Invited Comment

Quantum optics and frontiers of physics: the third quantum revolution

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Abstract

The year 2015 was the International Year of Light. However, it also marked, the 20th anniversary of the first observation of Bose–Einstein condensation in atomic vapors by Eric Cornell, Carl Wieman and Wolfgang Ketterle. This discovery could be considered as one of the greatest achievements of quantum optics that has triggered an avalanche of further seminal discoveries and achievements. For this reason we devote this essay for the focus issue on ‘Quantum Optics in the International Year of Light’ to the recent revolutionary developments in quantum optics at the frontiers of all physics: atomic physics, molecular physics, condensed matter physics, high energy physics and quantum information science. We follow here the lines of the introduction to our book ‘Ultracold atoms in optical lattices: Simulating quantum many-body systems’ (Lewenstein et al 2012 Ultracold Atoms in Optical Lattices: Simulating Quantum Many-body Systems (Oxford: University Press)), and to a lesser extent the review article M Lewenstein et al (2007 Adv. Phys. 56 243). The book, however, was published in 2012, and many things has happened since then—the present essay is therefore upgraded to include the latest developments.

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In 1995 Bose–Einstein condensation (BEC) in dilute alkali gases was realized experimentally 1995 [3–5]. This has evidently started a new era. The AMO community—comprising physicists working in atomic, molecular, optics and quantum optics—realized very quickly that studies of ultracold atomic systems will allow for completely novel access to degenerate quantum many body systems. An AMO physicist expected that ultacold atomic gases will soon allow one to study problems analogue to those studied in condensed matter physics. The physicists working in condensed matter physics, remained, however, very sceptical initially. Their argument was that, at the very end, what had been achieved experimentally was a regime of weakly interacting Bose gases. These kind of systems were thoroughly investigated by the condensed matter theorists in the 1950s and 1960s [6, 7]. Of course, AMO experiments dealt with confined systems of finite size and typically inhomogeneous densities. For solid state/condensed matter experts this fact was a technical issue rather than a question of fundamental importance. Despite these early controversies, the Nobel foundation appreciated the discovery of BEC and awarded its yearly prize in 2001 to Cornell, Wieman and Ketterle ‘for the achievement of Bose–Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates’ [8, 9]. Today, from the perspective of more than 20 years, it is clear that due to the efforts of the whole AMO community these fundamental studies have enriched amazingly the standard ‘condensed matter’ understanding of static
and dynamical properties of ultracold Bose gases in particular, and ultracold matter in general [10].

The AMO community from the early days was trying to push the BEC physics towards new regimes and new challenges. The progress in these directions was indeed spectacular. At the beginning of the third millennium it became clear to AMO and condensed matter physicists, that we were entering a truly new quantum era with unprecedented possibilities of control of many body systems. The first theoretical proposal and experimental studies of the regime of strongly correlated systems with ultracold atoms and/or molecules appeared. Only a few years after the first observation of BEC, atomic degenerate Fermi gases were achieved [11–13] paving the way towards the observations of Fermi superfluidity. While in the weak interaction limit fermionic Fermi gases are well understood and described by the Bardeen–Cooper–Schrieffer theory (BCS) [7], its fate in the limit of strong correlations remains a mystery. Studies of the so called BEC–BSC crossover with ultracold atoms shed new light on this problem. For recent reviews reporting the large activity in this field see [14, 15] and [1]. In 2002 Greiner et al [16] observed the signatures of the quantum phase transition from the BEC (i.e. a bosonic superfluid) to the, so called, Mott phase (i.e. bosonic insulator) for bosons confined in an optical lattice. This experiment was based on the seminal proposal by Jaksch et al [17], and opened the in-depth-studies of strongly correlated quantum many body systems using ultracold atomic gases.

Nowadays, ultracold atomic and molecular physics address the same challenges and frontiers of modern quantum physics, as condensed matter and high energy physics. Yet, AMO systems are considered to be the most controllable ones to study many body physics. They are already finding highly nontrivial applications in quantum information, quantum metrology and quantum sensing; particularly impressive are their applications for powerful quantum simulators and quantum annealers. At the theory level, studies of ultracold atoms concern so many different fields, that one can talk nowadays about a ‘grand unification’ where AMO, condensed matter, nuclear physics, and even high energy physics theorists are joining efforts—for reviews see [2, 15], and [1]—.

Quoting out book [1]: ‘After the first quantum revolution at the beginning of the XX century, the second one associated with the name of John Bell and the experimental quest for non-locality of quantum mechanics together with the experimental control over single, or few particle systems—see in particular Alain Aspect’s introduction to [18]—, we are witnessing the third one: the quantum revolution associated to the control over macroscopic quantum systems and the rise of quantum technologies.’ Indeed this quantum revolution is not limited to the ultracold atoms, ions and molecules, but includes other platforms like NV-centers [19] and superconducting qubits [20] where quantum optics also plays a central role.

1. Historical perspective: cold atoms and quantum optics

The third quantum revolution does not arrive completely unexpected: the developments of atomic physics and quantum optics in the last 30 years have inevitably led to it. In the seventies and eighties of the 20th century, atomic physics was considered to be a very well established, developed and respectful area of physics. It was, however, by no means a ‘hot’ area. In particular, in theoretical atomic physics the open problems were very complex technically (for instance those involving many electron systems), from the methodological point of view nearly everything was already developed. The major problems that were considered concerned mainly questions related to optimization of these methods. AMO physics was undergoing an evolutionary progress—there was no sign for the approaching revolution and for discoveries of the totally new phenomena. On the other hand, the seventies and eighties mark the beginning of the Golden Age of experimental quantum optics. Spectacular discoveries took place in laser physics and nonlinear optics; they led to the Nobel prize for Schawlow and Bloembergen in 1981 ‘for their contribution to the development of laser spectroscopy’. On the other hand, many leading groups started to study quantum physics with individual quantum particles. These studies culminated in 1989 with the Nobel prize for Dehmelt and Paul ‘for the development of the ion trap technique’, and for Ramsey ‘for the invention of the separated oscillatory fields method and its use in the hydrogen maser and other atomic clocks’.

The beginning of the theoretical quantum optics goes back to the 1960s; in these years Glauber [21, 22], the winner of the Nobel Prize 2005, formulated the quantum coherence theory. At the end of the 1960s Haken; Weidlich, and by Scully and Lamb (Nobel laureate of 1955) developed the quantum theory of the laser. In 1983 one of the prominent theorists and a disciple of Weidlich and the Stuttgart school, Haake told Lewenstein: ‘In the 1970s and 1980s, however, theoretical quantum optics was not considered to be a separate, established area of theoretical physics—it was rather considered to be a part of statistical physics.’ There were many reasons for this. First, quantum optics of this period was dealing with single particle problems. If it considered the many body challenges, such as laser theory, or more generally optical instabilities [23], it would solve it using linear models, or employing relatively simple versions of various mean field approximations. Clearly, the most challenging intellectually theoretical works of that period dealt problems of quantum statistical physics, such as the understanding of the role of quantum fluctuations and quantum noise in optical instabilities [23, 24].

Drastic and dramatic changes of this situation occurred in the 1980s due to the unprecedented level of quantum engineering, defined as ‘preparation, manipulation, control and detection of quantum systems’. Quantum engineering was mostly developed in the field of atomic physics and quantum
The techniques of particle trapping and cooling have reached regimes of very low temperatures (today down to nanokelvin!) and precision that were considered unattainable just two decades ago. These techniques are nowadays routinely applied for atoms, ions, molecules and even nano-objects. The Nobel Foundation recognized the unprecedented successes of laser cooling in 1997, and gave its annual Prize in physics to Chu [25], Cohen-Tannoudji [26] and Phillips [27] for the development of methods to cool and trap atoms with laser light. Progress in laser cooling (and related methods of mechanical control of particles using light [28]) opened completely new paths and perspectives in AMO physics and quantum optics: it allowed one to develop atom optics and interferometry [29], and revolutionised precision metrology and quantum engineering.

It was the combination of laser cooling and evaporation cooling (invented by Hess in 1986) that led in 1995, to the experimental observation Bose–Einstein condensation (BEC) [3–5], a phenomenon which was theoretically discovered by Bose and Einstein in 1924. As we said before, these experiments marked evidently the birth of a new era. The Nobel Prize was given to Cornell and Wieman [8] and Ketterle [9] in 2001 ‘for the achievement of Bose–Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates’. Ketterle described it as ‘a true breakthrough, in which atomic physics and quantum optics has met condensed matter physics’ [9]. As we mentioned above, condensed matter physicists in the first moment underestimated the importance of this breakthrough. They claimed that BEC was observed in weakly interacting systems, which is perfectly described by the mean field Bogoliubov-de Gennes theory [10], developed for homogeneous systems in the 50’s.

The study of quantum correlations and entanglement. Theoretical quantum information science starts with the works of the late Peres [30, 31], with quantum cryptography, proposed by Bennett and Brassard [32], and Ekert [33], with more sophisticated quantum communication proposals by Bennett and Wiesner [34] and Bennett, Brassard, Crépeau, Jozsa, Peres, and Wootters [35], the discovery of the Shor’s quantum algorithm for number factorization [36], and the first proposal for experimental realization of a quantum computation Cirac and Zoller [37] (for general reviews see [38, 39]; for the latest version of the quantum information road map see [40]). These developments, and the following progress in the theory, allows us nowadays to understand much better and deeper quantum correlations and entanglement. In particular, over the last 20 years we have learned how to prepare and use entangled states as a resource. Today, quantum information stimulates new approaches and is in permanent and robust superposition with the physics of cold atoms, molecules and ions. The first quantum computers are being realized and commercialized: as suggested already by Feynman [41, 42], these are computers of special purpose— quantum simulators (QS) [43]. These are quantum devices that efficiently mimic quantum many body systems that can hardly be simulated using ‘classical’ computers [39]. A special kind of QS, the, so called, quantum annealers solve computationally hard classical optimization problems using quantum effects.

Optical lattices and Feshbach resonances. Perhaps the most fascinating is the fact that ultracold atoms can be brought to the regime of strongly correlated systems. This possibility was formulated first the seminal proposal of Jaksch et al.’ [17], who predicted the possibility of realizing the superfluid-Mott insulator transition using cold atoms in an optical lattice. The proposal was based on the bosonic Mott insulator transition proposed in condensed matter [44]. It is important to stress that Jaksch and colleagues wanted to use the Mott insulator (with fixed number of atoms per lattice site) for a preparation of a quantum register that could subsequently be used to perform quantum computing with cold atoms in a lattice. Entanglement between atoms could be achieved via controlled collisions [45]. Bloch–Hänsch group observed the superfluid-Mott insulator transition in the seminal experiment in 2001 [16]; this was without any doubts a benchmark in the studies of strongly correlated systems with ultracold atoms [46]. A Pandora’s box was open and several world leading groups studied similar quantum phase transitions: bosonic superfluid-Mott insulator transitions in pure Bose systems [47], in disordered Bose systems [48], in Bose-Fermi [49, 50], and Bose-Bose mixtures [51]. The possibility to change the collision properties of ultracold atoms by means of tuning the Feschbach resonances of the atomic species has been another tool of inestimable value. Such a tool has lead to the fermionic Mott insulator [52, 53]. Also, exotic Mott insulators were achieved in various experiments, such as Mott insulator of molecules [54], or an insulator of bound repulsive pairs of atoms, i.e. ‘pairs of atoms at a site that cannot release their repulsive energy due to the band structure of the spectrum in the lattice’ [55].

Cold trapped ions were initially investigated to realize the Cirac-Zoller quantum gate [37], and to attempt to build an scalable quantum computer using a bottom-up approach [56, 57]. Recent progress in this research line can be found in [58]. A new stream of ideas using cold ions for quantum simulators have recently appeared. First proposals have shown that ion-ion interactions mediated by phonons can be manipulated to implement various spin models [59–61], where the spin states correspond to internal states of the ion. In tight linear traps, one could realize the spin chains with interactions decaying as (distance)$^{-1}$, i.e. as in the case of dipole-dipole interactions. Such spin chains may serve as quantum neural network models and may be used for adiabatic quantum information processing [62, 63]. More interestingly, ions can be employed as quantum simulators, both in 1D and in 2D, where the ions form self-assembled Coulomb crystals [64]. The first steps towards the experimental...
realization of these ideas has been reported [65]. These results have been recently extended in [66–71]. Alternatively to spins, one could look at phonons in ion self-assembled crystals; these are also predicted to exhibit interesting collective behaviour from Bose condensation to strongly correlated states [72, 73]. Trapped ions may be used also to simulate mesoscopic spin-boson models [74]. Combined with optical traps or ion microtrap array methods can be used to simulate a whole variety of spin models with tunable interactions in a wide range of spatial dimensions and geometries [75]. Finally, they are promising candidates for the realization of many-body interactions [76]. All these theoretical proposals and the spectacular progress in the experimental trapped ion community pushes trapped ions physics to be soon used as widely as cold atoms to mimic condensed matter physics and beyond, as for instance in [77].

2. Quantum simulators

Before we proceed we would like to make a short detour to explain in a little more detail the concept of quantum simulators. The 20th century was the age of information and computers. But, even supercomputers have their limitations—as we well know for instance their ability to predict weather, a paradigmatic classical chaotic phenomenon, is very restricted. For this reason already in classical computer science the concept of special purpose computers was developed. These ‘classical simulators’ are not like universal classical computers—they can only simulate or calculate certain restricted class of models describing Nature. The best simulations of classical disordered systems, such as spin glasses, are nowadays obtained with such computers of special purpose—‘classical simulators’.

The situation is even more dramatic in quantum physics and chemistry. We know very well that an universal quantum computer will revolutionize our technology in the future. Unfortunately, this future does not seem to be very close at hand, and definitely concerns tens of years. However, quantum computers with a special purpose, i.e. quantum simulators, have existed for 5 years already (cf [78]).

Various quantum phenomena such as high-$T_c$ superconductivity or quark confinement are still awaiting universally accepted explanations because of the computational complexity of solving the simplified theoretical models designed to capture the relevant physics. Richard Feynman suggested in 1982 [41, 42], solving such models by ‘quantum simulation’. He pointed out that it might be possible sometimes to find a simpler and experimentally more accessible system to mimic the quantum system of interest. Feynman’s idea was motivated by the complexity of classical simulations of quantum systems. Suppose we want to study a system of $N$ spins $1/2$; then the dimension of the Hilbert space is $2^N$, and the number of coefficients we need to describe the wave function of the system may be in principle just as large. As $N$ grows, this number quickly becomes larger than the number of atoms in the Universe, so classical simulations are evidently impossible. Of course, in practice this number may be very much reduced by using clever representations of the wave functions, but in general there are many quantum systems, which are very hard to simulate classically. The modern concept of quantum simulators is not exactly the same as that of Feynman. Since condensed matter systems are typically very complex, theorists construct simplified models. In this simplification, symmetries of the original problem are kept intact, and the concept of universality is used: different Hamiltonians with similar symmetry properties belong to the same universality classes, i.e. they exhibit the same phase transitions and have the same critical exponents. Unfortunately, even these simplified models are often difficult to understand. A paradigmatic example of such a situation concerns high-$T_c$ superconductivity of cuprates, where it is believed that the basic physics is captured by an array of weakly coupled 2D Hubbard models for electrons, i.e. spin-1/2 fermions. Even this simple model reduced to one 2D plane does not allow for accurate treatment, and there is much controversy concerning, for instance, the phase diagram or the character of the transitions. The role of quantum simulators, as proposed in many quantum information projects, will be to simulate simple models, such as 2D Hubbard models, and obtain a better understanding of them, rather than try to simulate the full complexity of the condensed matter.

A ‘working’ definition of a quantum simulator could be as follows [1, 79]:

- A quantum simulator is an experimental system that mimics a simple model, or a family of simple models, of condensed matter (or high-energy physics, or quantum chemistry...).
- The simulated models have to be of some relevance for applications and/or our understanding of the challenges of the above-mentioned areas of physics, and of the new challenges in Physics, Chemistry and Biology that are still to be identified.
- The simulated models should be computationally intractable for classical computers. Note that this statement may have two meanings: (i) an efficient (scalable to large system size) algorithm to simulate the model might not exist, or might not be known; (ii) the efficient scalable algorithm may be known, but the size of the simulated model is too large to be simulated under reasonable time and memory restrictions. The latter situation, in fact, occurs with classical simulations of the Bose or Fermi Hubbard models as compared to their experimental quantum simulators. There might also be exceptions to the general rule. For instance, it is desirable to realize quantum simulators to simulate and to observe novel, hitherto only theoretically predicted phenomena, even though it might be possible to simulate these phenomena efficiently with present-day computers. Simulating and observing is more than just simulating.
- A quantum simulator should allow for broad control of the parameters of the simulated model, and for control of the preparation, manipulation and detection of the states

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of the system. In particular, it is important to be able to set the parameters in such a way that the model becomes tractable using classical simulations. This provides the possibility of validating the quantum simulator.

Fueled by the prospect of solving a broad range of long-standing problems in strongly-correlated systems, the tools to design, build, and implement QSs [1, 41, 79] have rapidly developed and are now reaching very sophisticated levels [78]. Many proposals exist and already many realizations of quantum simulators employing ultracold atoms and molecules in traps and in optical lattices (cf [80–91], for a review see also [15]), probing quantum dynamics (cf [92, 93]), employing ultracold trapped ions (cf [65, 67, 94–98], for recent reviews see [99, 100]), atoms in single or in arrays of cavities (cf [101]), ultracold atoms/ions in arrays of traps, ultracold atoms near nano-structures and multidimensional plasmonic traps [102–105], photons [106–110], arrays of quantum dots, circuit quantum electrodynamics (QED) and polaritons (cf [111–115], for a recent review see [116]), artificial lattices in solid state [117], nuclear magnetic resonance (NMR) systems [118–121] and superconducting qubits.

In addition to our book [1], a number of excellent recent reviews exist, in particular in the focus issue of Nature Physics Insight [122–127]. At the current pace, it is expected that we will soon reach the ability to finely control many-body systems whose description is outside the reach of a classical computer. For example, modeling interesting physics associated with a quantum system involving 50–100 spin-1/2 particles—whose general description requires $2^{50} \approx 10^{15}$ amplitudes—is out of the reach of current classical supercomputers, but perhaps within the grasp of a quantum simulator.

Obviously, the real-world implementations of a quantum simulation will always face experimental imperfections, such as noise due to finite precision instruments and interactions with the environment. The quantum simulator—as envisioned by Feynman—is fundamentally an analog device, in the sense that all operations are carried out continuously. However, errors in an analog device (also continuous, like temperature in the initial state, or the signal-to-noise ratio of measurement) can propagate and multiply uncontrollably [128]. Indeed, Landauer, a father of the studies of the physics of information, questioned whether quantum coherence was truly a powerful resource for computation because it required a continuum of possible superposition states that were ‘analog’ in nature [129]. In contrast to digital quantum simulators, where the error correction schemes might be in principle applied, the question of validation, calibration and reliability of analog quantum simulators is very subtle and very ‘hot’ [130].

3. Cold atoms and quantum optics at the frontiers of physics

Physics of ultracold quantum matter and quantum optics address nowadays, among others, common challenges of contemporary many body physics with condensed matter physics, nuclear physics, quantum field theory, high energy physics, and even astrophysics. In particular, many of these important challenges can be addressed within cold atoms and quantum optical wizardry:

• **1D systems.** In one dimension (1D) the role of quantum fluctuations and correlations is spectacular, making the theory in 1D particularly challenging. Still, the theory of 1D systems is very well developed. In 1D various, very strong methods exist such as Bethe Ansatz and quantum inverse scattering theory (cf [131]), powerful approximate approaches, such as bosonization, and conformal field theory—cf [132]. Moreover in 1D truly efficient numerical methods exist, such as density matrix renormalization group technique (DMRG)—cf [133]. Many of the predictions of the theory, however, have never been directly and clearly observed in experiments in condensed matter. On the other hand, they can be studied with cold atoms—for a review see [134, 135]. Among such effects that still call for experimental observation are: atomic Fermi, or Bose analogues of spin-charge separation. More generally, microscopic properties of of the, so called, Luttinger liquids provide a similar challenge [136–138]. 1D gas in the deep Tonks-Girardeau regime, i.e. the regime of hard-core bosons, was first realized in experiments by Paredes et al [139]—see also [47, 140, 141] and [142]—this was evidently a milestone toward further studies of strongly correlated 1D systems. The Tonks regime was also achieved in 2008 employing dissipative processes (two body losses) is perhaps the first experimental example of how to control a system by making use of its coupling to the environment [143–146]. See also the recent progress towards deep Tonks-Girardeau regime in [147].

• **Spin-boson model.** The spin-boson model is a paradigmatic model in condensed matter theory—for a review see [148]—. It consists of a two-level system coupled to a bosonic reservoir as such it is also a paradigmatic model of quantum dissipation theory in quantum optics Recati et al proposed [149]. To realize spin-boson model using an atomic quantum dot, i.e., a single atom in a tight optical trap, Laser-coupled to a BEC. In particular, an atomic quantum dot embedded in a 1D Luttinger liquid corresponds to a spin-boson model with Ohmic coupling. As such it undergoes a dissipative phase transition; moreover atomic physics techniques allow to determine Luttinger parameters in experiment. Also, it has been shown that in the low-energy limit the system of a driven quantum spin coupled to a bath of ultracold fermions can be mapped onto the spin-boson model with an Ohmic bath [150].

• **2D systems.** The celebrated Mermin–Wagner–Hohenberg theorem, says that 2D systems with continuous symmetry do not exhibit long range order at temperatures $T > 0$. For a superfluid the order is destroyed by phase fluctuations. 2D systems may, however, undergo
Kosterlitz–Thouless–Berezinskii transition (KTB) at low temperatures. In 2 dimensional BEC above the critical temperature, there is a proliferation of topological free excitations—vortices. Below the critical temperature, the vortex–antivortex form bound pairs, and the correlations decay is algebraic, rather than exponential. KTB transition has been observed in liquid Helium [151, 152], but observation of its microscopic nature (binding of vortex pairs) was possible only in the recent experiments [153–159]. It is worth mentioning that Thouless and Kosterlitz obtained the Nobel Prize in 2016 for the prediction of the KTB transition.

- **Hubbard and spin models.** In condensed matter important examples of strongly correlated states in condensed matter physics are described in theory by various types of Hubbard models [131, 160]. In condensed matter physics, obviously, Hubbard models are ‘reasonable caricatures’, or better to say approximations of real systems. This is very different in the physics Of ultracold atomic in optical lattices: here practically perfect realizations of a whole variety of Hubbard models are possible [161]! Hubbard models frequently reduce to various spin models, in certain limits, obviously. Again, cold atoms and ions allow to reach such limits, and provide perfect realizations of desired spin models—see for instance [162–165] and [66, 68, 70, 71, 87]. Specifically, spin–spin interactions between neighboring atoms can be implemented by bringing the atoms together on a single site and carrying out controlled collisions [45, 166, 167], on-site exchange interactions [80] or superexchange interactions [168]. Physics of ultracold atoms in optical lattices offers thus a fantastic playground for quantum simulations of specific condensed matter models.

- **Disordered systems: Interplay localization–interactions.** Disorder is practically always present in condensed matter systems: the effects of disorder are thus a central interest of condensed matter physics. One of the most important quantum effects of disorder is Anderson localization [169]. It is a single particle effect, in which wave function of single particles in a random potential undergoes exponential (or algebraic) localization in space due to the destructive interference effects during the scattering in the random potential. Amazingly, a controlled disorder, or pseudo–disorder may be created in ultracold atomic systems in traps or in optical lattices! This is usually achieved by adding an optical speckle potential, or by superposing several lattices with incommensurate periods of spatial oscillations [170, 171]. Other proposed methods are the admixture of different atomic species randomly trapped in sites distributed across the sample and acting as impurities [172], or the use of an inhomogeneous magnetic field which modifies randomly, close to a Feshbach resonance, the scattering length of the atoms [173]. More recently, the use of appropriately designed holographic mask was strongly advocated. In fact, recently, experimental realization of Anderson localization of matter waves has been reported in a non-interacting BEC of $^{39}\text{K}$ in a pseudo-random potential [174], and in a small condensate of $^{87}\text{Rb}$ in a truly random potential in the course of expansion in a one-dimensional waveguide [175]. Three dimensional Anderson localization has also been reported for a spin-polarized atomic Fermi gas of $^{40}\text{K}$ [176] and for $^{87}\text{Rb}$ ultracold atoms [177]. The experiment reported in [175], as well as the appearance of the effective mobility edge due to the finite correlation length of the speckle induced disorder, was precisely predicted in [178]. The interplay between disorder and interactions is a very active research area, also in ultracold gases. In Fermi systems with attractive interactions, disorder might destroy the possibility of BCS transition (‘dirty’ superconductors). Both in Fermi and Bose systems the presence of weak repulsive interactions disorder in general Leads to delocalization; strong interactions in such systems lead to localization à la Mott [179], and insulating behaviour. Between these two extreme, for intermediate interactions, there might exist exotic delocalized ‘metallic’ phases. Cold atoms in optical lattices also provide a perfect playground to study the crossover between Anderson-like (Anderson glass) glass and and Bose (Mott-like) glass. First experimental signatures of a Boseglass [48, 180, 181], of the Anderson glass crossover [182] and of glassy behaviour in binary atomic mixtures [183] have been reported. Theory indicates [184–187] indicates Anderson localisation and Anderson-like glass should survive the presence of weak nonlinear interactions and quasi-disorder in BEC. In fact, one expects that the systems will enter a novel Lifshits glass phase [188], where bosons condense in a finite number of states; these state belong to the low energy tail of the single particle spectrum, a so called Lifshits tail. Very recently, the emergence of a disorder-induced insulating state [189] and the observation of many-body localization [190] in interacting fermions have been reported.

- **Disordered systems: spin glasses.** Since the 1970s the nature of spin glasses is the subject of never ending debate. It has attracted a lot of attention [193–195] since the seminal papers of Edwards and Anderson [191], and Sherrington and Kirkpatrick [192]. The debate in short can concentrates on a competition between two competing pictures: the replica symmetry breaking picture of Parisi, and the droplet model of Fisher and Huse. These two very different are clearly applicable in some situations (Parisi for long range models, and droplet for short range ones), but they not applicable always. Ultracold atoms in optical lattices might contribute to resolve this controversy by, for instance, studying independent copies with the same disorder, the so called replicas. A measurement scheme for the determination of the disorder-induced correlation function between the atoms of two independent replicas with the same disorder has been proposed [196]. Cold atoms can also provide a deeper understanding of the role of quantum effects in spin glasses, such as quantum tunnelling. For instance behaviour of Ising spin glasses in transverse fields—i.e. in a truly quantum mechanical situation [197, 198]—
could be in principle investigated with ultracold atoms. More recently, the realization of the Dicke model with ultracold atoms in a single mode cavity [101], stimulated recently intensive studies of the connection between multi-mode Dicke models with random couplings and the spin glass physics [199, 200]. Also, the possibility of addressing NP versions of the number partitioning were mentioned and investigated in this context [201, 202].

- **Spin glasses and D-Wave computers.** In fact spin models are paradigms of multidisciplinary science. They are most relevant for various fields of physics, reaching from condensed matter to high energy physics, but they also find several applications beyond the physical sciences. In neuroscience, brain functions are modeled by interacting spin systems, going back to the famous Hopfield model of associative memory [203]. This directly relates to computer and information sciences, where pattern recognition or error-free coding can be achieved using spin models [204]. Importantly, many optimization problems, like number partitioning or the famous traveling salesman problem, belonging to the class of NP-hard problems, can be mapped onto the problem of finding the ground state of a specific spin model [205, 206]. This implies that solving a spin model itself is a task for which no general efficient classical algorithm is known to exist. A controversial development, supposed to provide also an exact numerical understanding of spin glasses, regards the D-Wave machine. These devices employing arrays of superconducting junctions, were recently introduced on the market. What they do is in fact that they solve (i.e. find the ground state of) classical spin glass models by using the, so called quantum annealing approach. Within this approach, if the system is trapped in a local minimum of energy, it can leave this configuration via quantum tunneling. Many researchers agree that D-Wave computers are genuine quantum simulators, because despite decoherence and coupling to a thermal bath, they are consistent with performing quantum annealing [207, 208], and with open quantum system dynamics [209], but the underlying mechanisms are not clear, and it remains an open question whether these machines provides a speed-up advantage over the best classical algorithms [210–212]. All this triggers interest in alternative quantum systems, in particular quantum optical systems, designed to solve general spin models via quantum simulation. A noteworthy system for this goal are trapped ions: Nowadays, spin systems of trapped ions are available in many laboratories [65–68, 70, 71], and adiabatic state preparation, similar to quantum annealing, is experimental state-of-art.

- **Disordered systems: Large effects by small disorder.** Weak disorder can have dramatic effects on the physical systems, both at the classical and the quantum levels. For instance, in the Random Ising model in 2D spontaneous magnetization is lost already for an infinitesimal but not zero disorder: this can be regarded as paradigmatic example of drastic changes arbitrary disorder can bring in classical statistical physics. At the quantum level, the most celebrated phenomena is Anderson localization that appears for arbitrary small disorder in 1D systems, and it is expected to appear for arbitrary small disorder also in 2D systems. Similar phenomena, in fact more general class of them can be studied and experimentally realized in cold atoms, e.g. [213, 214] and [215], where continuous symmetries in spin systems can be broken by disorder, such to allow for the appearance of long range order.

- **High Tc superconductivity.** After many years of research, the origin of high Tc superconductivity is still debated [216]. However, there is a consensus—see, for instance, P.W. Anderson’s contribution in [216]—that by understanding the $t - J$ limit of the 2D Hubbard model [160, 217–219] for spin 1/2, or two-component fermions we could at least explain part of the phenomenon. Simulating these models numerically is extremely hard and the results obtained are controversial. Quantum simulators based on cold fermionic atoms with spin (or pseudospin) 1/2 loaded in optical lattices can be used as an alternative tool for solving these models [220]—see also [221]—. As preliminary results, both ‘spinless’, i.e. polarized, as well as spin 1/2 unpolarized ultracold fermions [47, 222] have been realized in experiments; the recent observations of fermionic Mott insulator states [52, 53] are a spectacular progress to the goal. It is also worth noticing that Bose-Fermi mixtures have already been intensively studied in optical lattices [49, 50, 223]; these systems might exhibit superconductivity due to boson-mediated fermion-fermion interactions. Supercurrence interactions, demonstrated very recently in the context of ultracold atoms in optical lattices [168], are believed also to play an important role in the context of high-temperature superconductivity [224]. Remarkably, antiferromagnetic correlations for Fermion has been demonstrated in [225] in dimerized lattices via radio frequency band transfer and observed in momentum space via spin sensitive Bragg scattering of light [226] and very recently also in situ via quantum gas microscopy [227–230].

- **BCS-BEC crossover.** Trapped ultracold gases are also relevant for studying high Tc superconductivity. At (very) low temperatures, weakly attracting spin-1/2 fermions become superfluid and this superfluid phase is well described in terms of weakly bounded Cooper pairs as explained by the Bardeen–Cooper–Schrieffer (BCS) theory. Weakly repulsive fermions, On the other hand, weak and repulsive interactions between fermions favor the formation of bosonic molecules that can condense in a BEC. When interaction are strong, fermions undergo also a transition to the superfluid state, but at much higher T. This regime can be accessed in cold atoms by the use of Feshbach resonances and has allowed for the observation BCS-BEC crossover by several groups [231–233]—for a recent review on this technique see [234]—to observe such BCS-BEC crossover. Further experimental developments for the spin-balanced case can be found in [15] and [14]. There is much more controversy regarding...
imbalanced spin mixtures [235, 236]. First experimental signatures supporting pairing with finite momentum in spin-imbalance mixtures, the so called FFLO state [237, 238], have been observed in one-dimensional Fermi gases [86]. Interestingly, topological nontrivial flat bands have been proposed for increasing the critical temperature of the superconducting transition [239].

- **Frustrated antiferromagnets and spin liquids.** It is generally expected that in the neighborhood of a high $T_c$ superconducting phase, there is a (frustrated) antiferromagnetic phase. Also for this reason, frustrated antiferromagnets and quantum magnetism have attracted so much interest in condensed matter physics for decades. The challenge and the interest in quantum magnetism is in the possibility of realizing novel, exotic quantum phases, e.g. valence bond solid states, resonating valence bond states, and various kinds of quantum spin liquids—spin liquids of I and II kind, according to Lhuillier [240, 241], and topological and critical spin liquids, according to Fisher [242, 243]—. Cold atoms allows one to engineer different frustrated spin models in triangular, or even kagomé lattices [164] that should display such interesting phases. In particular, Damski et al [244, 245] proposed that cold dipolar Fermi gases, or Bose-Fermi mixtures in kagomé might allow for quantum spin liquid crystal, characterized by Néel like order at low $T$—see also [246] —. This novel quantum state of quantum matter turned out to show an unexpectedly high density of low energy excitations, similar to the one displayed by a liquid.

- **Topological order and quantum computation.** Several very unusual spin models displaying topological order have been pushed forward recently [247, 248] as candidates for realizing robust quantum computation (for a recent review see [249]). In spite of their exotic nature, some of these models can be engineered in cold atom systems [163, 250]. The latter proposal by Micheli et al [250] is particularly interesting [251]. They propose to load in a lattice hetero-nuclear polar molecules, excited to the lowest rotational level by microwaves, as effective spins that interact strongly at distance due to dipole-dipole interactions. As the range and spatial anisotropy of couplings are tunable, the method becomes an universal ‘toolbox’ for spin models. Experimental achievements in cooling of heteronuclear molecules that may have a large electric dipole moment [49, 252] have opened possibilities in this direction. Recently, a gas of ultracold ground state Potassium-Rubidium molecules has been realized [253].

- **Higher spin models.** There are many open questions also about Hubbard models, or lattice systems with higher spins. The Haldane conjecture about the existence of a gap, or its lack for the 1D antiferromagnetic spin chains with integer or half-integer spins, respectively, is perhaps the most famous. Ultracold spinor gases [254, 255] allows one to study these questions. The strong interacting limit of spinor gases loaded in optical lattices is particularly interesting [256–259], as their effective Hamiltonian becomes a generalized Heisenberg Hamiltonian. Spinor condensates in optical lattices have been addressed experimentally for ferromagnetic [260–263] and antiferromagnetic [264] interactions. Using Feshbach resonances [234] and varying the lattice geometry, it is in principle possible to prepare many regimes and quantum phases, the most most interesting of them being antiferromagnetic (AF) one. García-Ripoll et al [165] proposed of taking advantage of duality between the antiferromagnetic (AF) and ferromagnetic (F) Hamiltonians in order to prepare adiabatically AF states. Indeed, as dissipation and decoherence are practically negligible in such systems, and affect equally both ends of the spectrum, AF physics can be studied by preparing maximal energy states of $H_F = -H_{AF}$. Ytterbium fermionic atoms with N different spin states in an optical lattices implementing the fermionic SU(N) Hubbard model have been recently experimentally investigated [265].

- **Fractional quantum Hall states.** Since the seminal work of Laughlin [266], our understanding of the fractional quantum Hall effect (FQHE) [267] has dramatically improved. However, there are still many challenges to overcome as, for instance, a direct demonstration of the anyonic character of excitations or of other strongly correlated states. Trapped ultracold rotating gases [268, 269] allow in principle to study FQHE. The effects of rotation on the atoms is equivalent to ones produced by an ‘artificial’ constant magnetic field parallel the rotation axis on charged particles. There are proposals about how to detect directly fractional excitations [270] that exploits this equivalence. Loading the gases in an optical lattice increases the possibility of observing the phenomenon in two ways. On one hand, one can use a lattice with rotating site potentials, or an array of rotating microtraps to observe FQHE states of small systems of atoms—cf [271–273] and [274] and references therein—. On the other hand, ‘artificial’ magnetic field can be created in the lattice by using alternative techniques to rotation, e.g. by controlling the tunneling (hopping) matrix element in the corresponding Hubbard model [275]. Such techniques are very promising for creating FQHE type states [276–279]. Klein and Jaksch [280] have recently proposed to immerse a lattice gas in a rotating Bose condensate; tunneling in the lattice becomes then partially mediated by the phonon excitations of the BEC and mimics the artificial magnetic field effects. Last, but not least, a direct approach employing lattice rotation has been developed both in theory [281, 282], and in experiments [283]. The recent wave of very successful experiments creating artificial or synthetic magnetic fields employing laser induced gauge fields [81, 82] that are achieved by using spatial dependent optical coupling between different internal states of atoms. Such an approach is free from rotational restrictions. Very recently, this successful wave have invested also lattice experiments. Paradigmatic 2D models displaying a topological insulating behavior like the Hofstadter [284] and the Haldane [285] models have been realized experimentally in shallow harmonic traps in
[286, 287] and [288], respectively, by exploiting superlattices and Bragg pulses. The Hofstadter model has been also realized in ladders and slabs, that to say in lattices with sharp boundaries in one narrow dimension. Such dimension can be real as in the experiment by [289] or synthetic [290], that to say with the rungs of the ladder formed by the internal spin states of atoms, which provide an extra dimension to the system [291]. The synthetic Hofstadter slabs has been realized experimentally for bosons [292] and fermions [293] and the edge currents observed for the first time via spin-dependent measurements. Real and synthetic Hofstadter ladders and slabs offer a very promising route to the observation of many-body quantum Hall physics, both for bosons [294, 295] and fermions [296, 297].

- **Lattice gauge theories.** Gauge theories, and in particular lattice gauge theories (LGT) [298] are fundamental for both high energy physics and condensed matter physics. In spite of the progress in our understanding of LGT, mainly due to MonteCarlo calculations, many questions about the phase diagram and non-equilibrium dynamics of such theories remain unanswered (see e.g. [299]). Cold atoms can be helpful in two directions. Background classical gauge fields can be studied as ‘artificial’ non-Abelian magnetic fields that can be created in/off the lattice via appropriate control of the hopping matrix elements [300] or using effects of electromagnetically induced transparency [301], respectively. Non-Abelian background gauge fields are especially interesting because may led to the realization of generalized Laughlin states with non-Abelian fractional excitations. The other challenge is the engineering of dynamical quantum gauge fields. In fact, realizations of U(1) Abelian gauge theory, based on ring exchange interaction in square lattices [302], or involving three particle interactions in triangular lattices [303, 304] have been proposed already sometime ago. In the last four years the effort of simulating LGT in optical lattices has received new impulse. Most of the attention has focused on gauge magnets or link models [305–307], which can be viewed as spin version of ordinary Hamiltonian LGT and, as such, are easier to be simulated. Both Abelian and non-Abelian LGT can be simulated by exploiting angular momentum conservation [308, 309], SU(N)-invariant interaction in earth-alkali like atoms [310, 311], or the long-range interaction induced by Rydberg atoms [312, 313]. The proposed simulators would be capable to probe the confinement of charges, for instance, by measuring the string tension. Similar simulators have been proposed also for superconducting qubit [314] and trapped ions [315], where string breaking in Schwinger model has been very recently experimentally demonstrated with four ions [77]. These developments in quantum simulation has triggered also parallel developments in classical simulation both in 1D [316–318] and 2D or more [319, 320].

- **Superchemistry.** The challenges that can be tackled with cold atoms are not restricted to ones in condensed matter and high energy physics. Performing a controlled chemical reaction is a challenge of quantum chemistry. By the use of photoassociation or Feshbach resonances it is possible to drive the system from a desired initial state to a desired final quantum state of the reaction. Jaksch et al [321] proposed to start with a Mott insulator (MI) of two identical atoms, and to induce via photoassociation, first a MI of homonuclear molecules, and then a molecular SF via ‘quantum melting’. A similar process was consider for heteronuclear molecules [322] as an alternative route to molecular SF. Indeed, photoassociation of $^{87}$Rb molecules in a MI with two atoms per site have been observed in Bloch’s group [323], while Rempe’s group has achieved the first molecular MI employing Feshbach resonances [344]. Similarly, Grimm’s group has demonstrated the formation of three-body Efimov trimer states in trapped Cs atoms [324]. In a optical lattice, the same phenomena is predicted to be even more favorable [325]. An overview of the subject of cold chemistry can be found in [326], and in particular about cold Feshbach molecules in [327]. Control and creation of deeply bound molecules in the presence of an optical lattice has been demonstrated in [328].

- **Ultracold dipolar gases.** Within modern atomic and molecular physics, ultracold dipolar quantum gases offer one of the richest playgrounds for fascinating theoretical and experimental challenges—see [329, 330] and [331] for reviews. The experimental realization of the dipolar Bose gas of Chromium [332], and progresses in trapping and cooling of dipolar molecules [253] have paved the way towards ultracold quantum gases with dominant dipole interactions. More recently, also dipolar gases of Dysprosium [333] and Erbium [334] have been cooled up to reach quantum degeneracy. Very recently, dipolar gases [335] and polar molecules [336] have been used to simulate quantum magnetism. Due to long range interactions, dipolar BECs and BCS states of trapped gases can be affected by the trap geometry in peculiar ways. In optical lattice, dipolar ultracold gases realize extended Hubbard models that display interesting quantum insulating ‘solid’ phases, like phase checkerboard (CB), and superfluid phases like supersolid (SS) phase [337–339]. Rotating dipolar gases (RDG) are especially interesting. Bose–Einstein condensates of RDGs display novel vortex lattice geometries: square, ‘stripe crystal’, and ‘bubble crystal’ lattices [340]. In [341] it has been demonstrated that the existence of pseudo-hole gap for the Fermi RDGs also in the thermodynamic limit in the number of atoms $N \to \infty$. This property thus suggests that already mesoscopic RDGs, with $N \approx 50–100$, can be ideal for exploring the strongly correlated regime and achieving Laughlin liquids at filling $\nu = 1/3$, and quantum Wigner crystals at $\nu \leq 1/7$ [342].

- **Wigner crystals or self-assembled lattices.** Wigner, or Coulomb type of crystals are predicted to be formed due to long range repulsive dipolar atom-atom or molecule-molecule interactions [343–345], or ion-ion Coulomb interactions in the absence of a lattice [64].
Many of the above mentioned challenges are discussed discussed in the subsequent chapters of our book [1]. The interested reader should, however, extend her/his knowledge by turning toward the excellent books of Pitaevskii and Stringari [10] and Pethick and Smith [346] on Bose–Einstein Condensation in the weakly interacting regime, or to the books of Fetter and Walecka [347] or Wen [348] for many body and quantum field theory, among others.

4. Conclusions

The EU countries as well many others have recently concentrated considerable efforts to support quantum information science and technology. The EU in particular will launch a Quantum Flagship, which will seek for technological advances in the four pillars of the QI science; Quantum Computing and Simulations, Quantum Communications, Quantum Sensing and Metrology, and Quantum information Theory and Software.

While universal quantum computers still remain very challenging, quantum simulators already exist in the labs. Many of those are analogue or digital simulators of ‘interesting’ quantum phenomena. Recently, however, a lot of effort have been devoted to quantum annealers, like D-wave machines that are designed to solve classical NP-complete or at least ultra-complex problems [202, 349, 350]. Quantum annealers are perhaps the first quantum computing devices that have the chance to enter real technology and change our everyday life. In fact, decisive progress towards the definitive success of quantum computing devices may be achieved by integrating the different platforms [351] that are best suited for different tasks. The development of such an integrated structure would incarnate fully the spirit of the Quantum Flagship, and have quantum optics at its heart. One of the most intriguing applications of quantum simulators are towards problems that are not restricted to condensed matter, and even to physics in stricto sensu [352]. Among them, simulations of Lattice Gauge Theories are especially fascinating and may lead to breaking the 20th century distinction between low and high energy physics. Such simulators may serve also for testing holographic principle [353–356]. In fact, quantum simulation of high energy phenomena and gravitation [354–356] is not the only way in which quantum optics and quantum technologies may have a revolutionary impact over our understanding of Nature at all scales. Indeed, in addition to the celebrate success of the interferometer LIGO in detecting gravitational waves [357] as predicted by Einstein about a century ago [358, 359], the impressive progresses in the optical clock standards [360] are expected to lead in the next decade to direct tests of fundamental laws of physics by proving tighter and tighter bounds on the stability of fundamental constants [361].

Thus, exciting time and challenges are expecting the quantum optics community in the following years that require a common effort.

Quantum workers of the world, unite!

This is not the final struggle, but...

Let us group together, and tomorrow
The Quantum Internationale
Will be the human race.

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