Suppression of the horizon effect in pairing correlation functions of $t$-$J$ chains after a quantum quench

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We investigate the time evolution of density, spin, and pairing correlation functions in one-dimensional $t$-$J$ models following a quantum quench using the time-dependent density matrix renormalization group (tDMRG). While density and spin correlation functions show the typical light-cone behavior over a wide range of parameters, in pairing correlation functions it is strongly suppressed. This is supported by time-dependent BCS theory, where the light-cone in the pairing correlation functions is found to be at least two orders of magnitude weaker than in the density correlator. These findings indicate that in global quantum quenches not all observables are affected by the excitations equally.

I. INTRODUCTION

The spread of correlations and information in a quantum lattice system is constrained by the existence of a maximal speed. This was proven for the commutator of observables for short-ranged systems by Lieb and Robinson [1], and later also for correlation functions [2, 3]. This corresponds to the findings in global quantum quenches, where light-cone behavior in correlation functions has been reported in theoretical investigations (see, e.g., [4–8]), as well as in experiments with ultracold gases on optical lattices [9, 10]. As pointed out by Calabrese and Cardy [4, 5], a way to picture such an horizon effect is to see the quench as a local source for quasiparticles, which then move ballistically through the system, i.e., at a constant speed. They leave the typical linear signature in the time evolution of correlation functions, which travels with twice the velocity of the quasiparticles, since it is caused by two of them moving in opposite directions. An interesting question is if this behavior is typical for all quantum systems, and one direction considered in recent research is to treat systems with long-range interactions [11–26].

Here, we ask the question whether the light-cone signal is expected to behave generically in all propagators. In the spirit of Calabrese’s and Cardy’s considerations, one is lead to ask, if in superconducting systems Cooper-like pairs with twice the mass of bare electrons might take the role of the quasiparticles. In Ref. 27 it is reported that in the center of mass motion the frequency of oscillations doubles when entering a Luther-Emery like phase [28] with dominant singlet pairing correlation functions in a one-dimensional $t$-$J$ model. For the sake of simplicity, in the following we will refer to this phase as ‘superconducting’ (SC), although according to the Mermin-Wagner-Hohenberg theorem [29–31] no true formation of pairs is possible. This indicates that SC quasi-long-range order can affect the time evolution of observables on a qualitative level. Hence, having a larger mass, in the time evolution of correlation functions, a light-cone with a smaller velocity would be expected. The scope of this paper is to address this question by considering quantum quenches in a variant of the $t$-$J$ model with only transverse spin interactions in one spatial dimension, which has such a SC phase of significant size [32]. This model realizes a quantum simulator for magnetism and superconductivity using ultracold polar molecules [33–53] on optical lattices [54, 55].

In this paper, we focus on one-dimensional systems, for which one can efficiently compute the time evolution following a quantum quench by using the time dependent density matrix renormalization group (tDMRG) [56–64]. The main findings are that in contrast to the above expectation, at the filling considered, the slope of the light cone is always comparable to the one of non-interacting electrons, and that the light-cone is strongly suppressed or absent in pairing correlation functions. To further elucidate these findings, we compute the time evolution of the correlation functions also using a BCS approach. The main results of this mean-field approach agree well with the results of the tDMRG. In particular, the BCS theory shows that the amplitude of the light cone in the pairing correlation functions is suppressed up to two orders of magnitude compared to the one in the density-density correlators. Similar behavior is also found in the BCS-treatment of two-dimensional systems. As this approach is independent of the choice of the microscopic lattice model, this gives strong indications for the light-cone in pairing correlation functions to be generally suppressed in the time evolution following a global quantum quench.

The paper is organized as follows: In Sec. II we introduce the variant of the $t$-$J$ model and the observables treated in this paper. In Sec. III we present our tDMRG results for the local densities and for the correlation functions. In Sec. IV we discuss our BCS theory and its main results for the density-density and pairing correlation functions. Sec. V provides a summary. Appendix A contains details to the BCS calculations, and Appendix B exemplifies the mean-field results for the simplest two-dimensional case.
II. MODEL AND OBSERVABLES

A. t-J Model with Transverse Spin Exchange Interactions

In Refs. 27 and 65 the Hamiltonian

\[ H^{tJW} = -t_{\text{hop}} \sum_{i,\sigma} [c^\dagger_{i,\sigma} c^\sigma_{i+1,\sigma} + \text{h.c.}] + \frac{1}{|i-j|^3} \left[ \frac{J_{\perp}}{2} \left( S^+_{i} S^-_{j} + S^-_{i} S^+_{j} \right) + J_2 S_z^+ S^- \right] + V n_i n_j + W n_i S^z_j \]  \tag{1}

is derived as quantum simulator for quantum magnetism and superconductivity in systems of ultracold polar molecules [33–52] on optical lattices [54, 55]. The operators \( c^\dagger_{i,\sigma} \) are fermionic annihilation (creation) operators for a particle with spin \( \sigma \) on lattice site \( i \), the Hilbert space is the usual fermionic Hilbert space projected onto the space with no doublons (as in the usual \( t-J \) model), \( S^+_i = c^\dagger_{i,\uparrow} c^\downarrow_{i,\dagger} \) and \( S^-_i = c^\dagger_{i,\downarrow} c^\uparrow_{i,\dagger} \) are the spin raising and lowering operators, \( S^z_i = (c^\dagger_{i,\uparrow} c^\downarrow_{i} + c^\dagger_{i,\downarrow} c^\uparrow_{i})/2 \) is the \( z \) component of the spin operator, and \( n_i = \sum_{\sigma} c^\dagger_{i,\sigma} c_{i,\sigma} \) is the total density on site \( i \). In Ref. 53 it is found that spin-exchange interactions are indeed realized in such experiments, paving the way for further developments.

Model (1) is a generalization of the standard \( t-J \) model [66–70], in which one dimension (1D) reads

\[ H^{tJ} = -t_{\text{hop}} \sum_{i,\sigma} [c^\dagger_{i,\sigma} c^\sigma_{i+1,\sigma} + \text{h.c.}] + J \sum_i \left[ \frac{\vec{S}_i \cdot \vec{S}_{i+1}}{4} - \frac{1}{2} n_i n_{i+1} \right], \]  \tag{2}

and which usually is obtained via second-order degenerate perturbation theory from the Hubbard model [68]. In perturbation theory, one finds \( J = 4t_{\text{hop}}^2/U \), with \( U \) the strength of the Hubbard interaction, and it is not possible to tune the parameters \( t, J_\perp, J_2, V \) and \( W \) independently from each other. In contrast, in the polar molecules setup [27, 65] the values of the parameters are fully tunable. Note that model (2) is obtained from Eq. (1) by considering only nearest neighbor interactions and setting \( J_2 = J_\perp \equiv J \), \( V = -J/4 \) and \( W = 0 \). Here, we treat model (1) with \( J_2 = V = W = 0 \) and for simplicity we consider only nearest neighbor interactions, leading us to the \( t-J_\perp \) chain

\[ H^{tJ_\perp} = -t_{\text{hop}} \sum_{i,\sigma} [c^\dagger_{i,\sigma} c^\sigma_{i+1,\sigma} + \text{h.c.}] + \frac{J_\perp}{2} \sum_i \left[ S^+_i S^-_{i+1} + S^-_i S^+_i \right]. \]  \tag{3}

The ground state phase diagram of model (3) in one spatial dimension is discussed in Refs. 27 and 32 and is found to be similar to the one of model (2) discussed in Ref. 71. An important difference is that the singlet superconducting (SC) phase at low fillings is significantly enhanced.

In this paper we want to study the effect of a SC phase on the horizon effect, this is favorable, since it allows us to stay farther away from phase transition points, so that their possible effect will have less influence on the dynamics.

B. Observables

We treat the following observables in this paper. First, we consider the behavior of on-site quantities like the local density \( n_i \) and the local magnetizations \( S^\sigma_i \) on lattice site \( i \). We consider systems with zero total magnetization \( \langle \sum_i S^\sigma_i \rangle = 0 \). According to the Mermin-Wagner-Hohenberg theorem [29–31], also the local magnetizations are zero, since the corresponding continuous symmetry cannot be broken spontaneously in one spatial dimension. This is found to be true also in the course of the time evolution.

The connected density-density correlation function is given by

\[ N_{ij} = \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle. \]  \tag{4}

For the spin correlation functions, one in principle has to treat the longitudinal component

\[ C^\text{spin,long}_{ij} = \langle S^z_i S_j \rangle - \langle S^z_i \rangle \langle S^z_j \rangle \]  \tag{5}

and the transverse component

\[ C^\text{spin,trans}_{ij} = \langle S^+_i S^-_j \rangle \]  \tag{6}

independently, as the \( t-J_\perp \) model lacks SU(2) invariance. However, since the features in the time evolution are very similar, we will mainly discuss \( C^\text{spin,long}_{ij} \).

The superconducting properties are probed by the pairing correlation functions

\[ P^T_{ij} = \langle \Delta^+_i \Delta^-_j \rangle. \]  \tag{7}

In this paper we treat

\[ \Delta^+_i = \frac{1}{\sqrt{2}} \left( c^\dagger_{i,\uparrow} c^\dagger_{i+1,\dagger} - c^\dagger_{i,\downarrow} c^\dagger_{i+1,\dagger} \right) \]  \tag{8}

for singlet pairing and

\[ \Delta^+_i = c^\dagger_{i,\uparrow} c^\dagger_{i+1,\dagger} \]  \tag{9}

for triplet pairing.

Throughout the paper we work in units, in which \( \hbar = 1 \) and we set \( t_{\text{hop}} = 1 \).
C. Details for the time-dependent DMRG

We obtain the ground state using the density matrix renormalization group (DMRG) [56–58, 63, 64, 72] performing 5 sweeps and keeping up to \( n = 1000 \) states. We treat systems with \( N = 16 \) particles on \( L = 80 \) lattice sites, corresponding to a filling of 0.2, and zero total magnetization. We apply the adaptive time-dependent DMRG method using a Trotter time evolution scheme [59, 60, 73] for computing the dynamics of the system (3). The time evolution is initiated by a quantum quench, in which we keep \( t_{\text{hop}} = 1 \) fixed and suddenly change the value of \( J_\perp \). We keep up to \( m = 1400 \) states during the time evolution and find in the worst case a discarded weight \( \sim 10^{-8} \) at the maximal time explored. In all cases, we apply open boundary conditions (OBC).

III. LOCAL OBSERVABLES AND CORRELATION FUNCTIONS

A. Local density

In Fig. 1 we display a typical result for the time evolution of the local density following a quench starting from the gapless Luttinger liquid (LL) [28] phase with dominant spin-density wave correlations [27, 32]. As can be seen, in the initial state typical Friedel-like oscillations are obtained [74, 75], which are due to the open boundary conditions used. The wave number of these oscillations is the Fermi momentum \( k_F \) and is associated to the filling \( n \). For free spinful electrons, \( k_F = k_F^0 + k_F^\perp \), where in the initial state \( k_F^\perp = m n \). The Fourier transform displayed in Fig. 1(b) shows that in the initial state the wave vector of the density oscillations is \( k \approx 0.4 \pi \), in agreement with this expectation. However, soon after the quench, an additional wave vector at \( k \approx 0.2 \pi \) appears, which in the course of time becomes dominant. As this value is approximately half the original one, this insinuates that the Friedel-like density oscillations are now at half the Fermi momentum, as if one would have halved the number of particles causing these density oscillations. Such behavior has been observed in the \( t-J \) model in equilibrium [71], where the wave vector of the Friedel oscillations when increasing \( J/t \) smoothly goes to half the original value upon entering the singlet-superconducting phase at low fillings [71], and it is also known from spin systems, where the doubling of the period can be associated to the formation of pairs of magnons [76]. Taking on this picture, the quench seems to induce similar behavior, leading to the coexistence of two wave vectors in the oscillations of the local density.

When starting from the SC phase, instead, already in the initial state the wave vector of the Friedel-like oscillations is half the one of the LL phase at low \( J_\perp/t_{\text{hop}} \). The question arises, if the reverse effect might be realized, and a second wave vector with twice the value is obtained. However, this is not the case: as shown in Fig. 2, in a quench from the SC phase to a smaller value of \( J_\perp/t_{\text{hop}} \) the density oscillations instead are strongly suppressed in the bulk of the system.

B. Correlation functions

In Fig. 3 we contrast the time evolution of \( N_{ij} \) and \( C_{ij}^{\text{spin,long}} \) to the one obtained for \( P_{ij}^{T,S} \) for a quench from \( J_\perp = 1 \) to \( J_\perp = 6 \) at filling \( n = 0.2 \). As can be seen, a clear light-cone signal is visible in \( N_{ij} \) and \( C_{ij}^{\text{spin,long}} \), and similarly also in \( C_{ij}^{\text{spin,trans}} \) (not shown). The slope of the lightcone for quenches starting from \( J_\perp = 1 \) to values \( J_\perp = 2, 3, 6 \) is comparable and lies between 3 and 4. Hence, the velocities of the quasiparticles, which one can associate to the light-cone, are comparable to the ones of free electrons, where the slope of the light-cone has the value four [7]. This is also seen for quenches starting...
from $J_\perp = 5$ to the same final values. Furthermore, the slope of the lightcones for charge correlation functions and for spin correlation functions has comparable values. This indicates that the quasiparticles involved in the formation of lightcones in these observables are not related to pairs of fermions, for which one could expect a slower velocity, as the mass of such an object would be twice the mass of a free particle.

Next, we ask the question if such a change of velocity might be solely realized in pairing correlation functions. Figure 3 contrasts the time evolution of the singlet and triplet pairing correlation functions to the ones of density after a quench in the $t$-$J_\perp$ model (3) from $J_\perp,\text{initial} = 5$ to $J_\perp,\text{final} = 2$. (a) Time evolution as a function of position. (b) Fourier transform of the results of (a) to $k$-space by only considering the bulk region of (a).

These results indicate that in both pairing correlation functions the light-cone signal is strongly suppressed. Similar behavior is found when quenching from a SC phase to the LL phase at small $J_\perp$. However, in this case, a faint linear signal can be obtained. Again, its slope is $\sim 4$. This indicates that the light-cone in principle can also be realized in the pairing correlation functions, but is substantially weaker than in the other correlation functions, an issue we will address in more detail in Sec. IV.

Summarizing the results so far, the velocity of the quasiparticles causing the lightcone in SC phases is not half the one of non-interacting particles. This is an interesting finding and stands in contrast to the doubling of the frequency of the center-of-mass motion reported in Ref. [27] for a similar one-dimensional system upon entering the SC phase. It would be interesting to see if such a smaller velocity could be realized in systems with true SC long-range order, like two-dimensional systems at zero temperature. The data presented in Fig. 3 (a) allow also the observation of a pattern in the dynamics inside the correlated region. There, it is possible to see at least another ballistic signal identified by a local maximum spreading with a velocity slower than $V_\text{lc}$. This is reminiscent of findings for exactly solvable models [18], where internal patterns in the dynamics have been predicted and connected to phase velocities of the spectrum, in contrast to the lightcone given by the group velocity of excitations.

**IV. SUPPRESSION OF THE LIGHTCONE IN PAIRING CORRELATION FUNCTIONS IN BCS THEORY**

The findings of the previous section may be specific to the $t$-$J$-model and to one spatial dimension. In order to test their validity beyond this set-up, we now turn to a simple, analytically tractable model, which contains superconductivity. A simple approach is to recall BCS theory of superconductivity and to compute the time evolution of correlation functions after a quench in this framework. This leads us to the simple toy Hamiltonian

$$\mathcal{H} = \sum_k \left( \epsilon_k - \mu \right) \left( c_{k\uparrow}^\dagger c_{k\uparrow} + c_{k\downarrow}^\dagger c_{k\downarrow} \right) - \sum_k \left( \Delta c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \Delta^* c_{-k\downarrow} c_{k\uparrow} \right), \quad (10)$$

where $\epsilon_k = -2 \cos(k)$ is the dispersion of non-interacting electrons on a one-dimensional lattice, the operators $c_{k\sigma}^\dagger$ denote fermionic annihilation (creation) operators for a particle with momentum $k$ and spin $\sigma$ (note that the restriction of no double occupancy of the $t$-$J$-model is not valid here), and the gap $\Delta$ is assumed to be independent of momentum $k$ and of time $t$. This toy Hamiltonian can
be diagonalized using a Bogoliubov transformation
\[ c_{k\uparrow} = u_k \gamma_{k\uparrow} + v_k \gamma_{-k\downarrow}, \]
\[ c_{k\downarrow} = u_k^* \gamma_{k\uparrow} + v_k^* \gamma_{-k\downarrow}, \]
\[ c_{-k\uparrow} = u_k \gamma_{-k\downarrow} + v_k \gamma_{k\uparrow}, \]
\[ c_{-k\downarrow} = u_k^* \gamma_{-k\downarrow} - v_k \gamma_{k\uparrow}. \] (11)

The spectrum
\[ E_{k\uparrow} = E_{k\downarrow} = \sqrt{(\epsilon_k - \mu)^2 + |\Delta|^2} \] (12)
is gapless for $\Delta = 0$ and gapped otherwise.

In the following we treat quenches, in which we start from the ground state of model (10) with initial value of the gap $\Delta_i$ and at time $t = 0$ suddenly switch to the final value $\Delta_f$, which is used to compute the time evolution. This allows us to obtain exact results for the time evolution for the quench $\Delta_i = 0 \to \Delta_f = 1/2$, at $\mu = 0$, and to compute the observables $N_{ij}(t)$, $P^{ST}_{ij}(t)$, and $P^{SS}_{ij}(t)$ defined in Eqs. (4) and (7) – (9). Going from $\Delta = 0$ to a finite value opens a gap in the spectrum of the excitations. The computation of the analytical expressions for the time evolution of these observables using the BCS Hamiltonian is straightforward and the results are reproduced in App. A.

We expect that if the behaviour of the different correlation functions obtained for the $t$-$J$ model is typical for superconductivity, we will also observe it in this simple BCS model. In Fig. 4 we present the time evolution of the connected density-density correlation function Eq. (4) under the Hamiltonian (10). The time evolution exhibits a linearly increasing correlation edge, which can be identified as the lightcone. Since the system can be described by quasi-particles with a well described spectrum, Eq. (12), the lightcone velocity $V_{lc}$ can be compared to the maximum group velocity of the excitations $V_{lc} = 2 \max \partial_k E_{k\uparrow}$. As indicated by the red solid line in Fig. 4, the slope of the lightcone indeed is in agreement with this prediction.

For the specific case $|\Delta_f| = 1/2$ the velocity is $V_{lc} \approx$...
3.1. This value is comparable to the ones of non-interacting lattice fermions, as well as to the ones obtained after the quench in the $t$-$J$ model. Hence, BCS theory gives a similar picture as the strongly correlated lattice model.

In Figs. 5 and 6 we plot the time evolution of the singlet and triplet correlation functions as defined in Eqs. (7)–(9) (since in the BCS treatment the expectation values for $\langle \Delta^{(1)}(i)_{T,S} \rangle$ can be finite, we consider the connected correlation functions). Again as before, the lightcone is present also in these cases, and the speed of the signal is again in agreement with the maximal group velocity of the excitations. Hence, also in BCS theory the speed of the signal in the lightcone is not affected by the presence of pairs. However, for both pairing correlation functions the values of the observables inside the lightcone at distances larger $\sim 20$ are much smaller than the corresponding value of $N_{ij}$. In the cases shown, it is around two orders of magnitude smaller. Note that at smaller distances, there is a large signal, which does not move towards larger distances, which is similar to the findings in the $t$-$J$ model.

Hence, in BCS theory we see a lightcone in all correlation functions treated, but in the pairing correlation functions it is strongly suppressed. The analytical expressions for the dynamics in the BCS treatment allow us to further investigate this behavior. A closer look at the results for $N_{ij}$ shows that the main contribution to the strong signal in the lightcone in the density-density correlation function is due to the term

$$\langle n_{i\uparrow} n_{j\uparrow} \rangle + \langle n_{i\uparrow} n_{j\downarrow} \rangle.$$ (13)

The dynamics of this observable alone is plotted in Fig. 7. If we compare it to the density-density correlations, Fig. 4, the difference of the values between the two in the vicinity of the lightcone is extremely small. From the expressions in App. A one obtains that the term (13) does not contribute to the pairing correlation functions, which explains why their values in the vicinity of the lightcone are substantially smaller.

At the end of this section we mention that the same BCS approach leads to a similar picture in two spatial dimensions, see App. B for more details.

V. CONCLUSIONS

We considered the time evolution of local observables and different correlation functions following a quantum quench of the $t$-$J_L$ chain Eq. (3) at fillings $n = 0.2$. For quenches from a gapless initial state to a spin-gapped singlet-superconducting phase, the original Friedel oscillations change their wave vector from $k_F$ to $k_F/2$ at short times, and then a coexistence of both wave vectors is seen. Instead, for quenches in the reverse direction, the Friedel oscillations in the initial state are with wave vector $k_F/2$, and then are suppressed in the course of the time evolution.
In the correlation functions one clearly observes lightcone behavior in the density-density and spin correlation functions, but a strongly suppressed lightcone in the pairing correlation functions. The slope of the lightcone is approximately the same in all cases, and similar to the one of non-interacting fermions on a lattice. This indicates that superconducting pairing does not modify the time scales, at which correlations move through the system. This is in contrast to the behavior of center-of-mass oscillations in similar situations, for which the time scales are strongly affected by the SC phase, as seen in the doubling of the frequency of this oscillation [27].

Similar behavior is obtained in quenches in a BCS treatment. In one dimension, the values of the pairing correlations in the vicinity of the lightcone are strongly suppressed compared to the ones of the density-density correlation function. This can be understood by the lack of contribution of the term (13) in the pairing correlation functions, which carries most of the weight of the density correlation function in the vicinity of the lightcone.

The superconducting phases in the one-dimensional systems treated here are not true SC phases according to the Mermin-Wagner-Hohenberg theorem. It would be interesting to see if similar behavior is found in true superconducting phases, e.g., in two dimensional systems at zero temperature.

FIG. 6. Absolute value of the connected singlet correlation function Eq. (A11) as function of distance and time after a quench in the BCS Hamiltonian (10) from $\Delta_i = 0 \rightarrow \Delta_f = 1/2$. The red solid line has the slope of the maximal group velocity of the spectrum Eq. (12) multiplied by two and agrees very well with the border of the light-cone region. The colormap has been set to enhance the contrast in the vicinity of the lightcone, where the maximal value is $\sim 0.001$. Note that close to the lightcone, the strength of the signal is between one and two orders of magnitude smaller than the one of the density-density correlation function in Fig. 4.

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Appendix A: Analytical expressions for the time evolution of the correlation functions in time-dependent BCS theory

We study the BCS-Hamiltonian (10) using the Bogoljubov-transformation (11). In the set of equations (11), $u_k$ and $v_k$ are complex parameters that can be chosen to diagonalize the Hamiltonian (10) using $\xi = \xi_k - \mu$,

$$|u_k|^2 = \frac{1}{2} \left( 1 + \frac{\xi_k}{\sqrt{\xi_k^2 + |\Delta_f|^2}} \right) \quad (A1)$$

$$|v_k|^2 = \frac{1}{2} \left( 1 - \frac{\xi_k}{\sqrt{\xi_k^2 + |\Delta_f|^2}} \right). \quad (A2)$$

To study the time evolution, we promote the amplitudes $u_k$ and $v_k$ to be time dependent complex variables. Their explicit form is then determined solving the Heisenberg equation of motion, see Ref. [77, 78]

$$i\partial_t \begin{pmatrix} u_k(t) \\ v_k(t) \end{pmatrix} = \begin{pmatrix} \xi_k & \Delta_f \\ -\xi_k & -\xi_k \end{pmatrix} \begin{pmatrix} u_k(t) \\ v_k(t) \end{pmatrix}. \quad (A3)$$

FIG. 7. Absolute value of the dynamics of the observable Eq. (13) as function of distance and time after a quench in the BCS Hamiltonian (10) from $\Delta_i = 0 \rightarrow \Delta_f = 1/2$. The red solid line has the slope of the maximal group velocity of the spectrum Eq. (12) multiplied by two and agrees very well with the border of the light-cone region. The colormap has been set to enhance the contrast in the vicinity of the lightcone, where the maximal value is $\sim 0.01$, which is comparable to the one of the density-density correlation function in Fig. 4.
The initial conditions are then given by $u_k(t = 0) = 1$ and $v_k(t = 0) = 0$. This leads to
\[
\begin{align*}
    u_k(t) &= \cos \left( \frac{E_k^f t}{E_k^f} \right) - \frac{i \xi_k}{E_k^f} \sin \left( \frac{E_k^f t}{E_k^f} \right) \\
    v_k(t) &= -i \frac{\Delta_k^f}{E_k^f} \sin \left( \frac{E_k^f t}{E_k^f} \right).
\end{align*}
\]
These results can be used to compute the time evolution of expectation values of different observables. We find it useful to introduce the following functions, in terms of the correlator Eq. (4).

In time-dependent BCS theory, we consider only one type per function.

These results can be used to compute the time evolution of more complicated observables can be expressed:

\[
\begin{align*}
    \mathcal{U}_\uparrow (R, t) &= \frac{1}{L} \sum_k e^{ikR} |u_k|^2 n_{k\uparrow}^0 \\
    \mathcal{U}_\downarrow (R, t) &= \frac{1}{L} \sum_k e^{ikR} |u_k|^2 (1 - n_{k\uparrow}^0) \\
    \mathcal{V}_\uparrow (R, t) &= \frac{1}{L} \sum_k e^{ikR} |u_k|^2 n_{k\downarrow}^0 \\
    \mathcal{V}_\downarrow (R, t) &= \frac{1}{L} \sum_k e^{ikR} |u_k|^2 (1 - n_{k\downarrow}^0) \\
    \mathcal{U} \mathcal{V} (R, t) &= \frac{1}{L} \sum_k e^{ikR} u_k^* v_k (1 - n_{k\uparrow}^0 - n_{k\downarrow}^0),
\end{align*}
\]

where all of them depend on $R = |i - j|$ because the system has translational invariance. If our initial state is invariant under a spin flip, $n_{k\uparrow} = n_{k\downarrow}$, then the subscripts $\uparrow$ and $\downarrow$ can be suppressed. We hence consider in the following only one type per function.

1. Density-density correlations

In this section, we compute the time evolution of the correlator Eq. (4). In time-dependent BCS theory, we can rewrite this observable using the expressions of the previous section and obtain
\[
\mathcal{C} (R, t) = (\mathcal{U}_\uparrow + \mathcal{V}_\uparrow) (\mathcal{U}_\downarrow^* + \mathcal{V}_\downarrow^*) + (\mathcal{U}_\downarrow^* + \mathcal{V}_\uparrow^*) (\mathcal{U}_\uparrow + \mathcal{V}_\downarrow) + 2i [\mathcal{U} \mathcal{V}]^2, \tag{A4}
\]
where the functions $\mathcal{U}$ and $\mathcal{V}$ depend on the distance $R$ and on time $t$. We then consider a quench $\Delta_i = 0 \rightarrow \Delta_f = 1/2$ for a system at zero chemical potential $\mu = 0$, with initial conditions $n_{k\uparrow} = n_{k\downarrow} = \theta (k_F - |k|)$, and filling 0.2, i.e., $k_F = \frac{2\pi}{\sqrt{5}}$. The result is displayed in Fig. 4.

2. Triplet-pairing correlation function

We can now study the connected triplet-pairing correlation function, which we define as
\[
P_T (R, t) = \left\langle \Delta^\uparrow (i) \Delta^\downarrow (j) \right\rangle - \left\langle \Delta^\uparrow (i) \right\rangle \left\langle \Delta^\downarrow (j) \right\rangle \tag{A5}
\]
with $\Delta_T (i) = c_{i\uparrow} c_{i+1 \downarrow}^\dagger$.

Using the same simplifications used for the density-density correlations, we obtain
\[
P_T (R = |i - j|) = \mathcal{F} (R, t) \mathcal{F} (R + 1, t) - \mathcal{F}^2 (R, t), \tag{A7}
\]
where $\mathcal{F} (R, t) = [\mathcal{U}_\uparrow (R, t) + \mathcal{V}_\uparrow (R, t)]$.

This result shows that the time evolution of the triplet-pairing correlation function is obtained by subtracting the product of the same function at neighboring lattice points $R, R - 1, \text{and } R + 1$. This explains why its value is so small, as the total contribution is in fact a quartic contribution in the Taylor expansion.

3. Singlet-pairing correlation function

Finally we can write down the time evolution of the connected singlet-pairing correlation function
\[
P_S (R, t) = \left\langle \Delta^\uparrow (i) \Delta^\downarrow (j) \right\rangle - \left\langle \Delta^\downarrow (i) \right\rangle \left\langle \Delta^\uparrow (j) \right\rangle \tag{A9}
\]
with $\Delta_S^\uparrow (i) = \frac{1}{\sqrt{2}} \left( c_{i\uparrow} c_{i+1 \downarrow}^\dagger - c_{i\downarrow} c_{i+1 \uparrow}^\dagger \right)$.

As before, the exact expression can be written as function of the building blocks presented before, leading to
\[
P_S (R, t) = \mathcal{F} (R - 1, t) \mathcal{F} (R + 1, t) + \mathcal{F}^2 (R, t), \tag{A11}
\]
where $\mathcal{F}$ has been introduced in the previous section. In this case there is no subtraction of terms that could explain why this correlation function is so small compared to the density-density one. However, a closer look at the different fundamental building blocks involved in the expressions (A4) and (A11) shows that the difference is indeed the expression (13). As this is found to carry most of the weight in the density-density correlation function, the values of the singlet-pairing correlation functions are strongly suppressed.

Appendix B: Time-dependent BCS approach in two dimensions

The toy model of Hamiltonian (10) can easily be generalized to two-dimensional situations. The spatial indices become vectors, $i = (i_x, i_y)$, and correspondingly the Fourier vector $k = (k_x, k_y)$. The equation of motion for the amplitudes $u_k$ and $v_k$ takes exactly the same form
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FIG. 8. Time evolution of the absolute value of the connected density-density correlation function Eq. (A4) after a quench in the BCS Hamiltonian (10) from $\Delta_i = 0 \to \Delta_f = 1/2$ in a two-dimensional system. The coordinate $R$ labels the points on the line $(R, R)$, bisectrix of the plane. The red solid line has the slope of the maximal group velocity of the spectrum Eq. (12) multiplied by two and agrees very well with the border of the light-cone region. The colormap has been set to enhance the contrast in the vicinity of the lightcone, where the maximal value is $\sim 0.008$.

and it can be solved analytically again. For the quench $\Delta_i = 0 \to \Delta_f = 1/2$ we study the density-density correlation function, which takes the same general expression of Eq. (A4) where $R$ labels the two dimensional distance.

Because of the 2D geometry, the possibilities to define pair correlation functions are richer. Here, for the sake of simplicity, we consider only on-site pairing correlation functions defined as:

$$S(R, t) = \langle c_i^{\dagger} (t) c_{i+} (t) c_j (t) c_{j+} (t) \rangle = |\mathcal{F}(R, t)|^2, \tag{B1}$$

which correspond to s-wave pairing. For $D = 1$, the lightcone in $S$ has the same suppression as the correlators $P^T$ and $P^S$ compared to the density-density correlation function.

In Fig. 8 we plot the time evolution of the connected density-density correlation function along the line $(R, R)$ as function of time. The time evolution of $S(R, t)$ along the same line is plotted in Fig. 9. As in the 1D case, we observe a suppression of the amplitude of the lightcone from the density-density to the on-site correlation function.

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FIG. 9. Absolute value of the on-site correlation function Eq. (B1) as function of distance and time after a quench in the BCS Hamiltonian (10) from $\Delta_{c} = 0 \to \Delta_{f} = 1/2$ in a two-dimensional system. The coordinate $R$ labels the points on the line $(R, R)$, bisectrix of the plane. The red solid line has the slope of the maximal group velocity of the spectrum Eq. (12) multiplied by two and agrees very well with the border of the light-cone region. The colormap has been set to enhance the contrast in the vicinity of the lightcone, where the maximal value is $\sim 8 \times 10^{-6}$. Note that close to the lightcone, the strength of the signal is three orders of magnitude smaller than the one of the density-density correlation function in Fig. 8.

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