Abstract : The anomalous dimension of single and multi-trace composite operators of scalar fields is shown to vanish at all orders of the perturbative series. The proof holds for theories with $\mathcal{N} = 2$ supersymmetry with any number of hypermultiplets in a generic representation of the gauge group. It then applies to the finite $\mathcal{N} = 4$ theory as well as to non-conformal $\mathcal{N} = 2$ models.

Keywords: Extended Supersymmetry, BRS quantization, AdS-CFT Correspondence
1 Introduction

It has been known for a long time that quantum field theories with extended supersymmetry display a set of remarkable nonrenormalization properties. The $\beta$–function of the gauge coupling was argued to vanish to all orders of perturbation theory for the $\mathcal{N} = 4$ Super Yang–Mills theory (SYM) [1] and to receive only one–loop corrections for the $\mathcal{N} = 2$ case [2]. The conjectured AdS/CFT correspondence [3] has renewed the interest on the finiteness properties of these theories; in fact, many of the tests of the correspondence have relied on nonrenormalization properties, which are crucial in order to ensure a meaningful comparison between the strong coupling regime, accessible by type IIB supergravity computations, and the weak coupling one, where the usual field theory techniques are reliable.

In particular, the prototype example of the correspondence [3] establish a duality between type IIB superstring on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM in the superconformal phase. In this context, it has been shown that chiral gauge invariant operators belonging to short multiplets of the superconformal group $SU(2,2|4)$ have vanishing anomalous dimensions [4]. The analysis of correlation functions of Chiral Primary Operators (CPO), which are the lowest scalar components of such short superconformal multiplets, provided a highly nontrivial check for the AdS/CFT correspondence [4, 5, 6]. Moreover, it recently led to discover new double-trace operators which are protected despite they do not obey any of the known shortening conditions [8, 9].

We remark that the nonrenormalization properties of some CPO’s does not necessarily rely on superconformal algebra, and it can actually be shown also for theories with less number of supersymmetries. This is indeed the case for the gauge–invariant operator $\text{Tr}\phi^2$, where in the $\mathcal{N} = 2$ formalism $\phi$ corresponds to the complex scalar field of the vector multiplet. The anomalous dimension of this operator has been shown to vanish to all orders of the perturbative series in pure $\mathcal{N} = 2$ SYM [10]; this allowed to provide a rigorous proof of the celebrated nonrenormalization theorems for the beta functions of $\mathcal{N} = 2$ and $\mathcal{N} = 4$ theories [10, 11].

In this work we extend the analysis of [10] to higher rank gauge–invariant polynomials of the scalar field $\phi$ and $\bar{\phi}$, including both single and multi-trace operators. It is worth to underline that our proof holds for $\mathcal{N} = 2$ theories with any number of hypermultiplets in a generic representation of the gauge group. It then applies to the finite $\mathcal{N} = 4$ theory as well as to nonconformal models, which are at present object of intensive research in extensions of the AdS/CFT duality to phenomenologically more interesting gauge theories with a nontrivial renormalization group flow [12, 13].

A geometrical interpretation of such nonrenormalization properties can be given by resorting to the topologically twisted formulation of the $\mathcal{N} = 2$ theory. In fact, in this context, the operators we consider can be written as
components of the Chern classes of the universal bundle defined in [23].

This work is organized as follows. In Section 2 we sketch the basic features of the twisting procedure. In Section 3 we give the proof of finiteness of the operators \( \text{Tr} \phi^k, \ k \geq 2 \) (single–traced) and \( \Pi_i \text{Tr} \phi^{k_i}, \ \Sigma_i k_i = k \) (multi–traced).

Our conclusions are drawn in Section 4, while we confined into appendices the unavoidable technicalities of our proofs and the explicit examples \( k = 2, 3, 4 \).

## 2 The Twist

It is well known that \( \mathcal{N} = 2 \) SYM theory can be given a “twisted” formulation [14], which on flat manifolds is completely equivalent to the conventional one. Nonetheless, the use of the twisted variables makes more evident and easily understandable some important features, like the topological nature of a subset of observables [14] and the nonrenormalization theorem concerning the beta function of the gauge coupling constant [10]. In view of these advantages, we choose to work in the twisted version of \( \mathcal{N} = 2 \) SYM theory. We address the interested reader to the existing literature [14, 16] for what concerns the many and deep aspects of the twisting procedure. In this Section, we prefer rather to set up the field theoretical background in which we shall work.

The usual \( \mathcal{N} = 2 \) supercharges \( (Q^i_\alpha, \overline{Q}_{\dot{\alpha}}) \) are characterized by an index \( i = 1, 2 \), which counts the number of supersymmetries, and a Weyl spinor index \( \alpha \), which runs on the same values as \( i: \alpha = 1, 2 \). At the origin of the twist, is Witten’s idea of identifying the indices

\[
i \equiv \alpha .
\]

The resulting twisted supercharges \( Q^\beta_\alpha \) and \( \overline{Q}_{\dot{\alpha} \dot{\alpha}} \) can then be rearranged, with usual conventions [17], into a scalar \( \delta \), a vector \( \delta_\mu \) and a selfdual tensor \( \delta_{\mu \nu} \):

\[
\delta \equiv \frac{1}{\sqrt{2}} \varepsilon^{\alpha \beta} Q_{\alpha \beta} \quad (2.2)
\]

\[
\delta_\mu \equiv \frac{1}{\sqrt{2}} Q_{\alpha \dot{\alpha}} (\sigma^\mu)^{\dot{\alpha} \alpha} \quad (2.3)
\]

\[
\delta_{\mu \nu} \equiv \frac{1}{\sqrt{2}} (\sigma_{\mu \nu})^{\alpha \beta} Q_{\beta \alpha} . \quad (2.4)
\]

The \( \mathcal{N} = 2 \) supersymmetry algebra, written for the twisted supercharges, contains a subalgebra, formed by \( \delta, \delta_\mu \) and by the translations \( \partial_\mu \):

\[
\delta^2 = 0 \quad \{ \delta, \delta_\mu \} = \partial_\mu \quad \{ \delta_\mu, \delta_\nu \} = 0 . \quad (2.5)
\]
We call the subalgebra (2.5) “topological”, since it appears to be common to all known topological field theories [18], both of the Witten and Schwartz type [19]. The three twisted supercharges $\delta_{\mu \nu}$ turn out to be redundant, since they do not play any role either in the classical definition or in the quantum extension of the theory [15]. For this reason, we shall discard them throughout the rest of our reasoning.

Analogously to supercharges, the fields of the $\mathcal{N} = 2$ gauge multiplet $(A_\mu, \psi^i_\alpha, \overline{\psi}^i_\dot{\alpha}, \phi, \overline{\phi})$, which belong to the adjoint representation of the gauge group $G$, twist to

$$(A_\mu, \psi^i_\alpha, \chi_{\mu \nu}, \eta, \phi, \overline{\phi})$$

where

$$\eta \equiv \varepsilon^{\alpha \beta} \psi^{[\alpha \beta]}$$
$$\psi_\mu \equiv (\overline{\sigma}_\mu)^\alpha \dot{\alpha} \psi_\alpha$$
$$\chi_{\mu \nu} \equiv (\sigma_{\mu \nu})^{\alpha \beta} \psi_{(\alpha \beta)}$$

are the anticommuting fields resulting from the twist of the spinor fields $(\psi^i_\alpha, \overline{\psi}^i_\dot{\alpha})$ into $(\psi_{\alpha \dot{\alpha}}, \overline{\psi}_{\dot{\alpha} \alpha})$. Notice that the scalar fields $(\phi, \overline{\phi})$ and the gauge boson $A_\mu$ are not altered by the twisting operation.

The $\mathcal{N} = 2$ SYM pure gauge action twists to the TYM action

$$S_{\mathcal{N} = 2 \text{ SYM}}(A_\mu, \psi^i_\alpha, \overline{\psi}^i_\dot{\alpha}, \phi, \overline{\phi}) \longrightarrow S_{\text{TYM}}(A_\mu, \psi^i_\alpha, \chi_{\mu \nu}, \eta, \phi, \overline{\phi})$$

which, in the 4D flat euclidean spacetime reads [15]:

$$S_{\text{TYM}} = \frac{1}{g^2} \text{Tr} \int d^4x \left( \frac{1}{2} F^+_{\mu \nu} F^{+ \mu \nu} - \chi^{\mu \nu}(D_\mu \psi_\nu - D_\nu \psi_\mu)^+ + \eta D_\mu \psi_\mu - \frac{1}{2} \bar{\phi} D_\mu D^\mu \phi + \frac{1}{2} \bar{\phi} \{\psi^\mu, \psi_\mu\} - \frac{1}{2} \phi \{\chi^{\mu \nu}, \chi_{\mu \nu}\} - \frac{1}{8} [\phi, \eta] \eta - \frac{1}{32} [\phi, \overline{\phi}] [\phi, \overline{\phi}] \right)$$

where $F^+_{\mu \nu} = F_{\mu \nu} + \frac{1}{2} \varepsilon_{\mu \rho \sigma \nu} F^{\rho \sigma}$, $\tilde{F}^+_{\mu \nu} = \tilde{F}^+_{\mu \nu} = \frac{1}{2} \varepsilon_{\mu \rho \sigma \nu} F^{+ \rho \sigma}$ and $(D_\mu \psi_\nu - D_\nu \psi_\mu)^+ = (D_\mu \psi_\nu - D_\nu \psi_\mu) + \frac{1}{2} \varepsilon_{\mu \rho \sigma \nu} (D^\rho \psi^\sigma - D^\sigma \psi^\rho)$.

The symmetries of the action (2.9) are the usual gauge invariance

$$\delta_{\text{gauge}} S_{\text{TYM}} = 0$$

and the relevant twisted supersymmetries

$$\delta S_{\text{TYM}} = \delta_{\mu} S_{\text{TYM}} = 0$$

It is apparent from (2.7), that the 4D flat euclidean twist simply corresponds to a linear change of variables, therefore it is not felt by the partition function

$$Z = \int D\phi \ e^{-S[\phi]}$$
Moreover, the stress-energy tensor of the theory is modified by the twist only by a total derivative term which do not affect the translation generators \([10]\). This observation underlies the equivalence of the two, twist–related, theories \(\mathcal{N} = 2\) SYM and TYM when formulated on flat manifolds. The theory has been extended to the quantum level both in its untwisted \([20]\) and twisted \([15]\) formulation. Moreover, in \([10]\) has been demonstrated that a renormalization scheme does exist, in which the \(\beta\)–function of the gauge coupling constant \(g\) receives one loop contributions only.

3 Protected Gauge Invariant Operators

Let us analyze the class of composite operators of the type \(\text{Tr} \phi^k\), \(k \geq 2\) and \(\Pi_i \text{Tr} \phi^{k_i}\), \(\Sigma_i k_i = k\). The aim of this Section is to show that these operators are protected, in the sense that are perturbatively finite, or, equivalently stated, they have vanishing anomalous dimensions.

In order to prove this statement, let us consider the operator \(Q\) defined as

\[
Q = s + \omega \delta ,
\]

where \(s\) is the nilpotent BRS operator, \(\delta\) is the twisted scalar supercharge \((2.2)\), and \(\omega\) is a global commuting ghost.

The operator \(Q\) describes an invariance of the action \(S_{TYM}\)

\[
Q S_{TYM} = 0 ,
\]

and it is nilpotent

\[
Q^2 = 0 ,
\]

since \(s^2 = \delta^2 = \{s, \delta\} = 0\).

For our purposes, the \(Q\)–variations of the scalar field \(\phi\) and of the Faddeev–Popov ghost \(c\) are sufficient

\[
Q \phi^a = f^{abc} c^b \phi^c ,
Q c^a = \frac{1}{2} f^{abc} c^b c^c - \omega^2 \phi^a ,
\]

where the index \(a = 1, \ldots, \text{dim } G\) counts the adjoint representation matrices of the gauge group \(G\), and \(f^{abc}\) are the structure constants of the corresponding Lie algebra.

The operators \(\text{Tr} \phi^k\) and \(\Pi_i \text{Tr} \phi^{k_i}\), \(\Sigma_i k_i = k\) are the observables of TYM (hence \(\mathcal{N} = 2\) SYM) theory \([14, 15]\); we can write them as

\[
D^{a_1 \ldots a_k} \phi^{a_1} \ldots \phi^{a_k} ,
\]

where the \(D^{a_1 \ldots a_k}\) are completely symmetric invariant tensors of rank \(k\).
\[
\begin{align*}
  k = 2 & \quad D^{a_1a_2} = \delta^{a_1a_2}; \\
  k = 3 & \quad D^{a_1a_2a_3} = d^{a_1a_2a_3}; \\
  \vdots
\end{align*}
\]

In particular, the \( k = 2 \) operator \( \text{Tr} \, \phi^2 \) can be thought to as a kind of prepotential of the theory, since the whole action can be written as

\[
\Sigma \approx -\frac{1}{3g^2} \varepsilon^{\mu\nu\rho\sigma} \delta_\mu \delta_\nu \delta_\rho \delta_\sigma \int d^4x \, \text{Tr} \frac{\phi^2}{2}, \quad (3.6)
\]

where this relation holds modulo BRS trivial cocycles. The equation (3.6) reflects the fact that \( \text{Tr} \, \phi^2 \) contains all the physical information of the theory, as it constitutes its inner bulk (for a detailed discussion of this point, see [13, 10]).

A crucial point is that, in the space of local field polynomials which are not necessarily analytic in the constant parameter \( \omega \), the operators (3.5) can be written as \( Q \)-variations, i.e. in this space the cohomology of \( Q \) is empty. We stress that quantum field theory rules require that the 1PI generating functional \( \Gamma \) must be an analytic function in \( \omega \) as well as in any other parameter of the theory, whereas this condition can obviously be relaxed for quantum insertions, like for instance (3.5), which can be rendered analytic by a multiplication for a suitable power of \( \omega \), as we shall see.

The following result holds (see proof in Appendix A)

\[
\begin{align*}
  D^{a_1..a_k} \phi^{a_1}..\phi^{a_k} &= \mathcal{Q} \left[ D^{a_1..a_k} \left( f^{(k)}_0 \bar{c}^{a_1} \bar{c}^{a_2}..\bar{c}^{a_k} + f^{(k)}_1 \bar{c}^{a_1} \bar{c}^{a_2}..\bar{c}^{a_{k-1}} \phi^{a_k} \\
  &\quad + \ldots + f^{(k)}_{k-1} \bar{c}^{a_1} \phi^{a_2}..\phi^{a_k} \right) \right] \\
  &= \mathcal{Q} \left[ D^{a_1..a_k} \sum_{p=0}^{k-1} f^{(k)}_p \bar{c}^{a_1} \bar{c}^{a_2}..\bar{c}^{a_{k-p}} \phi^{a_{k-p+1}}..\phi^{a_k} \right],
\end{align*}
\]

or, in shorthand notation,

\[
\Phi^{(k)}(\phi) = \mathcal{Q} \, \mathcal{P}^{(k)}(c, \phi). \quad (3.8)
\]

The coefficients \( f^{(k)}_p \) are given by

\[
f^{(k)}_p = \frac{(-1)^{k-p}}{\omega^{2(k-p)}} \frac{(k-1)!k!}{p!(2k-p-1)!}, \quad 0 \leq p \leq k - 1, \quad (3.9)
\]

and \( sc^a \) is the ordinary BRS variation of the Faddeev–Popov ghost \( c \)

\[
sc^a = Qc^a|_{\omega=0} = \frac{1}{2} f^{abc} c^b c^c. \quad (3.10)
\]

Notice that the coefficient of the last term on the r.h.s. of (3.7) is universal

\[
f^{(k)}_{k-1} = -\frac{1}{\omega^2}. \quad (3.11)
\]
It is worth to remark that the operators (3.5) we are considering have an interesting geometrical interpretation [23]. Let us consider for the moment \( k = 2 \); by redefining the scalar field as \( \phi \to \omega^2 \phi \), (3.8) can be written as
\[
\text{Tr} \, \phi^2 = Q \text{Tr} \left( cQC - \frac{2}{3} ccc \right) .
\] (3.12)

If we define the universal connection \( \hat{A} = A + c \), (3.12) can be seen as the \((p = 0, q = 4)\) component of the equation
\[
\text{Tr} (\hat{F} \hat{F}) = \hat{dA}_{CS} = \hat{d} \text{Tr} \left( \hat{A} \hat{d} \hat{A} - \frac{2}{3} \hat{A} \hat{A} \hat{A} \right) ,
\] (3.13)
where by \((p, q)\) we indicate the grading of the forms with respect to the extended exterior derivative \( \hat{d} = d + Q \), \( p \) being the space-time form degree and \( q \) the ghost number. Thus \( \mathcal{P}^{(2)}(c, \phi) \) can be understood as the \((0, 3)\) component of the Chern-Simons form \( A_{CS} \) of the universal connection \( \hat{A} \), while \( \phi \) as the \((0, 2)\) component of the corresponding curvature \( F \). Analogously, higher rank single-trace operators (3.5) can be expressed in terms of \((0, k)\) components of higher Chern classes \( \text{Tr} \, \hat{F}^k \), while multi-trace operators are products of Chern classes.

We now come to the proof of finiteness; what is actually important in this sense is that operators (3.5) can be expressed as the \( Q \)-variations of polynomials in the fields \( \phi \) and \( c \), and, in particular, that they are related to \( D^{a_1..a_k}c_{a_1}sc_{a_2}..sc_{a_k} \) (the first term on the r.h.s. of (3.7), which belongs to the cohomology of the ordinary BRS operator, and is known to be finite to all orders of perturbation theory [21]). It is then natural to expect that the nonrenormalization property of the BRS invariant odd polynomials in the ghost \( c \) translates to (3.7) as well. As we shall see, this is indeed the case.

Let us consider now the classical action
\[
\Sigma = S_{TYM} + S_{gf} + S_{ext} + S_k .
\] (3.14)

\( S_{TYM} \) is the twisted \( \mathcal{N} = 2 \) SYM action (2.9), \( S_{gf} \) is the (Landau) gauge fixed term
\[
S_{gf} = Q \text{ Tr } \int d^4x \bar{c} \partial^\mu A_\mu ,
\] (3.15)
\( \bar{c} \) being the Faddeev–Popov antighost. According to the usual BRS procedure to implement at the quantum level classical field theories, in \( S_{ext} \) the nonlinear \( Q \)-variations of the fields \( \varphi \) are coupled to external fields \( \varphi^* \), sometimes called “antifields”
\[
S_{ext} = \sum_\varphi \text{Tr} \int d^4x \varphi^* Q \varphi , \quad \varphi = A_\mu , \chi_{\mu\nu} , \psi_\mu , \eta , \phi , \bar{\phi} , c ,
\] (3.16)
and finally \( S_k \) is given by
\[
S_k = \omega^{2k} \int d^4x \left( \alpha_0 \Phi^{(k)} + \alpha_{2k-1} \mathcal{P}^{(k)} + \sum_{p=2}^{2k-2} \alpha_{2k-p} a_{a_1..a_{p-1}} \partial_{a_1}..\partial_{a_{p-1}} \mathcal{P}^{(k)} \right) ,
\] (3.17)
In the above expression for $S_k$, the $\alpha$’s are external sources coupled to $\Phi^{(k)}(\phi)$, $P^{(k)}(c, \phi)$ (defined by (3.8)), and its derivatives with respect to the ghost $c$ ($\partial_\alpha \equiv \partial / \partial c^\alpha$). They are antisymmetric in their color indices $a$’s, and are commuting or anticommuting objects depending from the number of ghosts $c$ contained in the composite operators they are coupled to. Such a number is represented by the lower indices, hence their statistics is given by $(-1)^{2k-p}$. Notice that, thanks to the overall multiplying factor $\omega^{2k}$, the functional $S_k$ is, as it should for being part of the action, an analytic expression in $\omega$.

The whole action $\Sigma$ satisfies the Slavnov–Taylor (ST) identity

$$S(\Sigma) = S^{old}(\Sigma) + S^{(k)} \Sigma = 0,$$

where the usual ST operator

$$S^{old}(\Sigma) = \sum_\phi \text{Tr} \int d^4x \left( \frac{\delta S}{\delta \phi} \frac{\delta S}{\delta \phi^*} + \frac{\delta S}{\delta b} \right)$$

has been implemented to the $\alpha$–sector as follows (see Appendix)

$$S^{(k)} \Sigma = \int d^4x \left( \alpha_{2k-1} \frac{\delta}{\delta \alpha_0} + \sum_{p=2}^{2k-3} \frac{p(1-p)}{2} f^{mna_1..a_{p-1}} \alpha_{2k-p-1} \frac{\delta}{\delta a_{2k-p-1}} \right) \Sigma. \tag{3.20}$$

Notice that the $\alpha$–sources, transforming one into the other, enter the ST identity (3.18) as BRS doublets, which have the nice property of keeping unaltered the cohomology of the (linearized) ST operator.

For the proof of finiteness, it is crucial that an additional constraint on the theory is satisfied: the ghost equation, peculiar of the Landau gauge [22]. Its main consequence is a nonrenormalization theorem, which states that, in the Landau gauge, the ghost field $c$ enters the quantum theory only if differentiated ($\partial_\mu c$). It is precisely for the purpose of being able to take advantage of this additional symmetry of the theory, that in $S_k$ external sources have been coupled not only to $\Phi^{(k)}(\phi)$ and $P^{(k)}(c, \phi)$, but also to the operators $\partial_1..\partial_{p-1} P^{(k)}(c, \phi)$. The ghost equation reads (see Appendix)

$$F^a \Sigma = \Delta^a, \tag{3.21}$$

where

$$F^a = \int d^4x \left( \frac{\delta}{\delta c^a} + f^{abc} c^b c^c \frac{\delta}{\delta c^c} \right)$$

and $\Delta^a$ is a classical breaking, linear in the quantum fields, given by

$$\Delta^a = \int d^4x \left( \sum_\phi f^{abc} \phi^b \phi^c + \alpha_2^{a_1..a_{2k-3}} \partial_0 \partial_1..\partial_{2k-3} P^{(k)} \right). \tag{3.22}$$
indeed, the operator \( \partial_\alpha \partial_{\alpha_1} \cdots \partial_{\alpha_{2k-3}} \mathcal{P}^{(k)}(c, \phi) \) is independent from the field \( \phi \), and it is linear in the ghost field \( c \).

Once observed that both the ST identity (3.18) and the ghost equation (3.21), being anomaly–free, can safely be extended to the quantum level [15]

\[ S(\Gamma) = 0 \]
\[ \mathcal{F}^a \Gamma = \Delta^a \]  
(3.24)

where \( \Gamma \) is the 1PI generating functional \( \Gamma = \Sigma + O(\hbar) \), we are able to prove our main result, \textit{i.e.} that the quantum insertions \( \text{Tr} \phi^k \cdot \Gamma = \text{Tr} \phi^k + O(\hbar) \) have vanishing anomalous dimensions, namely satisfy the Callan–Symanzik (CS) equation

\[ C \left[ \text{Tr} \phi^k(x) \cdot \Gamma \right] = 0 \]  
(3.25)

where \( C \) is the CS operator

\[ C \equiv \mu \frac{\partial}{\partial \mu} + \hbar \beta_g \frac{\partial}{\partial g} - \hbar \sum_\phi \gamma_\phi N_\phi \]  
(3.26)

satisfying

\[ C \Gamma = 0 \]  
(3.27)

In (3.26), \( \beta_g \) is the \( \beta \)–function of the gauge coupling constant \( g \), \( \gamma_\phi \) is the anomalous dimension of the generic field \( \phi \), the operator \( N_\phi \) is the counting operator \( \text{Tr} \int d^4x \phi^* \delta_{\phi^*} \), and \( \mu \) is the renormalization scale.

Let us take the derivative of the quantum ST identity (3.18) with respect to the external source \( \alpha_{2k-1} \), coupled in (3.17) to \( \mathcal{P}^{(k)}(c, \phi) \)

\[ \frac{\delta}{\delta \alpha_{2k-1}} S(\Gamma) \bigg|_{\alpha=0} = 0 \]  
(3.28)

where we put to zero the set of external sources \( \alpha \) appearing in \( S_k \) (3.17).

From the expression of the ST operator, Eq. (3.28) writes

\[ b_\Gamma \frac{\delta \Gamma}{\delta \alpha_{2k-1}} \bigg|_{\alpha=0} = \frac{\delta \Gamma}{\delta \alpha_0} \bigg|_{\alpha=0} \]  
(3.29)

The operator \( b_\Gamma \) appearing in the above equation is the (off–shell nilpotent) linearized quantum ST operator

\[ b_\Gamma = \int d^4x \left[ \text{Tr} \sum_\phi \left( \frac{\delta \Gamma}{\delta \phi^*} \frac{\delta}{\delta \phi} + \frac{\delta \Gamma}{\delta \phi} \frac{\delta}{\delta \phi^*} + \frac{\delta \Sigma}{\delta b} \right) \right. 
\]

\[ + \alpha_{2k-1} \frac{\delta}{\delta \alpha_0} + \sum_{p=2}^{2k-3} \frac{p(1-p)}{2} \sum_{m=0}^{2k-p-1} \alpha_{2k-p-1}^m \alpha_{2k-p-1}^m \frac{\delta}{\delta \alpha_{2k-p-1}} \left( \frac{\delta}{\delta \alpha_{2k-p-1}} \right) \]  
(3.30)

and it can be recognized as the quantum functional translation of the operator \( \mathcal{Q} \). The relation (3.29) is the quantum extension of the classical one (3.8) between the composite operators \( \text{Tr} \phi^k \) and \( \mathcal{P}^{(k)}(c, \phi) \)

\[ D^{a_1 \cdots a_k} \phi^{a_1} \cdots \phi^{a_k} \cdot \Gamma = b_\Gamma \left( \mathcal{P}^{(k)}(c, \phi) \cdot \Gamma \right) \]  
(3.31)
Owing to the ghost equation (3.21), the quantum insertion $\mathcal{P}^{(k)}(c, \phi) \cdot \Gamma$ has vanishing anomalous dimension, since the undifferentiated ghost field $c$ does not get quantum corrections

$$\mathcal{C} \left( \mathcal{P}^{(k)}(c, \phi) \cdot \Gamma \right) = 0 .$$

Now, since the CS operator commutes with the linearized quantum ST operator $b_T$

$$[\mathcal{C}, b_T] = 0 ,$$

from Eq. (3.32) and Eq. (3.33) we derive our result (3.25): the composite operators $\text{Tr} \, \phi^k$ are finite to all orders of perturbation theory. This result can be trivially extended to the class of operators $\text{Tr} \, \overline{\phi}^k$.

We would like to stress once again that the presence of matter does not alter in any way our result (3.25). Indeed, we needed only to consider the symmetry transformations (3.4) of the scalar field $\phi$ of the $\mathcal{N} = 2$ gauge supermultiplet and of the Faddeev–Popov ghost field $c$. These transformations are not altered by the presence of matter fields, hence the matter sector is completely decoupled in our proof of existence of protected operators. Under this respect, matter is only an avoidable graphical charge, which we preferred to omit. The class of protected operators discussed in this paper refers therefore to $\mathcal{N} = 2$ SYM theories coupled to matter in an arbitrary representation of the gauge group, hence to $\mathcal{N} = 4$ theory as well, since for the particular choice of matter in the adjoint representation, $\mathcal{N} = 4$ SYM is recovered from the $\mathcal{N} = 2$ case.

4 Conclusions

We have shown the vanishing to all orders of the perturbative expansion of the anomalous dimension of local single and multi-trace composite operators of scalar fields in theories with $\mathcal{N} = 2$ supersymmetry. Although the proof has been given explicitly for the pure gauge case, it holds unaltered for $\mathcal{N} = 2$ SYM coupled to matter belonging to a generic representation of the gauge group $G$, hence in particular it remains valid for the $\mathcal{N} = 4$ case, obtained by putting matter multiplets in the adjoint representation of $G$. Nonrenormalization properties of these operators can be better displayed using the twisted formulation of the theory, where they have a nice geometrical interpretation as components of the Chern classes of a suitable universal bundle \[23\]. The study of correlation functions of such operators play a relevant role in tests of the AdS/CFT correspondence \[4, 9\]. In addition, it recently led to conjecture the existence of a new class of gauge invariant operators which are protected despite they do not obey any of the known shortening conditions \[9, 8\].

We stress that our analysis does not rely on the superconformal algebra. It would then be interesting to extend it to the above mentioned operators.
Moreover, the approach we followed can be applied also to nonconformal $\mathcal{N} = 2$ theories which have been recently considered in extensions of the AdS/CFT duality [12, 13]. Even if a precise map between bulk supergravity fields and boundary gauge-invariant field theory operators, as that of the $\mathcal{N} = 4$ superconformal theory, is not presently known for these cases, the appearance of protected operators in theories with a nontrivial renormalization group flow is a very interesting feature by itself, and could motivate further analysis on the correlation functions of these operators.

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A. \( \Phi^{(k)}(\phi) = Q\mathcal{P}^{(k)}(c, \phi) \)

The fields $\phi$ and $c$ and the parameter $\omega$ are assigned the following quantum numbers

|        | $\phi$ | $c$ | $\omega$ |
|--------|--------|-----|----------|
| dim.   | 1      | 0   | $-1/2$   |
| $\mathcal{R}$-charge | 2      | 0   | $-1$     |
| $\Phi\Pi$-charge | 0      | 1   | 1        |
| $\mathcal{R} + \Phi\Pi$ | 2      | 1   | 0        |

(A.1)

Therefore the most general expression for $\Phi^{(k)}(\phi) = D^{a_1..a_k} \phi^{a_1}..\phi^{a_k}$ having homogeneous $\mathcal{R} + \Phi\Pi$ quantum number is the following

$$D^{a_1..a_k} \phi^{a_1}..\phi^{a_k} = Q \left[ D^{a_1..a_k} \sum_{p=0}^{k-1} f_p^{(k)} c^{a_1} sc^{a_2}..sc^{a_k-p} \phi^{a_k-p+1}..\phi^{a_k} \right].$$

(A.2)

In (A.2), the coefficients $f_p^{(k)}$ depend on the global ghost $\omega$ appearing in the definition of the symmetry operator $Q = s + \omega \delta$ (3.1). The first term of the sum (A.2) is proportional to $D^{a_1..a_k} c^{a_1} sc^{a_2}..sc^{a_k}$, which is the $c^{(2k-1)}$ cocycle, belonging to the cohomology of the BRS operator [24], and known to be finite to all orders of perturbation theory [21]. The properties of the invariant polynomials $\Phi^{(k)}(\phi)$ are mostly determined by the relationship existing between them and the BRS cocycles $c^{(2k-1)}$.

Our first goal is to find the explicit expression for the coefficients $f_p^{(k)}$ in (A.2).

In order to obtain it, let us observe that

$$Q sc^a = \omega^2 s \phi^a,$$

(A.3)
and that

\[ D^{a_1\ldots a_n} c^{a_1} sc^{a_2} \ldots sc^{a_m} \phi^{a_{m+1}} \ldots \phi^{a_{n-1}} s \phi^{a_n} = \frac{2}{n-m} D^{a_1\ldots a_n} sc^{a_1} \ldots sc^{a_m} \phi^{a_{m+1}} \ldots \phi^{a_n} . \]  

(A.4)

Eq. (A.4) is a consequence of the nilpotency of the BRS operator and of the fact that the completely symmetric rank \(-k\) tensor \(D^{a_1\ldots a_k}\) is invariant, hence it satisfies the Jacoby–like identity

\[ D^{p_1 a_1} + D^{a_1 p_2 a_3} a_k f^{p_2 a_2} + \ldots + D^{a_1\ldots a_k-1 p_k} f^{p_k a_k} = 0 . \]  

(A.5)

Performing the \(Q\)-variation in (A.2) and exploiting the identities (A.3), (A.4), we get

\[ D^{a_1\ldots a_k} \phi^{a_1} \ldots \phi^{a_k} = - \sum_{p=1}^{k-1} D^{a_1\ldots a_k} f_p^{(k)} sc^{a_1} \ldots sc^{a_{k-p}} \phi^{a_{k-p+1}} \ldots \phi^{a_k} \]

\[ + \sum_{p=0}^{k-2} D^{a_1\ldots a_k} f_p^{(k)} \omega^2 \left(1 - \frac{2k}{p+1}\right) sc^{a_1} \ldots sc^{a_{k-p}} \phi^{a_{k-p}} \ldots \phi^{a_k} \]

\[ - f_{k-1}^{(k)} \omega^2 D^{a_1\ldots a_k} \phi^{a_1} \ldots \phi^{a_k} \]

Hence, reorganizing terms, we have

\[ D^{a_1\ldots a_k} \phi^{a_1} \ldots \phi^{a_k} = - f_{k-1}^{(k)} \omega^2 D^{a_1\ldots a_k} \phi^{a_1} \ldots \phi^{a_k} \]

\[ + \sum_{p=1}^{k-1} D^{a_1\ldots a_k} sc^{a_1} \ldots sc^{a_{k-p}} \phi^{a_{k-p+1}} \ldots \phi^{a_k} \left[ \omega^2 \left(1 - \frac{2k}{p}\right) f_p^{(k)} \right] . \]  

(A.6)

We must therefore impose

\[ f_{k-1}^{(k)} = - \frac{1}{\omega^2} \]  

(A.7)

\[ f_p^{(k)} = \omega^2 \left(1 - \frac{2k}{p}\right) f_p^{(k-1)} 1 \leq p \leq k - 1 . \]  

(A.8)

Eqs (A.7) and (A.8) can be solved by a closed formula

\[ f_p^{(k)} = \frac{(-1)^{k-p}}{\omega^{2(k-p)}} \frac{(k-1)!k!}{p!(2k-p-1)!} , 0 \leq p \leq k - 1 , \]  

(A.9)

as can be verified by induction. This proves the relations (3.7) and (3.9).

For future reference, it may be useful to write (3.7) for the lowest values \(k = 2, 3, 4\).

For \(k = 2\), the invariant tensor \(D^{ab}\) is just the Kronecker delta \(\delta^{ab}\), and from (3.7) and (3.9) we immediately get

\[ \phi^a \phi^a = Q \left( f_0^{(2)} c^a sc^a + f_1^{(2)} c^a \phi^a \right) = Q \left( \frac{1}{3 \omega^4} c^a sc^a - \frac{1}{\omega^2} c^a \phi^a \right) . \]  

(A.10)
For $k = 3$, there is only one completely symmetric tensor $D^{abc} = d^{abc}$, and
\[
d^{abc} \phi^a \phi^b \phi^c = QD^{abc} \left( f_0^{(3)} e^a s^b s^c + f_1^{(3)} e^a s^b \phi^c + f_2^{(3)} e^a \phi^b \phi^c \right) \quad (A.11)
\]
\[
= QD^{abc} \left( -\frac{1}{10} \omega^a e^a s^b s^c + \frac{1}{2 \omega^4} e^a s^b \phi^c - \frac{1}{\omega^2} e^a \phi^b \phi^c \right).
\]
For $k = 4$, there are more than one symmetric tensors $D^{abcd}$
\[
D^{abcd} \phi^a \phi^b \phi^c \phi^d = QD^{abcd} \left( f_0^{(4)} e^a s^b s^c s^d + f_1^{(4)} e^a s^b s^c \phi^d + f_2^{(4)} e^a s^b \phi^c \phi^d + f_3^{(4)} e^a \phi^b \phi^c \phi^d \right) \quad (A.12)
\]
\[
= QD^{abcd} \left( \frac{1}{35 \omega^8} e^a s^b s^c s^d - \frac{1}{5 \omega^6} e^a s^b s^c \phi^d + \frac{3}{5 \omega^4} e^a s^b s^c \phi^d - \frac{1}{\omega^2} e^a \phi^b \phi^c \phi^d \right).
\]

\section{The Slavnov–Taylor identity}

Our task is to extend the action of $Q$ on the set of external $\alpha$–sources introduced in
\[
S_k = \omega^{2k} \int d^4 x \left( \alpha_0 \Phi^{(k)} + \alpha_{2k-1} \mathcal{P}^{(k)} + \sum_{p=2}^{2k-2} \alpha_{a_{2k-p}} \partial_{a_1} \ldots \partial_{a_{p-1}} \mathcal{P}^{(k)} \right), \quad (B.1)
\]
by demanding
\[
Q S_k = 0. \quad (B.2)
\]
We have
\[
QS_k = \omega^{2k} \int d^4 x \left\{ (Q \alpha_0) \Phi^{(k)} \right. \\
\left. + \sum_{p=1}^{2k-2} \left[ (Q \alpha_{a_1} \ldots \alpha_{a_{2k-p}}) \partial_{a_1} \ldots \partial_{a_{p-1}} \mathcal{P}^{(k)} \right] \right. \\
\left. + (-1)^{2k-p} \alpha_{a_1} \ldots \alpha_{a_{2k-p}} (Q \partial_{a_1} \ldots \partial_{a_{p-1}} \mathcal{P}^{(k)}) \right\}, \quad (B.3)
\]
where we took into account the $Q$–invariance of the polynomials $\Phi^{(k)}(\phi)$ and the statistics of the $\alpha$–sources, given by $(-1)^{2k-p}$.

In order to evaluate $[B.3]$, we observe that the anticommutator between the derivative with respect to the ghost field $c (\partial_a)$ and the symmetry operator $Q$, gives the generators of the rigid gauge transformations $H^a$
\[
\{ \partial_a, Q \} X^b = -H^a X^b = -f^{abc} X^c, \quad (B.4)
\]
where $X^a$ is a generic function of the fields $\phi$ and $c$ and belongs to the adjoint representation of the gauge group. From the obvious relations
\[
H^a \Phi^{(k)}(\phi) = H^a \mathcal{P}^{(k)}(c, \phi) = \partial_a Q \mathcal{P}^{(k)}(c, \phi) = 0, \quad (B.5)
\]
we derive
\[ Q \partial_a \mathcal{P}^{(k)} = 0 \] (B.6)
and, in general,
\[ \alpha^{a_1 \cdots a_m} Q \partial_{a_1} \cdots \partial_{a_m} \mathcal{P}^{(k)} = (-1)^{m+1} \frac{m(m-1)}{2} f^{a_1 a_2 p} \partial_{a_3} \cdots \partial_{a_m} \partial_p \mathcal{P}^{(k)} \alpha^{a_1 \cdots a_m}. \] (B.7)

Hence (B.3) writes
\[ QS_k = \omega^{2k} \int d^4x \left\{ (Q \alpha_0) \Phi^{(k)} - \alpha_{2k-1} \Phi^{(k)} + \sum_{p=1}^{2k-2} \left[ (Q \alpha_{2k-p}^{a_1 \cdots a_{p-1}}) \partial_{a_1} \cdots \partial_{a_{p-1}} \mathcal{P}^{(k)} + \frac{(p-1)(p-2)}{2} \alpha_{2k-p}^{a_1 \cdots a_{p-1}} f^{a_1 a_2 q} \partial_{a_3} \cdots \partial_{a_{p-1}} \partial_q \mathcal{P}^{(k)} \right] \right\} \]
\[ = \omega^{2k} \int d^4x \left\{ (Q \alpha_0) \Phi^{(k)} - \alpha_{2k-1} \Phi^{(k)} + (Q \alpha_{2k-1}) \mathcal{P}^{(k)} + (Q \alpha_{2k-3}^{a_1 \cdots a_{2k-3}}) \partial_{a_1} \cdots \partial_{a_{2k-3}} \mathcal{P}^{(k)} \right. \]
\[ + \sum_{p=2}^{2k-3} \left( Q \alpha_{2k-p}^{a_{1-1}} + \frac{p(p-1)}{2} \alpha_{2k-p-1}^{a_{1-1}} r s a_{1-1} a_{p-2} f^{r s a_{p-1}} \right) \partial_{a_1} \cdots \partial_{a_{p-1}} \mathcal{P}^{(k)} \right\}. \]

In order that the whole expression \( QS_k \) vanish, by identifying terms containing equal number of ghost fields \( c \), we have
\[ Q \alpha_0 = \alpha_{2k-1} \] (B.9)
\[ Q \alpha_{2k-3}^{a_1 \cdots a_{2k-3}} = 0 \] (B.10)
\[ Q \alpha_{2k-1} = 0, \] (B.11)
and
\[ Q \alpha_{2k-p}^{a_1 \cdots a_{p-1}} = -\frac{p(p-1)}{2} f^{r s a_{p-1}} \alpha_{2k-p-1}^{a_1 \cdots a_{p-2}} 2 \leq p \leq 2k-3. \] (B.12)

Hence, the implementation of the ST identity to the \( \alpha \)-sector is given by
\[ S^{(k)} \Sigma = \int d^4x \left( \alpha_{2k-1} \frac{\delta}{\delta \alpha_0} + \sum_{p=2}^{2k-3} \frac{p(p-1)}{2} f^{m n a_{p-1}} \alpha_{2k-p-1}^{m n a_{p-2}} \frac{\delta}{\delta \alpha_{2k-p}} \right) \Sigma, \] (B.13)
which coincides with (3.20).

\section{The Ghost Equation}

To extend the ghost equation, let us take the functional derivative with respect to ghost field \( c \) of the action functional \( S_k \)
\[ \delta_a S_k = \sum_{p=1}^{2k-2} (-1)^{2k-p} \alpha_{2k-p}^{a_1 \cdots a_{p-1}} \partial_a \partial_{a_1} \cdots \partial_{a_{p-1}} \mathcal{P}^{(k)} \] (C.1)
\[ = \sum_{p=1}^{2k-3} (-1)^{2k-p} \alpha_{2k-p}^{a_1...a_{p-1}} \frac{\delta \sum}{\delta \alpha_{2k-p}^{a_1...a_{p-1}}} + \int d^4 x \ \alpha_2^{a_1...a_{2k-3}} \partial_a \partial_{a_1}...\partial_{a_{2k-3}} \mathcal{P}^{(k)} \]

Notice that, according to the expression (3.7), the maximum number of ghosts \( c \) present in \( \mathcal{P}^{(k)}(c, \phi) \) is \( 2k - 1 \), in the term \( c^{(2k-1)} \), whereas the maximum number of its ghost derivatives in the last term at the r.h.s of (C.1) is \( 2k - 2 \), hence this term is linear in \( c \) and independent from \( \phi \), i.e. it represents a classical breaking.

Therefore, the complete ghost equation reads

\[ \mathcal{F}^a \Sigma = \Delta^a \]  

(C.2)

where

\[ \mathcal{F}^a = \int d^4 x \left( \frac{\delta}{\delta c^a} + f^{abc} \frac{\delta}{\delta c^b} - \sum_{p=1}^{2k-3} (-1)^{2k-p} \alpha_{2k-p}^{a_1...a_{p-1}} \frac{\delta}{\delta \alpha_{2k-p}^{a_1...a_{p-1}}} \right) \]  

(C.3)

and \( \Delta^a \) is the classical breaking given by

\[ \Delta^a = \int d^4 x \left( \sum_{\phi} f^{abc} \phi^a \phi^b \phi^c + \alpha_2^{a_1...a_{2k-3}} \partial_a \partial_{a_1}...\partial_{a_{2k-3}} \mathcal{P}^{(k)} \right) \]  

(C.4)

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