CONSTRAINTS ON THE PROGENITORS OF TYPE Ia SUPERNOVAE AND IMPLICATIONS FOR THE COSMOLOGICAL EQUATION OF STATE

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ABSTRACT

Detailed stellar evolution calculations have been performed to quantify the influence of the main-sequence mass $M_{\text{MS}}$ and the metallicity $Z$ of the progenitor on the structure of the exploding white dwarf (WD), which are thought to be the progenitors of Type Ia supernovae (SNe Ia). In particular, we study the effects of progenitors on the brightness-decline relation $M(\Delta M_{15})$, which is a cornerstone for the use of SNe Ia as cosmological yardsticks. Both the typical $M_{\text{MS}}$ and $Z$ can be expected to change as we go back in time. We consider the entire range of potential progenitors with $1.5-7 M_\odot$ and metallicities between $Z = 0.02$ and $1 \times 10^{-10}$. Our study is based on the delayed detonation scenario with specific parameters that give a good account of typical light curves and spectra. Based on the structures for the WD, detailed model calculations have been performed for the hydrodynamical explosion, nucleosynthesis, and light curves.

The main-sequence mass has been identified as the decisive factor to change the energetics of the explosion and, consequently, dominates the variations in the rise-time–decline relation of light curves. $M_{\text{MS}}$ has little effect on the color index $B-V$. For similar decline rates $\Delta M_{15}$, the flux at maximum brightness relative to the flux on the radioactive tail decreases systematically with $M_{\text{MS}}$ by about $0.2^{m}$. This change goes along with a reduction of the photospheric expansion velocity $v_{\text{ph}}$ by about 2000 km s$^{-1}$. A change in the central density of the exploding WD has similar effects but produces the opposite dependency between the brightness-to-tail ratio and $v_{\text{ph}}$ and therefore can be separated.

The metallicity alters the isotopic composition of the outer layers of the ejecta. Selective line blanketing at short wavelengths decreases with $Z$ and changes systematically the intrinsic color index $B-V$ by up to $-0.06^{m}$, and it alters the fluxes in the $U$ band and the UV. The change in $B-V$ is critical if extinction corrections are applied. The offset in the calibration of $M(\Delta M_{15})$ is not monotonic in $Z$ and, in general, remains $\leq 0.07^{m}$.

We use our results and recent observations to constrain the progenitors and to discuss evolutionary effects of SNe Ia with redshift. The narrow spread in the fiducial rise-time–decline relation in local SNe Ia restricts the range of main-sequence masses to a factor of 2. The upper limit of 1 day for the difference between the local and distance sample supports the need for a positive cosmological constant. The size of evolutionary effects is small ($\Delta M \approx 0.2^{m}$) but is absolutely critical for the reconstruction of the cosmological equation of state.

Subject headings: cosmological parameters — distance scale — supernovae: general

On-line material: color figures

1. INTRODUCTION

The last decade has witnessed an explosive growth of high-quality data for supernovae both from the space and ground observatories with spectacular results, as well as new perspectives for the use of Type Ia supernovae (SNe Ia) as cosmological yardsticks and for constraining the physics of supernovae. One of the most important new developments in observational supernova research was to establish the long-suspected correlation between the peak brightness of SNe Ia and their rate of decline, $M(\Delta M_{15})$, by means of modern CCD photometry (Phillips 1993). SNe Ia have provided new estimates for the value of the Hubble constant ($H_{0}$) based on a purely empirical procedure (Hamuy et al. 1996a, 1996b; Riess, Press, & Kirshner 1996) and on a comparison of detailed theoretical models with observations (Höflich & Khokhlov 1996, hereafter HK96; Nugent et al. 1997). The values obtained are in good agreement with one another. More recently, the routine successful detection of supernovae at large redshifts, $z$ (e.g., Perlmutter et al. 1995, 1997; Riess et al. 1998; Garnavich et al. 1998), has provided an exciting new tool to probe cosmology. This work has provided results that are consistent with a low matter density in the universe and, most intriguing of all, yielded hints for a positive cosmological constant $\Omega_{\Lambda}$ of $\approx 0.7$. It is worth noting that the difference in the maximum magnitude between $\Omega_{\Lambda} = 0$ and 0.7 is $\approx 0.25^{m}$ for redshifts between 0.5 and 0.8. These results prompted the quest for the nature of the “dark” energy, i.e., cosmological equation of state. Current candidates include a network of topological defects such as strings, evolving scalar fields (i.e., quintessence), or the classical cosmological constant. For a recent review, see Ostriker & Steinhardt (2001) and Perlmutter, Turner, &
White (1999b). To separate between the candidates by SNe Ia, the required accuracy has to be better than 0.05\(^n\)–0.1\(^n\) (Albrecht & Weller 2000). The results on \(\Omega_\Lambda\) and future projects to measure the cosmological equation of state depend on the empirical \(M(\Delta M_{15})\), which is calibrated locally. This leaves systematic effects as the main source of concern.

Indeed, there is already some evidence that SNe Ia undergo evolution. It has been argued that the local SN Ia sample covers all the possible variations that may come from different progenitors, different explosion mechanisms and environments, etc. For local SNe Ia, the observational and statistical characteristics depend on their environment. They occur less often in ellipticals than in spirals, and the mean peak brightness is dimmer in ellipticals (Branch, Romanishin, & Baron 1996a, 1996b; Wang, Höflich, & Wheeler 1997; Hamuy et al. 2000). In the outer part of spirals the brightness is similar to ellipticals while, in more central regions, both intrinsically brighter and dimmer SNe Ia occur (Wang et al. 1997). These dependencies show us that SNe Ia likely depend on the underlying population and may undergo evolution. If the evolution realizes, then we have to know it and take it into account going back in time. Otherwise we cannot safely use a local calibration. In principle, a more distant sample could come from younger and more metal-poor progenitors, or the dominant explosion scenario may change.

There is general agreement that SNe Ia result from some process of combustion of a degenerate white dwarf (WD) (Hoyle & Fowler 1960). Within this general picture, three classes of models have been considered: (1) an explosion of a CO WD, with mass close to the Chandrasekhar mass, which accretes mass through Roche lobe overflow from an evolved companion star (Whelan & Iben 1973); the explosion is then triggered by compressional heating near the WD center; (2) an explosion of a rotating configuration formed from the merging of two low-mass WDs, caused by the loss of angular momentum due to gravitational radiation; and (3) an explosion of a low-mass CO WD triggered by the detonation of a helium layer (Nomoto 1980; Woosley, Weaver, & Taam 1980; Woosley & Weaver 1986). Only the first two models appear to be viable. The third, the sub-Chandrasekhar WD model, has been ruled out on the basis of predicted light curves and spectra (Höflich et al. 1996a; Nugent et al. 1997).

Delayed detonation (DD) models (Khokhlov 1991; Woosley & Weaver 1994; Yamaoka et al. 1992) have been found to reproduce the optical and infrared light curves and spectra of “typical” SNe Ia reasonably well (Höflich 1995, hereafter H95; Höflich, Khokhlov, & Wheeler 1995, hereafter HKW95; HK96; Fisher et al. 1998; Nugent et al. 1997; Wheeler et al. 1998; Höflich et al. 2000; Lentz et al. 2000; Gerardy et al. 2001). This model assumes that burning starts as subsonic deflagration and then turns to a supersonic, detonative mode of burning. Because of the one-dimensional nature of the model, the speed of the subsonic deflagration and the moment of the transition to a detonation are free parameters. The moment of deflagration-to-detonation transition (DDT) is conveniently parameterized by introducing the transition density, \(\rho_{\text{tr}}\), at which DDT happens. The amount of \(^{56}\text{Ni}\), \(M_{\text{\^{56}Ni}}\), depends primarily on \(\rho_{\text{tr}}\) (H95; HKW95; Umeda et al. 1999) and to a much lesser extent on the assumed value of the deflagration speed, initial central density of the WD, and initial chemical composition (ratio of carbon to oxygen). Models with smaller transition density give less nickel and hence both lower peak luminosity and lower temperatures (HKW95; Umeda et al. 1999). In DDs, almost the entire WD is burned, i.e., the total production of nuclear energy is almost constant. This and the dominance of \(\rho_{\text{tr}}\) for the \(^{56}\text{Ni}\) production are the basis of why, to first approximation, SNe Ia appear to be a one-parameter family. The observed \(M(\Delta M_{15})\) can be well understood as an opacity effect (Höflich et al. 1996a), namely, as a consequence of the rapidly dropping opacity at low temperatures (Khokhlov, Müller, & Höflich 1993). Less Ni means lower temperature and, consequently, reduced mean opacities because the emissivity is shifted from the UV toward longer wavelengths with less line blocking. A more rapidly decreasing photosphere causes a faster release of the stored energy and, as a consequence, steeper declining light curves with decreasing brightness. The DD models thus give a natural and physically well-motivated origin of the \(M(\Delta M_{15})\) relation of SNe Ia within the paradigm of thermonuclear combustion of Chandrasekhar mass CO WDs. Nonetheless, variations of the other parameters lead to some deviation from the \(M(\Delta M_{15})\). For example, a change of the central density results in an increased binding energy of the WD and a higher fraction of electron capture close to the center that reduce the \(^{56}\text{Ni}\) production (Höflich et al. 1996a). Because DD models allow us to reproduce the observations, we use this scenario to test the influence of the underlying stellar population on the explosion.

We note that detailed analyses of observed spectra and light curves indicate that mergers and deflagration models such as W7 may contribute to the supernova population (Höflich & Khokhlov 1996; Hatano et al. 2000). In particular, the classical “deflagration” model W7 with its structure similar to DD models has been successfully applied to reproduce optical light curves and spectra (e.g., Harkness 1987). The evidence against pure deflagration models for the majority of SNe Ia includes IR spectra that show signs of explosive carbon burning at high expansion velocities (e.g., Wheeler et al. 1998) and recent calculations for three-dimensional deflagration fronts by Khokhlov (2001), which predict the presence of unburned and partially burned material down to the central regions. Currently, pure deflagration models may be disfavored for the majority of SNe Ia, but clearly they cannot be ruled out either.

Previously, Höflich, Wheeler, & Thielemann (1998, hereafter HWT98) studied evolutionary effects induced by the progenitor. They calculate differences in the light curves and non-LTE spectra as a function of parameterized values of the integrated C/O ratio \(C/O_{\text{Mch}}\) and metallicity of the exploding WD. This study showed that a change of \(C/O_{\text{Mch}}\) alters the energetics of the explosion, which results in an offset of the brightness-decline relation. Most prominently, this effect can be identified by a change in the fiducial rise-time–decline relation \(t_{\text{Fmax}}/t_{\text{FAM15}}\). The offset in \(M(\Delta M_{15})\) is given by \(\Delta M_{\nu} \approx 0.1\Delta t\), where \(\Delta t\) is the dispersion in the rise time of the “fiducial” light curve. Aldering, Knop, & Nugent (2000) showed that \(t_{\text{Fmax}}/t_{\text{FAM15}}\) are identical within \(\Delta t = 1d\) for the local and distant sample, lending strong support for the notion that we need a positive \(\Omega_\Lambda\). A change in the metallicity \(Z\) causes a change in the burning conditions at the outer layers of the WD, and it alters the importance of the line blanketing in the blue to the UV. Based on detailed calculations, effects of similar order have been
found for both the DD and the deflagration scenario (HWT98; Lentz et al. 2000). Recent studies showed the additional effect that \( Z \) will influence the final structure of the progenitor and the resulting light curves (Umeda et al. 1999; Dominguez et al. 2000; Höflich et al. 2000). However, the former two studies were restricted to the progenitor evolution, whereas the latter included the connection between the progenitor and the light curve but was restricted to a progenitor of \( M_{\text{MS}} = 7 M_{\odot} \) and two metallicities, \( Z = 0.02 \) and 0.004.

A more comprehensive study may be useful to eliminate potential problems due to evolution of the progenitors for the determination of the cosmological equation of state, and it may provide a direct link to the progenitors of SNe Ia. In this work we connect \( M_{\text{MS}} \) and the initial metallicity of the WD to the light curves and spectral properties of SNe Ia for the entire range of potential progenitors. In §2 we discuss the evolutionary properties of our models. In §3 the results are presented for the explosion, nucleosynthesis, the light curves, and spectral properties. In §4 our model calculations are related to observations, and we discuss constraints for the progenitors and implications for the cosmological equation of state.

### 2. THE FORMATION OF A CO WD

CO WDs are the remnants of the evolution of low- and intermediate-mass stars (Becker & Iben 1980). Their progenitors are stars less massive than \( M_{\text{up}} \), which is the lower stellar mass for which a degenerate carbon ignition occurs after the central helium exhaustion. The precise value of \( M_{\text{up}} \) depends on the chemical composition (see Dominguez et al. 1999 for a recent evaluation of this mass limit). It ranges between 6.5 and 8 \( M_{\odot} \). On the basis of updated theoretical stellar models of intermediate-mass stars, Dominguez et al. (1999) found final CO core masses in the range 0.55–1.04 \( M_{\odot} \), in good agreement with semiempirical evaluations of the WD masses (see, e.g., Weidemann 1987).

In this paper we use CO WD structures obtained by evolving models with main-sequence masses \( M_{\text{MS}} \) between 1.5 and 7 \( M_{\odot} \) and metallicities \( Z \) between \( 10^{-10} \) and 0.02. In the following a label identifies a particular progenitor model, namely, \( ApBzCD \) for a progenitor with a main-sequence mass of \( AB \) \( M_{\odot} \) and \( Z = 10^{-10} \). These models have been obtained by means of the Frascati Raphson-Newton Evolutionary Code (FRANEC), which solves the full set of equations describing both the physical and chemical evolution of a star by assuming hydrostatic and thermal equilibrium and a spherical geometry (Chieffi & Straniero 1989; Chieffi, Limongi, & Straniero, 1998). For a detailed description of the adopted input physics see Straniero, Chieffi, & Limongi (1997) and Dominguez et al. (1999).

Because we are interested in the final chemical structure of a CO WD, let us recall the main properties and the major uncertainties of the evolutionary phases during which the CO core forms. In Table 1 we show some properties of our models. In columns (1)–(8) we give (1) the initial composition \( (Z \) and \( Y \)), (2) the model name, (3) the main-sequence mass (in \( M_{\odot} \)), (4) the mass of the CO core at the beginning of the thermal pulse phase (in \( M_{\odot} \)), (5) the C abundance (mass fraction) at the center, (6) the mass (in \( M_{\odot} \)) of the homogeneous carbon-depleted central region, (7) the averaged \( C/O \) ratio within the final, \( \approx 1.37 \) \( M_{\odot} \), CO WD after accretion, and (8) the \(^{56}\text{Ni}\) mass (in \( M_{\odot} \)) synthesized during the explosion. The C and O profiles (mass fraction) of the CO core for selected thermally pulsing models are shown in Figures 1–4. In particular, Figure 1 shows the changes induced by a different initial mass, while Figures 2 and 3 illustrate the effect of the metallicity.

The internal C and O profiles of a WD are generated in three different evolutionary phases of the progenitor, namely, (1) the central He burning, (2) shell He burning during the early asymptotic giant branch (AGB) phase, and (3) shell He burning during the thermally pulsing AGB phase. As illustrated in the figures, they produce three distinct layers.

The central He burning produces the innermost homogeneous layer. This phase is initially dominated by the carbon production via the \( \alpha \) reaction occurring in the center of a convective core. Once sufficient \(^{12}\text{C}\) is synthesized, the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reactions become competitive with \( \alpha \) reactions. Carbon is partially burned into \(^{16}\text{O}\). Since the opacity of a C/O mixture is larger than that of an He mixture, the extension

\[ ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \]

\[ \alpha \Omega \] stands for \( Z = 10^{-10} \).

| Initial Composition (1) | Model (2) | \( M_{\text{MS}} \) (\( M_{\odot} \)) (3) | \( M_{\text{Ti}}^{\text{MF}} \) (\( M_{\odot} \)) (4) | \( C_{\text{cen}} \) (\( M_{\odot} \)) (5) | \( M_{\text{cen}} \) (\( M_{\odot} \)) (6) | \( C/O_{\text{ cen}} \) (7) | \( ^{56}\text{Ni} \) (\( M_{\odot} \)) (8) |
|-----------------------|-----------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| \( Z = 0.02 \) ............ | 1p5z22 | 1.5                             | 0.55                            | 0.21                            | 0.27                            | 0.75                            | 0.589                           |
| \( Y = 0.28 \) ............ | 3p0z22 | 3.0                             | 0.57                            | 0.21                            | 0.28                            | 0.76                            | 0.584                           |
| \( Z = 10^{-3} \) ............ | 7p0z22 | 7.0                             | 0.99                            | 0.28                            | 0.70                            | 0.60                            | 0.516                           |
| \( Y = 0.23 \) ............ | 1p5z13 | 1.5                             | 0.59                            | 0.24                            | 0.31                            | 0.76                            | 0.587                           |
| \( Z = 10^{-4} \) ............ | 3p0z13 | 5.0                             | 0.91                            | 0.29                            | 0.58                            | 0.66                            | 0.541                           |
| \( Y = 0.23 \) ............ | 5p0z14 | 6.0                             | 0.98                            | 0.29                            | 0.71                            | 0.60                            | 0.522                           |
| \( Z = 10^{-10} \) ............ | 6p0z14 | 5.0                             | 0.89                            | 0.32                            | 0.49                            | 0.70                            | 0.549                           |
| \( Y = 0.23 \) ............ | 7p0z00 | 7.0                             | 0.99                            | 0.31                            | 0.59                            | 0.62                            | 0.525                           |
(in mass) of the convective core increases with time. When the He mass fraction in the convective core is reduced down to \( \approx 0.1 \), the He burning is mainly controlled by \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \), and most of the oxygen in the convective core is synthesized during the late He burning. The final abundances in the innermost region of a WD are strongly dependent on the duration of the last 5\%–10\% of the entire He-burning lifetime. In column (6) of Table 1, we report the size (in \( M_\odot \)) of this innermost homogeneous region, which corresponds to the maximum extension of the convective core.

The intermediate region of the final C/O structure is characterized by a rising carbon abundance. It is produced during the early AGB when the He-burning shell advances in mass until it approaches the H-rich envelope. The amount of carbon (oxygen) left behind increases (decreases) as a result of the progressive growth of the temperature in the shell, which favors the 3\( \alpha \) reactions with respect to the \( \alpha \) capture on \( ^{12}\text{C} \). In addition, the short lifetime does not allow a substantial conversion of carbon into oxygen.

Finally, a thin external layer is built up during the thermally pulsing AGB. At the beginning of a thermal pulse, the large energy flux is locally produced by the 3\( \alpha \) reactions. It induces the formation of a convective shell that rapidly overlaps the whole intershell region. Owing to the large He reservoir, a huge amount of carbon is produced at the base of the convective shell. After a few years (10–100 yr depending on the core mass) the convective shell disappears and a quiescent He burning takes place. It is during this longer phase that the \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) reactions convert a certain part

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**Fig. 1.**—Final chemical carbon (solid line) and oxygen (dotted line) profiles in the central region of stars between 1.5 and 7 \( M_\odot \) for solar abundances \( Z = 0.02 \).

**Fig. 2.**—Same as Fig. 1, but for stars with 3 \( M_\odot \) and metallicities \( Z \) between \( 10^{-10} \) and 0.02.

**Fig. 3.**—Same as Fig. 1, but for stars with 5 \( M_\odot \) and metallicities \( Z \) between \( 10^{-10} \) and 0.02.

**Fig. 4.**—Influence of the nuclear reaction rate of \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) on the final chemical profiles of C (solid line) and O (dotted line) for a star with 3 \( M_\odot \) and \( Z = 0.001 \). The high and low rates are taken from Caughlan et al. (1985) and Caughlan & Fowler (1988), respectively.
of the carbon produced during the pulse into oxygen. The C/O ratio left below the He-rich layer by the He-burning shell depends on the rate of the $\alpha$ captures on carbon. Note that the outer "blip" in the carbon profile is the result of the last thermal pulse where the He has not yet fully depleted. The size, in mass, of this third layer depends on the duration of the thermally pulsing AGB phase. Although it is influenced by the assumed mass-loss rate, it is generally believed that for $M < 3 M_\odot$ the CO core cannot increase more than 0.1–0.2 $M_\odot$ during the AGB. An even smaller increase of the CO core is expected for larger $M_{\text{MS}}$.

The subsequent phase has been calculated by accreting H/He-rich material on the resulting CO WD. Note that we have assumed that the progenitor ejects its H/He-rich envelope prior to the onset of the accretion epoch. The accreted matter has a final C/O ratio of $\approx 1$. When the star reaches a mass close to 1.37 $M_\odot$, ignition occurs close to the center. $\dot{M}$ has been adjusted to enforce that the thermonuclear runaway occurs at the same central density $\rho_c$ in all models.

### 2.1. Dependence on the Main-Sequence Mass

In Figure 1 the chemical structures of our models are shown as a function of $M_{\text{MS}}$ (for $Z = 0.02$). For low stellar masses, the core He burning happens under lower central temperatures (see, e.g., Dominguez et al. 1999). This favors the $\alpha$ captures on $^{12}\text{C}$, which are in competition with the 3$\alpha$ resulting in a slightly smaller central C/O ratio for low $M_{\text{MS}}$.

However, the size of the region of central He burning, $M_{\text{cen}}$, is increasing with $M_{\text{MS}}$. In a star with 7 $M_\odot$, the maximum size of the convective core is about 0.7 $M_\odot$, while in the 1.5 $M_\odot$ model, it is only 0.25 $M_\odot$. This is the dominating factor for changes in the mean C/O ratio, which, in general, produces the monotonic relation that C/O decreases with increasing $M_{\text{MS}}$ (Table 1).

### 2.2. Dependence on the Initial Metallicity

In Figures 2 and 3 the chemical structures are given for various $Z$ for stars with 3 and 5 $M_\odot$, respectively. The size of the innermost homogeneous region is not a monotonic function of $Z$. This is due to the peculiarity of very low metallicity intermediate-mass stars (see Chieffi et al. 2001). In these stars ($Z \leq 10^{-10}$), the central hydrogen burning proceeds via the $p$-$p$ chain instead of the CNO cycles, as it happens for larger metallicity. It results in a smaller He core (Ezer & Cameron 1971; Tornambè & Chieffi 1986). For solar metallicity, the higher opacities and the steeper temperature gradients produce a smaller He core (Becker & Iben 1979, 1980). In all cases the dependence on $Z$ of the final averaged C/O ratio (col. [7] of Table 1) is small compared to the effect of main-sequence mass. The variation with $Z$ ranges between 5% and 10%.

### 2.3. Uncertainties in the Final Chemical Structure

For the final chemical structure, the most important uncertainty is due to the ambiguity in the nuclear reaction rate of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. The innermost region is also sensitive to the treatment of turbulent convection, which may affect the duration of the late central He-burning lifetime and the size of the convective core.

The rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ at astrophysical energies is not well established (see, e.g., Buchman 1996, 1997). The cross section around the Gamow peak is dominated by ground-state transitions through four different processes: the E1 amplitudes due to the low-energy tail of the $^1\text{S}$ resonance at $E_{\text{cen}} = 2.42$ MeV and to the subthreshold resonance at $-45$ keV, and the E2 amplitudes due to the $^2\text{S}$ subthreshold resonance at $-245$ keV and to the direct capture to the $^{16}\text{O}$ ground state, both with the corresponding interference terms. Besides ground-state transitions, cascades, mainly through the E2 direct capture to 6.05 MeV $0^+$ and 6.92 MeV $2^+$ states, also have to be considered. Obviously, a higher rate (approximately a factor of 2) of this reaction reduces the carbon abundance left by both the core and the shell He burning. Some indications in favor of a high value for this reaction rate come from the rise times to maximum light in SNe Ia (HWT98) and from recent studies of pulsating WDs (Metcalfe, Winget, & Charbonneau 2001).

Concerning turbulent convective mixing, it only affects the region of the WD structure produced during the core He burning. The two major uncertainties are related to the possible existence of breathing pulses (see Castellani et al. 1985; Caputo et al. 1989; Renzini & Fusi Pecci 1988) and the possible existence of a sizeable convective core overshoot. As recently pointed out by Imbriani (2001, private communication), since breathing pulses increase the late core He-burning lifetime, they significantly reduce the central C/O. In this regard, they mimic the effects of a high $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate. On the contrary, convective core overshoot does not affect the central C/O, but it may enlarge the size of the convective core and, in turn, may produce a larger central, C-depleted region.

In the models presented in this paper we have neglected breathing pulses and convective core overshoot. All the models, except 3p0z13LR, have been obtained by means of a high rate for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction (as given by Caughlan et al. 1985). The 3p0z13LR model has been obtained by adopting the alternative low rate presented by Caughlan & Fowler (1988). A comparison between the two models with different $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate is shown in Figure 4. Changing the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate from the high to the low value drastically alters the chemical profiles of the progenitor. The carbon abundance increases by about a factor of 2. At the time of the explosion, the average composition of the WD changes from oxygen-rich (C/O$_{\text{Meq}} = 0.74$) to carbon-rich (C/O$_{\text{Meq}} = 1.22$) The consequences for the light curves are discussed below. Although a variation in the assumed convective mixing scheme may change the quantitative result of our analysis, the overall conclusions and tendencies cannot be significantly altered because its influence is limited to the innermost part of the preexplosive structure.

### 3. Explosions, Light Curves, and Spectral Properties

Spherical dynamical explosions and corresponding light curves are calculated. We consider DD models because these have been found to reproduce the optical and infrared light curves and spectra of SNe Ia reasonably well (Höflich 1995; HKW95; HK96; Nugent et al. 1997; Wheeler et al. 1998; Lentz et al. 2000; Höflich et al. 2000; Gerardy et al. 2001). Model parameters have been chosen that allow us to reproduce light curves and spectra of "typical" SNe Ia.

For our set of models, the differences can be attributed to changes in the progenitor structure of the CO WD. As a reference, we use the explosion of a progenitor with 5 $M_\odot$ at the main sequence and solar metallicity (model 5p0z22). At the time of the explosion of the WD, its central density is
\[2.0 \times 10^9 \text{ g cm}^{-3}\] and its mass is close to \(1.37 M_\odot\). The transition density \(\rho_{\text{tr}}\) from deflagration to detonation is chosen to be \(2.3 \times 10^7 \text{ g cm}^{-3}\).

3.1. Explosion Models

Explosion models are calculated using a one-dimensional radiation hydro code (HK96) that solves the hydrodynamical equations explicitly by the piecewise parabolic method (Colella & Woodward 1984). Nuclear burning is taken into account using an extended network of 606 isotopes from neutron, proton to \(^{74}\text{Kr}\) (Thielemann, Nomoto, & Hashimoto 1996 and references therein). The propagation of the nuclear burning front is given by the velocity of sound behind the burning front in the case of a detonation wave and in a parameterized form during the deflagration phase calibrated by detailed three-dimensional calculations (e.g., Khokhlov 1995, 2001; Niemeyer & Hillebrandt 1995). We use the parameterization as described in Dominguez & Höflich (2000). For a deflagration front at distance \(r_{\text{burn}}\) from the center, we assume that the burning velocity is given by \(v_{\text{burn}} = \max (v_l, v_t)\), where \(v_l\) and \(v_t\) are the laminar and turbulent velocities with

\[
v_l = 0.15 \sqrt{z_f g L_f},
\]

\[
\text{with } z_f = \frac{\alpha - 1}{\alpha + 1} \text{ and } \alpha = \frac{\rho^+ (r_{\text{burn}})}{\rho^-(r_{\text{burn}})}.
\]

Here \(z_f\) is the Atwood number, \(L_f\) is the characteristic length scale, and \(\rho^+\) and \(\rho^-\) are the densities in front of and behind the burning front, respectively. The quantities \(\alpha\) and \(L_f\) are taken directly from the hydro at the location of the burning front, and we take \(L_f = r_{\text{burn}}\). The transition density is treated as a free parameter. The description of the deflagration front does not significantly influence the final structure of the explosion (Dominguez & Höflich 2000). The total \(^{56}\text{Ni}\) production is governed by the preexpansion of the WD and, consequently, is determined by the transition density \(\rho_{\text{tr}}\) at which the burning front switches from the deflagration to the detonation mode (H95). From the physical point of view, \(\rho_{\text{tr}}\) should be regarded as a convenient way to adjust the amount of material burned during the deflagration phase. The value \(\rho_{\text{tr}}\) can be adjusted to produce a given amount of \(^{56}\text{Ni}\). This code includes the solution of the frequency-averaged radiation transport implicitly via moment equations, expansion opacities, and a detailed equation of state (see § 3.2). As expected from previous studies (see § 1), the overall density, velocity, and chemical structures are found to be rather insensitive to the progenitor, including the production of elements.

Although explosions and light curves have been calculated for the entire set of stellar cores, we will concentrate our detailed discussion on the extreme cases and the reference model. Results for intermediate models can be understood accordingly and interpolated using the quantities given in Table 1.

The final density, velocity, and chemical structures and detailed production of isotopes are shown in Figures 5, 6, and 7 for the extreme cases in metallicity \((Z = 0.02\) and \(10^{-10}\) \(5 M_\odot\)) and the extremes in \(M_{\text{MS}}\) \((1.5 - 7 M_\odot, Z = 0.02)\). Between 0.511 and 0.589 \(M_\odot\) of \(^{56}\text{Ni}\) is produced (Table 1). The production of individual isotopes varies only by about 10\% (Fig. 7). For the reference model 5p0z22, the final element abundances are given in Table 2. Variations in the final density and velocity structure are correspondingly small (Fig. 5).

In DDs, almost the entire WD is burned. The total release of nuclear energy depends mainly on the fuel, i.e., on the integrated C/O ratio C/O_{\text{Meh}} (HWT98). However, as usual for DD models, the deflagration phase is key for our understanding of the final results. During the deflagration phase, about 0.33 \(M_\odot\) of fuel is burned in our models (Figs. 5 and 6). In all explosions but the progenitors with \(M_{\text{MS}} = 1.5\) and \(3 M_\odot\) with \(Z = 0.02\) and \(M_{\text{MS}} = 1.5 M_\odot\) with \(Z = 0.001\), the deflagration front will propagate in the carbon-depleted layers. The amount of total energy produced during the deflagration phase and the binding energy of the progenitor determines the preexpansion of the outer layers and, consequently, the overall chemical structure. The binding energy of the WD is dominated by the central

![Fig. 5.—Final density (left scale) relative to \(\rho\) at the center and velocity profiles (right scale, in 1000 km s\(^{-1}\)) of the DD models. The results are given for progenitors with \(M_{\text{MS}} = 5 M_\odot\) with \(Z = 10^{-10}\) and 0.02 (5p0z22 + 5p0x00; left panel) and for progenitors with \(M_{\text{MS}} = 1.5\) and 7 \(M_\odot\) with \(Z = 0.02\) (1p5z22 + 7p0x22; right panel). The velocity and density of model 1p5z22 correspond to the higher and lower function, respectively. For the same \(M_{\text{MS}}\) but different \(Z\), the curves are indistinguishable. [See the electronic edition of the Journal for a color version of this figure.]](image)
density $\rho_c$ at the time of the explosion. Note that the C/O ratio has little influence on the structure of the WD because the pressure is dominated by degenerate electrons and the electron-to-nucleon ratio $Y_e$ is identical for $^{12}$C and $^{12}$O. The total energy production during the deflagration phase is governed by $\rho_c$ and by the nuclear energy release per gram, i.e., the composition. Both $\rho_c$ and $\rho_e$ have been kept the same in all models. Variations can be understood by the change of the mean C/O ratio and the mass $M_{\text{cen}}$ of the central, carbon-depleted region. The latter influences the temperature and, consequently, the laminar speed and the Atwood number (eq. [1]). At the central layers, all the material is burned up to iron group elements (Fig. 6). Some additional variation in the total $^{56}$Ni mass is caused by the C/O ratio of the matter burned during the detonation phase. For all models, the transition between $^{56}$Ni- and Si-rich layers is between 0.58 and 0.98 $M_\odot$. For $M_{\text{MS}} = 7$ $M_\odot$ and, to a much lesser extent, for $M_{\text{MS}} = 5$ $M_\odot$, this transition region overlaps with layers of the lower C-depleted region (Fig. 1). From the nuclear physics, the transition between complete and incomplete Si burning occurs in a narrow temperature range around $5 \times 10^9$ K. A locally lower C/O ratio results in lower burning temperatures. Consequently, the $^{56}$Ni/Si boundary is shifted inward and $M_{\text{56Ni}}$ is reduced. Overall, differences between the models remain small because the nuclear energy production for C and O burning differs only by $\approx 10\%$.

3.1.1. Dependence on the Main-Sequence Mass

An increasing $M_{\text{MS}}$ for a given metallicity leads to changes in $C_{\text{cen}}$ and $C/O_{\text{Mech}}$ by 33% and $-25\%$, respectively (Table 1). Increasing $M_{\text{MS}}$ from 1.5 to 7.0 $M_\odot$ alters the expansion velocity of a given mass element (Fig. 5, right panel), and it results in a shift of the chemical interfaces between complete, incomplete Si, and explosive C burning by $\approx 1500$ km s$^{-1}$ (Fig. 6).

3.1.2. Dependence on the Initial Metallicity $Z$

If we decrease the metallicity from solar ($Z = 0.02$) to $Z = 10^{-10}$ for stars with $5 M_\odot$, $C_{\text{cen}}$ and $C/O_{\text{Mech}}$ change by as little as 10% and $-7\%$, respectively. This means that the density and velocity structure of the chemical structures are virtually indistinguishable (Fig. 5). For stars with $M_{\text{MS}} \leq 3$ $M_\odot$, variations in $C_{\text{cen}}$ increase up to 30%, but, still, varia-

![Fig. 6.—Final chemical profiles for DD models of progenitors with main-sequence masses between 1.5 and 7 $M_\odot$. [See the electronic edition of the Journal for a color version of this figure.]](image-url)
tions in C/O\textsubscript{Meb} remain at a level of 10%. The overall energetics, density, and velocity structure remain mostly unchanged, but the preexpansion and consequently the chemical interfaces between different regimes of burning change by \(\approx 200\ \text{km s}^{-1}\). For all \(M\text{\textsubscript{MS}}\), the most noticeable difference is the increasing \(^{54}\text{Fe}\) production with \(Z\) in the layers of incomplete Si burning, which changes the spectra in the blue and in the UV (see HWT98 and below).

3.1.3. Influence of the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) Rate

As mentioned in § 2.3, there is some indirect evidence for a high cross section of this key reaction, but a low rate cannot be ruled out either. The consequences of a low rate are strong. For example, for a progenitor with \(M\text{\textsubscript{MS}} = 3.0\ M_\odot\) and \(Z = 0.001\), the low rate suggested by Caughlan & Fowler (1988) will increase \(\text{C}\text{\textsubscript{cen}}\) and \(C/O\text{\textsubscript{Meb}}\) from 0.26 to 0.51 and from 0.74 to 1.22 when compared to our favorite rate (Caughlan et al. 1985). The explosion becomes more energetic by about 20%, and the deflagration front propagates faster. The result is an increase of the \(^{56}\text{Ni}\) production by about 10% and a shift in the chemical interfaces by about 2500 km s\(^{-1}\).

3.2. Light Curves and Spectral Properties

Based on the explosion models, the subsequent expansion, bolometric, and monochromatic light curves are calculated (Höflich et al. 1998 and references therein). The light-curve code is the same used for the explosion, except that \(\gamma\)-ray transport is included via a Monte Carlo scheme and nuclear burning is neglected. In order to allow a more consistent treatment of the expansion, we solve the time-dependent, frequency-averaged radiation moment equations. The frequency-averaged variable Eddington factors and mean opacities are calculated by solving the frequency-dependent transport equations. About 1000 frequencies (in 100 frequency groups) and about 900 depth points are used. At each time step, we use \(T(r)\) to determine the Eddington factors and mean opacities by solving the frequency-dependent radiation transport equation in a comoving frame and integrate to obtain the frequency-averaged quantities. The averaged opacities have been calculated assuming LTE. Both the monochromatic and mean opacities are calculated in the narrow-line limit. Scattering, photon redistribution, and thermalization terms used in the light-curve opacity calculations are taken into account. In
previous works, the photon redistribution and thermalization terms have been calibrated for a sample of spectra using the formalism of the equivalent two-level approach (H95). Here, for increased consistency, we use the same equations and atomic models but solve the rate equations simultaneously with the light-curve calculation at about every 100th time step, on the expense of some simplifications in the non-LTE part compared to H95. For the opacities we use the narrow-line limit, and for the radiation fields we use the solution of the monochromatic radiation transport using $\times 1000$ frequency groups. Both the old and new approaches are about equivalent in accuracy with consistent results. Most noticeable, now, $B - V$ is bluer by about $-0.03^m$.

The following discussion is based on the same set of models used in the previous section. In Figures 8 and 9 we show the $B$ and $V$ light curves and some quantities at maximum light. Overall the different phases of light curves can be understood in the usual way including the bump at about day 35, which can be attributed to the change in the opacities between the layers of complete and incomplete burning (Dominguez 1991, 1994). For the reference model 5p022, a maximum brightness $M_B$ of $-19.20^m$ is reached at about 18.25 days after the explosion. The color index $B - V$ is $-0.02^m$.

As discussed in § 1, the amount of $^{56}$Ni, its distribution, and the expansion rate of the envelope are the dominant factors that determine the absolute magnitude at maximum and the light-curve shape. With all model parameters fixed except the progenitor mass and the metallicity, the differences of the light curves can be understood based on the previous discussion of the explosion models.

3.2.1. Dependence on the Main-Sequence Mass

By increasing $M_{\text{MS}}$ from 1.5 to 7.0 $M_\odot$ for $Z = 0.02$, both $M_B$ and $M_V$ decrease by $\approx 0.15^m$, consistent with a change in the $^{56}$Ni mass by 14% (Fig. 8, upper panel, and Fig. 9). The similarity in the density and velocity structures produces almost identical conditions at the photosphere. Thus, $B - V$ is insensitive to a change in $M_{\text{MS}}$ [$\Delta(B - V)_{\text{model}} \leq 0.01^m$]. Relative to the reference model, the rise times vary between $\pm 0.5$ (1p5z22) and 1.2 days (7p0z22). The decline rate $\Delta M_{14}$ is hardly affected. A change in $M_{\text{MS}}$ will result in an offset/dispersion in $M(\Delta M_{13})$ by up to $0.15^m$. Interestingly, the fluxes on the radioactive tail are much more similar than could be expected from the spread in the $^{56}$Ni masses by 14% (Fig. 8). The change in $M_{\text{MS}}$ is almost compensated by the differences in the energy deposition of $\gamma$-rays from the radioactive decay. In Figure 10 the escape probability for hard radiation is shown as a function of time. A significant fraction of $\gamma$ photons is thermalized up to about 150–200 days. The actual value of thermalization depends on the expansion rate, which is decreasing with mass (see above). For example, the fraction of thermalized $\gamma$ photons for the models 1p5z22, 5p0z22, and 7p0z22 are 24.2%, 25%, and 27%, respectively. The increase in the efficiency for the thermalization amounts to 11% over the mass range and almost compensates for the decrease in the $^{56}$Ni mass. Note that the ratio between maximum and tail
brightness is decreasing with $M_V$. This effect is opposite to the observed $M(\Delta M_{15})$ relation (e.g., Hamuy et al. 1996a). If realized in nature, a wide range in $M_{\text{MS}}$ would increase the dispersion in $\delta M(\Delta M_{15})$ by about 0.15 mag. The presence of this effect would reveal itself by an additional change in the expansion velocity measured by the Doppler shift of lines. For example, at maximum light, weak lines would indicate an expansion velocity at the photosphere that is smaller by $\approx 2000$ km $\text{s}^{-1}$ if we compare models $7p0z22$ and $1p5z22$.

The discussion above applies to all metallicities because $C_{\text{cen}}$ and $M_{\text{C/O}}$ vary in a similar range.

### 3.2.2. Dependence on the Initial Metallicity $Z$

For progenitors with $M_{\text{MS}} = 5 M_\odot$, a change in the metallicity has little effect on the energetics and, consequently, on the light curves (Fig. 8, middle panel). The most important effect is the systematic decline of $B-V$ by $\approx 0.05$ mag when $Z$ is changed from 0.02 to $10^{-10}$. This effect can be attributed to a change of the line blending by Fe in the decoupling region of photons, i.e., the atmosphere (HWT98; Lentz et al. 2000). In $B$ and $V$ at maximum light, opacities are dominated by electron scattering (HKM93), but iron lines are more important in $B$ compared to $V$. Consequently, a lower metallicity results in an increase of the flux ratio between $B$ and $V$. In the $U$ band and the UV, the opacities are dominated by lines. A change in the line blending will cause a change of both the radius of the flux formation and the specific flux. Therefore, a decrease in $Z$ may result in either an increase or a decrease of the monochromatic flux depending on the density structure. These findings apply to all main-sequence masses in our sample. To some extent, the exceptions are the models with $M_{\text{MS}} = 3 M_\odot$ for which the central carbon concentration varies with metallicity and produces a change in $M_{\text{SNII}}$ by 3% and a corresponding change in $M_\mu$ and $M_V$.

### 3.2.3. Influence of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ Rate

A low nuclear rate $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ increases the explosion energy compared to our standard rate ($3p0z13LR$ vs. $3p0z13$; see Fig. 8, lower panel). The rise time in $V$ is reduced by 2.7 days ($15.3$ days for $3p0z13LR$ vs. $18.0$ days for $3p0z13$). The enhanced escape probability for $\gamma$-rays (Fig. 10) explains the remaining differences, including the increased maximum brightness-to-tail ratio and the moderate increase of $M_V$ and $M_\mu$. These results are consistent with previous findings, which identified the importance of the $C/O$ ratio for the change in the rise time of “typical” SNe Ia (HWT98).

### 4. Final Discussion, Observational Constraints, and Conclusions

Using a DD model and realistic structures for the exploding WD, we have studied the influence of the progenitor star on the light curves and spectral properties of SNe Ia.

**Stellar models.** We considered stars between 1.5 and 7 $M_\odot$ and metallicities between $Z = 0.02$ (solar) and $10^{-10}$, which covers the full range of potential progenitors. The progenitor structures are based on detailed calculations for
the stellar evolution starting at the pre-main sequence up to the thermal pulses when most of the stellar envelope is ejected and a WD is formed with a mass between 0.5 and 1.0 $M_\odot$. Its size increases with $M_{\text{ms}}$ and, to a lesser extent, changes with metallicity. The subsequent accretion and burning at the surface of the WD let it grow to $M_{\text{ch}}$. As a final chemical structure, the WD shows a central region of reduced C abundance between 0.21 and 0.32 originating from the convective He burning, a layer of increased C abundance from the He shell burning, and a layer originating for the accretion phase. The mean C/O ratio decreases from the convective He burning, a layer of increased C burning carbon and oxygen to iron group elements differs by about 30% over the entire mass range. The sensitivity on the metallicity is much weaker (< 10%) and not monotonic.

**Supernovae.**—Our study of SNe Ia is based on DD models because they have been found to reproduce the monochromatic light curves and spectra of SNe Ia reasonably well, including the brightness-decline relation $M(\Delta M_{1.5})$. Deviations from a perfect relation are due to variations in the central density, properties of the deflagration front, and the progenitor structure. All parameters except the progenitors have been fixed to produce light curves and spectra typical for “normal” SNe Ia. In this paper, rise times to maximum light are between 17.7 and 19.4 days, $M_V = -19.25^{\text{m}}$ to $-19.11^{\text{m}}$, and $B - V = 0.02^{\text{m}}$ to $-0.07^{\text{m}}$. Differences between the models and light curves remain small because the nuclear energy production by burning carbon and oxygen to iron group elements differs by as little as $\approx 10\%$.

The change of $M_{\text{ms}}$ is the decisive factor to change the energetics. The $^{56}$Ni production varies by about 14%, and the velocities of the various chemical layers differ by up to 1500 km s$^{-1}$. A change in the metallicity hardly affects the overall structure of the progenitor. As already discussed in detail in HWT98, its main effect is a change in the production of $^{56}$Fe in the outer layers of incomplete Si burning.

As one of the main results of our study, we find that variations in $M_{\text{ms}}$ change the shape of the light curves but hardly affect $B - V$, whereas a change in $Z$ affects $B - V$. $M_{\text{ms}}$ alters the $M(\Delta M_{1.5})$ relation, which may be offset by up to 0.2$^\text{m}$. In addition, $M_{\text{ms}}$ changes the flux ratio between maximum light and the radioactive tail, and it alters the photospheric expansion velocities $v_{\text{ph}}$ measured by the Doppler shift of lines. For example, a change in $m_V(t_{\text{max}}) - m_V(t_{\text{max}} + 40d)$ by 0.2$^\text{m}$ is coupled to a decrease in $v_{\text{ph}}$ at maximum light by approximately $-2000$ km s$^{-1}$. Note that a change in the central density $\rho_c$ of the WD has a similar effect on $m_V(t_{\text{max}}) - m_V(t_{\text{max}} + 40d)$ but with the opposite sign for $\Delta v_{\text{ph}}$ (P. Höflich 2001, in preparation). In principle, this allows us to decide whether differences in $m_V(t_{\text{max}}) - m_V(t_{\text{max}} + 40d)$ between supernovae with similar $M(\Delta M_{1.5})$ are related to a change in the progenitor or the central density at the thermonuclear runaway, which is sensitive to the accretion rate.

In contrast to $M_{\text{ms}}$, the metallicity $Z$ hardly changes the light-curve shapes [$\delta M(\Delta M_{1.5}) \leq 0.06^{\text{m}}$]. It alters the line blocking by iron group elements at the photosphere mainly in the UV, $U$, and $B$ but hardly in $V$ (HWT98). In the models presented here, $B - V$ becomes systematically bluer with decreasing $Z$ (up to $\approx 0.07^{\text{m}}$). Because $B - V$ is the basic color index used to correct for interstellar extinction, the metallicity effect can systematically alter the estimates for the absolute brightness by up to 0.2$^\text{m}$.

$^{12}$C($\alpha, \gamma$)$^{16}$O. — For the example of a progenitor with $M_{\text{ms}} = 3 M_\odot$ and $Z = 0.001$, we have tested the influence of the low nuclear rate $^{12}$C($\alpha, \gamma$)$^{16}$O on the outcome. Using the lower rate suggested by Caughlan & Fowler (1988) instead of that by Caughlan et al. (1985) results in more energetic explosions because $^{16}$O production varies by about 14%, and $m_V$ are related to a change in the progenitor or the accretion phase. The mean C/O ratio decreases by about 30% over the entire mass range. The sensitivity on the metallicity is much weaker (< 10%) and not monotonic.

The theoretical light curves peak at $M_V = -19.21^{\text{m}}$ and $-19.30^{\text{m}}$ for 3p0z13 and 3p0z13LR, respectively. From the empirical fit of Riess, we would expect a rise time between 17.6 and 18.4 days, which favors the high rate $^{12}$C($\alpha, \gamma$)$^{16}$O of Caughlan et al. (1985). Note that the uncertainties in absolute values for the rise times are $\approx 1$–2 days (HWT98).

**Constraints on the progenitor.**—Empirically, the $M(\Delta M_{1.5})$ has been well established with a rather small statistical error $\sigma$ (0.12$^{\text{m}}$, Riess et al. 1996; 0.16$^{\text{m}}$, Schmidt et al. 1998; 0.14$^{\text{m}}$, Phillips et al. 1999; 0.16$^{\text{m}}$, Riess et al. 1999; 0.17$^{\text{m}}$, Perlmutter et al. 1999a). From theoretical models, a spread of 0.3$^{\text{m}}$–0.5$^{\text{m}}$ can be expected (Höflich et al. 1996b). This may imply a correlation between free model parameters, namely, the properties of the progenitors, the central density, or the transition density $\rho_c$. In this study, we find a spread in $M(\Delta M_{1.5})$ of about 0.2$^{\text{m}}$ for progenitors with $M_{\text{ms}}$ between 1.5 and 7.0 $M_\odot$. This may suggest a more narrow range in $M_{\text{ms}}$ for realistic progenitors. From the fiducial rise time, progenitors with $M_{\text{ms}} \geq 3 M_\odot$ are favored. This number should be regarded as a hint because of uncertainties in the light-curve models. Another constraint can be obtained from the observed spread in the rise-time–decline relation. Riess et al. (1999) find a spread in $t_V$ of $\pm 0.4$ days, whereas our models show a spread of $\approx 1.7$ days for 1.5 $\leq M_{\text{ms}} \leq 7 M_\odot$. To be consistent with the observations, the range of main-sequence masses has to be reduced by a factor of 2. This range is an upper limit because additional variations in the population of SNe Ia such as explosion scenarios or the central density of the WD at the time of ignition will likely result in lower correlations.

**The cosmological equation of state.**—In the following, we want to discuss our results in context of SNe Ia as probes for cosmology and for the determination of the cosmological equation of state. We limit the discussion to the effects due to different progenitors. For a discussion of other systematic effects such as gray dust, gravitational lensing, the influence of a change in the importance of different possible scenarios (e.g., merger vs. $M_{\text{ch}}$ models), etc., we want to refer to the growing literature in this field (e.g., Schmidt et al. 1998; HWT98; Perlmutter et al. 1999a).

Recently, there has been strong evidence for a positive cosmological constant (e.g., Perlmutter et al. 1999a; Riess et al. 1999). This evidence is based on observations that SNe Ia in the redshift range between 0.5 and 1.2 appear to be dimmer by about 0.25$^{\text{m}}$, which is comparable to the variations produced by different progenitors. However, both the internal spread in $M(\Delta M_{1.5})$ (see above) and the similarity in $M(\Delta M_{1.5})$ between the local SNe Ia and the high-z sample ($\Delta t \leq 1d$; Aldering et al. 2000) limit the likely range of models and, consequently, evolutionary effects to $< 0.1^{\text{m}}$ up to redshifts of 1. In addition, we do not expect a drastic change in the metallicity between local and supernovae at $\approx 0.06^{\text{m}}$. Not that $\delta M(\Delta M_{1.5}) \leq 0.06^{\text{m}}$. It alters the line blocking by iron group elements at the photosphere mainly in the UV, $U$, and $B$ but hardly in $V$ (HWT98). In the models presented here, $B - V$ becomes systematically bluer with decreasing $Z$ (up to $\approx 0.07^{\text{m}}$). Because $B - V$ is the basic color index used to correct for interstellar extinction, the metallicity effect can systematically alter the estimates for the absolute brightness by up to 0.2$^{\text{m}}$.

$^{12}$C($\alpha, \gamma$)$^{16}$O. — For the example of a progenitor with $M_{\text{ms}} = 3 M_\odot$ and $Z = 0.001$, we have tested the influence of
$z \leq 1$. Taking the linear dependence of $B-V$ on the metallicity (Fig. 9) and realistic ranges for $z$, reddening of "nongray" dust also will not change the conclusion on the need for a positive cosmological constant.

As discussed in §1, the quest for the nature of the "dark" energy is one of the central questions to be addressed in the future (e.g., White 1998; Perlmutter et al. 1999b; Albrecht & Weller 2000; Ostriker & Steinhardt 2001). For $z \geq 1$, we can expect both very low metallicities and a significant change of the typical $M_{\text{MS}}$. From this study, systematic effects due to different progenitors are limited to $\approx 0.2\sigma$. Therefore, without further corrections for the progenitor evolution, some of the alternatives for the nature of the "dark energy" may be distinguished without correction for evolution. However, for a more detailed analysis, an accuracy of about 0.05$\sigma$ (Weller & Albrecht 2001) is required. In this paper we have shown how a combination of spectral and light-curve data or different characteristics of the light curve can help to achieve this goal.

Limitations—Finally, we also want to mention the limitations of this study. Qualitatively, our results on the $Z$ dependence agree with a previous study (Höflich et al. 2000), which was based on a progenitor with $M_{\text{MS}} = 7 M_\odot$ calculated by Nomoto’s group (Umeda et al. 1999). The relation between $C/O_{\text{MS}}$ and the offset in the $M(\Delta M_{15})$ relation has been confirmed. However, the influence of $Z$ on $C/O_{\text{MS}}$ was found to be about twice as large, and the central C concentrations are systematically larger. The differences point toward a general problem. The details of the central structure and evolution in the convective He-burning core depend sensitively on the treatment of convection, semiconvection, overshooting, and breathing pulses (Lattanzio 1991; Schaller et al. 1992; Bressan et al. 1993; Vassiliadis & Wood 1993; for a detailed discussion see Dominguez et al. 1999). In particular, the central C concentration may vary between 0.1 and 0.5. We want to note that our value is consistent with direct measurements of the central C/O ratio found by the analysis of pulsational modes of WDs (Metcalfe et al. 2001). These uncertainties will affect the efficiency to separate the contribution of $M_{\text{MS}}$ and $Z$, respectively, i.e., the reason for $C/O_{\text{MS}}$ but not the observable relations for light curves and spectra.

We provide limits on the size of evolutionary effects due to $M_{\text{MS}}$ and metallicity of the progenitor ($\Delta M \approx 0.2\sigma$). Other evolutionary effects may be due to a systematic change in the dominant progenitor scenario (e.g., mergers vs. single degenerate) or the typical separation in binary systems that contribute to the SNe Ia at a given time. The latter may alter the accretion rates and, consequently, the central densities of the WD at the time of the thermonuclear runaway.

We did not consider the effect of the progenitor and of its preconditioning just prior to the explosion on the propagation of the burning front and, in particular, on the DDT, or, alternatively, the phase of transition from a slow to a very fast deflagration (W. Hillebrandt 1999, private communication). Although the model parameters have been chosen to allow for a representation of "typical" SNe Ia, more comprehensive studies and detailed fitting of actual observations are needed, e.g., to detangle effects due to a change in the ignition density versus the progenitor. In particular, observations of local SNe Ia have to be employed to narrow down and test for the proposed range of flux ratio between maximum and tail, as well as its relation to the expansion velocities.

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