A unified theory of superconductivity

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(Dated: September 22, 2008)

In this paper, we study the reliability of BCS theory as a scientific explanation of the mystery of superconductivity. It is shown clearly that the phonon-mediated BCS theory is fundamentally incorrect. Two kinds of glues, pairing (pseudogap) glue and superconducting glue, are suggested based on a real space Coulomb confinement effect. The scenarios provide a unified explanation of the pairing symmetry, pseudogap and superconducting states, charge stripe order, spin density wave (SDW), checkerboard-type charge-ordered phase, magic doping fractions and vortex structures in conventional and unconventional (the high-$T_c$ cuprates and MgB$_2$) superconductors. The theory agrees with the existence of a pseudogap in high-temperature superconductors, while no pseudogap feature could be observed in MgB$_2$ and most of the conventional superconductors. Our results indicate that the superconducting phase can coexist with an inclined hexagonal vortex lattice in pure MgB$_2$ single crystal with a charge carrier density $\rho_s = 1.49 \times 10^{22}/\text{cm}^3$. Finally, the physical reasons why the good conductors (for example, Ag, Au, and Cu) and the overdoped high-$T_c$ superconductors are non-superconducting are also explored.

PACS numbers: 74.20.-z, 74.20.Mn, 74.25.Qt, 74.20.Rp, 74.25.Dw

I. INTRODUCTION

Since the first discovery of superconductivity in mercury in 1911 by H. Kamerlingh Onnes,\textsuperscript{1} scientists around the world have been trying hard to find (or synthesize) the superconducting materials. Through nearly a century of efforts, it is now clear that superconductivity is an extremely common natural phenomenon occurring in a wide variety of materials, for example, pure metals, metallic alloys, heavily-doped semiconductors, a family of cuprate-perovskite ceramic materials,\textsuperscript{2,3} MgB$_2$\textsuperscript{4} and the newly synthesized iron-based systems.\textsuperscript{5,6} Soon after the discovery of the superconductivity, the search for a theoretical understanding of this mysterious phenomenon has always been one of the hottest topics in condensed matter physics. There are now thousands of theories on how superconductivity would work but none of these are definite (including the famous BCS theory\textsuperscript{7}). The new experimental evidence in favor of the localized Cooper pairs has just been reported\textsuperscript{8}, the discovery shakes the very foundation of the BCS theory. The new family of superconductors\textsuperscript{5} also strongly challenge the BCS theory based on the electron-phonon coupling mechanism.\textsuperscript{9,10} In other words, the mechanism of superconductivity (both conventional and non-conventional superconductors) remains unsettled. This raises two questions: (i) What is the main reason of superconductivity in various superconductors? (ii) Should the mechanisms responsible for different superconductors be different? In my opinion, any electronic pairing and superconducting phenomena should share exactly the same physical reason.

In the earlier works,\textsuperscript{11} we propose a real space mechanism of high-$T_c$ superconductivity which can naturally explain the complicated problems, such as pairing mechanism, pairing symmetry, charge stripes, optimal doping, magic doping fractions, vortex structure, phase diagram, Hall effect, etc. I am confident that the research may shed light on the fundamental of superconductivity. In the present paper, we try to extend the application of the theory in conventional superconductors and MgB$_2$\textsuperscript{4}.

FIG. 1: (a) In BCS theory, a single tiny electron can lead to a serious deformation of lattice structure. (b) We consider that the BCS scenario of (a) is physically unreasonable, the electron’s trajectory will be changed constantly due to the Coulomb attraction between the electron and ions.
II. THE BCS THEORY: SCIENCE OR MYTHOLOGY?

Recently, Anderson pointed out that the need for a bosonic glue (phonon) in cuprate superconductors is folklore rather than the result of scientific logic\textsuperscript{12}. In this Section, we would like to tell my physics colleague that the BCS theory is merely a mythology story which even doesn’t work for the conventional superconductors.

A. Who attracts who?

Fig. 1(a) shows an electron traveling inside a periodic lattice, as suggested by BCS, this electron will attract nearby positive charges in the material. Is this hypothesis physically valid?

The electron is a fundamental particle that carries a negative electric charge, and its mass is approximately $1/1836$ of that of the proton. Normally, an atom (or ion) has a mass that is more than 10000 the electron’s mass. All the forces involved in interactions between the electron and ions can be traced to the electromagnetic interactions. Here, we would like to raise one question: After exerted by the same amount of force, why the massive ions get a big-displacement, while the state of the electron remains almost unchanged. Obviously, the BCS recommended picture of Fig. 1(a) violates the most basic physical principles. A reasonable physical picture of the electron-ions interactions is presented in Fig. 1(b), where the electron’s state is changed constantly and there are no perceptible changes in the ions’ state.

B. Can “Ant” command “Elephant”?

According to BCS theory, a tiny electron 1 with a momentum $k$ can cause a collective vibration of the entire lattice (the generation of a quantized phonon $q$), as shown in Fig. 2(a). The scenario implies that, if the entire universe is a single-crystal superconductor, a single electron can excite a vibration of whole universe. This sounds like a gigantic “Butterfly Effect”. What’s even more confusing is when electron 2 with same $k$ and opposite spin appears, the phonon will annihilate instantly, as shown in Fig. 2(b). Or in BCS language, the phonon is absorbed integrality by electron 2. In order to continue the story, BCS further assume that the moment the electron 2 absorbs the phonon, an exact the same phonon has been generated by the electron 1 again, then the new phonon will be absorbed by electron 2, ... Science? or science fiction?

I really don’t know why such a absurd theory can become to be the cornerstone of modern physics. Supposing we now have an “ant” (electron) and an “elephant” (atomic lattice) (see Fig. 2), is it true that the “ant” is the “commander”? Beside, what mechanism can ensure the leading electron 1 never collide with atoms or other electrons?

C. Do electrons have eyes and ears?

In BCS theory (dynamic screening), the paired electrons are not physically close together or never in the same place at the same time. It is not clear how can these
extended pairs be crammed together to create a superconducting medium without getting disrupted. Schrieffer had try to explain how the loose Cooper pairs can finally lead to the superconductivity. He compared the concept to the Frug (a popular dance)\textsuperscript{13}, where dance partners (every male has an up spin and a female has a down spin) could be far apart and never touch each other (may be a couple of hundred feet apart) on the dance floor, yet remain a pair, as shown in Fig. 3 (a).

Although this analogy may sound interesting for audiences lacking of basic physics knowledge, from the viewpoint of physics, this comparison is meaningless. Because these two systems are completely different. The main differences are as follows:

1. The dance partners have healthy eyes and ears, do electrons have eyes and ears?
2. All dancers look different each other, but all electrons are identity.
3. The dancers are well trained and the choreography well rehearsed, who have told the electrons how to dance “Frug”, God?
4. In a superconductor, there are many “huge” (comparing to electron) atomic oscillators. But there are not any analogous “oscillator” on the dance floor.
5. The electrons of the Cooper pair should be momentum opposite, if the dance partners are also “momentum” opposite, can the dance go on forever?

I’m sure without eyes and ears [see Fig. 3 (b)], electrons cannot dance “Frug”!

D. Equidirectional momentum or opposite momentum?

1. Why two electrons so different?

To represent the k-space’s BCS theory in real space, two visual models of the Cooper pair attraction have been suggested (see Figs. 4 and 5). It is shown here that the real-space structures of Figs. 4 and 5 cannot follow directly from the BCS theory and the efforts to explain the

![FIG. 4: (a)-(b) A traditional model of Cooper pair attraction in real space where two electrons move in the same direction, (a) a passing electron 1 attracts the lattice (the positive ions), causing a slight increase of positive charge center due to Coulomb attraction, (b) and the trailing electron 2 is attracted by it. Here, we argue that the basic physical pictures described in (a) and (b) are physically untrue. As seen in the two subfigures, the ripple induced by electron 2 has been completely ignored in this analysis. (c) The actual situation where both electrons can distort the positively charged ions, independently. In this case, the phonon induced attraction between the two Cooper pairing electrons becomes invalid, because the two forces acting on electron 2 normally satisfy $f_2 \gg f_1$.](image)
condensation of Cooper pairs are proved to be unreliable or even physically unreasonable. Figs. 4 (a) and (b) show the model of Cooper pair attraction in real space where two electrons move in the same direction. In this case, the leading electron 1 attracts the lattice (the positive ions) and causes a slight increase of positive charge around it, as shown in Fig. 4(a). This increase in positive charge will, in turn, attract the trailing electron 2, as shown in Fig. 4(b). From BCS theory, we know that this coupling between two electrons is viewed as an exchange of phonons (the quanta of lattice vibration energy). However, this real space picture is totally inconsistent with the k-space’s BCS theory in many aspects. As is well known, the BCS theory asserts that the two paired electrons must have opposite spin and opposite momentum. But, Figs. 4 (a) and (b) show clearly that the real space representation of the bound Cooper pair electrons are in the same momentum. Furthermore, this approach fails to explain why the two paired electrons should be spin antiparallel.

In fact, the major flaw of the BCS theory is that the lattice distortion caused by electron 2 has been completely ignored in this analysis. In our opinion, a complete picture of the real space description of BCS theory must take into account not only the electron 1 but also the electron 2, as illustrated in Fig. 4(c). From this figure it is clear that there are two forces acting on electron 2: the attractive force $f_1$ produced by the positive charge center of electron 1 and the drag force $f_2$ exerted by the positive charge center of electron 2 itself. BCS theory suggests that electron pairs can couple over a range of hundreds of nanometers, three orders of magnitude larger than the lattice spacing, therefore, the drag force $f_2$ is generally much larger than the attractive force $f_1$. This further implies phonon-mediated BCS theory is not valid in physics.

2. How to avoid electron-electron repulsion and how to maintain a instantaneous “attraction” forever?

Figure 5 shows another visual model of the Cooper pair attraction with two electrons moving in the opposite direction. Compared with Figures 4, although now the two electrons have opposite momentum as suggested by BCS theory, apart from the spin and phonon issues discussed above, there are a number of fatal problems with this explanation. First, even if the positive charge center of Fig. 5 (a) can attract another electron passing in the opposite direction [see Fig. 5 (b)], apparently, the attraction is instantaneous. Second, when two electrons approach each other, a strong electron-electron repulsion is unavoidable [Fig. 5 (b)]. All these factors indicate the Cooper pair should split up rather than stay together when the pair is formed by two electrons with opposite momentum.

In a word, two real space electron-phonon mechanisms are examined and they cannot give a satisfactory expla-

\begin{equation}
F_c = \frac{e^2}{4\pi\varepsilon_0\Delta^2},
\end{equation}

where $e$ is the electron charge and $\Delta$ is the distance between two electrons.

It is known that study of superconducting correlations in conventional superconductors is always performed in momentum-space (dynamic screening), where the paired electrons are seldom or never in the same place at the same time. In the case of dynamic screening, only the long-range Coulomb interaction $e^2/\Delta$ is considered while the short-range electron-electron magnetic interactions is completely ignored. We argue here that, in the case of real-space screening, the magnetic forces among the electrons (see also Fig. 6) should be taken into account. Approximately, the magnetic dipolar interaction forces $F_m$ exerted on the electrons are given by

\begin{equation}
F_m \approx \frac{3\mu_0 \mu^2}{2\pi\Delta^4} \cos \theta_1 \cos \theta_2,
\end{equation}
where $\mu_0$ is the permeability of free space and $\mu_B$ is the Bohr magneton.

The forces $F_m$ of Eq. (2) can be attractive and repulsive depending on the orientation ($\theta_1$ and $\theta_2$) of electron magnetic moment $m_s(j)$, ($j = 1, 2$). When $\theta_1 = \theta_2 = 0$ (or $\pi$), the magnetic poles of the paired electrons are lined up in parallel (spin-parallel pair correlation), contrary to the spin antiparallel BCS theory. Consequently, the attractive magnetic force reaches its maximum value $F_m^{\text{max}} = 3\mu_0\mu_B^2/2\pi\Delta^4$, and the electron pair corresponds to the most stable and energy minimum state. When $|\theta_1 - \theta_2| = \pi$, the two electrons are spin antiparallel obeying the BCS theory, but the corresponding pair is in the most unstable and maximum energy state. Such a construal has significant implication that the BCS theory is physically unreasonable.

**IV. PAIRING GLUE AND PSEUDOGAP**

At high temperatures, the vibrational motion of the material’s lattice becomes so stiff that it tends to break up the electron pairs instead of holding them together. So what could possibly provide the glue that keeps the carriers bound in Cooper pairs? Although many candidates for this glue (including spin fluctuations, phonons, polaron, charge stripes and spin stripes) have been proposed, what the pairing glue in high-$T_c$ cuprates is still an open question. In this Section, we would like to discuss the issue from the point of view of real-space confinement effect. To describe this, two spin parallel electrons of Fig. 6 with a joint paired-electron magnetic moment $M_s = m_s(1) + m_s(2) = 2m_s$ are embedded into a CuO plane of the cuprate superconductor, as shown in Fig. 7.

Looking at the figure, just a simplification, only nearest-neighbor and next-nearest-neighbor interactions are considered. Inside the unit cell, the possible paired-electrons with the magnetic moment $M_s$ along the $\theta$ direction, and the corresponding distance between the electrons is reexpressed as $\Delta(\theta)$. From the figure, one can easily conclude that the pair with the $M_s$ oriented in (100), (010), (001) and (100) directions is generally considered to be more stable (minimum energy) due to the suppression of the four oxygen ions ($O^{2-}$ for hole-doped, $O^{2-}$ for electron-doped), as opposed to the cases, in (110), (101), (110) and (110) directions where the bound pair tends to be separated by Coulomb forces of the Cu$^{2+}$. As a consequence, the distance $\Delta(\theta)$ between the two electrons of the pair has a minimum (maximum binding energy) at $\theta = 0, \pi/2, \pi$ and $3\pi/2$, while at $\theta = \pi/4, 3\pi/4, 5\pi/4$ and $7\pi/4$, $\Delta(\theta)$ will reach its maximum value (minimum binding energy). Obviously, the unified model (see Fig. 7) for both hole- and electron-doped cuprates has essentially the same pairing mechanisms (pairing glue). In the previous paper, a more detailed study was done based on the Coulomb’s equation and the results suggested the dominant $d$-wave symmetry in hole-doped cuprates and a possible mixed ($s + d$)-wave symmetry in electron-doped systems. The results revealed that the localized electromagnetic interactions are indeed the source (glue) of localized cooper pairs characterized by the pseudogap.
FIG. 8: (a) Due to the magnetic phase-coherence among the electron pairs, a helical dynamical spin-density-wave (SDW) is inspired in the metallic charge stripe (vortex line) and the superconductivity and SDW can coexist along this stripe, (b) when the electron pairs are highly coherent, the charge stripe can be considered as a ‘supersolid’ where any electron pair inside always experiences a pair of compression forces (a repulsive force pairing mechanism). If background ions are positive, pairing and superconducting will occur at the same time.

The nature of the normal-state gap (pseudogap) phase of HTSC is still highly controversial. ARPES and tunneling measurements show a clear pseudogap which was seen to persist even at room temperature.20–22 There are many models attempt to describe the mysterious pseudogap state. Strictly speaking, none of the proposed models is completely satisfactory. As discussion above, here we present a new approach based on the simple and natural picture of the real-space confinement effect, and the pseudogap is associated with the local structure of unit cell in CuO$_2$ plane. Thus it should not be surprising about the pseudogap behavior which indicate the formation of pairs (localized cooper pairs) below $T^* > T_c$.

In our viewpoint, pseudogap phenomenon is merely a real space confinement effect in the superconductors if electrons were confined inside one unit cell. Besides, to have a stable pseudogap phase, pair-pair interactions should be suppressed. Hence, decreasing the charge carrier density is a useful way to open and maintain a pseudogap in the superconductors. In other words, the pseudogap is associated with the local structure and the charge carrier density in the superconductors. Therefore, the localized Cooper pairs (pseudogap) are likely to survive in insulating or nonmetallic materials.

Nowadays, more and more beautiful experimental results suggest that stripes are common in cuprates and may be important in the mechanism for HTSC. In the paper,11 based on the GL theory formalism, we argued that the dimerized charge stripes can contribute to the mechanism of superconductivity in cuprate superconductors and the dynamical spin density wave (SDW) coherent phases can be established along the stripes. As can be seen, the high-$T_c$ superconducting order is also inherently related to the a real space collective confinement . Consequently, the superconductivity has an origin different from pseudogap in high-$T_c$ superconductors. Here a similar real space collective confinement picture is introduced into the conventional superconductors, as shown in Fig. 8, very different from the localized pairing mechanism (see Fig. 7) of the high-$T_c$ superconductors. In this case, a real space helical dynamical spin-density-wave [Fig. 8 (a)] and superconductivity coexist to form a dimerized charge supersolid (a charge-Peierls dimerized transition), as shown in Fig. 8 (b). Indeed, both the pairing and superconducting (phase coherence) occur simultaneously at $T_c$, as generally accepted experimental facts. In the real space collective confinement picture, the so-called spin density wave (SDW), superconducting charge stripe and the vortex line are exactly the same thing. Anyway, the spin correlation of Fig. 8 is a general phenomenon in superconductors, and it must be the fundamental to the mechanism (superconducting “glue”) of superconductivity in conventional and unconventional superconductors.

V. A COLLECTIVE CONFINEMENT AND SUPERCONDUCTING GLUE

Physically, pairing in cuprates is an individual behavior characterized by pseudogap, while superconductivity is a collective behavior of many coherent electron pairs.
and long-range repulsive forces. The formation of stripe patterns is generally attributed to the competition between short-range attractive forces and long-range repulsive forces. We argued that, in the proper doped LSCO superconductor, the electron pairs can self-organize into a ‘superlattice’ (Wigner crystal of electron pairs) with the primitive cell \((A, B, C) = (ha, kb, lc)\), as shown in Fig. 9. Consequently, the “material” composed of electron-pair “atoms” will undergo a structure transition from random to order phase (LTO, LTT). Thus, the doping level \(x\) is given by

\[
x = p(h, k, l) = 2 \times \frac{1}{h} \times \frac{1}{k} \times \frac{1}{l},
\]

and the corresponding charge carrier density is

\[
\rho_s = \frac{2}{ABC} = \frac{2}{hkl} \frac{1}{abc} = \frac{x}{abc},
\]

where \(h, k, \) and \(l\) are integral numbers. Note that, from the viewpoint of energy, it is also possible that the ‘superlattice’ exhibits two simple hexagonal structures (see below).

**A. LTT1\((h, k, l)\) non-superconducting phase**

We found that there are only five abnormal phases (the so-called “magic doping phases”) in LSCO, which are related to the anomalous suppression of superconductivity. They are LTT1\((6, 6, 1)\) of \(x = 1/18\) , LTT1\((4, 4, 1)\) \((x = 1/8)\), LTT1\((4, 4, 2)\) \((x = 1/16)\), LTT1\((3, 3, 2)\) \((x = 1/9)\) and LTT1\((2, 2, 2)\) \((x = 1/4)\) where the nondispersive superlattices of \(6a \times 6a, 4a \times 4a, 4a \times 4a, 3a \times 3a\) and \(2a \times 2a\) in CuO planes can be expected, respectively. At \(x = 1/8, LTT1(4,4,1)\) can also coexists with the LTO original lattice \((a = b)\) of the LSCO [see Fig. 10(a) and (b)]. This may explain the famous “1/8 anomaly” in various high-\(T_c\) superconductors.\(^{17,24-28}\) Note that although the nondispersive \(4a \times 4a\) superstructure [see Fig. 10(a)] seems to be exactly the same in both samples \((x = 1/8\) and \(1/16)\).\(^{29,30}\) We show, for the first time, that two samples are in fact very different: in the sample of \(x = 1/8\) indicated by LTT1\((4,4,1)\) in this paper, where all CuO planes are doped [Fig. 10(b)]; while at \(x = 1/16\) of LTT1\((4,4,2)\), only half of the CuO planes (every two planes) are doped [Fig. 10(c)].

Encouragingly, apart from the \(x = 1/8\), some unusual results have already been observed at \(x = 1/16, x = 1/9\) and \(x = 1/4\) of the doped LSCO crystals. For instance, by high resolution ARPES experiments on \(x \sim 1/16\) sample, an anomalous change at ~ 70 mev in the nodal scattering rate was reported,\(^{31}\) and the observations of intrinsic anomalous superconducting properties at magic doping levels of \(x = 1/16\) and \(x = 1/9\) had been found by \(dc\) magnetic measurements.\(^{32}\) The experimental verification of the strong-correlation fluctuations in a non-superconductive \(x = 1/4\) sample has been noted.\(^{33}\) Most recently, Wakimoto et al.\(^{34}\) reported the
the superlattice constants have the following relation

$$A/C = \frac{ha}{lc} = \frac{2\sqrt{3}}{3} \approx 1.154700.$$  

Fig. 12 (b) shows the LTT3(h, k, l), the vortex lattice has a tetragonal symmetry in XZ plane with a orientation 45° and the superlattice constants:

$$A/C = \frac{ha}{lc} = 2.$$  

While in simple hexagonal (SH) phases, as shown in Figs. 12 (c) and (d), the charge stripes possess identical trigonal crystal structures. In the SH1(h, k, l) phase [see Fig. 12 (c)], the superlattice constants have the following relation

$$A/C = \frac{ha}{lc} = \frac{2\sqrt{3}}{3} \approx 1.154700.$$  

For the SH2(h, k, l) phase of Fig. 12 (d), this relation is given by

$$A/C = \frac{ha}{lc} = 2\sqrt{3} \approx 3.46410.$$  

It is commonly accepted that samples of La2−xSrxCuO4 have the highest $T_c$ at Sr concentration (optimal doping) $x \sim 0.16$ with the experimental lattice constants: $a = 3.79A$ and $c = 13.25A$. In this subsection, basing on the above analysis, we will attempt to provide a general description of the stable superconducting phase (metallic stripe) in LSCO and give a possible relationship between the lattice constants and optimal doping phase. Note that although the LTT2 and SH phases have rather different spatial structure, Eqs. (3) and (4) are still valid for the SH phases due to the appropriate definition of the superlattice constants ($A$ and $C$) in Figs. 12 (b) and (c). In LSCO, based on the experimental lattice constants ($a = 3.79A$ and $c = 13.25A$), the relationship between the doping level $x = 1/hkl$ and $A/C$ is shown in Fig. 13. We argue that the SH1(12, 1, 3) vortex phase of $x = 1/18 \approx 0.05555$ is most likely the lowest doped superconducting LSCO sample, in favor of the experimental result that superconductivity emerges at $x \sim 0.056$ in LSCO superconductor35–37. The superconducting SH1(8, 1, 2) ($x = 1/8$) is completely suppressed by the non-superconducting phases of LTT1(4, 1, 2) ($x = 1/8$). We find several candidates for the optimal doping phase in LSCO system (see
and $LTT_{3(14,1,2)}$. Among the residual phases $[SH_{2(12,1,1)}]$ of $x = 1/6$ and $LTT_{2(7,1,2)}$ of $x = 1/7$, we consider that the maximum high-$T_c$ phase (optimal doping) may be relevant to $SH_{2(12,1,1)}$. Using Eqs. (3) and (4), one arrives at the analytical values of the optimal doping density $x = 1/6 \approx 0.1667$ and charge carrier density $\rho_s \sim 8.76 \times 10^{20}/cm^3$, in reasonable agreement with the experiments ($x \sim 0.16$ and $\rho_s \sim 9 \times 10^{20}/cm$). Two other superconducting phases $LTT_{2(10,1,3)}$ of $x = 1/15$ and $LTT_{3(14,1,2)}$ of $x = 1/14$ are also analytically determined.

### C. Phase diagram

From the discussion of our results, we summarize the doping dependence of $T_c$ for LSCO in a schematic phase diagram in Fig. 14. It is well known that the antiferromagnetic Mott insulator phase is found near the origin of $La_2CuO_4$. For doping beyond a few percent, the material enters the disordered phase (spin glass). At $x = 1/18$, the material will undergo an insulator-to-metal transition, at the same time displaying superconductivity at low temperature. According to Eq. (3), the "magic effect" is possibly taking place at rational doping levels $1/4, 1/8, 1/9, 1/16$ and $1/18$, where the $LTT_{1}$ superlattice phases ($A = B$) can coexist with the $LTT_{1}$ original lattices ($a = b$) in the LSCO. In these specific situations, the paired electrons are localized, hence the corresponding charge orders appear to be completely destructive to superconductivity.

We note here that the bosonic theory predicts all magic doping fractions at $x = (2m + 1)/2n$, where $m$ and $n$ are integers, which implies the possibility of an infinite magic doping fractions in LSCO, while our theory predicts commensurate effect only at four magic doping fractions $1/4, 1/8, 1/9$ and $1/16$ (see Fig. 14). The reported measurements find a tendency towards charge ordering at four particular rational doping fractions of $1/4, 1/8, 1/9, 1/16$ and $1/18$ and is most consistent with our theoretical prediction. In view of the intriguing agreement of the experimental data with our model, it would be desirable to systematically perform direct measurements of the charge order in the underdoped LSCO materials, where the nondispersive checkerboard-type ordering with periodicity $3a \times 3a$ can be experimentally observed at the doping level $x = 1/9$.

While at $x = 1/15, 1/14, 1/7$ and $1/6$, the stable quasi-one-dimensional metallic charge stripe orders can coexist with superconductivity. Consequently, the high $T_c$ stable superconducting phases (vortex lattices) are associated with the special doping levels ($1/15, 1/14, 1/7$ and $1/6$) of LSCO, as shown in Fig. 13 and Fig. 14.
TABLE I: Lattice constants, charge carrier density, the optimal doping levels (analytical values $x = 2/hkl$ and experimental data $x'$) and the possible optimal superconducting charge-stripe phases (vortex lattices) in cuprate and MgB$_2$.

| Superconductors          | $a$ (Å) | $a'$ (Å) | $b$ (Å) | $c$ (Å) | $h$  | $k$  | $l$  | $A/C$ | Vortex phase    | $\rho_s$ ($cm^{-3}$) | $x$      | $x'$   |
|--------------------------|---------|----------|---------|---------|------|------|------|------|----------------|----------------------|----------|--------|
| La$_{2-x}$Sr$_x$CuO$_4$  | 3.79    | 3.80     | 13.25   | 12      | 1    | 1    | 3.433|       | SH2(12,1,1)    | $8.73 \times 10^{20}$ | 1/6      | 0.1667 |
| YBCO$_{124}$             | 3.84    | 3.87     | 27.24   | 7       | 1    | 1    | 0.988|       | LTT2(7,1,2)    | $7.48 \times 10^{20}$ | 1/7      |        |
| YBCO$_{247}$             | 3.85    | 3.87     | 50.29   | 13      | 1    | 1    | 0.995|       | LTT2(13,1,1)   | $3.04 \times 10^{20}$ |         |        |
| BSCCO$_{2212}$           | 3.80    | 3.80     | 30.80   | 8       | 1    | 1    | 0.989|       | LTT2(8,1,1)    | $7.33 \times 10^{20}$ |         |        |
| BSCCO$_{2223}$           | 3.80    | 3.80     | 37.82   | 10      | 1    | 1    | 1.004|       | LTT2(10,1,1)   | $3.66 \times 10^{20}$ |         |        |
| TBCO$_{2201}$            | 3.90    | 3.90     | 23.20   | 6       | 1    | 1    | 1.008|       | LTT2(6,1,1)    | $9.44 \times 10^{20}$ |         |        |
| MgB$_2$                  | 3.086   | 3.524    | 2       | 1      | 2    | 0.876|       | SH2(2,1,2)    | $1.49 \times 10^{22}$ |         |        |

VII. VORTEX LATTICES IN CUPRATE, MgB$_2$ AND PURE METALLIC SUPERCONDUCTORS

The existence of LTT2 and SH charge-stripe phases in superconductors is most likely a universal feature as shown clearly in Table I. We believe that there is an intrinsic relationship between the vortex structure and the LTT2 and SH charge-stripe phases of superconductors, in other words, they are exactly the same thing. This (see Fig. 12) may explain why in some cases the Abrikosov flux lattices are experimentally observed in conventional type II superconductors, high-$T_c$ superconductors, and MgB$_2$. From these data, it becomes evident that both hexagonal and square vortex lattices can be observed in many conventional and non-conventional superconductors. In BSCCO$_{2212}$, the results suggest a possible tetragonal LTT2(8,1,1) phase which may explain the observation of short-range vortex phase having square symmetry.

In MgB$_2$, the corresponding data show that the absolute value of the carrier density of MgB$_2$ is about two orders larger than that of YBa$_2$Cu$_3$O$_7$, as suggested by experimental studies. The analytical result confirms the existence of the hexagonal vortex lattice [SH2(2,1,2) phase] in MgB$_2$. Figure 15 shows the vortex line (charge stripe) structures in the superconducting plane (B plane) of the SH2(2,1,2) inclined hexagonal vortex lattice. The hexagonal vortex lattice possessing similar structure as Fig. 12(c) can be experimentally observed in XZ plane, it should be noted that vortex lines are non-perpendicular to the XZ plane (with a included angle 60$^\circ$). In addition, MgB$_2$ is a non-pseudogap superconductor due to a much higher charge carrier density, as shown in Table I.

To end this section, we would like to present a quali-
A tentative interpretation of the hexagonal vortex lattice and superconducting vortex lines in the pure metallic superconductors. As shown in Fig. 16, a vortex line (charge stripe) is coherently built up in the body-centered cubic lattice because of the real space confinement effect, as shown in Fig. 16 (a). To maintain a stable superconducting phase, different vortex lines should organize themselves into a periodic vortex lattice, for instance, the hexagonal vortex lattice of Fig. 16 (b).

**A. Why the good conductors are non-superconducting**

The real space confinement pictures (see Figs. 8 and 12) imply that, to be a superconductor, some periodic and stable quasi-one-dimensional “freeways” [see Fig. 16 (a)] for the superconducting electron pairs should be built naturally in the system. Any superconducting behavior is always accompanied by the formation of the vortex lattice in the materials. A higher superconducting transition temperature only mean the existence of some more stable “freeways” and vortex lattice in the superconductor. Therefore, to get higher $T_c$ superconductors, the crystal structure and the charge carrier density of the materials should be taken into account. According to the above discussions, it is obvious that a appropriate charge carrier density (not too high, not too low) is helpful for a higher $T_c$. Excess charge carrier concentrations in a material is harmful for superconductivity. As shown in Fig. 17, in a system with high concentrations of charge carriers, the crowded vortex lattice is unstable owing to the strong electromagnetic interactions between vortex lines. In this case, the charge carriers are more likely to be formed in a random and stable phase. This may explain why the good conductors (for example, Ag, Au, and Cu) and the overdoped high-$T_c$ superconductors are non-superconducting.

**VIII. CONCLUDING REMARKS AND FURTHER EXPERIMENTS**

Without Hamiltonian, without wave function, without quantum field theory, our scenario has provided a beautiful and consistent picture for describing the myriad baffling microphenomena which had previously defied explanation. The encouraging agreement of our results with the experiments implies a possibility that our theory would finally open a new window in physics. The new ideas presented in this paper may change the way we view our world. We insist that any electronic pairing and superconducting phenomena should share exactly the same physical reason. We argue that the $k$-space quasiparticle picture is very difficult to provide a convincing explanation of the superconductivity and the famous BCS theory may be incorrect. Finally, we would like to mention that many results in this paper could be verified by further experiments.

**Acknowledgments**

The author would like to thank Dr. Kezhou Xie and Dr. Ken C. Lai for many useful suggestions.
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