Conflict Management of Evidence Theory Based on Belief Entropy and Negation

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ABSTRACT Discount coefficient is an efficient method to address conflicting evidence combination in Dempster-Shafer evidence theory. However, how to determine the discount coefficient of each evidence is an open issue. In this paper, considering both the influence of the amount of information contained in the evidence itself and the fuzziness of the evidence based on the negation of basic belief assignment, a new discount coefficient is presented. The proposed discount coefficient is a fractional form. The numerator is Deng entropy, and the denominator is entropy difference between initial body of evidence (BOE) and its negation. The more information contained in the evidence, the more value is obtained. And the lower fuzziness of evidence, the less value is obtained. A numerical example is given to illustrate the application of this proposed method in the combination of highly conflicting evidence.

INDEX TERMS Dempster-Shafer evidence theory, conflict management, discount coefficient, Deng entropy, negation, target recognition.

I. INTRODUCTION

Dempster-shafer theory, (D-S theory), as an important and widely used method in data fusion, has played a crucial role in objects classification [1], target recognition [2], [3], decisions making [4], uncertainty measure [5], [6] and so on [7]–[9]. Although, Dempster-shafer combination rule expand the probability into the power set instead of individual elements and has some compelling properties such as associativity and commutativity, when the collected evidence is highly conflicting with each other, the classical Dempster combination rule can lead to counter-intuitive results caused by the normalization step [10].

In order to solve this problem, methods such as Smet’s unnormalized combination rule [11], Yager’s combination rule [12], Dubois and Prade’s disjunctive combination rule [13] and so on have been proposed to modify Dempster-shafer combination rule. Another kind of approaches focus on modifying evidence sources. Murphy proposed an alternative approach—averaging, which provides an accurate record of contributing beliefs, but lacks convergence [14]. Hence a weighted average method based on a distance function of evidence is proposed by Yong et al. [15]. The modified averaging method not only maintain the association relationship of the evidence, but also has a better performance in convergence. Recently, more and more innovative methods have been proposed, such as [16], [17].

Among the previous studies, discounting is an important category of solutions, but how to define reasonable discount coefficient is open to discussion. The main difficulty is how to estimate the reliability of evidence [18]. At origin, researchers usually looked for a single criterion to measure the reliability of evidence [19]. However, in order to evaluate the reliability of the evidence more accurately and reasonably, mono-criterion has been superseded by multi-criterion little by little [20]. For example, in [21], both intrinsic and extrinsic reasons of conflict are taken into consideration. In [22], [23], some measures to evaluate reliability degrees by a sequential discounting approach are proposed.

In this paper, the main contribution is to present a discount coefficient generated by the quotient of Deng entropy and entropy difference between initial body of evidence (BOE) and its negation. The setting of discount coefficient corresponding to each BOE is mainly influenced by two factors, which is similar to the uncertainties with Atanassov’s intuitionistic fuzzy sets (AIFSs) in [24], [25]. One of the factors
is the amount of information contained in the evidence itself, that is, the uncertain degree of basic belief assignment (BBA) which can be measured by Deng entropy [26], [27]. Another is entropy difference between initial BBA and its negation. Since negation provides more information from the opposite side, it can enable to treat the issue in a more comprehensive and objective way. Negation can be applied to many fields such as fuzzy knowledge representation [28], [29] and reasoning [30], [31]. Researchers also find that the fuzziness of BBA could be measured by the conflict between the initial BBA and its negation. The more difference between the initial BBA and its negation stands for the lower fuzziness of evidence, thus acquiring a smaller weight. In this research, the discount coefficient is a fractional form in which Deng Entropy is treated as a numerator and entropy difference between initial BOE and its negation as a denominator.

The rest of this article is structured as follows: In Section 2, preliminaries of basic D-S theory, Deng entropy, negation and discounting procedures are introduced. The detailed calculation steps of the proposed method are presented in section 3. In section 4, some numerical examples are used to illustrate the applicability of the method. The strengths and weaknesses of the proposed method will also be discussed in this section. Finally, some conclusions are given in section 6.

II. PRELIMINARIES

In this section, the preliminaries of basic D-S theory, Deng entropy, a matrix-based method of BBA negation and discounting procedures will be briefly introduced.

A. DEMPSTER-SHAFER EVIDENCE THEORY

Uncertainty is inevitable in many practical applications [32]–[36]. To handle this kind of problem, many methods were exploited in the past few years, like probabilistic linguistic [37], [38], D number [39], [40], grey prediction model [41], complex network [33], and fuzzy sets [42]–[46]. As one of the most useful tools to handle uncertainty, the main characteristic of Dempster-Shafer theory [47], [48] is that it can satisfy weaker conditions than Bayesian probability theory, and has the ability to express uncertainty and unknown directly [49]. Therefore, evidence theory has been extensively studied and applied in many fields [50]–[52], including classification [53], [54], supply chain management, reliability analysis [55], especially in the field of evidence reasoning [56]–[58]. Here, some basic concepts of Dempster-Shafer evidence theory are introduced as follows:

Definition 1: Let $\Theta$ be a fixed set of $N$ mutually exclusive and exhaustive elements, The $\Theta$ is referred to the frame of discernment (FOD) which is defined as [47], [48]:

$$\Theta = \{H_1, H_2, \ldots, H_N\}. \quad (1)$$

And let us denote $P(\Theta)$ as the power set of $2^N$ elements $A$ of $\Theta$

$$P(\Theta) = \{\phi, H_1, H_2, \ldots, H_N, (H_1, H_2), (H_1, H_3) \cdots \} \quad (2)$$

The elements of $P(\Theta)$ that have a nonzero mass are called focal elements. A body of evidence (BOE) is the set of all the focal elements, which can be expressed as follows:

$$\emptyset, m = \{[A, m(A)] \in AP(\Theta) \text{ and } m(A) > 0\}. \quad (3)$$

Definition 2: A basic belief assignment (BBA) is a function from $P(\Theta)$ to $[0,1]$, which is defined by [47], [48]

$$m : P(\Theta) \rightarrow [0,1], \quad (4)$$

and which satisfies the following conditions:

$$\sum_{A \in P(\Theta)} m(A) = 1, \quad (5)$$

$$m(\phi) = 0. \quad (6)$$

The mass $m(A)$ represents how strongly the evidence supports $A$ if $A \subseteq \Theta$ and $A \neq \Theta$, while $A = \Theta$ means have no idea about how to allocate it to the whole power set. The BBA is the central concept with many corresponding operations such as correlation [59], [60], weight [61], [62], belief rule-based [63], [64], and so on.

Definition 3: Assume there are two independent BBAs from different sources indicated by $m_1$ and $m_2$, the Dempster combination rule is used to combine them as follows [47], [48]:

$$m_\oplus(A) = \frac{\sum_{B \cap C = A} m_1(B)m_2(C)}{1 - k}, \quad \forall A \subseteq \Theta, A \neq \emptyset \quad (7)$$

$$k = \sum_{B,C \subseteq \Theta, B \cap C = \emptyset} m_1(B)m_2(C), \quad (8)$$

where $\oplus$ represents the operator of combination and $k$ reflects the degree of the conflict.

The combination rule is the base of evidential reasoning [65]–[67].

B. DENG ENTROPY

Entropy has been seen as an indicator to measure uncertainty of some probability density function (PDF) [68], which has been applied to many fields [69], [70]. As for BBA, the Deng entropy is adopted [71]. Deng entropy is a generalization of Shannon entropy since it is degenerated as Shannon entropy when the BBA is degenerated as a probability distribution. What’s more, Deng entropy will increase monotonously with the rise of the size of a particular set [26]. So, if an evidence itself contains more information, that is, the greater the degree of uncertainty, the greater the corresponding Deng entropy value will be obtained. Deng entropy is used in many applications such as decision tree [72], data fusion [73], [74] and so on [75], [76]. As a result, Deng entropy is constantly improving and evolving in the continuous exploration of many researchers [77], [78].

Definition 4: Deng entropy is defined as follows [26]:

$$E_d(m) = - \sum_{A \subseteq \Theta} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1}, \quad (9)$$
where \( m \) is a mass function defined on the frame of discernment \( \Theta \), and \( A \) is the focal element of \( m \), \( |A| \) is the cardinality of \( A \). For more detailed information, please refer to Ref. [26].

C. THE NEGATION OF BPA BASED ON MATRIX

Negation provides a new perspective of probability distribution [79]. A lot of optional approaches to calculate the negation of probability distribution are presented by researchers. As a result, it is of great significance to explore the negation of BBA based on matrix is presented [81].

Definition 5: Suppose there is \( N \) elements on the frame of discernment, thus the power set can be represented as \([81]\):

\[
2^\Theta = \{A_1, A_2, A_3, \ldots, A_{2^N}\} \text{ with } A_1 = \emptyset, A_{2^N} = \Theta.
\]

Let BBA is donated by \( m \) satisfying the constraints \( m(A_i) = m_i \), and \( \sum_{i=1}^{2^N} m_i = 1, m_i \in [0, 1] \), thus the BBA vector (BBAV) of \( m, M \), is defined as follows:

\[
M = (m_1, m_2, m_3, \ldots, m_{2^N})^T.
\] (10)

In the same way, the BBAV of \( m_{not} \), \( M_{not} \) can be represented as:

\[
M_{not} = (\bar{m}_1, \bar{m}_2, \bar{m}_3, \ldots, \bar{m}_{2^N})^T.
\] (11)

Definition 6: Given a BBA vector \( M \) and it’s corresponding \( M_{not} \), the \( 2^N \) – dimension matrix \( G \) which considered as the negation operator is defined as follows [81]:

\[
G_{i,j} = \begin{cases} 
0 & i = j, \\
A_i \cap \bar{A}_j & i \neq j,
\end{cases}
\] (13)

where \( j \neq 2^N \) and \( \bar{A}_j = \Theta - A_j \in \Theta \). Especially, while \( j = 2^N \), that is, \( A_j = \emptyset \), the element \( G_{i,j} \) in the negation operator \( G \) is defined as [81]:

\[
g_{i,j} = \begin{cases} 
1 & i = j, \\
0 & i \neq j.
\end{cases}
\] (14)

D. DISCOUNTING PROCEDURES

The discounting procedures of the reliability of the BOEs is defined as follows:

Definition 8: Suppose the BBA given by the source \( S_j \) over \( \Theta = m_j \). Let \( m_{alpha,j} \) represents the BBA \( m_j \) discounted by a discounting coefficient \( \alpha_j \in [0, 1] \) and is defined as follows [82]:

\[
m_{alpha,j} = \begin{cases} 
(1 - \alpha_j)m_j(A) + \alpha_j & A = \Theta, \\
(1 - \alpha_j)m_j(A) & A \subset \Theta.
\end{cases}
\] (15)

III. THE PROPOSED METHOD

In evidence theory, D-S Combination rule can be used to fuse knowledge from different sources to obtain more relevant information, which has been applied to many fields; however, one of the main difficulties lies in unreasonable results may be obtained if evidence from different sources are highly conflicting with each other. In this section, a new method based on Deng entropy and entropy difference between evidence and its negation is proposed to manage and combine conflicting evidence, in which both the information that the evidence itself contained and the degree of fuzziness of the evidence is taken into consideration. To be more intuitive, the whole process of the proposed method is shown in Fig.1.

Let \( \Theta \) donate the frame of discernment that contains a set of \( N \) mutually exclusive and exhaustive elements: \( \Theta = \{H_1, H_2, \ldots, H_N\} \). There are \( k \) BBAs corresponding \( k \) independent bodies of evidence (BOE), which could be donated as: \( M = \{m_1, m_2, m_3, \ldots, m_k\} \). A BOE is the set of all the focal elements donated as \( A_1, A_2, \ldots, A_{2^N} \). The remaining part of this section will describe the calculation process of each steps in detail.

Step 1: Calculate the Deng entropy of each body of evidence. Deng entropy is an efficient and accepted method to measure the uncertain degree of BBA.

FIGURE 1. The flow chart of the processes of the proposed method.
Step 4: Calculate Deng entropy difference of initial BOE

Step 3: Calculate Deng entropy of each negation of BOE by

\[ E_d(m_i) = - \sum_{A \subseteq P(\theta)} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1}, \quad i = 1, 2, \ldots, k. \] (16)

Step 2: Construct the negation of BOE, \( M_{not} \), for each evidence. First calculate the negation operator \( G \) (a matrix) based on Eq. 13 and Eq. 14. Since there are \( k \) bodies of evidence, \( k \) \( G \) can be obtained. Then, by using Eq. 12, the values of \( M \) and \( G \) for each evidence can be brought into it, and the values of \( M_{not} \) for each evidence can be obtained. \( M_{not} \) can be donated as:

\[ M_{not} = \{\bar{m}_1, \bar{m}_2, \bar{m}_3, \ldots, \bar{m}_k\}^T. \] (17)

Step 3: Calculate Deng entropy of each negation of BOE by using the Eq.9 again, that is:

\[ E_d(\bar{m}_i) = - \sum_{A \subseteq P(\theta)} m(\bar{A}) \log_2 \frac{m(\bar{A})}{2^{|\bar{A}|} - 1}, \quad i = 1, 2, \ldots, k. \] (18)

The negation allows us to further analyze propositions from the opposite perspective. For instance, the meaning of \( Ed(\bar{m}_i) \) is the amount of information contained in the \( \bar{A} \) proposition. The main purpose of this step is to prepare for the next calculation of the entropy difference.

Step 4: Calculate Deng entropy difference of initial BOE and its negation. The difference between them reflects the fuzzy degree of the proposition, which can be considered as another factor. It can be donated as follows:

\[ |E_d(\bar{m}_i) - E_d(m_i)| = \left| - \sum_{A \subseteq P(\theta)} m(\bar{A}) \log_2 \frac{m(\bar{A})}{2^{|\bar{A}|} - 1} \right. \]
\[ + \sum_{A \subseteq P(\theta)} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1}|, \quad i = 1, 2, \ldots, k. \] (19)

In this case, if the bigger value of \( |E_d(\bar{m}_i) - E_d(m_i)| \), the greater uncertainty difference between proposition \( A \) and proposition \( \bar{A} \), which means the lower fuzzy degree of the proposition. To be straightforward, assuming that proposition \( A \) means that it will rain tomorrow, \( A \) means that it will not rain tomorrow. If the probability of proposition \( A \) is 90% and that of \( \bar{A} \) is 10%, it is obvious that the probability of rain tomorrow is relatively high, that is, proposition \( A \) has less uncertainty and lower fuzziness.

Step 5: After calculating the values of the two major factors affecting the weight allocation, we define a weight \( W_i \). The weight of each proposition is equal to the quotient of Deng entropy of initial BOE obtained in step 1 and the entropy difference obtained in step 4, which is written as follows:

\[ W_i = \frac{Ed(m_i)}{|Ed(\bar{m}_i) - Ed(m_i)|}, \quad i = 1, 2, \ldots, k. \] (20)

As Deng entropy is approximately proportional to the amount of information contained in the proposition itself, the proposition with the larger Deng entropy is assigned more weight. However, due to the greater the difference of entropy, the lower the fuzziness, the smaller weight ought to be assigned to the proposition. Therefore, there is an inverse relationship between the entropy difference and the weight we defined.

Step 6: Normalize the weights obtained in step 5. First, choose the largest value of \( k \) weights. Second, by dividing the \( k \) weight values obtained in step 5 by the maximum values, the discount coefficients \( D_i \) corresponding to the \( k \) propositions are obtained. It is worth noting that at least one of these values is 1, which is the discount coefficient of the proposition with the largest weight. The Fig.2 shows the detailed process.

Step 7: Use the Discounting procedures and Dempster-Shafer combination rule to fuse the evidence. First, according to Eq.15, for each BOE, multiply the discount coefficient \( D_i \) with each BBA \( (m_1, m_2, \ldots, m_k) \) in vector \( M \) respectively. Second, since multiplied with the discount coefficient,
the sum of all BBAs \((D_1 \cdot m_1 + D_2 \cdot m_2 + \ldots + D_k \cdot m_k)\) do not add up to 1 and it is not clear how the remaining BBAs should be allocated. In the simplest way, the remaining BBA is assigned to the universal set. As a result, for each BOE, a new element \((1 - D_1 \cdot m_1 - D_2 \cdot m_2 - \ldots - D_k \cdot m_k)\) will be added. Then \(M_i\) of each BOE becomes the following form:

\[
M_i = [D_1 \cdot m_1, D_2 \cdot m_2, \ldots, D_i \cdot m_i, 1 - D_1 \cdot m_1 - D_2 \cdot m_2 - \ldots - D_k \cdot m_k]^T. \quad (21)
\]

Finally, the final fusion result can be obtained by using D-S combination rules based on Eq.7 and Eq.8.

### IV. NUMERICAL EXAMPLES AND DISCUSSION

In this section, an example is given to illustrate the use of the proposed combination rule and some discussion about the results will also be presented.

**Example 1**: An example from [15] is adopted. In an automatic target recognition system based on multi-sensor, it is assumed that the actual target is A. From four different sensors, the system collects four bodies of evidence, as shown below:

\[
\begin{align*}
\langle \text{H}_1, m_1 \rangle & = ([A], 0.5), \langle [B], 0.2 \rangle, \langle [C], 0.3 \rangle), \\
\langle \text{H}_2, m_2 \rangle & = ([A], 0), \langle [B], 0.9 \rangle, \langle [C], 0.1 \rangle), \\
\langle \text{H}_3, m_3 \rangle & = ([A], 0.55), \langle [B], 0.1 \rangle, \langle [A, C], 0.35 \rangle), \\
\langle \text{H}_4, m_4 \rangle & = ([A], 0.55), \langle [B], 0.1 \rangle, \langle [A, C], 0.35 \rangle).
\end{align*}
\]

**Step 1**: Calculate Deng entropy for each body of evidence based on Eq.16.

\[
\begin{align*}
Ed(m_1) & = -0.5 \times \log_2 \frac{0.5}{2^1 - 1} - 0.2 \times \log_2 \frac{0.2}{2^1 - 1} \\
& \quad - 0.3 \times \log_2 \frac{0.3}{2^1 - 1} = 1.48548, \\
\end{align*}
\]

\[
\begin{align*}
Ed(m_2) & = 0 - 0.9 \times \log_2 \frac{0.9}{2^1 - 1} - 0.1 \times \log_2 \frac{0.1}{2^1 - 1} \\
& = 0.46899, \\
\end{align*}
\]

\[
\begin{align*}
Ed(m_3) & = -0.55 \times \log_2 \frac{0.55}{2^1 - 1} - 0.1 \times \log_2 \frac{0.1}{2^1 - 1} \\
& \quad - 0.35 \times \log_2 \frac{0.35}{2^1 - 1} = 1.6779, \\
\end{align*}
\]

\[
\begin{align*}
Ed(m_4) & = -0.55 \times \log_2 \frac{0.55}{2^1 - 1} - 0.1 \times \log_2 \frac{0.1}{2^1 - 1} \\
& \quad - 0.35 \times \log_2 \frac{0.35}{2^1 - 1} = 1.6779.
\end{align*}
\]

**Step 2**: Construct the negation of BOE, \(M_{not}\), for each evidence. First, the negation operator \(G\) is obtained based on Eq.13 and Eq.14. It can be predicted that since the first two bodies of evidence contain the same focal elements, the negation operator \(G\) obtained should also be the same. In the same way, the last two bodies of evidence corresponds to the same \(G\)

\[
G1 = G2 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix},
\]

\[
G3 = G4 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 0 \end{pmatrix}.
\]

Then based on Eq.12, the values of \(M_{not}\) for each BOE is obtained as follows:

\[
\begin{align*}
M_{not1} & = (0.25, 0.4, 0.35), \\
M_{not2} & = (0.5, 0.05, 0.45), \\
M_{not3} & = (0.033, 0.625, 0.342), \\
M_{not4} & = (0.033, 0.625, 0.342).
\end{align*}
\]

**Step 3**: Calculate Deng entropy of each negation of BOE by using Eq.9 again.

\[
\begin{align*}
Ed(\overline{m}_1) & = -0.25 \times \log_2 \frac{0.25}{2^1 - 1} - 0.4 \times \log_2 \frac{0.4}{2^1 - 1} \\
& \quad - 0.35 \times \log_2 \frac{0.35}{2^1 - 1} = 1.55887, \\
\end{align*}
\]

\[
\begin{align*}
Ed(\overline{m}_2) & = -0.5 \times \log_2 \frac{0.5}{2^1 - 1} - 0.05 \times \log_2 \frac{0.05}{2^1 - 1} \\
& \quad - 0.45 \times \log_2 \frac{0.45}{2^1 - 1} = 1.02920, \\
\end{align*}
\]

\[
\begin{align*}
Ed(\overline{m}_3) & = -0.033 \times \log_2 \frac{0.033}{2^1 - 1} - 0.625 \times \log_2 \frac{0.625}{2^1 - 1} \\
& \quad - 0.342 \times \log_2 \frac{0.342}{2^1 - 1} = 1.65796, \\
\end{align*}
\]

\[
\begin{align*}
Ed(\overline{m}_4) & = -0.033 \times \log_2 \frac{0.033}{2^1 - 1} - 0.625 \times \log_2 \frac{0.625}{2^1 - 1} \\
& \quad - 0.342 \times \log_2 \frac{0.342}{2^1 - 1} = 1.65796.
\end{align*}
\]

**Step 4**: Calculate Deng entropy difference of initial BOE and it’s negation based on Eq.19

\[
\begin{align*}
|Ed(\overline{m}_1) - Ed(m_1)| & = |1.55887 - 1.48548| = 0.07339, \\
|Ed(\overline{m}_2) - Ed(m_2)| & = |1.0292 - 0.46899| = 0.56021, \\
|Ed(\overline{m}_3) - Ed(m_3)| & = |1.65796 - 1.6779| = 0.0994, \\
|Ed(\overline{m}_4) - Ed(m_4)| & = |1.65796 - 1.6779| = 0.0994.
\end{align*}
\]

**Step 5**: Based on Eq.20, the weight of each proposition is calculated as follows:

\[
W_1 = \frac{1.48548}{0.07339} = 20.24090,
\]
Table 1. Results of different combination rules of evidence.

| Rule                     | \( m_1, m_2 \)          | \( m_1, m_2, m_3 \)          | \( m_1, m_2, m_3, m_4 \)          |
|-------------------------|-------------------------|-----------------------------|-----------------------------------|
| Dempster-Shafer's rule  | \( m(A)=0 \)            | \( m(A)=0 \)               | \( m(A)=0 \)                      |
|                         | \( m(B)=0.8571 \)       | \( m(B)=0.6316 \)          | \( m(B)=0.3288 \)                |
|                         | \( m(C)=0.1429 \)       | \( m(C)=0.3684 \)          | \( m(C)=0.6712 \)                |
| Murphy's rule           | \( m(A)=0.1543 \)       | \( m(A)=0.3500 \)          | \( m(A)=0.6027 \)                |
|                         | \( m(B)=0.7469 \)       | \( m(B)=0.5224 \)          | \( m(B)=0.2627 \)                |
|                         | \( m(C)=0.0988 \)       | \( m(C)=0.1276 \)          | \( m(C)=0.1346 \)                |
| Deng et al.'s rule      | \( m(A)=0.1543 \)       | \( m(A)=0.4861 \)          | \( m(A)=0.7773 \)                |
|                         | \( m(B)=0.7469 \)       | \( m(B)=0.3481 \)          | \( m(B)=0.0628 \)                |
|                         | \( m(C)=0.0988 \)       | \( m(C)=0.1637 \)          | \( m(C)=0.1600 \)                |
| The proposed rule       | \( m(A)=0.1193 \)       | \( m(A)=0.5852 \)          | \( m(A)=0.8491 \)                |
|                         | \( m(B)=0.0550 \)       | \( m(B)=0.09097 \)         | \( m(B)=0.0112 \)                |
|                         | \( m(C)=0.0724 \)       | \( m(C)=0.0284 \)          | \( m(C)=0.0122 \)                |
|                         | \( m(ABC)=0.7533 \)     | \( m(AC)=0.2957 \)         | \( m(AC)=0.1275 \)               |

Step 6: Normalize the weights obtained in step 5 according to the flow chart Fig. 2. The final discount coefficients are as follows:

\[
D_1 = \frac{20.24090}{84.14744} = 0.240541, \\
D_2 = \frac{0.46899}{0.56021} = 0.837168, \\
D_3 = \frac{1.6779}{0.01994} = 84.14744, \\
D_4 = \frac{0.01994}{1.6779} = 0.009949, \\
D_5 = \frac{84.14744}{84.14744} = 1, \\
D_6 = \frac{84.14744}{1} = 1.
\]

Step 7: Use the Discounting procedures and Dempster-Shafer combination rule to fuse the evidence. Compared with other combination rules, the final results are shown in the Table 1 and Fig. 3.

From the result, it can be clearly seen that all the approaches have obtained reasonable results except the classical Dempster-Shafer’s combination rule. Due to the normalization problem, the value of \( m(A) \) is always 0, which is contrary to the real situation. In this case, as long as there is a “bad” evidence in the collected evidence, such as \( m_2 \), even if more pieces of evidence that support the target A is collected later, the final results will not be improved.

The remaining three methods have effectively solved the evidence conflict problem and identified target A to some extent. However, among the three methods, the convergence of the proposed method is the best. This means that when the number of evidence is limited, the proposed method can make the final decision more quickly. For instance, when the system collects only two pieces of evidence \( m_1 \) and \( m_2 \), only the current method can deduce the correct target A, while other methods support the wrong target B. Moreover, when the system collects three pieces of evidence, the BBA allocated by the current method to target A is the largest among all the methods, which indicates the support to target A is the strongest on this condition. The main cause of these phenomena mentioned above is that, by taking advantage of both Deng entropy and entropy difference between the initial BOE and its negation, the importance of each piece of evidence can be judged. The importance of “bad” evidence is low, and correspondingly less weight is allocated. On the contrary, reliable evidence, such as \( m_3, m_4 \), obtains greater weight. Therefore, the proposed method is suitable for decision-making when the amount of evidence is limited compared with other methods.

On the other side, it is undeniable that the final results still contain some uncertainty. As can be seen from the above results, a small portion of BBA is still allocated to multi-subset \( A \cup C \) in the final result. The root of this problem lies in the last step of our computational process. In the last step, since the discount coefficient multiplied by BBAs in each BOE, the sum of all BBAs in the BOE does not satisfy the condition of equal to 1, so the remaining BBA is allocated to the complete set and then fused by D-S combination rule. Fortunately, even if this uncertainty persists, the degree of uncertainty decreases with the increase of the number of evidence, so that the impact on final decision-making can be neglected. Another shortcoming of this method is computationally expensive. As a result, the proposed method is not suitable for application scenarios with high real-time
requirements and large amount of data. Furthermore, it can not be ignored a particular case that the entropy difference between the initial BOE and its negation (denominator of Discount coefficient) is zero. Here, we look at a simple example to explain this special case.

Example 2: In an automatic target recognition system based on multi-sensor, it is assumed that the actual target is A. There are three bodies of evidence collected from three different sensors:

\( \emptyset_1, m_1 = (\{A\}, 0.6), \{B\}, 0.4) \),
\( \emptyset_2, m_2 = (\{A\}, 0.1), \{B\}, 0.9) \),
\( \emptyset_3, m_3 = (\{A\}, 0.7), \{B\}, 0.3) \).

First of all, solve the issue based on the steps that proposed in section 3.

Step 1: Calculate Deng entropy for each BOE based on Eq.16.

\[ Ed(m_1) = -0.6 \times \log_2 \frac{0.6}{2^1 - 1} - 0.4 \times \log_2 \frac{0.4}{2^1 - 1} = 0.79408, \]
\[ Ed(m_2) = -0.1 \times \log_2 \frac{0.1}{2^1 - 1} - 0.9 \times \log_2 \frac{0.9}{2^1 - 1} = 0.46899, \]
\[ Ed(m_3) = -0.7 \times \log_2 \frac{0.7}{2^1 - 1} - 0.3 \times \log_2 \frac{0.3}{2^1 - 1} = 0.88129. \]

Step 2: Construct the negation operator \( G \) of each BOE.

\[ G_1 = G_2 = G_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

Then the values of \( M_{not} \) for each bodies of evidence is obtained as follows:

\( M_{not1} = (0.4, 0.6), \)
\( M_{not2} = (0.9, 0.1), \)
\( M_{not3} = (0.7, 0.3). \)

Step 3: Calculate Deng entropy of each negation of BOE.

\[ Ed(\bar{m}_1) = -0.4 \times \log_2 \frac{0.4}{2^1 - 1} - 0.6 \times \log_2 \frac{0.6}{2^1 - 1} = 0.79408, \]
\[ Ed(\bar{m}_2) = -0.9 \times \log_2 \frac{0.9}{2^1 - 1} - 0.1 \times \log_2 \frac{0.1}{2^1 - 1} = 0.46899, \]
\[ Ed(\bar{m}_3) = -0.7 \times \log_2 \frac{0.7}{2^1 - 1} - 0.3 \times \log_2 \frac{0.3}{2^1 - 1} = 0.88129. \]

Step 4: Calculate Deng entropy difference of initial BOE and its negation based on Eq.19.

\[ |Ed(\bar{m}_1) - Ed(m_1)| = |0.79408 - 0.79408| = 0, \]
\[ |Ed(\bar{m}_2) - Ed(m_2)| = |0.46899 - 0.46899| = 0, \]
\[ |Ed(\bar{m}_3) - Ed(m_3)| = |0.88129 - 0.88129| = 0. \]

It is clearly to be observed that entropy difference between the initial BOE and its negation which is also the denominator of the weight we defined is 0, so that the above calculation had to be terminated in the step 4. In this scenario, it seems that our proposed method can not address the problem when target recognition system has only two probable targets.

However, in this special case, only a small transformation of the formula is needed. Once the denominator is modified from \(|Ed(\bar{m}_i) - Ed(m_i)|\) to \(|e(Ed(\bar{m}_i) - Ed(m_i))|\), the proposed method is effective again, and the final discount coefficients for these three BOEs are 0.90104, 0.53216, and 0.88129 respectively. It can be seen that the coefficient factor for the second BOE is the lowest, which is in line with the actual situation.

V. CONCLUSION

Since classical D-S evidence theory has some problems in dealing with highly conflicting evidence from different sources, this paper attempted to propose a new method to solve this issue. The main contribution is the information that the evidence itself contains and the fuzziness of the evidence are both taken into consideration based on the Deng entropy and the entropy difference between the initial BOE and its negation. The proposed method has a better performance at convergence. It provides an effective method for making correct decisions when the amount of evidence is limited. However, it should be pointed that there is still some uncertainty in the final results, which is caused by the normalization step, though it does not affect our decision-making with the increase of the number of evidence. It is one of our ongoing works.

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