A revision of the generalized uncertainty principle

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Received 30 January 2008, in final form 17 March 2008
Published 29 April 2008
Online at stacks.iop.org/CQG/25/105003

Abstract
The generalized uncertainty principle arises from the Heisenberg uncertainty principle when gravity is taken into account, so the leading order correction to the standard formula is expected to be proportional to the gravitational constant \( G_N = L_{Pl}^2 \). On the other hand, the emerging picture suggests a set of departures from the standard theory which demand a revision of all the arguments used to deduce heuristically the new rule. In particular, one can now argue that the leading order correction to the Heisenberg uncertainty principle is proportional to the first power of the Planck length \( L_{Pl} \). If so, the departures from ordinary quantum mechanics would be much less suppressed than what is commonly thought.

PACS numbers: 03.65.Ta, 04.60.Bc

1. Introduction

The search for a common description of particle physics and gravity and for a quantum theory of the gravitational sector is certainly one of the most outstanding and longstanding problems in physics. Despite important discoveries, at present a reliable theoretical framework is lacking and even the very meaning of quantum spacetime is not clear. However, from heuristic considerations, quite general arguments and simple analogies, we expect a set of model-independent features which should occur in any consistent theory of quantum gravity. Among other things, it is a common belief that the Heisenberg uncertainty principle (HUP) \( \Delta x \Delta p \gtrsim 1 \) has to be replaced by the so-called generalized uncertainty principle (GUP) [1–3]:

\[
\Delta x \Delta p \gtrsim 1 + \alpha L_{Pl}^2 (\Delta p)^2,
\]

where \( \alpha \) is a positive dimensionless coefficient (which in general may depend on \( x \) and \( p \)) and \( L_{Pl} \sim 10^{-33} \) cm is the Planck length. Since \( \alpha > 0 \), equation (1) implies the existence of a minimum observable length, very probably close to the Planck one. On the other hand, the GUP is deduced from heuristic arguments which use standard concepts of different frameworks, such as quantum mechanics from one hand and general relativity from the other hand, so the final result can be at most ‘reasonable’, but cannot be ‘fully reliable’. In particular, one should
see equation (1) as a first approximation to a more complex inequality, whose exact form is probably impossible to derive from general grounds and outside a well-defined theory.

An important feature of equation (1) is that there is no term linear in the Planck length. This is exactly what one can find in the literature, since no derivation provides it: the reason is basically that the modification of the HUP arises when gravitational interactions are taken into account, so the leading order correction is expected to be proportional to the gravitational constant $G_N = L_P^2$. On the other hand, the GUP implies the existence of a minimum length of order $L_P$ and the latter is promoted to fundamental parameter in the new picture. Revising the derivation of the GUP, it is now natural to expand the GUP in powers of $L_P$ rather than of $G_N$, and to include the term linear in $L_P$ into equation (1). We can arrive at the same conclusion by noticing that the GUP requires a modification of the standard de Broglie relation (indeed a wavelength smaller than the minimum length cannot make sense) and this is always neglected when the GUP is derived, because at a first approach one uses standard physics only. On the other hand, a modified de Broglie relation could, in turn, generate a term proportional to $L_P$ in the GUP. This is basically the result of the present work. If so, the departures from standard quantum mechanics would be much less suppressed than what is commonly believed and most consequences of the GUP discussed in the literature would have to be reconsidered.

The content of the paper is as follows. In sections 2 and 3, I review the basic ingredients which are used respectively in the derivation of the HUP and of the GUP. In section 4, I show that the leading order correction to the HUP could be proportional to the first power of the Planck length. In sections 5 and 6, I briefly discuss the implications of the proposal of the present work with two simple examples. In section 7, there is a summary and the conclusions.

2. Heisenberg uncertainty principle

The Heisenberg’s microscope is a gedanken experiment whose purpose is to measure the position and momentum of a particle, say an electron, by using some probe, for example a photon. The setup of the experiment is sketched in figure 1. Since the photon has a wavelength $\lambda$, we are able to resolve at best length scales of order $\lambda$ itself. More precisely, wave optics theory requires that the projected electron position uncertainty is\(^1\)

$$\Delta x \gtrsim \frac{\lambda}{2\theta},$$

(2)

\(^1\) For the sake of simplicity, throughout the paper I focus attention on one space dimension only. In the case of two or more space dimensions, the exact picture becomes more complicated and model dependent, but the basic features are essentially the same. I always use $\hbar = c = 1$ units.
where $\theta \ll 1$ is the angle defined in figure 1. Of course, in order to be detected by the microscope, the photon must be scattered within the cone of angle $2\theta$. The two extreme cases are reported in figure 1, where the scattered photon is indicated respectively with $\gamma'$ and $\gamma''$. In the first case, the photon hits the right edge of the lens of the microscope: the momentum in the $x$-direction of the electron after the collision is $p'_x$, while the one of the photon is $q' \sin \theta$, where $q'$ is the photon momentum. In the second case, the photon hits the left edge of the lens of the microscope and we have respectively $p''_x$ for the electron and $-q'' \sin \theta$ for the photon. Since the total momentum of the system is conserved, the two limiting cases have equal final momentum in the $x$-direction:

$$p'_x + q' \sin \theta = p''_x - q'' \sin \theta. \tag{3}$$

For $\theta \ll 1$, $\sin \theta \approx \theta$ and $q' \approx q''$. On the other hand, since we cannot distinguish experimentally the two cases, the electron momentum uncertainty in the $x$-direction is

$$\Delta p'_x \gtrsim p''_x - p'_x \sim 2q' \theta. \tag{4}$$

Lastly, we use the de Broglie relation to connect the momentum $q'$ of the photon with its wavelength $\lambda$

$$q' = \frac{2\pi}{\lambda}. \tag{5}$$

and we multiply equation (2) by equation (4), finding the well-known result $\Delta x \Delta p \gtrsim 1$.

Let me remark the crucial assumptions leading to the HUP: wave optics theory, momentum conservation and de Broglie relation.

### 3. Generalized uncertainty principle

When one takes gravitational interactions into account, the ordinary HUP must be revised, since the classical concept of spacetime breaks down as we approach distances close to the Planck length $L_{Pl}$. The result is the GUP in equation (1), which can be deduced heuristically from several different frameworks (string theories [1], black hole physics [2] and even more general considerations on the gravitational interactions [3]), so it is a common belief that it must hold in any consistent picture of quantum gravity. Consequences of the GUP can be found, e.g., in [4].

A simple derivation of equation (1) can be obtained even from dimensional grounds. In addition to the uncertainty due to the wave–particle duality, there must be one caused by the gravitational interaction between the electron and the photon, which increases as the photon energy increases. This is an inevitable effect that cannot be shielded and that is independent of the nature of the probe, as follows from the equivalence principle. Hence, the leading order of the extra uncertainty is at least proportional to the gravitational coupling constant $G_N = L_{Pl}^2$.

Then, for dimensional reasons, one finds equation (1).

Another simple and intuitive derivation is the following. From the ordinary HUP, we know that the electron position accuracy could be improved using more and more energetic photons. However, it is also clear that this approach has a lower bound, because if we use too many energetic photons, a black hole is formed in the collision between the photon and the electron. This statement can also be rephrased by saying that it is impossible to localize an amount of energy with a space resolution better than the radius of the black hole with the same mass. So, the position uncertainty cannot decrease below the corresponding black hole radius and the minimum space resolution is

$$(\Delta x)_{\text{min}} \sim \max[1/(\Delta p) , \frac{G_N}{\Delta p}]. \tag{6}$$

Combining linearly the two uncertainties in equation (6), we find the GUP of equation (1).
Even if it is often ignored, the GUP should be seen only as a first approximation to a more complicated formula. Indeed, it is deduced using standard physics, while it is well known that in the emerging picture several rules of our current theoretical frameworks break down.

Before proceeding, let us note two important points:

1. The basic ingredients which are used to find the leading order correction to the HUP are the equivalence principle, according to which all the forms of energy couple to gravity with the same strength, and ordinary special relativity kinematics, in particular the standard dispersion relation for massless particles, i.e. \( P = E \), and the standard energy–momentum conservation. In models where the minimum length is independent of the reference frame, at least the last two assumptions could be wrong, see e.g. [5]. However, if we use standard expressions, we introduce only errors of order higher than \( L_{\text{Pl}}^2 \).

2. Since the modification of the HUP arises from the gravitational interaction between the photon and the electron, the leading order correction is expected to be at least proportional to \( G_N \) or, in other words, there is no term linear in the Planck length. This is what one finds in all the presently available literature on the GUP.

4. Revision of the GUP

Since the GUP implies the existence of a minimum length, the wavelength of every particle has now a natural lower bound and a modification of the standard de Broglie relation seems inevitable, see [6] and note 2. In particular, we can expect that the wavelength of a particle can be written as \( \lambda = f(p) \), where \( f(p) \) is some model-dependent function of the momentum \( p \) such that, at low energies (\( p \ll L_{\text{Pl}}^{-1} \)), we recover the standard de Broglie relation

\[
\lambda \approx \frac{2\pi}{p},
\]

while at high energies (beyond the Planck scale) \( f(p) \) goes asymptotically to a minimum wavelength, quite naturally equal or somewhat larger than the minimum length of the theory,

\[
f(p) \to \lambda_{\text{min}}.
\]

So, for \( p \ll L_{\text{Pl}}^{-1} \), we can write

\[
\lambda = \frac{2\pi}{p} \left(1 + \frac{a L_{\text{Pl}} p + b L_{\text{Pl}}^2 p^2}{p} + \cdots\right),
\]

where \( a, b, \ldots \) are dimensionless model-dependent parameters.

If we now repeat the argument leading to the HUP with the new de Broglie relation \( \lambda = f(p) \), we find

\[
\Delta x \Delta p \gtrsim q f(q) \sim 1 + a L_{\text{Pl}} \Delta p + b L_{\text{Pl}}^2 (\Delta p)^2 + \cdots
\]

and, in general, we can expect the term linear in the Planck length. Then, when we take gravity into account

\[
\Delta x \Delta p \gtrsim 1 + a L_{\text{Pl}} \Delta p + \alpha L_{\text{Pl}}^2 (\Delta p)^2 + \cdots.
\]

In the simplest case \( \alpha' = b + \alpha \), where \( b \) comes from the modified de Broglie relation, while \( \alpha \) is due to the gravitational interaction between the electron and the photon. The ellipsis on the right-hand side of equation (11) means that surely we are neglecting higher order corrections

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2 In principle, there may be the possibility that the de Broglie relation holds up to a maximum momentum \( p_{\text{max}} \) and minimum wavelength \( \lambda_{\text{min}} \) (not smaller than the minimum length of the theory) and that the momentum of a single particle cannot exceed \( p_{\text{max}} \). However, even if there exists this maximum momentum, as in the frameworks of [5], such an explanation cannot work for non-elementary objects.
in $L_{Pl}$. Even if in principle the dimensionless parameter $a$ could be positive or negative, we can reasonably guess that $f(p)$ is a monotonic function and hence $a \geq 0$. As for the sign of higher order coefficients in equation (9), we do not have any indication, but anyway the expression must be consistent with the existence of a minimum length.

For instance, a modified de Broglie relation which leads to $a \neq 0$ and seems consistent with general requirements is

$$\lambda = \frac{2\pi L_{Pl}}{1 - \exp(-L_{Pl}p)} = \frac{2\pi}{p} \left( 1 + \frac{1}{2} L_{Pl} p + \cdots \right).$$

(12)

A common choice which can be found in the literature is instead

$$\lambda = \frac{2\pi L_{Pl}}{\tanh(L_{Pl}p)} = \frac{2\pi}{p} \left( 1 + \frac{1}{3} L_{Pl}^2 p^2 + \cdots \right).$$

(13)

Equation (13) is derived for example in [7], where the authors assume, among other things, the following canonical commutation relation:

$$[x, p] = i \left( 1 + \alpha L_{Pl}^2 p^2 \right).$$

(14)

Even if it is not the unique possibility, equation (14) leads to the GUP in equation (1). However, if we relax the requirement that the new commutation relation has to reproduce exactly the GUP in equation (1), because we believe that the latter is only an approximated expression, both in the canonical commutation relation and in the de Broglie relation the leading order corrections to their standard expressions can be proportional to the first power of the Planck length. Unfortunately, at least at present, there are no model-independent arguments which can suggest a particular high energy generalization of the relation between momentum and wavelength and hence we have a large number of reasonable candidates.

The derivation of the modified de Broglie relation from the assumed canonical commutation relation is non-trivial and depends on several assumptions, in particular on the fate of Lorentz invariance at short distances (since a minimum length implies the deformation or violation of the latter) and how it is realized in the model. However, if we knew the ‘true’ commutation relation $[x, p]$, we would not need any gedanken experiment to find the new uncertainty principle, but we could use the general formula

$$(\Delta A)^2 (\Delta B)^2 \geq \frac{1}{4} |\langle [A, B] \rangle|^2,$$

(15)

which holds for any observable $A$ and $B$ (in our case $A, B = x, p$) and basically relies only on the definition of uncertainty and expectation value of observables. From equation (15) we clearly see that $a \neq 0$ demands just that the leading order correction of $[x, p]$ is linear in $L_{Pl}$. If it is not, then $a = 0$.

The outcome of the Heisenberg’s experiment may also be affected by a deviation from standard conservation rules. Indeed, even if not strictly compulsory, in some theoretical frameworks with a minimum length the usual energy–momentum conservation is ‘deformed’. An example can be found in the third paper of [5], where the authors consider a two-body collision $1 + 2 \rightarrow 3 + 4$ and find

$$\frac{p_1}{1 - L_m E_1} + \frac{p_2}{1 - L_m E_2} = \frac{p_3}{1 - L_m E_3} + \frac{p_4}{1 - L_m E_4}.$$  

(16)
where $L_m$ is the minimum and universal length scale of the theory, which in our case is $L_{Pl}$ or of order $L_{Pl}$. Here the counterpart of equation (3) is

$$p'_x(1 + L_m E' + \cdots) + q' \sin \theta (1 + L_m q' + \cdots) = p''_x(1 + L_m E'' + \cdots) - q'' \sin \theta (1 + L_m q'' + \cdots),$$

where $E'$ and $E''$ are the electron energies after the collision with the photon in the two limiting cases. Equation (4) has now to be replaced with

$$\Delta p \gtrsim p''_x - p'_x \sim 2q' \theta + L_m (2q'^2 \theta + p'_x E' - p''_x E'') + \cdots,$$

and we get equation (11) with $a L_{Pl} \sim L_m$ if there are no other corrections linear in $L_{Pl}$.

Here I would like to stress that when one deduces heuristically for the first time the GUP, the term proportional to $L_{Pl}$ cannot appear, because the Planck length has no intrinsic meaning in Newtonian gravity or in general relativity. On the other hand, in the emerging picture $L_{Pl}$ is promoted to the fundamental parameter of the theory, which controls the minimum space resolution, and revising self-consistently the GUP, we cannot exclude the possibility that $a \neq 0$ in equation (11). More precisely, it is clear from equation (6) that the GUP is obtained when we combine quantum mechanics, which provides a good description of physical phenomena at energies much smaller than the Planck scale, with standard gravity, which works for macroscopic objects, whose masses are many orders of magnitude larger than the Planck one. On the other hand, we are unable to describe the microscopic physics near the Planck scale and for this reason we do not know strong arguments which require us to write equation (11) with $a \neq 0$. One could try to find indications from string theory, but even the latter is not fully under control near the Planck (string) scale. However there may be arguments which point to this direction: indeed, the usual description of black holes is expected to break down below a minimum mass $M_{\text{min}} \sim M_s/g_s^2$, where $M_s$ is the string scale (usually somewhat smaller than the Planck mass) and $g_s$ is the string coupling, often assumed smaller than one. There may be an energy range around the Planck/string scale where physics is intrinsically stringy and before the interaction between the electron and the photon is disturbed by black hole production (see the second argument in section 4), the formation of ‘string balls’ could dominate the uncertainty of the electron position, see, e.g., [8] and references therein. These string balls would be objects whose size at the moment of their formation is at the level of the string length $L_s = 1/M_s$, independently of their mass (by contrast, black hole radius increases linearly with the mass). If so, there would be an energy range where the spatial resolution is limited by the creation of such extended objects and we could be tempted to replace equation (6) with

$$\Delta x_{\text{min}} \sim \max[1/(\Delta p), L_s, G_N \Delta p].$$

Then, combining linearly the three uncertainty in equation (19), we would get equation (11) with $L_s \approx a L_{Pl}$ and $\alpha' = 1$ ($L_s$ is usually only a little larger than $L_{Pl}$, say $L_s \sim L_{Pl}/g_s$).

5. Hydrogen atom

It is well known that the hydrogen atom (and every atom in general) should quickly decay classically, but that the HUP makes it stable. Indeed, the energy of the electron orbiting around the proton is

$$E \sim \frac{p^2}{2m_e} = \frac{e^2}{r},$$

(20)
and classically it has no lower bound: for $r \to 0$, $E \to -\infty$. On the other hand, from the HUP follows
\[ E \gtrsim \frac{1}{2m_{\varphi}r^2} - \frac{e^2}{r} \geq -\frac{me^4}{2} = -E_0 \sim -10 \text{ eV}, \tag{21} \]
where $E_0$ is roughly the energy of the ground state. In the case of the GUP in (1), we can proceed as above and consider the term proportional to the square of the Planck length as a small correction to the standard formula. After trivial passages, one finds the order of magnitude of the correction to the ground state:
\[ \Delta E_0 \sim L_{Pl}^2 \rho^2 e^{8} \]
\[ \sim m_{\varphi} \left( \frac{E_0}{E_{Pl}} \right) \sim 10^{-48} \text{ eV}, \tag{22} \]
where $E_{Pl} = 1/L_{Pl} \sim 10^{19}$ GeV is the Planck energy. On the other hand, if the GUP has the term linear in $L_{Pl}$, the induced energy shift of the ground state would be
\[ \Delta E_0 \sim m_{\varphi}^{1/2} \omega^{1/2} \left( \frac{E_0}{E_{Pl}} \right) \sim 10^{-23} \text{ eV.} \tag{23} \]
Such a correction is surely still too small to be experimentally tested: at present, theoretical predictions and laboratory measurements can agree at best at the level of some kHz, about $10^{-11}$ eV, as in the case of the Lamb shift in the hydrogen atom [9]. However, the effect would be anyway not so much suppressed as it is commonly thought and, hopefully, future investigations on the subject may point to some phenomenon where corrections to the HUP could be experimentally interesting in the case where the leading order term is proportional to $L_{Pl}$, while could not in the case where the latter is proportional to $L_{Pl}^2$.

6. Harmonic oscillator

Another interesting system with a lot of possible applications is represented by the harmonic oscillator. For example, it can provide a crude picture of heavy meson systems such as the charmonium and the bottomonium [10], where the confining force can be described by a linearly rising potential at large distances. The Hamiltonian is
\[ E = \frac{p^2}{2m_{\varphi}} + \frac{1}{2}m_{\varphi}\omega^2 r^2, \tag{24} \]
where $m_{\varphi}$ is the constituent quark mass, i.e. $m_c \approx 1.3$ GeV for the $c$-quark and $m_b \approx 4.5$ GeV for the $b$-quark. From the HUP $p \gtrsim 1/r$, we find
\[ E \gtrsim \frac{1}{2m_{\varphi}r^2} + \frac{1}{2}m_{\varphi}\omega^2 r^2 \geq \omega, \tag{25} \]
where $\omega$ is the binding energy of the system and is roughly equal to the energy gap separating adjacent levels, so $\omega \sim 0.3$ GeV. The correction of the standard GUP is
\[ \Delta E \sim m \left( \frac{\omega}{E_{Pl}} \right)^2 \sim 10^{-36} \text{ MeV}, \tag{26} \]
while in the case the leading order correction to the HUP is linear in $L_{Pl}$ we have
\[ \Delta E \sim m^{1/2} \omega^{1/2} \left( \frac{\omega}{E_{Pl}} \right) \sim 10^{-17} \text{ MeV.} \tag{27} \]

3 To be more precise, here $m_{\varphi}$ would be the reduced mass of the system, i.e. $m_{c}/2$ or $m_{b}/2$, but for our order of magnitude estimate this is not relevant.
Experimental uncertainties are typically something less than 1 MeV, but in the case of $J/\psi$ the accuracy is at the level of $10^{-2}$ MeV [11]. Theoretical predictions are much more difficult than the hydrogen atom because they are based on semi-phenomenological models. So, these quantum gravity effects are clearly well beyond experimental tests for the foreseeable future.

7. Conclusions

There are essentially no doubts that gravity implies a revision of the Heisenberg uncertainty principle, but it is always assumed that the first-order correction to the standard formula is proportional to the gravitational constant $G_N$, that is to the square of the Planck length $L_{Pl}$. In this paper I have shown that this is not necessarily true and that, once $L_{Pl}$ has acquired a clear physical meaning, associated with the minimum observable length of the theory, there are apparently no reasons to exclude that the leading order correction is proportional to the first power of $L_{Pl}$. Such a result cannot be deduced at a first approach, because common derivations of the generalized uncertainty principle use the standard formulation of quantum mechanics and general relativity, where the Planck length is only the square root of the gravitational coupling constant $G_N$. On the other hand, the emerging picture necessarily requires a revision of the ingredients used to deduce it. For example, the existence of a minimum length implies a modification of the standard de Broglie relation and this affects the final expression of the generalized uncertainty principle. Similar consequences may also arise from some ‘deformed’ energy–momentum conservation. Stringy effects at the Planck/string scale may also play a fundamental role. Unfortunately, the presence or absence of the linear term is model dependent and at present there are no firm theoretical arguments that can suggest what really happens. On the other hand, such an observation can make the phenomenology more interesting and hopefully within the reach of future experiments. Lastly, it should be clear that, anyway, the generalized uncertainty principle in equation (1) cannot be an exact formula, but at best a good approximation valid at some (not yet well-understood) regime.

Acknowledgments

I would like to thank Federico Urban for helpful comments and suggestions. This work is supported in part by NSF under grant PHY-0547794 and by DOE under contract DE-FG02-96ER41005.

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