On the scaling laws for low-temperature plasmas at macro and micro scales

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Abstract. The theoretical background and historical development of the similarity theory during the past decades are reviewed. We demonstrate similar discharges in local and nonlocal kinetic regimes, taking the low-pressure capacitive radio frequency (rf) discharges and microdischarges as examples. By using fully kinetic particle-in-cell simulations, we verify the similarity law (SL) and show a violation of frequency scaling (f-scaling) in the low-pressure capacitive rf plasmas. The similarity relations for electron density and electron power absorption are confirmed in similar rf discharges. With only the driving frequency varied, the f-scaling for electron density is also validated, showing almost the same trend as the similarity scaling, across most of the frequency regime. However, violations of the f-scaling are observed at lower frequencies, which are found to be relevant to the electron heating mode transition from stochastic to Ohmic heating. The scaling characteristics have also been comprehensively studied for microdischarges with dimensions from hundreds to several microns, with transition from secondary electron dominated regime to field emission regime. Finally, practical applications of the similarity and scaling laws are summarized.

1. Introduction

Similarity (or scaling) transformations in gas discharges help map out how discharge conditions change to retain the same characteristics, which are essential for discharge devices scaled to a variety of dimensions [1]. Discharge similarities can also be utilized to reduce the number of independent discharge parameters and group discharges at different scales, which provides a possibility of making confident parameter predictions of a prototype device from scaled ones [6]. Early work on gas discharge similarity was contributed by Townsend [7], Holm [8], von Engel [9], Margenau [10], and many others. The usage of combined parameters, such as the reduced gap length \( pd \) (gas pressure times gap distance) and reduced electric field \( E/p \) (electric field divided by gas pressure) are essential for simplify the characterization of discharge behaviors, e.g., utilizing Townsend coefficient with local-field approximation to describe the ionization process under direct current voltage [7]. In 1948, Margenau [10] established the similarity principle for high frequency discharges and proposed an additional combined parameter, reduced frequency \( f/p \) (driving frequency divided by gas pressure), which was later experimentally verified by Jones and Morgan [11]. Lisovskiy et al [12] validated the
breakdown scaling (e.g., Paschen’s law) for radio frequency (rf) discharges in both atomic and molecular gases, e.g., argon, nitrogen, and hydrogen.

The similarity laws are important tools for understanding discharge phenomena in novel or extreme regimes, which in more recent years are receiving growing interest in exploring the scalability of discharges at various scales. We hereby make a statement to distinguish the similarity and scaling laws. The former hold with multiple control parameters scaled simultaneously, e.g., maintaining the similarity invariants \( pd \) and \( f/p \) to achieve similar discharges. The later usually determine the discharge parameter dependence on only one of the controlled parameters, e.g., discharge characteristics scaling with gas pressure, gap dimension, and driving frequency, respectively.

Although the similarity laws have been demonstrated under many specific conditions, researches on the validity of similarity laws and their relations to fundamental discharge processes are still incomplete. Here, we take two cases, i.e., low-pressure rf discharge [13, 14] and high-pressure microdischarge [15, 16], as examples to show the applicability of similarity laws in the corresponding parameters regimes.

2. Similarity in low-pressure rf discharges

Figure 1 shows the similar rf discharge models. The rf plasmas are geometrically symmetric between two parallel-plate electrodes. Under the similar discharge conditions (see left in figure 1 (a)), the gas pressure \( p \), gap distance \( d \), and driving frequency \( f \) are simultaneously tuned by the scaling factor, i.e.,

\[
k = p_k / p_1 = d_k / d_1 = f_k / f_1.
\]

The steady-state electron density and scaled ionization degree from the particle-in-cell (PIC) simulations as the scaling factor \( k \) increases from 1 to 10 are shown in figure 1 (b). Theoretical electron densities are predicted from the base case using \( k^{-2} n_e(k) = n_e(1) \). The simulated electron densities for scaled gaps are in a good agreement with the theoretical predictions. The scaled ionization degree \( k^{-1} \chi \) (where \( \chi = n_e / N_n \) with \( N_n \) being the gas neutral number density) is constant, which also confirms the similarity relation for the ionization degree, i.e., \( k^{-1} \chi(k) = \chi(1) \).

The simulation results explicitly confirm the validity of low-pressure rf discharges in nonlocal kinetic regimes. More detailed results can be found in [13], in which we show the electron heating rates are also in proportional and can be scaled in similar rf plasmas. In [14], we further compare the SL scaling to the \( f \)-scaling and demonstrate the electron kinetic invariance in similar discharge systems. However, violations of the \( f \)-scaling are observed at lower frequencies, which are found to be relevant to the electron heating mode transition from stochastic to Ohmic heating [14]. Generally, with the similarity laws being valid, discharge properties in one gap can be extrapolated another among similar discharge systems.

Figure 1. (a) Schematic of rf discharges in scaled gaps and the discharge condition parameters \((p, d, f)\) are tuned through the scaling factor \(k\); for the base case \((p_1, d_1, f_1) = (5\text{ mTorr, } 10\text{ cm, } 13.56\text{ MHz});\) (b) electron densities from the PIC simulations are in good agreement with theoretical predictions; the scaled ionization degree is constant with \(k = 1 \sim 10\) [13].
3. Similarity in high-pressure microdischarges

The scaling characteristics are studied for microdischarges with dimensions from hundreds to several microns, with transition from secondary electron dominated regime to field emission regime [17]. Here, we show the similarity laws in Townsend breakdown when the microgaps have multiple concentric surface protrusions on the cathode.

The schematic slice of the microgap in the $r$-$z$ plane is shown in figure 2 (a). The microgap consists of two plane-parallel circular electrodes of radius $R$. A DC voltage $U_{dc}$ is applied to the anode through a ballast resistor $R_b = 100$ kΩ and the cathode is grounded. The concentric protrusions are introduced on the cathode surface with hemi-elliptical cross-section. The shape of protrusions and their configuration are defined by protrusion height $a$, radial dimension (width) $b$, and $X \geq 2b$ is the spacing from tip to tip between two neighboring protrusions. The cathode protrusions result in the minimum gap distance $d_{\text{min}} = d_{\text{max}} - a$ from the anode to the protrusion tip and the maximum gap distance $d_{\text{max}}$ from the anode to the cathode substrate.

We have four cases (A1, A2, B1, B2) to check the validity of the breakdown similarity in geometrically similar gaps, in which all linear dimensions are proportional via a scaling factor $k$. Cases A1 and B1 are the original microgaps with $k = 1$ and cases A2 and B2 are the corresponding scaled up microgaps with $k = 2$. The parameters are as follows: case A1: $d_{\text{max}} = 200$ μm, $R = 500$ μm, $a = 50$ μm, $b = 25$ μm, and $X = 50$ μm; case A2: $d_{\text{max}} = 400$ μm, $R = 1000$ μm, $a = 100$ μm, $b = 50$ μm, and $X = 100$ μm; case B1: $d_{\text{max}} = 200$ μm, $R = 500$ μm, $a = 100$ μm, $b = 50$ μm, and $X = 100$ μm; and case B2: $d_{\text{max}} = 400$ μm, $R = 1000$ μm, $a = 200$ μm, $b = 100$ μm, and $X = 200$ μm. It can be seen that although four breakdown curves versus the gas pressure are separated (see figure 2 (b)), the breakdown curves versus the scaled gas pressure ($k \times \text{pressure}$) are overlapping (see figure 2 (c)). Even though the gap distance ranges from $d_{\text{min}}$ to $d_{\text{max}}$ due to the surface protrusions, the breakdown curve $U_b = f(k \times \text{pressure})$ can be referred to a general Paschen’s law when the cathode is not flat. More detailed discussion can be referred from [15], and the effect of single surface protrusion, such as electric field distortion and polarity effect, on the breakdown can be further found in [18]. The breakdown scaling law still holds for microgaps with multiple surface protrusions despite the electric field distortion near the cathode, through which the applicability of the scaling laws can be extended.

4. Conclusion

We have briefly shown the recent studies on the similarity laws in discharges of macro and micro scales. Taking the low-pressure rf discharges for example, it is confirmed that the similarity laws also
hold for macroscale discharges in nonlocal kinetic regimes. The fundamental mechanisms which maintain the similarity relations are found to be the electron kinetic invariance in compared systems. Under the microdischarge conditions, although the nonlinear collision processes (e.g., three-body collision [20]) may violate the similarity scaling, the breakdown scaling still holds in microgap with multiple cathode surface protrusion, which extends Paschen’s laws conventionally for uniform field toward nonuniform conditions. Other effects, e.g., the effect of the nonlinear collisions, on discharges across the macro to micro scales should be more comprehensively investigated to further extend the application of similarity laws.

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