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Atmospheric Excitation of Polar Motion

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Abstract  The polar motion excited by the fluctuation of global atmospheric angular momentum (AAM) is investigated. Based on the global AAM data, numerical results demonstrate that the fluctuation of AAM can excite the seasonal wobbles (e.g., the 18-month wobble) and the Chandler wobble, which agree well with previous studies. In addition, by filtering the dominant low frequency components, some distinct polar wobbles corresponding to some great diurnal and semi-diurnal atmospheric tides are found.

Keywords  polar motion; atmospheric excitation; 18-month wobble; Chandler wobble; atmospheric tides

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Introduction

Polar motion refers to the displacement of the Earth’s rotation axis in a frame tied to the Earth (or more precisely, the mantle). It is a response to the mass redistribution and relative motions within the Earth system, as well as torques from both celestial bodies and Earth’s internal layers (such as the inner and outer cores) and superficial fluid envelopes (such as the atmosphere and ocean)¹ ² . Any process that can perturb the rotation axis is an excitation source of polar motion. In this study, only the effects of atmospheric excitation will be investigated.

The atmosphere, with strong mobility forced primarily by the diurnal and seasonal cycles, is the primary excitation source for the Earth’s rotation on intraseasonal and seasonal timescales³ ⁷ . The atmospheric excitation is usually separated into two parts: the “wind” terms due to the atmospheric motion relative to the crust plus mantle; and the “pressure” terms due to the variations of the atmospheric mass distribution, evident through surface pressure changes. At seasonal and intraseasonal timescales, the wind has been shown to be the dominant excitation source to the length of day (LOD) change, whereas it is comparable to the surface pressure change in exciting the Earth’s polar motion⁵ ⁸ ¹ ² .

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In the present study, the global atmospheric angular momentum (AAM) data\[^{8-11}\] ranging from 1948 to 2006, are adopted to analyze the AAM’s influence on the rotational axis. We find that the atmospheric excitation can be quite significant in exciting the polar motion, especially the 18-month wobble and the Chandler wobble.

1 Theory of nonrigid earth rotation

As is well known, the governing equations of polar motion, namely the Liouville equations, can be written as\[^{1-2}\]

\[
i m + m = \psi \quad (1)
\]

where \(\sigma = \frac{C - A}{A} \Omega\) is the Euler frequency, \(C\) and \(A\) are the principal moments of inertia, and \(\Omega\) is the mean rotation rate of the Earth. In Eq.(1), the traditional complex notations are adopted

\[
m = m_1 + im_2 \\
c = c_1 + ic_{23} \\
h = h_1 + ih_2 \\
L = L_i + iL_2 \\
\psi = \psi_1 + i\psi_2
\]

In Eq.(2), \(m, c, h\) and \(L\) are the pole coordinate, product of inertia (caused by the mass redistribution within the Earth system), relative angular momentum (caused by the relative motion with respect to the mantle) and torques from celestial bodies, respectively, while \(\psi\) is the so called excitation functions including all the factors perturbing the rotation state of the Earth. The excitation function might be written as\[^{1-2}\]

\[
\psi = \frac{Q^2 c - iQ\dot{c} + Qh - ih + iL}{\Omega^2 (C - A)} 
\]

(3)

The rotation of the Earth will give rise to a centrifugal force, which leads to a deformation of the nonrigid Earth. That is the Earth’s principal departure from the rigid body. Also, there are some other non-rotational deformations, such as the luni-solar tidal deformation. We can decompose the total deformation as

\[
c = c^R + c^{NR}
\]

and correspondingly

\[
\psi = \psi^R + \psi^{NR}
\]

(5)

where the superscripts \(R\) and \(NR\) correspond to the rotational and non-rotational deformations, respectively.

Concerning the Lagrangian displacement field \(s(r,t)\), a particle with an initial position \(r\) will get placed to \(r + s(r,t)\), then the total acceleration vector can be expressed as

\[
a = \left( \frac{\partial}{\partial t} + \omega \times \right) \left( \frac{\partial}{\partial t} + \omega \times (r + s) \right)
\]

(6)

where \(\omega\) is the angular velocity of the Earth. Noting \(\omega = (m_1 + m_2 + m_3)\Omega\) and neglecting terms of order \(m_1 m_j\) and \(m_i m_j\) \((i,j = 1,2,3)\), Eq.(6) can be simplified to

\[
f = -\nabla \phi = -\nabla \frac{1}{2} \left[ m_1 [\cos \lambda + m_1 \sin \lambda] P_{20} (\cos \theta) \right.
\]

\[
- m_2 [2P_{20}(\cos \theta) + 1] \right]
\]

(7)

is the centrifugal potential. In Eq.(8), \(P_{20}(\cos \theta)\) is the un-normalized associated Legendre polynomials. According to Love’s principle, the induced perturbation of the Earth’s gravitation potential measured at the Earth’s surface might be expressed as

\[
\delta V(r) = k \phi(r)
\]

(9)

where \(k\) is the second-order Love number. On the other hand, \(\delta V(r)\) might be decomposed as (only the second-order component is retained)

\[
\delta V(r) = \frac{GMa^2}{r^2} \sum_{m=0}^{\infty} \left( \delta \overline{c}_{2m} \cos m\lambda + \delta \overline{s}_{2m} \sin m\lambda \right) \overline{P}_{2m}(\cos \theta)
\]

(10)

where \(\overline{P}_{2m}(\cos \theta)\) is the fully-normalized associated Legendre polynomials, while \(\delta \overline{c}_{2m}\) and \(\delta \overline{s}_{2m}\) are the corresponding spherical harmonic coefficients. From Eqs.(18) to (10) and using the relationship between un-normalized and fully-normalized associated Legendre polynomials\[^{13}\], one can get

\[
\left\{ \begin{array}{l}
\delta \overline{c}_{21} = \frac{-2ka^3 \Omega^2}{3\sqrt{15GM}} m_2 \\
\delta \overline{c}_{21} = \frac{ka^3 \Omega^2}{\sqrt{15GM}} m_1 \\
\delta \overline{s}_{21} = \frac{-ka^3 \Omega^2}{\sqrt{15GM}} m_2
\end{array} \right.
\]

(11)
Physically speaking, $\delta \mathbf{C}_{21}$ and $\delta \mathbf{S}_{21}$ are related with the product of inertia, namely \cite{1-2}

$$
\begin{align*}
\delta \mathbf{C}_{20} &= -\frac{c_3^r -(c_3^r + c_3^r)}{\sqrt{5Ma^2}} \\
\delta \mathbf{C}_{21} &= -\frac{c_3^r}{\sqrt{15Ma^2}} \\
\delta \mathbf{S}_{21} &= -\frac{c_3^r}{\sqrt{15Ma^2}}
\end{align*}
$$

(12)

Then from Eqs.(11) and (12), the rotational-induced perturbation of tensor might be written as

$$
c^r = c_3^r + ic_3^r = \frac{ka^r}{3G} \Omega^2 m = \frac{k}{k_s} (C - A)m
$$

(13)

where $k$ is the second-degree Love number and $k_s = \frac{3G(C - A)}{a^r \Omega^2}$ is the secular Love number. Thus, the excitation function might be expressed as

$$
\psi = \psi^p + \psi^{NR} = \frac{\Omega^2 c^r - i\Omega c^{NR} + \Omega^2 e^{NR} - i\Omega e^{NR} + \Omega h - ih + L}{\Omega^2 (C - A)}
$$

(14)

Substituting Eqs.(13) and (14) into Eq.(1), one gets

$$
i \frac{k_s}{\sigma_c} \Omega^2 m + (k_s - k)m = \frac{\Omega^2 c^{NR} - i\Omega c^{NR} + \Omega h - ih + L}{\Omega^2 (C - A)} = \psi^{NR}
$$

(15)

noting that $\frac{1}{\sigma_c} = 305 \frac{1}{\Omega}$ and $\frac{k_s}{k} = 0.94 \approx 3.13$, thus $\frac{k}{\Omega} = 10^{-3} \frac{k_s}{\sigma_c}$ and is quite negligible. Thus, Eq.(15) can be simplified to

$$
i \frac{1}{\sigma_c} m + (1 - \frac{k_s}{k_s})m = \psi^{NR}
$$

(16)

Let $\sigma_c = (1 - \frac{k_s}{k_s})\sigma_c$ ($\sigma_c$ is in fact the Chandler frequency), Eq.(16) might be rewritten as

$$
i \frac{1}{\sigma_c} m + m = \frac{k_s}{k_s - k} \psi^{NR} = \psi^{eff}
$$

(17)

where $\psi^{eff}$ might be called the effective excitation function \cite{1-2}. As is well known, the solution to Eq.(17) is

$$
m = e^{i\sigma_c} \left[ m_0 - i\sigma_c \left( \frac{1}{\sigma_c} \right)^{-1} \psi^{eff}(\tau) e^{-i\sigma_c \tau} d\tau \right]
$$

(18)

In fact, $c^r$ denotes the permanent rotational deformation of the Earth due to the rotation with a constant angular velocity $\Omega$. We can limit our interest in the non-rotational deformations since we can always adopt Eq.(17) to include this permanent deformation.

Thus, the supscript “NR” will be omitted in the following text, but one should keep in mind that the quantities such as $c$ and $\psi$ are actually $c^{NR}$ and $\psi^{NR}$, respectively.

## 2 Atmospheric excitation of polar motion

Barnes et al. (1983) \cite{4} had introduced the so called angular momentum functions which make the treatment of the influence of atmosphere on the rotation of the Earth much more convenient. The complex angular momentum function is

$$
\chi = \frac{\Omega c + h}{\Omega(C - A)} \equiv \chi_p + \chi_w
$$

(19)

where the subscripts $p$ and $w$ denote the pressure term (relevant with $\Omega c$) and the wind term (relevant with $h$) respectively.

The atmospheric pressure will load the Earth, and the pressure term $\chi_p$ will give rise to an additional term $k' \chi_p$, which denotes the loading deformation of the Earth ($k'$ is the second-order load Love number). Then, the total angular momentum function will be

$$
\chi_{total} = (1 + k') \chi_p + \chi_w
$$

(20)

Considering Eqs.(17), (19) and (20) (noting that only the non-rotational deformations are needed and concerned here), the effective angular momentum might be expressed as

$$
\psi^{eff} = \frac{k_s}{k_s - k} \left( \chi_{total} - i \frac{\chi_{total}}{\Omega} \right)
$$

(21)

By substituting the numerical values $k = 0.30$, $k_s = 0.94$ and $k' = -0.30$ \cite{1-2} to Eq.(21), one gets

$$
\psi^{eff} = \chi_p + 1.43 \chi_w - i \frac{\chi_p + 1.43 \chi_w}{\Omega}
$$

(22)

where $\chi_{eff} = 1.00 \chi_p + 1.43 \chi_w$. Substituting Eq.(22) into Eq.(18), one gets the atmospheric-excited polar motion

$$
m = e^{i\sigma_c} \left[ m_0 - i\sigma_c (1 + \frac{\sigma_c}{\Omega}) \int_0^t \chi_{eff}(\tau) e^{-i\sigma_c \tau} d\tau \right] - \frac{\sigma_c}{\Omega} \left[ \chi_{eff}(t) - e^{-i\sigma_c \tau} \chi_{eff}(0) \right]
$$

(23)
3  Polar motion excited by AAM fluctuations

Here we adopt the NCEP (National Centers for Environmental Prediction) values of global atmospheric angular momentum (AAM) as calculated from NCEP/NCAR (National Center for Atmospheric Research) re-analyses archived on pressure surfaces[8-11]. Data are given up to four times daily from 1948-1-1 to 2006-12-31, and are provided by the Global Geophysical Fluids Data (GGFD) Center of International Earth rotation and Reference systems Service (IERS) (the AAM data are available at http://www.iers.org/). The AAM data contain values of inverted barometer (IB) pressure (or, mass) terms, Non-IB pressure terms, and wind (or, motion) terms of the atmospheric angular momentum (see Fig.1). The wind terms are computed by integrating winds from the Earth’s surface to 10 hPa, the top of atmospheric model. The inverted barometer correction involves applying the mean atmospheric surface pressure over the whole world ocean to every point over the world ocean. More information about the data can be accessed at the above mentioned website.

Based on the AAM data and the theory provided in sections 2 and 3, the polar motion excited by the AAM fluctuations

\[ m_{AAM} = m_{AAM1} + im_{AAM2} \]  

(24)
can be obtained (see Fig.2. The \(X\) and \(Y\) components denote \(m_{AAM1}\) and \(m_{AAM2}\) respectively). One can see that the AAM-excited polar motion \(m_{AAM}\) is significant and can even exceed 100 mas sometimes (noting the observed polar motion is approximately on the order of 200 mas). Also, \(m_{AAM}\) contains variations of multiple frequencies. In order to obtain its frequency spectrum, a Fast Fourier Transformation (FFT) analysis is applied to \(m_{AAM}\). However, the frequency spectrum of \(m_{AAM}\) is dominated by some low frequency variations (such as the Chandler and annual wobbles as well as an 18-month fluctuation. Wherein, the Chandler wobble (CW) and the 18-month fluctuation are described in Fig.3 while the annual wobble is removed), and other components of \(m_{AAM}\) are all enshrouded by them. From Fig.3, one can see that the AAM-excited CW can reach about 30 mas, accounting for 19% of the observed amplitude of CW (with a mean value about 160 mas). This result is quite in agreement with the study of Wahr (1983)[3], which found that the atmosphere is not the primary excita-
tion source though it has a noticeable effect on the Chandler wobble excitation during 1900-1973. Besides, the variation of AAM might give rise to the 18-month polar motion with an amplitude of 15 mas. The 18-month wobble is known as one component of the seasonal variation of the pole position\cite{1,2}.

As stated above, some high frequency AAM-induced polar motion are enshrouded by the low frequency variations, so a high-pass filter should be applied to $m_{AAM}$. In the present study, Butterworth filter, one of the Infinite Impulse Response (IIR) filters, is chosen to remove the low frequency components of $m_{AAM}$. The results are shown in Fig.4 and Fig.5.

Fig.4 shows the original $m_{AAM}$ (the top figure) and its high frequency variations (the bottom figure) obtained by applying the Butterworth filter to $m_{AAM}$. One can see that the diurnal and semi-diurnal components of $m_{AAM}$ become evident after filtering. According to Fig.5, there are 3 main peaks both in the diurnal and semi-diurnal frequency bands. These peaks should correspond to some largest atmospheric tides (see Table 1): the 3 peaks in the diurnal frequency band correspond to the $O_1$, $P_1$ and $K_1$ tides respectively; the 3 peaks in the semi-diurnal frequency band correspond to the $N_2$, $M_2$ and $S_2$ tides respectively. Because of the low sampling rate of the AAM data, the semi-diurnal signals (just corresponds to the Nyquist frequency) could be only marginally determined, and the diurnal signals are also mixed with much uncertainty. Thus, the frequency spectra shown in Fig.5 are rather sparse, and their corresponding frequencies might not coincide with the actual atmospheric tidal frequencies exactly.

**Table 1** Some significant atmospheric tides\cite{14}

| Frequency band | Angular frequency(°/h) | Origin  |
|----------------|------------------------|---------|
| Diurnal        | $O_1$                  | 13.943063 Moon |
|                | $P_1$                  | 14.958931 Sun     |
|                | $K_1$                  | 15.041069 Moon and sun |
| Semi-diurnal   | $N_2$                  | 28.439730 Moon |
|                | $M_2$                  | 28.984104 Moon |
|                | $S_2$                  | 30.000000 Sun |

The periodic luni-solar tidal generating force will attract the atmosphere and then give rise to periodic atmospheric redistribution as well as periodic winds. The two factors, corresponding to the mass and motion terms in the excitation function stated by Eq.(3), will...
contribute to polar wobbles with the same periods. That is why components with tidal frequencies appear in the polar motion. According to Fig.5, the atmosphere tides will lead to polar wobbles with a few mas in the diurnal frequency band and a few tenth of mas in the semi-diurnal frequency band.

4 Discussion and conclusion

In this paper, the atmospheric excitation of polar motion is studied both theoretically and numerically for the case of a biaxial Earth (just following the traditional theory). Our results demonstrate that the atmospheric excitation is significant to the seasonal wobble (e.g., the 18-month wobble) and the Chandler wobble, which coincide well with previous studies, as well as some distinct polar wobbles corresponding to some great diurnal and semi-diurnal atmospheric tides.

The role of the atmosphere in exciting polar motion has been relatively well quantified, mainly because the time variations of the atmospheric angular momentum (AAM) are well constrained by the available meteorological observations. However, the oceanic excitation, although expected to be important, have not been known as well as the atmospheric one, and the main
difficulty is the lack of observation data of the worldwide ocean bottom pressures and currents\cite{15}.

On the other hand, Shen et al. (2007)\cite{16} and Chen et al. (2009)\cite{17} predicted that the triaxiality of the Earth could give rise to a small fluctuation in the length of day (LOD) which does not present in the theory of biaxial Earth rotation. Chen et al. (2009)\cite{18} predicted that the geophysical excitations for the case of triaxial Earth will differ from the case of biaxial one. Our further studies demonstrate that the difference between the atmospheric-induced polar motions for the biaxial and triaxial cases is close to 1 mas, which should not be ignored within the present measurement accuracy\cite{19}. The details are beyond the scope of the present study and will be presented in a separate paper.

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