All-glass vacuum tube heat transfer simplified model

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Abstract. This paper addresses the design of an algorithm that estimates the irradiance and the selective layer temperature of the all-glass vacuum tube as a function of the ambient temperature in Bucaramanga, Northern Colombia. A Matlab code to meteorological National Aeronautics and Space Administration data processing and to find a linear relationship between the irradiance and ambient temperature on the location of study, was developed. The behavior of this relationship for several hours of the day was studied, and it was found that it is possible to use this relationship as a reliable preliminary approach in water heating systems design using all-glass vacuum tube. This facilitates the design calculations and would encourage the use of solar energy in water heating systems in Colombia.

1. Introduction
The Sun is the closest star to Earth, emits energy in the form of radiation due to the processes of fusion of hydrogen inside. The emitted solar radiation is practically the only source of energy that the planet receives, so in the first place, it is important to be able to measure or quantify it accurately. There are two ways to estimate solar radiation: theoretically, using mathematical models, and experimentally, using instruments that measure the amount of radiation. Direct measurements of solar radiation are a reliable source of information since they deliver data in real-time. However, measuring instruments have the disadvantage of not being available to most researchers due to their high cost, so it is sometimes convenient to estimate radiation data using mathematical models based on easier-to-measure input parameters, such as temperature, relative humidity, cloudiness, rainfall and hours of solar brightness. This article proposes the construction of a mathematical model that quantifies the irradiance and selective layer temperature on all-glass vacuum tube according to the ambient temperature of the place in which it is desired to know the value of these parameters.

Several investigations have developed models that allow quantifying solar radiation based meteorological data; [1] establishes a relationship between the intensity of solar radiation and the number of light hours, testing in several villages in Senegal, and extending to West Africa by extrapolation. In [2], the authors propose a direct relationship between solar radiation, the number of light hours and the relative ambient humidity, obtained by statistical methods. Other authors suggest that the fraction between direct solar radiation and global solar radiation is a function of the atmospheric clearness index, proposing analytical expressions whose parameters are obtained by adjusting least squares to the set of experimental data [3]. Some authors suggest that solar radiation can be estimated from the difference in maximum and minimum ambient temperature using a simple equation [4], using a linear regression model [5] or using ambient temperature and relative humidity data [6].

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In Colombia - due to its geographical position on the planet, within the torrid zone and in the Andean region - there are varied and special climatic conditions that allow the availability of the solar resource, which represents an opportunity for renewable energy for sustainable development. Having said that, this paper proposes a linear regression model to quantitatively estimate the incident heat flux, and consequently, the temperature in an all-glass vacuum tube based on the ambient temperature of the study place. On the design of water heating systems using all-glass vacuum tube, is important to know the glass temperature, because based on this parameter, is possible to calculate the temperature into the storage tank. For this reason, it is convenient to have a simple model to estimate the glass temperature in function of ambient temperature. This facilitates the design calculations and would encourage the use of solar energy in water heating systems in Colombia.

2. Theoretical framework

The geometric relationships between a plane of any particular orientation relative to the Earth at any time and the incoming beam solar radiation, can be described in terms of several angles [7]. Direction of beam radiation \( \theta \) is function of these angles, that is, \( \theta = f(\delta, \phi, \beta, \gamma, \gamma_s, \omega) \). Equation (1) shows how to calculate \( \theta \).

\[
\cos(\theta) = \cos(\theta_z) \cos(\beta) + \sin(\theta_z) \sin(\beta) \cos(\gamma_s - \gamma)
\]  

(1)

Where \( \theta_z \) is the zenith angle, \( \delta \) is the solar declination angle estimated by Cooper equation [8], \( \phi \) is the latitude, \( \beta \) is the angle between the tube and the horizontal, \( \gamma \) is the surface azimuth angle, \( \omega \) is the hour angle, and \( \gamma_s \) is the solar azimuth angle.

2.1. Natural convection

The present study considered a natural convective flow around a cylindrical tube. The surface temperature of external tube cylindrical is \( T_c \) and the selective layer temperature is \( T_s \). For calculations of heat transfer it is necessary to consider the steady condition, the tube and its cover are isothermal and the air is an ideal gas. \( T_c \) and \( T_s \) represent temperatures in steady operating conditions. The natural convection heat transfer performance on the tubes can be quantified using the heat transfer coefficient \( h \) [9] which is given as a function of fluid properties, temperatures and geometry, that is, \( h = f(g, B, v, T_a, T_c, T_s, r, L_c) \), where \( g \) is gravitational constant, \( B \) is the volumetric expansion coefficient, \( v \) is kinematic viscosity, \( T_a \) is ambient temperature and \( L_c \) is the characteristic length of the geometry. First, it is necessary to calculate The Rayleigh number \( R_a \), which is dimensionless. Equation (2) shows how to calculate \( R_a \).

\[
R_a = \frac{gB(T_s - T_a)(L_c)^3}{v^2 Pr \cdot F}
\]  

(2)

Where \( Pr \) is the Prandtl number evaluated at film temperature \( T_f = (T_c + T_a)/2 \) and \( F \) is a correction factor for tilted tubes. Then it is necessary to calculate the Nusselt number \( Nu \), which can by expressed in function of Prandtl number. Equation (3) shows how to calculate Nusselt number.

\[
Nu = \frac{hL_c}{k} = CR_a^n
\]  

(3)

Where \( R_a \) is the Rayleigh number, \( k \) is the air thermal conductivity evaluated at film temperature, \( C \) and \( n \) are constants which depend on the geometry of the surface and the flow regime. Finally, once the heat transfer convection coefficient is known, the rate of heat transfer by natural convection, \( \dot{Q}_{conv} \), can by estimate using the Equation (4).

\[
\dot{Q}_{conv} = hA_s(T_c - T_a)
\]  

(4)
Where $A_s$ is the heat transfer surface.

2.2. Radiation heat loss
The radiation heat loss $Q_{sc}$ on the selective layer in all-glass vacuum tube can by estimated according [10]. Equation (5) shows how to calculate $Q_{sc}$.

$$Q_{sc} = \frac{A_s \sigma (T_s^4 - T_c^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left(\frac{r_1}{r_2}\right)^2}$$  \hspace{1cm} (5)

Where $\sigma = 5.67 \times 10^{-8}$ [W/m²·K⁴], is the Stefan-Boltzmann constant, $\varepsilon_1$ and $\varepsilon_2$ are the emissivity on surface 1 and 2, and $r_1$ and $r_2$ are the radius of inner vacuum tube and outer vacuum tube respectively.

3. Data and methods
The location using for this study is (7°07′07″ N - 73°06′58″ W). Based on date provided by National Aeronautics and Space Administration (NASA) prediction of worldwide energy resources, the daily temperature data (maximum and minimum) and insolation clearness index data $k_T$ are obtained for 2018. The relationship between daily solar radiation $H$ and instantaneous insolation $I$ is estimate according the method as described by [3]. The hourly ambient temperature is estimated from the daily temperature using the method $\sin(14R-1)$ described by [11]. The high transmissivity evacuated borosilicate glass tube studied by [12] was taken as a reference. The properties of the evacuated glass tube are shown in Table 1. Because the heat transfer from the simple hot tube to the surroundings depends on the ambient temperature $T_a$, a theoretical calculation of the solar radiation $\dot{Q}$ through twin-glass tube is undertaken to simulate the effect of $T_a$ in transferred energy between the pipe and the surroundings, thus, firstly it is necessary to obtain the irradiance $\dot{q}$ which is a global and diffuse insolation function, that is, $\dot{q} = f(I,I_d)$ and is calculated according [3].

| Item | Parameter | Value / Ref | Unit |
|------|-----------|-------------|------|
| 1    | Material  | Borosilicate glass 3.3 | N/A |
| 2    | Coefficient of thermal expansion | $3.3 \times 10^{-6}$ | mm/°C |
| 3    | Outer vacuum tube diameter | 58 | mm |
| 4    | Inner vacuum tube diameter | 47 | mm |
| 5    | Length | 1800 | mm |
| 6    | Thickness | 1.8 | mm |
| 7    | Absorptivity | 0.925 | N/A |
| 8    | Emissivity | 0.900 | N/A |
| 9    | Transmissivity | 0.890 | N/A |

4. Result and discussion
Based on [6] is assumed a lineal model which is calculated by a linear regression, that is, $\dot{Q} = \beta_0 + \beta_1 T_a$. Using the least squares method described by [13], the value of $\beta_0$ and $\beta_1$ are calculated. The least squares fit to the heat incident data for $\omega = 0$ (12:00 at noon) and $\beta = 15°$ is shown in Equation (6).

$$\dot{Q} = 2.036 T_a + 9.186$$  \hspace{1cm} (6)

Ambient temperature, solar radiation and the scatter plot with the fitted line of ambient temperature versus solar incident heat is shown in Figure 1.
Figure 1 shows the fitted line of ambient temperature versus solar radiation. It is clear from Figure 1 that the solar radiation increases with the temperature, which is a logical result since these variables are closely related. A range of ambient temperature are established by 15 °C – 30 °C, therefore the heat incident on vacuum tube $\dot{Q}$ (W) is known for each ambient temperature. The problem involves heat transfer from the selective layer to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. This energy balance is shown in Equation (7).

$$\dot{Q}_{\text{layer-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q} = hA_s(T_c - T_a)$$  \hspace{1cm} (7)

So, it is necessary to find the glass temperature $T_c$ and selective layer temperature $T_s$ when steady operating conditions are established. To determine the heat transfer convection coefficient $h$ it is necessary to know $T_c$, which is not available. Therefore, the solution requires a trial-and-error approach. Once $T_c$ is found, $T_s$ is calculated using Equation (5). A Matlab code to calculate these temperatures is implemented for a range of $T_a$ established. Results are shown in Figure 2.

Figure 2 shows the variation of the selective layer temperature and the incident heat flux as a function of the ambient temperature, for different hours of the day. It is clear from Figure 2 for both lines that the cut of the fitted line $\hat{\beta}_0$ with the vertical axis, increases as the hour angle $\omega$ is near noon,
and decreases after that, which corroborates the fact that maximum solar radiation occurs at noon. The selective layer temperature fitted models obtained to several hour of day is shown in Table 2.

![Figure 2](image)

**Figure 2.** (a) $T_s$ and $\dot{Q}$ vs $T_a$ at 9 am, (b) 10 am, (c) 11 am, (d) 12 noon, (e) 1 pm, (f) 2 pm.

**Table 2.** Selective layer temperature fitted models.

| Item | hour | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $T_s = f(T_a)$ model [°C] |
|------|------|-----------------|-----------------|----------------------------|
| 1    | 8:00 | 55.840          | 1.173           | $T_s = 55.840 + 1.173T_a$  |
| 2    | 9:00 | 54.980          | 1.377           | $T_s = 54.980 + 1.377T_a$  |
| 3    | 10:00| 54.590          | 1.501           | $T_s = 54.590 + 1.501T_a$  |
| 4    | 11:00| 55.600          | 1.511           | $T_s = 55.600 + 1.511T_a$  |
| 5    | 12:00| 56.720          | 1.469           | $T_s = 56.720 + 1.469T_a$  |
| 6    | 13:00| 57.480          | 1.394           | $T_s = 57.480 + 1.394T_a$  |
| 7    | 14:00| 57.520          | 1.309           | $T_s = 57.520 + 1.309T_a$  |
| 8    | 15:00| 56.850          | 1.221           | $T_s = 56.850 + 1.221T_a$  |
| 9    | 16:00| 55.480          | 1.140           | $T_s = 55.480 + 1.140T_a$  |
5. Conclusions
The all-glass vacuum tube heat transfer simplified model is established in this paper. The results suggest a linear relationship between incident radiant heat flux and ambient temperature. According to results above, the linear regression model can be used to estimate the incident heat on vacuum tube in the location of study. A linear relationship of the form $Q = f(T_a) = \hat{\beta}_0 + \hat{\beta}_1 T_a$ could be determined. For this location, $Q = 9.186 + 2.036 T_a$ at noon. In the same way, the results suggest a linear relationship between selective layer temperature and ambient temperature. A linear relationship of the form $T_s = f(T_a) = \hat{\beta}_0 + \hat{\beta}_1 T_a$ for several hour of day, could be determined, for example, the linear relationship between selective layer temperature and ambient temperature was $T_s = 56.720 + 1.469 T_a$ at noon.

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