Gap dependent mass of photon in photonic topological insulator

Marcelo Vieira∗

Centro de Ciências Exatas e Sociais Aplicadas,
Universidade Estadual da Paraíba, Patos, PB, Brazil

Sergei Sergeenkov† and Claudio Furtado§

Departamento de Física, Universidade Federal da Paraíba,
Caixa Postal 5008, 58051-970, João Pessoa, PB, Brazil

Abstract

By using an analogy with axionic like systems, we study light propagation in periodic photonic topological insulator (PTI). The main result of this paper is an explicit expression for the PTI band structure. More specifically, it was found that for nonzero values of the topological phase difference \( \gamma = \theta_2 - \theta_1 \) a finite gap \( \delta \propto \gamma^2 \) opens in the spectrum which is equivalent to appearance of nonzero effective photon mass \( m^*(\delta) \propto \frac{\sqrt{\delta}}{\delta+2} \).
I. INTRODUCTION

Photonic crystals are artificial periodic structures in which electromagnetic waves (photons) can be manipulated in an analogous way to the electrons in semiconductors [1, 2]. In particular, photonic crystals can be used as devices for confinement and light curving [3] as well as a single frequency filter and a cavity for nanolasers [4]. Unlike the conventional optical fibers (where the light propagates by successive total reflections leading to a significant loss of energy), in the so-called photonic fibers the light follows a practically loss-free path.

The discovery of topological insulators (TI) [5, 6] has rekindled interest in existence of nontrivial topological phases in optical systems. In particular, photonic counterparts of quantum Hall edge states have been predicted [7, 8] and experimentally observed [9–11] in systems with broken time-reversal symmetry.

In this paper we introduce an exactly solvable model of the photonic topological insulator (PTI) and discuss its unusual properties. In particular, an explicit expression for the PTI band structure (spectrum) $\omega(k)$ with TI induced gap parameter $\delta$ has been obtained, leading to the gap dependent effective photon mass $m^*(\delta)$.

II. THE MODEL

Recall that the electromagnetic response of an ordinary (nontopological) insulator is described by the Maxwell action

$$S_0 = \int d^3xdt \left( \epsilon_0 \epsilon E^2 - \frac{1}{\mu_0 \mu} B^2 \right)$$

leading to conventional macroscopic fields describing electric polarization and magnetization

$$D = \frac{\delta S_0}{\delta E} = \epsilon_0 \epsilon(r) E$$

$$H = \frac{\delta S_0}{\delta B} = \frac{1}{\mu_0 \mu(r)} B$$

What would be the electromagnetic response of a topological insulator (TI)? The novel property of the TI is that it has a conductive surface. It means that for an incident electric field, a surface current will arise inducing a magnetic field in the material, that is magnetizing it. In other words, in the TI an electric field induces a magnetization. Likewise, the magnetic
field induces a dielectric polarization in the TI. These two phenomena result in appearance of the so-called magneto-electric effects. It is worthwhile to mention that similar effects occur in axion like systems [12]. Thus, from the electrodynamic point of view, we can consider a TI as an axionic medium [13–15] and, by analogy, add the following topological term to the conventional Maxwell action

$$S_\theta = \frac{\alpha}{2\pi} \int d^3x dt \left( E \cdot B \right) \theta(r)$$ (4)

where $\alpha$ is the fine structure constant, and $\theta$ is a phenomenological model parameter that characterizes the topological phase of TI. With the introduction of the above axionic term, we arrive at the constitutive relations describing the macroscopic properties of the TI, namely:

$$D = \varepsilon_0 \varepsilon(r) E - \frac{\alpha}{2\pi} \theta(r) B$$ (5)

$$H = \frac{1}{\mu_0 \mu(r)} B + \frac{\alpha}{2\pi} \theta(r) E$$ (6)

To model a periodic photonic crystal, we assume that the above-introduced parameters $\varepsilon$, $\mu$ and $\theta$ are periodic with a spacial period equal to $d = a + b$, that is:

$$\begin{pmatrix} \varepsilon(r) \\ \mu(r) \\ \theta(r) \end{pmatrix} = \begin{pmatrix} \varepsilon(z) \\ \mu(z) \\ \theta(z) \end{pmatrix} = \begin{pmatrix} \varepsilon(z + a + b) \\ \mu(z + a + b) \\ \theta(z + a + b) \end{pmatrix}$$ (7)

III. RESULTS AND DISCUSSION

For the normal incidence of plane waves (with the wave vector $k = k\hat{z}$), from the source-free Maxwell equations

$$\nabla \cdot D = 0$$ (8)

$$\nabla \cdot B = 0$$ (9)

$$\nabla \times E = -\frac{\partial B}{\partial t}$$ (10)
we obtain

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \tag{11}$$

we obtain

$$\mathbf{E}(\mathbf{r}, t) = e^{-i\omega t}(E_x(z)\hat{x} + E_y(z)\hat{y}), \quad \mathbf{B}(\mathbf{r}, t) = e^{-i\omega t}(B_x(z)\hat{x} + B_y(z)\hat{y}) \tag{12}$$

for the field configurations of the problem, where (in view of Eq. (10))

$$B_x = -\frac{i}{\omega} \frac{dE_y}{dz}, \quad B_y = \frac{i}{\omega} \frac{dE_x}{dz} \tag{13}$$

Notice that in the regions where $\theta$ is uniform and constant, the Maxwell equations are identical to the ones for the conventional dielectrics.

Since the parameters that characterize the medium are periodic, the fields should also obey the Bloch theorem

$$\mathbf{E}(z + a + b) = e^{ik(a+b)}\mathbf{E}(z), \quad \mathbf{B}(z + a + b) = e^{ik(a+b)}\mathbf{B}(z) \tag{14}$$

allowing us to restrict the problem to the unity cell defined as the region $0 \leq z \leq a + b$. It can be easily verified that the general solution for both fields in this region reads

$$E_x = c_x \cos q_2 z + d_x \sin q_2 z, \quad \frac{a}{2} \leq z < \frac{a}{2} + b$$

$$E_y = c_y \cos q_2 z + d_y \sin q_2 z, \quad \frac{a}{2} \leq z < \frac{a}{2} + b$$

$$E_x = c_x \cos q_2 z + d_x \sin q_2 z, \quad \frac{a}{2} \leq z < \frac{a}{2} + b$$

$$E_y = c_y \cos q_2 z + d_y \sin q_2 z, \quad \frac{a}{2} \leq z < \frac{a}{2} + b$$

where $q_i = \frac{\omega \sqrt{\varepsilon \mu}}{c}$ with $c$ being the velocity of light.
After applying the interface boundary conditions at $z = \frac{a}{2}$ and $z = \frac{a}{2} + b$, we finally obtain a transcendental equation as a condition for existence of nontrivial self-consistent solutions of the above system, namely

$$\cos k(a + b) = \cos q_1 a \cos q_2 b - \Gamma(\delta) \sin q_1 a \sin q_2 b$$

(19)

where

$$\Gamma(\delta) = \frac{1}{2} \left( \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} + \delta Z_1 Z_2 \right)$$

(20)

with $Z_i = \sqrt{\mu_i / \epsilon_i}$ being the impedance of the medium, and $\delta = \frac{\alpha \mu_0^2}{4 \pi^2} \gamma^2$ the topological insulator induced gap related parameter with $\gamma = \theta_2 - \theta_1$.

Recalling that $q_i = \frac{\omega}{c} \sqrt{\epsilon_i \mu_i}$, it becomes clear that Eq.(19) describes an implicit dependence of the frequency $\omega$ on the wave vector $k$. For a particular case when $\epsilon_1 = \epsilon_2 = \mu_1 = \mu_2 = 1$ and $b = a$, Eq.(19) can be resolved explicitly leading to the following expression

$$\omega(k) = \omega_0 \arccos \left[ \frac{\delta + 2 \cos \left( \frac{\pi k}{k_0} \right)}{\delta + 2} \right]$$

(21)

where $\omega_0 = \frac{c}{2a}$ and $k_0 = \frac{\pi}{2a}$.

The band diagram $\omega(k)$ for photonic crystal based on topological insulator for different values of $\delta$ is shown in Fig.1. Notice that for conventional (nontopological) insulator with $\delta = 0$ there is no gap in the spectrum. The gap opens only for non-zero values of $\delta$ and its width increases with increasing of $\delta$.

Furthermore, using the above spectral law $\omega(k)$, we can introduce an effective photonic mass $m^*$, namely

$$\frac{1}{m^*} = \frac{1}{\hbar} \left( \frac{\partial^2 \omega}{\partial k^2} \right)_{k = k_0}$$

(22)

From Eq.(21) we obtain

$$m^*(\delta) = m_0 \left( \frac{3\sqrt{\delta}}{\delta + 2} \right)$$

(23)

where $m_0 = \frac{4\hbar}{3mc}$.

The evolution of the normalized effective mass $m^*/m_0$ with the gap parameter $\delta$ is shown in Fig.2. It is also important to emphasize that (for a given value of $\delta$) the absolute value of the photon mass $m^*$ drastically depends on the period of the photonic crystal $d = 2a$, ranging from light photons (with $m^* \simeq 10^{-36} \text{kg}$ for $d = 1 \mu m$) to heavy photons (with $m^* \simeq 10^{-33} \text{kg}$ for $d = 1 \text{nm}$). The latter estimate is typical for nontopological photonic crystals [16]. It would be interesting to experimentally check these predictions using the existing PTI.
FIG. 1. The band diagram $\omega(k)$ of photonic crystal based on toplogical insulators for different values of the gap related parameter $\delta$ according to Eq.(21).
FIG. 2. The dependence of normalized effective photon mass on the gap related parameter $\delta$ according to Eq.(23).

IV. CONCLUSION

In summary, we studied propagation of light in a model system describing periodic photonic crystal comprised of topological insulators. By adding an axionic like term into conventional Maxwell equations, we obtained a closed system of linear equations on electric and magnetic fields. By resolving the resulting transcendental equation, the existence of nontrivial band structure in the photonic topological insulator (PTI) was revealed. Namely, it was found that for nonzero values of the topological phase parameter $\gamma = \theta_2 - \theta_1$ a finite gap $\delta \propto \gamma^2$ opens in the spectrum which is equivalent to appearance of nonzero effective photon mass $m^*(\delta)$. The latter was shown to quite noticeably increase with $\delta$ and decrease with the period of the photonic crystal $d = a + b$.

Acknowledgements

We thank Brazilian agencies CAPES, CNPQ and FAPESQ for financial support.

[1] E. Yablonovitch, Photonic crystals: Semiconductors of light, Scientific American 285, 47 (2001).
[2] K. M. Ho, C. T. Chan, and C. M. Soukoulis, Phys. Rev. Lett. 65, 3152 (1990).
[3] Shawn-Yu Lin, E. Chow, V. Hietala, P.R. Villeneuve and J.D. Joannopoulos, Science 282, 274 (1998).
[4] D. D. Marcenac and J.E. Carrol, IEE Proceedings J - Optoelectronics 140, 157 (1993).
[5] M.Z. Hasan and C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
[6] X. -L. Qi and S. C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).
[7] F.D.M. Haldane and S. Raghu, Phys. Rev. Lett. 100, 013904 (2008).
[8] S. Raghu and F.D.M. Haldane, Phys. Rev. A 78, 033834 (2008).
[9] Y. Poo, R. Wu, Z. Lin, Y. Yang, and C.T. Chan, Phys. Rev. Lett. 106, 093903 (2011).
[10] Mikael C. Rechtsman, Julia M. Zeuner, Yonatan Plotnik, Yaakov Lumer, Daniel Podolsky, Felix Dreisow, Stefan Nolte, Mordechai Segev, and Alexander Szameit, Nature 496, 196 (2013).
[11] Cheng He, Xiao-Chen Sun, Xiao-Ping Liu, Ming-Hui Lu, Yulin Chen, Liang Feng, and Yan-Feng Chen, PNAS 113, 4924 (2016).
[12] F. Wilczek, Phys. Rev. Lett. 58, 1779 (1987).
[13] Xiao-Liang Qi, Rundong Li, Jiadong Zang, and Shou-Cheng Zhang, Science 323, 1184 (2009).
[14] G. Rosenberg and M. Franz, Phys. Rev. B 82, 035105 (2010).
[15] A. Karch, Phys. Rev. Lett. 103, 171601 (2009).
[16] V.S. Gorelik, Phys. Scr. T140, 014046 (2010).