On the way from 0- to $P_i$-junction

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Abstract. The superconductor/ferromagnet/superconductor (S/F/S) junctions and s-wave/d-wave junctions provide the possibility to realize the $\pi$-state where the ground state phase difference between the adjacent superconducting layers is equal to $\pi$. Such $\pi$-junction incorporated in a superconducting circuit may generate a spontaneous current. We investigate theoretically how the transition from the usual 0-state to the $\pi$-state occurs. We demonstrate that under certain conditions it is possible to obtain a $\varphi_0$-junction with an arbitrary phase difference $\varphi_0$ ($0 < \varphi_0 < \pi$) at the ground state. We discuss the different mechanisms of the $\varphi_0$-junction realization and its properties.

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In the usual Josephson junctions (JJ) at equilibrium the phase difference of the superconducting order parameter on the two banks is zero. However, in the Josephson junctions with ferromagnetic interlayer (S-F-S junctions), the ground state may correspond to the phase difference equal to $\pi$ ("$\pi$-junctions") [1]. This phenomenon is related with the damping oscillatory behavior of the Cooper pair wave function in a ferromagnet [2]. Another realization of the $\pi$-junctions is possible at the contact of s-wave superconductor with a high-temperature superconductors where the order parameter have the d-wave symmetry [3]. Bulaevskii et al. [4], pointed out that the $\pi$-junction incorporated into the superconducting circuit would generate a spontaneous current.

Roughly the the transition from 0-state to the $\pi$-state may be considered as the change of the sign of the critical current in a current-phase relation

$$ j(\varphi) = j_{c1} \sin(\varphi). $$

(1)

Usually, the single harmonic current-phase relation provides a good basis for the description of the JJ. However at 0- $\pi$ transition $j_{c1}$ vanishes and the role of the second harmonic becomes predominant. The current-phase relation

$$ j(\varphi) = j_{c1} \sin \varphi + j_{c2} \sin 2\varphi $$

(2)

corresponds to the following phase dependent contribution to energy of the JJ

$$ E_J(\varphi) = \frac{\Phi_0}{2\pi c} \left[ -j_{c1} \cos \varphi - \frac{j_{c2}}{2} \cos 2\varphi \right]. $$

(3)

If we neglect the second harmonic term, then 0 state occurs for $j_{c1} > 0$. Near 0 – $\pi$ transition $j_{c1} \to 0$ and the critical current at the transition $j_c = |j_{c2}|$ and if $j_{c2} > 0$, the minimum energy
always occurs at $\varphi = 0$ or $\varphi = \pi$. In the opposite case ($j_c^2 < 0$) the transition from 0 to $\pi$ state is continuous and there is region where the equilibrium phase difference takes any value $0 < \varphi_0 < \pi$ [1]. The negative sign of the second harmonic may be for example realized in the S-F-S junctions with varying F layer thickness [5] or in the 45° grain-boundary JJs made of d-wave superconductors [7]. The properties of such a $\varphi_0 - JJ$ are rather peculiar [5], [6], [8], for example the magnetic field penetrates in the form of the two different type of the Josephson vortices.

The current-phase relation (2) is antisymmetric $j(-\varphi) = -j(\varphi)$ which is the consequence of the preserved time reversal symmetry. However a rather special situation is possible when the weak link in JJ is a non-centrosymmetric magnetic metal with broken inversion symmetry: such a junction provides the realization the unusual current-phase relations

$$j(\varphi) = j_c \sin(\varphi + \varphi_0), \quad (4)$$

which implies that the junction energy $E_J \sim -j_c \cos(\varphi + \varphi_0)$ and the minimum energy corresponds to the non-zero phase difference $\varphi = -\varphi_0$. The phase shift $\varphi_0$ is proportional to the magnetic moment, and therefore these $\varphi_0 -$ junctions serve example of systems with direct coupling between magnetic moment (internal exchange field) and superconducting current. This opens an interesting field of application of $\varphi_0 -$ junctions in superconducting spintronics. Varying the N layer thickness we may easily control the phase shift $\varphi_0$.

Note that recently superconductors with broken inversion symmetry attracted a lot of attention. Namely the heavy fermion superconductor CePt$_3$Si provides a famous example of the superconductivity and antiferromagnetism coexistence in the noncentrosymmetric compound [9]. Now the number of superconductors without inversion symmetry approaches one dozen and during the last years their properties were under intense studies from both theoretical and experimental points of view, see for example [10] and references cited therein. In the presence of the magnetic field the lack of inversion symmetry leads to the spatially modulated helical superconducting phase [11],[12]. On the microscopical level the special character of the electron spectrum in metal with broken inversion symmetry may be described by the Rashba-type spin-orbit coupling [13]: $\alpha (\sigma \times \mathbf{p}) \mathbf{n}$, where $\mathbf{n}$ is the unit vector along the asymmetric potential gradient. To illustrate the unusual properties of the Josephson junction with broken inversion symmetry we use a simple Ginzburg-Landau approach. The Rashba-type interaction in the presence of the field $\mathbf{h}$ acting on the electron spin leads to the following Ginzburg-Landau free energy density [11],[12],

$$F = a |\psi|^2 + \gamma |\mathbf{D} \psi|^2 + \frac{b}{2} |\psi|^4 - \varepsilon \mathbf{n} \cdot \left[ \hbar \times \left( \psi \left( \mathbf{D} \psi \right)^* + \psi^* \left( \mathbf{D} \psi \right) \right) \right], \quad (5)$$

where $\psi$ is the superconducting order parameter, $D_i = -i \partial_i - 2e A_i$ and the coefficient $a$ becomes zero at some temperature $T_{c0} : a \sim (T - T_{c0})$. The special character of the superconductivity with broken inversion symmetry is described by the last term in (5) with the coefficient $\varepsilon \sim \alpha$. Further on to concentrate on the special properties of this junction we neglect the orbital effect. This is justified for example for the geometry presented in Fig. 1 when the magnetization is along $z-$axis (due to the demagnetization effect) or it is lying in plane $x - y$ (in such a case the $\mathbf{n}$ vector must be along $z-$axis).

In the considered case of the weak link of the length $2L$, see Fig. 1, the order parameter depends only on the coordinate $x$ and the corresponding Ginzburg-Landau equation is

$$a\psi - \gamma \frac{\partial^2 \psi}{\partial x^2} + 2i \varepsilon h \frac{\partial \psi}{\partial x} = 0, \quad (6)$$
where we have neglected the non-linear term assuming the superconducting banks being at temperature slightly below their critical one.

For illustration we assume the continuity conditions for $\psi$ at $x = \pm L$: $\psi(\pm L) = |\Delta| \exp(\mp \frac{i \varphi}{2})$, with $|\Delta|$ being the modulus of the order parameter in the banks and $\varphi$ is the superconducting phase difference across the junction. Performing the calculations of the current we readily find in the limit of the large $L$

$$j = 4e\gamma |\Delta|^2 \sqrt{\frac{a}{\gamma} - \tilde{\varepsilon}^2} \sin(\varphi + 2\tilde{\varepsilon}L) \exp(-2\sqrt{\frac{a}{\gamma} - \tilde{\varepsilon}^2}L).$$

This current-phase relation is exactly of the type (4). The analysis on the basis of Eilenberger and Usadel equations also confirm the conclusion of the $\varphi_0-$ junction formation in the case when the weak link is a magnetic metal with broken inversion symmetry.

The S/F/S Josephson junction may have zero or $\pi$ phase difference in the ground state depending on the length of the weak link. In contrast in the $\varphi_0-$ junction the ground state is always different from zero and $\pi$ states (except the occasional events $\varphi_0 = \pi n$). In the superconducting ring with $\pi-$ junction the spontaneous current appears [4] if the parameter $k = \frac{c\Phi_0}{2\pi^2 j}\lesssim 1$, here $L$ is the inductance of the system. For the $\varphi_0-$ junction the system energy:

$$E(\varphi) = \frac{jc}{2e} \left( -\cos(\varphi + \varphi_0) + \frac{k\varphi^2}{2} \right),$$

Therefore the minimum energy is achieved for the phase difference satisfying the equation

$$\sin(\varphi + \varphi_0) + k\varphi = 0,$$

which always have a non zero solution and then the $\varphi_0-$ junction will always generate the spontaneous current with the flux $\Phi = -\Phi_0 \left( \frac{\varphi_0}{\pi} \right) (1 - k)$ in $k \ll 1$ limit. The SQUID with one normal and another $\varphi_0-$ junction would reveal the shift of the diffraction pattern by $\varphi_0$. The $\varphi_0-$ Josephson junctions may serves as a natural phase shifter in the superconducting electronics circuits.

The discussed junction with broken inversion symmetry provides a direct mechanism of the coupling between supercurrent and magnetic moment - indeed the phase shift $\varphi_0$ is proportional...
to z component of the spin field. This means that the precessing magnetization will be directly coupled with the current which opens new interesting perspectives to study the coupled magnetic and current dynamics in Josephson junctions. Applying the voltage to the $\phi_0$ junction we obtain the Josephson generation and the magnetic moment of the weak link will experience the effective field varying with Josephson frequency. If this frequency is close to the ferromagnetic resonance frequency, it may be an efficient way to generate the spin precessing. Inversely the spin precessing in the weak link would generate superconducting current in the circuit with $\phi_0 -$ junction.

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