Collision-free control strategy for on-ramp merging: A spatial-dependent constraint following approach

Meng Tianchuang1, Hu Zhanyi1, Huang Jin1,*, Yang Diange1, Yang Zeyu1 and Zhong Zhihua1

1School of Vehicle and Mobility, Tsinghua University
huangjin@tsinghua.edu.cn

Abstract. This paper focuses on the problem of on-ramp merging control under the cooperation of intelligent and connected vehicles. A decentralized collision-free control strategy is proposed for on-ramp merging control. Each vehicle in the virtual platoon constructed by all vehicles on the arterial road and the on-ramp is equipped with a spatial-dependent constraint following controller. Under nonlinear vehicle dynamics, the proposed controller is proved to be uniformly bounded, thus assuring that each vehicle can satisfy the safety requirements to avoid collision at any specific spatial location, especially at the most dangerous merging point. Compared with time-dependence, this spatial-dependence means much more stability because spatial conditions during the on-ramp merging process are more static and invariant. Finally, a simulation containing six vehicles with relatively extreme testing conditions is conducted to validate the effectiveness of the proposed approach. The results demonstrate that the spacing errors can converge to 0 with respect to varying spatial-dependent desired spacings. The spacing errors of the six vehicles are kept at a relatively low level with a maximum value of 3.0778m. The maximal acceleration is 0.6060 m/s² and the maximal deceleration is -1.4042 m/s². All vehicles can achieve collision-free safety for on-ramp merging with a smooth and non-saturated control input generated by the proposed controller.

1. Introduction
The transportation systems and the automotive industry have been experiencing a fast development around the whole world. With the estimation that the total ownership of motor vehicles all around the world is about to increase to exceed one billion, the experts predict that the number will double during the following ten or twenty years [1]. The dense transportation activities provide people convenience, however, together with resulting in many problems, including speed breakdown, extra fuel consumption, increase of wasted hours, traffic flow vibration and more traffic congestions in converging areas [2-3], where on-ramp merging scenarios have become main triggers [4].

With various kinds of sensing and communication equipment, intelligent and connected vehicles (ICVs) have achieved considerable momentum in dealing with aforementioned issues. Compared with traditional manually driven vehicles, ICVs can provide shorter spacings and faster responses for inter-vehicle communication [5-6]. The vehicle-to-vehicle (V2V) communication and vehicle-to-infrastructure (V2I) communication enable the vehicles to communicate with others to achieve cooperative driving by adopting new-generation techniques.

There is a large number of literature on cooperative on-ramp merging control of ICVs [7-12]. Rios-Torres et al. [6], Scarinci et al. [13] and Bevly et al. [14] have reviewed the research efforts on the on-ramp merging but with different focuses on control techniques, benefit evaluation and vehicular
technologies, respectively. Research efforts on coordinating ICVs’ merging can be roughly divided into two categories, i.e. centralized and decentralized approaches. Centralized approaches are used when there is at least one task in the system that is globally decided for all vehicles by a single central controller [15]. Meanwhile, in decentralized approaches, all vehicles are treated as autonomous agents that attempt to achieve safety through strategic interaction and maximize their own efficiency [15].

There have been many studies focusing on designing controllers to achieve merging safety and high efficiency, but most of them do not focus on binding the controller performance with static spatial locations in the environment. Our team’s previous work [16-23] mainly concentrates on the robust controller design and doesn’t deal with this spatial-dependence, neither. Spatial-dependence means that the controller can assure that each vehicle satisfies the safety requirements to avoid collision at specific spatial locations, especially at the most dangerous merging point. Compared with the time-dependent controller, which plans a velocity or acceleration profile based on specific time, the spatial-dependence means much more stability and reliability since spatial environment of the on-ramp merging scenario is not likely to change abruptly (the position of the merging point is just with no accident). By binding the controller performance with static spatial locations in the merging scenario, merging safety at specific spatial locations is more straightforward and foreseeable. This spatial-dependence is exactly the main goal and emphasis of this paper based on our previous work. The kernal contribution of this paper is proposing such a decentralized spatial-dependent constraint following controller with an analytical, closed-form solution to the on-ramp merging control problem.

In this paper, we propose a decentralized spatial-dependent collision-free control strategy. The vehicles on the arterial road and the on-ramp together construct a virtual platoon [24] based on the initial conditions. A spatial-dependent constraint following controller is equipped on each vehicle to follow the preceding vehicle in the virtual platoon, with the target of decreasing the spacing error with respect to the desired safe spacing. By constraint following, we mean that the controller is designed based on the equality constraint with expected performance. Through the performance of spatial-dependent safety assurance, we guarantee a safe and stable on-ramp merging process with the proposed method.

The rest of the structure of the paper is as follows. Section 2 formulates the problem, including nonlinear vehicle dynamics, spatial-dependent desired spacing design and state transformation. Section 3 designs the spatial-dependent controller with the uniform boundedness performance guaranteed. Section 4 shows the simulation results. Finally, concluding remarks are provided in Section 5.

2. Problem formulation

As demonstrated in Figure 1, the virtual platoon containing a variety of heterogenous vehicle models (e.g. sedans, vans, trucks) is constructed by vehicles on the two roads under the on-ramp merging scenario. The virtual platoon contains $N + 1$ vehicles, where the index 0 represents the leading vehicle and the indexes from 1 to $N$ represent the following vehicles from front to back. The position of the $i$th vehicle is expresses as $x_i, i = 0, 1, ..., N$. The $i$th vehicle’s length is denoted as $l_i$. The spacing between the $i$th vehicle and its preceding vehicle is denoted as $d_i$.

![Figure 1. Virtual platoon formation.](image)

In this problem, we presume that every vehicle owns specific sensors to detect its real-time spacing between itself and the preceding and other necessary state data. Besides, the follower can acquire information of its preceding vehicle with communication apparatus.

This section mainly describes the platoon’s dynamics modeling, the design of the spatial-dependent desired spacing and the state transformation from the bounded spacing error state to the unbounded.
2.1. System dynamics

In this on-ramp merging problem, the longitudinal dynamics model for the \(i\)th \((i = 0, 1, ..., N)\) vehicle is formulated as

\[
\begin{cases}
    \dot{x}_i(t) = v_i(t) \\
    M_i \cdot \dot{v}_i(t) = u_i(t) - c_i \cdot v_i(t) \cdot |v_i(t)| - F_i
\end{cases}
\]

where \(t\) is time, \(v_i\) is the velocity, \(M_i\) is the vehicle mass, \(u_i\) is the driving or braking force input. Besides, \(-c_i \cdot v_i(t) \cdot |v_i(t)|\) denotes the aerodynamic resistance and \(-F_i\) denotes the sum of the rolling resistance and gradient resistance.

The real-time spacing of the \(i\)th vehicle \((i = 1, ..., N)\) in real time is

\[d_i(t) = x_{i-1}(t) - x_i(t) - l_{i-1}\]

2.2. Spatial-dependent desired spacing

The target of this on-ramp merging control system is to control all inter-vehicle spacings in the virtual platoon to approach their respective desired spacing. In other words, the spacing error should be as small as possible for safety. Here the spacing error of the \(i\)th vehicle, i.e. \(e_i(t)\), is defined as

\[e_i(t) = d_i^{\text{des}}(x_i) - d_i(t)\]

where \(d_i^{\text{des}}(x_i)\) is the spatial-dependent desired spacing.

While small inter-vehicle spacing is critically dangerous, too large spacing will degrade the performances of fuel economy and traffic efficiency. Therefore, the spacing error need to be properly bounded. Let the expected boundedness of the spacing error be

\[e_i^{\text{min}} < e_i(t) < e_i^{\text{max}}, \quad i = 1, 2, ..., N\]

where the constants \(e_i^{\text{min}}\) and \(e_i^{\text{max}}\) denote the minimal and the maximal allowable spacing error(s) respectively. This system aims to control the spacing error to converge to 0 as time going while strictly satisfying this bidirectional constraint.

Next, it should be noted that the desired spacing is designed as spatial-dependent, which means the desired spacing is the function of the position \(x_i\).

\[d_i^{\text{des}} = g_i(x_i), \quad i = 1, 2, ..., N\]

where \(g_i(\cdot)\) is a function that maps the position to desired spacing. What’s more, the desired spacing satisfies the following boundary conditions

\[
\begin{cases}
    g_i(x_i^0) = d_i^0 \\
    g_i(x_m) = d_{\text{safe}}
\end{cases}
\]

where \(x_i^0\) is the initial position, \(d_i^0\) is the initial spacing, \(x_m\) is the position of the merging point and constant \(d_{\text{safe}}\) is the expected safe spacing at the merging point.

The first boundary constraint indicates that the initial spacing error \(e_i(x_i^0)\) is 0, which guarantees that the bidirectional constraint is satisfied at the initial time. The second boundary constraint indicates that the final desired spacing at the merging point \(e_i(x_m)\) is an expected proper safe value, which, at the same time, guarantees the safety at the merging point for each vehicle since the spacing error is bidirectionally constrained around \(d_{\text{safe}}\).

After that, we design the specific formula of the function \(g_i(\cdot)\). The trigonometric form is chosen as a candidate, that is,

\[
d_i^{\text{des}} = g_i(x_i) = k_i^0 \cdot \sin[k_i\left(x_i + k_i^2\right)] + k_i^3
\]

where \(k_i^0 = \frac{d_i^0 - d_{\text{safe}}}{2}\), \(k_i^1 = \frac{\pi}{x_m - x_i^0}\), \(k_i^2 = \frac{x_m - 3x_i^0}{2}\), \(k_i^3 = \frac{d_i^0 + d_{\text{safe}}}{2}\). No matter what the initial spacing \(d_i^0\)
is, and whether it is larger or smaller than \(d_{\text{safe}}\), this trigonometric form can satisfy the above boundary constraints. Besides, this trigonometric form has other good properties for this problem. At the initial position \(x_i^0\) and the merging point \(x_m\) (which signifies the ending of the merging control process), \(\frac{d d_i^\text{des}}{dx_i}\), the derivative of \(d_i^\text{des}\) with respect to position \(x_i\), equals 0. It means that the desired spacing has smooth changing at both the beginning and the ending of the merging control process. What’s more, it is known that the derivative of a trigonometric function also owns the trigonometric form. Therefore, the multi-order derivatives of \(d_i^\text{des}\) with respect to position \(x_i\) are bounded and relatively not large.

2.3. State transformation
To employ the Lyapunov stability analysis, this subsection discusses how to perform a transformation to convert the domain of the spacing error from bounded to unbounded.

We utilize a two times continuously differentiable bijective function \(f_i(\cdot): (e_i^{\text{min}}, e_i^{\text{max}}) \rightarrow \mathbb{R}\), which satisfies the following conditions:

\[
\begin{align*}
\lim_{e_i \rightarrow e_i^{\text{min}}} f_i(e_i) &= -\infty \\
\lim_{e_i \rightarrow e_i^{\text{max}}} f_i(e_i) &= +\infty \\
f_i(0) &= 0 \\
\frac{\partial f_i}{\partial e_i} &
eq 0, \text{ for all } e_i \in (e_i^{\text{min}}, e_i^{\text{max}})
\end{align*}
\]

What’s more, its inverse function \(f^{-1}_i(\cdot)\) should be at least two times continuously differentiable. In this problem, we set \(e_i^{\text{max}} = -e_i^{\text{min}} = b\). Thus, the bidirectional constraint becomes \(-b < e_i(t) < b\). Then we formulate the transform function as

\[
z_i = f_i(e_i) = \ln \frac{b + e_i}{b - e_i}
\]

where \(z_i\) is the transformed state with a domain of \(\mathbb{R}\). The inverse function is

\[
e_i = f^{-1}_i(z_i) = b \cdot \frac{e_i^{z_i} - 1}{e_i^{z_i} + 1}
\]

Let

\[
\varphi_{i,1} = \frac{\partial f^{-1}_i}{\partial z_i} = \frac{2be_i^{z_i}}{(e_i^{z_i} + 1)^2} \\
\varphi_{i,2} = \frac{\partial^2 f^{-1}_i}{\partial z_i^2} = \frac{2be_i^{z_i}(1 - e_i^{z_i})}{(e_i^{z_i} + 1)^3}
\]

From equations (2) and (3), it can be known that

\[
x_i = f^{-1}_i(z_i) + x_{i-1} - d_i^\text{des} - l_{i-1}
\]

Differentiating it with respect to time \(t\), we get

\[
v_i = G_i \cdot (\varphi_{i,1}\dot{z}_i + v_{i-1})
\]

\[
\dot{v}_i = G_i \cdot \left( \varphi_{i,2}\dot{z}_i^2 + \varphi_{i,1}\dot{z}_i + \dot{v}_{i-1} - \frac{\partial^2 d_i^\text{des}}{\partial x_i^2} \cdot v_i \right)
\]

where
\[ G_l = 1 + \frac{\partial d_i^{\text{des}}}{\partial x_i} \]  

Substitute equations (14) and (1) into (1), we obtain
\[ M_i G_i \ddot{\varphi}_{i,1} \dot{z}_i = u_i - M_i G_i \left[ \varphi_{i,2} \dot{z}_i^2 + \dot{v}_{i-1} - \frac{\partial^2 d_i^{\text{des}}}{\partial x_i^2} \cdot \dot{v}_i^2 \right] - c_i G_i |G_i| \left( \varphi_{i,1} \dot{z}_i + v_{i-1} \right) \cdot \left| \varphi_{i,1} \dot{z}_i + v_{i-1} \right| - F_i \]

\[ (17) \]

3. Controller design

Physically speaking, the square of \( z_i \) can be interpreted as a potential energy function [25] between the \( i \)th vehicle and its preceding vehicle, which approximates to a hazard coefficient for vehicles [20]. In this section, a controller is proposed to maintain the vehicle at the state with a low potential energy. What’s more, the boundedness of \( z_i \) under the controller means that the spacing error will satisfy the bidirectional constraint in equation (4) so that a high traffic efficiency and the merging safety are both achieved. This inspires us to construct a controller rendering the uniformly bounded (UB) performance.

3.1. Control input

This subsection designs the control input \( u_i \).

With the aim of improving ride comfort, the converging of the transformed state \( z_i \) should be gently regulated. An equation constraint for this expected performance is defined, that is,
\[ h_i z_i(t) + \dot{z}_i = 0 \]

where \( h_i > 0 \) is scalar constant to be prescribed. Equation (18) implies that
\[ z_i(t) = z_i(t_0) \cdot e^{-h_i t} \]  

Equation (19) implies that for any initial \( z_i(t_0) \), \( z_i(t) \) will converge to 0 as \( t \to \infty \). The time of \( z_i(t) \) approaching 0 can be modulated by \( h_i \).

Define
\[ \beta_i = h_i z_i(t) + \dot{z}_i \]

(20)

to represent the deviation from the equation constraint.

By differentiating equation (18) with respect to \( t \), we obtain
\[ h_i \ddot{z}_i(t) + \ddot{z}_i = 0 \]

By using the constraint-following control method [26], \( z_i \) is assured to follow this equation constraint. Through Udwadia-Kalaba approach [26], the following control force \( p_{i,1} \) is introduced.
\[ p_{i,1} = -h_i M_i G_i \varphi_{i,1} \dot{z}_i(t) + M_i G_i \left[ \varphi_{i,2} \dot{z}_i^2 + \dot{v}_{i-1} - \frac{\partial^2 d_i^{\text{des}}}{\partial x_i^2} \cdot \dot{v}_i^2 \right] + c_i G_i |G_i| \left( \varphi_{i,1} \dot{z}_i + v_{i-1} \right) \cdot \left| \varphi_{i,1} \dot{z}_i + v_{i-1} \right| + F_i \]

(22)

This control force is model-based. It is interpreted as a feedforward part.

Considering the initial condition offset, we design a feedback control part to modify the system to approach the expected equation constraint (18), that is,
\[ p_{i,2} = -\kappa_i M_i G_i \varphi_{i,1} \beta_i \]

(23)

where \( \kappa_i \) is a scalar constant.

Therefore, the overall control force is designed as
\[ u_i = p_{i1} + p_{i2} \]  

3.2. Performance analysis

In this subsection, we prove uniformly bounded (UB) performance of the designed controller. The UB performance is defined as follows.

**Definition 1.** Uniformly Bounded [26]: For any \( r_i > 0 \), there exists a \( \delta_i(r_i) < \infty \) such that if \( \| \beta_i(t_0) \| \leq r_i \), then \( \| \beta_i(t) \| \leq \delta_i(r_i) \) for all \( t \geq t_0 \).

The proofs are as follows.

First, the Lyapunov function candidate is chosen as

\[ V_i = \frac{1}{2} \beta_i^2 \]  

Then, the derivative of \( V_i \) with respect to \( t \) is

\[ \dot{V}_i = \beta_i \cdot \dot{\beta}_i = \beta_i \cdot [h_i \dot{z}_i(t) + \ddot{z}_i] \]  

Substitute \( \ddot{z}_i \) into equation (26), we can get

\[ \dot{V}_i = \beta_i \cdot \left( h_i \ddot{z}_i(t) + M_i^{-1} G_i^{-1} \phi_{i1}^{-1} \cdot p_1 - \phi_{i1}^{-1} \left( \phi_{i2} \ddot{z}_i^2 + v_{i-1} - \frac{\partial^2 q_{i1}^{\text{des}}}{\partial x_i^2} \cdot v_i^2 \right) \right. - c_i M_i^{-1} \phi_{i1}^{-1} G_i \left( \phi_{i1} \ddot{z}_i + v_{i-1}\right) \cdot \phi_{i1} \ddot{z}_i^2 + v_{i-1} \right) - M_i^{-1} G_i^{-1} \phi_{i1}^{-1} \cdot F_i \right) + \beta_i \cdot M_i^{-1} G_i^{-1} \phi_{i1}^{-1} \cdot p_2 \]  

The first term containing \( p_2 \) is 0. Thus, we get

\[ \dot{V}_i = \beta_i \cdot M_i^{-1} G_i^{-1} \phi_{i1}^{-1} \cdot p_2 = -\kappa_i \beta_i^2 \leq 0 \]  

where \( \dot{V}_i = 0 \) when and only when \( \beta_i = 0 \).

Therefore, we derive the uniform boundedness of the system:

\[ \delta_i(r_i) = r_i \]  

The UB performance of \( \beta_i \) is satisfied under any initial condition \( \beta_i(t_0) \). That is, if \( \| \beta_i(t_0) \| \leq r_i \), then \( \| \beta_i(t) \| \leq \delta_i(r_i) \) for all \( t \geq t_0 \).

The boundedness of \( \beta_i(t) \) indicates the boundedness of \( z_i(t) \). The proofs are as follows.

Since \( \| \beta_i(t) \| \leq \delta_i(r_i) \), we have

\[ -\delta_i(r_i) \leq h_i z_i(t) + \dot{z}_i \leq \delta_i(r_i) \]  

where \( \delta_i(r_i) \) is a constant depending on the initial conditions.

The differential inequalities are solved, and we obtain

\[ \begin{align*}
    z_i \geq -\frac{\delta_i(r_i)}{h_i} + \left[ z_i(t_0) + \frac{\delta_i(r_i)}{h_i} \right] \cdot e^{-h_i t} \\
    z_i \leq \frac{\delta_i(r_i)}{h_i} + \left[ z_i(t_0) - \frac{\delta_i(r_i)}{h_i} \right] \cdot e^{-h_i t}
\end{align*} \]  

which shows that \( z_i(t) \) is also bounded.

According to the state transformation, the boundedness of \( z_i(t) \) implies that if the initial spacing error \( e_i(t_0) \) satisfies the bidirectional constraint, the subsequent spacing error \( e_i(t) \) will also satisfy the bidirectional constraint. Therefore, compact stable formation and collision avoidance of the virtual platoon are realized, which also means that the safety of on-ramp merging control is realized.
4. Simulation results
In this section, a numerical simulation is conducted to verify the effectiveness of the proposed method.

Considering six vehicles ($N=5$), with three on the arterial road and the rest on the on-ramp, after forming a virtual platoon, the 1st, 3rd and 5th vehicles in the virtual platoon are originally from the arterial road, while the 2nd, 4th and 6th are originally from the on-ramp. The initial conditions are listed in Table 1 (The units of position, velocity, mass and vehicle length are m, m/s, kg and m, respectively). The initial positions of adjacent vehicles ($x_0^0$, $x_1^0$, $x_2^0$, $x_3^0$, $x_4^0$ and $x_5^0$) are the same, which forms a relatively extreme testing conditions. Some parameters are listed in Table 2.

**Table 1.** Initial conditions of 6 vehicles.

| $x_0^0$ | $x_1^0$ | $x_2^0$ | $x_3^0$ | $x_4^0$ | $x_5^0$ |
|---------|---------|---------|---------|---------|---------|
| 400     | 400     | 377     | 377     | 351     | 351     |
| $v_0^0$ | $v_1^0$ | $v_2^0$ | $v_3^0$ | $v_4^0$ | $v_5^0$ |
| 20.2    | 19.8    | 20.1    | 20.2    | 20.3    | 19.7    |
| $M_0$   | $M_1$   | $M_2$   | $M_3$   | $M_4$   | $M_5$   |
| 1000    | 950     | 1860    | 950     | 1860    | 1000    |
| $l_0$   | $l_1$   | $l_2$   | $l_3$   | $l_4$   | $l_5$   |
| 4       | 4       | 5.356   | 4       | 5.256   | 4       |

**Table 2.** Simulation parameters.

| $x_m$         | $h_i$ ($i = 1, ..., 6$) | $\kappa_i$ ($i = 1, ..., 6$) | $d_{\text{des}}^i$ ($i = 1, ..., 6$) | $b$ |
|---------------|------------------------|-------------------------------|-----------------------------------|-----|
| 1000m         | 0.5                    | 10                            | 20m                               | 5m  |

The simulation results are illustrated in Figures 2-7. The control input generated by the proposed controller is shown in Figure 2. The control input surges in the beginning for less than 2 seconds. After that, it is maintained at a stably regulating stage where the control inputs of all vehicles are bounded within a range of $[0, 1200]$ N.

**Figure 2.** Control input profiles of the proposed controller.

The position, velocity and acceleration profiles of the six vehicles are shown in Figures 3–5. From Figure 3, we can see that the initial positions of the vehicles in virtual platoon are very dangerous, where some initial spacings in the virtual platoon are 0, which means vehicles from two roads with the same sequence order are overlapped at the initial state. At the position of the merging point ($x_m = 1000m$), safe spacings are generated by the coordination of all vehicles under the proposed controller. Figure 4 demonstrates the velocities. It’s evident that the velocity regulation is very smooth, that is, all the velocities maintain at a range within 15~20 m/s. In Figure 5, the accelerations have a relatively sharp change in the beginning and then keep at a stable and low level within $-0.5$~$0.5$ m/s$^2$. Even though the acceleration increases sharply in the beginning, the maximum acceleration and deceleration are 0.6060 m/s$^2$ and $-1.4042$ m/s$^2$ respectively.

As for the spacing and spacing errors, we can refer to Figures 6–7. The spacings are successfully regulated from initial dangerous values to the final desired spacing. The spacing errors with respect to the spatial-dependent desired spacing are kept at a relatively low level, with a maximum value of...
3.0778m. The spacing errors associated with spatial positions are illustrated in Figure 8. It’s clear that all spacing errors remain within a safe range during the on-ramp merging process and converge to 0 finally.

The simulation results validate the effectiveness of the proposed method in the on-ramp merging control problem. The spacing errors corresponded with the spatial-dependent desired spacing can be controlled at a relatively low level, thus ensuring the safety of the on-ramp merging process.

5. Conclusion
This paper proposes a decentralized spatial-dependent collision-free control strategy for on-ramp merging. The designed spatial-dependent controller is proved to be uniformly bounded and assure that each vehicle satisfies the safety requirements to avoid collision at specific spatial locations. The spatial-dependence is one kernel point in the proposed strategy. It brings much more stability and reliability.
because the spatial environment in the on-ramp merging scenario (the length of the roads, the location of the merging point, et c.) is relatively static and invariant. Finally, a simulation containing six vehicles with relatively extreme testing conditions is conduct. The results validate the effectiveness of the proposed controller. The spacing errors of the six vehicles are kept at a relatively low level with a maximum value of 3.0778m. The maximum acceleration and deceleration are 0.6060 m/s² and -1.4042 m/s² respectively. The collision-free safety with smooth and non-saturated control inputs is achieved with the proposed controller. Future research work will be concentrated on the design of the robustness of the controller against the time-varying uncertainties in the vehicle parameters, as well as further comparison of other existing approaches.

Acknowledgments
This research is sponsored in part by the NSFC Program (No. 61872217, No. U1701262, No. U1801263), the research is also sponsored in part by the Guangdong Provincial Key Laboratory of Cyber-Physical Systems, as well as be sponsored in part by the Industrial Internet innovation and development project of ministry of industry and information technology.

References
[1] Jia, Dongyao, et al. "A survey on platoon-based vehicular cyber-physical systems." IEEE communications surveys & tutorials 18.1 (2015): 263-284. vol. 18, no. 1, pp. 263–284, Mar. 2015.
[2] Malikopoulos, Andreas A., and Juan P. Aguilar. "An optimization framework for driver feedback systems." IEEE Transactions on Intelligent Transportation Systems 14.2 (2013): 955-964.
[3] B. Schrank, B. Eisele, T. Lomax, and J. Bak, “2015 urban mobility scorecard,” Texas A & M Transp. Inst., College Station, TX, USA, Tech. Rep., 2015.
[4] Margiotta, Richard A., and Dena Snyder. An agency guide on how to establish localized congestion mitigation programs. No. FHWA-HOP-11-009. United States. Federal Highway Administration. Office of Operations, 2011. [Online]. Available: http://ops.fhwa.dot.gov/publications/fhwatchop11009/fhwatchop11009.pdf
[5] Li, Shengbo Eben, et al. "Dynamical modeling and distributed control of connected and automated vehicles: Challenges and opportunities." IEEE Intelligent Transportation Systems Magazine 9.3 (2017): 46-58.
[6] Rios-Torres, Jackeline, and Andreas A. Malikopoulos. "A survey on the coordination of connected and automated vehicles at intersections and merging at highway on-ramps." IEEE Transactions on Intelligent Transportation Systems 18.5 (2016): 1066-1077.
[7] R. C. Carlson, I. Papamichail, and M. Papageorgiou, “Local feedback-based mainstream traffic flow control on motorways using variable speed limits,” IEEE Trans. Intell. Transp. Syst., vol. 12, no. 4, pp. 1261–1276, Dec. 2011.
[8] G.-R. Iordanidou, C. Roncoli, I. Papamichail, and M. Papageorgiou, “Feedback-based mainstream traffic flow control for multiple bottlenecks on motorways,” IEEE Trans. Intell. Transp. Syst., vol. 16, no. 2, pp. 1–12, Apr. 2014.
[9] S. Agarwal, P. Kachroo, S. Contreras, and S. Sastry, “Feedback-coordinated ramp control of consecutive on-ramps using distributed modeling and Godunov-based satisfiable allocation,” IEEE Trans. Intell. Transp. Syst., vol. 16, no. 5, pp. 2384–2392, Oct. 2015.
[10] Ntousakis, Ioannis A., et al. "Assessing the impact of a cooperative merging system on highway traffic using a microscopic flow simulator." ASME 2014 International Mechanical Engineering Congress and Exposition. American Society of Mechanical Engineers Digital Collection, 2014.
[11] Pasquale, Cecilia, et al. "Two-class freeway traffic regulation to reduce congestion and emissions via nonlinear optimal control." Transportation Research Part C: Emerging Technologies 55 (2015): 85-99.
[12] Milanés, Vicente, et al. "Automated on-ramp merging system for congested traffic situations."
IEEE Transactions on Intelligent Transportation Systems 12.2 (2010): 500-508.

[13] Scarinci, Riccardo, and Benjamin Heydecker. "Control concepts for facilitating motorway on-ramp merging using intelligent vehicles." Transport reviews 34.6 (2014): 775-797.

[14] Bevly, David, et al. "Lane change and merge maneuvers for connected and automated vehicles: A survey." IEEE Transactions on Intelligent Vehicles 1.1 (2016): 105-120.

[15] Rios-Torres, Jackeline, and Andreas A. Malikopoulos. "Automated and cooperative vehicle merging at highway on-ramps." IEEE Transactions on Intelligent Transportation Systems 18.4 (2016): 780-789.

[16] Yang, Z., et al. "Adaptive Robust Control for a Heterogeneous Vehicular Platoon." 2019 Chinese Automation Congress (CAC) IEEE, 2019.

[17] Yang, Z., et al. "Design and Optimization of Robust Path Tracking Control for Autonomous Vehicles with Fuzzy Uncertainty." IEEE Transactions on Fuzzy Systems PP.99(2021):1-1.

[18] Yang, Z., et al. "Safety-guaranteed constraint-oriented modelling and control for bidirectional vehicular platoons." IET Control Theory and Applications 3(2020).

[19] Hu, Z., et al. "Safety guaranteed longitudinal motion control for connected and autonomous vehicles in a lane-changing scenario." IET Intelligent Transport Systems 15.1(2021).

[20] Yang, Z., et al. "Utilizing Bidirectional Inequality Constraints in Optimal Robust Control for Heterogeneous Vehicular Platoons." IET Intelligent Transport Systems 14.7(2020):802-811.

[21] Hu, Z., et al. "Embedding robust constraint-following control in cooperative on-ramp merging." IEEE Transactions on Vehicular Technology PP.99(2021):1-1.

[22] Yang, Z., et al. "Adaptive constraint-following control for uncertain nonlinear mechanical systems with measurement error." International Journal of Robust and Nonlinear Control 2(2021).

[23] Hu, Zhanyi, et al. "Cooperative-game-theoretic optimal robust path tracking control for autonomous vehicles." Journal of Vibration and Control (2021): 10775463211009383.

[24] Uno, Atsuya, Takeshi Sakaguchi, and Sadayuki Tsugawa. "A merging control algorithm based on inter-vehicle communication." Proceedings 199 IEEE/IEEJ/JSAM International Conference on Intelligent Transportation Systems (Cat. No. 99TH8383). IEEE, 1999.

[25] Yu, Jinwei, et al. "Region-based flocking control for networked robotic systems with communication delays." European Journal of Control 52 (2020): 78-86.

[26] Chen, Ye-Hwa, and Xinrong Zhang. "Adaptive robust approximate constraint-following control for mechanical systems." Journal of the Franklin Institute 347.1 (2010): 69-86.