Estimation of Machine Parameters in Superconducting Transformer using Differential Evolution

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Abstract. To analyze the inrush current in a superconducting transformer, the machine parameters for the transformer were estimated from the measured current using a search algorithm. To address the large rising edge error in estimations performed using a genetic algorithm (GA), a differential evolution (DE) was used in this study. As a result, the estimated time was reduced to about 1/10 that obtained with GA, and the evaluation value indicating the difference between the measured value and the estimated value was reduced to about 1/2. Thus, it was possible to estimate with higher accuracy by using DE.

1. Introduction

When a transformer used in a power system is turned on, an inrush current is generated, which affects the power quality of other power systems. These effects can include unnecessary operation of protection relays due to overcurrents, malfunctions of control devices due to momentary voltage dips and flickering of lighting. In order to understand and take measures against these effects, it is necessary to analyze the behavior when the inrush current occurs under all conditions [1].

Future power systems may make use of superconducting transformers [2, 3]. When superconducting wires are used in transformer windings, the losses in the windings are very small and the energy efficiency is high. Reducing the energy loss also increases the electrical load, and reducing the core cross-sectional area reduces the size and weight. However, when superconducting transformers are introduced into power systems, the basic principles will be the same as for normal transformers, so it is necessary to analyze inrush currents.

It may be possible to measure the inrush current of a transformer that is affecting the power quality of a power system by installing an instrumental current transformer on the high voltage side of the transformer. The measurements can be performed in a non-contact manner by using a clamp in the output circuit. In this case, the transformer machine parameters and input condition values can be estimated from the inrush current waveform. In previous studies, an estimation method of the machine parameters that combines the Ralston’s optimal Runge-Kutta method [4] with a genetic algorithm (GA) [5] has been developed to enable analysis even when operating a superconducting transformer in a power system [6].

In this paper, we investigate an alternative method to estimate the machine parameters, combining the optimal Runge-Kutta method and differential evolution (DE), which is a kind of evolutionary algorithm. We assess the method by comparing GA and DE results.
2. Calculation Methods

The system for this analysis method is shown in Figure 1 as a single-line diagram, taken from Ref. [6]. The host system consists of a power source and a power transmission line. Assuming that the transformer to be analyzed is introduced into the system from the high-voltage side, the switch connected to the low-voltage side is always open. In an actual experiment, capacitors, in-house transformers, lightning arresters and other devices are always connected to the transformer’s low-voltage bus. However, it was fixed to the above model for easy analysis. The current obtained in the analysis is the instantaneous value of the ammeter in the illustrated model. The circuit in Figure 1 represents the equivalent circuit for a single-phase superconducting transformer. Since the target transformer does not quench by the inrush current, the winding resistance was set to 0 Ω. In addition, since the resistance component of the iron core is negligibly large, the loss component was ignored and only the inductance component, which reflects the magnetic saturation characteristics, was considered. The system and machine parameters necessary for the calculation in Figure 1 are as follows. \( R_B \) and \( L_B \) are the upper system impedance, \( L_{C1} = L_c / 2 \) is the high-voltage side leakage inductance, \( L_c \) is the transformer leakage inductance and \( L_L \) is the excitation inductance reflecting the excitation saturation characteristics.

![Figure 1. Equivalent circuit for single-phase superconducting transformer.](image)

In this study, simplified hysteresis of the iron core is used, as shown in Figure 2. \( L_L \) in the equivalent circuit of Figure 1 takes the exciting inductance \( L_m \) when the transformer is not saturated, and takes the self-inductance of only the winding when it is saturated, that is, the air-core inductance \( L_{air} \), shown in Figure 2. The optimal Runge-Kutta method uses simple magnetic saturation characteristics in which the excitation inductance \( L_L \) changes stepwise with respect to changes in the magnetic flux \( \phi \) of each phase. The values of \( L_m \) and \( L_{air} \) take a constant value for non-saturation and saturation, respectively. From the above, the circuit equations can be expressed by equations (1) to (3).

\[
\begin{align*}
L_k \frac{di_m}{dt} + R_A i_m &= V(t) \\
V(t) &= V_m \sin \left( \omega t + \frac{\pi}{180} \theta_0 \right) \\
\frac{di_m}{dt} &= \left( V(t) - R_A i_m \right) / L_k
\end{align*}
\]

Where, in the case of Figure 1, the total resistance is \( R_A = R_B \) and the total inductance is \( L_K = L_B + L_{C1} + L_L \). \( i_m \) is the inrush current, \( L_L \) is the excitation inductance representing the magnetic saturation characteristic, \( V_m \) is the peak value of the power supply voltage and \( \theta_0 \) is the voltage input phase. In addition, \( \omega = 2\pi f_e \), and \( f_e \) is the frequency of the power supply voltage.

The magnetic flux \( \phi \) can be obtained from equation (4) by modifying equation (1), and \( L_L \) can be obtained from equations (5) and (6).

\[
\phi = \int V(t) dt - R_A \int i_m dt
\]

\[
L_L = \begin{cases} 
L_m \left( |\phi| \leq \phi_{\text{max}} \right) \\
L_{air} \left( \phi_{\text{max}} > |\phi| \right)
\end{cases}
\]

\[
\phi_{\text{max}} = \phi_n \left( \frac{B_s}{B_n} \right)
\]

Where, \( \phi_{\text{max}} \) is the saturation magnetic flux, \( B_n \) is the rated magnetic density of the transformer, \( B_s \) is the saturation magnetic flux density and \( \phi_n \) is the rated magnetic flux. From the above, \( i_m \) can be obtained.
A numerical method of ordinary differential equations is used to calculate the inrush current in this study. It is a one-step method that gives an explicit solution, and is the optimal Runge-Kutta method improved by Ralston to give an error smaller than that of the Runge-Kutta method [4]. This calculation method can cope with a step-like change in the \( \phi-L \) characteristic because the coefficient changes with respect to the time step \( \Delta t \). When the current \( i \) is a function \( i(t) \) of time \( t \) and the time derivative \( \frac{di}{dt} \) of \( i \) is a function \( f(t, i) \), the optimal Runge-Kutta method is formulated by equation (7) in Ref. [6].

Measurements of the inrush current waveform were performed with a time interval of \( 1.0 \times 10^{-4} \) s up to \( t = 0.035 \) s. Therefore, the time increment of the optimal Runge-Kutta method is \( \Delta t = 1.0 \times 10^{-4} \) s, and for each time increment the difference between the measured value and the calculated value is calculated. The evaluation value, \( d \), is calculated from the average of each error obtained by dividing the sum by the number of samples, and is given by

\[
d = \frac{1}{N} \sum \left|i_{\text{MEA}} - i_{\text{SIM}}\right|
\]

Where, \( i_{\text{MEA}} \) represents the measured value, \( i_{\text{SIM}} \) represents the calculated value and \( N \) represents the number of samples.

The machine parameters estimated are \( L_c \), \( L_m \), \( L_{\text{air}} \), \( B_n \), initial phase \( \theta_0 \) and the residual magnetic flux \( \phi_r \). Substituting these 6 parameters into the above equations allows us to calculate \( i \). Table 1 shows the search range for the present machine parameters.

The differential evolution (DE) is a kind of evolutionary algorithm that performs a multipoint search using a probabilistic solution group [7, 8]. Since it is suitable for a wide range of multidimensional problems, it was used to search for solutions of the 6-dimensional problems in this study. The control parameters include \( N \), the number of generations, the scaling factor \( F \) and the crossover rate \( CR \). The values of those parameters affect the search for solutions.

### Table 1. Search range of machine parameters.

| Machine parameters             | Search range            |
|-------------------------------|-------------------------|
| Leakage inductance [mH]       | \( 51.3 < L_c < 73.8 \) |
| Excitation inductance [mH]    | \( L_c < L_m \)         |
| Air core inductance [mH]      | \( 0 < L_{\text{air}} < 10L_c \) |
| Rated magnetic flux density [T]| \( 1.55 < B_n < 1.7 \)  |
| Initial phase [degree]        | \( 0 \leq \theta_0 < 360 \) |
| Residual magnetic flux [%]    | \( -85 \leq \phi_r \leq 85 \) |
3. Results and Discussion
The machine parameters were estimated by DE with $CR = 0.9$, $F = 0.8$, 20 individuals and 2000 generations. Table 2 shows the machine parameters estimated by GA and DE. Differences were observed in $L_{\text{air}}$, $\phi_r$ and $L_m$. Table 3 shows the evaluation value $d$ and processing time for GA and DE. Compared with GA, the $d$ of DE was reduced to 1/2 and the calculation time was reduced to 1/10.

Figure 3 shows the waveform of the inrush current for the measured and estimated values by GA and DE. In the vicinity of 0 A, where the difference between the calculated value by GA and the measured value was large, the calculated value by DE was close to the measured value, and the estimation accuracy was improved.

Figure 4 shows $d$ as a function of generation for DE and GA. The $d$ of DE is initially larger than that of GA but becomes smaller at about 300 generations, becoming closer to the optimal value. On the other hand, while the $d$ of GA gradually decreases, it decreases slower than for DE. This is because the search range becomes narrower as the number of generations increases, and the optimum value can be searched more finely in DE.

Figure 5 shows the difference in $d$ due to changes in $F$ and $CR$. $d$ decreases as $CR$ increases because the closer $CR$ is to 1.0, the more crossover occurs and the easier it is to find a solution. $d$ decreases as $F$ increases, and $d$ is larger at $F = 1.0$ than at $F = 0.6$ and $F = 0.8$. This is because a larger $F$ can search a wider range. On the other hand, a detailed search is not possible around the solution.

Table 2. Machine parameters estimated by GA and DE.

|       | $L_{\text{air}}$ [mH] | $\phi_r$ [%] | $L_c$ [mH] | $\theta_0$ [degree] | $L_m$ [mH] | $B_c$ [T] |
|-------|------------------------|--------------|------------|--------------------|------------|-----------|
| GA    | 46.0                   | -28.6        | 70.0       | 195                | $6.40 \times 10^4$ | 1.67      |
| DE    | 146                    | -55.9        | 66.4       | 195                | $3.30 \times 10^4$ | 1.69      |

Table 3. Comparison of GA and DE evaluation value and processing time.

| Evaluation value $d$ [A] | Time [s] |
|--------------------------|----------|
| GA                       | 19.5     | 5.54     |
| DE                       | 9.57     | 0.515    |

Figure 3. Waveform of inrush current for measured and estimated values by GA and DE.
4. Summary
The machine parameters for a superconducting transformer were estimated using DE. The estimation performance is superior than the conventional method using GA. Specifically, the calculation time for the estimation is shorter, and \( d \) indicating the difference between the calculated value and the measured value is smaller. The difference in \( d \) can also be seen from the current waveform.

The estimation performance is improved because of the difference in the search method between DE and GA. Since GA searches for a solution randomly within a certain search range, an approximate solution can be found early. However, even if the generation advances, it may not be possible to find the optimal solution. On the other hand, since the search range of DE changes, the area around the optimal solution can be searched in detail as the generation advances.

In order to use DE properly, it is important to set \( CR \) and \( F \). In this study, we found that the larger the \( CR \), the smaller the \( d \). It seems that the possibility of a new solution can be found by performing many crossovers. The value of \( d \) was smallest when \( F = 0.6 \). If \( F \) is small, the search range at the beginning of the search becomes narrow and it is difficult to search for the optimum value. On the other hand, if \( F \) is large, a detailed search cannot be performed when the generation advances.

In the future, to further improve the estimation accuracy, we plan to perform calculations with models that are closer to reality and to review the evaluation function.

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