The Scalar Meson Sector and the $\sigma$, $\kappa$ Problem

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Abstract. In the light scalar meson sector ($M \lesssim 1.8$ GeV) one expects at least one $q\bar{q}$ nonet and a glueball, possibly also multi-quark states. We discuss the present phenomenological evidence for $\sigma$ and $\kappa$ particles; if real, they could be members of the lightest (quark or multi-quark) nonet together possibly with $a_0(980)$ and $f_0(980)$. Alternatively, the lightest nonet could include $f_0(980)$ but not $\sigma$ and $\kappa$. Future decisive experimental studies, concerning tests of symmetry relations, especially in B-decays, are outlined.

INTRODUCTION

The light scalar meson sector represents still a big puzzle. There should be a nonet of P wave $q\bar{q}$ states with $J^{PC} = 0^{++}$ besides the other rather well known $P$ wave nonets with $1^{++}$, $2^{++}$ and $1^{--}$. In addition one expects the lightest glueball in the same channel. Furthermore, it is possible that more complex multi-quark bound states exist. In Table 1 we list the light scalar mesons according to the Particle Data Group [1].

Of particular recent interest are the $\sigma$ and $\kappa$ particles in the $\pi\pi$ $I=0$ and $K\pi$ $I=\frac{1}{2}$ channels, the lightest particles for the given isospin. Because of their large width as compared to their mass $\Gamma \gtrsim M$ these states are difficult to identify experimentally and therefore their status is still under debate, also at this conference. Besides the question of their existence one has to enlighten their role in spectroscopy, i.e. to determine their constituent structure and the multiplet they belong to. The high interest in these states also originates from their important role they play in meson theories based on chiral symmetry (reviews [2, 3, 4]).

In this talk the status of the phenomenology and possible future tests will be discussed.

TWO ROUTES FOR SCALAR SPECTROSCOPY

Among the various approaches to scalar spectroscopy we will contrast two routes which are essentially different in the classification of the states, although not unique among

| I = 0 | $f_0(600)$ (or $\sigma$) | $f_0(980)$ | $f_0(1370)$ | $f_0(1500)$ | $f_0(1710)$ | $f_0(2020)$ |
| I = $\frac{1}{2}$ | $\kappa(900)$? | $K_0^*(1430)$ | $K^*(1950)$? |
| I = 1 | $a_0(980)$ | $a_0(1450)$ |
themselves.

**Route I:** two multiplets below 1800 MeV

The upper multiplet includes the uncontroversial $qq\bar{q}$ state $K^*$ (1430), then also the nearby $a_0(1450)$. In the isoscalar channel one observes $f_0(1370)$, $f_0(1500)$ and $f_0(1720)$. The $0^{++}$ glueball is assumed with mass around 1600 MeV as found in quenched lattice calculations. Then the glueball and two members of the nonet can mix and generate the three observed $f_0$ states. After the original proposal [6] several such mixing schemes have been considered (review [7]) using different phenomenological constraints.

The lower mass states are now left over. The $f_0(980)$ and $a_0(980)$ could be $K\bar{K}$ molecules [8] or 4 quark states [9], recently proposed [10] to explain radiative $\phi$ decays. An interesting possibility would be to combine these two states with $\sigma$ and $\kappa$ into a second light nonet, either of $qq\bar{q}\bar{q}$ or of $qq\bar{q}\bar{q}$ type. Such schemes appear in theories of meson meson scattering in a realization of chiral symmetry, for an outline, see [3].

**Route II:** one multiplet below 1800 MeV

In an alternative path one starts again with $K^*(1430)$ but takes $f_0(980)$ as the lightest isoscalar $qq\bar{q}$ state [11, 12, 13, 14]. The identification of the other members of the nonet differs; in the phenomenological approach [12] the lightest isovector is $a_0(980)$ and the second isoscalar is $f_0(1500)$ (as in [11]) with a flavour mixing as in the pseudoscalar sector with $f_0(980)$ and $\eta'$ near flavour singlet and $f_0(1500)$ and $\eta$ near octet. The glueball is a rather broad state (width of order of mass) [12,13, 14] centered around 1 GeV or a bit larger. In this case $\sigma$ is the low mass component of the glueball whereas $\kappa$ is not relevant for spectroscopy. Despite differences in detail $\sigma$ and $\kappa$ are not members of a nonet ($\sigma$ possibly a mixture of glueball and $qq\bar{q}\bar{q}$ [14]).

This leads to the important questions:

1. Are $\sigma$, $\kappa$ poles in the amplitude, genuine resonances and members of a nonet?
2. Is $f_0(980)$ a member of a low mass or high mass multiplet?

We adress the second question below considering symmetry relations. The means to answer the first question are the detailed investigation of the relevant amplitudes (finding resonances, typically as circles in the complex amplitude plane (“Argand diagram”) or from maximum phase variation), and the study of available production and decay channels to find out about the constituent structure.

In $2 \rightarrow 2$ scattering processes the standard method is the determination of the moments of the angular distribution from which the partial wave amplitudes can be determined ($\langle Y_L^0 \rangle \cong \sum c_{lm} \text{Re}(A_l A_m^*)$) up to an overall phase and discrete ambiguities.

In Dalitz plot analyses of 3-body decays $R \rightarrow 1 + 2 + 3$ it will be useful to compare the fits with phase sensitive quantities. A straightforward generalization of the above [15] would be the study the moments in the relevant non-exotic channels $(ij)$ which get the direct contribution from channel $(ij)$ as above but an additional contribution from the crossed channel(s)

$$\langle Y_L^0 \rangle \cong \sum c_{lm} \text{Re}(A_l A_m^*) + \text{crossed channel background.} \tag{1}$$

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1 The recent report by Bali [5] quotes the mass range 1.4 . . . 1.8 GeV; the unquenched results are typically around 20% lower and decrease with decreasing quark mass.
This additional contribution is slowly moving if the spin of the crossed channel resonances is low. The comparison of the full resonance model (usually a Monte Carlo) with the channel moments (say, up to L=4) should reveal the fine structure from the interfering partial wave amplitudes. Another possibility is the study of phases using a known resonance as analyser [16].

\[ \sigma \text{ POLE} \]

Summarizing a large variety of fits the PDG estimates the Breit-Wigner pole position as \( M = (400 - 1200) - i(300 - 500) \) MeV reflecting a considerable fluctuation of the results. The width \( \Gamma = 2 \text{Im} \, M \) is of the order of the mass itself. Next, we will discuss the recent results which are based on phase determinations using angular distributions.

1. Elastic \( \pi\pi \) Scattering

Data are obtained from single pion production using the One-Pion-Exchange model and from \( K \rightarrow \pi\pi\nu \) decays applying the Watson theorem. Recent studies using the Roy equations which implement analyticity, unitarity and crossing symmetry are found consistent with chiral symmetry constraints in the threshold region [17, 18]; also a unique phase shift solution “down flat” obtained from the polarized target data has been found [19], closely similar to the earlier results from unpolarized data [20]. In the analysis [17] also the \( \sigma \) pole is determined with remarkably small errors:

\[
M = 470 \pm 30, \quad \Gamma = 590 \pm 40 \text{ MeV.} \tag{2}
\]

The mass is much below \( \approx 850 \) MeV where the phase shift passes 90°. The origin of this pole and its connection to chiral symmetry can be understood by a qualitative argument (see [17]). Chiral symmetry leads to an amplitude zero in the isoscalar S wave amplitude \( T_{00}^0 \) near threshold (Adler zero) and results in a small scattering length. Unitarity then requires an imaginary part of the amplitude which rises more strongly than the real part, like \( s^2 = E_{CM}^4 \), in the chiral limit (\( m_\pi = 0 \))

\[
T_{00}^0 = s/(16\pi F_\pi^2), \quad \text{Im} \, T_{00}^0 = |T_{00}^0|^2 \sim s^2 \tag{3}
\]

with the pion decay constant \( F_\pi = 92.4 \) MeV. Within a certain unitarization method one obtains the unitary amplitude

\[
T_{00}^0 = s/(16\pi F_\pi^2 - is) \tag{4}
\]

which has the correct threshold behaviour [3] and a pole at \( \sqrt{s} = \sqrt{-16\pi F_\pi^2} = 463 - i463 \) MeV, not far away from the result of the full fit \( \sqrt{s} = 470 - i295 \) MeV in (2).

The errors in the pole determination take into account experimental errors and systematic errors from the parametrization, including a single pole (\( \sigma \)) or two poles (\( \sigma, f_0(980) \)). From the above qualitative argument one may deduce that this distant pole is generated by the unitarization procedure to manage the rapid increase of the imaginary part near threshold, given the small scattering length.

Whether this distant pole so generated correspond to a short living particle to be classified into a flavour multiplet has to be thought of further.
The \( \pi \pi \) S-wave amplitude in a larger mass range up to 1700 MeV (recent results \cite{22}) can be viewed as broad object centered around 1000 MeV which interferes destructively with \( f_0(980) \) and \( f_0(1500) \) \cite{21,12}. Fits of the \( \pi \pi \) elastic and various inelastic channels in a K-matrix formalism yield a mass \( M \sim (1450 - i800) \) MeV without including a \( \sigma \) pole explicitly \cite{13}. These K matrix results may not have satisfactory analytic properties at very low energies: if they are inserted into dispersion relations the \( \sigma \) pole reappears \cite{23}. However, in the large energy range the S wave \( \pi \pi \) amplitude (not counting the narrow resonances at 980 and 1500 MeV) describes only one circle in the Argand diagram corresponding to one broad state. This is most plausibly placed in the region 1.0-1.5 GeV; then the \( \sigma \) pole influences the low energy behaviour but does not generate an extra circle at low mass.

Investigating various production processes for this broad state it was concluded that it fulfills in all cases (except radiative \( J/\psi \) decays) the expectations for a glueball \cite{12}. The recent observation of a broad peak in \( K\bar{K} \) from \( B \rightarrow K\bar{K}K \) by Belle \cite{24} has also been taken as new evidence for this interpretation \cite{25}. The glueball interpretation is also favoured by the K matrix analysis \cite{13}, QCD sum rules require a broad glueball near 1000 MeV with the large decay into \( \pi \pi \) \cite{14}.

2. Decay \( D^+ \rightarrow \pi^+ \pi^+ \pi^- \)

A promising source of information is provided by the 3-body decays of D and B mesons. In the isobar model one considers the final state to proceed through intermediate resonances in any non-exotic channel. In the simplest way one takes a sum of Breit Wigner resonances \( A_i \), each multiplied by a constant amplitude and phase factor

\[
T = a_0e^{i\delta_0} + \sum_i a_i e^{i\delta_i} A_i(s_{12}) + \ldots
\]

where the dots refer to channels (13) and (23) if resonant.

Alternatively, one can express each channel (ij) by a multi-channel real K-matrix

\[
K_{ij} = \sum_{\alpha} g_{ij}^{\alpha} s_\alpha^{-1} + \ldots
\]

with poles \( s_\alpha \). The decay amplitude is then expressed by \( F_i = (I - iK\rho)^{-1} P_j \). The K matrix describes the propagation; with its poles it is universal in all processes and is determined from the multi-channel fit to 2-body collisions. The \( P \) vector contains the initial production amplitudes to be fitted for each process.

The decay \( D^+ \rightarrow \pi^+ \pi^+ \pi^- \) has been analysed two years ago by the E791 Collaboration applying the isobar model \cite{26}. They could not get a satisfactory fit without including the \( \sigma \) resonance in the fit to describe a peak at around 500 MeV. They obtained the parameters \( M_\sigma = 478 \) MeV and the width \( \Gamma_\sigma = 324^{+24}_{-40} \pm 21 \); this is considerably narrower than the elastic \( \pi \pi \) scattering result \cite{2}. Clearly, the phases here are more rapidly varying than in elastic scattering which would imply strong rescattering effects.

A new result by the FOCUS collaboration presented at this conference \cite{27} confirms the finding by E791 on the need for a \( \sigma \) contribution within the sum \cite{5}. An alternative fit has been carried out using the K matrix approach \cite{13} which does not include \( \sigma \) explicitly. This fit contains five \( f_0 \) states found from 2 body collisions with the
appropriate weights. The peak at low mass is then produced either by the S wave in this region itself or by the reflection from the crossed channel contributions. In any case, the phase variation over the peak region is smooth and does not cross 90° at the peak.

Further clarification should come from a careful study of phase sensitive quantities. As emphasized before [15] the angular moment \( \langle Y_0^0 \rangle \propto \langle \cos \theta \rangle \) is proportional to the S-P interference and therefore a \( \sigma \) resonance should show a characteristic interference with the tail of the \( \rho \) – above the smooth background from the crossed channel.

An alternative possibility has been studied by the E791 collaboration [28] using the \( \Delta A_2^2 \) method [16]. They selected the \( f_2(1270) \) resonance in \( s_{12} \) as analyser and compared the difference of densities above and below the resonance mass \( \Delta A_2^2 = A_2^+ - A_2^- \) as function of the conjugate mass \( s_{13} \) which is related to the amplitude phase

\[
\Delta A_2^2(s_{13}) \propto \sin(2\delta(s_{13})).
\]

They found a strong variation of the phase through the \( \sigma \) region by altogether 180° indicating a \( \sigma \) Breit Wigner resonance. As the \( f_2 \) is only rather weakly produced it will be important to confirm the effect with the clear \( \rho \) and \( f_0(980) \) resonances. In these cases, it is more convenient to compare the two models with and without rapid phase variation directly to the two stripes \( A_2^+ \) and \( A_2^- \) which contain the same phase sensitivity. Together with the \( \langle \cos \theta \rangle \) moment variation it should be possible to get the wanted information on the behaviour of phases.

3. Central production \( pp \rightarrow p(\pi \pi)p \)

This process is assumed to be dominated at small momentum transfers between the protons by double Pomeron exchange. Recent measurements determined the angular distributions and the relative phases of amplitudes [29, 30].

The centrally produced \( \pi \pi \) system peaks shortly above threshold below 400 MeV and this peak has been related to \( \sigma \) as well [29, 31]. In this case there is a simple dynamical explanation of the peak [15] in terms of the subprocess

Pomeron Pomeron \( \rightarrow \pi \pi \) \hspace{1cm} (8)

with one-pion-exchange. This interpretation is suggested by the close similarity of this process with \( \gamma \gamma \rightarrow \pi \pi \), not only with respect to the S wave peak near 400 MeV but also to the very unusual peak in the D wave near 500 MeV.

Concerning the behaviour of the S wave phase we note that both experiments find the phase difference \( \phi_S - \phi_{D_0^-} \) slowly rising from threshold to about 90° near 900 MeV very similar to elastic \( \pi \pi \) scattering. On the other hand, the phase difference \( \phi_S - \phi_{D_0^-} \) is rather energy independent below 1 GeV. This is difficult to explain assuming a common production mechanism. The presence of several production mechanisms with different spin couplings of the proton would invalidate the simple kind of analysis neglecting the spin effects. In any case, there is no rapid phase variation of the S wave near the peak of 400 MeV as expected from a simple Breit Wigner resonance. In this case a non-resonant mechanism can be identified.

4. Decay \( J/\psi \rightarrow \omega \pi \pi \)

There is a sizable peak around 500 MeV in this process studied first by DM2 [32] and now by BES [33]. Again one may ask whether the peak can be represented by a normal
Breit Wigner resonance. Studying the $\pi\pi$ angular distribution as measured by DM2 we concluded [15] that the $\pi\pi$ phase shift has to increase slowly through the peak region, otherwise the interference with the nearly real D-wave (assumed to be the tail of $f_2$) would lead to a sign change of the interference term $\langle \cos^2 \theta \rangle$.

Using the new high statistics data from BES on the $\pi\pi$ angular distributions and others it is actually possible to determine the S wave phase directly [33]; in this analysis the background D wave component is related not to $f_2$ but to the tail of $b_1(1235)$ in the crossed channel, albeit for a $b_1$ width considerably larger than given by the PDG. The resulting phase shifts behave smooth in the peak region as in elastic scattering but not like a local Breit-Wigner resonance. An explanation of such behaviour is suggested in terms of a $\sigma$ pole with strongly energy dependent width (from Adler zero).

5. Other results

There are other channels where broad low mass peaks are observed, in particular decays of charmonia $\psi', \psi''$ and $Y', Y''$ into $\pi\pi$ and the respective ground state. These peaks may be related to $\sigma$ as well [34]. Peaks are also seen in $\tau$ decays. As there are no phase studies available we do not discuss these further here.

6. Unsuccessful searches

Finally we emphasize that in some reactions searches have been negative. In particular, CLEO [35] did not find any $\sigma$ signal in the neutral $D^0$ decay $D^0 \rightarrow \pi^+ \pi^- \pi^0$. Here the $\pi^+ \pi^-$ mass spectrum also shows a peak at small masses which is entirely explained by the crossed channel resonances.

$\kappa$ POLE

At first sight, the low energy $K\pi$ scattering looks similar to $\pi\pi$: there is the possibility of a broad resonance close to threshold. However, there are some characteristic differences. In the following, we discuss the various observations.

1. elastic $K\pi$ scattering

The phase shifts of elastic scattering have been extracted from pion production experiments as in case of $\pi\pi$ scattering by the LASS collaboration [36]. The S wave in the region up to 1.6 GeV has been described by a superposition of a smooth background and the $K_0^*(1430)$ Breit-Wigner resonance

$$S = BG + BW \ e^{2i\delta_{BG}}, \quad BG = \sin \delta_{BG} e^{i\delta_{BG}}, \quad \cot \delta_{BG} = \frac{1}{aq} + \frac{bq}{2}. \quad (9)$$

Another measurement is obtained from the semileptonic decays $D^+ \rightarrow K^- \pi^+ \mu^+ \nu$ by the FOCUS collaboration [37]. The Watson theorem relates the final state phase shifts to those of elastic scattering in the elastic region. Data are consistent with a constant $K\pi$ phase of $\varphi_{K\pi} = 45^\circ$ in $800 < M_{K\pi} < 1000$ MeV. This is indeed in close agreement with the elastic scattering phase which varies between about $35^\circ$ and $50^\circ$ in this range [15].

The background phase in this parametrization rises up to about $50^\circ$ at 1.5 GeV near the first resonance. Alternative parametrizations yield $70^\circ$ [33]. This is quite different from $\pi\pi$ scattering, where the phase passes $90^\circ$ already at 850 MeV, below the first resonance $f_0(980)$. The need for an extra state in $K\pi$ is therefore not evident, contrary to $\pi\pi$. 
The question whether the elastic scattering data require a \( \kappa \) pole has been investigated by Cherry and Pennington \[38\]. They expand the scattering amplitude into a complete series of functions with correct branch cuts and truncate if no significant improvement is obtained. They find always the \( K^\ast(1430) \) but not the \( \kappa \). There is therefore no evidence for \( \kappa \) from the present data but the very low energy region below 800 MeV is not available yet in elastic scattering. Hopefully such data will be obtained from semileptonic D decays.

On the other hand, the data can also be described by models which include a \( \kappa \) pole. This is found in multi-channel fits using chiral symmetry constraints. Recent analyses in chiral perturbation theory yield acceptable fits to the \( K\pi \) phase shifts \[39, 40\]. The position of the \( \kappa \) pole is found as \( M_\kappa \sim 750 - i230 \) MeV \[40\].

2. Decay \( D^+ \rightarrow K^- \pi^+\pi^+ \)

The analysis by the E791 collaboration \[41\] proceeds similar at first to the corresponding one of the \( 3\pi \) final state: An isobar model fit including \( K^\ast(1430) \) and constant background but no \( \kappa \) does not give a good fit. A satisfactory fit is found if the \( K^\ast(1430) \) parameters are varied and a \( \kappa \) resonance is introduced with \( M \sim 797 - i205 \) MeV.

Further studies \[42\] have shown that the angular asymmetry \( \langle \cos \theta \rangle \) in the \( K^\ast(890) \) region between 800 and 1000 MeV is rather well fitted by both models with and without \( \kappa \) which implies that in this mass region both models have similar phases despite their different analytic expressions. At low masses \( M < 800 \) MeV a better description is obtained for angular distributions in a fit with \( \kappa \). These results show the importance of the study of more details of the final state in the determination of the partial waves. It will be interesting to compare directly the total \( S \) wave phase in the full mass region of both models and compare with the elastic phase.

3. Decay \( J/\psi \rightarrow K^\ast(890)K\pi \)

This channel has been investigated by the BES collaboration \[43\] and there is some similarity to the corresponding channel with \( \omega \pi\pi \). The \( K^\ast \) band, after subtraction of suitable side bands shows a \( \kappa \) peak of \( 3\sigma \) significance with mass \( M \sim (771^{+164}_{-221} \pm 55) - i(110^{+112}_{-64} \pm 48) \). At this conference two analyses of BES data have been presented, both finding the \( \kappa \) albeit with different parameters: the first analysis with \( M \sim (760 \pm 20 \pm 40) - i(420 \pm 45 \pm 60) \) MeV \[33\] also determines the \( K\pi \) phase shifts and finds results consistent with elastic \( K\pi \) scattering; the second one yields \( M = (882 \pm 24) - i(167 \pm 41) \) MeV \[44\]. The considerable differences in the width results indicate the difficulty in the determination of this quantity.

4. Unsuccessful searches

Again, in some other \( D \) decay channels the \( \kappa \) has been searched for but could not be confirmed: In the channel \( D^0 \rightarrow K^- \pi^+\pi^0 \) the \( \kappa \) fraction was \( 0.4 \pm 0.3 \% \) \[45\] and in \( D^0 \rightarrow K^0 K^- \pi^+ (15 \pm 12) \% \) \[46\].

**TEST OF SYMMETRIES - \( \sigma, \kappa \) IN A SCALAR NONET?**

As we have seen it is difficult to establish the \( \sigma, \kappa \) poles uniquely by fitting different parametrizations to the data, but in models respecting chiral symmetry these poles appear after unitarization. As they are quite far away from the real axis their interpretation
in terms of real particles is not without doubt. Therefore we consider it a crucial test whether these particles also obey the relations following from the assumed underlying flavour symmetries. Here we consider the attractive possibility that $\sigma, \kappa, f_0(980)$ and $a_0(980)$ form a nonet, either built from $q\bar{q}$ or from $qqq\bar{q}$. We consider here two such tests.

**Tests in $J/\psi$ decays**

Symmetry relations involving the decays $J/\psi \rightarrow$ tensor + vector particles have been tested successfully by the DM2 Collaboration [47]. There are deviations from $SU(3)$ symmetry from electromagnetic interactions and quark mass effects which are taken from similar analyses of the vector + pseudoscalar final states. For example, using PGD results, one finds for

$$R^T = \frac{J/\psi \rightarrow K^*_2(1430)\bar{K}^*(890)}{J/\psi \rightarrow f_2(1270)\omega} = \frac{(3.4 \pm 1.3) \times 10^{-3}}{(4.3 \pm 0.6) \times 10^{-3}} = 0.8 \pm 0.3.$$ (10)

where $R^T = 1$ in the $U(3)$ symmetry limit with ideal mixing.

We assume now that the scalars fulfill the same relations as the tensors and assume the quark composition $\sigma \leftrightarrow f_2 = (u\bar{u} + d\bar{d})/\sqrt{2}$. Then the ratio $R^T$ should equal the corresponding ratio for scalars $R^S$ if we assume the same symmetry breaking in both multiplets. With the data from DM2 [32] for $\sigma$ and the preliminary result from BES [43] for $\kappa$ we find after correction for neutrals

$$R^S = \frac{J/\psi \rightarrow \kappa\bar{K}^*(890)}{J/\psi \rightarrow \sigma\omega} = \frac{(0.19 \pm 0.18) \times 10^{-3}}{(2.4 \pm 0.45) \times 10^{-3}} = 0.08 \pm 0.08.$$ (11)

so the relation $R^T = R^S$ looks badly broken even if there are large errors. Adding an $s\bar{s}$ component to $\sigma$ would make the agreement worse. At this state of the $\kappa$ analysis we do not draw any final conclusion, but rather recommend the repetition of this exercise, once the data are considered firm. It would also be interesting to include results for $a_0(980)\rho$ and $f_0(980)(\phi/\omega)$ in the analysis.

**Tests in charmless $B$-meson decays**

Recently, a large production rate has been found for the decay $[24, 48]$

$$Br(B^+ \rightarrow K^+ f_0(980)) \sim 15 \times 10^{-6};$$ (12)

comparable to $B \rightarrow K \pi$, also the decay $K_0^*(1430)\pi^-$, see Fig. 1 for a recent result by BELLE [24]. The $f_0(980)$ is a very interesting particle from our present viewpoint (double faced “Janus-particle”), as it could belong either to a multiplet of lower mass with $\sigma, \kappa$ and $a_0(980)$ (Route I) or to one of higher mass with $a_0, K_0^*(1430)$ and $f'_0$ (Route II); in the scheme [12] $a_0 \equiv a_0(980)$ and $f'_0 \equiv f_0(1500)$ but other choices are possible.
Starting from this observation a strategy has been proposed [25] to find the members of the scalar nonet and to determine their flavour mixing. This involves the study of the decays

\[ B \rightarrow K + \text{Scalars}, \quad B \rightarrow K^+ + \text{ Scalars}. \]  

(13)

Some experience has been gained with the charmless decays as in (13) but with pseudoscalars instead of scalars. We follow here the phenomenological approach [49] which derives the 2 body decay rates from the processes \( b \rightarrow s u \bar{u}, b \rightarrow s d \bar{d}, b \rightarrow s s \bar{s} \) which get equal contributions from the QCD penguin diagrams and smaller contributions of order 20\% from the CKM suppressed tree diagrams. Furthermore, for the flavour singlet mesons, there is a purely gluonic contribution from the penguin process \( b \rightarrow s g \). In a simplified version [25] only the dominant penguin \( sq \bar{q} \) and flavour singlet amplitudes are kept and this has allowed already acceptable fits to the available decay rates.

This simplified model is then taken over for the scalars and allows predictions for all members of the nonet in terms of only few parameters. There is the penguin amplitude \( P_{AB} \) for decay into hadrons from multiplets \( A, B \) (\( s \rightarrow A \), spectator quark \( \rightarrow B \)), the gluonic amplitude \( \gamma_{AB} P_{AB} \) and the amplitude with hadron \( A \) and \( B \) exchanged \( \beta_{AB} P_{AB} \).

For simplicity, as in the PP and PV case we assume \( \gamma_S \equiv \gamma_{PS} \equiv \gamma_{VS} \) real, furthermore for the exchanged amplitude \( \beta_{PS} = -\beta_{VS} = 1 \). The (-) sign comes from the fact that the VS state is in a P wave and this leads to alternating decay patterns in PV and PP final states [50] and the same happens with scalars.

For illustration we give in Table 2 some predictions for the two schemes with high or low mass nonet using the results of Table 2 in [15]. For the higher multiplet (Route II) we choose \( f_0 \rightarrow f_0(980), K^+(1430) \) and \( f'_0 \rightarrow f_0(1500) \), mixed like \( \eta' \), \( \eta \) as in the pseudoscalar sector [11, 12]. Taking the new data on \( B^+ \rightarrow K^+ f_0(980) \) and \( K^0 \pi^+ \) as input [24, 48] (neglecting \( f_0 \rightarrow K \bar{K} \) decays) we determine \( \gamma_S \approx -0.36 \) and predict the rates for the other members of the nonet; in Table we show the predictions for 3 more
channels concerning \( f_0 \) and \( f'_0 \). Especially, \( f'_0 \) has a small rate; this seems to be born out by the BELLE data \([24]\) in the \( \pi\pi \) channel (see Fig. 1), but there may be some signal in the \( K\bar{K} \) channel which has to be analysed further. A clear prediction are the large rates with \( K^* \). Other predictions follow easily.

For the lower multiplet (Route I) we choose \( f_0 \to f_0(980) \sim s\bar{s} \) and \( f'_0 \to \sigma \sim (u\bar{u} + d\bar{d})/\sqrt{2} \). Now only the \( f_0(980) \) rate is known. We first assume (Ia) the absence of the gluonic process \( (\gamma_s = 0) \). Then we obtain a \( \sigma \) rate half as big as the \( f_0(980) \) rate whereas the data in Fig. 1 do not indicate any low mass effect at all. Next (Ib) we put \( \gamma_s = -0.5 \) to obtain a small \( \sigma \) rate, but then a large \( \kappa \) rate follows which is not easy to recognize in the data \([24]\) but it should be looked for quantitatively.

This serves only as a simple exercise to demonstrate the relevance of the symmetry relations. As the dominant penguin processes are flavour symmetric all the nonet members should be produced in some \( K \) or \( K^* \) channel with rate comparable to \( f_0(980) \). So it is interesting to find out which scalars are strongly produced and fulfill the approximate relations. In this way one may also learn whether \( a_0(980) \) or \( a_0(1430) \) is the isovector member.

### CONCLUSIONS

1. **Poles** \( \sigma, \kappa \) are not necessarily required from the acceptable fits to data but they appear commonly in parametrizations with the small scattering length from chiral symmetry and with unitarity.

2. **Elastic scattering:** In \( \pi\pi \) scattering besides \( f_0(980) \) and \( f_0(1500) \) there is apparently a slowly moving background amplitude describing an approximate circle in the complex plane; this could be a broad resonance with mass \( \gtrsim 1 \) GeV (a glue-ball?). In \( K\pi \) scattering there is only one resonance, \( K^*(1430) \), below 1800 MeV but no extra circle. The \( \sigma, \kappa \) represent distant poles in parametrizations \( (\Gamma \gtrsim \text{Mass}) \); they modify the behaviour of amplitudes near threshold but do not generate circles. They are not necessarily real propagating particles.

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**TABLE 2.** Predicted rates for scalars \( f'_0 \) associated with \( K \) and \( K^*(890) \), with rates \( K_{f_0}(980) \) \( (f_0 \to \pi\pi \) only) and \( K_0^*(1430)\pi^+ \) as input (underlined) \([24]\) (see also text).

| Route II: \( B^+ \to \) | \( f_0, f'_0, K_0^* \) | \( \gamma_s = -0.36 \) |
|---|---|---|
| amplitude | \( (3 + 4\gamma_s)/\sqrt{6} \) | \( \gamma_s/\sqrt{3} \) | \( (-1 + 4\gamma_s)/\sqrt{6} \) | \( (2 + \gamma_s)/\sqrt{3} \) | \( 1 \) |
| rate [10\(^{-6}\)] | 15 | 1.6 | 38 | 34 | 38 |

| Route Ia: \( B^+ \to \) | \( \sigma, f_0, \kappa \) | \( \gamma_s = 0 \) |
|---|---|---|
| amplitude | \( (1 + \gamma_s) \) | \( (1 + 2\gamma_s)/\sqrt{2} \) | \( (-1 + \gamma_s) \) | \( (1 + 2\gamma_s)/\sqrt{2} \) | \( 1 \) |
| rate [10\(^{-6}\)] | 15 | 7.5 | 15 | 7.5 | 15 |

| Route Ib: \( B^+ \to \) | \( \sigma, f_0, \kappa \) | \( \gamma_s = -0.5 \) |
|---|---|---|
| rate [10\(^{-6}\)] | 15 | 0 | 135 | 0 | 60 |
3. **Low mass peaks**: A new development are the phase studies of the low mass effects in 3-body decays. There is still a controversy on whether the phase in the $\pi\pi$ and $K\pi$ channels move differently from elastic scattering. This question should be clarified by further studies of phase sensitive quantities, the ultimate goal being an energy independent phase shift analysis. In some cases the peaks with slow phase movement can be explained by non-resonant mechanisms.

4. **Symmetry relations for decay rates**: They represent the crucial test for the particle interpretation and the flavour properties of $\sigma$ and $\kappa$ poles and other scalars. One possibility is offered by $J/\psi$ decays. A powerful approach is the study of decays $B \to K(K^*)+\text{scalars}$ which should allow to find the members of the lightest $0^{++}$ nonet and the flavour mixing of the isoscalars. In particular, if decay rates into $\sigma$ and $\kappa$ are measured, one may find out about their flavour symmetry and whether $f_0(980)$ belongs to a low mass or a high mass multiplet.

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