Piezoelectric Cylindrical Design for Harvesting Energy in Multi-Directional Vibration Source

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Abstract. Vibration Energy Harvester (VEH) has attracted a great attention recently both in academia and industry. One of the most challenging issues in VEH is the possibility to harvest vibration energy in multiple directions. In fact, Conventional VEH (CVEH) using cantilever beam’s structure may possibly become inefficient for the application under multi-directional vibration sources. To overcome this shortcoming of CVEH, this paper proposes a novel design of piezoelectric cylindrical energy harvester (PCEH) which is using patches of piezoelectric material attached to the surface of a cylindrical structure. The Finite Element Method (FEM) analysis using COMSOL Multiphysics software package showed that PCEH has a great potential for the applicability of VEH in the multi-directional vibrating applications such as wearable devices and biomedical devices.

1. Introduction
The development of electronic industry has led to a new generation of the small electronic device which has the micro-scale volume and requires lower power consumption [1]-[3]. However, the lack of scalable energy sources has been a challenge for the implementation of those devices in real-life applications. Batteries, which are conventionally the most common power sources, have not kept pace with the demand for the miniaturization of these devices [4]. In this context, the vibration energy harvesting (VEH), which can scavenge mechanical energy from surrounding and convert it into electricity, have received great interest from academia and industry. Within the vast field of micro-power source, the mechanical vibrations have been proved to be the most suitable energy source for the small scale devices due to its abundance, versatility and power density [5], [6]. With the scalable energy harvester, the electronic devices are expected to operate with a continuous and autonomous power source without any interaction from the surrounding. Among the different types of vibration energy harvester, the piezoelectric energy harvesting (PEH) has been reported to be the most common choice due to its significant advantages in the power density, the simple structure and the ease of application [7]. At the early stages of development, most of the piezoelectric energy harvesters were designed based on cantilever beam structure owing to its large deflection response under the purely bending mode shape. However, for the applications which are excited by a multi-directional vibration...
source, CVEH may possibly show a poor energy harvesting performance. To overcome this shortcoming of CVEH, several researchers have proposed a variety of multi-directional design for PEHs. Mei et al [8] presented a double-wall cylindrical energy harvester which can harvest vibration energy in various directions. The authors claimed that by using the double wall design, the number of resonant modes is increased, and thus the operating bandwidth of the energy harvester is broadened. Chen et al [9] proposed a dandelion-like multi-directional vibration energy harvester with a good performance in various testing directions. Wei-Jiun et al [10] presented a tri-directional harvester which comprises two piezoelectric cantilevers vibrating along two perpendicular directions and a spring-mass system under magnetic effect. In general, the existing designs of multi-directional PEH commonly require auxiliary structures. These structures increase the complexity and the size of the PEH and therefore may decrease the applicability of the device. In an attempt to treat multi-directional energy harvesting challenge, this paper proposes a simple design of piezoelectric cylindrical energy harvester (PCEH) which is using piezoelectric material attached to the surface of a cylindrical structure as shown in figure 1. The study using finite element method in COMSOL Multiphysics software package revealed that PCEH has a great potential for the VEH in the multi-directionally vibrating application.

![Figure 1. Design of piezoelectric cylindrical energy harvester.](image1)

![Figure 2. The meshing of PCEH in COMSOL Multiphysics.](image2)

2. Theoretical background of piezoelectric materials
Piezoelectric material has a long history of development. These materials specifically generate electric charges under mechanical deformation. Among various types of piezoelectric materials, the piezoelectric polymer such as polyvinylidene fluoride (PVDF) has attracted a great attention owing to its great flexibility and ease of application. In this research, this material will be laminated on the surface of the plastic rod. Under the deformation of the plastic rod, the curvature stress and strain will appear in the piezoelectric layer. The electric charges, which are generated during the bending deformation of the piezoelectric layers, will be collected through the electrodes. These electrodes are then connected to an external electrical circuit. And the output voltage will be measured through a resistor load. In general, a stress-charge form of the piezoelectric constitutive equations is given as follow,

\[
T = c^E S - e^E E, \quad (1)
\]

\[
D = e^S S + e^S E, \quad (2)
\]

where the \( T, S, E, D \) denote the stress component vector, the strain component vector, the electric field component vector, and the electric displacement component vector, respectively; the \( c^E, e, e^S \) denote the elasticity matrix, piezoelectric coupling matrix, and electric permittivity coefficient matrix, respectively; the superscripts \( E, S \) denote the parameter at constant electric field and at constant strain, respectively; the superscript \( t \) stands for the transpose.

The generated charges are calculated through the electric displacement \( D \) by,

\[
Q = \iint D d\psi, \quad (3)
\]

where \( \psi \) is the area of the effective electrode.
The voltage $V_{\text{output}}$ is measured through the resistor load $R_L$ as $V_{\text{output}} = i_{\text{cir}} R_L$, with $i_{\text{cir}} = \dot{Q}$, where $i_{\text{cir}}$ is the current across the circuit and the overdot denotes the derivative with respect to time. All of these mathematical equations are embedded into the COMSOL model.

3. Design and modeling

The design comprises a circular tip mass mounted to the tip of a cylindrical acrylic rod as shown in figure 1. The radius of the tip mass and the cylindrical rod are 7.5 mm and 10 mm, respectively. A piezoelectric polyvinylidene fluoride (PVDF) patch with the size 70x10x0.3 mm$^3$ is laminated on the surface of the cylindrical rod. The lengths of the cylindrical rod and the circular tip mass are 200 mm and 10 mm, respectively. Under the external excitation, the cylindrical rod will establish a bending mode and accordingly the piezoelectric layer will be compressed and stretched. The system is then connected to an electrical circuit. The voltage output will be measured through the resistive load of $10^6$ $\Omega$. A model of PCEH has been built in COMSOL Multiphysics 5.2 Software Package. The detailed material properties and parameters are provided in the Table. 1. Solid Mechanics, Electrostatics, Piezoelectric Effect, and Electrical Circuit Modules were used to model and simulate the design.

| Model Segment             | Materials       | Property          | Value   | Unit   |
|---------------------------|-----------------|-------------------|---------|--------|
| Cylindrical Rod           | Acrylic Plastic | Density           | 1190    | kg.m$^{-3}$ |
|                           |                 | Young’s Modulus   | 3.2e9   | Pa     |
|                           |                 | Poisson’s Ratio   | 0.35    |        |
| Circular Tip Mass         | Neodymium       | Density           | 7500    | kg.m$^{-3}$ |
| Piezoelectric layer       | PVDF            | Density           | 1780    | kg.m$^{-3}$ |
|                           |                 | Relative permittivity | 12     |        |

For the piezoelectric layer, polyvinylidene fluoride (PVDF) is using owing to its high mechanical flexibility compare to others piezoelectric material. The values in the elasticity matrix $E^c$ and the coupling matrix $e$ of the PVDF are provided as following: $E^c = \{10^{12}/39.5, -10^{12}/10.2, 10^{12}/42, -10^{12}/25.4, -10^{12}/19.5, 10^{12}/99.9, 0, 0, 0, 10^{12}/1.82, 0, 0, 0, 0, 10^{12}/1.69, 0, 0, 0, 0, 10^{12}/1.43\}$ Pa and $e = \{0, 0, 0.069, 0, 0, 0.069, 0, 0, -0.099, 0, -0.099, 0, -0.069, 0, -0.081, 0, 0, 0, 0\}$ C/m$^2$. Notes that for anisotropic elastic materials the stress-strain matrix is symmetric.

A three-dimensional finite element model in COMSOL is considered for this research. Figure 2 showed the meshing of the piezoelectric layer and the cylindrical rod with the circular tip mass. Tetrahedral elements were chosen for meshing, and a mesh refinement was applied to the region near the piezoelectric layer for a better calculating accuracy. The acrylic rod is fixed at one end and the other end with the attached circular tip mass is set free for vibration. The gravity is chosen in the opposite direction of the Z-direction, which is in the length direction of the rod. This gravity force will be taken into account during the calculation for a more realistic study.

4. Results and Discussion

4.1 Modal analysis
A stationary analysis was conducted using Static Study Module in COMSOL. The maximum stress is calculated with Von Mises criteria. The maximum stress was equal to 7.96 [MPa] at the fixed end of the rod and the minimum stress is equal 0.1178 [MPa] at the free end of the rod. The results in figure 3 showed the obvious tendency of the stress and strain distribution with the high stress concentrated at the fixed end and the maximum deflection was found at the free end of the rod. Based on that observation, the PVDF layers were attached near the fixed end of the rod to maximize the energy harvesting efficiency. As the PVDF layers were rolled on the surface of the cylindrical rod, it is essential to use the Rotated Coordinate System in COMSOL to specify the poling direction of the layer. In particular, the PVDF layer was polarized in radius direction as illustrated by the arrows in figure 4. In this figure, the radial blue arrows indicate the poling directions.

Eigenfrequency study showed the first seven eigenmodes obtained at various frequencies ranging from 57.891 Hz to 1254.9 Hz. These seven modes and their associated frequencies are shown in figure 4. It should be noted that the 1st and 2nd modes are purely bending modes while the 3rd, 4th, 5th, 6th and 7th modes are the combination of bending mode and torsional mode. The 1st and 2nd modes are under consideration in this research. These modes are more suitable for actual ambient vibration sources due to their low natural frequencies. The frequency response study was also conducted as shown in figure 6. The swept sine frequency test has been utilized with the frequency range from 10 Hz to 1300 Hz and the frequency step of 0.5 Hz. The peak amplitude was detected at around 65 Hz, 446.5 Hz, 739.5 Hz, 1207.5 Hz and 1255 Hz which showed a good agreement with the eigenfrequency study. Furthermore, it is worth noting in figure 6(b) that the output voltage of bending mode (at 65 Hz) and torsional mode (at 446.5 Hz) are comparable.

![Figure 3. Stress distribution of the PCEH under static force.](image1)

![Figure 4. The polarization and the static electric potential of PVDF layer.](image2)

![Figure 5. First seven modes of PCEH and the associated natural frequencies.](image3)
Figure 6. Frequency response curves for (a) the tip deflection and (b) the output voltage obtained with base excitation in X-direction and the base acceleration of 0.5 m/s².

4.2 Response to multi-directional vibration source

Figure 7. Frequency response curves for (a) the tip deflection and (b) the output voltage obtaining with the base excitation in X, Y, Z-direction and the base acceleration of 0.5 m/s².

As mentioned in the previous subsection, the purely bending mode will be the dominating mode within the frequency bandwidth where the frequency of actual vibration sources is usually found. Therefore, this study will focus on the purely bending mode from now on. Under each direction -X, Y, and Z direction of the base excitation, the frequency response test with the base acceleration of 0.5 m/s² and the frequency bandwidth from 30Hz to 100Hz are performed as shown in figure 7. The results showed that the output voltages of PCEH at three different excitation conditions are on similar scales. It should also be noted that there is a small discrepancy in the results for X-direction. Particularly, the peak output amplitudes were found at around 57 Hz and 65 Hz for the case of X-excitation. Meanwhile, in Y- and Z-excitation condition, only the peak value at 65 Hz is observed. This is resulted from the unsymmetrical design of the PCEH - due to the present of the piezoelectric layers. More precisely, the PVDF was only attached to the rod’s surface which is perpendicular to the Y-direction in this case. However, the overall results showed that in all three excitation condition, the peak voltage outputs are comparable. In other words, this indicated that PCEH can harvest energy in three different directions for the low base excitation at 0.5 m/s².

Figure 8. Design of the PCEH with 2 PVDF layers attached on the rod’s surface in X and Y direction: (a) normal view and (b) projected view.
For a higher base excitation, the effect of unsymmetrical design may possibly become larger. Therefore, to further examine the potential of PCEH in this case, another PVDF patch is attached to the rod’s surface which is perpendicular to X-direction as shown in figure 8. By conducting the same simulation, the frequency response results are shown in figure 9. Notes that the excitation acceleration has been increased to 5 m/s² in this case.

**Figure 9.** Frequency response curves for (a) the tip deflection and (b) the output voltage obtaining with the base excitation in X, Y, Z-direction and the base acceleration at 5 m/s².

In this simulation, the displacement and the voltage output in the Z-excitation condition are smaller when compared to the ones in X- and Y-excitation condition. However, it is shown that the results in X- and Y-direction are approximately in the same high displacement and voltage values (peak amplitude at 5.5 mm for both cases). This can be explained by the similar structure from the viewing direction in X- and Y-direction.

**Figure 10.** Time response of output voltage of the PCEH with the excitation in (a) X- and (b) Y-direction, obtained at the excitation frequency of 65 Hz and the base acceleration of 5 m/s².

In order to further verify this phenomenon, a time response study has been conducted with the excitation frequency of 65 Hz and the base acceleration of 5 m/s² as shown in figure 10. The results showed the output voltages are in harmonic form with the maximum amplitude at around 7.3 V and 7 V for the base excitation on X- and Y-direction, respectively. The output voltages obtained in both cases are totally in similar range. In conclusion, regardless of the base acceleration intensity, the results confirmed that the PCEH may harvest efficiently energy in more than one direction with a proper position of the PVDF layers.

### 4.3 Parameter optimization
Figure 11. The variation of the maximum of the tip displacement and the output voltage with the (a) cylindrical length and (b) radius at 0.3 m/s².

For the same PVDF layer with the size above, the maximum tip displacement and output voltage are calculated and plotted at different values of the length of the cylindrical rod from 100 mm to 200 mm as shown in figure 11. Notes that in this optimization process, we use the design of 1 PVDF layer. The results demonstrated that there exists a set of values for geometrical parameters with that the optimum energy harvesting performance can be obtained. In particular, the maximum displacement and output voltage have been achieved with the cylindrical rod’s length of 130 mm when the radius is fixed at 7.5 mm (as in figure 11(a)) and the optimum radius of 9.5 mm when the length is fixed at 120 mm (as in figure 11(b)). All the results above were obtained with the base acceleration of 0.3 m/s² and in Y-direction. In addition, it is also worth noting that the numbers of PVDF layer laminated on the surface of the cylindrical rod also affect the energy harvesting performance. The designs of PCEH with 1, 2, 3 and 4 PVDF layers laminated on the spherical surface of the cylindrical rod are presented in figure 12 (a). These designs were tested by conducting the swept sine frequency tests at the same condition (the base acceleration of 0.3 m/s² and in Y-direction). The frequency response curves for the output voltage of each case are compared in figure 12 (b). The maximum output voltage was obtained with the design using 3 PVDF layers.

Figure 12. (a) Schematic designs of PCEH and (b) variation of the output voltage with different numbers of PVDF layer.

To see the effect of resistive load on the performance of the PCEH, a series of parametric study has also been conducted for the design with 1 PVDF layer. The base acceleration was kept at 5 m/s² and in Y-direction. Figure 13 showed that while increasing the values of resistive load, the optimum power was determined at 10 MΩ. Furthermore, from figure 9(a), the output voltage across the load reached the saturated value at 100 MΩ.
5. Conclusion
A series of cylindrical-based energy harvesters were proposed in this study. The modeling and simulation study using Finite Element Method (FEM) were performed in COMSOL Multiphysics. The preliminary results pointed out that PVDF layers should be attached at the clamped end of the cylindrical rod for the sake of harvesting efficiency. By now, the PCEH could efficiently harvest energy with multi-directional excitation at low and high excitation intensity with a proper configuration of PVDF layers on the surface of the plastic rod. This finding may help to bypass the challenge of VEH in the multi-directionally vibrating applications. In addition, parametric studies have also been conducted, and the results indicated that there exists an optimal design of the cylindrical structure for PCEH to maximize energy harvesting power (geometry, the number of PVDF layers, and resistive load value). Further study will focus on experimental verification and the design of broadband piezoelectric cylindrical energy harvester.

6. References
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