I. INTRODUCTION

A quantum dot typically consists of $10^5$ atoms, electrons tunneling through such devices experience hyperfine interaction with the nuclear spins of the host material.\footnote{Combination of these nuclear spins can be described as a large external effective spin system which interacts with the single electron spin.} Combinations of huge numbers of spins can be described as a large external effective spin system which interacts with the single electron spin. In experiments with quantum dots, features like a large Overhauser field and self-sustained current oscillations have been observed.\footnote{The effects of spin-orbit and hyperfine coupling in nuclear spins are observed in the scope of recent research for various kinds of systems, like self-assembled quantum dots, carbon nanotubes or molecular magnets.} The latter are promising candidates for spintronics.\footnote{The interaction of electron spins with a large spin shows interesting nonlinear effects and the system is known to exhibit chaotic behavior. These effects appear for the closed system with anisotropic coupling and an external magnetic field. López-Moniz and co-workers discussed the coupling to two external leads, which were assumed to be polarized. Within a rate equation approach they found, that the chaotic behavior survives for small magnetic fields.}

The effects of spin-orbit and hyperfine coupling with the nuclear spins of the host material leads to a Zeeman splitting of the electronic level $\sigma$, corresponding to $\varepsilon_\sigma = \varepsilon_d \pm \frac{1}{2} B_2$, with $\sigma \in \uparrow, \downarrow$. Without further interactions this solely leads to two spin-dependent current channels. Only electrons with spin-up (down) can tunnel through the upper (lower) energy level. These energy levels are broadened due to the coupling to the leads and for small magnetic fields an overlap of both channels exists, but there is no communication arranged between the two energy levels and spin-flips cannot occur. Here we enable transitions between the two energy levels with the help of a large external spin.

We assume that a vertical magnetic field $B_2$ is applied to a Fano Anderson model.\footnote{We assume that a vertical magnetic field $B_2$ is applied to a Fano Anderson model. The magnetic field leads to a Zeeman splitting of the electronic level $\varepsilon_\sigma$ into two levels, corresponding to $\varepsilon_\sigma = \varepsilon_d \pm \frac{1}{2} B_2$, with $\sigma \in \uparrow, \downarrow$. Without further interactions this solely leads to two spin-dependent current channels. Only electrons with spin-up (down) can tunnel through the upper (lower) energy level. These energy levels are broadened due to the coupling to the leads and for small magnetic fields an overlap of both channels exists, but there is no communication arranged between the two energy levels and spin-flips cannot occur. Here we enable transitions between the two energy levels with the help of a large external spin.} The magnetic field leads to a Zeeman splitting of the electronic level $\varepsilon_\sigma$ into two levels, corresponding to $\varepsilon_\sigma = \varepsilon_d \pm \frac{1}{2} B_2$, with $\sigma \in \uparrow, \downarrow$. Without further interactions this solely leads to two spin-dependent current channels. Only electrons with spin-up (down) can tunnel through the upper (lower) energy level. These energy levels are broadened due to the coupling to the leads and for small magnetic fields an overlap of both channels exists, but there is no communication arranged between the two energy levels and spin-flips cannot occur. Here we enable transitions between the two energy levels with the help of a large external spin.

We investigate the Fano-Anderson model coupled to a large ensemble of spins under the influence of an external magnetic field. The interaction between the two spin systems is treated within a meanfield-approach and we assume an anisotropic coupling between these two systems. By using a nonadiabatic approach we make no further approximations in the theoretical description of our system, apart from the semiclassical treatment. Therewith we can include the short-time dynamics as well as the broadening of the energy levels arising due to the coupling to the external electronic reservoirs. We study the spin dynamics in the regime of low and high bias. For the infinite bias case we compare our results to ones obtained from a simpler rate equation approach, where higher order transitions are neglected. We show, that these higher order terms are important in the range of low magnetic field. Additionally, we analyze extensively the finite bias regime with methods from nonlinear dynamics and discuss the possibility of switching of the large spin.

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II. MODEL

We assume that a vertical magnetic field $B_2$ is applied to a Fano Anderson model. The magnetic field leads to a Zeeman splitting of the electronic level $\varepsilon_\sigma$ into two levels, corresponding to $\varepsilon_\sigma = \varepsilon_d \pm \frac{1}{2} B_2$, with $\sigma \in \uparrow, \downarrow$. Without further interactions this solely leads to two spin-dependent current channels. Only electrons with spin-up (down) can tunnel through the upper (lower) energy level. These energy levels are broadened due to the coupling to the leads and for small magnetic fields an overlap of both channels exists, but there is no communication arranged between the two energy levels and spin-flips cannot occur. Here we enable transitions between the two energy levels with the help of a large external spin.
which interacts with the electronic spin. Fig. 1 depicts a sketch of the considered model. There, $\hat{S}$ denotes the electronic spin operator for the levels, which components are defined via

$$
\hat{S}_x = \frac{1}{2} \left( \hat{d}^\dagger_+ \hat{d}^- + \hat{d}^\dagger_- \hat{d}^+ \right),
\hat{S}_y = \frac{1}{2i} \left( \hat{d}^\dagger_+ \hat{d}^- - \hat{d}^\dagger_- \hat{d}^+ \right),
\hat{S}_z = \frac{1}{2} \left( \hat{d}^\dagger_+ \hat{d}^- - \hat{d}^\dagger_- \hat{d}^+ \right),
$$

(1)

introducing the creation/annihilation operators $\hat{d}_±/\hat{d}_0$ of the electronic levels. The vertical magnetic field $B_z$ couples to the $\hat{S}_z$ operator, leading to the splitting of the initial single level. The Hamiltonian for the Fano-Anderson model reads

$$
\mathcal{H}_\text{FA} = \sum_\sigma \varepsilon_\sigma \hat{d}_\sigma^\dagger \hat{d}_\sigma + \sum_{k\alpha\sigma} \varepsilon_{k\alpha\sigma} \hat{c}_{k\alpha\sigma}^\dagger \hat{c}_{k\alpha\sigma} + \sum_{k\alpha\sigma} \left( V_{k\alpha\sigma} \hat{c}_{k\alpha\sigma}^\dagger \hat{d}_\sigma + V_{k\alpha\sigma}^* \hat{d}_\sigma^\dagger \hat{c}_{k\alpha\sigma} \right) + B_z \hat{S}_z.
$$

(2)

Note, that the left ($\alpha = L$) and right ($\alpha = R$) lead operators $\hat{c}_{k\alpha\sigma}^\dagger/\hat{c}_{k\alpha\sigma}$ are spin-dependent. This enables us to consider polarized leads, where the density of states for spin-up and spin-down electrons with energy $\varepsilon_{k\alpha\sigma}$ differ. This could be realized with ferromagnetic leads. The third term in the Hamiltonian describes the transitions between a state in the lead and the electronic levels with tunneling amplitude $V_{k\alpha\sigma}$. For simplicity, we include the prefactor of the last term in Eq. (2), containing the electronic $g$-factor, into the definition of the magnetic field.

As well as the electronic spin operators, the large external spins' $z$-component $\hat{J}_z$ couples to the magnetic field. The free motion for it is described by

$$
\mathcal{H}_J = B_z \hat{J}_z,
$$

(3)

where we assume the same $g$-factor and therewith the same magnetic field as for the electronic spin operators. This is a simplification in theoretical approaches, cf. \cite{13,19}, but a generalization is straightforward.

Here, a large external spin means an effective spin describing a big ensemble of spins, for example, the collective spin of the nuclei in a quantum dot. Electrons which tunnel through this device, experience an interaction with the effective spin of the whole system. This interaction is described by Fermi contact hyperfine coupling terms \cite{12}

$$
\hat{V} = \sum_i \lambda_i \hat{S}_i \hat{J}_i, \quad i = x, y, z;
$$

(4)

introducing the coupling constant $\lambda_i$. If these coupling constants are equal for all components $\lambda_i = \lambda$, one speaks of an isotropic coupling. There, the two spins decouple in the long time limit. In the anisotropic case, where at least for two components $\lambda_i \neq \lambda_j$ is valid, interesting dynamical behavior was observed.\cite{18}

We treat the interaction of the two spins in a semiclassical manner. Therefore, we have to assume that the quantum fluctuations in the system are small. This should be valid as long as the external spin is large and its fluctuation can be neglected. As a consequence of this assumption, we have no spin decay due to dissipation and the large spin’s length $J$ is conserved.

Using a mean-field approximation \cite{12} for Eq. (4) we obtain

$$
\hat{V}_\text{MF} = \sum_i \lambda_i \left( \hat{S}_i \langle \hat{J}_i \rangle + \hat{J}_i \langle \hat{S}_i \rangle - \langle \hat{S}_i \rangle \langle \hat{J}_i \rangle \right), \quad i = x, y, z.
$$

(5)

Thereby, the fluctuations $\delta \hat{A}_i = \hat{A}_i - \langle \hat{A}_i \rangle$, $\hat{A} \in \hat{S}, \hat{J}$ have been neglected. Now we can build up a closed system of equations for the considered system.

For the large spin we use the commutation relations to derive the Heisenberg equations of motion

$$
\langle \dot{\hat{J}}_x \rangle = - \left( \lambda_x \langle \hat{S}_z \rangle + B_x \langle \hat{J}_y \rangle + \lambda_y \langle \hat{S}_y \rangle \langle \hat{J}_y \rangle \right),
\langle \dot{\hat{J}}_y \rangle = \left( \lambda_y \langle \hat{S}_z \rangle + B_y \langle \hat{J}_x \rangle - \lambda_x \langle \hat{S}_x \rangle \langle \hat{J}_x \rangle \right),
\langle \dot{\hat{J}}_z \rangle = - \lambda_x \langle \hat{S}_x \rangle \langle \hat{J}_y \rangle - \lambda_y \langle \hat{S}_y \rangle \langle \hat{J}_y \rangle.
$$

(6)

This is a strongly nonlinear system, since the values of the electronic spin components depend on the large spin.

A decisive role in the theoretical description of this system plays the treatment of the electronic spin. By using a rate equation approach the electronic spin components are obtained from equations of motions similar to Eq. (6). Therewith the short-time dynamics is included, but the contributions of the leads come only in as rates. Following from that higher order transitions are neglected and one is restricted to the infinite bias regime. One way of including higher order transitions is by applying an adiabatic approximation, where the movement of the large spin is assumed to be slow compared to changes in the
The electronic spin operators can than be derived via Keldysh Green’s functions and are given in explicit expressions. The disadvantage of this method is that the short-time dynamics is missing.

In the next part of this paper we derive equations of motion for the electronic spin operators using a completely nonadiabatic approach. The latter enables us to include the short-time dynamics as well as higher order transitions. Following from that we can probe the rate equation and the adiabatic approach.

A. Nonadiabatic approach

In the framework of the nonadiabatic approach, we calculate all system quantities by considering the full time-dependence of all system quantities. In the following, we assume an anisotropic coupling \( \lambda_y = 0 \) and \( \lambda_x = \lambda_z = \lambda \), and together with the mean-field approach we obtain an effective Hamiltonian for the electronic levels

\[
\mathcal{H}_e(t) \equiv \sum_{\sigma} \varepsilon_\sigma(t) \dot{d}_\sigma \dot{\sigma}_\sigma + \frac{\lambda}{2} \langle \hat{J}_z(t) \rangle \left( \dot{d}_\uparrow \dot{d}_\downarrow + \dot{d}_\downarrow \dot{d}_\uparrow \right) + \mathcal{H}_T,
\]

where \( \mathcal{H}_T \) contains the coupling to the leads. Based on this effective Hamiltonian we can describe the effects arising due to the coupling to a large external spin. The component \( \langle \hat{J}_z(t) \rangle \) - component solely leads to an additional shift of the electronic levels, but the coupling to the \( \langle \hat{J}_z(t) \rangle \) - component enables transitions between both levels. This Hamiltonian corresponds to a two-level or a parallel double dot system.

The derivation of the equations of motion for the spin operators is performed by starting from the Heisenberg equations of motion. The derivation is similar to those used before for the description of nanoelectromechanical systems, for details see Ref. [20]. We use a flat band approximation leading to the spin dependent tunneling rates \( \Gamma_{\sigma\sigma} = 2\pi \sum_{\kappa} |V_{\kappa\sigma}|^2 \delta(\omega - \varepsilon_{\kappa\sigma}) \). Finally, the result for the spin operator expectation values yield (\( \Gamma \equiv \Gamma_{\sigma\sigma} = \sum_{\alpha} \Gamma_{\sigma\sigma} \))

\[
\frac{d}{dt} \langle \hat{S}_x(t) \rangle = -\Gamma \langle \hat{S}_x(t) \rangle - \left( B_x + \lambda \langle \hat{J}_z(t) \rangle \right) \langle \hat{S}_y(t) \rangle + \sum_{\alpha} \int d\omega \text{Re} \left[ B_{\alpha\uparrow}^x(\omega, t) + B_{\alpha\downarrow}^x(\omega, t) \right],
\]

\[
\frac{d}{dt} \langle \hat{S}_y(t) \rangle = -\Gamma \langle \hat{S}_y(t) \rangle + \left( B_z + \lambda \langle \hat{J}_z(t) \rangle \right) \langle \hat{S}_z(t) \rangle - \lambda \langle \hat{J}_x(t) \rangle \langle \hat{S}_z(t) \rangle + \sum_{\alpha} \int d\omega \text{Im} \left[ B_{\alpha\uparrow}^y(\omega, t) - B_{\alpha\downarrow}^y(\omega, t) \right],
\]

\[
\frac{d}{dt} \langle \hat{S}_z(t) \rangle = -\Gamma \langle \hat{S}_z(t) \rangle + \lambda \langle \hat{J}_x(t) \rangle \langle \hat{S}_y(t) \rangle + \sum_{\alpha} \int d\omega \text{Re} \left[ B_{\alpha\uparrow}^z(\omega, t) - B_{\alpha\downarrow}^z(\omega, t) \right],
\]

with the definition

\[
B_{\sigma\sigma'}^\alpha(\omega, t) = i \sum_k V_{k\sigma\sigma'} \delta(\omega - \varepsilon_{k\sigma\sigma'}) e^{i\varepsilon_{k\alpha} t} \langle \hat{S}_\alpha(0) | d_{\sigma}(t) \rangle,
\]

for the lead-transition functions

\[
\frac{d}{dt} B_{\sigma\sigma'}^\alpha(\omega, t) = -i(\varepsilon_\sigma(t) - \omega - \frac{i}{2} \Gamma) B_{\sigma\sigma'}^\alpha(\omega, t) - \frac{\lambda}{2} \langle \hat{J}_x(t) \rangle B_{\sigma\sigma'}^\alpha(\omega, t) + \frac{\Gamma_{\sigma\sigma}}{2\pi} f_\alpha(\omega),
\]

\[
\frac{d}{dt} B_{\sigma\sigma'}^\alpha(\omega, t) = -i(\varepsilon_\sigma(t) - \omega - \frac{i}{2} \Gamma) B_{\sigma\sigma'}^\alpha(\omega, t) - \frac{\lambda}{2} \langle \hat{J}_x(t) \rangle B_{\sigma\sigma'}^\alpha(\omega, t).
\]

The time-dependent \( z \)-component of the large spin is included into the effective levels \( \varepsilon_\sigma(t) \), cf. Eq. 11. Here, the Fermi function \( f_\alpha(\omega) \) does not depend on the spin due to the assumption, that the chemical potentials for spin-up and -down electrons in each lead are equal.

For the numerical calculations we decompose the \( B_{\sigma\sigma'}^\alpha \) into their real and imaginary part and discretize the integration over the lead energies in \( N \) intervals. Therefore, we obtain \([6 + 16 (N + 1)] \) coupled equations, including the equations of motion for the large spin, cf. Eq. 10.

Using the definitions for the lead-transitions function Eq. 10, the electronic current is obtained from

\[
\mathcal{J}_{\alpha\sigma}(t) = e \left\{ \int d\omega \, 2 \text{Re} \left[ B_{\sigma\sigma}^\alpha(\omega, t) \right] - \Gamma_{\alpha\sigma} \langle \hat{n}_\sigma(t) \rangle \right\},
\]

where \( e \) equals the electron charge and the equation of motion for the occupation yields

\[
\frac{d}{dt} \langle \hat{n}_\sigma(t) \rangle = -\Gamma \langle \hat{n}_\sigma(t) \rangle + \lambda \langle \hat{J}_x(t) \rangle \langle \hat{S}_y(t) \rangle + \sum_{\alpha} \int d\omega \, 2 \text{Re} \left[ B_{\alpha\sigma}^\alpha(\omega, t) \right].
\]

Here, the upper/lower lower sign refers to \( \uparrow / \downarrow \) - electrons. Note, that the total occupation number of the dot \( \langle \hat{N}(t) \rangle = \langle \hat{n}_\uparrow(t) \rangle + \langle \hat{n}_\downarrow(t) \rangle \) still couples to the spin operators via the transition functions. In the next section we derive a rate equation approach where this dependence is omitted.

B. Infinite bias: rate equation approach

We recover previous results for this model [19] by applying an adiabatic approximation to our approach. In concrete terms, we assume the large spin’s movement as slow compared to the electrons which are entering the system. As a consequence, we neglect the time-dependence of the large spin in the equations for the lead-transition functions Eq. 10, leading to \( \varepsilon_{\sigma}(t) \equiv \varepsilon_{\sigma} \) and a decoupling
from the equations for the large spin, \( \langle \hat{J}_x(t) \rangle \equiv \langle \hat{J}_x \rangle \). The remaining two coupled equation can be solved via a Laplace transformation. Note, that this method is not a complete adiabatic approach, because in the end we still solve equations of motions for the electronic spin components, which is not the case for a full adiabatic ansatz.

Starting from Eq. (11) we obtain for \( t \to \infty \)
\[
B^a_{a\sigma}(\omega) = \frac{\Gamma a}{2\pi} f_a(\omega) \frac{-i(\epsilon_a - \omega - \frac{1}{2}\Gamma)}{N(\omega)},
\]
\[
B^b_{a\sigma}(\omega) = \frac{\Gamma a}{2\pi} f_a(\omega) \frac{i\frac{1}{2} \langle \hat{J}_z \rangle}{N(\omega)},
\]
with \( N(\omega) = (\epsilon_a - \omega - \frac{1}{2}\Gamma)(\epsilon_a - \omega - \frac{1}{2}\Gamma) - \frac{\Gamma^2}{4} \langle \hat{J}_x \rangle^2 \). Before inserting these results into the equations for the spin operators Eq. (8), we separate them into real and imaginary part and perform the integrations over \( \omega \). Hence, the spin operator equations become
\[
\frac{d}{dt} (\hat{S}_x(t)) = -\Gamma \langle \hat{S}_x(t) \rangle - \left( B_z + \lambda \langle \hat{J}_z(t) \rangle \right) \langle \hat{S}_y(t) \rangle,
\]
\[
\frac{d}{dt} (\hat{S}_y(t)) = -\Gamma \langle \hat{S}_y(t) \rangle + \left( B_z + \lambda \langle \hat{J}_z(t) \rangle \right) \langle \hat{S}_x(t) \rangle - \lambda \langle \hat{J}_z(t) \rangle \langle \hat{S}_z(t) \rangle,
\]
\[
\frac{d}{dt} (\hat{S}_z(t)) = -\Gamma \langle \hat{S}_z(t) \rangle + \lambda \langle \hat{J}_z(t) \rangle \langle \hat{S}_y(t) \rangle + \frac{1}{2} (\Gamma_{L\uparrow} - \Gamma_{L\downarrow}).
\]
This result coincides with\(^{15}\): the whole system has now been reduced to six coupled equations and the current simplifies to
\[
J_{a\sigma}(t) = e \Gamma_{a\sigma} \left\{ \delta_{aL} - \langle \hat{n}_{a}(t) \rangle \right\},
\]
with the occupation
\[
\frac{d}{dt} \langle \hat{n}_{a}(t) \rangle = -\Gamma \langle \hat{n}_{a}(t) \rangle \pm \lambda \langle \hat{J}_x(t) \rangle \langle \hat{S}_y(t) \rangle + \Gamma_{L\sigma}.
\]
As mentioned before, the total occupation number of the dot \( \langle \hat{N}(t) \rangle \) decouples from the remaining equations and becomes constant in the long-time limit
\[
\langle \hat{N}(t \to \infty) \rangle = \frac{(\Gamma_{L\uparrow} + \Gamma_{L\downarrow})}{\Gamma}.
\]
The equations for the spin operators Eq. (13) and the large external spin Eq. (6) provide further possibilities for an analytic investigation. The fixed points of the system can easily be calculated, see\(^{16}\).

By varying the magnetic field \( B_z \) and the tunneling rate \( \Gamma \) three regions of different dynamical behavior were obtained from the rate equation approach. In Fig. 2 these regions are depicted. The solid black lines correspond to parameters used in\(^{16}\). There, the length of the large spin equals \( j = |\hat{J}| = 10 \) and the tunneling rates match \( 2\Gamma_{L\uparrow} = \Gamma_{L\downarrow} = \Gamma \). This choice of tunneling rates implies \( \Gamma_{R\uparrow} = 0 \), because in this case the current flows exclusively through the upper electronic level. Following from that, spin-down electrons get trapped into the lower level and only contribute to transport after a spin-flip.

![Fig. 2: Regions of different dynamical behavior concerning the parameters \( \Gamma \) and \( B_z \) using the rate equation approach. As a complement, the gray lines in Fig. 2 illustrate the change of the first two regions if the length of the large spin is varied, while the ratio of the tunneling rates are fixed. For a smaller spin (\( j = 10^2 \)), region II decreases to the benefit of region I, which increases. Equally, the reverse situation can be seen, if the length of the spin is increased (\( j = 10^2 \)). Modifying the ratio of the tunneling rates shifts the vertical line separating region II from region III.](image)

### C. Infinite bias: results

In Fig. 3 the results for regime I are depicted. The rate equation results describe damped oscillations as expected. For large times the \( z \)-component of the large spin becomes polarized parallel to the magnetic field and one spin-down electron gets trapped in the lower energy level. There the spin trajectories end up in one of the fixed points
\[
\langle \hat{J}_{1,2}^z \rangle = (0, 0, \pm j), \quad \langle \hat{S}_{1,2}^z \rangle = \left( 0, 0, \frac{\Gamma_{L\uparrow} - \Gamma_{L\downarrow}}{2\Gamma} \right).
\]
These points exist in the whole parameter regime and are independent of the magnetic field.

In the nonadiabatic case the dynamical behavior is quite different. For small times the spin trajectories follow the damped results from the rate equation approach. But the damping decreases strongly after the time step \( t \lambda \approx 1000 \). The same amount of time steps later, the amplitude drops down for one oscillation period, followed by a return to the initial oscillation. This behavior is similar for all spin components. If we consider the \( z \)-component for the large spin, the turning point appears when it approaches its fixed point value \( \langle \hat{J}_{z,1}^z \rangle = j \).

By varying the parameters we found no damped oscillations in region I at all. For a certain range for the magnetic field and the tunneling rate the dynamics appear as in Fig. 3. For \( \Gamma/\lambda \approx 8 \) the oscillations disappear and we enter region II.

In Fig. 4 the results for regime II are depicted. The
rate equation approach predicts three kinds of dynamical behavior, namely strongly damped, self-sustained and chaotic oscillations.

For $B_z/\lambda = 0.2$ and small tunneling rate $\Gamma/\lambda = 0.7$, the system performs strong damped oscillations and runs into one of the fixed points

$$\langle \hat{J}_z \rangle = \left( \frac{\Gamma}{B_z} B_{2,1,2} \pm \sqrt{j^2 - \frac{\Gamma^2}{B_z^2} B_{1,2}^2 - \frac{B_z^2}{\lambda}} \right),$$

$$\langle \hat{S}_z \rangle = \left( 0, B_{1,2} - \frac{B_z}{\lambda} \right),$$

where $\langle \hat{S}, \hat{J} \rangle$ and $\langle \hat{S}, \hat{J} \rangle_{\text{fixed}}$ correspond to $B_1/B_2$, which yield

$$B_{1,2} = \pm \sqrt{\frac{B_z}{\lambda} \left( \frac{\Gamma_L - \Gamma_{L_\perp}}{2\Gamma} - \frac{B_z}{\lambda} \right)}.$$

This result coincides perfectly with the nonadiabatic result, see left row in Fig. 3. The spin components run into the fixed point $\langle \hat{J}_z \rangle$, cf. Eq. (19). The large spin becomes almost completely polarized perpendicular to the magnetic field.

In the first instance, by further increasing the tunneling rate, the rate equations forecast self-sustained oscillations for the systems trajectories, followed by a chaotic oscillating behavior. Using the nonadiabatic approach, things change again. There, the chaotic behavior appears already in the parameter region, where self-sustained oscillations were predicted by the rate equation approach. Within the nonadiabatic approach we do not recover these self-sustained oscillations in region II, we solely observe strong damping or chaotic-like behavior.

![FIG. 3: Comparison of the nonadiabatic approach and the rate equation approach results for regime I. The magnetic field equals $B_z/\lambda = 0.1$ and the tunneling rate is chosen as $\Gamma/\lambda = 9$. Initial conditions are $\langle \hat{J}_x(0) \rangle = \langle \hat{J}_y(0) \rangle = 5(\sqrt{3} - 1)/(2\sqrt{2})$, $\langle \hat{J}_z(0) \rangle = 5\sqrt{2}\sqrt{5 + \sqrt{3}} \langle \hat{S}_x(0) \rangle = \langle \hat{S}_y(0) \rangle = 0$, $\langle \hat{S}_z(0) \rangle = 0.5$.](image)

![FIG. 4: Behavior of the spin components in regime II. Left graph depicts results for $B_z/\lambda = 0.2$ and $\Gamma/\lambda = 0.7$, where fast damping of the trajectories appear in both approaches. The middle graph shows results for $B_z/\lambda = 0.1$ and $\Gamma/\lambda = 0.16$ where the rate equations predict self-sustained oscillations, but for the nonadiabatic results already chaos appear. Just as in the right graph, were the tunneling rate is further increased ($\Gamma/\lambda = 0.015$). For clarity, we omitted the rate equation results for the chaotic-like regions. The lowest row depicts the corresponding current results, for the chaotic-like regions the green line denotes $-I_{xy}$, the light blue line $I_{y}$ and the dark blue line $I_{z}$. Initial conditions are chosen as for Fig. 3.](image)
parameters are relations, depicted in the upper graph on the right side. For lower tunneling rate, the \( \langle \hat{S}_i \rangle \) trajectories perform non-sinusoidal, but periodic oscillations, depicted in the upper graph on the right side. Parameters are \( B_u / \lambda = 1.0 \), \( \epsilon_d / \lambda = 0 \) and with the initial conditions \( \langle \hat{J}_y(0) \rangle = 3 \sqrt{5} \), \( \langle \hat{J}_z(0) \rangle = -1 \) and \( \langle \hat{S}_x(0) \rangle = 0 \). Not surprising, because this spin component corresponds to the occupation difference of the dot system \( 2 \langle \hat{S}_z \rangle = \langle \hat{n}_\uparrow \rangle - \langle \hat{n}_\downarrow \rangle \). Inserting the latter into the current equation Eq. (18) and using the solution for the total occupation number in the long-time limit, cf. Eq. (17), we obtain for the current through lead \( i \) in the rate equation frame

\[
\mathcal{J}_i(t) = e \left[ \frac{\delta_{\alpha L} - \Gamma_{\alpha L} + \Gamma_{\alpha L}}{2 \Gamma} \right] \left( \Gamma_{\alpha L} + \Gamma_{\alpha L} \right) - \langle \hat{S}_z(t) \rangle (\Gamma_{\alpha L} - \Gamma_{\alpha L}) \right],
\]

And with \( \Gamma = \Gamma_{\alpha L} + \Gamma_{\alpha R} \) it is obvious that \( \mathcal{I}_L(t) = -\mathcal{I}_R(t) \) for all times \( t \) and hence current conservation is ensured. The latter is also valid for the nonadiabatic approach.

From the above comparison we can already conclude, that higher order terms in the system-lead coupling as included in the nonadiabatic approach, do matter in the regime of low magnetic field.

### III. DYNAMICS IN THE FINITE BIAS REGIME

In the last section our investigation was focused on the infinite bias regime. There, the interaction between the large spin and the electronic system leads to interesting nonlinear effects. The rate equation approach is restricted to this regime of high external bias. With our nonadiabatic approach we learned, that differences appear if one includes higher order transition terms. In this section, we take the next step by studying the finite bias regime.

Due to the lack of an easy access to further analytic studies for the nonadiabatic approach, we use an adiabatic approach based on Keldysh Green’s functions for the dynamical analysis, for details see Sec. [3]. This enables us to clarify the effects of a finite transport window.

#### A. Dynamical analysis: adiabatic approach

Using a complete adiabatic approach for our system implies the assumption that the large spin’s movement is slower compared to the electrons jumping through the system. This is an additional assumption on top of the mean-field approach, where quantum fluctuations are neglected already. But even for the derivation of the rate equations one needs an adiabatic approximation for the electrons tunneling into the system. We performed the latter in the last section to decouple the lead-transition functions, cf. Eq. (19). A full adiabatic approach also assumes that the electronic spin changes on a time-scale which is much smaller than the one of the large spin. This approximation for the interaction between the electronic spin and the large spin reduces the number of dynamical equations to three, because solely the ones for the large spin remain. The equations of motion for the electronic spin operators are solved with the help of Green’s functions.

We can use the adiabatic approach to search for fixed points and also perform a rough characterization of them. The predictions for the classification of the fixed points coincide quite good with the actually obtained nonadiabatic results. However, a complete adiabatic treatment of the system cannot catch the whole dynamics of the system. Even in the infinite bias regime the adiabatic
results do not coincide with the ones obtained from the rate equation or the nonadiabatic approach. The reason therefore lies in the lack of additional friction terms, which could be gained by expanding the Green’s functions as done in Ref. [13] for a related system.

1. Fixed point analysis of $P^\pm_0$: center or saddle point

The fixed points $P^\pm_0$, where the large spin is completely polarized parallel to the magnetic field, cf. Eq. [18], appears also for the adiabatic system. But the corresponding value of the $z$-component for the electronic spin does now depend on several system parameters; for zero temperature it yields

$$
\langle \hat{S}_z^\alpha (\pm j) \rangle = \sum_\alpha \frac{\Gamma_\alpha}{2\pi^2} \arctan \left( \frac{\langle \mu_\alpha - \varepsilon_\pm \rangle}{\Gamma/2} \right)
- \frac{\Gamma_\alpha}{2\pi^2} \arctan \left( \frac{\langle \mu_\alpha - \varepsilon_\pm \rangle}{\Gamma/2} \right),
$$

(22)

The $x$- and $y$-component are zero as for the rate equation approach. For infinite bias the $z$-component solely depends on the tunneling rates $\langle \hat{S}_z^{\text{tot}} (\pm j) \rangle = (\Gamma_L - \Gamma_L)/(2\pi)$, which coincides with Eq. [18].

For the fixed points $P^\pm_0$, one eigenvalue of the Jacobian is zero and the two other read

$$
\mathcal{E}^{0,\pm}_{2,3} = \pm i \sqrt{T_0 \left( \frac{\lambda_j \partial \langle \hat{S}_x \rangle}{\partial \langle J_x \rangle} \left\{ \langle J_x \rangle = 0, \langle J_x \rangle = \pm j \right\} \right)},
$$

(23)

with $T_0 = \left[ \lambda \langle \hat{S}_z^\pm (\pm j) \rangle + B_j \right]$. Therewith two possible realizations can appear. If the root is real the eigenvalues are complex conjugate and the fixed point is therewith stable and can be classified as a stable center. This means, that periodic oscillations for the adiabatic spin dynamics are expected. The real part is equal to zero and therefore no damped oscillations should appear. This missing damping is characteristic for the adiabatic results, hence this approach cannot describe the complete dynamics of the system. This is valid even in the infinite bias regime, since the appearing dynamical features like self-sustained oscillations require an additional positive and negative damping.

In Fig. 7, $\langle \hat{S}_z^\pm \rangle$ as a function of the applied bias is depicted, together with the real (gray dashed-dotted line) and the imaginary (cyan solid line) part of the corresponding eigenvalue. The two upper graphs show the behavior in regime I for $\langle \hat{S}_x^\pm \rangle$ polarized parallel (left) and anti-parallel (right) to the magnetic field. We observe regions where the imaginary part of the eigenvalues is equal to zero and therewith the oscillations disappear.

The ranges of finite real part are slightly different for a parallel and a anti-parallel polarization of the large spin $z$-component. These differences appear for small bias, the intervals of finite real part are denoted in the caption of Fig. 7. For $P^\pm_0$ the intervals with finite real part are smaller as for $P^0_0$. Depending of the existence of other fixed points, the regions where only one fixed point has eigenvalues with finite imaginary parts, are promising for the occurrence of spin-flips.

Based on the chosen polarization of the leads and therewith the different tunneling rates for the spin-down electrons we obtain a system which is not symmetric. Hence, the dynamical behavior for negative detuning is different than for positive detuning $V_{\text{bias}} > 0$. This is clearly visible in Fig. 7 where for large negative bias the imaginary part in region I is always unequal to zero. The infinite bias result for region I equals $\langle \hat{S}_z^\pm \rangle = \pm 0.25$ as expected, whereby the sign depends on the detuning. For region I the system performs periodic oscillations for a bias $V_{\text{bias}}/2\lambda > 60$.

In contrast, in region II (2nd row in Fig. 7) no finite imaginary part for the high bias regime exists. This coincides with the infinite bias result for the nonadiabatic approach, where the concerned fixed points do not appear as stable centers or spirals. But here, for a small bias range and also for negative detuning, we find, that the eigenvalues can become complex and therewith the fixed points $P^\pm_0$ can exist as center. For positive detuning the fixed point $P^\pm_0$ has already turned into a saddle point, but the anti-parallel state is alive until $V_{\text{bias}}/2\lambda = 6.4$.

For region III, which means for $B_z/\lambda > 0.25$, the be-
havior of the fixed points is similar to region I, if the magnetic field and the tunneling rate are small. But for higher values of these parameters the real part of the eigenvalues is always zero.

2. **Two conditions for the fixed points** \( P^{\pm}_{SN} \)

The next fixed points obtained from Eq. \( \ref{eq:8} \) include the requirement that \( \langle \hat{S}_z \rangle = 0 \) and the condition \( \langle \hat{S}_z \rangle + B_z/\lambda = 0 \) has to be complied with as well. We define

\[
P^{\pm}_{SN} : \left( \langle J_x \rangle, \pm \sqrt{J^2 - (J_x)^2} - \langle J_z \rangle, \langle J_z \rangle \right). \tag{24}
\]

In the infinite bias regime \( P^{\pm}_{SN} \) coincide with the four fixed points in region II Eq. \( \ref{eq:19} \), obtained from the rate equation. The investigation of \( P^{\pm}_{SN} \) in the finite bias regime is possible only numerically. In Fig. 8 we present results obtained for region I. The cycle shapes originate from the conservation of the large spin, hence the radius equals \( j \). Outside of these cycles \( \langle \hat{J}_y \rangle \) becomes complex and no real solution for \( P^{\pm}_{SN} \) exists.

The stability is defined by the eigenvalues of \( P^{\pm}_{SN} \). One eigenvalue of \( P^{\pm}_{SN} \) is again zero, due to the fact, that we actually deal with a two-dimensional system. The remaining eigenvalues yield

\[
\mathcal{E}^{SN}_{2,3} = \frac{\lambda}{2} \langle \hat{J}_y \rangle \left\{ \frac{\partial(\langle \hat{S}_z \rangle)}{\partial(\langle J_x \rangle)} - \frac{\partial(\langle \hat{S}_z \rangle)}{\partial(\langle J_z \rangle)} \right\} \pm \sqrt{ \left( \frac{\partial(\langle \hat{S}_z \rangle)}{\partial(\langle J_x \rangle)} + \frac{\partial(\langle \hat{S}_z \rangle)}{\partial(\langle J_z \rangle)} \right)^2 - 4 \langle \hat{S}_z \rangle \frac{\partial(\langle \hat{S}_z \rangle)}{\partial(\langle J_x \rangle)} \frac{\partial(\langle \hat{S}_z \rangle)}{\partial(\langle J_z \rangle)} } \right\}. \tag{25}
\]

For a small bias, \( V_{bias}/2\lambda < 5 \), only one solution for \( P^{\pm}_{SN} \) exists, corresponding to the case \( \langle J_x \rangle = 0 \), which we name as \( P^{\pm}_{SN} \), the latter is stable until \( V_{bias}/2\lambda < 7 \) cf. Fig. 9. By further increasing the bias another bifurcation appears, where \( P^{\pm}_{SN} \) becomes unstable and two other fixed points are created. These points are symmetric to the \( \langle J_x \rangle \) - axis and move with higher bias values further to \( \langle J_z \rangle = \pm j \). When they reach the border of the cycle, they disappear and \( \langle \hat{S}_z \rangle + B_z/\lambda = 0 \) has no longer a real solution. The emerging fixed points have negative/positive real eigenvalues and can be characterized as stable/unstable nodes. For region II these points do not disappear for high bias and the nodes are the only remaining stable solutions of the system. These nodes correspond to fast damping behavior for the spin components.

For \( P^{\pm}_{SX} \), where \( \langle \hat{J}_y \rangle = 0 \), the last term in Eq. \( \ref{eq:24} \) is unequal to zero and the component \( \langle J_x^\star \rangle \) is obtained from the transcendental equation

\[
- \frac{B_z}{\lambda} = \sum_{\alpha} \frac{\Gamma_{\alpha}^+}{2\pi \Gamma} \arctan \left( \frac{\mu_{\alpha} - \varepsilon_{d}^\star}{\Gamma/2} \right) - \frac{\Gamma_{\alpha}^-}{2\pi \Gamma} \arctan \left( \frac{\mu_{\alpha} - \varepsilon_{d}^-}{\Gamma/2} \right), \tag{26}
\]

with \( \varepsilon_{d}^\star = \varepsilon_d \pm 0.5(B_z + \lambda J_y^\star) \). This equation has no solution for \( \langle J_x^\star \rangle \) in the infinite bias case and therefore the fixed points \( P^{\pm}_{SX} \) do not exist there. In the regime of a low magnetic field, the right side of Eq. \( \ref{eq:26} \) has to be small. Following from that, the argument of the arctan function has to be small as well, leading to the
of tunneling rate $\Gamma$ and magnetic field $B_z$.

FIG. 10: Fixed points for unpolarized leads as a function of bias is chosen as $V_{\text{bias}}/2\lambda = 5$. The right graphs depict the adiabatic results for the large spin, the small cycles on the top/bottom correspond to $P_0^\pm$ and the larger cycle to $P_0^\pm$ (region A). Magnetic field for region A/B is $B_z/\lambda = 0.1/0.3$.

estimate for the evolution of the $z$-component as linear to the applied bias $(J^z_0) \sim V_{\text{bias}}$.

This linear behavior is clearly visible in Fig. 9 where we plotted $P_{\sigma\sigma}^\pm$ and its eigenvalues as a function of the applied bias. The arcs corresponds to the $\langle J_i^\pm \rangle$ components, which determines whether the fixed point exists or not. Within this arc the point is real and therewith physically reasonable. The radius of the arc is limited by the length of the large spin.

For both regions we observe small ranges with finite imaginary part, where the fixed point can be classified as a stable center. As well as ranges, where a saddle point occurs. In regime I, the point $P_{\sigma\sigma}^+$ starts its existence for $V_{\text{bias}}/2\lambda \approx -5$. At the bias value, $P_0^+$ turns into a saddle point, cf. Fig. 4.

This behavior matches quite good those of $P_0^\pm$. We can interpret $P_{\sigma\sigma}^\pm$ as the complementary points in the region, where $P_0^+$ disappears. As we discussed before, for a certain bias region the stable solution $P_0^+$ turns into a saddle point and we can estimate, that then the fixed points appearing in Fig. 5 correspond to stable solutions of the dynamical system.

3. Fixed points for unpolarized leads

For unpolarized leads the tunneling rates become spin-independent $\Gamma = \Gamma_L + \Gamma_R$. There only the fixed points $P_0^\pm$ and $P_{\sigma\sigma}^\pm$ exist, as depicted in Fig. 10. In dependence of the tunneling rate and the magnetic field, two main regions appear. The first one, region A, has the stable fixed points $P_0^+$ and $P_{\sigma\sigma}^+$, these points can be characterized as centers. In this region spin-flips of the large spin can be possible, because $P_0^+$ is no stable solution. But the system is still multistable and therewith the trajectories can also end up in the fixed points $P_{\sigma\sigma}^\pm$. Note, that for an total adiabatic ansatz, a complete spin-flip is not observable, because the trajectories starting parallel to the magnetic field end up in $P_{\sigma\sigma}^\pm$, as shown in the right graphs of Fig. 10. If one reverses the direction of the magnetic field, region A contains $P_0^+$ instead of $P_0^-$. In region B solely $P_0^\pm$ are stable fixed points. By further increasing the bias, the region where $P_{\sigma\sigma}^\pm$ exist gets smaller and in the infinite bias case only the second region persists and all electronic spin components $\langle S_i \rangle$ become zero. Then the two systems decouple and the large spin oscillates with the Larmor frequency.

B. Results for the nonadiabatic approach in the finite bias regime

We expect to obtain the same fixed points for the nonadiabatic approach as for the adiabatic approach. For the case $P_0^\pm$, we want to show, that this fixed point also appears in the nonadiabatic regime. There, all lead-transition functions for different spin $B_{\sigma\sigma}$ vanish and we obtain for the ones with equal spins the simple result

$$B_{\sigma\sigma}^\pm(\omega) = i \frac{\Gamma_{\sigma\sigma}}{2\pi} \frac{\int_0^\infty d\epsilon f_0(\omega - \epsilon + i\frac{\Gamma}{4})}{\epsilon^2 + \frac{\Gamma^2}{4}} = i \frac{\Gamma_{\sigma\sigma}}{2\pi} G_{\sigma\sigma}^R(\omega),$$

(27)

containing the spin-dependent single-level Green’s function $G_{\sigma\sigma}^R(\omega)$ without coupling of the two electronic levels, due to $\langle J^\pm_0 \rangle = 0$. Also the equation for the $z$-component of the electronic spin is straightforwardly obtained from

$$\langle \dot{S}_z \rangle = \frac{1}{\Gamma} \sum_{\alpha \sigma} \int d\omega \Re \left[ B_{\alpha\sigma}^{n+}(\omega) - B_{\alpha\sigma}^{n-}(\omega) \right] = \frac{1}{2\pi} \sum_{\alpha \sigma} \frac{\Gamma_{\alpha\sigma}}{\Gamma} \arctan \left[ \frac{\mu_{\alpha\sigma} - \epsilon_{\sigma}}{\Gamma/2} \right].$$

(28)

This result coincides with Eq. (22) and the fixed point is identical to the one obtained with the adiabatic approach. In the same manner, we can construct also the other fixed points. In the general stationary case, all lead-transition functions decouple from the equation system and can be expressed by retarded Green’s function. The adiabatic expressions for electronic spin components can be reconstructed, too.

This equivalence is not valid considering the adiabatic eigenvalues, they are not directly transferable to our nonadiabatic approach. The fixed points are long-time quantities and stationary solutions of the dynamical system, in contrast the eigenvalues include higher order terms and their calculation requires the first derivations of the system’s variables.

But some predictions derived from the adiabatic eigenvalues also appear in the nonadiabatic approach. Therefore, we can estimate that the behavior for the full time-dependent solution is strongly influenced by the adiabatic eigenvalues.
All oscillations have disappeared for a bias in the range of the anti-parallel fixed point in the lower row of Fig. 11, which corresponds to a small bias $V_{\text{bias}} = \mu_{\uparrow}/2 = -\mu_{\downarrow}/2$. The electronic current, depicted in the lowest row, is separated into left/right and spin-up/don contributions. The magnetic field yield $B_z/\lambda = 0.1$ and $\Gamma/\lambda = 9$. Initial conditions as in Sec III for regime I.

1. Region I: disappearing of the oscillations

We start our presentation and discussion of the nonadiabatic results with the ones obtained in region I, corresponding to a low value of the external magnetic field and a large tunneling rate. In the case of infinite bias, we obtained an oscillating behavior between two cycles. This is different to the rate equation result, where damped oscillations appear.

From the adiabatic analysis of the last section, we obtained the prediction, that the fixed point $P^+_0$ of the system turns from a center into a saddle point for the bias range $V_{\text{bias}}/2\lambda \in [-5;60]$. When $P^+_0$ loses its stability at $V_{\text{bias}}/2\lambda \approx -5$, a supercritical pitchfork bifurcation appears and $P_{\text{SN}}^\pm$ are created, they exist until $V_{\text{bias}}/2\lambda \approx 7$ as centers. If the latter fixed points become unstable, two stable nodes are born and we expect the oscillations to disappear. Note, that these predictions originate from the adiabatic eigenvalues.

As depicted in Fig. 11, we obtain these features also in our nonadiabatic results. There, the spin components and the current are depicted for three different bias values. For $V_{\text{bias}}/2\lambda = 2$ we observe non-sinusoidal self-sustained oscillations. $\langle \hat{S}_z \rangle$ oscillates around $-B_z/\lambda$, which corresponds to $P_{\text{SN}}^-$. If we increase the applied bias for the initial conditions corresponding to the results depicted in Fig. 11, the oscillations disappear in the range of $V_{\text{bias}}/2\lambda \approx 8$. They run into one fixed point of the kind $P_{\text{SN}}^+$, which is clearly visible in Fig. 11 were $\langle \hat{S}_z \rangle = 0$ and $\langle \hat{S}_z \rangle = -B_z/\lambda$.

By varying the initial conditions, we observe that not all oscillations have disappeared for a bias in the range of $V_{\text{bias}}/2\lambda \approx 8$. Choosing the initial conditions near to the anti-parallel fixed point $P_0^-$, we observe oscillations, which are smoothly sinusoidal and run around $P_0^-$. For $V_{\text{bias}}/2\lambda \approx 10$, the oscillations disappear and the trajectories run in the same fixed point as depicted in the middle graph of Fig. 11. This result coincides perfectly with the border predicted in the adiabatic analysis.

The bifurcation point, where a revival of the oscillations appear, is also correctly predicted by the adiabatic analysis. For $V_{\text{bias}}/2\lambda \approx 60$ we observe again oscillations around $P_0^\pm$. These oscillations slightly show the two cycle behavior as in the infinite bias case, but the differences between the radii of the cycles is not large. The latter is visible in the third column of Fig. 11, where the results for $V_{\text{bias}}/2\lambda = 100$ are depicted.

The lower row in Fig. 11 depicts the electronic current $\langle I_{\sigma}(t) \rangle$, split into its constituent parts. The right tunneling amplitude is equal to zero and following from that, as well the corresponding current channel. Therefore, the current $I_{\text{RT}}$ should be equal to the total current through the system, hence current conservation remains valid. For the nonadiabatic current, this is ensured for the time-averaged current values, but can be different in the time-dependent case. There, we observed some accumulation of current in the central region.

When we consider the current channels for $V_{\text{bias}}/2\lambda = 2$ in Fig. 11, we notice that all channels reach their minima if the $\langle J_z \rangle$ component is maximal and in this case close to $j$. For this low bias regime, the shift of the energy level, due to the coupling to the large spin $\pm 1/2(B_z\lambda + \langle J_z \rangle)$, leads to a positioning of them slightly outside of the transport window. Therewith the current is minimal.

The current corresponding to the spin-up electrons flows in the direction of the bias. For the spin-down electrons the situation is more complicated, because they are not allowed to leave the system through the right lead.

FIG. 11: Nonadiabatic results obtained in region I for three different bias values, which is chosen in a symmetric manner $V_{\text{bias}} = \mu_{\lambda}/2 = -\mu_{\lambda}/2$. The electronic current, depicted in the lowest row, is separated into left/right and spin-up/down contributions. The magnetic field yield $B_z/\lambda = 0.1$ and $\Gamma/\lambda = 9$. Initial conditions as in Sec III for regime I.

FIG. 12: Nonadiabatic results for region II. The upper row corresponds to a small bias $V_{\text{bias}}/2\lambda = 2$, where the spin components oscillate around the coordinates of $P_0^-$. Increasing the bias, leads to a disappearing of the oscillations and the fixed point $P_{\text{SN}}^\pm$ is present, see lower row for $V_{\text{bias}}/2\lambda = 5$. The magnetic field yield $B_z/\lambda = 0.2$ and $\Gamma/\lambda = 0.7$. Note, that the time-scale of the current and the spin operators differs slightly. Initial conditions as in Sec III for regime I.
due to $\Gamma_{RL} = 0$. The electrons can either flip their spin, stay in the lower level or flow back into the left lead. If the last process appears the electron moves against the bias and the current becomes negative. This feature is slightly visible in the lowest graph of the first column in Fig. 11 there the current $I_{RL}$ drops below zero for a quite small region.

Again, we can address this effect to the position of the large spin’s $z$-component. In the case $I_{RL} < 0$, the shifting of the energy level is the other way around, because $\langle J_z \rangle$ is negative. Following from that, the spin-down level lays above the spin-down level and additionally in the neighborhood of the left Fermi edge ($\mu_L = 2/\lambda$) and electrons can occupy empty states in the left lead. After $\langle J_z \rangle$ passed its minima, the spin-down level moves down and therewith its current channel drops as well.

2. **Region II: Negative Detuning and spin-flip of the large spin**

The feature of negative current is better visible in region II, as we see in the current results for $V_{bias}/2\lambda = 2$, which are depicted in the upper row of Fig. 12. There, the large spin’s $z$-component oscillates close to $-j$ and therewith, both effective levels are clearly situated outside the transport window and the spin-down level lays again above the spin-up level. The oscillations of $\langle J_z \rangle$ are comparatively small and do not influence the behavior of the effective levels as much as for region I. They stay outside of the transport window for all times.

We try to interpret the evolution in time for the current channels focusing on transition between the levels. If $I_{RL}$ is negative, the right current reaches its minimum and the left current for spin-up electrons is maximal. Due to $I_{TR} < I_{RL}$, we can assume that spin-flips from the lower to the upper level happen and a depletion of the upper level into the left lead appears, due to the negativity of $I_{RL}$. This is in accordance with the evolution for the $\langle S_z \rangle$, which decreases in this range point.

The other way around, when $\langle S_z \rangle$ increases, the left current for spin-down electrons is maximal, as well as $I_{TR}$. But $I_{RL}$ is close to zero in this regime. Therefore, we interpret this kind of current cycle in the following way. A spin-down electron enters the upper level, flips into the lower lead, due to the interaction with the large spin, and finally leaves the central system to the right lead. This would explain why the current for spin-up electrons is maximal at the right lead and minimal at the left lead.

If the bias is increased, the regions of negative spin-down current vanish. As well as the oscillations of the spin components, as depicted in the lower row of Fig. 12. Remember, for the infinite bias regime, we found no periodic oscillations at all in this region. In accordance with the foregoing adiabatic analysis, the trajectories enter one fixed point of kind $P_{SN}^\pm$.

The adiabatic analysis, also predicts an earlier disappearing of the fixed point $P_0^+$, corresponding to a parallel alignment of the large spin, as the anti-parallel fixed point $P_0^-$. From that we propose the possibility of spin-flips for the large spin in the region, where only $P_0^-$ has finite imaginary part: $V_{bias}/2\lambda \in [-5; 6.4]$. 

To monitor how the first fixed point disappears we choose our initial conditions close to $P_0^+$ and assume negative detuning. The results are depicted in Fig. 14. For $V_{bias}/2\lambda = -10$ the trajectories perform smooth oscillations around $P_0^+$. As expected from Fig. 7, the $z$-component of the electronic spin $\langle S_z \rangle \approx 0.2$ and is therewith positive. As discussed in the last section, the observed system is not symmetric and therewith the fixed point $P_0^+$ stays alive by further decreasing the bias.

Increasing the bias we see a different behavior, $\langle J_z \rangle$ drops down when the bias passes the threshold $V_{bias}/2\lambda = -5$ as it is clearly visible in Fig. 13. At first the trajectory runs into the fixed point $P_{SN}^\pm$, laying in the middle of both points $P_0^\pm$. But even for $V_{bias}/2\lambda = -4$, the spin components enter the anti-parallel fixed point $P_0^-$.

If we decrease the tunneling rate, we observe a transition to chaotic-like oscillations as in the infinite bias.

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**FIG. 13:** The large spin’s $z$-component for several bias values in regime II. Parameter as for Fig. 12 and initial conditions are $\langle J_z \rangle = \langle J_y \rangle = \sqrt{50 - 9.9^2}/2, \langle J_z \rangle = 9.9$ and $\langle S_\parallel \rangle = \langle S_{\perp} \rangle = 0, \langle S_z \rangle = 0.5$.

**FIG. 14:** The graphs depict the different contributions to the current in regime III. The right graph presents a detail of the left graphs drawn together for a short time range. Initial conditions and parameters as in Sec. 11[13] for regime III.
\( V_{\text{bias}} = \infty \)

\[ P^\pm_{\text{SN}} X \]

\[ P^\pm_{\text{SN}} P^\pm_{\text{SN}} \]

\[ B_z / \lambda \]

\( V_{\text{bias}} = 2 \)

\( V_{\text{bias}} = 20 \)

\( V_{\text{bias}} = 140 \)

\( \Gamma / \lambda \)

\[ 0.1, 0.2, 0.3, 0.4 \]

\[ 10, 20, 30, 40, 50, 60 \]

FIG. 15: Stable fixed point as a function of tunneling rate and magnetic field. The external bias is increased from left to right, the explicit values are denoted in the graphs (in units of [\( \lambda \)]). Additionally, the infinite bias result is plotted in the last graph on the right side. The blue area denotes the regime where no oscillations appear and only the nodes from the \( P^\pm_{\text{SN}} \)-series exist.

3. Region III: oscillations and high frequency

In Sec. II B we presented results for larger values of the magnetic field, where the trajectories oscillate around \( P^\pm_0 \). This kind of behavior is recovered for the finite bias regime. But we observe slight differences in the regime of small bias. In Fig. 14 the current for regime III is depicted \((V_{\text{bias}}/2\lambda = 2)\), the results for the left and the right current are split into their contributions from spin-up and down electrons. We observe that the current channel \( I_{L\downarrow} \) oscillates around the zero axis and following from that, the current is flowing in both direction. The frequency of the oscillations is quite high and the spin components oscillate between two cycles, which signatures are clearly visible in the left graphs in Fig. 14.

IV. CONCLUSION

With our nonadiabatic approach we are able to probe the rate equation and the adiabatic approach and thus can discuss the advantages and disadvantages of these two methods.

The rate equation approach from [13,19] is a quite practical method. The numerical effort is low and the dynamical system can be investigated analytically. However, this method neglects higher order transitions and we learnt from the nonadiabatic results, that these are relevant in the regime of low magnetic field. In the latter region, the rate equation misses parts of the nonlinear dynamics. If the magnetic field increases, the results of both approaches coincide.

The limitation of the rate equation approach to high external bias is another disadvantage: at finite bias, we observe even richer dynamics within the nonadiabatic approach. There, features like spin-flips of the large spin and suppression of the oscillations as a function of the applied bias appear. The disadvantages belonging to the nonadiabatic approach are the high numerical effort and its inaccessibility for further analytic investigations.

In our work, we utilized the adiabatic Green’s function method to analyze our system. However, we omitted the presentation of time-dependent results, because the complete adiabatic approach does not capture a lot of the dynamics even in the infinite bias regime. Chaotic, damped or two cycle-like oscillations are not observable within this approach: the dynamical system reduces to three equations of motion, which is not suitable to describe the system in the range of low magnetic fields. Only if the system performs smoothed low-frequency oscillations or runs directly in a stable fixed point, the adiabatic approach can correctly describe the dynamics.

On the other hand, the adiabatic Green’s functions are suitable for the analysis of the system. For the finite bias regime, our initial adiabatic analysis matched quite well the behavior of the nonadiabatic approach. Therewith we can re-define the dynamical regions as depicted in Fig. 15. It is clearly visible that the external bias is an important parameter for this system. The regions where only one of the main fixed points \( P^\pm_0 \) exists are promising for switching of the large spin, but due to the fact that we deal with a multistable system, there are always additional stationary solutions present.

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Appendix A: Frequency doubling of \(\langle \hat{S}_z(t) \rangle\)

The equation of motion for the \(z\)-component of the electronic spin in Eq. (13) does not directly couple to the magnetic field, but it couples to the product of the \(x\)-component for the large spin and the \(y\)-component of the electronic spin. The latter develop sinusoidal in time with the same frequency \(\omega_s\) and without a phase shift, cf. Fig. 5. If we approximate their evolution in time with \(\langle J_x^{eff}(t) \rangle = a_s \sin \omega_s t\) and \(\langle J_y^{eff}(t) \rangle = a_e \sin \omega_e t\), we can estimate an effective solution for the \(z\)-component of the electronic spin \((A_s = a_s a_e)\)

\[
\langle \hat{S}_z^{eff}(t \to \infty) \rangle = \frac{(\Gamma_{lt} - \Gamma_{lt} + 2A_s)}{2\Gamma} - \frac{1}{8} \left[ \frac{\cos(2\omega_s t) + 2\omega_s / \Gamma \sin(2\omega_s t)}{(\Gamma_A)^{-1}(\omega_s^2 - \frac{\omega_e^2}{2})} \right].
\]

(A1)

Hence, the oscillation goes with twice the frequency of the other spin components. Note, that for a more general ansatz, p.e. \(\langle J_x^{eff}(t) \rangle = a_s \sin \omega_s t + b_j \cos \omega_j t\), the result is similar and differs solely in the prefactor.

Appendix B: Adiabatic approach: Green’s functions

The expectation values of the spin operators in frequency space are obtained from

\[
\langle \hat{S}_i \rangle(\omega) = \frac{-i}{2} \text{tr} [\hat{G}^{<}(\omega) \hat{\sigma}_i], \quad i = x, y, z,
\]

(B1)

containing the Pauli spin matrices

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

If we reconsider the effective Hamiltonian Eq. (7) for this system, we remember the similarity to a parallel two-level system and as in that case, the Green’s functions have matrix character

\[
\mathbf{G}(\omega) = \begin{pmatrix} G_{++}^{<}(\omega) & G_{+\downarrow}^{<}(\omega) \\ G_{\downarrow+}^{<}(\omega) & G_{\downarrow\downarrow}^{<}(\omega) \end{pmatrix}.
\]

The off-diagonal functions refer to the coupling to the \(x\)-component of the large spin. Without this coupling we end up with two independent levels. For the derivation of the electronic spin we need the lesser Green’s function whose calculation takes usage of the Keldysh equation

\[
G^{<}_{\sigma\sigma'}(\omega) = \sum_{\sigma''} G^R_{\sigma\sigma''}(\omega) \Sigma_{\sigma''\sigma'}^{<}(\omega) G^A_{\sigma''\sigma}(\omega),
\]

(B2)

with the lesser self energy

\[
\Sigma_{\sigma\sigma'}^{<}(\omega) = \delta_{\sigma,\sigma'} \sum_{\alpha} \Sigma_{\alpha\alpha}^{<} = \delta_{\sigma,\sigma'} \sum_{\alpha} i\Gamma_{\alpha\sigma} f_{\alpha\sigma}(\omega).
\]

(B3)

The retarded and the advanced Green’s function are obtained from their equation of motion and yield

\[
G^{R,A}_{\sigma\sigma'}(\omega) = \begin{pmatrix} \omega - \epsilon_{\sigma'} - \Sigma_{\sigma'\sigma}^{R,A} & \omega - \epsilon_{\sigma} - \Sigma_{\sigma'\sigma}^{R,A} - \frac{\lambda^2}{2} \langle \hat{J}_z \rangle^2 \\ \omega - \epsilon_{\sigma} - \Sigma_{\sigma'\sigma}^{R,A} & \omega - \epsilon_{\sigma'} - \Sigma_{\sigma'\sigma}^{R,A} \end{pmatrix},
\]

(B4)

with the retarded/advanced self energy

\[
\Sigma_{\sigma\sigma'}^{R,A} = \delta_{\sigma,\sigma'} \sum_{\alpha} \Sigma^{R,A}_{\alpha\alpha} = \delta_{\sigma,\sigma'} \sum_{k\alpha} \frac{|V_{k\alpha\sigma}|^2}{\omega - \epsilon_{k\alpha\sigma} \pm i0},
\]

(B5)

which real part \( \Lambda_{\sigma\sigma}(\omega) \) solely leads to a shift of the level energies \( \epsilon_{\sigma} \) and therefore can be neglected. The imaginary part corresponds to the tunneling rate \( \Gamma_{\alpha\sigma}(\omega) \equiv \Gamma_{\alpha\sigma} \).

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