NCOS and D-branes in Time-dependent Backgrounds

Rong-Gen Cai$^1$, Jian-Xin Lu$^2$ and Nobuyoshi Ohta$^3$

$^1$ Institute of Theoretical Physics, Chinese Academy of Sciences, P. O. Box 2735, Beijing 100080, China
$^2$ Interdisciplinary Center for Theoretical Study University of Science and Technology of China, Hefei, Anhui 230026, China
$^3$ Michigan Center for Theoretical Physics, Randall Physics Laboratory University of Michigan, Ann Arbor, MI 48109-1120, USA
$^3$ Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

Abstract

We study noncommutative open string (NCOS) theories realized in string theory with time-dependent backgrounds. Starting from a noncommutative Yang-Mills theory (NCYM) with a constant space-space noncommutativity but in a time-dependent background and making an S-dual transformation, we show that the resulting theory is an NCOS also in a time-dependent background but now with a time-dependent time-space noncommutativity and a time-dependent string scale. The corresponding dual gravity description is also given. A general $SL(2, \mathbb{Z})$ transformation on the NCYM results in an NCOS with a time-dependent time-space noncommutativity and a constant space-space noncommutativity, and also in a time-dependent background.

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*e-mail address: caig@itp.ac.cn
†e-mail address:jxlu@ustc.edu.cn
‡e-mail address: ohta@phys.sci.osaka-u.ac.jp
1 Introduction

String theory in various backgrounds is a subject of much interest. In particular, non-commutative theories emerge from the open strings on the D-branes in constant $B$-field background \cite{1}. This suggests that such theories may be directly relevant to understanding the space-time structure at short distances in quantum gravity. It has been known that these theories have a very useful description in terms of gravity dual solutions \cite{2,3}, which have clarified many interesting properties of these theories.

Most of the investigations to date are focused on static backgrounds. It is then natural to study string theories in time-dependent backgrounds. These theories are expected to be important in understanding the evolution of our universe. In fact, there have already appeared several papers on string theories in time-dependent backgrounds \cite{4}-\cite{13}. Various quotients of Minkowski space-time have been studied recently as concrete realizations of string theories on time-dependent backgrounds. Among others, one of the simplest examples of the space-time orbifold is that of the “null-brane” \cite{4}.

The dynamics of D-branes can be studied by the attached open strings. In a certain decoupling limit, the massive modes of open strings as well as bulk closed strings are decoupled from the theory, and one can study the dynamics without the complication of gravity. It is also possible to understand the theory in terms of the dual description by a bulk theory of gravity, in the spirit of AdS/CFT correspondence. In particular, it has been shown that D-branes in the null-brane background have an interesting decoupling limit and that the resulting theory corresponds to a noncommutative Yang-Mills theory (NCYM) with a constant space-space noncommutativity but in a time-dependent background\cite{9}. The dual description has been also given which is time-dependent. The theory is nonlocal in space in an interesting time-dependent manner, but no such a theory with a time-dependent noncommutativity in the time direction and in a time-dependent background has been considered so far. One is then naturally led to ask whether or not such a theory exists.

For this purpose, we study the S-dual of the above NCYM in this paper. We show that such a theory indeed exists in our simple setting of null-brane orbifold by using the dual gravity description and making S-duality transformation of the solution. We find that there is an interesting decoupling limit also in this time-dependent case. The

\footnote{As shown in \cite{9}, such a constant space-space noncommutativity can be transformed back to the original time-independent non-singular coordinate system and it becomes dependent on both space and time.}
resulting theory turns out to be a noncommutative open string theory (NCOS), in much
the same way as time-independent case [14, 15, 16], but now with a time-dependent
noncommutativity in the time direction and also in a time-dependent background. We also
examine the properties of the theory under a more general $SL(2, \mathbb{Z})$ transformation. We
find that the NCOS theory is transformed again into another NCOS in general, but under
certain circumstances it is transformed into the aforementioned NCYM, again similar to
static case [17, 18, 19, 20].

2 D3-branes in time-dependent backgrounds and NCYM

Let us first review the D3-branes in time-dependent null-brane backgrounds [4]. The
geometry of the null-brane is simply an orbifold of a Minkowski space-time
\[
    ds^2 = -2dx^+dx^- + dx^2 + dz^2 + dx_{\perp}^2
\]
by the identification
\[
    x^+ \sim x^+, \quad x \sim x + 2\pi x^+,
\]
\[
    x^- \sim x^- + 2\pi x + (2\pi)^2 x^2 / 2, \quad z \sim z + 2\pi R,
\]
where $dx_{\perp}^2$ denotes the line element of a six-dimensional Euclidean space. The resulting
space can also be described by the metric
\[
    ds^2 = -2d\hat{x}^+d\hat{x}^- + d\hat{x}^2 + (\hat{x}^2 + R^2) d\hat{z}^2 + 2(\hat{x}^+d\hat{x}^- - \hat{x}d\hat{x}^+)d\hat{z} + dx_{\perp}^2,
\]
where these coordinates have the relations to those in (1) as follows:
\[
    \hat{x}^+ = x^+, \quad \hat{x}^- = x^- - \frac{zx}{R} + \frac{z^2 x^+}{2R^2}, \quad \hat{x} = x - \frac{zx^+}{R}, \quad \hat{z} = \frac{z}{R}.
\]
In these coordinates the orbifold becomes simple
\[
    \hat{z} \sim \hat{z} + 2\pi.
\]
There is another set of coordinates [1]:
\[
    x^+ = y^+, \quad x = y^+y, \quad x^- = y^- + y^+y^2 / 2,
\]
in which the quotient identification is simple:
\[
    y^+ \sim y^+, \quad y \sim y + 2\pi,
\]
\[
    y^- \sim y^-, \quad z \sim z + 2\pi R,
\]
and the metric is also much simpler than the one (3)

\begin{equation}
    ds^2 = -2dy^+dy^- + (y^+)^2dy^2 + dz^2 + dx_1^2.
\end{equation}

Note that the coordinate transformation (3) is singular when \(y^+ = 0\). Since we will adopt the coordinates in (3), so in what follows \(y^+ \neq 0\) is assumed. In the coordinates (3) we write down the supergravity solution of D3-branes

\begin{equation}
    ds^2 = H^{-1/2}(-2dy^+dy^- + (y^+)^2dy^2 + dz^2) + H^{1/2}(dr^2 + r^2d\Omega_5^2),
\end{equation}

\begin{equation}
    A_4 = \frac{y^+}{Hg_s}dy^+ \wedge dy^- \wedge dy \wedge dz, \quad e^{2\phi} = g_s^2,
\end{equation}

where \(g_s\) is the coupling constant of closed string and

\begin{equation}
    H = 1 + \frac{4\pi g_s N}{\alpha'^2}
\end{equation}

with \(N\) being the number of D3-branes in the configuration. Following the steps enumerated in 9, we obtain the required D3-brane configuration

\begin{equation}
    ds^2 = H^{-1/2} \left[ -2dy^+dy^- + \frac{HR^2}{HR^2 + (y^+)^2} \left( (y^+)^2dy^2 + dz^2 \right) \right] + H^{1/2}(dr^2 + r^2d\Omega_5^2),
\end{equation}

\begin{equation}
    2\pi\alpha'B_{yz} = \frac{R(y^+)^2}{HR^2 + (y^+)^2}, \quad A_{y^+y^-} = \frac{H^{-1}}{g_sR}y^+,
\end{equation}

\begin{equation}
    A_{y^+y^-yz} = \frac{H^{-1}y^+}{g_s} \frac{HR^2}{HR^2 + (y^+)^2}, \quad e^{2\phi} = g_s^2 H R^2 / (HR^2 + (y^+)^2).
\end{equation}

Taking the decoupling limit 9:

\begin{equation}
    \alpha' \to 0, \quad u = \frac{r}{\alpha'} = \text{fixed}, \quad \tilde{R} = \frac{\alpha'}{R} = \text{fixed},
\end{equation}

and keeping \(g_s\) constant, the supergravity configuration reduces to

\begin{equation}
    ds^2 = \alpha' \frac{u^2}{\lambda^2} \left[ -2dy^+dy^- + h^{-1}((y^+)^2dy^2 + dz^2) + \frac{\lambda^4}{u^4}(du^2 + u^2d\Omega_5^2) \right],
\end{equation}

\begin{equation}
    2\pi\alpha'B_{yz} = \alpha' \frac{\tilde{R}(y^+)^2u^4}{\lambda^4} h^{-1}, \quad A_{y^+y^-} = \alpha' \frac{\tilde{R}y^+u^4}{g_s\lambda^4}, \quad e^{2\phi} = g_s^2 h^{-1},
\end{equation}

where \(\lambda^4 = 4\pi g_s N\), and

\begin{equation}
    h = 1 + \frac{\tilde{R}^2(y^+)^2u^4}{\lambda^4}.
\end{equation}

\footnote{For the 4-form gauge potential \(A_4\), we have used a gauge transformation which removes the pure gauge term from the 4-form potential in the usual D3-brane configuration.}
It is easy to confirm that under the decoupling limit (12), the resulting theory is indeed a noncommutative Yang-Mills theory in (3 + 1) dimensions. Naively using the Seiberg-Witten relation [1] to the solution (11) in the "flat" limit \( H = 1 \), we get the open string moduli \( G_s = g_s \),

\[
\Theta^{yz} = 2\pi \tilde{R},
\]

and

\[
G^{ij} = \begin{pmatrix}
0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & (y^+)^{-2} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

This indicates that we have a good open string moduli with constant noncommutativity parameter in the coordinates (13), although the closed string metrics and hence the theory itself is time-dependent [9]. It is quite interesting that even though the closed string metrics is time-dependent, they are so in an intricate way that the noncommutativity parameter is constant. However, it is worthwhile mentioning here that as shown in [9], if one uses the coordinates (1), the Yang-Mills theory lives in a flat static space with noncommutative parameter depending on space-time. We will see that the situation is drastically changed for our time-dependent NCOS theories.

### 3 S-duality and NCOS

In [14] it is shown that strongly coupled, spatially noncommutative \( \mathcal{N} = 4 \) Yang-Mills theory has a dual description as a weakly coupled noncommutative open string theory. What is the case of D3-branes in time-dependent backgrounds?

Under the S-duality, the supergravity dual (13) becomes

\[
ds^2 = \alpha' u^2 h^{1/2} \left[ -2dy^+dy^- + h^{-1}((y^+)^2dy^2 + dz^2) + \frac{\lambda^4}{u^4}(du^2 + u^2d\Omega_5^2) \right],
\]

\[
2\pi\alpha' B_{y^+y^-} = \alpha' \frac{\tilde{R}(y^+)^2u^4}{\lambda^4} h^{-1}, \quad A_{yz} = -\alpha' \frac{\tilde{R}(y^+)^2u^4}{g_s\lambda^4} h^{-1}, \quad e^{2\phi} = g_s^2h,
\]

\[\text{In obtaining the following, we actually rescale the coordinates such that the various fields have the desired dependences on the } g_s \text{ factor which is convenient for us to discuss the corresponding NCOS in other dimensions. They are: } y^+ \rightarrow g_s^{1/2}y^+, \ y^- \rightarrow g_s^{-3/2}y^-, \ y \rightarrow g_s^{-1}y, \ z \rightarrow g_s^{-1/2}z, \ u \rightarrow g_s^{-1/2}u, \ R \rightarrow g_s^{1/2}R, \ \tilde{R} \rightarrow g_s^{-1/2}\tilde{R}. \text{This is not important in the present case but it will be for other Dp-branes discussed in section 4 since the } g_s \text{ factor scales in the corresponding decoupling limit.} \]
where $\lambda^4 = 4\pi g_s N$ again.

What is the dual (open string) theory to the supergravity configuration (16)? To see this, let us first write down\footnote{We also rescale the coordinates as in footnote 3 with the replacement of $u \rightarrow g_s^{-1/2} u$ by $r \rightarrow g_s^{-1/2} r$.} the S-dual of the solution (11):

$$ds^2 = H^{-1/2} F^{-1/2} \left[ -2dy^+ dy^- + F((y^+)^2 dy^2 + dz^2) + H(dr^2 + r^2 d\Omega_5^2) \right],$$

$$2\pi\alpha'B_{y^+ y^-} = \frac{y^+}{HR}, \quad A_{yz} = -\frac{(y^+)^2 F}{g_s RH}, \quad e^{2\phi} = g_s^2 F^{-1}, \quad (17)$$

where

$$F = \frac{HR^2}{HR^2 + (y^+)^2},$$

and $g_s$ is the inverse of the original string coupling. Taking the decoupling limit (12), the resulting configuration from the solution (17) is identical to that of (16).

In the “flat” space limit $H = 1$, we can examine the open string moduli by using Seiberg-Witten relation again. From eq. (17) with $H = 1$, we find the open string parameters are given by

$$G^{ij} = \begin{pmatrix}
0 & -\sqrt{\frac{R^2 + (y^+)^2}{R^2}} & 0 & 0 \\
-\sqrt{\frac{R^2 + (y^+)^2}{R^2}} & 0 & 0 & 0 \\
0 & 0 & -\sqrt{\frac{R^2 + (y^+)^2}{(y^+)^2}} & 0 \\
0 & 0 & 0 & -\sqrt{\frac{R^2 + (y^+)^2}{R^2}}
\end{pmatrix}, \quad (18)$$

and

$$\Theta_{y^+ y^-} = 2\pi\alpha' \frac{y^+}{R} = 2\pi y^+ \tilde{R}. \quad (19)$$

Note that the noncommutativity parameter (19) is well-defined in the decoupling limit (12) but it is now time-dependent, in contrast to the space-space noncommutative case (14) for the NCYM! Also $\alpha' G^{ij}$ is finite in the decoupling limit (12), which means that although the modes of closed strings decouple in the limit (12), the massive modes from the open strings do not decouple, resulting in a noncommutative open string theory. Further, it is easy to find that in that case the effective open string scale is $\alpha'_\text{eff} = y^+ \tilde{R}$, depending on time and the coupling constant of open strings is still a constant, $G_s = g_s$.

The supergravity description (16) may be compared with the one dual to NCOS with constant time-space noncommutativity given in [14]. Write

$$h = \frac{u^4}{A^4} \left( 1 + \frac{A^4}{u^4} \right), \quad (16)$$
and we get
\[ ds^2 = \alpha' f^{1/2} \left[ \frac{u^4}{\lambda^2 A^2}(-2dy^+dy^-) + \frac{A^2}{\lambda^2} f^{-1}((y^+)^2dy^2 + dz^2) + \frac{\lambda^2}{A^2}(du^2 + u^2d\Omega_5^2) \right], \]
\[ 2\pi\alpha' B_{y^+y^-} = \alpha' \frac{\tilde{R}y^+u^4}{\lambda^4}, \quad A_{yz} = -\alpha' \frac{1}{g_sRf}, \quad e^{2\phi} = g_s^2u^4/A^4f. \] (21)

If $A^4$ were constant, this would be exactly the gravity solution dual to NCOS with constant noncommutativity \[14\]. For small $u$, we recover the AdS$_5 \times$S$^5$ but with an orbifolding of the flat 4-dimensional slice in AdS$_5$ since the open string theory reduces to $N = 2$ super Yang-Mills theory at low energies or long distance ($N = 2$ due to orbifolding). However, the theory significantly deviates from that for $u \sim A$, which determines the size of the noncommutativity.

The solution has actually time-dependent noncommutativity, and along constant light-cone coordinate $y^+$, the theory looks exactly as the NCOS with constant noncommutativity. Away from that region, the theory has different noncommutativity scale.

More generally we can consider an $SL(2, \mathbb{Z})$ transformation to the solution [11]. Since the axion field is zero, we have
\[ \tilde{\tau} = \frac{a\tau + b}{c\tau + d}, \quad \tau \equiv ie^{-\phi}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbb{Z}, \]
whose imaginary part gives
\[ e^{-\tilde{\phi}} = \frac{e^{-\phi}}{|c\tau + d|^2}. \] (23)

The Einstein-frame metric is unchanged under the $SL(2, \mathbb{Z})$, so $ds^2_E = e^{-\phi/2}ds^2 = e^{-\tilde{\phi}/2}d\tilde{s}^2$, giving
\[ d\tilde{s}^2 = |c\tau + d|ds^2. \] (24)

We find from the solution [11] the transformed configuration as\[5\]
\[ d\tilde{s}^2 = F^{-1/2}H^{-1/2} \sqrt{\frac{e^2 + g_s^2d^2F}{c^2 + g_s^2d^2}} \left[ -2dy^+dy^- + F((y^+)^2dy^2 + dz^2) \right] \]
\[
2\pi\alpha'\tilde{B} = \frac{d(y^+)^2F}{HR} \frac{g_s}{\sqrt{c^2 + d^2 g_s^2}} dy \wedge dz - \frac{cy^+}{HR} \frac{1}{\sqrt{c^2 + d^2 g_s^2}} dy^+ \wedge dy^-,
\]
\[
e^{2\tilde{\phi}} = g_s^{-2} F \left( c^2 + d^2 g_s^2 F \right)^2, \quad \tilde{x} = (ac + bd g_s^2 F) \left( c^2 + d^2 g_s^2 F \right)^{-1},
\]
where the function \( F \) is the same as before, \( g_s \) is the original string coupling and \( \tilde{g}_s = g_s^{-1}(c^2 + d^2 g_s^2) \) is the new one. The harmonic function \( H = 1 + 4\pi\alpha^2\tilde{g}_s N/r^4 \).

Using the Seiberg-Witten relation, we obtain the open string moduli: the open string metric

\[
G^{ij} = \begin{pmatrix}
0 & -\sqrt{\frac{g_s^2 d^2 + c^2 w}{c^2 + g_s^2 d^2}} & 0 & 0 \\
-\sqrt{\frac{g_s^2 d^2 + c^2 w}{c^2 + g_s^2 d^2}} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{(y^+)^2} \sqrt{\frac{g_s^2 d^2 + c^2 w}{c^2 + g_s^2 d^2}} & 0 \\
0 & 0 & 0 & \sqrt{\frac{g_s^2 d^2 + c^2 w}{c^2 + g_s^2 d^2}}
\end{pmatrix},
\]

the noncommutative parameter

\[
\Theta^{ij} = 2\pi\alpha' \begin{pmatrix}
0 & -\frac{cy^+}{R\sqrt{c^2 + g_s^2 d^2}} & 0 & 0 \\
\frac{cy^+}{R\sqrt{c^2 + g_s^2 d^2}} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{g_s d}{R\sqrt{c^2 + g_s^2 d^2}} \\
0 & 0 & \frac{g_s d}{R\sqrt{c^2 + g_s^2 d^2}} & 0
\end{pmatrix},
\]

and the open string coupling constant \( G_s = \tilde{g}_s = g_s^{-1}(c^2 + g_s^2 d^2) \). In the above, the function \( w = 1 + (y^+)^2/R^2 \). One can check that the above moduli give the correct ones when \( a, b, c, d \) are specialized to their corresponding values.

In the decoupling limit (12), assuming \( c \neq 0 \), we get

\[
d\tilde{s}^2 = \alpha' \sqrt{\frac{\tilde{f}}{1 + g_s^2 d^2/c^2}} \left[ \frac{u^4}{\lambda^2 A^2} (-2dy^+ dy^-) + \frac{A^2}{\lambda^2} f^{-1}(dy^- dy^+ + dz^2) + \frac{\lambda^2}{A^2} \left( du^2 + u^2 d\Omega_5^2 \right) \right],
\]
\[
2\pi\alpha'\tilde{B} = \alpha' \frac{1}{f R} \frac{d g_s}{\sqrt{c^2 + d^2 g_s^2}} dy \wedge dz - \alpha' \frac{R u^4 y^+}{\lambda^4} \frac{c}{\sqrt{c^2 + d^2 g_s^2}} dy^+ \wedge dy^-,
\]
\[
e^{2\tilde{\phi}} = g_s^{-2} \left( 1 + \frac{u^4}{A^4} \right) \left( c^2 + \frac{g_s^2 d^2}{1 + u^4/A^4} \right)^2,
\]
\[
\tilde{x} = \left( ac + \frac{g_s^2 b d}{1 + u^4/A^4} \right) \left( c^2 + \frac{g_s^2 d^2}{1 + u^4/A^4} \right)^{-1},
\]

(28)
where
\[ \tilde{f} = 1 + \frac{c^2 + g_s^2 d^2}{c^2 y^4} A^4. \]  
(29)

This corresponds to a general NCOS theory with a time-dependent time-space noncommutativity and a constant space-space noncommutativity. The effective string scale is now
\[ \alpha'_\text{eff} = y^+ \tilde{R} c / \sqrt{c^2 + d^2 g_s^2}. \]

The space-space noncommutativity is directly proportional to the original string coupling \( g_s \) if \( c \neq 0 \). For the special case of \( c = 0 \), it gives a gravity dual to NCYM. So we can see from (25) that in the case of \( c = 0 \), the space-space noncommutative NCYM in a time-dependent background is transformed back to NCYM. Thus we find that the space-space noncommutative NCYM theory in a time-dependent background is transformed into NCOS in general, but for the special case \( c = 0 \) it is transformed to NCYM again. Since we get an NCOS theory from the NCYM by an \( SL(2,\mathbb{Z}) \) transformation, which makes a group, by making a further \( SL(2,\mathbb{Z}) \) transformation to the obtained NCOS theory, we get another NCOS theory. This means that the NCOS theory itself transforms into NCOS in general, but of course it may transform into NCYM in special case. This is much the same as the time-independent noncommutative theories [19, 20].

4 The Cases for Other dimensions

The authors of [11] have extended the study in [9] to other dimensional \( Dp \)-branes, \( NS5 \)-branes and \( M5 \)-branes in time-dependent backgrounds, and discussed the supergravity duals of decoupled worldvolume theories. In the previous section we have discussed supergravity dual of (3 + 1)-dimensional NCYM with space-space noncommutativity but in a time-dependent background and its S-duality, resulting in NCOS with time-dependent noncommutativity. In this section we give the supergravity dual of other dimensional NCOS theory.

Applying T-duality to the solution (17), we can obtain \( Dp \)-brane solution with electric field in the "null-brane" geometry which can be viewed as deformed \((F, Dp)\) bound states discussed in [21],

\[ ds^2 = H^{-1/2} F^{-1/2} [-2 dy^+ dy^- + F((y^+)^2 dy^2 + dz^2 + \sum_{i=1}^{p-3} dx_i^2) + H(dr^2 + r^2 d\Omega_8^2)] , \]

\[ 2 \pi \alpha' B_{y^+ y^-} = \frac{y^+}{HR}, \quad e^{2\phi} = g_s^2 H^{3-p} F^{p-5}, \]

\[ A_{y^+ y^- y z_1 \cdots z_{p-3}} = - \frac{(y^+)^2 F}{g_s R H}, \quad A_{y^+ y^- y z_1 \cdots z_{p-3}} = \frac{y^+ F}{H g_s} , \]  
(30)
where
\[ F = \frac{H R^2}{H R^2 + (y^+)^2}, \quad H = 1 + \frac{c_p g_s N \alpha''(7-p)/2}{r^{7-p}} \]
with \( c_p = 2^{5-p} \pi \frac{2^p}{3} \Gamma[(7 - p)/2] \). Considering the following decoupling limit
\[ \alpha' \to 0, \quad \bar{g}_s = g_s \alpha'(p-3)/2 = \text{fixed}, \quad u = \frac{r}{\alpha'} = \text{fixed}, \quad \tilde{R} = \frac{\alpha'}{R} = \text{fixed}, \quad (31) \]
the supergravity solution becomes
\[

d s^2 = \alpha' \left( \frac{u}{\lambda} \right)^{(7-p)/2} h^{1/2} \left( -2dy^+dy^- + h^{-1}((y^+)^2 dy^2 + dz^2 + \sum_{i=1}^{p-3} dx_i^2) \right) \\
+ \left( \frac{\lambda}{u} \right)^{7-p} \left( du^2 + u^2 d\Omega^2_{8-p} \right), \\
2\pi \alpha' B_{y^+y^-} = \alpha' y^+ \tilde{R} \left( \frac{u}{\lambda} \right)^{7-p}, \quad e^{2\phi} = \bar{g}_s h^{(5-p)/2} \left( \frac{\lambda}{u} \right)^{(7-p)(3-p)/2}, \quad (32)
\]
where
\[ h = 1 + \frac{\tilde{R}^2 (y^+)^2 u^{7-p}}{\lambda^{7-p}} \]
and \( \lambda^{7-p} = c_p \bar{g}_s N \). Naively using Seiberg-Witten relation, we can easily show that the worldvolume theory on the Dp-brane \( (p < 6) \) in the solution (30) decouples from the closed string theory in the decoupling limit (31), resulting in a \((p+1)\)-dimensional NCOS theory with time-dependent noncommutativity and open string scale \( \alpha'_{\text{eff}} = \tilde{R} y^+ \). The open string coupling constant \( G_s = g_s (R^2 / (R^2 + (y^+)^2))^{(p-3)/4} = \bar{g}_s (\tilde{R} y^+)^{(3-p)/2} \). So only for \( p = 3 \), the \( G_s \) is time-independent. The supergravity solution (32) is just the gravity dual description of the decoupled NCOS theory within the region where the supergravity description is valid in the usual manner \[22\].

5 Conclusions

In this paper we have discussed time-dependent noncommutative theories obtained from string theories in time-dependent background on the simple null-brane orbifold. The resulting NCYM theory has only a constant space-space noncommutativity but in a time-dependent background. Using S-duality transformation, we have identified the resulting theory as a noncommutative NCOS in a time-dependent background but with a time-dependent time-space noncommutativity. We have also examined the transformation properties of the NCYM under a general \( SL(2,\mathbb{Z}) \) transformation, and find that it is
transformed into NCOS in general, but under a special circumstance it is transformed back to NCYM. This is quite similar to the case of noncommutative theories resulting from D-branes in the static backgrounds.

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