Radiation-Induced “Zero-Resistance State” at Low Magnetic Fields and near Half Filling of the Lowest Landau Level

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We theoretically predict the radiation-induced “zero-resistance state” near half filling of the lowest Landau level, which is caused by the photon-assisted transport in the presence of oscillating density of states due to composite fermion Landau levels, and is analogous to the radiation-induced “zero-resistance state” of electrons at low magnetic fields. Based on a non-perturbative theory, we show that the radiation field does not break the bound state of electrons and flux quanta, i.e. composite fermions. Our prediction is independent of the power and frequency of radiation field.

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Recently discovered states of a two-dimensional (2D) electron system with vanishingly small resistance in the presence of microwave radiation at low magnetic fields [1, 2] reveal a fundamentally new non-equilibrium state of matter. This is so not just because of the apparent zero resistance (which is the main reason why there has been intense attention to this problem), but also because of magnetoresistance oscillation which eventually leads to the vanishingly small resistance. This radiation-induced magnetoresistance oscillation sheds new light on the interaction of radiation and 2D electron system.

It is generally believed that the zero-resistance state is due to an instability caused by photon-assisted transport in the presence of an oscillating density of states, which leads to negative conductivity for sufficiently large radiation power. In spite of this overall consensus, it is not clear whether previous theoretical frameworks adequately treated the rather strong interaction between the radiation field and the 2D electron systems under a magnetic field. In this article, we develop a non-perturbative theory which treats the interaction between radiation and electrons exactly (though the scattering with impurities is treated perturbatively).

Using this non-perturbative theory, we would like to make two predictions. First, it is predicted that there is a resonantly enhanced magnetoresistance oscillation near \( \omega \simeq \omega_c \) which may reveal higher harmonic oscillations. Second, we predict similar “zero-resistance states” near half filling of the lowest Landau level (\( \nu = 1/2 \)) using the composite fermion theory [3]. In particular, we show that the radiation field does not break the bound state of electrons and flux quanta, i.e. composite fermions, regardless of the power and frequency of the radiation. Finally, note that we will not address the question how the negative resistance leads to the zero resistance [4]. Instead, we will focus on the nature of the magnetoresistance oscillations at low magnetic fields and near \( \nu = 1/2 \).

Let us begin by writing the Hamiltonian in the presence of microwave radiation. The radiation wavelength is long enough so that the electric field is coherent and uniform in space over the whole sample. We will choose the Landau gauge with \( \mathbf{A} = B(0, x) \) in which the momentum in \( y \) direction \( (k_y) \) is a good quantum number. Then, for eigenstates with \( k_y = -X_j/l_B^2 \) (where \( l_B = \sqrt{\hbar c/eB} \) is the magnetic length), the single particle Hamiltonian is written:

\[
H = \frac{p_x^2}{2M} + \frac{1}{2}M\omega_c^2(x - X_j)^2 - F(t)(x - X_j) - F(t)X_j = H_F(p_x, x - X_j) - F(t)X_j \tag{1}
\]

where \( F(t) = eE \cos \omega t \), \( E \) is the electric field, and \( H_F(p_x, x - X_j) \) is the Hamiltonian of a time-dependent forced harmonic oscillator centered at \( x = X_j \).

The time-dependent forced harmonic oscillator described by \( H_F(p_x, x) \) can be solved exactly for general \( F(t) \) without using any perturbation theory: the analytic solution is given as follows:

\[
\psi(x, t) = \chi(x - \xi(t), t)e^{i\theta(x, t)} \tag{2}
\]

where \( \theta(x, t) = \frac{M}{\hbar} \frac{dx(t)}{dt} (x - \xi(t)) + \frac{1}{\hbar} \int_0^t Ldt' \) satisfies the Schrödinger equation of the unforced harmonic oscillator, and can be taken to be either the usual number eigenstate or any other non-stationary solution such as a coherent state. The center position \( \xi(t) \) satisfies the classical motion of a forced harmonic oscillator: \( M\frac{d^2\xi}{dt^2} + M\omega_c^2\xi = F(t) \) where \( \omega_c = eB/mc \). \( L \) is the classical Lagrangian: \( L = \frac{M}{2} \left( \frac{dx}{dt} \right)^2 - \frac{M\omega_c^2}{2} \xi^2 + F(t)\xi \). The key point is that, other than the phase factor, the analytic solution in Eq. (2) is precisely the same wavefunction as an unforced harmonic oscillator with center position displaced by \( \xi(t) \). Since \( \xi(t) \) is just the classical solution of the forced harmonic oscillator, it is independent of any quantum numbers.

The analytic solution of the Hamiltonian in Eq. (1) is
fied because of the time-dependent phase factor, let us should be identical to that without a radiation field, if

\[ \exp \left( i \frac{eE}{\hbar} X_j \left( \frac{1}{\omega} + \frac{\omega}{\omega^2 - \omega'_2} \right) \sin \omega t \right) \]

\[ = \chi_n(x - X_j - \xi(t), t)e^{i\theta(x, t)} \]

Then given by:

\[ \phi_{n,j}(x, t) = \psi_n(x - X_j, t) \exp \left( i \frac{eE}{\hbar} \int_0^t dt' F(t') \right) \]

where \( n \) is the Landau level index. The phase factor

\[ \exp \left( i \frac{eE}{\hbar} \int_0^t dt' F(t') \right) \]

is due to the last term in Eq. (1). Strictly speaking, the energy is not conserved in the presence of radiation. However, the Landau level index \( n \) is an invariant quantum number which is connected to the usual Landau level index of unforced harmonic oscillator. It is easy to understand the meaning of invariance if one considers the situation where an electron is in the \( n \)-th Landau level initially, and the radiation field is turned on at \( t = 0 \). The electron will stay in the exact state in Eq. (3) with the same Landau level index \( n \) at \( t > 0 \).

Because the orbital part of exact solution, \( \phi_{n,j}(x, t) \), in Eq. (3) is quantized in the exactly same way as eigenstates in the absence of radiation, the density of states should be identical to that without a radiation field, if there were no time-dependent phase factor. In order to understand how the density of states should be modified because of the time-dependent phase factor, let us rewrite \( \phi_{n,j}(x, t) \) as follows:

\[ \phi_{n,j}(x, t) = \chi_n(x - X_j - \xi(t), t)e^{i\theta(x, t)} \]

\[ \times \exp \left( i \frac{eE}{\hbar} X_j \left( \frac{1}{\omega} + \frac{\omega}{\omega^2 - \omega' _2} \right) \sin \omega t \right) \]

\[ = \chi_n(x - X_j - \xi(t), t)e^{i\theta(x, t)} \]

\[ \times \sum_{m=-\infty}^{\infty} J_m \left( \frac{eE}{\hbar} X_j \left( \frac{1}{\omega} + \frac{\omega}{\omega^2 - \omega' _2} \right) \right) e^{im\omega t} \]

where the Jacobi-Anger expansion is used: \( e^{ix}\sin \phi = \sum_m J_m(z)e^{im\phi} \). Note that \( \theta(x, t) \) is independent of the center coordinate \( X_j \). Following Tien and Gordon [5], Eq. (4) can be viewed in such a way that the wavefunction contains components which have energies, \( \epsilon_n, \epsilon_n + \hbar\omega, \epsilon_n + 2\hbar\omega, \ldots , \epsilon_n \pm m\hbar\omega \), etc., and each component is weighted by \( J_m \). Therefore, the density of states should also contain contributions from each energy component:

\[ \rho'(\epsilon) = \sum_m \rho(\epsilon + m\hbar\omega)J_m^2 \left( \frac{eE}{\hbar} X_j \left( \frac{1}{\omega} + \frac{\omega}{\omega^2 - \omega' _2} \right) \right) \]

where \( \rho(\epsilon) \) is the density of states without radiation. Note that, though \( \theta(x, t) \) is dependent on time (and position), it does not affect any of the energy components because it does not depend on the center coordinates \( X_j \). So one can get rid of \( \theta(x, t) \) via a proper gauge transformation without affecting physical consequences.

Now, let us discuss the effect of a modified density of states in the context of transport which is formulated via the center-migration theory [6-8]. A schematic diagram is shown in Fig. 1. In the Landau gauge with \( A = B(0, x) \), electrons travel freely along the \( y \) direction, while confined in the \( x \) direction, which causes the Hall effect. Longitudinal resistance (in the \( x \) direction) appears when electrons are scattered by impurities so that there is a momentum transfer \( \Delta k \). This momentum transfer is equivalent to a spatial jump in the center coordinates along the \( x \) direction by the distance of \( |\Delta X_j| = |\Delta k|^2 \). The probability of jumps in the center coordinate is proportional to the overlap between the wavefunctions of initial and final center coordinates:

\[ P(\Delta X_j) = \left| \int_{-\infty}^{\infty} dx \chi_n(x - X_j)\chi_n'(x - X'_j) \right|^2 . \]

The Landau level index of the highest occupied level is sufficiently large for low magnetic fields: \( n \approx n' \approx 50 \) for systems studied in experiments. So, \( \chi_n(x) \) is sharply peaked near the edges of the wave packet around \( x = \pm \sqrt{2n + 1}B \). Therefore, \( P(\Delta X_j) \) is also sharply peaked around \( \Delta X_j \approx 2\sqrt{2n + 1}B \). Assuming that the transport occurs predominantly through this channel of momentum transfer \( |\Delta k| = |\Delta X_j|^2 \approx 2\sqrt{2n + 1}B \), we
can write the following formula for current in the presence of an external bias $eV$:

$$I = eD \sum_{m=-\infty}^{\infty} J_m^2(\alpha) \times \int d\epsilon \rho(\epsilon)\rho(\epsilon + mh\omega + eV)[f(\epsilon) - f(\epsilon + mh\omega + eV)]$$

(7)

where $\alpha = \frac{eE}{\pi} \Delta X_j \left(\frac{1}{2} + \frac{\omega}{\omega_c}\right)$, $f(\epsilon)$ is the usual Fermi-Dirac distribution function and $D$ is the transmission coefficient (which is proportional the overlap in Eq. (6)) between two eigenstates separated by the distance $\Delta X_j \approx 2\sqrt{2m + \Pi T}$. Then, the zero-bias conductivity $\sigma(\epsilon = \frac{dI}{dV}|V=0)$ can be written as follows:

$$\sigma = e^2 D \sum_{m=-\infty}^{\infty} J_m^2(\alpha) \times \int d\epsilon \{ - f'(\epsilon + mh\omega)\rho(\epsilon)\rho(\epsilon + mh\omega) + [f(\epsilon) - f(\epsilon + mh\omega)]\rho(\epsilon)\rho'(\epsilon + mh\omega) \}$$

(8)

Eq. (8) is similar to the formula obtained by Shi and Xie. However, it is important to note that there is a significant difference in the argument of Bessel function. The argument shows a resonance behavior which was not obtained in previous perturbative theories. Consequences of this resonant behavior will be discussed in detail later.

Until now, the effect of impurities has been included only in a way that they cause the scattering between states with different center coordinates. Another important effect of impurities is the broadening of Landau levels so that the density of states is not a series of delta functions. Following Ando et al. (6), we assume the following form for the density of states (in the absence of radiation):

$$\rho(\epsilon) = \rho_0 \left(1 - \lambda \cos \frac{2\pi \epsilon}{\hbar \omega_c}\right)$$

(9)

where $\lambda = 2e^{-\pi/\omega_c \tau f}$ and $\tau_f$ is the relaxation time.

It is now straightforward to compute the conductivity by plugging the density of states in Eq. (9) into Eq. (8). As it can be shown by full integration, the photon-assisted conductivity basically gives rise to a slow oscillation as a function of inverse of magnetic field: $\propto \sin(2\pi m \omega / \omega_c)$ with $m$ an integer, while the usual Shubnikov-de Haas oscillation is a fast oscillation $\propto \sin(2\pi \mu / \hbar \omega_c)$ with $\mu$ the chemical potential. One obtains a separation between the fast Shubnikov-de Haas oscillations and the slow oscillations in radiation-induced conductivity when (i) the chemical potential energy is much larger that the temperature: $\mu/k_B T \gg 1$, and (ii) the temperature is not too low, compared to the Landau level spacing: $k_B T / \hbar \omega_c \sim O(1)$. Note that the regime of our interest, $\hbar \omega_c \ll k_B T \ll \mu$, is consistent with the parameter range of experiments.

In this regime, the contribution of photon-assisted conductivity can be written as follows:

$$\frac{\sigma - \sigma_{\text{SdH}}}{\sigma_0} = \frac{\lambda^2}{2} \sum_{m=-\infty}^{\infty} J_m^2(\alpha) \times \left[ \cos \left(2\pi m \frac{\omega}{\omega_c}\right) - 2\pi m \frac{\omega}{\omega_c} \sin \left(2\pi m \frac{\omega}{\omega_c}\right) - 1 \right]$$

(10)

where $\sigma_0 = e^2 D \rho_0$ and $\sigma_{\text{SdH}}$ is the conductivity in the absence of radiation, which is nothing but the conductivity due to the usual Shubnikov-de Haas effect. In other words, $\sigma_{\text{SdH}}$ can be defined as the conductivity when $E = 0$. Also, as discussed previously, $\alpha = \frac{eE}{\pi} \Delta X_j \left(\frac{1}{2} + \frac{\omega}{\omega_c}\right)$ with $\Delta X_j \approx 2\sqrt{2m + \Pi T}$. There are several points to be emphasized.

First, the $m$-th harmonic of photon-assisted conductivity oscillation is weighted by $J_m^2(\alpha)$. When $\alpha \ll 1$, the only important term is the zeroth harmonic which is just a constant and can be absorbed into the conductivity in the absence of radiation. As $\alpha$ increases, weights of other harmonics become important because the sum of all weights is unity, i.e. $\sum_{m=-\infty}^{\infty} J_m^2(\alpha) = 1$, and the weight of zeroth harmonic decreases (though oscillatory). In particular, $\alpha$ is resonantly enhanced when $\omega$ is very close to $\omega_c$. Therefore, it is predicted that, in addition to the first harmonic ($m = 1$) oscillation, there should be prominent oscillations of other harmonics ($m \geq 2$) near $\omega \approx \omega_c$. This additional harmonic effect may have already been observed in experiments (2-10).

Second, Eq. (10) explicitly shows that the conductivity in the presence of radiation is identical to that without radiation if $\omega / \omega_c$ is an integer. In fact, this property is generally true for any periodic density of states. Therefore, it is a much more robust feature that the position of “zero-resistance state” (8-11).

Third, superficially, temperature dependence is absent in Eq. (10). The temperature dependence of radiation-induced resistance oscillations originates from the $T$ dependence of scattering, either through the broadening of the density of states (via the relaxation time $\tau_f$ in Eq. (9)) or through the $T$ dependence of scattering between states with different center coordinates. Similar arguments were given by Shi and Xie (8), and Durst et al. (11).

Fourth, Eq. (10) describes the longitudinal conductivity $\sigma_{xx}$ which is oscillatory. On the other hand, our theoretical framework naturally predicts that the Hall conductivity $\sigma_{xy}$ in the presence of radiation is identical to that without radiation. Therefore, $\rho_{xy} = \sigma_{xy}/(\sigma_{xx}^2 + \sigma_{xy}^2)$ should be roughly equal to the $\rho_{xy}$ without radiation if $\sigma_{xx} \ll |\sigma_{xy}|$, but may have some signature of oscillatory
behavior for intermediate values of $\sigma_{xx}$ \[\text{12}\]. Also, because $\rho_{xx} = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{xy}^2) \approx \sigma_{xx}/\sigma_{xy}^2$ for $\sigma_{xx} \ll \sigma_{xy}$, the longitudinal resistivity is proportional to the longitudinal conductivity.

Now we make connection between the states at low magnetic fields and near $\nu = 1/2$. In particular, we would like to predict the radiation-induced magnetoresistance oscillation near half filling of the lowest Landau level, based on the composite fermion (CF) theory \[\text{3}\]. The random phase approximation (RPA) of fermionic Chern-Simons gauge theory \[\text{13, 14, 15}\] predicts the following, simple relationship between the resistances of systems of strongly interacting electrons at filling factor $\nu = \nu^*/(2p\nu^* + 1)$ ($p$ is an integer) and that of weakly interacting composite fermions at $\nu^*$:

$$\rho = \rho_{\text{CF}} + \rho_{\text{CS}}$$

where $\rho_{\text{CF}}$ is the resistivity matrix of composite fermions and

$$\rho_{\text{CS}} = \frac{\hbar}{e^2} \begin{bmatrix} 0 & 2p \\ -2p & 0 \end{bmatrix},$$

where $2p$ is the number of flux quanta captured by composite fermions. In particular, the composite fermion theory tells us that, for $p = 1$, there is a mapping between the states in the vicinity of $\nu = 1/2$ and the states at low magnetic fields, i.e. $\nu^* \gg 1$. So, if the longitudinal resistance of weakly interacting fermion systems at low magnetic fields vanishes in the presence of microwave radiation, the composite fermion theory predicts a vanishing longitudinal resistance for systems of strongly interacting electrons near $\nu = 1/2$, provided that the radiation field does not break composite fermions which is a bound state of electron and flux quanta. Now we show that the radiation field does not break the composite fermions (as long as the electric field is coherent and uniform in space).

We begin by noting that the ground state in the presence of long-wavelength radiation is identical to the ground state without radiation (up to the center-of-mass motion) regardless of the inter-electron interaction. To this end, let us write the general Hamiltonian in a perpendicular magnetic field with inter-electron interaction as well as radiation:

$$H = \frac{1}{2M} \left( p - \frac{e}{c} A \right)^2 + \sum_{i<j} V(r_{ij}) + H_{\text{rad}},$$

where the radiation Hamiltonian $H_{\text{rad}} = -F(t) \sum_{i} x_i = -F(t)x_{\text{cm}}$ with $x_{\text{cm}}$ being the center-of-mass coordinate, $V(r_{ij})$ is the interelectron interaction, and $r_{ij}$ is the distance between the $i$-th and $j$-th electron. Since the kinetic energy term in the Hamiltonian is quadratic in $\mathbf{r}$ and $\mathbf{p}$ and is linear in $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, the center-of-mass and relative coordinates decouple, which is known as the generalized Kohn theorem \[\text{17}\]. Consequently, the interaction with radiation in the long-wavelength limit can not change the state of relative motion since $H_{\text{rad}}$ depends only on the center-of-mass coordinate.

A more explicit proof in the lowest Landau level can be obtained as follows. As shown previously, the Landau level index is invariant in the presence of radiation so that electrons stay in the exact states in Eq. \[\text{4}\] with the same Landau level index $n$ (and also with the same center coordinate index $X_j$, though the center position becomes $X_j + \xi(t)$). So, there is a one-to-one correspondence between the basis states of the lowest Landau level with and without radiation. More importantly, interaction with radiation does not change the shape or the relative distance between the basis states. Therefore, the Coulomb matrix elements do not change, nor does the relative-coordinate part of the ground state which originates from Coulomb correlation.

Now, it is important to note that Coulomb correlation in the lowest Landau level manifests itself as flux attachment because flux quanta are actually zeroes of the many-body wavefunction, and electrons try to minimize their Coulomb energy cost by forming a bound state with zeroes. Also, note that, after the flux attachment, electron systems near $\nu = 1/2$ are mapped onto weakly interacting systems of composite fermions which form effective Landau levels with low effective magnetic field $B^* = B - B_{1/2}$ where $B_{1/2}$ is the magnetic field at $\nu = 1/2$. Therefore, our theory of photon-assisted transport at low fields can be directly applied to the CF states near $\nu = 1/2$ \[\text{14}\].

We may also establish the period of radiation-induced magnetoresistance oscillation near $\nu = 1/2$. Remember that, at low fields, the magnetoresistance oscillation is periodic as a function of $\omega/\omega_c$. In the case of composite fermion systems, the cyclotron frequency should be modified to the effective cyclotron frequency of composite fermions:

$$\omega^*_c = \frac{eB^*}{m^*c} = \frac{e}{m^*c}(B - B_{1/2}) \quad (14)$$

where the CF mass $m^*$ is given as follows:

$$m^* = C\frac{\varepsilon \hbar^2}{e^2} \sqrt{\frac{eB}{\hbar c}} = C\frac{\varepsilon \hbar^2}{e^2} \frac{1}{l_B} \quad (15)$$

where $\varepsilon$ is the dielectric constant and $C \approx 3.0$ \[\text{12, 18, 19}\] is a dimensionless constant. For a typical value of magnetic field $B \approx 10$ T for $\nu = 1/2$, $m^*$ is roughly 3 to 4 times larger than the band mass of electron in GaAs \[\text{20}\]. Therefore, overall, the radiation-induced magnetoresistance oscillation near $\nu = 1/2$ will have 3 to 4 times shorter period as a function of $1/(B - B_{1/2})$ for a given value of radiation frequency $\omega$. However, it is important to note that the CF mass itself depends weakly on the magnetic field so that there will be a small correction to the period of oscillation: $\omega/\omega_c^* \approx \frac{B_{1/2} + B^*}{B^*}$. Finally, note that the measurement of radiation-induced magne-
toresistance can be regarded as a way of measuring the effective mass of composite fermion.

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