Simulation study of the filamentation of counter-streaming beams of the electrons and positrons in plasmas

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Abstract
The filamentation instability (FI) driven by two spatially uniform and counter-streaming beams of charged particles in plasmas is modelled by a particle-in-cell simulation. Each beam consists of electrons and positrons. The four species are equally dense and have the same temperature. The one-dimensional simulation direction is orthogonal to the beam velocity vector. The magnetic field grows spontaneously and rearranges the particles in space, such that the distributions of the electrons of one beam and the positrons of the second beam match. The simulation demonstrates that as a result no electrostatic field is generated by the magnetic field through its magnetic pressure gradient prior to its saturation. This electrostatic field would be repulsive at the centres of the filaments and limit the maximum charge and current density. The filaments of electrons and positrons in this simulation reach higher charge and current densities than in one with no positrons. The oscillations of the magnetic field strength induced by the magnetically trapped particles result in an oscillatory magnetic pressure gradient force. The latter interplays with the statistical fluctuations in the particle density and it probably enforces a charge separation, by which electrostatic waves grow after the FI has saturated.

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(Some figures in this article are in colour only in the electronic version)
1. Introduction

If a plasma is initially free of any net current, but has a nonequilibrium particle velocity distribution, then it can support the growth of magnetic fields through the redistribution of currents in space. This has been demonstrated first by Weibel [1] for a plasma with a bi-Maxwellian electron velocity distribution. The currents are rearranged through the growing plasma waves into filaments, which are separated by electromagnetic fields [2]. This is also the case for the instability driven by counter-streaming beams of charged particles, which is commonly referred to as the beam-Weibel instability or the filamentation instability (FI) [3–6].

The FI can generate strong magnetic fields in astrophysical environments such as the leptonic pulsar winds [7]. The FIs are also important for the generation of cosmological magnetic fields [8] and for inertial confinement fusion [9, 10], where laser pulses accelerate electron beams to relativistic speeds. Previous simulation studies have revealed various aspects of the growth and saturation of the FI. The FI driven by two counter-propagating beams of electrons has been examined by using particle-in-cell (PIC) and Vlasov simulations. Such studies have been performed in one spatial dimension (1D) [11–15], in two spatial dimensions (2D) [3, 16–18] and in 3D [19]. The effects of a guiding magnetic field on counter-streaming electron beams have also been examined [18, 20]. The simulation studies in [13, 21] have investigated the impact of the ions on the nonlinear stage of the FI.

The FIs are usually triggered by the electrons. The ion filamentation is slower and often coupled through electrostatic fields to the electron filamentation. The FI involving only electrons couples to electrostatic fields during its quasi-linear growth phase. We refer to this source mechanism of electrostatic waves as the quasi-linear electrostatic instability (QEI). The electrostatic field amplitude grows in response to the QEI at twice the exponential rate of the magnetic field amplitude [12–14] and it oscillates around an equilibrium value after the FI has saturated. This equilibrium amplitude is such that it exerts a force on the electrons that equals that of the average magnetic pressure gradient force (MPGF) [15]. This was demonstrated for the case of two counter-streaming, equally dense electron beams, for which the growth rate of the purely transverse FI is highest relative to those of the competing electrostatic two-stream instability and of the partially electromagnetic mixed mode instability [22]. The omission of wavevectors aligned with the beam velocity vector is thus most realistic.

The pulsar winds carry with them positrons [7]. The impact of the positrons on the initial growth phase of the FIs is understood, in principle, by solving the linear dispersion relation, as it has been done, for example, in [23, 24, 25] for the FI and in [26] for the Weibel instability. The nonlinear evolution of instabilities driven by beams of the electrons and positrons have been modelled with PIC simulations in 2D [27, 28] and in 3D [29].

In this paper, we consider two counter-streaming plasma beams, each of which contains an identical number of positrons and electrons. The FI is modelled in a short 1D simulation box, in which only one filament pair develops, as has been done in [12, 13, 15] for the electron beams. The nonlinear saturation of the FI is not captured correctly, since we exclude the filament merging and the multi-dimensional structure of the filaments [3]. However, the filaments are not circular if the beams are warm and if they have the same density. We find spatial intervals of the filaments that are planar over several electron skin depths [17, 18, 30]. A 1D simulation can give an insight into the dynamics of such planar boundaries. The 1D geometry allows us to freeze the filament pair just after the initial saturation and we can analyse the filaments in an almost time-stationary form. We can isolate a single filament pair to better understand its dynamics by omitting its collective interactions with the neighbouring filaments. The restriction to one spatial dimension furthermore permits us to use a good statistical representation of the plasma phase space distribution and we can reduce the simulation noise. Accurate measurements of the
fields and of the phase space distribution are thus possible. We choose initial conditions, which are identical to those in [14, 15], except for the positronic beam component that we include here. This allows us to compare directly the electromagnetic interaction of two counter-propagating electron beams with that of two counter-propagating beams of the electrons and positrons.

We summarize our key results. The symmetry between the electrons and the positrons suppresses the QEI during the quasi-linear growth phase of the FI. The magnetic trapping model [11], which does not consider an electric field, accurately describes the saturation magnetic field in the simulation. The electrostatic field driven by the QEI would repel the electrons at the centre of the filament and attract those farther away [15], thus limiting the charge density accumulation due to the FI. Its absence implies that the filament confinement is stronger, if the positrons are present; higher peak density values can be reached and the spacing between the filaments is larger. The magnetic fields should reach higher spatial gradients compared with a system of counter-propagating electron beams due to the stronger currents. We could separate the purely magnetic FI driven by counter-propagating electron–positron beams from a secondary electrostatic instability (SEI), which we show to be unrelated to the QEI. The waves the SEI drives have a broadband wavenumber spectrum. We bring forward evidence for a connection between the SEI and the spatio-temporal oscillations of the MPGF. These oscillations occur on time scales which are comparable to the inverse plasma frequency and on spatial scales of the order of a Debye length. The MPGF can thus interplay through these oscillations with the statistical fluctuations of the plasma, by which a charge separation can occur. We propose that this separation breaks the initial symmetry of electrons and positrons and destabilizes the filament. The power of the waves driven by the SEI, which have a frequency that is close to the plasma frequency, grows in time. Its power can be fitted as a function of time by two exponential functions, which are separated by a break in the growth rate.

The paper is organized as follows. Section 2 briefly describes the PIC simulation method and it discusses our initial conditions. Section 3 presents our simulation results, which are then discussed in section 4.

2. The PIC simulation method and the initial conditions

The standard PIC method [31] can model the processes in a collisionless kinetic plasma. It approximates the plasma phase space distribution by an ensemble of volume elements or computational particles (CPs). The ensemble properties of the CPs are an approximation to the ensemble properties of the corresponding physical plasma species. Each CP with index \( i \) of the species \( j \) can have a charge \( q_j \) and mass \( m_j \) that differ from those of the plasma species they represent, e.g. the mass \( m_e \) and the charge \( -e \) of an electron. However, the charge-to-mass ratio must be preserved.

The CPs follow trajectories in the simulation domain, which are determined by the electric \( \mathbf{E} \) and the magnetic \( \mathbf{B} \) fields. The electromagnetic fields and the global current \( \mathbf{J} \) are defined on a spatial grid. The equations that are solved by the PIC code are

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}, \quad (1)
\]

\[
\nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{E} = \rho, \quad (2)
\]

\[
\frac{dp_i}{dt} = q_j(E[x_i] + v_i \times B[x_i]), \quad p_i = m_j v_i \Gamma(v_i), \quad \frac{dx_i}{dt} = v_i. \quad (3)
\]

The electromagnetic fields are evolved in time using the Faraday law and the Ampere law (equation (1)). Equations (2) are typically fulfilled as constraints, or they are enforced by correction steps. Our code is based on the virtual particle method [32] and fulfils Poisson’s
equation as a constraint and $\nabla \cdot \mathbf{B} = 0$ is solved exactly in 1D and to round-off precision in higher dimensions. The trajectories of the CPs are updated using equations (3). We refer the interested reader to [31–36] for a more thorough discussion of numerical PIC schemes.

Equations (1)–(3) can be scaled to physical units with the total plasma frequency $\omega_p = (n_e e^2 / m_e \epsilon_0)^{1/2}$ and the skin depth $\lambda_e = c / \omega_p$, where the total particle number density $n_t = \sum_j n_j$ is summed over the four leptonic species, which are equally dense. The quantities in physical units denoted by the subscript $p$ are obtained from the normalized ones by substituting $E_p = \omega_p c m_e E / e$, $B_p = \omega_p m_e B / e$, $J_p = e c n_e \rho_p$, $v_p = v_c$ and $p_p = m_e c p$. The charge $q_j$, in this normalization, is 1 for the positrons and $-1$ for the electrons, while $m_j = 1$. We also normalize $\Omega = \omega / \omega_p$ and $k = k_p c / \omega_p$, where $\omega$, $k_p$ have physical units.

Both beams in our simulation study move in opposite $z$-directions with the speed modulus $v_b = 0.3$, giving the relative beam speed $2v_b/(1 + v_b^2) \approx 0.55$. This $v_b$ is about half of that used in [28]. Each of the two beams consists of one electron species and of one positron species and all four species are equally dense. Initially all beams are spatially uniform. The velocity distribution in the rest frame of each beam is a Maxwellian with the thermal speed $v_t \equiv (kT/m_e)^{1/2} = v_b/18$ in all directions. The 1D simulation box with its periodic boundary conditions is aligned along the $x$-direction. Only waves with wavevectors parallel to $x$ can grow and we use the scalar wavenumber $k$. The length $L = 1.25 \lambda_e$ of the simulation box is identical to that of the shortest one in [15], if we neglect the positron contribution to $\lambda_e$. The simulation box is subdivided into 500 grid cells of equal length $\Delta_x$. The ratio $\Delta_x / \omega_p / v_t = 0.15$ and the Debye length is resolved well. Each of the four plasma species is represented by $2.45 \times 10^5$ CPs. The $E = 0$ and $B = 0$ at the simulation’s start. The total simulation time is $t_{\text{sim}} = 177$, which is subdivided into $10^4$ time steps $\Delta t$.

3. Simulation results

The FI driven by a plasma flow along $z$ and a simulation box that is aligned along $x$ will lead to the initial growth of a magnetic field along $y$. The growing net current $J_z(x)$ will also result in a growing $E_z$ by Ampere’s law. An electrostatic $E_z$ field would grow in the case of a system that is composed of two electron beams.

Figure 1 displays the energy densities of these field components, which we denote as $E_{BY}$, $E_{EZ}$ and $E_{EX}$. The energy density $E_{BY}$ of the magnetic $B_y$ component dominates and it reaches a few per cent of the total energy, in line with previous simulations [28]. The exponential growth rate of $B_y$ is $\Omega_s \approx 0.25$, which is close to the expected value $\approx v_b/\gamma_b^{1/2}$ for cold beams. The $E_{EZ}$ grows at the same rate as $E_{BY}$, but its values are three orders of magnitude less. The energy density $E_{EX}$ of the electrostatic field grows after the FI has saturated. The growth rate of $E_{EX} \propto E_{EX}^2$ can be fitted with an exponential function with the growth rate $\Omega_s \approx 0.13$ between $45 < t < 90$ and with a second, slower growing one for $t > 100$. The growth of $E_{EX}$ in figure 1 cannot be attributed to the QEI, because then the $E_{EX}$ should grow at twice the exponential rate of $E_{BY}$ until $t \approx 50$ and oscillate around an equilibrium value after that time.

We look in more detail at the fields to better understand the cause of the oscillations of $E_{BY}$ around its equilibrium and the source mechanism of the growth of $E_{EX}$. The field components driven by the FI are investigated in figure 2. The $B_y$, $E_z$ saturate at $t \approx 50$. The $B_x(x,t)$ then remains practically stationary, while $E_z(x,t)$ is damped.

We compute the spatial power spectrum $P(k,t)$ of $B_y$, $E_z$ and $E_x$ by a 1D Fourier transform over the full box length $L$ and by taking the square of the amplitude modulus. The frequency
Figure 1. The energy densities in units of the total energy in the simulation: the uppermost curve $E_{BY}$ corresponds to $B_y$. Its exponential rate, which is twice the $\Omega_i$ of $B_y$, is $\approx 0.5$ during $10 < t < 50$. It oscillates around an equilibrium after that. The curves of $E_{EZ}$ and $E_{BY}$ grow initially at the same exponential rate, but $E_{EZ}$ decreases for $t > 50$. The electrostatic $E_{EX}$ starts to grow at $t \approx 45$, when $E_{BY}$ has saturated.

Figure 2. (Colour online.) The electromagnetic fields: panels (a) and (b) show $B_y$ and $E_z$, respectively. Both are correlated and their phase difference is $90^\circ$. The $E_z$ field is damped in time. Panel (c) shows the spatial power spectrum $\log_{10} P(k, t)$ of $B_y$. The $k_1$ mode dominates, but harmonics can be seen. Panel (d) shows the frequency power spectrum $\log_{10} P(k_1, \Omega)$ of $B_y$. Peaks are found at $\Omega = 0$ and $\Omega \approx 0.3$. Panel (e) evidences a $\log_{10} P(k, t)$ of $E_z$ resembling that of $B_y$ in (c) and the $\log_{10} P(k_1, \Omega)$ of $E_z$ in (f) also has a maximum at $\Omega \approx 0.3$. The power in (c), (e) and (d), (f) is normalized to the same value.

The power spectra (dispersion relation) are obtained through a 2D Fourier transform of the field data over the box length $L$ and for $t > 50$. The amplitude moduli are then squared to give $P(k, \Omega)$. The base-10 logarithm of $P(k, t)$ of $B_y$ in figure 2(c) evidences that most power is concentrated in mode $k_1$, with $k_s = 2\pi s/L$, and that the power in this mode is oscillating. Weaker harmonics with uneven $s$ also occur. The $P(k_1, \Omega)$ of $B_y$ reveals peaks at $\Omega = 0$ and $\Omega \approx 0.3$. The peak at $\Omega = 0$ dominates, because $B_y$ is practically stationary after $t = 50$. The $P(k, t)$ and the $P(k_1, \Omega)$ of $E_z$ in figures 2(e) and (f) resemble qualitatively those of $B_y$, but they are weaker and the peak with $\Omega = 0$ is absent in the $E_z$-field.
The $E_x(x, t)$ in figure 3 reveals no spatial correlation with $B_y(x, t)$. The $P(k, t)$ of the $E_x$-field in figure 3(b) and of the $B_y$-field in figure 2(c) show no link between the $k$ of the dominant waves. The spatial power spectrum $P(k,\Omega)$ in figure 3(c) reveals that the strongest waves have a $\Omega \approx 1$.

The particle phase space distributions provide more information. The electrons of beam 1 (moves in the positive $z$-direction) are species 1 and the positrons are species 3. The electrons and positrons of beam 2 are denoted as species 2 and 4, respectively. The current fluctuations of the PIC simulations imply that $B_z, E_y \neq 0$. These fields correspond to waves that propagate in the form of the high-frequency electromagnetic modes. However, the peak value of $E_{EZ}$ exceeds the energy densities of $E_y, B_z$ by six orders of magnitude and the latter remain practically constant during the simulation time. We can restrict our investigation to the phase space projections $f(x, v_x)$ and $f(x, v_z)$, because $B_x = 0, B_z \approx 0$ and $E_y \approx 0$. The $f(x, v_z)$ will reveal the electromagnetic structures, while the electrostatic structures are represented by $f(x, v_x)$. Figure 4 shows the phase space distributions of species 1, 2 and 4 at $t = 50$. The electrons of beam 1 and 2 are separated in space, which is typical of the F1 driven by counter-streaming electron beams. Species 4 shows an $f(x, v_z)$ that is identical to that of species 1. The $f(x, v_z)$ distribution of species 1 can be mapped onto that of species 4 by switching the sign of $v_z$. Both these observations are expected, because in the absence of a significant $E_z$ the opposite speed and charge of both species cancel. The Lorentz force thus displaces both species in the same way. The relation is also the same between species 2 and 3. The high degree of symmetry is also demonstrated by movie 1 (available at stacks.iop.org/PPCF/51/065015), which animates in time the projected phase space distributions of species 1, which is added to that of species 4. The colour scale shows the base-10 logarithm of the number of CPs. Figure 5 displays the number densities $N_i(x) = \int f_i(x, v_x) \, dv_x$ for each of the species $i$, normalized to the mean value $N_0 = \langle N_i(x) \rangle$. The $N_1 = N_4$ and $N_2 = N_3$ within the resolution of the image. The charge density modulations reach a peak value $\approx 4N_0$. This peak value is...
Figure 4. (Colour online.) The base-10 logarithm of the phase space densities at the time $t = 50$ in units of CPs: panels (a)–(c) display the $f(x, v_z)$ of species 1, 2 and 4 (positrons). (d)–(f) show the $f(x, v_x)$ corresponding to the panels above them. The distributions in (d), (f) are practically identical and (a), (c) can be mapped onto each other by changing the sign of $v_z$. The $f(x, v_x)$ of species 1 and 2 are shifted by $L/2$.

Figure 5. Panel (a): the number density distribution $N_j(x, t = 50)$ of the four species $j$ in units of their mean density $N_0$. The curves $N_1(x)$ and $N_2(x)$ match (their differences amount to less than the thickness of the curves) and they peak at $x = 0$. The also matching $N_3(x)$ and $N_4(x)$ have their maximum at $x = L/2$. Panel (b) plots $\Delta N = \sum_j q_j N_j(x)/4N_0$, with $q_j = 1$ and $-1$ being the positron and electron charge. The fluctuations amount to less than $10^{-2}$ and they are by their scale size $\approx\Delta x$ the statistical fluctuations due to the finite number of CPs per cell.

higher by a factor 3 and, accordingly, the filament confinement is thus stronger here than in [15], which did not consider positrons. Each species is represented by $N_p = 4.9 \times 10^4$ CPs per cell. The statistical fluctuations of the particle number are thus $N_p^{-0.5} \approx 5 \times 10^{-3}$, which is comparable to the measured charge density fluctuations $\Delta N = (4N_0)^{-1} \sum_i q_i N_i < 10^{-2}$ in figure 5(b). This observation, together with the fact that the $\Delta N$ oscillates on a scale $\Delta x$, implies that the fluctuations at $t = 50$ are due to the finite number of CPs per cell. These fluctuations have a $k$-spectrum that is qualitatively similar to that of thermal noise. The fluctuation amplitude $\Delta N$ is increased within the filaments, e.g. in the interval $0.5 < x < 0.75$, because we have not normalized it to the local particle number, but to the average density.

Figure 6 displays the phase space distributions of species 1, 2 and 4 at $t = 177$. The $f(x, v_z)$ are qualitatively unchanged compared with those in figure 4 and species 1 and 4 are still symmetric to a change in the sign of $v_z$. The distributions $f(x, v_x)$ reveal that the particles
Figure 6. (Colour online.) The base-10 logarithm of the phase space densities at the time $t = 177$ in units of CPs: panels (a)–(c) show $f(x, v_z)$ of species 1, 2 and 4, respectively. (d)–(f) show the $f(x, v_x)$ corresponding to the panels above. Species 1 and 4 reveal similar $f(x, v_z)$ and distributions $f(x, v_x)$ that are qualitatively symmetric to a change in the sign of $v_z$. The $f(x, v_x)$ of species 1 and 2 are shifted by $L/2$.

Figure 7. (Colour online.) Panel (a): the $N_j(x)$ at the time $t = 177$ of the four species $j$ in units of $N_0$. The electron distributions $N_1(x)$ and $N_2(x)$ are denoted by the solid curves and $N_3, N_4$ are dashed and blue. The curves $N_1(x)$ and $N_4(x)$ peak at $x = 0$. The curves for electrons and positrons do not match. Panel (b) plots the $\Delta N = \sum_j q_j N_j(x)/4N_0$, with $q_1 = 1$ and $-1$ being the positron and electron charge. The fluctuation amplitude is about 10% and they oscillate on scales $\gg \Delta x$.

are concentrated at the same positions as in figure 4. The particles have been heated up along the $x$-direction in between the dense filaments. The small scale structures, in particular in $f(x, v_z)$, differ for species 1 and 4 and we expect now clear charge density modulations. Figure 7 compares $N_1(x)$ with $N_4(x)$ and $N_2(x)$ with $N_3(x)$. Their differences can now be seen even from the $N_j$. The value $N_2(x = 0.08)$ exceeds that of $N_1(x = 0.08)$ by about 0.4 and fluctuations of this size occur in the entire box, which is demonstrated by the $\Delta N$ in figure 7(b). The typical length scale of the oscillations is about 0.1 $\gg \Delta x$ and they are caused by the waves in figure 3.

The peak amplitude of $E_x$ in figure 3 is $\approx 5 \times 10^{-3}$ at $t = 177$. The Lorentz force $v_b B_y$ for $v_b = 0.3$ and a maximum $B_y \approx 0.07$ at $t = 177$ (figure 2) is larger than the electrostatic force by a factor of 4. The electrostatic force is weaker before this time, while $B_y$ is constant after $t = 50$. Thus, the electrons experience for most of the simulation duration $E_x$ only as a perturbation and the gyrofrequency of the CPs is determined by $B_y$ and $v_z \approx v_b$, which is also demonstrated by movie 1 (available at stacks.iop.org/PPCF/51/065015). The rotation of
the dense filament in the distribution $f(x, v_x)$ has an approximately constant angular velocity in phase space. The structures in $f(x, v_x)$ and, thus, $J_z$ will change with this characteristic frequency and impose the modulation of $B_y$ and $E_z$ with $\Omega \approx 0.3$ in figures 2(d) and (f). This frequency is $\approx \Omega_{1i}$ and consistent with the magnetic trapping.

The charge density fluctuations in figure 5(b) at $t = 50$ were at noise levels, which was expected from the low $E_{EX}$ in figure 1 at this time and no QEI occurred. It is only after the FI has saturated that the electrostatic waves grow due to the SEI. The SEI yields the growth of $E_{EX}$ in the interval $50 < t < 177$ and the waves in figure 3 have no obvious correlation with the $B_y$ in figure 2. We will now identify one potential cause of the SEI. The force imposed by a magnetic pressure gradient on a current $J$ is

$$J \times B = -\nabla B^2/2.$$ (4)

Only $B_y$ is growing in our 1D simulation box and the only possible spatial derivative is $\partial_x$. Equation (4) simplifies to $J_y B_z = B_y \partial_x B_y/\partial x$. The $B_y$ is strong for $t > 50$ and $\partial_x B_y \neq 0$. The MPGF on the right-hand side does, therefore, not vanish and the particles are accelerated.

Figure 8 shows the MPGF after $t = 50$ and $A(x, \Omega)$, which is its spatial frequency spectrum obtained by a Fourier transform over time. The maxima and minima of $\partial_x P_{BF}$ are stationary in space but its magnitude oscillates in time. The time average of the MPGF is positive to the right of the positions $x = 0$ and $x = L/2$ and negative to the left of these positions. The $x = 0$ and $x = L/2$ coincide with the stable equilibrium points of the respective filaments. The MPGF is thus accelerating the particles away from the equilibrium points. The MPGF is, however, weaker than the drift force $q J_y v_B B_j$, which is responsible for the filament confinement (magnetic trapping).

Consider the filament formed by species 2 and 3 at $t = 50$, which is centred at $x = L/2$ in figures 5 and 8(a). The electrons of species 2 move with the velocity $\approx -v_B z$, while the positrons of species 3 move with $\approx v_B z$. Their currents have the same sign and the MPGF accelerates the electrons and the positrons in the same $x$-direction. As long as the positrons and the electrons have the same density everywhere, the $E_z$ does not grow, because $J_z = 0$.
in \( \partial_t E_x + J_x = 0 \). The term \( \partial_y B_z - \partial_z B_y = 0 \) in the 1D geometry. The MPGF thus does not result here in the \( E_x \)-field discussed in [13, 15], which has a wavelength that is half of that of the wave in \( B_y \). However, the finite number of CPs introduces statistical fluctuations in the charge density (figure 5) that imply that the MPGF accelerates locally (on Debye length scales) a different number of electrons and positrons, by which \( J_x \neq 0 \). An electric field grows, which tries to restore the charge neutrality. The MPGF can couple to these fluctuations, if the force gradient is high (comparable to the spatial scale of the fluctuations) and if the force oscillates with the characteristic frequency of the fluctuations. Their dominant oscillation frequency in the spatial intervals with \( B_y \approx 0 \) is the plasma frequency and it is the upper-hybrid frequency otherwise. Figure 3(c) shows that the growing fluctuations have a broad frequency band and peak at \( \Omega \approx 1 \). The spatial gradients of the plasma density will, however, influence the fluctuation spectrum [37]. A force interacting with such fluctuations should thus also have a broad frequency band.

The spectrum \( A(x, \Omega) \) in figure 8(b) reveals that the highest frequencies can be reached by the MPGF at \( x \approx 0.1, 0.5, 0.7 \) and at 1.1, although not with a high driving amplitude. The positions close to \( x = L/4 \) and \( x = 3L/4 \) experience stronger oscillations up to \( \Omega = 1 \). The MPGF also changes on spatial scales comparable to the Debye length, which is \( v_b/c = 1/60 \) in our normalization. An example is here a change at \( t \approx 70 \) and \( x \approx 0.325 \) in figure 8(a). The MPGF changes from 0.02 to -0.02 over a distance \( \approx 0.03 \). The connection between the growth of \( E_{EX} \) and the oscillations of the MPGF is also evidenced by a comparison of figure 1 with figure 8(a). The growth of \( E_{EX} \) slows down at \( t \approx 100 \). The oscillations in figure 8(a) close to \( L/4 \) and \( 3L/4 \) are more intense and shorter in duration before \( t = 100 \) and the spatial gradients are higher. The MPGF can thus couple easier energy to the charge density fluctuations. The spatio-temporal oscillations of \( \partial_x \tilde{P}_{BY} \) soften up after \( t = 100 \), and the growth of \( E_{EX} \) slows down.

4. Discussion

In this paper we have considered the FI driven by two counter-streaming beams, each consisting of the electrons and positrons. The beams have initially been spatially uniform and the electromagnetic fields were set to zero. The 1D simulation is aligned along the \( x \)-direction, which excludes the merging of filaments beyond a certain size and limits the physical realism of its nonlinear evolution [3]. However, the filaments are usually not circular but elongated [17, 18, 30]. A 1D geometry may thus be a valid approximation for those parts of filaments, which are quasi-planar such as the one investigated in [18]. The two beams move in opposite \( z \)-directions at the same speed modulus \( v_b = 0.3 \), which is sufficiently low to exclude significant relativistic effects. It is sufficiently high to obtain a linear growth rate of the FI that is comparable to those of the electrostatic modes and of the mixed modes. This may be further aided by the equal beam densities, which favour the FI if positrons are absent [22]. The strong electric fields with their oblique polarization in the PIC simulation in [28] demonstrate, however, that in particular the mixed modes would compete with the FI in a more realistic 2D or 3D simulation.

Our aim has been to obtain further insight into the dynamics of a filament pair formed by two counter-propagating beams of the electrons and positrons and to compare it with that of an electron filament pair. The simulation parameters are identical to those in the simulation in [15] that did not take into account the positrons. The short simulation box allows us to use a good statistical plasma representation and low noise levels. The charge density fluctuations inherent to the PIC simulation method provide noise over the full band of wavenumbers resolved by the simulation, out of which the wavemodes and any secondary instabilities can grow.
The FI redistributes the beams of charged particles in space into current filaments [3]. Our simulation box length $L$ and the periodic boundary conditions allow only a single pair of filaments to grow. The centres of the electron filaments are spatially separated by the distance $L/2$ and this is also the case for both positron filaments. The filament formed by the electrons of one beam coincides with the filament containing the positrons of the second beam. Their phase space distributions $f(x, v_x)$, which hold the information about the electrostatic structures, match and their phase space distributions $f(x, v_z)$ can be mapped from one to the other by a change in the sign of $v_z$. The currents and densities of both components add up. The symmetry between the electrons and positrons within the same filament implies that the MPGF accelerates them in the same direction. We have found that their partial currents cancel each other until the FI saturates, implying that no $E_x$ can grow due to the QEI.

The simulation in [15] evidenced that this $E_x$ would accelerate the electrons away from the centre of the filament. The presence of the positrons in the simulation in this work removes this repulsion. Higher charge and current densities can be reached by electron–positron beams compared with those containing only electrons and the spacing between the filaments is larger. The magnetic fields can reach higher spatial gradients. The magnetic fields grew to an amplitude set by magnetic trapping [11].

We have identified a SEI that is leading to a growing electrostatic energy density after the FI saturated, resulting in a broadband wave spectrum. The exponential growth rates of these waves remained well below that of the FI and they show no correlation with the MPGF. These observations are conflicting with the properties attributed to the QEI, implying that the SEI and the QEI must have different source mechanisms.

The finite number of CPs imply statistical fluctuations of the charge density on Debye length scales. We have found here that the oscillation spectrum of the MPGF involves a wide band of frequencies that reach a maximum that exceeds the plasma frequency and that the MPGF changes significantly on scales comparable to the Debye length of the plasma after the FI has saturated. The MPGF can thus couple to the statistical fluctuations of the charge density. We have proposed that this coupling amplifies the fluctuations of $J_x$ and $E_x$ and that this is the cause of the SEI.

This hypothesis can be tested and the properties of the SEI can be examined in more detail with Vlasov simulations. They solve the Vlasov–Maxwell equations directly and do not approximate the plasma by phase space blocks. They are thus free of noise due to statistical plasma density fluctuations. The SEI should not develop in Vlasov simulations, unless seed fluctuations are introduced. The filaments, which are plasma structures resulting out of magnetic instabilities, would be more stable in Vlasov simulations than in the PIC simulations. Previously, such an enhanced stability has only been reported for nonlinear structures (electron phase space holes) evolving out of purely electrostatic Buneman instabilities [38]. A localized charge density perturbation can be introduced into a Vlasov simulation and its interplay with the MPGF can be investigated. Such studies cannot be performed with PIC codes, where we always have an ensemble of charge density perturbations interacting with the MPGF.

The energy density $E_{EX}$ of the waves driven by the SEI remained below $10^{-2} E_{BY}$. The $E_x$-field will, however, not be negligible after $t = 177$, because the electric force $q_j E_x$ competes with the Lorentz force $q_j v_j B_j$ and $v_j < 1$. The ratio between the strongest Lorentz force and the strongest electric force has been 4 at $t = 177$ and the instability is growing. The SEI is also likely to play a more important role in the PIC simulations that use only a low number of particles per cell. The higher relative charge density fluctuations imply, that the SEI grows from higher initial amplitudes, which reduces its growth time. The initial electric
noise power scales approximately inversely proportional to the number of particles per cell. A reduction from our $4.9 \times 10^4$ particles per cell to 50 particles per cell will increase the initial power of $E_f$ by the factor $10^3$.

The energy density of the electric field has grown to a significant fraction of that of the magnetic field in the 1D and 2D simulations of counter-propagating electron beams [12–15, 18], and it should influence the interplay of the filaments during their nonlinear evolution. No significant electric fields are observed during the quasi-linear growth phase of the FI and immediately after its saturation, if each beam carries electrons and positrons with an equal density. Our future studies will thus assess the impact of an absent electric field on the filament size distribution. This distribution can be approximated by a Gumbel distribution in 1D if no positrons are present [14]. We will also examine with 2D simulations how the filament size distribution orthogonal to the beam velocity vector evolves in time when positrons are present. The characteristic filaments size increases in the absence of the positrons linearly with time [17]. The impact of the electrostatic and mixed modes on the evolution of the filaments will also be addressed by 2D simulation studies that contain the beam velocity vector.

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