Potential for inert adjoint scalar field in SU(2) Yang-Mills thermodynamics

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Abstract

A scalar adjoint field is introduced as a spatial average over (anti)calorons in a thermalized SU(2) Yang-Mills theory. This field is associated with the thermal ground state in the deconfining phase and acts as a background for gauge fields of trivial topology. Without invoking detailed microscopic information we study the properties of the corresponding potential, and we discuss its thermodynamical implications. We also investigate the gluon condensate at finite temperature relating it to the adjoint scalar field.
1 Introduction

The potential importance of topological field configurations in generating a finite correlation length in the dynamics of thermalized, nonabelian gauge fields [1] was emphasized a long time ago [2]. In particular, calorons, i.e. instantons at finite temperature $T$, play an important role in the description of pure SU(2) and SU(3) Yang-Mills theories in their deconfining phase. However, the inclusion of topologically nontrivial field configurations when evaluating thermodynamical quantities is complicated because of the nonlinearity of the theory [3]. Also, there is no a priori infrared-cutoff when integrating out residual quantum fluctuations about these finite-action configurations [4].

In [5] the possibility that a scalar adjoint field $\phi$ can act as a thermal average upon calorons and anticalorons was addressed. More precisely, BPS saturated and stable (trivial holonomy) configurations of topological charge modulus $|Q| = 1$, Harrington-Shepard solutions [6], were integrated into an adjoint scalar field $\phi$ in the deconfining phase. This field $\phi$, which is part of the thermal ground state, affects thermodynamical quantities. The scalar field $\phi$ induces an adjoint Higgs mechanism: some of the gauge modes acquire a (temperature-dependent) mass on tree-level. Notice that rotational symmetry in the thermal system together with the perturbative renormalizability of the fundamental Yang-Mills action [7] forbid the emergence of any other composite field, induced by BPS saturated, nonpropagating, fundamental field configurations, but an adjoint scalar field.

To be more definite, let us denote by $\mathcal{L}_{YM} = -\frac{1}{4}(F_{\mu\nu}^a)^2$ the fundamental SU(2) Yang-Mills Lagrangian, with $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$ ($g$ is the fundamental coupling). In the partition function all possible configurations of the field $A_\mu$ have to be considered, including the topologically nontrivial ones. Upon the emergence of the scalar adjoint field $\phi$ we are led to consider the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \text{Tr} \left( \frac{1}{2} G_{E,\mu}^{a\nu} G_{E,\nu}^{a\mu} + (D_\mu \phi)^2 + V(|\phi|^2) \right)$$

where $G_{E,\mu}^{a\nu} = \partial_\mu a_\nu^a - \partial_\nu a_\mu^a - e f^{abc} a_\mu^b a_\nu^c$. The fields $a_\mu^a$ refer to (coarse-grained) topologically trivial fluctuations and $e$ denotes the effective coupling constant. The effect of topology is embodied in the scalar adjoint field $\phi$, which acts as a background for the dynamics of propagating gauge fields. While the field $\phi$ acts as an infrared-cutoff in the spatial coarse-graining performed in the fundamental theory [5] it represents an ultraviolet cutoff in the coarse-grained theory, implying a rapidly converging loop expansion [4, 9]. The potential $V(|\phi|^2) = \Lambda^6 / |\phi|^2$, where $\Lambda$ is the Yang-Mills scale, was evaluated in [5] by performing the spatial average over calorons explicitly.

In this work we would like to show that apart from general properties such

\footnotetext{1}{For a more detailed discussion see [5]. The argument involves the fact that perturbative renormalizability implies that $\phi$ transforms homogeneously under changes of the gauge. On the fundamental level the only homogeneously transforming quantity is the field strength and (nonlocal) products thereof which can always be reduced to spin-0 and spin-1 representations of SU(2). While the former is irrelevant on BPS saturated configurations the latter plays an important role. A rigorous version of the here-sketched argument will be presented in [5].}
as gauge invariance and the periodicity of $\phi$ in the euclidean time the potential $V(|\phi|^2)$ follows by requiring BPS saturation. That is, no explicit calculation involving calorons is needed to derive $V(|\phi|^2)$. Over and above the existence of a Yang-Mills scale $\Lambda$, which needed to be assumed in [5], turns out to be redundant since $\Lambda$ is shown to be a purely nonperturbative constant of integration.

The article is organized as follows: In Sec. 2 we discuss the implications of BPS saturation for the dynamics of the field $\phi$. The properties of (coarse-grained) topologically trivial fluctuations and the combined effect of the ground state and the excitations on basic thermodynamical quantities are investigated in Sec. 3. In Sec. 4 we show that a gluon condensate emerges in the framework of the effective theory and that its $T$-dependence is in agreement with lattice results [10, 11, 12].

## 2 Gauge invariance, BPS saturation, and inertness

In the deconfining phase of thermalized SU(2) Yang-Mills theory we study the possibility that an adjoint scalar field $\phi$ describes (part of) the thermal ground state. Namely, we postulate that $\phi = \phi(\tau)$, where $0 \leq \tau \leq \beta \equiv 1/T$, emerges from by virtue of a spatial average over (anti)selfdual fundamental field configurations in euclidean spacetime. Intuitively, we assume that the nontrivial nature of the Yang-Mills ground state can be described by a scalar-adjoint field $\phi$.

Without the need to perform the average explicitely the field $\phi$ enjoys the following general properties as a consequence: (i) Since $\phi$ is obtained by a spatial coarse-graining over noninteracting, stable, BPS saturated field configurations (topology changing energy and pressure free fluctuations) it is itself BPS saturated, thus the associated energy density vanishes. (ii) Originating from periodic-in-$\tau$ field configurations (in a given gauge) it is itself periodic. (iii) The gauge invariant modulus $|\phi|$ does not depend on spacetime (trivial expansion into Matsubara frequencies due to coarse-graining over energy and pressure free configurations).

We will now show that conditions (i)-(iii) uniquely fix the potential $V$ for the field $\phi \equiv \phi^a(\tau) \lambda^a \ (\text{tr} \lambda_a \lambda_b = 2 \delta_{ab}, \ a = 1, 2, 3)$ when working with a canonical kinetic term in its euclidean Lagrangian density $L_\phi$:

$$L_\phi = \text{tr} \left( (\partial_\tau \phi)^2 + V(\phi^2) \right) \tag{1}$$

Since the coarse-graining is over exact solutions to the Yang-Mills equations the emerging field $\phi$ must minimize the effective action. Thus $\phi$ satisfies the Euler-

\[\text{As long as this term contains two powers of time derivatives this is not a constraint on generality due to (iii). Moreover, although we ignore the connection to the microscopic physics in the present work we surely can make an appropriate choice of gauge such that the coarse-graining over noninteracting topological defects generates } A_\mu = 0 \text{ on the macroscopic level.}\]
Lagrange equations subject to Eq. (1):

\[
\partial^2 \phi^a = \frac{\partial V(|\phi|^2)}{\partial |\phi|^2} \phi^a \leftrightarrow \partial^2 \phi = \frac{\partial V(\phi^2)}{\partial \phi^2} \phi ,
\]  

(2)

where \(|\phi| \equiv \sqrt{\frac{1}{2} \text{tr} \phi^2}.

The gauge invariance of the potential \(V = V(|\phi|^2)\) (central potential) in Eq. (2) implies that the solution has to describe motion in a plane of the three-dimensional vector space spanned by the Lie-algebra valued generators of SU(2) Yang-Mills theory. (The angular momentum is a constant of motion in a central potential.)

Without restriction of generality (a global gauge choice) we choose the plane \((\phi^1, \phi^2, 0)\). Thus the solution takes the following form:

\[
\phi = |\phi| \lambda_1 \exp(i \lambda_3 \theta(\tau)) = |\phi| (\lambda_1 \cos(\theta(\tau)) + \lambda_2 \sin(\theta(\tau))) ,
\]  

(3)

or in components

\[
(\phi^1, \phi^2, \phi^3) = |\phi| (\cos(\theta(\tau)), \sin(\theta(\tau)), 0).
\]  

(4)

According to (ii) the function \(\theta(\tau)\) needs to satisfy the following condition

\[
\theta(\tau + \beta) = \theta(\tau) + 2\pi n ,
\]  

(5)

where \(n\) is an integer. Finally, condition (i) implies the vanishing of the (euclidean) energy density \(H_E(\phi)\):

\[
H_E(\phi) = \text{tr} \left( (\partial_\tau \phi)^2 - V(\phi^2) \right) = 2 \left( (\partial_\tau \phi^a)^2 - V(|\phi|^2) \right) = 0 , \quad \forall \beta .
\]  

(6)

Substituting Eq. (3) into Eq. (6) we have

\[
|\phi|^2 (\partial_\tau \theta(\tau))^2 - V(|\phi|^2) = 0 .
\]  

(7)

According to (iii) the potential \(V(|\phi|^2)\) does not depend on \(\tau\). As a consequence of Eq. (7), we then have \(\partial_\tau \theta(\tau) = \text{const.}\) Together with Eq. (5) this yields:

\[
\theta(\tau) = \frac{2\pi}{\beta} n \tau,
\]  

(8)

up to an inessential constant phase (global gauge choice). Notice that the case \(n = 0\) is excluded if we impose that \(V \neq 0\). Now Eq. (8) implies that \(\partial_\tau^2 \theta(\tau) = 0\), and thus we obtain from Eqs. (3) and (2) that

\[
(\partial_\tau \theta(\tau))^2 = - \frac{\partial V(|\phi|^2)}{\partial |\phi|^2} .
\]  

(9)

\(^3\)In a similar way, the case \(Q = 0\) is excluded for BPS saturated, microscopic field configurations if we insist on a nonvanishing action.
Eliminating \((\partial_x \theta(\tau))^2\) from Eqs. (7) and (9) we have:

\[
\frac{V(|\phi|^2)}{|\phi|^2} = -\frac{\partial V(|\phi|^2)}{\partial |\phi|^2}.
\]  

(10)

Notice that Eq. (10) is valid for all values of \(\beta\). The unique solution to the first-order differential equation (10) is

\[
V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}
\]

(11)

where the mass scale \(\Lambda\) enters as a constant of integration. On dimensional grounds \(\Lambda\) has to appear with the sixth power. We interpret \(\Lambda\) as the Yang-Mills scale which, however, is not operational on the level of BPS saturated dynamics, see below. (On this level the energy-momentum tensor vanishes.)

Let us now determine the modulus \(|\phi|\). By inserting the potential \(V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}\) and Eq. (8) into Eq. (7) we have

\[
|\phi| = \sqrt{\frac{\Lambda^3}{2\pi|n|T}}.
\]

(12)

This implies that the field \(\phi\) vanishes in a power-like way with increasing temperature. The value of the integer \(n\) can not be determined within the macroscopic approach we have applied to deduce \(\phi\)'s potential. Microscopically, one observes that the definition of \(\phi\)'s phase does only allow for the contribution of Harrington-Shepard solutions [6] of topological charge modulus \(|Q| = 1\) which implies that \(n = \pm 1\) [5].

Finally, we point out the inertness of the field \(\phi\). According to Eqs. (11) and (11) the square of the mass \(M_\phi\) of potential (radial) fluctuations \(\delta \phi\) is given as (setting \(|n| = 1\))

\[
M_\phi^2 = 2 \frac{\partial^2 V}{\partial |\phi|^2}|_{|\phi|=\sqrt{\frac{\Lambda^3}{2\pi T}}} = 48 \pi^2 T^2.
\]

(13)

Thus \(\frac{M_\phi^2}{T^2} = 48 \pi^2 \gg 1\), and no thermal excitations exist. On the other hand, we have \(\frac{M_\phi^2}{|\phi|^2} = 12 \lambda^3\) where \(\lambda \equiv \frac{2\pi T}{\Lambda}\). For \(\lambda \gg 1\) one has that \(\frac{M_\phi^2}{|\phi|^2} \gg 1\). In practice, \(\lambda > \lambda_c = 13.87\), see [5]. Since \(|\phi|\) is the maximal resolving power allowed in the effective theory we conclude that quantum fluctuations of the field \(\phi\) do not exist.

3 Topologically trivial fluctuations

3.1 Effective Lagrangian and ground state

For the reader’s convenience we briefly repeat the derivation of [5] leading to the complete ground-state description of SU(2) Yang-Mills thermodynamics in its deconfining phase.
If topological fluctuations were absent then renormalizability \[7\] would assure that the action of the fundamental theory is form-invariant under the applied spatial coarse-graining. Since the topological part is integrated into an inert field \(\phi\) this still holds true for the part of the effective action induced by \(Q = 0\)-fluctuations \(a_\mu\). We thus are confronted with the following, gauge invariant effective Lagrangian for the dynamics of coarse-grained \(Q = 0\) fluctuations \(a_\mu\) subject to the background \(\phi\):

\[
L_{\text{eff}} = L[a_\mu] = \text{tr} \left( \frac{1}{2} G^{\mu\nu}_E G^{\mu
u}_E + (D_\mu \phi)^2 + \frac{\Lambda^6}{\phi^2} \right),
\]

where

\[
G^{\mu\nu}_E = \partial_\mu a_\nu - \partial_\nu a_\mu - i e [a_\mu, a_\nu] = G^{a,\mu\nu}_E \frac{\lambda_a}{2},
\]

\[
a_\mu = a_\mu^a \frac{\lambda_a}{2}, \quad D_\mu \phi = \partial_\mu \phi - i e [a_\mu, \phi],
\]

and \(e\) denotes an effective gauge coupling. According to Eq. (14) the equation of motion for the field \(a_\mu\) is:

\[
D_\mu G^{\mu\nu}_E = i e [\phi, D_\nu \phi].
\]

This is solved by the pure-gauge configuration \(a_\mu = a_\mu^{qs}\) given as

\[
a_\mu^{qs} = \mp \delta_4 \frac{2 \pi}{3} \frac{\lambda_3}{2} = \frac{i}{e} (\partial_\mu \Omega) \Omega^\dagger \text{ with } \Omega = e^{\pm i 2 \pi \frac{\lambda_3}{3}} \Rightarrow D_\mu \phi = 0
\]

The entire ground state thus is described by the \(a_\mu^{qs}, \phi\) implying a ground-state pressure \(P^{qs}\) and energy density \(\rho^{qs}\) given as \(P^{qs} = -4 \pi \Lambda^3 T = -\rho^{gs}\): The inclusion of gluon fluctuations, which contribute to the dynamics of the ground-state, by virtue of \(a_\mu = a_\mu^{gs}\) after coarse-graining shifts the vanishing results, obtained from BPS saturated configurations alone, to finite values. This makes the Yang-Mills scale \(\Lambda\) (gravitationally) visible. Turning to propagating fluctuations \(\delta a_\mu\) in the effective theory it is advantageous to work in unitary gauge.

### 3.2 Unitary gauge and Higgs mechanism

By performing a gauge rotation \[4\]

\[
U = e^{-i \frac{\pi}{3} \lambda_3} \Omega \quad \text{we have that} \quad a_\mu^{qs} \rightarrow U a_\mu^{qs} U^\dagger = \frac{i}{e} (\partial_\mu U) U^\dagger = 0 \quad \text{and} \quad \phi = \lambda_3 |\phi|.
\]

This is the unitary gauge. The field strength \(G^{\mu\nu}_E\) and the covariant derivative \(D_\mu \phi\) are functionals of the fluctuations \(\delta a_\mu\) only. We have

\[
L^{u,q}_{\text{eff}} = L[\delta a_\mu] = \frac{1}{4} (G^{a,\mu\nu}_E [\delta a_\mu])^2 + 2 e^2 |\phi|^2 \left( (\delta a^{(1)}_\mu)^2 + (\delta a^{(2)}_\mu)^2 \right) + 2 \frac{\Lambda^6}{|\phi|^2}.
\]

\[4\] Notice that \(U\) is smooth and antiperiodic. One can introduce a center jump to make it periodic by sacrificing its smoothness \[5\]. However, the associated electric center flux does not carry any energy or pressure and the periodicity of effective gluon fluctuations is maintained. These are the physical reasons why the transformation to unitary gauge is admissible.
Fluctuations $\delta a^{(1,2)}_\mu$ are massive in a temperature dependent way while the mode $\delta a^{(3)}_\mu$ remains massless representing the fact that SU(2) is broken to its subgroup U(1) by the field $\phi$. One has

$$m^2 = m_1^2 = m_2^2 = 4e^2 |\phi|^2, \quad m_3^2 = 0.$$  \hfill (19)

### 3.3 Energy density, pressure and running coupling

From the effective Lagrangian (18) we derive the energy density $\rho$ and the pressure $p$ on the one-loop level. This is accurate on the 0.1%-level as shown in \[5, 15\].

This strong suppression of the effects of residual $Q = 0$ quantum fluctuations in the effective theory takes place due to limited resolution, given by the modulus $|\phi|$, and due to emergent, temperature-dependent tree-level mass. Both phenomena introduce nonperturbative aspects into the loop expansion based on the tree-level action Eq. (18) which render the radiative corrections small.

On the one-loop level we have

$$\rho = \rho_3 + \rho_{1,2} + \rho_{gs}, \quad p = p_3 + p_{1,2} + p_{gs},$$  \hfill (20)

where the subscript 1,2 is understood as a sum over the two massive modes. Explicitly, we have:

$$\rho_3 = \frac{2\pi^2}{30} T^4, \quad \rho_{1,2} = 6 \int_0^\infty \frac{k^2 dk}{2\pi^2} \exp\left(\sqrt{m^2 + k^2} - \frac{1}{|\phi|^2}\right), \quad \rho_{gs} = \frac{2\Lambda^6}{|\phi|^2} = 4\pi\Lambda^3 T, \quad (21)$$

$$p_3 = \frac{2\pi^2}{90} T^4, \quad p_{1,2} = -6T \int_0^\infty \frac{k^2 dk}{2\pi^2} \ln\left(1 - e^{-\frac{\sqrt{m^2 + k^2}}{T}}\right), \quad p_{gs} = -\rho_{gs}. \quad (22)$$

The effective coupling constant $e$ is a function of the temperature $e = e(T)$, and so is $m$. The function $e = e(T)$ is deduced by requiring the validity of the Legendre transformation

$$\rho = T \frac{dp}{dT} - p.$$  \hfill (23)

in the effective theory.

By substituting the equations (20) into (23) we obtain:

$$4\pi\Lambda^3 = -6D(m) \frac{dm(T)}{dT}, \quad D(m) = \int_0^\infty \frac{k^2 dk}{2\pi^2} \frac{m}{\sqrt{m^2 + k^2}} e^{\frac{1}{\sqrt{m^2 + k^2}} e^{-\frac{\sqrt{m^2 + k^2}}{T}}} - 1. \quad (24)$$

Solving the differential equation (24) and inverting the solution, the function $e(T)$ follows by virtue of Eq. (19).

Eq. (24) is of first order. Thus a boundary condition needs to be prescribed. It was shown in \[5\] that the evolution at low temperature decouples from the boundary physics at high temperature. That is, there exists a low-temperature attractor to the evolution. This attractor is characterized by a logarithmic pole, $e \sim -\log(T - T_c)$.
where $T_c = 13.87 \frac{\Lambda}{2\pi}$, signalizing the presence of a phase transition, and by a plateau $e \equiv \sqrt{8\pi}$ for $T$ sufficiently larger than $T_c$, indicating magnetic charge conservation for screened monopoles. In Fig. 1 we indicate the (scaled) energy density $\rho$, pressure $p$, and entropy density $s$ as functions of temperature. A detailed comparison of these results with those obtained on the lattice (for both differential and integral method) is carried out in the first reference of [5]. While there is good agreement for the infrared safe quantity entropy density $s$ the pressure $p$ becomes negative close to the phase boundary which is qualitatively in agreement with the result of the differential method but not with that of the integral method.

4 Gluon condensate

We start with an intuitive discussion on the gluon condensate. As emphasized in [13], at $T = 0$ instantons are responsible for a nonzero gluon condensate [14]. In fact, the average $\langle F_{\mu\nu}^2 \rangle = -4 \langle \mathcal{L}_{YM} \rangle = 2 \left\langle B^2 - \overline{B}^2 \right\rangle$ is zero at any order in perturbation theory ($F_{\mu\nu}$ is the fundamental stress-energy tensor). Instantons are selfdual solutions in euclidean spacetime, which, in Minkowski space, are interpreted as tunnelling events implying $E^{i,a} = \pm i B^{i,a}$ and so generate a positive average $\langle F_{\mu\nu}^2 \rangle$. An estimate of the gluon condensate is thus obtained by evaluating the action density in euclidean spacetime $\langle F_{\mu\nu}^2 \rangle = \langle \mathcal{L}_{YM}^{\text{euc}} \rangle$ [13-14].

In the framework of the effective theory for thermalized, deconfining SU(2) Yang-Mills dynamics we evaluate the action density by virtue of Eq. (14). Thus the average $\langle F_{\mu\nu}^2 \rangle$ corresponds to

\[ \langle F_{\mu\nu}^2 \rangle = 4 \langle \mathcal{L}_{YM}^{\text{euc}} \rangle \propto 4 \langle \mathcal{L}_{\text{eff}}^{u.g.} \rangle = 4 \rho_{gs} = 16\pi\Lambda^3 T, \]

where a proportionality between the average $\langle \mathcal{L}_{YM}^{\text{euc}} \rangle$ over fundamental fields and the
average \( \langle \mathcal{L}_{\text{eff}}^{u,g} \rangle \) evaluated in the effective theory with coarse-grained fields holds \[16\]. We notice that the ground-state energy density \( \rho_{gs} \) is responsible for the emergence of a nonvanishing thermal average \( \langle F_{\mu \nu}^2 \rangle \).

More precisely, the gluon condensate should be defined as a renormalization-group invariant object. This holds for \( \langle \frac{\beta(g)}{2g} F_{\mu \nu}^a F_{a,\mu \nu} \rangle \) where \( \beta(g) \) is the full beta-function for the fundamental coupling \( g \), compare with \[16\]. By virtue of the trace anomaly we can evaluate this quantity as

\[
\langle \frac{\beta(g)}{2g} F_{\mu \nu}^a F_{a,\mu \nu} \rangle_T = \rho - 3p.
\] (26)

In \[17\] we have shown that within the effective theory a linear growth \( \rho - 3p = 6\rho_{gs} = 24\pi \Lambda^3 T \) for \( T \gg T_c \) follows. Such a linear growth has been found by lattice simulations \[10, 11, 12\] and also in an analytical approach \[18\]. In the latter a momentum-dependent, universal modification of the dispersion relation for propagating, fundamental gluon fields, motivated by the reduction of the physical state space a la Gribov, is introduced. While this is interesting a direct comparison of both approaches beyond the observation of a linear growth of the trace anomaly would need a coarse-graining over the modified gluon propagation of \[18\].

5 Conclusions

In this article we have derived the potential \( V(|\phi|^2) = \Lambda^6 / |\phi|^2 \) for an inert, adjoint scalar field \( \phi \) by solely assuming its origin to be a spatial average over noninteracting, BPS saturated topological field configurations in the underlying theory: SU(2) Yang-Mills thermodynamics being in its deconfining phase. That is, no detailed microscopic information on these configurations other than their stability and BPS saturation is needed to derive the potential for the effective field \( \phi \). The conceptually interesting implication of our present work is that the Yang-Mills scale \( \Lambda \) emerges as a constant of integration: \( \Lambda \)’s existence needs not be assumed as in \[5\]. For our presentation to be selfcontained we have repeated the derivation of the effective action, involving the field \( \phi \) as a background, for the coarse-grained, topologically trivial fluctuations. We also have pointed out that the (linear) temperature dependence of the gluon condensate agrees with that found in lattice simulations.

There is a host of applications of SU(2) Yang-Mills thermodynamics in particle physics \[19\] and cosmology \[20\].

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