On the dark energy rest frame and the CMB
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Abstract. Dark energy is usually parametrized as a perfect fluid with negative pressure and a certain equation of state. Besides, it is supposed to interact very weakly with the rest of the components of the universe and, as a consequence, there is no reason to expect it to have the same large scale rest frame as matter and radiation. Thus, apart from its equation of state \( w \) and its energy density \( \Omega_{DE} \), one should also consider its velocity as a free parameter to be determined by observations. This velocity defines a cosmological preferred frame, so the universe becomes anisotropic and, therefore, the CMB temperature fluctuations will be affected, modifying mainly the dipole and the quadrupole.

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INTRODUCTION

Recent observations [1, 2, 3] suggest that the universe could be dominated by a fluid with negative pressure [4, 5] which has been called dark energy (DE). This DE component is usually assumed to behave as a perfect fluid with (possibly) a time-evolving equation of state \( p_{DE} = w_{DE}(z)\rho_{DE} \) and whose interactions with the rest of components of the universe are very weak [5]. Therefore, apart from the density parameter \( \Omega_{DE} \) and equation of state \( w_{DE} \), a complete knowledge of its energy-momentum tensor requires the determination of its relative velocity with respect to the rest of components of the universe or, in other words, its large scale rest frame. In fact, recent measurements of peculiar velocities of matter bulks with respect to the CMB [6] have shown the existence of a coherent matter flow on scales \( \lesssim 300 h^{-1}\) Mpc. These observations suggest that matter and radiation rest frames could differ from each other at large scales even though they were strongly coupled before recombination. Thus, it makes sense to ask about DE large scale rest frame given that it is supposed to interact very weakly with the rest of particles of the universe. Moreover, in [7] it is shown that the presence of a moving DE component at the epoch when photons decouple from baryons could straightforwardly account for the observed dark flow of matter with respect to radiation. In this paper we show how the presence of a moving DE component would affect the CMB temperature fluctuations.

COSMOLOGY WITH MOVING FLUIDS

We consider a universe filled with four homogeneous perfect fluids, namely: baryons (B), radiation (R), dark matter (DM) and dark energy (DE), so that the total energy-momentum tensor reads \( T^{\mu\nu} = \sum_{\alpha} [(\rho_\alpha + p_\alpha)u_\alpha^\mu u_\alpha^\nu - p_\alpha g^{\mu\nu}] \), where \( \alpha = B, R, DM, DE \) and \( u_\alpha = \gamma_\alpha(1, v_\alpha) \) are the velocities of the fluids with \( \gamma_\alpha \) a normalization factor de-
term by \( u_{\alpha \mu} u_{\alpha}^{\mu} = 1 \). Besides, every fluid satisfies a barotropic equation of state: \( p_{\alpha} = w_{\alpha} \rho_{\alpha} \). We shall also study the case in which DE is a null fluid (a fluid moving at the speed of light) for which the previous energy-momentum tensor is still valid, but with \( u_{DE \mu} u_{DE}^{\mu} = 0 \). We can see that, in general, the \((0, i)\) component of Einstein equations yields the following algebraic relation:

\[
g_{0i} \equiv S_i = \frac{\sum_{\alpha} g_{\alpha}^{2} (\rho_{\alpha} + p_{\alpha}) v_{i \alpha}}{\sum_{\alpha} g_{\alpha} (\rho_{\alpha} + p_{\alpha})},
\]

The combination \( \rho_{\alpha} + p_{\alpha} \) appearing in this expression is usually interpreted in General Relativity as the density of inertial mass of the fluid so that we can interpret \( \bar{S} \) as the cosmic center of mass (CCM) velocity. Notice that a pure cosmological constant with equation of state \( w_{DE} = -1 \) has vanishing inertial mass and, therefore, does not contribute to the CCM velocity.

Since matter and radiation were coupled in the early universe, their velocities must lie along the same direction. On the other hand, in the CCM rest frame, where \( \bar{S} = 0 \), the DE velocity must lie along the opposite direction according to \((1)\) so that we have axial symmetry around the direction given by the velocities and, if we choose the velocities lying along the z-axis, the metric will be given by \( ds^{2} = dt^{2} - a_{\perp}^{2} (dx^{2} + dy^{2}) - a_{||}^{2} dz^{2} \).

However, since observations show that the anisotropy of the universe, if any, is small, we can assume that \( a_{\perp} = a (1 + \delta_{\perp}) \), with \( a \) the usual scale factor and \( \delta_{\perp} \ll 1 \). Moreover, one can define the degree of anisotropy by means of \( h = 2 (\delta_{\perp} - \delta_{\parallel}) \), whose solution according to Einstein equations to first order in \( \delta \)'s is given by \((2)\):

\[
h = 6 \int_{a_{*}}^{a} \frac{1}{d\tilde{a}} \left[ \int_{a_{*}}^{\tilde{a}} \sum_{\alpha} (\rho_{\alpha} + p_{\alpha}) \sinh^{2} \theta_{\alpha} \frac{d\tilde{a}}{\sqrt{\sum_{\alpha} \rho_{\alpha}}} \right] \frac{d\tilde{a}}{\sqrt{\sum_{\alpha} \rho_{\alpha}}},
\]

where we assume that the universe is initially isotropic, i.e. \( h(a_{*}) = h(a_{*}) = 0 \) and \( \theta_{\alpha} \) is the rapidity, defined by \( \tanh \theta_{\alpha} = a_{||} v_{\alpha} \equiv V_{\alpha} \). When the velocities are small we can approximate \( \sinh^{2} \theta_{\alpha} \simeq V_{\alpha}^{2} \) so that we conclude that the first contribution to the anisotropy is of second order in the velocities. The solution given by \((2)\) is still valid when we consider a null fluid if we set \( \sinh^{2} \theta_{DE} = 1 \) and, in such a case, the dominant contribution to the degree of anisotropy comes from the null fluid and we can neglect the rest of components in the sum. Apart from Einstein equation, we also need the energy and momentum conservation equations for each fluid which can be solved in two limits:

- **Slow-moving fluids.** When the velocities are small, the energy densities are unaffected by the motion and we have the usual evolution: \( \rho_{\alpha} = \rho_{0\alpha} a^{-3(w_{\alpha} + 1)} \) whereas the velocities evolve according to \( V_{\alpha} = V_{0\alpha} a^{3w_{\alpha} - 1} \).
- **Fast-moving fluids.** In the ultrarelativistic limit, the physical velocity remains constant and the energy density evolves according to \( \rho_{\alpha} = \rho_{0\alpha} a^{-2(1+w)/(1-w)} \).

For a null fluid the energy-momentum conservation equations allow us to obtain the solutions \( p_{N} = p_{N0} \) and \( \rho_{N} = \rho_{0N} (a_{||}/a_{\perp})^{-2} - p_{N0} \) without assuming any particular equation of state. Moreover, if we require the energy density to be positive at all times we have to impose \( p_{N0} < 0 \) so that a null fluid behaves as radiation during the early universe and as a cosmological constant at late times.
CONTRIBUTIONS TO THE CMB

The motion of the fluids has effects on CMB temperature fluctuations through the Sachs-Wolfe effect. To first order in the velocities it only affects the dipole according to the following expression $[9]$: 

$$ \langle \delta T / T \rangle_{\text{dipole}} = \bar{n} \cdot (\vec{S} - \vec{V})_{0}^{\text{dec}} $$

where 0 and dec denote present and decoupling times respectively. Thus, the dipole must be interpreted as a Doppler effect due to the relative motion of the observer with respect to the cosmic center of mass, and not just with respect to the CMB rest frame. That way, an observer who measures a vanishing dipole would be at rest with respect to the cosmic center of mass. This cosmological dipole contribution can be identified with the anomalous dipole detected in $[6]$ that gives rise to the dark flow of matter. In fact, its observed amplitude can be related to the present velocity of DE which, assuming $w_{DE}^{0} \approx -0.97$, can be estimated to be $v_{DE}(t_{0}) \sim 1 \text{km/s}$. $[7]$

In the case of small anisotropy, it is possible to see that the temperature fluctuation has two contributions $[10]$: 

$$ \delta T = \delta T_{I} + \delta T_{A} $$

where $\delta T_{I}$ is the fluctuation produced during inflation and $\delta T_{A}$ is due to the anisotropy generated by the motion of the fluids, which is given by $[8]$: 

$$ (\delta T_{A} / T) = 2 |h_{0} - h_{\text{dec}}| / (5 \sqrt{3}) $$

For an arbitrary direction of the velocities $(\theta, \phi)$ and assuming a statistically isotropic distribution for inflation fluctuations, we get 

$$ (\delta T_{I})^{2} = (\delta T_{A})^{2} + f(\theta, \phi, \alpha) \delta T_{I} \delta T_{I} $$

where $\alpha$ are random phase factors coming from inflation and $f$ is a function satisfying $|f| \leq 0.98$. Then, the total quadrupole lies between $(\delta T)_{-}^{2}$ and $(\delta T)_{+}^{2}$ with $(\delta T)_{-}^{2} = (\delta T)_{A}^{2} + (\delta T)_{I}^{2} \pm 0.98 \delta T_{A} \delta T_{I}$. On the other hand, the observed quadrupole is in the range $91.51 \mu K^{2} \leq (\delta T)_{obs}^{2} \leq 406.48 \mu K^{2}$ (including the cosmic variance). Thus, if we assume that inflation contribution agrees with the central measured value, i.e., $(\delta T)_{I}^{2} = 247 \mu K^{2}$, the anisotropic contribution should satisfy $(\delta T)_{A}^{2} \lesssim 1254 \mu K^{2}$, which defines the allowed region for the dark energy models. However, standard inflation predicts a larger value: $(\delta T)_{I}^{2} = 1252 \mu K^{2}$. In such a case, if $247.90 \mu K^{2} \lesssim (\delta T)_{A}^{2} \lesssim 2883.80 \mu K^{2}$ then the fluids motion could explain the low observed quadrupole for certain values of dark energy velocity and phase factors.

Model examples

- **Constant equation of state.** For a model with constant equation of state close to $-1$ (but different from $-1$) we find that the velocities are extremely small so that all the fluids are nearly at rest and the effect on the quadrupole is completely negligible.

- **Scaling models.** In these models the equation of state of DE mimics that of the dominant component of the universe eventually exiting from this regime and joining into one with constant equation of state close to $-1$. The quadrupole generated by scaling models is fixed by two parameters: the initial DE fraction $\epsilon$ and its initial velocity $V_{DE}^{*}$, and for small velocities it is given by $\delta T_{A} / T \approx 0.44 \epsilon (V_{DE}^{*})^{2}$. Then, if we take into account the bounds obtained above we get that the allowed region is: $\epsilon (V_{DE}^{*})^{2} \lesssim 2.9 \times 10^{-5}$. Moreover if $1.3 \times 10^{-5} \lesssim \epsilon (V_{DE}^{*})^{2} \lesssim 4.3 \times 10^{-5}$ these models could explain the low quadrupole.

- **Tracking models.** In tracking models the equation of state of DE is initially close to 1 so that the velocity grows as $a^{2}$ until it reaches the speed of light and then it
remains constant according to the solution for fast moving fluids. In that regime, the energy density starts falling extremely fast and eventually becomes completely negligible, giving rise to a fluid of vanishing energy density moving at the speed of light. Therefore, we can conclude that tracking models are unstable against velocity perturbations.

- **Null Dark Energy.** When DE is described by a null fluid, the quadrupole is fixed just by $\epsilon$. For small velocities, the quadrupole is approximately given by $\delta T^A_A/T \simeq 2.58\epsilon$ and the constraints on the anisotropic contribution lead to following allowed region: $\epsilon \lesssim 5 \times 10^{-6}$. Again, if $2.2 \times 10^{-6} \lesssim \epsilon \lesssim 7.6 \times 10^{-6}$, these models could explain the observed quadrupole with the standard contribution from inflation.

**CONCLUSIONS**

In this work we have shown how a moving DE fluid can generate large-scale anisotropy starting from an isotropic universe and how this can affect the CMB temperature fluctuations. In particular, we have seen that in such a case, the CMB dipole is due to the relative motion of the observer with respect to the cosmic center of mass and, besides, it could explain the observed dark flow of matter. Concerning the quadrupole, we obtain bounds on the anisotropic contribution by comparing with observations and found that scaling and null DE models succeed in explaining the low measured value starting from the standard contribution from inflation.

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