Recombination Models

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Abstract. We review the current status of recombination and coalescence models that have been successfully applied to describe hadronization in heavy ion collisions at RHIC energies. Basic concepts as well as actual implementations of the idea are discussed. We try to evaluate where we stand in our understanding at the moment and what remains to be done in the future.

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1. Introduction

Since the last Quark Matter conference, held in 2002, the idea of quark recombination or coalescence (ReCo) had a very rapid and successful career as a model for hadronization. Recombination of quarks was first formulated over two decades ago by R. C. Hwa and collaborators as a model for hadronization in the fragmentation region of hadron-hadron collisions [1]. There were early efforts to connect it to other descriptions of hadronization like fragmentation [2, 3]. For heavy ion collisions, quark coalescence ideas first appeared soon after that [4] and were later on successfully used in ALCOR [5].

The recent revival of recombination/coalescence began when it was realized [6] that the elliptic flow \( v_2 \) measured at RHIC obeys a simple valence quark scaling, that naturally arises from a recombination picture. Soon after that, it was pointed out [7, 8] that other RHIC puzzles, like the anomalous enhancement of baryons and the absence of nuclear suppression in baryon spectra can be explained by ReCo as well.

This talk seeks to review the current status of recombination/coalescence models. We will revisit the fundamental concepts and experimental findings that support recombination. We will then discuss different implementations of the model and its limitations. We close with an outlook on future developments.

2. Basic Ideas

Hadronization has always been a very difficult aspect of strong interaction processes. A lot of effort went into the invention of methods to work around this non-perturbative phenomenon, with the effect that our knowledge of hadronization dynamics is still very
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limited. One example is the formulation of fragmentation or \textquotedblleft{}quark decay\textquotedblright{} functions which rely on the common concept of separating long and short distance dynamics. The cross section of hadron production in $e^+e^-$, lepton-hadron or hadron-hadron collisions can be written as a convolution $d\sigma_H = d\sigma_a \otimes D_{a\rightarrow H}$ of a parton production cross section $d\sigma_a$ with a fragmentation function $D_{a\rightarrow H}(z, \mu)$ \cite{10}. $D_{a\rightarrow H}(z, \mu)$ gives the probability to find a hadron $H$ in the hadronizing parton $a$ with a momentum fraction $z$, $0 < z < 1$. $\mu \gg \Lambda_{QCD}$ is a perturbative scale that is set by the transverse momentum.

The $z$-dependence of fragmentation functions is not calculable in perturbation theory. However, they are universal, i.e. process independent, objects. Fragmentation functions for the most important hadrons have been measured, mainly in $e^+e^-$ collisions, and are available in parametrized form. Nevertheless, the availability of measured fragmentation functions should not lead to the impression that we have a good understanding of the underlying hadronization process.

One important question is the following: at which scales (i.e. in which $P_T$ region) is leading twist perturbative QCD (pQCD) reliable? It has been shown that next-to-leading order (NLO) calculations of $\pi^0$ production in $pp$ collisions at RHIC, using modern parametrizations of fragmentation functions \cite{11} are in good agreement with PHENIX data down to surprisingly low pion $P_T$ of 1 GeV/c \cite{12}.

On the other hand, there are clear signs that the fragmentation concept is not working at very low $P_T$. A good example is the leading particle effect. In the very forward (and low $P_T$) region of hadron-hadron collisions \cite{11}, the composition of particle species deviates from expectations in a fragmentation picture. This is impressively confirmed by recent experimental results, e.g. from the FNAL E791 collaboration \cite{13}. With a $\pi^-$ beam impinging on a fixed nuclear target, they measure a $D^-/D^+$ asymmetry that goes to 1 in the very forward direction. While fragmentation would predict this asymmetry to be very close to 0, one can understand this effect starting from a recombining $c\bar{c}$ pair produced in the collision. The recombination $\bar{c}d \rightarrow D^-$ is enhanced with respect to $cd \rightarrow D^+$, because the $d$ is a valence quark in the beam $\pi^-$ while the $\bar{d}$ is only a sea quark \cite{14}.

Other examples are provided by the RHIC experiments. A proton/pion ratio $\sim 1$ was observed in central Au+Au collisions between $P_T = 1.5$ and 4 GeV/c \cite{15}. This is in contradiction to pQCD calculations, that give $p/\pi \sim 0.1 \ldots 0.2$ \cite{16}. Similar results for $\Lambda/K^0$ \cite{17} suggest that there is a general pattern of baryon enhancement at RHIC energies. This enhancement is so strong that it neutralizes the strong jet quenching observed for mesons at RHIC \cite{18}. The nuclear modification factor $R_{AA}$ for baryons is close to 1 up to 4 GeV/c \cite{15,17,19}. Thus, although pQCD seems to work very well in $p + p$ for $P_T$ above 1 GeV/c, this is not the case in Au+Au even at 4 GeV/c.

The two examples have in common that hadronization takes place in a phase space filled with partons, either beam remnants or the hot and dense medium created in heavy ion collisions. This is very different from $e^+e^-$ collisions where phase space is nearly empty. One can include corrections to single parton fragmentation in terms of higher twist fragmentation. Such contributions include the process of two or more partons fragmenting into hadrons. However, nothing is known about these higher twist corrections. Instead of the rather complicated twist expansion, let us directly look at the limiting case of a phase space densely packed with partons. What will happen upon hadronization? The most simple picture is that the quarks and antiquarks that constitute the valence quark structure of a hadron (having the correct quantum numbers), recombine/coalesce together to form this hadron.
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Figure 1. Recombination and fragmentation for a meson at $P_T \approx 6$ GeV/c, starting from a parton spectrum with steep slope (solid line). Fragmentation requires a single parton with transverse momentum larger than $P_T$ to start with, while recombination is possible with two partons that have roughly $P_T/2$ each. The competition between both processes is decided by slope and normalization of the parton spectrum.

In fragmentation the entire parton content of the hadrons has to come from gluons and $q\bar{q}$ pairs emitted from the fragmenting parton inside the “black box” called fragmentation function. For recombination, it is assumed that no further branching of partons occurs. Apparently these are limiting cases. There must be a smooth transition between both extremes as a function of phase space density. On the other hand, one might speculate that the absence of additional branching is a hint that some sort of equilibrium is reached in the parton phase. Figure 1 shows how both processes form a meson.

Which partons enter the recombination process? In most implementations, quarks are assumed to be non-perturbative and to have an effective mass. This, and the fact that only valence quarks are involved, leads to the interpretation of these degrees of freedom as constituent quarks. Gluons do not participate in recombination and are used to dress the quarks. The alternative to the constituent quark picture is to explicitly convert gluons into $q\bar{q}$ pairs.

3. Mathematical Formulation

ReCo can be formulated in terms of Wigner functions. The yield of mesons $M$ coalescing from two partons $a, b$ is given by

$$\frac{dN_M}{d^3P} = \sum_{a,b} \int \frac{d^3R}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} W_{ab} \left( R - \frac{r}{2}, \frac{p}{2} - q; R + \frac{r}{2}, \frac{p}{2} + q \right) \Phi_M(r,q). \quad (1)$$

$W_{ab}$ and $\Phi_M$ are the Wigner functions of the partons and the meson respectively, $P$ and $R$ are the momentum and spatial coordinate of the meson and the sum runs over all possible combinations of quantum numbers, essentially leading to a degeneracy factor $C_M$. The generalization of this formula for baryons is straightforward [10]. The Wigner function for the partons is usually factorized into classical one-particle phase space distributions, $W_{ab}(r_a, p_a; r_b, p_b) = w_a(r_a, p_a) w_b(r_b, p_b)$. This assumes that the partons are completely uncorrelated before hadronization. We will come back to this point later.
We can immediately draw some conclusions. Suppose the parton spectrum is exponential in transverse momentum $p_T$, $w = A e^{-p_T/T}$, with some slope parameter $1/T$. Then fragmentation and recombination would provide meson spectra

$$dN_{\text{frag}}/d^2p_T D \propto A e^{-p_T / \langle z \rangle T}$$

$$dN_{\text{reco}}/d^2p_T \propto A^2 e^{-p_T / T}$$

respectively. Here $\langle D \rangle$ and $\langle z \rangle$ are average values of the fragmentation function and the scaling variable $z$. Since $\langle z \rangle < 1$, fragmentation is less effective than recombination on an exponential spectrum, as long as the normalization $A$ is not too small. On the other hand, if the parton spectrum has power law form, $w = B p_T^{-\alpha} T$, the yields are

$$dN_{\text{frag}}/d^2p_T \sim p_T^{-\alpha}$$

$$dN_{\text{reco}}/d^2p_T \sim p_T^{-2\alpha}$$

for mesons. This implies that fragmentation will dominate at large $p_T$ for power law spectra (which are predicted by perturbative QCD).

If the parton spectrum is exponential, one can check that with increasing $p_T$ the result is less and less sensitive to the momentum dependence of the hadronic Wigner function (i.e. the shape of the momentum space wave functions used to model it). In fact, it seems that for $p_T > 2$ GeV/c one can safely take the momentum space wave function to have zero width (i.e. to be a $\delta$-function) [16].

Let us assume that the parton phase exhibits an azimuthal anisotropy. The elliptic component of this asymmetry is described by the elliptic flow coefficient $v_2(p_T)$ [20]. ReCo predicts the resulting elliptic flow for hadrons to be

$$v_2^M(p_T) = 2v_2\left(\frac{p_T}{2}\right),$$

$$v_2^B(p_T) = 3v_2\left(\frac{p_T}{3}\right)$$

for mesons and baryons respectively [16]. One should note that the derivation of these scaling laws uses narrow, $\delta$-shaped wave functions. See [16] for the case of wave functions of finite width. As can seen be in Figure 2 the scaling laws are impressively confirmed by experimental data for $p_T > 1$ GeV/c, while at lower $p_T$ the mass of the hadron determines the elliptic flow, well described by hydrodynamics [19, 22]. Deviations for pions are discussed below. Above $\sim 5$ GeV/c a perturbative mechanism, driven by jet quenching, should take over from recombination, but experimental data are not yet conclusive [16].
4. Implementations

The development of ReCo in the past 18 months was mainly driven by four groups who published work using their individual implementations of the ReCo concept. These groups are (in parentheses the short form used thereafter) Duke/Minnesota/Kyoto (Duke) [7, 23, 16, 24, 25], Ohio State/Wayne State (Ohio) [21, 26, 27], Oregon [28, 29] and Texas A&M/Budapest (TAM) [8, 30, 31, 32].

The Duke and Oregon groups try to evaluate Equation (1) analytically using certain assumptions that essentially boil it down to a convolution of one-particle distributions and hadron wave functions in longitudinal momentum space (longitudinal with respect to the hadron momentum). As an example, the Duke group writes the meson spectrum from recombination as [16]

$$E dN M / d^3 p = C_M \int d\sigma (P^+ \cdot u) \int dx w_a(\sigma, x P) w_b(\sigma, (1-x) P) |\phi_M(x)|^2$$

(4)

where $d\sigma$ integrates over the hadronization hypersurface $\Sigma$, $u$ is the four vector orthogonal to $\Sigma$, $x$ is the momentum fraction of parton $a$ in the meson, $P^+$ is the light cone momentum and $\phi_M$ the wave function of the meson. The parton phase that undergoes hadronization is assumed to have an exponential part at low $p_T$ (soft partons) and a power law tail at high $p_T$ (hard partons). For central Au+Au collisions at RHIC, the Duke group uses a thermal distribution with temperature $T = 175$ MeV and average radial flow velocity $v = 0.55c$ for the soft partons, while the hard partons are taken from a minijet calculation [33] including energy loss [16].

The Ohio and TAM groups developed Monte Carlo implementations of the recombination process. These can be connected to string or parton cascade models that prepare the parton state before hadronization. One main difference lies in the treatment of the connection between soft and hard partons. While the Duke group strictly separates soft and hard physics, allowing only the soft partons to recombine and only the hard partons to fragment, the TAM group includes additional coalescence of soft and hard partons [8, 30]. The Oregon group carries this a step further and replaces fragmentation functions by a scenario where minijet partons develop a shower which subsequently recombines. In this model, they are able to describe fragmentation functions reasonably well. Applied to heavy ion collisions, this allows them to introduce recombination of soft partons with shower partons [29].

All four groups describe hadron data from RHIC at intermediate and large transverse momenta very well. The calculations for spectra reproduce the noticeable kink in the data around $p_T = 4$ GeV/c for mesons and 6 GeV/c for baryons coming from the transition from soft (recombination) to hard (perturbative) particle production. Soft-hard coalescence can improve the fit to data points in the transition region, which is then extended to even higher $p_T$. Figure 3 shows a result obtained by the Duke group compared with RHIC data. The baryon/meson ratios in ReCo are essentially given by the ratio of degeneracy factors $C_B/C_M$, leading naturally to an enhanced proton/pion ratio. The most recent results can be found in [16, 30, 29].

As already mentioned, the elliptic flow of all measured hadron species, $\pi$, $K$, $p$, $\Lambda$, $\Xi$ and $\Omega$, follows the scaling law (1) for $p_T > 1$ GeV/c. This permits to unambiguously extract $v_2(p_T)$ for partons. Surprisingly, the elliptic flow of strange quarks is the same as for light quarks [16, 24]. Slight deviations from the scaling can be seen for pions. This can be traced back to the small mass of the pion and the fact that most pions do not hadronize directly but are from $\rho$ decays [32].
Figure 3. Spectra of $\pi^0$, $p$, $K^0_s$ and $\Lambda + \bar{\Lambda}$ as a function of $P_T$ at midrapidity for central Au+Au collisions at $\sqrt{s} = 200$ GeV/c [18]. RHIC data are taken from PHENIX ($\pi^0, p$) [34, 15] and STAR ($K^0_s, \Lambda + \bar{\Lambda}$) [17].

5. ReCo Challenges and Answers

Despite its success, several aspects of ReCo are problematic. For one, recombination in its current implementation cannot describe the bulk production of hadronic matter. This seems to contradict the claim that ReCo is the right choice for very dense parton systems. However, one should note that the ReCo formula (1) is only for the average meson created by recombination (like the fragmentation formula is for the average hadron produced from one parton). None of these formulas describe the exclusive hadron content of the system. Note that in (1) the mesons scale with the square of the parton density, whereas the total number of mesons has to scale linearly, of course. While ReCo as a concept is certainly correct also for the bulk of hadron production, the simple formula (1) does not describe this.

The second issue is that Equation (1) conserves momentum, but not energy. In general, it is not possible to have energy conservation in $2 \rightarrow 1$ and $3 \rightarrow 1$ processes. In reality, particles in this strongly interacting environment will be off-shell, making energy conservation possible. However, instead of taking displacements from the mass shell into account, it is more convenient to have particles on the mass shell and permit small violations of energy conservation. This is a safe procedure as long as the violations are small compared to the total energy $E$ of the hadron, i.e. for $P_T$ above $\sim 2$ GeV/c [16]. A special role is played by Goldstone bosons. Their description is particularly difficult in a picture where the coalescing quarks have constituent masses.

The situation can be improved by taking into account coalescence of resonances and their subsequent decay. This introduces $2 \rightarrow 2$ and $3 \rightarrow 2$ processes and energy can be exactly conserved. Including the $\rho$ resonance, decaying into two pions, helps to cure the problem that pions are too heavy in a constituent quark picture [22]. Another critical question concerns entropy. Apparently, recombination reduces the degrees of freedom in the system, therefore leading to a decrease in entropy. We note that this
statement is not relevant as long as we do not address total particle numbers. But in any case, including resonances considerably improves this situation as well.

6. For the Future

One can think of a long list of problems that should be addressed in the near future. It was pointed out that the study of resonances within the recombination model provides new insights about interactions in the hadronic phase and could even be used to pin down properties of resonances and exotic states \[25\]. It also has to be decided whether charmed hadrons follow the recombination systematics. The elliptic flow of charm quarks has to be measured \[26, 31\]. First results on higher harmonics are available now \[35\]. They provide novel tests for the ReCo concept and can be used to further pin down the partonic phase \[36\]. Furthermore, the role of soft-hard coalescence has to be investigated in more detail, in particular with respect to hadron-hadron correlations.

RHIC data indicates that strong hadron-hadron correlations are present at intermediate \(P_T\), where ReCo dominates \[37\]. Such correlations are expected for the fragmentation process where several hadrons emerge from the same parton. It was argued that hadrons from recombination are emitted statistically so that no correlations should be observable. This is not true. Correlations in the parton phase are naturally translated into correlations in the hadron phase by the recombination process. We immediately conclude that there must be non-negligible correlations between soft partons.

Let us assume we want to describe the recombination of four partons 1,2,3,4 into two mesons \(A, B\). In analogy to Equation (1), a 4-parton Wigner function \(W_{1234}\) is needed to describe double meson production. We can include correlations in the parton phase by replacing the simple single particle factorization by

\[
W_{1234} = N \prod_{i=1}^{4} w_i \left( 1 + \sum_{i<j} C_{ij} \right) \tag{5}
\]

where \(C_{ij}\) is a 2-parton correlation function and \(N\) is a normalization factor. This will lead to non-trivial correlation functions between mesons. The correlations in the parton phase could originate from interactions between hard and soft partons and would be naturally present in soft-hard recombination. It remains to be seen whether the correlations measured at RHIC will consistently fit into this picture. But if this is the case, ReCo will provide a fascinating new picture of the partonic phase.

What will happen at LHC? Part of the ReCo success story is, that jet quenching is so strong at RHIC, suppressing hard processes in the final state by a factor of 4. Hard processes will be more abundant at LHC, but increased jet quenching and a brighter thermal source will probably overwhelm them up to even higher \(P_T\) than at RHIC \[38\].

7. Summary

Quark recombination/coalescence is a successful model to describe hadronization in dense parton systems. Central \(Au+Au\) collisions provide a partonic medium that is sufficiently dense for coalescence of soft partons to overcome fragmentation at intermediate transverse momenta between 2 and 5 GeV/c. Soft-hard coalescence could push this region to even higher \(P_T\).
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References

[1] K. P. Das and R. C. Hwa, Phys. Lett. B 68, 459 (1977); Erratum ibid. 73, 504 (1978); R. G. Roberts, R. C. Hwa and S. Matsuda, J. Phys. G 5, 1043 (1979).
[2] V. Chang and R. C. Hwa, Phys. Rev. Lett. 44, 439 (1980); Erratum ibid. 44, 1163 (1980).
[3] R. Migneron, L. M. Jones and K. E. Lassila, Phys. Lett. B 114, 189 (1982); Phys. Rev. D 26, 2235 (1982).
[4] C. Guptet al., Nuovo Cim. A 75, 408 (1983); J. A. Lopez, J. C. Parikh and P. J. Siemens, Phys. Rev. Lett. 53, 1043 (1989).
[5] B. A. Kniehl, G. Kramer and B. Potter, Nucl. Phys. B 582, 514 (2000); S. Kretzer, Phys. Rev. D 62, 054001 (2000).
[6] J. C. Collins and D. E. Soper, Nucl. Phys. B 194, 445 (1982).
[7] V. Greco, C. M. Ko and P. Levai, Phys. Rev. Lett. 89, 022301 (2002).