Improvement of angular rotation measurement resolution and sensitivity based on an SU(1,1) interferometer with intensity sum detection

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Abstract
In order to realize the enhancement of angular rotation measurement sensitivity, a new scheme based on an SU(1,1) interferometer with intensity sum detection has been proposed. A coherent beam carrying orbital angular momentum (OAM) enters into the SU(1,1) interferometer where the Dove prisms are inserted. By taking full advantage of the two outputs and employing intensity sum detection, the angular rotation measurement sensitivity can be enhanced by higher gain, photon number and OAM. Meanwhile, resolution can be improved while the pump beam carries OAM. The angular rotation measurement sensitivity of the SU(1,1) interferometer can beat standard quantum limit \(1/(2\sqrt{N})\), and it still can be optimal when photon depletion inside this interferometer is the same.

1. Introduction

For the photon with a Laguerre–Gaussian (LG) mode, it carries both spin angular momentum (SAM) and orbital angular momentum (OAM) [1–3]. For SAM, it can form a two-dimensional Hilbert space. And it is different for OAM. OAM is related to the transverse angular phase of the light. It is a Hilbert space that is discrete, orthonormal and high dimensional [4, 5]. The OAM eigenstates are characterized by helical phase fronts described by \(\exp(i\phi)\), where \(i\) is the quanta number of OAM and \(\phi\) is the azimuthal angle. OAM can form a high-dimensional Hilbert space due to that it is infinite. Meanwhile, OAM has been applied in quantum optics [6–8], quantum imaging [9, 10] and quantum information [11–13].

Recently, in the field of precision measurement, OAM has developed quickly. In 1998, the angular-displacement measurement based on OAM was realized by Miles et al. [14]. Barnett et al. claimed that in the above construction, they can realize the ultra-resolution of the angular rotation measurement [15]. In 2011, Jha et al. proposed that the sensitivity can reach \(1/2\nu\) while the inputs are \(N\) entangled photons carrying OAM based on the Mach–Zehnder interferometer [16]. When the inputs are \(N\) independent photon, the sensitivity can reach \(1/2\nu\) which is named as standard quantum limit (SQL). By injecting photons carrying OAM, D'Ambrosio et al. experimentally measured the small rotation angle of a Dove prism in 2013 [17]. Zhang et al. improved the resolution by employing the OAM based on the quantum measurement strategy, in which a narrow Full Width at Half Maximum (FWHM) of the normalized signal was obtained. In particular, the sensitivity of angular rotation measurement can be improved by using higher OAM [18, 19].

Meanwhile, Liu et al. theoretically analyzed the angular rotation measurement of SU(1,1) interferometers with homodyne detection [20, 21], and found that the optimal sensitivity of SU(1,1) interferometers can reach \(1/4\nu\). Hao et al. employed the parity detection to realize the resolution improvement of angular rotation measurement [22]. Jiao et al. used squeezing and parametric amplification to enhance the angular displacement estimation with homodyne detection [23]. Recently, the angular sensitivity was also enhanced in the hybrid interferometer by Zhang et al. [24]. In addition to the balance homodyne detection, another substract intensity...
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detection is also employed in the precision measurement which can boost the sensitivity [25]. For the interferometer, the intensity sum detection can get the two-outports photon number. It is larger than the single-outport photon number and can improve the phase sensitivity [26]. So, we wish to use the intensity sum detection to boost the sensitivity of angular rotation measurement. Then an SU(1,1) interferometer with the coherent state and intensity sum detection is proposed in this paper.

This paper is organized as follows. In section 2, the theoretical models of angular rotation measurement scheme are described. The resolutions and sensitivities of the SU(1,1) interferometers are discussed in section 3. We take the photon depletion and the variable gain coefficients into consideration in section 4. In section 5, we make a conclusion.

2. The theoretical analysis
As described in figure 1, this interferometer consists two parametric amplifier (PA) process and two mirrors. Different from the traditional SU(1,1) interferometer [27–29], it has two Dove prisms in the two arms which is similar to [21]. This interferometer is named as the SU(1,1) interferometer due to that it can be characterized by the SU(1,1) group. A coherent beam and an vacuum beam enter into the first PA and pass through the Dove prisms. Then the two beams after the first PA are combined by the second PA. The photon detectors are used to get the photon number after the interferometer. In order to show the inevitable photon depletion, fictitious beam splitters B(T) are employed with the transmission coefficient T1 and T2 as [19]. For convenience, the counter-clockwise light path is labelled as ‘a’ and the clockwise light path is labelled as ‘b’. According to [30, 31], four wave mixing (FWM) is one kind of PA and can be divided into two sorts. In [30], the probe beam carries OAM and the pump beam carries OAM in [31]. So, the scheme in figure 1 is divided into two models. In figure 1(a), the probe beam carries OAM "h and the pump beam carries OAM "h in figure 1(b).

2.1 Model A
In model A, the topological charge of the probe beam is 'l. The OAM conversion formula in the PA process is
\[ l_a + l_b = 2l_{\text{pump}} \]
and subscripts denote the corresponding beams[30]. So the topological charge in light path 'b
is −'l. The output of a PA process in terms of its input operators follows
\[
\hat{a}_l = \sqrt{G_1} \hat{a}_0 + \sqrt{G_1 - 1} \hat{b}_0^\dagger, \\
\hat{b}_l = \sqrt{G_1 - 1} \hat{a}_0^\dagger + \sqrt{G_1} \hat{b}_0,
\]
where G1 and the following G2 are used to denote gain coefficients of the two PA processes, respectively. \( \hat{a}_0 (\hat{a}_0^\dagger) \) and \( \hat{b}_0 (\hat{b}_0^\dagger) \) represent the annihilation operators(creation operators) and the subscripts ‘in’ and ‘out’ represents the input state and output state, respectively. When a beam with OAM "h passes through the Dove prism, it will have a phase shift of 2\( \theta \) and \( \theta \) is the rotation angle of prism. Then, the output operators in terms of input
operators $\hat{a}_0$ and $\hat{b}_0$ can be given by

$$\hat{a}_{\text{out}} = J_1\hat{a}_0 + J_2\hat{b}_0,$$

$$\hat{b}_{\text{out}} = J_3\hat{a}_0^\dagger + J_4\hat{b}_0,$$

and $J_1 = T_1\sqrt{G_1}G_2e^{2i\theta_1} + T_2\sqrt{(G_1 - 1)(G_2 - 1)}e^{2i\theta_2}$, $J_2 = (T_1\sqrt{G_1}G_2 - 1)G_2 - 1)N + 2(T_1^2 + T_2^2)(2G_1G_2 - G_1 - G_2)$.

Next, according to quantum estimation process, the angular rotation measurement sensitivity can be calculated by

$$\Delta\theta = \frac{\sqrt{\langle \hat{O}\rangle - \langle \hat{O}\rangle^2}}{\sqrt{\langle \hat{O}\rangle}} |\langle \hat{O}\rangle| \sin(2\theta)$$

which is an important criteria of the interferometer, can be de

$$\langle \hat{O}\rangle = \langle \hat{O}_a + \hat{O}_b \rangle = \langle \hat{a}_{\text{out}}\hat{a}_{\text{out}}^\dagger + \hat{b}_{\text{out}}\hat{b}_{\text{out}}^\dagger \rangle,$$

where $O_a$ and $O_b$ are the two-outport intensities after the interferometer, respectively. Then the total intensity can be expressed as

$$\langle \hat{O}\rangle = \langle T_1^2(G_1 + T_2^2(G_1 - 1))(2G_2 - 1)N + 2(T_1^2 + T_2^2)(2G_1G_2 - G_1 - G_2) \rangle + T_1T_2\sqrt{G_1G_2(G_1 - 1)(G_2 - 1)}(4N + 3)\cos(2\theta)\rangle.$$

Here, $\theta = \theta_1 - \theta_2$, $\langle \hat{a}\rangle = \sqrt{N} \cdot N$ is the photon number of the input coherent beam. The visibility, which is the maximum and the minimum values of $\langle \hat{O}\rangle$.

$$V = \frac{T_1T_2\sqrt{G_1G_2(G_1 - 1)(G_2 - 1)}(4N + 3)\cos(2\theta)}{(T_1^2 + T_2^2)(2G_1G_2 - G_1 - G_2)}.$$

2.2. Model B

For model B, the topological charge of the pump beam is $l$ and the probe beam does not carry OAM. According to phase matching condition $3\text{11}$, the topological charge of path $b$ is $2l$. Analogously, the operators of output signals in model B can be shown as

$$\hat{a}_{\text{out}} = M_1\hat{a}_0 + M_2\hat{b}_0,$$

$$\hat{b}_{\text{out}} = M_3\hat{a}_0^\dagger + M_4\hat{b}_0,$$

and $M_1 = T_1\sqrt{G_1G_2} + T_2\sqrt{(G_1 - 1)(G_2 - 1)}e^{-2i\theta_1}$, $M_2 = (T_1\sqrt{G_1G_2} - 1)G_2 - 1)N + 2(T_1^2 + T_2^2)(2G_1G_2 - G_1 - G_2)$.

Therefore, the signal in model B is

$$\langle \hat{O}\rangle = \langle \hat{O}_a + \hat{O}_b \rangle = \langle T_1^2(G_1 + T_2^2(G_1 - 1))(2G_2 - 1)N + 2(T_1^2 + T_2^2)(2G_1G_2 - G_1 - G_2) \rangle + T_1T_2\sqrt{G_1G_2(G_1 - 1)(G_2 - 1)}(4N + 3)\cos(4\theta)\rangle.$$

The visibility of the signal in model B is

$$V = \frac{T_1T_2\sqrt{G_1G_2(G_1 - 1)(G_2 - 1)}(4N + 3)\cos(4\theta)}{(T_1^2 + T_2^2)(2G_1G_2 - G_1 - G_2)}.$$

3. The resolution and sensitivity

First, we focus on the resolution which is evaluated by the FWHM of the normalized sign peak. The narrower FWHM, the better resolution. The FWHM of the two models’ normalized signal and the SU(2) interferometer
are displayed in figure 2. According to figure 2(a), the FWHM of model A is same to that of the SU(2) interferometer due to the same OAM. Here, the normalized signal of the SU(2) interferometer is 
\[
IN(l) = \frac{1}{\cos^2(l\theta)}.
\]
The FWHM of model B halves the peak-to-peak spacing of FWHM of model A. So, the resolution is boosted by a factor of 2. It can be explained by comparing \cos(2l\theta) with \cos(4l\theta) existing in equations (4) and (8), respectively. According to the phase match condition, when the OAM of model A is \(l\theta\), the OAM of model B is \(2l\theta\). So the FWHM of model B is half than that of model A. In figure 2(b), the FWHM of the two models and the SU(2) interferometer become better with the increase of topological charge \(l\) and the resolution of model A and the SU(2) interferometer are always the same. For model B, , the FWHM is half with the same topological charge. Moreover, the photon number \(N\) has no effect on the resolution. Then it can be arbitrary positive integer. Figure 3 presents the visibility. It can be seen that the visibility increases progressively and ultimately tend to be 1 with the increase of the photon number \(N\). In order to have a better visibility, larger input photon number \(N\) is preferred. And the visibility of model A is same to that of model B. In equations (5) and (9) \(l\) is missing. So, the topological charge has nothing to do with the visibility.

The sensitivities of angular rotation measurement of the two models are shown in figure 4. And we take the natural logarithm of sensitivity in z-axis. According to figure 4(a) and 4(b), with the same OAM, the sensitivities of the two models change with the angular rotation \(\theta\). And the sensitivity value can be lower with the larger photon number. The lower value, the better sensitivity. Different from figure 4(a) and 4(b), the two models have the optimal sensitivities at different angles in figure 4(c) and 4(d) and the rotation angle changes periodically. With the increase of the photon number, the sensitivities can be much better. However, the sensitivity is much worse when the rotation angle is near 0.

The optimal sensitivities versus the topological charge \(l\) are displayed in figure 5. The sensitivity curves of the two models are far lower than that of SQL which represents the phase sensitivity 
\[
\frac{1}{2N_1}.
\]
\(N_1\) is the total photon number inside the SU(1,1) interferometer and it is \((2G_1 - 1)N + 2(G_1 - 1)\). In figure 5, both the sensitivities of

![Figure 2](image2.png)

**Figure 2.** (a) The normalized signals versus angular rotation and (b) the FWHM of two models’ normalized signal versus the topological charge \(l\) with arbitrary average photon \(N\). Others’ parameters are \(G_1 = G_2 = 10, N = 10, l = 3\) and \(T_1 = T_2 = 1\).

![Figure 3](image3.png)

**Figure 3.** The visibility curve of this scheme for arbitrary topological charge \(l\). The visibility is 0.89 when \(N = 10\). Others’ parameters are figure 2.
two schemes can beat SQL and the sensitivities can be better with the increase of topological charge \( l \). Meanwhile, for mode A and model B, with the same OAM, the optimal sensitivity with \( N = 20 \) is better than that with \( N = 10 \). So better sensitivity can be achieved by employing higher OAM and larger photon number. In addition, the sensitivity of model A with the photon number \( N = 10 \) is better than SQL with \( N = 20 \). And the sensitivity of the model B with the photon number \( N = 10 \) can beat that of model A with the photon number \( N = 20 \). Then, model B performs better than model A in sensitivity.

4. The effects of gain and transmission coefficients

In this part, gain and transmission coefficients on sensitivities are shown in figure 6. Figures 6(a) and (b) present the optimal sensitivity for different coefficients \( G_1 \) and \( G_2 \). When \( G_1 \) and \( G_2 \) increase from 0 to 20, the sensitivity...
can be better and it is optimal with $G_1 = G_2 = 20$. In figure 6(a), the sensitivity with $G_1 = 10$ and $G_2 = 20$ is same to that with $G_1 = 20$ and $G_2 = 10$. In addition, the sensitivity of model B is better than that of model A with the same gain coefficients. And $G_1$ and $G_2$ play the same role in figure 6(b). Likewise, figures 6(c) and (d) show the optimal sensitivities on transmission coefficients $T_1$ and $T_2$. The larger transmissivity, the better sensitivity. The sensitivity reaches the worst when $T_1$ is approaching 0 and $T_2 = 1$, or inverse. Without the depletion, the sensitivity can be optimal. With the depletion, the sensitivity still is optimal only with $T_1 = T_2$ for model A and model B. So the scheme is robust against photon depletion when intensity sum detection is employed. Moreover, it is clear that the sensitivity of model B is better than that of model A with the same gain coefficients.

5. Conclusion

This paper has proposed to use the intensity sum detection to boost the angular rotation sensitivity based on the SU(1,1) interferometer. The resolutions and sensitivities of the SU(1,1) interferometers are displayed. When OAM is carried by a pump beam, the SU(1,1) interferometer has better resolution and sensitivity than that of the SU(2) interferometer. When OAM is carried by a probe beam, only the sensitivity can be improved. Meanwhile, the sensitivity can be boosted by increasing topological charge $l$ or the gain coefficient $G_1$ and $G_2$. Besides, due to the employment of intensity sum detection, the sensitivity is optimal for the photon depletion $T_1 = T_2$, which means that the scheme is robust against photon depletion. This scheme has many potential applications in the field of precision measurement, such as phase sensing [32, 33] and microcantilever displacement [34].

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Disclosure statement

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