Rotated Golden Codewords Based Space-Time Line Code Systems

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ABSTRACT The space-time line code (STLC) has been recently proposed in the literature. In STLC systems, the channel state information is assumed to be known at the transmitter, but not at the receiver. In the conventional STLC systems, the inputs of the STLC encoder are either $M$-ary phase shift keying symbols or $M$-ary quadrature amplitude modulation symbols and can achieve diversity order with very low detection complexity. In order to further improve the error performance of the STLC systems, in this paper, rotated Golden codewords based STLC (RGC-STLC) systems are proposed. We extend the $1 \times 2$ and $2 \times 2$ transmit-receive antenna structure in the conventional STLC into $1 \times 4$ and $2 \times 4$ transmit-receive antenna structures in the proposed RGC-STLC system. The inputs of the proposed RGC-STLC encoder are the rotated Golden codewords. Compared to the conventional $1 \times 2$ and $2 \times 2$ STLC systems, the $1 \times 4$ and $2 \times 4$ RGC-STLC systems achieve a higher diversity order. Based on the feature of the rotated Golden codewords, low complexity detection schemes for the $1 \times 4$ and $2 \times 4$ RGC-STLC systems are further proposed. Finally, the union bounds on the average symbol error probability for the $1 \times 4$ and $2 \times 4$ RGC-STLC systems are derived and shown to match the simulation results very well at high signal-to-noise ratios.

INDEX TERMS In-phase and quadrature maximum likelihood detection, reduced complexity detection, space-time block code, space-time line code (STLC), threshold based signal detection.

I. INTRODUCTION

There exists an ever-growing demand to increase data transmission rate and improve communication reliability in 5th and beyond-5th generation wireless communication systems. In general, data transmission rate can be increased by employing multiplexing techniques and the reliability of a wireless communication system can be improved by employing diversity techniques. Multiple-input multiple-output (MIMO) techniques can be used to increase data transmission rate and/or improve the reliability of a wireless communication system. Actually, there is a tradeoff between data transmission rate and communication reliability in terms of the diversity-multiplexing gain [1]. In this paper, the main focus is to improve on the reliability of communications in a single-input multiple-output (SIMO) or MIMO system setting.

In the literature, the encoding approach of a MIMO system enables the system to increase the data transmission rate and/or improve the reliability of a wireless communication system. For example, the space-time block code (STBC) is one type of MIMO system. A well-known STBC system with two transmit antennas is the Alamouti STBC which achieves full diversity at the receiver [2]. The encoding of the Alamouti STBC makes the codeword matrix orthogonal. The orthogonal codeword matrix further requires linearly combining the received signal to detect the transmitted symbol. Another example of an STBC is the Golden code [3]. The encoding of the conventional Golden code enables the Golden code system to simultaneously achieve diversity order and multiplexing gain. However, the maximum likelihood (ML) detection complexity of the Golden code is proportional to $O(M^4)$, which is very high.

In the above two examples, the channel state information (CSI) is assumed to be known or fully estimated at the receiver. When the CSI is fully known at the transmitter, not the receiver, the diversity order can also be achieved through the encoding of the space-time line code (STLC), which is recently proposed in [4]. In all STLC systems, the receiver only needs to linearly combine the received signals. No CSI or effective channel gain is required for detecting...
$M$-ary phase shift keying ($M$-PSK) symbols at the receiver. However, it is required to estimate the effective channel gain for detecting $M$-ary quadrature amplitude modulation ($M$-QAM) symbols. If the effective channel gain is fully estimated at the receiver, the STLC systems achieve diversity order with very low detection complexity. This is the key feature of the STLC systems.

Since the STLC was proposed, the technique has been applied in different communication systems. For example, STLC has been applied in massive MIMO and multiuser systems with antenna allocations [5]. STLC has also been applied in the uplink non-orthogonal multiple access (NOMA) random access system [6]. An STLC coded regenerative two-way relay system has been studied in [7]. Very recently, double STLC and generalized STLC have been proposed in [8] and [9], respectively. Compared to the conventional STLC, double STLC and generalized STLC provides the multiplexing gain and spatial diversity. Since the CSI is known at the transmitter, transmit antenna selection is an easy way to achieve a higher diversity order, and consequently improve error performance in an STLC system. On this note, transmit antenna selection for STLC systems and multiuser STLC has been discussed in [10] and [11], respectively.

In the literature, there are other approaches to achieve an increased diversity order. The simple way to achieve receive diversity order is that the receiver has more independent receive antennas. However, only two receive antennas are available in the STLC systems because of the special encoding of the STLC at the transmitter. Signal space diversity (SSD) is another technique to improve diversity order [12]. In SSD systems, the signal diversity is achieved by transmitting the in-phase and quadrature of the rotated multi-dimensional signal in independent fading channel. While no additional bandwidth, transmit power and receive antennas are required in SSD systems, ML detection is required to achieve optimal error performance. Recently, Golden codeword based SIMO (GC-SIMO) modulation has been proposed in [13]. In [13], an increased diversity order can be achieved through the transmission of the Golden codewords. The ML detection complexity of GC-SIMO is proportional to $O(M^2)$, where $M$ is the order of the modulation. Very recently, an alternative encoding of the Golden code has been proposed in [14]. The rotated Golden codeword is also proposed in the alternative encoding of the Golden code. Compared to the conventional Golden codeword, the rotated Golden codewords are able to convert one two-dimensional ML detection into two one-dimensional ML detections. Hence, in this paper, we propose to apply the rotated Golden codewords in the STLC system. We refer to the proposed system as a rotated Golden codewords based STLC (RGC-STLC) system. We extend $1 \times 2$ and $2 \times 2$ in the conventional STLC into $1 \times 4$ and $2 \times 4$ in the proposed RGC-STLC system. In order to achieve the optimal error performance, the in-phase and quadrature ML (IQML) detection is proposed in this paper. The IQML detection complexity of the RGC-STLC is only proportional to $O(2M)$.

The encoding of the rotated Golden codewords can be regarded as superposition encoding in the NOMA system. The threshold based signal detection (TSD) can also be applied to detect the transmitted symbols in the proposed RGC-STLC system. By aid of the TSD, in this paper, the reduced complexity IQML (RC-IQML) is further proposed. Compared to the IQML, the RC-IQML further reduces detection complexity. Typically, the detection complexity of the proposed RC-IQML is very low at high signal-to-noise ratios (SNRs). The error performance of the conventional STLC with and without transmit antenna selection has been analyzed in [10]. In order to derive a closed-form error performance expression, the union bounds on the average symbol error probability (ASEP) for the proposed $1 \times 4$ and $2 \times 4$ RGC-STLC systems are also derived in this paper.

The main contributions of this paper are summarized as:

- Both $1 \times 4$ and $2 \times 4$ RGC-STLC systems are proposed.
- The union bound of the ASEP of the proposed $1 \times 4$ and $2 \times 4$ RGC-STLC systems are derived;
- By aid of the TSD, the RC-IQML is proposed to detect the transmitted symbols for both $1 \times 4$ and $2 \times 4$ RGC-STLC systems.

The remainder of this paper is organized as follows: in Section II, the system models of the proposed $1 \times 4$ and $2 \times 4$ RGC-STLC systems are presented. The signal detection schemes of the proposed $1 \times 4$ and $2 \times 4$ RGC-STLC systems are presented in Section III. In Section IV, the union bounds of ASEP for the $1 \times 4$ RGC-STLC systems are derived. The simulation results for the $1 \times 4$ and $2 \times 4$ RGC-STLC systems are demonstrated in Section V. Finally, the paper is concluded in Section VI.

Notation: Bold lowercase and uppercase letters are used for vectors and matrices, respectively. $[\cdot]^T$ and $[\cdot]^*$ represent the transpose and conjugate operations, respectively. $(\cdot)^H$, $\cdot | \cdot$ represents Hermitian and magnitude operations, respectively. $D(\cdot)$ denotes the constellation demodulator function. Let $x$ be a complex valued symbol. $Re\{x\}$ denotes the operation to select the in-phase or real part of $x$. $Q(\cdot)$ is the Gaussian Q-function. $(\cdot)^I$, $(\cdot)^Q$ denote the in-phase and quadrature components, respectively.
[3]. The Golden code achieves full-rate and full-diversity. Its encoder takes four complex-valued symbols and generates four super-symbols. Let $x_i$ be an MQAM symbol with $E\{|x_i|^2\} = \varepsilon$, $x_i \in \Omega$, $i \in [1 : 4]$, where $\Omega$ is the signal set of MQAM. The Golden code transmission matrix is given by [3]:

$$X = [X_1 X_2] = \begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \end{bmatrix},$$  

(1)

where $x_{11} = \frac{1}{\sqrt{5}} \alpha (x_1 + x_2 \theta)$, $x_{22} = \frac{1}{\sqrt{5}} \alpha (x_1 + x_2 \bar{\theta})$, $x_{12} = \frac{1}{\sqrt{5}} \beta (x_3 + x_4 \theta)$ and $x_{21} = \frac{1}{\sqrt{5}} \beta (x_3 + x_4 \bar{\theta})$, with $\theta = \frac{1 + j\bar{\theta}}{2}$, $\bar{\theta} = 1 - \theta$, $\alpha = 1 + j\bar{\theta}$, $\bar{\alpha} = 1 + j(1 - \bar{\theta})$ and $\gamma = j$.

Since $E\{|x_i|^2\} = \varepsilon$, we have $E\{|x_i|^2\} = \varepsilon$, $i, k \in [1 : 2]$.

In (1), there are four super-symbols, $\frac{1}{\sqrt{5}} \alpha (x_1 + x_2 \theta)$, $\frac{1}{\sqrt{5}} \alpha (x_3 + x_4 \theta)$ and $\frac{1}{\sqrt{5}} \gamma \alpha (x_3 + x_4 \bar{\theta})$. In this paper, we refer to these super-symbols as the Golden codewords. These super-symbols form two pairs of Golden codewords $\{\frac{1}{\sqrt{5}} \alpha (x_1 + x_2 \theta), \frac{1}{\sqrt{5}} \alpha (x_3 + x_4 \bar{\theta})\}$ and $\{\frac{1}{\sqrt{5}} \gamma \alpha (x_3 + x_4 \bar{\theta}), \frac{1}{\sqrt{5}} \gamma \alpha (x_3 + x_4 \bar{\theta})\}$ in the conventional Golden code. In this paper, only the pair of Golden codewords $\{\frac{1}{\sqrt{5}} \alpha (x_1 + x_2 \theta), \frac{1}{\sqrt{5}} \alpha (x_1 + x_2 \bar{\theta})\}$ is applied in the proposed STLC system.

In the pair of Golden codewords, both $\alpha$ and $\bar{\alpha}$ are complex. Let $\alpha = |\alpha| e^{j\varphi_1}$ and $\bar{\alpha} = |\bar{\alpha}| e^{j\varphi_2}$.

For convenient discussion, we further let $\beta_1 = \frac{1}{\sqrt{5}} |\alpha|$ and $\beta_2 = \frac{1}{\sqrt{5}} |\bar{\alpha}|$. It is noted that $|\beta_1|^2 + |\beta_2|^2 = 1$. It is also easy to derive that $\frac{1}{\sqrt{5}} |\alpha|$ $\beta = \beta_2$. If we rotate $x_{11} = \frac{1}{\sqrt{5}} \alpha (x_1 + x_2 \theta)$ by angle $-\varphi_1$ and $x_{22} = \frac{1}{\sqrt{5}} \alpha (x_1 + x_2 \bar{\theta})$ by angle $-\varphi_2$, we then have:

$$g_1 = e^{-j\varphi_1} x_{11} = \beta_1 x_1 + \beta_2 x_2,$$

(2.1)

$$g_2 = e^{-j\varphi_2} x_{22} = \beta_2 x_1 - \beta_1 x_2.$$

Both $g_1$ and $g_2$ in (2.1) and (2.2) are referred to as rotated Golden codewords (RGC), which will be applied in the proposed STLC system.

The $N_t \times N_r$ STLC has been documented in [4], where $N_t = 1$ or $N_t = 2$ and $N_r = 2$ are the numbers of transmit antennas and the number of receive antennas, respectively. The encoding and linear combination detection (LCD) of the conventional STLC have been presented in [4]. However, [4] did not explain how the encoding and the LCD of the STLC are derived from theory. In the following subsection, we reveal how the encoding and the LCD may be derived from theory.

### B. Extending Alamouti Encoding into STLC Encoding and Decoding

Consider an $N_t \times N_r$ Alamouti scheme with $N_t = 2$ and $N_r = 1$. Let $h_{nt}, n_t \in [1 : 2]$, be the channel fading coefficient between transmit antenna $n_t$ and the receive antenna. Let $x_i, i \in [1 : 2]$ be MQAM symbols with $E\{|x_i|^2\} = \varepsilon$.

In terms of transmitting $x_1$ and $x_2$, the Alamouti codeword matrix is given by [2]:

$$X = \begin{bmatrix} x_1^* & x_2 \\ -x_2^* & x_1 \end{bmatrix}.$$  

(3)

At the receiver, the received signals are given by:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} x_1^* & x_2 \\ -x_2^* & x_1 \end{bmatrix} h_{nt} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix},$$  

(4)

where $n_i, i \in [1 : 2]$ represents the additive white Gaussian noise (AWGN) elements.

Ignoring the noise terms in (4), then (4) may be rewritten as:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ -h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix},$$  

(5)

(5) is further rewritten as:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}.$$  

(6)

In (6), let $H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$. $H$ is an orthogonal matrix. We further have $H^H H = (|h_{11}|^2 + |h_{12}|^2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Either (5) or (6) can be regarded as the encoding of the STLC system.

Suppose there is another $N_t \times N_r$ system with $N_t = 1$ and $N_r = 2$. The channel fading coefficient between the transmit antenna and receive antenna $n_r, n_r \in [1 : 2]$, is denoted as $h_{nr}$, which is known at the transmitter. (6) can be regarded as the encoding of the $1 \times 2$ system assuming that the CSI is known at the transmitter.

Multiplying $\begin{bmatrix} h_{11}^* & h_{12}^* \\ h_{21} & h_{22} \end{bmatrix} H$ in both sides of (6), we have:

$$\begin{bmatrix} h_{11}^* & h_{12}^* \\ h_{21} & h_{22} \end{bmatrix} H \begin{bmatrix} s_1^* \\ s_2 \end{bmatrix} = \begin{bmatrix} h_{11}^* & h_{12}^* \\ h_{21} & h_{22} \end{bmatrix} H \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix}.$$  

(7)

(7) is further rewritten as:

$$\begin{bmatrix} h_{11}^* s_1^* + h_{12}^* s_2 \\ h_{21} s_1^* - h_{22} s_2 \end{bmatrix} = \delta_2 \begin{bmatrix} x_1^* \\ x_2 \end{bmatrix},$$  

(8)

where $\delta_2 = |h_{11}|^2 + |h_{12}|^2$.

(8) can be regarded as the decoding or detection for the above scheme, which will be discussed in detail below.

Suppose $s_1^*$ and $s_2$ are transmitted in the above $1 \times 2$ system, the received signal $r_{n_r, n_r}$ in terms of the receive antenna $n_r$ and the transmitted symbol $s_1^*$, $i \in [1 : 2]$ are given by:

$$r_{1,1} = h_{11}^* s_1^* + n_1 s_1^*,$$

(9.1)

$$r_{1,2} = h_{12}^* s_1^* + n_1 s_2,$$

(9.2)

$$r_{2,1} = h_{21} s_1^* + n_2 s_1^*,$$

(9.3)

$$r_{2,2} = h_{22} s_1^* + n_2 s_2.$$  

(9.4)

If noise terms in (9.1) to (9.4) are ignored, then substituting (9.1) to (9.4) into (8), we have:

$$r_{1,1} + r_{2,2} = \delta_2 x_1,$$

(10.1)

$$r_{2,1} - r_{1,2} = \delta_2 x_2.$$  

(10.2)
Either \( r_{1,s_1}^x + r_{2,s_2}^x \) in (10.1) or \( r_{2,s_1}^y - r_{1,s_2}^y \) in (10.2) represents LCD. The LCD of \( x_1 \) and \( x_2 \) in (10.1) and (10.2) is the same as the ones in (16a) and (16b) in [4].

Hence, from the Alamouti orthogonal codeword matrix we have derived the encoding and decoding or detection of the STLC. In the following two subsections, we will describe in detail the proposed \( 1 \times 4 \) RGC-STLC and \( 2 \times 4 \) RGC-STLC systems.

### C. \( 1 \times 4 \) Rotated Golden Codewords Based Space-Time Line Code System

In this subsection, we only consider the \( 1 \times 4 \) RGC-STLC system. Based on (2.1) and (2.2), the rotated Golden codewords \( g_{2i-1} \) and \( g_{2i} \) are given by:

\[
g_{2i-1} = \beta_1 x_{2i-1} + \beta_2 x_{2i}, \tag{11.1}
\]

\[
g_{2i} = \beta_2 x_{2i-1} - \beta_1 x_{2i}, \tag{11.2}
\]

where \( i \in [1 : 2] \), \( x, k \in [1 : 4] \) are the squared M/QAM symbols with \( E[|x|^2] = \varepsilon \). The modulation order of the squared M-QAM is \( M \).

The proposed \( 1 \times 4 \) RGC-STLC system needs two block transmissions, where each block contains two time slots. Let \( h_i = [h_{1,2i-1} \ h_{1,2i}]^T \in \mathbb{C}^{2 \times 1} \), be the \( i \)th block channel fading vector, where \( h_{n,t} \) is defined as the channel fading coefficient between the transmit antenna \( n_t \) and the receive antenna \( n_r \). \( h_{n,t} \) is independent and identically distributed (i.i.d) complex Gaussian random variables (RVs) distributed as \( CN(0,1) \). Every \( h_{n,t} \) lasts one block transmission and takes on another independent value in the next one block transmission. In the proposed \( 1 \times 4 \) RGC-STLC system \( n_t = 1 \). Define \( \delta_{i,2} = |h_{1,2i-1}|^2 + |h_{1,2i}|^2 \), which is the sum of the channel gains in the \( i \)th block transmission. Similar to the definition in [4], \( \sqrt{\delta_{i,2}} \) is defined as the effective channel gain.

Based on (6), the encoder of the proposed \( 1 \times 4 \) RGC-STLC takes the rotated Golden codewords \( g_{2i-1}, g_{2i} \) and the channel coefficients \( h_{1,2i-1} \) and \( h_{1,2i} \) as inputs, and outputs RGC-STLC symbols, \( s_{1} \) and \( s_{2} \), which are given by:

\[
s_1 = \frac{\mathbb{E}[s_1]}{\sqrt{\delta_{1,2}}}, \tag{12.1}
\]

\[
s_2 = \frac{\mathbb{E}[s_2]}{\sqrt{\delta_{2,2}}}, \tag{12.2}
\]

\[
s_1 = \frac{\mathbb{E}[s_1]}{\sqrt{\delta_{1,2}}}, \tag{12.3}
\]

\[
s_2 = \frac{\mathbb{E}[s_2]}{\sqrt{\delta_{2,2}}}. \tag{12.4}
\]

It takes two block transmissions to transmit RGC-STLC symbols in the proposed \( 1 \times 4 \) RGC-STLC system. As discussed earlier, each block transmission contains two time slots. If \( s_{k} \) is transmitted in the \( k \)th time slot of the \( i \)th block transmission, then at receive antenna \( 2i-1 \) and \( 2i \) the received signals \( z_{k,2i-1} \) and \( z_{k,2i} \) are given by:

\[
z_{k,2i-1} = h_{1,2i-1}s_k + n_{k,2i-1}, \tag{13.1}
\]

\[
z_{k,2i} = h_{1,2i-1}s_k + n_{k,2i-0}, \tag{13.2}
\]

where \( n_{k,2i-1} \) and \( n_{k,2i-0} \) are distributed as \( CN(0,\rho) \), where \( \rho \) is the SNR at each receive antenna in the proposed \( 1 \times 4 \) RGC-STLC system.

### D. \( 2 \times 4 \) Rotated Golden Codewords Based Space-Time Line Code System

In this subsection, we consider the \( 2 \times 4 \) RGC-STLC system. The definitions of the channel fading vector, the effective channel gain and the rotated Golden codewords \( g_{2i-1} \) and \( g_{2i} \) are exactly the same as those in the \( 1 \times 4 \) RGC-STLC system. Again, define \( \delta_{i,4} = \sum_{n_r=1}^{2} \sum_{n_t=1}^{2} |h_{n_r,n_t}|^2 \), which is the sum of the channel gains in the \( i \)th block transmission.

Then, based on (6), the encoder of the proposed \( 2 \times 4 \) RGC-STLC takes the rotated Golden codewords \( g_{2i-1}, g_{2i} \) and the channel coefficients \( h_{n_t,(i-1)\times2+n_r}, n_t \in [1 : 2], n_r \in [1 : 2] \) as inputs, and outputs two pairs of RGC-STLC symbols, \( s_{1} \) and \( s_{2} \), which are given by:

\[
s_{1} = \frac{\mathbb{E}[s_1]}{\sqrt{\delta_{1,4}}}, \tag{14.1}
\]

\[
s_{2} = \frac{\mathbb{E}[s_2]}{\sqrt{\delta_{2,4}}}. \tag{14.2}
\]

\[
s_{1} = \frac{\mathbb{E}[s_1]}{\sqrt{\delta_{1,4}}}, \tag{14.3}
\]

\[
s_{2} = \frac{\mathbb{E}[s_2]}{\sqrt{\delta_{2,4}}}. \tag{14.4}
\]

Transmit antenna \( n_t \) transmits \( s_{1} \) and \( s_{2} \) during time slot 1 and time slot 2 of the \( i \)th block transmission, respectively. In the proposed \( 2 \times 4 \) RGC-STLC system, it takes four time slots to transmit two pairs of RGC-STLC symbols for each transmit antenna. During time slot \( k \) of the \( i \)th block transmission if transmit antenna \( n_t = 1 \) transmits \( s_{1} \) and transmit antenna \( n_t = 2 \) transmits \( s_{2} \), then at receive antennas \( 2i-1 \) and \( 2i \) the received signals \( z_{k,2i-1} \) and \( z_{k,2i} \) are given by:

\[
z_{1,2i-1} = h_{1,2i-1}s_{1} + h_{2,2i-1}s_{2} + n_{1,2i-1}, \tag{15.1}
\]

\[
z_{1,2i} = h_{1,2i}s_{1} + h_{2,2i}s_{2} + n_{1,2i-0}, \tag{15.2}
\]

\[
z_{2,2i-1} = h_{1,2i}s_{1} + h_{2,2i-1}s_{2} + n_{2,2i-1}, \tag{15.3}
\]

\[
z_{2,2i} = h_{1,2i}s_{1} + h_{2,2i}s_{2} + n_{2,2i-0}. \tag{15.4}
\]

where \( r = (i - 1) \times 2, n_{1,2i-1} \) and \( n_{1,2i-0} \) are exactly the same as those in (13.1) and (13.2) in the \( 1 \times 4 \) RGC-STLC system.

### III. Signal Detection Schemes for the \( 1 \times 4 \) and \( 2 \times 4 \) RGC-STLC Systems

Similar to the discussion in [4], at the receiver the effective channel gain is required to be estimated for detecting
MQAM symbols. In this paper, we assume that the effective channel gain is perfectly estimated at the receiver. The signal detection schemes of the $2 \times 4$ RGC-STLC systems are the same as the ones of $1 \times 4$ RGC-STLC systems except for different effective channel gain. The effective channel gain is $\delta_{k,2}, k \in [1 : 2]$ in the $1 \times 4$ RGC-STLC systems, while the effective channel gain is $\delta_{k,4}, k \in [1 : 2]$ in $2 \times 4$ RGC-STLC systems. Thus, in this section, we mainly focus on the signal detections for the $1 \times 4$ RGC-STLC systems.

A. LINEAR COMBINATION DETECTION FOR $1 \times 4$ RGC-STLC SYSTEM

LCD for the conventional STLC has been discussed in [4]. In Subsection II.A, we revealed how the LCD is derived in (10.1) and (10.2) from theory. In this subsection, we will apply (10.1) and (10.2) to detect the transmitted symbols in the proposed $1 \times 4$ RGC-STLC system. The received signals in (13.1) and (13.2) in the $1 \times 4$ RGC-STLC system can be further rewritten as:

\[
\begin{align*}
  z_{k,1} &= h_{1,1} r_{k,1}^1 + n_{k,1}, \\
  z_{k,2} &= h_{1,2} r_{k,2}^1 + n_{k,2}, \\
  z_{k,3} &= h_{1,3} r_{k,3}^1 + n_{k,3}, \\
  z_{k,4} &= h_{1,4} r_{k,4}^1 + n_{k,4},
\end{align*}
\]

where $k \in [1 : 2]$.

Based on the LCD, we further have:

\[
\begin{align*}
  r_1 &= \sqrt{\frac{1}{2} g_{11}} + n_{1,1} + n_{2,2}^*, \\
  r_2 &= \sqrt{\frac{1}{2} g_{12}} + n_{1,3} + n_{2,4}^*, \\
  r_3 &= \sqrt{\frac{1}{2} g_{21}} + n_{1,2}^* - n_{2,1}, \\
  r_4 &= \sqrt{\frac{1}{2} g_{22}} + n_{1,4}^* - n_{2,3},
\end{align*}
\]

where $r_1 = z_{1,1} + z_{2,2}^*$, $r_2 = z_{1,3} + z_{2,4}^*$, $r_3 = z_{1,2}^* - z_{2,1}$, and $r_4 = z_{1,4}^* - z_{2,3}$.

Let $w_1 = n_{1,1} + n_{2,2}^*$, $w_2 = n_{1,3} + n_{2,4}^*$, $w_3 = n_{1,2}^* - n_{2,1}$, and $w_4 = n_{1,4}^* - n_{2,3}$. $w_i, i \in [1 : 4]$, are distributed as $CN(0, \frac{2}{\rho})$.

Let $g_{11} = r_1/\sqrt{\delta_{11}^2}$, $g_{12} = r_2/\sqrt{\delta_{12}^2}$, $g_{21} = r_3/\sqrt{\delta_{21}^2}$, and $g_{22} = r_4/\sqrt{\delta_{22}^2}$. Based on the encoding of rotated Golden codewords in (11.1) and (11.2), the linearly combined signals are expressed as:

\[
\begin{align*}
  r_{x1} &= x_1 + \frac{\beta_1}{\sqrt{\delta_{11}}} w_1 + \frac{\beta_2}{\sqrt{\delta_{12}}} w_2, \\
  r_{x2} &= x_2 + \frac{\beta_1}{\sqrt{\delta_{11}}} w_1 - \frac{\beta_2}{\sqrt{\delta_{12}}} w_2, \\
  r_{x3} &= x_3 + \frac{\beta_1}{\sqrt{\delta_{11}}} w_3 + \frac{\beta_2}{\sqrt{\delta_{12}}} w_4, \\
  r_{x4} &= x_4 + \frac{\beta_1}{\sqrt{\delta_{11}}} w_3 - \frac{\beta_2}{\sqrt{\delta_{12}}} w_4,
\end{align*}
\]

where $r_{x1} = \beta_1 r_{g1} + \beta_2 r_{g2}$, $r_{x2} = \beta_2 r_{g1} - \beta_1 r_{g2}$, $r_{x3} = \beta_1 r_{g3} + \beta_2 r_{g4}$, and $r_{x4} = \beta_2 r_{g3} - \beta_1 r_{g4}$. In the derivation of (18.1) to (18.4), we apply $\beta_1^2 + \beta_2^2 = 1$.

Finally, the estimation of the transmitted symbols are given by:

\[
\hat{x}_i = D(r_{xi}).
\]

B. IN-PHASE AND QUADRATURE MAXIMUM LIKELIHOOD DETECTION

The above LCD algorithm also has very low detection complexity. However, the above detection scheme cannot achieve full diversity from $\delta_{1,2}$ and $\delta_{2,2}$ in the RGC-STLC system. In this subsection, we propose a detection scheme, IQML to achieve full diversity from $\delta_{1,2}$ and $\delta_{2,2}$.

Since $\beta_1$ and $\beta_2$ are real, (11.1) and (11.2) can be rewritten as:

\[
\begin{align*}
  g_{2i-1} &= (\beta_1 x_{2i-1}^I + \beta_2 x_{2i-1}^Q) + j(\beta_1 x_{2i-1}^Q + \beta_2 x_{2i-1}^I), \\
  g_{2i} &= (\beta_2 x_{2i}^I - \beta_1 x_{2i}^Q) + j(\beta_2 x_{2i}^Q - \beta_1 x_{2i}^I).
\end{align*}
\]

Let $g_i = g_i^I + j g_i^Q$, $x_i = x_i^I + j x_i^Q$, $i \in [1 : 4]$. (20.1) and (20.2) can be further rewritten as:

\[
\begin{align*}
  g_{2i-1}^p &= \beta_1 x_{2i-1}^p + \beta_2 x_{2i-1}^p, \\
  g_{2i}^p &= \beta_2 x_{2i}^p - \beta_1 x_{2i}^p,
\end{align*}
\]

where $p \in [I, Q]$.

Since only squared $MQAM$ is taken into account in this paper, we have $K = \sqrt{M}$. Let $\Omega_K^p$ be the signal set of $x_i^p$ and $\Omega_M^p$ be the signal set of $g_i^p$. The cardinalities of $\Omega_K^p$ and $\Omega_M^p$ are $K$ and $M$, respectively. Then in (21.1) and (21.2), $x_i^p \in \Omega_K^p$ and $g_i^p \in \Omega_M^p$.

The linearly combined signals in (17.1) to (17.4) can be rewritten as:

\[
\begin{align*}
  r_1^p &= \sqrt{\frac{1}{2} g_{11}^p} (\beta_1 x_1^p + \beta_2 x_2^p) + w_1^p, \\
  r_2^p &= \sqrt{\frac{1}{2} g_{12}^p} (\beta_2 x_1^p - \beta_1 x_2^p) + w_2^p, \\
  r_3^p &= \sqrt{\frac{1}{2} g_{21}^p} (\beta_1 x_3^p + \beta_2 x_4^p) + w_3^p, \\
  r_4^p &= \sqrt{\frac{1}{2} g_{22}^p} (\beta_2 x_3^p - \beta_1 x_4^p) + w_4^p.
\end{align*}
\]

The IQML detection of the transmitted $x_{2i-1}$ and $x_{2i}$ can be expressed as:

\[
[x_{2i-1} \hat{x}_{2i}] = \min_{x_{2i-1}, x_{2i}} (d_{p,2i-1}^p + d_{p,2i}^p),
\]

where

\[
\begin{align*}
  d_{p,2i-1}^p &= |r_{2i-1}^p - \sqrt{\frac{1}{2} g_{11}^p} (\beta_1 x_{2i-1}^p + \beta_2 x_{2i-1}^p)|^2, \\
  d_{p,2i}^p &= |r_{2i}^p - \sqrt{\frac{1}{2} g_{22}^p} (\beta_2 x_{2i}^p - \beta_1 x_{2i}^p)|^2.
\end{align*}
\]

The above IQML detection does achieve full diversity from $\delta_{1,2}$ and $\delta_{2,2}$, however the detection complexity in terms of two absolute operations is proportional to $O(M)$. In order to meet the aim of STLC, low complexity detection, we propose the RC-IQML detection in the following subsections.

C. THRESHOLD BASED SIGNAL DETECTION

Both $g_{2i-1}^p$ and $g_{2i}^p$ in (11.1) and (11.2) can be regarded as a pair of superposition encodings in NOMA systems. The TSD has been proposed to detect the transmitted symbols [15].
this subsection, we apply the TSD to detect \( x_{2i-1} \) and \( x_{2i} \).

Let \( i = 1 \), then (11.1) and (11.2) can be rewritten as:

\[
\hat{g}_1^i = \beta_1 x_{2i}^1 + \beta_2 x_{2i}^2, \tag{25.1}
\]

\[
\hat{g}_2^i = \beta_3 x_{2i}^3 - \beta_1 x_{2i}^1, \tag{25.2}
\]

where \( x_{2i}^p \in \Omega_K^p \) and \( g_{2i}^p \in \Omega_M^p \).

Given \( x_{2i}^i \in \Omega_K^p, i \in [1 : 2] \), based on either \( g_1^i \) in (25.1) or \( g_2^i \) in (25.2), \( \Omega_M^p \) can be constructed. Suppose \( \Omega_K^p = \Omega_{16}^p = [-3, -1, 1, 3] \) then we have \( \Omega_M^p = \Omega_{16}^p = [-4.129, -3.078, -2.428, -2.026, -1.376, -0.975, -0.727, -0.325, 0.257, 0.727, 0.975, 1.376, 2.026, 2.428, 3.078, 4.129] \).

For convenient discussion, we let \( x_1^i = -3, \ldots, x_4^i = 3 \) in the signal set \( \Omega_4^p \) and \( z_1^g = -4.129, \ldots, z_4^g = 4.129 \) in the signal set \( \Omega_{16}^p \). In \( \Omega_{16}^p \), it is easily seen that \( z_k^g \neq z_k^g \) for \( k_1 \neq k_2 \).

Suppose \( z_k^g \) is transmitted over an AWGN channel, which is given by:

\[
y = z_k^g + n, \tag{26}
\]

where \( n \) is distributed as \( CN(0, \sigma^2) \).

At the receiver, the transmitted symbol in (26) can be estimated by using the TSD. The thresholds play a key role in the TSD. Let \( t_{th}^k, k \in [1 : M - 1] \), represent these thresholds. These thresholds \( t_{th}^k \) are easily derived as:

\[
t_{th}^k = \frac{z_k^g + z_{k+1}^g - z_k^g}{2}. \tag{27}
\]

For convenient discussion, we set \( t_{th}^0 = -\infty \) and \( t_{th}^M = +\infty \). Now we apply the TSD in (27) to estimate the transmitted symbol. Suppose \( k_{th}^k < y < t_{th}^k \). Then the TSD estimates the transmitted signal as \( z_k^g \).

As discussed in [15], the superposition coding of (25.1) and (25.2) can be alternatively expressed as encoding functions \( f_1 \) and \( f_2 \). Given \( x_{2i}^p = z_{2i}^p, i \in [1 : 2] \) then \( g_1^i = z_1^g \) and \( g_2^i = z_2^g \) are expressed as:

\[
z_1^g = f_1 \left( z_{1i}^1, z_{1i}^2 \right), \tag{28.1}
\]

\[
z_2^g = f_2 \left( z_{1i}^1, z_{1i}^2 \right), \tag{28.2}
\]

where \( z_1^g, z_2^g \in \Omega_M^p \).

The signal detection at the receiver can be regarded as two inverse functions \( f_1^{-1} \) and \( f_2^{-1} \). Suppose that the \( g_1^i \) and \( g_2^i \) are estimated as \( \hat{g}_1^i = z_1^g \) and \( \hat{g}_2^i = z_2^g \) at the receiver, then the estimation of \( x_{2i}^p, i \in [1 : 2] \) can be expressed as two inverse functions \( f_1^{-1} \) and \( f_2^{-1} \), which are given by:

\[
\hat{x}_{2i}^1 = f_1^{-1} \left( z_{1i}^1 \right), \tag{29.1}
\]

\[
\hat{x}_{2i}^2 = f_2^{-1} \left( z_{1i}^2 \right). \tag{29.2}
\]

\[D. \text{ REDUCED COMPLEXITY IQML DETECTION}\]

In this subsection, we propose a RC-IQML algorithm. From the previous subsection, it is easily seen that the transmitted \( x_{2i}^p \) can be estimated based on either \( f_1^{-1} \) in (29.1) or \( f_2^{-1} \) in (29.2). If \( \hat{x}_{2i}^1 = \hat{x}_{2i}^2 \), the probability of transmitting \( x_{2i}^p = \hat{x}_{2i}^p \) is very large. If both \( \hat{x}_{2i}^1 = \hat{x}_{2i}^2 \) and \( \hat{x}_{2i}^1 = \hat{x}_{2i}^2 \), we directly output the estimate of the transmitted \( x_{2i}^p \) as \( \hat{x}_{2i}^p \).

and \( x_{2i}^p \) as \( \hat{x}_{2i}^p \). If \( \hat{x}_{2i}^1 \neq \hat{x}_{2i}^2 \), we perform IQML to estimate the transmitted \( x_{2i}^p \). The implementation of the RC-IQML is shown in Algorithm 1. In Algorithm 1, we only detect the transmitted symbols \( x_1 \) and \( x_2 \) by use of the RC-IQML. Similarly, we can also detect the transmitted symbols \( x_3 \) and \( x_4 \) by use of the RC-IQML.

\[\text{Algorithm 1: Implementation of the RC-IQML.}\]

\[\text{Input:\}\]

(1): \( r_1^p \) in (22.1) and \( r_2^p \) in (22.2);

(2): \( \beta_1 \) and \( \beta_2 \);

(3): Estimation of transmitted \( x_i; \hat{x}_{2i}^1, \hat{x}_{2i}^2 \) and \( \hat{x}_i, i \in [1 : 2] \), and \( \Omega_{16}^p \).

\[\text{Output:\}\]

Finally estimated \( \hat{x}_i, i \in [1 : 2] \).

Clear \( \Omega_1^1, \Omega_2^2 \); for \( i \leftarrow 1 \) to 2 do

if \( \hat{x}_i = \hat{x}_i^2 \) then \( \Omega_i^1 = \hat{x}_i^1 \); else

\( \Omega_{x} = \Omega_{16}^p \); end if

end for

\( \hat{x}_1 = \hat{x}_1^1 \); \( \hat{x}_2 = \hat{x}_2^1 \); end if

\[\text{end if}\]

end if

\[\text{end if}\]

\[\text{Algorithm 1: Implementation of the RC-IQML.}\]

In this section, we focus on the derivation of the union bound on the ASEP of the \( 1 \times 4 \) and \( 2 \times 4 \) RGC-STLC systems. We will derive the ASEP union bound for \( 1 \times 4 \) and \( 2 \times 4 \) RGC-STLC systems in Subsection IV.A and Subsection IV.B, separately.

\[\text{A. THE ASEP UNION BOUND OF THE } 1 \times 4 \text{ RGC-STLC SYSTEM}\]

The derivation of the ASEP union bound for the \( 1 \times 4 \) RGC-STLC system is based on one pair of received signals, either (17.1) and (17.2) or (17.3) and (17.4). In this section, (17.1) and (17.2) will be used to derive the ASEP union bound. Both (17.1) and (17.2) are rewritten as:

\[
r_1 = \sqrt{\beta_1 \left( \beta_1 x_1 + \beta_2 x_2 \right) + w_1}, \tag{30.1}
\]

\[
r_2 = \sqrt{\beta_2 \left( \beta_2 x_1 - \beta_1 x_2 \right) + w_2}. \tag{30.2}
\]

where \( w_i, i \in [1 : 4] \), are distributed as \( CN(0, \sigma^2) \).

Using (30.1) and (30.2), either \( x_1 \) or \( x_2 \) may be taken into account in the following derivation. We use only \( x_1 \) to derive
the union bound of the symbol error probability. Let \( p_x(e) \) be the ASEP of \( x_1 \) during the transmission. Since there are \( M \) signal values of \( x_1, p_x(e) \) may be expressed as:

\[
p_x(e) = \sum_{k_1=1}^{M} p(e|x_1 = z_{x_1}^{k_1})p(x_1 = z_{x_1}^{k_1}),
\]

(31)

where \( p(e|x_1 = z_{x_1}^{k_1}) \) is the conditional error probability when \( x_1 = z_{x_1}^{k_1} \) is transmitted. \( p(x_1 = z_{x_1}^{k_1}) \) is the probability for \( x_1 = z_{x_1}^{k_1} \).

If \( x_1 = z_{x_1}^{k_1} \) were directly transmitted, \( p(e|x_1 = z_{x_1}^{k_1}) \) would be easily derived. However, the proposed RGC-STLC system transmits either \( g_1 = \beta_1 x_1 + \beta_2 x_2 \) or \( g_2 = \beta_2 x_1 - \beta_1 x_2 \), not \( x_1 \). At the receiver only linearly combined received signals \( \sqrt{\delta_1} g_1 + w_1 \) and \( \sqrt{\delta_2} g_2 + w_2 \) are known, and \( x_1 + n_1 \) is not known. Thus, we need to derive the symbol error performance of the proposed RGC-STLC system based on the linearly combined received signals \( \sqrt{\delta_1} g_1 + w_1 \) and \( \sqrt{\delta_2} g_2 + w_2 \).

Based on the encoding of the rotated Golden codewords in (11.1) and (11.2), transmitting \( x_1 = z_{x_1}^{k_1} \) is equivalent to transmitting one pair of \( M \) rotated Golden codewords \( (g_1, g_2) \), where \( g_1 = \beta_1 x_1 + \beta_2 x_2 \) and \( g_2 = \beta_2 x_1 - \beta_1 x_2 \), \( x_1 = Z_{k_1}, x_2 \in \Omega \). Then \( p(e|x_1 = z_{x_1}^{k_1}) \) in (31), can be expressed as:

\[
p(e|x_1 = z_{x_1}^{k_1}) = \sum_{k_2=1}^{M} p(e|g_1, g_2)p(x_2 = z_{x_2}^{k_2}).
\]

(32)

Let \( g = (g_1, g_2) \) and \( \hat{g} = (\hat{g}_1, \hat{g}_2) \). The pairwise error probability (PEP) \( P(g \rightarrow \hat{g}) \) is defined as \( g \) which is detected as \( \hat{g} \) at the receiver. Then the \( p(e|g_1, g_2) \) in (32) is the PEP \( P(g \rightarrow \hat{g}) \). For \( p(x_1 = z_{x_1}^{k_1}) = \frac{1}{M} \) and \( p(x_2 = z_{x_2}^{k_2}) = \frac{1}{M} \), \( p_x(e) \) may be further expressed as:

\[
p_x(e) = \frac{1}{M^2} \sum_{k_1=1}^{M} \sum_{k_2=1}^{M} P(g \rightarrow \hat{g}).
\]

(33)

The conditional PEP \( P(g \rightarrow \hat{g}|h_1, h_2) \) is given by:

\[
P(g \rightarrow \hat{g}|h_1, h_2) = P(A \geq B),
\]

(34)

where

\[
A = |r_1 - \sqrt{\delta_1} g_1|^2 + |r_2 - \sqrt{\delta_2} g_2|^2;
\]

\[
B = |r_1 - \sqrt{\delta_1} \hat{g}_1|^2 + |r_2 - \sqrt{\delta_2} \hat{g}_2|^2.
\]

Substituting (30.1) and (30.2) into \( A \) and \( B \) we further have:

\[
A = |w_1|^2 + |w_2|^2;
\]

\[
B = |\sqrt{\delta_1} d_1 + w_1|^2 + |\sqrt{\delta_2} d_2 + w_2|^2.
\]

where \( d_1 = g_1 - \hat{g}_1 \) and \( d_2 = g_2 - \hat{g}_2 \).

\( P(A \geq B) \) in (34) is further derived as:

\[
P(A \geq B) = P(C \geq D),
\]

(35)

where

\[
C = Re\{\sqrt{\delta_1} d_1^* w_1\} + Re\{\sqrt{\delta_2} d_2^* w_2\};
\]

\[
D = \frac{1}{2}|\sqrt{\delta_1} d_1|^2 + |\sqrt{\delta_2} d_2|^2.
\]

\( C \) is the equivalent AWGN component distributed as \( N(0, \{\delta_1|d_1|^2 + \delta_2|d_2|^2\}) \).

Finally, we have:

\[
P(g \rightarrow \hat{g}|h_1, h_2) = P(A \geq B) = Q\left(\frac{P}{4\delta_1|d_1|^2 + \delta_2|d_2|^2}\right).
\]

(36)

Define the instantaneous SNR at the receive antenna \( l \) during time slot \( i \) as \( \gamma_{i,l} = \{|h_{i,l}|^2\}/\rho \). Since \( h_{i,l} \) are i.i.d complex Gaussian RVs distributed as \( CN(0, 1) \) we have \( \gamma_{i,l} \). Let \( \gamma_i = \gamma_{i,1} + \gamma_{i,2} \). Then the PDF of \( \gamma_i \) is given by [15]:

\[
f_{\gamma_i}(\gamma_i) = \frac{1}{2\gamma_i^2} e^{-\gamma_i},
\]

(37)

where \( \tilde{\gamma}_i = \gamma_{i,1} \).

Finally, the PEP \( P(g \rightarrow \hat{g}) \) is given by:

\[
P(g \rightarrow \hat{g}) = \int_{0}^{+\infty} \int_{0}^{+\infty} P(g \rightarrow \hat{g}|h_1, h_2) f_{\gamma_1}(\gamma_1) f_{\gamma_2}(\gamma_2) d\gamma_1 d\gamma_2.
\]

(38)

In order to derive a closed-form expression for PEP \( P(g \rightarrow \hat{g}) \), we need to approximate the Gaussian Q-function in (38) for integration. Taking into account conciseness of the ASEP expressions and approximation accuracy, we approximate \( Q(\cdot) \) in (36) using the trapezoidal rule as:

\[
Q(x) \approx \frac{1}{2} e^{-\frac{x^2}{2}} + \sum_{k=1}^{c} e^{-\frac{x^2}{2k}},
\]

(39)

where \( s_k = \sin^2(2\pi/k) \) and \( c \) is the number of partitions for the integration in the Q-function.

Substituting (37) and (39) into (38), \( P(g \rightarrow \hat{g}) \) may be derived as:

\[
P(g \rightarrow \hat{g}) = \frac{1}{4\sqrt{\pi} c} \left( \prod_{k=1}^{2} \left( \frac{1 + \frac{\rho c}{\delta_k}}{\delta_k} \right)^2 \right) + \sum_{k=1}^{c-2} \left( \prod_{k=1}^{c-2} \left( \frac{1 + \frac{\rho c}{\delta_k}}{\delta_k} \right)^2 \right).
\]

(40)

B. THE ASEP UNION BOUND OF THE 2 x 4 RGC-STLC SYSTEM

(30.1) and (30.2) are the equivalent received signal models for the 1 x 4 RGC-STLC ASEP union bound analysis. Since the signal detection schemes of the 2 x 4 RGC-STLC systems are the same as the ones of the 1 x 4 RGC-STLC systems except for different effective channel gain based on (30.1) and (30.2), the equivalent received signal models for the 2 x 4 RGC-STLC ASEP union bound analysis is given by:

\[
r_1 = \sqrt{\delta_1} (\beta_1 x_1 + \beta_2 x_2) + w_1,
\]

(41.1)

\[
r_2 = \sqrt{\delta_2} (\beta_2 x_1 - \beta_1 x_2) + w_2.
\]

(41.2)

In (30.1) and (30.2), \( \delta_{1,2} = |h_{1,2}|^2 - |h_{1,2}|^2 \), while in (41.1) and (41.2), \( \delta_{1,4} = \sum_{n=1}^{2} \sum_{n'=1}^{2} |h_{n,n'}|^2 \). It is easily seen that the diversity order of \( \delta_{1,2} \) is 2, while the diversity order of \( \delta_{1,4} \) is 4. Similar to the error performance
analysis for the $1 \times 4$ RGC-STLC system, (33) is also the union bound on the ASEP for the $2 \times 4$ RGC-STLC system. But the $P(\mathbf{g} \rightarrow \hat{\mathbf{g}})$ for $2 \times 4$ RGC-STLC system is given by:

$$P(\mathbf{g} \rightarrow \hat{\mathbf{g}}) = \frac{1}{4c} \left[ \prod_{k=1}^{2} \left( 1 + \frac{\rho d_k}{8 c} \right)^4 + 2 \sum_{c=1}^{2} \prod_{k=1}^{2} \left( 1 + \frac{\rho \delta_k}{8 c} \right)^4 \right]. \quad (42)$$

where the definition of $\rho$ is the same as earlier.

V. SIMULATION RESULTS

In this section, we present the simulation results for the proposed $1 \times 4$ and $2 \times 4$ RGC-STLC systems for 16QAM and 64QAM. As discussed in Section II system model, it is assumed that the CSI is fully known at the transmitter. It is also assumed that the effective channel gains are known at the receiver. Rayleigh frequency-flat fading channels with AWGN, as described in (13.1) and (13.2) are considered in all simulations. For comparison, we also simulate the conventional $1 \times 2$ and $2 \times 2$ STLC systems.

In all figures, the legend, "$N_t \times 2$ STLC $M$QAM stands for conventional $N_t \times 2$ STLC $M$QAM; the legend, "$N_t \times 4$ RGC-STLC $M$QAM", stands for the proposed $N_t \times 4$ RGC-STLC $M$QAM.

In this section, we firstly discuss the detection complexity of the IQML and RC-IQML, then analyze the error performance of the proposed RGC-STLC systems, and finally compare the error performance between the proposed RGC-STLC system and the conventional STLC system.

A. DETECTION COMPLEXITY ANALYSIS

As discussed in Section III.C, the IQML achieves optimal error performance with detection complexity $M$, where detection complexity is in terms of the number of absolute operations per symbol in either (24.1) or (24.2). However, it is difficult to quantify the number of absolute operations in the RC-IQML detection. We analyze the detection complexity for the RC-IQML detection through simulations.

FIGURE 1: Detection complexity of RGC-STLC 16QAM vs SNR

FIGURE 2: Detection complexity of RGC-STLC 64QAM vs SNR

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B. ERROR PERFORMANCE ANALYSIS

In this subsection, we analyze the symbol error performance of the proposed RGC-STLC systems. FIGUREs 3 and 4 show the error performance of the $N_t \times 4$ RGC-STLC with 16QAM and 64QAM, respectively. The ASEP union bounds of the proposed $N_t \times 4$ RGC-STLC $M$QAM in (33), are also shown
in FIGUREs 3 and 4, respectively. From FIGUREs 3 and 4, it is observed that:

1) Compared to the $1 \times 4$ RGC-STLC $M$QAM systems, $2 \times 4$ RGC-STLC $M$QAM systems achieve at least 7 dB at the symbol error rate (SER) of $2 \times 10^{-5}$;

2) Until the SER of $2 \times 10^{-5}$, the RC-IQML achieves the error performance of IQML;

3) The ASEP union bounds match the simulated SER at high SNRs. Typically, the ASEP union bounds of $2 \times 4$ RGC-STLC $M$QAM systems overlap the simulated SER at high SNRs.

![FIGURE 4: Error performance of the RGC-STLC 64QAM vs SNR](image)

![FIGURE 5: Error performance comparison between RGC-STLC and STLC 16QAM](image)

C. RGC-STLC $M$QAM VS CONVENTIONAL STLC $M$QAM

In this subsection, we compare the symbol error performance between the proposed $N_t \times 4$ RGC-STLC $M$QAM systems and the conventional $N_t \times 2$ STLC $M$QAM systems. FIGUREs 5 and 6 show the error performance comparison between the proposed RGC-STLC and the conventional STLC with 16QAM and 64QAM, respectively. In the following discussion, all comparisons are at the SER of $2 \times 10^{-5}$. From FIGUREs 5 and 6 it is observed that:

1) Compared to the $1 \times 2$ STLC $M$QAM systems, $1 \times 4$ RGC-STLC $M$QAM systems achieve at least 7 dB. However, $2 \times 4$ RGC-STLC $M$QAM systems achieves only around 3 dB compared to $2 \times 2$ STLC $M$QAM systems;

2) Simulation results in FIGUREs 5 and 6 validate that both $2 \times 2$ STLC $M$QAM systems and $1 \times 4$ RGC-STLC $M$QAM systems achieve the same diversity order.

![FIGURE 6: Error performance comparison between RGC-STLC and STLC 64QAM](image)

VI. CONCLUSION

In order to further improve the error performance of the STLC system, in this paper, the RGC-STLC system was proposed. The configuration of two receive antennas in the conventional STLC system was extended to the configuration of four receive antennas in the proposed RGC-STLC system. A reduced complexity detection scheme, RC-IQML, was proposed to detect the transmitted symbols in the RGC-STLC systems. In order to validate our simulations, the ASEP union bounds of the proposed RGC-STLC systems were derived. Simulation results and union bounds showed that $1 \times 4$ RGC-STLC $M$QAM systems achieve at least 7 dB compared to $1 \times 2$ STLC $M$QAM systems. However, $2 \times 4$ RGC-STLC $M$QAM systems only achieves around 3 dB compared to $2 \times 2$ STLC $M$QAM systems.

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