New mechanism to cross the phantom divide

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Recently, type Ia supernovae data appear to support a dark energy whose equation of state \( w \) crosses \(-1\), which is a much more amazing problem than the acceleration of the universe. We show that it is possible for the equation of state to cross the phantom divide by a scalar field in the gravity with an additional inverse power-law term of Ricci scalar in the Lagrangian. The necessary and sufficient condition for a universe in which the dark energy can cross the phantom divide is obtained. Some analytical solutions with \( w < -1 \) or \( w > -1 \) are obtained. A minimal coupled scalar with different potentials, including quadratic, cubic, quartic, exponential and logarithmic potentials are investigated via numerical methods, respectively. All these potentials lead to the crossing behavior. We show that it is a robust result which is hardly dependent on the concrete form of the potential of the scalar.

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I. INTRODUCTION

Over the course of the past decade, the evidence for the acceleration of the universe [1] is one of the most striking cosmological discoveries. A number of approaches, namely the existence of dark energy, have been adopted to try and explain the remarkable observation. An incomplete list includes: quintessence, phantom, k-essence, modified gravity, and so on [2]. The quintessence field models correspond to an equation of state \( w \geq -1 \). The region where the equation of state is less than \(-1\) is typically refer to as a presence due to something called phantom dark energy. Furthermore, in the wake of the more accurate data, a surprising result emerges: the recent study of the type Ia supernovas data show that the time varying dark energy provides a better fit than the cosmological constant, in addition, especially, the equation of state parameter \( w \), defined as the ratio of pressure to energy density, may cross the phantom divide \( w = -1 \). This crossing to the phantom region is neither possible for an ordinary minimally coupled scalar field nor for a phantom field. It clearly shows that this transition is difficult to be realized by only a single-field models of dark energy. In this article, we try to explore new ways and means to obtain the consistency for the theoretical outcome with the present observation.

The contribution of the matter content of the universe is represented by the energy momentum tensor on the right hand side of Einstein equations, whereas the left hand side is represented by pure geometry. Thus, there are two ways to give rise to current acceleration: (i) supplementing the energy momentum tensor by an exotic matter with negative pressure such as a cosmological constant or a scalar field; (ii) modifying the geometry itself. Since the dark energy models mentioned previously have not gotten fully satisfactory explanations of cosmic acceleration, it is worthwhile considering the possibility that acceleration of universe is not due to some kind of stuff, but rather is caused by modified gravitation. Lately, there is a very interesting operation on the general relativity: a modification to the Einstein-Hilbert action by involving new terms of inverse powers of the curvature scalar (in the following text we call it inverse-R gravity), of the form \( \sqrt{-g} R^{-n} \) (with \( n > 0 \)) [3]. Terms of this form would become important in the late universe and can bring about self-accelerating solutions, supplying a purely geometric candidate for dark energy. But, as shown in [3], the effect of extra term \( \sqrt{-g} R^{-n} \) equals to a scalar field with a smooth positive potential. Therefore, the EOS of dark energy still fails to cross the phantom divide yet. Given the challenge of this problem, we investigate a novel model to pursue the challenge. In general \( f(R) \) gravity (even without scalar field), the phantom divide crossing behavior of dark energy is shown in [4, 5], and \( R^2\)-term can yield a transient crossing [4, 5].

In the next section we shall make a brief review of inverse-R gravity and its relation to the scalar theory in Einstein gravity. In the section III we investigate the mechanism for a single scalar to cross the phantom divide by an analytical method. In section IV, we present the numerical results of the crossing behavior of a scalar with different potentials in frame of inverse-R gravity in detail. And in Section V, we will present the main conclusion and some discussions.

II. INVERSE-R GRAVITY

There are varies modifications of Einstein-Hilbert action. Basically, we classify them into two categories: one is ultraviolet modification which will be important in the high curvature region (early universe) while the other is infrared modification which plays significant role in the low curvature region (late universe). The ultraviolet modification has
been investigated widely in the inflation models, for example the starobinsky inflation \[\text{[1]}\]. Recently, the infrared modification arouses much interests because of the discovery of cosmic acceleration. The inverse-R gravity is one of the leading model in the model with infrared modifications. It is noted that the inverse-R gravity is equivalent to a class of scalar tensor theory with \(\omega = 0\), which is not compatible with solar system observations \[\text{[2]}\]. However, the physical meaning of this approach is not clear enough as the first sight. For example, it is shown that a specific \(f(R)\) gravity implying cosmic acceleration does not physically equal to scalar tensor theory, though it do equal scalar tensor theory mathematically \[\text{[8]}\]. Thus, it is still sensible to study modified gravity and scalar tensor theory, respectively.

Moreover, a scalar curvature squared term to the action can save the inverse-R gravity and help it to pass the tests in solar system \[\text{[9]}\]. We study the behavior of a scalar field in the inverse-R gravity with \(R^2\) correction, whose action \(S\) reads,

\[
S = \frac{m_{pl}^2}{2} \int d^4x \sqrt{-g}(f(R)) + \int d^4x \sqrt{-g}(\mathcal{L}_M + \mathcal{L}_\phi).
\]  

(1)

Here \(m_{pl}\) is Planck mass, \(g\) denotes the determinant of the metric, \(R\) marks the Ricci scalar, \(\alpha, \beta\) are constants with mass dimension, \(\mathcal{L}_M\) is the matter Lagrangian,

\[
f(R) = R - \frac{\alpha^4}{R} + \beta^2 R^2,
\]  

(2)

and \(\mathcal{L}_\phi\) labels the Lagrangian of a scalar field with potential \(V(\phi)\),

\[
\mathcal{L}_\phi = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi).
\]  

(3)

In this set-up, \(\beta\) is at the scale of inflation and \(\alpha\) is at the scale of the present universe, hence \(\beta \gg \alpha\). Note that \(R^2\) term aids the original inverse-R gravity to pass the tests in solar system and to ensure the universe undergoes a matter-dominated era, though we should ensure its effects in the late universe are tiny. The action \(\text{[1]}\) yields field equation,

\[
- g_{\mu\nu} f + 2 R_{\mu\nu} \frac{\partial f}{\partial R} + 2 g_{\mu\nu} \nabla^2 \frac{\partial f}{\partial R} - 2 \nabla_\mu \nabla_\nu \frac{\partial f}{\partial R} = \frac{T^M_{\mu\nu} + T^\phi_{\mu\nu}}{m_{pl}^2},
\]  

(4)

where \(T^M_{\mu\nu}, T^\phi_{\mu\nu}\) denote the energy-momentums of matter and scalar field, respectively. In the present article, we suppose that matter is just dust. By assuming an FRW universe with scale factor \(a\),

\[
ds^2 = -dt^2 + a^2 d\Sigma^2,
\]  

(5)

where \(d\Sigma^2\) denotes the metric of a 3-dimensional maximally symmetric space, we obtain the friedmann equation corresponding to \(\text{[1]}\),

\[
H^2 + \frac{k}{a^2} - \frac{F(H, \dot{H}, \ddot{H})}{3m_{pl}^2} = \frac{\rho_M + \dot{\phi}^2/2 + V}{3m_{pl}^2},
\]  

(6)

where \(H \equiv \dot{a}/a\) is the Hubble parameter, a dot denotes derivative with respect to the cosmic time \(t\), and

\[
F = \frac{m_{pl}^2 \alpha^4}{12(\dot{H} + 2H^2)^3} \left( 2H \ddot{H} + 15 H^2 \dot{H} + 2 \dot{H}^2 + 6H^4\right) - 18(6H^2 \dot{H} - \dot{H}^2 + 2H \ddot{H}) \frac{m_{pl}^2}{\beta^2}.
\]  

(7)

Another equation to close this system is the acceleration equation, which can be replaced by the equation of motion for \(\phi\),

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{dV}{d\phi} = 0.
\]  

(8)

In this construction the scalar field together with the extra geometric term plays the role of dark energy. First we present a concise note on the definition of dark energy. In the inverse-R gravity theory, there is a surplus \(F\)-term in the modified Friedmann equation \(\text{[6]}\). However, almost all observed properties of dark energy are obtained in frame of general relativity. To explain the the observed evolving EOS of the effective dark energy, we introduce the
concept “equivalent dark energy” or “virtual dark energy” in the modified gravity models [10]. We derive the density of virtual dark energy caused by the non-minimal coupled scalar by comparing the modified Friedmann equation in the inverse-R gravity with the standard Friedmann equation in general relativity. The generic Friedmann equation in the 4-dimensional general relativity can be written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3m_{pl}^2}(\rho_{dm} + \rho_{de}),$$  \hspace{1cm} (9)

where the first term of RHS in the above equation represents the dust matter and the second term stands for the dark energy. Comparing (9) with (6), one obtains the density of virtual dark energy in STG,

$$\rho_{de} = \rho_\phi + F,$$  \hspace{1cm} (10)

where

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V.$$  \hspace{1cm} (11)

We see that the $F$-term makes a key difference from the standard model. In the next section, we shall study the dynamics of a universe in inverse-R gravity, especially the dynamics of the virtual dark energy (dark energy for short).

### III. EXACT SOLUTIONS IN INVERSE-R COSMOLOGY

To find exact solutions is an important but difficult topic in such a high non-linear system (6) and (8). In this section we study some exact solutions to explore the mechanism to cross the phantom divide for the dark energy in inverse-R cosmology. In this section we only consider an ideal case, that is, $\beta = 0$, to see some qualitative behavior of the late universe in inverse-R gravity. And we leave the more realistic case with $R^2$-term in the numerical analysis.

#### A. Power-law solution

In a spatially flat universe, the Friedmann equation (6) reduces to

$$H^2 + \frac{k}{a^2} - \frac{F(H, \dot{H}, \ddot{H})}{3m_{pl}^2} = \frac{\dot{\phi}^2/2 + V}{3m_{pl}^2},$$  \hspace{1cm} (12)

during the dark energy dominated stage. Square-law solution arises when the potential is chosen to take the exponential form,

$$V = m_e^4 e^{m_e \phi},$$  \hspace{1cm} (13)

where $m_e$ is a constant. With this potential, [12] and [8] have a particular solution as follows,

$$a = a_0 t^2,$$  \hspace{1cm} (14)

$$\phi = 2m_{pl} \ln \left( \frac{m_e^2 t}{10m_{pl}} \right).$$ \hspace{1cm} (15)

It is easy to calculate that $w = -2/3$ in this case. One sees that this universe is “uniformally accelerating”, since $\ddot{a} = \text{constant}$. An interesting point in this uniformly accelerating universe is that the extra term $F$ vanishes just in time, and hence it is the solution with which we are familiar in standard general relativity by coincidence.

#### B. De Sitter solution

The dynamical system of the scalar filed with canonical and non-canonical Lagrangian has been widely studied [11], among which the global structure of the phase space has been investigated and various critical points and their physical significance have been identified and manifested. We have shown that if the potential of the model with
kinetic energy (quintessence) has a non-vanishing minimum [12], or the potential of a model with negative energy (phantom) has non-vanishing maximum [13], the dynamical system of the model admits late-time attractor solutions corresponding to \( w_\phi = -1 \) and \( \Omega_\phi = 1 \), where \( \Omega \) denotes the fraction of a stuff of the universe.

For a class of model that admits late-time de Sitter attractor, we have a solution,

\[
a = a_0 e^{\left[ V_0/6 m_{pl}^2 + (V_0^2/4 m_{pl}^4 + \alpha^4/12)^{1/2}/2 \right]^{1/2} (t-t_0)},
\]

(16)

\[
\phi = \phi_0.
\]

(17)

From (10) and (17), we derive

\[
F = \frac{\alpha^4 m_{pl}^4}{8 \left[ V_0/3 m_{pl}^2 + (V_0^2/9 m_{pl}^4 + \alpha^4/12)^{1/2} \right]}
\]

(18)

Thus \( \frac{d\rho_\phi}{da} = \frac{dF}{da} = 0 \) and the crossing phenomenon cannot happen near the late-time de Sitter attractor.

\section*{C. Big rip solution}

Present observation data indicate the possibility of dark energy with \( w < -1 \), dubbed as phantom [14]. In the ordinary case, the realization of \( w < -1 \) could not be achieved. Unfortunately, it suffers from the well known quantum stability problem. Though there are a few discussions to evade the stability problem for phantom model, we are far from “solving it” [15]. A good \( w < -1 \) model should avoid the negative kinetic energy as much as possible.

Under the potential,

\[
V(\phi) = V_0 \left( 1 - \frac{\phi}{m_{pl}} \int_{\phi_0 \over m_{pl}} du \frac{d}{1 + \gamma |I(u)|^4} \right),
\]

(19)

where \( I(u) \) is the inverse function of the function \( u = m_{pl} (\beta \ln v + \frac{3 \gamma v^4}{2}) \), and

\[
\beta = 7 + \sqrt{73},
\]

(20)

\[
\gamma = \frac{\alpha^4}{200(25 + \sqrt{73})},
\]

(21)

we have a big rip solution,

\[
a = a_0 (t-t_{br})^{-2},
\]

(22)

\[
\phi = m_{pl} \left[ \beta \ln(t_{br} - t) + \frac{3 \gamma (t-t_{br})^4}{2} \right],
\]

(23)

where \( t_{br} \) is the rip epoch. For this solution, we have

\[
w = -\frac{4}{3}
\]

(24)

\section*{D. The crossing mechanism}

Since there are both \( w > -1 \) and \( w < -1 \) solutions, the crossing phenomenon is bounded to happen in our model.

Before studying the crossing behavior in this model, we say a bit more on the relation between density evolution and the equation of state. In fact, we can know that dark energy behaves as quintessence, vacuum or phantom only
from its rate of change of the density. We reach this point from the following arguments. Since the dust matter obeys the continuity equation and the Bianchi identity keeps valid, dark energy itself satisfies the continuity equation

$$\frac{d\rho_{de}}{dt} + 3H(\rho_{de} + p_{eff}) = 0,$$

(25)

where \(p_{eff}\) denotes the effective pressure of the dark energy. And then we can express the equation of state for the dark energy as

$$w = \frac{p_{eff}}{\rho_{de}} = -1 - \frac{\alpha}{3\rho_{de}} \frac{d\rho_{de}}{da},$$

(26)

From the above equation we find that the behavior of \(w_{de}\) is determined by the term \(\frac{d\rho_{de}}{da}, \frac{d\rho_{de}}{da} = 0\) (cosmological constant) bounds phantom and quintessence. More intuitively, if \(\rho_{de}\) increases with the expansion of the universe, the dark energy behave as phantom; if \(\rho_{de}\) decreases with the expansion of the universe, the dark energy behave as quintessence; if \(\rho_{de}\) decreases and then increases, or increases and then decreases, we are certain that EOS of dark energy crosses phantom divide. A more important reason why we use the density to describe property of dark energy is that the density is more closely related to observables, hence is more tightly constrained for the same number of redshift bins used \([16]\). With data accumulation, observations which favor dynamical dark energy become more and accurate. Furthermore, a significant possibility appears recently: the EOS of dark energy may cross \(-1\) (phantom divide) \([17]\), which is a serious challenge for theoretical physics. The theoretical explore of the crossing phenomenon was proposed in \([18]\). This interesting topic is under intensively studying very recently \([19]\), see also \([10]\) and for some earlier references therein.

\(26\) can be rewritten as

$$w = -1 - \frac{1}{3H\rho_{de}}(\dot{\rho}_{\phi} + \ddot{F}).$$

(27)

The behaviors of \(F\) and \(\rho_{\phi}\), increase or decrease with expansion of the universe, depend on the rate of expansion. A simple calculation presents,

$$\dot{F} = \frac{Hm_{pl}^2\alpha^4}{6(2H^2 + \dot{H})^4} \left[ \dot{H}(2H^2 + \dot{H}) + 3\dot{H}(2H^2 - 7H\dot{H} - \ddot{H}) + 3\dot{H}(\dot{H}^2 - 16H^2\dot{H} - 4H^4) \right].$$

(28)

For a power-law universe,

$$a = a_0 \left( \frac{t}{t_0} \right)^{\xi},$$

(29)

where \(t_0\) is the present age of the universe, \(\xi\) is a constant, \(\xi > 0\) denotes an expanding universe; \(\xi > 1\) implies an accelerating one; \(\xi > 2\) implies \(\ddot{a} > 0\), hence \(\dot{F} > 0\) and \(\ddot{F} > 0\), which presents a possibility to cross the phantom divide for the dark energy. Substituting (29) into (28), we derive,

$$\dot{F} = \frac{-(\xi - 2)m_{pl}^2\alpha^4}{2\xi^3(2\xi - 1)^2}.$$  

(30)

In the \(\xi = \frac{11 + \sqrt{17}}{2}\) case, we have \(\dot{F}_{max} = 1.75 \times 10^{-6}\). If \(\xi >> 1\), we have \(\dot{F} << 1\). It is consistent that \(\dot{F} = 0\) in the de Sitter expansion, because de Sitter expansion is faster than any power-law accelerating expansion.

From (8) and (12), we find that

$$\dot{H} = \frac{3H\dot{\phi}^2 + \ddot{F}}{6m_{pl}^2H},$$

(31)

$$\ddot{H} = \frac{-3\dot{H}\dot{\phi}^2 - 6H\dot{\phi}\ddot{\phi} + \dddot{F} - 6m_{pl}^2\dot{H}^2}{6m_{pl}^2H}.$$  

(32)

The universal argument is that the crossing phenomenon happens at \(t = t_c\) iff \(\dot{H}_{t=t_c} = 0\) and \(\dddot{H}_{t=t_c} \neq 0\). In the quintessence case in standard model, ie, \(F \equiv 0\), \(\dot{H}_{t=t_c} = 0\) implies \(\dddot{H}_{t=t_c} \neq 0\). Therefore, the crossing behavior never happens. In inverse-R gravity, \(\dot{H}_{t=t_c} = 0\) and \(\dddot{H}_{t=t_c} \neq 0\) iff,

$$\dddot{F} = 3H\dot{\phi}^2,$$  

(33)
and
\[ \dot{F} \neq 3H\dot{\phi}^2. \]  
(34)

We now have proved two theorems as follows,

**Theorem 1** The crossing phenomenon happens iff \( \dot{F} = 3H\dot{\phi}^2, \dot{F} \neq 6H\dot{\phi}\dot{\phi}, \dot{\phi} \neq 0 \) and \( \ddot{\phi} \neq 0 \).

**Theorem 2** If \( \dot{F} < 3H\dot{\phi}^2 \), we have the model \( w < -1 \); if \( \dot{F} > 3H\dot{\phi}^2 \), we have the model with \( w > -1 \).

**IV. NUMERICAL EXAMPLES**

In the above section, we discuss the evolution of the universe in the scalar dominated stage via an analytical way. For a realistic universe, the pressure-less dust is a necessary component. However, it is difficult to study the evolution of the universe when we introduce a dust in inverse-R gravity with \( R^2 \) corrections. In this section, we investigate the dynamics of scalars with five different potentials, including quadratic, cubic, quantic, inverse power-law, exponential and logarithmic potentials in a universe with dust component via numerical methods, respectively. In the following text, we always assume a spatially flat universe.

**A. quadratic potential**

Quadratic potential is the most widely-investigated potential in field theory, which represents a mass term. Any potential around its minimum (if it has a minimum) can be treated as quadratic potential. In this subsection we study the case of quadratic potential,

\[ V = m^2\phi^2, \]  
(35)

where \( m \) is a parameter with dimension of mass.

For convenience, we introduce two dimensionless variables

\[ x \equiv H/H_0, y \equiv \phi/M_p, \]  
(36)

with which we rewrite Friedman equation (9) and the equation of motion (8) for \( \phi \) into the following form,

\[ x^2 = \Omega_{m0}e^{-3s} + \frac{1}{6}x^2y^2 + \frac{1}{3}k_2y^2 + \frac{b(4x^2x'^2 + 2x^3x'' + 15x^3x' + 6x^4)}{36(xx' + 2x^2)} + 18c(6x^3x' + 2x^2x'^2 + 2x^3x''), \]  
(37)

\[ 0 = xx'y' + x^2y'' + 3x^2y' + 2k_2y, \]  
(38)

where a prime denotes the derivative with respect to the so-called e-folds \( s \equiv \ln a, b \equiv \frac{a^4}{H_0^2}, c \equiv \frac{H_0^2}{2x} \) is a dimensionless constant, the coefficient \( k_2 \equiv \frac{m^2}{H_0^2} \) and

\[ \Omega_{m0} \equiv \frac{\rho_{m0}}{H_0^2m_{pl}^2}. \]  
(39)

Here \( H_0 \) is the current Hubble parameter, \( \rho_{m0} \) represents the current density of dust. Generally, \( c \) is a tiny number since the inflation scale \( \beta \) is much higher than \( H_0 \). Hence, the effects of \( R^2 \)-term in (2) can be omitted in late universe, although it becomes important in the early universe and structure formation era. This also implies that the exact solutions discussed in the last section make physical sense in the late universe.

Though we get the sufficient condition for the \( F \)-term to help the dark energy to cross the phantom divide, it is difficult to apply this condition before we obtain the exact solution. However in a universe with multiple components, one hardly obtain an exact solution. Under this situation, we present a numerical result about the dark energy density in Fig.1. For the sake of convenience, we introduce the dimensionless density as below,

\[ \beta = \frac{\rho_{de}}{3H_0^2m_{pl}^2} = \frac{1}{6}x^2y^2 + \frac{1}{3}k_2y^2 + \frac{b(4x^2x'^2 + 2x^3x'' + 15x^3x' + 6x^4)}{36(xx' + 2x^2)} + 18c(6x^3x' + 2x^2x'^2 + 2x^3x''). \]  
(40)
The most important parameter from the aspect of observation is the deceleration parameter $q$, which carries the total effects of the cosmic fluids,

$$q = \frac{\ddot{a}}{a^2} = -1 - \frac{x'}{x}. \quad (41)$$

We also displays $q$ in Fig. 1.

From Fig. 1, obviously, the equation of state of dark energy crosses -1. Simultaneously, the deceleration parameter is consistent with observation. As is known to all, the equation of state of a single scalar in standard general relativity never crosses the phantom divide, the $F$-term, induced from the $R^{-1}$ term in the Lagrangian, plays an essential role in this crossing.

### B. cubic potential

Next, we study the potential $V = m_3 \phi^3$. This potential has been investigated in classical theory, especially in the dynamics of an oscillator. As for $V = m_3 \phi^3$, the Friedman equation (6) and the equation of motion (8) become

$$x^2 = \Omega_{m0} + \frac{1}{6} x^2 y'^2 + \frac{1}{3} k_3 y^3 + \frac{b(4x^2 x'' + 2x^3 x''' + 15x^3 x' + 6x^4)}{36(x x' + 2x^2)^3} + 18c(6x^3 x' + x^2 x'^2 + 2x^3 x''), \quad (42)$$

$$0 = xx'y' + x^2 y'' + 3x^2 y' + 3k_3 y^2, \quad (43)$$

where $k_3 = m_\text{pl} m_3/H_0^2$.

The results in this situation is presented in Fig. 2. One can see easily the crossing behavior as mentioned above.

### C. quantic potential

In this subsection we explore the dynamics of the universe for a scalar with $V = c \phi^4$ ($c$ is a dimensionless constant) in inverse-R gravity. The quantic potential is extremely important in modern physics. Due to Higgs mechanism, every massive particle get mass through a quantic potential. Then, for $V = c \phi^4$, the two principle equations become

$$x^2 = \Omega_{m0} + \frac{1}{6} x^2 y'^2 + \frac{1}{3} k_4 y^4 + \frac{b(4x^2 x'' + 2x^3 x''' + 15x^3 x' + 6x^4)}{36(x x' + 2x^2)^3} + 18c(6x^3 x' + x^2 x'^2 + 2x^3 x''), \quad (44)$$
FIG. 2: This figure is for the evolutions of $\beta$, $w$, and $q$ for $V = m_3 \phi^3$, in which $\Omega_{m0} = 0.3$, and initial values for coefficients: $k_3 = 6, b = 20, x_0 = 1, y_0 = 0, x_0' = -0.3, y_0' = 0, c = 10^{-116}$.

FIG. 3: $\beta$, $w$ and $q$ as functions of $s$. In this figure we set $V = c\phi^4$, $\Omega_{m0} = 0.3$, and initial values for coefficients: $k_4 = 11, b = 20, x_0 = 1, y_0 = 0, x_0' = -0.3, y_0' = 0, c = 10^{-116}$.

\[ 0 = xx'y' + x^2y'' + 3x^2y' + 4k_4y^3, \tag{45} \]

in which $k_4 = cm_{pl}^2/H_0^2$.

We give the results in Fig.3.
D. exponential potential

In this subsection, we study the exponential potential. In the standard inflation models, the exponential potential is an important example which can be solved exactly in the standard model. In addition, we know that such exponential potentials of scalar fields occur naturally in some fundamental theories such as string/M theories. We present the results about $V = m^4_e e^{\phi/\mpl}$ in following content.

Concerning $V = m^4_e e^{\phi/\mpl}$, the equations (6) and (8) to be

$$x^2 = \Omega_{m0} + \frac{1}{6} x^2 y'^2 + \frac{1}{3} k_e e^y + \frac{b(4x^2x'^2 + 2x^3x'' + 15x^3x' + 6x^4)}{36(xx' + 2x^2)^3} + 18c(6x^3x' + x^2x'^2 + 2x^3x''), \tag{46}$$

$$0 = xx'y' + x^2y'' + 3x^2y' + k_e e^y, \tag{47}$$

where $k_e = m^4_e H_0^{-2} m_{pl}^{-2}$.

The results are given by Fig.4.

E. logarithmic potential

The logarithmic potential is not widely studied in particle physics. To fit parts of elementary particle mass spectra by involving logarithmic potentials comes into physicists’ view only in recent decades \cite{20}. In this paper, we study it phenomenologically. With regard to $V = m^4_e \ln \frac{\phi}{\mpl}$, the corresponding equations (6) and (8) will be

$$x^2 = \Omega_{m0} + \frac{1}{6} x^2 y'^2 + \frac{1}{3} k_l \ln y + \frac{b(4x^2x'^2 + 2x^3x'' + 15x^3x' + 6x^4)}{36(xx' + 2x^2)^3} + 18c(6x^3x' + x^2x'^2 + 2x^3x''), \tag{48}$$

$$0 = xx'y' + x^2y'' + 3x^2y' + \frac{k_l}{y}, \tag{49}$$

where $k_l = m^4_l H_0^{-2} m_{pl}^{-2}$.
FIG. 5: This figure for $V = m_l^4 \ln \frac{x}{m_{pl}}$, $\Omega_{m0} = 0.3$, and initial values for coefficients: $k_l = 1$, $b = 20$, $x_0 = 1$, $y_0 = 1$, $x'_0 = -0.3$, $y'_0 = 0$, $c = 10^{-116}$.

We present the results in Fig. 5.

From figs. 1-5, one sees that all the $w$ of the virtual dark energy with different potentials are successfully to make the equation of state of dark energy cross $-1$. And the 3 curves in all the figures have a similar shape, which implies that the crossing behavior is determined by the extra geometric term. Consequently, we have to say that it is geometric property of the inverse-R gravity itself, which is independent of the individual potentials.

V. CONCLUSION

In conclusion, the results of this paper demonstrate convincingly that it is possible to realize the crossing $w = -1$ for the equation of state of the dark energy by a single scalar. We find the necessary and sufficient condition for a universe in which the dark energy cross the phantom divide in inverse-R gravity. And then we investigated different potentials to minimally coupled scalar field $\phi$, including quadratic, cubic, quartic, exponential, logarithmic potentials in inverse-R gravity with $R^2$ correction. And the results state clearly that different potentials lead to the crossing behavior, respectively. Therefore, we conclude that it is a robust property of inverse-R gravity with $R^2$ correction, not controlled by a special potential.

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[1] A. G. Riess et al., Astron. J. 116, 1009 (1998); S. Perlmutter et al., Astrophys. J. 517, 565 (1999); E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006) [arXiv:hep-th/0603057].
[2] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D 70 (2004) 043528 [arXiv:astro-ph/0306438].
[3] S. Nojiri and S. D. Odintsov, eConf C0602061, 06 (2006) [Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007)] [arXiv:hep-th/0601213]; M. C. B. Abdalla, S. Nojiri and S. D. Odintsov, Class. Quant. Grav. 22, L35 (2005) [arXiv:hep-th/0409177]; S. Nojiri and S. D. Odintsov, Phys. Rev. D 74, 086005 (2006) [arXiv:hep-th/0608008]; S. Nojiri and S. D. Odintsov, J. Phys. Conf. Ser. 66, 012005 (2007) [arXiv:hep-th/0611071]; S. Nojiri and S. D. Odintsov, Phys. Rev. D 78, 046006 (2008) [arXiv:0804.3519 [hep-th]] ; K. Bamba, S. Nojiri and S. D. Odintsov, JCAP 0810, 045 (2008) [arXiv:0807.2575 [hep-th]]; K. Bamba, C. Q. Geng, S. Nojiri and S. D. Odintsov, Phys. Rev. D 79, 083014 (2009) [arXiv:0810.4296 [hep-th]].
[5] S. Nojiri and S. D. Odintsov, arXiv:1011.0544 [gr-qc].
[6] A. Starobinsky, Phys. Lett. B 91, 99 (1980).
[7] T. Chiba, Phys. Lett. B 575, 1 (2003) arXiv:astro-ph/0307338.
[8] S. Capozziello, S. Nojiri and S. D. Odintsov, Phys. Lett. B 634 (2006) 93 arXiv:hep-th/0512118; S. Capozziello, S. Nojiri, S. D. Odintsov and A. Troisi, arXiv:astro-ph/0604431.
[9] S. Nojiri and S. D. Odintsov, Phys. Rev. D 68, 123512 (2003) arXiv:hep-th/0307288; S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 36, 1765 (2004) arXiv:hep-th/0308176.
[10] H Zhang, Crossing the phantom divide, to appear in Dark Energy: Theories, Developments and Implications, Nova Science Publisher, 2010.
[11] J. G. Hao and X. Z. Li, Phys. Rev. D 68 (2003) 033512; J. G. Hao and X. Z. Li, Phys. Rev. D 68 (2003) 033501 (2003) arXiv:hep-th/0305207.
[12] R. R. Caldwell, Phys. Lett. B 545 (2002) 23 arXiv:astro-ph/9908168; J. G. Hao and X. Z. Li, Phys. Rev. D 67, 107303 (2003) arXiv:gr-qc/0302100; R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003) arXiv:astro-ph/0302506; S. Nojiri and S. D. Odintsov, Phys. Lett. B 562, 147 (2003) arXiv:hep-th/0303117; E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. D 70, 043539 (2004) arXiv:hep-th/0405034.
[13] S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 36, 975 (2004) [arXiv:gr-qc/0310027]; S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 38, 1285 (2006) arXiv:hep-th/0506212; S. Nojiri and S. D. Odintsov, Phys. Rev. D 72, 023003 (2005) arXiv:hep-th/0505215; S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 38, 1285 (2006) arXiv:hep-th/0506212; S. Nojiri and S. D. Odintsov, Phys. Lett. B 632, 597 (2006) arXiv:hep-th/0507182; S. Nojiri and S. D. Odintsov, Phys. Lett. B 637, 139 (2006) arXiv:hep-th/0603062; H. S. Zhang and Z. H. Zhu, Phys. Rev. D 73, 043518 (2006); X. Z. Li, C. B. Sun and P. Xi, Phys. Rev. D 79, 023501 (2009) arXiv:0903.3088 [gr-qc]; J. G. Hao and X. Z. Li, Phys. Lett. B 606, 7 (2005) arXiv:astro-ph/0404154; X. H. Zhai, X. D. Xi and X. Z. Li, Int. J. Mod. Phys. D 15, 1151 (2006) arXiv:astro-ph/0511814; H. Mohseni Sadjadi, Phys. Lett. B 687, 114 (2010) arXiv:0909.1002 [gr-qc]; M. Jamil and M. U. Farooq, JCAP 1003, 001 (2010) arXiv:1002.1434 [gr-qc]; F. Cannata and A. Y. Kamenshchik, arXiv:1005.1878 [gr-qc]; E. A. Lim, I. Sawicki and A. Vikman, JCAP 1005, 012 (2010) arXiv:1003.5751 [astro-ph.CO]; R. G. Cai, Q. Su and H. B. Zhang, JCAP 1004, 012 (2010) arXiv:1001.2207 [astro-ph.CO]; I. Y. Aref’eva, N. V. Bulatov and S. Y. Vernov, arXiv:0911.5105 [hep-th]; M. Jamil, Int. J. Theor. Phys. 49, 144 (2010) arXiv:0912.4468 [hep-th]; K. Nozari and T. Azizi, arXiv:0911.3333 [gr-qc]; S. Y. Vernov, Class. Quant. Grav. 27, 035006 (2010) arXiv:0907.0468 [hep-th]; E. O. Kahya, V. K. Onemli and R. P. Woodard, Phys. Rev. D 81, 023508 (2010) arXiv:0904.4811 [gr-qc]; T. Qiu, Mod. Phys. Lett. A 25, 909 (2010) arXiv:1002.3971 [hep-th]; L. L. Honorob, B. A. Reid, O. Menu, L. Verde and R. Jimenez, arXiv:1006.0877 [astro-ph.CO]; G. Izquierdo and D. Pavon, Phys. Lett. B 688, 115 (2010) arXiv:1004.2360 [astro-ph.CO]; M. Bouhmadi-Lopez, Y. Tavakoli and P. V. Moniz, JCAP 1004, 016 (2010) arXiv:0911.1428 [gr-qc]; H. S. Zhang and Z. H. Zhu, Phys. Rev. D 75, 023510 (2007) arXiv:astro-ph/0611834; H. S. Zhang and H. Noh, Phys. Lett. B 679, 81 (2009) arXiv:0904.0067 [gr-qc]; R. G. Cai, H. S. Zhang and A. Wang, Commun. Theor. Phys. 49, 948 (2005) arXiv:hep-th/0505186; H. S. Zhang and Z. H. Zhu, Mod. Phys. Lett. A, V24, No.7 (2009) 541, arXiv:0704.3121; H.S. Zhang and X. Z. Li, arXiv:1006.3970 [gr-qc].
[20] K. Paasch, Progress in physics, V1, 36(2009), and reference therein.