A NEW METHOD FOR THE DETERMINATION OF THE GROWTH RATE FROM GALAXY REDSHIFT SURVEYS

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ABSTRACT

Given a redshift survey of galaxies with measurements of apparent magnitudes, we present a novel method for measuring the growth rate $f(\Omega)$ of cosmological linear perturbations. We use the galaxy distribution within the survey to solve for the peculiar velocity field which depends in linear perturbation theory on $b = f(\Omega)/b$, where $b$ is the bias factor of the galaxy distribution. The recovered line-of-sight peculiar velocities are subtracted from the redshifts to derive the distances, which thus allows an estimate of the absolute magnitude of each galaxy. A constraint on $b$ is then found by minimizing the spread of the estimated magnitudes from their distribution function. We apply the method to the all sky $K = 11.25$ 2MASS Redshift Survey and derive $b = 0.35 \pm 0.1$ at $z \sim 0$, remarkably consistent with our previous estimate from the velocity–velocity comparison. The method could easily be applied to subvolumes extracted from the Sloan Digital Sky Survey to derive the growth rate at $z \sim 0.1$. Further, it should also be applicable to ongoing and future spectroscopic redshift surveys to trace the evolution of $f(\Omega)$ to $z \sim 1$. Constraints obtained from this method are entirely independent from those obtained from the two-dimensional distortion of $\xi(s)$ and provide an important check on $f(\Omega)$, as alternative gravity models predict observable differences.

Key words: dark matter – large-scale structure of universe

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1. INTRODUCTION

Large-scale density perturbations in the universe are gravitationally unstable and grow via linear theory. The growing mode of large-scale density perturbations, $D(a)$, is characterized by the more observationally relevant growth rate

$$f(\Omega) = \frac{d \ln D}{d \ln a}, \quad (1)$$

where $a$ is the scale factor of the universe and $\Omega$ is the matter density parameter. It is found that the growth index $\gamma = d \ln f/d \ln \Omega$ is very well approximated by $\gamma = 0.55 + 0.05[1+w(z = 1)]$ (Linder 2005) for a cosmological background dominated by dark energy with an equation of state, $P = w\rho c^2$. The growth rate is not only sensitive to the background cosmology, but also to the theory of gravitation invoked as the driver for structure formation. Geometric $R^n$ (e.g., Gannouji et al. 2009) and Dvali–Gabadadze–Porrati (DGP; Dvali et al. 2000; Wei 2008) gravity models give substantially different behaviors of $f$. Models with dark sector long-range forces, in addition to gravity, even introduce a scale dependence into $f$ (e.g., Keselman et al. 2010).

Here we present a new method for constraining $f(\Omega)$ from redshift surveys of galaxies with measured apparent magnitudes. Redshifts of galaxies systematically differ from the actual distances by the line-of-sight components of their peculiar velocities. Hence, the directly measurable intrinsic luminosities or absolute magnitudes of galaxies inferred from the observed flux using redshifts rather than distances will show larger spread than the true values.

Gravitational instability theory allows a prediction of the peculiar velocity field from observed galaxy distribution given $f(\Omega)$ and the biasing relation between galaxies and mass. The method uses this predicted velocity field to get distances for deriving true absolute magnitudes. That is, the catalog redshift distribution is used to make the linear correction to the redshift space distortion. Constraints on $f$ can then be derived by minimizing the scatter of the estimated absolute magnitudes from a reference distribution. Since the galaxy distribution could be biased relative to the mass density field, the constraints on $f$ are degenerate with the assumed biasing. Adopting linear biasing, $\delta_z = b \delta_m$, between the galaxy number density contrast, $\delta_g$, and the mass density contrast, $\delta_m$, yields constraints on $b = f/b$ that are independent from those obtained from the apparent anisotropy in the observed galaxy clustering (Kaiser 1988).

We note that the peculiar velocity field is dominated by large-scale structure and is insensitive to small-scale issues such as the biasing of luminous, massive field ellipticals versus typical galaxies. Since in linear theory, $v(k) = \delta(k)/k$, and $\delta(k) \propto k^{-2}$, the velocity is dominated by large scale (small $k$) and not by infall to clusters. Thus it is expected to be insensitive to the small-scale smoothing.

In Section 2 we describe the method in detail, presenting general expressions and deriving the relevant approximations. In Section 3 we offer general analytic assessments of the method. In Section 4 we apply the method to the 2MASS Redshift Survey (2MRS) of galaxies limited to magnitude $K = 11.25$ (Huchra et al. 2005). We conclude in Section 5 with a general discussion of the results and of the prospects for the application of the method to future data.
2. THE METHOD

The method is largely based on formalism developed in Nusser et al. (2011) to estimate large-scale bulk flows. Here we extend the method to derive constraints on an assumed model for the peculiar velocity field. The model adopted here relies on a linear theory prediction of $v$ from the observed distribution of galaxies in a redshift survey and depends on the single parameter $\beta$.

We are given a flux limited survey of galaxies with observed apparent magnitudes $m < m_1$, angular positions, and redshifts $cz$ (in $\text{km \ s}^{-1}$). Let $r$ (also in $\text{km \ s}^{-1}$) be the luminosity distance to the galaxy in the sample. For simplicity of notation and description we assume here that the distance and spatial extent of the survey are small so that $r$ is well approximated by the physical distance. Therefore, $cz = r + v$ where $v = \hat{r} \cdot u$ is the line-of-sight component of three-dimensional peculiar velocity $u$ of the galaxy. The results can readily be extended to the general case once we specify the underlying cosmological model. Since $r = cz - v$ this prediction allows an estimate of the true absolute magnitude,

$$M = m - 15 - 5 \log r = M_0 - 5\log(1 - v/cz),$$

where the measurable absolute magnitude $M_0 = m - 15 - 5 \log cz$ is determined from observations. As a model for $v$ we use here the velocity field $\nu$ associated with the number density fluctuations field inferred from the galaxy distribution in a redshift survey. We emphasize that the redshift catalog used to predict $v$ does not need to be the same as the catalog of galaxies used to compute the luminosity function. This model velocity field depends on the biasing relation between mass and galaxies and on cosmological parameters. Because the peculiar velocity of a galaxy is uncorrelated with its true absolute magnitude, unbiased constraints on cosmology galaxy biasing can be derived by demanding that the distribution of the magnitudes, $M$, is consistent with a reference distribution function (i.e., luminosity function).

The equations of gravitational instability theory relate the underlying mass density contrast, $\delta_m$, to the peculiar velocity field $v$. Here we adopt linear theory in which the relation solely depends on $f(D)$. For linear biasing, $\delta = b \delta_m$, between mass and galaxies the appearance of $f$ is replaced by the single parameter $\beta = f/b$. Therefore, the method presented here will focus on constraints on $\beta$ only. In practice we follow the linear theory procedure outlined by Nusser & Davis (1994). The peculiar velocity is irrotational in redshift space (Chodorowski & Nusser 1999) and can be expressed as $v(s) = -\nabla \Phi(s)$ where $\Phi(s)$ is a potential function. If we expand the angular dependence of the potential function $\Phi(s)$ and the density contrast $\delta_m(s)$, spherical harmonics in the form $\Phi(s) = \sum \Phi_{lm}(\theta, \phi)$ and similarly for $\delta_m$, then, to first order, $\Phi_{lm}$ and $\delta_{lm}$ satisfy,

$$\frac{1}{s^2} \frac{d}{ds} \left( s^2 \frac{d \Phi_{lm}}{ds} \right) - \frac{l(l+1)}{1+\beta} \frac{d \Phi_{lm}}{ds} = \frac{\beta}{1+\beta} \left( \delta_{lm} - \kappa(s) \frac{d \Phi_{lm}}{ds} \right),$$

where $\kappa = d \ln \phi / ds$ represents the correction for the bias introduced by the Kaiser rocket effect due to the fact that galaxies are weighted by the selection function estimated at the redshift position rather than the true distance.

More sophisticated models for the velocity field involving additional parameters will not be discussed here. In principle, the constraint can be obtained without resorting to the luminosity function simply by minimizing the variance of $M_0 - 5 \log(1 - v/\nu/cz)$ with respect to $\beta$. However, much tighter constraints are obtained from the full distribution. We define the luminosity function, $\Phi(M)$, expressed in terms of the absolute magnitudes, as the number density of galaxies per unit magnitude. The probability, $P(M_0|cz, v)$, of observing a galaxy having an observed magnitude $M_0$ in a flux limited sample, depends on its redshift, $cz$, and its radial peculiar velocity, $v$, and is well approximated as (Nusser et al. 2011)

$$P(M_0|cz, v) = \frac{\Phi(M)}{\int_{-\infty}^{M_0} \Phi(M) dM},$$

where $M_l = M_0 - 5 \log(1 - v/cz)$ and $M_0 = m_l - 15 - 5 \log cz$. The expression is valid as long as the relative errors on the measured redshifts are small ($\sigma_{cz}/\nu \ll 1$). The probability distribution of the whole sample of galaxies is the product of the single probabilities

$$P_s = \prod_i P(M_{0i}|cz_i, v_i),$$

where $i$ runs over all galaxies of the sample. Given a form for $\Phi(M)$, the parameter $\beta$ is then constrained by maximizing $P_s$ in which the dependence on $\beta$ is via the $v(\beta)$ as inferred from the spatial distribution of galaxies. In principle one could use the “nonparametric” fit methods (Efstathiou et al. 1988; Davis & Huchra 1982) to approximate $\Phi(M)$. However, here we will only apply the method to the 2MRS sample which is reasonably approximated by a Schechter luminosity function (Westover 2007). Therefore, we assume $\Phi(M)$ is well approximated by a Schechter form (Schechter 1980)

$$\Phi(M) = 0.4 \ln(10) \Phi^* 10^{5.4(M_0 + 100)/M_0} \exp\left(-10^{5.4(M_0 + 100)/M_0}\right).$$

The normalization $\Phi_s$ does not concern us here. The shape parameters $M_\star$ and $\alpha$ generally depend on the galaxies’ type, redshift and band of observation. In terms of the luminosity ($M = -2.5 \log L + \text{const}$), this function acquires the simpler form

$$\Phi(M(L)) = 0.4 \ln(10) \Phi^* \left(\frac{L}{L_\star}\right)^{1+\alpha} \exp\left(-\frac{L}{L_\star}\right).$$

Inserting all of this expression into Equation (4) gives

$$P(M_0|cz; v) = \frac{0.4 \ln(10) \left(\frac{L}{L_\star}\right)^{1+\alpha} e^{-L/L_\star}}{\Gamma(1+\alpha, L_\star/L_\star)},$$

where $L/L_\star = (1 - v/cz)^2 10^{0.4(M_0 - M_\star)}$ and $L_\star/L_\star = (1 - v/cz)^2 10^{0.4(M_0 - M_\star)}$.

To summarize, given the observed absolute magnitudes $M_0$ for a sample of objects, a Schechter model for the luminosity function and linear theory prediction for the underlying velocity field $v(\beta)$, we determine $\beta$, $\alpha$, and $M_\star$ by minimizing Equation (5).

The effects of differential bias in the luminosities of the sample have not yet been studied but are expected to be modest. The more luminous galaxies are indeed at the centers of attraction, but motion of cluster centers on larger scales is
present as well. Fainter galaxies would see this same motion, plus the infall into a nearby cluster. That is, fainter galaxies see the effects of a larger range of wavenumber, but since \( v(k) \propto \delta(k)/k \) and with \( \delta(k) \) approximately \( k^{-2} \), the shorter \( k \) values, longer wavelengths, dominate. All galaxies are sensitive to them and therefore we expect the bias to be unimportant.

3. GENERAL ASSESSMENTS OF THE METHOD

There are three sources of error which affect the derivation of \( \beta \) in the method presented here. (1) “shot-noise” error resulting from the finite number galaxies, (2) cosmic variance due to variations in the large-scale structure in volumes of the universe comparable in size to the volume probed by the redshift survey at hand, and (3) inaccuracies in the peculiar velocity reconstruction. In this section we offer a general assessment of the applicability of the method to data at higher redshifts.

To compensate for the degrading of the signal (\( \sim v/cz \)) with redshift, surveys with a large number of galaxies need to be invoked. We will only consider here shot-noise error. Peculiar velocity reconstruction errors and cosmic variance (of surveys with similar volumes) scale with redshift in a similar manner to the signal and hence their corresponding relative error will not depend on redshift.

To the list of errors above we do not add biases introduced by adopting a specific form for the luminosity function since, as we will argue in Section 3.2, these biases are expected to be insignificant. Further, the issue becomes completely irrelevant for future surveys where the number of galaxies is large enough to allow the use of “nonparametric fit” techniques for modeling the luminosity function, relaxing the need for specific parametric forms.

3.1. Sensitivity to “Shot-noise” and Redshift

Assume a cubic\(^6\) region at high redshift \( cz \) containing a flux limited sample of \( N \) galaxies with redshifts \( cz_i \approx cz \). From the galaxy distribution in this region, one can derive the peculiar velocity field as a function of \( \beta \). Consider a Schechter form for the luminosity function and assume \( cz \) is large enough that the limiting luminosity \( L_l \) of a galaxy that could be observed is significantly larger than \( L_* \). In the limit \( L_l \gg L_* \), the \( \Gamma \) function in the expression for \( P(M_0|cz, v) \) in Equation (8) is well approximated as \( \Gamma(1+\alpha, L_l/L_*) = (L_l/L_*)^\alpha \exp(-L_l/L_*) \) so that,

\[
P(M_0|cz; v) = 0.4 \ln(10) \frac{L}{L_*} \left( \frac{L_*}{L_l} \right)^\alpha e^{-(L_l/L_*)}.
\]

This approximation will allow an analytic expression for \( \beta \) by minimizing \( \ln P_s = -\sum \ln P(M_0|cz_i, v(\beta)) \). For simplicity, we further approximate \( v_i(\beta) = F(\beta)v_i \) where \( v_i \) is the line of sight velocity reconstructed with \( \beta = 1 \). For linear velocity reconstruction from the galaxy distribution in real space \( F(\beta) = \beta \). But for reconstruction from redshift space data a significantly better scaling is \( F = 2.5\beta/(1 + 1.5\beta) \) (Davis et al. 2011). This is not an exact result, but it suffices here since we are only interested in a general assessment of the expected error on \( \beta \). Since \( \beta \) appears only via \( F(\beta) \), we will perform the minimization with respect to \( F \) and write the result in terms of

\[
\beta \text{ at the end of the calculation. Using } L = (1 - F v_{1i}/cz_i)^2 L_0 \text{ and } L_l = (1 - F v_{1i}/cz_i)^2 L_{0l} \text{ we get }
\]

\[
\delta \ln P_s = \frac{\partial \ln P_s}{\partial F} \sum F \ln \left( 1 - F v_{1i}/cz_i \right) - \frac{L_0 - L_{0l}}{L_0} \left( 1 - F v_{1i}/cz_i \right)^2
\]

\[
= 2 \sum \left[ \frac{\Delta L_i}{L_*} v_{1i}/cz_i - F(v_{1i}/cz_i)^2 \left( 1 + \frac{\Delta L_i}{L_*} \right) \right],
\]

where \( \Delta L_i = L_0 - L_{0l} \) and in the last step we have neglected \( O(v_{1i}/cz_i)^2 \) terms and assumed that the Hubble flow-like \( \sum v_{1i}/cz_i \) is negligible compared to the other terms. The 1\( \sigma \) shot-noise error on \( F \) is, therefore,

\[
\delta F = \left( \frac{2}{\delta^2 \ln P/\partial F^2} \right)^{1/2}
\]

\[
= \left[ \sum \left( \frac{v_{1i}/cz_i}{L_0} \right)^2 \left( 1 + \frac{\Delta L_i}{L_*} \right) \right]^{-1/2}.
\]

This expression can be easily estimated when the luminosity, \( L_0 \), is computed from the actual distances, i.e., \( cz_i = r_i \). This means that \( L_0 \) is the true intrinsic luminosities and, therefore, \( \Delta L \) and \( v_1 \) are uncorrelated. Further, in the limit \( L_l \gg L_* \), the average \( \Delta L/L_* \) is unity. For \( F = 2.5\beta/(1 + 1.5\beta) \) we get, \( \delta\beta \approx 0.4(1 + 1.5\beta)\delta F \). With all this, Equation (11) gives

\[
\delta\beta = 0.4(1 + 1.5\beta)(2N)^{-1/2} \frac{cz^2}{\sigma_{v1}}
\]

\[
= 0.044(1 + 1.5\beta) \left( \frac{10^5}{N} \right)^{1/2} \frac{\bar{z}}{0.1 \sigma_{v1}},
\]

where \( \bar{z} = (1/z^2)^{-1/2} \) and \( \sigma_{v1} \) is the rms value of \( v_1 \) in km s\(^{-1}\). The analysis of Davis et al. (2011) gives, on large scales, \( \sigma_{v1} \sim 600 \) km s\(^{-1}\) for \( \beta = 1 \), at \( z = 0 \).

The scaling \( \delta\beta \propto N^{-1/2} \bar{z} \) in Equation (12) should be approximately valid also when the condition \( L_l \gg L_* \) is not strictly satisfied. For 2MRs this scaling gives roughly the same error on \( \beta \) using distant galaxies with \( cz > 4000 \) km s\(^{-1}\) as using galaxies with the lower redshifts.

3.2. Sensitivity to Assumed Form of the Luminosity Function

It is instructive to assess the sensitivity of \( \beta \) on the assumed form of the luminosity function. We give here a simple example in which the assumed luminosity function differs greatly from the true form, yet the estimate for \( \beta \) is unbiased. Consider an ideal volume-limited sample of galaxies with a true luminosity function of a Schechter form with \( \alpha = 0 \), i.e., an exponential distribution. Let us try to recover \( \beta \) assuming a Gaussian form for the luminosity function, \( P(M_0|cz, v) \propto \sigma_L^{-1}(1 - F v_1)^2 e^{-(L - L_0)^2/2\sigma_L^2} \), where, like in the previous section, \( v_1 = F(\beta)v_1 \). Minimizing the quantity, \( -\sum \ln P(M_0|cz_i, F v_i) \), with respect to \( L_m, \sigma_L, F \), yields, respectively,

\[
0 = L_m - \langle L \rangle,
\]

\[
0 = \sigma_L^2 - \langle (L - L_m)^2 \rangle,
\]

\[
0 = \sum_i \left[ \frac{\delta L_i}{\sigma_L} - \frac{L_0(L_m - L_m)}{\sigma_L^2} \frac{v_{1i}}{cz_i} \left( 1 - F v_{1i}/cz_i \right)^3 \right].
\]

\[\]
For simplicity we further assume that \( L_0 \) is computed with \( cz_t = r_t \), so that the true solution \( \beta = 0 \) (the generalization to cases where the true \( \beta \) is different from zero is trivial). Hence, \( L_0 \) are equal to the true luminosities and, therefore, follow an exponential distribution for which \( (L_0) = L_\alpha \) and \( (L_0^2) = 2L_\alpha^2 \). Further, there is no correlation between \( L_0 \) and \( u_t/cz_t \), meaning that the average of products of powers of \((1 - F v_i/cz_i)\) and \( L_0 \) is the product of the averages. In the limit \( N \to \infty \), straightforward algebraic manipulation then yields \( F(\beta) = 0 \), \( L_m = L_\alpha \), and \( \sigma^2 = L_\alpha^2 \). Therefore, in this example, where the assumed and true luminosity functions differ grossly, the best-fit \( \beta \) is unbiased. This is not surprising since the underlying principle of the method is a reduction in the spread of \( \beta \). Therefore, the assumed form of the luminosity function should only affect the weighting given to galaxies in a certain luminosity range rather than the best-fit \( \beta \). Assuming a wrong luminosity function increases the random error on \( \beta \) but does not introduce any systematic biases.

### 4. APPLICATION TO 2MRS

In this section we apply the method outlined above to the all sky 2MRS consisting of 23,200 galaxies down to the magnitude \( K = 11.25 \). Details about the catalog, including the precise completeness, sky coverage and selection effects can be found in Huchra et al. (2005). The preparation of the catalog for the application of the method is done similarly to Davis et al. (2011). The peculiar velocity field is derived from the galaxy distribution in the 2MRS for an array of \( \beta \) values using the linear theory methodology of Nusser & Davis (1994). The derived velocity field is robust within \( cz < 10^4 \) km s\(^{-1}\), above that redshift discreteness effects become important. Hence we limit the analysis to the 18,000 galaxies with \( cz < 10^4 \) km s\(^{-1}\). In the derivation of the velocity fields, the galaxy distribution is smoothed with a Gaussian window of constant width \( 400 \) km s\(^{-1}\). To further remove strong nonlinearities the derived three-dimensional velocity fields are smoothed with a Gaussian window of constant width, \( R_\delta \). Linear theory recovers the flow pattern reasonably well even at \( \delta \lesssim 1 \), but not beyond (Nusser et al. 1991; Branchini et al. 2002). Therefore, although the peculiar velocity field is predicted from the distribution of all galaxies, in the maximization of \( P_\beta \) to assess the robustness of the method we remove galaxies in regions with density contrast higher than \( \delta_{\text{cut}} \) as listed in Table 1 for both values of \( R_\delta, R_v = 6, 10 \) Mpc. Using expression (8) we minimize \(-\ln P_\beta = -\sum_i \ln P(M_0 | cz_i, v(\beta))\) (the summation is over all galaxies) with respect to \( \beta \) and the Schechter parameters, \( \alpha \) and \( M_* \).

### 4.1. Error Estimation Based on Mock 2MRS Catalogs

The overall expected errors in \( \beta \), including possible biases, are based on mock catalogs designed to match the general properties of the 2MRS. For this purpose we use 135 2MRS mock catalogs very similar to those compiled by Davis et al. (2011). These catalogs are extracted from a parent mock catalog of the Two Micron All Sky Survey (2MASS; Skrutskie et al. 2006). The parent catalog is generated from the Millennium simulation (Springel et al. 2005) using semi-analytic galaxy formation models (De Lucia & Blaizot 2007). All 135 2MRS catalogs satisfy the following conditions. (1) The central “observer” in each mock is selected to reside in a galaxy with a quiet velocity field within \( 500 \) km s\(^{-1}\), similar to the observed universe. (2) The motion of the central galaxy is 500 to 700 km s\(^{-1}\). (3) The density in the environment of the local group, averaged over a sphere of \( 400 \) km s\(^{-1}\) radius, is less than twice the normal. These conditions select central observers that are similar to the conditions of our own Local Group.

The luminosity function in each mock is approximated by a Schechter form. Using the galaxy distribution in each mock, the corresponding peculiar velocity field is generated for an array of \( \beta \) values. The mean and standard deviation of those best values from the 135 catalogs is \( \beta = 0.49 \pm 0.13 \). The cosmological density parameters of the Millennium simulation are \( \Omega_m = 0.25 \) and \( \Lambda = 0.75 \) for matter and cosmological constant. This yields \( f = \Omega_m^{0.55} = 0.47 \) (Linder 2005). A calculation of the rms galaxy density fluctuations of the mocks yields \( b = 1 \pm 0.1 \) for the mocks. The 1\( \sigma \) rms scatter of the best-fit \( \beta \) values is 0.13. This scatter reflects “shot-noise” errors due to the finite number of galaxies, cosmic variance due to the limited volume covered by the 2MRS, and inaccuracies in the reconstruction of the peculiar velocity field by means of linear theory. Shot-noise is subdominant when the method is applied to all 2MRS galaxies within \( 10^4 \) km s\(^{-1}\). Since \( P_\beta \) contains no information about cosmic variance and reconstruction errors, the width of \(-2 \ln P_\beta \) vs. \( \beta \), reflects shot-noise errors only. From the width of \(-2 \ln P_\beta \) we get 1\( \sigma \) shot-noise error of \( \delta_{\text{sn}} \approx 0.055 \) in the application to 2MRS within \( cz < 10^4 \) km s\(^{-1}\). Cosmic variance error could be estimated by running the method with the actual velocity field of galaxies in the mock. This amounts to \( \delta_{\text{cv}} \approx 0.09 \). Adding errors in quadratures, we infer a velocity reconstruction error of \( \delta_{\text{rec}} \approx 0.1 \), comparable to \( \delta_{\text{sn}} \).

We will apply the method to various cuts of the 2MRS. All corresponding errors listed in Table 1 are based on similar cuts taken from the mocks. Shot-noise errors, \( \delta_{\text{sn}} \), will be treated as (nearly) independent of \( \beta \) (see Section 3.1), while cosmic variance, \( \delta_{\text{cv}} \) and velocity reconstruction errors, \( \delta_{\text{rec}} \), are assumed to be proportional to \( \beta \), because of the \( \beta \) dependence in the reconstructed field.

### 4.2. \( \beta \) from the Real 2MRS

The results are presented in Table 1 and the top panel of Figure 1. The plotted quantity, \( \Delta \chi^2 \), is \(-2 \ln P_\beta \) evaluated as a function of \( \beta \) minus its value at the minimum. In the figure, the parameters \( \alpha, M_* \) are fixed at their best-fit values. The overall errors listed in Table 1 are based on the analysis of the mocks in Section 4.1. The 1\( \sigma \) shot-noise, \( \delta_{\text{sn}} \), given by the

### Table 1

| Sky     | \( c_{z\text{cut}} \) (km s\(^{-1}\)) | \( \delta_{\text{cut}} \) Fraction | \( (\alpha, M_\ast) \) | \( \beta \pm 1\sigma \) Error |
|---------|-----------------------------------|----------------------------------|------------------------|--------------------------|
| All     | 0                                 | 100% (−0.92, −23.14)              | 0.30 ± 0.10            |
| All     | 0                                 | 79% (−0.90, −23.19)               | 0.31 ± 0.10            |
| All     | 0                                 | 58% (−0.89, −23.14)               | 0.35 ± 0.10            |
| All     | 4000                              | 61% (−0.84, −23.15)               | 0.26 ± 0.15            |
| All     | 4000                              | 44% (−0.81, −23.09)               | 0.29 ± 0.15            |
| North   | 0                                 | 41% (−0.88, −23.16)               | 0.23 ± 0.10            |
| North   | 0                                 | 30% (−0.85, −23.09)               | 0.26 ± 0.11            |
| South   | 0                                 | 38% (−0.93, −23.20)               | 0.40 ± 0.17            |
| South   | 0                                 | 29% (−0.93, −23.17)               | 0.41 ± 0.17            |

**Notes.** Galaxies in regions with density contrast larger than \( \delta_{\text{cut}} \) and redshifts less than \( c_{z\text{cut}} \) are excluded from the maximization of \( P_\beta \). The quoted errors on \( \beta \) are based on mock catalogs and include shot-noise, cosmic variance, and inaccuracies in the peculiar velocity reconstruction. The results given here correspond to \( R_\delta = 6 h^{-1} \) Mpc.

The results are presented in Table 1 and the top panel of Figure 1. The plotted quantity, \( \Delta \chi^2 \), is \(-2 \ln P_\beta \) evaluated as a function of \( \beta \) minus its value at the minimum. In the figure, the parameters \( \alpha, M_* \) are fixed at their best-fit values. The overall errors listed in Table 1 are based on the analysis of the mocks in Section 4.1. The 1\( \sigma \) shot-noise, \( \delta_{\text{sn}} \), given by the
width of the curves at $\Delta \chi^2 = 1$ are significantly smaller than the corresponding total errors quoted in Table 1, i.e., $\delta \beta_{\text{tot}}$ is not the main source of error for the 2MRS catalog. Table 1 lists results only for $R_s = 6 h^{-1}$ Mpc, however, an application of the method with $R_s = 10 h^{-1}$ Mpc and $12 h^{-1}$ Mpc yields consistent results within the total $1\sigma$ errors in the table. The derived $\beta$ for both choices of $\delta_{\text{cut}}$ are consistent with Davis et al. (2011) and the values of $\alpha$ and $M_*$ agree well with Westover (2007).

It is useful to examine how the best-fit $\beta$ changes when in the maximization of $P_r$ we include only distance galaxies with $cz > 4000$ km s$^{-1}$. The results are given in Table 1 and in the bottom panel of Figure 1, showing $\Delta \chi^2$ versus $\beta$ for this case. Shot-noise, as indicated by the width of the curves, increases with respect to the full sample (see the top panel of the same figure). The constraints from the distant cut are still reasonably tight and, within the total $1\sigma$ errors, are fully consistent with those obtained from the whole sample.

We also applied the method to galaxies in the northern and southern Galactic hemispheres, separately. The results, listed in Table 1, show a non-negligible difference between the derived $\beta$ in the two hemispheres. The corresponding error on $\beta$ is based on the application of the method to “northern” and “southern” hemispheres in the mocks. We attribute the difference between the north and the south to cosmic variance and confirm that it is consistent with the mocks. For each mock catalog we compute $\beta_{\text{north}}$ and $\beta_{\text{south}}$, and find an rms value $(\langle (\beta_{\text{north}} - \beta_{\text{south}})^2 \rangle)^{1/2} = 0.2$.  

5. DISCUSSION AND CONCLUSIONS

We have presented a new method to determine $\beta$ from galaxy redshift surveys. The method is entirely independent of distance indicators such as the Tully–Fisher relation and of analyses of anisotropic correlation functions ($\xi(r_p, \pi)$; Kaiser 1987). As a preliminary application of the method, we have resorted to the $K = 11.25$ flux limited 2MRS all sky survey. In the maximization procedure of $P(M_\beta | cz, v(\beta))$, galaxies in dense regions should be excluded since the flow pattern in these regions is not well recovered by linear theory. For our adopted density contrast cut $\delta_{\text{cut}} = 1$, we get a best-fit $\beta = 0.35 \pm 0.1$ for velocities smoothed with a Gaussian window of $6 h^{-1}$ Mpc in width. However, the results obtained with $\delta_{\text{cut}} = 2$ and $\delta_{\text{cut}} = \infty$ are consistent with this best-fit value and are reported in Table 1. These constraints on $\beta$ agree very well with those of Davis et al. (2011) who compared the peculiar velocities of the Spiral Field I Band (SFI++) catalog of spiral galaxies (Masters et al. 2006; Springob et al. 2007) with the velocity field predicted from the 2MRS.

There are three sources of uncertainties which contribute to the error budget on the estimated $\beta$:

1. “shot-noise” due to the finite number of galaxies;
2. cosmic variance which reflects the variation of the large-scale structure in random volumes comparable in size to the volume covered by the data set under consideration;
3. inaccuracies in the linear methodology for reconstructing the peculiar velocity from the galaxy distribution.

For the 2MRS sample of $\sim 18,000$ galaxies within $cz = 10^4$ km s$^{-1}$, shot-noise is subdominant. Increasing the number of galaxies in the sample without probing larger volumes will not tighten the constraint significantly. Cosmic variance can only be reduced by pushing toward deeper and larger surveys. The main galaxy sample of the Sloan Digital Sky Survey (SDSS; Strauss et al. 2002) already offers this opportunity. Let us consider the $\sim 7500$ deg$^2$ patch around the Northern Galactic cap in the SDSS-DR7 release (Abazajian et al. 2009). A shell with $\Delta z = 0.1$ centered at $z \sim 0.135$ close to the peak of the galaxy $dn/dz$ has a comoving volume $\sim 20$ times larger than that of the 2MRS and it could be divided into $\sim 12$ independent cubes of $200 h^{-1}$ Mpc, each containing as many galaxies as 2MRS. Applying our method to each of them would dramatically decrease cosmic variance and errors in the reconstructed velocities. According to Equation (12) the now dominant shot-noise error would be $\delta \beta \sim 0.06$, twice as small as in the 2MRS case, as shown in Table 2.

Shot-noise errors increase linearly with redshift and they will be the limiting factor in limiting the precision of constraining $\beta$ from the method presented in this paper. Next-generation, large redshift surveys like BOSS, BigBOSS, and EUCLID will allow an application of our method to measure $\beta$ out to $z \sim 1$. We have applied Equation (12) to estimate the expected shot-noise error on $\beta$ for all these surveys. The results are listed in Table 2. The ongoing SDSS-III (BOSS) survey will target highly luminous galaxies with a nearly constant number density $n \sim 3 \times 10^{-4} h^3$ Mpc$^{-3}$ (Eisenstein et al. 2011) over the redshift range $[0.2-0.6]$. Because of the relatively low number density of objects our method will measure $\beta$ with uncertainties twice as large as for the 2MRS case. The BigBOSS survey will expand BOSS, both in sky coverage and depth. The expected number of emission line galaxies (Schlegel et al. 2011) will be large enough to decrease the errors on $\beta$ significantly. In fact, the reduction of the relative errors will be even larger since, in this redshift
range, $\beta$ is an increasing function of $z$. Finally, the EUCLID survey (Laureijs 2009) will reduce errors even further in the redshift range $z = [0.7 - 1.0]$.

How do the $\beta$ estimates obtained with our method compare with those obtained from the analysis of the anisotropy pattern of the two-point correlation function $\xi(r_p, \pi)$? At low ($z < 0.2$) redshift, the $\xi(r_p, \pi)$ method has been applied to SDSS (Tegmark et al. 2006) and 2dF (Hawkins et al. 2003) allowing $\beta$ to be estimated with an error $\delta \beta = 0.15$. The application of our method to 2MRS already gives $\beta$ with a similar precision and the upcoming application to SDSS-II data (Bernardi et al. 2003) will allow us to estimate $\beta$ with a precision sufficient to test the validity of popular alternative gravity models, like the five-dimensional brane-world of Dvali et al. (2000) and Wei (2008) as well as the possibility of a coupling between the dark energy and the dark matter sectors (di Porto & Amendola 2008).

The first measurement of the growth rate at larger redshifts has been performed by Guzzo et al. (2008). From the observed $\xi(r_p, \pi)$ of VVDS galaxies they obtained $\beta(z = 0.77) = 0.70 \pm 0.26$. More recently, the Wiggle-z experiment (Blake et al. 2011) has measured the normalized growth rate $f(\Omega)\sigma_8$ in four redshift bins to $z = 0.9$ with an error $\delta f(\Omega)\sigma_8 \sim 0.1$. This is already comparable with the expected performance of our method on future data sets. Indeed, our method does not compare favorably with the analysis of galaxy clustering in future surveys. In the case of EUCLID, the goal is to estimate the growth rate from $\xi(r_p, \pi)$ with a precision of 0.01 at $z = 1$, if the rms mass fluctuation $\sigma_8$ can be determined accurately (Song & Percival 2009). In this case our alternative method to measure $\beta$ will constitute an effective way to keep systematic errors below $\delta \beta = 0.1$. It is important to note that if galaxy bias will only be constrained at the 10% level then both methods, the analysis of $\xi(r_p, \pi)$ and the one proposed here, will constrain the growth rate $f(\Omega)$ with similar precision. Further, planned redshift surveys will deliver velocity dispersion for all the elliptical galaxies will be obtained. Using the Faber–Jackson relation (Faber & Jackson 1976), between luminosity and velocity dispersion of elliptical galaxies, in conjunction with our method will produce even tighter constraints. Additional constraints on $f$ could also be obtained from the expected large-scale supernova survey (e.g., Bhattacharya et al. 2011).

Quasi-linear (i.e., the mildly nonlinear regime) dynamical reconstruction methods offer a substantial improvement in the accuracy of the predicted peculiar velocities. In particular, the fast action method (Nusser & Branchini 2000; Branchini et al. 2002) adaptation of Peebles’ least action principle (Shaya et al. 1995) is fast and easy to implement. This method is significantly better than linear theory for the reconstruction of the peculiar velocity field on small scales and also in dense regions. The linear theory relation between mass and velocity solely depends on the growth rate $f(\Omega) = \Omega^\gamma$. In the quasi-linear regime, there is an additional explicit dependence on $\Omega$, raising the possibility of separate constraints on $\Omega$ and the growth index $\gamma$. However, this does not seem a promising route to break the $\Omega - \gamma$ degeneracy since the explicit $\Omega$ dependence is very weak ($\sim \Omega^{-2}$, Nusser & Colberg 1998).

The method could easily be extended to account for systematic differences in the luminosity functions of different types of galaxy populations. This could be done by assigning different luminosity functions for these populations, provided that the sample is large enough to allow robust determination of the respective increase in the number of free parameters. We did not use this approach for 2MRS since late-type galaxies outnumber the early types, especially that galaxies in dense regions are excised from the analysis because of the failure of linear velocity reconstruction in dense regions. In addition, in the analysis of the luminosity function of 2MRS galaxies, objects were removed because the linear reconstruction of the velocity field is unreliable in high-density environment. A further possible improvement could be achieved by the use of nonparametric fit techniques for modeling the galaxy luminosity distribution. Although the Schechter form is a good approximation for the 2MRS, it is likely less successful for larger and deeper data sets. A variety of such nonparametric fit methods (e.g., Efstathiou et al. 1988; Davis & Huchra 1982) could easily be incorporated in the method presented here.

Angular coherent photometric miscalibrations are likely to contaminate the data at some level. However, as long as the density field inferred from the galaxy distribution is not significantly affected, the method should yield an unbiased $\beta$. This is because the underlying velocity field should be uncorrelated with observational miscalibrations. If systematic biases introduce serious spurious modes in the density field then it is doubtful if the data could be useful for any analysis of large-scale clustering.

The tests show, in the figure, that we are quite insensitive to the degree of nonlinear clustering, which is reassuring. Changing the smoothing scale or $\delta_{\text{cut}}$ does not change the best fit at more than $1\sigma$. This is as expected because the velocity field is itself a linear phenomenon dominated by large-scale structure.

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