Breathing mode frequencies of a rotating Fermi gas in the BCS-BEC crossover region

Theja N. De Silva
Department of Physics, Applied Physics and Astronomy,
The State University of New York at Binghamton, Binghamton, New York 13902, USA.

We study the breathing mode frequencies of a rotating Fermi gas trapped in a harmonic plus radial quartic potential. We find that as the radial anharmonicity increases, the lowest order radial mode frequency increases while the next lowest order radial mode frequency decreases. Then at a critical anharmonicity, these two modes merge and beyond this merge the cloud is unstable against the oscillations. The critical anharmonicity depends on both rotational frequency and the chemical potential. As a result of the large chemical potential in the BCS regime, even with a weak anharmonicity the lowest order mode frequency increases with decreasing the attractive interaction. For large enough anharmonicities in the weak coupling BCS limit, we find that the excitation of the breathing mode frequencies make the atomic cloud unstable.

I. INTRODUCTION

The rapid progress of ultra-cold atomic gas experiments provides unique opportunities for well controlled studies of quantum many body physics. For the case of Fermi atomic systems, the possibility of controlling the s-wave scattering length ($a$) between two different spin components allows to control the interaction by using a magnetically tuned Feshbach resonance \[1\]. This unique capability allows one to investigate the cross-over between the weakly interacting BCS regime (the regime where $a \to 0^-$) and Bose-Einstein condensate of dimers (the regime where $a \to 0^+$) \[2\]. These two regimes meet in strongly interacting limit where the scattering length is divergent and at this unitarity limit, the physics is expected to be universal \[3\].

The appearance of quantized vortices of a quantum fluid under rotation offers direct evidence of superfluidity. For example, the observation of quantized circulation in a rotating superfluid $^4$He \[4\] and the observation of vortex lattice in a rotating Fermi gas of $^6$Li \[5\] are two classic demonstrations of phase coherence in a superfluid. These are analog to the vortex lattice in type-II superconductors in the presence of a magnetic field. These vortices melt as the magnetic field increases and then the superconductors turn into normal at sufficiently large magnetic fields. For the case of rapidly rotating Fermi gases, the force due to the trapping frequency almost balances the centrifugal force and superfluid cloud spreads in the plane perpendicular to the rotation axis. At the limit of very large rotation, the theory predicts that the atomic system enters into the fractional quantum Hall regime \[6, 7\]. However, the fractional quantum Hall window is expected to be very small and inversely proportional to the number of atoms in the trap. A possible way of stabilizing the fractional quantum Hall regime is to add a positive quartic trapping potential.

In this paper, we study the collective breathing mode frequencies of a rotating Fermi gas in the presence of a quartic trapping potential by using a hydrodynamic approach. A negative, but small quartic term is always present with the Gaussian optical potentials in current experimental setups while added positive quartic term ensures the stability of the fast rotating regime. Thermodynamic properties of a Bose gas confined in harmonic plus quartic potential trap can be found in ref. \[5\]. As breathing mode frequencies are very sensitive to the equation of state, these dynamical quantities can be used as tests for various theories. The breathing mode frequencies have been measured for non-rotating Fermi systems in the BCS-BEC region \[6, 7\]. In most of the parameter regions, experimental results agree well with the hydrodynamic approaches, variational approaches and sum rule approaches \[11\]. However, the measured finite temperature axial and radial breathing mode frequencies show a striking increase in the intermediate BCS regime \[8, 10\], in contrast to zero temperature theoretical calculations in a harmonic trap. Furthermore, experimentalists were unable to measure the breathing mode frequencies in the weak coupling BCS limit. The deviation of the experimental data from theoretical results and the lack of experimental data in the weak coupling BCS limit were believed to be due the large Landau damping when the superfluid energy gap is much smaller than the collective oscillation energies. With inclusion of a positive quartic term in the trapping potential, we find somewhat similar deviation of the breathing mode frequencies in the intermediate regions of BCS regime. We find that the breathing mode frequencies deviate significantly from the modes frequencies calculated in a harmonic trap and the atomic cloud is unstable against the breathing mode oscillations at larger chemical potentials. As the chemical potential is larger in the weak coupling BCS limit, excitation of breathing mode frequencies make the atomic cloud unstable. It should be noted that the Gaussian optical trap potential provides a negative quartic term in the trapping potential in experimental setups. We investigate the effect of negative quartic term and find that the breathing mode frequencies tend to decrease in the entire BCS-BEC crossover region.

This paper is organized as follows. In section II, we present the derivation of breathing mode frequencies us-
We consider a rotating Fermi atomic system trapped in a harmonic plus radial quartic potential in the BCS-BEC crossover region. The trapping potential is

\[ V_{ex}(r, z) = \frac{1}{2} M \omega_r^2 r^2 + \frac{1}{2} M \omega_z^2 z^2 + \frac{K}{4} r^4 \]  

where \( M \) is the atom mass, \( \omega_r \)'s are the harmonic trapping frequencies, and \( r^2 = x^2 + y^2 \). The Fermi atomic system rotates about the z-axis at frequency \( \Omega \).

We restrict ourselves to the case of large number of vortices in the system where the wavelength of the oscillation frequencies is much larger than the inter-vortex distance. This condition always satisfies as the typical wavelength of the lowest mode oscillations is of the order of the system size. Further, we assume that the vortices are uniformly distributed in the superfluid so that we do not have to consider microscopic details of the single vortices. These assumptions are valid at the limit of large rotations where the atomic cloud spreads in the plane perpendicular to the rotation. Within this diffused vorticity approximation\[12\], diffuse vorticity is given by \( \nabla \times \mathbf{v} = 2 \Omega \mathbf{r} \), where superfluid velocity is given by \( \mathbf{v} = (\hbar/2M) \nabla \theta \). The local superfluid density \( n \) and the local phase \( \theta \) are related through the wave function \( \psi = \sqrt{n} e^{i\theta} \). The uniform vortex density is given by \( n_v = 2M \Omega/\hbar \).

Assuming local equilibrium, we start with the continuity and Euler equations of rotational hydrodynamics,

\[ \frac{\partial n}{\partial t} = -\nabla \cdot [n(r) \mathbf{v}] \tag{2} \]

and

\[ M \frac{\partial \mathbf{v}}{\partial t} = -\nabla \left[ \frac{1}{2} M \mathbf{v}^2 + V_{ex}(r, z) - \frac{1}{2} \Omega^2 r^2 + \mu(n) \right] + 2M \mathbf{v} \times \Omega + M \mathbf{v} \times \nabla \times \mathbf{v} \tag{3} \]

This hydrodynamic description is valid as long as the collisional relaxation time \( \tau \) is much smaller than the inverse of the oscillation frequencies; \( \omega_r \ll 1 \). The equation of state enters through the density dependent local chemical potential \( \mu(n) \). We fix the local chemical potential by introducing the equation of state in the form of \( \mu(n) \propto n^{(\gamma - 1)/\gamma} \). As we will discuss in the next subsection, the polytropic index \( \gamma \) is calculated by the method proposed by Manini and Salasnich\[13\] in the entire BCS-BEC crossover region. Linearizing the density \( n \) and the superfluid velocity \( \mathbf{v} \) around their equilibrium values as \( n = n_0(r) + \delta n, \mathbf{v} = \mathbf{v}_0 + \delta \mathbf{v} \) and \( \mu(n) = \mu(n_0) + \delta \mu \) with \( \delta \mu = (\partial \mu/\partial n)|_{n=n_0} \delta n \), we obtain the linearized version of the hydrodynamic equations.

\[ \frac{\partial \delta n}{\partial t} = -\nabla \cdot [n_0(r) \delta \mathbf{v}] \tag{4} \]

and

\[ M \frac{\partial \delta \mathbf{v}}{\partial t} = -\nabla \delta \mu + 2M \delta \mathbf{v} \times \Omega \tag{5} \]

Starting from these two linearized equations, collective breathing mode frequencies have been calculated in ref.[14] and ref.[15] for a harmonic trap. As the authors have used two different ansatz for the velocity fluctuation, they produce two different results for the breathing mode frequencies. In this paper we closely follow the approach adopted in ref.[15], generalizing the theory to an anharmonic trap. The ansatz used in ref.[15] ensures the conservation of angular momentum properly. In order to solve the linearized equations for the breathing mode frequencies, we take the equilibrium density in the lowest mode approximations as

\[ \{ \delta \Omega_1 \times r + \delta \Omega_2 \times r^2 \mathbf{r} + \nabla [\alpha \pm r^2 + \alpha_{\perp} z^2 + \beta r^4] \} \times \exp[-i\omega t] \tag{6} \]

and

\[ \delta n = n_0^{1-\gamma} (a_0 + a_{\perp} r^2 + a_{\perp} z^2 + b r^4) \exp[-i\omega t] \tag{7} \]

The first two terms \( \delta \Omega_1 \) and \( \delta \Omega_2 \) in Eq. (6) are parallel to the axis of rotation and guarantee that angular momentum is conserved during the oscillations. Substituting these two ansatz into Eq. (1) and Eq. (5), we derive four linear equations for the variational parameters \( a_0, a_{\perp}, a_{\perp}, a_{\perp}, \) and \( b \). These linear equations yield three non-zero solutions for the breathing mode frequencies \( \omega_m \approx \omega/\omega_r \) as roots of the following equation.

\[ A + B \omega_m^2 + C \omega_m^4 + \omega_m^6 = 0 \]

with,

\[ A \equiv -(1 - \zeta^2)^2 \delta^2[64d\gamma(\gamma + 1) + 48\gamma^2 + 56\gamma + 61] - (1 - \zeta^2)^2 \delta^2(32\gamma^2 + 104\gamma + 48) - 16\zeta^2 \delta^2(\gamma + 2), \]

\[ B \equiv (1 - \zeta^2)^2[8(\gamma + 1)(2\gamma + 1) + 16\gamma(\gamma + 2)] + (1 - \zeta^2)[\delta^2(8\gamma^2 + 26\gamma + 12) + \gamma^2(40\gamma + 24)] + 8\gamma \delta^2(\gamma + 2) + 16\gamma^4 \] and

\[ C \equiv (1 - \zeta^2)(2 - 10\gamma) - \delta^2(\gamma + 2) - 8 \]

The constants, \( \zeta \equiv \Omega/\omega_r, \delta \equiv \omega_z/\omega_r, \) and \( d \equiv 1/(1 - \zeta^2) \)[\(Kh/M^2\omega_r^6\)(\mu_0/\hbar \omega_r)] are a set of dimensionless parameters. The three solutions of Eq. (8) are the lowest order axial breathing mode frequency \( \omega_1 \) and the lowest and next lowest order radial breathing mode frequencies \( \omega_2 \) and \( \omega_3 \).
The Effective polytropic index and the chemical potential in the BCS-BEC crossover region

We use the proposal made by Manini and Salasnich [13] to calculate the effective polytropic index $\gamma$ and the chemical potential $\mu$ in the BCS-BEC crossover region. In the weak coupling BCS limit ($a \rightarrow 0^-$) and the unitarity limit ($a \rightarrow \infty$), the polytropic index is $\gamma = 2/3$. In the deep BEC limit ($a \rightarrow 0^+$), the polytropic index is $\gamma = 1$. In the BCS-BEC crossover regime, the scattering length’s dependence on $\gamma$ is given by [13]

$$\gamma = \frac{2/3 - 2y\epsilon'(y)/5 + y^2\epsilon''(y)/15}{\epsilon(y) - y\epsilon'(y)/5}$$  \hspace{1cm} (9)

with the parameter $y = 1/(k_f a)$ is the interaction parameter with $k_f$ being the Fermi wave vector. The function $\epsilon(y)$ is related to the energy per atom given by $E = (3/5)E_F(y)$, where $E_F = \hbar^2 k_F^2 / 2M$ is the Fermi atomic energy of a non-interacting Fermi system in the trap. Above $\epsilon'(y) = \partial \epsilon(y) / \partial y$ and the double prime indicates the second derivative of the function on its argument. Using the data presented in reference [16], Manini and Salasnich [13] used a data fitting scheme to derive an analytical form of the function $\epsilon(y)$ in the entire BCS-BEC region,

$$\epsilon(y) = \alpha_1 - \alpha_2 \arctan \left( \frac{\alpha_3 y \beta_1 + |y|}{\beta_2 + |y|} \right)$$  \hspace{1cm} (10)

Two different sets of parameters are proposed for $\alpha_i$’s and $\beta_i$’s in the BCS regime ($y < 0$) and the BEC regime ($y > 0$). In the BCS regime, the parameters are $\alpha_1 = 0.42$, $\alpha_2 = 0.3692$, $\alpha_3 = 1.044$, $\beta_1 = 1.4328$, and $\beta_2 = 0.5523$. In the BEC regime, the parameters are $\alpha_1 = 0.42$, $\alpha_2 = 0.2674$, $\alpha_3 = 5.04$, $\beta_1 = 0.1126$, and $\beta_2 = 0.4552$. The expression for the chemical potential $\mu$ is given by [13]

$$\mu = E_F \epsilon(y) - y\epsilon'(y)/5$$  \hspace{1cm} (11)

We determine the Fermi energy $E_F$ of a non-interacting Fermi system in a harmonic plus radial quartic potential through the number equation.

$$N = \frac{1}{15\delta} \sqrt{\frac{M^2 \omega_r^2}{\hbar K}} \left( \frac{E_F}{\hbar \omega_r} \right)^{5/2} f(E_F)$$  \hspace{1cm} (12)

We define the function $f(E_F)$ as

$$f(E_F) = \pm 8 \left( \pm 1 + \left(1 - \zeta^2\right)^2 \frac{1}{4 \left| K E_F \right|} \right)^{1/2} \mp \sqrt{\frac{(1 - \zeta^2)^2}{4 \left| K E_F \right|} \frac{1}{4 \left| K E_F \right|}}$$

$$\times \left[ 15 + 5(1 - \zeta^2)^2 \frac{1}{K E_F} + \frac{1}{2} (1 - \zeta^2)^4 \left( \frac{1}{(K E_F)^2} \right) \right]$$  \hspace{1cm} (13)

FIG. 1: Central chemical potential ($\mu_0$) of a non-rotating Fermi gas as a function of interaction parameter $1/(k_f a)$ for various values of $\bar{K}$. From top to bottom $\bar{K} = 0.05$ (long black dashed line), and 0.01 (short black dashed line), 0 harmonic trap (black solid line), $-0.005$ (short gray dashed line), and $-0.01$ (long gray dashed line). For the calculation, we use $N = 2.0 \times 10^{5}$ and $\delta = 0.045$.

FIG. 2: The lowest order radial breathing mode frequencies of a rotating Fermi gas in a harmonic trap with aspect ratio $\delta = 0.045$. The rotation frequencies are $\zeta = 0$ (solid line), 0.5 (long dashed line) and 0.9 (short dashed line).

where the scaled parameters are $\bar{K} \equiv \hbar K/(M^2 \omega_r^4)$ and $\bar{E}_F \equiv E_F/\hbar \omega_r$. The upper and lower signs are corresponding to $K > 0$ and $K < 0$ respectively. For the case of harmonic trap, the Fermi energy is $E_F = \hbar \omega_r [3N\delta(1 - \zeta^2)]^{1/3}$.

III. RESULTS AND DISCUSSION

In FIG. 1 we plot the central chemical potential of a non-rotating Fermi system calculated from Eq. (11) as a function of inverse scattering length for two different representative values of anharmonicity. We use $N = 2.0 \times 10^{5}$ number of atoms in the trap with $\delta = 0.045$. Our calculation shows a kink at unitarity limit due to the discontinuity in the function $\epsilon(y)$ proposed by Manini and Salasnich [13] (This kink appears in all the calculated macroscopic quantities).

Solving Eq. (13) for the case of harmonic potential trap...
(K = 0) in a rotating (ζ ≠ 0) Fermi system, the lowest axial and radial breathing modes frequencies are given by

\[ \omega_m^2 = \frac{1}{2}(\gamma (2 + \delta^2 - 2 \zeta^2) + 2(1 + \delta^2 + \zeta^2) \pm \sqrt{\{\gamma (2 + \delta^2 - 2 \zeta^2) + 2(1 + \delta^2 + \zeta^2)\}^2 - 8\delta^2 (\gamma (-3 + \zeta^2) - 2(1 + \zeta^2))} \}^{1/2}. \]  

(14)

For the isotropic trap (δ = 1), at non-interacting limit and at unitarity limit (γ = 2/3), the mode frequencies are \( \omega_m = 2 \) and \( \omega_m = \sqrt{2 + 2 \zeta^2} / 3 \). In the deep BEC limit (γ = 1), the mode frequencies are \( \omega_m = \sqrt{(1/2) (7 \pm \sqrt{9 - 8 \zeta^2})} \). For the case of harmonic potential (K = 0) in a non-rotating (ζ = 0) limit, Eq. (14) reduces to

\[ \omega_m^2 = \frac{1}{2}(2(1 + \delta^2) + \gamma (2 + \delta^2) \pm \sqrt{[2(1 + \delta^2) + \gamma (2 + \delta^2)^2 - 8(2 + 3\gamma)\delta^2]}. \]  

(15)

For the case of highly anisotropic limit (δ << 1), the two mode frequencies at \( \gamma = 2/3 \) are \( \omega_1/\omega_2 = \sqrt{12/5} \) and \( \omega_2/\omega_2 = \sqrt{\Omega^2 / 10 \gamma} / 3 \) as expected. For this case, the two mode frequencies at \( \gamma = 1 \) are \( \omega_1/\omega_2 = \sqrt{5} / 2 \) and \( \omega_2/\omega_2 = 2 \). In the BCS-BEC crossover region, the lowest order breathing modes frequencies are calculated from Eq. (15) by using the \( \gamma \) from Eq. (9). The results for several representative values of \( \gamma \) are given in FIG. 2 and FIG. 3.

We solve Eq. (8) for the breathing mode frequencies for various values of K in both rotating and non-rotating Fermi systems. We calculate the central chemical potential \( \mu_0 \) for fixed number of atoms \( N = 2.0 \times 10^6 \) in the trap. We find that as \( d = \sqrt{1/(1 - \zeta^2)^2} (\hat{K}\hbar / M \omega_2^2) (\mu_0 / \hbar \omega_r) = [1/(1 - \zeta^2)^2] \hat{K}\hat{\mu} \) increases, the lowest order radial breathing mode frequency increases while the next lowest order breathing mode frequency decreases. Then at a critical value of \( d = d_c \), these two modes merge and beyond this critical \( d_c \), the atomic cloud is unstable against the oscillations. FIG. 4 shows the lowest and the next lowest order radial mode frequencies at \( \gamma = 2/3 \) (non-interacting limit and unitarity limit) and \( \gamma = 1 \) (deep BEC limit).

As evidenced by FIG. 5, the lowest order axial breathing mode frequencies as a function of \( \hat{K} \) for \( \zeta = 0 \) (black), 0.3 (long dashed), 0.6 (short dashed) and, 0.9 (dotted) at unitarity. We use the values \( \delta = 0.045 \) and \( N = 2.0 \times 10^6 \).

In the BEC limit where \( \gamma = 1 \), we calculate the lowest and next lowest radial breathing mode frequencies as a function radial anharmonicities at unitarity.
function of radial anharmonicity $\tilde{K}$ for three representative values of rotational frequencies $\zeta$. We fixed the number of atoms to be $2.0 \times 10^6$ and $\tilde{\delta} = \tilde{\omega}_z/\omega_r = 0.045$. As shown in FIG. [4] as we increase $\tilde{K}$ the lowest order radial breathing mode frequency increases, while the next lowest order radial breathing mode frequency decreases. Further increase of $\tilde{K}$ merges these two modes and beyond this merging point the atomic cloud is unstable against the oscillations.

The lowest order and next lowest order radial breathing mode frequencies as a function of the interaction parameter $[1/(k_f a)]$ are shown in FIG. [7]. In the presence of radial anharmonicity, the lowest order mode frequency tends to increase in the BCS regime while the next lowest order breathing mode frequency tends to decrease. This deviation becomes large as the anharmonicity increases.

As we have discussed before, at larger $K\mu_0$ values the two lowest order modes merge and the cloud is unstable against the breathing mode oscillations beyond this point. The data in the FIG. [8] shows the same information as FIG. [4] but we plot only the lowest order radial breathing mode frequencies for small quartic potentials together with experimental data from ref. [10].

**IV. CONCLUSIONS**

We have discussed the breathing mode frequencies of a rotating Fermi gas trapped in a harmonic plus radial quartic potential. We find that the radial breathing mode frequencies strongly depend on the rotation and anharmonicity through parameter $d = [1/(1 - \zeta^2)^2](\hbar^2/(\Lambda^2 \omega_z^3})(\mu_0/\hbar \omega_r)$. As $d$ increases, the lowest order radial breathing mode’s frequency increases and the next lowest order mode decreases. Beyond some critical $d_c$, these two modes merge and the cloud is unstable against the oscillations.

As the chemical potential is large in the intermediate BCS regime, even with a very weak quartic potential the parameter $d$ is large. As a result, the lowest order breathing mode frequency increases in the intermediate BCS regime. Even though the Gaussian optical trap potential provides a negative anharmonic term in the trapping potential, this positive anharmonic behavior has been seen in recent experiments [9, 10]. In the weak coupling BCS limit, the chemical potential is even larger so that we find the atomic cloud is unstable against the oscillations at large positive anharmonicities. For negative quartic potentials, the breathing mode frequencies tend to decrease in the BCS-BEC crossover region.
V. ACKNOWLEDGEMENTS

This work was supported by the Binghamton University. We are very grateful to Kaden Hazzard for very enlightening discussions and critical comments on the manuscript.

[1] U. Fano, Phys. Rev. A 124, 1866 (1961); H Feshbach, Ann. Phys. 5, 357 (1961).
[2] C. A. Regal et al., Phys. Rev. Lett. 92, 040403 (2004); M. W. Zwierlein et al., Phys. Rev. Lett. 92, 120403 (2004); C. Chin et al., Science 305, 1128 (2004); T. Bourdel et al., Phys. Rev. Lett. 93, 050401 (2004); J. Kinast et al., Phys. Rev. Lett. 92, 150402 (2004); G. B. Partridge, et al., Phys. Rev. Lett. 95, 020404 (2005).
[3] H. Heiselberg Phys. Rev. A 63, 043606 (2001); K. M. OHara, S. L. Hemmer, M. E. Gehm, S. R. Granade, J. E. Thomas, Science 298, 2179 (2002); G. M. Bruun, Phys. Rev. A 70, 053602 (2004); Tin-Lun Ho, Phys. Rev. Lett. 92, 090402 (2004); J. Carlson, S.-Y. Chang, V. R. Pandharipande, and K. E. Schmidt, Phys. Rev. Lett. 91, 050401 (2003).
[4] S.C. Whitmore and W. Zimmermann, Jr., Phys. Rev. 166, 181 (1968).
[5] M. Zwierlein et al., Nature 435, 1047 (2005).
[6] H. Zhai and T.-L. Ho, Phy. Rev. Lett., 97, 180414 (2006); M. Y. Veillette, D. E. Sheehy, L. Radzihovsky, and V. Gurarie, Phys. Rev. Lett. 97, 250401 (2006); G. Moller and N. R. Cooper, Phys. Rev. Lett. 99, 190409 (2007).
[7] N. R. Cooper, N. K. Wilkin, and J. M. Gunn, Phys. Rev. Lett. 87, 120405 (2001); B. paredes, P. Zoller, and J. I. Cirac, Sol. St. Com. 127, 155 (2003).
[8] E. Lundh, Phys. Rev. A 65, 043604 (2002); K. Kasamatsu, M. Tsubota, and M. Ueda, Phys. Rev. A 66, 053606 (2002); G. Kavoulakis and G. Baym, New J. Phys. 5, 51.1 (2003); E. Lundh, A. Collin, and K. A. Suominen, Phys. Rev. Lett. 92, 070401 (2004); T. K. Ghosh, Phys. Rev. A 69, 043606 (2004); T. K. Ghosh, Eur. Phys. J. D 31 101 (2004); G. M. Kavoulakis, A. D. Jackson, and Gordon Baym, Phys. Rev. A 70, 043603 (2004); Ionut Danaila, Phys. Rev. A 72, 013605 (2005); A. Collin, Phys. Rev. A 73, 013611 (2006); S. Bargi, G. M. Kavoulakis, and S. M. Reimann, Phys. Rev. A 73, 033613 (2006); Michiel Snoek and H. T. Stoof, Phys. Rev. A 74, 033615 (2006); S. Gautam, D. Angom, Eur. Phys. J. D 46, 151155 (2008).
[9] J. Kinast, A. Turlapov, and J. E. Thomas, Phys. Rev. A 70, 051401(R) (2004); J. Kinast, A. Turlapov, and J. E. Thomas, Phys. Rev. Lett. 94, 170404 (2005); M. Bartenstein, A. Altmeier, S. Riedl, S. Jochim, C. Chin, J. H. Denschlag, R. Grimm, Phys. Rev. Lett. 92, 203201 (2004).
[10] J. Kinast, S. L. Hemmer, M. E. Gehm, A. Turlapov, and J. E. Thomas, Phys. Rev. Lett. 92, 150402 (2004).
[11] Theja N. De Silva and Erich J. Mueller, Phys. Rev. A 72, 063614 (2005); H. Heiselberg, Phys. Rev. Lett. 93, 040402 (2004); S. Stringari, Europhys. Lett. 65, 749 (2004); H. Hu, A. Minguzzi, X. J. Liu, and M. P. Tosi, Phys. Rev. Lett. 93, 190403 (2004); Y. E. Kim and A. L. Zubarev Phys. Rev. A 70, 033612 (2004); N. Manini and L. Salasnich, Phys. Rev. A 71, 033625 (2005); G. E. Astrakharchik, R. Combescot, X. Leyronas and, S. Stringari, Phys. Rev. Lett. 95, 030404 (2005); A. Bulgac and G. F. Bertsch, Phys. Rev. Lett. 94, 070401 (2005); M. Manini and L. Salasnich, Phys. Rev. A 71, 033625 (2005); Y.Ohashi and A. Griffin, e-print cond-mat/0503641.
[12] R. P. Feynman, edited by C. J. Gorter Progress in Low Temperature Physics, North- Holland, Amsterdam, (1955).
[13] N. Manini and L. Salasnich, Phys. Rev. A 71, 033625 (2005).
[14] T. K. Gosh and K. Machida, Phys. Rev. A 73, 025601 (2006).
[15] M. Antezza, M. Cozzini, and S. stringari, Phys. Rev. A 75, 053609 (2007).
[16] G. E. Astrakharchik, J. Boronat, J. Casulleras, and S. Giorgini, Phys. Rev. Lett. 93, 200404 (2004).