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Model mismatch analysis and compensation for modal phase measuring deflectometry

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Abstract: The correspondence residuals due to the discrepancy between the reality and the shape model in use are analyzed for the modal phase measuring deflectometry. Slope residuals are calculated from these discrepancies between the modal estimation and practical acquisition. Since the shape mismatch mainly occurs locally, zonal integration methods which are good at dealing with local variations are used to reconstruct the height residual for compensation. Results of both simulation and experiment indicate the proposed height compensation method is effective, which can be used as a post-complement for the modal phase measuring deflectometry.

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References and Links

1. M. C. Knauer, J. Kaminski, and G. Hausler, "Phase measuring deflectometry: a new approach to measure specular free-form surfaces," in 2004), 366-376.
2. J. Balzer and S. Werling, "Principles of shape from specular reflection," Measurement 43, 1305-1317 (2010).
3. L. Huang, C. S. Ng, and A. K. Asundi, "Dynamic three-dimensional sensing for specular surface with monoscopic fringe reflectometry," Opt. Express 19, 12809-12814 (2011).
4. L. Huang, J. Xue, B. Gao, C. McPherson, J. Beverage, and M. Idir, "Modal phase measuring deflectometry," Opt. Express 24, 24649-24664 (2016).
5. W. Li, P. Huke, J. Burke, C. von Kopylow, and R. B. Bergmann, "Measuring deformations with deflectometry," in 2014), 92030F-92030F-92012.
6. G. Li, Y. Li, K. Liu, X. Ma, and H. Wang, "Improving wavefront reconstruction accuracy by using integration equations with higher-order truncation errors in the Southwell geometry," J. Opt. Soc. Am. A 30, 1448-1459 (2013).
7. L. Huang, M. Idir, C. Zuo, K. Kazznatcheev, L. Zhou, and A. Asundi, "Comparison of two-dimensional integration methods for shape reconstruction from gradient data," Optics and Lasers in Engineering 64, 1-11 (2015).
8. W. H. Southwell, "Wave-front estimation from wave-front slope measurements," J. Opt. Soc. Am. 70, 998-1006 (1980).
9. H. Ren, F. Gao, and X. Jiang, "Least-squares method for data reconstruction from gradient data in deflectometry," Appl. Opt. 55, 6052-6059 (2016).10.
10. H. Ren, F. Gao, and X. Jiang, "Improvement of high-order least-squares integration method for stereo deflectometry," Appl. Opt. 54, 10249-10255 (2015).11.
11. L. Huang, C. S. Ng, and A. K. Asundi, "Fast full-field out-of-plane deformation measurement using fringe reflectometry," Optics and Lasers in Engineering 50, 529-533 (2012).

1. Introduction

As a shape measurement technique for free-form specular surfaces, Phase Measuring Deflectometry (PMD) is well studied during the past decades [1-3]. Unlike the conventional PMD, the Modal Phase Measuring Deflectometry (MPMD) [4] was recently proposed to handle the PMD data for a simultaneous reconstruction of the slope and height using a pre-defined shape model and a post optimization of screen pose.
In MPMD, a better fitting of the Surface Under Test (SUT) could generally be achieved using more orders to describe the SUT, however, this comes with a sacrifice in memory cost and computing time. Moreover, even if the number of orders can be increased to offer more degrees of freedom to the fitting process, it is still difficult to perfectly represent local structures and high frequency waviness, especially when using a pre-selected model. This model-SUT mismatch issue is one of the major error sources in MPMD. It can be critical and vital while the surface details are of interest, and of course it limits the MPMD application.

This work aims to analyze the issue of model-SUT mismatch mentioned above and provide a solution which can be applied in general cases for practical MPMD measurements. The proposed approach combines the modal and zonal shape reconstruction methods. The fact that PMD is good at deformation measurement [5] is employed to reconstruct the global shape with the modal method and the correspondence residuals are used to reconstruct the local details with the zonal method as if we are measuring the deformation from the model-described surface to the real SUT.

2. Introduce to MPMD and the proposed compensation approach

As shown in Fig. 1, the PMD setup is mainly composed with a screen, one or several camera(s), a specular SUT, and of course a computer to control and process the measurement.

In MPMD, the surface shape is described by a model, e.g. Chebyshevs, Zernikes, or B-splines. The surface height and slope can be adjusted by changing the model coefficients $c$ to minimize the discrepancy between ray tracing $\hat{m}$ and actual acquisition $m$ to best explain the captured images. If the system calibration is not good enough to provide a reliable screen pose, the screen pose ($\omega, T$) can also be included into the optimization as described in Eq. (1).

$$\left[\omega, T, c\right] = \arg \min_{\omega, T, c} \sum_{n=1}^{N} \| \hat{m}_n(\omega, T, c) - m_n \|^2,$$  \hspace{1cm} (1)

where $N$ is the total number of valid correspondence pairs. By using a nonlinear least squares solver such as the Levenberg-Marquardt algorithm, the parameters can be estimated through iterations. Due to a trade-off between less computational load and better surface fitting, the model with limited number of coefficients cannot match perfectly the SUT, and has to leave the correspondence residuals $r$ as discrepancies.

$$r = m - \hat{m}.$$  \hspace{1cm} (2)

The shape difference between the reconstructed surface and the SUT is hidden in these residuals, and can be further reduced through a compensation procedure. The surface normal at sampling points can be calculated by using the ray tracing result $\hat{m}$ as
\[ \hat{N} = \frac{\mathbf{m} - \mathbf{\hat{X}}}{\|\mathbf{m} - \mathbf{\hat{X}}\|} - \mathbf{P}, \tag{3} \]

where \( \mathbf{\hat{X}} = (\hat{x}, \hat{y}, \hat{z})^T \) stands for the model-estimated coordinates of the sampling points on SUT. \( \mathbf{P} \) is the camera probe ray which is determined via camera calibration. On the other hand, the measured \( \mathbf{m} \) also provides the other set of surface normal at the same point \( \mathbf{\hat{X}} \) as

\[ \mathbf{N} = \frac{\mathbf{m} - \mathbf{\hat{X}}}{\|\mathbf{m} - \mathbf{\hat{X}}\|} - \mathbf{P}. \tag{4} \]

The slope residuals \( (\Delta s_x, \Delta s_y) \) can be calculated as

\[
\begin{aligned}
\Delta s_x &= \tan \left( \arctan(\hat{s}_x) - \theta_x \right) - \hat{s}_x, \\
\Delta s_y &= \tan \left( \arctan(\hat{s}_y) + \theta_x \right) - \hat{s}_y, 
\end{aligned}
\tag{5}
\]

where \( \hat{s}_x \) and \( \hat{s}_y \) are the model-estimated slopes. From the known vectors \( \mathbf{\hat{N}} \) and \( \mathbf{N} \), we can calculate \( \theta_x \) and \( \theta_y \), which are the rotation angles of the surface normal from \( \mathbf{\hat{n}} = \frac{\mathbf{\hat{N}}}{\|\mathbf{\hat{N}}\|} \) towards \( \mathbf{n} = \frac{\mathbf{N}}{\|\mathbf{N}\|} \) along the \( x \)- and \( y \)-axes following the right hand rule. The angles \( \theta_x \) and \( \theta_y \) can be determined from the rotation matrix \( \mathbf{R} \) which is calculated as

\[ \mathbf{R} = \mathbf{I} + \mathbf{w} + \mathbf{w}^2 \frac{1 - \mathbf{n} \cdot \mathbf{\hat{n}}}{\|\mathbf{v}\|^2}, \tag{6} \]

where \( \mathbf{I} \) is the identity matrix, \( \mathbf{v} \equiv \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \mathbf{n} \times \mathbf{\hat{n}} \), and \( \mathbf{w} = \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{pmatrix} \). The signs before \( \theta_x \) and \( \theta_y \) in Eq. (5) are explained by Fig. 2 to show the relations between the angle and slope variation in Cartesian coordinates system following the right hand rule.

Fig. 2. The \( x \)- and \( y \)-slope changes based on the rotations in right hand rule along \( y \)- and \( x \)-axes, respectively. (a) Rotation along \( y \)-axis with a positive angle gives a negative change to \( x \)-slope, and (b) rotation along \( x \)-axis with a positive angle offers a positive change to \( y \)-slope.
Two-dimensional integration methods can be employed to reconstruct the compensatory height from slope residuals \( (\Delta s_x, \Delta s_y) \). Since the residuals after model-based optimization generally occur at the local structures on SUT, the zonal integration methods \([6-8]\) are preferred and use as equation (7) because of their good performance in handling local shape variations.

\[
\Delta z = f_{\text{zonal}}(\hat{x}, \hat{y}, \Delta s_x, \Delta s_y).
\] (7)

The higher-order finite-difference-based least-squares integration (HFLI) method proposed by Li et al. \([6]\) is selected to reconstruct the compensatory height form residual slopes in this work, as it outperforms in a comparison study \([7]\). Finally the height is compensated to the MPMD-estimated height as a final result: \( z = \hat{z} + \Delta z \). Because the local height variation is commonly smaller than the global height, the changes on lateral coordinates are ignored and not compensated.

3. Simulation

In order to verify the feasibility of the proposed compensation method, a monoscopic PMD system shown in Fig. 3(a) is simulated to measure a concave specular surface. We add on the SUT a scratch-like crossline defect as describe in in Fig. 3(b). The system configuration in simulation is similar to the setup configuration of our real experiment described in the next section. Gaussian noise with standard deviation of \(2\pi/100\) is added as phase noise.

![Fig. 3. A PMD setup is simulated in world coordinates (a) to measure an SUT (b).](image)

Uniform cubic B-splines with \(17 \times 17\) control points are selected as the shape model in MPMD. Because the local structures (e.g. crossline scratches here) on the SUT are hardly represented by the selected shape model, larger correspondence residuals are expected.

![Fig. 4. MPMD provides the model-based construction (a) which is the global shape of the SUT, correspondence residuals (b), and the vector map of the residuals caused by local shapes.](image)

After several iterations in MPMD, the model coefficients are optimized while the correspondence error is minimized as show in Fig. 4. However, obvious discrepancies in Eq. (2) can still be easily observed in the vector map of correspondence residuals in Fig. 4(c). The proposed method is then applied to reconstruct the local details on the SUT. As shown in Fig. 5 (a-b), the slope residuals are calculated according to Eqs. (3)-(5). The height for compensation shown in Fig. 5 (c) is determined by integrating these slope residuals.
Comparing the shape errors before and after the height compensation in Fig. 6(a) and (b), the proposed method is effective in local shape compensation for MPMD.

Furthermore, the slopes of the SUT can be determined from the surface normal calculation in Eq. (4). Theoretically it should be able to directly integrate the slope to reconstruct the final height results and it should be equivalent to the MPMD with zonal compensation. However, owning to the perspective and distortion effects, the slope data from PMD are usually in quadrilateral grids instead of rectangular ones, and the reconstruction with HFL or other zonal methods can be affected by quadrilateral grids [7]. In the same grids, smaller slope values yield lower height errors. Because the residual slopes are much smaller than the entire surface slopes, the reconstruction error in Fig. 6(b) is smaller than that in Fig. 6(c). Some recent improvements were studied in handling slopes in quadrilateral grids [9, 10], applying which should further reduce the reconstruction errors.

4. Experiment

In our experiment, the SUT is a 200 mm x 95.3 mm concave mirror. There are several defects on the mirror surface, which can be observed in the marked region in Fig. 7(a). An LCD screen (Dell P2414H with 1920×1080 pixels and 0.2745 mm×0.2745 mm pixel pitch) and a CCD camera (Manta G-145 with 1388×1038 pixels and 12-bit pixel depth) compose a monoscopic PMD setup.

Fig. 7. Local defects are obvious in the SUT photo (a) as well as the captured vertical (b) and horizontal (c) fringe patterns. (d) is the reconstructed shape with MPMD, and (e) is the correspondence residual visualized as vectors.
From the captured fringe pattern in Fig. 7(b-c), those defects are easier to be identified through the deflection of fringes as marked in the rectangles. The MPMD using uniform cubic B-splines with $17 \times 17$ control points successfully reconstructs the global concave shape of the mirror shown in Fig. 7(d). The correspondence residuals shown in Fig. 7(e) imply that there are local details not yet reconstructed.

![Fig. 8. The local height (c) is reconstructed from the residuals of x-slope (a) and y-slope (b), and then compensate to the MPMD reconstructed height as a final result (d).](image)

The residual slopes in x- and y-direction calculated by using Eq. (5) are shown in Fig. 8(a-b), and Fig. 8(c) displays the height for compensation integrated from residual slopes, which shows the local detailed variations on the globally concave SUT. The propose height compensation is implemented via adding the integrated local height (as illustrated with colors in Fig. 8(d)) onto the MPMD result.

5. Discussion

The major goal of the modal optimization is to find out the global profile of the SUT, and leave its local details to the following zonal reconstruction. Because zonal reconstruction only handles the residual slopes, it is similar to the deformation measurement with PMD [5, 11]. However, this work is based on MPMD, so the rotational vectors in slope residual have already been minimized before applying the zonal integration method.

Initial values are still important for the nonlinear least squares optimization in MPMD. Improper initial coefficients of the shape model may mainly introduce a spherical shape error as a result, and this sort of error cannot be compensated with the proposed method because the correspondence residuals in MPMD are not sensitive to it.

6. Summary

We analyze the correspondence residuals in MPMD, which is mainly at the local height variations on SUT due to the mismatching with the shape model. A method is then proposed to calculate the slope residuals and further reconstruct the local structures by using zonal shape reconstruction method. Finally, the reconstructed local shape can compensate to the MPMD result. Simulation and experiment are carried out to demonstrate the feasibility of the proposed method, which is a compliment for MPMD approach.

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