A Bi-Gaussian Acoustic Emission Model for Sliding Friction

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Abstract. Acoustic emissions (AEs) generated by the sliding friction has been an effective media for analyzing the tribological interaction between surfaces. The existing AE model has revealed the relationship between the AEs generated by sliding friction and the surface parameters from a single-stratum Gaussian surface perspective, but researches have confirmed that two-process or worn surfaces show bi-Gaussian stratified characteristics. A bi-Gaussian AE model was established through analyzing the elastic deformation energy of the contacting asperities on the faces, which were represented using the bi-Gaussian stratified theory. Two numerically generated surfaces and four real worn surfaces were employed for analysis. The bi-Gaussian AE model and the existing Gaussian AE model, along with the deterministic AE model as the reference, were respectively applied to the surfaces to obtain the AE power variation with separation. The results show that the bi-Gaussian model has good accuracy for all the surfaces involved, exhibiting better stability than the Gaussian model, because the bi-Gaussian model better reflects the real contact status of the surfaces.

1. Introduction

Acoustic emission (AE) refers to the acoustic wave generated by the local energy release of materials. Besides the classical application of material damage detecting [1-3], it has been an effective approach for analyzing the tribological interaction between surfaces, exhibiting excellent sensitivity, information density and engineering convenience. Applications involved machining [4,5], bearings[6-9], and seals[10-15], etc.

In order to analyze the AEs generated by friction, it is imperative to model the correlation between the tribological behavior and the AEs. Boness et al [16] looked into the AE signals generated by friction, establishing an empirical relationship between the integrated RMS and the wear volume. Lingard et al [17] found a linear relationship between the AE count and the wear volume, together with a relationship between the AE count and the friction-pair condition. Although the abovementioned research has revealed the relationship between the friction AE and the tribological behavior, their conclusions were not based on a physical foundation. Fan et al [18] took a step forward by establishing an analytic relationship between the AE RMS and surface topography under the assumption that the energy of the AE arose from the elastic energy release of the asperity contact, modeling the AE RMS model under a slight sliding condition of Gaussian surfaces.

Fan’s work assumed the surface analyzed to be Gaussian. However, the existing research has shown that the two-process [19-21] or worn [22-25] surfaces exhibit the bi-Gaussian stratified characteristic, which means that the surfaces consist of two Gaussian components with different
roughness scales, other than the Gaussian characteristic assumption commonly used. For characterizing a bi-Gaussian surface, the segmented separation method was firstly proposed [19-23,26]. Hu et al [27] proposed a continuous separation method, which has better performance than the segmented separation. The bi-Gaussian characterization tools were employed for practical problems, including establishing elastic asperity contact model [28-29], analysing the cavitation in water-film lubrication [30], and describing the surface topography evolution [24,31-35]. In regard of the AEs, Hu [36] compared the simulated AEs generated by the original surfaces and the reconstructed surfaces with bi-Gaussian stratified theory, showed that the bi-Gaussian characteristics do make difference.

In this study, the bi-Gaussian AE model is established based on the modeling of elastic energy release and the bi-Gaussian stratified theory. Two different forms were used in defining the statistics of the upper strata. Two numerically generated surfaces and four real worn surfaces are analyzed by the Gaussian AE model, the bi-Gaussian model and a deterministic model, respectively.

2. The Gaussian AE model

When under light load, AEs arise from the release of the elastic deformation energy of the contacting asperities during a sliding friction. The energy stored by the summit indexed \( k (k = 1, 2, \ldots, N_s) \) can be expressed as [18]

\[
U_{Ek} = \frac{2}{5} W_k \delta_k.
\]  

Here \( W_k \) refers to the squeezing force on summit \( k \), and \( \delta_k \) refers to the exceeding height of summit \( k \) beyond the matching ideal plane.

The time needed for the release of an individual summit contact \( \tau_k \) can be expressed as [18]

\[
\tau_k = \frac{a_k}{v} = \left( \frac{W_k R_{sk}}{E} \right)^{\frac{1}{3}}. \tag{2}
\]

The equivalent radius \( R_{sk} \) can be yielded from equivalent radius in \( x \) and \( y \) directions [29].

In the following deduction, we made a revision to the existing model, as the contact load and energy release rate of individual summit were not clarified by Fan [18]. According to the Hertz theory, the squeezing force of summit \( k (k = 1, 2, \ldots, N_s) \) can be expressed as

\[
W_k = E \delta_k^2 R_{sk}^{\frac{1}{2}}. \tag{3}
\]

The energy release rate of the individual summit \( k \) is yielded as below:

\[
\dot{U}_{Ek} = \frac{U_{Ek}}{\tau_k} = \frac{2}{5} E \delta_k^2 v. \tag{4}
\]

Under the assumption that the summit height obeys a Gaussian distribution, a statistical model can be established:

\[
\dot{U}_E = \frac{2}{5} EAv \eta_s \int_h^\infty (z_s - h)^2 \varphi_{\mu, s}(z_s) dz_s. \tag{5}
\]

Here \( \varphi_{\mu, s}(\cdot) \) is the PDF (probability density function) of a Gaussian distribution with mean value \( \mu \) and height distribution RMS \( s \):

\[
\varphi_{\mu, s}(x) = \frac{1}{\sqrt{2\pi}s} \exp \left( -\frac{(x - \mu)^2}{2s^2} \right). \tag{6}
\]

By employing a constant \( K \) that serves as the represent of the conversion rate from the elastic energy to the stress wave energy detected, the AE power, represented by the RMS of AE wave, can be expressed as
\[
RMS_G = K \sqrt{U_E} = K \sqrt{\frac{2}{5} E Av \eta_s \int_h^\infty (z_s - h)^2 \varphi_{zsm, \sigma_s}(z_s) dz_s.}
\]  
(7)

This model will be mentioned as the Gaussian AE model.

3. The bi-Gaussian AE model

3.1. Representing surface topography

The probability material ratio (PMR) is an effective representation of surface height distribution. The PMR, which is obtained from the surface height distribution \( z_{i,j} (i = 1, 2, ..., N_x, j = 1, 2, ..., N_y) \), is defined as:

\[
MR(z) = \frac{\text{count}(z_{i,j} > z)}{N_x N_y} = 1 - CDF(z),
\]

(8)

\[
PMR(z) = \sqrt{2 \text{inverf}(2MR(z) - 1)}.
\]

(9)

Here, \( \text{count}(z_{i,j} > z) \) is the count of the nodes with height \( z_{i,j} \) higher than a certain height \( z \). \( \text{inverf}(\cdot) \) is the inverse of error function \( \text{erf}(\cdot) \).

For a Gaussian surface, there is

\[
PMR(z) = \sqrt{2 \text{inverf}}(1 - 2\Phi\left(\frac{Z - z_m}{\sigma_u}\right)).
\]

(10)

\( \Phi(\cdot) \) refers to the CDF of the standard Gaussian distribution

\[
\Phi(x) = 0.5 + 0.5 \text{erf}\left(\frac{x}{\sqrt{2}}\right).
\]

(11)

A bi-Gaussian surface is the mixture (obeying surface combination theory) of two Gaussian surfaces: \( z_u \) with mean height \( z_{mu} \) and height distribution RMS (root-mean-square) \( \sigma_u \); \( z_l \) with mean height \( z_{ml} \) and height distribution RMS \( \sigma_l \). According to the continuous separation method [27], the probability material ratio of the bi-Gaussian surface can be yielded as:

\[
PMR(z) = \sqrt{2 \text{inverf}}\left(2 \left(1 - \Phi\left(\frac{Z - z_{mu}}{\sigma_u}\right)\right) \left(1 - \Phi\left(\frac{Z - z_{ml}}{\sigma_l}\right)\right) - 1\right).
\]

(12)

The undetermined parameters \( z_{mu}, \sigma_u, z_{ml}, \sigma_l \) can be obtained by fitting the measured PMR with Eq. (12).

3.2. Establishing the bi-Gaussian AE model

For a bi-Gaussian stratified surface, if not under a heavy load, the actual contact occurs only on the summits within the upper component [28,29], indicating that the entire-surface summit parameters (i.e., \( \sigma_s \) and \( \eta_s \)) in Eq. (7) should be replaced by those of the upper component, as below:

\[
RMS_{bi-G} = K \sqrt{U_E} = K \sqrt{\frac{2}{5} E Av \eta_{su} \int_h^\infty (z_s - h)^2 \varphi_{zsmu, \sigma_su}(z_s) dz_s.}
\]

(13)

Note that it is the parameters of summit height distribution \( z_{sk}(k = 1, 2, ..., N_s) \) instead of those of surface height distribution \( z_{i,j}(i = 1, 2, ..., N_x, j = 1, 2, ..., N_y) \) that are used here, which is not directly obtained by the original continuous separation method above. Two adjustments are considered in order to define bi-Gaussian summit distribution parameters:

**Definition A**

Find all summits \( z_{sk}(k = 1, 2, ..., N_s) \) on the entire surface, and then fit the summit PMR with Eq. (16) so that \( z_{smu}^A, \sigma_{su}^A, z_{sml}^A \) and \( \sigma_{si}^A \) will be obtained[29].
\[ MR_s(z_s) = \frac{\text{count}(z_{sk} > z_s)}{N_s}, \]  
\[ PMR_s(z_s) = \sqrt{2} \text{inverf}(2MR_s(z_s) - 1), \]  
\[ PMR_s^*(z_s) = \sqrt{2} \text{inverf}\left(2\left(1 - \Phi\left(\frac{z_s - z_{smu}}{\sigma_{su}}\right)\right)\left(1 - \Phi\left(\frac{z_s - z_{sml}}{\sigma_{sl}}\right)\right) - 1\right). \]

The knee point concept from the segmented separation method is still remained, even though there is not an actual corner on the PMR curve yielded by continuous separation method [27]. The knee point for summit distribution is defined below:

\[ r_{sknee} = \frac{z_{smu} - z_{sml}}{\sigma_{su} - \sigma_{sl}}, \]  
\[ z_{sknee} = \frac{z_{sml} \sigma_{su} - z_{smu} \sigma_{sl}}{\sigma_{su} - \sigma_{sl}}. \]

The \( z_{sk} (k = 1, 2, ..., N_s) \) satisfying \( z_{sk} > z_{sknee} \) are considered to belong to the upper component, thus obtaining the summit density \( \eta^A_{zu} \), while the rest belonging to the lower component, obtaining \( \eta^A_{zl} \).

**Definition B**

Divide the surface with the knee point of surface height distribution [27]

\[ r_{knee} = \frac{z_{mu} - z_{ml}}{\sigma_u - \sigma_l}, \]  
\[ z_{knee} = \frac{z_{ml} \sigma_u - z_{mu} \sigma_l}{\sigma_u - \sigma_l}. \]

Then, find all summits \( z_{sk} (k = 1, 2, ..., N_s) \) on the entire surface, and the \( z_{sk} \) satisfying \( z_{sk} > z_{knee} \) are distinguished as the upper component to fit a Gaussian distribution to yield \( z_{smu}^B \) and \( \sigma_{su}^B \), and obtain summit density \( \eta^B_{zu} \), while the rest belonging to the lower component and obtaining \( z_{sml}^B \), \( \sigma_{sl}^B \) and \( \eta^B_{zl} \).

### 4. Results and discussion

The analysis was proceeded on the following six surfaces (see Figure 1): (a) Simulated Gaussian surface; (b) Simulated bi-Gaussian surface; (c) SiC (a measured worn surface, and so were the following surfaces); (d) TC; (e) RiC; (f) MiC. Surfaces (c) to (f) were real worn surfaces of a mono-spring face seal, each of which had undergone a 24h running-in and a 100h service, and surface (b) was a simulated bi-Gaussian surface yielded by bi-Gaussian reconstruction[27]. Surface (a) was a simulated Gaussian surface reconstructed for reference.

The PMR curves of the target surfaces above were calculated in terms of the surface height and summit height, respectively, as shown in Figure 2. It can be seen that the PMR curves of the Gaussian surface (a) exhibit notable linear feature while those of the real worn surfaces do not. Instead, the PMR curves of real worn surfaces exhibit characteristic similar to that of the simulated bi-Gaussian surface, that is, consisting of two connecting sloping line (or being bilinear) approximately, revealing that the real worn surfaces do consist of two strata and should be characterized using bi-Gaussian stratified theory rather than a traditional entire-surface statistic. Then, the continuous separation method was applied to the surface height PMR and summit height PMR, respectively. The results are displayed in Figure 2, and listed in Table 1.

After obtaining surface statistics, the Gaussian model and bi-Gaussian model, along with a deterministic model as a reference, are applied to the surfaces above. The deterministic model can be simply described as summing up the \( \hat{U}_{Ek} \) in Eq. (4). The dimensionless AE RMS, defined as

\[ RMS^* = \frac{RMS}{\sqrt{EAv}} \]

is yielded and illustrated in Figure 3.
**Figure 1.** The analyzed surfaces

**Table 1.** Bi-Gaussian surface statistics

| Surface data     | (b)     | (c)     | (d)     | (e)     | (f)     |
|------------------|---------|---------|---------|---------|---------|
| Surface          |         |         |         |         |         |
| $\sigma_{\mu}/\mu m$ | 0.429   | 0.092   | 0.034   | 0.064   | 0.199   |
| $\sigma_{u}/\mu m$  | 0.1009  | 0.0548  | 0.0115  | 0.0353  | 0.0488  |
| $z_{mu}/\mu m$     | 0.1979  | 0.0419  | 0.0025  | 0.0090  | 0.0379  |
| $\sigma_{l}/\mu m$  | 1.0570  | 0.1985  | 0.4246  | 0.3513  | 1.4473  |
| $z_{ml}/\mu m$     | 0.7697  | 0.1421  | 0.8507  | 0.5364  | 2.2888  |
| $r_{knee}$         | 0.5981  | 0.6973  | 2.0534  | 1.6690  | 1.6095  |
| $z_{knee}/\mu m$   | 0.1375  | 0.0037  | -0.0211 | -0.0499 | -0.0406 |
| Summit            |         |         |         |         |         |
| $\sigma_{\mu}/\mu m$ | 0.276   | 0.067   | 0.015   | 0.048   | 0.172   |
| $z_{sm}/\mu m$     | 0.161   | 0.058   | 0.013   | 0.029   | 0.039   |
| $\eta/(\mu m)^{-2}$ | 0.491   | 0.110   | 0.136   | 0.107   | 0.100   |
| Entire-surface    |         |         |         |         |         |
| $\sigma/\mu m$     | 0.0986  | 0.0569  | 0.0080  | 0.0406  | 0.0524  |
| $z_{smu}/\mu m$    | 0.2416  | 0.0780  | 0.0130  | 0.0338  | 0.0634  |
| $\eta/(\mu m)^{-2}$ | 0.3775  | 0.0959  | 0.1314  | 0.1043  | 0.0911  |
| Summit            |         |         |         |         |         |
| Definition A      |         |         |         |         |         |
| $\sigma_{s}/\mu m$ | 1.0580  | 0.1617  | 0.2065  | 0.5061  | 1.6785  |
| $z_{sm}/\mu m$     | 1.3558  | 0.2252  | 0.4775  | 1.0856  | 3.0428  |
| $\eta/(\mu m)^{-2}$ | 0.1132  | 0.0142  | 0.0046  | 0.0027  | 0.0085  |
| $r_{s,knee}$      | 1.1612  | 1.4052  | 2.3398  | 2.2599  | 1.8322  |
| $z_{s,knee}/\mu m$ | 0.1271  | -0.0021 | -0.0057 | -0.0580 | -0.0326 |
| Summit            |         |         |         |         |         |
| Definition B      |         |         |         |         |         |
| $\sigma_{s}/\mu m$ | 0.1052  | 0.0603  | 0.0104  | 0.0412  | 0.0543  |
| $z_{sm}/\mu m$     | 0.2194  | 0.0610  | 0.0114  | 0.0322  | 0.0589  |
| $\eta/(\mu m)^{-2}$ | 0.3671  | 0.0938  | 0.1347  | 0.1038  | 0.0917  |
| $\sigma_{s}/\mu m$ | 1.2482  | 0.1736  | 0.6372  | 0.8851  | 2.9599  |
| $z_{sm}/\mu m$     | 1.0133  | 0.1412  | 0.2629  | 0.4338  | 1.6467  |
| $\eta/(\mu m)^{-2}$ | 0.1237  | 0.0163  | 0.0013  | 0.0033  | 0.0079  |
Figure 2. The PMR curves of the analyzed surfaces

For the Gaussian surface (a), the Gaussian model and the deterministic model agrees perfectly. However, for the simulated bi-Gaussian surface (b), the Gaussian model yields obviously greater AE power, while the result of the bi-Gaussian model, with either upper strata statistic definition A or B, shows perfect agreement with that of the deterministic model. This significant error of the Gaussian model is caused by taking the entire-surface statistics to analyze a friction process which only involves the higher summits, whose height distribution RMS is quite smaller than that of the entire surface. However, the bi-Gaussian model can correctly characterize the distribution of the summits involved.

For the real worn surfaces (c), (d), (e) and (f), analysis shows that:

1) The deterministic model and the bi-Gaussian model result in similar results for either the surface (d) or (f), while the result of the Gaussian model shows obvious error.
The Gaussian model and the bi-Gaussian model both agree with the deterministic model for surfaces (c) and (e). The Gaussian model doesn’t show an obvious error like what it shows for surfaces (b), (d) and (f), because that the two strata of either surface (c) or (e) do not differ significantly.

(3) For surface (e), both the Gaussian model and the bi-Gaussian model failed to capture the AE characteristics accurately, because of the relatively poor regularity of the highest summits.

As for the comparison of the two upper strata statistic definitions, the bi-Gaussian model performs well no matter using either A or B, but the definition A leads to slightly better performance. Therefore, the definition A is more recommended.

Figure 3. Dimensionless AE RMS
5. Conclusion
A bi-Gaussian model was established based on the existing Gaussian model and bi-Gaussian stratified theory. As a reference, a deterministic model is also employed.

The AE characteristics of several simulated surfaces and real worn surfaces are analyzed. The following can be concluded from the results:

1. The bi-Gaussian model exhibits good accuracy for the bi-Gaussian surfaces, including the simulated bi-Gaussian surface and the real surfaces worn in experiments, thus exhibiting good stability.

2. Although the existing Gaussian model shows small error for surfaces which don’t possess strong stratified characteristic, the error, brought by the ignorance of stratified characteristic, increases with the enlarging disparity between strata, which means that a bi-Gaussian modeling for the AEs generated from bi-Gaussian surfaces is necessary.

3. Between the two potential definitions for the upper strata statistics, the one which define the statistics by directly fitting the probability material ratio is more recommended to be used in the bi-Gaussian model.

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