Algorithm for designing building constructions expressed by nonlinear functions

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Abstract. The aim of the work is to develop an algorithm for designing building constructions, namely their geometric construction (the geometrical location of points) in the form of an expression using the formula (equation) of geometric property of construction expressed in the projection by nonlinear functions. The article gives an algorithm for finding analytical dependencies of nonlinear functions used in applications. You must select a coordinate system before composing the curve equation. The type of the desired equation depends on the choice of the coordinate system. There are no rules that could guide one in choosing a coordinate system, and the ability to make the most rational choice is given only by experience. In this paper, the examples of the choice of coordinate systems show the form of functions, namely, their simplification. In some cases, geometric construction is the only way to present a graphic image of the building structure itself, no matter how complex it may be in the Cartesian coordinate system. The polar coordinate system is presented here as an alternative for unique structures and structures of increased complexity in the Cartesian coordinate system. This is achieved by establishing a relationship between the current coordinates of the point located on the projection of the construction and constants characterizing the properties of this construction, called a function on the plane and a surface in space in researches. The prospect of this approach has proven itself: in the construction of exquisite palace temples, beautiful architectural ensembles that make the space surrounding a person comfortable.

1. Introduction.

Developing the ideas of V.G. Shukhov, the modern world gets innovative solutions where it is necessary to use not monoliths, but connecting links, like a radio mast in Moscow, Shukhov towers in the form of oil and gas pipelines, water tanks, domed roofs, etc. Therefore, the development of design algorithms for strong and lightweight building constructions due to reviving methods is especially relevant in the era of the digital economy, when increasing the accuracy of calculations of complex structures expressed by nonlinear functions allows you to save unaccounted materials and make future architecture projects strong and beautiful, with predetermined properties.

It is known that the complexity of the task of the designed construction depends on the choice of the coordinate system. Even in the case of choosing a polar coordinate system, the type of structure and its projection will depend on the choice of the location of the pole and the polar axis in it. For example, if the construction is represented by a circle in the projection, then the equation of the circle has a simpler form if the pole coincides with the center of the circle than if it is on the circle itself, namely \( \rho = a \) and \( \rho = 2a \cdot \cos \varphi \), respectively [1].

But in the case of a projection of a construction in the form of an ellipse, on the contrary, the view will be simpler for second-order curves if the pole is in one of the focal points of the ellipse and the
polar axis passes through the foci. Thus, in polar coordinates, the same line can be determined by both an algebraic and a transcendental equation, depending on the choice of the pole and the polar axis. As a result, we cannot classify construction lines on the basis of their equations in polar coordinates, but these lines can be used to obtain a more convenient form compared to the Cartesian coordinate system in which lines such as the Archimedes spiral and the logarithmic spiral, as well as the four-petal rose, are specified by implicit functions. There is no algorithm for choosing a coordinate system, only experience allows us to make the most convenient choice.

2. Methods.
The widespread use of building constructions made up of metal beams arranged along rectilinear generators, combining high strength and lightness has determined their wide distribution in our country and abroad. The research method consists in formation of an equation of the desired construction, we choose an arbitrary point M with x and y coordinates lying on the desired curve in projection onto the Oxy plane. This means that x and y will be the current coordinates. To make a geometrical place of the points lying on the curve and satisfying a certain condition means to find the equation of this curve i.e. to express the geometric property of the curve using the formula. We have found the following equations using this method: Cassini oval, for the case m > n, as well as Bernoulli lemniscates, for the case m = n in Cartesian and polar coordinate systems on the plane, as well as their cylindrical surfaces in space with the possibility of their hardening along rectilinear generators.

3. Results and Discussion.
We begin the presentation of the material with the statement of tasks. According to the developed algorithm, you need to find the equations of geometric location of points of the building construction by the condition: the product of distances for which up to two given points A and B is a constant value equal to m^2. The distance between A and B is 2n. It is necessary to consider the construction option for the case when m = n and find building constructions that meet the specified conditions and build them.

To solve the above problems, namely, compiling the desired projections onto the Oxy plane, it is convenient to take the straight line passing through the A and B points through the A axis (then the coordinates of the A and B points will be simple), and take the straight line bisecting segment AB as the oy axis (equal points of points A and B allow us to assume the symmetry of the curve) (Figure 1).

![Figure 1. Coordinate system selection: AM ∙ BM = m^2](image)

Let x and y be the coordinates of an arbitrary point M lying on the desired curve (Fig. 1), i.e. M (x, y) - point M with current coordinates. By establishing the relationship between the current coordinates and the constant values characterizing the given curve, we will compose the equation of the curve (geometric location). In this case, we need to establish the relationship between x, y and the constants m^2 and n. From the definition of the geometric location it follows (Fig. 1): AM ∙ BM = m^2. Expressing
the lengths of the segments AM and BM according to the formula for the distance between two points (the coordinates of points A and B with respect to the coordinate system chosen by us will be A (-n, 0), B (n, 0)), we get (1):

\[ \sqrt{(x + n)^2 + y^2} \cdot \sqrt{(x - n)^2 + y^2} = m^2 \]  

(1)

This is the equation of this geometrical place. After simplification: freeing from radicals, making possible reductions and elementary transformations, we get:

\[ (x^2 + y^2 + n^2 + 2nx) \cdot (x^2 + y^2 + n^2 - 2nx) = m^4, \text{ or } (x^2 + y^2)^2 - 4n^2 \cdot x^2 = m^4 - n^4 \]  

(2)

To build a curve, we examine the found equation. Since x and y are squared in equation, which does not change their sign to the opposite when changing their sign, the curve is symmetric about the coordinate axes. Further, from the geometric definition of the geometric place in question, it is obvious that this curve is a closed curve.

When \( y = 0 \), we find the points of intersection with the axis ox:

\[ x^4 - 2n^2x^2 = m^4 - n^4 \]

When \( x = 0 \), we find the points of intersection with the axis oy:

\[ y^4 + 2n^2y^2 = m^4 - n^4 \]

Having built a series of curve points, the coordinates of which can be found, calculating one of the coordinates from the given arbitrary values of the other from the found equation, and connecting them with a smooth line, we get (for \( m > n \)) a curve (Figure 2).

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**Figure 2. Oval Cassini**

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a) With the same choice of coordinate axes, the lemniscate equation takes the form (3):

\[ (x^2 + y^2)^2 = 2m^2(x^2 - y^2) \]  

(3)

b) If the pole is combined with the beginning of the Cartesian coordinate system, and the polar axis is combined with the axis ox, then the lemniscate equation will have the form (4):

\[ \rho^2 = 2m^2 \cdot \cos 2\varphi \]  

(4)

First of all, the position of the coordinate system is established to derive the equation in polar coordinates: we combine the pole with the origin of the selected Cartesian system in the plane, and we align the polar axis with the axis ox. Then we get (Figure 3):
Figure 3. Polar system selection: AM·BM=m^2

Evaluating the segments AM and BM according to the well-known trigonometry formula \( x = \rho \cos \varphi; \ y = \rho \sin \varphi; \ x^2 + y^2 = \rho^2 \), we obtain equation (5):

\[
\sqrt{\rho^2 + m^2} + 2m \cdot \rho \cos \varphi \cdot \sqrt{\rho^2 + m^2} - 2m \cdot \rho \cos \varphi = m^2
\]

(5)

Performing elementary transformations, we obtain:

\[
(p^2 + m^2)^2 - 4m^2p^2 \cdot \cos^2 \varphi = m^4; \ \rho^4 = 4m^2 \cdot p^2 \cdot \cos^2 \varphi - 2m^2 \cdot \rho^2,
\]

or \( \rho^4 = 2m^2p^2 \cdot (2\cos^2 \varphi - 1) \) and finally (6):

\[
\rho^2 = 2m^2 \cdot \cos 2\varphi
\]

(6)

This curve is shown in Figure 4.

Figure 4. Lemniscata Bernoulli

A comparative analysis of the executed lines and their equations shows us that the accuracy of their calculations is the same, but it is easier to make calculations in the polar coordinate system for the lemniscate, since explicitly setting \( \rho = f(\varphi) \) reduces the order of the function for calculations (the Cartesian system is fourth order, the polar system is second order).

This approach finds a wide range of practical application, starting from a summer residence, a platform in front of a house or school to the architecture of the future, for breaking a flower garden or an exotic arrangement of plants to building castles, windows, roofs in space, these lines form surfaces of interesting cylindrical shape in the Cartesian coordinate system, when the Z-axis takes any value. (Figure 5).

Figure 5. Cylindrical surfaces: a) Oval Cassini; b) Lemniscata Bernoulli of straight lines, called rectilinear generators of these surfaces.
The surfaces defined by equations (2) and (3) are cylindrical and called the Cassini and Bernoulli cylinders, respectively. Their generators are parallel to the oz axis, and the guides are the Cassini oval and the lemniscate Bernoulli in the oxy plane.

And with the help of structural hardening by means of straight-line generators (Figure 6), interest in them will increase in the near future, perhaps, they will open up a universal approach to composing equations and surfaces to predetermined geometric parameters for high-order nonlinear functions, with research on the possibility of their hardening [2].

Figure 6. One-sheeted hyperboloid composed of straight lines called rectilinear generators

4. Conclusion
1. You should choose a coordinate system for designing building constructions of a specific geometric location with predetermined properties. (Figure 1, Figure 2).
2. After choosing a coordinate system, you should determine the location of the parameters specified in the project for the given values in these coordinates by designing them on a plane.
3. On the desired line, you should select an arbitrary point with the current coordinates (Figure 1, Figure 2, point M).
4. Having established the relationship of current coordinates with constant values (parameters) according to the project condition, compose the equation: formulas (2), (3), (4).
5. After simplifying the compiled equation, construct a building structure (surface) by examining it for properties of interest.
6. By activating your intuition, your experience will tell you which coordinate system to choose and how to place the given parameters in them in order to simplify the finding of the equation for constructing the construction.
7. If you design the surface and the location of constant values (parameters) in it, the correctness of the selected coordinate system on a plane or in space will make it possible to make a less complex function, taking into account all the relationships between the geometrical location of the points and the parameters under study.

In conclusion, we note that having mastered this algorithm for compiling building structures, according to predetermined conditions, you can design constructions of any degree of complexity for their construction, which is especially in demand in construction equipment. Considering the problems encountered in the design as additional features [3-11], we open the curtain of endless transformations of some forms of lines and surfaces into others, as well as the variety of methods for their hardening, by connecting links of one chain of rectilinear generators. Having created a database of elements of building structures and their connecting links, we come to the possibility of automating the process of designing structures with predetermined properties.

References
[1] Grigorenko N L, Grigorieva É V, Khailov E N and Roi P K 2019 Optimal Control Problems for a Mathematical Model of the Treatment of Psoriasis *Computational Mathematics and Modeling* 30 pp 352–363

[2] Mohapatra J and Mahalik M K 2018 An Initial Value Method for Solving Singularly Perturbed Boundary Value Problems Using Adaptive Grids *Computational Mathematics and Modeling* 29 pp 48–58

[3] Voskoglou M A 2017 Note On The Graphical Representation Of The Derivatives *Physical and Mathematical Education: scientific journal* 2(12) pp 9-16

[4] Pandey P K 2018 A numerical technique for the solution of general eighth order boundary value problems: a finite difference method *Ural Mathematical Journal* 4 (1) pp 56-62

[5] Pandey P K 2013 Fourth Order Finite Difference Method for Sixth Order Boundary Value Problems *Computational Mathematics and Mathematical Physics* 53 (1) pp 57-62

[6] Viswanadham K N S K and Ballem S 2014 Numerical solution of eighth order boundary value problems by Galerkin method with quintic B-splines *International Journal of Computer Applications* 89 (15) pp 7-13

[7] Reddy S M 2016 Numerical solution of eighth order boundary value problems by Petrov-Galerkin method with quintic B-splines as basic functions and septic B-splines as weight functions *International Journal of Engineering and Computer Science* 5 (09) pp 17894-17901

[8] Jiang Z W 2014 A meshfree method for numerical solution of nonhomogeneous time dependent problems *Abstract Appl. Anal* 978310

[9] Voloshinov D V 2010 *Constructive geometric modeling. Theory, practice, automation: monograph* (Saarbrucken: Lambert Academic Publishing) p 355

[10] Vabishchevich P N, Vasil’ev V I and Vasil’eva M V 2015 Computational identification of the right hand side of a parabolic equation *Comput. Math. Math. Phys.* 55 (9) pp 1015-1021

[11] Schmitz K S 2018 Chapter 1: Philosophy of Science *Physical Chemistry* pp 183-367