Spin-tensor decomposition: 
A useful tool for shell model effective interaction

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Abstract. The spin-tensor decomposition is employed to construct a new interaction, named CKHeN, for 0p-shell. This new interaction is used to calculate the effective single-particle energies of πp3/2 and πp1/2 orbitals in Li isotopes, and the level structures of ⁷Li and ⁸Li isotopes. The calculated level structures are found in good agreement with experimental data.

1 Introduction
Spin-tensor decomposition (STD) is a useful tool to decompose the model-space dependent shell model effective two-nucleon interaction into its central, spin-orbit and tensor force structure [1]. For last one and half decades, it has been used with the aim to understand the role of different components of two-nucleon interaction in the shell evolution in neutron-rich nuclei [2–4]. It has also been used to show why microscopic shell model interactions fail to describe the shell evolution in neutron-rich nuclei [3]. In this study, we use STD for the CK(8-16) interaction derived for 0p-shell nuclei, and examine the properties of its total spin \( J \) averaged proton-neutron central and tensor force matrix elements;

\[
\mathcal{V}^{\pi \nu}_{jj'} = \frac{\sum J (2J+1) \mathcal{V}^{\pi \nu}_{jj'}}{(2J+1)(2j'+1)},
\]

where, sum runs only over the Pauli principal allowed \( J \) values.

For bare-tensor force, Otsuka et. al. [6], showed that proton-neutron interaction \( \mathcal{V}^{\pi \nu}_{jj'} \) corresponding to proton spin-up orbital \( (j_\pi = l + 1/2) \) and neutron spin-down orbital \( (j'_n = l' - 1/2) \) (or vice-versa) is attractive, whereas, if both proton and neutron orbitals are spin-up (or spin-down), the interaction is repulsive. It is also demonstrated that bare-tensor force matrix elements barely change and hold its nature after dealing with short-range repulsion part of two-nucleon problem and in-medium effects [7]. Furthermore, the numerical analysis shows that tensor interaction of well-established shell model effective interaction, e.g., USDB, has same nature as for bare-tensor force.

The proton-neutron central \( \mathcal{V}^{\pi \nu} \) of shell model effective interaction is found to possess strong-orbital node property of central interaction has been analytically demonstrated by Smirnova et. al., [9] using the spin-exchange zero-range δ potential.

In Fig. 1, we show proton-neutron central and tensor matrix elements of the CK(8-16) interaction. Here, central force matrix elements do not have similar strength which manifest that central component of CK(8-16) lacks weak-spin dependency. Further, tensor force matrix elements are present with opposite nature than its regular nature. In present work, we construct a new interaction in which these discrepancies are not present.

2 Spin-tensor decomposition and New effective interaction - CKHeN
Spin-tensor decomposition: Nucleons are intrinsic spin 1/2 fermions; therefore, the interaction between two-nucleon can be written as the linear sum of scalar product of configuration space operator \( Q \) and spin space operator \( S \) of...
3 Shell model calculations

The CKHeN interaction consists of $p_{3/2}$ and $p_{1/2}$ orbitals for protons and neutrons above $^4$He core, and is tested for the effective single-particle energies (ESPEs) of $\pi_0p_{3/2}$ and $\pi_0p_{1/2}$ orbitals in Li isotopes and the level structures of $^{7,8,9}$Li isotopes. The ESPE ($\varepsilon'$) of $\pi$-orbit orbital in Li isotopes is given by [8]:

$$\varepsilon'_p = \varepsilon_p + \sum_{j'} n_{j'} V^{\pi}_{jj'} \tag{3}$$

where, $\varepsilon_p$ is unperturbed single-particle energy of orbital $j$, and $n_{j'}$ is number of neutrons in orbital $j'$. The level structures of $^{7,8,9}$Li isotopes are calculated using shell model code-NUSHELLEX@MSU [12]. The calculated ESPEs and level structures are shown in Fig. 1 and 2, respectively. The results obtained using the CK(8-16) interaction are also shown in these figures.

With the CKHeN interaction, the energy gap between spin-orbital partners, $\pi_0p_{3/2}$ and $\pi_0p_{1/2}$, in Li isotopes is found to remain nearly constant (see Fig. 1). This is similar to the observation seen in F and Sc isotopes where the energy gap between $\pi_0d_{5/2}$ and $\pi_0d_{3/2}$ orbitals, and $\pi_0f_{5/2}$ and $\pi_0f_{3/2}$ orbitals, respectively, remains nearly constant [8, 13]. However, with the CK(8-16) interaction, the energy gap between spin-orbital partners in Li isotopes increases when neutrons occupy $\pi_0p_{3/2}$ orbital, which is not consistent with the systematic.

In Fig. 2, the CKHeN interaction is shown to reasonably predict the experimental low-lying states of $^{7,8,9}$Li relative to the CK(8-16) interaction.

4 Summary

In this work, spin-tensor decomposition is employed to discuss the discrepancies of CK(8-16) interaction and to construct a new effective interaction, named CKHeN, for $0p$-shell. The new interaction is tested for Li isotopes and found reasonably predicting their spectroscopic properties.

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