Hassling as Money Burning: A Note

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Abstract

Why do states sometimes permit military activities that are minor, costly, and yet inconclusive? We argue that these low-level conflicts known as “hassling” are useful commitment tactics to prevent preventive wars. While states have incentives to cooperatively and permanently refrain from hassling, they recognize that doing so might beget a preventive war due to the rising power’s inability to make appeasing offers prior to the power shift. Maintaining the hassling activities that might persist after a greater war would reduce the spoils of the victor, thus dampening the declining power’s preventive war incentive. As such, costly hassling activities are efficient in the sense that they preclude costlier large-scale wars.

1 Model

Setup. There are two states, A and D, engaging in a full-information, infinite-horizon crisis bargaining game in the shadow of hassling activities. Here we interpret D as a rising power and A as a declining one. States are endowed with the capacity to hassle. Heuristically, we conceive hassling as low-level mutually costly activities that states have to manage in a cooperative way. If one side suffers decisively from a total war but preserves its hassling capacity, then hassling activities would last forever. For example, bloodshed would continue in the form of guerrilla after a central government has been overthrown. Prior to the start of this repeated game, nature unleashes a level of hassling potential $h_0$ from the hassling capacity. $h_0$ is a random variable drawn from a commonly known $F([0, d])$ with mean $\mu$ and an upper bound $d \in (0, 1)$.
Now we describe the stage game. D begins in control of a unit size resource. States have two agendas for a typical period $t$. At the beginning of this period, if the hassling capacity has yet to be destroyed, they recognize the hassling potential $h_{t-1}$ inherited from the preceding period. They then simultaneously choose between 1) refusing to cooperate, and 2) cooperatively and irrevocably destroying the hassling capacity. If both say yes to cooperation, then hassling is eliminated now and forever, and the unit size resource remains intact. If either one of them says no, then the hassling capacity persists; hassling occurs at the level of $h_{t-1}$ and affects states’ period-$t$ payoff.

After the cooperation decision, two sides play an ultimatum game with D proposing an offer $x_t \in [0, 1]$ to A. If A accepts the offer, then the period-$t$ interaction between states ends and the payoffs realize. Otherwise, no agreement is reached and both states would declare war. A total war ensues if at least one state makes this declaration; it imposes a pair of costs $c_A$ and $c_D$ to two states, and selects a winner probabilistically. The winner takes the resource left on the table and ends the state-level interaction forever. At the end of this period, if the hassling capacity persists, nature independently draws a level of potential $h_t$ from the same distribution $F$. The game then proceeds to period $t + 1$. States interact again conditional on no war.

Hassling, if allowed, would do several things to the outcome of the ultimatum game. Conditional on peace, hassling costs a relatively minor share $\epsilon$ of the resource. We make this assumption for the salient sake of reality that either maintaining the hassling capacity or the hassling activity per se is costly. Conditional on a total war, hassling destroys a fraction $1 - h_{t-1}$ of the period-$t$ resource. Here we can take $1 - h_{t-1}$ at its face value, but we may also interpret it as the probability that valuable assets are completely destroyed (e.g. the probability of escalating into a nuclear war). Both interpretations describe chance events that would inflict a state after one expends the cost of deploying the conventional force, $c_i$. Moreover, we assume $\epsilon < 1 - d$. The most straightforward interpretation is that $\epsilon$ is the cost of maintaining the hassling capacity between two meetings of a particular period $t$, while $1 - d$ measures the total cost of hassling. Three assumptions together reflect the following ideas: hassling is more unpredictable and damaging to the assets at war than at peace, and we never know for sure if minor aggression like hassles might escalate into mutual destruction.
Let $\tau_t$ indicate if a war has occurred by period $t$ and $\chi_t$ indicate if hassling has been eliminated forever starting from period $t$. Then each state’s period $t+1$ strategy maps from history $H_t = \cup_{s \leq t}(\tau_s, \chi_s, h_{s-1}, x_s)$ to its own action described above. In case of a total war at period $t$, we label two states’ strategies as $\emptyset$ for period $t+1$ onward.

To capture power shift, we suppose that $p_1 > p_2 = p_3 = \ldots = p$ where $p_t$ is A’s war winning probability at period $t$. This assumption implies that the rising power D is more likely to win subsequent wars after period 1.

**Equilibrium concept.** Following Schram (2021), we focus on the subgame perfect equilibria (SPNE). A SPNE is peaceful if $\tau_t = 0$ for all $t$. Within the peaceful equilibria, we further identify a class of hassling equilibria in which $\chi_t \neq 0$ for some $t$; a non-hassling peaceful equilibrium may thus be viewed as a degenerate hassling equilibrium with all $\chi_t$ being 0. A SPNE is called war equilibrium if $\tau_t = 1$ for some positive $t$.

Since a state cannot unilaterally avert a total war or eliminate hassling, there always exists a “harsh” Nash equilibrium in which both sides just go straight to fight without cooperating or negotiating\(^1\). This sort of equilibrium nonetheless fails subgame perfection: anticipating a rearing total war, both sides would cooperate to eliminate hassling today to preserve the valuable asset. We thus identify a more benign war equilibrium:

**Proposition 1.** There exists a war equilibrium (that is subgame perfect): both states cooperate to eliminate hassle, and fights a total war afterwards.

This proposition is important because it identifies a “Nash reversion” type of punishment strategy that would support more complex equilibrium strategies of this repeated game.

2 Analysis

The first-best outcome that states could hope for is that they cooperate to eliminate hassling once-for-all, and strike a peaceful deal to preclude the war possibility. But the power shift at the end of period 1 may prevent this from happening, due to the classic preventive war rationale. That

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\(^1\)To fit the model description, they can do so after insincere ultimatum bargaining.
is, the rising power must appease the declining power by transferring enough resource prior to
the power shift, anticipating its constrained capacity to promise future transfers. But the rising
power’s resource constraint today may deem such a large transfer infeasible.

To illustrate why hassling may help, we look at conditions under which a non-hassling peaceful
equilibrium fails to exist, but a hassling equilibrium exists. Decompose the entire game into the
period-1 hassling-elimination decision and its continuation game $G$ thereafter. By construction, “no
war” for the game $G$ is a necessary criterion for peaceful equilibria. Then we shall 1) characterize a
set of conditions that preclude war in $G$, showing the conditions are more benign if hassling occurs
in the entire game. 2) confirm that the payoff of the conjectured hassling equilibrium is indeed
higher than that of the best war equilibrium.

We examine the no-war condition for $G$ under the following assumption:

**Assumption.** $p_1 + \frac{\delta}{1-\delta}(p_1 - p) - (c_A + \delta c_D) > 1.$

Phrase the problem in a relatively general manner. For each state within $G$, there are two sorts
of constraints that prevent a peaceful settlement. The first one is the participation constraint (IR):
each side must be willing to talk rather than triggering a preventive war before the power shift
starts. The second one is the promise keeping constraints (PK): each side must honor the promise
made on day 1, including whether to cooperate to eliminate hassle and refrain from using force for
day 2, 3, ...

For a typical period $t$, two sides’ hassling-elimination decision is a function of history $\mathcal{H}_{t-1}$
if they haven’t cooperated yet. D’s offering strategy is a mapping $x_t : \mathcal{H}_{t-1} \times \chi_t \to [0, 1]$. A’s
acceptance strategy maps from $\mathcal{H}_{t-1} \times \chi_t \times [0, 1]$ to a binary decision of rejection or acceptance.
A total war occurs conditional on no agreement. For presentation clarity, we simply use $\chi_t$ to
indicate if hassling is gone and $x_t$ the offering strategy. Define $y_t = (1 - \chi_t)(1 - \epsilon) + \chi_t$ to be
the resource being divided at peace and $z_t = (1 - \chi_t)\mu + \chi_t$ the resource being divided at war.
Following a total war at period $t$, the average per-period payoff following a war victory is fixed at
$w_t = (1 - \chi_t)\mu + \chi_t$ because the state of hassling-elimination stays there forever.
We list the constraints below.

(AIR) \[ y_1 x_1 + \sum_{i=2}^{\infty} \delta^{i-1} y_i x_i \geq p_1 [z_1 + \frac{\delta}{1-\delta} w_t] - c_A \]

(APK) \[ \forall t \geq 2, \sum_{i=t}^{\infty} \delta^{i-1} y_i x_i \geq \delta^{t-1} [p(z_t + \frac{\delta}{1-\delta} w_t) - c_A] \]

(DIR) \[ y_1 (1 - x_1) + \sum_{i=2}^{\infty} \delta^{i-1} y_i (1 - x_i) \geq (1 - p_1) [z_1 + \frac{\delta}{1-\delta} w_t] - c_D \]

(DPK) \[ \forall t \geq 2, \sum_{i=t}^{\infty} \delta^{i-1} y_i (1 - x_i) \geq \delta^{t-1} [(1 - p)(z_t + \frac{\delta}{1-\delta} w_t) - c_D] \]

Given the the issue at stake for peace and war \((y_t, z_t)\), AIR says A prefers accepting the sequence of offers \(\{x_i\}_{i \geq 1}\) to triggering the preventive war. APK says that A prefers accepting the sequence of offers \(\{x_i\}_{i \geq 2}\) to triggering a total war starting from period 2. DIR says D is willing to make the sequence of offers \(\{x_i\}_{i \geq 1}\) rather than engaging in the preventive war. DPK says that D is willing to make the sequence of stationary offers \(\{x_i\}_{i \geq 2}\) rather than engaging in war from period 2 on.

### 2.1 No peaceful equilibrium without hassling

We start by analyzing the existence conditions for a non-hassling peaceful equilibrium. Such an equilibrium has the property that \((y_t, z_t, w_t) = (1, 1, 1)\) for all \(t \geq 1\). Multiplying the DPK constraints by \(\delta\) and adding to the AIR constraint, we arrive at an inequality described by the Assumption. Therefore,

**Proposition 2.** Under the Assumption, there does not exist a non-hassling peaceful equilibrium.

It is useful to fit this proposition in the context of the canonical preventive war model. The LHS of the Assumption, \(p_1 + \frac{t}{1-\delta}(p_1 - p) - (c_A + \delta c_D)\), has two parts. One has to do with the power shift \(p_1 - p\), the other has to do with the aggregated war cost \((c_A + \delta c_D)\). The constraint says that it is harder to prevent preventive wars if the power shift is larger or if the aggregated war cost is lower.
2.2 Hassling as peace-keeper

To show hassling helps, we aim to identify a more benign condition than Assumption that would avert war in $G$, assuming hassling is on path for the entire game.

Restrict attention to strategies of the following kind. First, both sides permit hassling in period 1, and cooperate to eliminate hassling from period 2 onward. Second, the rising power D makes stationary offers from period 2 on. This means that $(y_1, z_1, w_1) = (1 - \epsilon, h_0, \mu)$ and $(y_t, z_t, w_t) = (1, 1, 1)$ for all $t \geq 2$, and $x_t = x$ for all $t \geq 2$. The peace constraints for the continuation game $G$ thus boil down to

\[
\text{(AIR)} \quad (1 - \epsilon)x_1 + \delta \frac{x}{1 - \delta} \geq p_1[h_0 + \frac{\delta}{1 - \delta}\mu] - c_A
\]
\[
\text{(APK)} \quad \frac{x}{1 - \delta} \geq \frac{p}{1 - \delta} - c_A
\]
\[
\text{(DIR)} (1 - \epsilon)(1 - x_1) + \delta \frac{1 - x}{1 - \delta} \geq (1 - p_1)[h_0 + \frac{\delta}{1 - \delta}\mu] - c_D
\]
\[
\text{(DPK)} \quad \frac{1 - x}{1 - \delta} \geq \frac{1 - p}{1 - \delta} - c_D
\]

To avert war, it must be that state D can make an offer $x_1 \in [0, 1]$ such that all constraints hold. Combining AIR and DPK,

**Proposition 3.** A necessary condition for the existence of a hassling equilibrium is $x_1 := p_1 h_0 + \frac{\delta}{1 - \delta}(\mu p_1 - p) - (c_A + \delta c_D) \leq 1 - \epsilon$.

*Proof.* The necessity follows from adding AIR to DPK multiplied by $\delta$.

Introducing the possibility of hassling does two things. On the one hand, it directly affects A’s preventive war incentive by imposing a cost to negotiated settlement $\epsilon$ and a war attrition factor $h_0$; the war incentive is enhanced directly by the quantity $\epsilon - p_1(1 - h_0)$. On the other hand, hassling reduces the war incentive via the channel of power shift. Together, hassling dampens the war incentives because even a war victory would forgo the benefit of cooperating to eliminate mutually harmful hassling activities in the future. That said, we don’t preclude the possibility
that certain levels of hassling may enhance the war incentive\(^2\).

Move to the sufficiency part. We show first that the same condition on \(x_1\) suffices to support a peaceful equilibrium in the continuation game \(G\). So far, we have not considered the deviations regarding the bilateral promise to eliminate hassle and D’s proposing strategy in the crisis bargaining. To do so, it is necessary to specify a punishing strategy from period 2 onward. The following one works:

**Definition 1.** Given a promise to end hassling and sequence of offers from period 2 onward, define a punishment strategy to be “firm” with respect to \(\hat{x}\) if both sides declare a total war whenever 1) D proposes any \(x' \neq \hat{x}\), or 2) at least one side does not cooperate to eliminate hassling.

With the firm punishment strategy in the background, we claim:

**Lemma 1.** Suppose states allow for hassling in period 1. Then \(x_1 \leq 1 - \epsilon\) is also sufficient to support a peaceful SPNE without subsequent hassling in the continuation game \(G\).

**Proof.** Set \(x_D\) to be the solution to \(\frac{1-x}{1-\delta} = \frac{1-p}{1-\delta} - c_D\) and \(x_A\) to be the solution to \(\frac{x}{1-\delta} = \frac{p}{1-\delta} - c_A\). We see that \(x_D = p + (1-\delta)c_D > x_A = p - (1-\delta)c_D\). Construct a peaceful SPNE in game \(G\) as follows: D makes an offer \(x_1 = x_A\) to A. If the game moves to period 2, both states remove hassling and D makes a stationary offer \(x^* = x_D\) to A every period thereafter. A accepts any offer \(x \geq x_D\), and triggers war if observing \(x < x_D\). Off the path, states play the firm punishment strategy.

Check the incentive constraints. AIR and DPK would bind by construction; APK holds because D offers more than \(x_A\) from period 2 on. To see why DIR holds, we add up AIR to DIR side by side (since AIR binds). The LHS of the new inequality simplifies to \((1 - \epsilon) + \frac{\delta}{1-\delta}\); the RHS simplifies to \(h_0 + \frac{\delta}{1-\delta} \mu - (c_A + c_D)\). Because \(1 - \epsilon > d > h_0\) by assumption, the LHS is clearly larger.

Next check if A or D can do better. A cannot benefit from period 2 on because the proposed strategy has held D indifferent between triggering a war and making the offer. For the same reason, D cannot benefit by proposing anything different at stage 1 because it has held A indifferent between triggering a preventive war and accepting the sequence of offer. The only deviation that D may gain is to propose some \(x' \in [x_A, x_D]\) with the hope that A may accept. This deviation

\(^2\)To illustrate, consider the following extreme example: hassling on average damages little to assets at war (\(\mu \approx d \approx 1\); A’s winning probability is bounded above by \(\hat{p}_1 := \frac{1}{1-\eta}\).
would encounter the firm punishment strategy and beg a total war, which does not benefit D. The credibility of such a punishment follows from the fact that the war-war equilibrium survives subgame perfection (see Mailath and Samuelson (2006) for a technical discussion and Schram (2021) for an application).

While the condition indeed insures a peaceful SPNE, one may nonetheless find its associated proposing strategy starting from period 2 unsatisfying; after all, it is entirely possible that the declining state A accepts a revisionist offer $x' \in [x_A, x_D)$ that is better than the war option.

We make two justifications here. Crucially, this condition establishes the limits for the existence of a peaceful equilibrium. For example, we think of two states being able to write a contract at period 1 specifying the transfers from period 2 onward that is enforced by a third party. While at first glance this sort of enforceability goes against the rationale of preventive war, it is in fact more benign. To see this, we introduce a mediator who could credibly enforce a war between two states so long as D attempts to revise the offer from period 2 on. Suppose in period 1 states have eliminated hassling forever. Even assuming enforceability, there is still no non-hassling peaceful equilibrium due to the canonical commitment problem: the declining power would ask for more than the entire pie prior to the power shift. But if instead states allow for hassling in period 1, then peace is possible thanks to the enforceability. The reason is that, hassling has relaxed D’s resource constraint to make transfers prior to the power shift; the main hurdle to sustaining peace becomes D’s wish to renegotiate for better terms from period 2 onward. Enforceability eliminates this renegotiation possibility, because any revisionist offer would encounter war enforced by the mediator. This stark comparison already indicates that hassling could be useful to prevent preventive wars.

We now further justify the model robustness by introducing a stronger requirement to SPNE that would preclude any such renegotiation possibility. Basically, renegotiation-proofness requires D’s per-period offer to exactly match that of A’s war payoff from period 2 on (which is lower than what Proposition 4 demands). This requirement does not affect the distribution of the discounted flow of payoffs from a hassling equilibrium (if it exists), but it does further constrain D’s capacity to make appeasing offers at period one. With the new solution concept, we would identify the
existence condition by binding A’s participation and promising-keeping constraints (as opposed to A’s participation and D’s promise-keeping constraints in the benchmark).

After straightforward computation, we show that

**Lemma 2.** Suppose states allow for hassling in period 1. Then\[ p_1 h_0 + \frac{\delta}{1-\delta} (\mu p_1 + p - 1) - (1+\delta)c_A \leq 1 - \epsilon \]is sufficient to support a peaceful, renegotiation-proof SPNE without subsequent hassling in the continuation game G.

The shortcut to establish a similar condition for a non-hassling peaceful equilibrium is by sending \( h_0 \) and \( \mu \) to 1. Since the LHS of the condition above is increasing in both arguments, we see again that hassling effectively dampens the preventive war incentives.

### 2.3 Hassling equilibrium: existence condition

Now we return to the question: when is it rational for states to allow for inefficient hassling activities?

Under the Assumption, a non-hassling peaceful equilibrium is impossible. If two sides cooperate to eliminate hassling, they should anticipate a preventive war. In this situation, the rising power D would expect a discounted payoff \( U_W = \frac{1-p_1}{1-\delta} - c_D \) and the declining power A would expect \( \frac{p_1}{1-\delta} - c_A \). For hassling to emerge as a part of equilibrium strategies, it must be that at least one state would expect a better outcome than war (because the other side cannot unilaterally eliminate hassling). Note that A would be kept indifferent between triggering a preventive war and playing the equilibrium strategy. This means that D is the state that would benefit more from hassling-as-money-burning. All that remains is to characterize the critical level of hassling \( h_0 \) that makes A indifferent.

To do so, we can compute the total discounted size of flow minus A’s required share. It is straightforward to show that D would obtain a discounted payoff of \( U_H = (1 - \epsilon) + \frac{\delta}{1-\delta} - [p_1(h_0 + \frac{\delta}{1-\delta}\mu) - c_A] = (1 - \epsilon - p_1 h_0) + \frac{\delta}{1-\delta}(1 - p_1 \mu) + c_A \). Then \( U_H \geq U_W \Leftrightarrow p_1(1 - h_0) - \epsilon + (c_A + c_D) + \frac{\delta}{1-\delta}(p_1 - \mu) \geq 0 \). Let \( \bar{h}_0 \) solve \( p_1(1 - h_0) - \epsilon + (c_A + c_D) + \frac{\delta}{1-\delta}(p_1 - \mu) = 0 \) and \( h_0 \) solve the \( p_1 h_0 + \frac{\delta}{1-\delta}(\mu p_1 - p) - (c_A + \delta c_D) = 1 - \epsilon \). An initial hassling potential \( h_0 \) lower than both would support a hassling equilibrium.
Proposition 4. Define $h^* = \min\{\underline{h}_0, \bar{h}_0\}$, which is strictly decreasing in $\mu$. Then a hassling equilibrium exists whenever $h_0 \leq h^*$. Ex ante, this event occurs with a probability $F(h^*)$ if $h^* < d$ and 1 if $h^* \geq d$.

We argue that an interior $h^*$ can be supported by certain parameter values. To illustrate, suppose $\epsilon$ is small and $p_1 > \mu > \frac{p}{p_1}$, which suggests that $\bar{h}_0 > 1$. We can adjust $\delta$ such that the term $\frac{\delta}{1-\delta}(\mu p_1 - p)$ can be made arbitrarily large or small. Then an interior $h_0$ follows from a continuity argument on $\delta$.

Why is hassling effective? We may understand this by appealing to the logic of preventive war. Basically, a preventive war is attractive if its rent – the war payoff prior to the power shift minus the most benign credible present value transfer – is positive. Leaving the hassling issue unsettled essentially commits the rising power to burn this rent today and tomorrow, rendering a preventive war unprofitable.

2.4 Empirical Implications

Now we elaborate two empirical implications from the analysis.

Remark 1. Hassling never persists over period 1. That is, the inefficient hassling capacity shall be destroyed no later than the start of the first crisis bargaining after the power shift.

Absent the power shift, both states would cooperate to eliminate inefficiencies due to hassling in the first period. The rationale behind hassling is that it may prevent an even more inefficient preventive war. So long as both states survive the preventive war risk and the power shift is no longer a concern, they have no reason to permit hassling activities any more.

This remark perhaps speak to Powell (2019) as to why some unsettled issues remain unsettled. Leaving them as they are deters preventive wars arising from rapid power shifts.

Remark 2. The probability of a preventive war is decreasing in the average damage of hassling, $1 - \mu$.

This observation is a corollary of Proposition 4. The deterrence effect of hassling on preventive wars comes from its future damages to the declining power. The larger the damage is, the more
likely the declining power is willing to sit down to resolve the issue no later than period 2.

Reference

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