Investigations into Reconstruction Techniques for the National Ignition Facility Neutron Imaging System

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Abstract. Neutron imaging is currently being developed as a primary diagnostic for inertial fusion studies at the National Ignition Facility (NIF). It is an attractive diagnostic for measuring asymmetries in the burn region and will be able to operate at neutron fluences found during ignition scale implosions. The most straightforward technique for imaging of the spatial distribution of deuterium-tritium (DT) fusion neutrons utilizes a simple pinhole aperture, which blocks all neutrons outside of the solid angle defined by the pinhole and results in a blurred image at the detector. We are currently investigating source image reconstruction techniques from detector images. Source reconstructions from Monte Carlo neutron transport (MCNP) calculations are shown to emulate hydrodynamic simulations with imposed Legendre asymmetries to high accuracy.

1. Introduction
With the completion of the National Ignition Facility (NIF), full-scale ignition attempts will soon be made to prove the viability of inertial confinement fusion (ICF) as a demonstration of controlled thermonuclear fusion opening the door for future fusion-based energy sources. Prior to these ignition attempts, ICF designs must rely upon the results from smaller scale experiments and complex numerical simulations to predict performance at the NIF. Some open questions standing in the way of ignition relate to implosion symmetry and our ability to optimize fusion burn.

Unfortunately, spatially-resolved x-ray diagnostics will not be able to operate at the neutron yields expected for ignition shots. The NIF neutron imaging system (NIS) is currently being developed to overcome this problem by directly imaging the spatial variation of deuterium-tritium (DT) fusion neutrons originating from the implosion [1, 2, 3]. (A description of the NIF-NIS design can be found elsewhere in these proceedings [Wilke].)

The basic idea of neutron imaging is that source neutrons originating outside of the field of view (FOV) of the aperture or originating in the FOV, but directed away from the aperture, are attenuated so that they make a negligible contribution to the recorded signal. For neutron imaging, the source fluence incident outside of the aperture opening must be attenuated using several centimeters of high-Z material given the large mean free path of neutrons in solid density matter. Image reconstruction techniques are then needed to reconstruct the source geometry from recorded images given knowledge of the neutron transport through the aperture.
The choice of aperture geometry has a great effect on image quality and often the type of reconstruction method needed. Penumbral apertures [1] have the advantage of higher signal-to-noise (SNR) ratio over that of pinhole imaging; however, pinholes can achieve a better spatial resolution. Reconstruction techniques designed for the former aperture type have used autocorrelation methods [4, 5] among others, which are limited to spatially invariant aperture point spread functions (PSF) or require the source to be centered in the FOV. This kind of precision alignment will be a difficult task at the NIF, which warrants the development of flexible and robust reconstruction techniques that are not limited to these conditions. Furthermore, it is expected that the NIF will produce several orders of magnitude higher neutron yields than any current laser facility along with improved background shielding so arrays of pinhole apertures should provide adequate SNR. Ring apertures [5] may be an alternative, but they must satisfy the ring delta function requirement [6] to high accuracy and may be limited by manufacturability.

For general imaging techniques not limited to the aforementioned conditions, formation of an image at the detector (in this case, scintillator) can be described mathematically by the product of a (4-dimensional) matrix containing the PSF from all points in the FOV to all points in the detector plane times a matrix containing the actual object (neutron source) (see Fig. 1).

![Figure 1. Schematic of image formation from neutron source](image)

The image plane at the NIF is displaced by 28 m from the source plane resulting in a \( \sim 100 \) times magnification. Christensen et al. [7] applied a method based on singular value decomposition (SVD) to the linear system described in Fig. 1. The technique demonstrated the ability to reconstruct asymmetric neutron sources from experiments performed at the Omega Laser, however, the reconstructions were limited to \( \sim 6 \) \( \mu \)m pixels in the source plane for a \( \sim 185 \times 185 \) \( \mu \)m FOV. Our current goal is to have \( 2 \) \( \mu \)m pixels with an equivalent or larger FOV to improve the attainable resolution with this technique. This paper describes the progress made to reach that goal. In the next section we discuss the reconstruction techniques currently being pursued along with the procedure used to form the reconstructed object. The techniques are demonstrated using numerical simulations of ICF implosions.

2. Procedure for NIF-NIS source reconstruction and relevant mathematics

The goal of reconstruction is to form the matrix \( o \) from knowledge of the transfer matrix \( M \) and detected image \( i \). The problem with this is that \( M \) cannot be inverted directly because it is extremely ill-conditioned and possibly rank deficient. Alternative methods can be used to approximate the matrix inversion, two of which have been implemented for the NIF-NIS. Both are based on finding solutions to the normal equations, which are the result of minimizing the residual in a least squares sense [8] giving

\[
M^T o = M^T i
\]  

(1)
or the damped least squares (regularization) equivalent

\[
\left( M^T M + \lambda^2 L^T L \right) \mathbf{o} = M^T \mathbf{i}
\]  

(2)

where the second term inside the parentheses damps high frequency noise in the solution.

To perform a reconstruction, knowledge of the spatial variation of the PSF is required. This information is obtained by propagating point sources of neutrons through the pinhole and onto a detector using a Monte Carlo particle transport code (MCNP). A single point source at any location in the source plane contributes a non-zero fluence to all points in the detector plane so if the calculation is repeated for all points in the source plane, a 4-dimensional transfer matrix \( M_{ijkl} \) is generated. The next step is to create the image \( i_{ij} \) of the object to be reconstructed. In the future, this will be the recorded image from implosion experiments, but to prove the technique, known objects are currently simulated using MCNP.

Once \( M_{ijkl} \) and \( i_{ij} \) have been formed, they are concatenated and sent to either reconstruction routine, which are described next. Once the reconstruction for \( o_{kl} \) has been performed the array is converted back to \( o_{kl} \) resulting in the reconstructed source. The numerical methods are outlined below and focus is placed on their specific implementation for neutron image reconstruction.

2.1. SVD

The solution to Eqn. 1 in terms of the singular values and singular vectors is \([9, 10, 11]\)

\[
\mathbf{o} = \sum_{j=1}^{r} \left( \frac{1}{\sigma_j} \right) \mathbf{u}_j^T \mathbf{v}_j
\]  

(3)

where \( \sigma_j \) are the singular values of \( M \). A similar equation can be found for the regularized solution in [9]. The idea is to cut-off this expansion prior to reaching small singular values, which would otherwise result in noise amplification.

In our implementation, we use the CLAPACK (v. 3.1.1.1 [12]) function DGESVD to perform the decomposition and then reconstruct the solution using Eqn. 3. Calls to the DGESVD function require significant memory resources so all arrays are allocated dynamically. Upon output of the solution, the number of terms kept is chosen to optimize the reconstruction by balancing the norm of the solution and the norm of the residual [9].

2.2. Iterative

Besides the SVD, an iterative scheme has also been implemented that uses a biconjugate gradient (BCG) algorithm [13]. The BCG is the generalization of CG [8] for non-symmetric and non-positive definite systems. In our implementation, the BCG recursion is applied to the system in Eqn. 2 since it will reduce to Eqn. 1 by setting \( \lambda = 0 \).

The BCG is guaranteed to converge in \( N \) iterations where \( N \) is the dimension of the square matrix \( M \), although a stopping condition is placed on the recursion if the residual norm reaches a small enough value. This property of the BCG allows for much faster reconstructions and less memory requirements than for the SVD for large systems.

3. Simulated Reconstructions

We have recently adapted MCNP to accept source definitions of simulated implosions from radiation hydrodynamics calculations. With this tool, simulations of the NIF-NIS from the imploding capsule to the scintillator plane can be performed followed by reconstructions of known objects. Implosions have been simulated and used for image reconstruction consisting of low-mode Legendre asymmetries as shown in Fig. 2 a). After running the MCNP transport calculation the detector image shown in Fig. 2 b) is formed and is used in the reconstruction.
Figure 2. a) Simulated source with low-mode Legendre asymmetries b) Image of source at detector and c) contour plot of source reconstruction using 1000 term SVD expansion.

Note the triangular shape of this image, which is due to the triangular geometry of the current NIF pinhole. The transport calculations and reconstructions were performed using 74 × 74 pixels in both the source and image planes and the FOV of the source plane was 185 × 185 μm².

The original source shown in Fig. 2 a) was reconstructed using the SVD routine keeping 1000 terms in the expansion (BCG with 100 iterations gives very similar result) and the reconstruction is shown as a contour plot in c), which is seen to compare well with the input source. We are currently pursuing the development of a figure of merit for closeness of the reconstruction.

4. Conclusions
In this paper, we have demonstrated the ability to reconstruct objects from pinhole neutron images given knowledge of neutron transport through the aperture. We have found that SVD and BCG methods give very similar reconstructions although the BCG method is more efficient for large problems. The inclusion of noise in the reconstructions has begun with promising early results, but further tests and development are needed.

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