Stress Intensity Factors of Slanted Cracks in Bi-Material Plates

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Abstract. In this study, the stress intensity factors (SIF) of slanted cracks in bi-material plates subjected to mode I loading is numerically solved. Based on the literature survey, tremendous amount of research works are available studying the normal cracks in both similar and dissimilar plates. However, lack of SIF behavior for slanted cracks especially when it is embedded in bi-material plates. The slanted cracks are then modelled numerically using ANSYS finite element program. Two plates of different in mechanical properties are firmly bonded obliquely and then slanted edge cracks are introduced at the lower inclined edge. Iso-parametric singular element is used to model the crack tip and the SIF is determined which is based on the domain integral method. Three mechanical mismatched and four slanted angles are used to model the cracks. According to the present results, the effects of mechanical mismatch on the SIF for normal cracks are not significant. However, it is played an important role when slanted angles are introduced. It is suggested that higher SIF can be obtained when the cracks are inclined comparing with the normal cracks. Consequently, accelerating the crack growth at the interface between two distinct materials.

1. Introduction

The evaluation of stress intensity factor (SIF) to determine the structural reliability and durability is an important aspect in modern engineering world. There are many methods used to characterize the components or materials containing cracks such as J-integral, COD, CTOD, SIF and others [1-5]. For linear elastic materials, SIF is widely and successfully used. Generally, in order to determine the SIF, there are two groups of methods used such displacement extrapolation and the energy release rate methods. J-integral is also used and then it is converted to SIF. Another type of works can also be found in analyzing the cracks using SIFs [6-8].

For the crack positioned between two different materials, the displacement extrapolation method is difficult to use. Then, energy based method is frequently used. Matsumoto et al. [9] used an interaction energy release rate based on the boundary element method (BEM) to compute the SIF for the interfacial cracks. They found that the proposed approach is applied to several cracks in dissimilar media and the results are compared with those obtained by the conventional approach based on the extrapolation method. The analytical displacement and stress solutions for an interface crack between two infinite dissimilar media subjected to uniform stresses at infinity are used to give the auxiliary field, in which the values of the stress intensity factors are known. It is demonstrated that the present
The method can give accurate results for the stress intensity factors of various bi-material interface cracks under coarse mesh discretization. Ikeda et al. [10] developed the mixed mode fracture criterion in order to characterize the crack between dissimilar materials. Various types of crack geometries are experimentally tested. They found that the developed criterion is successfully characterized by the SIF determined at the interface cracks. Noda and Xu [11] investigated the controlling parameter affecting the SIF for a planar interfacial crack. The problem is formulated as a system of singular integral equations on the basis of the body force method. In the numerical analysis, unknown body force densities are approximated by the products of the fundamental density functions and power series, where the fundamental density functions are chosen to express singular behavior along the crack front of the interface crack exactly. The numerical results show that this numerical technique is successful, and the boundary conditions are satisfied precisely.

Noda [12] presented the SIF formulas for three-dimensional crack in homogenous and bonded dissimilar materials. The generalized stress intensity factors at the interface are expressed as a function of Dundurs parameters $\alpha$ and $\beta$. The generalized integral formulation uses the fundamental density functions and power series, where the fundamental density functions are chosen to express singular behavior along the crack front of the interface crack exactly. The numerical results show that this numerical technique is successful, and the boundary conditions are satisfied precisely.

Noda [12] presented the SIF formulas for three-dimensional crack in homogenous and bonded dissimilar materials. The generalized stress intensity factors at the interface are expressed as a function of Dundurs parameters $\alpha$ and $\beta$. Proposed formulas are usefully evaluating defects with any aspect ration under any combinations of the materials. Lan et al. [13] numerically studied the effect of mechanical mismatches and relative crack size to the SIFs at the crack tip of a bi-material bonded strip. The variations of stress intensity factors of a bi-material bonded strip are discussed with varying the relative crack size and material combinations. This investigation may contribute to a better understanding of the initiation and propagation of the interfacial cracks.

Therefore, this work presents the stress intensity factors when the cracks are inclined obliquely with the presence of mechanical mismatches. The preliminary works have been conducted [16, 17] and analyzed where the cracks are aligned eccentrically and on the multiple crack interactions in dissimilar materials. There are four slanted angles are used including the normal cracks and three types of mechanical mismatches are applied. The goal is to examine the SIFs response when these two important parameters are varied.

### 2. Calculating stress intensity factors

The evaluation of J-integral around the crack tip is based on the domain integral method initially introduced by Shih et al. [14]. This integral formulation uses area integration for 2D and volume integration for 3D problems which offer much better accuracy than contour integral and it is also much easier to implement numerically. Eq. (1) represents the 2D domain J-integral taking accounts the absence of thermal strain, path dependent plastic strain, body forces occur within the integration area.

$$
J = \int_A \left[ \sigma_{ij} \frac{\partial u_i}{\partial x_j} - w \delta_{ij} \right] \frac{\partial q_i}{\partial x_j} dA
$$

where $q_{ij}$ is the stress tensor, $u_i$ is the displacement vector, $w$ is the strain energy density, $\delta_{ij}$ is the Kronecker delta, $x_i$ is the coordinate axis and $q$ is referred to as the crack extension vector. The direction of $q$ is similar with $x$-axis of the local coordinate specified at the crack tip and it is normally chosen as zero at nodes along the contour, $\Gamma$. It is also a unit vector for all nodes inside $\Gamma$ except the mid-side nodes and known as virtual crack extension nodes. For 3D problems, the principal is similar to the 2D problems. However, domain integral representation of the J-integral becomes volume integration [15].

Two approaches for calculating stress intensity factor (SIF) are available in ANSYS software, interaction integral and displacement extrapolation methods. The first method is used because much easier to implement numerically and it also offers better accuracy and fewer mesh requirement. This method is similar to the domain integral method for J-integral evaluation describe previously. The
discussion on the domain integral methods can be found elsewhere [15]. The interaction integral is defined as in Eq. (2).

\[
I = - \int_{\gamma} q_i \left[ \tau_{ij} + e_{ij} \delta_{ij} - \sigma_{ij}^{aux} - \sigma_{ij}^{aux} u_{k,j,i} - \sigma_{ij}^{aux} u_{l,j,k} \right] \sqrt{r} \int_{\gamma} \delta_{ij} ds
\]  

(2)

where \( \sigma_{ij}, e_{ij} \) and \( u_i \) are the stress, strain and displacement and \( \sigma_{ij}^{aux}, e_{ij}^{aux} \) and \( u_{ij}^{aux} \) are the stress, strain and displacement of the auxiliary field and \( q_i \) is the crack extension vector.

\[
\sigma_{ij}^{aux} = \frac{K_{ij}^{aux}}{\sqrt{2\pi r}} f_{ij}^{\prime} (\theta) + \frac{K_{ii}^{aux}}{\sqrt{2\pi r}} f_{ii}^{\prime\prime} (\theta) + \frac{K_{III}^{aux}}{\sqrt{2\pi r}} f_{III}^{\prime\prime\prime} (\theta)
\]  

(3)

\[
uamma = \frac{K_{I}^{aux}}{2\mu} \sqrt{\frac{r}{2\pi}} g_{I}^{\prime} (\theta,\nu) + \frac{K_{II}^{aux}}{2\mu} \sqrt{\frac{r}{2\pi}} g_{II}^{\prime\prime} (\theta,\nu) + \frac{2K_{III}^{aux}}{\mu} \sqrt{\frac{r}{2\pi}} g_{III}^{\prime\prime\prime} (\theta,\nu)
\]  

(4)

\[
e_{i,j}^{aux} = \frac{1}{2} (u_{i,j}^{aux} + u_{j,i}^{aux})
\]  

(5)

An expression for the energy release rate in terms of mixed-mode SIFs is defined as for plane strain condition as in Eqs. (6)-(8).

\[
J = \frac{\left( K_{I}^{2} + K_{II}^{2} \right)}{E} \left( 1 - \nu^2 \right) + \frac{K_{III}^{2}}{E} \left( 1 + \nu \right)
\]  

(6)

\[
J = \frac{\left[ \left( K_{I} + K_{II}^{aux} \right)^2 + \left( K_{II} + K_{III}^{aux} \right)^2 \right]}{E} \left( 1 - \nu^2 \right) + \frac{\left( K_{III}^{aux} + K_{III}^{aux} \right)}{E} \left( 1 + \nu \right)
\]  

(7)

\[
J = J + J^{aux} + I
\]  

(8)

The interaction integral can be associated with the SIFs as Eq. (9)

\[
I = \frac{2\left( 1 - \nu^2 \right)}{E} \left( K_{I} K_{II}^{max} + K_{II} K_{III}^{max} \right) + \frac{1}{\mu} \left( K_{III}^{max} K_{III}^{max} \right)
\]  

(9)

By setting \( K_{II}^{aux} = 1 \) and \( K_{III}^{max} = 0 \),

\[
K_{I} = \frac{E}{2\left( 1 - \nu^2 \right)} I
\]  

(10)

By setting \( K_{II}^{aux} = 1 \) and \( K_{I}^{max} = K_{III}^{max} = 0 \) and \( K_{III}^{max} = 1 \) and \( K_{I}^{max} = K_{III}^{max} = 0 \) leads to the relationship between modes II and III SIFs with I, respectively.

\[
K_{II} = \frac{E}{2\left( 1 - \nu^2 \right)} I
\]  

(11)

\[
K_{III} = \mu I
\]  

(12)

where, \( K_i \) is a stress intensity factor with \( i \) is a loading mode, \( i = 1, 2 \) and 3. J-integral can be represented as J and I is a interaction domain integral. While, E and \( \mu \) is a modulus of elasticity and modulus of rigidity, respectively.
3. Methodology

Two inclined ends of plates of different mechanical properties are perfectly bonded. At a lower end of connection, there is an edge crack with a length of $a$ as shown in Figure 1(a). The edge point of the crack can be represented as a middle point of the total length of $2L$. The crack angles $\theta$ are varied in the range of 0° to 40° and the crack length of also varied. The relative crack length, $a/L$ is selected in the range of 0.1 to 0.7. In order to investigate the influence of mechanical mismatch on the stress intensity factors (SIF), three different types of relative mechanical mismatches are used such as 1.0, 0.5 and 5.0.

In this work, ANSYS finite element program is used to model and solve the crack problems. It is also assumed that the plate is fulfilled the plain strain condition. The constructions of the finite element model (FEM) started with the coordinate identifications of each point. Once all points are identified, all points are properly connected to form the plate with a crack. Then, the connected lines are converted into an area or domain. Two areas are used, at the upper and lower regions of the inclined crack. It is also used since this work dedicated to study the effect of mechanical mismatch on the SIF. Before the model is meshed, the crack tip is firstly identified. It is important since at this point, an iso-parametric singular element is used and on the other hand, the plate is modelled with irregular elements.

![Figure 1. (a) Schematic diagram of the plate with a crack and (b) FE model with an enlarged area of the crack tip.](image)

The FEM is shown in Figure 1(b) with an enlarged area of the crack tip. The lower end of the plate is constrained in y-direction and in order to prevent the body rotation, at a bottom lower left end point is also constrained. The mode I loading is then applied at the upper end of the plate. In this paper, $J$-integral based on the domain integral method is used. Then, the calculated $J$-integral is converted into SIFs using the interaction integral formulation. This technique is used instead of the traditional technique which is the displacement extrapolation method due to ease of use. In order to generalize the values of SIFs, it is converted into normalized values or it is also known as the geometrical correction factor, $F$ as in Eqs. (13) and (14):

$$F_I = \frac{K_I}{\sigma \sqrt{\pi a}}$$  \hspace{1cm} (13)

$$F_{II} = \frac{K_{II}}{\sigma \sqrt{\pi a}}$$  \hspace{1cm} (14)
where, $F$ is geometrical correction factor, $K$ is a stress intensity factor, $\sigma$ is a applied stress and $a$ is a crack depth. Before further analysis is carried out, the present model must be validated with the existing model. Since it is hard to find the behavior of inclined crack in mechanical mismatch condition, then the crack in a similar mechanical properties condition is used. Figure 2 shows the comparison between the present works with the existing model. It is found that the present model is well agreed for all crack ranges.

![Model validations.](image)

Figure 2. Model validations.

4. Results and discussion

4.1 Effect of Mechanical Mismatch on the SIFs

Two types of SIFs can be obtained simultaneously such as mode I and II SIFs. The values of SIFs are presented as normalized values $F_I$ and $F_{II}$, respectively. Figure 3 shows the behavior of $F_I$ when mechanical mismatches, $\alpha$ and inclined angles, $\theta$ are varied. For normal cracks, the values of $F_I$ are almost flattened when the cracks are extended as in Fig. 3(a). It is also showed that the mechanical mismatches have no significant effect on the distribution of $F_I$. When slanted angles, $\theta$ are introduced, the effect of $\alpha$ are more pronounced as in Figs. 3(b)-3(d). In the region of relatively smaller cracks ($a/L < 0.3$), mechanical mismatch is also not contributed to increase the $F_I$ where the SIFs are almost similar to each other's. When the slanted angles are increased, the distributions of $F_I$ are diverged. It is seemed that the divergence of $F_I$ is wider when slanted angle increased.

Fig. 4 shows the behavior of $F_{II}$ for different slanted angles when mechanical mismatches are varied. The pattern of $F_{II}$ is identical compared with $F_I$ however they are in opposite directions. For normal cracks ($\theta = 0^\circ$), the values of $F_{II}$ are also flattened but it is slightly decreased when higher value of $\alpha$ and $a/L$ is used as in Fig. 4(a). If slanted angles are introduced when modelled the cracks, the $F_{II}$ is increased inversely. There is an interesting point occurs where if $\alpha \geq 5$, the values of $F_{II}$ seemed not to alter for both slanted angles $30^\circ$ and $40^\circ$. 
Figure 3. Effect of relative crack length on the dimensionless SIFs, $F_1$ for different slanted angles, (a) $0^\circ$, (b) $10^\circ$, (c) $30^\circ$ and (d) $40^\circ$ when mechanical mismatched are varied.
Figure 4. Effect of relative crack length on the dimensionless SIFs, $F_{II}$ for different slanted angles, (a) $0^\circ$, (b) $10^\circ$, (c) $30^\circ$ and (d) $40^\circ$ when mechanical mismatched are varies.
4.2 Effect of Slanted angles on the SIFs
When mode I SIF, $F_I$ is plotted against relative crack depth, $a/L$ and the slanted angle is varied as shown in Fig. 5. The influence of $F_I$ for similar mechanical properties is showed in Fig. 5(a) for various slanted angles. For normal cracks, the $F_I$ is not affected when $a/L$ is varied. If the slanted angle is introduced, the values of $F_I$ are gradually increased. For 100 angle increment, $F_I$ increased almost uniformly. This is probably due to the symmetrical effect of the cracks.

![Graph](image)

Figure 5. Effect of relative crack length on the dimensionless SIFs, $F_I$ for different mechanical mismatch, (a) $\alpha = 1$, (b) $\alpha = 5$ and (d) $\alpha = 10$ when slanted angles are varied.

Figs. 5(b) and 5(c) reveal the distributions of $F_I$ for two mechanical mismatches, $\alpha = 5$ and 10. As the relative crack depth, $a/L$ increased, the curves of $F_I$ for different slanted angles diverged. It is also clearly showed that once the slanted angles are increased the $F_I$ is slightly lowered and it is probably if $\theta > 40^\circ$, the SIFs obtained from slanted angles approached the SIFs of normal cracks.
4.3 Crack Mechanisms in Bi-Material Plates

Figs. 6 and 7 reveal the crack mechanisms of the $10^\circ$ and $40^\circ$ slanted angle cracks in bi-material plates. The mechanical properties of the bottom are higher than the upper ends. It is mean that the upper plate experienced relatively high mechanical deformation when compared with the lower end. According to Fig. 5, the $F_I$ for $10^\circ$ is higher than $40^\circ$ regardless of their slanted angles. This behavior of cracks is due to the fact that for the almost flattened, the crack mount is capable to open wider than the higher value of slanted angle cracks. On the other hand, the stresses at the crack tip are nicely symmetrical. Comparing to the Fig. 7, the stresses at the crack tip is not symmetrical where the formation of mode II SIF, $F_{II}$ is significant. It is also revealed that when higher mechanical mismatch is used, the $F_I$ is significantly reduced where $F_I$ is approached the value of $F_I$ for normal cracks.

![Figure 6 Crack mechanisms of the $10^\circ$ slanted cracks in bi-material plates for different mechanical mismatches, $\alpha$, (a) 1.0, (b) 5.0 and (c) 10.0.](image)

![Figure 7 Crack mechanisms of the $40^\circ$ slanted cracks in bi-material plates for different mechanical mismatches, $\alpha$, (a) 1.0, (b) 5.0 and (c) 10.0.](image)

5. Conclusion

The following conclusions are drawn from this study:

i. For the normal crack, the distributions of $F_I$ and $F_{II}$ are not greatly dependent on the mechanical mismatched.

ii. When slanted angle cracks are introduced, higher SIFs can be obtained. However, when slanted angle is high, the SIFs approached the SIFs for normal cracks.

iii. The SIFs are reduced when higher slanted angles are used due to different crack mechanisms. Relatively, wider crack month occurred when lower slanted angle is used comparing with higher angle.

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