Running of the Charm-Quark Mass from HERA Deep-Inelastic Scattering Data

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work partially done within scope of PROSA, ZEUS and H1 collaborations

Introduction: running of $\alpha_s$ and $m_b$

Final results on charm mass running: DESY-17-048 (on arXiv soon)

Interpretation in terms of Higgs Yukawa couplings: PoS CHARM2016 (2017) 012
Running of strong coupling „constant“ $\alpha_s$

- e.g. from jet production at e+e-, ep, and pp at DESY, Fermilab and CERN

$\alpha_s(Q) = 0.1171^{+0.0075}_{-0.0050}$ (3-jet mass)

$\alpha_s(M_Z) = 0.1185 \pm 0.0006$ (World average)

Yes, it runs!

EPJC 75 (2015) 186

reminder

updates see talks
K. Rabbertz
and D. Britzger
Running of $\alpha_s$ and quark masses $m_Q$

- $\alpha_s$ running depends on number of coulours $N_C$ and number of quark flavours $n_f$

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \alpha_s \times (11N_C - 2n_f)/12\pi \ln(\mu^2/\mu_0^2)}$$

- Quark mass running depends on $\alpha_s$, e.g.

$$m_Q(\text{pole}) = m_Q(m_Q) \left(1 + \frac{4}{3} \frac{\alpha_s}{\pi}\right)$$

$$= m_Q(\mu) \left(1 + \frac{\alpha_s}{\pi} \left(\frac{4}{3} + \ln\left(\mu^2/m_Q^2\right)\right)\right)$$

or

$$m_Q(\mu) = m_Q(m_Q) \times \left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{c_0}$$

with

$$c_0 = \frac{4}{(11 - 2n_f/3)} = \frac{4}{9}$$

for $N_C$ and $n_f = 3$

- Part of gluon field around quark not 'visible' any more when 'looking' at smaller distances/larger energy scales

$\Rightarrow$ effective quark mass decreases
$m_b(m_b) = 4.07 \pm 0.14_{\text{fit}} +0.01^{+0.05}_{-0.07} \text{ mod } +0.08_{-0.05} \text{ par } +0.08^{+0.08}_{-0.05} \text{ th } \text{GeV}$

PDG: $4.18 \pm 0.03 \text{ GeV}$ (lattice QCD + time-like processes)
The running beauty quark mass

translate to $2m_b$

![Graph showing the running beauty quark mass with data points from PDG, ZEUS, and LEP, along with their respective uncertainties. The graph illustrates the trend of $m_b(\mu)$ with $\mu$ in GeV.]
Fixed Flavour Number Scheme (FFNS)

- no charm in proton
- full kinematical treatment of charm mass
  (multi-scale problem: $Q^2, p_T, m_c \rightarrow \text{logs of ratios}$)
- no resummation of logs
- no extra matching parameters

$e^+ \rightarrow \gamma \rightarrow c \bar{c}$

LO

$\sqrt{\alpha_s} \rightarrow \gamma \rightarrow c \bar{c}$

920 GeV

$p$

$e^- + e^+ \rightarrow c \bar{c} + NLO \ (+\text{partial NNLO})$ corrections,

“natural” scale:

$\mu^2 = Q^2 + 4m_c^2$

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**Measurement of \( \overline{MS} \) charm mass**

**simultaneous mass + PDF fit of combined charm data and inclusive HERA I DIS data**

\[ m_c(m_c) = 1.26 \pm 0.05_{\text{exp}} \pm 0.03_{\text{mod}} \pm 0.02_{\text{as}} \text{ GeV} \]

**PDG:** \( 1.275 \pm 0.025 \text{ GeV} \) (lattice QCD + time-like processes)

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**Measurement of $m_c$ running**

Step 1: extract $m_c(m_c)$ separately for 6 different kinematic ranges in $\mu^2 = Q^2 + 4m_c^2$ (take log average for central scale)
**mc** fit and uncertainties

use appropriate PDF set for each mass (from inclusive DIS data only), fit charm data

Fit uncertainty
- Was estimated by taking $\Delta \chi^2 = 1$ (dominant uncertainty)

Parametrisation
- Adding extra parameter in the PDF parametrisation

Model uncertainty
- Variation of the strangeness suppression factor
- Lower cut on $Q^2$ for inclusive data
- The evolution starting scale
- The b-quark mass

Theory
- Variation of $\alpha_s$
- Variation of the factorisation and renormalization scales of heavy quarks by factor 2

sensitivity to $m_c(m_c)$ decreases with increasing scale $\mu^2 = Q^2 + 4m_c^2$

'in reality', have measured $m_c(\mu)$ at each scale

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The running charm quark mass

Step 2: translate back to $m_c(\mu)$, which was actually measured, using LO formula consistent with NLO $\overline{MS}$ QCD fit

(OpenQCDrad, Alekhin et al.)

Running mass concept in QCD is self-consistent!
### Numerical details

| Subset | $N_{dat}$ | $Q^2$ range [GeV$^2$] | $\mu$ [GeV] | $m_c(m_c)$ [GeV] | $m_c(\mu)$ [GeV] |
|--------|-----------|------------------------|-------------|------------------|------------------|
| 1      | 15        | 2.5–7                  | 3.3         | 1.256 $^{+0.078}_{-0.070}$ $^{+0.054}_{-0.000}$ | 0.984 ± 0.061   |
| 2      | 12        | 12–18                  | 4.5         | 1.192 $^{+0.075}_{-0.073}$ $^{+0.043}_{-0.000}$ | 0.867 ± 0.055   |
| 3      | 13        | 32–60                  | 7.0         | 1.208 $^{+0.092}_{-0.088}$ $^{+0.045}_{-0.000}$ | 0.830 ± 0.063   |
| 4      | 7         | 120–200                | 12.7        | 1.344 $^{+0.130}_{-0.131}$ $^{+0.073}_{-0.074}$ | 0.895 ± 0.087   |
| 5      | 4         | 350–650                | 21.9        | 1.143 $^{+0.222}_{-0.221}$ $^{+0.133}_{-0.163}$ | 0.676 ± 0.132   |
| 6      | 1         | 2000                   | 44.8        | 1.050 $^{+0.684}_{-0.760}$ $^{+0.400}_{-0.149}$ | 0.562 ± 0.412   |

Table 1: Values of $m_c(m_c)$ at different scales $\mu$, determined from six different subsets, and corresponding values of $m_c(\mu)$. The first uncertainty (fit) corresponds to the uncertainty $\delta_{\text{fit}}$ added in quadrature with the symmetrised systematic uncertainties $\delta_1 - \delta_6$. The second uncertainty (scale) of $m_c(m_c)$ corresponds to the scale variation uncertainty $\delta_7$. No scale uncertainty is quoted for $m_c(\mu)$ (see text). The range of $Q^2$ values contributing to the six data subsets shown in Fig. 1 is given. Also given is the corresponding logarithmic average scale $\mu$ for each subset according to Eq. (2), and the number $N_{dat}$ of charm data points contributing to each measurement.
Breakdown of uncertainties on $m_c$

| Subset | $\delta_{\text{fit}}^{\text{exp}}$ | $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ | $\delta_6$ | $\delta_7$ |
|--------|-------------------------------------|------------|------------|------------|------------|------------|------------|------------|
|        | [m_b] [%]                           | [\alpha_s] | [f_s] [%]  | (Q_0) [%]  | (Q_{min}^2) [%] | (param) [%] | (scale) [%] |
| 1      | ± 5.4                              | +0.1 -0.4  | -1.2 +2.6  | -0.4 +0.2  | +0.5       | +1.4       | +0.5       | +3.1 +4.3  |
| 2      | ± 6.0                              | +0.2 -0.5  | -0.9 +0.7  | -0.5 +0.2  | +0.3       | +1.0       | +0.9       | +2.4 +3.6  |
| 3      | ± 7.2                              | +0.3 -0.7  | -0.4 +0.3  | -0.8 +0.3  | +1.7       | +0.3       | +1.8       | +0.1 +3.7  |
| 4      | ± 9.6                              | +0.5 -0.8  | +0.7 -0.6  | -0.8 +0.5  | +0.5       | -1.2       | +0.1       | -5.5 +5.4  |
| 5      | ± 19.2                             | +0.5 -1.2  | +1.6 -1.8  | -1.2 +0.5  | -0.5       | +2.1       | -1.7       | -14.3 +11.6|
| 6      | ± 63.8                             | -7.4 -2.9  | +5.9 -5.7  | -3.0 -7.6  | +6.5       | -33.3      | +9.5       | +38.1 -14.2|

Table 2: Summary of the systematic uncertainties in the $m_c(m_c)$ determinations. The definitions of the uncertainty sources, the meaning of the symbols in the first and second row and related details are given in the text. In cases where opposite variations of a variable yield uncertainties with the same sign, only the larger one is considered for the uncertainty combination in Table 1. Except for $\delta_7$, these uncertainties also apply to $m_c(\mu)$.
Subdividing HERA DIS charm data into 6 kinematic intervals, running of charm-quark mass in MSbar scheme has been determined for the first time (conceptually similar to running of $\alpha_s$ from jets)

Interplay/treatment of correlations between mass and PDF fits nontrivial, details see DESY-17-048

Charm-quark mass running consistent with QCD
Higgs Yukawa couplings from $m_Q$

relate $m_t$, $m_b$, $m_c$ to associated Higgs Yukawa couplings

LO EW (+NLO QCD) formula:

$$y_Q = \sqrt{2m_Q/v}$$

use Higgs/EW scheme in which this relation is exact!
Direct measurements of Higgs Yukawa couplings

ATLAS and CMS, JHEP08 (2016) 045

**Graph:**
- **Axes:**
  - **Y-axis:** $\frac{m_F}{k_{FV}}$ or $|v_{FV}|$
  - **X-axis:** Particle mass [GeV]

- **Data Points:**
  - $\tau$, $b$, $\mu$,
  - ATLAS+CMS

- **Lines:**
  - SM Higgs boson
  - $[M, \epsilon]$ fit

- **Confidence Levels:**
  - 68% CL
  - 95% CL
relate $m_t$, $m_b$, $m_c$ to associated Higgs Yukawa couplings

LO EW (+NLO QCD) formula:

$$y_Q = \sqrt{2m_Q/v}$$

for top see backup

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Conclusion

Part II

Experimental representation of running Yukawa couplings obtained for the first time.

Heavy quark physics is also QCD + Higgs physics.

So far, Higgs couplings and their running as obtained from quark masses are consistent with directly measured Higgs couplings.

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Backup
The HERA ep collider and experiments

HERA I: $\sim 130 \text{ pb}^{-1}$ (physics)

HERA II: $\sim 380 \text{ pb}^{-1}$ (physics)

combined: $\sim 2 \times 0.5 \text{ fb}^{-1}$

up to 30% of cross section

HERA:

$318 \text{ GeV}$

$p (920 \text{ GeV})$  $e (27.6 \text{ GeV})$
Deep Inelastic ep Scattering at HERA

HERA:

\( (\ell) \) Electron

\( \gamma, Z \)

Proton (P)

Electron (\( \ell' \))

\( q = \ell - \ell' \)

kinematic variables:

\[ Q^2 = -q^2 \]

photon (or Z) virtuality, squared momentum transfer

\[ X = \frac{Q^2}{2Pq} \]

Bjorken scaling variable,

for \( Q^2 \gg (2m_q)^2 \): momentum fraction of p constituent

\[ \gamma = \frac{qP}{\ell P} \]

inelasticity,

\( \gamma \) momentum fraction (of e)

\( Q^2 \lesssim 1 \text{ GeV}^2; \) photoproduction

\( Q^2 \gtrsim 1 \text{ GeV}^2; \) DIS
Heavy flavour contributions to $F_2$

Measure cross section
$$\frac{d^2\sigma}{dx\,dQ^2} \sim \frac{2\pi\alpha^2}{Q^4x} \left\{ \left[ 1 + (1 - y)^2 \right] \sigma_{R}(x, Q^2) \right\}$$

QCD

$e^+$

$Q^2$, $x = Q^2/2pq$

$\sigma_R^{bb}, \sigma_R^{cc}$

$p$

Detect

Anything

$e^+$

$Q^2, x$

$b, \bar{b}$ or $c, \bar{c}$

$p$

$\sigma_R^{bb}, \sigma_R^{cc}$

$27.6$ GeV

$Q^2$

$\gamma$

$920$ GeV

$g(x)$

$p$
comparison to ABM FFNS

very good description of data in full kinematic range

unambiguous treatment of $m_c$ in all terms of calculation

here: $\overline{\text{MS}}$ running mass

(similar predictions for pole mass)
$m_c(m_c)$ from FONLL fit of HERA data

V. Bertone et al., arXiv 1605.01946, JHEP 1608 (2016) 050

**Table 1**: Summary of $m_c(m_c)$ results from various schemes.

| Scheme                          | $m_c(m_c)$ [GeV]                        |
|--------------------------------|----------------------------------------|
| FONLL (this work)              | $1.335 \pm 0.043^{+0.019}_{-0.000}$ (param) $+0.011^{+0.008}_{-0.000}$ (mod) $+0.033^{+0.006}_{-0.000}$ (th) |
| FFN (this work)                | $1.318 \pm 0.054^{+0.010}_{-0.010}$ (param) $+0.015^{+0.009}_{-0.009}$ (mod) $+0.005^{+0.005}_{-0.005}$ (th) |
| FFN (HERA) [9]                 | $1.26 \pm 0.05^{+0.03}_{-0.02}$ (exp) $+0.02^{+0.02}_{-0.02}$ (param) $+0.02^{+0.02}_{-0.02}$ (th) |
| FFN (Alekhin et al.) [24]     | $1.24 \pm 0.03^{+0.03}_{-0.02}$ (exp) $+0.02^{+0.02}_{-0.02}$ (scale) $+0.006^{+0.006}_{-0.006}$ (th) (approx. NNLO) |
| S-ACOT-χ (CT10 str. 1) [29]   | $1.22^{+0.05}_{-0.11}$ (strategy 1) |
| S-ACOT-χ (CT10 str. 2) [29]   | $1.18^{+0.05}_{-0.11}$ (strategy 2) |
| S-ACOT-χ (CT10 str. 3) [29]   | $1.19^{+0.06}_{-0.15}$ (strategy 3) |
| S-ACOT-χ (CT10 str. 4) [29]   | $1.24^{+0.06}_{-0.15}$ (strategy 4) |
| World average [53]            | $1.275 \pm 0.025$                      |

**Figure**: Graph showing the fit results for $m_c(m_c)$ from various schemes.
top quark mass running

very preliminary procedure (with caveats, “cheated” a bit):

- use (conceptually constant) LO MC mass measured as function of scale-dependent quantity (e.g. $m_{\tau\tau}$)
- check self-consistency of cross section measurements with data used for mass determination
- ‘convert’ LO MC mass to NLO pole mass by comparing MC and pole mass extractions from same data
- convert pole mass to $\overline{MS}$ mass using 3-loop QCD
- use 1-loop evolution for actual running (NLO QCD)

(in the future, like for $m_c$ and $m_b$, extract NLO (or NNLO) running mass directly from data, e.g. cross section, in each kinematic bin)
top quark mass as function of $m_{tt}$

"MC mass"

deviation from average of

$172.35 \pm 0.16_{\text{stat}} \pm 0.48_{\text{sys}}$ GeV
differential top cross section shape consistent with NLO

NLO theory uses pole mass scheme

use CMS to be consistent with previous slide

similar results for lepton+jets channel only

-> measurements and LO+PS/NLO theory are self-consistent and consistent with ATLAS and NNLO
convert top MC mass to pole mass

ATLAS, JHEP10 (2015) 121, CMS-TOP-12-022, Phys. Lett. B728 (2014) 526
CMS-TOP-13-004, JHEP 1608 (2016) 029

**ATLAS**

\[ m_t^{(pole)} = 173.7 \pm 1.5 \text{ stat} \pm 1.4 \text{ syst} +1.0-0.5 \text{ th} \text{ GeV} \]

**CMS**

\[ m_t^{(pole)} = 173.8 +1.7-1.8 \text{ total} \text{ GeV} \leftrightarrow m_t^{(MC_{CMS,l+jets})}=172.35 \pm 0.16 \text{ stat} \pm 0.48 \text{ syst} \text{ GeV} \]

**PDG**

\[ m_t^{(pole)} = 176.7 \pm 4.0-3.4 \text{ GeV}, \quad \text{“} m_t^{(MC)} \text{”} =173.21 \pm 0.51 \text{ stat} \pm 0.71 \text{ syst} \text{ GeV} \]

“MC” and pole masses almost the same

ATLAS, JHEP10 (2015) 121, CMS-TOP-12-022, Phys. Lett. B728 (2014) 526
CMS-TOP-13-004, JHEP 1608 (2016) 029

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convert pole masses to running mass

caveat:
not all uncertainties from conversion included
(needs theoretically better defined procedure!)
-> take with grain of salt, for illustration purposes
Discussion

of future conceptual improvements

- avoid MC mass and pole mass intermediate steps for top
  -> extract $m_t(\mu)$ directly from data, as already done for $b,c$
  (e.g. from absolute $m_{tt}$ cross sections in CMS-TOP-16-008)
  need NLO QCD theory for LHC using running mass

- extend LO EW + NLO QCD approach
  (running of Higgs couplings is purely QCD-induced!)
  to NLO EW + NNLO QCD + interference
  highly non-trivial but eventually necessary
  (Standard Model is not QCD only)