An Orbifold and an Orientifold of Type IIB Theory on K3 × K3

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ABSTRACT

We consider the compactification of type IIB superstring theory on K3 × K3. We obtain the massless spectrum of the resulting two dimensional theory and show that the model is free of gravitational anomaly. We then consider an orbifold and an orientifold projection of the above model and find that their spectrum match identically and are anomaly-free as well. This gives a dual pair of type IIB theory in two dimensions and can be understood as a consequence of SL(2, Z) symmetry of the ten dimensional theory. We also point out the M-theory duals of the type IIB compactifications considered here.

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I. Introduction:

Recent developments of string dualities [1–4] have enhanced our understanding of the non-perturbative behavior of various string theories considerably. By assuming the existence of an eleven dimensional quantum theory, known as M-theory [3,5–11], the string dualities can be understood more or less in a unified way with the exception of certain symmetries of type IIB theory in ten dimensions [12,13]. It has been argued in ref.[13] that the symmetries of type IIB theory can be made more transparent by postulating the existence of a twelve dimensional quantum theory called as F-theory. Given the close relationships among F-theory, M-theory, and string theories, it has been pointed out in [15] that certain four dimensional compactifications of F-theory are related to M-theory and string theory compactifications below four dimensions. Thus, a deeper understanding of string theory compactifications in two dimensions will shed light in our understanding of compactifications of F-theory in four dimensions and holds the promise of solving the cosmological constant problem along the lines proposed by Witten [16].

Having pointed out some motivations, we study in this paper a two dimensional compactification of type IIB superstring theory on $K^3 \times K^3$. Six dimensional compactifications of type IIB theory on $K^3$ and its orientifold have been studied before. In the first case [17], one gets a model with chiral $N=2$ supersymmetry (counting in terms of 6d Weyl spinors) which contains a gravity multiplet along with twenty-one antisymmetric tensor multiplets as its massless spectrum and is shown to be free of gravitational anomaly. This model has been shown [18,19] to be equivalent to M-theory compactification on an orbifold $T^5/Z_2$ where five tensor multiplets come from the untwisted sector and the remaining sixteen tensor multiplets come from the twisted sector by placing sixteen Ramond-Ramond (R-R) five-branes on the internal space as dictated by the condition of gravitational anomaly cancellation. In the second case [20], when one considers the orientifold projection of compactification of type IIB theory on $K^3$, one gets a chiral $N=1$ supersymmetric model which contains in addition to a gravity multiplet, nine tensor multiplets and twelve hypermultiplets in the untwisted sector and eight vector as well as eight hypermultiplets in the twisted sector by placing eight R-R five-branes on the internal space. An identical spectrum can also be obtained [21,22] from an orbifold model of M-theory on $(K^3 \times S^1)/Z_2$, but in this case eight tensor and eight hypermultiplets come from the twisted sector. Gravitational anomalies have been shown to be cancelled in all
these models constructed. In fact, in some cases the condition of gravitational anomaly cancellation provides a useful guide for recovering the states in the twisted sector. In this paper, we extend the previous cases and consider the compactification of type IIB theory further on another K3. This two dimensional model will be shown to possess a chiral N=8 supersymmetry (counted in terms of 2d Majorana-Weyl (M-W) spinors). We compute the complete massless spectrum of the model by making use of the index theory and the Dolbeault cohomology of the simplest Calabi-Yau manifold K3 [23]. We show that this gives a consistent compactification of type IIB theory in 2d, since the model is free of gravitational anomaly. We then project out the above model by an orbifold group \( \{1, (-1)^{F_L} \cdot \sigma\} \) and an orientifold group \( \{1, \Omega \cdot \sigma\} \). Here \( F_L \) denotes the space-time fermion number operator on the left moving sector of the world-sheet, \( \sigma \) denotes the involution on the K3 surface and \( \Omega \) is the orientation reversal of the world sheet. Type IIB theory is invariant under these operations. We compute the massless spectrum of this orbifold and the orientifold models both from the untwisted sector and from the twisted sector. We find that the spectrum, which has chiral N=4 supersymmetry, match precisely for the two models and thus we obtain a dual pair of type IIB theory in two dimensions. This can be understood as a consequence of the SL(2, Z) symmetry of the ten dimensional type IIB theory. M-theory duals of the type IIB compactification on K3 × K3 as well as the orbifold and orientifold models, considered here, have also been pointed out.

This paper is organized as follows. In section II, after briefly reviewing the compactification of type IIB theory on K3, we compute the spectrum for the K3 × K3 reduction. We also show that the model is anomaly-free. We describe the orbifold projection of this model in section III. We compute the spectrum of this orbifold model for both the untwisted and the twisted sectors and show that the model is anomaly-free as well. The orientifold projection is considered in section IV. In this case we find that the spectrum matches identically with the previous orbifold model as a consequence of the SL(2, Z) invariance of the ten dimensional theory. We also mention in brief the M-theory duals of the two dimensional compactifications of type IIB theory that are considered here. Finally, we present our conclusions in section V.

II. Compactification of Type IIB Theory on K3 × K3:

The massless spectrum of type IIB theory in ten dimensions are given by the tensor
product of a left-moving and a right-moving super Yang-Mills multiplet of the same chirality as follows [24]:

\[(8_v + 8_c) \otimes (8_v + 8_c) = (1 + 28 + 35_v + 1 + 28 + 35_c)_B + (8_s + 8_s + 56_c + 56_c)_F\]  

(1)

Here ‘v’, ‘s’ and ‘c’ represent the vector and two inequivalent spinor representations of SO(8). ‘B’ and ‘F’ represent respectively the bosonic and the fermionic states. So, the bosonic sector of this theory contains a graviton \(35_v\), denoted by \(\hat{g}_{\mu\nu}(\hat{x})\) an antisymmetric tensor \(28\), denoted by \(\hat{B}^{(1)}_{\mu\nu}(\hat{x})\) and a dilaton \(1\), denoted by \(\hat{\phi}^{(1)}(\hat{x})\) from the Neveu-Schwarz–Neveu-Schwarz (NS-NS) sector \(8_v \otimes 8_v\) and another scalar \(1\), denoted by \(\hat{\phi}^{(2)}(\hat{x})\), another antisymmetric tensor \(\hat{B}^{(2)}_{\mu\nu}(\hat{x})\) \(28\) and a four-form antisymmetric tensor \(35_c\), denoted by \(\hat{A}_{\mu\nu\rho\sigma}(\hat{x})\), whose field-strength is anti-self-dual, from the R-R sector \(8_c \otimes 8_c\). In the fermionic sector, there are two spin \(3/2\) \(56_c\) Rarita-Schwinger fields which are antichiral and are denoted as \(\hat{\psi}^{(1)}(\hat{x})\) and \(\hat{\psi}^{(2)}(\hat{x})\). Also, there are two spin \(1/2\) \(8_s + 8_s\) chiral Majorana-Weyl fermions, denoted as \(\hat{\lambda}^{(1)}(\hat{x})\) and \(\hat{\lambda}^{(2)}(\hat{x})\). The two sets of spinors come from NS-R \(8_v \otimes 8_c\) and R-NS \(8_c \otimes 8_v\) sectors.

We now briefly review the K3 reduction of type IIB theory to set up our notations and conventions. A ten dimensional field will be decomposed into a six dimensional (R\(^6\)) external field and a four dimensional (K3) internal field as \(\hat{\Phi}(\hat{x}) = \tilde{\Phi}(\tilde{x}) \otimes \bar{\Phi}(\bar{y})\) where \(\tilde{x}^\mu\) is the coordinate on R\(^6\) and \(\bar{y}^m\) is the coordinate on K3. \(\tilde{\Phi}(\tilde{x})\) will correspond to the massless particle on R\(^6\) if \(\tilde{\Phi}(\bar{y})\) satisfies certain differential equations on the internal manifold determined by the equations of motion of the ten dimensional fields. Massless spectrum of the reduced theory then corresponds to the number of solutions of these differential equations. We also need to know certain properties of K3 surface, for example, it admits a Ricci-flat, anti-self-dual Riemann curvature and has the following non-zero Betti numbers, \(b_0 = b_4 = 1\), \(b_2^+ = 3\), \(b_2^- = 19\), where \(b_2^+ (b_2^-)\) denotes the number of self-dual (anti-self-dual) harmonic \((1, 1)\) forms. Using this general strategy and the properties of the K3 surface [25], we find that the ten dimensional metric \(\tilde{g}_{\mu\nu}(\tilde{x})\) yields one six dimensional graviton \(\tilde{g}_{\mu\nu}(\tilde{x}) \otimes I\) (corresponding to one \((b_0 = 1)\) constant zero mode on K3), no gauge fields \(\tilde{A}_\mu(\tilde{x}) \otimes \tilde{f}_m(\tilde{y})\) (since \(b_1 = 0\)) and 58 scalars \(\tilde{a}_1(\tilde{x}) \otimes \tilde{h}_{mn}(\tilde{y})\). Here \(\tilde{h}_{mn}(\tilde{y})\) satisfies the Lichnerowicz equation and the Lichnerowicz operator has \(b_2^+ \times b_2^- + 1 = 58\) zero modes.

\*We will denote the ten dimensional fields and coordinates with a ‘hat’, six-dimensional objects with a ‘tilde’ and the two dimensional fields without any accent. The objects on the internal manifold will be denoted with a ‘bar’.
[26,27] on K3 corresponding to the number of gauge invariant deformations of the metric preserving the Ricci flatness condition. Similarly, from $\hat{B}_{\mu\nu}^{(1)}(x)$ we get one antisymmetric tensor $\tilde{B}_{\mu\nu}^{(1)}(x) \otimes I$ in six dimensions and 22 scalars $\tilde{b}_1(x) \otimes \tilde{f}_{\bar{m}n}(\bar{y})$, since there are 22 harmonic two-forms on K3 altogether. The ten dimensional dilaton $\hat{\phi}^{(1)}(x)$ simply gives a scalar $\tilde{\phi}^{(1)}(x) \otimes I$ in six dimensions. Having thus described the reduction of the NS-NS sector, we now turn to the R-R sector. The scalar $\hat{\phi}^{(2)}(x)$ gives a six dimensional scalar $\tilde{\phi}^{(2)}(x) \otimes I$. The antisymmetric tensor $\tilde{B}_{\mu\nu}^{(2)}(x)$ gives one antisymmetric tensor $\tilde{B}_{\mu\nu}^{(2)}(x) \otimes I$ and 22 scalars $\tilde{b}_2(x) \otimes \tilde{f}_{\bar{m}n}(\bar{y})$, whereas, the four-form anti-self-dual tensor $\hat{A}_{\mu\nu\rho\sigma}^-(x)$ gives 19 self-dual two-forms $\hat{A}_{\mu\nu}^-(x) \otimes \tilde{f}_{\bar{m}n}(\bar{y})$ and 3 anti-self-dual two forms $\hat{A}_{\mu\nu}^-(x) \otimes \tilde{f}_{\bar{m}n}(\bar{y})$ in six dimensions. Finally, the anti-self-dual four-form will give one scalar $\tilde{a}_2(x)$ half of which comes from $\hat{A}_{m\bar{n}\bar{p}\bar{q}}^-$ and another half by taking the Hodge-dual of $\hat{A}_{\mu\nu\rho\sigma}^-$. We will not get any gauge field (since $b_3 = 0$) or any three-form tensor (since $b_1 = 0$) from it.

Now, we consider the fermionic sector. Here we will make use of the index theory on K3 surface. It is well-known that the Dirac operator on K3 surface has two zero modes corresponding to two (0, p) forms (0, 0) and (0, 2), whereas the Rarita-Schwinger operator has 40 zero modes corresponding to twice the number of 20 harmonic (1, 1) forms [23] i.e. $I_{1/2}(K3) = 2$ and $I_{3/2}(K3) = -40$. Here $I$ counts the number of positive chirality minus the number of negative chirality zero modes. So, the ten dimensional gravitino of negative chirality $\tilde{\psi}_\mu^{(1)-}(x)$ will yield one six dimensional gravitino of negative chirality $\tilde{\psi}_\mu^{(1)-}(x) \otimes \tilde{\eta}_i^{1/2}(\bar{y})$, counted in units of Weyl spinor and 20 positive chirality spin 1/2 Weyl fermions $\tilde{\chi}^{(1)+}(x) \otimes \tilde{\eta}_m(x)$. Finally, one positive chirality spin 1/2 M-W fermion $\tilde{\lambda}^{(1)+}(x)$ will give a positive chirality Weyl fermion $\tilde{\lambda}^{(1)+}(x) \otimes \tilde{\eta}_j^{1/2}(\bar{y})$ in six dimensions. Thus we have obtained the six dimensional fermions from the NS-R sector. From the R-NS sector we get the identical spectrum i.e. one gravitino of negative chirality and 21 positive chirality spin 1/2 Weyl fermions.

So, the complete spectrum of type IIB theory on K3 consists of one graviton $(\tilde{g}_{\mu\nu}(\bar{x}))$, two gravitinos of negative chirality $(\tilde{\psi}_\mu^{(1)-}(\bar{x}), \tilde{\psi}_\mu^{(2)-}(\bar{x}))$, 105 scalars $(58 \tilde{a}_1(x) + 22 \tilde{b}_1(x) + 22 \tilde{b}_2(x) + 1 \tilde{\phi}^{(1)}(x) + 1 \tilde{a}_2(x) + 1 \tilde{\phi}^{(2)}(x))$, 21 self-dual antisymmetric tensors $(1 \tilde{B}_{\mu\nu}^{(1)+}(x) + 1 \tilde{B}_{\mu\nu}^{(2)+}(x) + 19 \tilde{A}_{\mu\nu}^+(x))$, 5 anti-self-dual antisymmetric tensors $(1 \tilde{B}_{\mu\nu}^{(1)-}(x) + 1 \tilde{B}_{\mu\nu}^{(2)-}(x) + \tilde{A}_{\mu\nu}^-(x))$ and 42 positive chirality spin 1/2 Weyl fermions $(20 \tilde{\chi}^{(1)+}(x) + 1 \tilde{\lambda}^{(1)+}(x) + 20 \tilde{\chi}^{(2)+}(x) + 1 \tilde{\lambda}^{(2)+}(x))$. It can be easily checked that the spectrum is free of gravitational anomaly [28] since they satisfy $I_{3/2} : I_{1/2} : I_A = -2 : 42 : 16 = 1 : -21 : -8$. Here $I_{3/2}$, $I_{1/2}$ and $I_A$ are respectively the gravitational anomalies for spin 3/2 (posi-
tive chirality minus negative chirality) spin 1/2 and antisymmetric tensor (self-dual minus anti-self-dual) fields. Note that the spectrum has chiral N=2 supersymmetry and it contains one gravity multiplet (˜\(\mathcal{M}\)-theory compactification on the orbifold T_\(g\) it contains one gravity multiplet (˜\(\mathcal{N}\)us anti-self-dual) fields. Note that the spectrum has chiral N=2 supersymmetry and negative chirality minus negative chirality) spin 1/2 and antisymmetric tensor multiplets (\(\hat{A}^{+}_{\mu\nu}, \hat{A}^{-}_{\alpha}\), 5 \(\hat{\phi}^{(I,J)}\)) where I, J are USp(4) indices and \(\alpha\) is the spinor index for which up(down) means antichiral(chiral) spinors. Also note that the 105 scalars parametrize the moduli space O(21, 5)/(O(21) \(\times\) O(5)). Identical spectrum has also been obtained from M-theory compactification on the orbifold T^5/Z_2 [18, 19].

We now follow this procedure to obtain the massless spectrum of the two dimensional reduction of type IIB theory on K3 \(\times\) K3. We note that in 2d, vectors or higher rank tensors do not have any propagating degree of freedom and therefore, we will not count them in the spectrum. Also, both the graviton and the gravitino have formally \(-1\) degree of freedom and so, graviton has to be compensated by a scalar whereas the gravitinos would have to be compensated by spin 1/2 M-W fermions. Keeping these in mind, we find that the six dimensional graviton will give a 2d graviton \(\hat{g}_{\mu\nu}(\vec{x}) \rightarrow g_{\mu\nu}(x) \otimes \mathbf{1}\) and 58 scalars \(\hat{g}_{\mu\nu}(\vec{x}) \rightarrow c_{1}(x) \otimes \bar{h}_{mn}(\vec{y})\). 58 six dimensional scalars \(\hat{a}_{1}(\vec{x})\) will give 58 two dimensional scalars \(c_{2}(x) \otimes \mathbf{1}\). The dilaton \(\tilde{\phi}^{(1)}(\vec{x})\) will yield a single scalar in 2d \(\phi_{1}(x) \otimes \mathbf{1}\).

The antisymmetric tensor \(\hat{B}_{\mu\nu}^{(1)}(\vec{x})\) gives 22 two dimensional scalars \(c_{3}(x) \otimes \bar{f}_{mn}(\vec{y})\) and 22 six dimensional scalars \(\hat{b}_{1}(\vec{x})\) give another set of 22 scalars \(c_{4}(x) \otimes \mathbf{1}\) in 2d. Similarly, in the R-R sector, we get from \(\tilde{\phi}^{(2)}(\vec{x})\), one two dimensional scalar \(\phi_{2}(x) \otimes \mathbf{1}\) and from \(\hat{B}_{\mu\nu}^{(2)}(\vec{x})\), we get 22 scalars \(c_{5}(x) \otimes \bar{f}_{mn}(\vec{y})\) and 22 six dimensional scalars \(\hat{b}_{2}(\vec{x})\) yield one more set of 22 scalars \(c_{6}(x) \otimes \mathbf{1}\). Now from 19 self-dual antisymmetric two-forms \(\hat{A}^{+}_{\mu\nu}(\vec{x})\) we get \(19 \times 19 = 361\) chiral bosons \(A_{1}^{+}(x) \otimes \bar{f}_{mn}(\vec{y})\) and \(19 \times 3 = 57\) antichiral bosons \(A_{1}^{-}(x) \otimes \bar{f}_{mn}(\vec{y})\) in 2d. Similarly, from 3 six dimensional anti-self-dual antisymmetric two-forms \(\hat{A}_{\mu\nu}^{-}(\vec{x})\) we obtain \(3 \times 19 = 57\) antichiral bosons \(A_{2}^{+}(x) \otimes \bar{f}_{mn}(\vec{y})\) and \(3 \times 3 = 9\) chiral bosons \(A_{2}^{-}(x) \otimes \bar{f}_{mn}(\vec{y})\). Finally, from the six dimensional scalar \(\hat{a}_{2}(\vec{x})\), we get one more scalar \(c_{7}(x) \otimes \mathbf{1}\).

So, collecting all the massless particles in the bosonic sector we have one graviton \(g_{\mu\nu}(x)\), 577 chiral bosons (58 \(c_{1}^{-}(x)\) + 58 \(c_{2}^{-}(x)\) + 1 \(\phi_{1}^{-}(x)\) + 22 \(c_{3}^{-}(x)\) + 22 \(c_{4}^{-}(x)\) + 1 \(\phi_{2}^{-}(x)\) + 22 \(c_{5}^{-}(x)\) + 22 \(c_{6}^{-}(x)\) + 361 \(A_{1}^{-}(x)\) + 9 \(A_{2}^{-}(x)\) + 1 \(c_{7}^{-}(x)\)) and 321 antichiral bosons (58 \(c_{1}^{+}(x)\) + 58 \(c_{2}^{+}(x)\) + 1 \(\phi_{1}^{+}(x)\) + 22 \(c_{3}^{+}(x)\) + 22 \(c_{4}^{+}(x)\) + 1 \(\phi_{2}^{+}(x)\) + 22 \(c_{5}^{+}(x)\) + 22 \(c_{6}^{+}(x)\) + 57 \(A_{1}^{+}(x)\) + 57 \(A_{2}^{+}(x)\) + 1 \(c_{7}^{+}(x)\)). In counting the bosons, note that, we have split the 2d bosons into chiral and antichiral components. Also note that if we compensate \(-1\) degree of freedom of the two dimensional graviton by a scalar, we will be left with 576 chiral and 320 antichiral bosons.
We next turn our attention to the fermionic sector. In this case we will apply exactly the same procedure as the six dimensional reduction on K3. Since in 2d, we have M-W spinors we will count the spinors in terms of them unlike the case in 6d, where they were counted in terms of Weyl spinors. Since the Dirac operator and the Rarita-Schwinger operator on K3 has 2 and 40 zero modes respectively, we get from two six dimensional gravitinos $\tilde{\psi}_\mu^{(1)}(\bar{x})$ and $\tilde{\psi}_\mu^{(2)}(\bar{x})$, eight two dimensional gravitinos of negative chirality $\psi^{(1)}_\mu(\bar{x}) \otimes \bar{n}^{+}_{1/2}(\bar{y})$ and $\psi^{(2)}_\mu(\bar{x}) \otimes \bar{n}^{-}_{1/2}(\bar{y})$. Note that there is a pair of $\bar{n}^{+}_{1/2}(\bar{y})$ and each Weyl spinor splits up into two M-W spinors. Also, we get 160 spin 1/2 M-W fermions of positive chirality $\chi^{(3)}(x) \otimes n^{-}_{1/2}(\bar{y})$ and $\chi^{(4)}(x) \otimes n^{-}_{1/2}(\bar{y})$. Note here, that there are 40 $\bar{n}^{-}_{1/2}(\bar{y})$ and Weyl spinors split up into two M-W spinors. Also, from 40 positive chirality spin 1/2 Weyl fermions $\chi^{(1)}(x) \otimes \bar{n}^{+}_{1/2}(\bar{y})$ and $\chi^{(2)}(x) \otimes \bar{n}^{-}_{1/2}(\bar{y})$ of positive chirality in 2d. Finally, from two Weyl fermions of positive chirality $\tilde{\lambda}^{(1)}(\bar{x})$ and $\tilde{\lambda}^{(2)}(\bar{x})$, we get eight positive chirality spin 1/2 M-W fermions $\lambda^{(1)}(x) \otimes \bar{n}^{+}_{1/2}(\bar{y})$ and $\lambda^{(2)}(x) \otimes \bar{n}^{-}_{1/2}(\bar{y})$. So, now counting all the fermions including NS-R and R-NS sectors we have eight gravitinos of negative chirality $(4 \psi^{(1)}_\mu(\bar{x}) + 4 \psi^{(2)}_\mu(\bar{x}))$ and 328 positive chirality spin 1/2 M-W fermions $(80 \chi^{(3)}(x) + 80 \chi^{(4)}(x) + 80 \chi^{(1)}(x) + 80 \chi^{(2)}(x) + 4 \lambda^{(1)}(x) + 4 \lambda^{(2)}(x))$. Note that we have a chiral N=8 supersymmetry since we got eight gravitinos in 2d, which is consistent, because we started out with a chiral N=32 supersymmetric theory in ten dimensions which would have given a chiral N=32 supersymmetry if we considered a toroidal compactification. But since K3 is half-flat it preserves only half of the original supersymmetry and therefore on K3 × K3 reduction only 1/4 th of the original supersymmetry should remain and thus we got a chiral N=8 supersymmetry in 2d. In terms of chiral N=8 supermultiplets the spectrum can be arranged as a gravity multiplet $(g_{\mu\nu}, 8 \psi^-_\mu, 8 \lambda^+, \phi)$ and 40 matter multiplets $(8 \lambda^+, 8 \phi^+)$. The rest of the chiral bosons 576 $\phi^-$ remain inert under supersymmetry. We have generically denoted the scalars as $\phi$ and spin 1/2 M-W fermions as $\lambda$.

Now we show that the 2d model we have obtained is free of gravitational anomaly [28]. The gravitational anomaly associated with spin 3/2 field (chiral minus antichiral) is given by $I_{3/2} = \frac{23}{24} p_1$, that of spin 1/2 field is $I_{1/2} = -\frac{1}{24} p_1$ and for the chiral minus antichiral boson is $I_s = -\frac{1}{24} p_1$, with $p_1$ being the anomaly polynomial. One can form an anomaly-free combination from them as $I_{3/2} - m I_{1/2} + (23 + m) I_s = 0$, where $m$ is any integer. So, the spectrum will be anomaly-free if they satisfy $I_{3/2} : I_{1/2} : I_s = 1 : -m : (23 + m)$. Note
here that the fermions in this formula are complex (Weyl) fermions and their number will be half of the M-W fermions we have counted. The two dimensional spectrum we have obtained satisfies $I_{3/2} : I_{1/2} : I_s = -4 : 164 : -256 = 1 : -41 : (23 + 41)$. We thus have a consistent two dimensional compactifications of type IIB theory on $K3 \times K3$.

III. An Orbifold Projection:

In this section, we consider an orbifold projection of the above two dimensional model of type IIB theory on $K3 \times K3$. The orbifold group of transformation we consider consists of the product $(-1)^{F_L} \cdot \sigma$, where $F_L$ is the space time fermion number operator in the left moving sector of the world-sheet and $\sigma$ denotes the involution on the $K3$ surface. Since type IIB theory is invariant under these operations we find the orbifold model by projecting out this symmetry where only the massless states which remain invariant under these operations will be retained. We will first compute the massless spectrum originating in the untwisted sector and then point out how the twisted sector states could be obtained in analogy with ref.[29]. Finally, we will show that the spectrum thus obtained is free of gravitational anomaly.

By constructing a special $K3$ surface it has been shown in ref.[30] how the involution $\sigma$ acts on various harmonic forms. Making use of the Lefschetz fixed point theorem, it is found that out of twenty harmonic $(1, 1)$ forms only twelve remain invariant and eight change sign under the action of $\sigma$. Out of these twelve, eleven are anti-self-dual and the remaining one Kahler $(1, 1)$ form and the other two $(2, 0)$ and $(0, 2)$ two-forms are self-dual. Thus we have $b_2^+ = 3$, $b_2^- = 11$, which remain invariant and the other eight $b_2^-$ change sign. Now we consider the action of $(-1)^{F_L}$ on various fields of type IIB theory. Since $(-1)^{F_L}$ has the effect of changing the sign of a fermion in the left moving sector, it is clear from (1), that all the states in the R-R sector will change sign, leaving the NS-NS sector invariant. Whereas the fermions originating in the R-NS sector will change sign leaving the NS-R sector invariant. Thus summarizing:

\[
(-1)^{F_L} : \quad \hat{g}_{\mu\nu} \rightarrow \hat{g}_{\mu\nu}; \quad \hat{\phi}^{(1)} \rightarrow \hat{\phi}^{(1)}; \quad \hat{B}^{(1)}_{\mu\nu} \rightarrow \hat{B}^{(1)}_{\mu\nu} \\nonumber \\
\hat{\phi}^{(2)} \rightarrow -\hat{\phi}^{(2)}; \quad \hat{B}^{(2)}_{\mu\nu} \rightarrow -\hat{B}^{(2)}_{\mu\nu}; \quad \hat{A}_{\mu\nu\rho\sigma} \rightarrow -\hat{A}_{\mu\nu\rho\sigma} \\nonumber \\
\hat{\psi}_{\mu}^{(1)} \rightarrow \hat{\psi}_{\mu}^{(1)}; \quad \hat{\lambda}^{(1)} \rightarrow +\hat{\lambda}^{(1)} \quad \nonumber \\
\hat{\psi}_{\mu}^{(2)} \rightarrow -\hat{\psi}_{\mu}^{(2)}; \quad \hat{\lambda}^{(2)} \rightarrow -\hat{\lambda}^{(2)} \quad \nonumber 
\]

(2)
It is now straightforward to compute the massless spectrum of this orbifold model which remain invariant under the combined operations \((-1)^{F_L} \cdot \sigma\). We have seen before that the ten dimensional metric gives 58 scalars on K3 reduction corresponding to the number of gauge invariant deformations of the metric. In this case, the number of scalars which would remain invariant under the involution \(\sigma\) would be 
\[b_2^+ \times b_2^- + 1 = 34.\]
So, on K3 \(\times\) K3 reduction we will get \((34 + 34)\) scalars, denoted by \(d_1(x)\) from \(\hat{g}_{\hat{\mu}\hat{\nu}}(\hat{x})\) which will remain invariant under \((-1)^{F_L} \cdot \sigma\). We will also get a two dimensional graviton \(g_{\mu\nu}(x)\).
From \(\hat{\phi}^{(1)}(\hat{x})\) we will get one scalar \(d_2(x)\) in 2d. Also, from \(\hat{B}^{(1)}_{\hat{\mu}\hat{\nu}}(\hat{x})\), we get \((14 + 14)\) scalars \(d_3(x)\), since 14 of the 22 harmonic two-forms are invariant. \(\hat{\phi}^{(2)}(\hat{x})\) will not give a scalar since the \((0, 0)\) form is invariant under \(\sigma\). From \(\hat{B}^{(2)}_{\hat{\mu}\hat{\nu}}(\hat{x})\) we get \((8 + 8)\) scalars \(d_4(x)\), since eight harmonic \((1, 1)\) forms change sign under \(\sigma\). To count the number of invariant massless states from \(\hat{A}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}(\hat{x})\), we first note that there will not be any scalar in 6d or in 2d coming from it since it changes sign under \((-1)^{F_L}\). However, scalars would be formed out of the six dimensional two-forms. Under first K3 reduction we found that there are 19 self-dual two-forms and 3 anti-self-dual two-forms out of which 8 self-dual two-forms change sign under \(\sigma\). So, in the next K3 reduction we will obtain \(8 \times 11 = 88\) chiral bosons \(d_5^-(x)\) and \(8 \times 3 = 24\) antichiral bosons \(d_6^+(x)\). Also, from the 11 self-dual two-forms which remained invariant under \(\sigma\) in the first K3 reduction, we will get \(11 \times 8 = 88\) chiral bosons \(d_7^-(x)\) and from 3 anti-self-dual two-forms, which also remained invariant under \(\sigma\) in the first K3 reduction, we will get \(3 \times 8 = 24\) antichiral bosons \(d_8^+(x)\). So, counting all the states in the bosonic sector we have apart from a two dimensional graviton, 289 chiral bosons \((68 d_1^-(x) + 1 d_5^-(x) + 28 d_5^+(x) + 16 d_4^-(x) + 88 d_5^-(x) + 88 d_7^-(x))\) and 161 antichiral bosons \((68 d_1^+(x) + 1 d_2^+(x) + 28 d_3^-(x) + 16 d_4^+(x) + 24 d_6^+(x) + 24 d_8^+(x))\).
If we compensate \(-1\) degree of freedom of two dimensional graviton by a scalar we will be left with 288 chiral and 160 antichiral bosons.

In order to count the fermionic states, we make use of the index theory of K3 surface as before. Corresponding to two invariant \((0, p)\) forms \((0, 0)\) and \((0, 2)\), the Dirac operator has two zero modes and therefore we get 4 gravitinos from \(\hat{\psi}_{\hat{\mu}}^{(1)}(\hat{x})\) of negative chirality in 2d. Also, from the 12 invariant harmonic \((1, 1)\) forms we get \((48 + 48)\) spin 1/2 M-W fermions of positive chirality \(\chi^{(1)}(x)\) in 2d. From \(\hat{\lambda}^{(1)}(\hat{x})\) we get 4 spin 1/2 M-W fermions of positive chirality \(\lambda^{(1)}(x)\). The gravitino from R-NS sector \(\hat{\psi}_{\hat{\mu}}^{(2)}(\hat{x})\) does not give any gravitino in 2d since it changes sign under \((-1)^{F_L}\), whereas it gives \((32 + 32)\) spin 1/2 M-W fermions of positive chirality \(\chi^{(2)}(x)\), corresponding to the 8 harmonic \((1, 1)\)
forms which change sign under $\sigma$. Finally, $\hat{\lambda}^{(2)}(\hat{x})$ does not give rise to any M-W spin 1/2 fermions. So, altogether we have 4 gravitinos of negative chirality and 164 spin 1/2 M-W fermions of positive chirality $(96\chi^{(1)}+(x)+4\lambda^{(1)}+(x)+64\chi^{(2)}+(x))$. Taking into account only the untwisted sector states, we verify that the spectrum is already anomaly-free since in this case we have, $I_{3/2} : I_{1/2} : I_s = -2 : 82 : -128 = 1 : -41 : 64 = 1 : -41 : (23 + 41)$. We also note that this model has a chiral N=4 supersymmetry because of the presence of 4 gravitinos of negative chirality.

It is known from M-theory compactifications [21] that the condition of gravitational anomaly cancellation by itself is not always powerful enough to determine the massless spectrum completely. Even in the case of some orientifold models of type IIB theory on K3, the untwisted sector itself becomes anomaly-free [20]. So, one has to rely on some other principle to obtain the massless states from the twisted sector. The orbifold model of type IIB theory on $K3 \times K3$, that we considered is also anomaly-free, as we have seen, if we consider only the untwisted sector states. In order to find the twisted sector states, we will follow closely the arguments given by Sen in ref.[21,29]. We first note that orbifold of $K3 \times K3$ has 64 fixed points — eight from each K3. Near each of these fixed points the space would look like $T^8/(-1)^{F_L} \cdot I_8$ and therefore the physics in the neighborhood of those fixed points would be the same for type IIB theory either on $(K3 \times K3)/(-1)^{F_L} \cdot \sigma$ or on $T^8/(-1)^{F_L} \cdot I_8$. Here $I_8$ has the effect of changing the sign of all the coordinates of $T^8$. Fortunately, the twisted sector states of type IIB theory on $T^8/(-1)^{F_L} \cdot I_8$ have already been computed by Sen in the second reference [29] just mentioned. It has been argued there by converting this theory to type IIA theory on $T^8/I_8$ with an $R \to 1/R$ transformation on one of the circles of $T^8$ and through a tadpole calculation [31, 29] that the twisted sector states of this theory live on the sixteen elementary type IIB strings (NS one-branes) each of which supports a vector multiplet of two dimensional N=16 supersymmetry algebra with eight scalar components labelling the location of these strings on the internal space. Since the orbifold of $K3 \times K3$ has 1/4th as many fixed points as the orbifold of $T^8$, the twisted sector states in this case live on four elementary type IIB strings. It is, therefore, clear that just like what happens for the orientifold model of type IIB theory on K3, the spectrum is anomaly-free, even when we include the twisted sector states, as expected.
IV. An Orientifold Projection:

We now consider an orientifold model [32–35] of type IIB theory on K3 × K3. The orientifold group that we consider consists of a product of the involution on K3 surface and the orientation reversal of the world-sheet which is denoted as Ω. The orientation reversal basically interchanges the left moving modes of the string with the right moving modes [36]. Under the operation Ω, it is clear from (1), that the graviton and the dilaton in the NS-NS sector will remain invariant, whereas the antisymmetric tensor $\hat{B}_{\mu\nu}(\hat{x})$ will change sign since it involves an antisymmetric combination of the left moving and the right moving oscillators. On the other hand, the bosonic fields in the R-R sector which are formed out of symmetric combination of the left-moving and the right-moving modes will change sign because the vertex operator contains a product of spin-fields in the left and the right-moving sectors and gives a sign under Ω. So, the scalar $\hat{\phi}^{(2)}(\hat{x})$ and $\hat{A}_{\mu\nu\rho\sigma}(\hat{x})$ will change sign whereas $\hat{B}_{\mu\nu}^{(2)}(\hat{x})$ will remain invariant. In the fermionic sector, the fermions formed out of the positive combination of the NS-R and the R-NS sector $(\mathbf{8}_L \otimes \mathbf{8}_c^R + \mathbf{8}_L \otimes \mathbf{8}_c^R)$ remain invariant whereas the fermions formed out of negative combination $(\mathbf{8}_L \otimes \mathbf{8}_c^R - \mathbf{8}_c^L \otimes \mathbf{8}_c^R)$ change sign. So, summarizing:

$$\Omega : \hat{g}_{\mu\nu} \rightarrow \hat{g}_{\mu\nu}; \quad \hat{\phi}^{(1)} \rightarrow \hat{\phi}^{(1)}; \quad \hat{B}_{\mu\nu}^{(1)} \rightarrow -\hat{B}_{\mu\nu}^{(1)}$$
$$\hat{\phi}^{(2)} \rightarrow -\hat{\phi}^{(2)}; \quad \hat{B}_{\mu\nu}^{(2)} \rightarrow \hat{B}_{\mu\nu}^{(2)}; \quad \hat{A}_{\mu\nu\rho\sigma} \rightarrow -\hat{A}_{\mu\nu\rho\sigma}$$
$$\left(\hat{\psi}_\mu^{(1)} - \hat{\psi}_\mu^{(2)}\right) \rightarrow \left(\hat{\psi}_\mu^{(1)} + \hat{\psi}_\mu^{(2)}\right); \quad \left(\hat{\psi}_\mu^{(1)} - \hat{\psi}_\mu^{(2)}\right) \rightarrow -\left(\hat{\psi}_\mu^{(1)} - \hat{\psi}_\mu^{(2)}\right)$$
$$\left(\hat{\lambda}^{(1)} + \hat{\lambda}^{(2)}\right) \rightarrow \left(\hat{\lambda}^{(1)} + \hat{\lambda}^{(2)}\right); \quad \left(\hat{\lambda}^{(1)} + \hat{\lambda}^{(2)}\right) \rightarrow -\left(\hat{\lambda}^{(1)} + \hat{\lambda}^{(2)}\right)$$

As in the orbifold model we will first compute the states in the untwisted sector under the combined operation $\Omega \cdot \sigma$ and then mention, in analogy with ref.[29], how to compute the twisted sector states. We again find that this orientifold model is free of gravitational anomaly.

The counting of states in this case proceeds exactly the same way as in the orbifold model with a few differences. As mentioned before, the ten dimensional graviton $\hat{g}_{\mu\nu}(\hat{x})$ gives a two dimensional graviton and $(34 + 34)$ scalars $e_1(x)$ from two K3 which remains invariant under $\sigma$. We get one more scalar $e_2(x)$ from $\hat{\phi}^{(1)}(\hat{x})$ in 2d. Now since $\hat{B}_{\mu\nu}^{(1)}(\hat{x})$ changes sign under $\Omega$ we will get $(8 + 8)$ scalars $e_3(x)$ from the reduction on K3 × K3, corresponding to the eight harmonic (1, 1) forms which change sign under $\sigma$. In the R-R sector, $\hat{\phi}^{(2)}(\hat{x})$ will not give a scalar, whereas from $\hat{B}_{\mu\nu}^{(2)}(\hat{x})$, we get $(14 + 14)$ scalars $e_4(x)$
corresponding to 14 two forms on K3 that remain invariant under \( \sigma \). Finally, the counting of states from \( \hat{A}_{\mu\nu\rho\sigma}(\hat{x}) \) is exactly the same as in the orbifold model and therefore, we get \((8 \times 11 + 11 \times 8) = 176\) chiral bosons \( e^{-5}_7(x) \) and \((8 \times 3 + 3 \times 8) = 48\) antichiral bosons \( e^{+5}_6(x) \) in 2d. Collecting all the bosonic states we have one graviton \( g_{\mu\nu}(x) \), 289 chiral bosons \((68 e^{-1}_1(x) + 1 e^{-2}_1(x) + 16 e^{-3}_3(x) + 28 e^{-4}_4(x) + 176 e^{-5}_5(x))\) and 161 antichiral bosons \((68 e^{+1}_1(x) + 1 e^{+2}_1(x) + 16 e^{+3}_3(x) + 28 e^{+4}_4(x) + 48 e^{+5}_6(x))\).

The massless spectrum for the fermionic sector can again be obtained by applying index theory on K3 surface. Thus, we get from the combination \((\hat{\psi}_{\mu}^{(1)} - \hat{\psi}_{\mu}^{(2)})(\hat{x})\), four M-W gravitinos \( \psi^-_{\mu}(x) \) of negative chirality in 2d, corresponding to the two invariant \((0, p)\) forms on K3. Corresponding to 12 invariant harmonic \((1, 1)\) forms, we get \((48 + 48)\) spin 1/2 M-W fermions of positive chirality \( \chi^{(1)+}(x) \) in 2d. Similarly, we also get 4 spin 1/2 M-W fermions of positive chirality from \((\hat{\lambda}^{(1)+} + \hat{\lambda}^{(2)+})(\hat{x})\), denoted as \( \lambda^{(1)+}(x) \), corresponding to two invariant \((0, p)\) forms. Finally, from \((\hat{\psi}_{\mu}^{(1)} - \hat{\psi}_{\mu}^{(2)})(\hat{x})\), we get \((32 + 32)\) spin 1/2 M-W fermions of positive chirality, denoted as \( \chi^{(2)+}(x) \), corresponding to 8 harmonic \((1, 1)\) forms which change sign under \( \sigma \). Collecting all the fermionic states we have four negative chirality gravitinos \( \psi^-_{\mu}(x) \) and 164 spin 1/2 M-W fermions of positive chirality \((96 \chi^{(1)+}(x) + 4 \lambda^{(1)+}(x) + 64 \chi^{(2)+}(x))\). Thus we again have a chiral N=4 supersymmetry and the cancellation of the gravitational anomaly is exactly the same as shown before.

Now in order to construct the twisted sector states, we again use the similar argument as was considered for the orbifold model. In this case, the space near the 64 fixed points would look like \( T^8/\Omega \cdot I_8 \). The twisted sector states of type IIB theory on \( T^8/\Omega \cdot I_8 \) have also been obtained by Sen [29]. In this case, it has been found that the twisted sector states live on 16 R-R one-branes (type IIB strings) and each of them supports a vector multiplet of two dimensional N=16 supersymmetry algebra in which the eight scalars label the positions of the R-R one-branes on the internal space. Therefore, viewed in terms of type IIB theory on \((K3 \times K3)/\Omega \cdot I_8\), the twisted sector states live on four R-R one-branes and thus, once again we find that the gravitational anomaly cancels even if we include the twisted sector states as expected. The massless spectrum matches precisely for both the orbifold and the orientifold models including the untwisted as well as the twisted sector states. Note that this is quite expected since under the ten dimensional SL(2, \(Z\)) symmetry of type IIB theory, \((-1)^F_L\) gets precisely converted to the world-sheet parity transformation \( \Omega \).

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We here briefly point out how the M-theory duals of the various type IIB compactifications, we have considered in this paper, can be obtained by using a chain of duality arguments [21]. Let us first consider the type IIB theory on $K3 \times K3$. By choosing a special $K3$ surface, this theory is equivalent to type IIB theory on $(K3 \times T^4)/\{1, \mathcal{I}_4\}$ and then making an $R \to 1/R$ duality transformation on one of the circles of $T^4$, we convert this to type IIA theory where $\mathcal{I}_4$ changes to $(-1)^{F_L} \cdot \mathcal{I}_4$. So, the above model is dual to type IIA theory on $(K3 \times T^4)/\{1, (-1)^{F_L} \cdot \mathcal{I}_4\}$. Since M-theory on $S^1$ is the strong coupling limit of type IIA string theory when the radius of the circle is large, this type IIA model is dual to M-theory on $(K3 \times T^5)/\{1, J_5\}$, where $J_5$ has the effect of changing the sign of all the coordinates of $T^5$ and the antisymmetric three-form present in M-theory.

It is easy to verify that $J_1$ in M-theory has the same effect as $(-1)^{F_L}$ in type IIA theory. The bosonic sector of M-theory compactification on $(K3 \times T^5)/\{1, J_5\}$ has been obtained in ref.[37] and indeed was found to have the same spectrum as type IIB on $K3 \times K3$. Next we consider the orbifold model of type IIB theory on $(K3 \times K3)/\{1, (-1)^{F_L} \cdot \sigma\}$. Since the orientifold model we have considered in this section is dual to the orbifold model, they will give the identical M-theory dual. By using a chain of duality arguments it has been shown in ref.[21] that the type IIB theory on $K3/\{1, (-1)^{F_L} \cdot \sigma\}$ is dual to type IIA theory on the same orbifold where the massless states from the twisted sector get interchanged with those of the untwisted sector. Since $(-1)^{F_L}$ in type IIA theory has the same effect as $J_1$ in M-theory, we conclude that type IIB theory on $(K3 \times K3)/\{1, (-1)^{F_L} \cdot \sigma\}$ is dual to M-theory compactification on $(K3 \times K3 \times S^1)/\{1, J_1 \cdot \sigma\}$. It will be interesting to verify how the massless spectrum precisely arises in this M-theory compactification. Certain relevant two dimensional compactifications of M-theory have been considered in [38].

V. Conclusions:

To summarize, we have considered in this paper a two dimensional reduction of type IIB theory on $K3 \times K3$. We found that this gives a consistent compactification of type IIB theory since the resulting spectrum is free of gravitational anomaly. We have also considered an orbifold and an orientifold projection of the above model and computed the massless spectrum for both the untwisted and the twisted sectors. We found that

* This duality has also been conjectured in [19].
the orbifold and the orientifold models have chiral N=4 supersymmetry and the resulting spectrum in the two cases match precisely including the untwisted and the twisted sector states. We have also shown that in both cases, the models are free of gravitational anomaly. The matching of the spectrum for these models is a consequence of the SL(2, Z) symmetry of the ten dimensional type IIB theory. We have also briefly pointed out the M-theory duals of the various type IIB compactifications considered in this paper. By considering other orbifold/orientifold of type IIB theory on K3 × K3, it will be interesting to see how to obtain models with less number of supersymmetries and study their M-theory as well as F-theory duals.

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References:

1. M. Duff, R. R. Khuri and J. X. Lu, Phys. Rep. 259 (1995) 213.
2. A. Sen, Int. J. Mod. Phys. A9 (1994) 3707.
3. E. Witten, Nucl. Phys. B443 (1995) 85.
4. C. Hull and P. Townsend, Nucl. Phys. B438 (1995) 109.
5. J. H. Schwarz, Phys. Lett. B367 (1996) 97.
6. P. Horava and E. Witten, Nucl. Phys. B460 (1996) 506.
7. F. Aldabe, *Heterotic and Type I Strings from Twisted Supermembranes*, hep-th/9603183.
8. O. Aharony, *String Theory Dualities from M-theory*, hep-th/9604103.
9. M. J. Duff, R. Minasian and E. Witten, *Evidence for Heterotic/ Heterotic Duality*, hep-th/9601036.
10. B. S. Acharya, *N=1 Heterotic/M-Theory Duality and Joyce Manifolds*, hep-th/9603034.
11. A. Das and S. Roy, *On M-Theory and the Symmetries of Type II String Effective Actions*, hep-th/9605073.
12. J. H. Schwarz, Phys. Lett. B360 (1995) 13.
13. C. M. Hull, Phys. Lett. B357 (1995) 545.
14. C. Vafa, *Evidence for F-Theory*, hep-th/9602022.
15. S. Sethi, C. Vafa and E. Witten, *Constraints on Low Dimensional String Compactifications*, hep-th/9606122.
16. E. Witten, Int. J. Mod. Phys. A10 (1995) 1247.
17. P. K. Townsend, Phys. Lett. B139 (1984) 283.
18. K. Dasgupta and S. Mukhi, *Orbifolds of M-Theory*, hep-th/9512196.
19. E. Witten, *Five-Branes and M-theory on an Orbifold*, hep-th/9512219.
20. A. Dabholkar and J. Park, *An Orientifold of Type IIB Theory on K3*, hep-th/9602030; *Strings on Orientifolds*, hep-th/9604178.
21. A. Sen, *M-Theory on (K3 × S^4)/Z_2*, hep-th/9602010; *Orbifolds of M-Theory and String Theory*, hep-th/9603113.
22. A. Kumar and K. Ray, *M-Theory on Orientifolds of K3 × S^4*, hep-th/9602144.
23. M. A. Walton, Phys. Rev. D37 (1988) 377.
24. M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory*, Vol. I, Cambridge University Press, 1987.
25. T. Eguchi, P. Gilkey and A. Hanson, Phys. Rep. 66 (1980) 213.
26. D. Page, Phys. Lett. B80 (1978) 55.
27. S. Hawking and C. Pope, Nucl. Phys. B146 (1978) 381.
28. L. Alvarez-Gaume and E. Witten, Nucl. Phys. B234 (1984) 269.
29. A. Sen, *Duality and Orbifolds*, hep-th/9604070.
30. J. Schwarz and A. Sen, Phys. Lett. B357 (1995) 323.
31. C. Vafa and E. Witten, Nucl. Phys. B447 (1995) 261.
32. A. Sagnotti, in Cargese '87, *Non-perturbative Quantum Field Theory*, eds. G. Mack et.al. (Pergamon Press, 1988), 521; G. Pradisi and A. Sagnotti, Phys. Lett. B216 (1989) 59; M. Bianchi and A. Sagnotti, Phys. Lett. B247 (1990) 517; Nucl. Phys. B361 (1991) 519.
33. P. Horava, Nucl. Phys. B327 (1989) 461.
34. J. Dai, R. G. Leigh and J. Polchinski, Mod. Phys. Lett A4 (1989) 2073.
35. E. Gimon and J. Polchinski, *Consistency Conditions for Orientifolds and D-Manifolds*, hep-th/9601038.
36. J. Polchinski, S. Chaudhuri and C. V. Johnson, *Notes on D-Branes*, hep-th/9602052.
37. B. S. Acharya, *M-theory Compactification and Two-Brane/Five-Brane Duality*, hep-th/9605044.
38. A. Kumar and K. Ray, *Compactifications of M-Theory to Two Dimensions*, hep-th/9604164.