Universal behaviour in the floating up of quantum Hall extended states as $B \to 0$

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We investigate the relationship between the quantum Hall extended states and the apparent $B = 0$ ‘metal’-insulator transition in extremely low density, high quality two-dimensional n-GaAs systems ($\mu_{\text{peak}} \sim 2 \times 10^7 \text{cm}^2\text{V}^{-1}\text{s}^{-1}$). The combination of small effective mass and high quality allow us to resolve the continued floating up in energy of the extended states down to very low magnetic fields, $B = 0.015\text{T}$, despite an apparent ‘metal’-insulator transition at $B = 0$. We present a modified global phase diagram which brings together conflicting data in the literature from different material systems, and reconciles the differences as to the fate of the extended states as $B \to 0$.

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There has been intense interest in the unexpected metallic behaviour observed in dilute two dimensional (2D) electron and hole systems. While the one parameter scaling theory of localization [1] predicts that in the absence of a magnetic field all 2D systems are insulating at $T = 0$, an anomalously strong ‘metallic’-like behaviour ($dR/dT > 0$) has been observed in various high quality, strongly interacting (large $r_s$) systems over the last few years [2–12]. Although this temperature dependence suggests the possibility of a ‘metallic’ ground state, recent experiments [13–15] have shown that even in the ‘metallic’ phase there are logarithmic localising corrections which appear to drive the conductivity to zero as $T \to 0$. If these corrections are not overcome by strong interaction effects [16] the results indicate that the system is insulating at $T = 0$. Despite numerous experimental and theoretical proposals the questions of what causes ‘metallic’ behaviour at finite temperatures, and if a 2D system can be a true metal at $T = 0$, remain a subject of controversy.

Experimentally it is not easy to attain temperatures below a few tenths of millikelvin, making the detection of logarithmic insulating behaviour in high quality samples where ‘metallic’ behaviour is observed difficult. Another way to determine the ground state is to relate what happens to the extended states in the quantum Hall regime at finite magnetic fields to the apparent ‘metal’-insulator transition (MIT) at $B = 0$ [17]. If there are extended states below the Fermi energy at $B = 0$ then the system will be metallic at $T = 0$, otherwise it is an insulator. Since the one parameter scaling theory [1] shows that non-interacting 2D systems are always localised, Khmelnitskii [18] and Laughlin [19] argued that the quantum Hall extended states ‘float up’ in energy as $B \to 0$, so that only localised states remain below the Fermi energy. This floating up has been observed in disordered n-GaAs and p-Ge systems [20,21] that are insulating at $B = 0$. However, recent studies of high quality p-GaAs systems, which exhibit ‘metallic’ behaviour at $B = 0$, indicate that the Landau levels remain at a finite energy as $B \to 0$ [17,22], suggesting a link between the $B = 0$ ‘metallic’ behaviour and the quantum Hall extended states.

In this paper we track the evolution of the quantum Hall extended states as $B \to 0$ in extremely high quality GaAs electron systems that exhibit an apparent MIT at $B = 0$. Two different samples from undoped GaAs/AlGaAs heterostructures, where carriers are induced with a gate bias [23], were studied. These samples are amongst the highest quality systems in which the 2D MIT has been observed ($\mu_{\text{peak}} \sim 2 \times 10^7 \text{cm}^2\text{V}^{-1}\text{s}^{-1}$). The main difference between the samples is the lowest carrier density reached: $5 \times 10^9 \text{cm}^{-2}$ in sample A and $3 \times 10^9 \text{cm}^{-2}$ in sample B. Most significantly the electron effective mass ($0.067m_e$) is low compared to other semiconductor systems, and since the Landau level separation scales as $eB/m^*$ we are able to follow the Landau levels to very low magnetic fields – precisely where ‘floating up’ should be most pronounced.

Figure 1 shows the resistance of sample A as a function of temperature at $B = 0$ for different carrier densities on both sides of the ‘metal’-insulator transition. At low densities $n_s = (5–7.1) \times 10^9 \text{cm}^{-2}$, the sample shows insulating behaviour where $R_{xx}$ increases as $T \to 0$. As the density is increased to $7.8 \times 10^9 \text{cm}^{-2}$ the resistance exhibits non-monotonic behaviour, with insulating-like behaviour ($dR/dT < 0$) at high temperatures and ‘metallic’-like behaviour ($dR/dT > 0$) at lower temperatures (Fig. 1(c)). The change in sign of $dR/dT$ occurs at $T_f/T_F \sim 0.1$, where $T_F$ is the Fermi temperature. This non-monotonic behaviour is due to the system becoming non-degenerate [24] as has been observed in other material systems when $T/T_F \sim 0.1–0.7$ [17,21,22]. As the density is increased further, to $9.8 \times 10^9 \text{cm}^{-2}$, the sample shows ‘metallic’ behaviour over the complete measurement range with the resistance dropping as $T \to 0$ (Fig. 1(d)). The ‘metallic’-
like drop in resistance in this sample (∼ 5%) is consistent with other studies where a decrease of anything between 3% \cite{12} and a factor of eight \cite{2} has been observed.

Fig. 1. Longitudinal resistance of sample A as a function of temperature at \( B = 0 \) for the carrier densities indicated.

Having demonstrated the existence of an apparent ‘metal’-insulator transition at \( B = 0 \), we now turn to examine the evolution of the extended states in the quantum Hall regime as \( B \rightarrow 0 \). Measurements of the magnetoresistance \( R_{xx} \) at different temperatures are used to identify the quantum Hall liquid-insulator transitions using two well-established methods \cite{17,20}. This approach has the advantage that the transitions between quantum Hall and insulating states define a quantum critical point and provides an independent means of determining the ground state of the system at \( B = 0 \). In the first method, \( R_{xx}(B) \) is measured at a fixed carrier density for different temperatures. Typical traces are shown in Figs. 2(a-c) for densities that are insulating at \( B = 0 \). The arrows mark the temperature independent points in Fig. 3(d).

To track the evolution of the Landau levels in magnetic field, we have repeated these measurements at many different carrier densities and magnetic fields, and plot the position of the \( T \)-independent points in Fig. 3(d). Solid symbols mark points determined from the fixed density measurements while open symbols are obtained from fixed field measurements. Good agreement is obtained between these two techniques, and the data in Fig. 3(d) clearly shows that for \( B < 0.05 \) T the extended states float up rapidly as \( B \rightarrow 0 \). The power of this technique is that these transition points are \( T \)-independent and should therefore persist down to \( T = 0 \) since they mark a quantum phase transition between quantum Hall liquid and insulating states. However at extremely low magnetic fields it is no longer possible to resolve the Landau levels and this technique is not valid. In particular there are no Landau levels at \( B = 0 \), and whilst there is an apparent transition in Fig. 3(f) it is not known if it persists to \( T = 0 \). It is therefore crucial to track the quantum Hall transitions to as low a magnetic field as possible to shed light on the \( B = 0 \) ground state.
FIG. 3. Position of the extended states as a function of density and magnetic field, from studies in different material systems. The open and closed squares in (d) mark the position of the extended states with corresponding error bars obtained by the two different methods discussed in the text. The dotted lines in (a-d) mark the magnetic field below which thermal broadening becomes significant, \( \hbar \omega_c/4k < 50 \text{ mK} \).

In these experiments we have tracked the extended states down to the lowest fields that they can be resolved \( (B = 0.015 \text{ T}) \), and find that they float up continuously with no signs of saturation as \( B \rightarrow 0 \). Furthermore the slope \( dn_e/dB \) of the Landau level trajectory at these small fields \( (0.015 < B < 0.05 \text{ T}) \) is more than an order of magnitude larger than previous studies in p-GaAs [17]. The observation that the extended states start to float up so rapidly as \( B \rightarrow 0 \) makes it difficult to relate the quantum Hall extended states in finite magnetic fields to the apparent \( B = 0 \text{ MIT at } n_e^* = (8.0 \pm 0.25) \times 10^9 \text{ cm}^{-2} \) [23], since this would require a sharp change in the trajectory of the extended states below 0.015T.

To compare these results with previous studies we show in Figs. (a-c) similar data from disordered n-GaAs [20], disordered p-Ge [21], and high quality p-GaAs [17]. In both of the disordered systems no apparent ‘metallic’ behaviour is observed at \( B = 0 \), and the Landau levels float up rapidly as \( B \rightarrow 0 \). However in p-GaAs, where ‘metallic’-like behaviour is observed at \( B = 0 \), the Landau levels start to float up, but then appear to saturate to a finite density as \( B \rightarrow 0 \). This observation has led to the suggestion that there is a relation between the quantum Hall effect and the ‘metallic’ behaviour observed at \( B = 0 \) in these systems [17].

To resolve the discrepancy between the various experiments as to the fate of the extended state as \( B \rightarrow 0 \) we must consider both the effects of disorder and finite temperature. It is known that the Landau levels start to float up when \( \omega_c \tau = \mu B \sim 1 \) [22]. In disordered n-GaAs (Fig. 3(a)) this floating up occurs at relatively high magnetic fields \( B \sim 2.8 \text{ T} \) such that the small effective mass makes it easy to resolve the Landau level separation \( \hbar B/m^* \). For disordered p-Ge in Fig. 3(b) the effective mass is higher, but the strong disorder still means the floating up occurs at large fields, \( B \sim 2 \text{ T} \), where the Landau levels are easily resolved.

The floating up becomes more difficult to observe experimentally in high quality samples (where the apparent \( B = 0 \) metallic behaviour occurs) since we require very low temperatures, \( kT \ll \hbar B/m^* \), to avoid thermal smearing [24]. In the high quality p-GaAs of Fig. 3(c) the low disorder means that the levels only start to float up at low magnetic fields, \( B = 0.4 \text{ T} \), where due to the large hole mass the Landau level spacings are already inherently small and thermal broadening becomes significant. The vertical dotted lines in Fig. 3(a-d) mark the magnetic fields below which it is not possible to track the Landau levels, because the level separation becomes comparable to the thermal broadening at 50mK. This line clearly shows that resolving the Landau levels below \( B \sim 0.08 \text{ T} \) is problematic for the high quality p-GaAs sample in Fig. 3(c). However for low disorder n-GaAs the small electron mass makes it possible to safely track the Landau levels to much smaller magnetic fields, as can be seen in Fig. 3(d). Here we can reach magnetic fields as low as \( B \sim 0.015 \text{ T} \) before thermal broadening sets in, and down to these low magnetic fields there is no sign of saturation. For completeness we have also marked the apparent \( B = 0 \) transition, but re-iterate that unlike the quantum Hall transitions at finite \( B \), it is not known if this transition persists to \( T = 0 \). Significantly, this is the first time that continuous floating up of the Landau levels has been observed in high quality samples that exhibit ‘metallic’ behaviour at \( B = 0 \).

Although both p- and n-GaAs show floating up of the Landau levels, an apparent discrepancy still remains. In Fig. 3(c) it might appear that at intermediate fields \( (B = 0.2 \text{ T}) \) the Landau levels start to saturate before thermal broadening sets in. However, we will show that it is not simply how low in magnetic field we can track the Landau levels, but how small \( 1/\nu \) is \( (\text{where } 1/\nu = eB/\hbar n_e) \), that defines the trajectory of the extended states as \( B \rightarrow 0 \). In Figure 4 we present a global phase diagram which reconciles the experimental data from these different material systems with differing degrees of disorder. This diagram...
is based on the global phase diagram (GPD) of Kivelson, Lee, and Zhang [27] (shown in the inset) modified to highlight the floating up of the Landau levels in energy. In the GPD (where the x and y-axis are 1/ν and disorder) ‘floating up’ of the Landau levels in energy as 1/ν → 0 is equivalent to a ‘floating down’ of the quantum Hall effect: (1) Using high quality n-GaAs systems we have observed that the extended states that exist in finite magnetic field float continuously up in energy as B → 0, despite an apparent ‘metal’-insulator transition at B = 0. (2) Most importantly, when plotted on a modified form of the Kivelson, Lee and Zhang global phase diagram, the Landau level trajectories from this and previous studies in different material systems are in fact consistent with each other, showing that the extended states always float up in energy as B → 0, irrespective of whether there is a B=0 MIT or not. The phase diagram highlights the need to go to extremely low values of 1/ν (1/ν ≪ 0.1) in order to determine the ultimate fate of the extended states as B → 0.

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