Fermions tunneling from higher-dimensional charged AdS black hole in dRGT massive gravity within modified dispersion relation

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The tunneling behavior of fermions with half-integral spin from higher-dimensional charged anti-de Sitter (AdS) black hole in de Rham, Gabadadze and Tolley (dRGT) massive gravity is investigated via the modified Hamilton-Jacobi equation and modified dispersion relation (MDR). The results show the modified thermodynamic quantities do not only related to the properties of higher-dimensional charged AdS black hole in dRGT massive gravity, but also depended on the coupling constant $\sigma$ and the mass of emitted particles $m$. It also finds that the modified Hawking temperature is higher than the original one, which indicates the effect of MDR can significantly enhance the evolution of black hole. Besides, our results can be verified by the modified Stefan-Boltzmann law.

\section{I. INTRODUCTION}

Based on the classical viewpoint, people once thought that black holes can only absorb objects [1]. However, this situation has been changed after Hawking proved that black hole can radiate particles. In the theory of black hole radiation (now we call it “Hawking radiation theory”), Hawking introduced the quantum mechanism into gravity theory of curved spacetime, and showed that the black hole can radiate particles [2, 3]. This theory profoundly reveals the deep connection between quantum theory, gravitation theory, thermodynamics, and statistical physics. Therefore, one can obtain the temperature of black holes with the help of Hawking radiation. Now, the theory of Hawking radiation is considered as the most important tool for investigating the thermodynamics properties of black holes.

In addition to the original research method of studying the radiation of black holes, many new methods have been proposed in recent years [4–10]. In Refs. [6, 8], Kraus, Parikh and Wilczek pointed out that the black hole radiation process can be considered as the quantum tunneling. Therefore, when regarding the horizons as the tunneling barrier, one can easily obtain the tunneling rate of the emitted particle and the thermodynamic properties of black holes. Subsequently, Srinivasan and Padmanabhan developed the tunneling method, and put forward the Hamilton-Jacobi ansatz, which greatly simplifies the research process and promotes people’s understanding of black holes [7]. Then, Kerner and Mann used the Hamilton-Jacobi ansatz to discuss the fermion tunneling from spherically symmetric black holes [9, 10]. In Refs. [11–13], Yang et al. successfully derived the Hamilton-Jacobi equation from Klein-Gordon equation, Dirac equation, and Maxwell equation. Their works indicate that Hamilton-Jacobi ansatz can be used to describe tunneling behavior of particles with any spin on the horizons of black holes. Hence, Hamilton-Jacobi ansatz is an effective way to study the Hawking radiation. Up to now, by using the Hamilton-Jacobi ansatz, investigations on many kinds of black holes have been done extensively [14–23].

On the other hand, the classical theory of Hawking radiation has some defects [24]. It has been found that the radiation derived from classical theory is pure thermal state, which implies the black hole would emit their all information at the end of evaporation and leads to the “Information loss paradox”. Besides, the singularities of spacetimes would be exposed to the universe since the horizons disappear. For solving this paradoxical situation, a lot of methods have been proposed. Recent years, a lot of works claimed that there is a minimum measurable length in the nature [25–27]. In Refs. [28, 29], Amelino-Camelia showed that the standard energy-momentum dispersion relation need be changed to modified dispersion relation (MDR) when it nears the minimum measurable length. According to the MDR, people find a new way to ameliorate the problems of classical theory of Hawking radiation. In Refs. [30–38], the authors studied the modified thermodynamic properties of black holes via the MDR, they results implied the effect of MRD has significant effect on the evolution of black holes, it prevents black holes from total evaporation and leads to remnant.

So far, most of the works have been limited to study the tunneling behavior of particles with spin 0 or 1/2. However, it should be noted that the black hole radiate the particles of both integral spin and half-integral spin. Therefore, in this paper, we would like to discuss the modified tunneling behavior fermions with half-integral spin. First, using a new kind of MDR that proposed by Amelino-Camelia, we derived the modified Hamilton-Jacobi equation from Rarita-Schwinger equation in the curved spacetime. Then, we study the fermion tunneling from D-dimensional charged AdS black hole in dRGT massive gravity via the modified Hamilton-Jacobi equation. Finally, the MDR corrected Hawking temperature of D-dimensional charged AdS black hole in dRGT massive gravity is obtained.
The outline of this paper is as follows. In Sec. II, according to the new kind of MDR and WKB approximation, we derived the modified Hamilton-Jacobi equation from Rarita-Schwinger equation. In Sec. III, by using the modified Hamilton-Jacobi equation, the MDR corrected tunneling rate of fermion with half-integral spin and the MDR corrected Hawking temperature of D-dimensional charged AdS black hole in dRGT massive gravity is obtained. The paper ends with conclusions in Sec. IV.

II. THE MODIFIED HAMILTON-JACOBI EQUATION FOR FERMIONS WITH HALF-INTEGRAL SPIN

In this section, we derive the modified Hamilton-Jacobi equation from Rarita-Schwinger equation. In Refs. [28, 29], Amelino-Camelia has proposed a kind of MDR, which is given by

\[ \psi_0 = E^2 + m^2 - (\ell_p p_0)^2 \beta E^2, \]

where \( p_0, E \) and \( m \) are the energy, momentum and mass of particles, respectively, \( \ell_p \) is the minimum measurable length, which equals the order of Planck scale. \( \beta \) is a key characteristic of the magnitude of the effects to be expected. Meanwhile, the Rarita-Schwinger equation is given by as follows:

\[ (\gamma^\mu \partial_\mu + m) \psi_{a_1 \cdots a_k} = 0. \]  

The above equation can be used to describe the kinematics of fermions with half-integral spin in Minkowski spacetime, which satisfies the supplementary conditions \( \gamma^\mu \psi_{a_2 \cdots a_k} = \partial_\mu \psi_{a_2 \cdots a_k} = \psi_{\mu a_2 \cdots a_k} = 0. \) When \( \psi_{a_1 \cdots a_k} = \psi \), that is \( \kappa = 1 \), one has \( \partial_\mu \psi_{a_2 \cdots a_k} = \psi_{\mu a_2 \cdots a_k} = 0 \), Eq. (2) becomes the Dirac equation of spin 1/2. However, when considering \( \kappa = 1 \), \( \psi_{\mu a_2 \cdots a_k} \) vanishes, and Eq. (2) becomes reduces to the kinematics equation for fermions with spin 3/2 [39].

It should be noted that the spacetimes around the black holes are extremely curved. Therefore, in order to investigate the tunneling behavior of fermions on event horizon of black holes, one needs generalize the Rarita-Schwinger equation to the curved spacetime. According to Refs. [13, 40], the Rarita-Schwinger equation in the curved spacetime is given by

\[ (\gamma^\mu D_\mu + m) \psi_{a_1 \cdots a_k} = 0, \]

with supplementary conditions

\[ \gamma^\mu \psi_{a_2 \cdots a_k} = D_\mu \psi_{a_2 \cdots a_k} = \psi_{\mu a_2 \cdots a_k} = 0, \]

where \( \gamma^\mu \) denotes the gamma matrix in curved spacetime, which satisfies the commutation relation \( \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu \nu}I \). Notably, \( D_\mu \) in Eq. (4) is defined as \( D_\mu = \partial_\mu + \Omega_\mu + i e A_\mu / \hbar \) with the spin connection in curved spacetime \( \Omega_\mu \). For \( \Omega_\mu = 0 \), the spacetime becomes flat.

By using Eq. (2), the Rarita-Schwinger equation in curved spacetime can be modified as follows:

\[ \left[ \gamma^\mu D_\mu + \frac{m}{\hbar} - \sigma h^2 \left( \sqrt{g^{\mu \nu}} D_\mu \gamma^\nu \right)^{-1} (\gamma^j \partial_j) \right] \psi_{a_1 \cdots a_k} = 0, \]

where the coupling constant \( \sigma \ll 1 \) is the correction term. For deriving the Hamilton-Jacobi equation, the wave function \( \xi \) of fermions takes the form as

\[ \psi_{a_1 \cdots a_k} = \xi_{a_1 \cdots a_k} e^{\pm \hbar}, \]

where \( \xi_{a_1 \cdots a_k} \) and \( S \) are a matrices and the action of fermion, respectively. The angular momentum parameters and radiation energy parameters of radiation particles can be denoted as \( \partial_\mu S = -\omega \). Now, substituting Eq. (5) and Eq. (6) into Eq. (4), and then ignoring the higher-order term \( O (\hbar) \), one has

\[ \{ i \gamma^\mu (\partial_\mu S + e A_\mu) + m - \sigma \left[ \sqrt{g^{\mu \nu}} (-i \omega + i e A_\nu) \right]^{\beta - 1} \gamma^\nu (\omega - e A_\nu) \gamma^\beta (\partial_\beta S + e A_\beta) \} \xi_{a_1 \cdots a_k} = 0. \]

By considering the relation \( \gamma^\mu (\partial_\mu S + e A_\mu) = -\gamma^\nu (\omega - e A_\nu) + \gamma^\beta (\partial_\beta S + e A_\beta) \), the above-mention equation can be rewritten as

\[ \Gamma^\mu (\partial_\mu S + e A_\mu) \xi_{a_1 \cdots a_k} + \left[ m + \sigma (g^{\mu \nu})^{\beta + \frac{1}{2}} (-i \omega + i e A_\nu)^{\beta + 1} \right] \xi_{a_1 \cdots a_k} = 0, \]

where \( \Gamma^\mu = \gamma^\mu - \sigma (g^{\mu \nu})^{\beta + \frac{1}{2}} (-i \omega + i e A_\nu)^{\beta + 1} \gamma^\nu. \)

Now, multiplying \( \Gamma^\nu (\partial_\nu S + e A_\nu) \) by Eq. (8), the results is

\[ \Gamma^\mu \Gamma^\nu \Delta \xi_{a_1 \cdots a_k} + \left[ m + \sigma (g^{\mu \nu})^{\beta + \frac{1}{2}} (-i \omega + i e A_\nu)^{\beta + 1} \right]^2 \cdot \xi_{a_1 \cdots a_k} = 0, \]

where \( \Gamma^\mu \Gamma^\nu = \gamma^\mu \gamma^\nu - 2 \sigma (g^{\mu \nu})^{\beta + \frac{1}{2}} (-i \omega + i e A_\nu) \gamma^\nu + O (\sigma^2) \) and \( \Delta = (\partial_\mu S + e A_\mu)(\partial_\nu S + e A_\nu) \). Next, interchanging the subscript \( \mu \) and \( \nu \). Then, adding the result with Eq. (9), and multiply it by 1/2 [40], one yields

\[ \left\{ \frac{\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu}{2} - \sigma (g^{\mu \nu})^{\beta + \frac{1}{2}} (-i \omega + i e A_\nu)^{\beta + 1} \right\} \Delta - \sigma \left[ m + \sigma (g^{\mu \nu})^{\beta + \frac{1}{2}} (-i \omega + i e A_\nu)^{\beta + 1} \right]^2 \]

\[ + O (\sigma^2) \} \xi_{a_1 \cdots a_k} = \left\{ \gamma^{\mu \nu} \Delta + m^2 + 2 \sigma m (g^{\mu \nu})^{\beta + \frac{1}{2}} \cdot (-i \omega + i e A_\nu)^{\beta + 1} - 2 \sigma (g^{\mu \nu})^{\beta + \frac{1}{2}} (-i \omega + i e A_\nu) \right\} \gamma^{\mu \nu} \Delta \]

\[ + O (\sigma^2) \} \xi_{a_1 \cdots a_k} = 0. \]

Moreover, by using the anti-commutation relations of gamma matrices, Eq. (10) can be simplified as follows:

\[ i \sigma \gamma^\mu (\partial_\mu S + e A_\mu) \xi_{a_1 \cdots a_k} + \mathcal{M} \xi_{a_1 \cdots a_k} = 0, \]
where
\[ M = \frac{g^{\mu\nu} \Delta + m^2 + 2\sigma m (g^{tt})^{\frac{\Delta+1}{2}} (-i\omega + ieA_t)^{\Delta+1}}{-2i (g^{tt})^{\frac{\Delta+1}{2}} (-i\omega + ieA_t)^{\Delta+1} (\partial_\nu S + eA_\nu)} . \] (12)

Here one needs multiply \(-i\sigma\gamma^\nu (\partial_\nu S + eA_\nu)\) by the left side of Eq. (11), then, interchanging the subscript \(\mu\) and \(\nu\), adding the result with Eq. (11) and divided by 2, the final results is
\[
\left[ g^{\mu\nu} \Delta + m^2 + 2\sigma m (g^{tt})^{\frac{\Delta+1}{2}} (-i\omega + ieA_t)^{\Delta+1} \right]^2
+ 2\sigma^2 g^{tt} (\partial_\mu S + eA_\mu) (\partial_\nu S + eA_\nu) = 0 .
\] (13)

Here the higher order terms \(O (\sigma^2)\) need be ignoring since they are too small, and one gets
\[
g^{\mu\nu} (\partial_\nu S + eA_\nu) (\partial_\mu S + eA_\mu) + m^2 + 2\sigma m (g^{tt})^{\frac{\Delta+1}{2}} (-i\omega + ieA_t)^{\Delta+1} = 0 .
\] (14)

In Ref. [41], Kruglov get a kind of modified Dirac equation by setting \(\beta = 1\), namely, \((\tilde{\gamma}^\mu \partial_\mu + m/h - i\tilde{\epsilon} \tilde{\gamma}^i \partial_3 \tilde{\gamma}^i) \psi = 0\). For researching convenience, we also take \(\beta = 1\) and Eq. (14) can be rewritten as follows:
\[
g^{\mu\nu} (\partial_\nu S + eA_\nu) (\partial_\mu S + eA_\mu) + m^2
- 2\sigma mg^{tt} (\omega - eA_t)^{\Delta+1} = 0 .
\] (15)

Obviously, Eq. (14) and Eq. (15) are the modified Hamilton-Jacobi equation for fermions with half-integral spin that derived from the Rarita-Schwinger equation in curved spacetime. However, according to the previous work, it finds that the Hamilton-Jacobi equation can be derived not only from Rarita-Schwinger equation or Dirac equation, but also from Klein-Gordon equation, Maxwell equations, and gravitational wave equation, \textit{et al}. Therefore, the Hamilton-Jacobi equation can be used to describe the kinematic properties of particles with any spin. In the next section, we use modified Hamilton-Jacobi equation to investigate the fermions tunneling from the D-dimensional charged AdS black hole in dRGT massive gravity.

### III. FERMIONS TUNNELING FROM THE D-DIMENSIONAL CHARGED ADS BLACK HOLE IN DRGT MASSIVE GRAVITY

In this section, the tunneling behavior of fermions on the horizon of D-dimensional charged AdS black hole in dRGT massive gravity is studied. In higher-dimensional dRGT massive gravity, the action with a negative cosmological constant can be given by
\[
I = \frac{1}{16} \int d^D x \sqrt{-g} \left[ \mathcal{R} - 2\Lambda + m^2 \sum_{i=1}^{4} c_i u_i (g, f) \right] ,
\] (16)

where \(c_i\), \(\mathcal{R}\) and \(f\) are a series of constants, the scalar curvature and a fixed rank-2 symmetric tensor, respectively. The last four terms of above equation represent the massive potential \([42-49]\). Meanwhile, are the \(D \times D\) matrix’s \(K^\mu_\nu = \sqrt{g^{\alpha\beta} f_{\alpha\beta}}\) symmetric polynomials of the eigenvalues
\[
U_1 = [\mathcal{K}] , \quad U_2 = [\mathcal{K}]^2 - [\mathcal{K}^2] ,
\]
\[
U_3 = [\mathcal{K}]^3 - 3 [\mathcal{K}] [\mathcal{K}^2] + 2 [\mathcal{K}^3] ,
\]
\[
U_4 = [\mathcal{K}]^4 - 6 [\mathcal{K}^2] [\mathcal{K}^2] + 8 [\mathcal{K}^3] [\mathcal{K}] + 3 [\mathcal{K}^2]^2 - 6 [\mathcal{K}^3] ,
\] (17)

where \(\mathcal{K}\) is the matrix square root \((\sqrt{A})^\mu_\nu (\sqrt{A})^\nu_\kappa = A^\mu_\kappa\), and the rectangular brackets denote the traces, namely, \([K] = K^\mu_\mu\) and \([K^n] = (K^n)^\mu_\mu [50]. Based on the action Eq. (1), the line element of D-dimensional charged AdS black hole in dRGT massive gravity is given by
\[
ds^2 = -f (r) dt^2 + f (r)^{-1} dr^2 + r^2 d\Omega^2_{D-2} ,
\] (18)

where \(d\Omega^2_{D-2} = h_{ij} dx^i dx^j\) represents the line element of a \((D-2)\)-dimensional space with constant curvature \((D-2) (D-3) k\) and \(k\) denotes the spatial curvature constant. When \(k = 0\), the horizon hypersurface of space-time is flat. If the spatial curvature constant takes value \(k = 1\), the geometric property of black hole horizon hypersurface has positive curvature, whereas it takes negative curvature for \(k = -1\). When considering reference metric \(f_m = \text{diag} (0, 0, c^2_0 h_{ij})\), the metric function \(f (r)\) can be expressed as follows:
\[
f (r) = k + c_0^2 c_2 m^2 + \frac{r^2}{l^2} + \frac{c_0 c_2 m^2}{D-2} - \frac{16\pi M}{V_{D-2} r^{D-3}}
+ \frac{(D-3) c_0^2 c_2 m^2}{r} + \frac{(D-3) (D-4) c_2^2 c_4 m^2}{r^2}
+ \frac{1}{2} (D-2) (D-3) r^2 (D-3) ,
\] (19)

where \(V_{D-2}\) is the volume of space spanned by coordinates \(x_i\), \(M\) is the mass of black hole [43]. \(l\) denotes the AdS radius. The electric field in D-dimensions is \(F_{tr} = 1/r^{(D-2)}\) and the electromagnetic potential becomes \(A_t = -q (D-3) r^{D-3}\). It should be mentioned that \(c_1 > 0\), and terms \(c_3 m^4\) and \(c_4 m^2\) vanish when \(D < 5\) and \(D < 6\), respectively. One can obtain the line element of D-dimensional Schwarzschild AdS black hole solution when \(m \to 0\) and \(q = 0\). Based the geometric properties of D-dimensional charged AdS black hole in dRGT massive gravity, its event horizon is located at \(f (r)|_{r=r_H} = 0\).

Now, substituting the inverse tensors of D-dimensional charged AdS black hole in dRGT massive \(g^{\mu\nu}\) into Eq. (14), the modified Hamilton-Jacobi equation can be
rewritten as follows
\[-f(r)^{-1} \left( \frac{\partial S}{\partial t} \right)^2 + f(r) \left( \frac{\partial S}{\partial r} \right)^2 + r^{-2} \left( \frac{\partial S}{\partial \omega} \right)^2 + m^2 + 2\sigma m f(r)^{-1} \left[ \omega + \frac{eq}{(D-3)r^{(D-3)}} \right]^2 = 0. \quad (20)\]

Considering the properties of Eq. (18), the action can be represented as \( S = -\omega t + W(r) + \Theta(\Omega_{D-2}) \). Hence, the radial part of Eq. (20) is
\[-f(r)^{-1} \omega^2 + 2\sigma m f(r)^{-1} \left[ \omega + \frac{eq}{(D-3)r^{(D-3)}} \right]^2 + f(r) \left( \frac{\partial W(r)}{\partial r} \right)^2 + m^2 + \lambda_0 = 0, \quad (21)\]
and the non-radial part of modified Hamilton-Jacobi equation becomes
\[ r^{-2} \left( \frac{\partial \Theta}{\partial \Omega_{D-2}} \right)^2 - \lambda_0 = 0. \quad (22)\]

For a spherically symmetric black hole, the non-radial part modified Hamilton-Jacobi equation dose not devote the tunneling rate of emitted particles. So, here we only keep Eq. (21), which leads to the integral function
\[ W_\pm = \pm \int f(r)^{-1} \sqrt{(1 - 2\sigma m)(\omega - eA_t)^2 - f(r)(m^2 + \lambda_0)} dr, \quad (23)\]
where the \( +(-) \) denote the outgoing (incoming) solutions of Eq. (23), respectively. Now, solving the above integral on the event horizon, the result is
\[ W_\pm = \pm i\pi \left( \frac{1 - m\sigma}{f'(r_H)} \right) (\omega - \omega_0), \quad (24)\]
where we denote that \( \omega_0 = -eq/[ (D-3)r_H^{D-3}] \). Based on the tunneling theory of black holes, the tunneling rate of fermions with half-integral spin at event horizon is given by
\[ \Gamma = \exp \left[ -\frac{2}{\hbar} \left( \text{Im} W_+ - \text{Im} W_- \right) \right] \]
\[ = \exp \left[ -\frac{4\pi}{\hbar} \left( 1 - m\sigma \right) (\omega - \omega_0) \right]. \quad (25)\]

Since Eq. (25) is similar to the Boltzmann formula. Therefore, the modified Hawking temperature of D-dimensional charged AdS black hole in dRGT massive becomes
\[ T_H = \frac{\hbar}{4\pi r_H} \Xi (1 + m\sigma) = T_0 (1 + m\sigma), \quad (26)\]

where
\[ \Xi = (D-3)k + \frac{(D-1)r_H^2}{l^2} + c_0 c_1 m^2 r_H + (D-3) c_2 c_0^2 m^2 + \frac{(D-3) (D-4) c_3 c_1 m^2}{r_H} - \frac{q^2}{2 (D-3) r_H^{2(D-3)}} \]
\[ + \frac{(D-3) (D-4) (D-5) c_4 c_0^2 m^2}{r_H^2} \quad (27)\]
In Eq. (26), we express the mass of D-dimensional charged AdS black hole in dRGT massive gravity in terms of the event horizon as follows: \[ M = \frac{(D-2) V_{D-2} r_H^{D-3}}{16\pi} \left[ k + \frac{r_H^2}{l^2} + \frac{c_0 c_1 m^2 r_H}{D-2} + c_0 c_2 m^2 + \frac{(D-3) c_3 c_1 m^2}{r_H} + \frac{(D-4) (D-3) c_4 c_0^2 m^2}{r_H^2} \right] + \frac{q^2}{2 (D-2) (D-3) r_H^{2(D-3)}} \quad (28)\]
Eq. (26) shows that \( T_0 \) is the original Hawking temperature of D-dimensional charged AdS black hole in dRGT massive gravity, whereas \( T_H \) is the modified case.

Furthermore, by using Eq. (26) and the first law of black hole thermodynamics \( dS = (dM - \Omega_H dJ - \Phi dQ)/T \) with the electromagnetic potential \( \Omega_H \) and rotating potential \( \Phi \), the modified entropy of D-dimensional charged AdS black hole in dRGT massive gravity is given by
\[ S_H = \int \frac{dM - \Phi dQ}{(1 + \sigma m) T_0} = S_0 - m \sigma \int dS_0 + O(\sigma^2), \quad (29)\]
where \( S_0 = V_{D-2} r_H^{D-2}/4 \) is the original entropy of D-dimensional charged AdS black hole in dRGT massive gravity. It is clear both modified Hawking temperature \( T_H \) and entropy \( S_H \) do not only related to the properties of D-dimensional charged AdS black hole in dRGT massive gravity, but also depended on the coupling constant \( \sigma \) and the mass of emitted particles \( m \). When \( \sigma = 0 \), one obtains \( T_0 \) and \( S_0 \).

By analyzing Eq. (28), one can see that the modified Hawking temperature is higher than the original one, which indicates the effect of MDR can significantly enhance the evolution of black hole. As we know, the emission rate of black holes can be reflected by Stefan-Boltzmann law. Therefore, it is interesting to study the effect of MDR on Stefan-Boltzmann’s law. Generally, the Stefan-Boltzmann law can be expressed as follows:
\[ \frac{dE}{dt} = \sigma_S A T^4, \quad (30)\]
where \( E \) is the total energy of black hole, \( \sigma_S \) is the Stefan-Boltzmann constant, \( A \) and \( T \) represent the area and
temperature of black holes, respectively. Now, inserting Eq. (28) into Eq. (30), the Stefan-Boltzmann law can be rewritten as

\[
\left(\frac{dE}{dt}\right)_{\text{modified}} = \sigma_S A T_0^4 (1 + m\sigma)^4 = \left(\frac{dE}{dt}\right)_{\text{original}} (1 + m\sigma)^4 , \tag{31}
\]

where \((dE/dt)_{\text{original}}\) is the original Stefan-Boltzmann law. Eq. (31) shows that the modified emission rate is higher than the original case, which is in line with previous analysis of temperature.

IV. DISCUSSION

In this work we studied the fermions tunneling from D-dimensional charged AdS black hole in dRGT massive gravity via the modified Hamilton-Jacobi equation for fermions with half-integral spin. First of all, by using a new kind of MDR and the WKB approximation, we derived the modified Hamilton-Jacobi equation from Rarita-Schwinger equation. Then, with the help of the modified Hamilton-Jacobi equation, the MDR corrected Hawking temperature and entropy of D-dimensional charged AdS black hole in dRGT massive gravity, and fermion behavior on its event horizon are obtained. The results show that the modified thermodynamic quantities do not only related to the properties of black hole, but also depended on the coupling constant \(\sigma\) and the mass of emitted particles \(m\). Meanwhile, by analyzing the emission rate of black holes via Stefan-Boltzmann law, it finds that the modified emission rate is higher than the original case, which indicates that the MDR can obviously affect the evolution of black holes. It should be noted that we only set \(\beta = 1\) in this paper, however, one may obtain some interesting results when \(\beta\) takes other values.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 11575022) and the Fundamental Research Funds of China West Normal University (Grant Nos. 17E0903 and 17YC518).

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