Jets at Hadron Colliders at Order $\alpha_s^3$: A Look Inside

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Abstract

Results from the study of hadronic jets in hadron-hadron collisions at order $\alpha_s^3$ in perturbation theory are presented. The focus is on various features of the internal structure of jets. The numerical results of the calculation are compared with data where possible and exhibit reasonable agreement.

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Recent advances, both theoretical\cite{1,2} and experimental\cite{3}, in the study of jet production in hadron collisions have made possible detailed comparisons of theory with experiment. In the Standard Model our general understanding of the high energy collisions of hadrons suggests that jets arise when short distance, large momentum transfer interactions generate partons (quarks and gluons) that are widely separated in momentum space just after the hard collision. In a fashion that is not yet quantitatively understood in detail these configurations are thought to evolve into hadronic final states exhibiting collimated sprays of hadrons, which are called jets. These jets are then the observable signals of the short distance parton configurations.

This general qualitative picture is characteristic of both perturbative QCD and the data. When one proceeds to a quantitative confrontation of theory and experiment, a precise definition of a jet must be supplied, and measured jet cross sections depend on the definition used. For instance, when one defines a jet as consisting of all the particles whose momenta lie inside a cone of radius $R$, then measured jet cross sections depend on $R$. On the theoretical side, in an order $\alpha_3^s$ calculation a jet can consist of two partons instead of just one. At this level, then, a jet can have internal structure and jet cross sections calculated at order $\alpha_3^s$ will depend on the jet definition applied. Two questions arise. First, how well does the dependence on the jet definition exhibited by the theoretical jet cross section match that of the experimental jet definition? Second, how well does the internal structure calculated at order $\alpha_3^s$ compare to the internal structure of experimentally observed jets? We address these questions in this Letter.

We consider the inclusive single jet cross section in $p\bar{p}$ collisions. This cross section is a function of the physical variables $s$, the total energy, $E_T$, the transverse energy of the jet ($E_T = E \sin \theta$), and $\eta$, the pseudorapidity of the jet ($\eta = \ln \cot(\theta/2)$) and, as suggested above, the definition of the jet. The theoretical inclusive jet cross section also depends on the unphysical renormalization/factorization scale $\mu$ and on the specific choice of parton distribution functions, $f_{a/A}(x, \mu)$. In a Born level ($\alpha_2^s$) calculation the dependence on the scale $\mu$ arises from both the parton distribution functions and the parton scattering cross section through the dependence of the latter on the strong coupling constant $\alpha_s(\mu)$. At order $\alpha_3^s$ explicit factors of $\ln(\mu)$ appear that serve to cancel a part of this $\mu$ dependence. If higher order contributions were calculated, they would tend to eliminate, successively, more of the $\mu$ dependence. At any fixed order in perturbation theory the residual $\mu$ dependence acts as an estimator of the theoretical uncertainty associated with the truncated perturbation series.

We find that the situation for the jet cross section at order $\alpha_3^s$ is a major improvement over the order $\alpha_2^s$ Born result both because the $\mu$ dependence is reduced and because the cross section exhibits a reasonable dependence on the jet definition. While the Born cross section exhibits monotonic dependence on $\mu$, the higher order result is relatively insensitive to the value of $\mu$ in a broad region near $\mu \simeq E_T/2$. We estimate\cite{1} that the residual theoretical uncertainty, as indicated by the residual $\mu$ dependence, is $\sim 10\%$. The uncertainty in the cross section due to the current uncertainty in the parton distribution functions is estimated\cite{1} to be somewhat larger, $\sim 20\%$. In the perturbative calculations described here,
the effects of the long distance fragmentation processes and of the soft interactions of the “spectator” partons are ignored. We estimate\[1\] that these uncalculated power suppressed effects constitute a correction of order $\sim 6 \text{ GeV/}E_T$ to the cross section. Thus for jet $E_T$’s of order 100 GeV, as discussed here, the nonperturbative uncertainty is of the same order or smaller than that due to perturbative effects.

At hadron colliders jets are typically defined\[4\] in terms of the particles $n$ whose momenta $\mathbf{p}_n$ lie within a cone centered on the jet axis $(\eta_J, \phi_J)$ in pseudorapidity $\eta$ and azimuthal angle $\phi$, \( [(\eta_n - \eta_J)^2 + (\phi_n - \phi_J)^2]^{1/2} < R \). The jet angles $(\eta_J, \phi_J)$ are the averages of the particles’ angles,

$$
\eta_J = \sum_{n \in \text{cone}} p_{T,n} \eta_n / E_{T,J},
$$
$$
\phi_J = \sum_{n \in \text{cone}} p_{T,n} \phi_n / E_{T,J}
$$

(1)

with $E_{T,J} = \sum_{n \in \text{cone}} p_{T,n}$. This process is iterated so that the cone center matches the jet center $(\eta_J, \phi_J)$ computed in Eq. (1). It is important to note that this jet algorithm is not yet fully defined since jets can overlap. In particular, it is possible for the constraints above to be satisfied by configurations where some particles are common to more than one cone. In the $\alpha_3^s$ calculation it is possible for a one parton jet to lie within the cone of a two parton jet.

In the theoretical calculations described here\[1\], we use the rule that only the two parton jets are included and the overlapping one parton jets are discarded. The precise definitions used by the various experiments differ to a greater or lesser extent from this form\[5\].

While the Born cross section with only a single parton per jet is $R$ independent, the order $\alpha_3^s$ cross section can have 2 partons inside a jet and is $R$ dependent. The theoretical expectation for the $R$ dependence is shown in Fig. 1 along with results from CDF\[3\]. The inclusive single jet cross section is evaluated at $E_T = 100 \text{ GeV}$ using the HMRS(B)\[6\] parton distributions. Since dependence on $R$ is not present in the Born cross section, this dependence is a lowest order result at order $\alpha_3^s$, that is, $d\sigma/dR = 0 + \mathcal{O}(\alpha_3^s)$. One therefore expects that, although the cross section itself is relatively $\mu$ independent, its slope $d\sigma/dR$ will be quite strongly $\mu$ dependent. Just this behavior is indicated in Fig. 1 by the curves for the cross section versus $R$ for 3 different $\mu$ values. (This figure is essentially Fig. 3 of Ref. 3 but with the correct theoretical result. The fourth, dot-dash curve and the parameter $R_{\text{sep}}$ will be explained below. The values of the corresponding $R$-independent but strongly $\mu$-dependent Born cross section are also indicated.) A correlated feature is that the $\mu$ dependence of the jet cross section changes as we vary $R$. While the order $\alpha_3^s$ jet cross section is relatively independent of $\mu$ for $R \approx 0.7$, for large $R$, $R > 0.9$, it is dominated by the order $\alpha_3^s$ real emission process and becomes a monotonically decreasing function of $\mu$ much like the Born result. At small $R$, $R < 0.5$, the $\mu$ dependence of the order $\alpha_3^s$ cross section is dominated by the negative contribution from the virtual correction associated with the collinear singularity and becomes a monotonically increasing function of $\mu$. In either regime we expect higher order corrections to be important so that the usefulness of fixed order perturbation theory is compromised. Thus we conclude that, at this order in perturbation theory, the results
are most stable for $R \approx 0.7$. It is precisely this size that was used in the inclusive single jet cross section analysis published by CDF\cite{3}. In some sense the perturbation theory is telling us that $R = 0.7$ is the “optimal size” for a jet cone, at least from the standpoint of making comparison with the order $\alpha_s^3$ result.

The comparison with the data in Fig. 1 suggests that, while the agreement between theory and data for $R = 0.7$ is quite good, the strong dependence on $R$ exhibited by the data favors a small $\mu$ value, i.e., larger $\alpha_s$ and more radiation. To make this comparison more quantitative we can characterize both the data and the theory curves in terms of 3 parameters,

$$\sigma = A + B \ln R + CR^2. \tag{2}$$

The parameterizations of the data and the theory for $\mu = E_T/2$ and $\mu = E_T/4$ are indicated in the first three rows of Table 1 (the parameter $R_{sep}$ will be defined below). We see that the theoretical value for the $R$-independent $A$ parameter is not too sensitive to $\mu$. This is expected since it is a true one loop quantity, containing both the order $\alpha_s^2$ contribution and contributions from real and virtual graphs at order $\alpha_s^3$. The $B$ and $C$ terms, however, are subject to larger theoretical uncertainty since these terms express the $R$ dependence of the cross section and this appears first at order $\alpha_s^3$. This is indicated by the sensitivity to $\mu$ found in the Table. We can naively associate the $B$ term with correlated (approximately collinear) final state parton emission that is important near the jet direction and the $C$ term with essentially uncorrelated initial state parton emission that is important far from the jet direction.

The theoretical value of $A$ agrees quite well with the experimental value of $A$. The agreement between the data and the $\mu = E_T/2$ theory for $C$ is also quite good, but the agreement for $B$ is worse than one would expect. This suggests that for the $\mu = E_T/2$ theory, the amount of initial state emission far from the jet direction is about right but that there is not enough correlated radiation near the jet center. If we change $\mu$ to $E_T/4$, then the effective $\alpha_s$ is larger and there is more radiation in all parts of phase space. Now $B$ is larger, although still smaller than indicated by the data, while $C$ is larger than indicated by the data.

To examine this issue further and to analyze the internal structure of jets in detail, it is useful to consider the fractional $E_T$ profile, $F(r, R, E_T)$ (we suppress the dependence on the jet direction $(\eta_J, \phi_J)$). Given a sample of jets of transverse energy $E_T$ defined with a cone radius $R$, $F(r, R, E_T)$ is the average fraction of the jets’ transverse energy that lies inside an inner cone of radius $r < R$ (concentric with the jet defining cone). Said another way, the quantity $1 - F(r, R, E_T)$ describes the fraction of $E_T$ that lies in the annulus between $r$ and $R$. It is this latter quantity that is most easily calculated in perturbation theory as it avoids the collinear singularities at $r = 0$. Computing the $E_T$ weighted integral of the $p\bar{p} \to 3$ partons $+X$ cross section over the annulus and normalizing to $E_{T,J}$ times the Born cross section yields the order $\alpha_s$ contribution to $1 - F$ (the numerator is purely order $\alpha_s^3$ while the denominator is purely order $\alpha_s^2$). The result for $F$ is plotted in Fig. 2 versus the inner radius $r$ with $R = 1.0$ for $E_T = 100$ GeV and compared to preliminary CDF data\cite{7}. Again curves for three choices of $\mu$ are exhibited. (The fourth, dot-dash curve
and the parameter $R_{sep}$ will be explained below.) As with the $R$ dependence of the cross section discussed above, $F$ is being calculated to lowest nontrivial order and thus exhibits monotonic $\mu$ dependence. While there is crude agreement between theory and experiment, the theory curves are systematically below the data for all interesting values of $\mu$. This situation suggests that the theoretical jets have too large a fraction of their $E_T$ near the edge of the jet ($r \simeq R$).

We have seen that the $R$ dependence and the $B$ parameter suggest the importance of higher orders to increase the level of associated radiation, at least near the center of the cone. At the same time our detailed considerations of the parameter $C$ and of $F$ suggest that the data favor a reduction of the $E_T$ fraction near the edge of the cone. Although these conclusions seem contradictory, there may be a consistent explanation based on a detailed but important physical point concerning how the jets are defined. We will present a preliminary discussion of this point here and present a more detailed study in a separate note. The issue is that of merging, how close in angle should two partons be in order to be associated as a single jet. In a real experiment such a situation is presumably realized as two sprays of hadrons, each with finite angular extent due to both fragmentation effects and real experimental angular resolution effects. If the angular separation is large enough, there is a valley in the $E_T$ distribution between the two sprays and experimental jet finding algorithms will tend to recognize this situation as two distinct jets. Recall that we expect for jets of $E_T > 100$ GeV that the angular extent of fragmentation effects will be small compared to the defined jet cone sizes. However, the theoretical jet algorithm we are using will merge two partons into a single jet whenever it is mathematically possible. This includes the limiting configuration when two equal transverse energy partons (each with $E_T/2$) are just $2R$ apart. The calculation counts this as a single jet of transverse energy $E_T$ with its cone centered between the two partons, i.e., centered on the valley. The treatment of this configuration in a real experiment will depend in detail on the implementation of the jet algorithm. To simulate the experimental algorithm in a simple way we add an extra constraint in our theoretical jet algorithm. When 2 partons, $a$ and $b$, are separated by more than $R_{sep}(\leq 2R)$, $R_{ab} = [(\eta_a - \eta_b)^2 + (\phi_a - \phi_b)^2]^{1/2} \geq R_{sep}$, we no longer merge them into a single jet. The theoretical jet algorithm used above corresponds to $R_{sep} = 2R$ as noted in Table 1. As an example, the results of calculating both the $R$ dependence and the $E_T$ fraction $F$ with $R_{sep} = 1.3R$ and $\mu = E_T/4$ are illustrated by the dot-dash curves in Figs. 1 and 2. Clearly the extra constraint of $R_{sep}$ has ensured that there is approximately the observed fraction of $E_T$ near the edge of the cone while the reduced $\mu$ value has increased the amount of associated radiation near the center of the cone and produced a larger variation with $R$. This conclusion is also verified by the last line in Table 1 where we observe that, compared to the first line of theoretical results, $B$ has increased while $C$ has remained the same. The values of both $B$ and $C$ that arise with these parameter choices are in reasonable agreement with the data. The jet cross section itself is relatively insensitive to the parameter $R_{sep}$, decreasing by $\leq 10\%$ as $R_{sep}$ is reduced from $2R$ to $1.3R$ with fixed $\mu$ for $E_T = 100$ GeV.

In summary, the agreement between data and QCD perturbation theory at order $\alpha_s^3$ for
the question of the dependence on the jet definition is a vast improvement over the situation that obtained at the Born level. There is good agreement between theory and experiment, at least for $E_T \geq 50$ GeV and $R$ near 0.7. On the question of the detailed structure within jets the qualitative agreement is good but there are important quantitative issues that seem to be dependent on the details of the implementation of the jet definition, especially the question of jet merging. Further study, both theoretical and experimental, is required to obtain a full understanding of this problem. This is particularly interesting since there is some indication that such detailed internal jet structure can be invoked to differentiate quark jets from gluon jets.

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Table 1: 3 parameter fits to data and calculated curves in Figs. 1, 3 and 4

|                  | A  | B  | C  |
|------------------|----|----|----|
| CDF data         | 0.54 | 0.28 | 0.22 |
| $\mu = E_T/2, R_{sep} = 2R$ | 0.52 | 0.13 | 0.19 |
| $\mu = E_T/4, R_{sep} = 2R$ | 0.47 | 0.19 | 0.30 |
| $\mu = E_T/4, R_{sep} = 1.3R$ | 0.49 | 0.22 | 0.19 |

Figure Captions

**Fig. 1:** Inclusive jet cross section versus $R$ for $\sqrt{s} = 1800$ GeV, $E_T = 100$ GeV and $0.1 < |\eta| < 0.7$ with $\mu = E_T/4, E_T/2, E_T$ compared to data from CDF; the dot-dash curve is explained in the text. Also indicated for the three $\mu$ choices are the values of the $R$-independent Born cross section.

**Fig. 2:** $F(r, R, E_T)$ versus $r$ for $R = 1.0$, $\sqrt{s} = 1800$ GeV, $E_T = 100$ GeV and $0.1 < |\eta| < 0.7$ with $\mu = E_T/4, E_T/2, E_T$ compared to data from CDF; the dot-dash curve is explained in the text.