On Lagrangian Formulation for Spin \((2, 1, 1)\) Massive Tensor Field on Minkowski Backgrounds

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Abstract

We derive the Lagrangian for a new model of a massive rank-4 tensor field with generalized spin \((2, 1, 1)\) in Minkowski spacetime of any dimension \(d > 5\), by using dimensional reduction applied to a reducible gauge model of a massless rank-4 tensor field with generalized spin \((2, 1, 1)\) in Minkowski spacetime of dimension \(d + 1\).

Keywords: higher-spin fields; gauge theories; Lagrangian formulation; higher-spin symmetry algebra.

1 Introduction

It is well known that massive and massless higher-spin fields are high-energy excitations of superstring models \([1]\), including completely symmetric \([2, 3, 4, 5, 6, 7]\) and mixed-symmetric \([8, 9]\) fields of integer and half-integer spins. These models imply the existence of interactions and particles with spins higher than 2, whose experimental discovery, including the recent discovery of the Higgs boson, having the mass of 125 GeV \([10]\), indirectly confirming the minimal \(N = 1\) supersymmetric extension of the Standard Model (involving the mass of a light Higgs boson), is one of the research tasks (for instance, search for dark matter and supersymmetric partners) being carried out at the LHC. Experimental search for these particles and interactions calls for an adequate mathematical apparatus for their description within the formalism of scattering amplitudes.

Lagrangian description for higher-spin fields is one of the basic ingredients for computing the vacuum mean values of physical quantities involved in high-energy processes within the Feynman diagram technique using the Lagrangian quantum action provided by the Batalin–Vilkovisky rules \([11]\). It should be noted that the currently available literature (for a review, see, e.g., \([12, 13, 14, 15, 16, 17]\)) contains quite a large number of results devoted to completely

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symmetric and completely antisymmetric tensor and spin-tensor fields in spaces of constant curvature in both the metric-like [2, 3] and the frame-like [4] descriptions, whereas the case of mixed-symmetric higher-spin fields has not been studied so well. Exceptions include the recently constructed Lagrangian descriptions with traceless constraints within the frame-like approach [18, 19] and the metric-like approach [20] using the concept of Bianchi identities in Minkowski spacetime, as well as the (spin-)tensor fields subject to two-row [21, 22] and two-column [23] Young diagrams in anti-de Sitter spaces. Amongst the Lagrangian descriptions without constraints and with specific values of generalized integer spin in the configuration space involving no auxiliary fields, there have been known only certain examples of fields described by two-row Young diagrams of spin \( s = (2, 1) \) in constant-curvature spaces [24], \( s = (2, 2) \) in Minkowski space [25], as well as the case of 4-th rank massive tensors in flat and (anti-)de Sitter spaces subject to the Young tableaux \( Y[3, 1], Y[2, 2] \) with two columns in terms of an antisymmetric basis [26], accompanied by a study of the massless and partially massless limits.

The papers [27, 28, 29] of one of the authors of the present work have been the first ones to obtain unconstrained Lagrangian descriptions for arbitrary free particles of integer and half-integer generalized spin on a basis of the BRST (Becchi—Rouet—Stora–Tyutin) – BFV (Batalin–Fradkin–Vilkovisky) formalism, applied in its original setting [30] to the generalized canonical quantization of arbitrary gauge theories with constraints, and reflecting the presence of a special one-parameter supersymmetry, i.e., the BRST-symmetry\(^1\) [31]. In particular, the article [27] was the first to obtain, by solving the cohomological gauge complex, such a gauge-invariant Lagrangian description for a massless 4-th-rank tensor field \( \Phi_{\mu\nu,\rho,\sigma} \) of spin \((2, 1, 1)\) in a flat \( d \)-dimensional spacetime \( \mathbb{R}^{1,d-1} \) that is defined entirely in terms of the tensor components of \( \Phi_{\mu\nu,\rho,\sigma} \). At the same time, no Lagrangian description for a free massive 4-th-rank tensor of spin \((2, 1, 1)\) in \( \mathbb{R}^{1,d-1} \) has been presented. Such a description is important due to the fact that it may be able to provide a basis for constructing a physically meaningful theory of interacting massive higher-spin fields having no auxiliary fields and interacting with lower-spin fields. In this respect, the present work intends to find a Lagrangian for the model [27] of a massive 4-th-rank tensor field of spin \((2, 1, 1)\). The construction may be carried out in two different ways: either by repeating the procedure [27] of solving the cohomological gauge complex in a special Fock space with the use of a massive tensor field \( \Phi_{\mu\nu,\rho,\sigma} \), or by applying dimensional reduction to an already known massless gauge model for a similar tensor field, determined, however, in a \((d+1)\)-dimensional flat spacetime. The present work intends to solve the above problem on a basis of the latter approach.

The main part of the present work is organized as follows. First, we recall the equations that express the fact that a massive field with generalized spin \( s = (2, 1, 1) \) belongs to the relevant irreducible representation of the Poincare group in terms of a mixed-symmetric 4-th-rank tensor \( \Phi^{\mu\nu,\rho,\sigma} \) in a \( d \)-dimensional Minkowski spacetime. Second, we present a gauge-invariant Lagrangian for a mixed-symmetric 4-th-rank tensor with generalized spin \( s = (2, 1, 1) \), defined in a \((d+1)\)-dimensional Minkowski spacetime, and describe reducible gauge transformations for this Lagrangian. Third, based on the rules of dimensional reduction that transform a massless theory in \( \mathbb{R}^{1,d} \) into a massive theory in \( \mathbb{R}^{1,d-1} \), we apply these rules to a Lagrangian model of a massless tensor field in \( \mathbb{R}^{1,d} \). We fix the reducible gauge invariance for a massive model, obtained by eliminating the Stueckelberg auxiliary fields and lower-spin gauge parameters, and make a transfer from a Lagrangian gauge model to a non-gauge model, defined entirely in terms of a massive mixed-symmetric 4-th-rank tensor \( \Phi_{\mu\nu,[\rho,\sigma]} \). In Conclusion, we summarize the obtained results and outline the perspectives of their application.

\(^1\)In this respect, we recall that the first results involving a constrained Lagrangian description for completely symmetric bosonic fields have been presented in [32, 33].
2 Lagrangian Description of Massless Model in $\mathbb{R}^{1,d}$

Let us recall that, based on the approach of E. Wigner, a massive particle of generalized spin $s = (2, 1, 1)$ is described within the metric formalism by a 4-th-rank tensor $\Phi^{\mu\nu,\rho,\sigma}(x) \equiv \Phi_{\mu\nu,\rho,\sigma}$, being symmetric in its first two indices, $\Phi_{\mu\nu,\rho,\sigma} = \Phi_{\rho\sigma,\mu\nu}$, and satisfying the Klein–Gordon equation (2.1), as well as the divergentless (2.2), traceless (2.3) and mixed-symmetry (2.4) equations:

$$
(\partial^\mu \partial_\mu + m^2) \Phi_{\mu\nu,\rho,\sigma} = 0, \tag{2.1}
$$

$$
\partial^\mu \Phi_{\mu\nu,\rho,\sigma} = \partial^\rho \Phi_{\mu\nu,\rho,\sigma} = \partial^\sigma \Phi_{\mu\nu,\rho,\sigma} = 0, \tag{2.2}
$$

$$
\eta^{\mu\nu} \Phi_{\mu\nu,\rho,\sigma} = \eta^{\nu\rho} \Phi_{\mu\nu,\rho,\sigma} = \eta^{\rho\sigma} \Phi_{\mu\nu,\rho,\sigma} = 0, \tag{2.3}
$$

$$
\Phi_{\{\mu, \nu\}};\rho,\sigma = 0, \quad \Phi_{\{\rho, \sigma\}};\mu = 0, \quad \Phi_{\{\mu, \rho\}};\nu = 0, \tag{2.4}
$$

Eqs. (2.1)–(2.4) reflect the property that the tensor field $\Phi^{\mu\nu,\rho,\sigma}$ belongs to an irreducible massive representation of the Poincare group in $\mathbb{R}^{1,d-1}$, and, in particular, Eqs. (2.4) imply that the tensor field $\Phi^{\mu\nu,\rho,\sigma}$ belongs to the Young tableaux $Y(2, 1, 1)$, whereas the last equation in (2.4) implies that it is only the part of $\Phi^{\mu\nu,\rho,\sigma}$ which is antisymmetric, $\Phi_{\mu\nu,[\rho,\sigma]} = -\Phi_{\mu\nu,[\sigma,\rho]}$, in the two final indices $\rho, \sigma$, that survives, where $\mu, \nu, \rho, \sigma = 0, 1, \ldots, d-1$.

Our starting point is a gauge-invariant Lagrangian for a massless 4-th-rank tensor $\Phi^{MN,[R,S]}$ with generalized spin $s = (2, 1, 1)$, defined in a $(d+1)$-dimensional Minkowski spacetime, being antisymmetric with respect to the permutation of the third and fourth indices, $\Phi^{MN,[R,S]} = -\Phi^{MN,[S,R]}$, and subject to the (off-shell) Young symmetry conditions

$$
\Phi^{MN,[R,S]} + \Phi^{RM,[N,S]} + \Phi^{NR,[M,S]} = 0, \tag{2.5}
$$

which accompany equations of the form (2.1)–(2.3), however, in a $(d+1)$-dimensional spacetime, and also with $m = 0$. For tensor transformations, we use the mostly minus signature for the metric tensor $\eta_{MN} = \text{diag}(+,-,\ldots,-)$, with $(M, N, P, R, S)$ being $(d+1)$-dimensional Lorentz indices with the structure

$$(M, N, P, R, S) = ((\mu, d), (\nu, d), (\pi, d), (\rho, d), (\sigma, d)), \quad \eta_{MN} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}, \tag{2.6}
$$

where $\mu, \nu, \pi, \rho, \sigma$ are $d$-dimensional Lorentz indices.

The action has the form [27]:

$$
S_{(2,1,1)} = \int d^{d+1}x \left[ \frac{1}{2} \Phi^{MN,[R,S]} \left( \square \Phi^{MN,[R,S]}_{\mu\nu,\rho,\sigma} + \partial_M [\partial_R \Phi^{T[N],[T,S]}_{\mu\nu,\rho,\sigma} + \partial_N \Phi^{T[R],[T,S]}_{\mu\nu,\rho,\sigma} \right) ight.
\right.
\left. + \partial_S [\partial_T \Phi^{T[N],[R,T]}_{\mu\nu,\rho,\sigma}] + 2 \partial_R \left[ \partial_M [\partial_T \Phi^{T[N],[S,N]}_{\mu\nu,\rho,\sigma}] + \partial_N [\partial_T \Phi^{T[M],[S,T]}_{\mu\nu,\rho,\sigma}] - 2 \Phi^{T[M],[T,S,N]}_{\mu\nu,\rho,\sigma} \right] \right]
\left. - \frac{1}{4} \Phi^{T[M,N]}_{\mu\nu,\rho,\sigma} \left( \square \Phi_{S[M,N],[M,N]} + 2 \partial_M \partial_R \left[ \Phi_{S[M,N],[M,N]} - \Phi_{R[M,N]} \right] \right) \right]
\left. - 2 \Phi_{S[M,N],[M,N]} \left( \partial_R \partial_T \Phi_{T[M,N],[S,N]} + \Phi_{S[M,N],[M,N]} \partial_R \partial_T \Phi_{T[M,N],[S,N]} + 2 \Phi_{S[M,N],[M,N]} \partial_R \partial_T \Phi_{T[M,N],[R,S]} \right) \right]
\left. + \frac{1}{2} \Phi_{S[M,N],[M,N]} \partial_T \partial_R \left( \Phi_{T[R,T],[N]} - \Phi_{T[R,T],[N]} \right) \right].
\tag{2.7}
$$

Due to relations (2.5), the action (2.7) is invariant with respect to Abelian gauge transformations:

$$
\delta \Phi_{MN,[R,S]} = - \frac{1}{2} \partial_M \partial_{\{\mu,\nu\}}_{\{R,S\}} + \frac{1}{2} \partial_R \partial_{\{\rho,\sigma\}}_{\{M,[N]\}} + \frac{1}{2} \partial_S \partial_{\{\mu,\nu\}}_{\{M,[R]\}}, \tag{2.8}
$$
where the gauge parameters $\xi_{R,[M,N]}$ are defined by the notation of $[27]$ as $\phi_{1[R,[M,N]}^{(1)}$, and the symbol $\wedge$ over the index $R$ in the function $\xi_{[M,[R,N]}$ implies that $R$ is not involved in symmetrization, $\{M,[R,N]\}$, defined by $\xi_{[M,[R,N]} = \xi_{M,[R,N]} + \xi_{N,[R,M]}$. In turn, the gauge transformations for the dependent gauge parameters $\xi_{R,[M,N]}$, being antisymmetric under permutations of the indices $M, N$ given by the rule $\xi_{R,[M,N]} = \xi_{R,M,N} - \xi_{R,N,M}$, and not belonging to any specific Young tableau, have the form

$$\delta \xi_{M,[N,R]} = 2\partial_N \epsilon_{M,R}^{(1)}$$

and

$$\delta \xi_{M,R} = -\partial_R \epsilon_{M}^{(2)},$$

where it has been taken into account that the first-stage gauge parameters (second-rank tensor) $\xi_{M,R}^{(1)}$ are also gauge-dependent, which is reflected in the presence of gauge transformations for them in terms of (by now independent) second-stage gauge parameters $\xi_{M}^{(2)}$. By means of the notation $[27]$, the quantities $\epsilon_{M,R}^{(1)}$, $\xi_{M,R}^{(2)}$ are defined as $\xi_{M,R}^{(1)} \equiv \phi_{1[M,R]}^{(2)}$ and $\xi_{M}^{(2)} \equiv \phi_{1,M}^{3}$. Relations (2.8) and (2.9) determine the model of a massless tensor field $\Phi_{M,N,[R,S]}$ with the action (2.7) as a gauge theory of a massless tensor field of second-stage reducibility.

In view of (2.4), the field tensor $\Phi_{M,N,[R,S]}$ is doubly traceless. Notice that one may equivalently use the tensor $\hat{\Phi}_{M,N,[R,S]}$,

$$\hat{\Phi}_{M,N,[R,S]} \equiv \Phi_{M,N,[R,S]} - \frac{1}{2} \Phi_{[RM],[N,S]} - \frac{1}{2} \Phi_{N,[R,M,S]},$$

which satisfies the Young symmetry conditions (2.5) identically, with the same action (2.7) and transformations (2.8), after a simple substitution $\Phi_{M,N,[R,S]} \rightarrow \hat{\Phi}_{M,N,[R,S]}$.

3 Lagrangian Description of Massive Model in $\mathbb{R}^{1,d-1}$

First, the dimensional reduction of a massless theory in $\mathbb{R}^{1,d}$ to a massive one in $\mathbb{R}^{1,d-1}$ is given by the rules for derivatives: $\partial_{\mu} = (\partial_{\mu}, im)$. Second, there holds the following representation for the dynamical field and gauge parameters:

$$\Phi_{M,N,[R,S]} = (\Phi_{\mu\nu,[\rho,\sigma]}, \Phi_{\mu[\rho,d]}^{\nu}, \Phi_{\mu[d,\rho]}^{\nu}, \Phi_{\mu[d],[\rho,\sigma]}, \Phi_{d[\rho,d]}^{\nu}, \Phi_{d[d],[\rho,\sigma]}),$$

(3.11)

$$\xi_{M,[N,R]} = (\xi_{\mu,[\nu,\rho]}, \xi_{\mu[d,\nu]}^{\rho}, \xi_{\mu[d],[\nu,\rho]}^{\rho}, \xi_{d[d],[\nu,\rho]}^{\rho}, \xi_{d[d],[\rho,\sigma]}^{\rho}),$$

(3.12)

$$\xi_{M,N}^{(1)} = (\xi_{\mu,d}^{(1)}, \xi_{\mu,d}^{(1)}, \xi_{d,d}^{(1)}, \xi_{d,d}^{(1)}),$$

$$\xi_{M}^{(2)} = (\xi_{d}^{(2)}, \xi_{d}^{(2)}).$$

(3.13)

The Young symmetry condition for the tensor $\Phi_{M,N,[P,S]}$ (2.5) implies that after a projection onto $\mathbb{R}^{1,d-1}$ the scalar $\Phi_{d[\rho,d]}^{\nu}$ and the $d$-dimensional first-rank tensor $\Phi_{d[\rho,d]} = \frac{1}{3} \Phi_{[d][d],[\rho]}$ vanish identically, and there hold the following conditions for the remaining tensor projections (3.11):

$$\Phi_{d[\rho,d]}^{\nu} = -2\Phi_{d[\rho,d]}^{\nu}, \quad \Phi_{\mu\nu,[\rho,\sigma]} = -2\Phi_{d[\mu,[\nu,\rho]}^{\nu}, \quad \Phi_{\mu[d],[\rho,\sigma]} = 0.$$  

(3.14)

Therefore, the total configuration space of a massive free particle with spin $s = (2, 1, 1)$ in $\mathbb{R}^{1,d-1}$ contains one fourth-rank massive mixed-symmetric tensor, $\Phi_{\mu[\nu,\rho]},$ an auxiliary third-rank tensor, $\varphi_{\mu\nu,\rho} \equiv \Phi_{\mu[d],[\nu,\rho]}^{d};$ and an auxiliary second-rank antisymmetric tensor, $\varphi_{[\rho\sigma]} \equiv \Phi_{d[\rho,d]}^{\nu}$. The two latter tensors play the role of Stueckelberg fields.

The set of gauge parameters (3.12) consists of one third-rank tensor, $\xi_{\mu,[\nu,\rho]}$, two antisymmetric second-rank tensors, $\xi_{\nu,[\rho,\sigma]} \equiv \xi_{d,[\nu,\rho]}$ and $\xi_{\mu,\nu} \equiv \xi_{\mu[d],[\nu,\rho]}$, and one vector, $\xi_{\rho} \equiv \xi_{d[,\rho]}$. In its turn, the set of first-stage gauge parameters (3.13) contains a second-rank tensor, $\xi_{\mu,d}^{(1)}$, two vectors, $\xi_{\mu,d}^{(1)}, \xi_{\mu,d}^{(1)} \equiv \xi_{\mu,d}^{(1)}$, $\xi_{\mu,d}^{(1)}$, and a scalar, $\xi_{d}^{(1)} \equiv \xi_{d}^{(1)}$. At the same time, the vector $\xi_{d}^{(2)}$ and scalar $\xi_{d}^{(2)}$ are second-stage gauge parameters.
The corresponding gauge-invariant action with the given auxiliary fields can be deduced from (2.7) by dimensional reduction, $\mathbb{R}^{1,d} \to \mathbb{R}^{1,d-1}$, and must be invariant with respect to the gauge transformations, implied by (2.8),

$$
\delta \Phi_{\mu\nu,[\rho,\sigma]} = -\frac{1}{2} \partial_\mu \xi_{,[\rho,\sigma]} + \frac{1}{2} \partial_\rho \xi_{,[\mu,\sigma]} + \frac{1}{2} \partial_\sigma \xi_{,[\mu,\rho]},
$$

(3.15)

$$
\delta \varphi_{\mu,\rho} = \frac{1}{2} \partial_\mu \xi_{,\rho} - \frac{1}{2} \partial_\rho \xi_{,\mu} - \frac{1}{2} m \xi_{,[\mu,\rho]},
$$

(3.16)

$$
\delta \xi_{,[\mu,\sigma]} = -m \xi_{,[\mu,\sigma]},
$$

(3.17)

which, in consequence of (2.9), are reducible:

$$
\delta \xi_{,[\mu,\rho]} = 2 \partial_\mu \xi^{(1)}_{,\rho}, \quad \delta \xi_{,[\mu,\nu]} = 2 \partial_\mu \xi^{(1)}_{,\nu},
$$

(3.18)

$$
\delta \xi_{,\mu,\nu} = 2 \partial_\nu \xi^{(1)}_{,\mu} - 2 m \xi^{(1)}_{,\mu}, \quad \delta \xi_{,\mu} = 2 m \xi^{(1)}_{,\mu} - 2 \partial_\mu \xi^{(1)}.
$$

(3.19)

At the same time, the second-stage reducibility implies

$$
\delta \xi^{(1)}_{,\mu,\nu} = -\partial_\nu \xi^{(2)}_{,\mu}, \quad \delta \xi^{(1)}_{,\mu} = -\partial_\mu \xi^{(2)}, \quad \delta \xi^{(1)} = -m \xi^{(2)}.
$$

(3.20)

Transition to a non-gauge description is made by fixing the gauge parameters $\xi^{(1)}_{,\mu}, \xi^{(1)}$ in (3.20), (3.21) with the help of shift transformations, with the corresponding independent gauge parameters $\xi^{(2)}_{,\mu,\nu}, \xi^{(2)}$ of second stage, so that there arises an intermediate reducible gauge theory of first-stage reducibility with independent gauge parameters $\xi^{(1)}_{,\mu,\nu}, \xi^{(1)}_{,\mu}$. In a similar way, one can remove the gauge tensors $\xi_{,\mu,\nu}, \xi_{,\mu}$ by using respective gauge shifts with the help of $\xi^{(1)}_{,\mu,\nu}, \xi^{(1)}_{,\mu}$ in (3.19). In the gauge theory with the remaining independent gauge parameters $\xi_{,\mu,\nu}, \xi_{,\mu}$ one finally eliminates the fields $\varphi_{\mu,\rho}, \varphi_{[\rho,\sigma]}$ in relations (3.16), (3.17) by using gauge shift transformations with the respective parameters $\xi_{,\mu,\nu}, \xi_{,\mu}$, so that the theory becomes a non-gauge one, defined entirely in terms of the massive mixed-symmetric 4-th-rank tensor $\Phi_{\mu\nu,[\rho,\sigma]}$.

As a result, the Lagrangian of a massive 4-th-rank tensor $\Phi^{\mu\nu,[\rho,\sigma]}$ with generalized spin $s = (2, 1, 1)$ in a $d$-dimensional Minkowski spacetime, for the tensor $\Phi^{\mu\nu,[\rho,\sigma]}$ subject to conditions (2.10), acquires the form

$$
\mathcal{L}_{(2,1,1)} = \frac{1}{2} \tilde{\Phi}^{\mu\nu,[\rho,\sigma]} \left\{ \left( \Box + m^2 \right) \Phi_{\mu\nu,[\rho,\sigma]} + \partial_{[\mu} \left[ \partial_{\nu} \tilde{\Phi}^{[\tau,\rho],+[\sigma,\tau]} \right] + \partial_{[\nu} \tilde{\Phi}^{[\tau,\rho],+[\sigma,\tau]} + \partial_{[\rho} \tilde{\Phi}^{[\tau,\sigma],+[\mu,\tau]} \right) \\
-2 \partial_{\tau} \tilde{\Phi}^{\tau,[\rho,\sigma]} \} + 2 \delta_{\rho} \left( \partial_{[\mu} \tilde{\Phi}^{\tau,[\nu,\sigma]} + \partial_{[\nu} \tilde{\Phi}^{\tau,[\rho,\sigma]} - \partial_{\tau} \tilde{\Phi}^{\tau,[\rho,\sigma]} \right) \} \\
- \frac{1}{4} \tilde{\Phi}^{\tau,[\rho,\sigma]} \left\{ \left( \Box + m^2 \right) \tilde{\Phi}^{\tau,[\rho,\sigma]} - 2 \partial_{\rho} \partial_{\sigma} \left( \tilde{\Phi}^{\tau,[\rho,\sigma]} - \tilde{\Phi}^{[\rho,\sigma]} \right) \right\} \\
- \frac{1}{2} \tilde{\Phi}_{\rho,\nu}^{[\mu,[\nu,\nu]]} \partial_{\rho} \partial_{\sigma} \tilde{\Phi}_{\mu\nu,[\rho,\sigma]} - \tilde{\Phi}_{\rho,\nu}^{[\mu,[\nu,\nu]]} \partial_{\rho} \partial_{\nu} \tilde{\Phi}_{\mu\sigma,[\rho,\nu]} + 2 \tilde{\Phi}^{[\mu,[\rho,\sigma]]} \partial_{\rho} \partial_{\nu} \tilde{\Phi}_{\mu\sigma,[\rho,\nu]} \\
+ \frac{1}{2} \tilde{\Phi}_{\rho,\sigma}^{[\mu,[\nu,\nu]]} \partial_{\mu} \partial_{\nu} \left( \tilde{\Phi}_{\sigma,\rho,\nu}^{[\mu,[\nu,\nu]]} - \tilde{\Phi}_{\rho,\nu,\nu}^{[\mu,[\nu,\nu]]} \right). \right\}
$$

(3.22)

4 Conclusion

In this work, we have obtained a new Lagrangian for a free particle of mass $m$ and spin $s = (2, 1, 1)$ in a $d$-dimensional Minkowski spacetime, by applying dimensional reduction to a gauge-invariant model for a massless free particle of the same spin in a $(d + 1)$-dimensional Minkowski spacetime, which has been obtained earlier on a basis of the universal BRST–BFV approach in [27]. It should be noted that the resulting Lagrangian is applicable to the dimensions $d \geq 6$ [27, 28] subject to the inequality $d \leq 10$, implied by string theory.
In future perspective, we find it promising to use the respective Lagrangian descriptions for massive and massless free particles as initial models for the construction of interacting Lagrangian theories, in particular, those interacting with an external electromagnetic field. We also suggest extending the Lagrangian description to the case of spin $s = (2, 1, \ldots, 1)$ with $(k-1)$ projections of generalized spin equal to 1, on condition that $k \leq \lfloor d/2 \rfloor$ [27].

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