Moduli stabilization and the pattern of sparticle spectra

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Abstract. We discuss the pattern of low energy sparticle spectra which appears in some class of moduli stabilization scenario. In case that light moduli are stabilized by non-perturbative effects encoded in the superpotential and a phenomenologically viable de Sitter vacuum is obtained by a sequestered supersymmetry breaking sector, the anomaly-mediated soft terms become comparable to the moduli-mediated ones, leading to a quite distinctive pattern of low energy sparticle masses dubbed the mirage mediation pattern. We also discuss low energy sparticle masses in more general mixed-mediation scenario which includes a comparable size of gauge mediation in addition to the moduli and anomaly mediations.

Keywords: supersymmetry breaking, moduli stabilization

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INTRODUCTION

Low energy supersymmetry (SUSY) is one of the prime candidates for physics beyond the standard model at TeV scale [1]. One of the key questions on low energy SUSY is the origin of soft SUSY breaking terms of the visible gauge/matter superfields in the low energy effective lagrangian [2]. Most of the phenomenological aspects of low energy SUSY are determined by those soft terms which are presumed to be induced by the auxiliary components of some messenger fields. In string theory, moduli fields including the string dilaton are plausible candidates for the messenger of SUSY breaking [3]. In addition to string moduli, the 4-dimensional supergravity (SUGRA) multiplet provides a model-independent source of SUSY breaking, i.e. the anomaly mediation [4], which induces a soft mass $m_{soft} \sim m_{3/2}/8\pi^2$.

To identify the dominant source of soft terms, one needs to compute the relative ratios between different auxiliary components including the auxiliary component of the 4D SUGRA multiplet. This requires an understanding of how the messenger moduli are stabilized at a nearly 4D Poincare invariant vacuum. In this talk, we discuss the pattern of low energy sparticle spectra which appears in some class string compactifications which realize the low energy SUSY at TeV scale while stabilizing all moduli [5, 6].

4D EFFECTIVE SUGRA WITH SEQUESTERED SUSY BREAKING

Our theoretical framework is an effective SUGRA of string compactification with a sequestered SUSY breaking sector. To be specific, we will be focusing on KKLT-type compactification with a warped throat which is produced by stringy flux [7, 8]. The internal space of KKLT-type compactification consists of a bulk space which might be approximately a Calabi-Yau (CY) manifold, and a highly warped throat attached at CY with SUSY-breaking brane stabilized at its IR end. In such geometry, the bulk CY can be identified as the UV end of throat. To realize the high scale gauge coupling unification, the visible gauge and matter fields are assumed to live on D branes stabilized within the bulk CY. In the following, we will assume that the SUSY breaking at the IR end of throat is provided by an anti-brane [8], which might be the simplest way to realize $N = 1$ SUSY breaking at a meta-stable vacuum in string theory. Although we are taking the SUSY breaking by anti-brane for simplicity, the resulting sparticle spectra in the visible sector are independent of this particular choice, and valid even for generic type of SUSY breaking at the IR end of throat [4, 9].

The 4D effective theory of the KKLT-type compactification includes the UV superfields $\Phi_{UV} = \{T, U, \Sigma\}$ and $V^a, Q^i$, where $T$ and $U$ are the Kähler and complex structure moduli of the bulk CY, $V^a$ and $Q^i$ are the gauge and matter superfields confined on the visible sector D branes, and $\Sigma$ denotes the open string moduli on those D branes at the UV side. There are also 4D fields localized at the IR end of throat, e.g. $\Phi_{IR} = \{Z, \xi^a\}$, where $Z$ is the throat (complex structure) modulus superfield parameterizing the size of 3-cycle at the IR end, and $\xi^a$ is the Goldstino fermion living on the SUSY breaking brane. If SUSY is spontaneously broken by a chiral superfield $Y$ on SUSY breaking brane at the IR end, the Goldstino corresponds to the fermion component of $Y$:

$$ Y = Y_0 + \xi^a \theta_\alpha + F^a \theta_\alpha \theta_\alpha, $$

where $\langle F^a \rangle \equiv M_{SUSY}^2 \theta_\alpha \theta_\alpha$ sets the scale of SUSY breaking. After integrating out $Y_0$ and $F^a$, SUSY appears to be non-linearly realized. As is well known, low energy effective
action with non-linearly realized SUSY can be written on the \(N = 1\) superspace with the Goldstino superfield:

\[
\Lambda^a = \frac{1}{M_{\text{SUSY}}} \xi^a + \theta^a + \ldots,
\]

where the ellipses stand for the Goldstino-dependent higher order terms in the \(\theta\)-expansion. If SUSY is explicitly broken by an anti-brane as in the original KKLT proposal \[8\], there is no degree of freedom corresponding to the \(N = 1\) superpartner of \(\xi^a\). However still the low energy dynamics can be described by an effective action on \(N = 1\) superspace with the Goldstino field as in the low energy limit of spontaneously broken SUSY \[5\].

In addition to the above UV and IR fields, there is of course the 4D SUGRA multiplet which is quasi-localized in the bulk CY, and also the string dilaton superfield \(S\) whose wavefunction is approximately constant over the whole internal space.

Generic 4D effective SUGRA action can be written as

\[
\int d^4x \sqrt{g} \left[ \int d^4 \theta C C^* \left\{ -3 \exp \left( -\frac{K}{3} \right) \right\} + \left\{ \int d^2 \theta \left( \frac{1}{4} f_a W^{\alpha \mu \nu} W_a^{\alpha \mu \nu} + C^3 W \right) + h.c. \right\} \right]
\]

where \(g_{\mu \nu}\) is the 4D metric in the superconformal frame, \(C = C_0 + F^C \theta^2\) is the 4D SUGRA compensator, \(K\) is the Kähler potential, and \(f_a = T + i S\) (\(l = \) rational number) are holomorphic gauge kinetic functions which are assumed to be universal to accommodate the high scale gauge coupling unification. The UV and IR fields are geometrically separated by warped throat, thus are sequestered from each other in \(e^{-K/3}\):

\[
-3 \exp \left( -\frac{K}{3} \right) = \Gamma_{UV} + \Gamma_{IR},
\]

where

\[
\Gamma_{UV} = \Gamma_{IR}^{(0)} (S + S^*, \Phi_{UV}, \Phi_{UV}^*) + \beta (S + S^*, \Phi_{UV}^*) Q^a \bar{Q}^a,
\]

\[
\Gamma_{IR} = \Gamma_{IR}^{(0)} (S + S^*, Z, Z^*) + \frac{C_{\Sigma^2}}{C} \Lambda^2 \Gamma_{IR}^{(1)} (S + S^*, Z, Z^*) + h.c.
\]

\[
+ C C^* \Lambda^2 \Lambda^2 \Gamma_{IR}^{(2)} (S, S^*, Z, Z^*) + \ldots,
\]

where \(\Phi_{UV} = \{ T, U, \Sigma \}, \) and \(\Gamma_{IR}\) is expanded in powers of the Goldstino superfield \(\Lambda^a\) and the superspace derivatives \(D_A = \{ \partial_A, D_A, \bar{D}_A \}\). The above effective action is written on flat superspace background and the SUSY-breaking auxiliary component of the 4D SUGRA multiplet is encoded in the \(F\)-component of the compensator \(C\). In the superconformal gauge in which \(C = C_0 + F^C \theta^2\), the 4D action is invariant under the rigid Weyl transformation under which

\[
C \rightarrow e^{-2\sigma} C, \quad g_{\mu \nu}^C \rightarrow e^{2(\sigma + \sigma')} g_{\mu \nu}^C, \\
\theta^a \rightarrow e^{-\sigma + 2\gamma} \theta^a, \quad \Lambda^a \rightarrow e^{-\sigma + 2\gamma} \Lambda^a,
\]

where \(\sigma\) is a complex constant, and this determines for instance the \(C\)-dependence of \(\Gamma_{IR}\).

The effective superpotential of KKLT compactification contains three pieces:

\[
W = W_{\text{flux}} + W_{np} + W_{\text{Yukawa}},
\]

where the flux-induced \(W_{\text{flux}}\) stabilizing \(S, U, Z, \Sigma\) includes the Gukov-Vafa-Witten superpotential \(W_{GVW} = \int (F_3 - 4\pi i SH_3) \wedge \Omega\), where \(\Omega\) is the holomorphic \((3, 0)\) form of the underlying CY space, \(W_{np}\) is a non-perturbative superpotential stabilizing \(T\), and finally \(W_{\text{Yukawa}}\) denotes the Yukawa couplings of the visible matter fields. Generically, each piece takes the form:

\[
W_{\text{flux}} = \left( \mathcal{F}(U, \Sigma) + \frac{N_{RR}}{2\pi i} Z \ln Z + \mathcal{O}(Z^2) \right)
- 4\pi i \mathcal{S}(H(U, \Sigma) + N_{NS} Z + \mathcal{O}(Z^2)),
\]

\[
W_{np} = \mathcal{A}(U, \Sigma) e^{-8\pi^2 (k_1 T + l_1 S)},
\]

\[
W_{\text{Yukawa}} = \frac{1}{6} \lambda_{ijk}(U, \Sigma) Q^i Q^j Q^k,
\]

where \(k_1, l_1\) are rational numbers, \(N_{RR}, N_{NS}\) are integers defined as \(N_{RR} = \int_S F_3, N_{NS} = -\int_H H_3\), where \(S\) is the 3-cycle collapsing along the throat, \(\Sigma\) is its dual 3-cycle, and \(F_3\) and \(H_3\) are the RR and NS-NS 3-forms, respectively. Here, we assumed that the axionic shift symmetry of \(T\), i.e. \(T \rightarrow T + \text{imaginary constant}\), is preserved by \(W_{\text{flux}}\) and \(W_{\text{Yukawa}}\), but is broken by \(W_{np}\). Note that \(Z\) is defined as \(\int_S \Omega = Z\), and then \(\int_S \Omega = \frac{1}{m_{3/2}} Z \ln Z + \) holomorphic \([7]\).

The above 4D effective action of KKLT-type compactification involves many model-dependent functions of moduli, which are difficult to be computed for realistic compactification. Fortunately, the visible sector soft terms can be determined by only a few information on the compactification, e.g. the rational parameters \(l, k_1, l_1\) in \(f_a\) and \(W_{np}\) and the modular weights which would determine the \(T\)-dependence of \(\mathcal{A}\), which can be easily computed or parameterized in a simple manner. In particular, soft terms are practically independent of the detailed forms of \(\Gamma_{UV}^{(0)}, \Gamma_{IR}, \mathcal{F}, \mathcal{S}, \mathcal{A}\) and \(\lambda_{ijk}\). This is mainly because (i) the heavy moduli \(\Phi = \{ S, U, \Sigma \}\) stabilized by flux have negligible \(F\)-components, \(F^S/\Phi \sim m_{3/2}^2/m_{3/2} < m_{3/2}/8\pi^2\), thus do not participate in SUSY-breaking, and (ii) the SUSY-breaking IR fields \(Z\) and \(\Lambda^a\) are sequestered from the observable sector.
The vacuum value of $Z$ is determined by $W_{\text{flux}}$, and related to the expm warp factor $e^{2A}$ at the tip of throat as

$$Z \sim \exp \left( -8\pi^2 N_{RR} S_0 / N_{NS} \right) \sim e^{2A},$$

(9)

where $S_0$ is the vacuum value of $S$ determined by $D_3 W = 0$. Since the scalar component of $CC^\star$ corresponds to the conformal factor of $g^{H\bar{H}}$, which can be read off from the Weyl transformation (6), $C$ in $\Gamma_{IR}^{(0)}$ should appear in the combination $Ce^A \sim CZ^{1/3}$. Then the $C$-dependence determined by the Weyl invariance (6) suggests that

$$\Gamma_{IR}^{(0)} \sim (ZZ^e)^{1/3} \sim e^{2A},$$

$$\Gamma_{IR}^{(1)} \sim Z \sim e^{3A},$$

$$\Gamma_{IR}^{(2)} \sim (ZZ^e)^{2/3} \sim e^{4A}$$

(10)

for which

$$m_Z \sim \frac{F_Z}{Z} \sim e^A$$

(11)

as anticipated. Here and in the following, unless specified, we use the unit with the 4D Planck scale $M_{pl} = 1/\sqrt{8\pi G_N} = 1$.

The SUSY breaking at the tip of throat provides a positive vacuum energy density of the order of $M_{SUSY}^4 \sim e^{4A}$. This positive vacuum energy density should be cancelled by the negative SUGRA contribution of the order of $m_{3/2}^2$, which requires

$$m_{3/2} \sim e^{2A}.$$  

(12)

One then finds the following pattern of mass scales [5]:

$$m_{S,U,\Sigma} \sim \frac{1}{M_\alpha R^3} \sim 10^{15} \text{ GeV},$$

$$m_Z \sim e^A M_\alpha \sim 10^{10} \text{ GeV},$$

$$m_T \sim m_{3/2} \ln(M_{pl}/m_{3/2}) \sim 10^9 \text{ GeV},$$

$$m_{3/2} \sim m_{\text{soft}} \ln(M_{pl}/m_{3/2}) \sim 10^9 \text{ GeV}$$

(13)

where $m_{\text{soft}}$ denotes the soft masses of the visible fields, e.g. the gaugino masses, and the string scale $M_\alpha$ and the CY radius $R$ are given by $M_\alpha \sim 1/R \sim 10^{17} \text{ GeV}$.

The heavy moduli $S, U, \Sigma$ and the throat modulus $Z$ couple to the light visible fields and $T$ only through the Planck scale suppressed interactions. Those hidden sector fields can be integrated out to derive an effective action of $\mathcal{V}^a, Q^i, T$ and the Goldstino superfield $\Lambda^a$ renormalized at a high scale near $M_{GUT}$. After this procedure, the effective action can be written as [5, 8]

$$\int d^4\sqrt{g} \int d^2\theta \left( \frac{1}{4} f_a^{\text{eff}} W^a W^a + C^3 W_{\text{eff}} \right) + \int d^4\theta CC^\star \Omega_{\text{eff}},$$

(14)

where

$$f_a^{\text{eff}} = T + i S_0,$$

$$\Omega_{\text{eff}} = -3 \epsilon e^{-k_0/3} + \mathcal{V}^i Q^i Q^i - e^{4A} CC^\star C^2 \mathcal{P}_{\text{lift}}^a \left( e^{3AC} \Lambda^2 C \right) + \mathbb{J}_{ij} \mathcal{Q}^i \mathcal{Q}^j \mathcal{Q}^k,$$

with $S_0 = \langle S \rangle$, $K_0 = K_0(T + T^*)$ is the Kähler potential of $T$, $e^{k_0/2} \mathcal{Y}_i$ is the Kähler metric of $Q^i$, $\mathcal{P}_{\text{lift}}^a$ and $C_0$ are constants of order unity, and finally $w_0$ is the vacuum value of $W_{\text{flux}}$. Note that at this stage, all of $e^{2A}$, $\mathcal{P}_{\text{lift}}$, $\Gamma_0, S_0, w_0$, and $\mathcal{J}$ correspond to field-independent constants obtained after $S, U, \Sigma$ and $Z$ are integrated out. As we have noticed, the condition for vanishing cosmological constant requires

$$w_0 \sim e^{2A} \sim -\pi^2 \epsilon_{i}^{0} S_0 \left( l_0 = \frac{2N_{GR}}{3N_{NS}} \right).$$

(15)

and the weak scale SUSY can be obtained for the warp factor value $e^{2A} \sim 10^{-14}$. For such a small value of warp factor, one finds that the SUSY-breaking $F$ components are determined as follows independently of the moduli Kähler potential $K_0$ [5, 6, 9]:

$$\frac{F_C}{C} = \frac{m_{3/2}}{l_0} \left( 1 + \mathcal{O} \left( \frac{1}{4\pi^2} \right) \right),$$

$$\frac{F_T}{T + T^*} = \frac{l_0}{l_0 - l_1} \frac{m_{3/2}}{\ln(M_{pl}/m_{3/2})} \left( 1 + \mathcal{O} \left( \frac{1}{4\pi^2} \right) \right),$$

$$\frac{F_{S,U,\Sigma}}{m_{3/2}} < \frac{m_{3/2}}{8\pi^2}.$$  

(16)

Basically same pattern of $F$ components is obtained in more general set-up with arbitrary number of Kähler moduli $T_i$ [10]. Without loss of generality, one can choose a field basis $T_i = \{ T_s, T_a \}$, for which the superpotential is given by

$$W_{\text{eff}} = W_0 + \sum_{i} \gamma_i e^{-8\pi^2 (k_i T_s + l_i S_0)},$$

(17)

where $\gamma_i$ are constants of order unity, while $w_0 \sim e^{2A}$ as required to tune the cosmologically constant to be nearly zero. If the moduli Kähler potential admits a solution for $\partial K_0 / \partial T_a = 0$ in this field basis, one finds that $T_s$ is stabilized by nonperturbative terms in $W_{\text{eff}}$ with a mass $m_{T_s} \sim m_{3/2} \ln(M_{pl}/m_{3/2})$, while $\Re(T_a)$ are stabilized essentially by the uplifting potential $V_{\text{lift}} = e^{4A} \mathcal{P}_{\text{lift}} e^{2K_0/3}$ with a mass $m_{\Re(T_a)} \sim m_{3/2} \ln(M_{pl}/m_{3/2})$. (Im($T_a$) are nearly massless axions one of which might solve the strong CP problem.) Still the $F$-components follow the pattern [10]

$$\frac{F_{T_s}}{T_s + T_s^*} \sim \frac{F_{T_a}}{T_a + T_a^*} \sim \frac{m_{3/2}}{\ln(M_{pl}/m_{3/2})}.$$
\[ f^{SU} \sim \frac{m^2_{3/2}}{m_{SU}} \] (18)

which is basically same as (16).

One of the interesting features of SUSY breaking at the IR end of throat is the sequestering property, i.e. there is no sizable Goldstino-matter contact term:

\[ \Delta m^2_{3/2} \sim \lambda^2 \Xi^2 Q^i Q^j \] (19)

in \( \Omega_{\text{eff}} \) of (14), which would give an additional contribution \( \Delta m^2 \) to the soft scalar mass-squares. This amounts to that there is no operator of the form \( (ZZ^*)^{1/3} Q^i Q^j \) or \( (ZZ^*)^{2/3} \lambda^2 \Xi^2 Q^i Q^j \) in \( e^{-k/3} \) of (4). Since \( Q^i \) and \( \lambda^a \) are geometrically separated by warped throat, such contact term can be generated only by the exchange of bulk field propagating through the throat. Simple operator analysis assures that the exchange of chiral multiplet can induce only a higher order operator in the superspace derivative expansion, while the exchange of light vector multiplet \( \tilde{V} \) can generate the Goldstino-matter contact term with \( \Delta m^2 \sim \langle D \rangle \), where \( D \) is the D-component of \( \tilde{V} \). Quite often, throat has an isometry symmetry providing light vector field which might generate the Goldstino-matter contact term. However, in many cases, the isometry vector multiplet does not develop a nonzero D-component, and thereby not generate the contact term (4, 12). As an example, let us consider the SUSY breaking by anti-D3 brane stabilized at the tip of Klebanov-Strassler (KS) throat which has an SO(4) isometry (13). Adding anti-D3 at the tip breaks SUSY and also SO(4) down to SO(3). However the unbroken SO(3) assures that the SO(4) vector multiplets have vanishing D-components, thus do not induce the Goldstino-matter contact term. In fact, this is correct only up to ignoring the isometry-breaking deformation of KS throat, which is caused by attaching the throat to compact CY. Recently, the effect of such deformation has been estimated (12), which found

\[ \Delta m^2 \lesssim 0.008 \times m^2_{3/2}. \] (20)

This is small enough to be ignored compared to the effects of \( F^C \) and \( F^D \) obtained in (16).

**MIRAGE MEDIATION PATTERN OF SPARTICLE MASSES**

The results (16) and (18) on SUSY-breaking \( F \)-components indicates that

\[ F^T / T \sim \frac{m^2_{3/2}}{4\pi} \gg |F^{SU}|, \] (21)

where \( T \) denotes generic Kähler moduli. In such case, soft terms are determined dominantly by the Kähler moduli-mediated contribution and the one-loop anomaly mediated contribution which are comparable to each other. For the canonically normalized soft terms:

\[ -\frac{1}{2} M_{\alpha} \lambda^a \lambda^a - \frac{1}{2} m^2_i |\phi^i|^2 - \frac{1}{6} A_{ijk} y_{ijk} \phi^i \phi^j \phi^k + \text{h.c.}, \] (22)

where \( \lambda^a \) are gauginos, \( \phi^i \) are sfermions, \( y_{ijk} \) are the canonically normalized Yukawa couplings, the soft parameters at energy scale just below \( M_{\text{GUT}} \) are given by

\[ M_{\alpha} = M_0 + \frac{b_{\alpha}}{16\pi^2} g^2_{\text{GUT}} m_{3/2}, \]

\[ A_{ijk} = \frac{1}{16\pi^2} \gamma_{ijk} g^3 \]

\[ m^2_{\tilde{l}} = m^2_{\tilde{l}} - \frac{1}{32\pi^2} \frac{d g_{\tilde{l}}}{d \ln m_{3/2}} m^2_{3/2} \]

\[ + \frac{1}{4\pi^2} \left( \sum_{jk} \frac{1}{4} y_{ijk}^2 A_{ijk} - \sum_a g^2_{\tilde{A}} C^a_{ij}(\phi^i) M_0 \right) m_{3/2}, \]

where the moduli-mediated soft masses \( M_0, \tilde{A}_{ijk} \) and \( m^2_{\tilde{l}} \) are given by

\[ M_0 = F^T \frac{\partial \ln (\text{Re}(f_{\alpha}))}{\partial f_{\alpha}}, \]

\[ \tilde{A}_{ijk} = F^T \frac{\partial \ln (\phi^i \phi^j \phi^k)}{\partial f_{\alpha}}, \]

\[ m^2_{\tilde{l}} = -|F^T|^2 \frac{\partial \ln (\phi^i)}{\partial f_{\alpha}} \] (23)

and \( b_{\alpha} = -3 \text{tr}(T^2_{\alpha} \text{Adj}) + \sum_j \text{tr}(T^2_{\alpha} \phi^j) \), \( \gamma = 2 \sum_a g^2_{\tilde{A}} C^a_{i}(\phi^i) - \frac{1}{2} \sum_{jk} \tilde{A}_{ijk} y_{ijk} \), where \( C^a_{ij}(\phi^i) = (N^2 - 1)/2N \) for a fundamental representation \( \phi^i \) of the gauge group \( SU(N) \), \( C^a_{ij}(\phi^i) = q^a_i \) for the \( U(1) \) charge \( q_i \), and \( \omega_{ij} = \sum_{k} y_{ijkl} y^*_l \) is assumed to be diagonal.

Taking into account the 1-loop RG evolution, the above soft masses at \( M_{\text{GUT}} \) lead to the following low energy gaugino masses

\[ M_{\alpha}(\mu) = M_0 \left[ 1 - \frac{1}{8\pi^2} b_{\alpha} g^2_{\text{GUT}} \left( \frac{M_{\text{mir}}}{\mu} \right) \right], \] (24)

showing that the gaugino masses are unified at the mirage messenger scale (6):

\[ M_{\text{mir}} = \frac{M_{\text{GUT}}}{\langle M_{\text{mir}} / m_{3/2} \rangle^{1/2}}, \] (25)

where

\[ \alpha = \frac{m^2_{3/2}}{M_0 \ln (M_{\text{mir}} / m_{3/2})}, \]

while the gauge couplings are still unified at \( M_{\text{GUT}} = 2 \times 10^{16} \text{ GeV} \). With this feature of mirage unification, the SUSY breaking scheme discussed above has been named as mirage mediation (14). The low energy values
of $A_{ijk}$ and $m_i^2$ generically depend on the associated Yukawa couplings $y_{ijk}$. However if $y_{ijk}$ are negligible or if $\tilde{A}_{ijk}/M_0 = (\tilde{m}_i^2 + \tilde{m}_j^2 + \tilde{m}_k^2)/M_0^2 = 1$, their low energy values also show the mirage unification feature [3]:

$$A_{ijk}(\mu) = \tilde{A}_{ijk} + \frac{M_0}{8\pi^2}(\gamma + \gamma_{ij} + \gamma_k) \ln \left( \frac{M_{\text{mir}}}{\mu} \right),$$

$$m_i^2(\mu) = \tilde{m}_i^2 - \frac{M_0^2}{8\pi^2} Y_i \left( \sum_j c_j Y_j \right) g_i^2 \ln \left( \frac{M_{\text{GUT}}}{\mu} \right) + \frac{M_0^2}{4\pi^2} \left[ \gamma - \frac{1}{2} \frac{d\gamma}{d \ln \mu} \ln \left( \frac{M_{\text{mir}}}{\mu} \right) \right] \ln \left( \frac{M_{\text{mir}}}{\mu} \right). (26)$$

where $Y_i$ is the $U(1)_y$ charge of $\phi^i$. Quite often, the moduli-mediated squark and slepton masses have a common value, i.e. $\tilde{m}_0^2 = \tilde{m}_7^2$, and then the squark and slepton masses of the 1st and 2nd generation are unified again at $M_{\text{mir}}$.

Mirage mediation can be generalized in a way which includes a comparable size of gauge mediation [16,17,18]. As an example, one can consider a model with exotic vector-like matter fields $\Phi + \Phi^c$ described by

$$\int d^4 \theta CC^a \left( \gamma_\Phi \Phi^c \Phi + \gamma_\Phi \Phi \Phi^c \Phi^c \right) + \int d^2 \theta C^3 \left( \kappa X^a + \lambda X \Phi^c \Phi \right) \quad (n > 3), \quad (27)$$

where $\kappa \sim 1/M_{\text{GUT}}$. One then finds [17]

$$F^X/X = - \frac{2}{\alpha - 1} \frac{F^C}{C}, \quad \langle X \rangle \sim (m_{3/2} M_{\text{GUT}}^{-3})^{1/(n-2)}. \quad (28)$$

Integrating out the massive $\Phi + \Phi^c$ gives rise to the following gauge threshold correction to soft parameters at the gauge messenger scale $\langle X \rangle$:

$$\Delta M_a(\langle X \rangle) = - \frac{N_\Phi}{16\pi^2} g_a^2 \left( \frac{F^X}{X} + \frac{F^C}{C} \right),$$

$$\Delta m_i^2(\langle X \rangle) = \frac{2N_\Phi}{16\pi^2} \sum_a g_a^2 C_i^a(\Phi^c) \left( \frac{F^X}{X} + \frac{F^C}{C} \right)^2,$$

$$\Delta A_{ijk}(\langle X \rangle) = 0, \quad (29)$$

where $N_\Phi$ denotes the number of $\Phi + \Phi^c$ which is assumed to be 5 + 5 of SU(5). These gauge mediation contributions are comparable to the moduli and anomaly mediations, and alter the shape of low energy particle spectra.

For the gauginos and matter families with small Yukawa coupling and $\sum c_i Y_i = 0$, one can find the approximate analytic expression of low energy running masses at $\mu < \langle X \rangle$ [18]:

$$M_\alpha(\mu) = M_{\text{eff}} \left[ 1 - \frac{1}{8\pi^2} b_\alpha g_a^2(\mu) \ln \left( \frac{M_{\text{mir}}}{\mu} \right) \right],$$

$$m_i^2(\mu) = \left( \frac{M_{\text{eff}}}{M_{\text{mir}}} \right)^2 \left\{ \gamma - \frac{1}{2} \frac{d\gamma}{d \ln \mu} \ln \left( \frac{M_{\text{mir}}}{\mu} \right) \right\} \times \ln \left( \frac{M_{\text{mir}}}{\mu} \right) \left( \frac{M_{\text{eff}}}{M_{\text{mir}}} \right)^2 \left( \frac{M_{\text{mir}}}{\mu} \right)^{\frac{1}{4\pi^2}} . \quad (30)$$

where

$$M_{\text{eff}}^0 = R M_0,$$

$$M_{\text{eff}}^0 = \frac{M_{\text{GUT}}}{(M_{\text{GUT}}/m_{3/2})^{\alpha/2R}},$$

$$\langle m_i^2 \rangle^2 = m_i^2 + \left[ \frac{2}{N_\Phi} \sum_a C_i^a(\Phi^c) \frac{g_a^2(\langle X \rangle)}{g_0^2} \right] (1 - R)^2 M_0^2$$

$$\frac{\Delta m_i^2}{M_0} \left( \frac{m_{3/2}}{M_0} \right),$$

$$\beta = \frac{m_{3/2}}{F^X/X}. \quad (31)$$

Here $b_\alpha$ are the one-loop beta function coefficients at a scale $\mu < \langle X \rangle$ and $g_0^2 \simeq 1/2$ corresponds to the unified gauge coupling constant in the absence of exotic matter fields $\Phi + \Phi^c$. Note that the gaugino masses are still unified at a mirage scale $\langle X \rangle$ even when there exists a sizable extra gauge mediation contribution, while the mirage unification of sfermion masses is generically lost. Although the mirage unification of sfermion masses is generically lost in the presence of gauge mediation, the deviation is not so significant for the class of models giving $\beta < 0$ [18]. For instance, for the models of [27], $|R - 1| = O(0.1)$ for a reasonable range of $\alpha, N_\Phi$ and $n > 3$, indicating that sfermion masses show a mirage unification at the same scale as the gaugino masses up to small deviations of $O(10\%$).

In regard to phenomenology, the most interesting feature of mirage mediation is that it gives rise to significantly compressed low energy SUSY spectrum compared to other popular schemes such as mSUGRA, gauge mediation and anomaly mediation. This feature can be easily understood by noting that soft parameters are unified at $M_{\text{mir}} \approx M_{\text{GUT}} (m_{3/2}/M_{\text{GUT}})^{\alpha/2R}$ which is hierarchically lower than $M_{\text{GUT}}$ as $\alpha$ has a positive value of order unity.

In fact, mirage mediation provides more concrete prediction under a rather plausible assumption. In the fol-
lowing, we present some predictions of the minimal mirage mediation yielding the low energy soft parameters given by \(\mathcal{M}_{\text{GUT}}\). Assuming that \(f_s\) are (approximately) universal, which might be required to realize the gauge coupling unification at \(M_{\text{GUT}}\) through 

\[
M_1 \cong M_0 (0.42 + 0.28 \alpha), \\
M_2 \cong M_0 (0.83 + 0.085 \alpha), \\
M_3 \cong M_0 (2.5 - 0.76 \alpha),
\]

leading to

\[
M_1 : M_2 : M_3 \\n\simeq (1 + 0.66 \alpha) : (2 + 0.2 \alpha) : (6 - 1.8 \alpha).
\]

The TeV scale masses of the 1st and 2nd generations of squarks and sleptons are also easily obtained to be

\[
m_Q^2 \simeq m_D^2 + M_Q^2 (5.0 - 3.6 \alpha + 0.51 \alpha^2), \\
m_D^2 \simeq m_D^2 + M_Q^2 (4.5 - 3.3 \alpha + 0.52 \alpha^2), \\
m_L^2 \simeq m_L^2 + M_Q^2 (0.49 - 0.23 \alpha - 0.015 \alpha^2), \\
m_E^2 \simeq m_E^2 + M_Q^2 (0.15 - 0.046 \alpha - 0.016 \alpha^2),
\]

where \(Q, D, L, E\) denote the \(SU(2)_L\) doublet squark, singlet down-squark, doublet lepton, and singlet lepton, respectively. Assuming that the matter Kähler metrics obey simple unification (or universality) relations such as \(\mathcal{K}_Q = \mathcal{K}_E\) and \(\mathcal{K}_D = \mathcal{K}_L\), we find

\[
M_1^2 : (m_Q^2 - m_E^2) : (m_D^2 - m_L^2) \\n\simeq (0.18 + 0.24 \alpha + 0.09 \alpha^2) : \\
(4.9 - 3.5 \alpha + 0.53 \alpha^2) : \\
(4.0 - 3.1 \alpha + 0.54 \alpha^2).
\]

If the idea of low energy SUSY is correct and the gluino or squark masses are lighter than 2 TeV, some superparticle masses, e.g., the gluino mass and the first two neutralino masses as well as some of the squark and slepton masses, might be determined at the LHC by analyzing various kinematic invariants of the cascade decays of gluinos and squarks. It is then quite probable that the LHC measurements of those superparticle masses are good enough to test the above predictions of mirage mediation \[19\].

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