I. INTRODUCTION

The understanding of the IR dynamics of QCD has been very much boosted in the past years by the endeavors in obtaining a very detailed picture for the fundamental Green’s functions of the theory in both lattice and continuum QCD. Namely, a consensus has been reached about both the fact that the gluon propagator takes a non-zero finite value at vanishing momentum (corresponding to a dynamical generation of an effective gluon mass) and the fact that the ghost propagator behaves essentially as its tree-level expression dictates. These findings have recently contributed, for instance, to establish a striking connection between the effective gluon mass and continuum QCD, as well as to the construction of a process-independent strong running coupling which agrees very well with the Bjorken sum-rule effect of a process-independent strong running coupling.

Very recently, the authors of [1] have performed a thorough study of the effect of lattice artifacts on pure Yang-Mills SU(3) gluon and ghost propagators. In particular, they found that the low-momentum behavior of the gluon propagator which, as explained in [1], is to be renormalized on the lattice by applying the MOM prescription,

\[ D_{\mu \nu}(p) = \langle A_\mu^a(p)A_\nu^b(-p) \rangle = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \]

(2.1)

where \( A_\mu^a \) is the gauge field in momentum space, latin (greek) indices correspond to color (Lorentz) degrees of freedom, \( \langle \cdot \rangle \) expresses the integration over the gauge fields, which is replaced by the average over gauge field configurations in lattice QCD, and \( D(p^2) \) is the so-called gluon propagator which, as explained in [1], is to be renormalized on the lattice by applying the MOM prescription,

\[ D_R(p^2, \zeta^2) \mid_{p^2=\zeta^2} = Z_3^{-1}(\zeta^2) D(\zeta^2) = \frac{1}{\xi^2} \]

(2.2)

where \( \zeta^2 \) is the renormalization point, fixed at 4 GeV in ref. [1]. The details of the computation of the gluon propagator on the lattice can be found in the literature, for instance in some previous works of the authors of [1], as [35], or in previous works of some of us as [8].

In a very recent lattice analysis of the three-gluon vertex and running coupling [37], we have also computed the gluon propagator for different lattice bare couplings. In particular, we obtained the results displayed in Fig. [1]
employed such a kinematical cut that hypercubic artifacts [40–42]. In addition, we have also as a very efficient prescription to cure the data from the remaining discretization artifact.

As a consequence of this, the largest accessible momentum for the simulation at $\beta=5.6$ is not much above the momentum, $\zeta = 1.3$ GeV, which we take here for the renormalization point. Indeed, imposing the renormalization condition at $\zeta = 4$ GeV, for which $a\zeta \sim 1.5\pi$ at $\beta=5.6$ and $\sim \pi$ at $\beta=5.8$, might imply to incorporate sizable discretization artifacts and, as the propagators are thus required to take there the same value, $1/\zeta^2$, propagate these artifacts down to low IR momenta.

The latter is a possible source, partially at least, for the lattice spacing effect reported in [1]. However, our propagators displayed in the upper panel of Fig. 1 renormalized at $\zeta = 1.3$ GeV, show the same effect: the data obtained with a larger value of the lattice spacing (lower $\beta$) appear to deviate upwards when the momentum decreases. Alternatively, we claim that this striking feature cannot be a discretization artifact but the consequence of a systematic uncertainty in the lattice scale setting. Indeed, if one admits a small deviation in the lattice scale, $\alpha(\delta) = a(1+\delta)$, the “recalibrated” gluon propagator would stand for

$$D(\delta)(p^2) = (1 + \delta)^2 D((1 + \delta)^2 p^2) , \quad (2.3)$$

and, after renormalization at $p^2 = \zeta^2$,

$$D_R(\delta)(p^2, \zeta^2) = \frac{D((1 + \delta)^2 p^2)}{\zeta^2 D((1 + \delta)^2 \zeta^2)} ; \quad (2.4)$$

where $D$ stands for the bare lattice propagator obtained with the lattice spacing $a$. Therefore, the systematic deviation in the scale setting expressed by $\delta$ would result in a non-trivial transformation of the data that might well account for the low-momentum discrepancies shown by the upper panel of Fig. 1.

In order to check the validity of this conjecture, we just consider the results obtained at $\beta=5.6$ as non-deviated and estimate the deviation parameter $\delta$ at $\beta=5.8$ required to get rid of the low-momentum discrepancies and get the data from both simulations lying on top of each other. This can be strikingly seen in the lower panel of Fig. 1 to be left with which one needs to apply $\delta = -0.05$. Properly interpreted, the latter means that all the discrepancies can be explained if we accept a $5\%$ of deviation in the ratio between the lattice spacings at $\beta=5.8$ and at $\beta=5.6$, with respect to the values quoted in Tab. I. These values have been obtained in [39] by using the Sommer parameter, $r_0$, and are compatible with those used in [1] and set by the string tension in [33]. In both cases, the scale setting procedures refer to the force between external static charges. The relative accuracy of $r_0/a$ resulting from the thorough statistical analysis of [39] is of the order 0.3–0.6%, but a cut-off-dependent systematical uncertainty of 2–3% can be sensibly conceived and might be enough to explain the lattice spacing effects at low-momentum shown here and previously reported in [1]. Other scale setting prescriptions as the more precise one grounded on the Wilson flow [41,46] could presumably result on reduced systematic uncertainties. The comparison of the running of renormalized propagators can anyhow be of

![FIG. 1: Upper panel.- Lattice gluon propagator results for the set-ups given in Tab. I. Lower panel.- The same gluon propagator results after applying to the data at $\beta=5.8$ the “recalibration” described in the text through Eqs. (2.3,2.4), with $\delta=-0.05$ for the deviation parameter.](image)

| $\beta$ | $N$ | $a$ [fm] | confs |
|---|---|---|---|
| 5.6 | 48 | 0.236 | 960 |
| 5.8 | 48 | 0.147 | 960 |

TABLE I: Lattice set-ups specifying the bare lattice coupling $\beta = 6/g^2$, the number of lattice sites in any of the directions, $N$, the lattice spacing, $a$, and the number of gauge-field configurations exploited. The lattice scales has been taken from [39].

for $\beta=5.6$ and $\beta=5.8$ from quenched simulations with the Wilson action in $48^4$ lattices. Details of the lattice set-ups can be found in Tab. I. The statistical errors have been estimated by applying the jackknife method. The propagators are displayed as a function of the lattice momenta $p_\mu = 2\pi/(Na)n_\mu$, with $n_\mu = 0,1,\ldots,N/4$, instead of the tree-level improved $\bar{p}_\mu = 2/a\sin(a p_\mu/2)$. We have applied the $H(4)$-extrapolation [40], which has been proven as a very efficient prescription to cure the data from the hypercubic artifacts [40,42]. In addition, we have also employed such a kinematical cut that $ap \leq \pi/2$, thus lessening the impact of any remaining discretization artifact. As a consequence of this, the largest accessible momenta...
III. CONCLUSIONS

We suggest that the lattice spacing effects discussed by the authors of [1], taking place in the low-momentum domain of the quenched gluon and ghost propagators, can be better justified by invoking small systematic deviations in the lattice scale setting based on the definition of the force between external static charges.

Acknowledgements

We thank the support of Spanish MINECO FPA2014-53631-C2-2-P research project, SZ acknowledges support by the National Science Foundation (USA) under grant PHY-1516509 and by the Jefferson Science Associates, LLC under U.S. DOE Contract #DE-AC05-06OR23177.

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