Re-examination of log-periodicity observed in the seismic precursors of the 1989 Loma Prieta earthquake

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Abstract. Based on several empirical evidence, a series of papers has advocated the concept that seismicity prior to a large earthquake can be understood in terms of the statistical physics of a critical phase transition. In this model, the cumulative seismic Benioff strain release $\epsilon$ increases as a power-law time-to-failure before the final event. This power law reflects a kind of scale invariance with respect to the distance to the critical point: $\epsilon$ is the same up to a simple rescaling $\lambda^z$ after the time-to-failure has been scaled by a factor $\lambda$. A few years ago, on the basis of a fit of the cumulative Benioff strain released prior to the 1989 Loma Prieta earthquake, Sornette and Sammis [1995] proposed that this scale invariance could be partially broken into a discrete scale invariance, defined such that the scale invariance occurs only with respect to specific integer powers of a fundamental scale ratio. The observable consequence of discrete scale invariance takes the form of log-periodic oscillations decorating the accelerating power law. They found that the quality of the fit and the predicted time of the event are significantly improved by the introduction of log-periodicity. Here, we present a battery of synthetic tests performed to quantify the statistical significance of this claim. We put special attention to the definition of synthetic tests that are as much as possible identical to the real time series except for the property to be tested, namely log-periodicity. Without this precaution, we would conclude that the existence of log-periodicity in the Loma Prieta cumulative Benioff strain is highly statistically significant. In contrast, we find that log-periodic oscillations with frequency and regularity similar to those of the Loma Prieta case are very likely to be generated by the interplay of the low-pass filtering step due to the construction of cumulative functions together with the approximate power law acceleration. Thus, the single Loma Prieta case alone cannot support the initial claim and additional cases and further study are needed to increase the signal-to-noise ratio if any. The present study will be a useful methodological benchmark for future testing of additional events when the methodology and data to construct reliable Benioff strain function become available.

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1. Introduction

The idea that earthquakes are somewhat analogous to critical phenomena of statistical mechanics has been gaining ground in the last few decades [Chelidze, 1982; Allègre et al., 1982; Sornette and Sornette, 1990; Tumarkin and Shirman, 1992; Sornette and Sammis, 1995; Newman et al., 1995; Bowman et al., 1998; Jaume and Sykes, 1999]. One of the consequences of this new point of view is that events occurring even several decades before a large main shock can be considered as seismic precursors, and that a study of this precursory seismicity might give a fairly good indication of when the impending major earthquake will take place, and how big it is going to be. Although such considerations are still in infancy, and the usual caveats about earthquake prediction must be kept in one’s mind, a lot of work has already been devoted to “post-diction”, sometimes with impressive success. One of the pioneering cases in that direction was the 1989 Loma Prieta earthquake, where Sornette and Sammis [1995] proposed to see the empirical power law used by Bufe and Varnes [1993] in the perspective of criticality in the sense of statistical physics [Sornette, 2000]. They found that the cumulative Benioff strain starting about 50 years ago could be well-fitted with a power law, giving rise to a post-diction for the main event of (1990.3 ± 4.1), a reasonably satisfactory result.

Things got even more exciting after the paper of Sornette and Sammis [1995] where it was pointed out that the strong oscillations around the power law could be fitted as well using a complex exponent correction to scaling:

\[
\epsilon(t) = A + B(t_f - t)^2 \{1 + C \cos[\omega \log(t_f - t) + \phi]\}.
\]

(1)

In this formula, the parameter \(t_f\) is the time of the main shock (a pure power law would correspond to \(C = 0\)), and the best fit gave rise to an estimate \(t_f = 1989.9 \pm 0.8\), considerably closer to the real date than the one in [Bufo and Varnes, 1993], and with much less uncertainty.

The existence of complex correction to scaling exponents could be linked to an underlying discrete scale invariance, a very appealing property from a theoretical point of view: the initial observation in [Sornette and Sammis, 1995] therefore spurred a lot of development [Saleur et al., 1996a, b; Johansen et al., 1996; Huang et al., 1998; Sornette, 1998; Bowman et al., 1998; Jo-

Hansen et al., 2000; Sammis and Smith, 1999; Jaume and Sykes, 1999].

Although the quality of the fit in Sornette and Sammis [1995] is, to the naked eye, impressively good, the suspicion arose recently that the oscillations in the Loma Prieta Benioff strain could merely be the result of noise. The synthetic tests performed in [Sornette and Sammis, 1995] being somewhat incomplete, we have decided to reanalyze this question much more carefully in the present work.

We have performed two types of synthetic tests to do this re-analysis.

In the first type, we consider random power laws (explained in Section 3), with parameters that match those of the real data, and study whether noise can give rise to log-periodic structures after integration. The advantage of this approach is that the two key ingredients of the analysis of the real data (power law and integration) are captured in a simple way. Its drawback is that the synthetic data (a sequence of random numbers drawn from a power law probability distribution, from which a cumulative quantity is constructed) is not quite of the same nature than the real data (a sequence of times and magnitudes, from which the cumulative Benioff strain is constructed). In particular, the sampling for the synthetic data is essentially periodic in log scale, and this effect, combined with integration, is expected to give rise to spurious log-periodic oscillations indeed. Nevertheless, we find the consideration of these synthetic tests quite useful, as it complement the considerations in [Huang et al., 2000].

The second type of synthetic tests is devised especially to avoid this issue of sampling: we generate data for both time and magnitude, in such a way that the probability distributions of the synthetic (\(t^*, m^*\)) and the real (\(t, m\)) quantities are the same. There is a problem in doing so; although the synthetic data and the real data have the same distribution, there is no guarantee that the cumulative Benioff strain constructed form the synthetic sequences is really a power law, because a power law dependence involves higher-order statistics (i.e., correlation and dependence) not captured by the one-point distribution functions. To preserve the feature of the real data that events are more frequent and with higher magnitudes when closer to the main shock (or the last data point), we added a reordering procedure which shuffles the synthetic sequences in such a way that the event with the \(j^\text{th}\) magnitude is at the same position as the real event with the \(j^\text{th}\) magnitude in the real sequence.

We find that for both type of synthetic tests, it is,
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surprisingly, highly possible to get spurious (that is, entirely due to noise) log-periodic oscillations which are as good as those observed in the real Loma Prieta data. This conclusion is made quantitative in a variety of ways, in particular by studying the highest peak of the spectrum of oscillations around the power law for the real data, and building the probability distribution of such peaks for synthetic data. We thus conclude that, at the present time, it is not possible to distinguish the log-periodic oscillations observed in [Sornette and Sammis, 1995] from noise.

The present study is related to [Huang et al., 2000]. The common theme is the investigation of the conditions under which log-periodicity can be created spontaneously by noise. In [Huang et al., 2000], the goal is to study in details the underlying mechanism, relying solely on the manipulation of data: the generally found non-uniform sampling together with a low pass filtering step, as occurs in constructing cumulative functions, in maximum likelihood estimations and detrending, is enough to create apparent log-periodicity. A detailed exploration of this mechanism has been offered in [Huang et al., 2000] together with extensive numerical simulations to demonstrate all its main properties. It was shown that this “synthetic” scenario for log-periodicity relies on two steps: 1) the fact that approximately logarithmic sampling in time corresponds to uniform sampling in the logarithm of time; 2) integration reddens the noise and, in a finite sample, creates a maximum in the spectrum leading to a most probable frequency in the logarithm of time. In [Huang et al., 2000], this insight was then used to analyze the 27 best aftershock sequences studied by [Kisslinger and Jones, 1991] and search for traces of genuine log-periodic corrections to Omori’s law, which states that the earthquake rate decays approximately as the inverse of the time since the last main shock. The observed log-periodicity was shown to almost entirely result from the “synthetic scenario” due to the data analysis. From a statistical point of view, resolving the issue of the possible existence of log-periodicity in aftershocks will be very difficult as Omori’s law describes a point process with a uniform sampling in the logarithm of the time. By construction, strong log-periodic fluctuations are thus created by this logarithmic sampling. In contrast, in the present paper, we apply the insight obtained in [Huang et al., 2000], to study accelerated power laws culminating in a finite-time singularity at time $t_f$.

To be complete, we should also point out the following. Sornette and Sammis [1995] paper contains a forward prediction of an earthquake in the Komandorski Island region at time $t_f = 1996.3 \pm 1.1$ year. Forward predictions provide a much larger statistical significance since the model parameters are estimated independently and outside their domain of application (see below the discussion in the section on the analysis procedure). Forward prediction has also the quality of increasing the number of cases. The prediction of a critical time is not enough, one must specify the magnitude of the predicted earthquake. In [Sornette and Sammis, 1995], the magnitude was not specified but can probably be taken following the specification of Bufe et al. [1994] of a magnitude in the range 7.5-8.5 occurring in a zone originally outlined by Nishenko [1991]. The largest earthquake in the Harvard catalog during the time period 1994-1998 has a moment magnitude $M_W = 6.6$ (1996/07/16, 56.16N, 164.98E). The same event has the magnitude $M_S = 6.4$ in the PDE catalog. If the prediction is considered correct only if both its time and magnitude range is as predicted, this prediction is a failure. However, it is hard to draw any firm conclusion based on this single case with respect to usefulness of the critical earthquake concept and of log-periodicity as the methodology has evolved significantly since the initial paper of Sornette and Sammis [1995] (see for instance [Bowman et al., 1998; Ouillon and Sornette, 2000]).

The plan of this paper is as follows. In Section 2 we present some general observations on synthetic tests and their interpretations. Details on our two types of tests, together with their results, are presented in Sections 3 and 4. Our conclusions are collected in Section 5.

2. Analysis Procedure

Usually, the major problem in establishing the statistical significance of a forecasting procedure is its retrospective character involving a limited number of data and a significant number of explicit (the parameters of the fit) and implicit (the total time and space windows used, etc.) degrees of freedom. In such situations, the calculation of statistical significance becomes very difficult and uncertain as soon as the adjustable parameters are determined from the data. The conclusions can then be artifacts of the processing technique or of a selection bias. The paper of Sornette and Sammis [1995] certainly suffers from this problem. Since there are no general methods or techniques that would allow us to overcome these difficulties, we stick to a more modest approach, which turns out to be sufficient to draw a clear and meaningful conclusion.
We develop two types of synthetic tests. For both types, the key part is the comparison of the log-periodic oscillations in the real seismicity to those in the synthetic sequences. Similar oscillations should have similar frequencies and similar regularities, and this can best be quantified by considering the spectrum of these oscillations. We focus on the highest peak (characterized by angular frequency $\omega$ and peak height $h$) in the spectrum, which quantifies the most significant frequency component in the oscillations as a function of $\log(t_f - t)$ (since we are looking for log-periodicity). The Lomb method [Press et al., 1992] is used instead of the usual Fourier Transform, since the data points are not equidistantly spaced.

The ultimate purpose of synthetic tests is to get the significance level (the probability of getting the same thing by accident) of the real observation. One natural way of evaluating the significance level would be, it seems, to count the number of synthetic peaks (defined as spectral components with spectral power higher than neighboring frequencies, in other words, local maximums in the spectrum) within the intervals $\omega^s \in [\omega^r - \Delta, \omega^r + \Delta]$ and $h^s \geq h^r$ (superscript $s, r$ denote synthetic and real data respectively), and to normalize this by the total number of synthetic peaks. This might however lead to incorrect conclusions. For instance, suppose that, from 10000 synthetic peaks, only one peak with peak height higher than 10 (the height of the real peak) and angular frequency $\omega$ in the range of $[\pi, 2\pi]$ was found: could we conclude that the significance level is 0.01% (or confidence level of 99.99%)? Probably not. To see why, suppose the distribution of $\omega$’s were uniform in $[0, 40]$, and the distribution in peak height uniform in $[6.862, 10.004]$; then the probability of observing one peak of height above 10 and $\omega \in [\pi, 2\pi]$ would roughly be $1/10000$. But this small probability does not mean that the peak of height above 10 and $\omega \in [\pi, 2\pi]$ is highly significant! Due to the uniform distribution in both $\omega$ and peak height, any other pair of $(\omega, h)$ would have, in fact, been observed with the same probability. To draw positive conclusions from such a simple counting analysis, one would need to have in advance a theory indicating where the peak should be found, and approximately at what height. It is not clear that there is such a thing at present, and therefore we prefer to use a more conservative approach.

In the language of statistics, this question is related to the difference between first-order and second-order statistics. In first-order statistics, we ask: “what is the probability to observe the peak we see just by chance?” In second-order statistics, we ask: “what is the probability to observe some (first-order significant) peak somewhere (whatever its position)?” In our context, the determination of the confidence level within first-order statistics requires that we have an a priori understanding of where the peak should be found.

In the absence of any theoretical predictions, it seems natural, to quantify the significance level of the log-periodic oscillations, to rely somehow on the probability distribution of $\omega$ and peak height in the synthetic samples, i.e. to rely on second-order statistics. We have not managed to come up with a totally satisfactory, objective way to use this probability distribution however. A useful quantity we came up with—-but it should not be trusted blindly—is the ratio $R$ of the probability of observing a given peak to the probability of observing the most probable peak: if the ratio is close to 1, this surely indicates that the peak is not very significant.

To obtain this ratio, one needs the probability density function $p(\omega, h)$ of the synthetic peaks: the latter can be constructed using the Kernel Density method [Silverman, 1986; Beardah, 1995]. We then set $R = \frac{p(\omega^r, h^r)}{\int p(\omega^s, h^s) d\omega dh}$ where $(\omega^r, h^r)$ characterize the real peak, and $(\omega^{mp}, h^{mp})$ is the most probable synthetic peak. The ratio quantifies how frequently in synthetic sequences we can observe log-periodicity similar to that observed in the real sequence. There are some technical advantages in using this ratio. For instance, there is in fact no arbitrariness in choosing the intervals $d\omega$ and $dh$, since they cancel out between the numerator and the denominator: it is then enough to just use the function value of $p(\omega, h)$ at $(\omega^r, h^r)$ and $(\omega^{mp}, h^{mp})$. Also, the special choices (e.g., the degree of smoothing and the type of kernels) made in constructing the probability density function $p(\omega, h)$ hopefully also cancel out in the ratio, and have little influence on the final result.

The generation of synthetic data and extraction of oscillations depend on the type of synthetic tests and will be explained separately in the following sections.

3. Synthetic Test I

3.1. Generation of Synthetic Data

We start from a simple method which takes into account the most crucial features of the real data. We then refine this method in several ways in Section 4.

The crucial features of the real data are power law and integration. The latter—considering the cumulative Benioff strain—is necessary for numerical reasons, as there are not enough data points to study directly the rate at which energy is released (moreover, consid-
erating the rate leads to other difficulties, like the influence of the binning intervals etc). We therefore take a pure power law as the null hypothesis, and generate data with a probability density fitting the power law part of \( \frac{dc}{dt} \): we mimic real data as closely as possible, taking in particular the same number of points. We then construct a synthetic cumulative Benioff strain by numerical integration, and investigate whether noise in the sampling of the power law can give rise to spurious log-periodic oscillations.

Again, the power law is

\[
\frac{dc(t)}{dt} \propto (t_f - t)^{m-1}
\]

following (1) in [Sornette and Sammis, 1995]. We assume the range for \( t \) is \([t_0, t_1]\) with \( t_1 < t_f \) to avoid the singularity at \( t_f \). After normalization, we have

\[
\frac{dc(t)}{dt} = \frac{m}{(t_f - t_0)^m - (t_f - t_1)^m}(t_f - t)^{m-1}.
\]

For the real seismic precursors, \( t \) is the time of occurrence of an earthquake, for synthetic events \( t \) is a random variable with probability function \( p(t) = \frac{dc(t)}{dt} \). To make sure the synthetic events and the real seismic precursors have the same power law distribution, we chose the same parameters \( t_0, t_1, t_f, m, N \) (from Table I and Figure 1 of [Sornette and Sammis, 1995]), where \( N \) is the number of events. (Since the original data was not available at the time of this study, we retrieved data from the CNSS catalog using their space-time-magnitude window. However, due to some unknown reason, our data set was slightly different from their data set. We got only 27 events instead of 31.) The random variable \( t \) with given \( p(t) \) can be transformed from a random variable \( x \) uniformly distributed on \([x_0, x_1]\) by solving [Press et al., 1992]

\[ p(x)dx = p(t)dt. \]

The transformation is

\[
t = t_f - [(t_f - t_0)^m - \frac{x - x_0}{x_1 - x_0}((t_f - t_0)^m - (t_f - t_1)^m)]^{\frac{1}{m}}.
\]

We then construct the cumulative distribution function of \( t \) and use this function to mimic the cumulative Benioff strain of the real sequence. They have the same power law parameters and they are both, indeed constructed from integration.

### 3.2. Extraction of Oscillations

The original analysis procedure used in [Sornette and Sammis, 1995] was to fit the cumulative Benioff strain to a power law with log-periodic oscillations ((1), the same as (8) in [Sornette and Sammis, 1995]). The quality of the observed log-periodicity was not quantified in [Sornette and Sammis, 1995] other than by showing that the quality of the fit measured by the residue as well as the predicted critical time \( t_f \) were both substantially improved compared to those from the fit with the simple power law, but for our study, it is crucial to do so.

To get the oscillations, we first fit a power law with log-periodic oscillations (1) to both the real and synthetic data. The pure power law part (obtained by setting \( C = 0 \) in (1)) is then subtracted, and we obtain the remaining oscillations. These oscillations are in turn analyzed using the procedure outlined in Section 2.

### 3.3. Results

#### 3.3.1. The real sequence.

We first characterize the log-periodicity observed in the real sequence. The fit of (1) to the real data is remarkable (Figure 1). The oscillations around the power law part show approximately 2.5 cycles of regular oscillations (Figure 2), the spectrum of which has a peak near \( \omega' \sim 6.1 \) with height \( h' \sim 7.5 \) (Figure 3), which is significantly different from Gaussian noise (the chance of observing such a peak from Gaussian noise of the same number of data points is less than 2% according to (13.8.7) of [Press et al., 1992]). However, since we are dealing with oscillations around a cumulative quantity, integrated Gaussian noise would be a more appropriate null hypothesis. We will study the chance of observing such a peak from integrated Gaussian noise in Section 3.3.2.

#### 3.3.2. Synthetic sequences.

300 synthetic sequences were generated using the parameters of the seismic precursors for the 1989 Loma Prieta earthquake. They were analyzed in the same way as the real precursor sequence. See Figures 4, 5, and 6 for typical results. We note that it is possible to observe synthetic sequences which are remarkably similar to the real sequence: similar amplitude of oscillations, similar frequency, and similar regularity. In the following part, we quantify how frequently such sequences can actually be observed.

The distribution function of the frequencies and peak heights of the synthetic sequences were constructed using the Kernel Density method [Silverman, 1986; Beardsah, 1995] (Figure 7). The real peak is not far from
Figure 1. The fit of a power law with log-periodic oscillations to the normalized cumulative Benioff strain of the seismic precursors of the 1989 Loma Prieta earthquake.

Figure 2. The oscillations around the power law of the normalized cumulative Benioff strain of the seismic precursors of the 1989 Loma Prieta earthquake.

Figure 3. The Lomb Periodogram of the oscillations shown in Figure 2.

Figure 4. Plot of a quantity similar to that in Figure 1 from a synthetic sequence.
Figure 5. Plot of a quantity similar to that in Figure 2 from a synthetic sequence.

Figure 6. Plot of a quantity similar to that in Figure 3 from a synthetic sequence.

Figure 7. The distribution function of frequencies and peak heights from the synthetic sequences. The position of the real peak is marked by the vertical line.

The most probable synthetic peak. From the distribution function (Figure 7), we find that the probability density function at the most probable synthetic peak is proportional to 0.031, while it is proportional to 0.016 at value of $\omega$ and $h$ corresponding to the real peak. The ratio of these two quantities is close to one half. If we look at the separate distribution of frequencies and peak heights (Figures 9 and 10), we see that the frequency from the real sequence is slightly higher than the most probable synthetic peak, and the peak height of the real sequence is almost the most probable synthetic peak height. Note that the frequencies from synthetic sequences have a rather narrow distribution, as expected from [Huang, 1999; Huang et al., 2000]. The regularity of oscillations observed in the real sequence (Figure 1, quantified by the peak height) is not surprising due to the strong smoothing effect of integration [Huang, 1999; Huang et al., 2000]. When the same analysis procedure was applied to both the real data and the synthetic data, this kind of regularity is observed in both the real data and the synthetic data.

4. Synthetic Test II

4.1. Generation of Synthetic Data

For synthetic tests to be effective, the synthetic data should differ from the real ones by only one characteristic—the characteristic to be tested. Since in our problem we want to test log-periodicity in the oscillations around a power law, the synthetic data should
Figure 8. 2D map view of Figure 7. Each circle represents one synthetic peak. The frequency of the real peak is marked by the vertical line, peak height by the horizontal line.

Figure 9. The distribution of the synthetic frequencies. The vertical line marks the position of the real frequency. The two diamonds (♦) mark the FWHM (full-width-half-maximum) of the distribution (the same for all subsequent similar plots).

Figure 10. The distribution of the synthetic peak heights. The vertical line marks the position of the real peak height.

be the same power law but with known noise (could be additional data errors or random fluctuations.). Since the power law of the cumulative Benioff strain is in fact implied by the magnitude distribution and the temporal spacing between events and their correlations, we have to generate synthetic magnitude $m^*$ and time $t^*$ to get the same power law. $t^*$ and $m^*$ should be random numbers having the same probability distribution as $t$ and $m$, because in our observations we had control over neither $t$ nor $m$. In this light, the analysis in Section 3 is thus a bit oversimplified; we now refine it.

One difficulty in generating synthetic $t^*$ and $m^*$ is that the theoretical probability density function (pdf) of magnitudes and times for our real data is unknown. This problem can be solved by using the empirical pdf constructed from the real data. However, we should not use the empirical pdf directly, otherwise all features of the real data would be reproduced in the synthetic data. For example, the empirical distribution of the real time sequence (Figure 11) shows some regular oscillations (that could well be genuine physically-based log-periodic oscillations) around its general trend.

If we used exactly this empirical distribution to generate the synthetic time sequences, all synthetic sequences would show similar regular oscillations: this is of course not appropriate, since what we want to test is precisely whether these oscillations are the result of noise. Therefore, we decided to use only the general trend of the experimental data to generate our synthetic samples. This general trend is obtained by smoothing
Figure 11. The normalized cumulative number of events up to time \( t \) of the time sequence of the seismic precursors of the 1989 Loma Prieta earthquake (solid line connecting circles). The other solid line is the empirical line repeatedly smoothed (50 times) by 3-point moving average.

The empirical distribution. Three-point moving average was applied repeatedly 50 times (10 for smoothing the cumulative distribution function of magnitudes). The number of times is not crucial as long as the oscillations were wiped out. The criteria for our choice are closeness to the empirical curve and lack of oscillations. Similar considerations were applied to the generation of synthetic magnitude sequence. The assumption for the general trend was not crucial. A reasonable one, close to the empirical cumulative distribution curve but without the fluctuations, would suffice.

The next difficulty is that, for a sequence of times and magnitudes generated using this method, there is no guarantee that the cumulative Benioff strain will follow a power law. The time sequence more or less follows a power law (we have verified that the cumulative number of events of the seismic precursors of the 1989 Loma Prieta earthquake is similar to the cumulative Benioff strain in power law and log-periodic oscillations), but, when combined with the magnitude sequence to construct the whole Benioff strain curve, there is no obvious reason why we should always get a power law. It is natural to expect that the power law of the real sequence comes mainly from the fact that events occur more frequently with increasing magnitude (trend only) when closer to the main shock. To preserve this feature in the synthetic sequences, we decided to re-order the events in the synthetic sequence such that the event of the \( k^{th} \) magnitude would occur at the same position in both the real and the synthetic cases (for example, both the real sequence and the synthetic sequences have the event of the second biggest magnitude being the \( k^{th} \) one in the sequence). This reordering scheme is applied only to the magnitude sequence (time sequence is an ordered sequence by definition). One example is shown in Figures 12 and 13.

We performed synthetic tests both with and without the reordering scheme. In fact, the re-
results turned out to be almost identical.

4.2. Extraction of Oscillations

The method of extracting oscillations from the cumulative Benioff strain is slightly different from that of Section 3.2, however the difference turns out to be insignificant.

Two ways are possible to obtain the oscillations: the first involves extracting the best-fit power law from the real data. The drawback of this approach is that power law fits are often not as stable as fits including the log-periodic corrections. Sometimes (around 8% of all cases) the fit even converges to a \( t_f \) smaller than the time of the last data point, thus \( t_f - t \) is negative and \( (t_f - t)^{(m-1)} \) is complex since \( m < 1 \), which is of course unphysical. The advantage of this approach is that log-periodicity is not assumed in the first place. The second approach involves extracting the power law obtained from a best-fit power law with log-periodic oscillations [Johansen and Sornette, 1999a, b; Johansen et al., 1999]. The advantage here is that the fit always converges well, but now log-periodicity is somewhat assumed from the very beginning. The more positive view point advocated in [Johansen and Sornette, 1999a, b; Johansen et al., 1999] to justify this procedure is that fitting with log-periodicity allows one to take the most probably noise into account [Huang et al., 2000] and thus to obtain a good pure power law representation by putting the coefficient \( C = 0 \). In practice, the results of either approach were very similar: in the following, we report only the results using the second one.

It is of course crucial to use exactly the same analysis procedure for both the real data and the synthetic data, otherwise features generated by the analysis procedure for the real data may not be detected by the synthetic tests.

The cumulative Benioff strain was first constructed from the magnitude sequence \( m_i \):

\[
e_i = \sum_{j=1}^{i} 10^{0.75m_j}
\]

(5)

and then normalized such that \( \epsilon_{\text{max}} = 1 \) (the unit was changed without influencing the conclusion). As in [Sornette and Sammis, 1995], the cumulative Benioff strain was then fitted to a power law with log-periodic oscillations.

\[
detrn = \frac{\epsilon(t) - A}{B(t_f - t)^z}
\]

(6)

which should be either noise or pure log-periodic cosine according to (1), and then analyzed by the procedure explained in Section 2.

4.3. Results

4.3.1. The real sequence. The fitting of the real data showed good agreement between the real data and the theoretical curve (Figure 14). There was a peak at \( \omega = 6.1 \) in the spectrum of the de-trended data (Figure 16), close to the best-fit parameter \( \omega = 5.7 \).

4.3.2. Synthetic sequences. 1000 synthetic sequences (one example in Figures 12 and 13) were analyzed using the same procedure as that for the real sequence. The cumulative Benioff strain of the synthetic sequence showed obvious similarity to the real data (Figure 17).

We first checked whether the synthetic data gave rise to power laws. The method to do this was to fit these data to a power law shape, and measure the summed square of error (SSE) from the fitting: in about 50% of the cases, the synthetic sequences had an SSE smaller than that of the real sequence, indicating that they were roughly as good power laws as the real data.
Figure 15. The de-trended data of the normalized Benioff strain of the 30 seismic precursors of the 1989 Loma Prieta earthquake.

Figure 16. Spectrum of the de-trended data in Figure 15.

Figure 17. The fitting of the normalized cumulative Benioff strain of one synthetic sequence to (1).

We note here that the ratio of the probability of observing a synthetic sequence with SSE similar to that from the real sequence divided by the probability of observing a synthetic sequence with the most probable SSE is around 79%. If we used SSE to measure the goodness of the power law model for our synthetic data, the real sequence would be very close to the most probable synthetic sequence.

We then compared the fit of a power law with log-periodic oscillations to the real and the synthetic sequences.

The SSE of the synthetic sequences are in the range [0.005, 0.05], centered around the SSE from the real sequence (∼ 0.013). There are around 1/4 of the synthetic sequences that have a SSE smaller than that from the real sequence. The ratio of the probability of observing a synthetic sequence with SSE similar to that from the real sequence and the probability of observing a synthetic sequence with the most probable SSE is around 96%. If we use SSE to measure the regularity of log-periodic oscillations, the real sequence is thus very close to the most probable synthetic sequence.

The $t_f$ from the synthetic sequences is distributed in [1988,1995] (Figure 18). The ratio of the probability of observing a synthetic sequence with $t_f$ similar to that from the real sequence divided by the probability of observing a synthetic sequence with the most probable $t_f$ is
around 95%. Thus the apparent accurate prediction of the actual main-shock time for the 1989 Loma Prieta earthquake [Sornette and Sammis, 1995] might be due to chance. The width of the distribution (FWHM, full width at half maximum) is 6.8 years, narrower than that from the fit of a pure power law (7.8 years), suggesting that log-periodicity may improve power law fits by accounting for the most probable noise [Huang et al., 2000].

There is a well-defined peak in the distribution of the frequencies of the log-periodic oscillations from the fitting of the synthetic sequences (Figure 19). The ratio of the probability of observing a synthetic sequence with frequency similar to that from the real sequence divided by the probability of observing a synthetic sequence with the most probable frequency is around 81%.

We summarize the above results in Table 1.

We now turn to the statistics from the characterization of log-periodicity by spectral analysis of the de-trended data. If we only look at the distribution of peak heights, the ratio of the probability observing a synthetic peak similar to the real peak divided by the probability of observing the most probable synthetic peak is around 99.9% (Figure 20).

If we only look at the frequencies, the ratio of the probability of observing a synthetic fre-
Table 1. Parameters From the Fit of the Synthetic Data With PW (Pure Power Law) and PWLG (Power Law With Log-Periodic Oscillations)

|            | pr  | pmp | ratio | mp  | r   | left | right | FWHM |
|------------|-----|-----|-------|-----|-----|------|-------|------|
| PW m       | 0.72| 1.99| 0.36  | 0.34| 0.69| 0.17 | 0.57  | 0.40 |
| PW tc      | 0.073| 0.13| 0.56  | 1994.3| 1988.7| 1988.4| 1996.2| 7.8  |
| PW SSE     | 20.0| 24.9| 0.79  | 0.028| 0.037| 0.016| 0.052| 0.036|
| PWLG m     | 1.94| 2.23| 0.87  | 0.45 | 0.52 | 0.25 | 0.63  | 0.38 |
| PWLG tc    | 0.13| 0.14| 0.95  | 1990.5| 1989.8| 1988.1| 1994.8| 6.8  |
| PWLG w     | 0.21| 0.26| 0.81  | -4.28| 5.67 | 3.16 | 6.99  | 3.83 |
| PWLG C     | 2.84| 6.71| 0.42  | -0.039| -0.083| -0.078| 0.075| 0.15 |

*a* Value of the probability density function at a synthetic peak similar to the real peak.

*b* Value of the probability density function at the most probable synthetic peak.

*c* The most probable value from synthetic data.

*d* The value from real data.

*e* Value at the left point of the FWHM of the distribution of synthetic values.

*f* Value at the right point of the FWHM of the distribution of synthetic values.

*g* Full width at Half Maximum of a peak.

Figure 21. The distribution of frequencies from the de-trended data. The vertical line marks the value from the real sequence.

Figure 22. The distribution of peak heights and frequencies from the de-trended data.

Not using the reordering scheme did not sig-
significantly change the above results, except that, for some synthetic sequences, the power law was not good.

Also, recall that the foregoing results involved fitting data to a power law with log-periodic oscillations, then de-trending. Fitting data to a pure power law instead produced very similar results.

We summarize all the results in the previous sections in Table 2.

Table 2. Synthetic Tests of the Log-Periodicity Observed in the Benioff Strain of the Seismic Precursors of the 1989 Loma Prieta Earthquake

|                        | EOPW<sup>a</sup> | De-trended data |
|------------------------|------------------|-----------------|
|                        | pr<sup>b</sup>   | pmp<sup>c</sup> | ratio | pr  | pmp | ratio |
| ω                      | 0.11             | 0.11            | 0.99  | 0.12| 0.22| 0.56  |
| h                      | 0.19             | 0.20            | 0.93  | 0.12| 0.12| 1.00  |
| (ω, h)                 | 0.027            | 0.031           | 0.89  | 0.016| 0.029| 0.56  |
| (ω, h)<sup>d</sup>     | 0.020            | 0.025           | 0.81  | 0.16| 0.23| 0.73  |

<sup>a</sup>Extracted oscillations using the best-fit pure power law.

<sup>b</sup>Value of the probability density function at a synthetic peak similar to the real peak.

<sup>c</sup>Value of the probability density function at the most probable synthetic peak.

<sup>d</sup>No reordering.

5. Discussion

These synthetic tests were performed to determine whether it is possible to observe the reported log-periodicity [Sornette and Sammis, 1995] from integrated noise in power laws, and if possible, how big the probability is. We found that, if we use the highest peak of the spectrum of the oscillations around the power law of the cumulative Benioff strain to quantify the log-periodicity in the oscillations, peaks similar to the peak observed from the real sequence (the real peak) were indeed frequently observed from the synthetic sequences. The odds of observing a synthetic peak similar to the real peak is more than 50% of the odds of observing the most probable synthetic peak.

It is reasonable to use the highest peak in the spectrum of a signal to quantify the most significant frequency component in the signal. The position of the peak is the frequency (ω), and the peak height (h) quantifies the regularity of that frequency component. If two signals have similar peaks in their spectra, they must have oscillations of similar frequency and regularity.

Peaks similar to the real peak were observed frequently from the synthetic sequences. To quantify this frequency of observation, we constructed the probability density function p(ω, h) of the synthetic peaks in the space of (ω, h). From p(ω, h), we were able to obtain the probability of observing a synthetic peak similar to the real peak (ω<sup>r</sup>, h<sup>r</sup>), that is p(ω<sup>r</sup>, h<sup>r</sup>) dω dh. This probability might be a small number, which is meaningful only when compared with the probability of observing the most probable synthetic peak, which is p(ω<sup>mp</sup>, h<sup>mp</sup>) dω dh. The ratio of the two probabilities quantifies well how frequently we observe the feature from synthetic data.

The mechanism at the origin of log-periodicity in the synthetic data sets has been discussed in [Huang, 1999; Huang et al., 2000]. Briefly, log-periodicity results from the fact that taking the cumulative of a power law involves a low pass filtering step (reddening of the noise) which, in a finite sample, creates a maximum in the spec-
trum leading to a most probable log-frequency corresponding approximately to 1.5 cycles over the full sampled interval.

We looked into two quantities for log-periodicity in the oscillations around the power law of the cumulative Benioff strain. The extracted oscillations are the difference between the data and the best-fit power law. The de-trended data were obtained using (6). For both of them, the ratio of the two probabilities is bigger than 50%, which means that it is not only possible to observe that kind of log-periodicity in synthetic data, but also highly probable.

Our synthetic events and the real events have the same distribution in time and magnitude, and they were analyzed in exactly the same way. Since discrete scale invariance is not present in the synthetic data, the log-periodicity observed in the real sequence cannot be used as evidence for discrete scale invariance.

In fact, even the power law—used as evidence of ordinary scale invariance—could also be explained by other mechanisms. Indeed, the cumulative Benioff strains of the synthetic sequences in Section 4 do follow power laws similar to that of the real sequence. For the real sequence, the cumulative distribution function of event times is not significantly different from a straight line. The cumulative distribution of moments is not a power law either (S shape instead of the usual power law shape) (this is an ad hoc statement. The reason is that for this sequence of magnitudes, the number of data points is small (only 30) and the magnitude cut-off is very high (5.0). So even if in general the moment distribution is a power law, large statistical fluctuations may make the moment distribution of this sequence non power law. We use an empirical distribution function instead of the usual power law assumption to avoid dependence on that assumption. In fact our method will still be valid no matter what the underlying distribution is.). But for the real sequence, magnitudes tends to be bigger when closer to the main shock, especially for the last several events [Jones, 1994]. Since small difference in magnitudes will be translated into quite big difference in the cumulative Benioff strain, the last several events would make the would-be linear trend bend upward which happens to be well described by a power law of small exponent. Since this increasing tendency of magnitudes was preserved in the generation of synthetic magnitudes, power laws are also good for describing the synthetic data. In fact, without the re-ordering scheme to preserve that feature of magnitudes, we were still able to obtain similar results. As long as the magnitudes are not exactly uniform, the largest magnitude will bend the would-be linear trend somewhere, and power law with small exponents can still describe the data well.

The improved accuracy of the prediction of the main shock time of the 1989 Loma Prieta earthquake by consideration of log-periodic oscillations was regarded as a evidence supporting the hypothesis in [Sornette and Sammis, 1995]. However, from the study presented here, it is not rare to obtain $t_f$ near that value from our synthetic data containing events in the range of [1940,1988]. If we use a power law to describe the data, by definition $t_f$ should be slightly bigger than the time of the last data point. Indeed, from our simulations, we found that $t_f$ is distributed in [1988,1995], and the chance of observing a synthetic $t_f$ similar to the real $t_f$ is around 95% of the probability of observing the most probable synthetic $t_f$. The point is, given a sequence of events in that time range and a power law assumption, that kind of $t_f$ is not hard to find.

It is important to emphasize that the present study does not alter the usefulness of studying seismic precursors. However, the physical interpretation associated with the observations in [Sornette and Sammis, 1995] does not seem to be warranted, at least on the face of the Loma Prieta case only. However, as pointed out in [Huang et al., 2000] and also found in this paper, log-periodic oscillations are robust features of power laws. The present analysis as well as those given in [Huang et al., 2000] suggests that, whatever their origin (noise or physical), they might still be used to improve the prediction of the main event. This is clearly what we observe in our synthetic tests performed on pure power laws without log-periodicity: a power law fit with log-periodicity has a better estimate for $t_f$ than a pure power law without the log-periodic oscillations. The reason may be that, by fitting the most probable form of noise, the fit is more stable. It seems worthwhile to investigate this possibility further in future studies.
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