One-Way Hyperbolic Metasurfaces Based on Synthetic Motion

Yarden Mazor, Student Member, IEEE, and Andrea Alù, Fellow, IEEE
(Invited Paper)

Abstract—Moving metasurfaces support guided waves exhibiting unusual optical properties, including strong anisotropy, nonreciprocity, and hyperbolic dispersion. However, for these phenomena to be noticeable, high speeds are typically required, challenging their practical implementation. Here we show a viable route toward the realization of strongly nonreciprocal hyperbolic propagation in metasurfaces placed synthetically in motion through traveling-wave space–time modulation. In addition to nonreciprocal hyperbolic propagation, the proposed time-modulation scheme induces additional exotic opportunities for nanophotonic systems, such as efficient nonreciprocal frequency conversion and mode transfer.

Index Terms—Hyperbolic, moving media, nonreciprocal, space–time modulation.

I. INTRODUCTION

HYPERBOLIC dispersion enables the propagation of directional waves with high spatial momentum over broad bandwidths [1], particularly useful for imaging, enhanced light–matter interactions, and optical emission [2]–[7]. This unusual regime is conventionally achieved using metamaterials composed of oppositely signed permittivity multilayers in the case of bulk media [2] or nanostraps for hyperbolic metasurfaces [4], [8]–[10]. To realize nonreciprocal hyperbolic transport, leading to nonreciprocal hyperbolic surface wave propagation analogous to moving at velocity \( \gamma \) along \( \hat{z} \), shown in Fig. 1(a). The moving surface response satisfies the boundary condition

\[
\hat{x} \Delta H = Z_s^{-1} \text{diag}(\gamma, \gamma^{-1}) E_{\text{tan}} + Z_s^{-1} \gamma (\hat{v} \times \mu_0 H_0) + (\hat{v} \hat{z}) \hat{x} \cdot \varepsilon_0 \Delta E
\]

(1)

Manuscript received May 8, 2019; revised September 25, 2019; accepted October 10, 2019. Date of publication October 29, 2019; date of current version March 3, 2020. This work was supported in part by the Air Force Office of Scientific Research with MURI under Grant FA9550-18-1-0379, in part by the Office of Naval Research under Grant N00014-19-1-2011, and in part by the Simons Foundation. (Corresponding author: Andrea Alù.)

Y. Mazor is with the Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX 78712 USA.
A. Alù is with the Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX 78712 USA, and also with the Advanced Science Research Center, The City University of New York, New York, NY 10031 USA (e-mail: aalu@sci.cuny.edu).

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Digital Object Identifier 10.1109/TAP.2019.2949134

IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 68, NO. 3, MARCH 2020

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where $\Delta H$, $\Delta E$ are the field discontinuities across the boundary in the lab frame (not moving with the surface) and $\gamma = \sqrt{1 - v^2/c^2}$. This boundary condition, derived in detail in [17], modifies the standard $\mathbf{x} \times \Delta \mathbf{H} = Z_s^{-1} E_{\text{tan}}$ to capture several new physical processes stemming from mechanical motion. The first term corresponds to motion induced anisotropy -- electric fields in the direction of motion induce different currents than fields perpendicular to it. The second is a current term induced by the magnetic Lorentz force operating on the mechanically moving charges, and the third term is a convection term, where current is generated by the mechanical motion of charges. If $\bar{X}_s > 0$, a stationary surface is known to support transverse-magnetic (TM) surface waves with phase velocity $v_p(\bar{X}_s) = c/\sqrt{1 + 4X_s^2}$ [33]. When $v > v_p$, it supports nonreciprocal hyperbolic dispersion [17], as calculated in Fig. 1(b) for $X_s = 4.5$ with normalized velocity $\beta = v/c = 0.114$. Fig. 1(c) shows that the response of the moving surface to a stationary point-source excitation located $\sim 0.02 \lambda_0$ above the surface, featuring highly asymmetric plasmon excitation due to nonreciprocity. Combined with the broad bandwidth over which an impedance surface supports confined wave propagation, this response can be of interest for several nanophotonic applications benefiting from strong Purcell enhancement over broad bandwidths combined with nonreciprocal features, which is characteristic of hyperbolic metamaterials and metasurfaces [9], [10], [34]. Yet, given the impractically large velocities required, in the following we explore synthetic motion established by spatiotemporal traveling wave modulation, as a way to provide a robust, tunable platform for the implementation of these exotic wave phenomena.

II. FORMULATION

The response of a space–time-modulated metasurface can be described using various models, as a function of the actual modulated parameters (permittivity, permeability, effective capacitance, or inductance) and the modulation technique (electro/magneto-optical, mechanical/ acoustic, all-optical, etc.). Here we start by focusing on one of these options -- a space–time-modulated metasurface implemented using modulated variable capacitors and inductors. One can generally describe such a surface using the impedance operator $\hat{Z}_s$ [25]

$$E_{\text{tan}}(r_t, t) = \hat{Z}_s[J_s(r_t, t)]$$

$$= L_s(r_t, t) \frac{\partial J_s(r_t, t)}{\partial t} + C_s(r_t, t) \int J_s(r_t, t)$$

(2)

where $r_t$ is a point on the surface, $E_{\text{tan}}$ is the electric field component tangent to the surface, $L_s$ is the surface inductance, and $C_s$ the inverse capacitance, both assumed to be periodic functions of space and time. If we assume that the surface impedance experiences negligible temporal dispersion in the relevant frequency band of interest, and that the surface elements are modulated by a cosine function, as shown schematically in Fig. 2(a), the impedance operator can be simplified to the scalar expression [19]

$$Z_s = j \eta_0 \bar{X}_s \left( 1 - M \cos \left[ \Omega t - \frac{2\pi}{\lambda_0} z \right] \right)$$

(3)

where $a$ being the modulation period, $\Omega$ is the angular frequency, $M$ is the modulation depth, and $v_m = \Omega a/2\pi$ is the modulation wave velocity. The assumption of negligible dispersion encapsulates an important trade-off. It facilitates a more straightforward analysis, and working in this regime can ease a broadband implementation of this scheme. On the contrary, it dictates that we do not operate very close to resonances that may enhance the discussed phenomena at the expense of narrower bandwidths.

Unlike a mechanically moving surface, space–time modulation produces frequency mixing and generates new harmonics. The 2-D dispersion of the supported surface waves can be calculated after representing them as an infinite sum of TM
Floquet harmonics [20, 25]

$$H = \sum_{n=-\infty}^{\infty} H_{n}^{1,2}(\cos \varphi_n \hat{z} - \sin \varphi_n \hat{y}) e^{-j\alpha_n x - j\kappa_n(y,z)} e^{j\omega_n t} + C.C$$

(4)

where (2) refers to above (below) the surface, \( \kappa = 2\pi/a \), \( k_n = k_{1n} \hat{\gamma} + (k_{1z} + n\kappa) \hat{z}, k_{1n} = |k_{1n}| \) and we define, \( \cos \varphi_n = (k_{1z} + n\kappa)/k_{1n}, \sin \varphi_n = k_{1y}/k_{1n} \), \( \omega_n = \omega + n\Omega \), \( k_{1n} = \omega_n/c \), and we must satisfy \( k_{1n}^2 + \alpha_n^2 = k_{1n}^2 \).

III. DISPERSION AND SPECTRAL CHARACTERISTICS

By applying the boundary conditions on the modulated surface, we obtain a recursive relation between the harmonic amplitudes

$$H_{n-1,(y,z)}^1 + \frac{2}{M} D_n H_{n,(y,z)}^1 + H_{n+1,(y,z)}^1 = 0$$

(5)

with \( D_n = 1 - \frac{2X_s k_{0n} a}{1 + 4(1 + M)^2 X_s^2} \) the value of \( v_p \) associated with the maximal impedance \( Z_s, \max = j\eta_0 \tilde{X}_s(1 + M) \). Beyond this value, the interaction between the modulation wave and the propagating wave may evoke instabilities and parametric amplification [35]. Due to the complicated nature of the continued fraction dispersion, we first take a qualitative, approximate look at its main properties [20, 36] to provide physical insights into the effects of space–time modulation on the surface wave dispersion.

Surface wave propagation over an isotropic impedance surface obeys a conical dispersion – the central orange cone in Fig. 2(b). Space–time modulation replicates this cone along a diagonal line corresponding to the transformation \( (k_z, \omega) \to (k_z + 2\pi/a, \omega + \Omega) \), whose slope is determined by \( v_m \). An isofrequency cross section is shown in Fig. 2(c) for \( X_s = 4.5, a = 0.15\lambda_0 \), modulation frequency \( f_m = \Omega/2\pi = 0.155 f_0 \) (dashed lines show the case of \( f_m = 0 \) for comparison), yielding \( v_p = c(1 + 4X_s^2)^{-1/2} \approx 0.11c \), and \( v_m \approx 0.028c \). The circles correspond to \( k_{1z} a + 2\pi n a^2 + (k_{1y} a)^2 = k_{1n}^2 (1 + 4X_s^2) \) (to reduce cluttering, not all harmonics are shown), and the smaller, blue, thick circles show the light cone cross sections. For small modulation depths \( M \), the isofrequency dispersion of the modulated system differs from this schematic picture only close to the circle intersections, generating spatial bandgaps due to a strong harmonic interaction near these points.

This is illustrated in Fig. 3(a) displaying the exact solution of the dispersion equation (6) with spatial modulation but without temporal one, and Fig. 3(b) with both modulations, respectively, for \( M \in [0.1, 0.4] \) (a surface wave solution must reside outside the light cones shown in thick black lines). In the stationary case [see Fig. 3(a)], the dispersion is flat and symmetric with \( k_z \), as expected, while in the synthetically moving scenario [see Fig. 3(b)], the dispersion shows a slope consistent with the approximate plot in Fig. 2(c), and a nonreciprocal hyperbolic branch is obtained. The slope \( s \) of the common tangent, which yields the hyperbolicity of the synthetically moving surface, is given by

$$s = \frac{1}{\sqrt{\frac{v_p^2}{v_m^2} - 1}} = \frac{1}{\sqrt{\frac{\alpha_n^2}{c^2} [1 + 4X_s^2]}} - 1$$

(7)

The bandgaps induced by coupling neighboring harmonics can be used to tailor the desired dispersion contour properties with large flexibility. For instance, in the channelization regime [1], [10] of particular interest in the context of hyperbolic metasurfaces to channel subdiffactive details of an image in the far field [37], [38], all surface waves with a wide range of transverse momenta travel in the same direction manifested as a straight line in the isofrequency contour.

By tailoring the modulation signal, we can adjust the flatness of the dispersion, as seen in Fig. 3(a) and (b). As the modulation depth increases, the outer curve flattens, and the internal, closed contours draw apart, realizing a response analogous to [1], corresponding to subdiffraction imaging on the surface, additionally endowed with nonreciprocal properties in Fig. 3(b). As a result, large \( k_y \) propagation is possible only toward certain directions denoted by the tangent line. For small \( M \), we can extend the perturbative approach in [39] to show that the width of the spectral gap created around the strongly coupled with regions of the dispersion, where \( n, n+1 \)
IV. EXCITATION

Hyperbolic propagation is of particular interest for the possibility of channeling high-momentum surface waves along specific directions. We examine this opportunity by studying a point source excitation of the proposed time-modulated metasurfaces. When time modulation is not present, coupling between all spatial harmonics supported by the surface occurs necessarily at the same frequency. Therefore, while coupling to the flat part of the dispersion using wave-vector-matched excitation is possible [11], [10], for instance through a carefully designed grating coupler, trying to excite a specific attribute of the dispersion using a localized emitter proves to be generally difficult, as most of the energy will couple to unwanted, typically low, wave numbers.

Fig. 4(a) shows the x component of the electric field distribution when exciting a spatially modulated surface by a unit, point electric dipole $\mathbf{p} = p_0 \hat{x}$ directed in the $\hat{x}$-direction, oscillating with angular frequency $\omega$ ($\omega p_0 = \omega p_0 = 1 [A \cdot s]$ [41]). For moderate modulation depths ($M = 0.15$ here), the induced field profile looks almost isotropic, despite the complex band diagram associated with this surface [see Fig. 3(a)]. Adding synthetic motion access not only enables nonreciprocal hyperbolic propagation but interestingly also allows isolating the extreme features of the dispersion diagram in Fig. 3(b), as it yields a more complex system of coupling between spatial and temporal harmonics [18]. While the fundamental harmonic remains almost isotropic for excitation with a point source [see Fig. 4(b)], similar to the static scenario, it is now possible to excite higher order harmonics through parametric mixing. These harmonics present pronounced nonreciprocal features and directional excitation of surface waves, as seen in Fig. 4(c) for $n = 3$, corresponding to $f = f_0 + 3 f_m$, resembling the excitation of the moving metasurface shown in Fig. 1 [17], with large efficiency. The inset shows the electric field magnitude as a function of angle $\theta$, when sampled on a circle around the source with radii $R = 0.15 \lambda_0$ (blue) and $R = 0.2 \lambda_0$ (orange). The field is concentrated around the angle predicted by the slope of the dispersion in Fig. 3(b), about 75° [schematically shown by the purple dashed line in Fig. 2(c)], and it is emitted only toward the right of the source, due to the surface nonreciprocity.

We can use the same concept to couple the source to symmetric canalization of waves, using a standing wave modulation $Z_s = j \eta_0 \chi_r (1 + M \cos[kx] \cos[\Omega t])$. In this case, Fig. 4(d) shows again nearly isotropic emission into the fundamental harmonic, while $n = 3$, presented in Fig. 4(e), shows strong in-plane confinement and canalization. In Fig. 4(c), to synthesize nonreciprocal hyperbolic propagation toward a slanted angle, we were bound to relatively high modulation frequencies, since the slope $s$ increases with $f_m$ based on (7). Here we do not have this constraint and can achieve canalization for smaller modulation frequencies ideal for a photonic implementation. More examples of modulated and synthetically moving metasurfaces and details regarding our full-wave simulations are provided in Appendix B.

V. DISCUSSION

While both mechanical and synthetic motions enable nonreciprocal hyperbolic response, important differences between the cases exist. Mechanical motion requires $v > v_p$ to support hyperbolic propagation, and the motion introduces local anisotropy to the surface or media properties
In the case of synthetic motion, hyperbolic propagation is supported in principle even for low values of \( v_m \), and arbitrarily large modulation velocities may be achieved even with low modulation frequencies, making this approach attractive for practical purposes.

However, when space–time modulation is considered, we introduce periodicity into the problem, which needs to be comparable to the wavelength of the surface wave on the unmodulated surface impedance (otherwise the circles in Fig. 2(b) would be too far apart, and different harmonics would not interact). The effect is thus inherently nonlocal, and it is associated with asymmetric coupling of adjacent harmonics. Interestingly, the hyperbolic regime exists for complementary ranges of velocities \((v, v_m)\); for a moving surface, the velocity must be greater than the phase velocity of the surface waves at rest, while in the modulated case, the modulation velocity must be smaller than the phase velocity \(v_{p,\min}\) to avoid instabilities (although stable regimes for higher velocity exist, but they require fast modulation frequencies). We can define an effective velocity \(v_{\text{eff}}\) induced by the synthetic modulation velocity

\[
v_{\text{eff}} = v_p \sqrt{\frac{1}{1 - 4X^2(v_m/c)^2}}.
\]  (9)

defined as the mechanical velocity that provides the same slope of the hyperbolic dispersion. This expression, combined with the limitation \(v_m < v_{p,\min}\), reveals a relevant trade-off of synthetically moving hyperbolic metasurfaces: on the one hand, larger \(M\) enables stronger coupling to higher order harmonics that sustain stronger excitation of nonreciprocal plasmons, as presented in Fig. 5(a). At the same time, \(v_{p,\min}\) decreases, limiting the effective synthetic velocity \(v_{\text{eff}}\), so we can achieve in a stable regime. The black curve in the inset of Fig. 5(a) shows \(v_{\text{eff}}\) as a function of \(v_m\), with maximum possible \(v_m\) depicted as colored vertical lines corresponding to a given value of \(M\).

Since the effect is based on the coupling between harmonics, propagation in the opposite direction is not completely suppressed, but it is significantly less dominant. To shed light on the directional nature of propagation over the proposed metasurface (as shown for \(n = 3\) in Fig. 4), we calculate \(H_n/H_0\) along the dispersion curves. The results are shown in Fig. 5(b) for \(M = 0.15\), where the color coding corresponds to the most dominant harmonic (\(\max(|H_n/H_0|)\)) at every point along the dispersion. We can see that the higher harmonics are the dominant ones in the flatter regions of the dispersion. Further, if we examine the most dominant regions in \(n = 3\) (where the radio \(H_3/H_0\) is the largest), we will notice that this occurs in the outer regions of the dispersion (shown with larger markers). Hence, we can expect that a confined source exciting a broad range of spatial harmonics, such as the point source discussed, couples strongly to highly directional waves for the upconverted harmonics – as shown in Fig. 4 of the main article. In Appendix A, we show additional field harmonics, highlighting an increase in directionality for higher harmonics. For comparison, in blue we show the dispersion presented in Fig. 1(b), corresponding to \(v_{\text{eff}}\) for the chosen parameters, highlighting the strong analogy between the two systems in terms of overall response.

VI. Conclusion

In summary, we have shown that space–time modulation of a surface impedance provides a flexible platform to mimic mechanical motion, enabling access to intriguing dispersion features such as extreme anisotropy, flat dispersion, canalization, and nonreciprocal hyperbolic propagation. Implementation at lower frequencies can involve a 2-D periodic arrangement of varactors or switched capacitors [23], [25], [27], and using cascaded or N-way schemes may also improve the coupling efficiency to higher harmonics [42], [43]. At optical frequencies, the implementation may be comparatively more challenging due to the fast response times needed from the materials. Many of the current state-of-the-art techniques, including ultra-fast modulation speeds, are discussed in [30]. These include acoustic/mechanical wave modulation, for example, using stimulated Brillouin scattering [44]–[46], or electrical gating for low to moderate modulation frequencies. Higher modulation frequencies can be achieved by modulation of ITO films and graphene [47], [48] and optical modulation based on material nonlinearities [32]. These findings may open interesting opportunities in the realization of nonreciprocal subdiffractive imaging systems, highly unusual plasmon transport structures, and enhanced nonreciprocal light–matter interactions based on the proposed platform.
APPENDIXES

A. Dispersion, Spatial Bandgap, and Directionality for Space–Time-Modulated Surfaces

If we substitute the field expansion [see (4)] into the appropriate boundary condition, together with the assumption of cosine traveling wave modulation, we obtain (5). The solutions to the equation $D_n(k_x, k_z) = 0$ yield the circles shown in Fig. 2(c). Following the same steps shown in [18], [19], and [39], we arrive at a dispersion equation represented using continued fractions

$$D_n - \frac{M^2/4}{D_{n-1} - \frac{M^2/4}{D_{n-2} - \frac{M^2/4}{D_{n-3}}}} = 0.$$  \hspace{1cm} (10)

And the solutions $(k_x, k_z)$ to this equation yield the dispersion curve. Throughout this article, the continued fractions were evaluated using the modified Lentz algorithm [49], [50]. A compact notation uses the symbol $K$ in the following form

$$b_0 + \frac{a_1}{b_1 + \frac{a_2}{b_2 + \cdots}} = b_0 + \sum_{n=1}^{\infty} \frac{a_n}{b_n}$$  \hspace{1cm} (11)

which lets us rewrite (10) as

$$D_n + \sum_{m=1}^{\infty} \frac{-M^2/4}{D_{n+m}} = 0.$$  \hspace{1cm} (12)

For the continued fraction to converge, we must satisfy $|2D_n/M| > 2$, for some finite $n_0$. This yields the constraint

$$v_m = \frac{\Omega a}{2\pi} < \frac{c}{\sqrt{1 + 4(1 + M^2 \bar{X}^2)}}$$  \hspace{1cm} (13)

meaning that the modulation wave velocity must be smaller than the local TM wave velocity guided on any point of the surface. For $M \ll 1$, we can get a perturbative solution for the dispersion by using the following equivalent representation to (10)

$$D_n + \sum_{m=1}^{\infty} \frac{-M^2/4}{D_{n+m}} = 0.$$  \hspace{1cm} (14)

and following the considerations presented in [33], we may represent the dispersion around the intersection point approximately as

$$D_n D_{n+1} = \frac{M^2}{4}.$$  \hspace{1cm} (15)

Now, let us denote the in-plane wave vector corresponding to the intersection of the circles in Fig. 2(c) as $k_{i,1}$ and represent the solution to (15) as $k_i = k_{i,1} + k'$ with $|k'| \ll |k_{i,1} + 2n\pi \hat{z}|$. It is convenient to use the vectors shown in Fig. 2(c) using black lines. These vectors are $k_{i,1} = k_{i,1} + 2n\pi \hat{z}$ and $k_{i,2} = k_{i,1} + 2(n + 1)\pi \hat{z}$. If we substitute these assumptions into (15), we obtain

$$(k_{i,1} \cdot k')(k_{i,2} \cdot k') = 4M^2 \bar{X}^2 (k_{0,n} a^2) (k_{0,n+1} a)^2.$$  \hspace{1cm} (16)

Equation (16) represents a hyperbola which approximates the dispersion around the spatial bandgaps. By using geometric properties of this hyperbola [51], we may extract its major axis – being an accurate measure of the spatial bandgap width shown in (8) of this article, for small modulation depths.

As discussed in Section V, directional excitation in the hyperbolic regime is obtained for higher harmonics. Fig. 6 shows how the excitation of the surface becomes more and more directional as we look at higher order harmonics excited by the point source.

B. Numerical Modeling and Convergence of Space–Time-Modulated Surfaces in COMSOL Multiphysics

Assume we have a surface with an impedance that is a periodic function of time and may include also dependence on the spatial coordinates. It can be represented by its temporal Fourier series

$$\sigma = \sum_{n=-\infty}^{\infty} \sigma_n(y, z) e^{-jn\Omega t}.$$  \hspace{1cm} (17)

The electromagnetic fields may also be represented in the same manner

$$E(r, t) = \sum_{n=-\infty}^{\infty} E_n(r) e^{j\omega t + jn\Omega t} \hspace{1cm} (18)$$

$$H(r, t) = \sum_{n=-\infty}^{\infty} H_n(r) e^{j\omega t + jn\Omega t}.$$
which can be rearranged into
\[\vec{n} \times \left( \sum_{n=-\infty}^{\infty} \mathbf{H}_{n,0}(\mathbf{r}) e^{j\Omega t} - \sum_{n=-\infty}^{\infty} \mathbf{H}_{n,0}(\mathbf{r}) e^{j\Omega t} \right) = (\sigma_0 + \sigma_1 e^{j\Omega t} + \sigma_{-1} e^{-j\Omega t}) \sum_{n=-\infty}^{\infty} \mathbf{E}_{n,0}(\mathbf{r}) e^{j\Omega t}\]
which can be rearranged into
\[\sum_{n=-\infty}^{\infty} \vec{n} \times (\mathbf{H}_{n,0}(\mathbf{r}) - \mathbf{H}_{n,0}(\mathbf{r})) e^{j\Omega t} = \sum_{n=-\infty}^{\infty} (\sigma_0 \mathbf{E}_{n,0} + \sigma_1 \mathbf{E}_{n-1,0} + \sigma_{-1} \mathbf{E}_{n+1,0}) e^{j\Omega t}\]
\[= \sum_{n=-\infty}^{\infty} \mathbf{J}_{n,0} e^{j\Omega t}. \] (20)

This equation can be used to model this system in full-wave simulations using the following scheme:

1) The solution is truncated to a finite number of harmonics. The validity of this step can be later checked by adding additional harmonics and verifying that the solution in the frequency of interest is not significantly altered. We will refer to the number of harmonics in the truncated solution as \(N\). Naturally, the amount of variables to be solved by the simulation software is associated with the value of \(N\), which is limited by the computational resources. In our case, we could go up to \(N = 5\) and maintain reasonable calculation times, and therefore this value is regarded as the reference for error calculations.

2) Each harmonic \(n\), corresponding to frequency \(f_0 + nf_m\) (where \(f_m = \Omega/2\pi\)) is assigned an electromagnetic wave solver in the solution domain.

3) The point source (or any other source of interest) is placed in the instance corresponding to the fundamental frequency \(f_0\).

4) The surface current for each harmonic is defined by (21) in the “surface current density” boundary condition, using the electric field expressions from the relevant instances.

To examine the convergence of the numerical full-wave solution, let us look at the fields obtained for the \(n = 0\) and \(n = 3\) harmonic for various values of \(N\), as seen in Fig. 7. We see that there are almost no visible changes when adding more harmonics to the truncation, indicating convergence. To get a more quantitative look at this, we examine the electric field norm along a circular arc around the source location, with radius of 0.15 \(\lambda_0\) [similar to the sampling inset shown in the inset of Fig. 4(c)].

We can define the relative error for a specific truncation \(N\) and for a field harmonic \(n\) [corresponding to the harmonics in the expansion in (18)] as
\[\varepsilon_n(N) = \frac{|\mathbf{E}_n(N)| - |\mathbf{E}_n(N_{\text{ref}})|}{|\mathbf{E}_n(N_{\text{ref}})|},\] (22)
where \(\mathbf{E}_n(N)\) is the \(n\)th harmonic of the electric field for a solution truncated after \(N\) terms, and \(N_{\text{ref}} = 5\). The errors are shown in Fig. 8(a) and (b) for \(n = 3\) and \(n = 0\), respectively, and we see that even for \(N = 3\) the errors are small and reduce further as we add additional harmonics.

C. Canalization for Smaller Modulation Frequencies

As mentioned in this article, if we are interested in achieving canalization of waves in a symmetric way, without incorporating nonreciprocal hyperbolic propagation, we can use smaller modulation frequencies than the ones discussed. Fig. 9 shows the obtained response for \(f_m \approx 0.02f_0\) (in this article \(\approx 0.15f_0\) is used) in the same way presented in Fig. 4.
Using lower modulation frequency renders the directional response at a frequency much closer to the base harmonic.

ACKNOWLEDGMENT

The authors thank Dr. D. Sounas for assistance with full-wave electromagnetic modeling.

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Yarden Mazor (S’16) was born in Nazareth Illit, Israel, in 1981. He received the B.Sc. degree (summa cum laude) in biomedical engineering from the Technion – Israel Institute of Technology, Haifa, Israel, in 2009, and the Ph.D. degree in electrical engineering from Tel-Aviv University, Tel-Aviv, Israel, in 2017, under the supervision of Prof. B. Z. Steinberg, with a focus on nonreciprocal and asymmetric wave propagation in 1-D and 2-D plasmonic particle arrays.

In 2017, he joined the Group of Prof. A. Alù at The University of Texas at Austin, Austin, TX, USA, as a Post-Doctoral Fellow. His current research interests include wave physics, analytical methods and modeling, metamaterials and metasurfaces, wave propagation in time-varying, wave interaction with rotating systems, and asymmetric wave excitation and propagation in complex media.

Dr. Mazor was a recipient of the Honorable Mention at the Student Paper Competition in IEEE AP-S/URSI International Conference, Puerto Rico, in 2016, and the Third Prize at the Student Paper Competition in Metamaterials Conference, Oxford, in 2015.