Study in Network Stability based on MC

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Abstract. Monte Carlo is a convenient method for statistics sampling theory of mathematical and physical. It is widely applied to the field of staff performance in geological, electric, medical, optical, water conservancy and so on. In order to effectively assess the stability and reliability of the network, this paper put forward an improved Monte Carlo evaluation method on the basis of the traditional Monte Carlo method. First of all, the paper elaborated the principle of improved Monte Carlo method and the specific method of how to calculate the stability of the network. The improved Monte Carlo method employed the recursive variance reduction method to get a smaller sample in the original sample, to ensure the estimate is unbiased. Secondly, the paper applied the ideology of permutation and combination to the network nodes and links in the model, and combined with the upper and lower boundary of network performance, to calculate the connected stability. Finally, the paper carried out experiments between the improved and traditional Monte Carlo method in Matlab simulation to contrast and analyze. The results show that the improved Monte Carlo method to evaluate the stability of the network has higher accuracy and smaller variance. So the experiment proved that it has the high feasibility and validity.

1. Introduction

In recent years network has become an important platform for people’s work and living. Requirements and dependence of network quality is getting higher and higher for People, and network reliability has become safety cornerstone of global science and technology, economic and political stability. At the same time, network reliability has become a hot research in academic research, and the current research on network reliability at home and abroad mainly includes the study of network reliability calculating algorithm, the study of the upper and lower bound of the stability and reliability simulating evaluation methods, etc[1].

Monte Carlo method is widely used in geology, power, medicine, optics, water conservancy and other fields in reliability study. So the Monte Carlo principle is also introduced in network reliability evaluation. However, there are some limitations in the traditional Monte Carlo assessment of network stability method, when the network is extremely stable and extremely unstable, error of the evaluation results becomes larger[2]. After analysis, it is found that the unbiased of variance of the sample with smaller variance is better in Monte Carlo. Therefore, obtain sub-samples with small variance by recursive variance attenuation method based on the traditional Monte Carlo method, and calculate the network link and node connectivity stability through the permutations, and ultimately calculate the network stability threshold combined with extreme situation of the upper and lower bounds and other conditions[3]. In this way, the method avoids the error assessment in the extreme case, and improves the accuracy and efficiency of the evaluation.
2. Network Model

Network stability is often considered to the network's ability for interfering and damaging. That is, when local failure of the network occurs, network can reduce the network loss to a minimum in time, so as to ensure the stable using of network[4].

The connectivity stability of network is the most commonly used concept of network stability in recent years. Reliability of network is the probability of anon empty network, which indicates that the link is normal in undirected network with faulty nodes. For example, there are many the connected links between two nodes, one of which fail, and can still be connected through other links and maintain the normal operation of the network, but may affect the transmission efficiency because of changing the length of the link. But in reality, the link may also be a failure, this will affect the rest of the nodes and cannot connect, in which case the network stability will be greatly reduced. Therefore, in the actual situation, it is necessary to consider the undirected network that the link and the node are faulty. The connectivity stability is the probability of the normal connection of the remaining nodes in the normal link in this topology.

A complex network can be modeled by an undirected graph $G(S,E)$, where $S$ and $E$ are the set of vertices and edges (arcs) of $G$. Moreover the probabilities of failure of the network components could be represented by assigning probabilities of failure to the vertices and/or edges (arcs) of its underlying graph (digraph). It is a set of the nodes (or vertices) $S=\{S_1, \ldots, S_n\}$ that represents all nodes of a network, connect together by a set of the edges(or links) $E=\{E_1, \ldots, E_m\}$,where $n$ and $m$ are number of the vertices and the number of the edges in the graph. Network reliability $R(G)$ is the probability that the network is in the connected state. Any node and link may be compro nodes and links, and abnormal indicates a damaged fault condition. The probability of a normal working state of a node is mised when a burst occurs in the network, where Normal is used to indicate the normal working state of $P_{S}$,Then the probability of the fault state is $Q_{S} = 1 - P_{S}$.

The probability of the link in the normal and fault state of the node is $P_{V}$, $Q_{V} = 1 - P_{V}$, respectively. If the probability that the two nodes are in the Normal state is the same, the two nodes are homogeneous. For example, If any network node $S_1, S_2 \in S$, and $S_1, S_2$ are homogeneous links.

According to the performance principle of link, it is know that the network stability is the sum of lengths of all shortest link path between all nodes that can transmit information each other. The total length of the shortest path $C_i$ indicates network performance, that is, \[ N = \sum_{i=1}^{m} C_i \], Where $m$ is The total number of links, $C_i$ is the performance of the i-th link. The definition of the stability of the network connection is known, the smaller $N$ value is, the better network performance is, because the better the work state, the shorter the link between nodes, suitable for fast transmission of information in the network. According to the definition of network performance can calculate the network performance of the minimum and maximum, respectively, that is, when the link is in normal work state and all links are in fault status.

In the above network model, the network performance is linear relationship with the link, and it is convenient to apply Monte Carlo method to calculate network stability. The Monte Carlo method belongs to the statistical sampling method, so obtain the approximate value of the network stability by sampling of multiple network stability and calculating its mean value.

3. The Monte Carlo Method

The Monte Carlo method is a statistical sampling theory for mathematical or physical, is used to the stability study of geological, electrical, medical, optical, water and other areas. Monte Carlo can solve the problem of randomness[6].
3.1. Monte Carlo Principles

The Monte Carlo method is to obtain the probability or random variable expected value of solving the problem based on the random sampling test, then its probability or expected value as a solution of the problem. So Monte Carlo is based on a probabilistic model.

To calculate the graphics area of the irregular shape as an example to illustrate Monte Carlo method. The irregular shape B exists in a square A with a side length of a, and randomly take n points in the square, if there are m points in the irregular graph, and the area of the irregular graph is \( s = \frac{m}{n} \cdot a^2 \) approximately. The irregular pattern is shown in Figure 1.

![Figure 1. Area of the irregular graph](image)

3.2. Calculation Method of Monte Carlo

Solve the problem using Monte Carlo method is divided into the following four processes[7].

First, establish a random sequence that satisfies the probability distribution of the characteristics of the random variable, and the probability model is established for the random sequence. For deterministic problems that do not satisfy the random characteristics, need to be transformed by parameters, the parameter is transformed into the random sequence of the calculating problem.

Secondly, carry out the distribution sampling for the probability model. Random samples are taken by satisfying the total sample of the probability distribution in order to get the sample value for the solving problem.

Then set the estimated parameters, that is, set a random variable as the solution of the problem.

Finally, calculate the value of the estimated parameters according to the above probability model, can get the approximate solution.

4. Evaluation of Network Stability

4.1. Traditional Monte Carlo Assessment Method

Carry out sampling for the probability model, and calculate the value of the estimated parameters, which is considered to be an approximation of the network stability in Monte Carlo method. Extract a set of samples from all random variables using a random number method \( x^{(1)}, x^{(2)} \ldots x^{(i)} \), and calculate the network stability value \( G_{x^{(i)}} \), and obtain n relatively independent network stability sample values after repeating n times \( G_{x^{(1)}}, G_{x^{(2)}}, \ldots, G_{x^{(n)}} \). \( \Phi_{G}(X) \) is a random event of network stability, and the estimated parameter of \( R(G) \) can be obtained after N sampling[8].

\[
\hat{R}_{1} = 1/N \cdot \sum_{i=1}^{N} \Phi_{G}(x^{(i)})
\]
Where, $x^{(i)}$ is a sample of $x$.

According to the statistical central limit theory we can see, $G_{x^{(i)}}$, $G_{x^{(2)}}$, ..., $G_{x^{(n)}}$ can estimate the distribution of network stability. When $\Phi_G(X)$ is a bernoulli random variable and $N \to \infty$, and calculate mean $E(\hat{R}_i)$ and variance $\text{Var}(\hat{R}_i)$, where,

$$E(\hat{R}_i) = R(G)$$

The estimated parameter is approximately equal to the network stability value.

$$\text{Var}[\Phi_G(X)] = E[\Phi_G(X)]\left[1 - E(\Phi_G(X))\right] = R(G)\left[1 - R(G)\right]$$

And variance of the estimated parameter is,

$$\text{Var}(\hat{R}_i) = \text{Var}[\Phi_G(X)]/N = R(G)\left[1 - R(G)\right]/N$$

$a$ is its stability index, and is defined as follows.

$$a = [E(\hat{R}_i) - 1]/\text{Var}(\hat{R}_i)$$

Then network stability probability $P_G$ corresponds to $a$. If $a$ tends to infinitely small, that is, $1 - a \to \infty$, has

$$N/\text{Var}[\Phi_G(X)] \leq Z^2\sigma^2/\text{Var}(\hat{R}_i)$$

From the above formula we can see that the smaller the variance of the statistic $\Phi_G(X)$, the higher the accuracy of the sampling experiment.

4.2. Improved Monte Carlo Assessment Method

From the above formula we can see the smaller the variance of the sample, the higher the accuracy of the estimated value, so error of the network approximation stability is smaller.

Data is recursively decomposed using a recursive variance attenuation method in the paper [9]. Replace the original sample by sub-sample sampling. That is, the original sample is decomposed into sub-samples and recursively call the sampling method. The termination condition can be set according to the parameter and the variance relationship of the approximate value. This method reduces the variance of the sample to improve the accuracy of the estimate so that the result is closest to the actual network stability value. Therefore, to improve the Monte Carlo method, it is important how to determine the sample of the smaller variance. Firstly, the link state in the network is sampled, and to determine its sub-network through the link state sample, and the link is in a normal state, but the node may be fault. Then the node state in the sub-network carry out sampling.

The link state of network $G$ is sampled to obtain network connected stability $R(G)$, which is expressed as follows.

$$\hat{R}_2 = 1/N_1 \sum_{i=1}^{N_1} \psi_G(y^{(i)})$$

Where, $y^{(i)}$ is a sample of $y$, and $N_1$ is the number of samples after the link state is sampled. $\psi_G(y) = E[\Phi_G(X) | X_E = y]$. $\psi_G(y)$ indicates the stability of the connected network when the link
state is $y$, if $G_y$ and $G_y'$ are homogeneous network model, and the links are in normal working condition in $G_y'$, get

$$R(G_y) = \psi_G(y)$$

(8)

And get its estimated parameters using sub-sample recursive decomposition method for $\psi_G(y)$

$$\hat{\psi}_G(y) = 1 / N_2 \cdot \sum_{i=1}^{N_2} Z^{(i)} G_{y,i}$$

(9)

Where, $Z^{(i)}$ is state of connected, and $t$ is the number of recursive calls. Get,

$$E[Z^{(i)}(G_y)] | x_E = y] = \psi_G(y)$$

(10)

$$Var[Z^{(i)}(G_y)] | x_E = y] \leq \psi_G(y)[1 - \psi_G(y)]$$

(11)

The final result of the estimated parameters is

$$\hat{R}_2 = 1 / N_1 N_2 \cdot \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} Z^{(i)} G_{y,j}$$

(12)

We can see

$$E[\psi_G(y)] = E[\Phi_G(X)] = R(G) \text{ and } E\left[ E(X \mid Y) \right] = E(X)$$

(13)

Get $E[Z^{(i)}(G_{y,j})] = E[Z^{(i)}(G_y)] | x_E = y]$, that is $E[Z^{(i)}(G_y)] = R(G) = E(\hat{R}_2)$. So prove that the estimated parameters $\hat{R}_2$ is unbiased, can be used as a network stability approximation. Also because of $Var[\Phi_G(X)] = R(G) [1 - R(G)]$ and $Var[Z^{(i)}(G_{y,j})] = E[(Z^{(i)}(G_{y,j}))^2] - E[Z^{(i)}(G_{y,j})]^2]$, we have

$$E^2[Z^{(i)}(G_{y,j})] = R^2(G),$$

(14)

we get

$$Var[Z^{(i)}(G_y)] | x_E = y] = E[(Z^{(i)}(G_{y,j}))^2] - \psi_G^2(y)$$

$$\leq \psi_G^2(y)[1 - \psi_G^2(y)]$$

That is, $E[(Z^{(i)}(G_{y,j}))^2] = E\left[ E[Z^{(i)}(G_y)] | x_E = y^2 \right] \leq E[\psi_G(y)] = R(G)$, get

$$Var[Z^{(i)}(G_y)] \leq Var[\Phi_G(X)]$$

(14)

We prove that the variance of sample $G \left( y^{(i)} \right) \in$ the improved Monte Carlo method is smaller than the variance of sample $G_x \in$ the traditional Monte Carlo method, get the estimated parameters of $\hat{R}_2$. 

5
After the samples with smaller variance are obtained by recursive call decomposition, the nodes and links are calculated by permutation and combining method to obtain the final network stability value [10]. The stability calculation method of network $G$ is $R(G) = \sum_{X \in \Omega} f(X)$.

Where,

$$f(X) = \sum_{i=0}^{n} \frac{P_s Q^{n-i}}{i! (n-i)^r}$$ (16)

The upper and lower bounds of the stability of the network are

$$f_{\min} (X) = P^r(X) Q^{n-r(X)} / r(X)! (n-r(X))!$$

$$f_{\max} (X) = P^r(X) Q^{n-r(X)} / (n-i)!$$

For $f_{\min} \leq f(X) \leq f_{\max}$

And get,

$$\sum_{X \in \Omega} f_{\min} (X) \leq R(G) \leq \sum_{X \in \Omega} f_{\max} (X)$$ (17)

In summary, the improved Monte Carlo estimation network stability algorithm specific steps are as follows.

Input: $G = (V,E)$, each link reliability $p(e_i)$

Output: Calculate network reliability $R(G)$

Algorithm: Improved Monte Carlo Estimation Network Stability Algorithm

Input the relevant parameters of the network $G$.

Step1: Input the relevant parameters of the network $G$, Set the number of samples are $r_U$, $r_L$, $(i=1,\ldots,S)$, $(j=1,\ldots,E)$ respectively, and initialize $R$ and $V$.

Step2: The working status of all links is randomly sampled, and constitute a random vector $y$.

Step3: According to corresponding to the network $G_{y'}$ of the random vector $y'$, and delete isolated nodes that rows and columns are 0, and delete the corresponding isolated node in the target node, and the stability of the node is constant.

Step4: Recursively invoke sub-sample samples Recursively call sampling function $Z(G)$ of the sub sample, and calculate the corresponding $G_{y'}$ for each calling, and let $R = R + Z(G_{y'})$, $V = V + [Z(G_{y'})]^2$.

Step5: To determine whether the $N'$ sampling is completed, and its termination condition is, (1) Target node is empty or has only one node. (2) All nodes are stable, and connectedness of the network $G$ is $K$. 

\[
\text{Var} \left( R_2 \right) = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \left( Z^{(i)} G_{y'} - R(G)^2 \right)
\]

$$= \frac{1}{N_1 N_2} \left( N_1 N_2 \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} Z^{(i)} G_{y'} - N_1 N_2 R(G)^2 \right)$$ (15)
Step 6: Calculate the stability of the network $R(G)$.

5. Simulation Test

In the simulation experiment, there are seven nodes in the network undirected model, which the network contains unstable nodes or links. The simulation experiment was simulated by Matlab[11]. The network contains unstable nodes or links. It is assumed that the connectivity of a node and link in the network model is stable, and their values were 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, respectively. We simulate the network model shown in Figure 2 below and run the simulation program.

![Undirected network model](image)

**Figure 2.** Undirected network model

(1) Firstly, the variance of the stability of the two Monte Carlo methods is compared and analyzed in order to verify the unbiased of the two methods. The two methods take the same network nodes, link connection stability and sampling times to carry out the simulation experiment, and the sampling times are 100 times. The variance and standard deviation of the traditional Monte Carlo method and the improved Monte Carlo method are calculated in the network stability are shown in Figure 3 (a) and (b).

![Comparison of variance of the two methods](image)

**Figure 3(a).** Comparison of the standard deviation of the stability of the network by using the two Monte Carlo methods
Figure 3(b). Comparison of the variance of the two Monte Carlo methods for network stability

It can be seen from the simulation results in Figure 3 (a) and (b) that the variance and the standard deviation decrease gradually as the number of link nodes increases. And in the case of the same link node, the improved is better than the traditional Monte Carlo method because the variance and the standard deviation of the former are smaller. Variance and standard deviation of the results are basically consistent, and it shows that the test method has a strong credibility and feasibility. This shows that variance unbiased of improved Monte Carlo is better. The improved Monte Carlo method has a smaller variance and standard deviation than the traditional Monte Carlo method in the case of the same sampling times

(2) In addition, we verify the relationship between the accuracy of two Monte Carlo Methods and the number of sampling, and take the same network connectivity stability and different sampling times. The error results of the two methods are shown in Figure 4.

Figure 4. The error rate varies with the number of sample
It can be seen from Figure 4. that the error rate of the two methods decreases gradually with the increase of the sampling times, and the error rate of the improved Monte Carlo method is small under the same sampling times, which shows the superiority of the improved Monte Carlo method. When the stability of the link and the node is equal, the improved Monte Carlo method is lower than the traditional Monte Carlo method with the increasing of the sampling times, and the improved Monte Carlo method is used to calculate the network stability estimating are more accurate. This is also consistent with the actual situation, with the increasing in the number of sampling, the system collects the effective data steadily increasing, the effective evaluation data is also increasing, so error rate will reduce with increasing number of sampling. The improved Monte Carlo method obtains the sub-sampling samples of a smaller variance using recursive decomposition of the sample, thus ensures the estimated unbiased, it shows error rate is the smaller than the traditional Monte Carlo method.

By comparing and analyzing the simulation results of two Monte Carlo methods, we can see that the improved Monte Carlo method uses the recursive decomposition sample method to get the sub-sampling samples with smaller variance. When calculating the permutation stability, we apply the permutation and combination theory The experimental results also verify the theoretical part, which fully shows that the improved Monte Carlo method has better unbiasedness and accuracy than the traditional Monte Carlo method.

6. Conclusion
The Monte Carlo method estimates sample stability approximations based on part of the sample constants in the probability model. However, the traditional Monte Carlo method has some limitations in two extreme cases where the network is extremely stable and extremely unstable, and its accuracy is greatly affected by the number of samples. The analysis shows that the Monte Carlo method with smaller variance can get a higher accuracy rate. Therefore, this paper recursively decomposes the traditional method based on the recursive variance attenuation method to get the sample with the smaller variance and combines the permutation and combination method Node and link connectivity stability calculation. In addition, the upper and lower bounds of the network stability are calculated and finally the network stability value is determined. In order to validate the method, the Matlab simulation of improved Monte Carlo method and the traditional Monte Carlo method is carried out in this paper. The error rate and the variance result of the two methods are compared and analyzed. The experimental results show that the modified Monte Carlo method has a smaller variance And the error is lower, and less affected by the number of samples. This shows that the method has better unbiasedness and accuracy.

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