Parametric simulator of cyclic and non-cyclic
impulsive vibration signals for diagnostic research
applications

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Abstract.
In recent years cyclostationary analysis of vibration signals is considered to be one of the
most potent approaches for diagnostics of machines with rotating components. However, it is a
subject of an extensive research towards extending its robustness due to its significant inefficiency
in the presence of non-cyclic impulsive components in measured data. This problem is especially
visible in datasets measured on machines such as ore crushers, where the high-energy impacts
are a natural phenomenon. Unfortunately, due to practical inaccessibility, real-life datasets
necessary to properly study this problem are extremely difficult to obtain. To address this
issue, the authors propose an easy to use simulator of impulsive components. It covers both
cyclic components that can describe various types of fault signatures, and non-cyclic ones that
can represent impacts occurring naturally due to the nature of machine operation. Simulated
signals have been compared with real ones to ensure a high similarity degree, which in turn
guarantees a relatively high level of realism.

1. Introduction
Impulsive content in vibration signals can have different sources of origin:

- **External artifacts**: Rare accidental impacts of random occurrence and energy, typically
  strong in energy in comparison to the rest of the signal, i.e., the object was randomly hit
  by a person passing by [1];

- **External impulsive noise**: Expected noise, external to the process in origin, however
  random in occurrence and/or energy, i.e. loose bolt, vehicles passing in close proximity,
  instability of the object of interest or a neighboring object [2];

- **Internal impulsive noise**: Process-related noise random in occurrence and/or energy but
  often relatively strong in energy compared to the rest of components in the signal, i.e. ore
  falling into the crushing or sieving machine, the crushing process itself, ore pieces randomly
  bouncing in the sieving screen [3, 4];

- **Internal cyclic impulsive component**: The best candidate for fault-related signal of
  interest. The cycle of impulses corresponds directly to a rotation of a particular element
  in the machine (i.e., the frequency of consecutive balls passing a fixed point on the bearing
  race that can indicate a crack in the race). It may not be perfectly periodic with a fixed
period because of rotational speed fluctuations in the machine (i.e., in the electric motor, the speed decreases with the increased external load) [5].

Signals contaminated with impulsive noise pose a certain difficulty for the analysis if the component of interest is also impulsive, which is often the case for local damage occurrences in machines with rotating elements. Especially when the frequency bands of two different impulsive components overlap, it is difficult to separate them without constructing complicated analytical techniques [6, 7, 3, 8, 9]. Unfortunately, very often it may not be possible to have access to real-life signals measured on industrial machinery in operation, that contain fault information and display complicated structure. Industrial companies avoid releasing such data and very few people are allowed to get or measure such signals. This is why researchers in the field of signal processing very often have to rely on simulated signals in order to construct analytical algorithms [10, 11, 12, 13, 14, 15, 16]. Although it is always better to work on real-life signals, it is common that the particular features of the signal, that are relevant for a given analysis, are often known and understood. Hence, it is possible to simulate test signals having in mind particular features.

Vibration signals containing impulsive behavior can be simulated in many different ways (i.e. as a stochastic process drawn from certain distribution, as a result of exciting an impulse response of some system with a process describing excitation pattern, as an assembly from elementary features etc.). Even when one considers using statistical distributions for the simulation purposes, there are a lot of them to choose from, which broadens the scope of possibilities even more. Such methods utilize for example α-stable distribution [17], Poisson process [18], Cox Process, Gaussian distribution or Uniform distribution [19].

Unfortunately, many scripts that allow to simulate certain signals that are available online are in the form of source code without any interface, require specific expertise to be used properly, are badly commented and are heavily fragmented (prepared for a certain specified task and nothing else). To address those issues, authors prepared an easy to use simulator of cyclic and non-cyclic impulsive vibrations signals equipped with graphical user interface. In this paper several different simulation scenarios are demonstrated to showcase specific features of the program.

2. Usage
General workflow of usage is presented in the Figure 1. Additionally, the authors provide a step-by-step user manual that explains each function in detail.

3. Signal construction
3.1. Impulse generation
A signal of interest (SOI) is constructed as a series of individual impulses distributed in time with a given period $T$. A single impulse is defined as a decaying harmonic oscillation:

$$g(t) = A \cdot \sin (2\pi f_c t) e^{-mt},$$  \hspace{1cm} (1)

where $A$ is the amplitude, $t$ is time, $f_c$ is the center frequency in the carrier frequency domain and $m$ is a decay coefficient for the exponential function.

An elementary impulse is generated this way, because real impulses that originate from the unitary (instantaneous) impacts in real-life scenarios when the energy is dissipated, can be modeled as decaying harmonic oscillations, since the response of any particular machine is not a factor in this case [20].

3.2. Bandwidth limitation
Impulses for particular components are generated using parameters provided by the user, as described in section 3.1. As a consequence, they are characterized with their natural bandwidth,
Figure 1. Flowchart of program usage.

Figure 2. Example of a single impulse $g(t)$.

which is a width observable i.e. on the spectrogram, as the impulse visible above the background noise. However, it is common that user desires to replicate conditions observed in real-life measurements, that contain components occupying narrower bands that occur naturally.
To enable the functionality of bandwidth limitation, the application is equipped with the possibility to enable additional bandpass filter for the particular component (cyclic or non-cyclic, parameterized separately).

In this application authors implemented bandpass finite impulse response (FIR) filter [21]. To simplify the parameterization, the filter order has been set to 40. In the program, user specifies the lower and upper cutoff frequencies relatively to the center carrier frequency of a given component.

3.3. Time randomization

The option of time randomization has been introduced to avoid the perfectly time-invariant frequency of the impulses in cyclic component. Such situation does not occur in reality, and in particular causes cyclostationary analysis of such signal to return perfect results. It is not the desirable outcome when one tries to simulate the real-life signal. In order to avoid this, time location of each impulse of cyclic component can be modulated with a random value in samples, such as:

$$t_{loc_i} = t_{loc_i} + U(-0.5r, 0.5r),$$

(2)

where \(t_{loc_i}\) is the time stamp of the \(i^{th}\) impulse, \(U\) denotes uniform distribution, and \(r\) is the value provided by the user that denotes the total possible range of modulation, given in samples.

3.4. Autoregressive modeling of reference signal

When the user selects the option to use reference signal for the simulation, it may happen that a given signal already contains some impulses, either originating from the real-world fault present in the signal, or caused by some random events that happened during the measurement. User may want to clear the reference signal from such imperfections. To address that issue, authors implemented the option to use autoregressive model (AR) of the reference signal, instead of the signal itself. The AR(p) model of the provided reference signal \(X\) is defined as follows [22]:

$$X(t) = c + \sum_{k=1}^{p} \varphi_k X(t - k) + \varepsilon(t)$$

(3)

where \(p\) is model order, \(\varphi_1, \ldots, \varphi_p\) denote coefficients of the model, \(\{\varepsilon(t)\}\) is the white Gaussian noise with the mean 0 and the variance of \(\sigma^2\) and \(c\) is the constant value.

Technically speaking, the AR model is constructed as a custom IIR (infinite impulse response) filter with dedicated magnitude response, that mimics the frequency response function of the given input data [23]. The \(\varphi_1, \ldots, \varphi_p\) coefficients and the white noise input variance \(\sigma^2_{AR}\) are estimated using Yule-Walker approach [22]. Because of the formula of the AR(p) model, the vector of estimated parameters has the form \(\hat{C} = [1, -\hat{\varphi}_1, \ldots, -\hat{\varphi}_p]\).

Obtained parameters of the model are used to construct the transfer function of the effective filter. The transfer function \(H\) is represented as:

$$H(t) = \frac{1}{1 - \hat{\varphi}_1 t^{-1} - \ldots - \hat{\varphi}_p t^{-p}},$$

(4)

Finally, modeled reference signal \(\hat{X}\) based on the provided reference signal \(X\) is simulated as:

$$\hat{X}(t) = \varepsilon_{AR}(t) * H(t)$$

(5)

where the "\(*\)" operator means convolution of the input white noise \(\{\varepsilon_{AR}(t)\}\) with mean 0 and variance \(\hat{\sigma}^2_{AR}\) and \(H(t)\).
3.5. Construction of cyclic component
In the first place, the time instances of the occurrence of impulses are defined as

\[ t_{loc_i} = i \times dt \]

where \( t_{loc_i} \) is the location of \( i^{th} \) impulse given in samples, \( i \in \{1, \ldots, \lfloor nx/dt \rfloor \} \), \( nx \) is the signal length given in samples and \( dt = 1/f_{imp} \) where \( f_{imp} \) is the frequency of cyclic impulses.

Then, impulses generated according to the description in section 3.1 are placed at those time instances.

3.6. Construction of non-cyclic component
The construction of the non-cyclic component consists of two steps. Firstly, the time instances of the occurrence of impulses are calculated as time stamps consecutively separated from each other by the time segments, the values of which are drawn from the distribution \( t_{dist} \) and its respective parameters selected by the user. As a result, the vector \( t_{loc} \) is obtained that serves the same function as in section 3.5, however in this case its elements are not separated with constant distance.

Secondly, when the amount of impulses in non-cyclic component is known based on \( t_{loc} \) derivation, the value of amplitude of each of them is drawn from the distribution \( a_{dist} \) and its respective parameters selected by the user. Those values, since varying, are used in generation of each individual impulse according to the procedure described in section 3.1.

The distributions \( t_{dist} \) and \( a_{dist} \) can be chosen by the user from Uniform, Gaussian or Poisson.

4. Results
In this section authors present the exemplary results of the simulations in three different scenarios:

(i) Simulation of a single cyclic but desynchronized component with autoregressive model of a reference signal from rolling-element bearing operating in the drive pulley of a belt conveyor;
(ii) Simulation of both cyclic and non-cyclic component with only Gaussian noise as a background;
(iii) Simulation of both cyclic and non-cyclic component with autoregressive model of a reference signal from rolling-element bearing operating in the copper ore crusher.

For all scenarios, authors provided figures containing the full GUI of the simulator program so that Reader can observe all of the used parameters.

4.1. Scenario 1: cyclic component with reference signal
In this section the results of the first scenario are presented. For this example, the objective was to simulate local damage of the inner race of the bearing (see Fig. 3). To make the resulting signal realistic in terms of its characteristic structure, a real signal has been used as a reference. However, since it already contained a similar fault, authors used the autoregressive model of the reference signal so that the original fault can be omitted, but the general structure remains. For this simulation, authors used the AR order of 256. The fault itself has been modeled as a cyclic impulsive component at the center carrier frequency of 3000 \( HZ \), however the impulses have been desynchronized for two reasons. Firstly, the original reference signal (see Fig. 4) exhibits impulsive behavior that is not perfectly periodic. Secondly, perfectly periodic modulations are very easy to be detected (i.e. using cyclostationary analysis), so that scenario would be too perfect and not realistic at all. One can observe that the reference signal is amplitude-modulated, especially with respect to the lower parts of its frequency spectrum (approximately
below 1 kHz), and although not perfectly, the aspect of amplitude modulation is also somehow translated to the simulated signal via AR model.

![Signal parameters](image1)
![Signal parameters](image2)

**Figure 3.** Example of a signal with cyclic components presented with the autoregressive model based on the reference signal.

![Reference signal](image3)

**Figure 4.** Reference signal
4.2. Scenario 2: Cyclic and non-cyclic component without reference signal

In Fig. 5 the exemplary signal with cyclic and non-cyclic impulses with presence of Gaussian noise is presented. In this example no reference signal is used, so the option to introduce a background noise becomes useful. Although it is not necessary for the signal and its visualization on the time series plot, the spectrogram of signal containing only the impulses becomes distorted and unusable. This is also the reason to give the user the option to select if they want to export the simulated signal with noise or without it. Possibility of tailoring the simulated signal precisely to the requirements is another advantage of using only low-energy noise as a background instead of a reference signal.

![Figure 5. Example of a signal with cyclic and non-cyclic components presented with the Gaussian noise.](image)

4.3. Scenario 3: Cyclic and non-cyclic component with reference signal

In this example authors show how the reference signal measured on a bearing of ore crusher (see Fig. 7) can be modeled with AR model in order to remove the impulsive behavior, and in this form used as a reference so that different impulsive components can be introduced. Authors designed a cyclic impulsive component that simulates an outer race fault of the bearing at the center frequency of 7000 Hz, and also a non-cyclic component that describes ore pieces randomly falling into the machine at different center frequency of 6000 Hz (see Fig. 6).

5. Conclusion

In this paper, the authors present an easy-to-use simulator of impulsive signals equipped with an intuitive graphical user interface. With the access to many useful parameters, it allows to precisely control the conditions of the constructed signal. It is helpful for simulating fault scenarios in the presence of non-cyclic impulsive noise, however it can also be useful in simple scenarios, such as introducing cyclic fault-related component to signal from machine in good condition. Monitoring of results as time series plot and time-frequency representation allows to
arrange signal components exactly as needed. The feature of using a signal for reference allows
not only to tailor the generated components specifically for a given problem (especially in terms
of placing the components properly in the spectrum), but also allows to use autoregressive model
of the reference signal instead of the signal itself.

Figure 6. Example of a signal with cyclic and non-cyclic components presented with the
autoregressive model based on the reference signal.

Figure 7. The reference signal.
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