Smooth matching of $\hat{q}$ from hadronic to quark and gluon degrees of freedom

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One of the key signatures of the Quark Gluon Plasma (QGP) is the energy loss of high momentum particles as they traverse the strongly interacting medium. The energy loss of these particles is governed by the jet transport coefficient $\hat{q}/T^3$, wherein it has been thought that there is a large jump as the system transitions between the hadron gas and Quark Gluon Plasma phases. Here we calculate $\hat{q}/T^3$ within the Hadron Resonance Gas (HRG) model with the particle list PDG16+ and find that, if one incorporates the experimental error in the hadronic calculation of $\hat{q}/T^3$ and assumes a higher pseudo-critical temperature, then a smooth transition from the hadron gas phase into the Quark Gluon Plasma phase is possible. We also find a significant enhancement in $\hat{q}/T^3$ at finite baryon chemical potential and find issues with the relationship between the shear viscosity and the jet transport coefficient within a hadron gas phase.

I. INTRODUCTION

While jets are primarily thought of as a tool that probe the early stages of heavy-ion collisions [13–19], the jet transport coefficient $\hat{q}$ is influenced by the contributions of $\hat{q}_{HRG}$ from the hadron resonance gas as well. In the well-know JET Collaboration paper [4] a summary of $\hat{q}(T)$ calculations is shown, using a variety of models benchmarked against the same medium. A primary conclusion from that work was that there is a large jump in $\hat{q}$ from the hadron gas phase at low temperature where $\hat{q}/T^3 \lesssim 1$, up to the QGP phase where $\hat{q}/T^3 \approx 4.5$. The argument for this behavior is that the QGP phase is strongly interacting, wherein high $p_T$ partons would lose a significant amount of energy (hence a large $\hat{q}/T^3$), whereas the hadron resonance gas phase is weakly interacting and the high-$p_T$ partons would pass through it largely unaffected.

However, our current picture of the QGP/hadron resonance gas phase transition has evolved significantly over the years and this may no longer hold. For instance, we know that the QGP/hadron resonance gas transition is a smooth cross-over with no critical point/first-order phase transition anywhere near the $\mu_B = 0$ range that is probed at the LHC and top RHIC energies [3][12]. Additionally, this implies that there may well be different hadronization and freeze-out temperatures for light vs. strange hadrons [13][19]. Furthermore, there is no reason to believe that the inflection temperatures of different transport coefficients should occur at a single temperature either. In fact, in a bottom-up picture from holography [20] that has been tuned to precisely fit lattice QCD [21][22] (and even make predictions for high-order susceptibilities), it was found that there is a large range ($T_{pc} \sim 130 – 200$ MeV) of pseudo-critical temperatures for the relevant transport coefficients. In this work, it was found that the inflection point for $\hat{q}/T^3$ is located around $T_{pc} \sim 195$ MeV, which implies that calculations of $\hat{q}/T^3$ in a hadron resonance gas model may be relevant up to high temperatures. Additionally, it also implies that it may be possible to smoothly transition from $\hat{q}/T^3$ in a hadron resonance gas picture to a QGP picture.

In this manuscript, we calculate $\hat{q}/T^3$ within a hadron resonance gas model and assume a higher $T_{pc}$ as well as incorporate all experimental error bars from the nuclear saturation density and the jet transport parameter for a quark at the center of a nucleus. Once these two effects are incorporated in our calculations, we find that $\hat{q}/T^3$ can smoothly connect to the extracted $\hat{q}/T^3$ from the JET collaboration [1]. We then extend this work to finite baryon densities where $\hat{q}/T^3$ is approximately double in value at $\mu_B \sim 400$ MeV. We also find that large differences exist at $\mu_B > 400$ MeV, when excluded volume effects are included in the calculation. Finally, we test the quasi-empirical relationship between the shear viscosity and the jet transport coefficient derived in Ref. [23] and find that it does not seem to hold in the hadronic phase.

II. HADRON RESONANCE GAS MODEL

In the hadron resonance gas model, one can calculate the pressure and energy density of a system of hadrons, assuming that they are point-like particles, according to the following formulas:
where \( g_j \) is the degeneracy of each hadron, \( m_j \) is the mass, and \( B_j, S_j, \) and \( Q_j \) are the baryon number, strangeness and electric charge carried by each hadron. Additionally, \( \mu_B, \mu_S, \) and \( \mu_Q \) are the corresponding chemical potentials for each conserved charge. The sums run over all possible baryons and mesons belonging to a given particle list. We will discuss our particle list choices below.

In order to calculate \( \hat{q}_{HRG} \) we use Eq. (9) from Ref. [16]

\[
\hat{q}_{HRG}(T) = \frac{\hat{q}_N}{\rho_N} \left[ \frac{2}{3} \rho_M(T) + \rho_B(T) \right]
\]

where \( \rho_N \sim (0.15 - 0.17) fm^{-3} \) is the nucleon density in the center of a large nucleus and the jet transport parameter for a quark at the center of a large nucleus, \( \hat{q}_N \sim (0.024 \pm 0.008) \) GeV\(^2\)/fm, is extracted [24] from HERMES data [25]. If we take the range of error-bars into account, the prefactor in the above equation lies in the range \( \hat{q}_N / \rho_N = 2.4 - 5.8 \). Throughout the rest of this section, the bands shown in the plots for \( \hat{q} \) take this variation into account. The factor of 2/3 in the above equation assumes that mesons are composed of 2 quarks and baryons contain 3 quarks. While this assumption is certainly true for valence quarks (namely in the high \( x \) regime), at the \( x \) and \( Q^2 \) values probed in heavy-ion and LHC energies sea quarks play a significant role (see e.g. Fig. 19 from HERAPDF1.0 [26]). Thus, it is not clear whether this factor of 2/3 should still remain, but we leave it in the calculation for lack of a better definition.

In Eq. (3), \( \rho_M(T) \) is the total particle density of mesons and \( \rho_B(T) \) is the total particle density of baryons, both calculated within the HRG model. The total particle densities can be calculated using the partial pressures of mesons \( \rho_M \) and baryons \( \rho_B \), such that \( \rho_M = \rho_M(T)/T \) and \( \rho_B = \rho_B(T)/T \). The partial densities are then defined as:

\[
\rho_M(T, \mu_B, \mu_S, \mu_Q) = \sum_{j \in M} \frac{-g_j T^3}{2 \pi^2} \int_0^{\infty} p^2 \left[ \exp \left( \frac{\sqrt{p^2 + m_j^2}}{T} - \frac{B_j \mu_B + S_j \mu_S + Q_j \mu_Q}{T} \right) + 1 \right]^{-1} dp
\]

\[
\rho_B(T, \mu_B, \mu_S, \mu_Q) = \sum_{j \in B} \frac{g_j T^3}{2 \pi^2} \int_0^{\infty} p^2 \left[ \exp \left( \frac{\sqrt{p^2 + m_j^2}}{T} - \frac{B_j \mu_B + S_j \mu_S + Q_j \mu_Q}{T} \right) - 1 \right]^{-1} dp
\]

Both quantities will be influenced by the number of resonances in our system, especially close to the cross-over phase transition. In this paper we compare two different lists from the Particle Data Group, one from 2005 (PDG05) and another developed in Ref. [27] from 2016 that includes all *-**** states (PDG16+).

We will also compare the ideal HRG \( \hat{q}/T^3 \) to that from an excluded volume picture. Since the prefactor \( \frac{\hat{q}_N}{\rho_N} \) is unchanged in an excluded volume picture, we only need to include excluded volume effects in the total particle density. First we define the ideal total density (where mesons are weighted by \( \frac{2}{3} \)):

\[
\rho_q = \frac{2}{3} \rho_M + \rho_B.
\]

Then the excluded volume version follows from the transcendental equation

\[
\rho_{v,q}(T, \mu_i) = \rho_q(T, \mu_i) \exp(-v \rho_{v,q}(T, \mu_i)).
\]

which can be easily solved using a Lambert W function

\[
\rho_{v,q}(T, \mu_i) = \frac{1}{v} W(v \rho_q(T, \mu_i)).
\]

Finally, the excluded volume \( \rho_{v,q}(T, \mu_i) \) can be used to calculate the excluded volume version of \( \hat{q} \)

\[
\hat{q}_v(T, \mu_i) = \frac{\hat{q}_N}{\rho_N} \rho_{v,q}(T, \mu_i).
\]

III. RESULTS: CONTINUOUS \( \hat{q}/T^3(T) \)

In Fig. 1 we show a comparison between \( \hat{q}_{HRG}(T) \) for the PDG05 and PDG16+, calculated in the ideal HRG model. We find, indeed, that there is a small increase in \( \hat{q}_{HRG}(T) \) close to the phase transition when more resonances are included in the hadronic spectrum. However, the inclusion of additional resonances has only a
small effect compared to the uncertainties that remain in $\hat{q}_{N}/\rho_N$. In fact, once one considers the entire error band from $\hat{q}_{N}/\rho_N$, a continuous transition from $\hat{q}_{HRG}(T)$ into $\hat{q}_{QGP}(T)$ is possible. Comparing our results to Fig. 10 in Ref. [4], we find a much smoother transition of $\hat{q}_{HRG}(T)$ into the QGP phase, which may affect energy loss models. Our results match quite well the JET collaboration’s point from Ref. [4]. The holography curve is taken from Ref. [20] assuming $\lambda = g^2 \times N_c = 3$. The JET collaboration point is from Ref. [3].

The consequences of a smoother $\hat{q}(T)$ would imply that it is possible to still lose energy even when hadronic degrees of freedom begin to appear. Thus, one should systematically study a variety of decoupling temperatures from the medium, as has been done in Refs. [2] [31).

We note that there are subtleties when determining $\hat{q}$ in a hadronic transport model noted in Ref. [32], that go beyond the purpose of this work. In such an approach, one can better study the influence of $2 \leftrightarrow 2$ interactions and the influence of heavier resonances may play a larger role, but we leave that study for a future work. One must first incorporate the additional resonances from the PDG16+ into SMASH, which is beyond the scope of this paper.

We now proceed to study the effect of an excluded volume approach in the hadron resonance gas model. In Ref. [33] it was shown that the maximum radius for a volume with the PDG16+ is $r = 0.25$ fm, which corresponds to the maximal effect on $\hat{q}$. We find that this correction leads to a suppression of $\hat{q}/T^3$ at high temperatures, but we still find a reasonable match to the result of the JET collaboration, as shown in Fig. 2. While in the case of the ideal HRG model the difference in $\hat{q}(T)/T^3$ between PDG16+ and PDG05 was not very significant, we point out that the PDG05 barely supports any excluded volume at all when comparing to lattice QCD results (as shown in Ref. [34]), so that only the PDG16+ list would allow any effects from excluded volume. Thus, for the rest of the paper we only show PDG16+ results, for which we consider the ideal and excluded volume approaches.

In Fig. 2, we compare ideal and excluded volume results for $\hat{q}(T)/T^3$ to both the holographic result from Ref. [21] and the formulas from hadronic transport depending on $p_T$. In the case of holography, the results are given in terms of $\hat{q}(T)/T^3/\sqrt{\lambda}$, where $\lambda = 4\pi \alpha N_c = Ng^2$ is the ‘t Hooft coupling. For illustrative purposes we assume $\lambda = 3$, which seems a reasonable estimate. Additionally, we also compare to SMASH for $p_T = 1$ GeV and $p_T = 10$ GeV (with the caveat that in Ref. [32] it is pointed out that their calculations might not be directly equivalent to $\hat{q}$) and find that $p_T = 1$ GeV is much closer to our estimate. We note that, in the HRG model formula from this paper, there is no explicit $p_T$ dependence of a jet, so it is difficult to make an apples-to-apples comparison.

**IV. RESULTS: ENERGY LOSS AT THE BEAM ENERGY SCAN - $\hat{q}(T, \mu_B)$**

At lower beam energies, generally one does not expect signatures of energy loss because of the shorter lifetime of the QGP (e.g. see Fig. 9 in Ref. [34] for lifetime estimates). However, recent measurements from STAR may indicate jet energy loss [35]. To understand energy loss better at low beam energies, we can easily extend Eq. (3) to finite $\{\mu_B, \mu_S, \mu_Q\}$ within a hadron resonance gas picture.

In Fig. 3, we show $\hat{q}_{HRG}(T, \mu_B = 500$ MeV)/$T^3$ and find a considerable enhancement, compared to the result at zero chemical potential discussed before. Additionally, since it is expected that the transition temperature decreases at finite $\mu_B$, we only plot $\hat{q}_{HRG}/T^3$ up to $T_{pc} = 160$ MeV, using holography at large $\mu_B$ as guidance [20]. Since $\hat{q}_{HRG}/T^3$ is nearly double than the one at $\mu_B = 0$, one can conclude that even with hadronic degrees of freedom, one can anticipate energy loss at low beam energies, which is consistent with results from STAR [35]. This is, of course, not a comment on the production of jets, which we would intuitively expect to be suppressed at low beam energies, but we are unaware of any calculations that show this explicitly.

In order to have a better connection to experiments, we use isentropes to demonstrate a possible trajectory across

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**FIG. 1.** $\hat{q}_{HRG}(T)$ calculations within the HRG model for PDG05 (red band) and PDG16+ (blue band). The band corresponds to the uncertainty in $\hat{q}_N/\rho_N$. The holography curve is taken from Ref. [20] assuming $\lambda = g^2 \times N_c = 3$. The JET collaboration point is from Ref. [4].
the QCD phase diagram. Here we calculate \( \hat{q}/T^3 \) along the isentropes extracted from the BSQ EoS from Ref. [36], because it also uses the PDG16+ (note these isentropes were only extracted for the ideal HRG so we do not show the excluded volume results in the following). We show a comparison between the simplistic QCD EoS that is only 2D i.e. \( \{T, \mu_B\} \) vs. the realistic one that contains all 3 conserved charges within heavy-ion collisions: baryon number \( B \), strangeness \( S \), and electric charge \( Q \). On average one expects strangeness neutrality i.e. \( \langle \rho_S \rangle = 0 \) and the fraction of protons to neutrons to be fixed by the type of nuclei collided such that \( \langle \rho_Q \rangle \sim 0.4 \langle \rho_B \rangle \). We note that the picture is, of course, much more complex once one considers local charge fluctuations and out-of-equilibrium effects [37–42], but this at least provides us with a general idea of the consequences of just considering baryon number conservation vs. the full BSQ EoS.

First we consider a beam energy of \( \sqrt{s_{NN}} = 27 \) GeV, where the entropy per baryon number at freeze-out is \( s/\rho_B = 70 \), as shown in Fig. 4 (a). At small enough \( \mu_B \) there is no effect from the inclusion of strangeness neutrality on \( \hat{q}/T^3 \). Thus, we do not consider higher beam energies. However, once one reaches low enough beam energies such as \( \sqrt{s_{NN}} < 14.5 \) GeV where \( s/\rho_B = 30 \), one can see that the impact from the 4D EoS plays some role. However, the biggest role is played by the isentrope itself, that gives a bend in \( \hat{q}/T^3 \) due to the assumption that entropy is conserved throughout the evolution of the system.

V. RESULTS: TESTING \( \eta T / w \sim T^3 / \hat{q} \)

In Ref. [20], a quasi-empirical formula was established, which relates the shear viscosity to the inverse of \( \hat{q} \) such that

\[
\eta T / w \sim T^3 / \hat{q}.
\]  (8)

In the absence of direct calculations of transport coefficients from QCD, we must turn to phenomenological approaches, as performed in this paper. Here we test our two approaches for calculating \( \eta T / w \) and \( \hat{q}/T^3 \) in the hadron resonance gas model against each other, to see if this relationship leads to reasonable values of \( \eta T / w \).

The results of the extracted \( \eta T / w \) from \( \hat{q} \) are shown in Fig. 5 and their magnitudes appear to be reasonable at \( \mu_B = 0 \). They have a value of \( \eta T / w \sim 0.2 \) for an ideal hadron resonance gas and around \( \eta T / w \sim 0.3 \) for the excluded volume approach (assuming \( T_{pc} = 200 \) MeV). Additionally, at low temperatures \( \eta T / w \) increases, which is to be expected. An issue arises when one compares an ideal hadron resonance gas to the excluded volume one. If one uses an excluded volume approach, as was done in Refs. [28–44], one finds that interactions reduce \( \eta T / w \), which is in contrast to what we find using the \( \hat{q}/T^3 \) formalism from Ref. [4]. Additionally, we can compare the direct result from an excluded volume calculation from Ref. [33] (assuming the same radius in both calculations of

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**FIG. 2.** \( \hat{q}_{HRG}(T) \) calculations for PDG16+ with an ideal HRG vs. excluded volume with \( r = 0.25 \) fm, where the band takes into account the error in \( \hat{q}_N / \rho_N \). The holography curve is taken from Ref. [20] assuming \( \lambda = g^2 * N_c = 3 \). The JET collaboration point is from Ref. [34] and the SMASH calculations from Ref. [32].

**FIG. 3.** \( \hat{q}_{HRG}(T) \) at \( \mu_B = 500 \) MeV calculations within the hadron resonance gas for PDG16+ with an ideal HRG vs. excluded volume approach where the band corresponds to the error in \( \hat{q}_N / \rho_N \). The holography curve is taken from Ref. [20] assuming \( \lambda = g^2 * N_c = 3 \).
Because of the issue with the relationship between $\eta T/w$ and $\hat{q}$, we looked at recent calculations from the hadronic transport code SMASH for guidance. The code has been used to calculate both $\eta T/w$ [46] and $\tilde{q}$ [32], that we then convert into $\hat{q}/T^3$. We compare the direct $\eta T/w$ calculation to $\eta T/w \sim T^3/\hat{q}$ in Fig. 4. We find a somewhat similar result as our HRG model calculation if we consider $\hat{q}$ for $p_T = 1$ GeV. The relationship $\eta T/w \sim T^3/\hat{q}$ approximately holds at high temperatures, while deviations arise at low temperatures. However, if one considers a larger $p_T$, the estimated $\eta T/w$ is significantly smaller than the directly calculated one. Since both the HRG model and hadronic transport cannot consistently reproduce $\eta T/w \sim T^3/\hat{q}$, further investigations into this mismatch within a hadron resonance gas model might be needed.

Finally, we also explore $\eta T/w \sim T^3/\hat{q}$ at finite $\mu_B$. Qualitatively, we find that the extracted $\eta T/w$ at $\mu_B = 500$ MeV behaves as expected, as it is smaller at large $\mu_B$ as its fixed temperature. We observe that the excluded volume approach demonstrates less sensitivity to $\mu_B$ compared to the ideal HRG model case.
VI. CONCLUSIONS

In this paper we calculate $q/T^3$ within the hadron resonance gas model formalism for both an ideal gas of hadrons and an excluded volume approach. Additionally, if we incorporate the experimental uncertainty into that calculation and assume a higher pseudo critical temperature for the maximum of $q/T^3$, we are able to smoothly connect our results to those of the JET collaboration. At the time of finalizing this paper JETSCAPE released new $q/T^3$ results that would also smoothly match to our hadronic results [17]. Already a significant effort has been invested to determine this coefficient within the Quark Gluon Plasma phase [18,19]. Our results indicate that dynamical models should also study the effect of energy loss more thoroughly within the hadron gas phase.

Furthermore, we find that within hadronic models (both for an excluded volume hadron resonance gas and hadronic transport- SMASH) it is not clear whether the relationship $q/T^3$ holds, which warrants further investigation.

Finally, we find an enhancement of $q/T^3$ at finite $\mu_B$. At large $\mu_B$ we find that the results in an excluded volume approach deviate more strongly from the ideal hadron resonance gas ones (which is expected). While we anticipate a low chance of jets being produced at the beam energy scan (although we do not exclude the possibility of mini-jets/low $p_T$ jets), our results demonstrate that any jets produced would be heavily suppressed.

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