Signatures of Chaos in the Statistical Distribution of Conductance Peaks in Quantum Dots

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Abstract

Analytical expressions for the width and conductance peak distributions of irregularly shaped quantum dots in the Coulomb blockade regime are presented in the limits of conserved and broken time-reversal symmetry. The results are obtained using random matrix theory and are valid in general for any number of non-equivalent and correlated channels, assuming that the underlying classical dynamic of the electrons in the dot is chaotic or that the dot is weakly disordered. The results are expressed in terms of the channel correlation matrix which for chaotic systems is given in closed form for both point-like contacts and extended leads. We study the dependence of the distributions on the number of channels and their correlations. The theoretical distributions are in good agreement with those computed in a dynamical model of a chaotic billiard.

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I. INTRODUCTION

One of the most interesting aspects of electron transport in submicron scale devices is the interplay between quantum coherence and aperiodic but reproducible conductance fluctuations. Over the past decade the phenomenon of universal conductance fluctuations in disordered systems (where impurity scattering dominates) has been understood through the use of stochastic models. More recently, a new generation of experiments was designed to measure conductance fluctuations in the ballistic regime where the dynamics of the electrons in the device is determined by the geometry of its boundary. The stochastic approach to these systems is justified by the underlying classical chaotic dynamics. This situation is distinct from the diffusive case, where the corresponding classical limit of the quantum problem is not fully understood.

In this paper we discuss the conductance fluctuations in quantum dots. These are semiconductor devices in which the electrons are confined to a two-dimensional region whose typical linear dimension is in the submicron range. In particular we are interested in the Coulomb blockade regime where the leads are weakly coupled to the dot, either because the leads are very narrow, or due to the presence of potential barriers at the lead-dot interface. The electrons inside the dot are characterized by isolated resonances whose width is smaller than their average spacing, and conductance occurs through resonant tunneling. As a consequence, the conductance peaks when the Fermi energy matches a resonance energy of the electrons inside the dot and an additional electron tunnels into the dot. Such a system resembles the compound nucleus in its region of isolated resonances. The macroscopic charging energy required to add an electron to a dot is determined by its capacitance \( C \) and is given by \( e^2/C \). Since \( C \) is a constant which is determined essentially by the geometry of the dot, the conductance exhibits equally spaced oscillations as a function of the gate voltage (or Fermi energy). At low temperatures \( \Gamma \ll kT < \Delta \) the width of the conductance peaks is \( \sim kT \), but the heights exhibit order of magnitude variations.

When the electron-impurity mean free path is larger than the size of the dot, the classical dynamics of the electron inside the dot is determined by the scattering from the dot’s boundary. Due to small irregularities in the dot’s shape, the electron displays chaotic motion, and its quantum transport through the dot can be described by statistical S-matrix theory. Since the Coulomb blockade regime is dominated by resonances, the conductance peaks can be used to probe the chaoticity of the underlying resonance wavefunctions. A statistical theory of the conductance peaks was originally developed in Ref. By using \( R \)-matrix theory, the conductance peak amplitude was expressed in terms of the electronic resonance wavefunction across the contact region between the dot and the leads. When the dynamics of the electron inside the dot is chaotic, the fluctuations of the wavefunction inside the dot are assumed to be well described by random matrix theory (RMT). In Ref. the conductance distribution was derived in closed form for one-channel leads. These results were rederived in Ref. and later extended to the case of two-channel leads in the absence of time-reversal symmetry through the use of the supersymmetry technique. However, the calculations required by this technique become too complicated to apply in the general case of any number of possibly correlated and/or non-equivalent channels.

The conductance distributions for one-channel leads were recently measured and found to be in agreement with theory for both cases of conserved and broken time-reversal
symmetry. This indicates that the dephasing effect, which plays an important role in open dots, is of little importance for closed dots.

In this paper we discuss in detail the width and conductance peak distributions for leads with any number of channels that are in general correlated and non-equivalent. Exact closed expressions for these distributions are derived for both cases of conserved and broken time-reversal symmetry. We find that these distributions are entirely characterized by the eigenvalues of the channel correlation matrices $M^l$ and $M^r$ in the left and right leads, respectively. The strength of our approach is in its simplicity, since it relies solely on standard RMT techniques. To test our predictions we compare our analytical findings to numerical simulations of a chaotic dynamical model, the conformal billiard. Statistical width and conductance distributions of one-channels leads were recently studied in detail in this model. Although our paper deals mainly with ballistic dots whose classical dynamics is chaotic, our results should also be valid in the diffusive regime of weakly disordered dots, where random matrix theory is applicable.

We note that under certain conditions the partial width is analogous to the wavefunction intensity at a given point. Therefore our width distributions can also be tested by microwave cavity experiments, where the intensities are measured at several points that are spatially correlated.

The plan of the paper is as follows: In Section II we briefly review the conductance in quantum dots in their Coulomb blockade regime. In Section III we discuss the statistical model and derive analytic results for the partial and total width distributions in each lead, for the channel correlation matrix and for the conductance distribution. We investigate the variation of these distributions as a function of the number of channels and their sensitivity to the degree of correlations between them. Those findings are compared in Section IV with numerical results obtained for the conformal billiard. Finally, in Section V we discuss the validity of our assumptions in the the context of typical experiments.

II. CONDUCTANCE IN QUANTUM DOTS

In this section we briefly review the formalism and introduce the notation used throughout this paper. In particular, we express the conductance peak heights in terms of the channel and resonance wavefunctions of the dot.

For $\Gamma \ll kT \ll \Delta$, which is typical of many experiments, the observed on resonance conductance peak amplitude is given by

$$G_\lambda = \frac{e^2}{h} \frac{\pi}{2kT} g_\lambda \quad \text{with} \quad g_\lambda = \frac{\Gamma^l_\lambda \Gamma^r_\lambda}{\Gamma^l_\lambda + \Gamma^r_\lambda}, \quad (1)$$

where $\Gamma^l_\lambda$ is the partial decay width of the resonance $\lambda$ into the left (right) lead. Since each lead can support several open channels we have $\Gamma^l_\lambda = \sum_c \Gamma^l_{c\lambda}$, where $\Gamma^l_{c\lambda}$ is the partial width to decay into channel $c$ in the left (right) lead.

In the $R$-matrix formalism, the partial widths are related to the resonance wavefunction inside the dot. More specifically, introducing the partial amplitudes $\gamma_{c\lambda}$, such that $\Gamma_{c\lambda} = |\gamma_{c\lambda}|^2$, one can write
\[ \gamma_{c\lambda} = \sqrt{\frac{\hbar^2 k_c P_c}{m}} \int dS \Phi_c^*(r) \Psi_\lambda(r). \]  

(2)

Here \( \Psi_\lambda(r) \) is the \( \lambda \)-th resonance wavefunction in the dot, \( \Phi_c(r) \) is the transverse wavefunction in the lead that corresponds to an open channel \( c \), and the integral is taken over the contact area between the lead and the dot. \( k_c \) and \( P_c \) are the longitudinal wavenumber and penetration factor in channel \( c \), respectively.

Eq. (2) shows that the contributions to the partial width amplitude from the internal and external regions of the dot factorize. The information from the region external to the dot is contained in \( k_c \) and \( P_c \). These quantities are determined by the wave dynamics in the leads and are non-universal. They affect the average widths and enter explicitly in the correlation matrix \( M \). However, the fluctuation properties of the conductance are generic and depend only on the statistical properties of the electronic wavefunction at the dot-lead boundary inside the barrier region.

A different physical modeling of a quantum dot assumes point-like contacts and each lead is composed of several such point contacts. In this model the conductance is also given by (2) with each point contact \( r_c \) considered as one channel. The corresponding partial width is

\[ \gamma_{c\lambda} = \sqrt{\frac{\alpha_c A \Delta}{\pi}} \Psi_\lambda(r_c), \]  

(3)

where \( A \) is the area of the dot, \( \Delta \) is the mean spacing and \( \alpha_c \) is a dot-lead coupling parameter.

Both models can be treated by our formalism. This becomes apparent after the following considerations. A resonance eigenfunction with eigenenergy \( E = E_\lambda \) can be approximated by an expansion in a fixed basis \( \rho_\mu \) of wavefunctions with the given energy \( E \) inside the dot

\[ \Psi_\lambda(r) = \sum_\mu \psi_{\lambda\mu} \rho_\mu(r). \]  

(4)

The sum over \( \mu \) is truncated at \( N \) basis states, where \( N \) is large and determined by precision requirements. The partial width in channel \( c \) can then be expressed by the scalar product

\[ \gamma_{c\lambda} = \langle \phi_c | \psi_\lambda \rangle = \sum_\mu \phi_{c\mu}^* \psi_{\lambda\mu}, \]  

(5)

where

\[ \phi_{c\mu} \equiv \sqrt{\frac{\hbar^2 k_c P_c}{m}} \int dS \Phi_c^*(r) \rho_\mu(r) \]  

(6)

for the extended leads model, and

\[ \phi_{c\mu} \equiv \sqrt{\frac{\alpha_c A \Delta}{\pi}} \rho_\mu^*(r_c) \]  

(7)

for the point contact model. Thus, we are led to similar formulations of both the extended leads and point-like contacts problems; in the corresponding \( N \)-dimensional space the partial width amplitudes of a level are simply the projections of its corresponding eigenstate vector \( \psi_\lambda \) on the channel vectors \( \phi_c \). The only difference between the two models is the explicit expression for the channel vector \( \phi_c \). We note that the scalar product (5) (that will be used throughout this paper) is different from the original scalar product defined in the spatial region extended by the dot.
III. STATISTICAL MODEL

Due to the irregularity of the dot’s shape, the motion of the electron inside the dot is expected to be chaotic. In Ref. 8 we have developed a statistical theory of the conductance peaks by assuming that the vectors \( \psi_\lambda \) that correspond to the resonance wavefunctions inside the dot have the same statistical properties as the eigenvectors of a random matrix ensemble. Here we study the limits of conserved time-reversal symmetry, corresponding to the Gaussian Orthogonal Ensemble (GOE), and of broken time-reversal symmetry, corresponding to the Gaussian Unitary Ensemble (GUE). The transition from one symmetry to another occurs when an external magnetic field is applied. The width distribution (or equivalently the wavefunction intensity distribution) was derived in the crossover regime between symmetries for the case of one channel leads only.

A. The Joint Distribution of Partial Width Amplitudes

In RMT the eigenvector \( \psi \equiv (\psi_1, \psi_2, \ldots, \psi_N) \) (here and in the following we omit the eigenvector label \( \lambda \)) is distributed randomly on a sphere \( P(\psi) \propto \delta(\sum_{\mu=1}^N |\psi_\mu|^2 - 1) \). The joint distribution of the partial width amplitudes \( \gamma = (\gamma_1, \gamma_2, \ldots, \gamma_\Lambda) \) for \( \Lambda \) channels is then given by

\[
P(\gamma) = \frac{\Gamma(\beta N/2)}{\pi^{\beta N/2}} \int D[\psi] \left[ \prod_{c=1}^\Lambda \delta(\gamma_c - \langle \phi_c | \psi \rangle) \right] \delta \left( \sum_{\mu=1}^N |\psi_\mu|^2 - 1 \right),
\]

where \( D[\psi] \equiv \prod_{\mu=1}^N d\psi_\mu \) for the GOE and \( D[\psi] \equiv \prod_{\mu=1}^N d\psi_\mu^* d\psi_\mu / 2\pi i \) for the GUE. To evaluate (8) we transform the \( \Lambda \) channels to a new set of orthonormal channels \( \hat{\phi}_c \)

\[
\phi_c = \sum_{c'} \hat{\phi}_{c'} F_{c'c} \quad \text{with} \quad \langle \phi_c | \hat{\phi}_{c'} \rangle = \delta_{cc'}.
\]

We then take advantage of the invariance of the corresponding Gaussian ensemble under an orthogonal (unitary) transformation to rotate the eigenvector \( \psi \) such that its first \( \Lambda \) components are along the new orthonormal channels. Denoting by \( O \) the orthogonal (unitary) matrix whose first \( \Lambda \) rows are the orthonormal vectors \( \hat{\phi}_c (c = 1, \ldots, \Lambda) \), we change variables in (3) to \( \hat{\psi}_\mu = \sum_\nu O_{\mu\nu} \psi_\nu \). Using \( \psi_c = \langle \hat{\phi}_c | \psi \rangle \) we find

\[
P(\gamma) = \frac{\Gamma(\beta N/2)}{\pi^{\beta N/2}} \det F \int D[\hat{\psi}] \delta \left( \hat{\gamma}^\dagger \hat{\gamma} + \sum_{\mu=\Lambda+1}^N |\hat{\psi}_\mu|^2 - 1 \right),
\]

The integration over these first \( \Lambda \) components is now easily done and gives

\[
P(\gamma) = \frac{\Gamma(\beta N/2)}{\pi^{\beta N/2} |\det F|} \int D[\hat{\psi}] \delta \left( \hat{\gamma}^\dagger \hat{\gamma} + \sum_{\mu=\Lambda+1}^N |\hat{\psi}_\mu|^2 - 1 \right),
\]
where \( \hat{\gamma}_c \equiv \langle \hat{\phi}_c | \psi \rangle = \sum_{c'} \gamma_{c'} F_{c'c}^{-1} \) are the partial widths to decay to the new channels and the metric is as before but excluding the first \( \Lambda \) components of \( \psi \). Finally, the latter integral is easily done by introducing spherical coordinates in the \( N - \Lambda \) dimensional space. We obtain

\[
P(\gamma) = \frac{\Gamma(\beta N/2)}{\pi^{\beta/2} \Gamma(\beta(N-\Lambda)/2)|\det F|} \left[ 1 - \gamma^\dagger (F^\dagger F)^{-1} \gamma \right]^{\frac{N-\Lambda}{2}-1}.
\] (12)

For \( \Lambda \ll N \) and in the limit \( N \to \infty \), we recover a simplified expression

\[
P(\gamma) = (\det M)^{-\beta/2} e^{-\frac{\beta}{2} \gamma^\dagger M^{-1} \gamma},
\] (13)

where the matrix \( M \equiv (NF^\dagger F)^{-1} \) is just the metric defined by the original channels

\[
M_{cc'} = \frac{1}{N} \langle \phi_c | \phi_{c'} \rangle.
\] (14)

The distribution (13) is normalized with the measure \( D[\gamma] \equiv \prod_{c=1}^\Lambda d\gamma_c/2\pi \) for the GOE and \( D[\gamma] \equiv \prod_{c=1}^\Lambda d\gamma_c d\gamma_c^*/2\pi i \) for the GUE. Note that for both ensembles the joint partial width amplitudes distribution is Gaussian, the main difference being that the partial amplitudes are real for the GOE and complex for the GUE. Such a Gaussian distribution is also obtained by assuming that the distribution is form-invariant under an orthogonal (unitary) transformation.\(^{27}\)

It follows from (13) that the matrix \( M \) is just the correlation matrix of the partial widths

\[
M_{cc'} = \frac{\gamma_c \gamma_{c'}}{\gamma_c^2}.
\] (15)

In general the channels are correlated and non-equivalent, i.e. non-equal average partial widths. According to (14) this is equivalent to assuming channels that are non-orthogonal and have non-equal norms.

### B. The Channel Correlation Matrix \( M \)

We shall now derive explicit expressions for the correlation matrix \( M \) in a chaotic quantum dot. Using Eq. (14) and the definition of the scalar product (5) we find

\[
M_{cc'} = \frac{\hbar^2}{2m} \sqrt{k_c k_{c'} P_c P_{c'}} \int dS \int dS' \Phi^*_c(r) \left[ \frac{1}{N} \sum_{\mu} \rho_{\mu}(r) \rho_{\mu}(r') \right] \Phi_c(r').
\] (16)

We first discuss the case where there is no magnetic field so that the motion inside the dot is that of a free particle. Therefore, a resonance eigenstate inside the dot at energy \( E = \hbar^2 k^2 / 2m \) can be expanded in a basis of free particle states at the given energy \( E \). Since RMT is applicable on a local energy scale, this is the fixed basis \( \rho_\mu \) for which the eigenvector coefficients \( \psi_\mu \) are distributed randomly (on the sphere). Using polar coordinates, such a basis of free waves is given by \( \rho_\mu(r) \propto J_\mu(kr) \exp(i\mu \theta) \) with \( \mu = 0, \pm 1, \pm 2, \ldots \), where \( J_\mu \) are Bessel functions of the first kind. Denoting by \( N \) the number of such waves on the energy shell, we find
\[
\frac{1}{N} \sum_{\mu} \rho_\mu^*(r) \rho_\mu(r') = \frac{1}{\mathcal{A}} \sum_{\mu} J_\mu(kr) J_\mu(kr') e^{i\mu(\theta'-\theta)} = \frac{1}{\mathcal{A}} J_0(k|r-r'|),
\]

where we have used the addition theorem for the Bessel functions. A similar relation holds if we choose a plane waves basis \( \rho_\mu(r) = \mathcal{A}^{-1/2} \exp(i k_\mu \cdot r) \) at a fixed energy \( \hbar^2 k^2/2m \) but with random orientation of \( k_\mu \) and use the integral representation of \( J_0 \). With help of Eq. (17) we obtain for the correlation matrix

\[
M_{cc'} = \frac{\hbar^2}{2m\mathcal{A}} \sqrt{k_c k_{c'}} P_c P_{c'} \int dS \int dS' \Phi^*_c(r) J_0(k|r-r'|) \Phi_{c'}(r'),
\]

for extended leads, while for the point contact model we find

\[
M_{cc'} = \frac{\alpha \Delta}{\pi} J_0(k|r_c - r_{c'}|).
\]

We remark that Eq. (19) is equivalent to \( C(k|\Delta r|) \equiv \frac{\Psi^*(r)\Psi(r')}{|\Psi(r)|^2} = J_0(k|r-r'|) \). This result was first derived in Ref. 29 based on the assumption that the Wigner function of a classically chaotic system is microcanonical on the energy surface, and recently studied extensively in the Africa billiard. However, in these references the average is taken for a fixed wavefunction over a local region around \( (r + r')/2 \).

When an external magnetic field \( B \) is present, the electronic classical underlying dynamics undergoes a transition from chaotic to integrable as the field gets stronger, regardless the shape of the billiard. In this paper, however, we only discuss the case of weak fields for which the motion is chaotic, and we are interested in the transition from orthogonal to unitary symmetry. While in the unitary case the wavefunctions become complex, the arguments that lead to Eq. (17) are still valid and the wavefunction correlator \( C(k|\Delta r|) \) is unchanged.

The wavefunction correlation \( C(k|\Delta r|) \) has been also derived for weakly disordered systems using the supersymmetry technique \( \ref{31} \) in the unitary and orthogonal symmetries. In addition Ref. \( \ref{31} \) derives the joint probability distribution for the intensity of an eigenfunction at two different points. We remark that the joint distribution of the wavefunction amplitude at \( \Lambda \) points \( r_c \) is a special case of (13) obtained for \( \gamma_{\Lambda} \equiv \Psi_{\Lambda}(r_c) \) (see the point contact case \( \ref{3} \) except that the points \( r_c \) can be chosen anywhere within the dot and not only on the boundary). We then obtain

\[
P(\Psi_{\Lambda}(r_1), \Psi_{\Lambda}(r_2), \ldots, \Psi_{\Lambda}(r_{\Lambda})) = (\det M)^{-\beta/2} \exp \left[ -\frac{\beta}{2} \sum_{cc'}^{\Lambda} \Psi^*_{\Lambda}(r_c) \left( M^{-1} \right)_{cc'} \Psi_{\Lambda}(r_{c'}) \right],
\]

where \( M_{cc'} = \mathcal{A}^{-1} J_0(k|r_c - r_{c'}|) \). The distributions of Ref. \( \ref{31} \) are then easily obtained from (20) when \( \Lambda = 2 \).}

### C. Total Width Distribution

We calculate next the total width distribution \( P(\Gamma) \) in a given lead that supports \( \Lambda \) channels and is characterized by a correlation matrix \( M \). Although this quantity is not
directly measurable in experiments with quantum dots, it appears very often in resonant scattering by complex objects. We remark that for a dot with reflection symmetry \( \Gamma' = \Gamma^r \equiv \Gamma \) the conductance peak \( g \) in (1) is proportional to \( \Gamma \). Such dots are, however, difficult to fabricate.

Using \( \Gamma = \sum_c |\gamma_c|^2 = \gamma^\dagger \gamma \), the characteristic function of \( P(\Gamma) \) is given by
\[
\tilde{P}(t) = \int_{-\infty}^{\infty} d\Gamma \exp(it\Gamma) P(\Gamma) = \int_{-\infty}^{\infty} D[\gamma] \exp(it\gamma^\dagger \gamma) P(\gamma). \tag{21}
\]

Since \( P(\gamma) \) is a Gaussian, we readily obtain \( \tilde{P}(t) = [\text{det}(I - 2it\Gamma/\beta)]^{-\beta/2} \). The distribution itself is then given by an inverse Fourier transform
\[
P(\Gamma) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \frac{e^{-it\gamma}}{[\text{det}(I - 2it\Gamma/\beta)]^{\beta/2}}. \tag{22}
\]

The matrix \( M \) is Hermitian and positive definite (since \( x^\dagger M x = |x \cdot \gamma|^2 > 0 \) for any \( x \neq 0 \)) and therefore its eigenvalues \( w_c^2 \) are all positive. According to (22), \( P(\Gamma) \) depends only on \( w_c^2 \). This is a consequence of the invariance of \( \Gamma \) under an orthogonal (unitary) transformation of the \( \Lambda \) partial width amplitudes.

We first discuss the simpler GUE case, for which the integrand has poles \( -i/w_c^2 \) along the negative imaginary axis. Taking a contour integration along the real line and a half-circle that encloses all the poles in the lower half of the plane, we can calculate (22) by residues. Assuming that all eigenvalues of \( M \) are non-degenerate, the poles are all simple and we find
\[
P_{\text{GUE}}(\Gamma) = \left( \prod_c \frac{1}{w_c^2} \right) \sum_{c=1}^{\Lambda} \left[ \prod_{c' \neq c} \left( \frac{1}{w_c^2} - \frac{1}{w_{c'}^2} \right) \right]^{-1} e^{-\Gamma/w_c^2}. \tag{23}
\]

The distribution \( P_{\text{GUE}}(\Gamma) \) given by (23) must be positive, which can be directly verified by using the concavity of the exponential function.

For two channels (\( \Lambda = 2 \)) which are in general non-equivalent (\( M_{11} \neq M_{22} \)) and correlated (\( M_{12} \neq 0 \)), the eigenvalues are given by \( w_{1,2}^2 = (M_{11} + M_{22})/2 \pm \sqrt{(M_{11} - M_{22})/2 + |M_{12}|^2} \). Then, Eq. (23) reduces to
\[
P_{\text{GUE}}^{\Lambda=2}(\hat{\Gamma}) = \frac{2a_{+}}{\sqrt{a_{-}^2 + |f|^2}} e^{-2a_{+}^2 \hat{\Gamma} / (1 - |f|^2)} \sinh \left( \frac{2a_{+} \sqrt{a_{-}^2 + |f|^2}}{1 - |f|^2} \hat{\Gamma} \right), \tag{24}
\]
where \( \hat{\Gamma} = \Gamma/\Gamma_0 \) is the width in units of its average value, \( f = M_{12}/\sqrt{M_{11}M_{22}} \) measures the degree of correlation between the two channels and \( a_{\pm} = 1/2 \left( \sqrt{M_{11}/M_{22}} \pm \sqrt{M_{22}/M_{11}} \right) \) are dimensionless parameters such that for equivalent channels \( a_{+} = 1 \) and \( a_{-} = 0 \). In the latter case, we reproduce the result of Ref. [12].

For degenerate eigenvalues, we can calculate (22) by using the residue formula for higher order poles. Alternatively we can slightly break the degeneracy of the eigenvalues by \( \eta \) and take the limit \( \eta \to 0 \). For example, for two channels Eq. (23) gives
\[
P(\Gamma) = \left( e^{-\Gamma/w_1^2} - e^{-\Gamma/w_2^2} \right) / (w_2^2 - w_1^2). \]
By taking \( w_2^2 = w_1^2 + \eta \), in the limit \( \eta \to 0 \) we recover
\[ P(\Gamma) = \frac{\Gamma}{w^4} e^{-\Gamma/w^2}, \]  
(25)

which is the \( \chi^2 \) distribution in four degrees of freedom. More generally, when all \( \Lambda \) channels are uncorrelated and equivalent \( (M = w^2 I) \) we recover the well-known \( \chi^2 \) distribution in \( 2\Lambda \) degrees of freedom:\[ P_{GUE}^{(0)}(\Gamma) = \frac{1}{w^{2\Lambda}(\Lambda - 1)!} \Gamma^{\Lambda-1} e^{-\Gamma/w^2}. \]  
(26)

We have denoted this limiting distribution in (26) by \( P_{GUE}^{(0)} \) as it will serve as our reference distribution against which to compare the distributions in the general case of correlated and/or inequivalent channels.

For the GOE case, the integral of Eq. (22) is more difficult to evaluate since the singularities of the integrand along the negative imaginary axis \( t = -i\tau \) are of the type \( (\tau - 1/2w_c^2)^{-1/2} \). In this case the semi-circle part of the contour (in the lower half of the plane) is deformed to go up and then down along the negative imaginary axis so as to exclude all the singularities. When going around a singularity of the above type the function changes sign. Therefore, after sorting the inverse eigenvalues of \( M \) in ascending order \( w_1^{-2} < w_2^{-2} < \ldots \), we have

\[ P_{GOE}(\Gamma) = \frac{1}{\pi^{2\Lambda/2}} \left( \prod_c \frac{1}{w_c} \right) \sum_{m=1}^{\Lambda/2-1} \int_{1/2w_{2m-1}^2}^{1/2w_{2m}^2} d\tau \frac{e^{-\Gamma\tau}}{\prod_{r=1}^{2m-1}(\tau - \frac{1}{2w_r^2}) \prod_{s=2m}^{\Lambda}(\frac{1}{2w_s^2} - \tau)}, \]  
(27)

where for an odd number of channels \( \Lambda \), we define \( 1/2w_{\Lambda+1}^2 \rightarrow \infty \). The integrand of each term on the r.h.s. of (27) is singular at the two endpoints of the integration interval, but this singularity is integrable. For the case of two channels that are in general non-equivalent but correlated, Eq. (27) reduces to

\[ \begin{align*}
P_{GOE}^{\Lambda=2}(\hat{\Gamma}) &= \frac{a_+}{\sqrt{1 - |f|^2}} e^{-a_+^2 \hat{\Gamma}/(1 - |f|^2)} I_0 \left( \frac{a_+ \sqrt{a_+^2 + |f|^2}}{1 - |f|^2} \right),
\end{align*} \]  
(28)

where \( f \) and \( a_\pm \) are defined as before (see following Eq. (24)) and \( I_0 \) is the Bessel function of order zero. The case of equivalent channels is obtained in (28) by substituting \( a_+ = 1 \) and \( a_- = 0 \).

The reference distribution \( P_{GOE}^{(0)} \), defined as before for the case where all \( \Lambda \) channels are equivalent and uncorrelated, is found directly from (22) to be the \( \chi^2 \) distribution in \( \Lambda \) degrees of freedom:\[ P_{GOE}^{(0)}(\Gamma) = \frac{1}{(2w^2)^{\Lambda/2} (\Lambda/2 - 1)!} \Gamma^{\Lambda/2-1} e^{-\Gamma/2w^2}. \]  
(29)

The top panels (a and b) in Fig. 1 show the width distributions for a two-channels lead in the GOE statistics. The left panel is for equivalent channels \( (M_{22}/M_{11} = 1) \) and for various degrees of correlations \( f = 0.25, 0.5, 0.75, \) and \( 0.95 \). The right panel is for uncorrelated \( (f = 0) \) but non-equivalent channels: \( M_{22}/M_{11} = 2, 3, 4, \) and \( 5 \). The bottom panels (c and
d) in Fig. 1 are similar to (a and b) except that they correspond to the GUE case. We note that in all figures we display as $\Gamma$ the normalized total width $\frac{\Gamma}{\Gamma}$.

The correlation matrix in the point contact model is fully determined by $k|\Delta r|$ and the number of channels $\Lambda$. The left panels in Fig. 2 show the GOE width distributions for $k|\Delta r| = 0.25, 1, 4$ and for different number of channels $\Lambda = 2, 4, 6$. The right panels of Fig. 2 show similar results but for the GUE statistics. The deviation of the width distribution from the reference distribution $P(0)(\Gamma)$ which corresponds to equivalent and uncorrelated channels (dashed lines in Fig. 2) becomes larger as the number of channels increases for a given $k|\Delta r|$.

**D. Conductance Peaks Distribution**

To calculate the conductance distribution $P(g)$ in the general case, we assume that the left and right leads are far enough from each other and thus uncorrelated. The left and right leads are characterized by their own correlation matrix $M_l$ and $M_r$, respectively. Under this assumption

$$P(g) = \int d\Gamma_l d\Gamma_r \delta \left( g - \frac{\Gamma_l \Gamma_r}{\Gamma_l + \Gamma_r} \right) P(\Gamma_l) P(\Gamma_r),$$

(30)

where $P(\Gamma)$ is given by (23) in the unitary case and by (27) in the orthogonal case.

The distribution $P(g)$ can be evaluated by the following identity

$$\int_0^\infty d\Gamma_1 \int_0^\infty d\Gamma_2 e^{-\Gamma_1/\delta_1} e^{-\Gamma_2/\delta_2} \delta \left( g - \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \right) = 4g e^{-\left(\frac{1}{\sqrt{\delta_1}} + \frac{1}{\sqrt{\delta_2}}\right)^2} \left[ K_0 \left( \frac{2g}{\sqrt{\delta_1 \delta_2}} \right) + \frac{1}{2} \left( \sqrt{\frac{\delta_2}{\delta_1}} + \sqrt{\frac{\delta_1}{\delta_2}} \right) K_1 \left( \frac{2g}{\sqrt{\delta_1 \delta_2}} \right) \right],$$

(31)

provided $\delta_1, \delta_2 > 0$. To obtain this identity we have used the integral representation of the Bessel function $K_\nu(z) = 1/(2z/\nu) \int_0^\infty dt t^{-\nu-1} e^{-z^2/4t}$.

For the unitary case, Eqs. (23) and (31) give

$$P_{GUE}(g) = 16g \left( \prod_c \frac{1}{v_c^2} \right) \left( \prod_d \frac{1}{w_d^2} \right) \sum_{c,d} \left[ \prod_{c' \neq c} \left( \frac{1}{v_c^2} - \frac{1}{v_{c'}^2} \right) \prod_{d' \neq d} \left( \frac{1}{w_d^2} - \frac{1}{w_{d'}^2} \right) \right]^{-1} \times e^{-\left(\frac{1}{v_c} + \frac{1}{w_d}\right)^2} \left[ K_0 \left( \frac{2g}{v_c w_d} \right) + \frac{1}{2} \left( \frac{v_c}{w_d} + \frac{w_d}{v_c} \right) K_1 \left( \frac{2g}{v_c w_d} \right) \right],$$

(32)

where $v_c^2$ and $w_d^2$ are the eigenvalues of the left and right lead correlation matrices $M_l$ and $M_r$, respectively. The previous published results are special cases of Eq. (18) for one channel leads with $\Gamma' = \Gamma'$ (i.e. $v_1 = w_1$), while the distribution of Ref. 12 is obtained for two (equivalent) channels leads whose matrices are related by an overall asymmetry factor $M' = aM_l$.

A similar calculation for the orthogonal limit gives
whose shape is defined by the image of the unit circle in the complex 
conformal mapping 
by 
c 5, where the nearest-neighbors level spacing distribution 
P converged levels by diagonalizing a matrix of order 1000). This is 
demonstrated in Fig. verified that the corresponding spectrum exhibits GOE-like spectr al fluctuations (we used 
measures the spectral rigidity, are shown.

The parameters 
5, respectively, for symmetric Λ-point leads with \( k |\Delta r| = 0.25, 1, 4 \) and for \( \Lambda = 2, 4, \) and 
6 (the same cases shown in Fig. 3). In analogy to \( P(\Gamma) \), all figures depicting \( P(g) \) display 
the normalized conductance \( g \) defined as \( g/\mathcal{G} \). By comparing Fig. 3 with Fig. 2 we conclude 
that, as \( \Lambda \) increases, the conductance distribution shows stronger deviation from its limiting 

case of uncorrelated equivalent channels (dashed lines) than the width distribution does.

Fig. 4 shows the GOE (left) and GUE (right) conductance peak distribution (33) and 
(32), respectively, for symmetric Λ-point leads with \( k |\Delta r| = 0.25, 1, 4 \) and for \( \Lambda = 2, 4, \) and 
6 (the same cases shown in Fig. 3). In this limit 
the smaller width in (1) and the conductance peak 
\( g \) is proportional to the partial width 
in the dominating lead. In this limit \( P(g) \) is reduced to \( P(\Gamma) \) shown by the dashed lines in 
Fig. 3. The asymmetry effect becomes larger for an increasing number of channels. This 
effect can be noticed by comparing the GOE and GUE cases, since for the same number of 
physical channels \( \Lambda \) the GUE has a larger number of “effective” channels.

IV. DYNAMICAL MODEL

To test the RMT predictions for the statistical distributions, we modeled a quantum dot 
by a system whose classical dynamics is chaotic. The model is the conformal billiard 
whose shape is defined by the image of the unit circle in the complex \( z \)-plane under the 
conformal mapping

\[
w(z) = \frac{z + bz^2 + c e^{i\delta} z^3}{\sqrt{1 + 2b^2 + 3c^2}}.
\] (34)

The parameters \( b, c \) and \( \delta \) control the billiard shape. Eq. (34) ensures that area \( \mathcal{A} \) enclosed 
by \( w(z) \) is normalized to \( \pi \) and is independent of the shape. We analyze the case \( b = 0.2, 
c = 0.2 \) and \( \delta = \pi/2 \), for which the classical phase space is known to be chaotic. We have 
verified that the corresponding spectrum exhibits GOE-like spectral fluctuations (we used 
300 converged levels by diagonalizing a matrix of order 1000). This is demonstrated in Fig. 
4, where the nearest-neighbors level spacing distribution \( P(s) \) and the \( \Delta_3 \) statistics, which 
measures the spectral rigidity, are shown.

To investigate the effect of an external magnetic field, we consider the same billiard 
threaded by an Aharonov-Bohm flux line, which does not affect the classical dynamics. 
The flux is parametrized by \( \Phi = \alpha \Phi_0 \) where \( \Phi_0 \) is the unit flux. We use the same set of 
values for \( b, c, \) and \( \delta \) to insure classical chaotic motion, and choose \( \alpha = 1/4 \) for maximal 
time-reversal symmetry breaking. The statistical tests shown in Fig. 5 confirm that this
choice of $\alpha$ corresponds to the unitary limit. We remark that the $\Delta_3$-statistics is a better measure to distinguish between the GOE and GUE cases than the level spacing distribution $P(s)$ (used in Ref. [20]).

A. Spatial Correlations

The eigenfunction amplitude correlation $C(k|\Delta r|) = \frac{\Psi^*(r)\Psi(r')}{|\Psi(r)|^2}$ was recently investigated thoroughly for the conformal billiard [30]. The results agree fairly well with the theoretical prediction, namely $C(k|\Delta r|) = J_0(k|r-r'|)$ if one averages over the orientation of $\Delta r$. This result is obtained based on semiclassical arguments and the eigenfunctions studied in Ref. [30] were chosen accordingly to be highly excited states (deep in the semiclassical region).

In order to apply this result to quantum dots, further considerations are in order. First, a typical semiconductor quantum dot in the submicrometer range contains several hundred electrons, and it is therefore not obvious that the eigenstates around the Fermi level are necessarily semiclassical. Second, scars associated with isolated periodic orbits give corrections to $C(k|\Delta r|)$ which depend on the orientation of $\Delta r$ and are of order $O(\hbar^{1/2})$. The fluctuations of the spatial correlation of the billiard eigenfunctions were recently studied [37] and found also to be suppressed by $O(\hbar^{1/2})$. These corrections are negligible if one averages over all orientations around a given point $r$, keeping the modulus $|\Delta r|$ fixed, but this is difficult to implement experimentally. At a fixed orientation the fluctuations of the spatial correlations seem to be rather small if $k|\Delta r| \lesssim 3$ so that (19) is a good approximation. For larger values of $k|\Delta r|$, there could be significant fluctuations from (19) but in this region the width and conductance distributions are closer to their limiting case of independent channels and are not very sensitive to the exact correlations.

Our results were obtained by using the billiard eigenfunctions with Neumann boundary conditions where the normal derivative of the wavefunctions vanishes on the boundaries. We analyze eigenfunctions in the vicinity of the 100th excited level which resembles the experimental situation. By moving the points around the circle we generate more statistics and average over orientations. The results are shown in Fig. 6 where the correlations in the model (solid line) compare well with the theoretical result (dashed line) for both cases with and without magnetic flux. The agreement is fair, particularly for $k|\Delta r| \lesssim 5$. For $k|\Delta r| \gg 1$, the deviations from the theoretical value of $C(k|\Delta r|)$ are not important since the channels are weakly correlated and the distributions are very close to those describing uncorrelated channels. Thus, corrections to our analytical findings should not be large, as is supported by the numerical evidence presented below.

In our model studies we imposed Neumann boundary conditions around the entire billiard and not just at the dot-lead boundary. To mimic the experimental situation we would have to impose mixed boundary conditions, which makes the calculations much more computationally intensive. However, we now argue that our simplified situation still provides reasonable results. For extended leads, the length of the dot-lead contact region $D$ must satisfy $kD \gg 1$ in order to support open transverse channels (in dots containing several hundred electrons). Therefore, deviations from $C(k|\Delta r|)$ at the edge of the dot-lead contact region (where our boundary conditions are unrealistic) are averaged out. For point-like
contacts, the physical picture is that the conductance is probing the wavefunction in the vicinity of the constriction (the region that couples the dot to the external lead). We then need to know the characteristic properties of the wavefunction inside the dot where our model is quite satisfactory.

**B. Coupling to Leads and Distributions**

We first studied the point-like contacts model by describing the lead as a sequence of \( \Lambda \) equally spaced points on the boundary of the billiard (in the \( w \)-plane). According to Eq. (19) the correlation matrix \( M \) is then completely determined by

\[
|k|\Delta r| \approx k\delta \theta |w'(r = 1, \theta)|
\]

(where \( \delta \theta \) is the angle that spans the arc between two neighboring points in the \( z \)-plane) and \( \Lambda \). In this model it is easy to generate strong correlations by choosing the points close enough, unlike the (discretized) Anderson model where the channels are weakly correlated even if the lead is composed of nearest neighboring points. The eigenvalues \( w_c^2 \) are found by diagonalizing the matrix \( M \).

In Figs. 7 and 8 we compare for the unitary and orthogonal limits, respectively, the total width distribution \( P(\Gamma) \) obtained by solving the conformal billiard (histograms) with the theoretical predictions (solid lines), for several values of \( |k|\Delta r| = 0.5, 1, 2 \) and \( \Lambda = 2, 4, 6 \). The distributions \( P^{(0)}(\Gamma) \) for equivalent and uncorrelated channels are indicated by the dashed lines, and are just the \( \chi^2 \) distributions in \( \Lambda (2\Lambda) \) degrees of freedom for the GOE (GUE). The agreement between the model and the analytic RMT predictions confirms the validity of the statistical model for a chaotic dot. We observe from Figs. 7 and 8 that for the larger values of \( |k|\Delta r| \), the distributions get closer to those for uncorrelated channels. This is consistent with the decrease in spatial correlations (see Fig. 4). Another interesting observation is that, for a constant \( k|\Delta r| \) (i.e. fixed correlations), the deviation from the limiting case of independent channels becomes larger with an increasing number of channels.

Figs. 9 and 10 show a comparison between the theoretical conductance peaks distributions for symmetric leads, as given by Eqs. (32) and (33) for the unitary and orthogonal cases, respectively, and those calculated for the Africa billiard with symmetric \( \Lambda \)-point leads (\( \Lambda = 2, 4, \) and 6) and for different values of \( |k|\Delta r| \). The dashed lines are again the limiting case of uncorrelated and equivalent leads. Observations that are similar to the ones made above for the width distributions, can be made with respect to the conductance peaks distributions. Comparing the width and conductance peaks distributions, we note that the conductance distribution shows stronger deviation from its limit for uncorrelated equivalent channels than does the width distribution.

We also studied extended leads by taking the contact region of the lead and the dot to have a finite length \( D \approx |w'|\Delta \theta \) on the dot’s boundary (in the \( w \)-plane) where \( w' \) is evaluated at the corresponding angle where the lead is located. In this case the channels are defined by the allowed quantized transverse momenta \( \kappa_c = \pi n_c/D \) with \( n_c = 1, 2, \ldots, \Lambda \), where \( \Lambda = \text{int}[kD/\pi] \). To calculate the partial amplitude for the conformal billiard, the integral in Eq. (2) (defined in the \( w \)-plane) is mapped into an integral along an arc in the \( z \)-plane which is spanned by an angle \( \Delta \theta \)

\[
\gamma_{c\lambda} = \frac{\hbar^2}{2m} \int_{\Delta \theta} d\theta |w'(r = 1, \theta)| \Phi^*_\lambda(\theta) \Psi_\lambda(r = 1, \theta), \quad (35)
\]
where $\Phi_c(\theta) = \sqrt{2/D} \sin(\kappa_c w' |\theta|)$ are the transverse channel wavefunction and for simplicity we have set $k_c P_c = 1$. The resonance eigenfunction $\Psi_\lambda$ is given in terms of its expansion in $e^{i m \theta}$ (with $m = 0, \pm 1, \pm 2, \ldots$

$$
\Psi_\lambda(r = 1, \theta) = N_\lambda \sum_j c_\lambda^j \sqrt{\pi (\gamma_j^2 - |\ell_j - \alpha|^2)} e^{i \ell_j \theta},
$$

(36)

where $N_\lambda$ is a normalization constant, $\gamma_j$ are the zeros of $J'_{|\ell_j - \alpha|}$ and $c_j$ are expansion coefficients as in Ref. 20.

To guarantee that the correlation matrix $M$ in (10) is the same for eigenfunctions of the billiard which belong to different energies, we choose $D$ such that $kD$ = constant and scale the partial amplitude (2) by $k_1/2$. The resulting matrix is

$$
kM = \frac{\hbar^2}{2 m k D} \int_{\Delta \theta} d\theta \int d\theta' |w'(r = 1, \theta)| |w'(r = 1, \theta')| 
\times \sin \left( \frac{\pi n_c}{k D} w' |\theta| \right) J_0 \left( |w'| |\theta - \theta'| \right) \sin \left( \frac{\pi n_c'}{k D} w' |\theta'| \right).
$$

(37)

This scaling is desirable in order to be consistent with the theoretical approach presented above, but experimentally it is very hard to accomplish. Fortunately, this scaling of $D$ is insignificant for present experiments[14,15] that deal with dots containing several hundred electrons $N$. Indeed, from the Weyl formula we have $k_F \propto N^{1/2}$ so that $\delta k_F/k_F = \delta N/2N \ll 1$. The latter inequality is obtained when we estimate $\delta N$ to be the number of observed Coulomb blockade peaks (since each Coulomb blockade peak corresponds to the addition of one electron into the dot). The relative variation of $k_F$ is thus small and can be neglected.

We find that the channels in the extended leads model are weakly correlated and that the average partial widths in the various channels exhibit a moderate variation. In such a case the total width distribution is not very different from the case of uncorrelated equivalent channels. Our model calculations for extended leads are shown in Fig. 11 and are in agreement with the RMT predictions for uncorrelated channels (dashed lines). An interesting effect is that with an increasing number of channels even small deviations in $P(\Gamma)$ give rise to relatively large deviations in $P(g)$, as can be seen in Fig. 11.

**V. CONNECTION TO EXPERIMENTS AND CONCLUSIONS**

We have discussed both the cases of orthogonal and unitary symmetries. To relate to actual experimental situations, it is important to estimate the minimal strength of the magnetic field $B_c$ which ensures complete time-reversal symmetry breaking. For a ballistic electron[39,40,20] $B_c A \propto \sqrt{\tau_{cr}/\tau_0}$, where $\tau_{cr}$ and $\tau$ are, respectively, the time it takes the electron to cross the dot and the Heisenberg time $\tau = \hbar/\Delta$. For an electron at the Fermi energy $B_c A \propto N^{-1/4} \Phi_0$, where $N$ is the number of electrons in the dot. The proportionality factor is non-universal and depends on the exact geometry of the dot. In a semiclassical analysis[39,40] it can be expressed in terms of classical quantities. For the dots used in some recent experiments,[41,42] $B_c$ is of order of a few mT. Such small values of $B_c$ do not alter significantly the classical dynamics of the electron[41] and our assumption that the correlation
$C(k|\Delta r|)$ is unchanged is justified. Nevertheless, these small variations in the magnetic field have appreciable quantum mechanical effects, i.e. the crossover from orthogonal to unitary symmetry.

In conclusion, we have derived closed expressions for the width and conductance peak distributions in quantum dots in the Coulomb blockade regime. The main assumption is that the electron’s dynamics is chaotic for a ballistic dot or weakly diffusive for a disordered dot. For given correlation matrices that characterize the left and right leads, these distributions are universal and distinct for conserved and broken time-reversal symmetry. While recent experiments have measured the conductance distributions in symmetric one-channel leads, it would be interesting to measure and compare with theory the conductance distributions in more general situations of dots with multi-channel leads.

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FIGURES

FIG. 1. Total width distributions $P(\Gamma)$ for a two-channel lead. Panels (a), (b) correspond to the orthogonal symmetry and (c), (d) to the unitary symmetry. $P(\Gamma)$ for equivalent but correlated channels with $f = 0.25, 0.5, 0.75$ and $0.95$ are shown in (a) and (c), while uncorrelated but non-equivalent channels $M_{22}/M_{11} = 2, 3, 4, 5$ are shown in (b) and (d). Descending values of $f$ (or $M_{22}/M_{11}$) correspond to distributions that extend to larger values of $\Gamma$.

FIG. 2. Total width distributions for a Λ-point lead and $k|\Delta r| = 0.25, 1$ and 4 in a quantum dot with orthogonal symmetry (GOE) and with unitary symmetry (GUE). (a) GOE; Λ = 2; (c) GOE; Λ = 4; (e) GOE; Λ = 6; (b) GUE; Λ = 2; (d) GUE; Λ = 4; (f) GUE; Λ = 6. The dashed lines correspond to uncorrelated and equivalent channels. Increasing values of $k|\Delta r|$ correspond to curves which approach the case of uncorrelated channels.

FIG. 3. Same as Fig. 2 but for the conductance peak distributions $P(g)$ in dots with symmetric Λ-point leads.

FIG. 4. Conductance peak distributions $P(g)$ for asymmetric four-point leads with $k|\Delta r| = 1$ and an asymmetry factor of $a = 1$ and $a = 10$. (a) GOE; (b) GUE. The dashed curves describe the limit $a \to \infty$ where $P(g)$ reduces to $P(\Gamma)$.

FIG. 5. The nearest neighbors level spacing distribution $P(s)$ and the $\Delta_3$-statistics for the conformal billiard with $b = 0.2, c = 0.2$ and $\delta = \pi/2$. We consider the states between the 50th and the 350th. Left: no magnetic flux ($\alpha = 0$). Right: with magnetic flux of $\alpha = 1/4$.

FIG. 6. The spatial wavefunction correlation $C(k|\Delta r|)$ calculated for the conformal billiard (squares) compared with the theoretical prediction $J_0(k|\Delta r|)$ (solid line). Panel (a) displays the case where $\alpha = 0$ and (b) corresponds to $\alpha = 1/4$.

FIG. 7. Total width distributions $P(\Gamma)$ for the unitary case for several values of $k|\Delta r| = 0.5, 1, 2$ and for various number of channels $\Lambda = 2, 4, 6$. The solid lines are the theoretical distributions, while the dashed lines correspond to uncorrelated and equivalent channels. The histograms are the results from the Africa billiard ($b = 0.2, c = 0.2, \delta = \pi/2$) where a magnetic flux line ($\alpha = 1/4$) breaks the time-reversal symmetry.

FIG. 8. Total width distributions $P(\Gamma)$ for conserved time-reversal symmetry (GOE). Conventions are as in Fig. 7, with solid lines describing the orthogonal prediction.

FIG. 9. Conductance peak distributions $P(g)$ for the unitary symmetry. The histograms display the results obtained from the Africa billiard for symmetric Λ-point leads and different values of $k|\Delta r|$. The solid lines are the RMT prediction and the dashed lines correspond to uncorrelated and equivalent channels. The cases presented are the same as in Fig. 7.

FIG. 10. Same as Fig. 9 but for the orthogonal symmetry where the theoretical distribution is given by Eq. (33).
FIG. 11. Comparison of conformal billiard results for extended leads with \( \text{int}[kD/\pi] = 6 \) (histograms) and theoretical predictions. The panels represent total width distribution \( P(\Gamma) \) for orthogonal (a) and unitary (b) limits and conductance distribution \( P(g) \) for orthogonal (c) and unitary (d) limits. The dashed lines correspond to uncorrelated equivalent channels.
\[ C(\frac{k}{|\Delta r|}) \]

(a)

(b)
$P(\Gamma)$ vs $\Gamma$

- $\Lambda = 2$, $k\Delta r = 2.0$
- $\Lambda = 4$, $k\Delta r = 2.0$
- $\Lambda = 6$, $k\Delta r = 2.0$

- $\Lambda = 2$, $k\Delta r = 1.0$
- $\Lambda = 4$, $k\Delta r = 1.0$
- $\Lambda = 6$, $k\Delta r = 1.0$

- $\Lambda = 2$, $k\Delta r = 0.5$
- $\Lambda = 4$, $k\Delta r = 0.5$
- $\Lambda = 6$, $k\Delta r = 0.5$
