Resonant Electron-Plasmon Interactions in Drifting Electron Gas

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(Dated: December 1, 2020)

Abstract

In this paper we investigate the resonant electron-plasmon interactions in a drifting electron gas of arbitrary degeneracy. The kinetic corrected quantum hydrodynamic model is transformed into the effective Schrödinger-Poisson model and driven coupled pseudoforce system is obtained via the separation of variables from the appropriately linearized system. It is remarked that in the low phase-speed kinetic regime the characteristic particle-like plasmon branch is profoundly affected by this correction which is a function of the electron number density and temperature. We also present an alternative explanation of the quantum wave-particle duality as a direct consequence of resonant electron-plasmon interaction (electron murmuration). In this picture drifting electrons are resonantly scattered by spatial electrostatic energy distribution, characterizing them by the de Broglie’s oscillations. The phase-shift and amplitude of excitations in damped driven pseudoforce system is derived and their variations in terms of normalized chemical potential and electron temperature is studied. In particular we investigate the kinetic correction effect on energy dispersion relation in the electron gas in detail. It is revealed that only the low phase-speed branch of the dispersion curve is significantly affected by the kinetic correction. It is also found that increase in the electron number density leads to increase in effective mass and consequently decrease in electron mobility while the increase in the electron temperature has the converse effect. The kinetic correction also significantly lowers the plasmon conduction band. Current model may be further elaborated to investigate the beam-plasmon interaction and energy exchange in multispecies quantum plasmas.

PACS numbers: 52.30.-q,71.10.Ca, 05.30.-d
I. INTRODUCTION

Plasma oscillations play fundamental role in many physical phenomena associated with collective interaction of charged species. Due to complex nature of electromagnetic interactions between variety of charges in multispecies plasmas a wide spectrum of interesting linear and nonlinear phenomena such as different instabilities, wave-particle interaction, solitons, double layer, shock waves, etc. can exist in these environments. Plasma theories such as kinetic and hydrodynamic models have been developed over the past decade in order to study various interesting collective effects. In plasmas the electron fluid is almost the main ingredient of dielectric response to electromagnetic waves and dynamic structure factor, leading to many interesting phenomena such as Thomson, Compton, stimulated Raman and Brillouin scattering which are used as efficient plasma diagnostic tools. Electrons are also responsible for other important physical properties of plasmas such as charge shielding, ionic bound states, electrical and energy transport phenomena, heat capacity and many others. Dielectric response of electron gas in solid state plasmas plays the prominent role in optical properties of metals, semiconductors, nanometric structures, low dimensional system and liquid crystals. However, in order to understand many of the physical properties of plasmas one has to have detailed information on collective aspects of electron species in these environments. In hydrodynamic plasma theories the electron dynamic is simply coupled to electromagnetic fields through Maxwell equations. Therefore, hydrodynamic and magnetohydrodynamic theories constitute a cost effective and analytic means to investigate a wide range of plasma phenomena in a straightforward manner. These theories, if properly formulated, are also believed to be able to capture many collective aspect such as collisionless or collisional damping previously known to be purely kinetic effects.

Due to evergrowing necessity of developments in miniaturized low-dimensional semiconductor and nanostructured devices for futuristic electronic purposes, a renewed effort has been devoted to discover physical properties of dense quantum electron gas over the past few years. Although the theories of collective quantum electron gas dielectric response has been fully developed more than half a century ago, many interesting properties of quantum electron fluid is yet to be discovered. Many renewed and enhanced quantum kinetic and quantum hydrodynamic theories have emerged over the recent years. Application of these
theories predicts a very insightful future for the science of quantum plasmas and have helped to discover a broad range of new interesting collective phenomena of quantum electron gas unknown up until the recent decade \cite{55, 61, 63, 66, 86}. Application of Schrödinger-Poisson system \cite{67} and coupled pseudoforce method has shown a double-tone wave-particle dynamics of the quantum electron gas in a unique picture \cite{68-74}. We strongly believe that this dual-lengthscale theory of quantum plasmas provides a better picture for wave-particle phenomena in collective environments such as plasmas and fluids. The quantum fluid theory may also provide appropriate answer to some of the most fundamental quantum mechanical questions regarding the matter-waves and mysterious duality nature of particles \cite{75} and nonlocality effect \cite{76}. It has been recently shown that the model of Bohmian mechanics and its relation to the quantum electron diffraction potential appearing in the hydrodynamic formulation is more fundamental than the single-particle Schrödinger wave equation \cite{77, 78}. It was also concluded that the Bohm’s quantum potential makes a basis for the Schrödinger equation instead of being a consequence of this equation. However, the proof of de Broglie’s hypothetical matter-wave relation given in current research is original in the sense that it does not rely on the fundamental equation of the quantum mechanics.

In current research starting from quantum hydrodynamic theory of an electron gas a one dimensional model of pseudoforce for arbitrary degenerate electron beam is developed and the de Broglie’s hypothetical matter-wave equation is mathematically described as a pseudoresonance effect between the particle-like branch of the plasmon excitation in the beam and the electron drift. A generalized dual wavenumber de Broglie equation is obtained which reduces to the original equation the classical dilute electron beam limit. Some interesting spatial matter-wave instability in the beam propagation is introduced for the first time and discussed in terms of the average beam speed and electron screening effect.

II. THE QUANTUM FLUID MODEL

Dynamic properties of a free electron gas can be studied through the hydrodynamic model to a great extent. Quantum hydrodynamic model may be used to study the dynamic properties of electron gas with arbitrary degree of degeneracy. A closed set of conventional
quantum hydrodynamic equations for isothermal electron gas read

\[ \frac{\partial n}{\partial t} + \frac{\partial nv}{\partial x} = 0, \quad (1a) \]
\[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{e}{m} \frac{\partial \phi}{\partial x} - \frac{1}{m} \frac{\partial \mu}{\partial x} + \frac{\gamma \hbar^2}{2m^2} \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{n}} \frac{\partial^2 \sqrt{n}}{\partial x^2} \right), \quad (1b) \]
\[ \frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n - n_0), \quad (1c) \]

where the dependent variables \( n, v, \mu \) and \( \phi \) refer to the number density, average speed, chemical potential and electrostatic potential, respectively. The last term in the momentum equation is due to the Bohm potential which is the origin of nonlocal effects in de Broglie-Bohm pilot wave theory \[76\]. The prefactor \( \gamma \) is the kinetic correction to the Bohm potential the origin of which has been the subject of intense investigations during recent years \[79–83\].

The parametric equation of state (EoS) for isothermal electron gas with arbitrary degree of nonrelativistic degeneracy is given in terms of the Fermi integrals

\[ n(\eta, T) = \frac{2^{7/2} \pi m^{3/2}}{h^3} F_{1/2}(\eta) = -\frac{2^{5/2} (\pi mk_B T)^{3/2}}{h^3} \text{Li}_{3/2}[- \exp(\eta)], \quad (2a) \]
\[ P(\eta, T) = \frac{2^{9/2} \pi m^{3/2}}{3h^3} F_{3/2}(\eta) = -\frac{2^{5/2} (\pi mk_B T)^{3/2}}{h^3} (k_B T) \text{Li}_{5/2}[- \exp(\eta)], \quad (2b) \]

where \( \eta = \beta \mu \) with \( \beta = 1/k_B T \) and \( F_k \) is the Fermi integral of order \( k \)

\[ F_k(\eta) = \int_0^\infty \frac{x^k}{\exp(x - \eta) + 1} dx. \quad (3) \]

The function \( \text{Li}_k \) is called the polylog function defined as

\[ F_k(\eta) = -\Gamma(k + 1) \text{Li}_{k+1}[- \exp(\eta)], \quad (4) \]

in which \( \Gamma \) is the gamma function. Note that the isothermal EoS is not the only choice and for fast processes the adiabatic EoS \[23\] may be employed. However, for illustrative simplified purpose we have chosen to use the isothermal electron EoS in current model. The kinetic correction has been studied in many recent literature \[84–89\] and for electron gas with arbitrary degeneracy at finite temperature may be written as \[89\]

\[ \gamma = \frac{\text{Li}_{3/2}[- \exp(\beta \mu_0)] \text{Li}_{-1/2}[- \exp(\beta \mu_0)]}{3 \text{Li}_{1/2}[- \exp(\beta \mu_0)]^2}, \quad (5) \]

where, \( \mu_0 \) is the equilibrium chemical potential. It is well known that the quantum hydrodynamic model \[1\] can be readily transformed \[67\] to the following Schrödinger-Poisson
system using the Madelung transformations \cite{41}

\[
\begin{align*}
\frac{i\hbar}{\partial t} N & = -\frac{\gamma \hbar^2}{2m} \frac{\partial^2 N}{\partial x^2} - e\phi N + \mu(n,T) N, \\
\frac{\partial^2 \phi}{\partial x^2} & = 4\pi e(|N|^2 - n_0),
\end{align*}
\]

where \( N = \sqrt{n(x,t)} \exp[iS(x,t)/\hbar] \) is the state function characterizing spatiotemporal evolution of the electron gas with \( N N^* = n(x,t) \) being the number density and \( v(x,t) = (1/m)\partial S(x,t)/\partial x \) the electron fluid speed. Also, \( n_0 \) represents the equilibrium electron gas number density. The coupled system (6) can be used to study the propagation of an electron beam of arbitrary degeneracy with arbitrary degree of degeneracy in an equilibrium temperature. Let us consider an arbitrary degenerate electron beam drifting at constant fluid velocity, \( v \). The value of the electron beam chemical potential may vary depending on the degeneracy degree and the temperature in a wide range. However, in a classical regime it can have the negative values up to zero electronvolts and in quantum regime it is positive few electronvolts such as for fully degenerate typical metals. Therefore, the Schrödinger-Poisson system (6) can model a wide range of phenomenon for electron beam excitations from classical regime up to the fully degenerate quantum regime. For our case the beam of electron gas drifting with constant speed \( v \) leads to \( S(x,t) = px + f(t) \) in which \( p = mv \) is the electron fluid beam momentum and \( f(t) \) is an arbitrary function of time. Therefore, the state function becomes \( N(x,t) = \psi(x,t) \exp(i\phi(x)/\hbar) \) where \( \psi(x,t) = \sqrt{n(x)} \exp[i f(t)/\hbar] \). It is interesting that in the limit \( \mu_0 = 0 \) we have \( \gamma = 1/3 \) while in the fully degenerate electron gas limit it has the value \( \gamma = 1/9 \).

To this end, let us now consider separable solution \( N(x,t) = \psi(x)\psi(t) \). After the separation of spatial and temporal parts in first equation (6), we arrive at

\[
\begin{align*}
\frac{\gamma \hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + e\phi(x)\psi(x) - \mu(n,T)\psi(x) & = \epsilon\psi(x) \exp(i\phi(x)/\hbar), \\
\frac{d^2 \phi(x)}{dx^2} & = 4\pi e(|\psi(x)|^2 - n_0),
\end{align*}
\]

where \( \epsilon \) is the energy eigenvalue of the electronic system, \( \psi(x) = \sqrt{n(x)} \) and \( \psi(t) = \exp[i f(t)/\hbar] \). Here we are interested in finding the solution for spatial variations. Therefore, we have the first two coupled equations in (7) to solve. The linear system for small
perturbations is

\[
\frac{\gamma d^2 \Psi(x)}{dx^2} + \Phi(x) + E \Psi(x) = -E \exp(ik_d x),
\]

(8a)

\[
\frac{d^2 \Phi(x)}{dx^2} - \Psi(x) = 0,
\]

(8b)
in which \(k_d = \hbar l_p/p\) is the normalized de Broglie’s wavenumber in which \(l_p = \hbar/\sqrt{2m_eE_p}\) is the plasmon length and \(E_p = \hbar \omega_p\) being the plasmon energy and \(\omega_p = \sqrt{4\pi e^2 n_0/m}\) the electron plasma frequency. Also, \(E = \epsilon/E_p\) with \(\epsilon\) being the energy eigenvalue of the system. Note that in the linearization of dependent quantities we have used \(\psi \simeq \psi_0 + \psi_1\), \(\phi \simeq 0 + \phi_1\) and \(p = 0 + p_1\) We have also used the normalization scheme \(\Psi(x) = \psi(x)/\psi_0\) \((\psi_0 = \sqrt{n_0}\) is the unperturbed value of \(\psi(x)\) and \(\Phi(x) = e\phi/E_p\). The spacial coordinate \(x\) is normalized to the plasmon wavelength \(\lambda_p = 2\pi/k_p\) with \(k_p = \sqrt{2mE_p/\hbar}\) being the characteristic plasmon wavenumber. On the other hand, the time is scaled with respect to inverse plasmon frequency, \(\omega_p = E_p/\hbar\). Note that the solution for \(N(x,t)\) would be quasi-stationary for the beam since \(\psi(x,t)\psi^*(x,t)\) does not explicitly depend on time. Therefore, the spacial distribution of electron density remains constant over the time whereas the electron beam flows with uniform speed.

III. PSEUDORESONANT PLASMON EXCITATIONS

Let us now study the solution to the coupled pseudoforced differential system (8). The general solution to the homogenous system read

\[
\begin{bmatrix}
\Phi_g(x) \\
\Psi_g(x)
\end{bmatrix} = \gamma \alpha \begin{bmatrix}
\Psi_0 + k_2^2 \Phi_0 & -(\Psi_0 + k_1^2 \Phi_0) \\
-(\Phi_0/\gamma + k_1^2 \Psi_0) & \Phi_0/\gamma + k_2^2 \Psi_0
\end{bmatrix} \begin{bmatrix}
\cos(k_1x) \\
\cos(k_2x)
\end{bmatrix},
\]

(9)
in which \(\Phi_0\) and \(\Psi_0\) characterize the initial values where we have assumed \(d\Psi(x)/dx|_{x=0} = d\Phi(x)/dx|_{x=0} = 0\), for simplicity. The characteristic plasmon excitation wavenumbers \(k_1\) and \(k_2\) are given as

\[
k_1 = \sqrt{\frac{E - \alpha}{2\gamma}}, \quad k_2 = \sqrt{\frac{E + \alpha}{2\gamma}}, \quad \alpha = \sqrt{E^2 - 4\gamma}.
\]

(10)

Note that the complementarity relation \(k_1k_2 = 1\) always holds. It can be seen the plasmon excitations satisfy the energy dispersion \(E = (1 + \gamma k^4)/k^2\) with a minimum at \((E_m, k_m) = \ldots\)
The particular solution, on the other hand, can be written in the following simplified form

\[
\Phi_p(x) = \frac{E \left[ \gamma (k_d^2 - k_1^2) \cos (k_2 x) - \gamma (k_d^2 - k_2^2) \cos (k_1 x) - \alpha \cos (k_d x) \right]}{\alpha \gamma (k_d^2 - k_1^2) (k_d^2 - k_2^2)}.
\]  

(11a)

\[
\Psi_p(x) = \frac{E \left[ (1 - \gamma k_d^2 k_1^2) \cos (k_2 x) - (1 - \gamma k_d^2 k_2^2) \cos (k_1 x) + \alpha k_d^2 \cos (k_d x) \right]}{\alpha \gamma (k_d^2 - k_1^2) (k_d^2 - k_2^2)}.
\]  

(11b)

The kinetic correction in our normalization scheme takes the following form

\[
\gamma = \frac{L_i^{3/2} \left[ - \exp(2\sigma/\theta) \right] L_{i-1/2} \left[ - \exp(2\sigma/\theta) \right]}{3L_{i/2} \left[ - \exp(2\sigma/\theta) \right]^2},
\]  

(12)

in which \( \sigma = \mu_0/E_p \), \( \theta = T/T_p \) is the fractional electron to plasmon energy \( T_p = E_p/k_B \).

Figure 1 depicts various parameters involved in plasmon excitations. Figure 1(a) shows the characteristic wavenumber branches of plasmon excitations (wave-like \( k_1 = 1/k^2 \) and particle-like \( k_2 = \gamma k^2 \)) for different values of normalized chemical potential, \( \sigma = \mu_0/E_p \) at given normalized temperature \( \theta = T/T_p \). Note that the plasmon wavenumbers depend on the chemical potential and electron temperature via the kinetic correction, \( \gamma(\sigma, \theta) \). The lower/upper branches correspond to wave/particle excitation in the electron gas. It is clearly remarked that chemical potential has significant effect on particle branch at higher kinetic energy values \( E \). The parameter dependence of kinetic correction factor is depicted in Fig. 1(b). It is shown that increase in the normalized temperature increases the correction while increase in the normalized chemical potential leads to decrease in this correction parameter.

It is seen that in the fully degenerate regime the kinetic correction approaches the value \( \gamma = 1/9 \) \(^{85, 88}\). Moreover, Fig. 1(c) shows the dependence of plasmon energy on chemical potential and temperature of the electron gas. It is remarked that increase in values of normalized chemical potential and temperature lead to increase in the plasmon energy. Note that all these parameter depend on the number density of the electron gas. The number density of electron gas in normalized for is given as

\[
n(\sigma, \theta) = \frac{16e^6m^2\theta^6}{\pi^3h^6} L_{i/2} \left[ \exp \left( \frac{2\sigma}{\theta} \right) \right]^4.
\]  

(13)

The variations of plasmon length in micrometers is depicted in Fig. 1(d). It is seen that the plasmon length at classical limit is few micrometers and decreases rapidly to nanometer scale at degenerate regime. It is also remarked that this characteristic lengthscale decreases significantly at higher temperature only in classical regime.
FIG. 1: (a) Effect of normalized chemical potential $\sigma$ on the characteristic plasmon wavenumbers for given normalized electron temperature $\theta$. (b) Effect of normalized chemical potential $\sigma$ and normalized electron temperature $\theta$ on the kinetic correction factor $\gamma$. (c) Variations in plasmon energy of electron gas in terms of the normalized chemical potential $\sigma$ for given normalized electron temperature $\theta$. (d) Variations in plasmon length of electron gas in terms of the normalized chemical potential $\sigma$ for given normalized electron temperature $\theta$. The increase in thickness of curves indicate increase in the varied parameter value above each panel.
The solution (11) becomes resonant solution at electron drift conditions \( k_d = \frac{\hbar}{p} = \frac{k_1}{k_p} \) and \( k_d = \frac{\hbar}{p} = \frac{k_2}{k_p} \), in dimensional units. Note that the later condition reduces to de Broglie’s hypothetical assumption, hence, the particle branch of plasmon excitation becomes resonant at condition \( p = \hbar k_2 \). In the single-particle limit or at elevated kinetic energy values \( E \gg 1 \) the particle-like branch of plasmon excitations becomes identical to the free electron branch and we obtain the relation \( p = \hbar k \). Note also that in such a limit the wave-like branch vanishes according to Eqs. (10). However, current plasmon theory introduces the second de Broglie wavenumber \( k_1 \) to the electron gas particles at high wavelength limit.

Due to the gradient correction to kinetic energy of the electron gas the principal de Broglie’s wavenumber is affected strongly by the electron gas density and temperature. However, the second or wave-like branch is not affected by these parameters. These effects may be confirmed by different experiments revealing the wave and particle aspects of a dense electron beam. It can be seen that the de Broglie’s wavenumbers depend on chemical potential and temperature of the electron gas via the kinetic correction factor. However, the dependence on the chemical potential and temperature in fully degenerate regime is insignificant according to Fig 1(b).

Figure 2(a) and 2(b) show the electrostatic energy and wavefunction profiles for the driven plasmon excitations in drifting electron gas with given normalized chemical potential, \( \sigma = \mu_0/E_p \), and electron temperature, \( \theta = T/T_p \), values. The oscillations are evidently triple mode consisting of dual plasmon mode in addition to the de Broglie, s mode caused by the electron drift. The de Broglie parameter \( k_d = v/v_p \) is the normalized electron drift speed to that of the plasmon, \( v_p = \hbar k_p/m_e \). Note that \( \Psi(x)\Psi^*(x) \) gives the normalized electron number density distribution along the electron beam. Figure 2(c) shows the stability region for plasmon excitations of arbitrary degenerate electron gas as dashed areas for given normalized parameter values and for two different electron temperatures. The excitations are stable for energy eigenvalues, \( E > 2\sqrt{\gamma} \). It is remarked that the increase in chemical potential gives rise to the stability of excitations, whereas, the increase in the temperature leads to destabilization effect. Finally, Fig. 2(d) depicts the dual pseudoresonance effect due to de Broglie’s wavenumber matching \( k_d = k_1, k_2 \), which is the plot of amplitude of plasmon excitations driven by electron drift in terms of normalized drift speed for given other normalized parameters. Therefore, for any given energy eigenvalue, \( E \), the wavenumbers \( k_1 \) and \( k_2 \) given by dispersion relation \( E(\sigma, \theta) = k_1^2 + k_2^2 \), are in fact the normal stationary
FIG. 2: (a) The electrostatic energy distribution for given values of energy eigenvalue, chemical potential, temperature and de Broglie wavenumber. (b) The wavefunction profile for given values of energy eigenvalue, chemical potential, temperature and de Broglie wavenumber. (c) Effect of normalized electron temperature on the plasmon excitation stability in arbitrary degenerate electron gas. (d) Amplitude of driven plasmon excitations with dual resonance in the absence of plasmon damping.
modes of plasmon excitations in the electron gas. In other words, an existing stationary plasmon mode can be excited in the drifting electron gas by the de Broglie’s wavenumber matching condition. Note also that for \( k_d = v/v_p = \gamma^{1/4} \) de Broglie’s condition coincides with the quantum beating condition for which \( k_1 = k_2 \).

The hypothetical wave nature of electrons proposed by de Broglie [90], which was soon confirmed by many experiments, has been one of the most intriguing phenomena for the life of quantum theory. The problem is the key to understanding of the theory which has been called not well-understood by the Nobel laureate Richard Feynman: "I think I can safely say that nobody really understands quantum mechanics". The quantum nonlocality effects caused by the wave-particle phenomenon has led to some unphysical interpretations of quantum phenomena [75] such as the double-slit experiments and quantum measurements. There has been early attempts to survive the deterministic nature of particle dynamics in the quantum theory such as the proposal of de Broglie-Bohm pilot wave theory [76]. However, the later theory does not actually provide a physical description of the pilot wave origin. Current hydrodynamic model of plasmon excitation provides a clear picture of the phenomenon. The dual lengthscale plasmonic excitations in an electron gas are caused by collective effects due to the electrostatic interactions, we may call it the electron murmuration phenomenon. On the other hand, it was shown that the de Broglie’s wave-particle relation is a direct consequence of resonant electron-plasmon interaction in the driven pseudoforce system [8]. Therefore, it is seen that electrons are piloted by electron murmuration wave which is produced by their electrostatic interactions. However, the murmuration phenomenon is not the actual cause of quantum wave. The quantum de Broglie wave is caused by the resonant interaction of electron drift with the collective plasmon excitations (the electron murmurations).

IV. TRANSIENT AND PERSISTENT PLASMON EXCITATIONS

We will now consider the pseudodamped system in which the spacial damping of the wavefunction and electrostatic energy may have various origins. One of the cases is the screening effect around the positively charged ions in the gas. Such screening is due to local changes in the chemical potential of the gas which characteristic normalized wavenumber 

\[
\xi(\sigma, \theta) = (E_p/2)\partial n/\partial \mu = (1/2\theta)Li_{1/2}[-\exp(2\sigma/\theta)]/Li_{3/2}[-\exp(2\sigma/\theta)].
\]

In a pure electron
gas the plasmon damping may also occur due to various channels such as the electron-neutral collisions and Landau damping. Therefore, the appropriate model for plasmon excitations in electron gas with screening effect is as follows

\[
\frac{d^2 \Psi(x)}{dx^2} + 2\gamma \xi \frac{d\Psi(x)}{dx} + \Phi(x) + E\Psi(x) = -E \cos(k_d x), \tag{14a}
\]

\[
\frac{d^2 \Phi(x)}{dx^2} + 2\xi \frac{d\Phi(x)}{dx} - \Psi(x) = 0, \tag{14b}
\]

in which \(\xi\) is the normalized pseudodamping parameter

\[
\Phi_{gd}(x) = \frac{\gamma e^{-\xi x}}{\alpha} \left\{ \begin{array}{l}
(\kappa_2^2 \Phi_0 + \Psi_0) \left[ \cos(\beta_1 x) + \frac{\xi}{\beta_1} \sin(\beta_1 x) \right] - \\
(\kappa_1^2 \Phi_0 + \Psi_0) \left[ \cos(\beta_2 x) + \frac{\xi}{\beta_2} \sin(\beta_2 x) \right] \end{array} \right\},
\]

\[
\Psi_{gd}(x) = \frac{\gamma e^{-\xi x}}{\alpha} \left\{ \begin{array}{l}
(\Phi_0/\gamma + k_2^2 \Psi_0) \left[ \cos(\beta_2 x) + \frac{\xi}{\beta_2} \sin(\beta_2 x) \right] - \\
(\Phi_0/\gamma + k_1^2 \Psi_0) \left[ \cos(\beta_1 x) + \frac{\xi}{\beta_1} \sin(\beta_1 x) \right] \end{array} \right\},
\]

where \(\beta_1 = \sqrt{k_1^2 + \xi^2}\), \(\beta_2 = \sqrt{k_2^2 + \xi^2}\) are the characteristic pseudodamped plasmon wavenumbers. The pseudodamped plasmon excitations follow the energy dispersion \(E = [1 + \gamma(k^2 + \xi^2)]/2(k^2 + \xi^2)\). It is evident that the general solution (15) is transient and does not extend to long ranges. The particular solution, on the other hand, reads

\[
\Phi(x) = \frac{Ee^{-\xi x}}{\alpha \eta_1 \eta_2} \left[ \eta_1 (k_d^2 - k_1^2) \cos(\beta_2 x) - \eta_2 (k_d^2 - k_2^2) \cos(\beta_1 x) \right] - \tag{16a}
\]

\[
\frac{Ee^{-\xi x}}{\alpha \eta_1 \eta_2} \left[ \xi \eta_1 (k_d^2 + k_1^2) \frac{\sin(\beta_2 x)}{\beta_2} - \xi \eta_2 (k_d^2 + k_1^2) \frac{\sin(\beta_1 x)}{\beta_1} \right] - \frac{E}{\gamma \eta_1 \eta_2} \times \tag{16b}
\]

\[
\left\{ [ (k_d^2 - k_1^2) (k_d^2 - k_2^2) - 4k_d^2 \xi^2 ] \cos(k_d x) + 2k_d \xi (k_1^2 + k_2^2 - 2k_d^2) \sin(k_d x) \right\},
\]

\[
\Psi(x) = -\frac{Ee^{-\xi x}}{\alpha \eta_1 \eta_2} \left[ \eta_1 k_2^2 (k_d^2 - k_2^2) \cos(\beta_2 x) - \eta_2 k_1^2 (k_d^2 - k_1^2) \cos(\beta_1 x) \right] + \tag{16d}
\]

\[
\frac{Ee^{-\xi x}}{\alpha \eta_1 \eta_2} \left[ \xi \eta_1 k_2^2 (k_d^2 + k_2^2) \frac{\sin(\beta_2 x)}{\beta_2} - \xi \eta_2 k_1^2 (k_d^2 + k_1^2) \frac{\sin(\beta_1 x)}{\beta_1} \right] + \frac{E}{\gamma \eta_1 \eta_2} \times \tag{16e}
\]

\[
\left\{ [1 - (k_1^2 + k_2^2 - k_d^2) (k_1^2 + 4\xi^2) ] k_d^2 \cos(k_d x) + 2k_d \xi (1 + k_1^4 - 4\xi^2 k_d^2) \sin(k_d x) \right\},
\]

where \(\eta_1 = (k_d^2 - k_1^2)^2 + 4k_d^2 \xi^2\) and \(\eta_2 = (k_d^2 - k_2^2)^2 + 4k_d^2 \xi^2\). The persistent solution may be written in the forms \(\Phi(x) = A \cos(k_d x - \phi)\) and \(\Psi(x) = -A \cos(k_d x - \psi)\) with amplitude and phase given as \(A = E/\gamma \eta_1 \eta_2\) and

\[
\tan \phi = \left[ \frac{2\xi k_d (k_1^2 + k_2^2 - 2k_d^2)}{(k_1^2 - k_d^2)(k_2^2 - k_d^2) - 4k_d^2 \xi^2} \right], \quad \tan \psi = \left[ \frac{2(\xi/k_d) (1 + k_1^4 - 4\xi^2 k_d^2)}{1 - (k_1^2 + k_2^2 - k_d^2)(k_1^2 + 4\xi^2)} \right]. \tag{17}
\]

Note that the excitation amplitude and phases depend on the chemical potential and the temperature of the electron gas via the kinetic correction and damping parameters. It is
also remarked that the presence of damping leads to a phase shift between the electrostatic energy and density distribution in the system.

Figure 3 shows the amplitude of damped plasmon excitations \( \Omega \) for various normalized parameter values in terms of normalized electron drift. Figures 3(a) and 3(b) show that two set of resonant amplitude peaks occur for given parameters. The long wavelength resonance in Fig. 3(a) are due to de Broglie’s matching condition at wave-like branch while small wavelength ones in Fig. 3(b) are due to de Broglie’s matching condition at particle-like branch in dispersion relation. It is revealed that increase in the energy eigenvalue of excitations caused the amplitude height to decrease and move to smaller wavenumbers for wave-like resonances, whereas, it causes the amplitude to move to higher wavenumbers for particle-like resonances. It is noted that the wave-like resonance amplitudes are relatively higher than that of particle-like ones. Figure 3(c) shows that increases in the normalized chemical potential of electron gas leads the particle-like resonance peak to sharply decrease and shift to higher wavenumber values. However, the change in chemical potential and electron temperature, which have not been shown, does not affect the wave-like resonance peaks. Figure 3(d) shows that the effect of electron temperature variation on the particle-like resonance peak is insignificant but opposite to the case of chemical potential changes.

Figure 4 Depicts the electrostatic energy and wavefunction profiles of persistent solution of resonant plasmon excitations for given parameters. The presence of pseudodamping has led to filtration of other excitations than the forcing oscillations. Therefore, resonant plasmon excitations became single-tone with de Broglie wavelength distribution. In is remarkable that a drifting single-momentum electron gas leads to a uniform periodic distribution of electron density over the space wavelength of which is give by \( \lambda = h/p \). It is also seen that there is phase difference in oscillations between these two profiles which caused by the damping (screening) effect. Figures 4(c) and 4(d) show the phase variations in the electrostatic energy in terms of the de Broglie wavenumber. Effects of electron temperature and chemical potential is shown respectively in Figs. 4(c) and 4(d). These variations have the pronounced effect around the particle-like resonance and clearly have the characteristic features of dual resonant wave phenomena. Note that when \( \phi = 0 \) the electric field spacial distribution becomes in-phase with driver (electron flow). This parameter is therefore a key measure for the electron beam-plasmon interaction quality. In classical resonance phenomenon this is the point for which the maximum power in delivered to the system.
FIG. 3: (a) Amplitude of high phase-speed damped driven plasmon excitations in terms of nor-
malized electron drift speed for different energy eigenvalues. (b) Amplitude of low phase-speed
damped driven plasmon excitations in terms of normalized electron drift speed for different energy
eigenvalues. (c) Amplitude of low phase-speed damped driven plasmon excitations in terms of
normalized electron drift speed for different normalized chemical potential. (d) Amplitude of low
phase-speed damped driven plasmon excitations in terms of normalized electron drift speed for
different normalized electron temperature. The increase in thickness of curves indicate increase in
the varied parameter value above each panel.
FIG. 4: (a) The electrostatic energy distribution for given values of energy eigenvalue, chemical potential, temperature, de Broglie wavenumber and damping parameter. (b) The wavefuntion profile for given values of energy eigenvalue, chemical potential, temperature, de Broglie wavenumber and damping parameter. (c) Variation of the electrostatc energy distribution phase shift with respect to the driving pseudoforce for different values of the normalized electron temperature. (d) Variation of the electrostatc energy distribution phase shift with respect to the driving pseudoforce for different values of the normalized chemical potential. The increase in thickness of curves indicate increase in the varied parameter value above each panel.
To this end, we study the effects of kinetic correction and electron gas parameters on the energy dispersion of plasmon excitations. Figure 5(a) depicts the energy dispersion in the presence (solid curve) and the absence (dashed curve) of kinetic correction for given chemical potential and electron temperature parameter values. The effect of kinetic correction is huge on the low phase-speed particle-like branch (which we call it the kinetic regime) with almost no effect on the high phase-speed wave-like branch which are separated at the quantum beating point, $k = 2\sqrt{\gamma}$. The effect of kinetic correction is evidently increases sharply as the energy eigenvalue increases. The group speed of low phase-speed branch has significantly decreased due to the kinetic correction. This feature has been previously discussed in thermodynamic analysis of one component quantum plasmas [85, 89] where the kinetic correction only applies to low phase-speed plasma phenomenon. It is remarkable that the kinetic correction profoundly lowers the energy minimum (plasmon conduction levels) of excitations accessible to Fermi electrons. Figure 5(b) reveals the effect of normalized chemical potential of electron gas on the low phase-speed dispersion branch indicating that increase of this parameter leads to lower group-speed in this branch. Note that the second derivative of energy is a measure of effective mass and mobility of electrons at a given wavenumber at plasmon level. It is therefore concluded that increase in the chemical potential leads to increase in the effective mass of electrons and decrease in their mobility. Moreover, Fig. 5(c) shows the effect of electron temperature on the dispersion curve. It is evident that increase in the electron gas temperature increases the electron mobility in contrast to the case of chemical potential increase. Finally, Fig. 5(d) shows the effect of chemical potential on dispersion curve of damped plasmons. The similar effect as in Fig. 5(b) is also apparent in this case. However, the presence of strong dampind (charge screening) leads to truncation of wave-like branch.

V. CONCLUSION

We studied the resonant interaction between drifting electrons and plasmon excitations in an electron gas of arbitrary degenerate electron gas using the effective kinetic corrected Schrödinger-Poisson system. It was found that the kinetic correction leads to significant effect of electron number density and temperature on the plasmon excitations. We gave a physical description of hypothetical de Broglie wave-particle theory in terms of resonant
drifting electron iteration by the collective distribution of electrostatic charge in space. We also found the solution of the damped driven pseudoforce system in the presence of kinetic correction and calculated the amplitude and phase shifts of electrostatic energy and wavefunction relative to that of the driver pseudoforce. The kinetic correction was found to profoundly affect the dispersion curve in the low phase-speed (small wavelength) regime which we call it the kinetic regime. It was further revealed that electron density leads to increase in effective electron mass and decrease in its mobility in plasmon conduction band while the increase in the temperature has the inverse effect. Current mode of drifting quantum electron gas can be used to investigate various other phenomena such as the beam-plasma interactions.

VI. DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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FIG. 5: (a) The plasmon dispersion in the absence (dashed) and presence (solid) of the kinetic correction. (b) The effect of normalized electron chemical potential on the plasmon dispersion curve. (c) The effect of normalized electron temperature on the plasmon dispersion curve. (d) The effect of normalized electron chemical potential on the damped plasmon dispersion curve. The increase in thickness of curves indicate increase in the varied parameter value above each panel.