Spin Correlations in top quark pair production near threshold at the $e^-e^+$ Linear Collider

Qing Jun Zhang, Chong Sheng Li, Jian Jun Liu, and Li Gang Jin
Department of Physics, Peking University, Beijing 100871, China

February 7, 2008

Abstract

We investigate the spin correlations in top quark pair production near threshold at the $e^-e^+$ linear collider. Comparing with the results above the threshold region, we find that near the threshold region the off-diagonal basis, the optimized decomposition of the top quark spins above the threshold region, does not exist, and the beamline basis is the optimal basis, in which there are the dominant spin components: the up-down (UD) component for $e^-_Le^+$ scattering and the down-up (DU) component for $e^-_Re^+$ scattering can make up more than 50% of the total cross section, respectively.

Keywords: top quark, spin correlations, threshold.
PACS numbers: 12.38-t, 13.88.+e, 14.65.Ha

*csli@pku.edu.cn
The top quark is the heaviest known particle in the standard model (SM) and rapidly decays before it hadronizes [1], and the spin information of the top quark is preserved from production to decay. Thus we can expect the spin orientation of the top quark to be observable experimentally. The spin correlations for the top quark pair production above the threshold at $e^+e^-$ colliders have been extensively discussed [2]. Parke and Shadmi [3] proposed the generic spin basis and found that the “off-diagonal” basis, a special case of the generic spin basis, is a more optimized decomposition of the top quark spins for $e^+e^-$ colliders. As shown in Refs. [4, 5], the off-diagonal basis is indeed the optimal spin basis even after the inclusion of $\mathcal{O}(\alpha_s)$ QCD corrections: at $\sqrt{s} = 400$ GeV using the off-diagonal basis the dominant spin components in both $e_L^-L$ and $e_R^+R$ scattering make up more than 99% of the total cross section at both tree and one-loop level, but such fraction is only $\sim 53\%$ in the helicity basis.

Up to now, all studies of spin correlations of the top quark pair production at the $e^-e^+$ collider are limited only to the process of $tt$ production above the threshold region. In the future $e^-e^+$ linear collider, threshold production of the top quark pair will allow to study their properties with extremely high precision. Because of large top quark mass and decay width, the bound-state resonances lose their separate identify and smear together into a broad threshold enhancement [6], and as a result, the nonpertubative QCD effects induced by the gluon condensate are small, allowing us to calculate the cross section with high accuracy by using perturbative QCD even in the threshold region. And it is interesting to investigate the spin correlations of the top quark pair production near threshold. Some methods used to deal with the behavior of top quark pair production near threshold have been established. The Green function technique was demonstrated suitable to calculate the total cross section and to predict the top quark momentum distribution, and independent approaches were developed for solving the Schrödinger equation in position space and the Lippmann-Schwinger equation in momentum space [6, 7, 8, 9].

In this letter, we study the spin correlations in the top quark pair production near threshold. In order to include effects of the quark-antiquark potential and the decay width in our calculation, we use two ingredients: the first is the vertex function, which represents
the QCD binding effects and the anomalous interactions, and can be obtained through solving Lippmann-Schwinger equation [10]; the second is the two unstable-particle phase space in the nonrelativistic limit [7], which, combined with the vertex function, describes the top quark momentum distribution. As a cross check of our calculation, after using above ingredients, also we obtain the total cross section and the momentum distribution which are consistent with ones given in Ref. [8].

In the SM, we consider the process

$$e^-e^+\rightarrow t\bar{t}$$  \hspace{1cm} (1)$$

at the LC with $\sqrt{s} \simeq 2m_t$. The tree level $Ve^-e^+$ vertex can be written as

$$\Gamma_{Ve}^\mu = \gamma^\mu(K_L^V P_- + K_R^V P_+),$$  \hspace{1cm} (2)$$

where $P_\pm = (1 \pm \gamma_5)/2$, and the SM values for these coupling factors are $K_{L,R}^\gamma = -e$ for $V = \gamma$, $K_{L,R}^Z = e(2\sin^2\theta_W - 1)/2\sin\theta_W\cos\theta_W$ and $K_{R}^Z = e\sin\theta_W/\cos\theta_W$ for $V = Z$, and the $\theta_W$ is the Weinberg angle.

The $Vt\bar{t}$ vertex function $\Gamma_{Vtt} (V = \gamma, Z)$ can be generally written as

$$\Gamma_{Vtt}^\mu = \gamma^\mu(A_V P_- + B_V P_+) + \gamma^5 p^\mu/m_t C_V,$$  \hspace{1cm} (3)$$

where $p^\mu$ is the momentum of the outgoing top quark, and $A_V, B_V, C_V$ are the form factors. As shown in Fig.1, the vertex function $\Gamma_V (\Gamma_{Vtt})$ satisfies following integral equation [7]:

$$\Gamma_V = X_V + \int \frac{d^4k}{(2\pi)^4}(-4\pi C_F \alpha_s)D_{\mu\nu}(p-k)\gamma^\mu S_F(k + \frac{q}{2})\Gamma_V(k, q)S_F(k - \frac{q}{2})\gamma^\nu,$$  \hspace{1cm} (4)$$

where $X_\gamma = \gamma_\mu$ and $X_Z = \gamma_\mu\gamma_5$, $S_F$ is the top quark propagator, and $D_{\mu\nu}$ is the gluon propagator. In the nonrelativistic limit, the propagators are replaced by

$$-4\pi C_F \alpha_s D_{\mu\nu}(p) \rightarrow iV(\vec{p})\delta_{\mu0}\delta_{\nu0},$$  \hspace{1cm} (5)$$

$$S_F(k + \frac{q}{2}) \rightarrow i\frac{1+\gamma^0}{2} - \frac{\vec{k}\cdot\vec{E}}{2m_t},$$  \hspace{1cm} (6)$$

$$S_F(k - \frac{q}{2}) \rightarrow i\frac{1-\gamma^0}{2} - \frac{\vec{k}\cdot\vec{E}}{2m_t}.$$  \hspace{1cm} (7)
where \( E_{NR} = \sqrt{s} - 2m_t \). With above approximations, when the QCD binding effects and the CP-violating anomalous couplings are included [10], the form factors \((A_V, B_V, C_V)\) in the vertex function \( \Gamma_V \) are given by

\[
A_\gamma = \frac{2}{3}e(1 - \frac{2C_F\alpha_s}{\pi})G(|\vec{p}_1|)\varphi, \tag{8}
\]

\[
A_Z = \frac{e}{\sin \theta_W \cos \theta_W} \left[ (\frac{1}{4} - \frac{2}{3} \sin^2 \theta_W)(1 - \frac{2C_F\alpha_s}{\pi})G(|\vec{p}_1|)\varphi + \frac{1}{4}(1 - \frac{C_F\alpha_s}{\pi})F(|\vec{p}_1|)\varphi \right], \tag{9}
\]

\[
B_\gamma = \frac{2}{3}e(1 - \frac{2C_F\alpha_s}{\pi})G(|\vec{p}_1|)\varphi, \tag{10}
\]

\[
B_Z = \frac{e}{4\sin \theta_W \cos \theta_W} \left[ (1 - \frac{8}{3} \sin^2 \theta_W)(1 - \frac{2C_F\alpha_s}{\pi})G(|\vec{p}_1|)\varphi - (1 - \frac{C_F\alpha_s}{\pi})F(|\vec{p}_1|)\varphi \right], \tag{11}
\]

\[
C_\gamma = -i\epsilon\tau_\gamma (1 - \frac{C_F\alpha_s}{\pi})F(|\vec{p}_1|)\varphi + \frac{2}{3}i\epsilon\tau_g D(|\vec{p}_1|)\varphi, \tag{12}
\]

\[
C_Z = \frac{e}{\sin \theta_W \cos \theta_W}(-i\epsilon\tau_\gamma (1 - \frac{C_F\alpha_s}{\pi})F(|\vec{p}_1|)\varphi + i\epsilon\tau_g (\frac{1}{4} - \frac{2}{3} \sin^2 \theta_W)D(|\vec{p}_1|)\varphi) \tag{13}
\]

where \( \epsilon_\tau_\gamma, \epsilon_\tau_z \) and \( \epsilon_\tau_g \) are the anomalous couplings of the top quark, and \( \varphi = \frac{|\vec{p}_1|^2}{m_t} - (E_{NR} + i\Gamma_t) \). The Green-functions \((G, F \text{ and } D)\) in Eqs.(8)-(13) are the solutions of the Lippmann-Schwinger equations [10]:

\[
\left( \frac{|\vec{p}|^2}{m_t} - (E_{NR} + i\Gamma_t) \right)G(E_{NR}, |\vec{p}|) + \int \frac{d^3k}{(2\pi)^3} [V(|\vec{p} - \vec{k}|)G(E_{NR}, |\vec{k}|)] = 1, \tag{14}
\]

\[
\left( \frac{|\vec{p}|^2}{m_t} - (E_{NR} + i\Gamma_t) \right)\vec{p}^\dagger F(E_{NR}, |\vec{p}|) + \int \frac{d^3k}{(2\pi)^3} [V(|\vec{p} - \vec{k}|)\vec{k}^2 F(E_{NR}, |\vec{k}|)] = \vec{p}^\dagger, \tag{15}
\]

\[
\left( \frac{|\vec{k}|^2}{m_t} - (E_{NR} + i\Gamma_t) \right)\vec{k}^\dagger D(E_{NR}, |\vec{p}|) + \int \frac{d^3k}{(2\pi)^3} [V(|\vec{p} - \vec{k}|)\vec{k}^2 D(E_{NR}, |\vec{k}|)] = \int \frac{d^3k}{(2\pi)^3} [V(|\vec{p} - \vec{k}|)(\vec{p} - \vec{k})^i G(E_{NR}, |\vec{k}|)], \tag{16}
\]

which can be derived from Eq.(4), and the QCD potential \( V(|\vec{p} - \vec{k}|) \) is given by [11]

\[
V(|\vec{p} - \vec{k}|) = V_C(|\vec{p} - \vec{k}|) + V_{BF}(|\vec{p} - \vec{k}|) + V_{NA}(|\vec{p} - \vec{k}|) \tag{17}
\]

with

\[
V_C(|\vec{p} - \vec{k}|) = -\frac{4\pi C_F\alpha_s(|\vec{p} - \vec{k}|^2)}{|\vec{p} - \vec{k}|^2}[1 + \frac{\alpha_s(|\vec{p} - \vec{k}|^2)}{4\pi}a_1 + \frac{\alpha_s(|\vec{p} - \vec{k}|^2)}{4\pi}^2a_2], \tag{18}
\]
\[
V_{BF}(|\vec{p} - \vec{k}|) = -4\pi C_F \alpha_s(|\vec{p} - \vec{k}|^2) \frac{1}{m_t^2|\vec{p} - \vec{k}|^4} \left[ \frac{(\vec{p} \times \vec{k})^2}{m_t^2|\vec{p} - \vec{k}|^4} + \frac{1}{4m_t^2} + \frac{3i(\vec{p} \times \vec{k}) \cdot (\vec{S}_t + \vec{S}_\bar{t})}{2m_t^2|\vec{p} - \vec{k}|^2} \right] + \frac{1}{2m_t^2}((\vec{S}_t + \vec{S}_\bar{t})^2 - \frac{(\vec{S}_t + \vec{S}_\bar{t}) \cdot (\vec{p} - \vec{k})^2}{|\vec{p} - \vec{k}|^2})]
\]

\[
V_{NA}(|\vec{p} - \vec{k}|) = -\frac{3\pi^2 C_F \alpha_s^2(|\vec{p} - \vec{k}|^2)}{m_t|\vec{p} - \vec{k}|},
\]

\[
\alpha_s(|\vec{p} - \vec{k}|^2) = \frac{4\pi}{\beta_0 \ln(|\vec{p} - \vec{k}|^2/\Lambda^2)} \left[ 1 - \frac{2\beta_1 \ln(\ln(|\vec{p} - \vec{k}|^2/\Lambda^2))}{\beta_0^2 \ln(|\vec{p} - \vec{k}|^2/\Lambda^2)} \right],
\]

where \(\vec{S}_t\) and \(\vec{S}_\bar{t}\) are the top and antitop spin operators, and

\[
a_1 = \frac{43}{9}, \quad a_2 = 155.842, \quad \Lambda = 226 \text{ MeV},
\]

\[
\beta_0 = 23/3, \quad \beta_1 = 58/3, \quad \mu = 20 \text{ GeV}.
\]

Using above vertex functions, we calculate the spin correlations. In the \(t\bar{t}\) center of mass frame (CMS), the scattering plane is defined to be the X-Z plane where the electron is moving along the +Z direction and \(\theta_t\) is defined as the scattering angle of the top quark, and we also set \(\phi_t = 0\). The Born helicity amplitudes for the process (1) are obtained by summing the contributions from both the Z and \(\gamma\):

\[
M(h_{e-}, h_{e+}, h_t, h_{\bar{t}}) = M(h_{e-}, h_{e+}, h_t, h_{\bar{t}})^\gamma + M(h_{e-}, h_{e+}, h_t, h_{\bar{t}})^Z R(s),
\]

where \(s = 4E_e^2\) is the total energy in CMS, \(E_e\) is the energy of the electron, and \(R(s) = s/(s - m_Z^2)\). \(^1\)

In the generic spin basis \([3]\) the top quark (anti-top quark) spin states are defined in the top quark (anti-top quark) rest-frame, where one decomposes the top (anti-top) spin along the direction \(\hat{s}_t\) (\(\hat{s}_{\bar{t}}\)), which makes an angle \(\xi\) with the anti-top (top) momentum in the clockwise direction. Thus, the state \(t_{\uparrow \bar{t}_{\uparrow}}\) (\(t_{\downarrow \bar{t}_{\downarrow}}\)) refers to a top with spin in the +\(\hat{s}_t\) (-\(\hat{s}_t\)) direction in the top rest-frame, and an anti-top with spin +\(\hat{s}_{\bar{t}}\) (-\(\hat{s}_{\bar{t}}\)) in the anti-top rest-frame.

In the generic spin basis, the amplitudes \(M(h_{e-}, h_{e+}, \hat{s}_t, \hat{s}_{\bar{t}})\) for the process \(e^-e^+ \to t\bar{t}\) can be generally written as

\[
M(- + t_{\uparrow \bar{t}_{\uparrow}} \text{ or } t_{\downarrow \bar{t}_{\downarrow}}) = \pm 2E_e|m_t(A_L + B_L)| \sin \theta \cos \xi - (|\vec{p}_t|/(A_L - B_L))
\]

\(^1\)At NLC with \(\sqrt{s} \approx 2m_t\), the imaginary part of the Z propagator can be neglected safely.
\[ M(- + t_1 \bar{t}_1 \text{ or } t_1 \bar{t}_1) = 2E_e [m_t (A_L + B_L) \sin \theta \sin \xi \pm (E_e (A_L + B_L) + |\vec{p}_t| (A_L - B_L) \cos \theta) + \cos \xi (|\vec{p}_t| (A_L - B_L) + E_e (A_L + B_L) \cos \theta)], \]
\[ M(+ - t_1 \bar{t}_1 \text{ or } t_1 \bar{t}_1) = \pm 2E_e [m_t (A_R + B_R) \sin \theta \cos \xi \big( (A_R - B_R) |\vec{p}_t| - \cos \theta E_e (A_R + B_R) \big) \sin \xi \mp 2E_e C_R \frac{|\vec{p}_t|}{m_t} \sin \theta], \]
\[ M(+ - t_1 \bar{t}_1 \text{ or } t_1 \bar{t}_1) = 2E_e [m_t (A_R + B_R) \sin \theta \sin \xi \mp (E_e (A_R + B_R) + |\vec{p}_t| (B_R - A_R) \cos \theta) + (|\vec{p}_t| (B_R - A_R) + E_e (A_R + B_R) \cos \theta) \cos \xi], \]

where \( A_{L,R}, B_{L,R}, C_{L,R} \) are the form factors and defined as
\[ A_{L,R} = \frac{1}{s} \left( K_{L,R}^\gamma A_{\gamma} + K_{L,R}^Z A_Z R(s) \right), \]
\[ B_{L,R} = \frac{1}{s} \left( K_{L,R}^\gamma B_\gamma + K_{L,R}^Z B_Z R(s) \right), \]
\[ C_{L,R} = \frac{1}{s} \left( K_{L,R}^\gamma C_\gamma + K_{L,R}^Z C_Z R(s) \right). \]

Because the \( Vt\bar{t} \) vertex (\( V=\gamma,Z \)) has complex structures near threshold, we can not find such a spin angle \( \xi \) that makes Eq.(24) equal to zero, and then there is not the off-diagonal basis, in contrast to the case of the above threshold. The amplitudes in the helicity basis and the beamline basis can be obtained by setting \( \cos \xi = \pm 1 \) and \( \cos \xi = (\cos \theta + \frac{|\vec{p}_t|}{m_t})/(1 + \frac{|\vec{p}_t|}{m_t} \cos \theta) \) in Eqs.(24)-(27), respectively.

In the nonrelativistic limit, the differential cross sections of two unstable particle production [7] in the generic spin basis can be expressed as
\[ \frac{d\sigma(h_{e^-}, h_{e^+}, \hat{s}_t, \hat{s}_l)}{d \cos \theta} = \frac{\Gamma_t}{8\pi^2 M_t^2} \int \frac{|\vec{p}_t|^2}{(E_{NR} - |\vec{p}_t|^2)^2 + \Gamma_t^2} |M(h_{e^-}, h_{e^+}, \hat{s}_t, \hat{s}_l)|^2 d|\vec{p}_t|. \]

In the numerical calculation, we use the following parameters as standard input [13]:
\[ \alpha_s(M_Z) = 0.117, \quad m_Z = 91.188 \text{ GeV}, \quad m_t = 174 \text{ GeV}, \]
\[ \Gamma_t = 1.43 \text{ GeV}, \quad \sin^2 \theta_W = 0.2311. \]

We define the fractions of the total cross sections for the different spin components in the beamline spin basis as following:
\[ R(e_{L,R}^-, e^+, \hat{s}_t, \hat{s}_l) = \frac{\sigma(e_{L,R}^-, e^+, \hat{s}_t, \hat{s}_l)}{\sigma_{total}(e_{L,R}^+ \rightarrow t\bar{t})}. \]
With $E_{NR} = 5$ GeV, we have

$$R(e^- e^+ \to t\bar{t}_1 \text{ or } e^+_R e^+ \to t\bar{t}_1) \simeq 50\%,$$  \hspace{1cm} (34)

$$R(e^- e^+ \to t\bar{t}_1 \text{ or } e^+_R e^+ \to t\bar{t}_1) \simeq 24\%,$$  \hspace{1cm} (35)

$$R(e^- e^+ \to t\bar{t}_1 \text{ or } e^+_R e^+ \to t\bar{t}_1) \simeq 13\%,$$  \hspace{1cm} (36)

$$R(e^- e^+ \to t\bar{t}_1 \text{ or } e^+_R e^+ \to t\bar{t}_1) \simeq 13\%.$$  \hspace{1cm} (37)

These results show that in the $t\bar{t}$ threshold region all spin components cannot be neglected. In Fig.2 we show the differential cross sections for the process $e^-_{L,R} e^+ \to t\bar{t}$ in the beamline basis with $E_{NR} = 5$ GeV. One can see that there is a dominant spin component when scattering angle $\theta$ ranges between $\pi/3$ and $2\pi/3$. More precisely, according to the definition of $R$ in Eq.(33), we integrated the $\theta$ from $\pi/3$ to $2\pi/3$, instead of 0 to $\pi$, and then have $R(e^- e^+ \to t\bar{t}_1 \text{ or } e^+_R e^+ \to t\bar{t}_1) \simeq 79\%$. But, as shown in Fig.3, in the helicity basis there are not such dominant spin components.

Moreover, in our calculation, we considered the Higgs potential effect on the vertex functions. Our numerical results show that such effect is very small and can be neglected. The anomalous couplings ($d_{t\gamma}$, $d_{tz}$, $d_{tg}$) are one of several sources to provide CP-violation [12], and the numerical results show that these effects on the spin correlations in the $t\bar{t}$ threshold region are very small, too. For example, with taking $d_{t\gamma} = d_{tz} = d_{tg} = 10^{-3}$ [10], the corresponding changes of the spin correlations are smaller than 0.1%.

To summarize, we have calculated the spin correlations in the top quark pair production near threshold at the $e^-e^+$ Linear Collider in the SM. We start from the general form of the $Vt\bar{t}$ vertex ($V=\gamma, Z$) near threshold, derive out the amplitudes in the generic spin basis, and give the differential cross sections in the NNLO QCD potential. Comparing with the previous results above the threshold region in Refs. [4, 5], we find:

(a) The most important difference between the two regions is that in the above threshold region we can find the off-diagonal basis in which only one spin component is appreciably non-zero, but in the threshold region the off-diagonal basis does not exist.
(b) Near threshold the beamline basis is the optimal basis, in which there are the dominant spin components: the up-down (UD) component for $e_L^- e^+$ scattering and the down-up (DU) component for $e_R^- e^+$ scattering can make up more than 50% of the total cross section, respectively.

(c) The observables of the spin correlations near threshold are less advantageous than ones of above the threshold region. Nevertheless, because of the extremely high measurement precision of the top quark pair threshold production, it is still valuable to study the spin correlations in the top quark pair production near threshold.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China.

References

[1] I. Bigi, H. Krasemann, Z. Phys. C7, 127(1981); J. Kühn, Acta. Phys. Austr. (Suppl.) XXIV, 203 (1982); I. Bigi, Y. Dokshitzer, V. Khoze, J. Kühn, and P. Zerwas, Phys. Lett. 181B, 157(1986).

[2] D. Atwood and A. Soni, Phys. Rev. D45(1992)2405; G.L. Kane, G.A. Ladinsky and C.P. Yuan, Phys. Rev. D45(1992)124; C. P. Yuan, Phys. Rev. D45(1992)782; G.A. Ladinsky, Phys. Rev. D46(1992)3789; W. Bernreuther, O. Nachtmann, P. Overmann and T. Schröder, Nucl. Phys. B388(1992)53, erratum B406(1993)516; T. Arens and L.M. Seghal, Nucl. Phys. B393(1993)46; Carl R. Schmidt, Phys. Rev.
D54(1996) 3250; A.Brandenburg, M. Flesch, P. Uwer, Phys. Rev. D59(1999) 014001; A.Brandenburg, M. Flesch, P. Uwer, hep-ph/9911249.

[3] S. Parke and Y. Shadmi, Phys. Lett. B 387(1996)199.

[4] H.X. Liu, C.S. Li, Z.J. Xiao, Phys. Lett. B458(1999)393.

[5] Michihiro Hori, Yuichiro Kiyo, Jiro Kodaira, Takashi Nasuno, Stephen Parke, hep-ph/9801370; J. Kodaira, T. Nasuno, S. Parke, Phys. Rev. D 59(1999)014023; Y. Kiyo, J. Kodaira, K. Morii, T. Nasuno, S. Parke, Nucl.Phys.Proc.Suppl. 89 (2000) 37.

[6] M.J. Strassler, M.E. Peskin, Phys. Rev. D43(1991)1500; V.S. Fadin and V.A. Khoze, Yad. fiz. 48(1988) 487; JEPT Lett. 46(1987)525.

[7] R. Harlander, M. Ježabek, J. H. Kühn, M. Peter, Z. Phys. C73 (1997)477.

[8] M. Ježabek, J.H. kün, T. Teubner, Z. Phys. C56(1992) 653; B.A. Kniehl, A. Sirlin, DESY 92-102; Y. Sumino, K. Fujii, K. Hagiwara, H. Murayama, C-K. Ng, Phys. Rev. D47(1993) 56; M. Ježabek, T. Teubner, Z. Phys. C59(1993) 669; H. Murayama, Y. Sumino, Phys. Rev. D47(1993) 82.

[9] A.H. Hoang, T. Teubner, Phys. Rev. D58 (1998) 114023; A.H. Hoang, T. Teubner, Phys. Rev. D60 (1999)114027; T. Nagabno, A. Ota, Y.Sumino, Phys. Rev. D60 (1999)114014; A.H. Hoang et. al., CERN-TH/99-415; M.van Iersel, C.F.M van der Burgh, B.L.G. Bakker, hep-ph/0010243; M. Beneke, CERN-TH/99-281; S. Su, M. B. Wise, Phys. Lett. B510(2001)205; A.A. Penin, A.A. Pivovarov, Phys. Atom. Nucl. 64(2001)275; Yad. Fiz. 64(2001)323; L.W. Stewart, AIP Conf.Proc. 618 (2002) 395.

[10] M. Ježabek, T. Nagano, Y. Sumino, Phys. Rev. D62 (2000)014034.

[11] W. Fisher, Nucl. Phys. B129(1977)157; A. Billoire, Phys. Lett. B92(1980)343; S.N. Gupta and S. Radford, Phys. Rev. D24(1981)2309; Phy. Rev. D25(1982)3430 (erratum); S.N. Gupta, S.F. Radford and W.W. Repko, Phys. Rev. D26(1982)3305; M.
Peter, Phys. Rev. Lett. 78(1997)602; Nucl. Phys B501(1997)471; Y. Schröder, Phys. Lett. B447(1999)321.

[12] D. Atwood, S. Bar-Shalom, G. Eilam, A. soni Phys. Rept. 347(2001)1; S. Khalil, IPPP/02/78, DCPT/02/156; S.D. Rindani, hep-ph/0202045.

[13] Particle Data Group, K. Hagiwara et al., Phys. Rev. D 66, (2002)010001.
Figure 1: Lippmann-Schwinger equation in diagrammatical form.

\[
\Gamma_\mu(q) = \Gamma_\mu(q) + \Gamma_\mu(q)
\]

Figure 2: The differential cross sections in the beamline basis for the \(e^-_Le^+\) and \(e^-_Re^+\) processes with the \(t\bar{t}(UU), t\bar{t}(DD), t\bar{t}(UD)\) and \(t\bar{t}(DU)\) productions, assuming \(E_{NR} = \sqrt{s} - 2m_t = 5\) GeV.
Figure 3: The differential cross sections in the helicity basis for the $e_L^- e^+ \rightarrow t_{L,R}\bar{t}$ and $e_R^- e^+ \rightarrow t_{L,R}\bar{t}$ processes at $E_{NR} = \sqrt{s} - 2m_t = -1$ GeV.