A Note on the Transverse Five-Brane in M(atrix) Theory

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Abstract
We describe a way to compute scattering amplitudes in M(atrix) quantum mechanics, that involve the transverse five-brane. We then compute certain scattering processes and show that they have the expected $SO(5)$ invariance, give the correct transverse-five-brane mass, and agree with the supergravity result.
1 introduction

Recently there has been a proposal that M-theory \cite{1, 2} in the infinite momentum frame can be described as the large $N$ limit of $SU(N)$ Yang-Mills quantum mechanics\cite{3}. In subsequent work the brane \cite{4, 5, 6} content of type IIA theory have been discussed in this framework \cite{7, 8, 9, 10, 11, 12}. The branes are represented as classical backgrounds. For the transverse five-brane or the type IIA $NS$-five-brane, one does not have such a representation. In this note we will describe how to compute scattering amplitudes involving transverse five-brane (we will often call it $5NS$-brane), and by that confirming its existence.

Starting with the $0 + 1$ theory (quantum mechanics), one can T-dualise to other $d + 1$ dimensional theory and then use some specific duality of that field theory, T-dual back and get some new information in the $0 + 1$ dimensional theory. Here we will employ this strategy in order to compute some properties of the transverse five-brane. One does not have a description of the transverse five-brane in $0 + 1$ dimensional theory, however as noticed in \cite{8} there is an indirect way of describing it. Start with the configuration of the $(2 + 0)$-brane (sometimes called the membrane of matrix theory) in $0+1$-dimensions, and have extra three dimension be compact (not necessarily small) call them $X_1, X_2, X_3$. Now if we start with the $(2 + 0)$ in the direction $(4, 5)$ and T-dualise along $X_1, X_2, X_3$, then we end up with a $(5 + 3)$-brane in $3+1$ dimensions. Now one can use the S-duality\footnote{Branes in the IKKT \cite{13} model were discussed in \cite{14, 15}} of $(3+1)$ Yang-Mills, which is equivalent to the S-duality of type IIB string to get to the configuration $(5NS + 3)$-brane, then T-dual back to get the configuration $(5NS + 0)$-brane in the $0 + 1$ dimensional theory stretching in directions $(1 − 5)$. Unfortunately we do not know how the S-duality acts on the commutator $[X_4, X_5] = ic$, so one does not have the description of the transverse five-brane.

Let us instead focus on some scattering amplitudes involving the transverse five-brane, as the procedure above takes us from $0 + 1$ theory to another $0 + 1$ theory (albeit with possible different parameters) one can map a scattering amplitude involving the transverse five-brane to a known amplitude, computable in $(0 + 1)$-dimensional matrix theory. In this fashion one can get certain information on the transverse five-brane in the context that we want. In order to do so all we need is to map known configurations in one theory to the other via the transformation $T^3ST^3$, and also to know how the parameters (coupling and size of the space) change. Of course one needs to know the exact result in the original theory in order to be able to do this as typically the coupling will change, luckily we expect the long distance scattering to be exactly computable in the matrix theory.

We start in section (2) with a review of T-duality in the context of ma-
tatrix theory, then in section (3) we describe how various configurations are transformed under the $T^3ST^3$ transformation. We calculate in section (4) scattering of zero-branes from the transverse five-brane $(5^{NS} + 0)$, and show that it has an $SO(5)$ invariance and agrees with the super-gravity result. From these calculation we extract the transverse-five-brane mass. We also give the results of the scattering of two Five-branes and a five-brane and an anti-five-brane.

2 Review of T Duality

In this section we review the description of compactification in matrix theory on a $d$ dimensional torus\footnote{Discussion of compactifications on other manifolds can be found in \cite{17,18,19,20,21}}. We start with the $(0 + 1)$ dimensional field theory. If one of the nine direction is compact (with length $L$) one has in the theory new sectors of states, namely open strings that wind around the compact directions \cite{22}. If instead of describing those states this way one just Fourier transform to another description, then one sees that the same theory is naturally described as a $(1 + 1)$ dimensional theory with the extra dimension being compactified on a circle of length $l_s = (\frac{l_p}{R_{11}})^{1/2} l_p$, where $R_{11}$ is the radius of the eleventh direction and $l_p$ is the Planck length in eleven dimensions.) Given that one has $d$ dimensions compact (on a torus) one ends with a $d + 1$ dimensional super Yang-Mills compactified on the dual $d$-torus \cite{22}. Now this procedure is just the T-duality of type IIA (at strong coupling), as replacing winding modes with momenta modes (Fourier transform) is T-duality. In fact in the Lagrangian given a field $X_i$, if we Fourier transform (T-dualise) in that direction then \footnote{Discussion of compactifications on other manifolds can be found in \cite{17,18,19,20,21}}

$$X_i \rightarrow i \partial x_1 + A_i(x_i) = D_i,$$  (1)

where $x_i$ is now a parameter ranging on the dual circle and $A_i(x_i)$ captures the degrees of freedom in the $i$ direction of the original brane (position, momentum).

Let us see how the T-duality acts on the branes of the theory. Starting in $(0 + 1)$ dimensions the basic objects can be thought as zero-branes, while in $(d + 1)$ dimensions the basic objects are branes of dimension $d$, as one expects. Starting with a membrane configuration $[X_1, X_2] = i c$ in $(0 + 1)$ dimension, if one T-dualise in directions other that $(1, 2)$ then we just end with a configuration $[X_1, X_2] = ic$ in a higher dimensional super Yang-Mills theory. This configuration can easily be seen to describe a $(d + 2)$-brane (of course bound to many $d$-branes), as one expects from T-duality. If however we Fourier transform on coordinate $X_1$ then one ends in a $(1 + 1)$ dimensional filed theory with a configuration

$$[i \partial x_1 + A_i(x_i), X_2] = ic \rightarrow D_{x_1}X_2 = ic.$$  (2)
Now as $x_1$ is the world sheet coordinate along the 1-brane (which is the elementary object of this theory) this configuration describes a 1-brane at an angle with respect to the $x_1$ direction. If we just T-dualise the configuration of a membrane bounded to zero-brane in Type IIA string theory then we will get a 1-brane with an angle, just like we have. What we mean when we say a string at an angle is the following. As we are on a compact space, a string configuration that is wrapped $n$ times in one direction and $m$ times in another direction is a string at an angle $\sim m/n$ for $n \gg m$, in the non compact space.

T-dualising now in the $X_2$ direction one ends up with $[D_1, D_2] = iF_{12} = ic$ a magnetic field on the world volume of a $2 + 1$ theory [3]. As is well known this represents zero-branes bounded to the two-branes [23], which is the T-dual of the configuration we started with.

Let us now start with a configuration of a moving zero-brane in the $0 + 1$ dimensional theory. This is described by a background $X_1 = vt$ which can be recast in the form $[D_1, X_1] = iv$, Fourier transform along $X_1$ to get a configuration in $1 + 1$ dimensions of the form $[D_1, D_1] = iF_{01} = iv$ which implies the existence of a constant electric field in the $1 + 1$ dimensional theory as one expects from T-duality.

Start with the configuration of a bound state of a four-brane bounded to two-branes and zero-brane in the theory in $0+1$ dimensions. This is described by $[X_1, X_2] = ic_1$, $[X_4, X_3] = ic_2$ where $c_1 = \frac{2\pi R_1 R_2}{n_1}$, $c_2 = \frac{2\pi R_3 R_4}{n_2}$, and the number of zero-brane is $N = n_1 n_2$. The number of membrane in the $(1, 2)$ direction is $\frac{1}{i2\pi R_1 R_2} Tr[X_1, X_2] = N c_1 = n_2$ and the number of membranes in the $(3, 4)$ direction is $n_1$. Now T-dualise along $X_1$ one gets a configuration in $1 + 1$, which is $X_2 = c_1 x_1$, $[X_3, X_4] = ic_2$. This describes in the matrix theory $n_1$ 3-branes bounded to $N$ 1-strings in the $x_1$ direction and one string at an angle $\sim c_1$, exactly what one expects from the string theory.

3 The Action of $T^3ST^3$

In this section we will describe how configuration change under the $T^3ST^3$ transformation and how the parameters of the theory change.

3.1 Mapping of Configurations

In order to make things clear let us always call the three directions we T-dualise in $X_1, X_2, X_3$, any different direction will be labeled as $X_i$. As was explained before under the transformation $T^3ST^3$ a $(0+1)$ dimensional field theory is transformed back to a $(0+1)$ dimensional field theory.

Under this transformation a membrane, described by the classical background, $[X_i, X_j] = ic$ is transformed to a transverse five-brane in directions $(1, 2, 3, i, j)$. Take a membrane $[X_i, X_i] = ic$ under $T^3$ one gets $D_1 X_i = ic$
\( c = \frac{2\pi R_i R_k}{N} \) which is a configuration of \( N \) three-branes in directions \((1, 2, 3)\) and one three-brane in directions \((2, 3, i)\). Then acting by \( S \) one gets the same configuration and acting again by \( T^3 \) we are back in a configuration of a membrane \([X_1, X_i] = ic\). Similar transformation can be made on other configuration and we will just give the results:

- Membrane in \( i, j \leftrightarrow \text{transverse five-brane in directions } (1, 2, 3, i, j) \), or in symbols \( 2_{(i,j)} + 0 \leftrightarrow 5^{NS}_{(1,2,3,i,j)} + 0 \)
- \( 2_{(1,i)} + 0 \leftrightarrow 2_{(1,i)} + 0 \)
- \( \text{zero-brane moving in } i \text{ direction } (0 + v_i) \leftrightarrow \text{same} \)
- \( 2_{(2,3)} + 0 \leftrightarrow \text{zero-brane moving in direction } 1 \ (0 + v_1) \)
- \( 4_{(1,i,j,k)} + 2_{(i,j)} + 2_{(1,k)} + 0 \leftrightarrow 5^{NS}_{(2,3,i,j,k)} + 5^{NS}_{(1,2,3,i,j)} + 2_{(1,k)} + 0 \)
- \( 4_{(1,2,i,j)} + 2_{(1,i)} + 2_{(2,j)} + 0 \leftrightarrow \text{same}. \)
- \( 4_{(1,2,i,j)} + 2_{(1,2)} + 2_{(i,j)} + 0 \leftrightarrow 4_{(1,2,i,j)} + v_3 + 5^{NS}_{(1,2,3,i,j)} + 0 \)
- \( 4_{(1,2,3,i)} + 2_{(12)} + 2_{(3,i)} + 0 \leftrightarrow 1^{NS}_i + v_3 + 2_{(3,i)} + 0 \)

Given the results for the potentials between these configurations given in \([10, 12]\) and taking care to take into account the effect of the extra compact direction one can read off the potentials between configurations that one does not have access directly in the \(0 + 1\) dimensional field theory.

Now as \( T \) and \( S \) duality does not change the supersymmetry of a configuration we can also see that we can get new configurations that preserve a quarter of the supersymmetry that has relative velocity between the brane configuration. For example two stationary orthogonal \((2 + 0)\)-branes is a supersymmetric configuration if there parameter \( c \) are the same. This implies that a zero-brane moving with a certain velocity parallel to a \((5_{NS} + 0)\) bound state is also supersymmetric. Similarly because a configuration of \((4 + 2 + 2 + 0)\) and a stationary zero-brane is supersymmetric \([23, 24]\) for a specific choice of parameter, then all the configuration on the right-hand side off the table above plus a zero-brane are supersymmetric\(^4\).

### 3.2 Parameter transformation

Let us start with a string theory at coupling \( g \), the lengths for the \((1, 2, 3)\) directions labeled by \( L_1, L_2, L_3 \), and lengths for the other directions \( X_j \). Under T-duality in a \( k \) direction the length \( L'_k = \frac{L_k}{L_k} \) and the coupling \( g' = g \frac{L_k}{L_k} \).

\(^4\)Some related classical supergravity solutions can be found in \([27, 26, 28]\)
Under S duality $g' = g^{-1}$ and all lengths transform $L' = L(g')^{1/2}$. Starting with the above parameters under $T^3ST^3$ one ends up with

$$
\begin{align*}
g' &= gV_p^{1/2} \\
L_i' &= L_iV_p^{-1/2} \\
X_i' &= X_iV_p^{1/2}
\end{align*}
$$

(3)

Where $V_p = \frac{L_1L_2L_3}{g_t^3}$ is the volume in Planck units of the space $L_1, L_2, L_3$.

Notice that we have used the S-duality transformation for the parameter from string theory, of course with the philosophy at hand this should come from the S-duality symmetry of the Yang-Mills theory. In $(3+1)$ Yang-Mills under S-duality the three-torus stays the same, the coupling is inverted and the scalars are multiplied by $V_p$ (the scalars represent the transverse space). Using then the conformal invariance we scale the torus sides by $V_p^{1/2}$ which is accompanied multiplying the scalars by $V_p^{-1/2}$ which is then just the S-duality of the string theory.

Due to the $T^3ST^3$ symmetry of the theory the phase shift $[29]$ of a configuration must be mapped to the phase shift of the mapped configuration.

$$A = -\int dtV(b^2 + v^2t^2) = A' = -\int dt'V'(b'^2 + v'^2t'^2)$$

(4)

where $b^2 = b^2V_p$ and $t' = tV_p^{1/2}$.

4 Transverse five-brane scattering

In this section we will calculate various scattering process involving the transverse five-brane, and show that we get the correct long distance result.

We will calculate here the scattering of a zero-brane (or graviton) from the transverse five-brane. There are three different directions the zero-brane can move. First its motion can be transverse to the five-brane, this is mapped to a calculation of a zero-brane scattering off a membrane with velocity in the $(6-9)$ directions. Second the zero-brane can move parallel to the five-brane. This case is actually two-case that should agree. Movement along the $(1-3)$ directions is mapped to a computation involving two relatively stationary orthogonal two-branes, and movement along the $(4,5)$ direction is mapped to a zero-brane moving parallel to a two-brane. The last two-computations although they are different should give the same answer due to the $SO(5)$ symmetry of the transverse five-brane.

Now when computing the scattering one must take into account the three compact directions $(1, 2, 3)^[5]$.  

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[5]Compact brane in matrix theory were also considered in [30], and in string theory in [24]
4.1 Zero-brane scattering off a transverse five-brane

Starting with a zero-brane scattering off a transverse five-brane with velocity transverse to the five-brane, we calculate a zero-brane scattering off a two-brane (described by \([X_4, X_5] = i c\)) with three orthogonal directions much smaller that the distance between the zero-brane and two-brane. We get the long range potential \([10]\)

\[
V = -\frac{1}{2} \int \frac{ds}{s} e^{-b^2 s} \left( \frac{c^2 + v^2}{2c\sqrt{\pi}} \right)^2 s^4 \left( \frac{\pi^{3/2}}{L_1 L_2 L_3 s^{3/2}} \right)
\]  

(5)

Where the last factor in parenthesis is from the compactified directions, \(c = \frac{L_4 L_5}{2\pi N}\), and \(b\) is the distance between the membrane and the zero-brane.

Let us interpret this result using five-brane variables. and using equation \([4]\). Labeling by prime, quantities after the \(T^{3}ST^{3}\) transformation and using equation \([3]\) we find

\[
c = \frac{L_4 L_5}{2\pi N} = \frac{L_1' L_2' L_3' L_4' L_5'}{2\pi N g' l^3_{T^3}} = \frac{L_1' L_2' L_3' L_4' L_5'}{(2\pi)^{5/2} N g'}
\]  

(6)

Further \((2\pi)^{3/2} gL_1L_2L_3 = g'(2\pi)^3\).

The potential between the zero-brane and transverse five-brane is then

\[
V_{0,5}' = -\pi(\frac{c^2 + v^2}{2c\sqrt{\pi}})^2 b'^{-2}
\]  

(7)

We will now calculate the scattering of a zero-brane off the transverse five-brane when the zero-brane has velocity along direction 1 (or directions 2 or 3). the corresponding configuration is two orthogonal stationary \((2 + 0)\)-branes. The potential for this configuration was discussed in \([10]\). The potential is

\[
V = -\frac{1}{8\sqrt{\pi}} \int \frac{ds}{s^{3/2} \sinh c_1 s \sinh c_2 s} [4 + 2 \cosh 2c_1 s + 2 \cosh 2c_2 s - 4 \cosh(c_1-c_2) s - 4 \cosh(c_1+c_2) s]
\]  

(8)

Where \(c_1 = \frac{L_4 L_5}{2\pi N_1}\) and \(c_2 = \frac{L_4 L_5}{2\pi N_2}\). The long range potential when taking into account one extra transverse (to both) compact direction is,

\[
V_{long} = -\int \frac{ds}{s^{3/2} 8\sqrt{\pi} c_1 c_2 s} (c_1^2 - c_2^2)^2 s^4 \left( \frac{\sqrt{\pi}}{L_1 s^{1/2}} \right) = -\frac{(c_1^2 - c_2^2)^2}{8c_1 L_1 c_2 b^2}.
\]  

(9)

Now after the \(T^{3}ST^{3}\) transformation the two-brane in the \((2,3)\) direction is mapped to one unit of momentum along direction 1, while the zero-branes that were bound to it are mapped to zero-branes. The velocity of the cluster of \(N_1\) zero brane will then have the velocity

\[
v = \frac{2\pi}{N_1 L_1 T_0}
\]  

(10)
Where $L'_1$ is the length of direction 1 after the transformation and $T'_0 = \frac{2\pi}{\theta}L'_s$ is the zero-brane mass after the transformation. Using equation (3) one finds

$$v = \frac{L_2 L_3}{2\pi N_1} = c_1$$  \hspace{1cm} (11)

Thus the potential between a zero-brane moving along direction 1 and a transverse five-brane stretched in directions $(1 - 5)$

$$V'_{5,0} = -N_1 \frac{\pi (v_1^2 - c_2^2)^2}{4(2\pi)^{3/2}c_2 g'^2} b'^{-2}.$$  \hspace{1cm} (12)

Where $c_2 = \frac{L'_1 L'_2 L'_4 L'_5}{2\pi^{3/2} N_g}$.  

4.2

If the zero-brane is moving in directions $(4, 5)$ then this is mapped to a zero-brane moving in directions $(4, 5)$ along a bound state of $(2 + 0)$-branes stretched in directions $(4, 5)$. So let us calculate the later configuration in the matrix theory.

In order to calculate the phase shift one has to calculate the one-loop vacuum energy of the zero-brane quantum mechanics in the presence of a background representing the membrane and the moving zero-brane. We follow the notation of [10, 12]. One needs to calculate determinants of the operator $(-\partial_0 + M^2)$, where $M^2$ is the mass squared of the off diagonal elements of the matrices. The membrane is defined by the coordinates $X_5 = P$, $X_4 = Q$ and $[Q, P] = ic = i L'_4 L'_5$, and the zero-brane is traveling with velocity $v$ in direction 4. Define

$$H = P^2 + (Q - vt)^2 + Ib^2$$  \hspace{1cm} (13)

Then (in Euclidean space), for the complex bosons one has six with $M^2 = 2H$ one with $M^2 = 2H - 4\sqrt{c^2 - v^2}$ and one with $M^2 = 2H + 4\sqrt{c^2 - v^2}$ For the fermions (in Euclidean space) one finds

$$M^2_f = H - ic\gamma_5\gamma_4 - iv\gamma_4$$  \hspace{1cm} (14)

thus giving eight fermions with $M^2_f = H - \sqrt{c^2 - v^2}$ and eight with $M^2_f = H + \sqrt{c^2 - v^2}$.

Evaluating the determinants we find that the potential takes the form

$$V = -\frac{1}{4\sqrt{\pi}} \int \frac{ds}{s^{3/2} \sinh cs} [6 + 2 \cosh 2\sqrt{c^2 - v^2}s - 8 \cosh \sqrt{c^2 - v^2}s]$$  \hspace{1cm} (15)

This gives a long range potential (after taking into account three transverse compact directions)

$$V_{\text{long}} = -\frac{\pi (c^2 - v^2)^2}{4c L_1 L_2 L_3} b^{-2}$$  \hspace{1cm} (16)
We can now calculate the potential between a transverse five-brane and a cluster of $N_1$ zero-branes moving with velocity in the 4-th direction $V'_5, 0 = -\frac{N_1 \pi (c^2 - v^2)^2}{4c'g'(2\pi)^{3/2}}b'^{-2}$ (17)

We see that equations (12) and (17) (remembering $c = c_2$) agree thus confirming the SO(5) invariance expected from the transverse five-brane. We will shortly also compute the mass of the transverse five-brane.

4.3 Supergravity calculation

Let us compare this to a supergravity calculation of a scattering of a zero-brane off a transverse five brane moving in the 11 direction (i.e from the point of view of type IIA string theory, this is the $(5_{NS} + 0)$ bound state). The metric for the transverse five brane in eleven-dimensions was given in \[31\] (we slightly changed the notation),

$$ds^2 = H^{1/3}[H^{-1}(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dy_4^2 + dy_5^2) + dy_1^2 + dx^i dx_i]$$ (18)

Here $\tilde{t} = \frac{1}{\cos \theta}(t - \sin \theta y_{11}), \tilde{y}_{11} = \frac{1}{\cos \theta}(y_{11} - t \sin \theta), H = 1 + \cos^2 \theta \frac{Q}{r^2}$, and $Q = \frac{M_5}{\cos \theta}$.

Computing the potential for null geodesics \[32\] on this metric we find

$$V \sim -\frac{M_5 (\tilde{P}_{11}^2 + P_1^2) \cos^2 \theta}{\cos \theta r^2}$$ (19)

Where

$$\tilde{P}_{11} = \frac{1}{\cos \theta}(P_{11} - E \sin \theta)$$ (20)

and $E, P_{11}, P_1, P_\perp$ are the energy, momentum in the 11 direction, momentum in direction 1, and momentum in the perpendicular direction of the scattered zero-brane, respectively.

We are interested in the case where $P_\perp = 0$ but $P_1 \neq 0$. The velocity in direction 1 is then

$$\frac{E^2}{P_{11}^2} = 1 + \frac{v^2}{1 - v^2} = 1 + (\gamma v)^2 = \cosh^2 \nu$$ (21)

The potential then becomes

$$V \sim -\frac{M_5(1 - \sin \theta \cosh \nu)^2}{\cos \theta} r^{-2}$$ (22)

To compare to the matrix calculation we should go to the limit where the membrane velocity in the 11 direction approaches 1, and the velocity of the
zero brane in direction 1 is small. In this case we write $\theta = \pi/2 - c'$. The potential at this limit is

$$V \sim -\frac{M_5 ((c')^2 - v^2)^2}{c'} r^{-2}$$

(23)

This is the same as equations (12) and (17) if we identify $c = c'$.

Let us determine $c'$. From the supergravity solution one has

$$\sin \theta = \frac{P_{11}}{E} = \frac{NM_0}{E}$$

(24)

$$E^2 = M_5^2 + N^2 M_0^2$$

(25)

Where $E, P_{11}$ are the energy, and the momentum in the 11 direction of the moving five-brane (which is the eleven dimensional description of the bound state of a transverse five-brane and zero-branes) and $M_5, M_0, N$ are the mass of the five-brane mass of a zero-brane and the number of zero-brane respectively. Then we find

$$c' = \frac{M_5}{NM_0}$$

(26)

From this identification and the identification from the scattering in string theory $c = c'$ we can read off the transverse five-brane mass in the matrix calculation ($l_s^2 = 2\pi$).

$$M_5 = \frac{L_1' L_2' L_3' L_4' L_5'}{(2\pi)^2 (g')^2}$$

(27)

Which is the correct transverse five-brane mass.

If we now calculate the scattering in the supergravity with $P_{\perp} \neq 0$ and $P_1 = 0$ we will get the result from the matrix model equation (7). The agreement may also be viewed as an additional evidence for S-duality.

Notice that if we wanted to check that an elementary string will acquire a Berry phase once it is transported around the $NS$ five-brane, this would correspond to the transportation of a four-brane around a two-brane in the original theory which does acquire the berry-phase.

4.4

Similar calculations can be done for the scattering of two transverse five-branes ($5^{NS} + 0$), this is mapped to a computation of scattering two membranes ($2 + 0$). Taking care of the compact directions, the resulting potential between two membranes and between a membrane and an anti-membrane are

$$V_{2,2} = -\frac{L_4 L_5 v^4}{8c^2 L_1 L_2 L_3} r^{-2}$$

(28)

$$V_{2,\bar{2}} = -\frac{2L_4 L_5 v^2}{L_1 L_2 L_3} r^{-2}$$

(29)
Using the transformation $T^3 S T^3$ this gives the potential between two moving transverse five-branes in matrix theory, and the potential between a transverse five-brane and an anti transverse five-brane, to be $(l_s^2 = 2\pi)$

\[
V_{5,5}' = -\frac{L_1' L_2' L_3' L_4' L_5'}{g^2 (2\pi)^{5/2}} \frac{v^4}{4c^2 \pi^{1/2}} r'^{-2} \quad (30)
\]

\[
V_{5,\bar{5}}' = -\frac{L_1' L_2' L_3' L_4' L_5'}{g^2 (2\pi)^{5/2}} \frac{4c^2}{\pi^{1/2}} r'^{-2} \quad (31)
\]

and $c$ is as in equation (3). Of course one can continue and calculate scattering processes with all the states described in section (3), and probably others.

In general this way of calculating scattering amplitudes is no less powerful than the way one does that for configurations which we know how to represent them as classical backgrounds. However observing the connection between parameters given in section (3.2), one sees that we are unable to describe the limit in which the five-brane are un-compactified. This maybe related to the absence of the corresponding charge in the supersymmetry algebra of zero-branes in [11].

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