FUNDAMENTAL STEPS OF GROUP VELOCITY
FOR SLOW SURFACE POLARITON UNDER
THE QUANTUM HALL EFFECT CONDITIONS

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Abstract

A new type of collective electromagnetic excitations, namely surface polaritons (SP) — in a 2D electronic layer in a high magnetic field under Quantum Hall Effect (QHE) conditions is predicted. We have found the spectrum, damping, and polarization of the SP in a wide range of frequencies $\omega$ and wavevectors $k$. It is shown that near the Cyclotron Resonance (CR) ($\omega \sim \Omega = eB/mc$) the phase velocity of the SP is drastically slowed down and the group velocity undergoes fundamental steps defined by the Fine Structure Constant $\alpha = e^2/\hbar c$. In the vicinity of a CR subharmonic ($\omega \sim 2\Omega$) the negative (anomalous) dispersion of the SP occurs. The relaxation of electrons in the 2D layer gives rise to a new dissipative collective threshold-type mode of the SP. We suggest a method for calculating the kinetic coefficients for the 2D electronic layer under QHE condition, using the Wigner distribution function formalism and determine their spatial and frequency dispersion. Using this method we have calculated the line-shape of the CR and the d.c.
conductance under the QHE condition, which are in good agreement with experimental data.

73.40.Hm, 72.30+q, 73.20.Mf
I. INTRODUCTION

Since the discovery of the Quantum Hall Effect (QHE) [1–3], a number of authors have investigated the weak damping of collective electromagnetic waves in 2D electronic layers in a strong magnetic field $B$ [3,4]. The quantization of the Hall conductivity and the vanishingly small dissipative (longitudinal) conductivity lead, under the QHE conditions, to spatial and frequency dispersion in the system and hence to the generation of an unusually slow collective wave, whose dispersion characteristics are also quantized.

In this paper a new type of collective electromagnetic excitations in a 2D electronic system under the QHE conditions is predicted, viz., the slow surface polaritons (SP). We calculate the spectrum, damping, and polarization of that wave in a wide range of frequencies $\omega$ and wavevectors $k$. The phase velocity of the SP is drastically slowed down near the principal cyclotron resonance ($\omega \sim \Omega$, where $\Omega = eB/mc$ is the cyclotron frequency) and their group velocity undergoes fundamental jumps, whose magnitude is determined by the Fine Structure Constant $\alpha = e^2/\hbar c$. The number of the slow SP modes is conditioned be the magnitude of the Landau-level filling factor $\mathcal{N} = \pi \ell^2 n$ (where $\ell = (\hbar/eB)^{1/2}$ is the magnetic length, and $n$ the density of 2D electrons), i.e., by the value of the quantized Hall conductivity. In the vicinity of the CR subharmonic ($\omega \sim 2\Omega$), the negative (anomalous) dispersion of SP (see Fig. [4]) occurs. Besides, a new type of SP appears near the CR, which is dissipative in nature. The condition of existence of that additional SP is determined by the quantized threshold criterion, which allows determining the relaxation frequency at low temperature to an accuracy of $\alpha$ (see Eq. (3.17)).

The QHE has been intensely studied with the use of methods of modern condensed-matter theory [3]. The Integer QHE (IQHE) is thought to be caused by localization of electrons in the two-dimensional systems, and the Fractional QHE (FQHE) is due to electron-electron interaction, which leads to generation of correlated many-particle ground state [3] at distinct fractional values of the Landau-level filling factor $\mathcal{N}$. As this takes place, the paradox lies in the fact that the presence of "dirt" is the necessary condition for the localization-
delocalization phase transition effect; although it is well known that for the observation of QHE, and, particularly FQHE, the use of perfect samples with a high mobility is required. QHE is one of the problems of the *postmodern quantum mechanics* (to use the term of R.Harris), discussed in four papers in Physics Today in 1993 \[5\]. The technological advances of the last decade have succeeded in fabrication of two-dimensional systems in which the ballistic (or quasiballistic) transport with large mean free paths can be realized.

From the above reasoning we suggest a simple method, which uses almost the Pauli principle alone, for the description of the kinetics of electrons in 2D systems placed in a strong quantizing magnetic field \( B \). By using the Wigner distribution function \[6\] we derive the kinetic characteristics of a 2D electronic gas under the QHE conditions and find their spatial and frequency dispersion. By means of these results we can adequately describe the d.c. effects of IQHE, as well as the line-shape of the CR under the IQHE condition and hence the dynamics of collective electromagnetic excitations under QHE conditions. The article is organized as follows. In Section II we use the Wigner distribution function for describing the transport phenomena in a 2DES under QHE conditions. We calculate the conductivity tensor with a spatial and frequency dispersion. In Section III we present the electrodynamics in 2DES in the high magnetic fields (QHE effect). We derive the dispersion relation for electromagnetic surface waves and discuss the dispersion, polarization and damping for the quantized SP in this system. We conclude the paper with a brief summary of results and possible applications (Section IV).

**II. TRANSPORT IN THE QUANTUM HALL EFFECT**

To find the conductivity tensor accounting for the spatial and frequency dispersion in a 2D electronic gas placed in a high quantizing magnetic field \( B \) (under QHE conditions) oriented normally to the 2D layer (see Fig. [3]), we will apply the Wigner distribution function \[6-8\]:

\[
f^W_p (r) = \int \text{d} r' T r \{ \hat{r} \exp[-i(p + e/c A(r)) r'] \psi^+(r - r'/2) \psi(r + r'/2) \}. \tag{2.1}
\]
Here $\hat{\rho}$ is the statistical operator of the system; $\psi^+(\mathbf{r})$ and $\psi(\mathbf{r})$ are the Fermi operators of generation and annihilation respectively, of particles at point $\mathbf{r}$; $\mathbf{A}$ the vector-potential of the electromagnetic field. In the case when the scale size of the spatial inhomogeneity exceeds both the radius of interaction between the particles and the de Broglie electron wavelength, the kinetic equation for the Wigner distribution function Eq. (2.1) takes the form [6–8] equivalent to the classical kinetic equation:

$$\frac{\partial f^W_p}{\partial t} + \mathbf{v} \frac{\partial f^W_p}{\partial \mathbf{r}} + e\{\mathbf{E} + \frac{1}{c}[\mathbf{v}, \mathbf{B}]\} \frac{\partial f^W_p}{\partial \mathbf{p}} = \hat{I}\{f^W_p\}. \quad (2.2)$$

Here $\mathbf{E}$ and $\mathbf{B}$ are the electric field and the magnetic induction vectors; $e$ the electron charge, and $\mathbf{v}$ the velocity of conduction electrons.

In the case under consideration, if the 2D electronic system is infinite in the $xy$–plane (see Fig. 1), then the typical scale of inhomogeneity is the wavelength $k^{-1}$ of the collective electromagnetic wave. Thus, the existence criteria for Eq. 2.2 are $k \ll n^{1/2}$, (since with weak screening $n^{-1/2}$ is the characteristic length interaction between the particles) and $k\ell \ll 1$ (since in a strong magnetic field the magnetic length $\ell = (\frac{c\bar{h}}{eB})^{1/2}$ represents the de Broglie wavelength of electrons). The collision integral, $\hat{I}\{f^W_p\}$, differs essentially from the classical collision integral, since the quantum transitions accounted for by, $\hat{I}\{f^W_p\}$, reflect the character of statistics, obeyed by the particles, and the distinction of the Wigner distribution function from the classical one [1]. The equilibrium Wigner distribution function sets the collision integral, $\hat{I}\{f^W_p\}$, to zero. The equilibrium Wigner function can be expressed via its value for an equilibrium ensemble of quantum states of an electron in a magnetic field $\mathbf{B}$. By using the definition Eq. (2.1) and substituting the wavefunctions of an electron in an electromagnetic field into Eq (2.1), we obtain for the spinless electrons [7,8]

$$f_0(\epsilon) = \sum_{s=\frac{1}{2}}^{\infty} n_F \left[ \frac{\hbar \Omega (s + \frac{1}{2}) - \mu}{T} \right] \Gamma_s(\frac{\epsilon}{\hbar \Omega}), \quad (2.3)$$

$$\Gamma_s(x) = 2(-1)^s \exp(-2x)L_0^0(4x),$$

$$n_F(x) = (1 + e^x)^{-1}.$$
Here $\epsilon = \frac{p^2}{2m}$ is the energy of 2D electrons and $L_s^{(0)}(x)$ the Laguerre polynomial. If we replace the summation over $s$ by integration, then for $\hbar \Omega \ll T$ ($T$ is the temperature) Eq. (2.3) transforms into an equilibrium Fermi distribution function ($\mu$ is the chemical potential).

With the knowledge of the equilibrium Wigner distribution function we can describe all the thermodynamical relations. In this paper we will consider the 2DES when the chemical potential $\mu$ is constant over the entire system. The relation between the electron density $n$ and the chemical potential $\mu$ in a strong magnetic field can be found, as usual, from the normalization condition. The density $n$ of 2D electrons is the average of $f_0(\epsilon)$ Eq. (2.3). As a result we obtain after the averaging

$$\mathcal{N} = \pi \ell^2 n = \sum_{s=0}^{\infty} n_F \left[ \frac{\hbar \Omega (s + \frac{1}{2}) - \mu}{T} \right].$$

(2.4)

It is shown that the Landau-level filling factor $\mathcal{N}$ assumes only positive integer values if $T \ll \hbar \Omega, \mu$. The fact that the filling factor $\mathcal{N}$ can assume only integer values leads to IQHE. However, Eq. (2.4) contains a contradiction. Indeed, why should the ratio of two independent values which the electron density $n$ and the magnetic induction $B$ in the sample are take only integer values? It is well-known that the mean value of a microscopic magnetic field is the magnetic induction $B$. The value of $B$ in the sample should be found from the formula $H = B - 4\pi M(B)$, where $H$ is the external magnetic field and $M(B)$ the magnetic moment. If the temperature is not too low, then $M(B) \ll B$ and the magnetic induction value $B$ differs but slightly from $H$. As temperature is decreased, the amplitude of oscillations of the magnetic moment $M(B)$ increases and a situation appears, when the regions of $H$-values corresponding to three various values of $B$, show up. This ambiguity indicates an instability of states similar to that taking place on the Van-der-Waals curve of the equation of state [9–11]. In other words, at such values of the external magnetic fields diamagnetic phase transitions take place in the system, at which an inhomogeneous state (domain type and(or) periodic structures) appear in the system, when the magnetic induction $B$ and the electron density $n$ become coordinate-dependent functions. In the vicinity of such phase transitions and at the inhomogeneous states, the scaling-type dependencies of the
conductivity on the magnetic field and singularities at the fractional values of the Landau-level filling factor $\aleph$ should appear due to the scaling-type invariance (see [9–11]). This kind of behavior should result in singularities in the Hall conductivity and in a longitudinal (dissipative) conductivity of the 2D electronic gas at the fractional values of the filling factor $\aleph$, i.e., to FQHE. In particular, when the magnetic induction and the electron density are characterized by an inhomogeneous structure, the splitting of states and the additional gaps might originate in the electron spectrum, and the spectrum degeneracy in the orbit centre coordinate in the magnetic field may become removed.

Besides, an additional drift of electrons [12] can arise in a weakly inhomogeneous magnetic field which can give an additional contribution to the Hall current. However, far apart from the diamagnetic phase transition [9–11] equilibrium states exist; the electron density is independent of coordinates, and the magnetic induction $B$ assumes a sufficient value to satisfy relation Eq. (2.4), and the Landau-level filling factor is an integer value, i.e., the IQHE condition is met. Thus, in this paper we will analyze the electron kinetics under the IQHE condition.

The form of the electron-phonon and electron-impurity collisions integral Eq. (2.3) is too complicated [7]. However, we will consider here the effects determined by the linear response to the electric field. In this case the distribution function, $f^W_p$, can be found in an approximation linear in the external field, $E$. It is well-known [7] that for sample with a high electron mobility the collision integral can be represented in the $\tau$–approximation, where the mean free path time $\tau$ is determined by the momentum relaxation frequency, being a function of the electron energy, $\epsilon$. In other words, for this quasiballistic regime we can find the Wigner distribution function in the form:

$$f^W_p = f_0(\epsilon) + f_1,$$

where $f_0(\epsilon)$ is the equilibrium distribution function in a high magnetic field Eq. (2.3), and $f_1(t, p, r)$ the correction to the Wigner distribution function, which is determined by the electric field $E$. The collision integral, $\hat{I}\{f^W_p\}$, will be written as
\[
\hat{I}\{f_p^W\} = -\nu(\epsilon) f_1. \tag{2.6}
\]

In the general case we will assume the electric field to be a function of coordinates and time. Then for the Fourier-transform of the current density, \(j\), and the electric field, \(E\), one obtains the linear relations

\[
j_\alpha(\omega, k) = \sigma_{\alpha\beta}(\omega, k) E_\beta(\omega, k); \tag{2.7}
\]

\[
E_\alpha(r, t) = \int \frac{d^3k}{(2\pi)^3} E_\alpha(\omega, k) e^{i(kr - \omega t)}; \tag{2.8}
\]

\[
E_\alpha(\omega, k) = \int d^2k d\omega \sigma_{\alpha\beta}(\omega, k) e^{i(kr - \omega t)}. \tag{2.9}
\]

The Fourier-transform of the conductivity tensor, \(\sigma_{\alpha\beta}(\omega, k)\), accounting for the spatial and frequency dispersion can be found by using the Wigner distribution function Eqs.(2.2),(2.3),(2.5). Thus, in the general case we have

\[
\sigma_{\alpha\beta}(\omega, k) = \frac{2e^2}{\hbar} \sum_{s=0}^{\infty} n_F \left\{ \frac{\hbar \Omega (s + \frac{1}{2}) - \mu}{T^2} \right\} \int_0^\infty d\xi \frac{(\pi^2 \xi / 2)}{sh[\pi \gamma(\xi)]} \frac{\partial \Gamma_s(\xi)}{\partial \xi} D_{\alpha\beta}(\xi) \tag{2.10}
\]

Here the tensor components, \(D_{\alpha\beta}(\xi)\), are given by

\[
D_{xx}(\xi) = m_1(\xi) + m_2(\xi) + 2m_3(\xi) \cos 2\beta;
\]

\[
D_{yy}(\xi) = m_1(\xi) + m_2(\xi) - 2m_3(\xi) \cos 2\beta; \tag{2.11}
\]

\[
D_{xy}(\xi) = -im_1(\xi) + im_2(\xi) + 2m_3(\xi) \sin 2\beta;
\]

\[
D_{yx}(\xi) = im_1(\xi) - im_2(\xi) + 2m_3(\xi) \sin 2\beta.
\]

The functions \(m_1(\xi), m_2(\xi)\) and \(m_3(\xi)\) are defined by the expressions:

\[
m_1(\xi) = J_{-i\gamma-1}(z)J_{i\gamma+1}(z);
\]

\[
m_2(\xi) = J_{-i\gamma+1}(z)J_{i\gamma-1}(z); \tag{2.12}
\]

\[
m_3(\xi) = J_{i\gamma+1}(z)J_{-i\gamma+1}(z),
\]

where \(z = \sqrt{2\xi} kl; \gamma = \frac{\nu(\epsilon \hbar \Omega)}{\Omega} - i\omega / \Omega; k = \sqrt{k_x^2 + k_y^2}; \cos \beta = \frac{k_x}{k}; \) and \(\sin \beta = -\frac{k_y}{k}.\)
Note that the Fourier-transform of conductivity, Eqs.(2.10)-(2.13) contains terms proportional to \( \cos 2\beta \) and \( \sin 2\beta \), whose appearance results from the two-dimensionality of the electronic gas. They violate the symmetry of the kinetic coefficients for the Fourier-transform of, \( \sigma_{\alpha\beta}(\omega, k) \). Such polarizable terms may be important for the polarization of electromagnetic eigenoscillations in 2D electronic systems. The integration over \( d^2r \) re-establishes the symmetry.

One can easily see that generally the conductivity tensor experiences resonance oscillations, viz., of the cyclotron-resonance type in the case of a strong spatial dispersion (when \( k\ell^* \gg 1 \), where \( \ell^* \) is the mean free path) on the multiple harmonics (subharmonics of the CR when \( \omega = s\Omega, s = 1, 2, \ldots \)). However, the case of a strong spatial dispersion can be practically realized only at very high frequencies. As is easy to see from Eq. (2.3) and from the expression for the conductivity tensor the strong quantization in a magnetic field "destroys the Fermi surface" and the conductivity is due to all electrons with various energies. Thus, the frequency dispersion of the conductivity (and the spatial dispersion as well) is in essence defined by the form of the function \( \nu = \nu(\epsilon) \). At low frequencies, when \( \omega \leq \nu(\epsilon) \), it might be an "indicator" of the function \( \nu(\epsilon) \). It is also clear that the lineshape of the CR can be essentially dependent on the function \( \nu = \nu(\epsilon) \) \[11,12\]. The form of the longitudinal d.c. conductivity \( (\sigma_{xx} = \sigma_{yy}, \text{when} \omega = 0, k = 0) \) is also drastically dependent on the type of the function \( \nu = \nu(\epsilon) \) \[13\]. The lineshape of the CR can be changed if the electron effective mass \( m = m(\epsilon) \) depends on the electron energy \( \epsilon \), when the dispersion law of conduction electrons differs from the quadratic one \[14\]. In this paper we will assume that the mobility is very high and we will find the conductivity when the relaxation frequency is an effective constant, i.e., \( \nu=\text{const} \) \[15\]. We will consider the most realistic case, when the spatial dispersion is sufficiently weak, i.e.,

\[
k\ell \ll 1. \tag{2.13}
\]

Then the Fourier-transform of the conductivity tensor can be obtained in the form:

\[
\sigma_{\alpha\beta} = \frac{2e^2}{h} \frac{N}{1 + \gamma^2} \left\{ B_{\alpha\beta} - \frac{(k\ell)^2}{2\gamma} \left(1 + \frac{N}{2}\right) C_{\alpha\beta} \right\}, \tag{2.14}
\]
where

\[ \begin{align*}
B_{xx} &= B_{yy} = \gamma; \quad B_{xy} = -B_{yx} = 1; \\
C_{xx} &= a + \cos 2\beta; \quad C_{yy} = a - \cos 2\beta; \\
C_{xy} &= -b - \sin 2\beta; \quad C_{yx} = b - \sin 2\beta.
\end{align*} \]

(2.15)

Let us summarize the formulas for the resistance tensor in the d.c. case, when \( \omega = 0 \), \( k = 0 \) (IQHE)

\[ \begin{align*}
\rho_{xx} &= \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{\hbar}{2e^2 N} \gamma; \\
\rho_{xy} &= \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{\hbar}{2e^2 N} \frac{1}{\gamma^2 + 4}.
\end{align*} \]

(2.16) (2.17)

Fig. 2 shows the graphs of \( \rho_{xx} \) and \( \rho_{xy} \) as functions of the magnetic field (\( \omega = 0 \)) calculated by Eqs. (2.16) and (2.17). (The chemical potential \( \mu = 20meV \), the mean frequency of a momentum relaxation \( \nu = 10^{12}s^{-1} \), for the temperature \( T = 100mK \)). As can be seen, Eqs.(2.16) and (2.17) are good enough to describe the classical picture of IQHE [3,16], even at \( \nu=\text{const} \). Eq. (2.14) and Eq. (2.16) show that if \( \nu=\text{const} \) then the relation \( \frac{\rho_{xx}}{\rho_{xy}} = \frac{\gamma}{\nu} = \frac{\nu}{\Omega} \) should exist, which can be observed under the IQHE condition. The deviation from this simple relation demonstrates the energy dependence of the relaxation frequency, \( \nu = \nu(\epsilon) \) [16]. Here we will show the lineshape of the CR (the high-frequency absorption \( \sim \Re \sigma_{xx} \)) for various frequencies , \( \omega \), as a function of the magnetic field (see Fig. 3). Obviously, the lineshape of the CR is highly sensitive to the CR position. In the case when the line centre is located at the centre of the IQHE plateau, it shows kinks at the points, where the jumps of QHE occur. Fig.3(a) shows the lineshape of cyclotron resonance in 2DES calculated for GaAs/GaAlAs by formula Eq. 2.14, for the temperature \( T = 50mK, \) (\( k \to 0 \)) and \( \nu = 2 \cdot 10^{11}s^{-1} \). The wide resonance line, which captures the several steps of QHE for the rather low frequency \( \omega \), when the CR takes place at the quite small value of magnetic field.
It is shown that the additional structure of the line, which came about from the interference of singularities of the CR and QHE. If the centre of CR line is located near the QHE step, then the amplitude of the CR is increased. Fig. 3(b) shows the narrow resonance lines, which take place for the higher frequencies $\omega$ located in the range of one step of the QHE.

It is shown that the structure of line, whose centre is in the centre of the Hall plateau, exists on the wings of line at the values of magnetic field $B$ corresponding to the jump of the Landau-level filling factor $\aleph$. The amplitude of CR line is increased, when the center of CR line is located at the point of jump of the Landau-level filling factor $\aleph$ (The lines for the frequencies $\omega_3 = 1.00 \cdot 10^{13}\text{s}^{-1}$, $\omega_5 = 1.35 \cdot 10^{13}\text{s}^{-1}$). Such features of the CR line were observed by a number of authors [17].

### III. ELECTRODYNAMICS OF 2DES UNDER THE QHE CONDITION

The propagation of electromagnetic waves through systems with a 2D electronic gas in the dielectric environment, placed in a strong magnetic field (see Fig. 1), is described by the Maxwell equations for the scalar and vector potentials in the Lorentz gauge, viz.

$$\nabla A + \frac{\epsilon}{c} \frac{\partial \varphi}{\partial t} = 0. \tag{3.1}$$

The potentials satisfy the usual wave equation [18,19]

$$\begin{align*}
\left[ \nabla^2 - \frac{\epsilon}{c^2} \left( \frac{\partial}{\partial t} \right)^2 \right] \varphi (\mathbf{r}, t) &= -\frac{4\pi}{\epsilon} \rho_{\text{tot}} (\mathbf{r}, t), \quad \tag{3.2} \\
\left[ \nabla^2 - \frac{\epsilon}{c^2} \left( \frac{\partial}{\partial t} \right)^2 \right] \mathbf{A} (\mathbf{r}, t) &= -\frac{4\pi}{c} \mathbf{j}_{\text{tot}} (\mathbf{r}, t). \quad \tag{3.3}
\end{align*}$$

These values are related to the fields in Eq. (2.2) as

$$\mathbf{E} = -\nabla \varphi - c^{-1} \frac{\partial \mathbf{A}}{\partial t}; \quad \mathbf{B} = \text{rot} \mathbf{A}. \tag{3.4}$$

Here $\rho_{\text{tot}} = \rho_{\text{ex}} + \rho$ is the total charge density in the system and $\mathbf{j}_{\text{tot}} = \mathbf{j}_{\text{ex}} + \mathbf{j}$ the total current density; $\rho_{\text{ex}}$ and $\mathbf{j}_{\text{ex}}$ are the external charge and current densities, respectively. In the
system under consideration $\rho \sim j \sim \delta(z)$ \textsuperscript{[13]}, so that the charges and currents exist only in the 2D electronic layer, and the external currents and charges in the system are absent: $\rho_{ex} = j_{ex} = 0$. In this case the potentials $A$ and $\varphi$ can be found from the homogeneous equation set Eqs.(3.2), (3.3). Using the Fourier transformation Eq. (2.7) and taking into account $j(\omega, k, z) = j(\omega, k, 0) \delta(z)$, we obtain:

$$A(\omega, k, z) = A_0(\omega, k) e^{-p|z|}, \quad (3.5)$$

where $p = \sqrt{k^2 - \frac{\omega^2}{c^2} \varepsilon}$. In other words, the system supports eigenoscillations in the form of a surface wave pressed up against the 2D electronic layer, (see Fig. 1). In this case the component $A_z = 0$, while the scalar potential can be found from the Lorentz gauge (2.17):

$$kA = \frac{\omega \varepsilon}{c} \varphi. \quad \text{The current density in the 2D electronic layer can be represented in the form}$$

$$j_\alpha(\omega, k) = i\frac{\omega}{c} \sigma_{\alpha\beta}(\omega, k) \left[ A_\beta(\omega, k, 0) - \frac{c^2}{\varepsilon \omega^2} k_\beta k_\gamma A_\gamma(\omega, k, 0) \right]. \quad (3.6)$$

Thus, the dispersion relation is found from the condition

$$A(\omega, k, 0) = \frac{2\pi}{cp} j(\omega, k, 0). \quad (3.7)$$

This dispersion relation takes the form:

$$D = Det \left\{ \delta_{\alpha\beta} - \frac{2\pi i\omega}{c^2 p} \left[ \sigma_{\alpha\beta}(\omega, k) - \frac{c^2}{\varepsilon \omega^2} \sigma_{\alpha\gamma}(\omega, k) k_\gamma k_\beta \right] \right\} = 0, \quad (3.8)$$

where $\delta_{ab}$ is the Kronecker delta.

Fig. 1 presents the dispersion curves of the SP propagating in the system of Fig. 1. The dispersion curves for the surface polariton on the boundary of 2DES calculated for the various values of the Landau-level filling factor $\mathfrak{g}$ ($\mathfrak{g} = 1; \mathfrak{g} = 5$, and $\mathfrak{g} = 10$). 2DES is realized by heterostructure GaAs/GaAlAs, the effective mass $m = 0.068m_0$, $\varepsilon = 12$. The $y$–axis gives the real part of frequency, and the $x$–axis gives the wave number. It is seen that the spectrum of the SPs is gapless at low frequencies ($\omega \gg \Omega$) and they exist both in the low-frequency region $\omega < \Omega$ and in the high-frequency region $\omega > \Omega$. In the low-frequency region, far away from the CR, the phase velocity of the SP is close to the light
velocity \( v_d = c/\sqrt{\varepsilon} \) in the dielectric, which surrounds the 2D electronic layer. In the high-frequency region and in the vicinity of the principal CP (\( \omega \sim \Omega \)) the phase velocity of the SPs drastically decreases and they are transformed into slow waves. In the frequency range \( \Omega < \omega < 2\Omega \), where \( \ell^{-1} \gg k \gg \frac{\omega}{c}\sqrt{\varepsilon} \), one can neglect the retardation effect and the spatial dispersion in the conductivity tensor of Eq. (2.14). The dispersion relation can be brought to the form

\[
\omega^2 = \Omega^2 + \frac{2\pi v_n k \Omega}{\varepsilon},
\]  

(3.9)

where \( v_n = \frac{2e^2}{\hbar} \Re \). It is easy to see that with \( \Omega \gg \frac{v_n k}{\varepsilon} \) (\( \Re \sim 1 \)) the dispersion law of the SP is linear (\( \omega \sim k \)), while in the opposite case \( \Omega \gg \frac{v_n k}{\varepsilon} \) (or when \( \Re \gg 1 \)) the dispersion law is of a square-root type (\( \omega \sim \sqrt{k} \)).

Near the CR (\( \omega \sim \Omega \)) the retardation effect cannot be neglected and the dispersion law of the SP becomes

\[
\omega = \Omega + \frac{v_n}{c} \frac{\pi \Omega}{c p_1} \left\{ \frac{p_1^2 c^2}{\varepsilon p_1^2 - 1} + \frac{2\pi^2 v_n}{\varepsilon c} \right\} - i\nu.
\]  

(3.10)

Here \( p_1 = \sqrt{k^2 - \frac{\Omega^2}{c^2} \varepsilon} \). The value of the relative deceleration of the SP is determined by the Fine Structure Constant \( \alpha \). The group velocity, \( \mathbf{v}_g = \frac{\partial \omega}{\partial \mathbf{k}} \), of the SP undergoes the fundamental steps in the vicinity of the CR:

\[
\frac{\mathbf{v}_g}{v_d} = \frac{2\sqrt{2} \alpha \Re}{\sqrt{\varepsilon}}.
\]  

(3.11)

In other words, the deceleration of the wave near the CR is considerable, and the reason for the quantization of the group velocity is the quantization of the Hall conductivity, i.e., in fact the system possesses a fundamental parameter of the velocity dimension, the conductance quantum \( \frac{2e^2}{\hbar} \). At the point of the CR the character of conductivity is changed and it becomes imaginary, i.e., reactive one, and therefore the conductance becomes nondissipative, and the slow wave (the slow SP) appears. With a further increase of frequency \( \omega \), near the doubled CR (\( \omega \sim 2\Omega \)), the spatial dispersion effects of conductivity Eq. (2.14) becomes noticeable,
the group velocity changes its sign and takes negative values. In this region the SP shows
the anomalous (negative) dispersion.

The SP spectrum near the CR subharmonic \((ω \sim 2Ω)\) is obtained in the form

\[
ω = 2Ω - \Omega \frac{b}{d} \left( \frac{k\ell}{2} \right)^2 \left( 1 + \frac{\Omega}{2} \right) - i\nu. \tag{3.12}
\]

Here

\[
b = \frac{v_n}{c} \left( \frac{4\pi}{3} \right) \left( \frac{Ω}{cp} \right) \left( 1 - \frac{p^2c^2}{4εΩ} \right); \tag{3.13}
\]

\[d = 1 + 2b\] and \(p_2 = \sqrt{k^2 - \frac{4Ω^2}{c^2}ε}.\) At \(ω > 2Ω\) the SP propagates through the system
at a velocity close to that of the SP far away from the doubled CR (see Fig. 4). It can be
see from Eq. (29) that the spectrum of the SP is ”strongly pressed” to the line of the CR
subharmonic \((ω = 2Ω)\). The relative dispersive width of the SP has a scale of the small
parameter \(hΩ/mc^2 \ll 1\) near the CR. This dispersion curve (see the cut-in in the Fig. 4) of
the SP starts near the fundamental mode of the light line \((ω = kv_d)\), then it is branched (the
number of branches is equal to the Landau-level filling factor \(ν\)) and the group velocity is
quantized in the same way as near the principal CR \((ω = Ω)\). But the group velocity is very
low \(v_g \sim v_d(hΩ/mc^2)\), in the wavenumber region where the dispersion curve is ”strongly
pressed” to the line \(ω = 2Ω\). The spectrum of the SP ”slides” near the line \(k \simeq εΩ/v_n\).
At such values of \(k\) the spectral curve is detached from the line \(ω = 2Ω\) (see Fig. 4). The
relative attenuating rate of the SP is of the order \(ν/ω\) and is small for samples with a high
electron mobility. At low frequencies \((ω \ll Ω)\) the SP is a wave of the TE type, changing in
to TM at \(ω > Ω\). The SP polarization is defined by the following expression:

\[
A = A_0 \begin{pmatrix} 1 & q & 0 \end{pmatrix} e^{-|z|} e^{i(kr - ωt)}, \tag{3.14}
\]

The vector potential components are interrelated as \(A_y = qA_x, A_z = 0\), the scalar
potential \(φ\) is given by the Lorentz gauge \((3.1)\). The polarization parameter \(q\) is
\[ q = \left\{ 1 - \frac{2\pi i \omega}{c^2 \rho} \left[ \sigma_{xx} (1 - \frac{c^2 k_x^2}{\epsilon \omega^2}) - \frac{c^2}{\epsilon \omega^2} \sigma_{xy} k_x k_y \right] \right\} \left/ \left\{ \sigma_{xy} (1 - \frac{c^2 k_y^2}{\epsilon \omega^2}) - \frac{c^2}{\epsilon \omega^2} \sigma_{xx} k_x k_y \right\} \right. \].

(3.15)

It is easy to see that the SP polarization under the QHE condition is sensitive to the terms \( \sin(2\beta) \) and \( \cos(2\beta) \) of Eqs.(2.14 and 2.16). This leads to rotation of polarization plane as a function of the magnetic field \( B \). Far from the CR, where the spatial dispersion of conductivity is negligible, the polarization parameter \( q \) takes the simpler form

\[ q = \frac{c p}{\omega} \left\{ -i \frac{c}{v_n} \frac{(1 + \gamma^2)}{2\pi} + \frac{c p}{\epsilon \omega \gamma} \right\}. \]

(3.16)

It is evident that the polarization parameter \( q \) is quantized due to the Hall quantization.

In the system under consideration (see Fig. 1) another SP exists, which appears near the CR (\( \omega \sim \Omega \)). This SP is a dissipative-type wave. The surface wave exists when \( \text{Re} \rho > 0 \). When the relaxation frequency \( \nu \neq 0 \), the frequency \( \omega \) is a complex value. A straightforward analysis shows that the additional (dissipative) SP mode is practically nondispersional and the condition for its existence is determined by the threshold condition

\[ \frac{\nu}{\Omega} > 2\alpha \frac{\mathcal{N}}{\sqrt{\varepsilon}}. \]

(3.17)

The dispersion curve of the additional (dissipative) SP is shown in Fig. 5.

Thus, the observation of such a wave specifies the relaxation frequency to an accuracy up to the Fine Structure Constant \( \alpha \).

Fig. 5(a) (\( \mathcal{N} = 1; \ \nu/\Omega = 0.01 \)) shows that at such values of the relaxation frequency \( \nu \) the Additional Surface Polariton (ASP) is appears, which has an endpoint of the spectrum defined by the condition \( \text{Re} \rho = 0 \). The SP damping increases sharply near the CR, where the SP drastically decelerates. The damping of the ASP is sharply decreased and approaches zero, when it tends the endpoint of the spectrum. The ASP exists on the left side of the "light line" \( \omega = k v_d \), being in fact the delocalization wave because it is "weakly pressed" to the 2DES. The SP is the proper surface wave, which is "strongly pressed" to the 2DES far from the principal mode (the "light line") \( \omega = k v_d \).

Fig. 5(b) (\( \mathcal{N} = 1; \ \nu/\Omega = 0.2 \)) shows the spectra of the ASP and SP, when \( \nu \) is increased. At such values of \( \nu \) the spectrum is essentially modified. A gap opens in the spectrum. This
gap is brought about by the endpoint in the spectrum of the low frequency SP \((\omega < \Omega)\) and the blending of the ASP with the decelerated SP. The low frequency SP becomes a completely delocalized wave, it is practically not connected to the 2DES. The damping of the new blended mode (ASP and SP) has a fixed value \(\nu/\Omega\) in the over a wide range of the wave numbers \(k\).

Fig. 5(c) \((\mathcal{N} = 5; \; \nu/\Omega = 0.1)\) shows the spectral curves for lower magnetic fields (when the Landau-level filling factor is \(\mathcal{N} = 5\)). The ASP is blended with the decelerated part of the SP near the principal CR. The low frequency part of the principal mode of the SP \((\omega = kv_d)\) is separated from the ASP, and its spectrum \((\text{Re}\omega = \omega' = \omega(k))\) ends at the point where \(\text{Re} p = 0\). The principal mode \(\omega = kv_d\) is "weakly pressed" to the 2DES for all the values of \(\Omega\) and \(k\) because \(\text{Re} p \ll \text{Im} p\). But the decelerated part of the SP and ASP (the upper curve in 5(c)) are "strongly pressed" to the 2DES, since for this part of the spectrum \(\text{Re} p \gg \text{Im} p\). In other words, the slow SP and ASP are proper surface waves.

Fig. 5(d) \((\mathcal{N} = 5; \; \nu/\Omega = 0.2)\) shows the picture of spectral curves, when the relaxation frequency \(\nu\) is greater than that of 5c. The endpoint for the principal SP mode \((\omega = kv_d)\) is moved down off the ASP and slow SP spectral curve. In that picture only the real parts of the spectral curves cross, while the imaginary parts \(\omega'\) of the frequencies assume different values [20].

The spectral picture of the ASP changes crucially at larger values of the Landau-level filling factor (see Fig. 5(e), \(\mathcal{N} = 10 \; \nu/\Omega = 0.1\)). First, the curves of the ASP and slow SP are separated, second the ASP curve acquires an endpoint of the spectrum \((\text{Re} p = 0)\). It is significant that the spectrum shows an anomalous (negative) dispersion near the CR and an endpoint of ASP. At high values of the relaxation frequency (see Fig. 5(f), \(\mathcal{N} = 10; \nu/\Omega = 0.2\)) the picture of the dispersion curves is of the same kind as in Fig. 5(d), when \(\mathcal{N} = 5; \nu/\Omega = 0.2\).

The curves of the SP damping in the series of pictures in Fig. 5 are qualitatively similar. The negative damping of the SP becomes essential and is of order \(\nu/\Omega\) near the CR, where the SP is drastically decelerated. The damping of the ASP is sharply diminished in the
vicinity of the point where Re$p = 0$, and turns to zero. The damping of the principal SP mode ($\omega = kv_d$) becomes vanishingly small when the relaxation frequency $\nu$ is increased. This is due to the fact that the 2DES has a small conductivity (it transforms to a dielectric) and the surface wave is separated from the 2DES to become a quasibulk mode.

IV. CONCLUSION

To conclude it is well to emphasize that the phase velocity of the SP is a remarkably small value near the CR. In other words, the 2D electronic layer under the QHE condition is an effectively decelerating system. This fact can be used for various applications in microelectronics. For example, it can be used for the excitation of surface electromagnetic waves by a beam of charged particles passing near a 2D electronic layer and for efficient conversion of the beam energy into the energy of waves.

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FIGURES

FIG. 1. The geometry of structure of 2D electronic layer embedded in dielectric medium with dielectric constant $\varepsilon$.

FIG. 2. The Hall ($\rho_{xy}$) and longitudinal ($\rho_{xx}$) resistance as a function of magnetic field $B$ for the typical parameters of 2D electronic structure GaAs/GaAlAs.

FIG. 3. The lineshape of cyclotron resonance in 2DES under the QHE condition.

FIG. 4. The dispersion curves for the surface polariton on the boundary of 2DES calculated for the various values of the Landau-level filling factor $\mathcal{N}$ ($\mathcal{N} = 1; \mathcal{N} = 5$, and $\mathcal{N} = 10$).

FIG. 5. SP and ASP spectrum ($\omega'$) (solid line); SP damping ($\omega''$) (dashed line); and ASP damping ($\omega''$) for various values of $\mathcal{N}$ and $\nu/\Omega$: 