Analysis and Prediction of Subway Settlement Deformation Based on Grey Model

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Abstract. In this paper, the monitoring data of settlement and deformation along the western section of a metro in H city are collated and analyzed, the actual situation of settlement and deformation is grasped, and the long-term settlement and deformation trend are predicted by using grey prediction model. In the process of forecasting, a non-equidistant grey forecasting model is established by using the data of the first several groups of monitoring points, and the predicted values are compared with the last group of monitoring data to verify the reliability of the model and further optimize the model. Then, the optimized model is used to predict the long-term settlement deformation of the subway, which provides a basis for the deformation control and management of the subway.

1. Introduction
With the rapid development of China’s economic construction, the development of underground space has become an important way to accelerate the process of urbanization, and the rapid development of the subway plays an important role in the underground transportation[1]. However, due to the influence of load, soil quality and complicated hydrogeological conditions, the longitudinal settlement deformation of subway tunnel structure will occur. When the longitudinal deformation is too large, the phenomenon of leakage of water, mud and even tensile failure of segments will occur, which is likely to cause the deformation of subway track. If the track is not maintained reasonably and effectively, it will affect the comfort and safety of train operation, and even cause loss of life and property [2].

At present, the research on the settlement of subway tunnel mainly focuses on the construction period, while the research on the long-term settlement deformation of the tunnel during the operation period is less. The long-term settlement monitoring needs a long time span and a large cost. Therefore, it is particularly important to analyze and sort out the settlement deformation data obtained from the short-term monitoring of subway tunnel, and study and accurately predict its long-term settlement.

For the prediction of long-term deformation of tunnel, after obtaining the monitoring data of settlement deformation, the data can be simply processed, and then the mathematical model is established for analysis and prediction, and then the long-term unknown data can be obtained. The commonly used mathematical modeling methods include time series analysis model, regression analysis method, gray system analysis model, spectrum analysis method, etc[3-6]. In view of the small amount of data required for the theoretical analysis of the grey system, the stable prediction can be made according to the incomplete information, and then the important information contained in the original data can be mined out. However, the monitoring data of the project in this case is small and the monitoring time interval is different, so the grey system analysis method is selected for analysis and prediction.
2. Project overview

2.1. Project introduction
The first phase project along the western line of a subway is not only the core component of the urban rail transit network framework, but also the basic project for the development of rail transit from single line operation to network operation. It connects the main city and the sub City, with a total length of 47.97 km, including 41.36 km underground line, 6.14 km elevated line and 0.3 km ground line, with 31 stations, 2 parking lots and vehicle bases Seat.

2.2. Hydrogeological conditions
The area where the project is located is mainly alluvial plain, no obvious structural features are found in the area, and the neotectonic movement is not obvious. The upper soil layer is silty sand and sandy silt, which is loose, easy to collapse and easy to produce quicksand under the action of groundwater; the middle soil layer is silt and mucky soft soil with high water content, which is easy to produce rheology or thixotropy; the lower soil layer is silty clay with medium compressibility and good engineering performance, but its thickness is not large and its distribution is uneven. The bottom is moderately weathered bedrock with moderate depth, wide distribution and good engineering performance, which is an ideal bearing stratum for pile foundation.

The drilling depth along the project is mainly pore phreatic water of Quaternary loose rock, and the deep part is bedrock fissure water. Groundwater mainly exists in the surface soil, silty clay and muddy soil, with shallow depth. It is mainly recharged by precipitation and surface water runoff, and belongs to pore phreatic water. Bedrock fissure water mainly exists in the lower part of the strongly weathered and moderately weathered bedrock fissures, with slow flow rate, downward drainage, and small water volume.

2.3. Layout of settlement monitoring points
Tianbao din03 precise level and corresponding indium tile level are used to monitor the settlement of subway station and tunnel. The monitoring range is K0 + 000-k21 + 803.316, with a total of 21.8 km. It is divided into 38 monitoring sections, each of which is divided into up line and down line. There are 150 ~ 250 monitoring points in the interval. The monitoring work started on June 20, 2012 and ended on August 31, 2015, during which a total of 7 monitoring data were collected.

3. Settlement deformation analysis

3.1. Longitudinal settlement variation along the line
There are 38 monitoring sections in the project, each section has 150 ~ 250 monitoring points, and each monitoring point obtains 7 monitoring data. Due to the large number of monitoring data, only the monitoring value of the maximum deformation point in each interval is used to represent the settlement and deformation of the interval. Based on the summary of the original data of each period in each section, the maximum settlement deformation curves along the up line and down line of the subway are obtained, as shown in Fig. 1 and Fig. 2 (the subsidence deformation value is negative, and the uplift deformation value is positive).
It can be seen from the above two figures that the settlement deformation along the line is particularly significant in the interval 3, section 25, section 29 and section 37, and the situation of the up line and the down line is consistent. The reasons for the significant settlement deformation may be the uneven distribution of soft soil in the lower part of the area, the large local load on the upper part of the tunnel, the influence of adjacent engineering construction around the tunnel, and the vibration generated by the subway itself.

3.2. Settlement analysis and summary
In order to further study the settlement and deformation of each monitoring point in the four sections with significant deformation, the distribution curves of the accumulated settlement along the line of each monitoring point are drawn according to the original monitoring data, and the original data of the maximum settlement deformation point in each section are selected to sort out and draw the monitoring data curve. Due to the limited space, only the distribution curve of accumulated settlement along interval 3 and the monitoring data curve of XHBK-CJ-R42, the largest settlement deformation point in the interval, are shown in Fig. 3 and Fig. 4 respectively.
Finally, the cumulative settlement deformation curve of the maximum settlement deformation points of the four sections is drawn, as shown in Figure 5.

4. Grey system modeling

4.1. Introduction to the model

The grey model established based on grey theory is a dynamic fuzzy prediction model. Starting from a time series of data, the nonlinear sequence of the original data is transformed into a linear sequence through one accumulation, so as to weaken the randomness of the original data, and then the potential laws hidden in the original data are obtained [7]. The typical model GM (1,1) of grey theory is the time series prediction model with equal time interval, while the settlement deformation monitoring data of the project is a kind of time series with non-equal time interval, so the model can not be directly used, and the non-equal time interval grey model must be used for modeling and prediction analysis [8]. The calculation principle is as follows: firstly, a set of 7 original monitoring data is selected as a sequence $X^{(0)}$, $X^{(0)}(K_i) = \{X^{(0)}(K_1), X^{(0)}(K_2), \ldots, X^{(0)}(K_n)\}$ At this time, the time interval $\Delta K_i = K_i - K_{i-1} \neq$ constant. After multiplying the two, the sequence is obtained by one-time accumulation $X^{(1)}$, $X^{(1)}(K_i) = \{X^{(1)}(K_1), X^{(1)}(K_2), \ldots, X^{(1)}(K_n)\}$ Then the time response function is obtained

$$X^{(1)}(K_i) = \left[ X^{(0)}(K_i) - \frac{u}{a} \right] e^{-u(k_i-K_i)} + \frac{u}{a}.$$ Restore the model function:

$$X^{(0)}(K_{i+1}) = \frac{1}{\Delta K_{i+1}} \left( 1 - e^{-uK_{i+1}} \right) \left[ X^{(0)}(K_i) - \frac{u}{a} \right] e^{u(K_{i+1} - K_i)}.$$ The parameters $a$ and $u$ satisfy the formula
\[ \begin{bmatrix} a, u \end{bmatrix} = \left( B^T B \right)^{-1} B^T Y \] \[
B = \begin{bmatrix}
-\bar{y}(K_x) & 1 \\
-\bar{y}(K_y) & 1 \\
\vdots & \vdots \\
-\bar{y}(K_y) & 1
\end{bmatrix}, \quad Y = \begin{bmatrix}
x_0(K_x) \\
x_0(K_y) \\
\vdots \\
x_0(K_y)
\end{bmatrix}
\]

### 4.2. Modeling

The non-equidistant GM (1,1) model function obtained from the above section can establish the grey model for the four monitoring points with the maximum settlement deformation in the project. Here, the modeling process of XHBK-CJ-R42 monitoring point in interval 3 is listed in detail, and the modeling process of the other three monitoring points is similar.

According to 7 groups of original settlement deformation monitoring data obtained from monitoring point XHBK-CJ-R42 from July 4, 2012 to January 9, 2015, a non-equidistant grey prediction model is established, and relevant parameters in the model are listed, as shown in Table 1. (the sign used to indicate uplift or descent is omitted in the modeling process)

| Number (k) | Monitoring time | Time interval | \( x_0(k_i) \) | \( x^{(1)}(k_i) \) |
|-----------|-----------------|---------------|----------------|-----------------|
| 1         | 2012/7          | \( \Delta k \cdot 0 \) | 10.9485        | 0               |
| 2         | 2012/8          | 0.9           | 10.9494        | 9.8545          |
| 3         | 2012/11         | 3.2           | 10.9544        | 44.9085         |
| 4         | 2013/4          | 4.8           | 10.9603        | 97.5180         |
| 5         | 2013/8          | 4.2           | 10.9641        | 143.2383        |
| 6         | 2014/5          | 9.2           | 10.9809        | 244.2626        |
| 7         | 2015/1          | 7.9           | 11.0034        | 331.1894        |

Using MATLAB programming, we can get the following results:

\[ \begin{bmatrix} a, u \end{bmatrix} = \begin{bmatrix} 1.84876858469945e-04 \\
9.47015367416324 \end{bmatrix} \]

Substituting the time response function, we get the following results:

\[ \hat{x}^{(1)}(k_i) = 5.922341813721481 e^{04e^{-1.84876858469945e-04(k_i-k_i)}} - 5, 921246963721481 e^{04} \]

After substituting the restored model function, we get the following results:

\[ \hat{x}^{(0)}(k_{i+1}) = 5.922341813721481 e^{04e^{-1.84876858469945e-04(k_{i+1}-k_i)}} \]

The grey model fitting results of monitoring point XHBK-CJ-R42 can be obtained by calculating the model function, as shown in Table 2.

| Observation time | Original observed settlement data (m) | Model fitting data (m) |
|------------------|---------------------------------------|------------------------|
| 2012/7           | 10.9485                               | 10.9485                |
| 2012/8           | 10.9494                               | 10.9479                |
| 2012/11          | 10.9544                               | 10.9520                |
| 2013/4           | 10.9603                               | 10.9601                |
| 2013/8           | 10.9641                               | 10.9692                |
| 2014/5           | 10.9809                               | 10.9828                |
| 2015/1           | 11.0034                               | 11.0002                |

Through calculation, the residual error test, correlation test and posterior error test of the fitting value of the grey model are all qualified. The grey model parameters of the remaining three monitoring points are shown in Table 3.
Table 3. Model parameter table of each monitoring point

| Model parameter | a       | u       | Max (e) | C       | P |
|-----------------|---------|---------|---------|---------|---|
| XHBK-CJ-R42     | -0.00018488 | 10.9470 | 0.0051  | 0.1426  | 1 |
| LXFQ-CJ-L71     | -0.00029097  | 6.7116  | 0.0022  | 0.0781  | 1 |
| WLWG-CJ-R27     | -0.00014114  | 14.6295 | 0.0071  | 0.1976  | 1 |
| FJDZ-CJ-R72     | -0.00016356  | 15.2390 | 0.0084  | 0.2064  | 1 |

It can be seen from table 3 that the residual value of the grey model of each monitoring point is small, and the maximum value is only 8.4mm, with excellent accuracy level. By substituting the parameters in the table into the reduced model function, the fitting values of the settlement deformation model of the other three monitoring points can be calculated.

5. Settlement prediction

5.1. Model verification and optimization

In addition to the seven groups of monitoring data obtained during the monitoring period, the project conducted a field monitoring in July 2015. Here, we will use the last set of new data to further test the accuracy and reliability of the model function. The comparison results between the measured data and the model predicted value are shown in table 4.

Table 4. Comparison table of the last group of data of each monitoring point

| Monitoring point | Original monitoring settlement value (m) | Model predictive value (m) | e (mm) |
|------------------|----------------------------------------|---------------------------|-------|
| XHBK-CJ-R42      | 11.0153                                 | 11.0144                   | -0.9  |
| LXFQ-CJ-L71      | 6.7675                                  | 6.7802                    | 12.7  |
| WLWG-CJ-R27      | 14.6934                                 | 14.6998                   | 5.4   |
| FJDZ-CJ-R72      | 15.3019                                 | 15.3218                   | 19.9  |

It can be seen from table 4 that there is a certain deviation between the predicted value and the measured value, and it is necessary to further optimize the model to reduce the deviation.

The model optimization can be achieved by recalculating the last group of monitoring data. Here, the model optimization process of XHBK-CJ-R42 monitoring point in interval 3 is listed in detail. Using MATLAB programming, we can get the following results:

\[
\begin{bmatrix}
\hat{a} \\
\hat{u}
\end{bmatrix} = \begin{bmatrix}
-1.866969047078504 \times 10^{-4} \\
10.946872397189496
\end{bmatrix}
\]

Substituting the time response function, we can get the following results:

\[
\hat{x}^{(1)}(k_t) = 58645.40960324931 e^{-1.866969047078504 \times 10^{-4}(k_t - k_0)} - 58634.46110321e^{-1.866969047078504 \times 10^{-4}(k_t - k_0)}
\]

The optimized model function of monitoring point XHBK-CJ-R42 can be obtained by substituting the reduced model function.

\[
\hat{x}^{(0)}(k_{i+1}) = 58645.40960324931 \frac{1}{\Delta k_{i+1}} (1 - e^{-1.866969047078504 \times 10^{-4} \Delta k_{i+1}}) e^{-1.866969047078504 \times 10^{-4}(k_t - k_i)}
\]

The model fitting results can be obtained by replacing the monitoring data of XHBK-CJ-R42, as shown in Table 5.

Table 5. Data comparison table of XHBK-CJ-R42 monitoring point prediction model

| Observation time | Original settlement data(m) | Model fitting data(m) |
|------------------|----------------------------|-----------------------|
| 2012/7           | 10.9485                    | 10.9485               |
| 2012/8           | 10.9494                    | 10.9478               |
| 2012/11          | 10.9544                    | 10.9520               |
| 2013/4           | 10.9603                    | 10.9602               |
| 2013/8           | 10.9641                    | 10.9693               |
| 2014/5           | 10.9809                    | 10.9830               |
The optimized model still meets the residual test, correlation test and posterior difference test, and the fitting value is closer to the original data, and the optimization effect is good. The optimization process of the other three monitoring points is similar, and the grey model parameters of each monitoring point after optimization are shown in Table 6.

Table 6. Parameter table of prediction model for each monitoring point

| model parameter | a       | u        | C        | P |
|-----------------|---------|----------|----------|---|
| XHBK-CJ-R42     | -0.00018670 | 10.9469 | 0.1038 | 1 |
| LXFQ-CJ-L71     | -0.00025769 | 6.7132  | 0.1371 | 1 |
| WLWG-CJ-R27     | -0.00013446 | 14.6303 | 0.1150 | 1 |
| FJDZ-CJ-R72     | -0.00013825 | 15.2417 | 0.2383 | 1 |

5.2. Long term settlement deformation prediction

According to table 6, the model fitting values of each monitoring point can be calculated after optimizing the prediction model parameters of each monitoring point. The fitting value is compared with the monitoring data, and the curves of the original monitoring data and model fitting data of each monitoring point are drawn, as shown in FIG. 6 to FIG. 9.

It can be seen from the above figures that the deviation between the optimized model fitting data and the original monitoring data is small, the fitting effect is good, and the curve is highly consistent. Therefore, the optimized model can be used to predict the settlement deformation value of each monitoring point at any time point in the future. Here, only the predicted values of the model in June 2020 are listed, and the corresponding accumulated settlement deformation and deformation rate are calculated, as shown in Table 7.
Table 7. Prediction model parameters of each monitoring point

| Monitoring point | model prediction value (m) | accumulated settlement deformation (mm) | settlement deformation rate (mm/d) |
|------------------|----------------------------|----------------------------------------|-----------------------------------|
| XHBK-CJ-R42      | 11.0837                    | 135.2                                  | 0.047                             |
| LXFQ-CJ-L71      | 6.8293                     | 115.8                                  | 0.041                             |
| WLWG-CJ-R27      | 14.7617                    | 126.7                                  | 0.044                             |
| FJDZ-CJ-R72      | 15.3825                    | 145.0                                  | 0.050                             |

According to the settlement deformation rate in Table 7, it can be converted into the annual settlement deformation value of each monitoring point in the future (the deformation value is reserved to one decimal place, calculated as 360 days per year). Taking the monitoring point XHBK-CJ-R42 as an example, the future annual average settlement deformation value is 0.047x360=16.9mm. According to the calculation, the average annual change values of the future settlement and deformation of the four monitoring points in the above table are 16.9mm, 14.4mm, 18.575mm and 18.125mm respectively, which are all less than the long-term monitoring and control indexes of the structure by 20mm, which belong to the safe range. However, the average annual deformation value of monitoring points XHBK-CJ-R42 and FJDZ-CJ-R72 exceeds 80% of the control value, which has reached the alarm state. Therefore, attention should be paid to the settlement and deformation of the monitoring section where the two monitoring points are located, and the daily monitoring and maintenance of the tunnel in the section should be strengthened.

6. Conclusion
The original monitoring data of 7 groups of settlement deformation of 34 monitoring sections along the western line of a subway are summarized and sorted out, and it is found that 4 sections have significant settlements. The monitoring points with the largest settlement deformation are selected from each of the four sections for further analysis. It is found that the settlement deformation of the monitoring points increases rapidly with the passage of time during the whole monitoring period, and the cumulative settlement value increases approximately linearly with time. Based on this trend, the settlement of monitoring points will continue to occur in the future, and the deformation is large.

In order to better monitor and manage the settlement and deformation of subway section in the later stage, a non-equidistant grey prediction model is established to predict the long-term settlement and deformation. Based on the monitoring data of the four monitoring points with the largest settlement deformation, the non-equidistant grey prediction model is accurate and reliable, and the fitting results are in good agreement with the original monitoring data. It can accurately predict the settlement deformation at a certain time point in the future, and provide a reliable basis for the safe operation of the subway.

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