Quantized Microwave Faraday Rotation

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The interaction of light with matter in the presence of a static magnetic field gives rise to a wide variety of phenomena, including the Faraday effect whereby the polarization of light is rotated by magnetically induced circular birefringence. We report quantitative microwave Faraday rotation measurements conducted with a high-mobility two-dimensional electron gas in a GaAs/AlGaAs semiconductor heterostructure. Cyclotron motion of charge carriers in a two-dimensional electron gas leads to a large Faraday effect in the microwave frequency range. The magnitude of Faraday rotation is suppressed at higher frequencies by the kinetic inductance that arises from electron inertia. As with the Hall effect, a continuous classical Faraday effect is observed as well as a quantized Faraday effect. The high electron mobility enables a large single-pass Faraday rotation of \( \theta_F \approx 45^\circ \) (±0.8 rad) to be achieved at a modest magnetic field of \( B \approx 100 \) mT. In the quantum regime, the Faraday rotation \( \theta_F \) is naturally quantized in units of the fine structure constant \( \alpha \approx 1/137 \), giving a geometric prescription for determining \( \alpha \). Electromagnetic confinement leads to Faraday rotation that is quantized in units of an effective \( \alpha^* \) whose value is on the order of its free space value. Engineering of the electromagnetic field distribution and impedance, including resonant cavity effects, could hence be used to purposefully enhance or suppress Faraday rotation.

**Keywords:** Faraday effect | quantum Hall effect | fine structure constant

INTRODUCTION

Faraday rotation is the phenomenon whereby the polarization state of linearly polarized light is rotated by matter under the influence of a magnetic field applied along the direction of propagation [1]. Faraday rotation manifests itself in a wide range of physical settings, from the passage of radio frequency waves through interstellar gas [2] to X-ray transmission through iron films [3]. Beyond electromagnetic waves alone, the acoustic analog of Faraday rotation has been used as a probe of the superfluid properties of \(^3\)He-B [4], wherein spin-orbit coupling couples acoustic response with magnetic field. More recently, Faraday rotation has been used in the THz domain as a probe of the topological properties of low-dimensional electron systems [5–9]. Here, we report on quantitative microwave measurements of Faraday rotation in a high-mobility two-dimensional electron gas (2DEG). The rotation angle is shown experimentally to be quantized at multiple filling factors of the integer quantum Hall effect in units of an effective fine structure constant \( \alpha^* \). This work is paving the way for contactless measurements of other condensed matter systems where the bulk transverse conductivity is quantized.

Both the classical Hall effect and the Faraday effect arise from the action of Lorentz force upon charge in the presence of an applied magnetic field \( B \). As depicted in Fig.1(A), the classical Hall effect is the net accumulation of charge and the generation of an electric field \( E_H \) transverse to the direction of current flow \( I \) and magnetic field \( B \). The Hall effect is usually quantified by the transverse Hall resistivity \( \rho_{xy} = V_H/I = B/ne \) (see Fig.1(C)), where \( n \) is the electron sheet density and \( e \) the electric charge. The classical Hall effect can also be described with a Hall angle \( \theta_H = \rho_{xy}/\rho_{xx} \), where \( \rho_{xx} \) is the longitudinal resistivity of the conducting sheet. Similarly, the Lorentz force can also lead to the classical Faraday effect depicted in Fig.1(B), whereby the direction of a linearly polarized electromagnetic wave is rotated as it passes through a medium of thickness \( d \) under the influence of a magnetic field \( B \) parallel to the optical axis. The Faraday rotation is the angle of linear polarization rotation and is linearly dependent upon the magnetic field, \( \theta_F = V d B \) (see Fig.1(D)), where \( V \) is the Verdet constant for Faraday rotation. Other phenomena can also lead to Faraday rotation, such as Larmor precession of electron spins, as in the case of ferrite components used in microwave isolators and circulators.

In the case of a two-dimensional electron gas, the Faraday effect and Hall effect have a common origin with the cyclotron motion of charge carriers. Faraday rotation induced by a 2DEG under a static magnetic field can be readily understood by Fresnel analysis for transmission of right- and left-handed circularly polarized mi-
crowaves. The complex conductivity of a 2DEG under right- and left-hand circularly polarized excitation is \( \sigma_{\pm} = \sigma_{xx} \pm i\sigma_{yx} \). The Fresnel transmission coefficients are,

\[
t_{\pm} = \frac{2}{2 + Z_0\sigma_{xx} \pm iZ_0\sigma_{yx}}
\]
leading to a phase delay between right- and left-handed waves. Here, \( \sigma_{xx} \) and \( \sigma_{yx} \) are the longitudinal and transverse conductivities of the 2DEG at the frequency of the incident wave with the conductivity tensor the inverse of the resistivity tensor \( \sigma = \rho^{-1} \), and \( Z_0 \) is the impedance of free space. The rotation of a linearly polarized wave is thus \( \theta_F = \frac{1}{2} \arg (t_-/t_+) \), leading to the Faraday rotation of a sheet conductor given by

\[
\tan(\theta_F) = \frac{Z_0\sigma_{yx}}{2 + Z_0\sigma_{xx}}.
\]

In the classical Drude limit for a highly conductive 2DEG with \( Z_0\sigma_{xx} \gg 1 \), the Faraday rotation is small and well approximated by the Hall angle \( \theta_H \approx \theta_F = \mu B \), where \( \mu \) is the charge carrier mobility of the 2DEG.

A strong magnetic field can give rise to the quantum Hall effect (QHE), where the Hall resistance is quantized in units of \( h/e^2 \), the resistance quantum \( 10 \). In the high magnetic field limit of the integer \( 10 \) (or fractional \( 11 \)) quantum Hall regime, the conductivity \( \sigma_{xx} = 0 \) and \( \sigma_{yx} = ie^2/\hbar \) where \( i \) is the integer filling factor (\( \nu \) in the fractional regime). The Faraday rotation angle itself becomes quantized,

\[
\tan(\theta_F) = \frac{ie^2}{2\hbar} = i\alpha.
\]

Here, the fine structure constant \( \alpha = Z_0e^2/2\hbar \) sets the natural angular scale for Faraday rotation \( 12 \). Electromagnetic confinement with dielectric and/or conductor structures can be used to enhance the interaction between electromagnetic wave and 2DEG \( 16 \), thereby making it possible to enhance Faraday rotation above that of free space.

Interestingly, Faraday rotation is a bulk probe of the quantum Hall state. In the QHE at integer filling factors \( i \), charge transport experiments probe edge currents, but it is important to recall that the bulk transverse conductivity \( \sigma_{xy} \) is quantized in the quantum Hall regime \( 13 \), and Faraday rotation explicitly probes the conductivity quantization of the bulk. Understanding the microwave Faraday rotation of the integer quantum Hall regime is an important step towards understanding Faraday rotation in the fractional quantum Hall (FQH) regime \( 11 \) hosted in ultra-high-mobility 2DEGs. The FQH states of a 2DEG are governed by incompressible Laughlin-like liquids, and perhaps host even more exotic quantum states such as the Moore-Read Pfaffian \( 19 \), for example.

FIG. 1. Classical Hall/Faraday effects and experimental setup. A schematic representation of the classical Hall (A) and Faraday effects (B) is shown together with the magnetic field dependence of the Hall angle \( \theta_H \) (C), and Faraday rotation angle \( \theta_F \) (D). Here, \( \mu \) denotes the charge carrier mobility, \( V \) is the Verdet constant and \( d \) the sample thickness. The incident and transmitted electric field amplitudes are denoted by \( E_{\text{in}} \) and \( E_{\text{tran}} \), respectively. (E) Experimental setup to measure the Faraday rotation for microwaves. A linearly-polarized electromagnetic wave is injected into a circular hollow waveguide (Port 1) that supports two orthogonally-polarized TE\(_{11}\) modes. The output power is measured using an orthomode transducer in a direction parallel (Port 3) and perpendicular (Port 4) to the incoming electromagnetic wave.

EXPERIMENTAL SETUP

The experimental setup is illustrated in Fig.1(E), consisting of a circular hollow waveguide assembly designed for polarization sensitive microwave scattering measurements at cryogenic temperatures with a magnetic field oriented along the waveguide axis. The silver-plated hollow waveguide of diameter 23.825 mm supports two orthogonally-polarized TE\(_{11}\) modes. A 30 mm thick high-mobility 2DEG hosted in an AlGaAs/GaAs heterostructure grown by molecular beam epitaxy on a \( d=0.55 \) mm thick GaAs substrate with square dimensions 10 mm \( \times \) 10 mm was inserted within the waveguide using a copper plate with a 9 mm diameter aperture. The mobility of the 2DEG was determined to be \( \mu \approx 1 \times 10^6 \) cm\(^2\)V\(^{-1}\)s\(^{-1}\) by way of quasi-DC
transport measurements at $T \simeq 20$ mK on a piece cut from the same wafer (during a separate cool down). The electronic density $n$ of the 2DEG was determined from the Landau level sequence observed in the Faraday rotation (see below), and found to be $1.92(2) \times 10^{11}$ cm$^{-2}$. A coaxial-to-circular waveguide adapter (port 1) was used to excite the 2DEG with a linearly polarized TE$_{11}$ mode. The perpendicular (port 4) and parallel (port 3) polarized TE$_{11}$ mode fields were collected with an orthomode transducer, which consists of orthogonally polarized electric dipole couplers to coaxial transmission lines. The entire assembly was thermally anchored to the cold plate of a dilution refrigerator with a base temperature of $\sim 7$ mK. All temperatures quoted in this work correspond to the temperature of the mixing chamber of the dilution refrigerator. While the incident microwave illumination and/or imperfect thermalization will raise the temperature of the 2DEG electronic bath above that of the mixing chamber, our temperature dependence study of the Faraday rotation angle suggests the electrons are cooled down to at least $\sim 200$ mK. Finally, a $\pm 6$ T magnetic field was applied along the waveguide axis using a superconducting solenoid with the positive (+) direction aligned with the direction of propagation of the incident microwave.

The incident microwaves at 11.2 GHz were generated by a vector network analyzer (VNA) that was also used to measure the transmitted microwaves, thus enabling measurement of the scattering parameters (see Fig. 2). Low-loss, high-frequency coaxial assemblies were used to couple the VNA to the hollow waveguide assembly in the dilution refrigerator. A low-temperature switch was used to transmit the microwaves from ports 3 and 4 of the hollow waveguide to the VNA using the same coaxial assembly, thereby limiting differences in transmission to the hollow waveguide apparatus. A cryogenic preamplifier was also used at the $\sim 4$ K stage of the dilution refrigerator together with filters and attenuators to minimize microwave induced Joule heating of the 2DEG and suppress spurious reflections within the coaxial assembly.

**EXPERIMENTAL RESULTS**

The measured scattering parameter amplitudes $|S_{41}|$ and $|S_{31}|$ are shown in Fig. 2(A) and (B) for perpendicular and parallel polarized transmission, respectively, versus applied magnetic field $B$. The difference in the scattering parameter amplitudes of $\sim 0.1$ dB for positive and negative magnetic fields arises from a slight misalignment in excitation and detection ports. This corresponds to a systematic error of approximately $\sim 1\%$ in the field amplitude. The perpendicular polarization transmission amplitude $|S_{41}(B)|$ plotted versus $B$ in Fig. 2(A) reveals a staircase pattern hinting at a quantization of perpendicular polarization transmission related to Landau level formation in the 2DEG. The magnetic field dependent Faraday rotation, $\theta_F(B)$, is determined from the scattering parameter amplitudes via $\tan(\theta_F(B)) = |S_{41}(B)/S_{31}(B)|$. The Faraday rotation $\theta_F(B)$ is shown in Fig. 2(C) and a maximum Faraday rotation $\theta_F^{max} \simeq 45^\circ$ ($\simeq 0.8$ rad) is observed at a modest applied magnetic field of $B \simeq 100$ mT. This peak in $\theta_F$ demarks the low magnetic field regime where $\theta_F$ increases with $B$ and the high-field regime where $\theta_F$ decreases with increasing $B$. The maximum Faraday rotation observed is exceptionally large, and a colossal Verdet constant $V \sim 2.7 \times 10^8$ rad T$^{-1}$ m$^{-1}$ is inferred, considering the effective thickness $d$ of the active medium is the 30 nm thickness of the 2DEG.

The high-electron mobility of the 2DEG leads to large Faraday rotation at a modest magnetic field because the maximum rotation occurs at a magnetic field scale in the vicinity of the onset of Landau level formation. Well defined Landau levels are formed when charge carriers in the 2DEG can complete cyclotron orbits with a low probability of scattering, corresponding to $\omega_c \tau = \mu B \simeq 1$. We observe a saturation in Faraday rotation, and the primary effect of increasing electron
mobility of the 2DEG is to reduce the magnetic field at which Faraday rotation saturates.

Examining the Faraday rotation in the low field regime \( B \lesssim 50 \text{ mT} \) further, the 2DEG is well approximated with a Drude conductivity that includes kinetic inductance corrections to the Faraday rotation of Eq. [2]. The inductive response is proportional to \( \omega \tau \) of \( \frac{\gamma}{2} \) the 2DEG at finite frequency \( \omega \), with \( \tau = m^* \mu_e / e \simeq 38 \text{ ps} \) being the charge carrier scattering time, \( m^* = 0.067 m_e \) is the effective mass in GaAs, and \( \omega \tau \simeq 2.7 \) for our experiments. The normalized, zero-field conductivity \( Z_0 \sigma_{\perp}(B = 0) \simeq 12 \gg 1 \) so that in this high-conductivity limit the Faraday rotation can be found by Fresnel analysis to be \( \theta_F \approx \omega_e \tau (1 + \omega^2 \tau^2) \). Kinetic inductance, which accounts for the inertia of charge carriers, can thus suppress Faraday rotation if \( \omega \tau \gg 1 \) in the Drude limit.

We now turn our attention to the high magnetic field regime \( B \gtrsim 100 \text{ mT} \). At sufficiently large magnetic field, the normalized Hall conductivity \( Z_0 \sigma_{\parallel} \propto \frac{1}{B} \) of the 2DEG is much larger than the normalized longitudinal conductivity \( Z_0 \sigma_{\parallel x} \), and in this case \( \tan(\theta_F) \sim Z_0 \sigma_{\parallel x} / 2 \gg \frac{1}{B} \). To exemplify this behaviour, the Faraday rotation angle \( \theta_F \) is plotted in Fig. 3 as a function of \( 1/B \) (solid red line). Six plateaus are clearly observed in \( \theta_F \) versus \( 1/B \) with the lowest three plateaus evenly spaced along both axes, and a further three more evenly space plateaus are observed with twice the step-height. These plateaus originate with the occurrence of Landau levels leading to quantized Hall conductivity and thus a quantization of the Faraday effect (Eq. [3]) at integer filling factor \( i = 2, 3, 4, 6, 8, \) and 10, as shown on the upper \( x \)-axis of Fig. 3. From the observed sequence, an electron density \( n = 1.92(2) \times 10^{11} \text{ cm}^{-2} \) is extracted using the Landau level filling factor relation, \( i = nh/eB \), and is consistent with quasi-DC transport studies performed on samples of the same wafer. Here, the expected spin degeneracy lifting of the Landau levels occurs in between integer filling \( i = 4 \) and 6, at a magnetic field value \( B \sim 1.8 \text{ T} \) consistent with previous quasi-DC charge transport studies of 2DEGs hosted in similar heterostructures with comparable electron mobility and density.

A hallmark of the quantum Hall effect is a vanishing longitudinal conductivity, \( \sigma_{\parallel x} = 0 \), concomitant with the quantization of the transverse Hall conductivity \( \sigma_{\parallel y} = ie^2/h \). Quantization of the Faraday rotation can be understood by considering the relation between 2DEG microwave current density \( \vec{J}(\omega) \) and microwave electric field \( \vec{E}(\omega) \),

\[
\vec{J}(\omega) = \sigma \vec{E}(\omega) = \begin{pmatrix} 0 & -ie^2/h \\ +ie^2/h & 0 \end{pmatrix} \vec{E}(\omega), \]  

(4)

here shown at integer \( i \) filling. In the quantum Hall regime, the 2DEG current density \( \vec{J}(\omega) \) is orthogonal to the electric field \( \vec{E}(\omega) \). The current density \( \vec{J}(\omega) \) results in scattered microwave fields, in transmission and reflection, that include quantized field contributions orthogonal to the incident field \( \vec{E}(\omega) \). As a result, the Faraday rotation \( \theta_F \) is itself quantized by the quantized Hall conductivity. The quantization of rotation disappears in the absence of well-formed quantum Hall states, as the Hall conductivity is not quantized and the longitudinal conductivity \( \sigma_{\parallel x} \gg 0 \).

The Faraday rotation was also measured during a separate cool down in a slightly different experimental configuration employing two coaxial assemblies instead of a single assembly with a cryogenic switch. These measurements are shown in the inset of Fig. 3 with the temperature of the dilution refrigerator at \( \sim 10 \text{ mK} \) (red line) where quantization is visible, and at 3.2 K (dotted green line) where quantization is absent. In the quantum Hall regime, at temperatures \( k_B T \) approaching the Landau level energy gap \( \Delta \), thermal excitation of electrons across \( \Delta \) gradually smears out conductivity quantization until it is ultimately absent. In our measurements, the plateaus of Faraday rotation \( \theta_F \) are barely visible at 3.2 K, consistent with orbital quantization of the 2DEG in the presence of a strong magnetic field at the origin of Faraday rotation quantization.

FIG. 3. Quantized Faraday rotation. Main panel: Faraday angle \( \theta_F \) versus \( 1/B \) (solid red line) at the base temperature of the dilution refrigerator. The expected position of each observed Faraday plateau is shown by horizontal markers with the quantization condition \( \tan(\theta_F) = ia^2 \). Inset: comparison of Faraday angle measurements at \( \sim 10 \text{ mK} \) (solid line) and 3.2 K (dashed line) temperature of the dilution refrigerator in a slightly different experimental configuration employing two coaxial assemblies instead of a single assembly with a cryogenic switch. The Faraday angle \( \theta_F \) at 3.2 K is offset vertically for clarity.
DISCUSSION

In the quantum Hall regime, Faraday rotation is quantized in units of the fine structure constant $\alpha$, a consequence of the dimensionless impedance mismatch between the impedance of free space $Z_0 = \sqrt{\mu_0/\epsilon_0}$ and the conductance quantum $2e^2/h$, wherein $\tan(\theta_F) = iZ_0e^2/2h = i\alpha$ at integer $i$ filling. We thus arrive at a geometric prescription for measurement of the fine structure constant by Faraday rotation of a 2DEG in the quantum Hall regime interrogated with plane waves in free space. In practice, a high-mobility 2DEG will inevitably be hosted in a semiconductor heterostructure while a wave guiding structure is used to bring electromagnetic waves to the 2DEG. The dielectric response of the semiconductor host, and the modification of wave impedance and field distribution by a wave guide [16] will lead to a modification of the quantized Faraday rotation away from the vacuum fine structure constant $\alpha \approx 1/137$. The effect of frequency-dependent electromagnetic confinement can be accounted for with an effective fine structure constant, $\tan(\theta_F) = i\alpha^*$. Fitting the experimentally measured $\tan(\theta_F)$ versus $1/B$ by considering solely the mid-point of each plateau, the effective constant is found to be $\alpha^* = 2.80\alpha$. Engineering of the electromagnetic field distribution and impedance, including resonant cavity effects, could be used to purposefully enhance or suppress Faraday rotation.

The bulk quantized transverse conductivity $\sigma_{yx}$ in the quantum Hall regime in a high-mobility 2DEG can thus be observed by means of quantized Faraday rotation. Interest in novel, low-dimensional electronic phenomena such as the quantum spin Hall effect [20] and the quantum anomalous Hall effect [21] continues to grow. Microwave Faraday rotation is a contactless method that may prove useful in probing these phenomena. Further, as a consequence of the high mobilities achievable in the GaAs/AlGaAs 2DEG system, large Faraday rotations can be obtained at modest applied magnetic fields of $\sim 100$ mT at the cross-over between the classical and quantum Hall regimes. It is foreseeable that the Faraday effect arising from cyclotron motion of high mobility charge carriers in semiconductor materials and heterostructures could be used to isolate and circulate microwave signals, in lieu of conventional bulk ferrites that rely on off-resonant Larmor precession to impart Faraday rotation.

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