Scalar self-force on static charge in a long throat

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We compute the self-force on a scalar charge at rest in the space–time of long throat. We consider arbitrary values of the mass of the scalar field and the constant of nonminimal coupling of the scalar field to the curvature of space–time. We also show the coincidence of explicit calculations of self-force in the limit of large mass of the field with known results.

Keywords: Self-force; wormhole.

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1. Introduction

A very well-known phenomenon that occur with a charge in a curved space–time, is that it may become subjected to the self-interactions. The origin of this induced self-interaction resides on the nonlocal structure of the field caused by the space–time curvature or nontrivial topology.

In flat space–time, this effect is produced by a local distortion of the field lines associated with the particle’s acceleration. For electrically charged particles in flat space–time, the self-force is given by the Abraham–Lorentz–Dirac formula. In the gravitational field, the self-energy problem becomes more complicated. The reason is that contribution to the self-energy in this case is nonlocal. The self-force problem for an electric charge in a curved space background was first investigated by DeWitt and Brehme and later by Hobbs. The gravitational self-force was first calculated almost simultaneously by Mino, Sasaki and Tanaka and by Quinn and Wald. Later, Quinn derived the equivalent formula for a charge coupled to a minimally-coupled massless scalar field.

A number of simple static configurations has been analyzed, including the self-force acting on scalar or electric charges held static in the space–time of a Schwarzschild black hole, electric or magnetic dipoles which are static outside...
that the approximate solution (18) of Eq. (14) does not depend on the conditions at infinity and in considered situation, the effect of self-action is a local one even in the limit of massless field.

We also note that the asymptotic behavior of the function $F(\mu)$ for $\mu \gg 1$ is of the form (Fig. 1).

![Curve representing $|\mu - F(\mu)|24\mu$ for $\mu \gg 1$.](image)

This means that the limit of $\phi_{\text{ren}}$ at $mr \to \infty$ is equal to (see Ref. 28)

$$
\phi_{\text{ren}}(x) \approx \frac{q}{2m} \left[ -\frac{g_{tt,i}}{12g_{tt}} + \frac{5g_{tt,i}g_{tt}}{48g_{tt}^2} - \left( \xi - \frac{1}{6} \right) R \right] - \frac{q}{mr^2} \left( \xi - \frac{1}{6} \right) + O\left( \frac{q}{mL^2} \right).
$$

(38)

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