Accounting for running $\alpha_s$ for the non-singlet components of the structure functions $F_1$ and $g_1$ at small $x$.

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Infrared evolution equations incorporating the running QCD coupling are constructed and solved for the non-singlet structure function $f_{NS}$. Accounting for dropped logs of $x$ in DGLAP it leads to a scaling-like small $x$ behaviour:

$$f_{NS} \sim \left(\frac{\sqrt{Q^2}}{x}\right)^a.$$ 

In contrast to the leading logarithmic approximation, intercepts $a$ are numbers and do not contain $\alpha_s$. It is also shown that the leading logarithmic approximation may be unreliable for predicting $Q^2$-dependence of the DIS structure functions in the HERA range.

Non-singlet structure functions, i.e. flavour-dependent contributions to the deep inelastic structure functions, have been the object of intensive theoretical investigation. First, they are interesting because they are experimentally measurable quantities; second, they are comparatively technically simple for analysis, and can be regarded as a starting ground for a theoretical description of DIS structure functions. In the present talk we discuss the explicit expressions for the non-singlet contribution $f_{NS}^+$ to the structure function $F_1$ and for the non-singlet contribution $f_{NS}^-$ to the spin structure function $g_1$ at $x$. These expressions account for both leading (double-logarithmic) and sub-leading (single-logarithmic) contributions to all orders in QCD coupling and include the running $\alpha_s$ effects. Contrary to DGLAP and to some other works on small $x$, we do not use a priori the standard parametrisation $\alpha_s = \alpha_s(Q^2)$ in our evolution eqs. Indeed it has been shown recently that such a dependence is a good approximation at large $x$ but is not correct when $x$ is small.

As we account for double-logarithmic (DL) and single-logarithmic (SL) contributions to all orders and regardless of the arguments, we cannot use the DGLAP eqs. Instead, we construct and solve two-dimensional infrared evolution equations (IREE) for $f_{NS}$ appreciating evolution with respect to $x$ and to $Q^2$. In the context of this method, $f_{NS}^\pm$ evolves with respect to the infrared cut-off $\mu$ in the transverse momentum space: $k_{t\perp} > \mu$ for all virtual particles. In doing so, we provide $f_{NS}^\pm(x,Q^2)$ with $\mu$ dependence too. However,
it’s unavoidable when $\alpha_s$ is running because the standard expression

$$\alpha_s(t) = \frac{1}{b \ln(t/\Lambda_{QCD}^2)},$$ (1)

is valid only when $t \gg \Lambda_{QCD}^2$ and therefore if we introduce the infrared cut-off as

$$k_{i\perp} > \mu > m_{\text{max}} \gg \Lambda_{QCD},$$ (2)

with $m_{\text{max}}$ being the mass of the heaviest involved quark, we can neglect quark masses and still do not have infrared singularities. Besides the restrictions imposed by Eq. (2) $\mu$ is not fixed, so $f_{NS}$ can evolve with respect to $\mu$, eventually arriving at the following expressions for the non-singlet structure functions:

$$f_{NS}^\pm = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} C \left( \frac{1}{x} \right) \omega \exp \left[ \left( 1 + \lambda \omega \right) F_0^\pm \right] y$$ (3)

where $C$ is an (non-perturbative) input and $F_0^\pm$ are the new anomalous dimensions. They account for the total resummation of the most essential at small $x$ NLO contributions of the type $(\alpha_s/\omega^2)^n$ and $(\alpha_s/\omega)^n$ $(n = 1, ...,)$,

$$F_0^\pm = 2 \left[ \omega - \sqrt{\omega^2 - (1 + \lambda \omega)(A(\omega) + \omega D^\pm(\omega))}/2\pi^2 \right]$$ (4)

where

$$A(\omega) = \frac{4C_F \pi}{b} \left[ \frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty \frac{d\rho \exp(-\rho \omega)}{(\rho + \eta)^2 + \pi^2} \right].$$ (5)

and

$$D^\pm(\omega) = \frac{2C_F}{\omega b^2 N} \int_0^\infty d\rho \exp(-\rho \omega) \ln \left( \frac{\rho + \eta}{\eta} \right) \left[ \frac{\rho + \eta}{(\rho + \eta)^2 + \pi^2} \mp \frac{1}{\rho + \eta} \right].$$ (6)

We have used in Eqs. (3,4,5) the following notations: $\eta = \ln(\mu^2/\Lambda_{QCD}^2)$, $\rho = \ln(s/\mu^2)$, $\lambda = 1/2$ and the first coefficient of the $\beta$-function $b = (11N - 2n_f)/12\pi$. $A$ corresponds to accounting for running $\alpha_s$. $\pi^2$ in denominators appears due to analytical properties of $\alpha_s(t)$: it must have a non-zero imaginary part when $t$ is time-like. $D$ contains the signature-dependent contributions.

Expanding the resummed anomalous dimension $F_0^\pm$ into series in $1/\omega$ we reproduce the singular in $\omega$ terms of LO and NLO DGLAP- anomalous dimensions where $\alpha_s(Q^2)C_F/2\pi$ is replaced by $A$.

It is shown in Refs. 3 that $A$ can be approximated by $\alpha_s(Q^2)C_F/2\pi$ only at large $x$. Concerning the small- $x$ and large $Q^2$ asymptotics of $f_{NS}^\pm$, Eq. (4) reads that

$$f_{NS}^\pm \sim x^{-\omega_0^\pm} (Q^2/\mu^2)^{\omega_0^\pm}/2,$$ (7)

and

$$A = ...$$
with the intercepts $\omega_0^{\pm}$ being the leading, i.e. the rightmost, singularities of $F_0^{\pm}$. Eqs. (3,4) read that $\omega_0^{\pm}$ are the rightmost roots of

$$\omega^2 - (1 + \lambda \omega)(A(\omega) + \omega D^{\pm}(\omega))/2\pi^2 = 0.$$  

(8)

Eq. (8) contains $n_f, \Lambda_{QCD}$ and $\mu$ as parameters. Choosing e.g. $n_f = 3$ and $\Lambda_{QCD} = 0.1$ GeV one can solve Eq. (8) numerically and obtain $\omega_0^{\pm}$ as a function of $\mu$. The solutions are given in Fig. 1. Both $\omega_0^{+}$ and $\omega_0^{-}$ acquire imaginary parts at $\mu < 0.4$ GeV. As besides, for applicability of Eq. (1), $\mu$ must be much greater than $\Lambda_{QCD}$, we think that the region $\mu < 0.4$ GeV is beyond control of our approach. Both $\omega_0^{+}$ and $\omega_0^{-}$ are maximal at $\mu \approx 1$ Gev and slowly decrease with $\mu$ increasing. Therefore we can estimate values of the intercepts as

$$\Omega_0^{+} = 0.37, \quad \Omega_0^{-} = 0.4.$$  

(9)

It is interesting that this result was independently confirmed recently by extrapolating of fits for $f_3$ into small $x$ region. Eq. (8) was obtained from Eq. (4) which contains $\pi^2$ -terms. Basically, they are beyond of control of logarithmic accuracy and might be dropped. With $\pi^2$ -terms dropped, we obtain the smooth curves for $\omega_0^{\pm}$ depicted in Fig. 1. These curves show that $\pi^2$ -terms can be easily neglected for the values of $\mu$ greater than $\mu_0 = 5.5$ GeV. However, $\mu_0^2 = 30$ Gev$^2$ corresponds to the HERA $Q^2$ range. Then Eq. (8) immediately implies that, with such a big $\mu_0$, the logarithmic accuracy is not enough to obtain a correct $Q^2$ dependence in the HERA range. On the other hand, it also explains why DLA estimates $\alpha_s = \alpha_s(Q^2)$ may be correct for predicting the $x$ dependence: indeed, in DLA where the coupling is fixed, one should use rather $\alpha_s = \alpha_s(\mu_0^2)$ than $\alpha_s = \alpha_s(Q^2)$ as taken from DGLAP, but as it happens that $\mu_0^2 = Q^2$ in the HERA range, both estimates coincide.

Acknowledgments

This work supported in part by grants INTAS-97-30494, RFBR 00-15-96610 and the EU QCDNET contract FMRX-CT98-0194.

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Figure 1: Dependence of the intercept $\omega_0$ on infrared cutoff $\mu$ at $\Lambda_{QCD} = 0.1$ GeV: 1– for $f_1^{NS}$; 2– for $g_1^{NS}$; 3– and 4– for $f_1^{NS}$ and $g_1^{NS}$ respectively without account of $\pi^2$-terms.

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