Nonlinear acoustics determination of phase characteristics of PVDF membrane hydrophones

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Abstract. When an ultrasonic pressure wave propagates through a nonlinear medium, the relative phasing of the generated harmonics causes a distinct asymmetry between the positive and negative pressure levels and between the rise and fall time of examined waveforms. A faithful quantitative reproduction of the source transducer's pressure field requires amplitude and phase measurements by calibrated hydrophone probes. Nonlinear hydrophone calibration provides amplitude and phase information at discrete multiples of an acoustic source's fundamental frequency. Two PVDF bilaminar membrane hydrophones were first calibrated in terms of their amplitude sensitivity to the pressure levels generated by two different HIFU (High Intensity Focused Ultrasound) circular source transducers operating at 5 MHz and 10 MHz, enabling phase studies up to 105 and 100 MHz, respectively. Introducing two newly-developed phase-dispersion representations, the phase responses of the two membrane hydrophones were determined with respect to the phase of the complex frequency response extracted from the nonlinear field simulated by a semi-empirical computer model which predicts the near and the far field pressure distributions. These phase differences compared favorably with the results obtained from the commercially available PiezoCAD simulation model. The protocol for specifying the complex pressure field of source transducers through measurements using the calibrated hydrophones is described. The results obtained indicate that the membranes exhibit close to linear decay of phase against the frequency.

1. Introduction and Background
In an ultrasound field where the acoustic spectrum contains a significant spread of frequencies the output voltage waveform, which is the acoustic spectrum convolved with the complex frequency response of the measurement system, will differ in shape from the true pressure waveform since the amplitude and phase response of PVDF membrane hydrophones are frequency dependent. The determination of peak rarefractional and peak compressional pressure amplitudes requires phase response data in the frequency range considered. Hence, a complete description of a source transducer's pressure field should consist of the phase transfer function (PTF) as well as the amplitude transfer function (ATF). To recover the true pressure-time waveform of a source transducer, the measured signal's spectrum must be deconvolved from the hydrophone's spectral response by means of its calibration data available as complex sensitivity versus frequency response. Cooling and Humphrey [1] proposed utilizing nonlinear wave propagation and a time-shifted relative phases approach to phase characterize membrane hydrophones. This work verifies their approach and provides evidence that their approach can be replaced by a direct comparison of the phases relative to the phase values at the fundamental. In the following sections, the semi-empirical methodology used to extract the characteristic phase spectra of two PVDF membrane hydrophones from their voltage-time measurements of the nonlinear pressure-time waveform generated by HIFU sources is presented. The amplitude and phase spectra of a 5 MHz and a 10 MHz HIFU source were measured by a Marconi and a Sonora bilaminar hydrophone, respectively, whose phase characteristics were obtained and displayed in the 100 MHz bandwidth by utilizing two different schemes for extracting hydrophone

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phase spectra characteristics. This work supports using the nonlinear propagation method can be used to calibrate PVDF membrane hydrophones allowing faithful reproduction of the measured pressure-time waveform, providing the hydrophone’s complex sensitivity is known at discrete frequencies, which are multiples of the fundamental of an acoustic source.

2. Methodology
Two different schemes for extracting hydrophone phase spectra characteristics from nonlinear acoustic field measurements were used. In section 2.2 the steps to obtain the results of section 3 are given.

2.1. Hydrophone phase characteristics and phase spectrum of source transducer.
The measured voltage signal is the response of the source's pressure output convolved with the hydrophone's response characteristic. The JW non-linear program, summarized in section 2.3, predicts the complex pressure spectrum produced by the transducer source. Thus, the hydrophone sensitivity spectrum is obtained from the ratio of the measured voltage spectrum and the non-linear spectrum.

\[
A_h e^{i\phi_h} \frac{[mV/MPa]}{[mV]} = A_m e^{i\phi_m} \frac{[mV]}{[MPa]}/A_j e^{i\phi_j}
\]  

(1)

\[
\frac{A_m}{A_j} e^{i(\phi_m-\phi_j)} = A_m \left( \frac{A_m R + iA_m I}{A_j R + iA_j I} \right) = A_m \left( \frac{A_m R A_j R + A_m I A_j I}{A_m A_j R - A_m A_j I} \right) + i\left( \frac{A_m A_j I - A_m A_j R}{A_m A_j R - A_m A_j I} \right)
\]  

(2)

Then, with (N+H) symbolizing the hydrophone measurement of a new source transducer, its pressure output spectrum can be determined from the hydrophone's response sensitivity spectrum via:

\[
A_N e^{i\phi_N} \frac{[MPa]}{[mV]} = A_{N+H} e^{i\phi_{N+H}} \frac{[mV]}{[MPa]}/A_h e^{i\phi_h}
\]  

(3)

The quantities in the brackets in (2) are identified as the cosine and sine difference formulae. Frequency spectra are provided by the F.T. programs in terms of the Real and Imaginary parts of the amplitude or the Amplitude and phase. Thus hydrophone characteristic phase spectra can be specified in two equivalent ways: via spectra of the intensive measures Re/Ampl and Im/Ampl or by the phase spectrum. Either characterization may be used to deconvolve the pressure field of a source transducer.

2.2. Step by step semi-empirical methodology for acquiring characteristic hydrophone phase spectrum from pressure-time waveform

**Acquire Pressure-Time Waveform & Run JW nonlinear model program ⇒**

Obtain Fourier transform spectra of: measured voltage signal (M), the JW nonlinear field prediction (J), and hydrophone modeling (PC), for verification.

⇒ R + i*I ⇐ Amplitude = A = (R^2 + I^2)^(1/2) at source harmonics, f_n.

**Two alternative pathways [1a)-1c) or 2a)-2c)] to obtain the hydrophone phase spectrum ⇒**

1a) Phase (\(\phi\)) = tan^(-1)(I/R) or sin^(-1)(I/A) or cos^(-1)(I/A); assign \(\phi\) to quadrant determined by R + i*I

1b) Calculate hydrophone (H) phase spectrum as: \(\phi_H = \phi_{M,J} - \phi_{Meas-\phi}\).

1c) Compare the calculated differences between the FFT measured (M) phases and the nonlinear JW model phases: \(\phi_{M,J}(f_n) - \phi_{PC}(f_i)\), to the PiezoCAD predicted phases: \(\phi_{PC}(f_n) - \phi_{PC}(f_i)\)

2a) Calculate: cos(\(\phi\)) = R/A and sin(\(\phi\)) = I/A for M, and JW

2b) Determine hydrophone phase spectrum directly via cosine and sine phase difference formulae applied to the measured and the JW nonlinear program’s predicted phase.

2c) Compare the above 2b) responses to the PiezoCAD hydrophone cosine and sine predictions.
It is appropriate to note that the spatial averaging errors due to the underestimate of the pressure amplitude do not affect the phase measures which are calculated from the ratio of extensive quantities.

2.3 Non-linear Propagation Model
The “JW” nonlinear acoustic simulation model, used as a reference to compare with the experimental values, requires as a boundary condition the apodization curve, which is a mathematical reproduction of the surface-pressure amplitude profile of the acoustic source that is measured by a 1D scan at the surface of the source along its diameter. This semi-empirically determined representation of the pressure distribution at the surface of the source transducer is distinct from the Gaussian distribution, as in the parabolic solution to the KZK model used by some other simulation models [1,2,3]. After the needed parameters have been input, the JW program theoretically propagates the sound field away from the transducer’s surface to an arbitrary field point. The output is available in the form of the complex Fourier spectra of the field generated by the source. The desired phase and amplitude information is extracted from this complex spectrum. The details of the JW model, which uses the hyperbolic propagation operator to calculate the acoustic field generated in a nonlinear and lossy medium, were explained in [4,5,6]. In this work, the JW model calculated the amplitude and phase spectra at the focal points of the 5 MHz, 20 mm diameter, focal number 1.9, Sonic Concepts acoustic source and the 10 MHz, 10 mm diameter focal number 4.21 Panametrics acoustic source. The FFT programs provided real and imaginary amplitudes as well.

3. Results
As outlined in section 2.2 the hydrophone phase spectrum is acquired from selected differences between the discrete Fourier transforms of the acquired pressure-time waveform and the nonlinear program’s calculated pressure field. Selected experimental results are shown in figures 1 to 4. The phase spectrum, $\phi_N = \phi_{Meas-JW}$, shown in figure 1 specifies the Sonora hydrophone's phase characteristic. Figure 2 shows the characteristic phase spectra for both the Marconi and Sonora bilaminar hydrophones in comparison to the PiezoCAD modeled phase spectra in comparison to the PiezoCAD modeled phase spectra in terms of the phases themselves.

![Figure 1. The 10 MHz JW and measured phases and their calculated difference, $\phi_{Meas-JW}$](image-url)
The characteristic hydrophone phase spectra \( \phi_H(f_n) = \phi_M(f_n) - \phi_{JW}(f_n) \) and \( \phi_{PC}(f_n) \) were normalized to zero phase at the principal frequency as indicated in figure 2 where the comparison of the measured bilaminar hydrophone characteristic phase spectra to the PiezoCAD modeled phase spectra shows reasonable validation of the phase dispersion representation presented here. The PiezoCAD model amplitude and phase spectra were determined at the harmonic frequencies. The PVDF thicknesses of the (2*9\( \mu \)m) Sonora and the (2*25\( \mu \)m) Marconi bilaminar hydrophones were modeled as (2*10\( \mu \)m) and (2*25.5\( \mu \)m) membranes to account for the adhesive thicknesses.

![Figure 2. The bilaminar hydrophone characteristic phase spectra \( \phi_H = \phi_M - \phi_{JW} \) compared to \( \phi_{PC} \).](image_url)

![Figure 3. The Sonora hydrophone cos(\( \phi_H \)) = cos(\( \phi_M - \phi_{JW} \)) and sin(\( \phi_H \)) = sin(\( \phi_M - \phi_{JW} \)) phase spectra compared to the PiezoCAD calculated cosine and sine, cos(\( \phi_{PC} \)) and sin(\( \phi_{PC} \)).](image_url)
The phase representations in figures 3 and 4 (including the phase shift to zero phase at 10 and 5 MHz, respectively) were calculated using the cosine and sine difference formulae as discussed in sections 2.1 and 2.2. The figure comparing the Marconi spectra, \( \sin(\phi_H) \) to \( \sin(\phi_{PC}) \), has the format of figure 4 shifted 20 MHz lower; i.e., the 45 and 85 MHz min and max for the cosine phase representations occur at 25 and 65 MHz, respectively, for the sine phase representations.

![5 MHz HiFU Phases: Marconi Hydrophone with 65cm cable](image)

Comparing cosine DATA: Measured - JW to PiezoCAD Model

**Figure 4.** Marconi hydrophone \( \cos(\phi_H) \) phase spectrum compared to the PC calculated \( \cos(\phi_{PC}) \).

4. Summary and Conclusions
A complete description of a transducer's pressure field should consist of the phase transfer function (PTF) as well as the amplitude transfer function (ATF). The faithful quantitative reproduction of a source transducer's pressure field requires amplitude and phase measurements by calibrated hydrophone probes. An ideal hydrophone should exhibit behavior close to that of an ideal point receiver. However, the available probes are not fully adequate for this purpose because the probe's frequency response is usually insufficient to cover the whole range of needed frequencies and the probe's finite aperture introduces spatial averaging errors. A probe area larger than the pressure field area leads to an underestimate of the pressure amplitude. To recover the true pressure-time waveform of a source transducer, the measured signal's spectrum must be deconvolved from the hydrophone's spectral response by means of its calibration data available as complex sensitivity versus frequency response. The nonlinear propagation method of hydrophone calibration, which provides amplitude and phase calibration information at discrete frequencies which are multiples of the fundamental of an acoustic source, was examined. The results presented were limited to examining and verifying the applicability of the non-linear propagation method to determine the phase characteristics of membrane hydrophones. Detailed algorithms for extracting phase values from hydrophone pressure measurements were given. The data presented were based on the analysis and measurement of the pressure field at the focal point of two different HIFU circular source transducers energized by 5 and 10 MHz 10-cycle burst signals, enabling phase studies up to 105 and 100 MHz, respectively by two different bilaminar PVDF membrane hydrophones. It was pointed out that the measured response included the phase characteristics of the hydrophone plus the phase of the acoustic (HIFU) source. In
order to phase calibrate the membrane hydrophone, one must subtract the source's non-linear acoustic contribution to the measured field. This could be done by using a fiber-optic hydrophone implemented as a point receiver in 100 MHz bandwidth and exhibiting virtually uniform amplitude response and a zero phase shift [7]. Then the phase of the tested hydrophone signal could be directly compared with the signal measured by the fiber optic hydrophone. However, in this study the phase responses of the two membrane hydrophones were determined with respect to the phase of the complex frequency response extracted from a computer-simulated nonlinear field by subtracting the calculated harmonic frequency phase values from the corresponding measured Fourier phase values. The JW finite amplitude computer model, which assumes the hydrophone in the field behaves as a point receiver, incorporated the characteristics of the HIFU transducer as measured by a pressure scan at the surface of the acoustic source by a needle hydrophone. The PVDF hydrophone phase characteristics obtained from the difference between the calculated (finite amplitude computer model) harmonic frequency phase values and the corresponding measured Fourier phase values were compared to the phase output of the commercial program, PiezoCAD, which predicted the hydrophone’s frequency characteristics. Alternatively, the real and imaginary parts of the Fourier spectra directly yielded the cosines and sines corresponding to the phase differences. The earlier discussed time-shifted relative phases approach of Cooling and Humphrey [32] was replaced here by a direct comparison of the phases relative to the phase values at the fundamental. The Measured–JW phase values (as well as the cosine and sine values) compared very well with the corresponding PiezoCAD model values. Based on this outcome it can be concluded that reliable PVDF membrane hydrophone phase calibration characteristics can be obtained by the non-linear propagation method using the algorithm presented. The algorithm allows the determination of the complex pressure field of source transducers through measurements using the calibrated hydrophone. The overall uncertainty of the amplitude sensitivity calibration was dependent on frequency and determined to be about ±12% up to 40 MHz, ±20% from 40 to 60 MHz and ±25% from 60 to 100 MHz.

Fundamental limitations of the approach were also discussed and these included discrete versus continuous method. The results obtained indicate that the PVDF membrane hydrophones exhibit close to linear decay of phase against the frequency. In a subsequent work the influence of the attached cable length on the hydrophone’s phase and amplitude will be presented.

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