Worldvolume and target space anomalies in the D=10 super–fivebrane sigma–model

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Abstract

The fields of the conjectured “heterotic” super–fivebrane sigma–model in ten dimensions are made out of a well known gravitational sector, the $X$ and the $\vartheta$, and of a still unknown heterotic sector which should be coupled to the Yang–Mills fields. We compute the one–loop $d = 6$ worldvolume and $D = 10$ target space Lorentz–anomalies which arise from the gravitational sector of the heterotic super–fivebrane sigma–model, using a method which we developed previously for the Green–Schwarz heterotic superstring. These anomalies turn out to carry an overall coefficient which is $1/2$ of that required by the string/fivebrane duality conjecture. As a consequence the worldvolume anomaly vanishes if the heterotic fields consist of 16 (rather than 32) complex Weyl fermions on the worldvolume. This implies that the string/fivebrane duality conjecture can not be based on a “heterotic” super–fivebrane sigma–model with only fermions in the heterotic sector. Possible implications of this result are discussed.

* Supported in part by M.P.I. This work is carried out in the framework of the European Community Programme Gauge Theories, Applied Supersymmetry and Quantum Gravity” with a financial contribution under SC1–CT–92–D789.
1. Introduction

It is known from longtime that $N=1$ Supergravity in ten dimensions exists in two variants: the $B_2$–version \[1\] which involves a two–form $B_{mn}$, naturally coupled to superstrings, and the $B_6$–version \[2\] which involves a six–form $B_{m_1\ldots m_6}$, naturally coupled to super–fivebranes. The two versions are dual to each other in the sense that the field strength $H_3$ of $B_2$ is the dual of the field strength $H_7$ of $B_6$. When these Supergravities are coupled to a Super–Yang–Mills theory the Green–Schwarz anomaly cancellation mechanism \[3\] works in both cases \[3,4\] provided the gauge group is $E_8 \otimes E_8$ or $SO(32)$ and the field strengths $H_3$ or $H_7$ are modified by suitable Chern–Simons terms. More recently it has been discovered that the $B_2$ Supergravity admits non singular solitonic fivebrane solutions \[5,6\] and, viceversa, the $B_6$ Supergravity admits non singular heterotic string solutions \[7\].

These results led to the conjecture that the heterotic string and the “heterotic” fivebrane in ten dimensions are dual to each other, meaning that the strong coupling regime of the string is described by the weak coupling regime of the fivebrane and viceversa \[5–14\]. This implies that quantum loop effects of the string should correspond to sigma–model loop effects (tree level) of the fivebrane. The strong/weak coupling duality between heterotic strings and fivebranes in $D=10$ is supported also by the fact that their low energy bosonic effective actions are related by a rescaling of the metric which gives rise to an inversion of the quantum loop expansion parameter.

On the other hand this duality conjecture presents also some problematic aspects, see e.g. refs. \[15,16\], and it is, moreover, difficult to test since a consistent formulation of the “heterotic” super fivebrane is still lacking: the classical $\kappa$–invariant action for the gravitational sector of the $D=10$ super–fivebrane is well known\[17\], but a $\kappa$–invariant action for the heterotic sector, which couples to the $D=10$ target Super–Yang–Mills fields, is not yet known. Indeed, it appears a difficult if not impossible task to find it since every simply minded action would destroy $\kappa$–invariance.

In spite of these difficulties, it is, however, clear that fivebranes are fated to play a role in string duality and that it is worthwhile to analyse “heterotic” super–fivebrane models with the ultimate goal of finding a consistent formulation for them. An important step in this direction, and this is the purpose of the present paper, would be constituted by the knowledge of their worldvolume and target space anomalies: the formers are analogous to the worldsheet conformal anomalies.
(central charge) in the string and have to cancel in a consistent model; actually, the requirement of their cancellation constrains the field content of the heterotic sector for the string and for the fivebrane. The latters are “genuine sigma–model” effects and determine heavily the structure of the $N = 1, D = 10$ supergravity theory, in which the sigma–model is embedded, in that they should cancel via the (dual of) the Green–Schwarz mechanism.

A first attempt in this direction has been performed in [13]. Here, with a somehow conjectural calculation which has, however, been questioned in [18], the target space anomaly polynomial for the $D = 10$, “heterotic” fivebrane with gauge group $SO(32)$ has been evaluated and found to be in agreement with the string/fivebrane duality conjecture if one assumes that the heterotic sector is made out of 32 complex $d = 6$ Weyl fermions.

Recently we performed a systematic analysis [19,20] of the one–loop anomalies of the Green–Schwarz heterotic string sigma–model. In ref. [20] we determined, in particular, the worldsheet “genuine string” anomaly (i.e. the one which survives in the flat limit) in a covariant background gauge, generalizing a method first proposed by Wiegmann [21], and we have shown that it vanishes in ten dimensions.

In this paper we apply the methods used in [19,20] to compute the one–loop target space and worldvolume Lorentz anomalies stemming from the gravitational sector of the $D = 10$, super–fivebrane sigma–model as described by the $\kappa$–invariant action of ref. [17]. We perform the calculation in the framework of the background field method combined with a normal coordinate expansion [22]. This allows us to keep the invariance under target space $SO(1, 9)$ transformations manifest at the classical level and to impose a covariant background gauge to fix $\kappa$–symmetry.

We compute first the target space Lorentz anomaly following the approach of ref. [19]. This anomaly receives contributions only from the functional integration over the quantum counterparts, $y^\alpha$, of the chiral fermionic fivebrane $\vartheta$–fields. Once this anomaly is known, we can perform a finite target space local $SO(1, 9)$ rotation, as in [20], to transform the kinetic term for the $y$'s, which depends also on the target fields, to a canonical kinetic term supplemented by a Wess–Zumino action. At this point it becomes rather straightforward to compute the total worldvolume Lorentz anomaly arising from the gravitational sector of the fivebrane.

Since the field content and the corresponding action of the heterotic fivebrane sector is not yet known, the target space (gauge) and worldvolume anomalies coming from this sector can not be computed from first principles. However, by assuming that this sector is made out of $N_\psi$ complex $d = 6$ Weyl fermions
coupled to the target space gauge fields and the worldvolume metric (as assumed also in the canonical string/fivebrane duality conjecture[13]) these anomalies can be computed via the index theorem even in the absence of an explicit form of the action. Therefore, under this assumption, we get an explicit expression of the total worldvolume and target space anomaly polynomial for the would–be heterotic fivebrane, see formula (34).

It happens that the worldvolume anomaly cancels if \( N_\psi = 16 \) meaning that the heterotic fermions have to be 16 rather than the 32 which are expected in the canonical duality conjecture. On the other hand, the remaining purely target space anomaly polynomial, which should be cancelled via the dual Green–Schwarz mechanism, does not match the polynomial predicted by duality in that the relative coefficients of the terms \( trR^4 \) and \( trF^4 \) differ by a factor of 1/2. This is due to the fact that the target space Lorentz anomaly arising from the gravitational fivebrane sector, which is triggered essentially by the term \( trR^4 \), turns out to be 1/2 of that usually expected. The possible meaning of these results will be discussed at the end of the paper.

2. The Super–fivebrane \( \sigma \)–model

The action for the gravitational sector of the super–fivebrane sigma–model embedded in an \( N = 1, D = 10 \) target space supergravity background[17] is given by

\[
S_6 = -\frac{1}{(2\pi)^3\beta'} \int d^6\sigma \left( \frac{1}{2} \sqrt{g} g^{ij} v_i^a v_j^a - \frac{1}{6!} \varepsilon^{j_1\cdots j_6} V_{j_1}^{A_1} \cdots V_{j_6}^{A_6} B_{A_6\cdots A_1} - 2\sqrt{g} \right).
\]

(1)

The fivebrane fields are the supercoordinates \( Z^M = (X^m(\sigma), \vartheta^\mu(\sigma)) \), \( (m = 0, 1, \ldots, 9 \) and \( \mu = 1, \ldots, 16) \) and the worldvolume metric \( g^{ij}(\sigma) \) where \( i, j = 0, 1, \ldots, 5 \) are curved worldvolume indices. In what follows it will be useful to write the metric in terms of sechsbeins, \( g^{ij} = e^i_\alpha e_j^\alpha \eta^\alpha \), where \( \widehat{a}, \widehat{b} = 0, 1, \ldots, 5 \) are flat \( SO(1,5) \) indices. We set \( V_i^A(Z) = \partial_i Z^M E_M^A(Z) \) and \( v_i^A = e^{-1/3\varphi} V_i^A \) where \( E_M^A \) is the target space superzehnbein, \( \varphi \) is the dilaton superfield and the flat \( SO(1,9) \) index \( A \) stands for ten bosonic and sixteen fermionic entries, \( A = (a, \alpha) \) \( (a = 0, 1, \ldots, 9 \) and \( \alpha = 1, \ldots, 16). \) The \( D = 10 \) six–superform \( B_6 = \frac{1}{6!} E^{A_1} \cdots E^{A_6} B_{A_6\cdots A_1}, \) where \( E^A = dZ^M E_M^A, \) appears in (1) through its pullback on the \( d = 6 \) worldvolume of the fivebrane.

The symmetries of the action are given by the \( d = 6 \) diffeomorphisms with parameter \( c^j, \kappa–\)transformations with parameter \( \kappa^\alpha \) \( (a D = 10 \) spinor and \( d = 6 \)
scalar) and, if the metric is replaced by the sechsbeins, local worldvolume $SO(1,5)$ Lorentz transformations with parameter $\ell^a b$

$$\delta Z^M = \Delta^\alpha E^M_\alpha + c^j \partial_j Z^M$$

$$\delta g^{ij} = 2X^{ij} - \frac{1}{2} g^{ij} X^{hk} g_{hk} + c^k \partial_k g^{ij} - 2 \partial_k c^{(i} g^{j)k}$$

$$\delta e^i_a = \ell^b a \hat{e}^i_b + c^j \partial_j e^i_a - \partial_j c^i e^j_a + (\kappa - \text{transformations}).$$

We have set:

$$\Delta^\alpha = (\mathbb{I} + \Gamma)^\alpha_\beta \kappa^\beta$$

$$\Gamma^\alpha_\beta = \frac{1}{6! \sqrt{g}} \varepsilon^{j_1 \ldots j_6} v^{a_1}_{j_1} \ldots v^{a_6}_{j_6} (\Gamma_{a_1 \ldots a_6})^\alpha_\beta$$

and $X^{ij}$ is given in the appendix.

Actually, $\kappa$–invariance of the action is achieved once the target space superforms satisfy suitable constraints. Introducing the $SO(1,9)$ superconnection

$$\Omega_A^B = E^C \Omega_{CA}^B, \quad \Omega^\alpha_\beta = \frac{1}{4} (\Gamma_{ab})^\alpha_\beta \Omega_a^b, \quad \Omega^a_\alpha = \Omega_a^a = 0, \quad \Omega^{ab} = -\Omega^{ba}, \quad \text{the supertorsion} \quad T^A = dE^A + E^B \Omega_B^A = \frac{1}{2} E^B E^C T_{CB}^A \quad \text{and the } B_6–\text{supercurvature}$$

$$H_7 = dB_6, \quad \text{following the conventions of ref. [27], these constraints read}$$

$$T_{\alpha\beta}^a = 2(\Gamma^a)_{\alpha\beta}$$

$$T_{\alpha\alpha}^b = 0$$

$$H_{\alpha\beta a_1 \ldots a_5} = -2 e^{-2\varphi} (\Gamma_{a_1 \ldots a_5})_{\alpha\beta}$$

$$H_{\alpha a_1 \ldots a_6} = -2 e^{-2\varphi} (\Gamma_{a_1 \ldots a_6})_{\alpha} D_{\beta} \varphi, \quad \text{and the components of } H_7 \text{ with more than two spinorial indices are zero.}$$

The action (1) is also invariant under $SO(1,9)$ “external” local Lorentz transformations, with parameter $L_A^B, L^\alpha_\beta = \frac{1}{4} (\Gamma_{ab})^\alpha_\beta L^{ab}, \quad L_a^\alpha = L_a^a = 0 \quad L_{ab} = -L_{ba}$, under which we have

$$\delta L \Omega_A^B = dL_A^B + L_A^C \Omega_C^B - \Omega_A^C L_C^B$$

$$\delta L E^A = -E^B L_B^A \quad \text{(6)}$$

To determine the worldvolume anomalies we choose to keep the effective action invariant under $d = 6$ diffeomorphisms, at the expense of local $SO(1,5)$ Lorentz anomalies, and we trigger the “genuine sigma–model” $\kappa$–anomalies (the ones which go to zero when the target fields are switched off) through the $SO(1,9)$ local Lorentz–anomalies; the latters are tied to the formers by a coupled cohomology problem (see the introduction in ref. [19]).
We apply the background field method supplemented by a normal coordinate expansion to keep the classical action manifestly \( SO(1,9) \) invariant. So we write \( Z = Z_0 + \Pi(Z_0, y) \), treat the \( Z_0 \) as classical fields and perform the functional integration over the (flat) quantum variables \( y^A = (y^a, y^\alpha) \). The worldvolume \( \text{sechsbein} \) \( e^a_\hat{a} \) is treated as purely classical and constrained to satisfy the classical equations of motion

\[
v_i^a v^a_j = g_{ij}.
\]

Here, and in what follows, the \( v_i^a \) and all target space fields are evaluated at \( Z_0 \).

Since (non trivial) local Lorentz anomalies can arise only from the integration over the fermionic \( y^\alpha \) we concentrate now on these variables only. First of all, the constraint (7) allows to perform an \( SO(1,9) \) covariant \( \kappa \)–gauge fixing. Due to (7), in fact, the matrix \( \Gamma \) defined in (4) satisfies \( \Gamma^2 = 1 \) and \( \text{tr} \ \Gamma = 0 \), such that the constraint

\[
\frac{1}{2} + \Gamma y = 0
\]

eliminates just half of the sixteen \( y^\prime s \) and defines an \( SO(1,9) \) covariant background gauge fixing\[20]. Moreover, (7) allows also to write an embedding equation for the \( SO(1,5) \) (torsion free) spin connection one–form \( \omega^\hat{a} \hat{b} = d\sigma^j \omega^\hat{a} \hat{b}_j \)

\[
\omega^\hat{a} \hat{b} = \omega^{(0)}^\hat{a} \hat{b} + e^{\frac{1}{2}} \varphi \left( \frac{1}{2} v^B v^C v^a_j a + v^B v^a_j v^C_j T^{BC}_a + \frac{2}{3} e^i v^a_j \partial_i \varphi \right)
\]

where

\[
\omega^{(0)}^\hat{a} \hat{b} = \left( \partial_j v^a_{[a} - \Omega^{ab}_j v^b_{[a]} \right) v^a_{j]}
\]

\[
v^a_j = e^j_a v^a_j
\]

\[
\Omega^{ab} = V^A_j \Omega^{Aab}.\]

Notice that \( \omega - \omega^{(0)} \) is an \( SO(1,5) \) tensor one–form, meaning that the anomaly polynomials associated to \( \omega \) and \( \omega^{(0)} \) fall in the same \( SO(1,5) \)–anomaly cohomology class. We will take advantage of this fact below. The \( SO(1,5) \) and \( SO(1,9) \) curvature two–forms are given respectively by

\[
\mathcal{R}^\hat{a} \hat{b} = d\omega^\hat{a} \hat{b} + \omega^\hat{a} \hat{c} \omega^\hat{b} \hat{c}
\]

\[
R^a_b = d\Omega^a_b + \Omega^a_c \Omega^c_b.
\]

To be precise, the former is an intrinsic \( d = 6 \) form while the latter, as it stands, is a superform of the ten dimensional target superspace, whose pullback on the \( d = 6 \) worldvolume is naturally induced by \( dZ^M = d\sigma^i \partial_j Z^M \). In what follows we
will not indicate this pullback explicitly since its occurrence will be clear from the context.

3. The target space Lorentz anomaly

Upon performing the normal coordinate expansion\textsuperscript{[19]} of the action (1) and using (5) one gets for the $SO(1,5)$ and $SO(1,9)$ invariant kinetic term of the $y^\alpha$ the expression

\[ \frac{1}{2} \int d^6\sigma \sqrt{g} \, e^{-\frac{1}{2} \varphi} e^j_a \, \hat{v}^a_y \, \Gamma_{\beta} a \frac{\Gamma}{2} D_j y \]

where $D_j = \partial_j - \frac{1}{4} \Gamma_{cd} \Omega_{jcd}$. Enforcing the gauge fixing and rescaling the $y'$s this becomes

\[ I(v, \Omega, y) = \frac{1}{2} \int d^6 \sigma \sqrt{g} \, e^j_a \, \hat{v}^a_y \, y \, \Gamma \frac{\Gamma}{2} \frac{D_j y}{2} \]

(10)

Actually, the normal coordinate expansion of the action (1) gives rise to additional terms quadratic in the $y^\alpha$'s which in eq. (10) we did not write. The only effect of these additional terms is a redefinition of the $SO(1,9)$ Lorentz connection by Lorentz covariant terms. Therefore they give rise at most to trivial anomalies (trivial cocycles) and can be neglected. Eq. (10) is the starting point of our perturbative analysis of the super–fivebrane anomalies. As for the string, the non–canonical dependence of the propagator on the $v^a_\alpha$ can be eliminated by an $SO(1,9)$ rotation of the $y^\alpha$ which is, however, expected to be anomalous. So the first step consists in deriving the $SO(1,9)$ anomaly associated to (10). Since the form of the anomaly is strongly constrained by the consistency condition it is sufficient to determine the anomaly under an $SO(4)$ subgroup of $SO(1,9)$, and for a particular configuration of the background fields. We choose a configuration for which

\[ e^j_a = \delta^j_a \]

(11)

\[ v^a_\alpha(\sigma) = \text{constant} \]

(12)

\[ v^a_\alpha \Omega_{jab} = 0. \]

(13)

This implies in particular that the 16x16 matrices $\Gamma$ and $\Gamma_j \equiv e^j_a \, \hat{v}^a_\alpha \, \Gamma^a = g^{ij} v_{ia} \Gamma^a$, appearing in (10), are now constant matrices and satisfy a six–dimensional Dirac algebra:

\[ \{ \Gamma^i, \Gamma^j \} = 2\eta^{ij} \]

\[ \{ \Gamma, \Gamma^i \} = 0. \]

(14)
Moreover, due to (13)
\[ \Omega_{jab} \cdot [\Gamma^{ab}, \Gamma^i] = 0. \] (15)
For this particular configuration the action (10) is invariant under a local \(SO(4)\) subgroup of \(SO(1,9)\) specified by the constraint
\[ v^a_a L_{ab} = 0. \] (16)
For the configuration (11)–(13) the \(y\)–propagator becomes just \(k_j \Gamma_j\) and the \(y-y-\Omega\) vertex is \(i \frac{1}{2} \Gamma^i (\frac{1}{4} \Gamma_{ab})\). Then the leading one–loop anomaly diagrams, which in six dimensions are box diagrams with four external \(\Omega\), can be easily evaluated. Calling \(R^{(0)}_{ab} = d\Omega_{ab}\), the anomaly coming from the box diagrams can be computed to be
\[
\frac{1}{384} \frac{1}{(2\pi)^3} \int \left( tr \left( LR^{(0)} R^{(0)} R^{(0)} \right) - \frac{3}{4} tr \left( LR^{(0)} \right) tr \left( R^{(0)} R^{(0)} \right) \right),
\] (17)
where the traces are in the fundamental representation of \(SO(1,9)\). To get this one has to use that in the Feynman diagrams, thanks to (14,15), one can replace the \(\Gamma\)–matrix traces
\[
tr \left[ (\Gamma^i \Gamma^j \cdots)(\Gamma^{ab} \Gamma^{cd} \cdots) \right] \to \frac{1}{16} tr(\Gamma^i \Gamma^j \cdots) tr(\Gamma^{ab} \Gamma^{cd} \cdots)
\] (18a)
and that
\[
tr \left( \bar{H} \bar{H} \bar{H} \bar{H} \right) = -tr \left( H^4 \right) + \frac{3}{4} \left( tr \left( H^2 \right) \right)^2
\] (18b)
for any antisymmetric matrix \(H^{ab}\) where \(\bar{H} \equiv \frac{1}{4} \Gamma_{ab} H^{ab}\), and the traces at the r.h.s. of (18b) are in the fundamental representation of \(SO(1,9)\). From (17) one can read the anomaly polynomial associated to \(SO(1,9)\) transformations as
\[
X^L_s = \frac{1}{384} \frac{1}{(2\pi)^3} \left( tr R^4 - \frac{3}{4} \left( tr R^2 \right)^2 + \gamma \ tr R^2 tr R^2 \right).
\] (19)
We included a term proportional to \(tr R^2\) which could not be derived by the method above since in the configuration (11)–(13) \(R^{\sim}_{a b} = 0\). The unknown coefficient \(\gamma\) will be determined below.

The result (19), for \(\gamma = 0\), can actually also be derived using the index theorem\(^{[23]}\). For a complex Weyl fermion in \(d = 6\) the anomaly polynomial is given by
\[
X^I_s = \frac{1}{384} \frac{1}{(2\pi)^3} \left( -16 \ tr F^4 + 4 \ tr F^2 tr R^2 - \frac{N}{12} \left( tr R^2 \right)^2 - \frac{N}{15} \ tr R^4 \right),
\] (20)
where the Yang–Mills trace over the F’s is in whatever representation the fermions are and N is its dimensionality. In the configuration (11)–(13) the action (10) corresponds indeed to one real chirally projected fermion with sixteen components, \(y^\alpha\), which is equivalent to one complex \(d = 6\) Weyl fermion. The “Yang–Mills” matrices F are replaced by \(\frac{1}{4} \Gamma_{ab} R^{ab}\), but, since these matrices and the \(d = 6\) Dirac matrices \(\Gamma^i\) live in the same 16–dimensional representation space, the traces over the two kinds of matrices factorize with a factor of \(\frac{1}{16}\) (see (18a)). In summary, one has to set in (20)

\[
\begin{align*}
N &= 1 \\
\mathcal{R} &= 0 \\
tr F^4 &\to \frac{1}{16} tr \left( \frac{1}{4} \Gamma_{ab} R^{ab} \right)^4,
\end{align*}
\]

and, due to (18b), one gets again (19) with \(\gamma = 0\).

4. The worldvolume Lorentz anomaly

Eq. (19) parametrizes the non–invariance of the measure \(\int \{Dy\}\) in the functional integral which defines the effective action, under a generic finite \(SO(1,9)\) rotation on the \(y^\alpha\), with transformation matrix \(\Lambda^a_b \in SO(1,9)\). Performing such a rotation we can rewrite the effective action \(\Gamma\) as

\[
e^{i\Gamma} = \int \{Dy\} e^{iI(v,\Omega,y)} = e^{-i\Gamma_{WZ}(\Lambda)} \int \{Dy\} e^{iI(v^\Lambda,\Omega^\Lambda,y)} \equiv e^{-i\Gamma_{WZ}(\Lambda)} e^{i\Gamma_0},
\]

where

\[
\Omega^\Lambda = d\Lambda\Lambda^T + \Lambda\Omega\Lambda^T
\]

\[
(v^\Lambda)^a_a = \Lambda^a_b \, v^b_a,
\]

and the Wess–Zumino term \(\Gamma_{WZ}(\Lambda)\) can be read off from (19) as follows. If we define for a generic \(SO(M)\)–connection one–form \(\tilde{\Omega}\) and the related curvature two–form, \(\tilde{R} = d\tilde{\Omega} + \tilde{\Omega}\tilde{\Omega}\), canonical Chern–Simons forms,

\[
\begin{align*}
tr \tilde{R}^2 &= dY_3(\tilde{\Omega}) \\
tr \tilde{R}^4 &= dY_7(\tilde{\Omega}),
\end{align*}
\]

then, due to (19) and (22), we have

\[
\Gamma = \Gamma_0 - \int_{M^7} \left( U_7(\Omega^\Lambda) - U_7(\Omega) \right).
\]
where
\[ U_7(\Omega) = \frac{1}{384(2\pi)^3} \left( Y_7(\Omega) - \frac{3}{4} Y_3(\Omega) \, \text{tr}R^2 + \gamma Y_3(\Omega) \, \text{tr}R^2 \right) . \] (25)

The boundary of \( M_7 \) is the fivebrane worldvolume. Until now we considered a generic \( \Lambda \in SO(1,9) \). We want now choose a \( \Lambda \) for which the kinetic term of the \( y' \)s in \( I(v^\Lambda, \Omega^\Lambda, y) \) becomes canonical, allowing to read the worldvolume Lorentz anomaly of \( \Gamma_0 \) directly from the index theorem. To do this we choose a basis in the four–dimensional space orthogonal to the six \( v_{\hat{a}}^a \), introducing four \( SO(1,9) \) vectors \( \{ N_r^a \} \), \( r, s = 6, 7, 8, 9 \), satisfying
\[ N_a^r N_s^a = -\delta^{rs} \]
\[ N_a^r v_a^b = 0, \] (26)
and set \( \Lambda^b_a \equiv \{ \Lambda^b_a, \Lambda^r_a \} \)
\[ \Lambda^b_a = v_a^b \]
\[ \Lambda^r_a = N_a^r. \] (27)

This \( \Lambda \) belongs indeed to \( SO(1,9) \) in that \( \Lambda_a^b \Lambda_c^d \eta^{bd} = \eta^{ac} \), due to (7) and (26).

This procedure introduces an “intermediate” external local symmetry group \( SO_E(4) \) in the game, with \( SO(1,5) \) and \( SO(1,9) \)–invariant connection one–form given by
\[ d\sigma^j W_{jrs} = W_{rs} = \left( dN_{ra} - \Omega_{ab} N_b^r \right) N_s^a, \]
whose curvature is
\[ T_{rs} = dW_{rs} + W_{r}^t W_{ts}. \]

We proceed now to the determination of the \( SO(1,5) \), \( SO_E(4) \) and \( SO(1,9) \) anomalies coming from each of the three terms in (24). \( U_7(\Omega) \) carries only \( SO(1,9) \) anomalies with anomaly polynomial given clearly by \( X_8^L \), see (19). \( \Gamma_0 \) and \( \int U_7(\Omega^\Lambda) \), on the contrary, carry only \( SO(1,5) \) and \( SO_E(4) \) anomalies and, moreover, since \( SO_E(4) \) is an intermediate symmetry group \( \Gamma_0 - \int U_7(\Omega^\Lambda) \) has to be \( SO_E(4) \) invariant (this last condition will allow us eventually to determine the coefficient \( \gamma \) in eq. (19)). Let us first give the results. The anomaly polynomial carried by \( -\int U_7(\Omega^\Lambda) \) turns out to be
\[ X_8^\Lambda = \frac{1}{384(2\pi)^3} \left( -\text{tr}R^4 - \text{tr}T^4 + \left( \frac{3}{4} - \gamma \right) (\text{tr}R^2)^2 + \frac{3}{4} (\text{tr}T^2)^2 + \left( \frac{3}{2} - \gamma \right) \text{tr}R^2 \text{tr}T^2 \right), \] (28)
while that associated to $\Gamma_0$ is
\[ X_8^{(0)} = \frac{1}{384(2\pi)^3} \left( -\frac{1}{15} tr R^4 + tr T^4 - \frac{1}{12} (tr R^2)^2 - \frac{3}{4} (tr T^2)^2 + \frac{1}{2} tr R^2 tr T^2 \right), \tag{29} \]
where all traces are in the fundamental representations of the $SO$–Lie algebras.

To get (28) one has to note that
\[ (\Omega^A)_{a \hat{b}} = \omega_a^{(0)} \hat{b} \]
\[ (\Omega^A)_{rs} = W_{rs}, \tag{30} \]
while $(\Omega^A)_{a r} = (d\nu^a - \Omega^{ab} v_{ba}) N_{ar}$ is not a connection but a tensor under $SO(1,5) \otimes SO_E(4)$, and to use the decompositions
\[ Y_7(\Omega^A) = Y_7(\omega^{(0)}) + Y_7(W) + X_7 + dX_6 \]
\[ Y_3(\Omega^A) = Y_3(\omega^{(0)}) + Y_3(W) + X_3 \tag{31} \]
\[ tr R^2 = tr R^2 + tr T^2 + dX_3. \]

Here $X_3$ and $X_7$ are completely invariant forms, given in the appendix, and $X_6$ is a local form which can, therefore, be disregarded. Taking (31) into account it is straightforward to compute $\delta \left(-\int U_7 (\Omega^A)\right)$ and to realize that the resulting anomaly, apart from trivial cocycles, is represented by $X_8^\Lambda$. To compute $X_8^{(0)}$ we observe that, with the choice (27), one has
\[ I(v^A, \Omega^A, y) = \frac{1}{2} \int d^6 \sigma \sqrt{g} \ y \ e^\hat{a}_A \Gamma^a \frac{1}{2} \left( \partial_j - \frac{1}{4} \omega^{(0)}_{\hat{b} \hat{c}} \Gamma^\hat{b} \Gamma^\hat{c} - \frac{1}{4} W_{jrs} \Gamma^{rs} \right) y, \tag{32} \]
where now all gamma matrices are constant matrices and $\hat{\Gamma} = \Gamma^0 \Gamma^1 \cdots \Gamma^5$ gives rise to a true chiral projector, $\frac{1 - \hat{\Gamma}}{2}$. The kinetic term for the $y$'s in (32) is now canonical and the anomalies carried by $\Gamma_0$ can be retrieved from the index theorem, eq. (20). Our fermion is sixteen–dimensional, but real, corresponding thus to one complex $d = 6$ Weyl fermion, hence we have to set $N = 1$. Our gauge–group is $SO_E(4)$, with generators $\frac{1}{4} \Gamma^{rs}$, hence we have to identify $F \rightarrow \frac{1}{4} \Gamma^{rs} T_{rs}$; but since the matrices $\hat{\Gamma}^a$ and $\Gamma^{rs}$, although being commuting, live in the same sixteen–dimensional space, the traces over the $F$'s have to be divided by 16. Taking the identity (18b) into account, (with $\Gamma_{ab} \rightarrow \Gamma_{rs}$), one gets indeed (29).

In summing up $X_8^L, X_8^A$ and $X_8^{(0)}$ one sees that the cancellation of the $SO_E(4)$ anomalies requires
\[ \gamma = 2. \]
This cancellation could, actually, also be used as an alternative procedure to determine the $SO(1,9)$ anomaly polynomial $X_8^L$ completely. The total anomaly polynomial arising from the gravitational sector of the super–fivebrane sigma–model is thus given by

$$X_8 = X_8^L + X_8^A + X_8^{(0)}$$

$$= \frac{1}{384(2\pi)^3} \left( -\frac{16}{15} tr\mathcal{R}^4 - \frac{4}{3} \left( tr\mathcal{R}^2 \right)^2 + 2 tr\mathcal{R}^2 tr\mathcal{R}^2 - \frac{3}{4} \left( tr\mathcal{R}^2 \right)^2 + tr\mathcal{R}^4 \right).$$

(33)

5. Adding the anomaly from the “heterotic” sector

Eq. (33) is our principal result. We see that even for a flat $D = 10$ background, i.e. for $R = 0$, the worldvolume $SO(1,5)$ anomaly is non vanishing. To cancel this anomaly one has necessarily to add a “heterotic” sector to the theory. As said in the introduction, it is still unknown how to couple such a sector in a $\kappa$–invariant way. If we assume, in analogy to the string, that such a sector is made out of $N_\psi d = 6$ complex Weyl fermions, with chirality opposite to that of the $\psi^\alpha$, which belong to an $N_\psi$–dimensional representation of a gauge group $G$, and that they are chirally coupled to the gauge fields $A$, with Lie algebra valued curvature two–form $\mathcal{F} = dA + AA$, and to the worldvolume connection $\omega$, then one has to add to $X_8$ just $-X_8^I$ with $N = N_\psi$ (see eq. (20)). The resulting anomaly polynomial of the “heterotic” fivebrane would then be given by $X_8^H = X_8 - X_8^I$ which can be written as

$$X_8^H = \frac{1}{(2\pi)^3} \frac{1}{384} \left( (N_\psi - 16) \left( \frac{1}{15} tr\mathcal{R}^4 + \frac{1}{12} \left( tr\mathcal{R}^2 \right)^2 \right) + (2 tr\mathcal{R}^2 - tr\mathcal{R}^2) \left( tr\mathcal{R}^2 - 2 tr\mathcal{F}^2 \right) \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. 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(34)

First of all we observe that for a trivial background ($R = F = 0$) the cancellation of the $SO(1,5)$ anomalies requires sixteen heterotic fermions,

$$N_\psi = 16.$$

(35)

For a non–trivial background the anomalies which do not involve $\mathcal{R}$, but only the target space curvatures $R$ and $F$, can be cancelled by choosing an appropriate transformation law for $B_6$ and modifying accordingly its field strength by an appropriate Chern–Simons form, (see ref. [13]). The resulting $D = 10$ supergravity
theory would contain the Yang-Mills field strength $F$ with values in a *sixteen-dimensional* representation of some gauge group $G$: this fact prevents already a direct comparison with heterotic string theory since neither $SO(32)$ nor $E_8 \otimes E_8$ admit a sixteen-dimensional irreducible representation.

The result (34) presents also a feature which is more dramatic for the fivebrane itself: the mixed term in the second line, proportional to $\text{tr} R^2 (\text{tr} R^2 - 2 \text{tr} F^2)$, cannot be cancelled by modifying the $B_6$-field strength, and the heterotic fivebrane sigma–model would be inconsistent at the quantum level.

To discuss our results more in detail we recall now the principal features of string/fivebrane duality.

6. The string/fivebrane duality conjecture

The pure $N = 1, D = 10$ supergravity theory in ten dimensions admits two dual formulations, one based on $B_2$ and one based on $B_6$, whose dynamics is described by symmetric formulations in that both field strengths are closed, $dH_3 = 0 = dH_7$. This symmetry disappears if one couples the pure Supergravity to a Super–Yang–Mills theory because in this case $dH_3 \neq 0$; in some sense it has been restored through the discovery of superstring theories, more precisely through the Green–Schwarz anomaly cancellation mechanism which implies $dH_3 \neq 0, dH_7 \neq 0$. The $N = 1, D = 10$ Supergravity–Super–Yang–Mills anomaly polynomial $I_{12}$ factorizes, in fact, for the gauge groups $SO(32)$ and $E_8 \otimes E_8$ into $I_{12} = \frac{1}{2\pi} I_4 I_8$ [3] where for $SO(32)$ one has

\begin{align*}
I_4 &= \frac{1}{(2\pi)^4} \left( \text{tr} R^2 - \text{tr} F^2 \right) \equiv d\omega_3 \quad (36) \\
I_8 &= \frac{1}{(2\pi)^3} \frac{1}{192} \left( \text{tr} R^4 + \frac{1}{4} (\text{tr} R^2)^2 - \text{tr} R^2 \text{tr} F^2 + 8 \text{ tr} F^4 \right) \equiv d\omega_7, \quad (37)
\end{align*}

where the traces over the $F$’s are in the fundamental representation of $SO(32)$. On the other hand, the cancellation of sigma–model $\kappa$–anomalies in the heterotic string, with action normalized in a standard way as $S_2 = -\frac{1}{2\pi\alpha'} \int d^2 \sigma \frac{1}{2} \epsilon^{ij} V_i^A V_j^C B_{CA} + \cdots$, requires the introduction of an invariant $B_2$–field strength given by\[19\]

\begin{align*}
H_3 &= dB_2 - (2\pi\alpha') \omega_3 \\
dH_3 &= -(2\pi\alpha') I_4.
\end{align*}

As a consequence $B_2$ transforms anomalously under $SO(1, 9)$ and $SO(32)$ and the $N = 1, D = 10$ anomaly can be cancelled by adding to the classical supergravity
action the term
\[ \Delta S_{10} = \frac{-1}{2\pi} \int \left( \frac{1}{2\pi \alpha'} B_2 I_8 + \frac{2}{3} \omega_3 \omega_7 \right). \]  (39)

Since the kinetic term of \( B_2 \) is given by \( S_{10} = \frac{-1}{2\kappa^2} \int \frac{1}{2} e^{-2\varphi} H_3 * H_3 \), where \( \kappa^2 \) is the ten dimensional Newton’s constant, the addition of (39) modifies the field equation of \( H_3 \) to
\[ d \left( *e^{-2\varphi} H_3 \right) = \frac{2\kappa^2}{(2\pi)^2 \alpha'} I_8, \]  (40)

which represents a one–loop string effect. According to the string–fivebrane duality conjecture, eq. (40) should arise from the cancellation of sigma–model anomalies in the heterotic fivebrane, as the Bianchi identity for the generalized \( B_6 \) field–strength, via the identification
\[ H_7 = *e^{-2\varphi} H_3, \]
which leads to
\[ dH_7 = \frac{2\kappa^2}{(2\pi)^2 \alpha'} I_8 \]  (41)
\[ H_7 = dB_6 + \frac{2\kappa^2}{(2\pi)^2 \alpha'} \omega_7, \]  (42)

where \( B_6 \) is identified with our fivebrane six–form in (1).

Another characteristic feature of this duality is the Dirac–Nepomechie–Teitelboim quantization condition\[24\] on the tensions of strings and fivebranes which reads
\[ 2\kappa^2 = n(2\pi)^5 \alpha' \beta', \]  (43)

where \( n \) is integer.

7. Discussion

The duality conjecture leads us to compare \( I_8 \) in eq. (37) with the target space anomaly polynomial arising from eq. (34). If we set \( N = 16 \) and disregard for the moment the second line of (34) we are led to compare its third line with \( I_8 \). As we mentioned already, this comparison is prevented by the fact that the representations of \( F \) and \( F' \) do not match, but apart from that we see also that the terms not involving \( F \) carry coefficients which differ by a factor of 1/2 w.r.t. the corresponding terms in \( I_8 \). So one is forced to conclude that a fundamental super–fivebrane described by the action (1), supplemented with a heterotic sector
of 16 complex Weyl fermions, with still unknown action, is not in agreement with string–fivebrane duality. We can, however, make the following observations.

Doubling the gravitational fivebrane sector? It is worthwhile to notice that, for the gauge group $SO(32)$, we could find complete agreement with the duality conjecture if the gravitational sector of the fivebrane would correspond to two, instead of one, complex Weyl fermions. For what concerns the anomaly this would just amount to multiply the anomaly polynomial $X_8$ in (33) by a factor of two, and, upon adding the heterotic sector, one would obtain for the total anomaly polynomial, instead of (34),

$$\tilde{X}_8^H = 2X_8 - X_8^I = \frac{1}{(2\pi)^3} \frac{1}{192} \left( (N_\psi - 32) \left( \frac{1}{15} tr R^4 + \frac{1}{12} (tr R^2)^2 \right) + (2 tr R^2 - tr R^2)(tr R^2 - tr F^2) + tr R^4 + \frac{1}{4} (tr R^2)^2 - tr F^2 tr R^2 + 8 tr F^4 \right).$$

(44)

In this case one needs indeed 32 heterotic fermions to cancel the pure worldvolume anomaly, the fivebrane gauge group can be taken to be $SO(32)$ and one can identify $F \equiv \mathcal{F}$. Moreover, the second line in (44) can now be eliminated by invoking the anomalous transformation law of $B_2$ resulting from (38) and adding to the classical fivebrane action the local term

$$\Delta S_6 = \frac{1}{(2\pi)^3} \frac{1}{192} \frac{4}{\alpha'} \int B_2(2 tr R^2 - tr R^2)
= \frac{1}{(2\pi)^3} \frac{1}{192} \frac{4}{\alpha'} \int (H_3 + 2\pi \alpha' \omega_3)(2Y_3(\omega) - Y_3(\Omega)).$$

(45)

The advantage of the second form of this counterterm is that it does not involve the “string” two–form $B_2$, but only the curvature $H_3 = *e^{2\phi} H_7$. Its variation cancels then the second (dangerous) line in (44) upon using the $H_3$–Bianchi identity in (38) which has to be interpreted as equation of motion for $H_7$. What remains of $\tilde{X}_8^H$ is then just a pure target space polynomial, the third line in (44), which coincides exactly with $I_8$. The corresponding anomaly can then be eliminated by setting

$$H_7 = dB_6 + (2\pi)^3 \beta' \omega_7
= (2\pi)^3 \beta' I_8.$$

(46)

(47)

These equations would then perfectly respect the duality conjecture since they coincide with (42,41) if (43) holds with $n = 1$. Notice, however, that as a consequence of the necessary subtraction of the local term (45), and contrary to what
happens for the string sigma–model, here the anomaly cancellation mechanism requires not only the Bianchi identity for $H_7$ but also its equation of motion (38). While the solutions of the Bianchi identity (38) in superspace are well known until now no solution is known for (47); apart from that one should also keep in mind that if one insists on both, regarding one as equation of motion and the other as Bianchi identity, even if one can solve them simultaneously in superspace, the resulting supergravity equations of motion can not be deduced from a local gauge– and Lorentz–invariant supergravity action.

It is certainly difficult to imagine that a consistent fivebrane sigma–model exists in which the gravitational anomaly (33) is just doubled; nevertheless our quantitative result – i.e. that the anomaly coming from the gravitational sector of the fivebrane is just half of what would be naively expected on the basis of the duality conjecture – for which at present we have no clear interpretation, may in the future help to cast the duality conjecture itself in a more concrete formulation.

**Particular configurations of the gauge fields.** It may be interesting to notice that if one insists on 16 heterotic fermions, and hence on (34), from a purely formal point of view one can find agreement with duality if one chooses particular configurations for the Yang–Mills fields. First of all, if on sets them to zero ($F = F = 0$), the second line in (34) can be eliminated by subtracting now from the classical action $1/2\Delta S_6$ and imposing $dH_7 = \beta'(48)$. This matches now with (41), for $F = 0$, if

$$2\kappa^2 = 1/2(2\pi)^5\alpha'/\beta'$$

which corresponds to (43), but with $n = 1/2$. Since the Dirac quantization condition arises from the requirement that the product of the charges of a single fivebrane and a single string are integer, (48) would amount to the existence of half–charged elementary fivebranes. Half–charged fivebranes arose, actually, in ref. [25] where they appear, however, always in pairs such that their total charge is always integer. Half integral magnetic charges have arisen also on fixed points of $Z_2$–orbifold compactifications of $N = 1, D = 11$ Supergravity in ref. [26].

Incidentally one may notice that one can cancel (34) also if one couples the 16 heterotic fermions to 16 abelian gauge fields and introduces an $H_3$ satisfying

$$dH_3 = -\frac{\alpha'}{4} (trR^2 - 2trF^2).$$

$X^H_8$ could then be cancelled subtracting $1/2\Delta S_6$ and setting $dH_7$ equal to the third line of (34). It is puzzling to notice that the resulting equation for $H_7$ and
(49) coincide with (41) and (38) respectively, imposing again (47), if one sets in \( \mathcal{F} \) the 480 non abelian gauge fields of \( SO(32) \) to zero and identifies the ones in its Cartan subalgebra with the 16 abelian gauge fields to which the heterotic fermions are coupled.

**Comment on the target space polynomial of ref. [13]** The results of the present paper, and of ref. [20], allow us to give a partial justification of the somehow conjectural derivation of the target space anomaly polynomials for the heterotic string and fivebrane sigma–models performed by Dixon et. al. in [13]. The method implied in that paper works, in fact, once one has made sure of the cancellation of worldsheet/worldvolume anomalies. For the string we showed in [20] that the worldsheet Lorentz anomalies get a contribution \( N_y = 8 \), from the eight physical Majorana–Weyl quantum \( \vartheta \)'s, and a contribution \( L = 24 \) induced by the \( D = 10 \) target space Lorentz anomaly, which has weight \( -L \), via a Wess–Zumino term. This anomaly is cancelled introducing \( N_\psi = 32 \) heterotic fermions such that

\[
L = N_\psi - N_y.
\]

This equation implies that one can compute the target space Lorentz anomaly by using formally the index theorem and counting the fermions, taking their chirality into account, as \( L = 32 - 8 \). This was, indeed, the procedure applied in [13] and the result was \( I_4 \) of eq. (36). The reason for why this works for the string exactly is that in two dimensions the anomaly polynomial contains only irreducible invariants i.e. \( trF^2 \) and \( trR^2 \).

For the fivebrane the same reasoning can be applied for what concerns the irreducible term \( trR^4 \). We saw that in this case one has \( N_y = 1 \) and \( L = 15 \), see eqs. (29,28), and the cancellation of the worldvolume anomalies required \( N_\psi = 16 \). This means that the weight of \( trR^4 \) can be computed formally using the index theorem (20), with \( tr\mathcal{R}^4 \to tr\mathcal{R}^4 \) and \( N = -L = 1 - 16 \), in agreement with (19). In ref. [13] an overcounting of the quantum \( \vartheta \)'s led to \( N_y = 2 \), and the heterotic fermions have been assumed to be 32 instead of 16; this led to \( -L = 2 - 32 \) and brought to a doubling of the coefficient of \( trR^4 \). For what concerns the factorized terms in the anomaly polynomial we can only observe that the polynomial (44), in which the \( y \)'s have been doubled by hand and one should set \( N_\psi = 32 \), reduces to the corresponding expression in [13] only if, instead of setting \( \mathcal{R} = 0 \) as one should, one identifies \( tr\mathcal{R}^2 \leftrightarrow tr\mathcal{R}^2 \).

In conclusion, our results add to the well known open problem of a \( \kappa \)-invariant action for the heterotic sector of the fivebrane a further one, namely, that the
worldvolume and target space anomalies of its gravitational sector are 1/2 of what would be expected on the basis of string/fivebrane duality. Sometimes two difficulties which at first sight seem unrelated conspire to shed new light on both of them. Our hope is that the analysis presented here will contribute in the future to find a consistent formulation for a heterotic super–fivebrane sigma–model.

Appendix

I) The symmetric tensor $X^{ij}$, which parametrizes the $\kappa$–transformation of the metric, is given by

$$X^{ij} = -2Y^{ij} (v^k \alpha \beta (\Gamma^a)_{\alpha \beta} + \lambda \alpha - \frac{1}{3} \kappa^i \lambda^j \Delta^\alpha$$

$$+ \frac{2}{5! \sqrt{g}} \epsilon^{j_1 \ldots j_5} (v^a \alpha \beta \gamma d \Gamma^a_{\alpha \beta} = 0$$

where

$$Y^{ij} = \frac{1}{6! \sqrt{g}} \frac{1}{\sqrt{g}} (Z_{i_1 j_1} \ldots Z_{i_5 j_5} + Z_{i_1 j_1} \ldots Z_{i_4 j_4} g_{i_5 j_5} + \ldots + g_{i_1 j_1} \ldots g_{i_5 j_5})$$

and

$$Z_{ij} = v^a \alpha \beta \gamma d \Gamma^a_{\alpha \beta}$$

II) The completely invariant forms $X_3$ and $X_7$ can be expressed in terms of the following $SO_E(4) \otimes SO(1, 5)$ tensor–forms:

$$C_{\alpha \beta} = (\Omega^a)_{\alpha \beta}$$

$$(C^T)_{\alpha \beta} = -C_{\alpha \beta}$$

$$P_{\alpha \beta} = dC_{\alpha \beta} + W_{s \alpha \beta} + \omega_{\beta}^a C_{\alpha \beta}$$

$$(P^T)_{\alpha \beta} = -P_{\alpha \beta}.$$
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