Concordance cosmology without dark energy

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ABSTRACT

According to the separate universe conjecture, spherically symmetric sub-regions in an isotropic universe behave like mini-universes with their own cosmological parameters. This is an excellent approximation in both Newtonian and general relativistic theories. We estimate local expansion rates for a large number of such regions, and use a scale parameter calculated from the volume-averaged increments of local scale parameters at each time step in an otherwise standard cosmological N-body simulation. The particle mass, corresponding to a coarse graining scale, is an adjustable parameter. This mean field approximation neglects tidal forces and boundary effects, but it is the first step towards a non-perturbative statistical estimation of the effect of non-linear evolution of structure on the expansion rate. Using our algorithm, a simulation with an initial $\Omega_m = 1$ Einstein–de Sitter setting closely tracks the expansion and structure growth history of the $\Lambda$CDM cosmology. Due to small but characteristic differences, our model can be distinguished from the $\Lambda$CDM model by future precision observations. Moreover, our model can resolve the emerging tension between local Hubble constant measurements and the Planck best-fitting cosmology. Further improvements to the simulation are necessary to investigate light propagation and confirm full consistency with cosmic microwave background observations.

Key words: cosmology: dark energy – cosmological parameters – methods: numerical.

1 INTRODUCTION

Gravitation being the only effective force on the largest scales, cosmological evolution is governed by general relativity (GR). To zeroth order, the homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) solutions of Einstein’s equations drive the expansion and growth history of the Universe. The concordance $\Lambda$CDM model (e.g., Planck Collaboration 2016) posits an unknown form of energy with negative pressure and an energy density about $10^{123}$ off from theoretical expectations. The $\Lambda$CDM paradigm reproduces most observations, although, to this day, no plausible candidate for dark energy has emerged and some tensions remain (see Buchert et al. 2016, for a recent comprehensive review). Most notably, the latest local measurements (Riess et al. 2016) of the Hubble constant are up to $3.4\sigma$ high compared to the value derived from Planck observations (Planck Collaboration 2016) of the cosmic microwave background.

The ubiquitous presence of clusters, filaments, and voids in the cosmic web manifestly violate the assumed homogeneity of $\Lambda$CDM. Given the non-linear nature of Einstein’s equations, it has been known for a while that local inhomogeneities influence the overall expansion rate, whereas the magnitude of such backreaction effect is debated. In particular, Green & Wald (2014, 2016) argued that the effect of inhomogeneities on the expansion of the Universe is irrelevant, while Buchert et al. (2015) disputed the general applicability of the former proof. More recently, Giblin, Mertens & Starkman (2016) used numerical relativity to show the existence of a departure from FLRW behaviour due to inhomogeneities, beyond what is expected from linear perturbation theory. Nevertheless, the spectacular successes of the homogeneous concordance model suggest that any effect of the inhomogeneities on the expansion rate should be weak, unless it mimics the $\Lambda$CDM expansion and growth history to a degree allowed by state of the art observations. In this spirit, we present a statistical non-perturbative algorithm, a simple modification to standard N-body simulations, that provides a viable alternative to dark energy while it can simultaneously resolve the Hubble constant puzzle.

In the late-time non-linear evolution of the Universe, coarse graining and averaging are both problematic (see Wiltshire 2007a, 2011, and references therein). The complex-
ity of Einstein’s equations prevents direct numerical modelling of backreaction, which ideally would require a general relativistic simulation of space-time seeded with small initial fluctuations. Such an ideal simulation would contain a hierarchy of coarse graining scales describing the space-time of stars, galaxies, galaxy clusters, intra-cluster medium and dark matter, etc., and their metric would be stitched together in a careful manner (see Wiltshire 2014, for a discussion of the hierarchy of coarse graining scales). As fluctuations grow due to non-linear gravitational amplification, space-time itself becomes complex, and even the concept of averaging becomes non-trivial.

Despite the difficulties, Buchert (2000, 2001) and others (Buchert & Rässänen 2012) realized that backreaction can be understood in a statistical fashion through the spatial averaging of Einstein’s equations on a hypersurface, leading to the Buchert equations. Second-order perturbative solutions to the equations are given by Kolb et al. (2005) and Rässänen (2010), while Wiltshire (2007a,b, 2009) presents an analytical solution to a two-scale (voids and walls) inhomogeneous universe.

We propose a non-perturbative, multi-scale statistical approach to study strong backreaction based on the general relativistic separate universe conjecture, which states that a spherically symmetric region in an isotropic universe behaves like a mini-universe with its own energy density \( \Omega = 1 + \delta \) (Weinberg 2008). The conjecture was proven by Dai, Pajer & Schmidt (2015) for compensated top hat over- and underdensities. In the quasi Newtonian framework, the separate universe conjecture is widely used in successful spherical collapse models (Bernardeau 1994; Mohayaee et al. 2006; Neyrinck 2016). We build on this conjecture to estimate the expansion rate as the volume average of local expansion rates, avoiding the calculation of any geometrical quantities such as the average curvature of the universe. Our algorithm neglects tidal forces, but the scales over which averages are calculated are solidly grounded in Newtonian physics, simplifying the interpretation of our results. We show that under our algorithm, the non-Gaussian distribution of matter arising from the non-linearities of the cosmological fluid equations causes the expansion rate to decrease at a slower rate than normally calculated from the global Friedmann equations, thereby mimicking the effect of dark energy.

In our scheme the coarse graining scale is an adjustable, phenomenological parameter corresponding to the best “particle size” to use when modelling the evolution of the Universe. When the coarse graining scale approaches the scale of homogeneity, our model, obviously, shows no effect: in this limit it is equivalent to the global Friedmann equations. Approaching very small scales, the assumptions of the model progressively break down due to the increasing anisotropy around, and inhomogeneity inside, the spherically symmetric regions. Somewhere between the extremes, there is an optimal scale that we expect to be around the size of virialized structures detached from the Hubble flow, therefore on the order of \( 10^9 - 10^{10} M_\odot \). The coarse graining scale is a semi-nuisance parameter to be fit, analogous to halo model parameters. While in this work we use a single, redshift-independent, comoving coarse graining scale, in principle, the optimal scale could depend on the state of the Universe and its constituents and thus, on redshift.

\[
\begin{bmatrix}
\Omega_{m,1} \\
\Omega_{m,2} \\
\vdots \\
\Omega_{m,N}
\end{bmatrix} \xrightarrow{\text{Friedmann eq.}} \Delta V \xrightarrow{} a(t + \delta t)
\]

\[
\begin{bmatrix}
\Omega_{m,1} \\
\Omega_{m,2} \\
\vdots \\
\Omega_{m,N}
\end{bmatrix} \xrightarrow{\text{Friedmann eq.}} \begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
\vdots \\
\Delta V_N
\end{bmatrix} \xrightarrow{} a(t + \delta t)
\]

Figure 1. Top: Standard cosmological N-body simulations evolve the Friedmann equations using the average density. Since the total mass is constant the scale factor increment is independent of density fluctuations. Bottom: We calculate the expansion rate of local mini-universes and average the volume increment spatially to get the global scale factor increment.

2 INHOMOGENEOUS MODEL BASED ON THE SEPARATE UNIVERSE CONJECTURE

Cosmological N-body simulations integrate Newtonian dynamics with a changing GR metric that is calculated from averaged quantities. There is a choice in how the averaging is done:

**Standard approach:** Traditional cosmological N-body simulations use the Friedmann equations with the average density (calculated as the total mass of particles divided by the volume) to determine the overall expansion rate at each time step. Implicit in this approach is the calculation of the average density, since the total mass of particles is constant, and so is the average comoving density.

**Average Expansion Rate Approximation (AvERA) approach:** Using the separate universe conjecture and neglecting anisotropies around spherically symmetric regions, we calculate the local expansion rate on a grid from the Friedmann equations using the local density, and then perform spatial averaging to calculate the overall expansion rate. The algorithm exchanges the order of averaging and calculating the expansion rate and, due to the non-linearity of the equations, the two operations do not commute, see Fig 1.

Our collisionless cosmological N-body simulation code (Rácz, G. et al. 2017, 2016) applies the Delaunay Tessellation Field Estimation (DTFE) method (Schaap & van de Weygaert 2000) to estimate the local density \( \rho_D \) from the discrete particles. The output of DTFE is the density field on a regular grid of small cubes with equal volume \( \mathcal{D} \). The code can compute the average scale factor increment of an inhomogeneous universe as described above, but it can also reproduce the standard \( \Lambda \)CDM simulation results obtained with Gadget2 (Springel 2005) when executed with the same initial conditions.

We estimate the expansion rate from the local average density

\[
\Omega_{m,D} = \frac{\rho_D}{\rho_{c,0}}
\]

using the matter-only Friedmann equations

\[
H_D = \frac{\dot{a}_D}{a_D} = H_0 \sqrt{\Omega_{m,D} a_D^{-3} + (1 - \Omega_{m,D}) a_D^{-2}}.
\]

Note that Eqs. 3 and 4 are identical to the Newtonian spherical collapse equations which provide a surprisingly accurate description of the full dynamics (Neyrinck 2016), and form
the basis of other successful approximations, such as halo models and the Press–Schecter formalism.

The volumetric expansion of mini-universes is the cube of the linear expansion, assuming statistical isotropy. Ignoring the boundary conditions and the local environment of touching Lagrangian regions, one can average the volume increment of the independent domains to get the total volume increment of the simulation cube, i.e. the global increment of a homogeneous, effective scale factor, cf. Eq. 2. This is equivalent to neglecting correlations between regions and non-sphericity caused by tidal forces, not unlike in the case of halo models. The statistical approach means that we can avoid stitching together regions of space-time. We use a global simulation time step size and, while the corresponding infinitesimal changes of local redshift may vary from region to region, the expansion rate is averaged in every simulation step, hence distances and velocities are rescaled homogeneously using the effective scale factor. As a result, similarly to standard N-body simulations, time is kept homogeneous and in one-to-one correspondence with redshift.

We ran simulations with up to 1.08 · 10⁶ particles of mass \( M = 1.19 \cdot 10^{13} M_\odot \) in a volume of 147.62³ Mpc³. The initial redshift was set to \( z = 9 \) for both the standard ΛCDM and the AvERA simulations. At this redshift, backreaction and the effect of \( \Lambda \) are both expected to be negligible. Since we focus on the expansion rate, Zel’dovich transients from the late start are insignificant. Initial conditions were calculated using LPTic (Crocce, Pueblas & Scoccimarro 2006) with a fluctuation amplitude of \( \sigma_8 = 0.8159 \) which is defined assuming the ΛCDM growth function. The initial expansion rate was set to match the current value of \( H_0 = 67.74 \, \text{km/s/Mpc} \) (Planck Collaboration 2016) for the ΛCDM model, yielding \( H_{\text{no99}} = 1191.9 \, \text{km/s/Mpc} \). Except for the value of \( \Lambda \), AvERA simulations were run with parameters derived from the latest Planck CMB observations.

As a consistency test, the initial conditions exactly reproduce the ΛCDM expansion history when inhomogeneities are not accounted for and \( \Lambda \) is non-zero. Similarly, with \( \Lambda = 0 \) and homogeneous expansion, the initial conditions reproduce the expansion history of a flat, matter only (\( \Omega_m = 1 \), \( \Omega_\Lambda = 0 \)) FLRW model with \( H_0 = 37.69 \, \text{km/s/Mpc} \). Fig. 2 summarizes the main results of our paper, where the expansion history \( a(t) \), the Hubble parameter \( H(t) \), the redshift \( z(t) \) and the average density \( \rho(t) \) are plotted for the AvERA model (blue), ΛCDM (red) and EdS (green) with the same initial conditions at \( z = 9 \). The evolution of the parameters from AvERA mimic ΛCDM remarkably well, while the EdS model deviates more and more at later epochs. We emphasize that, despite the overall similarity, there are small numerical differences between the former two models which can be tested in future high precision observations.

As it was mentioned before, in AvERA simulations the expansion history and the resulting present day Hubble parameter depend on the particle mass, which corresponds to the coarse graining scale. To explore the effects of the coarse graining scale, we executed simulations with different particle masses between \( 1.17 \cdot 10^{13} - 3 \cdot 10^{12} M_\odot \). The resulting \( z = 0 \) Hubble parameters, as a function of particle mass, are summarised in Tab. 1. The sensitivity of \( H_0 \) to the coarse graining scale is relatively minor: a factor of 10 change in the particle mass causes about a 10 per cent change in the present time Hubble parameter, see Fig. 4. The detailed investigation of this effect will be presented in a future paper.

3 COMPARISON WITH OBSERVATIONS

Given the close similarity of the expansion history of the AvERA model with that of ΛCDM, and the fact that linear growth history is driven by the time evolution of the expansion rate, the AvERA model provides an adequate framework for the interpretation of many observations supporting the concordance model, despite the fact that the current version of the simulation is not suitable yet to compute light propagation across the curved space-time regions. Luckily, luminosity distance at low redshift (but beyond the statistical scale of homogeneity) is primarily determined by the expansion history and is only slightly sensitive to curvature. In what follows, we do not attempt to fit any data, we simply plot our fiducial model with different coarse graining scales against select key observations.

One of the first and strongest observational proofs of accelerating cosmic expansion came from type Ia supernova distance modulus measurements (Riess et al. 1998; Perlmutter et al. 1999; Scolnic et al. 2015). Fig. 3 shows the distance moduli from the observations overplotted with curves from the EdS, Planck ΛCDM, and our model. We used the Supercal compilation (Scolnic et al. 2015; Scolnic & Kessler 2016) of supernova observations, with magnitudes corrected to the fiducial color and luminosity, and set the zero point of the absolute magnitude scale to match the Cepheid-distance-based absolute magnitudes as determined by Riess et al. (Riess et al. 2016). Both the Planck ΛCDM and our AvERA model follow the observed deviation from EdS. If we choose the coarse graining scale such that the local Hubble
µ
68
74
graining scale, treated as a nuisance parameter, is related to assumptions of the approximation. Since virialised objects de-
as has an ambiguity associated with the optimal coarse grain-
across curved regions, our approach is equivalent to a non-
connect space-time regions or compute light propagation
rate universe approximation. While we do not attempt to
cal expansion rates of mini-universes based on the sepa-
we estimated the global expansion rate by averaging lo-
4 CONCLUSIONS AND DISCUSSION
scales smaller than the statistical homogeneity scale which
we cannot account for the effect of inhomogeneities on
the Hubble parameter as a spatial average over the universe,
constant, respectively. At the same time, since we calculate
z the high and low
constraints, our model can naturally
Our modified Ω
m = 1 simulation mimics the ΛCDM ex-

Figure 3. The relative distance modulus \( \Delta \mu = DM - DM_{\text{Las}} \) as a function of redshift. The green line corresponds to the reference \( \Omega_m = 1 \) flat Universe, while the red curve shows the standard ΛCDM model with the concordance cosmological parameter set. The observed values from the SuperCal supernova compilation (Scolnic et al. 2015; Scolnic & Kessler 2016), calibrated using Cepheid distances by Riess et al. (Riess et al. 2016), are shown with gray errorbars, and their binned values with darker dots. The result of our second \( \Omega_m = 1 \) AvERA simulation with a particle mass of \( 2.03 \times 10^{11} M_\odot \) (see also Fig. 4 and Tab. 1) is shown in blue. It differs from the homogeneous EdS solution, and fits better the supernova data than the concordance ΛCDM model.

constant is reproduced (see Fig. 4), the AvERA model is favoured: \( \chi^2_{\text{Av}} = 1347.8 \) vs. \( \chi^2_{\text{LCDM}} = 2485.7 \), see Fig 3.

The tension between the locally measured (Riess et al. 2016) value of the Hubble constant \( H_0 = 73.24 \pm 1.74 \) \( \text{km/s/Mpc} \) and the estimate from Planck data (Planck Collaboration 2016) \( H_0 = 67.27 \pm 0.66 \) \( \text{km/s/Mpc} \) is worthy of special attention, given that the significance of the difference is over \( 3\sigma \) within the ΛCDM paradigm. According to Fig. 4 which displays the high and low \( z \) constraints, our model can naturally incorporate both. In particular, our two highest resolution simulations that are consistent with the Planck constraints yield the values of 71.38 \( \text{km/s/Mpc} \) and 73.14 \( \text{km/s/Mpc} \) for the Hubble constant, respectively. At the same time, since we calculate the Hubble parameter as a spatial average over the universe, we cannot account for the effect of inhomogeneities on \( H_0 \) on scales smaller than the statistical homogeneity scale which could explain the tension between local and CMB observations.

Table 1. Summary of simulation input parameters and the resulting values of \( H_0 \). In all cases the linear size of the simulation box was \( L = 147.62 \) Mpc and the early epoch value of the Hubble parameter was set to \( H(z = 9) = 1191.9 \) \( \text{km/s/Mpc} \), complying with Planck ΛCDM best-fit parameters.

| \( N \) | \( M \) [\( 10^{11} M_\odot \)] | \( H_0 \) [\( \text{km/s/Mpc} \)] |
|-------|------------------|------------------|
| 135,000 | 9.40 | 65.4 |
| 320,000 | 3.96 | 68.9 |
| 625,000 | 2.03 | 71.4 |
| 1,080,000 | 1.17 | 73.1 |

4 CONCLUSIONS AND DISCUSSION
We have presented a modified \( N \)-body simulation where we estimated the global expansion rate by averaging local expansion rates of mini-universes based on the separate universe approximation. While we do not attempt to connect space-time regions or compute light propagation across curved regions, our approach is equivalent to a non-perturbative statistical backreaction calculation. Our approximation neglects tidal forces due to anisotropies, and has an ambiguity associated with the optimal coarse graining scale. For a large enough scale, the effect is negligible, while on small scales anisotropies break the underlying assumptions of the approximation. Since virialised objects detach from the expansion, we expect that the optimal coarse graining scale, treated as a nuisance parameter, is related to the size of the typical virialised regions.

Figure 4. Left: The relation between the Hubble parameter values for the local (vertical axis) and the distant (horizontal axis) Universe. The horizontal stripe corresponds to the \( 1\sigma \) range al-

\[ H(z=9) = 1191.9 \text{ km/s/Mpc} \]

Left: The relation between the Hubble parameter values for the local (vertical axis) and the distant (horizontal axis) Universe. The horizontal stripe corresponds to the \( 1\sigma \) range al-

\[ H(z=9) = 1191.9 \text{ km/s/Mpc} \]

Right: The effect of coarse graining on \( H_0 \) at a given initial Hubble parameter \( H(z = 9) \).
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using elliptical collapse equations for the mini-universes we consider. This refinement of our calculations would quantify to first order the effect from tidal forces, but is left for future work. Our simple model with a reasonable coarse graining scale yields results that are consistent with the observations, and the model also has a simple physical interpretation. Further studies are needed both on the theoretical front and on fitting cosmological parameters, nevertheless, our approach is not only a viable alternative to dark energy models, but appears to be flexible enough to resolve some tensions in a natural way.

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REFERENCES

Barkana R. and Loeb A., 2001, Nature, 412, 699
Buchert T., 2000, Gen. Relativity and Gravitation, 32, 105
Buchert T., 2001, General Relativity and Gravitation, 33, 1381
Buchert T. et al., 2015, Classical and Quantum Gravity, 32, 215021
Buchert T., Coley A. A., Kleiner H., Routkema B. F., Wiltshire D. L., 2016, International Journal of Modern Physics D, 25, 1630007
Buchert T., Riisäinen S., 2012, Annual Review of Nuclear and Particle Science, 62, 57
Crocce M., Puelas S., Scoccimarro R., 2006, MNRAS, 373, 369
Dai L., Pajer E., Schmidt F., 2015, J. Cosmology Astropart. Phys., 10, 059
Giblin J. T., Mertens J. B., Starkman G. D., 2016, Phys. Rev. Lett., 116, 251301
Green S. R., Wald R. M., 2014, Classical and Quantum Gravity, 31, 234003
Green S. R., Wald R. M., 2016, Classical and Quantum Gravity, 33, 125027
Kolb E. W., Matarrese S., Notari A., Riotto A., 2005, Phys. Rev. D, 71, 023524
Mohayaee R., Mathis H., Colombi S., Silk J., 2006, MNRAS, 365, 939
Neyrinck M. C., 2016, MNRAS, 455, L11
Perlmutter S. et al., 1999, ApJ, 517, 565
Planck Collaboration, 2016, A&A, 594, A13
Rácz, G. et al., 2016, source code available at http://www.vo.elte.hu/papers/2016/nbody/
Rácz, G. et al., 2017, in prep.
Riess A. G. et al., 2016, ApJ, 826, 56
Schaap W. E., van de Weygaert R., 2000, A&A, 363, L29
Scornic D. et al., 2015, ApJ, 815, 117
Scornic D., Kessler R., 2016, ApJ, 822, L35
Springel V., 2005, MNRAS, 364, 1105
Weinberg S., 2008, Cosmology. Oxford University Press, Oxford, UK

Figure 5. Left: Evolution of the distribution of $1 + \delta$ during the AvERA N-body simulation. Right: The normalized line of sight comoving distance $D_L^*(z) = \int_0^z \frac{H_0}{H(z)} dz$ at each time step has been calculated from a matter-only FLRW model with $\Omega_m$ corresponding to the actual peak $1 + \delta$ (blue). The curve deviates from the $\Omega_m = 1$ model (green) and closely follows the $\Lambda$CDM model (red).
Wiltshire D. L., 2007a, New Journal of Physics, 9, 377
Wiltshire D. L., 2007b, Physical Review Letters, 99, 251101
Wiltshire D. L., 2009, Phys. Rev. D, 80, 123512
Wiltshire D. L., 2011, Classical and Quantum Gravity, 28, 164006
Wiltshire D. L., 2014, in Proceedings of the 15th Brazilian School on Cosmology and Gravitation, Perez Bergliaffa S.E.; Novello M., ed., Cambridge Scientific Publishers, Cambridge, UK, pp. 203–244