Fission of $^{240}$Pu with Symmetry-Restored Density Functional Theory

P. Marević and N. Schunck

Nuclear and Chemical Sciences Division, Lawrence Livermore National Laboratory, Livermore, CA 94551, USA

(Dated: April 24, 2020)

Nuclear fission plays an important role in fundamental and applied science, from astrophysics to nuclear engineering, yet it remains a major challenge to nuclear theory. Theoretical methods used so far to compute fission observables rely on symmetry-breaking schemes where basic information on the number of particles, angular momentum, and parity of the fissioning nucleus is lost. In this work, we analyze the impact of restoring broken symmetries in the benchmark case of $^{240}$Pu.

Introduction. Nuclear fission, the process of splitting an atomic nucleus into two or more fragments, is a key ingredient for modeling nucleosynthesis, as it prevents the production of superheavy elements and its products (fission fragments, neutrons, and photons) impact the astrophysical reaction rates [1–5]. Detailed knowledge of the fission channels (spontaneous, neutron-induced, etc.) of selected actinide nuclei is also at the heart of important societal applications in medicine, energy production, nuclear forensics and safeguards, etc. [6]. In spite of recent advances in experimental techniques [7–8], measurements are not always possible and theoretical simulations that model the entire process leading to the formation and decay of fission fragments are mandatory.

Despite formidable efforts over the past eighty years, a fully microscopic description of the fission phenomenon based on nuclear forces among protons and neutrons and quantum many-body methods remains a challenge [9–10]. Nuclear density functional theory (DFT) is currently the only fully quantum-mechanical framework that can be used to compute fission observables, such as spontaneous fission half-lives [11] or fission fragment distributions [12–18]. Time-dependent DFT provides a natural framework to explain the energy sharing among the fragments [19–20], which is key to predicting their deexcitation.

Following the initial insight of Bohr and Wheeler, most DFT-based approaches to fission are built upon the assumption that a small set of collective degrees of freedom (typically related to the deformation of the nuclear shape) drive the fission process [10–22]. This description is formalized through the concept of spontaneous symmetry breaking: the intrinsic nuclear density does not conserve symmetries of the nuclear Hamiltonian [23–24]. In particular, the geometrical deformation of a nucleus is manifested by the breaking of rotational, axial, or reflection symmetry; nuclear superfluidity [25] by the breaking of particle number symmetry, etc. The corresponding potential energy surfaces (PES) encode the total energy as a function of order parameters associated with breaking each symmetry. They can be used to infer important quantities of interest, from tunneling probabilities for spontaneous fission half-lives [11], to the determination of initial states for time-dependent approaches [19–20], or basis states for quantum configuration mixing with the generator coordinate method (GCM) [26–27]. The most advanced PES for fission studies are based on solving the Hartree-Fock-Bogoliubov (HFB) equation with Skyrme [28–29], Gogny [30–31], or relativistic [32–33] functionals.

However, such approaches conceal the basic information on quantum numbers related to each broken symmetry, such as the particle number, angular momentum, and parity of a nucleus. Restoring these symmetries is especially important to model different fission channels. For example, symmetry-breaking theory is incapable of distinguishing between the neutron-induced fission of $^{235}$U and the photofission of $^{236}$U [34]. In both cases, the compound nucleus is the same, $^{236}$U, but the spin-parity distribution can be substantially different since $^{235}$U(n,f) involves coupling the $^{235}$U ground-state angular momentum $J = 7/2$ with the spin distribution of the neutron beam, while $^{236}$U($\gamma,f$) couples the spin 1 of the photon with $J = 0$ of an even-even nucleus. Symmetry restoration techniques are also essential to obtain more realistic estimates of fission fragment characteristics, as was shown in the simplest case of particle number restoration [16–55]. Finally, correlation energies induced by symmetry restoration modify the overall PES, which could impact fission dynamics.

With the exception of a couple of pioneering works [56–57], there has been no attempt at examining the impact of symmetry restoration in the context of fission. In addition to formal difficulties with symmetry restoration for standard functionals [24], the computational cost of probing a large number of extremely deformed configurations in heavy nuclei is prohibitively high, especially when simultaneously restoring multiple symmetries. In fact, symmetry-breaking PES can usually only be computed by employing large harmonic oscillator (HO) bases with many incomplete shells which are not closed under spatial rotations and for which conventional algorithms of rotational symmetry restoration are inapplicable [38].

In this work, we implement for the first time the technique of rotational symmetry restoration in incomplete bases originally proposed in [38], and perform the first symmetry restoration in $^{240}$Pu from the ground state to scission. High-performance computing capabilities enable us to quantify the effect of particle number, angu-
lar momentum, and parity projections on the underlying PES and on the fission fragment mass distributions. Method. Symmetry-restored DFT is a two-step method. In the first step, we generated a set of axially-symmetric HFB configurations with the HFBTHO package [39], using the SkM* parameterization of the Skyrme energy functional [40], a mixed volume-surface contact pairing force [41], and constraints on the values of the quadrupole ($q_{20}$) and octupole ($q_{30}$) moments. These quantities correspond to the elongation and the mass asymmetry of a nuclear shape, respectively, and arguably represent the most pertinent collective degrees of freedom for describing the fission phenomenon. The HFB equations were solved by expanding the solution in the HO basis. In order to ensure a proper convergence, $N_{\text{max}} = 31$ incomplete shells with the corresponding lowest $N_{\text{osc}} = 1100$ oscillator states were taken into account.

In the next step, collective correlations related to the restoration of symmetries were incorporated by projecting the HFB configurations onto good values of angular momentum $J$, particle numbers ($N, Z$), and parity $\pi$. The projected kernels $K_{q}^{J^*NZ}$ play the central role in this procedure,

$$ K_{q}^{J^*NZ} = \int \sum_{\varphi_{\omega}, \tau} \mathcal{O}_{\varphi_{\omega}, \tau}^{q, J^*NZ}, $$

where $\int_{\beta} = \frac{2J+1}{2} \int_{0}^{\pi} d\beta \sin \beta \mathrm{d}J_{00}(\beta)$ denotes integration over the rotational angle $\beta$ with the small Wigner matrix $d_{J_{00}}^{(J)}(\beta)$ as the corresponding weight, while $\sum_{\varphi_{\omega}, \tau} \equiv \sum_{\varphi_{\omega}, \tau} \int_{0}^{\pi} e^{-i N_{\varphi} \varphi_{\omega}} e^{-i Z_{0} \varphi_{\tau}} \mathrm{d}\varphi_{\omega} \mathrm{d}\varphi_{\tau}$ denotes Fomenko sums [42] over gauge angles $\varphi_{\omega} = l_{\omega} \frac{\pi}{12}$ ($l_{\omega} = 0, ..., N_{\varphi} - 1$) for neutrons ($\tau = n$) and protons ($\tau = p$). For the sake of compactness, we introduce an abbreviation for rotational and gauge angles, $x \equiv \{\beta, \varphi_{\omega}, \varphi_{\tau}\}$. The integrand of Eq. (1) can then be explicitly written as

$$ \mathcal{O}_{q}^{x} = \langle \Phi(q) | \hat{O} e^{-i \beta J_{y} + i \varphi_{\omega} N + i \varphi_{\tau} Z} | \Phi(q) \rangle, $$

where $e^{-i \beta J_{y} + i \varphi_{\omega} N + i \varphi_{\tau} Z} \equiv \hat{R}$ is the rotation operator, and $J_{y}, N,$ and $Z$ correspond to the $y$-component of the total angular momentum, the neutron number, and the proton number operators, respectively. The norm overlap kernel $N_{q}^{J^*NZ}$ is obtained by using Eq. (2) with the identity operator, $\hat{O} = 1$, and the Hamiltonian kernel $H_{q}^{J^*NZ}$ by using it with the nuclear Hamiltonian $\hat{H}$. When computing large-scale PES, it is customary to improve convergence by truncating and adjusting at each point $q$ the characteristics of the underlying HO basis. However, the resulting basis is not closed under spatial rotations. Formally, given the rotational symmetry transformation $T$ and a single-particle basis defined by the creation and annihilation operators $\{c_{i}^{\dagger}, c_{k}\}$, the rotated basis $T^{-1}\{c_{i}^{\dagger}, c_{k}\}T$ will contain states that are not present in the original basis. This prevents us from using conventional symmetry-restoring algorithms, which all assume closure of the basis under rotations. The elegant solution to this hurdle was proposed 25 years ago by Robledo [38], who extended the Wick theorem [43, 44] to encompass bases not closed under symmetry transformations. Based on the formalism of Ref. [38], we can write the rotated norm overlap as

$$ N_{q}^{x} = \sqrt{\det A_{q}^{x} \times \det R^{x}}, $$

where

$$ A_{q}^{x} = U_{q}^{T} (R_{q}^{x} T)^{-1} U_{q}^{T} + V_{q}^{T} R_{q}^{x} V_{q}^{*}. $$

Here, $U_{q}$ and $V_{q}$ are the Bogoliubov matrices corresponding to the HFB configuration $|\Phi(q)\rangle$, and $R_{q}$ denotes the matrix of the rotation operator $\hat{R}$ in the HO basis. Note that in the case of a basis closed under rotations $|\det R_{q}| = 1$, and the expression (3) reduces to the conventional Onishi formula [45]. In the symmetry-restored DFT framework, the Hamiltonian kernel $H_{q}^{J^*NZ}$ is a functional of the one-body, transition density $\rho_{q}^{x}$ and pairing tensor $\kappa_{q}^{x}$. When the basis is not closed under rotations, these read

$$ \rho_{q}^{x} = R_{q}^{x} V_{q}^{*} A_{q}^{x-1} V_{q}^{T}, $$

$$ \kappa_{q}^{x} = R_{q}^{x} V_{q}^{*} A_{q}^{x-1} U_{q}^{T}, $$

where the mixed-density prescription was used in both the angular momentum projection (AMP) and the particle number projection (PNP) procedure [47]. The symmetry-restored energy is simply the ratio $E_{q}^{J^*NZ} = H_{q}^{J^*NZ} / N_{q}^{J^*NZ}$.

Least-Energy Fission Pathway. Although distinct from the most probable fission path [48, 49], the least-energy fission pathway provides valuable information about fission dynamics such as the existence and energies of fission barrier heights or fission isomers. These pseudodata are important to predict the stability of superheavy elements or evaluate neutron-induced fission cross sections, especially in regions of the nuclide chart where no experimental data is available [50, 51]. The goal of the present analysis is to assess the effect of symmetry restoration on such data along the entire fission pathway. In this regard, it represents an extension of the early work by Bender and coworkers who studied the effect of symmetry restoration along the reflection-symmetric ($q_{30} = 0$) pathway and up to moderate deformations only [30].

In the upper panel of Fig. 4 we plot the deformation energy of $^{240}$Pu along the least-energy fission pathway as calculated in the HFB approximation (turquoise squares). 95 configurations along the pathway were determined by constraining quadrupole moments within a range $21 \leq q_{20} \leq 397$ (in b) with steps $\Delta q_{20} = 4$ b, while $q_{30}$ moments were left unconstrained and determined self-consistently. We then projected these configurations onto good values of particle numbers (PNP, red triangles) and
onto good values of particle numbers, angular momentum 
($J = 0$), and parity (PNP&AMP, blue circles). The two 
inersects in the upper panel of Fig. 1 show the convergence 
of the PNP and PNP&AMP procedures with respect to the 
umber of integration points for the pre-scission configura-
tion, $q_{20} = (345.0 \text{ b}, 42.6 \text{ b}^{3/2})$, where the 
underlying basis is the most incomplete. In order to en-
sure proper numerical convergence across all considered 
configurations, we set $N_x = 9$ and $26 \leq N_y \leq 30$.

![FIG. 1. (a): least-energy fission pathway in $^{240}$Pu calculated in 
the HFB approximation (turquoise squares); projected onto 
good values of particle numbers (PNP, red triangles); projected 
on good values of particle numbers, angular momentum 
($J = 0$) and parity (PNP&AMP, blue circles); obtained by 
adding the rotational correction $E_{RC}$ on top of the PNP 
values (PNP+RC, empty squares). The insets show the con-
vergence of the PNP and PNP&AMP for the pre-scission con-
figuration with respect to the number of gauge angles $N_x$ and 
the number of rotational angles $N_y$, respectively. (b): ratio 
$R_{RC} = (E_{PNP}^{0+} - E_{PNP})/E_{PY}$; see text for details.

The potential energy curve of $^{240}$Pu is characterized by 
two minima, the ground state and a fission isomer, and 
two fission barriers [52]. The scission point is marked by 
a sharp drop in energy, which occurs here at $q_{20} \approx 345$ b. 
Table I lists the corresponding energies of these configura-
tions. Although the HFB energy of the inner barrier 
(9.37 MeV) is about 3.3 MeV higher than the empirical 
value inferred from fission cross sections [43, 54], this 
is mostly caused by the omission of triaxial effects in our 
calculations [53, 55]: including them lowers the 
effective the first barrier by about 1.7 MeV [29]. This effect 
is amplified by symmetry restoration, which lowers the 
barrier by an additional 1.3 MeV, pushing the theoretical 
value well within the uncertainty limits of the em-
pirical value (typically about 1 MeV). The outer barrier 
is axially-symmetric and reflection-asymmetric [29]. Its 
height is lowered by as much as 2.3 MeV by the sym-
metry restoration, again pushing the theoretical value 
within the 1 MeV limit of the empirical value, 5.15 MeV [54]. On the other hand, the HFB energy of the fission 
isomer is already in decent agreement with the empirical 
value of 2.25 $\pm$ 0.20 MeV [57]: symmetry restoration 
degrades this agreement. These numbers are consistent 
with those reported in [30].

| Configuration       | HFB  | PNP | PNP&AMP |
|---------------------|------|-----|---------|
| Inner barrier       | 9.37 | 8.78| 8.05    |
| Fission isomer      | 2.67 | 2.27| 1.02    |
| Outer barrier       | 6.75 | 6.28| 4.58    |
| Pre-scission        | 11.68| 10.88|11.85   |

While previous work in Refs. [38, 39] was exclusively 
focused on the potential energy curve near the two barri-
ers, we extend this study all the way to the scission point. 
Of particular interest is the pre-scission energy, which is 
defined as the energy difference between the outer bar-
rier and the scission configuration, and which gives an 
important contribution to the total energy released during 
fission. Interestingly, even though the corrections to 
the barriers are significant, we find that the total corre-
lation energy beyond the outer barrier saturates, with the 
result that symmetry restoration has a negligible impact 
on the value of pre-scission energy.

In many studies of spontaneous fission, the effect of 
AMP is simulated by what is known as the rotational 
energy correction [10]. It was observed in [58] that this 
term is well approximated by $E_{RC} = 0.7 \times E_{PY}$ where 
$E_{PY} = -\langle J^2 \rangle/(2J_{PY})$, $\langle J^2 \rangle$ is the total angular mo-
moment dispersion, $J_{PY}$ is the Peierls-Yoccoz moment 
of inertia [59], and the phenomenological quenching fac-
tor $0.7$ is included to account for approximations intro-
duced in calculating $J_{PY}$. In panel (a) of Fig. 1 we 
also show the curve obtained by adding $E_{RC}$ on top of 
the calculated PNP values (PNP+RC), while the ratio 
$R_{RC} = (E_{PNP}^{0+} - E_{PNP})/E_{PY}$ is shown in the lower panel 
of Fig. 1. Our calculations confirm that $E_{RC}$ is an excel-
ent approximation to the exact model at very large de-
formation and all the way to the scission point. However, 
we also observe that for configurations with $q_{20} \lesssim 150$ b 
the quenching factor of 0.7 is not sufficient, leading to 
differences in energy up to 2.5 MeV. This discrepancy 
could have a severe impact on observables that are very 
sensitive to details of the underlying PES, such as the 
spontaneous fission half-lives [11, 40].

2D PES and Fission Fragment Distributions. While one-dimen-
sional fission paths can be sufficient to compute obser-
vables such as half-lives and cross sections, quantities such 
as fission fragment distributions require probing at least two dimensions in the collective space. 
Starting from the PES of $^{240}$Pu in the HFB approxima-
tion reported in [29], we thus selected a total of 1150 
configurations within 20 MeV of the ground state en-
ergy. They cover a very broad range of quadrupole and
octupole deformations, with $20 \leq q_{20} \leq 567$ (in b) and $0 \leq q_{30} \leq 70$ (in b$^{3/2}$). We then projected each of these configurations onto good values of particle numbers, angular momentum, and parity using the above method.

FIG. 2. (a) Two-dimensional symmetry-restored PES of $^{240}$Pu in the $(q_{20}, q_{30})$ plane for the $J^\pi = 0^+$ configuration. (b) Same for $J^\pi = 1^-$. Both surfaces are normalized with respect to the energy of their respective ground states. (c) Difference between the symmetry-restored and the HFB surface for the $J^\pi = 0^+$ configuration. (d) Same for the $J^\pi = 1^-$.  

Figure 2 shows the PES for the $J^\pi = 0^+$ (a) and $J^\pi = 1^-$ (b) states with the exact number of particles $N = 146$ and $Z = 94$. Overall, the PES retains its main features such as the fission isomer and the main fission valley, which extends from $(q_{20}, q_{30}) \approx (90 \text{ b}, 0 \text{ b}^{3/2})$ to $(q_{20}, q_{30}) \approx (340 \text{ b}, 40 \text{ b}^{3/2})$. Panels (c) and (d), which show the energy difference between the symmetry-restored and HFB surfaces, provide an additional insight. In particular, for the $0^+$ state, a pronounced gain in energy is observed at low $q_{30}$ values for a wide range of configurations, pointing to the possible enhancement of symmetric fission. For the $1^-$ state, the correlation energy is large along and around the least-energy pathway, suggesting broader fission fragment distributions.

To estimate the actual effect on fission fragment distributions, we used the FELIX solver [60] to solve the collective Schrödinger equation originating from the Gaussian overlap approximation of the time-dependent GCM. The inputs to FELIX were the PES (HFB and PNP&AMP for the $0^+$ and $1^-$ states), the GCM inertia tensor computed at the perturbative cranking approximation, and scission configurations defined by the HFB expectation value of the Gaussian neck operator $q_N$ with a folding factor of width $\sigma_A = 5$; see [13] for a discussion. To simulate the neutron-induced fission for thermal neutrons, the energy of the initial state was set at 1 MeV above the inner barrier. Figure 3 demonstrates the impact on the fragment mass distributions: fission becomes more symmetric after projections, and the symmetric fission mode is indeed enhanced for the $0^+$ state. Furthermore, the distribution for the $1^-$ state is significantly broadened and favors less asymmetric fragmentations. Note that, by definition, symmetric fission is not possible for the $1^-$ configuration (indicated by a gap in the curve). These results represent the first attempt to quantify the effect of symmetry restoration on actual fission observables. A fully consistent determination of fission fragment distributions will require developing a projected theory of collective inertia, estimating the spin distribution of the fissioning nucleus, and generating PES with a much higher resolution in $(q_{20}, q_{30})$.

Conclusion. Restoring broken symmetries is a necessary step for nuclear models to describe different fission channels. In this work, we reported the first symmetry-restoring description of fission from the ground state to scission. Our analysis of the benchmark case of $^{240}$Pu indicates that projection correlation energies cannot be approximated by phenomenological formula across the entire range of deformations relevant for fission, and that symmetry restoration may have a substantial impact on the mass distribution of fission fragments. These conclusions should be validated by developing a projected theory of collective inertia.

Acknowledgments. We thank R. Vogt for her careful reading of the manuscript. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344 (NS). Computing support for this work came from the Lawrence Livermore National Laboratory and the National Nuclear Security Administration.
National Laboratory (LLNL) Institutional Computing Grand Challenge program.

[1] J. Beun, G. C. McLaughlin, R. Surman, and W. R. Hix. Fission Cycling in a Supernova R-Process. Phys. Rev. C, 77(3):035804, 2008.

[2] M. R. Mumpower, R. Surman, G. C. McLaughlin, and A. Aprahamian. The Impact of Individual Nuclear Properties on R-Process Nucleosynthesis. Prog. Part. Nucl. Phys., 86-86, 2016.

[3] M. R. Mumpower, T. Kawano, T. M. Sprouse, N. Vassh, E. M. Holmbeck, R. Surman, and P. Möller. β-Delayed Fission in r-Process Nucleosynthesis. ApJ, 869(1):14, 2018.

[4] M. R. Mumpower, P. Jaffke, M. Verriere, and J. Randrup. Primary Fission Fragment Mass Yields across the Chart of Nuclides. arXiv:1911.06344, 2019.

[5] N. Vassh, R. Vogt, R. Surman, J. Randrup, T. M. Sprouse, M. R. Mumpower, P. Jaffke, D. Shaw, E. M. Holmbeck, Y. Zhu, and G. C. McLaughlin. Using Excitation-Energy Dependent Fission Yields to Identify Key Fissioning Nuclei in r-Process Nucleosynthesis. J. Phys. G: Nucl. Part. Phys., 46(6):065202, 2019.

[6] A. C. Hayes. Applications of Nuclear Physics. Rep. Prog. Phys., 80(2):026301, 2017.

[7] A. N. Andreyev, K. Nishio, and K.-H. Schmidt. Nuclear Fission: A Review of Experimental Advances and Phenomenology. Rep. Prog. Phys., 81(1):016301, 2017.

[8] Karl-Heinz Schmidt and Beatriz Jurado. Review on the Progress in Nuclear Fission – Experimental Methods and Theoretical Descriptions. Rep. Prog. Phys., 81(10):106301, 2018.

[9] H. J. Krappe and K. Pomorski. Theory of Nuclear Fission. Springer, 2012.

[10] N. Schunck and L. M. Robledo. Microscopic Theory of Nuclear Fission: A Review. Rep. Prog. Phys., 79(11):116301, 2016.

[11] A. Baran, M. Kowal, P.-G. Reinhard, L. M. Robledo, A. Staszczak, and M. Warda. Fission Barriers and Probabilities of Spontaneous Fission for Elements with Z ≥ 100. Nucl. Phys. A, 944:442, 2015.

[12] H. Goutte, P. Casoli, and J.-F. Berger. Mass and Kinetic Energy Distributions of Fission Fragments Using the Time Dependent Generator Coordinate Method. Nucl. Phys. A, 734:217, 2004.

[13] D. Regnier, N. Dubray, N. Schunck, and M. Verriere. Fission fragment charge and mass distributions in 239Pu(n,f) in the adiabatic nuclear energy density functional theory. Phys. Rev. C, 93(5):054611, 2016.

[14] H. Tao, J. Zhao, Z. P. Li, T. Nikšić, and D. Vretenar. Microscopic Study of Induced Fission Dynamics of 226Th with Covariant Energy Density Functionals. Phys. Rev. C, 96(2):024319, 2017.

[15] D. Regnier, N. Dubray, and N. Schunck. From Asymmetric to Symmetric Fission in the Fermium Isotopes within the Time-Dependent Generator-Coordinate-Method Formalism. Phys. Rev. C, 99(2):024611, 2019.

[16] Marc Verrière, Nicolas Schunck, and Toshihiko Kawano. Number of Particles in Fission Fragments. Phys. Rev. C, 100(2):024612, 2019.

[17] Jie Zhao, Tamara Nikšić, Dario Vretenar, and Shan-Gui Zhou. Microscopic Self-Consistent Description of Induced Fission Dynamics: Finite-Temperature Effects. Phys. Rev. C, 99(1):014618, 2019.

[18] Jie Zhao, Jian Xiang, Zhi-Pan Li, Tamara Nikšić, Dario Vretenar, and Shan-Gui Zhou. Time-Dependent Generator-Coordinate-Method Study of Mass-Asymmetric Fission of Actinides. Phys. Rev. C, 99(5):054613, 2019.

[19] Aurel Bulgac, Piotr Magierski, Kenneth J. Roche, and Ionel Stetcu. Induced Fission of 249Pu within a Real-Time Microscopic Framework. Phys. Rev. Lett., 112(12):122504, 2016.

[20] Aurel Bulgac, Shijin, Kenneth J. Roche, Nicolas Schunck, and Ionel Stetcu. Fission Dynamics of 249Pu from Saddle to Scission and Beyond. Phys. Rev. C, 100(3):034615, 2019.

[21] Niels Bohr and John Archibald Wheeler. The Mechanism of Nuclear Fission. Phys. Rev., 56(5):426, 1939.

[22] Abraham Klein, Niels R. Walet, and G. Do Dang. Classical Theory of Collective Motion in the Large Amplitude, Small Velocity Regime. Ann. Phys., 208(1):90, 1991.

[23] Nicolas Schunck. Energy Density Functional Methods for Atomic Nuclei. IOP Expanding Physics. IOP Publishing, Bristol, UK, 2019. OCLC: 1034572493.

[24] J. A. Sheikh, J. Dobaczewski, P. Ring, L. M. Robledo, and C. Yannouleas. Symmetry Restoration in Mean-Field Approaches. arXiv:1901.06992, 2019.

[25] D.M. Brink and R.A. Broglia, editors. Nuclear Superfluidity - Pairing in Finite Systems. Cambridge University Press, 2005.

[26] P.-G. Reinhard and K. Goeke. The Generator Coordinate Method and Quantised Collective Motion in Nuclear Systems. Rep. Prog. Phys., 50(1):1, 1987.

[27] J. F. Berger, M. Giord, and D. Gogny. Time-Dependent Quantum Collective Dynamics Applied to Nuclear Fission. Comput. Phys. Commun., 63(1):365, 1991.

[28] A. Staszczak, A. Baran, J. Dobaczewski, and W. Nazarewicz. Microscopic Description of Complex Nuclear Decay: Multimodal Fission. Phys. Rev. C, 80(1):014309, 2009.

[29] N. Schunck, D. Duke, H. Carr, and A. Knoll. Description of Induced Nuclear Fission with Skyrme Energy Functionals: Static Potential Energy Surfaces and Fission Fragment Properties. Phys. Rev. C, 90(5):054305, 2014.

[30] M. Warda and J. L. Egido. Fission Half-Lives of Super-heavy Nuclei in a Microscopic Approach. Phys. Rev. C, 86(1):014322, 2012.

[31] R. Rodriguez-Guzmán and L. M. Robledo. Microscopic Description of Fission in Uranium Isotopes with the Gogny Energy Density Functional. Phys. Rev. C, 89(5):054310, 2014.

[32] Bing-Nan Lu, En-Guang Zhao, and Shan-Gui Zhou. Potential Energy Surfaces of Actinide Nuclei from a Multidimensional Constrained Covariant Density Functional Theory: Barrier Heights and Saddle Point Shapes. Phys. Rev. C, 85(1):011301, 2012.

[33] Jie Zhao, Bing-Nan Lu, Dario Vretenar, En-Guang Zhao, and Shan-Gui Zhou. Multidimensionally Constrained Relativistic Mean-Field Study of Triple-Humped Barriers in Actinides. Phys. Rev. C, 91(1):014321, 2015.

[34] Krishichayan, Megha Bhike, C. R. Howell, A. P. Tomchev, and W. Tornow. Fission Product Yield Measurements Using Monoenergetic Photon Beams. Phys. Rev. C,
[35] Guillaume Scamps and Denis Lacroix. Effect of Pairing on One- and Two-Nucleon Transfer below the Coulomb Barrier: A Time-Dependent Microscopic Description. *Phys. Rev. C*, 87(1):014605, 2013.

[36] M. Bender, P.-H. Heenen, and P. Bonche. Microscopic Study of $^{240}$Pu: Mean Field and Beyond. *Phys. Rev. C*, 70(5):054304, 2004.

[37] M. Samyn, S. Goriely, and J. Pearson. Further Explorations of Skyrme-Hartree-Fock-Bogoliubov Mass Formulas. V. Extension to Fission Barriers. *Phys. Rev. C*, 72(4):044316, 2005.

[38] L. M. Robledo. Practical Formulation of the Extended Wick’s Theorem and the Onishi Formula. *Phys. Rev. C*, 50(6):2874, 1994.

[39] R. Navarro Perez, N. Schunck, R.-D. Lasserri, C. Zhang, and J. Sarich. Axially Deformed Solution of the Skyrme–Hartree–Fock–Bogolyubov Equations Using the Transformed Harmonic Oscillator Basis (III) HFBTHO (v3.00): A New Version of the Program. *Comput. Phys. Commun.*, 220(Supplement C):363, 2017.

[40] J. Bartel, P. Quentin, M. Brack, C. Guet, and H.-B. Häkansson. Towards a Better Parametrisation of Skyrme-like Effective Forces: A Critical Study of the SKM Force. *Nucl. Phys. A*, 386(1):79, 1982.

[41] J. Dobaczewski, W. Nazarewicz, and M. V. Stoitsov. Contact Pairing Interaction for the Hartree-Fock-Bogoliubov Calculations. In *The Nuclear Many-Body Problem 2001*, number 53 in Nato Science Series II, page 181. Springer Netherlands, 2002.

[42] V. N. Fomenko. Projection in the Occupation-Number Space and the Canonical Transformation. *J. Phys. A: Gen. Phys.*, 3(1):8, 1970.

[43] R. Balian and E. Brezin. Nonunitary Bogoliubov Transformations and Extension of Wick’s Theorem. *Nuovo Cim. B*, 64(1):37, 1969.

[44] K. Hara and S. Iwasaki. On the Quantum Number Projection. *Nucl. Phys. A*, 332(1):61, 1979.

[45] R. G. Nazmitdinov, L. M. Robledo, P. Ring, and J. L. Egido. Representation of Three-Dimensional Rotations in Oscillator Basis Sets. *Nucl. Phys. A*, 596(1):53, 1996.

[46] Naoki Onishi and Shiro Yoshida. Generator Coordinate Method Applied to Nuclei in the Transition Region. *Nucl. Phys.*, 80(2):367, 1966.

[47] L. M. Robledo. Remarks on the Use of Projected Densities in the Density-Dependent Part of Skyrme or Gogny Functionals. *J. Phys. G: Nucl. Part. Phys.*, 37(6):064020, 2010.

[48] Jhilam Sadhukhan, K. Mazurek, A. Baran, J. Dobaczewski, W. Nazarewicz, and J. A. Sheikh. Spontaneous Fission Lifetimes from the Minimization of Self-Consistent Collective Action. *Phys. Rev. C*, 88(6):064314, 2013.

[49] Samuel A. Giuliani and Luis M. Robledo. Fission properties of the Barcelona-Catania-Paris-Madrid energy density functional. *Phys. Rev. C*, 88:054325, 2013.

[50] M. Sin, R. Capote, A. Ventura, M. Herman, and P. Obložinský. Fission of Light Actinides: Th$^{232}$(n,f) and Pa$^{231}$(n,f) Reactions. *Phys. Rev. C*, 74(1):014608, 2006.

[51] S. Goriely, S. Hilaire, A. J. Koning, M. Sin, and R. Capote. Towards a Prediction of Fission Cross Sections on the Basis of Microscopic Nuclear Inputs. *Phys. Rev. C*, 79(2):024612, 2009.

[52] S. Bjørnholm and J. E. Lynn. The Double-Humped Fission Barrier. *Rev. Mod. Phys.*, 52(4):725, 1980.

[53] G. N. Smirenkin. Preparation of Evaluated Data for a Fission Barrier Parameter Library for Isotopes with $Z=82-98$, with Consideration of the Level Density Models Used. Technical Report INDC(CCP)--359, International Atomic Energy Agency (IAEA), 1993.

[54] R. Capote, M. Herman, P. Obložinský, P. G. Young, S. Goriely, T. Belgya, A. V. Ignatyuk, A. J. Koning, S. Hilaire, V. A. Pljuko, M. Avrigeanu, O. Bersillon, M. B. Chadwick, T. Fukahori, Zhigang Ge, Yinlu Han, S. Kailas, J. Kopecky, V. M. Maslov, G. Refo, M. Sin, E. Sh. Soukhovitskii, and P. Talou. RIPL – Reference Input Parameter Library for Calculation of Nuclear Reactions and Nuclear Data Evaluations. *Nucl. Data Sheets*, 110(12):3107, 2009.

[55] S. E. Larsson, I. Ragnarsson, and S. G. Nilsson. Fission Barriers and the Inclusion of Axial Asymmetry. *Phys. Lett. B*, 38(5):269, 1972.

[56] M. Girod and B. Grammaticos. Triaxial Hartree-Fock-Bogolyubov Calculations with D1 Effective Interaction. *Phys. Rev. C*, 27(5):2317, 1983.

[57] M. Hunyadi, D. Gassmann, A. Krasznahorkay, D. Habs, M. Girod, B. Grammaticos. Triaxial Hartree-Fock-Bogolyubov Calculations Using the D1 Effective Interaction. *Phys. Rev. C*, 27(5):2317, 1983.

[58] M. Hunyadi, D. Gassmann, A. Krasznahorkay, D. Habs, P. G. Thirolf, M. Csatlós, T. Faestermann, G. Graw, J. Gulyás, R. Hertenberger, H. J. Maier, Z. Máté, A. Metz, and M. J. Chromik. Excited Superdeformed $K^\pi=0^+$ Rotational Bands in $^{26}$Vibrational Fission Resonances of $^{240}$Pu. *Phys. Lett. B*, 505(1):27–35, 2001.

[59] L. M. Robledo. Remarks on the Use of Projected Densities in the Density-Dependent Part of Skyrme or Gogny Functionals. *J. Phys. G: Nucl. Part. Phys.*, 37(6):064020, 2010.

[60] J. L. Egido, J. Lessing, V. Martin, and L. M. Robledo. On the Solution of the Hartree-Fock-Bogoliubov Equations by the Conjugate Gradient Method. *Nucl. Phys. A*, 594(1):70, 1995.