The off-forward Quark-Quark Correlation Function

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Abstract

The properties of the non-forward quark-quark correlation function are examined. We derive constraints on the correlation function from the transformation properties of the fundamental fields of QCD occurring in its definition. We further develop a method to construct an ansatz for this correlator. We present the complete leading order set of generalized parton distributions in terms of the amplitudes of the ansatz. Finally we conclude that the number of independent generalized parton helicity changing distributions is four.

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I. INTRODUCTION

In quantum field theory the non-perturbative nature of composite particles is described by matrix elements of operators of the fundamental fields of the theory evaluated between the initial and the final state of the particle under consideration. Accordingly, in QCD, the information on the internal partonic structure of hadrons is entirely given by matrix elements of all possible quark and gluon operators evaluated between hadronic states, forward and non-forward.

Forward matrix elements of quark and gluon operators, i.e. operators between hadronic states of equal momenta, are investigated in hard inclusive processes, while non-forward matrix elements, i.e. operators between hadronic states of different momenta, have been investigated in recent years in the context of non-forward high-energy exclusive processes such as Compton scattering in the deeply virtual kinematical limit (DVCS) and hard diffractive vector-meson production. Experimental results \cite{1,2,3,4} indicate the feasibility of measuring these processes much more precisely with dedicated experiments in the future.

In this article we will focus on hadronic matrix elements of bilinear quark operators, since these are the ones involved in the description of dominant contributions to non-forward hard processes. We consider their Fourier transforms, i.e. the quark-quark correlation functions, which contain all information concerning the non-perturbative nature of hadrons. From non-forward quark-quark correlation functions, for convenience, the so called generalized parton distributions (GPDs) are defined. \cite{5,6}

Following and generalizing the method developed for the ordinary forward parton distributions \cite{7,8}, we will formulate the most general ansatz for the quark-quark correlation functions starting from general principles, and we will then analyze leading order GPDs by projecting the ansatz with different Dirac matrices. In this way we will be able to express the GPDs in terms of the amplitudes entering the ansatz and establish a formal method for the determination and classification of the independent quark GPDs.

The number of independent GPDs occurring in a given Dirac projection is not self-evident. For instance it has been debated in the literature \cite{9,10} whether there are two or four independent quark helicity changing GPDs corresponding to the forward transversity distribution. With the method developed in this article we will show unambiguously that there are indeed four independent helicity changing GPDs at leading twist.
The outline of the work is as follows: in Section II we define the non-forward quark-quark correlation function and we introduce a new method to build an ansatz for it. We also derive constraints on the non-forward quark-quark correlator imposed by the hermiticity properties of the quark fields and their well-known behavior under parity and time reversal operations. In Section III we relate the non-forward correlation function to leading twist quark GPDs by tracing the ansatz with various Dirac matrices and by integrating over quark momentum components. Different Dirac structures probe different spin properties of the hadrons. Finally, in Section IV we draw conclusions and discuss the outlook for this subject.

II. THE NON-FORWARD QUARK-QUARK CORRELATION FUNCTION

We define the non-forward quark-quark correlation function \( \Phi_{\Lambda, j\Lambda}(k, k', P, P') \), depending on the hadron and quark momenta (see Fig. I for notation), by Fourier transforming the hadronic matrix elements of quark-quark operators

\[
\Phi_{\Lambda, j\Lambda}(k, k', P, P') = \frac{1}{(2\pi)^{4}} \int d^{4}z \ e^{i(k+k') \cdot z/2} \langle P', \Lambda' | \bar{\psi}_{i}(\frac{z}{2}) \psi_{j}(\frac{z}{2}) | P, \Lambda \rangle .
\]  

(1)

We assume the hadron is a spin \( \frac{1}{2} \) particle, say a nucleonic target, which is in an eigenstate of light-cone helicity characterized by the initial and final light-cone helicity, \( \Lambda \) and \( \Lambda' \) respectively, which are defined from the spin vectors \( S_{\mu} \) and \( S'_{\mu} \)

\[
S_{\mu} = \frac{\Lambda}{m}(P_{\mu} - \frac{m^{2}}{P^{+}} v'^{\mu}) , \quad S'_{\mu} = \frac{\Lambda'}{m}(P'^{\mu} - \frac{m^{2}}{P'^{+}} v'^{\mu}) ,
\]  

(2)

where \( m \) is the hadron mass, and \( v'^{\mu} \) is the null vector on the light-cone \( v'^{\mu} = [0, 1, \vec{0}_{T}] \).

Throughout the paper we use the component notation \( u = [u^{+}, u^{-}, \vec{u}_{T}] \) with \( u^{\pm} = (u^{0} \pm u^{3})/\sqrt{2} \) and the transverse part \( \vec{u}_{T} = (u^{1}, u^{2}) \). Explicit representations of spinors for light-cone helicity eigenstates are given for instance by Kogut and Soper \[13\] or Brodsky and Lepage \[14\]. The non-forward correlation function in Eq. (1) represents a transition matrix element of quark-quark operators and not an expectation value like the forward quark-quark correlation function \[15\]. Therefore it provides more general information about hadrons compared to the ordinary quark-quark correlation function.

The known properties of the quark-quark correlation function determine the structure of an ansatz, which will be the starting point for our analysis. The quark-quark correlation
FIG. 1: Diagrammatic representation of the correlation function \( \Phi_{i\Lambda, j\Lambda}(k, k', P, P') \). From an initial hadron with momentum \( P \) a quark with momentum \( k \) is taken out, and reinserted with momentum \( k' = k + \Delta \) to form the final state hadron with changed momentum \( P' = P + \Delta \).

The function in the helicity basis, defined in Eq. (1), is a \( 4 \times 4 \) matrix in the partonic Dirac space, labelled by \( i \) and \( j \), and a \( 2 \times 2 \) matrix in the hadronic helicity space, labelled by \( \Lambda \) and \( \Lambda' \). Therefore we formulate an ansatz given by the product of a partonic and a hadronic sector separately. The partonic sector is spanned by the set \( \hat{\Gamma}_{ij} \) of the 16 independent \( 4 \times 4 \) partonic Dirac matrices, and the hadronic sector is represented by all possible independent spinorial products, \( \bar{u}_k(P', \Lambda') \Gamma_{kl} u_l(P, \Lambda) \), evaluated between final and initial light-cone helicity states. Since the correlation function in Eq. (1) is a scalar in Lorentz space, we saturate the open indices of the tensorial product of hadronic and partonic sectors with all possible independent tensors, \( t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_h} \), constructed from the kinematical variables \( \bar{k} = (k + k')/2 \), \( \bar{P} = (P + P')/2 \), and \( \Delta = P' - P \), the metric tensor \( g_{\alpha\beta} \) and one antisymmetric tensor \( \epsilon_{\alpha\beta\rho\sigma} \), leading to the ansatz of the form

\[
\Phi_{i\Lambda, j\Lambda}(\bar{k}, \bar{P}, \Delta) = \hat{\Gamma}_{ij}^{\mu_1\cdots\mu_p} \left[ \bar{u}_k(P', \Lambda') \Gamma_{kl}^\nu u_l(P, \Lambda) \right] t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_h}(\bar{P}, \bar{k}, \Delta). \tag{3}
\]

In the hadronic sector the spinorial products (spinor indices suppressed)

\[
\bar{u}(P', \Lambda') u(P, \Lambda), \quad \bar{u}(P', \Lambda') \gamma_5 u(P, \Lambda), \quad \bar{u}(P', \Lambda') \sigma^{\alpha\beta} u(P, \Lambda), \tag{4}
\]

\[
\bar{u}(P', \Lambda') \gamma^\alpha u(P, \Lambda), \quad \bar{u}(P', \Lambda') \gamma^\alpha\gamma_5 u(P, \Lambda) \tag{5}
\]

occur. From the Gordon identities we can deduce that only the three spinor products in (4) are independent. These spinorial products are evaluated in a specific frame in Appendix A. Furthermore, not all possible contractions of indices have to be taken into account in the construction of the ansatz for the correlation function. The relation

\[
0 = \bar{u}(P', \Lambda') u(P, \Lambda) \Delta^\alpha + \bar{u}(P', \Lambda') i \sigma^{\alpha\beta} 2P_\beta u(P, \Lambda) \tag{6}
\]
shows that contractions of the tensorial spinor product $\bar{u} \sigma^{\alpha \beta} u$ with $P_\beta$ reduce to the scalar spinor product multiplied with $\Delta^\alpha$, and the relation

$$0 = \bar{u}(P', \Lambda') \gamma_5 u(P, \Lambda) 2 \bar{P}^\alpha + \bar{u}(P', \Lambda') i \sigma^{\alpha \beta} \gamma_5 \Delta_\beta u(P, \Lambda)$$

(7)

together with the known identities

$$\sigma^{\alpha \beta} \epsilon_{\alpha \beta \rho \sigma} = -2i \sigma_{\rho \sigma} \gamma_5$$
$$\sigma^{\alpha \beta} \epsilon_{\alpha \rho \sigma \tau} = -i \gamma_5 (\sigma_{\rho \sigma} g^\beta_\tau - \sigma_{\rho \tau} g^\beta_\sigma + \sigma_{\sigma \tau} g^\beta_\rho)$$

(8)

entails

$$\bar{u}(P', \Lambda') \sigma^{\alpha \beta} u(P, \Lambda) \epsilon_{\alpha \beta \rho \sigma} \Delta^\rho = 4 \bar{u}(P', \Lambda') \gamma_5 u(P, \Lambda) \bar{P}_\sigma ,$$
$$\bar{u}(P', \Lambda') \sigma^{\alpha \beta} u(P, \Lambda) \epsilon_{\alpha \rho \sigma \tau} \Delta^\rho = \bar{u}(P', \Lambda') \gamma_5 u(P, \Lambda) \left(2 \bar{P}_\sigma g^\beta_\tau - 2 \bar{P}_\tau g^\beta_\sigma\right)$$
$$+ \frac{1}{2} \bar{u}(P', \Lambda') \sigma^{\gamma \delta} u(P, \Lambda) \epsilon_{\gamma \delta \sigma \tau} \Delta^\rho ,$$

(9)

which reveals that the tensorial spinor product $\bar{u} \sigma^{\alpha \beta} u$ contracted with either $\epsilon_{\alpha \beta \rho \sigma} \Delta^\rho$ or $\epsilon_{\alpha \rho \sigma \tau} \Delta^\rho$ is proportional to structures already accounted for. Note that the tensors $t_{\mu_1 \cdots \mu_p\nu_1 \cdots \nu_q}(\bar{P}, \bar{k}, \Delta)$ contain at most one Levi-Civita symbol since tensors with more than one do not result in new structures. The product of two Levi-Civita tensors reduces to the product of Kronecker symbols, from which no new structure arises. In a similar way the case of tensors $t_{\mu_1 \cdots \mu_p\nu_1 \cdots \nu_q}(\bar{P}, \bar{k}, \Delta)$ with more than two Levi-Civita symbols can be excluded.

The correlation function (1) fulfills the following constraints derived from properties of the Dirac quark fields and of the hadronic states under hermitian conjugation, parity and time reversal transformations (cf. [8, 16])

$$\Phi_{i\Lambda, j\Lambda}^f(k, k', P, P') = \gamma_0 \Phi_{i\Lambda, j\Lambda'}(k', k, P', P) \gamma_0 \quad \text{(hermiticity constraint)}$$
$$\Phi_{i\Lambda', j\Lambda}(k, k', P, P') = \gamma_0 \Phi_{i-\Lambda', j-\Lambda'}(\bar{k}, \bar{k}', \bar{P}, \bar{P}') \gamma_0 \quad \text{(parity constraint)}$$
$$\Phi^*_{i\Lambda', j\Lambda}(k, k', P, P') = (-i \gamma_5 C) \Phi_{i\Lambda', j\Lambda}^f(\bar{k}, \bar{k}', \bar{P}, \bar{P}') (-i \gamma_5 C) \quad \text{(time reversal constraint)} ,$$

(10)

where the notation $\tilde{u} = (u^0, -\vec{u})$ for momenta and spin vectors is used. These constraints can be implemented in building the ansatz for the correlation function. As far as the time
reversal constraint is concerned, Collins has shown that the time reversal constraint is not applicable if a Wilson line is inserted in the quark correlation function. Therefore imposing the time reversal constraint corresponds to defining the correlator without a Wilson line.\[12\]

Taking into account all the above information leads to the most general ansatz for $\Phi$. The lengthy expressions are an intermediate result of our investigation and are explicitly given in Appendix B.

### III. GENERALIZED PARTON DISTRIBUTIONS

The generalized parton distributions $\Phi_{\Lambda',\Lambda}(x, \xi; t)$ are obtained as traces of the quark-quark correlation function with the Dirac matrices $\Gamma$, integrated over the transverse and minus components of the quark momentum, $\vec{k}_T$ and $\bar{k}$, respectively,

$$
\Phi_{\Lambda',\Lambda}^{[\Gamma]}(x, \xi; t) = \frac{1}{2} \int d^2 \vec{k}_T d\bar{k}^- \, Tr[\Phi_{\Lambda',\Lambda}^{\Lambda,\Gamma}],
$$

where $x$ and $\xi$ are the light-cone momentum fractions $x \equiv \bar{k}^+ / P^+$ and $\xi \equiv -\Delta^+ / (2 P^+)$ and the Mandelstam variable $t = \Delta^2$ denotes the momentum transfer squared. The projections $\Phi_{\Lambda',\Lambda}^{[\Gamma]}$ carry only hadron helicity indices since the parton Dirac indices have been saturated in taking the trace. Following \[7, 8\] we rewrite the integral in Eq. (11) with respect to the covariant integration variables, $\sigma = 2 \bar{P} \cdot \bar{k}$ and $\tau = \bar{k}^2$, and the azimuthal angle $\phi$

$$
\Phi_{\Lambda',\Lambda}^{[\Gamma]}(x, \xi; t) = \int d\sigma d\tau d\phi \, \theta(x \sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau) \, Tr[\Phi_{\Lambda',\Lambda}^{\Lambda,\Gamma}] \frac{4 \bar{P} +}{4 \bar{P}^+}.
$$

The projections of the quark-quark correlation function $\Phi$ with the various Dirac matrices $\Gamma$ in Eq. (12) correspond to the different generalized distribution functions and determine also which distribution functions occur in a non-forward hard process at different orders in $1/\bar{P}^+$, $\bar{P}^+$ scaling with the hard scale in the process. In particular the leading order (twist 2) distribution functions \[7, 8, 17\] are obtained by projecting out the ansatz for the correlation function with the Dirac matrices $\gamma^+, \gamma^+ \gamma_5$ and $i \sigma^{+i} \gamma_5$.

#### A. Unpolarized parton distribution

Let us consider the unpolarized generalized distribution functions of the proton. Thus we project the non-forward quark-quark operator with the matrix $\gamma^+$. Following \[19\] using
the notation of \[18\] the GPDs $H(x, \xi; t)$ and $E(x, \xi; t)$ are defined by

$$\Phi_{\Lambda'} = \frac{1}{2} \int \frac{dz}{2\pi} e^{iz \cdot \hat{P}^+} \langle P', \Lambda' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | P, \Lambda \rangle \big| z^+ = 0, z^- = 0$$

$$= \frac{\bar{u}(P', \Lambda') \gamma^+ u(P, \Lambda)}{2P^+} H(x, \xi; t) + \frac{\bar{u}(P', \Lambda') i \sigma^+ \Delta \Delta u(P, \Lambda)}{4mP^+} E(x, \xi; t).$$

(13)

Evaluating the quantity $\Phi_{\Lambda'}$ for both proton helicity flip ($\Lambda' = -\Lambda$) and helicity non-flip ($\Lambda' = \Lambda$), the generalized distribution functions, $H$ and $E$, result from the following set of two equations

$$\Phi_{++} = \Phi_{--} = \sqrt{1 - \xi^2} \left( H - \frac{\xi^2}{1 - \xi^2} E \right),$$

$$\Phi_{+-} = - \left( \Phi_{+}^{\gamma^+} \right)^* = \eta \frac{\sqrt{t_0 - t}}{2m} E$$

with $t_0$ defined as

$$t_0 = \frac{4\xi^2m^2}{1 - \xi^2},$$

(14)

and a phase $\eta$ given by

$$\eta = \frac{\Delta^1 + i\Delta^2}{|\Delta|}.$$  

(15)

Substituting the expression of the ansatz (B7) and (B8) for the helicity non-flip and helicity flip correlation functions, $\Phi_{++}$ or $\Phi_{--}$, and $\Phi_{+-}$ or $\Phi_{-+}$, respectively, in (12) and tracing with $\gamma^+$ we have a set of two equations for the two unknown distribution functions $H$ and $E$, which is solvable and gives the two unpolarized generalized parton distribution functions as

$$H = \frac{1}{\sqrt{1 - \xi^2}} \left( A^{(1)}(x, \xi; t) - \frac{2m\xi^2}{\sqrt{1 - \xi^2} \sqrt{t_0 - t}} A^{(3)}(x, \xi; t) \right),$$

$$E = - \frac{2m}{\sqrt{t_0 - t}} A^{(3)}(x, \xi; t),$$

(17)

where we have introduced the function $A^{(1)}(x, \xi; t)$ and $A^{(3)}(x, \xi; t)$ defined through the coefficients $d^{(n)}_m$ of Eqs. (B7) and (B8)

$$A^{(1)}(x, \xi; t) = \frac{1}{\sqrt{1 - \xi^2}} \int d\sigma d\tau d\phi (x \sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau) [d^{(1)}_5 + x d^{(1)}_7 + 4\xi^2 d^{(1)}_{10}].$$

(18)
and

\[
A^{(3)}(x, \xi; t) = \int d\sigma d\tau d\phi \theta(x \sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau) \eta \\
\left\{ \frac{m}{\sqrt{t - t_0} \sqrt{1 - \xi^2}} (-2 \xi \left[ d_6^{(3)} + x d_9^{(3)} + d_{11}^{(3)} + 4 \xi^2 d_{13}^{(3)} \right] \\
- 2 \xi \frac{\sqrt{t_0 - t}}{m} d_{112}^{(3)} - 2 \xi \frac{m}{\sqrt{t - t_0}} d_{96}^{(3)} \right\} .
\]  

(19)

The expressions for the generalized distribution functions \( H \) and \( E \) in Eq. (17) do not contain any term in the amplitudes \( d_m^{(4)} \) and \( d_m^{(2)} \), since these amplitudes are related to the amplitudes \( d_m^{(1)} \) and \( d_m^{(3)} \), respectively, as shown in Eqs. (B4), (B5), (B6), (B9), (B10) and (B11).

In the forward case rotational invariance around the collinear axis implies the conservation of the longitudinal component of angular momentum, i.e. it requires total helicity to be conserved (refer to Fig. 1)

\[
\Lambda + \lambda' = \Lambda' + \lambda .
\]  

(20)

Helicity conservation in Eq. (20) then shows a link between the quark and nucleon helicity degrees of freedom. Through the projection of the quark-quark correlation function with the matrix \( \gamma^+ \) the nucleon helicity does not flip and the only possible hadron helicity combinations in the forward limit are ++ and −−.

B. Polarized parton distributions

The polarized quark distributions, \( \tilde{H}(x, \xi; t) \) and \( \tilde{E}(x, \xi; t) \), are defined by the Fourier transforms of the axial vector matrix element

\[
\Phi_{\Lambda'\Lambda}^{[\gamma^+ \gamma_5]} \equiv \frac{1}{2} \int \frac{dz_-}{2\pi} e^{ix \cdot z_-} \langle P', \Lambda' | \bar{\psi}_q(-\frac{z_-}{2}) \gamma^+ \gamma_5 \psi_q(\frac{z_-}{2}) | P, \Lambda \rangle |_{z_-=0, z_+^+} = \frac{\bar{u}(P', \Lambda') \gamma^+ \gamma_5 u(P, \Lambda)}{2P^+} \tilde{H}(x, \xi; t) + \frac{\bar{u}(P', \Lambda') \Delta^+ \gamma_5 u(P, \Lambda)}{4m P^+} \tilde{E}(x, \xi; t) .
\]  

(21)

For the different proton helicity combinations we now find

\[
\Phi_{++}^{[\gamma^+ \gamma_5]} = -\Phi_{--}^{[\gamma^+ \gamma_5]} = \sqrt{1 - \xi^2} \left( \tilde{H} - \frac{\xi^2}{1 - \xi^2} \tilde{E} \right) ,
\]  

(22)

and

\[
\Phi_{-+}^{[\gamma^+ \gamma_5]} = \left( \Phi_{-+}^{[\gamma^+ \gamma_5]} \right)^* = \eta \frac{\sqrt{t_0 - t}}{2m} \xi \tilde{E} .
\]  

(23)
Substituting the ansätze (B7) and (B8) for the correlation functions we obtain a set of two equations in the two unknown functions $\tilde{H}$ and $\tilde{E}$

$$\tilde{H} = \frac{1}{\sqrt{1 - \xi^2}} \left( B^{(1)}(x, \xi; t) - \frac{2m\xi}{\sqrt{1 - \xi^2} \sqrt{t_0 - t}} B^{(3)}(x, \xi; t) \right),$$

$$\tilde{E} = -\frac{2m}{\xi \sqrt{t_0 - t}} B^{(3)}(x, \xi; t),$$

where we have introduced the function $B^{(1)}(x, \xi; t)$ and $B^{(3)}(x, \xi; t)$ defined as

$$B^{(1)}(x, \xi; t) = \frac{1}{\sqrt{1 - \xi^2}} \int d\sigma d\tau d\phi \theta(x \sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau)[d^{(1)}_{12} + x d^{(1)}_{14} + 4\xi^2 d^{(1)}_{17}],$$

and

$$B^{(3)}(x, \xi; t) = \int d\sigma d\tau d\phi \theta(x \sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau) \eta \left\{ \frac{m}{\sqrt{t - t_0} \sqrt{1 - \xi^2}} [d^{(3)}_{16} + 4\xi^2 d^{(3)}_{18} + x (d^{(3)}_{19} + 4\xi^2 d^{(3)}_{21}) + 4\xi^2 d^{(3)}_{23}] ight\} + \frac{m}{\sqrt{t - t_0} \sqrt{1 - \xi^2}} \left[ 2\xi (d^{(3)}_{97} - x d^{(3)}_{98}) - \frac{\sqrt{t_0 - t}}{m}[d^{(3)}_{113} + x d^{(3)}_{114}] \right].$$

respectively.

**C. Parton helicity changing distributions**

There are also twist 2 generalized distributions that change the helicity of the active parton. The corresponding quark distributions are constructed from the operator $\bar{\psi}_q i \sigma^i \gamma_5 \psi_q$. By counting the helicity amplitudes Hoodbhoy and Ji [9] introduce two independent quark helicity changing generalized distributions corresponding to hadron helicity flip and non-flip terms. However, Diehl [10] claims that there are four independent parton helicity changing
generalized distributions defined by

\[ \mathcal{G}^i_{\Lambda\Lambda} \equiv \frac{\sqrt{1 - \xi^2}}{2\sqrt{1 - \xi^2}} \int \frac{dz^-}{2\pi} e^{ix P^+ z^-} \langle P', \Lambda' \vert \bar{\psi}_q(\frac{-z}{2}) i \sigma^i \gamma_5 \psi_q(\frac{z}{2}) \vert P, \Lambda \rangle \bigg|_{z^+=0, z_T=0} \]

\[ = \bar{u}(P', \Lambda') i \sigma^i \gamma_5 u(P, \Lambda) H_T(x, \xi; t) + \bar{u}(P', \Lambda') \frac{i \epsilon^{+i\alpha\beta} \Delta_{\alpha} P_{\beta}}{m^2} u(P, \Lambda) \tilde{H}_T(x, \xi; t) \]

\[ + \bar{u}(P', \Lambda') \frac{i \epsilon^{+i\alpha\beta} \Delta_{\alpha} \gamma_{\beta}}{2m} u(P, \Lambda) E_T(x, \xi; t) + \bar{u}(P', \Lambda') \frac{i \epsilon^{+i\alpha\beta} \gamma_{\beta}}{m} u(P, \Lambda) \tilde{E}_T(x, \xi; t). \]

(27)

In Eq. (27) the Lorentz index \( i \) takes the values 1,2). In the following we show that the same conclusion concerning the number of independent helicity changing GPDs is obtained by tracing the ansatz for the quark-quark correlation given in Eq. (B7) with the matrix \( \psi_q i \sigma^i + i \gamma_5 \). Following Diehl [10] we define

\[ \mathcal{G}^i_{\Lambda\Lambda} = \frac{\Phi^{(\sigma^i \gamma_5)}_{\Lambda\Lambda}}{\sqrt{1 - \xi^2}}. \quad (28) \]

By comparing the definitions for \( \mathcal{G}^i_{\Lambda\Lambda} \) in Eq. (28) and the projections of the non-forward correlation function \( \Phi \) with matrix \( i \sigma^i \gamma_5 \), the GPDs, \( H_T, \tilde{H}_T, E_T, \) and \( \tilde{E}_T \), arise. In fact the trace of the ansatz for the non-forward quark-quark correlation function with the matrix \( i \sigma^i \gamma_5 \) gives a system of four linear independent equations

\[ \Phi^{(i \sigma^i \gamma_5)}_{++}(x, \xi; t) = C^{(1)i} + D^{(1)i} \]

\[ \Phi^{(i \sigma^i \gamma_5)}_{+-}(x, \xi; t) = C^{(1)i} - D^{(1)i} \]

\[ \Phi^{(i \sigma^i \gamma_5)}_{-+}(x, \xi; t) = C^{(3)i} + D^{(3)i} \]

\[ \Phi^{(i \sigma^i \gamma_5)}_{--}(x, \xi; t) = C^{(3)i*} - D^{(3)i*} \]

(29)

where \( C^{(1)i} \), expressed in terms of the coefficients of the ansatz, reads

\[ C^{(1)i} = \frac{1}{\sqrt{1 - \xi^2}} \int d\sigma d\tau d\phi \theta(x \sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau) \left\{ - \frac{\epsilon^{+i\sigma \rho} \Delta_{\sigma} d^{(1)}_{21}}{m P^+ P^+ \Delta_\rho d^{(1)}_{23}} + \frac{\epsilon^{+i\sigma \rho} \Delta_{\sigma} d^{(1)}_{23}}{m P^+ P^+ \Delta_\rho d^{(1)}_{23}} \right\} \]

(30)
and $D^{(1)i}$ reads

$$
D^{(1)i} = \frac{1}{\sqrt{1-\xi^2}} \int d\sigma d\tau d\phi \theta(x\sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau) \left\{ \Delta^i \left[ \frac{i}{m^3} (\xi\sigma - 2m^2 \xi x + \vec{k} \cdot \Delta) d_{56}^{(1)} \right. \\
+ \frac{i}{m^3} (\xi x\sigma - 2\xi \tau - x \vec{k} \cdot \Delta) d_{57}^{(1)} + \frac{2i\xi}{m} (d_{31}^{(1)} - x d_{33}^{(1)}) \right\},
$$

where $\vec{k} \cdot \Delta = x(m^2 - \frac{t}{4}) (\xi + 1) - \xi \sigma - \vec{k}_T \cdot \Delta_T$.

The function $C^{(3)i}$ can be written as

$$
C^{(3)i} = \int d\sigma d\tau d\phi \theta(x\sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau) \eta \left\{ \frac{m}{\sqrt{t-t_0} \sqrt{1-\xi^2}} \left[ \Delta^i m \sqrt{1-\xi^2} \right. \\
+ \frac{\Delta^i}{m^3} \left. d_{88}^{(3)} (2m^2 \xi x - 2\xi \vec{k} \cdot \vec{P} - \vec{k} \cdot \Delta) + \frac{\Delta^i}{m^3} 2\xi d_{90}^{(3)} (2\xi \vec{k} \cdot \vec{P} - 2\xi \vec{k}^2 - x \vec{k} \cdot \Delta) \\
- 2 \frac{\epsilon^{\rho\sigma+i}}{m P^+} \vec{P}_\rho \Delta_\sigma \xi d_{31}^{(3)} - 2 \frac{\epsilon^{\rho\sigma+i}}{m P^+} \vec{k}_\rho \Delta_\sigma \xi d_{34}^{(3)} + \frac{\Delta^i}{m^3} \right] \right\}
$$

(32)

and $D^{(3)i}$ in terms of the amplitudes $d_{m}^{(3)}$ is

$$
D^{(3)i} = \int d\sigma d\tau d\phi \theta(x\sigma - x^2 m^2 + \frac{x^2 t}{4} - \tau) \eta \left\{ \frac{m}{\sqrt{t-t_0} \sqrt{1-\xi^2}} \Delta^i m \sqrt{1-\xi^2} \right. \\
+ \frac{\Delta^i}{m^3} \left. 2\xi d_{86}^{(3)} (x^2 m^2 - 2x \vec{k} \cdot \vec{P} + \vec{k}^2) \right] \right\}
$$

(33)

Inserting in Eq. (27) the results for the spinorial products, given in the Appendix A, we obtain the four possible helicity combinations

$$
G^j_{++} = H_T \frac{\epsilon^{+j\rho\sigma} \vec{P}_\rho \Delta_\sigma \xi \sqrt{1-\xi^2}}{(1+\xi)m} \\
- \bar{H}_T 2m \epsilon^{+j\rho\sigma} \vec{P}_\rho \Delta_\sigma \sqrt{1-\xi^2} \\
+ E_T \frac{2 (\vec{P}^+ \Delta^j + \xi \epsilon^{+j\rho\sigma} \vec{P}_\rho \Delta_\sigma)}{\sqrt{1-\xi^2}} \\
+ \bar{E}_T \frac{-i \vec{P}^+ \Delta^j + \xi \epsilon^{+j\rho\sigma} \vec{P}_\rho \Delta_\sigma}{\sqrt{1-\xi^2}}
$$

(34)
\[ G_{i-}^j = H_T \frac{\epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma \xi \sqrt{1 - \xi^2}}{(1 + \xi)m} \]

\[ - i \tilde{H}_T 2m \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma \sqrt{1 - \xi^2} \]

\[ + E_T \left( - \tilde{P}_+ \Delta^j \xi + i \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma \right) \]

\[ + \tilde{E}_T i \tilde{P}_+ \Delta^j + \xi \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma \sqrt{1 - \xi^2} \]

\[ (35) \]

\[ G_{i+}^j = -\eta \left[ -2 H_T \frac{-i \tilde{P}_+ \Delta^j + \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma}{\sqrt{t_0 - t}} \right. \]

\[ - \tilde{H}_T \frac{\epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma (4m^2\xi^2 - \xi^2 t + t)}{\sqrt{1 - \xi^2} \sqrt{t_0 - t}} \]

\[ + 4m E_T \xi^2 \frac{- \tilde{P}_+ \Delta^j + i \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma}{\sqrt{1 - \xi^2} \sqrt{t_0 - t}} \]

\[ - 2m \xi \tilde{E}_T \left( - \tilde{P}_+ \Delta^j + i \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma \right) \]

\[ (36) \]

\[ G_{i-}^j = \eta^* \left[ -2 H_T \frac{i \tilde{P}_+ \Delta^j + \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma}{\sqrt{t_0 - t}} \right. \]

\[ - \tilde{H}_T \frac{\epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma (4m^2\xi^2 - \xi^2 t + t)}{\sqrt{1 - \xi^2} \sqrt{t_0 - t}} \]

\[ + 4m \xi^2 E_T \frac{\tilde{P}_+ \Delta^j + i \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma}{\sqrt{1 - \xi^2} \sqrt{t_0 - t}} \]

\[ - 2m \xi \tilde{E}_T \left( - \tilde{P}_+ \Delta^j + i \epsilon^{+j\rho} \bar{P}_\rho \Delta_\sigma \right) \]

\[ (37) \]

Since we are only interested in the number of independent GPDs we refrain from isolating \( H_T, \tilde{H}_T, E_T, \) and \( \tilde{E}_T \) in Eqs. (34), (35), (36) and (37). Instead, from the fact that Eq. (29) constitutes a set of four linearly independent equations, we conclude that one can define four independent GPDs from it. Thus, we have shown that the number of independent helicity changing generalized parton distributions is four, as claimed by Diehl [10].

IV. CONCLUSIONS

We have presented a detailed analysis of the non-forward quark-quark correlation function. Constraints on the correlation function, not yet in the literature, were obtained by
implementing the known properties of the fundamental fields of QCD, quarks and gluons, under parity and time reversal transformations and applying hermiticity. We developed a new method to construct an ansatz for the correlation function. The quark-quark correlation function could then be expressed in terms of tensorial structures formed by the independent dynamical vectors and by Dirac matrices. The constraints obtained were implemented to reduce the number of independent amplitudes multiplying these tensorial structures in the ansatz.

Finally we projected out the leading order GPDs, i.e. we expressed the unpolarized, polarized and parton helicity flip distributions in terms of the amplitudes occurring in the ansatz. The formalism adopted allowed us to conclude that the number of independent parton helicity changing distributions is four, in agreement with Diehl’s argument [10]. We stress that the result about the number of the independent GPDs was obtained by Diehl in a completely different way and this is a confirmation of both methods used to approach the problem. On one hand we wrote the most general ansatz which can describe non-forward quark-quark correlation functions, i.e. we represented matrix elements of non-local non-forward quark-quark operators in terms of tensorial structures, built from the involved momenta on the basis of general properties of invariance. Then we traced the ansatz for the non-forward correlation function with different Dirac matrices and we could read off which of these structures contribute to each GPD. On the other hand Diehl’s approach was to count the number of independent helicity amplitudes occurring in DVCS cross sections on the basis of time reversal and parity invariance which these amplitudes have to fulfill.

The advantage of having built an ansatz for the non-forward quark-quark correlation function is that, by tracing it with the different \( \Gamma \) Dirac matrices, we gain the different generalized distribution functions in terms of some of the amplitudes occurring in the ansatz, and we are thus able to predict the dependence of GPDs upon the different fundamental structures entering the ansatz.

Note that in Eq. (11) we integrate over \( d^2 \tilde{k}_T d \tilde{k}^- \) and thus we consider only distribution functions which do not depend on the transverse momentum of the quarks \( \tilde{k}_T \). We remark that, in principle by having an ansatz for the off-forward quark-quark correlator, one could extract generalized profile functions which depend additionally on the transverse momentum \( \tilde{k}_T \) of quarks. For instance the investigation of \( \tilde{k}_T \)-depending ordinary parton distributions has been extensively carried out by many groups theoretically and experimental investiga-
tions are currently under way. For non-forward processes no formalism for the systematic
study of $\tilde{k}_T$-dependence has yet been attempted and an experimental program on
$\tilde{k}_T$ effects seems far beyond present abilities. In the foreseeable future there are good prospects to
acquire some knowledge on GPDs depending on $(x, \xi, t; Q^2)$, as well as possibly additional
$\tilde{k}_T$-dependence.

We have worked out a powerful method of analysis which in the present paper was applied
completely to the leading twist level. The same method can be implemented to investigate
twist 3 and twist 4 generalized distribution functions. For instance one could expect that
useful relations between leading and next to leading order generalized distributions could
emerge as suggested by similar experience in the forward case. In this sense the present
work represents a valuable starting point for further investigations.

APPENDIX A: SPINORIAL PRODUCTS

The independent spinorial products of ansatz (3) listed in Eq. (4) are most easily evaluated
by using explicit expressions for light-cone helicity spinors

\begin{equation}
\bar{u}_{LC}(P,+) \frac{1}{(2\sqrt{2}P^+)^{1/2}} \begin{pmatrix}
\sqrt{2} P^+ + m \\
P^1 + i P^2 \\
\sqrt{2} P^+ - m \\
P^1 + i P^2
\end{pmatrix},
\bar{u}_{LC}(P,-) \frac{1}{(2\sqrt{2}P^-)^{1/2}} \begin{pmatrix}
-P^1 + i P^2 \\
\sqrt{2} P^- + m \\
P^1 - i P^2 \\
-\sqrt{2} P^- + m
\end{pmatrix}.
\end{equation}

(A1)

normalized according to

\begin{align}
\bar{u}(P,+) u(P,+ &= \bar{u}(P,-) u(P,-) = 2 m \\
\bar{u}(P,-) u(P,+ &= \bar{u}(P,+) u(P,-) = 0.
\end{align}

(A2)

In a frame of reference where the longitudinal direction is defined by the proton average
momentum $\bar{P}$, the momenta of incoming and outgoing hadrons are parameterized as (in
the light-cone component notation $z^\mu = [z^+, z^-, \vec{z}_\perp]$, where $z^\pm = (z^0 \pm z^3)/\sqrt{2}$ and $\vec{z}_\perp$ is a
two-dimensional transverse vector)

\begin{align}
P^\mu &= (\bar{P} - \Delta/2)^\mu = \left[ (1 + \xi) \bar{P}^+, \frac{m^2 + \vec{\Lambda}_1^2 / 4}{2(1 + \xi) P^+}, -\frac{\vec{\Lambda}_1}{2} \right] \\
P'^\mu &= (\bar{P} + \Delta/2)^\mu = \left[ (1 - \xi) \bar{P}^+, \frac{m^2 + \vec{\Lambda}_1^2 / 4}{2(1 - \xi) P^+}, +\frac{\vec{\Lambda}_1}{2} \right].
\end{align}

(A3)
With this parametrization one obtains for the spinorial products in the helicity non-flip case, i.e. \( \Lambda' = \Lambda = \pm 1 \)

\[
\bar{u}(P', \Lambda') \ u(P, \Lambda) = \frac{2m}{\sqrt{1-\xi^2}} \\
\bar{u}(P', \Lambda') \gamma_5 \ u(P, \Lambda) = \Lambda \frac{2m \xi}{\sqrt{1-\xi^2}} \\
\bar{u}(P', \Lambda') \sigma^{\mu\nu} \ u(P, \Lambda) = \frac{i m }{\bar{P}^+} (\Delta^{\mu\nu} - \Delta^{\nu\mu}) + \frac{\Lambda m \sqrt{1-\xi^2}}{\sqrt{1-\xi^2}}.
\]

(A4)

The spinorial products if the helicity is flipped and \( \Lambda' = -\Lambda = 1 \) are

\[
\bar{u}(P', \Lambda') \ u(P, \Lambda) = -\eta \sqrt{t_0 - t} \\
\bar{u}(P', \Lambda') \gamma_5 \ u(P, \Lambda) = -\eta \Lambda \sqrt{t_0 - t} \\
\bar{u}(P', \Lambda') \sigma^{\mu\nu} \ u(P, \Lambda) = -\eta \left[ -\frac{2i}{\sqrt{t_0 - t}} \frac{\bar{P}^+ \Delta^{\nu\mu} - \Delta^{\nu\mu}}{P^+ (1 - \xi^2) \sqrt{t_0 - t}} - \frac{2i m^2 (\Delta^{\mu\nu} - \Delta^{\nu\mu})}{\bar{P}^+ (1 - \xi^2) \sqrt{t_0 - t}} - \frac{4 \Lambda m^2 \xi \epsilon^{\mu\nu\rho\sigma} \bar{P}_\rho v'_\sigma}{\bar{P}^+ (1 - \xi^2) \sqrt{t_0 - t}} + \frac{2 \Lambda \epsilon^{\mu\nu\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{t_0 - t}} \right],
\]

(A5)

while if \( \Lambda' = -\Lambda = -1 \)

\[
\bar{u}(P', \Lambda') \ u(P, \Lambda) = \eta^* \sqrt{t_0 - t} \\
\bar{u}(P', \Lambda') \gamma_5 \ u(P, \Lambda) = \eta^* \Lambda \sqrt{t_0 - t} \\
\bar{u}(P', \Lambda') \sigma^{\mu\nu} \ u(P, \Lambda) = \eta^* \left[ -\frac{2i}{\sqrt{t_0 - t}} \frac{\bar{P}^+ \Delta^{\nu\mu} - \Delta^{\nu\mu}}{P^+ (1 - \xi^2) \sqrt{t_0 - t}} - \frac{2i m^2 (\Delta^{\mu\nu} - \Delta^{\nu\mu})}{\bar{P}^+ (1 - \xi^2) \sqrt{t_0 - t}} - \frac{4 \Lambda m^2 \xi \epsilon^{\mu\nu\rho\sigma} \bar{P}_\rho v'_\sigma}{\bar{P}^+ (1 - \xi^2) \sqrt{t_0 - t}} + \frac{2 \Lambda \epsilon^{\mu\nu\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{t_0 - t}} \right]
\]

(A6)

where the phase factor is given as

\[
\eta = \frac{\Delta_1 + i \Delta_2}{|\Delta_\perp|},
\]

(A7)

and

\[
|\Delta_\perp| = \sqrt{\frac{-4 \xi^2 m^2}{1-\xi^2} + \frac{4 \xi^2 m^2 + \bar{\Lambda}^2}{1-\xi^2}} \sqrt{1-\xi^2} = \sqrt{t_0 - t} \sqrt{1-\xi^2},
\]

(A8)

which contains the implicit definition of the quantity \( t_0 \). Studying the form factor decomposition of the tensor current of the proton [10], we need additionally the following Dirac
bilinears. For the helicity non-flip case, i.e., $\Lambda' = \Lambda = \pm 1$, we have
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \bar{P}_\rho \gamma_\sigma u(P, \Lambda) = \frac{-i \bar{P}^+ \Lambda \Delta^j + \xi \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{1 - \xi^2}}
\]
\[
\bar{u}(P', \Lambda') \sigma^{+j} \gamma^5 u(P, \Lambda) = \frac{\epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma \xi \sqrt{1 - \xi^2}}{(1 + \xi) m}
\]
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma u(P, \Lambda) = -2m i \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma \sqrt{1 - \xi^2}
\]
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \Delta_\rho \gamma_\sigma u(P, \Lambda) = \frac{2(\Lambda \bar{P}^+ \Delta^j \xi + i \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma)}{\sqrt{1 - \xi^2}}
\]
\[
(A9)
\]

The results for the helicity flip case with $\Lambda' = -\Lambda = -1$ are
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \bar{P}_\rho \gamma_\sigma u(P, \Lambda) = -\eta \left[ -2 \frac{m^2 \xi (\bar{P}^+ \Delta^j \Lambda + i \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma)}{\sqrt{1 - \xi^2 \sqrt{t_0 - t}}} \right]
\]
\[
\bar{u}(P', \Lambda') \sigma^{+j} \gamma^5 u(P, \Lambda) = -\eta \left[ -2 \frac{i \bar{P}^+ \Lambda \Delta^j + \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{t_0 - t}} \right]
\]
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma u(P, \Lambda) = -\eta \left[ -\frac{\epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma (4m^2 \xi^2 - \xi^2 t + t)}{\sqrt{1 - \xi^2 \sqrt{t_0 - t}}} \right]
\]
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \Delta_\rho \gamma_\sigma u(P, \Lambda) = -\eta \left[ 4m \xi^2 \frac{\bar{P}^+ \Lambda \Delta^j + i \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{1 - \xi^2 \sqrt{t_0 - t}}} \right],
\]
\[
(A10)
\]

and for $\Lambda' = -\Lambda = 1$
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \bar{P}_\rho \gamma_\sigma u(P, \Lambda) = \eta^* \left[ -2 \frac{m^2 \xi (\bar{P}^+ \Delta^j \Lambda + i \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma)}{\sqrt{1 - \xi^2 \sqrt{t_0 - t}}} \right]
\]
\[
\bar{u}(P', \Lambda') \sigma^{+j} \gamma^5 u(P, \Lambda) = \eta^* \left[ -2 \frac{i \bar{P}^+ \Lambda \Delta^j + \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{t_0 - t}} \right]
\]
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma u(P, \Lambda) = \eta^* \left[ -\frac{\epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma (4m^2 \xi^2 - \xi^2 t + t)}{\sqrt{1 - \xi^2 \sqrt{t_0 - t}}} \right]
\]
\[
\bar{u}(P', \Lambda') \epsilon^{+j\rho\sigma} \Delta_\rho \gamma_\sigma u(P, \Lambda) = \eta^* \left[ 4m \xi^2 \frac{\bar{P}^+ \Lambda \Delta^j + i \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{1 - \xi^2 \sqrt{t_0 - t}}} \right],
\]
\[
(A11)
\]

where the phase is again given as in Eq. (A7). Note that latin indices $i, j = 1, 2$, while greek indices run from 0 to 3 and $\epsilon^{+1-2} = 1$, since we have adopted the convention $\epsilon^{0123} = 1$.

**APPENDIX B: ANSATZ FOR THE NON-FORWARD QUARK-QUARK CORRELATION FUNCTION**

For the sake of convenience, in view of the numerous coefficients, we now switch over from explicitly displaying the hadron helicity indices to a more dense notation using the index...
\((\kappa)\) defined as \((1) = ++, (2) = +-, (3) = --, and (4) = --\) and suppressing spinor indices \(ij\). Moreover contractions of momenta with Levi-Civita tensors will be written in the compact notation \(\epsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta \equiv \epsilon_{abcd}\).

In order to write the ansatz for the quark-quark correlation function as indicated in Eq. \((3)\) we now multiply the independent spinorial products, given in Eqs. \((A4), (A5)\) and \((A6)\), with the 16 independent partonic Dirac matrices and saturate the free indices with the following tensors (indices \(\alpha\) and \(\beta\) are intended saturated with spinorial products, while indices \(\mu, \nu\) with indices from partonic Dirac matrices):

\[
\begin{align*}
1 & \quad \bar{P}_\mu \quad \bar{k}^\mu \quad \Delta^\mu \quad \epsilon^{\mu \bar{P} \Delta} \quad \bar{P}_\mu \bar{k}^\nu \quad \bar{P}_\mu \Delta^\nu \quad \bar{k}^\mu \Delta^\nu \quad \epsilon^{\mu \nu \bar{P} \Delta} \quad \epsilon^{\mu \nu \bar{k} \Delta} \\
\bar{k}^\alpha \Delta^\beta & \quad \epsilon^{\alpha \beta \bar{P} \Delta} \\
g & \quad \epsilon^{\alpha \beta \bar{P} \Delta} \\
g & \quad \epsilon^{\alpha \beta \bar{P} \Delta} \\
g & \quad \epsilon^{\alpha \beta \bar{P} \Delta} \\
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g & \quad \epsilon^{\alpha \beta \bar{P} \Delta} \\
g & \quad \epsilon^{\alpha \beta \bar{P} \Delta} \\
g & \quad \epsilon^{\alpha \beta \bar{P} \Delta} \\
\end{align*}
\]

(B1)

The method introduced in Eq. \((3)\) produces an ansatz containing Lorentz tensorial structures multiplied with various amplitudes that are functions of all possible Lorentz scalars. In the ansatz we choose to indicate explicitly only the dependence on scalar products \(\Delta \cdot v'\) involving the vector \(v'\) which occurs in the definition of the helicity eigenstates of the hadrons. Through
For the helicity non-flipped case with \( \kappa = 1, 4 \) the ansatz reads

\[
\Phi^{(\kappa)}(k, P, \Delta) = \frac{(\delta_{\kappa1} + \delta_{\kappa4})}{\sqrt{1 - \xi^2}} \left[ m a_1^{(\kappa)} + \frac{m \Delta \cdot v' + v m a_3^{(\kappa)}}{P^+} a_4^{(\kappa)} + \frac{m \Delta \cdot v'}{P^+} a_5^{(\kappa)} + \frac{\Delta \cdot v' + v m^2}{P^+} a_{10}^{(\kappa)} + \gamma_5 \Delta a_{16}^{(\kappa)} + \frac{\Delta \cdot v'}{P^+} a_{12}^{(\kappa)} + \gamma_5 \Delta a_{12}^{(\kappa)} + \frac{\Delta \cdot v'}{P^+} a_{13}^{(\kappa)} + \gamma_5 \frac{\Delta \cdot v'}{P^+} a_{14}^{(\kappa)} + \frac{\Delta \cdot v'}{P^+} a_{15}^{(\kappa)} + \gamma_5 \Delta a_{16}^{(\kappa)} + \frac{\Delta \cdot v'}{P^+} a_{17}^{(\kappa)} + \gamma_5 \frac{\Delta \cdot v'}{P^+} a_{18}^{(\kappa)} + \frac{\sigma_{\mu\nu} P^\mu k^\nu}{m} a_{19}^{(\kappa)} + \frac{\sigma_{\mu\nu} \Delta k^\nu}{m} a_{20}^{(\kappa)} + \frac{\sigma_{\mu\nu} \Delta k^\nu}{m} a_{21}^{(\kappa)} + \frac{\sigma_{\mu\nu} \Delta k^\nu}{m} a_{22}^{(\kappa)} + \frac{\sigma_{\mu\nu} \Delta k^\nu}{m} a_{23}^{(\kappa)} + \frac{\sigma_{\mu\nu} \Delta k^\nu}{m} a_{24}^{(\kappa)} + \frac{\sigma_{\mu\nu} P^\mu k^\nu}{m} a_{25}^{(\kappa)} + \frac{\sigma_{\mu\nu} \Delta k^\nu}{m} a_{26}^{(\kappa)} + \frac{\sigma_{\mu\nu} \Delta k^\nu}{m} a_{27}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{28}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{29}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{30}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{31}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{32}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{33}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{34}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{35}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{36}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{37}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{38}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{39}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{40}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{41}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{42}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{43}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{44}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{45}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{46}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} P^\mu k^\nu}{m} a_{47}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{48}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{49}^{(\kappa)} + \frac{\gamma_5 \sigma_{\mu\nu} \Delta k^\nu}{m} a_{50}^{(\kappa)} + \Delta a_{51}^{(\kappa)} + \gamma_5 \frac{\Delta a_{52}}{P^+} a_{53}^{(\kappa)} + \gamma_5 \frac{\Delta a_{54}}{P^+} a_{55}^{(\kappa)} + \frac{\sigma_{\mu\nu} P k^\nu}{m} \frac{\sigma_{\mu\nu} \Delta k^\nu}{m} a_{56}^{(\kappa)} + \frac{\sigma_{\mu\nu} \Delta \Delta k^\nu}{m} a_{57}^{(\kappa)} \right] .
\]

(B3)

For the helicity non-flipped case we implement the constraints on the non-forward quark-
quark correlator imposed by the hermiticity properties of the quark fields and their well-known behavior under parity and time reversal operations as stated in Eq. (10). In particular parity invariance imposes the following relations

\[ a^\kappa_m = a^\kappa_m \quad m = 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 42, 43, 44, 45, 46, 48, 52, 53, 54 \]
\[ a^\kappa_m = -a^\kappa_m \quad m = 3, 4, 12, 13, 14, 15, 16, 17, 18, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 47, 49, 50, 51, 55, 56, 57 \]

(B4)

Assuming hermiticity

\[ a^\kappa_m = (a^\kappa_m)^* \quad m = 1, 4, 5, 6, 7, 10, 11, 12, 14, 17, 18, 19, 22, 24, 25, 26, 29, 30, 32, 36, 37, 39, 42, 45, 46, 48, 51, 54, 56, 57 \]
\[ a^\kappa_m = -(a^\kappa_m)^* \quad m = 2, 3, 6, 8, 9, 13, 15, 16, 20, 21, 22, 23, 27, 28, 31, 33, 34, 35, 38, 40, 41, 43, 44, 47, 49, 50, 52, 53, 55 \]

(B5)

Imposing the time reversal constraint reduces the number of independent amplitudes in the ansatz since

\[ a^\kappa_m = (a^\kappa_m)^* \quad m = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 55, 56, 57 \]
\[ a^\kappa_m = -(a^\kappa_m)^* \quad m = 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54 \]

(B6)

From Eqs. (B4), (B5) and (B6) the diagonal amplitudes \( a^\kappa_m \) are either real or pure imaginary. For the case in which the hadron helicity is conserved, \( \kappa = 1, 4 \), the ansatz for the
amplitudes \( a_n^{\kappa} \) by the imaginary unity \( i \)

\[ \Phi^{(\kappa)}(\vec{k}, \vec{P}, \Delta) = \frac{(\delta_{\kappa 1} + \delta_{\kappa 4})}{\sqrt{1 - \xi^2}} \left[ m a_1^{(\kappa)} + m \frac{\Delta \cdot v'}{P_+} a_2^{(\kappa)} + m a_5^{(\kappa)} + \bar{P} a_6^{(\kappa)} + \Delta \frac{\Delta \cdot v'}{P_+} a_4^{(\kappa)} + \bar{P} a_7^{(\kappa)} + \gamma^5 \frac{m^2}{P_+} a_8^{(\kappa)} + \bar{P} \left( \frac{\Delta \cdot v'}{P_+} \right)^2 a_9^{(\kappa)} + \frac{\Delta \cdot v'}{P_+} a_{10}^{(\kappa)} + \bar{P} a_6^{(\kappa)} + \frac{\Delta \cdot v'}{P_+} a_4^{(\kappa)} + \gamma^5 \frac{m^2}{P_+} a_8^{(\kappa)} + \gamma^5 \frac{m^2}{P_+} a_9^{(\kappa)} + \gamma^5 \frac{m^2}{P_+} a_{10}^{(\kappa)} + \bar{P} a_6^{(\kappa)} \right] \]

If the hadron helicity is flipped from Eqs. (A5) and (A6) one deduces that the ansatz has a factor \(-\eta (\eta^*)\), given in Eq. (16), which reflects the difference in phase of the initial and final hadronic spin. From the method introduced in Eq. (3) the ansatz for the off-diagonal components of the quark-quark correlation function \((\kappa = 2, 3)\) reads

\[ \Phi^{(\kappa)}(\vec{k}, \vec{P}, \Delta) = \frac{(\eta^* \delta_{\kappa 2} - \eta \delta_{\kappa 3})}{\sqrt{t - t_0} \sqrt{1 - \xi^2}} \left[ m a_1^{(\kappa)} + m \frac{\Delta \cdot v'}{P_+} a_2^{(\kappa)} + m a_5^{(\kappa)} + \gamma^5 \frac{m \Delta \cdot v'}{P_+} a_4^{(\kappa)} + \bar{P} \left( \frac{\Delta \cdot v'}{P_+} \right)^2 a_9^{(\kappa)} + \gamma^5 \frac{m^2}{P_+} a_8^{(\kappa)} + \gamma^5 \frac{m^2}{P_+} a_9^{(\kappa)} + \gamma^5 \frac{m^2}{P_+} a_{10}^{(\kappa)} + \bar{P} a_6^{(\kappa)} \right] \]
We implement again hermiticity, parity and time reversal invariance. In particular parity invariance imposes the following relations

\[ d_m^2 = \eta^2 d_m^3 \quad m = 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 63, 64, 65, 66, 67, 68, 69, 78, 79, 80, 81, 82, 83, 84, 85, 92, 94, 95, 96, 100, 101, 102, 107, 108, 110, 111, 112, 116, 117, 118, 123 \]

\[ d_m^2 = -\eta^2 d_m^3 \quad m = 3, 4, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 93, 97, 98, 99, 103, 104, 105, 106, 109, 113, 114, 115, 119, 120, 121, 122. \]
Applying hermiticity

\[ d_m^2 = (d_m^3)^* \quad m = 1, 4, 5, 7, 8, 10, 12, 14, 16, 18, 19, 21, 23, 25, 27, 29, 31, \]
\[ 34, 36, 37, 38, 39, 41, 43, 45, 47, 48, 51, 53, 54, 57, 58, 61, \]
\[ 64, 65, 68, 73, 75, 76, 78, 79, 81, 83, 84, 87, 88, 90, 92, 94, \]
\[ 95, 97, 98, 100, 104, 105, 108, 110, 111, 113, 114, 116, 120, 121 \]

\[ d_m^2 = -(d_m^3)^* \quad m = 2, 3, 6, 9, 11, 13, 15, 17, 20, 22, 24, 26, 28, 30, 32, 33, \]
\[ 35, 40, 42, 44, 46, 49, 50, 52, 55, 56, 59, 60, 62, 63, 66, 67, \]
\[ 69, 70, 71, 72, 74, 77, 80, 82, 85, 86, 89, 91, 93, 96, 99, 101, \]
\[ 102, 103, 106, 107, 109, 112, 115, 117, 118, 119, 122, 123 \] .

\[ \text{(B9)} \]

Imposing the time reversal constraint reduces the number of independent amplitudes in the ansatz since

\[ d_m^\kappa = \eta^2 (d_m^\kappa)^* \quad m = 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, \]
\[ 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, \]
\[ 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, \]
\[ 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 100, 101, 102, 103, \]
\[ 104, 105, 106, 107, 116, 117, 118, 119, 120, 121, 122, 123 \]

\[ d_m^\kappa = -\eta^2 (d_m^\kappa)^* \quad m = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 \]
\[ 19, 20, 21, 22, 23, 24, 25, 26, 28, 36, 87, 88, 89, 90, 91, 92, 93, 94, \]
\[ 95, 96, 97, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, \]

\[ \text{(B10)} \]

Because of the relations in Eq. (B9), (B10) and (B11) some of the amplitudes are zero. We refrain from rewriting the lengthy expression of the off-diagonal ansatz.
In the forward limit $\Delta = 0$ the spinorial products in Eq. (A5) and Eq. (A6) cannot be built through the trace method since the product $\bar{u}(P, -) u(P, +) = \bar{u}(P, +) u(P, -) = 0$. This implies that the off-diagonal part of the ansatz does not converge. The method here developed is therefore applicable only to the cases for which $\Delta$ is different from zero.

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