{0}^{\infty} dy J_0(myr) \left\{ -\frac{2}{y} + \frac{(y^2 + 4)}{y^2} \arccot \left( \frac{2}{y} \right) \right\}, \tag{65}
\]
we expand the Bessel function at large argument (large \( k \)) as follows
\[
J_0(myr) \approx \sqrt{\frac{2}{\pi myr}} \cos \left( myr - \frac{\pi}{4} \right), \tag{66}
\]
and the resulting error function as follows
\[
\text{erf} \left( \sqrt{2\sqrt{myr}} \right) \approx \frac{2e^{-2my}}{\sqrt{\pi} \left( \sqrt{2myr} + 2 + \sqrt{2myr} \right)}. \tag{67}
\]
Then we have
\[
V_{\text{first}}(r) = \frac{\alpha}{r} - \frac{\alpha^2 \sqrt{\pi} e^{-2myr}}{12} \left\{ 3\sqrt{mr} + 1 - \sqrt{mr} \left[ 8myr \left( 2myr - 2\sqrt{mr} + 1 \right) + 1 \right] \right\}. \tag{68}
\]
Eq.\((65)\) is displayed in Fig.\((12)\) by the red-dashed line, and shows that at the lowest order, the interaction is well described by a Coulomb interaction with intensity given by the fine structure constant. The one-loop correction provides a polarization effect coming from the virtual particle-antiparticle creation when bosonic fields approach at distances of the order of \( r \approx 1/2m \). We point out that this effect is well known in the usual treatment of QED in four dimensions, where similar results are found \cite{[22][23]).

To treat the large-distance behavior, we set \( |k|^2 \ll 4m^2 \) in the polarization tensor. Then Eq. \((62)\) can be written analytically by
\[
\pi_{\text{NR}}(|k|^2) = \frac{\alpha}{6m} |k|^2. \tag{69}
\]
Consequently, the potential has the form of the well-know Keldish potential \cite{[24]}
\[
V_{\text{Keld}}(r) = \frac{\alpha \pi}{2 \omega} \left[ H_0 \left( \frac{r}{\omega} \right) - Y_0 \left( \frac{r}{\omega} \right) \right], \tag{70}
\]
where \( r_0 = \frac{\alpha}{12m} \), \( H_0(r/\omega) \) and \( Y_0(r/\omega) \) are the Struve and the Neumann functions, respectively. This expression has a similar structure in the non-relativistic regime of PQED coupled to a charged Dirac field in \((2+1)D \) \cite{[24]} with \( r_0 = 2\alpha/3M \) and \( M \) is the fermion mass. To analyze the behavior of potential at long distances, we rewrite Eq. \((70)\) in terms of a new variable \( \ell = r/r_0 \) resulting into
\[
r_0 V(\ell) = \frac{\alpha \pi}{2} \left[ H_0(\ell) - Y_0(\ell) \right]. \tag{71}
\]
For \( r \gg r_0 \) we can write \cite{[20]}
\[
V(r) = \frac{\alpha}{r} - \frac{\alpha^3}{144m^2} \frac{1}{r^3} + O \left( \frac{\alpha^5}{(12m)^5} \frac{1}{r^5} \right). \tag{72}
\]
Therefore, the potential is still given by the Coulomb potential at lowest order. The first correction is given by an isotropic dipolar potential whose strength is proportional to \( \alpha^3 \).

The blue line in Figure\((12)\) shows the Keldish potential of Eq.\((70)\), which captures very well the long-distance behavior of the exact result.
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models of relativistic field theories with ultracold atoms. Dissipation, or in the quantum simulation of analogue systems, a bosonic description might be more adequate than the fermionic one [22, 23]. These results are of importance for a series of applications in condensed matter physics, where the Coulomb potential gets modified by an isotropic dipolar potential. Whereas at short distances a polarization effect arises, Eq. (64), at large distances the potential has an exponential behavior, expressed in Eq. (65), typical of a polarization effect of the charges. At large distances the Coulomb potential is corrected by an isotropic dipolar potential.

V. CONCLUSIONS

In this work we studied the scalar version of the projected Quantum Electrodynamics in two spatial dimensions. We first introduced the model and defined its Feynman rules. Then we provided explicit calculations of the radiative corrections of the relevant interactions vertices. Many of the results were obtained by conveniently applying dimensional regularization to the model. We then computed the short- and large-distance behavior of such potential. Whereas at short distances a polarization effect arises, similarly to the four-dimensional QED, at large distances the Coulomb potential gets modified by an isotropic dipolar potential. These results are of importance for a series of applications in condensed matter physics, where a bosonic description might be more adequate than the correspondent fermionic counterpart. Examples may be found in superconductivity, in the exciton-polariton condensation, or in the quantum simulation of analogue models of relativistic field theories with ultracold atoms.

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Appendix A: USEFUL IDENTITIES

In this Appendix we review some useful identities involving the so-called Feynman parameters often encountered in QED calculations [23] that also appear in our one-loop calculations discussed in the main text as well as integrals arising from dimensional regularization.

We start with the following two identities

$$\frac{1}{a^{\alpha}b^{\beta}} = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_{0}^{1} dx \frac{x^{\alpha - 1}(1 - x)^{\beta - 1}}{[a + b(1 - x)]^{\alpha + \beta}}, \quad (A1)$$

$$\frac{1}{a^{\alpha}b^{\beta}c^{\gamma}} = \frac{\Gamma(\alpha + \beta + \gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} \times \int_{0}^{1} dx \int_{0}^{1 - x} dy \frac{(1 - y)^{-1}y^{-1}(x - y)^{\beta - 1}}{[ax + by + c(1 - x - y)]^{\alpha + \beta + \gamma}}. \quad (A2)$$

When performing dimensional regularization we make use of the integrals

$$\int \frac{d^{D}k}{(2\pi)^{D}} \frac{k^{2}}{|k^{2} - \Delta|^{\alpha}} = \frac{i^{2\alpha - 1}}{(4\pi)^{\frac{D}{2}}} \frac{D}{2} \frac{\Gamma(\alpha - \frac{D}{2} - 1)}{\Gamma(\alpha)} \frac{\Delta^{\frac{D}{2} - \alpha}}{\Delta}, \quad (A3)$$

$$\int \frac{d^{D}k}{(2\pi)^{D}} \frac{1}{|k^{2} - \Delta|^{\alpha}} = \frac{i^{2\alpha + 1}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\alpha - \frac{D}{2})}{\Gamma(\alpha)} \Delta^{\frac{D}{2} - \alpha}, \quad (A4)$$

$$\int \frac{d^{D}k}{(2\pi)^{D}} \frac{k^{\mu}k^{\nu}}{|k^{2} - \Delta|^{\alpha}} = \frac{g^{\mu\nu}}{D} \int \frac{d^{D}k}{(2\pi)^{D}} \frac{k^{2}}{|k^{2} - \Delta|^{\alpha}}, \quad (A5)$$

where $\Delta$ is a parameter with the dimension of a mass. Finally it is useful to recall the expansion of the Gamma function for a small argument

$$\Gamma\left(\frac{\varepsilon}{2}\right) = \frac{2}{\varepsilon} - \gamma + O(\varepsilon), \quad (A6)$$

where $\gamma = 0.5772...$ is the Euler-Mascheroni constant. Also, for an arbitrary quantity $Q$ in the limit of vanishing $\varepsilon$, we have

$$Q^{\varepsilon} \simeq 1 + \frac{\varepsilon}{2} \ln Q. \quad (A7)$$
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