Wormhole and Hawking Radiation

M.Hotta and M.Yoshimura
Department of Physics, Tohoku University
Sendai 980 Japan

Abstract

It is shown in a variant of two dimensional dilaton gravity theories that an arbitrary, localized massive source put in an initially regular spacetime gives rise to formation of the wormhole classically, without accompanying the curvature singularity. The semiclassical quantum correction under this wormhole spacetime yields Hawking radiation. It is expected, with the quantum back reaction added to the classical equation, that the information loss paradox may be resolved in this model.

---

*Work supported in part by the Grant-in-Aid for Science Research from the Ministry of Education, Science and Culture of Japan No. 0
†E-mail address: YOSHIM at JPNTUV0.M. BITNET.
The end point of the black hole evaporation poses a challenging conceptual problem that must be solved before one embarks on a full-fledged theory of quantum gravity. If the black hole is an inevitable consequence of the gravitational collapse, then one appears to have no alternative choice but to accept the thermal spectrum of Hawking radiation [1]. The question is then: does a quantum mechanical pure state develop into a mixed thermal state? To answer this question in a definitive way, one presumably has to deal with the problem of quantum back reaction to the background spacetime, in this case, caused by Hawking radiation. Some interesting toy model in two dimensions has been proposed recently to discuss this issue at depth, however with inconclusive results so far with regard to the end point of the black hole evaporation [2], [3].

Among others, an interesting alternative resolution to the clash between gravity and quantum mechanics has been suggested in Ref. [4] and Ref. [5], which take the view that the apparent loss of information in the Hawking radiation is actually recovered by information flow into a disconnected world in the interior of the black hole inaccessible to this world. If this is the solution, the curvature singularity inside the black hole, which normally exists, must be replaced by something like a wormhole. Whether and how this might be realized is not known to the best of our knowledge.

In this paper we would like to provide such a mechanism related to the wormhole formation and its subsequent evaporation in a concrete two dimensional model. The model is a variant of two dimensional dilaton gravity theories considered by Callan, Giddings, Harvey, and Strominger (CGHS) [2]. A unique classical solution without singularity is found for an arbitrary localized massive source, and is shown to yield the wormhole geometry (Lorentzian) with global event horizon. Moreover, in the large N semiclassical limit the wormhole background gives rise to Hawking radiation. Although we still lack a quantitative semiclassical analysis with the back reaction included in the form of the usual Polyakov term, it is guaranteed in this model that time evolution does not lead to the curvature singularity even when quantum back reaction is incorporated. Thus either of the two possibilities with regard to the end point of the wormhole evaporation exists; a complete disappearance leaving behind no horizon, or evolution to a still unknown quantum remnant of the wormhole structure remaining behind event horizon.
We consider the two dimensional dilaton gravity defined by the classical action,

\[ S_r = \frac{1}{2\pi} \int \! d^2 x \sqrt{-g} \left[ e^{-2\varphi} \left( -R - 4 \partial_\mu \varphi \partial^\mu \varphi + 4\lambda^2 \right) + L^{(m)} \right]. \quad (1) \]

This model was called the reversed CGHS model \cite{6}, because it differs from the original CGHS model \cite{2}, in the two signs of the curvature and the dilaton kinetic terms. Despite the sign reversal this model describes a reasonable toy model of gravity in the weak field limit \cite{7}. The cosmological constant \( \lambda^2 \) sets a length scale in the theory. A novel feature of the dilaton gravity is that the coupling factor \( e^{2\varphi} \) acts as a varying gravitational constant, as in the Brans-Dicke theory in four dimensions. For the matter part \( L^{(m)} \) we take a massive field as the source of the gravity force.

The field equation in the conformal gauge, \( ds^2 = -e^{2\varphi} dx^+ dx^- \) with \( x^\pm = x^0 \pm x^1 \), is written as

\[ \partial_+ \partial_- e^{-2\varphi} - \lambda^2 e^{2\rho - 2\varphi} = \pi T_+ , \]
\[ \partial_+ \partial_- (\rho - \varphi) = -\frac{\pi}{2} e^{2\varphi} T_+ , \]
\[ \partial_\pm^2 e^{-2\varphi} - 2 \partial_\pm (\rho - \varphi) \partial_\pm e^{-2\varphi} = -\pi T_{\pm\pm} . \quad (2) \]

The stress tensor \( T_{\mu\nu} \) for the general localized massive source consistent with the conservation is given by

\[ T_{++} = T_{--} = T_{+-} = -\frac{M}{4\pi} e^\rho \delta (x^1) , \quad (5) \]

with \( M = \pi m \), \( m \) a point source mass \cite{7}. The massive source is located at \( x^1 = 0 \).

Exact classical solution, along with the scalar curvature \( R \), is then given by

\[ e^{-2\varphi} = e^{-2\lambda x^0 \left( A - \frac{M}{2\lambda B} \sqrt{1 + Be^{2\lambda(x^0 - |x^1|)} - 1} \right) + AB} , \quad (6) \]
\[ e^{2\rho} = e^{2\varphi} e^{-2\lambda x^0 \left( A - \frac{M}{2\lambda B} \sqrt{1 + Be^{2\lambda(x^0 - |x^1|)} - 1} \right)} , \quad (7) \]
\[ R = \frac{2Me^{2\lambda x^0}}{A \sqrt{1 + Be^{2\lambda(x^0 - |x^1|)}} - \frac{M}{2\lambda B} \delta (x^1) - \frac{4\lambda^2 AB e^{2\lambda x^0}}{A \sqrt{1 + Be^{2\lambda(x^0 - |x^1|)}} + AB e^{2\lambda x^0}} . \]

The parameters, \( A \) and \( B \), are integration constants. This set of solutions exhausts all classical solutions with the reflection symmetry under \( x^1 \to -x^1 \) when the localized source is given by Eq.\( 5 \), as will be shown in a subsequent paper \cite{8}. The divergent curvature at the source \( \propto \delta (x^1) \) should be regarded as an artifact of the localized
matter distribution. With an extended source this component of the curvature is expected to be smeared out.

To uncover the spacetime structure of these classical solutions, it is important to extend the coordinate maximally. For this purpose, we introduce a new coordinate system according to \( \tilde{x}^\pm = -\frac{1}{\lambda} e^{-\lambda x^\pm} \), for \( x^1 > 0 \). To obtain the form of solution in \( x^1 < 0 \), we replace the coordinates by the rule, \( \tilde{x}^+ \leftrightarrow \tilde{x}^- \). The range of the new coordinates, \( \tilde{x}^\pm \), is then extended to the whole real axis.

Unless \( A = 0 \), all these solutions have both time-like curvature and dilaton singularity at the same location,

\[
\tilde{x}^+ = -\frac{AB}{\lambda^2 A\tilde{x}^- + \frac{M}{2B} \sqrt{(\lambda \tilde{x}^-)^2 + B}},
\]

for \( \tilde{x}^+ < \tilde{x}^- \), and at a similar location in the other region, \( \tilde{x}^+ > \tilde{x}^- \). We reject these singular solutions as a model of the gravitational collapse, since one cannot set up a regular boundary condition at the null past infinity for these solutions.

When \( A = 0 \), two types of solutions with different signs of \( B \) exist, but are related to each other by a coordinate transformation. By a suitable choice of the origin of time coordinate \( x^0 \), one may set \( B = -\frac{M}{2\lambda} \) and gets the unique solution without singularity,

\[
e^{-2\varphi} = e^{-2\lambda x^0} \sqrt{1 - \frac{M}{2\lambda} e^{2\lambda(x^0 - |x^1|)}},
\]

\[
e^{2\rho} = \frac{1}{1 - \frac{M}{2\lambda} e^{2\lambda(x^0 - |x^1|)}},
\]

\[
R = 2Me^{2\lambda x^0} \delta(x^1).
\]

It is not necessary to extend the coordinate patch in this case.

The spacetime described by this metric is everywhere flat, except at the location of the source. To see this and detailed spacetime structure more clearly, it is useful to introduce the asymptotically flat coordinate with \( ds^2 = -d\sigma^+ d\sigma^- \). First in the region of \( x^1 > 0 \), \( \rho = \rho(x^-) \), and the equation, \( e^{2\rho} dx^- = d\sigma^- \), leads to

\[
\sigma^+ = x^+, \quad \sigma^- = x^- - \frac{1}{2\lambda} \ln(1 - \frac{M}{2\lambda} e^{2\lambda x^-}).
\]

The range of the new coordinates \( \sigma^\pm \) is bounded by \( x^+(\sigma^+) > x^-(\sigma^-) \), while the old coordinate is limited by \(-\infty < x^- < x_H \) with

\[
x_H = -\frac{1}{2\lambda} \ln \frac{M}{2\lambda}.
\]
The inversion of the coordinate is given by

\[ x^+ = \sigma^+, \quad x^- = -\frac{1}{2\lambda} \ln\left(\frac{M}{2\lambda} + e^{-2\lambda\sigma^-}\right). \]  

(15)

In this new coordinate \( e^{-2\varphi} = e^{-2\lambda\sigma^0} \), clearly indicating the linear dilaton vacuum. On the other hand, the asymptotically flat coordinate for \( x^1 < 0 \) is given by

\[ \chi^- = x^-, \quad \chi^+ = x^+ - \frac{1}{2\lambda} \ln\left(1 - \frac{M}{2\lambda} e^{2\lambda x^+}\right), \]  

(16)

again with \(-\infty < x^+ < x_H\).

The spacetime geometry described by this metric represents the wormhole structure. First, note that the trajectory of the source in terms of the flat coordinate is the boundary of the region, \( x^1 > 0 \),

\[ \sigma^- = \sigma^+ - \frac{1}{2\lambda} \ln\left(1 - \frac{M}{2\lambda} e^{2\lambda\sigma^+}\right). \]  

(17)

This trajectory approaches asymptotically the null line \( \sigma^+ = x_H \). Thus to an observer far away from the source sitting at rest in the Minkowski coordinate system, the source appears to move with acceleration. A similar equation of the source trajectory in the left region may be written in terms of \( \chi^\pm \) coordinates, with the null asymptote at \( \chi^- = x_H \). Since the worldline of the source is unique, the two forms of the trajectory must be identified. The two coordinate patches, \((\sigma^+, \sigma^-)\) and \((\chi^+, \chi^-)\), are thus joined at the source.

The Penrose diagram of this spacetime is depicted in Fig.1. The world line of the localized source is designated by the solid arrow at the center. The spacetime is bounded by the lines, \( x^\pm = \pm\infty \) and \( x^\pm = x_H \). These boundary lines are uniquely characterized by the equation, \( (\nabla e^{-2\varphi})^2 = 0 \), and form the dilaton singularity, mostly harmless in much the same way as the asymptotic region of the linear dilaton vacuum. Two portions of the spacetime, \( R \) and \( L \), are disconnected causally, and are separated by the two global event horizons, \( x^\pm = x_H \). For instance, an observer in the region \( R \) can influence only within the forward light cone entirely confined in \( R \), and he cannot receive any information from a person in the region \( L \). It should be clear then that this spacetime has a wormhole structure with the global event horizon, furthermore being everywhere flat, except being curved at the source.

We now consider the quantum effect under this wormhole background. One loop path integral due to \( N \) massless fields yields the well known trace anomaly. Combined with the energy-momentum conservation, it determines the form of the
stress tensor components up to the two unknown functions, \( t_{\pm}(x^{\pm}) \), which must be fixed by the boundary condition of the problem at hand \([3], [4]\). The relevant stress tensor corresponding to the right moving flux at \( I_{-}^{L} \) is

\[
\langle T_{x^{-},x^{-}} \rangle = -\frac{N}{12\pi} \left( \partial_{-}^{2} \rho - (\partial_{-} \rho)^{2} + t_{-}(x^{-}) \right).
\] (18)

The boundary condition appropriate to the gravitational collapse is that no incoming flux exists at the past null infinity, \( x^{+} = -\infty \), which implies \( t_{-}(x^{-}) = 0 \) since \( \rho = O[e^{2\lambda x^{+}}] \).

The energy-momentum tensor in the right half of the spacetime \( R \) is then expressed in terms of the flat \( (\sigma^{+}, \sigma^{-}) \) coordinate as

\[
\langle T_{\sigma^{-},\sigma^{-}} \rangle = \left[ \frac{\partial x^{-}}{\partial \sigma^{-}} \right]^{2} \langle T_{x^{-},x^{-}} \rangle = -\frac{N\lambda^{2}}{12\pi} \frac{1 + \frac{4\lambda}{M}e^{-2\lambda\sigma^{-}}}{(1 + \frac{2\lambda}{M}e^{-2\lambda\sigma^{-}})^{2}}.
\] (19)

Near the horizon, \( \sigma^{-} \to \infty \), and

\[
\langle T_{\sigma^{-},\sigma^{-}} \rangle \sim -\frac{N\lambda^{2}}{12\pi}.
\] (20)

Combined with \( \langle T_{\sigma^{+},\sigma^{-}} \rangle = \langle T_{\sigma^{-},\sigma^{+}} \rangle = 0 \), this can be understood as a thermal flux with the temperature of \( T = \frac{\lambda}{\pi} \); \( \rho_{\text{thermal}} = \frac{N\pi}{12}T^{2} \). The precise form of the thermal spectrum for the right moving emitted particles may be derived by the standard method of Bogoliubov transformation, as in Ref.[1]. The particle emission thus derived is an indication of evaporation of the classical wormhole.

To fully incorporate the back reaction in the semiclassical approximation, it is imperative to analyze the semiclassical equations with the one loop quantum correction added to the classical equations. It is difficult to analyze this set of equations by analytic methods for the Polyakov one loop effective action, \( \sqrt{-g} R \frac{1}{16\pi} R \propto \rho \partial_{+} \partial_{-} \rho \), but as a matter of principle there should be no problem to follow the spacetime evolution. In particular, our reversed model does not give rise to any curvature singularity even when the quantum effect is included, as shown in Ref.[3] by looking into the coefficient matrix of the kinetic terms.

It is then reasonable to expect either of the following two possibilities on the end point of Hawking radiation to occur. In one case the classical spacetime bounded by the dilaton singularity is modified only at the quantitative level by the quantum effect, and the wormhole geometry is unchanged leaving behind the event horizon. Since we have shown the Hawking radiation to be independent of the mass \( M \) of
an arbitrary classical wormhole, it is likely that the end point of the evaporation in
this case is a still unknown quantum remnant of the wormhole geometry. The infor-
mation loss paradox is then resolved by the leak of information to the disconnected
world inaccessible to us; the principle of quantum mechanics is unchanged, but the
usual rule of quantum mechanics restricted to the one side of the spacetime must be
modified. In another case the wormhole is completely melted by the quantum back
reaction, and the final spacetime is a flat linear dilaton vacuum. In this case the
thermal nature of Hawking radiation is superficial or may even not exist for a very
small mass hole, and quantum mechanical correlation is retained as a whole.

In summary, we demonstrated in a variant of two dimensional dilaton gravity the-
ories that the classical end point of the collapse due to an arbitrary localized massive
body is the wormhole. The standard one loop quantum effect induces Hawking radi-
ation under the wormhole background. With the quantum correction included into
the semiclassical set of equations, it is expected that the information loss paradox
at the end point of the gravitational collapse may be resolved in this model.
References

[1] S.W.Hawking, Commun.Math.Phys. 43, 199(1975).

[2] C.G.Callan, S.B.Giddings, J.A.Harvey and A.Strominger, Phys.Rev. D45, 1005(1992).

[3] J.G.Russo, L.Susskind and L.Thorlacius, Phys.Lett.292B,13
   T.Banks, A.Dabholkar, M.R.Douglas and M.O'Loughlin, Phys.Rev.D45, 3607(199
   S.W.Hawking, Phys.Rev.Lett. 69, 406(1992);
   B.Birnir, S.B.Giddings, J.A.Harvey, and A.Strominger, Phys.Rev.D46, 638(19
   L.Susskind and L.Thorlacius, Nucl.Phys. B382, 123(1992).

[4] F.Dyson, unpublished (1976).

[5] S.W.Hawking, Phys.Rev. D37, 904(1988); K.Sato, H.Kodama, M.Sasaki, and
   K.Maeda, Phys.Lett. 108B, 103(1982); and references therein.
   For a recent review, J.Preskill, Do Black Holes Destroy Information?, Caltech
   preprint, CALT-68

[6] M.Yoshimura, Phys. Rev.D47, 5389(1993).

[7] M.Hotta and M.Yoshimura, Classical and Quantum Aspects of Dilaton Force in
   Two Dimensions, Tohoku University preprint, TU/93/437, June 1993.

[8] M.Hotta and M.Yoshimura, in preparation.

[9] S.M.Christensen and S.A.Fulling, Phys.Rev.D15, 2088(1977).
Figure Caption

Figure 1

The Penrose diagram of the classical wormhole spacetime. The world line of the localized source is designated by the solid arrow at the center. The global event horizon at $x^\pm = x_H$ separates the right and the left regions $R$ and $L$ causally.