Abstract. Several recent papers -using effective QCD chiral Lagrangians- reproduced results obtained with the general QCD parameterization (GP). These include the baryon $8+10$ mass formula, the octet magnetic moments and the coincidental nature of the “perfect” $(\mu_p/\mu_n) = -(3/2)$ ratio. Although we anticipated that the GP covers the case of chiral treatments, the above results explicitly exemplify this fact. Also we show by the GP that -in any model or theory (chiral or non chiral) reproducing the results of exact QCD- the Franklin (Coleman Glashow) sum rule for the octet magnetic moments must be violated.

1.Introduction: Chiral results and GP results

1- In a recent publication Durand et al.[1] (see also [2]) derived, by an effective chiral Lagrangian, the same $8+10$ baryon mass formula that, using the general QCD parameterization (GP) [3], [4], had been obtained in [5] and re-discussed in [6]. Also they derive in Ref.[1] an expression for the baryon octet magnetic moments similar to that obtained with the GP in Refs.[3, 4, 6, 9]. They kindly acknowledged all this in [1].

2- These interesting results of Durand et al. add to one by Leinweber et al. In Ref.[7] (see also [8]) they show that, in a chiral description, the fact that the ratio $\mu(p)/\mu(n)$ is so near to $-(3/2)$ (the NR quark model result) is coincidental. We agree: In fact we reached the same conclusion by the GP method [4, 6, 9] independently of a specific chiral description.

3- Another point discussed recently (e.g. [10], [11]) is the Franklin sum rule [12] for the octet baryon magnetic moments. The revived interest in the rule is due to the fact [10] that according to a chiral quark model, the Franklin rule should be exact, while experimentally is not. The GP shows that, in fact, the rule is violated by two specific 1st order flavor breaking terms, present in the QCD but not in the $\chi$QM.

4- The GP explains the Pondrom fit [13] of the magnetic moments.

2. The general parameterization of QCD.

The method (GP) [3, 6] is derived exactly from the QCD Lagrangian exploiting only few general properties. Although non covariant, the GP is relativistic. Also [6]
the renormalization point for the quark masses can be selected at will in the QCD Lagrangian. The GP is compatible, in particular, with a quasi-chiral Lagrangian (with light u,d quarks) if the latter does not violate the properties of the QCD Lagrangian. By integrating over the virtual $q\bar{q}$ and gluon variables, the method parameterizes exactly the results of the QCD calculations of various hadron properties, expressing them in a few body language. It allows to write, almost at first sight, the most general expression for the spin-flavor structure of quantities relevant to the lowest baryons ($8+10$) and mesons. Unexpectedly one finds that, for the lowest hadrons, the GP is characterized, usually, by a rather small number of terms.

A consequence of the GP is that it allows to know if a constituent quark model is consistent with QCD. For any given property under study (masses, magnetic moments, etc.) it displays the exact result of QCD as a parameterized spin-flavor expression. The terms (and only the terms) present in the GP are compatible with QCD.

Of course solving QCD (if one could do that) would express all parameters in the GP, in terms of $\Lambda_{QCD}$ and the masses of the quarks. But much can be understood even if one is unable to calculate by QCD the values of the parameters. This goes as follows: Consider e.g. the masses of the lowest baryons ($8+10$). Neglecting the e.m corrections and the u, d mass difference these are 8. But there are only 8 GP parameters in this case; so they can be empirically determined. After doing this a hierarchy in the parameters emerges: The parameters multiplying spin flavor structures of increasing complexity are smaller and smaller. This is true for any quantity (not only the masses). Often this hierarchy allows to neglect some terms in the GP; in particular it explains why the non relativistic quark model NRQM \cite{14} works.

3. The $8+10$ mass formula. A comparison with the chiral results.

The parameterization of the masses $M_B$’s of the 8 and 10 baryons is (for the notation and use of Eq.\(\ref{eq:1}\) see \cite{5,6}; only the combination $a+b$ enters in the masses):

\[
\text{“parameterized mass” = } A + B \sum P_i^s + C \sum (\sigma_i \cdot \sigma_k) + \\
+ D \sum_{i>k} (\sigma_i \cdot \sigma_k)(P_i^s + P_k^s) + E \sum_{i \neq k \neq j} (\sigma_i \cdot \sigma_k)P_j^s + a \sum_{i>k} P_i^s P_k^s + \\
+ b \sum_{i>k} (\sigma_i \cdot \sigma_k)P_i^s P_k^s + c \sum_{i \neq k \neq j} (\sigma_i \cdot \sigma_k)(P_i^s + P_k^s)P_j^s + d P_1^s P_2^s P_3^s
\]  

(1)

In Eq.\(\ref{eq:1}\) the flavor breaking term $\Delta m \bar{\psi}P^s \psi$ in the QCD Lagrangian is taken into account to all orders in $\Delta m = m_s - m$, no matter how large is $\Delta m$; thus Eq.\(\ref{eq:1}\) includes all orders in flavor breaking. The values (in MeV) of the parameters in Eq.\(\ref{eq:1}\), obtained fitting the baryon masses, are \cite{5,6}:

\[
A = 1076, \quad B = 192, \quad C = 45.6, \quad D = -13.8 \pm 0.3, \quad (a+b) = -16 \pm 1.4, \quad E = 5.1 \pm 0.3, \quad c = -1.1 \pm 0.7, \quad d = 4 \pm 3
\]  

(2)
The hierarchy is evident. The values decrease rather strongly with increasing complexity of the accompanying spin-flavor structure so that one can neglect \( c \) and \( d \) in Eq. (1) and obtain the following mass formula [5], a generalization of the Gell-Mann Okubo formula including octet and decuplet:

\[
\frac{1}{2}(p + \Xi^0) + T = \frac{1}{4}(3\Lambda + 2\Sigma^+ - \Sigma^0)
\]

(3)

The symbols stay for the masses and \( T \) is the following combination of decuplet masses:

\[
T = \Xi^* - \frac{1}{2}(\Omega + \Sigma^*)
\]

(4)

Because of the accuracy reached, we wrote so as to be free of electromagnetic effects before comparing it to the data. (The combinations in (3) are independent of electromagnetic and of \( m_d - m_u \) effects, to zero order in flavor breaking \( (m_s - m) \).) The data satisfy as follows (using the pole values, in MeV, of the masses)

\[
l.h.s. = 1133.86 \pm 1.25 \quad r.h.s. = 1133.93 \pm 0.04
\]

(5)

an agreement confirming the smallness of the \( c, d \) terms neglected in (1) (With the conventional values of the masses the agreement is similar).

Of course a full QCD calculation, if feasible, would express each \( (A, B \ldots c, d) \) in terms of the quantities in the QCD Lagrangian, the running quark masses and the dimensional (mass) parameter \( \Lambda \equiv \Lambda_{QCD} \); for instance: \( A \equiv \hat{A}(m/\Lambda, m_s/\Lambda) \) where \( \hat{A} \) is some function. Similarly for \( B, C, D, E, a, b, c, d \).

The results of Durand et al. [1] (see Sect.1) are obtained using an effective chiral QCD Lagrangian and heavy baryon chiral perturbation theory. They so reobtain our mass formula (Eq.(3)) (our \( T \) is their \( \hat{\alpha}_{MM} \) in their Eq.(3.38); they did not, however, extract the e.m. corrections from their sum rule) 3. Of course this result of [1] was expected. As stated in Sect.2, the GP is compatible with any relativistic chiral description, satisfying the listed general properties of QCD (for the pion field in the chiral Lagrangian, compare the end of Sect.4). But it is interesting to see this in practice, especially in view of the heavy calculations in the chiral treatment of [1] 4.

Clearly the work of Durand et al. also confirms indirectly the existence of a hierarchy, as shown by the fact that they reobtain our mass formula (Eq.3) which is due to the smallness of \( c, d \) in (2).

As to the hierarchy in the GP, its field theoretical basis is discussed in [4]. The terms in the GP can be related to classes of Feynman diagrams in QCD (fig.1 of [4]). The

3It is only after doing this that the agreement becomes striking, as in Eq.5 above.

4In Ref.[1](E) “the parameter \( T \)” should be read as “the quantity \( T \)” (in fact \( T \) is defined by Eq.4). Also the statement from “so is not to be used” to “Our approaches differ in that respect” is not too clear to us, because our \( T \) is identical to their \( \hat{\alpha}_{MM} \) (except that we included the e.m. corrections). If the above statement means that, with the chiral effective Lagrangian, the individual baryon masses can be calculated in terms of the couplings introduced in the Lagrangian, this may be true in principle; but, as in many chiral treatments, uncertainties often arise in practice.
decrease of the coefficients that multiply terms with more quark indices is due: a) to
the increase of the number of gluons exchanged; b) to the fact that each flavor breaking
$P^i$ also carries a reduction factor. Above we saw this for the masses. Indeed the values
in Eq.(2) show that in Eq.(1) an additional pair of indices (corresponding to at least an
additional gluon exchange between quark lines) implies a reduction factor in the range
from 0.22 to 0.37 \[11\] \[6\]. (One gets 0.37 using the pole values of the decuplet masses as
we did in Eq.(2) and 0.22 using the conventional values). The range of values 0.20 to
0.37 covers all the hadron properties examined so far. We will adopt usually 0.3 for the
reduction factor due to “one gluon exchange more”. The flavor reduction factor is in
the range 0.3 to 0.33. Our reduction factor $\approx 0.3$ is just an empirical
number derivable in principle from QCD. Some papers relate this $\approx 1/3$ to the $1/N_c$
expansion. We do not see similarities between the basis of the GP (an exact QCD parameterization) and
the $1/N_c$ expansion (see \[15\]).

4. The magnetic moments of the baryon octet

To first order in flavor breaking the parameterized magnetic moments $M_z(B)$ of the
octet baryons $B$ derived from QCD are necessarily (notation in Ref. \[6\]):

$$M_z(B) = \sum_{\nu=1}^{7} \tilde{g}_\nu (G_\nu)_z$$

where:

$$G_1 = \sum_i Q_i \sigma_i ; \quad G_2 = \sum_i Q_i P^i \sigma_i ; \quad G_3 = \sum_{i\neq k} Q_i \sigma_k ; \quad G_4 = \sum_{i\neq k} Q_i P^i \sigma_k$$

$$G_5 = \sum_{i\neq k} Q_k P^i \sigma_i ; \quad G_6 = \sum_{i\neq k} Q_i P^k \sigma_i ; \quad G_7 = \sum_{i\neq j \neq k} Q_i P^j \sigma_k$$

It is understood that the expectation value of the r.h.s of \[6\] on the octet spin-flavor
states $W_B$ (compare \[6\]) must be taken. Eight $G_\nu$’s appear in Eq.(23) of Ref.\[6\]; but
due to the following Eq.(8)-holding for the expectation values of the $G_\nu$’s in the $W_B$’s
(see \[6\]) - $G_0 = Tr[Q P^* \sum_i \sigma_i]$ is expressed in terms of the $G_\nu$’s with $\nu = 1...7$; thus
the sum in Eq.(6) contains 7 terms; their coefficients $\tilde{g}_\nu$ differ inappreciably from the
$g_\nu$ multiplying the 8 $G_\nu$. This is a very general consequence of QCD (compare the
evaluation of the quark loop effect in \[17\]).

$$G_0 = -\frac{1}{3} G_1 + \frac{2}{3} G_2 - \frac{5}{6} G_3 + \frac{5}{3} G_4 + \frac{1}{6} G_5 + \frac{1}{6} G_6 + \frac{2}{3} G_7$$

Though the $G_\nu$’s look non relativistic, Eq.(6) is an exact consequence of full QCD
(to first order in flavor breaking). We repeat this to avoid misinterpreting the Eq.(6)
as a sort of generalized NRQM. Note the relative dominance of $\tilde{g}_1$ and $\tilde{g}_2$ in the sum
in Eq.(6) (see the Eq.(12) below); this is explains the fairly good 2-parameter fit of the
naive NRQM:

$$M_z(B) = \tilde{g}_1 (G_1)_z + \tilde{g}_2 (G_2)_z \quad (NRQM)$$

4
We now compare the above results with the chiral treatment of Durand et al. [1]. It is notable that the latter produces precisely 7 terms for the octet baryon magnetic moments, to first order flavor breaking, (their Eqs.(4.6) to (4.12)) written in terms of Pauli spin matrices, similarly to our equations (14)(their symbols $M$ are our $P^*$). Their seven $m$'s are linear combinations of our $G$'s. In their footnote 14 a relation appears similar to our Eq.(8).

In the following we will need the magnetic moments in terms of the $\tilde{g}_\nu$'s. From Eq.(6) we get (the baryon symbol indicates the magnetic moment):

\[ p = \tilde{g}_1 \]
\[ n = -(2/3)(\tilde{g}_1 - \tilde{g}_3) \]
\[ \Lambda = -(1/3)(\tilde{g}_1 - \tilde{g}_3 + \tilde{g}_2 - \tilde{g}_5) \]
\[ \Sigma^+ = \tilde{g}_1 + (1/9)(\tilde{g}_2 - 4\tilde{g}_4 - 4\tilde{g}_5 + 8\tilde{g}_6 + 8\tilde{g}_7) \]
\[ \Sigma^- = -(1/3)(\tilde{g}_1 + 2\tilde{g}_3 + (1/9)(\tilde{g}_2 - 4\tilde{g}_4 + 2\tilde{g}_5 - 4\tilde{g}_6 - 4\tilde{g}_7) \]
\[ \Xi^0 = -(2/3)(\tilde{g}_1 - \tilde{g}_3) + (1/9)(-4\tilde{g}_2 - 2\tilde{g}_4 + 4\tilde{g}_5 - 8\tilde{g}_6 + 10\tilde{g}_7) \]
\[ \Xi^- = -(1/3)(\tilde{g}_1 + 2\tilde{g}_3) + (1/9)(-4\tilde{g}_2 - 2\tilde{g}_4 - 8\tilde{g}_5 - 2\tilde{g}_6 - 2\tilde{g}_7) \]

and:

\[ \mu(\Sigma\Lambda) = -(1/\sqrt{3})(\tilde{g}_1 - \tilde{g}_3 + \tilde{g}_6 - \tilde{g}_7) \]

From the PDG values of the magnetic moments [16] we obtain:

\[ \tilde{g}_1 = 2.793 \quad \tilde{g}_2 = -0.934 \quad \tilde{g}_3 = -0.076 \quad \tilde{g}_4 = 0.438 \]
\[ \tilde{g}_5 = 0.097 \quad \tilde{g}_6 = -0.147 \quad \tilde{g}_7 = 0.154 \]

In Eq.(12) the hierarchy is apparent: The average value of the one gluon exchange reduction factor derived from the values of $|\tilde{g}_6|, |\tilde{g}_5|, |\tilde{g}_4|$ is 0.25, having adopted 0.3 for the flavor reduction factor (this is 0.33 from the ratio of $|\tilde{g}_2|$ and $|\tilde{g}_1|$). We go on here using 0.3 for both reduction factors; doing so, the maximum discrepancy between estimated and empirical values is 2.5 for each $|\tilde{g}_\nu|$ with $\nu = 4, 5, 6$.

An exception is $|\tilde{g}_3| \approx 0.08$. This is much too small: One expects from the hierarchy $2.79 \times 0.3 \approx 0.84$, a value 10 times larger. We discuss this in Sects. 5,6.

A comment to the Pondrom’s 4-parameters fit [13] of the baryon moments is appropriate here. Pondrom’s fit is based on the conjecture of assigning to the quarks different magnetic moments in different baryons and assume additivity. But these assumptions lead to the following approximate empirical relations (holding to ±0.1):

\[ -(2/3)p \simeq n \quad (1/2)(\Sigma^- - \Sigma^+) \simeq n \quad \Xi^- + (1/2)\Xi^0 \simeq 2\Lambda \]
The Eqs. (13), of course, reduce from 7 to 4 the number of the $\tilde{g}_\nu$’s. One thus finds that the GP plus the smallness of $\tilde{g}_3$ explain why a 4-parameters fit is rather good, independently of any symmetry (a question raised in [13]). Finally, the fit [13] gives $\mu(\Sigma\Lambda) = -1.61$. The GP formula (11) - gives instead $\mu(\Sigma\Lambda) = -1.48 \pm 0.04$. Experimentally it is $|\mu(\Sigma\Lambda)| = 1.61 \pm 0.08$; errors are still large.

To go on with the comparison to the work of Durand et al.[1], a remark on the pion exchange terms in a class of calculations of the baryon moments is necessary. For instance, for the $p$ and $n$ moments $M_z(p, n)$ a typical such term is:

$$M_z(p, n) = \ldots \ldots \ldots + \alpha \sum_{i \neq k} (\sigma_i \times \sigma_k)z(\tau_i \times \tau_k)$$

where the dots in Eq.(14) refer to the contributions other than pion exchange and $\alpha$ is some coefficient. Because in the exact QCD Lagrangian only the quark and gluon fields intervene (not those of pions), the question arises of the meaning of such pion exchange terms. The answer (see [18]) is that they simply duplicate terms already present in the GP; they can be always incorporated into them. It is:

$$\sum_{i \neq k} (\sigma_i \times \sigma_k)(\tau_i \times \tau_k) = -8G_1 + 4G_3$$

Durand et al. in [1] showed that all pion exchange magnetic moments could be rewritten in terms of their 7 quantities $m$’s (their Eqs.(4.6) to (4.12) in [1]) - which are simply certain linear combinations of our $G$’s. We found this result (from their Eq.(4.29) to their Eq.(4.36)) interesting also because it confirms the GP on this rather subtle point; the pion loops in the chiral treatment of Ref.[1] are eliminated by a mechanism that must be equivalent to that of Eq.(15).

5. The coincidental nature of the “perfect” 3/2 prediction for $|\mu(p)/\mu(n)|$

The Eq.(6) of the GP for the magnetic moments, applied to $p$ and $n$, gives

$$M(n, p) = \tilde{g}_1G_1 + \tilde{g}_3G_3 = \tilde{g}_1 \sum_i Q_i \sigma_i + \tilde{g}_3 \sum_{i \neq k} Q_i \sigma_k$$

In Sect.4 we noted that $\tilde{g}_3$ is ten times smaller than the value $2.79 \times 0.3 \simeq 0.84$ expected from the hierarchy; the other $\tilde{g}_\nu$’s (with $\nu=4,5,6$) differ by no more than 2.5 times from their expected values. Due to this we suggested in [4, 6, 9] that the early prediction of the NRQM [14] $|\mu(p)/\mu(n)| = 3/2$ is coincidental. Indeed (Eq.(16)) it is

$$|\mu(p)/\mu(n)| = -3/2[\tilde{g}_1/(\tilde{g}_1 - \tilde{g}_3)]$$

so that $|\mu(p)/\mu(n)|$ depends critically on $\tilde{g}_3$, the coefficient of the second (non additive) term in Eq.(16).

Recently Leinweber et al. [7] reached the same conclusion, that the almost perfect 3/2 prediction is coincidental. In Ref.[7] $|\mu(p)/\mu(n)|$ is calculated in a chiral QCD
perturbation theory, dynamically broken by pions; the above ratio varies from 1.37 to 1.55 as the pion mass varies from 0 to \( \simeq 280 \) MeV (corresponding to “a variation of current quark mass from 0 to just 20 MeV”). Again, as with the work of Durand et al., the chiral conclusion agrees with that from the GP. We try however to clarify some statements in Ref.\[7\] and in Cloet et al. \[8\]: 1) The assertion in Ref.\[7\] that “within the constituent quark model the ratio \(|\mu(p)/\mu(n)|\) would remain constant at 3/2, independent of the change of the quark mass” is correct only in an additive model, such as the original NRQM \[14\]. The GP expression (16) for the \( p,n \) magnetic moments in a constituent model obtained from QCD, shows that this is not additive (for the importance of non additivity see also Ref.(19)). 2) Cloet et al. \[8\], referring to the GP as “something a little more sophisticated than the simplest constituent quark model”, add a statement on “the need to incorporate meson cloud effects into conventional constituent quark models”. There is no doubt that constituent quarks must be dressed, but this was already there in the old additive NRQM \[14\]. The GP incorporates all \( q\bar{q} \) and gluon effects \[3\]; in particular the magnetic moments of the \( 8 \) baryons in the GP are those of \textit{any possible} constituent quark model compatible with QCD, endowed with the correct “dressing” of the (constituent) quarks.

To conclude: The chiral Lagrangians of Durand et al. and of Leinweber et al. produce results in agreement with the GP. As to the question: What about the pion field that appears in the chiral Lagrangians of \[1\] and \[7\], but not in the exact QCD Lagrangian (where the pion is not an independent field), this has been answered in Sect.4 for the magnetic moments and a similar argument should be true for the masses (as shown by the results of \[1\]).

6. Parameterizing the magnetic moments of the decuplet: \( p,n,\Delta \)

We apply now the GP to the magnetic moments of the \( \Delta \)’s in addition to \( p,n \). This clarifies further the mechanism producing accidentally a small value of \( \tilde{g}_3 \) (noted in the past section) and thus the coincidental nature of \(|\mu(p)/\mu(n)| = 3/2\). Also we obtain some results on the \( \Delta \)’s. The general QCD spin-flavor structure of the magnetic moments of \( p,n,\Delta \)’s is \[3\] \[8\]:

\[
\mu(B) = \sum_{\text{perm}} [\alpha Q_1 + \delta (Q_2 + Q_3)] \sigma_{1z} + [\beta Q_1 + \gamma (Q_2 + Q_3)] \sigma_{1z} (\sigma_2 \cdot \sigma_3) \tag{18}
\]

The Eq.(18) is the same as Eq.(62) of \[3\]; \( \alpha,\delta,\beta,\gamma \) are four real parameters. The sum over perm(utations) in (18) means that to the term \((123)\) one adds \((321)\) and \((231)\).\footnote{In \[3\] correct the following misprints: In Eq.(63) insert \((-2\gamma)\) in the second square brackets; in Eq.(66) write \( F=\delta - \beta - 4\gamma \); in Eq.(64) (Q term) replace \(-2\gamma\) with \(+4\gamma\).}

We adopt the “standard” hierarchy for the parameters \( \alpha,\delta,\beta,\gamma \) with the reduction factor \( 0.3 \) for one more gluon exchange; but because this factor for the magnetic moments is between 0.2 and 0.3 -see the remarks in Sect.4 after Eq.(12)- we widen the error in \( \delta \), the largest parameter after the dominant one \( \alpha \)

\[
|\delta/\alpha| = 0.2 \leftrightarrow 0.3; \quad |\beta/\delta| \approx 0.3; \quad |\gamma/\delta| \approx 0.3 \tag{19}
\]
From Eqs. (18, 16) we obtain for \( \tilde{g}_1 \) and \( \tilde{g}_3 \)
\[
\tilde{g}_1 = \alpha - 3\beta - 2\gamma \quad ; \quad \tilde{g}_3 = \delta - \beta - 4\gamma
\]  
(20)

From Eq. (16) one has for the the magnetic moments \((p, n)\), now indicated by the particle symbols, expressed in proton magnetons:
\[
p = (\alpha - 3\beta - 2\gamma) , \quad n = -(2/3)(\alpha - \delta - 2\beta + 2\gamma)
\]  
(21)

Hence:
\[
(n/p) = -2/3(1 + [-\delta + \beta + 4\gamma]/p)
\]  
(22)

Thus the deviation of \(|n/p|\) from \((2/3)\) is determined by the term in square brackets in Eq. (22). If it were not for the second order terms \((\beta + 4\gamma)\) with \(\beta\) and \(\gamma\) of order \((0.3)^2\) (three indices), the dominant deviation would be of order \(|\delta/p| = 0.25 \pm 0.05\); that is 20\% to 30\% of the “perfect” value 2/3. To summarize, the mechanism giving to \(|n/p|\) a value so near to \((3/2)\) is this: In Eq. (22) \((\beta + 4\gamma)\) almost cancels \((-\delta)\) (which is \(> 0\)), producing, accidentally, \((n/p) = -(3/2)\) to a few percent. One can also show that \(\beta\) and \(\gamma\) must have opposite signs and \(\gamma\) is \(< 0\). This is derived using the \((\Delta \rightarrow p\gamma)\) matrix element extrapolated to vanishing transferred photon momentum \((k = 0)\) that we know experimentally. We do not enter on this here (compare [9] where, however, some data must be changed). Here we just give the approximate values of \(\alpha, \delta, \beta, \gamma\). It is: \(\alpha \simeq 3\) and the values of \(\delta, \beta, \gamma\) (affected by the errors stated in Eq. (19)) are: \(\delta = -0.75, \beta = 0.25, \gamma = -0.25\).

One more point: From Eq. (18) one can also express in terms of \(\alpha, \delta, \beta, \gamma\) the magnetic moments \(\mu(\Delta)\) of the \(\Delta\)’s. It is:
\[
\mu(\Delta) = (\alpha + 2\delta + \beta + 2\gamma)Q_\Delta = (\mu(p) + 2\delta + 4\beta + 4\gamma)Q_\Delta
\]  
(23)

In view of the above, the magnetic moment of the singly charged \(\Delta^+\), \(\mu(\Delta^+)\) (the coefficient of \(J_z\)), is expected to be appreciably smaller than \(\mu(p)\), but the error on its value is large. (Of course it is \(\mu(\Delta^+) \simeq 2\mu(\Delta^+)\). We stress that, in general, it is \(\mu(\Delta^0) = kQ + \xi\), but \(\xi\) is negligible \([7]\) because it is a Trace term strongly depressed by the exchange of several gluons needed by the Furry theorem.

7. Some remarks on a sum rule for the baryon octet magnetic moments

Long ago Franklin [12] suggested a sum rule for the baryon octet magnetic moments that is often called the Coleman-Glashow rule since it has the same form as the Coleman-Glashow rule for the electromagnetic mass differences. Franklin’s rule is:
\[
\Sigma_\mu = 0
\]  
(24)

where it is
\[
\Sigma_\mu \equiv \mu(p) - \mu(n) + \mu(\Sigma^-) - \mu(\Sigma^+) + \mu(\Sigma^0) - \mu(\Xi^-)
\]  
(25)
Because in chiral models of baryons of the Manohar-Georgi type ($\chi$QM) the rule of Eq. (24) should be satisfied [10], but in reality it is violated:

$$\Sigma_\mu = 0.49 \pm 0.03$$

some work has been done [10, 11] to understand the reason for this fact. After showing that in a ($\chi$QM) the rule (24) is satisfied, Linde et al. consider several extensions of the $\chi$QM (several phenomenological models) that break the rule. We refer to [10, 11] for many references and a description of the models.

Here we will only show that it follows from the GP that the rule is necessarily broken by two specific first order flavor breaking terms, $\tilde{g}_5$ and $\tilde{g}_6$. Indeed, using the Eqs. (10), (12) it is:

$$\Sigma_\mu \equiv 2(\tilde{g}_5 - \tilde{g}_6) \quad (27)$$

With the $\tilde{g}_\nu$’s given in Eq. (12) the r.h.s. of Eq. (27) is in fact $(0.49 \pm 0.03)$.

The interest of the above deduction stays in its conclusion: Any Lagrangian or phenomenological model (chiral or non chiral) designed to reproduce the result of the exact QCD Lagrangian violates the Franklin (Coleman-Glashow) sum rule for the octet baryon magnetic moments if, and only if, the coefficients of the flavor breaking terms $G_5$ and $G_6$ do not vanish and their difference does not vanish. In such case the model must be built in such a way that $2(g_5 - g_6)$ is equal to the experimental value $0.49 \pm 0.03$. All other parameters, multiplying $G_\nu$ with $\nu \neq 5, 6$, even the flavor breaking ones with $\nu = 2, 4, 7$, do not produce violations of the rule.

8. Conclusion

In Ref. [6] we stated that the general QCD parameterization (GP) explains why a large variety of different theories and models, including relativistic chiral theories, may work successfully. Now, thanks especially to the treatment by Durand et al. [1] of the baryon masses and magnetic moments, we have an explicit detailed confirmation that the GP covers (see also [7]) the case of the relativistic chiral field theories, provided that such theories (or models) are compatible with the general properties of QCD.
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