Dyons near the Planck scale

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In the present letter we suggest a new model of preons-dyons making composite quark-leptons and bosons, described by the supersymmetric string-inspired flipped $E_6 \times \tilde{E}_6$ gauge group of symmetry. This approach predicts the possible extension of the Standard Model to the Family replicated gauge group model of type $G^{N_{\text{fam}}}$, where $N_{\text{fam}}$ is the number of families and $G$ is the symmetry group: $G = \text{SMG}$, $SU(5)$, $SO(10)$, $E_6$, etc. Here $E_6$ and $\tilde{E}_6$ are non-dual and dual sectors of theory with hyper-electric $g$ and hyper-magnetic $\tilde{g}$ charges, respectively. Starting with an idea that the most realistic model leading to the unification of all fundamental interactions (including gravity) is the “heterotic” string-derived flipped model, we have assumed that at high energies $\mu > 10^{16}$ GeV there exists the following chain of the flipped models:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X \rightarrow$$

$$SU(5) \times U(1)_X \rightarrow SU(5) \times U(1)_{Z1} \times U(1)_{X1} \rightarrow SO(10) \times U(1)_{X1} \rightarrow E_6,$$

ended by the flipped $E_6$ gauge group of symmetry at the scale $M_{\text{SSG}} \sim 10^{18}$ GeV. Suggesting $N = 1$ supersymmetric $E_6 \times \tilde{E}_6$ preonic model we have considered preons as dyons confined by hyper-magnetic strings in the region of energies $\mu \lesssim M_{\text{Pl}}$. Our model is based on the recent theory of composite non-Abelian flux tubes in SQCD – analog ANO-strings. Considering the breakdown of $E_6$ and $\tilde{E}_6$ at the Planck scale into the $SU(6) \times U(1)$ gauge group, we have shown that the six types of $k$-strings – composite $N = 1$ supersymmetric non-Abelian flux tubes – are created by the condensation of spreons-dyons near the Planck scale and have six fluxes quantized according to the $Z_6$ center group of $SU(6)$: $\Phi_n = n\Phi_0$ ($n = \pm 1, \pm 2, \pm 3$). These fluxes give three types of $k$-strings with tensions $T_k = kT_0$, where $k = 1, 2, 3$, and produce three (and only three) generations of composite quark-leptons and bosons giving a very specific type of “horizontal symmetry”. Thus, the present model predicts $N_{\text{gen}} = N_{\text{fam}} = 3$. It was shown that our preonic strings are very thin, with radius $R_{\text{str}} \sim 10^{-18}$ GeV$^{-1}$, and their tension $T_0$ is enormously large: $T_0 \sim 10^{38}$ GeV$^2$. It was shown that the condensation of spreons near the Planck scale gives the phase transition at some scales $M_{\text{crit}}$ and $\tilde{M}_{\text{crit}}$, which correspond to the following breakdowns of $E_6$ (or $\tilde{E}_6$) for preons: $E_6 \rightarrow SU(6) \times U(1)$, or $\tilde{E}_6 \rightarrow SU(6) \times \tilde{U}(1)$. We have calculated the critical values of gauge coupling constants: $\alpha^{-1}(M_{\text{crit}}) \approx 4.23$ and $\alpha^{-1}(M_{\text{crit}}) \approx 2.13$. It was investigated that in our world we have quark-leptons and gauge bosons $A_\mu$ in the region of energies $\mu \lesssim M_{\text{Pl}}$, but monopolic “quark-leptons” and dual gauge fields $\tilde{A}_\mu$ exist in the region $\mu \gtrsim M_{\text{Pl}}$. 
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1. Superstring theory is a paramount candidate for the ultimate theory unifying all fundamental interactions including gravity. It was shown in Refs. [1] that superstrings are free of gravitational and Yang-Mills anomalies if a gauge group of symmetry is $SO(32)$, or $E_8 \times E_8$. The “heterotic” superstring theory $E_8 \times E_8$ was suggested in [1] as a more realistic model for unification. This ten-dimensional Yang-Mills theory can undergo spontaneous compactification for which $E_8$ group is broken to $E_6$ in four dimensions, but $E_8'$ group remains unbroken and gives “hidden sector” of SUGRA.

In the present investigation we develop a new preonic $E_6 \times \tilde{E}_6$ model of composite quark-leptons and bosons, in which preons are dyons confined by hyper-magnetic strings. Here $E_6$ and $\tilde{E}_6$ are non-dual and dual sectors of theory with hyper-electric $g$ and hyper-magnetic $\tilde{g}$ charges, respectively.

Pati was first [2] who suggested to use the strong magnetic force to bind preons-dyons making the composite objects. This idea has an extension in our model [3] (see also the talk [4]), which was constructed in the light of recent investigations of composite non-Abelian flux tubes in SQCD [5–7].

We start with the ‘flipped’ supersymmetric group of symmetry $E_6 \times \tilde{E}_6$ and show that the dual sector of this theory described by the group $\tilde{E}_6$ is broken in our world up to the Planck scale $M_{Pl} \approx 1.22 \cdot 10^{19}$ GeV. The breakdown of the dual sector gives a very specific type of the “horizontal symmetry” predicting three generations of the Standard Model.

2. In Ref. [8] we have considered that only ‘flipped’ $SU(5)$ unifies $SU(3)_C$ and $SU(2)_W$ of the Standard Model (SM) at the GUT scale $M_{GUT} \sim 10^{16}$ GeV. An explanation of the discrepancy between the unification scale $M_{GUT}$ and string scale $M_{str} \sim 10^{18}$ GeV was given by the assumption that there exists a chain of extra intermediate symmetries between $M_{GUT}$ and $M_{Pl}$:

$$SU(5) \times U(1)_X \rightarrow SU(5) \times U(1)_{Z1} \times U(1)_{X1} \rightarrow SO(10) \times U(1)_{X1} \rightarrow E_6.$$  \hspace{1cm} (1)

We have considered such Higgs boson contents of the $SU(5)$ and $SO(10)$ gauge groups, which give the flipped $E_6$ final unification at the scale $\sim 10^{18}$ GeV and decreased running of the inversed gauge coupling constant $\alpha^{-1}$ near the Planck scale. Such an example, presented by Fig. [1] suits the purposes of our new model of preons. Here and below we consider the flipped models in which $SU(5)$ contains Higgs bosons $h, \tilde{h}$ and $H, \tilde{H}$ belonging to $5_h, \tilde{5}_h$ and $10_H + \tilde{10}_H$ representations of $SU(5)$, respectively, also 24-dimensional adjoint Higgs field $A$ and Higgs bosons belonging to additional higher representations. Correspondingly, the flipped $SO(10)$ (coming at the superGUT scale $M_{SG}$) contains $10_h + \tilde{10}_h$ and $45_H + \tilde{45}_H$, 45-dimensional adjoint $A$ and higher representations of Higgs bosons. As it was shown in [8], such Higgs boson contents lead to the flipped $E_6$ final unification at the supersuperGUT scale $M_{SSG} \sim 10^{18}$ GeV.

Fig. [1] presents an example of running of the inversed gauge coupling constants $\alpha^{-1}_i(\mu)$ ($\mu$ is the energy scale) for $i = 1, 2, 3, X, Z, X1, Z1, 5, 10$. It was shown that
at the scale $\mu = M_{GUT}$ the flipped $SU(5)$ undergoes the breakdown to the supersymmetric (MSSM) $SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X$ gauge group of symmetry, which is the supersymmetric extension of the MSSM originated at the seesaw scale $M_{SS} \approx 10^{11}$ GeV, where heavy right-handed neutrinos appear. A singlet Higgs field $S$ provides the following breakdown to the SM (see [9]):

$$SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y.$$  \hspace{1cm} (2)

Supersymmetry extends the conventional SM beyond the scale $M_{SUSY}$. In our example $M_{SUSY} = 10$ TeV.

The final unification $E_6$ assumes the existence of three 27-plets of $E_6$ containing three generations of quarks and leptons including the right-handed neutrinos $N^c_i$ (here $i = 1, 2, 3$ is the index of generations). Quarks and leptons of the fundamental 27 representation decompose under $SU(5) \times U(1)_X$ subgroup as follows:

$$27 \rightarrow (10, 1) + (\bar{5}, -3) + (\bar{5}, 2) + (5, -2) + (1, 5) + (1, 0).$$  \hspace{1cm} (3)

The first and second quantities in the brackets of Eq. (3) correspond to the $SU(5)$ representation and $U(1)_X$ charge, respectively. We consider charges $Q_X$ and $Q_Z$ in the units $1/\sqrt{40}$ and $\sqrt{3}/5$, respectively, using assignments: $Q_X = X$ and $Q_Z = Z$ [8].

The conventional SM family which contains the doublets of left-handed quarks $Q$ and leptons $L$, right-handed up and down quarks $u^c, d^c$, also $e^c$, is assigned to the $(10, 1) + (\bar{5}, -3) + (1, 5)$ representations of the flipped $SU(5) \times U(1)_X$, along with right-handed neutrino $N^c$. These representations decompose under

$$SU(5) \times U(1)_X \rightarrow SU(3)_C \times SU(5)_L \times U(1)_Z \times U(1)_X.$$  \hspace{1cm} (4)

This decomposition for the $E_6$ 27-plet is given by Dr C.R. Das in his talk [4].

It is necessary to notice that the flipping of our models:

$$d^c \leftrightarrow u^c, \quad N^c \leftrightarrow e^c$$  \hspace{1cm} (5)

distinguishes our ‘flipped $SU(5)$’ from the standard Georgi-Glashow $SU(5)$.

We have the following 16 spinorial representation of $SO(10)$:

$$F(16) = F(10, 1) + F(\bar{5}, -3) + F(1, 5).$$  \hspace{1cm} (6)

Higgs chiral superfields occupy the 10 representation of $SO(10)$:

$$h(10) = h(5, -2) + h^c(\bar{5}, 2).$$  \hspace{1cm} (7)

3. Why three generations exist in Nature? We suggest an explanation considering a new preonic model of composite SM particles. The model starts from the supersymmetric flipped $E_6 \times \tilde{E}_6$ gauge group of symmetry for preons.
Considering the $N = 1$ supersymmetric flipped $E_6 \times \widetilde{E}_6$ gauge theory for preons in 4D-dimensional space-time, we assume that preons $P$ and antipreons $P^c$ are dyons with charges $g$ and $\tilde{g}$, respectively, resided in the 4D hypermultiplets $\mathcal{P} = (P, P^c)$ and $\tilde{\mathcal{P}} = (\tilde{P}, \tilde{P}^c)$. Here “$\tilde{\mathcal{P}}$” designates spreons, but not the belonging to $\widetilde{E}_6$.

The dual sector $\widetilde{E}_6$ is broken in our world to some group $\widetilde{G}$, and preons and spreons transform under the hyper-electric gauge group $E_6$ and hyper-magnetic gauge group $\widetilde{G}$ as their fundamental representations:

\begin{equation}
P, \tilde{P} \sim (27, N), \quad P^c, \tilde{P}^c \sim (\overline{27}, \overline{N}),
\end{equation}

where $N$ is the $N$-plet of $\widetilde{G}$ group. We also consider scalar preons and spreons as singlets of $E_6$:

\begin{equation}
P_s, \tilde{P}_s \sim (1, N), \quad P_s^c, \tilde{P}_s^c \sim (1, \overline{N}),
\end{equation}

which are actually necessary for the entire set of composite quark-leptons and bosons. This idea was suggested in Ref. [10].

The hyper-magnetic interaction is assumed to be responsible for the formation of $E_6$ fermions and bosons at the compositeness scale $\Lambda_s$. The main idea of the present investigation is an assumption that preons-dyons are confined by hyper-magnetic supersymmetric non-Abelian flux tubes which are a generalization of the well-known Abelian Abrikosov-Nielsen-Olesen (ANO)-strings for the case of the supersymmetric non-Abelian theory developed in Refs. [5–7]. As a result, in the limit of infinitely narrow flux tubes (strings) we have the following bound states:

i. quark-leptons (fermions belonging to the $E_6$ fundamental representation):

\begin{equation}
Q^a \sim P^{aA}(y) \left[ \mathcal{P} \exp \left( i \tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]_A^B \left( P^c \right)_B(x) \sim 27,
\end{equation}

\begin{equation}
\tilde{Q}_a \sim (P_s^c)^A(y) \left[ \mathcal{P} \exp \left( i \tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]_A^B P_{aB}(x) \sim \overline{27},
\end{equation}

where $a \in 27$-plet of $E_6$, $A, B \in N$-plet of $\widetilde{G}$, and $\tilde{A}_\mu(x)$ are dual hyper-gluons belonging to the adjoint representation of $\widetilde{G}$;

ii. “mesons” (hyper-gluons and hyper-Higgses of $E_6$):

\begin{equation}
M_6^a \sim P^{aA}(y) \left[ \mathcal{P} \exp \left( i \tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]_A^B \left( P^c \right)_{bB}(x) \sim 1 + 78 + 650 \text{ of } E_6,
\end{equation}

\begin{equation}
S \sim (P_s)^A(y) \left[ \mathcal{P} \exp \left( i \tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]_A^B \left( P^c \right)_{B}(x) \sim 1,
\end{equation}

where $a, b \in 27$-plet of $E_6$, and $A, B \in N$-plet of $\widetilde{G}$. This idea was suggested in Ref. [10].
iii. “baryons” of $\tilde{G}$-triplet (see below):

$$D_1 \sim \epsilon_{ABC} P^{aA'}(z) P^{bB'}(y) P^{cC'}(x) \left[ \mathcal{P} \exp \left( i \tilde{g} \int_x^z A_\mu dx^\mu \right) \right]^{A'}_A \times \left[ \mathcal{P} \exp \left( i \tilde{g} \int_x^y \tilde{A}_\mu dx^\mu \right) \right]^{B'}_B \left[ \mathcal{P} \exp \left( i \tilde{g} \int_x^x \tilde{A}_\mu dx^\mu \right) \right]^{C'}_C,$$

and their conjugate particles.

The bound states (10)–(15) are shown in Fig. 3 as unclosed strings (a) and “baryonic” configurations (b). It is easy to generalize Eqs. (10)–(15) for the case of string constructions of superpartners – squark-sleptons, hyper-gluinos and hyper-higgsinos. Closed strings – gravitons – are presented in Fig. 3(c). All these bound states belong to the $E_6$ representations and they are in fact the \( N = 1 \ 4D \) superfields.

We assume that near the Planck scale preonic $E_6$ can be broken by Higgses belonging to the 78-dimensional representation of $E_6$ (see Ref. [3]):

$$E_6 \rightarrow SU(6) \times SU(2) \rightarrow SU(6) \times U(1),$$

(16)

where $SU(6) \times U(1)$ is the largest relevant invariance group of the 78.

If $SU(6) \times U(1)$ group of symmetry works near the Planck scale, then we deal just with the theory of non-Abelian flux tubes in $N = 1 \ SQCD$, which was developed recently in Refs. [5–7].

Let us consider the condensation of spreons-dyons at the Planck scale. One can combine the $Z_6$ center of $SU(6)$ with the elements $\exp(i\pi) \in U(1)$ to get topologically stable string solutions possessing both windings, in $SU(6)$ and $U(1)$. Now onwards we assume the dual sector of theory described by $\tilde{SU}(6) \times \tilde{U}(1)$, which is responsible for hyper-magnetic fluxes. Then, according to the results obtained in Refs. [5–7], we have a nontrivial homotopy group:

$$\pi_1 \left( \frac{SU(6) \times U(1)}{Z_6} \right) \neq 0,$$

(17)

and flux lines form topologically non-trivial $Z_6$ strings.

Besides $SU(6)$ and $U(1)$ gauge bosons, the model contains six scalar fields charged with respect to $U(1)$ and belong to the 6-plet of $SU(6)$. Considering scalar fields of spreons

$$\tilde{\mathcal{P}} = \{ \phi^a \},$$

(18)
which have indices $a$ of $SU(6)$ and $A$ of $\tilde{SU}(6)$ fundamental multiplets, we construct condensation of spreons in vacuum:

$$\tilde{P}_{\text{vac}} = \left\langle \tilde{P}^{aA} \right\rangle = v \cdot \text{diag}(1, 1, \ldots, 1), \quad a, A = 1, \ldots, 6. \quad (19)$$

Now we give the solution for the preonic $N=1$ supersymmetric non-Abelian flux tubes based on theory [5–7]. Dual symmetry included in our model slightly modifies theory [5–7] by consideration of the Zwanziger formalism [11, 12].

4. As it was shown in Refs. [11, 12], the aim to describe symmetrically non-dual and dual Abelian fields $A_\mu$ and $\tilde{A}_\mu$, covariantly interacting with electric $j_\mu^{(e)}$ and magnetic $j_\mu^{(m)}$ currents respectively, is realized by the following Zwanziger’s action:

$$S_{ZW} = \int d^4x \left( -\frac{1}{2} n^\mu n^\lambda \sqrt{-g} g^{\rho\sigma} \left( F_{\mu\nu} F_{\lambda\rho} + G_{\mu\nu} G_{\lambda\rho} + i F_{\mu\nu} \tilde{G}_{\lambda\rho} - i G_{\mu\nu} \tilde{F}_{\lambda\rho} \right) \right), \quad (20)$$

where

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu, \quad (21)$$

$$G_{\mu\nu} = \partial_\nu \tilde{A}_\mu - \partial_\mu \tilde{A}_\nu, \quad (22)$$

and

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}. \quad (23)$$

In Eq. (20) a constant unit vector $n^\mu$ denotes the direction of Dirac strings, which are frozen in time and parallel in space.

The generalized Zwanziger formalism for non-Abelian gauge theories was suggested in Refs. [12, 13] (see also references therein), and we consider the following Zwanziger-type action:

$$S^{NAZW} = -\frac{1}{4\pi} \int D\xi^\mu(s) ds \left\{ \text{Tr} \left[ \left( \dot{\xi}^\mu(s) F_{\mu\nu}(x) \right) \left( \dot{\xi}^\lambda(s) F^\nu_\lambda(x) \right) \right] \dot{\xi}^{-2} + \text{Tr} \left[ \left( \dot{\xi}^\mu(s) G_{\mu\nu}(x) \right) \left( \dot{\xi}^\lambda(s) G^\nu_\lambda(x) \right) \right] \dot{\xi}^{-2} + \text{Tr} \left[ \left( \dot{\xi}^\mu(s) F_{\mu\nu}(x) \right) \left( \dot{\xi}^\lambda(s) \tilde{G}^\nu_\lambda(x) \right) \right] \dot{\xi}^{-2} + \text{Tr} \left[ \left( \dot{\xi}^\mu(s) G_{\mu\nu}(x) \right) \left( \dot{\xi}^\lambda(s) \tilde{F}^\nu_\lambda(x) \right) \right] \dot{\xi}^{-2} \right\}, \quad (24)$$

where $\xi^\mu(s)$ represents an arbitrary disposition of string in $D$-dimensional space-time ($D = 4$ in our case). The functional integral $\int D\xi^\mu(s)$ introduces the sum over the string’s shape. The “tilde” denotes “dual”, but in this non-Abelian case it is not a simple “Hodge star duality” given by Eq. (23) (see [12, 13]).

In Eq. (24) we have:

$$F_{\mu\nu} = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) + ig \left[ A_\mu(x), A_\nu(x) \right], \quad (25)$$

and

$$G_{\mu\nu}(x) = \partial_\nu \tilde{A}_\mu(x) - \partial_\mu \tilde{A}_\nu(x) + ig \left[ \tilde{A}_\mu(x), \tilde{A}_\nu(x) \right]. \quad (26)$$
Here and below we do not consider the “absorption” of the coupling constants $g$ and $\tilde{g}$ by vector potentials $A_\mu$ and $\tilde{A}_\mu$.

5. Near the Planck scale the gauge group for preons is supersymmetric

$$[SU(6) \times U(1)] \times \left[ SU(6) \times \tilde{U}(1) \right],$$

revealing the generalized dual symmetry (see [11–13] and references therein). In the non-dual sector, besides $SU(6)$ and $U(1)$ gauge bosons, preons $P = (P_1, P_2^+)^T$ (Dirac fermions and $SU(2)_R$ singlets) and other matter fields, the model contains six scalar fields $\tilde{P}$ (spreons) charged with respect to $U(1)$ gauge group and belong to the 6-plet of $SU(6)$. In general, duality leads to the $6 \times 6$ matrix (18) for spreons. Also they are $SU(2)_R$ doublets: $\tilde{P} = (\tilde{P}_1, \tilde{P}_2)$. As in Refs. [5], we introduce a zero charge scalar field $f$ and $35 \times 35$ matrix of the complex adjoint scalar fields:

$$f = \{f^{jj}\} \quad \text{with} \quad j, J = 1, 2, ..., 35. \quad (27)$$

In terms of these fields the action of $\mathbb{N}=2$ supersymmetric theory can be deformed by the breaking to $\mathbb{N}=1$ terms containing mass parameters $\mu_1$ and $\mu_2$ for the fields $f^{jj}$ and $f$. The action takes the following form:

$$S = S^{ZW} + S^{NAZW} + S^{(\text{matter})} + \text{other terms (topological, gauge fixing, etc.)}, \quad (28)$$

where $S^{ZW}$ and $S^{NAZW}$ are given by Eqs. (20) and (24), and the third term in Eq. (28) is:

$$S^{(\text{matter})} = \int d^4x \left( |D_\mu f^{jj}|^2 + |\nabla_\mu \tilde{P}_1|^2 + |\nabla_\mu \tilde{P}_2|^2 + |\partial_\mu f|^2 + U(\tilde{P}, f^{jj}, f) \right) + \text{other matters}, \quad (29)$$

where $D_\mu$ is the covariant derivative in the adjoint representation defined as

$$D_\mu = \partial_\mu - ig_6 [A_\mu(x), ...] - i\tilde{g}_6 [\tilde{A}_\mu(x), ...], \quad (30)$$

while

$$\nabla_\mu = \partial_\mu - i \frac{g_6 A_\mu + \tilde{g}_6 \tilde{A}_\mu}{\sqrt{2N}} - i \left( g_6 A_\mu^j T^j + \tilde{g}_6 \tilde{A}_\mu^j T^j \right). \quad (31)$$

In our case $N = 6$, and $T^j, T^j$ are $SU(6)$ generators. The charge $g_1$ and $(\tilde{g}_1)$ belongs to the $U(1)$ and $\tilde{U}(1)$ gauge group respectively and $g_6$ and $(\tilde{g}_6)$ is the charge of the $SU(6)$ and $(SU(6))$ gauge group of theory respectively.
The potential $U(\tilde{P}, f^{jJ}, f)$ is a sum of various supersymmetric $D$ and $F$ terms:
\[
U(\tilde{P}, f^{jJ}, f) = \frac{1}{2g_6^2} \left( \frac{1}{g_5^2} (f^{jJ})^* f^{jJ} + \overline{P}_1 T^j T^j \tilde{P}_1 - \overline{P}_2 T^j T^j \tilde{P}_2 \right)^2 \\
+ \frac{g_1^2}{8} \left( \left| \tilde{P} \right|^2 - N\xi \right)^2 + g_6^2 \left( \left| \tilde{P}_2 T^j T^j \tilde{P}_1 \right| + \mu_2 f^{jJ} \right)^2 \\
+ g_1^2 \left( \tilde{P}_2 \tilde{P}_1 + \mu_1 f \right)^2 + \frac{1}{2} \sum_{a,A} \left( \left| f + f^{jJ} T^j T^j \right| P_{aA} \right)^2 \\
+ \left| \left( f + f^{jJ} T^j T^j \right) \overline{P}_{2,aA} \right|^2,
\]

where we have charges:
\[
g_1^2 = g_1^2 + \tilde{g}_1^2 \quad \text{and} \quad g_6^2 = g_6^2 + \tilde{g}_6^2,
\]
which are a result of the dual symmetry.

In Eq. (32) we have a parameter $\xi$ belonging to the Fayet-Iliopoulos $D$-term, which does not break $N=2$ supersymmetry. The breakdown to $N=1$ supersymmetric theory is realized with the help of parameters $\mu_1$ and $\mu_2$ (see [5]). However, the Fayet-Iliopoulos $D$-term triggers the spontaneous breaking of the gauge symmetry.

The action (28)–(33) gives the solution for the supersymmetric non-Abelian flux tubes in our preonic model. This solution was obtained by method developed in Ref. [5].

The vacuum expectation value (VEV) of spreons is given as
\[
v = \sqrt{\xi} \gg \Lambda_4,
\]
where $\Lambda_4$ is the 4-dimensional scale. This VEV is equal to
\[
v \sim M_{P1} \sim 10^{19} \text{ GeV},
\]
because we assume that spreons are condensed at the Planck scale.

Non-trivial topology (17) amounts to winding of elements of matrix (18), and we obtain string solutions:
\[
\tilde{P}_{\text{string}} = v \cdot \text{diag} \left( e^{ia(x)}, e^{ia(x)}, \ldots, 1, 1 \right), \quad \text{where} \quad x \to \infty.
\]

Three types of string moduli space
\[
\frac{SU(6)}{SU(5) \times U(1)} \times \frac{SU(6)}{SU(4) \times SU(2) \times U(1)} \quad \text{and} \quad \frac{SU(6)}{SU(3) \times SU(3) \times U(1)}
\]
give us solutions for three types of $Z_6$-flux tubes which are a non-Abelian analog of ANO-strings.
6. Assuming at the ends of strings the existence of the preon $P$ and antipreon $P^c$ with hyper-magnetic charges $n\tilde{g}$ and $-n\tilde{g}$, respectively, we obtain six types of strings having the fluxes $\Phi_n$ quantized according to the $Z_6$ center group of $SU(6)$:

$$\Phi_n = n\Phi_0, \quad n = \pm 1, \pm 2, \pm 3.$$  \hfill (38)

Indeed, $Z_6$ has six group elements:

$$Z_6 = \left\{ \exp \left( 2\pi \frac{n}{6} i \right) \right\| n \mod 6 \right\}.$$  \hfill (39)

So far as $n$ is given by modulo 6, the fluxes of tubes corresponding to the solutions with $n = 4, 5$ are equal to the fluxes with $n = -2, -1$, respectively (see also [6]).

String tensions of these non-Abelian flux tubes also were calculated in Refs. [5]. The minimal tension is:

$$T_0 = 2\pi \xi,$$  \hfill (40)

which in our preonic model is equal to:

$$T_0 = 2\pi v^2 \sim 10^{38} \text{ GeV}^2.$$  \hfill (41)

Such an enormously large tension means that preonic strings have almost infinitely small $\alpha' \rightarrow 0$, where $\alpha' = 1/(2\pi T_0)$ is a slope of trajectories in string theory [1]. Three types of the preonic $k$-strings have the following tensions:

$$T_k = kT_0, \quad \text{where} \quad k = 1, 2, 3.$$  \hfill (42)

If the Fayet-Iliopoulos term $\xi$ vanishes in the supersymmetric theory of preons, then spreon condensate vanishes too, and the theory is in the Coulomb phase. But for a non-vanishing $\xi$ the spreons develop their VEV, and the theory is in the Higgs phase. Then hyper-magnetic charges of preons and antipreons are confined by six strings which are oriented in opposite directions. By this reason, six strings have only three different tensions (42).

Also preonic strings are enormously thin. Indeed, the thickness of the flux tube depends on the mass of the dual gauge boson $\tilde{A}_\mu$ acquired in the confinement phase:

$$m_V = g v.$$  \hfill (43)

As it is shown below, in the region of energies $AB$ near the Planck scale, where spreons are condensed (see Fig. 2) we have:

$$\alpha = \frac{g^2}{4\pi} \approx 1, \quad g \approx 2\sqrt{\pi} \approx 3.5,$$  \hfill (44)

and the thickness of preonic strings given by the radius $R_{str}$ of the flux tubes is very small:

$$R_{str} \sim \frac{1}{m_V} \sim \frac{1}{gv} \sim 10^{-18} \text{ GeV}^{-1}.$$  \hfill (45)
Such infinitely narrow non-Abelian supersymmetric flux tubes remind us superstrings of Superstring theory. Having in our preonic model supersymmetric strings with \(\alpha' \to 0\) we obtain, according to the description [1], only massless ground states: spin 1/2 fermions (quarks and leptons), spin 1 hyper-gluons and spin 2 massless graviton, as well as their superpartners. The excited states belonging to these strings are not realized in our world as very massive: they have mass \(M > M_{Pl}\).

7. The hyper-flavor “horizontal” symmetry was suggested first in Refs. [14]. The previous Section gives a demonstration of a very specific type of the “horizontal symmetry”: three, and only three, generations of fermions and bosons present in the superstring-inspired flipped \(E_6\) theory, and also in each step given by Fig. up to the Standard Model. This number “3” is explained by the existence of three values of hyper-magnetic fluxes, which bind hyper-magnetic charges of preons-dyons. At the ends of these preonic strings there are placed hyper-magnetic charges \(\pm \tilde{g}_0\), or \(\pm 2\tilde{g}_0\), or \(\pm 3\tilde{g}_0\), where \(\tilde{g}_0\) is the minimal hyper-magnetic charge. Then all bound states of Fig. 3 form three generations – three 27-plets of \(E_6\) corresponding to the three different tube flux values. We also obtain the three types of gauge bosons \(A^i_{\mu}\) (\(i = 1, 2, 3\) is the index of generations) belonging to the \(27 \times 27 = 1 + 78 + 650\) representations of \(E_6\). Fig. 4 illustrates the formation of such hyper-gluons: Fig. 4(a) corresponds to the composite singlet, Fig. 4(b) and Fig. 4(c) correspond to the adjoint 78-plet and 650-plet of hyper-gluons, respectively.

Such a description predicts the Family replicated gauge group of symmetry \([E_6]^3\) for quark-leptons and bosons, which works near the Planck scale. Here the number of families is equal to the number of generations \(N_g = 3\). We assume that \([E_6]^3\) for quark-leptons takes place in the region of energies \(M_{FR} \leq \mu \leq M_{crit}\), where the scales \(M_{FR}\) and \(M_{crit}\) are indicated in Fig. by points \(D\) and \(A\), respectively.

The breakdown \([E_6]^3 \to E_6\) at the scale \(M_{FR}\) is provided by several Higgses. The analogous mechanism is described in reviews [15] and references therein. Indeed, in the Family replicated gauge group we have three types of gauge bosons \(A^i_{\mu}\), which produce linear combinations:

\[
A^{(i)}_{\mu,diag} = C_1^{(i)} A^{(1st fam.)}_\mu + C_2^{(i)} A^{(2nd fam.)}_\mu + C_3^{(i)} A^{(3rd fam.)}_\mu, \quad i = 1, 2, 3. \tag{46}
\]

The combination:

\[
A_{\mu,diag} = \frac{1}{\sqrt{3}} \left( A^{(1st fam.)}_\mu + A^{(2nd fam.)}_\mu + A^{(3rd fam.)}_\mu \right) \tag{47}
\]

is massless one, which corresponds to the 78 adjoint representation of hyper-gluons of \(E_6\). Two other combinations are massive and exist only in the region of energies \(\mu \geq M_{FR}\).

The assumption that only \([E_6]^3\) for quark-leptons exists in Nature may be not valid: we are not sure that the Family replicated gauge groups \([SO(10)]^3\), \([SU(5)]^3\), \([SMG \times U(1)]_{(B-L)}\)^3, or \([SMG]^3\) do not survive at lower energies \(\mu < M_{FR}\). Here we have used the following notation: \(SMG\) is the Standard Model group \(SU(3)_C \times SU(2)_L \times U(1)_Y\).
The values $\alpha^{-1}(M_{\text{crit}})$ and $\alpha^{-1}(\tilde{M}_{\text{crit}})$ can be calculated by the method developed in Ref. [16]. If $SU(N)$ group is broken by Abelian scalar particles belonging to its Cartan $U(1)^{N-1}$ subalgebra, then $SU(N)$ critical coupling constant $\alpha^\text{crit}_N$ is given by the following expression in the one-loop approximation (see [13, 16]):

$$\alpha^\text{crit}_N \approx \frac{N}{2} \sqrt{\frac{N+1}{N-1}} \alpha^\text{crit}_{U(1)};$$

(48)

where $\alpha^\text{crit}_{U(1)}$ is the critical coupling constant for the Abelian $U(1)$ theory.

Here it is necessary to distinguish $E_6$ gauge group of symmetry for preons from $E_6$ for quark-leptons. The points $A$ and $B$ of Fig. 2 respectively correspond to the breakdowns of $[E_6]^3$ and $[\tilde{E}_6]^3$ to the region $AB$ of spreon condensation. There we have the breakdown (see Ref. [3]) of the preon (one family) $E_6$ (or $\tilde{E}_6$):

$$E_6 \to SU(6) \times U(1)$$

(or

$$\tilde{E}_6 \to \tilde{SU}(6) \times \tilde{U}(1),$$

(49)

and the scale $M_{\text{crit}}$ (or $\tilde{M}_{\text{crit}}$) is the scale of breaking.

In our preonic model the group $SU(6)$ is broken by condensed Abelian scalar spreons-dyons belonging to the Cartan subalgebra $U(1)^5$. Then for the one family of preons we have:

$$\alpha^\text{crit}_6 \approx 3.55 \alpha^\text{crit}_{U(1)},$$

(49)

according to Eq. (48) for $N = 6$.

The behaviour of the effective fine structure constants in the vicinity of the phase transition point “Coulomb-confinement” was investigated in the compact lattice $U(1)$ theory by Monte Carlo method [18]. The following result was obtained:

$$\alpha^\text{crit}_{\text{lat.}U(1)} \approx 0.20 \pm 0.015, \quad \tilde{\alpha}^\text{crit}_{\text{lat.}U(1)} \approx 1.25 \pm 0.010.$$ (50)

The calculation of the critical coupling constants in the Higgs scalar monopole (dyon) model of dual $U(1)$ theory [16, 17] gave the following result:

$$\alpha^\text{crit}_{U(1)} \approx 0.21, \quad \tilde{\alpha}^\text{crit}_{U(1)} \approx 1.20 \quad \text{in the Higgs monopole model},$$ (51)

and

$$\alpha^\text{crit}_{U(1)} \approx 0.19, \quad \tilde{\alpha}^\text{crit}_{U(1)} \approx 1.29 \quad \text{in the Higgs dyon model}.\quad (52)$$

According to Eqs. (50)–(52), the condensation of spreons leads to the following critical constants:

$$\alpha^\text{crit}_6 \approx 3.55 \cdot 0.2 \approx 0.71, \quad \tilde{\alpha}^\text{crit}_6 = (\alpha^\text{crit}_6)^{-1} \approx 1.41.$$ (53)

For the point $C$ shown in Fig. 2 we have:

$$\alpha^\text{(one fam.)}_6(M_{\text{Pl}}) = 1.$$ (54)
This result for the one family of preons confirms the estimate (44). Here we have used the Dirac relation for non-Abelian theories (see explanation in Ref. [13]):

\[ g \tilde{g} = 4\pi n, \quad n \in \mathbb{Z}, \quad \alpha \tilde{\alpha} = 1. \]  

(55)

From the phase transition result (53) for the one family of preons, we obtain the following inverted critical coupling constant for \([E_6]^3\) at the phase transition point \(A\) (see reviews [15]):

\[ \alpha_A^{-1}(M_{\text{crit}}) = 3\tilde{\alpha}_6^{\text{crit}}(M_{\text{crit}}) \approx 3 \cdot 1.41 \approx 4.23. \]  

(56)

At the phase transition point \(B\), when we have the breakdown of \([\tilde{E}_6]^3\) into the confinement phase, the one-family value \(\tilde{\alpha}_6^{\text{crit}}(\tilde{M}_{\text{crit}}) \approx 0.71\) gives the following result:

\[ \alpha_B^{-1}(\tilde{M}_{\text{crit}}) \approx 3 \cdot 0.71 \approx 2.13. \]  

(57)

The values (56) and (57) were used for the construction of the curve \(AB\) presented in Fig. 2. The point \(C\) corresponds to the Planck scale and \(\alpha^{-1}(M_{Pl}) = 3\), according to Eq. (54).

The condensation of spreons at the Planck scale predicts the existence of the second minimum of the effective potential \(V_{\text{eff}}(\mu)\) at the scale \(\mu = M_{Pl}\). The behaviour of this potential and its relation with the Multiple Point Principle – theory of degenerate vacua by D.L. Bennett, C.D. Froggatt and H.B. Nielsen (see reviews [15] and references therein) will be considered in our future investigation.

Fig. 2 shows two points \(A\) and \(B\) near the Planck scale. The point \(A\) corresponds to the breakdown of \(E_6\) for preons according to the chain (16). The group \(E_6\) is broken in the region of energies \(\mu \geq M_{\text{crit}}\) producing hyper-electric strings between preons. The point \(B\) in Fig. 2 indicates the scale \(\tilde{M}_{\text{crit}}\) corresponding to the breakdown \(\tilde{E}_6 \rightarrow SU(6) \times \tilde{U}(1)\). At the point \(B\) hyper-magnetic strings are produced and exist in the region of energies \(\mu \leq \tilde{M}_{\text{crit}}\) confining hyper-magnetic charges of preons. As a result, in the region \(\mu \leq M_{\text{crit}}\) we see quark-leptons with charges \(ng\) (\(n \in \mathbb{Z}\)), but in the region \(\mu \geq \tilde{M}_{\text{crit}}\) monopolic “quark-leptons” – particles with dual charges \(m\tilde{g}\) (\(m \in \mathbb{Z}\)) – may exist. Since \(\tilde{M}_{\text{crit}} > M_{Pl}\), monopoles are absent in our world.

In the region of energies \(M_{\text{crit}} \leq \mu \leq \tilde{M}_{\text{crit}}\) around the Planck scale, both hyper-electric and hyper-magnetic strings come to play: preons, quarks and monopolic “quarks” are totally confined giving heavy neutral particles with mass \(M \sim M_{Pl}\), but closed strings – gravitons – survive there.

The dotted curve in Fig. 2 describes the running of \(\alpha^{-1}(\mu)\) for monopolic “quark-leptons” created by preons, which are bound by supersymmetric hyper-electric non-Abelian flux tubes. We assume that such monopoles do not exist in our world, however, they can play an essential role in the Universe vacuum (Cosmological Constant) [19].

**Conclusions**
i. In the present paper starting with an idea that the most realistic model based on the superstring theory is the ‘flipped’ $E_6$ gauge group of symmetry, we have assumed that at high energies $\mu > 10^{16}$ GeV there exists the following chain of the flipped models:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X \rightarrow$$

$$SU(5) \times U(1)_X \rightarrow SU(5) \times U(1)_{Z1} \times U(1)_{X1} \rightarrow SO(10) \times U(1)_{X1} \rightarrow E_6,$$

with the flipped $E_6$ final unification. We have chosen such Higgs boson contents of the $SU(5)$ and $SO(10)$ gauge groups, which give the flipped $E_6$ at the scale $\sim 10^{18}$ GeV and the decreased running of $\alpha^{-1}(\mu)$ near the Planck scale.

ii. We have shown that the final unification $E_6$ assumes the existence of three 27-plets of the flipped $E_6$ gauge group of symmetry.

iii. Suggesting $N = 1$ supersymmetric $E_6 \times \tilde{E}_6$ preonic model of composite quark-leptons and bosons, we have assumed that preons are dyons confined by hyper-magnetic strings in the region of energies $\mu \lesssim M_{Pl}$. This approach is an extension of the old idea by J. Pati to use the strong magnetic forces which may bind preons-dyons in composite particles – quark-leptons and bosons. Our model is based on the recent theory of composite non-Abelian flux tubes in SQCD [5–7].

iv. Considering the breakdown of $E_6$ (or $\tilde{E}_6$) at the Planck scale into the $SU(6) \times U(1)$ (or $\tilde{SU}(6) \times \tilde{U}(1)$) gauge group we have shown that the six types of $k$-strings – $N = 1$ supersymmetric non-Abelian flux tubes – are created by the condensation of spreons near the Planck scale.

v. It was shown that the six types of strings-tubes having six fluxes quantized according to the $Z_6$ center group of $SU(6)$:

$$\Phi_n = n\Phi_0, \quad n = \pm 1, \pm 2, \pm 3,$$

create three types of $k$-strings with tensions:

$$T_k = kT_0, \quad k = 1, 2, 3,$$

which produce three (and only three) generations of composite quark-leptons and bosons. We have obtained a specific type of the “horizontal symmetry” explaining a flavor.

vi. It was investigated that in the present model preonic strings are very thin, with radius

$$R_{str} \sim 10^{-18} \text{ GeV}^{-1},$$

and their tension is enormously large:

$$T_0 \sim 10^{38} \text{ GeV}^2.$$
vii. The model predicts the existence of three families of 27-plets and also gauge bosons \( A^i_\mu \) (with \( i = 1, 2, 3 \)) belonging to the 78-plets of \( E_6 \). Then near the Planck scale we have the Family replicated gauge group of symmetry \([E_6]^3\). In the present paper we have assumed that the breakdown \([E_6]^3 \rightarrow E_6\) occurs near the Planck scale leading to the \( E_6 \) unification at the scale \( \sim 10^{18} \text{ GeV} \).

viii. We have considered that the condensation of spreons-dyons near the Planck scale gives the phase transitions at the scales \( M_{\text{crit}} \) and \( \widetilde{M}_{\text{crit}} \) shown in Fig. 2. These scales (points A, B of Fig. 2) correspond to the breakdown of \( E_6 \) and \( \widetilde{E}_6 \) for preons, respectively:

\[
E_6 \rightarrow SU(6) \times U(1) \quad \text{and} \quad \widetilde{E}_6 \rightarrow \widetilde{SU}(6) \times \widetilde{U}(1).
\]

ix. It was investigated that hyper-magnetic strings are produced and exist at \( \mu \leq \widetilde{M}_{\text{crit}} \), and hyper-electric strings are created and exist at \( \mu \geq M_{\text{crit}} \). As a result, in our world we have quark-leptons and gauge bosons \( A_\mu \) in the region of energies \( \mu \lesssim M_{\text{Pl}} \), but monopolic “quark-leptons” and dual gauge fields \( \widetilde{A}_\mu \) exist in the region \( \mu \gtrsim M_{\text{Pl}} \) (at the trans-Planckian scales, see [19]). We have calculated the critical values of gauge coupling constants at the scales \( M_{\text{crit}} \) and \( \widetilde{M}_{\text{crit}} \):

\[
\alpha^{-1}(M_{\text{crit}}) \approx 4.23, \quad \text{and} \quad \alpha^{-1}(\widetilde{M}_{\text{crit}}) \approx 2.13.
\]

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Fig. 1: This figure presents the running of the inverted gauge coupling constants $\alpha_i^{-1}(x)$ for $i = 1, 2, 3, X, Z, X1, Z1, 5, 10$ of the chain $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X \rightarrow SU(5) \times U(1)_X \rightarrow SU(5) \times U(1)_Z \times U(1)_X \rightarrow SO(10) \times U(1)_X \rightarrow E_6$, corresponding to the breakdown of the flipped $SU(5)$ to the supersymmetric (MSSM) $SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_X$ gauge group of symmetry with Higgs bosons belonging to the $5_h + \bar{5}_h, 10_H + \bar{10}_H$, 24-dimensional adjoint $A$ and higher representations of the flipped $SU(5)$. The final unification is $E_6$ at the supersuperGUT scale $M_{SSG} \approx 10^{18}$ GeV with $\alpha^{-1}(M_{SSG}) \approx 22$ for $M_{SUSY} = 10$ TeV and seesaw scale $M_S = 10^{11}$ GeV.
Fig. 2: The figure illustrates a qualitative description of the running of $\alpha^{-1}(x)$ near the Planck scale predicted by the present preonic model. The region $AD$ corresponds to the Family replicated gauge group of symmetry $[E_6]^3$, which comes at the scale $M_{FR}$ given by point $D$. The point $A$ at the scale of energy $\mu = M_{crit}$ shows that hyper-electric preonic strings exist for $\mu \geq M_{crit}$. The point $B$ corresponds to the scale $\mu = \tilde{M}_{crit}$ and indicates that hyper-magnetic preonic strings exist for $\mu \leq \tilde{M}_{crit}$. The curve $AB$ corresponds to the region of energies, where spreons are condensed near the Planck scale giving both, hyper-electric and hyper-magnetic, preonic strings. For $\mu \geq \tilde{M}_{crit}$ we have the running of $\alpha^{-1}(x)$ for monopolic "quark-leptons". The point $C$ corresponds to the Planck scale and gives $\alpha^{-1}(M_{Pl}) = 3$. 

$\alpha_3(M_Z) = 0.117$

$M_t = 174$ GeV
Fig. 3: Preons are bound by hyper-magnetic strings: (a,b) correspond to the string configurations of composite particles belonging to the 27-plet of $E_6$ gauge group of symmetry; (c) represents a closed string describing a graviton.
Fig. 4: Vector gauge bosons belonging to the $1 + 78 + 650$ representations of $E_6$. Gluons are composite objects created by fermionic preons $P, P^c$: (a) corresponds to the singlet; (b) is the 78-plet and (c) is the 650-plet of $E_6$. 