MATRIX FOURIER TRANSFORM WITH DISCONTINUOUS COEFFICIENTS

O. YAREMKO, E.ZHURAVLEVA

Abstract. The explicit construction of direct and inverse Fourier’s vector transform with discontinuous coefficients is presented. The technique of applying Fourier’s vector transform with discontinuous coefficients for solving problems of mathematical physics. Multidimensional integral transformations with non-separated variables for problems with discontinuous coefficients are constructed in this work. The coefficient discontinuities focused on the of parallel hyperplanes. In this work explicit formulas for the kernels in the case of ideal coupling conditions are obtained; the basic identity of the integral transform is proved; technique of integral transforms is developed.

Keywords: Fourier’s vector transform, integral transforms, discontinuous coefficients

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1. INTRODUCTION

Different representations of the solutions of the equilibrium equation through functions of tension are used when solving problems by the variable separation method. The required problem is taken to the solution of differential equations of a more simple structure with the help of such representations. Each functions of tension in these equations "is not fastened" with others, but it enters into boundary conditions together with the others. A.F.Ulitko [7] has offered rather effective method of research of problems of mathematical physics - a method Eigen vector-valued functions. This method is the vector analogue of the Fourier method.

The method of integral transformations is also an analytical method of the decision of solution of problems theory of elasticity. The method of integral transformations we consider and develop in this article. we come to the most simple problem in space of images with the help of the integral transformations (Fourier, Laplace, Hankel, etc.). The finding of the formula of direct is the main difficulty in solving of problems of this approach. Extensive enough bibliography of works on use of this method in problems of the theory of elasticity is resulted in J.S.Ufljand’s monography [2]. Method of the vector integral transforms of Fourier is equivalent the method Eigen vector-valued functions, however, unlike the last it can to be applied successfully be used, applied to the solution of problems of the theory of elasticity in a piece-wise homogeneous medium. The theory of integral transforms of Fourier with piece-wise constant coefficients in a scalar case was studied by Ufljand J.S. [16], [17], Najda L.S. [11], Protsenko V. S [12], [13], Lenjuk M. P [8],
The vector variant of a method adapted for the solution of problems in piece-wise homogeneous medium is developed by the author in [2], [19]. Unknown tension in the boundary conditions and in the internal conditions of conjugation don’t commit splitting in a considered dynamic problem, so the application of the scalar integral transforms of Fourier with piece-wise constant coefficients does not lead to success. Method of the vector integral transforms of Fourier with discontinuous coefficients is used for its solution in the present work. Conformable theoretical bases of a method are presented in item 4 for granted. The necessary proofs by the method of contour under the scheme developed in [2] and [19]. The closed form solution of the dynamic problem found in the use of this method in item 4. Integral transforms arise in a natural way through the principle of linear superposition in constructing integral representations of solutions of linear differential equations. First note that the structure of integral transforms with the relevant variables are determined by the type of differential equation and the kind of media in which the problem is considered. Therefore decision of integral transforms are the problem for mathematical physics piecewise-homogeneous (heterogeneous) media. It is clear this method is an effective for obtaining the exact solution of boundary-value problems for piecewise-homogeneous structures mathematical physics.

The author together with I.I.Bavrin has proposed integral transforms with non-separate variables for solving multidimensional problems in the work [?].

Let $V$ from $R^{n+1}$ be the half-space

$$ V = \{(y_1, ..., y_n, x) \in R^{n+1}: x > 0\}, $$

then solution of the Dirichlet’s problem for the half-space is expressed by Poisson formula takes the form: [?]

$$ u(x, y) = \Gamma \left( \frac{n+1}{2} \right) x^{\frac{n+1}{2}} \int_{y=0}^{x} \frac{x}{[\eta-y+2x^{\frac{n+1}{2}}]} \eta f(\eta) d\eta. $$

Obviously Poisson’s kernel is the form of integral Laplace transform and therefore expansion of the function $f(y)$ for the eigenfunctions of the Laplace operator $\Delta$ is obtained from the reproduce properties of the Poisson kernel:

$$ f(y) = \lim_{\tau \to 0} \int_{0}^{\infty} e^{-\lambda \tau} \left( \frac{1}{(\sqrt{2\pi})^n} \int_{R^n} \frac{J_{\frac{n+1}{2}} (\lambda |y-\eta|)}{|y-\eta|^{\frac{n+1}{2}}} f(\eta) d\eta \right) d\lambda, $$

where $J_\nu$ is Bessel’s function of order $\nu$ [?]. We may assume that integral transforms with non-separate variables are defined as follows [?] on the basis of this expansion: direct integral Fourier transform has the form

$$ F[f](y, \lambda) = \frac{1}{(\sqrt{2\pi})^n} \int_{R^n} \frac{J_{\frac{n+1}{2}} (\lambda |y-\eta|)}{|y-\eta|^{\frac{n+1}{2}}} f(\eta) d\eta \equiv \hat{f}(y, \lambda), $$

inverse Fourier integral transform has the form

$$ F^{-1}[\hat{f}](y) = \lim_{\tau \to 0} \int_{0}^{\infty} e^{-\lambda \tau} \hat{f}(y; \lambda) d\lambda \equiv f(y). $$

In our case the construction of multi-dimensional analogues for integral transforms (1)-(2) with discontinuous coefficients is the purpose of this research.
2. ONE-DIMENSIONAL INTEGRAL TRANSFORMS WITH DISCONTINUOUS COEFFICIENTS

In this paper integral transforms with discontinuous coefficients are constructed in accordance with author’s work \[10\]. Let \( \varphi (x, \lambda) \) and \( \varphi^* (x, \lambda) \) be eigenfunctions of primal and dual problems Sturm-Liouville for Fourier operator on sectionally homogeneous axis \( I_n \),

\[
I_n = \left\{ x : x \in \bigcup_{j=1}^{n+1} (l_{j-1}, l_j), \ l_0 = -\infty, \ l_{n+1} = \infty, \ l_j < l_{j+1}, \ j = 1, n \right\}.
\]

Let us remark that eigenfunction \( \varphi (x, \lambda) \),

\[
\varphi (x, \lambda) = \sum_{k=2}^{n} \theta (x - l_{k-1}) \theta (l_k - x) \varphi_k (x, \lambda) + \theta (l_1 - x) \varphi_1 (x, \lambda) + \theta (x - l_n) \varphi_{n+1} (x, \lambda)
\]

is the solution of separated differential equations system

\[
\left( a_m^2 \frac{d^2}{dx^2} + \lambda^2 \right) \varphi_m (x, \lambda) = 0, \ x \in (l_m, l_{m+1}); \ m = 1, ..., n + 1,
\]

by the coupling conditions

\[
\left[ \alpha_{m1} \frac{d}{dx} + \beta_{m1} \right] \varphi_k = \left[ \alpha_{m2} \frac{d}{dx} + \beta_{m2} \right] \varphi_{k+1}, \quad x = l_k, \ k = 1, ..., n; \ m = 1, 2,
\]

on the boundary conditions

\[
\varphi_1 |_{x=-\infty} = 0, \quad \varphi_{n+1} |_{x=\infty} = 0.
\]

Similarly, the eigenfunction \( \varphi^* (x, \lambda) \),

\[
\varphi^* (\xi, \lambda) = \sum_{k=2}^{n} \theta (\xi - l_{k-1}) \theta (l_k - \xi) \varphi^*_k (\xi, \lambda) + \theta (l_1 - \xi) \varphi^*_1 (\xi, \lambda) + \theta (\xi - l_n) \varphi^*_{n+1} (\xi, \lambda)
\]

is the solution of separate differential equations system

\[
\left( a_m^2 \frac{d^2}{dx^2} + \lambda^2 \right) \varphi^*_m (x, \lambda) = 0, \ x \in (l_m, l_{m+1}); \ m = 1, ..., n + 1,
\]

by the coupling conditions

\[
\frac{1}{\Delta_{1,k}} \left( \alpha_{m1} \frac{d}{dx} + \beta_{m1} \right) \varphi^*_k = \frac{1}{\Delta_{2,k}} \left( \alpha_{m2} \frac{d}{dx} + \beta_{m2} \right) \varphi^*_{k+1}, \ x = l_k,
\]

where

\[
\Delta_{i,k} = \det \left( \begin{array}{c c} \alpha_{k1}^i & \beta_{k1}^i \\ \alpha_{2i}^k & \beta_{2i}^k \end{array} \right) k = 1, ..., n; \ i, m = 1, 2,
\]

on the boundary conditions

\[
\varphi^*_1 |_{x=-\infty} = 0, \quad \varphi^*_{n+1} |_{x=\infty} = 0.
\]

Further normalization eigenfunctions is accepted by the following:

\[
\varphi_{n+1} (x, \lambda) = e^{ia_{n+1}x\lambda}, \quad \varphi^*_{n+1} (x, \lambda) = e^{-ia_{n+1}x\lambda}.
\]
Let direct $F_n$ and inverse $F_n^{-1}$ Fourier transforms on the Cartesian axis with $n$ division points be defined by the rules in [10]:

$$F_n [f] (\lambda) = \sum_{m=0}^{n+1} \int_{m}^{m+1} u_m^*(\xi, \lambda) f_m (\xi) d\xi \equiv \hat{f} (\lambda),$$

(3)

$$f_k (x) = \frac{1}{\pi i} \int_{0}^{\infty} u_k (x, \lambda) \hat{f} (\lambda) \lambda d\lambda.$$  

(4)

3. Vector Fourier transform with discontinuous coefficients

Let’s develop the method of vector Fourier transform for the solution this problem. Let’s consider Sturm–Liouville vector theory [1] about a design bounded on the boundary conditions.

$$(\alpha_k + \lambda^2 \delta_{j1}) \frac{d}{dx} + (\beta_{j1} + \lambda^2 \gamma_{j1}) y_k = (\alpha_k + \lambda^2 \delta_{j2}) \frac{d}{dx} + (\beta_{j2} + \lambda^2 \gamma_{j2}) y_{k+1},$$

(5)

and conditions of the contact in the points of conjugation of intervals

$$(\alpha_k \frac{d}{dx} + (\beta_{j1} + \lambda^2 \gamma_{j1}) y_k = (\alpha_k \frac{d}{dx} + (\beta_{j2} + \lambda^2 \gamma_{j2}) y_{k+1},$$

$x = l_k, \ k = \Gamma, n, \ j = 1, 2, \ldots,$ where

$$y_m (x, \lambda) = \begin{pmatrix} y_{1m} (x, \lambda) \\ \vdots \\ y_{rm} (x, \lambda) \end{pmatrix}, \ |y_m| = \sqrt{y_{1m}^2 + \ldots + y_{rm}^2}, \ m = \Gamma, n + 1.$$

Let for some $\lambda$ the considered the boundary problem has a non-trivial solution

$$y (x, \lambda) = \sum_{k=1}^{n} \theta (x - l_{k-1}) \theta (l_k - x) y_k (x, \lambda) + \theta (x - l_n) y_{n+1} (x, \lambda).$$

The number $\lambda$ is called an Eigen value in this case, and the corresponding decision $y (x, \lambda)$ is called Eigen vector-valued function.

$$\alpha_{11}^0, \beta_{11}^0, \gamma_{11}^0, \delta_{11}^0, \alpha_{j1}^k, \beta_{j1}^k, \gamma_{j1}^k, \delta_{j1}^k, \alpha_{j2}^k, \beta_{j2}^k, \gamma_{j2}^k, \delta_{j2}^k, A_j - (j = 1, 2; \ m = 1, n + 1; \ k = 1, n)$$

are matrixes of the size $r \times r$. We will required invertible

$$\det M_{mk} \neq 0, \ \lambda \in [0, \infty)$$

(8)

for matrixes

$$M_{mk} = \begin{pmatrix} \beta_{1m}^k + \lambda^2 \gamma_{1m}^k & \alpha_{1m}^k + \lambda^2 \delta_{1m}^k \\ \beta_{2m}^k + \lambda^2 \gamma_{2m}^k & \alpha_{2m}^k + \lambda^2 \delta_{2m}^k \end{pmatrix}, \ m = 1, 2; \ k = \Gamma, n.$$  

Matrices $A_n^2$ and $\Gamma_n^2$, are is $m = \Gamma, n + 1$ -positive-defined [6]. We denote

$$\Phi_{n+1} (x) = e^{q_{n+1} x}; \ \Psi_{n+1} (x) = e^{-q_{n+1} x}; \ q_{n+1}^2 = A_{n+1}^2 (\lambda^2 E + \Gamma^2).$$

(9)
Let us introduce the following notation
\[(\Phi_k, \Psi_k), \quad k = 1, n:
\]
\[
\left[ (\alpha^k_{j1} + \lambda^2 \delta^{k}_{j1}) \frac{d}{dx} + (\beta^k_{j1} + \lambda^2 \gamma^{k}_{j1}) \right] (\Phi_k, \Psi_k) =
\]
(9) \[
\left[ (\alpha^k_{j2} + \lambda^2 \delta^{k}_{j2}) \frac{d}{dx} + (\beta^k_{j2} + \lambda^2 \gamma^{k}_{j2}) \right] (\Phi_{k+1}, \Psi_{k+1}), \quad k = 1, n, \quad j = 1, 2.
\]

Let us introduce the following notation
\[
\begin{align*}
0 \Phi (x, \lambda) & = \left[ (\alpha^0_{11} + \lambda^2 \delta^{0}_{11}) \frac{d}{dx} + (\beta^0_{11} + \lambda^2 \gamma^{0}_{11}) \right] \Phi_1 (x, \lambda) \bigg|_{x = l_0}, \\
0 \Psi (x, \lambda) & = \left[ (\alpha^0_{11} + \lambda^2 \delta^{0}_{11}) \frac{d}{dx} + (\beta^0_{11} + \lambda^2 \gamma^{0}_{11}) \right] \Psi_1 (x, \lambda) \bigg|_{x = l_0}, \\
\Omega_k & = \begin{pmatrix} \Phi_k \\ \Phi_k^t \\ \Psi_k \\ \Psi_k^t \end{pmatrix}, \quad i = 1, n, + 1.
\end{align*}
\]

**Theorem 1.** The spectrum of the problem (9), (10), (11) is a continuous and fills all semi axis \((0, \infty)\). Sturm–Liouville theory time is degenerate. To each Eigen value \(\lambda\) corresponds to exactly \(r\) linearly independent vector-valued functions. As the last it is possible to take \(r\) columns matrix-importance functions.

\[
u (x, \lambda) = \sum_{k=1}^{n} \theta (x - l_{k-1}) \theta (l_k - x) u_k (x, \lambda) + \theta (x - l_n) u_{n+1} (x, \lambda),
\]

(10) \[
u_j (x, \lambda) = \Phi_j (x, \lambda) \Phi^{-1}_1 (\lambda) - \Psi_j (x, \lambda) \Psi^{-1}_1 (\lambda).
\]

That is
\[
y^m (x, \lambda) = \begin{pmatrix} u_{1m} (x, \lambda) \\ \vdots \\ u_{rm} (x, \lambda) \end{pmatrix}.
\]

Dual Sturm–Liouville theory consists in a finding of the non-trivial solution of separate simultaneous ordinary differential equations with constant matrix coefficients.

(11) \[
\left( A^2 m_{dx^2} + \lambda^2 E + \Gamma^2_n \right) y_m = 0, \quad q_m^2 = \lambda^2 E + \Gamma^2_m, \quad m = 1, n + 1
\]
on the boundary conditions

(12) \[
\left. \left( \frac{d}{dx} y^*_1 (\beta^0_{11} + \lambda^2 \gamma^{0}_{11})^{-1} + y^*_1 (\alpha^0_{11} + \lambda^2 \delta^{0}_{11})^{-1} \right) \right|_{x = l_0} = 0, \quad ||y^*_{n+1}|| < \infty,
\]
and conditions of the contact in the points of conjugation of intervals

(13) \[
\left( - \frac{d}{dx} y_k^*, y_k^* \right) \left( \beta^k_{11} + \lambda^2 \gamma^{k}_{11} \alpha^k_{11} + \lambda^2 \delta^{k}_{11} \right) =
\]
\[
= \left( - \frac{d}{dx} y_{k+1}^*, y_{k+1}^* \right) \left( \beta^k_{22} + \lambda^2 \gamma^{k}_{22} \alpha^k_{22} + \lambda^2 \delta^{k}_{22} \right)^{-1}, \quad x = l_k, \quad k = 1, n.
The solution of the boundary value problem we write in the form of
\[
y^* (\xi, \lambda) = \sum_{k=2}^{n} \theta (\xi - l_{k-1}) \theta (l_{k} - \xi) y^*_k (\xi, \lambda) + \theta (l_{1} - \xi) y^*_1 (\xi, \lambda) + \theta (\xi - l_{n}) y^*_{n+1} (\xi, \lambda),
\]
\[
y^*_m (\xi, \lambda) = \left( y^*_{m1} (\xi, \lambda) \cdots y^*_{mr} (\xi, \lambda) \right),
\]
\[
\|y^*_m\| = \sqrt{(y^*_{m1})^2 + \cdots + (y^*_{mr})^2}, m = 1, n + 1.
\]

**Theorem 2.** The spectrum of the problem \([3, 4, 7]\) is a continuous and fills semi axis \((0, \infty)\). Sturm–Liouville theory \(r\) time is degenerate. To each Eigen value \(\lambda\) corresponds to exactly \(r\) linearly independent vector-valued functions. As the last it is possible to take \(n\) rows matrix-importance functions.

\[
u^* (x, \lambda) = \sum_{k=1}^{n} \theta (x - l_{k-1}) \theta (l_{k} - x) u^*_k (x, \lambda) + \theta (x - l_{n}) u^*_{n+1} (x, \lambda),
\]
\[
u^*_j (x, \beta) = \begin{pmatrix} 0 & \xi (\beta) \\ \Phi (\beta) & 0 \end{pmatrix} \Omega_j^{-1} (x, \beta) \begin{pmatrix} 0 \\ E \end{pmatrix} A_{j}^{-2},
\]

That is
\[
y^* j (\xi, \lambda) = \left( u^*_0 (\xi, \lambda) \cdots u^*_r (\xi, \lambda) \right), j = 1, r.
\]

The existence of spectral functions \(u (x, \lambda)\) and the conjugate spectral function \(u^* (x, \lambda)\) allows to write a vector decomposition theorem on the set of \(I^*_n\).

**Theorem 3.** Let the vector-valued function \(f (x)\) is defined on \(I^*_n\) continuous, absolutely integrated and has the bounded total variation. Then for any \(x \in I^*_n\) true formula of decomposition
\[
f (x) = -\frac{1}{\pi j} \int \infty \theta (x, \lambda) (\int \infty \theta (\xi, \lambda) f (\xi) d \xi + \left( \gamma f_1 (l_0) + \delta f_1 (l_0) \right) + \sum_{k=1}^{n} \left( \phi (\lambda), \psi (\lambda) \right) \Omega k^{-1} (l_k, \lambda) M^{-1} k_1 (\lambda). \)
\]

\[
\left( \gamma_0 f_1 (l_0) + \delta_0 f_1 (l_0) \right) + \sum_{k=1}^{n} \left( \phi (\lambda), \psi (\lambda) \right) \Omega k^{-1} (l_k, \lambda) M^{-1} k_1 (\lambda).
\]

The decomposition theorem allows to enter the direct and inverse matrix integral Fourier transform on the real semi axis with conjugation points:
\[
F_{u+} [f] (\lambda) = \int \infty \theta (\xi, \lambda) f (\xi) d \xi + \left( \gamma f_1 (l_0) + \delta f_1 (l_0) \right) + \sum_{k=1}^{n} \left( \phi (\lambda), \psi (\lambda) \right) \Omega k^{-1} (l_k, \lambda) M^{-1} k_1 (\lambda).
\]

\[
\left( \gamma_0 f_1 (l_0) + \delta_0 f_1 (l_0) \right) + \sum_{k=1}^{n} \left( \phi (\lambda), \psi (\lambda) \right) \Omega k^{-1} (l_k, \lambda) M^{-1} k_1 (\lambda).
\]
\begin{align}
F_{n+1}^{-1}[\tilde{f}](x) &= -\frac{1}{\pi i} \int_0^\infty \lambda u(x, \lambda) \tilde{f}(\lambda) \, d\lambda = f(x),
\end{align}

when
\[
f(x) = \sum_{k=1}^n \theta(l_k - x) \theta(x - l_{k-1}) f_k(x) + \theta(x - l_n) f_{n+1}(x).
\]

Let’s result the basic identity of integral transform of the differential operator
\[
B = \sum_{j=1}^n \theta(x - l_{j-1}) \theta(l_j - x) \left( A_j^2 \frac{d^2}{dx^2} + \Gamma_j^2 \right) + \theta(x - l_n) \left( A_{n+1}^2 \frac{d^2}{dx^2} + \Gamma_{n+1}^2 \right).
\]

**Theorem 4.** If vector-valued function
\[
f(x) = \sum_{k=1}^n \theta(x - l_{k-1}) \theta(l_k - x) f_k(x) + \theta(x - l_n) f_{n+1}(x),
\]
is continuously differentiated on set three times, has the limit values together with its derivatives up to the third order inclusive
\[
f^{(m)}_k(l_{k-1}) = f^{(m)}_k(l_{k-1} + 0), \quad m = 0, 1, 2, 3; \quad k = 1, n + 1
\]
Satisfies to the boundary condition on infinity
\[
\lim_{x \to \infty} \left( u^*(x, \lambda) \frac{d}{dx} f(x) - \frac{d}{dx} u^*(x, \lambda) \, f(x) \right) = 0
\]
Satisfies to homogeneous conditions of conjugation [7], that basic identity of integral transform of the differential operator \( B \) hold
\[
F_{n+1}[B(f)](\lambda) = -\lambda^2 \tilde{f}(\lambda) - \left\{ \left( \beta_{11}^0 f_1(l_0) + \alpha_{11}^0 f_1'(l_0) \right) - \left( \gamma_{11}^0 A_{11}^2 f_{1//}(l_0) + \delta_{11}^0 A_{11}^2 f_{1///}(l_0) \right) \right\}.
\]

The proof of theorems 1, 2, 3, 4 is spent by a method of the method of contour integration. Similarly presented to work of the author [19].

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