Hawking Radiation of Dirac Particles in an Arbitrarily Accelerating Kinnersley Black Hole

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Quantum thermal effect of Dirac particles in an arbitrarily accelerating Kinnersley black hole is investigated by using the method of generalized tortoise coordinate transformation. Both the location and the temperature of the event horizon depend on the advanced time and the angles. The Hawking thermal radiation spectrum of Dirac particles contains a new term which represents the interaction between particles with spin and black holes with acceleration. This spin-acceleration coupling effect is absent from the thermal radiation spectrum of scalar particles.

PACS numbers: 04.70.Dy, 97.60.Lf

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I. INTRODUCTION

It has been more than a quarter century since Hawking’s remarkable discovery that a black hole is not completely black but can emit radiation from its event horizon. An important subject in black hole physics is to reveal the thermal properties of various black holes. Last few decades has witnessed much progress on investigating the thermal radiation of scalar fields or Dirac particles in some stationary axisymmetry black holes. Nevertheless most of these efforts (see [2–4], for examples) were concentrated on studying the thermal properties of static or stationary black holes. Because a realistic black hole in astrophysics can radiate or absorb matter surrounding it, it is nonstationary and evolves in the time. Thus the thermal properties of non-stationary spacetimes are more interesting than that of static or stationary black holes, and worth much more studies about them. A well-known method to determine the location and the temperature of the event horizon of a dynamical black hole is to calculate vacuum expectation value of the renormalized energy momentum tensor [5]. But this method is very complicated, it gives only an approximate value of the location and that of the temperature. Thus it is of limited use and meets great difficulties in many cases.

To study the Hawking evaporation of the non-stationary black holes, Zhao and Dai [6] suggested a new method of the generalized tortoise coordinate transformation (GTCT) which can give simultaneously the exact values of the location and the temperature of the event horizon of a non-stationary black hole. By generalizing the common tortoise-type coordinate \( r_* = r + \frac{1}{2\kappa} \ln(r - r_H) \) in a static or stationary spacetime [2,7] (where \( \kappa \) is the surface gravity of the studied event horizon) to a similar form in a non-static or non-stationary spacetime [6] and allowing the location of the event horizon \( r_H \) to be a function of the advanced time \( v = t + r_* \) and/or the angles \( \theta, \varphi \), the GTCT method reduces Klein-Gordon or Dirac equation in a known black hole spacetime to a standard wave equation near the event horizon. For instances, the location of the event horizon is a constant \( r_H = 2M \) in the Schwarzschild black hole while it is a function of the advanced time \( r_H = r_H(v) \) in a Vaidya-type spacetime. This method has been applied to investigate the thermal radiation of scalar particles in the non-uniformly accelerating Kinnersley “photon rocket” solution [8] and in the non-uniformly accelerating Kerr black hole [9] as well.
However, it is very difficult to investigate the quantum thermal effect of Dirac particles in the non-stationary black holes. The difficulty lies in the non-separability of variables for the Chandrasekhar-Dirac equation [10] in the most general spacetimes. The Hawking radiation of Dirac particles has been studied only in some non-static black holes [11]. Recently we [12] have tackled with the evaporation of Dirac particles in a non-stationary axisymmetric black hole. Making use of the GTCT method, we consider the asymptotic behaviors of the first-order and second-order forms of Dirac equation near the event horizon. Using the relations between the first-order derivatives of Dirac spinorial components, we eliminate the crossing-terms of the first-order derivatives in the second-order equations and recast each equation to a standard wave equation near the event horizon. Not only can we re-derive all results obtained by others [13], but also we find that the Fermionic spectrum of Dirac particles displays another new effect dependent on the interaction between the spin of Dirac particles and the angular momentum of black holes. This spin-rotation effect is absent from the Bosonic spectrum of Klein-Gordon particles.

It is natural to see whether or not our method can work effective in other cases. In this paper, we apply it to deal with the Hawking effect of Dirac particles in a non-spherically symmetric and non-stationary Kinnersley black hole, namely Kinnersley “photon rocket” solution [14]. The local event horizon of a dynamical black hole is determined here by the null hypersurface condition. We use different methods to deduce the location of the event horizon and find that they all give the same result. By means of a GTCT, we can also derive the event horizon equation from the limiting form of the first-order Dirac equation near the event horizon. The location and the shape of the Kinnersley black hole is not spherically symmetric [8]. Then we turn to the second-order Dirac equation. With the aid of a GTCT, we adjust the temperature parameter in order that each component of Dirac spinors satisfies a simple wave equation after being taken limits approaching the event horizon. We demonstrate that both the shape and the temperature of the event horizon of Kinnersley black hole depend on not only the time, but also on the angles. The location and the temperature coincide with those obtained by investigating the Hawking effect of Klein-Gordon particles in the accelerating Kinnersley black hole [8]. But the thermal radiation spectrum of Dirac particles shows a new effect dependent on the interaction between the
spin of Dirac particles and the angular acceleration of black holes. This effect displayed in the Fermi-Dirac spectrum is absent in the Bose-Einstein distribution of Klein-Gordon particles. We find that this spin-acceleration coupling effect does not exist in a non-uniformly rectilinearly accelerating Kinnersley black hole.

The paper is outlined as follows: In Sec. 2, we introduce the most general GTCT and derive the equation that determines the location of the event horizon from the null surface condition. Sec. 3 is devoted to discussing the Hawking radiation of Dirac particles in the Kinnersley spacetime. First, we work out the explicit form of Dirac equation in the Newman-Penrose formalism and investigate the asymptotic behavior of the first-order Dirac equation near the event horizon. The equation that determines the location of the event horizon can be inferred from the vanishing determinant of the coefficients of the first-order derivative terms. Next, we use the relations between the first-order derivative terms to eliminate the crossing-term of the first-order derivatives in the second-order Dirac equation near the event horizon, and adjust the parameter $\kappa$ introduced in the GTCT so as to recast each second-order equation into a standard wave equation near the event horizon. In the meantime, we can get an exact expression of the Hawking temperature. Then the second-order equation is manipulated by separation of variables and the thermal radiation spectrum of Dirac particles are obtained by Damour-Ruffini-Sannan’s method. In Sec. 4, we give a brief discussion about the new effect which represents the interaction between the spin of particles with spin-1/2 and the angular acceleration of black holes.

II. KINNERSLEY BLACK HOLE AND ITS EVENT HORIZON

The Kinnersley metric, generally called as the “photon rocket” solution, is interpreted as the external gravitational field of an arbitrary accelerating mass. In the advanced Eddington-Finkelstein coordinate system $[v; r; \theta; \varphi]$, the line element of Kinnersley’s rocket solution reads

$$ds^2 = 2dv(Gdv - dr - fr^2d\theta + gr^2\sin^2\theta d\varphi) - r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

where $2G = 1 - 2M(v)/r - 2ar\cos\theta - r^2W^*W$, $W = f - ig\sin\theta$, $W^* = f + ig\sin\theta$, $f = b\sin\varphi + c\cos\varphi - a\sin\theta$, and $g = (b\cos\varphi - c\sin\varphi)\cot\theta$. The metric is of type-D
under Petrov classification. The arbitrary function $M(v)$ describes the change in the mass of the source as a function of the advanced time; $a = a(v), b = b(v)$ and $c = c(v)$ are acceleration parameters: $a$ is the magnitude of acceleration, $b$ and $c$ are the rates of change of its direction. The co-moving spherical coordinate system is oriented in such a way that the direction $\theta = 0$ to the north pole always coincides with the direction of the acceleration.

The spacetime geometry of an evaporating black hole is characterized by three surfaces: the timelike limit surface, the apparent horizon and the event horizon. According to York [16], the horizons of a dynamical black hole may be obtained to first order in luminosity by note that (i) the apparent horizons are the outermost “trapped” surfaces that the expansion of null-geodesic congruences (or null rays parametrized by $v$) $\vartheta \approx 0$; (ii) the event horizons are null surfaces where the acceleration of null-geodesic congruences $d^2 r/dv^2 \approx 0$, or equivalently they are determined via the Raychadhuri equation by the requirement that $d\vartheta/dv \approx 0$ as they must be strictly null, and (iii) the timelike limit surfaces are defined as surfaces such that $g_{\nu\nu} = 0$.

It is generally accepted that the event horizon is necessarily a null surface and is defined by the outermost locus traced by outgoing photons that can “never” reach arbitrarily large distances [16]. In a nonstationary black hole spacetime, the event horizon should still be a null surface that satisfies the null surface condition: $g^{ij} \partial_i F \partial_j F = 0$, and the event horizon determined by the above null hypersurface condition is, in fact, a local event horizon. We shall adopt this definition and use different methods to derive the equation that determines the location of local event horizon of an arbitrarily accelerating Kinnersley black hole. We find that each method can give the same result consistently.

First, let’s seek the local event horizon of Kinnersley spacetime (1) by using of the null surface condition. From the null surface equation $F(v, r, \theta, \varphi) = 0$, namely $r = r(v, \theta, \varphi)$, one can easily obtain

$$\partial_v F + \partial_r F \partial_v r = 0, \quad \partial_\theta F + \partial_r F \partial_\theta r = 0, \quad \partial_\varphi F + \partial_r F \partial_\varphi r = 0.$$  \hspace{1cm} (2)

Substituting (2) into the explicit expression of the null surface condition $g^{ij} \partial_i F \partial_j F = 0$ in the Kinnersley metric (1)

$$(2G + r^2 W^* W)(\partial_v F)^2 + 2\partial_v F(\partial_v F - f \partial_\theta F + g \partial_\varphi F)$$
\[
\frac{1}{r^2}(\partial_\theta F)^2 + \frac{1}{r^2 \sin^2 \theta} (\partial_\varphi F)^2 = 0, \quad (3)
\]

one gets
\[
2G + r^2 W^* W - 2\partial_v r + 2f \partial_\theta r - 2g \partial_\varphi r + \frac{(\partial_\theta r)^2}{r^2} + \frac{(\partial_\varphi r)^2}{r^2 \sin^2 \theta} = 0.
\]
The local event horizon is the hypersurface \( r = r_H(v, \theta, \varphi) \) that satisfies the above equation or
\[
1 - 2M \frac{2}{r_H} - 2a r_H \cos \theta - 2r_{H,v} + 2f r_{H,\theta} - 2g r_{H,\varphi} + \left( \frac{r_{H,\theta}}{r_H} \right)^2 + \left( \frac{r_{H,\varphi}}{r_H \sin \theta} \right)^2 = 0, \quad (4)
\]
in which \( r_{H,v} = \partial_v r_H, r_{H,\theta} = \partial_\theta r_H \) and \( r_{H,\varphi} = \partial_\varphi r_H \) can be viewed as parameters depicting the evolution of the event horizon.

When \( a = b = c = 0 \) but \( M \neq 0 \), the event horizon of Vaidya black hole is located at \( r_H = 2M/(1 - 2r_{H,v}) \); When \( M = b = c = 0 \) and \( a = \text{const} \), the Rindler event horizon of the uniformly rectilinearly accelerating observer satisfies
\[
1 - 2a r_H \cos \theta - 2a r_{H,\theta} \sin \theta + \frac{r_{H,\theta}^2}{r_H^2} = 0,
\]
it is a paraboloid of revolution \( r_H = 1/a(\cos \theta \pm 1) \).

In the case where \( M(v), a(v), b(v) \) and \( c(v) \) are not equal to zero, Eq. (4) has in general three roots. In this general case, the analysis is a little involved, and will not be discussed here. There should exist two kinds of event horizon: Rindler-type horizon and Schwarzschild-type horizon. All of them depend not only on \( v \), but also on \( \theta, \varphi \). It means that the location of the event horizon and the shape of the black hole change with time. The location of event horizon is in accord with that obtained in the case of discussing about the thermal effect of Klein-Gordon particles in the same spacetime [8].

Next, we adopt the GTCT method to deduce the equation of local event horizon. As the Kinnersley metric is lack of any symmetry, we introduce the most general form of the GTCT [3] as follows
\[
r_* = r + \frac{1}{2\kappa} \ln (r - r_H), \quad v_* = v - v_0,
\]
\[
\theta_* = \theta - \theta_0, \quad \varphi_* = \varphi - \varphi_0, \quad (5)
\]
namely,
\[ dr_* = dr + \frac{1}{2\kappa(r - r_H)}(dr - r_{H,v}dv - r_{H,\theta}d\theta - r_{H,\phi}d\phi), \]
\[ dv_* = dv, \quad d\theta_* = d\theta, \quad d\phi_* = d\phi, \]

where \( r_H = r_H(v, \theta, \phi) \) is the location of the event horizon, \( \kappa \) is an adjustable parameter and is unchanged under tortoise transformation. All parameters \( v_0, \theta_0 \) and \( \varphi_0 \) are arbitrary constants characterizing the initial state of the hole.

Applying the GTCT (5) to the null hypersurface equation (3) and taking the \( r \to r_H(v_0, \theta_0, \varphi_0), \quad v \to v_0, \quad \theta \to \theta_0 \) and \( \varphi \to \varphi_0 \) limits, the event horizon equation is then obtained by letting the term in the bracket before \( \left( \frac{\partial}{\partial r_*} F \right)^2 \) to be zero,

\[ 2G - 2r_{H,v} + r_{H}^2 W^* W + 2f r_{H,\theta} - 2g r_{H,\phi} + \frac{r_{H,\theta}^2}{r_H^2} + \frac{r_{H,\phi}^2}{r_H^2 \sin^2 \theta_0} = 0. \] (6)

Eq. (6) is just the same equation (4) when \( v = v_0, \theta = \theta_0 \) and \( \varphi = \varphi_0 \). Because here we deal with the case of a slow evaporation of black holes, we need later only consider the situation very close to the initial state of the event horizon, namely \( r_H \approx r_H(v_0, \theta_0, \varphi_0) \) when \( v \approx v_0, \theta \approx \theta_0 \) and \( \varphi \approx \varphi_0 \). This assertion is due to that the GTCT approach is a local analysis method, the latter originating from that Hawking radiation comes from vacuum fluctuations near the event horizon.

### III. HAWKING EVAPORATION OF DIRAC PARTICLES

Now we turn to investigating the quantum feature of the Kinnersley spacetime, especially the thermal radiation of electrons, that is, we must derive the Hawking temperature of the event horizon and the thermal radiation spectrum of Dirac particles from the event horizon. To this end, we work out the spinor form of Dirac equation in the Newman-Penrose (NP)

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1Throughout the paper, we make a convention that all coefficients in the front of each derivatives term take values at the event horizon \( r_H = r_H(v_0, \theta_0, \varphi_0) \) when a GTCT is made and followed by taking limits approaching the event horizon, e.g., \( G, f, g \) and \( W \) take their values at \( v = v_0, \theta = \theta_0 \) and \( \varphi = \varphi_0 \).
formalism. We choose a complex orthogonal null-tetrad system in the Kinnersley black hole such that its directional derivatives are

\[
D = -\partial_r, \quad \Delta = \partial_v + G\partial_r, \\
\delta = \frac{1}{\sqrt{2r}} \left( -r^2 W\partial_r + \partial_\theta + \frac{i}{\sin \theta} \partial_\phi \right), \\
\bar{\delta} = \frac{1}{\sqrt{2r}} \left( -r^2 W^*\partial_r + \partial_\theta - \frac{i}{\sin \theta} \partial_\phi \right).
\] (7)

It is not difficult to determine the non-vanishing complex NP spin coefficients in the above null-tetrad as follows 2

\[
\rho = \frac{1}{r}, \quad \mu = \frac{G}{r} + ig \cos \theta, \quad \gamma = (-G_{,r} + ig \cos \theta)/2, \\
\tau = \frac{W}{\sqrt{2}}, \quad \bar{\pi} = -\frac{W^*}{\sqrt{2}}, \quad \alpha = -\frac{\cot \theta}{2\sqrt{2r}} + \frac{W^*}{\sqrt{2}}, \quad \beta = \frac{\cot \theta}{2\sqrt{2r}}, \\
\nu = \frac{1}{\sqrt{2r}} \left[ (2rG - r^2 G_{,r})W^* + r^2 W^* + G_{,\theta} - \frac{ig_{,\phi}}{\sin \theta} \right].
\] (8)

The dynamical behavior of spin-1/2 particles in curved spacetime is described by the four coupled Chandrasekhar-Dirac equations 2 expressed in the following spinor form

\[
(D + \epsilon - \rho)F_1 + (\bar{\delta} + \bar{\pi} - \alpha)F_2 = \frac{i\mu_0}{\sqrt{2}} G_1, \\
(\Delta + \mu - \gamma)F_2 + (\delta + \beta - \tau)F_1 = \frac{i\mu_0}{\sqrt{2}} G_2, \\
(D + \epsilon^* - \rho^*)G_2 - (\delta + \bar{\pi}^* - \alpha^*)G_1 = \frac{i\mu_0}{\sqrt{2}} F_2, \\
(\Delta + \mu^* - \gamma^*)G_1 - (\bar{\delta} + \beta^* - \tau^*)G_2 = \frac{i\mu_0}{\sqrt{2}} F_1,
\] (9)

where \(\mu_0\) is the mass of Dirac particles. Inserting the needed NP spin coefficients and making substitutions \(P_1 = \sqrt{2r}F_1, P_2 = F_2, Q_1 = G_1, Q_2 = \sqrt{2r}G_2\) into Eq. (9), we obtain

\[
-\mathcal{D}_0 P_1 + (\mathcal{L} - r^2 W^* \mathcal{D}_0) P_2 = i\mu_0 r Q_1, \\
2r^2 B_1^I P_2 + (\mathcal{L}^\dagger - r^2 W \mathcal{D}_0) P_1 = i\mu_0 r Q_2, \\
-\mathcal{D}_0 Q_2 - (\mathcal{L}^\dagger - r^2 W \mathcal{D}_2) Q_1 = i\mu_0 r P_2, \\
2r^2 B_1 Q_1 - (\mathcal{L} - r^2 W^* \mathcal{D}_0) Q_2 = i\mu_0 r P_1,
\] (10)

2Here and hereafter, we denote \(G_{,r} = dG/dr\), etc.
in which we have defined operators

\[ B_n = \partial_v + GD_n + (G_r - ig \cos \theta)/2, \quad D_n = \partial_r + n/r, \]
\[ \mathcal{L} = \partial_\theta + \frac{1}{2} \cot \theta - \frac{i}{\sin \theta} \partial_\varphi, \quad \mathcal{L}^\dagger = \partial_\theta + \frac{1}{2} \cot \theta + \frac{i}{\sin \theta} \partial_\varphi. \]

Because the Chandrasekhar-Dirac equation (9) can be satisfied by identifying \( G_1, G_2 \) with \( F_2^*, -F_1^* \), respectively, so one may deal with a pair of components \((P_1, P_2)\) only. Although Eq. (10) can not be decoupled, to deal with the Hawking radiation, one should be concerned about the asymptotic behavior of Eq. (10) near the horizon only. Under the transformation (5), Eq. (10) with respect to \((P_1, P_2)\) can be reduced to the following limiting form near the event horizon

\[
\frac{\partial}{\partial r_*} P_1 + \left( r_{H,\theta} - \frac{i}{\sin \theta_0} r_{H,\varphi} + r_H^2 W^* \right) \frac{\partial}{\partial r_*} P_2 = 0, \\
\left( r_{H,\theta} + \frac{i}{\sin \theta_0} r_{H,\varphi} + r_H^2 W \right) \frac{\partial}{\partial r_*} P_1 - 2r_H^2 \left( G - r_{H,v} \right) \frac{\partial}{\partial r_*} P_2 = 0, \tag{11}
\]

after being taken the \( r \rightarrow r_H(v_0, \theta_0, \varphi_0) \), \( v \rightarrow v_0 \), \( \theta \rightarrow \theta_0 \) and \( \varphi \rightarrow \varphi_0 \) limits. If the derivatives \( \frac{\partial}{\partial r_*} P_1 \) and \( \frac{\partial}{\partial r_*} P_2 \) in Eq. (11) are not equal to zero, the existence condition of non-trivial solutions for \( P_1 \) and \( P_2 \) is that its determinant vanishes, which gives the above-head equation (6). This treatment can be thought of as another derivation of the location of event horizon. It is interesting to note that a similar procedure applying to another pair of components \((Q_1, Q_2)\) will bring about the same result.

To investigate the Hawking radiation of spin-1/2 particles, we need deal with the behavior of the second-order Dirac equations near the event horizon. A direct calculation gives the second-order form of Dirac equations for \((P_1, P_2)\) components as follows

\[
\begin{aligned}
&\left[ 2r^2 B_0 D_0 + (\mathcal{L} - r^2 W^* D_{-1} \mathcal{L}^\dagger - r^2 W D_0) - \mu_0^2 r^2 \right] P_1 \\
&+ 2r^2 \left[ (\mathcal{L} - r^2 W^* B_1) - B_0 (\mathcal{L} - r^2 W^* D_2) \right] P_2 = 0, \tag{12}
\end{aligned}
\]

and

\[
\begin{aligned}
&\left[ 2r^2 D_1 B_1^\dagger + (\mathcal{L}^\dagger - r^2 W D_1) (\mathcal{L}^\dagger - r^2 W^* D_2) - \mu_0^2 r^2 \right] P_2 \\
&+ \left[ D_{-1} (\mathcal{L}^\dagger - r^2 W D_0) - (\mathcal{L}^\dagger - r^2 W D_1) D_0 \right] P_1 = 0. \tag{13}
\end{aligned}
\]
Given the GTCT in Eq. (5) and after some lengthy calculations, the limiting form of Eqs. (12,13), when $r$ approaches $r_H(v_0, \theta_0, \varphi_0)$, $v$ goes to $v_0$, $\theta$ goes to $\theta_0$ and $\varphi$ goes to $\varphi_0$, yields

$$\mathcal{K}P_1 + \left[ -A + r_H^2 G, r + 2r_H^2 G + 2r_H^2 W^* W + 2i g \cos \theta_0 - 2r_H f \cot \theta_0 
+ \cot \theta_0 \left( -r_{H, \theta} + \frac{ir_{H, \varphi}}{\sin \theta_0} \right) \frac{\partial}{\partial r_*} P_1 + 2r_H^2 G, \theta 
- \frac{i}{\sin \theta_0} G, r + r_H^2 W^* + W^* \left( 2r_H G - r_H^2 G, r \right) \frac{\partial}{\partial r_*} P_2 = 0, \right.$$ (14)

and

$$\mathcal{K}P_2 + \left[ -A + 3r_H^2 G, r + r_H (6G - 4r_H, v) + 6r_H^3 W^* W - 2r_H^2 f \cot \theta_0 
+ (4r_H f - \cot \theta_0) r_{H, \theta} - \left( 4g r_H + i \cot \theta_0 \right) \frac{r_{H, \varphi}}{\sin \theta_0} 
- \left( r_{H, \theta} + \frac{r_{H, \varphi}}{\sin^2 \theta_0} \right) \frac{\partial}{\partial r_*} P_2 = 0. \right.$$ (15)

In the above, the operator stands for the term involving the second-order derivatives

$$\mathcal{K} = \left[ \frac{A}{2\kappa} + 2r_H^2 (2G - r_H, v) + 2r_H^2 W^* W + 2r_H^2 (f r_H, \theta - g r_H, \varphi) \right] \frac{\partial^2}{\partial r_*^2} + 2r_H^2 \frac{\partial^2}{\partial r_* \partial v_*} - (f r_H^2 + r_{H, \theta}) \frac{\partial^2}{\partial r_* \partial \theta_*} + 2 \left( g r_H^2 - \frac{r_{H, \varphi}^2}{\sin^2 \theta_0} \right) \frac{\partial^2}{\partial r_* \partial \varphi_*}.$$

By the event horizon equation (4) or (8), we know that the coefficient $A$ is an infinite limit of 0/0-type. Using the L’Hôpital rule, we arrive at its obvious result

$$A = \lim_{r \to r_H} \frac{2r^2 (G - r_{H, v}) + r^4 W^* W + 2r^2 (f r_{H, \theta} - g r_{H, \varphi}) + r_{H, \theta}^2 + \frac{r_{H, \varphi}^2}{\sin^2 \theta}}{r - r_H} = 2r_H^2 G, r + 2r_H^3 W^* W - 2r_H^3 (r_{H, \theta} + r_{H, \varphi}^2 / \sin^2 \theta_0). \quad (16)$$

Now let us select the adjustable parameter $\kappa$ in the operator $\mathcal{K}$ such that

$$r_H^2 \equiv \frac{A}{2\kappa} + 2r_H^2 (2G - r_H, v) + 2r_H^2 W^* W + 2r_H^2 (f r_{H, \theta} - g r_{H, \varphi}),$$

which means the surface gravity of the horizon is

$$\kappa = \frac{r_H^2 G, r + r_H^3 W^* W - r_H^3 (r_{H, \theta} + r_{H, \varphi}^2 / \sin^2 \theta_0)}{r_H^2 (1 - 2G) - r_H^4 W^* W + r_{H, \theta}^2 + r_{H, \varphi}^2 / \sin^2 \theta_0}. \quad (17)$$

With such a parameter adjustment and using relations (11), Eqs. (14,15) can be recast into the following standard wave equation near the horizon in an united form.
\[
\left[ \frac{\partial^2}{\partial r_*^2} + 2 \frac{\partial}{\partial r_* \partial v_*} - 2C_3 \frac{\partial^2}{\partial r_*^2 \partial \theta_*} + 2\Omega \frac{\partial^2}{\partial r_* \partial \varphi_*} + 2(C_2 + iC_1) \frac{\partial}{\partial r_*} \right] \Psi = 0, \tag{18}
\]

where \( \Omega, C_3, C_2, C_1 \) shall be regarded as finite real constants. The physical meaning of \( \Omega \) is that it can be interpreted as the angular velocity of the black hole due to deformation. For completeness, they are listed as follows

\[
\Omega = g - \frac{r_{H,\varphi}}{r_H^2 \sin^2 \theta_0}, \quad C_3 = f + \frac{r_{H,\theta}}{r_H^2},
\]

while both \( C_1 \) and \( C_2 \) are real,

\[
2(C_2 + iC_1) = \frac{2G}{r_H} - G_{,r} + 2i g \cos \theta_0 - \frac{\cot \theta_0}{r_H^2} \left( r_{H,\theta} - \frac{ir_{H,\varphi}}{\sin \theta_0} \right)
\]

\[
- 2 f \cot \theta_0 + \frac{2}{r_H^2} \left( r_{H,\theta}^2 + \frac{r_{H,\varphi}^2}{\sin^2 \theta_0} \right) - \frac{1}{r_H^2} \left( r_{H,\theta\theta} + \frac{r_{H,\varphi\varphi}}{\sin^2 \theta_0} \right)
\]

\[
+ \frac{C_3 - i\Omega \sin \theta_0}{(G - r_{H,v}) r_H^2} \left[ r_{H}^2 W_{,v}^* + W^* (2r_H G^* - r_{H,\theta} G_{,r}) + G_{,\theta} - \frac{i}{\sin \theta_0} G_{,\varphi} \right]
\]

for \( \Psi = P_1 \), and

\[
2(C_2 + iC_1) = \frac{2G}{r_H} + G_{,r} + 2r_H W^* W - 2f \cot \theta_0
\]

\[
- \frac{\cot \theta_0}{r_H^2} \left( r_{H,\theta} + \frac{ir_{H,\varphi}}{\sin \theta_0} \right) - \frac{1}{r_H^2} \left( r_{H,\theta\theta} + \frac{r_{H,\varphi\varphi}}{\sin^2 \theta_0} \right)
\]

for \( \Psi = P_2 \). The expression in Eq. (19) is very complicated, but one can notice that the last term in the square bracket is proportional to \( \nu(r_H) \) (i.e. the value of the spin coefficient \( \nu \) at the event horizon \( r_H \)).

Because all coefficients in Eq. (18) can be viewed as constants approximately, the wave equation can be manipulated like an ordinary differential one. Now separating variables as \( \Psi = R(r_*) \Theta(\theta_*) \Phi(\varphi_*) e^{-i\omega r_*} \) and substituting it into Eq. (18), one gets

\[
\Theta' = \lambda \Theta, \quad \Phi' = (\sigma + im) \Phi,
\]

\[
R'' = 2i(\omega - m\Omega - C_1 + iC_0) R', \tag{21}
\]

where \( C_0 = C_2 - \lambda C_3 + \sigma \Omega, \) \( \lambda \) is a real constant introduced in the separation variables, \( \omega \) the energy of electrons, \( m \) the quantum number of its azimuthal angular momentum. The solutions are

\[
\Theta = e^{\lambda \theta_*}, \quad \Phi = e^{(\sigma + im) \varphi_*},
\]

\[
R = R_1 e^{2i(\omega - m\Omega - C_1) r_* - 2C_0 r_*} + R_0. \tag{22}
\]
The ingoing wave and the outgoing wave to Eq. (18) are, respectively,

\[
\Psi_{\text{in}} = e^{-i\omega v_+ + (\sigma + im)\varphi_+ + \lambda \theta_+},
\]

\[
\Psi_{\text{out}} = \Psi_{\text{in}}e^{2i(\omega - m\Omega - C_1)r_+ - 2C_0r_+}, \quad (r > r_H).
\] (23)

Near the event horizon, we have \( r_+ \sim (2\kappa)^{-1}\ln(r - r_H) \). Clearly, the outgoing wave \( \Psi_{\text{out}}(r > r_H) \) has a logarithm singular and is not analytic at the event horizon \( r = r_H \), but can be analytically extended from the outside of the hole into the inside of the hole through the lower complex \( r \)-plane (i.e., \( (r - r_H) \rightarrow (r_H - r)e^{-i\pi} \)) to

\[
\tilde{\Psi}_{\text{out}} = \Psi_{\text{in}}e^{2i(\omega - m\Omega - C_1)r_+ - 2C_0r_+e^{ipC_0/\kappa}e^{\pi(\omega - m\Omega - C_1)/\kappa}}, \quad (r < r_H).
\] (24)

According to the method suggested by Damour and Ruffini [2] and developed by Sannan [7], the relative scattering probability of the outgoing wave at the horizon is easily obtained

\[
\left|\frac{\Psi_{\text{out}}}{\tilde{\Psi}_{\text{out}}}\right|^2 = e^{-2\pi(\omega - m\Omega - C_1)/\kappa}. \quad (25)
\]

The thermal radiation spectrum of Dirac particles from the event horizon of the hole is given by the Fermionic distribution

\[
\langle N(\omega) \rangle = \frac{\Gamma(\omega)}{e^{(\omega - m\Omega - C_1)/T_H + 1}}, \quad (26)
\]

in which \( \Gamma(\omega) \) is the barrier factor of certain modes, and the Hawking temperature \( T_H = \kappa/(2\pi) \) is obviously expressed as

\[
T_H = \frac{1}{4\pi r_H} \times \frac{Mr_H - r_H^2 a \cos \theta_0 - r_H^2 \theta - r_H^2 \varphi}{Mr_H + r_H^2 a \cos \theta_0 + 2^{-1}(r_H^2 \theta + r_H^2 \varphi)/\sin^2 \theta_0}. \quad (27)
\]

It follows that the temperature depends not only on the time \( v \), but also on the angles \( \theta \) and \( \varphi \) because it is determined by the surface gravity \( \kappa \), a function of \( v, \theta \), and \( \varphi \). The temperature is consistent with that derived from the investigation of the thermal radiation of Klein-Gordon particles in the Kinnersley black hole [8].

**IV. SPIN-ACCELERATION COUPLING EFFECT**

There are two parts in the thermal radiation spectrum (26), one is the rotational energy \( m\Omega \); another is \( C_1 \) due to the coupling between the spin of electrons and the angular momentum of the black hole. Comparing the thermal spectrum of spin-1/2 particles with that
of scalar particles [3], we find that an extra term $C_1$ appears in the former and is absent in the latter. From its explicit expression,

$$C_1 = \frac{C_3 \sin \theta_0}{2(G - r_{H,v})} \left[ r_{H,v}^2 g + (2r_H G - r_{H,v}^2 G_r) g - \frac{G_{,\varphi}}{\sin^2 \theta_0} \right]$$

$$- \frac{\Omega \sin \theta_0}{2(G - r_{H,v})} \left[ r_{H,v}^2 f + (2r_H G - r_{H,v}^2 G_r) f + G_{,\varphi} \right]$$

$$+ g \cos \theta_0 + \frac{r_{H,\varphi} \cos \theta_0}{2r_H^2 \sin^2 \theta_0}, \quad (\Psi = P_1)$$

$$C_1 = -\frac{r_{H,\varphi} \cos \theta_0}{2r_H^2 \sin^2 \theta_0}, \quad (\Psi = P_2) \quad (28)$$

one can find that $C_1 = 0$ for $\Psi = P_1, P_2$ in the non-uniformly rectilinearly accelerating black hole ($b = c = 0$ and $r_{H,\varphi} = 0$). Further more, if we only consider the second term in Eq. (28), then we can rewrite the “spin-dependent” term as

$$\omega_s \sim \frac{s r_{H,\varphi} \cos \theta_0}{r_H^2 \sin^2 \theta_0}, \quad (s = 1/2, \quad \Psi = P_1; \quad s = -1/2, \quad \Psi = P_2) \quad (30)$$

for different spin states $s = \pm 1/2$ respectively. Here we are interested especially in this term because it is obviously related to the spin of electrons in different helicity states. The factor $r_{H,\varphi}$ describes the deformation of black hole during its evolution, while the factor $\cos \theta_0$ comes from the scalar product between the spin vector of electrons and the “deformation angular momentum” $r_{H,\varphi}/(r_H^2 \sin^2 \theta_0)$ of the black hole. One can notice that this “deformation angular momentum” is also contained in the expression of $\Omega$. Thus this new term represents the spin-acceleration coupling effect which can be interpreted as the interaction between the angular variation of the black hole and the spin of the particles.

One can observe that $\omega_s$ vanishes at the equator $\theta = \pi/2$ and diverges in $s \theta^{-2}$ near the north pole $\theta \approx 0$ and in $-s(\pi - \theta)^{-2}$ near the south pole $\theta \approx \pi$. In general, the dependence of $C_1$ upon the mass $M$ and the acceleration parameters $(a, b, c)$ is indirect and non-trivial because they are involved in the expression of $r_H$. To understand this point better, we consider a particular case where we take $M, a, b$ and $c$ as constants and set $\theta = \theta_0 = \pi/2$. In this case, we have $2G = 1 - 2M/r - r^2 f^2$, $f = b \sin \varphi + c \cos \varphi - a$ and $g = 0$ as well as $r_{H,v} = r_{H,\varphi} = 0$. Thus the event horizon equation becomes

$$r_H^2 - 2Mr_H + r_{H,\varphi}^2 = 0, \quad (31)$$
and it has solutions \( r_H = M(1 \pm \cos \varphi); M(1 \pm \sin \varphi) \), which demonstrate that the event horizon is a triangle function of \( \varphi \). Without loss of generality, let’s take one solution as
\[ r_H = M(1 + \cos \varphi). \]
Then the Hawking temperature is
\[ T_H = \frac{\cos \varphi}{2\pi M(1 + \cos \varphi)(3 - \cos \varphi)}. \]
Further, we have \( C_3 = 0 \) and \( \Omega = \sin \varphi / M(1 + \cos \varphi)^2 \). For \( \Psi = P_2 \), \( C_1 \) vanishes; while for \( \Psi = P_1 \) due to \( f_{,v} = G_{,\theta} = 0 \), it becomes
\[ C_1 = \frac{\sin \varphi(\cos \varphi - 2)(b \sin \varphi + c \cos \varphi - a)}{\sin^2 \varphi + M^2(1 + \cos \varphi)^4(b \sin \varphi + c \cos \varphi - a)^2}. \]
Obviously \( C_1 \) is linearly dependent on \( a, b \) and \( c \) if \( M = 0 \). We note that \( r_H = 2M \), \( T_H = 1/(8\pi M) \) and \( \Omega = C_1 = 0 \) at \( \varphi = \varphi_0 = 0 \).

In the general case where \( M, a, b \) and \( c \) are arbitrary functions of the advance time \( v \), to analysis \( C_1, \Omega \) and \( T_H \) is, however, very complicated. It needs us first to determine the explicit expression of \( r_H \) in terms of these parameters, but this topic apparently exceeds the context of this paper. We wish to discuss them in other circumstances.

## V. CONCLUSIONS

Equations (3) and (27) give the location and the temperature of the event horizon of the Kinnersley black hole, which depend not only on the advanced time \( v \) but also on the angles \( \theta, \varphi \). Eq.(26) shows the thermal radiation spectrum of Dirac particles in the arbitrarily accelerating Kinnersley spacetime, in which an extra term \( C_1 \) appears. We find that \( C_1 \) vanishes when \( b = c = 0 \) and \( r_{H,\varphi} = 0 \). This means that this term does not exist in the rectilinearly accelerating Kinnersley black hole. We contend that this new effect probably arise from the interaction between the spin of Dirac particles and the acceleration of the evaporating black hole. Besides, we notice that this spin-acceleration coupling effect appearing in the Fermionic thermal spectrum of Dirac particles is not shown in the Bosonic spectrum of scalar particles.

In summary, we have studied the Hawking radiation of Dirac particles in an arbitrarily accelerating Kinnersley black hole whose mass changes with time. The location and the temperature of the event horizon of the accelerating Kinnersley black hole are just the same
as those obtained in the discussion on thermal radiation of Klein-Gordon particles in the same spacetime. But the thermal spectrum of Fermi-Dirac distribution of particles with spin-1/2 displays an extra interaction effect between the spin of Dirac particles and the angular acceleration of black holes. The character of this term is its obvious dependence on the different spin states. This spin-acceleration coupling effect is absent from the thermal radiation spectrum of Klein-Gordon particles. In addition, our discussion here is easily generalized to the most general case of a non-uniformly accelerating Kinnersley black hole with electric charge $Q(v)$, magnetic charge $P(v)$ and cosmological constant $\Lambda$ ($2G = 1 - 2M(v)/r + (Q^2 + P^2)/r^2 - 4a \cos \theta (Q^2 + P^2)/r - 2ar \cos \theta - r^2 (f^2 + g^2 \sin^2 \theta) - \Lambda r^4/3$).

ACKNOWLEDGMENT

This work was supported in part by the NSFC in China. We especially thank our referee for his several advice on improving this manuscript and for clarifying the incorrect statement on the observational possibility of the spin-acceleration coupling effect in astrophysics.

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