**Abstract**

Theoretical $g$-factor calculations for the first excited $^2P_{3/2}$ state of boronlike ions in the range $Z=10-20$ are presented and compared to the previously published values. The first-order interelectronic-interaction contribution is evaluated within the rigorous QED approach in the effective screening potential. The second-order contribution is considered within the Breit approximation. The QED and nuclear recoil corrections are also taken into account.
I. INTRODUCTION

Significant progress in the \( g \)-factor studies in highly charged ions has been achieved in the last two decades [1, 2]. Contemporary experiments have reached the precision of \( 10^{-9} - 10^{-11} \) for hydrogenlike and lithiumlike ions [3–7]. One of the highlights in this field is the most accurate determination of the electron mass from the combined experimental and theoretical studies of the \( g \) factor of hydrogenlike ions [8]. Extension of these studies to lithiumlike ions has provided the stringent test of the many-electron QED effects [7, 9–11]. The high-precision \( g \)-factor measurement of the two isotopes of lithiumlike calcium [10] and the most elaborate evaluation of the nuclear recoil effect for this system [12] have demonstrated a possibility to study the bound-state QED effects beyond the Furry picture in the strong field regime [13]. It is expected that \( g \)-factor studies in few-electron ions will be able to provide an independent determination of the fine structure constant \( \alpha \) [14–16].

The ALPHATRAP experiment at the Max-Planck-Institut für Kernphysik (MPIK) is capable of the ground-state \( g \)-factor measurements for wide range of few-electron ions, including boronlike ones [1]. The ARTEMIS project at GSI implements the laser-microwave double-resonance spectroscopy of the Zeeman splitting in both ground \( [(1s)^2(2s)^22p] \, ^2P_{1/2} \) and first excited \( [(1s)^2(2s)^22p] \, ^2P_{3/2} \) states of middle-\( Z \) boronlike ions [17, 18]. In particular, boronlike argon is chosen as the first candidate for these measurements. Theoretical investigations of the \( g \) factor of boronlike ions were performed recently in Refs. [19–24]. Various methods have been used in these works for evaluation of the interelectronic-interaction contribution, including the large-scale configuration-interaction approach in the basis of the Dirac-Fock-Sturm orbitals (CI-DFS) [19, 21], the GRASP2K [20] and MCDFGME [22] packages based on relativistic multi-configuration Dirac-Hartree-Fock (MCDHF) method, the second-order perturbation theory (PT) in effective screening potential [21, 23], and the high order coupled cluster (CC) method [24]. For the ground-state \( g \) factor of boronlike argon the results of the CI-DFS, PT, and CC approaches are in agreement, while the both MCDHF results reveal a deviation on the level of \( 10^{-4} \). In the present work, we extend the second-order perturbation-theory calculations to the \( ^2P_{3/2} \) state. The QED and nuclear recoil corrections are also taken into account. The results for boronlike ions in the range \( Z=10–20 \) are presented and compared to the previously published values [19, 22, 24]. We use the relativistic units \( (\hbar = c = 1) \) and the Heaviside charge unit \( (\alpha = e^2/(4\pi), e < 0) \).
II. METHODS AND RESULTS

The total $g$-factor value of boronlike ion with zero nuclear spin can be written as

$$g = g_D + \Delta g_{\text{int}} + \Delta g_{\text{QED}} + \Delta g_{\text{rec}} + \Delta g_{\text{NS}} , \tag{1}$$

where $\Delta g_{\text{int}}$, $\Delta g_{\text{QED}}$, $\Delta g_{\text{rec}}$, and $\Delta g_{\text{NS}}$ are the interelectronic-interaction, QED, nuclear recoil, and nuclear size corrections, respectively. The Dirac value $g_D$ for the $2p_{3/2}$ state is

$$g_D = \frac{4}{15} \left[ 2\sqrt{4 - (\alpha Z)^2} + 1 \right] = \frac{4}{3} - \frac{2}{15} (\alpha Z)^2 - \ldots . \tag{2}$$

The interelectronic-interaction correction is considered within the perturbation theory. The first-order term $\Delta g_{\text{int}}^{(1)}$ (one-photon exchange) is calculated within the rigorous QED approach, i.e., to all orders in $\alpha Z$. The second-order term $\Delta g_{\text{int}}^{(2)}$ (two-photon exchange) is considered within the Breit approximation. The general formulae for this contribution can be found from the complete quantum electrodynamical formulae for the two-photon-exchange diagrams presented in Ref. [25]. Care should be taken to account properly for the contribution of the negative-energy states, since it is comparable in magnitude to the positive-energy counter-part.

We incorporate the effective screening potential in the zeroth-order approximation. This improves the convergence of the perturbation theory and provides a reliable estimation of the higher-order remainder. The corresponding counter-terms should be considered in calculations of the first- and second-order contributions. The difference between the $g$-factor values in the screening and pure Coulomb potentials is termed as the zeroth-order contribution $\Delta g_{\text{int}}^{(0)}$. We use the following well-known screening potentials: core-Hartree (CH), Dirac-Hartree (DH), Kohn-Sham (KS), and Dirac-Slater (DS), see, e.g., Ref. [26] for more details.

In Table I we present the interelectronic-interaction contributions to the $g$-factor multiplied by $10^6$. The total value of $\Delta g_{\text{int}}$ is found as,

$$\Delta g_{\text{int}} = \Delta g_{\text{int}}^{(0)} + \Delta g_{\text{int}}^{(1)} + \Delta g_{\text{int}}^{(2)} , \tag{3}$$

where the first-order correction $\Delta g_{\text{int}}^{(1)}$ is divided into the following three parts:

$$\Delta g_{\text{int}}^{(1)} = \Delta g_{\text{int}}^{(1)[+]} + \Delta g_{\text{int}}^{(1)[-]} + \Delta g_{\text{int}}^{(1)\text{[QED]}} . \tag{4}$$
The positive-energy-states ($\Delta g_{\text{int}}^{(1)[+]}$) and negative-energy-states ($\Delta g_{\text{int}}^{(1)[−]}$) contributions are calculated in the Breit approximation. The QED contribution ($\Delta g_{\text{int}}^{(1)\text{[QED]}}$) is the difference between the rigorous QED and the Breit-approximation values.

As the final results for $\Delta g_{\text{int}}$, we take the values calculated in the Kohn-Sham potential. The uncertainty due to unknown higher-order contributions can be estimated as the spread of the obtained results for different potentials. As one can see from the Table I, the maximal difference of the values of $\Delta g_{\text{int}}$ varies between $1.8 \times 10^{-6}$ for $Z=10$ and $0.8 \times 10^{-6}$ for $Z=20$. Interelectronic-interaction corrections of the third and higher orders have been evaluated for lithiumlike ions within the CI-DFS [9] and CI [11] methods. The results obtained in these papers suggest that this estimation of the uncertainty is quite reliable.

The one-loop QED correction $\Delta g_{\text{QED}}^{(1)}$ is given by the sum of the self-energy and vacuum-polarization contributions,

$$\Delta g^{(1)}_{\text{QED}} = \Delta g_{\text{SE}} + \Delta g_{\text{VP}}.$$  \hspace{1cm} (5)

The self-energy correction was calculated to all orders in $\alpha Z$ for both $2p_{1/2}$ and $2p_{3/2}$ states in the range $Z=1$–$12$ in Ref. [27]. This values can be extrapolated to a good accuracy by the following $\alpha Z$-expansion [27, 28],

$$\Delta g_{\text{SE}} = \frac{\alpha}{\pi} \left[ b_{00} + \frac{(\alpha Z)^2}{4} b_{20} + \frac{(\alpha Z)^4}{8} \{\ln[(\alpha Z)^{-2}]b_{41} + b_{40}\} \right].$$  \hspace{1cm} (6)

The values $b_{00}(2p_{1/2}) = −1/3$ and $b_{00}(2p_{3/2}) = 1/3$ have long been known [29, 30]. The values $b_{20}(2p_{1/2}) = 0.48429$ and $b_{20}(2p_{3/2}) = 0.59214$ have been found in Ref. [28]. Our fitting procedure based on the least squares method reproduces these coefficients if they are taken as unknown, which serves as a check of its consistency. In this way we extrapolate the results of Ref. [27] up to $Z=20$. In addition, we estimate the screening correction for the $2p_{3/2}$ state employing the effective nuclear charge $Z_{\text{eff}}$ instead of $Z$ in Eq. (6). The effective nuclear charge $Z_{\text{eff}}$ is found from our rigorous calculations of the self-energy correction for the $2p_{1/2}$ state with an effective screening potential [23]: Eq. (6) with $Z_{\text{eff}}$ should reproduce the result obtained with the Kohn-Sham potential. The screening shift $Z - Z_{\text{eff}}$ lies in the range 1.3–1.7 for the ions under consideration. We ascribe the 100% uncertainty to the screening correction obtained in this rather approximate way.

The dominant contribution of the vacuum polarization is given by the two-electron diagrams where the vacuum-polarization potential acts on the $1s$ and $2s$ electrons. This contribution was estimated as $5.5 \times 10^{-9}$ for $Z=18$ in Ref. [19], which is much smaller than
the total theoretical uncertainty. The two-loop contribution $\Delta g_{\text{QED}}^{(2)}$ is represented by its zeroth-order term of the $\alpha Z$-expansion \[30\].

The nuclear recoil effect in boronlike argon was calculated in Refs. \[19, 21\] within the Breit approximation to zeroth and first orders in $1/Z$. Systematic calculations of this effect for the $2p_{1/2}$ state in the range $Z=10–20$ were performed in Ref. \[31\]. Recently, these calculations have been extended to $Z=20–92$ including the leading-order QED contributions beyond the Breit approximation \[32\]. In the present paper, we evaluate this effect for the $2p_{3/2}$ state with the relativistic recoil operators to zeroth order in $1/Z$ with the Kohn-Sham effective screening potential. The leading-order term of the finite-nuclear-size correction can be written as \[33\]

$$\Delta g_{\text{NS}} = \frac{(\alpha Z)^6}{720} m^4 \langle r^4 \rangle. \quad (7)$$

For $Z=10–20$ it gives the values of the order $10^{-18}–10^{-16}$ which is negligible at the present level of accuracy.

The individual contributions and the total $g$-factor values for the $2p_{3/2}$ state of boronlike ions in the range $Z=10–20$ are presented in Table \[II\]. The values of $\Delta g_{\text{int}}$ calculated in the Kohn-Sham potential are used. Our results for argon are in agreement with the PT results from Refs. \[19, 21\] and with the CC results from Ref. \[24\]. The difference between the data from Ref. \[20\] and those of the present work ranges from $0.000042$ for $Z=10$ to $0.000094$ for $Z=20$. The difference between the data from Ref. \[22\] and those of the present work ranges from $0.000067$ for $Z=14$ to $0.000102$ for $Z=20$. The origin of this disagreement is not clear at present. We suppose that the negative-energy-states contribution was not taken into account completely in Refs. \[20, 22\].

Zeeman splitting of the $2p_j$ states acquires significant nonlinear contributions. In particular, the second- and third-order terms in magnetic field can be observed in forthcoming measurements for boronlike argon \[17, 19\]. Recently, the systematic calculations of these terms for the wide range of boronlike ions have been presented by our group \[34\]. The most important contribution for the $2p_{3/2}$ state is the shift of the levels with $m_j = \pm 1/2$ proportional to $B^2$. It can be represented as the $m_j$-dependent $g$-factor contribution varying from $\pm 1.52 \times 10^{-4}$ for $Z=10$ to $\pm 5.68 \times 10^{-6}$ for $Z=20$ at the field of 1 T (it scales linearly with $B$). For more detailed description of the second- and third-order contributions see Ref. \[34\].
III. CONCLUSION

In conclusion, the $g$ factor of the $^2P_{3/2}$ state of boronlike ions in the range $Z=10–20$ has been evaluated with an uncertainty on the level of $10^{-6}$. The leading interelectronic-interaction correction has been calculated to all orders in $\alpha Z$. The higher-order interelectronic-interaction and nuclear-recoil effects have been taken into account within the Breit approximation. The one-loop self-energy correction has been found from extrapolation of the previously published high-precision results for $Z=1–12$ with an approximate account for screening.

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Table I: Interelectronic-interaction correction to the $g$ factor of boronlike ions in the $^{2}P_{3/2}$ state.

The terms of the zeroth ($\Delta g_{\text{int}}^{(0)}$), first ($\Delta g_{\text{int}}^{(1)}$), and second ($\Delta g_{\text{int}}^{(2)}$) orders of perturbation theory obtained with the core-Hartree (CH), Dirac-Hartree (DH), Kohn-Sham (KS), and Dirac-Slater (DS) screening potentials. First-order term is split into the contributions of the positive-energy ($\Delta g_{\text{int}}^{(1)+}$) and negative-energy ($\Delta g_{\text{int}}^{(1)-}$) spectra calculated within the Breit approximation and the QED part ($\Delta g_{\text{int}}^{(1)[\text{QED}]}$). All numbers are in units of $10^{-6}$.

|        | CH     | DH     | KS     | DS     |
|--------|--------|--------|--------|--------|
|        | $Z = 10$ |        |        |        |
| $\Delta g_{\text{int}}^{(0)}$ | 302.983 | 376.227 | 312.152 | 276.157 |
| $\Delta g_{\text{int}}^{(1)+}$ | −22.611 | −117.851 | −34.824 | 17.678  |
| $\Delta g_{\text{int}}^{(1)-}$ | −22.371 | −2.806  | −19.623 | −29.741 |
| $\Delta g_{\text{int}}^{(1)[\text{QED}]}$ | −0.141  | −0.111  | −0.141  | −0.158  |
| $\Delta g_{\text{int}}^{(2)}$ | 6.256   | 6.889   | 6.580   | −0.758  |
| $\Delta g_{\text{int}}$ | 264.116 | 262.348 | 264.143 | 263.178 |
|        | $Z = 12$ |        |        |        |
| $\Delta g_{\text{int}}^{(0)}$ | 368.346 | 462.044 | 379.402 | 334.176 |
| $\Delta g_{\text{int}}^{(1)+}$ | −27.393 | −148.063 | −41.786 | 22.005  |
| $\Delta g_{\text{int}}^{(1)-}$ | −29.402 | −4.746  | −26.187 | −38.614 |
| $\Delta g_{\text{int}}^{(1)[\text{QED}]}$ | −0.281  | −0.233  | −0.282  | −0.308  |
| $\Delta g_{\text{int}}^{(2)}$ | 6.288   | 7.275   | 6.400   | −0.441  |
| $\Delta g_{\text{int}}$ | 317.559 | 316.277 | 317.547 | 316.819 |
|        | $Z = 14$ |        |        |        |
| $\Delta g_{\text{int}}^{(0)}$ | 433.852 | 547.842 | 446.756 | 392.322 |
| $\Delta g_{\text{int}}^{(1)+}$ | −32.164 | −178.105 | −48.754 | 26.389  |
| $\Delta g_{\text{int}}^{(1)-}$ | −36.370 | −6.680  | −32.691 | −47.407 |
| $\Delta g_{\text{int}}^{(1)[\text{QED}]}$ | −0.489  | −0.420  | −0.492  | −0.528  |
| $\Delta g_{\text{int}}^{(2)}$ | 6.306   | 7.516   | 6.289   | −0.265  |
| $\Delta g_{\text{int}}$ | 371.135 | 370.154 | 371.108 | 370.511 |
|                | CH       | DH       | KS       | DS       |
|----------------|----------|----------|----------|----------|
| **Z = 16**     |          |          |          |          |
| $\Delta g^{(0)}_{\text{int}}$ | 499.514  | 633.710  | 514.250  | 450.613  |
| $\Delta g^{(1)}_{\text{int}}[+]$ | $-36.888$ | $-208.003$ | $-55.691$ | $30.839$ |
| $\Delta g^{(1)}_{\text{int}}[-]$ | $-43.298$ | $-8.610$  | $-39.151$ | $-56.142$ |
| $\Delta g^{(1)}_{\text{int}}[\text{QED}]$ | $-0.780$  | $-0.686$  | $-0.783$  | $-0.833$  |
| $\Delta g^{(2)}_{\text{int}}$ | 6.315    | 7.682    | 6.204    | $-0.166$ |
| $\Delta g_{\text{int}}$ | 424.863  | 424.092  | 424.828  | 424.311  |
| **Z = 18**     |          |          |          |          |
| $\Delta g^{(0)}_{\text{int}}$ | 565.355  | 719.702  | 581.912  | 509.070  |
| $\Delta g^{(1)}_{\text{int}}[+]$ | $-41.544$ | $-237.766$ | $-62.572$ | $35.370$ |
| $\Delta g^{(1)}_{\text{int}}[-]$ | $-50.195$ | $-10.541$ | $-45.575$ | $-64.829$ |
| $\Delta g^{(1)}_{\text{int}}[\text{QED}]$ | $-1.167$  | $-1.043$  | $-1.171$  | $-1.235$  |
| $\Delta g^{(2)}_{\text{int}}$ | 6.316    | 7.804    | 6.132    | $-0.116$ |
| $\Delta g_{\text{int}}$ | 478.765  | 478.155  | 478.726  | 478.259  |
| **Z = 20**     |          |          |          |          |
| $\Delta g^{(0)}_{\text{int}}$ | 631.397  | 805.864  | 532.824  | 567.713  |
| $\Delta g^{(1)}_{\text{int}}[+]$ | $-46.116$ | $-267.395$ | $-69.382$ | $39.996$ |
| $\Delta g^{(1)}_{\text{int}}[-]$ | $-57.065$ | $-12.472$ | $-51.964$ | $-73.472$ |
| $\Delta g^{(1)}_{\text{int}}[\text{QED}]$ | $-1.661$  | $-1.507$  | $-1.668$  | $-1.747$  |
| $\Delta g^{(2)}_{\text{int}}$ | 6.311    | 7.896    | 6.065    | $-0.101$ |
| $\Delta g_{\text{int}}$ | 532.866  | 532.386  | 532.824  | 532.134  |
Table II: Individual contributions to the $g$ factor of the $^2P_{3/2}$ state of boronlike ions in the range $Z=10–20$. The values obtained with the Kohn-Sham potential are used for the interelectronic-interaction correction $\Delta g_{\text{int}}$ (see Table I). The $g$-factor values from Refs. [20–22, 24] are given for comparison.

| Ion  | $2_{10}^\text{Ne}^{5+}$ | $2_{12}^\text{Mg}^{7+}$ |
|------|-------------------------|-------------------------|
| Dirac value $g_D$ | 1.332623 079 | 1.332310 417 |
| Interelectronic interaction $\Delta g_{\text{int}}$ | 0.000264 1 (18) | 0.000317 5 (13) |
| One-loop QED $\Delta g_{\text{QED}}^{(1)}$ | 0.000775 7 (5) | 0.000776 3 (7) |
| Two-loop QED $\Delta g_{\text{QED}}^{(2)}$ | −0.000001 2 | −0.000001 2 |
| Nuclear recoil $\Delta g_{\text{rec}}$ | −0.000008 9 (15) | −0.000007 8 (11) |
| Total value $g$ | 1.333652 8 (23) | 1.333394 1 (17) |
| $g$ from Ref. [20] | 1.333695 | 1.333448 |

| Ion  | $2_{14}^\text{Si}^{9+}$ | $3_{16}^\text{S}^{11+}$ |
|------|-------------------------|-------------------------|
| Dirac value $g_D$ | 1.331940 789 | 1.331544 136 |
| Interelectronic interaction $\Delta g_{\text{int}}$ | 0.000371 1 (10) | 0.000424 8 (8) |
| One-loop QED $\Delta g_{\text{QED}}^{(1)}$ | 0.000777 2 (9) | 0.000778 2 (10) |
| Two-loop QED $\Delta g_{\text{QED}}^{(2)}$ | −0.000001 2 | −0.000001 2 |
| Nuclear recoil $\Delta g_{\text{rec}}$ | −0.000006 8 (8) | −0.000006 1 (6) |
| Total value $g$ | 1.333081 1 (16) | 1.332709 8 (14) |
| $g$ from Ref. [20] | 1.333143 | 1.332783 |
| $g$ from Ref. [22] | 1.333148 (7) | 1.332788 (8) |
|                      | \(^{40}\text{Ar}^{13+}\) | \(^{40}\text{Ca}^{15+}\) |
|----------------------|---------------------------|---------------------------|
| Dirac value \(g_D\)  | 1.331 030 389             | 1.330 489 471             |
| Interelectronic interaction \(\Delta g_{\text{int}}\) | 0.000 478 7 (6)          | 0.000 532 8 (7)          |
| One-loop QED \(\Delta g_{\text{QED}}^{(1)}\) | 0.000 779 5 (12)         | 0.000 780 9 (13)         |
| Two-loop QED \(\Delta g_{\text{QED}}^{(2)}\) | -0.000 001 2 (1)         | -0.000 001 2 (1)         |
| Nuclear recoil \(\Delta g_{\text{rec}}\) | -0.000 004 9 (4)         | -0.000 004 9 (4)         |
| Total value \(g\)    | 1.332 282 5 (14)          | 1.331 797 1 (15)          |
| \(g\) from Ref. [20] | 1.332 365                 | 1.331 891                 |
| \(g\) from Ref. [22] | 1.332 372 (1)             | 1.331 899 (7)             |
| \(g\) from Ref. [21] | 1.332 282 (3)             |                           |
| \(g\) from Ref. [24] | 1.332 286                 |                           |