Decoherence and equilibration under nondestructive measurements

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Abstract

The evolution of observable quantities of finite quantum systems is analyzed when the latter are subject to nondestructive measurements. The type and number of measurements characterize the level of decoherence produced in the system. A finite number of instantaneous measurements leads to only a partial decoherence. But infinite number of such measurements yields complete decoherence and equilibration. Continuous measurements result in partial decoherence in finite time, but produce complete decoherence and equilibration as time tends to infinity. Resulting equilibrium states are characterized by representative statistical ensembles that, generally, retain information on initial conditions. Any system, to be observable, necessarily requires the presence of measurements, whose large number leads to the system equilibration and decoherence.

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1 Introduction

Decoherence and equilibration in quantum systems are the problems that has been studied in numerous papers. An extensive list of related literature can be found in the recent review articles [1-13]. Nowadays, this problem has gained much interest with regard to finite quantum systems. Such finite systems are now intensively studied both theoretically and experimentally because of their role in a variety of applications ranging from quantum electronics to quantum information storage, processing, and computing [14-17]. Equilibration and decoherence from a strongly nonequilibrium initial state have been studied, e.g., for such quantum systems as spin assemblies [7,18-21], trapped atoms [6,13,22-27], and quantum dots [28].

One, generally, distinguishes two kinds of system dynamics: one is when the finite quantum system is isolated and another when it is connected to some environment. It is known that if the system is coupled to a sufficiently large equilibrium environment, it equilibrates and decoheres due to its interaction with surrounding [2,4,29-32]. But if the system is isolated, its motion is quasi-periodic and, thus, the system cannot equilibrate in the strict sense, displaying instead the Poincaré recurrences [33]. However, an isolated system can equilibrate on average, relaxing to a quasi-equilibrium state defined by an ergodic average [34] and staying close to it most of the time [5]. Dynamics of isolated quantum systems depends on their closeness to integrability, revealing pre-equilibration effects [18,19,35,36] and displaying essential dependence on the way of their preparation, especially on the presence of defects [37,38].

In the present paper, we consider the intermediate case, when a finite quantum system is quasi-isolated, being almost isolated, except the action of nondestructive measurements, so that they do not destroy the system properties. We analyze the system decoherence under such nondestructive measurements. The main difference of the present consideration from the previous works is in the following.

(i) The studied quantum system interacts not with an equilibrium bath, but with a nonequilibrium measuring device.

(ii) The interaction part of the Hamiltonian is time-dependent, while a bath is usually described by a time-independent Hamiltonian. This essentially complicates mathematics and results in rather different consequences. The evolution operator now is not the standard exponential form $\exp(-Ht)$. Solving the evolution equation for this operator requires now to invoke the Lappo-Danilevsky theory.

(iii) In the case of measurements, decoherence is not necessarily complete, as it would be for the bath, but the level of decoherence depends on the type and number of measurements. Specifically, nondestructive measurements are analyzed, but not arbitrary external perturbations of the system. In that sense, the consideration is limited by exactly this type of measurements. Under this restriction, the overall treatment can be done for quite general conditions: the nature of the system can be arbitrary; it is not required that its spectrum be nondegenerate; the system states can be either pure or mixed; it is not required that the initial states be of the product type; no time averaging is needed; interactions of the system with the measuring device can be of arbitrary strength, but not necessarily weak; the measuring device can also be of rather arbitrary nature, provided it is nondestructive.

As is stressed above, the main point is the consideration of the time-dependent interaction of the system with an external device, which makes the principal difference from the
standardly treated case of an equilibrium bath. Time-dependent interactions are typical of measurement procedures, when a measuring device is switched on and off. This is why it is possible to interpret such nonequilibrium interactions as measurements. Throughout the paper, the word "measurement" is used as a conditional term describing time-dependent influence on the system of an external nonequilibrium source. In addition, two limiting cases of the source influence, instantaneous and continuous, which will be analyzed in the paper, have the forms that are commonly associated with measurements [39-41]. Therefore the interpretation of the time-dependent source influence as measurement seems to be justified. This, however, is not compulsory and one can treat the word "measurement" just as a brief term for the considered time-dependent interactions possessing several of the properties typical of measurement process.

2 Quantum system under nondestructive measurements

Let the quantum system of interest be described by a Hamiltonian \( H_A \) acting on a Hilbert space \( \mathcal{H}_A \). The system is subject to a measurement procedure. The measuring device is characterized by a Hamiltonian \( H_B \) acting on a Hilbert space \( \mathcal{H}_B \). The total Hamiltonian is the sum

\[
H_{AB} = H_A + H_B + H_{\text{int}},
\]

where \( H_{\text{int}} \) is the term describing the interaction between the system and the measuring device. Hamiltonian (1) is defined on the Hilbert space

\[
\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B.
\]

Strictly speaking, the Hamiltonian (1) should be written as

\[
H_{AB} = H_A \otimes \hat{1}_A + \hat{1}_A \otimes H_B + H_{\text{int}},
\]

with \( \hat{1}_A \) and \( \hat{1}_B \) being the unit operators on the corresponding spaces. However, it is commonly accepted to omit the unit operators, writing, for simplicity, the Hamiltonian in form (1).

The measurement is called *nondestructive* when it does not disturb the system, in the sense that the system Hamiltonian is conserved,

\[
[H_A, H_{AB}] = 0.
\]

Conversely, the studied system does not destroy the measuring device, so that its Hamiltonian is also conserved,

\[
[H_B, H_{AB}] = 0.
\]

As a consequence of this definition, one has

\[
[H_A, H_{\text{int}}] = [H_B, H_{\text{int}}] = 0, \quad [H_{\text{int}}, H_{AB}] = 0.
\]

The measuring procedure, enjoying these properties, can also be termed minimally disturbing measurement [42-44] or nondemolition measurement [45-48].
The system Hamiltonian defines a complete orthonormal basis \{\ket{n}\} by the eigenproblem
\[ H_A \ket{n} = E_n \ket{n} . \] (7)
Respectively, the device Hamiltonian, by the eigenproblem
\[ H_B \ket{k} = \beta_k \ket{k} , \] (8)
defines a complete orthonormal basis \{\ket{k}\}.

Because of properties (4) and (5), the total Hamiltonian (1) possesses the eigenfunctions \( \ket{nk} \equiv \ket{n} \otimes \ket{k} \). A measurement procedure is a nonequilibrium process, which leads to a complication coming from the fact that the interaction Hamiltonian \( H_{\text{int}} = H_{\text{int}}(t) \), generally, depends on time. Therefore the eigenvalues of the total Hamiltonian also depend on time, being given by the eigenproblem
\[ H_{AB}(t) \ket{nk} = [E_n + \varepsilon_{nk}(t)] \ket{nk} . \] (9)

The temporal evolution of the total statistical operator is prescribed by the rule
\[ \dot{\rho}_{AB}(t) = \hat{U}_{AB}(t) \hat{\rho}_{AB}(0) \hat{U}_{AB}^+(t) , \] (10)
where the evolution operator is a unitary operator satisfying the Schrödinger equation
\[ i \frac{d}{dt} \hat{U}_{AB}(t) = H_{AB} \hat{U}_{AB}(t) . \] (11)

The total Hamiltonian \( H_{AB} = H_{AB}(t) \) depends on time through the interaction term \( H_{\text{int}}(t) \). This does not allow us to represent the evolution operator in a simple exponential form as it is usually accepted in the case of an equilibrium bath. However, we may notice that
\[ \braket{nk}{H_{AB}(t) \int_0^t H_{AB}(t') \, dt'}{np} = \delta_{mn} \delta_{kp} \int_0^t [E_n + \varepsilon_{nk}(t')] \, dt' . \] (12)
Consequently, the Lappo-Danilevsky condition [49]
\[ \left[ H_{AB}(t), \int_0^t H_{AB}(t') \, dt' \right] = 0 \] (13)
holds true, at least in the weak sense. Owing to this condition (13), one can represent the solution of Eq. (11) as
\[ \hat{U}_{AB}(t) = \exp \left\{ -i \int_0^t H_{AB}(t') \, dt' \right\} . \] (14)
The possibility of representing the evolution operator in form (14) is due to the Lappo-Danilevsky condition (13) that becomes valid when the measurement procedure is nondestructive in the sense of conditions (4) and (5).

Thus the total composite system is characterized by the statistical ensemble \{\mathcal{H}_{AB}, \hat{\rho}_{AB}(t)\}, with the statistical operator satisfying the normalization condition
\[ \text{Tr}_{AB} \hat{\rho}_{AB}(t) = 1 , \] (15)
where the trace is over the total space (2).
3 Temporal evolution of observable quantities

What one is interested in any physical problem is the behavior of the system observable quantities that are represented by self-adjoint operators $\hat{A}$ composing the algebra of local observables $\mathcal{A} \equiv \{\hat{A}\}$ defined on the system space $\mathcal{H}_A$. The measurable observables are given by the averages

$$\langle \hat{A}(t) \rangle \equiv \operatorname{Tr}_{AB} \hat{\rho}_{AB}(t) \hat{A},$$

in which the trace is over $\mathcal{H}_{AB}$. But, since $\hat{A}$ is given on $\mathcal{H}_A$, the latter average (16) reduces to

$$\langle \hat{A}(t) \rangle = \operatorname{Tr}_A \hat{\rho}_A(t) \hat{A},$$

with the partial statistical operator

$$\hat{\rho}_A(t) \equiv \operatorname{Tr}_B \hat{\rho}_{AB}(t),$$

in which the degrees of freedom of $\mathcal{H}_B$ are traced out.

Passing to a matrix representation transforms average (17) to

$$\langle \hat{A}(t) \rangle = \sum_{mn} \rho^A_{mn}(t) A_{nm},$$

with $A_{mn} \equiv \langle m | \hat{A} | n \rangle$ and a density matrix

$$\rho^A_{mn}(t) \equiv \langle m | \hat{\rho}_A(t) | n \rangle.$$

The latter can be written as

$$\rho^A_{mn}(t) = \sum_k \rho^{AB}_{mnk}(t),$$

where

$$\rho^{AB}_{mnk}(t) \equiv \langle mk | \hat{\rho}_{AB}(t) | nk \rangle.$$

In general, any basis could be used for such a matrix representation [50]. But, as far as the system observables are of interest, it is convenient to employ the basis $\{|nk\rangle\}$ composed of the eigenvectors $|n\rangle$ of the system Hamiltonian $H_A$ and eigenvectors $|k\rangle$ of $H_B$. Then, because of Eqs. (9) and (14), one has

$$\hat{U}_{AB}(t)|nk\rangle = \exp \left\{ -i E_n t - i \int_0^t \epsilon_{nk}(t') \, dt' \right\} |nk\rangle.$$

This, for matrix (22), gives

$$\rho^{AB}_{mnk}(t) = \rho^{AB}_{mnk}(0) \exp \left\{ -i \omega_{mn} t - i \int_0^t \epsilon_{mnk}(t') \, dt' \right\},$$

with the initial condition

$$\rho^{AB}_{mnk}(0) \equiv \langle mk | \hat{\rho}_{AB}(0) | nk \rangle,$$

transition frequencies

$$\omega_{mn} \equiv E_m - E_n,$$
and the notation
\[ \varepsilon_{mnk}(t) \equiv \varepsilon_{mk}(t) - \varepsilon_{nk}(t) . \] (26)

By definitions (25) and (26), one has
\[ \omega_{nn} = 0, \quad \varepsilon_{nnk}(t) = 0 , \] (27)
because of which the diagonal element
\[ \rho^{A}_{nn}(t) = \sum_{k} \rho^{AB}_{nnk}(0) \equiv \rho_{nn} \] (28)
does not depend on time. Because of normalization (15), we have
\[ \sum_{n} \rho^{A}_{nn}(t) = \sum_{n} \rho_{nn} = 1 . \] (29)

Separating in sum (19) the terms with \( m = n \) and \( m \neq n \) yields
\[ \langle \hat{A}(t) \rangle = \sum_{n} \rho_{nn} A_{nn} + \sum_{m \neq n} \rho^{A}_{mn}(t) A_{nm} . \] (30)

As an initial expression \( \hat{\rho}_{AB}(0) \) one usually takes a disentangled factor product of \( \hat{\rho}_{A}(0) \) and \( \hat{\rho}_{B}(0) \). Here we do not assume this simplification, but keep the general form of \( \hat{\rho}_{AB}(0) \). Whether the latter is entangled or not does not play a principal role for what follows.

4 Measurement procedure of several measurements

In general, a measurement procedure can include multiple acts of measurement. Let there be \( M \) such measurement acts, so that the action of the measuring device be represented as the sum
\[ H_{\text{int}}(t) = \sum_{j=1}^{M} f_{j}(t) \hat{X}_{j} , \] (31)
in which \( f_{j}(t) \) is a real function and \( \hat{X}_{j} \) is a self-adjoint operator on \( \mathcal{H}_{AB} \).

According to the definition of nondestructive measurements in Eqs. (4) and (5) and its consequence (6), we have the commutators
\[ [H_{A}, \hat{X}_{j}] = [H_{B}, \hat{X}_{j}] = 0 . \] (32)

Therefore the eigenproblem for \( \hat{X}_{j} \) reads as
\[ \hat{X}_{j} |nk\rangle = \xi_{jk} |nk\rangle , \] (33)
with a real eigenvalue \( \xi_{jk} \). As a result, the eigenproblem for the interaction operator (31) takes the form
\[ H_{\text{int}}(t) |nk\rangle = \alpha_{nk}(t) |nk\rangle , \] (34)
with the eigenvalue
\[ \alpha_{nk}(t) = \sum_{j=1}^{M} \xi_{jnk} f_j(t) . \] (35)

In view of Eqs. (8) and (9), we get
\[ \varepsilon_{nk}(t) = \alpha_{nk}(t) + \beta_k , \]
which leads to
\[ \varepsilon_{mk}(t) - \varepsilon_{nk}(t) = \alpha_{mk}(t) - \alpha_{nk}(t) . \]
Then Eq. (26) gives
\[ \varepsilon_{mnk}(t) = \sum_{j=1}^{M} x_{jmnk} f_j(t) , \] (36)
where
\[ x_{jmnk} \equiv \xi_{jmk} - \xi_{jnk} . \] (37)

Introducing the notation
\[ R_{mn}(t) \equiv \sum_k \rho_{mnk}^{AB}(0) \exp\left\{ -i \sum_{j=1}^{M} x_{jmnk} \varphi_j(t) \right\} , \] (38)
with
\[ \varphi_j(t) \equiv \int_0^t f_j(t') dt' , \] (39)
for matrix (21), we obtain
\[ \rho_{mn}^A(t) = R_{mn}(t) \exp(-i \omega_{mn} t) . \] (40)

5 Evolution after last measurement

Function (39) characterizes the integral impact of the \( j \)-th measurement during the period of time \([0, t]\). Suppose that after the last \( M \)-th measurement, occurring at time \( t_M \), the integral impact (39) becomes
\[ \varphi_j(t) = \varphi(t) \quad (t > t_M) . \] (41)
As is shown below, property (41) is valid for different types of measurements, including discrete measurements, whose action is equivalent to instantaneous kicking [51,52], as well as in the opposite case of continuous permanent measurements, acting uniformly in time [41,47].

Under condition (41), Eq. (38) reduces to
\[ R_{mn}(t) = \sum_k \rho_{mnk}^{AB}(0) \exp\{-ix_{mnk} M \varphi(t)\} , \] (42)
where
\[ x_{mnk} \equiv \frac{1}{M} \sum_{j=1}^{M} x_{jmnk} . \] (43)
Let us introduce the density function
\[ g_{mn}(x) \equiv \sum_k \rho_{mnk}^A(0) \delta(x - x_{mnk}) . \] (44)

This function incorporates the properties of the measuring device, which affect the measured quantity. Therefore it can be called the *effect density* [39]. Definition (44) makes it possible to rewrite Eq. (42) as the integral transformation
\[ R_{mn}(t) = \int g_{mn}(x) \exp\{-ixM\varphi(t)\} \, dx . \] (45)

The effect density (44) is normalized as
\[ \int g_{mn}(x) \, dx = \sum_k \rho_{mnk}^A(0) . \] (46)

Invoking the definition
\[ \rho_{mn} \equiv \rho_{mn}^A(0) = \sum_k \rho_{mnk}^A(0) \] (47)

reduces normalization (46) to the form
\[ \int g_{mn}(x) \, dx = \rho_{mn} . \] (48)

Therefore the effect density can be represented as
\[ g_{mn}(x) = \rho_{mn} p_{mn}(x) , \] (49)

where the distribution \( p_{mn}(x) \) is normalized to one,
\[ \int p_{mn}(x) \, dx = 1 . \] (50)

Thus for Eq.(45), we come to the expression
\[ R_{mn}(t) = \rho_{mn} D_{mn}(t) , \] (51)

with the *decoherence factor*
\[ D_{mn}(t) \equiv \int p_{mn}(x) \exp\{-ixM\varphi(t)\} \, dx . \] (52)

Respectively, matrix (40) reads as
\[ \rho^A_{mn}(t) = \rho_{mn}(t) D_{mn}(t) , \] (53)

where
\[ \rho_{mn}(t) \equiv \rho_{mn} \exp(-i\omega_{mn}t) . \] (54)

As a result, for the evolution of observable quantities (30), we have
\[ \langle \hat{A}(t) \rangle = \sum_n \rho_{mn} A_{nn} + \sum_{m \neq n} \rho_{mn}(t) A_{nm} D_{mn}(t) . \] (55)

This formula describes the evolution of observables after the system has been subject to a measurement consisting of a series of nondestructive measurement acts.
6 Decoherence caused by nondestructive measurements

The second term in Eq. (55) is due to interference effects typical of coherent quantum systems. Measurement procedure destroys coherence. In order to derive an explicit expression for the decoherence factor (52) one has to model the distribution function $p_{mn}(x)$ and to define the type of the measurement procedure characterized by function $\varphi(t)$. The measuring device is a macroscopic system, because of which its spectrum can be treated as continuous, similarly to the density of states of macroscopic statistical systems [53]. Therefore, the summation over $k$ in the above formulas should be understood as integration over this multi-index. We shall consider two typical distributions, the Gaussian and Lorentz ones, and two opposite measurement procedures, instantaneous and continuous. This choice is based on the following arguments. A measuring device, being a macroscopic object, contains a large number of elements acting randomly on the system. As is known from the central limit theorem, the action of a large number of random elements is well described by Gaussian distribution. Because of this, the effect density of measuring devices is commonly represented by Gaussians [39,54-56]. Lorentzian distribution arises when measurement is realized by means of optical beams [40]. Two usually considered types of measurements are instantaneous [39,40] and continuous [39,41,55] measurements.

In the case of the Gaussian distribution

$$p^G_{mn}(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{x^2}{2\sigma^2} \right) ,$$

we have the decoherence factor

$$D^G_{mn}(t) = \exp \left\{ -\frac{\sigma^2}{2} M^2 \varphi^2(t) \right\} .$$

Here the standard deviation $\sigma$, in general, can depend on the indices $m$ and $n$. But for the simplicity of notation, this dependence is not shown explicitly.

While for the Lorentz distribution

$$p^L_{mn}(x) = \frac{\sigma}{\pi(x^2 + \sigma^2)} ,$$

the decoherence factor is

$$D^L_{mn}(t) = \exp\{-\sigma M \varphi(t)\} .$$

When each act of the measurement procedure is instantaneous, such that

$$f_j(t) = \delta(t - t_j) ,$$

then Eq. (39) is the unit-step function

$$\varphi_j(t) = \Theta(t - t_j) .$$

Consequently, after the last measurement at time $t_M$,

$$\varphi(t) = 1 \quad (t > t_M) .$$
Then for the decoherence factor (57), in the case of the instantaneous measurements, we get

\[ D_{\text{inst}}^G(t) = \exp \left( -\frac{\sigma^2}{2} M^2 \right), \]  

(63)

where the indices \( m \) and \( n \) are omitted. And for the decoherence factor (59), we find

\[ D_{\text{inst}}^L(t) = \exp (-\sigma M). \]  

(64)

Another situation happens in the opposite case of a single but continuous measurement, when

\[ f_j(t) = 1, \quad M = 1, \]  

(65)

so that

\[ \varphi_j(t) = \varphi(t) = t. \]  

(66)

Then the decoherence factor (57) is

\[ D_{\text{cont}}^G(t) = \exp \left\{ -\frac{1}{2} \left( \frac{t}{t_{\text{dec}}} \right)^2 \right\}, \]  

(67)

with the decoherence time

\[ t_{\text{dec}} \equiv \frac{1}{\sigma}. \]  

(68)

And the decoherence factor (59) becomes

\[ D_{\text{cont}}^L(t) = \exp \left( -\frac{t}{t_{\text{dec}}} \right), \]  

(69)

with the decoherence time as in Eq. (68).

In this way, a finite number of instantaneous measurements or continuous measurements during a finite interval of time lead to a partial decoherence in expression (55) of observable quantities. If the number of instantaneous measurements infinitely increases or the time of accomplishing continuous measurements tends to infinity, then decoherence becomes complete:

\[ \lim_{M \to \infty} \langle \hat{A}(t) \rangle = \lim_{t \to \infty} \langle \hat{A}(t) \rangle = \sum_n \rho_{nn} A_{nn}. \]  

(70)

Since the limit in Eqs. (70) does not depend on time, this also implies equilibration.

The matrix \( \rho_{mn} \), according to Eqs. (22) and (28), generally, depends on the initial state of the total composite object, system plus device, since

\[ \rho_{mn} = \sum_k \langle mk \mid \hat{\rho}_{AB}(0) \mid nk \rangle. \]

The dependence on the measuring device disappears if at the initial time the studied quantum system and the measuring device are not entangled, so that

\[ \hat{\rho}_{AB}(0) = \hat{\rho}_A(0) \otimes \hat{\rho}_B(0). \]
Then because of the normalization condition

$$\sum_k \langle k | \hat{\rho}_B(0) | k \rangle = 1,$$

the matrix element

$$\rho_{mn} = \langle m | \hat{\rho}_A(0) | n \rangle$$

contains information only on the system initial state.

In the limit of infinite number of instantaneous measurements or large time of continuous measurement, the observable quantities reduce to the averages involving only the diagonal elements $\rho_{nn}$, which demonstrates the importance of such diagonal terms [57]. The values of $\rho_{nn}$ can be found by defining the corresponding representative ensemble uniquely describing the system [58]. The diagonalization occurs not because of imposing some additional averaging over the random phases of initial states [59], or because of time-averaging resulting in ergodic averages [34], but happens naturally as a consequence of repeated measurement actions.

7 Discussion

The evolution of a quantum system, subject to the action of nondestructive measurements, is considered. Two types of measurement procedures are analyzed, instantaneous measurements and continuous measurements. A finite number of instantaneous measurements or continuous measurements during a finite time interval lead to partial decoherence. But if instantaneous measurements are repeated infinite number of times or a continuous measurement lasts infinite time, then there happens complete decoherence and equilibration.

The interaction of a quantum system with a measuring device is principally different from its interaction with a bath. The latter is usually represented by a large equilibrium system. Contrary to this, the measurement procedure is a nonequilibrium process. Therefore the interaction part of the total Hamiltonian is time-dependent. This does not allow one to write down the evolution operator in the standard exponential form. To solve the evolution equation, we have to invoke the Lappo-Danilevsky theory [49].

We have considered the evolution of quantum systems under the action of nondestructive measurements. Then the system, starting form an initial nonequilibrium state, evolves into a state that is partially decoherent. The level of decoherence depends on the type and number of nondestructive measurements. Real measurements are usually supposed to be nondestructive in order not to destroy the studied system. Of course, a measurement, being an external perturbation, could also be destructive. Moreover, it is possible to study the situation opposite to that treated in the present paper. It is admissible to investigate the problem of how an equilibrium system develops being subject to the action of a time-dependent external perturbation driving the system out of equilibrium [60]. But such destructive perturbations, generally, are not characterized as measurements. And this opposite process of system destabilization is a principally different problem.

It is worth stressing that absolutely isolated systems do not exist. The notion of an absolute isolation is self-contradictory, since the statement that a system is isolated implicitly assumes that the fact of the system isolation is proved by observations. In the other case, such a statement cannot be true. To be true, a statement must be confirmed by observations. But
the latter, even being accomplished by nondestructive measurements, influence the system, leading to its, at least partial, decoherence. The absence of absolutely isolated systems and their decoherence, caused by observations, result in the system equilibration.

The fact that the system isolation must be proved by measurements does not contradict the possibility of treating a system as being isolated during a finite time duration. For instance, one can prepare a system in a well defined initial state by a prescribed preparation method. Then, after a finite time, one accomplishes a measurement by a nondestructive state tomography and determines the final system state. Supposing that in between these two measurements, the initial and final one, the system has not been influenced by any other disturbances, it could be possible to assume that it has been isolated during this finite time interval. Such an assumption is nothing but a particular case of the finite number of measurements, as has been considered above. But, in any case, one has to accomplish at least two measurement procedures in order to assume that the system has been isolated for the finite time between these measurements. In such a case of a finite temporal observation, the system, as is explained above, does not equilibrate and exhibits only partial decoherence.

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