Manipulating Light Pulses via Dynamically controlled Photonic Bandgap

A. André and M. D. Lukin

Physics Department and ITAMP, Harvard University, Cambridge, Massachusetts 02138

(October 26, 2018)

When a resonance associated with electromagnetically induced transparency (EIT) in an atomic ensemble is modulated by an off-resonant standing light wave, a band of frequencies can appear for which light propagation is forbidden. We show that dynamic control of such a bandgap can be used to coherently convert a propagating light pulse into a stationary excitation with non-vanishing photonic component. This can be accomplished with high efficiency and negligible noise even at a level of few-photon quantum fields thereby facilitating possible applications in quantum nonlinear optics and quantum information.

PACS numbers 03.67.-a, 42.50.-p, 42.50.Gy

Techniques for coherent control of light-matter interaction are now actively explored for storing and manipulating quantum states of photons. In particular, using electromagnetically induced transparency (EIT) and adiabatic following of “dark-state polaritons”, the group velocity of light pulses can be dramatically decelerated and their quantum state can be mapped onto metastable collective states of atomic ensembles.

In contrast to such a coherent absorption process, the present Letter describes how a propagating light pulse can be converted into a stationary excitation with non-vanishing photonic component. This is accomplished via controlled modification of the photonic density of states in EIT media by modulating the refractive index with an off-resonant standing light wave. By varying the properties of the resulting photonic band structure in time, the original light pulse can be converted into an excitation inside the bandgap where its propagation is forbidden. Long storage of excitations with non-vanishing photonic component may open interesting prospects for enhancement of nonlinear optical interactions. In particular, an intriguing and practically important application of this effect for interactions between few-photon fields is discussed in the concluding paragraph of this Letter.

Before proceeding, we note that there exists a substantial literature on photonic bandgap materials. Recently photonic bandgap structures have been investigated theoretically for strong coupling of single atoms with photons. Photonic bandgap based on interaction with atoms in an optical lattice were also investigated. We also note other related work on EIT-based control of the propagation properties of light in atomic media.

The key idea of the present approach can be qualitatively understood by first considering a medium consisting of stationary atoms with a level structure shown in Fig. 1a. The atoms are interacting with a weak signal field and two strong fields. The running wave control field \( \Omega_c \) is tuned to resonant frequency of the \( |b\rangle \rightarrow |c\rangle \) transition. In the absence of the field \( \Omega_c \), this situation corresponds to the usual EIT: in the vicinity of a frequency corresponding to two-photon resonance the medium becomes transparent for a signal field. This transparency is accompanied by a steep variation of the refractive index.

The dispersion relation can be further manipulated by applying an off-resonant standing wave field with Rabi frequency \( \Omega_s(z) = \Omega_s \cos(k_z z) \) and a frequency detuning \( \Delta \). This field induces an effective shift of resonant frequency (light shift) that varies periodically in space, resulting in a spatial modulation of the index of refraction according to \( \delta n(z) = (c/v_g) \Omega_s^2 \cos^2(k_z z) \), where \( c/v_g \) is the ratio of speed of light in vacuum to group velocity in the medium. When the modulation depth is sufficiently large, signal light propagating near atomic resonance in the forward \( z \) direction with wavenumber \( k \) near \( k_s \) may undergo Bragg scattering into the backward propagating mode with wavenumber \( -k \). In direct analogy to e.g., optical interferometers, the scattering of the counterpropagating fields into each other can modify the photonic density of states. In particular, a range of frequencies (“photonic bandgap”) can appear for which light propagation is forbidden. According to a standard technique to analyze the resulting band structure, Bloch’s theorem can be applied so that the propagating solutions obey \( E(z+a) = e^{iK a} E(z) \), where \( K \) is the Bloch wave vector. Imposing this condition and assuming that the wave vectors of the fields are close \( (k \simeq k_s) \), we can solve for the band structure and obtain near two-photon resonance

\[
\cos(K a) = \cosh \left( \frac{g^2 N}{\Omega_c^2} a \sqrt{\Delta_s^2 - (\omega - \omega_{ba})^2} \right),
\]

where \( g = \sqrt{\frac{\varphi}{2 \Delta_0 V}} \) is the atom-field coupling constant, \( N \) is the number of atoms, \( \Delta_s = \frac{\Omega_c^2}{\Delta} \) is the amplitude of the light shift modulation, \( \varphi \) is the dipole moment of the \( a \rightarrow b \) transition, \( V \) the quantization volume and the factor \( \frac{g^2 N}{\Omega_c^2} \) corresponds to \( c/v_g \). For frequencies such that \( |\omega - \omega_{ba}| < |\Delta_s| \) a bandgap is created: the Bloch wavevector acquires an imaginary part and the propagation of waves in the medium is forbidden. For an outside observer such a medium can be viewed as a mirror: an incident wave with frequency inside the bandgap would
we use collective slowly varying atomic operators \[14\] Fig. 1a. classical driving fields with Rabi-frequencies \(\Omega_s\) of the optical field being \(k\) direction. In the presence of a bandgap, the forward component of \(\hat{a}\) state \(c\) couples resonantly to the transition between the ground state \(g\) and the excited state \(e\), and \(\sigma\) appear as well as a Raman coherence \(\sigma_{bc}\). Using slowly varying envelopes, we then have the equations of motion for the forward and backward modes

\[
\left( \frac{\partial}{\partial t} \pm c \frac{\partial}{\partial z} \right) \hat{E}_\pm(z, t) = igN \hat{\sigma}_{ba}^\pm(z, t). \tag{3}
\]

Assuming weak quantum fields and solving perturbatively, we finally find lowest order in the weak fields and in an adiabatic approximation (assuming \(\Omega_s(t)\) and \(\Omega_e(t)\) change in time slowly enough \[3\])

\[
\hat{\sigma}_{ba}^\pm(z, t) = - \frac{i}{\Omega_c} \left[ \frac{\partial}{\partial t} \hat{\sigma}_{bc}^\pm(z, t) - i \Delta s e^{\pm i 2 \Delta k z} \hat{\sigma}_{ab}^\mp(z, t) \right] \tag{4}
\]

\[
\hat{\sigma}_{bc}^\pm(z, t) = - \frac{i}{\Omega_c} \left[ \frac{\partial}{\partial t} \hat{\sigma}_{ba}^\pm(z, t) - i \hat{F}_{ba}^\pm(t) \right], \tag{5}
\]

where \(\Delta k = k_s - k_0\), \(\Delta s = |\Omega_a|/\Delta\) is the amplitude of the spatially modulated light shift caused by the standing wave field \(\Omega_s\) and \(\hat{F}_{ba}^\pm(t)\) are \(\delta\)-correlated noise forces. Note that in the adiabatic limit the noise forces are negligible \[3\]. The propagation equations are thus

\[
\left( \frac{\partial}{\partial t} \pm c \frac{\partial}{\partial z} \right) \hat{E}_\pm(z, t) = - \frac{g^2 N}{\Omega_c} \frac{\partial}{\partial t} \hat{E}_\pm(z, t) \frac{\partial}{\partial \Omega_c} \Omega_c e^{\pm i 2 \Delta k z}, \tag{6}
\]

which indicates that the forward and backward slowly propagating modes become coupled. Specifically, the first term on the right-hand side gives rise to propagation at the group velocity \(v_g = c/(1 + g^2 N/\Omega_c^2)\) \[3\] while the second term gives rise to coupling between the forward and backward propagating modes. This coupling is optimum when the effective phase matching \(\Delta k = k_s - k_0 = 0\) is achieved \[3\]. Note that both the “control” field \(\Omega_c\) and the standing wave amplitude \(\Omega_s\) can be time-dependent and that as long as changes are slow enough (adiabatic limit \[3\]) the above equations describe the correct dynamics of the coupled modes.

To obtain a solution in the case of time-dependent fields \(\Omega_s(t)\) and \(\Omega_e(t)\), we introduce new quantum fields \(\hat{\Psi}_+(z, t)\) and \(\hat{\Psi}_-(z, t)\) (forward and backward propagating dark-state polaritons \[3\]) \(\hat{\Psi}_\pm(z, t) = \cos \theta(t) \hat{E}_\pm(z, t) - \sin \theta(t) \sqrt{N} \hat{\sigma}_{bc}^\pm(z, t)\), where \(\tan^2 \theta(t) = \frac{g^2 N}{\Omega_c^2} \) is the mixing angle between the photon and matter components of the polariton. The polaritons then obey the coupled equations

\[
\left( \frac{\partial}{\partial t} \pm c \frac{\partial}{\partial z} \right) \hat{\Psi}_\pm = i \Delta_s \tan^2 \theta(t) \hat{\Psi}_\mp, \tag{7}
\]

where \(\tau(t) = \int dt' \cos^2 \theta(t')\). Eq. (7) describes propagation with velocity \(v_g(t) = c \cos^2 \theta(t)\) of the two polaritons (traveling in opposite directions) and coupling...
with rate $\Delta_r(t) \sin^2 \theta(t)$. Note that in the limit $c \gg v_g$, the photonic component $\hat{E}_\pm \simeq (\Omega_c/g\sqrt{N})\hat{\Psi}_\pm$ is finite for non-zero control field $\Omega_c$.

We consider now the scenario in which the standing wave beams are initially off and the control field is on, with Rabi frequency $\Omega_c^{in}$ (corresponding to a group velocity $v_g^{in}$). A forward propagating photon wavepacket can then be stored in the medium in the form of a Raman coherence $\hat{\sigma}_{kk}^c(z,t)$ and subsequently released [4]. We consider the case when the standing wave field is first switched on, establishing the bandgap, followed by the control field (with Rabi frequency $\Omega_c^0$ corresponding to a group velocity $v_g^0$, possibly different from $v_g^{in}$), releasing the pulse in the bandgap medium. For simplicity we consider the case when the standing wave is switched on before or simultaneously with the control field, so that, the coupling rate $\Delta_s(\tau) \tan^2 \theta(\tau)$ does not depend on $\tau$. In this case, we solve [5] by Fourier transforming $\hat{\Psi}_\pm(z) = \frac{1}{2\pi} \int dk e^{ikz} \hat{\Psi}_\pm(k)$ to obtain

$$
\dot{\hat{\Psi}}_+(k,\tau) = \left[ \cos(\chi(\tau)) - i \frac{kc}{\zeta} \sin(\chi(\tau)) \right] \hat{\Psi}_+(k,0),
$$

$$
\dot{\hat{\Psi}}_-(k,\tau) = -i \frac{\chi}{\zeta} \sin(\chi(\tau)) \hat{\Psi}_+(k,0), \quad (8)
$$

where $\chi \equiv \Delta_r(\tau) \tan^2 \theta(\tau)$ and $\zeta = \sqrt{(kc)^2 + \chi^2}$. According to [5], the various Fourier components (wavenumber $k$) of the pulse cycle back and forth between the corresponding forward and backward modes at a rate which depends weakly on $k$. In particular when the spatial extent of the pulse inside the medium is large enough to give a relevant range of wavenumber negligible compared to the strength of the coupling of forward and backward modes, pulse distortion is negligible and the spatial envelopes have the time dependence $\hat{\Psi}_+(z,\tau) = \cos(\chi(\tau))\hat{\Psi}_+(z,0)$ and $\hat{\Psi}_-(z,\tau) = -i \sin(\chi(\tau))\hat{\Psi}_+(z,0)$. The wavepacket periodically cycles between a forward and backward propagating component, the result of which is trapping of the pulse in the medium as shown in Fig. 3. The wavepacket is trapped as a combination of light pulse and Raman coherence.

The above analysis involves an adiabatic approximation and ignores the decay of Raman coherence. In order to ignore the motion compared to the coupling we require that $\chi \gg kc$ and since the maximum $k$ can be estimated from the initial length of the pulse in the medium we find that this is equivalent to requiring that $\Delta_s T \gg \frac{v_g^0}{\gamma}$, where $T$ is the duration of the initial pulse. As seen from [5] the effect of non-zero values of $k$ is that the trapped pulse become spatially distorted. Expanding $\sqrt{\chi^2 + (kc)^2} \simeq \sqrt{\chi^2 + (kc)^2/2} \tau$ we need $\tau \ll \chi/(kc)^2$, which gives after expressing $\tau$ in terms of real time $t$ that $\Delta_r(T(v_g^0/\gamma))^2$, where $t_{int}$ is the maximum time during which the pulse may be trapped without suffering distortion. Furthermore, taking into account the limits imposed by adiabaticity (i.e., modulation of index occurs within the transparency window $\Delta_s \ll (\Omega_c^0)^2/\gamma$) and the fact that the trapped pulse must fit inside the medium when travelling at the reduced group velocity, we find that the trapping or interaction time is limited by

$$t_{int} \lesssim \frac{g^2 N}{\gamma_{ab} (c/L)} \frac{v_g^0}{\gamma_c} T, \quad (9)$$

where $L$ is the length of the medium. This limiting quantity corresponds to the density length-product and can be rather large for optically dense medium.

To summarize, we have shown that by spatially modulating the dispersive feature of the EIT resonance it is possible to induce a photonic bandgap. By dynamically controlling the resulting band structure, a propagating light pulse can be converted into a stationary excitation which is effectively trapped in the medium.

To conclude, we note some interesting avenues opened by this work. First, we note that the present work is not restricted to the use of stationary or cold atoms, for example a Doppler-free configuration involving pairs of copropagating fields is shown in Fig. 1b. In this case, the two polaritons are associated with distinct atomic states $| c \rangle$ and $| c' \rangle$. Each polariton corresponds to a Doppler-free Raman configuration and they are coupled by a Doppler-free two-photon transition. Second, this work may open interesting prospects for nonlinear optics. For example, a trapped photonic excitation can be used to induce a light shift via interaction with another atom-like polariton. Large nonlinear phase shifts at the single-photon level can be expected and open up the way for possible applications in quantum non-linear optics and quantum information without the limitations associated with traveling wave configurations [6] and without invoking cavity QED techniques [17]. Finally, it is intriguing to consider extension of these ideas to manipulate photonic bandgap in condensed matter.

This work was supported by the NSF through ITR program and the grant to the ITAMP.

[1] M. O. Scully and M. S. Zubairy, Quantum Optics, (Cambridge University Press, Cambridge, England, 1997).
[2] S. E. Harris, Phys. Today 50, No. 7, 36 (1997).
[3] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. 84, 5094 (2000); M. D. Lukin, S. F. Yelin and M. Fleischhauer, Phys. Rev. Lett. 84, 4232 (2000).
[4] C. Liu et al., Nature 409, 490 (2001); D. Phillips et. al., Phys. Rev. Lett. 86, 783 (2001).
[5] S. E. Harris and Y. Yamamoto, Phys. Rev. Lett. 81, 3611 (1998).
[6] S. E. Harris and L. V. Hau, Phys. Rev. Lett. 82, 4611 (1999).
FIG. 1. Atomic level configuration for EIT-induced photonic bandgap. a) Stationary atoms scheme, b) moving atoms scheme. The standing wave of Rabi frequency $\Omega_s$ is detuned by $\Delta$ from resonance with the $|c\rangle \rightarrow |d\rangle$ transition, giving rise to a spatially modulated light shift $\Delta_s(z,t) = |\Omega_s(z,t)|^2/\Delta$.

FIG. 2. a) Dispersion relation $\omega(K)$ for waves propagating in EIT-induced bandgap medium. Plotted is the detuning from two-photon resonance $\omega = \nu - \nu_0$ in units of linewidth $\gamma$ vs. the Bloch wavevector $K$ in units of $\gamma/c$. For $|\omega| \gg \gamma$, the dispersion relation corresponds to the usual $\nu \approx cK$ while near resonance the slope is changed to the group velocity $\nu = v_g K$. For $|\omega| \leq \Delta_c$ a bandgap appears, as shown in the inset (full line: real part of $K - k_s$, dashed line: imaginary part). b) Reflection coefficient vs. frequency near EIT resonance for a sample of $^{87}$Rb atoms corresponding to a density $n \sim 10^{13}$ cm$^{-3}$ and length of medium $L \sim 4$ cm ($\gamma_{bc} = 1$ kHz, $\gamma \sim 10$ MHz, $g\sqrt{N} \sim 400$ MHz, $c/v_g \sim 10^3$ and $\Delta_c \sim 400$ kHz), giving a peak reflectivity of 98.9%.

FIG. 3. a) Amplitude of forward and backward propagating polaritons $|\Psi_+(z,t)|^2$ and $|\Psi_-(z,t)|^2$. b) Corresponding electric fields and c) total intensity (forward and backward components) averaged over the optical wavelength. Also shown is the time-dependence of the “control” field $\Omega_s(t)$ (dotted line) and of standing wave field $\Omega_s(t)$ (dashed line). Note that $v_g^{\text{in}}/v_0^{\text{in}} \approx 15$ here so that initial motion of the pulse is noticeable on these plots. Axes are in arbitrary units.