Large eddy simulations in MHD: the rise of counter-rotating vortices at the magnetopause

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Abstract. A study of the magnetohydrodynamic (MHD) development of coherent structures in compressible, inhomogeneous, mixing layers due to the velocity shear instability is reported. The non-linear evolution of the original vorticity sheet is computed with 3-D large eddy simulations (LES) of temporal mixing layers tailored to represent distinctive conditions at the terrestrial magnetopause. We find that the boundary layer is characterized by the growth of large-scale vortices and becomes a site of mass mixing and enhanced plasma diffusion. In MHD the Lorentz force and its associated baroclinic term, together with the ordinary baroclinic term, and stratified entropy across the mixing layer, conspire to hinder vorticity flux conservation. In our LES the non-conservation of vorticity becomes manifest after ~ one rollover time when in addition to vortices with positive rotation (the same as the original vorticity sheet) other coherent structures with strong negative vorticity also arise, a noteworthy effect examined here. It is found that the vorticity is concentrated in cores of both signs with absolute values ~ 4-5×ωi, (maximum vorticity of the initial shear layer). Concomitant with 3-D vortex stretching, the kinetic helicity also rises at vorticity cores. Furthermore, high temperature occurs in the cores, ~ 3×Ti (magnetospheric temperature) correlated with local density depletion, ~ 0.4×ni (magnetospheric density), while gas and magnetic pressure remain close to surrounding values. The study is intended as a contribution to the understanding of solar wind interaction with the magnetosphere during periods of northward interplanetary magnetic field.

1. Introduction
We report a study of magnetohydrodynamic (MHD) development of vortices in compressible, non-homogeneous boundary layers, due to the velocity shear instability (Kelvin-Helmholtz, KH). Transition layers between flows of different velocity, density, and magnetic fields, are prominent aspects of space plasmas. In a magnetosphere context the magnetopause boundary layer comes to mind separating the stagnant, hot and tenuous magnetosphere plasma threaded by the Earth's magnetic field from the cold and dense magnetosheath plasma flowing past it. We present results of the non-linear dynamics of the KH instability in MHD obtained with large eddy simulations (LES) using a new 3D+t MHD code of an arbitrary Lagrangian - Eulerian and finite volume class (ALE+FV). The code was developed at our Institute (For technical details see [1]). The study assumes periodic boundary conditions, both along, as well as transverse to, the average flow. The MHD simulations show the roll-
off of large-scale vortices in the boundary layer, stretched across the main stream and oriented along the magnetic field lines.

The simulation reveals, among other features, the evolution of pairs of intense counter-rotating vortices of similar strength, which develop from a velocity gradient of much less vorticity all in the same direction. Hot and tenuous plasma is entrained in the core of these vortices. We argue that the rise of counter-rotating vortices in the 3-D simulations is due to the joint presence of non-uniform density and magnetic forces. We also confirm that the addition as an initial disturbance of a trigger velocity field pattern (of sizable amplitude) associated with a fast growing mode (according to linear theory) shortens the time it takes for the KH instability to fully develop, thereby hastening the appearance of large-scale coherent structures.

Fluid dynamics is a significant aspect of our problem, particularly when it deals with mixing of matter. In fluid dynamics a mixing layer is the region between two flows of different velocity, often also of different density (see, e.g., [2], [3]). An important example in a magnetospheric context is the boundary layer, which is often found to have a magnetic field orientation typical of the magnetosphere at the place of observation, but with a plasma population that contains both particles of high energies typical of the magnetosphere, as well as low energy magnetosheath plasma, i.e., a mixture of plasmas from both sides of the layer [4].

![Simulation](image)

**Figure 1.** The linear KH instability for a December 7, 2000, boundary layer model. Growth rate versus wavenumber as normalized quantities, for the optimal $k$ angle for the instability $\phi = 32^\circ$. The dashed line indicates the growth rate and wavenumber of the most unstable mode. The solid line indicates the mode used to start the numerical simulation (see text).

In fluid mechanics a mixing layer is frequently turbulent due to the velocity gradient, which renders it prone to instability. The turbulent layer is characterized by the presence of large-scale vortices, and it becomes a site of enhanced mass diffusion (e.g., [2]). Turbulent flow implies one disordered in space and time, but this specification does not necessarily imply a chaotic behavior in all three dimensions. Mixing layers in ordinary fluids are usually a source of quasi 2-D turbulence. Large-scale vortices arise from the rolling-up of the original vortex sheet that separates motions with different speeds. Likewise, quasi 2-D vortical structures also occur in mixing layers in space plasmas, as shown here and in many other studies (e.g., [5], [6], [7]). However, in MHD, vortex generation depends also on the direction and strength of the magnetic field.
In spite of what the term may seem to imply, fluid turbulence is a deterministic phenomenon, i.e., it is governed by deterministic laws and equations, even though the complexity of the non-linear evolution is such that predictability in practice is lost after a finite time. As the Reynolds number $Re$ increases, direct numerical simulation usually can deal only with the large scale aspects of the flow, and contain errors introduced by numerical schemes, the absence of small scale effects, and imprecise description of initial and boundary conditions. All the errors are amplified by fluid dynamic non-linearities, and after some time the predicted and the actual turbulent flow can be very different. Therefore, as is widely known, unpredictability and randomness appear even in deterministic systems.

Vortices arising in KH--unstable layers have been extensively studied in ordinary fluid dynamics, both in experiments as well as in computations. A similar body of knowledge on the plasma version of the KH instability has grown also in space physics. Many authors provided observational, theoretical, and computational insights. The progress in our understanding of magnetospheric KH can be followed in surveys, for example, [8], [9], [10], [11]. Still, further research on dynamical properties of MHD mixing layers is desirable in order to attain a deeper understanding comparable to that of classical fluids.

In the magnetospheric context computational MHD of the KH instability began with the pioneering work of Miura [12]. Recent studies using computer simulations (e.g., [13], [14]) strive to understand in what manner precisely does KH lead to mass exchange. In [6] a main thrust was to show that KH rolled up vortices can foster mass transfer, and in the same spirit is [13]. However, the sonic and magnetic Mach numbers, $M_s$ and $M_A$, respectively, of our numerical experiments correspond to a physical regime different from that of these works. Our simulation models the near flanks where the flow is still subAlfvenic. Their studies are for more distant sites where the plasma is superAlfvenic. While in our study the magnetic field lines show “rigidity”, in their works the flow can easily bend the magnetic field lines.

Figure 2. Initial configuration of numerical simulation. Panel $a$ (left) gives vorticity contours $\omega = |\omega| \text{sign}(\omega_z) = \text{const.}$ on three planes $Z = \text{const.}$, and on the back of the computational box $X = \text{const.}$; sets of streamlines (black solid lines) on $Z = \text{const.}$ planes are also represented. Panel $b$ (middle) shows (on the same planes as in panel $a$) the initial temperature distribution with $T = \text{const.}$ contours. A set of magnetic field lines (yellow solid lines) is also shown here. Panel $c$ (right) gives the isodensity surface $n = 9 \text{ p cm}^{-3}$ (with a rotated view). The blue cones indicate the velocity field, which shows an initial perturbation as the vorticity field in panel $a$ (see text). The color scales in the panels indicate the intensity of the corresponding scalar quantity. All axes tick marks are in units of Earth radii, $R_E$.

The computational study of a boundary layer MHD flow presented here is modeled with parameters belonging to the duskside magnetopause, derived from data recorded by Cluster 3 on December 7, 2000. The event is in the class of northward pointing interplanetary magnetic fields (IMF), on which more is said in [16]. Cluster was crossing the low latitude boundary layer (LLBL) of the magnetosphere near the dusk terminator. The phenomenology of that event, which developed after the impact of a tangential discontinuity/vortex sheet (TD/VS) on the magnetosphere, was studied in
another paper [15]. We gave there also results of a stability analysis based on input parameters measured in situ. The theoretical study was in agreement with observations interpreted as the growth of surface waves of KH origin. The main physical properties of plasma and fields at the Cluster locale are the subsonic $M_s = 0.57$, and sub-Alfvenic $M_A = 0.70$, flow, and a very small magnetic shear across the boundary layer.

The LES allows us to achieve a better understanding of the boundary layer physics after the onset of the KH instability in the particular event considered, which was characterized by moderate values of $M_s$ and $M_A$, both smaller than one. Because of their relevance to energy, momentum, and possibly also mass transfer into the magnetosphere, KH simulations can be particularly instructive. Summing up: besides being of concern for magnetofluidynamics in revealing aspects of vortex formation in boundary layers, our simulations are significant also for magnetospheric physics.

The plan of the paper is as follows. In section 2 we review briefly the bases and the limitations of LES. We comment on some fluid dynamics and MHD aspects of the problem, and explain the reasons for the non-conservation of the vorticity flux in our computational study. In section 3 we report the initial and boundary conditions for the 3D+t numerical simulation. We give the parameters derived from Cluster data for December 7, 2000, on which the boundary layer model is tailored to. We report also on the linear stability results for the same model. Section 4 shows the results of the numerical experiment. The computations bring forward novel features of the process that broadens, and strain the low latitude boundary layer (LLBL), the outer coat through which external plasma enters the magnetosphere. Section 5 contains a summary and conclusions.

2. LES, MHD aspects, and non-conservation of vorticity flux
The precise details of the KH instability conditions in compressible MHD are intricate, but speaking in general one can say that the boundary layer becomes unstable when the magnetic shear angle across
the layer is small, and/or when the Alfvenic Mach number $M_A = U/V_A$ (where $U$ denotes the velocity of the external flow and $V_A$ is the Alfvén velocity) is large, both factors limiting the stabilizing $B$-forces. The rolling-up of the initial vorticity stratum by the Kelvin-Helmholtz mechanism generates large vortices, and the vorticity of these structures is further modulated, and increased by a 3-D vortex stretching mechanism.

Figure 4. Vorticity contours, $\omega = |\omega| \text{sign}(\omega) = \text{const.}$, and shape of vorticity lines at $t = 36$ s ($\approx 2/5$ rollover time $\tau_r$). The pattern of the initial perturbation is still visible albeit with a larger amplitude.

Let $l$ denote a typical scale length of large vortices, while $v$ is a characteristic velocity of the turbulence (for instance, a typical rotational velocity of the eddy), and let $\nu_{eff}$ stand for the effective diffusion coefficient for momentum transfer in the plasma. Then we can introduce a large-scale Reynolds number with $Re = l/v/\nu_{eff}$. When the rollover time $\tau_r = l/v$ of the large eddies is much smaller than the diffusion time $l^2/\nu_{eff}$, i.e., when $Re \gg 1$, which is normally the case in space plasmas, a range of large-scale dynamics exists that can be studied with the ideal MHD theory, supported during the nonlinear stage by computational fluid dynamics (CFD). It is only at the small scale length that dissipative effects become important. But their influence on large-scale phenomena is reflected after lapses much longer than the turnover time, which is not the case of this study where the simulations last up to $t \sim \tau_r$. The CFD of large structures ignoring dissipative effects is technically known as large eddy simulations (LES) [2].

The computational work is based on the set of compressible, ideal (non-resistive, and inviscid) MHD equations. It is not necessary to report the MHD set here: the equations are shown in the integral form adequate for the numerical ALE+FV code in [1], and are written in differential form in [16] (this JOP Conf. Series).

For any vector field $\vec{X}$ it is customary to define the helicity density, and the total helicity (over a volume), of $\vec{X}$ as

$$h_X = \vec{X} \cdot \text{curl}(\vec{X}), \quad H_X = \int_{\text{Vol}} \vec{X} \cdot \text{curl}(\vec{X}) d^3x \tag{1}$$
When \( \vec{X} = \vec{v} \), \( \vec{\omega} \equiv \text{curl}(\vec{v}) \) is the vorticity, and the kinetic helicity density and the kinetic helicity are introduced as

\[
h_v = \vec{V} \cdot \vec{\omega}, \quad H_v = \int_{\text{vol}} \vec{V} \cdot \vec{\omega} \, d^3 x, \tag{2}
\]

quantities considered in the analysis of the simulations. The kinetic helicity density \( h_v \) is an indicator of the presence of swirling flow. Note that the kinetic helicity \( H_v \) is not conserved in MHD, and that in classical fluids it is an invariant only when the density \( \rho \) is uniform (see, for instance, [19]).

We may also mention that when \( \vec{X} = \vec{A} \), where \( \vec{A} \) is the vector potential of the magnetic field \( \vec{B} \equiv \text{curl}(\vec{A}) \), the magnetic (or potential) helicity is introduced. The total magnetic helicity is conserved in ideal MHD, while the total cross helicity, i.e., the integral over a volume of the cross helicity density \( \vec{B} \cdot \vec{\omega} \), is an MHD invariant only under special conditions. However, we do not examine magnetic or cross helicities in this work.

**Figure 5.** Vorticity contours and some streamlines at \( t = 89 \) s, near to \( \frac{1}{2} \tau_r \). The maximum vorticity value increased, and the vorticity sheet begins to rollover. Dotted arrows point to a spot where a seed of negative vorticity begins to grow.

While the large-scale dynamics of vortices can be studied ignoring dissipation, further vorticity changes take place in a MHD mixing layer because the Lorentz force \( \vec{F}_L = \vec{J} \times \vec{B} / c \) is not in general curl-free, and because its associated baroclinic term \(-\left(1/\rho^2\right)\nabla \rho \times \vec{F}_L\) is not zero in a perturbed LLBL. Moreover, note that in a stratified LLBL the plasma entropy \( S \) across the boundary is non-uniform. Together with \( \rho, p, T, \) and \( \vec{B}, S \) is stratified in the direction transverse to the main flow. And due to the pressure balance condition \( p + B^2/8\pi = c_1 \), the isentropic condition \( p\rho^{\gamma} = c_2 \) (where \( c_1, c_2 \), denote constants) does not hold across the boundary layer, regardless of the fact that the entropy \( S \) of all plasma elements is preserved along their trajectories during the ensuing dynamic evolution, in accord with ideal MHD. Therefore, even the baroclinic term \(-\left(1/\rho^2\right)\nabla \rho \times \nabla p\) of classical fluid dynamics ([20]) is active in this context. And since it can be shown that
\[ \text{curl} \left( -\frac{1}{\rho} \nabla \rho \right) = \nabla T \times \nabla S, \] (3)

then, although in equilibrium the gradients of temperature and entropy are initially parallel, the RHS of this equation does not stay at zero value during the evolution. For instance, under perturbed conditions the appearance of a streamwise component of \( \nabla T \) in conjunction with the transverse component of \( \nabla S \), generates a spanwise \text{curl} component.

All the mentioned terms appear in the MHD vorticity evolution equation, and may hinder the conservation of the vorticity flux. For compressible, ideal MHD plasma, the following equation holds

\[ \frac{d}{dt} \left( \frac{\omega}{\rho} \right) - \left( \frac{\omega}{\rho} \right) \nabla \cdot v = \frac{1}{\rho} \nabla \rho \times \nabla \rho + \frac{1}{\rho} \text{curl} \left( \nabla \times \vec{B} / \rho \omega \right), \] (4)

where the left hand side terms lead to the well known conservation of the flux of \( \omega / \rho \) when the right hand side is zero. The latter terms are responsible instead for the non-conservation of the vorticity flux during the large eddies’ evolution. In our computational study, the non-conservation of vorticity becomes manifest since in addition to the vortices with positive rotation (with the same sign as the original vorticity sheet) other vortices with strong negative vorticity also arise, a striking effect that we report in this paper. Vorticity concentrated in cores of both signs, with absolute value several times larger than the maximum vorticity of the initial shear layer, are found in our LES.

\[ \text{Figure 6.} \] Vortex development at \( t = 89 \) s in two panels: left a) isovorticity surface \( \omega = 0.0485 \) s\(^{-1}\) (equal to \( \omega_i \), the maximum value of the initial vorticity layer) with a rotated view that helps to reveal a twofold structure; right b) isovorticity surface \( \omega = 0.02 \) s\(^{-1}\) (\( \approx 2/5 \omega_i \)), regular view.

In CFD a mixing layer with periodic boundary conditions in the streamwise (or mean flow) direction is called a temporal mixing layer, which is the kind studied here. As mentioned, unstable temporal mixing layers develop large vortex-like structures. In fluid dynamics these are called coherent structures because they last for long periods of time, and can be found far downstream with roughly similar shapes (see, e.g., [20], [2]). As found also in preceding MHD computational studies, and again in this paper, the vortex core is a concentration of vorticity around which convected plasma
layers of different density values wind-up, mixing outer with inner matter. The coherent structures have an important influence on the boundary layer properties, like the downstream broadening of the layer width, or the lengthening of the transit time of plasma elements through the local site. As Moffat liked to state “vortices are the sinews of turbulence” [21].

3. Local boundary layer model and stability

We study the evolution of a planar stratified flow in which the physical quantities are initially constant in $(x, z)$ planes, and vary only perpendicular to these planes ($y$-direction). Initially we assume for the velocity and magnetic fields, $\mathbf{v} = (v_x(y), 0, 0)$, $\mathbf{B} = (B_x(y), 0, B_z(y))$, respectively. Similarly, we write $\rho(y) = m_p n(y)$ for the mass density, and $T(y)$ for the temperature ($n$ is the number of particle density, while $\rho$ is the mass density; $m_p$ being the proton mass). The planar flow is intended to model a local site of the magnetopause boundary layer. The initial field profiles, that represent a steady (or average) state, are built with hyperbolic tangents like $\tanh(y/d)$, where $d$ is a normalization length for the $y$ coordinate.

The physical parameters that define the initial and boundary conditions of the problem are the values on either sides of the transition (with subindex 1 for the magnetosheath, and 2 for the magnetosphere) of the vector fields intensities $v \equiv |\mathbf{v}|$, $B \equiv |\mathbf{B}|$, the scalar fields, $\rho$ (or $n$), $T$, the angle $\theta$ of $\mathbf{B}$ with the $x$ axis (aligned with the main flow), and the width $\Delta = 2d$ of the gradient layer as a basic scale length. We could keep $\Delta$ as a free quantity, but in view of the geophysical application we assume $\Delta = 0.5 \text{ RE}$ for the output of the simulations, and measure distances taking the Earth radius $\text{RE}$ as unit. With these elements, using $Y = y/d$ as a normalized coordinate, and assuming zero velocity on the magnetospheric side of the boundary layer, the initial field profiles of the model are as follows,

$$V_x = \frac{1}{2} U_1 (1 + \tanh(Y)),$$

$$\rho = \frac{1}{2} (\rho_1 + \rho_2) + \frac{1}{2} (\rho_1 - \rho_2) \tanh(Y),$$

$$B = \frac{1}{2} (B_1 + B_2) + \frac{1}{2} (B_1 - B_2) \tanh(Y),$$

$$\theta = \frac{1}{2} (\theta_1 + \theta_2) + \frac{1}{2} (\theta_1 - \theta_2) \tanh(Y),$$

$$B_z = B \cos(\theta), \quad B_x = B \sin(\theta),$$

The temperature function $T(y)$ is a consequence of the preceding profiles and the pressure balance equation across the boundary layer, which lead to

$$T(y) = \frac{m_p}{2 k_b \rho(y)} \left[ p_1 + \frac{B_1^2}{8 \pi} - \frac{B(y)^2}{8 \pi} \right].$$

Following up the findings of [15], we show results of MHD LES tailored to the December 7, 2000 event. Specifically, we apply boundary conditions derived from data observed by Cluster 3 at a dusk flank of the MP on December 7, 2000, from ~14:00 to 14:29 UT, as in [15]. While the initial profiles develop and change spanwise completely during the LES, the asymptotic values $Y \to \pm \infty$ of the quantities on either side are fixed as boundary conditions.

The main parameters for the case of December 7, 2000 are listed in Table 1. From these values it follows that at the Cluster position the sonic Mach number ($M = U_1/c_s$) is $M = 0.57$, and the Alfvenic Mach number ($M_A = U_1/V_A$) is $M_A = 0.70$, both evaluated (conventionally) on the magnetosheath side.
Furthermore, the magnetic shear angle $\delta \theta = \theta_2 - \theta_1$ (difference of magnetic field lines direction on either sides) turns out to be very small $\approx 5^\circ$, i.e., they are nearly aligned. This is an important property of the local configuration, quite favorable to the excitation of KH activity, since the stabilizing influence of the magnetic field critically depends on the existence of a substantial shear angle.

Table 1. Parameters for the case of December 7, 2000

| Time     | Magneto sheath | Magnetosphere |
|----------|----------------|---------------|
| $n_1$ (cm$^{-3}$) | 13.6           | 2.3           |
| $B_1$ (nT)     | 34.6           | 22.1          |
| $U_1$ (km s$^{-1}$) | 139         | $\sim 0$      |
| $T_1$ (keV)    | 0.38           | $\sim 2.6$    |
| $\theta_1$    | $121^\circ$    | $126^\circ$   |

Figure 1 illustrates the growth rate $\gamma$ of the unstable modes, represented by $g = \gamma d/U_1$, as a function of $kd$, where $k$ stands for the absolute value of the wave number of the modes. Both quantities are embodied in dimensionless numbers using $d = \Delta/2$ ($\Delta$ ~ width of the velocity gradient layer), and $U_1$ the plasma velocity at the magnetosheath close to the MP. The results correspond to a particular value of the angle $\phi \equiv \angle \vec{k} \cdot \vec{v}_1 = 32^\circ$, the most favorable for the growth of the perturbation. The maximum growth rate is given by $g_m = \gamma_m d/U_1 = 0.079$, and corresponds to the mode with $k_m d = 0.55$ [15].

Figure 7. Complexity of the 3-D flow: a close-up of vorticity contours with streamlines shows the formation of a swirling flow in the large eddy.

Since $M_A^2 \approx 0.5$ the magnetic tensions are strong and govern the development of the instability. As a consequence, the wavevector $\vec{k}$ of the most unstable mode is nearly perpendicular to the average direction of the magnetic field (both in magnetosheath and magnetosphere) because magnetic stabilization is then minimal (almost null since $\delta \theta \sim 0$). We found that changing the $\vec{k}$ direction by $\sim \pm 10^\circ$ from $\phi = 32^\circ$ (or from $\phi = 32^\circ + 180^\circ$) the modes are stable. Therefore, perturbations with a specific orientation on the magnetopause, determined by the local direction of the magnetic field, grow faster than any other and become the dominant pattern of the KH excitation with the passage of time.
The e-folding time of the instability was evaluated as $\tau_e \approx 89$ s (or somewhat more, depending on estimates of the boundary layer thickness) [15]. Since the KH activity was observed soon after a sudden change of direction of the IMF that turned due north, in a time lapse comparable to the theoretical exponentiation time, we conjectured that the TD/VS impact on the magnetopause, by adding substantial perturbations to the boundary, acted as a trigger that accelerated the development of the KH instability. By analogy, we introduce a similar feature in the simulation: a significant perturbation in the initial conditions of the LES. This is done adding a $k$-mode with $\varphi = 32^\circ$, and 10% velocity amplitude, to the initial steady state velocity field. The other fields are left unperturbed.

The computational box is periodic in $X, Z$, streamwise and transverse to the flow, so that the LES described here correspond to a temporal mixing layer. In the computational box $k_x, k_z$, can take only restricted values, so that the modes are quantified. Due to the choice of side lengths the fastest growing mode for $\varphi = 32^\circ$ that fits in the numerical box corresponds to $kd = 0.79$, as indicated by the dashed line in figure 1 with $g = 0.066$, somewhat smaller than $g_m$. The computational box and the initial state are described in the three panels of figure 2: $a$) vorticity contours $\omega = |\partial | \text{sign} (\omega_z)$, and streamlines; $b$) temperature contours $T$(eV) and magnetic field lines; $c$) isodensity surface $n = 9$ p/cm$^{-3}$, and velocity cones. The level of initial perturbation at $t = 0$ s can be appreciated in figure 3 where we show the isovorticity surface with $\omega = 0.032$ s$^{-1}$. The maximum initial $|\text{curl}(\vec{v})|$ is $\omega = 0.0485$ s$^{-1}$. The colour scale indicates the value of the quantities. As time goes by, the same kind of plots that represent pictorially the non-linear evolution will be shown and commented.

4. Numerical experiments

The numerical experiments run for about one turnover time. Controls by increasing the number of cells give satisfactory results, significant features and values are reproduced with increased precision.

Figure 8. The vorticity contour plot at $t = 132$ s ($\approx 3/4 \pi$) shows the rise of counter-rotating vortices, and the pairing of negative and positive vorticity cores of similar intensity.
Figure 4, at $t = 36$ s, about $2/5$ of $\tau$, shows the aspect of vorticity contours, $\omega = \text{const.}$, and the shape of a set of vorticity lines. We observe that the pattern of the linear perturbation is still present, although with a larger amplitude.

Significant changes appear in the following plots, drawn at $t = 89$ s, about $1/2 \tau$. Figure 5 shows vorticity contours and a few streamlines. The maximum vorticity has increased, and the vorticity sheet begins to rollover. The dotted arrows point to a spot where a seed of negative vorticity just starts to grow.

Also at $t = 89$ s, figure 6 gives details of the vortex development in two panels: a) shows the isovorticity surface $\omega = 0.0485$ s$^{-1}$ (equal to $\omega_b$, the maximum value of the initial vorticity layer) displayed with a rotated view that helps to exhibit a twofold structure; b) back to the regular view, shows the isovorticity surface $\omega = 0.02$ s$^{-1}$ ($\approx 2/5 \omega_b$). The changes in the flow and the complexity of the 3-D dynamics are illustrated in a close-up, figure 7, where we can observe in the pattern of the streamlines the formation of a swirling flow in the large eddy.

At $t = 132$ s ($\approx 3/4 \tau$) the rise of negative vorticity is evident in the vorticity contours of figure 8. There is pairing (or coupling) of negative and positive vorticity cores. The evolution of temperature and density can be appreciated in the two panels of figure 9, a and b, where the progress of mixing of hot and tenuous magnetospheric matter with the cold and dense plasma of the magnetosheath is apparent. Also at $t = 132$ s, figure 10 shows kinetic helicity density contours together with streamlines, which furnish evidence for the complex swirling pattern associated with the vortex core. The rise of

![Figure 9](image-url)

Figure 9. Two panels show at $t = 132$ s from left to right contour plots with the evolution of: a) temperature (eV), and b) density $p$ cm$^{-3}$ (rotated view). Mixing of hot - tenuous with cold - dense plasma is apparent.

these coherent structures has important consequences for the properties of the mixing layer. There is a broadening of the boundary layer's width, and a substantial lengthening of the transit time of plasma, compared with that of a stable laminar boundary layer.

At about one turnover time, $t = 180$ s the 3-D stretching mechanism has intensified the initial vorticity level by a 4-5 fold factor. This can be noted in figure 11 a) with the shape of the vorticity contours and the change of values in the scale of color. Figure 11 b) shows a correlated advance of mixing via the isodensity surface $n = 9$ p cm$^{-3}$. It is evident that convected layers of different density wind-up around the large vortices, mixing the outer with the inner matter.

An important characteristic of the physical regime of the simulation can be seen in figure 12 that shows temperature contours, and a set of magnetic field lines. At $t = 180$ s, as at earlier times, the magnetic field lines are straight and do not bend with the time-varying plasma flow. The slope of the
lines shows only moderate oscillations during the simulation. Another noteworthy feature of figure 12 is the magnetic line associated with the positive vorticity center: it shows that the core of the vortex is aligned with the magnetic field.

**Figure 10.** Time \( t = 132 \) s, shows kinetic helicity density contours (\( \text{km s}^{-2} \)), and streamlines. Evidence of a complex 3-D swirling pattern at the vortex core.

Also for \( t = 180 \) s figure 13 shows the left panel a) where density contours reveal that the same vortex core is considerably depleted with respect to the magnetopause density (\( \text{p cm}^{-3} \)). It also shows a high density patch in the vorticity structure, ~40% larger than the magnetosheath density. The site is correlated with a local increase of gas pressure, whereas temperature remains near to the surrounding values. In figure 13 the right panel b) gives another close-up, where temperature contours (eV) show that the positive vortex core is very hot, with temperatures several times larger than those of the

**Figure 11.** At \( t = 180 \) s (\( \approx \) one turnover time), left to right, two panels: a) with vorticity contours (scale of color in the interval -0.2 to 0.2 \( \text{s}^{-1} \)), and b) progress of mixing via the isodensity surface \( n = 9 \text{ p cm}^{-3} \) (rotated view).
magnetopause plasma. Gas and magnetic pressure are roughly constant across the vortex, and close to the surrounding values.

Figure 12. Temperature contours (eV) and magnetic field lines (solid yellow) at t =180 s

Finally we briefly comment on other numerical runs with the same configuration, but starting from random perturbations. The experiments did not show a development of the instability. Neither the $k_r$ directions nor the wavelengths ($2\pi/k$) of the random perturbations that fit into the computational box were in proper conditions to excite the KH process. The perturbations could not comply with the favorable orientation of $k_r$, which is close to $32^\circ$ (≈ flute modes). After a turnover time, $t \sim 180$ s only stable modes could be observed, with a small fluctuation level.

5. Summary and discussion

MHD simulations have been very fruitful in elucidating features of the KH instability at the magnetopause [12], [8], [17], [6], [13], [18]. Our work is intended as a contribution to this line of research, modeling the boundary layer with data derived from the Dec. 7, 2000, event, during a period of northward IMF [15]. The study complements works done for a different physical regime with $MA >> 1$. In the limit $MA \rightarrow \infty$ the magnetic field behaves like a passive “contaminant”, and yields to flow convection. In [6] the magnetic field lines where found to wind up inside the vortex structure, and small scale current sheets developed with a potential to generate tearing processes.

We reported here a study of the MHD development of vortices in compressible, non-homogeneous, mixing layers due to the velocity gradient instability. The non-linear evolution was computed with 3-D LES of temporal mixing layers tailored to represent distinctive conditions of the terrestrial magnetopause. The boundary layer is characterized by the growth of large-scale vortices, and becomes a site of mass mixing favoring enhanced plasma diffusion. The rise of counter-rotating vortices from the evolution of the original vortex sheet is one of the noteworthy processes revealed by the LES. The nonlinear numerical simulation of the KH excitation was performed with a new 3D+t MHD code of ALE-FV type. The main results obtained can be summarized as follows.
i) The time for the development of a significant deformation of the boundary layer and, with it, the growth of important non-linear characteristics, is shorter than, or comparable to, the e-folding time of the linear theory when the initial structure starts from a condition already significantly perturbed: “trigger and acceleration of the instability”.

ii) Our simulations show the formation of counter-rotating plasma cores with a concentrated vorticity much larger than the vorticity initially stored in the boundary layer. Non conservation of vorticity becomes manifest at about (or even before) ~ one rollover time in our LES, where in addition to vortices with positive rotation (the same sign of the original vorticity sheet) other coherent structures with strong negative vorticity also arise. It is found that vorticity concentrates in cores of both signs with absolute values ~ 4-5 $\times \omega_1$, the maximum vorticity of the initial shear layer. Concomitant with 3-D vortex stretching, also kinetic helicity density rises at the vorticity cores.

iii) The plasma in the vortex centers is hotter ~ 3 times $T_i$ (magnetospheric temperature), and more tenuous ~ 1/3 $n_i$ (magnetospheric particle density) than that of the adjacent magnetosphere. The longer-term evolution of these singular structures, which would be particularly relevant to cases when the IMF is northward-pointing for a long time, is the subject of future work.

iv) We observed a kind of “rigidity” of magnetic field lines, at least for the $M_A = 0.7$ value examined, which is evident in the strong influence of the magnetic field direction on the formation and shape of the non-linear features.

Although the overall aspect may look 2-D at first sight, on closer inspection many 3-D properties become manifest, so that the study of the process requires in fact a 3-D treatment. This aspect of the non-linear stage takes a different tack with respect to linear theory, which can be reduced to a 2-D picture, as indicated by the stability equation of KH modes (see [16]).

![Figure 13. Time $t = 180$ s, two panels, left to right, a) close-up of density contours (p cm$^{-3}$) showing a depleted vortex core and patches of particle concentration; b) close-up of temperature contours (eV) indicating a very hot vortex core (positive $\omega$).](image-url)

v) There is a clear indication, yet to be quantified, of an increment in the time the flowing plasma stays in the boundary layer region deformed by the instability due to the complex pattern of the streamlines.

vi) The broadening of the boundary layer, the plasma mixing in the rolling-off of the vorticity layers, and the increased transit time of the plasma elements, are all factors conducive to an enhancement of ion diffusion through the magnetopause.

All these factors bear on the issue of how effective the development of the KH instability is in fostering mass transfer into the magnetopause. In our numerical experiments the runs have not been extended to lapses much longer than one turnover time in view of the absence of dissipative terms, or subgrid complements, in the present version of the code. The computation of particle trajectories and
an estimate of diffusion is a desirable goal, albeit not yet achieved. This, and several other issues need further work, which is partly underway. Our study is intended as a contribution to the knowledge of vortex formation in MHD, and to the understanding of solar wind interaction with the magnetosphere during periods of northward IMF.

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