Abstract A stable cyclicity of correlation coefficients $K_{corr}$ for some solar activity indices versus $F_{10,7}$ was found after monthly averages values analysis. These indices are: Wolf numbers, 10,7 cm radio flux $F_{10,7}$, 0,1-0,8 nm background, the total solar irradiance, Mg II UV-index (280 nm core to wing ratio) and counts of flares. The correlation coefficients of the linear regression of these solar activity indices versus $F_{10,7}$ were analyzed for every year in solar cycles 21 - 23. We found out that the values of yearly determined correlation coefficients $K_{corr}$ for solar activity indices versus $F_{10,7}$ show the cyclic variations with stable period closed to half length of 11-year cycle (5,5 years approximately).

Key words: Solar cycle, Observations, Solar activity indices

1 Introduction

We have studied monthly averaged values of some global solar activity indices in cycles 21, 22 and 23. We analyzed the observed data of Wolf numbers W, 10,7 cm radio flux $F_{10,7}$, the X-ray 0,1-0,8 nm background, the UV Mg II 280 nm core to wing ratio, the total solar irradiance and counts of flares. Most of these observed data we used in our paper were published in Solar-Geophysical Data bulletin. All the indices studied in this paper are very important not only for analysis of solar radiation formed on the different altitudes of solar atmosphere, but also for solar-terrestrial relationships as the key factors of the solar radiation influence on the different layers of terrestrial atmosphere.

In this paper we studied the interconnection between activity indices characterized the solar fluxes from different atmosphere’s layers in cycles 21 - 23. The unusual cycle 23 was examined carefully: the rise, decline, minimum and maximum phases were studied separately. This made it possible to determine that correlation coefficients of linear regression $K_{corr}$ have the different values for different cycle’s phases.
(Vitinsky et al., 1986) analyzed solar cycles 18, 19 and 20 and pointed out that correlation for spot numbers versus radio flux $F_{10,7}$ does not show the linear behavior during all the activity cycle. Also it was emphasized the importance of statistical study in our understanding of solar activity. To achieve to best agreement in approximation of spot numbers by $F_{10,7}$ observations (Vitinsky et al., 1986) proposed to approximate the dependence $W - F_{10,7}$ by two linear regressions: the first one - for the low solar activity ($F_{10,7}$ less than 150) and the second one - for the high activity ($F_{10,7}$ more than 150).

In our paper we found out than the linear correlation was violated not only for maximums of solar activity cycles but for minimums of the cycles too.

In this paper we also analyzed the yearly determined correlation coefficients $K_{corr}$ for solar activity indices versus $F_{10,7}$.

![Figure 1: Wolf numbers from direct observations (1850 to 2005) and indirect estimated data (1750 to 1849). Solar activity cycles 1 - 23. The smoothed curve enveloping the cycle’s maximums shows how the century cyclicity exerts influence on spots number values](image)

2 Solar activity indices in the cycle 23

The recent solar cycle 23 was the outstanding cycle for authentic observed data from 1849 year. It lasted 12.7 years and was the longest one for two hundred years of direct solar observations. This cycle is the second component in the 22-year Hale magnetic activity cycle but the 23rd cycle was
the first case of modern direct observations (from 1849 to 2008 years) when Gnevyshev-Ohl rule was violated: activity indices in cycle 23 had their maximum values less then the values in cycle 22 (but according to Gnevyshev-Ohl rule the cycle 23 must dominate), see Figure 1.

Then we have to say a few words about solar indices studied in this paper.

The Wolf numbers is a very popular, widely used solar activity index: the series of Wolf numbers observations continue more than two hundred years.

The solar radio microwave flux at wavelengths 10.7 cm $F_{10,7}$ has also the longest running series of observations started in 1947 in Ottawa, Canada and maintained to this day at Penticion site in British Columbia. This radio emission comes from high part of the chromosphere and low part of the corona. $F_{10,7}$ radio flux has two different sources: thermal bremsstrahlung (due to electrons radiating when changing direction by being deflected by other charged particples) and gyro-radiation (due to electrons radiating when changing direction by gyrating around magnetic fields lines). These mechanisms give rise to enhanced radiation when the temperature, density and magnetic fields are enhanced. So $F_{10,7}$ is a good measure of general solar activity.

The Mg II 280 nm is important solar activity indicator of radiation. (Scupin et al., 2005) showed that the Mg II index derived from daily solar observations of the core-to-wing ratio of the Mg II doublet at 279.9 nm provides a good measure of the solar UV variability and can be used as a reliable proxy to model extreme UV (EUV) variability during the solar cycle. The Mg II observation data were obtained from several satellite’s (NOAA, ENVISAT) instruments. NOAA started in 1978 (during the 21st, 22nd and the first part of the 23rd solar activity cycles), ENVISAT was launched on 2002 (last part of the 23rd solar activity cycle). Comparison of the NOAA and ENVISAT Mg II index observation data shows that both the Mg II indexes agree to within about 0.5%. We used both the NOAA and ENVISAT Mg II index observed data.

Data of GOES observations of the X-ray 0.1-0.8 nm background were taken from Solar Geophysical Data bulletin. This permanent monitoring of solar disk at the 0.1-0.8 nm range is a good indicator of solar corona activity without flares.

Data of the total solar irradiance SOHO observations for the 23rd cycle were taken from Solar-Geophysical Data bulletin. Earlier from 1985 to 2000 the total solar irradiance was observed by Earth Radiation Budget Satellite (EBRS).

We also analyzed rapid processes on the Sun - monthly counts of grouped solar flares (according Solar-Geophysical Data the term ‘grouped’ means observations of the same event by different sites were lumped together and
counted as one).

Figure 2: The monthly averages for (a) Wolf numbers, (b) $F_{10.7}$ radio flux (low corona), (c) Mg II core to wing ratio (chromosphere), (d) total irradiance (photosphere), (e) counts of flares and (f) 0.1-0.8 nm background (corona) in cycles 21, 22 and 23.

Figures 2 demonstrates that for all activity indices (monthly average values of the Wolf numbers, 10.7 cm radio flux $F_{10.7}$, total solar irradiance, UV MgII 280 nm flux, count of flares and 0.1 - 0.8 nm background in the 23rd solar activity cycles one can see two maximums separated one from another on 1.5 year approximately. We see the similar double-peak structure for in cycles 22 and 23 but for cycle 21 the double-peak structure is not evident. We see that there are displacements in both maximum occurrence time of all
Table 1: Correlation coefficients for activity indices versus $F_{10,7}$ at rise phase, decline phase and maximum phases of the cycle 23

| Activity Indices | Rise Phase of 23$^{rd}$ Cycle | Decline Phase of 23$^{rd}$ Cycle | Maximum Phase of 23$^{rd}$ Cycle | All Over the Cycle 23 |
|------------------|-------------------------------|----------------------------------|----------------------------------|-----------------------|
| W - $F_{10,7}$   | 0.919                         | 0.961                            | 0.742                            | 0.939                 |
| Mg II - $F_{10,7}$ | 0.963                         | 0.964                            | 0.757                            | 0.879                 |
| Total Sol Irr - $F_{10,7}$ | 0.879                         | 0.949                            | 0.743                            | 0.920                 |
| 0.1-0.8nm - $F_{10,7}$ | 0.899                         | 0.814                            | 0.773                            | 0.812                 |
| Flares Counts/10 - $F_{10,7}$ | 0.905                         | 0.895                            | 0.785                            | 0.890                 |

these indices in the 23$^{rd}$ solar cycle.

(Ishkov, 2009) pointed that in this unusual cycle 23 near the 2$^{nd}$ maximum the monthly averages for Wolf number values during 8 months exceeded 113 and also noted that near the 2$^{nd}$ maximum the most of sunspot groups were less in size (than near the 1$^{st}$ maximum), their magnetic fields were less composite and characterized by the greater lifetime.

The double-peak structures we see in the 23$^{rd}$ cycle clearly. The probable reason of such double-peak structures is a modulation of the 11-year fluxes variations by both of the quasi-biennial and 5.5 year cyclicity. The different time of 1$^{st}$ and 2$^{nd}$ maximum appearance may be caused by the difference in fluxes formation conditions (for our indices) at different atmosphere’s altitudes of the Sun.

Figures 2 also shows that for all solar indices in the cycle 23 the relative depth of the cavity between two maximums is about 10 – 15%.

In cycle 23 the $F_{10,7}$ radio flux and the total solar irradiance have the lowest values from 2007 to 2009 (the beginning of the cycle 24) all over of these indices observation period.

When studied six activity indices in 23$^{rd}$ solar activity cycle we separate out rise phase (from October 1997 to November 1997), cycle maximum phase (from November 1997 to July 2002) and decline phase (from Jul 2002 to Jan 2006).

The results of correlation study of solar activity indices versus $F_{10,7}$ in cycle 23 are demonstrated in Table 1.

Table 1 demonstrates that the maximum values of correlation coefficients $K_{corr}$ reached for the rise and decline phases of the cycles. According to our calculations the highest values of correlation coefficients $K_{corr}$ we see in connection between W and $F_{10,7}$. Correlation coefficients $K_{corr}$ between 0.1-0.8 nm background flux versus $F_{10,7}$ are minimal of all correlation coefficients determined here. The values of linear regression coefficients also differed among themselves for the different cycle’s phases, see Figure 3.
Figure 3: Correlation between monthly averages of solar indices versus $F_{10.7}$ radio flux in cycle 23. (a) Wolf numbers, (b) total irradiance, (c) Mg II core to wing ratio and (d) 0.1 - 0.8 nm background

The cyclic variation of fluxes in different spectral ranges and lines at the 11-year time scale are widely spread phenomenon for F, G and K stars (not only for Sun).

The chromospheric activity indices (radiative fluxes at the centers of the H and K emission lines of Ca II - 396.8 and 393.4 nm respectively) for solar-type stars were studied during HK project by (Baliunas et al., 1995) at Mount Wilson observational program during 45 years, from 1965 to the present time. Authors of the HK project supposed that all the solar-type stars with well determined cyclic activity about 25% of the time remain in the Maunder minimum conditions.

Some scientists proposed that the solar activity in following cycles will be very low similar to activity in the Maunder minimum period. Unlike this mention (Volobuev, 2009) predicted the main parameters of the new 24th cycle as usual activity cycle’s parameters not similar to the cycles during Maunder minimum. At Figure 1 we also see (if we continue the smoothed curve) the influence of the century cyclicity to the 24th cycle’s maximum.
3 Changed relation between activity indices and $F_{10,7}$ in cycles 21-23

Figure 4: Correlation between monthly averages for solar indices versus $F_{10,7}$ radio flux in cycles 21 - 23. (a) Wolf numbers, (b) Mg II core to wing ratio, (c) total irradiance and (d) counts of flares.

We analyzed the interconnection between activity indices $W$, Mg II core to wing ratio, total solar irradiance and counts of flares versus $F_{10,7}$ for the 21st, 22nd and 23rd solar cycles and calculated yearly averaged values of this linear regression correlation coefficients $K_{corr}$.

Figure 4 demonstrates correlation between solar indices and radio flux $F_{10,7}$ in cycles 21 - 23 (Wolf numbers, Mg II UV-index, total solar irradiance and counts of flares).

We see that close correlation between activity indices takes place for cycles 21, 22 and 23. This interconnection corresponds to the linear regression equation:

$$F_{ind}^{cycle} = a_{ind}^{cycle} + b_{ind}^{cycle} \cdot F_{10,7}$$

were $F_{ind}^{cycle}$ is the activity index flux, $a_{ind}^{cycle}$ is the intercept of linear regression,
Table 2: Coefficients of linear regression (intercept - $a_{\text{cycle ind}}$ and slope - $b_{\text{cycle ind}}$) for solar activity indices versus $F_{10,7}$.

| activity indices | cycle 21 | cycle 22 | cycle 23 | error $\sigma$ |
|------------------|----------|----------|----------|----------------|
| $W_{\text{cycle}}$ | $a = -66.4$ | $a = -56.43$ | $a = -67.27$ | $\sigma_a = 3.0$ |
|                  | $b = 1.11$  | $b = 1.012$  | $b = 1.001$  | $\sigma_b = 0.02$ |
| $F_{\text{MgII}}$ | $a = 0.257$ | $a = 0.256$ | $a = 0.257$ | $\sigma_a = 5.25E-04$ |
|                  | $b = 1.20E-04$ | $b = 1.09E-04$ | $b = 1.20E-04$ | $\sigma_b = 4.0E-06$ |
| $F_{\text{tot.sol Irr}}$ | $a = 1364.312$ | $a = 1364.044$ | - | $\sigma_a = 0.058$ |
|                  | $b = 0.011$ | $b = 0.013$ | - | $\sigma_b = 4.4E-04$ |
| $F_{\text{fl.counts/10}}$ | $a = -35.38$ | $a = -21.33$ | $a = -25.88$ | $\sigma_a = 2.05$ |
|                  | $b = 0.58$ | $b = 0.39$ | $b = 0.33$ | $\sigma_b = 0.014$ |

$b_{\text{cycle ind}}$ is the slope of linear regression.

The results from Table 2 show that coefficients of linear regression ($a_{\text{cycle ind}}$ and $b_{\text{cycle ind}}$) for solar activity indices versus $F_{10,7}$ differ for different activity cycles (21 - 23). This difference of regression coefficients is the most significant for cycle 23, see also Figure 4, where one can compare the linear regressions for cycles 21 - 23.

(Ishkov, 2009) pointed that there was very high level of flared activity in cycle 21 and very low level of flared activity in cycle 23. We see (Table 2) that the difference of $a_{\text{fl.counts/10}}^{23}$ and $b_{\text{fl.counts/10}}^{23}$ values from these values, determined for cycles 21 and 22, is more significant among all the different cycle’s coefficients of regression. Figure 4 also demonstrates that the flared activity in the 23rd cycle almost twice weaker (count of flares versus $F_{10,7}$) in comparison to 21st cycle.

We have to point out that close interconnection between radiation fluxes characterized the energy release from different atmosphere’s layers is the widespread phenomenon among the stars of late-type spectral classes. (Bruevich and Alecseev, 2007) confirmed that there exists the close interconnection between photospheric and coronal fluxes variations for solar-type stars of F, G, K and M spectral classes with widely varying activity of their atmospheres. It was shown that the summary areas of spots and values of X-ray fluxes increase gradually from the sun and HK project stars with the low spotted discs to the highly spotted K and M-stars for which (Alecseev and Gershberg, 1996) constructed the zonal model of the spots distribution.

We’ve calculated yearly averaged values $K_{\text{corr}}$ of linear regression for solar activity indices versus $F_{10,7}$ for cycles 21, 22 and 23. Figure 5 demonstrates the results of yearly calculations of these solar activity indices versus $F_{10,7}$ $K_{\text{corr}}$ variations during the cycles 21 - 23. We can see that all the $K_{\text{corr}}$ values
Figure 5: Yearly calculated correlation coefficients of linear regression $K_{corr}$ for (a) Wolf numbers, (b) total irradiance, (c) counts of flares and (d) Mg II UV-index versus $F_{10.7}$ in solar cycles 21, 22 and 23.

have the maximum values at rise and at decline phases. We see the cyclic behavior of $K_{corr}$ yearly values in these solar activity cycles and estimated the value of period of $K_{corr}$ cyclic variations as 5.5 years approximately. We assumed that this new cyclicity (characterized with period’s value equal to half length of 11-year cycle) is very important for the successful forecasts of the global activity indices.

The cyclic behavior of $K_{corr}$ can be explained by next assumption: we imagine that activity index flux depends on time $t$ (similar to two-component models conceptions) by the following expression:

$$F_{ind}(t) = F_{ind}^{background}(t) + \Delta F_{ind}^{AR}(t)$$

were $F_{ind}^{background}(t)$ is the background flux rising continuously with increasing of solar activity and $\Delta F_{ind}^{AR}(t)$ is the additional flux to the overall flux from the active regions.

The previous correlation study allows us to consider that $F_{ind}^{background}(t)$ and $\Delta F_{ind}^{AR}(t)$ are the linear functions of the background and activity regions level of solar activity. In our case we choose the radio flux $F_{10.7}$ as the best indicator of solar activity level:
\[ F_{\text{ind}}^{\text{background}}(t) = a_1 + b_1 \cdot F_{10,7}^{\text{background}}(t) \]

\[ \Delta F_{\text{ind}}^{\text{AR}}(t) = a_2 + b_2 \cdot \Delta F_{10,7}^{\text{AR}}(t) \]

The coefficients \(a_1\) and \(b_1\) vary from \(a_2\) and \(b_2\) in different power for our different activity indices. For Wolf numbers this difference is small, but for 0.1 - 0.8 nm background and count of flares the difference between \(a_1\), \(b_1\) and \(a_2\), \(b_2\) is significant enough.

During the rise and decline cycle’s phases the dependence \(F_{\text{ind}}(t)\) versus \(F_{10,7}(t)\) is approximately linear because coefficients \(a\) and \(b\) from Table 2 (which described all the cycle) are close to \(a_1\) and \(b_1\) (see Figure 3 and Figure 4) and relative addition flux from active regions \(\Delta F_{\text{ind}}^{\text{AR}}(t)\) is neglect with respect to \(F_{\text{ind}}^{\text{background}}(t)\). So additional flux from active regions cannot destroy a balance in the close linear correlation between \(F_{\text{ind}}(t)\) and \(F_{10,7}(t)\) and respective values of \(K_{\text{corr}}\) reach the maximum values from all the cycle.

During the minimum of activity cycle both values \(F_{\text{ind}}^{\text{background}}(t)\) and \(\Delta F_{\text{ind}}^{\text{AR}}(t)\) are small, but additional flux from active regions is not neglect in relation to background flux that has the minimum values from all the activity cycle. Therefore values \(a\) and \(b\) from Table 2 cannot describe the linear regression in cycle’s minimum and values of \(K_{\text{corr}}\) reach the minimum values in the cycle.

During the maximum of activity cycle \(\Delta F_{\text{ind}}^{\text{AR}}(t)\) often exceeds \(F_{\text{ind}}^{\text{background}}(t)\) so disbalance in linear regression between activity indices increases and values of \(K_{\text{corr}}\) reach the minimum values in the activity cycle too.

### 4 Conclusions

For a long time the scientists were interested in the simulation of processes in the earth’s ionosphere and upper atmosphere. It’s known that the solar radiance at 30.4 nm is very significant for determination of the Earth high thermosphere levels heating. (Lukyanova and Mursula, 2011) showed that the for solar 30.4 nm radiance fluxes forecasts (very important for Earth thermosphere’s heating predictions) there were more prefer to use Mg II 280 nm observed data unlike usual \(F_{10,7}\) observations. (Bruevich and Nusinov, 1984) for these purposes have developed a two-component model of solar emission in the EUV range (10 - 105 nm). It was shown that the intensity ratios of individual spectral lines depend only on the solar activity level.

The close interconnection between activity indices make possible new capabilities in the solar activity indices forecasts. For the successful forecasts of maximum values and other parameters of future activity cycles it has
been required to take into account the century component as was shown in forecasts developed by (Volobuev, 2009). At Figure 1 we see the smoothed curved described the maximums of cycles which shows the century cyclicity dependence on Wolf numbers cycle’s maximums.

In this paper we found out the cyclic behavior of yearly values of correlation coefficients $K_{corr}$ of linear regression for W, total solar irradiance, Mg II 280 nm and counts of flares versus $F_{10.7}$ during solar activity cycles 21,22 and 23 (see Figure 5). Since we show that yearly values of $K_{corr}$ have the maximum values at the rise and decline phases so the linear connection between indices is more strong in these cases. It means that the prognoses of solar indices, based on $F_{10.7}$ observations, will be more successful at rise and decline cycle’s phases.

We also determined that the yearly values of $K_{corr}$ are characterized by cyclic variations with the period that is equal to half length of period at 11-year time scale (5.5 years about). Our study of linear regression between solar indices and $F_{10.7}$ confirms the fact that at minimum and at maximum cycle’s phases the nonlinear state of interconnection between solar activity indices (characterized the energy release from different layers of solar atmosphere) increases.

**Acknowledgements** The authors thank the RFBR grant 09-02-01010 for support of the work.

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