Change in the Orbital Period of a Binary System Due to an Outburst in a Windy Accretion Disk

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Received October 26, 2020; revised March 5, 2021; accepted March 30, 2021

Abstract—We consider a new mechanism for the removal of angular momentum from an X-ray binary system and the change in its orbital period—the mass loss in the form of a wind from an accretion disk. A powerful wind from a disk is observed in X-ray transients and is predicted by models. We have obtained an analytical estimate for the increase in the orbital period of a binary system with a wind from the disk during an outburst; quantitative estimates are given for the systems XTE J1118+480, A0620−00, and GRS 1124−68. The rates of increase in the period are comparable in absolute value to the observed rates of secular decrease in the period. We also compare the predicted rates of change in the period of a binary system due to the mass transfer into the disk and the outflow from the second Lagrange point with the observed ones. We conclude that the above-mentioned mechanisms cannot explain the observed secular decrease in the period, and it is necessary to consider a circumbinary disk that removes the binary’s angular momentum.

DOI: 10.1134/S1063773721050017

Keywords: X-ray binaries, wind, transients, period, accretion.

INTRODUCTION

The data on the changes in the orbital period may contain important information about the parameters of a close binary system and the processes inside it: the mass transfer between the components, the evolution of the companion star, etc. In close binaries the change in the orbital period is related mainly to a decrease in the angular momentum (Cherepashchuk 2013). This decrease in the angular momentum of a close binary system can be caused by various mechanisms, among which there are three main ones: mass loss by the system (Rappaport et al. 1982), magnetic braking (Verbunt and Zwaan 1981), and gravitational waves (Landau and Lifshitz 1975).

Low-mass X-ray binaries (LMXBs) are one of the types of close binary systems in which a change in the orbital period is observed. In these systems mass is transferred from an optical star to a compact component, which is accompanied by the formation of a hot accretion disk that is an X-ray emission source. Gonzalez Hernandez et al. (2012) (hereafter GH12) were able for the first time to detect a change in the orbital period of a LMXB when analyzing the observations of the system XTE J1118+480. The orbital period turned out to decrease in the system. In later papers (Gonzalez Hernandez et al. 2014, 2017; hereafter GH14 and GH17) a decrease in the period was also found for A0620−00 and GRS 1124−68. The authors believe that the main source of the rapid decrease in the period in these LMXBs is magnetic braking. However, the model values obtained by them, even under the assumption of a sufficiently strong magnetic field, are lower than the observed ones by one or two orders of magnitude.

In this paper we consider another mechanism for the removal of angular momentum from a binary system—the mass loss by the system. It is important to note that LMXBs exhibit recurrent outbursts caused by instabilities in the disk or unsteady mass transfer between the components. During an outburst the accretion rate onto the compact component rises by several orders of magnitude, with almost all of the LMXB radiation falling into the X-ray range (see, e.g., Chen et al. 1997). Presumably, during outbursts...
there is a mass outflow in the form of a wind from the accretion disk around the compact object. The presence of such a wind in LMXBs is supported by present-day observations pointing to an expansion of ionized matter. An outflow is observed both in the X-ray range from narrow blueshifted absorption lines (Díaz Trigo and Boirin 2016) and in the optical range from line broadening and shift (Muñoz-Darias et al. 2019; Casares et al. 2019). In most cases, X-ray absorption lines are observed in systems with an inclination more than 50°. Consequently, the absorbing plasma has a higher density closer to the disk, suggesting that the observed matter outflows from the disk (Higginbottom and Proga 2015). The observed mass loss rates in the wind were obtained for the intermediate mass X-ray binary Her X-1 in Kosec et al. (2020), where the estimates depend strongly on the wind geometry: if the wind spreads along the disk, then the mass loss rate in the wind is approximately equal to the central accretion rate onto the compact object; if, however, the wind is spherically symmetric, then the mass loss rate increases by an order of magnitude. Observational estimates of the rate of mass outflow for several X-ray transients are presented in another paper (Ponti et al. 2012). It follows from these estimates that the ratios of the mass loss rate in the wind to the central accretion rate onto the compact object lie in the range from 1 to 10. In simulations (Higginbottom et al. 2017, 2019; Luketic et al. 2010) the estimates of this ratio are in the range from 2 to 15. Thus, both simulations and observations argue for a powerful mass flow from the disk.

The question about the change in the orbital period of a binary system as a result of mass loss by a particular component (or both) has already been studied previously. The simplest problem of isotropic gas ejection describing this process without taking into account the effect of the ejected matter on the motion of the binary system was solved by Jeans (1928). He assumed that the ejected matter was removed very quickly without affecting the orbital motion. This may be true in the case of strong nova and supernova outbursts. If the ejection velocity is not high enough, then the ejected matter affects the orbital motion of the binary system not only due to the change in its mass and angular momentum, but also due to the gravitational effect on the system. In addition, the ejected matter can return into the system to any of the stars, causing additional changes in the period. For example, a viscous toroidal disk around a binary system (circumbinary disk) can be formed, which will effectively remove the angular momentum that it gets through a tidal interaction (Chen and Podsiadlowski 2019; hereafter CP19).

In addition to the wind from the accretion disk, a mass outflow from the outer Lagrange point $L_2$ located behind the less massive component (behind the optical star in the case of a LMXB) can occur in the system. In contrast to the wind from the disk, which is most active only during an outburst, the mass flow from $L_2$ can persist and lead to a permanent decrease in the period of the system.

A detailed solution of the problem of mass loss by a binary system is given in Kruszewski (1964). In his formulation it is necessary to know the velocity components and the coordinates of the ejected matter, which we do not know with a sufficient accuracy. We considered a simpler problem in which the effect of the ejected matter on the orbital motion of the system as it goes to infinity is ignored.

MODEL

Wind from the Accretion Disk

Let the matter from the companion star be transferred through the Lagrange point $L_1$ into the Roche lobe of the compact object and become part of the accretion disk there. Under the influence of irradiation, magnetic or radiation pressure, some part of the matter can be carried away from the disk in the form of a wind, taking with it both mass and angular momentum. Thus, the total mass and total angular momentum of the binary system change. We will assume that the entire wind starts from a fixed radius in the disk.

Let us calculate the change in the system’s orbital period. We will use Kepler’s third law in the form

$$P = 2\pi\sqrt{\frac{A^3}{GM}},$$

where $M$ is the total mass in the binary system, and $P$ and $A$ are the period and semimajor axis, respectively. Next, let us find the relative change in the orbital period of the binary $\Delta P/P$ by varying (1) as a function of $A$ and $M$. As a result, we will obtain

$$\frac{\Delta P}{P} = \frac{3}{2} \frac{\Delta A}{A} - \frac{1}{2} \frac{\Delta M}{M},$$

where $P$ and $\Delta P$ are the orbital period of the system and its change, respectively. Our ultimate goal is to directly relate $\Delta M/M$ and $\Delta P/P$. Consequently, it is necessary to express $\Delta A/A$ via $\Delta M/M$.

To calculate this change in the semimajor axis, we use the law of conservation of angular momentum. We assume that before the outburst the disk was a ring of mass $M_{\text{disk}}$, with a characteristic radius $R_{\text{disk}}$. Below, all of the “primed” and “unprimed” variables (when considering the effect of the wind) will denote the system’s state after and before the outburst, respectively.
The total angular momentum of the binary system before the ejection of matter to infinity $J$ is

$$J = J_{\text{opt}} + J_x + J_{\text{disk}},$$

where $J_{\text{opt}}$, $J_x$, and $J_{\text{disk}}$ are the angular momenta of the optical star, the compact object, and the accretion disk around it, respectively. They are defined as follows:

$$J_{\text{opt}} = M_{\text{opt}}\omega_{\text{orb}}R_{\text{opt}}^2,$$

$$J_x = M_x\omega_{\text{orb}}R_x^2,$$

$$J_{\text{disk}} = M_{\text{disk}}(\omega_{\text{orb}}R_x^2 + \sqrt{GM_xR_{\text{disk}}}).$$

It can be seen that the angular momentum of the disk consists of the orbital angular momentum and the angular momentum associated with the Keplerian rotation around the relativistic star. $M_{\text{opt}}$ and $M_x$ are the masses of the optical and compact components, $R_{\text{opt}}$ and $R_x$ are the distances from the centers of mass of the stars to the center of mass of the binary system, and $\omega_{\text{orb}}$ is the angular velocity of orbital rotation (see Fig. 1):

$$R_{\text{opt}} = A(M_x + M_{\text{disk}})/M,$$

$$R_x = AM_{\text{opt}}/M,$$

$$\omega_{\text{orb}} = \sqrt{GM/ \Lambda^3},$$

$$M = M_{\text{opt}} + M_x + M_{\text{disk}}.$$

We assume that after the outburst the accretion disk was completely used up: part of the disk mass, $\Delta M_{\text{acc}}$, fell onto the compact object, causing its mass to increase to $M'_x = M_x + \Delta M_{\text{acc}}$, while the rest of the disk mass, $|\Delta M_{\text{wind}}| = M_{\text{disk}} - \Delta M_{\text{acc}}$ ($\Delta M_{\text{wind}} < 0$), escaped from the system in the form of a wind. The angular momentum of the system after the outburst is then

$$J' = J'_{\text{opt}} + J'_x,$$

$$J'_{\text{opt}} = M_{\text{opt}}\omega'_{\text{orb}}(R'_{\text{opt}})^2,$$

$$J'_x = (M_x + \Delta M_{\text{acc}})\omega'_{\text{orb}}(R'_x)^2 + \Delta M_{\text{acc}}\sqrt{GM_xR_{\text{in}}},$$

where $R_{\text{in}}$ is the inner radius of the disk (the radius of the last stable orbit). We neglect the angular momentum $\Delta M_{\text{acc}}\sqrt{GM_xR_{\text{in}}}$ added to the angular momentum of the compact object as a result of accretion, because the inner radius of the disk $R_{\text{in}}$ is much smaller than other characteristic radii of the problem. After the outburst $R_{\text{opt}}$, $R_x$, $\omega_{\text{orb}}$, and $M$ acquire new values:

$$R'_{\text{opt}} = A'(M_x + \Delta M_{\text{acc}})/M',$$

$$R'_x = A'M_{\text{opt}}/M',$$

$$\omega'_{\text{orb}} = \sqrt{GM'/ (\Lambda')^3},$$

$$M' = M_{\text{opt}} + M_x + \Delta M_{\text{acc}},$$

where $\Lambda'$ is the semimajor axis after the outburst. The change in the system’s angular momentum $J' - J$ is equal to the angular momentum carried away in the wind $\Delta J_{\text{wind}}$ ($\Delta J_{\text{wind}} < 0$). If the wind started from the radius $R_{\text{esc}}$, then the angular momentum of the ring with mass $|\Delta M_{\text{wind}}|$ involved in the orbital motion and rotation around the compact object was carried away in it:
where $k_{\text{esc}} \equiv R_{\text{esc}}/A$. Here, we take into account the possibility that the characteristic radius of the disk changes during the outburst. Thus, the wind can outflow from a radius larger than the characteristic radius of the disk in quiescence ($R_{\text{disk}}$).

By substituting (8) into the equation $J = J' - \Delta J_{\text{wind}}$ ($\Delta J_{\text{wind}} < 0$), we will find the relation of the change in the semimajor axis due to the wind $\Delta A_{\text{burst}} \equiv A' - A$ to the decrease in the total mass of the system and accretion:

$$\frac{1}{2} \frac{\Delta A_{\text{burst}}}{A} = \left[ \frac{(M_{\text{opt}} + M_x)^{3/2}}{M_{\text{opt}} \sqrt{M_x}} \right] \times \left( \sqrt{k_{\text{esc}}} - \sqrt{k_{\text{disk}}} \right) - \frac{1}{2} \frac{\Delta M_{\text{wind}}}{M_{\text{opt}} + M_x} + \frac{(M_{\text{opt}} + M_x)^{3/2}}{M_{\text{opt}} \sqrt{M_x}} \sqrt{k_{\text{disk}}} \frac{\Delta M_{\text{acc}}}{M_{\text{opt}} + M_x},$$

(9)

where $k_{\text{disk}} \equiv R_{\text{disk}}/A$, by analogy with $k_{\text{esc}}$.

Having divided (9) by the outburst duration, we can express the mean rate of change in the semimajor axis due to the wind $\Delta J_{\text{wind}}$ ($\Delta J_{\text{wind}} < 0$); we will find the relation of the change in the semimajor axis due to the wind $\Delta A_{\text{burst}} \equiv A' - A$ to the decrease in the total mass of the system and accretion:

$$\frac{1}{2} \frac{\Delta A_{\text{burst}}}{A} = \left[ \frac{(M_{\text{opt}} + M_x)^{3/2}}{M_{\text{opt}} \sqrt{M_x}} \right] \times \left( \sqrt{k_{\text{esc}}} - \sqrt{k_{\text{disk}}} \right) - \frac{1}{2} \frac{\Delta M_{\text{wind}}}{M_{\text{opt}} + M_x} + \frac{(M_{\text{opt}} + M_x)^{3/2}}{M_{\text{opt}} \sqrt{M_x}} \sqrt{k_{\text{disk}}} \frac{\Delta M_{\text{acc}}}{M_{\text{opt}} + M_x},$$

where $k_{\text{esc}} \equiv R_{\text{esc}}/A$, by analogy with $k_{\text{esc}}$.

It then follows from (12) that $|\Delta M_{\text{wind}}| \times \sqrt{GM_x R_{\text{esc}}} \leq M_{\text{disk}} \sqrt{GM_x R_{\text{in}}}$. Taking this into account and introducing the parameter $C_w \equiv |\Delta M_{\text{wind}}| / \Delta M_{\text{acc}}$, we will get the constraint

$$C_w \leq C_w^* \equiv \frac{\sqrt{k_{\text{disk}}}}{\sqrt{k_{\text{esc}}} - \sqrt{k_{\text{disk}}}} \text{,}$$

(13)

Hence it follows that at $C_w = C_w^*(q)$, i.e., when all of the entire initial Keplerian angular momentum of the disk $J_{\text{disk}}^{\text{Kepl}}$ is carried away by the wind, the change in the orbital period of the binary is described by the following formula (Jeans model):

$$\frac{\dot{P}_{\text{burst}}}{P} = -\frac{2}{1 + q} \frac{\langle M_{\text{wind}} \rangle}{M_{\text{opt}}}. \quad (14)$$

In particular, Eq. (14) is applicable when all of the ring matter goes away from the system without an accretion event. However, this approximation (a wind without accretion) for X-ray nova outbursts is unjustified.

Note that $C_w^*(q)$ imposes a constraint on the parameter $C_w(q)$, i.e., on the wind power, only under the scenario where the wind outflow radius is greater than the initial disk radius, i.e., $k_{\text{esc}} > k_{\text{disk}}$ (and, hence, Eq. (13) makes sense). In other cases, the wind from the disk can be arbitrarily powerful.
Using the introduced parameter $C_w$, we will finally rewrite Eq. (10) as
\[
\frac{\langle P_{\text{burst}} \rangle}{P} = 3 \left[ \sqrt{\frac{1+q}{q}} k_{\text{disk}} (C_w + 1) \right] + 2 \frac{C_w}{31 + q} \left[ \sqrt{\frac{1+q}{q}} k_{\text{esc}} C_w \right] \frac{\langle \dot{M}_{\text{acc}} \rangle}{M_{\text{opt}}}.
\]
(15)

It should be noted that, in view of the derived constraint (13) on the ratio of the mass loss rate in the wind to the accretion rate, the combined effect always leads to an increase in the period.

**Mass Transfer into the Disk**

Let us now turn to another source of the change in the period or, more specifically, to the mass transfer from the optical component of the system into the Roche lobe of the compact object. In this case, it is also necessary to use Eq. (2), but now under the assumption that matter of mass $\Delta M_{\text{tr}}$ ($\Delta M_{\text{tr}} > 0$) was transferred from the star with a point mass $M_{\text{opt}}$ and fell into the accretion disk at the radius $R_{\text{disk}}$, without changing the system’s total mass ($\Delta M = 0$):
\[
\frac{\Delta P_{\text{tr}}}{P} = 3 \frac{\Delta A_{\text{tr}}}{A}.
\]
(16)

Here, $\Delta P_{\text{tr}}$ and $\Delta A_{\text{tr}}$ are the changes in the orbital period and semimajor axis, respectively. Let us also write out the angular momenta before and after the mass transfer from the optical component into the accretion disk:
\[
J = M_x M_{\text{opt}} \sqrt{\frac{G A}{M_x + M_{\text{opt}}}},
\]
(17)

\[
J'' = M_x \omega''_{\text{orb}} (R''_x)^2 + (M_{\text{opt}} - \Delta M_{\text{tr}}) \omega''_{\text{orb}} (R''_{\text{opt}})^2 + \Delta M_{\text{tr}} (\omega''_{\text{orb}} (R''_{\text{opt}})^2 + \sqrt{G M_x R_{\text{disk}}}) ,
\]
where
\[
R''_x = A'' (M_{\text{opt}} - \Delta M_{\text{tr}}) / M,
\]
(19)

\[
R''_{\text{opt}} = A'' (M_x + \Delta M_{\text{tr}}) / M,
\]
\[
\omega''_{\text{orb}} = \sqrt{\frac{G M}{(A'')^3}},
\]
\[
A'' = A + \Delta A_{\text{tr}},
\]
\[
M = M_x + M_{\text{opt}}.
\]

Then, taking into account $J = J''$ (the total orbital angular momentum does not change) and Eq. (16), we will obtain a formula for the change in the period of the binary system due to the mass transfer into the disk around the compact object and will write everything in terms of $\langle \dot{P}_{\text{tr}} \rangle$, $q$, and $\langle \dot{M}_{\text{tr}} \rangle$:
\[
\frac{\langle \dot{P}_{\text{tr}} \rangle}{P} = \frac{3}{(1+q)} \left[ q - \frac{1}{q} - (1+q)^{3/2} \right] \frac{\langle \dot{M}_{\text{tr}} \rangle}{M_{\text{opt}}},
\]
(20)

The derived expression is reduced to the classical formula describing the mass transfer between point-mass components (see, e.g., Cherepashchuk 2013) by setting $k_{\text{disk}}$ equal to zero.

Since in this paper we consider low-mass X-ray binaries, in which the lower-mass star is the donor, the relative rate of change in the period due to the mass transfer into the disk will always be positive ($\langle \dot{P}_{\text{tr}} \rangle / P > 0$) and will lead to an increase in the orbital period. Note that the change in the period caused by the wind (instantaneous value) is greater than the change in the period due to the mass transfer by 2–3 orders of magnitude.

**Outflow from the Lagrange Point $L_2$**

As has been asserted above, mass can outflow not only from the disk, but also from the outer Lagrange point $L_2$, which is located behind the less massive component of the system. We will use the paper by Soberman et al. (1997) to estimate the change in the period:
\[
\frac{\dot{P}_{L_2}}{P_1} = \frac{3 M_{L_2}}{M} \left( \frac{M^2}{M_{\text{opt}} M_x} \sqrt{\frac{R_{L_2}}{A}} \right),
\]
(21)

where $\dot{M}_{L_2}$ ($\dot{M}_{L_2} < 0$) is the rate of mass outflow from the Lagrange point $L_2$ and $R_{L_2}$ is the distance from $L_2$ to the system’s center of mass (Emelianov and Salyamov 1983):
\[
R_{L_2} / A \approx \frac{1}{1+1/q} \left[ \frac{1}{3(1+1/q)} \right]^{1/3} + \frac{1}{3} \left[ \frac{1}{3(1+1/q)} \right]^{2/3} - \frac{1}{9} \left[ \frac{1}{3(1+1/q)} \right] + \frac{50}{81} \left[ \frac{1}{3(1+1/q)} \right]^{4/3}.
\]
(22)

It can be seen that the outflow from the second Lagrange point also leads to a decrease in the period.
RESULTS

Using Eq. (15), we can determine the change in the orbital period due to the wind from the disk and the accretion of matter by specifying the masses of the system’s components \(M_x\) and \(M_{\text{opt}}\), the mean rates of mass outflow and accretion \(\langle \dot{M}_{\text{wind}} \rangle\) and \(\langle \dot{M}_{\text{acc}} \rangle\), the disk size before the outburst \(R_{\text{disk}}\), and the effective disk radius \(R_{\text{esc}}\) from which the outflow occurs. For the characteristic radii of the problem, it is convenient to introduce the parameters \(k_{\text{disk}} \equiv R_{\text{disk}}/A\) and \(k_{\text{esc}} \equiv R_{\text{esc}}/A\).

If the angular momentum of the matter passing through the Lagrange point \(L_1\) does not change during the disk formation (which is accurate enough for typical values of \(q\) in LMXBs), then the matter forms a ring with a circularization radius \(R_{\text{circ}}\) rotating with the Keplerian speed. We assume that the disk in quiescence before the outburst does not spread out, and its characteristic radius \(R_{\text{disk}}\) remains equal to \(R_{\text{circ}}\). Then, according to (Frank et al. 2002, Eq. (4.20)), we have

\[
R_{\text{disk}} = R_{\text{circ}} = \left(1 + \frac{1}{q}\right)^4 A. \tag{23}
\]

For example, using (23), we find: \(k_{\text{circ}} \equiv R_{\text{circ}}/A = 0.226, 0.307,\) and \(0.420\) for \(q = 5, 7,\) and 20, respectively.

At the onset of the outburst, due to the abrupt heating of matter in the ring and the increase in its viscosity, a redistribution of angular momentum begins: a part of the matter with a decreasing specific angular momentum falls along a spiral to the center and the other part with a large specific angular momentum moves away, causing the disk to expand to its maximum sizes. In conservative accretion disks without a wind, almost all of the angular momentum from the disk is pumped from its outer radius into the orbital angular momentum of the binary system (Ichikawa and Osaki 1994).

To estimate the maximum possible effect of the wind, suppose that all of the wind starts from the outer disk boundary, where the matter has the greatest specific angular momentum. The outer disk boundary is acted upon by the tidal forces from the neighboring component and, therefore, the accretion disk does not reach the boundaries of the Roche lobe of the compact object. Following Suleimanov et al. (2008), we will choose the outer radius of the accretion disk \(R_{\text{esc}}\) to be equal to the tidal radius \(R_{\text{tid}}\), which accounts for 90% of the Roche lobe radius \(R_{\text{RL}}\) determined from the Eggleton (1983) formula:

\[
R_{\text{esc}} = R_{\text{tid}} = 0.9 \times R_{\text{RL}}. \tag{24}
\]

where \(q \equiv M_x/M_{\text{opt}}\) is the ratio of the masses of the relativistic (for which the Roche lobe is considered) and optical components of the close binary system. Using (24), we will obtain \(k_{\text{tid}} \equiv R_{\text{tid}}/A = 0.469, 0.520,\) and 0.567 for \(q = 5, 7,\) and 20, respectively.

Since Eqs. (23) and (24) are approximation ones and were derived from various considerations, at sufficiently large values of the mass ratio \(q\) (starting from \(q \approx 47.8\)) the circularization radius begins to exceed the outer disk radius, which is unrealistic. Therefore, we impose the following condition on the disk radius before the outburst: \(R_{\text{disk}} \leq R_{\text{esc}}\).

Having obtained the estimates (23) and (24) for the characteristic radii of the problem, let us study how the effect of a change in the period of the binary system depends on the relative power of the wind from the disk. Suppose that the mean accretion rate onto the compact object is \(\langle \dot{M}_{\text{acc}} \rangle = 10^{18}\) g s\(^{-1}\), which is a typical accretion rate during an LMXB outburst in order of magnitude and is about one tenth of the critical Eddington accretion rate onto a nonrotating black hole with a mass \(M_x = 10 M_\odot\).

We will assume the mass loss rate in the wind to be proportional to the accretion rate onto the black hole: \(\left\langle \dot{M}_{\text{wind}} \right\rangle = C_w \times \left(\dot{M}_{\text{acc}}\right)\). In Fig. 2 the rates of relative change in the orbital period according to (15) are plotted against the ratio of the mass loss rate in the wind to the accretion rate \(C_w\) for various LMXB component mass ratios \(q\) at \(R_{\text{esc}} = R_{\text{tid}}\) and \(R_{\text{disk}} = R_{\text{circ}}\). The curves end on the right at \(C_w = C_w^* (q)\). However, in the half-interval \(q \gtrsim 47.8\) no values of \(C_w^*\) exist, because the approximation formulas for the tidal and circularization radii give a nonphysical relation (see Eq. (13)). Therefore, the curves in this range of \(q\) (\(q \gtrsim 47.8\)) are indefinitely increasing ones. Note also that in the intervals \(q \lesssim 0.1\) and \(1.1 \lesssim q \lesssim 42.7\) the curves are decreasing functions of \(q\), because at these values of the binary component mass ratio the condition \(k_{\text{esc}} > k_{\text{tid}}^{1/2} + 2/3[q^{1/2}/(1 + q)^{3/2}]\) is met.

In Fig. 3 the normalized change in the period \(\Delta P/P (\Delta M_{\text{acc}}/M_{\text{opt}})^{-1}\) is plotted against the binary component mass ratio \(q\). As can be seen from the figure, the presence of a wind for a range of \(q\) typical of LMXBs leads to a decrease of the predicted increase in the period due to the outburst compared to the conservative model.
Fig. 2. Relative change in the orbital period of a close binary system versus ratio of the mass loss rates due to the wind and accretion for various mass ratios at $R_{\text{esc}} = R_{\text{tid}}$. The mean accretion rate $\langle \dot{M}_{\text{acc}} \rangle$ is $10^{18}$ g s$^{-1}$.

Fig. 3. Normalized change in the period $(\Delta P_{\text{burst}}/P)(\Delta M_{\text{acc}}/M_{\text{opt}})^{-1}$ versus logarithm of the component mass ratio $q = M_x/M_{\text{opt}}$. The solid curve indicates the change in the period if there is no wind from the disk ($C_w = 0$), the dotted red curve indicates the case where the mass loss in the wind is equal to the accreted mass ($C_w = 1$), but the effective radius of the wind outflow is equal to the characteristic disk radius before the outburst; the remaining curves indicate the cases where the wind starts from the tidal disk radius $R_{\text{tid}}$ and $C_w = 1$ and $C_w = C_w^*$. The region lying below the minimum possible change in the period is painted gray.
Fig. 4. Evolution of the simulated outbursts for the systems XTE J1118+480, A0620−00, and GRS 1124−68. The initial accretion rate was taken to be equal to the Eddington one. The ratio of the mass loss rates due to the wind and accretion is \( C_w = 7, 8.45, \) and \( 3.87, \) respectively. The parameters of the systems are listed in Table 1.

**Application to Real Systems**

Using the FREDDI code\(^1\) (Malanchev and Lipunova 2016; Lipunova and Malanchev 2017), we simulated outbursts for the systems XTE J1118+480, A0620−00, and GRS 1124−68 (Fig. 4). The model parameters for the systems are given in Table 1. Note that the initial accretion rate for all three systems was taken to be equal to the critical Eddington one.

The FREDDI code computes the viscous evolution of a disk zone that is fully ionized. The code was developed to compute the light curves of soft X-ray transients with a fast rise and quasi-exponential decay and was modified for this work to take into account the effect of the wind from the disk (Avakyan et al. 2019).

The accretion rates derived through our simulations were used to estimate the change in the orbital period (Figs. 5–7). The ratio of the rate of mass outflow in the wind to the accretion rate onto the black hole was chosen to be \( C_w = 7, 8.45, \) and 3.87 for XTE J1118+480, A0620−00, and GRS 1124−68, respectively. For the systems A0620−00 and GRS 1124−68 we chose the maximum possible of \( C_w = C^*_{\text{w}}(q) \) within the model with \( R_{\text{esc}} = R_{\text{tid}} \) and \( R_{\text{disk}} = R_{\text{circ}}. \) Such values lead to a lower limit on \( \Delta P. \) However, it is impossible to determine \( C^*_{\text{w}} \) for the close binary XTE J1118+480, since the tidal radius \( R_{\text{tid}} \) and the circularization radius \( R_{\text{circ}} \) are equal (or, in other words, \( k_{\text{esc}} = k_{\text{disk}} \)). This is because the component mass ratio in this system \( q \) exceeds the boundary value of 47.8. Therefore, \( C_w \) was set equal to 7 (Luketic et al. 2010). The lower limit on \( \Delta P \) for this system is realized at \( C_w = 0 \) (Fig. 3).

At such parameters the mean values of the change in the orbital period during the outburst are \( \sim 4.3 \times 10^{-10}, 9.6 \times 10^{-11}, \) and \( 8.2 \times 10^{-11} \) \( \text{s}^{-1} \) for XTE J1118+480, A0620−00, and GRS 1124−68, respectively. Such a strong influence of the wind on the orbital period occurs only during the outburst itself (\( \sim 40−90 \) days, we chose 45 days for XTE J1118+480 and A0620−00 and 85 days for GRS 1124−68). However, in quiescence (from several to tens of years) the accretion and wind mass loss rates in the disk are much lower, and the wind formation mechanism can be completely switched off.

Table 2 gives the calculated changes in the periods due to the wind from the disk and the accretion of matter for the systems XTE J1118+480, A0620−00, and GRS 1124−68 under the assumption that one outburst emerges in \( T_q = 6, 30, \) and 20 years, respectively. In other words, the systems, according to the model, stay for 45 days in a state with high accretion and wind mass loss rates (Fig. 4), when the period changes significantly, whereupon there comes a quiescent state lasting \( \Delta T_q. \) Such times were not chosen by chance. Specifically, XTE J1118+480 flared up quite often, in 2000 and 2005 (Vojtech

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\(^1\)The FREDDI code can be downloaded from the web page [http://xray.sai.msu.ru/~malanchev/freddi](http://xray.sai.msu.ru/~malanchev/freddi).
Table 1. The parameters of the binary systems XTE J1118+480, A0620−00, and GRS 1124−68 used in our simulations

| Parameters       | XTE J1118+480 | A0620−00 | GRS 1124−68 | References* |
|------------------|---------------|----------|-------------|-------------|
| Black hole mass, $M_\bullet$ | 7.06 $M_\odot$ | 6.5 $M_\odot$ | 11.0 $M_\odot$ | [1], [2], [3] |
| Optical star mass, $M_{\text{opt}}$ | 0.10 $M_\odot$ | 0.26 $M_\odot$ | 0.89 $M_\odot$ | [1], [2], [3] |
| Binary period, $P$ | 0.1699 d | 0.3230 d | 0.4326 d | [4], [5], [6] |
| Inclination, $i$, deg | 74.0 | 51.0 | 43.2 | [2], [7], [3] |
| Viscosity parameter, $\alpha$ | 0.1 | 0.1 | 0.1 | [8] |

* [1] Chereshchuk et al. (2019a); [2] Chereshchuk et al. (2019b); [3] Wu et al. (2016); [4] GH12; [5] GH14; [6] GH17; [7] Cantrell et al. (2010); [8] Shakura and Sunyaev (1973).

The peak accretion rate is equal to the Eddington one: $\dot{M}_{\text{acc},0} = \dot{M}_{\text{Edd}} = 1.4 \times 10^{18} (M_\bullet/M_\odot)$ g s$^{-1}$.

Table 2. Changes in the orbital period due to the outburst $\Delta P_{\text{burst}}/\Delta T_{\text{d}}$ [s s$^{-1}$] for the systems XTE J1118+480, A0620−00, and GRS 1124−68 according to the values from our computations (wind and accretion) as well as the observed and model values from GH and CP19

| Parameters                  | XTE J1118+480 | A0620−00 | GRS 1124−68 |
|-----------------------------|---------------|----------|-------------|
| Lower limit in model        | $6.6 \times 10^{-11}$ | $3.9 \times 10^{-13}$ | $9.5 \times 10^{-13}$ |
| Observations (GH)           | $-6.0 \times 10^{-11}$ | $-1.9 \times 10^{-11}$ | $-6.6 \times 10^{-10}$ |
| Model 1 (GH)                | $-5.4 \times 10^{-13}$ | $-7.6 \times 10^{-13}$ | $-8.9 \times 10^{-13}$ |
| Model 2 (GH)                | $-2.7 \times 10^{-11}$ | $-8.6 \times 10^{-12}$ | $-3.5 \times 10^{-12}$ |
| Gravitational waves (CP19)  | $-3.0 \times 10^{-13}$ | $-2.0 \times 10^{-13}$ | $-4.0 \times 10^{-13}$ |
| Magnetic braking (CP19)     | $-7.8 \times 10^{-12}$ | $-3.8 \times 10^{-12}$ | $-2.2 \times 10^{-12}$ |

Simón 2020), which cannot be said about A0620−00 and GRS 1124−68, in which no outbursts have been observed since 1975 and 1991 (Connors et al. 2017; Wu et al. 2016), respectively. The observations of XTE J1118+480 used in GH12 and GH14 to determine the decrease in the system’s period were carried out from 2000 to 2012. In the case of A0620−00, the last measurement of the period was made in 2006 (GH14), while the last outburst occurred in 1975. For GRS 1124−68 the last measurement of the period was in 2012 (GH17), more than twenty years after the 1991 outburst.

Apart from our model estimates, Table 2 also gives the values from the series of papers GH12, GH14, and GH17 (both theoretical ones and those determined from observations; hereafter “GH”) and the estimates from CP19. The model of a decrease in the period of a binary from Johannsen et al. (2009), which takes into account the magnetic braking and the mass loss due to the black hole evaporation, was used in GH. Two sets of parameters for this model are used in GH: “realistic” and “extreme” (the maximum possible effect of mass loss and magnetic braking). In Table 2 the designations Model 1 and Model 2 were chosen for the realistic and extreme sets of parameters, respectively. All of the values are given in order to qualitatively compare the powers of the various mechanisms affecting the evolution of the orbital periods of binary systems.

The changes in the period due to outbursts are hardly noticeable against the background of the observed secular trend in XTE J1118+480, A0620−00, and GRS 1124−68 on time scales of 6, 30, and 20 years, respectively. Note that the values in the first row of Table 2 ignored the other effects causing secular changes in the orbital period described above, namely the outflow from the Lagrange point $L_2$ and the mass transfer from the optical component into the disk.

For the three systems Figs. 5−7 show the changes in the periods in one year during which one outburst occurs. The lines with “steps” indicate the combined effect of the mass transfer into the disk (which manifests itself as a hardly noticeable secular increase in the period for $\langle \dot{M}_{\text{tr}} \rangle = 10^{16}$ g s$^{-1}$) and the outburst
Fig. 5. Annual change in the period for the system XTE J1118+480 according to the model (blue curves: mass transfer and outburst), observations (black curve), and persistent outflow from the Lagrange point $L_2$ (green and red curves).

Fig. 6. Annual change in the period for the system A0620−00 according to the model (blue curves: mass transfer and outburst), observations (black curve), and persistent outflow from the Lagrange point $L_2$ (green and red curves).

The effect of the outflow from the point $L_2$ is shown for two values of the mass loss rate differing by a factor of 10. The outflow from the point $L_2$ leads to a secular decrease in the period, but the observed rate requires too high mass loss rates. It can be seen that for each of the three systems the outflow from $L_2$ could explain a decrease in the period of the order of the observed one only at very high, unrealistic mass loss rates through $L_2$, especially in the case of GRS 1124−68.

Observations of the orbital period of an X-ray binary immediately before and after an X-ray nova out-
burst would be good evidence for a dramatic change in the period during the outburst due to the wind from the disk. According to our model, the orbital periods of XTE J1118+480, A0620−00, and GRS 1124−68 during their outbursts at the chosen values of \( C_w \) can increase by \( \gtrsim 1.6 \), \( \gtrsim 0.4 \), and \( \gtrsim 0.6 \) ms, respectively. However, no such observations of these systems have been carried out so far.

**DISCUSSION**

According to the GH14 and GH17 observations, the three LMXBs with black holes considered demonstrate a dramatic decrease in the period in quiescence. The rate of decrease in the period due to the gravitational radiation (CP19) is less than the observed one by 2−3 orders of magnitude, while the estimates for magnetic braking are smaller by an order of magnitude, even for very favorable parameters (Table 2). Out of the mechanisms considered by us, only the mass outflow from \( L_2 \) with an impossibly high rate could explain the observed rates (Figs. 5−7).

In addition to these braking mechanisms, the following is also possible. The matter ejected by the wind from the accretion disk or escaped through \( L_2 \) can form a toroidal disk around the binary system (circumbinary disk). Such a ring structure can remove the tidal torque acting on it from the binary system due to viscosity and, thus, can reduce the angular momentum of the binary system (and its orbital period). The characteristic viscous time of the evolution of this ring of matter exceeds considerably the viscous time of the hot disk around the compact object, which allows it to efficiently remove the angular momentum without continuous feeding with matter. Munu and Mauerhan (2006) and X. Wang and Z. Wang (2014) provide observational evidence for the existence of such a disk around A0620−00 and XTE J1118+480. The influence of such a ring on the period of a close binary system was studied in CP19 based on the results from Artymowicz and Lubow (1994). According to CP19, the observed decrease in the period of A0620−00 and XTE J1118+480 can be explained if the mass of such a ring around the systems is \( \sim 10^{-9} M_\odot \). It follows from our simulations that for A0620−00 and XTE J1118+480 during an outburst (even for \( C_w = 2 \)), the mass ejected by the wind from the disk is \( 3.4 \times 10^{-8} M_\odot \) and \( 2.4 \times 10^{-8} M_\odot \), respectively. Most likely, not all of the matter ejected by the wind during the outburst settles into the ring around the binary, but even under this assumption such an estimate may be enough to explain the observed decrease in the period for A0620−00 and XTE J1118+480. However, for GRS 1124−68 the ring around the binary system must be heavier by two orders of magnitude or, more specifically, \( 10^{-7} M_\odot \). The required mass can be reached after one outburst, but under the assumption of a very strong wind (for \( C_w = 10 \) the mass ejected in the wind is...
1.1 \times 10^{-7} M_\odot). The fact of such a dramatic change in the period of GRS 1124–68 makes it even more interesting for a further study.

CONCLUSIONS

An outburst in an LMXB leads to a significant change of its orbital period. We derived a general analytical formula to estimate this effect by taking into account the wind from the accretion disk and gave quantitative estimates of the change in the period for three LMXBs.

Observations of various LMXBs and measurements of their orbital period immediately before and after outbursts can reveal the predicted changes in the orbital period and make it possible to compare them with the mass loss rates due to the wind.

We also estimated the changes in the periods due to the mechanisms that also operate in the quiescent state of LMXBs: the mass transfer from the donor into the disk around the black hole and the mass outflow from the Lagrange point \( L_3 \). The latter leads to a secular decrease in the orbital period. However, it cannot explain the observed rates of decrease in the period in the sources considered either. Apparently, either extremely strong magnetic braking or the removal of angular momentum into the ring of matter surrounding the binary system needs to be invoked to explain them.

ACKNOWLEDGMENTS

We are grateful to K.A. Postnov for his valuable remarks, the seminar of the Relativistic Astrophysics Department for the fruitful discussion, and I.I. Antokhin for his comments. One of us (A.L. Avakyan) was supported by the “BASIS” Foundation for the Advancement of Theoretical Physics and Mathematics (grant no. 20–2–1–106–1). This study was supported by the Interdisciplinary Scientific and Educational School of the Moscow State University “Fundamental and Applied Space Research.” The development of the FREDI code was supported by RFBR grant no. 18–502–12025.

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*Translated by V. Astakhov*