Chain inflation and the imprint of fundamental physics in the CMBR

Diego Chialva$^1$ and Ulf H. Danielsson$^2$

Institutionen för fysik och astronomi
Uppsala Universitet, Box 803, SE-751 08 Uppsala, Sweden

$^1$diego.chialva@fysast.uu.se  $^2$ulf.danielsson@fysast.uu.se

Abstract

In this work we investigate characteristic modifications of the spectrum of cosmological perturbations and the spectral index due to chain inflation. We find two types of effects. First, modifications of the spectral index depending on interactions between radiation and the vacuum, and on features of the effective vacuum potential of the underlying fundamental theory. Second, a modulation of the spectrum signaling new physics due to bubble nucleation. This effect is similar to those of transplanckian physics. Measurements of such signatures could provide a wealth of information on the fundamental physics at the basis of inflation.

September 2008
The current most popular models for inflation are based on chaotic inflation. In these models a scalar field rolls slowly subject to Hubble friction in a shallow potential. In [1] we proposed an alternative scenario that shares many of the features of slow roll chaotic inflation, but also differs in several important aspects. Our model is based on chain inflation and makes use of a series of several first order phase transitions. More precisely, we imagine a potential with a large number of small barriers that separate local, metastable minima. The barriers prevent the field from rolling, and without quantum mechanical tunneling the inflaton is stuck in a local minimum. By appropriately choosing the heights and widths of the barriers, one can obtain tunneling probabilities such that the field effectively rolls slowly down the potential through repeated tunneling events. In this way we can achieve a slow roll in the sense of having a slow change in $H^2 \sim \rho V$ ($\rho V$ being the vacuum energy density), even if the potential for the fields is steep. The details of this process were worked out in [1], and it was also shown that suitable potentials might be find in flux compactified string theory.

The main features of the model introduced in [1] is as follows. We assume that the bubbles, after their formation through tunneling, rapidly percolate and collide. The energy difference between two subsequent minima is temporarily stored in the bubble walls, and we assume that this energy is rapidly converted into radiation as the bubbles collide. In this way we obtain a coarse grained picture where the main effect of the barriers and the tunneling is to introduce a source term for radiation in the Friedmann equations.

For earlier attempts on models featuring inflation through chain of first order decays, see also [2, 3, 4, 5].
A scalar field can be understood as a fluid consisting of two components: a component corresponding to the kinetic energy, \( T \), and a component corresponding to the potential energy, \( V \). In the case of slow roll we have \( T \sim \varepsilon V \ll V \), where \( \varepsilon \) is the slow roll parameter, and as a consequence the dynamics is dominated by the potential energy leading to accelerated expansion and inflation. In our version of chain inflation the kinetic component is further suppressed relative to the potential energy. On the other hand, radiation is produced through the tunneling leading to \( \rho_{\text{rad}} \sim \varepsilon V \). As a result we effectively have, to first order in \( \varepsilon \), a model consisting of a decaying cosmological constant and a coupled component of radiation. For the case of chaotic inflation, it is important to understand that it is the sub-dominant kinetic energy that determines the spectrum of the fluctuations. The kinetic energy corresponds to a hydrodynamical fluid with an effective speed of sound that is equal to the speed of light. In contrast, the potential energy does not correspond to a hydrodynamical fluid and lacks a well defined speed of sound.

The amplitude of the primordial fluctuations are set by the speed of sound. The general result is

\[
P \sim \frac{H^2}{c_s \varepsilon},
\]

where \( c_s \) is the speed of sound of the hydrodynamical component. For chaotic inflation \( c_s = 1 \). In our model for chain inflation, the role of the kinetic energy is taken over by the radiation where \( c_s = \frac{1}{\sqrt{3}} \). As a result, the primordial spectrum is corrected to

\[
P \sim \frac{\sqrt{3}H^2}{\varepsilon}.
\]

The result differs from chaotic inflation through a simple factor of \( \sqrt{3} \). While this simple argument captures the main physics of the model, there are many important points of the derivation that are carefully discussed in [1].

In the present paper we discuss the possibility of further effects on the primordial spectrum from various sources. In the first part of the paper we will discuss the modifications to the spectrum of cosmological perturbations due to the presence of the non-negligible interaction between radiation and vacuum energy. We will discuss how they arise and appear to be specific to our model of chain inflation. In the second part of the paper we will instead consider how bubble nucleation affect the spectrum of perturbations, and in particular we will study the imprint of the size of the bubbles on the CMB.

## 2 Effects due to interactions

In [1] we derived a system of equations that determine the evolution of the comoving curvature perturbations during a period of chain inflation. The approach was based

\[2\text{We will leave aside the interesting possibility of having sizable contributions from isocurvature perturbations and/or non-gaussianities.}\]
on the traditional analysis of scalar perturbations in field (slow-roll/chaotic) models, as presented in [6]. We start with a brief review of the approach used in [1], and show that the method of [6] is not the most convenient one in the case of chain inflation. We will therefore propose another way of analyzing and re-writing the system of equations that is better suited for our model.

We start from equation (97) in [1],

\[
\begin{aligned}
\dot{\xi} &= \frac{a\left(\rho + p\right)}{H^2} R \\
\dot{R} &= \frac{1}{3} a^2 \left(\rho + p\right) \left(-k^2 - a^2 4\pi G \frac{Q_V}{3H}\right) \xi - \frac{4}{3} HS_{V,r} 
\end{aligned}
\]  

(3)

where

- \(a\) is the cosmological scale factor, \(H\) is the Hubble rate, \(\phi\) is the gravitational potential, and \(G\) is Newton’s constant
- \(Q_V\) is the energy-momentum transfer vector
- \(a\phi = 4\pi GH \xi\),
- \(R\) is the comoving curvature perturbation
- \(\rho, p\) are the total energy and pressure density
- \(\varepsilon\) is the slow-roll parameter
- \(k\) is the comoving wavenumber for the perturbation
- \(S_{V,r}\) is the relative entropy perturbation between vacuum \((V)\) and radiation \((r)\).

In the following we will neglect the term proportional to \(S_{V,r}\). As a result our conclusions apply only to models with negligible contributions from isoentropic perturbations, or, alternatively, just to the adiabatic component of the whole spectrum of perturbations.

Comparing equations (3) with the analogous equations in [6] (in flat space), we see the importance of interactions in our multicomponent system, as represented by the term \(-a^2 4\pi G \frac{Q_V}{3H} \xi\). Note also that this can be conveniently re-written as

\[\dot{\rho}^V = Q_V\]  

(4)

\[\dot{\rho}^r = -4H\rho^r - Q_V\]  

(5)

where \(\rho^{V/r}\) is the energy density for vacuum/radiation.
2.1 The equation of motion

We will now expand \( w'' \) in slow-roll parameters such as \( \varepsilon \) and the decay rates in such a way that we make sure to include contributions from derivatives of the slow-roll
parameters up to order $O(\varepsilon)$. We will study the general case of a vacuum energy $\epsilon_n$ in the form of a power law,
\[ \epsilon_n \sim \frac{m^4}{c!} n^c, \]
(12)
where the integer $n$ labels the vacuum, and $m_f$ is an energy scale (depending on couplings, extra-dimensions and similar features of the microscopical theoretical model). We will need the following formula (see appendix for derivation)
\[ \dot{\epsilon} \sim \left(1 + \frac{2}{c}\right) H \varepsilon^2 - D\tilde{\Gamma}, \]
(13)
where $D\tilde{\Gamma}$ depends on the decay rates per unit time $\tilde{\Gamma}$ and is defined in the appendix. Note that we do not restrict to equal rates at every step. We find
\[ \frac{w''}{w} \sim \mathcal{H}^2 \varepsilon + \frac{1}{2} \mathcal{H}^2 \left(1 + \frac{2}{c}\right) \varepsilon - D\tilde{\Gamma}, \]
(14)
having neglected terms of order $\varepsilon^2, \varepsilon D\tilde{\Gamma}, D\tilde{\Gamma}^2$ and higher. Our equation of motion now becomes
\[ u'' + \left(\frac{1}{3} k^2 - \frac{1}{3} \mathcal{H}^2 \varepsilon - \mathcal{H}^2 \varepsilon - \frac{1}{2} \left(1 + \frac{2}{c}\right) \mathcal{H}^2 \varepsilon - a\mathcal{H} D\tilde{\Gamma}\right) u = 0. \]
(15)
In a quasi-deSitter space, where
\[ a \sim -\frac{1}{H \eta(1-\varepsilon)} \quad (\eta < 0), \]
(16)
we have
\[ \frac{1}{\eta^2} \left(\nu^2 - \frac{1}{4}\right) = \mathcal{H}^2 \left(\frac{\varepsilon}{3} + \varepsilon + \frac{1}{2} \left(1 + \frac{2}{c}\right) \varepsilon - \frac{D\tilde{\Gamma}}{H}\right), \]
(17)
which allow us to read off
\[ \nu^2 \sim \frac{1}{4} + \frac{\varepsilon}{3} + \varepsilon + \frac{1}{2} \left(1 + \frac{2}{c}\right) \varepsilon - \frac{D\tilde{\Gamma}}{H}. \]
(18)
The general solution for the equation
\[ u'' + \left(\frac{1}{3} k^2 - \frac{1}{\eta^2} \left(\nu^2 - \frac{1}{4}\right)\right) u = 0 \]
(19)
reads
\[ u = \sqrt{-\eta} \left[c_1(k) H^{(1)}(\nu) (-k\eta) + c_2(k) H^{(2)}(\nu) (-k\eta)\right], \]
(20)
where $H^{(1)}_{\nu}(x)$ and $H^{(2)}_{\nu}(x)$ are Hankel functions of the first and second kind, respectively. In the limit $-k\eta \gg 1$ we have that

$$H^{(1)}_{\nu} \sim \sqrt{\frac{2}{-\pi k\eta}} e^{i(-k\eta - \frac{\pi}{4})} \quad H^{(2)}_{\nu} \sim \sqrt{\frac{2}{-\pi k\eta}} e^{-i(-k\eta - \frac{\pi}{4})}. \quad (21)$$

Following standard procedure we match the solution with the Bunch-Davies vacuum, finding

$$u = \frac{1}{2} \sqrt{\frac{\pi}{c_s}} e^{i(\nu + \frac{1}{2}) \frac{\pi}{2}} \sqrt{-\eta} H^{(1)}_{\nu}(-k\eta). \quad (22)$$

For superhorizon scales $(-k\eta \ll 1)$

$$H^{(1)}_{\nu} \sim \sqrt{\frac{2}{\pi}} e^{-i\frac{\pi}{4}} 2^{-\nu - \frac{3}{4}} \frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)} (-k\eta)^{-\nu} \quad (23)$$

so that, finally,

$$u \sim e^{i\frac{\pi}{2}(\nu + \frac{1}{2})} 2^{-\nu - \frac{3}{4}} \frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)} (-k\eta)^{-\frac{1}{2}}. \quad (24)$$

This is what we need in order to obtain the spectrum of perturbations.

### 2.2 Spectrum of perturbations

The spectrum of perturbations is conveniently expressed through the comoving curvature perturbation, which is constant on superhorizon scales during inflation. We can obtain it using the first equation in (3), which we repeat here for convenience

$$\mathcal{R} = \frac{H^2}{a(\rho + p)} \dot{\xi}. \quad (25)$$

As a result we obtain

$$\mathcal{R} = \frac{H}{\sqrt{2} M_{\text{Plank}} \varepsilon} \frac{1}{\sqrt{2k^3c_s}} \left(\frac{k}{aH}\right)^{\frac{3}{2}} (1 + O(\varepsilon)), \quad (26)$$

and the spectrum becomes

$$P_k^R = \frac{H^2}{8\pi^2 M_{\text{Plank}}^2 \sqrt{\frac{1}{3}} \varepsilon} \left(\frac{k}{aH}\right)^{1-2\nu} (1 + O(\varepsilon)). \quad (27)$$

From this we read off the spectral index

$$n_s - 1 = 1 - 2\nu \sim -\frac{2}{3} \varepsilon - 2\varepsilon - \left(1 + \frac{2}{c}\right) \varepsilon + \frac{D\bar{\Gamma}}{H}. \quad (28)$$

---

7In the following section we will discuss this choice thorough, investigating the possibility of new fundamental physics at a scale larger than the Plank one.
which alternatively can be written as
\[ n_s - 1 \sim \frac{c_s^2}{3H^3M_{\text{Plank}}^2}Q_V - 2\varepsilon - \left(1 + \frac{2}{c}\right)\varepsilon + \frac{D\tilde{\Gamma}}{H}, \] (29)

which is our final result. It is evident from this formula that the corrections to the spectral index due to the interactions between vacuum and radiation imply an extra tilt to the spectrum (blue or red depending on the value of \(D\tilde{\Gamma}\)). (Note that the result for \(D\tilde{\Gamma} = Q_V = 0\) is precisely the same as the usual chaotic/slow-roll result in the case of \(c = 2\) as observed in [1].) It appears to us that this feature of the spectrum is strongly characteristic of a first order transition, since in deriving (3) in [1], we made use of specific aspects of first order transitions (such as the fact that the momentum perturbation for the vacuum was zero).

3 Effects of a new intermediate scale

3.1 The choice of vacuum

In deriving our results for the spectrum of cosmological perturbations and the spectral index in the previous section, we have followed the standard procedure of matching our solution to the Bunch-Davies vacuum. Essentially this means that we resolve the issue of the non-uniqueness of the vacuum in a cosmological space time by tracking the modes to infinitely short scales, where the effect of cosmological scales such as the horizon can be ignored. At such scales there is a unique vacuum just as in Minkowsky space. This is the Bunch-Davies vacuum.

As is well known there is a potential problem with this procedure since one can not reliably track the modes to scales shorter than the Planck (or string) scale without taking into account effects of string theory and quantum gravity.\(^8\) Hence there are likely corrections to the choice of vacuum of order \(\frac{H}{\Lambda}\), where \(\Lambda\) is the scale of new physics. This is known as the transplanckian problem. Actually, it represents more of an opportunity than a problem since it could be an observational window to new physics.

In our case we have yet another scale that enters. We have assumed a coarse graining over the nucleating bubbles that is valid only for scales substantially larger than the size of the bubbles, \(r_b\). Hence, we have full control over the evolution of the perturbations only while their wavelength is larger than the size of the bubbles. If we follow the evolution of a specific mode backwards in time it will eventually reach a scale as short as the size of the bubbles, and our picture breaks down.

What is the effective quantum state that should be used as an initial condition at this point? It is in general very difficult to give a precise answer to this question both because of the usual difficulties due to quantization in curved spaces, and also

\(^8\)See for example [8, 9, 10, 11, 12]
because of the great generality of our model of chain inflation (no field theory model is specified). Without a detailed model, we have no other option than to impose an effective initial condition for the perturbations, and to postulate their creation out of the vacuum. This is formally very similar to the case of the transplanckian problem. In several works ([8, 9, 10, 11, 12]) it has been argued that initial conditions must be imposed at the Planck scale (or string scale) due to our ignorance of physics at higher energies. In our model the scale for new physics will instead be the size of the bubbles, but the analysis, that we now review, will be more or less the same.

We begin by noting that the physical momentum $p$ and the comoving momentum $k$ are related through

$$k = ap = -\frac{p}{\eta H},$$

(30)

where $\eta$ is conformal time, $p$ is the physical momentum, $k$ is the comoving momentum and $a$ is the scale factor. We impose the initial conditions when $p = \Lambda$, where $\Lambda$ is the energy scale important for the new physics given by $\Lambda \sim 1/r_b$. In our case $r_b$ is just the size of the bubbles. We find that the conformal time when the initial condition is imposed to be

$$\eta_0 = -\frac{\Lambda}{H k}.$$  

(31)

As we see, different modes will be created at different times, with a smaller linear size of the mode (larger $k$) implying a later time.

In our case we would in principle be able to calculate the form of the perturbations at the the scale $r_b$, by tracing the evolution backwards in time, through the nucleating bubbles, to even smaller scales. Presumably the result would depend on the fine details of the physics of bubble nucleation, which is beyond the scope of the present paper.

Instead we will take the same attitude as in [10] and encode the unknown new physics into the choice of the vacuum. The claim of [10] is that the primordial spectrum is corrected through a modulating factor. These results can be directly taken over to our case with the result that

$$P(k) \sim \frac{H^2}{\varepsilon c_s} \left( 1 - \frac{H}{\Lambda} \sin \left( \frac{2\Lambda}{H} \right) \right),$$

(32)

where $c_s = \frac{1}{\sqrt{3}}$ is the speed of sound. In the transplanckian case, $\Lambda$ is typically constant and equal to the Planck scale or string scale. The modulation of the spectrum comes from the rolling inflaton that leads to a changing $H$. In our case, $\Lambda$ will also be changing, but the amplitude $\frac{H}{\Lambda}$ is nevertheless expected to be small, and not to change very much during inflation. The argument of the sine, on the other hand, is a large number and can easily change by several times $2\pi$ during the relevant time period for the generation of the primordial perturbations.

Let us now investigate in more detail what the effect will be in the case of chain inflation.
3.2 The size of the bubbles

To proceed we need to know more about the process of nucleation of bubbles. The rate of bubble formation per unit time and physical volume, in the analysis of [14], [15], is given by the exponential of the euclidean instanton action, $S_E$, responsible for the tunneling (the so-called "bounce"),

$$\Gamma \sim e^{-S_E}.$$  \hfill (33)

The action evaluated on the bounce is given by

$$S_E = \frac{\pi^2}{2} r^4 \Delta \epsilon + 2\pi^2 r^2 S,$$ \hfill (34)

where $r$ is the radius of the bubble, $\Delta \epsilon$ the change in energy due to the nucleation of the bubble, and $S$ is the bubble’s wall tension.\(^9\) In principle, there is also a third term present due to the effect of gravity, but we assume the size of the bubbles to be much smaller than the Hubble scale so that we can ignore it. The critical radius that allows for the nucleation of a bubble that will successfully expand, and therefore enables tunneling, is obtained by extremizing the above Euclidean action. The result is

$$r_b \equiv r_{\text{critical}} = \frac{3S}{\Delta \epsilon},$$ \hfill (35)

$$S_E |_{r=r_{\text{critical}}} = \frac{27\pi^2}{2} \frac{S^4}{(\Delta \epsilon)^3}.$$ \hfill (36)

The setup outlined in these formulas is a static one. The tunneling occurs between two vacua of the theory, and any possible time evolution of the background is not taken into account. In our scenario, on the other hand, we have a chain of tunneling events occurring through time. The length scale signaling new physics (corresponding to the radius of the nucleated bubbles) depends on the time when the particular mode of interest is produced. For simplicity, however, we will assume that the change in the radius is slow enough that we can use the above analysis.

Accounting for the time evolution of the background, when computing the critical radius of the bubbles at a given time, is most easily achieved by expressing the variation of the energy density due to the nucleation as

$$\Delta \epsilon \sim -\frac{d\rho^V}{dt} \langle \tau \rangle = 6H^3 M^2_{\text{Plank}} \langle \tilde{\Gamma} \rangle \epsilon,$$ \hfill (37)

where we have used the Friedman equation, and defined $\langle \tau \rangle \equiv (\langle \tilde{\Gamma} \rangle)^{-1}$ to be the average tunneling time (we recall that $\tilde{\Gamma}$ is the decay rate per unit time).\(^{11}\) Also, the surface

\(^9\)We recognize in this the variation of the Gibbs energy for the nucleation of a bubble: with a different normalization for what concerns energy density and surface tension.

\(^{10}\)If the tension is due to a scalar field, $\phi$, we have that $S = \int d\phi \sqrt{V}$, where $V$ is the potential.

\(^{11}\)All averages are taken with the distribution of vacuum phases $\rho^V_m = \epsilon_m p_m(t)$, see appendix and
tension $S$ needs to have a time dependence. It is more convenient, though, to express $S$ through the extremized action, as

$$S = \left( \frac{2S_E}{27\pi^2} \right)^{\frac{1}{4}} \Delta \epsilon^{\frac{3}{4}},$$

(38)

Eventually we find

$$r_b H = \left( \frac{2S_E}{c\pi^2} \right)^{\frac{1}{4}} \langle \tau \rangle^{-\frac{1}{4}} \langle \tilde{\Gamma} \rangle^{-\frac{1}{4}} \left( \frac{H}{M_{pl}} \right)^{\frac{1}{4}}$$

$$= \left( \frac{2S_E}{c\pi^2} \right)^{\frac{1}{4}} \langle \tau \rangle^{-\frac{1}{4}} \langle \tilde{\Gamma} \rangle^{-\frac{1}{4}} \left( \frac{8\pi^2 \eta}{\sqrt{3}} \right)^{\frac{1}{4}},$$

(39)

where $\eta \sim 2.5 \cdot 10^{-9}$ from the normalization of the spectrum. With $\varepsilon = 10^{-2}$ we find

$$r_b H \sim 3.9 \cdot 10^{-3} S_E^{\frac{1}{4}} e^{-\frac{1}{4} \langle \tau \rangle} \langle \tilde{\Gamma} \rangle^{-\frac{1}{4}}.$$  

(40)

In our calculation we have ignored time dependent corrections to, e.g., $S_E$ that are suppressed by $1/n$.

Let us now consider the possible observational implications of the above effects. Successful chain inflation requires that while the vacuum undergoes the transitions, the phase distribution in the universe is peaked consecutively on the various phases. That is, the transitions occur consecutively, and in a short time (shorter than the Hubble time). Rapid tunneling implies that $S_E$ should be at most of order one, in order for $\langle \tau \rangle H \ll 1$. If we then use $\varepsilon = \frac{c}{2H} \langle \tilde{\Gamma} \rangle = \frac{c}{2\langle \tau \rangle H} \langle \tilde{\Gamma} \rangle$, we find that $\langle \tau \rangle \langle \tilde{\Gamma} \rangle$ needs to be small. With a peaked distribution we have, to a good approximation,

$$\langle \tau \rangle \langle \tilde{\Gamma} \rangle \sim 1/n,$$

and we see that $n$ needs to be large.

Turning back to equation (32), and the corrections to the spectrum from the presence of a new scale, we know from the work on transplanckian physics that values of the order $\varepsilon \sim 10^{-2}$ and $\frac{H}{\Lambda} \sim 10^{-3}$ could possibly yield an observational effect. The restriction on $\varepsilon$ comes from the requirement that $H$ changes in an appreciable way in order for there to be a modulation. Using $H \sim k^{-\varepsilon}$ and (32) we have, following [13],

$$\frac{\Delta k}{k} \sim \frac{\pi H}{\varepsilon \Lambda},$$

(41)

where, in our case, $\Lambda \sim 1/r_b$. We see that $\frac{\Delta k}{k}$ of order one, and a reasonable amplitude on the order of percent are easily obtainable within our model using values of $n \sim 10^4$.

4 Discussion

As we have argued, chain inflation will lead to several new effects on the spectrum of primordial perturbations. In particular, our calculations show that the spectral index
is changed from the naive one due to the presence of interactions between radiation and the vacuum energy (the contribution proportional to $Q_V \propto \varepsilon$ in formulas (28,29)).

The detailed predictions depend in a sensitive way on the distribution of the decay rates between the different minima. If it would be possible to measure these decay rates in a precise way, they would provide us with a wealth of information on features of the (effective) potential of the underlying fundamental theory. One needs to keep in mind, though, that it is necessary to distinguish these effects from other similar effects that could arise from non-standard potentials in other models of inflation.

Another, possibly more characteristic prediction, is the existence of signatures similar to those that could be generated through transplanckian physics. That is, a modulation of the spectrum due to the presence of a fundamental scale. In case of transplanckian physics, it is the Planck scale (or string scale) that determines the effect, while in the case of chain inflation it is instead the size of the nucleating bubbles. It is interesting to note that the model quite naturally, without much fine tuning, gives rise to effects of a reasonable magnitude that possibly could be detected. As in the case of transplanckian physics we have only been able to make a very rough estimate of the size of the effect.

In order to make better predictions of observational signatures, a precise model with an explicit potential and field content needs to be specified. In [II], based on work in [16] and [17], we proposed that flux compactified type IIB string theory provides such models in a natural way. In that work we focused on the stabilization of the complex structure moduli using fluxes. With the help of monodromy transformations, generated by going around singular points in the moduli space of Calabi-Yau compactifications, we were able to show the existence of long sequences of minima of the necessary form. A quadratic behaviour, with $c = 2$, typically arises when the axiodilaton is stabilized independently of the complex structure moduli, while the linear behaviour with $c = 1$ arises when the axiodilation is stabilized together with the complex structure moduli. The detailed form of the potentials depends heavily on the choice of Calabi-Yau manifolds and fluxes, but the overall features seem to be rather generic. In particular, the barriers in between the minima are expected to be such that an effective slow roll behaviour arises.

It would be interesting to further explore the possibility of generating potentials for chain inflation through string theory. Given the difficulty in finding appropriate potentials for standard inflation, we believe this to be a worthwhile enterprise.

\footnote{In our simplified model the Kähler moduli, i.e. the moduli determining the sizes of the extra dimensions, were assumed to be fixed by other physics.}
A The slow-roll parameter and its first derivative in a power-law chain inflation model

During inflation we naturally expect:

\[ H^2 \sim \frac{8 \pi G}{3} \rho^V \]  

(42)

and

\[ \rho^V \sim \sum_m \epsilon_m p_m. \]  

(43)

Here \( p_m(t) \) is the fraction of volume occupied by the vacuum \( m \) at time \( t \) and its time evolution is given by (see [1])

\[ \dot{p}_m = -\tilde{\Gamma}_m p_m + \tilde{\Gamma}_{m+1} p_{m+1}. \]  

(44)

From this, we find

\[ \dot{\rho}^V = -\sum_m \Delta \epsilon_m \tilde{\Gamma}_m p_m, \]  

(45)

and from [12] we see

\[ \Delta \epsilon_m \sim \frac{\epsilon_m}{m}. \]  

(46)

Then, using this and [12], we find for the slow-roll parameter

\[ \epsilon = -\frac{\dot{H}}{H^2} = \frac{c}{2H} \frac{\sum_m \epsilon_m \tilde{\Gamma}_m p_m m^{-1}}{\sum_n \epsilon_n p_n}. \]  

(47)

If we now define an average \( \langle \tilde{\Gamma}_n \rangle \) as

\[ \langle \tilde{\Gamma}_n \rangle = \frac{\sum_m \epsilon_m \tilde{\Gamma}_m p_m m^{-1}}{\sum_n \epsilon_n p_n}, \]  

(48)

the slow-roll parameter is given by

\[ \epsilon \sim \frac{c}{2H} \langle \tilde{\Gamma}_n \rangle. \]  

(49)

The time derivative of the slow-roll parameter is as follows. From (47) and using (46)

\[ \dot{\epsilon} = \frac{\epsilon}{2} \langle \tilde{\Gamma}_n \rangle + \frac{c}{2H} \left( \frac{\sum_m \epsilon_m \tilde{\Gamma}_m \dot{p}_m m^{-1}}{\sum_n \epsilon_n p_n} + c \left( \frac{\sum_m \epsilon_m \tilde{\Gamma}_m p_m m^{-1}}{\sum_n \epsilon_n p_n} \right)^2 \right) \]  

(50)

13Here and in the following \( \rho^V \) represents the energy density of the vacuum in the interior of the bubbles. We in general expect this formula for the following reason. First of all during inflation the total energy density is dominated by the vacuum component. The latter is then given by the contributions respectively of the interior of the bubbles and the walls. But the energy density of uncollided walls is proportional to the energy difference between two consecutive vacua, while the one of the interior of bubbles is proportional to the energy level, which is greater than the difference.
Let us focus on the two terms in each of the brackets. The second one is simply

$$c\langle \sum_m \epsilon_m \tilde{\Gamma}_m \rho_m m^{-1} \rangle^2 = c\langle \tilde{\Gamma} \rangle^2. \quad (51)$$

It is easy to see that using (44), the numerator in the first term reads

$$\sum_m \epsilon_m \tilde{\Gamma}_m \dot{\rho}_m m^{-1} = \sum_m \tilde{\Gamma}_{m+1} p_{m+1} \left( \frac{\epsilon_m \tilde{\Gamma}_m}{m} - \frac{\epsilon_{m+1} \tilde{\Gamma}_{m+1}}{m+1} \right)$$

$$= -\sum_m \tilde{\Gamma}_{m+1} p_{m+1} \left( \Delta \left( \frac{\epsilon_{m+1}}{m+1} \right) \tilde{\Gamma}_{m+1} + \frac{\epsilon_m}{m} \Delta(\tilde{\Gamma}_{m+1}) \right) \quad (52)$$

where we have defined, for any quantity $f_m$

$$\Delta (f_m) \equiv f_m - f_{m-1}. \quad (53)$$

We find that:

$$\Delta \left( \frac{\epsilon_{m+1}}{m+1} \right) = (c-1) \frac{\epsilon_{m+1}}{(m+1)^2} \quad (54)$$

If we now define

$$\sigma_{\langle \tilde{\Gamma} \rangle}^2 = \left\langle \left( \frac{\tilde{\Gamma}}{n} \right)^2 \right\rangle - \left\langle \frac{\tilde{\Gamma}}{n} \right\rangle^2. \quad (55)$$

Then, from (50) and (49), we have

$$\dot{\epsilon} \sim \left( 1 + \frac{2}{c} \right) H \epsilon^2 - (c-1) \epsilon \sigma_{\langle \tilde{\Gamma} \rangle}^2 \left( \frac{\tilde{\Gamma}}{n} \right)^{-1} - \epsilon \left( \frac{\tilde{\Gamma} \Delta\tilde{\Gamma}}{n} \right)^{-1} - (c-1) \epsilon \left( \frac{\tilde{\Gamma} \Delta\tilde{\Gamma}}{n n-1} \right)^{-1} \quad (56)$$

where all the averaging has been made using the distribution $\rho_m = \epsilon_m \rho_m$.

For ease of notation, we define

$$D \tilde{\Gamma} \equiv (c-1) \epsilon \sigma_{\langle \tilde{\Gamma} \rangle}^2 \left( \frac{\tilde{\Gamma}}{n} \right)^{-1} + \epsilon \left( \frac{\tilde{\Gamma} \Delta\tilde{\Gamma}}{n} \right)^{-1} - (c-1) \epsilon \left( \frac{\tilde{\Gamma} \Delta\tilde{\Gamma}}{n n-1} \right)^{-1}. \quad (57)$$

**Acknowledgments**

The work was supported by the Swedish Research Council (VR) and by the EU Marie Curie Training Site contract: MRTN-CT-2004-512194.
References

[1] D. Chialva and U. H. Danielsson, “Chain inflation revisited,” arXiv:0804.2846 [hep-th].

[2] K. Freese and D. Spolyar, “Chain inflation: 'Bubble bubble toil and trouble',' JCAP 0507 (2005) 007 [arXiv:hep-ph/0412145].

[3] K. Freese, J. T. Liu and D. Spolyar, “Chain inflation via rapid tunneling in the landscape,” arXiv:hep-th/0612056.

[4] Q. G. Huang, “Simplified Chain Inflation,” JCAP 0705 (2007) 009 arXiv:0704.2835 [hep-th].

[5] Q. G. Huang and S. H. Tye, “The Cosmological Constant Problem and Inflation in the String Landscape,” arXiv:0803.0663 [hep-th].

[6] J. Garriga and V. F. Mukhanov, “Perturbations in k-inflation,” Phys. Lett. B 458 (1999) 219 arXiv:hep-th/9904176.

[7] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, “Theory of cosmological perturbations.” Phys. Rept. 215 (1992) 203.

[8] J. Martin and R. H. Brandenberger, “The trans-Planckian problem of inflationary cosmology,” Phys. Rev. D 63 (2001) 123501 arXiv:hep-th/0005209.

[9] J. C. Niemeyer, “Inflation with a high frequency cutoff,” Phys. Rev. D 63 (2001) 123502 arXiv:astro-ph/0005533.

[10] U. H. Danielsson, “A note on inflation and transplanckian physics,” Phys. Rev. D 66 (2002) 023511 arXiv:hep-th/0203198.

[11] U. H. Danielsson, “Inflation, holography and the choice of vacuum in de Sitter space,” JHEP 0207 (2002) 040 arXiv:hep-th/0205227.

[12] R. Easther, B. R. Greene, W. H. Kinney and G. Shiu, “A generic estimate of trans-Planckian modifications to the primordial power spectrum in inflation,” Phys. Rev. D 66 (2002) 023518 arXiv:hep-th/0204129.

[13] L. Bergstrom and U. H. Danielsson, “Can MAP and Planck map Planck physics?,” JHEP 0212 (2002) 038 arXiv:hep-th/0211006.

[14] S. R. Coleman, “The Fate Of The False Vacuum. 1. Semiclassical Theory,” Phys. Rev. D 15 (1977) 2929 [Erratum-ibid. D 16 (1977) 1248].

[15] S. R. Coleman and F. De Luccia, “Gravitational Effects On And Of Vacuum Decay,” Phys. Rev. D 21 (1980) 3305.
[16] U. H. Danielsson, N. Johansson and M. Larfors, “The world next door: Results in landscape topography,” JHEP 0703 (2007) 080 [arXiv:hep-th/0612222].

[17] D. Chialva, U. H. Danielsson, N. Johansson, M. Larfors and M. Vonk, “Deforming, revolving and resolving - New paths in the string theory landscape,” JHEP 0802 (2008) 016 [arXiv:0710.0620 [hep-th]].

[18] J. Hamann, S. Hannestad, M. S. Sloth and Y. Y. Y. Wong, “Observing trans-Planckian ripples in the primordial power spectrum with future large scale structure probes,” arXiv:0807.4528 [astro-ph].