Resource-Aware Distributed Submodular Maximization: A Paradigm for Multi-Robot Decision-Making

Zirui Xu,† Vasileios Tzoumas‡

Abstract—Multi-robot decision-making is the process where multiple robots coordinate actions. In this paper, we aim for efficient and effective multi-robot decision-making despite the robots’ limited on-board resources and the often resource-demanding complexity of their tasks. We introduce the first algorithm enabling the robots to choose with which few other robots to coordinate and provably balance the trade-off of centralized vs. decentralized coordination. Particularly, centralization favors globally near-optimal decision-making but at the cost of increased on-board resource requirements; whereas, decentralization favors minimal resource requirements but at a global suboptimality cost. All robots can thus afford our algorithm, irrespective of their resources. We are motivated by the future of autonomy that involves multiple robots coordinating actions to complete resource-demanding tasks, such as target tracking, area covering, and monitoring. To provide closed-form guarantees, we focus on maximization problems involving monotone and “doubly” submodular functions. To capture the cost of decentralization, we introduce the notion of Centralization Of Information among non-Neighbors (COIN). We validate our algorithm in simulated scenarios of image covering.

I. INTRODUCTION

In the future, robots will be coordinating actions over inter-robot communication networks to complete tasks such as:

• Image Covering: How mobile robots can collaboratively map indoor areas? [1]
• Area Monitoring: How robot swarms can collaboratively detect rare events? [2]
• Target Tracking: How mobile robot networks can collaboratively track multiple evading targets? [3]

The success of such complex tasks depends on the ability of the robots to coordinate actions efficiently and effectively.

But efficiency and effectiveness are opposing performance goals: (i) efficiency requires decentralized coordination: the robots have limited on-board communication, computation, and memory resources [1], thus they can only afford to coordinate with a few other robots, instead of all; and (ii) effectiveness requires centralized coordination instead: optimal action coordination is guaranteed when all robots coordinate with all others simultaneously, instead of a few others. In light of (i) and (ii), to achieve efficiency and effectiveness means to balance centralization vs. decentralization.

Research Question. How to balance centralized vs. decentralized coordination to achieve efficient and effective multi-robot decision-making despite the robots’ limited resources? Particularly, centralization favors globally optimal decision-making but at the cost of increased on-board resource requirements; whereas, decentralization favors minimal resource requirements but at a global suboptimality cost.

*Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109 USA; ziruixu@umich.edu
†Department of Aerospace Engineering and Robotics Institute, University of Michigan, Ann Arbor, MI 48109 USA; vtzoumas@umich.edu

### Resource-Aware Distributed Decision-Making: A Paradigm

| Computations per Agent | Proportional to the number of the agent’s available actions |
|------------------------|----------------------------------------------------------|
| Communication Rounds  | Proportional to the number of agents                      |
| Memory per Message    | Memory length of storing an action                        |
| Communication Topology| Directed and even disconnected                            |
| Suboptimality Guarantee| Gracefully balances the trade-off of centralization vs. decentralization |

TABLE I: Resource-Aware Distributed Decision-Making. A multi-robot decision-making algorithm is resource-aware when: (i) it requires near-minimal computation, communication and memory requirements, in particular, requirements that are proportional to the consumed resources when each robot acts alone, disconnected from all others; (ii) it is valid for even disconnected networks, i.e., even when robots lack the resources to coordinate with others; and (iii) it has a suboptimality guarantee that captures the cost of decentralization, i.e., the cost due to robots choosing to coordinate only with a few others to satisfy their on-board resources.

In this paper, we provide the first algorithm addressing the Research Question. To this end, we focus on the resource-aware coordination paradigm defined in Table I, shifting away from the current paradigm that focuses on achieving distributed communication but ignoring the robots’ computation, communication, and memory limitations.

To provide closed-form guarantees, we focus on multi-robot tasks that are captured by objective functions that are monotone and “doubly” submodular [4], [5], a diminishing returns property. Such functions appear in tasks of image covering [6] and vehicle deployment [7], among others. The said tasks require the robots to distributively

$$\max_{s_i \in V_i, \forall i \in N} f(\{s_i\}_i \in N),$$

where $N$ is the set of robots, $s_i$ is robot $i$’s action (e.g., a motion primitive), $V_i$ is robot $i$’s set of available actions, and $f : \prod_{i \in N} V_i \mapsto \mathbb{R}$ is the objective function.

Related Work. Problem 1 is NP-hard [8] and has been actively researched in the optimization, control, and operations research literature [3]–[26]. Although near-optimal algorithms have been achieved, they focus on achieving distributed communication only, ignoring the robots’ on-board resource constraints. Hence, they often require high communication rounds, computation, and memory overheads: robots with limited on-board resources, such as the resource-limited Crazyflie drones, cannot afford the algorithms [1]. No submodular optimization algorithms exist that balance centralization vs. decentralization. Although [12], [27] study how limited information impacts the approximation performance of the sequential greedy algorithm [10], the impact is quantified with respect to global properties of the communication network such as its clique number, instead of with
respect to the coordination-neighborhoods chosen locally by each robot to satisfy its own on-board resources. Further, the sequential greedy algorithm [10] and its variants [6], [9], [11]–[13] require increasing memory as the network size increases. The state-of-the-art consensus algorithms [23]–[25], although they achieve distributed communication, require excessive computations and communication rounds.

Contributions. We provide the first algorithm addressing the Research Question. We name the algorithm Resource-Aware distributed Greedy (RAG). RAG enables the robots to choose independently with which robots to coordinate balancing global near-optimality with minimality of resource requirements, i.e., balancing centralization with decentralization. All robots can thus afford RAG, even Crazyflies.

RAG can cover the spectrum from a fully centralized to a fully decentralized algorithm, i.e., from an algorithm where all robots coordinate with all others to where all robots coordinate with none. Across the spectrum:

- RAG enjoys near-minimal resource requirements per Table I, on par or superior to the state of the art (Section IV).
- RAG enjoys the first suboptimality bound that captures the centralization vs. decentralization trade-off (Section V). Particularly, the bound gracefully decreases with increasing decentralization. At the two ends of the spectrum:
  - Suboptimality Under Full Centralization: RAG enjoys the suboptimality bound 0.5, which is close to the best possible bound 1 − 1/e ≃ 0.63 [23].
  - Suboptimality Under Full Decentralization: RAG’s suboptimality bound becomes \((1 − \kappa_f)/(2 − \kappa_f)\), where \(\kappa_f\) is \(f\)’s total curvature which captures the worst-case information overlap between any single robot and \(\) all other robots [28]. Specifically, \(\kappa_f\) takes values in \([0, 1]\), and when \(\kappa_f = 0\) then there is no information overlap, whereas when \(\kappa_f = 1\) then at least one robot is redundant. In sum, when \(\kappa_f = 0\) then RAG is near-optimal, as expected, and when \(\kappa_f = 1\) then RAG is arbitrarily bad, which is also expected in this worst-case since all agents have chosen to ignore all others.

We introduce the notion of Centralization Of Information among non-Neighbors (COIN) to capture the cost of decentralization. COIN captures the information overlap between a robot \(i\) and the robots that robot \(i\) does not coordinate with.

COIN innovates compared to the total curvature \(\kappa_f\) [28] which instead captures the worst-case information overlap between any robot and all other robots. \(\kappa_f\) cannot capture the suboptimality cost due to decentralization since it (i) ignores the structure of each robot’s coordination neighborhood and (ii) focuses on the worst-case robot. The same holds true for the extensions of \(\kappa_f\) to subsets of robots [29, Eq. (2)].

Evaluation on Robotic Application. We evaluate RAG in simulated scenarios of image covering (Section VI). RAG demonstrates comparable or superior performance, requiring, e.g., (i) 5 orders of magnitude less computation time vs. the consensus algorithm in [23], (ii) comparable communication rounds vs. the greedy algorithm in [13], and (iii) 40% less memory vs. the consensus algorithm in [23]. Still, RAG demonstrates the best observed covering performance.

II. DISTRIBUTED SUBMODULAR MAXIMIZATION: A MULTI-ROBOT DECISION-MAKING PERSPECTIVE

We define the Distributed Submodular Maximization problem of this paper (Problem I). We use the notation:

- \(\mathcal{G} \triangleq \{\mathcal{N}, \mathcal{E}\}\) is a communication network with nodes \(\mathcal{N}\) and edges \(\mathcal{E}\). Nodes represent agents (e.g., robots), and edges represent communication channels.
- \(\mathcal{N}^-_i \triangleq \{j \in \mathcal{N} : (j, i) \in \mathcal{E}\}\), for all \(i \in \mathcal{N}\); i.e., \(\mathcal{N}^-\) is the in-neighbors of \(i\).
- \(\mathcal{N}^+_i \triangleq \{j \in \mathcal{N} : (i, j) \in \mathcal{E}\}\), for all \(i \in \mathcal{N}\), given graph \(\mathcal{G}\); i.e., \(\mathcal{N}^+_i\) is the out-neighbors of \(i\).
- \(\mathcal{V}_N \triangleq \bigcap_{i \in \mathcal{N}} \mathcal{V}_i\), given a collection of sets \(\mathcal{V}_i\)\(\mathcal{N}\); i.e., \(\mathcal{V}_N\) is the cross-product of the sets in \(\mathcal{V}_i\)\(\mathcal{N}\).
- \(f(s | A) \triangleq f(A \cup \{s\}) − f(A)\), given a set function \(f : 2^\mathcal{V} \rightarrow \mathbb{R}\), \(s \in \mathcal{V}\), and \(A \subseteq \mathcal{V}\); i.e., \(f(s | A)\) is the marginal gain in \(f\) for adding \(s\) to \(A\).

The following preliminary framework is also required.

Agents. The terms “agent” and “robot” are used interchangeably in this paper. \(\mathcal{N}\) is the set of all robots. The robots cooperate for a task, such as image covering.

Actions. \(\mathcal{V}_i\) is a discrete and finite set of actions available to robot \(i\). For example, \(\mathcal{V}_i\) may be a set of \(i\) motion primitives that robot \(i\) can execute to move in the environment [30] or \(i\)’s discretized control inputs [17].

Communication Network. The communication network \(\mathcal{G}\) among the robots may be directed and even disconnected. If \((j, i) \in \mathcal{E}\), then a communication channel exists from robot \(j\) to robot \(i\); i.e., \(i\) can receive, store, and process the information from \(j\). The set of all robots that can send information to \(i\) is \(\mathcal{N}^-\), i.e., \(i\)’s in-neighborhood. The set of all robots that \(i\) can send information to is \(\mathcal{N}^+_i\), i.e., \(i\)’s out-neighborhood.

Objective Function. The robots coordinate their actions to maximize an objective function. In tasks, such as image covering, target tracking, and persistent monitoring, typical objective functions are the covering functions [6], [7], [23]. Intuitively, these functions capture how much area/information is covered given the actions of all robots. They satisfy the properties defined below (Definition 1 and Definition 2).

Definition 1 (Normalized and Non-Decreasing Submodular Set Function [10]). A set function \(f : 2^\mathcal{V} \rightarrow \mathbb{R}\) is normalized and non-decreasing submodular if and only if

\[
\begin{align*}
& f(\emptyset) = 0; \\
& f(A) \leq f(B), \text{ for any } A \subseteq B \subseteq \mathcal{V}; \\
& f(s | A) \geq f(s | B), \text{ for any } A \subseteq B \subseteq \mathcal{V} \text{ and } s \in \mathcal{V}.
\end{align*}
\]

Normalization \((f(\emptyset) = 0)\) holds without loss of generality. In contrast, monotonicity and submodularity are intrinsic to the function. Intuitively, if \(f(A)\) captures the area covered by a set \(A\) of activated cameras, then the more sensors are activated, the more area is covered; this is the non-decreasing property. Also, the marginal gain of covered area caused by activating a camera \(s\) drops when more cameras are already activated; this is the submodularity property.

Definition 2 (2nd-order Submodular Set Function [4], [5]). \(f : 2^\mathcal{V} \rightarrow \mathbb{R}\) is 2nd-order submodular if and only if

\[
f(s) − f(s | A) \geq f(s | B) − f(s | A, B), \tag{2}
\]

5960
for any disjoint $A \subseteq V$ and $B \subseteq V$ ($A \cap B = \emptyset$) and $s \in V$.

The 2nd-order submodularity is another intrinsic property to the function. Intuitively, if $f(A)$ captures the area covered by a set $A$ of sensors, then marginal gain of the marginal gains drops when more sensors are already activated.

**Problem Definition.** In this paper, we focus on:

**Problem 1** (Distributed Submodular Maximization). Each robot $i \in N$ independently selects an action $s_i$, upon receiving information from and about the in-neighbors $N_i^-$ only, such that the robots’ actions $\{s_i\}_{i \in N}$ solve the

$$\max_{s_i \in V_i, \forall i \in N} f(\{s_i\}_{i \in N}),$$

where $f : 2^V \to \mathbb{R}$ is a normalized, non-decreasing submodular, and 2nd-order submodular set function.

Problem 1 requires each robot $i$ to independently choose an action $s_i$ to maximize the global objective $f$, only based on local communication and information. If robot $i$ decides to select an action $s_i$ after a number of communication rounds (i.e., iterations of information exchange), then

$$s_i = \phi_i\left(\text{Information received from robot } i \text{'s sensors, } \{\text{Information received from robot } j \text{ till now}\}_{j \in N_i^-}\right),$$

for a decision algorithm $\phi_i$ to be found in this paper.

In Section III we introduce RAG, an algorithm that runs locally on each robot $i$, playing the role of $\phi_i$.

**Assumption 1.** The communication network $G$ is fixed between the executions of the algorithm.

That is, we assume no communication failures once the algorithm has started, and till its end. Still, $G$ can be dynamic across consecutive time-steps $t = 1, 2, \ldots$, when Problem 1 is applied in a receding-horizon fashion (Remark 1).

**Remark 1** (Receding-Horizon Control, and Need for Minimal Communication and Computation). Image covering, target tracking, and persistent monitoring are dynamic tasks that require the robots to react across consecutive time-steps $t = 1, 2, \ldots$. Then, Problem 1 must be solved in a receding-horizon fashion [31]. This becomes possible only if the time interval between any two consecutive steps $t$ and $t + 1$ can contain the required number of communication rounds to solve Problem 1. Thus, for faster reaction, the smaller the number of communication rounds must be, and the smaller the computation effort per round must be. Otherwise, real-time performance will be compromised by latency.

**III. RESOURCE-AWARE DISTRIBUTED GREEDY (RAG) ALGORITHM**

We present the Resource-Aware distributed Greedy (RAG) algorithm, the first resource-aware algorithm for Problem 1. RAG’s pseudo-code is given in Algorithm 1. The algorithm requires only local information exchange, among only neighboring robots and about only neighboring robots. Each agent $i$ starts by selecting the action $s_i$ with the largest marginal gain $g_i$ (lines 3–4). Then, instead of exchanging information with all other agents $N \setminus \{i\}$, $i$ exchanges information only with the in-neighbors $N_i^-$ and out-neighbors $N_i^+$ (line 5). Afterwards, $i$ checks whether $i = \arg \max_{j \in N_i^- \cup \{i\}} g_j$, i.e., whether $i$ is the “best agent” among the in-neighbors only, instead of all agents in the network (line 6). If yes, then $i$ selects $s_i$ as its action (line 7), and lets only its out-neighbors know its selection (line 8). Otherwise, $i$ receives the action(s) from the agent(s) $j \in N_i^- \setminus \{i\}$ that just selected action(s) in this iteration (i.e., set $T_i^{\text{new}}$ in line 10), and continues onto the next iteration (lines 9–15). Notably, $T_i^{\text{new}}$ may contain multiple agents, and may even be empty.

**Remark 2** (Directed and Disconnected Communication Topology). RAG is valid for directed and even disconnected communication topologies, in accordance to the paradigm Table I. For example, if $N_i^- = N_i^+ = \emptyset$ in Algorithm 1, then agent $i$ is completely disconnected from the network.

**IV. COMPUTATION, COMMUNICATION, AND MEMORY REQUIREMENTS OF RAG**

We present RAG’s computation, communication, and memory requirements, summarized in Table II. We use the additional notation:

- $\text{length}_{\phi_i}$ and $\text{length}_s$ are the lengths of a message containing a real number or an action $s \in V_N$, respectively.
- $\text{diam}(\mathcal{G})$ is the diameter of a network $\mathcal{G}$, i.e., the longest shortest path among any pair of nodes in $\mathcal{G}$ [33];
- $|X|$ is the cardinality of a discrete set $X$.

**Proposition 1** (Computation Requirements). Each agent $i$ performs $O(|V_i| |N_i^-|)$ function evaluations during RAG.
Continuous Domain

| Method                        | Du et al. [22] | Robey et al. [23] | Rezazadeh and Kia [24] | RAG (this paper) |
|-------------------------------|----------------|-------------------|-------------------------|-----------------|
| Computations per Agent        | ~ \( \Theta(M |N|^2) \) | \( \Omega(M |N|^2.5 / \epsilon) \) | \( \Omega(M |N|^2 \text{diam}(\mathcal{G}) / \epsilon) \) | \( \Omega(|V_i| |N_i^-|^2) / \epsilon) \) |
| Communication Rounds          | \( \sim \Theta(|V|^2) \) | \( \Omega(|V|^2.5 / \epsilon) \) | \( \Omega(|V|^2 \text{diam}(\mathcal{G}) / \epsilon) \) | \( \Omega(|V| |N|^3) \) |
| Memory per Message            | \( M \text{length}_\# \) | \( |V_i| \text{length}_\# \) | \( |V_i| \text{length}_\# \) | \( \max (\text{length}_\#, \text{length}_h) \) |
| Communication Topology        | connected, undirected | connected, undirected | connected, undirected | even disconnected, directed |
| Suboptimality Guarantee       | \((1/2 - \epsilon) \text{OPT}\) | \( (1 - 1/e) \text{OPT} - \epsilon \) | \((1 - 1/e - \epsilon) \text{OPT}\) | \(1/2(\text{OPT} - \sum_{i \in N} \text{coin}_i)\) |

Discrete Domain

| Method                        | Corah and Michael [6] | Liu et al. [32] | Konda et al. [13] | RAG (this paper) |
|-------------------------------|-----------------------|-----------------|-------------------|-----------------|
| Computations per Agent        | \( O(|V_i| |N_i|^3) \) | \( O(|V_i| |V|^2) \) | \( |V_i| \) | \( O(|V_i| |N_i^-|^2) \) |
| Communication Rounds          | \( \Omega(1 / \epsilon) \) | \( \leq (2 |N| + 2 \text{diam}(\mathcal{G})) \) | \( 2 |N| - 2 \) | \( 2 |N| - 2 \) |
| Memory per Message            | \( \max (|V_i|, \Omega(1/\epsilon)) \text{length}_\# \) | \( |V_i| (\text{length}_\# + \text{length}_h) \) | \( (|N| - 1) \text{length}_\# \) | \( \max (\text{length}_\#, \text{length}_h) \) |
| Communication Topology        | fully connected       | connected, directed | connected, undirected | even disconnected, directed |
| Suboptimality Guarantee       | \(1/2(\text{OPT} - \epsilon)\) | \(1/2 \text{OPT}\) | \(1/2 \text{OPT}\) | \(1/2(\text{OPT} - \sum_{i \in N} \text{coin}_i)\) |

Table II: RAG vs. State of the Art. The state of the art is divided into algorithms that optimize (i) in the continuous domain, employing a continuous representation of \( f \) [15], and (ii) in the discrete domain. The continuous-domain algorithms need to compute the continuous representation’s gradient via sampling; \( M \) denotes the sample size (\( M = 3 \) in [22], 10 in [23], and 1000 in [24], in numerical evaluations with 10 or fewer agents).

Each agent \( i \) needs to re-evaluate the marginal gains of all \( v \in V_i \), every time an in-neighbor \( j \in N_i^- \) selects an action. Therefore, \( i \) will perform \(|N_i^-||V_i|\) evaluations in the worst case (and \(|V_i|\) evaluations in the best case).

**Proposition 2** (Communication Requirements). RAG’s number of communication rounds is at most \( 2|N| - 2 \).

Each iteration of RAG requires two communication rounds: one for marginal gains, and one for actions. Also, RAG requires \(|N| - 1\) iterations in the worst case (when only one agent selects an action at each iteration). All in all, RAG requires at most \( 2|N| - 2 \) communication rounds.

**Proposition 3** (Memory Requirements). RAG’s largest inter-agent message length is \( \max (\text{length}_\#, \text{length}_h) \).

Any inter-agent message in RAG contains either a marginal gain, or an action \( s \). Thus, the message’s length is either \( \text{length}_\# \) or \( \text{length}_h \). The total on-board memory requirements for each agent \( i \) are \(|N_i^-| \max (\text{length}_\#, \text{length}_h) \).

**Remark 3** (Near-Minimal Resource Requirements). RAG has near-minimal computation, communication, and memory requirements, in accordance to Table I. (i) Computations per Agent: the number of computations per agent is indeed proportional to the size of the agent’s action set, and, in particular, is \( O(|V_i| |N_i^-|) \), i.e., decreasing as \(|N_i^-|\) decreases. The number of computations would have been minimal if instead it was \( O(|V_i|) \), since that is the cost for agent \( i \) to compute its best action in \( V_i \). (ii) Communication Rounds: the number of communication rounds is indeed proportional to the number of agents, and, in particular, is at most \( 2|N| - 2 \). The number is near-minimal, since, in the worst case of a line communication network, \(|N| - 1\) communication rounds are required for information to travel between the most distant agents. (iii) Memory per Message: the length per message is indeed equal to the length of a real number or of an action.

Besides, each agent \( i \) can afford to run RAG, in accordance to Table I, by adjusting the size of its in-neighborhood \( N_i^- \). For example, by decreasing the size of \( N_i^- \): (i) agent \( i \)’s computation effort decreases, since the effort is proportional to the size of \( N_i^- \) (Proposition 1); (ii) the per-round communication effort decreases, since the total communication rounds remain at most \( 2|N| - 2 \) (Proposition 2) but the number of received messages per round decreases (RAG’s lines 5–6); and (iii) the on-board memory-storage requirements decrease, since the inter-agent message length remains constant (Proposition 3) but the number of received messages per round decreases (RAG’s lines 5–6).

**Remark 4** (vs. State-of-the-Art Resource Requirements). RAG has comparable or superior computation, communication, and memory requirements (Table II).

**Context and notation in Table II.** We divide the state of the art into algorithms that optimize (i) in the continuous domain, employing the continuous representation multi-linear extension [15] of \( f \) [22]–[24], and (ii) in the discrete domain [6], [13], [32]. The continuous-domain algorithms employ consensus-based techniques [23], [24] or algorithmic game theory [22], requiring the computation of the multi-linear extension’s gradient. The computation is achieved via sampling; \( M \) in Table II denotes that sample size. \( M \) is equal to 3 in [22], 10 in [23], and 1000 in [24], in numerical evaluations with 10 or fewer agents.

The computations per agent and communication rounds reported in Table II for [22] are based on the numerical evaluations therein, since a theoretical quantification is unknown and appears non-trivial to derive as a function of \( N \), \( \epsilon \), or other of the problem parameters. Further, all continuous-domain algorithms’ resource requirements depend on additional problem-dependent parameters (such as Lipschitz constants, the diameter of the domain set of the multi-linear extension, and a bound on the gradient of the multi-linear extension), which here we make implicit via the big \( O \), Omega, and Theta notation. \( \epsilon \) determines the approximation performance of the respective algorithms.
Computations. Konda et al. [13] rank best with $|V_i|$ computations per agent. RAG ranks 2nd-best with $O(|N_i^-|)$ computations. The continuous-domain algorithms require a higher number of computations, proportional to $|N|^2$ or more.

Communication. For undirected networks, Konda et al. [13] and RAG rank best, requiring in the worst-case the same communication rounds; but RAG is also valid for directed networks. For appropriate $\epsilon$, the algorithm by Corah and Michael [21] may require fewer communication rounds but in [21], a pre-processing step with a fully connected network is required. The remaining algorithms require a significantly higher number of communication rounds.

Memory. RAG ranks best when $\text{length}_i \leq |V_i|$ length. Otherwise, RAG ranks after Robey et al. [23] and Rezazadeh and Kia [24], which then rank best (tie); but RAG is also valid for directed networks.

The approximation guarantee of RAG, along with its comparison to the state of the art, are discussed next.

V. APPROXIMATION GUARANTEE OF RAG:
CENTRALIZATION VS. DECENTRALIZATION PERSPECTIVE

We present RAG’s suboptimality bound (Theorem 1). To this end, we first introduce the notion of Centralization of Information among non-Neighbors to quantify the bound.

We also use the notation:
- $N_i^- \triangleq N \backslash \{N_i^- \cup \{i\}\}$ is the set of agents beyond the in-neighborhood of $i$ (see Fig. 1), i.e., $i$’s non-neighbors;
- $S_{N_i^c} \triangleq \{s_j\}_{j \in N_i^c}$ is the agents’ actions in $N_i^c$;
- $S_{OPT} \subseteq \arg \max_{s_i \in V_i, s_j \in N_i^-} f(\{s_i\}_i \in N_i)$, i.e., $S_{OPT}$ is an optimal solution to Problem 1;
- $S_{RAG} \triangleq \{s_i^{RAG}\}_{i \in N}$ is RAG’s output for all agents.

A. Centralization of Information: A Novel Quantification

We use the notion of Centralization Of Information among non-Neighbors (COIN) to bound RAG’s suboptimality.

Definition 3 (Centralization Of Information among non-Neighbors (COIN)). Consider a set function $f : 2^{V_i} \mapsto \mathbb{R}$, and an agent $i \in N$ that has chosen an in-neighborhood $N_i^- \subseteq N$ and an action $s_i \in V_i$. Then, agent $i$’s Centralization Of Information among non-Neighbors is defined by

$$\text{coin}_i(s_i, N_i^-) \triangleq \max_{s_j \in V_j, s_j \in N_i^-} \left[ f(s_i) - f(s_i | S_{N_i^c}) \right].$$

If $f$ were entropy, then coin$_i$ looks like the mutual information between the information collected by agent $i$’s action $s_i$, and the information collected by agents $N_i^c$’s actions $S_{N_i^c}$, i.e., the actions of agent $i$’s non-neighbors.

coin$_i$ captures the centralization of information within the context of a multi-agent network for information acquisition: coin$_i = 0$ if and only if $s_i$’s information is independent from $S_{N_i^c}$, i.e., if and only if the information is decentralized between agent $i$ and its non-neighbors $N_i^c$.

Remark 5 (Monotonicity). coin$_i$ is non-increasing larger $N_i^c$ is: when $N_i^c$ is maximal, i.e., agent $i$’s in-neighborhood is the whole network (full centralization), then coin$_i = 0$; whereas, when $N_i$ is empty (full decentralization), then coin$_i$ takes its maximum value, which we upper bound in eq. (6).

Computing coin$_i$ can be NP-hard [8], or even impossible since agent $i$ may be unaware of its non-neighbors’ actions; but upper-bounding it can be easy. For an image covering task, we obtain such a bound next.

Remark 6 (Upper Bounds for COIN: Image Covering Example). Consider an image covering task where each agent carries a camera with a round field of view of radius $r_s$ (Fig. 2(a)). Consider that each agent $i$ has fixed its in-neighbor $N_i^-$, i.e., its communication range $r_i$ for receiving information. Then, coin$_i$ is equal to the overlap of the field of views of agent $i$ and its non-neighbors, assuming, for simplicity, that the bound remains the same across two consecutive moves. Since the number of agent $i$’s non-neighbors may be unknown, an upper bound to coin$_i$ is the gray ring area in Fig. 2(b), obtained assuming an infinite amount of non-neighbors around agent $i$, located just outside the boundary of $i$’s communication range. That is,

$$\text{coin}_i(s_i, N_i^-) \leq \max(0, \pi r_i^2 - (r_i - r_s)^2),$$

for any $s_i \in V_i$. The bound in eq. (5) as a function of the communication range $r_i$ is shown in Fig. 2(c). As expected, it tends to zero for increasing $r_i$, since when $r_i > 2r_s$ then the field of views of robot $i$ and of each of the non-neighboring robots $N_i^c$ are non-overlapping, and thus coin$_i = 0$.

Remark 7 (vs. Total Curvature (over Sets) [28, 29]). coin’s definition generalizes the notion of total curvature $\kappa_f$ of a set function $f : 2^V \mapsto \mathbb{R}$, introduced by Conforti and Cornuéjols [28]. $\kappa_f$ captures the worst-case information overlap between any single action and all other actions, and is formally defined as $\kappa_f \triangleq 1 - \min_{s \in V} \frac{f(s \setminus \{s\})}{f(s)}$. Instead, coin$_i$ captures the information overlap between the action of a specific agent $i$, instead of any agent in $N$, and the actions of its non-neighbors $N_i^c$ only, instead of all other agents in $N \setminus \{i\}$, capturing directly the decentralization of information across the network with respect to agent $i$. A similar comparison holds for the recent generalization of the total curvature to subsets $X$ of $V$ [29, Eq. (2)]. From coin$_i$’s and $\kappa_f$’s definitions, their relationship is given by

$$\text{coin}_i(s_i, N_i^-) \leq \text{coin}_i(s_i, \emptyset) \leq \kappa_f f(s_i).$$

Remark 8 (vs. Pairwise Redundancy [6]). coin$_i$’s definition generalizes the notion of pairwise redundancy $w_{ij}$ between two agents $i$ and $j$, introduced by Corah and Michael [6]. The notion was introduced in the context of parallelizing the execution of the sequential greedy [10], by ignoring the edges between pairs of agents in an a priori fully connected
network. The comparison of the achieved parallelized greedy in [6] and RAG is found in Table II. Besides, \( w_{ij} \) captures the mutual information between the two agents \( i \) and \( j \), defined as \( w_{ij} \triangleq \max_{s_i \in V_i} \max_{s_j \in V_j} [f(s_i) - f(s_i | s_j)] \); whereas \( \text{coin}_i \) captures the mutual information between an agent \( i \) and all its non-neighbors, capturing directly the decentralization of information across the network.

B. Approximation Guarantee of RAG

**Theorem 1** (Approximation Performance of RAG). Consider that each robot \( i \) has fixed its in-neighborhood \( N_i^- \) (Assumption 1). RAG selects \( S_{\text{RAG}} \) such that \( s_i^{\text{RAG}} \in V_i, \forall i \in \mathcal{N} \), and

\[
 f(S_{\text{RAG}}) \geq \frac{1}{2} \left[ f(S_{\text{OPT}}) - \sum_{i \in \mathcal{N}} \text{coin}_i(N_i^-) \right].
\]  

(7)

Equation (7)'s suboptimality bound is computable a posteriori (after RAG's termination) since it relies on the knowledge of \( \{s_i^{\text{RAG}}\}_{i \in \mathcal{N}} \). Nonetheless, an a priori computable bound is trivially implied by eq. (7):

\[
 f(S_{\text{RAG}}) \geq \frac{1}{2} \left[ f(S_{\text{OPT}}) - \sum_{i \in \mathcal{N}} \text{coin}_i(N_i^-) \right].
\]  

(8)

where \( \text{coin}_i(N_i^-) \triangleq \max_{s_i \in V_i} \text{coin}_i(s_i, N_i^-) \) is the worst-case \( \text{coin}_i(s_i, N_i^-) \) among all available actions \( s_i \) to agent \( i \).

**Remark 9** (Centralization vs. Decentralization). RAG’s suboptimality bound in Theorem 1 captures the trade-off of centralization vs. decentralization:

- Near-optimality requires large \( N_i^- \), i.e., centralization: the larger \( N_i^- \), the smaller \( \text{coin}_i \), is, resulting in an increased global near-optimality for RAG.

- Minimal on-board resource requirements require instead a small \( N_i^- \), i.e., decentralization: the smaller \( N_i^- \), the less the communication-round, computation, and memory-storage effort, since the number of received messages per round decreases.

All in all, RAG covers the spectrum from fully centralized to fully decentralized, i.e., from all robots coordinating with all others to all robots coordinating with none. RAG’s guarantee in eq. (7) (and eq. (8)) captures the suboptimality cost due to decentralization throughout the spectrum:

- **Suboptimality under Full Centralization**: When all robots coordinate with all others, RAG matches the .5 suboptimality bound of the classical greedy [10]; then, \( N_i^c = \emptyset \) for all \( i \), i.e., \( \text{coin}_i = 0 \). For a centralized algorithm, the best possible bound is \( 1 - 1/e \simeq 63 \% \).

- **Suboptimality under Full Decentralization**: When all robots are disconnected from all others, then RAG’s suboptimality bound in eq. (7) takes its worst value, becoming \( (1 - \kappa_f)/(2 - \kappa_f) \). Thus, when \( \kappa_f = 0 \) then RAG is near-optimal, as expected, and when \( \kappa_f = 1 \) then RAG is arbitrarily bad, which is also expected since all agents have chosen to ignore all others.

- **Suboptimality in-between the Above Two Extremes**: For any coordination topology, RAG’s suboptimality bound lies in-between the above two extreme values; particularly, it is non-increasing as we move from full centralization to full decentralization, since \( \text{coin} \) is monotone (Remark 5).

**Remark 10** (vs. State-of-the-Art Approximation Guarantees). RAG is the first algorithm to quantify the trade-off of centralization vs. decentralization, enabling the agents to independently decide their in-neighborhood to balance the trade-off and, thus, balance near-optimality vs. on-board resources. Instead, the state of the art tunes near-optimality via a globally known hyper-parameter \( \epsilon \), without accounting for the balance of smaller vs. larger neighborhoods. [12], [27] study how limited information impacts the approximation performance of the sequential greedy [10] but the impact is quantified with respect to global properties of the communication graph such as its clique number, instead of with respect to the in-neighborhoods chosen locally by each robot.

VI. EVALUATION IN IMAGE COVERING WITH ROBOTS

We evaluate RAG in simulated scenarios of image covering with mobile robots (Fig. 2). We first compare RAG with the state of the art (Section VI-A; see Table III). Then, we evaluate the trade-off of centralization vs. decentralization with respect to RAG’s performance (Section VI-B; see Fig. 3).

---

1RAG’s worst-case suboptimality bound \((1 - \kappa_f)/(2 - \kappa_f)\) under full decentralization is derived from eq. (7) by first accounting for the facts that \( \text{coin}_i(s_i^{\text{RAG}}, \emptyset) \leq \kappa_f f(s_i^{\text{RAG}}) \) and \( f(S_{\text{RAG}}) \geq (1 - \kappa_f) \sum_{i \in \mathcal{N}} f(\{s_i^{\text{RAG}}\}) \) [29, Lemma 2.1] and, then, by rearranging terms.
We performed all simulations in Python 3.9.7, on a MacBook Pro with the Apple M1 Max chip and a 32 GB RAM. Our code is open-sourced here: https://gitlab.umich.edu/iral-cdc2022/resource-aware-code.

### A. RAG vs. State of the Art

We compare RAG with the state of the art in simulated scenarios of image covering (Fig. 2). To enable the comparison, we set up undirected and connected communication networks (RAG is valid for directed and even disconnected networks but the state-of-the-art methods are not). RAG demonstrates comparable or superior performance (Table III).

**Simulation Scenario.** We consider 50 instances of the setup in Fig. 2(a). Without loss of generality, each agent has a communication range of 15, a sensing radius of 10, and the action set (“forward”, “backward”, “left”, “right”) by 1 point. The agents seek to maximize the number of covered points.

**Compared Algorithms.** We compare RAG with the methods by Robey et al. [23] and Konda et al. [13] since, among the state of the art, they achieve top performance for at least one resource requirement and/or for their suboptimality guarantee (Table II)—the method by Corah and Michael [6] may achieve fewer communication rounds, yet it requires a fully connected network. To ensure the method in [23] achieves a sufficient number of covered points, we set the sample size \( M = 10 \), as is also set in [23], and the number of communication rounds \( T = 100 \).

**Results.** The results are reported in Table III, and mirror the theoretical comparison in Table II. RAG demonstrates superior or comparable performance, requiring, (i) 5 orders of magnitude less computation time vs. the state-of-the-art consensus algorithm in [23], and comparable computation time vs. the state-of-the-art greedy algorithm in [13], (ii) 1 order of magnitude fewer communication rounds vs. the consensus algorithm in [23], and comparable communication rounds vs. the greedy algorithm in [13], and (iii) the least memory (e.g., 40% less than the consensus algorithm). Still, RAG achieves the best approximation performance.

**B. The Trade-Off of Centralization vs. Decentralization**

We demonstrate the trade-off of centralization vs. decentralization, with respect to RAG’s performance.

**Simulation Scenario.** We consider the same setup as in Section VI-A, yet with the communication range increasing from 1 to 50. That is, the communication network starts from being fully disconnected (fully decentralized) and becomes fully connected (fully centralized). The communication range is assumed the same for all robots, for simplicity.

### Table III: RAG vs. State of the Art

| Method          | Robey et al. [23] | Konda et al. [13] | RAG (this paper) |
|-----------------|-------------------|------------------|------------------|
| **Total Computation Time (s)** | 1434.11 | 0.02 | 0.05 |
| **Communication Rounds** | 100 | 13.44 | 7.76 |
| **Peak Total Memory (MB)** | 290.43 | 181.07 | 167.07 |
| **Total Covered Points** | 1773.54 | 1745.18 | 1816.4 |

TABLE III: RAG vs. State of the Art. Averaged performance over 50 image covering instances involving 10 robots in a 50 points × 50 points map.

![Fig. 3: Centralization vs. Decentralization: Resource requirements and coverage performance of RAG for increasing communication range, in an image covering scenario with 10 robots in a 50 points × 50 points map.](image)

**Results.** The results are reported in Fig. 3. When a higher communication range results in more in-neighbors, then (i) each agent executes more iterations of RAG before selecting an action, resulting in increased computation time and communication rounds (1 iteration of RAG corresponds to 2 communication rounds; see RAG’s lines 5 and 11), and (ii) each agent needs more on-board memory for information storage and processing. In contrast, with more in-neighbors, each agent coordinates more centrally, and, thus, the total covered points increase. All in all, Fig. 3 captures the centralization vs. decentralization trade-off: for increasing communication range, the required on-board resources increase but also the total covered points increase. To balance the trade-off, the communication range may be set to 6.

### VII. Conclusion

**Summary.** We aimed for efficient and effective multi-robot decision-making despite the robots’ limited resources and the resource-demanding complexity of their tasks. We thus introduced the first algorithm (RAG) for distributed decision-making that balances centralization vs. decentralization. Particularly, centralization favors globally near-optimal decision-making but at the cost of increased on-board computation, communication, and memory requirements; whereas, decentralization favors minimal resource requirements but at a global suboptimality cost. We are motivated by complex tasks taking the form of Problem 1, such as image covering. To capture the trade-off, we introduced the notion of Centralization of Information among non-Neighbors (COIN). We validated RAG in simulations.

**Future Work.** RAG assumes synchronous communication. Besides, the communication topology has to be fixed and failure-free across communication rounds (Assumption 1). Our future work will enable RAG beyond the above limitations. We will also consider multi-hop communication. Correspondingly, we will quantify the trade-off of near-optimality vs. resource-awareness based on the depth of the multi-hop communication. We will also extend our results to any submodular function (instead of “doubly” submodular).
ACKNOWLEDGEMENTS

We thank Robey et al. [23] and Konda et al. [13] for sharing with us the code of their numerical evaluations. We also thank the anonymous reviewers for their comments.

REFERENCES

[1] K. McGuire, C. De Wagter, K. Tyuil, H. Kappen, and G. C. de Croon, “Minimal navigation solution for a swarm of tiny flying robots to explore an unknown environment,” Science Robotics, vol. 4, no. 35, p. 9710, 2019.

[2] V. Kumar, D. Rus, and S. Singh, “Robot and sensor networks for first responders,” IEEE Pervasive computing, vol. 3, no. 4, pp. 24–33, 2004.

[3] M. Corah and N. Michael, “Scalable distributed planning for multi-robot, multi-target tracking,” in IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2021, pp. 437–444.

[4] Y. Crana, P. L. Hammer, and R. Holzman, “A characterization of a cone of pseudo-boolean functions via supermodularity-type inequalities,” in Quantitative Methods in den Wirtschaftswissenschaften. Springer, 1989, pp. 53–55.

[5] S. Foldes and P. L. Hammer, “Submodularity, supermodularity, and higher-order monotonicities of pseudo-boolean functions,” Mathematics of Operations Research, vol. 30, no. 2, pp. 453–461, 2005.

[6] M. Corah and N. Michael, “Distributed submodular maximization on partition matroids for planning on large sensor networks,” in IEEE Conference on Decision and Control (CDC), 2018, pp. 6792–6799.

[7] A. Downie, B. Gharesifard, and S. L. Smith, “Submodular maximization with limited function access,” arXiv preprint:2201.00724, 2022.

[8] U. Feige, “A threshold of ln(n) for approximating set cover,” Journal of the ACM, vol. 45, no. 4, pp. 634–652, 1998.

[9] M. Sviridenko, J. Vondrak, and J. Ward, “Optimal approximation for submodular and supermodular optimization with bounded curvature,” Math. of Operations Research, vol. 42, no. 4, pp. 1197–1218, 2017.

[10] M. L. Fisher, G. L. Nemhauser, and L. A. Wolsey, “An analysis of approximations for maximizing submodular set functions – II,” in Polyhedral combinatorics, 1978, pp. 73–87.

[11] B. Gharesifard and S. L. Smith, “Distributed submodular maximization with limited information,” IEEE Transactions on Control of Network Systems (TCNS), vol. 5, no. 4, pp. 1635–1645, 2017.

[12] D. Grimsman, M. S. Ali, J. P. Hespanha, and J. R. Marden, “The impact of information on distributed submodular maximization,” IEEE Transactions on Control of Network Systems (TCNS), vol. 6, no. 4, pp. 1334–1343, 2018.

[13] R. Konda, D. Grimsman, and J. Marden, “Execution order matters in greedy algorithms with limited information,” arXiv preprint:2111.09154, 2021.

[14] A. Krause, J. Leskovec, C. Guestrin, J. VanBriesen, and C. Faloutsos, “Efficient sensor placement optimization for securing large water distribution networks,” Journal of Water Resources Planning and Management, vol. 134, no. 6, pp. 516–526, 2008.

[15] G. Calinescu, C. Chekuri, M. Pál, and J. Vondrak, “Maximizing a monotone submodular function subject to a matroid constraint,” SIAM Journal on Computing, vol. 40, no. 6, pp. 1740–1766, 2011.

[16] Z. Wang, B. Moran, X. Wang, and Q. Pan, “An accelerated continuous greedy algorithm for maximizing strong submodular functions,” J. of Combinatorial Optimization, vol. 30, no. 4, pp. 1107–1124, 2015.

[17] A. Krause, J. Leskovec, C. Guestrin, J. VanBriesen, and C. Faloutsos, “Efficient sensor placement optimization for securing large water distribution networks,” Journal of Water Resources Planning and Management, vol. 134, no. 6, pp. 516–526, 2008.

[18] G. Calinescu, C. Chekuri, M. Pál, and J. Vondrak, “Maximizing a monotone submodular function subject to a matroid constraint,” SIAM Journal on Computing, vol. 40, no. 6, pp. 1740–1766, 2011.

[19] Z. Wang, B. Moran, X. Wang, and Q. Pan, “An accelerated continuous greedy algorithm for maximizing strong submodular functions,” J. of Combinatorial Optimization, vol. 30, no. 4, pp. 1107–1124, 2015.

[20] A. Krause, J. Leskovec, C. Guestrin, J. VanBriesen, and C. Faloutsos, “Efficient sensor placement optimization for securing large water distribution networks,” Journal of Water Resources Planning and Management, vol. 134, no. 6, pp. 516–526, 2008.

[21] G. Calinescu, C. Chekuri, M. Pál, and J. Vondrak, “Maximizing a monotone submodular function subject to a matroid constraint,” SIAM Journal on Computing, vol. 40, no. 6, pp. 1740–1766, 2011.