Research Article

Quasi-Static Solution for ELF/SLF near-Zone Field of a Vertical Magnetic Dipole near the Surface of a Lossy Half-Space

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Dipole antennas over the boundary between two different media have been widely used in the fields of geophysics exploration, oceanography, and submerged communication. In this paper, an analytical method is proposed to analyse the near-zone field at the extremely low frequency (ELF)/super low frequency (SLF) range due to a vertical magnetic dipole (VMD). For the lack of feasible analytical techniques to derive the components exactly, two reasonable assumptions are introduced depending on the quasi-static definition and the equivalent infinitesimal theory. Final expressions of the electromagnetic field components are in terms of exponential functions. By comparison with direct numerical solutions and exact results in a special case, the correctness and effectiveness of the proposed quasi-static approximation are demonstrated. Simulations show that the smallest validity limit always occurs for component $H_{2z}$, and the value of $k^2\rho$ should be no greater than 0.6 in order to keep a good consistency.

1. Introduction

The propagation properties of extremely low frequency (ELF: 3–30 Hz) and super low frequency (SLF: 30–300 Hz) waves over the boundary between two different media have been extensively investigated for many years because of their useful applications in geophysics exploration, oceanography, overwater or underwater communication, and so on. For example, the US system Seafarer, operated at the frequency of 78 Hz [1], while the Russian one, called ZEVs, operated at the frequency of 82 Hz [2]. These systems have been used to communicate from a fixed station on the sea surface with a submarine traveling close to the ocean floor since 1990s [3, 4]. The original expression of this problem can date back to 1909 by Sommerfeld in his classic literature [5]. Subsequently, Norton proposed simple formulas and graphs in 1936 [6]. Following their works, many other progresses were made on numerical [7, 8] and analytical solutions [9–15] at a large propagation distance of $k\rho \gg 1$, where $k$ represents the wave number and $\rho$ is the propagation distance.

Evidently, in practical physical models such as a dipole source on the interface between two different media like air and Earth or sea water and rock, the electric propagation distance for the ELF/SLF ranges is usually very small, namely, $k\rho \ll 1$. Sommerfeld integrals are used for field components which can be expressed in terms of the derivatives of integrals including the Bessel function. Due to the divergence terms in Sommerfeld integrals, the investigation of ELF/SLF near-zone field is still not well developed yet. In the recent work by Xu et al. [16], a new quasi-static technique is used for analyzing the near-zone field radiated by a horizontal electric dipole near the sea-rock boundary. This new development rekindled our interest in investigating the magnetic-dipole excitation problem.

Several investigations of simplifying formulas had been made by Banos and Wesley [17]. They obtained approximation solutions suitable for both near and far fields in terms of Hertz potentials. Based on their efforts, Durrani [18, 19] addressed the sufficiently accurate but relatively simple field components in the sea produced by vertical and horizontal dipoles (both electric and magnetic types) under reasonable limitations. Also of interest is the satisfactory communication range between air and sea. Later on, in a series of papers by Bannister [20, 21], previous
theoretical work like [22] was well summarized, and all
field components for these four different types of dipoles
located on the boundary were presented over the quasi-
static range. In recent years, some near-field research has
gradually been carried out both analytically [23–25] and
numerically [26]. Particularly in 2010, Parise [27] estab-
lished the exact closed-form expressions of the electromag-
netic field components excited by a vertical magnetic
dipole lying on the surface of a flat and homogeneous lossy
half-space, but unfortunately \( H_\rho \) is still in terms of Bessel
functions, and these formulas are valid only when both of
the dipole source and the observation point are embedded
on the boundary. In many practical applications, both the
dipole source and the receiving point may be located near
the boundary. However, the corresponding study on this
near-field radiation of a vertical magnetic dipole is still in
the dark. Since it is a formidable task to calculate it ac-
curately by existing analytical methods, in this paper, we
will attempt to extend the work by Xu et al. [16] and
evaluate the integrals in purpose of deriving new ap-
proximation formulas.

Note that the propagation distance of interest is up to
dozens of kilometers, in this case, the propagation path via
the reflection of the ionosphere is much longer than that
along the Earth’s surface. Hence, the effect of the ionosphere
can be neglected and a simple half-space flat model is
established. At ELF/SLF ranges, media such as sea water/
ground/Earth are always regarded as highly lossy ones, while
the air is lossless. So, the ratio of the wave numbers is far
smaller than 1, i.e., \( k_{\text{air}}/k_{\text{medium}} \ll 1 \). Furthermore, usually
the heights of both the dipole source and the receiving point
are much smaller than the horizontal distance, i.e., \( d \ll \rho \) and
\( z \ll \rho \). The following are some concrete evaluations and
simplifications. Comparisons with direct numerical simu-
lations and exact solutions are also carried out from both the
amplitude and phase angle aspects eventually. The time
dependence \( e^{i\omega t} \) is assumed and suppressed throughout the
analysis.

\[ E_{2\rho}(\rho, z) = \frac{-\omega \mu_0 I dS}{2\pi} \int_0^\infty \left[ \frac{e^{i\gamma_1 |z-d|}}{2\gamma_2} - \frac{e^{i\gamma_2 (z+d)}}{2\gamma_1 + \gamma_2} \right] J_1(\lambda \rho) \lambda^2 d\lambda, \]

\[ H_{2\rho}(\rho, z) = \frac{I dS}{2\pi} \int_0^\infty \left[ \pm \frac{e^{i\gamma_1 |z-d|}}{2} - \frac{e^{i\gamma_2 (z+d)}}{2\gamma_1 + \gamma_2} \right] \frac{J_1(\lambda \rho) \lambda^2}{y_1 + y_2} d\lambda, \]

\[ H_{2z}(\rho, z) = \frac{i I dS}{2\pi} \int_0^\infty \left[ \frac{e^{i\gamma_1 |z-d|}}{2\gamma_2} - \frac{e^{i\gamma_2 (z+d)}}{2\gamma_1 + \gamma_2} \right] J_0(\lambda \rho) \lambda^3 d\lambda. \]

Note that the upper sign “+” in (2) corresponds to
\( z > d \), and the lower sign “−” corresponds to \( 0 < z < d \),
respectively. And the square root in (4) is taken in the first
quadrant.

The first two terms in formulas (1)–(3) stand for the
direct wave and the ideal reflected wave. Meanwhile, the
third term represents the lateral wave. Thus, these formulas
can be rewritten in the following forms:

\[ E_{2\rho}(\rho, z) = E_{2\rho}^d(\rho, z) + E_{2\rho}^r(\rho, z) + E_{2\rho}^l(\rho, z), \]

\[ H_{2\rho}(\rho, z) = H_{2\rho}^d(\rho, z) + H_{2\rho}^r(\rho, z) + H_{2\rho}^l(\rho, z), \]

\[ H_{2z}(\rho, z) = H_{2z}^d(\rho, z) + H_{2z}^r(\rho, z) + H_{2z}^l(\rho, z). \]

In the above integrals, the direct-field and reflect-field
terms can be derived via the method presented in the
monograph by King et al. [15].
The direct-field components are

\[ E_{2p}^d (\rho, z) = -\frac{\omega \mu_0 I d S}{4\pi} \int_0^\infty e^{i\rho_2 (z-d)} \frac{e^{i\rho_2^2}}{\gamma_2} J_1 (\lambda \rho) \lambda^2 d\lambda \]

\[ = -\frac{\omega \mu_0 I d S}{4\pi} \rho k_2 \left( \frac{1}{r_2^2} + i \frac{k_2}{r_2^2} \right) e^{i k_2 r_2} \]

\[ H_{2p}^d (\rho, z) = \pm \frac{I d S}{4\pi} \int_0^\infty e^{i\rho_2 (z-d)} \frac{e^{i\rho_2^2}}{\gamma_2} J_1 (\lambda \rho) \lambda^2 d\lambda \]

\[ = \pm \frac{I d S}{4\pi} \rho k_2 \left( \frac{d_2}{r_2^2} + i \frac{k_2}{r_2^2} \right) e^{i k_2 r_2} \]

The reflect-field components are

\[ E_{2p}^r (\rho, z) = -\frac{\omega \mu_0 I d S}{4\pi} \int_0^\infty e^{i\rho_2 (z+d)} \frac{e^{i\rho_2^2}}{\gamma_2} J_1 (\lambda \rho) \lambda^2 d\lambda \]

\[ = -\frac{\omega \mu_0 I d S}{4\pi} \rho k_2 \left( \frac{1}{r_2^2} - i \frac{k_2}{r_2^2} \right) e^{i k_2 r_2} \]

\[ H_{2p}^r (\rho, z) = \frac{I d S}{4\pi} \int_0^\infty e^{i\rho_2 (z+d)} \frac{e^{i\rho_2^2}}{\gamma_2} J_1 (\lambda \rho) \lambda^2 d\lambda \]

\[ = \frac{I d S}{4\pi} \rho k_2 \left( \frac{d_2}{r_2^2} - i \frac{k_2}{r_2^2} \right) e^{i k_2 r_2} \]

where

\[ d_1 = |z - d|, \]

\[ d_2 = z + d, \]

\[ r_1 = \sqrt{\rho^2 + d_1^2}, \]

\[ r_2 = \sqrt{\rho^2 + d_2^2}. \]

The rest lateral-wave terms are defined by

\[ E_{2p}^l (\rho, z) = -\omega \mu_0 I d S \frac{1}{2\pi} \int_0^\infty e^{i\rho_2 (z+d)} J_1 (\lambda \rho) \lambda^2 d\lambda \]

\[ = -\omega \mu_0 I d S \frac{F_{2p}(\rho, d_1)}{2\pi} \]

\[ H_{2p}^l (\rho, z) = \frac{I d S}{2\pi} \int_0^\infty \frac{\gamma_2}{\rho} e^{i\rho_2 (z+d)} J_1 (\lambda \rho) \lambda^2 d\lambda \]

\[ = \frac{I d S}{2\pi} \cdot F_{2p}(\rho, d_2). \]
\[ H_{2z}^l (\rho, z) = \frac{i dS}{2\pi} \int_0^\infty \frac{1}{M} e^{iy_2(z+d)} J_0 (\lambda \rho) \lambda^3 d\lambda \]

\[ = \frac{i dS}{2\pi} \cdot F_{2z}^l (\rho, d_z), \]

where \( M = y_1 + y_2 \). It should be pointed out that these formulas contain Bessel functions, whose integrands are highly oscillatory in the near-zone. Therefore, a novel approximation approach is required to be taken into consideration in the following subsection.

2.2. Simplications of the Lateral-Wave Terms under the Equivalent Infinitesimal Theory. For the beginning of simplification, we should figure out the definition of "quasi-static" or "near-field" involved in this paper. It is defined as the case where the distance from the source to the observation point is far less than a wavelength \((k\rho \ll 1)\) [15, 18]. Typically, if the frequency remains invariable, distances satisfying this condition contain the "near-field"; otherwise, if the distance is fixed, qualified frequencies form the "quasi-static" state.

It is readily understood from the definition, when the "quasi-static" assumption is considered, \(\omega \rightarrow 0\) and \(k_5 \rho \ll 1\) hold. Under this circumstance, contribution of the Sommerfeld integrals in (9)–(11) is mainly from the integration path where \(\lambda \gg k_2\). Therefore, the parameter \(y_2\) can be expressed as follows:

\[ y_2 = \sqrt{k_2^2 - \lambda^2} \approx i\lambda. \]

Normally, the medium in Region 1 is highly conductive when compared with air in Region 2, namely, \(k_2 \ll k_1\). Also, \(d\) and \(z\) are usually far less than the horizontal distance \(\rho\), i.e., \(\rho \ll d\) and \(z \ll \rho\). Thus, we have

\[ i |y_1 - y_2 - k_1| d_m \approx i \left[ k_2^2 - \lambda^2 - i\lambda - k_1 \right] d_m \ll 1, \]

where \(m = 1, 2\). It is seen from (13) that the limit of \(i |y_1 - y_2 - k_1| d_m\) can be approximated as zero. The equivalent infinitesimal theory, a special case of Taylor’s formula, could be subsequently applied here as follows:

\[ e^{i (y_1 - y_2 - k_1) d_m} - 1 \approx i (y_1 - y_2 - k_1) d_m \]

Then, by some deformation, a useful alternative form of \(y_1 - y_2\) is correspondingly obtained, so is the variable \(M\). We write

\[ y_1 - y_2 = i \frac{1 - e^{i (y_1 - y_2 - k_1) d_m}}{d_m} + k_1, \]

\[ \frac{1}{M} = \frac{y_1 - y_2}{(y_1 + y_2) (y_1 - y_2)} \]

\[ = \frac{-e^{i (y_1 - y_2 - k_1) d_m}}{(k_1^2 - k_2^2) d_m} + 1 + i k_1 d_m. \]

Hence, by substituting (16) into (9), we have

\[ F_{2z}^l (\rho, d_z) = \frac{-i}{(k_1^2 - k_2^2) d_z} \left[ e^{-ik_1 d_z} \int_0^\infty e^{i (y_1 - k_1) d_z} J_0 (\lambda \rho) \lambda^3 d\lambda \right] \]

\[ = \frac{-i}{(k_1^2 - k_2^2) d_z} \left[ e^{-ik_1 d_z} J_0 (\lambda \rho) \lambda^3 d\lambda \right]. \]

Similarly, (10) is equal to

\[ F_{2z}^l (\rho, d_z) = \frac{-i}{(k_1^2 - k_2^2) d_z} \left[ e^{-ik_1 d_z} \int_0^\infty e^{i (y_1 - k_1) d_z} J_0 (\lambda \rho) \lambda^3 d\lambda \right] \]

\[ + (ik_1 d_z - 1) \int_0^\infty e^{iy_2 d_z} J_0 (\lambda \rho) \lambda^3 d\lambda. \]

\[ \frac{\partial}{\partial d_z} \left[ \frac{1}{M} \right] \int_0^\infty \left[ e^{i (y_1 - y_2 - k_1) d_z} + 1 - ik_1 (z + d) \right] e^{iy_2 (z+d)} J_0 (\lambda \rho) \lambda^2 d\lambda \]

\[ = \frac{1}{k_1^2 - k_2^2} \frac{\partial}{\partial d_z} \int_0^\infty \left[ e^{i (y_1 - k_1) d_z} + (1 - ik_1 d_z) e^{iy_2 d_z} J_0 (\lambda \rho) \lambda^2 d\lambda. \right] \]
Finally, with the Appendix in [15], the final analytical results for \( F_{2\nu}^d, F_{2\nu}^d, F_{2\nu}^d \) can be obtained readily. We write

\[
F_{2\nu}^d (\rho, d_z) = \frac{\rho k_1}{r_z (k_1^2 - k_2^2)} \left[ \frac{k_1}{r_z} + \frac{3i}{k_1 r_z^2} \right] e^{ik_1 (r_z - d_z)} + \frac{\rho k_2 (ik_1 d_z - 1)}{r_z (k_1^2 - k_2^2)} \left[ \frac{k_2}{r_z} + \frac{3i}{k_2 r_z^2} \right] e^{ik_2 r_z},
\]

(20)

\[
F_{2\nu}^d (\rho, d_z) = \frac{-i \rho}{(k_1^2 - k_2^2)} \times \left[ \frac{k_2}{r_z} + \frac{3i}{k_1 r_z^2} \right] e^{ik_1 (r_z - d_z)} - k_1 k_2 \left( \frac{k_2}{r_z} + \frac{3i}{k_1 r_z^2} \right) e^{ik_2 r_z} + k_2 d_z \left( \frac{k_2}{r_z} + \frac{3i}{k_1 r_z^2} \right) e^{ik_2 r_z} - k_1 k_2 \left( \frac{k_2}{r_z} + \frac{3i}{k_1 r_z^2} \right) e^{ik_2 r_z}.
\]

(21)

\[
F_{2\nu}^d (\rho, d_z) = \frac{k_1}{k_2^2 - k_1^2} \left[ \frac{k_2}{r_z} + \frac{4ik_1}{r_z^2} - \frac{9i}{k_1 r_z^2} \right] e^{ik_1 (r_z - d_z)} + k_2 (ik_1 d_z - 1) \left( \frac{k_2^2}{r_z^2} + \frac{4ik_1}{r_z} - \frac{9i}{k_1 r_z^2} \right) e^{ik_2 r_z}.
\]

(22)

So far, we have provided the process of calculating the field components for a VMD source over the near-field range in Region 2 in detail. The complete approximation solutions for the field components in Region 1 can be evaluated with a similar method as well.

2.3. Solutions for the near-Zone Field with \( z = 0 \) and \( d = 0 \).

The complete quasi-static field components are derived readily with (9)–(11) and (20)–(22). For comparison purpose, in the following, we will attempt to reduce these formulas to the special case when both the dipole source and the receiving point are embedded on the boundary. For brevity and clarity, only \( H_{2z} \)'s derivation procedure is given as a demonstration. The magnetic component \( H_{2z} \) can be expressed as

\[
H_{2z} = \frac{1}{4\pi} k_2 \left( \frac{k_2}{r_z} + \frac{i}{r_z^2} - \frac{1}{k_2 r_z^4} \right) e^{ik_2 r_z} - i \frac{k_2}{r_z} \left[ \frac{k_2}{r_z} + \frac{3i}{k_2 r_z^2} \right] e^{ik_2 r_z} + \frac{2i k_2 d_z}{k_2^2 - k_1^2} \left( \frac{k_2}{r_z} + \frac{4ik_1}{r_z^2} - \frac{9i}{k_1 r_z^2} \right) e^{ik_2 r_z}.
\]

(23)

Since \( k_{2\rho} \ll 1 \) is assumed, with Maclaurin series, we have

\[
e^{ik_{2\rho}} = 1 + ik_{2\rho} - \frac{(k_{2\rho})^2}{2}.
\]

(24)

By substituting (24) into (23) and setting \( z = 0, d = 0 \), it converts to:

\[
H_{2z} = \frac{1}{2\pi} \left[ \frac{ik_1}{\rho^2} \left( \frac{1}{\rho^2} - \frac{4i}{k_1^2 \rho^4} - \frac{9}{k_1^2 \rho^6} \right) e^{ik_{2\rho}} + k_2^2 \left( \frac{1}{\rho^2} + \frac{9}{\rho^6} \right) \right].
\]

(25)
employed and then compared with the analytical result obtained in this paper. We assume that Region 1 is seawater characterized by the permeability $\mu_0$, relative permittivity $\varepsilon_r = 80$, and conductivity $\sigma_1 = 4 \text{ S/m}$. Region 2 is air characterized by the permeability $\mu_0$ and the permittivity $\varepsilon_0$. For a general case, the dipole source and receiving point are placed at the height $d = 1 \text{ m}$ and $z = 5 \text{ m}$, respectively. The amplitudes and phase angles of the three nonzero field components with respect to the propagation distance $\rho$ are evaluated in Figures 2–4 at operating frequencies $f = 3 \text{ Hz}$ and $300 \text{ Hz}$.

It is seen that both the amplitudes and phase angles obtained by analytical and direct numerical methods have a consistent trend in general. Figures 2(a) and 2(b) show that at the beginning there exists a relatively apparent difference between these two computed results. This is probably because when $\rho$ is too small, $d \ll \rho$ and $z \ll \rho$ are not very restrictive. But when $\rho > 10 \text{ m}$, two curves shed light on a good correspondence with each other. The other two curves, $E_{2\rho}$ and $H_{2\rho}$ in Figures 3(a), 3(b), 4(a), and 4(b) also show similar trends with $H_{2\rho}$. Obviously, in most cases of practical interest, $\rho \gg z$ and $\rho \gg d$, so the results confirm the high reliability and validity of the proposed method. Furthermore, it is also noted that the operating frequency has little influence on the amplitude and phase obtained by the two methods remaining the same even if the amplitudes and phase obtained by the two methods remain similar. For a sudden change to $\pi$, two curves start to deviate from each other. Eventually, for a sudden change to $\pi$, two curves start to deviate from each other and trajectories are not in full accord, similar conclusions can still be drawn.

It should also be born in mind that with large distances, $k_2\rho$ is more close to “1,” which then also leads to a larger difference between the numerical and analytical results. In this paper, attention is restricted to the near field, and $k_2\rho \sim 1$ is out of consideration. Computations show that different components have different validity limits of $k_2\rho$ for maintaining a consistency between the proposal and numerical solutions. For $E_{2\rho}$ and $H_{2\rho}$ components, results of both the amplitude and phase obtained by the two methods remain the same even if $k_2\rho$ is approaching 1. However, for $H_{2\rho}$, when the distance reaches 100 km (i.e., $k_2\rho \approx 0.6$, as can be seen in Figure 2(b)), two curves start to deviate from each other.

3.2. Comparison with Exact Solutions When $z = 0$ and $d = 0$. When both the dipole source and receiving point are located at the interface of a half-space, the exact field components excited by a vertical magnetic dipole have been addressed by Parise already [27]. The exact solutions of $H_{2\rho}$, $E_{2\rho}$, and $H_{2\rho}$ can be expressed in terms of

$$\begin{align*}
(\text{exact})_x &= \frac{i \rho}{2 \pi (k_2^2 - k_1^2)} (\hat{Q}_2 (\rho) - \hat{Q}_1 (\rho)), \\
(\text{exact})_y &= \frac{i \rho}{2 \pi (k_2^2 - k_1^2)} (Q_2 (\rho) - Q_1 (\rho)), \\
(\text{exact})_z &= \frac{1}{\pi \rho} \left[ \alpha^2 + \beta^2 \right] (K_1 (\alpha \rho) I_1 (\beta \rho) - \alpha \beta K_2 (\alpha \rho) I_2 (\beta \rho)),
\end{align*}$$

where

$$\begin{align*}
\hat{Q}_n (\rho) &= (ik_n^3 - 4k_n^2 \rho^2 - 9ik_n \rho + 9) \frac{e^{ik_n \rho}}{\rho^3}, \\
Q'_n (\rho) &= (k_n^2 \rho^2 + 3i k_n \rho - 3) \frac{e^{ik_n \rho}}{\rho^3}.
\end{align*}$$

Note that $\alpha$ and $K_n$ are the nth order modified Bessel functions of the first and second kind, respectively, where $n = 1, 2$.

Since it is a special case of the dipole radiation problem discussed in this paper, we can actually better validate the manner if we plot how that field changes as a function of the distance $\rho$ by our approximation method, the direct numerical calculation and this exact analytical solution. Comparisons are depicted in Figures 5–7. All the parameters involved are the same with those in Figures 2–4 except with different $d$ and $z$.

3.3. Evaluations for These Analytical Formulas. For further investigation about the features of the proposed method, several other evaluations are performed. As a natural extension, just to make it clear and to see how the locations of the dipole source and receiving point influence the propagation strength of electromagnetic fields, we consider the following simulations presented in Figure 8, with the relative permittivity $\varepsilon_r = 80$ and conductivity $\sigma_1 = 4 \text{ S/m}$ in Region 1. The operating frequency is taken as $f = 10 \text{ Hz}$.

Figure 8 illustrates the relationship between $|H_{2\rho}|$ and $z$ with different heights of the source at a constant propagating distance, where all other parameters are held unchanged. The constant is chosen as $50 \text{ m}$, $100 \text{ m}$, $500 \text{ m}$, and $1000 \text{ m}$,
Figure 2: Amplitudes and phase angles of field component $H_{zz}$ versus the propagation distance $\rho$ with the proposed and numerical solutions. (a, c) $f = 3$ Hz. (b, d) $f = 300$ Hz.

Figure 3: Continued.
Figure 3: Amplitudes and phase angles of the field component $E_{2\phi}$ versus the propagation distance $\rho$ with the proposed and numerical solutions. (a, c) $f = 3$ Hz. (b, d) $f = 300$ Hz.

Figure 4: Amplitudes and phase angles of the field component $H_{2\rho}$ versus the propagation distance $\rho$ with the proposed and numerical solutions. (a, c) $f = 3$ Hz. (b, d) $f = 300$ Hz.

Figure 5: Continued.
respectively. It is seen that, with the increasing of the radial distance, $|H_2z|$ will exhibit quite different characteristics. When $\rho$ is relatively small (see Figures 8(a) and 8(b)), the amplitude decreases at first and then increases gradually as the height of the observing point becomes higher. Typically, the minimum point of the curve will have a shift to a farther position from the origin if $\rho$ is larger. However, for sufficiently large values of $\rho$ (Figures 8(c) and 8(d)), the amplitude increases monotonically as $z$ increases. We also know that, when the dipole source is closer to the interface, the attenuation will be larger. Perhaps it is due to the different degrees of the effect by the lossy medium in Region 1 at different heights.

When the receiving point is at an arbitrary place on the $x$-$z$ plane in Region 2, and the dipole source is located at $d = 0$ m, 50 m, 150 m, respectively, with the operating frequency taken as $f = 10$ Hz, the strength distributions of the electric and magnetic components are revealed in Figures 9–10. We can actually see that both the electric and magnetic fields decay rapidly in all directions. As the position of the source is higher, the attenuation of the electromagnetic components will become slower. It is also noted that the distribution in
Figure 7: Amplitudes and phase angles of field component $H_{2\rho}$ versus the propagation distance $\rho$ with the proposed, exact, and numerical solutions. (a, c) $f = 3$ Hz. (b, d) $f = 300$ Hz.

Figure 8: Continued.
Figures 9(c) and 10(c) is very likely to that in free space, which implies that when the height of the dipole is high enough away from the interface, the radiation result is nearly not affected by the sea water.

4. Conclusions

This paper deals with the integrals for the electromagnetic field of a vertical magnetic dipole lying above the boundary of a lossy half-space in order to derive analytical formulas for the near-zone field. On the basis of the feature of “quasi-static” state and the equivalent infinitesimal theory, we can induce two rational assumptions. One is $c^2 \approx i\lambda$, and the other is about the substitution of $c_1 - c_2$. Comparisons with numerical solutions show that both the amplitude and phase are quite consistent with each other. Curves for contrast with the exact formulas are also drawn when the dipole source and observing point are both on the boundary, which is a further proof of the high accuracy and reliability of our manner. Simulations also show that different components have different validity limits of $k_2 \rho$ for maintaining a consistency between the proposal and numerical solutions. The
smallest validity limit always occurs for the component $H_{2z}$, and the value of $k_p\rho$ should be no greater than 0.6 in order to keep a good consistency. Subsequent computations, which indicate that operating frequencies and the positions of the dipole source and the receiving point will all affect the field components, are also performed. In addition, it should be pointed out that these formulas are also suitable for other cases where the condition $k_1 \ll k_2$ is satisfied. And the media in two regions are interchangeable. Under this circumstance, $\gamma_1 = il$ is acquired, and the approximation in (13) changes to $|\gamma_2 - \gamma_1 - k_3|$. Then, by a similar method presented in this paper, the final field expression can be obtained.

**Data Availability**

All data are available as MATLAB code.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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