Hall current and radiation effect with mass diffusion on transient rotating MHD flow

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Abstract
Radiation effect and hall current on electrically conducting rotating fluid with variable mass diffusion on an infinite vertical plate which is an isothermal has been analyzed. Using Laplace transform to find exact solution of non linear partial differential equation such as momentum, energy, and mass with boundary conditions. The result of velocity, temperature and concentration represent by graphically with different parameter like Hall parameter (m), thermal radiation (R), rotation parameter (Ω), Hartmann number(M). Combined analysis of Hall current, thermal radiation and rotating fluid plays very important role in science and technology like space research, fluid flow sensor, current sensor, geo physics etc.

Keywords
Thermal radiation, variable mass diffusion, rotating fluid.

AMS Subject Classification
76A99, 76B99.

1. Introduction
Considering MHD flow along with HMT (heat and mass transfer) on rotating fluid with radiation(R) on an IVP - isothermal vertical (Channel) plate is having variable mass diffusion has attracted the researchers to make more attention of its industrial, scientific, and engineering application viz. Involving high temperature, solar power technology, space technology, space vehicles etc. magneto hydrodynamic flow with Thermal radiation and Hall Effect is very important in science and technology like Hall Effect sensors can be used in pressure sensors, fluid flow sensors, rotating speed sensors, and electric sensors. Hall Effect principal used in brushless DC motors, nuclear power reactors, control of hypersonic flows, efficient Hall thrusters in magnetic propulsion, measure magnetic field, development of plasma actuator, control hydraulic valves by Hall effect joysticks, Hall current accelerators etc.

Analytic solution for transient free convection flow MHD-magneto hydrodynamic with steady heat flux was studied by Sacheti et al.\textsuperscript{[1]}. Magneto hydrodynamic (MHD) flow on semi infinite horizontal plate with Hall currents was given by Katagiri\textsuperscript{[2]}. An accelerated plate effect of Hall current on MHD flow was researched by Dekha\textsuperscript{[3]}. Hall effects on hydro magnetic free convection flow along a porous flat plate with mass transfer were given by Hossian et al.\textsuperscript{[4]}. Free convection MHD flow in presence of Hall effects on semi infinite vertical plate was given by Pop et al.\textsuperscript{[5]}. MHD flow with continuous moving plate by effect of Hall current by Pop et al.\textsuperscript{[6]}. Dileep singh et al.\textsuperscript{[7]} described MHD slip flow together heat transfer an accelerated plate or channel on porous medium in a rotating system. HMT flow over an inclined plate with free convection and Hall current was given by Mohammad Shah Alam et al.\textsuperscript{[8]}. In a rotating system with horizontal accelerated plate and MHD transient flow with Hall current in porous medium given by Nazibuddin Ahmed et al.\textsuperscript{[9]}. Gupta S. Studied Hall current in porous medium with hydro magnetic flow.\textsuperscript{[10]} Muthucumarswamy and Jeyanthi studied Hall current effect on MHD flow in the rotating sys-
tem on an infinite vertical plate about variable temperature in
presence of mass diffusion chemical reaction [11]. Hall effect
on hydro magnetic flow with free convection in the rotating
system analysed by Ram [12] Radiation effect and magnetic
effect on parabolic isothermal vertical plate with Hall
current studied by Laksmi Velu and Muthucumaraswamy
[13]. Muthucumaraswamy et al [14] solved for a Hall E-
effect problem with velocity profiles. Thamizhsudar et al. [15]
extended the problem, radiation effects on an exponentially
accelerated vertical plate with hall effect and absence of che-
miical reaction. The solution of Laplace inverse transforms of
exponential was given by Hetnarski [16].

2. Formation of Mathematical Model
Consider \(x', y'\) and \(z'\) are coordinate Cartesian system in
which infinite vertical plate fixed along axis \(-x'\) and axis \(-y'\)
is perpendicular to \(x'\) and \(z'\) axis. The fluid and plate have
same temperature in stationary condition at initial level. Here
incompressible transient viscous fluid flow with variable mass
diffusion on infinite vertical plate has been considered. A
constant magnetic field \(B_0\) is acting parallel to axis \(z'\) and have
influence on fluid flow. Simultaneously both the plate and
fluid are rotating about the \(z'\) axis in anticlockwise direction
by constant angular velocity \(\Omega'\). At initially the temperature
\(T_0'\) and fluid concentration \(C_0'\) of fluid maintained as constant.
With velocity \(u_0\), the plate is accelerated, the body temperature
\((T_0')\) and species concentration \((C_0')\) levels close to the vertical
plate are raised by liner with time \(t' > 0\) and maintained
constant.

An analysis of electrically conducting fluid with small
magnetic Reynolds number like plasma and ionized gas. Hence,
the applied magnetic field becomes \((0, 0, B_0)\). Excluding
pressure all physical parameter dependent on \(z'\) and \(t'\). Considering
the above assumption that the transient effect on MHD flow
of rotating fluid on vertical isothermal plate with radiation
and Hall Current is presented by governing equation by usual
Boussinesq Approximation as follows

\[
\frac{\partial u}{\partial t} = v \frac{\partial^2 u}{\partial z^2} + 2\Omega' v + s\beta (T' - T_0') \left( \frac{u + mv}{1 + m^2} \right) \frac{\sigma B_0^2 \mu_z^2}{\rho} \frac{z}{\rho} \tag{2.1}
\]

\[
\frac{\partial v}{\partial t'} = v \frac{\partial^2 v}{\partial z^2} - 2\Omega' u \left( \frac{v - mu}{1 + m^2} \right) \frac{\sigma B_0^2 \mu_z^2}{\rho} \tag{2.2}
\]

\[
\frac{\partial T'}{\partial t'} = A \frac{\partial^2 T'}{\partial z^2} - \frac{1}{\rho C_p'} \frac{\partial q_r}{\partial z'}, \text{ where } A = \frac{k}{\rho C_p'} \tag{2.3}
\]

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z^2} - \chi (C' - C_0') \tag{2.4}
\]

Fluid velocity \(u\) and \(v\) taken along \(x'\)-axis and \(y'\)-axis respectively, \(\beta'\)-thermal expansion coefficient, \(\omega_c\)-cyclotron
frequency, \(\tau_c\)-electron collision time \(\beta'\)-coefficient of vol-
umetric expansion, Specific heat at constant pressure \((C_p)\)
fluid thermal conductivity \((k)\) and fluid density \((\rho)\), radiative
heat flux \((q_r)\), conductivity \((\sigma)\) Hall parameter \((m = \omega_c \tau_c)\).

Initial condition and boundary conditions are taken as

\[
\begin{align*}
 u &= 0, v = 0, T' = T_0', C' = C_0' \text{ For } \ z' > 0 \text{ & } t' \leq 0 \\
 u &= u_0, v = 0, T' = T_0', C' = C_0' \text{ At } \ z' = 0 \text{ & } t' > 0 \\
 u &\rightarrow 0, T' \rightarrow T_0, v \rightarrow 0, C' \rightarrow C_0' \text{ As } \ z' \rightarrow \infty \text{ & } t' > 0
\end{align*}
\tag{2.5}
\]

The normal radiant expression in thin gray optical is

\[
\frac{\partial q_r}{\partial z} = -4 (T_0'^4 - T''^4) a^* \sigma^* \tag{2.6}
\]

Where \(\sigma^*\) denote Stefan-Boltzmann constant and \(a^*\) represent
coefficient of heat absorption. By using Taylor’s expression
about \(T_0'\) and neglecting higher term. Then \(T''^4\) is express in
linear function of \(T'\).

\[
T''^4 + 3T_0'^4 = 4T_0'^4 \tag{2.7}
\]

From (2.6) and (2.7) in (2.3) then

\[
\rho C_p' \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} + 16a^* \sigma^* T_0'^4 (T_0' - T_0') \tag{2.8}
\]

The following non dimensional variables and parameter are
used to reduce equation (2.1), (2.2), (2.4) and (2.8) in dimen-
sionless form.

\[
\begin{align*}
 U &= \frac{u}{u_0}, V = \frac{v}{v_0}, T = \frac{T'}{T_0'}, \Omega = \frac{\Omega'}{v_0}, G_r = \frac{Gr}{G_{r0}}, C_v = \frac{C_v}{C_{v0}}, R = \frac{16a^* \sigma^* T_0'^4}{k}, \\
 M^2 &= \frac{\sigma B_0^2 \mu_z^2}{\rho C_p'}, \theta = \frac{T - T_0}{T_0}, \ K = \frac{K_l}{v_0}, C = \frac{C_{oc} - C_0}{C_v - C_0}
\end{align*}
\tag{2.9}
\]

Where \(u\) and \(v\) represent primary and secondary fluid
velocity, \(M^2, G_r, G_c, S_c, K, R, \Omega\) are denoted as magnetic
parameter, \(t\) Grashof number, thermal radiation, Mass Grashof
number, chemical reaction parameter, Schmidt number , rotation
parameter and Prandtl number respectively.

Equation (2.1),(2.2),(2.4) and (2.8), reduce to no di-
mensonal form by using (2.9) as follows

\[
\begin{align*}
\frac{\partial U}{\partial t} &= 2\Omega V + G_r \theta + \frac{\partial^2 U}{\partial Z^2} + G_c C - M^2 \left( \frac{U + mV}{1 + m^2} \right) \tag{2.10}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial V}{\partial t} + 2\Omega U &= \frac{\partial^2 V}{\partial Z^2} + M^2 \left( \frac{mU - V}{1 + m^2} \right) \tag{2.11}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \theta}{\partial t} + R \theta &= \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial Z^2} \right) \tag{2.12}
\end{align*}
\]
\[ \frac{\partial C}{\partial t} + KC = \frac{1}{Sc} \left( \frac{\partial^2 C}{\partial Z^2} \right) \]  
\[ \text{(2.13)} \]

In non-dimensional initial and boundary conditions are

\[ U = 0, V = 0, \theta = 0, C = 0 \text{ for } Z > 0 \text{ and } t \leq 0 \]
\[ U = 1, V = 0, \theta = 1, C = t \text{ at } Z = 0 \text{ and } t > 0 \]
\[ U \rightarrow 0, V \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } Z \rightarrow \infty \text{ and } t > 0 \]
\[ \text{(2.14)} \]

Combining the equations (2.10) and (2.11) then

\[ \frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial Z^2} - \frac{a}{\sqrt{Z}} + Gr\theta + GeC \]  
\[ \text{(2.15)} \]

Where

\[ a = \frac{2M^2}{1 + m^2} + 2i \left( \Omega - \frac{M^2 m}{1 + m^2} \right) \text{ and } F = U + iV \]

\[ \frac{\partial \theta}{\partial t} + R\theta = \frac{1}{P_r} \left( \frac{\partial^2 \theta}{\partial Z^2} \right) \]

\[ \frac{\partial C}{\partial t} + KC = \frac{1}{Sc} \left( \frac{\partial^2 C}{\partial Z^2} \right) \]

The rotation parameter \( \Omega \) is \( \frac{mM^2}{1 + m^2} \), as a result of this the transverse velocity vanishes.

Conditions (2.14) transform in to

\[ \theta = 0, C = 0, F = 0 \text{ for } Z > 0 \text{ and } t \leq 0 \]
\[ \theta = 1, F = 1, C = t \text{ at } Z = 0 \text{ and } t > 0 \]
\[ \theta \rightarrow 0, C \rightarrow 0, F \rightarrow 0 \text{ as } Z \rightarrow \infty \text{ and } t > 0 \]
\[ \text{(2.16)} \]

### 3. Discussion of Exact Solution

Laplace inverse algorithm is used to find a solution of the equations (2.12),(2.13) and (2.15) along with initial and boundary condition (2.16).

\[ \theta(Z,t) = \frac{1}{2} (\exp(-c \sqrt{PTt}) \text{erfc}(0.5c \sqrt{PT} - \sqrt{Rt}) + \exp(c \sqrt{PTt}) \text{erfc}(0.5c \sqrt{PT} + \sqrt{Rt})) \]  
\[ \text{(3.1)} \]

\[ C(Z,t) = \frac{1}{2} \left( \exp(-c \sqrt{KS}t) \text{erfc}(0.5c \sqrt{S} - \sqrt{Kt}) + \exp(c \sqrt{KS}t) \text{erfc}(0.5c \sqrt{S} + \sqrt{Kt}) \right) - \frac{\eta}{2} \sqrt{\frac{Sc}{K}} \left[ \exp(-c \sqrt{KS}t) \text{erfc}(0.5c \sqrt{S} - \sqrt{Kt}) - \exp(c \sqrt{KS}t) \text{erfc}(0.5c \sqrt{S} + \sqrt{Kt}) \right] \]  
\[ \text{(3.2)} \]

\[ F(Z,t) = (0.5) \left[ \exp(-c \sqrt{at}) \text{erfc}(\eta - \sqrt{at}) + \exp(0.5c \sqrt{at}) \text{erfc}(\eta + \sqrt{at}) \right] \]

### 4. Results and Discussion

Considering the physical view of the problem, the numerical solution of the axial velocity (\( F \)) has been analysed from (3.1). Effect of different parameters like a Hartmann number (\( M \)), rotation parameter (\( \Omega \)), thermal radiation, thermal and mass...
Grashof number\((Gr, Gc)\), Schmidt number\((Sc)\) and time on fluid temperature, concentration and axial velocity analysed by graphical method. Fig.1 and Fig.2 depict the concentration profiles for different value of Schmidt number \((Sc)\) and reaction parameter \((k)\). When \(Sc = 0.16, 0.6, \text{ and } 2.01\) represent Schmidt number of Hydrogen, Water vapour and Ethyl Benzene respectively. From Fig.1 and Fig. 2 it is noticed that the concentration profile is decreasing when the increasing value of \(Sc\) and \(K\). Diffusivity relation between momentum and mass given by Schmidt number.

The temperature profiles for different values of Prandtl number \((Pr)\) and radiation parameter \((R)\) are depicted by Fig. 3 and Fig.4. From the above it is inferred that the temperature profiles rises as Prandtl number and radiation are decreases and also heat transfer is more in air\((Pr = 0.71)\) than water\((Pr = 7)\). The effect of thermal radiation is important in temperature profiles.

Fig. 5 to Fig. 8 shows that the behaviour of axial velocity under the effect of Hall parameter \((m)\), thermal and mass Grashof number \((Gr&Gc)\), Hartmann number \((M)\) and Rotation parameter \((\Omega)\). Fig. 5 represent the effect of axial velocity profiles for various value of \(\Omega\) when \(Sc = 2.01, Gr = 5, Gc = 10, m = 0.5, R = 2\) and \(t = 0.2\). It is noticed that the axial velocity falls by increases of rotation parameter. Fig. 6 shows that the axial velocity decreases with decreases of different value of \(Gc\) and \(Gr\). Fig. 7 and 8 depict the profiles of axial velocity for different value of Hartmann number and Hall parameter respectively when \(R = 2, Pr = 7, Gr = 5, Gc = 10 k = 0.2, Sc = 2.01\) and \(t = 0.2\). It is observed that rises as decreases of Hartmann number and increases of Hall parameter.

5. Conclusion

An analysis of concentration, temperature and axial velocity profilers are discussed with the effect of radiation and hall effect on an infinite vertical plate under first order chemical reaction in the presence of rotating fluid with varying mass diffusion.
• Concentration increases for decreasing value of Schmidt number ($S_c$) and Reaction parameter ($K$)
• Temperature decreases for increasing value of Prandtl number and Radiation.
• Axial velocity decreases by increasing of rotational parameter and Hartmann number.

It decreases with decreasing of Hall parameter. The result of the study plays an important role in applications of science and engineering, as well as in many transport processes in nature.

**NOMENCLATURE:**

\( (u, v, w) \) - Components of velocity
\( (U, V, W) \) - Non-dimensional velocity
\( (x', y', z') \) - Cartesian co-ordinates

- \( Gc \): Mass Grashof’s number
- \( Pr \): Thermal Prandtl number
- \( \tau \): Time
- \( \nu \): Kinematic viscosity
- \( Sc \): Schmidt number
- \( \rho \): Fluid density
- \( \sigma \): Electrical conductivity
- \( Gr \): Thermal Grashof’s number
- \( C' \): Fluid concentration
- \( c_w \): Wall concentration
- \( K \): Chemical reaction parameter
- \( J_c \): Component of current density
- \( \Omega \): Component of angular viscosity
- \( t \): Non-dimensional time
- \( \Omega \): Non-dimensional angular velocity
- \( B_0 \): Imposed magnetic field
- \( m \): Hall parameter
- \( M \): Hartman number
- \( \mu \): Coefficient of viscosity
- \( T \): Temperature of the fluid near the plate
- \( T_a \): Temperature of the plate
- \( \theta \): Dimensionless temperature
- \( T_{in} \): Temperature of the fluid far away from the plate
- \( C \): Dimensionless concentration
- \( k \): Thermal conductivity
- \( \beta \): Volumetric coefficient of thermal expansion
- \( \beta^* \): Volumetric coefficient of expansion with Concentration
- \( C_{in} \): Concentration for away from the plate
- \( K \): Chemical reaction parameter

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