Supersymmetric Lepton Flavour Violation at the LHC and LC

F. Deppisch\(^1\), J. Kalinowski\(^2\), H. Päs\(^1\), A. Redelbach\(^1\), R. Rückl\(^1\)

\(^1\) Institut für Theoretische Physik und Astrophysik, Universität Würzburg, D-97074 Würzburg, Germany

\(^2\) Institute of Theoretical Physics, Warsaw University, Warsaw, Poland

Abstract

In supersymmetric extensions of the Standard Model, the Yukawa and/or mass terms of the heavy neutrinos can generate lepton flavour violating slepton mass terms. These new supersymmetric sources of lepton flavour violation may both enhance the rates of charged lepton flavour violating processes, \(l_\alpha \rightarrow l_\beta \gamma\), and generate distinct final states, like \(l_\beta l_\alpha + \text{jets} + E_T\), at future colliders. First, we discuss the sensitivity of future \(e^+e^-\) colliders to the SLFV independently of the lepton flavour violating mechanism. Second, we study lepton flavour violating slepton pair production and decay at a future \(e^+e^-\) linear collider in the context of the seesaw mechanism in mSUGRA post-LEP benchmark scenarios. We investigate the correlations of these signals with the corresponding lepton flavour violating rare decays \(l_\alpha \rightarrow l_\beta \gamma\), and show that these correlations are particularly suited for probing the origin of lepton flavour violation.
1 Introduction

Neutrino oscillations imply the violation of individual lepton flavours and raise the interesting possibility of observing lepton flavour violation in processes with charged leptons, such as $\mu \rightarrow e\gamma$ or $\tau \rightarrow \mu\gamma$. In the Standard Model these processes are strongly suppressed due to small neutrino masses. In the supersymmetric extension of the Standard Model, however, the situation may be quite different. For example, the slepton mass matrices need not simultaneously be diagonalized with the lepton mass matrices. When sleptons are rotated to the mass eigenstate basis, the slepton mass diagonalization matrices $W_i^a$ enter the chargino and neutralino couplings

$$\tilde{e}_i(W_i^*)_{ia}\tilde{e}_a\tilde{\chi}^0 + \tilde{\nu}_i(W_\nu^*)_{ia}\tilde{e}_a\tilde{\chi}^- + \ldots$$

and mix lepton flavour (Latin and Greek subscripts refer to the mass-eigenstate and flavour basis, respectively). Contributions from virtual slepton exchanges can therefore enhance the rates of rare decays like $\mu \rightarrow e\gamma$. Furthermore, once superpartners are discovered, the supersymmetric lepton flavour violation (SLFV) can also be searched for directly at future colliders where the signal will come from the production of real sleptons (either directly or from chain decays of other sparticles), followed by their subsequent decays. Searches for SLFV at colliders have a number of advantages: superpartners can be produced with large cross-sections, flavour violation in the production and decay of sleptons occurs at tree level and therefore is suppressed only by powers of $\Delta m_{\tilde{l}}/\Gamma_{\tilde{l}}$ [1] in contrast to the $\Delta m_{\tilde{l}}/m_{\tilde{l}}$ suppression in radiative lepton decays, where SLFV occurs at one-loop ([2] and references therein). Generally, respecting the present bounds on rare lepton decays, large SLFV signals are possible both at the LHC [3] and at $e^+e^-$ colliders [1, 4–8]. This suggests that in some cases the LHC and future $e^+e^-$ colliders may provide competitive tools to search for and explore supersymmetric lepton flavour violation.

In this note we first discuss the sensitivity at future $e^+e^-$ colliders to SLFV independently of the lepton flavour violating mechanism. The simulation has been performed assuming a simplified situation with a pure 2-3 intergeneration mixing between $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$, and ignoring any mixings with $\tilde{\nu}_e$. In the analysis the mixing angle $\tilde{\theta}_{23}$ and $\Delta m_{23} = |m_{\tilde{\nu}_2} - m_{\tilde{\nu}_3}|$ have been taken as free, independent parameters [6].

In the second part, SLFV generated by the seesaw mechanism is considered. The heavy right-handed Majorana neutrinos give rise not only to light neutrino masses but also to mixing of different slepton flavours due to the effects of the heavy neutrinos on the renormalization-group running of the slepton masses. The implications of recent neutrino measurements on this mixing are investigated. Moreover we emphasize the complementarity of the radiative decays $l_\alpha \rightarrow l_\beta\gamma$ and the specific lepton flavour violating processes $e^\pm e^- \rightarrow l^\pm l_\alpha\tilde{\nu}_b\tilde{\chi}_0^0$ involving slepton pair production and subsequent decay [8].

2 Sensitivity at future $e^+e^-$ colliders to SLFV

In discussing the SLFV collider signals at future colliders, one has to distinguish two cases in which an oscillation of lepton flavour can occur: in processes with slepton pair production and in processes with single slepton production, which differ in the interference of the intermediate sleptons [1]. Slepton pair production is the dominant mechanism at lepton colliders, but it may also occur at hadron colliders via the Drell-Yan process. Single
sleptons may be produced in cascade decays of heavier non-leptonic superparticles. Such processes are particularly important for hadron colliders, but they may also be relevant for lepton colliders where a single slepton can be the decay product of a chargino or neutralino.

The amplitudes for pair production, $\bar{f}f \rightarrow \tilde{l}_i^+ \tilde{l}_i^- \rightarrow l_i^+ X l_i^- Y$, and single production, $f f' \rightarrow l_i^+ X \tilde{l}_i^- \rightarrow l_i^+ X \tau_i^- Y$, read, e.g.,

$$\mathcal{M}_{\alpha\beta}^\text{pair} = \sum_i \mathcal{M}_P^\text{pair} \frac{i}{q^2 - m_i^2 + i\bar{m}_i \Gamma_i} W_{\alpha\alpha}^i \mathcal{M}_D^i \frac{i}{p^2 - m_i^2 + i\bar{m}_i \Gamma_i} W_{\alpha\beta}^i \mathcal{M}_D^- \quad (\text{s-channel}) \quad (2)$$

$$\mathcal{M}_{\alpha\beta}^\text{sin} = \sum_i \mathcal{M}_P^\text{sin} W_{\alpha\alpha}^i \frac{i}{q^2 - m_i^2 + i\bar{m}_i \Gamma_i} W_{\alpha\beta}^i \mathcal{M}_D^- \quad (3)$$

where $\mathcal{M}_P$ and $\mathcal{M}_D$ are the respective production and decay amplitudes for sleptons in the absence of SLFV, and $W_{\alpha\beta}$ stands for the lepton flavour mixing matrix element.

For nearly degenerate in mass and narrow sleptons, $\Delta \bar{m}_{ij} \ll \bar{m}$ and $\bar{m}\bar{\Gamma}_{ij} \simeq (\bar{m}_i \Gamma_i + \bar{m}_j \Gamma_j)/2 \ll \bar{m}^2$, the products of slepton propagators can be simplified as follows

$$\frac{i}{q^2 - m_i^2 + i\bar{m}_i \Gamma_i} \frac{-i}{q^2 - m_j^2 - i\bar{m}_j \Gamma_j} \sim \frac{1}{1 + i \Delta \bar{m}_{ij}/\bar{m}\bar{\Gamma}_{ij}} \frac{\pi}{\bar{m}\bar{\Gamma}_{ij}} \delta(q^2 - \bar{m}^2). \quad (4)$$

Then, in the case of 2-3 intergeneration mixing, the cross-sections for the above processes (2 3), take a particularly simple form [9]:

$$\sigma_{\alpha\beta}^\text{pair} = \chi_{23} (3 - 4\chi_{23}) \sin^2 2\bar{\theta}_{23} \sigma(f f \rightarrow \tilde{l}_i^+ \tilde{l}_i^-) Br(\tilde{l}_i^+ \rightarrow l_i^+ X) Br(\tilde{l}_i^- \rightarrow l_i^- Y) \quad (5)$$

$$\sigma_{\alpha\beta}^\text{sin} = \chi_{23} \sin^2 2\bar{\theta}_{23} \sigma(f f' \rightarrow l_i^+ X \tilde{l}_i^-) Br(\tilde{l}_i^- \rightarrow l_i^- Y) \quad (6)$$

where $\sigma(f f' \rightarrow l_i^+ X \tilde{l}_i^-)$, $\sigma(f f \rightarrow \tilde{l}_i^+ \tilde{l}_i^-)$ and $Br(\tilde{l}_i^- \rightarrow l_i^- Y)$ are the corresponding cross-sections and branching ratios in the absence of flavour violation. The slepton flavour violating mixing effects are encoded in

$$\chi_{23} = \frac{x_{23}^2}{2(1 + x_{23}^2)} \quad \text{and} \quad \sin^2 2\bar{\theta}_{23} \quad (7)$$

where $x_{23} = \Delta \bar{m}_{23}/\bar{\Gamma}_{23}$. In the limit $x_{23} \gg 1$, $\chi_{23}$ approaches 1/2, the interference can be neglected and the cross-sections behave as $\sigma \sim \sin^2 2\bar{\theta}_{23}$. In the opposite case, the interference suppresses the flavour changing processes, and $\sigma \sim (\Delta \bar{m}_{23} \sin 2\bar{\theta}_{23})^2$.

To assess the sensitivity of a 500 GeV $e^+e^-$ linear collider to the SLFV, the following processes have been analysed

$$e^+e^- \rightarrow \tilde{\nu}_2 \tilde{\nu}_2 \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^0 \tilde{\chi}_1^0 \quad (8)$$

$$e^+e^- \rightarrow \tilde{\chi}_2^+ \tilde{\chi}_1^- \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^0 \tilde{\chi}_1^0 \quad (9)$$

$$e^+e^- \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^- \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^0 \tilde{\chi}_1^0 \quad (10)$$

Here $\tilde{\chi}_1^0 \rightarrow \tilde{\chi}_0^0 f f'$, and $\tilde{\chi}_0^0$ escapes detection. The signature of SLFV would be $\tau^\pm \mu^\mp + 4 \text{ jets} + E_T^*, \tau^\pm \mu^\pm + \ell + 2 \text{ jets} + E_T^*$, or $\tau^\pm \mu^\pm + E_T^*$, depending on the hadronic or leptonic $\tilde{\chi}_1^0$ decay mode. The purely leptonic decay modes are overwhelmed by background. In particular, the neutralino pair production process (10), which could still be open if
Figure 1: Various $3\sigma$ significance contours in the $\Delta m_{23} - \sin 2\theta_{23}$ plane, for the SUSY point mentioned in the text. The contours A and B show the integrated signals (8-9) at $\sqrt{s} = 500$ GeV and for 500 fb$^{-1}$ and 1000 fb$^{-1}$, respectively. The contour C shows the $\tilde{\nu}\tilde{\nu}'$ contribution separately for 500 fb$^{-1}$ [6]. The dotted lines indicate contours for $\text{Br}(\tau \rightarrow \mu\gamma) = 10^{-7}, 10^{-8}$ and $10^{-9}$ [11].

the second chargino and sleptons were too heavy for (8) and (9), is difficult to extract from background. On the other hand, with charginos decaying hadronically, the signal $\tau^+\mu^+ + 4$ jets + $E_T$ comes from both processes (8) and (9) and is SM-background free. The flavour-conserving processes analogous to (8) and (9), but with two $\tau$'s in the final state where one of the $\tau$'s decays leptonically to $\mu$, contribute to the background. On the other hand, if jets are allowed to overlap, an important SM background to the final states with $\tau^+\mu^+ + \geq 3$ jets + $E_T$ comes from $e^+e^- \rightarrow t\bar{t}g$.

The simulation of the signal and background has been performed for one of the MSSM representative points chosen for detailed case studies at the ECFA/DESY Workshop [10]: a mSUGRA scenario defined by $m_0 = 100$ GeV, $M_{1/2} = 200$ GeV, $A_0 = 0$ GeV, $\tan\beta = 3$ and $\text{sgn}(\mu) = +$. A simple parton level simulation has been performed with a number of kinematic cuts listed in [6]. For the processes (8) and (9) we find after cuts the following cross-sections, $\chi_{23}(3-4\chi_{23}) \sin^2 2\theta_{23} \times 0.51$ fb and $\chi_{23} \sin^2 2\theta_{23} \times 0.13$ fb, respectively, while the background amounts to 0.28 fb.

In Fig. 1 the significance is given by $\sigma_d = \frac{S}{\sqrt{S+B}}$ where S and B are the numbers of signal and background events, respectively, for a given luminosity. Shown is the region (to the right of the curves) in the $\Delta m_{23} - \sin 2\theta_{23}$ plane that can be explored or ruled out at a $3\sigma$ level at a linear collider of energy 500 GeV for the given integrated luminosity. The contour A is for 500 fb$^{-1}$ and B for 1000 fb$^{-1}$. For comparison, the boundary C shows the reach in the process $\tilde{\nu}_i\tilde{\nu}'_i$ alone (previously studied in [1,5]) using our cuts and assuming a luminosity of 500 fb$^{-1}$. The chargino contribution increases the sensitivity range to $\sin 2\theta_{23}$ by 10-20%, while the sensitivity to $\Delta m_{23}$ does not change appreciably.

In the same figure, the contour lines for constant branching ratios of $\tau \rightarrow \mu\gamma$ are shown for comparison [11]. In the limit of small mass splitting, $\text{Br}(\tau \rightarrow \mu\gamma)$ can be calculated
in the flavour basis using the mass insertion technique [12]. In our 2-3 intergeneration mixing scenario the radiative process $\tau \rightarrow \mu \gamma$ constrains the combination of parameters

$$\delta_{\mu\tau} = \sin 2\tilde{\theta}_{23} \Delta \tilde{m}_{23}/\tilde{m}. \quad (11)$$

The contours in Fig. 1 have been obtained from the approximate formula of Ref. [13], normalized to the current experimental limit,

$$Br(\tau \rightarrow \mu \gamma) \sim 1.1 \times 10^{-6} \left( \frac{\delta_{\mu\tau}}{1.4} \right)^2 \left( \frac{100 \text{ GeV}}{\tilde{m}} \right)^4. \quad (12)$$

This approximation only provides an order of magnitude estimate of the upper limit for the supersymmetric contribution to the radiative lepton decay. The exact result, which is sensitive to the details of mass spectra and mixings, can in fact be much smaller due to cancellations among different contributions [2]. Fig. 1 demonstrates that information from slepton production and decay could be competitive to the radiative lepton decays. In particular a LC can help to explore the small $\Delta \tilde{m}_{23}$ region. It should be stressed, though, that in a given model for lepton flavour violation also the correlation with $\mu \rightarrow e\gamma$ has to be considered [8], which in many cases can yield a more severe bound, as discussed in the next section.

### 3 Case study for the supersymmetric seesaw model

As a definite and realistic example for SLFV we consider the seesaw mechanism in mSUGRA models. In supersymmetric theories with heavy right-handed Majorana neutrinos, the seesaw mechanism [14] can give rise to light neutrino masses at or below the sub-eV scale. Furthermore, the massive neutrinos affect the renormalization group running of the slepton masses, generating flavour off-diagonal terms in the mass matrix. These in turn lead to SLFV in scattering processes at high energies and in rare decays. For illustration of the potential and complementarity of such SLFV searches we focus on the LC processes $e^\pm e^- \rightarrow l_\alpha^+ l_\beta^- \tilde{\chi}_0^0 \tilde{\chi}_0^0$ involving slepton pair production and subsequent decay, and on the corresponding radiative decay $l_\alpha \rightarrow l_\beta \gamma$. In particular, in an early ATLAS note [15] $\tau \rightarrow \mu \gamma$ is estimated to be observable at the LHC for a branching ratio of order $10^{-7}$. However, the limit one can reasonably expect may be an order of magnitude better [16].

For our study we use the mSUGRA benchmark scenarios proposed in [17] for LC studies, concentrating on those which predict charged left-handed sleptons that are light enough to be pair-produced at the center-of-mass energy $\sqrt{s} = 500 \text{ GeV}$. Furthermore, we implement the seesaw mechanism assuming degenerate Majorana masses for the right-handed neutrinos and constrain the neutrino Yukawa couplings by the measured masses and mixings of the light neutrinos. Further sources of SLFV exist in other models such as GUTs [18]. However, no realistic three generation case study of effects for collider processes has been performed so far, so that we restrict the discussion to the minimal seesaw model, here.
3.1 Supersymmetric seesaw mechanism

If three right-handed neutrino singlet fields $\nu_R$ are added to the MSSM particle content, one has the additional terms [19]

$$W_\nu = -\frac{1}{2} \nu_R^c T M \nu_R + \nu_R^c Y_{\nu} L \cdot H_2$$

(13)

in the superpotential. Here, $Y_{\nu}$ is the matrix of neutrino Yukawa couplings, $M$ is the right-handed neutrino Majorana mass matrix, and $L$ and $H_2$ denote the left-handed lepton and hypercharge $+1/2$ Higgs doublets, respectively. At energies much below the mass scale $M_R$ of the right-handed neutrinos, $W_\nu$ leads to the following mass matrix for the light neutrinos:

$$M_\nu = Y_{\nu}^T M^{-1} Y_{\nu} (v \sin \beta)^2.$$  (14)

From that the light neutrino masses $m_1, m_2, m_3$ are obtained after diagonalization by the unitary MNS matrix $U$. The basis is chosen such that the matrices of the charged lepton Yukawa couplings and Majorana masses are diagonal, which is always possible.

Furthermore, the heavy neutrino mass eigenstates give rise to virtual corrections to the slepton mass matrix that are responsible for lepton flavour violating processes. More specifically, in the mSUGRA models considered, the mass matrix of the charged sleptons is given by

$$m_i^2 = \begin{pmatrix} m_{1L}^2 & (m_{1R}^2)^T \\ m_{1R}^2 & m_{2R}^2 \end{pmatrix}$$  (15)

with

$$(m_{1L}^2)_{ij} = (m_{1L}^2)_{ij} + \delta_{ij} \left( m_{1L}^2 + m_{2R}^2 \cos 2\beta \left( -\frac{1}{2} + \sin^2 \theta_W \right) \right)$$

$$(m_{1R}^2)_{ij} = (m_{1R}^2)_{ij} + \delta_{ij} \left( m_{1L}^2 - m_{2R}^2 \cos 2\beta \sin^2 \theta_W \right)$$

$$(m_{1R}^2)_{ij} = A_{ij} v \cos \beta - \delta_{ij} m_{L} \mu \tan \beta.$$  

When $m_i^2$ is evolved from the GUT scale $M_X$ to the electroweak scale characteristic for the experiments, one obtains

$$m_L^2 = m_0^2 1 + (\delta m_L^2)_{\text{MSSM}} + \delta m_L^2$$  (16)

$$m_R^2 = m_0^2 1 + (\delta m_R^2)_{\text{MSSM}} + \delta m_R^2$$  (17)

$$A = A_0 Y_{l} + \delta A_{\text{MSSM}} + \delta A,$$  (18)

where $m_0$ is the common soft SUSY-breaking scalar mass and $A_0$ the common trilinear coupling. The terms $(\delta m_{L,R}^2)_{\text{MSSM}}$ and $\delta A_{\text{MSSM}}$ are well-known flavour-diagonal MSSM corrections. In addition, the evolution generates the off-diagonal terms $\delta m_{L,R}^2$ and $\delta A$ which, in leading-log approximation and for degenerate right-handed Majorana masses $M_i = M_R, i = 1, 2, 3$, are given by [20]

$$\delta m_L^2 = -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) (Y_{\nu}^T Y_{\nu}) \ln \left( \frac{M_X}{M_R} \right)$$  (19)

$$\delta m_R^2 = 0$$  (20)
\[ \delta A = -\frac{3A_0}{16\pi^2} (Y_i Y_\nu Y_\nu) \ln \left( \frac{M_X}{M_R} \right). \]  

(21)

In order to determine the product \( Y_\nu^\dagger Y_\nu \) of the neutrino Yukawa coupling matrix entering these corrections, one uses the expression

\[ Y_\nu = \frac{\sqrt{M_R}}{v \sin \beta} R \cdot \text{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}) \cdot U^\dagger, \]  

(22)

which follows from \( U^T M_\nu U = \text{diag}(m_1, m_2, m_3) \) and (14) [19]. Here, \( R \) is an unknown complex orthogonal matrix parametrizing the ambiguity in the relation of Yukawa coupling and mass matrices. In the following we will assume \( R \) to be real which suffices for the present purpose. In this case, \( R \) drops out from the product \( Y_\nu^\dagger Y_\nu \),

\[ Y_\nu^\dagger Y_\nu = \frac{M_R}{v^2 \sin^2 \beta} U \cdot \text{diag}(m_1, m_2, m_3) \cdot U^\dagger. \]  

(23)

Using existing neutrino data on the mass squared differences and the mixing matrix \( U \) together with bounds and assumptions on the absolute mass scale one can calculate \( Y_\nu^\dagger Y_\nu \). The only free parameter is the Majorana mass scale \( M_R \). The result is then evolved to the unification scale \( M_X \) and used as an input in the renormalization group corrections (19) to the slepton mass matrix. Finally, diagonalization of (15) yields the slepton mass eigenvalues \( \tilde{m}_i \) and eigenstates \( \tilde{l}_i \) (\( i = 1, 2, ..., 6 \)).

### 3.2 Lepton flavour violating processes

The flavour off-diagonal elements (19) in \( m_l^2 \) (\( \delta A = 0 \) in the mSUGRA scenarios of [17]) induce, among other SLFV effects, the processes \( e^+ e^- \to \tilde{l}_j^+ \tilde{l}_i^- \to l_\beta^- l_\alpha^- \tilde{\chi}_0^0 \tilde{\chi}_0^0 \), where SLFV can occur in the production and decay vertices. The helicity amplitudes for the pair production of \( \tilde{l}_j^+ \) and \( \tilde{l}_i^- \), and the corresponding decay amplitudes are given explicitly in [8]. In the approximation (5) for \( \sigma_{\alpha\beta}^{\text{pair}} \) one finds

\[ \sigma_{\alpha\beta}^{\text{pair}} \propto \alpha^4 \frac{|(\delta m_L)_{\alpha\beta}|^2}{\tilde{m}^2 \Gamma^2} \sigma(f \to \tilde{l}_j^+ \tilde{l}_i^-) Br(l_\alpha^- \to l_\alpha^+) Br(l_\alpha^- \to l_\alpha^- \tilde{\chi}_0^0) \]  

(24)

In the numerical evaluation no slepton degeneracy has been assumed as in (4), and the amplitude for the complete \( 2 \to 4 \) processes is summed coherently over the intermediate slepton mass eigenstates.

Similarly, the terms (19) are responsible for SLFV radiative decays \( l_\alpha \to l_\beta \gamma \) induced by photon-penguin type diagrams with charginos/sneutrinos or neutralinos/charged sleptons in the loop. Again schematically, the decay rates are given by [19,20]

\[ \Gamma(l_\alpha \to l_\beta \gamma) \propto \alpha^3 \frac{|(\delta m_L)_{\alpha\beta}|^2}{\tilde{m}^8} \tan^2 \beta, \]  

(25)

where \( \tilde{m} \) stands for the relevant sparticle masses in the loop.
Figure 2: Cross-sections at $\sqrt{s} = 500$ GeV for $e^+e^- \rightarrow \mu^+\mu^- + 2\tilde{\chi}_1^0$ (circles) and $e^+e^- \rightarrow \tau^+\tau^- + 2\tilde{\chi}_1^0$ (triangles) in scenario B.

![Figure 2: Cross-sections at $\sqrt{s} = 500$ GeV for $e^+e^- \rightarrow \mu^+\mu^- + 2\tilde{\chi}_1^0$ (circles) and $e^+e^- \rightarrow \tau^+\tau^- + 2\tilde{\chi}_1^0$ (triangles) in scenario B.](image)

Table 1: Parameters of selected mSUGRA benchmark scenarios (from [17]). The sign of $\mu$ is chosen to be positive and $A_0$ is set to zero. Given are also the mass and total width of the heaviest charged slepton and the mass of the lightest neutralino.

| Scenario | $m_{1/2}$/GeV | $m_0$/GeV | $\tan \beta$ | $\tilde{m}_6$/GeV | $\Gamma_6$/GeV | $m_{\tilde{\chi}}^0$/GeV |
|----------|----------------|-----------|--------------|-------------------|--------------|------------------|
| B        | 250            | 100       | 10           | 208               | 0.32         | 98               |
| C        | 400            | 90        | 10           | 292               | 0.22         | 164              |
| G        | 375            | 120       | 20           | 292               | 0.41         | 154              |
| I        | 350            | 180       | 35           | 313               | 1.03         | 143              |

3.3 Signals and background

Among the mSUGRA benchmark scenarios proposed in [17] for LC studies, the models B, C, G, and I (see Tab.1) predict left-handed sleptons which can be pair-produced at $e^+e^-$ colliders $\sqrt{s} = 500 \div 800$ GeV cms energies. In the following we will confine ourselves to these models.

Most likely, at the time when a linear collider will be in operation, more precise measurements of the neutrino parameters will be available than today. In order to simulate the expected improvement, we take the central values of the mass squared differences $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$ and mixing angles $\theta_{ij}$ from a global fit to existing data [21] with errors that indicate the anticipated 90 % C.L. intervals of running and proposed experiments as further explained in [2]:

$$\tan^2 \theta_{23} = 1.40^{+1.37}_{-0.66}, \quad \tan^2 \theta_{13} = 0.005^{+0.001}_{-0.005}, \quad \tan^2 \theta_{12} = 0.36^{+0.35}_{-0.16},$$

$$\Delta m_{12}^2 = 3.30^{+0.3}_{-0.3} \cdot 10^{-5} \text{ eV}^2, \quad \Delta m_{23}^2 = 3.10^{+1.0}_{-1.0} \cdot 10^{-3} \text{ eV}^2. \tag{26} \tag{27}$$

Furthermore, for the lightest neutrino we assume the mass range $m_1 \approx 0 - 0.03$ eV, which
at the lower end corresponds to the case of a hierarchical spectrum. Towards the upper end, it approaches the degenerate case.

In Fig. 2, the cross-sections for $e^+e^- \rightarrow \mu^+\mu^- + 2\chi^0_1$ and $e^+e^- \rightarrow \tau^+\tau^- + 2\chi^0_1$ are plotted for model B. The channel $\tau^+e^- + 2\chi^0_1$ is not shown since it is strongly suppressed by the small mixing angle $\theta_{13}$, and therefore more difficult to observe. As can be seen, for a sufficiently large Majorana mass scale the SLFV cross-sections can reach several fb. The spread of the predictions reflects the uncertainties in the neutrino data.

The Standard Model background mainly comes from $W$-pair production, $W$-production with $t$-channel photon exchange, and $\tau$-pair production. A 10 degree beam pipe cut and cuts on the lepton energy and missing energy reduce the SM background cross-sections to less than 30 fb for $(\mu e)$ final states and less than 10 fb for $(\tau \mu)$ final states. If one requires a signal to background ratio, $S/\sqrt{S+B} = 3$, and assumes a typical signal cross-section of 0.1 fb, one can afford a background of about 1 fb. Here an integrated luminosity of 1000 fb$^{-1}$ has been assumed. Whether or not the background process estimate above can be further suppressed to this level by applying selectron selection cuts, for example, on the acoplanarity, lepton polar angle and missing transverse momentum has to be studied in dedicated simulations. For lepton flavour conserving processes it has been shown that the SM background to slepton pair production can be reduced to about 2-3 fb at $\sqrt{s} = 500$ GeV [22].

The MSSM background is dominated by chargino/slepton production with a total cross-section of 0.2-5 fb and 2-7 fb for $(\mu e)$ and $(\tau \mu)$ final states, respectively, depending on the SUSY scenario and the collider energy. The MSSM background in the $(\tau \mu)$ channel can also contribute to the $\mu e$ channel via the decay $\tau \rightarrow \mu\nu_\mu
u_\tau$. If $\tilde{\tau}_1$ and $\tilde{\chi}_1^+$ are very light, like in scenarios B and I, this background can be as large as 20 fb. However, such events typically contain two neutrinos in addition to the two LSPs which are also present in the signal events. Thus, after $\tau$ decay one has altogether six invisible particles instead of two, which may allow to discriminate the signal in $\mu^+e^- + E_T$ also from this potentially dangerous MSSM background by cutting on various distributions. But also here one
needs a dedicated simulation study, in order to make more definite statements.

The corresponding branching ratios, \(Br(\mu \to e \gamma)\) and \(Br(\tau \to \mu \gamma)\), in model B are displayed in Fig. 3 [2]. One sees that a positive signal for \(\mu \to e \gamma\) at the minimum branching ratio observable in the new PSI experiment, \(Br(\mu \to e \gamma) \approx 10^{-13}\) [23] would imply a value of \(M_R\) between \(2 \cdot 10^{-12}\) GeV and \(2 \cdot 10^{-13}\) GeV. In comparison to \(\mu \to e \gamma\) the channel \(\tau \to \mu \gamma\) is less affected by the neutrino uncertainties. If the sensitivity goal \(Br(\tau \to \mu \gamma) = 10^{-8}\) [16] at the LHC is reached one could probe \(M_R = 10^{15}\) GeV.

Particularly interesting and useful are the correlations between SLFV in radiative decays and slepton pair production. Such a correlation is illustrated in Fig. 4 for \(e^+e^- \to \tau^+\mu^- + 2\tilde{\chi}_1^0\) and \(Br(\tau \to \mu \gamma)\). One sees that the neutrino uncertainties drop out, while the sensitivity to the mSUGRA parameters remains. An observation of \(\tau \to \mu \gamma\) with the branching ratio \(10^{-8}\) at the LHC would be compatible with a cross-section of order 10 fb for \(e^+e^- \to \sum_{i,j} \tilde{l}^+_i \tilde{l}^-_j \to \tau^+\mu^- + 2\tilde{\chi}_1^0\), at least in model C. However, there are also correlations of different flavor channels. This is illustrated in Fig. 5 where the correlation of \(e^+e^- \to \tau^+\mu^- + 2\tilde{\chi}_1^0\) and \(\mu \to e \gamma\) is shown. Despite of the uncertainties from the neutrino sector, already the present experimental bound \(Br(\mu \to e \gamma) < 1.2 \cdot 10^{-11}\) yields a stronger constraint on \(\sigma(e^+e^- \to \tau^+\mu^- + 2\tilde{\chi}_1^0)\) than the one obtained from Fig. 4 making cross-sections larger than a few \(10^{-1}\) fb at \(\sqrt{s} = 800\) GeV very unlikely in model B. If this scenario is correct, non-observation of \(\mu \to e \gamma\) at the new PSI experiment will exclude the observability of this channel at a LC. As a final remark we stress that in the channel \(e^+e^- \to \mu^+e^- + 2\tilde{\chi}_1^0\) cross-sections of 1 fb are compatible with the present bounds, while no signal at the future PSI sensitivity would constrain this channel to less than 0.1 fb.

However we want to emphasize again that these statements are very model dependent, and much bigger cross-sections are possible in general, as shown in section 2.

Figure 4: Correlation of \(\sigma(e^+e^- \to \tau^+\mu^- + 2\tilde{\chi}_1^0)\) at \(\sqrt{s} = 800\) GeV with \(Br(\tau \to \mu \gamma)\) in scenario (from left to right) C, G (open circles), B and I.
4 Summary and outlook

If superpartners are discovered at future colliders, we advocate the search for SUSY lepton flavour violation as a high priority topic of the experimental programme. At a LC, the most favourable signals are expected to come from the production and decay of sleptons and charginos. Considering only LFV in the $\mu - \tau$ sector, a case motivated by the large atmospheric neutrino mixing but more difficult to detect than LFV in the $e - \mu$ sector due to the presence of decaying taus, we have shown that the LC measurements may be complementary to searches for the radiative $\tau$ decay at the LHC. For example, a measurement of $Br(\tau \rightarrow \mu \gamma) = 10^{-8}$ at the LHC combined with the SLFV signal at a LC would point to $\sin^2 \theta_{23} \geq 0.4$ and $\Delta \tilde{m}_{23} \simeq 0.3 - 1$ GeV.

In the context of the SUSY seesaw mechanism of neutrino mass generation, correlations between SLFV in radiative decays and slepton pair production have been found particularly interesting. For instance, in a given MSSM scenario the measurement of $\tau \rightarrow \mu \gamma$ at the LHC would imply a definite cross section for $e^+ e^- \rightarrow \tau^+ \mu^- + 2\tilde{\chi}^0_1$ at the LC. Assuming a reasonable set of MSSM benchmark scenarios and $Br(\tau \rightarrow \mu \gamma) = 10^{-8}$ and using the present neutrino data, one predicts $\sigma(e^+ e^- \rightarrow \tau^+ \mu^- + 2\tilde{\chi}^0_1)$ in the range 0.05 to 10 fb. However, the non-observation of $\mu \rightarrow e \gamma$ with a branching ratio of about $10^{-13}$ at the new PSI experiment would exclude the observability of $\sigma(e^+ e^- \rightarrow \tau^+ \mu^- + 2\tilde{\chi}^0_1)$ at a linear collider. While the former correlation involving the same lepton flavours is insensitive to the uncertainties in the neutrino data (Fig. 4), the latter correlation is somewhat smeared out (Fig. 5). However, both types of correlations remain sensitive to the mSUGRA parameters and, hence, provide very useful tools for probing the origin of lepton flavour violation.

The complementarity of the LHC and LC (and of low-energy experiments) in the context of lepton flavour violation is far from being exhausted by the present study. Quantitative analyses of the impact of precise mass measurements at the LC on identifying the LFV decay chains at the LHC (and vice-versa) and other important features call for detailed Monte Carlo simulations which should be undertaken in the next round of the
LHC/LC studies.

Acknowledgements
This work was performed in the framework of the LHC/LC study group and supported by the Bundesministerium für Bildung und Forschung (BMBF, Bonn, Germany) under the contract number 05HT4WWA2. The work of JK was supported by the KBN Grant 2 P03B 040 24 (2003-2005) and 115/E-343/SPB/DESY/P-03/DWM517/2003-2005.

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