A Genetic Algorithm for the Multi-Pickup and Delivery Problem with Time Windows

Harbaoui Dridi1,2, R. Kammarti1,2, M. Ksouri2, P. Borne1

1. LAGIS: Ecole Centrale de Lille, Villeneuve d’Ascq, FRANCE
2. LACS: Ecole Nationale des Ingénieurs de Tunis, Tunis - Belvédère. TUNISIE

imenharbaoui@gmail.com, kammarti.ryan@planet.tn, Mekki.Ksouri@insat.rnu.tn, p.borne@ec-lille.fr

Abstract: In this paper, we present a genetic algorithm for the multi-pickup and delivery problem with time windows (m-PDPTW). The m-PDPTW is an optimization vehicles routing problem which must meet requests for transport between suppliers and customers satisfying precedence, capacity and time constraints. This paper proposes a brief literature review of the PDPTW, present our approach based on genetic algorithms to minimizing the total travel distance and thereafter the total travel cost, by showing that an encoding represents the parameters of each individual.

Keywords: vehicles routing, pickup and delivery problem, time windows, optimization, genetic algorithm.

Harbaoui Dridi was born in 1981 in TUNISIA; she graduated in engineering from the National Institute of Applied Science and Technology (INSAT) in 2006 and Masters Degree in Computer Science and Industrial Automation (INSAT) in 2007. Currently, it is doctoral student (in Electrical and Computer Engineering and Industrial Automation) at the National School of Engineers of Tunis (ENIT - TUNISIA) and the Central School of Lille (EC LILLE - FRANCE) and contractual assistant at the Higher Institute of Informatics (TUNISIA).

Ryan Kammarti was born in Tunis, Tunisia in 1978. He received the M.E. and the Master degree in Automatics and Industrial Computing from the National Institute of Applied Sciences and Technology (INSAT) in 2003 and the Ph.D. degree in automatics and industrial computing from the Ecole Centrale de Lille, Villeneuve d’Ascq, France, and the University of Lille and the National School of Engineers of Tunis in 2006. He currently occupies a assistant teacher position with the University of Tunis EL Manar, Tunis, Tunisia. He also a team head in the ACS research laboratory in the National School of Engineers of Tunis (ENIT). Ryan Kammarti was born in Tunis, Tunisia in 1978. He received the M.E. and the Master degree in Automatics and Industrial Computing from the INSAT in 2003 and the Ph.D. degree in automatics and industrial computing from the Ecole Centrale de Lille, Villeneuve d’Ascq, France, and the University of Lille and the National School of Engineers of Tunis (ENIT) in 2006. He currently occupies a assistant teacher position with the University of Tunis EL Manar, Tunis, Tunisia. He also a team head in the ACS research laboratory in the ENIT.

Mekki Ksouri was born in Jendouba, Tunisia in 1948. He received the M.A. degree in physics in the FST in Tunis in 1973, the M.E. degree Ingenieur SupElec from the High School of Electricity (Ecole Suprieure d’Electricite French Grande Ecole) in Paris in 1973, degree, the D.Sc. degree, and the Ph.D. degree from the University of Paris VI, Paris, France, in 1975 and 1977 respectively. He is Professor at the National School of Engineers of Tunis (ENIT). He was principal of the National Institute of Applied Sciences and Technology (INSAT) from 1999 to 2005, principal and founder of the High School of Statistics and Information Analysis from 2001 to 2005 and the High Institute of Technologic Studies from 1996 to 1999 and principal of The High Normal School of Technological Education from 1978 to 1990. Pr. Ksouri is the author or coauthor of many journal articles, book chapters, and communications in international conferences. He is also the author of 6 books. His activities focus on automatic control, robust control, and optimization in planning and scheduling, including implementation of fuzzy logic, neural nets, and genetic algorithms. Pr. Ksouri had received many distinctions like the Outstanding Contribution Award for Leadership in on the Organization of the CIFA 2004 Symposium, Chevalier dans l’Ordre des Palmes Académiques; France decret en date du 19/4/2002, Outstanding Contribution Award ; IEEE SMC 1998 and Ordre national du merite de l’education; Tunisie decret n97- 1369 du 18/07/97. Pr. Ksouri is a senior member IEEE and he’s the head and founder of many scientific associations like ATTNNA and ASET.

Pierre Borne received the Master degree of Physics in 1967, the Masters of Electronics, of Mechanics and of Applied Mathematics in 1968. The same year he obtained the Diploma of "Ingénieur IDN" (French "Grande Ecole"). He obtained the PhD in Automatic Control of the University of Lille in 1970 and the DSc of the same University in 1976. He became Doctor Honoris Causa of the Moscow Institute of Electronics and Mathematics (Russia) in 1999, of the University of Waterloo (Canada) in 2006 and of the Polytechnic University of Bucharest (Romania). He is author or co-author of about 200 Journal articles and book chapters, and of 38 plenary lectures and of more than 250 communications in international conferences. He has been the supervisor of 71 PhD thesis and is author of 20 books.
1. Introduction

With the time and economic constraints implications of the transport goods problem, its resolution becomes very difficult, requiring the use of tools from different disciplines (manufacturing, information technology, combinatorial optimization, etc.)... Indeed, the process from transport systems and scheduling are becoming more complex by their large size, by the nature of their relationship dynamics, and by the multiplicity of which they are subjected.

Many studies have been directed mainly towards solving the vehicle routing problem (VRP). It’s an optimization vehicle routing problem to meet travel demands. Other researchers became interested on an important variant of VRP which is the PDPTW (Pickup and Delivery Problem with Time Windows) with capacity constraints on vehicle.

The PDPTW is divided into two: 1-PDPTW (single-vehicle) and m-PDPTW (multi-vehicle).

Our object is to design a tool for m-PDPTW resolution based on genetic algorithms and Pareto dominance method to give a set of satisfying solutions to this problem minimizing the total travel distance and thereafter the total travel cost, by showing that an encoding represents the parameters of each individual.

2. Literature Review

2.1 Vehicle routing problem

The Vehicle Routing Problem (VRP) represents a multi-goal combinatorial optimization problem which has been the subject of much work and many variations in the literature. It belongs to the NP-hard class. [1] [Christofides, N and al., 1979], [2][Lenstra, J and al., 1981].

Heuristic methods have been used by Matsinen, T and al to solve the general problem in this area as the Travelling Salesman Problems [3] [Matsinen, T and al., 1992]. The Meta heuristics were as well applied to solving the vehicle routing problem. Among these methods, we can include ant colony algorithms, which were used by Montamenni, R and al for the resolution of DVRP and used also by Sorin C, N and al to solving the multiple constraints problem: vehicle route allocation. [4] [Montamenni, R and al., 2002], [5] [Sorin C, N and al., 2008].

The VRP principle is: given a depot D and a set of customers orders C = (c1, ... , Cn), to build a package routing, for a finite number of vehicles, beginning and ending at a depot. In these routing, a customer must be served only once by a single vehicle and vehicle capacity transport for a routing should not be exceeded. [6] [Nabaa, M and al., 2007].

Savelsbergh shown that the VRP is a NP-hard problem [7] [Savelsbergh, M.P.W and al., 1995]. Since the m-PDPTW is a generalization of the VRP it’s a NP-hard problem.

2.2 The PDPTW: Pickup and Delivery Problem with Time Windows

The PDPTW is a variant of VRPTW where in addition to the existence of time constraints, this problem implies a set of customers and a set of suppliers geographically located. Every routing must also satisfy the precedence constraints to ensure that a customer should not be visited before his supplier. [8] [Psaraftis, H.N., 1983].

A dynamic approach for resolve the 1-PDP without and with time windows was developed by Psaraftis, H.N considering objective function as a minimization weighting of the total travel time and the non-customer satisfaction. [9] [Psaraftis, H.N., 1980].

Jih, W and al have developed an approach based on the hybrid genetic algorithms to solve the 1-PDPTW, aiming to minimize combination of the total cost and total waiting time. [10] [Jih, W and al., 1999].

Another genetic algorithm was developed by Velasco, N and al to solve the 1-PDP bi-
objective in which the total travel time must be minimized while satisfy in prioritise the most urgent requests. In this literature, the method proposed to resolve this problem is based on a No dominated Sorting Algorithm (NSGA-II). [11] [Velasco, N and al., 2006].

Kammarti, R and al deal the 1-PDPTW, minimizing the compromise between the total travel distance, total waiting time and total tardiness time, using an evolutionary algorithm with Special genetic operators, tabu search to provide a set of viable solutions. [12] [Kammarti, R and al., 2004], [13] [Kammarti, R and al 2005a].

This work have been extended, in proposing a new approach based on the use of lower bounds and Pareto dominance method, to minimize the compromise between the total travel distance and total tardiness time. [14] [Kammarti, R and al 2006], [15] [Kammarti, R and al 2007].

About the m-PDPTW, Sol, M and al have proposed a branch and price algorithm to solve the m-PDPTW, minimizing the vehicles number required to satisfy all travel demands and the total travel distance. [16] [Sol, M and al., 1994].

Quan, L and al have presented a construction heuristic based on the integration principle with the objective function, minimizing the total cost, including the vehicles fixed costs and travel expenses that are proportional to the travel distance. [17] [Quan, L and al., 2003].

A new metaheuristic based on a tabu algorithm, was developed by Li, H and al to solve the m-PDPTW. [18] [Li, H and al., 2001].

Li, H and al have developed a “Squeaky wheel” method to solve the m-PDPTW with a local search. [19] [Li, H and al., 2002].

A genetic algorithm was developed by Harbaoui Dridi, I and al dealing the m-PDPTW to minimize the total travel distance and the total transport cost. [20] [Harbaoui Dridi, I and al., 2008]

3. Mathematical Formulation

Our problem is characterized by the following parameters:

- $N$: Set of customers and supplier vertices,
- $N^+$: Set of supplier vertices,
- $N^-$: Set of customers vertices,
- $K$: Vehicle number,
- $d_{ij}$: Euclidian distance between the vertex $i$ and the vertex $j$. If $d_{ij} = \infty$, then the road between $i$ and $j$ doesn’t exist,
- $t_{ijk}$: Time used by the vehicle $k$ to travel from the vertex $I$ to the vertex $j$,
- $[e_i,l_i]$: Time window of the vertex $i$,
- $s_i$: Stopping time at the vertex $i$,
- $q_i$: Goods quantity of the vertex $i$ request. If $q_i > 0$, the vertex $i$ is a supplier; if $q_i < 0$, the vertex $i$ is a customer and if $q_i = 0$ then the vertex was served.
- $Q_k$: Capacity of vehicle $k$,
- $i = 0..N$: Predecessor vertex index,
- $j = 0..N$: Successor vertex index,
- $k$: 1..K: Vehicle index,
- $x_{ijk} = \begin{cases} 1 & \text{If the vehicle travel from the vertex } i \text{ to the vertex } j \\ 0 & \text{Else} \end{cases}$
- $A_i$: Arrival time of the vehicle to the vertex $i$,
- $D_i$: Departure time of the vehicle from the vertex $i$,
- $y_{ik}$: The goods quantity in the vehicle $k$ visiting the vertex $I$,
- $C_k$: Travel cost associated with vehicle $k$,
- A vertex is served only once,
- There is one depot,
- The capacity constraint must be respected,
- The depot is the start and the finish vertex for the vehicle,
- The vehicle stops at every vertex for a period of time to allow the request processing,
- If the vehicle arrives at a vertex $i$ before its time windows beginning date $e_i$, it waits.

The function to minimize is given as follows:

$$\text{Minimiser } f = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} C_k d_{ij} x_{ijk}$$ (1)
Subject to:

\[ \sum_{k=1}^{K} \sum_{j=1}^{J} x_{ijk} = 1, j = 2, \ldots, N \]  \hspace{1cm} (2)

\[ \sum_{i=1}^{N} \sum_{j=1}^{J} x_{ijk} = 1, i = 2, \ldots, N \]  \hspace{1cm} (3)

\[ \sum_{i \in N} x_{i0k} = 1, \forall k \in K \]  \hspace{1cm} (4)

\[ \sum_{j \in N} x_{0jk} = 1, \forall k \in K \]  \hspace{1cm} (5)

\[ \sum_{i \in N} x_{ijk} - \sum_{j \in N} x_{ijk} = 0, \forall k \in K, \forall u \in N \]  \hspace{1cm} (6)

\[ x_{ijk} = 1 \Rightarrow y_{jk} = y_{ik} + q_i, \forall i, j \in N; \forall k \in K \]  \hspace{1cm} (7)

\[ y_{0k} = 0, \forall k \in K \]  \hspace{1cm} (8)

\[ q_k \geq y_{ik} \geq 0, \forall i \in N; \forall k \in K \]  \hspace{1cm} (9)

\[ D_w \leq D_v, \forall i \in N; \forall w \in N_i^+; \forall v \in N_i^- \]  \hspace{1cm} (10)

\[ D_0 = 0 \]  \hspace{1cm} (11)

\[ x_{ijk} = 1 \Rightarrow e_i \leq A_i \leq l_i, \forall i, j \in N; \forall k \in K \]  \hspace{1cm} (12)

\[ x_{ijk} = 1 \Rightarrow e_i \leq A_i + s_i \leq l_i, \forall i, j \in N; \forall k \in K \]  \hspace{1cm} (13)

\[ x_{ijk} = 1 \Rightarrow D + t_{ijk} \leq (l_j - s_j), \forall i, j \in N; \forall k \in K \]  \hspace{1cm} (14)

The constraint (2) and (3) ensure that each vertex is visited only once by a single vehicle. The constraint (4) and (5) ensure that the vehicle route beginning and finishing is the depot. The constraint (6) ensures the routing continuity by a vehicle. (7), (8) and (9) are the capacity constraints. The precedence constraints are guaranteed by (10) and (11). The constraints (12), (13) and (14) ensure compliance time windows.

4. Genetic Algorithm for Minimizing the Total Travel Cost

In this part, we present our approach based on genetic algorithms to minimize the total travel cost.

4.1 Solution coding

A chromosome is a succession (permutation) vertex, which indicates the order in which a vehicle is to visit all the vertices. Fig.1 represents the solutions under form of chromosomes.

| Vertex (i) | 0 | 5 | 8 | 2 | 6 | 4 | 3 | 10 | 7 | 9 | 1 |
|-----------|---|---|---|---|---|---|---|----|---|---|---|

Figure 1. Solution coding

The vertex "0" represents the depot.

4.2 Decoding the vehicles passage

The decoding allows, from every chromosome of the population to obtain an initial solution sign the passage of each vehicle on the corresponding vertex. (Fig.2)

| Vehicle | C1 | C2 | C3 |
|---------|----|----|----|
| V1      | 0  | 2  | 4  | 1  | 5  | 0  |
| V2      | 0  | 3  | 6  | 7  | 10 | 0  |
| V3      | 0  | 8  | 9  | 0  |    |    |

Figure 2. Order of vehicles passage

4.3 Crossover operator

Following the generation of the initial population, we proceed to crossover phase which ensures the recombination of parental genes for train new descendants. To do this, we choose the one point crossover. (Fig.3)

| Chromosome 1 | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|--------------|----|----|----|----|----|----|----|----|----|----|
| Chromosome 2 | 1  | 2  | 8  | 9  | 10 | 6  | 7  | 3  | 4  | 5  |

Figure 3. Crossover operator

4.4 Mutation operator

Mutation operator (Fig.4) is to choose two positions at random, within a chromosome and exchange their respective values.

| Chromosome 1 | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|--------------|----|----|----|----|----|----|----|----|----|----|
| Chromosome 2 | 1  | 2  | 3  | 1  | 5  | 4  | 2  | 3  | 1  | 5  |

Figure 4. Mutation operator

Note here that we must respect the constraint precedence, to ensure that a customer is not visited before his supplier, and capacity to ensure the non-overload of each vehicle.
4.5 Procedure of the proposed approach for minimizing the total travel cost

4.5.1 Generation of initial population

The choice of the initial population is important because it can make a genetic algorithm more or less fast to converge towards the global optimum.

In our case, we will generate two types of populations. A first population noted $P_{node}$, which represents all nodes to visit with all vehicles, according to the permutation list coding (Fig.1). The second population noted $P_{vehicle}$ indicates nodes number visited by each vehicle. Knowing that $k$ varies between 1 and $\frac{N'}{2}$ vehicles. Fig.5 shows an individual example of $P_{vehicle}$ with $N'=10$.

![Figure 5. Individual example of $P_{vehicle}$](image)

4.5.2 Correction procedure

The principle of correction precedence and capacity is to ensure that a customer is not visited before his supplier while respecting the vehicles capacity.

Whereas couples customer / supplier following: (1.5), (2.8), (9.7), (10.3) and (4.6), noting that $Q_{\text{max}} = 60$, we present, respectively, in Fig.6 and Fig.7 the principle of correction precedence and capacity.

![Figure 6. Correction precedence](image)

![Figure 7. Correction capacity](image)

4.5.3 Generation of population $P_{node/vehicle}$

Considering the population $P_{node}$ given by Fig.1, correction procedures and $P_{vehicle}$ population, given by Fig.5, we illustrated in Fig.8 an individual of the population $P_{node/vehicle}$.

![Figure 8. Individual of the population $P_{node/vehicle}$](image)

4.5.4 Calculation procedure

Each vehicle will serve vertices in the assignment order. Once returned to the depot, we calculate the transport cost corresponding to the travel distance, and we repeat this until all the vertices are served.

We reproduce this work for each individual to the population $P_{vehicle}$, taking into account the different possible combinations between $P_{vehicle}$ and $P_{node}$, to obtain thereafter the correspondent individual of the population $P_{node/vehicle}$ that minimizes our objective function. The algorithm of this procedure is shown by the Fig.9.

![Figure 9. Approach algorithm](image)
With:

- \( n \): size of initial population,
- \( f_i \): objective function to minimize (given by equation 1) for iteration \( i, i = 1 \ldots 2n \times 2n \).

We determine thereafter:

\[
\text{f}_{\text{research}} = \min_{i} f_i \quad (15)
\]

4.5.5 Simulation

We present in Table 1, the simulation results given by our approach.

![Figure 10. Calculation Procedure](image)

**Table 1. Simulation results**

| \( N' \) | \( n \) | \( k \) | \( \min_{i} \left( \sum_{k \in k \in N} \sum_{j \in j \in N} d_{ijk} \right) \) | \( \min f_i \) |
|---|---|---|---|---|
| 20 | 100 | 2 | 878.84 | 59425.95 |
| 20 | 500 | 2 | 843.39 | 52296.84 |
| 50 | 100 | 2 | 2379.22 | 145534.61 |
| 50 | 500 | 2 | 2234.06 | 140743.17 |
| 100 | 100 | 4 | 4581.24 | 265197.12 |
| 100 | 500 | 4 | 4439.64 | 257383.70 |

We observe that our approach reduces the
vehicles number (2 vehicles to serve up to 50 vertices and 4 vehicles to serve 100 vertices). Knowing that a vehicle will cause an augmentation travel cost on the addition of a driver, the travel fixed costs, to depreciation of the vehicle used. Hence the interest of our approach that aims to minimize the total travel cost.

5. Conclusion

In this paper we have proposed, following a study of the different approaches proposed for the resolution of 1-PDPTW and m-PDPTW, a genetic algorithm for minimizing the total travel cost. To do this, we presented the mathematical formulation of our problem, and we have detailed the calculation procedure for determine the optimal solution, that minimizes our objective function.

REFERENCES

1. CHRISTOFIDES, N., A. MINGOZZI, and P. TOTH, The vehicle routing problem. In Combinatorial Optimization, volume 11, John Wiley, 1979, pp. 315–338.

2. LENSTRA, J. and A. RINNOOY KAN, Complexity of the vehicle routing and scheduling problems, In Networks, volume 11, Springer, 1981, pp. 221–228.

3. MATSINEN, T., SAVOLAINEN, V., Efficient Combinatorial of heuristics for solving Travelling Salesman Problems, Published in: Studies in Informatics and Control, Vol. 1, No. 1, March 1992.

4. MONTAMENNI, R., L.M. GAMBARDELLA, A.E. RIZZOLI, A.V. DONATI, A new algorithm for a Dynamic Vehicle Routing Problem based on Ant Colony System, IDSIA, Switzerland, 2002.

5. SORIN C., N., CLAUDIU V, KIFOR, and CONSTANTIN, O., Ant Colony Solving Multiple Constraints Problem: Vehicle Route Allocation, Int. J. of Computers, Communications & Control, ISSN 1841-9836, E-ISSN 1841-9844 Vol. III, 2008.

6. NABAA, M., B. ZEDDINI and P. TRANOUZE, Approche décentralisée pour résoudre le problème du transport à la demande, Schedae, prépublication no. 23, fascicule no. 2, 2007, pp. 133-140.

7. SAVELSBERGH, M.P.W., SOL M., The general pickup and delivery problem, Transportation Science 1995.

8. PSARAFTIS, H.N., An exact algorithm for the single vehicle many to many immediate request dial a ride problem with time windows. Transportation Science, 17, 1983, pp. 351-357.

9. PSARAFTIS, H.N., A dynamic programming solution to the single vehicle many to many immediate request dial a ride problem., Transportation Science, 14(2):130–154, 1980.

10. JIH, W. and HSU, J., Dynamic vehicle routing using hybrid genetic algorithms, International Conference on Robotics and Automation, 1999, pp. 453–458.

11. VELASCO, N., P. DEJAX, C. GUERET and C. PRINS. Un algorithme génétique pour un problème de collecte bi-objectif, MOSIM 2006.

12. KAMMARTI, R., HAMMADI, S., BORNE P., and KSOURI M., A new hybrid evolutionary approach for the pickup and delivery problem with time windows, IEEE International Conference on Systems, Man and Cybernetic, 2004, Volume 2, Oct. 2004, pp. 1498-1503.

13. KAMMARTI, R., HAMMADI, S., BORNE P., and KSOURI M., Improved tabu search in an hybrid evolutionary approach for the pickup and delivery problem with time windows, Intelligent Transportation Systems, 2005. Proceeding 2005 IEEE, 2005, pp. 148-153.

14. KAMMARTI, R., HAMMADI, S., BORNE P., and KSOURI M. Solving the Real Time dynamic Pickup and Delivery Problem with an hybrid evolutionary approach, Multiconference on Computational Engineering in Systems Application. Volume 2, Oct. 2006, pp. 1520-1525.
15. KAMMARTI, R., BORNE P., and KSOURI M., Hybrid Evolutionary Approaches and new Benchmark for the 1-PDPTW, 2007 IFAC. MCPL.

16. SOL, M. and M. SAVELSBERGH, A branch-and-price algorithm for the pickup and delivery problem with time windows, Memorandum COSOR 94-22, Dept. of Mathematics and Computing Science, Eindhoven University of Technology, Eindhoven, The Netherlands, 1994.

17. QUAN, L. and M. MAGED, A new insertion-based construction heuristic for solving the pickup and delivery problem with time windows, Science direct, European Journal of Operational Research 2003.

18. LI, H. and A. LIM, A metaheuristic for the pickup and delivery problem with time windows, In IEEE International Conference on Tools with Artificial Intelligence, volume 13, 2001, pp. 160–167.

19. LI, H., A. Lim and B. Rodrigues. Solving the pickup and delivery problem with time windows using squeaky wheel optimization with local search, SMU Business Conference Paper Series, 2002.

20. HARBAOUI, I., DRIDI R., KAMMARTI, P. BORNE and M. KSOURI, Un Algorithme génétique pour le problème de ramassage et de livraison avec fenêtres de temps à plusieurs véhicules. CIFA 2008.