Dynamics of the 2d Potts model phase transition*
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The dynamics of 2d Potts models, which are temperature driven through the phase transition using updating procedures in the Glauber universality class, is investigated. We present calculations of the hysteresis for the (internal) energy and for Fortuin-Kasteleyn clusters. The shape of the hysteresis is used to define finite volume estimators of physical observables, which can be used to study the approach to the infinite volume limit. We compare with equilibrium configurations and the preliminary indications are that the dynamics leads to considerable alterations of the statistical properties of the configurations studied.

1. Introduction

Lattice gauge theory investigations of the deconfining phase transition, for reviews see \cite{1}, have mainly been limited to studies of their equilibrium properties. In contrast to that, their occurrence in nature is governed by temperature driven dynamics. In the early universe it is a cooling process, which is estimated to proceed slowly on the scale set by hadronic relaxation times. In heavy ion collisions a rapid heating of the nuclei is followed by a much slower cooling process and it is no longer clear whether this cooling is slow or fast on the relaxation time scale. In addition, the small sizes of the nuclei systems impose complications which require to be studied.

What dynamics should we study? Time evolution in nature would correspond to some molecular dynamics simulation of full QCD in Minkowski space, which is out of the question as already Euclidean space Monte Carlo (MC) calculations of QCD stretch computer resources to their limits. One may, however, argue that the computer-time evolution of a MC simulation reveals the correct physical features, as long as the updating procedure stays in the universality class of Glauber dynamics \cite{2}. These are local MC updating schemes, which imitate the thermal fluctuations of nature. Examples are single- and multi-hit Metropolis algorithms, as well as the heat-bath algorithm.

Also to simulate the dynamics of the deconfining phase transition within Euclidean lattice QCD is out of reach for an exploratory study. To gain some qualitative understanding, we have decided to investigate \(q\)-state Potts models, whose simulations are far less computer time intensive. The relationship to QCD is that an effective model for the Polyakov loop is provided by the 3\textsuperscript{d}, 3-state Potts model in an external magnetic field \cite{3}. Even this simplification is not yet a convenient starting point. To get confidence in our numerical methods, we limit our first round of simulations to the \(q\)-state Potts models in 2\textsuperscript{d}, for which a number of rigorous results exist \cite{4,5}.

In the following we drive our systems many times through various heating and cooling schedules and measure physical observables along the way. Averages and their error bars are calculated with respect to the repetitions of the schedule. Our observables are the internal energy and properties of Fortuin-Kasteleyn \cite{6} clusters.

Details of our investigations are reported in the next section and a brief summary with conclusions is given in the final section.

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2. Results

We have performed simulations on $L \times L$ lattices for the 2d Potts model with $q = 2, 5$ and 10, to have an example of a second order transition ($q = 2$), a weak ($q = 5$) and a strong ($q = 10$) first order transition. Due to space restrictions we limit the following presentation to $q = 10$.

For suitably chosen values of $\beta_{\text{min}}$ and $\beta_{\text{max}}$, we run in the range $\beta_{\text{min}} \leq \beta \leq \beta_{\text{max}}$ through each hysteresis cycle at least 640 times with $\beta$ changed by $\pm \Delta \beta$ after every sweep. The stepsize is

$$\Delta \beta = \frac{2(\beta_{\text{max}} - \beta_{\text{min}})}{n_{\beta} L^2}$$

(1)

where $\beta_{\text{min}}$ and $\beta_{\text{max}}$ define the terminal temperatures and the integer $n_{\beta} = 1, 2, \ldots$ is varied. In the limit where we send $L^2 \to \infty$ first and then $\Delta \beta \to 0$, the hysteresis is expected to survive, whereas it disappears if we send $\Delta \beta \to 0$ first and then $L^2 \to \infty$.

For a first order phase transition, the slowing down of the Markov process is exponential in computer time, $\sim \exp[2 f_s L]$, where $f_s$ is the interfacial tension. In the following we study the limit $L \to \infty$ and $\Delta \beta(L; n_{\beta}) \to 0$ for $n_{\beta}$ fixed. For 1-hit Metropolis updating we give in figure 1 our energy hysteresis data for the 10-state Potts model on a 80 $\times$ 80 lattice. Towards large $n_{\beta}$ we see convergence to the equilibrium curve, which is obtained by means of a multicanonical [7] simulation. To analyze the physical content of such a hysteresis, we define for each $n_{\beta}$ finite volume estimators for a physical observable as its value at the maximum opening of the corresponding hysteresis.

Figure 2 shows, for various fixed values of $n_{\beta}$, the thus obtained latent heat estimates versus the lattice size $L$. For sufficiently large $L$ all the estimators should, independently of their $n_{\beta}$ values, converge towards the infinite volume value, which is also given in the figure. For each $n_{\beta}$ we have performed fits to extrapolate towards $L \to \infty$, but only some of them give good estimates. For our small and medium sized lattices the results appear to suffer from finite size corrections in $L$ as well as in $\Delta \beta$, with the additional twist that $\Delta \beta$ can also be too small for some of the lattice sizes used. The best $L \to \infty$ estimate is

$$\Delta E = 0.3485(12)$$

at $n_{\beta} = 16$.

For 1-hit Metropolis updating we give in figure 1 the equilibrium curve and the values of $n_{\beta}$ given in the figure.

Figure 1. Energy hysteresis curves for the 10-state $d = 2$ Potts model on a 80 $\times$ 80 lattice for $\beta_{\text{min}} = 0.4$, $\beta_{\text{max}} = 1.2$ (not the entire range is shown) and the values of $n_{\beta}$ as given in the figure.

Figure 2. Finite size latent heat estimates for the 10-state $d = 2$ Potts model versus $L$ for fixed values of $n_{\beta}$, obtained from the maximum opening of energy hysteresis curves like those of figure 1. The lines are just to guide the eyes.
To focus on a detailed physical understanding of the finite volume transitions, we have begun a cluster analysis of the configurations. The appropriate cluster definition is the statistical one of Fortuin and Kasteleyn [6], see Ref. [8] for a review of this and related topics. We are interested in the effects of heating and cooling on the cluster structure, compared to their equilibrium structure. Whereas nucleation theory holds in the mean field approximation, phase transitions can also proceed via spinodal decomposition as the result of an off-equilibrium quench, which may still lead to observable signals, even when there is no longer a transition in the strict thermodynamical sense. This is somewhat different from the program of Satz [9], who focuses on geometric clusters and would like to use them as a signal when there is no proper transition.

3. Summary and Conclusions

Our energy hysteresis method allows for dynamical estimates of the latent heat and works similarly for other physical observables. While the precision of these estimates is not competitive with those of equilibrium investigations [7], the hysteresis method provides information about deviations which are rooted in the dynamics of the heating and cooling process. From our ongoing analysis of the Fortuin-Kasteleyn clusters we gain insight into the dynamics of the transition, which may help to identify signals observable in heavy ion experiments.

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