Tensor form of magnetization damping

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A tensor form of phenomenological damping is derived for small magnetization motions. This form reflects basic physical relaxation processes for a general uniformly magnetized particle or film. Scalar Landau-Lifshitz damping is found to occur only for two special cases of system symmetry.

I. INTRODUCTION

The dynamics of magnetization $\mathbf{M}$ of a single-domain ferromagnetic particle is usually described by the Landau-Lifshitz (LL) equation $^{[1]}$:

$$
\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\alpha}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}). \tag{1}
$$

Here $\mathbf{H}_{\text{eff}}$ is the effective field, $\gamma$ is the gyromagnetic ratio, $M_s = |\mathbf{M}|$ is the saturation magnetization and $\alpha$ is a dimensionless damping parameter. The first term in (1) directly follows from microscopic equations and describes an averaged precession of a large number of spins. The second term in (1) was introduced phenomenologically in Ref. $^{[1]}$ just from a simple geometric consideration to describe the magnetization damping. Later Gilbert $^{[2]}$ declared that this damping term can be rewritten as a “dry friction”. Here we focus on the case of small damping ($\alpha \ll 1$) and neglect non-uniform magnetization motions.

More complicated forms of phenomenological damping were proposed by Bar’yakhtar and co-authors $^{[3, 4]}$ on the base of general symmetry considerations of exchange and relativistic relaxation processes. According to these papers, for a uniform magnetization motion “the crystal symmetry should influence the form of the relaxation terms” and, therefore the phenomenological damping term should contain a damping tensor with several damping parameters (“hierarchy of dissipative terms”) instead of one (isotropic) damping.

However, a general problem of magnetization damping cannot be solved using just symmetry considerations. The magnetization relaxation process appears as a result of microscopic interactions of spins with each other and with phonons, conduction electrons and so on. In other words, a direct connection with microscopic physics must be found for the damping terms. The microscopic derivation of damping has been performed, for example, in the case of “valence-exchange” relaxation $^{[5]}$ and magnetization relaxation on paramagnetic impurities $^{[6]}$. A tensor structure of damping term follows from the results of Refs $^{[3]}$ and $^{[5]}$.

The aim of this paper is to demonstrate that the conventional relaxation term in the LL equation is inconsistent (with the exception of two special cases) with a form that follows from basic equations and derive a new form of phenomenological damping. This new tensor form of damping contains only one phenomenological parameter, which can be found experimentally. The dynamic equation with tensor damping reflects physical relaxation processes for a general uniformly magnetized particle or film. The length of the magnetization vector is conserved and a rate of change of the energy proportional to the square of the torque.

II. SMALL MAGNETIZATION MOTION

Let us consider a uniformly magnetized ferromagnetic particle. We shall study small-amplitude motions of magnetization in the vicinity of equilibrium state $\mathbf{M} = M_s \hat{z}_1$, where $\hat{z}_1$ is the unit vector in the equilibrium direction. It is well-known that if we neglect the loss of energy, the magnetization rotation around effective field in the vicinity of equilibrium, in general, is elliptical. If $\hat{x}_1$ and $\hat{y}_1$ are the unit vectors corresponding to principal ellipse axes, the magnetic energy $\mathcal{E}$ can be written in the form:

$$
\mathcal{E} / V = \frac{H_1}{2M_s} M^2_{x_1} + \frac{H_2}{2M_s} M^2_{y_1}. \tag{2}
$$

Here $V$ is the particle volume, $H_1$ and $H_2$ are positive fields, which include both microscopic and shape anisotropies and the external magnetic field.

Our aim now is to derive equations of motion for $M_{x_1}$ and $M_{y_1}$. We can do this by two different methods: a) with the help of Landau-Lifshitz Eq. (1) and b) with the help of a normal mode approach, where the relaxation is introduced from basic equations.

A. Linearized LL equations

Using (2) and (1), we can calculate the effective field $\mathbf{H}_{\text{eff}} = -\partial (\mathcal{E} / V) / \partial \mathbf{M}$ and write down the linearized equations for the transverse magnetization components ($M_{x_1} \simeq M_s$):
\[
\frac{d}{dt} \begin{pmatrix} M_{x_1} \\ M_{y_1} \end{pmatrix} = \begin{pmatrix} -\alpha \gamma H_1 & -\gamma H_2 \\ \gamma H_1 & -\alpha \gamma H_2 \end{pmatrix} \begin{pmatrix} M_{x_1} \\ M_{y_1} \end{pmatrix}.
\]

(3)

In the absence of relaxation (\(\alpha = 0\)), the diagonal terms in (3) are equal to zero and the frequency of ferromagnetic resonance is:

\[
\omega_0 = \gamma \sqrt{H_1 H_2}.
\]

(4)

According to Eq. (3), the diagonal terms, responsible for relaxation, in general, are different (\(H_1 \neq H_2\)). The damping coefficients are equal only in two special cases when \(H_1 = H_2\): 1) the case of spherical symmetry and 2) the case of uniaxial symmetry when the external magnetic field and equilibrium magnetization are oriented along the easy axis.

B. Normal mode approach

In this approach it is convenient to introduce the classical spin \(S = -MV/\hbar \gamma\). Thus the energy (2) becomes

\[
\mathcal{E}/\hbar = \frac{\gamma H_1}{2S} S_x^2 + \frac{\gamma H_2}{2S} S_y^2.
\]

(5)

We shall describe small oscillations of the magnetization in terms of complex variables \(a^*, a\) which are classical analogs of creation and annihilation operators of a harmonic oscillator and can be introduced by a Holstein-Primakoff transformation for \(S_z \approx -S\):

\[
S_x \approx (a^* + a)\sqrt{2S}/2, \quad S_y \approx (a^* - a)\sqrt{2S}/2i.
\]

(6)

The energy (5) now can be rewritten in the form

\[
\mathcal{E}/\hbar = Aa^*a + (B/2)(aa + a^*a^*),
\]

(7)

where \(A = \gamma(H_1 + H_2)/2\) and \(B = \gamma(H_1 - H_2)/2\).

The dynamic precession equations are given by

\[
da/\!\!\!dt = -iAa - iB a^*, \quad da^*/\!\!\!dt = iAa^* + iBa.
\]

(8)

The mixed terms in (8) can be eliminated by the linear canonical transformation

\[
a = uc + vc^*, \quad a^* = uc^* + vc,
\]

\[
u = \sqrt{\frac{A + \omega_0}{2\omega_0}}, \quad v = -\frac{B}{\sqrt{|A|}} \sqrt{\frac{A - \omega_0}{2\omega_0}}.
\]

(9)

Thus we describe the precession in terms of the normal mode \((c, c^*)\) with energy of an harmonic oscillator

\[
\mathcal{E}/\hbar = \omega_0 c^* c,
\]

(10)

where \(\omega_0 = \sqrt{A^2 - B^2}\) is the frequency of ferromagnetic resonance equivalent to Eq. (2) \((A + B = \gamma H_1\) and \(A - B = \gamma H_2\)). The dynamic equations for \(c\) and \(c^*\) are now independent:

\[
dc/\!\!\!dt = -i\omega_0 c, \quad dc^*/\!\!\!dt = i\omega_0 c^*.
\]

(11)

In order to construct a damped motion for this oscillator it is necessary to consider the interaction with a thermal bath. This implies an introduction of microscopic interactions with magnons, phonons, etc. and, in general, this problem must be solved using quantum statistical methods. Here, for simplicity, we will not focus on a particular microscopic relaxation mechanism. Such a problem in a general form was solved by Lax [8]. We shall use this result with some brief explanation.

Let us consider the Hamiltonian of harmonic oscillator interacting with a thermal bath:

\[
H = \hbar \omega_0 b^\dagger b + i\hbar (b^\dagger g - bg^\dagger) + H_{TB}.
\]

(12)

Here \(b^\dagger\) and \(b\) are the creation and annihilation boson operators, \(H_{TB}\) is the thermal bath Hamiltonian, \(g\) and its hermitian conjugate \(g^\dagger\) denote the thermal bath operators, which describe a weak interaction with the oscillator. Lax analyzed the density matrix equation with the Hamiltonian (12) and obtained dynamic equations for classical amplitudes \((b)\) and \((b^\dagger)\) to second order in the interaction, where \((\ldots)\) is the thermal bath averaging. Denoting \(c = (b)\) and \(c^* = (b^\dagger)\), we can write these equations as

\[
(d/\!\!\!dt + \eta) c = -i(\omega_0 + \Delta \omega) c, \quad (d/\!\!\!dt + \eta) c^* = i(\omega_0 + \Delta \omega) c^*,
\]

(13)

where

\[
\eta - i\Delta \omega = \int_0^\infty du \exp(-iu\omega_0) \langle [g(0), g^\dagger(u)] \rangle u \exp(-iuH_{TB}/\hbar).
\]

(14)

\([\ldots, \ldots]\) is the commutator, \(\eta\) is the relaxation rate and \(\Delta \omega\) is the frequency shift due to interaction with the thermal bath (usually \(|\Delta \omega| \ll \omega_0\)). The most important fact is that the equations (13) for classical complex amplitudes of damped harmonic oscillator are general. We can use these equations even if we do not know a microscopic relaxation mechanism and find \(\eta\) and \(\Delta \omega\) from experiment as phenomenological parameters.

From Eq. (13), utilizing (2) and (3) with \(M = -\hbar \gamma S/V\), we derive linearized equations for \(M_{x_1}\) and \(M_{y_1}\):

\[
\frac{d}{dt} \begin{pmatrix} M_{x_1} \\ M_{y_1} \end{pmatrix} = \begin{pmatrix} -\eta & -\gamma H_2 \\ \gamma H_1 & -\eta \end{pmatrix} \begin{pmatrix} M_{x_1} \\ M_{y_1} \end{pmatrix}.
\]

(15)

According to (2), the damping of \(M_{x_1}\) and \(M_{y_1}\) is identical. Such intrinsic isotropy of the transverse damping components is seen in the Bloch-Bloembergen relaxation term (see, e.g., [7]). We also see that the non-diagonal terms in (2) and (13) as expected, coincide with each other. The diagonal terms, responsible for relaxation, are different. This means that the damping term in Landau-Lifshitz equation (2) is inconsistent with the basic physics of a damped harmonic oscillator (excluding two special cases mentioned above).
III. CONSTRUCTING THE DAMPING TERM

We can generalize Eq. (1) to the form:
\[
\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} - \gamma \frac{\mathbf{M}}{M_s} \times \left[\frac{1}{v_c} \cdot (\mathbf{M} \times \mathbf{H}_{\text{eff}})\right].
\] (16)

Here a dimensionless damping tensor \( \alpha \) is introduced. Note that the new tensor damping conserves the length of the magnetization vector \((\mathbf{M}) = M_s\) and gives a rate of change of the energy proportional to the square of the torque \(\mathbf{M} \times \mathbf{H}_{\text{eff}}\).

The tensor \( \alpha \) should contain all necessary information about symmetry of the system. Such information is included in the expression for the energy of the system and can be expressed as a tensor \( \frac{\partial^2 (E/V_0)}{\partial \mathbf{M} \partial \mathbf{M}} \). Thus, we can consider
\[
\alpha = \kappa = \frac{\partial^2 (E/V_0)}{\partial \mathbf{M} \partial \mathbf{M}} = -\kappa \frac{\partial \mathbf{H}_{\text{eff}}}{\partial \mathbf{M}},
\] (17)
where \( \kappa \) is a dimensionless parameter. Substituting (2) into (17), we obtain the damping tensor (17) in the vicinity of equilibrium in the form:
\[
\alpha = \kappa \begin{pmatrix}
H_1/M_s & 0 & 0 \\
0 & H_2/M_s & 0 \\
0 & 0 & 0
\end{pmatrix}.
\] (18)

Equation (16) with the tensor \( \alpha \) (15) must be consistent with the Eqs. (3) for small magnetization oscillations. Linearizing (16) and comparing with (15), we obtain
\[
\kappa = \eta \gamma M_s/\omega_0^2.
\] (19)

In the case of two (or more) stable stationary states in the vicinity of equilibrium we have, in general, different FMR frequencies and relaxation rates \( \omega_{01}, \eta_1 \) and \( \omega_{02}, \eta_2 \), correspondingly.

IV. DISCUSSION

In this paper a new, tensor form of damping is derived that reflects symmetry in the magnetic system energy. The damping tensor appears from a general form of interaction of the normal modes of the magnetic system with a thermal bath. This leads to the identical relaxation of transverse magnetization components, as in the Bloch-Bloembergen equations. Our analysis is exact for small oscillations about equilibrium (10), but the tensor form (as an isotropic damping) may also apply to large magnetization motions. The damping tensor is scaled by only one phenomenological damping parameter \( \eta \), which can be obtained from the experiment. It is demonstrated that the conventional damping term in the Landau-Lifshitz equation applies only for two cases of high symmetry. One of the most important applications of an anisotropic damping is sure to be the case of thermal magnetization fluctuations (11), (12). The form of damping affects, so-called, fluctuation-dissipation relation and therefore changes the estimated noise level. More detailed study of anisotropic magnetization damping will be published elsewhere (13).

Some indications that the LLG equation does not agree well with experiment in a magnetic thin film is shown in Ref. (4). In order to check the validity of the above new damping form it is necessary to study experimentally in detail the ferromagnetic resonance (frequency and linewidth) in anisotropic magnetic systems, e.g., films. The aim is to demonstrate that \( \eta \) (instead of \( \alpha \)) is a primary relaxation rate which can depend on frequency \( \omega \), temperature \( T \), external magnetic field and so on.

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