Spectral Properties of Quarks at Finite Temperature in Lattice QCD

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Abstract
We analyze the quark spectral function above and below the critical temperature for deconfinement and at finite momentum in quenched lattice QCD. It is found that the temporal quark correlation function in the deconfined phase near the critical temperature is well reproduced by a two-pole ansatz for the spectral function. The bare quark mass and momentum dependences of the spectral function are analyzed with this ansatz. In the chiral limit we find that even near the critical temperature the quark spectral function has two collective modes corresponding to the normal and plasmino excitations in the high temperature ($T$) limit. The pole mass of these modes at zero momentum, which should be identified to be the thermal mass of the quark, is approximately proportional to $T$ in a rather wide range of $T$ in the deconfined phase.

1. Introduction

After the advent of Relativistic Heavy Ion Collider (RHIC), much attention has been paid for the properties of matter near and above the critical temperature of QCD phase transition, $T_c$. To understand the structure of the matter in this region, it is desirable to identify the basic degrees of freedom of the system and their excitation properties. In the present study, we analyze the spectral properties of quarks, which are one of the fundamental degrees of freedom of QCD, at finite temperature above and below $T_c$ using quenched lattice QCD with clover improved fermions \cite{1,2}.

At asymptotically high temperatures, one can calculate the quark propagator using perturbative techniques. It is known that the collective excitations of quarks in this limit develop a mass gap (thermal mass) that is proportional to $gT$, where $g$ and $T$ denote the gauge coupling and temperature, respectively \cite{3}. Moreover, in this limit the number of poles in the quark propagator is doubled; in addition to the normal modes, which reduce to poles in the free particle propagator, plasmino modes appear.

In order to see the properties of quark spectral function $\rho(\omega, p)$ in this limit, let us first consider the spectral function at zero momentum. In this case, the Dirac structure of $\rho(\omega, p)$ can be decomposed using the projection operators $L_\pm = (1 \pm \gamma^0)/2$ as

$$\rho(\omega, 0) = \rho_+^M(\omega)L_+\gamma^0 + \rho_-^M(\omega)L_-\gamma^0.$$  \hspace{1cm} (1)

While $\rho_\pm^M(\omega)$ for free quarks are given by $\rho_\pm^M(\omega) = \delta(\omega \mp m)$, in the high temperature limit one obtains $\rho_\pm^M(\omega) = [\delta(\omega - m_T) + \delta(\omega + m_T)]/2$, where $m_T = gT/\sqrt{6}$ is the thermal mass. The two poles at $\omega = \pm m_T$ correspond to the normal and plasmino modes.
The Dirac structure of $\rho(\omega, p)$ in the high temperature limit is also decomposed as

\[
\rho(\omega, p) = \rho_+^p(\omega, p)P_+(p)\gamma^0 + \rho_-^p(\omega, p)P_-(p)\gamma^0,
\]

with the projection operators $P_+(p) = (1 \pm \gamma^0 \cdot \hat{p} \cdot \gamma) / 2$ and $p = |p|$. The spectral functions $\rho_\pm^p(\omega, p)$ in the high temperature limit read

\[
\rho_\pm^p(\omega, p) = \rho_{\text{cont}}(\omega, p),
\]

where $\rho_{\text{cont}}(\omega, p)$ represents the contribution of the continuum taking non-zero values in the space-like region, and poles at $E_1(p) > 0$ and $E_2(p) > 0$ corresponds to the normal and plasmino modes, respectively. The dispersion relation of the plasmino has a minimum at nonzero quark spectral function in this case is decomposed into previously by the Bielefeld group to study screening masses and spectral functions [4, 2]. The quark mass on the lattice. We use gauge field ensembles which have been generated and used 2. Quark spectral function at zero momentum

In this section, we analyze the quark spectral function for zero momentum but with finite bare quark mass on the lattice. We use gauge field ensembles which have been generated and used previously by the Bielefeld group to study screening masses and spectral functions [4, 2]. The quark spectral function in this case is decomposed into $\rho_\gamma^M(\omega)$ as in Eq. (1). In order to extract the spectral function $\rho_\gamma^M(\omega)$ from the lattice correlation function, we assume a simple ansatz for the shape of $\rho_\gamma^M(\omega)$ including few fitting parameters. We found that the two-pole ansatz for $\rho_\gamma^M(\omega)$,

\[
\rho_\gamma^M(\omega) = Z_1(\omega E_1 + Z_2(\omega E_2),
\]

reproduces the lattice correlator quite well, where $Z_{1,2}$, $E_{1,2} > 0$ are fitting parameters determined from the correlated fit. The pole at $\omega = -E_2$ in Eq. (4) corresponds to the plasmino mode. The success of two-pole ansatz for the quark correlation functions suggests that the positions of the poles of the quark propagator would be near the real axis at $\omega = E_1$ and $-E_2$ with small imaginary parts. Provided that the positions of poles of the propagator are gauge independent, this also indicates that our results on the fitting parameters $E_1$ and $E_2$ have small gauge dependence.

In Fig. 1 we show the dependence of $E_1, E_2$ and $Z_2/(Z_1 + Z_2)$ on the bare quark mass, $m_p$, for $T/T_c = 1.25, 1.5$, and $3$ obtained from calculations on lattices of size $64^3 \times 16$. The figure shows that the ratio $Z_2/(Z_1 + Z_2)$ becomes larger with decreasing $m_p$ and eventually reaches 0.5 irrespective of $T$. The numerical result for each $T$ shows that $E_1 = E_2$ is satisfied within statistical errors there. These results mean that the quark propagator is chirally symmetric at this point [2]. Moreover, $\rho_\gamma^M(\omega)$ at this point has the same form as the spectral function in the high temperature limit. We therefore define the thermal mass of the quark on the lattice as $m_T \equiv (E_1 + E_2)/2$ at this point. One finds that the ratio $m_T/T$ is insensitive to $T$ in the range analyzed in this work, although it becomes slightly larger with decreasing $T$, which would be in accordance with the expected parametric form at high temperature, $m_T \sim gT$.

Figure 1 also shows that the relative strength of the plasmino pole, $Z_2/(Z_1 + Z_2)$, decreases with increasing values of the bare mass, $m_p$. The spectral function $\rho_\gamma^M(\omega)$ thus will eventually be dominated by a single-pole. This result agrees with the perturbative result in Yukawa models that $\rho_\gamma^M(\omega)$ approaches the spectral function of free quarks as the bare quarks mass becomes larger [5, 6].

For $T < T_c$, we found that the lattice correlator is concave in the log-scale plot. This behavior indicates that the positivity condition of $\rho_\gamma^M(\omega)$ is violated below $T_c$. In fact, we have checked that the two-pole ansatz Eq. (4) gives unacceptably large $\chi^2$/dof below $T_c$. It is also found that the
Figure 1: Bare quark mass dependence of fitting parameters $E_{1,2}$ and the relative strength of the plasmino mode, $Z_2/(Z_1+Z_2)$, at $T/T_c = 1.25, 1.5$ and $3$ obtained from calculations on lattice of size $64^3 \times 16$.

Quark correlator does not approach the chirally symmetric one even in the chiral limit, in contrast to the previous result for $T > T_c$. These results for $T < T_c$ seem consistent with a naïve picture that quark excitations are confined and the chiral symmetry is spontaneously broken below $T_c$.

3. Quark spectral function at finite momentum

Next let us analyze the quark spectral function at finite momentum on lattices with size $64^3 \times 16$ for $T/T_c = 1.5$ and $3$. Throughout this section we consider the quark propagator in the chiral limit. The quark propagator in the chiral limit is decomposed into $\rho^P(\omega, p)$ according to Eq. (2). Following the same approach used in the previous section, we adopt the two-pole ansatz

$$\rho^P(\omega, p) = Z_1 \delta(\omega - E_1) + Z_2 \delta(\omega + E_2),$$

and determine four parameters from a correlated fit. The $\delta$-functions at $\omega = E_1$ and $-E_2$ correspond to the normal and plasmino modes, respectively. We found that $\chi^2$/dof with this ansatz is always smaller than $1.5$ for all momenta analyzed in this study. This result means that the two-pole ansatz again reproduces the lattice correlation function well.

In Fig. [2] we show the momentum dependence of the fitting parameters $E_1$, $E_2$, and $Z_2/(Z_1+Z_2)$ for $T/T_c = 1.5$ and $3$. The horizontal axis represents the momentum on the lattice $\hat{p} = (1/a) \sin pa$. The figure shows that for large momentum $Z_2/(Z_1+Z_2)$ rapidly decreases and $E_1$ approaches the light cone. The spectral function at large momentum therefore approaches that of a free quark, consistent with the perturbative result. One also finds that $E_2$ is always smaller than $E_1$, in contrast to the results in the previous section. This behavior qualitatively agrees with the behavior of poles in the high $T$ limit [3]. One also observes from Fig. [2] that $E_2$ enters the space-like region at high momentum.
An interesting property of the quark propagator in the high temperature limit Eq. (3) is that the dispersion relation of the plasmino has a minimum at finite momentum. In Fig. 2, one sees that the value of $E_2$ at lowest non-zero momentum on our lattice, $p_{\text{min}} = 2\pi T (N_{\tau}/N_{\sigma}) \approx 1.5 T$, is slightly larger than that at zero momentum, and the existence of such a minimum is suggested but not yet confirmed in the present analysis.

In summary, we analyzed the dependence of the quark spectral function on temperature $T$, bare quark mass $m$, and momentum $p$ in quenched lattice QCD with Landau gauge fixing. Above $T_c$, we found that the two-pole approximations for the spectral functions in the projected channels, $\rho^{M}(\omega)$ and $\rho^{P}(\omega, p)$, can well reproduce the lattice correlation functions. Although further studies on the volume dependence is needed, this result indicates that the excitations of quarks have small decay width even near $T_c$. Below $T_c$, on the other hand, the two-pole ansatz fails completely.

The lattice simulations presented in this work have been carried out using the cluster computers ARMINIUS@Paderborn, BEN@ECT* and BAM@Bielefeld.

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