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COVID-19 and credit risk: A long memory perspective

Jie Yin \(^{a}\), Bingyan Han \(^{b}\), Hoi Ying Wong \(^{a,\ast}\)

\(^{a}\) Department of Statistics, The Chinese University of Hong Kong, Hong Kong
\(^{b}\) Division of Science and Technology, BNU-HKBU United International College, Zhuhai, Guangdong, China

Abstract

The COVID-19 pandemic shows significant impacts on credit risk, which is the key concern of corporate bondholders such as insurance companies. Credit risk, quantified by agency credit ratings and credit default swaps (CDS), usually exhibits long-range dependence (LRD) due to potential credit rating persistence. With rescaled range analysis and a novel affine forward intensity model embracing a flexible range of Hurst parameters, our studies on Moody’s rating data and CDS prices reveal that default intensities have shifted from the long-range to the short-range dependence regime during the COVID-19 period, implying that the historical credit performance becomes much less relevant for credit prediction during the pandemic. This phenomenon contrasts sharply with previous financial-related crises. Specifically, both the 2008 subprime mortgage and the Eurozone crises did not experience such a great decline in the level of LRD in sovereign CDS. Our work also sheds light on the use of historical series in credit risk prediction for insurers’ investment.

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1. Introduction

The COVID-19 pandemic has triggered market turmoil in an unprecedented way. Whereas the previous subprime mortgage and European debt crises originated from the financial system, the pandemic crisis has caused global and all-around damage to human lives and halted the economies. Almost no area has been left unscathed. During the initial shock in around March 2020, the global stock market experienced a tremendous downturn, of at least 25%. The U.S. Treasury market saw a surge in trade demand, and the Fed had to make $1 trillion of Treasury purchases over several weeks to restore market liquidity (Duffie, 2020). West Texas Intermediate (WTI) crude oil futures turned negative for the first time on April 20, 2020. Our experiences of such far-reaching and widespread events are very rare, and most of us were unprepared. The hidden implication of COVID-19 needs to be quantified and interpreted (Chen et al., 2021). Thus, it is urgently necessary to obtain a deep understanding of the economic and financial impact of the coronavirus outbreak.

This paper focuses on the effects of COVID-19 on credit risk, a key factor driving the prices of sovereign and corporate bonds. Insurance companies are arguably the major institutional investors in the sovereign and corporate bond markets, especially for long-term bonds (Campbell and Taksler, 2003; Ellul et al., 2011). In addition, they also participate into the credit derivatives markets directly. Table 1 shows that insurers are non-negligible players in the credit default swaps (CDS) market. In recent years, the CDS market size has stabilized. Insurers usually have over $100 billion notional amounts outstanding in CDS positions, while hedge funds have roughly $300 billion. Additionally, with a beneficial role of CDS in the bond market (Kim et al., 2016), corporate bond holders, even for those who do not trade CDS directly, still regard CDS as a key indicator of credit risk for the liquidity of the CDS trading (Eckert et al., 2016; Palmowski and Surya, 2020).

The pandemic has threatened the credit conditions of individuals, firms, and even countries. The decrease in potential labor income has lowered consumption demand, and many business sectors have shut down or experienced financial distress. Furthermore, high debt burdens and leverage ratios have put many firms’ solvency into question. Possible clusters of bankruptcies threaten economic stability and recovery in the future. Banks encounter balance sheet challenges when borrowers’ financial conditions deteriorate. Several recent works examine credit risk under COVID-19. Aramonte and Avalos (2020) document that credit rating downgrades are concentrated in the energy, retail, and entertainment sectors. Default correlations rise sharply for high-yield bonds and...
Table 1
Semiannual CDS over-the-counter market statistics. This table reports notional amounts outstanding in billions of US dollars. CCPs stand for central counterparties. SPVs are special purpose vehicles. Insurers include insurance and financial guaranty firms. Source: The Bank for International Settlements (BIS) derivatives statistics. https://www.bis.org/statistics/derstats.htm.

| Period | Total | Reporting dealers | CCPs | Banks and securities | Insurers | SPVs | Hedge funds | Other financials | Non-financials |
|--------|-------|-------------------|------|----------------------|----------|------|-------------|-----------------|---------------|
| H1 2021 | 8813  | 1112              | 5615 | 634                  | 112      | 75   | 406         | 616             | 243           |
| H2 2020 | 8359  | 1261              | 5222 | 412                  | 109      | 63   | 277         | 715             | 301           |
| H1 2020 | 8609  | 1446              | 5295 | 539                  | 110      | 50   | 297         | 762             | 311           |
| H2 2019 | 7578  | 1424              | 4239 | 387                  | 116      | 49   | 329         | 753             | 281           |
| H1 2019 | 7809  | 1571              | 4217 | 409                  | 111      | 46   | 371         | 778             | 305           |
| H2 2018 | 8141  | 1809              | 4445 | 400                  | 127      | 48   | 383         | 658             | 271           |
| H1 2018 | 8345  | 1999              | 4519 | 406                  | 125      | 56   | 381         | 624             | 236           |

more leveraged firms. Aldasoro et al. (2020) warn that the current performance of banks is similar to that in 2008. Brunnermeier and Krishnamurthy (2020) design a corporate finance framework for credit policies to avoid undue scarring from the pandemic and a large wave of bankruptcies. Agca et al. (2020) consider the credit risk from a supply chain perspective and find that household demand is an important factor for understanding the impact of the supply chain on credit risk. Haddad et al. (2021) show that the bond market suffered from much more liquidity issue than the CDS market did during the pandemic period. As we focus on the credit risk alone, the CDS market is more relevant for our investigation during the COVID-19 period.

Different from the literature mentioned above, we measure the impact of the COVID-19 crisis by long-memory patterns of credit risk. This is related to the usefulness of historical credit information for credit risk prediction and credit derivatives pricing during and after the pandemic crisis period. As a measure of the persistence in time series innovation, long-range dependence (LRD) or long-memoryness is a stylized fact found in many time series data. Hurst (1951) first discovered LRD in time series of river flows with the rescaled range (R/S) statistic. The Hurst parameter \( H \in (0, 1) \) is an index for the level of LRD. When \( H \in (0, 1/2) \), the process is usually referred to as short memory; when \( H \in (1/2, 1) \), it is long memory. Economically, LRD is related to credit rating persistence and thus the ability to use historical credit quality for credit prediction. Therefore, we pay attention to the change in the level of range dependence, especially the switch between long and short memory regimes. In the finance literature, LRD is found in many economic series, such as volatility (Comte and Renault, 1998; Cont, 2001; Rossi and Santucci de Magistris, 2013; Dark, 2018), interest rate (Lai, 2004), exchange rate (Andersen et al., 2001), oil price (Mensi et al., 2017), and even mortality rate (Yan et al., 2021; Wang et al., 2021). There are also a few related works on credit risk data. Martin et al. (2003) conduct R/S analysis on the returns of bond indices from January 1988 to April 2000 and find the long-memory pattern. Biagini et al. (2013) apply fractional Brownian motions to capture LRD in bond default rates.

In this paper, we validate the LRD pattern and examine the impact of COVID-19 under both physical and risk-neutral world. Under the physical measure, for the time series of Moody’s credit transition matrices and credit default swaps (CDS), we not only confirm the long-memory patterns found in them but also discover that the estimated Hurst parameters of low-rated issuers and CDS drop during the COVID-19 pandemic period. The classic rescaled range analysis is only used for exploration. In Section 2, we estimate the Hurst parameter \( H \) for credit rating data from Moody’s, described in Table 2 and Figs. 1–2. Extensive study reveals that \( H \) is usually estimated above 0.7, indicating a significant LRD in credit rating data; see Table 3. Most of the \( H \) estimates drop in the pandemic crisis period, as shown in Fig. 3. Because credit ratings are an important determinant of CDS spreads, we further examine the LRD in CDS quotations in Section 2.4. As expected, the CDS spread series exhibits long-memory behavior, with the Hurst parameter estimated close to 0.8. Considering the log-return series of CDS spreads, the estimated Hurst parameter is still larger than 0.5 except for the pandemic crisis period, showing a potential shift from the long-memory regime to the short-memory regime during the late COVID-19 period, as depicted in Fig. 4. The pandemic has caused considerable credit and outlook downgrades for vulnerable business in the energy, retail, and entertainment sectors (Aramonte and Avalos, 2020) and in banks with low profitability (Aldasoro et al., 2020). The weakened credit persistence seems consistent with the recent studies.

Credit ratings react slowly to market turmoil. In contrast, CDS are liquid credit derivatives that offer rapid updates on market implied credit risk. CDS are usually traded more actively than the underlying corporate bonds. Moreover, the aforementioned empirical study only shows the presence of LRD in credit-related stochastic processes under the physical probability measure; however, the financial industry is also interested in credit risk modeling under the pricing measure. Therefore, we develop a novel reduced-form model of credit risk that allows for both short- and long-memory patterns. The specification of default intensities plays a key role in reduced-form models for credit pricing. Examples of successful default intensity models include the use of Cox-Ingersoll-Ross (CIR) process (Brigo and Alfonsi, 2005), lognormal process (Pan and Singleton, 2008; Longstaff et al., 2011), and Hawkes process (Alt-Sahalia et al., 2014). However, most if not all of them fail to take LRD into account. The success of range dependent models for the volatility process (Gatheral et al., 2018; El Euch and Rosenbaum, 2019; Abi Jaber et al., 2019; Morelli and Santucci de Magistris, 2019) inspires us to develop a credit analogue in the present paper. We extend the affine forward intensity (AFI) model (Gatheral and Keller-Ressel, 2019) to incorporate range dependence for both short- and long-memory patterns. Compared with models based on fractional Brownian motions in Biagini et al. (2013); Morelli and Santucci de Magistris (2019), our AFI model with LRD provides an explicit exponential transform formula, which is derived in Theorem 3.2. It enables us to obtain a relatively efficient calibration on CDS spreads while maintaining the ability to detect short- and long-range dependence.

Based on the CDS pricing formula with LRD developed in Section 3.2, we calibrate the Hurst parameters from sovereign CDS data for the COVID-19 period in Section 4. Although the Hurst parameter is constant in AFI model, we can adopt a similar idea from implied volatility calibration and obtain the term structure of Hurst parameters by the rolling window method. The impact of the novel coronavirus is likely to be more pronounced in countries with severe outbreaks and more rapidly reflected in the sovereign CDS. We use CDS data of Italy and Brazil in this study because these countries have suffered severe outbreaks, with each ranking second in the number of infections at different times. The US has ranked first, and we have examined the LRD of US corporate credit ratings and CDS in the R/S analysis. The risk-neutral default inten-
sities implied by CDS spreads also have a smaller Hurst parameter \( H \) during the pandemic. This is consistent with the results of the \( R/S \) analysis. The calibrated Hurst parameter drops below 0.5 for certain months; see Fig. 7. Default intensities switch from the long-memory to the short-memory type for both economies, which can be detected by the AFI model.

A main conclusion of this paper is the credit market behaves differently during the COVID-19 crisis, in contrast to the 2008 subprime mortgage crisis and the Eurozone crisis. In the banking industry, Hassler et al. (2014) provide evidence that the collapse of the Lehman Brothers bank and the Eurozone crisis had differing effects on memory parameters. Whereas these two crises are directly associated with the economic system, COVID-19 is a public health issue, and it is likely to affect the credit market in a rather different manner. The sovereign CDS of Brazil have not been affected by the Eurozone crisis, as no obvious trend is found in the Hurst parameter. This is not surprising, as the Eurozone crisis is a regional matter. In contrast, Italy, one of the centers of Eurozone crisis (Alt-Sahalia et al., 2014), has a rapid change in the calibrated Hurst parameter. Unlike the COVID-19 pandemic, the Eurozone crisis has strengthened the long-memory characteristic, as shown in Fig. 9, indicating the important role of historical data in pricing. For the 2008 subprime mortgage crisis, the calibration results are more complicated, as this crisis quickly became a global one, affecting both Italy and Brazil. Before 2008, CDS prices were very flat with little variations; see Jorion and Zhang (2007) and Fig. 11. LRD is extremely strong with the calibrated Hurst parameter close to one. With the evolution of the subprime mortgage crisis, we observe a steep decline in \( H \) after February 2008. This pattern is similar to the results in the first half of 2020, as shown in Fig. 7. However, the calibrated \( H \) in 2008 is still larger than 0.5 and retains LRD. After Lehman Brothers filed for bankruptcy, the calibrated \( H \) gradually grew to over 0.7. The market returned to the situation of predicting credit in a significant long-memory manner.

Finally, to supplement the in-sample analysis, we conduct out-of-sample forecasting at the end of Section 4 to compare our AFI model and the Markovian model with fixed \( H = 1/2 \). The dataset is the same as that used for CDS long memory detection in Section 2.4. Under the criteria of the mean absolute error (MAE) and mean ratio of error to price (MRE), our model performs better in most cases. It provides another motivation to apply AFI model in the calibration and prediction for CDS spreads.

The rest of this paper is organized as follows. We review the \( R/S \) method to test the long-memory pattern and present the empirical results of the credit rating transitions and CDS spreads in Section 2. Section 3 proposes the AFI model with the explicit CDS pricing formula. Calibration is then conducted in Section 4 to assess the effects of COVID-19 and compare them with the Eurozone crisis and the 2008 financial crisis. An out-of-sample prediction is also included. Section 5 concludes the paper. Supplementary details are given in the Appendix.

2. Long memory in physical world

A credit rating is an evaluation of the credit risk assessed by credit rating agencies such as Moody’s, Standard & Poor’s, and Fitch for different issuers (e.g. individuals, companies, and governments). Rating actions, including upgrades and downgrades, constitute rating transition matrices, to which the public always pays attention. In many existing models, rating transition matrices are typically assumed to be generated by a Markov chain (Wozabal and Hochreiter, 2012), mixture of Markov chains (Frydman and Schuermann, 2008), or higher order Markov chains (Baena-Mirabete and Puig, 2018). However, the Markovian assumption is usually invalid if the long memory exists.

The long-memory feature of human mortality data is documented in Yan et al. (2021) using lifetime data. We recognize the similarity between the life table and the credit transition matrices published by rating agencies. A default event is similar to a death, and downgrading is similar to aging, as a person moves from a younger age to an older one. However, upgrading is possible in credit transition, whereas it is impossible to return to a younger age. Nevertheless, the methodologies for estimating the Hurst parameter from the collected tables or matrices for the two cases have many similarities.

After the long-memory detection in credit ratings, we extend to include credit default swaps (CDS), the important derivatives that have attracted widespread attention in credit risk. Additionally, Ismailescu and Kazemi (2010) reveal that upgrades and downgrades of credit ratings materially affect CDS spreads. We are curious to find out the LRD of these two credit-related objects in physical world.

2.1. Hurst parameter

Hurst (1951) proposes the Hurst parameter, \( H \), as the index of long-range dependence in a time series. A value of \( H = 0.5 \) corresponds to the classical Brownian motion. If \( H > 0.5 \), it indicates that a high or low value is likely to continue and that a long-memory pattern is present. When \( H < 0.5 \), the time series is prone to switching between high and low values, which is a short-memory process.

There are many ways to estimate \( H \) and provide statistical evidence for hidden structures, such as rescaled range (\( R/S \)) analysis, developed by Mandelbrot and Wallis (1969); detrended fluctuation analysis (DFA), proposed by Peng et al. (1994); and periodogram regression methods, put forward by Geweke and Porter-Hudak (1983). Inevitably, the estimated Hurst parameter is kind of noisy and volatile, especially for short time series like credit ratings. However, we are concerned about the level and fluctuation of Hurst parameters, instead of their exact values. Therefore, these methods are still useful and meaningful for long-memory pattern detection. Due to the similarity of the conclusions of these methods and the simplicity of the paper, we only discuss the most common and well-known one, \( R/S \) analysis, and then show the empirical results.

2.2. Rescaled range analysis

To test the evidence of long memory in a process, we illustrate the rescaled range analysis here. Consider a time series \( Y_{t} \) and divide it into sub series of length \( n \), written as \( Y_{t} = Y_{(m,m+1,...,m+n-1)} \), where \( m \) represents the starting point with the condition \( 1 \leq m \leq T + 1 - n \).

Let \( t = m, m + 1, ..., m + n - 1 \) and calculate

\[
X_{t} = Y_{t} - \frac{1}{n} \sum_{j=m}^{m+n-1} Y_{j}, \quad Z_{t} = \sum_{j=m}^{t} X_{j}. \tag{2.1}
\]

Denote the cumulative range as

\[
R_{m} = \max(0, Z_{m}, ..., Z_{m+n-1}) - \min(0, Z_{m}, ..., Z_{m+n-1}). \tag{2.2}
\]

The standard deviation is given by

\[
S_{m} = \sqrt{\frac{1}{n-1} \sum_{j=m}^{m+n-1} X_{j}^2}. \tag{2.3}
\]

Then, the following theorem (Mandelbrot and Wallis, 1969) provides a classical formulation for calculating the Hurst parameter.
Theorem 2.1. With the notations and definitions in (2.1)-(2.3), there exists a constant \( C \in \mathbb{R} \) such that the rescaled range \( R/S \) satisfies
\[
(R/S)_n := E(R_m/S_m) \sim Cn^H, \quad \text{as } n \to \infty. \tag{2.4}
\]

When the data length is short, a modified formulation is adopted to improve the performance in Annis and Lloyd (1976); Peters (1994). Due to the significant deviation of \( H \) from 0.5 for white noise especially when \( n \) is small, the theoretical values of \( R/S \) statistics are approximated. Weron (2002) points out the procedure of obtaining \( E(R/S)_n \), as shown in the following formulas:
\[
E(R/S)_n = \begin{cases}
\frac{n-1}{n} \sum_{i=1}^{n-1} \frac{n}{H(i)} \frac{n-1}{i} \sigma_i, & \text{for } n \leq 340 \\
\frac{n-1}{n} \sum_{i=1}^{n-1} \frac{n}{H(i)} \frac{n-1}{i} \sigma_i, & \text{for } n > 340 
\end{cases}
\tag{2.5}
\]
where \( \Gamma(\cdot) \) is the Gamma function. Then, the value of \( H \) is estimated by running a simple linear regression over a sample of increasing time horizons:
\[
\log(R/S)_n - \log E(R/S)_n = \log C + (H - 0.5) \log n. \tag{2.6}
\]

This modified equation provides more accurate expression. The slope of \( \log(R/S)_n - \log E(R/S)_n \) versus \( \log n \) plus 0.5 is then the estimate of \( H \).

In this paper, we use the R package called pracma to conduct the estimation. It provides the function hurstexp(\( \cdot \)), which is based on the rescaled range analysis and calculates different \( H \) using slightly different approaches (modified versions of \( R/S \) analysis). Due to the small sample size, we choose the corrected empirical Hurst parameter that follows the method in Weron (2002), as shown in (2.5)-(2.6).

We admit the limitations of this rescaled range analysis. It cannot provide accurate Hurst parameters and is only a preliminary method for rough estimation. It usually requires a long data length which can not be satisfied by datasets related to credit risk. However, applying it to the empirical study for a brief idea of LRD and showing the change of long memory are meaningful and enlightening.

2.3. Demonstrating long memory in credit ratings

Credit rating data are downloaded from Moody’s Default & Recovery Database (DRD).\(^1\) It contains historical rating data for global corporate and sovereign entities. Moody is one of the major credit rating agencies, and its global long-term rating scales are Aaa, Aa, A, Baa, Ba, B, Caa, Ca, and C. For each generic rating classification from Aa through Caa, there are modifiers 1, 2, and 3. Hence, Moody’s classifies all issuers by these 21 rating categories.

The choice of contained issuers in the empirical analysis is largely determined by data availability. We choose the rating data in the table called Senior Rating Standard in DRD, which is based on the full debt structure of the issuer. By July 21, 2020, 29,139 issuers were rated more than once. We select those who did not withdraw during the given rating period from January 1, 2014 to July 1, 2020 and obtain a sample of 3,472 issuers with monthly credit rating data.

Issuers’ credit rating transition matrices are further processed to obtain the transition of every specific rating symbol. The transition data can be expressed as the time series \( Y_{it}, i \in (1, 2, 3, \ldots, 21), t \in (1, 2, 3, \ldots, T) \), where \( i \) is the rating symbol, \( t \) is the rating time, and \( Y_{it} \) is the number of issuers with rating \( i \) at time \( t \). Table 2 summarizes overall information and descriptive statistics of the credit rating data applied.

To obtain some insights into the dataset, Fig. 1 and Fig. 2 visualize the changes in different credit ratings. We compare the performance of high- and low-level ratings during the COVID-19 crisis in 2020. Although the scales on the y-axis in the two plots are different, the percentage changes are comparable. It is obvious that the number of high-rated issuers is much more stable than of the low-rated ones. Furthermore, the numbers of issuers in the lowest three ratings increase dramatically after the outbreak of COVID-19.

We provide more details on the \( R/S \) estimations of the Hurst parameter across different periods and different rating symbols. Consider the same time length (window size) and roll the window to obtain Table 3. The Hurst parameter estimated from monthly data over 6 years is used to approximate the \( H \) of the last month in that period (window). For example, the \( H \) estimated from January 1, 2014 to January 1, 2020 is regarded as the Hurst estimate for December 2019. This serves our empirical purpose because we focus on the level and movement of \( H \) over a period rather than the exact values at a particular time point. To show aggregate information for all 21 rating symbols, we plot the mean of the estimated \( H \) accompanied by three representative ratings in Fig. 3.

In Table 3, all 21 credit ratings demonstrate a high degree of long memory. A large percentage of \( H \) values are greater than 0.7. Although not reported here, we also try other rolling window sizes, such as longer than 10 years or shorter than 2 years, and draw the same conclusion. The credit rating exhibits strong LRD. More importantly, credit rating data maintain a high level of LRD pattern even during the COVID-19 crisis period. Although the Hurst parameter is quite stable, with small changes, a slight downward trend of average \( H \) in Fig. 3 is recognized. There are several reasons for the small range of variability for the Hurst parameter \( H \). First, the differences in \( H \) are related to the step of rolling windows. Compared with the previous window, the data covered by the current window only differ in two months. Second, credit ratings are crucial for financial stability and therefore react slowly to the pandemic. It is reasonable that the change is quite small under the low frequency and magnitude of credit rating adjustments.

A closer look at the trend of every rating reveals another two prominent behaviors. The first one is that the estimated \( H \) of C-rated issuers drops dramatically and severely. The Ca-ranking category next to the C-rating also has an obvious drop in April 2020. Secondly, the highest rating Aaa also has a decreasing trend in \( H \) until April 2020, but increases afterward. For ratings from Aa1 to A2, all of the estimated \( H \) values increase at different levels. Combined with the comparative performance in Figs. 1-2, COVID-19 crisis does not alter the credit persistence of high-rated issuers. According to Kisgen (2006), high-rated issuers have a better capital structure and financial properties to resist risk based on Moody’s rating definitions. Thus, historical credit information is more reliable in the credit prediction of high-rated issuers.

2.4. Demonstrating long memory in CDS spreads

Credit default swaps (CDS) are liquid assets with more rapid updates on market implied credit risk than credit ratings. Moreover, changes in credit ratings have anticipated and lagged effects on CDS spreads. Typically, the CDS market also reacts fast to changes in credit ratings (Daniels and Jensen, 2005). Therefore, with this clear relationship between the credit default swap spreads and the credit ratings, we anticipate the existence of LRD in CDS spreads. In this section, we extend the \( R/S \) analysis to historical CDS spread data.

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\(^1\) DRD is not an open source database. We have subscribed to this service from Moody’s Analytics. See information on DRD at https://www.moodys.com/Pages/Default-and-Recovery-Analytics.aspx.
Table 2
Information and descriptive statistics of the credit rating data.

| Description                      | The total number of issuers | The number of ratings |
|----------------------------------|-----------------------------|-----------------------|
| Credit rating transition         | 3,472                       | 21                    |
| Time duration                    |                             |                       |
| 2014-01-01 to 2020-07-01         |                             |                       |
| Frequency                        |                             |                       |
| Monthly                          |                             |                       |
| The number of observations       |                             |                       |
| 21 × 79                          |                             |                       |
| Variable                         | Range of mean*              | Range of std. dev.**  |
| Yit                             | (3.8, 389)                  | (2.93, 45.2)          |

* Mean \( \bar{Y}_{it} = \frac{1}{T} \sum_{t=1}^{T} Y_{it} \) is the average number of issuers with rating \( i \). Range of mean returns the minimum and maximum of the set \( \{ \bar{Y}_{i1}, \bar{Y}_{i2}, ..., \bar{Y}_{i21} \} \).

** Std. dev. is the standard deviation of \( Y_{it} \), equal to \( \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (Y_{it} - \bar{Y}_{it})^2} \).

Fig. 1. The number of issuers with high-level credit ratings from Aaa to A2.

Fig. 2. The number of issuers with low-level credit ratings from Caa3 to C.
We clean the CDS data collected from Bloomberg as follows. Using Bloomberg’s equity screening function ‘EQS’ and other API, the first step is to screen the issuers according to whether they have CDS tickers and Moody's credit ratings. Then select from this set CDS contracts with 5-year maturity, which is the most liquid type of long-term CDS. The third step is to eliminate CDS contracts that have missing data points for the period, January 2, 2015 to July 3, 2020. Because CDS contracts are prone to missing data points in certain periods, it is difficult to find consecutive CDS spread data. Ultimately, we have 35 corporate CDS contracts with weekly historical data to test the long-memory pattern and the explicit CDS tickers are disclosed in Appendix Table A.9. As the spread of one CDS contract at time t is denoted by $S_t$, Table 4 summarizes overall information and descriptive statistics of the CDS spread sample applied.

We use R/S analysis to estimate the Hurst parameter across different periods by the rolling window method. The mean and standard deviation of the estimated $H$ are given in Table 5 for all of the CDS contracts. The first quartile ($Q_1$) and the third quartile ($Q_3$) are included to illustrate the distribution of $H$. Similar as the approximation method in Section 2.3, the Hurst parameter estimated from 5 years of weekly data is used to represent the $H$ of the last month. For a more intuitive presentation, the trend of the mean $H$ is presented in Fig. 4.
Statistics of estimated Hurst parameter for CDS contracts by the rolling window method (No transformation means that we directly estimate $H$ from CDS spreads $S_t$, while transformation means that the estimated $H$ is from log-return data $R_t$.)

| Period                  | Statistics of estimated Hurst parameter | Transformation |
|-------------------------|----------------------------------------|----------------|
|                         | No transformation                      |                |
|                         | Mean   | SE    | Q1   | Q3   | Mean | SE    | Q1   | Q3   |
| 2015-01-02 to 2020-01-03| 0.8124 | 0.0262| 0.7920| 0.8341| 0.6000| 0.1432| 0.5157| 0.7029|
| 2015-02-06 to 2020-02-07| 0.8127 | 0.0263| 0.7924| 0.8359| 0.6438| 0.1478| 0.5405| 0.7420|
| 2015-03-06 to 2020-03-06| 0.8130 | 0.0266| 0.7926| 0.8366| 0.5919| 0.1382| 0.4982| 0.6981|
| 2015-04-03 to 2020-04-03| 0.8073 | 0.0302| 0.7818| 0.8319| 0.5362| 0.1140| 0.4552| 0.6128|
| 2015-05-01 to 2020-05-01| 0.8066 | 0.0319| 0.7837| 0.8329| 0.5217| 0.0949| 0.4489| 0.6022|
| 2015-06-05 to 2020-06-05| 0.8077 | 0.0291| 0.7897| 0.8325| 0.4890| 0.0892| 0.4375| 0.5741|
| 2015-07-03 to 2020-07-01| 0.8070 | 0.0274| 0.7884| 0.8327| 0.4741| 0.0995| 0.4068| 0.5521|

Fig. 4. Estimated Hurst parameter for CDS contracts (It plots the average $H$ of 35 corporate CDS contracts for every period. One is estimated from CDS spreads (blue) and the other is from log-return (green).)

In addition to testing the LRD on original CDS spread data, we perform the same analysis on the log-return series of CDS spreads. The latter $R/S$ analysis is consistent with a similar procedure on the returns of bond indices in Martin et al. (2003). The weekly changes of the CDS log-prices are given by

$$I_t = \log(S_{t+\Delta}) - \log(S_t).$$

(2.7)

where $S_t$ is the CDS spread at time $t$ and $\Delta = 5/252$ represents the weekly interval. Here we assume that there are 252 trading days in a year and 5 trading days in a week.

Statistics in Table 5 show that the level of long-memoryness in CDS spread is fairly strong, with $H$ around 0.8. Although the Hurst parameter is smaller in the log-return series, it is close to 0.6 during the periods before March 2020, which also indicates a long-memory pattern.

When the COVID-19 outbreak occurred globally in March 2020, $H$ dropped. $H$ of the log-return series are even smaller than half. Generally, $R/S$ method overestimates the Hurst parameter. Therefore, the short-range dependence is likely to exist due to the impact of COVID-19. Regime shifted from long-range dependence to short-range dependence. Besides, comparing Fig. 4 with Fig. 3, average $H$ during the COVID-19 crisis is similar between CDS and credit ratings. There exists a relative decline probably in response to the pandemic. In addition, Q1 and Q3 exhibit analogous decreases. Overall, COVID-19 has a non-negligible influence on the persistence of credit time series.
This R/S method is only for motivating. To better understand the impact of COVID-19 on credit risk, we need to examine the dynamics of CDS spreads. Commonly, CDS spreads are modeled by credit default intensities. In the next section, we further provide a thorough analysis on default intensities with a model embracing both long- and short-range dependence in risk-neutral world. Calibration of $H$ based on the pricing model will not be limited by data length.

3. Default intensities with long memory

3.1. Risk-neutral pricing of CDS spreads

The dynamics of CDS spreads are described by the default intensities given under risk-neutral measure. However, R/S analysis does not directly work on default intensities and is unlikely to have an explicit pricing formula with mathematical tractability. Nevertheless, based on the observed persistent long-memory behavior in the empirical study, we need a new credit risk model to capture this stylized fact and maintain theoretical tractability at the same time. Although the R/S analysis is not directly connected with our proposed model, it validates the existence of LRD and motivates us to incorporate short- or long-range dependence into the model.

First, we briefly review the risk-neutral pricing framework for CDS spreads. Consider a complete probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ with a filtration $\mathcal{F} = \{\mathcal{F}_t\}_{t \leq T}$ satisfying the usual conditions. Denote by $T$ a fixed positive time horizon. $\mathbb{Q}$ is the market-implied pricing probability measure or the risk-neutral measure. $\mathbb{E}^{\mathbb{Q}}_{\cdot, [\cdot]}$ and $\mathbb{E}^{\mathbb{Q}}_{\cdot, [\cdot]}$ are the corresponding expectation and conditional expectation, respectively.

Consider a CDS contract initiated at time $t$ with maturity $M$. We denote the corresponding CDS spread by $S_t(M)$. Let $r_t$ be the risk-free interest rate and $\lambda_t$ be the credit default intensity. The CDS contract consists of the spread leg and the protection leg. When the protection premium is paid continuously, we write the present value of the CDS spread as

$$
\mathbb{E}^{\mathbb{Q}} \left[ \frac{t}{t} + M \int_{s} \exp \left( - \int \left( r_u + \lambda_u \right) du \right) ds | \mathcal{F}_t \right]. \tag{3.1}
$$

Furthermore, the time $t$ present value of the CDS protection leg with a fixed recovery rate $\delta$ of its par value is given by

$$
\mathbb{E}^{\mathbb{Q}} \left[ \frac{t}{t} + M \int_{s} \lambda_s \exp \left( - \int \left( r_u + \lambda_u \right) du \right) ds | \mathcal{F}_t \right]. \tag{3.2}
$$

The order of conditional expectation and integration can be interchanged by Fubini's theorem. Under a perfectly competitive market without arbitrage, the time $t$ present values of the two legs are equal. We assume that the default intensity $\lambda_t$ and the risk-free rate $r_t$ are independent, as in Pan and Singleton (2008); Longstaff et al. (2011); Ait-Sahalia et al. (2014). Then, the CDS spread $S_t(M)$ is solved as

$$
S_t(M) = \frac{(1-\delta) \int_{s}^{t+M} \mathbb{E}^{\mathbb{Q}} \left[ \lambda_s \exp \left( - \int \left( r_u + \lambda_u \right) du \right) | \mathcal{F}_t \right] ds}{\int_{s}^{t+M} \mathbb{E}^{\mathbb{Q}} \left[ \exp \left( - \int \lambda_u du \right) | \mathcal{F}_t \right] ds} \tag{3.3}
$$

$$(1-\delta) \int_{s}^{t+M} \mathbb{E}^{\mathbb{Q}} \left[ \lambda_s \exp \left( - \int \lambda_u du \right) | \mathcal{F}_t \right] ds \frac{D(t,s) \mathbb{E}^{\mathbb{Q}} \left[ \exp \left( - \int \lambda_u du \right) | \mathcal{F}_t \right] ds}{\int_{s}^{t+M} \mathbb{E}^{\mathbb{Q}} \left[ \exp \left( - \int \lambda_u du \right) | \mathcal{F}_t \right] ds} \tag{3.4}
$$

with $D(t,s) = \mathbb{E}^{\mathbb{Q}} \left[ \exp \left( - \int \lambda_u du \right) | \mathcal{F}_t \right]$, which is the time $t$ price of a zero-coupon bond with maturity date $s$.

If the protection premium is paid semiannually, then the formula of $S_t(M)$ is similar to (3.3) and can be written as

$$
\frac{2(1-\delta) \int_{t}^{t+M} D(t,s) \mathbb{E}^{\mathbb{Q}} \left[ \lambda_s \exp \left( - \int \lambda_u du \right) | \mathcal{F}_t \right] ds}{\sum_{j=1}^{2M} \int_{t}^{t+j/2} D(t,s) \mathbb{E}^{\mathbb{Q}} \left[ \exp \left( - \int \lambda_u du \right) | \mathcal{F}_t \right] ds} \tag{3.4}
$$

To evaluate the numerator in (3.3) and (3.4), we can use the property of characteristic function because

$$
\mathbb{E}^{\mathbb{Q}} \left[ \lambda_s \exp \left( - \int \lambda_u du \right) | \mathcal{F}_t \right] = \frac{\partial}{\partial \phi} \mathbb{E}^{\mathbb{Q}} \left[ \exp \left( \phi \lambda_s - \int \lambda_u du \right) | \mathcal{F}_t \right] \bigg|_{\phi=0}. \tag{3.5}
$$

Therefore, to price the CDS spread $S_t(M)$ in (3.3) or (3.4), the key point is to evaluate the following exponential transform formula

$$
\mathbb{E}^{\mathbb{Q}} \left[ \exp \left( \phi \lambda_T - \int \lambda_u du \right) | \mathcal{F}_t \right] \bigg|_{\phi=0}. \tag{3.6}
$$

where $\phi = i\omega$ with $\omega \in \mathbb{R}$. When $\phi = 0$, it matches the expectation in the denominator of $S_t(M)$. The numerator relies on the derivative in (3.5).

Considering models for the default intensities, the most simple one is constant $\lambda$ and then the default is the first jump of a Poisson process. Several more sophisticated models have been proposed, such as CIR process in Brigo and Alfonso (2005), Lognormal process in Pan and Singleton (2008); Longstaff et al. (2011), and Hawkes process in Ait-Sahalia et al. (2014). These models are popular mainly thanks to the fact that the explicit formula (3.6) is available for them. However, most models in the literature ignore the long-memoryness in CDS spread. In the next section, we adopt an affine forward intensity model to capture this stylized fact.

3.2. Affine forward intensity model

An affine specification of default intensities is inspired by Abi Jaber et al. (2019). Under the risk-neutral measure $\mathbb{Q}$, consider the credit default intensity $\lambda_t$ as a univariate Volterra square-root process (Abi Jaber et al., 2019),

$$
\lambda_t = \lambda_0 + \kappa \int_0^t \left( \theta - \lambda_s \right) ds + \int_0^t \sqrt{\lambda_s} dW^\mathbb{Q}_s, \tag{3.7}
$$

where $W^\mathbb{Q}$ is a standard Brownian motion under $\mathbb{Q}$, $K \in L^2_{\mathbb{Q}}(\mathbb{R}_+ \times \mathbb{R})$ is the kernel function, and $\lambda_0, \kappa, \theta, \sigma$ are positive constants. To incorporate long- and short-range dependence, a specific choice is the fractional kernel $K(t) = t^{1/2-1/2H}$, where $H$ is the Hurst parameter. Under a fractional kernel with $H \in (0, 1/2)$, (3.7) is first introduced in El Euch and Rosenbaum (2019) for option pricing with rough volatility. When $H = 1/2$, the model reduces to the classic Markovian CIR process.

The well-posedness of the Volterra square-root process is partially explained by the following theorem from Abi Jaber et al. (2019). Note that the assumptions in Theorem 3.1 hold for the fractional kernel $K(t) = t^{1/2-1/2H}$ with $H \in (0, 1/2)$.  

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Theorem 3.1 (Abi Jaber et al. (2019, Theorem 7.1)). Suppose that the kernel $K$ is strictly positive and completely monotone. In addition, there is $t \in (0, 2)$ such that $\int_0^t K(t) dt = 0$ and $\int_0^t (K(t) - K(t+h)) dt = 0$ for every $t < \infty$. Then, Volterra square-root process (3.7) has a unique in law $\mathbb{R}_+$-valued continuous weak solution for any initial condition $\xi_0 \in \mathbb{R}_+$.

For Volterra square-root process (3.7), it is shown in Abi Jaber et al. (2019, Lemma 4.2) that the conditional $\mathbb{Q}$-expected default intensity $\mathbb{E}^\mathbb{Q}[\xi_t | \mathcal{F}_t] = \xi_t(s)$ follows

$$\xi_t(s) = \xi_0(s) + \int_0^t \frac{1}{K} R_x(s-u) \sigma \sqrt{\lambda_u} dW_u^\mathbb{Q}, \quad (3.8)$$

where

$$\xi_0(s) = \left(1 - \int_0^{\frac{s}{K}} R_k(u) du\right) \lambda_0 + \theta \int_0^{\frac{s}{K}} R_k(u) du, \quad (3.9)$$

and $R_k$ is the resolvent of $\kappa K$ such that

$$\kappa K \ast R_k = R_k \ast (\kappa K) = \kappa K - R_k. \quad (3.10)$$

The notation $\ast$ denotes the convolution operation:

$$K \ast f(t) = \int_0^t K(t-s) f(s) ds. \quad (3.11)$$

Note that the resolvent always exists and is unique by Gripenberg et al. (1990, Theorem 2.3.1).

Remarkably, it is unnecessary to impose any specific models on $\lambda$ directly (Gatheral and Keller-Ressel, 2019). Instead, one can directly consider the family of forward intensity processes under $\mathbb{Q}$ as $\xi_t(s)$. The default intensity $\lambda$ is then a given $\mathcal{F}_t$-adapted continuous, integrable, and non-negative process. $\xi_t(s)$ is a martingale on $[0, s]$ and by the martingale representation theorem, there exists a predictable process $\eta_t(s)$ for each $s > 0$ such that $\xi_t(s)$ has a form of

$$\xi_t(s) = \xi_0(s) + \int_0^s \eta_t(u) dW_u^\mathbb{Q}, \quad t \in [0, s]. \quad (3.12)$$

We adopt the same convention from Gatheral and Keller-Ressel (2019) to extend the definition such that $\xi(s) = \lambda_0$ and $\eta_t(s) = 0$ for $t > s$. Suppose that $\eta_t(\cdot)$ satisfies

$$\int_0^T \int_0^t \eta_t(s)^2 ds dt < \infty, \quad Q \text{-a.s..} \quad (3.13)$$

(3.13) validates the integrability and is a mild assumption. There are many possible choices for the process $\eta_t(s)$.

Thus, Volterra square-root process (3.7) is a special forward density model (3.12), with

$$\eta_t(s) = \frac{1}{K} R_x(s-u) \sigma \sqrt{\lambda_u}. \quad (3.14)$$

Mathematically, we can take (3.8)-(3.9) as our model for $\xi_t(s)$ directly, without discussing Volterra square-root process (3.7). For convenience, we refer to (3.8)-(3.9) as affine forward intensity (AFI) model. To capture the long-memory feature, we have to consider the Hurst parameter $H \in (1/2, 1)$; see Comte and Renault (1998); Gatheral et al. (2018). However, when $H \in (1/2, 1)$, the fractional kernel becomes non-decreasing, and being completely monotone is unclear. Thus, the well-posedness of Volterra square-root process (3.7) remains an open question. Fortunately, the AFI model (3.8)-(3.9) is still valid. For a fractional kernel with any $H > 0$, the resolvent

$$R_e(t) = \kappa t^{\alpha-1} E_{\alpha,\alpha}(-\kappa t^\alpha), \quad \alpha = H + 1/2. \quad (3.15)$$

where $E_{\alpha,\alpha}(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma(n+1)}$ denotes the Mittag-Leffler function. See El Euch and Rosenbaum (2019, Appendix A1) for its properties. The AFI model (3.8)-(3.9) with (3.15) is still well-defined when $H > 1/2$ because assumptions are made on $\eta_t(s)$ and condition (3.13) still holds. Putting all of these theoretical results together, the AFI model (3.8)-(3.9) defines our proposed reduced-form credit risk model with flexible range dependence that allows for all values of $H \in (0, 1)$. It is worth mentioning that although $H$ is assumed to be constant during $[0, T]$, we can still employ it to obtain the term structure of Hurst parameter by the rolling window method. The rationale is analogous to the implied volatility calibration. The Black-Scholes model assumes a constant volatility. However, one can still regard it as a function of strike price and time to maturity. Similarly, we can think $H$ is different for data in different windows. Thus, a time series of $H$ is obtained by rolling the window.

We are now ready to present the exponential transform formula for (3.8), which enables a fast calibration of CDS spreads. Consider $\psi$ as a solution to the Riccati-Volterra equation

$$\psi = \phi K + \left( f - \psi + \frac{\sigma^2}{2} \psi^2 \right) * K, \quad (3.16)$$

where $\phi \in \mathbb{C}$ is a generic complex number and $f \in L^1([0, T], \mathbb{C})$. The following theorem provides the exponential transform formula for the AFI model (3.8)-(3.9), in an explicit form up to the solution to the Riccati-Volterra equation (3.16) without restriction to fractional kernels. The proof is given in Appendix B.

Theorem 3.2. Suppose that Assumption (3.13) holds and the Riccati-Volterra equation (3.16) has a unique complex-valued solution $\psi \in L^2([0, T], \mathbb{C})$. With the AFI model (3.8)-(3.9), define the process $(Y_t, 0 \leq t \leq T)$ as

$$Y_t = \phi \xi_t(T) + \int_0^T f(T-s) \xi_t(s) ds + \frac{1}{2} \int_0^T \sigma^2 \psi^2(T-s) \xi_t(s) ds. \quad (3.17)$$

If $\{\exp(Y_t), 0 \leq t \leq T\}$ is a true martingale, then

$$\mathbb{E}^\mathbb{Q} \left[ \exp \left( \phi \lambda_T + \int_0^T f(T-s) \lambda ds \right) \mathcal{F}_T \right] = \exp(Y_t), \quad 0 \leq t \leq T. \quad (3.18)$$

In the calibration, we consider $\phi = i\omega$ with $\omega \in \mathbb{R}$ and $f = -1$. For fractional kernels, if $H \in (0, 1/2)$, Abi Jaber et al. (2019, Lemma 6.3) prove that the Riccati-Volterra equation (3.16) has a unique global solution $\psi$, which is also continuous by Gripenberg et al. (1990, Theorem 12.11). $\exp(Y_t)$ in Theorem 3.2 is a true martingale by Abi Jaber et al. (2019, Theorem 6.1). When $H \in (1/2, 1)$, $K(t) = \frac{t^{H-1/2}}{\Gamma(H+1)}$ is increasing and Abi Jaber et al. (2019, Lemma 6.3) is not applicable. However, by Abi Jaber et al. (2019, Theorem B1), there exists $T_{\max} \in (0, \infty)$ such that (3.16) has a unique non-continuable solution $\psi$ on $[0, T_{\max})$. For $0 < T < T_{\max}$, Gripenberg et al. (1990,
Theorem 12.11) prove that $\psi$ is continuous and therefore bounded on $[0, T]$. Then, by selecting a sufficiently small $T > 0$ that with non-positive $f$ and $\phi = i o$, given $\xi_t(s) \geq 0$, we can obtain

$$\text{Re} [Y_t] = \int_0^T f(T - s)\xi_t(s)ds$$

$$+ \frac{\sigma^2}{2} \int_0^T [(\text{Re} \psi)^2(T - s) - (\text{Im} \psi)^2(T - s)]\xi_t(s)ds$$

$$\leq 0. \quad (3.19)$$

Then, exp($Y_t$) is a bounded local martingale and therefore a true martingale. Thus, Theorem 3.2 is applicable with a suitable time horizon $T > 0$ when exp($Y_t$) does not explode.

4. Empirical study on default intensities

In this section, we apply the AFI model (3.8)-(3.9) with the fractional kernel $K(t) = \frac{t^{H-1/2}}{\Gamma(H+1/2)}$, $H \in (0, 1)$, to the CDS spread data. Our main goal is to estimate the Hurst parameter $H$ for default intensities and detect the impact of the COVID-19 pandemic on long-memoryness in default intensities. A comparison between the pandemic crisis and other crises (the 2008 subprime mortgage crisis and Eurozone crisis) reveals the unique characteristic of the pandemic. Besides, we acknowledge that there are other methods to capture the structural impact of COVID-19. However, our methodology is sufficient to detect the break and demonstrates essential differences between several crises.

4.1. CDS spreads and COVID-19 in 2020

To investigate the impact of COVID-19 on credit risk implied by CDS prices, we focus on the comparison between countries. Each country has been impaired to a different degree by the pandemic and has imposed different lockdown regulations during the outbreak. It is easier to extract the effect of COVID-19 on countries than on companies.

Italy was the first European nation to be affected by COVID-19 and was at one point the center of the pandemic in Europe. On January 31, 2020, the first two confirmed cases were reported in Rome. Italy's sovereign CDS price remained flat during the first three weeks after the reported cases, as shown in Fig. 5. When the outbreak started in the late February 2020 and daily new confirmed cases soared in Italy, the CDS market reacted because of concerns about the pandemic, and the spreads first jumped to over 250 bps. During March and April 2020, the market experienced a sharp rise and fall, probably due to the uncertainty brought by COVID-19. In May 2020, Italy reported a decrease in new infections. The CDS market also calmed down, and spreads declined in May and June 2020. In contrast, credit rating agencies had relatively mild reactions during the COVID-19 outbreak. Fitch cut Italy's credit rating to BBB-minus on April 28, 2020, reflecting the impact of the pandemic outbreak.\(^2\) On May 8, 2020, Moody's kept Italy's outlook at "stable" and the rating at Baa3, while DBRS Morningstar changed Italy's trend to "negative" from "stable".\(^3\) Brazil has also suffered severely from the outbreak of COVID-19. The novel coronavirus is believed to have been first confirmed in Brazil on February 25, 2020. Different from those for Italy, the confirmed case data from Roser et al. (2020) indicate that the pandemic had not passed its peak at the time that this paper was being prepared, as illustrated in Fig. 6. Brazil's sovereign CDS prices rose sharply in March 2020, even when the number of daily new confirmed cases in Brazil was low. There are several possible reasons for this phenomenon. The COVID-19 pandemic is a global public health crisis, and outbreaks in other countries may have also influenced investors' positions on Brazil's sovereign CDS. Moreover, the low number of daily new confirmed cases may be due to the initial lack of testing for the novel coronavirus. S&P Global Ratings downgraded its outlook on Brazil's sovereign debt to "stable" on April 6, 2020,\(^4\) and Fitch downgraded its outlook on Brazil's credit rating to "negative" from "stable" on May 5, 2020.\(^5\)

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\(^2\) Source: Reuters, https://reut.rs/3f6Od9.

\(^3\) Source: Reuters, https://reut.rs/3fw4N5.

\(^4\) Source: Reuters, https://reut.rs/3B7Gq1R.

\(^5\) Source: Reuters, https://reut.rs/3fuvOt1.
4.2. Methodology

Under the AFI model (3.8)-(3.9) with resolvent (3.15), we calibrate the Hurst parameter $H$ for the sovereign CDS spreads of the aforementioned two economies during the COVID-19 period in 2020. The estimation procedures are as follows. We adopt a rolling window method, and the Hurst parameter for each month is calibrated on the most recent 78 weeks (1.5 years) of data points, and data from the current month are inclusive. For each month, we minimize the squared pricing errors over the weekly CDS quotation data with maturity $M = 5$ years; that is,

$$
\sum_{t=0}^{77\Delta} \left( \hat{S}_t(M) - S_t(M) \right)^2,
$$

where $t = 0, \Delta, \ldots, 77\Delta$ correspond to the observed weekly CDS quotations and $\Delta = 5/252$ represents the weekly interval. $\hat{S}_t(M)$ is the model implied CDS spread and $S_t(M)$ is the market observed data. Suppose that the protection premium is paid semiannually and formula (3.4) for $S_t(M)$ is adopted. We use 78 weeks of data mainly due to the trade-off between computational burden and estimation accuracy. In this paper, we follow Pan and Singleton (2008) and assume a deterministic constant risk-free rate $r$, which works reasonably well compared with more sophisticated stochastic risk-free rate for a low interest rate environment. The recovery rate $\delta$ is set to 0.4. CDS contracts with 5-year maturity are usually the most liquid credit derivatives in the market; see Pan and Singleton (2008); Longstaff et al. (2011). We focus on the univariate model for the default rate, which is sufficient. More technical details are given in the Appendix C. The calibration error is usually less than 0.1 bps and the real value is often around 100 bps.

Notably, we find that the calibrated $H$ is around 0.6 for most periods under the exact exponential transform formula (3.18) in Theorem 3.2. It is much smaller than the direct estimation result on CDS spreads but comparable to that using the log-return of CDS spreads. We have two explanations for this observation. First, the default intensity model is defined under the risk-neutral measure $Q$, which may have a different long-memory feature from the CDS spreads observed in the physical measure $P$. Second, $R/S$ analysis and the AFI model are distinct, which may introduce systematic estimation differences.

Note that when $H = 1/2$, the exponential transform formula (3.18) reduces to the classic Markovian form. With $f = -1$ and canceling the integral starting from 0 to $t$, it is given by

$$
\mathbb{E}^Q \left[ \exp \left( \phi \lambda T - \int_0^T \lambda_s ds \right) \bigg| \mathcal{F}_t \right] = \exp \left( k \theta \int_0^{T-t} \psi(s) ds + \psi(T-t) \lambda_s \right),
$$

where $\psi$ is the solution to (3.16) with $K = \text{id}$, identity function. The calibration with (4.2) is usually more efficient than (3.18), which is non-Markovian and depends on the past whole path of $\lambda$. Since $H$ is not far from 1/2 based on the calibration results with the exact (3.18), we propose the following approximation when $H$ is around 1/2:

$$
\mathbb{E}^Q \left[ \exp \left( \phi \lambda T - \int_0^T \lambda_s ds \right) \bigg| \mathcal{F}_t \right] \approx \exp \left( k \theta \int_0^{T-t} \psi(s) ds + \psi(T-t) \lambda_s \right),
$$

where $\psi$ is the solution to (3.16) with $K(t) = \frac{H-1/2}{|H+1/2|^2}$. This approximation has several advantages. The calibration with (4.3) is much faster than the use of (3.18). When $H > 1/2$, the exact formula (3.18) still depends on the Brownian motion $W^D_t$ via $\xi(t)$, so that the calibration with it usually relies on a computationally intensive simulation. When $H < 1/2$, Abi Jaiber et al. (2019, Theorem 4.5) shows that $V_t$ has an alternative form as (4.8), which has several more terms converging to zero for $H \rightarrow 1/2$ than our approximation. It also demonstrates another motivation for the approximation. In addition, we are interested in the trend of $H$ and whether $H > 1/2$ or $H < 1/2$. The exact value of $H$ becomes less

Fig. 6. CDS spreads and daily new COVID-19 cases in Brazil from January 1, 2020 to June 26, 2020. The red vertical dashed line shows the first cases reported on February 25, 2020. The green dash-dotted line represents the economic stimulus package announced by the Brazilian government on March 26, 2020. CDS price data are from Bloomberg. Daily case data are from the online source of Roser et al. (2020).
important. Therefore, we strike the balance between accuracy and computational efficiency through the approximation.

We, however, fully recognize the approximation error. To provide the convergence rate of our approximation to the exact value for $H \to 1/2$, we consider $H > 1/2$ and $H < 1/2$ separately. Let $| \cdot |$ be the modulus of complex numbers. To highlight the dependence on Hurst parameter, denote $\psi(\cdot; H) := \psi(\cdot)$ and $\xi(t; H) := \xi_t(\cdot)$. Given $0 \leq t < T$, define the difference between the modulus of the exact value and our approximation as

$$U_t := \left| \mathbb{E}^{Q} \left[ e^{j\theta \int_{0}^{t} \lambda_t ds} | F_t \right] - e^{j\theta \int_{0}^{t} \psi(s;H)ds + \psi(T-t;H)\xi_t} \right|.$$  \hfill (4.4)

For $H > 1/2$, consider

$$\delta_{R,H} := \left\| \int_{0}^{t} |R_t(u) - ke^{-\kappa u}| du \right\| \hfill (4.5)$$

$$\delta_{\psi,H} := \max \left\{ |\psi(T - u; H) - \psi(T - u; \frac{1}{2})|, \right. \hfill (4.6)$$

$$\left. \int_{0}^{T} |\psi(T - u; H) - \psi(T - u; \frac{1}{2})| du, \right.$$ 

$$\int_{0}^{T} \left| \psi^2(T - u; H) - \psi^2(T - u; \frac{1}{2}) \right| du \right\}$$

where the resolvent $R_t(\cdot)$ is given in (3.15) and $\psi(s; \frac{1}{2})$ is the solution to (3.16) with $H = 1/2$. $\delta_{R,H}$ and $\delta_{\psi,H}$ measure the convergence speeds of $R_t(\cdot)$ to $ke^{-\kappa u}$ and $\psi(T-t; H)$ to $\psi(T-t; 1/2)$, respectively. Although explicit forms of $\delta_{R,H}$ and $\delta_{\psi,H}$ are unavailable, they can be evaluated numerically with accuracy. Thus, we build the convergence result based on $\delta_{R,H}$ and $\delta_{\psi,H}$ in Theorem 4.1. The proof is given in Appendix B.

Theorem 4.1. Given $0 \leq t < T$. Suppose assumptions in Theorem 3.2 hold. Consider $f = -1$ and $\phi = 10$. $\omega \in \mathbb{R}$. For $H > 1/2$ and $H$ sufficiently close to $1/2$, there exists a constant $c > 0$ such that

$$\mathbb{E}^{Q}[|U_t|] \leq c \sqrt{e^{\delta_{R,H} + \delta_{\psi,H}} - 2e^{-\delta_{R,H} + \delta_{\psi,H}} + 1} = O(\sqrt{\delta_{R,H} + \delta_{\psi,H}}). \hfill (4.7)$$

When $H < 1/2$, $Y_t$ in (3.17) can be expressed as (Abi Jaber et al., 2019, Theorem 4.5),

$$Y_t = \kappa \theta \int_{0}^{t} e^{\sum_{k=0}^{\infty} \Delta_k f * \lambda_k t} + \int_{0}^{t} \Delta_k \psi * L(\xi_t + d\pi_t(\lambda_t \xi_t), \hfill (4.8)$$

where

$$h = T - t, \quad \Delta_k f(t) = f(t + h), \quad L(dt) = \frac{t^{-\eta}}{\Gamma(1 - \alpha)} dt,$$

$$\pi_t = \Delta_0 \psi * L - \Delta_0 (\psi * L). \hfill (4.9)$$

Similarly, we have the following convergence result.

Theorem 4.2. Given $0 < t < T$. Suppose assumptions in Theorem 3.2 hold. Consider $f = -1$ and $\phi = 10$. $\omega \in \mathbb{R}$. For $H < 1/2$ and $H$ sufficiently close to $1/2$, there exists a constant $c > 0$ such that

We consider the conclusion and p-value for calibrated Hurst parameter from January 2019 to June 2020 shown in Fig. 7.

|            | Mann-Kendall test | Pettit Test |
|------------|-------------------|-------------|
| UniCredit  | decreasing (0.006) | true change point (0.00115) |
| Brazil     | no trend          | true change point (0.006635) |

$$\mathbb{E}^{Q}[|U_t|] \leq c \sqrt{e^{\delta_{R,H} + \delta_{\psi,H}} - 2e^{-\delta_{R,H} + \delta_{\psi,H}} + 1} = O(\sqrt{\delta_{R,H} + \delta_{\psi,H}}), \hfill (4.10)$$

with $\pi_t(\delta t, \delta\pi_t, H, \delta\pi_t, H)$ defined in (4.9), (8.7), and (8.9). $\delta\pi_t$ and $\delta\pi_t, H$ converge to zero when $H \to 1/2$.

4.3. Empirical results

4.3.1. COVID-19

We first calibrate $H$ for each month from January 2019 to June 2020, 18 months in total. However, due to the missing Italy’s sovereign CDS quotation data in 2019 from Bloomberg, we are forced to consider the CDS data of UniCredit, Italy’s biggest bank, as a proxy. It is observed that Italy’s sovereign CDS and UniCredit’s CDS co-move and have almost the same trend in Fig. 8. Practitioners also often compare CDS data for these two identities. Fig. 7 illustrates the estimated Hurst parameter for each month from January 2019 to June 2020. In 2019, the Hurst parameters for both UniCredit and Brazil were larger than 0.5, indicating a clear long-memory characteristic during that period. In addition, UniCredit’s CDS has a stronger long-memory property than Brazil’s CDS during 2019. However, the Hurst parameter $H$ for UniCredit is more volatile. CDS prices in 2017 to 2019 had fewer fluctuations compared with those in 2020, as shown in Fig. 8. Indeed, the movements in CDS prices during the first half of 2020 are of almost the same order of magnitude as the total changes in the previous three years.

When the COVID-19 pandemic occurred in 2020, the calibrated Hurst parameters were significantly smaller. After February 2020, when the COVID-19 pandemic continued to spread, the corresponding Hurst parameters were less than 0.5 for both Brazil and UniCredit, showing a short-memory pattern. This reveals the significant impact of COVID-19 on CDS spreads and default risk on sovereign or corporate debts. We add two statistical tests to support the conclusions. The Mann-Kendall non-parametric test is used to determine whether a time series has an upward or downward trend (Mann, 1945). The non-parametric test proposed by Pettitt (1979) is to test a shift in the central tendency of a time series. Results in Table 6 show that the Mann-Kendall test reveals the decreasing trend of $H$ calibrated from UniCredit, while the Pettitt test detects the existence of true change points in both calibrated Hurst parameter trends.

Investors were concerned about the uncertainty effect of COVID-19 on the economy, making little reference to the historical credit performance, leading to the shift from the long-memory to the short-memory regime. In 2019, CDS prices for Brazil, Italy, and UniCredit showed a declining trend. In other words, investors would have expected a lower default risk of sovereign debts for Brazil and Italy if there had been no coronavirus outbreak. In the next two sections, we provide a quantitative comparison of the COVID-19 recession, Eurozone crisis, and the 2008 subprime mortgage crisis.
Fig. 7. Calibrated Hurst parameter from January 2019 to June 2020. For each month, we use 18 months of weekly data to estimate the Hurst parameter $H$. The data for the current month are inclusive.

Fig. 8. CDS spreads from January 1, 2017 to June 26, 2020. Weekly data are downloaded from Bloomberg. The dashed line for Italy indicates the period when data are missing, and solid lines indicate complete data. CDS data for UniCredit and Italy show clear co-movement behavior.

4.3.2. Eurozone crisis

In Fig. 9 (see also Table 7), we plot the calibrated Hurst parameters for Brazil, Italy, and UniCredit from January 2011 to February 2012. The estimation procedures are the same as in the previous subsection. Several insights can be drawn from the calibration results.

First, because the CDS data for both Italy and UniCredit are complete during this period, we conduct calibrations for both time series. The Hurst parameters for Italy and UniCredit show consistent movement for most of the period. This result supports our previous method of considering UniCredit’s CDS as a proxy for Italy’s sovereign CDS when there are missing data. As Italy’s biggest lender, it is reasonable that common underlying factors drive the CDS spreads.

Table 7

| Country   | Trend     | p-value     |
|-----------|-----------|-------------|
| UniCredit | increasing| (0.0086)    |
| Italy     | increasing| (0.016)     |
| Brazil    | no trend  |             |

6 See, for example, https://reut.rs/2T11tZE.
default risk for UniCredit and Italy. It also validates the capability of our calibration procedures to detect the trend of the long-memory property when the two time series have similar realized paths.

Second, the Hurst parameter for Brazil fluctuates around 0.55 ~ 0.6 and has no obvious trend during this time horizon supported by the Mann-Kendall test. This is consistent with Brazil’s CDS prices depicted in Fig. 10, which also have no significant trends. As expected, the Eurozone crisis has a limited impact on the default risk of Brazil’s sovereign debt. Overall, the debt crisis in Europe is relatively regional, while the COVID-19 pandemic is a global crisis with impacts on default risk for more countries.

Third, the results for Italy and UniCredit show an upward trend in the Hurst parameter around 2011. Mann-Kendall test rejects the null hypothesis and indicates increasing trends of both. As Fig. 10 shows, Italy and UniCredit’s CDS prices surge at the end of 2011. The upward trend for CDS prices started in May 2011. The large Hurst parameter $H$ shows the strong persistence in the innovations of default intensities.

In Figs. 7 and 9, the Hurst parameter for Brazil is usually smaller than that of Italy or UniCredit. This may be related to the differences in fiscal policy and economic growth between Brazil and Italy. A natural research direction is to explore this cross-sectional difference in long-memory property; however, this is
4.3.3. 2008 subprime mortgage crisis

Credit derivatives played a pivotal role in the 2008 subprime mortgage crisis, which involved the bankruptcies of several major financial institutions. The outstanding notional value of CDS contracts was at $61.2 trillion at the end of 2007 (Aldasoro and Ehlers, 2018). However, the changes in CDS spreads were relatively small before the crisis; see Fig. 11. Jorion and Zhang (2007) attribute this to the lack of sufficient new information for changing CDS prices. However, it may also indicate the underestimation of counterparty credit risk. When we conduct the calibration with the 78 weeks of data from August 2006 to January 2008, the estimated Hurst parameter $H$ is near 1, as shown in Fig. 12. This indicates extremely strong persistence in CDS quotations before the financial crisis.\footnote{The approximation in (4.3) may not be accurate enough when $H$ is near 1. However, it still provides a rough estimate and indicates strong persistence.} As the subprime mortgage crisis evolved, some financial institutions with considerable exposure to mortgage-related products faced liquidity issues or even filed for bankruptcy. Italy and Brazil’s CDS became volatile and climbed during the first quarter of 2008. The

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**Fig. 11.** CDS spreads from January 1, 2006 to December 31, 2008. The vertical green dash-dotted line indicates March 18, 2008, when the Fed cut the federal funds rate by 75 bps and allowed Fannie Mae & Freddie Mac to take on another $200 billion in debt. The red dashed line indicates September 15, 2008, when Lehman Brothers filed for bankruptcy. Weekly data source: Bloomberg.

**Fig. 12.** Calibrated Hurst parameter from January 2008 to December 2008. The same rolling window method is adopted, with a window size of 78 weeks.
actions taken by the Fed on March 18, 2008, initially calmed the global financial market. Italy and Brazil’s CDS dropped slightly during April and May 2008. However, we observe a steep decline in the Hurst parameter after February 2008. This pattern is similar to the results for the first half of 2020, as shown in Fig. 7. However, the estimated $H$ for 2008 is still larger than 0.5. We note that the long-memory characteristic in Italy and Brazil’s CDS markets was reduced after 2008. The calibrated Hurst parameter was not close to 1 in either 2011 or 2020.

After Lehman Brothers filed for bankruptcy on September 15, 2008, CDS spreads for Italy and Brazil soared to historical highs. Actions taken by the Fed could not prevent this great financial crisis. The estimated $H$ gradually climbed due to this strong upward trend in CDS spreads. This phenomenon is consistent with the estimation results for Italy during the Eurozone crisis. When Italy and UniCredit’s CDS spreads rose during the second half of 2011, the Hurst parameter was larger. For these two periods, investors had strong expectations of much higher default risk, which increased persistence in CDS spread innovations.

We do not have significant evidence that Italy had a higher Hurst parameter than Brazil during 2008. Their Hurst parameters are close to each other, in contrast with the distinct results observed in 2011–2012 and 2019–2020. This is reasonable, because the 2008 financial crisis is global and as Fig. 11 shows, these two time series roughly move together during 2007–2008 under the common shock from the US financial market. The CDS markets have been more efficient since the 2008 financial crisis. Italy and Brazil also faced distinct deficits and economic outcomes during 2011–2012 and 2019–2020, resulting in different long-memory characteristics.

4.3.3.4. Comparison results

The economic recessions caused by COVID-19 are different from those of previous crises. Recessions stemming from substantial deficits or house price bubbles are more tied to the economic system, so are more economically predictable and persistent. In contrast, COVID-19 related uncertainty is much well beyond the knowledge of financial experts. Investors can react and sometimes even overreact for a much longer period, such as the sharp rises and falls observed in Fig. 5. Thus, the persistence was reduced in credit markets, represented by the Hurst parameter $H$.

In contrast, the Eurozone crisis presents the strong persistence in the innovations of default intensities. One possible explanation is that the debt crisis is a financial issue and therefore somehow more economically predictable than the pandemic. Investors have confidence in applying historical data to project the movements of CDS prices. However, predicting and controlling of a pandemic are orthogonal to financial knowledge, making this situation more uncertain and unpredictable for investors. Investors no longer fully rely on the history of CDS market movements, and therefore the values of $H$ are reduced for the COVID-19 pandemic.

Comparing the movements in CDS spreads during the first half of 2020 with those in the second half of 2011 and the second half of 2008, the movements during the Eurozone crisis and the 2008 financial crisis have significantly larger orders of magnitude. A similar conclusion is derived for U.S. major banks’ CDS prices in 2008 and 2020; see Brunnermeier and Krishnamurthy (2020, Figure 2). Brunnermeier and Krishnamurthy (2020) conclude that it is too early to state that the U.S. is experiencing a financial crisis. However, Aldasoro et al. (2020) empirically find that banks’ performance in equity and debt markets during the pandemic has been on a similar path since the bankruptcy of Lehman Brothers in 2008; see Graph 1 therein. Comparing 2020 with 2008, our results for the long-memory characteristic indicate that the first half of 2020 is similar to the period of January 2008 to June 2008. However, the Hurst parameters are much smaller in 2020. Another distinction is the markets’ reactions to economic stimulus policies. In Figs. 5, 6 and 11, the vertical green lines indicate the actions of Italy, Brazil, and the Fed. Unfortunately, CDS markets seem to have shrugged off the economic stimulus measures adopted by Italy and Brazil in 2020. In contrast, CDS markets calmed down in April and May 2008 after the Fed’s actions. Overall, investors may be uncertain about the effectiveness of Italy and Brazil’s economic policies in response to COVID-19 and are waiting to see the outcomes.

Optimistically, it is possible that the coronavirus pandemic will end when a vaccine or effective therapy is available. People can expect a relatively quick economic recovery once the pandemic is past. However, lack of experience in handling the chaos of the pandemic and concerns about subsequent waves of coronavirus infection have reduced the persistence and stability of credit markets. Our studies reveal distinct features in the current COVID-19 crisis compared with previous crises. Therefore, governments and companies may have to adopt different strategies than in previous recessions to avoid the cluster of default events and bankruptcies; see also recent discussions of and proposals for government policies in Brunnermeier and Krishnamurthy (2020); Bartik et al. (2020); Duffie (2020); Nozawa and Qiu (2020); Skeel (2020); Hadad et al. (2021).

4.4. Out-of-sample forecasting

Previous empirical studies discuss in-sample calibrations. The calibration errors are very small, usually less than 0.1 bps, which shows the good fit of our model. To make our findings more convincing, we add an out-of-sample forecasting comparison. Based on our AFI model with flexible range dependence and Markovian model with fixed $H = 1/2$, we conduct one-step prediction on the previous CDS spread dataset introduced in Section 2.4. Concrete information on this dataset is presented in Table 4. By Abi Jaber et al. (2019, Theorem 4.5), when $H = 1/2$,

$$
\xi(t) = \mathbb{E}^Q [\lambda_t | F_t] = \theta \int_0^{1-\lambda_t} R_k(s) ds + \frac{1}{\kappa} R_k(s-t) \Delta_t, \quad (4.11)
$$

and $R_k(t)$ given in (3.15) is reduced to $ke^{-kt}$. Since $H$ is not far from 1/2, we use (4.11), where $R_k$ is (3.15) with $H \in (0, 1)$ to approximate the one-step forward default intensity $\xi(t+\Delta)$ in our AFI model forecasting.

For all 35 CDS quotations, we first use 50 weeks of historical data, where $T = 49\Delta$, $\Delta = 5/252$, and $M = 5$ years, for calibration by the same method mentioned in Section 4.1. The initial values of $H$ are adjusted according to the economic situation. The calibrated parameters are applied to calculate $\xi(t+\Delta)$ for each quotation. Next, the forecasting for the following week spread $\hat{S}_{t+\Delta}(M)$ is performed. In terms of performance measurement, we calculate the mean absolute error (MAE) and mean ratio of error to price (MRE) to summarize the effects. The unit of error is basis points (bps), the same as for CDS spreads. The results are shown in Table 6, where Fractional means flexible range dependence and Markovian means fixed $H = 1/2$.

Due to the COVID-19 crisis, almost all of the CDS spreads increased rapidly from the end of February in 2020. Accordingly, the errors are also much larger than before. Although the pandemic impaired the predictive ability of the models, in most cases, our fractional model has superior performance in out-of-sample forecasting. Therefore, taking LRD into account, we could have a more accurate forecast of CDS spreads and ameliorate insurance investments in credit risk market.
Table 8
One-step forecasting comparison between the AFI model and Markovian model.

| Period      | Predicted Day | Fractional MAE | MRE  | Markovian MAE | MRE  |
|-------------|---------------|----------------|------|---------------|------|
| 2019-02-22 to 2020-01-31 | 2020-02-07   | 2.9070        | 0.0538 | 3.1320        | 0.0585 |
| 2019-03-01 to 2020-02-07 | 2020-02-14   | 1.9766        | 0.0406 | 3.2017        | 0.0513 |
| 2019-03-08 to 2020-02-14 | 2020-02-21   | 3.2859        | 0.0623 | 3.1218        | 0.0572 |
| 2019-03-15 to 2020-02-21 | 2020-02-28   | 16.5266       | 0.2276 | 17.1674       | 0.2321 |
| 2019-03-22 to 2020-02-28 | 2020-03-06   | 14.5442       | 0.1240 | 15.9133       | 0.1419 |

5. Conclusion

This paper investigates the impact of the COVID-19 outbreak on credit risk from the perspective of the long-memory characteristic. We propose a novel affine reduced-form model for credit risk with flexible range dependence. Using classic $R/S$ analysis and the new AFI model, we thoroughly examine the long-range dependence in credit risk by considering credit ratings, CDS prices, and risk-neutral default intensities. We find that long memory has been weakened under the pandemic. In the case studies of Italy and Brazil, we contrast the estimated Hurst parameters during the COVID-19 crisis period with those of the Eurozone crisis and the 2008 subprime mortgage crisis. There are similarities and discrepancies in the trend of calibrated $H$ for these crises. We identify the hidden patterns in financial markets and shed light on the use of historical credit information for forecasting and pricing credit. Moreover, we show the good performance of our AFI model in both in-sample calibrations and out-of-sample predictions. Interesting future questions include cross-sectional differences in long-range dependence.

Declaration of competing interest

We declare that there is no competing interest.

Acknowledgements

The authors would like to thank the anonymous referees and the editors for their careful reading and valuable comments, which have greatly improved the manuscript. Jie Yin is grateful to participants for their insightful comments at the virtual 24th International Congress on Insurance: Mathematics and Economics, July 2021. H.Y. Wong acknowledges the Research Matching Grant (RMG project code: 8601495) received from the Research Grants Council of Hong Kong.

Appendix A. Details of the CDS spread data

Table A.9
35 CDS tickers for the CDS spread data described in Table 4.

| CSUM1J5 currency | CVABA1J5 currency | CVW1E5 currency | CRW1E5 currency | CTYS1E5 currency |
|------------------|-------------------|-----------------|-----------------|-----------------|
| CSOF1J5 currency | CSNE1E5 currency  | CPEU1E5 currency | CNEP1E5 currency | CMTO1E5 currency |
| CMSL1J5 currency | CMAR1J5 currency  | CLF1T1E5 currency | CGL1S1J5 currency | CHEI1E5 currency |
| CTB1E5 currency  | CRB1H2E5 currency | CTET1P1J5 currency | CT370372 currency | CSEI1E5 currency |
| CNEC1J5 currency | COK1J5 currency   | CJOR1J5 currency | CPM1U5 currency  | CJEI1E5 currency |
| CCONT1E5 currency| CCNGN1J5 currency | CCGT1E5 currency | CAXP1U5 currency | CACOM1J5 currency |

Appendix B. Proof of results

B.1. Proof of Theorem 3.2

Proof. The proof is inspired by Abi Jaber et al. (2019, Theorem 4.3) and Gatheral and Keller-Ressel (2019, Theorem 2.6). Applying the Itô formula to the AFI model (3.8) on time $t$, we obtain

$$d\xi_t(s) = -\frac{1}{k} R_k(s-t) \sigma \sqrt{\lambda_t} dW_t^Q.$$  \hspace{1cm} (B.1)

Then

$$dY_t = \frac{\phi}{k} R_k(T-t) \sigma \sqrt{\lambda_t} dW_t^Q + (f(T-t))\lambda_t dt - f(T-t)\lambda_t dt$$

$$+ \frac{1}{2} \left( \int_t^T f(T-s) \frac{1}{k} R_k(s-t) \sigma \sqrt{\lambda_t} dW_t^Q ds - \frac{\phi}{k} R_k(T-t) \sigma \sqrt{\lambda_t} dW_t^Q ds \right)$$

$$+ \frac{1}{2} \left( \int_t^T \sigma^2 \psi^2(T-s) \frac{1}{k} R_k(s-t) \sigma \sqrt{\lambda_t} dW_t^Q ds \right).$$  \hspace{1cm} (B.2)

The second equality applies the stochastic Fubini theorem (see Veraar (2012)) under Assumption (3.13) and the integrability of $\psi$. From Abi Jaber et al. (2019, Lemma 4.4), being completely monotone of kernels is not required, and we have

$$\frac{\phi}{k} R_k(T-t) + \int_t^T f(T-s) + \frac{\sigma^2}{2} \psi^2(T-s) \frac{1}{k} R_k(s-t) ds$$

$$= \psi(T-t).$$  \hspace{1cm} (B.3)

Therefore,

$$dY_t = -\frac{1}{2} \sigma^2 \psi^2(T-t) \lambda_t dt + \sigma \psi(T-t) \sqrt{\lambda_t} dW_t^Q.$$  \hspace{1cm} (B.4)
This implies that \(\exp(Y_t)\) is a local martingale. If \(\exp(Y_t)\) is a true martingale, then the transform formula (3.18) holds since \(Y_T = \phi\lambda_T + (f * \lambda(T))\).

**B.2. Proof of Theorem 4.1**

**Proof.** Denote the exact value and our approximation as

\[
e^l := \mathbb{E}^Q \left[ e^{\phi\lambda_T - \int_0^T \lambda_t ds} \right]_{T_2}
\]

\[
e = \exp \left[ \phi \xi_t(T; H) - \int_0^T \xi_t(s; H) ds \right]
\]

\[
+ \frac{1}{2} \int_0^T \sigma^2 \psi^2(T - s; H) \xi_t(s; H) ds,
\]

\[
e^{ll} := \mathbb{E}^Q \left[ e^{\kappa\theta - \int_0^T \lambda_t ds} \right]_{T_1}
\]

\[
e = \exp \left[ \kappa \theta \int_0^T \psi(s; H) ds + \psi(T - t; H) \lambda_t \right].
\]

respectively. Let \(e^l := \exp \left[ \kappa \theta - \int_0^T \psi(s; H) ds + \psi(T - t; \frac{1}{2}) \lambda_t \right].\) In other words, we have I, II and III defined as the logarithm on the right-handed side of the respective terms. A bound for the approximation error can then be expressed as follows.

\[
\mathbb{E}^Q[I_1] = \mathbb{E}^Q[\mathbb{E}^Q(e^{1/2} - 1)]/2\mathbb{E}^Q[(e^{1/2} - 1)^2]/2 + \mathbb{E}^Q[\mathbb{E}^Q(e^{1/2} - 1)^2]/2.
\]

By Abi Jaber et al. (2019, Lemma 6.3), \(\Re[\psi(\cdot; 1/2)] \leq 0\) so that \(\mathbb{E}^Q[e^{1/2}]\) is bounded. When \(H = 1/2, e^l = e^{ll}.\) Otherwise,

\[I - II
\]

\[= \phi \xi_t(T; H) - \phi \xi_t(T; 1/2) + \int_0^T \xi_t(s; 1/2) - \xi_t(s; H) ds
\]

\[+ \frac{1}{2} \int_0^T \psi^2(T - s; H) \xi_t(s; H) - \psi^2(T - s; 1/2) \xi_t(s; 1/2) ds
\]

Denote \(\Delta_R(u) := R_e(u) - \kappa e^{-x^2u}\) and \(\Delta_{\Delta^2}(u) := \psi^2(u; H) - \psi^2(u; 1/2).\) By definition (3.8)-(3.9),

\[\xi_t(s; H) - \xi_t(s; 1/2)
\]

\[= (\theta - \mu) \int_0^s \Delta_R(u) du + \int_0^s \frac{\sigma^2}{K} \Delta_R(s - u) \lambda_u W^Q_u.
\]

Then the last term of (I - II) becomes

\[\frac{\sigma^2}{2} \int_0^T \int_0^T \left[ \psi^2(T - s; H) \xi_t(s; H) - \psi^2(T - s; 1/2) \xi_t(s; H) ds
\]

\[+ \frac{\sigma^2}{2} \int_0^T \left[ \psi^2(T - s; 1/2) \xi_t(s; H) - \psi^2(T - s; 1/2) \xi_t(s; 1/2) ds
\]

\[= \frac{\sigma^2}{2} \int_0^T \Delta_{\Delta^2}(T - s) \xi_t(s; H) ds.
\]

Let \(G_t := \frac{\sigma^2}{2} \int_0^T \Delta_{\Delta^2}(T - s) \xi_t(s; H) ds.\) Then,

\[dG_t = -\frac{\sigma^2}{2} \Delta_{\Delta^2}(T - t) \lambda_t dt
\]

\[+ \frac{\sigma^2}{2} \int_0^T \Delta_{\Delta^2}(T - s) \frac{1}{K} R_e(s - t) \sigma \sqrt{\lambda_{i}} dW^Q_i ds.
\]

\[= G_0 - \frac{\sigma^2}{2} \int_0^T \Delta_{\Delta^2}(T - u) \lambda_u du
\]

\[+ \int_0^T \frac{\sigma^2}{2} \Delta_{\Delta^2}(T - s) \frac{1}{K} R_e(s - u) \sigma \lambda_u dW^Q_u.
\]

\[= G_0 - \frac{\sigma^2}{2} \Delta_{\Delta^2}(T - u) \lambda_u du + \int_0^T b(\Delta_{\Delta^2}, u) \sqrt{\lambda_u} dW^Q_u,
\]

where \(G_0\) is a constant. Note that \(R_e(\cdot)\) is bounded on \([0, T]\) when \(H > 1/2.\) Thus, \(\Re[\Delta_{\Delta^2}(u)] \leq c\delta_{\psi, H}\) for some \(c > 0,\) which may change from line to line. Furthermore,

\[\Re[\mu - II]
\]

\[= \Re[G_0] - \int_0^T \frac{\sigma^2}{2} \Re[\Delta_{\Delta^2}(T - u)] \lambda_u du
\]

\[+ \int_0^T \Re[b(\Delta_{\Delta^2}, u)] \sqrt{\lambda_u} dW^Q_u
\]

\[+ \int_0^T \Re[\frac{\sigma^2}{2} \psi^2(T - s; 1/2) - 1] \left[ \theta - \mu \right] \int_0^s \Delta_R(u) du
\]

\[+ \int_0^T \frac{\sigma^2}{K} \Delta_R(s - u) \lambda_u dW^Q_u ds
\]

\[\leq c(\delta_{\psi, H} + \delta_{\psi, H}) + c\delta_{\psi, H} \int_0^T \lambda_u du
\]

\[+ \int_0^T F(\Delta_R, \Delta_{\Delta^2}, u) \lambda_u dW^Q_u,
\]

where \(F(\Delta_R, \Delta_{\Delta^2}, u) := \Re[b(\Delta_{\Delta^2}, u)] + \int_0^T \Re[\frac{\sigma^2}{2} \psi^2(T - s; 1/2) - 1] \frac{\sigma^2}{K} \Delta_R(s - u) du\) satisfying \(F(\Delta_R, \Delta_{\Delta^2}, u) \leq c(\delta_{\psi, H} + \delta_{\psi, H}).\) Thus,

\[\mathbb{E}^Q[\text{e}^{\mu - II}] \leq \text{e}^{(\delta_{\psi, H} + \delta_{\psi, H})} \left[ \mathbb{E}^Q[\text{e}^{(\delta_{\psi, H} + \delta_{\psi, H})} \int_0^T \lambda_u du] \right]^{1/2} \times
\]

\[\left\{ \mathbb{E}^Q \left[ \exp \left( - \int_0^T \frac{1}{2} F^2(\Delta_R, \Delta_{\Delta^2}, u) \lambda_u du \right) \right] \right\}^{1/2}
\]

\[+ \int_0^T F(\Delta_R, \Delta_{\Delta^2}, u) \lambda_u dW^Q_u \right\}^{1/2}
\]

\[= 32
\]
Similarly, we have
\[
\mathbb{E}_Q\left[ \frac{1}{2} \right] = \mathbb{E}_Q\left[ e^{-\text{Re}[\pi_{[1-\Theta]]}] \right] \leq e^{c(\delta_H, \delta_H + \delta_{\phi,H})}.
\]
As \( \mathbb{E}_Q[\pi_{[1-\Theta]]}] \leq \mathbb{E}_Q[\pi_{[1-\Theta]]}] \),
\[
e^{-c(\delta_H, \delta_H + \delta_{\phi,H})} \leq \mathbb{E}_Q\left[ e^{-\text{Re}[\pi_{[1-\Theta]]}] \right] \leq e^{c(\delta_H, \delta_H + \delta_{\phi,H})}.
\]
In the same way, we have
\[
e^{-c(\delta_H, \delta_H + \delta_{\phi,H})} \leq \mathbb{E}_Q\left[ e^{2\text{Re}[\pi_{[1-\Theta]]}] \right] \leq e^{c(\delta_H, \delta_H + \delta_{\phi,H})}.
\]
Therefore,
\[
\left( \mathbb{E}_Q[\pi_{[1-\Theta]]}] - 1 \right)^2 \leq \sqrt{e^{c(\delta_H, \delta_H + \delta_{\phi,H})}} - 2e^{-c(\delta_H, \delta_H + \delta_{\phi,H})} + 1.
\]
Repeating the steps above, we deduce that
\[
\left( \mathbb{E}_Q[\pi_{[1-\Theta]]}] - 1 \right)^2 \leq \sqrt{e^{c\delta_H}} - 2e^{-c\delta_H} + 1. \quad \Box
\]

B.3. Proof of Theorem 4.2

Proof. Since \( f = -1 \), the exact value and the approximation in (B.6) become
\[
e_{II} = \exp \left[ \zeta_\theta \int_{0}^{T-t} \psi(s; H) ds + (\Delta_H \psi \ast L)(0) \lambda_0 - \pi_H(t) \lambda_0 \right]
\]
\[
+ (d\pi_H \ast \lambda_0) t \right],
\]
\[
e_{III} = \exp \left[ \zeta_\theta \int_{0}^{T-t} \psi(s; H) ds + \psi(T - t; H) \lambda_0 \right],
\]
Then
\[
\mathbb{E}_Q[\pi_{[1-\Theta]]}] = \mathbb{E}_Q[\pi_{[1-\Theta]]}] \frac{1}{2} \left( \mathbb{E}_Q[\pi_{[1-\Theta]]}] - 1 \right)^2 \right)^{1/2}
\]
\[
= \mathbb{E}_Q \left[ \left( e^{c\delta_H} - 2e^{-c\delta_H} + 1 \right)^{1/2} \right].
\]
\[
\mathbb{E}_Q[\pi_{[1-\Theta]]}] \text{ is bounded by } c > 0 \text{ since } \mathbb{E}[\psi(\cdot; H)] \leq 0 \text{ by Abi Jaber et al. (2019, Lemma 6.3).}
\]
To apply Theorem 3.2, define \( \xi \) as the solution to the following Riccati-Volterra equation with constant \( u \) and function \( d\pi_H \):
\[
u := \text{Re}[\Delta_H \psi \ast L(0)] - \psi(T - t; H),
\]
\[
\xi := u + \text{Re}[d\pi_H] - \kappa \xi + \frac{\sigma^2}{2} \xi^2 \ast K,
\]
\[
\delta_{\pi,H} := \upsilon_\lambda \lambda_0 + \int_{0}^{t} \left[ \text{Re}[d\pi_H(s)] \lambda_0 + \kappa (\psi - \lambda_0) \xi(s) \right]
\]
\[
+ \frac{1}{2} \sigma^2 \xi^2 (s) \lambda_0 ds. \quad \Box
\]

When \( H \) is sufficiently close to \( 1/2 \), \( u \) and \( \delta_{\pi,H} \) are sufficiently small. Therefore, by Theorem 3.2,
\[
\mathbb{E}_Q[\text{Re}[\pi_{[1-\Theta]]}]] = e^{\text{Re}[\pi_{[1-\Theta]]}] \lambda_0.
\]
Similarly, define \( \xi' \) with \( u \) replaced by \( 2u \) and \( \text{Re}[d\pi_H] \) replaced by \( 2\text{Re}[d\pi_H] \). And
\[
\xi' := 2u + \int_{0}^{t} \left[ \text{Re}[2d\pi_H(s)] \lambda_0 + \kappa (\psi - \lambda_0) \xi'(s) \right]
\]
\[
+ \frac{1}{2} \sigma^2 \xi'(s) \lambda_0 ds.
\]
Then
\[
\mathbb{E}_Q[\pi_{[1-\Theta]]}] = e^{\text{Re}[\pi_{[1-\Theta]]}] \lambda_0.
\]

The claim follows in the same way as in the proof of Theorem 4.1.

Appendix C. Technical details of calibration

The detailed procedure of calibration is given as follows. The unknown parameters are Hurst parameter \( H \), mean-reversion speed \( \kappa \), volatility of default intensity \( \sigma \), and the whole path of default intensity \( \lambda \). Obviously, we have to consider the path evaluated at a set of discrete points. To calculate the exponential transform formula (3.18) for a specific default intensity path, the Riccati-Volterra equation (3.16) has to be solved numerically. We adopt the Adam's method, detailed in El Euch and Rossenbaum (2019). The integral in (3.18) is then evaluated by Simpson's rule. Therefore, the model implied CDS spread \( \tilde{S}_t(M) \) is available as a function of unknown parameters given above. We minimize the mean-square errors (4.1) by optimization methods given in the software, e.g., least squares in SciPy.

It is well documented that calibration of CDS data is sensitive to the initial values (Ait-Sahalia et al., 2014), Longstaff et al. (2011) point out that the CDS spread \( \tilde{S}_t(M) \) is approximately equal to \( 1 - \delta \lambda_0 \) for many cases. Therefore, we set the initial value for \( \lambda_0 \) as \( \tilde{S}_t(M)/(1 - \delta) \). Another remedy is to have several trials on \( H \) to reduce the calibration errors induced by the choices of the initial values.

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