Lepton Mass Matrices with Four Texture Zeros

Zhi-zhong Xing *
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
and Institute of High Energy Physics, Chinese Academy of Sciences,
P.O. Box 918 (4), Beijing 100039, China †

He Zhang
Physics Department, Jilin University, Changchun 130023, China

Abstract

We propose two ansätze of lepton mass matrices with four texture zeros, and confront them with current experimental data on neutrino oscillations. The parameter space of each ansatz is carefully explored. We find that both ansätze can accommodate the normal hierarchy of neutrino masses and the bi-large pattern of lepton flavor mixing. Their predictions for the effective mass of the tritium beta decay and that of the neutrinoless double beta decay are too small to be detectable, but leptonic CP violation at the percent level is allowed. Some discussions are also given about the seesaw invariance of the four-zero texture of Dirac and Majorana neutrino mass matrices.

*Electronic address: xingzz@mail.ihep.ac.cn
†Mailing address
1 Introduction

The recent KamLAND [1] and SNO [2] experiments have provided us with very compelling evidence that the solar neutrino deficit is due to the matter-enhanced $\nu_e \rightarrow \nu_\mu$ oscillation with a large mixing angle $\theta_{\text{sun}} \sim 32^\circ$. Meanwhile, the K2K [3] and Super-Kamiokande [4] experiments have convinced us that the atmospheric neutrino anomaly is attributed to the $\nu_\mu \rightarrow \nu_\tau$ oscillation with another large mixing angle $\theta_{\text{atm}} \sim 45^\circ$. In contrast, the non-observation of the $\nu_e \rightarrow \nu_e$ oscillation in the CHOOZ experiment [5] indicates a rather small (even vanishing) mixing angle $\theta_{\text{chz}} < 13^\circ$. These three mixing angles may straightforwardly be related to the elements of the $3 \times 3$ lepton flavor mixing matrix $V$, which links the neutrino mass eigenstates ($\nu_1, \nu_2, \nu_3$) to the neutrino flavor eigenstates ($\nu_e, \nu_\mu, \nu_\tau$). To a good degree of accuracy, $\theta_{\text{sun}}$ and $\theta_{\text{atm}}$ describe the flavor mixing effect between the 1st and 2nd lepton families and that between the 2nd and 3rd lepton families, respectively; while the small mixing angle $\theta_{\text{chz}}$ is responsible for the flavor mixing effect between the 1st and 3rd lepton families. Thus the lepton flavor mixing matrix $V$ performs a bi-large mixing pattern, which is quite different from the tri-small mixing pattern of the quark flavor mixing matrix [6].

To interpret the bi-large lepton flavor mixing pattern, many phenomenological ansätze of lepton mass matrices have recently been proposed [7]. A particularly interesting category of the ansätze focus on texture zeros of charged lepton and neutrino mass matrices in a given flavor basis ‡, from which some nontrivial relations between flavor mixing angles and lepton mass ratios can be derived. The typical example is the Fritzsch ansatz [9] for symmetric lepton mass matrices,

$$M_{l,\nu} = \begin{pmatrix} 0 & C_{l,\nu} & 0 \\ C_{l,\nu} & 0 & B_{l,\nu} \\ 0 & B_{l,\nu} & A_{l,\nu} \end{pmatrix},$$

(1)

in which six texture zeros are included §. It has been shown by one of the authors [11] that this type of lepton mass matrices can naturally predict a normal hierarchy of neutrino masses and a bi-large pattern of lepton flavor mixing angles. Furthermore, Fukugita, Tanimoto and Yanagida [12] have demonstrated that very similar phenomenological predictions can also be achieved from a simple but interesting ansatz of lepton mass matrices based on both the Fritzsch texture and the seesaw mechanism [13].

The present paper aims to analyze the generalized Fritzsch ansatz of lepton mass matrices with four texture zeros,

$$M_{l,\nu} = \begin{pmatrix} 0 & C_{l,\nu} & 0 \\ C_{l,\nu} & 0 & B_{l,\nu} \\ 0 & B_{l,\nu} & A_{l,\nu} \end{pmatrix},$$

(2)

and its consequences on the neutrino mass spectrum, flavor mixing and CP violation. It is well known that the four-zero texture of quark mass matrices is more successful than the six-zero texture of quark mass matrices to interpret the strong hierarchy of quark masses and the smallness of flavor mixing angles. The spirit of lepton-quark similarity motivates us to conjecture that the lepton mass matrices might have the same texture zeros as the quark mass matrices. Such a conjecture is indeed reasonable in some specific models of grand unified

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‡ For instance, a lot of interest has recently been paid to possible two-zero textures of the neutrino mass matrix in the flavor basis where the charged lepton mass matrix is diagonal [8].

§ A pair of off-diagonal texture zeros of the charged lepton ($M_l$) or neutrino ($M_\nu$) mass matrix have been counted, due to symmetry, as one zero [10].
theories [7], in which the mass matrices of leptons and quarks are related to each other by a new kind of flavor symmetry. That is why the four-zero texture of lepton mass matrices has been considered as a typical example in some model-building works [14]. However, a careful and complete analysis of its phenomenological implications has not been done in the literature.

Naively, there is no doubt that the four-zero texture of $M_{l,\nu}$ in Eq. (2), which has more free parameters than the Fritzsch texture of $M_{l,\nu}$ in Eq. (1), must be able to interpret the observed bi-large pattern of lepton flavor mixing. To improve the analytical calculability and numerical predictability, one may follow a realistic strategy to concentrate on part of the whole parameter space of the four-zero texture of lepton mass matrices. The same strategy has actually been adopted in the study of the four-zero texture of quark mass matrices [15].

In this paper, we shall consider two simplified versions of $M_{l}$ and $M_{\nu}$ given in Eq. (2):

1. $|\tilde{B}_{l}| = m_{\mu}$ and $|\tilde{B}_{\nu}| = m_{2}$, where $m_{\mu}$ and $m_{2}$ stand respectively for the physical masses of $\mu$ and $\nu_{2}$. This interesting case, to be referred to as Ansatz (A), has been briefly discussed in Ref. [14].

2. $|\tilde{B}_{l}| = |B_{l}|$ and $|\tilde{B}_{\nu}| = |A_{\nu}|$. This specific case, to be referred to as Ansatz (B), has not been discussed in the literature.

Our main purpose is to explore the allowed parameter space of each ansatz and its implications on the neutrino mass spectrum and lepton flavor mixing measurable. We find that both ansätze are favored by current experimental data on neutrino oscillations. Their predictions for the effective mass of the tritium beta decay and that of the neutrinoless double beta decay are too small to be detectable, but leptonic CP violation at the percent level is definitely allowed. Finally, we present some brief discussions about the seesaw invariance of the four-zero texture of Dirac and Majorana neutrino mass matrices.

2 Framework

Note that all non-zero elements of $M_{l}$ and $M_{\nu}$ in Eq. (2) are in general complex. If the condition

$$\arg(A_{l,\nu}) + \arg(\tilde{B}_{l,\nu}) = 2 \arg(B_{l,\nu})$$

is satisfied, then $M_{l,\nu}$ can be decomposed as

$$M_{l} = P_{l}^{T}M_{l}P_{l}, \quad M_{\nu} = P_{\nu}^{T}M_{\nu}P_{\nu},$$

where

$$\overline{M}_{l,\nu} = \begin{pmatrix} 0 & |C_{l,\nu}| & 0 \\ |C_{l,\nu}| & |B_{l,\nu}| & |B_{l,\nu}| \\ 0 & |B_{l,\nu}| & |A_{l,\nu}| \end{pmatrix},$$

$$P_{l,\nu} = \begin{pmatrix} e^{i\alpha_{l,\nu}} & 0 & 0 \\ 0 & e^{i\beta_{l,\nu}} & 0 \\ 0 & 0 & e^{i\gamma_{l,\nu}} \end{pmatrix},$$

with $\arg(A_{l,\nu}) = 2\gamma_{l,\nu}$, $\arg(B_{l,\nu}) = \beta_{l,\nu} + \gamma_{l,\nu}$, $\arg(\tilde{B}_{l,\nu}) = 2\beta_{l,\nu}$ and $\arg(C_{l,\nu}) = \alpha_{l,\nu} + \beta_{l,\nu}$. The real symmetric matrices $\overline{M}_{l}$ and $\overline{M}_{\nu}$ can be diagonalized by use of the following unitary
transformations:

\[
U^T_i \mathcal{M}_i U_i = \begin{pmatrix}
  m_e & 0 & 0 \\
  0 & m_\mu & 0 \\
  0 & 0 & m_\tau
\end{pmatrix},
\]

\[
U^T_\nu \mathcal{M}_\nu U_\nu = \begin{pmatrix}
  m_1 & 0 & 0 \\
  0 & m_2 & 0 \\
  0 & 0 & m_3
\end{pmatrix},
\]

in which \((m_e, m_\mu, m_\tau)\) and \((m_1, m_2, m_3)\) denote the physical masses of charged leptons and neutrinos, respectively. The lepton flavor mixing matrix \(V\) arises from the mismatch between the diagonalization of the charged lepton mass matrix \(M_i\) and that of the neutrino mass matrix \(M_\nu\). Therefore, we obtain \(V = U^T_i \left(P^* \nu \nu\right) U^*_\nu\), whose nine matrix elements read explicitly as

\[
V_{pq} = U^T_{1p} U^*_{1q} e^{i\alpha} + U^T_{2p} U^*_{2q} e^{i\beta} + U^T_{3p} U^*_{3q} e^{i\gamma},
\]

where the subscripts \(p\) and \(q\) run respectively over \((e, \mu, \tau)\) and \((1, 2, 3)\), and the parameters \(\alpha, \beta\) and \(\gamma\) are defined as

\[
\alpha \equiv \alpha_\nu - \alpha_i, \quad \beta \equiv \beta_\nu - \beta_i, \quad \gamma \equiv \gamma_\nu - \gamma_i.
\]

Note that the overall phase of \(V\) has nothing to do with the experimental observables. Hence only two combinations of three phases \((\alpha, \beta, \gamma)\) are physically relevant. For simplicity, we take \(\gamma = 0\) in the following.

The matrix elements of \(V\) depend both on the ratios of lepton masses,

\[
x_i \equiv \frac{m_e}{m_\mu}, \quad y_i \equiv \frac{m_\mu}{m_\tau}, \quad x_\nu \equiv \frac{m_1}{m_2}, \quad y_\nu \equiv \frac{m_2}{m_3},
\]

and on the phase parameters \(\alpha\) and \(\beta\). As the values of \(m_e, m_\mu\) and \(m_\tau\) have precisely been measured [16], we have \(x_i \approx 0.00484\) and \(y_i \approx 0.0594\) to a good degree of accuracy. The other four free parameters can be determined or constrained from the present experimental data on neutrino oscillations. As the neutrino mass-squared differences of solar and atmospheric neutrino oscillations are given by

\[
\Delta m^2_{\text{sun}} \equiv m^2_2 - m^2_1 = m^2_2 \left|1 - x^2_\nu\right|,
\]

\[
\Delta m^2_{\text{atm}} \equiv m^2_3 - m^2_2 = m^2_3 \left|1 - y^2_\nu\right|,
\]

the observed hierarchy \(\Delta m^2_{\text{sun}} \ll \Delta m^2_{\text{atm}}\) may impose a very strong constraint on the values of \((x_\nu, y_\nu)\):

\[
R_\nu \equiv \frac{\Delta m^2_{\text{sun}}}{\Delta m^2_{\text{atm}}} = y^2_\nu \left|\frac{1 - x^2_\nu}{1 - y^2_\nu}\right| \ll 1.
\]

On the other hand, the mixing factors of solar, atmospheric and CHOOZ reactor neutrino oscillations are related to the matrix elements of \(V\) in the following way:

\[
\sin^2 2\theta_{\text{sun}} = 4|V_{e1}|^2|V_{e2}|^2,
\]

\[
\sin^2 2\theta_{\text{atm}} = 4|V_{\mu3}|^2 \left(1 - |V_{\mu3}|^2\right),
\]

\[
\sin^2 2\theta_{\text{chz}} = 4|V_{e3}|^2 \left(1 - |V_{e3}|^2\right).
\]
In view of the KamLAND [1], SNO [2], K2K [3], Super-Kamiokande [4] and CHOOZ [5] data on neutrino oscillations, we have $\Delta m_{\text{sun}}^2 \in [5.9, 8.8] \times 10^{-5}$ eV$^2$, $\sin^2 \theta_{\text{sun}} \in [0.25, 0.40]$ [17]; $\Delta m_{\text{atm}}^2 \in [1.65, 3.25] \times 10^{-3}$ eV$^2$, $\sin^2 2\theta_{\text{atm}} \in [0.88, 1.00]$ [18]; and $\sin^2 2\theta_{\text{chz}} < 0.2$ at the 90% confidence level. With the help of these experimental results, the allowed ranges of $x_\nu$, $y_\nu$, $\alpha$ and $\beta$ can be obtained from Eqs. (11) and (12).

Once the values of $x_\nu$ and $y_\nu$ are determined or constrained from current experimental data, we are able to calculate the absolute values of three neutrino masses by use of Eq. (10):

$$m_3 = \frac{1}{\sqrt{|1 - y_\nu^2|}} \sqrt{\Delta m_{\text{atm}}^2},$$

$$m_2 = \frac{y_\nu}{\sqrt{|1 - y_\nu^2|}} \sqrt{\Delta m_{\text{atm}}^2},$$

$$m_1 = \frac{x_\nu}{\sqrt{|1 - x_\nu^2|}} \sqrt{\Delta m_{\text{sun}}^2},$$

(13)

In addition, interesting predictions can be achieved for the effective mass of the tritium beta decay $\langle m \rangle_e$ and that of the neutrinoless double beta decay $\langle m \rangle_{ee}$:

$$\langle m \rangle_e^2 = \sum_{i=1}^{3} (m_i^2 |V_{ei}|^2) = m_3^2 \left( x_\nu y_\nu |V_{e1}|^2 + y_\nu^2 |V_{e2}|^2 + |V_{e3}|^2 \right),$$

$$\langle m \rangle_{ee} = \sum_{i=1}^{3} (m_i V_{ei}^2) = m_3 \left| x_\nu y_\nu V_{e1}^2 + y_\nu V_{e2}^2 + V_{e3}^2 \right|.$$  

(14)

The present experimental upper bound on $\langle m \rangle_e$ is $\langle m \rangle_e < 3$ eV [16], while the sensitivity of the proposed KATRIN experiment is expected to reach $\langle m \rangle_e \sim 0.3$ eV [19]. In comparison, the upper limit $\langle m \rangle_{ee} < 0.35$ eV has been set by the Heidelberg-Moscow Collaboration [20] at the 90% confidence level*. The sensitivity of the next-generation experiments for the neutrinoless double beta decay is possible to reach $\langle m \rangle_{ee} \sim 10$ meV to 50 meV [22].

The strength of CP violation in neutrino oscillations, which is measured by the Jarlskog invariant $\mathcal{J}$ [23], can also be predicted from the four-zero texture of lepton mass matrices under consideration. Indeed, $\mathcal{J}$ is defined through the following equation:

$$\text{Im} \left( V_{ai} V_{bj}^* V_{aj}^* V_{bi} \right) = \mathcal{J} \sum_{c,k} (\epsilon_{ace} \epsilon_{ijk}),$$

(15)

where the subscripts $(a,b,c)$ and $(i,j,k)$ run respectively over $(e, \mu, \tau)$ and $(1, 2, 3)$. The magnitude of $\mathcal{J}$ depends both on $(x_\nu, y_\nu)$ and on $(\alpha, \beta)$. If $|\mathcal{J}| \sim 1\%$ is achievable, then leptonic CP- and T-violating effects could be measured in a variety of long-baseline neutrino oscillation experiments [24] in the future.

*If the reported evidence for the existence of the neutrinoless double beta decay [21] is taken into account, one has $0.05 \text{ eV} \leq \langle m \rangle_{ee} \leq 0.84 \text{ eV}$ at the 95% confidence level.
3 Ansatz (A)

Now let us consider Ansatz (A), in which the requirements $|\tilde{B}_l| = m_\mu$ and $|\tilde{B}_\nu| = m_2$ are imposed on $M_l$ and $M_\nu$ in Eq. (2). Similar conditions ($|\tilde{B}_u| = m_c$ and $|\tilde{B}_d| = m_s$) have actually been taken in some literature [14] for the four-zero texture of quark mass matrices. Following Eqs. (3) and (4), we factor out the complex phases of $M_{l,\nu}$. Then three elements of the real symmetric mass matrix $M_{l,\nu}$ can be expressed in terms of its three mass eigenvalues:

\[
|A_l| = m_\tau - m_e, \\
|B_l| = \left[ \frac{m_e m_\tau (m_\tau - m_e - m_\mu)}{m_\tau - m_e} \right]^{1/2}, \\
|C_l| = \left( \frac{m_e m_\mu m_\tau}{m_\tau - m_e} \right)^{1/2},
\]

and

\[
|A_\nu| = m_3 - m_1, \\
|B_\nu| = \left[ \frac{m_1 m_3 (m_3 - m_1 - m_2)}{m_3 - m_1} \right]^{1/2}, \\
|C_\nu| = \left( \frac{m_1 m_2 m_3}{m_3 - m_1} \right)^{1/2}.
\]

The elements of the unitary transformation matrix $U_{l,\nu}$, which is used to diagonalize $\overline{M}_{l,\nu}$ in Eq. (6), can in turn be expressed in terms of the ratios $x_{l,\nu}$ and $y_{l,\nu}$ as follows (the indices “$l$” and “$\nu$” are neglected for simplicity):

\[
U_{11} = +i \left[ \frac{1}{(1 + x)(1 - x^2 y^2)} \right]^{1/2}, \\
U_{12} = + \left[ \frac{x(1 - y - xy)}{(1 + x)(1 - y)(1 - xy)} \right]^{1/2}, \\
U_{13} = + \left[ \frac{x^2 y^3}{(1 - y)(1 - x^2 y^2)} \right]^{1/2}, \\
U_{21} = -i \left[ \frac{x}{(1 + x)(1 + xy)} \right]^{1/2}, \\
U_{22} = + \left[ \frac{1 - y - xy}{(1 + x)(1 - y)} \right]^{1/2}, \\
U_{23} = + \left[ \frac{xy}{(1 - y)(1 + xy)} \right]^{1/2}, \\
U_{31} = +i \left[ \frac{x^2 y(1 - y - xy)}{(1 + x)(1 - x^2 y^2)} \right]^{1/2}, \\
U_{32} = - \left[ \frac{xy}{(1 + x)(1 - y)(1 - xy)} \right]^{1/2}, \\
U_{33} = + \left[ \frac{1 - y - xy}{(1 - y)(1 - x^2 y^2)} \right]^{1/2}.
\]
Note that $U_{1i}$ (for $i = 1, 2, 3$) are imaginary, and their nontrivial phases are due to the negative determinant of $M_{l,\nu}$.

The four free parameters $x_\nu$, $y_\nu$, $\alpha$ and $\beta$ can be constrained by use of Eqs. (11) and (12) as well as current data on neutrino oscillations. Their allowed ranges are shown in Fig. 1. We see that $x_\nu \sim 0.86$ and $y_\nu \sim 0.35$ typically hold. Thus the neutrino mass spectrum satisfies $m_1 < m_2 < m_3$. For $\alpha$ and $\beta$ varying from 0 to $2\pi$, we find that about half of the whole $(\alpha, \beta)$ parameter space can be excluded. Note that the correlation between $\alpha$ and $\beta$ in Ansatz (A) is not as strong as the phase correlation in the Fritzsch ansatz of lepton mass matrices [11, 12]. The reason is simply that the contribution of $M_l$ to $V$ is much smaller in Ansatz (A) than in the Fritzsch ansatz. Hence the relative phases between $M_l$ and $M_\nu$ in the former cannot significantly affect the magnitudes of nine matrix elements of $V$.

Fig. 1 also shows the outputs of $\sin^2 2\theta_{\text{atm}}$ versus $\sin^2 \theta_{\text{sun}}$ and $\sin^2 2\theta_{\text{chz}}$ versus $R_\nu$ restricted by Ansatz (A). It can be seen that larger values of $\sin^2 \theta_{\text{sun}}$ roughly correspond to smaller values of $\sin^2 2\theta_{\text{atm}}$. In addition, the ansatz predicts $\sin^2 2\theta_{\text{chz}} \geq 0.08$, a lower bound which is easily accessible in the upcoming long-baseline neutrino oscillation experiments [24]. If the upper limit $\sin^2 2\theta_{\text{chz}} < 0.1$ instead of $\sin^2 2\theta_{\text{chz}} < 0.2$ is input in the numerical calculations, one will arrive at $\sin^2 2\theta_{\text{atm}} \leq 0.91$. Such a low value of $\sin^2 2\theta_{\text{atm}}$ is tolerable, but not favored by current data. It becomes clear that the mixing angles $\theta_{\text{sun}}$, $\theta_{\text{atm}}$ and $\theta_{\text{chz}}$ are strongly correlated with one another in Ansatz (A). Thus more precise data on three mixing angles may provide a sensitive test of this phenomenological scenario.

The result $y_\nu^2 \sim 0.1$ implies that $m_3 \approx \sqrt{\Delta m^2_{\text{atm}}}$ is an acceptable approximation. More exactly, we obtain $m_3 \approx (4.3 - 6.1) \times 10^{-2}$ eV, $m_2 \approx (1.4 - 2.3) \times 10^{-2}$ eV and $m_1 \approx (1.1 - 2.1) \times 10^{-2}$ eV from Eq. (13). In calculating the allowed ranges of $m_1$ and $m_2$, we have ignored their correlation induced by the model itself. This generous treatment has no conflict with the plot of $(x_\nu, y_\nu)$ in Fig. 1, in which $x_\nu < 1$ results from the correlation between $m_1/m_2$ and $m_2/m_3$. The sum of three neutrino masses is consistent with $m_1 + m_2 + m_3 < 0.71$ eV, an upper bound set by the recent WMAP data [25]. We compute the effective mass of the tritium beta decay and that of the neutrinoless double beta decay by use of Eq. (14), and present the numerical results in Fig. 1. One can see that $\langle m \rangle_e \sim 10^{-2}$ eV and $\langle m \rangle_{ee} \sim 10^{-3}$ eV typically hold. Both quantities are too small to be measured in practice. Similarly, there is no hope to detect the effective (kinematic) masses of muon and tau neutrinos [26]. The numerical results for the Jarlskog parameter $J$ and the smallest matrix element $|V_{e3}|$ are also shown in Fig. 1. We find that the magnitude of $J$ may nearly be 1%, if $|V_{e3}|$ is close to its upper bound. It is possible to measure leptonic CP violation of this order in the future neutrino factories, if the terrestrial matter effects can be under control.

Finally we illustrate the typical texture of $\overline{M}_{l,\nu}$ by taking $x_\nu \approx 0.86$, $y_\nu \approx 0.35$ and $m_3 \approx 0.05$ eV. The result is

$$
\overline{M}_l \approx 1.78 \text{ GeV} \times \begin{pmatrix}
0 & 0.0041 & 0 \\
0.0041 & 0.059 & 0.016 \\
0 & 0.016 & 1
\end{pmatrix},
$$

$$
\overline{M}_\nu \approx 3.50 \times 10^{-2} \text{ eV} \times \begin{pmatrix}
0 & 0.56 & 0 \\
0.56 & 0.50 & 0.55 \\
0 & 0.55 & 1
\end{pmatrix}. \tag{19}
$$

It becomes obvious that lepton flavor mixing is dominated by the neutrino sector, as the matrix elements of $\overline{M}_l$ have a very strong hierarchy.
4 Ansatz (B)

We proceed to consider Ansatz (B), in which the requirements \(|\tilde{B}_l| = |B_l|\) and \(|\tilde{B}_\nu| = |A_\nu|\) are imposed on \(M_l\) and \(M_\nu\) in Eq. (2). Note that the condition \(|\tilde{B}_l| = |B_l|\) is similar to \(|\tilde{B}_a| \approx |B_a|\) and \(|\tilde{B}_d| \approx |B_d|\) for the four-zero texture of quark mass matrices [15], in view of the fact that charged leptons have a strong mass hierarchy as quarks. Because the condition \(|\tilde{B}_l| = |B_l|\) leads to \(|B_l| \approx m_\mu\) in the leading-order approximation, it is essentially equivalent to the condition \(|\tilde{B}_l| = m_\mu\) taken in Ansatz (A). In contrast, the requirement \(|\tilde{B}_\nu| = |A_\nu|\) is motivated by the experimental fact that the mixing angle of atmospheric neutrino oscillations is about 45° [4] (namely, the diagonalization of the (2,3) subsector of \(M_\nu\) may give rise to a rotation angle of 45°, if the condition \(|\tilde{B}_\nu| = |A_\nu|\) is satisfied). Such a phenomenological hypothesis for the texture of \(M_\nu\) results in an apparent “structural asymmetry” between \(M_l\) and \(M_\nu\), but its consequences on lepton flavor mixing are simple and interesting. Following Eqs. (3) and (4), we can factor out the complex phases of \(M_\nu\). Although we are able to exactly express both \(|A_l|, |B_l|, |C_l|\) and \(|A_\nu|, |B_\nu|, |C_\nu|\) in terms of the corresponding mass eigenvalues, the formulas for the former are too complicated to be instructive. It is therefore better to make some analytical approximations in deriving \(|A_l|, |B_l|\) and \(|C_l|\). In view of the strong mass hierarchy in the charged lepton sector, we expect that \(|A_l| \gg |B_l| \gg |C_l|\) naturally holds. Then we obtain

\[|A_l| \approx m_\tau \left(1 - \frac{m_\mu^2}{m_\tau^2}\right),\]
\[|B_l| \approx m_\mu \left(1 + \frac{m_\mu}{m_\tau}\right),\]
\[|C_l| \approx \sqrt{m_e m_\mu} \left(1 + \frac{m_\mu^2}{2m_\tau^2}\right),\]

(20)

to a good degree of accuracy. In contrast, the expressions of \(|A_\nu|, |B_\nu|\) and \(|C_\nu|\) are exact:

\[|A_\nu| = \frac{m_3 + m_2 - m_1}{2},\]
\[|B_\nu| = \frac{1}{2} \left[\frac{(m_3 + m_2 + m_1)(m_3 - m_2 - m_1)(m_3 - m_2 + m_1)}{m_3 + m_2 - m_1}\right]^{1/2},\]
\[|C_\nu| = \left(\frac{2m_1 m_2 m_3}{m_3 + m_2 - m_1}\right)^{1/2}.\]

(21)

The unitary transformation matrix \(U_l\), which has been defined to diagonalize \(\overline{M}_l\) in Eq. (6), can approximately be given as

\[U_l \approx \begin{pmatrix}
-i\left(1 - \frac{x_l}{2}\right) & \sqrt{x_l} & y_l \sqrt{x_l} y_l \\
-i\sqrt{x_l} & 1 - \frac{x_l}{2} - \frac{y_l^2}{2} & y_l \\
iy_l \sqrt{x_l} & -y_l & 1 - \frac{y_l^2}{2}
\end{pmatrix},\]

(22)

where the fact of \(x_l \sim y_l^2\) has been taken into account. In addition, the matrix elements of \(U_\nu\) can be expressed in terms of \(x_\nu\) and \(y_\nu\) as follows:

\[U_{11}^\nu = +i \left[\frac{(1 + y_\nu + x_\nu y_\nu)}{(1 + x_\nu)(1 + x_\nu y_\nu)(1 + y_\nu - x_\nu y_\nu)}\right]^{1/2},\]
by current data on neutrino oscillations. The numerical results are shown in Fig. 2. We see \( \langle \sum_{\nu} \rangle \) are also illustrated in Fig. 2. One can see that \( \sin^2 \theta_{\text{atm}} \) for the effective mass of the tritium beta decay and that of the neutrinoless double beta decay is little restriction on the phase parameters \( \alpha \) and \( \beta \). This feature can easily be understood: the contribution of \( M_\nu \) to \( V \) is dominant over that of \( M_l \) to \( V \), thus the magnitudes of nine matrix elements of \( V \) are essentially insensitive to the relative phases between \( M_l \) and \( M_\nu \).

With the help of Eqs. (11) and (12), one may compute the ranges of \( x_\nu \), \( y_\nu \), \( \alpha \) and \( \beta \) allowed by current data on neutrino oscillations. The numerical results are shown in Fig. 2. We see that the allowed range of \( x_\nu \) in Ansatz (B) is much larger than that in Ansatz (A). As both \( x_\nu < 1 \) and \( y_\nu < 1 \) hold, the neutrino mass spectrum satisfies \( m_1 < m_2 < m_3 \). Note that there is little restriction on the phase parameters \( \alpha \) and \( \beta \). This feature can easily be understood: the contribution of \( M_\nu \) to \( V \) is dominant over that of \( M_l \) to \( V \), thus the magnitudes of nine matrix elements of \( V \) are essentially insensitive to the relative phases between \( M_l \) and \( M_\nu \).

The outputs of \( \sin^2 2\theta_{\text{atm}} \) versus \( \sin^2 \theta_{\text{sun}} \) and \( \sin^2 2\theta_{\text{chz}} \) versus \( R_\nu \) are also shown in Fig. 2. We see that \( \sin^2 2\theta_{\text{atm}} \geq 0.94 \) holds and \( \sin^2 2\theta_{\text{atm}} \approx 1 \) is particularly favored. The latter is a natural consequence of the specific texture of \( M_\nu \), whose (2,3) subsector can be diagonalized by a rotation of 45°. Thus \( |V_{\mu 3}| \approx 1/\sqrt{2} \) leads to \( \sin^2 2\theta_{\text{atm}} \approx 1 \) in Ansatz (B). Another feature of Ansatz (B) is that changing the upper bound of \( \sin^2 2\theta_{\text{chz}} \) from 0.2 to 0.1 does not significantly affect the allowed range of \( \sin^2 2\theta_{\text{atm}} \). However, the correlation between \( \sin^2 2\theta_{\text{chz}} \) and \( R_\nu \) is stronger in Ansatz (B) than in Ansatz (A).

The approximation \( m_3 \approx \sqrt{\Delta m_{\text{atm}}^2} \) is reasonably good in Ansatz (B). Numerically, we obtain \( m_3 \approx (4.2 - 5.8) \times 10^{-2} \) eV, \( m_2 \approx (0.8 - 1.4) \times 10^{-2} \) eV and \( m_1 \approx (0.3 - 1.1) \times 10^{-2} \) eV from Eq. (13). This neutrino mass spectrum is quite similar to that in Ansatz (A). The results for the effective mass of the tritium beta decay and that of the neutrinoless double beta decay are also illustrated in Fig. 2. One can see that \( \langle m_\nu \rangle \sim 10^{-2} \) eV and \( \langle m_{ee} \rangle \sim 10^{-3} \) eV hold in Ansatz (B). Both of them are too small to be detected in reality. In addition, Fig. 2 shows that the magnitude of the Jarlskog parameter \( J \) may nearly be 1.5%, if \( |V_{e3}| \) is larger than 0.15. This result implies that it is possible to observe leptonic CP violation in the future long-baseline neutrino oscillation experiments.

\[
U_{12}^\nu = + \left[ \frac{x_\nu(1 - y_\nu - x_\nu y_\nu)}{(1 + x_\nu)(1 - y_\nu)(1 + y_\nu - x_\nu y_\nu)} \right]^{1/2},
\]
\[
U_{13}^\nu = + \left[ \frac{x_\nu y_\nu^2(1 - y_\nu + x_\nu y_\nu)}{(1 - y_\nu)(1 + x_\nu y_\nu)(1 + y_\nu - x_\nu y_\nu)} \right]^{1/2},
\]
\[
U_{21}^\nu = -i \left[ \frac{x_\nu(1 + y_\nu + x_\nu y_\nu)}{2(1 + x_\nu)(1 + y_\nu - x_\nu y_\nu)} \right]^{1/2},
\]
\[
U_{22}^\nu = + \left[ \frac{(1 - y_\nu - x_\nu y_\nu)}{2(1 + x_\nu)(1 - y_\nu)} \right]^{1/2},
\]
\[
U_{23}^\nu = + \left[ \frac{(1 - y_\nu + x_\nu y_\nu)}{2(1 - y_\nu)(1 + x_\nu y_\nu)} \right]^{1/2},
\]
\[
U_{31}^\nu = + i \left[ \frac{x_\nu(1 - y_\nu - x_\nu y_\nu)(1 - y_\nu + x_\nu y_\nu)}{2(1 + x_\nu)(1 + y_\nu - x_\nu y_\nu)} \right]^{1/2},
\]
\[
U_{32}^\nu = - \left[ \frac{(1 + y_\nu + x_\nu y_\nu)(1 - y_\nu + x_\nu y_\nu)}{2(1 + x_\nu)(1 - y_\nu)(1 + y_\nu - x_\nu y_\nu)} \right]^{1/2},
\]
\[
U_{33}^\nu = + \left[ \frac{(1 + y_\nu + x_\nu y_\nu)(1 - y_\nu - x_\nu y_\nu)}{2(1 - y_\nu)(1 + x_\nu y_\nu)(1 + y_\nu - x_\nu y_\nu)} \right]^{1/2}.
\]
To illustrate the texture of $M_{l,\nu}$, we typically take $x_\nu \approx 0.6$, $y_\nu \approx 0.21$ and $m_3 \approx 0.05$ eV. The numerical result is

$$M_l \approx 1.77 \text{ GeV} \times \begin{pmatrix} 0 & 0.0042 & 0 \\ 0.0042 & 0.064 & 0.064 \\ 0 & 0.064 & 1 \end{pmatrix},$$

$$M_\nu \approx 2.71 \times 10^{-2} \text{ eV} \times \begin{pmatrix} 0 & 0.41 & 0 \\ 0.41 & 1 & 0.80 \\ 0 & 0.80 & 1 \end{pmatrix}. \quad (24)$$

The similarity and difference between two ansätze are therefore obvious.

5 Seesaw

Two ansätze of lepton mass matrices discussed above can self-consistently describe the observed features of lepton flavor mixing, but they give no interpretation about why the masses of three neutrinos are so tiny. A simple way to improve our phenomenological ansätze is to incorporate them with the elegant idea of seesaw [13]. In the seesaw mechanism, the smallness of left-handed Majorana neutrinos is attributed to the existence of heavy right-handed Majorana neutrinos,

$$M_\nu \approx M_D M_R^{-1} M_D^T, \quad (25)$$

where $M_D$ and $M_R$ denote the Dirac neutrino mass matrix and the heavy Majorana neutrino mass matrix, respectively. In some grand unified theories (such as the SO(10) model [27]), one takes $[M_D, M_u] = 0$, where $M_u$ represents the up-type quark mass matrix. The mass matrix $M_R$ is practically unknown in almost all reasonable extensions of the standard model. Hence specific textures of $M_R$ and $M_D$ have to be assumed, in order to determine the texture of $M_\nu$.

Given $M_\nu$ and $M_D$, on the other hand, one can calculate $M_R$ by use of Eq. (25):

$$M_R \approx M_D^T M_\nu^{-1} M_D. \quad (26)$$

The scale of $M_R$ stands for the scale of new physics in this simple seesaw picture.

To be specific, we assume that $M_D = M_u$ holds and it has the same texture zeros as $M_l$ and $M_\nu$ have:

$$M_D = \begin{pmatrix} 0 & C_u & 0 \\ C_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{pmatrix}. \quad (27)$$

Only if the condition $\text{Det}M_\nu \neq 0$ is guaranteed for $M_\nu$ in Eq. (2), one may obtain the inverse matrix of $M_\nu$ as follows:

$$M_\nu^{-1} = \frac{1}{A_\nu C_\nu^2} \begin{pmatrix} B_\nu^2 - A_\nu \tilde{B}_\nu & A_\nu C_\nu & -B_\nu C_\nu \\ A_\nu C_\nu & 0 & 0 \\ -B_\nu C_\nu & 0 & C_\nu^2 \end{pmatrix}. \quad (28)$$

Then the texture of $M_R$ can be determined from Eq. (26) with the help of Eqs. (27) and (28):

$$M_R = \begin{pmatrix} 0 & C_R & 0 \\ C_R & \tilde{B}_R & B_R \\ 0 & B_R & A_R \end{pmatrix}. \quad (29)$$
where

\[
A_R = \frac{A_u^2}{A_\nu},
\]

\[
B_R = \frac{A_u B_u}{A_\nu} + \frac{B_u C_u}{C_\nu} - \frac{A_u C_u B_\nu}{A_\nu C_\nu},
\]

\[
\tilde{B}_R = \frac{B_u^2}{A_\nu} + \frac{2 \tilde{B}_u C_u}{C_\nu} - \frac{C_u^2 \tilde{B}_\nu}{C_\nu^2} - \frac{2 B_u C_u B_\nu}{A_\nu C_\nu} + \frac{C_u^2 B_\nu^2}{A_\nu C_\nu^2},
\]

\[
C_R = \frac{C_u^2}{C_\nu}.
\] (30)

We see that the texture zeros of \(M_D\) and \(M_\nu\) manifest themselves again in \(M_R\), as a consequence of the inverted seesaw relation given in Eq. (26). Therefore, all four lepton mass matrices \((M_l, M_D, M_R\) and \(M_\nu\)) are structurally parallel to one another. Such a structural similarity of lepton mass matrices, which is seesaw-invariant, might follow from a universal flavor symmetry hidden in a more fundamental theory of fermion mass generation. In particular, the underlying flavor symmetry must be related to the texture zeros of lepton mass matrices. It is worth mentioning that the Fritzsch texture of lepton mass matrices in Eq. (1) does not have the interesting property of seesaw invariance. Thus we argue that the four-zero texture of lepton mass matrix might be more attractive for model building at the energy scale where the seesaw mechanism works.

Now let us give an order-of-magnitude estimate of the matrix elements of \(M_R\) by taking the following phenomenologically-favored pattern of \(M_u\) [15]:

\[
M_u \sim \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t \end{pmatrix},
\] (31)

in which the relevant complex phases have been neglected for simplicity. Typically taking \(m_u/m_c \sim m_c/m_t \sim 0.0031\) and \(m_t \approx 175\) GeV at the electroweak scale [16], we obtain

\[
M_R \sim 8.8 \times 10^{14}\ \text{GeV} \times \begin{pmatrix} 0 & 5.4 \times 10^{-8} & 0 \\ 5.4 \times 10^{-8} & 9.6 \times 10^{-6} & 3.1 \times 10^{-3} \\ 0 & 3.1 \times 10^{-3} & 1 \end{pmatrix}
\] (32)

from Eq. (19) for Ansatz (A); and

\[
M_R \sim 1.1 \times 10^{15}\ \text{GeV} \times \begin{pmatrix} 0 & 7.3 \times 10^{-8} & 0 \\ 7.3 \times 10^{-8} & 9.6 \times 10^{-6} & 3.1 \times 10^{-3} \\ 0 & 3.1 \times 10^{-3} & 1 \end{pmatrix}
\] (33)

from Eq. (24) for Ansatz (B). We see that the structure of \(M_R\) is strongly hierarchical in either case. The scale of \(M_R\) is about \(10^{15}\) GeV, close to the typical scale of grand unified theories \(\Lambda_{\text{GUT}} \sim 10^{16}\) GeV.

It is worth remarking that the phase parameters of \(M_D\) and \(M_\nu\) have been ignored in estimating the matrix elements of \(M_R\). If those complex phases are included, it is possible to get CP violation in the lepton-number-violating decays of heavy Majorana neutrinos [28]. However, a successful interpretation of the observed matter-antimatter asymmetry of the universe via the leptogenesis mechanism [28] is rather nontrivial, because the details of \(M_D\) and \(M_R\) have to be taken into account. Further discussions on this topic are interesting but beyond the scope of this paper.
6 Summary

We have proposed and discussed two phenomenological ansätze of lepton mass matrices with four texture zeros. The parameter space of each ansatz has been carefully analyzed by use of current experimental data on neutrino oscillations. We demonstrate that the normal hierarchy of neutrino masses and the bi-large pattern of lepton flavor mixing can be accommodated in both ansätze. Their predictions for the effective mass of the tritium beta decay and that of the neutrinoless double beta decay are too small to be detected in practice. However, we find that leptonic CP violation at the percent level is possible for either ansatz. The correlation of relevant observable quantities in each ansatz allows us to test its validity, once more accurate experimental data become available. This property may also allow us to distinguish between these two different ansätze.

For the purpose of illustration, we have presented some brief discussions about the seesaw realization of our phenomenological scenarios. It is clear that the existence of heavy right-handed Majorana neutrinos at the scale of $10^{15}$ GeV or so may naturally interpret the smallness of left-handed Majorana neutrino masses. This observation would be useful for model building, from which some deeper understanding of the neutrino mass generation and lepton flavor mixing could be gained.

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Figure 1: The parameter space and phenomenological predictions of Ansatz (A).
Figure 2: The parameter space and phenomenological predictions of Ansatz (B).