Near BPS Skyrmions: Non-shell configurations and Coulomb effects

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The Skyrme Model: Motivations

- Skyrme Model = low-energy effective mesonic field theory of QCD
- Baryons and nuclei = topological solitons: conserved topological charge = baryon number
- Success: Hadron properties predicted within 30% (some quantities within a few %)
- Difficulties: Numerically challenging, Link to QCD, ...AND...
- Multibaryon physics (nuclear binding energy, configurations),

\[1\text{[based on works in : Phys.Rev. D82 (2010) 054023, arXiv:1205.1414 (2012)]}\]
The Problem: Multibaryons

- Binding energies too large especially for small nuclei (e.g., deuteron $\approx 80 \times$ observed value)
- Shell-like baryon and mass density configurations

$A = 2$

$A = 3$

$A = 4$
1. The Problem
2. New Model
3. Conclusions

1.1 The Skyrme Model
1.2 Multibaryons

Multibaryons : Alternatives:

- Different potential (mass) terms
- Rotational deformations
- More higher order terms in derivatives
- More mesons (e.g. ω, ρ, ...)

⇒ Previous results: Similar configurations and binding energies
2. New Model

2.1 Near BPS Skyrme Model

Basic idea:

Nuclei: \( M \approx A \cdot M_{\text{nucleon}} \) and almost constant density

\[ \downarrow \]

Construct model where \textbf{Skyrmions} \( \approx \) \textbf{BPS-solitons} with NON-shell density
New Model: Near BPS Skyrme Model

- The pion fields described by matrix $U \in SU(2)$
  
  $$U = \exp\left(-\frac{2i}{F_\pi} \vec{\tau} \cdot \vec{\pi}\right) = \phi_0 + i \vec{\tau} \cdot \vec{\phi}$$

  such that $\phi_0^2 + \vec{\phi}^2 = 1$

- Our model: Lagrangian
  
  $$\mathcal{L}_{NBPS} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_0 + \mathcal{L}_6$$

  Skyrme and BPS-solitons
New Model: Near BPS Skyrme Model

1. Quadratic NL\(\sigma\) term: kinetic term (here \(L_\mu = U^\dagger \partial_\mu U\))

\[ \mathcal{L}_2 = -\alpha \text{Tr} [L_\mu L^\mu] \quad (\alpha = \frac{F_\pi^2}{16}) \]

2. Quartic Skyrme term (necessary to stabilize soliton: Derrick Theorem)

\[ \mathcal{L}_4 = \beta \text{Tr} \left( [L_\mu, L_\nu]^2 \right) \quad (\beta = \frac{1}{32e^2}) \]

3. Potential term (\(\chi\)SB term): responsible for pion mass

\[ \mathcal{L}_0 = -\mu^2 V(U) \quad (= \frac{m_\pi^2 F_\pi^2}{8} \text{Tr} (1 - U)) \]
New Model: Near BPS Skyrme Model

- **Sextic term:** quadratic in time derivatives \( \Rightarrow \) standard hamiltonian formulation.

\[
\mathcal{L}_6 = -\frac{3}{2} \frac{\lambda^2}{16^2} \text{Tr} \left( [L_\mu, L_\nu] [L_\nu, L^\lambda] [L^\lambda, L_\mu] \right) = \lambda^2 \pi^4 B^\mu B_\mu
\]

where \( B^\mu \) is the baryon current

\[
B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( L_\nu L_\rho L_\sigma \right)
\]

- **Winding number = baryon number = atomic number =** \( A \)

\[
A = \int d^3 r B^0 = -\frac{1}{24\pi^2} \epsilon^{ijk} \int d^3 r \text{Tr} \left( L_i L_j L_k \right).
\]
Near BPS Skyrme Model: The Strategy

Get closer to saturation of Bogomol’nyi bound without losing link with pion physics:

\[
\mathcal{L}_{\text{NBPS}} = \mathcal{L}_0 + \mathcal{L}_6 + \mathcal{L}_2 + \mathcal{L}_4
\]

- Assume \(\mathcal{L}_0 + \mathcal{L}_6\) dominate (\(\Rightarrow\) BPS-solitons) and
- Treat \(\mathcal{L}_2\) and \(\mathcal{L}_4\) are small as perturbations.

Then:
Near BPS Skyrme Model: The Strategy

1. Setting \( \alpha = \beta = 0 \), choose an appropriate potential \( V(U) \) for NON-shell configuration

\[
-\mu^2 V(U) = \frac{\mu^2}{576} \text{Tr} \left[ \frac{(2I-U-U^\dagger)(2I+U+U^\dagger)^3}{\ln((2I+U+U^\dagger)/4)} \right]
\]

2. Solution = BPS-soliton with axial symmetry

\[
U = \cos F(r) + i\hat{n} \cdot \tau \sin F(r)
\]

\[
\hat{n} = (\sin \theta \cos n\varphi, \sin \theta \sin n\varphi, \cos \theta)
\]

with \( n = A = \) integer and analytical form

\[
F(r) = \mp 2 |\arccos (\exp[-ar^2])| \quad \text{with} \quad a = \left( \frac{\mu}{18n\lambda} \right)^{2/3}
\]
Near BPS Skyrme Model: The Strategy

Figure 1: Baryon density $B(r)$ ($r$ in MeV$^{-1}$)
Near BPS Skyrme Model: The Strategy

3. Switch on the NLσ and Skyrme terms as small perturbations

4. Add rotational and iso-rotational energies $E_{\text{rot}}$: well-known procedure

$$E_{\text{rot}} = \frac{1}{2} \left[ \frac{j(j+1)}{V_{11}} + \frac{i(i+1)}{U_{11}} + \left( \frac{1}{U_{33}} - \frac{1}{U_{11}} - \frac{n^2}{V_{11}} \right) \kappa^2 \right]$$

$i, j, \kappa =$ lab. isospin, spin, max. of B.F. $3^{rd}$ comp. of isospin.
$U_{ij}, W_{ij}, V_{ij} =$ moments of inertia.
Near BPS Skyrme Model: The Strategy

5. Add Coulomb energy $E_C$ using charge density

$$\rho(r) = J^{0}_{EM} \equiv \frac{1}{2} B^0(r) + i_3 \frac{U_{33}(r)}{U_{33}}$$

where $B^0(r)$ and $U_{33}(r)$ have an analytical form.

6. Add isospin breaking term $E_I$ (p-n mass difference):

$$E_I = a_I i_3$$

7. Fit 4 parameters of the model $\mu, \alpha, \beta, \lambda$ w.r.t. nuclear data and mass of the multiskyrmions:

$$M = E_{\text{stat}} + E_{\text{rot}} + E_C + E_I$$

and compute other properties of the nuclei.
Near BPS Skyrme Model: The Strategy

Set I: \( \alpha = \beta = 0 \), input = H and He masses
Set II: input = masses of 144 most stable isotopes
Set III: input = \( B/A \) of 144 most stable isotopes

|          | Set I | Set II | Set III |
|----------|-------|--------|---------|
| \( \mu \) (\( 10^4 \) MeV\(^2\)) | 1.49  | 1.51   | 1.73    |
| \( \alpha \) (\( 10^{-3} \) MeV\(^2\)) | 0     | 5.88   | 22.1    |
| \( \beta \) (\( 10^{-6} \) MeV\(^0\)) | 0     | -1.85  | -5.81   |
| \( \lambda \) (\( 10^{-3} \) MeV\(^{-1}\)) | 6.41  | 6.34   | 5.54    |
2.2 Binding Energies per Nucleon ($B/A$)

Binding Energies per Nucleon ($B/A$) : Set I, II and III
Near-BPS Skyrme Model

- Non shell-like configurations for baryon and charge densities for all $A$ possible
- Surprisingly good fit for $B/A$
- Other important results: e.g. size of nuclei $\propto A^{1/3}$

Next:

- Improve choice of potential $\implies$ Constant charge density with constant skin thickness for nuclei
- Study more properties: magnetic moments, form factors, vibrational modes, ...
- etc...
Questions