Spin Dependence of the Fundamental Plane of Black Hole Activity

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Abstract

The Fundamental Plane (FP) of Black Hole (BH) Activity in galactic nuclei relates X-ray and radio luminosities to BH mass and accretion rate. However, there is a large scatter exhibited by the data, which motivated us for a new variable. We add BH spin as a new variable and estimate the spin dependence of the jet power and disk luminosity in terms of radio and X-ray luminosities. We assume the Blandford-Znajek process as the main source of the outflow, and find that the jet power depends on BH spin stronger than quadratically at moderate and large spin values. We perform a statistical analysis for 10 AGNs which have sub-Eddington accretion rates and whose spin values are measured independently, and find that the spin-dependent relation describes the data significantly better. This analysis shows not only the spin dependence of the FP relation, but also implies the Blandford-Znajek process in AGN jets.

INTRODUCTION

Although nearly every galaxy contains a supermassive Black Hole (SMBH) at its core [1], the Active Galactic Nuclei (AGNs) are a small portion of this large SMBH family, and jet producing BHs are a tenth of all AGNs [2, 3]. Such systems have been shown to be described by an approximate power law relation between BH mass, X-ray and radio luminosity, called Fundamental Plane (FP) of black hole activity [4–8]. For BHs with outflow and an accretion disk, it is usually considered that X-ray luminosity is often linked with the accretion power (though it also gets contribution from the jet power) and radio luminosity is considered as an indicator of jet power. Hence, the FP relation can equivalently be expressed in terms of BH mass, disk luminosity and jet power. The jet and disk connection in such BH systems result in the mutual scaling of the radio and X-ray luminosities [4–9].

The Fundamental Plane relation typically applies for radiatively inefficient AGNs with sub-Eddington accretion rates. However, AGN data shows large scatters relative to the FP relation. Since BH spin is expected to have an important role in both emitted radiation and jets [10] and expectedly the functional dependencies of the jet and disk power on the spin are different, we suggest that the deviations from the FP solution may result from the BH spin. Therefore, by adding BH spin as a new variable, we predict that the data lies approximately on this new 4-variable relation. In this context, we study the spin dependence of the jet and accretion power and translate the results to the FP quantities, radio and X-ray powers.

On top of the angular frequency and size of the BH horizon, BH spin controls the location of the inner layers of the accreting matter, which affects the released energy fraction per mass as radiation. Moreover, as inner layers of accreting matter comes closer to the horizon, the magnetic field around the black hole gets stronger. We assumed Blandford-Znajek (BZ) process [11] (see also [12] for a recent review) as the source of the jet luminosity, $L_{\text{jet}}$, and it predicts the jet power scales quadratically with BH spin for small values of the spin. Our estimation confirms this scaling and we further find that jet power has a stronger functional dependence on spin at higher values.

We use independent spin measurements from two well-established methods, namely the X-ray reflection and continuum fitting, to test our prediction [13, 14]. We found that the spin modified FP describes data remarkably better, reducing the $\chi^2$ error per degree of freedom from $\sim 12.27$ to $\sim 2.56$ for 10 AGNs. This result could possibly be improved further by taking into account the variation of the accretion disk thickness and the nonlinearities between $L_{\text{jet}} \sim L_R$ and $L_{\text{disk}} \sim L_X$. This result further implies the existence of the BZ in AGN jets on top of the spin dependence of the FP relation.

BH SPIN DEPENDENCE OF DISK LUMINOSITY AND JET POWER

Theoretical results and general relativistic magnetohydrodynamical (GRMHD) simulations, have demonstrated that the spin energy can be a dominant portion of the jet power [10, 15–18]. This is supported by
observations indicating jet power that is orders of magnitude larger than the accretion power (e.g. \cite{10}). Here we focus on the Blandford-Znajek process which is based on energy extraction from a Kerr BH in the presence of magnetic field set by accretion disk \cite{11}. We assume other processes will be subdominant such as wide-angle winds or the Blandford-Payne process \cite{19} (recent simulation results show that winds in low accretion rate systems carry a small portion of the outflowing energy \cite{20}, while BP may still have some important role \cite{21}).

We start by expressing jet luminosity, $L_{\text{jet}}$, and bolometric disk luminosity\footnote{We use disk luminosity and accretion power interchangeably throughout the text.}, $L_{\text{disk}}$ in terms of three physical parameters: i) BH mass, $M_{\text{BH}}$ (or equivalently, the Eddington luminosity $L_{\text{Edd}}$), ii) dimensionless accretion rate, $\dot{m} \equiv \frac{\dot{M}}{L_{\text{Edd}}}$, with $L_{\text{Edd}}$ being the Eddington accretion rate, and iii) the dimensionless BH spin, $\tilde{a} \equiv \frac{cJ}{GM}^2$, with $J$ being the angular momentum of the BH,

$$L_{\text{jet}} \sim \dot{m}^\gamma L_{\text{Edd}}^\beta \mathcal{F}(\tilde{a}) ;$$

$$L_{\text{disk}} \sim \dot{m}^\kappa L_{\text{Edd}}^\beta \mathcal{E}(\tilde{a}) ,$$

where $\mathcal{F}(\tilde{a})$ and $\mathcal{E}(\tilde{a})$ are the spin dependent functions for the jet power and disk luminosity, respectively. Dimensional arguments set $\theta = \beta = 1$. As shown in the next section, the matter inflow to the BH and electric current around the BH are interconnected (setting the external magnetic field around BH), and the jet power is related with the inflowing energy density, which sets $\gamma = 1$. Finally, $\kappa$ is set depending on the accretion state. There are three main regimes for the $\kappa$ parameter: i) in the extremely low accretion rates ($\dot{m} \ll 10^{-5}$) $3 \leq \kappa \leq 6$, ii) for mildly accreting system ($10^{-5} \lesssim \dot{m} \lesssim 0.1$) $\kappa \simeq 2$, and iii) for large inflow rates ($\dot{m} \sim \mathcal{O}(1)$) $\kappa \simeq 1$.

### Spin Dependence of the Disk Luminosity

The spin dependent term, $\mathcal{E}(\tilde{a})$, shows the radiation conversion efficiency of the accreting matter into BH. The energy released as radiation is some fraction of the difference in the energy of the accreting matter at large radius and at innermost stable circular orbit (ISCO). This radiation efficiency increases as the spin of the BH increases because the innermost stable orbit comes closer to the event horizon. Radiation efficiency can be expressed by using circular equatorial geodesic equation as (see \cite{22}).

$$\mathcal{E}(\tilde{a}) = 1 - \left( \frac{\tilde{r}}{\tilde{r}^{\frac{3}{2}} - 2\tilde{r}^{\frac{1}{2}} \pm \tilde{a}} \right) \Bigg|_{\tilde{r}=\tilde{r}_{\text{ISCO}}} ,$$

where $\tilde{r} = r/GM$. This formula produces familiar results such as $\mathcal{E}(\tilde{a} = 0) \simeq 0.057$ and $\mathcal{E}(\tilde{a} = 1) \simeq 0.423$.

### Spin Dependence of the Jet Power

Although, isolated BHs are characterized by their mass, electric charge (expectedly small) and spin, numerous astrophysical phenomena can be observed around them in the presence of accretion disk. We assume Blandford-Znajek (BZ) process is the dominant source of the jet power\footnote{Note that the other mechanisms freeing energy from the BH, such as Blandford-Payne and winds, exist but in numerous GRMHD simulations, BZ is the main outflow reason.}. This process can be interpreted as the radiative energy outflow from a material with finite resistance moving in external magnetic field. Assume a piece of material with finite resistance rotates in the magnetic field, the electromotive force induced by this rotating body is $\mathcal{E} \propto B^2 w R^2$, where $w$ is its angular frequency, $B$ is the external magnetic field, and $R$ is its characteristic size. A similar expression has been derived in BZ formalism as \cite{11, 23}.

$$L_{\text{jet}} = \int S^r \, dA = \int \Omega_A (\Omega_H - \Omega_A) \left( \frac{\dot{a} \sin \theta}{\Sigma} \right)^2 \left( r^2 + \tilde{a}^2 \right) \Sigma \sin \theta \, d\theta \, d\phi \bigg|_{r=r_H} ,$$

where $\Omega_A$ is the angular frequency of the field lines, $\Omega_H$ is the angular frequency of the horizon (defined below), $S^r$ indicates the radial energy flow ($T^{0r}$ component) and $dA = \Sigma \sin \theta$ with $\Sigma = r^2 + a^2 \cos^2 \theta$.

$$\Omega_H \equiv \frac{1}{2GM} \frac{\tilde{a}}{1 + \sqrt{1 - \tilde{a}^2}} ,$$

(4)
In order to extract energy out, the field line angular frequency should be less than horizon angular frequency, \( \Omega_A < \Omega_H \). Assuming the field line velocity is approximately the field line velocity near equipartition values, one can evaluate \( \Omega_A \approx \Omega_{\text{Keplerian}} \). This implies that for \( \tilde{a} \lesssim 0.36 \), \( \Omega_{\text{Keplerian}} \gtrsim \Omega_H \), no energy extraction is expected and this value approximately sets the threshold for jet production. For moderate and large spin values, the field line frequency is found approximately as half of the horizon frequency, \( \Omega_A \approx \Omega_H / 2 \), as in Ref [11].

One can define the magnetic field strength as \( \epsilon^{ij} A_{ij} = \sqrt{\gamma B} \) so that the pressure and energy density resulting from magnetic field can be expressed as \( \propto B^2 \) (without metric modifications on the indices). Given the above relations, the jet luminosity can be expressed as

\[
L_{\text{jet}} \approx \int \frac{w_{\tilde{H}}^2(\tilde{a})}{4} (B')^2 (2 \tilde{r}_H(\tilde{a})) \sin^3 \theta \Sigma d\theta d\phi \approx w_{\tilde{H}}^2(\tilde{a}) \tilde{r}_H^2 (GM)^2 (B')^2 I(\theta)
\]

where \( I(\theta) \) is an expectedly \( O(1) \) number which can be also modified by the polar dependence of the magnetic field, \( \tilde{r}_H(\tilde{a}) = r_H / GM = (1 + \sqrt{1 - \tilde{a}^2}) \), and \( w_{\tilde{H}}(\tilde{a}) \equiv \frac{\tilde{r}_H}{\sqrt{1 - \tilde{a}^2}} \) is the dimensionless angular frequency of the horizon. Note that \( w_{\tilde{H}}^2 \tilde{r}_H^2 = \tilde{a}^2 \).

The system is assumed to be axisymmetric (azimuthal) and stationary. In Ref. [24], it is shown that the weak magnetic fields develop a strong MHD instability (and further numerically studied in [25]), saturating the field strength near equipartition values. Hence we have,

\[
\frac{B^2}{8\pi} \equiv \beta \cdot P_{\text{gas}} = \beta \cdot (\rho c_s^2) \approx \mathcal{O} \left( \frac{\rho (L/\mu)^2}{r^2} \right) \propto \mu \cdot n \cdot \gamma \cdot (L/\mu)^2 / r^2,
\]

with \( \beta \) representing a factor \( O(0.1 - 1) \).

We employ the continuity equation for the particle number density, \( J^\mu = n \cdot u^\mu \), with \( n \) being the number density of particles, \( J^\mu = \frac{1}{\sqrt{-g}} (\nabla - g^\mu, \cdot J^\nu) \). Due to stationarity and axisymmetry, partial derivatives of time and azimuthal angle vanish. We also assume that number density is a variable averaged over polar angle. We end up with,

\[
(n \cdot \Sigma u^\nu, r = 0) \Rightarrow -\frac{\dot{M}}{\mu} \equiv n \cdot \Sigma u^\nu \quad \text{where} \quad u^\nu = \gamma \cdot v_r.
\]

where \( \gamma \) is Lorentz factor. We estimate the scaled specific angular momentum using the geodesic expression at the equator as [22]

\[
\mathcal{L} \equiv \frac{L}{GM \mu} = \frac{(\tilde{r}^2 \mp 2\tilde{a} \tilde{r}^{1/2} + \tilde{a}^2)}{\tilde{r}^{3/4} \left( \tilde{r}^{3/2} - 3\tilde{r}^{1/2} \pm 2\tilde{a} \right)^{1/2}},
\]

where upper/lower signs are for prograde/retrograde orbits. Note that at large large distances, \( (r/GM) \gg 1 \), we have \( \mathcal{L} \approx \tilde{r}^{1/2} \) which is the standard Keplerian result. With this, we estimate the strength of the magnetic field in the innermost layers of the disk as

\[
B_{\text{in}}^2 \approx \frac{\mathcal{L} \cdot GM^2 \dot{M}}{\Sigma \tilde{r}^2 v_r} \bigg|_{\tilde{r}_{\text{in}}},
\]

Next we consider the magnetic field near the horizon by assuming that the plunging region, the region between ISCO and event horizon, satisfies nearly vacuum properties. The layers of charged particles near the ISCO form a current loop threading the horizon. The standard dipole magnitude for a loop is

\[
B_H \approx \frac{J \cdot A_{\text{loop}}}{r^3} \bigg|_{\tilde{r}_{\text{in}}},
\]

One can integrate over various rings that contribute to magnetic dipole moment around the horizon, however since the number density of the disk drops rapidly, the ISCO gives the dominant contribution to the magnetic field around horizon which allows us to represent the disk as a loop for this part of the calculation.

The radiation pressure is assumed to be comparable with the magnetic force exerted on the disk. This gives

\[
B^2 \Sigma \propto P \Sigma \propto I^2
\]

yielding

\[
B_{\text{H}}^2 \propto \frac{\mathcal{L}_{\text{in}}^2 (GM)^2 \dot{m}}{r_{\text{in}}^2 v_r} L_{\text{edd}} \propto \frac{\mathcal{L}_{\text{in}}^2 \dot{m}}{S^{7/2}(\tilde{a})} \frac{(GM)^{-2}}{f(\alpha, \epsilon)} L_{\text{edd}}.
\]

Typically the radial velocity is expressed as \( v_r \approx v_{\text{Keplerian}} \cdot f(\alpha, \epsilon) \), where \( f \) is a coefficient set by gas specific heat ratio, \( \epsilon \), and viscosity parameter, \( \alpha \), described in Shakura-Sunyaev prescription [26, 27]. We neglect the potential spin dependence of \( f \). In the last step, we evaluate the expression near the ISCO, \( S(\tilde{a}) \equiv \tau_{\text{ISCO}} / GM \)

\[
S(\tilde{a}) \equiv \left( 3 + 2\tilde{a} \mp [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2} \right),
\]
FIG. 1: The expected nonlinear spin dependence of the jet power in the Blandford-Znajek process.

where \( \tilde{a} \) indicating the prograde/retrograde rotation respectively, with \( Z_1 = 1 + (1 - \tilde{a}^2)^{1/3}[(1 + \tilde{a})^{1/3} + (1 - \tilde{a})^{1/3}] \) and \( Z_2 = (3\tilde{a}^2 + Z_1^2)^{1/2} \). \( S(\tilde{a}) \) is a monotonically decreasing function of spin. We have \( S(0) = 6 (r_{\text{ISCO}}(\tilde{a} = 0) = 6GM) \) and \( S(1) = 1 (r_{\text{ISCO}}(\tilde{a} = 1) = GM) \).

It is important to note that similar results can be obtained following the arguments in Ref. [9] which indicate that the dynamically cooled systems have a magnetic field dependence in the form as \( \dot{m}/M \). Furthermore, detailed numerical simulations confirm approximately the radial dependence of the magnetic field estimated above [28]. Our jet power expression becomes

\[
L_{\text{jet}} \propto \frac{\dot{m} L_{\text{in}}^2}{S_{\tilde{a}}^{1/2}} w_H^2 r_H^2 \frac{\dot{m}}{\dot{m}_{\text{edd}}} L_{\text{edd}}.
\] (13)

This result shows an interesting behaviour at large values of BH spin. For fixed M and \( \dot{m} \), jet power depends on the amplitude of the angular frequency. At small spin values, \( \tilde{a} \lesssim 0.3 \), \( S(\tilde{a}) \) is nearly constant, hence \( L_{\text{jet}} \propto w_H^2 \). For moderate spin values, we have \( L_{\text{jet}} \propto w_H^3 \), which is valid in the regime \( 0.4 \lesssim \tilde{a} \lesssim 0.8 \), and finally, \( L_{\text{jet}} \propto w_H^6 \), for \( \tilde{a} \gtrsim 0.9 \), in the large spin regime. All these regimes are approximately fitted as dashed lines to the exact relation given in Figure 1, where \( \tilde{a} \) in lower horizontal axis with a maximum value of \( a_{\text{max}} = 0.998 \), and \( w_H \) in the upper horizontal axis with a maximum value of \( w_{H,\text{max}} = 0.939 \). It is remarkable that for geometrically thick and fast spinning BHs, similar results were found in detailed simulations [17].

EVIDENCE FOR SPIN MODIFICATION OF THE AGN FUNDAMENTAL PLANE

The Fundamental Plane of black hole activity is a scaling expression for outflowing BHs with sub-Eddington accretion rates relating the BH mass, X-ray and radio luminosities [4–7]. This relation implies a coupling between the jet and the disk power, frequently dubbed as jet-disk symbiosis [29, 30] since X-rays gives a measure of accretion power, and radio emission a measure of jet power. Remarkably, FP expression has been shown to extend from masses of order solar mass up to supermassive BHs, which might be considered as an indication of the universality of jet production. In a radiatively inefficient systems \( L_{\text{disk}} \propto L_X \propto \dot{m}^\alpha L_{\text{edd}} \) with \( \alpha \) being a number in the range 2–3, close to 2.3 [31–33] and \( L_R \propto (\dot{m} \cdot M)^{17/12} \) [34]. Hence, the fundamental plane equation is expressed as [4–6]

\[
\log L_R = 0.6 \log (L_X) + 0.78 \log (M_{BH}) + \text{constant}
\] (14)

where luminosities are in units of erg \( \cdot \) s\(^{-1} \) and the mass of BH in solar masses, \( M_{\odot} \).

This result assumes that \( L_X \) and \( L_R \) are only functions of \( M \) and \( \dot{m} \). However for moderate and large spin values, both luminosities also strongly depend on the BH spin. On general grounds, the \( L_R - L_{\text{jet}} \) relation is quantified as \( L_R \simeq 10^{40} \left( \frac{L_{\text{jet}}}{10^{39} \text{erg/s}} \right)^{17/12} \text{erg/s} \) [35, 36]. A similar relationship exists between the X-ray
power and the bolometric disk luminosity, $L_X \propto L_{\text{disk}}$. Since both accretion power and jet power have different spin dependencies, this fact results in many orders of magnitude deviations between data and the standard FP relation.

We aim to remove these strong discrepancies by incorporating BH spin into the Fundamental Plane. For that purpose, we express our findings for the jet power (Eq. (13)) and disk luminosity (Eq. (2)) in terms of FP variables, $L_X$ and $L_R$. Finally, we test our predictions for 10 AGNs, given in Table I, whose spin values are measured independently and which are all in sub-Eddington regime. We label these relations as SMFP, the abbreviation of, “Spin Modified Fundamental Plane”.

The SMFP relation can be expressed as

$$\log L_R = \frac{17}{12} \log \left( \frac{w_H^2 \tilde{r}_H^2 \mathcal{L}^2}{S^{7/2}(\bar{a})} \right) = 0.6(\log L_X - \log \mathcal{E}(\bar{a})) + 0.78 \log M + \text{constant} \quad (15)$$

Introducing the scalings $L_{R,38} = (L_R/10^{38}\text{erg} \cdot \text{s}^{-1})$, $L_{X,40} = (L_X/10^{40}\text{erg} \cdot \text{s}^{-1})$, and $M_{B,H,8} = (M_{BH}/10^8M_\odot)$, one can express all the data in the form

$$\log L_{R,38}^{(i)} = -0.37 + 0.6 \log \left( \frac{L_{X,40}^{(i)}}{L_{X,40}} \right) + 0.78 \log (M_{B,H,8}) + \Delta^{(i)}. \quad (16)$$

Superscript $(i)$ is the label of each AGN. In standard Fundamental Plane relation, spin is set to a constant value and spin information is not included. $\Delta$ quantifies the scatter of each data point with respect to the FP predictions as the variance of all the data points is minimized. Hence, by definition, $\Delta_{FP} = 0$ for the standard FP relation [4-6]. However, $\Delta$ changes between $-3$ to $2$ for various data points.

When BH spin is included, $\Delta$ becomes a function of the BH spin. In Figure 2, we show that the data fits this new prediction better by studying the suppression of the scatter. In order to show that the inclusion of the spin information decreases considerably the amount of scatter, we invert the equation above and define the scatter function as

$$\Delta \equiv \log \frac{L_{R,38}}{10^{-0.37} \cdot \left( \frac{M_{BH}}{10^8M_\odot} \right)^{0.78} \cdot (L_{X,40})^{0.6}} \quad (17)$$

In the SMFP, this function is predicted as

$$\Delta_{SMFP} \equiv \mathcal{D} + \frac{17}{12} \log \left( \frac{w_H^2 \tilde{r}_H^2 \mathcal{L}^2}{S^{7/2}} \right) - 0.6 \log (\mathcal{E}) \quad (18)$$

We obtain $\mathcal{D} \simeq 0.58$, after minimizing the error variance. We also find that the AGNs nearly on the standard FP prediction corresponds to $\bar{a} \simeq 0.9$. Figure 2 shows the data points for the 10 AGNs given in Table I as green dots. The blue dots and corresponding error bars are produced by employing the spin measurements and measurement errors in Eq (18). It is clearly seen that in almost all cases, the spin modified expression tracks the data points better than the spin-blind standard prediction.

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Table I: The mass, X-ray luminosity [2-10 keV], radio luminosity [~ 5 GHz] (the measurement frequencies may vary, typically in the range [1-100GHz]) and spin data of 10 AGNs.

| Object   | $\bar{a}$  | $\log M/M_\odot$ | $\log L_R/\text{erg} \cdot \text{s}^{-1}$ | $\log L_X/\text{erg} \cdot \text{s}^{-1}$ | References |
|----------|-------------|-------------------|------------------------------------------|------------------------------------------|------------|
| FAIRALL 9 | 0.65 ± 0.1  | 8.4 ± 0.12        | 39.11                                    | 43.97                                    | [40, [57], [39], [39]] |
| ARK 564  | 0.96±0.01   | 6.04 ± 0.13       | 38.59                                    | 43.38                                    | [58, [59], [4], [41]]  |
| NGC 4151 | 0.94 ± 05   | 7.57 ± 0.2        | 38.49                                    | 42.48                                    | [55, [42], [4], [43]]  |
| M87 (NGC 4486) | 0.9 ± 0.1  | 9.81 ± 0.12       | 39.85                                    | 40.46                                    | [61, [53], [4], [4]]   |
| 3C 120   | 0.994±0.004 | 7.74 ± 0.17       | 41.36                                    | 44.06                                    | [56, [57], [60], [60]] |
| NGC 3783 | 0.92 ± 0.4  | 7.47 ± 0.10       | 38.78                                    | 43.10                                    | [44, [57], [45], [46]] |
| IRAS 00521-7054 | 0.98±0.018 | 7.7 ± 0.18        | 40.64                                    | 43.60                                    | [47, [47], [48], [49]] |
| NGC 1365 | 0.97±0.01   | 6.7 ± 0.2         | 37.65                                    | 40.60                                    | [50, [51], [48], [4]]  |
| ARK 120  | 0.64 ± 0.15 | 8.18 ± 0.2        | 38.56                                    | 43.95                                    | [58, [57], [39], [39]] |
| MRK 79   | 0.7 ± 0.1   | 7.72 ± 0.2        | 38.35                                    | 43.12                                    | [52, [57], [39], [39]] |

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3 Note that the spin distribution of heavy BHs are found to be moderate and large [37] even if the disk is geometrically thick.
ANALYSIS WITH BH SPIN IN THE FUNDAMENTAL PLANE

We employ the data given in Table I, and compute two quantities that signify the importance of the spin dependence. First, we use the (face value of the) independently measured spin values and calculate standard deviation for $\Delta$ in both FP and Spin Modified FP (SMFP). Second, we compute $\chi^2$ error per degree of freedom, by taking into account the experimental error in spin measurements via using a top-hat probability distribution function for each spin value within a 1-$\sigma$ error range. In both cases, $N = 10$ for our analysis.

$$\sigma_\Delta = \sqrt{\frac{\sigma^2}{\# \text{ dofs}}} = \frac{1}{N} \sqrt{\sum_{i=1}^{N} \left( \frac{\Delta_{\text{data}} - \Delta_{\ell}^{(i)}}{\sigma_{\Delta,\text{th}}} \right)^2} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{\delta_{\ell}^{(i)}}{\sigma_{\Delta,\text{th}}} \right)^2} \quad (19)$$

As discussed in the previous section, $\Delta_{\ell} = 0$ for FP, and for Spin Modified FP, it is given in Eq. (18). We obtain $\sigma_\Delta(FP) \approx 0.98$ and $\sigma_\Delta(SMFP) \approx 0.49$. It should be noted that for AGNs with spin information, the variance $\sigma_\Delta(FP) \sim 1$ is consistent with the larger samples used in Refs [4, 6].

We suggest that the nonlinear effects of spin modification on the radiation efficiency and jet power is the main cause of this behaviour and our SMFP prediction shows a remarkable consistency with the data. Note that the selection of data might have a role at some degree and an analysis with higher precision spin and radiation data could allow us to test our predictions better.

We calculate $\chi^2$ parameter per degree of freedom as

\[
\hat{\chi}^2 = \frac{\chi^2}{\# \text{ dofs}} = \frac{1}{N} \left[ -\log \left[ \Pi_{i=1}^{N} \int_{\tilde{a}_{i,\text{min}}}^{\tilde{a}_{i,\text{max}}} d\tilde{a}_i P(\tilde{a}_i) \exp \left[ -\left( \frac{\delta_{\ell}^{(i)}}{\sigma_{\Delta,\text{th}}} \right)^2 \frac{(L_{R,i}, L_{X,i}, M_i, \tilde{a}_i)/\sigma_{\Delta,\text{th}}^2) \right] \right) \right] = \frac{1}{N} \left[ -\log \left[ \int_{\tilde{a}_{i,\text{min}}}^{\tilde{a}_{i,\text{max}}} d\tilde{a}_i \frac{\delta_{\ell}^{(i)}}{\sigma_{\Delta,\text{th}}} \exp \left[ -\left( \frac{\delta_{\ell}^{(i)}}{\sigma_{\Delta,\text{th}}} \right)^2 \frac{(L_{R,i}, L_{X,i}, M_i, \tilde{a}_i)/\sigma_{\Delta,\text{th}}^2) \right] \right) \right) \right].
\]

Here $\sigma_{\Delta,\text{th}}^2 = \Delta L_R^2 + 0.62\Delta L_X^2 + 0.782\Delta M^2$, and $\Delta \tilde{a}_i = \tilde{a}_{i,\text{max}} - \tilde{a}_{i,\text{min}}$, and $P(\tilde{a}) = \frac{1}{\Delta a_i}$ due to top hat distribution estimation. Nearly all measurements have mass, radio and X-ray measurements with an error of 0.2 in log base, so we set $\Delta L_R = \Delta L_X = \Delta M = 0.2$. We obtain $\hat{\chi}_{FP}^2 \approx 12.27$ and $\hat{\chi}_{SMFP}^2 \approx 2.56$. Hence the inclusion of spin correction improves the fitting considerably.

We define $L_R \equiv L_R - \Delta_{\text{SMFP}}(\tilde{a})$ and plot in Figure 3, this new spin modified variable, $\tilde{L}_R$ (on the right panel) and $L_R$ (on the left panel) as a function of $L_X = 0.6\log L_{X,40} + 0.78\log M_S$. In both cases the slope of the line is 1. For the standard FP plane, there is considerable scatter (shown in the left panel), but the

\[4\] This is also consistent with Ref. [4] ($\sigma_\Delta \approx 0.89$) and Ref. [7] ($\sigma_\Delta \approx 1$)

FIG. 2: The scatter parameter, $\Delta$, is defined in Eq (17) such that the standard Fundamental Plane predicts it to be 0, ie. $\Delta_{FP} = 0$. By including BH spin as a new variable, we calculate $\Delta$ as a function of spin based on Eq. (18). Data (green points), and Spin Modified Fundamental Plane (SMFP) predictions (blue points) follow the trends and fit the data well.
FIG. 3: Left: The standard Fundamental plane, Right: Modified Fundamental Plane with spin dependence.

scatter decreases considerably with this new variable. However, the prediction curve on the right panel is still not razor-thin. There are potentially some reasons for that including the thickness of the accretion disk enters as an independent variable, data uncertainties, $L_X - L_{disk}$ and $L_R - L_{jet}$ connection, etc and they are discussed in more detail in the next subsections.

**Spin Dependence of the Fundamental Plane**

As discussed in the previous section, there exists a natural cutoff value for the spin value in the Blandford-Znajek (BZ) process, $\tilde{a} \sim 0.36$. In the regime of low spins, winds and Blandford-Payne type processes dominate over BZ. This leads to a nearly constant and small outflow for small spins. However, we estimate that BZ quickly starts dominating the power outflow at moderate spin values and the jet power grows even more nonlinearly for large spins. At low spin values, the ISCO is nearly constant and the angular frequency grows quadratically. As the spin grows the location of the inner layers of the accretion disk (equivalently the location of the source for the magnetic field) gets closer to the horizon. As a result, the BZ output changes more dramatically with growing spin. Since the jet and radiation efficiency have different functional dependence on the spin value, this results in strong scatter of the data from the standard FP relation when the spin information is not included.

If one assumes only quadratic dependence to spin value, the jet power grows from $\tilde{a} \sim 0.4$ to $\tilde{a} \sim 1$ about a factor of 6, and interestingly, the radiation efficiency also grows by a factor of 6 from small spin values to large spin values. This can be easily seen either from Eq (2) or approximately from the variation of the location of the ISCO in inverse gravitational potential, $(G_N M/r_{ISCO})$. Transfering these results to the FP variables, we get that the spin dependent contribution to the X-ray and radio power is modest. When we repeat our statistical analysis for the $L_{jet} \propto a^2$, we find only slight improvement in deviations, namely $\sigma_{L_{jet} \propto \tilde{a}^2} \approx 0.89$ and $\chi^2_{L_{jet} \propto \tilde{a}^2} \approx 9$.

**The Need For a New Variable in FP**

Fundamental Plane variables (BH mass, radio and X-ray luminosities) are derived from two main variables: mass and accretion rate. However, the 3 AGNs in the Table I, 3C120, IRAS 00521-7054 and MRK 79, show that these two variables are not enough to explain the data. Namely, these 3 AGNs have masses of around $10^{7.75} M_\odot$ and X-ray power of around $10^{43-44}$ erg $\cdot$ s$^{-1}$ implying these BHs have nearly same $M$ and $\dot{m}$, but their radio luminosities are different by more than 3 orders of magnitude. Therefore, it is difficult to explain this variation in the radio spectrum for AGNs by only mass and accretion rate. This fact motivates us to suggest spin as an additional missing variable (this point was also discussed in Ref. [4] and recently in [38]).

**Residual Scatter**

Here we note some potential reasons about the residual scatter in SMFP:

- $L_{bol} \neq$ const $\cdot L_X$ and $L_{jet} \neq$ const $\cdot L_R^{12/17}$.

We derived our results for the spin dependence of the jet power and bolometric disk luminosity, then transferred them to the standard definition of the Fundamental Plane relation which employs X-ray luminosity
and radio luminosities. Although there is a strong correlation between $L_X$ and $L_{\text{disk}}$, also between $L_R$ and $L_{\text{jet}}$, there might be slight deviations from the expressions adopted above [62].

- The ratio between the thickness of the accretion disk to the horizon size, $h/r_H$ enters in the expressions for both jet and disk luminosity. Throughout this analysis we implicitly assumed this ratio is nearly same for all the BH systems and this certainly introduces some fluctuations. However, these fluctuations could typically introduce scatter up to an order of magnitude which is subdominant with respect to spin dependence and could be reason for persistent small deviations in the spin modified version of the FP.

- There are observational uncertainties in the measurements of the spin, X-ray and radio luminosities. These uncertainties are again up to a factor of a few. Also it is known that jet power can also contribute X-ray luminosity but in the systems we focus on this effect is expected to be subdominant.

**SUMMARY AND CONCLUSIONS**

In accreting and outflowing BHs, the Fundamental Plane relation is one of the most important scalings relating three variables: the radio luminosity (indicating jet power), X-ray luminosity (indicating bolometric disk luminosity) and the BH mass. Although standard FP equation gives a good description of jet producing systems with sub-Eddington accretion rate, there are still considerable deviations around this relation. In order to explain these deviations, we investigated the spin dependence of the jet power and accretion power. We can summarize the effects of the BH spin on the jet and accretion power as follows: As the spin grows i) a larger fraction of the gravitational energy can be released as radiation since the stable orbits of the accreting matter could come closer to the horizon, ii) the magnetic field amplitude around the horizon grows since the source for the magnetic field comes closer, iii) the angular frequency of the horizon grows, iv) the size of the horizon decreases.

We derived three main results:

- We estimated the spin dependence of the dominant jet production mechanism, Blandford-Znajek process. We found that although for small spins, jet power is proportional to angular frequency quadratically as in the famous perturbative result, for moderate and large spins this is not the case. The jet power depends on the angular frequency more strongly (with sixth power) for very large values of spin parameter;

- We showed that standard Fundamental Plane cannot explain the data purely by assuming $L_X$ and $L_R$ are only functions of $M$ and $\dot{m}$. We gave an explicit example by using the data of 3 AGNs which have nearly same mass and X-ray power, but significantly different radio power;

- By using data on 10 AGN, we showed that BH spin has an important role in the black hole activity. Our Spin Modified Fundamental Plane (SMFP) relation shows significantly well agreement with the data. The many orders of magnitude scatter of the data in the standard FP relation drops to about an order of magnitude scatter in the SMFP. The remaining deviations can be explained by various sources, including the thickness of the accretion disk, uncertainties in the data, the environment around the AGN and uncertainties in the correlation between radio-jet power and X-ray-accretion power. In conclusion, our results do not only stress the vital role of BH spin in accreting and outflowing BHs, but also provide a strong observational indication for the existence of the Blandford-Znajek process.

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