Theory of Nuclear Reactions Leading to Superheavy Elements

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Abstract

Dynamical reaction theory is presented for synthesis of superheavy elements. Characteristic features of formation and surviving are discussed, which combinedly determine final residue cross sections of superheavy elements. Preliminary results on Z=114 are also given.

1 Introduction

Superheavy elements around Z = 114(or 126) and N = 184 have been believed to exist according to theoretical prediction of stability given by the shell correction energy in addition to average nuclear binding energy.\textsuperscript{[1]} This means that heavy atomic nuclei with fissility parameter \( x \geq 1 \) could be stabilized against fission by a huge barrier which is resulted in by the additional binding of the shell correction energy around the spherical shape. In other words, if superheavy compound nuclei(C.N.) are formed in such high excitation that closed shell structure is mostly destroyed, they have no barrier against fission and thus are inferred to decay very quickly. Therefore, the point is how to reach the ground state of the superheavy nuclei, or how to make a soft-landing at them. In order to minimize fission decays of C.N. or maximize their survival probabilities, so-called cold fusion reactions have been used, which succeeded in synthesizing SHEs up to Z = 112.\textsuperscript{[2]} They have the merit of large survival probabilities, but suffer from the demerit of small formation probabilities because of the sub-barrier fusion. On the other hand, so-called hot(warm) fusion reactions have the merit of expected large formation probabilities and the demerit of small survival probabilities due to relatively high excitation of C.N. formed. An optimum condition for large residue cross sections of SHEs is a balance or a compromise between formation and survival probabilities as a function of incident energy or excitation energy of C.N. formed.\textsuperscript{[3]} Therefore, the whole reaction process has to be described, from the encounter of incident ions to the formation of compound nuclei, and then to fission decays with residues left of a small probability. An optimum path has to be searched for over all possible incident channels including secondary beams becoming available sooner or later. Whether the reaction is “cold” or “hot” fusion is automatically determined by choosing combinations of projectiles and targets. The theoretical framework, thus, has to accommodate both of them. As is described in section\textsuperscript{[4]} the strong dissipation of energies of nuclear collective motions, which is well recognized by the fusion hindrance\textsuperscript{[5]} and the long fission life\textsuperscript{[6]} has to be taken into account in formation process as well as in survival process.
2 Theoretical framework

In reactions of massive systems, it is not clear that the compound nucleus theory can apply, i.e., that the formation and the decay of compound nuclei are independent. But for simplicity, we assume that it is valid, at least approximately. Then, the residue cross section is given by the following formula as usual,

\[ \sigma_{SHE} = \frac{\pi}{k^2} \sum_J (2J + 1) \cdot P_{for}^J(E_{c.m.}) \cdot P_{surv}^J(E^*) \] (1)

where \( P_{for}^J \) and \( P_{surv}^J \) denote the formation probability and the survival probability, respectively.

2.1 Formation probability

\( P_{for}^J \) is not equal to a simple transmission coefficient \( T_J(E_{c.m.}) \) of an optical potential, nor to a barrier penetration factor \( P_J(E_{c.m.}) \) of the combined potential of Coulomb repulsion and nuclear attraction. As is well known, there is the fusion hindrance in massive systems, roughly those with \( Z_1Z_2 \geq 1800 \), which could be interpreted by the overcome of so-called conditional saddle under the strong dissipation. The necessary additional energy is so-called extra-push energy. Therefore, \( P_{for}^J \) should take into account both the usual barrier penetration and the dissipative dynamics over the saddle. The former is approximated by the penetration factor for the parabolic barrier, while the latter is treated by Langevin equation, concerning the collective shape degrees of freedom of the compound systems starting with the contact configuration of the projectile and the target. In one-dimension, Langevin eq. is given as

\[ m \frac{d^2q}{dt^2} + \frac{\partial V}{\partial q} - \gamma \frac{dq}{dt} + R(t) = 0 \] (2)

The friction coefficient \( \gamma \) is taken to be equal to that of so-called one-body wall-and-window formula. The last time-dependent term \( R(t) \) in Eq. is a random force associated with the friction force. The last two terms are a phenomenological description of effects of the nucleonic degrees of freedom in excitation, considered as a heat bath. Thus, we assume the fluctuation-dissipation theorem,

\[ < R(t) \cdot R(t') > = \gamma \cdot T \cdot \delta(t - t') \] (3)

where \(<\>\) denotes an average over all the possible realizations of assumed gaussian noise of \( R \) and \( T \) does the temperature of the heat bath, i.e., the compound nucleus. The delta function in the r.h.s. is due to the Markovian assumption. Equivalently, one can use Kramers eq.

\[ \frac{\partial f(q,p,t)}{\partial t} = \{- \frac{\partial}{\partial q} p + \frac{\partial}{\partial p} \frac{\partial V}{\partial q} + \frac{\partial}{\partial p} (\beta \cdot p + m \beta T \frac{\partial}{\partial p})\} f(q,p,t) \] (4)

where \( p \) denotes the conjugate momentum to \( q \), and \( \beta \) is the reduced friction coefficient \( \gamma / m \). Eq. describes a time-evolution of distribution function in the phase space, while Eq. does trajectories of Brownian particle. Actually, Eq. was proposed by Kramers in order to interpret the fission width proposed by Bohr and Wheeler, from the dynamical view-point.
In realistic situations, processes are in many dimensions including mass asymmetry degree of freedom etc. in addition to the elongation or the separation between fragments. An important case that we will discuss below is that the incident channel is with $Z_1 Z_2 \geq 1,800$ and the compound nucleus is with $Z = 114$. In Fig. 1, an example of contour maps of energy surfaces is given for the symmetric mass partition ($Z_1 Z_2 \geq 3,000$). The axes are the elongation ($Z_0$), i.e., the relative distance between fragments and the deformation where fragment deformations are assumed to be proportional to their masses. It is seen that the contact point of two spheres is located below the conditional saddle point. The system has to climb up, governed by the fluctuation-dissipation dynamics (the multi-dimensional version of Eq. (2)) in order to reach the spherical configuration of the total system beyond the saddle point. Apparently, an additional incident energy is required, which corresponds to the extra-push energy. On the other hand, if we think about a very mass-asymmetric incident channel, such as $^{48}$Ca + $^{244}$Pu ($Z_1 Z_2 \simeq 1,800$), we may not suffer from the extra energy, because the contact point of two
incident nuclei can be located inside the conditional saddle point. Actually, the mass asymmetry parameter \( \alpha = 0.66 \) case is shown in Fig. 2, where the contact point is slightly inside and then the composite system evolves mainly over the flat region around the spherical configuration after touching each other. This aspect is surely favourable for formation probability. Anyhow, Langevin trajectories are calculated on the energy surfaces in order to obtain \( P_{J_{for}}^f \).

### 2.2 Survival probability

The survival probability \( P_{\text{surv}} \) is usually given as follows,

\[
P_{\text{surv}}(E^*) = \frac{\Gamma_n(E^*)}{\Gamma_n(E^*) + \Gamma_f(E^*)}
\]

, assuming that fission and neutron emission are dominant decay modes. The neutron emission width \( \Gamma_n \) can be calculated by Weiskopf formula, while the fission width by Bohr-Wheeler one or Kramers’ stationary limit.\[5\] If \( E^* \) is high enough for more than one neutron emission, then the expression at r.h.s. of Eq.(5) is repeatedly used for sequential \( E^* \) and factorized to obtain the final \( P_{\text{surv}} \). Anyhow, this method is valid only for cases with the fission barrier height being larger than the temperature \( T \). In SHE compound nuclei, especially in hot fusion reactions, there might be no fission barrier, because the barrier is given by the shell correction energy around the spherical configuration or somewhere in deformed region which is expected to melt in high excitation. The potential landscape changes depending on the temperature \( T \), i.e., on neutron emission. Then, a dynamical treatment is called for. In other words, final residue cross sections of SHE are to be determined by a competition between time scale of fission and that of restoration of the shell correction energy due to cooling by neutron evaporation. Therefore, a speed of cooling and a temperature dependence of the shell correction energy are crucial. The latter is shown in Fig. 3, the energy being normalized by the maximum value, i.e., that of \( T = 0 \). The calculations are made by the use of single particle levels in a Saxon-Wood finite potential well. The present results are in good agreement with the prescription proposed by Ignatyuk.\[8\] Using this temperature dependence and cooling obtained by the statistical code for evaporation, we obtain potential energy curves as a function of time. With this time-dependent potential, we calculate fission process with Eq.(2) or (4) or its simplified version and obtain a small fraction of probability captured by the pocket made by the restored shell correction energy, which gives a final survival probability \( P_{\text{sur}} \).

In Fig. 4, \( P_{\text{sur}} \)’s with \( J=10 \) are shown for several isotopes of compound nuclei with \( Z=114 \), as a function of initial excitation energy. We readily see that \( P_{\text{sur}} \) depends strongly on excitation energy as well as on mass number, i.e., neutron number of initial compound nucleus. This is due to differences in cooling speed due to different neutron separation energies \( B_n \)’s. Thus, we have to form a compound

\[\text{Figure 3:}\]

\[\text{Figure 4:}\]
nucleus which has as small \( B_n \) as possible, i.e., as large neutron number \( N \) as possible in order to obtain large survival probabilities. In cases with small \( N \) such as GSI experiments, higher excitation is extremely unfavored, thus “cold” fusion process is used. Note that the results in Fig. 4 are at the time \( 2,000 \times 10^{-21} \) sec and therefore they should be reduced by a reduction factor due to the last neutron emission for the final survival probabilities.

3 Preliminary results and discussion

In brief, using WKB approximation for tunneling and 3-dimensional Langevin equation for subsequent dynamical evolutions of the total system, we calculated formation probabilities, while one-dimensional Smoluchowski equation which is the approximation of Eq. (4) was used for fission decay or survival probabilities. Evaporations were taken into account in statistical model. Results are shown in Fig. 5 over possible incident channels leading to \( Z=114 \). Existences of the optimum energies in various channels are clearly seen, which resulted in by the compromise of the two factors. Rapid increases in the left hand side are due to barrier penetration and dissipative dynamical evolutions of shape degrees of freedom, while decreases in the right hand side due to energy dependences of survival probabilities against fission. It is worth to notice that the recent Dubna experiment observed an event which would be a signature for \( Z=114 \) element, at the energy predicted here to be most favourable in the incident channel \(^{48}\text{Ca} + ^{244}\text{Pu}\).[9]

As for a relation to so-called dinucleus system concept,[10] it would be helpful to have a look on contour map of energy surface in axes of the elongation (\( Z_0 \)), and their mass-asymmetry (\( \alpha \)), which is shown in Fig. 6. There is a Businaro-Gallone peak at the upper right corner. Touching points with asymmetries \( \alpha = 0.0 \) (symmetric case) and \( \alpha = 0.66 \) (close to \(^{48}\text{Ca} + ^{244}\text{Pu}\) case) are also shown, schematically. It should be noted here that the top horizontal line and the left vertical line both describe approximately the spherical configuration of the total system, thereby should converge to one point in a more realistic representation. The model based on the dinucleus system concept gives fusion probabilities over the Businaro-Gallone peak, so it evaluates evolutions along the vertical direction in Fig. 6, while the 3-D Langevin calculations take into account evolutions over all possible dynamically including the horizontal direction as well.

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Figure 6:

Potential Energy Surface MeV

- Mass Asymmetry $\alpha$
- Elongation $Z_0$

- Fusion

Businaro Gallone Point

Z=112 A=298 $\delta=0.00$