ON COOPERATIVE FUZZY BUBBLY GAMES

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(Communicated by Gerhard-Wilhelm Weber)

Abstract. The allocation problem of rewards/costs is a basic question for players namely individuals and companies that planning cooperation under uncertainty. The involvement of uncertainty in cooperative game theory is motivated by the real world where noise in observation and experimental design, incomplete information and further vagueness in preference structures and decision-making play an important role. In this paper we extend cooperative bubbly games to cooperative fuzzy bubbly games, where the worth of each coalition is a fuzzy bubble instead of an interval. Further, we introduce a set-valued concept called the fuzzy bubbly core. Finally, some results on fuzzy bubbly core are given.

1. Introduction. Cooperative game theory has been enriched in recent years with several models which provide decision making support in collaborative situations under uncertainty. These models are generalizations of the classical model regarding the type of coalition values. In classical cooperative game theory the payoffs to coalitions of players are known with certainty, but when uncertainty is taken into consideration the characteristic functions are not real-valued as in classical case. In this case they capture the uncertainty on the outcome of cooperation in its different forms such as stochastic uncertainty, fuzzy uncertainty, interval uncertainty, ellipsoidal uncertainty [13, 19, 28, 30].

Different types of uncertainty have generated different models in cooperative game theory. There are several models of cooperative games which consider stochastic uncertainty such as chance-constrained games ([10]), cooperative games in stochastic characteristic function form ([13, 28]) , cooperative games with random payoffs ([29]). Fuzzy uncertainty in the values of characteristic functions was considered in [21, 22, 23]. The model of cooperative grey games ([25]) combines grey uncertainty with interval uncertainty. Ellipsoidal uncertainty in coalition values

2020 Mathematics Subject Classification. Primary: 91A12, 91A86.
Key words and phrases. Cooperative games, uncertainty, bubble, core, fuzzy intervals.
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has led recently to cooperative ellipsoidal games ([30]), which can be seen as a generalization of cooperative interval games because they allow also to represent the players’ mutual dependencies, similarities and possible affinities to cooperate.

In game theoretical models fuzzy coalitions are used differently. For example, [5, 6] and [9] extend the domain of the characteristic function from subsets to fuzzy coalitions. In these work, the characteristic function assigns to each fuzzy coalition to a real number. On the other hand, [20] and [21] are concerned with the uncertainty in the values of characteristic functions. In these models, the domain of the characteristic function of a game remains to be the system of the deterministic coalitions but the worths of coalitions are fuzzy intervals. In [7, 14], and [27] fuzzy matrix games and different applications are discussed. Further, in [19], crisp coalitions for games are used, where the worth of any coalition is a fuzzy interval. In this current work we use the same characteristic function, which is defined in [19].

The (re)distribution of collective gains and costs is a central question for individuals and organizations contemplating cooperation under uncertainty. The theory of cooperative fuzzy interval games provides a new game theoretical angle and suitable tools for answering this question.

Cooperative fuzzy interval games have been proved useful for solving reward/cost sharing problems in situations with fuzzy data in cooperative environments (see [19, 20, 21, 22, 23]). A natural way to incorporate the uncertainty of coalition values into the solution of such reward/cost sharing problems is by using fuzzy interval solution concepts.

Cooperative fuzzy interval games are extensions of cooperative interval games (see [1, 2, 3, 4, 8]) from the point of view of the nature of payoffs, in the sense that each coalition value has the form of a fuzzy interval of real numbers rather than an interval. This means that one cannot evaluate a sharp value for the worth/cost obtainable by each coalition, but instead knows a degree of membership for each coalition value. Moreover, no probabilistic assumptions about the range of coalition values are known, as is usually the case in practice. Our primary goal is to place the model of cooperative fuzzy interval games within the cooperative interval game theory and to motivate continued interest in theory and application development.

On the other hand, bubbles can be represented as a mathematical object, so advantages of optimization, pattern recognition tools and machine learning can be used as different solution approaches to detect, forecast and control their size, shape and position in Operational Research, climate negotiations and policy, environmental management and pollution control, etc ([16, 17]).

Our study is motivated by the need to collaborate, e.g., for overcoming environmental challenges, by various causes of uncertainty which exist in the real world. In this paper, we extend a new class of cooperative games under fuzzy uncertainty and we suggest an interesting mathematical approach, where the logarithmic price process is represented with fuzzy intervals. To be precise the values of the coalitions are taken as fuzzy bubbles instead of real numbers. Further, we introduce a new solution concept, the bubbly core for cooperative fuzzy bubbly games. In accordance with this we overcome the bubbly uncertainty by using the model of cooperative fuzzy interval games.

The paper is organized as follows. Section 2 gives basic notions from cooperative fuzzy interval games and fuzzy interval calculus. The model of cooperative fuzzy bubbly games and the notion of the fuzzy bubbly core are introduced in Section 3.
Some properties of the fuzzy bubbly core are given in Section 4. Finally, Section 5 concludes this study.

2. Preliminaries. In this section, some useful results from the theory of fuzzy cooperative interval games and some preliminaries from fuzzy interval calculus are given.

2.1. Fuzzy intervals. A fuzzy set ([33]) \( F \) in \( \mathbb{R} \) is a function \( u_F : \mathbb{R} \to [0, 1] \) where \( u_F \) assigns to each point in \( \mathbb{R} \) a degree of membership. For any \( \alpha \in [0, 1] \), \( \alpha \)-level set (\( \alpha \)-cut) of \( F \) defined by as follows:

\[
[u_F]^{\alpha} = \{ x \in \mathbb{R} : u_F(x) \geq \alpha \} = [\bar{u}_F, u_F^+].
\]

If \( \alpha = 0 \) then \( [u_F]^{\alpha} = \text{cl} \{ x \in \mathbb{R} : u_F(x) > 0 \} = \text{supp}(u_F) \).

Here \( \text{cl} \{ x \in \mathbb{R} : u_F(x) > 0 \} \) is the closure of \( \{ x \in \mathbb{R} : u_F(x) > 0 \} \).

A fuzzy set \( F \) in \( \mathbb{R} \) is said to be a fuzzy interval, if the following conditions are satisfied [11]:

\begin{enumerate}
  \item \( [u_F]^{\alpha} \) compact for any \( \alpha \in [0, 1] \);
  \item \( [u_F]^{\alpha} \) convex for any \( \alpha \in [0, 1] \);
  \item \( [u_F]^{\alpha} \) normal i.e. there exist \( x \in \mathbb{R} \) such that \( u_F(x) = 1 \).
\end{enumerate}

We denote the set of all fuzzy intervals by \( F(\mathbb{R}) \). For any \( F \in F(\mathbb{R}) \) there exist \( a, b, c, d \in \mathbb{R} \) and \( L : [a, b] \to \mathbb{R} \) non-decreasing and \( R : [c, d] \to \mathbb{R} \) non-increasing such that the membership function \( u_F \) is given as below [11]:

\[
u_F(x) = \begin{cases}
L(x) & a \leq x \leq b \\
1 & b \leq x \leq c \\
R(x) & c \leq x \leq d \\
0 & \text{otherwise}
\end{cases}.
\] (1)

Specially if \( L \) and \( R \) are linear then \( u_F \) is called a trapezoidal fuzzy interval and its membership function is given as below [11]:

\[
u_F(x) = \begin{cases}
\frac{x-a}{b-a} & a \leq x \leq b \\
1 & b \leq x \leq c \\
\frac{x-d}{c-d} & c \leq x \leq d \\
0 & \text{otherwise}
\end{cases}.
\] (2)

This trapezoidal fuzzy interval denoted by \((a, b, c, d)\). We denote the set of all trapezoidal fuzzy intervals by \( F_T(\mathbb{R}) \). In this case if \( a = b \) and \( c = d \) then \((a, b, c, d)\) is compact interval, if \( a = b = c = d \) then \((a, b, c, d)\) is a real number.

For any \( \alpha \in [0, 1] \) the \( \alpha \)-level set of a trapezoidal fuzzy interval with membership function \( u \) is given by [11]:

\[
[u]^{\alpha} = \left[ L^{-1}(0) (1-\alpha) + \alpha L^{-1}(1), R^{-1}(0) (1-\alpha) + \alpha R^{-1}(1) \right]
= [a (1-\alpha) + \alpha b, (1-\alpha) d + \alpha c]
= [\bar{u}_F, u_F^+] .
\] (3)

Let \( F_1, F_2 \in F(\mathbb{R}) \) then we have,

\[
[u_{F_1+F_2}]^{\alpha} = [u_{F_1}]^{\alpha} + [u_{F_2}]^{\alpha} .
\] (4)

Let \( F_1, F_2 \in F(\mathbb{R}) \) then binary relation \( \succeq \) defined on \( F(\mathbb{R}) \) as recalled below ([11, 12]) . For all \( \alpha \in [0, 1] \)

\[
F_1 \succeq F_2 \iff [u_{F_1}]^{\alpha} \succeq [u_{F_2}]^{\alpha} \iff u_{F_1}^- \geq u_{F_2}^- \text{ and } u_{F_1}^+ \geq u_{F_2}^+ .
\] (5)
Let $\mathcal{F}_1 = (a_1, a_2, a_3, a_4)$ and $\mathcal{F}_2 = (b_1, b_2, b_3, b_4) \in \mathcal{F}_T (\mathbb{R})$ be two trapezoidal fuzzy intervals and $\lambda \in \mathbb{R}^+$, then following conditions are holds:

i. $\mathcal{F}_1 + \mathcal{F}_2 = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$,

ii. $\lambda \mathcal{F}_1 = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)$,

iii. $\mathcal{F}_1 \triangleright \mathcal{F}_2 \iff a_1 \geq b_1, a_2 \geq b_2, a_3 \geq b_3, \text{ and } a_4 \geq b_4$.

Let $\mathcal{F}_1, \mathcal{F}_2 \in \mathcal{F}(\mathbb{R})$. If there exist $\mathcal{F}_3 \in \mathcal{F}(\mathbb{R})$ such that $\mathcal{F}_1 = \mathcal{F}_2 + \mathcal{F}_3$, then $\mathcal{F}_3$ is said to be Hukuhara difference between $\mathcal{F}_1$ and $\mathcal{F}_2$ denoted by $\mathcal{F}_3 = \mathcal{F}_1 - H \mathcal{F}_2$ [12].

Let $\mathcal{F}_1 = (a_1, a_2, a_3, a_4)$ and $\mathcal{F}_2 = (b_1, b_2, b_3, b_4) \in \mathcal{F}_T (\mathbb{R})$ be two trapezoidal fuzzy intervals. If $a_i - b_i \leq a_j - b_j$ for all $i \leq j$ then the Hukuhara difference between $\mathcal{F}_1$ and $\mathcal{F}_2$ is given by:

$$\mathcal{F}_1 - H \mathcal{F}_2 = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4).$$

**Remark 2.1.** If $\mathcal{F}_1, \mathcal{F}_2 \in \mathcal{F}(\mathbb{R})$, then instead of $\mathcal{F}_1 - H \mathcal{F}_2$ we often write $\mathcal{F}_1 - \mathcal{F}_2$ whole paper.

### 2.2. Fuzzy interval games.

A cooperative fuzzy interval game is a pair $< N, \mathcal{U} >$ where $N = \{1, 2, ..., n\}$ is the set of players and $\mathcal{U} : 2^N \to \mathcal{F}(\mathbb{R})$ maps the coalitions $S \in 2^N$ into fuzzy intervals $\mathcal{U}(S) \in \mathcal{F}(\mathbb{R})$ with $\mathcal{U}(\emptyset) = 0$, here 0 is a fuzzy interval with membership function given by [19]:

$$u_0(x) = \begin{cases} 1, & x = 0 \\ 0, & x \neq 0. \end{cases}$$

(7)

It is obvious that above definition is extension of cooperative interval games sense of [1] and the classical TU-games. We denote by $\mathcal{F}(\mathbb{R})^N$ the set of all such fuzzy payoff vectors.

Let $< N, \mathcal{U} >$ be a cooperative fuzzy interval game. We call fuzzy interval core ($\mathcal{F}$-core in short) in the following set [19]:

$$C(\mathcal{U}) = \left\{ (\mathcal{F}_1, ..., \mathcal{F}_n) \in \mathcal{F}(\mathbb{R})^N : \sum_{i=1}^{n} \mathcal{F}_i = \mathcal{U}(N) \; \text{and} \; \sum_{i \in S} \mathcal{F}_i \triangleright \mathcal{U}(S) \; \forall S \subseteq N \right\}.$$  

### 3. On cooperative fuzzy bubbly games.

In this section, we inspired by [24]. Next, we introduce the fuzzy bubble.

**Definition 3.1.** Let $\mathcal{P}, \mathcal{P}^* \in \mathcal{F}(\mathbb{R})$. A fuzzy bubble is defined by $\mathcal{B} = \mathcal{P} - \mathcal{P}^*$, where $\mathcal{P}$ is the risky asset logarithmic fuzzy price and $\mathcal{P}^*$ is the fundamental logarithmic fuzzy price.

According to the Definition 3.1, we can say that a fuzzy bubble is a special kind of a fuzzy interval and $\mathcal{B} \in \mathcal{F}(\mathbb{R})$.

Now, we introduce a cooperative fuzzy bubbly game.

**Definition 3.2.** A cooperative fuzzy bubbly game is an ordered pair $< N, \tilde{\mathcal{U}} >$, where $N = \{1, ..., n\}$ is the set of firms, and $\tilde{\mathcal{U}} : 2^N \to \mathcal{F}(\mathbb{R})$ the characteristic function which assings to each coalition $S \in 2^N$ a bubble such that $\tilde{\mathcal{U}}(\emptyset) = 0$, where 0 is a fuzzy bubble with membership function given in (7).

In this study, the value of a coalition $S$ is considered to be the sum of rewards/costs of the coalition $S$ that can gain by means of an admissible rearrangement; then we obtain the corresponding fuzzy bubbly game $< N, \tilde{\mathcal{U}} >$, as follows:

$$\tilde{\mathcal{U}}(S) = \sum_{i \in S} \mathcal{B}_i.$$
Here $B_i$ is the fuzzy bubble of the $i$–th firm.

Next, we introduce the fuzzy bubbly core for cooperative fuzzy bubbly games.

**Definition 3.3.** Let $< N, \tilde{U} >$ be a cooperative fuzzy bubbly game. Its *fuzzy bubbly core* is defined by

$$C(\tilde{u}) = \left\{ (B_1, \ldots, B_n) \in \mathcal{F}(\mathbb{R})^N \mid \sum_{i \in N} B_i = \tilde{U}(N), \sum_{i \in S} B_i \supseteq \tilde{U}(S), \forall S \subseteq N \right\}. $$

The following example illustrates a cooperative fuzzy bubbly game arising from a real life situation.

**Example 3.4.** We consider three firms say Firm 1, Firm 2 and Firm 3 which consider cooperation. Here we use trapezoidal fuzzy intervals and they are chosen arbitrary. The risky asset logarithmic fuzzy prices of the three firms are $\mathcal{P}_1 = (4, 6, 8, 10)$, $\mathcal{P}_2 = (3, 5, 8, 11)$ and $\mathcal{P}_3 = (5, 9, 13, 16)$, the fundamental logarithmic fuzzy prices of the three firms are $\mathcal{P}_1^* = (1, 2, 3, 4)$, $\mathcal{P}_2^* = (2, 3, 5, 6)$ and $\mathcal{P}_3^* = (4, 6, 7, 9)$ respectively. Then, the bubbles of these firms are $B_1 = (3, 4, 5, 6)$, $B_2 = (1, 2, 3, 5)$ and $B_3 = (1, 3, 6, 7)$. This situation can be modeled by using cooperative fuzzy bubbly games such that $N = \{1, 2, 3\}$ is the set of firms and the fuzzy interval characteristic functions are:

$$u_{\tilde{U}(1)}(x) = \begin{cases} x - 3 & 3 \leq x < 4 \\ 1 & 4 \leq x \leq 5 \\ 0 & 0 \end{cases}, \quad u_{\tilde{U}(2)}(x) = \begin{cases} x - 1 & 1 \leq x < 2 \\ 1 & 2 \leq x \leq 3 \\ 0 & 0 \end{cases}$$

$$u_{\tilde{U}(3)}(x) = \begin{cases} \frac{x - 1}{2} & 1 \leq x < 3 \\ 1 & 3 \leq x \leq 6 \\ 0 & 0 \end{cases}, \quad u_{\tilde{U}(3)}(x) = \begin{cases} \frac{x - 4}{2} & 4 \leq x < 6 \\ 1 & 6 \leq x \leq 8 \\ 0 & 0 \end{cases}$$

The fuzzy bubbly core vector $(B_1, B_2, B_3)$ where $u_{B_1} = u_{\tilde{U}(1)}$, $u_{B_2} = u_{\tilde{U}(2)}$, $u_{B_3} = u_{\tilde{U}(3)}$ belongs to the fuzzy bubbly core and gives $B_1$ fuzzy bubbly payoff to firm 1, $B_2$ fuzzy bubbly payoff to firm 2 and $B_3$ fuzzy bubbly payoff to firm 3.

**Remark 3.5.** In economy and finance, it is a basic fact that the difference between the lower bound and the upper bound of the asset logarithmic price is always more than the difference between the lower bound and the upper bound of the fundamental logarithmic price. Consequently, the **Hukuhara difference** which is used throughout this paper is always defined for the class of cooperative fuzzy bubbly games.

Random fluctuation, or “diffusion”, appears if the asset price is under risk. Risky (logarithmic) prices are related with the particular assets regarded and, of course,
by their interrelations which are expressed by covariances, i.e., by first and second 
order and, if needed, even higher-order moments. Those particular and risky prices 
are compared with general prices which are often taken as some average related 
to a “population” of assets, in other words: to the asset prices in some reference 
economy. Another fundamental price is a one assessed from a neutral or risk-neutral 
position, where, e.g., the typical “fever” which accompanies a financial bubble and 
drives its trade price, is not taken into consideration.

4. On the fuzzy bubbly core. In this section, we inspired by [24]. Firstly, we 
define the fuzzy bubbly-balanced games.

Recall that a map \( \lambda : 2^N \setminus \{ \emptyset \} \rightarrow \mathbb{R}^+ \) is called a balanced map if

\[
\sum_{S \in 2^N \setminus \{ \emptyset \}} \lambda (S) e^S = e^N.
\]

Here \( e^S \) is the characteristic vector for coalition \( S \) with

\[
e^S_i = \begin{cases} 
1, & \text{if } i \in S, \\
0, & \text{if } i \in N \setminus S.
\end{cases}
\]

Definition 4.1. Let \( < N, \tilde{U} > \) be a cooperative fuzzy bubbly game. We say that 
\( < N, \tilde{U} > \) is \textbf{fuzzy bubbly-balanced} (in short: FB-balanced) if for each balanced 
map \( \lambda : 2^N \setminus \{ \emptyset \} \rightarrow \mathbb{R}^+ \) we have \( \sum_{S \in 2^N \setminus \{ \emptyset \}} \lambda (S) \tilde{U} (S) \leq \tilde{U} (N) \).

The proof of the following proposition is straightforward (see [1, 19]).

Proposition 4.2. If a cooperative fuzzy bubbly game has a non-empty bubbly core 
then it is FB-balanced.

Secondly, we illustrate the use of Proposition 4.2 in a real world situation which 
possesses fuzzy bubbly uncertainty.

Example 4.3. Consider the game in Example 3.4. Here, we can see that the game 
has a non-empty fuzzy bubbly core. By Proposition 4.2 we conclude that the game 
is FB-balanced.

Before closing Section 4, we investigate some basic properties of the bubbly core.

Proposition 4.4. Let \( < N, \tilde{U} > \) be a cooperative fuzzy bubbly game. Then, the 
fuzzy bubbly core \( \mathcal{C} (\tilde{U}) \) is a convex set.

Proposition 4.5. Let \( < N, \tilde{U} > \) be a cooperative fuzzy bubbly game. Then, the 
fuzzy bubbly core \( \mathcal{C} (\tilde{U}) \) is a superadditive map.

Proof. We have to prove that \( \mathcal{C} (\tilde{U}_1 + \tilde{U}_2) \geq \mathcal{C} (\tilde{U}_1) + \mathcal{C} (\tilde{U}_2) \). First, we note that 
the inclusion holds if \( \mathcal{C} (\tilde{U}_1) = \emptyset \) or \( \mathcal{C} (\tilde{U}_2) = \emptyset \). Otherwise, we take \((I_1, ..., I_n) \in \mathcal{C} (\tilde{U}_1) \) and \((J_1, ..., J_n) \in \mathcal{C} (\tilde{U}_2) \). Then,

\[
\sum_{k \in N} I_k + \sum_{k \in N} J_k = \tilde{U}_1 (N) + \tilde{U}_2 (N),
\]

\[
\sum_{k \in N} (I_k + J_k) = (\tilde{U}_1 + \tilde{U}_2) (N).
\]
For each $S \in 2^N \setminus \{\emptyset\}$, $\sum_{k \in S} u^+_I \geq \tilde{u}_1(S)$ and $\sum_{k \in S} u^+_J \geq \tilde{u}_2(S)$ implying that $\sum_{k \in S} u^+_I \geq u^+_\tilde{u}_1(S)$ and $\sum_{k \in S} u^+_J \geq u^+_\tilde{u}_2(S)$. Then, for each $S \in 2^N \setminus \{\emptyset\}$,

$$\sum_{k \in S} u^+_I + \sum_{k \in S} u^+_J \geq u^+_\tilde{u}_1(S) + u^+_\tilde{u}_2(S),$$

$$\sum_{k \in S} [u^+_I + u^+_J]^+ \geq u^+_\tilde{u}_1(S) + u^+_\tilde{u}_2(S).$$

Similarly,

$$\sum_{k \in S} [u^+_I + u^+_J]^+ \geq u^-\tilde{u}_1(S) + u^-\tilde{u}_2(S).$$

Hence, the fuzzy bubbly core $C(\tilde{u})$ is a superadditive map.

5. Conclusion and outlook. Our study is motivated by the need to collaborate, e.g., for overcoming environmental challenges, by various causes of uncertainty which exist in the real world. Further, we introduce a new solution concept, the bubbly core, for cooperative fuzzy bubbly games. Since bubbles allow including correlations among players, e.g., from the financial viewpoint and thus go beyond intervals this solution concept is a generalization of the fuzzy interval core for cooperative fuzzy interval games [19, 20, 21].

The authors of [15] introduce a new alternative geometrical approach to financial bubbles which is supported by modern optimization, the theory of inverse problems and machine learning. They also introduce early-warning signalling for financial bubbles by benefitting from the theory of optimization, inverse problems and clustering methods [16].

Whenever we want to particularly address items to the financial sector among the target variables or the environmental variables which, in fact, maybe be regarded in a dual relationship mutually, then we arrive at eco-finance networks [31]. This interpretation and variety of our studies also represents that the identification of dynamics related with the Kyoto Protocol, where financial expenditures and emissions reduced interact in time (TEM model) [15, 18, 26]. Financial negotiation processes, represented in the way of collaborative game theory [30], and the identification and dynamics of financial processes given by stochastic differential equations and their time-discretized versions [32], are an important part and an extension of our research.

In this paper we extend cooperative bubbly games to cooperative fuzzy bubbly games, where the worth of each coalition is a fuzzy bubble instead of an interval. Consequently, fuzzy bubbles could be represented as a mathematical object; new achievements in optimization, pattern recognition tools and machine learning can be used to prepare different solution techniques to detect, forecast and control their size, shape and position in Operational Research, climate negotiations and policy, environmental management and pollution control, etc. In this study we motivated by game theoretical extension of cooperative bubble games under fuzzy uncertainty. It may be interesting to do studies involving numerical tests and simulations for future work.

Acknowledgments. The authors are grateful to the referees for their suggestions, which have greatly improved the readability of the paper.
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Received July 2020; revised January 2021.

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