Advection errors in an orthogonal terrain-following coordinate: idealized 2-D experiments using steep terrains

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1. Introduction

The classic terrain-following coordinate (the CTF-coordinate; Phillips, 1957; Gal-Chen and Somerville, 1975) can turn a complex earth surface into a coordinate surface, so as to simplify the lower boundary of a numerical model. At present, the CTF-coordinate has been widely applied in atmospheric and oceanic models (Bleck, 2002; Wang et al., 2004; Davies et al., 2005; Madec, 2008; Skamarock et al., 2008, 2012; Wallcraft et al., 2009; Skamarock et al., 2012; Schättler et al., 2013). The steep vertical layers and the non-orthogonal vertical computational grids of the CTF-coordinate over steep terrain, however, induce significant advection errors in a model (Thompson et al., 1985; Sharman et al., 1988; Pielke, 2002; Sankaranarayanan and Spaulding, 2003; Steppeler et al., 2003; Ji et al., 2005; Li et al., 2012; Mesinger et al., 2012). Many methods have been proposed to reduce the advection errors in the CTF-coordinate. The most popular and successful one is the hybrid terrain-following coordinate (the HTF-coordinate; Arakawa and Lamb, 1977; Simmons and Burridge, 1981; Simmons and Strüfing, 1983; Schär et al., 2002; Klemp, 2011; Li et al., 2011), which has the smoothed vertical layers over steep terrain. An innovative method called cut-cell method featured with the orthogonal Cartesian grids above terrain and irregular grids near the terrain has been introduced to overcome the problem of non-orthogonal grids in the vertical (Adcroft et al., 1997; Yamazaki and Satomura, 2010; Lock et al., 2012; Adcroft, 2013; Steppeler et al., 2013; Good et al., 2014).

Recently, an orthogonal terrain-following coordinate (an OTF-coordinate; Li et al., 2014) has been designed to reduce the advection errors in the CTF-coordinate. In details, the OTF-coordinate can reduce the advection errors via smoothing the vertical layers, such as in a HTF-coordinate. More importantly, it can create orthogonal grids in the vertical, thereby further reducing the advection errors compared to the corresponding HTF-coordinate. The experiments with three kinds of terrains implemented by Li et al. (2015) indicated that the more complex the terrain is, the greater reduction of advection errors by the OTF-coordinate is. All the experiments implemented by Li et al. (2014, 2015) however, adopted terrains with gentle slopes only. Though the results consistently attested that both the smoothed vertical layers and orthogonal grids could reduce the advection errors, the former contributes more than the later.

This study aims to distinguish the relative importance between the smoothed vertical layers and orthogonal grids created by the OTF-coordinate over steep terrain. First, a new kind of terrain is designed with much steeper slope than the terrain employed by Li et al. (2014). Then, a set of 2-D linear advection experiments is conducted in the CTF-coordinate, HTF-coordinate and OTF-coordinate using the newly designed terrains. Finally, the relative importance of smoothed vertical layers and orthogonal grids in reducing the advection errors is shown by comparing the advection errors in these three coordinates. Moreover, the cause for different effects of orthogonal grids over different kinds of terrains is analyzed by comparing the number of

Keywords: terrain-following coordinate; advection errors; smoothed vertical layers; orthogonal computational grids; skewness; steep terrain

Abstract

This study illustrated the importance of smoothed vertical layers and orthogonal grids in an orthogonal terrain-following coordinate (an OTF-coordinate) in reducing advection errors over steep terrain. Three coordinates, namely, classic terrain-following-coordinate, hybrid terrain-following coordinate (HTF-coordinate) and OTF-coordinate, were employed in Schär-type advection experiments for various terrains. The results demonstrated that the OTF-coordinate could diminish the grids with high skewness in the HTF-coordinate over steep terrain; the orthogonal grids share the same importance in reducing advection errors with smoothed vertical layers. Therefore, the advection errors in the OTF-coordinate are considerably reduced than those in the corresponding HTF-coordinate over steep terrain.

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In order to increase terrain slope, a new kind of terrain is designed based on the five-crest wavelike terrain proposed by Schär et al. (2002). The expression of the new terrain is given by

\[ h(x) = \sin^2 \left( \frac{n \pi x}{2a} \right) h^*(x), \]  

where

\[ h^*(x) = \begin{cases} 
  h_0 \sin^2 \left( \frac{\pi x}{2a} \right) & |x| \leq a \\
  0 & |x| \geq a 
\end{cases} \]  

\( h_0 \) is the maximum height of the terrain, \( a \) is the half-width of the terrain, and \( n \) is the number of terrain peaks. Note that the maximum \( h(x) \) equals \( h_0 \), only if \( n \) is twice of an odd number.

The three kinds of terrains proposed by Schär et al. (2002) are shown in Figure 1(a)–(c), the one used in Li et al. (2014) is shown in Figure 1(c), and three kinds of new terrains designed in this paper are illustrated in Figure 1(e)–(g). Specifically, the new terrains have the same width but double the number of the peaks, respectively, corresponding to the ones proposed by Schär et al. (2002), and then lead to the increase of the slope as well as the number of the maximum slope that is mostly relevant to the high skewness of the computational grids (Figure 1(d) and (h)).

3. Parameters in the 2-D linear advection experiments

For consistency, we use the same parameters as Li et al. (2014), except for the terrain, the time step and the time integration scheme. Specifically, the definition \( \sigma = H \frac{b}{H + b} \) proposed by Gal-Chen and Somerville (1975) is adopted in the CTF-coordinate, where \( H \) is the top of model and \( b \) represents the terrain. The rotation parameter \( b = \left( \frac{H - c}{H + b} \right)^n \) \((n = 20)\) is chosen for the OTF-coordinate. The definition \( \sigma = z - \left( \frac{H - c}{H + b} \right)^n \) \((n = 24)\) is used in the HTF-coordinate to create similar vertical layers as those in the OTF-coordinate. Note that, the vertical layers are similar, but not identical, to those in the HTF-coordinate and OTF-coordinate, and the slopes of the vertical layers in these coordinates are shown in Figure S1, Supporting Information.

Three kinds of terrains shown in Figure 1(e)–(g) are used in the following experiments, and named as two-, six- and ten-crest terrain, respectively. The ten-crest terrain is shown in Figure 2 as an example. The domain of the experiments, fixed with 0–300 km in the horizontal and 0–25 km in the vertical, is also shown in Figure 2.
Figure 1. Six kinds of terrains used in this study. (a)–(c) are the terrains proposed by Schär et al. (2002) with \( \lambda = 32, 16 \) and 8 km. (e)–(g) are the terrains designed in this paper with \( n = 2, 6 \) and 10 in Equation (1). In both types of terrains, \( h_0 = 3 \) km and \( a = 50 \) km. (d) and (h) are the slopes of one-, three- and five-crest terrains and two-, six- and ten-crest terrains, respectively.

Figure 2. The wind field, the analytical solution of tracer and terrain used in the 2-D linear advection experiments. The left panel is the vertical profile of the given \( u \) field. The colored contours in the right panel represent the tracer \( q \) with contour interval of 0.1, and the thick black curve represents the newly designed ten-crest wavelike terrain.

4. Comparison of advection errors in different coordinates over various terrains

First, the vertical computational grids of the CTF-coordinate, HTF-coordinate and OTF-coordinate over two-, six- and ten-crest terrains as well as their skewnesses are shown in Figures 3 and 4. For each coordinate, there are 301 grids in the horizontal and 51 grids in the vertical, and every other grid line is shown in Figure 3. The skewness is calculated by maximum \( ||\theta - 90^\circ|| \), where \( \theta \) is the angle between two neighboring faces of a grid. Note that the smaller the skewness is, the more orthogonal the grid is, and the higher the grid quality is.

Compared to the CTF-coordinate (black grid lines in Figure 3), the HTF-coordinate (red grid lines in Figure 3) enjoys the smoothed vertical layers (also shown in Figure S1); and compared to the HTF-coordinate, the OTF-coordinate employs the orthogonal grids as shown in Figures 3 and 4. Consequently, the effect of smoothed vertical layers on reducing the advection errors can be obtained through comparing the advection errors between the HTF-coordinate and CTF-coordinate; while the comparison of the advection errors between the OTF-coordinate and HTF-coordinate can explain the effect of orthogonal grids.

Second, the root mean square errors (RMSEs) of the advection term in the CTF-coordinate, HTF-coordinate and OTF-coordinate in the whole integration over two-, six- and ten-crest terrains are all shown in Figure 5 (the relevant absolute errors at the end of integration are shown in Figure S2). Over the two-crest terrain, the RMSE in the HTF-coordinate is smaller than that in the CTF-coordinate (Figure 5(a)); while the RMSE in the OTF-coordinate is comparable with that in the HTF-coordinate (Figure 5(b)). It reveals that the smoothed vertical layers contribute more to reduce the advection errors than the orthogonal grids over the two-crest terrain, which is consistent with Li et al. (2014, 2015) as shown in Figure S3.

However, over the six- and ten-crest terrains, the RMSEs in the HTF-coordinate are much smaller than
Figure 3. Computational grids of the CTF-coordinate, HTF-coordinate and OTF-coordinate in the vertical over three kinds of terrains. Black curves in (a)–(c) are the grid lines in the CTF-coordinate; red curves in (d)–(f) are the grid lines in the HTF-coordinate; and blue curves in (g)–(i) are the grid lines in the OTF-coordinate.

those in the CTF-coordinate (Figure 5(c) and (e)); while the RMSEs in the OTF-coordinate are also much smaller than those in the HTF-coordinate (Figure 5(d) and (f)). Therefore, the effect of orthogonal grids is as important as the effect of smoothed vertical layers over the six- and ten-crest terrains.

Third, the RMSEs in the CTF-coordinate, HTF-coordinate and OTF-coordinate at the end of the integration obtained in this study using the newly designed steep terrains as well as the results obtained by Li et al. (2014, 2015) using relative gentle terrains are summarized in Table 1. According to the order of magnitude of RMSE in each coordinate, these six experiments can be classified into three types (in black, light blue and orange in Table 1).

Type one includes the experiments using one- and two-crest terrains (in black in Table 1). The RMSEs in all three coordinates are of the same order of magnitude, namely, both effects of smoothed vertical layers and orthogonal grids on reducing the advection errors are relatively inconspicuous over gentle terrain.

Type two includes the experiments using three- and five-crest terrains (in light blue in Table 1). The RMSEs in the HTF-coordinate and OTF-coordinate are of the same order of magnitude, but both of them are at least one order of magnitude smaller than those in the CTF-coordinate. It shows that the effect of smoothed vertical layers on reducing the advection errors is evident over steep terrain; namely, the effect of smoothed vertical layers over steep terrain is greater than the effect of orthogonal grids.

Type three includes the experiments using six- and ten-crest terrains (in orange in Table 1). The RMSEs in the HTF-coordinate are one order of magnitude smaller than those in the CTF-coordinate; however, the RMSEs in the OTF-coordinate are also one order of magnitude smaller than those in the HTF-coordinate. Explicitly, the effect of orthogonal grids is as important as the effect of smoothed vertical layers on reducing the advection errors over very steep terrain.

Finally, in order to explain different effects of orthogonal grids over various terrains as categorized in Table 1, the numbers of grids with certain skewness in the HTF-coordinate and OTF-coordinate are counted (Table 2). The numbers of a certain value of skewness in the OTF-coordinate are constantly smaller than those in the HTF-coordinate; namely, the effect of orthogonal grids on reducing the advection errors is achieved through diminishing the skewness of the computational grids.

For the one- and two-crest terrain experiments (in black in Table 2), none of the grids has skewness greater than 0.3. However, Thompson et al. (1985) proposed that the truncation error due to the non-orthogonal grids could be significantly reduced when the grid angle is smaller than 45° (skewness greater than 0.5). Therefore, the orthogonal grids have small effect on reducing the advection errors over one- and two-crest terrains. For the three-crest terrain experiments (in light blue in Table 2), there are only eight grids with skewness greater than 0.3 in the HTF-coordinate. Again, the effect of orthogonal grids is inconspicuous.

For the five- and six-crest terrain experiments (in magenta and orange in Table 2, respectively), the numbers of grids with skewness greater than 0.5 are five and eight in the HTF-coordinate, respectively. Therefore, the effect of orthogonal grids should be conspicuous. There results reveal the considerable effect of orthogonal grids over six-crest terrain, but not consistent with the relatively low effect over five-crest terrain, indicating that the effect of orthogonal grids may be sensitive to the number of the grids with high skewness. For the ten-crest terrain experiments (in orange in Table 2), there are 12 grids with skewness greater than 0.6, and 32 grids with skewness greater than 0.5 in the HTF-coordinate. Accordingly, the effect of orthogonal grids is significant.
In addition, the RMSEs in Table 1 are consistent with the skewness shown in Figures 4 and S4, namely, having the maximum in the CTF-coordinate and the minimum in the OTF-coordinate. Therefore, just as the orthogonal grids, the smoothed vertical layers can also diminish the skewness of computational grids, and therefore reduce the advection errors.

In conclusion, both the smoothed vertical layers and orthogonal grids can diminish the skewness of the computational grids, therefore reducing the advection errors (Figures 4 and S4, Table 1). However, the effect of orthogonal grids is sensitive to the number of grids with high skewness (Tables 1 and 2). Specifically, the effect of orthogonal grids on reducing the advection errors is considerable only when it can diminish the number of grids with high skewness. As a result, over gentle terrain, the grids in the HTF-coordinate are all with small skewness, and then the effect of orthogonal grids on reducing the advection errors is very small. Over very steep terrain, numerous grids in the HTF-coordinate have high skewness; therefore, the effect of orthogonal grids is considerable. Accordingly, the effect of orthogonal grids is as important as the smoothed vertical layers on reducing the advection errors over very steep terrain.

5. Conclusion and discussion

In order to explore the advection errors in the OTF-coordinate associated with smoothed vertical layers as well as orthogonal grids over steep terrain, this study used the CTF-coordinate, HTF-coordinate and OTF-coordinate to implement a set of 2-D linear advection experiments with terrains of various slopes. First, a new kind of terrain was designed based on the wavelike terrain proposed by Schär et al. (2002) to increase steepness (Figure 1(e)–(g)). Then, the CTF-coordinate, HTF-coordinate and OTF-coordinate were implemented in the 2-D linear advection experiments with three kinds of newly designed terrains (two-, six- and ten-crest terrains).

Comparing the advection errors between the CTF-coordinate and HTF-coordinate, as well as those between the HTF-coordinate and OTF-coordinate, we
verified that both smoothed vertical layers and orthogonal grids can consistently reduce the advection errors over various terrains (Figure 5). However, the relative importance of these two approaches in reducing the advection errors depends on terrain slope (Figure 5 and Table 1). The experimental results showed that: (1) over gentle terrain, both effects of smoothed vertical layers and orthogonal grids are relatively inconspicuous; (2) over steep terrain, the effect of smoothed vertical layers becomes significant; therefore, it becomes a more important factor than the orthogonal grids; (3) over very steep terrain, the effect of orthogonal grids is as important as that of smoothed vertical layers. Therefore, the advection errors in the OTF-coordinate can be considerably reduced than those in the HTF-coordinate over very steep terrain (Figure 5(d) and (f); in orange in Table 1).

The effect of orthogonal grids is sensitive to the number of grids with high skewness (Tables 1 and 2) as proposed by Thompson et al. (1985). The effect is considerable only when the orthogonal grids can diminish the number of grids with high skewness, while the grids with high skewness are sensitive to the steepness of terrain. Therefore, the effect of orthogonal grids is considerable over steep terrain. The effect of smoothed vertical layers on reducing the advection errors is also achieved through diminishing the skewness of grids (Figures 4 and S4, Table 1).

In addition, the vertical layers in the OTF-coordinate are smoother than those in the HTF-coordinate in this
Table 1. RMSEs of advection term in three coordinates for six kinds of terrains at the end of the integration.

| Kind of terrain | CTF-coordinate | HTF-coordinate (representing the effect of smoothed vertical layers) | OTF-coordinate (representing the effect of orthogonal grids) |
|-----------------|----------------|-------------------------------------------------|-------------------------------------------------|
| One-crest       | 3.36\times10^{-3} | 2.18\times10^{-3} | 2.14\times10^{-3} |
| Three-crest     | 1.01\times10^{-2}  | 2.32\times10^{-3} | 2.15\times10^{-3} |
| Five-crest      | 1.59\times10^{-1}  | 6.78\times10^{-3} | 2.61\times10^{-3} |
| Two-crest       | 8.47\times10^{-3}  | 2.66\times10^{-3} | 2.15\times10^{-3} |
| Six-crest       | 1.54\times10^{-1}  | 2.46\times10^{-2} | 4.21\times10^{-3} |
| Ten-crest       | 4.97\times10^{-1}  | 6.80\times10^{-2} | 6.72\times10^{-3} |

Results in the CTF-coordinate, HTF-coordinate and OTF-coordinate in one-, three- and five-crest terrain experiments are calculated in this paper using the same parameters as in Li et al. (2015) except for the time integration scheme, which is to use the fourth-order Runge–Kutta scheme instead of leapfrog scheme.

Table 2. Numbers of grids with certain skewness in the HTF-coordinate and OTF-coordinate for six kinds of terrains.

| Kind of terrain | HTF-coordinate | OTF-coordinate |
|-----------------|----------------|----------------|
|                 | >0.1           | >0.2           | >0.3           | >0.4           | >0.5           | >0.6           |
|                 | >0.1           | >0.2           | >0.3           | >0.4           | >0.5           | >0.6           |
| One-crest       | 84             | 2              | 0              | 0              | 0              | 0              |
| Three-crest     | 93             | 38             | 8              | 0              | 0              | 0              |
| Five-crest      | 173            | 96             | 46             | 25             | 5              | 0              |
| Two-crest       | 162            | 58             | 0              | 0              | 0              | 0              |
| Six-crest       | 154            | 90             | 52             | 30             | 8              | 0              |
| Ten-crest       | 194            | 126            | 86             | 58             | 32             | 12             |
|                 | 178            | 88             | 48             | 26             | 10             | 0              |

Experiments indicated by light blue and orange are consistent with the category in Table 1, and those by magenta have differences with the category in Table 1.

study (Figure S1), which partly enhanced the effect of orthogonal grids. The experiments using identical vertical layers in the HTF-coordinate and OTF-coordinate are being implemented in an ongoing study. Further analyses are needed to explain the relationship between grid quality in the vertical and advection errors over various terrains, through using more indexes such as smoothness and aspect ratio. Moreover, the effect of orthogonal terrain-following grids created by the OTF-coordinate needs to be compared with that of the vertical grids created by the cut-cell method in terms of reducing the advection errors.

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Supporting information

The following supporting information is available:

Figure S1. Slopes of vertical layers in three coordinates for two-, six- and ten-crest terrains. Figure S2. Absolute errors of advection term in three coordinates for two-, six- and ten-crest terrains. Figure S3. RMSEs of advection term in three coordinates for one-, three- and five-crest terrains. Figure S4. Skewness of vertical grids in the CTF-coordinate, HTF-coordinate and OTF-coordinate for three kinds of terrains.

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