The QCD Equation of State with Thermal Properties of $\phi$ mesons

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Abstract. In this work a first attempt is made to extract the Equation of State (EoS) using experimental results of the $\phi$ meson produced in nuclear collisions at AGS, SPS and RHIC energies. The data are confronted to simple thermodynamic expectations and lattice results. The experimental data indicate a first order phase transition, with a mixed phase stretching energy density between $\sim 1$ and 3.2 GeV/$fm^3$.

1. Introduction

Quantum chromodynamics (QCD) predicts a phase transition from hadronic matter to the quark and gluon state at a critical temperature $T \sim 200$ MeV [1]. There are experimental evidences that nuclear collisions at ultrarelativistic energies induce this QCD phase transition and create a new state of matter where the properties of the matter are governed by quarks and gluons - such a system is called Quark Gluon Plasma (QGP). Current lattice QCD calculations indicate that the order of this transition depends on the quark masses, as well as on the baryochemical potential ($\mu_B$). In particular, a first order phase transition is predicted for large baryochemical potentials, while at zero and small baryochemical potentials a "cross over" is predicted. When studying nuclear collisions at ultra-relativistic energies the baryochemical potential at midrapidity decreases with increasing energy. Therefore, lattice QCD indicates that the order of the transition changes with energy ($\mu_B$) of the collision. However, lattice calculations for non-zero $\mu_B$ have large technical uncertainties and the best estimate of lattice remains at $\mu_B=0$.

In a first order phase transition the pressure increases with increasing temperature, until the transition temperature $T_c$ is reached, then remain constant during the mixed phase, and continue to increase after the end of the mixed phase. Related to this picture L. Van Hove [2] suggested to identify the deconfinement transition in high energy proton-antiproton collisions, looking at the variation of average transverse momentum ($\langle p_T \rangle$) of $\dagger$ Speaker
hadrons as a function of the hadron multiplicity at midrapidity \(\frac{dN}{dy}\) and searching for an increase of \(\langle p_T \rangle\) followed by a plateau-like behavior and again a subsequent increase. The \(\langle p_T \rangle\) is expected to reflect the thermal freeze-out temperature \((T_f)\) of hadrons and a flow component which can be related to the initial pressure, similarly the hadronic multiplicity reflect the entropy density of the system. While the purely thermal component of \(\langle p_T \rangle\) can not be related to the initial temperature which remains unmeasurable above \(T_c\), however the flow component in the inverse slope can reflect the plateau of the pressure during mixed phase.

It has been observed that the \(\langle m_T \rangle\) of pions, kaons and protons as a function of \(dN_{ch}/dy\) shows a Van-Hove-like behaviour as explained above for a wide range of collision energy and for the same centrality [3, 4]. Hydrodynamic calculations assuming a first order transition could reproduce these data [3, 5]. In this work, we study for the first time the variation of the inverse slope, \(T_{\text{eff}}\) extracted from the \(p_T\) distribution of the \(\phi\)-meson as a function of the initial energy density, \(\epsilon_{Bj}\) evaluated within the framework of Bjorken’s hydrodynamical model [7] and \(\sqrt{s_{NN}}\) for energies spanning from AGS, SPS to RHIC. The \(\phi\)-meson is of special interest [6] because of its small hadronic rescattering cross section of \(\sigma(\phi N)=10\ \text{mb}\) causing it to decouple earlier than other hadrons. Furthermore, due to its life time of \(\sim 45\ \text{fm/c}\), its main decay product \(K^+ K^-\) suffer less rescattering. Experimental results from Au+Au collisions at RHIC energies indicate that the \(\phi\) has a higher thermal freeze out temperature as compared to pions, kaons and protons. In particular their thermal freeze out temperature is within errors compatible with the chemical freeze out temperature of hadrons and the critical temperature. Another important experimental fact is the observation of a scaling of the elliptic flow of hadrons as a function of the transverse momentum when divided with the number of valence quarks. This observation is interpreted as an indication that elliptic flow builds in the partonic phase and can therefore be reflecting the initial conditions e.g. the initial pressure. Therefore the \(\phi\) and its flow phenomena are particularly interesting probes for studying the EoS and the nature of the phase transition.

Furthermore, we study here for the first time the inverse slope as a function of the initial energy density, \(\epsilon_{Bj}\). This is an important new feature of such studies, because it connects the inverse slope with a parameter characterizing the initial state of the collision build up after \(\sim 1\ \text{fm/c}\) and which reflects at the same time the collision energy, the stopping and the impact parameter of the collision. For example, at a given \(\sqrt{s_{NN}}\), different energy densities could be achieved by changing the colliding nuclei species or the impact parameter. The \(\epsilon_{Bj}\) is a meaningfull parameter also if equilibrium is not reached. It can be directly compared to the critical energy density \(\epsilon_c\) obtained in lattice QCD calculations- \(\epsilon_c \sim 1\ \text{GeV/fm}^3\). In the following the \(\phi\) data will be analysed and confronted to simple thermodynamic expectations which relate to the Van Hove signature and to lattice QCD predictions.
The QCD Equation of State with Thermal Properties of $\phi$ mesons

2. $\phi$ as a probe of the order of the phase transition

To study the dependence of the inverse slope of the $\phi$ meson on the collision energy and $\epsilon_{Bj}$, we have compiled the inverse slope of the transverse mass spectra ($m_T = \sqrt{m^2 + p_T^2}$) called ”effective temperature” $T^\phi_{\text{eff}}$ using data at mid-rapidity in the low $p_T$ domain from AGS - SPS to RHIC. The inverse slope has been estimated using an exponential function of the type: $\frac{1}{m_T} \frac{dN}{dy} = A \exp\left(\frac{m_T}{T_{\text{eff}}}\right)$. The use of slightly different fit functions leads to differences of the order of 10 MeV, which are small as compared to the experimental errors on the inverse slopes. The inverse slope of the $\phi$ meson extracted through the above fit includes a thermal component and a non-thermal component due to the collective transverse flow. At low $p_T$ (in non-relativistic domain, $p_T \ll m$), $T_{\text{eff}} = T_{\text{th}} + \frac{1}{2} m \langle \beta_T^2 \rangle$, where $\beta_T$ is the collective transverse velocity. We have used the above equation to estimate $T^\phi_{\text{th}}$ from $T^\phi_{\text{eff}}$, as a first approximation. The values of $\langle \beta_T \rangle$ as a function of $\sqrt{s_{NN}}$ has been obtained from Ref.[16] and as a function of centrality for $\sqrt{s_{NN}} = 200$ GeV AuAu collisions from Ref.[17].

In figure 1 the effective temperature of $\phi$, $T^\phi_{\text{eff}}$ (left) and the thermal component of the inverse slope $T^\phi_{\text{th}}$ (right) are shown as a function of the collision energy $\sqrt{s_{NN}}$. It is observed that from AGS to SPS energies these observables remain almost unchanged, showing a plateau-like structure. Going from SPS to RHIC energies, the inverse slope $T^\phi_{\text{eff}}$ exhibits a sudden jump while an increase is still observed in the $T^\phi_{\text{th}}$ component. This may be due to an imperfect transverse flow component subtraction at RHIC or other effect. It is observed that the $T^\phi_{\text{th}}$ is reaching at RHIC values compatible within errors with $T_c$. The observed plateau of $T^\phi_{\text{eff}}$ is a signature of a coexisting phase of quarks, gluons and hadrons for a first order phase transition, during which the initial pressure remains constant. The subsequent increase of $T^\phi_{\text{eff}}$ with $\sqrt{s_{NN}}$ at top RHIC energies indicates the end of the mixed phase and the entering into a pure QGP phase.

Now we study the dependence of the inverse slope and its thermal component on the initial energy density, $\epsilon_{Bj}$, estimated as $\epsilon_{Bj} = \langle \frac{dE_T}{dy} \rangle \frac{1}{\pi R^2} = \langle \frac{dN}{dy} \rangle \langle m_T \rangle \frac{1}{\pi R^2}$. Where...
The QCD Equation of State with Thermal Properties of φ mesons

\[ R = R_0 A^{1/3} \text{ and } A \sim \frac{N_{\text{part}}}{2}. \]  

From the experimental measurements of \( dE_T/dy \) and \( dN/dy \) with \( \langle m_T \rangle \), the observable \( \epsilon_{Bj} \) could be estimated for all centralities and center of mass energies at mid-rapidity.

In figure 2, \( T_{\text{eff}}^\phi \) (left) and \( T_{\text{th}}^\phi \) (right) are shown as a function of \( \epsilon_{Bj} \cdot \tau \). Note that the formation time, \( \tau \) is model-dependent and in general, in subsequent discussions we also assume \( \tau \sim 1 \text{ fm/c}. \) The above result reflects the properties of equation of state. We observe a plateau in \( T_{\text{eff}}^\phi \) stretching between \( \epsilon_{Bj} \sim 1 \) and \( 3.2 \text{ GeV}/f m^3 \), and increasing suddenly above \( 3.2 \text{ GeV}/f m^3 \). This behaviour as already discussed, suggests a first order transition, however from figure 2 we can now infer that the mixed phase is stretching between 1 and 3.2 GeV/fm³. The use of the \( \epsilon_{Bj} \) scale, allows us to establish here for the first time the \( \epsilon_{Bj} \) range of the mixed phase. More data on the \( \phi \) at \( \epsilon_{Bj} \) below 1 GeV/fm³ are needed to establish the increase of \( T_{\text{eff}}^\phi \) up to 1 GeV/fm³, as seen in other hadrons. The increase of the inverse slope at RHIC energies again indicate a pure phase of QGP, as is expected from a first-order phase transition. The thermal component \( T_{\text{th}}^\phi \) shows also a plateau while a smaller increase is still observed at 3.2 GeV/fm³.

The flow component of the inverse slope of the \( \phi \) reflecting the initial pressure, it is of interest to look directly observables linked to this initial pressure like the transverse flow velocity, \( \beta_T \) for all hadrons and the elliptic flow \( v_2 \) as a function of collision energy. The transverse flow velocity \( \beta_T \) shows exactly the same characteristic behaviour as the \( T_{\text{eff}}^\phi \) as a function of collision energy, namely increase, a plateau and subsequent increase at RHIC [16]. A similar pattern is suggested for the elliptic flow \( v_2 \) as a function of collision energy [18].

3. Comparison of φ data with lattice predictions

In the following we compare experimental data with lattice QCD predictions. In figure 3 the lattice prediction for the energy density is shown as a function of the temperature (left). This is compared to \( \epsilon_{Bj} \) as a function of \( T_{\text{th}}^\phi \) (middle) and \( T_{\text{eff}}^\phi \) (right) extracted.
from the slope of $\phi$ spectra. First we discuss the middle plot. The lattice prediction is for zero baryochemical potential therefore correspondig to the cross over region. The figure in the middle (data) is at non-zero $\mu_B$ and shows the variation of the initial energy density with $T_{th}^\phi$ temperature, which is measured at a later time than the energy density, namely at the thermal freeze out of the $\phi$. That is, the two variables in the data are a measure of the system at different times, while the lattice estimate is independent of time. This temperature is expected to be always below $T_c$ and does not reflect the initial $T$, which may exceed $T_c$ depending on the collision energy. This plot can be directly compared to figure 11 of $^{19}$, where the energy density from data has been studied as a function of $T$, while both x and y axis were estimated at the same time, namely at the hadronic chemical freeze out time and at $\mu_B = 0$. It is seen that the energy density increases approaching the $T_c$ from below, from which both the $T_c$ and critical exponents can be extracted $^{19}$.

If the transition occurs at $T_c \sim 200$ MeV as expected, the temperature of $\phi$ at the thermal freeze out should saturate below and near $T_c$, for all values of the initial density (up to infinity), as seen in figure $^3$ (middle panel). We observe a saturation in the value of $T_{th}^\phi$ near $T_C$, because $T_{th}^\phi \leq T_c$ as mentioned earlier. To measure temperatures above $T_c$ photons and dileptons are useful probes.

To avoid this saturation and to study an equivalent of the initial temperature, we use in the following figure (right) the inverse slope of $\phi$ instead of its temperature at hadronic freeze out. Doing that, we take advantage of the possible relationship between collective transverse flow and initial pressure.

We now discuss figure $^3$ (right panel), where the initial Bjorken energy density is shown as a function of the effective temperature of the $\phi$ for collisions from AGS, SPS to RHIC. Now both the x and y axis are reflecting parameters at initial times therefore this plot is
more appropriate to be compared to the lattice results. The effective temperature of the \( \phi \) here is a sum of a thermal freeze out temperature, which is expected to be always below \( T_c \), and a non-thermal component due to transverse flow, which relates to the initial pressure and reflects the initial conditions above \( T_c \). Therefore the variables in the two plots compared here are not exactly the same but they are correlated. Further analysis is needed to compare the data to lattice e.g. using exactly the same variables in both estimates. A study can be also done involving the elliptic flow \( v_2 \) and the transverse flow velocity \( \beta_T \) as a function of the initial energy density. Also a hydrodynamic calculation of the discussed variables is of interest. The above items are work in progress.

4. Summary

In summary, a first analysis of experimental data on the inverse slope parameter of \( \phi \) mesons as a function of \( \sqrt{s_{NN}} \) & energy density from AGS, SPS to RHIC, and their comparison to simple thermodynamic expectations, suggest a first order phase transition as predicted by lattice QCD at non-zero baryochemical potentials. The mixed phase of quarks, gluons and hadrons, is found to stretch between \( \epsilon_{B_j} \) of \( \sim 1 \) and \( 3.2 \text{ GeV/fm}^3 \). The latent heat density in a first order phase transition is

\[
\epsilon_Q(T_c) - \epsilon_H(T_c) = 4B,
\]

where \( \epsilon_Q(\epsilon_H) \) is the energy density of QGP (hadrons) at \( T_c \) and \( B \) is bag constant. With \( \epsilon_Q(T_c) \sim 3.2 \text{ GeV/fm}^3 \) and \( \epsilon_H(T_c) \sim 1 \text{ GeV/fm}^3 \), we get a reasonable value for \( B^{1/4} \sim 250 \text{ MeV} \). If we ignore \( B \), then \( \epsilon_Q/\epsilon_H \sim g_Q/g_H \sim 3.2 \) also a reasonable number comparable to results from lattice QCD [20], here \( g_Q(g_H) \) is statistical degeneracy of QGP (hadrons). The plateau and subsequent increase of the inverse slope parameter of \( \phi \) above \( \epsilon_{B_j} \sim 3.2 \text{ GeV/fm}^3 \) is in agreement with data from pions, kaons and protons, and is as well observed in the transverse flow velocity \( \beta_T \) reflecting the behaviour of the initial pressure. A first attempt to compare data to lattice QCD predictions is made.

The above results, while supporting the order of the transition predicted by lattice at large \( \mu_B \) up to \( \epsilon_{B_j} \sim 3.2 \text{ GeV/fm}^3 \), they do not exclude a change of order of the transition at smaller baryochemical potentials. Which points to further work, towards mapping out the order of the QCD phase transition as a function of energy and baryochemical potential.

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The QCD Equation of State with Thermal Properties of $\phi$ mesons

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