Novel model-based heuristics for energy optimal motion planning of an autonomous vehicle using A*

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Abstract: Predictive motion planning is the key to achieve energy-efficient driving, which is one of the main benefits of automated driving. Researchers have been studying the planning of velocity trajectories, a simpler form of motion planning, for over a decade now and many different methods are available. Dynamic programming has shown to be the most common choice due to its numerical background and ability to include nonlinear constraints and models. Although planning of optimal trajectory is done in a systematic way, dynamic programming doesn’t use any knowledge about the considered problem to guide the exploration and therefore explores all possible trajectories.

A* is an algorithm which enables using knowledge about the problem to guide the exploration to the most promising solutions first. Knowledge has to be represented in a form of a heuristic function, which gives an optimistic estimate of cost for transitioning between two states, which is not a straightforward task. This paper presents a novel heuristics incorporating air drag and auxiliary power as well as operational costs of the vehicle, besides kinetic and potential energy and rolling resistance known in the literature. Furthermore, optimal cruising velocity, which depends on vehicle aerodynamic properties and auxiliary power, is derived. Results are compared for different variants of heuristic functions and dynamic programming as well.

Keywords: energy-efficient driving, eco-driving, optimal velocity trajectory, motion planning, dynamic programming, A*, optimal cruising velocity, operational costs, automated driving, autonomous vehicles

1. INTRODUCTION

Increasing environmental awareness, strict regulations on greenhouse gas emissions and constant desire to increase the range of electric vehicles as well as the big economic benefits motivate a lot of research in the field of energy-efficient driving. So far, many different approaches addressing this topic exist. Some approaches are related to the vehicle design optimization, some to using alternative propulsion systems and some to the driving behavior optimization. In (Bingham, et al., 2012) authors present a study which shows that the driving behavior has a rather big influence on energy consumption. It is shown that energy consumption may vary in a range of approx. 30% depending on driving behavior. Knowledge about the upcoming driving route, the road slope profile and the ability to control the vehicle’s propulsion enables the optimization of the velocity trajectory of the vehicle with respect to the energy consumption. Discrete dynamic programming (DP) has been used for over a decade now for finding energy efficient velocity trajectories (E. Hellström, 2005). (E. Hellström, 2010). Because of high computational requirements in (Ozatay, et al., 2014) authors used a cloud to compute energy efficient velocity trajectories. By using model predictive control (MPC) to drive vehicles on free roads with up and down slopes, notable fuel savings are shown in (Kamal, et al., 2011). MPC was also used to control a hybrid vehicle driving over a hill and performing vehicle following in (Vajedi & L.Azad, 2016). In (Ajanović, et al., 2018) authors presented use of dynamic programming in an MPC-like framework. In (Sciarretta, et al., 2015) an overview of the existing approaches and the current state of the art can be found.

One of the first uses of A* algorithm for autonomous vehicle motion planning is presented in (Fraichard, 1993), the result of a Prometheus project. The author used A* to find shortest time motion in the presence of dynamic obstacles and introduced state-time space for dealing with dynamic
obstacles. A* was also used in DARPA Urban challenge by many teams, of special interest was the application from Stanford where authors introduce Hybrid-State A* search to deal with rounding error for finding shortest path (Dolgov, et al., 2008). Another recent application of A* for planning safe trajectories is shown in (Boroujeni, et al., 2017).

An application of A* for finding energy optimal velocity trajectory for an electric bicycle was presented in (Grossolleil & Mezel, 2012). Authors used kinetic and potential energy as well as a rolling resistance, but for estimation of air drag resistance, authors used an upper instead of a lower bound. The results were suboptimal with a difference of around 1.2% from another optimal control strategy. In (Chevrant-Breton, et al., 2014) authors introduced air drag and a time proportional cost without interdependence. For air drag, they used tunable minimum velocity, and for the time-proportional cost, they calculated a minimum time based on the maximum velocity. This approach leads to not so precise estimation. One case of using A* is also shown in (Flehmig, et al., 2015) but authors didn’t reveal the calculation of heuristics. From results, it is clear that some of the heuristics are giving a suboptimal solution.

In this work, novel heuristic functions for A* based vehicle velocity trajectory planning are introduced. The optimal cruising velocity, which minimizes air drag and time-proportional cost influence, is defined. The resulting, optimal cruising velocity is then used as a basis for the introduced heuristic functions.

2. PROBLEM DEFINITION

The problem of finding an energy efficient vehicle velocity represents an optimal control problem. This problem has differential constraints defined by vehicle dynamic model, state-dependent constraints which have to be satisfied and a cost function which is minimized.

2.1 System model

Since low model complexity is crucial for efficient optimization, the vehicle is modeled as a particle-mass. The vehicle is represented by two states: s represents traveled longitudinal distance and v represents longitudinal velocity of the vehicle in the lane and vehicle dynamics as in (1)-(3).

\[ \dot{s}(t) = v(t) \] (1)

\[ \dot{v}(t) = \frac{F_{m}(t) - F_{r}(t)}{m} \] (2)

\[ F_{r}(t) = \frac{1}{2} \rho c_{d} A_{r} v(t)^{2} + c_{m} mg \cos(\alpha(s(t))) + \cdots + mg \sin(\alpha(s(t))) \] (3)

The propulsion force of the electric motor is denoted by \( F_{m} \) and resistive forces by \( F_{r} \). It can be seen from (3) that resistive forces include air drag resistance, roll resistive forces and road inclination related gravity resistive forces. The propulsion element, electric motor, with inner torque \( T_{m} \) is modeled statically by:

\[ F_{m}(t) = k \frac{T_{m}(t) \eta \omega(t)}{r_{w}}, \text{ with } k = 2 r_{w} \pi \omega(t) \] (4)

(4) includes an efficiency coefficient \( \eta \), a combined transmission ratio of the powertrain \( k \), the radius of the wheels \( r_{w} \) and the rotational speed of the motor \( \omega_{m} \).

2.2 Cost function

The cost function has to reflect the initial requirement of minimal energy consumption. When considering propulsion power only, energy-efficient behavior results in zero velocity trajectory. To avoid this, some authors introduced a term to the cost function to weight the traveling time (Sciarretta, et al., October 2015). The weighting coefficient is then tuned such that the travel time is comparable to times achieved by human drivers. However, energy consumption includes energy used for auxiliaries \( P_{aux} \) (e.g. infotainment, air conditioning, etc.) which can be approximated with a constant load. This load corresponds to a weighting coefficient presented in other works, but it is determined experimentally and not tuned.

Consequently, the energy used is the integral of the power over time of the trip represented by:

\[ E_{\text{min}} = \int_{0}^{T_{f}} (\omega(t) T_{m}(t) + P_{aux}) dt \] (5)

Instead of using time as a variable for integration, the distance can be used too. This offers some advantages for solving as final time is not known, and final distance is, and road slope appears as a function of distance (Saerens, 2012).

\[ E_{\text{min}} = \int_{0}^{S} \left( \frac{k T_{m}(s)}{2 r_{w} \pi} + \frac{P_{aux}}{v(s)} \right) ds \] (6)

2.3 Complexity

To solve the optimization task numerically, using graph searching methods, state discretization is necessary. By increasing the number of system states considered in the optimization problem, complexity is increased rapidly. Additionally, and even more problematic, the number of possible transitions needed to be evaluated in each step is increased significantly. Bellman called this problem the „curse of dimensionality“ (Bellman, 1954).

3. OPTIMAL MOTION PLANNER

In this work two different motion planners are used, one based on DP as discussed in (Ajanović, et al., 2017) and one using A* which is introduced in this paper.
3.1 Search space

As the optimization task is solved by A*, graph searching method, state discretization is necessary and appropriate graph has to be constructed. The graph is constructed by taking distance and velocity as dimensions. Taking distance is useful when the final time is not fixed but it brings several disadvantages such as when one trajectory goes through zero velocity it is impossible to calculate time spent in that state. By assuming positive velocities, the graph is directed. To consider dynamic obstacles, an additional dimension for time \( t \) should be used (Fraichard, 1993).

\[
\begin{align*}
\text{Fig. 1. Search space constructed as a graph}
\end{align*}
\]

3.2 A* search

A* is one of the earliest yet one of the most used methods for path planning (P.E., et al., 1968). It is based on well-known, Dijkstra's algorithm but uses heuristics to guide exploration of the nodes which according to heuristic lead more quickly to the solution and therefore has better performance. It is an optimal method for finding the optimal path under certain conditions.

Starting from the initial node, which is chosen as the current node, all neighbors are determined and added to the OPEN list. From the OPEN list node, the one with the lowest cost is chosen to be the next current node. This is repeated until the goal node is reached or the whole graph is explored. The cost is calculated using (7) where \( g(n) \) represents the cost to travel from initial node to current node and \( h(n) \) the cost to travel from current node to goal node, estimated using some heuristics.

\[
f(n) = g(n) + h(n) \quad (7)
\]

There can be several issues when implementing A*. To speed up finding the minimum of all open nodes, the open list is usually organized as a priority queue. In this work, it is implemented as a binary heap with a hash table.

3.3 Accumulated cost

Accumulated cost \( g(n) \) is exact cost necessary to come from initial node to that node. It is usually calculated cumulatively. One step transition cost is added to parent node’s accumulated cost. To calculate transition costs the model (1)-(4) is used.

3.4 Heuristic function for cost-to-go estimation

The heuristic function is used to estimate the cost needed to travel from any node (point in state space), defined by initial velocity and position \( (v_i, s_i) \) to the goal node. As it is shown in (Hart, et al., 1968) if the heuristic function is underestimating the cost-to-go, the result of A* search is the optimal trajectory. For the shortest path search, the usual heuristic function is the Euclidian distance. To find the energy optimal velocity trajectory, the heuristic function must always underestimate the energy needed to drive from any node to the goal node.

\[
W = W_{acc} + W_{air} + W_{roll} + W_{a} + E_{aux}
\]

4. HEURISTIC FUNCTIONS

The energy needed to drive from the initial state \( (v_i, s_i) \) to the final state \( (v_f, s_f) \) based on model (1)-(3) can be presented as a sum of the work needed for acceleration, overcoming air drag, roll, road slope and the energy used by the auxiliary.

\[
W_{acc} \geq \Delta E_k = m\frac{v_f^2 - v_i^2}{2} \quad (9)
\]

If there are losses, the needed energy will be higher.

\[
W_{acc} + W_{air} \geq \Delta E_k + \Delta E_p = \Delta E_k + mg(h_f - h_i)
\]

Where  \( h_f \) and  \( h_i \) are elevations of a final and initial point on the road which are determined \( s_i \) and \( s_f \).
4.3 Heuristic function for rolling resistance related work

As this force is modeled as a constant, it does not depend on vehicle driving trajectory. It can be exactly calculated for the specific road segment and included in the heuristics.

4.4 Heuristic function for air-drag and auxiliary power

As opposed to $W_a$, $W_{rol}$ and $W_s$, which can be expressed by the initial the final values only, $W_{air}$ and $E_{aux}$ depend on velocity trajectory between the initial and the final state.

\[
W_{air} + E_{aux} = \int_{0}^{s_f} \left( \frac{1}{2} \rho c_d A_f v(s)^2 + \frac{P_{aux}}{v(s)} \right) ds
\]  
(11)

\[
W_{air} + E_{aux} = \int_{0}^{s_f} F_v(v(s)) ds
\]  
(12)

\[
F_v = \frac{1}{2} \rho c_d A_f v(s)^2 + \frac{P_{aux}}{v(s)}
\]  
(13)

$F_v$ is considered as a virtual force consisting of air drag force and virtual force by $P_{aux}$. $P_{aux}$ can include also some economic costs such as hourly rate (hr) of the operator, vehicle renting, etc. These are then divided by current electricity price (ep) to get the power equivalent.

\[
P_{aux} = P_{aux} + \frac{hr}{ep}
\]  
(14)

The optimal cruising velocity is unique for a vehicle. It depends on vehicle aerodynamic shape, air density and $P_{aux}$ (eventually, motor efficiency can be included). If the vehicle is driving faster than the optimal cruising velocity the air drag work will be dominant, and if driving slower $P_{aux}$ will be dominant.

The minimum virtual force, $F_{v,min}$ can be used to estimate the lower bound of $W_{air} + E_{aux}$, defined with the initial and the final states.

\[
W_{air} + E_{aux} \geq W_A = \frac{s_f}{s_i} \int_{s_i}^{s_f} F_{v,min} ds = F_{v,min} (s_f - s_i)
\]  
(17)

As it was shown in (17), air drag and auxiliary-related work, $W_{air} + E_{aux}$ is always greater or equal to the work when driving with constant velocity $v^*$. This work is calculated as a product of the minimum virtual force $F_{v,min}$ and the distance from the initial state $s_i$ to the final state $s_f$.

![Fig. 3. Different possible trajectories for driving from initial to final state, and lower bound trajectory for $W_A$ calculation.](image)

Similar results are obtained in (Xu, et al., 2017) where authors analyzed a conventional vehicle using approximated engine fuel injection rate map. As in conventional vehicles, auxiliary power comes from the alternator, which represents an additional load on the engine, the results are similar. In this work, a theoretical explanation why this is happening and what is influencing this behavior and how to calculate that velocity is outlined.

4.5 Improved heuristic function for air-drag and auxiliary power

For situations where initial or final velocity is not equal to optimal cruising velocity $v^*$ the air drag influence can be more precisely estimated by including acceleration periods to reach $v^*$. As acceleration is limited, this transition is not instantaneous. It is important to note that acceleration work should not be included; it is included in kinetic and potential energy. Only energy $W_{air} + E_{aux}$ as if the vehicle would move on this trajectory is considered. This is depicted on picture Fig 4.

For visualization purposes, t is used as the x-axis.

While driving with constant acceleration, from $v_1$ to $v_2$, the distance traveled can be calculated as:

\[
\text{distance} = \frac{v_2^2 - v_1^2}{2a}
\]
Fourth and fifth trajectory, estic,   aux 4 trajectory

\[ s = \frac{v_f^2 - v_i^2}{2a} \]  

(18)

For calculating \( s_1 \), general formula (18) is used, with \( v_f = v_i, v_2 = v^* \) and \( a = a_f \). For calculating \( s_2 \), it is with \( v_f = v^*, v_2 = v_i, \) and \( a = a_i \). Depending on the initial and the final velocity accelerations \( a_f \) and \( a_i \) are determined as:

\[
a_f = \begin{cases} 
  v_f > v^*, a_{min} \\
  v_f < v^*, a_{max} \\
  v_f > v^*, a_{max}
\end{cases} \quad \text{and} \quad a_i = \begin{cases} 
  v_i > v^*, a_{min} \\
  v_i < v^*, a_{max} \\
  v_i > v^*, a_{max}
\end{cases}
\]  

(19)

The work \( W_{AI} = W_{air} + E_{aux} \), when driving with constant acceleration, for time \( T \), from velocity \( v_i \) to \( v_f \):

\[ W_{AI} = \int_0^T \left( \frac{1}{2} \rho_c A_f v(t)^3 + P_{aux} \right) dt, \text{with } v(t) = v_i + at; \]  

(20)

\[ T = \frac{v_f - v_i}{a} \]  

(21)

\[ W_{AI} = \frac{1}{2} \rho_c c_d A_f \left( v_i^3 T + \frac{3v_i^2 a T^2}{2} + \frac{a^2 T^4}{4} \right) + P_{aux} T \]  

(22)

Using relations (21) and (22), the cost of driving with constant acceleration between two velocities can be calculated. The total work \( W_{AI} \), in this case, is the sum of two works (acceleration and deceleration) and the constant speed driving with \( v^* \) for distance \( s_f - s_i \) or \( s_i - s_f \).

If \( s_f + s_i > s_f - s_i \), the optimal velocity cannot be reached. There are two subcases in this case. 1) When \( v_i \) and \( v_f \) are on opposite sides of \( v^* \). This is a trivial case; as the final state is not reachable for limited acceleration and therefore the cost is infinite. This case is marked with dashed red line on Fig. 5. 2) When both \( v_i \) and \( v_f \) are either smaller or greater than \( v^* \). There will be no constant speed driving part as \( v^* \) will not be reached, it will accelerate until it reaches \( v_s \) and then decelerate, or the opposite. This is depicted with a solid red line on Fig. 5.

By using the \( s_f + s_i = s_f - s_i \), the velocity at which acceleration switches can be determined.

\[ v_s = \frac{2a d_s s + a_i v_i^2 - a_f v_f^2}{a_f - a_i} \]  

(23)

Fig. 4. Velocity trajectory for \( W_{AI} \) calculation.

\[ W_{AI} \] in the case as in Fig. 5, is the sum of two works (acceleration and deceleration).

4.6 Motor efficiency

Taking the maximum motor efficiency as a constant efficiency will result in underestimating heuristic. The motor efficiency can be applied only to power which flows through the motor. Therefore, it should not be applied to \( P_{aux} \) as this energy doesn’t flow through the motor.

5. SIMULATION RESULTS

Both, DP and A* are used to plan the optimal velocity trajectory for driving on a 1 km long segment of the A9 highway, in the vicinity of Graz, Austria. DP and A* result in the exact same solution, which verifies the optimality of both implementations.

Using different heuristics, the error of the estimation, as well as the number of examined nodes, are analyzed. The results are shown in Table 1. It is important to note that, the number of examined nodes for DP depends on the maximum velocity used; and for A* it does not. On the other hand, DP provides the exact cost-to-go calculation which is used as a reference for calculation of the error for different heuristics.

| DP                  | \( \Delta E \) / \( +W_{real} \) | \( +E_{pot} \) | Eff | \( +W_{AI} \) |
|---------------------|-------------------------------|--------------|-----|--------------|
| Nodes explored      | 30351                         | 26186        | 24304 | 21507        | 11188 | 11012        |
| Average error[\%]  | 0                              | -191.9       | -113.2 | -93.3        | -22   | -19.9        |

The first column contains results when using DP. The kinetic and potential energy, shown in the second column, are used in all heuristics, as results would be suboptimal if they would be omitted. The third column contains results when using roll resistance additionally. The fourth column contains results when efficiency is applied to previously used heuristic as described in 4.6. Fourth and fifth columns contain results when heuristics introduced in 4.4. and 4.5., are used additionally.
As it is shown in Table 1., the average error improved almost 5 times when proposed heuristic \(W_{d1}\) is added, while optimality is preserved. As it can be seen in Fig. 6, the error is always negative, which means that the heuristic always underestimates cost-to-go. A*, in general, examines a smaller number of nodes than DP, and it further decreases as the precision of heuristics increases.

6. CONCLUSIONS AND OUTLOOK

It was shown that proposed heuristics, based on including air drag and auxiliary power, significantly contributed to the improved precision of estimation, which decreased the number of examined nodes. Heuristics could be used for the estimation of a vehicle’s range. In future, machine learning algorithms could be also used to estimate costs more precisely and model-based heuristics proposed here as a correction. As proposed heuristics always underestimates the cost-to-go, it could eliminate overestimating the range. Additionally, the optimal cruising velocity, could be used to advise drivers to drive more efficiently.

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