Active Learning by Greedy Split and Label Exploration

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Abstract

Annotating large unlabeled datasets can be a major bottleneck for machine learning applications. We introduce a scheme for inferring labels of unlabeled data at a fraction of the cost of labeling the entire dataset. We refer to the scheme as greedy split and label exploration (GSAL). GSAL greedily queries an oracle (or human labeler) and partitions a dataset to find data subsets that have mostly the same label. GSAL can then infer labels by majority vote of the known labels in each subset. GSAL makes the decision to split or label from a subset by maximizing a lower bound on the expected number of correctly labeled examples. GSAL improves upon existing hierarchical labeling schemes by using supervised models to partition the data, therefore avoiding reliance on unsupervised clustering methods that may not accurately group data by label. We design GSAL with strategies to avoid bias that could be introduced through this adaptive partitioning. We evaluate GSAL on labeling of three datasets and find that it outperforms existing strategies for adaptive labeling.

1 Introduction

One of the key bottlenecks in modern machine learning is the annotation of datasets. While advances in technology have significantly increased the ability of computers to collect large amounts of unlabeled data, supervised learning requires annotation of this data. For example, in classification tasks, this annotation typically requires human experts to provide labels for the true class of each example. The effort and cost of this labeling process is often prohibitive for many applications. In this paper, we introduce a scheme for acquisition of labeled examples that is able to infer high-quality labels with limited labeling budgets. Our approach builds a hierarchy of data subsets via greedy split and label exploration (GSAL).

Schemes that acquire high-quality labels with lower cost can therefore have tremendous impact on the applicability of machine learning. In settings where there is a limited budget for annotation, tools are needed to identify which examples would be most informative to label. Such tools need to balance introduction of bias and coverage of the example space. For example, in datasets where different classes have nonuniform proportions, it can be important to ensure coverage of underrepresented classes while avoiding wasteful oversampling of overrepresented classes.

Random sampling introduces no bias, but it can provide a poor representation of the data space when the budget is low. Random sampling has difficulty finding examples in rare classes or in sparse regions of the input space. On the other hand, active learning approaches aim to acquire labels for data most useful for training specific model families. This goal can introduce significant bias. For example, methods such as uncertainty sampling prefer labeling points close to a model’s decision
boundary, so the distribution of labeled points will be highly dependent on the model family being trained. The acquired labels may not be as useful for training other model families.

Instead, a better approach is to use the structure of the data to determine which examples to label. Dasgupta and Hsu [5] introduced such an approach called hierarchical sampling for active learning (HSAL). They construct a hierarchical clustering and adaptively determine how to prune the clustering. The idea behind HSAL is that if examples sampled from a cluster exhibit high label uniformity, i.e., are mostly the same label, then it can be inferred that the rest of the cluster is likely to have that majority label. The effectiveness of HSAL thus relies on the quality of the clustering and how well it aligns with the true labels of the classification task. In many settings, a feature-based clustering can have low label uniformity, resulting in negligible gains when using HSAL.

Dasgupta and Hsu [5] use an unsupervised clustering approach to avoid introducing bias into their data partitioning. However, using some acquired data to guide the partitioning can drastically improve the uniformity of the partitions. Our approach is therefore to design a scheme that can use supervised splitting to partition the data but takes actions to remove the bias induced by such splitting. Supervised splitting allows the algorithm to adapt to information it obtains during labeling, leading to higher quality label inferences and data efficiency.

To the best of our knowledge, GSAL is the first work that partitions data guided by a supervised model. Using supervision is ideal for settings in which clustering does not partition the data well by label. One extreme example of this scenario is when data is uniformly distributed throughout most dimensions of the feature space, in which case even the best clustering methods will fail. Data is often collected with many measurements irrelevant to the target concept. Approaches that interactively learn the structure of data can be restricted by the clustering scheme used. Our approach breaks this restriction by partitioning data with supervised models that can find relevant partitionings.

Our contributions are that we introduce our GSAL strategy for actively choosing examples to label. We derive GSAL using probabilistic analysis to ensure that the actions chosen by GSAL greedily improve a lower bound on the expected number of correct labels by the current hierarchy. We define the overall method to use either unsupervised or supervised splits and a procedure to ensure that supervised splits do not introduce harmful bias to our labeling scheme. We evaluate GSAL on labeling of three datasets and test the ability of the inferred labels to train models. Through these experiments, we demonstrate that GSAL is able to acquire good estimates of labels with high data efficiency.

2 Related Work

Active learning is a popular approach for training a learner in the setting where obtaining labeled data is expensive and unlabeled data is abundant. The learner selects data sequentially to be labeled by an oracle, or human labeler. The learner chooses this data by predicting which examples could be most informative. There are several approaches for deciding the “most informative” data to label, including least-confidence [11], least-margin [9], and least-entropy [14]. Other approaches learn by querying informative and representative examples [8]. Unlike traditional active learning in which individual data is chosen to be the most informative, we seek to choose the most informative subset of data, then randomly choose a sample from that subset to label. We also perform an additional step where we infer labels of unlabeled data if we believe that most data in a subset has the same label, based on the true labels obtained by the oracle. While the typical goal in active learning is to label the data most useful for training a specific model, our goal is to output a labeling of the data that could be useful for training any downstream classifier.

Our method for choosing the most informative subset to sample from relies on classical concentration bounds on random variables [12]. These bounds follow the intuition that if we have seen more labeled samples from a subset of data, we have a more confident estimate of the subset’s label distribution. These bounds guarantee limits on the deviation between empirical measurements of statistics and their true expectations. Our analysis mainly uses Hoeffding’s Inequality [7], which is a distribution-free concentration bound—meaning it holds for any underlying distribution.

The work of Dasgupta and Hsu [5] on HSAL is foundational to ours. The key limitation that has prevented this innovative work from having massive impact on the data-hungry state of applied machine learning is that the fixed, unsupervised clusterings that HSAL is restricted to often do not fit the label patterns of data. And when they do not fit, the statistical assumptions that are needed for the
correctness of their approach prevent any adjustment. Our goal in this work is to build a workaround for this issue, allowing the data partitioning scheme to adapt to the labels it observes, catching data inefficiencies in the label inference scheme. Recent work by Tosh and Dasgupta \cite{tosh} aims to mitigate the effects of an incorrect clustering by presenting the oracle with a snapshot of a clustering and obtaining a corrected clustering back from the oracle. However, our approach avoids introducing more complex tasks for the oracle, only asking it to classify individual examples.

The output of our GSAL method is not only a set of inferred labels, but also bounded confidences in those inferences. This type of information makes the labels our method generates amenable to use in weakly supervised learning. Recent methods for weakly supervised learning include approaches that allow annotators to design noisy labeling functions or weak signals and methods that use confidence values to reason about dependencies among the weak signals \cite{1,13}.

| Table 1: Variables and Definitions |
|------------------------------|
| \( \tilde{f}_v \) & Lower bound of correct labels in dataset \( v \) |
| \( B \) & Budget of oracle calls |
| \( \hat{y}_v \) & Majority label of all empirically sampled data in dataset \( v \) |
| \( n_v \) & Sample size of dataset \( v \) |
| \( m_v \) & The number of samples of majority label \( \hat{y} \) in a single dataset \( v \) |
| \( N_v \) & The total number of data points in dataset \( v \), including labeled and unlabeled data |
| \( p_v \) & The true proportion of data points in dataset \( v \) with label \( \hat{y}_v \) |
| \( p'_v \) & The estimated proportion of data points in dataset \( v \) with majority label \( \hat{y}_v \), based on the sampled data points seen so far |
| \( t_v \) & \( p'_v - p_v \), the “buffer size” |
| \( k \) & The number of unique label classes |
| \( a \) & Possible actions, \( a \in \{ \text{split, label} \} \) |
| \( v \) & Node, or subset of the full dataset \( \{x_1, x_2, ... x_N\} \) |
| \( \rho \) & Probability of using labeled data to train a supervised splitting function (as opposed to for calculating bound \( \tilde{f} \)) |
| \( T \) & Set of leaf nodes in tree |

3 Active Learning by Greedy Split and Label Exploration

The algorithm we introduce aims to assign labels to an unlabeled dataset using a limited number of queries to an oracle, which returns the true label to a single data point. The algorithm performs a greedy search of the full dataset for uniform subsets—subsets for which it is believed that most true labels are the same. In the greedy search, we allow two types of operations:

1. query the oracle to label a data point,
2. split a dataset into subsets if we believe that a dataset is not uniform.

Due to the limited budget of oracle queries, we cannot obtain labels for the entire dataset and must guess the true labels for some data. Our algorithm decides to split a dataset or to query the oracle about a single data point based on whichever operation has a higher potential for finding uniform datasets. The output of our algorithm is a label assignment. Each subset has a majority label. We assign this majority label to all unlabeled points within the subset. This label assignment can then be used as input data toward a classification task or as weak supervision.

This algorithm can be viewed as a procedure that grows a tree, where nodes are subsets of data and leaves are assumed to be the most uniform subsets of data. A parent node is “split” when it is believed that a set of child nodes has the potential to induce a better label assignment than the parent node.
Algorithm 1 Pseudocode for Greedy Split and Label Exploration (GSAL)

Require: Dataset $X = \{x_1, x_2, \ldots, x_N\}$, $Y = \emptyset$, budget $B$, training ratio $\rho$, quality threshold $q$, splitting function $\text{split}$, and oracle labeler $\text{oracle}$

1: Initialize a leaf node with all data; $v_r \leftarrow \{x_1, x_2, \ldots, x_N\}$
2: Create a tree with root $v_r$; $T \leftarrow \{v_r\}$
3: while $B > 0$ do
4:   Estimate expected correct labels for all leaves; $S \leftarrow \tilde{f}(v_i)$ for leaf $v_i \in T$
5:   Estimate expected correct labels for all leaves if label; $S_{\text{label}} \leftarrow \tilde{f}_{\text{label}}(v_i)$ for leaf $v_i \in T$
6:   Estimate expected correct labels for all leaves if split; $S_{\text{split}} \leftarrow \tilde{f}_{\text{split}}(v_i)$ for leaf $v_i \in T$
7:   Choose best action $a^*$ and node $v^*$ corresponding to
   $$a^*, v^* \leftarrow \arg \max_{a \in \{\text{split}, \text{label}\}, v \in T} S_{a}[v] - S[v]$$
8:   if $a^* =$ label then
9:      Select $x_{v^*, i}$ randomly from $v^*$
10:     $y_{v^*, i} \leftarrow \text{oracle}(x_{v^*, i})$
11:     Choose with probability $(1 - \rho)$ if adding to the label set.
12:     if label set then
13:        Update majority label $\hat{y}^*$ for node $v^*$ with new label $y_{v^*, i}$
14:        $n_{v^*} \leftarrow$ number of non-training labeled points in $v^*$ of majority label $\hat{y}^*$
15:        Add new label to set of labels; $Y \leftarrow Y \cup \{y_{v^*, i}\}$
16:        $n_{v^*} \leftarrow n_{v^*} + 1$
17:     else
18:        Add $x_{v^*, i}, y_{v^*, i}$ to $v^*$’s isolated training set.
19:     end if
20:   end if
21: else
22:     Split parent node into set of child nodes; $U \leftarrow \text{split}(v^*)$
23:     For each $u \in U$, set $n_u = 0$, $m_u = 0$ and empty isolated training set
24:     Remove parent node from tree; $T \leftarrow T - v^*$
25:     Add child nodes to tree; $T \leftarrow T \cup U$
26: end if
27: end while
28: for $v \in T$ and $p_v > q$ do Set $y_{v, i} = \hat{y}_v$ and add label to return set $Y \leftarrow Y \cup \{y_{v, i}\}$ end for
29: return $Y$

Algorithm 1 contains pseudocode describing the overall algorithm, and we describe its steps in detail in the remainder of this section.

3.1 Algorithm Description

Let $T$ be the set of leaf nodes of our tree. For each node $v$, let $X_v = \{x_{v,1}, x_{v,2}, \ldots, x_{v,N_v}\}$ be the dataset of examples belonging to $v$. Let $Y$ be the set of labels of all examples. At all times during the GSAL algorithm, we assume that each data example is assigned the majority label of its leaf node, i.e., $\hat{y}_v$.

GSAL maintains estimates for the number of examples expected to be correctly labeled according to the current tree structure and majority labels. It uses these estimates to greedily decide whether to split subsets or query for new labels. The estimate for each leaf $v$ depends on the number of its labeled examples, $n_v$, its total number of examples, $N_v$, and the number of labels $m_v$ that are of the majority class. Given these quantities, GSAL’s estimate is

$$\tilde{f}(m, n, N) = \max_{t \in [0,1]} \left( n_v + \left(1 - \exp\left(-2n_v t^2\right)\right) \left(N_v - n_v\right) \left(\frac{m_v}{n_v} - t\right) \right). \quad (1)$$

As a shorthand, we sometimes write $\tilde{f}(v)$ to mean the formula applied with the statistics of node $v$.
We describe the derivation of this estimate formula in Section 4.1. Notably, this formula can be efficiently computed using ternary search [4, 10], and it represents a lower bound on an expected number of correctly labeled examples. Since the formula depends on quantities that are either known or can be estimated as the result of split and label actions, GSAL can use the formula to anticipate the resulting estimate after these actions.

When performing an oracle query, we anticipate that the oracle will return the leaf node’s majority label for the data point queried, thus increasing the majority count \( m_v \) and the label count \( n_v \). Therefore, the number of estimated correct labels for the query operation is

\[
\tilde{f}_{\text{label}}(v) = \tilde{f}(m_v + 1, n_v + 1, N).
\]  

(2)

For the split operation, a subroutine will divide the examples in \( X_v \) into a set \( U \) of child nodes. This splitting will also distribute the labeled examples among the child nodes. For each child node \( u \in U \), let \( m_u \) be the number of examples in the majority label of \( u \). (Importantly, this majority label \( \hat{y}_u \) need not be the same as the majority label \( \hat{y}_v \) of the parent \( v \).) Then the estimated correct labels for all the candidate children of \( v \) after splitting is

\[
\tilde{f}_{\text{split}} = \sum_{u \in U} \tilde{f}(m_u, n_u, N_u).
\]  

(3)

### Splitting Procedures
The split operation partitions a node \( v \) into a set of nodes \( U \) that uniquely cover the examples in \( X_v \). That is, for each example \( x_{v,i} \in X_v \), there is exactly one \( X_u \) where \( u \in U \) that contains \( x_{v,i} \). GSAL splitting can be done with an unsupervised clustering subroutine or with a supervised learning step.

When splitting with an unsupervised clustering method, all examples in \( v \) are provided to a clustering algorithm, which provides the resulting partition. Since unsupervised partitioning is unaffected by the known labels, the labeled examples can be recategorized into the child nodes \( U \) and used for further bound calculations.

Unfortunately, unsupervised clustering can often be poorly correlated with the factors that determine classification. To obtain a better partitioning of a node \( v \), we can use some of the labeled data to train a classifier to categorize the data into partitions. For training a supervised split, each node reserves an isolated, special set of labeled examples that are not used to calculate \( m_v, n_v, \) and \( \tilde{f}(v) \). This isolation is analogous to training and validation splits in traditional evaluation of supervised learning; we train a model on the isolated set and evaluate on the resulting partitioned labeled set.

We further analyze the effect of unsupervised and supervised splitting on our ability to estimate the number of correctly labeled examples in Section 4.

### 4 Theoretical Analysis

Our greedy split and label exploration (GSAL) procedure is designed to increase a pessimistic lower bound on an expected number of correctly labeled examples. As GSAL creates leaf nodes containing data subsets, it tracks a set of labeled examples for each leaf that are uniformly and independently selected from the leaf population. This unbiased sample allows us to derive a bound on the expected number of correctly labeled points. To avoid introducing bias into the labeled subset, GSAL “forgets” a node’s labeled points when it splits that node. The following sections discuss the bound we use and the reason this forgetting is necessary.

#### 4.1 Derivation and Analysis of Lower Bound \( \tilde{f} \)

Our goal is to provide a lower bound on the expected number of correct labels obtained by GSAL if we choose the majority label for each set of data. At any given point in GSAL, we perform the operation that is expected to most increase the lower bound. This bound should be pessimistic to ensure that the bound is rarely violated, but it should still provide a reasonable estimate on the expected number of correct labels. Most importantly, it should follow the same “curve” as the true count of correct labels for the algorithm to make the correct greedy choice of labeling a data point or splitting a set of data. Our bound is based on Hoeffding’s Inequality [7], a distribution-free concentration bound that becomes tighter to the true value as more labels are obtained. Hoeffding’s Inequality is also
pessimistic, which is ideal for the application of our algorithm because it will favor labeling data over splitting a dataset when the expected label uniformity is the same.

Let \( N \) be the total number of points in a set. We will omit the indexing subscript \( v \) for notational cleanliness in this analysis. Let \( n \) be the number of points with known labels, and let \( m \) be the number of points with the majority label. Let the true proportion of the majority label in this set be \( p \).

Hoeffding bounds are defined with a deviation \( t \). Since our current estimate of the probability of the majority label is \( p' = \frac{m}{n} \), Hoeffding’s Inequality provides this guarantee:

\[
\Pr \left( p \leq \frac{m}{n} - t \right) \leq \exp \left( -2nt^2 \right).
\]

(4)

The interpretation of Hoeffding’s Inequality is that the true probability of the random variable will be within \( t \) of the empirical estimate unless the bound fails. Probability of failure is bounded by \( \exp \left( -2nt^2 \right) \). We define the expected number of correct labels for when Hoeffding’s Inequality holds and when it fails.

In the case of failure, we pessimistically assume the number of correct labels is only \( n \), i.e., everything that has not already been labeled will be labeled incorrectly. In the case that the probability is bounded, we assume the number of correct labels is proportional to the lower bound \( \frac{m}{n} - t \). Then the expected number of correct labels \( f \) is

\[
f(m, n, N) = pN \geq n + \left( 1 - \Pr \left( p \leq \frac{m}{n} - t \right) \right) (N - n) \left( \frac{m}{n} - t \right)
\]

\[
\geq n + \left( 1 - \exp \left( -2nt^2 \right) \right) (N - n) \left( \frac{m}{n} - t \right)
\]

\[
:= \bar{f}(m, n, N, t),
\]

(5)

where the first inequality holds because we use pessimistic estimates of the quantities of correct labels, and the second inequality holds because we use pessimistic estimates of the probability.

Since the bound \( \bar{f}(m, n, N, t) \) holds for any buffer \( t \), we can optimize it to choose the \( t \) that yields the tightest lower bound. There is no closed-form solution for the maximum of \( \bar{f} \). However, since it is a univariate, unimodal function, ternary search \([4, 10]\) can approximate the maximum to the desired accuracy after logarithmic executions of \( \bar{f} \). The unimodality of the bound \( \bar{f} \) can be seen because it is the sum of a function linear in \( t \), and a product of a concave quadratic function and a log-concave function. Since such a product is itself log-concave and therefore unimodal, and the linear function has no effect on unimodality, the overall bound is unimodal and amenable to ternary search.

Importantly, since each node’s estimate is a lower bound on its expected number of correctly labeled examples, the sum of all nodes’ estimates also bounds the expected value of the entire tree’s number of correctly labeled examples. This fact follows the linearity of expectation. Thus, when GSAL chooses the node-action pair that most increases the estimated bound, it is greedily choosing to improve the overall estimate of expected total correct labels.

4.2 Supervised Splitting Procedure and Preserving Independence

Equation (5) uses Hoeffding’s Inequality, which depends on the fact that the random variables are drawn independently from a fixed distribution. Since the random variable in question is whether an example belongs to the majority class, the observed labels satisfy this requirement if they are randomly sampled from any subset \( v \). However, if the random points are sampled from a parent set \( v \), which is later split into child nodes \( \{u | u \in U\} \), special care is needed to ensure that the independence and uniform sampling probability hold.

When the partitioning \( U \) is determined by an unsupervised split, it is not affected by whether points are labeled or what their labels are. Instead, it can be thought of as a pre-determined partitioning. Therefore, a random sample from a parent node \( v \) that happens to be part of a child node \( u \) is also a random sample from \( u \).

In contrast, when partitioning \( U \) is determined by a supervised model, the data used to train the model is no longer randomly sampled from the resulting child nodes. Therefore, GSAL tracks an isolated
We then evaluate the test accuracy of a learned logistic regression model using the labels assigned by each approach as it uses a budget of 1,500. We control the quality of labels by only assigning labels if the empirical uniformity of each dataset is above $q = 0.85$ when our algorithm terminates. We compare the number of labels obtained, the quality of labels indicated by the true label accuracy, and if the empirical uniformity of each dataset is above $q = 0.85$ when our algorithm terminates. We compare the number of labels obtained, the quality of labels indicated by the true label accuracy, and

set of labeled examples for each node used to train splitting models. Since this isolated training set is independent of the data used to calculate $f_{\text{split}}$, it preserves the fact that the data in the partitioned subsets is randomly sampled.

Regardless of the method of splitting, the usage of the $f_{\text{split}}$ score to decide whether to split introduces a dependency. In practice, this dependency can be slight and may not have serious effects on the bounds. However, since the bound $f_{\text{split}}$ may be evaluated many times on a node—and using multiple possible partitionings—it is safer to have GSAL reset its counts after the split operation. Doing so ensures that the data considered for calculating future bounds is uniformly randomly sampled from the new child nodes. This assurance is the reason for the “forgetting” in line 20 of Algorithm 1. The impact of forgetting on data efficiency is somewhat mitigated by the fact that, if an example sampled for labeling has previously been labeled and forgotten, we can simply look up the known label and use it, with no need to invoke the costly oracle.

5 Experiments

We compare GSAL to a baseline of the HSAL algorithm proposed by Dasgupta and Hsu [5]. HSAL uses unsupervised splitting of data and a tree that does not adapt to new label queries from the oracle. We show the quality and the quantity of the labels obtained by our algorithm, and empirically show that our bound $F$ is rarely violated. We apply HSAL and GSAL to the 20 Newsgroups, MNIST [6], and Fashion-MNIST [16] datasets. We start each method with the training sets of completely unlabeled examples, and we simulate oracle calls by revealing the true label of the requested examples.

5.1 Setup

We first evaluate the number of correctly labeled examples by each approach as it uses a budget of 25% of each dataset. We plot the estimated bound and the actual number of correct labels if we use the majority votes of each leaf in Figure 1. For GSAL’s splitting, we train multi-class logistic regression models. For HSAL, we use a k-means clustering, with two means, to perform each hierarchical split.

We then evaluate the test accuracy of a learned logistic regression model using the labels assigned by each algorithm using a budget of 1,500. We control the quality of labels by only assigning labels if the empirical uniformity of each dataset is above $q = 0.85$ when our algorithm terminates. We compare the number of labels obtained, the quality of labels indicated by the true label accuracy, and

![Figure 1: The number of correctly labeled points by each approach applied to the three datasets. GSAL (top row) uses label information along the way to refine its hierarchy. HSAL (bottom row) uses a fixed hierarchy, which avoids bias but is restricted to an initial hierarchical clustering. On the 20 Newsgroups data, GSAL (top right) makes a significant adjustment after receiving enough labels to infer a data split better aligned to the class labels than the clustering used by HSAL (bottom right).](image)
the accuracy of using the labels in practice on the learned model. Table 2 lists the averages with standard deviations resulting from three trials. We present additional results with different budgets in the appendix.

### 5.2 Results

Our method returns significantly more labels than the unsupervised HSAL approach. The label accuracies are comparable on the Fashion-MNIST and 20 Newsgroup data, while GSAL has a clear advantage in label accuracy on the (digit) MNIST data. The increase in data quantity leads to better trained models on Fashion-MNIST and MNIST. On the 20 Newsgroups data, neither approach returns many new labels, mostly due to the quality-control mechanism filtering out inferred labels with insufficient confidence. Since the training set for 20 Newsgroups only contains around 500 examples per class, it is possible that the budget of 1,500 is too small for this dataset.

In empirical trials of our algorithm, our lower bound $\tilde{f}$ holds for most cases. Figure 1 plots our bound behavior over time for the different algorithms and datasets. Regardless of the splitting method, violations to our bound are rare and seem to occur when the guessed majority label is incorrect for a dataset. Any bound violations of our estimated score return back to normal when more data points are sampled from the subset of data where the majority label was incorrect. Bound violations may also occur immediately after a split when the child nodes have no information about the majority label. For this reason, a minimum size may be used to limit node splits when not much is known about the child node.

Figure 1 also plots the resulting bound and correct label counts when using our supervised splitting and the HSAL-like unsupervised splitting. There is a common trend in all datasets where the fixed hierarchy used by HSAL leads to fewer labeled examples. Our GSAL method’s amounts of correct labels always dominate HSAL’s in our experiments. Even in the 20 Newsgroup data, where our quality-control mechanism removed many of the labels inferred by GSAL, there is a clear inflection point in the curve where GSAL makes a supervised split that leads to a large jump in the number of correctly labeled points.

### 6 Conclusion

We present a method for greedily finding uniform subsets of an unlabeled dataset by querying labels from an oracle and splitting datasets apart in a supervised manner. By finding uniform subsets of data, we may assign labels to be used for future learning tasks without enduring the cost of labeling the full dataset. Our method uses statistical analysis to reason about when it can stop labeling a data subset with high enough confidence. Our framework hierarchically splits the dataset into subsets that are each assigned a label, seeking a hierarchy that contains uniformly labeled subsets. It decides on labeling or splitting actions to greedily increase a confidence-based estimate of the number of correctly labeled points. Our framework also uses strategies to allow the use of supervised learning to guide the splitting. Supervised splitting is preferred in many cases to unsupervised splitting because it is better at finding splits that correspond to the true labels of data, and it is therefore more likely to find uniform clusters. While supervised splitting induces a bias on the final label assignment, we remove this bias by isolating data to be used to train splits and data to be used to calculate the bounds.
that score the actions. In our experiments, our method is able to correctly label significant proportions of datasets while only observing the labels of a small fraction of examples. Moreover, it performs better than similar methods restricted to unsupervised splitting. With the introduction and evaluation of GSAL, we take a key step toward reducing the practical cost of machine learning.

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A Results of Trials with Different Budgets

We include here the resulting numbers of returned labels, accuracy of returned labels, and accuracy of model trained on returned labels for each method and each dataset. For each budget size, we run a single trial and report its statistics in these tables.

The quality control postprocessing removes all labels inferred from clusters with smaller than 0.85 uniformity. Therefore, small-budget trials often infer no labels, so we leave the label accuracy fields of those trials empty.

### Table 3: Fashion MNIST dataset, using GSAL (our method).

| Budget | Size of Y | Label Accuracy | Learned Model Accuracy |
|--------|-----------|----------------|-----------------------|
| 100    | 3707      | 0.722          | 0.532                 |
| 500    | 13096     | 0.640          | 0.627                 |
| 1000   | 11706     | 0.917          | 0.741                 |
| 1500   | 10450     | 0.920          | 0.759                 |

### Table 4: Fashion MNIST dataset, using HSAL.

| Budget | Size of Y | Label Accuracy | Learned Model Accuracy |
|--------|-----------|----------------|-----------------------|
| 100    | 1604      | 0.352          | 0.572                 |
| 500    | 12352     | 0.771          | 0.523                 |
| 1000   | 6828      | 0.924          | 0.726                 |
| 1500   | 5611      | 0.955          | 0.748                 |

### Table 5: MNIST dataset, using GSAL (our method).

| Budget | Size of Y | Label Accuracy | Learned Model Accuracy |
|--------|-----------|----------------|-----------------------|
| 100    | 5926      | 0.945          | 0.65                  |
| 500    | 4907      | 0.934          | 0.818                 |
| 1000   | 19907     | 0.843          | 0.697                 |
| 1500   | 21958     | 0.868          | 0.748                 |

### Table 6: MNIST dataset, using HSAL.

| Budget | Size of Y | Label Accuracy | Learned Model Accuracy |
|--------|-----------|----------------|-----------------------|
| 100    | 100       | —              | 0.748                 |
| 500    | 5470      | 0.875          | 0.726                 |
| 1000   | 4452      | 0.917          | 0.803                 |
| 1500   | 8151      | 0.854          | 0.7005                |
Table 7: 20 Newsgroups, using GSAL (our method).

| Budget | Size of $Y$ | Label Accuracy | Learned Model Accuracy |
|--------|-------------|----------------|------------------------|
| 100    | 100         | —              | 0.269                  |
| 500    | 500         | —              | 0.505                  |
| 1000   | 1033        | 0.998          | 0.576                  |
| 1500   | 1523        | 1.0            | 0.618                  |

Table 8: 20 Newsgroups, using HSAL.

| Budget | Size of $Y$ | Label Accuracy | Learned Model Accuracy |
|--------|-------------|----------------|------------------------|
| 100    | 100         | —              | 0.265                  |
| 500    | 513         | 0.979          | 0.452                  |
| 1000   | 1024        | 0.989          | 0.542                  |
| 1500   | 1624        | 0.985          | 0.564                  |