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Geometric analysis of the Yang-Mills-Higgs-Dirac model. (English) Zbl 07603380
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Summary: The harmonic sections of the Kaluza-Klein model can be seen as a variant of harmonic maps with additional gauge symmetry. Geometrically, they are realized as sections of a fiber bundle associated to a principal bundle with a connection. In this paper, we investigate geometric and analytic aspects of a model that combines the Kaluza-Klein model with the Yang-Mills action and a Dirac action for twisted spinors. In dimension two we show that weak solutions of the Euler-Lagrange system are smooth. For a sequence of approximate solutions on surfaces with uniformly bounded energies we obtain compactness modulo bubbles, namely, energy identities and the no-neck property hold.

MSC: 35Q41 Time-dependent Schrödinger equations and Dirac equations 35Q40 PDEs in connection with quantum mechanics 35B65 Smoothness and regularity of solutions to PDEs 35B44 Blow-up in context of PDEs 35D30 Weak solutions to PDEs 53C43 Differential geometric aspects of harmonic maps 58E15 Variational problems concerning extremal problems in several variables; Yang-Mills functionals 81T13 Yang-Mills and other gauge theories in quantum field theory

Keywords: super Yang-Mills; Kaluza-Klein geometry; harmonic sections; regularity; energy identity

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