Compact stars in Eddington-inspired Born-Infeld gravity: Anomalies associated with phase transitions

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We study how generic phase transitions taking place in compact stars constructed in the framework of the Eddington-inspired Born-Infeld (EiBI) gravity can lead to anomalous behavior of these stars. For the case with first-order phase transitions, compact stars in EiBI gravity with a positive coupling parameter \( \kappa \) exhibit a finite region with constant pressure, which is absent in general relativity. However, for the case with a negative \( \kappa \), an equilibrium stellar configuration cannot be constructed. Hence, EiBI gravity seems to impose stricter constraints on the microphysics of stellar matter. Besides, in the presence of spatial discontinuities in the sound speed \( c_s \) due to phase transitions, the Ricci scalar is spatially discontinuous and contains \( \delta \)-function singularities proportional to the jump in \( c_s^2 \) acquired in the associated phase transition.

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I. INTRODUCTION

Based on the gravitational action and nonlinear electrodynamics originally proposed by Eddington [1], and Born and Infeld [2], respectively, Bañados and Ferreira recently established a new Eddington-inspired Born-Infeld (EiBI) theory of gravity [3], which reduces to general relativity (GR) in vacuum (see, e.g., [4, 5] for relevant studies). However, in the presence of coupling between matter, EiBI theory demonstrates several distinctive features, including the avoidance of singularities in the early cosmology and in the Newtonian collapse of pressureless particles, the formation of stable pressureless stars, and the existence of pressureless cold dark matter with a nonzero Jeans length [3, 6–12]. All such features stem from the effective repulsive gravitational effect intrinsic to EiBI theory with a positive coupling constant \( \kappa \) (see the following discussion for the definition of \( \kappa \)). Hence, in comparison with GR, EiBI theory also allows compact stars to attain higher mass limits [6, 11].

Unlike GR, in EiBI theory gravity and matter are nonlinearly coupled together. Therefore, the distinctions between GR and EiBI theory are expected to emerge in the high-density regime. To this end, the static structure and the stability of compact stars in EiBI theory have become the focus of several recent papers [6, 7, 11]. As noted in these papers, the presence of terms proportional to the spatial derivatives of the density in the equations governing the hydrostatic equilibrium and radial oscillation of compact stars in EiBI theory could potentially hamper the related studies if nonanalytic (i.e., discontinuous or nondifferentiable) equations of state (EOSs) are considered. To bypass the problems arising from such derivatives, in the above-mentioned studies the EOSs employed in constructing compact stars in EiBI theory are either polytropic or analytic fits mimicking the behavior of realistic nuclear matter [13]. However, even for polytropic EOSs of the form \( P \propto \rho^\Gamma \), with \( P \), \( \rho \), and \( \Gamma \) being the pressure, rest-mass density, and a fixed constant, respectively, Pani and Sotiriou have recently discovered that the Ricci scalar diverges at the stellar surface if the constant \( \Gamma \) is greater than 3/2 [14]. Such singular behavior is attributable to the auxiliary field \( q_{\mu\nu} \) proposed in EiBI gravity. In the process of eliminating the auxiliary field in the equation of motion, higher derivatives of the matter field emerge and hence lead to a sensitive dependence on the matter distribution.

In the present paper we investigate how the static structure of compact stars in EiBI theory could be affected by drastic changes in the EOS. Specifically, we consider nuclear EOSs with first-order (second-order) phase transitions, where the energy density (the sound speed) is a discontinuous function of pressure. Even in GR, the effects of such discontinuities are known to be nontrivial [15, 16], and could lead to a third family of compact stars [17, 18]. For compact stars in EiBI gravity, we find that in the presence of first-order phase transitions, no equilibrium configuration exists if the parameter \( \kappa < 0 \). Furthermore, the Ricci scalar develops a \( \delta \)-function singularity and discontinuity at the radius where the speed of sound is discontinuous due to phase transitions. We note that such behavior is in stark contrast to the case in GR.

The plan of the paper is as follows. We briefly summarize EiBI gravity and the equilibrium configuration of compact stars in Secs. [11] and [111] respectively. In Sec. [114] we consider equilibrium stars in EiBI gravity constructed with EOSs with first-order phase transitions and investigate the effects and physical implications of such transitions. The singular

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behavior revealed in the Ricci scalar as a consequence of the discontinuity in the sound speed is studied in Sec. III. Finally, our conclusions are summarized in Sec. VI. We use units where \( G = c = 1 \) unless otherwise noted.

## II. EDDINGTON-INSPIRED BORN-INFELD GRAVITY

Based on the Eddington gravity action and the Born-Infeld nonlinear electrodynamics \([1, 2]\), Bañados and Ferreira established the EiBI gravity \([11]\), which is summarized here as follows. The starting point of the EiBI theory is the action \( S \) given by

\[
S = \frac{1}{16\pi} \frac{2}{16\pi} \int d^4x \left( \sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}|} - \lambda \sqrt{-g} \right) + S_M[g, \Psi_M].
\]  

(2.1)

Here \( R_{\mu\nu} \) is the symmetric part of the Ricci tensor and is constructed solely from the connection \( \Gamma^\alpha_{\beta\gamma} \). However, the matter action \( S_M \) depends only on the metric \( g_{\mu\nu} \) and the matter field \( \Psi_M \). Besides, the determinant of a tensor \( f_{\mu\nu} \) is denoted by \( |f_{\mu\nu}| \), and the convention \( f \equiv |f_{\mu\nu}| \) is used hereafter. The parameters \( \kappa \) and \( \lambda \) are related to the cosmological constant by \( \Lambda = (\lambda - 1)/\kappa \) such that, in the limit \( \kappa \to 0 \), the action (2.1) reduces to the Einstein-Hilbert action.

In the present paper we aim at asymptotic flat solutions and hence consider the case with \( \lambda = 1 \). Several constraints on the value and the sign of the remaining parameter \( \kappa \) have been obtained from solar observations, big bang nucleosynthesis, and the existence of neutron stars \([6, 7, 12, 19]\). In particular, for cases with positive \( \kappa \), effective gravitational repulsion prevails, leading to the existence of pressureless stars and increases in the mass limits of compact stars \([6, 11]\).

It is important to note that the spacetime metric \( g_{\mu\nu} \) and the connection \( \Gamma^\alpha_{\beta\gamma} \) are treated as independent fields in EiBI theory. The field equations are derived by minimizing the action (2.1) with respect to \( g_{\mu\nu} \) and \( \Gamma^\alpha_{\beta\gamma} \) separately, and read as \([2]\)

\[
q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu},
\]

(2.2)

\[
\sqrt{-q} q^{\mu\nu} = \sqrt{-g} g^{\mu\nu} - 8\pi\kappa\sqrt{-g} T^{\mu\nu},
\]

(2.3)

with \( q_{\mu\nu} \) being an auxiliary metric related to the connection,

\[
\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} q^{\alpha\sigma} (\partial_\sigma q_{\beta\gamma} + \partial_\beta q_{\sigma\gamma} - \partial_\gamma q_{\sigma\beta}).
\]

(2.4)

On the other hand, the stress-energy tensor \( T^{\mu\nu} \) still satisfies the standard conservation equations,

\[
\nabla_\mu T^{\mu\nu} = 0,
\]

(2.5)

where, as in GR, the covariant derivative refers to the metric \( g_{\mu\nu} \).

It is worthwhile to note that both \( R_{\mu\nu} \) and \( T_{\mu\nu} \) can lead to a difference in the two metrics \( g_{\mu\nu} \) and \( q_{\mu\nu} \). If somehow there are some discontinuities in \( T_{\mu\nu} \) (or its derivatives), then the field equation (2.3) implies that \( g_{\mu\nu} \) and/or \( q_{\mu\nu} \) will accordingly acquire the corresponding property. However, as \( R_{\mu\nu} \) is constructed from the connection \( \Gamma^\alpha_{\beta\gamma} \), given by (2.4), it contains the second-order derivatives of \( q_{\mu\nu} \). So it is difficult, if not impossible, to maintain the balance of the field equation (2.2) at these discontinuities if the second-order derivatives of \( q_{\mu\nu} \) do not exist. Thus, we expect that the discontinuities in \( T_{\mu\nu} \) (or its derivatives) will give rise to similar discontinuities in \( g_{\mu\nu} \), but not in \( q_{\mu\nu} \). On the other hand, in GR the density profile of compact stars (or its spatial derivatives) is in general discontinuous due to the presence of phase transitions. Therefore, in the following discussion we consider compact stars in EiBI gravity and study specifically how phase transitions could affect the equilibrium configuration and the Ricci scalars derived respectively from \( g_{\mu\nu} \) and \( q_{\mu\nu} \).

## III. STATIC EQUILIBRIUM OF COMPACT STARS

The structure of compact stars in EiBI theory has been studied by Pani et al. \([6, 7]\) and Sham et al. \([11]\). Here we briefly review and follow the approach developed in Ref. \([11]\). For a static and spherically symmetric spacetime, the spacetime metric \( g_{\mu\nu} \) and the auxiliary metric \( q_{\mu\nu} \) are taken to be

\[
g_{\mu\nu} dx^\mu dx^\nu = -e^{\phi(r)} dr^2 + e^{\lambda(r)} dr^2 + f(r)d\Omega^2,
\]

(3.1)

\[
q_{\mu\nu} dx^\mu dx^\nu = -e^{\beta(r)} dr^2 + e^{\alpha(r)} dr^2 + r^2 d\Omega^2.
\]

(3.2)
The compact star is made of a perfect fluid described by the standard stress-energy tensor

\[ T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu}, \]

(3.3)

where \( \epsilon \) and \( u^\mu \) are the energy density and four-velocity of the fluid, respectively.

The field equations (2.2) and (2.3) lead to a set of relations for the functions \( \phi, \lambda, f, \alpha, \) and \( \beta \) introduced in Eqs. (3.1) and (3.2),

\[
\frac{1}{\kappa} \left( 2 + \frac{a}{b^3} - \frac{3}{ab} \right) = \frac{2}{r^2} - \frac{2e^{-\alpha}}{r^2} + \frac{2e^{-\alpha}\alpha'}{r},
\]

(3.4)

\[
\frac{1}{\kappa} \left( \frac{1}{ab} + \frac{a}{b^3} - 2 \right) = \frac{2}{r^2} + \frac{2e^{-\alpha}}{r^2} + \frac{2e^{-\alpha}\beta'}{r},
\]

(3.5)

\[ e^\beta = e^\phi b^3 a^{-1}, \]

(3.6)

\[ e^\alpha = e^\lambda b, \]

(3.7)

\[ f(r) = \frac{r^2}{ab}, \]

(3.8)

where \( a \equiv \sqrt{1 + 8\pi\kappa\epsilon} \) and \( b \equiv \sqrt{1 - 8\pi\kappaP}, \) and hereafter primed quantities denote partial derivatives with respect to \( r. \) Besides, the conservation of the stress-energy tensor, Eq. (2.5), gives another relation,

\[ \phi' = -\frac{2P}{P + \epsilon}. \]

(3.9)

Combining Eqs. (3.4) - (3.9), one can obtain two first-order differential equations governing the structure of a compact star,

\[
dP/dr = -\Theta \left[ \frac{2}{\epsilon + P} + \frac{\kappa}{2} \left( \frac{3}{b^2} + \frac{1}{a^2c_s^2} \right) \right]^{-1} \left[ 1 - \frac{2m}{r} \right]^{-1},
\]

(3.10)

\[
dm/dr = \frac{1}{4\kappa} \left( 2 - \frac{3}{ab} + \frac{a}{b^3} \right) r^2,
\]

(3.11)

where

\[ \Theta \equiv \left[ \frac{1}{2\kappa} \left( \frac{1}{ab} + \frac{a}{b^3} - 2 \right) r + \frac{2m}{r^2} \right], \]

(3.12)

the speed of sound \( c_s \) is calculated from the EOS by \( c_s^2 = dP/d\epsilon, \) and the function \( m(r) \) is defined by

\[ e^{-\lambda} = \left( 1 - \frac{2m}{r} \right) ab. \]

(3.13)

Equations (3.10) and (3.11) are analogous and in the limit \( \kappa \to 0 \) reducible to the well-known Tolman-Oppenheimer-Volkov (TOV) equations in GR [20, 21]. With a given EOS \( P = P(\epsilon) \) and suitable boundary conditions (see below), these two equations completely determine the hydrostatic equilibrium configuration of a compact star in EiBI gravity. It follows directly from Eqs. (3.11) and (3.13) that \( m(r = 0) = 0, \) and \( m(r) \geq 0 \) increases monotonically with \( r. \) Besides, it can be shown that \( \Theta > 0 \) for positive \( \Theta \) and \( \Theta \approx 8\pi P\left[1 + \pi\kappa(\epsilon - 3P)^2/P + 8\epsilon \right] + O(\kappa^2) \geq 2m/r^2. \) As the typical value of \( 8\pi|\kappa|\epsilon \) at the center of a compact star is less than 0.4 [11], for specificity we assume in the present paper \( \Theta > 0. \) In fact, we have numerically verified that \( \Theta > 0 \) except for \( P < \epsilon. \)

The boundary conditions supplementing Eqs. (3.10) and (3.11) are as follows. First of all, the radius of the star \( R \) is as usual defined by the condition \( P(R) = 0. \) Outside the star where \( r > R, \) EiBI gravity is equivalent to GR, and \( g_{\mu\nu} \) is identical to the Schwarzschild metric (see, e.g., Refs. 22, 23). As a consequence, the appropriate boundary conditions at the stellar surface \( r = R \) are \( \epsilon = P = 0, a = b = 1, e^{-\alpha} = e^{-\lambda} = e^\beta = e^\phi = 1 - 2M/R, \) where \( M \equiv m(R) \) is the mass of the star.

The equilibrium configuration of compact stars has been studied in Refs. 6, 7, 11. However, the compact stars considered in these references are all characterized by the smooth EOSs \( P = P(\epsilon) \) where \( c_s^2 = dP/d\epsilon \) can be unambiguously defined, and is continuous and nonvanishing except at the stellar surface. Unlike the TOV equations in GR, the hydrostatic equilibrium equations (3.10) and (3.11) demonstrate an explicit dependence on \( c_s^2. \) Therefore, it is reasonable to expect that the behavior of \( c_s^2 \) could strongly affect the equilibrium configuration of a compact star. In the following we consider two different cases, namely, (i) \( c_s^2 = 0 \) (as in first-order phase transitions) and (ii) \( c_s^2 \) is discontinuous (as in both first- and second-order phase transitions).
IV. STARS WITH FIRST-ORDER PHASE TRANSITIONS

For an EOS with a first-order phase transition (see, e.g., Refs. [16, 24, 25]), there is an interval in which the energy density increases while the pressure remains the same, and consequently $c_s^2 = 0$. To examine the consequence and other related problems of such first-order phase transitions for compact stars in EiBI gravity, we first consider cases with $c_s^2 \approx 0$ and expand Eq. (3.10) up to order $c_s^2$,

$$
\frac{dP}{dr} \approx -\frac{2c_s^2 a^2 \Theta}{\kappa} \left[ 1 - \frac{2m}{r} \right]^{-1}.
$$

(4.1)

It should be noted that $d\epsilon/dr = c_s^{-2} dP/dr$ is still nonvanishing as $c_s^2 \to 0$. We study the implications for the cases $\kappa > 0$ and $\kappa < 0$ separately.

First of all, for $\kappa > 0$, we can still get a normal energy density profile in the region with $c_s^2 \approx 0$, where $d\epsilon/dr$ is approximately equal to a negative constant and $dP/dr \approx 0$. As shown in Fig. 1 where a first-order phase transition really takes place at a certain pressure (the EOS is adapted from Ref. [25]), there is a finite region with constant $d\epsilon/dr$. Therefore the energy density is now a continuous function. Physically, this corresponds to a "mixed phase" region (e.g., quark-nucleon mixed phase [16, 24]) with constant pressure. The thickness of this region is proportional to $\kappa$, and hence vanishes in GR, resulting in a discontinuity in the density profile (see Fig. 1). The existence of a constant pressure shell in compact stars built upon EiBI gravity is attributable to the effective repulsive gravity inherent in EiBI gravity with positive $\kappa$. Actually, the effective repulsion also leads to the formation of pressureless stars, as suggested in Refs. [6, 7].

On the other hand, if $\kappa < 0$ while $c_s^2 \approx 0$, both $dP/dr$ and $d\epsilon/dr$ will become positive, and in the limit of $c_s^2 \to 0$, $dP/dr \to 0^+$ and $d\epsilon/dr$ tends to a nonvanishing positive number. Such behavior is in stark contrast to the case of GR where both $dP/dr$ and $d\epsilon/dr$ are negative definite. If one starts with a certain energy density at the center of a star and integrates Eqs. (3.10) and (3.11) outward, one will find that the energy density drops initially and then rises again in the density region where a first-order phase transition occurs and $c_s^2 = 0$. As a result, the energy density cannot drop to zero and the surface of the star is undefined, implying that equilibrium compact stars characterized by EOSs with first-order phase transitions do not exist if $\kappa < 0$.

As a matter of fact, for stars with a sufficiently soft phase where $c_s^2$ is small, we note a similar problem for EiBI gravity with $\kappa < 0$. We focus on the last term of the modified TOV equation (3.10),

$$
\frac{dP}{dr} \propto -\left[ \frac{2}{\epsilon + P} + \frac{\kappa}{2} \left( \frac{3}{b^2} + \frac{1}{a^2 c_s^2} \right) \right]^{-1}.
$$

(4.2)

Given an equation of state, this term depends solely on the energy density. When $\kappa = 0$, i.e., in the case of GR, it is always negative. However, when $\kappa < 0$ and $c_s^2$ is small, this term may blow up at certain values of energy density.

**FIG. 1:** The density profile of a compact star with an EOS with a first-order phase transition as shown in the inset ($P$ and $\epsilon$ are in cgs units) is studied in both GR (i.e. $\kappa = 0$) and EiBI gravity ($8\pi \kappa \epsilon_0 = 0.1, 0.05$, with $\epsilon_0 = 1 \times 10^{15} \text{g cm}^{-3}$). The central density is also fixed at $\epsilon_0$.
regions, and consequently the modified TOV equation cannot be solved. This again implies that for $\kappa < 0$, compact stars with soft EOSs may not be able to support an equilibrium state.

The predicament encountered in solving Eq. (3.10) in the presence of first-order phase transitions can also be understood in terms of the concept of the apparent EOS proposed by Delsate and Steinhoff [26], who have shown that EiBI gravity can be recast as ordinary GR provided that the physical EOS is replaced with an apparent EOS $P_q(\epsilon_q)$, where for perfect fluids the apparent pressure $P_q$ and density $\epsilon_q$ are given respectively by $P_q = \pi \kappa P + \tau$, $\epsilon_q = \tau \epsilon - P$, with $\tau = [(1 + 8\pi \kappa \epsilon)(1 - 8\pi \kappa P)^3]^{-1/2}$ and $P = [\tau - 1 - \kappa \tau (3P - \epsilon)/2]/\kappa$. In first-order phase transitions, where $dP/d\epsilon = 0$,

$$\frac{dP_q}{d\epsilon_q} = \frac{8\pi \kappa (P + \epsilon)}{a^2 + 3b^2} \quad (4.3)$$

Therefore, for negative $\kappa$, $dP_q/d\epsilon_q < 0$. In this case, despite the fact that the TOV equations in GR guarantee that $P_q$ decreases monotonically towards the stellar surface, the apparent density $\epsilon_q$ increases instead in the region where a first-order phase transition takes place. Hence, by the same token, it is also impossible to construct the equilibrium configuration of a compact star governed by the TOV equations and the apparent EOS.

V. SINGULAR BEHAVIOR OF RICCI SCALAR

In the above discussion, we see that the equilibrium configuration of compact stars can generally be constructed in EiBI gravity with positive $\kappa$ even in the presence of phase transitions. In particular, the energy density $\epsilon$ is always a continuous function of the radial coordinate $r$. However, as noted in Ref. [14] and will be shown in the present paper, the associated Ricci scalar still acquires certain singular behaviors (e.g., discontinuities and divergence) due to higher derivatives of $\epsilon$.

In EiBI gravity, the Ricci scalar can be constructed using the metrics $g_{\mu\nu}$ and $g_{\mu\nu}^*$, which are respectively given by

$$R_q = -\frac{1}{2\epsilon_\alpha} \left[ - \beta' \alpha' r^2 + 2 \beta'' r^2 + 2 \beta' \lambda - 4r \beta' - 4e^\alpha + 4 \right], \quad (5.1)$$

$$R_g = -\frac{1}{2e^\lambda} \left[ - \phi' \lambda' + 2 \phi'' + 2(\phi' - \lambda') f' - (\frac{f'}{f})^2 + 4 \frac{f''}{f} - 4 \frac{e^\lambda}{f} \right]. \quad (5.2)$$

From the expressions of $R_q$ and $R_g$, Eqs. (5.1) and (5.2), one can show that $R_q$ depends on $c_s^2$ while $R_g$ depends on both $c_s^2$ and $(c_s^2)'$. In the presence of phase transitions of the first or second order, $c_s^2$ is in general discontinuous at the transition point (see, e.g., the inset in Fig. 1). Hence, it is likely that these Ricci scalars could become discontinuous or even blow up at radius $r_t$ where the phase transition occurs. However, it is readily shown that $\alpha, \alpha', \beta, \beta'$ and $\beta''$ are indeed continuous functions of $r$ even in the presence of phase transitions. Hence, $R_q$ is still well-behaved even in the vicinity of $r = r_t$, which agrees with our intuition as mentioned in Sec. II. Besides, as shown in Ref. [26] and discussed in the last section, $R_q$ is indeed the standard Ricci scalar in GR for a star constructed with the apparent EOS. Therefore, its regularity is also expected.

The situation is completely different for $R_g$. It contains a $\delta$-function singularity given by

$$R_g = \frac{\beta'}{e^\alpha} [A_1 (A_2 + 2A_3)] \delta(r - r_t) + \ldots \quad (5.3)$$

where the notation $[F]_\ldots = F(r = r_t^+) - F(r = r_t^-)$ for a physical quantity $F$ is introduced,

$$A_1 = \left\{ \frac{4}{a^2 - b^2} + \frac{3}{b^2} \right\} \left\{ c_s^2 + \frac{1}{a^2} \right\}^{-1}, \quad (5.4)$$

$$A_2 = \frac{3c_1^2}{b^2} + \frac{1}{a^2}, \quad (5.5)$$

$$A_3 = \frac{c_s^2}{b^2} - \frac{1}{a^2}, \quad (5.6)$$

and other finite terms have been suppressed. In addition to the $\delta$-function singularity, $R_g$ is also discontinuous there, with a jump given by
\[ [R_g]^{-} = -\frac{1}{2c^4} \left\{ \left( -\frac{4}{r} \beta' - 2\beta'' + \alpha' \beta' \right)[A_1(A_2 + 2A_3)]_{} - \beta^2[A_1(2A_2 + A_3)]_{} + \beta^2[A_1^2(A_2^2 + A_2A_3 + A_3^2)]_{} - 2\beta' \left[ \frac{d}{dr}A_1(A_2 + 2A_3) \right]_{} \right\}. \]  

(5.7)

For reference purpose, the detailed forms of \( A_i \) (\( i = 1, 2, 3 \)) are given below:

\[
A_1' = -A_1^2 \left\{ \frac{3a^2 + b^2}{b^2(a^2 - b^2)}(c_s^2)' 
+ \beta' A_1 \left( \frac{8(1 + c_s^2)c_s^2}{(a^2 - b^2)^2} - \frac{c_s^4}{b^4} - \frac{2}{a^4} \right) \right\},
\]

(5.8)

\[
A_2' = \frac{3}{b^6}(c_s^2)' - 2A_1\beta' \left( \frac{3c_s^4}{b^4} - \frac{1}{a^4} \right),
\]

(5.9)

\[
A_3' = \frac{1}{b^6}(c_s^2)' - 2A_1\beta' \left( \frac{c_s^2}{b^4} + \frac{1}{a^4} \right).
\]

(5.10)

In general, whereas the \( \delta \)-function singularity in \( R_g \) is directly proportional to the discontinuity in \( c_s^2 \), the jump \( [R_g]^{-}\) depends on both \( |c_s^2|^{-} \) and \( |(c_s^2)'|^{-} \). Thus, for compact stars where first- or second-order phase transitions take place, the Ricci scalar \( R_g \) is singular in the sense that it contains a \( \delta \)-function-type divergence and spatial discontinuity. As is well known, the Ricci scalar in GR is proportional \(-c + 3P\) and is always bounded. The \( \delta \)-function-type divergence in \( R_g \) discovered here is a distinctive feature of EiBI gravity.

We have numerically verified the said behavior of \( R_g \) and \( R_s \). For EOSs with phase transitions, either of first- or second-order (the EOSs are adapted from Refs. \([15, 25]\), respectively), \( R_g \) is found to be a continuous function, while \( R_s \) (see Fig. 2) is discontinuous and indeed blows up at the place where there is a jump in \( c_s^2 \). The discontinuity in \( R_g \) obtained numerically in fact agrees with the expression given by Eq. (5.7).

VI. DISCUSSION

In this paper, extending our previous work \([11]\), we have studied compact stars in EiBI gravity using realistic EOS models with phase transitions and discovered several anomalies in the behavior of such stars. For EOSs with first-order phase transitions, compact stars in EiBI gravity with \( \kappa > 0 \) exhibit an anomalous "mixed phase" region where \( c_s^2 = 0 \), the pressure is a constant and the energy density \( \epsilon \) is still a continuous function of the radius \( r \). This is in contrast to the situation in GR where \( \epsilon \) is discontinuous at the transition point. It is the effective pressure inherent in EiBI gravity with positive \( \kappa \) that leads to the existence of the constant pressure region. On the other hand, for \( \kappa < 0 \), the equilibrium configuration for compact stars with first-order phase transitions (or soft enough EOSs) cannot be constructed because \( \epsilon \) never vanishes and hence it is not possible to fulfill the boundary conditions \( (\epsilon = P = 0) \) at the stellar surface in order to match the interior solution smoothly to the Schwarzschild spacetime.

As a side remark, in general for \( \kappa \neq 0 \), we find that one cannot construct the simplest quark star model, described by the MIT bag model EOS, since \( \epsilon \) is finite when \( P = 0 \) for this EOS. Note that this does not pose a problem in GR because one only requires \( P = 0 \) at the surface in GR.

In GR, one can always construct a compact star model by solving the TOV equation with a prescribed EOS. The EOS model is required to satisfy only a few general conditions which are already imposed by the microscopic theory (e.g., the stability condition \( dP/d\epsilon \geq 0 \) and the causality limit \( c_s < c \)). On the contrary, EiBI gravity appears to put a more severe restriction on what kind of EOS one can use to build a stellar model. It is true that whether compact stars in nature exhibit a phase transition in their interiors is still a matter of debate. Nevertheless, it seems unreasonable that the underlying gravitational theory would put a constraint on which microscopic EOS model one can use to construct a theoretical stellar model in the first place. Even in the situation where a compact stars with a first-order phase transition can be constructed in EiBI theory (i.e., when \( \kappa > 0 \)), we still need to face the \( \delta \)-function singularity of the Ricci scalar \( R_g \) as we have shown in Sec. V.

Our discovery reported here complements the recent work of Pani and Sotiriou \([14]\) in which they demonstrated the singularity of \( R_g \) at the surface of a polytropic sphere for any \( \Gamma > 3/2 \). In fact, the singular behaviors of \( R_g \) discussed in the present paper and Ref. \([14]\) stem from a common physical origin, namely the emergence of the first- and second-order derivatives of the pressure in the Ricci scalar \( R_g \). These derivatives in the matter field arise as a
FIG. 2: The Ricci scalar $R_g$ (solid line, left-scale) and energy density $\epsilon$ (dashed line, right-scale) of a compact star where a first- (upper panel, central density = $1.2\epsilon_0$) or second-order (lower panel, central density = $3\epsilon_0$) phase transition occurs are plotted against the radius $r$. $8\pi\kappa\epsilon_0$ is equal to 0.1 for both cases. For $R_g$, $\delta$-function singularities (as indicated by the vertical lines in the figures) emerge at the places where $c_s^2$ is discontinuous.

In conclusion, EiBI gravity is appealing because it can avoid some of the singularities that plague GR by introducing nonlinear coupling between matter and gravity. However, it is also the same nonlinear coupling that leads to the anomalies of compact stars with phase transitions, as we have found in this paper. This renders the viability of EiBI gravity questionable.

Acknowledgments

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