INFLUENCE OF THE SOURCE EVOLUTION
ON PARTICLE CORRELATIONS

I.V.ANDREEV

P.N.Lebedev Physical Institute, Moscow, Russia

Abstract

Modification of the particles in the course of the source evolution is considered. Influence of this effect on multiplicities and correlations of the particles is displayed, including an enhancement of the production rates and identical particle correlations and also back-to-back particle-antiparticle correlations.

Classification codes: 13.85.Hd, 13.85.Ni
Keywords: multiple production, correlations, evolution, particle source.
1 Introduction

In this paper we consider effects which arise when particles are initially produced inside hadronic matter (particle source) and only afterwards, at the very last stage of the process, fly away as free particles. Being produced in the hadronic medium they represent a part of the medium, rather quasiparticles than free particles. So their spectrum $E(k)$ changes in the course of the source evolution (say expansion).

Such a picture leads to specific observable effects. First, production rates and identical pion correlations (HBT effect) can be amplified significantly. Second, back-to-back particle-antiparticle correlations (PAC) appear which are not necessarily small if quasiparticles differ essentially from free particles.

Let us remind in this connection that momentary transformation of quasiparticles into free particles was first considered [1] in the frame of simple oscillator model. Chaotic squeezed states of the particles arise in this model giving rise to enhancement of single-particle inclusive production and identical particle correlations, but antiparticles (and so PAC) are absent in the model. On the other hand, the presence of back-to-back PAC was noted [2] some time ago for particle production in the vacuum. However in this case PAC are suppressed (no source evolution effect in this case). Unsuppressed back-to-back correlations arising due to transformation of neutral quasiparticles into free particles were found recently in ref. [3]. This kind of correlations (including charged pions) was also suggested quite recently [4] as a probe of disoriented chiral condensate (DCC) formation. Here we consider pion creation inside a finite size source which undergoes time evolution (having finite lifetime). Both kinds of effects will be found, irrespective of DCC formation.

Below we consider multiple production processes at high energies. It is suggested that after collision an excited volume is formed which undergoes evolution. The last (hadronic) stage of the evolution is under consideration. So the specific effects discussed below are expected to be absent in those scenarios of particle production which do not treat the hadronization stage in detail. In particular one can hardly expect these effects in existing event generators though they are natural in hydrodynamical model.
Neutral pion production and evolution

Neutral pions are represented by real valued field $\varphi(x, t)$. Its decomposition has the form

$$\varphi(x, t) = \int \frac{d^3 k}{(2\pi)^{3/2}(2E_k)^{1/2}} \left[ a_k e^{-iEt+i\mathbf{k}\mathbf{x}} + a_k^\dagger e^{iEt-i\mathbf{k}\mathbf{x}} \right]$$

(1)

where annihilation and creation operators $a_k, a_k^\dagger$ satisfy canonical commutation relations.

To describe the field emission we introduce an effective classical current $j(x, t)$ (it is the practice in HBT effect study [2, 3]). The currents encode the space-time region of the particle production including time duration of the process. Clearly the currents represent some other particles producing pions. The current is considered as a random function and subject of averaging (chaotic source). Production cross-sections and correlations of identical particles can be now expressed through the mass shell Fourier transform of the currents:

$$j(E_k, \mathbf{k}) = \int dt e^{iE_k t} j_k(t) = \int d^4 x e^{iE_k t-i\mathbf{k}\mathbf{x}} j(x, t)$$

(2)

In particular the field correlators are:

$$< a_k^\dagger a_{k_2} >_0 = \frac{1}{2\sqrt{E_1 E_2}} < j^*(E_1, \mathbf{k}_1) j(E_2, \mathbf{k}_2) >$$

(3)

and, up to small corrections,

$$< a_{k_1} a_{k_2} >_0 = 0$$

(4)

where brackets include statistical averaging and index 0 in lhs of the equations indicates that the medium evolution is not taken into account.

If production takes place in a medium one has to modify the free propagation of the fields, say introduce some external potential. In our problem the time evolution of the medium is essential, so we confine ourselves to the homogenous sources. Then the problem is reduced to propagation of the quasiparticles having variable energy, $E_k = E_k(t)$. For example one may introduce variable mass $m(t)$ changing in the course of evolution with $m(\infty) = m_\pi$. 


Then our model is given by the Hamiltonian

\[
H = \frac{1}{2} \left[ \pi^2(x) + (\nabla \varphi)^2(x) + m^2(t) \varphi^2(x) \right] - j(x) \varphi(x), \quad \pi = \dot{\varphi}
\]  \(5\)

This Hamiltonian is similar to that of quantum oscillator with variable frequency in the presence of external force (which is the current now). Solution of the last problem is known for a long time \[6\]. It is helpful to represent the solution of the equations of motion in the form of canonical transformation of the creation and annihilation operators. The only new feature of the field theory model given by Eq.5 (in comparison with nonstationary quantum oscillator) is an appearance of two modes \(k\) and \(-k\) involved in the canonical transformation (as it was the case in the original Bogolubov transformation from particles to quasiparticles in superfluid \[7\]).

The resulting Bogolubov transformation solving the model given by Eq.5 has the form:

\[
\begin{pmatrix}
  a_k(t) \\
  a_{-k}(t)
\end{pmatrix}
= \begin{pmatrix}
  u & v \\
  v^* & u^*
\end{pmatrix}
\begin{pmatrix}
  a_k(0) + d_k(t) \\
  a_{-k}^*(0) + d_{-k}^*(t)
\end{pmatrix}
\]  \(6\)

with

\[
d_k(t) = \frac{i}{(2E_k)^{1/2}} \int_0^t dt_1 \xi(t_1) j_k(t_1)
\]  \(7\)

where \(\xi(t)\) is the classical solution for the oscillator with variable frequency and with initial conditions

\[
\xi(0) = 1, \quad \dot{\xi}(0) = iE_k(0)
\]  \(8\)

(we suggest that the currents are absent at \(t < 0\))

Coefficients of the transformation satisfy an equation

\[
|u|^2 - |v|^2 = 1
\]  \(9\)

and can be expressed through the same function \(\xi(t)\). So the form of \(\xi(t)\) is quite essential. In general it is an oscillating function with variable frequency and amplitude. What is more important, time reflected wave appears in \(\xi(t)\).
In particular keeping $E(t)$ constant in a small interval near some intermediate point $t = \bar{t}$ one gets

$$(2\bar{E})^{-1/2}\xi(\bar{t}) = \bar{u}^* e^{i\bar{E}\bar{t}} + \bar{v} e^{-i\bar{E}\bar{t}}, \quad \bar{E} = E(\bar{t})$$  \tag{10}$$

where $\bar{u}, \bar{v}$ are Bogolubov coefficients taken at $t = \bar{t}$ (with their oscillating time dependence canceled).

The appearance of time-reflected wave leads, through Eq.7 to two types of contributions to observable quantities like $< a^\dagger a >$. The first is of the form

$$\int \int dt_1 dt_2 e^{i(E_1 t_1 - E_2 t_2)} j(t_1) j(t_2)$$  \tag{11}$$

and the second is

$$\int \int dt_1 dt_2 e^{i(E_1 t_1 + E_2 t_2)} j(t_1) j(t_2)$$  \tag{12}$$

The contributions of the first type lead to unsuppressed correlations which depend on the difference $E_1 - E_2$ being maximal at $E_1 = E_2$. The second type contributions depend on the sum $E_1 + E_2$ (see ref. [2]) and they are always suppressed. Below we neglect the second type contributions keeping only "large" ones (of the first type). Note that expectation $< aa >$ also contains large contributions.

To get simple explicit results we simplify the above expressions. Firstly, the Bogolubov coefficients are taken to be real valued with

$$u = \cosh r, \quad v = \sinh r$$  \tag{13}$$

Secondly, the solution $\xi(t)$ is taken at a middle point $\bar{t}$, see Eq.10. We also suggest that the field expectations are absent at the initial moment $t = 0$ (no quasiparticles up to $t = 0$; presumably this is the moment of the phase transition to the hadronic phase). Then, using Eqs.6,7 we obtain expectation values at $t = \infty$:

$$\langle a^\dagger(k_1)a(k_2)\rangle = \cosh(r_1 + r_2 - \bar{r}_1 - \bar{r}_2) \frac{1}{2\sqrt{E_1 E_2}} \langle j^*(E_1, k_1) j(E_2, k_2) \rangle$$

$$+ \sinh r_1 \sinh r_2 F(k_1 - k_2)$$  \tag{14}$$

$$\langle a(k_1)a(k_2)\rangle_j = \sinh(r_1 + r_2 - \bar{r}_1 - \bar{r}_2) \frac{1}{2\sqrt{E_1 E_2}} \langle j^*(E_1, k_1) j(E_2, -k_2) \rangle$$
\[ \sinh(r_1 + r_2) F(k_1 + k_2) \] (15)

to be compared with Eqs. 3, 4.

In Eqs. 14, 15 \(r_i\) are full evolution parameters and \(\bar{r}_i\) are the parameters evaluated for medium evolution from initial moment \(t = 0\) up to a middle point \(t = \bar{t}\), see Eq. 10 (roughly \(\bar{r} = r/2\)). The first terms in rhs of Eqs. 14, 15 represent contributions of the pions produced by the currents (that is by some other particles) which are now enhanced in comparison with no evolution production, Eqs. 3, 4. The second terms in rhs of these equations contain form-factor \(F(k)\) which is Fourier transform of the initial volume \(V\),

\[ F(0) = V/(2\pi)^3 \] (16)

These terms represent the result of decay of the ground state of the medium. In another language, they represent parametric excitation of the field oscillators existing side by side with forced excitation if the oscillator parameters depend on time.

The evolution effect is determined by the evolution parameter \(r\) (the effect vanishes if \(r = 0\)). The value of \(r\) for every momentum \(k\) depends in general on initial energy \(E_{in}\), final energy \(E_f\) and characteristic time duration \(T\) of the hadronic stage of the process. The parameter \(r\) is maximal for small time duration

\[ r_{max} = \frac{1}{2} \ln \left( \frac{E_f}{E_{in}} \right), \quad ET \ll 1 \] (17)

and vanishes for \(ET \gg 1\) (adiabatic process).

As the final result we obtain expressions for inclusive production of neutral pions. Single-particle distribution is

\[ \frac{1}{\sigma} \frac{d\sigma}{d^3k}(k) = \langle a^\dagger(k)a(k) \rangle = \cosh(2r - 2\bar{r}) \frac{1}{2E} \langle |j(E, k)|^2 \rangle + \sinh^2 r \frac{V}{(2\pi)^3} \] (18)

where the first term is the usual production rate (up to pion energy modification) enhanced by \(\cosh\) factor arising due to source evolution. The second term gives an additional contribution due to ground state decay.

Two-particle inclusive distribution under consideration is given by

\[ \frac{1}{\sigma} \frac{d^2\sigma}{d^3k_1d^3k_2} = \langle a^\dagger_1a^\dagger_2a_1a_2 \rangle = \langle a^\dagger_1a_1 \rangle \langle a^\dagger_2a_2 \rangle + \langle a^\dagger_1a_2 \rangle \langle a^\dagger_2a_1 \rangle + \langle a^\dagger_1a_2 \rangle \langle a_1a_2 \rangle \] (19)
where the expectations in \( rhs \) are given by Eqs.14,15. The first term in \( rhs \) of Eq.19 is the product of single-particle distributions, the second term gives HBT effect and the third term describes back-to-back particle-antiparticle correlatios (PAC, see below) arising due to source evolution.

The relative correlation function which is measured in experiment is given by

\[
C(k_1, k_2) = \frac{\sigma d^2\sigma / d^3k_1 d^3k_2}{(d\sigma / d^3k_1)(d\sigma / d^3k_2)} = 1 + C_{HBT}(k_1, k_2) + C_{PAC}(k_1, k_2) \quad (20)
\]

The function \( C_{HBT}(k_1, k_2) \), describing HBT effect, reaches its maximum at \( k_1 - k_2 = 0 \) where

\[
C_{HBT}(k, k) = 1
\]

as usual though the slope of its momentum difference dependence is modified (it is diminished due to \( r \)-dependence, that is due to evolution effect, as one can see from Eqs.14,18).

The function \( C_{PAC}(k_1, k_2) \) describes PAC effect and gives an additional positive contribution to the correlation, this contribution being maximal at \( k_1 + k_2 = 0 \). The width of the PAC peak is expected to be close to that of HBT peak (compare Eq.14 and Eq.15). Its height depends crucially on evolution parameter \( r \), vanishing at \( r = 0 \) (see Eq.14) that is in the absence of the medium effect. The influence of the PAC effect on the function \( C(k_1, k_2) \) in Eq.20 at small momentum differences may be noticeable for soft neutral pions having small momenta \( k_1, k_2 \) and consequently small momentum differences \( k_1 - k_2 \). So one may expect an increase of the correlation function \( C(k_1, k_2) \) at all momenta \( k_i \) for soft neutral pions having \( |k| \leq 1/R \) where \( R \) is a characteristic size of the source. In particular from Eqs.14,15 and Eqs.18-20 we get an estimate:

\[
C(0, 0) \approx 2 + |\left( \frac{1}{\sigma} \frac{d\sigma}{dk} \right)_0 (k = 0) \sinh r + \frac{1}{2(2\pi)^3} \sinh^2 2r |^2
\]

(22)

where we put \( \bar{r} = r/2 \) and neglected pion energy modification in the current expectations so that the differential cross-sections in Eq.22 having index 0 are those without the evolution effect. Analogous equation is valid for \( \pi^+ - \pi^- \) correlations (see below). Numerical estimations of the PAC effect for different colliding particles will be given elsewhere.
3 Charged pion production

Consideration of charged pions is analogous to that of neutral ones. Now the field is complex valued and its decomposition reads

\[
\phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}(2E_k)^{1/2}} \left[ a_k e^{-iEt+ikx} + b_k^\dagger e^{iEt-ikx} \right] 
\]

(23)

containing annihilation operators \(a_k\) for particles (say \(\pi^-\)) and creation operators \(b_k^\dagger\) for antiparticles. Bogolubov transformation has a slightly different form in this case,

\[
a_k(t) = u [d_k + a_k(0)] + v [\tilde{d}_k + b_k(0)] \\
b_k(t) = u [\tilde{d}_k + b_k(0)] + v [d_k + a_k(0)] 
\]

(24)

containing function \(\tilde{d}_k\) which can be found from \(d_k\) by the substitution \(j(x,t) \rightarrow j^*(x,t)\).

As a result single-particle distributions of charged particles have the same form as those of neutral ones and two-particle distributions take the form

\[
\langle a_1^\dagger a_2^\dagger a_1 a_2 \rangle = \langle a_1^\dagger a_1 \rangle \langle a_2^\dagger a_2 \rangle + \langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_1 \rangle 
\]

(25)

for identical (like-sign) pions and

\[
\langle a_1^\dagger b_2^\dagger a_1 b_2 \rangle = \langle a_1^\dagger a_1 \rangle \langle b_2^\dagger b_2 \rangle + \langle a_1^\dagger b_2^\dagger \rangle \langle a_1 b_2 \rangle 
\]

(26)

for particle-antiparticle (\(\pi^+\pi^-\)) pair with

\[
\langle a_1 a_2 \rangle = \cosh(r_1 + r_2 - \bar{r}_1 - \bar{r}_2) \frac{1}{2 \sqrt{E_1 E_2}} (j^*(E_1, k_1) j(\bar{E}_2, k_2)) \\
+ \sinh r_1 \sinh r_2 F(k_1 - k_2) 
\]

(27)

\[
\langle a_1 b_2 \rangle = \sinh(r_1 + r_2 - \bar{r}_1 - \bar{r}_2) \frac{1}{2 \sqrt{E_1 E_2}} (j^*(E_1, k_1) j(\bar{E}_2, -k_2)) \\
+ \frac{1}{2} \sinh(r_1 + r_2) F(k_1 + k_2) 
\]

(28)
So like-sign pions have enhanced HBT correlations with additional ground state contribution (maximal at \( k_1 = k_2 \)) and unlike-sign pions acquire back-to-back correlations (maximal at \( k_1 = -k_2 \)) due to effect of source evolution. The terms of the form \(< a_1 a_2 >\) are suppressed for charged pions and they were omitted in the above equations. The resulting HBT and PAC contributions to the correlation functions of the charged pairs have the same form as for neutral pions in the previous section.

The physical interpretation of the above results is clear from the structure of the Bogolubov transformation. Quasiparticles in the source partly consist of free pairs (particle having momentum \( k \) and antiparticle having momentum \(-k\)). The pairs have opposite momenta of their constituents not to contribute to the total momentum of the quasiparticle (but influence its energy). The same is valid for the ground state of the hadronic medium (let us remind that we take into account time variation of the homogeneous source, say its selfsimilar expansion). The pairs release when the system decays into free particles. The HBT effect amplification is also dependent on these pairs.

The origin of doubled correlations between neutral pions is also clear. Neutral pions are identical particles and simultaneously they are antiparticles to themselves. So they show both types of correlations.

4 Conclusions

The evolution of the source in the course of the particle production process leads to enhancement of the production rates and to corresponding amplification of HBT effect characteristic for identical particles. Moreover the evolution leads to appearance of back-to-back particle-antiparticle correlations (PAC) which are not suppressed in general. The width of the corresponding peak (at \( k_1 = -k_2 \)) is close to that of the HBT peak (at \( k_1 = k_2 \)) and its height is given mainly by the evolution parameter \( r \). In turn the evolution parameter (which also determines the enhancement of single-particle distributions and HBT effect) depends on the spectrum of the quasiparticles in the source and on time duration of the production process, being larger for soft particles and for fast processes. This parameter can be determined as soon as PAC will be found in experiment.
Let us note in conclusion that the above effects are of rather general nature. Field evolution in the expanding Universe [8], Casimir effect in the volume with moving boundaries and particle production in strong electric field [9] are described in a similar way using Bogolubov transformation. These useful analogies are welcomed.

Acknowledgments

I would like to thank the participants of the 8th International Workshop on Multiparticle Production (Matrahaza, Hungary, June 14-21, 1998) for discussion of the work. I am also indebted to Prof. M.Biyajima who drew my attention to related paper by H.Hiro-Oka and H.Minakata. The work has been supported by the Russian Fund for Fundamental Research (grant 96-02-16210a).

References

[1] I.V.Andreev and R.M.Weiner, Phys.Lett. B 373, (1996) 159.
[2] I.V.Andreev, M.Plumer and R.M.Weiner, Phys.Rev.Lett. 67, (1991) 3475; Int.J.Mod.Phys. A 8, (1993) 4577.
[3] M.Asakawa and T.Csorgo, Heavy Ion Physics 4, (1996) 233; quant-ph/9708006.
[4] H.Hiro-Oka and H.Minakata, Phys.Lett. B 425, (1998) 129.
[5] M.Gyulassy, S.K.Kauffmann and L.R.Wilson, Phys.Rev.C 20 (1979) 2267
[6] K.Husimi, Progr.Theor.Phys. 9 (1953) 381
[7] N.N.Bogolubov,Proc.Acad.Sci.USSR, Ser.Phys. 11 (1947) 77
[8] N.D.Birrell and P.C.W.Davis,Quantum fields in curved space, Cambridge Univ.Press, Cammbridge (1982)
[9] A.A.Grib, S.G.Mamaev and V.M.Mostepanenko, Quantum effects in intensive external fields, Atomizdat, Moscow (1980)