Closed string tachyon: inflation and cosmological collapse

Celia Escamilla-Rivera\textsuperscript{1}, Gerardo García-Jiménez\textsuperscript{2}, Oscar Loaiza-Brito\textsuperscript{3} and Octavio Obregón\textsuperscript{3}

\textsuperscript{1} Fisika Teorikoaren eta Zientziaren Historia Saila, Zientzia eta Teknologia Fakultatea, Euskal Herriko Unibertsitatea, 644 Posta Kutxatila, E-48080 Bilbao, Spain
\textsuperscript{2} Facultad de Ciencias Físico Matemáticas, Universidad Autónoma de Puebla, PO Box 1364, 72000 Puebla, Mexico
\textsuperscript{3} Departamento de Física, División de Ciencias e Ingenierías, Campus León, Universidad de Guanajuato, PO Box E-143, 37150 León, Guanajuato, Mexico

E-mail: celia-escamilla@ehu.es, ggarcia@fcfm.buap.mx, oloaiza@fisica.ugto.mx and octavio@fisica.ugto.mx

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Abstract
Starting with a compactification of critical bosonic string theory on an internal space with constant curvature, we look for conditions upon which an assumed closed string tachyon potential drives an inflationary Universe to collapse as the tachyon reaches the minimum of the potential. We find that all these features are possible in a scenario in which the internal manifold is negatively curved with a constant volume. We restrict our study to the case in which the scale factor and the dilaton are not dynamically coupled. Near to the minima of the potential the dilaton slows down to a null or constant velocity while rolling towards strong values on a dynamical string metric. Finally, some comments are addressed about the existence of similar cosmological features in alternative tachyonic potentials.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Quantization of string theories in unstable backgrounds is broadly believed to be associated with the presence of tachyonic modes in the low-energy spectrum, as in the case of superstring theories, where the most well-known scenario concerns the presence of a tachyonic mode on the open string spectrum between pairs of D-branes and anti-D-branes. In such case, the tachyon reaches its minimal state of energy by rolling down to the minimum of the potential while the perturbative approach of the theory becomes reliable. This process is called tachyon
condensation, and broadly speaking, it yields the disappearance of the D-branes—the source of the tachyon modes—while leaving an excited state of closed strings carrying the energy of the original D-branes.

Nevertheless, tachyonic modes are not exclusively related to open strings, in fact, in the bosonic string spectrum there exist tachyonic modes propagating throughout spacetime usually known as bulk tachyons. In [1], the authors found evidence that in the context of closed string field theory, the tachyon potential has a critical point suggesting a relation with a closed string tachyon vacuum. Several considerations lead these authors to suggest that in such vacuum, closed string states will not propagate and spacetime would cease to be dynamical. Furthermore, the analysis of the physical decay turns out to be rather insensitive to the specific details of the tachyon potential about which little is known. Arbitrary tachyonic potentials of the form

\[ V(T) = -c^2 T^2 + \mathcal{O}(T^3), \]  

were studied in [2] considering the low-energy field equations that couple the metric, the dilaton \( \Phi \) and the tachyon field \( T \) and it was deduced that there exist conditions on \( T \) and \( V(T) \) for a big crunch to occur at finite time in the Einstein metric provided the bulk tachyon lives on an arbitrarily \( d \)-dimensional spacetime where the string metric has been taken constant. In the Einstein frame, the metric goes as \( e^{-2\Phi} \) times the string metric, with the dilaton rolling into strong coupling as the tachyon condensates and in turn, yielding a shrinking of the spacetime metric. Although a generic \( d \)-dimensional condensation of the tachyon field is linked to a collapsing Universe, a different approach is to consider a compactification on a \( (d-4) \)-dimensional space. In such case, many scalar fields would arise, playing an important role in the dynamical evolution of the Universe as well as the minimal required conditions on the extra dimensions to be compatible with a collapsing scenario driven by the rolling of the tachyon.

All these ideas lead us to raise our main goal which consists in studying whether, at least for a particular potential of the form above mentioned, it is possible to construct a model in which an accelerated expansion is present. Many interesting studies on this topic have been performed in the last few years (see for instance [3–7]) on which the classical closed string tachyon field plays an important role in the evolution of the Universe driving it to a collapse. However, in these attempts an expansion stage is absent. As a main result, we report a solution which describes a Universe which inflates prior to its collapsing, in a dynamical \( d \)-dimensional string metric scenario (instead of considering a fixed string metric as in [2]). Therefore, it is possible to establish a link between a dynamical evolution of the four-dimensional spacetime with the dynamics of the internal space, concerning for instance, the volume or the curvature.

As usual in this kind of models, we find that generically the effective theory contains five different scalar fields. Therefore, we concentrate our study to the particular case with a constant \( B \)-field and an internal space with variable volume but fixed curvature. With the objective to gain some insight about the role played by the tachyon field in the evolution of the Universe we restrict our analysis to a specific model where the internal volume evolves simultaneously with the closed string coupling. This last requirement allows us to decouple the dilaton from the scale factor dynamics and leads us to the fact that the dilaton keeps constant as time runs. Regarding the minimal conditions on the internal manifold, we observe that for the particular tachyon potential of the form (1) the internal space has a negative and constant curvature whose volume keeps constant as the Universe collapses. It is worth noting that compactifications on highly curved manifolds are expected within the context of string theory and that negative curved internal manifolds have been recently suggested in superstring compactification models to achieve a classical De Sitter vacuum [8–11].
A more general analysis involves a decoupling between the dilaton and the scale factor dynamics while leaving the dilaton and the internal volume to freely evolve from each other. In this case, we find that the dilaton goes towards strong coupling but slows down as the tachyon reaches the minimum of the potential. This behaviour although generic for non-compactification scenarios as in [2] arises in our case due to the very specific conditions we have assumed. By relaxing such conditions, it is possible to have different solutions. As perspective, we leave such analysis for further work.

The outline of this paper is as follows. We start by constructing the effective action of a closed string tachyon and a gravitational field from bosonic string theory under the already mentioned assumed conditions, where the four-dimensional Einstein metric is taken to be a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric. Using the Hamilton–Jacobi (H–J) formalism, we find solutions for the dilaton, the scale factor and the tachyon, as well as the minimal required conditions on the internal space. Our conclusions and comments are placed in section 3 and a brief discussion about other tachyon potentials is presented in the appendix.

2. Closed string tachyon cosmology

As a starting point, let us consider the low-energy dynamics of a critical bosonic string theory threaded with a closed string tachyon field \( T \), the dilaton \( \Phi \), the 3-form flux \( H_3 = dB_2 \) and the 26-dimensional metric \( g \):

\[
S = \frac{1}{2\kappa_0^2} \int d^{26}x \sqrt{-g_{26}} e^{-2\Phi} \left[ R + 4(\nabla \Phi)^2 - \frac{1}{12} H_3^2 - (\nabla T)^2 - 2V(T) \right],
\]

where \( V(T) \) is the corresponding closed string tachyon potential, and \( \kappa_0^2 = 8\pi G_{26} [1, 2] \). As a next step, we want to compactify the full theory on a 22-dimensional space \( X_{22} \) and to study the low-energy dynamics in four dimensions, by finding the minimal required conditions to have an inflationary cosmological model in which the universe evolves into a big crunch as the tachyon condensates.

Two important aspects emerge from here: the first is to decide how many parameters are kept free in the compactification, while a second concerns the election of a generic closed string tachyon potential. It is the last one which is much more arbitrary.

Let us start by studying the first aspect. In the most generic situation, a compactification on a 22-dimensional space leads to an effective 4-dimensional action depending on 5 different and independent fields:

1. the dilaton \( \Phi \),
2. the tachyon field \( T \),
3. the \( H \)-field,
4. the (non-necessary flat) internal curvature \( \mathcal{R} \) and
5. the internal volume denoted by \( \psi \).

However, we shall consider the simplest case in which \( H_3 \) vanishes and \( \mathcal{R} \) is constant. A more generic situation must undoubtedly consider a non-zero \( H_3 \)-flux [12] and a non-constant internal curvature as a function of the internal metric. Note that considering spaces with constant curvature and variable internal metric (volume) restricts the type of internal spaces on which the compactification is performed. The examples of such spaces are the Einstein manifolds.

After all these considerations in mind, we model the string metric as

\[
d\!s^2 = g_{\mu \nu} (x^\mu) \, dx^\mu \, dx^\nu + h_{mn} (x^\mu) \, dy^m \, dy^n.
\]
where $\mu, \nu$ run over the extended spacetime and $m, n$ run over $X_{22}$. Note that a four-dimensional coordinate dependence of the internal space has been assumed. Therefore, compactifying on $X_{22}$ leads to the effective four-dimensional action in the string frame:

$$S = \frac{1}{2\kappa_0^2} \int d^4x \sqrt{-g} e^{-2\Phi} \left[ R_4 (g^e) + \mathcal{R} + 4(\nabla \Phi)^2 - (\nabla T)^2 - 2V(T) \right].$$

(4)

To go into the Einstein frame, we take a conformal transformation [13]

$$\Omega^2(\Phi, \psi) = \omega^2 e^{2\xi}$$

where

$$\omega^2 = \kappa_0^2 m_p^2$$

and

$$\xi(t) = \Phi(t) - \frac{1}{2} \ln \psi(t).$$

Hence, the effective Lagrangian in the Einstein frame is given by

$$\mathcal{L} = \frac{m_p^2}{2} \sqrt{-g^E} \left[ R_4 + 6(\nabla \ln \Omega)^2 + 6\nabla^2 \ln \Omega + 4(\nabla \Phi)^2 - (\nabla T)^2 - 2\Omega^2(\Phi, \psi) \nabla^2 \mathcal{V}(T) \right].$$

(7)

where all contractions are with respect to $g^E$. The component $\mathcal{V}(T)$ of the effective tachyonic potential is consequently given by the tachyonic potential in the bosonic theory plus the constant curvature term:

$$\mathcal{V}(T) = V(T) - \frac{1}{2} \mathcal{R}.$$  

(8)

There is a large variety of tachyon potentials to be chosen, but with the purpose to describe a big crunch in a finite time in the Einstein frame, they are restricted to be of the form (1) [2, 1]. Recall that our goal is to incorporate such features in compactified models and look for the minimal conditions the internal manifold must possess to be compatible with such scenarios. Henceforth, we consider potentials of the above type particularly focusing on arbitrary tachyon potentials of the form

$$V(T) = -c_2^2 T^2 + c_4^2 T^4,$$

(9)

since it is in this case in which we find a richer scenario to look for solutions describing an inflationary Universe which evolves into a big crunch. There is certainly a width range of possible tachyonic potentials with the desired characteristics for the tachyon to condensate: a maximum at $T = 0$, a minimum for $V(T)$ at positive values of $T$ and $V(T) \to \infty$ as $T \to \infty$. For potentials of the form (1), we show some generic features in the appendix. Other potentials as exponentials or hyperbolic functions on the tachyon field do not lead to interesting cosmological features or they are too complicated to be implemented in our method. We leave them out of our study.

We are interested in looking for solutions of the equations of motion of time dependent fields accompanied by a rolling tachyon field. The time-dependence of the tachyon implies the absence of $\alpha'$ corrections [4]. The background is chosen to be a FLRW time-dependent Einstein metric:

$$ds^2_E = -dt^2 + e^{2\alpha(t)} (dr^2 + r^2 d\Omega^2),$$

(10)

where $\alpha(t) = e^{a(t)}$ is the scale factor.

Note that this is quite different than studying the low-energy dynamics of the bosonic string fields in a tachyonic potential with a higher dimensional FLRW-type metric (in the string frame). In such case, the extra dimensions are taken flat with a time-dependent warping factor [2, 14]. Our approach is therefore a little different. By considering the FLWR metric in the 4D spacetime, we are also looking for restrictions on the internal manifold on which the theory is compactified while the string metric remains dynamical.
Therefore, taking homogenous fields, the effective Lagrangian (7) reads,
\[
L = m^2_p e^{3\alpha} \left\{ -3\dot{\alpha}^2 - 3\xi^2 + 18\dot{\alpha}\dot{\xi} - 2\dot{\Phi}^2 + \frac{1}{4}\dot{T}^2 - \Omega^2 (\Phi, \psi) \psi'(T) \right\}, \quad (11)
\]
where \(\xi\) would play the role of an effective dilaton only if the internal volume keeps constant.

2.1. Constrained effective model

We observe from (11) that solving the full system seems to be a difficult task. Therefore, we proceed to study a simplified subsystem in which the field \(\xi(t)\) keeps constant for all \(t\). Physically this means that there is a dynamical pairing between the string dilaton and the internal volume yielding a constant \(\Omega\) which in turn allows us to decouple the velocities of the scale factor and the field \(\xi\). This is a strong constraint to put on the system. However, it is an interesting model to analyse the role played by the tachyon in the evolution of the Universe.

At the end of section 3, we shall relax the constraint to consider a non-constant \(\xi\).

In this very particular scenario, the equations of motion for the scale factor, the dilaton and the tachyon are
\[
\begin{align*}
\Phi : & \quad \ddot{\Phi} + 3\dot{\alpha}\dot{\Phi} = 0, \\
\alpha : & \quad 2\ddot{\alpha} + 3\dot{\alpha}^2 - 2\dot{\Phi}^2 + \frac{1}{4}\ddot{T}^2 - \Omega^2 \psi'(T) = 0, \\
T : & \quad \ddot{T} + 3\dot{\alpha}\dot{T} + \Omega^2 \psi'(T) = 0,
\end{align*}
\]
(12)
from which it is observed that it does not matter if the Hubble constant \(H = \dot{\alpha}\) is positive or negative, the dilaton \(\Phi\) runs into strong coupling or keeps constant. The former however indicates an interplay between the string coupling and the internal volume since \(\xi\) is constant.

For instance, it might be possible that in the strong coupling regime both, the string coupling and the internal volume, grow as the tachyon condensates, but the former rolls into large values faster than the last. We recall the reader that a complete analysis of the full system is left for future work.

To solve the corresponding equations of motion, we will make use of the H–J formalism, process which involves the usual definition for the momenta \(\pi_i = \partial_i L\) for \(i\) running over all fields \(\Phi, \alpha\) and \(T\), the corresponding canonical Hamiltonian reads
\[
\mathcal{H} = m_p^2 e^{-3\alpha} \left\{ -\pi_\alpha^2 - \frac{3}{2}\pi_\Phi^2 + 6\pi_T^2 + 12\Omega^2 e^{4\alpha} \psi'(T) \right\}.
\]
(13)
Using the standard H–J equation \(\mathcal{H}(\alpha, T; t) + \frac{\partial S}{\partial t} = 0\), where the Hamilton’s principal function is denoted by \(S\), and by making use of the standard identifications \(\partial_i S = \pi_i\), the H–J equation becomes [15],
\[
-2 \left( \frac{\partial H}{\partial T} \right)^2 + \frac{1}{2} \left( \frac{\partial H}{\partial \Phi} \right)^2 + 3H^2 = \Omega^2 \psi'(T),
\]
(14)
for \(S\) not explicitly depending on time and of the form
\[
S(\alpha, T) = e^{3\alpha} \Omega W(T),
\]
(15)
where the Hubble parameter is therefore only a function of \(T\):
\[
H = \dot{\alpha} = -\frac{1}{2} \Omega W(T).
\]
(16)
Note that having incorporated the constraint \(\dot{\xi} = 0\) at the level of the Lagrangian, we are able to study the role played by the tachyon field in the evolution of the scale factor, since the Hubble parameter depends only on the tachyon field which in turn yields a dynamically paired evolution of the string dilaton and the internal volume. On the other hand, observe that
selecting an $H$ not depending on $\Phi$ implies $\dot{\Phi} = 0$ for all $t$. We shall come back to this point at the end of the present section.

Our next purpose then is to look for the minimal conditions required for the effective tachyonic potential $V(T)$ being of the form $c_1^2 - c_2^2 T^2 + c_4^2 T^4$ (for other choices for $H$ see the appendix), to describe a Universe which inflates and collapses as the tachyon runs down to the minimum of its potential. We shall as well study the implications on the compactification model to be in accordance with such inflationary and collapsing Universe.

Let us then consider the particular solution of equation (14) in which $H$ does not depend on $\Phi$. The H–J equation (14) reduces to

$$-2 \left( \frac{\partial H}{\partial T} \right)^2 + 3H^2 = \mathcal{V}(T),$$

where $H = \Omega H$.

2.2. Inflation and collapse

As is shown by equation (14), choosing a specific potential $\mathcal{V}(T)$ constrains the Hubble parameter $H$ as a function of $T$. Choosing the Hubble parameter to be a polynomial on $T$ of the form

$$H(T) = -\frac{1}{2}(A + BT^2),$$

the effective potential $\mathcal{V}(T)$ is given by

$$\mathcal{V}(T) = \frac{3}{4}A^2 + \frac{B}{2}(3A - 4B)T^2 + \frac{3}{4}B^2 T^4.$$  

This potential is compatible with our assumed tachyonic potential (9) if the curvature of the internal 22-dimensional manifold is given by

$$R = -\frac{3}{2}A^2,$$

with $B(3A - 4B) < 0$.

We are interested in the possibility to have a richer scenario where the Universe starts small, expands till a maximum size and then contracts towards a big crunch. As it is well known, such features can be accomplished by adding a positive cosmological constant. However, as above mentioned, in order to keep the assumed tachyonic potential, the presence of a cosmological constant must be paired with a compactification on a negatively curved internal manifold with $R = -3A^2/2$, for $A \neq 0$, rendering the effective potential $\mathcal{V}(T)$ to have a non-zero value at its minimum. Following this line, the value of the curvature $R$ enters in the effective theory as a cosmological constant, from which we expect that it determines partially the evolution of the Universe. Nonetheless, such value not only enters as the cosmological constant but actually plays a secondary role in the dynamics of the Universe as we shall mention.

With the purpose to specifically determine such role let us describe some interesting features concerning different values for the parameters $A$ and $B$ and the corresponding solutions for the scalar factor, the tachyon and the dilaton fields.

(i) By substituting the function $H(T)$ back into the equations of motion, the tachyon field is found to be given by

$$T(t) = e^{2\Omega B t},$$

where we have fixed the integration constant to one and we have selected $B > 0$ such that time evolution is related to a tachyon which grows up. Using this expression, it is
Figure 1. Scale factor as a function of time \((A = -10, B = 1/10 \text{ and } \Omega = 1)\).

straightforward to compute the scale factor \(a(t) = e^{\alpha(t)}\) from the equation of motion for \(\alpha(t)\) which reads (see figure 1)

\[
a(t) = e^{-\frac{1}{2} (Ap + \frac{1}{2}e^{\Omega t})},
\]

(ii) The effective potential \(\mathcal{V}(T)\) has one or three extrema according to the following.

- For \(0 < A < 4B/3\), the effective potential \(\mathcal{V}(T)\) has three extrema at \(T_0 = (0, \pm \sqrt{(4B - 3A)/3B})\). There are conditions for tachyon condensation, but still an expansion stage is absent. In these scenarios, tachyon condensation accelerates the collapse of the Universe.
- For \(A < 0 < 4B/3\), expansion and contraction stages are present.

It is the latter case, we are interested in hereafter. Hence, the value of the potential at the minimum is in consequence given by

\[
\mathcal{V}(T_0) = \frac{2}{3}B(3A - 2B),
\]

from which it is observed that the vacuum energy is negative. See figure 2, where the tachyonic potential has been plotted with respect to \(t\) for some specific values for \(A\) and \(B\).

(iii) The time at which the tachyon field reaches the minimum of the potential is

\[
t_T = \frac{1}{4\Omega B} \ln \left( -\frac{A}{B} + \frac{4}{3} \right),
\]

while the time \(t_0\) at which the scale factor reaches a maximum, is given by

\[
t_0 = \frac{1}{4\Omega B} \ln \left( -\frac{A}{B} \right),
\]

from which it can be observed that the scale factor describes a Universe which expands from an asymptotically zero volume (i.e. \(a = 0\) at \(t \to -\infty\)), reaches a maximum size at the time \(t_0\) and contracts till the tachyon reaches its minimum at \(t_T\).
With this at hand, we are ready to study whether the collapse of the Universe is related to the tachyon condensation and the conditions on which inflation is present in these scenarios. An acceleration stage is present if the ratio $\dot{a}/a$ is positive, which in accordance with equations (21) and (22) is given by

$$\frac{\dot{a}}{a} = \frac{\Omega^2}{4} [\Omega^2 B^2 T^4 + 2\Omega B(A - 4B)T^2 + A^2].$$

The ratio is positive for $t \to -\infty$ till $t = t_a$ with

$$t_a = \frac{1}{4\Omega B} \ln \left( \frac{4B - A - 2\sqrt{A^2 - 2AB}}{B} \right),$$

time at which the acceleration stage finishes. Recall that we are considering the case in which $A < 0$ and $B > 0$ for which specific cosmological issues are dependent on their values while other are rather generic as the fact that $t_a < t_0 < t_T$, indicating that the acceleration stage ends before the Universe reaches its maximum, which occurs before the tachyon condensation.

Now, within this context, let us search for inflationary slow-roll conditions driven by the effective tachyon potential $V(T)$. Necessary and sufficient conditions for inflation are given by $\epsilon \ll 1$ and $\eta \ll 1$ where

$$\epsilon(T) = 2 \left( \frac{\mathcal{H}'}{\mathcal{H}} \right)^2 = 8 \frac{T^2}{(A/B + T^2)^2},$$

and

$$|\eta(T)| = \left| \frac{1}{V} \frac{\partial^2 V}{\partial T^2} \right| = \left| \frac{(3A - 4B)B + 9B^2T^2}{4(A + BT^2)^2 - 2B^2T^2} \right|. $$

We observe that there are several values of $A$ and $B$ for which such conditions are assured during a specific time interval. It is then possible to fix the values of $A$ and $B$ on which inflation (and the end of inflation) is guaranteed, which implies a fine-tuning on the ratio $A/B$ which
also determines how fast the Universe expands and/or collapses. It is important to mention that from string theory it is expected that $|A| >> 1$, i.e. high curvature of the internal space. This follows from the fact that $R \sim A^2 \sim R^{-6}$, where $R$ is the size of the extra dimensions.

The main question here is whether or not such conditions are compatible with a Universe which collapses as the tachyon field condensates. For $A = 0$, the assumed tachyonic potential (9) is compatible with the dynamics of an effective scalar field theory in a FLRW metric if and only if the internal manifold is Ricci-flat. Note that a compactification on a flat internal manifold with such tachyonic potential leads to a Universe with a scale factor given by

$$a(t) = \exp \left( -\frac{\Omega}{8} t \frac{\Omega}{B} \right),$$

which for $B > 0$ describes a Universe which starts at $t \to -\infty$ with $a = 1$ and it always contracts.

### 2.3. Tachyon condensation and the collapse of the Universe

We have seen that a model in which the extra dimensions are compactified on a negative curved manifold is compatible with the presence of inflation. Also we have shown that in such scenario, the maximum size of the Universe is reached before the tachyon condensates. In order to see if the Universe collapses as the tachyon condensates, we need to study the Equations of State (EoS) and show that the pressure is maximum at the minimum of the effective tachyonic potential.

Following the standard model of cosmology, we assume that the closed string tachyon is a field evolving in a FLRW Universe described by a perfect fluid with the standard EoS $p = \omega \rho$, where $\rho$ is the energy density and $p$ the pressure. The constant $\omega$ is given by

$$\omega = \frac{1}{2} \frac{T^2 - \mathcal{V}'(T)}{T^2 + \mathcal{V}'(T)} = -\frac{8B^2(\Omega^2 + 1)T^2 - 3(A + BT^2)^2}{8B^2(\Omega^2 - 1)T^2 + 3(A + BT^2)^2},$$

from which it is observed that at $t \to -\infty$, with the tachyon field at the top of the potential, $\omega_T = -1$, indicating that at early times the tachyon potential behaves as a cosmological constant. This also follows from the tachyon field potential $\mathcal{V}(T)$ from which we see that at early times, the only term which survives in the action is the constant $A^2$ which precisely behaves as a cosmological constant.

Let us comment on the complete evolution of $\rho$, $\rho$ and the EoS together (see figure 3). At the beginning, the density $\rho_T$ is positive, $\rho(t \to -\infty) = 3A^2/4$. As time runs, $\rho$ decreases indicating an expansion stage. The minimum value of $\rho$ is zero and it is reached at $t = t_\rho$ where

$$t_\rho = \frac{1}{4\Omega B} \ln \left( \frac{A}{B} \right),$$

which is precisely the time $t_0$ at which the Universe reaches its maximum size.

Note also that the pressure $p_T$ is negative at $t \to -\infty$ and increases its value as the Universe evolves until a maximum positive value at time $t = t_p$. This is consistent with a Universe which expands to a maximum size. After that, i.e. for times greater than $t_\rho$, the pressure decreases, and the Universe starts a contraction stage, becoming more and more negative as the time evolves, until the spacetime itself blows up [2]. The time at which the pressure is maximum is given by

$$t_p = \frac{1}{4\Omega B} \ln \left( \frac{8}{3} \frac{A}{B} \right).$$
Figure 3. Solid green line: the factor $\omega$ which for very early times is equal to $-1$. Dash blue line: energy density and dash-point red line: the pressure.

Hence, in the generic case, we have that $t_a < t_0 < t_f < t_p$, implying that in this scenario, acceleration finishes prior the time at which the Universe reaches its maximum size, which coincides with the time at which the density $\rho$ vanishes. After that, the Universe starts its contraction. Just before the Universe is in a stage in which the pressure is maximum (towards a big crunch), the tachyon reaches the minimum of its potential. We find that these features are rather generic for arbitrary potentials of the form (1) as briefly commented in the appendix.

Now, let us take the solutions for $\alpha$ and $T$ and substitute them back into the equation of motion for $\alpha$ in (34). It is straightforward to show that those solutions imply $\Phi = 0$ in accordance with our selection for the Hubble parameter not depending on $\Phi$.

2.4. Dilaton dynamics

Obtaining a constant solution for the dilaton points out how strong is our assumption on $\xi$ being constant. With the purpose to gain insight about the role played by the dilaton, let us consider the more general but still restricted case in which $\xi$ is not a constant but the fields $\xi$ and $\alpha$ are dynamically decoupled, i.e. $\xi \dot{\alpha} \ll 1$.

In such case, the equations of motion written in terms of the fields $\xi$, are given by

\begin{align*}
\xi : \quad & \ddot{\xi} + 3 \dot{\alpha} \dot{\xi} - \Omega^2 \gamma(T) = 0, \\
\Phi : \quad & \ddot{\Phi} + 3 \dot{\alpha} \dot{\Phi} + \Omega^2 \gamma(T) = 0, \\
\alpha : \quad & 2 \ddot{\alpha} + 3 \dot{\alpha}^2 - 3 \dot{\xi}^2 - 2 \dot{\Phi}^2 + \frac{1}{2} \dot{T}^2 - \Omega^2 \gamma(T) = 0, \\
T : \quad & \ddot{T} + 3 \dot{\alpha} \dot{T} + \Omega^2 \gamma(T) = 0, 
\end{align*}

and the corresponding Hamiltonian is given by

\begin{equation}
\mathcal{H} = \frac{e^{-3\alpha}}{1} \left( -\frac{\pi_\Phi^2}{12} + \frac{\pi_\Psi^2}{2} - \frac{\pi_\Phi^2}{8} + 2 \pi_\Phi \pi_\Psi + \frac{5}{12} \pi_\Psi^2 + \Omega^2 (\Phi, \psi) e^{6\alpha} \gamma(T) \right),
\end{equation}
Using the function $S$ proposed in (15) with $\Phi$ a function of $\Phi$ and $\psi$, the corresponding H–J equation reads

$$(3\mathcal{H}^2)_{\alpha} + \left(\frac{1}{2}\mathcal{H}^2\right)_{\Phi} + (-4\mathcal{H}^2)_{\psi, \Phi} + \left(\frac{5}{12}\mathcal{H}^2\right)_{\psi} - 2(\partial_T \mathcal{H})^2 = \Omega_1^2 \mathcal{Y}(T),$$

(36)

where the subindices denote the contribution for each field component in the Hamiltonian to the H–J equation. We immediately see that by considering a Lagrangian to be a function on both fields, $\Phi$ and $\psi$, the contribution coming from their coupling leads to an effective potential which goes to negative infinite values as the tachyon grows up. Such kinds of potentials do not support a tachyon condensation mechanism.

Therefore, we shall restrict our study to the case in which the Lagrangian depends only on the string dilaton $\Phi$ while the internal volume is kept constant. Note that a case in which the dilaton is kept constant with a dynamical internal volume, is not a solution of the equations of motion.

Let us start by considering $\Omega$ of the form

$$\Omega = \Omega_0 e^{\Phi(t)}.$$  

(37)

In such case, the H–J equation reduces to

$$\frac{7}{4} \mathcal{H}^2 - 2(\partial_T \mathcal{H})^2 = \mathcal{Y}(T).$$

(38)

Here we find appropriate conditions for a tachyonic potential of the form (1). In particular, let us consider again the very special case in which the Hubble parameter is given by (18). The effective potential is then

$$\mathcal{Y}(T) = \frac{7}{8} A^2 + 2B \left(\frac{7}{8} A - B\right) T^2 + \frac{7}{8} AB T^4,$$

(39)

with $8B > 7A$ to guarantee the presence of more than one extrema. It is also straightforward to check that the potential has a very flat region prior to tachyon condensation, indicating the existence of slow-roll conditions for inflation, although explicit solutions of $\Phi$ as a function of time still must be calculated. For other potentials of the form (1), it is also possible to find minimal conditions on $A$ and $B$ to have a tachyonic potential. In the present case, we also observe that for $A \neq 0$ the internal manifold is also negatively curved.

The tachyon field and the time derivative of the scale factor are given by

$$T = e^{2B\Omega_0} / e^{\alpha T},$$
$$\dot{\alpha} = -\frac{1}{2} (A + BT^2) \Omega_0 e^{\Phi}.$$  

(40)

For positive $B$ and negative $A$, the tachyon field goes towards strong values while $\dot{\alpha}$ vanishes at a finite $t$ meaning that there are conditions for the Universe to expand and collapse as the tachyon rolls down the potential.

Regarding the dilaton, we see from equation (34) that at $t \to -\infty$, since $\mathcal{Y}(T)$ is almost constant and positive, the quantity $\dot{\Phi} + 3\alpha \Phi \sim \Phi$ should be negative, indicating that as the tachyon starts rolling down its potential, the dilaton slows down. As the tachyon goes to the minima of the potential, $\mathcal{Y}$ becomes smaller and $\Phi$ increases its value. Finally, at the region in which the potential is very close to zero (for appropriate values of $A$ and $B$),

$$\dot{\Phi} + 3\alpha \Phi \sim \Phi \sim 0,$$

(41)

indicating that the dilaton has reached a zero or constant velocity. All together the above analysis tells us that $\Phi$ increases its value as the tachyon rolls down the potential, and keeps a constant or null velocity near to the minima.
3. Conclusions

In this paper, we have considered a compactification of critical bosonic string theory with a tachyonic potential into a four-dimensional spacetime by taking a constant $B$-field and a constant internal curvature. We have also studied two cases:

1. a dynamically paired evolution of the string coupling $e^\Phi$ and the internal volume and
2. a non-correlated evolution of them.

In both cases, the scale factor and the dilaton $\Phi$ are dynamically decoupled implying that the internal volume keeps constant through the whole process while the string and Einstein metric are time-dependent.

Our main goal was to study the conditions upon which an arbitrary closed string tachyon potential \[ 2, 1 \] effectively describes an inflationary Universe which collapses as the tachyon field reaches the minimum of its potential as well as the required conditions on the internal manifold to be compatible with it.

By choosing an arbitrarily specific potential with quadratic and quartic terms on the tachyon field, generated by a Hubble parameter of the form (14), we found a reliable model which considers a non-stable vacuum in which there are conditions for slow-roll inflation which finishes prior the time at which the Universe reaches its maximum size, coinciding to the time at which the density $\rho$ vanishes. After that, the Universe starts its contraction. Just before the Universe is in a stage in which the pressure is maximum (towards a big crunch), the tachyon reaches the minimum of its potential.

For the case 1, we found that the dilaton and the internal volume remain constant as the tachyon rolls down the potential. Also we found that quotients $-A/B \gg 1$ are more interesting since they allow a slower evolution of the Universe. In such scenarios, we have a universe which collapses as the tachyon reaches the minimum of its potential.

In addition, we have seen that in order to be consistent with the very specific tachyonic potential we have selected, the internal manifold must have a constant and negative internal curvature. Some comments about using other polynomial potentials on $T$ are shown in the appendix where it can be seen that all the cosmological features we have found for our model are rather generic. Nevertheless, it is worth mentioning that, on one hand, compactifications on highly curved manifolds are expected in the context of string theory suggesting high values for $A^2$. On the other hand, negative curved internal manifolds have been recently suggested in the context of superstring compactification in order to achieve a classical De Sitter Vacuum [8–10]. Both conditions are obtained with the tachyonic potential (9).

Note that the parameters $A$ and $B$ in the Hubble parameter (14) determines not only the evolution of the Universe, but the internal curvature and the internal volume dynamics as well. Deeper studies would consider the most generic case in which the evolution of the dilaton and the internal volume are not paired, as well as the presence of a non-constant $B$-field and a non-constant internal curvature. Undoubtedly, a more detailed analysis about different types of tachyonic potentials is as well very desirable. We leave some of these considerations for further work.

For the most interesting case 2, we find that just by assuming a very weakly coupling among the scale factor and the dilaton velocities, all the generic features we were interested in, as the expansion stage prior to the collapsing driven by the tachyon condensation, are kept in a model where the dilaton is dynamical. We found that the dilaton goes to strong coupling but slows down its velocity as the tachyon approaches the minima of the potential. All the above enforces the idea on which the closed string tachyon condensation is related to the collapse of the spacetime. It is still necessary to have a complete analysis of the system including a
non-vanishing coupling among $\dot{\alpha}$ and $\Phi$ as well as the inclusion of these types of models in a more realistic scenarios as type 0 superstrings.

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Appendix. Generic polynomial tachyonic potentials

Among the variety of tachyonic potentials reported in the literature, we have concentrated our study on those which can be written as polynomials on $T$. This is still a big set of possible tachyonic potentials and we restrict even more our study to some of the particular type.

As shown in equation (17), the tachyonic potential is computed throughout the Hubble parameter which is the generator function of the effective tachyonic potential $V(T)$. We consider $H(T)$ to be of the form

$$H(T) = \frac{1}{2} \sum_{n=0}^{\infty} C_n T^n,$$

(A.1)

with $n$ and $m$ integers. However, generic closed string tachyonic potentials $V(T)$ are expected to go as $T^2$ at leading order [1]. Therefore, we choose the Hubble parameter as

$$H(T) = \frac{1}{2} \left( \sum_{n=0}^{\infty} A_n T^n \right) \left( \sum_{m=2}^{\infty} B_m T^m \right).$$

(A.2)

For simplicity, let us concentrate just on the family of Hubble parameters given by

$$H(T) = \frac{1}{2} (A + BT^n) T^m.$$

(A.3)

The case studied in the body of the paper corresponds to $n = 2$ and $m = 0$. Here we report on the solutions obtained by considering other possible values for the numbers $n$ and $m$, leading to different tachyonic potentials of the type (1). We show some different cases in table A1, where we have also assumed a paired dynamical evolution of the string coupling and the internal volume. This means we have taken $\Omega$ to be a constant as in (16).

We see for instance that for $n = 3, m = 0$ and for $n = 0, m = 3$ the tachyon field grows into negative(positive) values for $B$ positive(negative). For the former case there are two contraction(expansion) stages while in the latter the Universe is always contracting (expanding). For $n = 0$ and $m = 2$, the Universe is always in a contraction or expansion stage. For $n = 2, m = 1$ and $n = 1, m = 2$, the specific values of $A$ and $B$ lead to a variety of different scenarios; a more detailed analysis is required here and we plan to incorporate such solutions in a future work. Finally, the case $n = 1, m = 1$ seems to have all the ingredients to describe an inflationary Universe which collapses as the tachyon reaches the minimum of its potential. In all cases, the rolling tachyon drives the Universe into a collapse. Note as well that for $n = 0$, the internal space is flat while for $n \neq 0$ the internal space has a non-zero internal curvature.
Table A1. Effective potential \( \mathcal{V} \), scale factor \( \alpha \) and the tachyon field \( T \) for different choices for the Hubble parameter \( H \).

| \( 2H \)                  | \( \mathcal{V}(T) \)                                                                 | \( \alpha(t) \)                                                                 | \( T(t) \)                        |
|---------------------------|-------------------------------------------------------------------------------------|---------------------------------------------------------------------------------|-----------------------------------|
| \( A + BT^2 \)            | \( \frac{1}{2}(A + BT)^2 - 2B^2T^2 \)                                             | \( -\frac{1}{2}At - \frac{1}{2}e^{At} \)                                     | \( e^{2At} \)                     |
| \( A + BT^2 \)            | \( \frac{1}{2}(A + BT)^2 - 2(B + A)^2T^2 \)                                        | \( -\frac{1}{2}e^{(A+B)t/2} \)                                                | \( e^{2t(A+B)/2} \)              |
| \( A + BT^3 \)            | \( \frac{1}{2}(A + BT)^3 - \frac{3}{2}B^2T^3 \)                                   | \( -\frac{1}{2}At - \frac{1}{2}\tan 3ABt \)                                   | \( \frac{\tan 3ABt}{3AB} \)      |
| \( A + BT^2T \)           | \( \frac{1}{2}(A + BT)^2T^2 - \frac{1}{2}(3BT + A)^2T^2 \)                        | \( -\frac{3}{2}B \ln(1 + \tan(3ABt^3)) \)                                    | \( -\frac{24\tan^3}{1 - 9\cos(3At)} \) |
| \( A + BT^3T \)           | \( \frac{1}{2}(A + BT)^3T^6 - \frac{9}{2}(A + BT)^3T^4 \)                        | \( \frac{2At}{A + BT} \)                                                      | \( -\frac{1}{3(A + BT)} \)       |

References

[1] Yang H and Zwiebach B 2005 A closed string tachyon vacuum? J. High Energy Phys. JHEP09(2005)054 (arXiv:hep-th/0506077)
[2] Yang H and Zwiebach B 2005 Rolling closed string tachyons and the big crunch J. High Energy Phys. JHEP08(2005)046 (arXiv:hep-th/0506076)
[3] McGreevy J and Silverstein E 2005 The tachyon at the end of the universe J. High Energy Phys. JHEP08(2005)090 (arXiv:hep-th/0506130)
[4] Swanson I 2008 Cosmology of the closed string tachyon Phys. Rev. D 78 066020 (arXiv:hep-th/0804.2262)
[5] Aref’eva I Y and Koshelev A S 2008 Cosmological signature of tachyon condensation J. High Energy Phys. JHEP09(2008)068 (arXiv:0804.3570 [hep-th])
[6] Farajollahi H, Salehi A, Tayebi F and Ravanpak A 2011 Stability analysis in tachyonic potential chameleon cosmology J. Cosmol. Astropart Phys. JCAP05(2011)017 (arXiv:1105.4045 [gr-qc])
[7] Farajollahi H and Salehi A 2011 A New approach in stability analysis: case study: tachyon cosmology with non-minimally coupled scalar field-matter Phys. Rev. D 83 124042 (arXiv:1106.0091 [gr-qc])
[8] Shiu G and Sumitomo Y 2011 Stability constraints on classical de sitter vacua J. High Energy Phys. JHEP09(2011)052 (arXiv:1107.2925 [hep-th])
[9] Danielsson U H, Haque S S, Koerber P, Shiu G, Van Riet T and Wrase T 2011 De Sitter hunting in a classical landscape Fortschr. Phys. 59 897–933 (arXiv:1103.4858 [hep-th])
[10] Danielsson U H, Haque S S, Shiu G and Van Riet T 2009 Towards classical de Sitter solutions in string theory J. High Energy Phys. JHEP09(2009)114 (arXiv:0907.2041 [hep-th])
[11] Hertzberg M P, Kachru S, Taylor W and Tegmark M 2007 Inflationary constraints on type IIA string theory J. High Energy Phys. JHEP12(2007)095 (arXiv:0711.2512 [hep-th])
[12] Suyama T 2007 Tachyon condensation with B-field J. High Energy Phys. JHEP02(2007)050 (arXiv:hep-th/0610127)
[13] Faraoni V, Gunzig E and Nardone P 1999 Conformal transformations in classical gravitational theories and in cosmology Fundam. Cosm. Phys. 20 121 (arXiv:gr-qc/9811047)
[14] Suyama T 2003 On decay of bulk tachyons arXiv:hep-th/0308030
[15] Muslimov A G 1990 On the scalar field dynamics in a spatially flat friedman universe Class. Quantum Grav. 7 231