Influence of friction forces in limiter-eyelet contact on twisting angles of the rubber elements of tracked system hinges

S A Korostelev¹, A V Gorbachev¹, A F Verbilov² and V V Kovalev²

¹ Department of land transport-technological systems, Polzunov Altai State Technical University, Barnaul, Altai Region, Russia
² Firearms and Technical Training Department, Barnaul Law Institute of the Ministry of Internal Affairs of Russia, Barnaul, Altai Region, Russia

Abstract. The article presents a study of the influence of the radial deformation limiters of rubber-metal compounds of tracked systems on the loading of rubber elements. Description of the mathematical model of the dynamic behaviour of a tracked propulsion unit with rubber-metal hinges is presented. Dynamic calculations using the software package based on the equations were performed. The analysis of the results of the computer calculation revealed the influence of friction forces and moments in the contact zone of the limiter-eyelet.

1. Introduction

The tracked system is widely used in construction and road vehicles. Elements of the tracked system are exposed to high dynamic loads affecting strength, reliability and durability. Reducing dynamic loads of the tracked systems is an important engineering task. One of the effective ways in this issue is the use of rubber elements in the mechanism of tracked system. Power rubber elements are applied in the constructions of the return and idler rollers, road wheels, suspension elements as well as in the tracked chain.

During the designing of the tracked systems it is very important to assure high requirements on metal intensity, durability, stiffness and strength of all the elements. One of the elements limiting the resource of the caterpillar engine is the connection of adjacent links. Structurally connecting links can be performed in a variety of ways, but most often used connections in the form of hinges. There are open and closed hinges, which can include rubber elements that act as an elastic bond or just seal the connection. Used rubber metal joint (RMJ) with force rubber elements have a high enough carrying capacity and good compensatory properties, high malleability, increased damping. The main drawback of the structures of the high-speed vehicles with the RMJ is that the rubber elements fully perceive the forces and moments acting on the hinge, which leads to large deformations of rubber and rapid destruction of rubber elements.

2. The rubber-metal hinge compound with rigid limiters of radial deformations
To increase the durability of the rubber elements of the connections of tracked chain the construction of rubber metal joint with a rigid limiter of radial deformation of rubber elements are used. When designing a sequentially rubber-metal hinge (Figure 1) it is necessary to ensure equal angular stiffness of rubber elements, located in the eyelets of the connected links. This condition ensures an equal twisting angle for all rubber elements and the same shearing stresses.

![Figure 1. Rubber-metal hinge compound of track chain links.](image)

1 - metal pin fitting; 2 - limiter of radial deformation; 3 - rubber elements of the lateral triple eyelets; 4 - rubber elements of double eyelets; 5 - rubber elements of the central eyelet; 6 - adjacent links.

When under the influence of a stretching force in the chain the magnitude of the radial deformation of rubber elements becomes equal to the magnitude of the radial gap between the limiter and the eyelet, the limiter comes into contact with the surface of the eyelet, with a mutual angular displacement of the link and pin. At this moment the friction force is caused, which is proportional to the force of normal pressure in the contact area (as shown in figure 2).

![Figure 2. Interaction between the track and the radial deformation limiter.](image)

Therefore, additional force factors are the friction forces and their moments, which additionally load adjacent elements [3]. The total radial and angular stiffness of the rubber elements of the adjacent links of the hinge are always differ. For example, for existing RMJ with 5 eyelets to ensure equal total angular stiffness of elements located in double and triple eyelets, the total radial rigidity of rubber elements of double eyelets will always be higher than triples [2]. Therefore, limiters of triple eyelets will always come into contact earlier, and the friction force and friction torque will also always be higher at the contact of the limiters of triple eyelets. Therefore, one of the tasks that must be solved when determining the loads acting on the rubber elements of the hinge connection is to evaluate the influence of the friction moment in the contact of the limiter-eyelet on the twisting angle of the rubber elements of the adjacent links [3].

3. The mathematical model of the dynamic behavior of a tracked propulsion unit with rubber-metal hinges

To assess the influence of friction forces in the contact of the limiter-eyelet, the methods to study the dynamics of solid systems are used [4 - 7].

The exact mathematical description of the phenomena occurring in a tracked vehicle is associated with great difficulties. The complication of mathematical models for describing the mechanical behavior of a tracked system requires a considerable amount of time when solving on a computer [8-10]. In this
paper, a number of assumptions are made that simplify the mathematical model of the dynamic behavior of the elements of a tracked propulsor [10 - 12].

-a mechanical system consisting of a body of the tracked vehicle, a sprocket wheel, a guide wheel and elements of the depreciation-tensioning device, suspension arms, road rollers, tracks makes a flat movement;

-links, road rollers, levers, wheels are absolutely rigid, non-deformable elements;

-connections between the elements of the tracked system are realized in the form of elastic, viscoelastic connections or absolutely rigid contact;

-dissipative forces in frictional bonds are dismissively small;

-elastic-plastic connections between the links of the tracked system and the ground are realized.

The motion of a mechanical system with holonomic constraints is described by the Lagrange equations, which are second-order differential equations for generalized coordinates

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} = Q_i + \sum \lambda_k \frac{\partial \Phi_k}{\partial q_i}.$$  \hspace{1cm} (1)

The kinetic energy of an individual element is determined by the equation

$$T = \frac{1}{2} m_i \left( \dot{x}_i^2 + \dot{y}_i^2 \right) + \frac{1}{2} J_i \dot{\phi}_i^2.$$  \hspace{1cm} (2)

The generalized forces acting on the $i$-element caused by the force connection with the $j$-element

$$Q_i^j = F_{ij}^x;$$

$$Q_i^r = F_{ij}^y;$$

$$Q_i^p = F_{ij}^r \left( \xi_j \sin \phi_i - \eta_j \cos \phi_i \right) + F_{ij}^y \left( \xi_j \cos \phi_i - \eta_j \sin \phi_i \right).$$  \hspace{1cm} (3)

The result is a system of differential algebraic equations connecting the vector of unknown displacements of the mechanical system

$$[M][\dot{q}] + [C][\ddot{q}] + [K][q] + [\Phi_q]^T \{\lambda\} = \{P(t)\}$$

$$\{\Phi(q,t)\} = \{0\},$$  \hspace{1cm} (4)

where $[M]$ is the global mass matrix of the system; $[K]$ is the global stiffness matrix of the system; $[C]$ is the global damping matrix of the system; $\{q\}$ is the vector of generalized coordinates; $\{\dot{q}\}$ is the vector of generalized velocities; $\{\ddot{q}\}$ is the vector of generalized accelerations; $\{\lambda\}$ - vector of Lagrange multipliers; $\{P(t)\}$ is the vector of external forces;

$$[\Phi_q] = \begin{bmatrix}
\frac{\partial \Phi_1}{\partial q_1} & \frac{\partial \Phi_1}{\partial q_2} & \cdots & \frac{\partial \Phi_1}{\partial q_n} \\
\frac{\partial \Phi_2}{\partial q_1} & \frac{\partial \Phi_2}{\partial q_2} & \cdots & \frac{\partial \Phi_2}{\partial q_n} \\
\cdots & \cdots & \cdots & \cdots \\
\frac{\partial \Phi_m}{\partial q_1} & \frac{\partial \Phi_m}{\partial q_2} & \cdots & \frac{\partial \Phi_m}{\partial q_n}
\end{bmatrix}.$$
The system of nonlinear differential-algebraic equations should be supplemented with initial and boundary conditions [4, 5]. That system of equations (4) includes $3N$ nonlinear differential equations of the second order ($N$ is the number of elements of the mechanical system) and $m$ kinematic connection equations, which are nonlinear algebraic equations. System (4) has a differentiation index of three [14]. To solve the system it is necessary to downgrade the index. By double-differentiation of the equations of the system (4) it is possible to transform it to the following form [5-7,10,11]

\[
\begin{bmatrix}
[M] & \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}
\begin{bmatrix}
\{\dot{q}\} \\
\{\lambda\}
\end{bmatrix}
=
\begin{bmatrix}
\{\dot{P}\} \\
\{\gamma\}
\end{bmatrix},
\]

(5)

where

\[
\{\gamma\} = -\{\Phi_q\} - \{\Phi_q q\} \{\dot{q}\} - 2\{\Phi_q q\} \{\dot{q}\}.
\]

In expression (5) the lower indexes denote the derivative of the function of the kinematic constraints by the generalized coordinates and time.

To solve the system of differential-algebraic equations (5) numerical methods are used [15, 16]. In order to stabilize the numerical algorithm when solving the system of equations (5) the vector \(\{\gamma\}\) is replaced by the expression [4-6]

\[
\{\gamma^{*}\} = \{\gamma\} - 2\alpha\{\Phi_q\} \{\dot{q}\} - (\beta) \{\Phi_q\} \{q\},
\]

(6)

where \(\alpha > \beta\) and \(\beta \neq 0\).

Large displacements of the elements of tracked systems from the initial state, the presence of kinematic equations of links, nonlinear characteristics of stiffness and damping of viscoelastic bonds, which are nonlinear functions of deformation and deformation rate, complicate the solution of the system and impose restrictions on the numerical methods used to solve it.

In the present work, as one of the stages of increasing the stability of the numerical solution of system (5), an algorithm of the linearization of the equations of motion and expressions describing holonomic kinematic constraints is used.

After linearization, the system of differential algebraic equations (5) takes the form as

\[
\begin{bmatrix}
[M] & \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}
\begin{bmatrix}
\{\Delta q\} \\
\{\Delta \lambda\}
\end{bmatrix}
=
\begin{bmatrix}
\{\Delta P\} \\
\{\gamma^{*}\}
\end{bmatrix},
\]

(7)

where

\[
\{\Delta P\} = [\overline{P}(t)] - [\overline{C}] \{\Delta q\} - [\overline{K}] \{\Delta q\}.
\]

From system (7), a following expression for determining the increment of the Lagrange multipliers at each step can be obtained [17]

\[
\{\Delta \lambda\} = \left(\overline{\Phi}_{\Delta q} M^{\frac{1}{2}} \left[\overline{\Phi}_{\Delta q}\right]^T \right)^{-1} \left(\overline{\Phi}_{\Delta q} M \left[\overline{\Phi}_{\Delta q}\right]^T \right) \{\Delta \lambda\}.
\]

(8)

The resulting expression is substituted into the first block of equations of system (7)

\[
[M] \{\Delta q\} = \{\overline{P}\} - \left(\overline{\Phi}_{\Delta q} M \left[\overline{\Phi}_{\Delta q}\right]^T \right) \left(\overline{\Phi}_{\Delta q} M \left[\overline{\Phi}_{\Delta q}\right]^T \right)^{-1} \left(\overline{\Phi}_{\Delta q} M \left[\overline{\Phi}_{\Delta q}\right]^T \right) \{\Delta P\} - \{\gamma^{*}\}.
\]

(9)
To solve the system of differential-algebraic equations, finite-difference formulas of the Newmark $\beta$-scheme are used [17]

$$
\begin{align*}
\{\Delta \ddot{q}_{1,\Delta t}\} &= \frac{1}{\beta \Delta t^2} \left( \{\Delta q_{1,\Delta t}\} - \{\Delta q_t\} \right) - \frac{1}{\beta \Delta t} \{\Delta \dot{q}_t\} - \left( \frac{1}{2\beta} - 1 \right) \{\Delta \ddot{q}_t\}, \\
\{\Delta \ddot{q}_{t+\Delta t}\} &= \frac{\alpha}{\beta \Delta t} \left( \{\Delta q_{t+\Delta t}\} - \{\Delta q_t\} \right) - \left( \frac{\alpha}{\beta} - 1 \right) \{\Delta \dot{q}_t\} - \Delta t \left( \frac{\alpha}{2\beta} - 1 \right) \{\Delta \ddot{q}_t\},
\end{align*}
$$

(10)

where $\alpha$, $\beta$ are the parameters of the method; $\Delta t$ - the length of the time interval.

Substituting expressions (10) into the system of linearized differential-algebraic equations (9), the initial system is transformed into a system of linear algebraic equations, for the solution of which direct methods are used. Using the values of displacements, as a result of solving this system, we determine the speed and acceleration using expressions (10). Then, using the expression (8), it is possible to determine the increments of the Lagrange multipliers.

Sequential step-by-step solution of the system of equations (7) allows to determine the position of the elements of a tracked propulsor at a specific point in time as well as displacements, speeds and accelerations.

4. The simulation analysis

The above equations are the basis of the software package for calculating the dynamic loads acting on the rubber elements of the track-type propulsion unit. The software package allows determining the displacements, speeds and accelerations of the elements of the tracked systems, which are used to determine the forces acting on the rubber elements.

In Figure 3 changes in the twisting angle of the rubber element when moving the hinge around the tracked propulsor’s perimeter are presented. The effect of the friction torque caused by the contact of the limiter and the eyelet is not taken into account. The results were obtained for a tracked tractor of class 3 when driving in first gear with a hook load. It can be seen from the figure that the rubber elements of the double eyelet both on the driving wheel and on the guides are twisted at a greater angle. In this case, the maximum value of the twisting angle does not exceed 6.0°.
**Figure 3.** Twisting angle of the rubber elements of the RMJ (the tractor’s speed of movement is 0.93 m/s, without the influence of the limiter): 1 - triple eyelets; 2 - double eyelets; 3 - the difference of the twist angles of the rubber elements of the triple and double eyelets.

In Figure 4 the change in the twist angle of rubber elements, taking into account the influence of friction in the contact limiter-eyelet is presented. The twisting angle of the rubber elements of the double eyelets when twisting the hinge after the engagement is 1.0° more than the rubber elements of the triple eyelets. It should be noted that in this area rubber elements are subject to maximum radial deformation, the value of which is limited by the radial clearance. The combined effect of radial deformation and twisting leads to greater damage to the rubber elements of the double eyelets in comparison with triple rubber elements [1, 2].

**Figure 4.** Twisting angle of the rubber elements of the RMJ (the tractor’s speed of movement is 0.93 m/s, taking into account the the influence of the limiter): 1 - triple eyelets; 2 - double eyelets; 3 - the difference of the twist angles of the rubber elements of the triple and double eyelets.

5. **Conclusion**

Friction forces in the contact area of the limiter-eyelet increase the twisting angle of the rubber elements of double eyelets on the leading section by 0.5°. For the RMJ with radial deformation limiters considered here rubber elements of the double eyelets on the leading section are twisted by 1.0° more than the rubber elements of triple eyelets. To create a uniform structure with an equal strength and increase the durability of the hinged connection of the links of a tracked propulsion units, the angular stiffness of the rubber elements located in the double eyelets must be greater than the angular stiffness of the rubber elements of the triple eyelets.

**References**

[1] Korostelev S A 2010 Assessment of the tense-deformed state of the rubber element of the rubber-metal joints of the tracked systems in assembly and torsion Tractors and agricultural machines (11) 26-9

[2] Korostelev S A 2012 Effect of the radial and axial deformation of the rubber element of the rubber-metal joint of the tracked systems on its stressed-strained state Izvestia of the Samara Research Center of the Russian Academy of Sciences 14(2) 378-80

[3] Korostelev S A, Verbilov A F and Kovalev V V 2012 Theoretical study of dynamic load of rubber-metal joints of the tracked systems with radial deformation limiters Izvestia of the Samara Research Center of the Russian Academy of Sciences 14(2) 381-3

[4] Shabana A A 2005 Dynamics of Multibody Systems (Cambridge: Cambridge University Press) 374
[5] Shabana A A 2010 Computational Dynamics (New York: John Wiley & Sons Ltd) 528
[6] Flores P, Koshy C S, Lankarani H M et al 2011 Numerical and experimental investigation on multibody systems with revolute clearance joints Nonlinear Dynamics 65(4) 383-98
[7] Flores P 2010 A parametric study on the dynamic response of planar multibody systems with multiple clearance joints Nonlinear Dynamics 61(4) 633-53
[8] Wallin M, Aboubakr A K, Jayakumar P et al 2013 A comparative study of joint formulations: application to multibody system tracked vehicles Nonlinear Dynamics 74(3) 783-800
[9] Choi J H, Lee H C and Shabana A A 1998 Spatial Dynamics of Multibody Tracked Vehicles Part I: Spatial Equations of Motion Vehicle System Dynamics 29(1) 27-49.
[10] Lee H C, Choi J H and Shabana A A 1998 Spatial Dynamics of Multibody Tracked Vehicles Part II: Contact Forces and Simulation Results Vehicle System Dynamics 29(2) 113-7
[11] Beck R R and Wehage R A 1979 The Modeling and Simulation of Two Coupled M-113 Armored Personnel Carriers Proceedings of the Tenth Annual Pittsburgh Conference on Modeling and Simulation 10(2) 353-9
[12] Haug E J, Wehage R A, Barman N C 1981 Design Sensitivity Analysis of Planar Mechanism and Machine Dynamics Journal of Mechanical Design (103) 561–70
[13] Choi J H, Campanelli M, Shabana A A and Wehage R A 1998 Approximation Methods in the Nonlinear Analysis of Planar Tracked Vehicles Vehicle System Dynamics 29(3) 181-211
[14] Harrier E 1999 Solution of ordinary differential equations (Moscow: Mir) 685
[15] Haug E J, Negrut D and Iancu M 1997 A State-Space-Based Implicit Integration Algorithm for Differential-Algebraic Equations of Multibody Dynamics Mechanics Based Design of Structures and Machines 25(3) 311-34
[16] Negrut D, Haug E J and German H C 2003 An Implicit Runge-Kutta Method for Integration of Differential Algebraic Equations of Multibody Dynamics Multibody System Dynamics 9(2) 121-42
[17] Yudakov A A and Boykov V G 2013 Numerical methods of integrating motion equations of multicomponent mechanical systems, based on methods of direct integration of equations of the dynamics of the finite elements method Bulletin of the Udmurt University. Mathematics.Mechanics. Computer science (1):131-44