Light-Cone Distribution Amplitudes of Doubly-Heavy Baryons

Alexander Parkhomenko and Alice Shukhtina
P. G. Demidov Yaroslavl State University, Sovietskaya 14, 150003 Yaroslavl, Russia
E-mail: parkh@uniyar.ac.ru, aliceshu@yandex.ru

Abstract. Doubly-heavy baryons, whose dynamics is determined by a light quark situated in a color field of a static pair of heavy quarks, are investigated. Non-local interpolation currents are introduced and corresponding matrix elements between the baryon and vacuum state are expressed in terms of light-cone distribution amplitudes. Model functions for baryon distribution amplitudes are suggested and their scale dependence is studied in the perturbative QCD framework. The similarity between the heavy meson and doubly-heavy baryon distribution amplitudes is discussed.

1. Introduction

Hadrons, being composite particles, are divided into two groups in dependence on their spin. Ordinary mesons, particles with an integer spin, are composed of a quark and antiquark. Taking into account their flavor content, there are light mesons constructed from $u$, $d$- and $s$-quarks, heavy mesons in which one light quark is replaced by heavy one — $c$- or $b$-quark, and quarkonia and $B_c$-mesons which consist of a heavy quark and heavy antiquark are doubly-heavy systems. Ordinary baryons, having a half-integer spin, are colorless systems of three quarks. They can be also classified based on a number of heavy quarks inside, in particular, light baryons are composed of $u$, $d$- and $s$-quarks only, heavy baryons contain one heavy quark, two heavy quarks are in doubly-heavy baryons, and in triply-heavy baryons all three quarks are heavy. In the present paper, we concentrate on heavy mesons and doubly heavy baryons which are related to each other by the heavy-quark symmetry.

$B$- and $D$-mesons are known very well since 70th of the last century while the situation with doubly-heavy baryons remains controversial till recently. At the beginning of this century the SELEX Collaboration declared about the discovery of the first double charmed baryon $\Xi_{cc}^+$ in the decay channel $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ with the statistical significance $6.3 \sigma$ [1]. Later on, this baryon was confirmed by the SELEX Collaboration in the decay $\Xi_{cc}^+ \to p D^+ K^-$ with the statistical significance $4.8 \sigma$ [2]. Combining together, the mass and lifetime of this baryon are $M_{\Xi_{cc}^+} = (3518.9 \pm 0.9) \text{ MeV}$ and $\tau_{\Xi_{cc}^+} < 3.3 \times 10^{-14} \text{ c}$ [3]. Note that the lifetime is much less than the existing theoretical predictions. None of other experiments (FOCUS, BaBar, Belle, LHCb) confirm the existence of this resonance. Recently, the LHCb Collaboration has found its isospin partner $\Xi_{cc}^{++}$ in the decay $\Xi_{cc}^{++} \to \Lambda_{cc}^+ K^- \pi^+ \pi^+$ with the significance $>12\sigma$ @ 13 TeV and $>7\sigma$ @ 8 TeV [4]. The existence of this state has been confirmed later in the decay $\Xi_{cc}^{++} \to \Xi_{cc}^{++} \pi^+$ with the statistical significance $6.3 \sigma$ [5]. The mass of the $\Xi_{cc}^{++}$ candidates reconstructed via these decay modes is $M_{\Xi_{cc}^{++}} = 3621.55 \pm 0.23(\text{stat}) \pm 0.30(\text{syst}) \text{ MeV}$ [6]. The measurements by
the SELEX and LHCb Collaborations show the large mass difference $\Delta M_{\Xi c c} = (103 \pm 2)$ MeV between the states which contradicts the isospin symmetry.

2. Motion of light quark in heavy meson

A heavy meson $M$ consists of the heavy antiquark $\bar{Q}$ and light quark $q$, where $q = u, d$ or $s$ is the flavor of the light quark, and it is one of hadronic system we interested in. At the quark level, one needs to introduce an appropriate interpolating current to which the heavy meson is coupled. To construct interpolating currents, we choose the reference system, where the heavy antiquark $\bar{Q}$ is the static source situated at the origin of its rest frame. The light quark $q$ is massless in this approach ($m_q = 0$), so it is moving on the light cone $z^2 = 0$, where $z$ is a separation between quarks. A gauge link between quarks, called the Wilson line, can be chosen as a straight light on the light cone:

$$E(0, z) = \mathcal{P} \exp \left\{ -ig_st \int_0^1 ds \, z^\mu A^\mu_a(s) \frac{\lambda^a}{2} \right\},$$  \hspace{1cm} (1)

where $A^\mu_a(s)$, with $a = 1, 2, \ldots, 8$ being the color, is the gluon four-potential, $\lambda^a$ are the Gell-Mann matrices, and $g_st$ is the strong coupling. Two light-like vectors $n^\mu_{\pm} = (1, 0, 0, \pm 1)/\sqrt{2}$, satisfying the conditions: $n^2_{\pm} = 0$ and $(n_+ n_-) = 1$, the basic vectors on the light cone. Meson-to-vacuum transition matrix element written in the Heavy Quark Effective Theory (HQET) [7–10]:

$$\langle 0 | \tilde{O}(z) | M(v) \rangle = f_M \left\{ \tilde{\varphi}_+(t) + \tilde{\varphi}_-(t) - \tilde{\varphi}_+(t) \tilde{\varphi}_-(t) \right\} \hat{z} \, u(v),$$  \hspace{1cm} (2)

is fully determined by two functions $\tilde{\varphi}_\pm(t)$ characterizing the motion of the light quark inside the meson and called Distribution Amplitudes (DAs). Here, $\hat{z} = z \mu \gamma^\mu$, $f_M$ is a constant with a dimension of mass, and $u(v)$ is wave function of the “spinor” meson (we assume that the spin of the heavy quark is decoupled and does not influence the heavy-meson dynamics). The following normalization of DAs, $\tilde{\varphi}_+(0) = \tilde{\varphi}_-(0) = 1$ is accepted. The constant $f_M$ is defined in the local limit ($t = 0$) of the matrix element [2]:

$$\langle 0 | \tilde{O}(0) | M(v) \rangle = f_M u(v).$$  \hspace{1cm} (3)

In particular, the $B$-meson leptonic decay constant is $f_B \simeq 200$ MeV.

The Fourier transforms of DAs $\phi_B^\pm(\omega)$:

$$\tilde{\varphi}_\pm(t) = \int_0^\infty d\omega \, e^{-i\omega t} \phi_B^\pm(\omega),$$  \hspace{1cm} (4)

are different from zero only at non-negative values of the variable $\omega$, which is the energy of the light quark in the heavy meson.

3. $B$-meson distribution amplitudes models

Exponential model. This DAs model was proposed by Grozin and Neubert [11]. In their approach DAs are approximated by the exponential function of the quark energy $\omega$:

$$\phi_B^+(\omega) = \frac{\omega}{\omega_0} e^{-\omega/\omega_0}, \quad \phi_B^-(\omega) = \frac{1}{\omega_0} e^{-\omega/\omega_0},$$  \hspace{1cm} (5)

where $\omega_0 = 2\Lambda/3$ is related to the meson effective mass $\Lambda = M - \bar{m}$, being the difference between the meson mass $M$ and heavy quark mass $\bar{m}$.

The dependence of the distribution amplitudes on the light quark energy is shown on the left panel in figure [1].
Figure 1. The energy dependence of the $B$-meson DAs in the exponential [11] (a) and linear [12] (b) models. The blue (solid) line corresponds to the leading DA, $\phi^+(\omega)$, and the red (dotted) one is for the non-leading, $\phi^-(\omega)$.

**Linear model.** The other model for DAs based on the DAs shape corresponding to light pseudoscalar mesons was suggested by Kawamura, Kodaira, Qiao and Tanaka [12]. DAs in this model have a linear dependence on $\omega$ in the momentum space, within the interval $[0, 2\bar{\Lambda}]$:

$$\phi_B^+(\omega) = \frac{\omega}{2\bar{\Lambda}^2} \theta(2\bar{\Lambda} - \omega), \quad \phi_B^-(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}^2} \theta(2\bar{\Lambda} - \omega),$$

(6)

where $\bar{\Lambda}$ is the same meson effective mass as in the exponential model. The light-quark energy dependence of the DAs is shown on the right panel in figure 1.

**Braun, Ivanov and Korchemsky (BIK) model.** A more complicated model of the $\phi_B^+(\omega, \mu)$ DA was proposed in [13]:

$$\phi_B^+(\omega, \mu) = \frac{4}{\pi \lambda B} \frac{\omega \mu}{\omega^2 + \mu^2} \left[ \frac{\mu^2}{\omega^2 + \mu^2} - \frac{2(\sigma_B - 1)}{\pi^2} \ln \frac{\omega}{\mu} \right],$$

(7)

which depends on the first inverse moments of the leading DA:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu), \quad \sigma_B(\mu) = \int_0^\infty \frac{d\omega}{\omega} \ln \frac{\mu}{\omega} \phi_B^+(\omega, \mu).$$

(8)

The explicit form of the second, non-leading distribution amplitude is as follows:

$$\phi_B^-(\omega, \mu) = -\frac{2}{\pi \lambda_B} \left( \frac{\omega \mu}{\omega^2 + \mu^2} + \arctan \frac{\omega}{\mu} - \frac{\pi}{2} + \frac{4(\sigma_B - 1)}{\pi^2} \left\{ \text{Im} \left[ \text{Li}_2 \left( \frac{i\omega}{\mu} \right) \right] - \arctan \frac{\omega}{\mu} \ln \frac{\omega}{\mu} \right\} \right).$$

(9)

The dependence of the distribution amplitudes on the light quark energy is shown on the left panel in figure 2.

**Lee and Neubert (LN) model.** A generalization of the exponential model was done by Lee and Neubert in [14]. The leading distribution amplitude in this model has two pieces — by the non-perturbative exponential distribution at $\omega < \omega_t$ and by the radiative tail above $\omega_t$:

$$\phi_B^+(\omega, \mu) = \frac{N\omega}{\omega_0} e^{-\omega/\omega_0} + \theta(\omega - \omega_t) \frac{C_F\alpha_s(\mu)}{\pi\omega} \left[ \left( \frac{1}{2} - \ln \frac{\omega}{\mu} \right) + \frac{4\bar{\Lambda}_{DA}}{3\omega} \left( 2 - \ln \frac{\omega}{\mu} \right) + ... \right],$$

3
where $C_F = 4/3$ in QCD and $\alpha_{\text{st}}$ is the strong coupling in NLO [3]:

$$\alpha_{\text{st}} = \frac{1}{b_0 t} \left( 1 - \frac{b_1 \ln t}{b_0^2 t} + \frac{b_1^2 (\ln^2 t - \ln t - 1) + b_0 b_2}{b_0^2 t^2} \right).$$

(10)

Here, $t = \ln(\mu_f^2/\Lambda^2)$ and $b_i$ ($i = 0, 1, 2$) are the coefficients of the QCD $\beta$-function [3] taken for $n_f = 4$ active quark flavors. $N$ in eq. (9) is the normalization coefficient which can be obtained from the condition:

$$\int_0^\infty \phi^+_{B}(\omega, \mu) d\omega = 1.$$ 

The effective mass $\bar{\Lambda}_{DA}$ of the heavy meson is defined as follows:

$$\bar{\Lambda}_{DA}(\mu_f, \mu) = \bar{\Lambda}_{SF}(\mu_*, \mu_*) \left[ 1 + \frac{C_F \alpha_{\text{st}}}{4\pi} \left( 6 \ln \frac{\mu_f}{\mu} - \frac{7}{4} \right) \right] - \mu_f \frac{C_F \alpha_{\text{st}}}{4\pi} \left( 3 \ln \frac{\mu_f}{\mu} - \frac{9}{2} + \frac{4\mu_*}{\mu_f} \right),$$

where $\bar{\Lambda}_{SF}(\mu_*, \mu_*) = (0.65 \pm 0.06)$ GeV at the scale $\mu_* = 1.5$ GeV, and

$$\omega_0 = \frac{2\Lambda_{DA}}{3} \left( 1 + 3 \frac{C_F \alpha_{\text{st}}}{4\pi} \right) + (2\sqrt{c} - 3) \mu_f \frac{C_F \alpha_{\text{st}}}{4\pi} + \ldots,$$

(11)

with $\mu_f = \mu$ for simplicity. The effective masses $\Lambda_{DA}$ and $\Lambda_{SF}$ are defined in the “distribution amplitudes” and “shape function” schemes, respectively.

The explicit form of the second distribution amplitude is calculated in the Wandzura-Wilczek approach [11]:

$$\phi^-_{B}(\omega, \mu) = \frac{N}{\omega_0} e^{-\omega/\omega_0} + \theta(\omega - \omega_t) \frac{C_F \alpha_{\text{st}}}{\pi \mu} \left( - \frac{\mu}{2\omega} \left[ 1 + 2 \ln \frac{\omega}{\mu} \right] + \frac{\Lambda_{DA} \mu}{3\omega^2} \left[ 3 - 2 \ln \frac{\omega}{\mu} \right] \right) +$$

$$+ \theta(\omega_t - \omega) \frac{C_F \alpha_{\text{st}}}{\pi \mu} \left( - \frac{\mu}{2\omega_t} \left[ 1 + 2 \ln \frac{\omega_t}{\mu} \right] + \frac{\Lambda_{DA} \mu}{3\omega_t^2} \left[ 3 - 2 \ln \frac{\omega_t}{\mu} \right] \right).$$

(12)

The dependence of the distribution amplitudes on the light quark energy is shown on the left panel in figure 2.

**Figure 2.** The energy dependence of the $B$-meson DAs in the Braun, Ivanov and Korchemsky [13] (a) and Lee and Neubert [14] (b) models. The blue (solid) line corresponds to the leading DA, $\phi^+(\omega)$, and the red (dotted) one is for the non-leading, $\phi^-(\omega)$.
4. Dynamics of light quark in doubly heavy baryon

Dynamics in the doubly heavy baryon is quite similar to the heavy meson. In this case, a heavy antiquark is replaced with a doubly-heavy diquark which looks the same according to strong interactions. Similar to the heavy-meson case, we choose the reference system, where the doubly-heavy diquark is the static source of strong interactions, situated at the frame origin. The light quark $q$ is again considered to be massless ($m_q = 0$), so it is on the light cone, $z^2 = 0$, where $z$ is a separation between light quark and doubly-heavy diquark which we assumed to be a point-like object.

For the description of the double heavy baryon, we should first introduce the local current:

$$J(x) = \epsilon_{abc}[Q_1(x)^a T \Gamma \tau Q_2(x)^b] \Gamma' q(x)^c,$$

(13)

where $a$, $b$, and $c$ are color indices, $\epsilon_{abc}$ is the totally antisymmetric Levi-Civita tensor, $C$ is the charge-conjugation matrix, and the set of Dirac matrices: $\Gamma^{(i)} = \{I, \gamma_5, \gamma_\mu \gamma_5, \sigma_{\mu \nu} = i[\gamma_\mu, \gamma_\nu]/2\}$, determines the quantum numbers of the current. The baryon-to-vacuum transition matrix element of a local operator with the spin-parity $J^P = 1/2^+$:

$$\langle 0 | J(0) | \Xi_{Q'Q} (v) \rangle = f_{\Xi_{Q'Q}} u(v),$$

(14)

is proportional to the baryon wave-function $u(v)$, being a bispinor, which is supplied by some dimensionful constant $f_{\Xi_{Q'Q}}$. The matrix elements of doubly-heavy baryons can be parameterized in complete analogy with the heavy meson ones. The matrix element of the transition from the baryonic state to the vacuum one is also determined by the two distribution amplitudes because of the heavy-quark symmetry.

The models of the meson DAs described above can be easily generalized to the case of doubly-heavy baryons. In particular, the Exponential and Linear models remain the same in their shapes but differ in the effective mass which is equal to $\Lambda = M - m_{Q_1} - m_{Q_2}$, where $M$ is the mass of the baryon and $m_{Q_1}$ and $m_{Q_2}$ are the masses of heavy quarks forming a doubly-heavy diquark in the baryon.

5. Scale Dependence of Doubly-Heavy-Baryons DAs

Scale dependence of physical quantities can be found by solving the renormalization group equations (RGE). In the case of the $B$-meson, the solution of RGE was found by Lange and Neubert [15].

In the previous section it was implicitly assumed that DAs do not depend on the energy scale $\mu$ and they considered at the fixed one, $\mu_0 = 1$ GeV. In fact, this dependence exists because both DAs models of the $B$-meson depend on effective meson mass $\Lambda(\mu) = M_B - m_B(\mu)$, where $M_B$ is the $B$-meson mass (it is fixed by experiments) and $m_B(\mu)$ is the $b$-quark mass which is scale dependent. Following, we will also consider the leading DA for the doubly heavy baryon.

The evolution equation for doubly heavy baryon can be written as follows:

$$\frac{d}{d \ln \mu} \phi_+(\omega, \mu) = -2 \int_0^\infty \gamma_+(\omega, \omega', \mu) \phi_+(\omega', \mu) d\omega',$$

(15)

which differs from the $B$-meson equation because the light quark is interacting with two heavy quarks instead of one in the $B$-meson. As the result, the evolution kernel should be modified respectively:

$$f(\omega, \mu, \mu_0, it) \sim \left(\frac{\omega}{\mu_0}\right)^{it + 2g} \left(\frac{\Gamma(1 - it - g)\Gamma(1 + it)}{\Gamma(1 + it + g)\Gamma(1 - it)}\right)^2, \quad g(\mu, \mu_0) = \frac{8}{3\mu_0^2} \ln \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}.$$  

(16)

As the initial fixed scale at which the shape of the DA is specified, we choose $\mu_0 = 1$ GeV. The scale dependence of $\phi_+(\omega, \mu)$ for $\Xi_{bb}$-baryon in the exponential (left plot) and linear (right plot) models is presented in figure [3].
\[ \mu = 1.0 \, \text{GeV} \]
\[ \mu = 1.5 \, \text{GeV} \]
\[ \mu = 2.0 \, \text{GeV} \]
\[ \mu = 2.5 \, \text{GeV} \]

\[ \omega, \, \text{GeV} \]

\[ \phi(\omega), \, \text{GeV}^{-1} \]

(a)

(b)

Figure 3. Scale dependence of \( \phi_+(\omega, \mu) \) for \( \Xi_{bb} \)-baryon in the exponential (a) and linear (b) models.

6. Conclusions and outlook

The theory describing heavy mesons on the light cone is studied and generalized to doubly-heavy baryons. DAs models for heavy baryons are proposed in the form of exponential and linear dependences. DA dependence on the energy scale is generalized to doubly-heavy baryon states assuming that the doubly-heavy diquark is a point-like object. DAs are of interest in the study of weak decays of heavy hadrons. In the case of the \( B \)-meson, decays like \( B \to \pi \ell \nu \ell \), \( \ell = e, \mu \) are studied both experimentally and theoretically very precisely. For doubly heavy baryons, similar semileptonic decays are \( \Xi_{bb} \to \Xi_{bc} + \ell^- + \bar{\nu}_\ell \) and \( \Xi_{bb} \to \Xi_b + \ell^- + \bar{\nu}_\ell \) which are waiting, first of all, for a discovery of \( \Xi_{bb} \)-baryon at the LHC. As far as it is found, an information about hadron wave functions (DAs) is needed to calculate hadronic transition matrix elements and corresponding decay ratios.

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