GAUGE COUPLING UNIFICATION VS. SMALL $\alpha_s \approx 0.11$

LESZEK ROSZKOWSKI

*Theoretical Physics Institute, University of Minnesota, Minneapolis, MN 55455, USA*

(On work done in collaboration with M. Shifman)

Abstract

I present a way of reconciling gauge coupling unification in minimal supersymmetry with small $\alpha_s(m_Z) \approx 0.11$ and discuss the ensuing consequences.

* Talk presented at SUSY-95, 15–19 May, 1995, Ecole Polytechnique, Palaiseau, France.
In this talk I would like to make two points:

• The value of $\alpha_s(m_Z)$ resulting from exact gauge coupling unification in the most popular version of the MSSM predicts the range of $\alpha_s(m_Z)$ significantly above the range of rather low values, not much above 0.11, implied by most experimental determinations and arguments from QCD; and

• One possible way of removing this discrepancy is to dramatically alter the usual relation between the masses of the wino and the gluino. This leads to important consequences for both phenomenology and GUT-scale physics. One solid prediction of this approach is the existence of a gluino below about 300 GeV, within the reach of the upgraded Tevatron.

Calculating $\alpha_s(m_Z)$ in the MSSM. It is well-known that, assuming that the gauge couplings unify, one can predict one of them treating the other two as input. Currently, most studies use the 2-loop renormalization group equations (RGE’s) $d\alpha_i/dt = b_i/(2\pi)\alpha^2_i + $ two-loops, where $i = 1, 2, 3$, $t \equiv \log(Q/m_Z)$ and $\alpha_1 \equiv 5/3\alpha_Y$. The one-loop coefficients $b_i$ of the $\beta$ functions for the gauge couplings change across each new running mass threshold. Their parameterization in the MSSM in the leading-log approximation can be found, e.g., in [2, 3].

The predicted value for $\alpha_s(m_Z)$ depends on the values of the input parameters: $\alpha$, $\sin^2\theta_W(m_Z)$, and $m_t$. It also receives corrections from: mass thresholds at the electroweak scale, the GUT-scale mass thresholds and non-renormalizable operators, the two-loop gauge and Yukawa contributions, and from scheme dependence ($\overline{\text{MS}}$ vs. $\overline{\text{DR}}$).

For the electromagnetic coupling we take $\alpha^{-1}(m_Z) = 127.9 \pm 0.1$. Recently, three groups have reanalyzed $\alpha(m_Z)$ [3] and obtained basically similar results which do not change the resulting value of $\alpha_s(m_Z)$ significantly [3, 4].

The input value of $\sin^2\theta_W(m_Z)$ is critical. This sensitivity is due to the fact that $\alpha_2(Q)$ does not change between $Q = m_Z$ and the GUT scale $Q = M_X$ as much as the other two couplings. Thus, a small increase in $\sin^2\theta_W(m_Z)$ has an enhanced (and negative) effect on the resulting value of $\alpha_s(m_Z)$. Following Ref. [5] we assume

$$\sin^2\theta_W(m_Z) = 0.2316 \pm 0.0003 - 0.88 \times 10^{-7} \text{ GeV}^2 \left[m_t^2 - (160 \text{ GeV})^2\right].$$  (1)

The global analysis of Ref. [7] implies that in the MSSM $m_t = 160 \pm 13$ GeV, consistent with the recently reported discoveries of the top quark: $m_t = 176 \pm 8 \pm 10$ GeV (CDF) and $m_t = 199 \pm 20 \pm 22$ GeV. Taking, instead of 160 GeV, even the D0 central value for $m_t$ would lower $\sin^2\theta_W(m_Z)$ and increase $\alpha_s(m_Z)$ by only 0.005. Shifting $\sin^2\theta_W(m_Z)$ up/down by 0.0003 shifts $\alpha_s(m_Z)$ down/up by only 0.0013.

Two-loop contributions in the RGE’s increase $\alpha_s(m_Z)$ by about 10%. The most important among them is the pure gauge term which yields $\Delta\alpha_s(m_Z) = 0.012$
if one assumes SUSY in both one- and two-loop coefficients of the \( \beta \) function all the way down to \( Q = m_Z \). Other corrections, from two-loop thresholds, Yukawa contributions, and due to changing from the conventional \( \overline{\text{MS}} \) scheme used here to the fully supersymmetric \( \overline{\text{DR}} \) scheme, are much smaller \cite{6,1}.

**\( \alpha_s(m_Z) \) from Constrained MSSM.** The MSSM, treated merely as the supersymmetrized Standard Model, contains a multitude of free parameters. One expects it to be a low-energy effective theory resulting from some GUT or, more generally, some more fundamental scenario (e.g. strings) valid at scales very much larger then the electroweak scale. Depending on one’s preferences for a more fundamental theory, one can make various additional assumptions relating the free parameters of the MSSM.

Perhaps the most commonly made assumption, other than the assumption of gauge unification itself, is the relation among gaugino masses: \( M_1(M_X) = M_2(M_X) = m_\tilde{g}(M_X) \equiv m_{1/2} \) which assigns at the GUT scale the same [common gaugino] mass to the bino, wino, and gluino states, respectively. This leads, due to renormalization effects, to the following well-known relations at the electroweak scale:

\[
M_1 = \frac{5}{3} \tan^2 \theta_W M_2 \simeq 0.5 M_2, \tag{2}
\]

\[
M_2 = \frac{\alpha_2}{\alpha_s} m_\tilde{g} \simeq 0.3 m_\tilde{g}. \tag{3}
\]

These relations, or at least the first of them, are commonly assumed in most studies of the MSSM, even though, strictly speaking, they are not necessary in the context of the model.

Another relation which stems from minimal supergravity and which is commonly assumed is the equality of all the (soft) mass parameters of all the sleptons, squarks, and typically also Higgs bosons, to some [common scalar] mass parameter \( m_0 \) at the GUT scale. Renormalization effects cause the masses of color-carrying sparticles to become, at the \( m_Z \) scale, typically by a factor of a few heavier than the ones of the states with electroweak interactions only.

Often one also imposes a very attractive mechanism of radiative electroweak symmetry breaking (EWSB), which provides additional constraint on the parameters of the model, in particular relates the SUSY Higgs/higgsino mass parameter \( \mu \) to the parameters of the model which break SUSY. This fully constrained framework is sometimes called the Constrained MSSM (CMSSM) \cite{10}. In practice, various groups have considered the MSSM with a varying number of additional assumptions, starting from adopting just the common gaugino mass to the CMSSM with additional constraints, e.g., from nucleon decay which requires specifying the underlying GUT, or string, model, the simplest \( SU(5) \) and \( SO(10) \) models being the most commonly studied. (A discussion of GUT physics is beyond the scope of this talk and I will only occasionally make references to expected corrections to low-energy variables,
like $\alpha_s(m_Z)$, from simplest GUT-models.) It is not always easy to discern what assumptions are actually responsible for what results.

At the level of accuracy described above, several studies \cite{3, 9, 10, 11, 12, 6} have generally agreed that, in the MSSM with additional assumptions of the common gaugino and scalar masses, and if one restricts oneself to masses roughly below 1 TeV then $\alpha_s(m_Z) \gtrsim 0.12$ (and $\gtrsim 0.13$ for SUSY masses below some 300 GeV). This is because $\alpha_s(m_Z)$ grows with decreasing masses of SUSY particles. The above prediction for $\alpha_s(m_Z)$ has been considered a success and the strongest evidence in favor of supersymmetric unification, especially in light of the range of $\alpha_s(m_Z) = 0.127 \pm 0.05$ resulting from the Z line shape at LEP \cite{7}.

**Is There an $\alpha_s$ Problem?** In spite of the existence of its several independent determinations, there is still no consensus on the experimental value of $\alpha_s(m_Z)$. Most low-energy measurements and lattice calculations of $\alpha_s$, when translated to the scale $m_Z$, generally give values much below the LEP range, between 0.11 and 0.117, with comparable or smaller error bars. (See, e.g., recent reviews \cite{13} for more detail.) The only indication (with small error bars) from low energies for larger $\alpha_s(m_Z)$ from $\tau$ decays \cite{14} has been questioned \cite{15}. A general tendency among various reviews on the value of $\alpha_s$ is to acknowledge the apparent discrepancy but adopt a sort of “wait-and-see” approach. Indeed, the world-average of $0.117 \pm 0.006$ (see Bethke in Ref. \cite{13}) is at least marginally consistent with most determinations, even though they seem to correspond to two disconnected (at 1σ) sets of values. In fact, three speakers \cite{1} at this meeting have assured us that, in this sense, there is no real $\alpha_s$ problem.

However, recently Shifman \cite{15} very vigorously argued that the internal consistency of QCD requires that $\alpha_s$ be close to 0.11. He gave some important reasons. Here I will quote only one: large $\alpha_s \approx 0.125$ would correspond to $\Lambda_{\overline{MS}} \approx 500$ MeV (in contrast to $\sim 200$ MeV for $\alpha_s(m_Z) \approx 0.11$). Such a large value is apparently in conflict with crucial features of QCD on which a variety of phenomena depend sensitively. Prompted by Shifman’s argument, Voloshin \cite{16} re-analyzed $\Upsilon$ sum rules claiming the record accuracy achieved so far: $\alpha_s(m_Z) = 0.109 \pm 0.001$. Also, a recent global fit \cite{17} to LEP data favors $\alpha_s(m_Z) = 0.112$. Clearly, small $\alpha_s(m_Z) \approx 0.11$ seems an increasingly viable possibility, while significantly larger values are predicted by the CMSSM.

**Can SUSY Unification Be Made Consistent with Small $\alpha_s(m_Z) \approx 0.11$?** Clearly, in the absence of large corrections from GUT-physics, the most popular version of the MSSM, with the additional mass relations between the gauginos and the scalars pre-

\footnote{One could say: As many as three speakers!}
predicts too large values of $\alpha_s(m_Z)$.

Several solutions to this problem can be immediately suggested. One is to remain in the context of the CMSSM but adopt a heavy SUSY scenario with the SUSY mass spectra significantly exceeding 1 TeV. This scenario would not only put SUSY into both theoretical and experimental oblivion, but is also, for the most part, inconsistent with our expectations that the lightest supersymmetric particle (LSP) should be neutral and/or with the lower bound on the age of the Universe at least some 10 billion years. (See Section 5 of Ref. [10].) Another possibility is to invoke large enough negative corrections due to GUT-scale physics. The issue was re-analyzed recently [6] and it was found that, under natural assumptions about the scale of GUT-scale corrections, $\alpha_s(m_Z) > 0.12$. Although it may well happen that the GUT-scale corrections are abnormally large, the predictive power is essentially lost. One can also employ [18] an intermediate scale for which there is good motivation from neutrino and axion physics.

Here, I would like to present a different approach. Its starting point is the question: Which ingredient of the CMSSM is mainly responsible for predicting large $\alpha_s(m_Z) > 0.12$? Is it the particle content of the MSSM, or rather some additional assumptions, which may not be a necessary ingredient of the model, and could therefore be relaxed.

To answer this question, in the first step let’s treat all the mass parameters entering [threshold corrections in] the RGE’s as completely unrelated from each other. Let’s thus assume no relations of any kind between the gauginos or between the scalars. (Since GUT-scale corrections are GUT model-dependent, let’s also turn them off, while keeping in mind that in reality they may be sizable, and that they may both contribute to increasing and decreasing $\alpha_s(m_Z)$.) By requiring only that all the masses be less than about 1 TeV, for comparison with the CMSSM case, we find that

$$\alpha_s(m_Z) > 0.106 \quad (4)$$

This shows that, in the MSSM itself, without imposing any additional mass relations, one can in principle easily obtain much smaller $\alpha_s(m_Z)$ than in the CMSSM. Since our task is to minimize $\alpha_s(m_Z)$, we take $m_t = 160$ GeV while fixing $\sin^2 \theta_W(m_Z)$ at its central value. Of course, the range of $\alpha_s(m_Z)$ values resulting from [exact] supersymmetric unification, still depends on the MSSM mass parameters. We thus arbitrarily choose them in such a way as to further minimize $\alpha_s(m_Z)$: we set $M_2, m_{\tilde{t}}, m_{\tilde{l}}, m_{\tilde{R}},$ and $m_{H_2}$ at 1 TeV, and $m_{\tilde{g}}, m_{\tilde{q}},$ and $m_{\tilde{t}_R}$ at 100 GeV. (See [1] for more details.)

How can such small values of $\alpha_s(m_Z)$ be consistent with supersymmetric unification? It can be easily seen that it is the gluino and the wino that play the dominant role in influencing $\alpha_s(m_Z)$. This can be done by examining the form of the 1-loop
mass threshold coefficients $b_i$ entering the RGE’s. Not only are the mass threshold corrections due to both the wino and the gluino the largest ($4/3$ and $2$, respectively) but also each of them affects the running of only one coupling, $\alpha_2$ and $\alpha_s$, respectively. By lowering $m_{\tilde{g}}$ and increasing $M_2$ one can easily descend to small $\alpha_s(m_Z)$ in the vicinity of $0.11$. Shifting other masses has a much smaller impact on the resulting range of $\alpha_s(m_Z)$. (See Fig. 1 of [1].)

**Implications.** The price that one has to pay is evident and can be seen in Fig. 1. Without invoking other effects, like large GUT-scale corrections, one has to have the wino actually heavier than the gluino. If $\alpha_s(m_Z) \approx 0.11$ and no large negative GUT-scale corrections are invoked, then $m_{\tilde{g}} \lesssim 300$ GeV and $M_2 \gtrsim 400$ GeV. In fact one finds $M_2 \gtrsim 3m_{\tilde{g}}$, thus violating the relation (3). (In order for the lightest neutralino to weight less than the gluino, and thus be the LSP, also the relation (2) has to be violated.) This is a clear prediction which, remarkably, can be tested even before the end of the millennium. If the upgraded Tevatron, which will search for gluinos up to about 300 GeV, finds a gluino below some 240 GeV, while LEP II does not find a wino-like chargino up to about 80 GeV, then (3) will likely be violated. (The chargino could still be of higgsino type but then so would be the lightest neutralino which would leave the MSSM without a viable candidate for DM in the Universe [19].) A vast majority of studies of all aspects of supersymmetry routinely assume (2)–(3).

There are also other important implications of this surprising scenario which have to do with some long-lasting anomalies in the $b\bar{b}$ system [1].
Finally, what implications for physics structure at the GUT scale follow from this bottom-up approach? In standard GUT's equal gaugino masses at $M_X$ are enforced by GUT gauge invariance. This is the most popular scenario in which equal gaugino masses are generated by coupling a SUSY GUT to $N = 1$ minimal supergravity (SUGRA) and choosing the simplest, delta-function, kinetic term for the gauge/gaugino fields. If instead one considers a general form of the kinetic term, one finds that relations among gaugino masses become arbitrary which could potentially give room to accommodate large gaugino mass non-degeneracy. However, it comes out that in this general case of non-minimal SUGRA also the gauge couplings become unequal at $M_X$. Since in our analysis we assume them equal (for the reasons of simplicity), we could attempt to implement the resulting gaugino mass ratios in non-minimal SUGRA by allowing some small blurring in gauge coupling unification (due to some small GUT-scale corrections) and still trying to generate large split in gaugino masses needed for lowering $\alpha_s(m_Z)$. However, at least in simplest GUT models (like $SU(5)$ and $SO(10)$) coupled to $N = 1$ SUGRA, it is probably impossible to accommodate such a large non-universality of gaugino masses while preserving almost exact gauge coupling unification. In this type of scenarios (even with general kinetic terms) the mechanism of lowering $\alpha_s(m_Z)$ presented here may play at best only a subdominant role. On the other hand, it may better fit an alternative approach in which non-zero gaugino masses are only generated below $M_X$.

Conclusions. There are a host of indications that the true value of $\alpha_s(m_Z)$ may be close to 0.11. The most popular form of the MSSM, with standard gaugino mass relations, predicts $\alpha_s(m_Z)$ at least some 10% larger. In that scenario, one would have to invoke large and negative GUT-scale corrections to rescue gauge coupling unification. This would provide a stringent constraint on GUT models while, at the electroweak scale, the predictive power would be significantly reduced.

We have made a simple observation that, even in the MSSM, one is able to obtain $\alpha_s(m_Z)$ in the vicinity of 0.11. This can be done at the expense of abandoning the usual mass relations among gaugino masses (2)–(3). One firm prediction of this approach is the existence of a relatively light gluino, $m_{\tilde{g}} \lesssim 300$ GeV and a heavy wino-like chargino, $M_2 \gtrsim 3m_{\tilde{g}}$, which can be tested in the upgraded Tevatron and LEP II. A relatively light gluino, in the ballpark of 100 GeV, may also help solving some long-lasting puzzles in $b$-quark physics. Implications for GUT-scale physics are also dramatic in the sense that this solution implies a large non-degeneracy of gaugino masses at $M_X$ in contrast with a standard approach in which GUT models are coupled to $N = 1$ supergravity.

References
[1] L. Roszkowski and M. Shifman, hep-ph/9503358, to appear in Phys. Rev. D.

[2] J. Ellis, S. Kelley, and D.V. Nanopoulos, Phys. Lett. B260 (1991) 131.

[3] R.G. Roberts and G.G. Ross, Nucl. Phys. B377 (1992) 571.

[4] L. Montanet, et al. (PDG), Phys. Rev. D50 (1994) 1173.

[5] A. Martin and D. Zeppenfeld, Phys. Lett. B345 (1995) 558; M.L. Swartz, hep-ph/9411353; S. Eidelman and F. Jegerlehner, hep-ph/9502298.

[6] P. Langacker and N. Polonsky, hep-ph/9503214.

[7] J. Erler and P. Langacker, hep-ph/9411203.

[8] F. Abe, et al. (CDF), hep-ex/9503002; S. Abachi, et al. (D0), hep-ex/9503003.

[9] R.G. Roberts and L. Roszkowski, Phys. Lett. B309 (1993) 329.

[10] G. Kane, C. Kolda, L. Roszkowski, and J. Wells, Phys. Rev. D49 (1994) 6173.

[11] V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. D47 (1993) 1093.

[12] P. Langacker and N. Polonsky, Phys. Rev. D47 (1993) 4028.

[13] P. Langacker, hep-ph/9412361 and hep-ph/9411247; S. Bethke, at Rencontres de Moriond, les Arcs, March 1995.

[14] See, e.g., F. Le Diberder and A. Pich, Phys. Lett. B286 (1992) 147.

[15] M. Shifman, Mod. Phys. Lett. A10 (1995) 605.

[16] M. Voloshin, Int. J. Mod. Phys. A10 (1995) 2865.

[17] G. Kane, R. Stuart, and J. Wells, hep-ph/9505207; G. Kane, talk at this meeting.

[18] D.-G. Lee and R.N. Mohapatra, hep-ph/9502210; B. Brahmachari and R.N. Mohapatra, hep-ph/9505347 and hep-ph/9508293.

[19] L. Roszkowski, Phys. Lett. B262, 59 (1991) and Phys. Lett. B278, 147 (1992).

[20] J. Ellis, K. Enqvist, D. Nanopoulos, and K. Tamvakis, Phys. Lett. 155B (1985) 381.

[21] T. Dasgupta, P. Mamales, and P. Nath, hep-ph/9501325; R. Arnowitt, private communication; K. Kounnas, private communication.

[22] M. Dine, A.E. Nelson, Y. Nir, and Y. Shirman, hep-ph/9507378.