Vortex correlations in a fully frustrated two-dimensional superconducting network

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Abstract. – We have investigated the vortex state in a superconducting dice network using the Bitter decoration technique at several magnetic frustrations \( f = \phi/\phi_0 = 1/2 \) and \( 1/3 \). In contrast to other regular network geometries where the existence of a commensurate state was previously demonstrated, no ordered state was observed in the dice network at \( f = 1/2 \) and the observed vortex-vortex correlation length is close to one lattice cell.

Introduction. – In the past decades, the vortex state of superconducting networks has been investigated by several groups using different imaging techniques (scanning Hall microscopy [1,2], scanning SQUID microscopy [3] or Bitter decoration [4–6]). The vortex configuration was studied in square or triangular lattices as a function of the magnetic field. In superconducting arrays the relevant variable is the magnetic frustration, \( f \), which represents the vortex filling factor. \( f \) is defined as \( f = \phi/\phi_0 \) with \( \phi_0 = h/2e \), the flux quantum, and \( \phi \), the magnetic flux per elementary plaquette. The vortex pattern reflects the spatial phase configuration of the superconducting order parameter resulting from the competition between the magnetic field and the underlying lattice [7]. For rational frustration \( f = p/q \), with \( p \) and \( q \) integer numbers, a commensurate ground state is generally expected as, for example, the checkerboard state which was imaged in square lattices at \( f = 1/2 \) [1,4].

The purpose of this letter is to present the detailed results of a magnetic-decoration experiment in a fully frustrated dice network. The unusual properties of the dice lattice were recently highlighted by the discovery of a peculiar destructive interference phenomenon occurring at \( f = 1/2 \) for the electronic wave function in the tight-binding model [8,9]. This phenomenon was shown later to manifest itself as a broadening of the superconducting transition and a suppression of the critical current in the corresponding superconducting wire network [10,11]. This observation was indicative of the absence of commensurate vortex state in the dice network, in contrast to the square network which was shown to exhibit a sharp critical current peak at \( f = 1/2 \) [12]. We report here new careful decoration experiments on a series of high-quality extended niobium dice networks under different magnetic fields. The analysis of the vortex correlation functions shows unambiguously that the vortex state is fully disordered at \( f = 1/2 \) and confirms the results suggested by our preliminary experiments on small networks [13].

The issue of the vortex configuration is of additional interest in the dice lattice as the vortices are located on the (dual) Kagomé lattice (fig. 1). Since the vortex variable is binary,
Fig. 1 – The dice lattice is shown here as full lines. Black points at the center of the cells illustrate the location of vortices at the nodes of the Kagomé lattice symbolized by dashed lines. Arrows show the 3 directions along which the correlation functions (eq. (1)) have been studied.

for $f$ smaller than 1 (in each cell there is either one vortex (+1) or no vortex (−1)), we can view this problem as Ising spins on the nodes of a Kagomé lattice. This general problem has been studied theoretically [14, 15] with first- and second-neighbour interaction. Our vortex experiment is the first observation of binary variables on the Kagomé lattice.

Samples and experimental technique. – The imaging was performed by the Bitter magnet-ic-decoration technique [16]. Since the magnetic contrast of vortices in superconducting wire networks [4] is very weak, we used the so-called flux compression technique [5]. The networks (fig. 2) were made of niobium wires (1 µm long, 100 nm wide) patterned from a 200 nm thick Nb layer epitaxially grown on a sapphire substrate at 550 °C. The patterning was achieved by reactive ion etching in $SF_6$ through an aluminum mask prepared by e-beam lithography [17]. Only the first 130 nm Nb was removed, leaving unpatterned a 70 nm thick niobium layer as a flux compression bottom layer. The total array size was 800 µm long and 600 µm wide. The networks contained about 550000 cells, with a wire width homogeneity better than 10%. The elementary cell area in all networks is 0.866 µm$^2$ and the matching field ($f = 1$) was

Fig. 2 – Left: partial view of the imaged network after decoration at $f = 1/2$. The length of each wire is 1 µm. The vertical bar on the left corresponds to 5 µm. The white points visible near the center of the cells are the images of the Ni clusters that decorate vortices. Right: the transcription of the vortex configuration. The black dots symbolize the observed vortices. The dice lattice is represented with full lines and the Kagomé (dual lattice) with dashed lines.
obtained for $B = 2.39$ mT. The decoration cell was placed inside a double $\mu$-metal shield. Before each run the magnetic field was calibrated using the magnetoresistance curve of a large SNS Josephson junction array. The flux accuracy is a few $10^{-3} \phi_0$ per cell. We always found the counted vortex density perfectly consistent with our field calibration. We first apply the magnetic field in the normal state at 10 K. Then the sample is slowly cooled down across the niobium transition temperature. Vortices nucleate at the niobium superconducting transition temperature, $T_c = 9.0$ K for our samples, and, because of the strong network pinning, their configuration freezes out as the temperature decreases a few mK below $T_c$ [18]. Flux compression takes place at the superconducting transition ($T_c = 8.93$ K) of the bottom layer when the network vortex loops convert into Abrikosov vortices. Once the temperature is stabilized at 4.2 K, Ni particles are flash-evaporated on the sample under a nominal residual helium pressure of 0.6 mbar. The vortex positions are then registered at room temperature using a scanning electron microscope (cf. fig. 2). This “one-shot” technique allows us to image a wide region and collect statistical information on the position of vortices.

**Experimental results.** – In this section we compare the vortex structures at $f = 1/3$ and at $f = 1/2$. The data are extracted from the SEM-micrographs and the vortex configuration is then regenerated on a computer. The pictures in figs. 3 and 5 (below) are parts of the complete images. Since the Bravais cell is composed of three different plaquettes, we represent the vortices with three different colors (blue, green, and red) depending on where they are located. To obtain a quantitative information on the degree of disorder, we have calculated linear correlation functions between vortex variables in the three equivalent directions of the lattice (see fig. 1). By making distinction between those three directions, we keep information on the domain shapes, which would be lost on averaging over the three contributions. The vortex variables $V_i$ are equal to +1 if a vortex is present in the $i$ cell and −1 if not. The three correlation functions, $C_\alpha$, $C_\beta$ and $C_\gamma$ are defined as

$$C_{\alpha,\beta,\gamma}(r) = \langle V_i V_{i+r} \rangle_{\alpha,\beta,\gamma},$$

(1)

where the $i+r$ cell is the $r$-th neighbour of the $i$ cell in the $\alpha$, respectively $\beta$ or $\gamma$, direction (see fig. 1) and $\langle ... \rangle_{\alpha,\beta,\gamma}$ is the mean product $V_i V_{i+r}$ for each $i$ having at least one $r$-th neighbour in the $\alpha$ (respectively, $\beta$ or $\gamma$) direction.

Fig. 3 – A part of the configurations observed in two different decoration runs: $f = 1/3 + 0.005$ (a) and $f = 1/3 + 0.016$ (b). The circles on the nodes of the Kagomé lattice represent the positions of the observed vortices. The blue, green and red colors indicate the three different positions in the Bravais cell. Single-colored domains correspond to one of the three equivalent commensurate states which are degenerate because of the 3-fold symmetry.
We first consider filling factor 1/3 where a stable commensurate state is expected. Indeed one can build a simple ordered configuration with 1 vortex every 3 cells, i.e. one vortex per Bravais cell. This ordered phase is three times degenerated due to the lattice symmetry. In fig. 3, we show two of those phases observed on two different runs performed at magnetic field $B = 0.797$ mT. We have analyzed each time a zone of approximately 3000 cells at the network’s center. The actual frustration, as determined from the observed number of vortices, is for the first run: $f = 1/3 + 0.005$ (fig. 3a). We observe a huge single domain extending over the whole picture. For the second run shown in fig. 3b, the actual frustration is $f = 1/3 + 0.016$. The observed effective domain size is approximately 10 cells. Excess vortices seem to gather near the domains walls, as can also be noticed in the former case (added vortices sit preferentially on the defect lines). This behavior is similar to the one observed in square arrays [1]. The correlation functions for $f = 1/3 + 0.005$ are shown in fig. 4. As can be seen, the correlation falls from 1 at $r = 0$ to approximately 0.85 at $r = 40$ in the three directions and this level of correlation is conserved beyond the 40th neighbour. The extended ordered vortex state at $f = 1/3$, is similar to that observed in the square or triangular lattices with rational frustrations [1, 4]. This structure remains ordered even within a few percent of excess vortices but the domain size decreases. This indicates the robustness of the 1/3-ordered state to extra vortices.

In contrast, at half-filling the vortex lattice is incommensurate with the dice lattice. One must consider two Bravais cells. We performed three different experiments at $f = 1/2$. The number of investigated cells was approximately 4000 for each experiment. The measured frustration was $\delta f = -0.0014, +0.0015, +0.0205$ around the ideal half-filling. A part of two regenerate images is displayed in fig. 5. Despite the fact that $f = 1/2$ is a rational filling factor, we did not observe simple ordered phases in any of the three experiments. Rather we observe strongly disordered states reminiscent of the incommensurate states occurring at irrational frustrations [20]. The correlation functions (see fig. 6) rapidly drop to approximately 0.05 beyond $r = 2$. This residual correlation is a real effect, larger than the experimental uncertainties. It is due to differences in the probability of populating the three different sites in the Bravais cell. It corresponds for this decoration experiment to a population ratio of 0.62, 0.32 and 0.56, respectively, for the different kind of sites. This difference could be due to some
slight inhomogeneity in the network temperature during cooling or in the wire geometry [21]. Presently, the origin of this small effect is not understood and remains an open problem. Fitting the correlation function with an exponential function $\propto e^{-r/\xi}$, we get a correlation length $\xi \approx 1.5$ for $f = 1/2$.

We also imaged a vortex lattice at $f = 1/6 + 0.0082$. We saw no ordered domains, but we could not make definitive conclusion on the 1/6-state with this experiment. This small deviation from the exact $f = 1/6$ indeed corresponds to 4.7% of the vortex population and might change the configuration more efficiently than for a higher filling factor such as $f = 1/3$ or $f = 1/2$. In the literature, no observation of ordered vortex state was ever reported in superconducting networks for such small filling factors.

**Discussion.** — Our observation of a very short correlation length at $f = 1/2$, is indicative of a strongly disordered state. This observation is consistent with previous transport measurements in superconducting Al networks with the same lattice [10]. A sharp peak was observed at $f = 1/3$, consistent with the strongly pinning commensurate state shown in fig. 3. On the other hand, the suppression of critical current at $f = 1/2$ was indicative of a weakening of the...
phase rigidity due to the localization of the superconducting wave function in the Aharonov-Bohm cages [8]. The absence of ordering is reminiscent of the problem of classical Ising spins (zero or one vortex per cell) on the nodes of the Kagomé lattice [22]. However, since the condition of nearest-neighbour interaction, which is central to the prediction of disordered state at all temperatures in the Ising model, is presumably not fulfilled for array vortices, one cannot draw conclusions from this analogy. Very recent theoretical studies of vortices in dice Josephson junctions arrays have more relevance as they directly address this issue of vortex ordering.

Korshunov [23] investigated the ground state of superconducting dice arrays with either cosine or quadratic dependence of the coupling energy upon the phase difference. He found a ground state made of an ordered configuration of vortex triads (clusters of three adjacent vortices). Because of the proliferation of zero-energy domain walls, the ground-state entropy is large but it is not extensive, in contrast to the case of Ising spins on a Kagomé lattice [22] where an extensive entropy was found. Therefore, in this model, the true ground-state configuration for \( f = 1/2 \) should be ordered. In contrast, the observed vortex correlation functions in our decoration experiments do not reveal any signature of order. Also, the histogram of vortex cluster sizes shows no preferred occurrence of triads. We do not believe that the geometrical irregularities present in the experimental sample [21] are responsible for the observed disordered state. Instead, we believe that it results from the combination of large thermal fluctuations and large entropy of low-energy configurational states at the vortex freezing temperature [18]. Indeed, a large number of low-energy vortex configurations are likely to exist in the dice lattice [24,25]. However, their difference in energy is much less than the energy barrier for vortex motion which, in the wire network [19], is mostly determined by the single wire length, independently of the network topology. On cooling down the vortex lattice may therefore be freezed out in a configuration different from the true ground state. A possible origin of the lack of ordering of the vortex lattice was recently pointed out by Cataudella and Fazio [25]. Their Monte Carlo simulations on the fully frustrated Josephson junction dice array suggest the existence of a low-temperature phase transition below which a glassy dynamics prevents the system from reaching the true ground state conjectured in ref. [23]. This scenario is consistent with our observation but cannot be tested in our strong vortex pinning wire network array.

Conclusion. – We performed magnetic imaging of the vortex configuration in Nb dice networks under several magnetic fields. We focused our study on magnetic frustrations \( f = 1/3 \) and \( f = 1/2 \). Numerical calculations of the correlations on the observed vortex positions lead to a quantitative characterization of the degree of disorder. While a robust commensurate state is found at \( f = 1/3 \), we observe a strongly disordered vortex configuration at \( f = 1/2 \) characterized by a vortex correlation length close to one cell size. It is worth noticing that this feature, particular to the dice lattice, appears here in a system with no geometrical disorder. The discrepancy with the Korshunov prediction that the ground state is ordered suggests that the true ground state is not accessible in our experiment. Further investigation of Josephson junction arrays on a dice lattice by transport and high sensitive magnetic microscopy [26] at much lower temperature are needed to reveal the true nature of the ground state.

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REFERENCES

[1] Hallen H. D., Seshadri R., Chang A. M., Miller R. E., Pfeiffer L. N., West K. W., Murray C. A. and Hess H. F., Phys. Rev. Lett., 71 (1993) 3007.
[2] Chang A. M. et al., Appl. Phys. Lett., 61 (1992) 1974.
[3] Vu L. N., Wistrom M. S. and Van Harlingen D. J., Phys. Rev. Lett., 63 (1993) 1693.
[4] Runge K. and Pannetier B., Europhys. Lett., 24 (1993) 737; J. Phys. (Paris), 3 (1993) 389.
[5] Bezryadin A., Ovchinnikov Y. and Pannetier B., Phys. Rev. B, 53 (1996) 8553.
[6] Pannetier B., Bezryadin A. and Eichenberger A., Physica B, 222 (1996) 253.
[7] Pannetier B., Chaussy J., Ramnal R. and Villégier J. C., Phys. Rev. Lett., 53 (1984) 1845.
[8] Vidal J., Mosseri R. and Douçot B., Phys. Rev. Lett., 81 (1998) 5888.
[9] Vidal J., Butaud P., Doucôt B. and Mosseri R., Phys. Rev. B, 64 (2001) 155306.
[10] Abilio C. C., Butaud P., Fournier Th., Pannetier B., Vidal J., Telesco S. and Dalzotto B., Phys. Rev. Lett., 83 (1999) 5102.
[11] The effect of confinement in Aharonov-Bohm cages was also observed in a dice array of quantum wires: Naud C., Faini G., Phys. Rev. Lett., 86 (2001) 5104.
[12] Buisson O., Giroud M. and Pannetier B., Europhys. Lett., 12 (1990) 727.
[13] Pannetier B., Abilio C. C., Serret E., Butaud P., Fournier Th. and Vidal J., Physica C, 32 (2001) 41.
[14] Kobayashi Y., Takagi T. and Mekata M., J. Phys. Soc. Jpn., 67 (1998) 3906.
[15] Moessner R. and Chalker J. T., Phys. Rev. B, 81 (1998) 12049.
[16] Traubé H. and Essmann U., Phys. Status Solidi, 20 (1967) 95.
[17] e-beam microfabricator Leica VBR6.
[18] Vortex pinning is strong in wire arrays [19] due to the large energy barrier for vortex crossing a superconducting wire. This barrier is suppressed in the vicinity of $T_c$ when the superconducting coherence length which defines the vortex core size becomes of order of the wire length. The condition $\xi(T) \approx a$ gives an indication of the onset of pinning by the wire array and determines the freezing temperature of the vortex lattice. It is reached at $\approx 4$ mK below $T_c$.
[19] Giroud M., Buisson O., Wang Y. Y. and Pannetier B., Low J. Temp. Phys., 87 (1985) 683.
[20] Halsey T. C., Phys. Rev. B, 31 (1985) 5728.
[21] The e-beam lithography process may lead to some defects which are repeated in each cell of the network. For example, we cannot discard a few percent difference in the widths of wires with different angles. The difference in the probability of populating the three different sites in the Bravais cell might be due to such systematic defects.
[22] Baxter R. J., J. Math. Phys., 11 (1970) 784; Huse D. A. and Rutenberg A. D., Phys. Rev. B, 45 (1992) 7536.
[23] Korshunov S., Phys. Rev. B, 63 (2001) 134503.
[24] Feigel’man M. V., Ioffe L. B., private communication (2001).
[25] Cataudella V. and Fazio R., cond-mat/0112307.
[26] Veauvy C., Mailly D. and Hasselbach K., cond-mat/0110196.