Summary: The joint distribution of a geometric Brownian motion and its time-integral was derived in a seminal paper by M. Yor [Adv. Appl. Probab. 24, No. 3, 509–531 (1992; Zbl 0765.60084)] using Lamperti’s transformation, leading to explicit solutions in terms of modified Bessel functions. In this paper, we revisit this classic result using the simple Laplace transform approach in connection to the Heun differential equation. We extend the methodology to the geometric Brownian motion with affine drift and show that the joint distribution of this process and its time-integral can be determined by a doubly-confluent Heun equation. Furthermore, the joint Laplace transform of the process and its time-integral is derived from the asymptotics of the solutions. In addition, we provide an application by using the results for the asymptotics of the double-confluent Heun equation in pricing Asian options. Numerical results show the accuracy and efficiency of this new method.

MSC:
91-XX Game theory, economics, finance, and other social and behavioral sciences
65-XX Numerical analysis

Keywords:
doubly-confluent Heun equation; geometric Brownian motion with affine drift; Lamperti’s transformation; asymptotics; boundary value problem

Full Text: DOI arXiv

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