On the Role of Low-Energy CP Violation in Leptogenesis

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The link between low-energy CP violation and leptogenesis became more accessible with the understanding of flavor effects. However, a definite well-motivated model where such a link occurs was still lacking. Adjoint SU(5) is a simple grand unified theory where neutrino masses are generated through the Type I and Type III seesaw mechanisms, and the lepton asymmetry is generated by the fermionic triplet responsible for the Type III seesaw. We focus exclusively on the case of inverted hierarchy for neutrinos, and we show that successful flavored leptogenesis in this theory strongly points towards low-energy CP violation. Moreover, since the range of allowed masses for the triplet is very restricted, we find that the discovery at the LHC of new states present in the theory, together with proton decay and unification of gauge couplings, can conspire to provide a hint in favor of leptogenesis.

I. INTRODUCTION

One of the main arguments in favor of the search for low-energy CP violation in the lepton sector is that this may provide an indication about the origin of the matter-antimatter asymmetry in the Universe. This is due to the connection between leptogenesis and neutrino masses within the seesaw mechanism. However, it turns out that such a link is difficult to establish without assuming particular structures of the mass matrices in the model. However, it turns out that such a link is difficult to establish without assuming particular structures of the mass matrices in the model. This was especially true in the case of "unflavored leptogenesis" (see e.g. the recent review and references therein). But even with flavored leptogenesis, the link can be obscure in the most general case, if one does not assume specific values for the parameters in the high-energy sector. For example, the assumption of subdominant CP-violation in the high-energy sector, in which case low-energy phases in the PMNS matrix are directly related to the size of the baryon asymmetry through leptogenesis, has recently attracted some attention.

In order to have a connection between leptogenesis and low-energy CP violation, one has to restrict the number of high-energy phases. This is achieved in the so-called two right-handed (2RH) neutrino model, or $N_3$-decoupling limit, where only one high-energy phase is present, instead of three for the model with three RH neutrinos. With two low-energy phases (the Dirac phase and one Majorana
phase) in the PMNS matrix, it is reasonable to guess that the low-energy phases can have an important impact. It was shown in [23] that, in the case of inverted hierarchy for neutrinos, the region of successful flavored leptogenesis was much larger when the low-energy phases were non-zero. Similar results were presented in [20, 24]. However, the model with two RH neutrinos, though simpler from the point of view of the number of parameters, is more difficult to motivate since in grand unified theories, in particular $SO(10)$, one has the same number of RH neutrinos as the number of families.

It is well-known that RH neutrinos and leptogenesis are naturally embedded in $SO(10)$-based theories. See [25, 26] for recent studies in this context. Recently, a realistic grand unified theory based on the $SU(5)$ gauge symmetry has been proposed [27], where neutrino masses are generated through a Type I plus Type III [28] seesaw mechanism. This case is similar to the 2RH neutrino model from the point of view of the number of parameters, but, in order to have viable unification [29], the fermionic triplet responsible for the Type III seesaw has to be much lighter than the singlet responsible for the Type I seesaw, implying different Boltzmann equations [30]. Therefore, in contrast to the usual renormalizable $SO(10)$ models where one has a Type I plus Type II seesaw mechanism, and one does not know whether the Higgs triplet or the fermionic singlet is responsible for leptogenesis, here we know which field in the theory generates the $B – L$ asymmetry.

Leptogenesis within the model [27] was investigated in detail in [31], where constraints on the parameter space of the theory were derived. In this letter, we study the particular role played by the low-energy $CP$-violating phases on the generation of asymmetry, and we find that in the case of inverted hierarchy for neutrinos, $CP$ violation at low energy is a crucial ingredient to have successful leptogenesis. Therefore, in this well-motivated model and assuming that the spectrum for neutrinos is inverted, low-energy phases naturally play a dominant role compared to the unknown high-energy phase. Finally, we show that the discovery at the LHC of new states present in the theory can lead to a prediction on the proton decay lifetime from gauge coupling unification and leptogenesis. It is actually remarkable that such relations can be obtained in this model.

This letter is organized as follows: In section II we present the model and its main predictions relevant for leptogenesis. In section III we discuss the crucial role of the low-energy Dirac and Majorana phases in leptogenesis. In section IV we discuss the correlation between the unification constraints, proton decay and leptogenesis. In the last section we summarize our findings.
II. ADJOINT $SU(5)$ UNIFICATION, NEUTRINO MASSES AND LEPTOGENESIS

In the context of renormalizable Adjoint $SU(5)$ [27], neutrino masses are generated through the Type I plus Type III seesaw mechanism. In this context the matter lives in the $\mathbf{5}$, $\mathbf{10}$ and $\mathbf{24}$ representations, while the Higgs sector is composed of $\mathbf{5}_H$, $\mathbf{24}_H$ and $\mathbf{45}_H$. See [29, 32] for the phenomenological and cosmological aspects of this proposal. The fields responsible for the Type I and Type III seesaw are $\rho_0 \sim (1, 1, 0) \subset \mathbf{24}$ and $\rho_3 \sim (1, 3, 0) \subset \mathbf{24}$, respectively. Integrating out these fields, the mass matrix for neutrinos reads

$$M^\nu_{\alpha \beta} = \left( \frac{h_{\alpha 1} h_{\beta 1}}{M_{\rho_3}} + \frac{h_{\alpha 2} h_{\beta 2}}{M_{\rho_0}} \right) v_0^2,$$  \hspace{1cm} (1)

where $M_{\rho_0}$ and $M_{\rho_3}$ are the masses of the fields responsible for Type I and Type III seesaw, respectively, $h$ is the Yukawa coupling matrix and $v_0$ the vacuum expectation value of the Standard Model Higgs. See [29] for more details. The theory predicts one massless neutrino at tree level. Therefore, one can have either a normal neutrino mass hierarchy: $m_1 = 0$, $m_2 = \sqrt{\Delta m^2_{\text{sol}}}$ and $m_3 = \sqrt{\Delta m^2_{\text{sol}} + \Delta m^2_{\text{atm}}}$, or an inverted one: $m_3 = 0$, $m_2 = \sqrt{\Delta m^2_{\text{atm}}}$ and $m_1 = \sqrt{\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sol}}}$, where $\Delta m^2_{\text{sol}} \simeq 8 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{\text{atm}} \simeq 2.5 \times 10^{-3}$ eV$^2$ are the mass-squared differences of solar and atmospheric neutrino oscillations, respectively [33].

A convenient parametrization of the $3 \times 2$ Yukawa coupling matrix $h$ is given by [34]

$$h = U D^\nu_{\nu}/v_0,$$  \hspace{1cm} (2)

where $U$ is the PMNS lepton mixing matrix, $D_{\nu} = \text{diag}(m_1, m_2, m_3)$ and $D_{\rho} = \text{diag}(M_{\rho_3}, M_{\rho_0})$. In this letter we will focus on the inverted spectrum of neutrinos since only in this case the predictions coming from leptogenesis depend crucially on the low-energy phases in the PMNS matrix. The $\Omega$ matrix takes then the well-known form corresponding to the Type I seesaw with two right-handed neutrinos [21]:

$$\Omega^{\text{III}} = \begin{pmatrix} \pm \sqrt{1 - \omega^2} & -\omega \\ \omega & \pm \sqrt{1 - \omega^2} \\ 0 & 0 \end{pmatrix},$$  \hspace{1cm} (3)

where $\omega$ is a complex parameter. For the PMNS matrix $U$, we adopt the usual parametrization [35]

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13} \end{pmatrix} \times \text{diag}(1, e^{i \Phi/2}, 1),$$  \hspace{1cm} (4)

where $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\delta$ is the Dirac $CP$-violating phase, $\Phi$ is the Majorana $CP$-violating phase.
In order to complete our discussion, we present the relevant interactions for leptogenesis:

\[ V_\nu = h_\alpha \ell_\alpha^T i \sigma_2 C \rho_3 H + h_\alpha^2 \ell_\alpha^T i \sigma_2 C \rho_0 H + M_{\rho_3} \text{Tr} \rho_3^T C \rho_3 + \frac{1}{2} M_{\rho_0} \rho_0^T C \rho_0 + \text{h.c.} \quad (5) \]

and

\[ \mathcal{L}_{\text{kin}} = i \text{Tr} \bar{\rho}_3 \gamma^\mu D_\mu \rho_3 + i \bar{\rho}_0 \gamma^\mu \partial_\mu \rho_0 \quad (6) \]

where \( D_\mu \rho_3 = \partial_\mu \rho_3 + ig_2 [W^\mu, \rho_3] \),

\[ \rho_3 = \frac{1}{2} \begin{pmatrix} T^0 & \sqrt{2} T^+ \\ \sqrt{2} T^- & -T^0 \end{pmatrix}, \quad \text{and} \quad W^\mu = \frac{1}{2} \begin{pmatrix} W^3_\mu & \sqrt{2} W^+_\mu \\ \sqrt{2} W^-_\mu & -W^3_\mu \end{pmatrix}. \quad (7) \]

As pointed out in [29], in this model the lightest field responsible for the seesaw mechanism is the triplet \( \rho_3 \) and the mass splitting with the singlet \( \rho_0 \) is very large, \( M_{\rho_0}/M_{\rho_3} > 40 \). In [31] leptogenesis in this theory was studied in detail. Our findings were that the \( CP \) asymmetry is given only by the vertex correction, and that the case of inverted spectrum for light neutrinos is only marginally allowed. In the next section, we will focus on this spectrum and show explicitly that the existence and size of the region of successful leptogenesis depends crucially on the presence of low-energy \( CP \) violation.

### III. LOW-ENERGY \( CP \)-VIOLENT PHASES AND LEPTOGENESIS

Without assuming any particular texture in the Dirac mass matrix, it is well-known that the low-energy phases in the PMNS matrix can only play a role when one considers \textit{flavored} leptogenesis, for \( M_{\rho_3} < 5 \times 10^{11} \text{ GeV} \) [36]. In the unflavored regime, for \( M_{\rho_3} > 5 \times 10^{11} \text{ GeV} \), the calculation is completely independent of the PMNS phases, because it involves only the combination \( (h^\dagger h)_{ij} \), where the PMNS matrix cancels out. This can be easily verified using the parametrization given in Eq. (2). It was shown in [31] that the case of inverted hierarchy in Adjoint \( SU(5) \) is only marginally allowed by successful flavored leptogenesis and that there is no region allowed in the unflavored regime. It is worth noting that this is in contrast to the 2RH neutrino model, where a region in the unflavored regime survives. In the case of normal hierarchy for neutrinos, there is a large region allowed in the unflavored regime [31], so the PMNS phases only change marginally the constraints. Therefore, as already mentioned previously, this letter will be exclusively devoted to the case of inverted hierarchy.

Let us recall the two fundamental quantities for leptogenesis, the decay parameter and the \( CP \) asymmetry parameter. In terms of the orthogonal parametrization given in Eq. (2), the decay parameters for the inverted
hierarchy are given by
\begin{align}
K_\alpha &= \frac{m_1 |U_{\alpha 1}|^2}{m_*} |1 - \omega^2| + \frac{m_2 |U_{\alpha 2}|^2}{m_*} |\omega|^2 \pm \frac{2 \sqrt{m_1 m_2}}{m_*} \Re \left( U_{\alpha 1} U^*_{\alpha 2} \sqrt{1 - \omega^2 \omega^*} \right), \\
K &= \frac{m_1}{m_*} |1 - \omega|^2 + \frac{m_2}{m_*} |\omega|^2,
\end{align}
where \( m_* \simeq 1.08 \times 10^{-3} \text{eV} \) is the equilibrium neutrino mass. As for the flavored \( CP \) asymmetries they can be written as
\begin{align}
\varepsilon_{\rho, \alpha} &\simeq -\frac{1}{8 \pi v_0^2} \frac{M_{\rho \alpha}}{m_1 |1 - \omega^2| + m_2 |\omega|^2} \left[ (m_2^2 |U_{\alpha 2}|^2 - m_1^2 |U_{\alpha 1}|^2) \Im(\omega^2) \\
&\pm \sqrt{m_1 m_2} (m_2 + m_1) \Re(U^*_{\alpha 1} U_{\alpha 2}) \Im(\omega \sqrt{1 - \omega^2}) \\
&\pm \sqrt{m_1 m_2} (m_2 - m_1) \Im(U^*_{\alpha 1} U_{\alpha 2}) \Re(\omega \sqrt{1 - \omega^2}) \right].
\end{align}
It can be noticed that the decay parameter and the \( CP \) asymmetry (up to a factor 3) are exactly the same as in the 2RH neutrino model.

As is well known in the 2RH neutrino case, the total \( CP \) asymmetry \( \varepsilon_{\rho 3} = \sum_\alpha \varepsilon_{\rho 3, \alpha} \) in the case of inverted hierarchy is suppressed by a factor \( \Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}} \) compared to normal hierarchy [37, 38]. Additionally, the total washout parameter \( K = \sum_\alpha K_\alpha \) is typically larger. Therefore, the lower bounds on the scale of leptogenesis in the case of inverted hierarchy are much more restrictive than in the case of normal hierarchy. In flavored leptogenesis, however, there exists the possibility of having both a smaller flavored decay parameter, and a flavored \( CP \) asymmetry that is larger than the total one, leading to a dramatic decrease of the lowest bound from \( 2 \times 10^{13} \text{ GeV} \) to \( 10^{10} \text{ GeV} \) [23]. The point is that the presence of low-energy phases is essential to keep the \( CP \) asymmetry in flavor \( \alpha \) large, i.e. \( \varepsilon_{\rho 3, \alpha} \gg \varepsilon_{\rho 3} \), and obtain at the same time a cancellation in the flavored decay parameter, leading to \( K_\alpha \ll K \). This is precisely what we shall obtain below.

Before turning to the actual evaluation of the role of the low-energy phases, we would like to point out that the allowed region obtained in [31] in the case of inverted hierarchy does not actually satisfy the condition of validity of the flavored Boltzmann equations specified in [36]. The reason is that the presence of such an allowed region relies crucially on the fact that the washout is largely reduced by flavor effects. In this particular situation, the reliability of classical Boltzmann equation is dubious. However, since a precise description of leptogenesis in the transition between the flavored and the unflavored regime, probably relying on a density matrix equation, is still missing, we shall simply assume here the validity of flavored Boltzmann equations below \( 5 \times 10^{11} \text{ GeV} \). This will enable us to show explicitly that, in the well-motivated model we consider, flavored leptogenesis does depend on the PMNS phases.

For the details of the leptogenesis computation and all definitions, we refer the reader to [31]. We just
FIG. 1: The allowed range for the mass of $\rho_3$ vs. the decay parameter $K$ is the part of the pink region below the hatched area defining the unflavored regime. Case of inverted hierarchy with $\Phi = 0$, $\delta = 0$ and $\sin \theta_{13} = 0.2$.

recall here that the border of the region of successful leptogenesis is defined by

$$\eta_B = 5.75 \times 10^{-10},$$

(11)
corresponding to the lower value allowed by WMAP5 at 3$\sigma$ [39]. Note moreover that we shall use a slightly different numerical factor compared to [31] for the conversion of a $B-L$ asymmetry into a $B$ asymmetry, 12/37 [40] instead of 29/78 [41], assuming that sphalerons remain in equilibrium until after the electroweak phase transition. This modifies the relation between the baryon-to-photon ratio $\eta_B$ and the relevant parameter in the Boltzmann equation $N^f_{\Delta_\alpha}$ as follows

$$\eta_B \simeq 3 \times 0.88 \times 10^{-2} \sum_{\alpha} N^f_{\Delta_\alpha}.$$

(12)

Now, in order to understand the role of the low-energy $C P$-violating phases let us define several scenarios:

- **Benchmark I.** First, we show in Fig. 1 the result of the flavored computation taking the $C P$-conserving values $\delta = 0$ and $\Phi = 0$. Note that, in order to see more clearly the transition with the next scenarios, we extended the flavored calculation (pink region) to $M_{\rho_3} = 10^{12}$ GeV. One notices from Fig. 1 that the pink region falls only in the unflavored regime, where our computation is not valid. Therefore, there is no allowed region. Changing the value of $\sin \theta_{13}$ does not change this conclusion. Therefore, low-energy $C P$ violation is necessary to extend the region below the line separating the unflavored from the flavored regime.
• **Benchmark II.** Now we turn on the Majorana phase $\Phi$, keeping $\delta = 0$. The result is shown in Fig. 2 for two choices of $\sin \theta_{13}$, 0.2 (left panel) and 0 (right panel). It can be seen that the pink region extends now considerably into the flavored regime. One actually recovers here the lowest bound found in \cite{31} $M_{\rho_3} > 2 \times 10^{11}$ GeV, for both values of $\sin \theta_{13}$. Note also that the allowed region is actually larger for $\sin \theta_{13} = 0$.

• **Benchmark III.** Finally, we set the Majorana phase to zero, and turn on the Dirac phase. The result is shown in Fig. 3 again for two values of $\sin \theta_{13}$, 0.2 (left panel) and 0.1 (right panel). An allowed region opens up below the hatched area also in this case, even though it remains smaller than in the case of non-zero Majorana phase, even for the maximal allowed value $\sin \theta_{13} = 0.2$.

In order to illustrate the effect of the PMNS phases in a more precise way, it is useful to show the allowed regions in the planes $(K, \Phi)$ and $(K, \delta)$, fixing the mass scale to the value $M_{\rho_3} = 3.5 \times 10^{11}$ GeV. The case with only the Majorana phase is depicted in Fig. 4 and the case with only the Dirac phase in Fig. 5. It
is not surprising that the region is much larger in the case of non-zero Majorana phase, and that values up to $K = 200$ are possible, in agreement with the right panel of Fig. 4 where the case $\sin \theta_{13} = 0$ is shown. In the case of non-zero Dirac phase with maximal allowed $\sin \theta_{13}$, only regions around $\delta = \pi/2$, $3\pi/2$ are possible, and at low values of the decay parameter, $K \lesssim 70$.

Before concluding, it is worth pointing out that our results for the shape (not the magnitude!) of the lower bounds for different values of the PMNS phases would be very similar in a 2RH neutrino model. The main difference comes from the fact that we have here an overall suppression of the final asymmetry.
by a factor roughly $3^{1.2}$, as well as an additional reduction of the efficiency factor due to the gauge interactions of the fermionic triplet. Altogether, we have a reduction of the final asymmetry by about one order of magnitude, leading to the interesting possibility of a lower bound on $M_{\rho_3}$ very close to the limit of the flavored regime at $5 \times 10^{11}$ GeV, unlike the 2RH neutrino case. Therefore, the sensitivity to low-energy CP violation, as we have shown, is all the more interesting.

**IV. PROTON DECAY, UNIFICATION AND LEPTOGENESIS IN ADJOINT SU(5)**

Leptogenesis is usually considered an elegant but hardly testable mechanism to explain the matter-antimatter asymmetry of the Universe due to the very high scale it involves. In this section we present an interesting scenario which could provide a hint for leptogenesis.

We start with the optimistic assumption that two states in the theory of Adjoint $SU(5)$ can be discovered at the LHC, namely the color scalar octet, $\Phi_1 \sim (8, 2, 1/2) \subset 45_H$ and the scalar $SU(2)_L$ triplet $\Sigma_3 \sim (1, 3, 0) \subset 24_H$. Then, as we have seen in the last section, leptogenesis for the inverted scheme of light neutrinos occurs for a very restricted mass range around $M_{\rho_3} = 3.5 \times 10^{11}$ GeV. So we can take the constraint from leptogenesis as a line in parameter space, instead of an extended region as in the case of normal hierarchy. Fixing the mass $M_{\Phi_1} = 1$ TeV, we show in Fig. 6 how the constraints from gauge coupling unification and leptogenesis when $M_{\Sigma_3} = 200$ GeV allow to pin down the GUT scale. We find $M_{GUT} = 5.62 \times 10^{15}$ GeV, from which the following proton lifetimes can be derived:

$$
\tau(p \to e^+\pi^0) = 3.2 \times 10^{34} \text{ years}, \quad \tau(p \to K^+\bar{\nu}) = 2.5 \times 10^{36} \text{ years}, \quad \tau(p \to \pi^+\bar{\nu}) = 8.1 \times 10^{34} \text{ years}.
$$

These lifetimes are potentially observable at future experiments.

It is very interesting that Adjoint $SU(5)$ provides a framework which allows to relate so different concepts as proton decay, leptogenesis and gauge couplings unification. We find that, assuming an inverted spectrum for neutrinos and unification of gauge couplings at a certain scale, the observation of proton decay in the predicted range can provide an indication for the high scale of leptogenesis.

**V. CONCLUSION**

We have investigated the effect of the low-energy CP-violating phases on leptogenesis in the context of Adjoint $SU(5)$. In this model neutrino masses are generated through the Type I and Type III seesaw mechanisms, and the lepton asymmetry is generated by the fermionic triplet responsible for the Type III

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1 The reduction of the $CP$ asymmetry by a factor 3 in this model is compensated by the 3 degrees of freedom of the fermionic triplet as shown in Eq. (12), but the washout is still a factor 3 larger, leading to the suppression by a factor $3^{1.2}$ in the strong washout.
FIG. 6: Constraints coming from unification of gauge couplings when $M_{\Phi_3} = 1$ TeV. The field $\Phi_3 \sim (3, 3, -1/3)$ mediates proton decay and its lower bound is $M_{\Phi_3} > 10^{12}$ GeV. The almost vertical line defines the constraint from leptogenesis in the inverted spectrum for neutrinos.

seesaw. We have found that, for an inverted spectrum of neutrinos, leptogenesis is disfavored without low-energy $CP$-violating phases assuming the validity of flavored Boltzmann equations below the usual scale of $5 \times 10^{11}$ GeV. Therefore, if one discovers that the spectrum of light neutrinos is inverted, Adjoint $SU(5)$ offers a nice example of a theory where leptogenesis is very sensitive to low-energy $CP$ violation. We have shown several numerical examples in order to understand the precise impact of each one of the PMNS phases. Finally, we have presented a scenario where the observation of new states at the LHC, together with unification of gauge couplings and proton decay in the predicted range, can provide an indication for the high scale of leptogenesis.

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