Intuitionistic fuzzy set-based time series forecasting model via delegation of hesitancy degree to the major grade de-i-fuzzification and arithmetic rules based on centroid defuzzification

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Abstract. De-i-fuzzification is a process of converting the intuitionistic fuzzy set into a fuzzy set. It becomes one of the core procedures in fuzzy time series forecasting model based on the intuitionistic fuzzy set. In this paper, we propose a fuzzy time series forecasting model based on intuitionistic fuzzy set via de-i-fuzzification. The de-i-fuzzification approach used is assigning the hesitancy degree to the major grade. The data are partitioned into a few intervals using the frequency density-based method. The data in the fuzzy set form is then transformed into an intuitionistic fuzzy set using the definition of intuitionistic fuzzy set. The arithmetic rules based on centroid defuzzification is used to obtain the forecasted output. The model is implemented on the data of student enrolment at the University of Alabama. The results are then compared to forecasting method using classical fuzzy set and similar de-i-fuzzification approach using max-min operation. The proposed method outperforms the other two methods, thus supports the fact that intuitionistic fuzzy set is a generalization of a classical fuzzy set and gives better performance in forecasting.

1. Introduction
In 1993, Song and Chissom [1] introduced the concept of fuzzy time series (FTS) by implementing it on the data of student enrollments at the University of Alabama [2]. Since then, the knowledge of FTS has been widely studied by researchers with some modification and improvement on the setting of the interval lengths, the order of the fuzzy logical relationships and the defuzzification methods to obtain the forecasted output of the data being predicted [3-10].

The aforementioned literatures used the classical fuzzy sets [11] as a basis in their FTS forecasting models. Atanassov [12] introduced the concept of intuitionistic fuzzy set (IFS) as a generalization of classical fuzzy set, by incorporating both the membership and non-membership functions for the fuzzy sets. The application of IFS in FTS forecasting model is proposed by Castillo et al. [13] in plant monitoring and diagnosis. Afterwards, many intuitionistic fuzzy set-based FTS forecasting models were developed [14-22].

Joshi and Kumar [14] used the algorithm for FTS forecasting by establishing fuzzy logical relationships using hesitation index. In 2014, Gangwar and Kumar [15] proposed an FTS forecasting model based on the interval lengths formed by partitioning the universe of discourse using the
cumulative probability distribution approach. The proposed model used the degree of hesitancy to establish fuzzy logical relationships. The FTS forecasting model based on the fuzzy sets induced by IFS was established by Kumar and Gangwar [16]. Fan et al. [17] proposed an intuitionistic FTS forecasting model based on order decision and adaptive partition algorithm. An intuitionistic FTS forecasting model based on dual hesitant fuzzy set was proposed by Bisht et al. [18] to handle non-determinism caused by multiple valid fuzzification approaches in the FTS model.

In 2018, Abhishekh et al. [20] proposed the intuitionistic FTS forecasting model using the score function-based method. The score function is used to measure the intuitionistic fuzzy values (IFV), in which the bigger score will always be chosen. Bisht and Kumar [21] proposed an IFS-based computational method to handle non-determinism in forecasting financial time series data by using intuitionistic fuzzy logical relationships. Recently, Abhishekh et al. [22] modified the intuitionistic FTS forecasting model by partitioning the universe of discourse using the average-length method since this method gives better forecasting performance [23]. The defuzzification method used in [22] is based on the maximum score function and the frequency of occurrence of the intuitionistic fuzzy logical relationships.

Alam et al. [24] proposed the intuitionistic FTS forecasting model based on delegation of hesitancy degree to the major grade de-i-fuzzification method with max-min composition operation. The forecasting performance was compared to the model which is based on classical fuzzy sets and it was shown that forecasting using IFS gives better performance. The result obtained is consistent with the findings presented in [21, 25]. However, the development of matrix logical relationship and the max-min composition operation in [24] involve large amount of computing time.

In the present paper, an intuitionistic FTS forecasting model based on delegation of hesitancy degree to the major grade de-i-fuzzification method with arithmetic rules based on centroid defuzzification is proposed. The arithmetic rules have simplified calculation compared to the max-min composition operator, and the centroid defuzzification considers the position of all the points since it describes the centre of the area or gravity for the membership function. This paper is organized as follows: section 2 presents some preliminaries on the fuzzy sets, fuzzy time series and intuitionistic fuzzy sets; section 3 proposes the intuitionistic FTS forecasting model; section 4 implements the proposed model on the student enrollment data at the University of Alabama; section 5 discusses the forecasted results; and finally the conclusion is given in section 6.

2. Preliminaries

In this section, some preliminaries on the fuzzy sets, fuzzy time series and intuitionistic fuzzy sets (IFS) are presented. The de-i-fuzzification approach by assigning the hesitancy degree to the major grade [26] and centroid defuzzification [27] are also reviewed.

2.1. Fuzzy set, fuzzy time series and IFS

The definition of fuzzy set is given in the following:

**Definition 2.1 [11]:** A fuzzy set $S$ on the universe of discourse, $D = \{d_1, d_2, d_3, ..., d_n\}$ can be written as

$$D = \sum_{i=1}^{n} \frac{\mu_S(d_i)}{d_i} \quad (1)$$

where $\mu_S(d_i)$ is the membership grade of $d_i$ in $D$ for $i = 1, 2, 3, ..., n$.

Next, the definitions that are related to fuzzy time series are reviewed.

**Definition 2.2 [1]:** Suppose $Y(t) \ (t = 0, 1, 2, ...)$ be the universe of discourse with fuzzy sets $f_i(t), \ (i = 0, 1, 2, ...)$ defined on it. Then the fuzzy time series $Y(t)$ is a collection of fuzzy sets $f_i(t)$. 

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Definition 2.3 [1]: If only \( Y(t) \) affects \( F(t) \), then \( F(t) = F(t-1) \circ R(t, t-1) \) represents \( F(t-1) \rightarrow F(t) \), where \( R(t, t-1) \) is a fuzzy relation between \( F(t) \) and \( F(t-1) \).

The intuitionistic fuzzy set is defined as follows:

Definition 2.4 [12]: An intuitionistic fuzzy set \( I \) in \( D \) can be written in the form

\[
I = \{(x, \mu_i(x), \nu_i(x)) \mid x \in D\}
\]

where \( \mu_i(x) : D \rightarrow [0, 1] \) is the degree of membership of \( x \) and \( \nu_i(x) : D \rightarrow [0, 1] \) is the degree of non-membership of \( x \). Note that for every \( x \) in \( D \),

\[
0 \leq \mu_i(x) + \nu_i(x) \leq 1. \tag{3}
\]

The degree of non-determinacy or hesitancy is defined by \( \pi_i(x) = 1 - \mu_i(x) - \nu_i(x) \).

In the following definition, the conversion of classical fuzzy sets into IFS by Jurio et al. [28] is given.

Definition 2.5 [28]: Let \( \beta \in \kappa \) where \( \kappa \) is the collection of all fuzzy sets in \( D \). Let \( \alpha : D \rightarrow [0, 1] \) and \( \beta : D \rightarrow [0, 1] \). \( f : [0,1] \times [0,1] \rightarrow L' \), where \( f(x, \alpha, \beta) = (f_\mu(x, \alpha, \beta), f_\nu(x, \alpha, \beta)) \) and \( f_\mu(x, \alpha, \beta) = x(1-\alpha \beta) \), \( f_\nu(x, \alpha, \beta) = 1 - \alpha \beta - f_\mu(x, \alpha, \beta) \). \( \tag{4} \)

2.2. De-i-fuzzification via delegation of hesitancy degree to the major grade

The method of de-i-fuzzification via equal distribution of hesitancy was proposed by Ansari et al. [26]. The proposed de-i-fuzzification method is defined in the following:

Definition 2.6 [26]: Let \( \pi_i \) be the degree of hesitancy for the IFS \( I = \{(x, \mu_i(x), \nu_i(x)) \mid x \in D\} \), then the new IFS obtained after the delegation of hesitancy degree to the major grade is given as follows:

\[
\tilde{I} = \begin{cases} 
(x, \mu_i(x) + \pi_i(x), \nu_i(x)) & \text{if } \mu_i(x) > \nu_i(x) \\
(x, \mu_i(x), \nu_i(x) + \pi_i(x)) & \text{if } \mu_i(x) < \nu_i(x).
\end{cases} \tag{5}
\]

2.3. Defuzzification Using Centroid Method

The defuzzification using centroid method was introduced by Pedrycz [27] based on the centre of gravity. The formula for the defuzzification of discrete fuzzy sets is given by

\[
\tilde{C} = \frac{\sum_{j=1}^{m} x_j \mu(x_j)}{\sum_{j=1}^{m} \mu(x_j)}. \tag{6}
\]

3. Proposed IFS-Based FTS Forecasting Model via Delegation of Hesitancy Degree to the Major Grade De-i-fuzzification and Arithmetic Rules based on Centroid Defuzzification

In this section, the authors present eight steps of the proposed model which involves the delegation of hesitancy degree to the major grade in the de-i-fuzzification process and the arithmetic rules of centroid-based defuzzification of fuzzy sets. The steps are given as follows:
Step 1: Define the universe of discourse, \( D = [D_{\text{min}} - \psi, D_{\text{max}} + \phi] \) where \( D_{\text{min}} \) and \( D_{\text{max}} \) are the minimum and maximum values of the historical data and \( \psi \) and \( \phi \) are two proper positive integers.

Step 2: Divide \( D \) into some intervals using the frequency density-based method [29].

Step 3: Fuzzify the historical data using triangular fuzzy numbers to obtain the fuzzy sets.

Step 4: Convert the fuzzy sets into IFS using definition 2.5.

Step 5: De-i-fuzzify IFSs, \( I \) into fuzzy sets by assigning the hesitancy degree to the major grade [26].

Step 6: Establish the fuzzy logical relationships (FLRs) based on the induced fuzzy sets and group them.

Step 7: Based on centroid method, defuzzify the induced fuzzy sets using equation (6).

Step 8: Calculate the forecasted output using the FLRs obtained in Step 6 and the crisp values of induced fuzzy sets from step 7.

The following shows the arithmetic rules for calculating the forecasted output:

1. If the fuzzified data of year \( n \) is \( \tilde{F}_a \) and there is a unique FLR, say \( \tilde{F}_a \rightarrow \tilde{F}_b \), where the defuzzified value for \( \tilde{F}_b \) is \( \tilde{C}_b \), then the forecasted data for year \( n + 1 \) is \( \tilde{C}_b \).

2. If the fuzzified data of year \( n \) is \( \tilde{F}_a \) and there are \( p \) unique FLRs, say \( \tilde{F}_a \rightarrow \tilde{F}_b, \tilde{F}_a \rightarrow \tilde{F}_c, \ldots, \tilde{F}_a \rightarrow \tilde{F}_k \), where the defuzzified values for \( \tilde{F}_b, \tilde{F}_c, \ldots, \tilde{F}_k \) are \( \tilde{C}_b, \tilde{C}_c, \ldots, \tilde{C}_k \) respectively, then the forecasted data of year \( n + 1 \) is given by \( \frac{\tilde{C}_b + \tilde{C}_c + \ldots + \tilde{C}_k}{p} \).

3. If the fuzzified data of year \( n \) is \( \tilde{F}_a \), where the defuzzified value for \( \tilde{F}_a \) is \( \tilde{C}_a \), and there is no FLR, then the forecasted enrollment of year \( n + 1 \) is \( \tilde{C}_a \).

4. Forecasting Student Enrollments at the University of Alabama

To illustrate the proposed FTS forecasting model, it is implemented in forecasting the student enrollments at the University of Alabama. The figure below shows the historical data of student enrollments since 1971 till 1992. This data is adopted from [2-3] and is widely used by researchers to improve FTS forecasting model such as [4-9], [14-20] and [22-24].

![Enrollment of Students at the University of Alabama (1971-1992)](image-url)

Figure 1. Student enrollments at the University of Alabama (1971-1992).
Step 1: From the data, $D_{\text{min}} = 13055$ and $D_{\text{max}} = 19337$. Then $\psi = 55$ and $\varphi = 663$ are chosen such that the universe of discourse, $D = [13000,20000]$. 

Step 2: Using the frequency density-based method [29], $D = [13000,20000]$ is divided into 14 intervals. These intervals are shown in table 1 with their corresponding triangular fuzzy sets.

Table 1. Intervals with corresponding triangular fuzzy numbers.

| Intervals     | Triangular fuzzy numbers | Intervals     | Triangular fuzzy numbers |
|--------------|--------------------------|--------------|--------------------------|
| $D_1 = [13000,13500]$ | $E_1 = (13000,13500,14000)$ | $D_8 = [16000,16333]$ | $E_5 = (16000,16333,16667)$ |
| $D_2 = [13500,14000]$ | $E_2 = (13500,14000,15000)$ | $D_9 = [16333,16667]$ | $E_6 = (16333,16667,17000)$ |
| $D_3 = [14000,15000]$ | $E_3 = (14000,15000,15250)$ | $D_{10} = [16667,17000]$ | $E_{10} = (16667,17000,18000)$ |
| $D_4 = [15000,15250]$ | $E_4 = (15000,15250,15500)$ | $D_{11} = [17000,18000]$ | $E_{11} = (17000,18000,18500)$ |
| $D_5 = [15250,15500]$ | $E_5 = (15250,15500,15750)$ | $D_{12} = [18000,18500]$ | $E_{12} = (18000,18500,19000)$ |
| $D_6 = [15500,15750]$ | $E_6 = (15500,15750,16000)$ | $D_{13} = [18500,19000]$ | $E_{13} = (18500,19000,20000)$ |
| $D_7 = [15750,16000]$ | $E_7 = (15750,16000,16333)$ | $D_{14} = [19000,20000]$ | $E_{14} = (19000,20000,20000)$ |

Step 3: The historical data is then fuzzified using the defined triangular fuzzy numbers in step 2. After fuzzification, the following fuzzy sets are obtained:

Step 4: The fuzzy sets are then converted into IFSs using Atanassov’s method of conversion as in definition 2.5. The obtained IFSs are as follows:

$I_1 = \{(13055,0.099,0.804),(13563,0.790,0.114),(13867,0.240,0.663)\}$

$I_2 = \{(13563,0.114,0.793),(13867,0.666,0.241),(14696,0.276,0.632)\}$

$I_3 = \{(14696,0.527,0.230),(15145,0.318,0.440),(15163,0.264,0.494)\}$

$I_4 = \{(15145,0.575,0.416),(15163,0.646,0.345),(15311,0.749,0.242),(15433,0.266,0.725)\}$

$I_5 = \{(15311,0.185,0.574),(15433,0.556,0.203),(15460,0.637,0.121),(15497,0.750,0.009)\}$

$(15603,0.446,0.313)\}$
The IFSs are then de-i-fuzzified into fuzzy sets by assigning the hesitancy degree to the major grade using definition 2.6 [26]. The induced fuzzy sets are obtained as follows:

\[
I_6 = \{(15603,0.397,0.567),(15861,0.536,0.428),(15984,0.062,0.903)\}
\]
\[
I_7 = \{(15861,0.259,0.325),(15984,0.547,0.037)\}
\]
\[
I_8 = \{(16388,0.253,0.050)\}
\]
\[
I_9 = \{(16388,0.149,0.755),(16807,0.524,0.380),(16859,0.383,0.522),(16919,0.220,0.685)\}
\]
\[
I_{10} = \{(16807,0.287,0.396),(16859,0.394,0.289),(16919,0.516,0.166)\}
\]
\[
I_{11} = \{(18150,0.357,0.153)\}
\]
\[
I_{12} = \{(18150,0.295,0.687),(18876,0.244,0.738),(18970,0.059,0.923)\}
\]
\[
I_{13} = \{(18876,0.283,0.093),(18970,0.354,0.023),(19328,0.253,0.124),(19337,0.250,0.127)\}
\]
\[
I_{14} = \{(19328,0.292,0.598),(19337,0.300,0.590)\}
\]

Step 5: The IFSs are then de-i-fuzzified into fuzzy sets by assigning the hesitancy degree to the major grade using definition 2.6 [26]. The induced fuzzy sets are obtained as follows:

\[
\tilde{F}_1 = 0.099/13055 + 0.886/13563 + 0.240/13867
\]
\[
\tilde{F}_2 = 0.114/13563 + 0.759/13867 + 0.276/14696
\]
\[
\tilde{F}_3 = 0.770/14696 + 0.318/15145 + 0.264/15163
\]
\[
\tilde{F}_4 = 0.584/15145 + 0.655/15163 + 0.758/15311 + 0.266/15433 + 0.159/15460 + 0.012/15497
\]
\[
\tilde{F}_5 = 0.185/15311 + 0.797/15433 + 0.879/15460 + 0.991/15497 + 0.687/15603
\]
\[
\tilde{F}_6 = 0.397/15603 + 0.572/15861 + 0.062/15984
\]
\[
\tilde{F}_7 = 0.259/15861 + 0.963/15984
\]
\[
\tilde{F}_8 = 0.950/16388
\]
\[
\tilde{F}_9 = 0.149/16388 + 0.620/16807 + 0.383/16859 + 0.220/16919
\]
\[
\tilde{F}_{10} = 0.287/16807 + 0.711/16859 + 0.834/16919
\]
\[
\tilde{F}_{11} = 0.847/18150
\]
\[
\tilde{F}_{12} = 0.295/18150 + 0.244/18876 + 0.059/18970
\]
\[
\tilde{F}_{13} = 0.907/18876 + 0.977/18970 + 0.876/19328 + 0.873/19337
\]
\[
\tilde{F}_{14} = 0.292/19328 + 0.300/19337
\]

Step 6: From the induced fuzzy sets obtained in the previous step, the fuzzy logical relationships (FLRs) are established, \( \tilde{F}_1 \rightarrow \tilde{F}_1, \tilde{F}_1 \rightarrow \tilde{F}_2, \tilde{F}_2 \rightarrow \tilde{F}_3, \tilde{F}_3 \rightarrow \tilde{F}_5, \tilde{F}_5 \rightarrow \tilde{F}_4, \tilde{F}_4 \rightarrow \tilde{F}_5, \tilde{F}_3 \rightarrow \tilde{F}_6, \tilde{F}_6 \rightarrow \tilde{F}_9, \tilde{F}_5 \rightarrow \tilde{F}_{10}, \tilde{F}_{10} \rightarrow \tilde{F}_8, \tilde{F}_8 \rightarrow \tilde{F}_9, \tilde{F}_5 \rightarrow \tilde{F}_5, \tilde{F}_5 \rightarrow \tilde{F}_4, \tilde{F}_4 \rightarrow \tilde{F}_7, \tilde{F}_7 \rightarrow \tilde{F}_{10}, \tilde{F}_{10} \rightarrow \tilde{F}_{11}, \tilde{F}_{11} \rightarrow \tilde{F}_{13}, \tilde{F}_{13} \rightarrow \tilde{F}_{13}, \tilde{F}_{13} \rightarrow \tilde{F}_{13} \) and \( \tilde{F}_{13} \rightarrow \tilde{F}_{13} \). The FLRs are then grouped as shown in table 2.

**Table 2. Fuzzy logical relationship (FLR) groups.**

| Group | IFLRs | Group | IFLRs |
|-------|-------|-------|-------|
| Group 1 | \( \tilde{F}_1 \rightarrow \tilde{F}_1, \tilde{F}_1 \rightarrow \tilde{F}_2 \) | Group 7 | \( \tilde{F}_7 \rightarrow \tilde{F}_{10} \) |
| Group 2 | \( \tilde{F}_2 \rightarrow \tilde{F}_3 \) | Group 8 | \( \tilde{F}_9 \rightarrow \tilde{F}_5 \) |
| Group 3 | \( \tilde{F}_3 \rightarrow \tilde{F}_5 \) | Group 9 | \( \tilde{F}_4 \rightarrow \tilde{F}_{10} \) |
| Group 4 | \( \tilde{F}_4 \rightarrow \tilde{F}_4, \tilde{F}_4 \rightarrow \tilde{F}_5, \tilde{F}_4 \rightarrow \tilde{F}_7 \) | Group 10 | \( \tilde{F}_{10} \rightarrow \tilde{F}_8, \tilde{F}_{10} \rightarrow \tilde{F}_{11} \) |
| Group 5 | \( \tilde{F}_5 \rightarrow \tilde{F}_4, \tilde{F}_5 \rightarrow \tilde{F}_3, \tilde{F}_5 \rightarrow \tilde{F}_6 \) | Group 11 | \( \tilde{F}_{11} \rightarrow \tilde{F}_{13} \) |
| Group 6 | \( \tilde{F}_6 \rightarrow \tilde{F}_9 \) | Group 12 | \( \tilde{F}_{13} \rightarrow \tilde{F}_{13} \) |
Step 7: The induced fuzzy sets are defuzzified using centroid method and 14 crisp values are
obtained, 
\[ \tilde{C}_1 = 13581.42, \quad \tilde{C}_2 = 14035.82, \quad \tilde{C}_3 = 14892.84, \quad \tilde{C}_4 = 15255.25, \quad \tilde{C}_5 = 15484.26, \]
\[ \tilde{C}_6 = 15768.92, \quad \tilde{C}_7 = 15957.88, \quad \tilde{C}_8 = 16388.00, \quad \tilde{C}_9 = 16793.99, \quad \tilde{C}_{10} = 16878.18 \]
\[ \tilde{C}_{11} = 18150.00, \quad \tilde{C}_{12} = 18527.05, \quad \tilde{C}_{13} = 19121.08 \quad \text{and} \quad \tilde{C}_{14} = 19332.56, \]
where \( \tilde{C}_i \) is the crisp value for \( F_i \) obtained using centroid formula for \( i = 1,2,...,n \) respectively.

Step 8: The forecasted output is finally calculated using the centroid values obtained in step 7
and based on FLR groups as in step 6.

5. Results and Discussion
In this section, the forecasted enrollments calculated from the steps in the previous section is
presented. The forecasted enrollments of the proposed model is compared to the method of forecasting
using fuzzy set and previously proposed method of de-i-fuzzification via delegation of hesitancy
degree to the major grade and max-min composition operation [24]. The comparison of the forecasted
enrollments is shown in Table 3.

| Year | Actual enrollment | Forecasting using fuzzy set [24] | Assigning hesitancy to major grade [24] | Proposed method |
|------|-------------------|----------------------------------|----------------------------------------|-----------------|
| 1972 | 13563             | -                                | -                                      | 13808.62        |
| 1973 | 13867             | 14562.5                          | 14562.5                                | 13808.62        |
| 1974 | 14696             | 14562.5                          | 14562.5                                | 14892.84        |
| 1975 | 15460             | 14562.5                          | 14562.5                                | 15484.26        |
| 1976 | 15311             | 16104.25                         | 16027.83                               | 15565.8         |
| 1977 | 15603             | 16104.25                         | 16027.83                               | 15565.8         |
| 1978 | 15861             | 16104.25                         | 16027.83                               | 15502.81        |
| 1979 | 16807             | 16104.25                         | 16027.83                               | 15502.81        |
| 1980 | 16919             | 16500                            | 16666.75                               | 15502.81        |
| 1981 | 16388             | 16500                            | 16666.75                               | 15502.81        |
| 1982 | 15422             | 17062.5                          | 17062.5                                | 15502.81        |
| 1983 | 15497             | 16104.25                         | 16027.83                               | 16793.99        |
| 1984 | 15145             | 16104.25                         | 16027.83                               | 16878.18        |
| 1985 | 15163             | 16104.25                         | 16027.83                               | 15484.26        |
| 1986 | 15984             | 16104.25                         | 16027.83                               | 16878.18        |
| 1987 | 16859             | 16104.25                         | 16027.83                               | 16878.18        |
| 1988 | 18150             | 16500                            | 16666.75                               | 17269           |
| 1989 | 18970             | 17062.5                          | 17062.5                                | 19121.08        |
| 1990 | 19328             | 18750                            | 18750                                  | 19121.08        |
| 1991 | 19337             | 18750                            | 18750                                  | 19121.08        |
| 1992 | 18876             | 18750                            | 18750                                  | 19121.08        |
| 1993 | -                 | 18750                            | 18750                                  | 19121.08        |
Table 4. MSE, RMSE, MAE and MAPE of the forecasted enrollments.

| Error      | Forecasting using fuzzy set [24] | Assigning hesitancy to major grade [24] | Proposed method |
|------------|----------------------------------|-----------------------------------------|-----------------|
| MSE        | 769925.3406                      | 723395.4344                             | 545566.5818     |
| RMSE       | 877.4538966                      | 850.526563                              | 738.624791      |
| MAE        | 717.9375                         | 690.4958                                | 530.6330        |
| MAPE       | 0.04354246                       | 0.041827848                             | 0.032652428     |

Referring to table 4, it is obviously shown that the proposed method outperforms the other forecasting methods. Referring to [21], [24] and [25], the obtained result supports the fact that the intuitionistic fuzzy set is a generalization of a fuzzy set and is capable of performing better in FTS forecasting model. The influence of centroid of induced fuzzy sets as the base of arithmetic rules has also improved the forecasting performance.

6. Conclusion

Comparing the MSE, RMSE, MAE and MAPE for each model, it can be obviously stated that the forecasting model based on the de-i-fuzzification via assigning hesitancy to major grade with centroid-based arithmetic rules performs better than the other models. The arithmetic rules based on centroid defuzzification have simplified calculation and consider overall information of the fuzzy sets compared to the max-min composition operation in [24]. This supports the fact that the proposed method is efficient and significantly closed to the actual observation data set.

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