Optimisation of multisine waveform for bio-impedance spectroscopy

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Abstract. The multisine excitation is widely used in impedance measurements to retain the advantages of the sine wave, while reducing the measurement time. Adding up sine waves increases the amplitude of the excitation signal, but, for the linearity assumption to be valid, the overall amplitude of the signal needs to be kept low. Thus, the crest factor (CF) of the excitation signal must be minimized. A novel empirical method for minimization of the CF is presented in this paper. As in case of other known methods, the computed CF may be guaranteed to be only a local minimum. However, a systematic variation of initial parameters, which is possible due to the sparing algorithm, ensures the CF value, very close or equal to the global minimum. A brief analysis of the calculation errors and comparison with the results from other sources is provided. In the DSP applications, initial phases of the waveforms must match the grid of sample points if separate signal sources are used to generate the components. An equation for calculation of the indexes of the sample points corresponding to the optimal initial phases is provided, and the time discreteness caused relative error of CF is illustrated.

1. Introduction
Sinusoidal waveforms can be readily measured using ordinary signal processing solutions. Use of the multisine excitation which is the sum of sinusoids, allows us retain the advantages of the sine wave, while reducing the measurement time. A multisine with \( k \) frequency components can be expressed as

\[
S_{\text{ms}}(t) = \sum_{i=1}^{i=n} A_i \cdot \sin(2\pi f_i + \Phi_i)
\]  

where \( A_i \) is the amplitude of the \( i \)-th component, and \( \Phi_i \) is its initial phase.

Adding up sine waves increases the excitation amplitude, however, for the linearity reasons, the overall amplitude of the signal must be kept low. On the other hand, a better signal-to-noise ratio (SNR) is also desired. To satisfy these contradictory requirements the amplitude extremes for a given energy of the signal should be minimized. The ratio of the peak value of a signal to its root mean square (RMS) level is called as crest-factor (CF). RMS value of a multisine signal is independent of initial phases \( \Phi_i \) of separate signal components, and thus, the CF minimization problem reduces to finding of the best set of phases, which ensures the minimum peak value of the multisine signal.

Much research has been done in this direction [1-7]. A simple formula proposed by Schroeder [1] produces better CF for the consecutive harmonic frequencies. However, in case of non-consecutive frequency distributions, e.g. with 10 prime harmonics (pos. 3 in table 1) the Shroeder formula gives \( CF = 2.24 \), when our optimization method gives \( CF = 1.48 \). Our calculation with the same frequency distribution but \( 1 \times 10^4 \) random phase sets gave a CF approximately 20% above the minimum value.
An exhaustive search of all possible phase combinations provides the best results, however, it is practical only for spectra having small numbers of harmonics \( k \) since the number of calculation steps \( n \) increases in power of \( k-1: n = (360 / \Delta \Phi)^{k-1} \), where \( \Delta \Phi \) is the phase step. More sophisticated iterative methods [2-5] and genetic algorithms (GA) [6] or a hybrid of GA with nonlinear iteration [7] provide a solution for large \( k \).

Minimization of the CF of multisine is not straightforward, it can be more formally stated as the minimization of a non-linear function and many local minima are present in the parameter space during the optimization process. There is no guarantee that iterative algorithms converge to a global minimum. GA has less probability to get stuck in the local minima since it uses a population of candidate solutions [6]. However, the converging to a global minimum is still not guaranteed and exploring various local optima takes more time.

2. A novel empirical method for the minimization of crest factor CF

Examination of amplitude minimums of the multisine signals shows that several equal minimums exist if the combinations of initial phases are changing within the full range from 0 to 360 degrees.

It was found that a phase combination corresponding to the amplitude minimum obtained with large phase steps, will appear also in the next pattern received with smaller phase steps (a fractal phenomenon). These observations led to the hypothesis that the initial phases of the frequency components that produce the minimal amplitude of the sum signal could be found using sequential searches with limited number of phase steps. Computer simulations with different frequency distributions proved that a recursive algorithm with 2 phase steps in each iteration could be used for finding the near-to-optimal set of initial phases of the included sine waves \( \Phi_{opt}(i) \). Figure 2 gives a flowchart describing the algorithm.

![Flowchart of the developed algorithm for calculation of optimal phases of sine waves \( \Phi_{opt}(i) \) of a multisine.](image)

As with the other known methods, the computed phases can only be guaranteed to be local optima since the result of the calculations depends also on a frequency distribution and the initial values of the parameters \( \Phi_{min} \) and \( \Phi_{max} \). The performance of the new method was further tested by a variation of these initial parameters. The results of two tests are presented in figure 2. \( \Phi_{min} \) was fixed to 0º and \( \Phi_{max} \) was varied with a step of 1º. The tests showed that choosing a proper value for \( \Phi_{max} \) allows detecting \( CF_{min} \) with relative error below 1%. In the case of randomly chosen \( \Phi_{max} \), the relative error is typically less than 10 %.

Figure 1. Flowchart of the developed algorithm for calculation of optimal phases of sine waves \( \Phi_{opt}(i) \) of a multisine. \( A_{min} = \min \sum_{i=1}^{k} |A_i| \sin(2\pi f_i + \Phi_i) \) where \( j = 1 \) or 2.
For the new method, the number of calculation steps $n^*$ for searching of optimal initial phases is expressed as

$$n^* = \log_2 \left( \frac{2^{2k} \Delta \Phi}{\Phi_{\text{max}} - \Phi_{\text{min}}} \right) + 1$$

The results obtained with the new method were compared with other published data (table 1).

Table 1. Comparison of calculated CF with reference data, CF_ref

| Pos. | List of frequency components $i$ | $k$ | CF | CF_ref | Reference |
|------|---------------------------------|-----|----|--------|-----------|
| 1    | $5, 21, 37, 53, 69, 85, 101$    | 8   | 1.59 | 1.64  | (7)       |
| 2    | $1, 2, 3, 4, 5...10$           | 10  | 1.40 | 1.51  | (5)       |
| 3    | $3, 5, 7, 11, 13, 17, 19, 23, 29, 31$ | 10  | 1.50 | 1.57  | (5)       |
| 4    | $10, 12, 15, 18, 22, 27, 33, 40, 48, 58, 70, 84, 100$ | 13  | 2.00 | 1.96  | (4)       |
| 5    | $1, 2, 3, 4, 5...15$           | 15  | 1.40 | 1.80  | (5)       |
| 6    | $3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 43$ | 15  | 1.65 | 1.77  | (5)       |
| 7    | $1, 2, 3, 4, 5...16$           | 16  | 1.41 | 1.42  | (4)       |

In figure 4, the optimization results for some typical frequency distributions are shown. To note in figure 4, the CF of equally distributed multisine components are close to the value of $\sqrt{2}$ if $k > 3$. In case of very sparse distribution, as a decimal logarithmic one, CF tend to the worst case of $\sqrt{2k}$. For all other frequency distributions, the optimized CF falls in between these two limiting values. It was found that the CF of optimized multisine with the binary logarithmic frequency distribution tends to the value of $\pi \sqrt{k} / 2\sqrt{2}$.

3. Accuracy of CF calculations and recalculation of optimal phases

Despite the method used, the accuracy of the calculated CF values depends on the sampling rate. It was found that the number of samples per period of the highest frequency component must be at least 10, in order to achieve relative error near 1 %. Close to the Nyquist sampling-rate, the relative error could increase over 20 %. The error depends also on the number of frequency components and their distribution as illustrated in Fig. 5. Optimized initial phases of the frequency components with some phase range and resolution can be recalculated to another suitable phase range (e.g., from 0 to 360°) and resolution. Figure 6 gives a flowchart describing this algorithm. An example: optimized multisine (pos. 2 in table 1) has CF 1.402 and $\Phi_{\text{opt}}(i)$ in the range from $-64.00°$ to $230.37°$. After recalculation $\Phi_{\text{opt}}(i)$ to the phase range from 0 to 360° with the new phase resolution $\Delta \Phi = 1°$, CF is 1.405 and in case of $\Delta \Phi = 10°$ CF increases to 1.459. Initial phases can be also recalculated to the corresponding indices of sample points $I_i(i)$ as

$$I_i(i) = (1 - (\Phi_i / 360)) \times (n_s / i)$$

where $n_s$ is a total number of sample points. $\Phi_i$ must be in the range from 0 to 360°.
4. Conclusions
The developed new method for the minimization of CF is fast, but gives the same results as the full exhaustive search if the number of frequency components is below 6. For a higher number of components, the results were compared with other published data and found to be similar or better.

An algorithm for recalculating of optimized initial phases to another phase range and resolution is presented, which is practical but highly preserves the previously obtained optimization results (CF_{min}).

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