Estimation of electricity storage capacity of compact SMESs composed of stacks of Si-wafers loaded with superconducting thin film coils in spiral trenches formed by MEMS process

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Abstract. A method to estimate the electricity storage capacity of our compact superconducting magnetic energy storage system composed of stacks of Si-wafers loaded with superconducting thin film coils in spiral trenches formed by MEMS process was developed and tested for initial 6 designs of wafer coils. The results show energy storage density as low as around 6 Wh/ℓ. This is not caused by the limitation of the hoop stress but by the limitation of the maximum magnetic flux density. Further calculations for optimum wafer coil designs will be continued in which the effect of the maximum magnetic flux energy and the effect of the maximum hoop stress on the limit of the amount of the electrical energy storage come into a good balance.

1. Introduction

We have been developing superconducting thin film coils in a spiral trench on a Si wafers for compact SMESs as shown in figure 1. A proof of concept has been performed using NbN thin films showing energy storage of 0.01 mJ [1]. Increasing NbN thickness by mitigating film stress [2], the stored energy increased up to 0.1 mJ [3]. For further improvement, we moved on to replacement of NbN by YBa₂Cu₃O₇₋δ (Y123) [4]. This replacement is to enable use of a cold source of liquid hydrogen: 20K or even of liquid nitrogen: 77K instead of use of a cryogenic refrigerator at 13 K or lower in the case of NbN. Another benefit of this replacement is that the critical superconducting current density of Y123 can be about 2 × 10¹⁰ A/m² which is about 18 times larger than that of NbN [5]. Since the amount of magnetic energy storage is proportional to the square of the current, we may expect 18 × 18 = 324 times larger energy storage in the case of Y123 in comparison to the case of NbN [6]. Since the amount of magnetic energy storage is proportional to the square of the current, we may expect 18 × 18 = 324 times larger energy storage in the case of Y123 in comparison to the case of NbN. Here, it should be noted that the electromagnetic stress imposed on the superconducting coil becomes also 324 times larger. In a typical favourable design of the spiral coil in the case of NbN, the four units system shown in figure 1 together with a cryogenic refrigerator, gives the approximate estimate of the magnetic energy storage volume density as high as 1.8Wh/ℓ and the highest electromagnetic stress near the
central axis of the coil unit as high as 0.11 GPa which is well below the mechanical strength limit of a single crystalline Si: 4 GPa. However, these values are estimated to be 583 Wh/ℓ and 35.6 GPa, respectively, by the replacement of NbN by Y123. This means the maximum superconducting current in the coil must be much reduced from the value derived from the critical current density. Then, we came on to more precise estimations of magnetic field, hoop stress and energy storage density for more concrete design of the wafer coil, and some initial results are presented here.

2. Method of estimation

2.1. Design factors of the spiral wafer coil and relationships of the related variables

Figure 2 shows a schematic illustration of about one half of the cross-section of a wafer coil design.

The lengths: \(a, b, c\) define the geometry of the coil with a spiral trenches of fixed 0.3 mm in depth. Number of turns of the spiral coil: \(n\) was taken as large as possible within the condition that \(d = a + n (b+c) - c \leq 46.5\) mm. The value: 4.3 mm was taken at the rim of the wafer coil for ease of handling. There are two types of the spiral wafer coil under the same design factors: \(a, b, c\): one is left-handed and the other is right-handed. Two types of the wafer coil are to be stacked alternately by wafer-bonding technology, connecting the coils in series to form a cylindrical unit of 101.6 mm in diameter and \(ℓ = 0.3\) m (=0.5 mm × 600) in length, as illustrated in figure 1 (b), being composed of 600 stacked wafer coils of 0.5 mm in thickness. The total inductance \(L_H\) was obtained using the following expression (1) for \(L\) for a cylindrical single-layer solenoid:

\[
L_H = L \times \frac{n}{n+1}
\]

\(L\): inductance of a single-layer solenoid with total number of turns \(n\).
Here, $\mu_0$ stands for space permeability $4\pi \times 10^{-7}$ H/m, $r = (a+d)/2$ stands for a mean radius of the spiral coil, $N (= 600 \cdot n)$ stands for total number of turns of the coil, and $\ell (= 0.3 \text{ m})$ stands for the length of the solenoid, and $K$ stands for Nagaoka coefficient which is approximately unity in the present case: $r \sim 33 \text{ mm} \ll \ell (= 0.3 \text{ m})$. If $s (\text{ m}^2)$ stands for the cross-section of the superconducting film buried in the trench, $\rho (\text{ m}^{-3})$ stands for free electron density and $e$ stands for elementary charge: $1.60 \times 10^{-7} \text{ C}$, the charge amount $q (\text{ C})$ in the superconducting film per length $dx$ along the trench can be expressed as follows;

$$q = e \cdot \rho \cdot s \cdot dx$$  \hspace{1cm} (2)

If $v$ stands for the velocity of electrons, the electric current $I (\text{ A})$ is expressed as follows;

$$I = e \cdot \rho \cdot s \cdot v$$  \hspace{1cm} (3)

If $B (\text{ T})$ stands for the magnetic flux density at the trench, the Lorentz force $F$ imposed on the current $I$ can be expressed as follows:

$$F = q \cdot v \cdot B = I \cdot dx \cdot B$$  \hspace{1cm} (4)

Since the corresponding surface area on the trench wall is $0.0003 \text{ m} \times dx$, the hoop stress $S (\text{ Pa})$ on the trench wall can be expressed as follows:

$$S = I \cdot dx \cdot B / (0.0003 \times dx) = I \cdot B / 0.0003$$  \hspace{1cm} (5)

The amount of electric energy $E (\text{ J})$ stored in a cylindrical unit illustrated in figure 1 (b) can be expressed as follows;

$$E = L \cdot I^2 / 2$$  \hspace{1cm} (6)

Figure 3 illustrates the magnetic flux density distribution on the spiral wafer coil having a steep peak at the center of the wafer. The peak value $B_{\text{peak}}$ may exceed the threshold value $B_{\text{threshold}}$ which is determined by the maximum hoop stress or by critical magnetic flux density of the superconductor thin film.

![Figure 3 Schematic illustration of the magnetic flux density distribution having a steep peak $B_{\text{peak}}$ at the center of the spiral wafer coil and required coil design to reduce the maximum magnetic flux density below the threshold value $B_{\text{threshold}}$ which is determined by the maximum hoop stress or by critical magnetic flux density of the superconductor thin film.](image)

determined by the maximum hoop stress endurable by the mechanical strength of Si wafer or determined by the critical magnetic flux density of the superconductor thin film. To reduce the maximum flux density below $B_{\text{threshold}}$, optimum coil design must be explored. To do this, calculation of magnetic flux density distribution at the innermost coil at the radius $a$ shown in figure 2 was performed changing the design parameters $a$, $b$ and $c$ in figure 2. An original calculation code (Python 3.6) based on the Biot-Savart law was developed and checked in a typical case of a circular ring current as shown in figure 4. The magnetic flux density at the center of the circular ring current can be analytically derived as $\mu_0 \cdot I / (2a)$, where $a$ is the radius of the circular current. Inserting the value for $I$ and $a$, the $2.85 \text{ mT}$ was obtained. The Python code also gave $2.85 \text{ mT}$, separately. Therefore, we thought it confirmed that the developed calculation code was working correctly.
2.2. Numerical calculation of spacial distribution of magnetic flux density

As for the calculation of the magnetic flux density distribution generated by the wafer-coil, we replaced the spiral coil by concentric circular currents with the same sizes and spacing for a calculation model. First, the magnetic flux density distribution along the line normal to the wafer coil surface on the innermost ring were calculated as shown in figure 5(a). Then, the calculated magnetic flux density distributions for 600 wafers were summed up as typically shown in figure 5(b).

Figure 4 An example of calculation of magnetic flux density distribution indicated with blue vectors around the circular ring coil with the size and current indicated in the figure by developed Python 3.6 code.

![Figure 4](image)

Figure 5 (a) Calculation of magnetic flux density distribution along the line normal to the one wafer coil on the innermost circular ring current, the calculated magnetic flux densities for all the concentric circular ring current were summed up. (b) Summing up of all the magnetic flux density caused by 600 stacked wafer coils.

3. Results of estimation

Figure 6 shows a typical example of the calculated result. The value of $E$ obtained from the expression (6) was 1186 Wh/ℓ in the case of the system illustrated in figure 1 (e). Here, the critical current density of Y123 is assumed to be $2 \times 10^{10} \text{ A/m}^2$. Therefore,

$$I = 2 \times 10^{10} \cdot \sigma$$
However, the hoop stress exceeded 5 GPa in the most of the region between 0 m and 0.3 m. Taking the safety factor:1/3 into account, to reduce the maximum hoop stress down to one third of the limit of the maximum stress:4 GPa for the Si wafer, that is, 1.3 GPa, the corresponding magnetic flux density is 137 T. Although the critical current density of Y123 shows a slow decrease with applied magnetic flux density, the reported data under the magnetic field does not exceed 25 T, which is far below the 137 T. To reduce this magnetic flux density down to, for example, 20 T, \( E \) is reduced to 5.92 Wh/ℓ. This is because \( E \) is proportional to \( I \) while \( B \) is proportional to \( I \). Table 1 summarizes the calculated results for six different design factors for \( a, b, c \).
including the result described above.

4. Conclusion
Method to estimate the electricity storage capacity of compact SMESs composed of stacks of Si-wafers loaded with superconducting thin film coils in spiral trenches formed by MEMS process was developed using Python 3.6 code. In initially estimated 6 designs, the calculated results show energy storage density as low as around 6 Wh/ℓ. This is because the maximum magnetic flux density limited the amount of electric energy storage rather than the maximum hoop stress. Further calculations for optimum wafer coil designs will be continued in which the effect of the maximum magnetic flux density and the effect of the maximum hoop stress on the limit of the amount of the electrical energy storage come into a good balance.

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