SUSY dark matter and lepton flavor violation

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Abstract: We study lepton flavor-violating (LFV) processes within a supersymmetric type-I seesaw framework with flavor-blind universal boundary conditions, properly accounting for the effect of the neutrino sector on the dark matter relic abundance. We consider several possibilities for the neutrino Yukawa coupling matrix and show that in regions of SUSY parameter space that yield the correct neutralino relic density, LFV rates can differ from naive estimates by up to two orders of magnitude. Contrary to common belief, we find that current LFV limits do not exclude neutrino Yukawa couplings larger than top Yukawa couplings. We introduce the ISAJET-M program that was used for the computations.
1. Introduction

It is now a firmly established experimental fact that neutrinos are massive and mix [1]. A global fit to current neutrino oscillation data [2] gives the following $3\sigma$ ranges/limits for the mixing parameters (in the “standard parametrization”, Eq. A.14),

$$
\sin^2 \theta_{12} = 0.304^{+0.066}_{-0.054},
\sin^2 \theta_{23} = 0.50^{+0.17}_{-0.14},
\sin^2 \theta_{13} < 0.056.
$$

The phases are not constrained by current data.

The most elegant and popular explanation for small neutrino masses is offered by the seesaw mechanism [3]. Here the Standard Model (SM) is augmented by three right-handed neutrinos (RHNs). The RHNs transform as singlets under the SM gauge group and thus can have a Majorana mass, which is taken to be very large, $M_{Maj} \sim 10^{15}$ GeV. One of the implications of neutrino mixing is the possibility for charged lepton flavor-violating processes. In the SM-seesaw these rates are highly suppressed by the very large scale $M_{Maj}$, so that LFV processes are a good probe of new physics.

Supersymmetry (SUSY) is a well motivated possibility for new physics [5, 6] because, among other things, it stabilizes the seesaw mechanism [3] and ameliorates the hierarchy problem appearing from the presence of the very high scale $M_{Maj}$ [8]. SUSY must be a broken symmetry that is parametrized by soft SUSY breaking (SSB) terms in the Lagrangian [9]. In general these terms can have arbitrary flavor structures that would induce unacceptably large flavor violating effects – this is the well-known SUSY flavor problem. The simplest solution is to assume a flavor-blind SUSY breaking mediation mechanism that will generate flavor universal SSB terms at some high scale. However, this does not mean that SSB terms will also remain flavor-universal at the weak scale: flavor-violating terms will be generated by the Yukawa couplings during the evolution from the high to the weak scale. As a consequence, SUSY contributions to LFV processes are suppressed by the characteristic mass scale of SUSY particles $M_{SUSY} \sim 1$ TeV (instead of $M_{Maj}$) and thus LFV can be observable. Many authors have studied these processes under various SUSY model assumptions and seesaw parameters (see e.g., [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]).

The prediction of LFV rates requires knowledge of the neutrino Yukawa coupling matrix. However, experimentally measured light neutrino masses and mixings do not provide sufficient information to determine it [10]. A top-down approach is frequently adopted in which the neutrino Yukawa matrix is set by a specific SUSY-Grand Unified Theory (GUT), with an $SO(10)$ gauge group being the favorite choice. In this work we take a general approach, and consider two cases of Yukawa unification parameters (defined in Eq. 2.13) inspired by $SO(10)$ relations. For both of these cases we study two scenarios with small and large mixings in the neutrino Yukawa matrix. Our analysis is based on the type-I seesaw mechanism [3].

The existence of a massive, electrically and color neutral, stable weakly interacting particle that can serve as a cold Dark Matter (DM) candidate is perhaps the most compelling
feature of the $R-$parity conserving Minimal Supersymmetric Standard Model (MSSM). In most cases the DM particle is the lightest neutralino, $\tilde{Z}_1$ [20]. The mass density of DM has been precisely determined by cosmological measurements: a combination of WMAP CMB data with the baryon acoustic oscillations in galaxy power spectra gives [21]

$$\Omega_{DM} h^2 = 0.1120_{-0.0074}^{+0.0074}$$

(2σ),

where $\Omega \equiv \rho/\rho_c$ with $\rho_c$ the critical mass density of the Universe, and $h$ is the scaled Hubble parameter. Such a precise determination places severe constraints on new physics scenarios.

In the simplest SUSY model with universal SSB values at the high scale, mSUGRA (or CMSSM) [22], only a few regions of parameter space survive: the bulk region [23, 24], the stau [25, 26] or stop [27] coannihilation regions, the hyperbolic branch/focus point (HB/FP) region [28], and the $A$ or $h$ resonance annihilation (Higgs funnel) regions [24, 29, 30]. The benchmark values of mSUGRA input parameters for these regions are listed in Table 1.

| Point | $m_0$ | $m_{1/2}$ | $A_0$ | $\tan\beta$ | Region |
|-------|-------|-----------|-------|-------------|--------|
| A     | 80    | 170       | -250  | 10          | bulk   |
| B     | 100   | 500       | 0     | 10          | $\tau$-coan. |
| C     | 150   | 300       | -1095 | 5           | $\tilde{t}$-coan. |
| D     | 500   | 450       | 0     | 51          | $A$-funnel |
| E     | 1370  | 300       | 0     | 10          | HB/FP |
| F     | 3143  | 1000      | 0     | 10          | HB/FP |
| G     | 2000  | 130       | -2000 | 10          | $h$-funnel |

Table 1: Parameters for benchmark points and corresponding DM-allowed regions of mSUGRA. The dimensionful parameters $m_0$, $m_{1/2}$, and $A_0$ are in GeV. For all points $\mu > 0$ and $m_t = 171$ GeV.

In previous work [32], some of us demonstrated that these neutrino-induced changes in the SUSY mass spectrum can significantly alter the DM (co)annihilation mechanisms with concomitant changes in $\Omega_{\tilde{Z}_1} h^2$ and DM direct and indirect detection rates.

The aim of the present work is to study predictions for LFV rates while correctly taking into account the aforementioned effect on the neutralino relic density. We take a model-independent approach and only consider effects from RGE running below the unification scale $M_{GUT}$. These two important points distinguish our work (which is closer in spirit to the study of Ref. [1]) from the study of Ref. [33]. We find that proper consideration of the interplay between the neutrino and SUSY sectors can change the predictions for the LFV rates in WMAP-allowed regions by a factor between 2 − 100 compared with naive estimates, and that contrary to common belief, large neutrino Yukawa couplings are not ruled out by current LFV bounds.

The rest of the paper is organized as follows. In the next section we briefly review LFV processes in SUSY-seesaw framework and motivate our ansatz for the neutrino Yukawa matrix. Results from numerical analyses are presented in Section 3. Section 4 is devoted to the discussion of our findings. Finally, we present our conclusions in Section 5. Our notation, a description of our code, and Yukawa RGEs with thresholds can be found in the appendices.
2. SUSY-seesaw and LFV processes

We begin with a brief discussion of our formalism. Details about our notation and conventions are relegated to Appendix A. The superpotential for the MSSM augmented by singlet right-handed neutrinos $\tilde{N}_i^c$ is

$$\hat{f} = \hat{f}_{MSSM} + (f_\nu)_{ij} \epsilon_{ab} \hat{L}_i^a \hat{H}_u^b \tilde{N}_j^c + \frac{1}{2} (M_N)_{ij} \tilde{N}_i^c \tilde{N}_j^c,$$

(2.1)

where $\hat{f}_{MSSM}$ is the MSSM superpotential shown in Eq. (A.1), $\hat{L}$ and $\hat{H}_u$ are, respectively, lepton doublet and up-higgs superfields, and $M_N$ represents the Majorana mass matrix for the (heavy) right-handed neutrinos. Above the scale $M_{\text{GUT}}$ the light neutrino mass matrix is given by the well-known type-I seesaw formula [3],

$$\mathcal{M}_\nu = -f_\nu M_N^{-1} f_\nu^T v_u^2,$$

(2.2)

where $v_u$ is the vacuum expectation value (VEV) of the neutral component $h_u^0$ of the up-type Higgs doublet $H_u$. At low energies the RHNs decouple from the theory and the light neutrino mass matrix is $\mathcal{M}_\nu = -\kappa v_u^2$. Here $\kappa$ is a coupling matrix of the dimension-5 effective operator generated by RHNs (see Eq. (A.11) for the definition), which is determined by matching conditions (A.12) at the RHN decoupling thresholds.

A common solution to the SUSY flavor problem is to assume that at some high scale sfermion SSB mass-squared matrices are diagonal and universal in flavor, and the trilinear couplings are proportional to the Yukawa couplings. In the framework of mSUGRA extended by right-handed neutrinos (mSUGRA-seesaw) the SSB boundary conditions at Grand Unification Scale $M_{\text{GUT}}$ take the particularly simple form:

$$m_{Q,U,D,L,E,R}^2 = m_0^2 \mathbb{I}, \quad m_{H_u}^2 = m_{H_d}^2 = m_0^2, \quad a_{u,d,e,\nu} = -A_0 f_{u,d,e,\nu}.$$

(2.3)

Since the Yukawa couplings $f_\epsilon$ and $f_\nu$ cannot be simultaneously diagonalized, non-vanishing off-diagonal elements will be generated in the left-handed slepton mass matrix $m_L^2$ via renormalization group evolution. In the leading-logarithmic approximation with universal boundary conditions (2.3) the off-diagonal elements are

$$(m_L^2)_{i \neq j} \simeq -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) \sum_k (f_L^T)_{ik} (f_L^*)_{kj} \log \frac{M_{\text{GUT}}}{M_N} ,$$

(2.4)

where $M_{N_k}$ are RHN decoupling scales that are approximately equal to the eigenvalues of the Majorana mass matrix $M_N$. At low energies, these off-diagonal terms induce LFV processes such as $l_i \to l_j \gamma$, $l_i \to 3l_j$ and $l_i \to l_j$ conversion in nuclei. Current bounds on LFV processes as well as the projected sensitivities of future experiments are summarized in Table 2.

| Process | Present | Future |
|---------|---------|--------|
| $\text{BR}(\mu \to e \gamma)$ | $1.2 \times 10^{-11}$ | $10^{-13}$ |
| $\text{BR}(\tau \to \mu \gamma)$ | $4.5 \times 10^{-8}$ | $10^{-9}$ |
| $\text{BR}(\tau \to e \gamma)$ | $3.3 \times 10^{-8}$ | $10^{-9}$ |
| $\text{BR}(\mu \to eee)$ | $1.0 \times 10^{-12}$ | $10^{-14}$ |
| $\text{BR}(\tau \to \mu \mu)$ | $3.2 \times 10^{-8}$ | $10^{-9}$ |
| $\text{BR}(\tau \to eee)$ | $3.6 \times 10^{-8}$ | $10^{-9}$ |
| $\text{CR}(\mu \text{Ti} \to e \text{Ti})$ | $4.3 \times 10^{-12}$ | $10^{-18}$ |
| $\text{CR}(\mu \text{Al} \to e \text{Al})$ | - | $10^{-16}$ |

Table 2: Present bounds and projected sensitivities for LFV processes.
The branching ratio for the flavor-violating radiative decay of a charged lepton is given by
\[
BR(l_i \rightarrow l_j \gamma) = \frac{\alpha}{4\Gamma(l_i)} m_i^8 (|A_L|^2 + |A_R|^2).
\]
Here \(\alpha\) is the electromagnetic fine structure constant, \(\Gamma(l_i)\) is the total decay width of the initial lepton, and \(A_{L,R}\) are form factors for left and right chiralities of the incoming lepton whose full expressions in SUSY were obtained in Ref. [12]. Because \(m_{l_i} \gg m_{l_j}\) one has \(A_R \gg A_L\) in the case of initial universality such as (2.3) [14, 13]. In the mass-insertion approximation, the branching ratio can be related to the corresponding off-diagonal element of the left-handed slepton mass matrix [13],
\[
BR(l_i \rightarrow l_j \gamma) \simeq BR(l_i \rightarrow l_j \bar{\nu}_j \nu_i) \frac{\alpha^3}{G_F m_s} |(m_{L_i})_{i \neq j}|^2 \tan^2 \beta,
\]
where \(G_F\) is the Fermi coupling constant and \(m_s\) is the characteristic mass scale of the SUSY particles in the loop. In the case of universal boundary conditions (2.3), this expression used in conjunction with the leading-log result (2.4) well approximates the full expression (2.5), if one sets [14]
\[
m_s^8 \simeq 0.5m_0^2m_{1/2}^2 \left( m_0^2 + 0.6m_{1/2}^2 \right)^2.
\]

LFV \(l_i \rightarrow 3l_j\) decays and \(l_i \rightarrow l_j\) conversion occur via \(\gamma\)-, \(Z\)- and Higgs-penguins as well as squark/slepton box diagrams [12]. Higgs-penguins dominate in the regime of large \(\tan \beta \simeq 60\) and light \(H/A\) Higgs boson mass \(\sim 100\) GeV, and enhance rates by up to a few orders of magnitude [13]. However, the latter condition cannot be realized in the universal scenario (2.3) [45]. It was shown in Ref. [14] that these LFV rates are well described in the universal scenario by the same \(\gamma\)-penguins that contribute to the radiative decays. The branching ratio for trilepton decays is approximately given by
\[
BR(l_i \rightarrow 3l_j) \simeq \frac{\alpha}{3\pi} \left( \log \frac{m_i}{m_j} - \frac{11}{4} \right) BR(l_i \rightarrow l_j \gamma).
\]

For \(\mu \rightarrow e\) conversion a similar relation holds:
\[
CR(\mu N \rightarrow eN) \equiv \frac{\Gamma(\mu N \rightarrow eN)}{\Gamma_{\text{capt}}} = \frac{16\alpha^4 Z}{\Gamma_{\text{capt}}} Z_{\text{eff}}^4 |F(q^2)|^2 BR(\mu \rightarrow e\gamma),
\]
where \(Z\) is the proton number of the nucleus \(N\), \(Z_{\text{eff}}\) is an effective atomic charge obtained by averaging the muon wave function over the nuclear density [16], \(F(q^2)\) denotes the nuclear form factor at momentum transfer \(q\) [17] and \(\Gamma_{\text{capt}}\) is the measured total muon capture rate [18]. In this work we consider two target materials that will be used by future experiments. For \(^{48}\text{Ti}\), which will be used by the proposed PRIME experiment at J-PARC [13], \(Z_{\text{eff}} = 17.6, F(q^2 \simeq -m_{\mu}^2) \simeq 0.54\) and \(\Gamma_{\text{capt}} = 2.590 \times 10^6\text{sec}^{-1}\). For \(^{48}\text{Al}\), the target material for the proposed Mu2e experiment at Fermilab [14], \(Z_{\text{eff}} = 11.48, F(q^2 \simeq -m_{\mu}^2) \simeq 0.64\) and \(\Gamma_{\text{capt}} = 7.054 \times 10^6\text{sec}^{-1}\).
2.1 $SO(10)$ GUTs

As discussed in the previous section, LFV rates crucially depend on the neutrino Yukawa coupling matrix $f_\nu$. However, this matrix cannot be reconstructed from experimental data by inverting the seesaw formula (2.2): $f_\nu$ and $M_N$ together depend on 18 parameters, while $\mathcal{M}_\nu$ contains only 9 observables. A common solution is to turn to GUTs where $f_\nu$ is related to the known Yukawa matrices of SM fermions.

$SO(10)$ GUTs unify all SM fermions and the right-handed neutrino of each generation in a single 16-dimensional spinor representation. The Higgs representation assignments are determined by the following decompositions of the direct products:

$$16 \otimes 16 = 10 \oplus 120 \oplus 126,$$

$$16 \otimes \bar{16} = 1 \oplus 45 \oplus 210.$$  

Many $SO(10)$ models exist in the literature with different choices of Higgs representations and, frequently, with additional flavor symmetries. These models can be divided into two general classes \cite{49}. The first uses only low dimensional Higgs multiplets $10, 16, 45$ with some nonrenormalizable operators constructed from them. This necessarily leads to large $R$–parity violation so that these models cannot provide a DM candidate. Models in the other class involve $10, 120$ or $126$ Higgs representations, have renormalizable couplings, preserve $R$–parity, and are often referred to as minimal Higgs models. The resulting set of sum-rules for the mass matrices are

$$M_{\nu,LR} = f_\nu v_u = f_{10} v_{10}^{10} + f_{126} v_{126}^{126} + f_{120} v_{120}^{120},$$

$$M_{\nu,RR} = M_N = f_{126} V_R,$$

$$M_{\nu,LL} = f_{126} v_L,$$

where $f_R$ are $SO(10)$ Yukawa coupling matrices, $v_{u,d}^R$ are VEVs of the various $SU(2)_L$ doublets (with Higgs fields residing in $R = 10, 126, 120$), and $v_L$ and $V_R$ are the $B – L$ breaking VEVs of the $SU(2)_L$ triplet and singlet respectively. In the type-I seesaw, which we consider in this paper, $v_L = 0$ and SUSY prevents it reappearance via loop diagrams \cite{7}.

From Eq. (2.12), if Higgs superfields reside in $10$, as they do in the simplest scenarios, then the neutrino Yukawa matrix will be identical to the up-quark Yukawa at $M_{GUT}$. If the higgses are in $126$, then $f_\nu = -3 f_u$. A dominant $120$ Higgs would lead to at least a pair of degenerate heavy up-quarks \cite{50} and thus is phenomenologically excluded. Motivated by the above, we introduce a neutrino Yukawa unification parameter as

$$f_\nu \propto R_{\nu u} f_u,$$

and consider two cases,\footnote{Henceforth, we denote $|R_{\nu u}|$ by $|R_{\nu u}|$.} $|R_{\nu u}| = 1$ and $|R_{\nu u}| = 3$. Note that $f_\nu$ and $f_u$ need not be aligned:}

the subdominant contributions from other higgs multiplets and/or flavor group structure
can lead to different diagonalization matrices. To keep our discussion as simple as possible, we consider two extreme cases of the mixing present in $f_\nu$.

**Large mixing:** The measured values of the neutrino mixing angles $[1, 2]$ are consistent with the so-called tri-bimaximal pattern $[51, 52]$, where $\sin^2 \theta_{12} = \frac{1}{3}$, $\sin^2 \theta_{23} = \frac{1}{2}$, $\sin^2 \theta_{13} = 0$. Thus it is reasonable to postulate that mixing in the neutrino Yukawa matrix at the GUT scale also has a tri-bimaximal form. In other words, we assume that the observed MNS mixing matrix arises only from the left-handed rotation matrix (i.e., we set $V_{\nu R} = 1$). We take a neutrino Yukawa matrix of the form

$$f_\nu = R_{\nu u} V_{\nu L} f_{u}^{\text{diag}},$$

with

$$V_{\nu L}^\dagger = U_{\nu}^\dagger = \begin{pmatrix}
    \sqrt{\frac{2}{3}} c_\chi c_\phi + \frac{1}{\sqrt{6}} c_\chi s_\phi & - \frac{1}{\sqrt{3}} c_\chi c_\phi - \frac{1}{\sqrt{2}} s_\chi s_\phi & \frac{1}{\sqrt{6}} c_\chi c_\phi + \frac{1}{\sqrt{2}} s_\chi s_\phi \\
    \frac{1}{\sqrt{6}} c_\chi c_\phi + \frac{1}{\sqrt{2}} s_\chi s_\phi & \frac{1}{\sqrt{3}} c_\chi c_\phi - \frac{1}{\sqrt{2}} s_\chi s_\phi & \frac{1}{\sqrt{6}} c_\chi c_\phi - \frac{1}{\sqrt{2}} s_\chi s_\phi \\
    \frac{1}{\sqrt{3}} c_\chi c_\phi - \frac{1}{\sqrt{2}} s_\chi s_\phi & \frac{1}{\sqrt{6}} c_\chi c_\phi - \frac{1}{\sqrt{2}} s_\chi s_\phi & \frac{1}{\sqrt{3}} c_\chi c_\phi + \frac{1}{\sqrt{2}} s_\chi s_\phi \\
\end{pmatrix},$$

where $s_\chi = \sin \chi$, $c_\chi = \cos \chi$, $s_\phi = \sin \phi$, $c_\phi = \cos \phi$ and the parameters $\chi, \phi \in [0, 2\pi]$. This is the simplest generalization of a tri-bimaximal mixing ($\chi = \phi = 0$) that allows CP violation $[52]$. A fit to experimental values $[1, 2]$ reveals that the Harrison-Scott parameters $\chi$ and $\phi$ are restricted to the vicinity of $\chi + \phi \simeq n\pi$, with the upper bound on $\theta_{13}$ imposing the strongest constraint. Ignoring RGE effects, we invert the seesaw formula and obtain the approximate form for the Majorana mass matrix

$$M_N \simeq \text{diag} \left( \frac{m_{\nu_1}^2}{m_{\nu_2}}, \frac{m_{\nu_2}^2}{m_{\nu_3}}, \frac{m_{\nu_3}^2}{m_{\nu_3}} \right) \times R_{\nu u}^2.$$

We will consider this case as representative of the large mixing scenario.

**Small mixing:** For the small mixing scenario, we take the neutrino and up-quark Yukawa matrices to be exactly aligned with each other at the GUT scale,

$$f_\nu = R_{\nu u} f_u,$$

so that neutrino mixing is given by the CKM matrix. Then, in the absence of significant RGE magnification effects, the Majorana mass matrix cannot be diagonal. Assuming tri-bimaximal mixing in $M_\nu$ and neglecting the small mixing in $f_\nu$ we can estimate eigenvalues $M_N$ for the normal hierarchy of light neutrinos ($m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$),

$$M_{N_1} \simeq \frac{3m_{\nu_2}^2}{m_{\nu_2}} R_{\nu u}^2, ~ M_{N_2} \simeq \frac{2m_{\nu_3}^2}{m_{\nu_3}} R_{\nu u}^2, ~ M_{N_3} \simeq \frac{m_{\nu_3}^2}{6m_{\nu_1}} R_{\nu u}^2.$$

For the inverted mass hierarchy ($m_{\nu_1} \simeq m_{\nu_2} \gg m_{\nu_3}$), a similar procedure yields

$$M_{N_1} \simeq \frac{3m_{\nu_2}^2}{m_{\nu_2}} R_{\nu u}^2, ~ M_{N_2} \simeq \frac{2m_{\nu_1}^2}{3m_{\nu_1}} R_{\nu u}^2, ~ M_{N_3} \simeq \frac{m_{\nu_3}^2}{2m_{\nu_3}} R_{\nu u}^2.$$
Notice that the largest RHN mass is controlled by the smallest light neutrino mass. From Eqs. (2.16)-(2.19), we see that RHNs have a very strong mass hierarchy “quadratic” to the one in up-quark sector: $M_{N_1} : M_{N_2} : M_{N_3} \sim m_2^2 : m_1^2 : m_0^2$. For this reason, only the spectrum with normal hierarchy of light neutrinos is feasible. A quasi-degenerate spectrum ($m_{\nu_1} \simeq m_{\nu_2} \simeq m_{\nu_3}$) would require the lightest Majorana mass to be in the $10^2 - 10^3$ GeV range with significant L-R mixing in the sneutrino sector which is in conflict with our approximations (see the discussion pertaining to Eq. A.15). Moreover, such light Majorana masses make successful thermal leptogenesis impossible. The inverse hierarchical case would require the heaviest Majorana mass to be of order $10^{17}$ GeV, which is well above the GUT scale. This type of spectrum also suffers instabilities under very small changes to $M_N$ and RGE evolution [53]. In the rest of the paper we concentrate on the normal hierarchy case, which is also favored by GUT model building [54].

3. Procedure and results

We extensively modified ISAJET [55] by including the neutrino sector and by implementing RGE evolution in matrix form to incorporate flavor effects in both the quark and lepton sectors. The resulting program ISAJET-M performs RGE evolution in the MSSM augmented with RHNs at 2-loop level taking into account various thresholds and computes sparticle spectra including radiative corrections. The computation of the neutralino relic density is done using the IsaReD code [26] and LFV rates are computed using the full one-loop formulae from Ref. [12]. A graphical outline of our procedure is presented in Fig. 1 and details of the program are provided in Appendix B.

In the neutrino sector we employ a “top-down” approach in which $f_{\nu}$ and $M_N$ are inputs at the scale $M_{GUT}$. Physical neutrino masses and mixings are derived results which we require to be within the experimental bounds (1.1). We consider the two cases for neutrino Yukawa unification parameter, $R_{\nu u} = 1$ and $R_{\nu u} = 3$ that were introduced earlier. For each case we consider scenarios with large and small neutrino mixings using Eqs. (2.14) and (2.17) to set the neutrino Yukawa matrix at $M_{GUT}$. This also restricts Majorana masses to be below that scale, i.e. $\text{max}(M_N) \lesssim M_{GUT}$.

In the quark sector we choose a basis at the weak scale in which CKM rotation is entirely in the up-quark sector: we set the fermion rotation matrices (A.3) to be $V_{uL} = V_{uR} = V_{CKM}$ and $V_{dL} = V_{dR} = 1$. Note that this does not mean that $f_d$ remains diagonal at all scales – off-diagonal terms will be generated at higher scales due to RGE effects. Similarly, for charged leptons we set $V_{eL} = V_{eR} = 1$ at $M_Z$.

Regarding the SUSY sector we work in the well-studied scenario specified by the parameter set,

$$m_0, \ m_{1/2}, \ A_0, \ \tan \beta, \ \text{sgn}(\mu), \ (3.1)$$

where GUT-scale boundary conditions are universal and defined by Eq. (2.3). This choice of boundary conditions is frequently referred to as mSUGRA-like. Instead of scanning over the full parameter space which would be exceedingly computational intensive, we study specific points for each DM-allowed region. Throughout this work we take $\mu > 0$.
as suggested by measurements of the muon anomalous magnetic moment \cite{56, 57, 58} and set the pole mass of top quark $m_t = 171$ GeV in accord with Tevatron data \cite{59}. For the DM relic density, we consider the conceptually simplest scenario in which the DM is

\textbf{Figure 1}: Code flowchart. $\mathcal{P}_i$ represent sparticles and Higgses arranged in the ascending order of their masses. The code is described in Appendix 3.
comprised only of the lightest neutralino $\tilde{Z}_1$ that is thermally produced in the standard ΛCDM cosmology. We first calculate the neutralino Relic Density (RD) and LFV rates in our framework with SUSY-seesaw using points from Table I that have WMAP-allowed values in the mSUGRA framework without seesaw – a procedure commonly used in the literature. Then, if the RD turns out to be too high, as is frequently the case, we find new points consistent with the WMAP range (1.2) by adjusting SSB parameters and calculate the corresponding LFV rates.

3.1 Large mixing

We begin by considering the $R_{\nu u} = 1$ case. Numerically we find that the GUT-scale Majorana mass matrix,

$$M_N = \text{diag} \left(4.75 \times 10^{-6}, 4.75 \times 10^{-5}, 1\right) \cdot 1.398 \times 10^{14} \text{GeV}, \quad (3.2)$$

produces the spectrum $m_{\nu_1} \sim 10^{-5} \text{eV}, m_{\nu_2} \simeq \sqrt{\Delta m^2_{21}} \simeq 8 \times 10^{-3} \text{eV}$ and $m_{\nu_3} \simeq \sqrt{\Delta m^2_{31}} \simeq 5 \times 10^{-2} \text{eV}$ that is in accord with experimental limits (1.1). We chose the mass of the lightest RHN to be far above $M_{\text{SUSY}}$ to prevent the unwanted mixing in the sneutrino sector. Equation (3.2) is in good agreement with our estimate (2.16) up to a factor of $\sim 2$ reduction of up-quark Yukawa couplings (see for example Ref. [60]) due to the RGE effects. This is because $f_{\nu}, M_N$ and the spectrum of light neutrinos are hierarchical and such experience very little change in RGE evolution [61].

In the Harrison-Scott parameterization (2.15), that we use for the mixing in the neutrino Yukawa matrix at the GUT scale, the Dirac phase $\delta$ and mixing angles are a function of $\phi$ and $\chi$. $\delta = \pi/2$ for $\phi = 0$ – see the left frame of Fig. 2. It is known that LFV rates depend on the value of the unknown Dirac phase $[6]$. In addition, most LFV rates are very sensitive to the value of $\theta_{13}$ [17] for which only an upper bound exists. To quantify these dependences, in the middle frame of Fig. 2 we show the branching ratio for $\mu \rightarrow e\gamma$ as a function of the Harrison-Scott parameters. We present them as enhancement factors relative to the rates at $\{\phi, \chi\} = \{0, 0.294\}$ for which $\theta_{13} = 0.239$ (or $\sin^2 \theta_{13} = 0.056$) and $\delta = \pi/2$. We see that with $\theta_{13}$ fixed, the branching ratio changes by up to $\sim 35\%$ under variation of $\delta$. The dependence on $\theta_{13}$ is greater and more complex: rates change by about two orders of magnitude for $\theta_{13}$ ranging from 0.239 to 0.045 (or $\sin^2 2\theta_{13} = 0.008$), with the latter being the ultimate reach of the Daya Bay experiment [63]. Closer to $\phi = \chi = 0$, the rates change much faster and drop by several more orders of magnitude. The rates for the other $\theta_{13}$-dependent LFV processes follow the same pattern as expected from Eqs. (2.8) and (2.9). On the other hand, rates for $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow 3\mu$ are relatively independent of $\theta_{13}$ [17]. In the right frame of Fig. 2 we show enhancement factor contours for $\tau \rightarrow \mu\gamma$. We see that rates change with $\theta_{13}$ by only $\sim 13\%$: this variation is an artifact of the parameterization (2.13), in which $\sin^2 \theta_{12} = 0.33/\cos^2 \theta_{13}$. Also variation with respect to $\delta$ is smaller – only up to $\sim 10\%$. We numerically confirmed that $\tau \rightarrow 3\mu$ rates behave similarly, as expected from Eq. (2.8).

In Figs. 3, 4, 5 and 6, we show the LFV rates along with current experimental bounds (horizontal dashed lines) and projected future sensitivities (dash-dotted lines). To account
Figure 2: Dependence of the weak-scale Dirac phase (left), $BR(\mu \rightarrow e\gamma)$ (middle) and $BR(\tau \rightarrow \mu\gamma)$ (right) on GUT-scale values of the Harrison-Scott parameters for benchmark point A with Yukawa unification parameter $R_{\nu u} = 1$. The contours for LFV branching ratios represent enhancement factors with respect to the point with $\delta = \pi/2$, $\{\phi, \chi\} = \{0, 0.294\}$ that is marked by the blue cross; all angles are in radians. The thick red lines are iso-$\theta_{13}$ contours for $\theta_{13} = 0.239$ (at the CHOOZ bound \[62\]; see Eq. 1.1) and for the ultimate 90% C. L. reach of the Daya Bay experiment $\theta_{13} = 0.045$ \[63\]. The contours remain essentially unchanged for all other points with large mixing and for $R_{\nu u} = 3$.

for the aforementioned dependences on $\theta_{13}$ and $\delta$, we show rates for $\{\phi, \chi\} = \{0, 0.294\}$ (for which $\sin^2 \theta_{13} = 0.056$ and $\delta = \pi/2$) as diagonally hatched bars and for $\phi = \chi = 0$ (resulting in $\sin^2 \theta_{13} = 0$) as solid bars. We also present rates for $\{\phi, \chi\} = \{0, 0.022\}$ (giving $\sin^2 \theta_{13} = 0.002$ and $\delta = \pi/2$) as cross-hatched bars to indicate the upper limit on the rates if the Daya Bay \[63\] and Double Chooz \[64\] experiments produce a null result.

In the bulk \[23, 24\] and the stau-coannihilation \[25, 26\] regions, the neutralino RD is within the WMAP range due to $\tilde{Z}_1$ interactions with the the lighter stau $\tilde{\tau}_1$. Under universal boundary conditions $\tilde{\tau}_1$ is a dominantly right-handed state and as such remains unaffected by the neutrino Yukawa coupling\(^3\). Therefore, the use of mSUGRA values (points A and B) in models with RHNs still produces the correct RD. At point A in the $R_{\nu u} = 1$ case, $BR(\mu \rightarrow e\gamma)$ changes from $1.77 \times 10^{-16}$ for $\phi = \chi = 0$, to $8.57 \times 10^{-8}$ for $\{\phi, \chi\} = \{0, 0.294\}$; intermediate allowed $\phi$ and $\chi$ values produce rates between these values. For $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow 3\mu$, the dependence on $\phi$ and $\chi$ parameters is reversed: larger $\phi$ and $\chi$ produce smaller rates. For example, at point A in the $R_{\nu u} = 1$ case, the $\tau \rightarrow \mu\gamma$ branching fraction is $2.09 \times 10^{-7}$ for $\phi = \chi = 0$ and is $1.77 \times 10^{-7}$ for $\{\phi, \chi\} = \{0, 0.294\}$. We see that $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow 3\mu$ are excellent probes of LFV: the current experimental

\(^3\)A detailed discussion of neutrino Yukawa coupling effects on SUSY spectrum and DM observables can be found in Ref. \[32\].
Figure 3: Radiative LFV decay rates in the large mixing case for two values of $R_{\nu u}$ (c.f. Eqs. 2.4 and 2.7). The heights of the solid bars show rates for exact tri-bimaximal mixing $\chi = \phi = 0$ at the GUT scale. Diagonally hatched bars represent $\{\phi, \chi\} = \{0, 0.294\}$ yielding the maximum allowed $\theta_{13}$ and $\delta = \pi/2$. Cross-hatched bars represent $\{\phi, \chi\} = \{0, 0.022\}$ which has $\delta = \pi/2$ and $\theta_{13} \simeq 0.045$ that is the ultimate reach of the Daya Bay experiment. Dashed lines represent the current bound and dash-dotted lines the projected sensitivity listed in Table 2. The letters denote various benchmark points presented in Tables 1, 3 and 4. Points C through G have a RD above the WMAP bound. The corresponding WMAP-consistent points indicated by subscripts $s, t$ and $g$ are obtained by adjusting $m_0, A_0$ and $m_{1/2}$, respectively.

bound of $\tau \to \mu \gamma$ rules out the bulk region for all values of $\phi$ and $\chi$.

In the stop-coannihilation region [27] the picture is radically different. A naive use of input parameters for point C gives a neutralino relic density $\Omega_{\tilde{Z}_1}h^2 = 0.34$, well above the WMAP bound. This is because $\tilde{t}_1$ is pushed to a higher mass causing stop-coannihilation to cease. To restore the stop-coannihilation mechanism and bring $\Omega_{\tilde{Z}_1}h^2$ down to the WMAP range, one can counteract the effect of $f_\nu$ by adjusting SSB parameters. Adjusting the common scalar mass parameter $m_0$, with the rest of SSB parameters held fixed, can lower the stop mass back to the desired mass range leading to a new WMAP-consistent point that we denote $C_s$ with parameter values shown in Table 3. Increasing $m_0$ also makes sleptons lighter and increases their mixing, as can be seen from Eqs. (2.4) and (2.7), leading to a factor of $\sim 2.8$ increase in LFV rates as compared to point C. The stop mass can also be lowered by dialing the trilinear A-term, resulting in another modified point $C_t$. The rates increase with respect to values at point C by only about 10%. Alternatively, one can raise the $\tilde{Z}_1$ mass closer to that of the $\tilde{t}_1$ by adjusting the common gaugino mass parameter $m_{1/2}$ producing a correct RD at the point $C_g$. Since the required increase is small, rates

$^4$Hereafter we use subscripts $s, t$ and $g$ to denote points obtained from those in Table 1 by adjusting the value of one the model parameters: scalar mass $m_0$, trilinear coupling $A_0$ or gaugino mass $m_{1/2}$, respectively. Single and double primes are used to further distinguish modified points in the large mixing with $R_{\nu u} = 3$ and the small mixing with $R_{\nu u} = 1$ scenarios.
Figure 4: Similar to Fig. 3 for $\tau$ radiative LFV decays. For $\tau \to \mu \gamma$, the $\{\phi, \chi\} = \{0, 0.022\}$ case is not shown as it produces rates very close to those for $\{\phi, \chi\} = \{0, 0.294\}$ (cross-hatched).

again grow only by $\sim 10\%$.

In the A-funnel region [24, 29], $f_\nu$ pushes the mass of the CP-odd Higgs boson $A$ up and away from the resonance, resulting in a larger RD of 0.144 for point D. This can be reduced by decreasing the scalar mass parameter (point $D_s$) or increasing the value of the trilinear A-term (point $D_t$), which lower the $A$ mass back to the resonance regime. In both cases LFV rates increase by about 8%. Increasing the gaugino mass parameter can increase the $\tilde{Z}_1$ mass to the resonance value $m_{\tilde{Z}_1} = 0.5 m_A$. But the $A$ mass also grows with $m_{1/2}$ although slower than the neutralino mass, so a large dialing is required, resulting in point $D_g$. This large change in $m_{1/2}$ increases the masses of the charginos in the loop resulting in about an order of magnitude drop in LFV rates.

In the lower part of the HB/FP region [25], at point E the RD is two orders of magnitude above the WMAP range. This is due to an increased value of $\mu$ from the $f_\nu$ effect
that can be counteracted by increasing $m_0$, resulting in new WMAP-allowed point $E_s$. The heavier sfermion spectrum causes LFV rates to decrease by about a factor of two. Adjustment of the trilinear parameter can somewhat lower $\mu$ \cite{6}, but not enough to get back into the HB/FP regime. It would be possible to reduce the RD by lowering $m_{1/2}$ to 189 GeV,
but at this value the chargino mass falls below the LEP2 bound of 103.5 GeV [65].

In the upper portion of the HB/FP region, the neutrino Yukawa couplings have an extremely large effect – the RD at point F becomes 12.3. Similarly to point E, the RD can be lowered by increasing the scalar mass parameter. In this part of the HB/FP region the chargino mass is sufficiently high that $m_{1/2}$ can be lowered without violating the LEP2 chargino bound, resulting in a WMAP-allowed value at point $F_g$. Since charginos become lighter with this adjustment, LFV rates increase by about 40%.

In the light Higgs resonance region [24, 30], the neutrino Yukawa coupling also destabilizes the $\tilde{Z}_1$ annihilation mechanism producing too large a RD at point G. Although neither the $\tilde{Z}_1$ nor $h$ masses are moved away from the desired regime $2m_{\tilde{Z}_1} = m_h$, the resonance mechanism ceases to function because $\tilde{Z}_1$ becomes bino-like and can no longer couple to the higgs. The neutralino-higgs coupling can be restored by lowering $\mu$ by increasing the scalar
Table 3: Modified benchmark points for mSUGRA-seesaw in case of the large mixing with $R_{\nu u} = 1$

| Point | $m_0$ | $m_{1/2}$ | $A_0$ | $\tan \beta$ | Region |
|-------|-------|-----------|-------|---------------|--------|
| $C_s$ | 94    | 300       | -1095 | 5             | $t$-coan. |
| $C_t$ | 150   | 300       | -1120 | 5             | $t$-coan. |
| $C_g$ | 150   | 294       | -1095 | 5             | $t$-coan. |
| $D_s$ | 440   | 450       | 0     | 51            | A-funnel |
| $D_t$ | 500   | 450       | 150   | 51            | A-funnel |
| $D_g$ | 500   | 724       | 0     | 51            | A-funnel |
| $E_s$ | 1722  | 300       | 0     | 10            | HB/FP   |
| $F_s$ | 3607  | 1000      | 0     | 10            | HB/FP   |
| $F_g$ | 3143  | 806.5     | 0     | 10            | HB/FP   |
| $G_s$ | 2435  | 130       | -2000 | 10            | $h$-funnel |
| $G_t$ | 2000  | 130       | -1680 | 10            | $h$-funnel |

mass. This decreases LFV rates by about 20%. The desired value of the neutralino-higgs coupling can also be achieved by adjustment of the trilinear parameter resulting in point $G_t$. In this case, the LFV rates increase only marginally compared to point G.

For $R_{\nu u} = 3$, we find that experimental limits (1.1) are satisfied for a GUT-scale Majorana mass matrix of the form,

$$M_N = \text{diag} \left( 4.5 \times 10^{-6}, 4.5 \times 10^{-5}, 1 \right) \cdot 1.3 \times 10^{15} \text{ GeV},$$

(3.3)

which we use in subsequent computations. This is a simple rescaling of Eq. (3.2) as expected from the seesaw formula. We have numerically verified that the dependence of the neutrino mixing parameters and LFV rates follow the same pattern shown in Fig. 2. Thus, rates on Figs. 3-6 are presented for the same choice of $\phi$ and $\chi$ discussed earlier.

As for $R_{\nu u} = 1$, the RD in points A and B remain unaffected by the presence of additional neutrino Yukawa couplings. However, the very large neutrino Yukawa generates large off-diagonal terms in the slepton mass matrix, as can be seen from Eq. (2.4), that boost LFV rates by more than an order of magnitude as compared to the $R_{\nu u} = 1$ case. Nevertheless, this is still not enough to rule out point B for the whole range of $\chi$ and $\phi$ values.

At point C, the RD is too large: the $\tilde{t}_1$ mass is pushed further away from the $\tilde{Z}_1$ due to larger neutrino Yukawa effects. Thus restoration of the stop coannihilation mechanism requires larger adjustments of scalar mass and trilinear coupling parameters, leading to new WMAP-consistent points $C'_s$ and $C'_t$ listed in Table 4. Unlike the $R_{\nu u} = 1$ case, adjustment of $m_{1/2}$ cannot restore the stop-coannihilation: effects of $f_\nu$ make $m_{\tilde{\tau}_1} < m_{\tilde{t}_1}$ and the RD is lowered to the WMAP range at $m_{1/2} = 725 \text{ GeV}$ by the stau-coannihilation mechanism. Increasing $m_{1/2}$ further makes $\tilde{\tau}_1$ the LSP before the stop-coannihilation regime can be reached.
In the A-funnel, larger neutrino Yukawas push \( m_A \) away from the resonance resulting in \( \Omega_{Z_1} h^2 = 0.33 \) at point D. Lowering the scalar mass parameter can bring the \( A \) mass back into the resonance regime at point \( D' \). The RD can also be lowered by either adjusting \( A_0 \) to \(-692 \) GeV or raising \( m_{1/2} \) to \( 745 \) GeV. However, either of these bring the \( \tilde{\tau}_1 \) mass close to the \( \tilde{\tau}_1 \) mass and activate the stau-coannihilation mechanism; further dialing of either parameter makes \( \tilde{\tau}_1 \) the LSP.

In the HB/FP point E, \( \mu \) is pushed by neutrino Yukawa coupling to very large values resulting in large RD, \( \Omega_{Z_1} h^2 = 25 \). To compensate, one needs to dial the scalar mass parameter to very high values (point \( E'_s \)). At such a large \( m_0 \), sleptons become very heavy causing LFV rates to drop by about two orders of magnitude. Consequently, rates for LFV muon decay and \( \mu - e \) conversion fall below current limits for all \( \chi \) and \( \phi \) values. For this point, \( \mu \) is so large that the HB/FP regime cannot be recovered by adjusting \( A_0 \) or \( m_{1/2} \).

Similarly, for point F we get an extremely high relic density \( \Omega_{Z_1} h^2 = 72 \) that can be compensated by a very large scalar mass at point \( F'_s \). The LFV rates are lowered by a factor of \( \sim 15 \) so that muon LFV rates are below experimental bounds for all allowed mixing angles. As in the lower part of the HB/FP region, dialing \( A_0 \) or \( m_{1/2} \) cannot bring \( \Omega_{Z_1} h^2 \) back in accord with WMAP.

In the \( h \)-resonance point G, \( \mu \) is pushed so high that \( \tilde{Z}_1 \) becomes a pure bino state unable to couple to the Higgs boson. This can be compensated only by significantly increasing the scalar mass yielding a new WMAP-consistent point \( G'_s \) with LFV rates that are smaller by two orders of magnitude.

### 3.2 Small mixing

For the case of small mixing, we set \( f_\nu(M_{GUT}) \) according to Eq. (2.17) and choose the neutrino spectrum to be \( m_{\nu_1} = 6 \times 10^{-4} \) eV, \( m_{\nu_2} = 8 \times 10^{-3} \) eV and \( m_{\nu_3} = 5 \times 10^{-2} \) eV. Our choice for \( m_{\nu_3} \) is constrained by the fact that we need \( M_{N_1} \gg M_{SUSY} \) for our approximation to remain valid. In this scenario LFV rates do not depend on neutrino mixing angles: Eq. (2.17) fixes the neutrino Yukawa matrix completely and perturbations of the structure of \( M_N \) do not produce significant changes in RGE evolution as can be seen from Eq. (2.4).

For \( R_{\nu\mu} = 1 \), full RGE evolution with our code yield the following eigenvalues of the Majorana mass matrix:

\[
M_{N_1} \simeq 8 \times 10^4 \text{ GeV} , \quad M_{N_2} \simeq 3.5 \times 10^9 \text{ GeV} , \quad M_{N_3} \simeq 2.5 \times 10^{15} \text{ GeV} .
\]  

(3.4)
This would appear to be in conflict with thermal leptogenesis, which requires the lightest Majorana mass to be heavier than about $10^9$ GeV \[^{66}\]. Nevertheless, successful leptogenesis is possible through the decay of the next-to-lightest RHN \[^{67}\]. Also notice that $M_{N_3}$ is closer to $M_{GUT}$ than it was in the case of large mixing. This, combined with small mixing in $f_\nu$, lead to rates that are several orders of magnitude smaller, putting them all significantly below current experimental bounds, as shown in Figs. \[^{8}\] and \[^{9}\].

As discussed in Section \[^{12}\], the use of mSUGRA values of points A and B still produces the correct RD. For example, at point A we have $BR(\mu \to e\gamma) = 1.1 \times 10^{-13}$ that is barely above the reach of the future MEG experiment. In the A-funnel, neutrino Yukawa couplings do affect the annihilation mechanism, but the effect is small and the RD remains within the WMAP range. For the other regions, adjustment of SSB parameters is necessary; the modified points are listed in Table \[^{5}\].

| Point | $m_0$ | $m_{1/2}$ | $A_0$ | $\tan \beta$ | Region   |
|-------|-------|-----------|-------|--------------|----------|
| $C_s''$ | 126   | 300       | -1095 | 5            | $t$-coan. |
| $C_g''$ | 150   | 300       | -1106 | 5            | $t$-coan. |
| $E_1''$ | 1505  | 300       | 0     | 10           | HB/FP    |
| $F_s''$ | 3300  | 1000      | 0     | 10           | HB/FP    |
| $F_g''$ | 3143  | 943       | 0     | 10           | HB/FP    |
| $G_t''$ | 2205  | 130       | -2000 | 10           | $h$-funnel |
| $G_t''$ | 2000  | 130       | -1895 | 10           | $h$-funnel |

Table 5: Similar to Table \[^{3}\] for the small mixing and $R_{\nu u} = 1$ case.

Even a decoupling scale as high as $M_{N_3} \sim 10^{15}$ GeV is enough to destabilize the stop-coannihilation mechanism, resulting in too large a RD for point C. Lowering the scalar mass parameter decreases the $\tilde{t}_1$ mass to the desired RD value at point $C_s''$, leading to $\sim 50\%$ increase in the LFV rates. The stop coannihilation mechanism can also be restored by adjusting the trilinear A-term (point $C_t''$) or the gaugino mass (point $C_g''$). In both cases the LFV rates increase only slightly from those for point C. At all points $\mu \to e\gamma$ rates are slightly below the MEG reach, so this region will only be probed by $\mu \to e$ conversion experiments.

At point E, the RD is also too high. For the reasons discussed in Section \[^{8}\], only adjustment of $m_0$ is possible, leading to consistency with WMAP for the values at point $E_s''$. Since the required shift is not as significant as in the large mixing case, the LFV rates drop only by $\sim 30\%$.

In the upper part of the HB/FP region at point D, the RD exceeds the WMAP value by two orders of magnitude. The desired higgsino content of $\tilde{Z}_1$ can be restored by adjusting the scalar or gaugino mass parameters, giving points $F_s''$ and $F_g''$. As a result, the LFV rates change by about 10% with respect to those at point D.

At point F, the higgsino content of $\tilde{Z}_1$ is diminished by neutrino Yukawa RGE effects resulting in too high a RD. The $h$-resonance mechanism can be restored by adjusting the scalar mass (point $G_s''$) or the trilinear A-term (point $G_t''$). These adjustments change the LFV rates approximately by a factor of two with respect to the prediction for point F.

For $R_{\nu u} = 3$, we find the eigenvalues of the Majorana mass matrix to be

$$M_{N_1} \simeq 7 \times 10^5 \text{ GeV}, \quad M_{N_2} \simeq 3 \times 10^{10} \text{ GeV}, \quad M_{N_3} \simeq 2.2 \times 10^{16} \text{ GeV}. \quad (3.5)$$
Figure 7: Radiative LFV decay rates in the small mixing case for benchmark points presented in Tables 1 and 5. All rates are below current experimental bounds. Dash-dotted lines represent the projected future sensitivity listed in Table 2.

The heaviest Majorana mass value is very close to $M_{GUT} \approx 2.3 \times 10^{16}$ GeV. Because of this, the effect of $f_\nu$ on RD is negligible and WMAP-consistent values are obtained for the
Figure 8: Similar to Fig. 7 for trilepton LFV decays.

mSUGRA points in Table I. From the resultant LFV rates in Figs. 7-9, we see that the larger neutrino Yukawa coupling produces LFV rates that are smaller by almost an order of magnitude. This is opposite to what we saw in the large mixing scenarios where $R_{\nu u} = 3$.
Figure 9: Similar to Fig. 7 for LFV conversion rates in titanium (upper frames) and aluminum (lower frames) targets.

rates were more than an order of magnitude greater that their $R_{\nu u} = 1$ cousins. This is also a direct consequence of the proximity of $M_{N_3}$ to $M_{GUT}$: the largest neutrino Yukawa decouples almost immediately and off-diagonal elements in the slepton doublet matrix are generated by the much smaller Yukawas of the first and second generations.

4. Discussion

We performed a detailed study of the LFV rates in the RD-allowed benchmark points, and demonstrated that the interconnection between the neutrino sector and neutralino dark matter is very important for predictions of LFV rates. Proper consideration of these effects change LFV rates by factor of a few to up to two orders of magnitude. We emphasize that although we used $SO(10)$ models to set the structure of the neutrino Yukawa matrix
at the GUT scale, our results are generic; they hold in any type-I SUSY-seesaw scenario with large neutrino Yukawa couplings.

The results in Section 3 imply the following about models with universal (or mSUGRA-like) SSB boundary conditions stipulated at the GUT scale:

- The small mixing scenario is completely consistent with present experimental bounds on LFV. Upcoming $\mu \to e\gamma$ experiments will probe only a very small corner of parameter space where both $m_0$ and $m_{1/2}$ are small and $R_{\nu\mu} = 1$. Future $\mu \to e$ conversion experiments, although suppressed by a factor $\sim Z\alpha/\pi$ with respect to $\mu \to e\gamma$, have better prospects due to the very well defined experimental signal. The PRIME experiment will be able to probe the entire bulk and stop-coannihilation regions as well as a significant portion of the A-funnel region, while the Mu2e experiment will only be able to probe the bulk and stop-coannihilation regions for $R_{\nu\mu} = 1$.

- Contrary to naive expectations, in the small mixing case, order of magnitude smaller LFV rates are expected for $R_{\nu\mu} = 3$ than for $R_{\nu\mu} = 1$. Such small rates will only be probed by the PRIME $\mu \to e$ conversion experiment with a Ti target.

- Future $\mu \to e\gamma$ measurements will not rule out large mixing scenarios for any values of $\phi$ and $\chi$ in the mixing matrix of Eq. (2.13) because of the high sensitivity of this channel to $\theta_{13}$. On the other hand, if $\theta_{13}$ is close to the CHOOZ bound, only the HB/FP and $h$-funnel regions are consistent with current limits.

- The $\tau \to \mu\gamma$ channel is an excellent probe as it is not sensitive to $\theta_{13}$. Current experimental limits exclude the bulk region and the stop-coannihilation regions. For large $R_{\nu\mu}$, part of the A-funnel region is also excluded. Future experiments at Super Flavor factories should be able to probe the A-funnel region almost entirely.

- Trilepton decays are weaker probes due to a factor $\sim \alpha$ suppression of the rates as compared to the two body modes. Nevertheless, current data rule out $\theta_{13}$ close to the CHOOZ bound for some regions of SUSY parameter space.

- $\mu \to e$ conversion in nuclei is the best probe of LFV. Future experiments will have sensitivity to almost the entire parameter space. In the large mixing case, $\mu \to e$ conversion is highly complementary to collider searches: it can probe large parts of the HB/FP region which can not be probed at the LHC.

The upcoming Daya Bay and Double Chooz experiments will soon be able to probe $\theta_{13}$ independently of the Dirac phase down to $\sin^2\theta_{13} = 0.002$. A signal of nonzero $\theta_{13}$ will significantly reduce uncertainties in the predictions of LFV rates if the observed neutrino mixing arises dominantly from $f_\nu$ as in the large mixing case. The $\theta_{13}$-dependent LFV rates will be within about two orders of magnitude of the maximum values shown in Figs. 3-6 thus further constraining model parameter space with current LFV data. For instance, the current $\mu \to e\gamma$ bound rules out a significant portion of the A-funnel for the case of large mixing and either value of $R_{\nu\mu}$. With future $\mu \to e\gamma$ measurements we will be able to tell
if the type-I SUSY-seesaw can be realized in the stau-coannihilation region with large $f_\nu$ mixing regardless of $R_{\nu u}$; see point B in Fig. 3.

If the LHC finds a signal of SUSY, then GUTs become very appealing. One might be able to combine the knowledge of the sparticle spectrum from the LHC with results from LFV experiments to determine the value of $R_{\nu u}$ and/or get some information about the mixing pattern in the neutrino Yukawa. For example, if SUSY is found to be realized in the bulk region (point A), then a type-I SUSY-seesaw can only exist if the mixing in $f_\nu$ is small. Then measurements from the PRIME experiment could be used to identify $R_{\nu u}$.

The situation becomes even more favorable if $\theta_{13}$ is known. For example, if SUSY is found to be consistent with the $A$-funnel region, then with the value of $\theta_{13}$ in hand, PRIME measurements will be able to test if the type-I SUSY-seesaw is operative for all mixing patterns and $R_{\nu u}$ values.

5. Conclusions

In previous work \[32\] we demonstrated that large neutrino Yukawa couplings can significantly affect the neutralino relic density. This effect can be counteracted by the adjustment of SSB parameters, with concomitant changes in the low-energy phenomenology. In this work, we studied LFV processes in the type-I SUSY-seesaw properly taking into account neutrino Yukawa effects on the neutralino RD. For simplicity, we considered a scenario with flavor-blind universal (or mSUGRA-like) boundary conditions defined at the GUT scale. In the neutrino sector we utilized the “top-down” approach in which neutrino Yukawa and Majorana mass matrices are inputs at $M_{\text{GUT}}$. We considered two cases for the neutrino-upquark unification parameter $R_{\nu u}$ (see Eq. 2.13) that are inspired by SO(10) models. For each scenario we examined two extreme cases for the mixing in the neutrino Yukawa matrix. We found that the common practice of using WMAP allowed points of mSUGRA in models with RHNs overestimates the LFV rates in most regions of parameter space.

In the $R_{\nu u} = 1$ case we found that the neutrino-neutralino interplay can result in significant changes in LFV predictions. The rates can change by a factor of up to 5 in the WMAP-allowed regions as compared to naive estimates. Effects are most prominent in regions with a large scalar mass parameter $m_0$ such as HP/FP and $h$-funnel regions. If the mixing in the neutrino Yukawa matrix is small, then all LFV rates are below current experimental bounds. In the future, this case can be probed to some extent by the MEG experiment and by the PRIME and Mu2e conversion experiments.

The case of very large unification parameter $R_{\nu u} = 3$, contrary to common lore, is not ruled out by current bounds on LFV processes even if the mixing in the neutrino Yukawa matrix is large. In the large mixing case a proper treatment of the neutralino-neutrino interplay leads to LFV rates that are smaller by about two orders of magnitude than naively expected. As result, many rates fall below current limits. Surprisingly, we found that if mixing in the neutrino Yukawa matrix is small, then for $R_{\nu u} = 3$, the LFV rates are an order of magnitude smaller than for $R_{\nu u} = 1$. If this scenario is realized, then only the future PRIME experiment will have sensitivity to some regions of the parameter space.
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A. Notation and conventions

Here we briefly list the most relevant equations to establish our formalism; we follow the notation and conventions of Ref. [5].

The soft SUSY breaking part of the Lagrangian is

\[
\mathcal{L}_{\text{soft}}^{\text{MSSM}} = - \left[ \hat{Q}_i^a (\tilde{m}_Q^2)_{ij} \hat{Q}_j^b H^c_i \hat{U}_j^c + \hat{d}_i \tilde{m}_D^2 \hat{d}_j \tilde{U}_j^{aib} \right]
\]

\[
+ \hat{L}_i \tilde{m}_L^2 \hat{L}_j + \hat{e}_i \tilde{m}_E^2 \hat{e}_j + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2
\]

\[
- \frac{1}{2} \left[ M_1 \tilde{\lambda}_0 \lambda_0 + M_2 \tilde{\lambda}_A \lambda_A + M_3 \tilde{g}_B \tilde{g}_B \right]
\]

\[
+ \left[ (a_u)_{ij} \epsilon_{ab} \hat{Q}_i^a H^b_{u} d^a_{Rj} + (a_d)_{ij} \hat{Q}_i^a H^a_{d \bar{d}^a_{Rj}} + (a_e)_{ij} \hat{Q}_i^a H^a_{e \bar{e}^a_{Rj}} + \text{h.c.} \right]
\]

\[
+ \left[ bH_a^0 H_{da} + \text{h.c.} \right] .
\]

(A.2)

The gaugino fields \( g_B \) (\( B = 1, 3 \)), \( \lambda_A \) (\( A = 1, 3 \)) and \( \lambda_0 \) transform according to the adjoint representations of \( SU(3)_c \), \( SU(2)_L \) and \( U(1)_Y \), respectively.

We use the RL convention for the fermion mass term, \( \mathcal{L}_{\text{mass}} = - (\bar{\psi}_R^i m_{ij} \psi_L^j + \text{h.c.}) \), in which physical (real and diagonal) mass matrices read

\[
m_u = v_u V_{ur} f_{uR}^T V_{ur}^T , \quad m_d = v_d V_{dr} f_{dR}^T V_{dr}^T , \quad m_e = v_e V_{er} f_e^T V_{er}^T .
\]

(A.3)

where \( v_u, v_d \) are up- and down-higgs VEVs with \( v = \sqrt{v_u^2 + v_d^2} \approx 174 \) GeV. The unitary rotation matrices \( V \) transform gauge eigenstates (unprimed) to mass eigenstates (primed) as follows:

\[
u_{Li}^J = (V_{ul})_{ij} u_{Lj} , \quad u_{Ri}^J = (V_{ur})_{ij} u_{Rj} , \]

\[
d_{Li}^J = (V_{dl})_{ij} d_{Lj} , \quad d_{Ri}^J = (V_{dr})_{ij} d_{Rj} , \]

\[
e_{Li}^J = (V_{el})_{ij} e_{Lj} , \quad e_{Ri}^J = (V_{er})_{ij} e_{Rj} .
\]

(A.4)
The Cabibbo-Kobayashi-Maskawa (CKM) matrix is $V_{CKM} = V_{ul}V_{dl}^\dagger$. Due to different matrix diagonalization conventions, our rotation matrices $V_*$ are hermitian conjugates of those in Ref. [63, 70).

We work in the Super-CKM (SCKM) basis where gluino vertices remain flavor diagonal. Here, the diagonalization of sfermion mass matrices proceeds in two steps. First, the squarks and sleptons are rotated “in parallel” to their fermionic superpartners

$$
\tilde{u}'_i = (V_{ul})_{ij} \tilde{u}_{Lj},
\tilde{d}'_i = (V_{dl})_{ij} \tilde{d}_{Lj},
\tilde{e}'_i = (V_{el})_{ij} \tilde{e}_{Lj},
\tilde{\nu}'_i = (V_{e\ell})_{ij} \tilde{\nu}_{Lj},
$$
(A.5)

where the SCKM scalar fields (primed) form supermultiplets with the corresponding fermion mass eigenstates, i.e., SCKM basis preserves the superfield structure after diagonalization of fermions. Next, $6 \times 6$ sfermion mass squared matrices in the SCKM basis are constructed:

$$
\mathcal{M}^2_u = \begin{pmatrix}
M^2_{uLL} + m^2_u + D(\tilde{u}_L)\mathbb{1} & -M^2_{uLR} + \mu \cot \beta m_u \\
-M^2_{uLR} + \mu^* \cot \beta m_u & M^2_{uRR} + m^2_u + D(\tilde{u}_R)\mathbb{1}
\end{pmatrix},
$$

$$
\mathcal{M}^2_d = \begin{pmatrix}
M^2_{dLL} + m^2_d + D(\tilde{d}_L)\mathbb{1} & -M^2_{dLR} + \mu \tan \beta m_d \\
-M^2_{dLR} + \mu^* \tan \beta m_d & M^2_{dRR} + m^2_d + D(\tilde{d}_R)\mathbb{1}
\end{pmatrix},
$$

$$
\mathcal{M}^2_e = \begin{pmatrix}
M^2_{eLL} + m^2_e + D(\tilde{e}_L)\mathbb{1} & -M^2_{eLR} + \mu \tan \beta m_e \\
-M^2_{eLR} + \mu^* \tan \beta m_e & M^2_{eRR} + m^2_e + D(\tilde{e}_R)\mathbb{1}
\end{pmatrix},
$$
(A.6)

where $D(\tilde{f})$ are the hypercharge $D$-term contributions to the corresponding sfermions, $\mathbb{1}$ is the $3 \times 3$ unit matrix, $m_f$ are the diagonal fermion mass matrices of Eq. (A.3), and the flavor-changing entries are contained in rotated SSB matrices,

$$
M^2_{uLL} = V_{ul}\mathcal{M}^2_u V_{ul}^\dagger,
M^2_{uRR} = V_{ur}\mathcal{M}^2_u V_{ur}^\dagger,
M^2_{uLR} = V_{ul}a_u V_{ur}^\dagger,
M^2_{dLL} = V_{dl}\mathcal{M}^2_d V_{dl}^\dagger,
M^2_{dRR} = V_{dr}\mathcal{M}^2_d V_{dr}^\dagger,
M^2_{dLR} = V_{dl}a_d V_{dr}^\dagger,
M^2_{eLL} = V_{el}\mathcal{M}^2_e V_{el}^\dagger,
M^2_{eRR} = V_{er}\mathcal{M}^2_e V_{er}^\dagger,
M^2_{eLR} = V_{el}a_e V_{er}^\dagger.
$$
(A.7)

Note that the squark doublet mass-squared SSB matrix $m^2_Q$ is rotated differently for $\mathcal{M}^2_u$ and $\mathcal{M}^2_d$. Due to the absence of right-handed neutrino states in the MSSM, the neutrino mass-squared matrix is a $3 \times 3$ matrix of the form,

$$
\mathcal{M}^2_\nu = V_{el}m^2_e V_{el}^\dagger + D(\tilde{\nu}_L)\mathbb{1}.
$$
(A.8)

Finally, the mass squared matrices (A.7) and (A.8) are diagonalized to obtain sfermion mass eigenstates. These mass eigenstates are labelled in ascending mass order.

To incorporate neutrino masses, we employ type-I seesaw mechanism where the MSSM is extended by three gauge singlet superfields $N_i^c$ each of whose fermionic component is the left-handed anti-neutrino and scalar component is $\tilde{\nu}_{Ri}$. The extended superpotential has the form shown in Eq. (3.1). The neutrino Yukawa and Majorana mass matrices are
diagonalized in analogy with Eq. \((A.3)\) by unitary matrices \(V_{\nu L}, V_{\nu R}\) and \(V_N\) according to

\[
\mathbf{m}_D = v_u V_{\nu R} \mathbf{f}^T_{\nu R} V^\dagger_{\nu L}, \quad V_N M_N V^T_N = \text{diag}(M_{N_1}, M_{N_2}, M_{N_3}),
\]

where \(\mathbf{m}_D\) is the Dirac neutrino mass matrix. Apriori the eigenvalues \(M_{N_i}\) are arbitrary, but the observed large difference between neutrino and charged lepton masses strongly suggests that the scale of Majorana masses \(M_{\text{Maj}} \equiv \max(M_{N_i})\) is much higher than the weak scale. Additional soft SUSY breaking terms should also be included so that the Lagrangian becomes,

\[
\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{soft}}^{\text{MSSM}} - \bar{\nu}_{Ri}(\mathbf{m}^2_{\nu R})_{ij} \nu_{Rj} + [ (a_{\nu})_{ij} \epsilon_{ab} \tilde{L}_i^a \tilde{H}_u^b \nu_{Rj} + \frac{1}{2} (b_{\nu})_{ij} \tilde{L}_i^a \tilde{H}_u^b \nu_{Rj} + \text{h.c.}].
\]

The eigenvalues of matrices \(a_{\nu}\) and \(b_{\nu}\) and the square roots of the eigenvalues of \(\mathbf{m}^2_{\nu R}\), are assumed to be of the order of the weak scale \(M_{\text{weak}}\).

Above the scale \(M_{\text{Maj}}\), the mass matrix for left-handed neutrinos is simply \(M_{\nu} = m_{\nu R}^2\). The Maki-Nakagava-Sakata (MNS) mixing matrix of physical neutrinos \([72]\) then has the form

\[
\mathbf{V}_{\nu L} = \mathbf{V}_{\nu R} \mathbf{V}_{\nu L}^T = M_N \equiv \text{diag}(M_{N_1}, M_{N_2}, M_{N_3})
\]

\(\mathbf{V}_{\nu R}\) is a 3 \(\times\) 3 complex symmetric coupling matrix that breaks lepton number explicitly, and is determined by matching conditions at RHN thresholds,

\[
(k)_{ij} |_{N_k}^- = (k)_{ij} |_{N_k}^+ + (F_{\nu})_{ik} \frac{1}{M_{N_k}} (f_{\nu}^T)_{ik} |_{N_k}^+
\]

where \(M_{N_k}^+ (M_{N_k}^-)\) denotes the value as the scale of decoupling of the \(k\)-th generation RHN \(M_{N_k}\) is approached from above (below).

After electroweak symmetry breaking, the Majorana mass matrix for light left-handed neutrinos is simply \(M_{\nu} = -\kappa v_u^2\). The symmetric 3 \(\times\) 3 matrix \(M_{\nu}\) can be diagonalized as

\[
U_\nu M_{\nu} U_{\nu}^T = \mathbf{m}_{\nu},
\]

where \(U_\nu\) is a unitary matrix and \(\mathbf{m}_{\nu}\) is a diagonal matrix of the physical neutrino masses \(m_1, m_2\) and \(m_3\). In labeling the (real non-negative) mass eigenstates we follow the usual convention that 1 and 2 denote states with the smallest mass-squared difference and \(m_1 < m_2\). The Maki-Nakagava-Sakata (MNS) mixing matrix of physical neutrinos \([72]\) then has the form \(V_{\nu L} = V_{\nu R} \mathbf{U}_{\nu L}^T\). Note that as a consequence of the seesaw mechanism, \(U_{\nu}\) is in general different from the neutrino Yukawa diagonalization matrix \(V_{\nu L}\) defined in Eq.\((A.9)\).

The MNS matrix can be parametrized as

\[
V_{\nu L} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i \delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i \delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i \delta} & -s_{23} c_{12} - c_{23} s_{13} s_{12} e^{i \delta} & c_{23} c_{13} \end{pmatrix} \times \text{diag}(e^{i \delta_F}, e^{i \delta_T}, 1),
\]

\((A.14)\)
with \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \), \( \theta_{ij} \) are mixing angles and the Dirac and Majorana CP-violating phases are \( \delta, \phi_{1,2} \in [0, 2\pi] \), respectively.

The addition of the \( \tilde{N}_i \) superfields also leads to an expansion of the sneutrino mass-squared matrix: it is now a \( 12 \times 12 \) matrix, so that the relevant part of Lagrangian is

\[
\mathcal{L} \supset -\frac{1}{2} \tilde{n}^T \begin{pmatrix}
M^{2}_{L1L} & 0 & \mathcal{M}^{2}_{L1R} & v_u f_{\nu}^* M^T_N \\
0 & (\mathcal{M}^{2}_{L1L})^T & v_u M^T_N & (\mathcal{M}^{2}_{L1R})^* \\
v_u M^T_N & v_u f_{\nu}^* & \mathcal{M}^{2}_{R1R} & -b^T_{\nu} \\
(\mathcal{M}^{2}_{L1R})^T & (\mathcal{M}^{2}_{L1R})^T & \mathcal{M}^{2}_{R1R} & -b^T_{\nu} \\
\end{pmatrix} \tilde{n},
\] (A.15)

where \( \tilde{n}^T \equiv (\tilde{\nu}_L^T, \tilde{\nu}_L^T, \tilde{\nu}_R^T, \tilde{\nu}_R^T) \), \( 0 \) is the \( 3 \times 3 \) null matrix and

\[
\mathcal{M}^2_{L1L} = m_f^2 + v^2_\nu f_{\nu}^* f_{\nu}^* + D(\tilde{\nu}_L)^{\dagger},
\]
\[
\mathcal{M}^2_{R1R} = m_f^2 + v^2_\nu f_{\nu}^* f_{\nu}^* + M_N^2,
\]
\[
\mathcal{M}^2_{L1R} = -v_u a_\nu^* + \mu v_d f_{\nu}^*.
\]

From this structure we see that there is sneutrino-antisneutrino mixing for right-handed states introduced by \( b_{\nu} \), with no corresponding terms in the left-handed sector. The Majorana mass matrix \( M_N \) contributes to the mass of the right-handed states and also results in the mixing of right-handed anti-sneutrino states with left-handed sneutrinos. Since \( M_N \) eigenvalues are much larger than the rest of the SSB parameters the matrix exhibits a seesaw type behavior, similar to the one for neutrinos: the \( 6 \times 6 \) L-L block is of \( \mathcal{O}(M^2_{\text{weak}}) \), while R-L blocks are of \( \mathcal{O}(M_{\text{weak}} M_{\text{Maj}}) \) and the R-R block is \( \mathcal{O}(M^2_{\text{Maj}}) \). Therefore the right-handed sneutrinos decouple and the phenomenologically relevant left-handed sneutrinos have a mass-squared matrix of the familiar MSSM form (A.8).

**B. Description of ISAJET-M**

In this section we describe the algorithm used to perform the calculation. Our code, named ISAJET-M, is ISAJET 7.78 ([55]) modified to evolve all couplings and SSB parameters, including the neutrino sector, in full matrix form at the 2-loop level. The following Standard Model parameters are used as inputs: fermion masses, the Z-boson pole mass \( M_Z \), the fine structure constant \( \alpha_{\text{MS}}(M_Z) \), the strong coupling constant \( \alpha_s^{\text{MS}}(M_Z) \) and CKM angles in the “standard parametrization”; see Table 6.

| parameter | value | parameter | value |
|-----------|-------|-----------|-------|
| \( M_Z \) | 91.1876 | \( m_t \) | 171 |
| \( 1/\alpha_{\text{MS}}(M_Z) \) | 127.918 | \( m_b - m_c \) | 3.42 |
| \( \alpha_s^{\text{MS}}(M_Z) \) | 0.1176 | \( m_e \times 10^3 \) | 0.511 |
| \( \sin^2 \theta_W(M_Z) \) | 0.23122 | \( m_{\mu} \) | 0.10566 |
| \( m_u(2 \text{GeV}) \) | 0.003 | \( m_{\tau} \) | 1.77699 |
| \( m_d(2 \text{GeV}) \) | 0.006 | \( \sin \theta^{CKM}_{12} \) | 0.22715 |
| \( m_s(2 \text{GeV}) \) | 0.095 | \( \sin \theta^{CKM}_{23} \) | 0.04161 |
| \( m_c(m_t)^{\text{MS}} \) | 1.25 | \( \sin \theta^{CKM}_{13} \) | 0.003682 |
| \( m_b(m_b)^{\text{MS}} \) | 4.20 | \( \delta^{CKM} \) | 1.0884 |

**Table 6:** SM input parameters ([53]) for ISAJET-M. All masses are in GeV and \( \delta^{CKM} \) is in radians.
In the first step \( \alpha_{\text{MS}}^{\overline{\text{MS}}} \) and \( \alpha_{\text{MS}}^{\overline{\text{MS}}} \) are evolved from \( Q = M_Z \) down to \( Q = 2 \) GeV using 2-loop QCD×QED RGEs with the additional 3rd QCD loop \([74, 75]\). We use step-function decoupling of fermions at the scale of their running mass and include finite threshold corrections at 2 loops according to Ref. \([76]\). Then we compute the running lepton masses \( m_{l}^{\overline{\text{MS}}} (2 \text{ GeV}) \) from their pole masses \( m_{l} \) using the 1-loop expression in the \( \overline{\text{MS}} \) scheme \([74]\):

\[
m_{l}^{\overline{\text{MS}}} (Q) = m_{l} \left[ 1 - \frac{\alpha_{\text{MS}} (Q)}{\pi} \left( 1 + \frac{3}{4} \ln \frac{Q^2}{m_{l}^2} \right) \right].
\]

Next, the two gauge couplings and all SM fermion masses (except for the top mass) are run up to \( M_Z \). Here fermion masses are converted from \( \overline{\text{MS}} \) to \( \overline{\text{DR}} \) scheme using formulae given in Ref. \([77, 78]\):

\[
m_{b}^{\overline{\text{DR}}} (M_Z) = m_{b}^{\overline{\text{MS}}} (M_Z) \left[ 1 + \frac{\alpha_{s}}{3\pi} - \frac{29\alpha_{s}^2}{126\pi^2} + \frac{3g_{s}^2}{128\pi^2} + \frac{13g_{s}^2}{1920\pi^2} \right]^{\overline{\text{MS}}},
\]

\[
m_{c}^{\overline{\text{DR}}} (M_Z) = m_{c}^{\overline{\text{MS}}} (M_Z) \left[ 1 + \frac{\alpha_{s}}{3\pi} - \frac{29\alpha_{s}^2}{126\pi^2} + \frac{3g_{s}^2}{128\pi^2} + \frac{g_{s}^2}{1920\pi^2} \right]^{\overline{\text{MS}}},
\]

\[
m_{\tau}^{\overline{\text{DR}}} (M_Z) = m_{\tau}^{\overline{\text{MS}}} (M_Z) \left[ 1 + \frac{3g_{s}^2}{128\pi^2} - \frac{9g_{s}^2}{640\pi^2} \right]^{\overline{\text{MS}}},
\]

For the lighter SM fermions these conversion corrections are neglected, i.e., we take \( m_{f}^{\overline{\text{DR}}} = m_{f}^{\overline{\text{MS}}} \) at \( M_Z \) for \( f = u, d, s, e, \mu \).

For the top quark, we obtain the running mass at \( Q = m_{t} \) using the 2-loop QCD expression \([78]\),

\[
m_{t}^{\overline{\text{DR}}} (m_{t}) = m_{t} \left[ 1 + \frac{5\alpha_{s}(m_{t})}{3\pi} + \left( \frac{\alpha_{s}(m_{t})}{\pi} \right)^2 \Sigma_{t}^{\text{2loop}} \right],
\]

where \( \Sigma_{t}^{\text{2loop}} \) is the 2-loop piece. Note that we activate the top quark at the scale of its mass, so we have a 5-flavor scheme below \( Q = m_{t} \) and all 6 flavors above it.

The obtained \( \overline{\text{DR}} \) values of SM fermion masses are then substituted into Eq. (A.3) to calculate the Yukawa matrices at \( M_Z \) in the gauge eigenbasis. A choice of basis is made by specifying fermion rotation matrices \( V_{\bullet} \) defined in Eq. (A.3). ISAJET-M also has options for performing calculations in the unmixed and dominant third-family approximations.

The obtained weak scale values of gauge couplings and Yukawa matrices are evolved to the Grand Unification scale \( M_{\text{GUT}} \) via MSSM RGEs in the \( \overline{\text{DR}} \) scheme. The GUT scale \( M_{\text{GUT}} \) is defined to be the scale at which \( g_{1} = g_{2} \), where \( g_{1} = \sqrt{5/3}g' \) is the hypercharge coupling in the GUT-scale normalization. We do not impose an exact unification of the strong coupling \( (g_{3} = g_{1} = g_{2}) \) at \( M_{\text{GUT}} \), assuming that the resulting few percent discrepancy comes from GUT-scale threshold corrections \([80]\).

For the evolution of gauge and Yukawa couplings, we use a multiscale effective theory approach proposed in Ref. \([51]\), where heavy degrees of freedom are integrated out at each particle threshold. In Appendix \([\ref{app:RGEs}]\) we list the corrected Yukawa 1-loop RGEs from an unpublished erratum of Ref. \([81]\). In the second-loop RGE terms we change from MSSM
formulae [62] to SM expressions [60, 70] at a single scale, \( Q = M_{SUSY} \equiv \sqrt{m_{L} m_{R}} \), thus introducing an error of 3-loop order which is negligible.

This “step beta-function approach” produces continuous matching conditions across thresholds. However, decoupling of a heavy particle also introduces finite shifts in RGE parameters [83]; a similar effect has long been known in QCD, where the decoupling of heavy quarks leads to shifts in the running masses of the light quarks [83]. Expressions for shifts induced by decoupling of each individual sparticle depend on the ordering of sparticle spectrum and are not yet known for the general case. Therefore, we implement these sparticle-induced finite shifts (to all three generations) collectively at a common scale \( Q = M_{SUSY} \) in the basis where Yukawa matrices are diagonal; we use 1-loop expressions of Ref. [85] without logarithmic terms that have already been resummed by the RGE evolution. For the top Yukawa coupling additional 2-loop SUSY-QCD corrections are included according to Ref. [79]. These finite threshold corrections are particularly important for GUT theories since they change ratios of Yukawa couplings from those at the weak scale, as was emphasized in Ref. [86]. This multiscale approach is a generalization of the one used by the standard ISAJET, as is described in detail in Ref. [87], and is preferred to single-scale decoupling when the sparticle mass hierarchy is large (as appears, for example, in the HB/FP region [28] of mSUGRA).

At \( M_{GUT} \), the SSB boundary conditions are imposed and all SSB parameters along with gauge and Yukawa couplings are evolved back down to the weak scale \( M_Z \). For the SSB parameters we use 2-loop RGEs from Ref. [82] with the following conversion between notations: \( f_\bullet \equiv Y_\bullet^T, a_\bullet \equiv -h_\bullet^T, b \equiv -B \). Unlike the gauge and Yukawa couplings, where beta-functions change at every threshold, the SSB beta-functions remain those of the MSSM all the way down to \( M_Z \). We do not take into account threshold effects from the appearance of new couplings in the region of broken supersymmetry introduced in Ref. [69, 70]. The entire parameter set is iteratively run between \( M_Z \) and \( M_{GUT} \) using full 2-loop RGEs in matrix form until a stable solution is obtained. After each iteration the Higgs potential is minimized and sparticle/Higgs masses are recalculated. The obtained mass spectrum is used in the next iteration to appropriately account for sparticle threshold effects on the RGE evolution.

The minimization of the RGE-improved potential is done using the tadpole method [88]. We include 1-loop contributions from third generation sfermions, which dominate, as well as contributions from charginos, neutralinos and Higgs bosons. Computation is done at an optimized scale \( Q = M_{SUSY} \), which effectively accounts for the leading 2-loop corrections. Since tadpoles that contribute to \( \mu \) and \( b \) strongly depend on the parameters themselves, an iterative procedure is employed. Calculations of \( \mu, b \) and tadpoles are iterated until consistent values with a precision of 0.1% are obtained; usually this requires 3-4 iterations.

In the computation of sparticle masses we use SSB parameters extracted at their respective mass scales. Then, the SSB matrices are assembled and rotated to the SCKM basis as in Eq. (A.7). The resultant matrices are plugged into the sfermion mass-squared matrices (A.6). Instead of diagonalizing the full \( 6 \times 6 \) matrices (A.6) we diagonalize three \( 2 \times 2 \) submatrices, thereby neglecting intragenerational mixings, which are required to be small by experimental limits on flavor changing neutral current processes [33]. For the
finite corrections, the full expressions of Ref. [85] for 1-loop self-energies are used.

For the SUSY-seesaw we also use the multiscale approach that is mandatory to obtain correct values in the neutrino sector in the case of hierarchical RHNe [89]. Our approach is identical to the one used in the REAP code [61]. Above the scale of the heaviest RH we have a full MSSM+RHN setup, with the MSSM RGEs extended to include full 2-loop equations for $f_\nu$, $a_\nu$, $m_{\tilde{\nu}R}^2$ and $M_N$ [10, 18]. As mentioned earlier, bilinear coupling matrix $b_\nu$ only introduces mixings for right-handed sneutrino states whose masses are $O(M_{\text{Maj}})$ and is thus irrelevant for our analysis. We also take into account additional contributions to RGEs of ordinary MSSM parameters due to the RHN superfields up to 2-loop order [10, 18, 61, 90]. For large neutrino Yukawa couplings these additional RGE terms cause changes to the MSSM sparticle spectrum that can have significant consequences for experimental rates [31, 32]. Below the scale of the lightest RHN, the RGEs are those of the MSSM, plus additional equations for the coupling $\kappa$ of the dimension-5 effective neutrino operator (A.11) that are included at 2-loop level [61]. In the intermediate region, both $\kappa$ and some elements of $f_\nu$ and $M_N$ are present. Transitions between them are done using matching conditions at RHN thresholds Eq. (A.12). We match only at tree-level and neglect small finite threshold corrections. The position of RHN thresholds $M_{N_k}$ are determined by the eigenvalues of the RGE-evolving Majorana mass matrix $M_N$ at those scales. Note that Eq. (A.12) is valid only in the basis where $M_N$ is diagonal at the threshold, which is different from the original basis at $M_{\text{GUT}}$ due to RGE effects. Below the scale $Q = M_{N_k}$, the $k$–th RHN superfields are absent from the theory so we remove the corresponding columns of $f_\nu$ and $a_\nu$ matrices and the $k$–th row and column of $m_{\tilde{\nu}R}^2$ and $M_N$ in the basis where $M_N$ is diagonal.

Since some of the RGE parameters (gauge and Yukawa couplings) are defined at the weak scale, while the rest are set at $M_{\text{GUT}}$, an iterative procedure is employed to solve the RGEs. Once a stable solution to the RGEs is obtained, the decay width and branching ratios for all the sparticles and Higgs bosons are calculated. This step as well as the previous computation of the mass spectrum is performed using standard ISAJET subroutines [55]. A graphical outline of our code, for a SUSY-seesaw model that uses top-down approach for the neutrino sector (i.e., $f_\nu$, $M_N$ are inputted at the GUT scale and physical light neutrino masses and mixing parameters are derived results), is shown in Fig. 1.

ISAJET-M has facility to calculate and return the neutralino relic density $\Omega_{Z_1} h^2$, neutralino-nucleon elastic cross sections, branching fractions for $b \to s\gamma$ and $B_s \to \mu\mu/\tau\tau$ decays, and supersymmetric contributions to the muon anomalous magnetic moment $\Delta a_\mu \equiv (g - 2)_{\mu}/2$. These computations are done, respectively, by the IsaReD [26], IsaReS [91], IsaBSG [92], IsaBMM [93] and IsaAMU [58] codes from the IsaTools package. In addition, for scenarios with massive neutrinos, branching fractions for lepton flavor violating decays $l_i \to l_j \gamma$ and $l_i \to 3l_j$ as well as rates for $\mu \to e$ conversion in several nuclei can be calculated using expressions from Ref. [12].

C. Yukawa RGEs

The following are the 1-loop RGEs for MSSM Yukawa coupling matrices with sparti-
cle/higgs thresholds from Ref. [81] with corrections from their unpublished erratum implemented:

\[
(4\pi)^2 \frac{d(f^T_{ij})}{dt} = \frac{3}{2} \left( s^2 \theta + c^2 \theta_H \right) (f^T_{ij} f^*_{ij}) + \frac{1}{2} \left( c^2 \theta + s^2 \theta_H \right) \sum_{k=1}^{N_d} (f^T_{ik} f^*_{kl}) \left( f^T_{ij} \right)
\]

\[
+ \frac{1}{2} \left( s^2 \theta + c^2 \theta_H \right) \left[ 2(f^T_{iu} f^*_{iu})_i + \frac{N_Q}{4} \sum_{k=1}^{N_d} (f^T_{ik} f^*_{kl})_i \right]
\]

\[
+ \frac{1}{2} \left( c^2 \theta + s^2 \theta_H - 4c^2 (\theta_h - \theta_H) \right) \left( f^T_{iu} f^*_{iu} \right)_i
\]

\[
+ (f^T_{ij}) \left[ (s^2 \theta + c^2) \sum \left( 3 f^T_{ij} + (f^T_{ij}) \right) \right]
\]

\[
(4\pi)^2 \frac{d(f^T_{ij})}{dt} = \frac{3}{2} \left( c^2 \theta + s^2 \theta_H \right) (f^T_{ij} f^*_{ij}) + \frac{1}{2} \left( s^2 \theta + c^2 \theta_H \right) \sum_{k=1}^{N_d} (f^T_{ik} f^*_{kl} f^T_{ij})
\]

\[
+ \frac{1}{2} \left( c^2 \theta + s^2 \theta_H - 4s^2 (\theta_h - \theta_H) \right) (f^T_{iu} f^*_{iu} f^T_{ij})
\]

\[
+ (f^T_{ij}) \left[ s^2 (\theta_h - 1) \sum \left( 3 f^T_{ij} + (f^T_{ij}) \right) \right]
\]

\[
- (f^T_{ij}) \left[ \frac{3}{5} g^2 \left\{ \frac{5}{12} + \frac{3}{4} \theta_h - \left( \frac{1}{36} \theta_{Q_j} + \frac{1}{9} \theta_{d_i} + \frac{1}{4} \theta_h \right) \theta_B \right\}
\]

\[
\left. + g^2 \left\{ \frac{9}{4} + \frac{9}{4} \theta_h - 3 \left( \theta_{Q_j} + \theta_{d_i} \right) \theta_W \right\} + g^2 \left\{ \frac{8}{3} \left( \theta_{Q_j} + \theta_{d_i} \right) \theta_q \right\} \right\}
\]

\[
\] where \( s = \sin \alpha, \ c = \cos \alpha, \) is Higgs mixing angle and the various \( \theta_H \)'s are equal to zero below the mass threshold of the respective particle and equal to one above it. The contributions from neutrino Yukawa couplings can be found in Refs. [10, 13].
References

[1] For a review of neutrino oscillations, see e.g., V. Barger, D. Marfatia and K. Whisnant, Int. J. Mod. Phys. E 12, 569 (2003) [arXiv:hep-ph/0308123].

[2] T. Schwetz, M. Tortola and J. W. F. Valle, New J. Phys. 10, 113011 (2008) [arXiv:0808.2016 [hep-ph]].

[3] H. Fritzsch and P. Minkowski, Phys. Lett. B 62, 72 (1976); P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity: proceedings of the workshop, Stony Brook, NY, (North-Holland, Amsterdam, 1979); T. Yanagida, KEK Report No.79-18, 1979; S. Glashow, in Quarks and Leptons, Cargese, France, (Plenum, New York, 1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[4] S. T. Petcov, Sov. J. Nucl. Phys. 25, 340 (1977) (1977 ERRAT,25,698.1977 ERRAT,25,1336.1977); S. M. Bilenky, S. T. Petcov and B. Pontecorvo, Phys. Lett. B 67, 309 (1977); T. P. Cheng and L. F. Li, Phys. Rev. Lett. 45, 1908 (1980); B. W. Lee, S. Pakvasa, R. E. Shrock and H. Sugawara, Phys. Rev. Lett. 38, 937 (1977) [Erratum-ibid. 38, 1230 (1977)]; V. D. Barger and D. V. Nanopoulos, Nuovo Cim. A 44, 303 (1978).

[5] H. Baer and X. Tata, Weak Scale Supersymmetry: From Superfields to Scattering Events, (Cambridge University Press, 2006).

[6] M. Drees, R. Godbole and P. Roy, Theory and phenomenology of sparticles: An account of four-dimensional $N=1$ supersymmetry in high energy physics, (World Scientific, 2004); S. P. Martin, arXiv:hep-ph/9709356.

[7] C. S. Aulakh, A. Melfo, A. Rasin and G. Senjanovic, Phys. Lett. B 459, 557 (1999) [arXiv:hep-ph/9902409].

[8] J. A. Casas, V. Di Clemente, A. Ibarra and M. Quiros, Phys. Rev. D 62, 053005 (2000) [arXiv:hep-ph/9904295]; J. A. Casas, J. R. Espinosa and I. Hidalgo, JHEP 0411, 057 (2004) [arXiv:hep-ph/0410298].

[9] L. Girardello and M. T. Grisaru, Nucl. Phys. B 194, 65 (1982).

[10] J. A. Casas and A. Ibarra, Nucl. Phys. B 618, 171 (2001) [arXiv:hep-ph/0103065].

[11] A. Masiero, S. K. Vempati and O. Vives, Nucl. Phys. B 649, 189 (2003) [arXiv:hep-ph/0209303].

[12] J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B 357, 579 (1995) [arXiv:hep-ph/9501407]; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D 53, 2442 (1996) [arXiv:hep-ph/9510309].

[13] J. Hisano and D. Nomura, Phys. Rev. D 59, 116005 (1999) [arXiv:hep-ph/9810479].

[14] S. T. Petcov, S. Profumo, Y. Takanishi and C. E. Yaguna, Nucl. Phys. B 676, 453 (2004) [arXiv:hep-ph/0306195].

[15] K. S. Babu and C. Kolda, Phys. Rev. Lett. 89, 241802 (2002) [arXiv:hep-ph/0206310]; A. Dedes, J. R. Ellis and M. Raidal, Phys. Lett. B 549, 159 (2002) [arXiv:hep-ph/0209207]; R. Kitano, M. Koike, S. Komine and Y. Okada, Phys. Lett. B 575, 300 (2003) [arXiv:hep-ph/0308021].

[16] E. Arganda and M. J. Herrero, Phys. Rev. D 73, 055003 (2006) [arXiv:hep-ph/0510405]; E. Arganda, M. J. Herrero and A. M. Teixeira, JHEP 0710, 104 (2007) [arXiv:0707.2955 [hep-ph]].
[17] S. Antusch, E. Arganda, M. J. Herrero and A. M. Teixeira, JHEP 0611, 090 (2006) [arXiv:hep-ph/0607263].
[18] A. Ibarra and C. Simonetto, JHEP 0804, 102 (2008) [arXiv:0802.3858 [hep-ph]].
[19] D. F. Carvalho, J. R. Ellis, M. E. Gomez and S. Lola, Phys. Lett. B 515, 323 (2001) [arXiv:hep-ph/0103256]; J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Rev. D 66, 115013 (2002) [arXiv:hep-ph/0206110]; F. Deppisch, H. Pas, A. Redelbach, R. Ruckl and Y. Shimizu, Eur. Phys. J. C 28, 365 (2003) [arXiv:hep-ph/0206122]; T. Blazek and S. F. King, Nucl. Phys. B 662, 359 (2003) [arXiv:hep-ph/0211368]; A. Masiero, S. K. Vempati and O. Vives, New J. Phys. 6, 202 (2004) [arXiv:hep-ph/0407325]; A. Masiero, S. Profumo, S. K. Vempati and C. E. Yaguna, JHEP 0403, 046 (2004) [arXiv:hep-ph/0401138]; S. T. Petcov, T. Shindou and Y. Takanishi, Nucl. Phys. B 738, 219 (2006) [arXiv:hep-ph/0508243]; S. T. Petcov, W. Rodejohann, T. Shindou and Y. Takanishi, Nucl. Phys. B 739, 208 (2006) [arXiv:hep-ph/0510404]; F. Deppisch, H. Pas, A. Redelbach and R. Ruckl, Phys. Rev. D 73, 033004 (2006) [arXiv:hep-ph/0511062]; L. Calibbi, A. Faccia, A. Masiero and S. K. Vempati, Phys. Rev. D 74, 116002 (2006) [arXiv:hep-ph/0605139]; S. Antusch, E. Arganda, M. J. Herrero and A. M. Teixeira, Nucl. Phys. Proc. Suppl. 169, 155 (2007) [arXiv:hep-ph/0610439]; S. Antusch and S. F. King, Phys. Lett. B 659, 640 (2008) [arXiv:0709.0666 [hep-ph]]; A. Ibarra, T. Shindou and C. Simonetto, JHEP 0810, 021 (2008) [arXiv:0809.0608 [hep-ph]].

[20] For reviews, see e.g. G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. 267, 195 (1996) [arXiv:hep-ph/9506380]; A. B. Lahanas, N. E. Mavromatos and D. V. Nanopoulos, Int. J. Mod. Phys. D 12, 1529 (2003) [arXiv:hep-ph/0308251]; M. Drees, arXiv:hep-ph/0410113; K. A. Olive, arXiv:astro-ph/0503065; Adv. Space Res. 42, 581 (2008).

[21] E. Komatsu et al., arXiv:0803.0547 [astro-ph]; see also http://lambda.gsfc.nasa.gov/product/map/current/parameters.cfm

[22] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982); R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B 119, 343 (1982); N. Ohta, Prog. Theor. Phys. 70, 542 (1983); L. J. Hall, J. D. Lykken and S. Weinberg, Phys. Rev. D 27 (1983) 2359; for reviews, see H. P. Nilles, Phys. Rept. 110, 1 (1984) and P. Nath, arXiv:hep-ph/0307123.

[23] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive and M. Srednicki, Nucl. Phys. B 238, 453 (1984); V. D. Barger and C. Kao, Phys. Rev. D 57, 3131 (1998) [arXiv:hep-ph/9704403].

[24] H. Baer and M. Brhlik, Phys. Rev. D 53, 597 (1996) [arXiv:hep-ph/9508321].

[25] J. R. Ellis, T. Falk and K. A. Olive, Phys. Lett. B 444, 367 (1998) [arXiv:hep-ph/9810360]; J. R. Ellis, T. Falk, K. A. Olive and M. Srednicki, Astropart. Phys. 13, 181 (2000) [Erratum-ibid. 15, 413 (2001)] [arXiv:hep-ph/9905481]; M. E. Gomez, G. Lazarides and C. Pallis, Phys. Rev. D 61, 123512 (2000) [arXiv:hep-ph/9907261] and Phys. Lett. B 487, 313 (2000) [arXiv:hep-ph/0004028]; A. B. Lahanas, D. V. Nanopoulos and V. C. Spanos, Phys. Rev. D 62, 023515 (2000) [arXiv:hep-ph/9909497]; R. L. Arnowitt, B. Dutta and Y. Santos, Nucl. Phys. B 606, 59 (2001) [arXiv:hep-ph/0102181].

[26] H. Baer, C. Balazs and A. Belyaev, JHEP 0203, 042 (2002) [arXiv:hep-ph/0202076].

[27] C. Boehm, A. Djouadi and M. Drees, Phys. Rev. D 62, 035012 (2000) [arXiv:hep-ph/9911496]; J. R. Ellis, K. A. Olive and Y. Santos, Astropart. Phys. 18, 395 (2003) [arXiv:hep-ph/0112113]; J. Edsjo, M. Schelke, P. Ullio and P. Gondolo, JCAP 0304, 001 (2003) [arXiv:hep-ph/0301106].
[28] K. L. Chan, U. Chattopadhyay and P. Nath, Phys. Rev. D 58, 096004 (1998) [arXiv:hep-ph/9710473]; J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. 84, 2322 (2000) [arXiv:hep-ph/9908309]; Phys. Rev. D 61, 075005 (2000) [arXiv:hep-ph/9909334]; H. Baer, C. h. Chen, F. Paige and X. Tata, Phys. Rev. D 52, 2746 (1995) [arXiv:hep-ph/9503271]; Phys. Rev. D 53, 6241 (1996) [arXiv:hep-ph/9512383]; H. Baer, C. h. Chen, M. Drees, F. Paige and X. Tata, Phys. Rev. D 59, 055014 (1999) [arXiv:hep-ph/9809223].

[29] M. Drees and M. M. Nojiri, Phys. Rev. D 47, 376 (1993) [arXiv:hep-ph/9207234]; H. Baer and M. Brhlilh, Phys. Rev. D 57, 567 (1998) [arXiv:hep-ph/9706509]; H. Baer, M. Brhlik, M. A. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, Phys. Rev. D 63, 015007 (2001) [arXiv:hep-ph/0005027]; J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Srednicki, Phys. Lett. B 510, 236 (2001) [arXiv:hep-ph/0102098]; L. Roszkowski, R. Ruiz de Austri and T. Nihei, JHEP 0108, 024 (2001) [arXiv:hep-ph/0106334]; A. Djouadi, M. Drees and J. L. Kneur, JHEP 0108, 055 (2001) [arXiv:hep-ph/0107316]; A. B. Lahanas and V. C. Spanos, Eur. Phys. J. C 23, 185 (2002) [arXiv:hep-ph/0106345].

[30] P. Nath and R. L. Arnowitt, Phys. Rev. Lett. 70, 3696 (1993) [arXiv:hep-ph/9302318]; A. Djouadi, M. Drees and J. L. Kneur, Phys. Lett. B 624, 60 (2005) [arXiv:hep-ph/0504090].

[31] H. Baer, C. Balazs, J. K. Mizukoshi and X. Tata, Phys. Rev. D 63, 055011 (2001) [arXiv:hep-ph/0010068].

[32] V. Barger, D. Marfatia and A. Mustafayev, Phys. Lett. B 665, 242 (2008) [arXiv:0804.3601 [hep-ph]].

[33] M. Ciuchini, A. Masiero, P. Paradisi, L. Silvestrini, S. K. Vempati and O. Vives, Nucl. Phys. B 783, 112 (2007) [arXiv:hep-ph/0702144].

[34] M. L. Brooks et al. [MEGA Collaboration], Phys. Rev. Lett. 83, 1521 (1999) [arXiv:hep-ex/9905013].

[35] K. Hayasaka et al. [Belle Collaboration], Phys. Lett. B 666, 16 (2008) [arXiv:0705.0650 [hep-ex]].

[36] B. Aubert et al. [BABAR Collaboration], arXiv:0908.2381 [hep-ex].

[37] U. Bellgardt et al. [SINDRUM Collaboration], Nucl. Phys. B 299, 1 (1988).

[38] Y. Miyazaki et al. [Belle Collaboration], Phys. Lett. B 660, 154 (2008) [arXiv:0711.2189 [hep-ex]].

[39] C. Dohmen et al. [SINDRUM II Collaboration], Phys. Lett. B 317, 631 (1993).

[40] S. Ritt [MEG Collaboration], Nucl. Phys. Proc. Suppl. 162, 279 (2006); T. Mori, Nucl. Phys. Proc. Suppl. 169, 166 (2007).

[41] M. Bona et al., arXiv:0709.0451 [hep-ex]; A. G. Akeroyd et al. [SuperKEKB Physics Working Group], arXiv:hep-ex/0406071.

[42] With no experiment planned, we use theoretical estimates based on the PSI beam intensity from W. J. Marciano, T. Mori and J. M. Roney, Ann. Rev. Nucl. Part. Sci. 58, 315 (2008).

[43] Y. Mori et al. [The PRIME Working Group], “An Experimental Search for the $\mu^- \rightarrow e^-$ Conversion Process at an Ultimate Sensitivity of the Order of $10^{-18}$ with PRISM”, LOI-25, http://www-ps.kek.jp/jhf-np/LOIlist/LOIlist.html
[44] E.C. Dukes et al. [Mu2e Collaboration], “Proposal to Search for $\mu^- N \to e^- N$ with a Single Event Sensitivity Below $10^{-16}$”, http://mu2e.fnal.gov/public/hep/index.shtml

[45] See e.g., A. Belyaev, S. Dar, I. Gogoladze, A. Mustafayev and Q. Shafi, arXiv:0712.1049 [hep-ph].

[46] H. C. Chiang, E. Oset, T. S. Kosmas, A. Faessler and J. D. Vergados, Nucl. Phys. A 559 (1993) 526.

[47] A. Czarnecki, W. J. Marciano and K. Melnikov, AIP Conf. Proc. 435, 409 (1998) [arXiv:hep-ph/9801218]; R. Kitano, M. Koike and Y. Okada, Phys. Rev. D 66, 096002 (2002) [Erratum-ibid. D 76, 059902 (2007)] [arXiv:hep-ph/0203110].

[48] T. Suzuki, D. F. Measday and J. P. Roalsvig, Phys. Rev. C 35, 2212 (1987).

[49] R. N. Mohapatra and A. Y. Smirnov, Ann. Rev. Nucl. Part. Sci. 56, 569 (2006) [arXiv:hep-ph/0603118], C. H. Albright and M. C. Chen, Phys. Rev. D 74, 113006 (2006) [arXiv:hep-ph/0608137].

[50] R. N. Mohapatra and B. Sakita, Phys. Rev. D 21, 1062 (1980).

[51] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B 530, 167 (2002) [arXiv:hep-ph/0202074]; Z. z. Xing, Phys. Lett. B 533, 85 (2002) [arXiv:hep-ph/0204049]; X. G. He and A. Zee, Phys. Lett. B 560, 87 (2003) [arXiv:hep-ph/0301092].

[52] P. F. Harrison and W. G. Scott, Phys. Lett. B 535, 163 (2002) [arXiv:hep-ph/0203209].

[53] C. H. Albright, Phys. Lett. B 599, 285 (2004) [arXiv:hep-ph/0407155].

[54] C. H. Albright, arXiv:0905.0146 [hep-ph].

[55] ISAJET, by H. Baer, F. Paige, S. Protopopescu and X. Tata, arXiv:hep-ph/0312045.

[56] G. W. Bennett et al. [Muon (g-2) Collaboration], arXiv:0811.1207 [hep-ex].

[57] A. Czarnecki and W. J. Marciano, Phys. Rev. D 64, 013014 (2001); [arXiv:hep-ph/0102122]. S. Komine, T. Moroi and M. Yamaguchi, Phys. Lett. B 506, 93 (2001) [arXiv:hep-ph/0102204].

[58] H. Baer, C. Balazs, J. Ferrandis and X. Tata, Phys. Rev. D 64, 035004 (2001) [arXiv:hep-ph/0103280].

[59] [Tevatron Electroweak Working Group and CDF Collaboration and D0 Collab], arXiv:0903.2503 [hep-ex].

[60] V. D. Barger, M. S. Berger and P. Ohmann, Phys. Rev. D 47, 1093 (1993) [arXiv:hep-ph/9209232].

[61] S. Antusch, J. Kersten, M. Lindner, M. Ratz and M. A. Schmidt, JHEP 0503, 024 (2005) [arXiv:hep-ph/0501272].

[62] M. Apollonio et al. [CHOOZ Collaboration], Eur. Phys. J. C 27, 331 (2003) [arXiv:hep-ex/0301017].

[63] X. Guo et al. [Daya-Bay Collaboration], arXiv:hep-ex/0701029.

[64] F. Ardellier et al. [Double Chooz Collaboration], arXiv:hep-ex/0606025.

[65] LEP2 SUSY Working Group, Combined LEP chargino results, up to 208 GeV for large m0, http://lepsusy.web.cern.ch/lepsusy/www/inos_moriond01/charginos_pub.html.
[66] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685, 89 (2004) [arXiv:hep-ph/0310123]; A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, JHEP 0609, 010 (2006) [arXiv:hep-ph/0605281].

[67] P. Di Bari and A. Riotto, Phys. Lett. B 671, 462 (2009) [arXiv:0809.2285 [hep-ph]].

[68] N. Cabibo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[69] A. D. Box and X. Tata, Phys. Rev. D 77, 055007 (2008) [arXiv:0712.2858 [hep-ph]]; Phys. Rev. D 79, 035004 (2009) [arXiv:0810.5765 [hep-ph]].

[70] A. D. Box, arXiv:0811.2444 [hep-ph].

[71] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267, 415 (1986).

[72] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

[73] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).

[74] H. Arason, D. J. Castano, B. Keszthelyi, S. Mikaelian, E. J. Piard, P. Ramond and B. D. Wright, Phys. Rev. D 46, 3945 (1992).

[75] O. V. Tarasov, A. A. Vladimirov and A. Y. Zharkov, Phys. Lett. B 93, 429 (1980); S. G. Gorishnii, A. L. Kataev and S. A. Larin, Sov. J. Nucl. Phys. 40, 329 (1984) [Yad. Fiz. 40, 517 (1984)].

[76] K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Phys. Rev. Lett. 79, 2184 (1997) [arXiv:hep-ph/9706430]; K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Nucl. Phys. B 510, 61 (1998) [arXiv:hep-ph/9708255].

[77] L. V. Avdeev and M. Y. Kalmykov, Nucl. Phys. B 502, 419 (1997) [arXiv:hep-ph/9701308].

[78] S. P. Martin and M. T. Vaughn, Phys. Lett. B 318, 331 (1993) [arXiv:hep-ph/9308222].

[79] A. Bednyakov, A. Onishchenko, V. Velizhanin and O. Veretin, Eur. Phys. J. C 29, 87 (2003) [arXiv:hep-ph/0210258].

[80] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B 402, 46 (1993) [arXiv:hep-ph/9207279]; Y. Yamada, Z. Phys. C 60, 83 (1993).

[81] D. J. Castano, E. J. Piard and P. Ramond, Phys. Rev. D 49, 4882 (1994) [arXiv:hep-ph/9308335].

[82] S. P. Martin and M. T. Vaughn, Phys. Rev. D 50, 2282 (1994) [arXiv:hep-ph/9311340].

[83] S. Weinberg, Phys. Lett. B 91, 51 (1980); L. J. Hall, Nucl. Phys. B 178, 75 (1981); B. A. Ovrut and H. J. Schnitzer, Phys. Lett. B 100, 403 (1981); B. A. Ovrut and H. J. Schnitzer, Nucl. Phys. B 179, 381 (1981).

[84] W. Wetz, Nucl. Phys. B 196, 259 (1982).

[85] D. M. Pierce, J. A. Bagger, K. T. Matchev and R. J. Zhang, Nucl. Phys. B 491, 3 (1997) [arXiv:hep-ph/9606211].

[86] G. Ross and M. Serna, Phys. Lett. B 664, 97 (2008) [arXiv:0704.1248 [hep-ph]].

[87] H. Baer, J. Ferrandis, S. Kraml and W. Porod, Phys. Rev. D 73, 015010 (2006) [arXiv:hep-ph/0511123].
[88] P. H. Chankowski, S. Pokorski and J. Rosiek, Nucl. Phys. B 423, 437 (1994) [arXiv:hep-ph/9303309]; V. D. Barger, M. S. Berger and P. Ohmann, Phys. Rev. D 49, 4908 (1994) [arXiv:hep-ph/9311269].

[89] S. Antusch, J. Kersten, M. Lindner and M. Ratz, Phys. Lett. B 538, 87 (2002) [arXiv:hep-ph/0203233].

[90] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Phys. Rev. D 63, 097302 (2001) [arXiv:hep-ph/0004166].

[91] H. Baer, C. Balazs, A. Belyaev and J. O’Farrill, JCAP 0309, 007 (2003) [arXiv:hep-ph/0305191].

[92] H. Baer and M. Brhlik, Phys. Rev. D 55, 3201 (1997) [arXiv:hep-ph/9610224]; H. Baer, M. Brhlik, D. Castano and X. Tata, Phys. Rev. D 58, 015007 (1998) [arXiv:hep-ph/9712305].

[93] J. K. Mizukoshi, X. Tata and Y. Wang, Phys. Rev. D 66, 115003 (2002) [arXiv:hep-ph/0208078].