Nonadditivity of Fluctuation-Induced Forces in Fluidized Granular Media

M. Reza Shaebani,1 Jalal Sarabadani,2 and Dietrich E. Wolf1
1Department of Theoretical Physics, University of Duisburg-Essen, 47048 Duisburg, Germany
2Department of Physics, University of Isfahan, Isfahan 81746, Iran
(Dated: January 19, 2013)

We investigate the effective long-range interactions between intruder particles immersed in a randomly driven granular fluid. The effective Casimir-like force between two intruders, induced by the fluctuations of the hydrodynamic fields, can change its sign when varying the control parameters: the volume fraction, the distance between the intruders, and the restitution coefficient. More interestingly, by inserting more intruders, we verify that the fluctuation-induced interaction is not pairwise additive. The simulation results are qualitatively consistent with the theoretical predictions based on mode coupling calculations. These results shed new light on the underlying mechanisms of collective behaviors in fluidized granular media.

PACS numbers: 45.70.Mg, 05.40.-a

Granular segregation has been extensively investigated during the last two decades aimed at revealing the underlying complex dynamics.1,2 Besides the scientific interest, understanding the mechanisms of segregation is of essential importance in geophysical3 and industrial4 processes. The behavior of granular mixtures, when mechan-ically agitated, depends on a long list of grain, container, and external driving properties.5,6 The control parameters can be tuned so that the demixing is initiated, reversed, or prevented.7,8,9 While the phase behavior of these systems is still a matter of debate, the nature of particle-particle interactions is known to play a crucial role; two extreme limits can be distinguished: (i) the fully fluidized regime where particles undergo only binary collisions, and (ii) the lasting contacts regime where durable frictional contacts exist during a considerable part of the agitation cycle. While in the latter case the relevant processes are, e.g., reorganization, inertia, and convection,10 some studies reveal the existence of another mechanism in the fluidized regime: in the presence of intruder particles, the hydrodynamic fields are modified especially in the inner regions between intruders, leading to effective long-range interactions.11,12,13,14 Cattuto et al.12 found that a pair of intruder particles experience an effective force in a driven granular bed, originating from the modification of the pressure field fluctuations due to the boundary conditions imposed by the intruders. Such Casimir-like interactions are expected in thermal noisy environments confined by geometrical constraints. Most reports, so far, are about either binary mixtures8,9 or one or few intruder particles in a bed of smaller ones11,10,13. An important question to address is how the collective behavior is influenced by the number and arrangement of the intruders.

In the present Letter, we study the effective interactions between immobile intruder particles immersed in a uniformly agitated granular fluid where all particles undergo inelastic binary collisions (Fig. 1). We show that the interaction between a pair of intruders exhibits a crossover from attraction to repulsion below a critical density, as predicted in.12 We here address the general conditions under which the transition happens, and present the phase diagram of the transition. Moreover, by comparing the behavior of two and multi intruder systems, we find that the fluctuation-induced force is not derived from a pair-potential; inserting a new intruder affects the previously existing interactions in a non-trivial way, depending on the relative positions of the intruders. Such a feature together with the possible sign change of the forces make the multi-body interactions more complicated and may lead to a variety of collective behaviors such as segregation, clustering, or pattern formation.

Analytical calculations using the theory of randomly driven granular fluids15 confirm our findings.

Simulation method — We consider a 2D granular fluid similar to the setup described in Refs.12,15,16 by means of molecular dynamics simulations. We have a reference system with two intruders A and B in which $L_0/r=200$, $R_0/r=10$, and $D_{AB,0}/r=30$ [see Fig. 1(a)].
The reference volume fraction $\phi_o$ is 0.66, and the normal restitution coefficient $\alpha_o$ is set to 0.8 for all collisions. Periodic boundary conditions are applied in both directions of the square-shaped cell to provide a spatially homogeneous state. The system is coupled to an external heat bath that uniformly transfers energy into the system; the acceleration of each particle follows a Gaussian white noise with zero mean and correlated form $\xi$. The system is considered to be in the nonequilibrium steady state, the hydrodynamic fields $\mathbf{F}$ are additive in this case; the driving strength $\xi$ is proportional to $h(L)$, which is the driving term $\propto\ln(L)$ versus $L$. By choosing $D_{\text{AB}}/r=30$ and $\phi=0.24$, the effective force $F_{\text{AB}}$ exerted on particle A in the triple configuration (A,B,D) is compared to $F_{\text{AB}}$ and $F_{\text{AD}}$ obtained from the binary systems (A,B) and (A,D), respectively. Note that the simulation is performed anew for each set of intruders. Figure 3(a) shows that $F_{\text{AB}}$ is nearly the vector sum of $F_{\text{AB}}$ and $F_{\text{AD}}$. However, a comparison between the sets (A,B,C), (A,B), and (A,C) in Figure 3(b) (with $D_{\text{AC}}/r=70$) reveals that the force is definitely not pairwise additive in this case; $F_{\text{AB}}$ is even smaller than $F_{\text{AB}}$.

In order to understand the mechanism behind the long-range interactions and the transition, we draw attention to the fact that the fluctuating hydrodynamic fields, e.g. density [see Fig. 3(c)], are notably influenced by the geometric constraints, resulting in pressure imbalance around the intruders and effective interactions between them. To establish a quantitative connection between the effective force and the hydrodynamic fluctuations, we first employ mode coupling calculations [12, 15, 21] to evaluate the two-body interactions. In the nonequilibrium steady state, the hydrodynamic fields $(p(r), T(r), n(r))$ fluctuate around their stationary values $(p_s, T_s, n_s)$. The average pressure fluctuation $p_r(r)$ in the presence of the boundary conditions imposed by the intruders behaves analogously to the Casimir effect, i.e., $p_r(r)$ in the boxed region of Fig. 3(a) differs from that of the cross-hatched region. Using the Verlet-Levesque
equation of state for a hard disks system \( p(n, T) = T f(n) \) (with \( f(n) = n(1 + \phi^2/8)(1 - \phi)^2 \), and \( n \) the number density) \(^{(22)}\), we expand the pressure up to second order around \((n_s, T_s)\), and take the statistical average over the random noise source: 
\[
p_j(r) = f'(n) n_s T \langle \delta n(r) \delta T(r) \rangle + \frac{1}{2} T_s f''(n) n_s \langle (\delta n(r))^2 \rangle.
\]
By Fourier transforming \( \delta n(r) \) and \( \delta T(r) \) one obtains

\[
p_j(r) = \int \left[ f'(n) n_s T \langle k \rangle + \frac{1}{2} T_s f''(n) n_s S_n(k) \right] dk, \tag{1}
\]
where the integral is taken over the \( k \) vectors allowed at position \( r \) by the boundary conditions, and \( S_n(k) \) is the pair structure factor defined as \( V^{-1} \langle \delta n(k) \delta n(-k) \rangle \).

The detailed description of the structure factor calculations will be reported elsewhere (see also Ref. \(^{(15)}\)). Here we denote the integrand of Eq. \((1)\) with \( g(k, \phi, \alpha) \) and compare \( p_j(r) \) for two surface points located on opposite sides of intruder \( A \) with the same \( y \) coordinates. The related \( k \) vectors in the \( x \) direction are confined to \( D_n(y) = D_{AB} - 2\sqrt{R^2 - y^2} \) and \( D_m(y) = L - D_{AB} - 2\sqrt{R^2 - y^2} \) in the cross-hatched and hatched regions, respectively; therefore, the pressure difference between these two points \( \Delta p(y) = p_j^{(in)} - p_j^{(out)} \) has \( y \) dependence. By integrating over \( y \), we arrive at the average pressure difference between the gap and outside region:

\[
\Delta p = \frac{1}{R} \int_{-R}^{R} \int_{2\pi l^*/D_{AB}(y)}^{2\pi l^*/2} \int_{2\pi l^*/D_{AB}(y)}^{2\pi l^*/2} \int_{2\pi l^*/2}^{2\pi l^*/2} \frac{g(k_x, k_y, \phi, \alpha)}{2R} dk_y dk_x dk, \tag{2}
\]

To ensure that the hydrodynamic description is valid, the integrals are restricted to the long wavelength range \( r^* = \text{max}(2r, l^*) \) (with \( l^* \) being the mean free path) and only small inelasticities are considered. Moreover, \( D_{AB} \) is always chosen large enough \((3R \leq D_{AB})\) so that the short-range depletion forces \(^{(23)}\) do not play a role. The effective force \( F_{AB} \) is calculated via Eq. \((2)\) for different values of \( \phi \) or \( D_{AB} \) and compared to the simulation results in Figs. \((1)\) and \((2)\). The forces are of the same order of magnitude as those obtained from the simulations. The deviations can be attributed to the fact that the hydrodynamic fluctuations are correlated in the gap and outside regions. The correction due to this effect is proportional to \( \partial^2 g / \partial \phi^2 \) which always has a sign opposite to that of \( g \), thus, Eq. \((2)\) overestimates the magnitude of the force. Using the mode coupling calculations, it is shown in Fig. \((1)\) that the transition point is sensitive to the choice of \( D_{AB} \) and \( \alpha \). The set of control parameters for sign switching of the Casimir force crucially depends on the physics of the system (see e.g. \(^{(24)}\)). Figure \((2)\) summarizes the calculations in a phase diagram in the \((\phi, \alpha, D_{AB})\) space, which appears to be in remarkable accord with the dynamical model. The phase diagram is not influenced by the choice of the steady-state temperature \( T_s \), while the magnitude of the force grows linearly with \( T_s \) as expected for Casimir forces in thermal fluctuating media \(^{(14)}\). Regarding the fact, that the leading term of \( g(k, \phi, \alpha) \) at small \( k \) is proportional to \( 1/k^2 \) \(^{(15)}\), one also finds from Eq. \((1)\) that \( p_j \), and therefore the force, in the thermodynamic limit behaves as \( 1/r \) in 3D while diverges logarithmically as \( \ln(L/r) \) in 2D.

**Multi-body effects** — In the triple system of Fig. \((3)\), loosely speaking, because of the independence of \( k \) vectors in \( x \) and \( y \) directions one expects that the vector sum holds. This is in agreement with the simulation results in Fig. \((3)\), neglecting the deviation due to hydrodynamic correlations. In the configuration of Fig. \((3)\), the effective force exerted on particle \( A \) results from the difference between the range of available \( k \) modes on its left and right sides. In the presence of particle \( C \), the range of allowed \( k \) modes decreases on the left side due to periodic boundary conditions, which causes a lowering of pressure difference between both sides of \( A \). The effective force \( F_{ABC} \) is thus smaller, compared to the binary interaction \( F_{AB} \). The calculated force on particle \( A \) using Eq. \((2)\) is shown with \( F_{AB} \) in Fig. \((3)\); the agreement is satisfactory. We introduce a measure \( \epsilon = |F_{AB} + F_{AC} - F_{ABC}| / |F_{AB}| \) for quantifying the deviation from the case that the interactions are derived from a pair-potential. Figure \((3)\) shows how \( \epsilon \), obtained from the mode coupling calculations, behaves when the position of particle \( C \) is varied.
on the $x$ axis. Figure 3(c) indicates that our analytical approach also provides reasonable estimates of the effective force acting on particle A in a chain configuration, while the deviation from pairwise summation of two-body forces grows with the number of intruders, $n$.

Finally, we investigate more complicated geometries by calculating the effective force field of a fixed square structure acting on a test intruder particle. To elucidate the impact of sign switching on the force pattern we compare the multi-body effects into account.

In conclusion, we focus on the problem of long-range fluctuation-induced forces between intruder particles immersed in an agitated fluid bed. The sign of the force can be reversed by tuning the control parameters. Furthermore, the multi-body interactions do not follow from two-body force descriptions. A newly inserted intruder, depending on its position, may affect the pressure balance around the other intruders. This suggests that the effective force is not derived from a pair-potential in agreement with our simulation results. Our findings represent a step forward in understanding the origin of collective behaviors in fluidized granular mixtures.

We would like to thank I. Goldhirsch for helpful discussions and J. Török and B. Farnudi for comments on the manuscript. Computing time was provided by John-von-Neumann Institute of Computing (NIC) in Jülich.

### TABLE I: Comparison between the theoretical (T) and simulation (S) results for the selected points in Figs. 5(left) and 4(right). Same force scales as Fig 4. The maximum error bars for the simulated $F$ and $\theta$ are $\pm 0.1$ and $\pm 7^\circ$, respectively.

|       | $F_{(A)}^\text{S}$ | $F_{(B)}^\text{S}$ | $F_{(C)}^\text{S}$ | $F_{(D)}^\text{S}$ | $\phi^\text{(A)}$ | $\phi^\text{(B)}$ | $\phi^\text{(C)}$ | $\phi^\text{(D)}$ |
|-------|-------------------|-------------------|-------------------|-------------------|------------------|------------------|------------------|------------------|
| left-S| 1.0               | 2.2               | 3.4               | 3.7               | 13°              | 92°              | 51°              | 44°              |
| left-T| 1.17              | 2.52              | 3.97              | 4.32              | 7°               | 90°              | 60°              | 45°              |
| right-S| 0.2               | 0.2               | 0.2               | 0.0               | 19°              | 88°              | 32°              | –                |
| right-T| 0.24              | 0.26              | 0.25              | 0.02              | 28°              | 90°              | 24°              | 225°             |

---

* Electronic address: reza.shaebani@uni-due.de

† present address: Max Planck Institute for Polymer Research, D-55128 Mainz, Germany.

[1] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, Rev. Mod. Phys. 68, 1259 (1996).
[2] A. Kudrolli, Rep. Prog. Phys. 67, 209 (2004).
[3] O. Pouliquen, J. Delour, and S. B. Savage, Nature 386, 816 (1997); L. Hsu, W. E. Dietrich, and L. S. Sklar, J. Geophys. Res. 113, F02001 (2008).
[4] J. C. Williams, Powder Technol. 15, 245 (1976); J. M. Ottino and D. V. Khakhar, Annu. Rev. Fluid Mech. 32, 55 (2000).
[5] M. P. Ciamarra, M. D. De Vizia, A. Fierro, M. Tarzia, A. Coniglio, and M. Nicodemi, Phys. Rev. Lett. 96, 058001 (2006).
[6] D. C. Hong, P. V. Quinn, and S. Luding, Phys. Rev. Lett. 86, 3423 (2001); M. Tarzia, A. Fierro, M. Nicodemi, and A. Coniglio, Phys. Rev. Lett. 93, 198002 (2004); M. Tarzia, A. Fierro, M. Nicodemi, M. P. Ciamarra, and A. Coniglio, Phys. Rev. Lett. 95, 078001 (2005); K. M. Hill and Y. Fan, Phys. Rev. Lett. 101, 088001 (2008).
[7] T. Shinbrot, Nature 429, 352 (2004).
[8] D. A. Sanders, M. R. Swift, R. M. Bowley, and P. J. King, Phys. Rev. Lett. 93, 208002 (2004).
[9] M. P. Ciamarra, A. Coniglio, and M. Nicodemi, Phys. Rev. Lett. 97, 038001 (2006); I. Zuriguel, J. F. Boudet, Y. Amarouchene, and H. Kelway, Phys. Rev. Lett. 95, 258002 (2005).
[10] T. Schnautz, R. Brito, C. A. Kruelle, and I. Rehberg, Phys. Rev. Lett. 95, 028001 (2005).
[11] E. Caglioti, A. Coniglio, H. J. Herrmann, V. Loreto, and...
M. Nicodemi, Europhys. Lett. 43, 591 (1998); T. Shinbrot and F. J. Muzzio, Phys. Rev. Lett. 81, 4365 (1998); T. Mullin, Phys. Rev. Lett. 84, 4741 (2000); G. Metcalfe, S. G. K. Tennakoon, L. Kondic, D. G. Schaeffer, and R. P. Behringer, Phys. Rev. E 65, 031302 (2002).

[12] C. Cattuto, R. Brito, U. M. B. Marconi, F. Nori, and R. Soto, Phys. Rev. Lett. 96, 178001 (2006).

[13] S. Aumaitre, C. A. Kruelle, and I. Rehberg, Phys. Rev. E 64, 041305 (2001).

[14] M. Kardar and R. Golestanian, Rev. Mod. Phys. 71, 1233 (1999).

[15] T. P. C. van Noije, M. H. Ernst, E. Trizac, and I. Pagonabarraga, Phys. Rev. E 59, 4326 (1999).

[16] G. Peng and T. Ohta, Phys. Rev. E 58, 4737 (1998).

[17] I. Goldhirsch and G. Zanetti, Phys. Rev. Lett. 70, 1619 (1993).

[18] T. P. C. van Noije, M. H. Ernst, and R. Brito, Phys. Rev. E 57, R4891 (1998).

[19] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-uniform Gases* (Cambridge University Press, Cambridge, 1970).

[20] D. Bartolo, A. Ajdari, J. B. Fournier, and R. Golestanian, Phys. Rev. Lett. 89, 230601 (2002).

[21] R. Brito and M. H. Ernst, Europhys. Lett. 43, 497 (1998).

[22] L. Verlet and D. Levesque, Mol. Phys. 46, 969 (1982).

[23] R. Roth, R. Evans, and S. Dietrich, Phys. Rev. E 62, 5360 (2000); C. N. Likos, Phys. Rep. 348, 267 (2001).

[24] C. Hertlein, L. Helden, A. Gambassi, S. Dietrich, and C. Bechinger, Nature 451, 172 (2008); M. Levin, A. P. McCauley, A. W. Rodriguez, M. T. Homer Reid, and S. G. Johnson, Phys. Rev. Lett. 105, 090403 (2010).

[25] P. Attard, J. Chem. Phys. 91, 3072 (1989); S. Jorge, E. Lomba, and J. L. F. Abascal, J. Chem. Phys. 116, 730 (2002).