Two-parameter neutrino mass matrices
with two texture zeros

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Abstract

We reanalyse Majorana-neutrino mass matrices \( M_\nu \) with two texture zeros, by searching for viable hybrid textures in which the non-zero matrix elements of \( M_\nu \) have simple ratios. Referring to the classification scheme of Frampton, Glashow and Marfatia, we find that the mass matrix denoted by A1 allows the ratios \((M_\nu)_{\mu\mu} : (M_\nu)_{\tau\tau} = 1 : 1\) and \((M_\nu)_{e\tau} : (M_\nu)_{\mu\tau} = 1 : 2\). There are analogous ratios for texture A2. With these two hybrid textures, one obtains, for instance, good agreement with the data if one computes the three mixing angles in terms of the experimentally determined mass-squared differences \( \Delta m_{31}^2 \) and \( \Delta m_{21}^2 \). We could not find viable hybrid textures based on mass matrices different from those of cases A1 and A2.

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1 Introduction

With the recent results of the Double Chooz, Daya Bay and RENO Collaborations [1] a non-zero reactor mixing angle $\theta_{13}$ has been established. Since the value of $\theta_{13}$ [1, 2, 3] is rather large, neutrino mixing may not just be a perturbation of tri-bimaximal mixing [4]. Therefore, in model building [5] one might dispense with seeking for models which have tri-bimaximal mixing at leading order.

The large reactor mixing angle has inspired renewed interest in texture zeros. It was already shown some time ago that, in the basis where the charged-lepton mass matrix is diagonal, there are only seven viable Majorana neutrino mass matrices with two texture zeros [6]. There are many investigations of two texture zeros in $M_\nu$—see [7, 8, 9, 10, 11, 12, 13] and references therein, but in this paper we concentrate on so-called hybrid textures [14] which have a richer structure: Apart from two elements being zero, there are non-zero elements of $M_\nu$ whose ratio is one or another small integer. In order to find viable hybrid textures, we use a numerical method to estimate the elements of $M_\nu$ [8, 13]. Then for each $M_\nu$ we make a hypothesis which ratios could be simple and accept or refute the hypothesis on the basis of a $\chi^2$-analysis.

In this investigation the textures denoted by A1 and A2 in the classification of Frampton, Glashow and Marfatia [6], given by

$$A1: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad A2: \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix},$$

(1)

will play a prominent role; in this equation, non-zero elements of $M_\nu$ are indicated by a cross. We find that these textures can be endowed with a viable hybrid structure. As a side remark, we note that the textures A1 and A2 allow only the normal ordering of neutrino masses [6]. Moreover, neutrinoless $\beta\beta$-decay by Majorana neutrino exchange is forbidden because the corresponding effective neutrino mass is given by $(M_\nu)_{ee} = 0$. Therefore, if such a decay is discovered, it must proceed via an alternative mechanism.

The paper is organized as follows. In section 2 we describe our numerical method for estimating the elements of $M_\nu$ and reanalyse two texture zeros. In section 3 we use these results for guessing possible hybrid textures. As anticipated above, we find two viable ones, which we denote by $\bar{A}1$ and $\bar{A}2$, based on textures A1 and A2, respectively. The remainder of section 3 is devoted to the discussion of $\bar{A}1$ and $\bar{A}2$. The conclusions are presented in section 4.

2 Two texture zeros and predictions for $| (M_\nu)_{ij} |$

In [13] a $\chi^2$-analysis was performed in order to investigate how well the seven allowed types of two texture zeros fit with the available neutrino data. In this approach, a $\chi^2$-function incorporates the experimental data and texture zeros have to be imposed. Here we will pursue an alternative approach. We define the function

$$F_{ijkl}(x) := | (M_\nu)_{ij}(x) | + | (M_\nu)_{kl}(x) | \quad (i, j, k, l = e, \mu, \tau),$$

(2)
where

\[ x = (m_0, \Delta m^2_{21}, \Delta m^2_{31}, s^2_{12}, s^2_{23}, s^2_{13}, \delta, \rho, \sigma). \]  

(3)

In the parameter set \( x \), \( m_0 \) is the lightest neutrino mass, \( \Delta m^2_{ab} \) are the mass squared differences, \( s^2_{cd} = \sin^2 \theta_{cd} \) are the sines squared of the mixing angles, \( \delta \) is the Dirac CP phase and \( \rho, \sigma \) are the two Majorana phases. For the lepton mixing matrix we use the parameterization of [15]. Evidently, minimizing \( F_{ijkl} \), taking into account the experimentally allowed ranges of the parameters in \( x \), should unveil the allowed two texture zeros. In order to minimize \( F_{ijkl} \) numerically, we make use of the Nelder–Mead algorithm (downhill simplex method) [16]. Since this algorithm is only capable of searching the whole parameter space \( \mathbb{R}^9 \), we add the function

\[ \Pi_D(x) := \begin{cases} 0, & \text{for } x \in D \\ 10^6, & \text{for } x \notin D \end{cases} \]  

(4)

to \( F_{ijkl}(x) \), in order to specify the allowed domain \( D \) for \( x \). To improve the chance of finding a good minimum, we started with 50000 different random start simplices and also performed perturbations around good candidates for minima.

The domain \( D \) of \( x \) was chosen as follows. For \( \Delta m^2_{ab}, s^2_{cd} \) and \( \delta \) we used the 1σ and 3σ ranges provided by the global fits of neutrino oscillation data [2, 3]. The Majorana phases \( \rho \) and \( \sigma \) were allowed to vary between 0 and 2π. Since most cosmological constraints on the sum of the neutrino masses [15] are of the order of \( \sum m_\nu < 1 \text{ eV} \), we allowed the lightest neutrino mass \( m_0 \) to vary between zero and 0.3 eV.

We minimized all 15 independent functions \( F_{ijkl} \), using the 1σ and 3σ ranges provided by the global fits of Forero et al. [2] (versions 2 and 3) and Fogli et al. [3]. The results for the 1σ range of Forero et al. (version 3) and Fogli et al. are shown in table 1. The results for the corresponding 3σ ranges can be found in table 2.

As can be read off from tables 1 and 2, due to the finite accuracy of the numerical procedure, none of the functions \( F_{ijkl} \) is found to have a minimum at exactly zero. However, the allowed texture zeros are readily distinguished from the disfavoured ones. For the allowed texture zeros, which are indicated in the tables by using boldface letters, we find numerical minima of the order of \( 10^{-9} \div 10^{-8} \text{ eV} \), while all other minima are at least four orders of magnitude larger.

Let us first discuss the results for the 3σ range—see table 2. At 3σ the data of Forero et al. (both versions 2 and 3) allow precisely those two texture zeros, which are also allowed following the original analysis [6], namely A1–C for the normal and B1–C for the inverted neutrino mass spectrum. The same holds for the data of Fogli et al.

This picture drastically changes, when we turn to the 1σ analysis—see table 1. The data of Forero et al. (version 3) now exclude B2 (inverted), B4 (inverted) and C (normal). The data of Fogli et al. allow only A1 (normal) at the 1σ level. The older global fit by Forero et al. (version 2) allows all “classical” two texture zeros A1–C (normal) and B1–C (inverted) also at the 1σ level.
Table 1: Minimal values of the functions \( F_{ijkl} \) for the normal and the inverted ordering of the neutrino masses. The results are obtained by allowing the input parameters taken for viable texture zeros (entries in boldface) the minima are typically of the order of \( \sigma \) according to [6] is found.

\[
\begin{array}{cccccc}
ij kl & \text{Forero et al. (version 3)} & \text{Fogli et al.} \\
 & \text{normal} & \text{inverted} & \text{normal} & \text{inverted} \\
A1 & ee e\mu & 9.01 \times 10^{-9} & 4.05 \times 10^{-2} & 2.06 \times 10^{-8} & 4.71 \times 10^{-2} \\
A2 & ee e\tau & 1.61 \times 10^{-8} & 4.70 \times 10^{-2} & 1.60 \times 10^{-3} & 4.34 \times 10^{-2} \\
 & ee \mu\mu & 1.51 \times 10^{-2} & 1.52 \times 10^{-2} & 1.29 \times 10^{-2} & 1.97 \times 10^{-2} \\
 & ee \mu\tau & 2.04 \times 10^{-2} & 1.56 \times 10^{-2} & 2.02 \times 10^{-2} & 1.67 \times 10^{-2} \\
 & ee \tau\tau & 1.32 \times 10^{-2} & 1.54 \times 10^{-2} & 2.60 \times 10^{-2} & 1.62 \times 10^{-2} \\
 & e\mu e\tau & 7.46 \times 10^{-4} & 7.44 \times 10^{-4} & 7.48 \times 10^{-4} & 7.57 \times 10^{-4} \\
B3 & e\mu \mu\mu & 1.32 \times 10^{-8} & 1.08 \times 10^{-8} & 4.14 \times 10^{-3} & 1.93 \times 10^{-2} \\
 & e\mu \mu\tau & 1.99 \times 10^{-3} & 2.02 \times 10^{-3} & 1.91 \times 10^{-3} & 1.96 \times 10^{-3} \\
B2 & e\mu \tau\tau & 6.49 \times 10^{-9} & 1.74 \times 10^{-2} & 2.38 \times 10^{-2} & 8.37 \times 10^{-4} \\
B1 & e\tau \mu\mu & 7.70 \times 10^{-9} & 7.01 \times 10^{-9} & 3.29 \times 10^{-3} & 1.86 \times 10^{-2} \\
 & e\tau \mu\tau & 1.95 \times 10^{-3} & 1.97 \times 10^{-3} & 2.14 \times 10^{-3} & 2.02 \times 10^{-3} \\
B4 & e\tau \tau\tau & 6.24 \times 10^{-9} & 1.84 \times 10^{-2} & 2.35 \times 10^{-2} & 1.23 \times 10^{-3} \\
 & \mu\mu \mu\tau & 4.11 \times 10^{-2} & 5.08 \times 10^{-3} & 3.79 \times 10^{-2} & 1.90 \times 10^{-2} \\
C & \mu\mu \tau\tau & 1.16 \times 10^{-2} & 4.21 \times 10^{-9} & 1.88 \times 10^{-2} & 5.56 \times 10^{-3} \\
 & \mu\tau \tau\tau & 3.85 \times 10^{-2} & 1.05 \times 10^{-3} & 5.01 \times 10^{-2} & 5.95 \times 10^{-3} \\
\end{array}
\]

3 Two simple hybrid textures for \( \mathcal{M}_{\nu} \)

As an additional gain of the analysis presented in section 2, we can calculate the absolute values of the elements of \( \mathcal{M}_{\nu} \) at the minima of \( F_{ijkl} \). In this way we find that some of the absolute values \( |\langle \mathcal{M}_{\nu} \rangle_{ij} | \) are approximately equal for textures of type B and C in the classification of [6], namely \( |\langle \mathcal{M}_{\nu} \rangle_{ee} | \approx |\langle \mathcal{M}_{\nu} \rangle_{\mu\tau} | \). It is interesting to note that this approximate equality is more pronounced in version 2 of [2] than in version 3 or in [3]. Anyway, replacing the approximate equality by an exact equality does not work for matrices of type B and C. We have checked this by using a \( \chi^2 \)-analysis and have always obtained a very large \( \chi^2 \), mostly because one or two mixing angles could not be reproduced.

It remains to discuss the textures of type A. For definiteness we show the results for
Table 2: Minimal values of the functions \( F_{ijkl} \) for the normal and the inverted ordering of the neutrino masses. The input parameters are allowed to vary in the 3\( \sigma \) range. For further information cf. table [1].

The 1\( \sigma \)-analysis of A1, using the data of Forero et al. (versions 2 and 3) and Fogli et al.:

\[
\begin{pmatrix}
2.101 \times 10^{-9} & 5.078 \times 10^{-9} & 1.199 \times 10^{-2} \\
5.078 \times 10^{-9} & 2.855 \times 10^{-2} & 2.245 \times 10^{-2} \\
1.199 \times 10^{-2} & 2.245 \times 10^{-2} & 2.604 \times 10^{-2}
\end{pmatrix}
\quad \text{Forero et al., version 2}, \quad (5)
\]

\[
\begin{pmatrix}
5.649 \times 10^{-9} & 3.358 \times 10^{-9} & 1.115 \times 10^{-2} \\
3.358 \times 10^{-9} & 2.586 \times 10^{-2} & 2.322 \times 10^{-2} \\
1.115 \times 10^{-2} & 2.322 \times 10^{-2} & 2.702 \times 10^{-2}
\end{pmatrix}
\quad \text{Forero et al., version 3}, \quad (6)
\]

\[
\begin{pmatrix}
1.700 \times 10^{-9} & 3.620 \times 10^{-9} & 1.073 \times 10^{-2} \\
3.620 \times 10^{-9} & 2.320 \times 10^{-2} & 2.246 \times 10^{-2} \\
1.073 \times 10^{-2} & 2.246 \times 10^{-2} & 2.936 \times 10^{-2}
\end{pmatrix}
\quad \text{Fogli et al.}, \quad (7)
\]

Looking at these matrices, we make the following observations. First of all, the texture zeros are represented by entries of the order of \( 10^{-9} \div 10^{-8} \text{eV} \), while all other entries are of the order of \( 10^{-2} \text{eV} \). Second, we see that |\( (M_\nu)_{\mu\mu} \)| \( \approx |(M_\nu)_{\tau\tau} | \) and 2|\( (M_\nu)_{e\tau} \)| \( \approx |(M_\nu)_{\mu\tau} | \) in equations [5] and [6], but these approximate relations are less pronounced with the data of Fogli et al.—see equation [7]. One finds analogous results for texture A2. Contrary to textures of type B and C, replacing the two approximate equalities by exact equalities, we find good to moderately good fits. This peculiar result conforms to the statement in [13] that textures of type A need less finetuning to reproduce the oscillation parameters than those of type B and C. Indeed, we find that changing the elements of \( \mathcal{M}_\nu \) slightly in order to achieve exact equalities works very well for type A, while for type B and C this procedure fails.

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Table 2

| ij kl | Forero et al. (version 3) | Fogli et al. |
|-------|--------------------------|-------------|
|       | normal                   | inverted    | normal | inverted |
| A1    | \( ee \, e\mu \) \( 5.51 \times 10^{-9} \) \( 3.47 \times 10^{-2} \) | \( 5.09 \times 10^{-9} \) \( 3.57 \times 10^{-2} \) | \( 6.61 \times 10^{-9} \) \( 3.63 \times 10^{-2} \) | \( 5.72 \times 10^{-9} \) \( 3.57 \times 10^{-2} \) |
|       | \( ee \, e\tau \) \( 1.20 \times 10^{-2} \) \( 1.14 \times 10^{-2} \) | \( 1.01 \times 10^{-2} \) \( 1.24 \times 10^{-2} \) | \( 1.85 \times 10^{-2} \) \( 1.15 \times 10^{-2} \) | \( 1.82 \times 10^{-2} \) \( 1.25 \times 10^{-2} \) |
|       | \( ee \, \mu\mu \) \( 9.86 \times 10^{-3} \) \( 1.14 \times 10^{-2} \) | \( 1.17 \times 10^{-2} \) \( 1.24 \times 10^{-2} \) | \( 9.07 \times 10^{-4} \) \( 5.96 \times 10^{-4} \) | \( 5.84 \times 10^{-4} \) \( 6.00 \times 10^{-4} \) |
|       | \( ee \, \tau\tau \) \( 6.86 \times 10^{-9} \) \( 6.47 \times 10^{-9} \) | \( 7.60 \times 10^{-9} \) \( 9.31 \times 10^{-9} \) | \( 1.77 \times 10^{-3} \) \( 1.79 \times 10^{-3} \) | \( 1.75 \times 10^{-3} \) \( 1.79 \times 10^{-3} \) |
| B3    | \( e\mu \, \mu\mu \) \( 9.63 \times 10^{-9} \) \( 5.73 \times 10^{-9} \) | \( 9.37 \times 10^{-9} \) \( 4.60 \times 10^{-9} \) | \( 1.82 \times 10^{-3} \) \( 1.83 \times 10^{-3} \) | \( 1.71 \times 10^{-3} \) \( 1.79 \times 10^{-3} \) |
|       | \( e\mu \, \mu\tau \) \( 6.86 \times 10^{-9} \) \( 6.47 \times 10^{-9} \) | \( 7.60 \times 10^{-9} \) \( 9.31 \times 10^{-9} \) | \( 1.77 \times 10^{-3} \) \( 1.79 \times 10^{-3} \) | \( 1.75 \times 10^{-3} \) \( 1.79 \times 10^{-3} \) |
| B2    | \( e\mu \, \tau\tau \) \( 9.49 \times 10^{-9} \) \( 8.89 \times 10^{-9} \) | \( 5.11 \times 10^{-9} \) \( 9.35 \times 10^{-9} \) | \( 3.64 \times 10^{-2} \) \( 6.24 \times 10^{-4} \) | \( 3.35 \times 10^{-2} \) \( 1.76 \times 10^{-3} \) |
|       | \( e\tau \, \mu\mu \) \( 9.90 \times 10^{-9} \) \( 8.85 \times 10^{-9} \) | \( 9.96 \times 10^{-9} \) \( 9.96 \times 10^{-9} \) | \( 1.77 \times 10^{-3} \) \( 1.79 \times 10^{-3} \) | \( 1.75 \times 10^{-3} \) \( 1.79 \times 10^{-3} \) |

\( M_{ij} \) for the normal and the inverted ordering (Forero et al. et al.)
The bottom line of this discussion is that in the following we will consider the mass matrices

$$\tilde{A}_1 : \mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & a \\ 0 & b & 2a \\ a & 2a & b \end{pmatrix}, \quad \tilde{A}_2 : \mathcal{M}_\nu = \begin{pmatrix} 0 & a & 0 \\ a & b & 2a \\ 0 & 2a & b \end{pmatrix}. \quad (8)$$

Actually, it suffices to discuss \( \tilde{A}_1 \), since \( \tilde{A}_2 \) emerges from \( \tilde{A}_1 \) through the exchange \( \mu \leftrightarrow \tau \). Therefore, we can use the following theorem [10, 11]:

Given two Majorana mass matrices \( \mathcal{M}_\nu \) and \( \mathcal{M}_\nu' \), in the basis where the charged-lepton mass matrix is diagonal, and

$$S\mathcal{M}_\nu S = \mathcal{M}_\nu' \quad \text{with} \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (9)$$

then the mixing parameters of \( \mathcal{M}_\nu' \) are related to those of \( \mathcal{M}_\nu \) by

$$\theta'_{12} = \theta_{12}, \quad \theta'_{13} = \theta_{13}, \quad \theta'_{23} = \frac{\pi}{2} - \theta_{23}, \quad \delta' = \pi + \delta. \quad (10)$$

The matrices (8) contain three parameters, the absolute values of \( a \) and \( b \), and the relative phase. However, fitting these mass matrices to the oscillation parameters gives a relative phase rather close to zero or \( \pi \). Therefore, we go one step further and assume that \( a \) and \( b \) are both real. Thus we end up with a two-parameter scheme for neutrino masses and mixing.

With real mass matrices (8), their eigenvalues \( \mu_j \) agree with the neutrino masses \( m_j \) apart from possible signs, i.e. \( m_j = |\mu_j| \). Both cases \( \tilde{A}_1 \) and \( \tilde{A}_2 \) lead to the equations

$$2b = \mu_1 + \mu_2 + \mu_3, \quad (11)$$
$$b^2 - 5a^2 = \mu_1\mu_2 + \mu_2\mu_3 + \mu_3\mu_1, \quad (12)$$
$$-a^2b = \mu_1\mu_2\mu_3. \quad (13)$$

Since there are three equations but only two real parameters, we find the consistency condition

$$\frac{1}{4} (\mu_1 + \mu_2 + \mu_3)^2 + 10 \frac{\mu_1\mu_2\mu_3}{\mu_1 + \mu_2 + \mu_3} - (\mu_1\mu_2 + \mu_2\mu_3 + \mu_3\mu_1) = 0. \quad (14)$$

By a rephasing of \( \mathcal{M}_\nu \) we can always achieve \( a > 0 \) and \( b > 0 \), which entails, due to equations (11) and (13),

$$\mu_1 + \mu_2 + \mu_3 > 0 \quad \text{and} \quad \mu_1\mu_2\mu_3 < 0, \quad (15)$$

respectively.

Since the textures A1 and A2 allow only the normal ordering of the neutrino mass spectrum, the masses \( m_2 \) and \( m_3 \) can be expressed as a function of the lightest mass \( m_1 \) and the mass-squared differences:

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}. \quad (16)$$

\( ^1 \)In this context it is interesting to note that in equation (24) of [17] a one-parameter hybrid texture for \( \mathcal{M}_\nu \) based on texture A1, which is not a special case of equation (8), has been proposed.
Table 3: Numerical results for texture $\bar{A}1$, using the best-fit values for $\Delta m_{21}^2$ and $\Delta m_{31}^2$ of Forero \textit{et al.} \cite{2} (version 3) and Fogli \textit{et al.} \cite{3}, respectively, as input. The values of the neutrino masses and of the parameters $a$ and $b$ are given in units of $10^{-2}$ eV.

|          | Forero \textit{et al.} | Fogli \textit{et al.} |
|----------|------------------------|----------------------|
| $m_1$    | 0.615                  | 0.601                |
| $m_2$    | 1.068                  | 1.056                |
| $m_3$    | 5.087                  | 5.006                |
| $\sin^2 \theta_{12}$ | 0.2912                | 0.2887               |
| $\sin^2 \theta_{23}$ | 0.4731                | 0.4731               |
| $\sin^2 \theta_{13}$ | 0.0239                | 0.0239               |
| $\cos \delta$ | $-1$                  | $-1$                 |
| $a$      | 1.098                  | 1.078                |
| $b$      | 2.770                  | 2.731                |

As a consequence, we can conceive equation (14) as condition to determine $m_1$, and, therefore, also $m_2$ and $m_3$, as a function of the measured mass-squared differences $\Delta m_{21}^2$ and $\Delta m_{31}^2$. However, then we also obtain $a$ and $b$ in terms of the mass-squared differences. All told, the neutrino mass spectrum and the mixing angles are, in this line of reasoning, functions of $\Delta m_{21}^2$ and $\Delta m_{31}^2$. In the following, we will discuss texture $\bar{A}1$. The results corresponding to texture $\bar{A}2$ are easily obtained via equation (10).

In the light of the above paragraph, we use as input the best fit values for $\Delta m_{21}^2$ and $\Delta m_{31}^2$, in order to determine the neutrino masses and the mixing angles. This input is $\Delta m_{23}^2 = 7.62 \times 10^{-5}$ eV$^2$ and $\Delta m_{31}^2 = 2.55 \times 10^{-3}$ eV$^2$ from \cite{2} (version 3) and $\Delta m_{21}^2 = 7.54 \times 10^{-5}$ eV$^2$ and $\Delta m_{31}^2 = 2.47 \times 10^{-3}$ eV$^2$ from \cite{3}; in the latter case we computed $\Delta m_{31}^2$ via $\Delta m_{3}^2 = \Delta m_{2}^2 + \delta m_{23}^2 / 2$ where $\delta m_{23}^2 \equiv \Delta m_{21}^2$ and $\Delta m_{23}^2 \equiv m_{3}^2 - (m_{1}^2 + m_{2}^2) / 2$. First we discuss the sign ambiguities. Due to our sign convention, the sign of $\mu_3$ is determined by the signs of $\mu_1$ and $\mu_2$ via the second inequality in equation (15). It turns out that equation (15) numerically admits only the signs such that $\mu_1 \mu_2 < 0$. Therefore, the two possibilities which remain are $\mu_1 < 0$, $\mu_2 > 0$ and $\mu_1 > 0$, $\mu_2 < 0$. Moreover, for each of these two possibilities the real solution of equation (14) is unique. After having obtained $m_1$ for each of the two sign possibilities, we compute the mixing angles. It turns out that the second sign possibility is ruled out because it gives a too big value for both $s_{12}^2$ and $s_{13}^2$. The numerical results for the signs $\mu_1 < 0$, $\mu_2 > 0$ are presented in table 3.

The mixing angles are practically insensitive to a variation of $\Delta m_{21}^2$ in the $3\sigma$ range. Under a variation of $\Delta m_{31}^2$ in the $3\sigma$ range, $s_{23}^2$ and $s_{13}^2$ change only insignificantly. However, if $\Delta m_{31}^2$ varies between $2.2 \times 10^{-3}$ eV$^2$ and $2.8 \times 10^{-3}$ eV$^2$, which covers the $3\sigma$ ranges of \cite{2} and \cite{3}, $s_{12}^2$ varies between 0.273 and 0.301. Correspondingly, $m_1$ changes from $5.4 \times 10^{-3}$ eV to $6.6 \times 10^{-3}$ eV.

The results for texture $\bar{A}2$ are obtained by applying equation (10), leading to the replacements $\cos \delta = -1 \rightarrow +1$ and $s_{23}^2 \rightarrow 1 - s_{23}^2$ in table 3. Consequently, in the case of texture $\bar{A}2$ the atmospheric mixing angle is in the second octant. It is amusing to observe that the analysis of \cite{3} slightly prefers $\cos \delta = -1$ and, therefore, texture $\bar{A}1$, but this
preference vanishes at the 2σ level.

4 Conclusions

In this paper we have assumed that the charged-lepton mass matrix is diagonal and the Majorana neutrino mass matrix $\mathcal{M}_\nu$ has two texture zeros. We have revisited the 15 possibilities of two texture zeros and confirmed the original seven viable texture zeros of [6] if we let the input parameters of the function $F_{ijkl}$ of equation (2) vary individually in the respective 3σ ranges provided by [2] and [3]. However, if we permit only the 1σ ranges, then in the case of Forero et al. [2] several of the previously viable textures cease to be so. Interestingly, if we take the input from Fogli et al. [3], at 1σ only one viable texture, $A_1$, remains. The reason for this is rooted mainly in the different 1σ ranges of $s_{23}^2$ in these papers. However, since it is even unclear in which octant the best fit value of $\theta_{23}$ lies, we should not put too much emphasis on that point.

Searching for viable hybrid textures, we have found two such textures, denoted by $\bar{A}_1$ and $\bar{A}_2$ in equation (8). Fitting these textures to the oscillation parameters suggests real parameters $a$ and $b$. Thus we end up with two two-parameter textures of the neutrino mass matrix. In other words, there are two parameters for three mixing angles and two mass-squared differences. In order demonstrate the viability of $\bar{A}_1$ and $\bar{A}_2$, we have adopted the strategy to compute the mixing angles from the fitted mass-squared differences. The mixing angles obtained in this way agree with the fit results of [2] at the 2σ level; this is also true for [3], with the exception of $s_{23}^2$ where the agreement is at 3σ.

Since the textures $\bar{A}_1$ and $\bar{A}_2$ are special cases of the textures of type A, they require normal ordering of the neutrino mass spectrum. Numerically, it turns out that the spectrum is hierarchical because $m_1/m_3 \simeq 1/8$. Moreover, $\bar{A}_1$ predicts an atmospheric mixing angle in the first octant and $\cos \delta = -1$, while for $\bar{A}_2$ $\theta_{23}$ lies in the second octant and $\cos \delta = +1$. As a function of the mass-squared differences, our two hybrid textures give $s_{23}^2$ fairly close to 0.5. This is in very good agreement with version 2 of [2], but it is not completely consistent with the mean values of $s_{23}^2$ in version 3 of the same authors and in [3]. But, as mentioned earlier, the experimental situation with respect to $s_{23}^2$ is a bit ambiguous.

If a texture of type A is realized in nature, then observation of neutrinoless $\beta\beta$-decay would signal new physics. In the case of our hybrid textures $\bar{A}_1$ and $\bar{A}_2$, there would be no CP violation in neutrino oscillations. This would be unfortunate, but at the moment the results of [3], although far from being significant, hint at $\delta \approx \pi$. As stressed in [13], the textures $A_1$ and $A_2$ need the least finetuning of all two texture zeros and they have no pronounced hierarchies in the matrix elements of $\mathcal{M}_\nu$. This is also borne out by the analysis of our hybrid textures—see table 3.

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2Hybrid textures with one texture zero, $(M_\nu)_{ee} = 0$, have recently been investigated in [18].
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