Can the effect of shear strength spatial variability be summarized as the pure spatial average?

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**ABSTRACT**

The purpose of this study is to examine in more detail under what conditions would spatial averaging over some prescribed region be sufficient to reproduce the response statistics arising from a spatially varying field. The spatially variable undrained shear strength will be first simulated by a random field. The active lateral force of the spatially variable soil mass to a retaining wall is simulated using the random field finite element method. This active lateral force is the actual active lateral force exhibited by the spatially variable soil. This actual active lateral force is compared to the active lateral force of a homogeneous soil mass whose shear strength equal to the spatial average over a prescribed area/line of interest. Based on these numerical studies, it is observed that the actual active lateral force (a random variable) and the spatial average active lateral force (a second random variable) are at most equal in “distribution”, but not “almost everywhere”.

**Key words:** spatial variability, spatial average, shear strength, random field

1 INTRODUCTION

Soil-structure interaction occurs over a finite volume of soil (influence zone). For a spatially variable soil mass, it is natural to examine if an equivalent homogeneous soil mass exist that can reproduce the same response statistics. It is equally natural to assume that the governing soil parameter in this homogeneous soil mass is the the spatial average (Vanmarcke 1977, 1984) over the influence zone. Fenton and Griffiths (2005) studied the settlement of a footing on a three-dimensional (3D) spatially variable soil mass with this practical objective in mind. They found that the settlement can be effectively simulated by considering the geometric average of the elastic modulus random field within a prescribed volume under the footing. Honjo and Otake (2013) studied the capacity of a footing on a two-dimensional (2D) spatially variable soil mass. They found that the capacity for the footing can be effectively simulated by considering the spatial average of the shear strength random field within a prescribed area under the footing.

In contrast to the spatial averaging over a prescribed region, Ching and Phoon (2013) found that the shear strength of a laboratory test specimen can NOT be effectively simulated by considering spatial averaging over any prescribed area or curve. Instead, they found that the shear strength can be effectively simulated by considering the average over the critical slip curve. Note that the critical difference here is that the critical slip curve is not a prescribed curve, but an emergent curve that depends on the random field realization. Hu and Ching (2015) also found that the active lateral force for a retaining wall can NOT be effectively simulated by considering the spatial average over any prescribed area or line.

Fenton and Griffiths (2005) and Honjo and Otake (2013) focused on the global response of a footing (settlement, capacity). Ching and Phoon (2013) and Hu and Ching (2015) focused at a more local level on the strength mobilized along an emergent critical slip curve. It is difficult to explain why these mechanical responses, which appear similar, would produce diametrically opposite conclusions. There is a strong practical motivation to examine the general validity of spatial averaging, because it is obviously easier to carry out reliability-based design using a random variable (spatial average) than a random field. The purpose of this study is to examine in more detail under what conditions would spatial averaging over some prescribed region be sufficient to reproduce the response statistics arising from a spatially variable field. A retaining wall problem is adopted to demonstrate this. The spatially variable field is restricted to the undrained shear strength in this paper. The method adopted by this paper is straightforward. Two sets of finite element method (FEM) analyses will be conducted. The first set considers a spatially variable soil mass...
whose shear strength is simulated by a random field. The outcome of this first set of random field finite element method (RFEM) is the actual active lateral force. It is the reference of the spatially variable soil mass. The same random field is then averaged over a prescribed area or line of interest to obtain the spatial average. The second set of FEM then considers a homogeneous soil mass whose shear strength is equal to the spatial average. Then, the outcome of this second set of FEM will be referred to as the spatial average active lateral force. It is then compared to the actual active lateral force simulated by the RFEM. The comparison will be made on the following two levels: Level I compares the probability distributions of the two sets of responses, whereas Level II compares the two sets of responses on the 1:1 line.

2 RANDOM FIELD AND ITS SIMULATION

In this study, the only random soil property is the soil shear strength \( \tau(x,z) \). The shear strength \( \tau(x,z) \) at a point in the soil mass is simulated by a random field, where \( x \) and \( z \) are respectively the horizontal and vertical coordinates. The friction angle is taken to be \( \theta \) for simplicity, i.e., \( \tau(x,z) = c(x,z) \), where \( c \) is the cohesion or undrained shear strength. The random cohesion \( c(x,z) \) is simulated as a stationary lognormal random field with inherent mean = \( \mu \) and inherent standard deviation = \( \sigma \). The coefficient of variation (COV) of this random field is equal to \( \sigma/\mu \). A stationary lognormal random field can be simulated by taking exponential of a stationary Gaussian random field. To define the correlation structure in \( c(x,z) \) between two locations with horizontal distance = \( \Delta x \) and vertical distance = \( \Delta z \), the single exponential auto-correlation model is considered (Vanmarcke 1977, 1984):

\[
\rho(\Delta x, \Delta z) = \exp\left(-\frac{2}{\Delta x/\delta_x} - 2 \frac{\Delta z/\delta_z}{2}\right)
\]

where \( \delta_x \) and \( \delta_z \) are the horizontal and vertical scales of fluctuation (SOFs), respectively. Jha and Ching (2013) developed the Fourier series method (FSM) for simulating stationary normal random fields (point process). A 2D stationary normal random field \( W(x,z) \) over a domain of size \( L_x \times L_z \) can be simulated by

\[
W(x,z) = \mu + Re\left[\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (a_{mn} + ib_{mn}) \exp\left(i2\pi \frac{m x}{L_x} + i2\pi \frac{n z}{L_z}\right)\right]
\]

where \( Re[.] \) denotes the real part of the enclosed complex number; \( a_{mn} \) and \( b_{mn} \) are independent zero-mean normal random variables with variance \( \sigma_{mn}^2 \) given by (Jha and Ching 2013)

\[
\sigma_{mn}^2 = \frac{\sigma^2}{q_r q_s} \left[\frac{1 - \exp(-q_r)}{1 + m^2 \pi^2 / q_r^2}\right] \times \left[\frac{1 - \exp(-q_s)}{1 + n^2 \pi^2 / q_s^2}\right]
\]

where \( q_r = L_x/\delta_x \) and \( q_s = L_z/\delta_z \). Besides simulating the point process of a normal random field, the FSM is also able to directly simulate the spatial average of the normal random field over a prescribed rectangular region in 2D and a prescribed line segment in 2D (Jha and Ching 2013).

3 RANDOM FIELD FINITE ELEMENT MODEL

The active lateral force (denoted by \( P_a \)) of a retaining wall can be simulated by the RFEM (Fenton et al. 2005; Hu and Ching 2015). The adopted RFEM model for the retaining wall is shown in Fig. 1a. In the RFEM, \( P_a \) can be simulated as the limiting soil force acting on the retaining wall when the wall moves away from the soil. The wall is assumed to be frictionless. The bottom boundary of the entire model is fixed, and the right boundary is composed of rollers. The height of the retaining wall is \( H \) is 5 m. The unit weight of the soil is equal to 20 kN/m\(^3\), the Young’s modulus is equal to 40 MN/m\(^2\), and the Poisson ratio is 0.3. The friction angle \( \phi = 0^\circ \). The cohesion \( c(x,z) \) is simulated as a lognormal random field using the FSM. The cohesion in each element is taken to be the spatial average of the \( c(x,z) \) random field over that element. There are two steps in the RFEM. The first step is geostatic equilibrium to build up the in-situ geostatic stress field, and the second step is to apply a horizontal wall displacement (away from the soil mass) to a failure state. \( P_a \) is defined as the lateral force when the displacement reaches 0.02H. This \( P_a \), denoted by \( P_a^m \), is the actual \( P_a \) for the RFEM. This RFEM has been analyzed in Hu and Ching (2015).

![Fig. 1. (a) RFEM model for the retaining wall and (b–d) three different spatial averaging domains](image-url)
area in Fig. 1c; and (c) geometrically averaging along the 45° line in Fig. 1d. Similarly, the spatial averages of the same c(x,z) random field over the above three area/line are first simulated using the FSM, and a homogeneous FEM is simulated to obtain the spatial average responses. The three spatial average responses are denoted by $P^{\text{RA}}$, $P^{\text{TA}}$ and $P^{\text{LA}}$, respectively (RA, TA, and LA mean ‘rectangular average’, ‘triangular average’, and ‘line average’, respectively).

4 ACTUAL VERSUS SPATIAL AVERAGE RESPONSES

The comparison between the actual ($P_a^m$) and spatial average responses ($P_a^{\text{RA}}$, $P_a^{\text{TA}}$ and $P_a^{\text{LA}}$) can be conducted on the following two levels (the number of random field realizations = 100 for all cases):

Level I – Comparison between the probability distributions of the actual and spatial average responses. The quantile-quantile (Q-Q) plot between the actual (from the RFEM) and spatial average responses (from the homogeneous FEM) will be used to compare the probability distributions. The good-of-fit is checked by the Kolmogorov-Smirnov test (K-S test). Based on the resulting p-values, the K-S test can determine whether the null hypothesis ($H_0$) that the two sets of responses are identically distributed or not. If the p-value is less than 0.05 (significance level), $H_0$ can be rejected at the customary 95% level of confidence. Otherwise, there is insufficient evidence to reject $H_0$.

In the Level I comparison, two statistics are also computed: (a) the ratio in the mean values $r_m = (\text{mean of the spatial average response})/(\text{mean of the actual response})$ and (b) the ratio in the COVs $r_{\text{COV}} = (\text{COV of the spatial average response})/(\text{COV of the actual response})$. If the two probability distributions are identical, $r_m = r_{\text{COV}} = 1$.

Level II – Comparison between the actual and spatial average responses on the 1:1 line. This level of comparison can identify whether the two sets of responses are equal to each other or not. The root mean square (RMS) of the normalized distance to the 1:1 line is used to quantify the deviation to the 1:1 line: $\text{RMS} = \left(\sum_{i=1}^{n} (d_i^2)\right)^{0.5}$, where $d_i = (\text{spatial average response} – \text{actual response})/\text{actual response}$ is the normalized distance for the i-th data point to the 1:1 line. RMS equals zero if and only if the two sets of responses lie exactly on the 1:1 line. Level I comparison is more strict than Level I. If the two sets of responses lie on the 1:1 line, they must have identical probability distributions – the fulfillment of Level II implies the fulfillment of Level I. However, the converse is not true.

For the Level I comparison, Fig. 2a and 2b show the Q-Q plots between the $P_a^m$ and $P_a^{\text{RA}}$ samples for two cases with $H = 5$ m, $\mu = 20$ kN/m², COV = 0.3 and isotropic SOF ($\delta/H = 0.5$ and 300). As mentioned previously, $P_a^{\text{RA}}$ refers to the spatial average response for the rectangular area (see Fig. 2b), and $P_a^m$ is the actual response. For the case with $\delta/H = 300$, the large p-value for the K-S test (Fig. 2b) indicates that the null hypothesis $H_0$ that the two sets of responses are identically distributed cannot be rejected. The ratio in the mean value ($r_m$) and the ratio in the COVs ($r_{\text{COV}}$) are both very close to 1.

The Level II comparison for the case with $\delta/H = 300$ further shows that the two sets of responses lie very close to the 1:1 line (RMS = 0.02) (Fig. 2d). This result is reasonable because the case with $\delta/H = 300$ is close to a homogeneous case.

![Fig. 2. Level I and II comparisons between the actual and spatial average responses for $\delta/H = 0.5$ and $\delta/H = 300$.](image)

However, for the case with $\delta/H = 0.5$, the small p-value (nearly zero) in the Level I comparison (Fig. 2a) indicates that $H_0$ can be rejected (i.e., $P_a^m$ and $P_a^{\text{RA}}$ are not identically distributed). Because $r_m = 0.86$, $P_a^{\text{RA}}$ has a mean value that is 1 – 86% = 14% lower than the mean of $P_a^m$ (P_{a^{\text{RA}}} is unconservative). The Level II comparison (Fig. 2c) further shows that $P_a^m$ and $P_a^{\text{RA}}$ are quite different: they are not close to the 1:1 line (RMS = 0.24).

Based on the results of the above Level I and Level II comparisons, it can be concluded that the spatial average is effective to characterize the shear strength spatial variability for $\delta/H = 300$ but not effective for $\delta/H = 0.5$. Moreover, it is on the unconservative side for $\delta/H = 0.5$. More detailed comparison results are present below.

Consider the case with $H = 5$ m, $\mu = 20$ kN/m², COV = 0.3 and isotropic SOF ($\delta/H = 0.1, 0.5, 2, 10, 50, 300$). As mentioned previously, three spatial averages
and $r_{\mu}$ and $r_{\text{COV}}$ are close to 1 for these cases, although some slight deviations from 1 are observed for $\delta/H > 2$ and 10. Even though the Level I comparison indicates that $\left(P^{\text{RA}}_a, P^{\text{TA}}_a, P^{\text{LA}}_a\right)$ and $P^m_a$ are roughly identically distributed for the four cases with $\delta/H > 1$, the large RMS values in the Level II comparison indicate that the $\left(P^{\text{RA}}_a, P^{\text{TA}}_a, P^{\text{LA}}_a\right)$ versus $P^m_a$ data are not close to the 1:1 line for the three cases with $\delta/H = 2$, 10, and 50. This is illustrated in Fig. 3, which shows the $P^m_a$ versus $P^m_a$ plot for the case with $\delta/H = 2$. It is clear that although $P^m_a$ and $P^m_a$ are roughly identically distributed, they are not close to the 1:1 line. That is, these three cases fulfill the Level I requirement but do not fulfill the Level II requirement. The only case that fulfills both requirements is the one with $\delta/H = 300$.

Table 1. Level I and II comparison results for various $\delta/H$

| $\delta/H$ | Level I | Level II |
|------------|---------|---------|
|            | p-value | $r_{\mu}$ | $r_{\text{COV}}$ | RMS |
| 0.1        | 0.00 (reject) | 0.85 | 0.68 | 0.17 |
| 0.5        | 0.00 (reject) | 0.86 | 1.02 | 0.24 |
| 2          | 0.68 (do not reject) | 0.92 | 1.07 | 0.46 |
| 10         | 0.96 (do not reject) | 0.97 | 1.02 | 0.17 |
| 50         | 1.00 (do not reject) | 1.00 | 1.00 | 0.13 |
| 300        | 1.00 (do not reject) | 1.00 | 1.00 | 0.02 |

| $\delta/H$ | Level I | Level II |
|------------|---------|---------|
|            | p-value | $r_{\mu}$ | $r_{\text{COV}}$ | RMS |
| 0.1        | 0.00 (reject) | 0.86 | 1.01 | 0.18 |
| 0.5        | 0.02 (reject) | 0.86 | 1.21 | 0.35 |
| 2          | 0.56 (do not reject) | 0.94 | 1.14 | 1.04 |
| 10         | 0.79 (do not reject) | 0.96 | 1.02 | 0.34 |
| 50         | 1.00 (do not reject) | 1.00 | 0.99 | 0.22 |
| 300        | 1.00 (do not reject) | 1.00 | 1.00 | 0.06 |

| $\delta/H$ | Level I | Level II |
|------------|---------|---------|
|            | p-value | $r_{\mu}$ | $r_{\text{COV}}$ | RMS |
| 0.1        | 0.03 (reject) | 0.95 | 1.01 | 0.12 |
| 0.5        | 0.34 (do not reject) | 0.93 | 1.21 | 0.19 |
| 2          | 0.99 (do not reject) | 0.97 | 1.14 | 0.21 |
| 10         | 0.99 (do not reject) | 0.99 | 1.02 | 0.17 |
| 50         | 1.00 (do not reject) | 1.00 | 0.99 | 0.05 |
| 300        | 1.00 (do not reject) | 1.00 | 1.00 | 0.01 |

For the case with $\delta/H = 0.1$, $H_0$ cannot be rejected in the Level I comparison, and $r_{\mu}$ is noticeably less than 1. This indicates that the means of $\left(P^{\text{RA}}_a, P^{\text{TA}}_a, P^{\text{LA}}_a\right)$ are smaller than the mean of $P^m_a$. $\left(P^{\text{RA}}_a, P^{\text{TA}}_a, P^{\text{LA}}_a\right)$ are unacceptable (RMS < 0.1) only for the case with $\delta/H = 300$ (there is an exception: RMS = 0.05 for $P^{\text{LA}}_a$ with $\delta/H = 50$).

4.1 Anisotropic case

The above results focus on isotropic random fields ($\delta_x = \delta_y = \delta$). It is interesting to compare the actual and spatial average responses under anisotropic random fields ($\delta_x \neq \delta_y$), which are common in reality. An anisotropic case will be examined in this section. More detailed comparison results are present below.

Consider the case with $H = 5$ m, $\mu = 20$ kN/m², COV = 0.3 and anisotropic SOF ($\delta_x/H = 0.1$ and $\delta_y/H = 0.5$, 2, 10, 50, 300). Similarly, three spatial averages ($P^{\text{RA}}_a, P^{\text{TA}}_a, P^{\text{LA}}_a$) are also considered. Table 2 shows the comparison results. For the three cases with $\delta/H > 2$, $H_0$ cannot be rejected in the Level I comparison (there is an exception: p-value = 0.047 for $P^{\text{TA}}_a$ with $\delta_x/H = 300$). $r_{\mu}$ and $r_{\text{COV}}$ are close to 1 for these cases, although some deviations from 1 are observed. Even though the Level I comparison indicates that $\left(P^{\text{RA}}_a, P^{\text{TA}}_a, P^{\text{LA}}_a\right)$ and $P^m_a$ are roughly identically distributed for the three cases with $\delta/H > 2$, the large RMS values in the Level II comparison indicate that the $\left(P^{\text{RA}}_a, P^{\text{TA}}_a, P^{\text{LA}}_a\right)$ versus $P^m_a$ data are not close to the 1:1 line for the three cases. It is clear that although $P^m_a$ and $P^m_a$ are roughly identically distributed, they are not close to the 1:1 line. That is, these three cases fulfill the Level I requirement but do not fulfill the Level II requirement, even for the cases with $\delta_x/H = 300$.

In summary, for the Level I point of view, $\left(P^{\text{RA}}_a, P^{\text{TA}}_a, P^{\text{LA}}_a\right)$ are acceptable representations (p-value > 0.05) for $P^m_a$ for the cases with $\delta/H > 1$ because they are roughly identically distributed. However, for the Level II point of view, none of them is acceptable.
The results in Table 3 imply the following conclusions. If the goal is to maintain the correct probability distribution of the actual response, the spatial average of the spatially variable shear strength over a prescribed region may be an acceptable representation for the reality (spatially variable field) as long as both \( \delta_s/H \) and \( \delta_r/H \) are sufficiently large (> 1). However, if the goal is to obtain the correct value of the actual response, the spatial average of the spatially variable shear strength is acceptable only when both \( \delta_s/H \) and \( \delta_r/H \) are very large (close to homogeneity).

Table 3. Cases that fulfill the Level I and Level II requirements

| Case     | Fulfill Level I | Fulfill Level II |
|----------|----------------|-----------------|
| Isotropic \( (\delta_r = \delta_s = \delta) \) | \( \delta/H = 2, 10, 50, 300^{(a)} \) | \( \delta/H = 300^{(b)} \) |
| Anisotropic \( (\delta_r \neq \delta_s) \) | \( \delta_r/H = 10, 50^{(c)} \) | none |

Note: (a) Isotropic case for \( P_{a}^{m} \) with \( \delta/H = 0.5 \) also fulfills the Level I requirement. (b) Isotropic case for \( P_{a}^{RA} \) with \( \delta/H = 50 \) also fulfills the Level II requirement. (c) Anisotropic cases for \( P_{a}^{RA} \) with \( \delta_r/H = 2 \) and 300, \( P_{a}^{m} \) with \( \delta_s/H = 300, P_{a}^{LA} \) with \( \delta_s/H = 0.5 \) and 2 also fulfills the Level I requirement.

Based on the above observations, we must point out that the Level I requirement is necessary but not sufficient. For a spatial average to be an accurate representation for the spatially variable shear strength, the Level I requirement must be fulfilled. However, if a spatial average fulfills the Level I requirement, it does not mean that the spatial average is an accurate representation. Consider an inappropriate spatial average and isotropic case with \( \delta/H = 2 \). The spatial average is taken over a rectangular region that is remote from the wall (see the red rectangular region in Fig. 1b) to obtain \( P_{a}^{RA} \). Figure 4 shows the Q-Q plot for \( P_{a}^{RA} \) and \( P_{a}^{m} \). \( P_{a}^{RA} \) and \( P_{a}^{m} \) are roughly identically distributed. However, it is inappropriate to assert that such a spatial average is an accurate representation, because the prescribed rectangular region is remote and can hardly affect the actual failure of the wall. The Level II comparison is more sensible for this case (see Fig. 4b): \( P_{a}^{RA} \) and \( P_{a}^{m} \) are not close to the 1:1 line. In fact, they are independent of each other. It is inappropriate to assert that such \( P_{a}^{RA} \) is an accurate representation for \( P_{a}^{m} \). In the same sense, the spatial average over the rectangular region right next to the wall (Fig. 1b) is also not 100% appropriate. Figure 3 shows the Q-Q plot and the 1:1 plot for this case: \( P_{a}^{RA} \) and \( P_{a}^{m} \) are roughly identically distributed but they are not very close to the 1:1 line. It is not 100% appropriate to assert that the \( P_{a}^{RA} \) based on this rectangular average is an accurate representation for \( P_{a}^{m} \). In our opinion, the Level II comparison is more suitable and meaningful than the Level I comparison. However, if we stick to the Level II comparison, \( P_{a}^{RA} \) is an accurate representation for \( P_{a}^{m} \) only for the isotropic
case with $\delta H = 300$.

Fig. 4. Comparisons between the actual and spatial average response (average over the red rectangular in Fig. 1a and isotropic case with $\delta H = 2$)

5 CONCLUSIONS

The purpose of this study is to examine in more detail under what conditions would spatial averaging over some prescribed region be sufficient to reproduce the response statistics arising from a spatially varying field. The answer is: it depends. If the goal is to maintain the probability distribution of the actual response (the Level I requirement), the answer is probably “Yes, as long as the scale of fluctuation is large enough”. Here “large enough” means the scale of fluctuation should not be smaller than the height of the retaining wall. If the goal is to maintain the correct value of the actual response (the Level II requirement), the answer is probably “No, unless both of the horizontal and vertical scales of fluctuation are very large (close to homogeneity)”. However, we argue that the Level II requirement is more sensible than Level I, because the Level I requirement is necessary but not sufficient.

This study is preliminary, but it has revealed complications to the attractive concept of smearing a spatial varying field into a homogeneous spatial average over a prescribed domain. The applicability of smearing merits more research, given its potential usefulness in reliability-based design.

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