Symmetry-related transport on a fractional quantum Hall edge

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Low-energy transport in two-dimensional topological insulators is carried through edge modes, and is dictated by bulk topological invariants and possibly microscopic Boltzmann kinetics at the edge. Here we show how the presence or breaking of symmetries of the edge Hamiltonian underlie transport properties, specifically d.c. conductance and noise. We demonstrate this through the analysis of hole-conjugate states of the quantum Hall effect, specifically the $\nu = 2/3$ case in a quantum point-contact (QPC) geometry. We identify two symmetries, a continuous $SU(3)$ and a discrete $Z_3$, whose presence or absence (different symmetry scenarios) dictate qualitatively different types of behavior of conductance and shot noise. While recent measurements are consistent with one of these symmetry scenarios, others can be realized in future experiments.

Descriptions of edges of fractional quantum Hall setups in terms of one-dimensional chiral Luttinger modes have proven to be more complex and exotic than first anticipated. This is a particularly pressing issue when it comes to multi-mode edges, e.g., the case of hole-conjugate states ($1/2 < \nu < 1$). One representative example is the edge of a bulk filling factor $\nu_{\text{bulk}} = 2/3$, where the interplay of disorder and interactions plays a crucial role. Less well understood is the role played by symmetries in determining observable transport properties such as conductance and noise, given the underlying topological structure.

Disorder-induced tunneling and inter-chiral-mode interactions at the edge of the phase $\nu_{\text{bulk}} = 2/3$ were predicted to renormalize the edge constituents to a charge mode and a counter-propagating coherent neutral mode. Indeed, upstream mode propagation has been observed experimentally. Subsequent experiments implementing quantum point contacts (QPCs), led to the conclusion that the original picture of the edge needed to be modified. Following a renormalization group analysis, the emerging picture comprises a fixed point with two downstream-running charged modes and two upstream neutral modes (cf. Fig. 1), whose underlying symmetries are at the focus of the present analysis. As much of the available experimental data is compatible with a picture of incoherent transport, direct experimental evidence of coherent upstream propagation, as discussed below) is yet to be found. It is clearly desirable to incorporate our current understanding into a general symmetry based framework. Not only can this be generalized to other topological states, but it can also serve as a guideline for searching for other symmetry scenarios with new experimental manifestations.

The present work demonstrates that, given the topological invariants of the bulk phase (dictated by the filling factor), different symmetries of the edge modes may underlie qualitatively different transport behavior, specifically the d.c. conductance and the low-frequency nonequilibrium noise. This facilitates the engineering of experimentally controlled setups (of the same bulk phase) with designed, symmetry-related, behavior. In order to demonstrate our approach, we consider a specific geometry, write down the most general fixed point action compatible with this setup, identity the relevant symmetries, and show how their presence (or absence) determine the resulting transport properties.

We start by discussing the two relevant symmetries of the action. The first describes the subspace spanned by the neutral modes. There are three most relevant types of neutral excitations, "down", "up" and "strange" (cf. Eq. (4); not to be confused with the $Z_3$ symmetry). In physical terms, the breaking of the $SU(3)$ symmetry takes place through charge equilibration in the charge sector of the chiral modes (see below). Secondly, there is a discrete $Z_3$ symmetry (cf. Eq. (4); not to be confused with the $Z_3$ subgroup of the $SU(3)$), related to three different sectors of the charge on $\phi_4$ with the fractional charge 0, 1/3 or 2/3. Breaking this $Z_3$ symmetry allows to connect the neutralon with the anti-neutralon sector, which is achieved by the tunneling of 1/3 charges into the innermost bare edge channel (cf. Fig. 1(a)).

We focus on a two terminal setup (cf. Fig. 2), consisting of the edge channels depicted in Fig. 1. The QPC is tuned such that the outermost charge channel is fully transmitted while the inner ones are fully reflected, naively leading to a quantized conductance of $(1/3)(e^2/h)$. Through charge equilibration between the
FIG. 1: (color online). Edge structures and excitations of the bulk filling factor $\nu_{\text{bulk}} = 2/3$. (a) Four bare edge channels are depicted with arrowheads indicating the direction of each mode (downstream (right movers) or upstream (left movers)). A quasi-particle tunneling from the outermost edge mode to the inner ones turns into two quasi-holes and one electron. Due to the presence of a vacuum region with $\nu = 0$, only electrons can tunnel to mode $\phi_3$, leaving the fractional charge on $\phi_3$ invariant, and giving rise to a $Z_3$ symmetry between sectors with fractional charge 0, 1/3, and 2/3. (b) Due to electron tunneling between the three inner edge channels ($\phi_2$, $\phi_3$, and $\phi_4$) and bare interaction between them, the inner three edge modes are renormalized to a downstream charge mode (denoted as a black solid line) with the filling factor discontinuity $\delta \nu = 1/3$ and two upstream neutral modes (denoted as yellow dashed lines). The same tunnel process as in (a) now creates a chargeon and neutral charges ($-1,1$). (c) Quasi-particles described by charges ($q_{n1}, q_{n2}$) form the fundamental representation of $SU(3)$. In analogy to the quantum numbers ($I_x,Y$) of isospin and hypercharge in flavor $SU(3)$ [29], the elementary excitations (neutralons) are labeled $u,d$, and $s$. The corresponding anti-neutralons form the conjugate representation; a conversion of neutralons into anti-neutralons is only possible if the $Z_3$ symmetry described above is broken.

charge channels (cf. Fig. 2(a) upper right) that breaks the $SU(3)$ symmetry of the fixed point, only one type of neutral excitations (neutralons, but not anti-neutralons) is created. These neutralons then decay converting into quasi-particles in the outermost channel and quasi-holes in the inner charge channel (cf. Fig. 2(b) lower right), generating an additional current at drain $D$, hence undermining conductance quantization (cf. the upper row in Table I). Breaking the $Z_3$ symmetry gives rise to the emergence of a quantized conductance, $(1/3)(e^2/h)$. The reason is that with the $Z_3$ symmetry broken, the generation of neutralons and anti-neutralons in the course of equilibration is equally probable, implying no additional d.c. current at drain $D$ yet a contribution to shot noise with a quantized Fano factor, $2e/3$.

Symmetries and tunneling operators.— We consider the reconstructed edge structure [23][25] compatible with experiment, with four chiral modes (1/3, -1/3, 1, -1/3) denoted as $\phi_i=1/3,2,3,4$, respectively. Assuming that the outermost edge mode is spatially far from the three inner ones, one obtains an intermediate fixed point supporting a 1/3 downstream charge mode $\phi_c \equiv (2\phi_2 + \phi_3 + \phi_4)/\sqrt{3}$, and two upstream neutral modes $\phi_{n1} \equiv (3\phi_2 + \phi_3)/\sqrt{2}$ and $\phi_{n2} \equiv (\phi_2 + \phi_3 + 2\phi_4)/\sqrt{2}$. The latter satisfy commutation relations $[\phi_{n1}(x),\phi_{n1}(x')] = -i\pi \text{sgn}(x-x')$, $[\phi_{n2}(x),\phi_{n2}(x')] = -i\pi \text{sgn}(x-x')/3$. At this intermediate fixed point, the neutral sector possesses a global $SU(3)$ symmetry (invariance under rotations in the $(u,d,s)$-space) with the Hamiltonian

$$H_0 = i\nu_n \int dx \Phi^\dagger(x) \partial_x \Phi(x) .$$

Here, $\Phi = e^{i \chi / \sqrt{3}}[\psi_u, \psi_d, \psi_s]^T$ is a three component chiral fermion field [30]: $\psi_u \equiv e^{-i(\phi_{n1} + \phi_{n2})/\sqrt{2}}$, $\psi_d \equiv e^{i(\phi_{n1} - \phi_{n2})/\sqrt{2}}$, and $\psi_s \equiv e^{i\phi_{n2}/\sqrt{2}}$ are flavors of neutralons, referred to as up, down, and strange neutralon, respectively (see Fig. 1(c)). $\chi$ is an auxiliary bosonic field with $[\chi(x),\chi(x')] = -i\pi \text{sgn}(x-x')$, providing a convenient way of representing $H_0$ in terms of the fermion field, and $\nu_n$ is the velocity of the neutral modes.

At the intermediate fixed point, there is neither charge nor thermal equilibration between the different modes, implying a non-zero thermal conductance. We now want to describe equilibration between the charge modes (which also includes the creation of neutral excitations), but still assume that the size of the system is shorter than the inelastic scattering length [31], at which the edge thermal conductance decays towards zero. To this end, we consider quasi-particle tunneling (“charge equilibration”) between the outer-most mode and the three inner modes, expressed in terms of the most relevant operators:

$$H_{\text{ch-\text{eq}}} = \int dx \left[ (t_u \psi_u^\dagger + t_d \psi_d^\dagger + t_s \psi_s^\dagger) e^{-i\phi_c/\sqrt{2}} e^{i\phi_{1/3}} + \text{H.c.} \right] .$$

Note that $H_{\text{ch-\text{eq}}}$ breaks the $SU(3)$ symmetry. A quasi-particle $e^{i\phi_{1/3}}$ is annihilated, creating a chargeon $e^{-i\phi_c/\sqrt{2}}$ (the charge sector of a quasi-particle excitation carrying a charge $e/3$ formed in the inner modes) and a neutralon. Conversely, while a quasi-particle is created, a chargeon is annihilated together with the creation of an anti-neutralon (the anti-particle of the neutralon).

We next recover the $SU(3)$ symmetry through disorder averaging. The tunneling amplitudes $t_{i,j=\text{u,d,s}}$ are random with white noise correlation $(t_{i,j}(x)t_{i,j}(x'))_{\text{dis}} = W_{\text{ch-\text{eq}}} t_{i,j} \delta(x-x')$. This is invariant under $SU(3)$ rotations owing to the random mixing of the neutral fields. The renormalization group scaling of the disorder variance $W_{\text{ch-\text{eq}}}$ gives rise to the elastic scattering length $\ell_{\text{ch-\text{eq}}} \sim 1/W_{\text{ch-\text{eq}}}^{3/5}$ [31]. After performing the disorder averaging, an effective action on the Keldysh contour $K$
This action is invariant under the $SU(3)$ transformation $\psi_j(x) \to U(x)\psi_j(x)$, unlike the Hamiltonian Eq. (2), the $SU(3)$ symmetry of the Hamiltonian Eq. (1) is thus restored in a statistical sense.

We next discuss a discrete transformation $T_3$ defined via $T_3(\phi_c/\sqrt{3}) T_3 = (\phi_c/\sqrt{3} - 2\pi/3)$, $T_3(\phi_{nz}/\sqrt{2}) T_3 = (\phi_{nz}/\sqrt{2} - 2\pi/3)$, and $T_3^3\phi_n T_3 = \phi_{n}$. In the original basis, $T_3^3\phi_n T_3 = \phi_n - 2\pi/3$ creates a kink, associated with the annihilation of a charge $e/3$ quasi-particle in $\phi_4$ (cf. Fig. 1(a)). In the basis of neutralons

$$T_3^3\psi_j T_3 = \psi_j e^{2\pi i/3} \quad T_3^3\psi^\dagger_j T_3 = \psi^\dagger_j e^{-2\pi i/3} \quad (4)$$

for $j = u, d, s$, reflecting the $Z_3$ nature of $T_3$. We find that $T_3$ is a symmetry of $H_0 + H_{\text{ch-eq}}$ (Eqs. (1) and (2)); here a quasi-particles is not allowed to tunnel into $\phi_4$.

Electron tunneling among the original chiral modes is strong near the intermediate fixed point [25]. Provided that the distance between those chiral modes is not too large, the $\nu = 0$ strip (Fig. 1(a)) will be smeared, and quasi-particle tunneling to/from $\phi_4$ will be facilitated. This, in terms of the renormalized modes, will give rise to the following Hamiltonian in the neutral sector (assumed to be a small perturbation):

$$H_{n\bar{n}} = \int dx(v_u\psi^\dagger_d\psi^\dagger_s v_d\psi^\dagger_u \psi_s + v_s\psi^\dagger_u \psi_d + \text{H.c.}) \quad (5)$$

This Hamiltonian describes neutralon-antineutralon mixing. In view of Eq. (2), $H_{n\bar{n}}$ breaks the $Z_3$ symmetry. Here, the annihilation operator of anti-neutralons is defined as $\psi^\dagger_j = \psi_j^\dagger = \psi_j^\dagger$. We note that Eq. (2) is the chargeon analogue of a BCS Hamiltonian. The Hamiltonian, Eq. (2), also breaks the $SU(3)$ symmetry. Note that the tunneling amplitudes $v_j = v_{u,d,s}$ are random with a white noise correlator $\langle \psi^\dagger_j(x) \psi^\dagger_j(x') \rangle_{\text{dis}} = W_{n\bar{n}}\delta(x - x')\delta_{j,j_2}$. This allows us to restore the $SU(3)$ symmetry by performing disorder averaging. The characteristic elastic length scale $\ell_{n\bar{n},0}$ for this process scales as $\ell_{n\bar{n},0} \sim 1/W^{3/7}_{n\bar{n}}$.

### Mechanism and Models

The QPC (cf. Fig. 2) we consider is set such that the outer-most channel is fully transmitted while the inner channels are fully reflected, contributing to the conductance of $e^2/(3h)$ to drain D. After transmission through or reflection from the QPC, the biased charge channels start to equilibrate with the unbiased ones via quasi-particle tunneling (described by Eq. (2)) in the upper right or lower left corner of Fig. 2(a).

Such tunneling events generate neutralons in the upper right corner or anti-neutralons in the lower left corner, which then propagate in a direction opposite to that of the charge channels through the QPC region with size $L_{\text{QPC}}$, and finally decay in the lower right or upper left corner (“decay region”) as depicted in Fig. 2(b). Depending on whether the system exhibits the $Z_3$ symmetry or not, we distinguish between two models: model (B) includes the terms of Eq. (5) to break the $Z_3$, while model (A) does not. This implies, that for (A) the fundamental representation furnished by neutralons is completely disconnected from the conjugate representation furnished by anti-neutralons [31], while neutralon/anti-neutralon mixing is allowed for model (B). In both models (A) and (B), we assume that the $SU(3)$ symmetry is statistically conserved, with $\ell_{\text{ch-eq},0} \ll L_{\text{arm}}$.

#### Tunneling current

The $Z_3$ symmetry of model (A) prevents the formation of a $e^2/(3h)$ conductance plateau. To understand this, suffice to consider a single impurity in the decay region in the lower/upper edge $r = 1/u$. The impurity is assumed to be at position $x_0$. Let us start with model (A). To leading order in $t_j$, the tunneling current at $x_0$ on the $r = 1,u$ edge can be expressed in terms of a local greater (lesser) Green’s function $g^>_j(x_0,t) = -i\langle \psi_j^\dagger(x_0,t)\psi^\dagger_j(x_0,0) \rangle$ ($g^<_j(x_0,t) = -i\langle \psi_j^\dagger(x_0,0)\psi^\dagger_j(x_0,t) \rangle$) of $j = u, d, s$ type neutralons
as

\[ I_{\text{tun},r} = \sum_{j=u,d,s} \frac{i e |t_j|^2}{3(\hbar)^2} \int_{-\infty}^{\infty} dt \left( \frac{a}{a + i v_c t} \right)^{1/3} \left[ g^{>}_{j,r}(t) - g^{<}_{j,r}(-t) \right]. \] (6)

Here, \( a \) is a short distance cutoff, and \( v_{1/3} \) and \( v_c \) are the velocity of the outermost channel and the inner charge channel, respectively. Information about the non-equilibrium state of neutralons is encoded in the local greater and lesser Green’s function \([32\, 37]\) via the decomposition

\[ g^{>}_{j,r}(t) = g_0(t) g^{\text{eq}}_{r}(t), \quad g^{<}_{j,r}(t) = g_0(-t) g^{\text{eq}}_{r}(t). \] (7)

Here, \( g_0(t) = -i [a/(a + iv_n t)]^{2/3} \) describes quantum correlations of neutralons, while \( g^{\text{eq}}_{r}(t) \) represents classical non-equilibrium aspects of neutralons, as

\[ g^{\text{eq}}_{r,\text{model A}}(t) = \left[ \frac{1}{2} + \frac{1}{2} e^{2\pi i \text{sgn}(t)/3} \right]^N \] (8)

with \( N = eV |t|/\hbar \). We briefly sketch the derivation of Eq. (7). In the limit of full charge equilibration, quasiparticles emanating from source \( S \) and then being transmitted through the QPC towards drain \( D \), reach the charge channel with probability 1/2. This is accompanied by the creation of neutralons with probability 1/2, cf. Eq. (7). Each of these neutralons arrives at \( x_0 \), described by a kink in the bosonic fields \( \phi_{n_{u}}(x_0) \) and \( \phi_{n_{d}}(x_0) \), and thus gives rise to a phase shift \( 2\pi \text{sgn}(t)/3 \) of the operator \( \psi_j(x_0,t)\psi_j^\dagger(x_0,0) \): following the arrival of a \( j' = u,d,s \) neutralon at \( x_0 \) and time \( t' \), \( \psi_{j'}(x_0,t) \psi_{j'}^\dagger(x_0,0) \) acquires a phase shift of \( 2\pi \text{sgn}(t)/3 \). This relies on

\[ \psi_{j'}(x_0,t') \left[ \psi_j(x_0,t) \psi_j^\dagger(x_0,0) \right] \psi_{j'}^\dagger(x_0,t') \]

\[ = \left[ \psi_j(x_0,t) \psi_j^\dagger(x_0,0) \right] e^{2\pi i \text{sgn}(t'-t) - \text{sgn}(0-t')/3}, \] (9)

provided \( \min(0,t) < t' < \max(0,t) \). For each arriving neutralon, the phase factor on the r.h.s. of Eq. (9) appears in one of the \( N \) factors of Eq. (8). We note that the phase factor reflects the anyonic statistics of neutralons and is identical for all flavors \( j' = u,d,s \). When no neutral excitation is generated (with a probability 1/2) during the equilibration process, \( \psi_j(x_0,t)\psi_j^\dagger(x_0,0) \) does not accumulate a phase factor, leading to the first term in the parenthesis of Eq. (8). Inserting Eqs. (7) and (8) into Eq. (8), we obtain the tunneling current as

\[ I_{\text{tun},r} = \sum_{j=u,d,s} \frac{2 e c |t_j|^2}{3 \hbar \Gamma(3/2)} \left( \frac{eV}{a^2 \hbar^2 v_c v_{1/3}} \right)^{1/3}, \] (10)

where \( c = \sin \left[ \tan^{-1} \left( \pi / (3 \ln(2)) \right) / 3 \right] \left( \sqrt{\pi / 3} + (\ln(2))^2 \right) \) and \( \Gamma(x) \) is the gamma function. The finite value of the tunneling current causes a deviation of the conductance from \( e^2/(3\hbar) \). If several impurities are taken into account and all the neutral excitations eventually decay \( (\ell_{\text{ch-eq.0}} < L_{\text{arm}}) \), it can be self-consistently shown that the conductance between source and drain is zero.

In the framework of model (B), the full mixing between neutralons and anti-neutralons in the QPC region \( (\ell_{\text{nn},0} < L_{\text{QPC}}) \) causes both types of particles to arrive with the same probability at \( x_0 \). As the phase shifts of neutralons and anti-neutralons are the complex conjugate of each other, the non-equilibrium part of the Green’s functions

\[ g^{\text{eq}}_{r,\text{model B}}(t) = \left[ \frac{1}{2} + \frac{1}{4} e^{2\pi i \text{sgn}(t)/3} + \frac{1}{4} e^{-2\pi i \text{sgn}(t)/3} \right]^N, \] (11)

is real and leads to a vanishing tunneling current when inserted into Eqs. (6)–(8), causing the conductance quantization of \( e^2/(3\hbar) \).

Non-equilibrium noise. — We now quantify the zero-frequency non-equilibrium noise measured at the drain for model (B). Using the non-equilibrium bosonization technique \([32\, 37]\), we compute generating functions \( g_{n_1} = \langle e^{i \lambda_1 \phi_{n_1}(x,t)} e^{-i \lambda_1 \phi_{n_1}(x',t')} \rangle \) and \( g_{n_2} = \langle e^{i \lambda_2 \phi_{n_2}(x,t)} e^{-i \lambda_2 \phi_{n_2}(x',t')} \rangle \) as

\[ g_{n_1} = \left( \frac{a}{a + iv_n \tau} \right)^{\lambda_1^2} \left[ \frac{2}{3} + \cos \left( \frac{\pi \sqrt{2} \lambda_1}{3} \right) \right]^M, \] (12)

\[ g_{n_2} = \left( \frac{a}{a + iv_n \tau} \right)^{\lambda_2^2} \left[ \frac{1}{2} + \cos \left( \frac{\sqrt{2} \lambda_2}{3} \right) + \cos \left( \frac{2\sqrt{2} \lambda_2}{3} \right) \right]^M, \] (13)

valid in the long time limit \( M \equiv eV |\tau|/\hbar \gg 1 \), with \( \tau \equiv (t - t') + (x - x')/v_n \). To derive Eqs. (12) and (13), we have used that up and down neutralons create a kink of height \( ±\sqrt{2}\pi / \phi_{n1} \) while the strange neutralons leave \( \phi_{n1} \) invariant. Likewise, up and down neutralons create a kink of height of \( \pm\sqrt{2}\pi /3 \) in \( \phi_{n2} \), while strange neutralons create a kink of height \( -2\sqrt{2}\pi /3 \).

By taking second derivatives, we obtain the correlation functions \( C_{n_i} = \langle \phi_{n_i}(x,t) \phi_{n_i}(x',t') \rangle - \langle \phi_{n_i}^2(x,t) \rangle (i = 1, 2) \) labels the two neutral modes as

\[ C_{n_1}(\tau) = \ln \left( \frac{a}{a + iv_n \tau} \right) - \frac{eV |\tau|^2}{3\hbar}, \]

\[ C_{n_2}(\tau) = \frac{1}{3} \ln \left( \frac{a}{a + iv_n \tau} \right) - \frac{eV |\tau|^2}{9\hbar}. \] (14)

The noise in the neutral currents \( I_{n_1}(x,t) = v_n \partial_x \phi_{n_1}/(\sqrt{2}\pi) \) and \( I_{n_2}(x,t) = 3v_n \partial_x \phi_{n_2}/(\sqrt{2}\pi) \) evaluated in the decay region of edge \( r = u/l \) is defined as \( S_{n_{i=1,2},r} = 2 \int dt \langle I_{n_i}(x,t) I_{n_i}(x,0) \rangle \). Employing
TABLE I: A summary of experimental manifestations of two symmetries, a continuous one $SU(3)$ and a discrete threefold one $Z_3$ (cf. Eq. (4)). Model (A) (model (B)) corresponds to the cell in the first (second) row and the third column.

| Symmetry | Conductance plateau | Zero noise on the plateau | Mesoscopic conductance fluctuation | Non-universal noise | Conductance plateau | Fano factor of $2e/3$ |
|----------|---------------------|---------------------------|------------------------------------|---------------------|---------------------|----------------------|
| Conserved $Z_3$ symmetry | $e^2/(3h)$ | $e^2/(3h)$ | $e^2/(3h)$ | $e^2/(3h)$ | $e^2/(3h)$ | $e^2/(3h)$ |
| Broken $Z_3$ symmetry ($\ell_{n,0} \ll L_{QPC}$) | $e^2/(3h)$ | $e^2/(3h)$ | $e^2/(3h)$ | $e^2/(3h)$ | $e^2/(3h)$ | $e^2/(3h)$ |

We emphasize that even though the second term of Eq. (14) only holds for the large $|\tau|$ limit, Eq. (15) is valid because it only depends on the values of $\partial_{x'} C_{n_1}(\tau)|_{x'=0}$ and $\partial_{x'} C_{n_2}(\tau)|_{x'=0}$ for $\tau \to \pm \infty$; it is insensitive to details of $\partial_{x'} C_{n_1}$ and $\partial_{x'} C_{n_2}$ at small $\tau$. Due to the decay of all neutral excitations, the electrical noise measured at drain D can be expressed as

$$S_{n_1,a/1} = \frac{\nu^2}{\pi^2} \int_{-\infty}^{\infty} dt \partial_x \partial_{x'} C_{n_1}(\tau)|_{x=x',t'=0}$$

$$= -\frac{1}{\pi^2} \int_{-\infty}^{\infty} \partial_{\tau} C_{n_1}(\tau) d\tau = \frac{2eV}{h},$$

$$S_{n_2,a/1} = \frac{2eV}{h}. \quad (15)$$

We have investigated the roles of two symmetries, a continuous $SU(3)$ and a discrete one $Z_3$, in influencing the two-terminal conductance and the dc noise of a quantum Hall strip at filling factor $\nu = 2/3$ with a single QPC. While recent measurements \[11, 13\] with a Fano factor $2e/3$ on the conductance plateau of $e^2/(3h)$ are explained relying on a broken $Z_3$ symmetry, other symmetry scenarios (summarized in Table I) can be realized in future experiments. We expect that our symmetry-based approach can be generalized to other edges of topological phases, most certainly to hole-conjugate quantum Hall states with a QPC or more complicated geometries (e.g. a double QPC).

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SUPPLEMENTAL MATERIAL

The organization of this Supplemental Material is as follows. In Sec. A, variances of random tunneling amplitudes \( t \) for different neutralon species are shown to be equal to each other at a \( SU(3) \)-symmetric fixed point. In Sec. B, it is shown that unless the \( Z_3 \) symmetry is broken, the fundamental representation furnished by neutralons is completely disconnected from the conjugate representation furnished by anti-neutralons. In Sec. C, an effective action after performing the disorder averaging is shown to be invariant under the \( SU(3) \) transformation. In Sec. D, we describe the important length scales, and discuss their dependence on an external voltage. In Sec. E, it is self-consistently shown that the two-terminal conductance is zero when \( Z_3 \) is completely intact and all of neutral excitations eventually decay \( (\tau_{eh\rightarrow 0}\ll \tau_{arm}) \). Finally, in Sec. F, for model (B), noise of neutral currents is shown to be converted into noise of charge currents following Eq. (16) in the main text.

A. \( SU(3) \)-SYMMETRIC FIXED POINT

In this section, we show that variances of random tunneling amplitudes for different neutralon species are equal to each other due to the presence of random \( SU(3) \) rotations at the \( SU(3) \)-symmetric fixed point. The fact that these variances are equal leads to a restoration of the \( SU(3) \) symmetry in a statistical sense as evident from Eq. (3) of the main text.

We follow the procedure of Ref. [SI]. Electron tunneling between the inner edge channels (cf. Fig. 1(a) in the main text) drives the RG flow to a disorder-dominated intermediate fixed point. The neutral sector at the intermediate
fixed point possesses a global SU(3) symmetry, and the corresponding action reads

\[
S_n = \int dxdt \left[ \frac{1}{4\pi} (\partial_x\phi_n_1(\partial_t - v_n\partial_x)\phi_n_1 + 3\partial_x\phi_n_2(\partial_t - v_n\partial_x)\phi_n_2) \right.
\]

\[
- \frac{1}{2\pi a} \left( \xi_1(x)e^{i\sqrt{2}\phi_n_1} + \xi_2(x)e^{-i\phi_n_1}/\sqrt{2}, e^{i\phi_n_2}/\sqrt{2} + \xi_3(x)e^{i\phi_n_1}/\sqrt{2}e^{i\phi_n_2}/\sqrt{2} + H.c. \right) \right].
\]

(S1)

The second line originates from electron tunneling between the inner channels. \(\xi_{i=1,2,3}\) are random variables. Note that the six electron tunneling operators written in Eq. (S1) form the non-diagonal generators of the SU(3) symmetry (cf. Fig. S1).

Including the action for an auxiliary field \(\chi\) (see Ref. [S2] for a similar procedure) and performing refermionization in terms of a three-component fermion field \(\Psi(x) = e^{i\chi/\sqrt{3}}[e^{-i(\phi_n_1 + \phi_n_2)/\sqrt{2}}, e^{i(\phi_n_1 - \phi_n_2)/\sqrt{2}}, e^{i\sqrt{2}\phi_n_2}/\sqrt{2}]/2\pi a = e^{i\chi/\sqrt{3}}[\psi_d, \psi_s, \psi_u]^T\), Eq. (S1) reads

\[
S_n = \int dxdt \left[ i\Psi^\dagger(-\partial_t + v_n\partial_x)\Psi - \left( \xi_1 \Psi^\dagger(\frac{\lambda_1 + i\lambda_2}{2})\Psi + \xi_2 \Psi^\dagger(\frac{\lambda_6 + i\lambda_7}{2})\Psi + \xi_3 \Psi^\dagger(\frac{\lambda_4 + i\lambda_5}{2})\Psi + H.c. \right) \right]. \quad (S2)
\]

\(\lambda_{1,2,4,5,6,7}\) are non-diagonal generators of the SU(3) group. The random terms are completely eliminated via performing a SU(3) gauge transformation of \(\Psi(x) = U(x)\Psi(x)\); the action becomes diagonal as

(S2)

\[
S_n = \int dxdt \left[ \frac{1}{4\pi} (\partial_x\phi_n_1(\partial_t - v_n\partial_x)\phi_n_1 + 3\partial_x\phi_n_2(\partial_t - v_n\partial_x)\phi_n_2) \right.
\]

\[
- \frac{1}{2\pi a} \left( \xi_1(x)e^{i\sqrt{2}\phi_n_1} + \xi_2(x)e^{-i\phi_n_1}/\sqrt{2}, e^{i\phi_n_2}/\sqrt{2} + \xi_3(x)e^{i\phi_n_1}/\sqrt{2}e^{i\phi_n_2}/\sqrt{2} + H.c. \right) \right].
\]

(S3)

Here \(\psi_u \equiv e^{-i(\phi_n_1 + \phi_n_2)/\sqrt{2}}, \psi_d \equiv e^{i(\phi_n_1 - \phi_n_2)/\sqrt{2}}, \) and \(\psi_s \equiv e^{i\sqrt{2}\phi_n_2}/\sqrt{2}\) are referred to as neutralons in the main text. The most relevant quasi-particle tunneling operators between the outer-most channel and the inner ones are given by

\[
H_{ch-eq} = \int dx \left[ u_0 e^{i(\phi_{1/3} + \phi_{2/3})} + d_0 e^{i(\phi_{1/3} - 2\phi_{2/3} - \phi_3)} + t_{u,0} e^{i(\phi_{1/3} - 2\phi_{2/3} - 3\phi_4)} + H.c. \right],
\]

(S4)
The bare tunneling amplitudes $t_{j,0}$ are random with white noise correlation $(t_{j,0}(x)t_{j',0}(x'))_{\text{dis}} = W_{\text{eq},j}^0 \delta(x-x') \delta_{j,j'}$. In the rotated basis of the $\psi_j$, the tunneling amplitudes $t_{j=u,d,s}$ are given by $t_j = \sum_{j'} U_{j,j'} t_{j',0}$. Then, the tunneling amplitudes $t_{j=u,d,s}$ are also random with the noise correlation as

$$
(t_j(x)t_{j'}^*(x'))_{\text{dis}} = \sum_{j,j'} \langle U_{j,j}(x)t_{j,0}(x)U_{j',j'}^*(x')t_{j',0}(x') \rangle_{\text{dis}} = \sum_{j,j'} \langle U_{j,j}(x)U_{j',j'}^*(x') \rangle_{\text{dis}} (t_{j,0}(x)t_{j',0}(x'))_{\text{dis}}
$$

$$
= \sum_j \langle U_{j,j}(x)U_{j',j'}^*(x') \rangle_{\text{dis}} W_{\text{eq},j}^0 \delta(x-x') = U_0 \delta(x-x') \delta_{j,j'} \sum_j W_{\text{eq},j}^0 = W_{\text{ch}-\text{eq}} \delta(x-x') \delta_{j,j'}.
$$

(S5)

The second equality in Eq. (S5) is due to the statistical independence of $U$ and $t_0$ with regards to the disorder average, and $\sum \langle U_{j,j}(x)U_{j',j'}^*(x') \rangle_{\text{dis}} = U_0 \delta_{j,j'} \delta(x-x')$ comes from the randomness of $U$. We defined $W_{\text{ch}-\text{eq}} \equiv U_0 \sum_j W_{\text{eq},j}^0$. Eq. (S5) shows that the variance of the tunneling amplitudes in the rotated basis does not depend on the neutralon flavor.

B. IMPLICATION OF THE $Z_3$ SYMMETRY: DISCONNECTED NEUTRALON AND ANTI-NEUTRALON SECTORS

In this section, we show that for conserved $Z_3$ symmetry the fundamental representation furnished by neutralons is completely disconnected from the conjugate representation furnished by anti-neutralons. To see this, we consider a correlator $M_{j \rightarrow j'} : \langle 0 | \psi_j(x_f,t_f) \psi_j^\dagger(x_i,t_i) | 0 \rangle$ between a neutralon $j = u,d,s$ at position $x_i$ and time $t_i$ and an anti-neutralon $j' = \bar{u}, \bar{d}, \bar{s}$ at position $x_f$ and time $t_f$. $\psi_j(x_f,t_f)$ and $\psi_j^\dagger(x_i,t_i)$ are operators in the Heisenberg picture. We assume that the $Z_3$ is a symmetry of system: $[H,T_3] = 0$. Then, the vacuum state $|0\rangle$ must be an eigenstate of the $T_3$, $T_3|0\rangle = t_0|0\rangle$. Because the $T_3$ is an unitary operator, $t_0$ must be a phase factor ($|t_0|^2 = 1$). Employing the symmetry condition $[T_3,H] = 0$ and Eq. (4) in the main text, $M_{j \rightarrow j'}$ is shown to be zero as

$$
M_{j \rightarrow j'} = \langle 0 | U(t_f)^\dagger \psi_j^\dagger(x_f)U(t_f - t_i) \psi_j^\dagger(x_i)U(t_i)|0\rangle
$$

$$
= \langle 0 | T_3 T_3^\dagger U(t_f)^\dagger \psi_j^\dagger(x_f)U(t_f - t_i) \psi_j^\dagger(x_i)U(t_i)|0\rangle
$$

$$
= e^{-\pi i/3} M_{j \rightarrow j'}.
$$

(S6)

Here, the operator $T_3^\dagger$ was moved from left to right, using the fact that it commutes with the time evolution operator $U(t) = e^{-iHt}$, and that $|t_0|^2 = 1$. Since Eq. (S6) can only be satisfied for $M_{j \rightarrow j'} = 0$, we conclude that in order to allow the mixing between neutralons and anti-neutralons, the $Z_3$ symmetry should be broken. Therefore, for the model (A) where the $Z_3$ is preserved, the fundamental representation furnished by neutralons is completely disconnected from the conjugate representation furnished by anti-neutralons.

C. SU(3) SYMMETRY RESTORATION BY DISORDER AVERAGING

In this section, it is shown that an effective action after performing the disorder averaging is invariant under the SU(3) transformation. To this end, we first consider $H_{n\bar{n}}$ (see Eq. (5) in the main text). The corresponding Keldysh action reads

$$
S_{n\bar{n}} = - \int dx \int dt (v_u \psi_d^\dagger \psi_s + v_d \psi_x^\dagger \psi_u + v_s \psi_u^\dagger \psi_d + \text{c.c.}).
$$

(S7)

Then, the disorder averaging of $e^{iS_{n\bar{n}}}$ is written as

$$
\langle e^{iS_{n\bar{n}}} \rangle_{\text{dis}} = \frac{\int Dv_j Dv_j^* D\psi_j D\psi_j^* e^{-\sum_j \int dx v_j(x) \psi_j^\dagger(x)/W_{\text{nn}} - i \int dx \int dt (v_u \psi_d^\dagger \psi_s + v_d \psi_x^\dagger \psi_u + v_s \psi_u^\dagger \psi_d + \text{c.c.})}}{\int Dv_j Dv_j^* e^{-\sum_j \int dx v_j(x) \psi_j^\dagger(x)/W_{\text{nn}}}}.
$$

(S8)

We used the fact that the random amplitudes $v_j=u,d,s$ satisfy the noise correlation of $\langle v_j(x) v_j^*(x') \rangle_{\text{dis}} = W_{n\bar{n}} \delta(x-x') \delta_{j,j'}$. Note that the variances do not depend on the neutralon flavor (see Sec. A of the Supplemental material for...
more detail). The integration over the amplitudes leads to
\[
\langle e^{iS_{nn}} \rangle_{\text{dis}} = \int D\psi \sum_{j} e^{-W_{nn}} \int dx \int_{K} dt dt' (\psi_j^\dagger(x,t)\psi_j(x,t) + \psi_j^\dagger(x,t)\psi_j(x,t) + \psi_j^\dagger(x,t)\psi_j(x,t))
\]
\[= \int D\psi \sum_{j} e^{iS_{nn,eff}}, \quad (S9)
\]
where the effective action after the disorder averaging is given as
\[
S_{n\bar{n},\text{eff}} = iW_{n\bar{n}} \int dx \int_{K} dt dt' (\psi_d^\dagger(x,t)\psi_d(x,t)\psi_{ch}(x,t') + \psi_d^\dagger(x,t)\psi_{ch}(x,t')\psi_d(x,t'))
\]
\[+ \psi_d^\dagger(x,t)\psi_{ch}(x,t)\psi_{ch}(x,t')\psi_d(x,t'). \quad (S10)
\]
It can be easily seen that this effective action is invariant under the SU(3) transformation \(\psi_j(x) \rightarrow \sum_{j'} U_{jj'}(x)\psi_{j'}(x)\) unlike the Hamiltonian \(H_{n\bar{n}}\). Thus the SU(3) symmetry is restored in the statistical sense. Furthermore, the same procedure through Eqs. (S7)-(S9) also leads to Eq. (3) in the main text.

D. LENGTH SCALES

Here we describe the relevant length scales, and discuss their scaling with an external voltage. We analyze it employing the renormalization group (RG) method of Ref. [S3]. The schematic dependence of the relevant length scales on the voltage is depicted in Fig. S2. We focus on the dependence on the voltage, but the voltage \(V\) is perfectly interchangeable with the temperature \(T\) when \(T \gg V\).

We first summarize all length scales considered in this work:

- \(L_{\text{arm}}\) : the length of the arms between the contacts and the QPC.
- \(L_{\text{QPC}}\) : the size of the QPC.
- \(\ell_0\) : the elastic scattering length beyond which disorder-induced electron tunneling mixes the inner modes. When \(L_{\text{arm}}\) exceeds \(\ell_0\), disorder becomes relevant and the system is driven to the intermediate fixed point [S1]. The intermediate fixed point is called the Wang-Meir-Gefen (WMG) fixed point in this section.
- \(\ell_{\text{ch-\,eq},0}\) : the elastic scattering length beyond which disorder-induced quasi-particle tunneling mixes the inner modes with the outer-most mode.
- \(\ell_{n\bar{n},0}\) : the elastic scattering length over which neutralons are mixed with anti-neutralons.
- \(\ell_{\text{ch-\,eq}}\) : the inelastic scattering length (the red curve in Fig. S2) over which the charge modes are equilibrated.
- \(\ell_{n\bar{n}}\) : the inelastic scattering length over which neutralons are equilibrated with anti-neutralons.
- \(\ell\) : the inelastic scattering length (the yellow curve in Fig. S2) over which the inner three modes equilibrate.
- \(\ell_{\text{in}}\) : the coherence length (the black thin curve in Fig. S2) at which the modes lose the coherence. It takes the smaller value between \(\ell\) and \(\ell_{\text{ch-\,eq}}\).
- \(L_V\) : the voltage length \(\sim 1/V\) (the black thick curve in Fig. S2) operating as an infrared cutoff.

We assume that the electron tunneling operators (Eq. (S1)) within the inner modes drive the system to the WMG intermediate fixed point [S1]. The length scale \(L_{\text{arm}}\) and \(L_{\text{QPC}}\) are much larger than \(\ell_0\) over which the inner modes are strongly mixed by disorder. In the vicinity of the WMG fixed point, we consider quasi-particle tunneling \(H_{\text{ch-\,eq}}\) between the outer-most mode and the inner modes (Eq. (2) in the main text) and neutral-antineutralon mixing term \(H_{n\bar{n}}\) (Eq. (5) in the main text). Below, we will find the scaling of the length scales associated with \(H_{\text{ch-\,eq}}\) and \(H_{n\bar{n}}\) using the RG method. \(\ell_0\) acts as the ultraviolet length cutoff of the RG analysis.

We first consider \(\ell_{\text{ch-\,eq},0}\), characterizing the elastic scattering between the outer-most mode and the inner ones. \(\ell_{\text{ch-\,eq},0}\) is determined by the following RG equation,
\[
\frac{d\tilde{W}_{\text{ch-\,eq}}}{d\ln L} = \tilde{W}_{\text{ch-\,eq}}(3 - 2\Delta_{\text{ch-\,eq}}), \quad (S11)
\]
Here $\tilde{W}_{\text{ch-eq}} \equiv W_{\text{ch-eq}} \ell_0^3/v_n^2$ (cf. see the inline equation above Eq. (3) in the main text for $W_{\text{ch-eq}}$) is the dimensionless disorder strength, and $\Delta_{\text{ch-eq}}$ is the scaling dimension of the $H_{\text{ch-eq}}$ (Eq. (2) of the main text) and is equal to $2/3$ at the fixed point. Assuming trivial density of states factors, $W_{\text{ch-eq}}$ can be thought of as the running dimensionless resistance. We now consider the voltage $V$ to be sufficiently small, such that the renormalized $W_{\text{ch-eq}}$ reaches one for $L = \ell_{\text{ch-eq},0} < L_V$ ($V < V^*$ in Fig. S2). The perturbative RG of Eq. (S11) breaks down when $\tilde{W}_{\text{ch-eq}}$ becomes unity and the corresponding length scale (at which $\tilde{W}_{\text{ch-eq}}$ renormalizes to unity) is $\ell_{\text{ch-eq},0} = \ell_0/\left(\tilde{W}_{\text{ch-eq}}^0\right)^{3/5}$. Here $\tilde{W}_{\text{ch-eq}}^0$ is the bare dimensionless disorder strength. Following the same procedure for the $H_{n\bar{n}}$ (Eq. (5) of the main text), we also obtain $\ell_{n\bar{n},0} = \ell_0/\left(\tilde{W}_{n\bar{n}}^0\right)^{3/7}$. Both $\ell_{n\bar{n},0}$ and $\ell_{\text{ch-eq},0}$ are elastic scattering lengths and do not depend on energy cutoffs (externally applied voltage $V$) of the system.

As $L_{\text{arm}}$ exceeds $\ell_{n\bar{n},0}$ and $\ell_{\text{ch-eq},0}$, the system is further renormalized away from the WMG fixed point, ultimately arriving at the low-energy fixed point. In the vicinity of the latter fixed point, counter-propagating neutral modes localize each other, leaving a charge mode and a neutral mode $S1$. The inelastic length scales (cf. the red curve at $V < V^*$ in Fig. S2) near the low-energy fixed point can exceed the elastic charge equilibration length $\ell_{\text{ch-eq},0}$ parametrically at sufficiently small voltages $S2$ $S3$. By continuity, there exists a regime ($\ell_{\text{ch-eq},0} < L_{\text{arm}} < \ell_{\text{ch-eq}}$) still governed by the WMG fixed point where our analysis is mostly performed.

When the voltage, on the other hand, is sufficiently large such that $\ell_{\text{ch-eq},0}$ is larger than $L_V$ ($V \gg V^*$ in Fig. S2), Eq. (2) in the main text with Eq. (S11) yields $\ell_{\text{ch-eq}}$. Eq. (S11) stops to be valid at the infrared cutoff $L_V$, leading to

$$\frac{\tilde{W}_{\text{ch-eq}}(L = L_V)}{\tilde{W}_{\text{ch-eq}}^0} = \left(\frac{L_V}{\ell_0}\right)^{5/3}.$$ (S12)

Beyond the scale $L_V$, the renormalization of the resistance, hence of $\tilde{W}_{\text{ch-eq}}$, continues classically (i.e., $\tilde{W}_{\text{ch-eq}}$ grows linearly as $L$ increases) $S3$, and in turn breaks down when $\tilde{W}_{\text{ch-eq}}$ becomes unity, leading to

$$\frac{\tilde{W}_{\text{ch-eq}}(L = L_V)}{L_V} = \frac{\tilde{W}_{\text{ch-eq}}(L = \ell_{\text{ch-eq}})}{\ell_{\text{ch-eq}}} \simeq \frac{1}{\ell_{\text{ch-eq}}}.$$ (S13)
Here, $\ell_{\text{ch-eq}}$ is defined as the length at which $\tilde{W}_{\text{ch-eq}}$ becomes unity. Employing Eqs. (S12) and (S13), we obtain

$$\ell_{\text{ch-eq}} = \ell_0 (\ell_0 / L V)^{2/3} / \tilde{W}^{0}_{\text{ch-eq}} \propto V^{2/3}$$

(depicted with the red curve at $V \gg V^*$ in Fig. (S2)). Following the same procedure for the $H_{\text{n-n}}$, we also obtain

$$\ell_{\text{n-n}} = \ell_0 (\ell_0 / L V)^{4/3} / \tilde{W}^{0}_{\text{n-n}} \propto V^{4/3}.$$ 

### E. THE $Z_3$ SYMMETRY AND THE TWO-TERMINAL CONDUCTANCE

In this section, we show self-consistently that the two-terminal conductance is zero if (i) $Z_3$ is present (model (A)) and (ii) all neutral excitations eventually decay ($\ell_{\text{ch-eq,0}} \ll L_{\text{arm}}$). The quasi-particle tunneling probability between the outer-most channel and the inner channels in each corner is defined as $P_{u,R}$, $P_{u,L}$, $P_{l,R}$, and $P_{l,L}$, respectively. All the electrical currents displayed in Fig. S3 are determined by the following rate equations

$$I_{1/3,u} = \frac{e^2 V_0}{3h} + \frac{e}{3} I_{n,L} P_{u,L}, \quad I_{1/3,l} = \frac{e}{3} I_{n,R} P_{l,R},$$

$$I_{c,L} = \frac{e^2 V_0}{3h} - \frac{e}{3} I_{n,L} P_{u,L}, \quad I_{c,R} = -\frac{e}{3} I_{n,R} P_{l,R},$$

$$\frac{e}{3} I_{n,L} = \frac{1}{2} P_{l,L} (I_{1/3,l} - I_{c,L}), \quad \frac{e}{3} I_{n,R} = \frac{1}{2} P_{u,R} (I_{1/3,u} - I_{c,R}). \quad (S14)$$

The conductance measured at D is calculated as

$$G_D = \frac{I_{c,R} + I_{1/3,R}}{V_0} = \frac{2e^2}{3h} \left( 1 - P_{l,L} P_{u,L} \right) \left( 1 - P_{l,R} P_{u,R} \right) / \left( 2 - P_{l,L} P_{u,L} - P_{l,R} P_{u,R} \right). \quad (S15)$$

When $P_{l,L} = P_{l,R} = P_{u,L} = P_{u,R} = 1$, i.e., all the neutral excitations eventually decay ($\ell_{\text{ch-eq,0}} \ll L_{\text{arm}}$), the conductance can be seen to be zero. When all the tunneling probabilities, on the other hand, go to zero, the conductance becomes $e^2/(3h)$.

### F. NON-EQUILIBRIUM NOISE

In this section, the noise of neutral currents is shown to be converted into the noise of charge currents by Eq. (16) in the main text when all neutral excitations decay ($\ell_{\text{ch-eq,0}} \ll L_{\text{arm}}$). All the calculations in this section are performed employing model (B).

The densities of neutral modes $n_1$ and $n_2$ are defined as $\rho_{n_1}(x) = \partial_x \phi_{n_1} / (\sqrt{2\pi})$ and $\rho_{n_2} = 3 \partial_x \phi_{n_2} / (\sqrt{2\pi})$ such that

FIG. S3: (color online). A two-terminal setup with a quantum point contact (QPC). Source S is biased by $V_0$. The two-terminal conductance is measured between S and D; the d.c. noise is measured at drain D. Black arrows denote the direction of tunneling charge currents at each corner.
creation of an up neutralon changes the density of neutral mode $n_1$ and $n_2$ by a delta function contribution since

$$
\psi_u(x') \left( \frac{\partial \phi_{n_1}(x)}{\sqrt{2\pi}} \right) \psi_u(x') = \left( e^{-i(\phi_{n_1}(x') + \phi_{n_2}(x'))/\sqrt{2}} \right) \left( \frac{\partial \phi_{n_1}(x)}{\sqrt{2\pi}} \right) \left( e^{i(\phi_{n_1}(x') + \phi_{n_2}(x'))/\sqrt{2}} \right)
$$

$$
= \frac{\partial \phi_{n_1}(x)}{\sqrt{2\pi}} + \delta(x - x'),
$$

$$
\psi_u(x') \left( 3\frac{\partial \phi_{n_2}(x)}{\sqrt{2\pi}} \right) \psi_u(x') = \left( e^{-i(\phi_{n_1}(x') + \phi_{n_2}(x'))/\sqrt{2}} \right) \left( \frac{3\partial \phi_{n_2}(x)}{\sqrt{2\pi}} \right) \left( e^{i(\phi_{n_1}(x') + \phi_{n_2}(x'))/\sqrt{2}} \right)
$$

$$
= 3\frac{\partial \phi_{n_2}(x)}{\sqrt{2\pi}} + \delta(x - x').
$$

Then, we define decay neutral current operators $I_{\text{dec},n_1}$ and $I_{\text{dec},n_2}$ in the decay region of neutralons from the equations of motion of neutral number operators $N_{n_1} = \int dx \rho_{n_1}(x)$ and $N_{n_2} = \int dx \rho_{n_2}(x)$ as

$$
I_{\text{dec},n_1} = \frac{dN_{n_1}}{dt} = \frac{i}{\hbar} [H_{\text{eq}}, N_{n_1}] = -\frac{i}{\hbar a} \sum_{\epsilon = \pm} \int dx \epsilon \left[ T_u(x) - T_d(x) \right]^*,
$$

$$
I_{\text{dec},n_2} = \frac{dN_{n_2}}{dt} = \frac{i}{\hbar} [H_{\text{eq}}, N_{n_2}] = -\frac{i}{\hbar a} \sum_{\epsilon = \pm} \int dx \epsilon \left[ T_u(x) + T_d(x) - 2T_s(x) \right]^*,
$$

where the tunneling operator $T_{j=u,d,s}$ of each neutralon is defined as

$$
T_j(x) = t_j(x) \psi_j^\dagger(x) e^{-i\phi_j(x)/\sqrt{\pi}} e^{i\phi_j(x)/\sqrt{2}},
$$

and a convenient notation $[AB]^\dagger = AB \ ([AB]^\dagger = B^\dagger A^\dagger)$ is used. Similarly, a charge tunneling current is defined as

$$
I_{\text{tun}} = \frac{ie}{\hbar} \left[ H_{\text{eq}}, \frac{1}{2\pi} \int dx \partial_x \phi_{1/3} \right] = \frac{ie}{3\hbar a} \sum_{\epsilon = \pm} \sum_{j=u,d,s} \int dx \epsilon \left[ T_u(x) + T_d(x) + T_s(x) \right]^*.
$$

The decay neutral currents and incoming neutral currents are zero, $\langle I_{\text{dec},n_1} \rangle = \langle I_{\text{dec},n_2} \rangle = 0$ and $\langle I_{n_1} \rangle = \langle I_{n_2} \rangle = 0$ (for their definitions, see in-line equations just below Eq. (15) in the main text); $\langle I_{\text{dec},n_1} \rangle = \langle I_{\text{dec},n_2} \rangle = 0$ can be derived using the fact that the Green’s function of neutralons (Eq. (12) in the main text) is real for the model (B) (through a similar procedure as Eq. (7)). $\langle I_{n_1} \rangle = \langle I_{n_2} \rangle = 0$ can be derived (i) taking the first derivative of Eqs. (13) and (14) (in the main text) by $\lambda_1$ and $\lambda_2$ and (ii) sending $\lambda_1$ and $\lambda_2$ to zero. Furthermore, the electrical tunneling current is zero as seen in Eq. (12) in the main text. Under the assumption that all neutral excitations eventually decay in the decay region, the noise $(S_{n_1}$ and $S_{n_2})$ of incoming neutral currents are identical to the noise of the decay neutral currents,

$$
S_{n_1} = S_{\text{dec},n_1} \equiv 2 \int_{-\infty}^{\infty} dt \langle I_{\text{dec},n_1}(t) I_{\text{dec},n_1}(0) \rangle \quad S_{n_2} = S_{\text{dec},n_2} \equiv 2 \int_{-\infty}^{\infty} dt \langle I_{\text{dec},n_2}(t) I_{\text{dec},n_2}(0) \rangle .
$$

respectively. Using Eqs. (S9)-(S12), the noise $(S_{\text{tun}})$ of the electrical tunneling current can be decomposed into the noise of the neutral decay currents as

$$
S_{\text{tun}} \equiv 2 \int dt \langle I_{\text{tun}}(t) I_{\text{tun}}(0) \rangle \approx \frac{e^2}{9\hbar^2 \pi a} \sum_{\epsilon = \pm} \int dt \int dx \int dx' \left( \langle T_u(x,t)^\dagger T_d(x',0)^\dagger \rangle + \langle T_d(x,t)^\dagger T_u(x',0)^\dagger \rangle + \langle T_d(x,t)^\dagger T_s(x',0)^\dagger \rangle \right)
$$

$$
= \frac{e^2}{9\hbar^2 \pi a} \sum_{\epsilon = \pm} \int dt \int dx \int dx' \left[ \frac{3}{4} \left( \langle T_u(x,t)^\dagger T_u(x',0)^\dagger \rangle + \langle T_d(x,t)^\dagger T_d(x',0)^\dagger \rangle \right) \right]
$$

$$
+ \frac{1}{4} \left( \langle T_u(x,t)^\dagger T_s(x',0)^\dagger \rangle + \langle T_d(x,t)^\dagger T_s(x',0)^\dagger \rangle \right) + 4 \langle [T_u(x,t)]^\dagger [T_d(x',0)]^\dagger \rangle
$$

$$
= \frac{e^2}{36} (3S_{\text{dec},n_1} + S_{\text{dec},n_2}) = \frac{e^2}{36} (3S_{n_1} + S_{n_2}).
$$

This charge noise of the electrical tunneling current is measured at drain D (see Fig. 2 in the main text). The charge noise in each of the upper and lower edges contributes to the zero-frequency noise $S_D$ as

$$
S_D = \frac{e^2}{36} (3S_{n_1,u} + S_{n_2,u} + 3S_{n_1,l} + S_{n_2,l}) = \frac{2ke^3V}{9\hbar}.
$$
We arrive at Eq. (16) in the main text.

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