The using of non-standard according to Hans Reichenbach’s clock synchronization in the integral covariant formulation of the laws of conservation

Valery Egorowich Stepanov
Physics and Industry Faculty, North Eastern Federal University, Yakutsk, 677000, Russia
E-mail: tritiy51@mail.ru

Abstract. The non-standard Reichenbach's synchronization is necessary for the coincidence of the numerical values of the integrals of the conserved quantities in two different inertial frames of reference, thereby ensuring an integral covariant formulation of the laws of conservation of physical quantities. The inclined axes of time in Minkowski's space have a physical meaning of time with non-standard clock synchronization according to Reichenbach.

1. Introduction
In quantum electrodynamics, the law of conservation of the electric charge is proved by the Noether's theorem, using condition of the invariance of the action for the Dirac's spinor field according to the one-parameter group of unitary transformations. Locally, this law corresponds to the equality to zero of the four-dimensional divergence of the electric current density vector. For a system of charges located in a bounded volume and vanishing at spatial infinity, the integral over the three-dimensional hypersurface of the space is equal to the total charge of the system and its magnitude is conserved in time as discussed by Michkewich [1].

The laws of conservation of energy-impulse and impulse momentum are formulated in the form of three-dimensional volume integrals connected by the Noether's theorem with the ten parametric Poincaré's group of continuous symmetry transformations of Minkowski's four-dimensional space-time as explained in [1]. Integrals over the spatial three-dimensional hypersurface from the densities of conserved quantities are constant magnitudes in time. Thus, integral conserved quantities are necessary parts in the theory of interactions of a system of particles and physical fields. However, the relativity of the simultaneity of different events, resulting from the Lorentz's transformations, raises the question of the compatibility of integral conservation laws with the principle of relativity of the special theory of relativity. We know the extreme point of view, which affirms the exclusively local (in the form of a four-dimensional differential equation of continuity) character of the conservation laws. For example, as explained by Feynman [2], where he considered a cylinder of finite length. In the own reference frame for a cylinder, an electron and a positron simultaneously emitted from the ends of the cylinder. In its own system, the full charge of the cylinder does not change, but in a different inertial reference frame the total charge varies at different moments of time on the clock at rest because of the relativity of the simultaneity of the different events. In this non-local incompatibility of the conservation law with the principle of relativity is expressed the Feynman’s paradox.
Here is considered the tensor formulation of the integral law of conservation of the total charge of a system bounded in space with is compatible respect to the Lorentz transformations.

2. Materials and Methods

2.1. Non-standard clock synchronization in distant simultaneity

The concept of time in the special theory of relativity is based on the dates of the space separated clocks with the same speed of motion, launched simultaneously. Simultaneous start-up of the clock is made by taking into consideration the time spent by the light on the path from the initial to the end points. In his famous work [3], Einstein proposed a condition about the equality of the speeds of light in the forward and backward directions and established the method of clock synchronization in the special theory of relativity. Einstein’s synchronization will be called standard.

In the famous Michelson–Morley’s experiment on an interferometer, light passes along a closed path, the absence of an ether wind is determined by the constancy of the average speed of light as it travels along a closed path C, the possible inequality of the light velocities in individual sections does not affect on the experimental result as discussed by Ugarov [4].

Reichenbach generalized Einstein's standard simultaneity to a topological one, compatible with the principle of causality and observability of the average speed of light along a closed route as explained in [5]. Nowadays in the scientific literature there is a discussion about the conventional status of the numerical value of the speed of light in one direction as discussed by Grunbaum [6].

Without entering into this discussion, we are going to prove here that by using non-standard clock synchronization according to Hans Reichenbach, we can formulate the law of charge conservation in a relativistically invariant manner, as the constancy of conserved quantities in their own frame of reference.

$L_{AB}$, at the instant $t_{a0}$ the light signal is emitted to the point B, which arrives at the moment $t_{b0}$ and returns back to the time $t_{a1}$ (Figure 1). With the standard synchronization according to A. Einstein, the moment of arrival at point B is at the middle of the interval of the total time of the trip back and forth, which is seen in formula (1).

\[ t_{b0} = t_{a0} + \frac{1}{2}(t_{a1} - t_{a0}). \] (1)

With nonstandard synchronization, the $1/2$ factor is replaced by the Reichenbach parameter $\epsilon_{AB}$, varying from zero to one

\[ u_{b0} = t_{a0} + \epsilon_{AB}(t_{a1} - t_{a0}) \] . (2)

The speed of light in one direction $C_{AB}$ is related with parameter of Reichenbach’s formula
\[ C_{AB} = \frac{L_{AB}}{u_{b0} - t_{a0}} = \frac{c}{2\varepsilon_{AB}}. \]  \hfill (3)

If at point B we shift back the time of starting the clock to an interval equal to \( t_{b0} - u_{b0} \), we will get an apparent increasing speed of light there, corresponding to formula (3). It is easy to see that
\[ u_{b0} = t_{b0} + u_{b0} - t_{b0} = t_{b0} + (\varepsilon_{AB} - \frac{1}{2}) \frac{2L_{AB}}{c}. \]  \hfill (4)

The origin of the Cartesian X-coordinate system is compatible with point A and put an arbitrary point B on the X-axis, this point will have the coordinate \( x \). Then the time \( u \) of a reference frame with non-standard synchronization will be related with the time of the reference frame with Einstein’s synchronization \( t \) by the relation
\[ u = t + (\varepsilon_{AB} - \frac{1}{2}) \frac{2x}{c}. \]  \hfill (5)

Let us consider a reference frame \( K' \) moving with a constant velocity \( v \) along the \( X \) axis. The stationary laboratory frame of reference is denoted by the symbol \( K \). These two systems are connected by special Lorentz’s transformations. We show that due to the shift the start of the remoted clocks the Feynman paradox can be resolved. Let it be a cylinder moving with velocity \( v \), the axis of symmetry of which is aligned with the axis \( x \). Let the electron emerge at the time \( t_{1}' \) from the point with the coordinate \( x_{1}' \) of the cylinder, and the positron emerge from the point \( x_{2}' \). These two events in the fixed system \( K \) have coordinates \( (t_{1}, x_{1}) \) and \( (t_{2}, x_{2}) \). By changing the clock synchronization in the laboratory system using formula (5) so that these two events are simultaneous in time \( u \), and also using special Lorentz’ transformations, we get that the Reichenbach’s parameter is determined by the condition
\[ \varepsilon_{AB} = \frac{1}{2} - \frac{v^2}{2c}, \]  \hfill (6)
\[ u = t - \frac{v}{c}x. \]  \hfill (7)

Consequently, due to the transition to non-standard clock synchronization, it is possible to resolve the Feynman's paradox in the sense that for both inertial reference systems the law of conservation of the total charge of the cylinder is simultaneously fulfilled in its self and in laboratory frames of reference.

2.2. The transition to non-standard synchronization leads to an oblique rectilinear time coordinate axis in the Minkowski’s space

In Minkowski’s space, the time coordinate \( x^0 \) having the dimension of length is equal to the product of the time \( t \) by the speed of light \( c \), the spatial coordinates following after it have the numbers 1, 2, 3. In the space-time plane, the coordinates \( u^0, u^1 \) for the laboratory inertial reference frame with non-standard clock synchronization are related to the old coordinates with Einstein’s synchronization by the following transformations
\[ u^0 = x^0 - \frac{v}{c}x^1, \]  \hfill (8)
\[ u^1 = x^1. \]  \hfill (9)

A set of simultaneous events forms a three-dimensional hypersurface in the Minkowski’s space that intersects the time axis orthogonally at the considered moment. When the reference frame \( K' \) moves along the axis \( x^1 \), the hypersurface of simultaneity of this proper reference frame of a charged cylinder cuts a line in the plane \( (x^0, x^1) \), which is defined by equation
\[ u^0 = x^0 - \frac{v}{c}x^1 = \text{const}. \]  \hfill (10)

Differentiation of this equation leads to the result
\[ \text{th} \theta = \frac{dx^0}{dx^1} = \frac{v}{c}. \]  

The time axis \( u^0 \) is orthogonal to the line defined by equation (9) (figure 2).

![Figure 2.](image)

**Figure 2.** Slope of the time axis of non-standard synchronization

Thus, the coordinate axes \( (u^0, u^1) \) form an angle \( \pi / 2 + \theta \). The inverse formulas to (9) for the transition to new coordinates have the form

\[
\begin{align*}
    x^0 &= u^0 + \frac{v}{c} u^1, \\
    x^1 &= u^1.
\end{align*}
\]

In the skew coordinate system, the nondiagonal component of the metric tensor differs from zero

\[ g_{01} = \mu_{00} \frac{\partial x^0}{\partial u^0} \frac{\partial x^0}{\partial u^1} = -\frac{v}{c}. \]

In addition, it turns out that

\[
\begin{align*}
    g_{00} &= \mu_{00} \frac{\partial x^0}{\partial u^0} \frac{\partial x^0}{\partial u^0} = 1, \\
    g_{11} &= -1 + \frac{v^2}{c^2}.
\end{align*}
\]

Thus, in the special theory of relativity, skew rectangular coordinate systems are uniquely associated with the shifts of the clock starting times equivalent to the non-standard clock synchronization according to Reichenbach.

### 2.3. Integral covariant formulation of conservation laws

Let us consider an insular system of mutually motionless particles. The law of conservation of charge for this system in the differential form is expressed as the vanishing of the divergence of the four-dimensional vector of the volumetric current density. To calculate the integral formulation of the integral over the four-dimensional tube world lines of particles insular system, forming a four-dimensional cylinder. This cylinder ends of two perpendicular to the time world lines of the surfaces of simultaneous events belonging to future and past moments of time of the observer in his own frame of reference insular system. According to the four-dimensional Gauss’s theorem the volume integral is converted to the integral over a closed surface, disappears on the lateral surface but is different from zero on the bases. The lateral surface of the cylinder tends to spatial infinity, where the charges are zero. Consequently, integral of lateral surface turns to zero. The result is an equality of two integrals over the three-dimensional hypersurfaces for different points in time. If we turn into the laboratory reference system, relatively to which insular system is moving inertially, then of the simultaneity hypersurfaces used in the insular system cease to be orthogonal to the time axis of the laboratory.
system. Because of the relativity of simultaneity, an unequal to one-half Reichenbach’s parameter arises from it (6), the speed of light will be equal to the fundamental constant only on the average. The hypersurfaces of simultaneity in the four-dimensional Minkowski’s space are invariant geometric objects. In the zone of relativity of simultaneity, any hypersurfaces can be chosen, while the numerical values of the integrals of the conserved quantities do not depend on the choice of the hypersurfaces of simultaneity.

3. Results

Thus, the non-standard Reichenbach's synchronization is necessary for the coincidence of the numerical values of the integrals of the conserved quantities in two different inertial frames of reference, thereby ensuring an integral covariant formulation of the laws of conservation of physical quantities.

The inclined axes of time in Minkowski's space have a physical meaning of time with non-standard clock synchronization according to Reichenbach.

References

[1] Michkewich N V 1969 (in Russian) Physical Fields in General Relativity (original Russian title: Fizicheskie poliya v obshey teorii otnositelnosti Moscow: Nauka)
[2] Feynman R 1987 (in Russian) The character of physical law (original Russian title: Karakter fizicheskix zakonov Moscow: Nauka)
[3] Einstein A 1905 Ann. Phys. 17 891
[4] Ugarov V A 1977 (in Russian) Special Relativity (original Russian title: Specialnaya Teoriya Otnositelnosti Moscow: Nauka)
[5] Reichenbach H 1924 Axiomatik der relativistischen Raum-Zeit-Lehre Braunschweig: Verlag
[6] Grunbaum A 2010 Found. Phys. 40 1285