Retrieving ideal precision in noisy quantum optical metrology

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Quantum metrology employs quantum effects to attain a measurement precision surpassing the limit achievable in classical physics. However, it was previously found that the precision returns the shot-noise limit (SNL) from the ideal Zeno limit (ZL) due to the photon loss in quantum metrology based on Mach-Zehnder interferometer. Here, we find that not only the SNL can be beaten, but also the ZL can be asymptotically recovered in long-encoding-time condition when the photon dissipation is exactly studied in its inherent non-Markovian manner. Our analysis reveals that it is due to the formation of a bound state of the photonic system and its dissipative noise. Highlighting the microscopic mechanism of the dissipative noise on the quantum optical metrology, our result supplies a guideline to realize the ultrasensitive measurement in practice by forming the bound state in the setting of reservoir engineering.

Introduction.— Pursuing high-precision measurement to physical quantities, metrology plays a significant role in advancing the innovation of science and technology. Restricted by the unavoidable errors, the metrology precision realized in classical physics is strongly bounded by the shot-noise limit (SNL) $N^{-1/2}$ with $N$ being the number of resource employed in the measurements. It was found that the SNL can be beaten by taking advantage of the quantum effects such as squeezing [1–3] and entanglement [4–6]. This inspires the newly emerged field, quantum metrology [7–9]. Many fascinating applications of quantum metrology have been proposed. The quantum effects of light can offer enhanced imaging resolution [10–12] in biological monitoring [13–15] and in optical lithography [16], and improved sensitivity in gravitational wave detection [17] and in radar [18]. The quantum characters of atoms or spins can provide an enhanced precision in sensing weak magnetic field [19–24] and ultimate accuracy for clocks [25–27].

A wide class of quantum metrology using quantized light as probe is generally based on the Mach-Zehnder interferometer (MZI). Caves first pointed out that the precision can beat the SNL and reach the Zeno limit (ZL) scaling as $N^{-3/4}$ with the mean photon number by using the squeezed light [1, 28]. Since then, a host of quantum states of light, such as N00N state [16], twin Fock state [29], two-mode squeezed state [30], and entangled coherent state [31, 32], have been found to perform well in quantum optical metrology. However, the decoherence caused by the unavoidable interactions of the optical probe with its environment degrades the real performance of quantum metrology [33–42], which hinders its practical application [43]. It was found that the NOON state losses its metrological advantage when even a single photon is absorbed by the environment [35]. Entangled coherent state can beat the SNL, but only when the photon loss is extremely small [31, 32]. This was proven to be true for the squeezed [33, 37] and definite-photon-number [35, 36] states. In these works, the decoherence of the optical probe was phenomenologically introduced by a transmissivity, which is equivalent to a continuous photon loss described by a master equation with constant loss rate under Born-Markovian approximation [35, 44, 45]. Given the inherent non-Markovian character of the decoherence dynamics [46–50], it is expected that such treatment is insufficient. It was really found in the Ramsey-spectroscopy-based quantum metrology that the non-Markovian effect can transiently make the metrology precision surpass the SNL in dephasing noises [51–53], which is in sharp contrast to the Markovian approximate result [54]. However, the precision gets worse and worse in long-encoding-time condition.

In this Letter, we reveal a mechanism to make the ideal precision of the MZI-based quantum metrology recovered when the exact non-Markovian dissipation dy-
namics of the optical probe is considered. Focusing on the long-encoding-time condition, which corresponds to the case with full loss of photon and the quantum superiority disappears completely in the previous phenomenological description [35, 44, 45], we show that the ideal ZL is asymptotically recovered when a coherent state in one input port and squeezed state in the other are employed. Our analysis reveals that such long-time recovery to the ideal precision is intrinsically determined by the formation of a bound state in the energy spectrum of the probe and its environment. On the one hand our result demonstrates that the phenomenological treatment overestimates the detrimental influences of the decoherence on quantum optical metrology, and on the other hand it supplies a guideline to realize the ultrasensitive measurement in practice by forming the bound state in the setting of reservoir engineering.

**Ideal quantum metrology.**—To estimate an unknown parameter of a system, one generally prepares a probe and couples it to the system to encode the parameter information. Then a series of measurement to certain observable is made to the probe. The value and the uncertainty of the parameter can be deduced from the measurement results [8]. Consider explicitly the estimation of a frequency parameter $\gamma$ of a system. We choose two modes of quantized optical fields with frequency $\omega_0$ as the probe. The encoding of $\gamma$ can be realized by the time evolution $\hat{U}_0(\gamma, t) = \exp(-i\hat{H}_0 t/\hbar)$ of the probe with

$$\hat{H}_0 = \hbar\omega_0 \sum_{m=1,2} \hat{a}_m^\dagger \hat{a}_m + \hbar\gamma \hat{a}_2^\dagger \hat{a}_2,$$

(1)

where the first term is the free Hamiltonian of the fields and the second one is the linear interaction of the second field with the system [31, 55, 56]. The evolution governed by $\hat{U}_0(\gamma, t)$ accumulates a phase difference $\gamma t$ between the two fields, which can be measured by the MZI. The MZI has two beam splitters $B_S, (i = 1, 2)$ separated by the phase shifter $\hat{U}_0(\gamma, t)$ and two detectors $D_i$ [see Fig. 1(a)] [57]. Its input-output relation reads

$$|\Psi_{\text{out}}\rangle = \hat{V}\hat{U}_0(\gamma, t)\hat{V}|\Psi_{\text{in}}\rangle,$$

(2)

where $\hat{V} = \exp[i\pi/4(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)]$ is the action of $B_S$ [30]. The detectors $D_i$ measure the photon difference $\hat{M} = \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2$ from $|\Psi_{\text{out}}\rangle$. Note although the measurement to $\hat{M}$ does not saturate the Cramér-Rao bound [58], it is sufficient to analyze the decoherence mechanism to such explicit protocol.

We consider the input state as $|\Psi_{\text{in}}\rangle = \hat{D}_0 \hat{S}_{\alpha}|0,0\rangle$, where $\hat{D}_0 = \exp(\alpha \hat{a}_1^\dagger - \alpha^* \hat{a})$ with $\alpha = |\alpha| e^{i\phi}$, $\hat{S}_\alpha = \exp\frac{1}{2}\xi(\hat{a}_1^2 - \hat{a}_1^2)$ with $\xi = r e^{i\phi}$, and $|0,0\rangle$ is the two-mode vacuum state. Its total photon number is $N = |\alpha|^2 + \sinh^2 r$, which contains the ratio $\beta \equiv \sinh^2 r/N$ from the squeezed mode and can be regarded as the quantum resource of the scheme. We can calculate from Eq. (2) that $\bar{M} = |\sinh^2 r - |\alpha|^2| \cos \gamma t$ and

$$\delta M = \{\cos^2 \gamma t[|\alpha|^2 + 2 \sinh^2 r \cosh^2 r] + \sin^2 \gamma t \times [\alpha \cosh r - \alpha^* \sinh r e^{i\phi}]^2 + \sinh^2 r]\}^{1/2},$$

(3)

where $\delta M = (\bar{N}^2 - \bar{N})^{1/2}$ and $\bar{N} = \langle \Psi_{\text{out}}|\hat{M}|\Psi_{\text{out}}\rangle$. They characterize the statistical distribution of the measurement results to $\bar{M}$. Then the best precision of estimating $\gamma$ can be evaluated by $\delta \gamma = \delta M / (\bar{M} \delta \bar{M})$ as

$$\min \delta \gamma = \frac{\left[(1 - \beta) e^{-2r} + \beta \frac{1}{2}\right]}{t\sqrt{N[1 - 2\beta]}},$$

(4)

which is called the ZL. It beats the SNL and manifests the significance of the squeezing character of light in the metrology scheme [1, 57]. Benefited from this squeezing-enhanced sensitivity, the squeezed state has been used in gravitational-wave observatory [17].

**Effects of dissipative noises.**—In reality, the decoherence caused by the unavoidable interaction of the probe with the environment would deteriorate the performance of the quantum metrology. Conventionally, the decoherence of the optical probe is analyzed in phenomenological manner by introducing a transmissivity [33–41]. This is equivalent to a continuous photon loss described by a master equation with constant loss rate under Born-Markovian approximation [35, 44, 45]. In recent years, people found that the interplay between the system and its environment caused by its inherent non-Markovian nature would induce diverse characters which cannot be captured by the Born-Markovian approximate treatment [46–50]. To uncover the constructive role play by the non-Markovian effect on quantum optical metrology, we here investigate the exact decoherence dynamics of the optical probe and evaluate its metrology scale, especially in the long-encoding-time condition.

Taking the dissipative noise into account, the Hamiltonian governing the parameter encoding reads

$$\hat{H} = \hat{H}_0 + \hbar \sum_k \omega_k [\hat{b}_k^\dagger \hat{b}_k + g_k (\hat{a}_2 \hat{b}_k^\dagger + \text{H.c.})],$$

(6)

where $\hat{b}_k$ is the annihilation operators of the $k$th environmental mode with frequency $\omega_k$ and $g_k$ is its coupling strength to the probe. The coupling is characterized by the spectral density $J(\omega) = \sum_k g_k^2 \delta(\omega - \omega_k)$. In the continuum limit, it reads $J(\omega) = \eta \omega |\omega/\omega_0|^{-s-1} e^{i\omega/\tilde{\omega}}$, where $\eta$ is a coupling constant, $\omega_0$ is a cutoff frequency, and the exponent $s$ classifies the noise into sub-Ohmic for $0 < s < 1$, Ohmic for $s = 1$, and super-Ohmic for $s > 1$ [59].
In the Heisenberg picture with \( \dot{\mathbf{b}}(t) = \hat{U}(\gamma, t) \mathbf{b}(t) \) and \( \hat{U}(\gamma, t) = \exp(-i\hat{H}t/\hbar) \), the equations of motion of the optical fields read \( \dot{a}_1(t) = i(\gamma + \omega_0)\dot{a}_2(t) - \int_0^t f(t - \tau)\dot{a}_2(\tau)d\tau \)

\[
\dot{a}_2(t) = -i(\gamma + \omega_0)\dot{a}_2(t) - \int_0^t f(t - \tau)\dot{a}_2(\tau)d\tau - \frac{i}{\hbar} \sum_k \dot{g}_k \dot{b}_k e^{-i\omega_k t},
\]

(7)

where \( f(t - \tau) = \int_0^\infty J(\omega) e^{-i\omega(t-\tau)} \) is the noise correlation function. The linearity of Eq. (7) implies that \( \dot{a}_2(t) \) can be expanded as \( \dot{a}_2(t) = c(t)\dot{a}_2 + \sum_k d_k(t)\dot{b}_k k \) [60]. One can check \( |c(t)|^2 + \sum_k |d_k(t)|^2 = 1 \) from \( |a_2(t), a_2'(t)| = 1 \). We obtain from Eq. (7) that \( c(t) \) obeys

\[
c(t) + i(\gamma + \omega_0)c(t) + \int_0^t f(t - \tau)c(\tau)d\tau = 0 \quad \text{(8)}
\]

under the initial conditions \( c(0) = 1 \). Containing all the backaction effect between the probe and the noise, the convolution in Eqs. (8) renders the dynamics non-Markovian. Assuming that the noise is initially in vacuum state \( |\Psi_E(0)\rangle = |\{0_k\}\rangle \) and repeating the same procedure as the ideal case, we can calculate

\[
\dot{M} = \text{Re} [e^{i\omega_0t}c(t)](\sinh^2 r - |\alpha|^2), \quad (9)
\]

\[
\delta M = \{\text{Im}[e^{i\omega_0t}c(t)]\}^2[|\alpha| \cos r - \alpha^* e^{i\phi} \sinh r]^2 + \sinh^2 r \quad \text{(9a)}
\]

\[
+ \{\text{Re}[e^{i\omega_0t}c(t)]\}^2[|\alpha|^2 + \sinh^2 r + \sinh^2 r] + \frac{1}{2} |c(t)|^2
\]

\[
\times (|\alpha|^2 + \sinh^2 r)^\frac{1}{2}
\]

(10)

which give the distribution of the measurement results to \( \dot{M} \) in the noisy situation. The precision \( \delta \gamma \) can be evaluated from Eqs. (9) and (10). It is analytically complicated. However, via analyzing the asymptotic behavior of Eq. (8), we can obtain its analytical form in the long-encoding-time limit.

When the coupling between the probe and the environment is weak and the typical time scale of \( f(t - \tau) \) is much smaller than the one of the field, we can apply the Markovian approximation to Eq. (8) and obtain \( c(t) = e^{-i[\kappa(\omega_0 + \gamma + \Delta)t]} \) with \( \kappa = 2\pi J(\omega_0 + \gamma) \) and \( \Delta = \frac{\chi}{2} \int_0^\infty \frac{d\omega}{\omega - \omega_0 - \gamma} \) [61]. We can calculate from Eqs. (9) and (10) that

\[
\min \delta \gamma \approx \left( \frac{e^{2\pi t} - 1}{2N^2} \right)^{1/2} \quad \text{when} \quad \beta = (2\sqrt{N})^{-1} \quad \text{and} \quad \varphi = 2\phi.
\]

Getting divergent with time, its minimum at \( t = \kappa^{-1} \) returns the SNL \( e\kappa(2N)^{-1/2} \). It shows that the quantum superiority of the scheme in the Markovian noise disappears completely. This is consistent to the noisy metrology result based on the Ramsey spectroscopy [54].

In the non-Markovian dynamics, Eq. (8) can be linearnized by Laplace transform \( \tilde{c}(s) = \Im \omega_0 + \gamma + \int_0^\infty \frac{J(\omega)}{\omega - \Im} d\omega = \Im \omega \), \( \omega = \Im s \).

It is interesting to note that the roots \( \omega \) multiplied by \( h \) is just the eigenenergy of the local system consisting of the probe and its environment in single-excitation subspace. To see this, we expand the eigenstate as \( \langle \Phi \rangle = \{x_k^1 + \sum_k y_k b_k^d|0_k, (0_k)\} \). From the stationary Schrödinger equation of the local system, we have \( [E - h(\omega_0 + \gamma)]x_k = \sum y_k b_k^d \) and \( y_k = h g_k x / (E - h\omega_k) \) with \( E \) being its eigenenergy, which readily lead to Eq. (11) with the replacement of \( \varphi \) by \( E/h \). It implies that, although the subspaces with more excitation number might be involved in the time evolution, the dissipative dynamics of the probe is essentially determined by the energy-spectrum character in the single-excitation subspace. Because \( y(\varphi) \) is a monotonically decreasing function with increasing \( \varphi \) in the regime \( \varphi < 0 \), Eq. (11) has one isolated root \( \varphi_0 \) in the regime \( \varphi < 0 \) provided \( y(0) < 0 \). It has infinite roots in the regime \( \varphi > 0 \), which form a continuous energy band. We call the discrete eigenstate with the eigenenergy \( h\varphi_0 \), falling out of the energy band bound state [47]. Its formation has profound influences on the dissipation dynamics of the probe. This can be seen by making the inverse Laplace transform

\[
c(t) = Ze^{-i\varphi t} + \int_{\varphi + i0}^{\varphi + i\infty} \frac{d\varphi}{2\pi} e^{(-i\varphi)c(\varphi)e^{-i\varphi t}}, \quad (12)
\]

where \( Z = [1 + \int_0^{\infty} \frac{J(\omega)}{\omega - \varphi} d\omega]^{-1} \) and the second term is contributed from the energy band. Oscillating with time in continuously changing frequencies, the second term behaves as a decay and tends to zero in the long-time condition due to out-of-phase interference. Therefore, if the bound state is absent, then \( \lim_{t \to \infty} c(t) = 0 \) characterizes a complete decoherence, while if the bound state is formed, then \( \lim_{t \to \infty} c(t) = Ze^{-i\varphi t} \) implies a dissipation suppression. For the Ohmic-type spectral density, it can be evaluated that the bound state is formed if \( \omega_0 + \gamma - \eta\omega_0 \Gamma(s) \leq 0 \). Here \( \Gamma(s) \) is the Euler’s \( \Gamma \) function. The dominate role of the bound state played in noncanonical thermalization [61] and quantum-correlation preservation [62] has been revealed.

We focus on the case in the presence of the bound state. Substituting the asymptotic solution \( Ze^{-i\varphi t} \) into \( \delta \gamma \) and using Eqs. (9) and (10), we obtain

\[
\min \delta \gamma |_{\beta = (2\sqrt{N})^{-1}} \approx \left( \frac{e^{2\pi t} - 1}{Z} \right)^{1/2} \quad \text{when} \quad t = \frac{2m + 1}{2\omega_0 - \omega_n} \quad \text{and} \quad \varphi = 2\phi.
\]

It reduces to the ZL (5) in the ideal case, where \( Z = 1 \) and \( \varphi_0 = \omega_0 + \gamma \). It is remarkable to see from Eq. (13) that, even in the long-encoding-time condition, \( \delta \gamma \) can still asymptotically tend to the ideal ZL with \( Z \) approaching 1, which can be controlled by manipulating the spectral density \( J(\omega) \). It is dramatically different from the phenomenological [33–41] and the Markovian approximate one [35, 44, 45], where \( \delta \gamma \) gets worse and worse in the long-encoding-time limit.
Indicating the significant role of the non-Markovian effect and the energy-spectrum character of the local system in the noisy mechanism of quantum metrology, our result supplies a guideline to retrieve the ideal precision in the noise case by engineering the formation of the bound state.

**Numerical results.**—To verify the distinguished role played by the formed bound state in the dissipation dynamics of the probe, we plot in Fig. 1(b) the long-time behavior of the $|c(t)|$ by numerically solving Eq. (8) for the Ohmic spectral density. We can obtain from our analysis that the bound state is formed when $\omega_\eta + \gamma - \eta \omega_c \leq 0$ for $s = 1$. We really observe in Fig. 1(b) an abrupt change of $|c(\infty)|$ from zero to finite values exactly coinciding with $Z$ in Eq. (12) with increasing $\omega_c$. Figure 1(c) reveals that the regime where $|c(\infty)|$ takes finite values well matches with the one where a bound state is formed in the energy spectrum of the local system. It is physically understandable from the fact that the bound state, as a stationary state of the local system, would preserve the quantum coherence in its superposed components during time evolution. It is also interesting to see that quite a large value of $|c(\infty)| = Z$ approaching unity can be achieved with increasing $\omega_c$. It readily implies from Eq. (13) that the ZL is asymptotically retrievable.

With the numerical result of Eq. (8) at hands, we can calculate the exact precision $\delta \gamma(t)$ from Eqs. (9) and (10). Figure 2(a) shows the evolution of $\delta \gamma(t)$ in different $\omega_c$. Oscillating with time, $\delta \gamma(t)$ takes its best values at the local minima marked by the blue blocks. It can be seen that the local minima become larger and larger with time when the bound state is absent, which is consistent with the Markovian result. It means that the noisy metrology scheme performs worse and worse with increasing the encoding time in this situation. However, as long as the bound state is formed, the profile of the local minima gets to be a decreasing function with time. Thus, the bound state makes the superiority of the encoding time as a resource in the ideal metrology case recovered. The red dashed line in Fig. 2(a) gives $\min(\delta \gamma)$ evaluated from Eq. (13), which matches well the local minima of $\delta \gamma(t)$ in the long-time condition. This verifies the validity of our result (13). Focusing on the case in the presence of the bound state, we plot in Fig. 2(b) $\min(\delta \gamma)$ in different $\omega_c$. It reveals that, with the formation of the bound state, not only the SNL can be surpassed, but also the ideal ZL can even be asymptotically retrieved. This can be further verified by $\min(\delta \gamma)$ as function of total photon number [see Fig. 2(c)]. Once again, it verifies the validity of the scaling (13). Furthermore, with the increasing of $Z$ accompanying the increasing of $\omega_c$, the precision gets nearer and nearer the ZL. All the results confirm our expectation that the precision asymptotically matching the analytical scaling (13) approaches the ZL with the formation of the bound state. Besides tuning $\omega_c$, the bound-state-favored retrieving of the ZL can also be obtained by tuning $\eta$. Figure 3 gives the evolution and scaling on $N$ of $\min\delta \gamma$ in different $\eta$. It confirms again our conclusion that the ZL is asymptotically recoverable when the bound state is formed.

Note that our finding on the bound-state-favored retrieving of the ideal precision in noisy situation is readily applicable to the optimal measurement protocol [58], where the Heisenberg-limit recovery is expected. Further, although only the Ohmic spectral density is considered, our result can be generalized to other spectral densities. With the rapid development of reservoir engineering tech-
nique, many structured environments with controllable spectral densities have been realized [63, 64]. The bound state has been observed in photonic band-gapped environment in circuit QED system [65]. The Ohmic spectral density is possible to be controlled in trapped ion system [66]. All these systems supply ideal experimental platform to verify the mechanism revealed in our work.

Conclusions.—In summary, we have microscopically studied the non-Markovian noise effect on the MZI-based quantum metrology scheme. An exact scaling relation of the precision to the photon number is derived in the long-encoding-time condition. It is remarkable to find that the non-Markovian effect and the formation of a bound state between the quantum probe and its environment are two essential reasons for retrieving the ZL: The bound state supplies the intrinsic ability and the non-Markovian effect supplies the dynamical way. Our result suggests a guideline to experimentation to implement the ultrasensitive measurement in the practical noise situation by engineering the formation of the bound state.

Acknowledgments.—The work is supported by the National Natural Science Foundation (Grant Nos. 11875150, 11474139, 11674139, and 11834005) and by the Fundamental Research Funds for the Central Universities of China.

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1. C. M. Caves, Phys. Rev. D 23, 1693 (1981).
2. J. Ma, X. Wang, C. Sun, and F. Nori, Physics Reports 509, 89 (2011).
3. N. J. Engelsen, R. Krishnakumar, O. Hosten, and M. A. Kasevich, Phys. Rev. Lett. 118, 140401 (2017).
4. T. Nagata, R. Okamoto, J. L. O’Brien, K. Sasaki, and S. Takeuchi, Science 316, 726 (2007).
5. Y. Israel, S. Rosen, and Y. Silberberg, Phys. Rev. Lett. 112, 103604 (2014).
6. X.-Y. Luo, Y.-Q. Zou, L.-N. Wu, Q. Liu, M.-F. Han, M. K. Tey, and L. You, Science 355, 620 (2017).
7. V. Giovannetti, S. Lloyd, and L. Maccone, Science 306, 1330 (2004).
8. V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. Lett. 96, 010401 (2006).
9. L. Pezzè, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, Rev. Mod. Phys. 89, 043832 (2013).
10. L. A. Lugiato, A. Gatti, and E. Brambilla, J. Opt. B 4, S176 (2002).
11. M. Tsang, R. Nair, and X.-M. Lu, Phys. Rev. X 6, 031033 (2016).
12. C. Lupo and S. Pirandola, Phys. Rev. Lett. 117, 190802 (2016).
13. A. Crespi, M. Lobino, J. C. F. Matthews, A. Politi, C. R. Neal, R. Ramponi, R. Osellame, and J. L. O’Brien, Appl. Phys. Lett. 100, 233704 (2012).
14. M. A. Taylor, J. Janousek, V. Daria, J. Knittel, B. Hage, H.-A. Bachor, and W. P. Bowen, Phys. Rev. X 4, 011017 (2014).
15. M. A. Taylor and W. P. Bowen, Physics Reports 615, 1 (2016).
16. A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. 85, 2733 (2000).
17. H. Grote, K. Danzmann, K. L. Dooley, R. Schnabel, J. Schluter, and H. Vahlbruch, Phys. Rev. Lett. 110, 181101 (2013).
18. S. Barzanjeh, S. Guha, C. Weedbrook, D. Vitali, J. H. Shapiro, and S. Pirandola, Phys. Rev. Lett. 114, 080503 (2015).
19. J. A. Jones, S. D. Karlen, J. Fitzsimons, A. Ardavan, S. C. Benjamin, G. A. D. Briggs, and J. J. L. Morton, Science 324, 1166 (2009).
20. G. Arrad, Y. Vinkler, D. Aharonov, and A. Retzker, Phys. Rev. Lett. 112, 150801 (2014).
21. W. Muessel, H. Strobel, D. Linnemann, D. B. Hume, and M. K. Oberthaler, Phys. Rev. Lett. 113, 103004 (2014).
22. C. Bonato, M. S. Blok, H. T. Dinani, D. W. Berry, M. L. Markham, D. J. Twitchen, and R. Hanson, Nature Nanotechnology 11, 247 (2015).
23. C. L. Degen, F. Reinhard, and P. Cappellaro, Rev. Mod. Phys. 89, 035002 (2017).
24. T. Chalopin, C. Bouazza, A. Evrard, V. Makhulov, D. Dreon, J. Dalibard, L. A. Sidorenkov, and S. Nascimbene, Nature Communications 9, 4955 (2018).
25. P. Kómir, E. M. Kessler, M. Bishof, L. Jiang, A. S. Srensen, J. Ye, and M. D. Lukin, Nature Physics 10, 582 (2014).
26. I. Kruse, K. Lange, J. Peise, B. Liécke, L. Pezzé, J. Arlt, W. Ertmer, C. Lisdat, L. Santos, A. Smerzi, and C. Klempt, Phys. Rev. Lett. 117, 143004 (2016).
27. O. Hosten, N. J. Engelsen, R. Krishnakumar, and M. A. Kasevich, Nature 529, 505 (2016).
28. It was proven that the optimal limit using squeezed state is the Heisenberg limit $N^{-1}$ with a full knowledge on the fluctuation of the photon number in the two output ports when the average photon number from the squeezed state is equal to the one of the coherent light [58].
29. M. J. Holland and K. Burnett, Phys. Rev. Lett. 71, 1355 (1993).
30. P. M. Anisimov, G. M. Raterman, A. Chiruvelli, W. N. Plick, S. D. Huver, H. Lee, and J. P. Dowling, Phys. Rev. Lett. 104, 103602 (2010).
31. J. Joo, W. J. Munro, and T. P. Spiller, Phys. Rev. Lett. 107, 083601 (2011).
32. Y. M. Zhang, X. W. Li, W. Yang, and G. R. Jin, Phys. Rev. A 88, 043832 (2013).
33. T. Ono and H. F. Hofmann, Phys. Rev. A 81, 033819 (2010).
34. P. A. Knott, T. J. Proctor, K. Nemoto, J. A. Dunningham, and W. J. Munro, Phys. Rev. A 90, 033846 (2014).
35. U. Dorner, R. Demkowicz-Dobrzenski, B. J. Smith, J. S. Lundeen, W. Wasilewski, K. Banaszek, and I. A. Walmsley, Phys. Rev. Lett. 102, 040403 (2009).
36. R. Demkowicz-Dobrzenski, U. Dorner, B. J. Smith, J. S. Lundeen, W. Wasilewski, K. Banaszek, and I. A. Walmsley, Phys. Rev. A 80, 013825 (2009).
37. Z. Huang, K. R. Motes, P. M. Anisimov, J. P. Dowling, and D. W. Berry, Phys. Rev. A 95, 053837 (2017).
38. S. D. Huver, C. F. Wildfeuer, and J. P. Dowling, Phys. Rev. A 78, 063828 (2008).
39. G. Gilbert, M. Hamrick, and Y. S. Weinstein, J. Opt. Soc. Am. B 25, 1336 (2008).
[40] M. A. Rubin and S. Kaushik, Phys. Rev. A 75, 053805 (2007).
[41] F. Hudelist, J. Kong, C. Liu, J. Jing, Z. Y. Ou, and W. Zhang, Nature Communications 5, 3049 (2014).
[42] Y.-S. Wang, C. Chen, and J.-H. An, New Journal of Physics 19, 113019 (2017).
[43] K. Banaszek, R. Demkowicz-Dobrzański, and I. A. Walmsley, Nat Photon 3, 673 (2009).
[44] J. J. Cooper, D. W. Hallwood, J. A. Dunningham, and J. Brand, Phys. Rev. Lett. 108, 130402 (2012).
[45] X.-M. Lu, S. Yu, and C. H. Oh, Nature Communications 6, 7282 (2015).
[46] J.-H. An and W.-M. Zhang, Phys. Rev. A 76, 042127 (2007).
[47] Q.-J. Tong, J.-H. An, H.-G. Luo, and C. H. Oh, Phys. Rev. A 81, 052330 (2010).
[48] W.-M. Zhang, P.-Y. Lo, H.-N. Xiong, M. W.-Y. Tu, and F. Nori, Phys. Rev. Lett. 109, 170402 (2012).
[49] H.-J. Zhu, G.-F. Zhang, L. Zhuang, and W.-M. Liu, Phys. Rev. Lett. 121, 220403 (2018).
[50] A. Strathearn, P. Kirton, D. Kilda, J. Keeling, and B. W. Lovett, Nature Communications 9, 3322 (2018).
[51] Y. Matsuzaki, S. C. Benjamin, and J. Fitzsimons, Phys. Rev. A 84, 012103 (2011).
[52] A. W. Chin, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. 109, 233601 (2012).
[53] K. Macieszczak, Phys. Rev. A 92, 010102 (2015).
[54] S. F. Huelga, C. Macchiavello, T. Pellizzari, A. K. Ekert, M. B. Plenio, and J. I. Cirac, Phys. Rev. Lett. 79, 3865 (1997).
[55] S. Boixo, A. Datta, M. J. Davis, S. T. Flammia, A. Shaji, and C. M. Caves, Phys. Rev. Lett. 101, 040403 (2008).
[56] S. Boixo, A. Datta, S. T. Flammia, A. Shaji, E. Bagan, and C. M. Caves, Phys. Rev. A 77, 012317 (2008).
[57] R. Demkowicz-Dobrzański, M. Jarzyna, and J. Kołodyński, Progress in Optics 60, 345 (2015).
[58] L. Pezzé and A. Smerzi, Phys. Rev. Lett. 100, 073601 (2008).
[59] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. 59, 1 (1987).
[60] K. W. Chang and C. K. Law, Phys. Rev. A 81, 052105 (2010).
[61] C.-J. Yang, J.-H. An, H.-G. Luo, Y. Li, and C. H. Oh, Phys. Rev. E 90, 052122 (2014).
[62] Y.-Q. Liu, J.-H. An, X.-M. Chen, H.-G. Luo, and C. H. Oh, Phys. Rev. A 88, 012129 (2013).
[63] C. J. Myatt, B. E. King, Q. A. Turchette, C. A. Sackett, D. Kielpinski, W. M. Itano, C. Monroe, and D. J. Wineland, Nature 403, 269 (2000).
[64] D. Kienzler, H.-Y. Lo, B. Keitch, L. de Clercq, F. Leupold, F. Lindenfelser, M. Marinelli, V. Negnevitsky, and J. P. Home, Science 347, 53 (2015).
[65] Y. Liu and A. A. Houck, Nature Physics 13, 48 (2016).
[66] D. Porras, F. Marquardt, J. von Delft, and J. I. Cirac, Phys. Rev. A 78, 010101 (2008).