Non-linear preheating after inflation and gravitational wave production

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Abstract. We present the evolution of the full set of Einstein equations during preheating after inflation, for a generic supersymmetric model of hybrid inflation. Preheating of the scalar metric fluctuations does affect the evolution of vector and tensor modes, and in particular they do enhance the induced stochastic background of gravitational waves during preheating. This gives an energy density in general an order of magnitude larger than that obtained by evolving the tensors fluctuations in an homogeneous background metric. This enhancement can improve the expectations for detection by planned gravitational waves observatories.

1. Introduction
Background (CMB) measurements [1], are consistent with an early period of inflation, which gives rise to the primordial curvature perturbation which seeds the large scale structure observed today. Inflation should be followed by a reheating period, during which the inflationary vacuum energy is converted into radiation. During the first stages of reheating, the evolution of the system may be dominated by non-perturbative effects as those of preheating, i.e., parametric amplification of quantum field fluctuations in a background of oscillating fields [2]. It does also enhance the tensor perturbations, sourced by the field anisotropic stress-energy tensor, giving rise to a stochastic background of gravitational waves (GW) [3].

In most of the studies of preheating of GW, fields and tensor fluctuations are evolved in a background Friedmann-Robertson-Walker (FRW) metric. However, beyond linear perturbation, tensors are also seeded by scalar and vector components of the metric. [4]. Given that metric and fields are non-linearly coupled through the Einstein equations, the parametric amplification of field fluctuations will be rapidly transferred to all metric perturbations. In [5] we showed that for the case of the scalar metric perturbations. And preheating of the scalar metric perturbations can then affect the amplification of tensors. Here we show that this is indeed the case for hybrid models, independently of model parameters [6].

2. Gravity waves from preheating
We have integrated the full set of Einstein equations, with the stress-energy tensor provided by the fields in hybrid inflation. Einstein equations are written in the so-called BSSN (Baumgarte-Shapiro-Shibata-Nakamura) formalism [7]. The spatial metric is given by $\gamma_{ij} = \exp(4\beta)\tilde{\gamma}_{ij}$, with $\det\tilde{\gamma}_{ij} = 1$. The metric dynamical variables are then $\beta$ and $\tilde{\gamma}_{ij}$, the trace of the extrinsic curvature $K = \tilde{\gamma}^{ij}K_{ij}$, and its traceless part $\tilde{A}_{ij} = \exp(-4\beta)(K_{ij} - \gamma_{ij}K/3)$. We choose to work in the synchronous gauge, where we have $\dot{K} = 6\beta$, with $\langle K \rangle/3 = H(t)$ being the average...
expansion rate, and \( \langle e^{2\beta} \rangle = a(t) \) the average scale factor ( "\( \langle \cdot \rangle \)" denotes spatial average). The spatial metric \( \tilde{\gamma}_{ij} \) encodes the two transverse and traceless degrees of freedom of the gravity waves, plus one additional scalar mode and 2 vector degrees of freedom, with \( \tilde{\gamma}_{ij} = 2 \tilde{A}_{ij} \).

The set of equations of motion (EOM) are then given by the Klein-Gordon equations for the fields, and those for the metric variables \([7]\) given by:

\[
\beta + 2\beta^2 = \frac{1}{6m_p^2} (V - 2T) - \frac{1}{24} \tilde{\gamma}_{ij} \tilde{\gamma}^{ij},
\]

\[
\tilde{\gamma}_{ij} + K \tilde{\gamma}^{ij} = 2e^{-4\beta} (M_{ij}^{TF} - R_{ij}^{TF}) + \tilde{\gamma}^k_{ij} \tilde{k}^j_{ki},
\]

where \( V \) is the potential for superradion hybrid inflation, \( V(\phi, \chi) = \tilde{V}_0 + g_0^2 \phi^4/4 + g_2(\Phi^2 - \phi^2)^2 \chi^2 + m_\phi^2 \Phi^2/2 \), and \( T = (\Phi^2 + \chi^2)/2 \) is the kinetic energy of the fields; \( \Phi \) is the inflaton field and \( \chi \) the waterfall field which triggers the phase transition at the end of inflation once the inflaton goes below the critical value \( \phi_c \) (both taken to be real scalar fields). The field dependent source is \( M_{ij} = m_p^{-2} (\partial_i \Phi \partial_j \Phi + \partial_i \chi \partial_j \chi) \), \( R_{ij} \) is the Ricci tensor of the metric \( \gamma_{ij} \), and the superscript "\( TF \)" denotes the trace-free part of the tensor. \( m_p \) is the reduced Planck mass.

The system is placed in a finite and discrete 3D comoving box of length \( L \) and \( N \) sites per spatial dimension. The procedure introduces a comoving ultraviolet cut-off in both space and momentum. For the problem of preheating after inflation, one tries to optimize the choice of the ratio \( O(N/L) \) to have an ultraviolet comoving momentum cut-off still larger than the preheating cut-off by the end of the simulation. This means that already at the start of the simulation our comoving box is smaller than the observable universe, but the relevant physical modes for preheating are all included.

We start the simulations some fraction of e-fold \( \Delta N_e = 0.05 \) after the end of inflation, with the background inflaton field still close to the critical point, \( \langle \Phi \rangle = \phi_c \exp(-\eta_0 \Delta N_e) \), and its background velocity given by the slow-roll conditions. The background values for the waterfall field are set to zero at this point. Classical inflaton field fluctuation in a spatial lattice with periodic boundary conditions are expanded as usual in Fourier modes \( \Phi_k \), with an initial vacuum amplitude \( |\Phi_k(0)| \approx 1/\sqrt{2\omega_k} \), where \( \omega_k = \sqrt{k^2 + m_\phi^2} \). The remaining variables must be chosen to satisfy the Einstein constraint equations, the momentum and the Hamiltonian constraint, at \( t = 0 \). This is the well known initial-value problem in general relativity \([8]\). We choose initially vanishing tensors and vectors, i.e., \( \tilde{\gamma}_{ij}(0) = \delta_{ij} \), such that the momentum constraint reduces to \( 2\partial_i K = -(3/2) \beta^2 (\tilde{\Phi} \partial_i \Phi + \tilde{\chi} \partial_i \chi) \). In order to fulfill this equation, we set the waterfall field as \( \chi(0) = \Phi(0) - \langle \Phi(0) \rangle \), and \( \chi(0) = \langle \Phi(0) \rangle - \tilde{\Phi}(0) \). This allows to solve the momentum constraint for the initial value of the expansion rate fluctuations, and use the Hamiltonian constraint to fix the fluctuations of the scale factor.

Preheating in hybrid inflation models has been extensively studied in the literature \([9]\). The parametric amplification of the fluctuations takes place first through an spinodal instability for the fields, during which the lower modes are quickly amplified. After a few oscillations, the amplitude of the fields has decayed enough to be out of the spinodal region, and tachyonic preheating ends. We just follow the evolution of the fields and metric variables up to the end of the resonance, before we loose the ultraviolet cut-off for the field modes. Metric variables will follow the same pattern of parametric amplification than the fields, with \( |\beta|^2 \sim m_p^{-2} |\Phi|^2 \). Keeping only the leading \( \beta \) terms in the Ricci tensor, we have:

\[
R_{ij}^{TF} \simeq [-4\partial_i \beta \partial_j \beta + 2\partial_i \partial_j \beta]^{TF},
\]

and thus \( R_{ij}^{TF} \) becomes comparable to the field contribution \( M_{ij}^{TF} \) in Eq. (2).

The metric variable \( \tilde{A}_{ij} \) is traceless but non-transverse, i.e., it contains more degrees of freedom than those two corresponding to GW. The traceless and transverse (TT) components
are projected by using the operator $[3] \Lambda_{ij,lm}(k) = P_{ij}(\hat{k})P_{lm}(\hat{k}) - P_{ij}(\hat{k})P_{lm}(\hat{k})/2$, where $P_{ij} = \delta_{ij} - k_i k_j$ and $k_i = k_i/k$. The energy density of the GW is then $\rho_{GW} = m_p^2 (\hat{A}_{ij}(t, x) \hat{A}_{ij}^*(t, x))^{TT} = m_p^2 \int d^3 k |\hat{A}_{ij}^{TT}(t, k)|^2 [3]$. On the LHS in Fig. 1, we have plotted $\rho_{GW}$, normalized to the initial vacuum energy $\rho_i = g^2 \phi_c^4$. We have taken as parameter models $g = 0.01$, $\phi_c = 0.005m_p$ and $\eta_0 = 0.05$. The value of $\rho_{GW}$ does not depend on the value of the coupling, which can be rescaled out from all the equations, but it does depend on the value of $\phi_c$ [3], which sets the scale for the field source term such that $\rho_{GW}/\rho_i \propto \phi_c^2$. We have included the results for different choices of the box size, to show that the final value of $\rho_{GW}$ does not depend on the choice of $L$ as far as we have all the relevant modes to start and end tachyonic preheating. With a larger comoving box we have more modes in the low momentum regime, and then tachyonic resonance starts slightly sooner, as can be seen in the plot. In this figure we have compared the results obtained when integrating the full Einstein equations (solid lines) with those obtained when integrating the tensor modes in a FRW background metric (dashed lines). The former are always roughly an order of magnitude larger due mainly to the contribution of the scalar modes of the metric fluctuations in Eq. (3). On the RHS in Fig. 1, we show that both source terms, $M_{ij}^{TF}$ and the leading term $R_{ij}^{TF}$ in Eq. (3), are of the same order by the end of the resonance. Scalar metric fluctuations start growing immediately after inflation due to the increase in the kinetic energy of the field (see Eq. (1)), and this effect leads the initial growth of the tensor perturbations.

In Fig. 2 we show the spectrum of GW per logarithmic frequency interval, $\rho_{GW}(k) = d\rho_{GW}/d\ln k$, at different times until the end of the tachyonic resonance. Comparing with the spectrum obtained in a FRW metric, the spectrum is enhanced and the peak is shifted towards lower values at around $k \simeq g\phi_c$. This effect at low momenta is again due to the scalar metric perturbations, which spectrum peaks below $g\phi_c$.

3. Summary

In summary, we have shown that non-linear effects due to metric perturbations enhance the amplitude of the GW stochastic background by an order of magnitude with respect to the calculations in a FRW background metric. Taking into account that $\rho_{GW}(k)$ is redshifted like radiation after the resonance, and assuming that entropy is conserved from reheating onwards, its
maximum present day value normalized by the critical density today is given by $h^2\Omega_{GW}^{peak} \approx 5.5 \times 10^{-9} \left(\phi_c/m_P\right)^2 \left(\frac{T_{RH}}{\rho_i}\right)^{1/3}$, where $T_{RH}$ is the reheating temperature. This amplitude is within the reach of the future GW observatory Advanced-LIGO for $\phi_c > 0.005m_P$, or BBO for $\phi_c < 0.005m_P$ [10]. However, today’s values for the frequency are $f = 6.4 \times 10^{10} \sqrt{g(k/(g\phi_c)})(T_{RH}/\rho_i^{1/4})^{1/3} \text{Hz}$, while the operating frequency range for Advanced-LIGO is $1 - 10^9 \text{Hz}$, and BBO will operate in the range $10^{-3} - 10^2 \text{Hz}$. Thus, to have the GW spectrum within the observable range, the coupling should be at most of the order of $g \approx 10^{-14}$. The numerical simulations so far can resolve the spectrum for the typical frequency of the resonance, $k \approx g\phi_c$ and above, but not the infrared (IR) tail for subhorizon and superhorizon modes. The behavior of the GW spectrum in this IR range and how they are affected by non-linear metric effects is still an open question. A tail of subhorizon modes at the time of preheating rising slower than $k^3$ could be detected by BBO, although $f_{\text{peak}}$ were in the range of $10^5 \text{Hz}$.

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