A finite spin system invariant under a symmetry group $G$ is a very illustrative example of the finite group action on a set of mappings $f: X \rightarrow Y$. In the case of spin systems $X$ is a set of spin carriers and $Y$ contains $2s+1$ $z$-components $-s \leq m \leq s$ for a given spin number $s$. Orbits and stabilizers are used as additional indices of the symmetry adapted basis. Their mathematical nature does not lead to smaller eigenproblems, but they label states in a systematic way. Some combinatorial and group-theoretical structures, like double cosets and transitive representations, appear in a natural way. In such a system one can construct general formulas for vectors of symmetry adapted basis and matrix elements of operators commuting with the action of $G$ in the space of states. Considerations presented in this paper should be followed by detailed discussion of different symmetry groups (e.g. cyclic of dihedral ones) and optimal implementation of algorithms. The paradigmatic example, i.e. a finite spin system, can be useful in investigation of magnetic macromolecules.

I. INTRODUCTION

Polynuclear clusters provide magnetic materials with a scale intermediate between that of isolated dimers or trimers and that of bulk materials. These magnetic materials exhibit features on a mesoscopic scale, so they may show quantum effects coexisting with classical behavior. In addition, large assemblies of spins are interesting as real objects on which one can test theoretical models with a finite number of spins. Polynuclear magnetic aggregates have well defined crystal structures, allowing quantitative comparison of theoretical results with experimental ones. Unlike other assemblies of small magnetic particles a typical sample of a molecular magnetic compound is composed of nominally identical non-interacting magnets with a unique set of chemically determined parameters. They are complex organometallic systems, too difficult to investigate them by ab initio methods, applicable to simple metal cluster.

The aim of this paper is to present and discuss combinatorial structure of finite spin models. Such investigation lead to general analytical formulas for the irreducible basis and matrix elements of any operator commuting with all symmetry operators $P(g)$, $g \in G$. The most important is a Hamiltonian of the system in question, but also other operators can be taken into account. Since $S^2$ commutes with the isotropic Heisenberg Hamiltonian, then the total spin $S$ can be used as an additional label of states. Finite spin models have been used in condensed matter physics for many years and have been applied to various models of magnetic materials: from the one-dimensional ferromagnetic Ising model to the three-dimensional Heisenberg antiferromagnets. Recent developments and discoveries have stimulated interests in finite spin models, since they can be applied to meso- and nanoscopic systems, especially magnetic macromolecules with well-determined symmetry groups like $[\text{Na}Fe_6(\text{OCH}_3)_2]_{12}([\text{C}_{17}\text{H}_{25}\text{O}_4]_{6}\text{ClO}_4 \text{ (Fe}_6\text{)}, \text{[Fe}_{10}(\text{OCH}_3)_{20}(\text{C}_2\text{H}_2\text{O}_2\text{Cl})_{10}] \text{ (Fe}_{10}\text{)}, \text{Mn}_{12}\text{acetate or [Mn(hfac)]_2NITPTh}_{6} \text{ (Mn}_6\text{)}}$. Considering macromolecules with many magnetic centers and large spin numbers (up to $s = 5/2$) one deals with a very large number of states even for a small number of magnetic ions $N$. Moreover, this number increases rapidly with increasing $N$ and $s$. However, group-theoretical and combinatorial methods yield decomposition of a space of states into subspaces with relatively small dimensions. Therefore, magnetic properties of macromolecules can be investigated very deeply and model parameters can be fitted in more accurate way. This paper, based on the results of some earlier papers, is one of the steps necessary in the study considerations of small (nanoscopic) rings with relatively large spin number.

It should be also stressed that the approach described and solutions suggested can be used in any problem where a Hamiltonian symmetry group $G$ (or, at least, its subgroup) permutes basis vectors. It means that a permutation representation $P$ of $G$ is determined or, in other words, a group action of $G$ on a basis $B$ of quantum space of states is introduced. This immediately gives rise to such structures and concepts as transitive representations, orbits, stabilizers, double cosets etc. Permutation representations themselves constitute a special case of induced representation and are applied in many branches of physics. Some problems related to permutation and transitive representations were studied by Lulek et al. and their results, in various extent, are used here.
II. FINITE SPIN MODELS AND SIMPLE COMBINATORIAL OBJECTS

A system of $N$ spin $s$ gives rise to $(2s + 1)^N$ Ising configurations

$$|m_1 m_2 \ldots m_N\rangle, \quad m_j \in \{-s, -s + 1, \ldots, s\}.$$  

They can be considered as mappings

$$\mu: \{1, 2, \ldots, N\} \rightarrow \{-s, -s + 1, \ldots, s\}; j \mapsto \mu(j) \equiv m_j.$$  

Magnetization of the system,

$$M = \sum_{j=1}^{N} \mu(j) = \sum_{j=1}^{N} m_j,$$  

is an example of an additive weight. Therefore, all combinatorial methods of enumeration with weights can be applied \[11\]. Moreover, each mapping (configuration) is also characterized by a content, i.e.

$$[k] := [k_{-s}, k_{-s-1}, \ldots, k_s]$$

being numbers of solutions of equations

$$\mu(j) = m; \quad m \in \{-s, -s + 1, \ldots, s\}.$$  

In fact $[k]$ is a non-ordered partition of $N$. A number of configurations with the same content is given by a polynomial coefficient

$$n([k]) = \frac{N!}{\prod_{m=-s}^{s} k_m!} = \frac{(\sum_{m=-s}^{s} k_m)!}{\prod_{m=-s}^{s} k_m!}. $$

Hence, a number of configurations (mappings) with a given weight (magnetization) is

$$n(N, M) = \sum_{[k]} \sum_{j k_j = M} n([k]). \quad (1)$$

Let

$$D_s^* \otimes D_s^* \otimes \ldots \otimes D_s^* = \bigoplus_{S=0 \text{ or } 1/2}^{Ns} n(S) D^S.$$  

be a decomposition of the $N$th tensor power of the (spin) irrep $D^s$. Then

$$n(S = Ns) = n(N, M = Ns) = 1; \quad n(S) = n(N, M = S) - n(N, M = S + 1) \text{ for } S < Ns.$$  

Some recurrences for $n([k])$ and $n(M)$ can derived from those for polynomial coefficients and depicted by analogs of the Pascal triangle. In Fig. 1b, the recurrence

$$n([k]) = \sum_{j=-s}^{s} n([k_{-s}, k_{-s+1}, \ldots, k_j - 1, \ldots, k_s])$$

is illustrated by arrows for $n[1, 1, 1] = n[0, 1, 1] + n[1, 0, 1] + n[1, 1, 0]$. Introducing this relation to Eq. (1) one obtains

$$n(N, M) = \sum_{m=-s}^{s} n(N - 1, M - m).$$

It is shown by arrows in Fig. 1b in the case $N = 3, M = -1$: $n(3, -1) = n(2, -2) + n(2, -1) + n(2, 0)$. The dotted lines in Fig. 1a indicate polynomial coefficients summed in Eq. (1). After summation one obtains numbers presented in the last row in Fig. 1b. Formal equations for the number of configurations with given $N$ and $M$ can be derived by methods presented in Kerber’s monograph [11].
III. FINITE GROUP ACTION

For finite both a group $G$ and a set $X$ an action $G \times X$ of $G$ on $X$ is defined as a mapping (an external multiplication rule in $X$)

$$(G \times X) \to X: (g, x) \mapsto gx \quad \text{with} \quad (g_1g_2)x = g_1(g_2)x \quad \text{and} \quad 1_Gx = x.$$ 

A stabilizer $G_x$ of $x \in X$ is a subgroup of $G$ such that

$$G_x := \{ g \in G \mid gx = x \}.$$ 

An orbit $G(x)$ of $x \in X$ is a subset of $X$ such that

$$G(x) := \{ gx \mid g \in G \}; \quad |G(x)||G_x| = |G|.$$ 

The action $G \times X$ can be raised to an action on the set $Y^X$ of all mappings $f: X \to Y$ ($|Y^X| = |Y|^{|X|}$ for finite sets):

$$(G \times Y^X) \to Y^X: f \mapsto f' = f \circ g^{-1}, \quad \text{i.e.} \quad f'(x) = f(g^{-1}x).$$ 

Enumeration of mappings needs another subset of $X$ — the so-called fixed points:

$$X_g = \{ x \in X \mid gx = x \}.$$ 

This set is used, for example, to determine number of mappings with given weight and stabilizer by means of the so-called Pólya substitution \cite{11}.

IV. TRANSITIVE REPRESENTATIONS, DOUBLE COSETS AND SO ON

Any action of $G$ on an orbit $G(x)$ is similar\cite{22} to the action of $G$ on the set of left cosets

$$G/G_x = \{ g_xG_x \mid 1 \leq r \leq |G(x)| \},$$
where $r$ labels representatives $g_r$ of left cosets; this action is defined as
\[ g(g_r G_x) = (gg_r) G_x. \]
If subgroups $G_x$ and $G_y$ are conjugated in $G$, then actions $c(G/G_x)$ and $c(G/G_y)$ are similar. It means that nonequivalent actions are labeled by subgroups $U$ representing classes of conjugated subgroups. These actions, $c(G/U)$, can be considered as building blocks of any action $cX$.

Let $B = \{|r| \leq |G/U|\}$ be a basis of a (formal) unitary space $L$ constructed as a linear closure (over the field of complex numbers) of $B$. The vectors in $B$ are in a one-to-one correspondence with elements of an orbit $G(U)$, i.e. with left cosets $g_r U$. Moreover, a representation $P$ of $G$ in the space $L$ can be defined in a natural way \[12].

\[ P(g)|r⟩ = |s⟩ \quad \text{if} \quad gg_r ∈ g_s U. \]

This representation is denoted as $R^{G:U}$ and called transitive.

A transitive representation $R^{G:U}$ is, in a general case, reducible and can be decomposed into a direct sum of irreducible representations (irreps) of $G$:
\[ R^{G:U} = \bigoplus n(\Gamma, U) \Gamma; \quad 0 ≤ n(\Gamma, U) ≤ [\Gamma], \]
where $[\Gamma]$ denotes dimension of $\Gamma$. Any action $cX$ determines a permutation representation of $P$ in a space spanned over vectors $|xr⟩$, where $x$ labels orbits and $r$ labels representatives of cosets $g_r G_x$. Since each finite action can be decomposed into action on orbits, then a permutation representation is decomposed into transitive representations.

A double coset is determined by two subgroups of $G$ and is defined as
\[ V g U = \{v gu \mid v ∈ V, u ∈ U\}. \]

Let us consider an action of $G$ on the Cartesian product $G(y) × G(x)$ of two orbits with the stabilizers $G_x = U$ and $G(y) = V$. This action is defined as
\[ (g, (y, x)) \mapsto (gy, gx) \]
and has the following properties ($x$ and $y$ are fixed):

- Each orbit contains an element of the form $(y, gx)$;
- The stabilizer of $(y, gx)$ is $G_y ∩ gG_x g^{-1}$;
- The mapping $G(y, gx) \mapsto G_y g G_x$ is a bijection, so orbits of the introduced action can be labeled by double coset representatives.

\section*{V. EQUATIONS FOR MATRIX ELEMENTS}

It can be shown \[13, 14\] that any operator $H$ commuting with all operators $P(g)$, $g ∈ G$, can be written in a quasi-diagonal form with blocks labeled by irreps $\Gamma$. In each block rows (columns) are indexed by a stability group $U$ ($V$, respectively), and by indices of vectors $|r⟩$ ($|s⟩$) of the irrep $\Gamma$. A general form of matrix elements is following:
\[ h_{Uxr, Vys}(\Gamma) = ([U]|V|)^{-1/2} \sum_{d=1}^{[V \setminus G/U]} [Vg_d U]|Ux g_d |H|V y e_G⟩B_{rs}^{UV, \Gamma}(g_d), \] \[ (2) \]
where $B_{rs}^{UV, \Gamma}(g_d)$ depends on the so-called reduction coefficients \[3\] and can be treated as group-theoretical parameters of a model under considerations. Analytic formulas for these coefficients can be determined in the case of cyclic or dihedral group $G \[13, 14\]$. This equation simplifies for the unit representation $\Gamma_0$:
\[ h_{Ux, V y}(\Gamma_0) = \left( \frac{|U|}{|V|} \right)^{1/2} \sum_{j=1}^{[G/U]} ⟨U x g_j |H|V y e_G⟩, \]
If there is only one double coset, represented by $e_G$, then all left coset representatives belong to it and
\[ h_{Ux, V y}(\Gamma_0) = \left( \frac{|G|}{|U||V|} \right)^{1/2} ⟨U x e_G |H|V y e_G⟩, \]
so it is enough to calculate matrix elements for orbit representatives.
TABLE I: Multiplicities \( n(\Gamma) \) for a ring of \( N = 12 \) spins \( s = 1 \) with \( G = D_{12} \) and \( M = 0 \)

| \( \Gamma \) | \( A_1 \) | \( A_2 \) | \( B_1 \) | \( B_2 \) | \( E_1 \) | \( E_2 \) | \( E_3 \) | \( E_4 \) | \( E_5 \) |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( n(\Gamma) \) | 3179        | 2987        | 3107        | 3056        | 6136        | 6158        | 6140        | 6160        | 6136        |

VI. SOME REMARKS ON NUMERICAL PROBLEMS

Application of the formula presented above is not immediate and easy. Of course, in very simple cases, e.g. \( N = 4 \) and \( s = 1 \), one can calculate all matrix elements by hand. Larger systems require proper construction and implementation of algorithms. Moreover, one can use additional quantum numbers (good quantum numbers as the magnetization and the total spin in the case of isotropic Heisenberg Hamiltonians) to obtain more detailed classification of states. Secondly, the labeling scheme of orbit representatives may include partitions of \( N \), which, in fact, label orbits of the symmetric group \( \Sigma_N \supset G \). The action of this group does not commute with the Hamiltonian, in a general case, but commutes with the operator \( S_i^2 = \sum_{j=1}^{n} s_j^2 \) of the total magnetization. Therefore, if one wants to use the quantum number \( M \) as an additional index, then it is natural to include partitions in the labeling scheme.

One of advantages of the formula presented is a fact, that in some cases (especially for one-dimensional irreps \( \Gamma \)) obtained matrix elements are integers. Therefore, considering the eigenproblems for \( S^2 \), which has integer eigenvalues (for \( N \) odd and half-integer \( s \) one can consider \( 4S^2 \) instead), we obtain a set of homogeneous linear equations with integer coefficients. This set can be solved exactly, but we have to cope with very large numbers. This can be done by means of any multiple precision library, for example GMP [17]. Prototypes provided by this library can be also used to store configurations in compact (packed) form. Moreover, there are as well procedures for high-precision float calculations, so the Hamiltonian eigenproblem can be solved, too. More detailed discussion of numerical problems can be found in other papers [13, 18].

VII. EXAMPLES

The proposed methods are especially useful in the case of ring-shaped magnetic molecules with cyclic of dihedral symmetry group. Below some results for two molecules are presented: (i) \( \text{Ni}_{12}(\text{O}_2\text{CMc})_{12}(\text{chp})_{12}(\text{H}_2\text{O})_6(\text{THF})_6 \) referred to as \( \text{Ni}_{12} \) and (ii) \( \text{Fe}_5 \). The first has the ferromagnetic ground state with the total spin \( S = 12 \) [6], whereas the second is an example of a molecular antiferromagnet with \( S = 0 \) in the ground state [3].

The \( \text{Ni}_{12} \) cluster contains a ring of twelve \( s = 1 \) spin carriers. There are \( 3^{12} = 531441 \) Ising configurations. Assuming isotropic interactions each subspace with a given magnetization \(-12 \leq M \leq 12 \) can be considered separately. The largest dimension, 73789, has the subspace with \( M = 0 \). Ising configurations spanning this space are labeled by the following non-ordered partitions of 12 into no more than three parts (number of configurations for each partition is given in the parenthesis): \([0,12,0]\) (1), \([1,10,1]\) (132), \([2,8,2]\) (2970), \([3,6,3]\) (18480), \([4,4,4]\) (34650), \([5,2,5]\) (16632), and \([6,0,6]\) (924). In the next step orbit of the symmetric group \( \Sigma_{12} \) are decomposed into orbits of the dihedral group \( D_{12} \), what leads to transitive representations; they are decomposed into irreps of \( D_{12} \). For each irrep one can construct a block of the Hamiltonian matrix according with Eq. (9). The dimensions of these blocks are collected in Table II, the largest one is 6160 for \( \Gamma = E_4 \). All eigenvalues (for \( M \neq 0 \), too) of the Hamiltonian

\[
\mathcal{H} = -\sum_{j=1}^{12} (J s_j s_{j+1} + g \mu_B B s_j^z)
\]

can be calculated with high numerical precision and thermodynamic properties can be determined. The best fit to experimental results presented by Blake et al. [19] is achieved for the exchange integral \( J/k_B = 8.5 \pm 0.5 K \) and the gyromagnetic ratio \( g = 2.23 \pm 0.01 \) [6].

The antiferromagnetic ring \( \text{Fe}_5 \) contains six magnetic ions with \( s = 5/2 \), so there are 46656 Ising configurations. Numbers \( n(S) \) and \( n(6, M) \) for \( M \geq 0 \) are presented in Table II. Considering the ground state one has to take into account 32 non-ordered partitions. Decomposition into orbits of the dihedral group involves only three stabilizers: \( C_1 \), \( D_1 \), and \( D_3 \). Taking into account decompositions of transitive representations into irreps one obtains results collected in Table III.

Our aim is to calculate spin correlation in the ground state assuming isotropic Heisenberg interactions. In the considered case \( N = 6, s = 5/2 \) the ground state has symmetry \( \Gamma = B_1 \), so the dimension of the eigenproblem is 385. However, we solve it in two steps: (i) at first we determine and orthonormalize eigenvectors of \( S^2 \) operator with the eigenvalue zero; (ii) next we transform the Hamiltonian matrix to obtain a block labeled by \( \Gamma = B_1 \) and \( M = S = 0 \). The eigenproblem for such small matrix (20 \times 20) can be easily solved with high numerical precision. We
TABLE II: Numbers \( n(S) \) and \( n(6, M) \) for \( S, M = 0, 1, \ldots, 15 \)

| \( S \) or \( M \) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|-----------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| \( n(S) \)     | 111 | 315 | 475 | 575 | 609 | 581 | 505 | 405 | 300 | 204 | 126 | 70  | 35  | 15  | 5  | 1  |
| \( 2S + 1)n(S) \) | 111 | 945 | 2375 | 4025 | 5481 | 6391 | 6565 | 6075 | 5100 | 3876 | 2646 | 1610 | 875 | 455 | 145 | 31 |
| \( n(6, M) \)  | 4332 | 4221 | 3906 | 3431 | 2856 | 2247 | 1666 | 1161 | 756  | 456  | 252  | 126  | 56  | 21  | 6  | 1  |

TABLE III: Numbers \( n(\Gamma) \) for \( M = 0, 1 \) and \( S = 0 \); the last one is simply calculated as the difference of the previous two

| \( \Gamma \) | \( n(\Gamma), M = 0 \) | \( n(\Gamma), M = 1 \) | \( n(\Gamma), S = 0 \) |
|----------|----------------|----------------|----------------|
| \( A_1 \) | 385 | 339 | 385 | 339 | 721 | 721 | 2 |
| \( A_2 \) | 339 | 385 | 339 | 385 | 721 | 721 | 20 |
| \( B_1 \) | 383 | 325 | 365 | 334 | 699 | 708 | 14 |
| \( B_2 \) | 325 | 365 | 334 | 699 | 708 | 13 |

Determine not only eigenvalues, but eigenstates, too, what allows to calculate spin correlations. The first three low-lying states have energies per spin (in the unit of \( J \)): \(-7.322\), \(-5.761\), \(-4.831\), what agrees with the results presented by Lascialfari et al.

The spin correlations per bond \( \omega_z(r) = \langle \sum_j s_j^z s_{j+r}^z \rangle / 6 \) in these states are following:

\[
\begin{align*}
    r = 1 & : -2.440817, +1.958689, -1.952410 \\
    r = 2 & : -1.920315, +1.056202, -1.188439 \\
    r = 3 & : -1.610233, +0.226289, -0.148779
\end{align*}
\]

Note that absolute value of the nearest-neighbor \( z-z \) correlation in the ground state is smaller than in the Néel state \( |N\rangle = |s, -s, s, -s, -s, s, -s, s, -s, s, -s, s, -s, s, -s\rangle \), when one obtains the value of 6.25. However, the total spin correlation, equal 3\( \omega_z(1) \), is stronger than in the Néel state, since \( x-x \) and \( y-y \) correlations are equal to zero in this state. This fact shows importance of other states with \( M = 0 \) and follows from quantum character of the model considered.

VIII. FINAL REMARKS

In this report we present briefly a way from (finite) group action to calculations of spin correlations. The group-theoretical and combinatorial object appearing in this course help us to label Ising configurations in the systematic way and to obtain relatively simple form of the matrix elements. To exploit all possible simplifications we have assume isotropic interactions. However, there is no limit imposed on its range: one can easily include next-nearest neighbors or biquadratic terms \( (s_j s_k)^2 \). In the case of uniaxial anisotropy the total spin \( S \) is not a good quantum number, but even in this case thermodynamic properties of small magnetic macromolecules can be determined (the total spin \( S \) was not used to obtain results presented in [3]). Such model is frequently assumed to achieve a better fit to experimental results, for example in the case of Mn\(_{12}\) acetate molecule [5]. The \( XYZ \) models, when the total magnetization is not a good number, are most difficult. However, the Ising configurations still can be labeled by partitions and stabilizers, but the Hamiltonian matrix cannot be decomposed into blocks labeled by \( M \); of course the label \( \Gamma \) can be used as long as the Hamiltonian commutes with all operators \( P(y) \). It should be also stressed that some results presented are obtained using procedures included in the GMP package. For example, we are able to solve exactly a set of homogeneous linear equation with integer coefficients. The methods proposed has been applied to magnetic macromolecules, however calculations in the thermodynamic limit are also very important, since there are still some open problems related with linear magnets (e.g. critical exponents near \( T = 0 \) K in the case of ferro-magnetic ordering [21]).
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[1] D. Gatteschi, A. Caneschi, L. Pardi, and R. Sessoli, Science 265, 1054 (1994); A. Müller, S. Q. N. Shah, H. Bögge, and M. Schmidtmann, Nature 397, 48 (1999).
[2] J. C. Bonner and M. E. Fisher, Phys. Rev. 135, A640 (1964); J. Oitmaa and D. D. Betts, Can. J. Phys. 56, 897 (1978).
[3] M. Affronte, J.C. Lasjaunias, A. Cornia, and A. Caneschi, Phys. Rev. B 60, 1161 (1999).
[4] A. Lascialfari, D. Gatteschi, A. Cornia, U. Balucani, M.G. Pini, and A. Rettori, Phys. Rev. B 57 (1998) 1115; G. Kamieniarz, R. Matysiak, W. Florek, and S. Walcerz, J. Magn. Magn. Mater. 203, 271 (1999).
[5] A. Caneschi, D. Gatteschi, C. Sangregorio, R. Sessoli, L. Sorace, A. Cornia, M.A. Novak, C. Paulsen, and W. Wernsdorfer, J. Magn. Magn. Mater. 200 (1999) 182.
[6] A. Caramico D’Auria, U. Esposito, F. Esposito, G. Kamieniarz, and R. Matysiak, J. Phys.: Condens. Matter 13, 2017 (2001).
[7] W. Florek, Acta Magnetica II, 43 (1985); ibid. V, 145 (1988); ibid. VIII, 25 (1991).
[8] W. Florek, Acta Phys. Polon. A 96, 699 (1999).
[9] B. Lulek, and T. Lulek, Acta Phys. Polon. A66, 149 (1984) and the references cited therein.
[10] B. Lulek, T. Lulek, R. Chatterjee, and J. Biel, Can. J. Phys. 63, 1061 (1985); B. Lulek, T. Lulek, J. Biel, and R. Chatterjee, Can. J. Phys. 63, 1065 (1985); B. Lulek and T. Lulek, Acta Phys. Polon. A71, 869 (1987); W. Florek and T. Lulek, J. Phys. A: Math. Gen. 20, 1921 (1987); T. Lulek, Acta Phys. Polon. A80, 769 (1991); S. Walcerz, T. Lulek, Acta Phys. Polon. A80, 781 (1991); S. Walcerz, Acta Magnetica X, 13 (1994/95).
[11] A. Kerber, Algebraic Combinatorics via Finite Group Actions (BI Wissenschaftsverlag, Mannheim-Wien-Zürich, 1991).
[12] S. L. Altmann, Induced Representations in Crystals and Molecules (Academic Press, London, 1977).
[13] W. Florek, Comp. Phys. Commun. 138, 264 (2001); W. Florek, Acta Phys. Polon. A 100, 3 (2001).
[14] W. Florek, Acta Phys. Polon. A 100, 3 (2001).
[15] T. Lulek, J. Physique (Paris) 45, 29 (1984); W. Florek and T. Lulek, J. Phys. A: Math. Gen. 20, 1921 (1987).
[16] S. Bucikiewicz, L. Dębski, and W. Florek, Acta Phys. Polon. A 100 (2001), in press.
[17] The GNU Multiple Precision (GMP) Library has been developed by T. Granlund and can be downloaded from GMP Home Pages at http://www.swox.com/gmp.
[18] S. Bucikiewicz and W. Florek, Comp. Meth. Sci. Tech. 7, 27 (2001); W. Florek, Comp. Meth. Sci. Tech. 7, 41 (2001).
[19] A. J. Blake, C. M. Grant, S. Parsons, J. M. Rawson, and R. E.P. Winpenny, J. Chem. Soc., Chem. Commun. 2363 (1994).
[20] A. Lascialfari, D. Gatteschi, F. Borsa, and A. Cornia, Phys. Rev. B 55 (1997) 14341.
[21] G. Kamieniarz, R. Dekeyser, G. Musiał, L. Dębski, and M. Bieliński, Phys. Rev. E 56, 144 (1997).
[22] The formal definition of similarity can be found in [11].