Student’s triangles congruence proving through flow proof strategy

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Abstract. The purpose of this study is to describe students proving congruence triangles. Proving congruence is a prerequisite to develop skills in constructing formal proof in mathematics. The subject in this study are 33 students of SMP Negeri 33 grade IX.1. The Learning process implemented with flow proof strategy, namely re-writing existed information, stating what is needed to be proved, using existed rules such as postulate and definition, and writing the consequence from used rules. Data were collected by a test with three questions and interviews. The results show that students know step by step in writing the proofs after the implementation of flow proof strategy and students’ ability on congruence proving in class nine SMP N 33 Palembang categorized satisfactory, with 4% of the students categorized as excellent, 16% in good, 48% in satisfactory, 24% in poor, and 8% in very poor category.

1. Introduction

In mathematics, geometry can be implemented to solve problems in human activity and assist in forming abstract thinking from concrete thinking [1-6]. One of the important topics in geometry is congruence because by learning congruence, several abilities such as spatial ability connection ability, and advanced thinking can be developed [7-10].

In topic of congruence, students must have the ability to prove because congruence proving can develop formal proof writing ability in mathematics [11]. Proof and proving are important parts in mathematics because they are the main points to construct mathematical knowledge [12-15] and can develop critical thinking ability, logic thinking, and improve mathematical understanding [16,17]. There are several advantages when solving mathematics proof, namely (1) verify the truth, (2) give knowledge why the statement is true, (3) communicate mathematics knowledge. For these reasons, proving in mathematics learning must be available [18].

Students unable to construct congruence proof with two column proof [19]. Therefore, a way to help to construct proof is needed. Besides using two column proof, there is another strategy for construct proof that is flow proof [20]. Flow proof is to construct proof with a flowchart [20]. Steps in proving with flow proof are analysis existed information and seek additional information needed for construct proof [21]. In constructing a proofs with flow proof students think up and back down for connecting between conjecture and conclusion [22]. Based on two opinions above, proving steps with flow proof in this research are (1) re-writing existed information, (2) stating what is needed to be proved, (3) using existed rules such as postulate and definition, and (4) writing the consequence from used rules. Flow proof facilitates students in constructing a proof [21]. This caused flow proof process started by analysis existed information in problems. Students more successful in constructing a proof with flow proof rather
than two column proof [23]. Applying flow proof in constructing a proof exercises students’ critical thinking and logic thinking [24]. Therefore based on the description above the purpose of this research is to find out students’ triangles congruence proving through flow proof strategy.

2. Method
This research is a descriptive qualitative study that aims to describe student’s triangles congruence proving through flow proof strategy. The subjects are grade IX.1 students of SMP Negeri 33 Palembang. This research implemented on August 8, 2019 until August 22, 2019. Data was collected by a result of tests and interviews. The research procedure consisted of the preparation phase, the implementation phase, and the data analysis. In the preparation stage, the researcher prepares an instrument such as pooron plan (RPP), students worksheet (LKPD) uses a flow proof strategy, a test consisting of three questions, an interview guide made based on mathematical proof indicators, and a scoring guide. Then, the researchers implemented learning with a flow proof strategy of two meetings. Before entering the core of learning, researchers explain the symbols in the diagram. The parallelogram is used to re-writing existed information, what is needed to be proved and existed rules such as postulate and definition. Rhombus is used for selecting the conditions of a triangle that is said to be congruent. The rectangle is used for the process when two triangles are congruent and the last arrow is used to represent an argument. Furthermore, at the core stage of learning there are 5 activities that implemented at meetings 1 and 2 namely problem exposure, group formation, use of flow proof which consists of re-writing existed information, stating what is needed to be proved, using existed rules such as postulate and definition, and writing the consequence from used rules, after the use of flow proof there are still two stages namely presentation and conclusion. At the third meeting, the activities were working on test questions. Problem number 1 proves three angles triangle are congruent if known that the sides are congruent. Problem number 2 is about proving three sides of a triangle that are congruent if the angles are congruent. And problem number 3 is about proving two triangles that are congruent due to the bisector of an equilateral triangle. After that, the test results are analyzed according to the scoring guidelines that have been made then the subject is chosen based on the level of ability to be interviewed.

3. Results and Discussion
After the test implemented, then the test data is analysed to see the mathematical proof ability. Students’ proving ability after being analysed and categorized in Table 1.

| Score  | Category       | Frequency | Percentage |
|--------|----------------|-----------|------------|
| 86 – 100 | Very good     | 1         | 4%         |
| 71 – 85   | Good          | 4         | 16%        |
| 56 – 70   | Satisfactory  | 12        | 48%        |
| 41 – 55   | Very poor     | 6         | 24%        |
| 0 – 40    | Very Very poor| 2         | 8%         |

Based on Table 1, students proving ability in class IX.1 of SMP Negeri 33 Palembang is categorized into five categories: very good, good, satisfactory, poor and very poor. The following is a description of the analysis of students' answers to the very good, good, satisfactory, poor and very poor categories. From 25 students, only 1 student was categorized as very good. This is due to mathematical proof is one of the activities that require a high level of ability so that only few students can complete it. This is in line with Nurrahmah's research [25] that most students have difficulty in proving. Students with very good categories get a score of 3 on each question for proving correctly without making mistakes. In addition, in questions number 1 to 3 students with a very good category show mathematical proof indicators starting with identifying information that is by writing what is known and what will be proven,
right in using concepts and principles, and using communicative language in constructing a proof and analysis of the information in the problem to be proven. Ability to analyse the information contained in the problem to be proven indicators appeared through an interview. The researcher asked the very good student of how the steps in writing proof and he answered that writing proof starts from writing what is known and what will be proven and choosing the rules. Students can determine the steps in writing because learning has been implemented flow proof strategy, so students know the flow in constructing a proof.

Table 2. Distribution of student answers type.

| Question Numbers | Value        | Categorized                  | Type of Answers                                                                 |
|------------------|--------------|------------------------------|---------------------------------------------------------------------------------|
| 1                | Good         |                              | Proving precisely                                                               |
| Satisfactory     |              |                              | Proving precisely                                                               |
|                  |              |                              | Proving incorrectly in identifying information                                |
|                  |              |                              | Proving incorrectly in choosing principles and rules                            |
| Poor             |              |                              | Proving correctly but not finished using principles and rules                   |
|                  |              |                              | Proving by writing inaccurate information                                       |
| Very poor        |              |                              | Proving that with no true statement                                             |
| 2                | Good         |                              | Proving by choosing the right principles and rules but is wrong in writing down the consequences of principles and rules |
| Satisfactory     |              |                              | Proving by mistaken symbols                                                    |
|                  |              |                              | Proving by writing inaccurate information                                       |
|                  |              |                              | Proving improperly in choosing principles and rules                             |
| Poor             |              |                              | Proving improperly in choosing principles and rules and writing symbols incorrectly |
|                  |              |                              | Does not proving                                                               |
|                  |              |                              | Proving only by writing information that is true and correct                    |
| Very poor        |              |                              | Proving that with no true statement                                             |
| 3                | Good         |                              | Proving with the right rules and principles but not right in identifying what will to prove |
| Satisfactory     |              |                              | Proving precisely                                                             |
|                  |              |                              | Proving with the right rules and principles but not right in identifying what is going to prove |
| Poor             |              |                              | Proving it by only writing down inaccurate information                          |
|                  |              |                              | Proving by only writing information that is correct but incomplete              |
| Very poor        |              |                              | Proving that with no true statement                                             |

Table 2 shows all students categorized good able to answer question number 1 and get a score of 3 because precisely in constructing a proof and nothing mistakes. Whereas in problem number two, there are 2 types of students' answers that have errors in using principles and rules. Errors in principles and rules are also in line with Nadlifah and Prabawanto research [26]. The study stated that in constructing a proof, students have mistaken in using concepts and definitions. Errors in principles and concepts can be seen in Figure 1.
Figure 1. The mistake in using principles and concepts.

Figure 1 shows students that categorized well having an error in writing the impacts of the rules and get score 2. It should be \( \angle A \cong \angle B \) and the impact is \( \overline{BC} \cong \overline{AC} \) and \( \angle C \cong \angle A \) the impact is \( \overline{AB} \cong \overline{BC} \). Students’ inaccuracy in writing the consequences of the used rules. Students in the good category are generally able to identify existing information. This was shown during the interview of the good category students saying they understood the symbols that exist so they can determine what was known from a theorem and only 1 student misidentified of what would be proven in question number 3. Besides that, good categorized students were also able in analysing the information contained in the problem that will be proven through interviews. Researchers ask good category students of how the steps in writing proof. They answer that to write proof start from writing what is known and what will be proven and choose the rules. Students can determine the steps in writing because learning has been implemented with a flow proof strategy, so students know the flow in constructing a proof. From the answers given by students, it was seen that students showed the indicators using communicative language in constructing a proof because they did not make mistakes in writing symbols.

Furthermore, students belong to satisfactory categories, in question number 1, there are 3 types of student answers. First the students proving precisely, second student failed in identifying the existing information, and third student mistaken in choosing the rules. Students who correctly write the proof in question number 1 have identified the information correctly by writing what is known, namely, \( \overline{AC} \cong \overline{BC} \cong \overline{AB} \) and what will be proven, that is \( \angle A \cong \angle B \cong \angle C \) as well as using the right principles and concepts namely using theorem 1 and transitive rules can be seen in Figure 2.

Figure 2. Satisfactory category student answers correctly number 1.

Figure 2 shows students in the satisfactory category fulfilled the indicators of using communicative proof language. This is indicated by the students being able to write the right symbols in constructing a proof. Therefore student in satisfactory category gets score 3 because give complete and correct answers with no errors. Furthermore, second type answers incorrectly identifying the information like writing
what will be proven, namely there are two triangles congruent which should be \( \angle A \cong \angle B \cong \angle C \). While the information that is known indicators all students in the category satisfactory correctly identify because they know the picture given is an equilateral triangle meaning that there are three sides of the same length. Then the third answer type of students is wrong in choosing the rules. Students use the rules of 2 triangles that are congruent which should be theorem 1 and transitive.

In question number 2 is the same as number one, there are 3 types of student answers: first, students write wrong symbols, second students are poor precise in identifying the information available, and third, students choose the rules incorrectly. While the information that is known to all students in the satisfactory category correctly identified because they know the picture given with the same measure of angle. Then the third answer type of students is wrong in choosing the rules. Students use the rules of two triangles that are congruent which should be theorem 2 and the transitive properties that happen because students have been wrong in identifying what will be proven to cause students to use the wrong rules. Difficulties in constructing a proof include understanding the existing problems and unable to apply existing concepts [27]. In question number 3 there are two types of students' answers, namely correct and poor precise in identifying existing information. In question number 3 the satisfactory category students correctly use the existing rules of definition and the rules of two triangles which are congruent, one of which is the side angle side. But there are students’ mistake in determining what will be proven, the same as students in good category. Some of the students write what would prove angles are congruent and others write the side is congruent that should just show whether two triangles are congruent or not. From the answers of students categorized as satisfactory, in general students are still difficult in determining what will be proven this is due to write what will be proven students must analyze from a statement. This is consistent with the results of Kartini and Suanto's research [28] which states that one of the students’ difficulties in constructing a proof is to understand what will be proven.

Most of students in the poor category make one error in each problem, like mistakes in writing what is known, what will be proven, symbols, and using rules. The last category is very poor, where each question does not have a correct statement. Students belong to very poor category get score 1 in every answer because they roof incorrectly. Besides all the indicators didn’t appear, when interviewed it was obtained that the students lack an understanding of the symbols so they could not identify what was known and caused difficulties in proving. This is in line with research conducted by Nadlifah and Prabawanto [26] states that the source of students’ difficulties in proving is the lack of understanding of the concept of existing definitions and images and limitations on language and symbols.

In general, students know the sequence in writing proof because flow proof strategy implemented from the first meeting with the problem theorem 2, namely ”If two angles of a triangle are congruent, then the sides opposite the congruent angles is congruent”.

Figure 3 shows with the flow proof strategy in the first step students are asked to rewrite what is known from the existing problem. In theorem 2 it is known that the two congruent angles are \( \angle A \cong \angle B \). Furthermore, with the guidance, students can determine what will be proven, namely \( AC \cong BC \). Furthermore, by using the existing rules in supporting information, students can determine that congruent triangles have 3 conditions, namely side-side-side, side-angle-side, and angle-side-angle. That means the side-side-side condition is not satisfying because of the known angle. Students use the symmetrical properties of the supporting information if \( \angle A \cong \angle B \) then \( \angle B \cong \angle A \). Because there are already two angles, meaning that there will be a congruent side. Students use the reflective properties found in supporting information if \( AB \cong BA \). In the final step, students write down the consequences of the rules used, namely fulfilling the congruent rules side-angle-side and \( \triangle ABC \cong \triangle BAC \) resulting in proven \( AC \cong BC \).
Figure 4 shows after completing the diagram, students are guided to construct proof, students are asked to construct a proof according to the steps in the diagram but asked to write in paragraphs. At this phase, students only copy what they write on the diagram which is originally in the form of a box into paragraph corresponding the existing steps. Starting from writing what is known, then what will be proven, what rules are used, and the consequences. From students’ answers, the flow proof step helps students in constructing a proof in line with the opinion of Larson [25] flow proof can facilitate students in constructing a proof.

After the implementation of the flow proof strategy, the ability of students to prove satisfactorily categorized. In the indicator analysis existing information will be proven, almost all students know the sequence of constructing a proof, this is due to the implementation of the flow proof strategy. From the students’ answers, there are some difficulties of students that cause errors, namely the indicators identifying existing information, this indicator can be shown by the first and second steps of flow proof strategies, namely writing what is known and what will be proven. In general, some students do not understand the symbols in the picture so it’s difficult to write what is known and students are confused in determining what will be proven. The next indicator is the use of concepts and principles, this indicator is shown in the flow proof step to determine the rules and consequences of the rules used. Some students do not use or choose the right rules in proving. Indicators are using communicative proof language at this phase students should be able to choose symbols that can meaningful construction proof. But 6 students are still incorrect in representing mathematical language. In line with Wang's research [11] that in proving theorems congruent triangles students still have errors in representing the mathematical language.
Figure 4. Constructing proofs with paragraphs.

4. Conclusion
Based on the results of the study, after the researcher applied the flow proof strategy for congruence topic, the mathematical proof students’ ability in class IX.1 of SMPN 33 Palembang was satisfactorily categorized. Some students that are categorized satisfactory already understand the sequence in writing proof that is writing existing information like what is known. When writing what will be proven and using principles and concepts, students confused in determining what is proven and in choosing which rules are used so cannot resolve until the statement is proven. As the student's mistake in proving that is wrong in writing symbols.

Researchers have some suggestions for teachers, flow proof can be used as a learning strategy in developing the student construct proof ability and before constructing a proof using flow proof, teachers should train students in verifying proof using flow proof so that students are accustomed to choosing rules in constructing a proof. And for other researchers, it is expected to carry out further research and perfect the flow proof strategy steps. Also, other researchers are expected to be able to innovate in the application of flow proof strategies, as well as use another topic in its application.

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