Synthesis of Channel Tracking for Random Process Parameters under Discontinuous Variation

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Abstract. The problem of synthesis of the channel structure to track the parameters of a random process under the conditions of their discontinuous variation using models of state and observation is considered. The parameters of tracking accuracy by digital modeling are obtained; the ways and possibilities of simplifying the obtained algorithm are considered with preservation of its accuracy.

1. Introduction
The analysis of the concepts of designing promising radioelectronic systems shows that these will be the systems of an integral type [1, 2, 3, 4]. In the structure of such systems, there must be an information system consisting of the channels that produce a functionally completed procedure for processing signals and information to solve a particular problem [5, 6]. The quality of signal and data processing (accuracy, immunity, reliability, integrity) in the information system can be improved if two mutually complementary ways are used.

The first way is the improvement of the devices and systems that make up the radioelectronic system, as well as the introduction of new, higher-quality performance characteristics into its composition. The second way is the development of the appropriate algorithmic support.

This implies the need to have an automated system consisting of several subsystems with different structures to support operation in various modes in conditions of natural and manmade clutter, possible violations of the regular tracking process, a drastic change in the behavior of signal parameters due to unforeseen reasons.

The need to forecast and track the trajectory of a vehicle, such as a car or aircraft, closed chemical reactor parameters, etc., requires solving the problem of continuous high-precision measurement in different conditions, including clutter. This is possible by using adaptive tracking meters, which can vary both the structure and parameters of the tracking filters depending on the situation [7].

2. Selection and justification of the system models
The aim of the study is the synthesis of a tracking filter with a simplified reconfigurable structure and an investigation of its accuracy characteristics.

Let the tracking system be exposed to two types of destabilizing factors: relatively slow changes and rapid changes in the tracked parameters [5].

It is possible to solve the problem of sufficiently accurate tracking, including in the presence of clutter, by using tracking meters in which different combinations of state models can be used.
A characteristic feature of this problem is the need to change the model of the system at random instants of time. This feature is an objective characteristic of the process of functioning of complex dynamical systems, which are called stochastic systems with a random change of the structure. When synthesizing the system, let us take into account that inertia-free discrete sensors are used for the primary measurement of parameters.

To ensure a sufficiently high accuracy of estimating and extrapolating the tracked parameters, and simultaneously, the ability to respond quickly to their changes, let us take for the first model (hereinafter model 1) a system of difference equations corresponding to the hypothesis of a constant rate of change of parameter \( x(k) \):

\[
\begin{align*}
    x(k+1) &= x(k) + v(k) \cdot T; \\
    v(k+1) &= (1 - \alpha \cdot T) \cdot v(k) + \xi(k),
\end{align*}
\]

where \( v(k) \) is a rate of variation \( x(k) \); \( T \) is a time step; \( \alpha \) is a time constant of slow relative variation; \( \xi(k) \) is a sequence of random variables with a Gaussian probability density.

It is known that the coordinates (zero derivatives) should be observed in any group of functionally connected spatially-estimated states of coordinate.

To account for quick (jumplike) variations of parameters, let us use a model with an extended band:

\[
\begin{align*}
    x(k+1) &= x(k) + v(k) \cdot T; \\
    v(k+1) &= (1 - \beta \cdot T) \cdot v(k) + \xi_2(k),
\end{align*}
\]

where \( \beta \) is a time constant of rapid relative variation, \( \xi_2(k) \) is a sequence of random variables with a Gaussian probability density.

Both models can be represented in matrix form:

\[
x(k+1) = \Phi^{(i)}(k) \cdot x(k) + \xi^{(i)}(k).
\]

The observation model has the form:

\[
z(k) = H^{(i)}(k) \cdot x(k) + \eta^{(i)}(k); \quad (i = 1,M; k = 0,1,...),
\]

where \( i \) is a system structure index; \( x(k) \) is an \( m \)-dimensional vector of phase coordinates; \( \Phi^{(i)}(k) \) is a state transition matrix; \( z(k) \) is an \( n \)-dimensional vector of observations; \( H^{(i)}(k) \) is an observation transition matrix; \( \xi^{(i)}(k), \eta^{(i)}(k) \) are sequences of stochastically and temporally independent random variables with Gaussian probability densities:

\[
\begin{align*}
    \xi^{(i)}(k) &\sim N[\xi^{(i)}(k) | \mu^{(i)}(k), Q^{(i)}(k)]; \quad (i = 1,M; k = 0,1,...); \\
    \eta^{(i)}(k) &\sim N[\eta^{(i)}(k) | m^{(i)}(k), R^{(i)}(k)]; \quad (i = 1,M; k = 0,1,...).
\end{align*}
\]

Here let us use the following notation for the Gaussian probability density of \( r \)-dimensional random variable \( \alpha \):

\[
N[\alpha | m,D] = (2 \cdot \pi)^{-r/2} \cdot |D|^{1/2} \cdot \exp\left\{ -\frac{1}{2} (\alpha - m)^T D^{-1} (\alpha - m) \right\}.
\]

The process of the structure change is described by the discrete-time Markov chain \( \{ \Theta(k), k = 0,1,... \} \), whose states are the indices of dynamic system structures (2.1), (2.2): \( \Theta(k) \in \Theta = \{1,2,...,M\} \). Random variables \( \{ x(k), \Theta(k), k = 0,1,... \} \) form a mixed Markov chain.
with a given probability description in the form of initial probabilities \( \pi(\Theta(0) = i), i, j = 1, M \)
conditional transition probabilities \( \pi(\Theta(k) = j | \Theta(k - 1) = i, x(k - 1)) \), \( i, j = 1, M \)
and also in the form of conditional initial and conditional transition probability densities \( q(x(0) | \Theta(0) = i), q(x(k) | x(k - 1), \Theta(k) = j, \Theta(k - 1) = i), i, j = 1, M \).

Each state \( i (i = 1, M) \) of the conditional Markov chain \( \{\Theta(k), k = 0,1,\ldots\} \) is related to some characteristics that determine the structure of the dynamic system (2.1), (2.2). Consequently, the structure of the dynamic system at each instant of time \( k \) is random and in the process of the system’s functioning varies in accordance with the probabilistic mechanism determined by the sequence \( \{\Theta(k), k = 0,1,\ldots\} \). The problem consists in the optimal estimation of the phase coordinates vector \( x(k) \), and the structure number \( i \) by observations \( z(k) \) if \( k = 0,1,\ldots \).

In accordance with the method of the synthesis of systems with random structure [9, 10], one can obtain an algorithm for the functioning of the system.

The estimates of the parameters at the output of the first filter channel are equal to:
\[
\hat{x}^{(1)}(k + 1) = \hat{x}^{(1)}(k) + \hat{v}^{(1)}(k) \cdot T + K^{(1)}(k) \cdot \nu^{(1)}(k); \quad (7)
\]
\[
\hat{v}^{(1)}(k + 1) = (1 - \alpha T') \cdot \hat{v}^{(1)}(k) T' + K^{(1)}(k) \nu^{(1)}(k); \quad (8)
\]
and for the second channel they are equal to:
\[
\hat{x}^{(2)}(k + 1) = \hat{x}^{(2)}(k) + \hat{v}^{(2)}(k) T' + K^{(2)}(k) \nu^{(2)}(k); \quad (9)
\]
\[
\hat{v}^{(2)}(k + 1) = (1 - \beta T') \hat{v}^{(2)}(k) T' + K^{(2)}(k) \nu^{(2)}(k); \quad (10)
\]
where \( \nu^{(i)} \) is the corresponding filter residual.

The a priori and a posteriori covariance matrix of error variances is determined for each model using the expressions given in [3].

Then, the covariance matrices of the one-step forecast of the observation vector are determined, and on their basis, the filter gain factors are calculated:
\[
K^{(i)}_{11}(k + 1) = -\frac{P^{(i)}_{11}(k + 1|k)}{V^{(i)}(k)}, \quad K^{(i)}_{12}(k + 1) = \frac{P^{(i)}_{21}(k + 1|k)}{V^{(i)}(k)}.
\]

After estimating the probability of system structure numbers \( W^{(i)}(k + 1) \), the resulting estimation of the parameter can be obtained:
\[
\hat{x}(k + 1) = \sum_{i} (\hat{x}^{(i)}(k + 1) \cdot W^{(i)}(k + 1)).
\]

3. Investigation of the algorithm for the operation of the range measuring device

Figure 1 shows true coordinate \( x(k) \) (curve 1) and resulting estimate \( \hat{x}(k) \) (curve 2) obtained at a moderate value of the observation clutter. As can be seen, with a sharp change in the parameter behavior, the accuracy of the estimate does not significantly deteriorate, unlike that shown in Figure 2. Figure 2 for a filter with a non-configurable structure shows true coordinate \( x(k) \) (curve 2) and resulting estimate \( \hat{x}(k) \) (curve 3) obtained with a moderate value of the observation noise, as well as true coordinate \( x(k) \) in the presence of the observation noise.
Figure 1. The resulting parameter estimate

Figure 2. The resulting parameter estimate for a normal filter

Figure 3. A block diagram of the filter functioning algorithm.

The resulting algorithm is quite complicated even with the two models used. This is due to the need to perform the following sequence of calculations at each step:
- calculate the first two conditional moments of phase coordinates, conditional residuals and covariance matrices of one-step forecast of the observation vector;
- using the results, calculate the conditional partial estimates and the corresponding covariance matrices of random variable \( x(k) \);
- calculate a posteriori probabilities of the system structure numbers;
- determine unconditional estimate \( \hat{x}(k) \) and the corresponding covariance matrix of the estimation error of the phase coordinate vector.

Figure 3 shows the corresponding block diagram of the algorithm. As can be seen from the diagram, the computation is carried out in two steps: forecast and correction. For each step \( k \) and for each index \( s_k \), information characteristics \( \hat{p}(s_k) \), \( \hat{x}(s_k) \), \( \hat{R}(s_k) \), determined at the stage of correction, depend on observations \( z_k \) and \( r_k \), and forecasted characteristics: probability \( \hat{p}(s_k) \) depends on \( \hat{p}(s_k) \), \( \hat{x}(s_k) \), \( \hat{R}(s_k) \); estimate \( \hat{x}(s_k) \) depends on \( \hat{x}(s_k) \), \( \hat{R}(s_k) \); covariance \( \hat{R}(s_k) \) depends on \( \hat{R}(s_k) \).

It is noteworthy that despite the linearity of the model of the object and the meter, the algorithm of filtering is not linear. This is explained by the dependence of the estimate of the phase coordinate vector on the random structure. However, as will be shown below, the implementation of the proposed method is quite possible for modern computing facilities.

4. A simplified algorithm and its characteristics
It is known that the residual value depends on the measurement noise value and the degree of model inaccuracy (forecast). Since in this case the input receives one signal, it can be concluded that the residuals variation caused by the noise will have similar values for both models \([6, 7, 8]\). They will differ mainly due to the inaccuracy of the forecast \([10, 11]\). Therefore, it is possible to construct a simplified algorithm based on the same two models (Figure 4). First, let us consider the estimates obtained using the conventional Kalman filters based on the two above mentioned models (Figure 5).

Since the filters have different bands, the estimates differ, despite the fact that their inputs are fed with the same signal and the same implementation of the observation noise. The solid line shows the estimation of the filter with a larger band (it responds more sharply to noises); the dotted line shows the estimation of the filter with a smaller band.

![Figure 4. A structure of a simplified coordinate estimation algorithm](image)

![Figure 5. Partial estimates obtained from the conventional Kalman filters based on two models](image)
Figure 6 shows true coordinate $x(k)$ (curve 1) and its resulting estimate $\hat{x}(k)$ of the synthesized and simplified algorithms (curves 2 and 3) obtained under the same conditions.

![Figure 6. Resulting estimates of angular coordinates](image)

5. Conclusion

Thus, an algorithm built on the basis of two models by the methods of the theory of systems with a random structure and a simplified algorithm has practically the same estimates. This is due to the presence of models with different characteristics, taking into account the different types of changes in the estimated parameter and the almost exact tracking of the signal behavior of either the first model or the second one. In general, the results of statistical modeling demonstrate the effectiveness of the proposed simplified algorithm.

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