Magnetic Helicity of Solar Wind Fluctuations at Ion-kinetic Scales

Lloyd D. Woodham,1✉ Robert T. Wicks,1,2 Daniel Verscharen,1,3
Christopher J. Owen,1 Bennett A. Maruca,4,5 and Benjamin L. Alteman6,7

1 Mullard Space Science Laboratory, University College London, Holmbury St. Mary, Surrey RH5 6NT, UK
2 Institute of Risk and Disaster Reduction, University College London, London WC1E 6BT, UK
3 Space Science Center, University of New Hampshire, Durham, NH 03824, USA
4 Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA
5 Bartol Research Institute, University of Delaware, Newark, DE 19716, USA
6 Department of Climate and Space Sciences and Engineering, University of Michigan, Ann Arbor, MI 48109, USA
7 Department of Applied Physics, University of Michigan, Ann Arbor, MI 48109, USA
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We use magnetic helicity to characterise solar wind fluctuations at ion-kinetic scales. For the first time, we separate the contributions to helicity from fluctuations propagating at angles quasi-parallel and oblique to the local mean magnetic field, \( \mathbf{B}_0 \). We find that the helicity of quasi-parallel fluctuations is consistent with Alfvén-ion cyclotron and fast magnetosonic-whistler modes driven by proton temperature anisotropy instabilities and the presence of a relative drift between \( \alpha \)-particles and protons. We also show that the helicity of oblique fluctuations has little dependence on proton temperature anisotropy and is consistent with kinetic Alfvén wave-like fluctuations from the anisotropic turbulent cascade. Our results provide evidence that the cascade does not produce quasi-parallel propagating fluctuations at ion-kinetic scales in the solar wind.

I. INTRODUCTION

The solar wind is a plasma that emanates from the solar corona and expands supersonically to form the heliosphere. This dynamic environment supports fluctuations such as turbulence, waves, and instabilities over a broad range of scales \([1]\). The coupling of electromagnetic fluctuations and particles over many scales is integral to energy transport and heating in plasmas. In situ measurements of the solar wind provide insights into these fundamental processes, making it a unique plasma laboratory to better understand other astrophysical plasmas that are inaccessible to spacecraft.

Solar wind fluctuations are predominately Alfvénic and exhibit a turbulent cascade of energy from large to small scales that is mediated by non-linear interactions \([2, 3]\). At wave-numbers \( k \ll 2 \pi /d_p \) and \( k \ll 2 \pi /\rho_p \), where \( d_p \) is the proton inertial length and \( \rho_p \), the proton gyroradius, the plasma behaves as a fluid. This range of scales is denoted the inertial range of turbulence and is characterised by fluctuations with increasing anisotropy \(( k_\perp \gg k_\parallel )\) towards smaller scales with respect to \( \mathbf{B}_0 \), the local mean magnetic field \([4, 8]\). At ion-kinetic scales, i.e., \( k \sim 2 \pi /d_p \) and \( k \sim 2 \pi /\rho_p \), Hall and Larmor-radius effects become important in mediating the physics of the cascade \([9]\), and the Alfvénic fluctuations show properties consistent with dispersive kinetic Alfvén waves (KAWs) \([10, 13]\). At these scales, KAWs are prone to collisionless damping via wave-particle interactions, which leads to fine structure in particle velocity distribution functions (VDFs) \([14]\). This fine structure increases the effective collision rate, enabling dissipation of the fluctuations and leading to plasma heating.

Solar wind particle VDFs often deviate from isotropic Maxwellian distributions due to a low rate of collisional relaxation \([15, 16]\). Non-Maxwellian features such as temperature anisotropies relative to \( \mathbf{B}_0 \), beams, and relative drifts between plasma species provide sources of free energy for instabilities \([13, 19, 20]\). One example is the proton temperature anisotropy, \( T_{p,\perp} / T_{p,\parallel} \), where \( T_{p,\perp} \) and \( T_{p,\parallel} \) are the proton temperatures perpendicular and parallel to \( \mathbf{B}_0 \), respectively. As the solar wind flows out into the heliosphere, local processes drive changes in \( T_{p,\perp} / T_{p,\parallel} \), leading to a deviation from Chew-Goldberger-Low theory for adiabatic expansion \([21, 22]\). If \( T_{p,\perp} / T_{p,\parallel} \) deviates far enough from unity, kinetic instabilities grow that act to limit this anisotropy. Measurements of the near-Earth solar wind show that the observed range of \( T_{p,\perp} / T_{p,\parallel} \) values is constrained by the increasing growth rates of these anisotropy-driven instabilities \([16, 22]\). In fact, Klein et al. \([21]\) show that over half of solar wind intervals support ion-scale kinetic instabilities, suggesting that they are ubiquitous in the solar wind.

Four kinetic instabilities driven by proton temperature anisotropy are relevant in the solar wind. The Alfvén ion-cyclotron (AIC) and mirror-mode instabilities are unstable at \( T_{p,\perp} \) sufficiently greater than \( T_{p,\parallel} \). On the other hand, the parallel and oblique firehose instabilities are unstable at \( T_{p,\parallel} \) sufficiently greater than \( T_{p,\perp} \). The AIC and parallel firehose instabilities have maximum growth rates for wave-vectors, \( \mathbf{k} \), that are parallel to \( \mathbf{B}_0 \), which respectively leads to growing AIC and fast magnetosonic-whistler (FMW) modes at \( k_\parallel d_p \lesssim 1 \). Conversely, the mirror-mode and oblique firehose instabilities, have maximum growth rates for \( \mathbf{k} \) at angles oblique to \( \mathbf{B}_0 \), and drive modes at \( k_\perp \rho_p \lesssim 1 \) that do not propagate in the
plasma frame. The two parallel instabilities can also be driven unstable by particle beams and drifts \cite{28, 30}, for example, the differential flow between $\alpha$-particles and protons, $v_d = v_A - v_p$. This drift velocity is about $v_d \simeq 0.6 v_A$, where $v_A$ the local Alfvén speed, and directed along $\mathbf{B}_0$ away from the Sun \cite{26, 34}. Podesta and Gary \cite{35, 36} show that the presence of a differential flow leads to a preferential driving of the AIC and parallel firehose instabilities in the direction of $v_d$ and $-v_d$, respectively.

Several studies \cite{35, 37–42} use magnetic helicity to characterise solar wind fluctuations at ion-kinetic scales. However, Taylor’s hypothesis \cite{43} limits single-spacecraft observations to the spacecraft frame, so that we can only measure a projection of $\mathbf{k}$ along the flow direction past the spacecraft, $k_r$. In this letter, we use a novel method to measure the wave-vector anisotropy of solar wind magnetic field fluctuations using magnetic helicity \cite{44}. For the first time, we separate the helicity of fluctuations propagating at quasi-parallel and oblique angles to $\mathbf{B}_0$. We find that periods of strong coherent helicity correspond to parallel-propagating fluctuations during intervals in which the plasma is unstable due to its proton temperature anisotropy. These fluctuations are preferentially driven due to the presence of a significant drift between $\alpha$-particles and protons. Furthermore, we show that the continual background helicity in the solar wind corresponds to fluctuations propagating oblique to $\mathbf{B}_0$. The amplitude of this signature shows little dependence on $\beta_p,||$ and $T_{||}/T_{\perp}$, and we attribute these fluctuations to the anisotropic turbulent cascade \cite{44–46, 47}. Our results suggest there is no strong parallel component of the turbulent cascade at ion-kinetic scales.

\section{Magnetic Helicity}

Magnetic helicity is a measure of the phase coherence between magnetic field components and serves as a useful indicator of the polarisation properties of solar wind fluctuations. The fluctuating magnetic helicity density in spectral form is defined as $H_m(k) = \mathbf{A}(k) \cdot \mathbf{B}^*(k)$, where $\mathbf{A}$ is the fluctuating magnetic vector potential, $\mathbf{B}$ is the fluctuating magnetic field, and the asterisk indicates the complex conjugate of the Fourier coefficients \cite{47}. From a single-spacecraft time series of magnetic field measurements, we can only determine a reduced form of the magnetic helicity density \cite{48–50}:

$$H_m^r(k_r) = 2 \text{Im} \left\{ P_{T N}(k_r) \right\},$$

where $P_{ij}(k_r) = B^*_i(k_r) \cdot B_j(k_r)$ is the reduced power spectral tensor in RTN coordinates. We define the normalised reduced fluctuating magnetic helicity density as:

$$\sigma_m(k_r) = \frac{k_r H_m^r(k_r)}{\left| \mathbf{B}(k_r) \right|^2} = 2 \text{Im} \left\{ P_{T N}(k_r) \right\} \frac{\text{Tr} \{ \mathbf{P}(k_r) \}}{\text{Tr} \{ \mathbf{P}(k_r) \}},$$

where $\text{Tr}\{\}$ denotes the trace. Here, $\sigma_m(k_r)$ is dimensionless and takes values between $[-1, 1]$, where $\sigma_m = -1$ indicates purely left-handed and $\sigma_m = +1$ indicates purely right-handed circular fluctuations, respectively. A value of $\sigma_m = 0$ indicates no overall coherence. We define the field-aligned coordinate system ($\hat{x}, \hat{y}, \hat{z}$),

$$\hat{z} = \frac{\mathbf{B}_0}{|\mathbf{B}_0|}, \quad \hat{y} = -\frac{\mathbf{v}_{sw} \times \mathbf{B}_0}{|\mathbf{v}_{sw} \times \mathbf{B}_0|}, \quad \hat{x} = \hat{y} \times \hat{z},$$

so that the solar wind velocity, $\mathbf{v}_{sw}$, lies in the $\hat{x}\hat{z}$ plane \cite{44}. We then separate the different contributions to magnetic helicity from fluctuations propagating quasi-parallel and oblique to $\mathbf{B}_0$:

$$\sigma_{ij}(k_r) = \frac{2 \text{Im} \left\{ P_{ij}(k_r) \right\}}{\text{Tr} \{ \mathbf{P}(k_r) \}},$$

where the indices $i, j = x, y, z$. Therefore, $\sigma_{xy}$ gives the helicity of fluctuations with $\mathbf{k} \times \mathbf{B}_0 \simeq 0$ and $\sigma_{yz}$ the helicity for fluctuations with $\mathbf{k} \times \mathbf{B}_0 \neq 0$. The component $\sigma_{xz}$ integrates to zero if the distribution of fluctuation power is gyrotropic. This novel analysis technique allows us to recover additional information about the wave-vector of the fluctuations using magnetic helicity.

\section{III. Method}

We analyse magnetic field and ion moment data from the MFI fluxgate magnetometer \cite{51, 52} and SWE Faraday cup \cite{53, 54} instruments on-board the \textit{Wind} spacecraft \cite{54} from Jun 2004 to Oct 2018. We neglect collisionally old wind, $A_r \geq 1$, where $A_r$ is the collisional age \cite{17}, which estimates the number of collisional timescales for protons. To account for heliospheric sector structure in the magnetic field measurements, we first calculate the Parker-spiral angle, $\theta_{PB} = \arctan (B_{0,T}/B_{0,R})$, where $B_{0,R}$ and $B_{0,T}$ are the average components of $\mathbf{B}_0$ over 92 s periods. If $\theta_{PB} > \pi/2$ for a day period exceeds 45° from the radial direction, we reverse the signs of the $B_{0,R}$ and $B_{0,T}$ components so that inwards fields are rotated outwards. This procedure removes the inversion of the sign of magnetic helicity due to the direction $\mathbf{B}_0$ with respect to the Sun.

We transform the 11 Hz magnetic field data into field-aligned coordinates (Equation\ref{3}) using $\mathbf{B}_0$ averaged over 92 s. We compute the continuous wavelet transform \cite{55} using a Morlet wavelet to obtain $P(f)$ as a function of the spacecraft-frame frequency, $f = k_r |\mathbf{v}_{sw}| / 2 \pi$. We then calculate magnetic helicity spectra, $\sigma_{xy}$ and $\sigma_{yz}$, using Equation\ref{4}. We average the spectra over 92 s so that a single spectrum overlaps with exactly one SWE measurement, giving a total of 1,696,270 observations, excluding data gaps. Following Woodham \textit{et al.} \cite{56}, we estimate the amplitude of $\sigma_{xy}$ and $\sigma_{yz}$ at ion-kinetic scales by fitting a Gaussian to the coherent peak in each spectrum at frequencies $f \sim 0.8$ Hz. We neglect any peak
at \( f > f_{\text{noise}} \), the frequency at which instrumental noise of the MFI magnetometer becomes significant \cite{57}. We designate the amplitude of the peak in each \( \sigma_{xy} \) and \( \sigma_{yz} \) spectrum as \( \sigma_\parallel \) and \( \sigma_\perp \), respectively.

We bin \( \sigma_\parallel \) and \( \sigma_\perp \) in \( \beta_{p,\parallel} T_{p,\parallel}/T_{p,\parallel} \) space using logarithmic bins \cite{58}, where \( \beta_{p,\parallel} = n_p k_B T_{p,\parallel}/(B^2/2\mu_0) \), \( n_p \) is the proton density and \( B_0 = |B_0| \). We use equal bin widths of \( \Delta \log_{10}(\beta_{p,\parallel}) = \Delta \log_{10}(T_{p,\parallel}/T_{p,\parallel}) = 0.05 \) and restrict our analysis to \( 0.01 \leq \beta_{p,\parallel} \leq 10 \) and \( 0.1 \leq T_{p,\perp}/T_{p,\parallel} \leq 10 \). In our plots, we neglect any bins with fewer than 10 data points to improve statistical convergence. In this parameter space we overplot contours of constant maximum growth rate, \( \gamma/\Omega_p \), where \( \Omega_p \) is the proton gyro-frequency, for the four kinetic instabilities driven by proton temperature anisotropy. We calculate these contours using linear Vlasov-Maxwell theory (see Maruca et al. \cite{22} and references therein).

IV. RESULTS & DISCUSSION

The presence of an \( \alpha \)-particle drift can break the symmetry of the proton VDFs, leading to a preferential driving of waves generated by anisotropy-driven AIC and parallel firehose instabilities. Linear Vlasov-Maxwell theory shows that the growth rates of AIC and FMW modes are greater in the anti-sunward and sunward directions, respectively, for \( v_d \) directed anti-sunward \cite{35, 36}. The propagation of AIC and FMW modes in different directions therefore leads to sign changes in the helicity of these waves when \( \sigma_\parallel \) is transformed from the plasma-frame to the spacecraft-frame. We summarise the possible cases for the sign of \( \sigma_\parallel \) in Table I. For example, if \( B_0 \) is directed anti-sunward, then left-handed AIC modes will have \( \sigma_\parallel < 0 \) or \( \sigma_\parallel > 0 \) if they propagate anti-sunward or sunward, respectively. By accounting for sector structure (see Section III), our resulting dataset is consistent with cases I and II from Table I removing ambiguity in the sign of \( \sigma_\parallel \) due to the direction \( B_0 \). Therefore, we hypothesise that \( \sigma_\parallel < 0 \) for both AIC and FMW modes present at ion-kinetic scales in the solar wind.

To test this hypothesis, we plot in Figure 1 the median \( \sigma_\parallel \)-value across the \( \beta_{p,\parallel} T_{p,\perp}/T_{p,\parallel} \) plane. The black dashed-lines show contours of constant \( \gamma/\Omega_i \) for the AIC and parallel firehose instabilities, which have greater growth rates along \( B_0 \). We see that the solar wind plasma occupies a significant extent of parameter space in the regions unstable to both the AIC and parallel firehose instabilities, as widely reported in the literature \cite{20, 22}. In these regions of parameter space, we see two distinct signatures at \( T_{p,\perp} > T_{p,\parallel} \) and \( T_{p,\perp} < T_{p,\parallel} \) where the median \( \sigma_\parallel \) assumes more negative values. These signatures indicate the presence of coherent fluctuations that we attribute to growing modes from these instabilities. The minimum helicity is about \( \sigma_\parallel \simeq -0.6 \) for the AIC modes and \( \sigma_\parallel \simeq -0.4 \) for the FMW modes. Since \( \sigma_\parallel < 0 \) corresponds to left-handed helicity in the spacecraft frame, Figure I indicates that AIC modes are preferentially driven anti-sunward, and that FMW modes are preferentially driven sunward. This result is consistent with our predictions as well as observations of quasi-parallel propagating waves in the solar wind \cite{35, 57, 42, 43, 67}.

Away from the unstable regions of the parallel instabilities in parameter space and close to \( T_{p,\perp} \simeq T_{p,\parallel} \), \( \sigma_\parallel \simeq 0 \), which indicates a lack of coherence in \( B \). In Figure 2 we plot the median value of \( |v_{d,\parallel}|/v_A \), the \( \alpha \)-particle parallel drift speed normalised by the Alfvén speed, across the \( \beta_{p,\parallel} T_{p,\perp}/T_{p,\parallel} \) plane. We define \( v_{d,\parallel} = v_{d,\parallel} - B_0 / |B_0| \). We include contours of constant \( \sigma_\parallel \) from Figure I to show the correlation between \( |v_{d,\parallel}|/v_A \) and \( \sigma_\parallel \) in this space. When a significant drift exists close to the unstable regions of the AIC and parallel firehose instabilities, a coherent signature in \( \sigma_\parallel \) also exists. The drift is stronger for \( T_{p,\perp}/T_{p,\parallel} > 1 \), reaching a maximum

| \( B_0 \) | I | II | III | IV |
|---|---|---|---|---|
| \( \beta_{p,\parallel} T_{p,\perp}/T_{p,\parallel} \) | Out | Out | In | In |
| \( k \) | Out | In | Out | In |
| \( \sigma_\parallel \) | - | + | - | + |
| \( \sigma_\perp \) | + | - | - | + |

* Here, \( \sigma_\parallel \) and \( \sigma_\perp \) give the sign of the magnetic helicity due to left-handed and right-handed fluctuations, respectively. The sign (+) designates a positive (negative) helicity.
of $|v_{d,\parallel}| \approx 0.6v_A$ at $\beta_{p,\parallel} > 0.1$. This peak in $|v_{d,\parallel}|$ occurs in the region of parameter space dominated by fast wind streams \cite{68}. For parallel firehose unstable regions in the parameter space, the drift is significantly weaker, reaching a maximum of $|v_{d,\parallel}| \approx 0.2v_A$. Therefore, the presence of a drift between ion species in the solar wind can explain the preferential driving associated with the AIC and FMW modes, which is consistent with previous studies \cite{29,30,36}.

Finally, in Figure 3 we plot $\sigma_\perp$ in the same parameter space. We include contours of constant $\gamma/\Omega_p$ for the mirror-mode and oblique firehose instabilities since these have higher growth rates at angles oblique to $B_0$. Throughout Figure 3 $\sigma_\perp > 0$ and peaks at $\sigma_\perp \approx 0.3$, close to $\beta_{p,\parallel} \approx 0.8$ and $T_{p,\perp} \approx T_{p,\parallel}$. This peak lies in a region of parameter space dominated by fast wind, which is typically more Alfvénic \cite{69}. There is also a small enhancement in the helicity in the unstable region of the oblique firehose instability, suggesting the presence of driven modes with a right-handed helicity in the spacecraft-frame. We do not expect to observe a signature from mirror-modes because they represent structures with $B$ directed along $B_0$, which will not be measurable using magnetic helicity. The lack of a strong dependence of the distribution of $\sigma_\perp$ on $\beta_{p,\parallel}$ and $T_{p,\perp}/T_{p,\parallel}$ implies that the dominant source of these fluctuations is unlikely to be related to kinetic instabilities. Instead, due to the anisotropic nature of the turbulent cascade at ion-kinetic scales, we expect KAW-like fluctuations to contribute to $\sigma_\perp$. From linear Vlasov-Maxwell theory, right-handed KAWs with $k_\perp \gg k_\parallel$ at kinetic scales ($k_\perp \rho_p \gtrsim 1$) have $\sigma_\perp \approx 1$ for $k \cdot B_0 > 0$ and $\sigma_\perp \approx -1$ for $k \cdot B_0 < 0$ \cite{70}. Therefore, Figure 3 is consistent with the presence of outward propagating right-handed fluctuations (Case I from Table I) that we interpret as KAW-like fluctuations from the turbulent cascades.

V. CONCLUSIONS

We use a novel analysis technique to recover information about the wave-vector of solar wind fluctuations using single-point spacecraft measurements. We separate the contributions to magnetic helicity into two components with respect to $B_0$: one for fluctuations propagating at quasi-parallel angles and the other for those propagating at oblique angles. We analyse over 1.6 million magnetic field and ion spectra from the Wind MFI and SWE instruments and quantify the amplitude of the helicity contributions $\sigma_\parallel$ and $\sigma_\perp$ to explore the sources of fluctuations at ion-kinetic scales.

By plotting $\sigma_\parallel$ across $\beta_{p,\parallel}T_{p,\perp}/T_{p,\parallel}$ space, we show that there is a significant negative enhancement in $\sigma_\parallel$ in unstable regions of both the AIC and parallel firehose instabilities. The median $\sigma_\parallel$ reaches a minimum of $\sigma_\parallel \approx -0.6$ at $T_{p,\perp}/T_{p,\parallel} > 1$. In the spacecraft-frame, these quasi-parallel propagating fluctuations are

![Figure 2](image2.png)

**FIG. 2.** Median parallel $\alpha$-proton drift, $|v_{d,\parallel}|/v_A$, across $\beta_{p,\parallel}$-space. We overplot contours of constant maximum growth rate, $\gamma/\Omega_p = 10^{-2}$, for the AIC and parallel firehose instabilities. We also show contours of constant $\sigma_\parallel$ from Figure 1 for reference.

![Figure 3](image3.png)

**FIG. 3.** Median $\sigma_\perp$ across $\beta_{p,\parallel}T_{p,\perp}/T_{p,\parallel}$ space. We overplot contours of constant maximum growth rate, $\gamma/\Omega_p$, for the mirror-mode and oblique firehose instabilities.
left-handed, consistent with left-handed AIC waves propagating anti-sunward for $T_{p,\perp}/T_{p,\parallel} > 1$ and right-handed FMW waves propagating sunward in the plasma-frame for $T_{p,\perp}/T_{p,\parallel} < 1$. In regions of a negative enhancement in $\sigma_\perp$, particularly for $T_{p,\perp}/T_{p,\parallel} > 1$, we also observe a substantial $\alpha$-particle drift with respect to the proton flow, consistent with the predictions of [33] and [39]. Elsewhere in $\beta_{\parallel}/T_{p,\parallel}$ space, $\sigma_\perp \approx 0$, which indicates no coherence in $\mathbf{B}$. This result suggests that fluctuations propagating quasi-parallel to $\mathbf{B}_0$ predominantly arise from ion instabilities, consistent with the background solar wind turbulence producing Alfvénic fluctuations with $k_\perp \gg k_\parallel$. These results show that instabilities are active and modes generated by them are common in the solar wind.

In addition, we show for the first time that $\sigma_\perp$ is distributed throughout the entire parameter space occupied by the solar wind and peaks at about $\sigma_\perp \approx 0.3$. This peak occurs at $T_{p,\perp} \approx T_{p,\parallel}$ and $\beta_{\parallel} \approx 0.8$, which is strongest in a region of $\beta_{\parallel}/T_{p,\perp}/T_{p,\parallel}$ space dominated by fast wind, suggesting that these fluctuations are more Alfvénic. Since $\sigma_\perp > 0$ and shows little dependence on $\beta_{\parallel}$ and $T_{p,\perp}/T_{p,\parallel}$, this signature is consistent with anisotropic KAW-like fluctuations from the turbulent cascade with significant $k_\parallel$ at ion-kinetic scales. We therefore conjecture that these fluctuations are insensitive to proton temperature anisotropy and instability growth, in agreement with Klein and Howes [71]. Furthermore, since the unstable AIC and FMW modes do not appear to interact with the turbulent cascade, and there is no evidence of helicity from turbulent fluctuations with significant $k_\parallel$, we provide evidence for a very limited role of quasi-parallel propagating fluctuations in turbulence and dissipation in the solar wind.

The method we employ here can be applied Parker Solar Probe and Solar Orbiter data to explore the role of fluctuations at kinetic scales in the corona and their evolution with increasing heliocentric distance. This will help us to diagnose the source and nature of the fluctuations that are crucial for the acceleration and heating of the solar wind.

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[1] D. Verscharen, K. G. Klein, and B. A. Maruca, Living Reviews in Solar Physics (to be published) arXiv:1902.03448.
[2] R. Bruno and V. Carbone, Living Reviews in Solar Physics 10, 2 (2013).
[3] C. H. K. Chen, Journal of Plasma Physics 82 (2016), 10.1017/S0022377816001124.
[4] R. T. Wicks, T. S. Horbury, C. H. K. Chen, and A. A. Schekochihin, Monthly Notices of the Royal Astronomical Society 407, L31 (2010).
[5] C. H. K. Chen, A. Mallet, T. A. Yousef, A. A. Schekochihin, and T. S. Horbury, Monthly Notices of the Royal Astronomical Society 415, 3219 (2011).
[6] C. H. K. Chen, A. Mallet, A. A. Schekochihin, T. S. Horbury, R. T. Wicks, and S. D. Bale, The Astrophysical Journal 758, 120 (2012).
[7] T. S. Horbury, R. T. Wicks, and C. H. K. Chen, Space Science Reviews 172, 325 (2012).
[8] C. Lacombe, O. Alexandrova, and L. Matteini, The Astrophysical Journal 848, 45 (2017).
[9] O. Alexandrova, C. H. K. Chen, L. Sorriso-Valvo, T. S. Horbury, and S. D. Bale, Space Science Reviews 178, 101 (2013).
[10] R. J. Leamon, C. W. Smith, N. F. Ness, and H. K. Wong, Journal of Geophysical Research: Space Physics 104, 22331 (1999).
[11] S. D. Bale, P. J. Kellogg, F. S. Mozer, T. S. Horbury, and H. Rene, Physical Review Letters 94, 215002 (2005).
[12] G. G. Howes, W. Dorland, S. C. Cowley, G. W. Hammett, E. Quataert, A. A. Schekochihin, and T. Tatsuno, Physical Review Letters 100, 065004 (2008).
[13] F. Sahraoui, M. L. Goldstein, G. Belmont, P. Canu, and L. Rezeau, Physical Review Letters 105, 131101 (2010).
[14] C. H. K. Chen, K. G. Klein, and G. G. Howes, Nature Communications 10, 740 (2019).
[15] J. C. Kasper, A. J. Lazarus, and S. P. Gary, Physical Review Letters 101, 261103 (2008).
[16] E. Marsch, Space Science Reviews 172, 23 (2012).
[17] B. A. Maruca, S. D. Bale, L. Sorriso-Valvo, J. C. Kasper, and M. L. Stevens, Physical Review Letters 111, 241101 (2013).
[18] J. C. Kasper, K. G. Klein, T. Weber, M. Maksimovic, A. Zaslavsky, S. D. Bale, B. A. Maruca, M. L. Stevens, and A. W. Case, The Astrophysical Journal 849, 126 (2017).
[19] J. C. Kasper, A. J. Lazarus, and S. P. Gary, Geophysical Research Letters 29, 1839 (2002).
[20] P. Hellinger, P. M. Trávníček, J. C. Kasper, and A. J. Lazarus, Geophysical Research Letters 33, L09101 (2006).
[21] S. D. Bale, J. C. Kasper, G. G. Howes, E. Quataert, C. S. Salem, and D. Sundkvist, Physical Review Letters 103, 211101 (2009).
[22] B. A. Maruca, J. C. Kasper, and S. P. Gary, The Astrophysical Journal 748, 137 (2012).
[23] S. Bourouaine, D. Verscharen, B. D. Chandran, B. A. Maruca, and J. C. Kasper, Astrophysical Journal Letters 777, L3 (2013).
In Figure 1 we show the probability density distribution of the solar wind data used in our study. We overplot contours of constant maximum growth rate, $\gamma/\Omega_p = 10^{-2}$, for each of the four kinetic instabilities under discussion: AIC, mirror-mode, parallel and oblique firehose, labelled separately in the plot. We also plot the median solar wind speed, $v_{sw}$ across $\beta_{p,\parallel} - T_{p,\perp}/T_{p,\parallel}$ space. See main text for definitions of parameters.

FIG. 1. Left: Probability density distribution of data across $\beta_{p,\parallel} - T_{p,\perp}/T_{p,\parallel}$ space. We overplot contours of constant maximum growth rate, $\gamma/\Omega_p = 10^{-2}$, for the proton temperature anisotropy instabilities: AIC, mirror-mode (M), parallel (PF) and oblique firehose (OF). Right: Median solar wind speed, $v_{sw}$ across $\beta_{p,\parallel} - T_{p,\perp}/T_{p,\parallel}$ space. We overplot contours of $\gamma/\Omega_p = 10^{-2}$ for the AIC and PF instabilities.

*woodhamlloyd@gmail.com