Sinusoidal motion of small particles through a Darcy-Brinkman-Forchheimer microchannel filled with non-Newtonian fluid under electro-osmotic forces

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ABSTRACT
This article deals with the electro-osmotic flow of non-Newtonian fluid in the presence of small particles moving in the sinusoidal form in a Darcy-Brinkman-Forchheimer medium. A transverse magnetic field is applied by ignoring the effects of an induced magnetic field with a small magnetic Reynolds number. To examine the non-Newtonian effects, Jeffrey fluid model is considered. The proposed mathematical modeling is based on the Poisson–Boltzmann equation, Debye length approximation, and ionic Nernst Planck equation. The modeled problem is formulated in a fixed frame, which is further transformed into a wave frame. The transport mechanism of positive ions in bordering fluid of negatively charged surface, fluid is also drawn with positive ions which are moving under the impact of the enforced electric field. Due to the superabundance of positive ions in bordering fluid of negatively charged surface, fluid is also drawn with positive ions which are moving under the impact of the enforced electric field. Electro-osmosis has a broad spectrum of applications in the medical and industrial fields. The phenomenon of peristalsis has attracted great attention of many researchers in view of its broad range of applications in the medical and industrial fields. Peristalsis is the expansion and compressions of the non-extendable tube in fluid produces waves which propagates with the extension of the tube. In the medical field, peristalsis plays a major role in the conveyance of urine from kidney to urinary urocyst via ureter, conveyance of gall from liver to the gall bladder, movement of female reproductive cells in the uterine duct, vasomotion of veins, etc. Many of the machines including blood pumps, heart-lung and dialysis machines are designed on the basis of peristalsis. In view of these applications, many authors undertook experimental and theoretical work. For instance, Mahmoud et al. [1] determined the consequences of the magnetic field on the peristaltic transportation of Jeffrey fluid through a porous medium. The effects of MHD with negligibly small Reynolds number and prolonged wavelength on the peristaltic propagation of non-Newtonian fluid through a rectangular tube were studied by Ellahi et al. [2]. Kothandapani and Srinivas [3] discussed the impact of the magnetic field on the peristaltic propagation of Jeffrey fluid. A parametrical study is carried out to compute the values of axial velocity, pressure gradient, and stream function. Tripathi et al. [4] studied the increment and decrement in EDL thickness by considering unsteady viscous fluid under the merged impacts of electro-osmosis and peristalsis. Hasona et al. [5] examined the effective methods to handle deadly diseases like cancer. They considered the combined impact of thermal radiation and MHD on nanofluid peristaltic flow. Zeeshan et.al [6] considered the peristaltic transportation of non-Newtonian Jeffrey fluid and studied the effect of MHD bi-phase flow. Pandey et al. [7] investigated the unsteady peristaltic procreation of particle-fluid hybridization and presented the conduction of particle-fluid that is engulfing through the gullet. Some important studies on the peristaltic in different geometrical configurations with different flow conditions are mentioned in the references [8–10].

Electro-osmosis is the motion of liquid that is bordering on the electrically charged walls in consequence of the enforced electric field. Due to the superabundance of positive ions in bordering fluid of negatively charged surface, fluid is also drawn with positive ions which are moving under the impact of the enforced electric field. Electro-osmosis has a broad spectrum of applications in biomedicine and biotechnology. Guo and Qi [11] analytically solved the electro-osmotic peristaltic percolation of Jeffrey fluid through a microchannel. They used retardation and relaxation time to express the visco-elasticity of the fluid. Entropy formation in a
microchannel inspired by a consistent peristaltic stream under the effect of the electrokinetic force and the magnetic field was explored by Ranjit and Shit [12]. Moatim et al. [13] probed the impression of Brownian and thermophoresis parameters of non-newtonian fluid in a microchannel through peristalsis and electro-osmotic phenomena. Few important studies related with the proposed topic can be found from the given references [14–22].

The ideology of particle-fluid hybridization is very practical in multiple physical problems like fluidization, the transmission of solid particles along with fluid, transport of liquid suspensions in chemical methods, etc. The accumulation of particles in the fluid has a vast range of applications in many engineering processes. There are many examples of particle-fluid abeyance in that can be observed in daily life such as in regular drinks contain gases, drinking water, and also the air which we breathe contains particles. Misra and Pandey [23] analyzed the peristaltic motion of particle fluid blending in a cylinder-shaped channel. The fluid in this case is viscous and incompressible. Sinnot et al. [24] made a relation of fluid transfer in the human bowel with peristaltic compression using smoothed particle hydrodynamics. Gad [25] considered the viscous inelastic fluid under the impact of the transversal magnetic field and explored the consequence of Hall current on the synergy of pulsating and peristaltic transport induced flow of the particle-fluid mixture. An investigation of slip impacts on the peristaltic actuation of viscous incompressible fluid between the walls of a 2D channel by Kamel et al. [26]. Bhatti and Zeeshan [27] explored the results of the fluctuating magnetic field and endoscopy on the blood flow of particle fluid interruption through an annulet based on peristalsis. Bhatti et al. [28] considered the particle-fluid mixture of blood flow that is generated by the peristaltic wave and studied the slip impacts and endoscopic survey. The flow passes through a non-uniform annulus. Kaimal [29] considered a dusty Newtonian fluid in which particles are suspended, and studied the peristaltic pumping flow between rigid plates of an axisymmetric wavy shaped tube.

Many problems are difficult and cannot be solved to obtain the exact solution. In order to break this complexity a technique popularly known as Homotopy Perturbation Method (HPM) was introduced. This method works in such a manner that it first reduces the non-linear problem into a set of linear equations along with boundary conditions and then obtains the closed-form solution. It can be used even without linearization. The HPM method is beneficial to solve nonlinear differential equations and provide better analytical results compared with other methods [30–33].

In the proposed model a parametric study is conducted on the electro-osmotic flow of particle fluid through a microchannel with the help of Darcy-Brinkman-Forchheimer medium. The nonlinear mathematical modelling is solved by using HPM. The Darcian and non-Darcian effects are examined in a microchannel. The mathematical outcomes for pressure gradient, velocity, and wall shear stress are obtained. Impacts of different parameters a presented with figures.

2. Mathematical framework

Let us consider the peristaltic actuation of electro-osmotic dusty abeyance of an incompressible viscoelastic fluid in a two-dimensional dissymmetric channel. The geometry of the flow is being depicted by using the Cartesian coordinate scheme. The X-axis is in the direction of the flow and the Y-axis is in a vertical direction which is normal to it. An effective stable uniform magnetic field is applied in the opposite direction with magnetic flux density. The induced magnetic field is omitted by putting on an insignificant magnetic Reynolds number. The pictorial depiction of the flow mechanism is shown in Figure 1 and mathematics of the geometry is given as:

\[ h_1(X, t) = l_1 + s_1 \cos^2 \left( \frac{\pi}{\lambda} (X - c_w t) \right), \]
\[ h_2(X, t) = -l_2 - r_1 \cos^2 \left( \frac{\pi}{\lambda} (X - c_w t) + \phi \right), \]

where \( s_1 \) and \( r_1 \) are the wave amplitudes of top and bottom walls respectively, \( \lambda \) symbolizes the wavelength, \( c_w \) the wave speed, \( t \) represents the time, and \( \phi \) is the phase difference, and \( X \) is the route in which the wave propagates. Fluid taken through the microchannel is viscoelastic Jeffrey fluid and will be examined under the mix impacts of peristalsis and electro-osmosis. The ruling equations of motion for both the phases are defined as:

2.1. Fluid-phase

\[ \nabla \cdot \mathbf{U}_f = 0, \]
\[ \left[ \frac{\partial \mathbf{U}_f}{\partial t} + \mathbf{U}_f (\mathbf{U}_f \cdot \nabla) \right] = -\frac{1}{\rho_f} \nabla P + \frac{1}{\rho_f} \nabla \cdot \tau + \frac{1}{(1 - c) \rho_f} \]

Figure 1. A diagrammatic illustration of the asymmetric flow mechanism.
\begin{equation}
\times \left( cs(U_p - U_f) + F_b - \frac{\mu}{K} U_f - \frac{C_p \rho_f K^2}{K^2} U_f^2 \right). \tag{4}
\end{equation}

2.2. Particulate-phase

\begin{align}
\nabla \cdot U_p &= 0, \\
\frac{\partial U_p}{\partial t} + U_p (U_p \cdot \nabla) U_p + \frac{1}{\rho_p} \nabla P &= \frac{1}{\rho_p} \left[ cs(U_f - U_p) + F_b \right]. \tag{6}
\end{align}

In which \(U_f(= U_f, V_f), U_p(= U_p, V_p), \rho_f, \rho_p, \rho, c, \mu, c_f, K \) and \( \tau \) are the velocity speed of the viscoelastic fluid, velocity speed of the particles, the density of the material that makes up the fluid, particle density, pressure, particle volume fraction, fluid dynamic viscosity, Forchheimer coefficient, porous medium permeability, and extra stress tensor of the Jeffrey fluid model respectively.

The whole physical force per unit quantity operating on the fluid \(F_b\) is expressed as:

\begin{equation}
F_b = J \times B + \rho_e E, \tag{7}
\end{equation}

where \( \rho_e \) is the entire density of the charges present in the aqua soltion, \( E \) is the concentration of electric field, \( J \) the current density, and \( B \) the concentration of magnetic field.

By using Ohm’s Law and Maxwell’s equation, the physical force in the momentum equation can be written as

\begin{equation}
F_b = -\sigma B_0^2 U_f + \rho_e E, \tag{8}
\end{equation}

where \( \sigma \) is the electrical conductivity, and \( B_0 \) the applied magnetic field.

The continuity equation and the equation of momentum for fluid- and particulate-phase in component form are written as:

2.3. Fluidic-phase

\begin{align}
\frac{\partial t}{\partial x} + \frac{\partial t}{\partial y} &= 0, \\
\frac{\partial t_f}{\partial x} + \frac{\partial t_f}{\partial y} &= 0, \\
\frac{\partial U_f}{\partial x} + \frac{\partial U_f}{\partial y} &= 0, \\
\frac{\partial U_f}{\partial t} + \frac{\partial U_f}{\partial y} &= 0, \\
\frac{\partial P}{\partial x} + \frac{\partial P}{\partial y} &= 0, \\
\frac{\partial P}{\partial t} + \frac{\partial P}{\partial y} &= 0, \\
\frac{\partial t}{\partial x} + \frac{\partial t}{\partial y} &= 0, \\
\frac{\partial t}{\partial x} + \frac{\partial t}{\partial y} &= 0, \\
\frac{\partial t_f}{\partial x} + \frac{\partial t_f}{\partial y} &= 0, \\
\frac{\partial t_f}{\partial x} + \frac{\partial t_f}{\partial y} &= 0.
\end{align}

\begin{align}
\frac{\partial t}{\partial x} + \frac{\partial t}{\partial y} &= 0, \\
\frac{\partial t_f}{\partial x} + \frac{\partial t_f}{\partial y} &= 0, \\
\frac{\partial t}{\partial t} + \frac{\partial t_f}{\partial t} &= 0, \\
\frac{\partial t_f}{\partial t} + \frac{\partial t_f}{\partial t} &= 0, \\
\frac{\partial t}{\partial t} + \frac{\partial t_f}{\partial t} &= 0, \\
\frac{\partial t_f}{\partial t} + \frac{\partial t_f}{\partial t} &= 0.
\end{align}

2.4. Particulate-phase

\begin{align}
\frac{\partial t_f}{\partial x} + \frac{\partial t_f}{\partial y} &= 0, \\
\frac{\partial t_f}{\partial x} + \frac{\partial t_f}{\partial y} &= 0, \\
\frac{\partial P}{\partial x} + \frac{\partial P}{\partial y} &= 0, \\
\frac{\partial P}{\partial t} + \frac{\partial P}{\partial y} &= 0, \\
\frac{\partial t}{\partial t} + \frac{\partial t_f}{\partial t} &= 0, \\
\frac{\partial t_f}{\partial t} + \frac{\partial t_f}{\partial t} &= 0.
\end{align}

The extra stress tensor for the Jeffrey fluid is expressed as \( [34] \):

\begin{equation}
\tau = \frac{\mu}{(1 + \lambda_1)} (\dot{n} + \lambda_2 \dot{n}), \tag{15}
\end{equation}

where \( \lambda_1 \) the ratio of relaxation to retardation time, \( \lambda_2 \) the retardation time, \( \dot{n} \) the shear rate and and dots over the quantities represent differentiation with respect to time. They are specified as:

\begin{align}
\dot{n} &= \nabla w + (\nabla w)^T, \\
\dot{\lambda} &= \left( \frac{\partial}{\partial t} + w \cdot \nabla \right) \lambda.
\end{align}

and

\begin{align}
\tau_{xx} &= \frac{2 \mu}{(1 + \lambda_1)} \left( 1 + \lambda_2 \left( \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial X} \right) \right), \tag{16} \\
\tau_{xy} &= \frac{2 \mu}{(1 + \lambda_1)} \left( \frac{\partial U}{\partial Y} - \frac{\partial U}{\partial X} \right) \left( 1 + \lambda_2 \left( \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial X} \right) \right), \tag{17} \\
\tau_{yy} &= \frac{2 \mu}{(1 + \lambda_1)} \left( \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial X} \right) \left( 1 + \lambda_2 \left( \frac{\partial U}{\partial Y} - \frac{\partial U}{\partial X} \right) \right). \tag{18}
\end{align}

We now will move from stationary to moveable frame, therefore, we define transformation variables which are as follows.

\begin{equation}
X = c_s X = \tilde{X}, Y = \tilde{Y}, V_f, V_p, \dot{c} = \tilde{U}_f, P = \tilde{P}. \tag{19}
\end{equation}

The Poisson-Boltzmann equation for the charge scattering is engaged due to the occupation of double-layered charge distribution mainly known as (EDL) in the flow medium and is illustrated as:

\begin{equation}
\nabla^2 \tilde{\phi} = -\frac{\rho_e}{\varepsilon}, \tag{20}
\end{equation}

where \( \varepsilon \) the dielectric constant, and, \( \rho_e \) the density of the total amount of ionic charge carriers. The total
charge density in behalf of the existence of Electric Double Layer is:
\[ \rho_e = ez(n_+ - n_-), \]  
(21)
where \( e \) is the electronic charge, \( z \) is the equity of the charges, and, \( n_+ \) and \( n_- \) are the densities of cathodic and anodic ions respectively. Boltzmann dispersion is written as:
\[ n^\pm = n_0 \exp \left( \frac{\pm e z \tilde{\phi}}{k_B T} \right), \]  
(22)
where \( n_0 \) is the strength of ions, \( k_B \) the Boltzmann constant, and \( T \) the average amount of temperature of the aqua solution.

We can determine the ionic distribution using the Nernst Planks Equation in a simplified form. that is.
\[ 0 = \frac{\partial^2 n^\pm}{\partial y^2} \pm \frac{\partial}{\partial y} \left( n^\pm \frac{\partial \tilde{\phi}}{\partial y} \right). \]  
(23)

Electro-osmotic flow basically emerges when an exterior electric field is applied on the charged particles that exist close to the bottom wall of the channel and in the vicinity of the combined surface. When the external electric field is applied, the charged particles get distributed, and this distribution is observed as potentials at the wall, on the border and the externally enforced electric field is applied, the charged particles get displaced on the wall and at the border, which are the dispersion of charged particles for small Debye thickness and are slightly affected by the applied electric field. Therefore, in this case we can determine the charge distribution without an externally enforced electric field. In view of low fluid velocity, the impact of fluid movement on charge dispersion can be ignored, that means, when the momentum equation has weak inertial terms or when Debye thickness is low.

For limited values of \( \tilde{\phi} \) we can write it as:
\[ \sinh \left( \frac{ez \tilde{\phi}}{k_B T} \right) \approx \frac{ez \tilde{\phi}}{k_B T}. \]  
(24)

Utilizing the above approximations, the Poisson Boltzmann equation takes the form
\[ \frac{\partial^2 \tilde{\phi}}{\partial y^2} = \beta^2 \tilde{\phi}, \]
where \( \beta \) is the electro-osmotic parameter that is associated with the thickness of Debye layer that is \( \lambda_D = \frac{1}{\beta} \). This approximation is true for small Debye thickness and should be limited.

We will solve Equation (21) with the boundary conditions stated below:
\[ \tilde{\phi} = \xi_1 \text{ at } y = h_1 \text{ and } \tilde{\phi} = \xi_2 \text{ at } y = h_2. \]  

The dimensionless potential field thus takes the form
\[ \tilde{\phi} = c_1 \cosh(\beta y) + c_2 \sinh(\beta y), \]
Where
\[ c_1 = \frac{\xi_2 \sinh(\beta h_1) - \xi_1 \sinh(\beta h_2)}{\sinh(\beta (h_1 - h_2))} \text{ and } \]
\[ c_2 = \frac{\xi_1 \cosh(\beta h_2) - \xi_2 \cosh(\beta h_1)}{\sinh(\beta (h_1 - h_2))}. \]

We use these non-dimensional parameters to model the governing equations:
\[ \tilde{x} = x \lambda, \tilde{y} = y l, \tilde{u}_{f,p} = \frac{u_{f,p}}{c_0}, \tilde{v}_{f,p} = \frac{v_{f,p}}{\delta c_0}, \tilde{p} = \frac{\gamma^2 p_0}{\lambda \mu c_0}, \]
\[ h_1 = \frac{H_1}{l_1}, h_2 = \frac{H_2}{l_1}, \delta = \frac{l_1}{\lambda}, l = \frac{l_2}{l_1}, s = \frac{s_1}{l_1}, r = \frac{r_1}{l_1}, \]
\[ R_e = \frac{\rho \mu c_0 l_1}{\mu}, m = \sqrt{\frac{\rho}{\mu}} B_{0} h_1, \]
\[ u_{hs} = \frac{e k_B T c_0}{\mu c e z}, n = \frac{sl_2^2}{\mu}, \]
\[ \tilde{\phi} = \frac{e z \tilde{\phi}}{k_B T} \xi_1 = \frac{e z \tilde{\phi}}{k_B T} \xi_2, \]
\[ D_a = K_{l_1^2} F_r = \frac{c_r \mu l_1^2}{\mu K l_1^2}. \]

Using the above dimensionless variables in the proposed modelling and using the approximation (smaller Reynolds number and long wavelength approximations) which are stated as \( R_e \to 0, \delta \to \infty \) we get the dimensionless form of the equations for fluid- and the particle-phase respectively as:
\[ \frac{1}{1 + \lambda_1} \frac{\partial^2 u_f}{\partial y^2} + \frac{1}{1 - c} \left[ n c (u_p - u_f) - m^2 (u_f + 1) \right] + \beta^2 u_{hs} \tilde{\phi} - D_a (u_f + 1) - F_r (u_f + 1)^2 \frac{dp}{dx} = \]  
(26)
\[ (u_f - u_p)n + \frac{1}{c} \left[ \beta^2 u_{hs} \phi - (u_p + 1) m^2 \right] \frac{dp}{dx}. \]
(27)

The corresponding no-slip boundary conditions in their dimensionless form is found as:
\[ u_f(h_1) = -1 \text{ where } y = h_1 = l + \cos^2(\pi x), \]  
(28)
\[ u_f(h_2) = -1 \text{ where } y = h_2 = -l - \cos^2(\pi x + \psi). \]
(29)

3. Solution of the problem
To solve the non-linear differential Equation (26) subjected to the boundary conditions stated in Equations (28) and (29), we use a powerful and efficient technique called Homotopy perturbation scheme to discover the series solutions. HPM is an efficient technique that works even without the linearization process [35]. It reduces the non-linear equation into a set of linear equations and then gives the asymptotic solutions. We
start the process with the following. Let us define homotopy in the following format:

\[ H(\psi, w) = [L(\psi) - L(\psi_0)](1 - w) + w[L(\psi) + N(\psi)], \]

(30)

\( N(\psi) \) is remaining part of the Equation (24) without the linear operator

\[ H(\psi, \zeta) = [L(\psi) - L(\psi_0)](1 - \zeta) 
+ \zeta \left( L(\psi) + \frac{d\psi}{dx} - \frac{1}{1-c} \right) 
\times \left[ c(\psi - \psi_0) - m^2(\psi - 1) 
+ \beta^2 \psi_h = D_a(\psi - 1) - F(\psi - 1)^2 \right], \]

(31)

where \( \zeta \in (0, 1) \), when \( \zeta = 0 \) it gives the initial guess and for \( \zeta = 1 \) it gives the original differential equation. The linear operator for equation

\[
L = \frac{\beta^2}{\alpha y^2},
\]

Also, we take the initial guess which as

\[ \psi_0 = -1 + (1 + \lambda_1)(y - h_1)(y - h_2). \]

Introducing the expansion to determine the solution of Equation (31), we have

\[ \psi = \psi_0 + \psi_1 + \ldots \ldots \]

(34)

By using the necessary condition of the homotopy perturbation technique that is when \( \zeta \rightarrow 1 \) the solution takes the form

\[ u_\ell = \lim_{\zeta \rightarrow 1} \psi = \psi_0 + \psi_1 + \ldots \ldots \]

(35)

The solution up to second-order for the fluid-phase can be written as:

\[
(36)
\]

The solution for the particulate-phase can be written as:

\[
(37)
\]
where \( M_1, M_2, M_3, M_4, M_5 \) are defined as

\[
M_1 = 1 + \left[ \frac{nc^2}{(1-c)(nc + m^2)} \right]
\]

\[
M_2 = \frac{1}{1-c} \left[ \frac{n^2c^2}{(nc + m^2)} - \frac{nc^2}{(nc + m^2)} - m^2 - D_0 \right]
\]

\[
M_3 = \frac{\beta^2u_n}{1-c} \left[ \frac{nc^2}{(nc + m^2)} + 1 \right]
\]

\[
M_4 = \frac{1}{1-c} \left[ \frac{ncm^2}{(nc + m^2)} + m^2 + D_0 \right]
\]

\[
M_5 = \frac{F_r}{1-c}
\]

The expression for wall shear stress is stated as

\[
\tau = \frac{\beta}{n} \left[ 1 - \frac{c}{m^2 + cn} \right] \frac{\partial}{\partial y} \left[ \frac{1}{1 + \lambda} \right]
\]

The volumetric rate of flow for particle- and fluid-phase is defined as

\[
dp = \frac{1}{(m^2 + cn)} \left[ \left(-\frac{1}{1 + \lambda} \right) \frac{\partial u_p}{\partial y} \right]_{y = h_1} = \tau_{wp}.
\]

From the Equations (38) and (39) the value of \( dp/dx \) is given as

\[
Q_f = (1-c) \left( u_f + 1 \right) dy,
\]

\[
Q_p = c \left( u_p + 1 \right) dy.
\]

Then the total volumetric rate of flow is given by

\[
Q_f + Q_p = Q.
\]

The expression for wall shear stress is stated as

\[
\tau = \frac{\beta}{n} \left[ 1 - \frac{c}{m^2 + cn} \right] \frac{\partial}{\partial y} \left[ \frac{1}{1 + \lambda} \right]
\]

\[
\frac{1}{1 + \lambda} \frac{\partial u_p}{\partial y} = \tau_{wp}.
\]

This can be calculated using Equations (36) and (37). The resulting expressions for fluid- and particulate-phase are interpreted as

\[
\tau_{wp} = \frac{1}{m^2 + cn} \left[ \frac{c}{(m^2 + cn)(1 + \lambda)} \right] \left( u_n \beta \left[ c_2 \text{Cosh}(y) + c_1 \text{Sinh}(y) \right] \right)
\]

and

\[
\tau_{wp} = \frac{1}{m^2 + cn} \left[ \frac{c}{(m^2 + cn)(1 + \lambda)} \right] \left( u_n \beta \left[ c_2 \text{Cosh}(y) + c_1 \text{Sinh}(y) \right] \right)
\]
The stream function $\tilde{\psi}$ defined for the present formulation read as
$$u_{pf} = \frac{\partial \tilde{\psi}_{pf}}{\partial y}, \quad v_{pf} = -\frac{\partial \tilde{\psi}_{pf}}{\partial x},$$

(42)

4. Results and discussion

The primary purpose of this presentation is to ascertain the impact of electric and magnetic field on the non-Newtonian Jeffery fluid mannequin extends in a curvy asymmetric channel which is shown in the Figures 2–9.

It is important to mention here that by taking $\lambda_1 \to 0$ in the resulting equations the results of viscous flow can be retrieved by the use of lubrication strategy. Furthermore, when $m \to 0$ the ruling model yields only electrokinetic flow. The same results are obtained when we take the electro-osmotic parameter $\beta \to \infty$. From the Equation (26) it can be seen that there is a non-linear term in the non-Darcian media. The resulting equation gains linearity if we take $F_r \to 0$. Different parameters engaged in the flow execute a critical function in changing the behaviour of flow, when we numerically compute velocity, pressure, shear stress and streamlines.

The values of physical parameters that are used for the graphical representation in the computational software Mathematica (10.3v) are given in Table 1.

Figure 2. Impacts of various parameters i.e $(F_r, \lambda_1, D_a, m, u_{hs})$ on velocity profile of the fluid-phase.
Figure 2(a-e) and Figure 3(a-e) exhibits the velocity curves of liquid and particles phase for Forchheimer parameter $F_r$, fluid parameter $\lambda_1$, Darcy number $D_a$, Hartmann number $m$ and Helmholtz-Smoluchowski velocity $u_{hs}$. Figure 2(a) and Figure 3(a) indicate the variation of Forchheimer number on velocity portrait for liquid and particle-phase respectively. $F_r$ exposes the non-Darcian effects in the porous medium. Physically $F_r$ demonstrates how the porous medium diverts from established Darcian-medium. As the distance from the wall increases the fluid and the particle velocity penetrates on increasing the values of $F_r$. No remarkable alteration in the conduct of fluid- and particulate-phase is observed. It is also certain from the figure that gains in Forchheimer number will increase the velocity distribution. Figure 2(b) and Figure 3(b) exhibits the variation of the fluid parameter $\lambda_1$ on fluid- and particle-phase, respectively. From the figure, it can be seen that when fluid discloses non-Newtonian conduct, the velocity portrait increment in a phenomenal way. However, the extent of the fluid-phase is greater when contrasted with the particle phase, which means $\lambda_1$ affects the fluid characteristics in an important manner. Figure 2(c) and Figure 3(c) intimate the Darcy reaction

![Figure 2](image-url)  
![Figure 3](image-url)

**Figure 3.** Impacts of different parameters i.e. on the velocity profile of the particle-phase.
Figure 4. Impacts of different parameters i.e \((F_r, \lambda_1, D_a, m)\) on the pressure gradient.

Figure 5. Impacts of different parameters i.e \((F_r, \lambda_1, m, \beta, u_{hs})\) on wall shear stress.
on pace distribution for both phases. Darcy number is related to the porosity. It is discernible from the graph that there is less penetration of velocity profile in both fluid- and particle-phase for high values of Darcy number. Figure 2(d) and Figure 3(d) represent the impact of Hartmann number (Magnetic field) on the speed configuration of fluid- and particulate-phase, respectively. The ratio between the electromagnetic and viscous hydrodynamics forces is designated as a magnetic field. A large gain in the opposite magnetic field $B_0$ causes a heavy resistance to slow down the fluid motion. The maximum velocity for both liquid and particle phase is accomplished at the focal point of the duct at $y = 0.2$. Figure 2(e) and Figure 3(e) represent the variation of Helmholtz-Smoluchowski velocity also known as ultimate electro-osmotic velocity. Helmholtz-Smoluchowski has a direct relation with the electric field. $u_{hs} < 0$ Cause improvement in the fluid motion while $u_{hs} > 0$ causes retardation. When $u_{hs} = 0$ there will be no electric field in the fluid motion. Fluid- and particulate-phase velocity diminish due to an increment in $u_{hs} > 0$.

Figure 4(a-d) displays the behaviour of the pressure gradient for various estimations of Forchheimer Number $F_r$, slip parameter $\lambda_1$, electro-osmotic parameter $\beta$, and Darcy number $D_a$. Figure 4(a) exhibits the impacts of $F_r$ on the pressure gradient. Forchheimer’s number depicts the wavy behaviour of pressure gradient all over the domain and shows a decreasing impact for higher values of $F_r$. Figure 4(b) shows that the increment in

![Figure 6](image)

Figure 6. Impacts of different parameters i.e ($F_r, \lambda_1, m, \beta, u_{hs}$) on wall shear stress for particulate-phase.
the fluid parameter $\lambda_1$ will diminish the pressure gradient. Also, we can see from the figure that the pressure gradient is minimal when the fluid behaves as non-Newtonian. Figure 4(c) shows that in the consequence of the vigorous impact of the magnetic field, the pressure gradient profile significantly reduces. Figure 4(d) shows the outcomes of the Darcian range of the pressure gradient. It is seen here that pressure gradient rises for higher values of Darcian parameter. Figure 5(a-e) and Figure 6(a-e) display the impacts of different parameters such as Forchheimer number $F_r$, fluid parameter $\lambda_1$, magnetic field $m$, electro-osmotic parameter $\beta$ and Helmholtz-Smoluchowski velocity $u_{hs}$ on shear stress profile for fluid-phase and particle-phase respectively. Figure 5(a) and Figure 6(a) display the impact of Forchheimer number $F_r$ on wall shear stress. It is seen here in both figures that wall shear stress for fluid- and particulate-phase increases when $x < 0.55$, although it shows decreasing mechanism for $x > 0.55$. Figure 5(b) and Figure 6(b) show the impacts of electro-osmotic parameter $\beta$ on the wall shears stress. For small values of $\beta$, impacts are insignificant whereas for values greater than 0.1 we can see significant increment on wall shear stress. Also, from figure Figure 5(c) and Figure 6(c) we can see a uniform increment in the shear stress configuration all over the realm whereas the shear stress profile gets parameter $m$ on the shear force of the wall. One can take a look at the substantial increment in the wall shear stress on increasing the magnetic field. On increasing the magnetic field, flow faces retardation and consequently shear stress of the wall increase. Figure 5(d) and Figure 6(d) show the impacts of fluid parameter $\lambda_1$ on the wall shear stress portrait. It can be seen that there is a uniform decrement in wall shear

**Figure 7.** Streamlines shape for various estimations of $F_r$, (a) $F_r = 0$, (b) $F_r = 0.7$, (c) $F_r = 0.9$. 

| $x$ | y | $x$ | y |
|-----|---|-----|---|
| 0.0 | 0.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 2.0 | 2.0 | 2.0 | 2.0 |

| $x$ | y | $x$ | y |
|-----|---|-----|---|
| 0.0 | 0.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 2.0 | 2.0 | 2.0 | 2.0 |

| $x$ | y | $x$ | y |
|-----|---|-----|---|
| 0.0 | 0.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 2.0 | 2.0 | 2.0 | 2.0 |
stress throughout the domain. Figure 5(e) and Figure 6(e) display the alteration of Helmholtz-Smoluchowski velocity. One can observe that for positive values of $u_{hs}$ wall shear stress increases in a significant manner while for negative values wall shear stress decreases. Also, the Helmholtz-Smoluchowski parameter with positive values is more effective than negative values.

The trapping phenomena illustrate the growth and downstream transmission of free vortices, referred to as fluid boluses. This phenomenon is of strong physiological importance, as it may be accounted for blood clotting and the medicinal transmission of bacteria. Figures 7–11 are presented to show the conduct of streamlines for various values of different parameters such as Forchheimer number $F_r$, Darcy number $D_a$, Hartmann number $m$, Helmholtz-Smoluchowski velocity $u_{hs}$ and fluid parameter $\lambda_1$. Streamlines seen are basically small tiny circulating boluses. From Figure 7 it can be concluded that the enhancement in the Forchheimer parameter $F_r$ will reduce the number of circulating bolus and magnitude gets effected against each value. Same behaviour is seen against the Darcy number in Figure 8. In Figure 9 one can observe that for small values of parameter $\lambda_1$ bolus formed are more significant and cleared and vary with different values of $\lambda_1$. In Figure 10 we can see that the magnetic field significantly effects the trapped bolus also effects the magnetitude of the trapped bolus, whereas a similar behaviour has been observed against Helmholtz-Smoluchowski velocity $u_{hs}$. 

Figure 8. The shape of streamlines for various estimations of $D_a$. (a) $D_a = 1$, (b) $D_a = 2$, (c) $D_a = 3$. 
Figure 9. The shape of Streamlines for various estimations of $\lambda_1$. (a) $\lambda_1 = 0.1$, (b) $\lambda_1 = 0.2$, (c) $\lambda_1 = 0.5$.

| Flow characteristics | Values of various physical parameters used in the computations. |
|----------------------|-----------------------------------------------------------------|
| velocity             | $s = 0.5, r = 0.5, l = 1, \beta = 0.2, x = 1, c = 0.6, n = 1.2, \psi = \frac{\pi}{3}, Q = 2, m = 0.8, u_{hs} = 2, f_r = 1, D_a = 0.6, \lambda_1 = 0.5, \zeta_1 = 1, \zeta_2 = 2$. |
| Pressure gradient     | $s = 0.5, r = 0.5, l = 1, \beta = 0.2, c = 1.9, n = 2, \psi = \frac{\pi}{3}, Q = 2, m = 0.8, u_{hs} = -2, f_r = 1, D_a = 1, \lambda_1 = 0.5, \zeta_1 = 1, \zeta_2 = 2$. |
| Shear stress          | $s = 0.5, r = 0.5, l = 1, \beta = 0.2, c = 0.6, n = 2, \psi = \frac{\pi}{3}, Q = 2, m = 0.8, u_{hs} = 2, f_r = 1, D_a = 1, \lambda_1 = 0.8, \zeta_1 = 1, \zeta_2 = 2$. |
| Streamlines           | $s = 0.5, r = 0.5, l = 1, \beta = 0.2, c = 5, n = 1.2, \psi = \frac{\pi}{3}, Q = 2, m = 0.8, u_{hs} = 2, f_r = 0.5, D_a = 0.6, \lambda_1 = 0.5, \zeta_1 = 1, \zeta_2 = 2$. |

5. Conclusion

In this paper, combined impacts of electromagnetic fields on the two-phase Jeffrey liquid model moving via asymmetric microchannel having Darcy-Brinkman-Forchheimer medium is considered. The proposed mathematical modelling is based on the Poisson-Boltzmann equation, Debye length approximation, and
ionic Nernst Planck equation. The modelled problem is formulated in a fixed frame, which is further transformed into a wave frame using the coordinate transformation. Lubrication theory also plays a significant role in the mathematical formulation of the problem. We use the estimation “prolonged wavelength and small Reynolds number”. Mathematical results are interpreted for fluid- and particle-phase velocity, pressure gradient, wall shear stress, and trapping mechanism. The main points of the proposed problem are given below:

(1) The Helmholtz-Smoluchowski parameter and magnetic-field display the retardation effects for the fluid- and particle-phase.

(2) The Jeffrey fluid parameter enhances the flow in a significant manner.

(3) The porosity parameter opposes the flow while the Forchheimer parameter enhances the velocity of the fluid.

(4) The magnetic field and fluidic and electro-osmotic parameters increases the Shear force of the wall while Helmholtz-Smoluchowski and the Jeffrey fluid parameters provide a reduction in the wall shear stress.

(5) The pressure gradient tends to increase due to an increment in porosity parameter, while it decreases against magnetic field, Jeffrey fluid parameter, and Forchheimer parameter.

Figure 10. The shape of Streamlines for various estimations of $m$. (a) $m = 0.1$, (b) $m = 0.8$, and (c) $m = 1.5$. 
Figure 11. The shape of Streamlines for various estimations of \( u_{hs} \). (a) \( u_{hs} = 1 \), (b) \( u_{hs} = 0 \), and (c) \( u_{hs} = -1 \).

(6) In trapping phenomena, boluses die out moderately due to enhancement in Forchheimer number and fluid-parameter.

The presented model grants effective results that could be very useful for learning more about electro-osmotic flows by adopting various approaches.

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No potential conflict of interest was reported by the author(s).

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