Atomic Bose-Fermi mixed condensates with Boson-Fermion quasi-bound cluster states

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The boson-fermion atomic bound states (composite fermion) and their roles for the phase structures are studied in a bose-fermi mixed condensate of atomic gas in finite temperature and density. The two-body scattering equation is formulated for a boson-fermion pair in the mixed condensate with the Yamaguchi-type potential. By solving the equation, we evaluate the binding energy of a composite fermion, and show that it has small $T$-dependence in the physical region, because of the cancellation of the boson- and fermion- statistical factors in the equation. We also calculate the phase structure of the BF mixed condensate under the equilibrium $B + F \leftrightarrow BF$, and discuss the role of the composite fermions: the competitions between the degenerate state of the composite fermions and the Bose-Einstein condensate (BEC) of isolated bosons. The criterion for the BEC realization is obtained from the algebraically-derived phase diagrams at $T = 0$.

1. Introduction

Since the experimental realization of the alkali-atom BEC, rapid progress has been made on the finite many-body physics of quantum atomic gas–tapped atoms cooled down to the ultra-low temperature where they show quantum-mechanical behaviors.

Characteristics of the system are summarized as follows:

- It is a system of $10^3 \sim 10^9$ atoms in $T = nK \sim \mu K$, which is made by the laser-cooling technique. The atoms are confined in a magnetically/optically-trapped potential of a harmonic oscillator shape (in many experiments); a typical scale of it is given by $a_{\text{HO}} \sim \mu m$ in the harmonic oscillator length.

- It is a gas of weak atomic-interaction in creation, but the interaction strength can be shifted at any rate by the experimental technique with Feshbach resonances (molecular resonances of two-body channels) as an external adjustable parameter. It is also possible to change the signature of the interaction (from repulsive one to attractive one and v. v.). It has made possible a systematic study of many-body quantum systems with weak/strong-correlations.

- A variety of precise experimental designs and observations are possible in the laser technology; for example, using quantum interference effects, phases of wave func-
tions have been observed for quantum-vortex states et.al. The time-development of systems is also traced (with the nsec ~ psec samplings), and has leaded to direct observations of BEC’s collective oscillations.

Quantum behaviors of atomic gas in ultra-low-$T$ depend on statistical properties of the constituent atoms (boson or fermion). A most interesting phenomenon in bosonic systems is BEC; it is the state where a large part of atoms occupies a lowest-energy one-body state so that their behavior is described by a wave field $\psi(x)$ (order parameter). The success of creating BEC of $^{87}$Rb by Colorado-JILA group in 1995 has been a break-up of the quantum atomic-gas physics, and a variety of phenomena has been observed on BEC \cite{1}: coherence and decoherence, quantum interference, collective oscillations, quantum vortices, atomic lasers and so on.

Quantum gas of fermionic atoms shows quite different behaviors. If the atomic interaction is enough weak, it shows fermi-degeneracy at low-$T$; it was first observed by JILA group in 1999 for the gas of $^{40}$K \cite{2} (the first “direct” observation of the fermi degeneracy). Recently, gas of fermionic atoms is a very hot topic in experimental and theoretical studies: especially systems with attractive interactions, where possible Cooper-like-pairs or molecule formations are expected to make a superconducting/superfluid transition.

Another interesting system is a bose-fermi mixed condensate (BF mixed condensate): gas of two kinds of atoms, bosonic and fermionic ones, trapped in the same potential. Studies by our group show that the interatomic interaction between bosonic and fermionic atoms (BF-interaction) causes many interesting phases and phenomena in the mixed condensates: collective excitations, induced instability & the collapse of the system (Super bose nova), phase separations & shell structures, Peierls instabilities & density waves \cite{3}. The first experiment for the BF mixed condensates has been done by Rice and ENS groups with $^{6,7}$Li isotopes \cite{4}. Now, many experiments are planed to realize the BF condensates of a variety of atom species: $^{39,41-40}$K, $^{84,86,88-87}$Sr, $^{168,170-174,176}$Yb, $^{87}$Rb-$^{40}$K et.al.

Let’s consider the BF mixed condensate with enough strongly-attractive BF-interaction. In such a system, a pair of bosonic and fermionic atoms (B and F) can be bound into a composite fermion (BF). In the physics of the mixed condensates, the existence of the composite fermion BF should be very important because a group of them can give new phases of clusterized matter, e.g. the fermi-degenerate state of composite fermions. In this paper, we discuss a structure of the BF as a bound state in the background of a mixed condensate in finite $T$ by solving a two-body scattering equation for the boson-fermion pair. We also solve an equilibrium condition for the clusterization process $B \leftrightarrow B + F$, and obtain phase structures of the mixed condensate. For a numerical calculation, we take a mixture of $^{39}$K (boson) and $^{40}$K (fermion) throughout this paper.

2. Binding energy of the composite fermion in BF-mixed condensate

We consider a uniform gas of BF mixed condensate with the same mass $m$ and the BF-interaction potential $V$. We omit the boson-boson and fermion-fermion interactions for simplicity. To study a possible composite-fermion formation, we set up a two-body (boson-fermion) scattering equation in a background of the mixed condensate with boson- and fermion-densities $n_{B,F}$ at $T$. The relevant equation can be derived using an equation of motion \cite{5} for a boson-fermion pair operator, and then by replacing the average (B,F)
Figure 1. $T$-dependence of the binding energy of the composite fermion. BF0~BF3 corresponds to the density $n_b = n_f = 10^{10}$, $10^{18}$, $10^{19}$, $10^{20}$ atoms/cm$^3$. The solid line shows the $E = -T$ which provides a stability measure of the composite fermion at $T$.

number by the statistical factors $g_{B,F}(\epsilon, T, \mu_b)$ with a kinetic energy $\epsilon$, where $\mu_{B,F}$ denote chemical potentials. The binding energy $B$ is given as a negative energy eigenvalue of the scattering equation.

For the BF-interaction, we assume a Yamaguchi type separable interaction \cite{6}

$$\langle p'|V|p\rangle = -\frac{\lambda}{\mu}g(p')g(p), \quad g(p) = (p^2 + \beta^2)^{-1},$$

(1)

where $p$ denotes a relative momentum. The strength $\lambda$ and the range parameter $\beta$ are in principle determined by the scattering length and the effective range of the boson-fermion scattering in free space. The binding energy $B$ should then be a solution of the equation

$$1 - 4\pi\lambda\int_0^\infty d\epsilon\sqrt{2m\epsilon} \left( \frac{1}{2m\epsilon + \beta^2} \right)^2 \frac{1 + g_b(\epsilon, T, \mu_b) - g_f(\epsilon, T, \mu_f)}{B + 2\epsilon} = 0.$$  

(2)

As it is now possible to adjust the size and the sign of the atomic interaction via Feshbach resonances, we here concentrate on the role of the background atomic gas for the bound state. We thus fix the interaction parameters to give a boson-fermion bound state in the free space, and study the solution of eq. (2) as a function of the density and temperature.

Fig. 1 shows the $T$-dependence of the BF binding energy. In the present formulation, the high-$T$ limit corresponds to the free space value. It is seen that the effect of the background matter is quite small even for a fairly low temperature; The deviation from the free space value is appreciable only below $T \sim B_{\text{free}}/k_B$ and at high densities. This is due to a cancellation of the boson and fermion statistical factors in eq. (2), which is specific for a composite fermion. For the same kind of particles, the statistical factor comes in the equation with the same sign, and the background has much larger effect; For the bosonic gas, the binding energy becomes larger, while, for the fermionic gas, the Pauli blocking hinders the binding.

3. Phase structure of the BF-mixed condensate

Let’s consider the uniform system of polarized bosons & fermions (B and F) and their composite fermions (BF), with the masses $m_{B,F,BF}$. The total number densities of B and F, $n_{Btot,Ftot}$, should be conserved.
In order to obtain the phase structure at $T$ under the clusterization process $B+F \leftrightarrow BF$, we consider the equilibrium condition

$$\mu_B + \mu_F = \mu_{BF} + \Delta mc^2,$$

where $\mu_{B,F,BF}$ are chemical potentials of atoms $B$, $F$, and $BF$ each other, and $\Delta m = m_{BF} - m_B - m_F$ is a binding energy of the BF state. Because the BF mixed condensate is a weak-interacting atomic gas, we take the ideal-mixing approximation, where the chemical potentials in (3) are obtained by the density formulae of the free bose/fermi gas:

$$n_B = \frac{(m_B)^{3/2}}{\sqrt{2\pi}} \int_0^{\infty} \frac{\sqrt{\epsilon}d\epsilon}{e^{(\epsilon-\mu_B)/k_BT} - 1}, \quad n_a = \frac{(m_a)^{3/2}}{\sqrt{2\pi}} \int_0^{\infty} \frac{\sqrt{\epsilon}d\epsilon}{e^{(\epsilon-\mu_a)/k_BT} + 1}, \quad (a = F, BF)$$

where $k_B$ is a Boltzmann constant, and $n_{B,F,BF}$ are the densities of the free (unpaired) $B$ & $F$ and the composite $BF$.

Solving eq. (3) with (4) under the atom-number conservation for $B$ and $F$: $n_B + n_{BF} = n_{B_{tot}}$ and $n_F + n_{BF} = n_{F_{tot}}$, we obtain the densities $n_{B,F,BF}$ as functions of $T$ and $n_{B_{tot},F_{tot}}$.

A special care should be paid when $T$ and $n_B$ satisfy $T < T_C \equiv \frac{2\hbar^2}{m_Bk_B} \left( \frac{n_B}{2.615} \right)^{2/3}$ (BEC criterion). In that case, a part of free bosons condensates into BEC and $\mu_B$ becomes zero; then, the equilibrium condition becomes $\mu_F = \mu_{BF} + \Delta mc^2$. When BEC exists, the condensed- and normal-component densities are given by $n_{BEC} = n_B \left[ 1 - \left( \frac{T}{T_C} \right)^{3/2} \right]$, and $n_{B_{nor}} = n_B - n_{BEC}$.

![Figure 1. $T$-dependence of boson density in BF-mixed condensate](image1)

![Figure 2. Phase diagram of BF mixed condensate at $T = 0$ K ($n_{B_{tot}} = 10^{15}$ atoms/cm$^3$)](image2)

In Fig. 2, we show the $T$-dependence of the free boson density $n_B = n_{BEC} + n_{B_{nor}}$ when $n_{B_{tot}} = n_{F_{tot}} = 10^{15}$ atoms/cm$^3$ as an typical example. The lines A0-A5 are for $\Delta m = (0, -3, -4, -4.71, -10) \times 10^{-6}$ K, and the oblique straight line is the critical border of the BEC region. The $n_B$ are found to decrease with decreasing $T$; it is because the number of composite fermions increases in low-$T$. In small $\Delta m$ cases (A0-A3), the $n_B$...
is still large in low-$T$ and free bosons can condensate into BEC, but, in large $\Delta m$ cases (A4,A5), free bosons are exhausted in making composite fermions and the $n_B$ becomes too small for the BEC realization. The line A6 is just to the critical case. In high-$T$, $n_B$ approaches to $n_{B\text{top}}$ in all cases; it is because all composite fermions dissociate into free bosons and fermions in the limit of $T \to \infty$.

When $n_{B\text{tot}} < n_{F\text{tot}}$, we can obtain the similar $T$-dependence in $n_B$ as in Fig. 1, but, when $n_{B\text{tot}} > n_{F\text{tot}}$, the BEC always occur in enough low-$T$ because, after all fermions are paired, the free bosons still remain.

Let’s consider the $T=0$ case, where the condition (3) becomes $0 + \epsilon_F = \epsilon_{BF} = \Delta mc^2$, where $\epsilon_a = \frac{(3\pi^2)^{2/3}}{2^{1/3}m_a}n_a^{2/3} (a = B,BF)$. We can solve this condition algebraically and obtain the phase structures at $T = 0$. In Fig. 3, we show the phase diagram in $n_{F\text{tot}} - \Delta m$ plane in the case of $n_{F\text{tot}} = 10^{15}$ atoms/cm$^3$, where the symbol (B,F,BF) means the coexistence of free bosons and free and composite fermions, and so on. In the same figure, we also show phases for the $\Delta m > 0$ case, which corresponds to those with the resonance BF state.

From this diagram, we can read off the criterion for BEC to occur; it should occur in the regions when free bosons exist at $T = 0$, e.g the ones with the symbol B in Fig. 3.

4. Summary and Discussion
In summary, we studied bound-state structure of the composite fermion in the BF mixed condensate in finite-$T$ and showed that the $T$-dependence of the binding energy is shown to be small because of the cancellation of boson- and fermion- statistical factors in the scattering equation. We also obtained the phase structure of the mixed condensate by solving the equilibrium condition for the clusterization process B + F $\leftrightarrow$ BF.

More details of the equilibrium calculation has been give in [7], and further applications of the present results should be discussed in the future publication [8].

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