I. INTRODUCTION

Transparent conductive electrodes are an essential component of smart electronics and optoelectronics, such as smart windows, touch panels and solar cells. Metal nanowire-based transparent conductive electrodes (TCE), such as Cu, Ag, and Au nanowire electrodes, are considered as a new generation of TCE which are capable of replacing indium tin oxide-based electrodes.1

A nanoring has been introduced as a new kind of TCEs. There are different methods to synthesize metallic nanorings. However, in all cases, a TCE includes both nanoring and nanowires. The latter may be bent or wavy. They fraction may vary. Table 1 presents some published data on synthesized metallic nanorings.

Notice that, for cylindrical wires of an isotropic metal, the electrical resistivity depends on the wire diameter, tending to the value of the bulk resistance as the wire diameter increases.9 Some experimental data are collected in Table 1. For comparison, the electrical resistivity of bulk silver is 15.9 nΩ/m.10 The temperature dependence of the resistivity should also be borne in mind. Selzer et al.15 reported that a value of the resistance of a single Ag nanowire (mean diameter of 90 nm) is 4.96 ± 0.18 Ω/μm (the values in corresponding row in Table 1 was calculated using these data) while the junction resistance is 25.2 ± 1.9 Ω (annealed junctions) and 52.9 ± 239 Ω (non-annealed ones). For AgNWs (average diameter of 70 ± 10 nm and average length of 8 ± 3 μm), Charvin et al.17 knowing the experimental sample sheet resistance, and expecting the simulated one to be the same, estimated the junction resistance as 14 ± 2 Ω. Gomes da Rocha et al.18 comparing simulations with experimental data, reported estimates of the junction resistance from 2.28 Ω to 152 Ω (45 ± 31 Ω) (see, Supplementary files in Ref. 14).

Monte Carlo simulations of the electrical conductance have been performed for 2D systems of conductive sticks and rings22–25. The electrical properties of such the systems have been considered theoretically using a percolation theory26 and a mean-field approach (MFA).5,19,22–25,27 Monte Carlo simulations of electrical percolation in thin films with conductive disks and sticks have been performed.31 The effective conductance of nanocomposites as a function of relative concentrations has been investigated. A synergistic effect has been reported when disks and sticks combine properly. Widely used junction resistance dominant assumption (JDA) (see, e.g., Refs. 20 and 21) has been used, i.e., both disks and sticks were assumed to have no electrical resistance, while a junction between any two conductive fillers was assumed to be a resistor. The resistance of the stick–stick was assumed to be five times larger than that of the disk–disk, since a contact between any two disks is an area, while a contact between any two sticks is a point contact. Dependencies of the electrical conductance with respect to mass ratio of the disk to stick and to stick length have been plotted.

Recently, MFA was successfully applied to pure nanoring- and to pure nanostick-based systems.23,24 However, to the best knowledge of us, MFA was not yet applied to a mixture of conductive fillers having different shapes. The goal of the present study is application of the MFA to a random system consisting of randomly deposited conductive nanorings and nanosticks.

The rest of the paper is constructed as follows. Section II presents our computational and analytical methods, viz., Sec. II A describes some technical details of our simulation, Sec. II B is devoted to the analytical consideration using a MFA. In Section III we present our main results and compare MFA predictions with computer simulations. Section IV summarizes the main results and suggests possible directions for further study.

Transparent electrodes based on a mixture of nanowires and nanorings

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We have studied the electrical conductance of two-dimensional random percolating networks of zero-width metallic nanowires (a mixture of rings and sticks). We took into account the nanowire resistance per unit length and the junction (nanowire/nanowire contact) resistance. Using a mean-field approximation (MFA), we derived the total electrical conductance of the nanowire-based networks as a function of their geometrical and physical parameters. The MFA predictions have been confirmed by our Monte Carlo (MC) numerical simulations. MC simulations were focused on the case when the circumferences of the rings and the lengths of the wires are equal. In this case, the electrical conductance of the network is almost insensitive to the proportions of rings and sticks when the wire resistance and the junction resistance are equal. When the junction resistance dominates over the wire resistance, a linear dependency of the electrical conductance of the network on the proportions of rings and sticks was observed.

Monte Carlo simulations were performed for 2D systems of conductive sticks22–25 and rings22–25. The electrical properties of such the systems have been considered theoretically using a percolation theory26 and a mean-field approach (MFA).5,19,22–25,27 network analysis25–27 and an effective medium theory (EMT).28

Monte Carlo simulations of electrical percolation in thin films with conductive disks and sticks have been performed.31 The effective conductance of nanocomposites as a function of relative concentrations has been investigated. A synergistic effect has been reported when disks and sticks combine properly. Widely used junction resistance dominant assumption (JDA) (see, e.g., Refs. 20 and 21) has been used, i.e., both disks and sticks were assumed to have no electrical resistance, while a junction between any two conductive fillers was assumed to be a resistor. The resistance of the stick–stick was assumed to be five times larger than that of the disk–disk, since a contact between any two disks is an area, while a contact between any two sticks is a point contact. Dependencies of the electrical conductance with respect to mass ratio of the disk to stick and to stick length have been plotted.

Recently, MFA was successfully applied to pure nanoring- and to pure nanostick-based systems.23,24 However, to the best knowledge of us, MFA was not yet applied to a mixture of conductive fillers having different shapes. The goal of the present study is application of the MFA to a random system consisting of randomly deposited conductive nanorings and nanosticks.

The rest of the paper is constructed as follows. Section II presents our computational and analytical methods, viz., Sec. II A describes some technical details of our simulation, Sec. II B is devoted to the analytical consideration using a MFA. In Section III we present our main results and compare MFA predictions with computer simulations. Section IV summarizes the main results and suggests possible directions for further study.

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TABLE I. Published data on metallic nanorings.

| Reference                        | Material | Ring diameter, µm | Wire diameter, nm |
|----------------------------------|----------|------------------|------------------|
| Zhou *et al.*                    | Ag       | 4.8–20           | ≈ 165            |
| Zhan *et al.*                    | Cu       | 23               | 200              |
| Azani and Hassanpour *et al.*    | Ag       | 15 ± 5           | 120 ± 20         |
| Li *et al.*                      | Ag       | 7.17–42.94       | 76               |
| Ning *et al.*                    | Ag       | 6.54–30.67       | 66.7–115.5       |
| Feng *et al.*                    | Ag       | 10               | 20–40            |

TABLE II. Published data on electrical resistivity of Ag nanowires, ordered in ascending order of nanowire diameter. We used original data on the electrical conductivity 11,12 to calculate the electrical resistivity. 2-P and 4-P are related to two-point probes method and four-point probes method, respectively.

| Reference                        | Method  | T, K     | Length, µm | Diameter (d), nm | ρ<sub>W</sub>, nΩ·m |
|----------------------------------|---------|----------|------------|-----------------|---------------------|
| Bid, Bora, and Raychaudhuri      | 2-P     | 295      | 6          | 15              | 41.1                |
| Bid, Bora, and Raychaudhuri      | 2-P     | 295      | 6          | 30              | 34.9                |
| Bellew *et al.*                  | 2-P     | room temp.| 7 ± 2      | 42 ± 12         | 20.3 ± 0.5          |
| Bid, Bora, and Raychaudhuri      | 2-P     | 295      | 6          | 50              | 29.2                |
| Gomes da Rocha *et al.*          | 4-P     | room temp.| 6.7        | 50 ± 13         | 22.6 ± 2.3          |
| Selzer *et al.*                  | 4-P     | room temp.| 25         | 90              | 31.6 ± 1.2          |
| Wang *et al.*                    | 4-P     | room temp.| 4.88       | 93.2            | 28.5                |
| Wang *et al.*                    | 4-P     | room temp.| 14.67      | 97.0            | 32.9                |
| Bid, Bora, and Raychaudhuri      | 2-P     | 295      | 6          | 100             | 27.8                |
| Kojda *et al.*                   | 4-P     | 293      | 15 ± 1     | 120 ± 20        | 37 ± 13             |
| Kojda *et al.*                   | 4-P     | 293      | 11 ± 1     | 107 ± 5         | 29 ± 5              |
| Kojda *et al.*                   | 4-P     | 293      | 13 ± 1     | 140 ± 10        | 27 ± 6              |
| Kojda *et al.*                   | 4-P     | 293      | 14 ± 1     | 150 ± 3         | 25.1 ± 1.3          |
| Bid, Bora, and Raychaudhuri      | 2-P     | 295      | 6          | 200             | 23.3                |
| Cheng *et al.*                   | 2-P     | 290      | 27.23      | 227             | 79.1                |

II. METHODS

A. Simulation

Two kinds of conductive fillers were used in our simulation, viz, rings with a given radius, r, and equiprobable orientated zero-width sticks with a given length, l. The electrical resistance per unit length of each filler is ρ<sub>W</sub>. The fillers of the both kinds were randomly placed on an insulating substrate. Their centers were independent and identically distributed within a square domain of size L × L. To reduce the finite-size effect, periodic boundary conditions (PBCs) were applied along both mutually perpendicular directions. The number density of sticks is

\[ n_s = \frac{N_s}{L^2}. \]  

(1)

while the number density of rings is

\[ n_r = \frac{N_r}{L^2}. \]  

(2)

Since the electrical conductance is our primary interest, the total number density of fillers

\[ n = n_r + n_s \]  

(3)

was above the percolation threshold, n ≥ n_c, in any case. For each value of the number density, simulation was performed for different proportions of rings and sticks. When the desired number density of the fillers was reached, the PBCs were removed, allowing us to consider the model as an insulating film of size L × L covered by conductive fillers. Then, superconductive busbars were attached to the opposite borders of the domain. A potential difference, V<sub>0</sub>, was applied to these busbars. The electrical resistance of each contact (junction) between any two fillers was R<sub>j</sub>. The electrical resistance of each contact (junction) between a filler and a busbar was R<sub>b</sub>. Both kinds of junctions where assumed to be ohmic. Consider a segment of the conductive filler (either a stick or a ring) between the two nearest junctions belonging to it. If a length of this segment is a, then its resistivity is ρ<sub>W</sub>a. Thus, there is a random resistor network (RRN). The network is irregular with different branch
resistances. Applying Ohm’s law to each branch and Kirchhoff’s point rule to each junction, a system of linear equations (SLEs) can be obtained. Although this SLE is huge, its matrix is sparse, therefore, the numerical solution of this SLE does not present significant difficulties. We used the EIGEN library\textsuperscript{32} to solve it.

We used domains of the fixed size $L = 32$, while the characteristic sizes of fillers were $r = 1$ and $l = 2\pi r$. To efficiently determine the percolation threshold (occurrence of a percolation cluster that spans the system in a given direction), the union-find algorithm\textsuperscript{33,34} was used. In our simulations, for the two limiting cases, when only one kind of fillers is presented, $n_c = 0.373 \pm 0.004$ for rings, while $n_c = 5.641 \pm 0.025$ for sticks.

![FIG. 1. Sample of randomly placed rings and sticks within a domain $L \times L$ with PBCs. All orientations of sticks are equiprobable. The domain size is $L = 32$. The ring radius is $r = 1$. The stick length is $l = 2\pi r$. The number of rings is $N_r = 200$. The number of sticks is $N_s = 200$. Busbars are shown as thick lines at top and bottom of the system. Effect of PBCs is demonstrated in colors. When a conductive filler intersects a border, its main part (containing the center of the filler) is shown in blue, a part obtained using a translation along only one direction is shown in red, while a part obtained using translations along the both directions is shown in magenta.](image)

The results of the computations were averaged over 10 independent runs. The error bars in the figures correspond to the standard deviation of the mean. When not shown explicitly, they are of the order of the marker size.

### B. Analytical consideration

The probability of the intersection of any two rings is

$$P_1 = 4\pi \left( \frac{r}{L} \right)^2$$  \hspace{1cm} (4)

(see, e.g., Ref.\textsuperscript{24}). The probability that a given ring intersects exactly $N$ other rings is described by the binomial distribution

$$\Pr(k = N) = \binom{N_r}{N} P_r^N (1 - P_r)^{N_r - N}.$$  \hspace{1cm} (5)

The expected number of intersection is

$$\langle N_{rr} \rangle = P_r \langle N_r - 1 \rangle \approx 4\pi r^2 n_r,$$  \hspace{1cm} (6)

since $N_r \gg 1$.

The probability of the intersection of any two sticks is

$$P_s = \frac{2}{\pi} \left( \frac{l}{L} \right)^2$$  \hspace{1cm} (7)

(see, e.g., Refs.\textsuperscript{18} and\textsuperscript{30}). The probability that a given stick intersects exactly $N$ other sticks is described by the binomial distribution

$$\Pr(k = N) = \binom{N_s}{N} P_s^N (1 - P_s)^{N_s - N}.$$  \hspace{1cm} (8)

The expected number of intersection is

$$\langle N_{ss} \rangle = P_s \langle N_s - 1 \rangle \approx \frac{2}{\pi} l^2 n_s,$$  \hspace{1cm} (9)

since $N_s \gg 1$.

The probability that a stick and a circle have one point of intersection is

$$P_1 = \frac{r^2}{L^2} \begin{cases} 4 \left( \arcsin z + z \sqrt{1 - z^2} \right), & \text{if } z \lesssim 1, \\ 2 \pi, & \text{if } z > 1, \end{cases}$$  \hspace{1cm} (10)

while the probability that a stick and a circle have two points of intersection is

$$P_2 = \frac{r^2}{L^2} \begin{cases} 4 z - \left\{ 2 \left( \arcsin z + z \sqrt{1 - z^2} \right), & \text{if } z \lesssim 1, \\ \pi, & \text{if } z > 1, \end{cases},$$  \hspace{1cm} (11)

where

$$z = \frac{l}{2r}.$$  \hspace{1cm} (12)

(see Supplement). The probability of the intersection of a stick and a ring is

$$P_{sr} = P_1 + P_2 = \frac{r^2}{L^2} \begin{cases} 4 z + \left\{ 2 \left( \arcsin z + z \sqrt{1 - z^2} \right), & \text{if } z \lesssim 1, \\ \pi, & \text{if } z > 1, \end{cases}.$$  \hspace{1cm} (13)
Thus, the expected number of intersections of a stick with a circle is
\[
\langle k \rangle = \frac{P_1 + 2P_2}{P_{rs}} = \frac{8z}{4z + \psi}, \quad \text{where}
\psi = \begin{cases} 
2 \left( \arcsin z + z\sqrt{1-z^2} \right), & \text{if } 0 < z \leq 1, \\
\pi, & \text{if } z > 1.
\end{cases}
\]

This quantity varies from \( \langle k \rangle = 1 \), when \( z = 0 \), through 1.6, when \( z = 1 \) (i.e., \( l = 2\pi r \)), to 2, when \( z \gg 1 \) (Fig. 2).

![Graph](image)

**FIG. 2.** Dependence of the average number of contacts between a stick and a ring, \( \langle k \rangle \), on the relative length of the stick, \( z \), (Eq. 14).

The probability that a given ring intersects exactly \( N \) sticks is described by the binomial distribution
\[
Pr(k = N) = \binom{N_s}{N} P_{rs}^N (1 - P_{rs})^{N_s - N}.
\]

The expected number of intersection is
\[
\langle N_{rs} \rangle = P_{rs}(N_s - 1) \approx P_{rs}N_s,
\]

since \( N_s \gg 1 \).

The probability that a given stick intersects exactly \( N \) rings is described by the binomial distribution
\[
Pr(k = N) = \binom{N_r}{N} P_{rs}^N (1 - P_{rs})^{N_r - N}.
\]

The expected number of intersection is
\[
\langle N_{sr} \rangle = P_{rs}(N_r - 1) \approx P_{rs}N_r,
\]

since \( N_r \gg 1 \).

Thus, the expected number of contacts per a ring is
\[
\langle k_r \rangle = 2\langle N_{rr} \rangle + \langle k \rangle\langle N_{rs} \rangle,
\]

while the expected number of contacts per a stick is
\[
\langle k_s \rangle = \langle N_{rs} \rangle + \langle k \rangle\langle N_{sr} \rangle.
\]

For any allowed value of \( z \),
\[
\langle k_r \rangle = 8\pi r^2 n_r + 4\pi n_s,
\]

\[
\langle k_s \rangle = \frac{2\pi^2}{\pi} n_s + 4\pi n_r.
\]

When the number density of the conductive fillers is large enough, the variation of the electrical potential along the film is close to linear. Only one conductive filler in the mean-field produced by all the other fillers may be considered instead of consideration of the whole system of fillers. This idea may be easily transferred to the case when the fillers of the two different shapes are presented.

Consider a linear conductive wire (stick) in external electric field. This stick is characterized by the resistivity, \( \rho_w \). Its lateral surface is covered by an isolator, which is characterized by the leakage conductivity
\[
G_s = \frac{R_1\langle k_s \rangle}{l}.
\]

According to Ref. 22, the fraction of the electrical conductance, which is due to all sticks, is equal to
\[
\sigma_s = \frac{n_s l}{2\rho_w} \left[ 1 - \sqrt{\frac{4}{\langle k_s \rangle \Delta}} \tanh \left( \sqrt{\frac{\langle k_s \rangle \Delta}{4}} \right) \right],
\]

where
\[
\Delta = \frac{\rho_w l}{R_1}.
\]

Likewise, consider a circular conductive wire (ring) in external electric field. This ring is characterized by the resistivity, \( \rho_w \). Its lateral surface is covered by an isolator, which is characterized by the leakage conductivity
\[
G_r = \frac{R_1\langle k_r \rangle}{2\pi r}.
\]

According to Ref. 24, the fraction of the electrical conductance, which is due to all rings, is equal to
\[
\sigma_r = \frac{\pi \lambda^2 l^3 n_r}{\rho_w (1 + \lambda^2 r^2)},
\]

where, in our case,
\[
\lambda^2 = \frac{\rho_w}{R_1} \left[ 4\pi n_r + \frac{2l}{\pi} n_s \right].
\]

The electrical conductance of the system of conductive rings and sticks is
\[
\sigma = \sigma_r + \sigma_s.
\]
When $R_w \ll R_j$ (JDA),
\[ \sigma \approx \frac{n_r l^2 k_n}{24R_j}. \] (30)

For not very large values of the number density, when $\lambda^2 \ll 1$,
\[ \sigma_r \approx \frac{\pi^3 n_r}{R_j} \left[ 4\pi n_r + \frac{2l}{\pi} n_s \right]. \] (31)

Thus,
\[ \sigma \approx \frac{1}{R_j} \left[ 4\pi^4 n_r^2 + 2l^3 n_r n_s + \frac{n_s^2 l^4}{12\pi} + \frac{n_r n_s l^3}{6} \right]. \] (32)

Let $n_s = xn$, $n_r = (1-x)n$, then
\[ \sigma \approx \frac{l^2 \pi^4}{R_j} \times \left[ 4\pi (1-x)^2 + x(1-x) \left( \frac{2l}{\pi} + \frac{l^3}{6\pi^3} \right) + x^2 \frac{l^4}{12\pi^4} \right]. \] (33)

In the particular case, when $l = 2\pi r$,
\[ \sigma \approx \sigma_0 \left( 1 - x + \frac{\pi^2}{3} \right), \quad \sigma_0 = \frac{4\pi n_r l^4}{R_j}. \] (34)

Here, $\sigma_0$ corresponds to the electrical conductance of a pure ring system in the case of JDA (31).

III. RESULTS

Figure 3 demonstrates dependency of the electrical conductance on the number density of fillers for the three different proportions of rings and sticks (pure ring-based system, pure stick-based system, and the equal part mixture of rings and sticks). The wire resistance and the junction resistance are of the same order. For all used values of the number density, MFA prediction slightly exceeds the simulated values of the electrical conductance.

Figure 4 presents dependencies of the electrical conductance on the proportions of rings and sticks, $x$, for the three different values of the number density of fillers, $n_r = xn$ and $n_s = (1-x)n$, when $n = 2$, $n = 5$, and $n = 10$. MFA a little bit overestimates the electrical conductance similar to pure stick and pure ring systems. When wire resistance and the junction resistance are of the same order, the electrical conductance is almost insensitive to the specific proportions of rings and sticks.

Figure 5 demonstrates dependency of the electrical conductance on the number density of fillers for the five different proportions of rings and sticks. The junction resistance dominates over the wire resistance. MFA predictions and simulations are in good agreement.

FIG. 3. Dependencies of the electrical conductance on the number density of fillers, $n = n_r + n_s$, when $n_r = 0$, $n_s = n_r$, and $n_s = 0$. MFA predictions are shown as curves, while markers correspond to the simulations. Here, $l = 2\pi r$, $r = 1$, $R_j = 1$, and $\rho_w = (2\pi)^{-1}$.

FIG. 4. Dependencies of the electrical conductance on the proportions of rings and sticks, $x$, for the three different values of the number density of fillers, $n_r = xn$ and $n_s = (1-x)n$, when $n = 2$, $n = 5$, and $n = 10$. MFA predictions and the least-squares fitting are shown as lines, while markers correspond to the simulations. Here, $l = 2\pi r$, $r = 1$, $R_j = 1$, and $\rho_w = (2\pi)^{-1}$.

FIG. 5. Dependencies of the electrical conductance on the proportions of rings and sticks, $x$, for the three different values of the number density of fillers, $n_r = xn$ and $n_s = (1-x)n$, when $n = 2$, $n = 5$, and $n = 10$, when the junction resistance dominates over the wire resistance. The dependencies are close to linear.

Coefficients of the least-squares fitting of the dependencies of the electrical conductance on the proportions of rings and sticks are collected in Table III. 
FIG. 5. Dependency of the electrical conductance on the number density of fillers, $n = n_s + n_r$, when $n_s = 0$, $n_s = n_r$, $n_s = 3 n_r$, and $n_s = 0$. MFA predictions are shown as curves, while markers correspond to the simulations. Here, $l = 2 \pi r$, $r = 1$, $R_l = 1$, and $\rho_w = (2 \pi r)^{-1}10^{-6}$.

FIG. 6. Dependency of the electrical conductance on the proportions of rings and sticks, $x$, for the three different values of the number density of fillers, $n_s = x n$ and $n_r = (1 - x) n$, when $n = 2$, $n = 5$, and $n = 10$. MFA predictions and the least-squares fitting are shown as lines, while markers correspond to the simulations. For the two larger values of $n$, the difference between MFA and LSF lines is invisible to the eye. Here, $l = 2 \pi r$, $r = 1$, $R_l = 1$, and $\rho_w = (2 \pi r)^{-1}10^{-6}$.

Figure 7 exhibits dependency of the normalized electrical conductance $\sigma/\sigma_0$ on the proportions of rings and sticks, $x$, for the three different values of the number density of fillers in the limiting case of JDA. A linear dependency predicted by MFA (34) is consistent with the simulations.

Figure 8 demonstrates the dependency of the electrical conductance on the proportions of rings and sticks, $x$, for the fixed values of the number density of fillers and different values of $l$, when the junction resistance dominates over the wire resistance (JDA). MFA predictions (33) are shown as curves, while markers correspond to the simulations.

IV. CONCLUSION

The electrical conductance of two-dimensional random percolating networks of zero-width metallic nanowires (a mixture of rings and sticks) using a mean-field approxi-
mation and computer simulation. We take into account the nanowire resistance per unit length and the junction (nanowire/nanowire contact) resistance. We derived the total electrical conductance of the nanowire-based networks as a function of their geometrical and physical parameters. MC simulations were focused on the case when the circumferences of the rings and the lengths of the wires are equal. Our Monte Carlo simulations confirmed the MFA predictions. In this case, the electrical conductance of the network is almost insensitive to the proportions of rings and sticks when the wire resistance and the junction resistance are equal. When the junction resistance dominates over the wire resistance, a linear dependency of the electrical conductance of the network on the proportions of rings and sticks was observed.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Yuri Yu. Tarasevich: Conceptualization (lead); Formal analysis (equal); Funding acquisition (lead); Methodology (lead); Project administration (lead); Supervision (lead); Visualization (lead); Writing – original draft (lead); Writing – review & editing (lead).

Andrei V. Eserkepov: Investigation (lead); Software (lead); Validation (equal); Visualization (supporting); Writing – original draft (supporting); Writing – review & editing (supporting).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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