A multiresolution inversion for imaging the ionosphere

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Abstract Ionospheric tomography has been widely employed in imaging the large-scale ionospheric structures at both quiet and storm times. However, the tomographic algorithms to date have not been very effective in imaging of medium- and small-scale ionospheric structures due to limitations of uneven ground-based data distributions and the algorithm itself. Further, the effect of the density and quantity of Global Navigation Satellite Systems data that could help improve the tomographic results for the certain algorithm remains unclear in much of the literature. In this paper, a new multipass tomographic algorithm is proposed to conduct the inversion using intensive ground GPS observation data and is demonstrated over the U.S. West Coast during the period of 16–18 March 2015 which includes an ionospheric storm period. The characteristics of the multipass inversion algorithm are analyzed by comparing tomographic results with independent ionosonde data and Center for Orbit Determination in Europe total electron content estimates. Then, several ground data sets with different data distributions are grouped from the same data source in order to investigate the impact of the density of ground stations on ionospheric tomography results. Finally, it is concluded that the multipass inversion approach offers an improvement. The ground data density can affect tomographic results but only offers improvements up to a density of around one receiver every 150 to 200 km. When only GPS satellites are tracked there is no clear advantage in increasing the density of receivers beyond this level, although this may change if multiple constellations are monitored from each receiving station in the future.

1. Introduction

Ionospheric tomography using Global Navigation Satellite Systems (GNSS) data is based on total electron content (TEC) observations along a series of station-satellite signal propagation paths in the inversion region to reconstruct the temporal and spatial variations of the ionospheric electron density [see, e.g., Pryse, 2003]. It has become one of most convenient and effective methods to monitor and detect the ionosphere in recent years and has demonstrated useful insights into the spatial and temporal variations of the ionosphere. Fremouw et al. [1992] proposed the inversion of TEC into two-dimensional electron density maps by using singular value decomposition for ionospheric tomography. Hernández-Pajares et al. [1999] constructed a two-layer model for detecting the global ionosphere. A multipass tomographic algorithm, named as Multi-Instrument Data Analysis System (MIDAS) was built to enable the ingestion of multiple data sources (e.g., ground-based GPS data, Low Earth Orbiter-based GPS data, and ionosonde measurements) into the inversion to produce images of ionospheric electron density [Mitchell and Spencer, 2003]. Bust et al. [2004] developed an ionospheric imaging technique based on data assimilations—Ionospheric Data Assimilation Three-Dimensional. A comprehensive review of ionospheric imaging is given in Bust and Mitchell [2008].

Usually, ionospheric tomography is applied to reconstruct large-scale ionospheric structures under different conditions to study the variability during both quiet and disturbed geomagnetic conditions. However, due to the limited number of GNSS satellites and the uneven ground station distributions, the spatial and temporal resolutions of tomographic images are rather limited, especially for medium- and small-scale structures. In recent years, the development of multi constellation GNSS and the increasing amount of ground-based GNSS stations in some regions provide a good opportunity for developing an ionospheric algorithm to image multiscale ionospheric structures. Liang et al. [2014] developed a data adaptive multiscale ionospheric model with high spatial resolution. Zheng et al. [2014] applied a three-dimensional multiscale ionospheric tomography to reconstruct the ionosphere in China. A new approach is proposed for computerized ionospheric tomography that aims to permit sparsity in the reconstruction coefficients by using wavelet basis functions [Panicciari et al., 2015].
For the purpose of imaging the ionosphere at a higher resolution than before, that is to push below the bounds of hundreds of kilometers and times of tens of minutes, a new multipass inversion approach which is adapted from the MIDAS algorithm is demonstrated in this paper. So far, MIDAS has been applied to a great deal of scientific research on the ionosphere at both storm time and quiet time. For instance, Yin et al. [2004] found the abnormal uplift of the ionospheric peak height over the United States for the period of the severe ionospheric storm on 15 July 2000, by comparing MIDAS electron density results with Millstone Hill radar observations and ionosonde data. Ionospheric imaging using GPS data with MIDAS is also conducted over Antarctica [Yin et al., 2009]. Giday et al. [2015] applied the optimized MIDAS algorithm to reconstruct the ionospheric structure in South Africa and verified the results with data from three local ionosondes.

In this paper, the ionospheric distribution of electron density over the U.S. western coast on 16–18 March 2015 when severe geomagnetic storms occurred has been reconstructed using the multipass tomographic method. For the validation of the tomographic results, ionosonde data and Center for Orbit Determination in Europe (CODE) TEC estimates are used to make a comparison. In addition, the impact of GPS ground data distribution on ionospheric imaging is also investigated so that the correlation between input data and final imaging resolution could be made.

2. Multipass Inversion Algorithm

Since the inverse problem is always underdetermined, certain basis functions are often needed to compensate for the lack of information by selecting the appropriate mapping matrix, so that the spatial and temporal distributions of ionospheric electron density can be represented separately. In general, different methods may be used in different dimensions to construct the matrix because the mapping matrix can be very large [Bust et al., 2007].

In MIDAS, for the horizontal variation of electron density, a grid with various resolutions is defined in latitude and longitude, while for the radial direction, a set of empirical orthogonal functions is generated from certain ionospheric models (such as International Reference Ionosphere (IRI) and Chapman function). Besides, a sliding window is applied in the temporal dimension. Using the recent version of MIDAS with only one pass inversion process [Spencer and Mitchell, 2007], coarse resolutions in space and time, such as 4° × 4° in latitude and longitude and 30 min interval, have generally to be adopted due to some limitation in GPS data coverage and the tomographic algorithm; therefore, only large-scale ionospheric structures can be reconstructed by that method [see, e.g., Yin and Mitchell, 2014].

In order to obtain ionospheric images with multiple resolutions, especially fine resolutions (e.g., 1° × 1° in latitude and longitude and less than 15 min in time), a multipass inversion approach is proposed in this paper on the basis of the current single pass inversion of MIDAS.

First of all, the number of levels of inversion is determined according to the required final resolution of ionospheric images, and electron density images are reconstructed on the first pass inversion with coarse resolutions.

Then inverse problem for the first pass is written as

$$Hx = z$$  \hspace{1cm} (1)

where $H$ is the GPS rays through the ionospheric grid intercept matrix, $z$ is the observations of slant TEC values over all the raypaths, and $x$ is the unknown electron density values corresponding each voxel. Equation (1) is also the formula of the single pass inversion algorithm. In MIDAS, the slant TEC values are calculated from geometry free GPS L1 and L2 phase measurements. Assuming that the unknown electron densities are constant within each grid voxel and the GPS signal travels in a straight line, the length of each raypath element through each intersected grid voxel is calculated from the known positions of the satellites and receivers.

In order to solve the inverse problem, a mapping function that transforms the problem into a set of basis functions with unknown coefficients is introduced in MIDAS. Equation (1) now can be represented by

$$HMy = z$$  \hspace{1cm} (2)
where $y$ stands for the basis functions and $M$ denotes unknown coefficients representing the relative contribution of the basis functions. Applying the coefficients $M$ to the basis functions, the solution of electron density is given by

$$x = My$$

(3)

Second, the resolutions in space and time are enhanced by a factor of 2 to the levels on the second pass inversion. For the second pass the inverse problem is formulated as

$$\hat{H}Hx = \hat{H}z + x_0$$

(4)

In which, $\hat{H}$ is the matrix transpose of $H$ and $x_0$ is the interpolated prior solution. For instance, if the levels of inversion are selected as one in both space and time, then the inversion is made with resolutions of $4^\circ \times 4^\circ$ in latitude and longitude and 30 min in time on the first pass inversion, and $2^\circ \times 2^\circ$ in latitude and longitude and 15 min in time on the second pass inversion.

In this study, GPS ground data over the western coast of the USA are provided by UNAVCO. The spatial range of the reconstruction is selected from 30° to 45° in north latitude, 100° to 125° in west longitude, and 100 to 1200 km in altitude. For the first pass inversion, the spatial and temporal resolutions are defined as $4^\circ \times 4^\circ$ and 30 min. Since the level of resolution enhancement for the multiinversion is chosen as two in latitude and longitude and one in time, the resolution for the second pass will be $1^\circ \times 1^\circ$ and 15 min.

3. Evidence for the Ionospheric Disturbance

The study in this paper is focused on the geomagnetic storm during solar cycle 24 that occurred mainly on 17 March 2015, whose source can be traced to the solar event on 15 March [Astafyeva et al., 2015; Ponomarchuk et al., 2015]. Figure 1 shows the variation of geomagnetic index $Dst$ and $Kp$ with UT obtained from the Space Physics Interactive Data Resource database [Zhizhin et al., 2008] between 15 and 18 March 2015. As illustrated in Figure 1, it can be seen that $Kp$ values remained very low (no more than 4) and $Dst$ data changed little and very smooth for the days of 15 and 16 March, indicating that the geomagnetic activity was not disturbed and the ionosphere was in the quiet condition. However, from 12:00 UT on the 17 March onward, $Kp$ started to increase while $Dst$ gradually reduced. Until about 23:00 UT $Dst$ reduced down to $-223$ nT and $Kp$ lifted up to 8, indicating a strong ionospheric storm reaching its main phase. This disturbance continued until the early morning of 18 March with $Dst$ lower than $-100$ nT and $Kp$ greater than 4.

Since independent ionospheric data such as ionosonde measurements will be used to validate our tomographic results in terms of electron density or TEC in this study, an ionosonde station in Point Arguello (code: PA836) (http://car.uml.edu/common/DIDBFastStationList) located in the west coast of the United States at 35.6° north latitude and 120.6° west longitude is selected to carry out the comparison between ionosonde data [Reinisch and Galkin, 2011] and ionospheric inversions over the storm period.

Variations of the autoscaled ionosonde data in terms of peak density at $F_2$ ($NmF_2$ in blue) and peak height at $F_2$ ($h_mF_2$ in green) with UT at Point Arguello between 16 and 18 March 2015 are shown in Figure 2. It should be noted that ionosonde measurements are not available at some times; therefore, zero values are used to represent those missing data in Figures 2a (16 March), 2b (17 March), and 2c (18 March). It can be seen from Figure 2a that the peak electron density increases around 13:00 UT (05:00 LT) and decreases around 02:00 UT (18:00 LT) under ionospheric quiet periods (such as on 16 March) driven primarily by the solar ionization and natural variability. However, this trend is not shown in Figures 2b and 2c, especially from 14:00 UT on 17 March to 03:00 UT on 18 March, when the ionosphere is mostly disturbed with $Kp$ above 6 and $Dst$ below $-100$ nT. On the contrary, the peak electron densities keep quite low during the local daytime between 17
March and 18 March, down to nearly one fourth of their quiet values, which indicates a negative ionospheric storm over the western USA.

The variation of $h_m F_2$ from the ionosonde observations is quite abnormal over the disturbed periods; for example, $h_m F_2$ remains at 240 km during the daytime on the 16 March while the ionosphere uplifts to almost 360 km at storm time on the 17 March. Yin et al. [2006] also observed the unusual increases of the peak height over the USA during several ionospheric storms. Nevertheless, due to the unavailability of ionosonde data at most disturbed times on 17 March, we have to concentrate our validation study on the ionospheric response to the more disturbed periods on 18 March and compare it with the normal behavior of the ionosphere on 16 March.

4. Results and Discussions

For the purpose of validation of our proposed new method, the experimental study is carried out in two steps. First, tomographic results obtained by the new multipass inversion approach and the no-multipass inversion method will be compared with ionosonde measurements and independent GPS TEC data provided by International GNSS Service IGS CODE center, respectively. For the convenience of description, tomographic results obtained by the multipass inversion algorithm are referred to as “multi” results, and tomographic results obtained by the single pass inversion algorithm is denoted as “no-multi” results hereafter. Second, various ground GPS data sets will be used as inputs to the multipass inversion algorithm in order to investigate the impact of data coverage on the inversion.

4.1. $N_m F_2$ Comparison With Ionosonde

The GPS ground station distribution of the GPS receivers used in this section is shown in Figure 3a. The input data into MIDAS are slant TECs along raypaths crossing the ionosphere from satellites to GPS ground stations, which can be calculated from GPS dual-frequency phase observations. Figure 3b illustrates tracks of Ionospheric Pierce Points (IPPs) along the path between GPS ground stations and GPS satellites at 03:00–04:00 UT on 16 March, with the assumption of the ionospheric shell height at 350 km. It demonstrates a good full coverage of measurements across the western USA. In order to test the reliability of
the tomography results, using ionosonde Point Arguello’s $N_{m}F_2$ observation data are compared with the tomographic results.

Figures 4a and 4b show the variations of ionosonde $N_{m}F_2$, multi-inversion $N_{m}F_2$ and no-multi inversion $N_{m}F_2$ with time at Point Arguello on 16 March where the tomographic resolutions are defined as $4^\circ \times 4^\circ$ in latitude and longitude with 30 min in time, and $1^\circ \times 1^\circ$ in latitude and longitude with 15 min interval for multi inversions, respectively. Figures 4c and 4d show the corresponding results on 18 March. In other words, Figures 4a and 4c show the first pass of the multireconstructions and the $4^\circ$ resolution no-multireconstructions.

**Figure 4.** Comparisons of $N_{m}F_2$ against UT between ionosonde measurements, multi-inversions, and no-multi-inversions with (a) $4 \times 4$ resolution on 16 March (b) $1 \times 1$ resolution on 16 March, (c) $4 \times 4$ resolution on 18 March, and (d) $1 \times 1$ resolution on 18 March.
Figures 4b and 4d show the second pass of the multireconstructions and a separate, higher-resolution (1°) no-multireconstructions. It should be noted that for the multipass inversion approach, tomographic results from the second pass are shorter in time than those from the first pass since the second pass inversion is processed on the basis of the first pass inversion. The time shortened is determined by the time window parameter setup in the algorithm at the beginning of the process. In this study, the temporal span observations used in the first pass inversion is 210 min longer than that of the second pass inversion throughout 24 h.

When comparing Figures 4a and 4b with Figures 4c and 4d, it can be seen that tomography results agree with ionosonde data much better on 16 March than on 18 March because the ionosphere was quiet on 16 March while kept disturbed on 18 March. On the other hand, multiresult and no-multiresult seen from Figures 4a and 4c are quite similar for the lower resolution rather than for the higher resolution in Figures 4b and 4d, where there is a larger divergence between multiresult and no-multiresult. In addition, as shown in Figure 4d, the unique ability of the higher time resolution multireconstruction to reproduce the type of variability can be seen in NmF2 from 18 March. While the time series does not exactly match the ionosonde data, it shows similar levels of variability on similar time scales that the no-multiresolution does not.

In order to analyze the statistical difference of NmF2 between multiresult and no-multiresult with ionosonde measurements over 2 days, mean errors and root-mean-square errors (RMSE) are computed for two resolutions and summarized in Table 1.

Table 1. NmF2 Errors Between Ionosonde Data, Multi-inversions, and No-multi-inversions

| Day     | Resolution | NmF2 Errors (10¹¹ el/m³) | Multi-inversions | No-Multi-inversions |
|---------|------------|--------------------------|------------------|---------------------|
| 16 March| 4° × 4°    | Mean                     | 0.8              | 0.8                 |
|         |            | RMSE                     | 0.5              | 0.5                 |
|         | 1° × 1°    | Mean                     | 0.7              | 1.1                 |
|         |            | RMSE                     | 0.5              | 0.6                 |
| 18 March| 4° × 4°    | Mean                     | 2.5              | 2.5                 |
|         |            | RMSE                     | 1.6              | 1.6                 |
|         | 1° × 1°    | Mean                     | 2.0              | 2.5                 |
|         |            | RMSE                     | 1.3              | 1.4                 |

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Table 1, the comparison with peak density, shows that the operation of the no-multi-inversion at 1° increases the error in the image in comparison with the operation of the no-multi-inversion at 4°, especially on 16 March. However, the two-stage operation of a 4° inversion and then a 1° inversion (based on the 4° result) achieves a lower overall error. This indicates that the multipass inversion algorithm is more suitable for high-resolution tomography than the no-multi-inversion algorithm and can be used for multiresolution reconstruction.

Electron density changes as a function of height and time at Point Argulleo from multiresults on 16 and 18 March are illustrated in Figures 5a through 5d with various resolutions. It is shown as Figures 5a and 5b that temporal variations of electron density in terms of NmF2 and hmF2 on 16 March are quite similar for both 4° × 4° and 1° × 1° resolutions, and coincident with ionosonde data well (Figure 2a). Nevertheless, on 18 March as shown in Figures 5c and 5d, there is an obvious divergence in peak density variations, especially from 00 to 08 UT and around 20 UT, though no difference in peak heights at F2 layer is seen from the comparison. Furthermore, it is interesting to note that hmF2 between 00 and 04 UT on 18 March is a bit higher than that on 16 March when compared both Figures 5d and 5b and Figures 5c and 5a, which is also seen from the ionosonde hmF2 variations in Figures 2a and 2c. Since the ionosphere was still disturbed over the early UT hours, i.e., 00–04 UT on 18 March, the uplift of peak heights from 240 km on 16 March to ~320 km on 18 March during the same period results in peak densities drop down, which may be attributed to neutral winds blowing plasma up into regions of lower recombination.

4.2. Comparison With CODE TEC

TEC data from one of IGS European data processing centers—CODE—can be used to provide an independent estimate to validate the absolute calibration level achieved in the tomographic TEC. The processing assumes that the ionosphere is an even distributed thin shell at the height of 450 km; a large number of
GNSS original data are applied to generate global CODE TEC data with the resolution of 2.5° by 5° in latitude and longitude at 1 h interval [Dach et al., 2016].

Figure 6 illustrates comparisons between CODE TEC and reconstructed TEC at 16:00UT on 16 and 18 March, and the resolution was set as 1° × 1° in longitude and latitude. Figures 6a and 6d are TEC provided by the CODE center on 16 and 18 March, respectively. Figures 6b and 6c are TEC obtained from multipass inversion and no-multipass inversion, respectively, on 16 March, and Figures 6e and 6f are those on 18 March.

When comparing Figures 6b and 6c with Figure 6a, it can be seen that the variations of TEC from south-east to north-west observed from multiresults agree better with those of CODE TEC than those from no-multiresults.

Figure 5. Temporal electron density variations (10^{11} \text{el/m}^3) at Point Argulleo for 1 × 1 resolution from (a) multi-inversions on 16 March, (b) no-multi-inversions on 16 March, (c) multi-inversions on 18 March, and (d) no-multi-inversions on 18 March.

Figure 6. Comparisons between CODE TEC maps at 16:00 UT on (a) 16 March and (d) 18 March and reconstructed TEC maps from (b) multi-inversions and (c) no-multi-inversions on 16 March, and (e) from multi-inversions and (f) no-multi-inversions on 18 March.
shown in Figure 6c. In addition, TEC enhancements around the area (about 30°N, 105°W) are separated from TEC variations over the surrounding area instead of showing continuous TEC increases seen in multiresults (Figure 6b) and CODE TEC (Figure 6a). This indicates that high-resolution TEC maps reconstructed by the no-multialgorithm may be creating some unreal ionospheric structures. This is also seen from the comparison among Figures 6d–6f on 18 March, where several small TECs appear in no-multial results rather than in multiresults. This will be examined further in a future study.

For the purpose of validating the above findings, vertical TEC variations with 24 UT hours at Point Arguello obtained from CODE estimates, multiresults and no-multial results are illustrated in Figures 7a and 7b for 16 and 18 March, respectively. From Figure 7a it can be seen that the temporal changes of TEC from multiresults coincide with those of CODE TEC better than those from no-multial results both on 16 and 18 March. In order to make a more detailed comparison of the differences, there is an error statistical analysis, as shown in Table 2.

In order to analyze the statistical difference of TEC from multiresult and no-multial result with CODE data over 2 days, mean errors and root-mean-square errors (RMSE) are computed and summarized in Table 2. It is interesting to note that the mean errors are always better for the multi-inversion. However, the RMS is not better on the disturbed day. It is not possible with the low resolution of the CODE data to determine the exact reason for this discrepancy at this stage; it may be real variability not captured at the resolution of the CODE analysis. This will be investigated in a future study.

### 4.3. Influence of GPS Ground Data Distribution

It is interesting to consider how this approach could be adapted to other regions of the world, where the density of GNSS stations is lower than in the USA. In order to analyze the impact of ground-based GPS data coverage on the proposed multipass inversion approach, three groups of GPS ground stations over the western coast of the USA are selected from the same data source as in section 3. The stations are grouped on the basis of the distance between two adjacent ground stations to ensure the uniformity of data coverage within the same reconstructed region. Namely, test 1 is for the sparse data coverage with only 14 ground stations, whose separation of two adjacent stations is about 450 km; test 2 is made for the medium density data coverage with 56 ground stations spacing about 300 km; and test 3 is formed for the dense data coverage with 205 ground stations spacing about 150 km.

Figure 8 illustrates the distribution of three sets of ground stations that will be used as data inputs into tomography for reconstructing the ionosphere over the same region at the same time. Figure 9 shows the

| Day       | Resolution | TEC Errors (TECU) | Multi-inversions | No-Multi-inversions |
|-----------|------------|-------------------|------------------|---------------------|
| 16 March  | 1° × 1°    | Mean: 2.3         | 4.1              |
|           |            | RMSE: 1.4         | 2.5              |
| 18 March  | 1° × 1°    | Mean: 4.5         | 5.8              |
|           |            | RMSE: 4.2         | 3.6              |
corresponding IPP tracks at 350 km of three groups of ground stations between 03:00 and 04:00 UT on 16 March. Figures 8a and 9a are for test 1, Figures 8b and 9b are for test 2, and Figures 8c and 9c are for test 3. It can be clearly seen that the density of GPS data coverage is high for test 3 and low for test 1 within the same area of 30°–45° north and 100°–125° west, while the data density for test 2 is in the middle.

Using the multipass inversion approach, tomographic images of electron density are reconstructed with final spatial resolution of 1° × 1° and temporal resolution of 15 min for three data groups on 16 and 18 March. In order to validate the reconstructed results of different data groups and investigate their impacts on the results, ionosonde measurement of $N_{mF_2}$ are applied here again. Figure 10 illustrates the comparison of $N_{mF_2}$ between ionosonde measurements and multi-inversions for tests 1, 2, and 3 on 16 and 18 March, respectively.

The experimental results of three groups are consistent with the ionosonde observations very well on 16 March when the ionosphere is quiet. The difference among tomographic results of three groups is very small, and the inversions agree with ionosonde data very well for most of the day, except those of test 1 has a little divergence from ionosonde data. However, comparisons of $N_{mF_2}$ among these results show quite deviated on 18 March, in particular, when the ionosphere appears disturbed from 00:00 to 06:00 UT. Additionally, experimental results of tests 2 and 3 are similar in terms of $N_{mF_2}$ values and temporal variations on both days, indicating that for the spatial resolution of 1° × 1°, tomographic results using test 2 ground data set can achieve the same accuracy as test 3 results which using the largest amount of GPS ground data.

However, among three data sets’ results on both days, it can be seen that the difference between test 1 results and ionosonde data is always greater than those of tests 2 and 3. For instance, there is a large

Figure 8. Selected GPS ground station distribution under different test conditions: (a) test 1, (b) test 2, and (c) test 3.

Figure 9. IPP tracks at 350 km of three GPS data groups between 3:00 and 4:00 UT on 16 March: (a) test 1, (b) test 2, and (c) test 3.
deviation between the test 1 inversions and ionosonde data which is up to $8 \times 10^{11}$ el/m$^3$ during disturbed ionospheric period between 00:00 to 06:00 UT on 18 March. This may be attributed to the sparse data coverage in test 1 that is not as suitable as tests 2 and 3 for the multipass inversion approach to imaging the ionosphere with high resolutions, such as 1° × 1°.

Mean errors and RMSEs between ionosonde data and tomographic results of tests 1, 2, and 3 on 16 and 18 March are shown in Table 3 so as to perform a quantitative analysis. From the calculated errors for tests 1, 2, and 3 on both days, it can be shown that test 1 tomographic results are the worst in terms of both errors. It may be concluded that more GPS data will be needed if ionospheric tomography is required for high resolutions. However, it is interesting to see that the errors of tests 2 and 3 data sets are nearly the same on both days even if the number of GPS ground stations in test 2 is much less than that of stations in test 3. It seems that the increase of data coverage may not provide any improvement in results. This is an important result because it gives an indication of the number density of ground stations needed for a given spatial-temporal image resolution.

### 4.4. Verification Through Simulation

The characteristic variations in the peak electron density and time scales seen in the experimental results could be consistent with the presence of large-scale traveling ionospheric disturbances (TIDs). Large-scale TIDs are described in Hargreaves [1992]. To test the new algorithm further, a simulation was run. The GPS satellite geometry repeats on a sidereal day, so a nominal day and time period were chosen to make the simulation with real receiver and satellite geometry. The receivers chosen were the same as those shown in Figure 3. Then a 4-D simulated ionosphere was created from the IRI model [Bilitza et al., 2014] and a TID was superimposed upon it. The TID was formed using the equations in the formulation by Hooke [1968] and was given large-scale TID parameters of wavelength 1000 km, perturbation amplitude 25%, and velocity 120 m/s.

The reconstructed TEC map for the same time frame is shown in Figure 11 with the color scale showing TEC in TECU (total electron content unit, 1 TECU = $10^{16}$ el/m$^2$) and the black square showing the location where the comparison is made. The arrow shows the direction of propagation. In more detail, Figure 12 shows the peak density in the $F$ region ionosphere in the simulation at the location shown in the previous figure, the reconstruction at 4° resolution, and at 1° resolution, that is using the methods outlined in this paper. It can be seen that the 4° resolution image does not really resolve the TID, whereas the 1° resolution image does resolve the TID but underestimates the amplitude. This simulation helps to confirm the interpretation of the results in

| Day     | N$_{mF2}$ Errors ($10^{11}$ el/m$^3$) | Test 1 | Test 2 | Test 3 |
|---------|--------------------------------------|--------|--------|--------|
| 16 March| Mean                                 | 0.9    | 0.8    | 0.8    |
|         | RMSE                                 | 0.5    | 0.5    | 0.5    |
| 18 March| Mean                                 | 3.1    | 2.1    | 2.0    |
|         | RMSE                                 | 2.5    | 1.4    | 1.3    |

Figure 10. $N_{mF2}$ comparisons between ionosonde data and multi-inversions of tests 1, 2, and 3 on (a) 16 March and (b) 18 March 2015.
section 4.1 and also shows that the imaging is showing sensible results but is still limited in the ability to fully resolve the amplitudes of large-scale TIDs. Nevertheless, the new two-stage algorithm shows good results in terms of wavelength and velocity.

Figure 13 shows a height versus time plot of the electron density profile in the model (Figure 13a) and the reconstruction (Figure 13b) for the 1° resolution grid. It can be seen that the reconstruction underestimates the amplitude of the TID and that the vertical profile shape on the reconstruction is slightly low on the bottomside. Nevertheless, there is some evidence of the change in the layer height being resolved during the passage of the TID.

5. Conclusions and Future Work

In this paper, for the purpose of imaging the ionosphere with multiple resolutions, a multipass tomographic algorithm is proposed to conduct the inversion using intensive ground GPS observation data over the U.S. West Coast for the days of 16 and 18 March 2015. First of all, by means of improving MIDAS algorithm from the single pass inversion process to a multipass inversion approach, tomographic results using the multipass inversion algorithm and the no-multimethods are validated with independent ionosonde $N_mF_2$ and CODE TEC data for different reconstruction resolutions including 4° × 4° and 1° × 1° in latitude and longitude, and 30 min versus 15 min in time. The comparisons show that for the low-resolution inversions, tomographic results from the first pass of the multipass approach have the same accuracy as those using the no-multimethod. However, for the high resolution, multipass results have significant advantages over no-multiresults in terms of mean errors and RMSEs. Therefore, the multipass inversion approach is suitable for multiresolution reconstructions.

In order to investigate the impact of the density of ground stations on ionospheric tomography results, three ground data sets with different GPS station distributions were grouped from the same data source over the western coast of the US, i.e., test 1 with the least ground stations, test 3 with the most stations, and test 2 in the middle. Tomographic results for three data groups using the multipass inversion approach are generated with the high resolution of 1° × 1° in space and 15 min in time. After validated with ionosonde data, it is clearly seen that the density of the ground station distribution has certain effect on the ionospheric tomography. For instance, inversions from test 1 are not as accurate as those from tests 2 and 3 on both days, especially when the ionosphere is disturbed. This means that more input data would be needed in ionospheric tomography required for high resolutions.

In addition, the errors between the ionosonde $N_mF_2$ and tomographic results of tests 2 and 3 are quite similar, indicating that too much ground data maybe not helpful to the improvement of tomographic accuracy for the given resolution in the algorithm.

There are two main conclusions that can be made from this study. First, the
multiresolution algorithm presented offers an improvement over a single-stage single resolution approach to ionospheric imaging. Second, the density of ground-based receivers needed can be evaluated experimentally and for the experiment shown it was only necessary to have a medium level of coverage of around 5–10 receivers in a 5 by 5° grid cell.

It is not straightforward to theoretically determine the density of an inhomogeneous distributed set of receivers needed to produce a 4-D inversion using a limited set of satellites in view. In fact, no publication has shown this in 3-D let alone 4-D. Consequently, the approach here offers a practical solution to determining a reasonable set of receivers based upon availability and actual measurements. Once this is determined, it can be inferred that that density would be applicable to other locations under similar geophysical conditions. Hence, it is a transferable result. The next stages of the research should include extending the study to other locations and in the future, to see the effects of multiple satellite constellations such as Global Navigation Satellite System, Galileo, and BeiDou.

For the example in this paper, when the spatial resolution was selected as 1° × 1°, the data group with medium density coverage in test 2 can reach the same accuracy as the one with most density coverage in test 3. The ground data density can affect tomographic results but only offers improvements up to a density of around one receiver every 150 to 200 km for this study. Furthermore, many data inputs can lead to the increasing of calculation time and the shortage of computer memory. The more general question of how much input data are needed for the tomographic algorithm with certain required resolutions, which will be our further study in the future.

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Figure 13. Height versus time plot of the electron density profile in (a) the model and (b) the reconstruction.

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