Crowdsourcing with Bounded Rationality: A Cognitive Hierarchy Perspective

Qi Shao, Man Hon Cheung, and Jianwei Huang

Abstract—Previous studies in crowdsourcing systems usually regard workers as fully rational players, who have infinite cognitive capabilities when reasoning about other players’ decisions. However, recent psychological studies have revealed that humans are often bounded rational with cognitive reasoning limits. In this paper, we present a first study regarding the impact of such worker bounded rationality in a crowdsourcing system, and characterize how the result obtained from this more practical assumption deviates from the fully rational benchmark. Specifically, we consider a simple two-stage crowdsourcing model, where a requester first determines the rewards for workers completing the tasks, and then workers make their task choices accordingly. First, we show that such a model is non-trivial to analyze even in the fully rational case, due to the integer constraints on workers’ choices. Nevertheless, we are able to characterize the closed-form solution of the optimal rewards and Nash equilibrium with full rationality by exploiting the special structure of the problem formulation. Next, we focus on the more practical bounded rational model, and apply the cognitive hierarchy theory from behavioral economics in the modeling of workers’ decisions. Comparing with the fully rational benchmark, we show that in practice the requester can receive a higher profit when considering the workers’ bounded rationality, especially when the number of workers is large or the workers’ average cognitive level is low. When the workers’ average cognitive level is high enough, however, the practical bounded rational model converges to the benchmark fully rational model.

I. INTRODUCTION

A. Motivations

Crowdsourcing [1] is a fast-growing outsourcing paradigm, where a large group of workers are recruited to accomplish some tasks together. Recent popular examples of online crowdsourcing platforms include Amazon Mechanical Turk [2] and Microworkers [3], where requesters can announce tasks together with the corresponding rewards, and workers can claim and complete the tasks to earn rewards. A properly designed incentive mechanism (i.e., how much reward to give and how to share the rewards among workers) is critical for the requester to achieve her desired objective.

Existing literature on incentive mechanism designs in crowdsourcing usually considered the fully rational workers, who have the same infinite cognitive levels of reasoning other workers’ decisions when choose their own. As a result, the workers’ interactions in terms of the task selections are formulated as a non-cooperative game, with the Nash equilibrium (NE) being the most widely used solution concept [4].

However, recent experimental studies in psychology have shown that people often have cognitive limits when making their reasoning decisions. Instead of considering the workers to be fully rational, it is more accurate to assume that the workers as bounded rational, who have heterogeneous and finite cognitive levels of reasoning about others’ decisions. One widely adopted theory that mathematically characterizes the bounded rational players’ interactions is the cognitive hierarchy (CH) theory [7] (also called the behavioral game theory). Under the CH theory, the players, who are categorized into a number of cognitive levels, reason in a progressive decision process to achieve the cognitive hierarchy equilibrium. More specifically, under the CH theory, the players with the lowest cognitive level (i.e., the “level-0” player) select their strategies randomly without considering the choices of the other players. A “level-1” player, who is one cognitive level higher than the level-0 player, makes his choice by assuming that all the other players are level-0 players with random choices. In general, a “level-k” player assumes that the other players are distributed according to a normalized Poisson distribution [7] from level-0 to level-(k − 1) in the population, and makes his decision by anticipating the other players’ decisions accordingly. In other words, a level-k player can accurately estimate the relative proportions of all the lower level players, but ignores the fact that there can be other players at the same or higher levels than his. Then we model the workers’ behaviors from level-0 to level-∞ progressively, and compute the total number of workers selecting the tasks. The CH theory has been very successful at explaining deviations from the NE for a wide range of games in practice [8].

In this paper, we aim to answer the following key questions:

- How would the workers select the tasks under the practical bounded rational model?
- In anticipating the bounded rational workers’ behaviors, how should the requester determine the optimal reward

1Such a limit has also been observed in some practical crowdsourcing campaigns. For example, in the Alipay Red Envelope Campaign [5], all participating workers could share 500 million RMB once they completed collecting red envelopes on the Alipay App. The workers know the exact number of participants completing the tasks in real time. A fully rational worker would only continue to complete the task if the potential benefit is larger than the cost. Nevertheless, about 251 million people finished the rather long collection process with a reward less than 2 RMB per worker [6], which might not even compensate a user’s mobile data and time cost involved.

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to maximize her profit?

- How does the bounded rational model relate to the fully rational benchmark and bring new insights?

B. Contributions

As the first paper on modeling workers’ limited cognitive reasoning capability in crowdsourcing, we consider a simple model where a requester recruits workers to complete two tasks. Different tasks involve different completion costs and different revenue functions. We derive the optimal requester rewards and workers task selection equilibrium in both fully rational (FR) and bounded rational (BR) models, where the FR serves as the performance benchmark.

We summarize the key results and contributions as follows:

- **Novel crowdsourcing model with the CH theory**: To the best of our knowledge, this is the first paper that applies the CH theory in crowdsourcing systems.
- **Closed-form analysis of the FR benchmark**: Despite of the integer constraints on workers’ task selections, we are able to derive the requester’s optimal reward and the workers’ NE in closed-form.
- **Theoretical comparison under the limiting case**: In the limiting case where the workers’ average cognitive level is high enough, we prove that the practical BR model converges to the benchmark FR model.
- **Exploiting workers’ limited reasoning capacity to improve profit**: Under the BR model, we show that the requester can exploit the “level-0” workers (with the lowest cognitive capabilities) as cheap labor and set a small reward to increase her profit (as compared with the FR benchmark), especially when the number of workers is large or the workers’ average cognitive level is low.

C. Related Works

There have been many excellent prior studies on the incentive mechanism design of crowdsourcing systems, where the players are assumed to be fully rational (e.g., [9]–[11]). Several recent work started to challenge this widely used assumption of full rationality. For example, Karaliopoulos et al. in [12] questioned the fully rational assumption in crowdsourcing systems with the support of empirical data, but the authors only conducted a comparative study of different behavioral theories. Peng et al. in [13] incorporated the bounded rationality of the contributors into an evolutionary competing game formulation. However, the authors mainly focused on iterative games and the evolutionary equilibrium solution, which requires a long period of time for the contributors to learn and adapt.

Camerer et al. proposed the CH theory in [7] to explain players’ behavioral deviation from the traditional equilibrium. Inspired by this idea, we apply the CH theory to model workers’ actual behaviors in one-shot crowdsourcing games. We characterize how the workers’ average cognitive level and the population size influence the requester’s and workers’ practical behaviors in the crowdsourcing system. These results have not been revealed in the earlier generic CH studies.

Fig. 1. A two-stage crowdsourcing model. In Stage I, the requester sets rewards $R_1$ and $R_2$ to incentivize the workers to put effort in two tasks. In Stage II, a worker could select either task 1, task 2, or not to participate. Workers selecting the same task equally share the corresponding reward.

II. System Model

In Section II-A, we provide a high level discussion of the crowdsourcing system. Then we introduce the decisions of the requester and the workers in two stages: The requester determines the rewards in Stage I (Section II-B), and the workers decide which task to select in Stage II (Section II-C).

A. Setting

As shown in Fig. 1, we consider a system where a requester announces two tasks on the crowdsourcing platform. The requester associates task $m \in \mathcal{M} = \{1, 2\}$ with a reward $R_m \geq 0$. Let $N = \{1, 2, \cdots, N\}$ be the set of workers. We refer to the requester as “she” and a worker as “he” in the rest of the paper.

The requester aims to attract enough workers to complete both tasks, with the goal of profit maximization (details in Section II-B). Each worker determines whether to participate and if yes which task to select, in order to maximize his payoff (details in Section II-C). We assume that a worker cannot work on both tasks.

As in most crowdsourcing systems, the requester has a larger market power and makes the decisions before the workers do. Hence we model the interactions in the system as a two-stage game.

B. Stage I: Requester’s Profit Maximization Problem

In Stage I, the requester is regarded as fully rational$^3$ and determines the rewards for both tasks to maximize her profit, as shown in the following optimization problem:

**Requester’s Profit Maximization Problem (Profit)**

$maximize \quad U_1(N_1(R_1, R_2)) + U_2(N_2(R_1, R_2)) - (R_1 + R_2) \quad \text{subject to} \quad R_1 \geq 0, R_2 \geq 0, \quad \text{variables:} \quad R_1, R_2. \quad (1)$

Here $N_m(R_1, R_2)$ is the number of workers selecting task $m \in \mathcal{M}$ given rewards $R_1$ and $R_2$ in Stage II, which will be derived under the FR and BR models in Section III and Section IV, respectively. We define $U_m(N_m)$ as the revenue function of task $m \in \mathcal{M}$, which increases with the number $N_m$.

$^3$For example, the two tasks involve the traffic estimation at two far-away locations at the same time, respectively [14].

$^3$With sufficient computational power, it is often reasonable to assume that the requester is fully rational and can choose an optimal strategy [13].
of workers selecting task \( m \). For example, more workers reporting traffic information leads to a more accurate travel time estimation in one region. In Section III-C, we will consider some concrete function forms of \( U_m(N_m) \).

C. Stage II: Workers’ Task Selection Problem

In Stage II, for each worker in set \( \mathcal{N} \), he needs to decide whether and which task to select. Let \( s_n \in \mathcal{S}_n = \{0, 1, 2\} \) denote worker \( n \)'s choice (strategy), with \( s_n = 0 \) denoting not selecting any task, \( s_n = 1 \) (or 2) denoting selecting task 1 (or 2). Let \( s_{-n} = (s_1, \ldots, s_{n-1}, s_{n+1}, \ldots, s_N) \) be the strategy profile of all other workers except worker \( n \). When a worker selects a task \( m \in \mathcal{M} \), he will incur a cost of \( c_m \geq 0 \). We assume that the cost is task dependent but does not depend on the identity of workers. \(^{4}\)

Let \( \mathcal{N}_m = \{n \in \mathcal{N} : s_n = m\} \) denote the set of workers who decide to select task \( m \in \mathcal{M} \) in Stage II, with a size of \( N_m = |\mathcal{N}_m| \). We have \( N_1 + N_2 \leq N \). When multiple workers select to complete the same task \( m \), the requester will equally divide\(^{5}\) the reward among all \( N_m \) workers. Hence the payoff of a worker \( n \) is

\[
\pi_n(s_n, s_{-n}, (R_1, R_2)) = \begin{cases} 
0, & \text{if } s_n = 0, \\
\frac{R_1}{N_1} - c_1, & \text{if } s_n = 1, \\
\frac{R_2}{N_2} - c_2, & \text{if } s_n = 2.
\end{cases}
\]

To maximize his payoff, a worker in Stage II will anticipate other workers’ strategies, and makes his decision accordingly. In the FR case, workers will have infinite reasoning capability and can accurately predict others’ behaviors. We will formulate the user interactions as a non-cooperative game, and derive the NE solution in Section III. In the BR case, however, we consider the more realistic assumption that workers have different cognitive levels and may have wrong beliefs about others’ strategies. We derive the corresponding cognitive hierarchy equilibrium (CHE) in Section IV.

III. FULLY RATIONAL MODEL

In this section, we analyze the case of fully rational workers. In Section III-A, we model workers’ behaviors in Stage II to obtain the sufficient and necessary conditions of the NE. By incorporating such conditions in Stage I, we reformulate the requester’s profit optimization problem in Section III-B, and present its closed-form solution in Section III-C.

A. Workers’ Non-cooperative Task Selection in Stage II

1) Non-cooperative Task Selection Game: When workers are fully rational, we formulate workers’ task selection decisions as a non-cooperative game, as one worker’s payoff depends on the choices of all other workers.

Definition 1: The Stage II task selection game is a tuple \( \Omega = (\mathcal{N}, \mathcal{S}, \Pi) \) defined by:

- Players: The set of workers \( \mathcal{N} \).
- Strategies: Each worker \( n \in \mathcal{N} \) selects a strategy \( s_n \) from the set \( \mathcal{S}_n = \{0, 1, 2\} \). The strategy profile of all workers is \( s = (s_n, \forall n \in \mathcal{N}) \). The set of feasible strategy profile of all workers is \( \mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n \).
- Payoffs: Each worker \( n \in \mathcal{N} \) maximizes his payoff as defined in (2). The vector \( \Pi(s, (R_1, R_2)) = (\pi_n(s, (R_1, R_2)), \forall n \in \mathcal{N}) \) contains the payoff functions of all workers.

For worker \( n \), given the rewards \( (R_1, R_2) \) and other workers’ strategy profile \( s_{-n} \), he aims to maximize his payoff:

\[
\text{Workers’ optimization problem: (Worker-Payoff)}
\]

\[
\text{maximize } \pi_n(s_n, s_{-n}, (R_1, R_2)) \quad (3)
\]

Assume under \( s_{-n} \), there are \( \hat{N}_m \) workers selecting task \( m \in \{1, 2\} \), respectively. After solving Problem (3), worker \( n \)'s best response task selection choice is

\[
s_n^*(s_{-n}, (R_1, R_2)) = \begin{cases} 
1, & \text{if } \frac{R_1}{N_1 + 1} - c_1 \geq \frac{R_2}{N_2 + 1} - c_2 \\
\text{and } R_1 - N_1 + 1 - c_1 \geq 0, \\
2, & \text{if } \frac{R_2}{N_2 + 1} - c_2 > \frac{R_1}{N_1 + 1} - c_1 \\
\text{and } R_2 - N_2 + 1 - c_2 \geq 0, \\
0, & \text{otherwise}.
\end{cases}
\]

2) Nash Equilibrium: The fixed point of all the workers’ best response choices is the Nash equilibrium (NE), where no worker can improve his payoff by deviating from his task choice unilaterally.

Definition 2 (Nash equilibrium): Under fixed rewards \((R_1, R_2)\), a strategy profile \( s^{NE} \) is an NE of game \( \Omega \) if

\[
\pi_n(s^{NE}, s^{NE}_{-n}, (R_1, R_2)) \geq \pi_n(s_n, s^{NE}_{-n}, (R_1, R_2)),
\]

\( \forall s_n \in \mathcal{S}_n, \forall n \in \mathcal{N} \). \( (5) \)

We can write the number of workers selecting a task \( m \in \mathcal{M} \) at the NE under rewards \((R_1, R_2)\) as

\[
N^\text{NE}_m (R_1, R_2) = \{n \in \mathcal{N} : s_n^{NE} = m\} \quad (6)
\]

3) Analysis: Proposition 1 characterizes the relationship between the requester’s rewards in Stage I and the workers’ NE choices in Stage I.

Proposition 1: Given any rewards \((R_1, R_2)\), the number of workers selecting the tasks at the NE is \( (N^\text{NE}_1, N^\text{NE}_2) \) if and only if\(^{6}\)

\[
R_1 - c_1 N^\text{NE}_1 \geq \max \left\{ 0, N^\text{NE}_1 \left( \frac{R_2}{N^\text{NE}_2 + 1} - c_2 \right) \right\},
\]

\( (7) \)

\[
R_2 - c_2 N^\text{NE}_2 \geq \max \left\{ 0, N^\text{NE}_2 \left( \frac{R_1}{N^\text{NE}_1 + 1} - c_1 \right) \right\},
\]

\( (8) \)

\[
(R_1 - c_1 (N^\text{NE}_1 + 1)) (N - N^\text{NE}_1 - N^\text{NE}_2) \leq 0,
\]

\( (9) \)

\[
(R_2 - c_2 (N^\text{NE}_2 + 1)) (N - N^\text{NE}_1 - N^\text{NE}_2) \leq 0.
\]

Proposition 1 illustrates how fully rational workers will behave in Stage II. Due to the page limit, the detailed proof is not given here.

\(^{4}\)For example, completing simple and small tasks (e.g., reporting traffic information) consumes a similar amount of cost for different workers. Different types of tasks, however, can lead to very different costs (e.g., reporting traffic speed vs. uploading pictures of traffic accident).

\(^{5}\)We assume that the task is easy to complete and there is no significant difference in term of the workers’ efforts/quality \([9]\), hence it is fair to equally split the reward \([13]\).

\(^{6}\)For notation simplicity, we write \((N^\text{NE}_1(R_1, R_2), N^\text{NE}_2(R_1, R_2))\) as \((N^\text{NE}_1, N^\text{NE}_2)\).
B. Bilevel Optimization

By incorporating the necessary and sufficient conditions (7)-(10) into Problem (Profit) in Stage I, the requester will be able to compute his optimal choices of rewards in Stage I considering the worker’s choices in Stage II. More specifically, the requester will solve the following problem that is equivalent to Problem (Profit) with fully rational workers:

**Requester’s Bilevel Profit Maximization (Profit-Bilevel)**

maximize $U_1(N_1) + U_2(N_2) - (R_1 + R_2)$  \hspace{1cm} (11a)
subject to $R_1 \geq 0$, \hspace{1cm} $R_2 \geq 0$, \hspace{1cm} (11b)
$R_1 - c_1N_1 \geq N_1 \left( \frac{R_2}{N_2 + 1} - c_2 \right)$, \hspace{1cm} (11c)
$R_2 - c_2N_2 \geq N_2 \left( \frac{R_1}{N_1 + 1} - c_1 \right)$, \hspace{1cm} (11d)
$R_1 - c_1N_1 \geq 0$, \hspace{1cm} $R_2 - c_2N_2 \geq 0$, \hspace{1cm} (11e)
$(N - N_1 - N_2)(R_1 - c_1N_1 - c_1) \leq 0$, \hspace{1cm} (11f)
$(N - N_1 - N_2)(R_2 - c_2N_2 - c_2) \leq 0$, \hspace{1cm} (11g)
$N_1 + N_2 \leq N$, \hspace{1cm} $N_1, N_2 \in \mathbb{N}$, \hspace{1cm} (11h)
variables: $R_1, R_2, N_1, N_2$. \hspace{1cm} (11i)

It should be noted that Problem (11) is a mixed integer programming problem, which is very challenging to solve because we cannot directly apply convex optimizations. Nevertheless, we can exploit the special structure of the crowd-sourcing problem and simplify the problem formulation. We will denote the optimal solution of Problem (Profit-Bilevel) as $(R_1^{NE*}, R_2^{NE*}, N_1^{NE*}, N_2^{NE*})$, where $(R_1^{NE*}, R_2^{NE*})$ are the optimal rewards in Stage I and $(N_1^{NE*}, N_2^{NE*})$ are the corresponding NE selections in Stage II.

**Proposition 2:** The rewards $(R_1^{NE*}, R_2^{NE*})$ in Stage I are optimal for Problem (Profit-Bilevel) only if

$R_1^{NE*} = c_1N_1^{NE*}$, \hspace{1cm} $R_2^{NE*} = c_2N_2^{NE*}$, \hspace{1cm} (12)

where

$N_1^{NE*} + N_2^{NE*} \leq N$ and $N_1^{NE*}, N_2^{NE*} \in \mathbb{N}$. \hspace{1cm} (13)

Due to the page limit, the proof of Proposition 2 is not given here.

Proposition 2 shows that (12) and (13) are the necessary conditions of the requester’s optimal rewards in Stage I. We note that if conditions (12) and (13) hold, then all the constraints in Problem (Profit-Bilevel) are satisfied. Thus, we can substitute $R_m = c_mN_m, \forall m \in \mathcal{M}$ in the objective function (11a), replace constraints (11b)-(11h) with conditions (12) and (13), and reformulate Problem (Profit-Bilevel) into Problem (Profit-Bilevel-Equivalent) as follows. Here we define the profit function for task $m \in \mathcal{M}$ as

$\Pi_m(N_m) = U_m(N_m) - c_mN_m$, \hspace{1cm} (14)

and the requester aims at maximizing the total profit:

**Requester’s Profit Maximization Problem (Profit-Bilevel-Equivalent)**

maximize $\Pi_1(N_1) + \Pi_2(N_2)$, \hspace{1cm} (15)
subject to $N_1 + N_2 \leq N$, \hspace{1cm} $N_1, N_2 \in \mathbb{N}$, \hspace{1cm} variables: $N_1, N_2$.

C. Optimal Rewards in Stage I and NE in Stage II

Our previous analysis applies to general increasing $U_1(N_1)$ and $U_2(N_2)$ functions. However, to capture the diminishing marginal return with respect to the number of workers on a task [11] and derive the closed-form solution, in the rest of this paper we will focus on a specific concave revenue function:

$U_m(N_m) = u_m \ln(1 + N_m)$, \hspace{1cm} (16)

where $u_m \geq 0$ is the revenue parameter of task $m \in \mathcal{M}$.

It should be noted that if $u_m \leq c_m$, for any positive $N_m$ value, we always have $\Pi_m(N_m) < 0$. As a result, the requester will never assign any positive reward to task $m \in \mathcal{M}$ to avoid such a trivial case, we make the following assumption for the rest of the paper.

**Assumption 1:** The revenue coefficients and costs satisfy: $u_m > c_m$, for each task $m \in \mathcal{M}$.

Next we derive the closed-form optimal solution of Problem (Profit-Bilevel-Equivalent). We denote $\lfloor x \rfloor$ and $\lceil x \rceil$ as floor and ceiling rounding function of variable $x$, respectively.

**Theorem 1:** The NE in Stage I under the FR model is $(N_1^{NE*}, N_2^{NE*}) = \arg \max_{(N_1,N_2) \in \mathcal{X}} \Pi_1(N_1) + \Pi_2(N_2)$, (17)

and the requester’s optimal rewards in Stage I are

$R_1^{NE*} = c_1N_1^{NE*}$ and $R_2^{NE*} = c_2N_2^{NE*}$. (18)

The set $\mathcal{X}^*$ of the feasible numbers of workers selecting the two tasks is defined differently for several cases below:

1) Case I (large worker population): $N > \frac{u_1 - c_1}{c_1}$ and $\mathcal{X}^* = \{ (N_1, N_2) : N_1 + N_2 \leq N, N_1 \in \mathcal{X}_1, N_2 \in \mathcal{X}_2 \}$.

2) Case II (small worker population): $N \leq \frac{u_1 - c_1}{c_1}$ and $\mathcal{X}^* = \{ (N_1, N_2) : N_1 + N_2 \leq N, N_1 \in \mathcal{X}_1, N_2 \in \mathcal{X}_2 \}$. We have three subcases here:

a) High utility coefficient $u_1: \frac{u_1}{N+1} - c_1 \geq u_2 - c_2$. In this subcase, $\mathcal{X}^* = \{ (N, 0) \}$;

b) High utility coefficient $u_2: \frac{u_2}{N+1} - c_2 \geq u_1 - c_1$. In this subcase, $\mathcal{X}^* = \{ (0, N) \}$;

c) Similar utility coefficients for task 1 and 2: $\frac{u_1}{N+1} - c_1 < u_2 - c_2$ and $\frac{u_2}{N+1} - c_2 < u_1 - c_1$. In this subcase, we define $\hat{c} = c_1 - c_2$, $\hat{u} = \sqrt{(\hat{c} + 1)^2 + 4\hat{u}}$, $N_1 = \left\{ \frac{u_1N + u_2 - u_1}{\hat{c} + \hat{u}}, \frac{N}{\hat{c} + \hat{u}} \right\}$, $N_2 = \left\{ \frac{u_2N + u_1 - u_2}{\hat{c} + \hat{u}}, \frac{N}{\hat{c} + \hat{u}} \right\}$.

Then $\mathcal{X}^* = \{ (\lfloor N_1 \rfloor, \lceil N_2 \rceil), (\lceil N_1 \rceil, \lfloor N_2 \rfloor) \}$.

Due to the page limit, the detailed proof of Theorem 1 is not given here. In the FR model, every worker who selects task $m$ at the NE will receive a reward of $\frac{R_m^{NE*}}{N_m^{NE*}} = c_m$, which equals to his operation cost of selecting task $m \in \mathcal{M}$. The key notations of the FR and BR models are listed in Table I.
TABLE I: KEY NOTATIONS

| Meaning                      | FR Model (Section III) | BR Model (Section IV) |
|------------------------------|------------------------|-----------------------|
| Optimal Rewards in Stage I   | $R_1^{CHE}, R_2^{CHE}$ | $R_1^{CHE}, R_2^{CHE}$ |
| Number of workers at Stage II’s equilibrium | $N_1^{CHE}(R_1, R_2)$ | $N_2^{CHE}(R_1, R_2)$ |

IV. BOUNDED RATIONAL MODEL

In this section, we consider the more practical BR model. We model workers’ behaviors based on the CH theory in Section IV-A, and compute the requester’s optimal rewards in Section IV-B.

A. Workers’ Task Selections in Stage II: Cognitive Hierarchy

To maximize his payoff in (2), a worker in Stage II compares three choices. If $R_m < c_m$, a worker will never select task $m \in M$, because his payoff will be negative even if he is the only one selecting the task. Hence we will focus on the non-trivial case as specified in Assumption 2, which is valid for the rest of the paper.

Assumption 2: The rewards and costs satisfy $R_m \geq c_m$ for each task $m \in M$.

Next, we model the BR case based on the CH theory [7], where workers with different cognitive thinking levels make decisions differently based on different beliefs of the other workers’ choices. Next we introduce the fundamental assumptions of the CH theory as in [7]:

1) The set of workers is divided into an infinite number of levels, indexed by $k = 0, 1, \cdots, \infty$. The number of workers at each level $k$ follows a Poisson distribution [7] $f(k) = \frac{e^{-\tau}}{\tau^k}$ with rate $\tau$ (which equals both the mean and the variance of the distribution). For any $\tau$ value, we have $\sum_{k=0}^{\infty} f(k) = 1$. The value of $\tau$ represents the average cognitive level of the population. For example, the value of $\tau$ may be larger if the average education level of the worker population is higher.

2) Level-$k$ ($k \geq 1$) workers know the accurate relative ratios among workers at lower rationality levels $f(0), f(1), \cdots, f(k - 1)$ and can accurately predict their behaviors, but ignore the existence of other level-$k$ and higher level workers. More specifically, a level-$k$ worker’s belief about the fraction of level-$h$ ($0 \leq h < k$) is $g_k(h) = \frac{f(h)}{\sum_{j=h+1}^{\infty} f(j)}$, where the subscript “$k$” denotes that such belief is unique to the level-$k$ worker. For any $h \geq k$, $g_k(h) = 0$. A level-$k$ worker believes that level-$h$ workers account for $g_k(h)$ proportion of the population.\(^7\)

3) Based on the discussions of 1) and 2), we can compute the choices of workers at different levels progressively.

\(^7\)Suppose there are two level-0, four level-1, and three level-2 workers in the platform. A level-1 worker believes that the workers except himself are at level-0. A level-2 worker has the belief that $g_2(0) = \frac{2}{2+4}$ and $g_2(1) = \frac{1}{2+4}$.\(^8\)

\(^8\)Recall that in the FR model, every worker can correctly anticipate all other workers’ decisions in Stage II. In the BR model, however, workers at different levels have different beliefs regarding the workers’ population composition and only anticipate the choices of workers at lower levels.

Under Assumption 2, a level-0 worker believes that he will get a non-negative payoff no matter which task to select (as he ignores the choices of any other worker in the population), hence he will select randomly. A level-$k$ worker will select a task that leads to the maximum payoff (including the choice of not participating), considering the task selections of all lower level workers.

Given any arbitrary rewards $(R_1, R_2)$ in Stage I, we propose a progressive task selection algorithm (i.e., Algorithm 1) based on [7], and computes the CHE solution $(N_1^{CHE}, N_2^{CHE})$ in Stage II:

- **Definition:** The function $e(k, m)$ represents the probability\(^10\) of level-$k$ workers selecting task $m$. The function $E(k, m)$ represents level-$k$ workers’ expected payoff of selecting task $m$, after taking into consideration of the participation strategies of level 0 to $k$ workers.
- **Initialization:** We define the number of workers selecting task $m$ as $n_m$, and the total fractions of workers that have been considered as $TF$. We initialize both values to be zero. For level-0 workers, they select randomly from two tasks, hence the probability will be $e(0, m) = \frac{1}{2}$ (line 1).
- **Expected payoffs:** We compute level-$k$ workers’ fraction $f(k)$ and the number of workers selecting task $m$ ($m = 3$ and 4). Then each level-$k$ worker believes that the number of workers selecting task $m$ is $\frac{m}{TF}$, and computes his expected payoff $E(k, m)$ accordingly (line 5).

\(^10\)For workers at some level, they may find that selecting either of the two tasks leads to the same non-negative payoff, in which case they can choose a mixed strategy by selecting both tasks with some probability distribution.

Algorithm 1: Task Selection Algorithm based on CH

Input: Rewards $R_1, R_2$; costs $c_1, c_2$; average cognitive level $\tau$; total number of workers $N$.

1. Initialization: Set $n_m = 0$, $TF = 0$, $MaxIters = 100N$, $e(0, m) = \frac{1}{2}$, $m \in M$.
2. For $k = 0$ to $MaxIters$: $k = k + 1$ do
   3. $f(k) = \frac{k}{k!} e^{-\tau}$, $TF = TF + f(k)$;
   4. $n_m = n_m + f(k) \cdot e(k, m) \cdot N$;
   5. $E(k, m) = \frac{R_m - c_m}{TF}$;
   6. if $E(k, 1) = E(k, 2) \geq 0$ then
      7. $e(k+1, 1) = e(k+1, 2) = \frac{1}{2}$;
   8. else
      9. if $E(k, 1) > E(k, 2)$ and $E(k, 1) \geq 0$ then
         10. $e(k+1, 1) = 1, e(k+1, 2) = 0$;
      11. else
         12. if $E(k, 2) > E(k, 1)$ and $E(k, 2) \geq 0$ then
            13. $e(k+1, 1) = 0, e(k+1, 2) = 1$;
         14. else
            15. $e(k+1, 1) = e(k+1, 2) = 0$;

Output: CHE Solution: $N_1^{CHE} = n_1$, $N_2^{CHE} = n_2$. 
• Selecting tasks: For level- \((k + 1)\) workers, if the payoffs of selecting these two tasks are equal and non-negative, they will select randomly (line 7). If the payoff of selecting task 1 (or 2) is non-negative and greater than the other one, they will select task 1 (or 2) (line 10 and 13). Otherwise, they will never select a task (line 15).

• Output: This process repeats until \(k \geq \text{MaxIters}\), when we have considered the reasoning of almost\(^{11}\) all the workers. The total number of workers selecting task \(m \in \mathcal{M}\) at the CHE is \(N^{CHE}_{m} = n_m\).

B. Optimal Rewards in Stage I

We have presented the requester’s optimization problem as in (1), which is the same for both the FR and BR cases in Stage II. Due to the progressive nature of Algorithm 1 in determining the task selection, it is difficult to compute the optimal reward in the BR model in closed-form. Nevertheless, we can still numerically compute the Stage I optimal rewards \((R^{CHE}_{1}, R^{CHE}_{2})\) through an exhaustive search, and we denote the corresponding Stage II CHE as \((N^{CHE}_{1}, N^{CHE}_{2})\).

1) Numerical Example: To gain a deeper understanding of the result, we choose the specific utility function \(U_m(N_m) = u_m \ln(1 + N_m), \forall m \in \mathcal{M}\), with utility parameters \(u_1 = 30\) and \(u_2 = 10\) as well as costs \(c_1 = 2\) and \(c_2 = 1\).

In Fig. 2, we plot the optimal task 1 reward \(R^{CHE}_{1}\) against \(N\) under different \(\tau\) values. As we can see, the three curves are not monotonically increasing. As an example, consider the case of \(\tau = 2\) (the red curve). When \(N\) increases from 20 to 80, \(R^{CHE}_{2}\) also increases, as the requester wants to attract more workers to complete the task.

However, when \(N\) reaches a threshold (i.e., \(N = 90\)), the requester is able to reduce the reward because she anticipates workers’ behaviors in Stage II. The requester aims to maximize her profit by attracting some higher level workers as the sharing reward per worker decreases (i.e., a smaller reward attracts similar number of workers, hence the profit increases).

As \(N\) becomes very large (e.g., \(N \geq 200\)), the optimal reward remains unchanged and equal to the cost \(c_1\). In this case, as there are many level-0 workers in the population, it is enough for the requester to focus on exploiting these cheap labors to complete the task.

Comparing the three curves with different \(\tau\) values, we can see that a higher \(\tau\) value (i.e., a more sophisticated population) makes it more difficult for the requester to take advantage of workers (i.e., by setting small rewards \(R^{CHE}_{m} = c_m\)).

V. COMPARISON OF THE FR AND BR MODELS

In this section, we compare the FR and BR models both theoretically and numerically. In Section V-A, we prove that the FR model is a limiting case of the BR model when the average cognitive level \(\tau\) approaches \(\infty\) (i.e., all users have an infinite reasoning capacity). Note that it is not generally true in the CH theory [7]. We further compare the two models numerically with a finite value of \(\tau\) in Section V-B.

A. What is the Connection between the FR and BR Models?

In this section, we show that the theoretically elegant FR model widely used in the literature is a limiting case of the more practical BR model (as \(\tau\) approaches \(\infty\)) in the context of our crowdsourcing system.\(^{12}\) This means that the insights derived by the vast incentive mechanism design literature for crowdsourcing are still good approximations of the reality when the workers have sufficient reasoning capabilities.

For a fair comparison between the FR and BR models, we will set \(R_m = R_{m}^{NE}\) in Stage I for both models, and then derive and compare the number of workers selecting task \(m \in \mathcal{M}\) in Stage II, which we will denote as \(N^{NE}_{m}\) and \(N^{CHE}_{m}(R_{1}^{NE}, R_{2}^{NE})\) in the FR and BR model, respectively.\(^{13}\) Since a worker cannot select two tasks simultaneously, we have \(N^{NE}_{1} + N^{NE}_{2} \leq N\), and we will theoretically discuss the case of \(N^{NE}_{1} + N^{NE}_{2} = N\) in Theorem 2.

Theorem 2: Consider the case where the optimal solution of Problem (Profit-Bilevel-Equivalent) satisfies \(N^{NE}_{1} + N^{NE}_{2} = N\). If we choose \((R_1, R_2) = (R_{1}^{NE}, R_{2}^{NE})\) in the BR case, then we have the following results: (a) every worker will participate by selecting one of the two tasks in the BR case; (b) as \(\tau\) approaches \(\infty\), the CHE \(N^{CHE}_{m}(R_{1}^{NE}, R_{2}^{NE})\) converges to the NE \(N^{NE}_{m}\) for \(m \in \{1, 2\}\).

Theorem 2 proves the convergence of CHE to NE under the case of \(N^{NE}_{1} + N^{NE}_{2} = N\). Although we are not able to theoretically prove the same result under the case of \(N^{NE}_{1} + N^{NE}_{2} < N\), we have numerically validated this property through extensive simulations under various system parameters. The above theoretical and empirical studies show that the FR model is a limiting case of the BR model, when the average cognitive level \(\tau\) approaches infinity.

B. Comparison of the FB and BR Models under a Finite \(\tau\)

Now we focus on the numerical comparison between the FR and BR models under the more realistic case of a finite \(\tau\). We will show that the requester can take advantage of the BR model and obtains a higher level of profit.

We choose the values of \(u_m\) and \(c_m\) similar as that of Fig. 2 in Section IV-B. In the FR model, we compute the profit maximizing rewards \((R_{1}^{NE}, R_{2}^{NE})\) and the corresponding NE \((N_{1}^{NE}, N_{2}^{NE})\) based on Theorem 1. Regarding the BR model, we set \(\tau = 5\) and consider two cases.

In the first scenario, we assume that the requester sets the same rewards \((R_{1}^{NE}, R_{2}^{NE})\) in Stage I (both for the FR and BR models), and focus on a fair comparison between the Stage II CHE and NE, as in Fig. 3. When \(N\) is small (i.e., \(N \leq 23\)), the numbers of workers selecting both tasks are similar under NE and CHE, and increase with \(N\). In this case, there are no two workers belonging to the same level, hence different workers make their decisions sequentially as in Algorithm 1.

\(^{12}\)In [7], Camerer et al. showed that as \(\tau \rightarrow \infty\), the CHE converges to an NE that can be reached by finitely many iterated deletions of weakly dominated strategies. However, our crowdsourcing model here does not belong to this category of games, hence the result in [7] does not apply.

\(^{13}\)Note that \(N^{CHE}_{m}(R_{1}^{NE}, R_{2}^{NE})\) is different from \(N^{CHE}_{m}\) defined in Section IV, which corresponds to the CHE when the requester chooses the rewards to optimize her profit, i.e., \(N^{CHE}_{m}(R_{1}^{CHE}, R_{2}^{CHE})\).
This means that higher level workers will be able to adjust their choices (i.e., if the number of workers selecting task $m$ exceeds $N_{m}^{NE*}$, higher level workers will no longer select the same task). As a result, given the same rewards in the FR and BR models, the $N_{m}^{NE*}$ and $N_{m}^{CHE*}$ in Stage II will be similar, and both increase with $N$ (Case II in Theorem 1).

As $N$ becomes large enough (i.e., $N \geq 65$), Fig. 3 shows that the $N_{m}^{NE*}$ under NE in the FR model no longer changes (Case I in Theorem 1), but the $N_{m}^{CHE*}$ under CHE in the BR model continues to increase. In the BR model, we can have $N_{m}^{CHE*} > N_{m}^{NE*}$ workers selecting task $m$. For example, before a particular level of workers’ task selection, the number of workers selecting task $m$ may approach $N_{m}^{NE*}$, and each worker in this level believes he will gain a positive payoff if he is the only one to select this task. However, there exist more than one workers selecting, hence we have $N_{m}^{CHE*} > N_{m}^{NE*}$. As $N$ increases, every cognitive level contains more workers, and the difference between $N_{m}^{CHE*}$ and $N_{m}^{NE*}$ will be larger.

Next we consider a second scenario, where we let the requester set the possibly different profit maximizing rewards in Stage I, and derive the corresponding CHE and NE as in Table I. Fig. 4 illustrates the results. When $N$ is small, the maximum profits in both models are the same and increase in $N$. This is because the optimal rewards in the BR and FR models are the same. Given the same rewards, workers behave similarly in the FR and BR models, hence the requester cannot increase her profit by setting any other reward.

As $N$ becomes large (i.e., $N \geq 23$), Fig. 4 shows that the profit in the FR model will not further increase because the optimal reward $R_{m}^{NE*}$ and the NE $N_{m}^{NE*}$ remain unchanged (Case I in Theorem 1), while the profit in BR model goes up unboundedly with the total number $N$. This is because although the optimal reward $R_{m}^{CHE*}$ in Stage I cannot compensate workers’ operation cost in Stage II (after reward sharing), level-0 workers still select task $m$ in the BR model. When $N$ increases, there are more level-0 workers, while the optimal total reward for each task remains unchanged and equal to the cost, hence the total profit increases unboundedly.

VI. CONCLUSION

In this paper, we presented the first study regarding how the bounded rationality (i.e., the cognitive limits of reasoning) affects the crowdsourcing incentive mechanism design. In the fully rational benchmark case, we derived the requester’s optimal rewards and workers’ Nash equilibrium choices in closed-form. In the bounded rational case, we modeled the workers’ belief formation and progressive decision process based on the cognitive hierarchy theory from behavioral economics. We analyzed how the requester can improve her profit by taking advantage of the workers’ imperfect reasoning. Simulation results showed that the requester is more likely to obtain a higher profit under a larger worker population with a lower average cognitive level.

In the future work, we will consider the impact of bounded rationality in more general and practical system scenarios, such as considering the heterogeneous worker costs that depend on the workers’ efforts levels and capabilities. We will also conduct field trials to validate our theory, e.g., justifying the assumptions of cognitive hierarchy theory and estimating the value of $\tau$ in a typical crowdsourcing worker population.

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