The design and verification of Mumax3

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We report on the design, verification and performance of MuMax³, an open-source GPU-accelerated micromagnetic simulation program. This software solves the time- and space dependent magnetization evolution in nano- to micro scale magnets using a finite-difference discretization. Its high performance and low memory requirements allow for large-scale simulations to be performed in limited time and on inexpensive hardware. We verified each part of the software by comparing results to analytical values where available and to micromagnetic standard problems. MuMax³ also offers specific extensions like MFM image generation, moving simulation window, edge charge removal and material grains.

Keywords: Micromagnetism, Simulation, Graphical Processing Unit

I. INTRODUCTION

MuMax³ is a GPU-accelerated micromagnetic simulation program. It calculates the space- and time-dependent magnetization dynamics in nano- to micro-sized ferromagnets using a finite-difference discretization. A similar technique is used by the open-source programs OOMMF[15] (CPU) and MicroMagnum[5] (GPU), and the commercial GpMagnet[25] (GPU).

MuMax³ is open-source software written in Go[4] and CUDA[6], and is freely available under the GPLv3 license on http://mumax.github.io. In addition to the terms of the GPL, we kindly request that any work using MuMax³ refers to the latter website and this paper. An nVIDIA GPU and a Linux, Windows or Mac platform is required to run the software. Apart from nVIDIA’s GPU driver, no other dependencies are required to run MuMax³.

Finite-element micromagnetic software exists as well like, e.g., NMag[17], TetraMag[19], MagPar[32] and FastMag[11]. They offer more geometrical flexibility than finite-difference methods, at the expense of performance.

In this paper we first describe each of MuMax³’s components and assert their individual correctness and accuracy. Then we address the micromagnetic standard problems [2], where all software components have to work correctly together to solve real-world simulations. We typically compare against OOMMF[15] which has been widely used and profoundly tested for over more than a decade. Finally, we report on the performance in terms of speed and memory consumption.

The complete input files used to generate the graphs in this paper are available in appendix A, allowing for each of the presented results to be reproduced independently. The scripts were executed with MuMax version 3.6.
FIG. 1: Each simulation cell is attributed a region index representing the cell’s material type. Material parameters like the saturation magnetization $M_{\text{sat}}$, anisotropy constants, etc are stored in 1D look-up tables indexed by the region index. Interfacial parameters like the exchange coupling $A_{\text{ex}}/M_{\text{sat}}$ are stored in a 2D lower triangular matrix indexed by the interface’s two neighbor region indices.

FIG. 2: Geometry obtained by logically combining an ellipsoid and rotated cuboid. The arrows depict the magnetization direction in this complex shape.

II. DESIGN

A. Material Regions

MuMax$^3$ employs a finite difference (FD) discretization of space using a 2D or 3D grid of orthorhombic cells. Volumetric quantities, like the magnetization and effective field, are treated at the center of each cell. On the other hand, interfacial quantities, like the exchange coupling, are considered on the faces in between the cells (Fig. 1).

In order to preserve memory, space-dependent material parameters are not explicitly stored per-cell. Instead, each cell is attributed a region index between 0 and 256. Different region indices represent different materials. The actual material parameters are stored in 256-element look-up tables, indexed by the cell’s region index.

Interfacial material parameters like the exchange coupling are stored in a triangular matrix, indexed by the region numbers of the interacting cells. This allows arbitrary exchange coupling between all pairs of materials (Section III C).

Time-dependent parameters In addition to region-wise space-dependence, material parameters in each region can be time-dependent, given by one arbitrary function of time per region.

Excitations like the externally applied field or electrical current density can be set region- and time-wise in the same way as material parameters. Additionally they can have an arbitrary number of extra terms of the form $f(t) \times g(x, y, z)$, where $f(t)$ is any function of time multiplied by a continuously varying spatial profile $g(x, y, z)$. This allows to model smooth time- and space dependent excitations like, e.g., an antenna’s RF field or an AC electrical current.

B. Geometry

MuMax$^3$ uses Constructive Solid Geometry to define the shape of the magnet and the material regions inside it. Any shape is represented by a function $f(x, y, z)$ that returns true when $(x, y, z)$ lies inside the shape or false otherwise. E.g. a sphere is represented by the function $x^2 + y^2 + z^2 \leq r^2$. Shapes can be rotated, translated, scaled and combined together with boolean operations like AND, OR, XOR. This allows for complex, parametrized geometries to be defined programmatically. E.g., Fig. 2 shows the magnetization in the logical OR of an ellipsoid and cuboid.
C. Interface

Input scripts  MuMAX³ provides a dedicated scripting language that resembles a subset of the Go programming language. The script provides a simple means to define fairly complex simulations. This is illustrated by the code snippet below where we excite a Permalloy ellipse with a 1 GHz RF field:

```plaintext
setgridsize(128, 32, 1)
setcellsize(5e-9, 5e-9, 8e-9)
setGeom(ellipse(500e-9, 160e-9))
Msat = 860e3
Aex = 13e-12
alpha = 0.05
m = uniform(1, 0, 0)
relax()

f := 1e9 // 1GHz
A := 0.01 // 10mT
B_ext = vector(0.1, A*sin(2*pi*f*t), 0)
run(10e-9)
```

Programming  The MuMAX³ libraries can also be called from native Go. In this way, the full Go language and libraries can be leveraged for more powerful input generation and output processing than the built-in scripting.

Web interface  MuMAX³ provides web-based HTML5 user interface. It allows to inspect and control simulations from within a web browser, whether they are running locally or remotely. Simulations may also be entirely constructed and run from within the web GUI. In any case an input file corresponding to the user’s clicks is generated, which may later be used to repeat the simulation in an automated fashion.

Data format  MuMAX³ uses OOMMF’s "OVF" data format for input and output of all space-dependent quantities. This allows to leverage existing tools. Additionally a tool is provided to convert the output to several other data formats like paraview’s VTK[3], gnuplot[1], comma-separated values (CSV), Python-compatible JSON, . . . , and to image formats like PNG, JPG and GIF. Finally, the output is compatible with the 3D rendering software MuView, contributed by Graham Rowlands[31].

III. DYNAMICAL TERMS

MuMAX³ calculates the evolution of the reduced magnetization $\mathbf{m}(\mathbf{r}, t)$, which has unit length. In what follows the dependence on time and space will not be explicitly written down. We refer to the time derivative of $\mathbf{m}$ as the torque $\mathbf{\tau}$ (units 1/s):

$$\frac{\partial \mathbf{m}}{\partial t} = \mathbf{\tau}$$

$\mathbf{\tau}$ has three contributions:

- Landau-Lifshitz torque $\mathbf{\tau}_{LL}$ (Section III A)
- Zhang-Li spin-transfer torque $\mathbf{\tau}_{ZL}$ (Section III G)
- Slonczewski spin-transfer torque $\mathbf{\tau}_{SL}$ (Section III H).
A. Landau-Lifshitz torque

MUMAX\textsuperscript{3} uses the following explicit form for the Landau-Lifshitz torque \cite{18, 20}:

\[
\vec{\tau}_{LL} = \gamma_{LL} \frac{1}{1 + \alpha^2} \left( \vec{m} \times \vec{B}_{\text{eff}} + \alpha \left( \vec{m} \times \left( \vec{m} \times \vec{B}_{\text{eff}} \right) \right) \right) \tag{2}
\]

with $\gamma_{LL}$ the gyromagnetic ratio (rad/Ts), $\alpha$ the dimensionless damping parameter and $\vec{B}_{\text{eff}}$ the effective field (T). The default value for $\gamma_{LL}$ can be overridden by the user. $\vec{B}_{\text{eff}}$ has the following contributions:

- externally applied field $\vec{B}_{\text{ext}}$
- magnetostatic field $\vec{B}_{\text{demag}}$ (III B)
- Heisenberg exchange field $\vec{B}_{\text{exch}}$ (III C)
- Dzyaloshinskii-Moriya exchange field $\vec{B}_{\text{dm}}$ (III D)
- magneto-crystalline anisotropy field $\vec{B}_{\text{anis}}$ (III E)
- thermal field $\vec{B}_{\text{therm}}$ (III F).

Fig. 3 shows a validation of the Landau-Lifshitz torque for a single spin precessing without damping in a constant external field.

B. Magnetostatic field

\textit{Magnetostatic convolution} A finite difference discretization allows the magnetostatic field to be evaluated as a (discrete) convolution of the magnetization with a demagnetizing kernel $\vec{K}$:

\[
\vec{B}_{\text{demag}} = \sum_i \vec{K}_{ij} \vec{M}_j
\]

where $\vec{M} = M_{\text{sat}} \vec{m}$ is the unnormalized magnetization, with $M_{\text{sat}}$ the saturation magnetization (A/m). This calculation is FFT-accelerated based on the well-known convolution theorem. The corresponding energy density is provided as:

\[
\mathcal{E}_{\text{demag}} = -\frac{1}{2} \vec{M} \cdot \vec{B}_{\text{demag}}
\]

FIG. 3: Validation of Eq. 2 for a single spin precessing without damping in a 0.1 T field along $z$, perpendicular to $\vec{m}$. Analytical solution: $m_x = \cos(0.1T\gamma_{LL} t)$.
TABLE I: Demagnetizing factors \( N_{ij} = H_i/M_j \) calculated for a cube discretized in the smallest possible number of cells with given aspect ratio along \( z \). The results lie close to the analytical value of 1/3, even for very elongated (aspect\( > 1 \)) or flat (aspect\( < 1 \)) cells. The off-diagonal elements (not shown) are consistent with zero within the single-precision limit.

| aspect | \( N_{xx} \) | \( N_{yy} \) | \( N_{zz} \) |
|--------|--------------|--------------|--------------|
| 8/1    | -0.333207    | -0.333207    | -0.333176    |
| 4/1    | -0.333149    | -0.333149    | -0.333144    |
| 2/1    | -0.333118    | -0.333118    | -0.333118    |
| 1/1    | -0.333372    | -0.333372    | -0.333372    |
| 1/4    | -0.333146    | -0.333146    | -0.333145    |
| 1/16   | -0.333176    | -0.333176    | -0.333280    |
| 1/64   | -0.333052    | -0.333052    | -0.333639    |

**Magnetostatic kernel** We construct the demagnetizing kernel \( \hat{K} \) assuming constant magnetization\[27\] in each finite difference cell and we average the resulting \( \mathbf{B}_{\text{demag}} \) over the cell volumes. The integration is done numerically with the number of integration points automatically chosen based on the distance between source and destination cells and their aspect ratios. The kernel is initialized on CPU in double precision, and only truncated to single before transferring to GPU.

The kernel’s mirror symmetries and zero elements are exploited to reduce storage and initialization time. This results in a 9 \( \times \) or 12 \( \times \) decrease in kernel memory usage for 2D and 3D simulations respectively, and is part of the reason for MuMax\(^3\)’s relatively low memory requirements (Section VII).

**Accuracy** The short-range accuracy of \( \hat{K} \) is tested by calculating the demagnetizing factors of a uniformly magnetized cube, analytically known to be -1/3 in each direction. The cube was discretized in cells with varying aspect ratios along \( z \) to stress the numerical integration scheme. The smallest possible number of cells was used to ensure that the short-range part of the field has an important contribution. The results presented in Table I are accurate to 3 or 4 digits. Standard Problem #2 (V B) is another test sensitive to the short-range kernel accuracy\[14\].

The long-range accuracy of the magnetostatic convolution is assessed by comparing kernel and the field of a single magnetized cell to the corresponding point dipole. The fields presented in Fig. 4, show perfect long-range accuracy for the kernel, indicating accurate numerical integration in that range. The resulting field, obtained by convolution of a single magnetized cell (\( B_{\text{sat}} = 1 \) T) with the kernel, is accurate down to about 0.01 \( \mu \)T — the single-precision noise floor introduced by the FFT’s.
TABLE II: Out-of-plane demagnetizing factors for a thin film with grid size $2 \times 2 \times 1$ and 2D PBC’s $\times P$ (column 1) or without PBC’s but with a corresponding grid size $\times 2P$ (column 2). Both give comparable results for sufficiently large $P$, verifying the PBC implementation.

| PBC | grid | Nzz |
|-----|------|-----|
| $\times 1$ | $2$ | -0.71257 |
| $\times 4$ | $8$ | -0.93879 |
| $\times 16$ | $32$ | -0.98438 |
| $\times 64$ | $128$ | -0.99514 |
| $\times 256$ | $512$ | -0.99713 |
| $\infty$ | $\infty$ | -1 |

TABLE III: in-plane demagnetizing factors for a long rod with grid size $1 \times 1 \times 2$ and 1D PBC’s $\times P$ (column 1) or without PBC’s but with a corresponding grid size $\times 2P$ (column 2). Both give comparable results for sufficiently large $P$, verifying the PBC implementation.

| PBC | grid | Nxx |
|-----|------|-----|
| $\times 1$ | $2$ | -0.251960 |
| $\times 4$ | $8$ | -0.476766 |
| $\times 16$ | $32$ | -0.498280 |
| $\times 64$ | $128$ | -0.499517 |
| $\times 256$ | $512$ | -0.499590 |
| $\infty$ | $\infty$ | -0.5 |

Periodic boundary conditions  MuMaX3 provides optional periodic boundary conditions (PBCs) in each direction. PBCs imply magnetization wrap-around in the periodic directions, felt by stencil operations like the exchange interaction. A less trivial consequence is that the magnetostatic field of repeated magnetization images has to be added to $\vec{B}_{\text{demag}}$.

In contrast to OOMMF’s PBC implementation[22], MuMaX3 employs a so-called macro geometry approach[16, 17] where a finite (though usually large) number of repetitions is taken into account, and that number can be freely chosen in each direction. MuMaX3’s setPBC($P_x$, $P_y$, $P_z$) command enables $P_x$, $P_y$, $P_z$ additional images on each side of the simulation box, given that $P$ is sufficiently large.

To test the magnetostatic field with PBC’s, we calculate the demagnetizing tensors of a wide film and long rod in two different ways: either with a large grid without PBC’s, or with a small grid but with PBC’s equivalent to the larger grid. In our implementation, a gridsize $(N_x, N_y, N_z)$ with PBC’s $(P_x, P_y, P_z)$ should approximately correspond to a gridsize $(2P_xN_x, 2P_yN_y, 2P_zN_z)$ without PBC’s. This is verified in tables II and III where we extend in plane for the film and along $z$ for the rod. Additionally, for very large sizes both results converge to the well-known analytical values for infinite geometries.

C. Heisenberg exchange interaction

The effective field due to the Heisenberg exchange interaction [10]:

$$\vec{B}_{\text{exch}} = 2 \frac{A_{\text{ex}}}{M_{\text{sat}}} \Delta \vec{m}$$

is evaluated using a 6-neighbor small-angle approximation[12, 13]:

$$\vec{B}_{\text{exch}} = 2 \frac{A_{\text{ex}}}{M_{\text{sat}}} \sum_i \frac{(\vec{m}_i - \vec{m})}{\Delta_i^2}$$
relative error
spin-spin angle (deg)
0
20
40
60
80
100
120
140
160

E/KmV (dimensionless)
mumax (small-angle)
exact (all angles)

FIG. 5: Numerical (Eq.6,8) and analytical (Eq.7) exchange energy denisty (in units $K_m = 1/2\mu_0 M_{sat}^2$) for spiral magnetizations as a function of the angle between neighboring spins (independent of material parameters). To ensure an accurate energy, spin-spin angles should be kept below $\approx 20–30^\circ$ by choosing a sufficiently small cell size.

where $i$ ranges over the six nearest neighbors of the central cell with magnetization $\vec{m}$. $\Delta_i$ is the cell size in the direction of neighbor $i$.

At the boundary of the magnet some neighboring magnetizations $\vec{m}_i$ are missing. In that case we use the cell’s own value $\vec{m}$ instead of $\vec{m}_i$, which is equivalent to employing Neumann boundary conditions [12, 13].

The corresponding energy density is provided as:

$$E_{exch} = A_{ex} (\nabla \vec{m})^2$$

$$= -\frac{1}{2} \vec{M} \cdot \vec{B}_{exch}$$

MuMax$^3$ calculates the energy from the effective field using Eqs. 6, 8. The implementation is verified by calculating the exchange energy of a 1D magnetization spiral, for which the exact form (Eq.7) is easily evaluated. Fig. 5 shows that the linearized approximation is suited as long as the angle between neighboring magnetizations is not too large. This can be achieved by choosing a sufficiently small cell size compared to the exchange length.

**Inter-region exchange** The exchange interaction between different materials deserves special attention. $A_{ex}$ and $M_{sat}$ are defined in the cell volumes, while Eq. 6 requires a value of $A_{ex}/M_{sat}$ properly averaged out between the neighboring cells. For neighboring cells with different material parameters $A_{ex1}$, $A_{ex2}$ and $M_{sat1}$, $M_{sat2}$ MuMax$^3$ uses a harmonic mean:

$$\vec{B}_{exch} = 2S \frac{2 A_{ex1} A_{ex2}}{A_{ex1} M_{sat1} + A_{ex2} M_{sat2}} \sum_i \frac{(\vec{m}_i - \vec{m})}{\Delta_i^2}$$

which can easily be derived, and where we set $S = 1$ by default. $S$ is an arbitrary scaling factor which may be used to alter the exchange coupling between regions, e.g., to lower the coupling between grains or antiferromagnetically couple two layers.

**D. Dzyaloshinskii-Moriya interaction**

MuMax$^3$ provides induced Dzyaloshinskii-Moriya interaction for thin films with out-of-plane symmetry breaking according to [7], yielding an effective field term:
FIG. 6: Simulated domain wall magnetization in a 250 nm wide, 0.6 nm thick Co/Pt film ($M_{\text{sat}}=1100\times10^3$ A/m, $A_{\text{ex}}=16\times10^{-12}$ J/m, $K_{u1}=1.27\times10^6$ J/m$^3$) as a function of the Dzyaloshinskii-Moriya strength $D_{\text{ex}}$. The left-hand and right-hand sides correspond to a Bloch and Néel wall, respectively. Results correspond well to [35].

\[ \vec{B}_{\text{DM}} = \frac{2D}{M_{\text{sat}}} \left( \frac{\partial m_z}{\partial x}, \frac{\partial m_z}{\partial y}, -\frac{\partial m_x}{\partial x} - \frac{\partial m_y}{\partial y} \right) \] (10)

where we apply boundary conditions[30]:

\[ \frac{\partial m_z}{\partial x} \bigg|_{\partial V} = \frac{D}{2A} m_x \] (11)

\[ \frac{\partial m_z}{\partial y} \bigg|_{\partial V} = \frac{D}{2A} m_y \] (12)

\[ \frac{\partial m_x}{\partial x} \bigg|_{\partial V} = \frac{D}{2A} m_y \] (13)

\[ \frac{\partial m_x}{\partial y} \bigg|_{\partial V} = \frac{D}{2A} m_z \] (14)

\[ \frac{\partial m_z}{\partial z} \bigg|_{\partial V} = \frac{\partial m_y}{\partial z} \bigg|_{\partial V} = \frac{\partial m_x}{\partial z} \bigg|_{\partial V} = 0 \] (15)

Numerically, all derivatives are implemented as central derivatives, i.e., the difference between neighboring magnetizations over their distance in that direction: $\partial \vec{m}/\partial i = (\vec{m}_{i+1} - \vec{m}_{i-1})/(2\Delta_i)$. When a neighbor is missing at the boundary ($\partial V$), its magnetization is replaced by $\vec{m} + \frac{\partial m}{\partial i} \big|_{\partial V} \Delta_i \hat{n}$ where $\vec{m}$ refers to the central cell and the relevant partial derivative is selected from Eq. 11–15.

In case of nonzero $D$, these boundary conditions are simultaneously applied to the Heisenberg exchange field.

The effective field in Eq.10 gives rises to an energy density:

\[ E_{\text{exch}(\text{DM})} = m_z (\nabla \cdot \vec{m}) - (\vec{m} \cdot \nabla)m_z \] (16)

\[ = -\frac{1}{2} \vec{M} \cdot \vec{B}_{\text{exch}(\text{DM})} \] (17)

Similar to the anisotropic exchange case, MuMax3 calculates the energy density from Eqns.17, 10. Eq.16 is the exact form, well approximated for sufficiently small cell sizes.

In Fig.6, the DMI implementation is compared to the work of Thiaville et al. [35], where the transformation of a Bloch wall into a Néel wall by varying $D_{\text{ex}}$ is studied.
FIG. 7: Uniaxial (top) and cubic (bottom) anisotropy energy density of a single spin as a function of its orientation in the $xy$-plane. The uniaxial axis is along $x$, the cubic axes along $x$, $y$ and $z$. The dots are computed with MuMax$^3$ (Eq.20,23), lines are analytical expressions (Eq.19,22). Positive and negative $K$ values denote hard and easy anisotropy, respectively.

E. Magneto-crystalline anisotropy

**Uniaxial** MuMax$^3$ provides uniaxial magneto-crystalline anisotropy in the form of an effective field term:

$$
\mathbf{B}_{\text{anis}} = \frac{2K_{u1}}{B_{\text{sat}}} (\mathbf{\hat{u}} \cdot \mathbf{\hat{m}}) \mathbf{\hat{u}} + \frac{4K_{u2}}{B_{\text{sat}}} (\mathbf{\hat{u}} \cdot \mathbf{\hat{m}})^3 \mathbf{\hat{u}}
$$

(18)

where $K_{u1}$ and $K_{u2}$ are the first and second order uniaxial anisotropy constants and $\mathbf{\hat{u}}$ a unit vector indicating the anisotropy direction. This corresponds to an energy density:

$$
\mathcal{E}_{\text{anis}} = -K_{u1} (\mathbf{\hat{u}} \cdot \mathbf{\hat{m}})^2 - K_{u2} (\mathbf{\hat{u}} \cdot \mathbf{\hat{m}})^4
$$

(19)

$$
= -\frac{1}{2} \mathbf{B}_{\text{anis}}(K_{u1}) \cdot \mathbf{M} - \frac{1}{4} \mathbf{B}_{\text{anis}}(K_{u2}) \cdot \mathbf{M}
$$

(20)

MuMax$^3$ calculates the energy density from the effective field using Eq.20, where $\mathbf{B}_{\text{anis}}(K_{ui})$ denotes the effective field term where only $K_{ui}$ is taken into account. The resulting energy is verified in Fig. 7. Since the energy is derived directly from the effective field, this serves as a test for the field as well.

**Cubic** MuMax$^3$ provides cubic magneto-crystalline anisotropy in the form of an effective field term:
\[ B_{\text{anis}} = -2K_{c1}/M_{\text{sat}}( (\hat{c}_2 \cdot \hat{m})^2 + (\hat{c}_3 \cdot \hat{m})^2)(\hat{c}_1 \cdot \hat{m})\hat{c}_1) + (\hat{c}_1 \cdot \hat{m})^2((\hat{c}_3 \cdot \hat{m})^2)(\hat{c}_2 \cdot \hat{m})\hat{c}_2 + (\hat{c}_1 \cdot \hat{m})^2((\hat{c}_2 \cdot \hat{m})^2)(\hat{c}_3 \cdot \hat{m})\hat{c}_3 ) \]
\[ -2K_{c2}/M_{\text{sat}}( (\hat{c}_2 \cdot \hat{m})^2(\hat{c}_3 \cdot \hat{m})^2)(\hat{c}_1 \cdot \hat{m})\hat{c}_1) + (\hat{c}_1 \cdot \hat{m})^2(\hat{c}_3 \cdot \hat{m})^2)(\hat{c}_2 \cdot \hat{m})\hat{c}_2 + (\hat{c}_1 \cdot \hat{m})^2(\hat{c}_2 \cdot \hat{m})^2)(\hat{c}_3 \cdot \hat{m})\hat{c}_3 ) \]
\[ -4K_{c3}/M_{\text{sat}}( (\hat{c}_2 \cdot \hat{m})^4 + (\hat{c}_3 \cdot \hat{m})^4)((\hat{c}_1 \cdot \hat{m})^2\hat{c}_1) + (\hat{c}_1 \cdot \hat{m})^4(\hat{c}_3 \cdot \hat{m})^4)((\hat{c}_2 \cdot \hat{m})^2\hat{c}_2 + (\hat{c}_1 \cdot \hat{m})^4(\hat{c}_2 \cdot \hat{m})^4)((\hat{c}_3 \cdot \hat{m})^2\hat{c}_3 ) \] (21)

where \( K_{cn} \) is the \( n \)th-order cubic anisotropy constant and \( \hat{c}_1, \hat{c}_2, \hat{c}_3 \) a set of mutually perpendicular unit vectors indicating the anisotropy directions. (The user only specifies \( \hat{c}_1 \) and \( \hat{c}_2 \). We compute \( \hat{c}_3 \) automatically as \( \hat{c}_1 \times \hat{c}_2 \).) This corresponds to an energy density:

\[ E_{\text{anis}} = \]
\[ K_{c1} ((\hat{c}_1 \cdot \hat{m})^2(\hat{c}_2 \cdot \hat{m})^2 + (\hat{c}_1 \cdot \hat{m})^2(\hat{c}_3 \cdot \hat{m})^2 + (\hat{c}_2 \cdot \hat{m})^2(\hat{c}_4 \cdot \hat{m})^2) + K_{c2} (\hat{c}_1 \cdot \hat{m})^2(\hat{c}_2 \cdot \hat{m})^2(\hat{c}_4 \cdot \hat{m})^2 + K_{c3} (\hat{c}_1 \cdot \hat{m})^4(\hat{c}_2 \cdot \hat{m})^4 + (\hat{c}_1 \cdot \hat{m})^4(\hat{c}_3 \cdot \hat{m})^4 + (\hat{c}_2 \cdot \hat{m})^4(\hat{c}_3 \cdot \hat{m})^4 ) \] (22)

which, just like in the uniaxial case, MuMax\(^3\) computes using the effective field:

\[ E_{\text{anis}} = -\frac{1}{4} B_{\text{anis}}(K_{c1}) \cdot \hat{M} - \frac{1}{6} B_{\text{anis}}(K_{c2}) \cdot \hat{M} - \frac{1}{8} B_{\text{anis}}(K_{c3}) \cdot \hat{M} \] (23)

which is verified in Fig. 7.

F. Thermal fluctuations

MuMax\(^3\) provides finite temperature by means of a fluctuating thermal field \( \hat{B}_{\text{therm}} \) according to Brown[9]:

\[ \hat{B}_{\text{therm}} = \bar{\eta}(\text{step}) \sqrt{\frac{2\mu_0 k_B T}{B_{\text{sat}} \gamma_{LL} \Delta V \Delta t}} \] (24)

where \( \alpha \) is the damping parameter, \( k_B \) the Boltzmann constant, \( T \) the temperature, \( B_{\text{sat}} \) the saturation magnetization expressed in Tesla, \( \gamma_{LL} \) the gyromagnetic ratio (1/\( T_s \)), \( \Delta V \) the cell volume, \( \Delta t \) the time step and \( \bar{\eta}(\text{step}) \) a random vector from a standard normal distribution whose value is changed after every time step.

Solver constraints \( \hat{B}_{\text{therm}} \) randomly changes in between time steps. Therefore, only MuMax\(^3\)’s Euler and Heun solvers (IV) can be used as they do not require torque continuity in between steps. Additionally, with thermal fluctuations enabled we enforce a fixed time step \( \Delta t \). This avoids feedback issues with adaptive time step algorithms.
FIG. 8: Arrhenius plot of the thermal switching rate of a 10 nm large cubic particle (macrospin), with $M_{\text{sat}}=1\text{MA/m}$, $\alpha=0.1$, $\Delta t=\times10^{-12}\text{s}$, $K_{u1}=1\times10^4$ or $2\times10^4\text{ J/m}^3$. Simulations were performed on an ensemble of $512^2$ uncoupled particles for $0.1\mu\text{s}$ (high temperatures) or $1\mu\text{s}$ (low temperatures). Solid lines are the analytically expected switching rates (Eq.25).

**Verification** We test our implementation by calculating the thermal switching rate of a single (macro-)spin particle with easy uniaxial anisotropy. In the limit of a high barrier compared to the thermal energy, the switching rate $f$ is known analytically to be [8]:

$$f = \gamma_{LL} \frac{\alpha}{1 + \alpha^2} \sqrt{\frac{8K_{u1}^3V}{2\pi M_{\text{sat}}^2kT}} e^{-K_{u1}/kT}$$  \hspace{1cm} (25)

Fig. 8 shows Arrhenius plots for the temperature-dependent switching rate of a particle with volume $V=10(\text{nm})^3$ and $K_{u1}=1\times10^4$ or $2\times10^4\text{ J/m}^3$. The MuMax$^3$ simulations correspond well to Eq.25.

**G. Zhang-Li Spin-transfer torque**

MuMax$^3$ includes a spin-transfer torque term according to Zhang and Li [38], applicable when electrical current flows through more than one layer of cells:

$$\vec{\tau}_{ZL} = \frac{1}{1 + \alpha^2} ((1 + \xi\alpha) \vec{m} \times (\vec{u} \cdot \nabla)\vec{m}) +$$
$$((\xi - \alpha) \vec{m} \times (\vec{u} \cdot \nabla)\vec{m})$$

$$\vec{u} = \frac{\mu_B \mu_0}{2e\gamma_0 B_{\text{sat}}} (1 + \xi^2) \vec{j}$$  \hspace{1cm} (26)

where $\vec{j}$ is the current density, $\xi$ is the degree of non-adiabaticity, $\mu_B$ the Bohr magneton and $B_{\text{sat}}$ the saturation magnetization expressed in Tesla.

The validity of our implementation is tested by Standard Problem #5 (Section V.E).

**H. Slonczewski Spin-transfer torque**

MuMax$^3$ provides a spin momentum torque term according to Slonczewski[33, 37], transformed to the Landau-Lifshitz formalism:

$$\vec{\tau}_{SL} = \beta \frac{\epsilon - \alpha \epsilon'}{1 + \alpha^2} (\vec{m} \times (\vec{m}_P \times \vec{m}))$$

$$-\beta \frac{\epsilon' - \alpha \epsilon}{1 + \alpha^2} \vec{m} \times \vec{m}_P$$

$$\beta = \frac{j_z \hbar}{M_{\text{sat}}^2 e d}$$  \hspace{1cm} (28)

$$\epsilon = \frac{P(\vec{r}, t) \Lambda^2}{(\Lambda^2 + 1) + (\Lambda^2 - 1)(\vec{m} \cdot \vec{m}_P)}$$

\hspace{1cm} (29)

\hspace{1cm} (30)
where $j_z$ is the current density along the $z$ axis, $d$ is the free layer thickness, $\vec{m}_P$ the fixed-layer magnetization, $P$ the spin polarization, the Slonczewski $\Lambda$ parameter characterizes the spacer layer, and $\epsilon'$ is the secondary spin-torque parameter.

**MuMax** only explicitly models the free layer magnetization. The fixed layer is handled in the same way as material parameters and is always considered to be on top of the free layer. The fixed layer’s stray field is not automatically taken into account, but can be pre-computed by the user and added as a space-dependent external field term.

As a verification we consider switching an MRAM bit in a 160 nm × 80 nm × 5 nm Permalloy ($M_{\text{sat}}=800 \times 10^3 \text{A/m}$, $A_{\text{ex}}=13 \times 10^{-12} \text{J/m}^2$, $\alpha=0.01$, $P = 0.5669$, $j_z=4.6875 \times 10^{11} \text{A}$, $\Lambda=2$, $\epsilon'=1$) by a total current of -6 mA along the $z$ axis using $\Lambda=2$, $\epsilon'=1$. These parameters were chosen so that none of the terms in Eq.28 are zero. The fixed layer is polarized at 20$^\circ$ from the $x$ axis to avoid symmetry problems and the initial magnetization was chosen uniform along $x$. The **MuMax** and OOMMF results shown in Fig. 9 correspond well.

**IV. TIME INTEGRATION**

**A. Dynamics**

**MuMax** provides a number of explicit Runge-Kutta methods for advancing the Landau-Lifshitz equation (Eq.2):

- **RK45**, the Dormand-Prince method, offers 5-th order convergence and a 4-th order error estimate used for adaptive time step control. This is the default for dynamical simulations.
- **RK32**, the Bogacki-Shampine method, offers 3-th order convergence and a 2nd order error estimate used for adaptive time step control. This method is used when relaxing the magnetization to its ground state in which case it performs better than RK45.
- **RK12**, Heun’s method, offers 2nd order convergence and a 1st order error estimate. This method is used for finite temperature simulations as it does not require torque continuity in between time steps.
- **RK1**, Euler’s method is provided for academic purposes.

These solvers’ convergence rates are verified in Fig. 10, which serves as a test for their implementation and performance. **Adaptive time step**: RK45, RK23 and RK12 provide adaptive time step control, i.e., automatically choosing the time step to keep the error per step $\epsilon$ close to a preset value $\epsilon_0$. As the error per step we use $\epsilon = \max|\tau_{\text{high}} - \tau_{\text{low}}| \Delta t$, with $\tau_{\text{high}}$ and $\tau_{\text{low}}$ high-order and low-order torque estimates provided by the particular Runge-Kutta method, and $\Delta t$ the time step. The time step is adjusted using a default headroom of 0.8.

In **MuMax**, $\epsilon_0$ is accessible as the variable MaxErr. Its default value of $10^{-5}$ was adequate for the presented standard problems. The relation between $\epsilon_0$ and the overall error at the end of the simulation is in general hard to determine. Nevertheless, we illustrate this in Fig. 11 for a single period of spin precession under the same conditions as Fig.10. It can be seen that the absolute error per precession scales linearly with $\epsilon_0$, although the absolute value of

![FIG. 9: Verification of the Slonczewski torque: average magnetization in a 160 nm × 80 nm × 5 nm rectangle with $M_{\text{sat}}=800 \times 10^3 \text{A/m}$, $A_{\text{ex}}=13 \times 10^{-12} \text{J/m}^2$, $\alpha=0.01$, $P = 0.5669$, $j_z=4.6875 \times 10^{11} \text{A}$, $\Lambda=2$, $\epsilon'=1$, $\vec{m}_P=(\cos(20^\circ), \sin(20^\circ), 0)$, initial $m=(1,0,0)$. Solid line calculated with OOMMF, points by MuMax.](image-url)
the error depends on the solver type and exact simulation conditions.

### B. Energy minimization

MuMax\(^3\) provides a `relax()` function that attempts to find the systems’ energy minimum. This function disables the precession term Eq.2, so that the effective field points towards decreasing energy. `Relax` first advances in time until the total energy cuts into the numerical noise floor. At that point the state will be close to equilibrium already. We then begin monitoring the magnitude of the torque instead of the energy, since close to equilibrium the torque will decrease monotonically and is less noisy than the energy. So we advance further until the torque cuts into the noise floor as well. Each time that happens, we decrease `MaxErr` and continue further until `MaxErr=10^{-9}`. At this point it does not make sense to increase the accuracy anymore (see Fig.11) and we stop advancing.

This `Relax` procedure was used in the presented standard problems, where it proved adequate. Typical residual torques after `Relax` are of the order of $10^{-4}–10^{-7}\gamma LL T$, indicating that the system is indeed very close to equilibrium. Nevertheless, as with any energy minimization technique, there is always a possibility that the system ends up in a saddle point or very flat part of the energy landscape.

`Relax` internally uses the RK23 solver, which we noticed performs better then RK45 in most relaxation scenarios. Near equilibrium, both solvers tend to take similarly large time steps, but RK23 needs only half as many torque evaluations per step as RK45.

---

**FIG. 10:** Absolute error on a single spin after precessing without damping for one period in a 0.1 T field, as a function of different solver’s time steps. The errors follow 1st, 2nd, 3rd or 5th order convergence (solid lines) for the respective solvers down to a limit set by the single precision arithmetic.

**FIG. 11:** Absolute error on a single spin after precessing without damping for one period in a 0.1 T field, as a function of different solver’s MaxErr settings. Solid lines represent 1st order fits. The same lower bound as in Fig. 10 is visible.
V. STANDARD PROBLEMS

In this section we provide solutions to micromagnetic standard problems #1–4 provided by the µMag modeling group[2] and standard problem #5 proposed by Najafi et al. [28]. Reference solutions were taken from[2] as noted, or otherwise calculated with OOMMF 1.2 alpha 5 bis[15].

A. Standard Problem #1

The first µMag standard problem involves the hysteresis loops of a 1µm × 2µm × 20 nm Permalloy rectangle ($A_{ex} = 1.3\times10^{-11}\text{J/m}$, $M_{sat} = 8\times10^5\text{A/m}$, $K_{u1} = 5\times10^2\text{J/m}^3$ uniaxial, with easy axis nominally parallel to the long edges of the rectangle) for the field approximately parallel to the long and short axis, respectively. Our solution is presented in Fig.12. Unfortunately the submitted solutions[2] do not agree with each other, making it impossible to assert the correctness in a quantitative way.

B. Standard Problem #2

The second µMag standard problem considers a thin film of width $d$, length 5$d$ and thickness 0.1$d$, all expressed in terms of the exchange length $l_{ex} = \sqrt{2A_{ex}/\mu_0M_{sat}^2}$. The remanence and coercitive field, expressed in units $M_{sat}$, are to be calculated as a function of $d/l_{ex}$.

The coercivity, shown in Fig.14, behaves interestingly in the small-particle limit where an analytical solution exists[14]. In that case the magnetization is uniform and the magnetostatic field dominates the behaviour. Of the solutions submitted to the µMag group [14, 24, 26, 34], the Streibl[34], Donahue[14] (OOMMF 1.1) and MuMax$^3$ results best approach the small-particle limit. It was shown by Donahue et al. [14] that proper averaging of the magnetostatic field over each cell volume is needed to accurately reproduce the analytical limit. Hence this standard problem serves as a test for the numerical integration of our demagnetizing kernel.

C. Standard Problem #3

Standard problem #3 considers a cube with edge length $L$ expressed in exchange lengths $l_{ex} = \sqrt{2A_{ex}/\mu_0M_{sat}^2}$. The magnet has uniaxial anisotropy with $K_{u1} = 0.1K_m$, with $K_m = 1/2\mu_0M_{sat}^2$, easy axis parallel to the z-axis. The
FIG. 13: Remanence for standard problem #2 as a function of the magnet size $d$ expressed in exchange lengths $l_{ex}$. The MuMax$^3$ calculations (points) use automatically chosen cell sizes between 0.25 and 0.5$l_{ex}$. OOMMF results (line) were taken from[2].

FIG. 14: Coercivity for standard problem #2 as a function of the magnet size $d$ expressed in exchange lengths $l_{ex}$. MuMax$^3$ calculations (points) use automatically chosen cell sizes between 0.25 and 0.5$l_{ex}$. OOMMF results (line) taken from[2]. The slight discrepancy at high $d$ is attributed to OOMMF’s solution using larger cells there. The analytical limit for very small size is by Donahue et al. [14].

critical edge size $L$ where the ground state transitions between a quasi-uniform and vortex-like state needs to be found, it is expected around $L$=8.

This problem was solved using a 16 × 16 × 16 grid. The cube was initialized with $\propto$3,000 different random initial magnetization states for random edge lengths $L$ between 7.5 and 9, and relaxed to equilibrium. Four stable states were found, shown in Fig.15: a quasi-uniform flower state (a), twisted flower state (b), vortex state (c) and a canted vortex (d). Then cubes of different sizes were initialized to these states and relaxed to equilibrium. The resulting energy for each state, shown in Fig.15, reveals the absolute ground states in the considered range: flower state for $L<8.16$, twisted flower for $8.16<L<8.47$ and vortex for $L>8.47$.

The transition at $L$=8.47 is in quantitative agreement with the solutions posted to µMag by Rave et al. [29] and by Martins et al. [2]. The existence of the twisted flower state was already noted by Hertel et al. [2], although without determining the flower to twisted flower transition point.

D. Standard Problem #4

Standard problem #4 considers dynamic magnetization reversal in a 500 nm × 125 nm × 3 nm Permalloy magnet ($A_{ex}$=1.3×10$^{-11}$ J/m, $M_{sat}$=8×10$^5$ A/m). The initial state is an S-state obtained after saturating along the (1,1,1) direction. Then the magnet is reversed by either field (a): (-24.6, 4.3, 0) mT or field (b): (-35.5, -6.3, 0) mT. Time-dependent average magnetizations should be given, as well as the space-dependent magnetization when $<m_x>$ first crosses zero.
FIG. 15: Standard problem #3: energy densities of the flower (a), twisted flower (b), vortex (c) and canted vortex (d) states as a function of the cube edge length $L$. Transitions of the ground state are marked with vertical lines at $L = 8.16$ and $L = 8.47$.

FIG. 16: MuMax$^3$ (dots) and OOMMF (lines) solution to standard problem #4a (top graph) and #4b (bottom graph), as well as space-dependent magnetization snapshots when $<m_x>$ crosses zero, for fields (a) and (b). All use a $200 \times 50 \times 1$ grid.

Our solution, shown in Fig.16, agrees with OOMMF.

E. Standard Problem #5

Standard problem #5 proposed by Najafi et al. [28] considers a 100 nm $\times$ 100 nm $\times$ 10 nm Permalloy square ($A = 13 \times 10^{-12}$ J/m, $M_{\text{sat}} = 8 \times 10^5$ A/m, $\alpha = 0.1$, $\xi = 0.05$) with an initial vortex magnetization. A homogeneous current $j = 10^{12}$ Am$^{-2}$ along $x$, applied at $t = 0$ drives the vortex towards a new equilibrium state. The obtained time-dependent average magnetization, shown in Fig.17, agrees well the OOMMF solution.
VI. EXTENSIONS

**MuMax**$^3$ is designed to be modular and extensible. Some of our extensions, described below, have been merged into the mainline code because they may be of general interest. Nevertheless, extensions are considered specific to certain needs and are less generally usable than the aforementioned main features. E.g., MFM images and Voronoi tessellation are only implemented in 2D and only qualitatively tested.

### A. Moving frame

**MuMax**$^3$ provides an extension to translate the magnetization with respect to the finite difference grid (along the $x$-axis), inserting new values from the side. This allows the simulation window to seemingly "follow" a region of interest like domain wall moving in a long nanowire, without having to simulate the entire wire. **MuMax**$^3$ can automatically translate the magnetization to keep an average magnetization component of choice as close to zero as possible, or the user may arbitrarily translate $\mathbf{m}$ from the input script.

When following a domain wall in a long in-plane magnetized wire, we also provide the possibility to remove the magnetic charges on the ends of the wire. This simulates an effectively infinitely long wire without closure domains, as illustrated in Fig.18.

Finally, when shifting the magnetization there is an option to also shift the material regions and geometry along. The geometry and material parameters for the "new" cells that enter the simulation from the side are automatically re-calculated so that new grains and geometrical features may seamlessly enter the simulation. This can be useful for, e.g., simulating a long racetrack with notches like illustrated in Fig.18, or a moving domain wall in a grainy material as published in [23].

### B. Voronoi Tessellation

**MuMax**$^3$ provides 2D Voronoi Tessellation as a way to simulate grains in thin films, similar to OOMMF[21]. It is possible to have **MuMax**$^3$ set-up the regions map with grain-shaped islands, randomly colored with up to 256 region numbers (Fig.19(a)). The material parameters in each of those regions can then be varied to simulate, e.g., grains with randomly distributed anisotropy axes or even change the exchange coupling between them (Fig.19(b)).

Our implementation is compatible with the possibility to move the simulation window. E.g., when the simulation window is following a moving domain wall, new grains will automatically enter the simulation from the sides. The new grains are generated using hashes of the cell coordinates so that there is no need to store a (potentially very large) map of all the grains beyond the current simulation grid. More details can be found in[23].
FIG. 18: Top frame: magnetization in a 1 µm wide, 20 nm thick Permalloy wire of finite length. The remaining frames apply edge charge removal to simulate an infinitely long wire. The domain wall is driven by a $3 \times 10^{12}$ A/m$^2$ current while being followed by the simulation window. So that it appears steady although moving at high speed (visible by the wall breakdown). While moving, new notches enter the simulation from the right.

FIG. 19: Example of a Voronoi tessellation with average 100 nm grains in a 2048 µm wide disk. Left: cells colored by their region index (0–256). Right: boundaries between the grains visualized by reducing the exchange coupling between them (Eq.9), and outputting MuMax$^3$’s ExchCoupling quantity, the average $M_{sat}/A_{ex}$ around each cell.

C. Magnetic force microscopy

MuMax$^3$ has a built-in capability to generate magnetic force microscopy (MFM) images in Dynamic (AC) mode from a 2D magnetization. We calculate the derivative of the force between tip and sample from the convolution:

$$\frac{\partial F_z}{\partial z} = \sum_{i=x,y,z} M_i(x, y) \ast \frac{\partial^2 B_{\text{tip},i}(x, y)}{\partial z^2}$$

(31)

where $\vec{B}_{\text{tip}}$ is the tip’s stray field evaluated in the sample plane. MuMax$^3$ provides the field of an idealized dipole or monopole tip with arbitrary elevation. No attempt is made to reproduce tip fields in absolute terms as our only goal is to produce output proportional to the actual MFM contrast, like shown in Fig.20.

Eq. 31 is implemented using FFT-acceleration similar to the magnetostatic field, and is evaluated on the GPU. Hence MFM image generation is very fast and generally takes only a few milliseconds. This makes it possible to watch ”real-time” MFM images in the web interface while the magnetization is evolving in time.
FIG. 20: (a) vortex magnetization in a 750 nm × 750 nm × 10 nm Permalloy square. (b), (c) are MuMax\(^3\)-generated MFM images at 50 nm and 100 nm lift height respectively, both using AC mode and a monopole tip model.

FIG. 21: MuMax\(^3\) throughput on GTX TITAN GPU, for all \(N \times N\) grid sizes up to 1024 \(\times\) 1024. Numbers with only factors 2,3,5,7 are marked with an open box, pure powers of two (corresponding to Fig.22) with a full box. Proper grid sizes should be chosen to ensure optimal performance.

VII. PERFORMANCE

A. Simulation size

Nowadays, GPU’s offer massive computational performance of several TFlop/s per device. However, that compute power is only fully utilized in case of sufficient parallelization, i.e., for sufficiently large simulations. This is clearly illustrated by considering how many cells can be processed per second. I.e., \(N_{\text{cells}}/t_{\text{update}}\) with \(t_{\text{update}}\) the time needed to calculate the torque for \(N_{\text{cells}}\) cells. We refer to this quantity as the throughput. Given the overall complexity of \(\mathcal{O}(N \log(N))\) one would expect a nearly constant throughput that slowly degrades at high \(N\). For all presented throughputs, magnetostatic and exchange interactions were enabled and no output was saved.

The throughput presented in Fig.21 for a square 2D simulation on a GTX TITAN GPU only exhibits the theoretical, nearly constant, behaviour starting from about 256 000 cells. Below, the GPU is not fully utilized and performance drops. Fortunately, large simulations are exactly where GPU speed-up is needed most and where performance is optimal.

MuMax\(^3\)’s performance is dominated by FFT calculations using the cuFFT library, which performs best for power-of-two sizes and acceptably for 7-smooth numbers (having only factors 2,3,5 and 7). Other numbers, especially primes should be avoided. This is clearly illustrated in Fig.21 where other than the recommended sizes show a performance penalty of up to about an order of magnitude. So somewhat oversizing the grid up to a nice smooth number may be beneficial to the performance.

Note that the data in Fig.22 is for a 2D simulation. Typically a 3D simulation with the same total number of cells costs an additional factor \(\propto 1.5\times\) in compute time and memory due to additional FFTs along the \(z\)-axis.

On the other hand, simulations with periodic boundary conditions will run considerably faster than their non-periodic counterparts. This is due to the absence of zero-padding which reduces FFT sizes by 2 in each periodic direction. Memory consumption will be considerably lower in this case as well.
B. Hardware

Apart from the simulation size, MuMax$^3$’s performance is strongly affected by the particular GPU hardware. We highlight the differences between several GPU models by comparing their throughput in Fig. 22. This was done for a 4 M cells simulation where all tested GPUs were fully utilized. So the numbers are indicative for all sufficiently large simulation sizes.

We also included OOMMF’s throughput on a quad-core 2.1 GHz core i7 CPU to give a rough impression of the GPU speed-up. The measured OOMMF performance (not clearly distinguishable in Fig. 22) was around $4 \times 10^6$ cells/s. So with a proper GPU and sufficiently large grid sizes, a speed-up of 20–45× with respect to a quad-core can be reached or, equivalently, a 80–180× speed-up compared to a single-core CPU. This is in line with earlier MuMax$^1$ and MicroMagnum benchmarks [5, 36]. It must be noted however that OOMMF operates in double-precision in contrast to MuMax$^3$’s single-precision arithmetic, and also does not suffer reduced throughput for simulations.

Finally, MicroMagnum’s throughput (not presented) was found to be nearly indistinguishable from MuMax$^3$. This is unsurprising since both MuMax$^3$’s MicroMagnum’s performance are dominated by CUDA’s FFT routines. In our benchmarks on a GTX650M, differences between both packages were comparable to the noise on the timings.

C. Memory use

In contrast to their massive computational power, GPUs are typically limited to rather small amounts of memory (currently 1–6 GB). Therefore, MuMax$^3$ was heavily optimized to use as little memory as possible. E.g., we exploited the magnetostatic kernel symmetries and zero elements and make heavy use of memory pooling and recycling.
Also, MuMax3 employs minimal zero-padding in the common situation of 3D simulations with only a small number of layers. For up to 10 layers there is no need to use a power of two, and memory usage will be somewhat reduced as well.

In this way, MuMax3 on a GPU with only 2 GB of memory is able to simulate about 9 million cells in 2D and 6 million in 3D, or about $2 \times 10^6$ more than MicroMagnum v0.2 [5] (see Fig.23). When using a lower-order solver this number can be further increased to $12 \times 10^6$ cells with RK23 (2D) or $16 \times 10^6$ cells with RK12(2D), all in 2 GB. Cards like the GTX TITAN and K20XM, with 6 GB RAM can store proportionally more, e.g., 31 M cells for 2D with the RK45 solver.

**VIII. CONCLUSION**

We have presented in detail the micromagnetic model employed by MuMax3, as well as a verification for each of its components. GPU acceleration provides a speed-up of 1–2 orders of magnitude compared to CPU-based micromagnetic simulations. In addition, MuMax3’s low memory requirements open up the possibility of very large-scale micromagnetic simulations, a regime where the GPU’s potential is fully utilized and where the speed-up is also needed most. E.g., depending on the solver type MuMax3 can fit 10–16 million cells in 2 GB GPU RAM — about $2 \times$ more than MuMax2 or MicroMagnum.

MuMax3 is open-source and designed to be easily extensible, so anybody can in principle add functionality. Some extensions like a moving simulation window, edge charge removal, Voronoi tessellation and MFM images have been permanently merged into MuMax3 and more extensions are expected in the future.

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Appendix A: Input scripts

1. Geometry (Fig. 2)

\[
\begin{align*}
\text{Nx} & := 256; \text{Ny} := 128; \text{Nz} := 32 \\
c & := 4e-9 \\
\text{setgridsize(Nx, Ny, Nz)} \\
\text{setcellsize(c, c, c)} \\
\text{s} & := \text{ellipsoid(c*Nx/2, c*Ny/2, c*Nz).transl(-Nx/7*c, 0, 0)} \\
\text{s} & = \text{s.add(rect(c*Nx/3, c*Ny/2).rotz(20*pi/180).transl(c*Nx/6, 0, 0))} \\
\text{setgeom(s)} \\
\text{msat} & = 800e3 \\
\text{aex} & = 13e-12 \\
\text{m} & = \text{uniform(1, 0, 0)} \\
\text{relax()} \\
\text{save(m)}
\end{align*}
\]

2. Precession (Fig. 3)

\[
\begin{align*}
\text{SetGridSize(1, 1, 1)} \\
\text{SetCellSize(1e-9, 1e-9, 1e-9)} \\
\text{m} & = \text{Uniform(1, 0, 0)} \\
\text{B_ext} & = \text{Vector(0, 0, 0.1)} \\
\text{EnableDemag} & = \text{false} \\
\text{TableAutosave(10e-12)} \\
\text{Run(3e-9)}
\end{align*}
\]

3. Cube demag tensor (Table I)

\[
\begin{align*}
\text{Msat} & = 1 / \mu_0 \quad // \ 1 \ \text{Tesla} \\
// \ \text{prolate cells} \\
\text{for} \ p := 0; \ p < 7; \ p++{ \\
\ \ \ \ \ \text{aspect} := \text{pow}(2, p) \\
\ \ \ \ \ \text{setGridSize(aspect, aspect, 1)} \\
\ \ \ \ \ \text{setCellSize(1/aspect, 1/aspect, 1)} \\
\ \ \ \ \ \text{m} & = \text{uniform(1, 0, 0)} \\
\ \ \ \ \ \text{Kx} & := \text{B_demag.Average()} \\
\ \ \ \ \ \text{m} & = \text{uniform(0, 1, 0)} \\
\ \ \ \ \ \text{Ky} & := \text{B_demag.Average()} \\
\ \ \ \ \ \text{m} & = \text{uniform(0, 0, 1)} \\
\ \ \ \ \ \text{Kz} & := \text{B_demag.Average()} \\
\ \ \ \ \ \text{fprintfn("result.txt", "1/", aspect, Kx.x(), Ky.y(), Kz.z())}
\}
\\
// \ \text{elongated cells} \\
\text{for} \ p := 0; \ p < 4; \ p++{ \\
\ \ \ \ \ \text{aspect} := \text{pow}(2, p) \\
\ \ \ \ \ \text{setGridSize(1, 1, aspect)} \\
\ \ \ \ \ \text{setCellSize(1, 1, 1/aspect)}
\}
m = uniform(1, 0, 0)
Kx := B_demag.Average()

m = uniform(0, 1, 0)
Ky := B_demag.Average()

m = uniform(0, 0, 1)
Kz := B_demag.Average()

fprintln("result.txt", aspect, "/1", Kx.x(), Ky.y(), Kz.z())

4. Long-range demag (Fig. 4)

setGridSize(512, 1, 1)
setCellSize(1e-9, 1e-9, 1e-9)
SetGeom(cell(0, 0, 0))
Msat = 1/mu0
m = uniform(1, 0, 0)
save(B_demag)

5. Sheet demag tensor with PBC (Table II)

c := 1e-9
SetCellSize(c, c, c)

Msat = 1/mu0
m = uniform(0, 0, 1)
N := 2

// small grid with PBC
for j := 0; j<10; j++{
  i := pow(2, j)
  SetGridSize(N, N, 1)
  SetPBC(i, i, 0)
  fprintln("pbc.txt", i, B_demag.Average())
}

// equivalent large grid without PBC
for j := 0; j<10; j++{
  i := pow(2, j)
  SetGridSize(i*2*N, i*2*N, 1)
  SetPBC(0, 0, 0)
  // evaluate over central slab
  defRegion(0, universe())
  defRegion(1, rect(c*N, c*N))
  fprintln("nopbc.txt", i, B_demag.Region(1).Average())
}

6. Rod demag tensor with PBC (Table III)

c := 1e-9
SetCellSize(c, c, c)
Msat = 1/\mu_0
m = \text{uniform}(0, 0, 1)
N := 2

// small grid with PBC
for j := 0; j<10; j++{
    i := \text{pow}(2, j)
    \text{SetGridSize}(N, N, 1)
    \text{SetPBC}(0, 0, i)
    \text{fprintln}("pbc.txt", i, B\text{demag}.Average())
}

// equivalent large grid without PBC
for j := 0; j<10; j++{
    i := \text{pow}(2, j)
    \text{SetGridSize}(N, N, 1*i*2)
    \text{SetPBC}(0, 0, 0)
    // evaluate over central part
    \text{defRegion}(0, \text{universe}())
    \text{defRegion}(1, \text{cuboid}(c*N, c*N, c*N))
    \text{fprintln}("nopbc.txt", i, B\text{demag}.Region(1).Average())
}

7. Exchange energy (Fig. 5)

N := 1024
c := 1e-9
\text{SetGridSize}(N, 1, 1)
\text{SetCellSize}(c, c, c)
\text{SetPBC}(1, 0, 0)
vol := N * c * c * c
A := 10e-12
Aex = A
Msat = 1e6
EnableDemag = false

\text{mag} := \text{newSlice}(3, N, 1, 1)
\text{for} k := 0; k<N/2; k++{
    \text{for} i := 0; i<N; i++{
        \text{phase} := (i * k * 2*pi) / N
        \text{my} := \sin(\text{phase})
        \text{mz} := \cos(\text{phase})
        \text{mag.set}(1, i, 0, 0, \text{my})
        \text{mag.set}(2, i, 0, 0, \text{mz})
    }
    \text{m.SetArray(mag)}
    \text{save(m)}
    \text{Energy} := vol * A * \text{pow}(2*pi*k / (N*c), 2)
    \text{fprintln}("output.txt", k, Energy, E\text{exch})
}

8. DM interaction (Fig. 6)

/*
 * Test for the DMI interaction, inspired by Thiaville et al. EPL 100 2012.
 * In this simulation, the DMI interaction converts a Bloch wall to a Néel wall.
 */
SetGridSize(128, 128, 1)
SetCellSize(250e-9/128, 250e-9/128, 0.6e-9)

Msat = 1100e3
Aex = 16e-12
alpha = 3
AnisU = vector(0, 0, 1)
Ku1 = 1.27E6
m = TwoDomain(0, 0, -1, 1, 1, 0, 0, 0, 1) // down-up domains with wall between Bloch and Néel type

tableadd(Dex)
for d:=0.0; d<0.3e-3; d+=0.001e-3{
    Dex = d
    relax()
    tablesave()
}

9. Uniaxial anisotropy (Fig. 7)

setGridSize(1, 1, 1)
setCellSize(1e-9, 1e-9, 1e-9)
V := pow(1e-9, 3)

Msat = 1000e3
AnisU = vector(1, 0, 0)

// vary magnetization direction
for i:=0; i<=360; i++{
    theta := i*pi/180
    m = uniform(cos(theta), sin(theta), 0)
    Ku1 = 1e6; Ku2 = 0
    fprintf("easy.txt", theta, E_anis.get()/V)
    Ku1 = -1e6; Ku2 = 0
    fprintf("hard.txt", theta, E_anis.get()/V)
    Ku1 = 0; Ku2 = 1e6
    fprintf("easy2.txt", theta, E_anis.get()/V)
    Ku1 = 0; Ku2 = -1e6
    fprintf("hard2.txt", theta, E_anis.get()/V)
}

10. Cubic anisotropy (Fig. 7)

setGridSize(1, 1, 1)
setCellSize(1e-9, 1e-9, 1e-9)
V := pow(1e-9, 3)

Msat = 1000e3
AnisC1 = vector(1, 0, 0)
AnisC2 = vector(0, 1, 0)

// vary magnetization direction
for i:=0; i<=360; i++{
    theta := i*pi/180
    m = uniform(cos(theta), sin(theta), 0)
    Kc1 = 1e6; Kc2 = 0; Kc3 = 0
    fprintf("easy.txt", theta, E_anis.get()/V)
11. Thermal fluctuations (Fig. 8)

c := 10e-9
setcellsize(c, c, c)
setgridsize(512, 512, 1)

Msat = 1e6
Aex = 0
alpha = 0.1
AnisU = vector(0, 0, 1)
m = uniform(0, 0, 1)
fixdt = 1e-12
Temp = 100 // varied in batch
Ku1 = 1e4 // varied in batch
enabledemag = false
setsolver(2)
run(10e-9) // reach equilibrium
autosave(m, .1e-6) // will count number of flips in post-processing
run(1e-6)

12. Slonzewski STT (Fig. 9)

length := 160e-9 ; Nx := 64
width := 80e-9 ; Ny := 32
thick := 5e-9 ; Nz := 1

Msat = 800e3 ; Pol = 0.5669
Aex = 13e-12 ; Lambda = 2
alpha = 0.01 ; EpsilonPrime = 1

SetGridSize(Nx, Ny, Nz)
SetCellSize(length/Nx, width/Ny, thick/Nz)
current:= -0.006 // Ampere
J = vector(0, 0, current/(length*width))

theta := pi*20/180 // Direction of fixed layer
FixedLayer = vector(cos(theta), sin(theta), 0)
m = uniform(1, 0, 0)

tableautosave(10e-12)
run(2e-9)
13. Solver convergence and MaxErr (Figs. 10 and 11)

SetGridSize(1, 1, 1)
SetCellSize(1e-9, 1e-9, 1e-9)

\[ B := 0.1 \text{ // Tesla} \]
\[ B_{\text{ext}} = \text{vector}(0, 0, B) \]
\[ Msat = 1/\mu_0 \]
\[ Aex = 10e-12 \]
\[ \text{EnableDemag} = \text{false} \]

SetSolver(5) //varied

for i:=1e-8; i<1e-2; i=i*sqrt(2){
    maxErr = i
    N0 := Step
    t = 0
    m = Uniform(1, 0, 1)
    runtime := 2*pi/(B*1.7595e11) // 1 period
    run(runtime)

    m1 := m.average()
    m0 := vector(1/sqrt(2), 0, 1/sqrt(2)) // analytical solution
    error := m0.sub(m1).len()
    fprintln("result.txt", i, runtime/(Step-N0), error)
}

14. Standard Problem 1 (Fig. 12)

setGridSize(512, 256, 1)
setCellSize(2e-6/512, 1e-6/256, 20e-9)

Aex = 1.3e-11
Msat = 8.0e5
Ku1 = 5.0e2
AnisU = vector(1, 0.001, 0) // easy axis nominally parallel to the long edge

// hysteresis sweep
Bmax := 75e-3
tableadd(B_{\text{ext}})

// virgin curve
m = vortex(1, 1).add(0.9, randommag())
for B := 0.0; B <= Bmax; B+=1e-3{
    B_{\text{ext}} = vector(B, 0, 0)
    relax()
    tableSave()
}

// sweep down
for B := Bmax; B >= -Bmax; B-=1e-3{
    B_{\text{ext}} = vector(B, 0, 0)
    relax()
    tableSave()
}

// sweep up
for B := -Bmax; B <= Bmax; B+=1e-3{
    B_{\text{ext}} = vector(B, 0, 0)
    relax()
15. Standard Problem 2 (Figs. 13 and 14)

// Msat and Aex should not matter
Msat = 1000e3
Aex = 10e-12

// exchange length
lex := sqrt(Aex.GetRegion(0) / (0.5 * mu0 * pow(Msat.GetRegion(0),2)))
mx := m.comp(0)
my := m.comp(1)
ms := m.comp(2)

// scan magnet size d
for d := 30; d >= 1; d--{
  Sizex := 5*lex*d
  Sizey := 1*lex*d
  Sizez := 0.1*lex*d

  // choose cells no larger than 0.5*lex
  nx := pow(2, ilogb(Sizex / (5*0.5*lex))) * 5
  ny := nx / 5
  SetGridSize(nx, ny, 1)
  SetCellSize(Sizex/nx, Sizey/ny, Sizez)

  // find remanence
  m = uniform(1, 0.3, 0)
  relax()
  remanence := m.average()

  // find coercivity by scanning applied field
  bc := 0.0
  Ms := Msat.GetRegion(0)
  for bc = 0.0445*Ms; (mx.average()+my.average()+ms.average()) > 0; bc += 0.00005*Ms{
    B_ext = vector(-bc*mu0/sqrt(3), -bc*mu0/sqrt(3), -bc*mu0/sqrt(3))
    relax()
  }
  B_ext = vector(0, 0, 0)

  fprintf("table.txt", d, remanence.x(), remanence.y(), bc/Ms)
}

16. Standard Problem 3 (Fig. 15)

N := 16
setGridSize(N, N, N)
setCellSize(1e-9, 1e-9, 1e-9) // will be varied

Msat = 500e3 // Msat and Aex should not matter
Aex = 20e-12

Ms := Msat.GetRegion(0)
A := Aex.GetRegion(0)
Km := 0.5 * mu0 * Ms * Ms
lex := sqrt(A / Km)
AnisU = Vector(0, 0, 1)
Ku1 = 0.1 * Km
// scan edge length L
for L := 8.0; L<=9.0; L+=0.02{
    c := L*lex/N
    setCellSize(c, c, c)
    m.loadFile("std3-flower.ovf") // load initial state
    V := pow(c*N, 3)
    relax()
    m_:= m.Average()
    fprintln("result.txt", L, m_.X(), m_.Y(), m_.Z(), E_total.Get(), E_exch.Get(), E_demag.Get(), E_anis.Get(), Km, V)
}

17. Standard Problem 4 (Fig. 16)

Nx:=200
Ny:=50
setgridsize(Nx, Ny, 1)
setcellsize(500e-9/Nx, 125e-9/Ny, 3e-9)

Msat = 800e3
Aex = 13e-12
alpha = 0.02

m = uniform(1, .1, 0)
relax()

tableautosave(5e-12)

B_ext = vector(-24.6E-3, 4.3E-3, 0)
mx := m.comp(0)
runWhile(mx.average() > 0)
save(m)
run(1e-9)

18. Standard Problem 5 (Fig. 17)

Nx := 50
Ny := 50
Nz := 5
setgridsize(Nx, Ny, Nz)
setcellsize(100e-9/Nx, 100e-9/Ny, 10e-9/Nz)

Msat = 800e3
Aex = 13e-12
alpha = 0.1
xi = 0.05

m = vortex(1, 1)
relax()
saveas(m, "relaxed.ovf")

J = vector(1e12, 0, 0)
Pol = 1

tableAutoSave(5e-12)
run(10e-9)
19. Extension: moving reference frame (Fig. 18)

\[\begin{align*}
Nx & := 512 \\
Ny & := 64 \\
c & := 2e-9 \\
\text{setGridSize}(Nx, Ny, 1) \\
\text{setCellSize}(c, c, 20e-9) \\
\end{align*}\]

\[\begin{align*}
w & := Nx \times c \\
h & := Ny \times c \\
Msat & = 800e3 \\
Aex & = 13e-12 \\
Xi & = 0.1 \\
alpha & = 0.02 \\
m & = \text{twomain}(1,1,0,0,1,0,-1,-1,0) \\
\end{align*}\]

\[\begin{align*}
\text{notch} & := \text{rect}(15*c, 15*c).\text{rotZ}(45*\pi/180).\text{transl}(0, Ny*c/2, 0).\text{repeat}(Nx*c/2 + 2*Ny*c, 0, 0).\text{inverse()} \\
\text{setgeom}(\text{notch}.\text{transl}(-2*Ny*c, 0, 0)) \\
\text{snapshotformat} & = "\text{PNG}\"
\end{align*}\]

\[\begin{align*}
\text{relax()} \\
\text{snapshot}(m) \\
\end{align*}\]

// Remove surface charges from left (mx=1) and right (mx=-1) sides to mimic infinitely long wire. We have to specify the region (0) at the boundaries.

\[\begin{align*}
\text{BoundaryRegion} & := 0 \\
\text{MagLeft} & := 1 \\
\text{MagRight} & := -1 \\
\text{ext_rmSurfaceCharge}(\text{BoundaryRegion}, \text{MagLeft}, \text{MagRight}) \\
\text{relax()} \\
\text{snapshot}(m) \\
\end{align*}\]

// Schedule output

\[\begin{align*}
\text{autosnapshot}(m, 50e-12) \\
\text{tableadd}(\text{ext.dwpos}) \\
\text{tableautosave}(10e-12) \\
\end{align*}\]

// Run the simulation with current through the sample

\[\begin{align*}
pol & = 0.56 \\
J & = \text{vector}(-3e12, 0, 0) \\
\text{Run}(5e-9) \end{align*}\]

20. Extension: Magnetic Force Microscopy (Fig. 20)

\[\begin{align*}
\text{setgridsize}(512, 512, 1) \\
\text{setcellsize}(2e-9, 2e-9, 10e-9) \\
\text{setgeom}(\text{rect}(750e-9, 750e-9)) \\
\text{msat} & = 800e3 \\
\text{aex} & = 13e-12 \\
\text{m} & = \text{vortex}(1, 1) \\
\text{relax()} \\
\text{save}(m) \\
\end{align*}\]

// MFM images for various lift heights

\[\begin{align*}
\text{for } i:=10; i<200; i+=10\{ \\
\end{align*}\]
MFMLift = i*1e-9
Snapshot(MFM)
}

21. Benchmark: throughput (Figs. 21 and 22)

msat = 800e3
aex = 13e-12
alpha = 0.01
b_ext = vector(0, 0.01, 0)
m = uniform(1, 0, 0)

setcellsize(4e-9, 4e-9, 4e-9)

// vary grid sizes
for n:=16; n<=1024; n++{
    setgridsize(n, n, 1)
    N2 := n*n
    steps(1) // warm-up kernel
    t = 0
    start := now()
    neval0 := Neval()
    steps(100)
    wall := since(start).Seconds()
    nevl := Neval() - neval0
    fprintln("benchmark.txt", N2, N2*nevl/wall, t/nevl)
}

22. Benchmark: memory (Fig. 23)

msat = 800e3
aex = 13e-12
alpha = 0.01
setcellsize(4e-9, 4e-9, 4e-9)

// increase grid size until we run out of memory
for e:=5; e<100; e++{
    n := 128*e
    setgridsize(n, n, 1)
    steps(1) // will crash here if out of memory
    fprintln("mem.txt", n*n) // save number of cells
}