Unbound geodesics from the ergosphere and potential observability of debris from ultrahigh energy particle collisions

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Particle collisions in black hole ergoregions may result in extremely high center of mass energies that could probe new physics if escape to infinity were possible. Here we show that some geodesics at high inclinations to the equatorial plane may be unbound. Hence a finite flux of annihilation debris plays a role. For a class of Penrose processes, we show that the Wald inequalities are satisfied, allowing the Penrose process to have a key role in high energy ejection. Hence the possibility of observing new physics effects from a black hole accelerator at unprecedentedly high particle collision energies remains a tantalizing, if futuristic, experimental vision.

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I. INTRODUCTION

It has been proposed that one consequence of a density spike of cold dark matter (DM) around a supermassive black hole (BH)\textsuperscript{1} is that particle collisions near the horizon can occur, producing centre of mass (c.m.) energy significantly larger than those obtained in the case of particle collisions in the absence of a BH\textsuperscript{2}. This effect (often referred to as the BSW effect) is especially interesting in the case of an extremal Kerr BH, where infinite energies can be realized, at least in principle\textsuperscript{3}, a suggestion that still remains valid despite several controversial issues.

One is a back-reaction argument against the acceleration given by\textsuperscript{4,5} but implicitly disputed by\textsuperscript{6,8}. Another, for us the most urgent issue to be addressed, is that of the limiting energy at infinity and the amount of any escaping flux of energetic annihilation debris\textsuperscript{9,10}. We believe that resolution of both of these issues is still incomplete. Specifically, previous treatments are restricted only to the BSW effect and most of the time only focus on geodesics in the equatorial plane. However it has been noted that there is the possibility of unbound geodesics with high energies from the ergoregion being preferentially directed near the rotation axis of a Kerr black hole\textsuperscript{11,13} and we believe that the Penrose process (PP)\textsuperscript{27} has a role to play in generating these high energies.

Here we develop a simple model in the test particle approximation that allows us to estimate the fraction of unbound geodesics collimated along the $z$-axis from within the ergoregion (sections II to IV). Our motivation is that collisions on near-horizon orbits around a Kerr black hole could contain unusual physics signatures, such as flavor violations, that may survive and yield a finite flux at infinity, even though highly redshifted, due to Penrose boosting of the energetics of collisional debris in the ergosphere\textsuperscript{16}. We reconsider in section V the possibilities offered by the PP.

The efficiency of energy extraction from the BH by the PP has been claimed to be low\textsuperscript{17,18}, mainly due to the strong restrictions imposed by the very general Wald inequalities\textsuperscript{28}. But this does not exclude the possibility of obtaining high energies observable at infinity, due for instance to the ratio of the mass ejected from the ergosphere to the mass falling into the BH as suggested in\textsuperscript{23}. We shall see in an example studied in section V that this is effectively a possibility. Moreover, there are possible additional contributions, via the decay of heavy DM particles\textsuperscript{25}, and of multiple collisions inside the ergosphere\textsuperscript{21}.

If escape to infinity is possible even for a small fraction of the debris orbits, then observability of new physics at energies unattainable in any terrestrial particle accelerator becomes an intriguing option.

II. BETWEEN THE ERGOREGION AND THE HYPERBOLIC LIMITING SURFACE

We place ourselves in the plane $(\rho, z)$ of Weyl coordinates $(\rho, z, \phi)$\textsuperscript{14,15} where any spatial figure (as in Fig. 2 below) can be easily completed in 3 dimensions by simple rotation around the $z$-axis. We restrict ourselves to unbound geodesics that are able to lead to some collimation, as studied in\textsuperscript{11,13}. We call ”collimated” geodesics the geodesics satisfying the condition $\rho/z \ll 1$ when $z \rightarrow \infty$. This requires in particular that their angular momentum is null $L_\phi = 0$ (see the leading term of equation (29), with equation (30), in\textsuperscript{11}). Let us recall that these papers demonstrate the existence of Kerr geodesics asymptotic to directions $\rho_1$ parallel to the $z$-axis, with $\rho_1^2 = a^2 + Q/(E^2 - 1)$, where $a$ is the spin (angular momentum by unit of mass) of the BH of mass $M$ (we put $M = 1$), $E$ the energy at infinity of the par-
particle and $Q$ the Carter constant.

Besides the preceding "perfectly" collimated geodesics, i.e. with asymptotes $\rho_1$, there is an infinity of (imperfectly) collimated geodesics defined from the set of parameters $(E, Q, L_z = 0)$, or equivalently in this case $(E, \rho_1, L_z = 0)$, even when $E$ and $\rho_1$ are fixed, depending on the initial conditions inside the ergosphere. We note that the geodesics are not linear equations. Though there is only one geodesic perfectly collimated for each value of $\rho_1$ (when $E$ is fixed), there is an infinity of (imperfectly) collimated geodesics for the same values of $E$ and $\rho_1$, converging towards the axis or diverging from it (e.g. see figures 2 and 5 in [13]).

We begin by determining the coordinates of the point $A$ of intersection of the trace in the plane $(\rho, z)$ of the ergosphere with the characteristic hyperbolic surface (when it exists). The trace in the plane $(\rho, z)$ of this hyperbolic surface is a limiting geodesic that bounds all the collimated unbound geodesics defined for the chosen parameters $(E, \rho_1)$ and we call this the hyperbolic limiting geodesic. Indeed, all of these geodesics cannot cross this hyperbolic surface, so that, starting from the ergosphere, they are generated inside the part of the ergosphere located between this hyperbolic surface and the ergosphere for $z > z_0$ where $z_0$ is the intersection of the hyperbolic surface and the $z$-axis.

The equation of the hyperbolic limiting surface is given by (see equation (39) in [12])

\[
\left(1 - \left(\frac{\rho_1}{a}\right)^2\right)^{-1} z^2 - \left(\frac{\rho_1}{a}\right)^{-2} \rho^2 = 1 - a^2, \tag{1}
\]

with

\[
\left(\frac{\rho_1}{a}\right)^2 = 1 - \mu_i^2, \tag{2}
\]

where $\mu_i^2 = -Q/[a^2(E^2 - 1)]$ is a special value of the variable $\mu \equiv \cos \theta$. The Carter constant $Q$ is necessarily negative (when positive, there is no hyperbolic limiting surface).

The point $A$ of intersection of the ergosphere [11, 13], defined by the equation

\[
z^2 = \left[1 - a^2 \left(1 - \frac{\rho_1}{a}\right)\right] \left(1 - \frac{\rho}{a}\right),
\]

with the hyperbolic limiting surface [1] is obtained from the equation

\[
\rho^2 - \left(2a - 1 \frac{a}{a}\right) \left(\frac{\rho_1}{a}\right)^2 \rho - (1 - a^2) \left(\frac{\rho_1}{a}\right)^4 = 0, \tag{3}
\]

with solutions

\[
\rho_A = \frac{\rho_1}{a}, \quad \rho_A = \left(1 - \frac{1}{a^2}\right) \frac{\rho_1^2}{a}. \tag{4}
\]

In [4] only the first solution is physically acceptable, since the second is negative, giving

\[
z_A^2 = (1 - a^2 + \rho_1^2) \left[1 - \left(\frac{\rho_1}{a}\right)^2\right], \tag{5}
\]

which is always positive ($Q < 0$).

We can now determine the surface $\Sigma$, in the plane $(\rho, z)$, extended between the ergoregion and the hyperbolic limiting surface, which is given by $\Sigma = \Sigma_1 - \Sigma_2$ where

\[
\Sigma_1 = \int_0^{\rho_A} z_{\text{ergo}}(\rho) d\rho, \quad \Sigma_2 = \int_0^{\rho_A} z_{\text{hyperbolic}}(\rho) d\rho. \tag{6}
\]

We now obtain

\[
\Sigma_1 = \frac{1}{16a^2} [\sin(4\rho_A) - \sin(4\nu_0) - 4(\nu_A - \nu_0)], \tag{7}
\]

with $\nu_0 = \arcsin(a)$ and $\nu_A = \arcsin(a\mu_i)$, and

\[
\Sigma_2 = (1 - a^2) \frac{\rho_1}{2a} \mu_i \left[\arcsinh \left(\frac{\rho_1}{\sqrt{1 - a^2}}\right) + \frac{\rho_1}{\sqrt{1 - a^2}} \left(1 + \frac{\rho_1^2}{1 - a^2}\right)^{1/2}\right]. \tag{8}
\]

III. THE ANGLE OF THE CONE FORMED BY PARTICLES LEAVING TO INFINITY

Let us look at the influence of the BH spin $a$ on the angle of the cone which limits the particles escaping to infinity.

The asymptote of the hyperbolic limiting surface is obtained for $\rho \to \infty$ and $z \to \infty$ and satisfies $(\rho/z)^2 \to (1 - \mu_i^2)/\mu_i^2$ or $\tan^2 \theta_i = \lim(\rho/z)^2$. Hence we see that, for $Q$ negative, which is the condition for $\rho_1$ to be associated with the angle $\theta_i$, (2) gives a direct link between $\rho_1$ and the angle $\theta_i$ of the outgoing cone.

To see how this angle $\theta_i$ behaves with respect to $a$, we need to express $\rho_1$ as a function of $a$. In order to do this we adopt the case of "double roots", $r_1 = r_2 = Y$, considered in [12]. The difference is that here we are not going to fix particular values of $a$ but leave it as a variable.

However we fix the energy $E$ by assuming $E \to \infty$ for two reasons. Firstly there exists a precise value for $\rho_1$ (for a fixed $a$) for $E \to \infty$ and a small range for $\rho_1$ with $E$ rapidly decreasing. This indicates a narrow beam that is very collimated in a domain near $\rho_1$ of particles moving to infinity and covering almost the entire spectrum of energies [12]. Secondly, the recently found BSW effect defined in [2] predicts the possibility of infinite energy for particles produced near the horizon [13]. We note that the possibility of a significant flux of these particles ever attaining infinite energies is highly controversial [9, 24]. Earlier work however has not considered detailed geodesics out of the equatorial plane. In fact, there are geodesics along or near the spin axis that are unbound.

Hence, considering $E \to \infty$ we have the following equation (13) in [12]

\[
a^4(Y + 1) + 2a^2(Y - 1)Y^2 + (Y - 3)Y^4 = 0. \tag{9}
\]
We find only one solution \( a^2 \) which is positive for \(-1 \leq Y \leq 3\),
\[
Y^3 - 3Y^2 + (Y + 1)a^2 = 0,
\]
producing three solutions for \( Y(a) \). If we introduce each of these solutions into equation (14) of [12] for \( \rho^2 /a^2 \), we obtain only one function of \( a \) positive and inferior to one, a condition equivalent to \( Q < 0 \). As a result, we have an expression for \( \theta_i \), the angle of the outgoing cone of geodesics attaining infinity, as a function of \( a \): see fig. 1.

**IV. THE ADMISSIBLE ERGOREGION**

We look for the surface \( \Sigma(a) \) when \( E \to \infty \) as considered previously, i.e. via the expression \( \rho_1(a) \) found in this case. We plot the ratio \( \Sigma(a)/S(a) \) in Fig. 1, where the total surface of the ergosphere region \( S(a) \) is given by equation (3) of [13]. This ratio can be interpreted as the probability of having a particle which can incur a PP and be ejected to infinity with a great energy inside the limiting cone among all the particles falling into the ergoregion. We can see that for \( a > 0.8 \) this probability becomes \( > 0.2 \) and reaches about 0.33 for \( a = 1 \).

Consider the following numerical example. For \( a = 0.997 \), and \( E \to \infty \) which, for a double root, corresponds to \( Y = -0.413334 \), we obtain \( \rho_1 = 0.825428, \sin(\theta_i) = 0.827911 \) or \( \theta_i = 0.975373 \text{ rad} = 55.848^\circ \) and the ratio \( (\Sigma/S) = 0.319328 \).

We plot in Fig. 2 the ergoregion and the hyperbolic limiting surface for several values of \( a \). See Table I for color code and corresponding values of \( \rho_1 \) and \( \Sigma/S \). The values of \( \rho_1 \) corresponds to \( E \to \infty \). \( \rho_1 \) is here always \( < a \) (i.e. the Carter constant \( Q < 0 \)). The admissible ergosphere region for the initial conditions of particles able to go to infinity inside the cone with a large energy is the region of intersection situated between any two curves of the same color. Note that \( a = 0.877 \) corresponds to the maximal cross section \( S \) of the ergovolume. The coordinates \( (\rho, z) \) are dimensionless. The "true" corresponding distances are \( \hat{\rho} = \rho M \) and \( \hat{z} = zM \) where the BH mass \( M \) is measured in number of solar masses \( M_{\odot} \), itself gravitationally equivalent to the distance: \( M_{\odot} = 1.48 \text{km} \).

| \( a \)     | \( \rho_1 \) | \( \Sigma/S \) |
|-----------|------------|---------------|
| 0.5 (Red) | 0.3466     | 0.11          |
| 0.7 (Magenta) | 0.5336 | 0.17          |
| 0.877 (Green) | 0.7062 | 0.24          |
| 0.95 (Blue) | 0.7785     | 0.28          |
| 0.999 (Black) | 0.8284 | 0.33          |

**TABLE I: Spin and hyperbolic limiting surface parameters**

(Fig. 2)

**V. PENROSE EFFECT AND WALD INEQUALITIES**

It now remains for us to determine the mechanism by which these high energy geodesics can be populated. The Penrose mechanism (PP) was suggested as a mechanism for fulfilling this role shortly after its discovery [27]. However this idea was abandoned once it was shown how the Wald inequalities [28], or their generalization to collisions [29], constrained this process. The modest Penrose mechanism efficiency [17] at best permits a gain of the same order of magnitude as the energy of the incoming particle [15] [19]. Thus the expected benefit of the Penrose
effect appears rather low when compared to any similar injection phenomenon (particle decay or collision) which would even occur in the absence of a Kerr BH [14, 23].

The idea of associating an external field to enlarge the range of the Wald inequalities and to increase the PP efficiency was then suggested, with, as the most likely example, an electromagnetic field [30], or magnetohydrodynamic effects [31]. The Penrose-type magnetically driven jet proposed by Blandford and Znajek is now the most commonly accepted Kerr BH acceleration mechanism [20, 32].

More recently, another effect, the so-called BSW effect was discovered [2]. At first sight, this seems to be a good candidate for fulfilling the role of an accelerator. This effect predicts the appearance of a large local energy (in c.m.) during a collision that would occur under specific conditions (extremal Kerr BH, very near to the horizon, in the equatorial plane, with a precise angular momentum derived from one of the incoming particles). It has since been generalized in various forms, including to the ergosphere outside the equatorial plane [25] or to multiple collisions [21] or to the case of particles with large angular momenta [7, 22]. These applications are almost always in the ergosphere, showing that the ergosphere plays an important (at least local) role as "accelerator" of particles (i.e., able to provide a high energy) [23]. However it has been shown that the energy provided by the latter effect, being local, undergoes, because of its proximity to the centre, a large redshift making it unobservable [9, 18].

We propose a new solution to this problem, combining the BSW effect and the PP. The remarkable feature of the PP that we put forward is its ability to retransmit the BSW effect and the PP. The remarkable feature of the PP is its ability to retransmit the redshift. Considering an initial particle with a negative energy to the locally produced high energy particles. The PP manages to overcome the redshift. We will show the PP that we put forward is its ability to retransmit the BSW effect and the PP. The remarkable feature of the centre, a large redshift making it unobservable [9, 18].

The tension between the PP and the Wald inequalities mainly comes from the fact that the (true) energy $E$ of the incoming particle is assumed to be positive. In the case where the particle comes from infinity at rest ($E = m$), the inequalities give a minimal speed of the falling particle greater than $c/\sqrt{2}$ (necessary to reach a state of negative energy) and a maximal speed for the outgoing particle lower than $c/\sqrt{2}$, not enough to thwart the redshift. Considering an initial particle with a negative energy permits reversal of these inequalities, allowing one to more easily obtain an ultrarelativistic outgoing particle and a falling particle maintained in a state of (more) negative energy.

Consider, in the ergosphere, a Penrose decay from an initial particle, with a mass $m$ and a weakly negative (true) energy $E = -m \gamma - m \epsilon$ ($0 < \epsilon \ll 1$), into a particle of mass $m_{\text{fall}} < m$ falling into a more bound state, i.e., with a more negative (true) energy $E_{\text{fall}} = -m_{\text{fall}} \epsilon$ and with, as we shall see, $m_{\text{fall}}/m_{\text{out}} = \epsilon^{-1} \sim \epsilon^{-2} \gg 1$, and a particle of lower mass $m_{\text{out}}$ ejected with a (true) high positive energy $E_{\text{out}}$.

Locally, in the c.m. frame, linked to the initial particle, the conservation of linear momentum reads:

$$m_{\text{fall}} \gamma_{1} v_{1} - m_{\text{out}} \gamma_{2} v_{2} = 0,$$

where $v_{i}$ is the relative velocity of the $i$th particle ($i = 1 = \text{fall}, i = 2 = \text{out}$) in the c.m. frame, and $\gamma_{i} = (1 - v_{i}^{2})^{-1/2}$ its associated Lorentz factor. Expressing the fact that the larger mass $m_{\text{fall}}$ has a slight recoil compared to the small mass $m_{\text{out}}$ by the relations

$$v_{1} = \epsilon, \quad \text{and} \quad \gamma_{2} = \epsilon^{-1},$$

the preceding equation becomes by noting that $v_{2} = 1/\gamma_{1} = (1 - \epsilon^{2})^{1/2}$,

$$\frac{m_{\text{out}}}{m_{\text{fall}}} = \frac{\gamma_{1}}{v_{2}} \epsilon^{2} = \frac{\epsilon^{2}}{1 - \epsilon^{2}} \simeq \epsilon^{2} + \epsilon^{4}.$$  

In all series expansions, we will limit ourselves to order $\epsilon^{4}$.

We also have

$$m = m_{\text{fall}} + m_{\text{out}} - |\Delta m| \approx m_{\text{fall}} + m_{\text{out}},$$

where the binding energy $|\Delta m|$ is assumed to be very small compared to the smallest mass, i.e., $|\Delta m| \ll m_{\text{out}} \ll m_{\text{fall}}$ or $m$. For example, it is sufficient to take

$$|\Delta m| = \epsilon.$$  

Hence, substituting (15) into (14) gives

$$\frac{m_{\text{fall}}}{m} = 1 - \frac{m_{\text{out}}}{m} = \frac{m_{\text{out}}}{m} \epsilon.$$  

On the other hand, using (13) and (16), we have

$$\frac{m_{\text{out}}}{m} = \frac{m_{\text{out}} m_{\text{fall}}}{m} = \frac{\epsilon^{2}}{1 - \epsilon^{2}} \left[ 1 - \frac{m_{\text{out}}}{m} (1 - \epsilon) \right],$$  

or

$$\frac{m_{\text{out}}}{m} \left[ 1 + \frac{\epsilon^{2} (1 - \epsilon)}{1 - \epsilon^{2}} \right] = \frac{\epsilon^{2}}{1 - \epsilon^{2}},$$  

i.e.,

$$\frac{m_{\text{out}}}{m} = \frac{\epsilon^{2}}{1 - \epsilon^{2}} \simeq \epsilon^{2}.$$  

From (16) we also obtain

$$\frac{m_{\text{fall}}}{m} = \frac{1 - \epsilon^{2}}{1 - \epsilon^{3}} \simeq 1 - \epsilon^{2} + \epsilon^{3}.$$  

Secondly, we have the Penrose equality

$$E_{\text{out}} = |E_{\text{fall}}| - |E|,$$

and to have a significant Penrose effect, the condition $|E_{\text{fall}}| \gg |E|$ is necessary, consistently with what we saw earlier, $m_{\text{out}}/m_{\text{fall}} \simeq \epsilon^{2} + \epsilon^{4}$. Thus, in this example,
the mass effect is the source of the Penrose effect. The Penrose equality becomes

$$\Gamma_{\text{out}} = \frac{E_{\text{out}}}{m_{\text{out}}} = \frac{|E_{\text{fall}}|}{m_{\text{fall}}} \frac{m_{\text{fall}}}{m_{\text{out}}} - \frac{|E|}{m_{\text{out}}} = \frac{1}{\epsilon} - 2\epsilon. \quad (22)$$

Finally, consider the Wald inequalities (equation (4) in [28]). For the particle $m_{\text{fall}}$ we have

$$-\gamma_1 \frac{m_{\text{out}}}{m} \epsilon - \gamma_1 v_1 \left( E^2 \frac{m_{\text{out}}^2}{m^2} + 1 \right)^{1/2} \leq -\epsilon$$

$$\leq -\gamma_1 \frac{m_{\text{out}}}{m} \epsilon + \gamma_1 v_1 \left( \frac{E^2}{m_{\text{out}}^2} \frac{m_{\text{out}}^2}{m^2} + 1 \right)^{1/2}, \quad (23)$$

$$\gamma_1 \epsilon^3 + \gamma_1 \epsilon \left( \epsilon^6 + 1 \right)^{1/2} \geq \epsilon \geq \gamma_1 \epsilon^3 - \gamma_1 \epsilon \left( \epsilon^6 + 1 \right)^{1/2}, \quad (24)$$

$$\gamma_1 \epsilon^2 - \gamma_1 \left( 1 + \frac{\epsilon^6}{2} \right) \leq 1 \leq \gamma_1 \epsilon^2 + \gamma_1 \left( 1 + \frac{\epsilon^6}{2} \right), \quad (25)$$

$$-\gamma_1 \leq 1 \leq \gamma_1, \quad (26)$$

which is always verified for any $v_1$. While for the particle $m_{\text{out}}$ we have

$$-\gamma_2 \frac{m_{\text{out}}}{m} \epsilon - \gamma_2 v_2 \left( 1 + \frac{\epsilon^6}{2} \right)^{1/2} \leq \frac{E_{\text{out}}}{m_{\text{out}}}$$

$$\leq -\gamma_2 \frac{m_{\text{out}}}{m} \epsilon + \gamma_2 v_2 \left( 1 + \frac{\epsilon^6}{2} \right)^{1/2}. \quad (27)$$

The first inequality of (27) is always verified (negative left, positive right), and the second one is

$$\Gamma_{\text{out}} \leq \gamma_2 v_2 - \gamma_2 \epsilon^3 + \gamma_2 \frac{\epsilon^6}{2}, \quad (28)$$

or

$$\Gamma_{\text{out}} = \frac{1}{\epsilon} - 2\epsilon < \frac{1}{\epsilon} \left( 1 - \frac{\epsilon^2}{2} \right) - \epsilon^2 = \frac{1}{\epsilon} - \frac{\epsilon}{2} - \epsilon^2, \quad (29)$$

or $2\epsilon > \epsilon/2$, which is always true. Therefore, the Penrose equality agrees with the Wald inequalities, and we can take

$$\Gamma_{\text{out}} \simeq \frac{1}{\epsilon} = \gamma_2. \quad (30)$$

Thus we see that for large values of $\gamma_2$, or equivalently values $v_2$ approaching 1, due to a mass effect, the particle "out" can have a high energy at infinity (which can be a bit lower than that obtained locally $m_{\text{out}}/\gamma_2$), in agreement with the Penrose equality and unrestricted by the Wald inequalities.

As an immediate consequence, if we define the efficiency of the PP as $\eta' = E_{\text{out}}/|E|$, we have $\eta' = m_{\text{out}} \Gamma_{\text{out}}/m_{\text{out}} \epsilon = 1/\epsilon^2 - 2 \simeq 1/\epsilon^2$, i.e., we obtain a large efficiency.

Let us stress the remarkable result that the same calculations made from the hypothesis of an initial particle with a slightly positive energy $E = +m_{\text{out}} \epsilon$ would lead to violated Wald inequalities. The limiting case $E = 0$ respects the inequalities.

The questions that remain concerning the initial particle are: (i) How was such an initial particle inside the ergosphere able to reach a state of negative energy (in our example above, slightly negative: $|E| \ll m_{\text{out}}$)? and (ii) is it possible in a decay to have high speed ejection?

It seems to us that we can obtain the first result from effects of BSW-type. They are purely gravitational and occur only inside the ergosphere. Though directly unobservable because of the redshift [9], the BSW effect locally produces (in the c.m. frame) high energy and can afford to send a particle, for example coming from infinity at rest $(E = m)$, at a relative speed greater than $1/\sqrt{2} = 0.7$ sufficient to enter a state of negative energy [28]. Thus, the BSW effect, or those of similar type, can be understood as possible "triggers" of the PP. More precisely, they are able to implement the necessary condition (state of negative energy) on a particle they produce to trigger a PP.

Concerning the second question, we note that local physical phenomena can happen, such as $\beta^-$-decays, which can eject particles of small mass (electron, antineutrino), relative to the mass of the expelling particle (radioactive nuclei), with ultrarelativistic speed $v_2 > 0.9$, and a weak effect of recoil ($v_1 \ll 1$). The recoil is sufficient to maintain the large particle in a more "connected" (with regard to the BH) state, that is to say with negative energy higher (in absolute value) than that of the initial particle, triggering the PP. Another example could be the annihilation of DM. For a hypothetical DM particle of mass 100GeV (see for example [26]), the Lorentz factor of a produced outgoing electron can be as high as $\sim 10^3$.

It is likely that Penrose injection will not uniformly fill the ergoregion but may be preferentially enhanced towards the equatorial plane as $a$ increases [24]. This would reduce the effect we consider, since the effective volume available for unbound orbits would be reduced.

VI. CONCLUSION

Our calculations demonstrate that the intersection of a cone along the spin axis, described by us as the hyperbolic limiting surface, with the ergoregion allows the launching of collimated unbound geodesics with high energies. The "two-step" process (BSW effect triggering PP) that we propose shows that it is possible to populate such geodesics. This would boost any potential signal at infinity. Suppose that the ergoregion is populated with particles. Then we can use the product of the ratio of the intersecting volume to the ergoregion with the total annihilating flux in the ergosphere as a crude measure of the flux generated by particle collisions. We conclude that a finite flux of annihilation debris is able to escape to infinity, in the case of near-extremal Kerr black holes. Hence the possibility of observing new physics effects from a
black hole accelerator at unprecedentedly high particle collision energies remains a tantalizing if futuristic experimental vision.

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