Research Article

Analysis of the Vibro-Acoustic Behavior of a Stiffened Double Panel-Cavity System

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An analytical solution for the vibro-acoustic behavior analysis of a stiffened double panel-cavity coupled system is presented. Unlike existing methods, this method is flexible with parameter analysis and can simulate the elastic boundary conditions of a stiffened double panel structure. The displacements of the stiffened double panel structures and the sound pressure of two inner acoustic cavities are expressed by two-dimensional (2D) and three-dimensional (3D) improved Fourier series methods, respectively. Then, the unknown coefficients of the vibro-acoustic control equation are solved by the Rayleigh–Ritz method based on the virtual work principle applied to the coupled system. In numerical results, the accuracy and effectiveness of the proposed method are validated by several comparison examples. Finally, the influence of some parameters on the vibro-acoustic behavior of the coupled system is investigated. Numerical results show that the external acoustic excitation at a certain angle can stimulate more resonant responses of the panel-cavity coupled system. This work can predict quickly the vibro-acoustic behavior of the stiffened double panel-cavity coupled system with a small truncation number. Some new results can be used as reference data for future work.

1. Introduction

Stiffened double panel-cavity coupled systems are widely used in engineering applications, such as transportation systems, aircraft fuselage shells, modern buildings, window glazing, and vehicles, due to their good sound insulation, heat preservation characteristics, and good rigidity. The study of the vibro-acoustic behavior of stiffened double panel-cavity coupled systems has been an important research topic in vibration and acoustic fields.

In the double panel-cavity coupled system, not only the vibration of the plates [1–7] and the sound pressure of the cavity [8–11] but also the coupled characteristics of the plate with the cavity should be considered. To provide some helpful insights into the sound transmission and energy transmission mechanism, some researchers made sufficient efforts [12–17] for panel-cavity coupled systems in the last few decades.

Research on double panel-cavity coupled systems attracted more attention from engineers. Kam et al. [18] investigated the vibro-acoustic characteristics of the acoustic cavity enclosed by a shear-deformed plate under elastic boundary conditions with the first Rayleigh integral and Rayleigh–Ritz method. Carneal and Fuller [19] used the experimental method to study the active control of sound transmission in a double panel-cavity system and then revealed the transmission mechanism of sound radiation in a double panel-cavity system. Xin et al. [20] used the Fourier series with the weighted residual method to study the vibro-acoustic characteristics of a double panel-cavity system with clamped boundary conditions. In the following work, they continued to study the sound transmission of a double
panel-cavity system with simply supported boundary conditions [21]. Raviprolu et al. [22] established an analytical model for the sound radiation behavior of a rectangular duct with flexible walls. Guo et al. [23] used the temperature field theory and modal superposition method to investigate the vibro-acoustic behavior of simply supported double partitions in a thermal environment. Lee [24] applied the multilevel residue harmonic balance method to establish the free vibration behavior analysis model of a nonlinear panel coupled with an extended cavity. Sadri and Younesian [25] studied the free vibration of a plate-cavity coupled system based on the Von Karman plate theory.

It is worth noting that the above-mentioned contributions to the vibro-acoustic behavior of plate-cavity coupled systems are limited to classical boundary conditions. To consider the elastically restrained boundary conditions of structures and the general impedance boundary conditions of a cavity, the improved Fourier series method was used by many researchers. Zhang and Li [26] adopted this method to analyze the vibration of a rectangular plate with arbitrary nonuniform elastic edge restraints. Chen et al. [27] also used this method to analyze the vibration and energy transmission characteristics of a plate structure with elastic edge restraints. Du et al. [28] solved the general impedance boundary condition simulation problem of a rectangular acoustic cavity by the improved Fourier series method; later, they established the vibro-acoustic analysis model of a panel-cavity coupled system [29]. Shi et al. [30] used this method to analyze the acoustic modal and steady-state responses of triangular and quadrangular prism acoustic cavities. Zhang et al. [31] used this method to analyze the vibro-acoustic analysis of an annular segment flexible panel-cavity coupled system. Shi et al. [32] used this improved Fourier series method to establish the theoretical model of a double panel-cavity coupled system and analyzed their vibro-acoustic behavior and sound transmission loss.

However, to the best of the authors’ knowledge, there is no related research on the acoustic and vibration characteristics of the stiffened double panel-cavity coupled system with elastically restrained boundary conditions. Therefore, it is very necessary to establish the vibro-acoustic model of stiffened double panel-cavity coupled system and analyze their acoustic and vibration characteristics. In this paper, the improved Fourier series method [33] is used to describe the displacement functions of the plate structure and the sound pressure functions in the acoustic cavity. Then the energy principle is introduced for coupled energy in panel-panel and panel-cavity coupled positions. Finally, the Rayleigh–Ritz method is used to solve the unknown coefficient of the equations. In numerical results, the accuracy of this method is verified comparison with the results obtained by the finite element method.

2. Theoretical Formulations

2.1. Description of the Coupled System. Consider the stiffened double panel-cavity coupled system model that is a composite of two enclosed cavities and three thin panels, as shown in Figure 1. To simplify the description, the coordinate systems and notations of the stiffened double panel-cavity coupled system are defined in Figure 1. Panels 1 and 2 represent the incident panel and the radiating panel, respectively. Panel 3 denotes the stiffener that is coupled with panels 1 and 2. The acoustic cavity $a$ and acoustic cavity $b$ are enclosed by these panels.

The boundary and structural coupled conditions of the system can be described in the form of elastic springs shown in Figure 2. The stiffness coefficients of normal and tangential springs can be represented by the symbols $k$ and $K$, respectively. Then arbitrary boundary conditions and arbitrary coupled conditions can be obtained by setting these boundary springs and coupled springs as appropriate values. For example, setting the stiffness coefficients of boundary springs of four sides to infinity (5e9) represents that four sides of the panel are clamped (CCCC). In the same way, the free boundary of four sides (FFFF) denotes setting the stiffness coefficients of boundary springs as zero. Except for these coupled interfaces in the panel-cavity coupled systems,
other acoustic boundaries of the acoustic cavity \( a \) and \( b \) are considered rigid walls.

2.2. Vibration and Acoustic Equations of the Coupled System. In this paper, the displacement functions of each panel and the acoustic pressure functions of each acoustic cavity can be described by two- and three-dimensional improved methods [33], respectively. For the sake of convenience and simplicity, only the examples using the improved Fourier method to describe panel 1 and acoustic cavity \( a \) are given here.

The displacement functions of the panel 1 can be expressed as follows:

\[
\begin{align*}
\omega_1 (x_1, y_1) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \cos(\lambda_{mLx} x_1) \cos(\lambda_{nLy} y_1) \\
&\quad + \sum_{m=0}^{\infty} \sum_{k=1}^{4} a_{km} \xi_{kLx}(y_1) \cos(\lambda_{mLx} x_1) \\
&\quad + \sum_{m=0}^{\infty} \sum_{k=1}^{4} \bar{a}_{km} \xi_{kLx}(x_1) \cos(\lambda_{nLy} y_1),
\end{align*}
\]

\[
\begin{align*}
u_1 (x_1, y_1) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_{mn} \cos(\lambda_{mLx} x_1) \cos(\lambda_{nLy} y_1) \\
&\quad + \sum_{m=0}^{\infty} \sum_{k=1}^{2} b_{km} \xi_{kLy}(x_1) \cos(\lambda_{nLy} y_1) \\
&\quad + \sum_{m=0}^{\infty} \sum_{k=1}^{2} \bar{b}_{km} \xi_{kLy}(y_1) \cos(\lambda_{mLx} x_1),
\end{align*}
\]

\[
\begin{align*}
\nu_1 (x_1, y_1) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{mn} \cos(\lambda_{mLx} x_1) \cos(\lambda_{nLy} y_1) \\
&\quad + \sum_{m=0}^{\infty} \sum_{k=1}^{2} c_{km} \xi_{kLx}(x_1) \cos(\lambda_{nLy} y_1) \\
&\quad + \sum_{m=0}^{\infty} \sum_{k=1}^{2} \bar{c}_{km} \xi_{kLx}(y_1) \cos(\lambda_{mLx} x_1),
\end{align*}
\]

where \( \lambda_{mLx} = m \pi/Lx_1 \) and \( \lambda_{nLy} = n \pi/Ly_1 \), \( A_{mn} \), \( a_{km} \), and \( \bar{a}_{km} \) represent the unknown coefficients of the bending displacement; \( B_{mn} \), \( b_{km} \), \( \bar{b}_{km} \), \( C_{mn} \), \( c_{km} \), and \( \bar{c}_{km} \) represent the unknown coefficients of in-plane displacement.

\( \xi_{kLx}(y_1) \) and \( \xi_{kLy}(x_1) \) represent the supplementary functions of the bending displacement. \( \xi_{kLx}(y_1) \) and \( \xi_{kLy}(x_1) \) represent the supplementary functions of the in-plane displacement. These supplementary functions are introduced to overcome the discontinuities with the displacement function and their partial derivatives at the boundary and coupled position of the panel. Considering computer speed, property truncation numbers \( M_x \), \( M_y \), and \( M_z \) should be selected in the actual calculation.

Similarly, the acoustic pressure functions of the acoustic cavity \( a \) can be expressed as follows:

\[
P_a(x_a, y_a) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \lambda_{mLx}^2 \sin(\lambda_{mLx} x_a) \cos(\lambda_{mLx} x_a) \\
+ \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \lambda_{mLx}^2 \sin(\lambda_{mLx} x_a) \cos(\lambda_{mLx} x_a)
\]

\[
\begin{align*}
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=1}^{4} \alpha_{km} \xi_{kLy}(x_a) \cos(\lambda_{mLx} x_a) \\
&\quad + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=1}^{4} \beta_{km} \xi_{kLy}(y_a) \cos(\lambda_{mLx} x_a) \\
&\quad + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=1}^{4} \gamma_{km} \xi_{kLy}(x_a) \cos(\lambda_{mLx} x_a) \\
&\quad + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=1}^{4} \delta_{km} \xi_{kLy}(y_a) \cos(\lambda_{mLx} x_a),
\end{align*}
\]

where \( \lambda_{mLx} = m \pi/Lx_a \), \( \lambda_{mLx} = m \pi/Ly_a \), and \( \lambda_{mLx} = m \pi/Lz_a \), \( \alpha_{km} \), \( \beta_{km} \), \( \gamma_{km} \), and \( \delta_{km} \) represent the unknown coefficients of the acoustic pressure. \( \xi_{kLy}(x_a) \), \( \xi_{kLy}(y_a) \), and \( \xi_{kLy}(z_a) \) represent the supplementary functions of the acoustic pressure. Property truncation numbers \( M_x \), \( M_y \), and \( M_z \) should be selected in the actual calculation.

The vibro-acoustic equations of the stiffened double panel-cavity coupled system can be written by the energy principle. The Lagrangian of the stiffened double panel structure and two acoustic cavities can be expressed as follows:

\[
L_s = U_s - T_s + W_s + W_{ca} + W_{ch},
\]

\[
L_a = U_a - T_a + W_a - W_{ca} - W_{ch},
\]

\[
L_b = U_b - T_b + W_b - W_{ca} - W_{ch},
\]

where \( U_s \) is the total potential energy of the stiffened double panel structure, including the bending and in-plane...
potential energy of panels, the potential energy of boundary springs, and the potential energy of the coupled springs at the structural coupled positions. \( T_s \) is the total kinetic energy of stiffened double panel structure. \( W_{ca \& bs} \) and \( W_{cb \& bs} \) are the work done by the acoustic cavities \( a \) and \( b \) acting on the panel-cavity coupled interface. \( W_i \) is the total work done by the external force. \( U_{a-c} \) and \( T_{a-c} \) are the potential and kinetic energy of acoustic cavity \( a \), respectively. \( W_{a-c} \) is the energy exchange between the outside parts with acoustic cavity \( a \), including the work done by the wall surfaces, the work done by the sound source in the acoustic cavity \( a \), and the energy exchange on the panel-cavity coupled surface. The mean of the expressions in equation (7) is similar to the one in equation (6).

These explicit energy and work expressions in the above equations can be expressed as follows:

\[
W_s = -F w_1(x_c, y_c),
\]

\[
W_{ca \& bs} = \int_s w_p \sigma ds,
\]

\[
U_s = \sum_{j=1}^{3} U_{jb} + \sum_{j=1}^{3} U_{ji} + \sum_{q=1}^{2} U_{qc} + \sum_{r=1}^{12} U_{rs},
\]

\[
T_s = \sum_{j=1}^{3} T_{jb} + \sum_{j=1}^{3} T_{ji},
\]

\[
W_{a-c} = W_{s,k,ca} + W_{swall} + W_{as},
\]

where \( U_{jb} \) and \( U_{ji} \) denote the bending and in-plane potential energy of the panel \( j \) structure, respectively. \( T_{jb} \) and \( T_{ji} \) denote the bending and in-plane kinetic energy of the panel \( j \) structure, respectively. \( U_{rs} \) denotes the potential energy of the coupled spring at the \( q \)-th panel-panel coupled position. \( U_{rs} \) denotes the potential energy of the boundary spring at the \( r \)-th side of the stiffened double panel structure. \( W_{s,k,ca} = -W_{ca \& bs} \) denotes the work done by the stiffened double panel structure on the acoustic cavity \( a \). \( W_{swall} = 0 \) and \( W_{as} = 0 \) denote the work done by the sound source in cavity \( a \) and the impedance wall surfaces, respectively.

The potential and kinetic energy of the panel \( j \) structure can be expressed as follows:

\[
U_{jb} = \frac{D_j}{2} \int_{0}^{L_{j,1}} \int_{0}^{L_{j,2}} \left( \frac{\partial^2 w_j}{\partial x_j^2} \right)^2 + \left( \frac{\partial^2 w_j}{\partial y_j^2} \right)^2 + 2\mu_j \left( \frac{\partial^2 w_j}{\partial x_j \partial y_j} \right)^2 dx_j dy_j,
\]

\[
U_{bj} = \frac{G_j}{2} \int_{0}^{L_{j,1}} \int_{0}^{L_{j,2}} \left( \frac{\partial u_j}{\partial x_j} + \frac{\partial v_j}{\partial y_j} \right)^2 - 2(1 - \mu_j) \frac{\partial u_j}{\partial x_j} \frac{\partial v_j}{\partial y_j} dx_j dy_j + \frac{(1 - \mu_j)}{2} \left( \frac{\partial v_j}{\partial x_j} + \frac{\partial u_j}{\partial y_j} \right)^2 dx_j dy_j,
\]

\[
T_{jb} = \frac{1}{2} \rho_j h_j \omega^2 \int_{0}^{L_{j,1}} \int_{0}^{L_{j,2}} w_j^2 dx_j dy_j,
\]

\[
T_{ji} = \frac{1}{2} \rho_j h_j \omega^2 \int_{0}^{L_{j,1}} \int_{0}^{L_{j,2}} [u_j^2 + v_j^2] dx_j dy_j,
\]

where \( D_j \) and \( G_j \) represent the bending and stretching stiffness of panel \( j \) structure, respectively. \( \rho_j, h_j, \) and \( \mu_j \) represent the density, thickness, and Poisson’s ratio of the panel \( j \) structure, respectively. \( \omega \) denotes the angular frequency of the system.

The coupled potential energy of the first structural coupled position and the boundary potential energy of the first boundary can be written as follows:

\[
U_{1c} = \frac{1}{2} \int_{0}^{L_{1,1}} K_{cl} \left( \frac{\partial u_1}{\partial x_1} \right)^2 dx_1 \quad \text{for} \quad C
\]

\[
+ k_{c1} \left( u_1 \big|_{x_1 = L_{1,1}} + u_2 \big|_{x_2 = L_{2,2}} \right)^2 \quad \text{and} \quad \text{for} \quad F
\]

\[
+ k_{c2} \left( u_1 \big|_{x_1 = L_{1,1}} - u_2 \big|_{x_2 = L_{2,2}} \right)^2 + k_{c3} \left( v_1 \big|_{x_1 = L_{1,1}} - v_2 \big|_{x_2 = L_{2,2}} \right)^2 dy_1,
\]

where \( k_{c1}, k_{c2}, k_{c3}, \) and \( K_{cl} \) represent the stiffness coefficients of coupled springs. \( k_{a1}, k_{a2}, k_{a3}, \) and \( K_{aw} \) represent the stiffness coefficients of boundary springs. Spring symbols \( K \) and \( k \) denote tangential and normal springs, respectively. \( L_{x1c} \) represents the coupled position in the panel 1 structure. The subscript 1 of the displacement and the coordinate symbols denotes the panel 1 structure. Other coupled potential energy of the first coupled position and the boundary potential energy can be obtained by changing corresponding subscripts. Through setting stiffness coefficients to each spring of these panels, different boundary and structure coupled conditions can be obtained. For example, the classical boundary conditions of the panel, clamped (C), free (F), and simply supported (S) boundary conditions can be easily realized by setting the stiffness of springs as proper values:

at the edge \( x_1 = 0 \) or \( x_1 = L_{x1}, \)

\[
\begin{align*}
\{ u & = \nu = \varphi_x = \varphi_y = 0, \quad \text{for} \quad C \n N_x & = N_{xy} = Q_x = M_x = M_{xy} = 0, \quad \text{for} \quad F, \n u & = \nu = \varphi_x = 0, N_{xy} = M_{xy} = 0, \quad \text{for} \quad S
\end{align*}
\]

at the edge \( y_1 = 0 \) or \( y_1 = L_{y1}, \)

\[
\begin{align*}
\{ u & = \nu = \varphi_x = \varphi_y = 0, \quad \text{for} \quad C \n N_x & = N_{xy} = Q_x = M_x = M_{xy} = 0, \quad \text{for} \quad F, \n \nu & = \varphi_y = 0, \quad N_{xy} = M_{xy} = 0, \quad \text{for} \quad S
\end{align*}
\]

The potential and kinetic energy of the acoustic cavity \( a \) can be expressed as follows:
\[ U_{a-c} = \frac{1}{2\rho_a \omega^2} \int_{V_a} \left( \frac{\partial p_a}{\partial x_a} \right)^2 + \left( \frac{\partial p_a}{\partial y_a} \right)^2 + \left( \frac{\partial p_a}{\partial z_a} \right)^2 \] dV_a \]

(21)

\[ T_{a-c} = \frac{1}{2\rho_a \omega^2} \int_{V_a} \left[ \left( \frac{\partial p_a}{\partial x_a} \right)^2 + \left( \frac{\partial p_a}{\partial y_a} \right)^2 + \left( \frac{\partial p_a}{\partial z_a} \right)^2 \right] dV_a, \]

(22)

where \( V_a \) denotes the volume of acoustic cavity \( a \). \( p_a \) is the sound pressure in acoustic cavity \( a \). \( \rho_a \) is the density of acoustic cavity \( a \). \( c_a \) is the acoustic velocity of cavity \( a \). The total work of acoustic cavity \( a \) to the stiffened double panel structure can be expressed as follows:

\[ W_{ca&k} = W_{ca&p1} + W_{ca&p2} + W_{ca&p3}. \]

(23)

and

\[
\begin{align*}
W_{ca&p1} &= \int_{Lya} \int_{Lya}^L w_1 \rho_a dx_a dy_a, \\
W_{ca&p2} &= -\int_{Lya} \int_{Lya}^L w_2 \rho_a dx_a dy_a, \\
W_{ca&p3} &= \int_{Lya} \int_{Lya}^L w_3 \rho_a dy_a dz_a,
\end{align*}
\]

(24)

where \( Lxa, Ly, \) and \( Lza \) represent the length, width, and height of the acoustic cavity \( a \), respectively. \( w_1, w_2, \) and \( w_3 \) represent the bending displacements of panels \( 1, 2, \) and \( 3 \). The kinetic energy, the potential energy of acoustic cavity \( b \), and the total work done on the stiffened double panel structure system can be obtained by changing corresponding subscripts in equations (21)–(24).

As above description, the Lagrange function equations of the stiffened double panel-cavity coupled system can be determined by the energy expressions of the acoustic cavity \( a \), the acoustic cavity \( b \), and the stiffened double panel structures. Substituting the displacement and sound pressure functions in equations (1)–(4) into the Lagrangian function equations in equations (5)–(7) and then using the Rayleigh–Ritz method against each unknown Fourier coefficient of the vibro-acoustic equations, the following matrix equations can be obtained:

\[
\begin{bmatrix}
K_s & C_{sba} & C_{sbb} \\
C_{sba} & K_b & M_b \\
C_{sbb} & M_b & K_b
\end{bmatrix}
\begin{bmatrix}
P_s \\
P_b \\
P_b
\end{bmatrix}
= \begin{bmatrix}
E_s \\
0 \\
0
\end{bmatrix},
\]

(25)

where \( K_s \) and \( M_s \) denote the stiffness and mass matrices of the stiffened double panel structure, respectively. \( K_b \) and \( M_b \) denote the stiffness and mass matrices of the acoustic cavity \( a \), respectively. \( K_b \) and \( M_b \) denote the stiffness and mass matrices of the acoustic cavity \( b \), respectively. \( C_{sba} = -C_{sbs} \) and \( C_{sbb} = -C_{sbb} \) denote the panel-cavity coupled matrices for the stiffened double panel structure acting on the acoustic cavity \( a \) and \( b \), respectively. \( \omega \) denotes the angular frequency of the system. \( P_s, P_b, \) and \( E \) denote the unknown Fourier coefficients of the sound pressure inner acoustic cavities and the displacements of all panels. The free vibro-acoustic behavior of this system can be obtained by setting the force vector at the right end of equation (25) to the zero vector.

2.3. Sound Transmission Loss. The incident acoustic power can be written as follows:

\[ \prod_{in} = \frac{1}{2} \text{Re} \int \rho_i \nu_i^* dA, \]

(26)

where \( \rho_i \) denotes the incident sound pressure and \( \nu_i = \rho_i / (\rho_{air} c_{air}) \) is the acoustic velocity. Item marked with an asterisk denotes the complex conjugate. \( \rho_{air} \) and \( c_{air} \) denote the density and acoustic velocity of air, respectively. When the incident wave is plane, the incident acoustic power in equation (26) can be also written as follows:

\[ \prod_{in} = \frac{\rho_i^2}{2 \rho_{air} c_{air} \pi} \cos(\varphi) \cdot Lx \cdot Ly \cdot \frac{1}{1 + i}, \]

(27)

where \( \varphi \) is the incident elevation angle.

The radiated acoustic power from the radiating panel can be written as follows:

\[ \prod_{r} = \frac{1}{2} \text{Re} \int \rho_r \nu_r^* dA, \]

(28)

where \( \nu_r \) and \( \rho_r \) denote the acoustic velocity and sound pressure on the surface of the radiating panel, respectively. The expression of \( \nu_r \) can be written as follows:

\[ \nu_r = j \cdot \omega \cdot \nu_s(x, y). \]

(29)

On the basis of Rayleigh’s integral, the sound pressure on the radiating panel \( \rho_r \) can be written as follows:

\[ \rho_r = \frac{j \cdot k \cdot \rho_{air} \cdot c_{air}}{2 \pi} \int_{s} v_r(x, y) e^{-jk \cdot r} ds, \]

(30)

where \( k = \omega / c_{air} \) denotes the wave number. \( r \) is the distance between the sound pressure point and the vibration source.

Substituting the acoustic velocity on the radiating panel \( v_r \) in equation (29) and the sound pressure on the radiating panel \( \rho_r \) in equation (30) into equation (28), the radiated acoustic power can be expressed as follows:

\[
\prod_{r} = \frac{\rho_{air} c_{air}}{2 \pi} \text{Re} \left[ \int_{s} \int_{s} v_r(x, y) \frac{e^{-jk \cdot r}}{r} v_r^* (x, y) dsds \right],
\]

(31)

After dividing the surface \( S \) of the radiating panel into discrete elements \( S_m (m = 1, 2, \cdots, M) \), the discrete distribution of the radiated acoustic power can be written as follows:

\[
\prod_{r} = \frac{\rho_{air} c_{air}}{2 \pi} k^2 \sum_{m=1}^{M} \sum_{n=1}^{M} \int_{s_m} \int_{s_n} \nu_m \sin(k \cdot r_{mn}) v_n^* vs d \bar{s} d \bar{s},
\]

(32)

\[
= \frac{M_s M_r}{2 \pi} \sum_{m=1}^{M_s} \sum_{n=1}^{M_r} \rho_{air} c_{air} k^2 \nu_m \sin(k \cdot r_{mn}) v_n^* r_n (S_m)^2,
\]
where $S_m$ denotes the area of the discrete element, which is assumed as equal. $r_{mn}$ denotes the distance between the center of the $m$th element with the center of the $n$th element. $v_{r_m}$ denotes the vibration velocity at the center of the $m$th element. The above radiated acoustic power expression can also be written as follows:

$$\prod_r = \sum_m \sum_n v_{r_m} R_{mn} v_{r_n}^*', \quad (33)$$

with

$$R_{mn} = \begin{cases} \frac{k^2 (\Delta s)^2 \rho \varepsilon_{air}}{4\pi} \sin(k \cdot r_{mn})}{k \cdot r_{mn}} & m \neq n, \\
\frac{k^2 (\Delta s)^2 \rho \varepsilon_{air}}{4\pi} & m = n, \end{cases} \quad (34)$$

The sound transmission loss (STL) of the stiffened double panel-cavity coupled system can be written as follows:

$$\text{STL} = 10 \log_{10} \left( \frac{\prod_{n}}{\prod_{r}} \right), \quad (35)$$

### 3. Numerical Results and Discussions

In this section, the validation of the numerical results and the influence of some key parameters on vibro-acoustic will be carried out. For convenience, three panels have the same material properties and medium constants in two acoustic cavities are assumed as the same, as shown in Table 1.

#### 3.1. Model Validation

The convergence and correction of the current method are verified in this subsection. Table 2 presents the first eight natural frequencies of the stiffened double panel-cavity coupled system. In this analysis, the constant Young's modulus of panels, Poisson's ratio of panels, thickness of panels, density of cavity mediums, density of panels, acoustic velocity of cavity mediums, Young's modulus of panels, Poisson's ratio of panels, thickness of panels, density of cavity mediums, are chosen as equal. Numerical results show that the method has good accuracy and fast convergence.

Based on the above frequency validations, we can obtain the vibration mode shapes of the stiffened panel structure in the stiffened double panel-cavity coupled system, as shown in Figures 3 and 4. It is easily seen that the vibration mode shapes of the stiffened double panel structure calculated by the current method agree well with those by the ANSYS from these figures. The differences between the modal shapes obtained by the current method and ones obtained by the ANSYS are mainly due to the selection of color map. In the first eight modes, the 1st, 6th, and 7th modes are mainly concentrated on the stiffened plate, and the rest modes are concentrated on the upper and lower plates. It is worth noting that all panels in the stiffened panel structure are coupled with acoustic cavities. Therefore, the sound pressure mode shapes of two acoustic cavities in the stiffened double panel-cavity coupled system should be considered. Figure 5 presents some sound pressure mode shapes of two acoustic cavities in the stiffened double panel-cavity coupled system. From this figure, it is easily seen that the sound pressure mode shapes of two acoustic cavities in the stiffened double panel-cavity coupled system calculated by the current method can agree with those by the ANSYS. It is worth noting that structural control modes are more than acoustic cavity control modes. For example, the 5th mode belongs to acoustic cavity control mode, while the rest of the first eight order modes belong to structure control mode.

Assume that the point $(x_1 = 0.7, y_1 = 0.3)$ on panel 1 is acted by the point force, whose amplitude is equal to 1 N, and its direction points to the negative direction of $z_1$. The damping ratios of all panel structures and acoustic cavities are set as 0.015. The reference value for the vibration velocity of panel structures and the acoustic pressure in cavities are $10^{-9}$ m/s and $2 \times 10^{-5}$ Pa, respectively. Figures 6–10 show vibration velocity responses at some points on panel structures under simply supported boundary conditions. Figures 9 and 10 show the sound pressure responses at some points in acoustic cavities. It is easily seen that the vibration velocity responses at some points on all panels and the sound pressure responses at some points in acoustic cavities calculated by the current method are in good agreement with those calculated by the commercial software ANSYS from these figures.

| Object | Value (units) |
|--------|--------------|
| Acoustic cavity, $a$ $(L_x a \times L_y a \times L_z a)$ | 0.5 m × 0.6 m × 0.8 m |
| Acoustic cavity, $b$ $(L_x b \times L_y b \times L_z b)$ | 0.5 m × 0.6 m × 0.8 m |
| The thickness of panels, $h$ | 0.004 m |
| The density of panels, $\rho$ | 7,800 kg/m$^3$ |
| Young’s modulus of panels, $E$ | 2.16 × 10$^{11}$ Pa |
| Poisson’s ratio of panels, $\mu$ | 0.28 |
| The density of cavity mediums, $\rho_{air}$ | 1.21 kg/m$^3$ |
| The acoustic velocity of cavity mediums, $\varepsilon_{air}$ | 344 m/s |

### 3.2. Parameter Analysis

The geometric parameters and material properties used for parameter analysis in this subsection are the same as those in previous examples, and the boundary conditions of the stiffened double panel structure and acoustic cavities remain unchanged. The
specific parameters of vibro-acoustic behavior can be studied with only changing corresponding parameters, such as thickness, length, material properties, or boundary conditions. The effects of the stiffener (panel 3) on the natural frequencies, responses, and sound transmission loss (STL) of the panel-cavity coupled system are studied in detail. When considering the influence of boundary springs on the vibration behavior, the response, and STL of the stiffened double panel-cavity coupled system, the stiffness amplitudes of all normal springs at the edges $y = 0$ and $y = Ly/3$ of the stiffener are equal to $k_{vs}$, while the stiffness amplitudes of other springs remain unchanged.

### Table 2: First eight natural frequencies of the stiffened double panel-cavity coupled system.

| Mode no | $M = N = 7$ | $M = N = 8$ | $M = N = 9$ | $M = N = 10$ | $M = N = 11$ | $M = N = 12$ | ANSYS |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|-------|
| 1       | 51.963      | 51.890      | 51.471      | 51.436      | 51.317      | 51.295      | 50.555 |
| 2       | 73.439      | 73.211      | 71.734      | 71.638      | 71.506      | 71.441      | 70.141 |
| 3       | 79.493      | 79.302      | 78.021      | 77.941      | 77.763      | 77.711      | 76.159 |
| 4       | 85.457      | 84.708      | 84.709      | 84.436      | 84.437      | 84.279      | 83.801 |
| 5       | 86.783      | 86.054      | 86.055      | 85.791      | 85.788      | 85.637      | 85.127 |
| 6       | 113.23      | 113.20      | 112.57      | 112.57      | 112.08      | 111.99      | 109.52 |
| 7       | 130.05      | 130.02      | 129.74      | 129.75      | 129.58      | 129.59      | 128.58 |
| 8       | 157.83      | 157.82      | 154.68      | 154.71      | 154.25      | 154.29      | 151.56 |

Figure 3: First four vibration mode shapes of the stiffened panel structure in the stiffened double panel-cavity coupled system: (a) results from ANSYS and (b) results from the current method.

Figure 4: The 5th–8th vibration mode shapes of the stiffened panel structure in the stiffened double panel-cavity coupled system: (a) results from ANSYS and (b) results from the current method.
Figure 5: Some sound pressure mode shapes of two acoustic cavities in the stiffened double panel-cavity coupled system: (a) results from ANSYS and (b) results from the current method.

Figure 6: Vibration velocity response at the position \((x_1 = 0.8\, \text{m}, y_1 = 0.1\, \text{m})\) on the surface of panel 1.

Figure 7: Vibration velocity response at the position \((x_2 = 0.2\, \text{m}, y_2 = 0.1\, \text{m})\) on the surface of panel 2.

Figure 8: Vibration velocity response at the position \((x_3 = 0.1\, \text{m}, y_3 = 0.1\, \text{m})\) on the surface of the stiffener.

Figure 9: Sound pressure response at the position \((x_a = 0.3\, \text{m}, y_a = 0.3\, \text{m}, \text{and } z_a = 0.4\, \text{m})\) in the acoustic cavity \(a\).
3.2.2. Vibration Velocity Response and Sound Pressure Response Analyses. In this subsection, the vibration velocity response and pressure response analyses are presented. The medium constants of the acoustic cavity, the geometric parameters, and the material properties of panel structures are the same as those used in Figures 6–10. The amplitude of the point force, the direction of the point force, and the position acted by the point force remain unchanged. In other words, the corresponding vibro-acoustic model of the stiffened double panel-cavity coupled system in Figures 6–10 is used in this subsection.

Figures 11–13 present the influence of the thickness, length, boundary conditions, and material properties of the stiffener on the vibration velocity response at the point \( x_2(x_2 = 0.2 \text{ m}, y_2 = 0.1 \text{ m}) \) on panel 2 and sound pressure response at the point \( (x_3 = 0.3 \text{ m}, y_3 = 0.3 \text{ m}, z_3 = L/2a) \) in the acoustic cavity \( a \). Except for the object being analyzed, the others remain unchanged in these figures. For example, the thickness \( h_3 \) that is equal to 0.002 m, 0.004 m, or 0.008 m is selected for the thickness analysis in Figure 11. The Al, ZrO\(_2\), and Al\(_2\)O\(_3\) material properties used in Figure 14 are respectively: \( E_{\text{Al}} = 7 \times 10^7 \text{ Pa}, \mu_{\text{Al}} = 0.3, \rho_{\text{Al}} = 2,700 \text{ kg/m}^3\); \( E_{\text{ZrO}_2} = 200 \times 10^9 \text{ Pa}, \mu_{\text{ZrO}_2} = 0.3, \rho_{\text{ZrO}_2} = 5,700 \text{ kg/m}^3\); and \( E_{\text{Al}_2\text{O}_3} = 380 \times 10^9 \text{ Pa}, \mu_{\text{Al}_2\text{O}_3} = 0.3, \rho_{\text{Al}_2\text{O}_3} = 3,800 \text{ kg/m}^3\). Some remarks can be given from these figures: (1) with the increase of the thickness of the stiffened panel, the resonance peaks of vibration velocity and sound pressure become less, and the resonance peaks move to higher frequencies. However, some peak values of sound pressure response remain unchanged with the increase in thickness. (2) When the height of the cavity and the length of the stiffened panel increase at the same time, the number of the response peaks of the vibration velocity of the bottom panel and sound pressure increases, and the sound pressure response become weak in general. (3) When the stiffness coefficients of the boundary springs change from 1E1 to 1E5, there is no obvious change in the vibration velocity response and acoustic pressure response. When the stiffness coefficients of the boundary springs increase from 1E5 to 1E9, some resonance peaks of the velocity response and sound pressure move to higher frequencies. (4) The response curve for the Al material is closer to the response curve for the ZrO\(_2\) material than one for the Al\(_2\)O\(_3\) material. The reason for the above phenomenon may be that the change of the geometric parameters and properties of the stiffened panel can cause the change of some natural frequencies; as a result, some vibration velocity and sound pressure response peaks will move. The change of acoustic cavity height can result in the distance between the top panel with the sound pressure response point in the acoustic cavity varying, and then the amplitude of the sound transmission loss will change.

The acoustic cavity plays an important role in the panel cavity coupled system. Therefore, the effects of the acoustic cavity on the vibration velocity and sound pressure should be investigated. The previous response analysis model of the stiffened double panel-cavity coupled system remain unchanged, and some parameters of acoustic cavities are changed for the next response analyses. Figure 15 presents the vibration velocity and sound pressure responses of the
Table 3: First seven natural frequencies of the stiffened double panel-cavity coupled system with the different thickness of the stiffener.

| Mode | $h_3 = 0.001\text{ m}$ | $h_3 = 0.002\text{ m}$ | $h_3 = 0.004\text{ m}$ | $h_3 = 0.006\text{ m}$ | $h_3 = 0.008\text{ m}$ |
|------|------------------|------------------|------------------|------------------|------------------|
| 1    | 28.956           | 34.529           | 51.435           | 66.297           | 77.855           |
| 2    | 32.128           | 57.074           | 71.640           | 78.322           | 83.472           |
| 3    | 32.382           | 65.343           | 77.945           | 84.449           | 84.454           |
| 4    | 47.504           | 70.485           | 84.440           | 85.798           | 85.801           |
| 5    | 54.079           | 70.618           | 85.790           | 89.355           | 101.10           |
| 6    | 66.829           | 84.411           | 112.56           | 150.29           | 161.50           |
| 7    | 68.532           | 85.770           | 129.74           | 159.13           | 161.55           |

Table 4: First seven natural frequencies of the stiffened double panel-cavity coupled system with different lengths of the stiffener.

| Mode | $L_x = 0.2\text{ m}$ | $L_x = 0.3\text{ m}$ | $L_x = 0.4\text{ m}$ | $L_x = 0.6\text{ m}$ | $L_x = 0.8\text{ m}$ |
|------|------------------|------------------|------------------|------------------|------------------|
| 1    | 77.969           | 74.750           | 72.187           | 63.692           | 51.435           |
| 2    | 78.370           | 76.336           | 75.047           | 73.312           | 71.639           |
| 3    | 84.681           | 84.688           | 84.658           | 84.563           | 77.943           |
| 4    | 88.875           | 87.553           | 86.872           | 86.164           | 84.438           |
| 5    | 154.72           | 154.52           | 137.34           | 87.783           | 85.788           |
| 6    | 157.60           | 156.63           | 153.99           | 145.49           | 112.56           |
| 7    | 160.95           | 161.22           | 156.09           | 155.41           | 129.74           |

Table 5: First eight natural frequencies of the stiffened double panel-cavity coupled system with the boundary springs of the stiffener.

| Mode | $k_{vs} = 1E1$ | $k_{vs} = 1E3$ | $k_{vs} = 1E5$ | $k_{vs} = 1E7$ | $k_{vs} = 1E9$ |
|------|----------------|----------------|----------------|----------------|----------------|
| 1    | 34.864         | 34.902         | 38.046         | 50.814         | 51.425         |
| 2    | 44.971         | 45.056         | 52.671         | 71.464         | 71.629         |
| 3    | 69.460         | 69.463         | 69.701         | 77.762         | 77.938         |
| 4    | 75.465         | 75.470         | 75.935         | 84.005         | 84.406         |
| 5    | 83.956         | 83.956         | 83.956         | 85.784         | 85.786         |
| 6    | 85.784         | 85.784         | 85.784         | 110.70         | 112.54         |
| 7    | 93.866         | 93.880         | 95.204         | 123.16         | 129.68         |

Figure 11: Vibration velocity response at the point on the surface of panel 2 and sound pressure response at the point in the acoustic cavity $a$ with different $h_3$: (a) vibration velocity response and (b) sound pressure response.
stiffened double panel-cavity coupled system with different acoustic cavity mediums. In this analysis, the acoustic cavity mediums, carbon dioxide (CO2), air, and chlorine (Cl2) gas are considered. Their medium constants are: $c_{CO2} = 270 \text{ m/s}$, $\rho_{CO2} = 1.83 \text{ kg/m}^3$, $c_{air} = 343 \text{ m/s}$, $\rho_{air} = 1.21 \text{ kg/m}^3$, $c_{Cl2} = 206 \text{ m/s}$, and $\rho_{Cl2} = 3.214 \text{ kg/m}^3$. It is easily seen that the acoustic cavity medium has a weak influence on the vibration velocity, but it has a strong influence on sound pressure. Figure 16 presents the vibration velocity responses of the stiffened double panel structure with acoustic and vacuum cavities. It is easily seen that the acoustic cavity can cause a slight shift of resonance peaks, and the vibration velocity response curve with acoustic cavities has one more resonance peak than that without acoustic cavities at the $f = 85.6 \text{ Hz}$. The reason for the above phenomenon is that the vibration velocity response is mainly determined by the structure, while the sound pressure response is mainly related not only to the structure but also to the medium constant of the acoustic cavity.

3.2.3. Sound Transmission Loss Analysis. In this subsection, the sound transmission loss analysis of the stiffened double panel-cavity coupled system under acoustic excitation is presented. The influence of the geometric parameters, material properties, and boundary conditions of the stiffened panel on the sound transmission loss of the stiffened double panel-cavity coupled system is investigated systematically. Then the influence of acoustic cavities on the sound transmission loss is considered. Except for the external

![Figure 12: Vibration velocity response at the point on the surface of panel 2 and sound pressure response at the point in the acoustic cavity a with different Lx3: (a) vibration velocity response and (b) sound pressure response.](image)

![Figure 13: Vibration velocity response at the point on the surface of panel 2 and sound pressure response at the point in the acoustic cavity a with different kvs: (a) vibration velocity response and (b) sound pressure response.](image)
excitation, other parameters of the system are consistent with the previous response analysis. The acoustic excitation with elevation angle $\phi = \pi/4$, azimuth angle $\theta = \pi/4$, and the amplitude of the incident sound pressure $p_i = 1$ Pa is used for the STL calculation.

Figures 17–20 present the sound transmission loss of the stiffened double panel-cavity coupled system with various thicknesses, lengths, boundary conditions, and materials of the stiffener, respectively. Some remarks can be given from these figures: (1) the stiffened double panel-cavity coupled system with thicker stiffened panel has less number of sound transmission loss valleys of the sound pressure loss, and its valleys move to higher frequencies. (2) When the height of the cavity and the length of the stiffened panel increase at the same time, the number of sound transmission loss valleys increases, and the amplitude of sound transmission loss becomes bigger out of sound transmission loss valleys. (3) Some sound transmission loss valleys move to a higher frequency with the increasing stiffness coefficients of the boundary springs $k_{ea}$ in a certain range. (4) The number of sound transmission loss valleys for the $\text{Al}_2\text{O}_3$ material is less than that for the $\text{Al}$ material or $\text{ZrO}_2$ material. The reason for the above phenomenon may be that the stiffened panel has an obvious influence on the natural frequency of the system, and changing the parameters of the stiffened panel will result in the movement of some sound transmission loss valleys.

Figure 21 presents the sound transmission loss of the stiffened double panel-cavity coupled system with different acoustic cavity mediums. From the figure, it is easily seen that the sound transmission loss for the air acoustic cavity is close to one for the carbon dioxide acoustic cavity, and the system with a chlorine cavity medium has the smallest sound transmission loss.
**Figure 16:** Vibration velocity response at the point on the surface of panel 2 with and without an acoustic cavity.

**Figure 17:** Sound transmission loss of the stiffened double panel-cavity coupled system with different thickness $h_3$.

**Figure 18:** Sound transmission loss of the stiffened double panel-cavity coupled system with different length $Lx_3$.

**Figure 19:** Sound transmission loss of the stiffened double panel-cavity coupled system with different stiffness amplitudes of boundary springs $k_{v_0}$.

**Figure 20:** Sound transmission loss of the stiffened double panel-cavity coupled system with different materials of the stiffener.

**Figure 21:** Sound transmission loss of the stiffened double panel-cavity coupled system with different acoustic cavity mediums.
transmission loss of three acoustic cavity mediums. Then the helium with lower density and higher acoustic velocity is used to replace the chlorine in the cavity to analyze the sound transmission loss, as shown in Figure 22. The medium constants of the helium are defined as $c_{\text{helium}} = 972 \text{ m/s}$ and $\rho_{\text{helium}} = 0.1664 \text{ kg/m}^3$. In this analysis, the acoustic excitations at a certain angle and perpendicular to the top panel are considered. It is easily seen that the sound transmission loss of the system with helium is the biggest in the three cavity mediums. The number of sound transmission loss valleys of the system excited by acoustic excitation at a certain angle is larger than 1 by acoustic excitation perpendicular to the top panel. The reason may be that the structure control mode resonance frequencies of the system are more easily excited by the acoustic excitation at a certain angle, while the acoustic excitation perpendicular to the top panel can excite easily the acoustic cavity control mode resonance frequencies of the system.

Figure 23 presents the sound transmission loss of the system with different acoustic cavity medium constants, in which only one of the density and acoustic velocity of acoustic cavities in the system is changed. From Figures 22 and 23, it is easily seen that the density of the acoustic medium can affect the amplitude of sound transmission loss of the system, and the system with a smaller density acoustic medium has a bigger amplitude of sound transmission loss. The acoustic velocity of the acoustic medium can affect the acoustic cavity control mode resonance frequency; in other words, it can cause the movement of some sound transmission loss valleys.

In order to achieve better sound insulation, heat insulation, and heat preservation effects, the medium of the double panel-cavity system is often vacuumed in practical engineering, such as vacuum double-layer glass and heat insulation wall. Therefore, it is very necessary to study the double-panel cavity system with vacuum mediums.
Figure 24 presents the sound transmission loss of the stiffened double panel structures with acoustic cavities and with vacuum cavities. The acoustic excitations at a certain angle and perpendicular to the top panel are considered in this analysis. From this figure, some remarks can be obtained: the stiffened double panel structure excited by the acoustic excitation at a certain angle has a bigger number of the sound transmission loss valleys than those excited by acoustic excitation perpendicular to the top panel. The difference between acoustic transmission loss of the system with acoustic cavities and with vacuum cavities for acoustic excitation perpendicular to the top panel is more obvious than one for acoustic excitation at a certain angle. The reason may be that the acoustic excitation at a certain angle can excite more structure control modes, and the corresponding sound transmission loss is mainly determined by structures in this case. Acoustic cavities can cause a slight shift of some valleys of the sound transmission loss of structures, and the amplitudes of sound transmission loss with vacuum cavities are bigger than those with acoustic cavities, especially at the low frequencies. It is worth noting that the sound transmission loss valley with acoustic cavities is smaller obviously than one with vacuum cavities, as the system with acoustic cavities has an acoustic cavity control mode at $f = 85.6$ Hz.

4. Conclusions

The vibro-acoustic behavior analysis model of an elastically restrained stiffened double panel-cavity coupled system has been proposed in this paper. The improved Fourier series method is used to describe the displacement functions of all panels and sound pressure functions in acoustic cavities. The unknown coefficients of displacement and sound pressure function are solved by the Rayleigh–Ritz method based on the energy principle of the structure-acoustic coupled system. The effectiveness and accuracy of the proposed model are validated by some comparisons in numerical examples. This method can overcome the differential discontinuities for various boundary and/or coupled conditions and be flexible with parameter analysis. However, it has limitations in the vibration behavior analysis of other irregular structure acoustical cavity coupled systems. From this study, some conclusions are as follows:

1. The thickness, length, boundary condition, and material properties of the stiffened panel have an obvious influence on the natural vibration frequencies of the stiffened double panel-cavity coupled system.

2. The geometric parameters and material properties of the stiffened panel can affect the vibration velocity response of the bottom panel and the sound pressure response in a cavity. The influence of the gas medium on the amplitude of acoustic pressure response in acoustic cavities is obvious, but that on the vibration velocity response of panels can be ignored.

3. The structure control mode resonance frequencies of the system are more easily excited by the acoustic excitation at a certain angle, while the acoustic excitation perpendicular to the top panel can excite easily the acoustic cavity control mode resonance frequency of the system.

4. The stiffened panel can affect the movement of the sound transmission loss valleys at structure control mode resonance frequencies; the acoustic velocity of acoustic cavities can cause the sound transmission loss valleys at acoustic cavity control mode resonance frequency to shift; and the density of acoustic cavity can affect the amplitude of the sound transmission loss.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no financial and personal relationships with other people or organizations that can
inappropriately influence our work, and there is no professional or other personal interest of any nature or kind in any product, service, and/or company that could be construed as influencing the position presented in, or the review of, this paper.

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