Identification of Acoustic Wave Propagation Pattern of a Supersonic Jet Using Frequency-Domain POD*

Yuta OZAWA,† Taku NONOMURA,‡ Masayuki ANYOJI,§ Hiroya MAMORI,∥ Naoya FUKUSHIMA,¶ Akira OYAMA,∥∥ Kozo FUJI,∥∥∥ and Makoto YAMAMOTO∥∥∥

∥Department of Mechanical Engineering, Tokyo University of Science, Tokyo 125–8585, Japan
‡Department of Aerospace Engineering, Tohoku University, Sendai, Miyagi 980–8579, Japan
§Department of Energy and Environmental Engineering, Kyushu University, Kasuga, Fukuoka 816–8580, Japan
∥Institute of Space and Astronautical Science, Japan Aerospace Exploration Agency, Sagamihara, Kanagawa 252–5210, Japan
¶Department of Information and Computer Technology, Tokyo University of Science, Tokyo 125–8585, Japan

Key Words: Supersonic Jet, Experiment, Frequency-domain POD

1. Introduction

The prediction of acoustic waves and reduction of their intensity level are of importance not only in scientific fields, but also in engineering fields. For example, strong acoustic waves emitted from a rocket plume might damage rocket payloads if the payload has one or more fragile items in it. Therefore, acoustic waves emitted from a supersonic jet have been studied for a long time using experiments and computations.1,2) The components of acoustic waves are classified as shown in Fig. 1.1) The Mach waves generated by a large-scale turbulent structure create a dominant noise from a supersonic jet. Tam and Burton3,4) explained the generation mechanism of Mach waves with a model that uses supersonic instability waves as a noise source. Shock associated noise is generated by interference between the turbulence and shock waves, and it is decomposed into screech tone and broadband shock associated noise. The former is a peaky noise, and the latter often appears with a frequency higher than the former and with the fundamental screech frequency. Powell5,6) explained the generation mechanism of screech using an acoustic feedback loop.

These acoustic waves have been investigated experimentally by many researchers for a long time. Tam et al.7,8) investigated the source and components of turbulent mixing noise using microphone measurements. Panda9) investigated the source of screech tone using schlieren visualization and the microphone measurement. Mercier et al.10) conducted high-speed schlieren visualizations and near-field acoustic investigations of three under-expanded jets. They assessed the screech-loop consistency using near-field acoustic phase analysis.

However, few studies have experimentally investigated the propagation pattern of these acoustic waves. The main objective of this research is to identify the acoustic propagation pattern from schlieren images. To identify the acoustic propagation pattern, a method that can characterize the acoustic directivity and source position from schlieren images can be a very powerful analysis tool for clarifying the mechanism of acoustic wave generation. The innovation of this research is to demonstrate the usefulness of the frequency-domain proper orthogonal decomposition (POD)11,12) which is an analysis method in computational fluid dynamics for analyzing schlieren images. In this study, we applied the frequency-domain POD11,12) to the experimental data of schlieren images for two different jet flows: a jet flow that has only weak shock waves and generates only turbulent mixing noise, and a jet flow that has strong shock cells in the potential core and generates screech tone as well as turbulent noise. Microphone measurements were conducted and the characteristics of these acoustic waves were confirmed to be the same as those reported previously. Then, the frequency-domain POD11,12) was applied to the schlieren images at the near field of the jet in order to characterize the acoustic wave propagation pattern, which has been rarely discussed with experimental data. By summarizing the present results, we concluded that the method proposed is effective for the extracting useful information of acoustic directivity and source position from schlieren images.

2. Materials and Methods

The present experiments were conducted using a supersonic wind tunnel installed in an anechoic room at Kyushu University. The high-pressure air was used for operation. We adjusted the pressure of the plenum chamber to realize an ideally expanded condition (jet Mach number: $M_j = 2.0$). The details of the nozzle geometries are described below.

2.1. Nozzle geometry

The diameter at the nozzle exit was 10 mm, and the Reynolds number of the supersonic jet was $5.0 \times 10^5$. Figure 2 shows the cross-sectional geometries of the axisymmetric nozzles. The air flows from left to right in this figure. The convergent-divergent nozzle (C-D nozzle) was designed to re-
produce the ideally expanded condition without shock waves. Therefore, the geometry downstream from the throat was optimized. The geometry on the upstream side was determined using the cosine function. The conical nozzle was designed to generate strong shock waves in the potential core. This nozzle had a flare angle of 15 degrees at the nozzle exit. Therefore, even under the ideally expanded condition, the flow of the conical nozzle contains shock waves owing to the nonuniformity at the nozzle exit. The design Mach number of all nozzles was 2.0. These nozzles are made using stereolithography.

2.2. Microphone measurement

The acoustic investigation was conducted using eight 1/4-inch microphones (TYPE4158N, ACO). The microphone sensitivity was 3.2 mV/Pa, the maximum sound pressure level (SPL) was 150 dB, and the sampling frequency was 192 kHz. Two DAQ analyzers (USB-6363, National Instruments) and shielded connector blocks (BNC-2120, National Instruments) were employed. In this study, near-field acoustic distribution was measured by grid search as shown in Fig. 3. The grid spacing is 20 mm ($x/D = 2$, $r/D = 2$), and the grid points are $16 \times 6$. We confirmed that this grid spacing is sufficient for us to identify the main acoustic wave propagation pattern from the previous study. The data obtained was processed using fast Fourier transformation (FFT). The data length for the FFT was 8192 points. The FFT overlapped 50% of the data length and was averaged 100 times. The frequency was nondimensionalized as a Strouhal number ($St = fD/V$). Here, $St$ is a Strouhal number, $f$ is the frequency, $D$ is the nozzle exit diameter, and $V$ is the nozzle exit velocity.

2.3. Schlieren visualization

We used the Z-type schlieren system with parabolic mirrors having diameters of 100 mm. The schlieren images were lit using a xenon light source of 300 W (LS-300, KatoKoken). They were recorded using a high-speed camera (FASTCAM SA-Z, Photron). Detailed images of the propagating acoustic waves near the nozzle exit were obtained at a sampling rate of 300,000 frames per second at a narrow angle of view (256 $\times$ 128 pixels). The exposure time of each frame was 0.25 $\mu$s, and the bit depth was 12 bits. These detailed images were analyzed using frequency-domain POD to extract acoustic wave features.

2.4. Frequency-domain POD

In recent years, it is easy to obtain detailed data rich in spatial and temporal resolutions thanks to the development of measuring instruments. In general, it is difficult to understand physical phenomena from such enormous data. Here, POD is an effective method for extracting the principle components from such data. This method is a linear decomposition method that gives us the most energetic modes of the unsteady flow and acoustic fields. In this study, frequency-domain POD was applied to acoustic fields in order to efficiently extract the acoustic wave propagation pattern from the schlieren images.

The schematic of standard time-domain POD is shown in Fig. 4. The gray points in Fig. 4 indicate the images of time series data. The standard time-domain POD simply processes these data in the time direction. However, this method is not suitable for complex flows or acoustic fields like a supersonic jet flow because several hundred modes are required for 90% reconstruction of the original flow fields in the case of broadband frequency phenomena. In this case, even the first mode has only a few percent of entire energy. This poor decomposition energy is due to the complexity of the acoustic fields containing the broadband frequency phenomena. Therefore, it is difficult for us to understand the main acoustic wave propagation pattern with a limited frequency range from POD modes that have poor energy.

Instead, frequency-domain POD gives us only several modes for the reconstruction of more than 90% of the original flow fields, and thus it is very simple to understand, though only a limited frequency range was analyzed. Another advantage is that the frequency-domain POD gives us phase-lag information while such information is lost for the time-domain POD. The phase information can also be obtained using conventional discrete Fourier transformation (DFT). However, it is necessary to adjust the phase of each data in order to perform ensemble averaging without losing the phase information. For this problem, Nakakita calculated the phase information between each pixel on the image and one reference pixel that was determined using the maximum signal strength. It is necessary to select the local reference pixel on the image in this way. This means that the phase information depends on the local reference pixel of the image. On the other hand, the frequency-domain POD calculates the phase information from the mode of the entire field. This point enables us to obtain the phase information without depending on the local reference pixel selection. Figure 5 shows a schematic of the frequency-domain POD. This method calculates Fourier coefficients from a certain length of time series data before POD analysis. In this study, the intensity of the schlieren image was used for the analysis. First, short-term DFT is conducted for the segments of the time series data. The length of the time series data for DFT was four time periods of the target Strouhal number (140 images at $St = 0.125$) based on previous studies, whereas the Hanning window for DFT was adopted. This data length

![Fig. 3. Cross-sectional geometries of the axisymmetric nozzles.](image)

![Fig. 4. Schematic of standard time-domain POD.](image)
for DFT is determined to obtain the phenomena, which are peaky and broadband corresponding to about one-sixth of the bandwidth. Second, a complex number POD is applied to the transformed Fourier coefficients. This procedure gives us the most energetic modes for each frequency.

3. Results and Discussion

3.1. Flow and acoustic fields

The far-field schlieren images that emphasize the density gradient in the flow direction are shown in Fig. 6. The results clearly show the flow structures and the existence of Mach waves. In the conical nozzle case, the schlieren image also shows strong shock cell structures in the potential core of the jet as compared to the C-D nozzle. Therefore, the geometry of the C-D nozzle is confirmed to eliminate or weaken the shock cells. Figure 7 shows the acoustic spectrum near nozzle exit ($x/D = 0, r/D = 4$). The conical nozzle has a characteristic peak at $St = 0.125$ and a strong acoustic intensity level over the whole frequency range as compared to the C-D nozzle. This is due to the shock-associated noise generated by the strong shock cells. This peak is considered to be a screech tone.

The distributions of the overall sound pressure level (OASPL) and the octave-band filtered sound pressure level (OBSPL) in the near field are shown in Fig. 8. OASPL shows that the strongest noise source is located near $x/D = 14–16$ at both nozzles. This position seems to be the end of the potential core. In the case of the C-D nozzle, OBSPL clearly shows directivity towards downstream of the Mach wave for all frequencies. The noise source position moves upstream as the Strouhal number increases. Moreover, the angle of Mach wave radiation increases as the Strouhal number increases. The tendencies of the Mach wave radiation angle and source location are in good agreement with the computational results of $M_x = 3.0^{15)}$. On the other hand, the OASPL of the conical nozzle near the nozzle exit is higher than that of the C-D nozzle. The OBSPL of the conical nozzle also has strong noise near the nozzle exit at $St = 0.125$. This Strouhal number corresponds to that of the screech tone shown in Fig. 7. Moreover, the conical nozzle has an acoustic wave with a different directivity from the Mach wave at $St = 0.25, 0.5$. This acoustic wave is considered to be a broadband shock-associated noise.

3.2. Results of frequency-domain POD

Frequency-domain POD was conducted and the results are discussed. Figure 9 shows the power ratio of the most energetic modes for $St = 0.125$ of both nozzles. Figure 10 shows the most energetic modes at $St = 0.125$. The monochrome image at the bottom is the schlieren image, and the color contours show the results of frequency-domain POD. The upper, middle and bottom figures show the distribution of the real part, amplitude and phase, respectively. The real part and phase are considered to represent an instantaneous acoustic propagation pattern, and the direction in which the phase increases corresponds to the direction of acoustic wave propagation. The amplitude is considered to represent the SPL. Therefore, the high-amplitude region seems to be an acoustic source position.

In the case of the C-D nozzle, 89 modes are required for 90% reconstruction of the DFT image processed at $St = 0.125$. The first mode shows the Mach wave propagates in the downstream direction. On the other hand, in the conical nozzle case, only 13 modes are required for 90% reconstruction of the image data processed using DFT. Especially, the first mode shows very different distribution compared to the C-D nozzle case. The real part shows omnidirectional acoustic waves, and the amplitude shows two strong noise sources that are located at the point of interaction between the stationary shock waves and the shear layer. The directivity and source position of the screech tone are clearly visualized by the schlieren images after post-processing using the frequency-domain POD. The difference in the modes in each
nzzle means that the screech tone is dominant for the conical nozzle because the first mode that indicates the screech tone has quite high energy. For the C-D nozzle, the peaky noise at $St = 0.125$ is not observed in the acoustic spectra, as shown in Fig. 7. Therefore, the number of modes in the C-D nozzle for reconstruction will increase because of the poor energy of each mode.

The frequency-domain POD for the other target Strouhal numbers ($St = 0.25, 0.5, 1.0$) is also conducted for both nozzles. Only the first mode of each nozzle is discussed because these results do not show a large difference from the first mode to the third mode. Figure 11 shows the real part of first mode at each target Strouhal number. For C-D nozzle, the Mach waves are clearly extracted at all the Strouhal numbers. As the Strouhal number increases, the wave length of the acoustic wave extracted becomes shorter and the angle of the Mach wave radiation increases. These characteristics are in good agreement with the results, as shown in Fig. 8. On the other hand, the results of the conical nozzle show a slip in acoustic wave propagation at $St = 1.0$. Moreover, the wave length suddenly becomes shorter at the edge of the Mach waves. These changes in the acoustic wave propagation pattern seem to be due to the influence of a screech tone. This is because position of these changes corresponds to the position of a screech, as shown in Fig. 10.

4. Conclusions

In this study, frequency-domain POD is proposed to be applied to aeroacoustic experimental data, and the results show that this method effectively extracts useful information for acoustic directivity and source position.

Specifically, high-speed schlieren visualization and the microphone measurement of ideally expanded supersonic jets which have weak shock waves or strong shock cells were conducted. The acoustic investigation revealed that Mach waves were observed for both nozzles, and screech tone was observed at $St = 0.125$ for only the conical nozzle. These characteristics are confirmed to be the same as those reported previously. The results of applying frequency-domain POD to the schlieren images illustrates that the first mode of the C-D nozzle at $St = 0.125$ shows a Mach wave that propagates downstream. On the other hand, the first mode of the conical nozzle observed a different pattern of screech tone. The real part shows omnidirectional acoustic waves, and the amplitude shows two strong noise sources located at the position of interaction between the stationary shock waves and the shear layer. The characteristics observed are in good agreement with the computational results of Nonomura and Fujii. Therefore, this is proof that frequency-domain POD is still effective for extracting acoustic waves from schlieren images. The acoustic propagation of the Mach wave and screech tone can be clarified.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Number 25709009, 17H03473.

References

1) Tam, C. K. W.: Supersonic Jet Noise, *Annu. Rev. Fluid Mech.*, 27 (1995), pp. 17–43.
2) Bailly, C. and Fujii, K.: High-Speed Jet Noise, *Mech. Eng. Rev.*, 3, 1 (2016), pp. 1–13.
3) Tam, C. K. W. and Burton, D. E.: Sound Generated by Instability Waves of Supersonic Flows. Part 1. Two Dimensional Mixing Layers, *J. Fluid Mech.*, 138 (1984), pp. 249–271.
4) Tam, C. K. W. and Burton, D. E.: Sound Generated by Instability Waves of Supersonic Flows. Part 2. Axisymmetric Jets, *J. Fluid Mech.*, 138 (1984), pp. 273–295.
5) Powell, A.: On the Mechanism of Choked Jet Noise, *Proc. Phys. Soc.*, B66 (1953), pp. 1039–1056.
6) Powell, A.: The Noise of Choked Jets, *J. Acoust. Soc. Am.*, 25 (1953), pp. 385–389.
7) Tam, C. K. W.: On the Two Components of Turbulent Mixing Noise from Supersonic Jets, Proc. AIAA Aerodynamics Conf., AIAA 96-1716, 1996, 18 pp.
8) Tam, C. K. W., Viswanathan, K., Ahuja, K. K., and Panda, J.: The Sources of Jet Noise: Experimental Evidence, *J. Fluid Mech.*, 615 (2008), pp. 253–292.
9) Panda, J.: An Experimental Investigation of Screech Noise Generation, *J. Fluid Mech.*, 378 (1999), pp. 71–96.
10) Mercier, B., Castelan, T., and Bailly, C.: A Schlieren and Nearsfeld Acoustic Based Experimental Investigation of Screech Noise Sources, Proc. 22nd AIAA/CEAS Aeroacoust. Conf., AIAA 2016-2799, 2016.
11) Suzuki, T., Rodony, D., Ryu, J., and Lele, S. K.: Noise Sources of High-Mach-Number Jets at Low Frequencies Studied with a Phased Array Approach Based on LES Data-Base, Center for Turbulence Research Annual Research Briefs, 2007, pp. 287–301.
12) Nonomura, T. and Fujii, K.: POD Analysis of Acoustic Waves from a Supersonic Jet Impinging on an Inclined Plate, Proc. 16th AIAA/CEAS Aeroacoust. Conf., AIAA 2010-4019, 2010.
13) Berkooz, G., Holmes, P., and Lumley, J. L.: The Proper Orthogonal Decomposition in the Analysis of Turbulent Flows, *Annu. Rev. Fluid Mech.*, 25 (1993), pp. 539–575.
14) Freund, J. B. and Colonius, T.: Turbulence and Sound-field POD Analysis of a Turbulent Jet, *Int. J. Aerocoust.*, 8 (2009), pp. 337–354.
15) Nakakita, K.: Detection of Phase and Coherence of Unsteady Pressure Field Using Unsteady PSP Measurement, Proc. AIAA Ground Testing Conf. Fluid Dynamics and Co-located Conf., AIAA 2013-3124, 2013.
16) Nonomura, T. and Fujii, K.: Overexpansion Effects on Characteristics of Mach Waves from a Supersonic Cold Jet, *AIAA J.*, 49, 10 (2011), pp. 2282–2294.

Nobuyuki Tsuboi
Associate Editor