Topological Thermal Hall Effect Induced by Magnetic Skyrmions

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Quantized transports of fermions are topological phenomena determined by the sum of the Chern numbers of all the energy bands below the Fermi energy. For bosonic excitations, e.g. phonons and magnons in a crystal, topological transport is dominated by the Chern number of the lowest energy band because the energy distribution of the bosons is limited below the thermal energy. Here, we demonstrate the existence of topological transport by bosonic magnons in a lattice of magnetic skyrmions – topological defects formed by a vortex-like texture of spins¹². We find a distinct thermal Hall signal when the ferromagnetic spins in an insulating polar magnet GaV₄Se₈ form magnetic skyrmions³⁴. Its origin is identified as the topological thermal Hall effect of magnons in which the trajectories of these magnons are bent by an emergent magnetic field produced by the magnetic skyrmions. Our theoretical simulations confirm that the thermal Hall effect is indeed governed by the Chern number of the lowest energy band of the magnons in a triangular lattice of magnetic skyrmions. Our findings lay a foundation for studying topological phenomena of other bosonic excitations.
Non-trivial band topology of quasiparticles in a crystal realizes unique transport phenomena protected by the topology. In anomalous quantum Hall effect, a celebrated example of such topological transport predicted by Haldane, the dissipationless quantized current of conduction electrons is realized even in zero field owing to the topology of the electrons characterized by the finite Chern number of the energy bands. In these topological transports of fermions, such as electrons in a metal and Majorana fermions in a Kitaev magnet, they occupy all the energy states below the Fermi energy, giving rise to a quantized current depending on the sum of the Chern numbers of the occupied bands.

Bosonic excitations in crystals can also exhibit topological transport when their energy bands acquire non-trivial topology. For example, magnons – collective excitations of spins – acquire non-trivial topology by the Dzyaloshinskii-Moriya (DM) interaction in ferromagnets. A topological transport is also theoretically suggested for phonons. The topological transport of charge-neutral excitations in insulators produces a thermal Hall effect, a thermal analogue of the topological Hall effects in metals. In an insulator, owing to the absence of the Hall effect by conduction electrons, the thermal Hall conductivity in an insulator directly reflects the Berry-phase effect on the heat carriers as well as the quantum statistics that the heat carriers obey. The thermal Hall effects in insulators have been observed in various magnetic insulators including ferromagnets, kagomé antiferromagnets, spin ices and Kitaev materials.

In contrast to fermions, two or more bosons are allowed to occupy the same quantum state at low energies. Therefore, unlike fermions, the topological transport of bosons is expected to be strongly governed by the Chern number of the lowest energy band. A prominent example of such topological transport of bosons is the topological magnon Hall effect by magnetic skyrmions. Each magnetic skyrmion produces an emergent magnetic field inside the core, resulting in a topological Hall effect for conduction electrons in metals. This emergent field is also theoretically suggested to affect magnons and to realize a topological thermal Hall effect of magnons in which thermal currents carried by magnons are deflected by the magnetic skyrmions. However, magnetic skyrmions in bulk materials are typically stable only near the magnetic ordering.
temperature, disabling the measurements down to low temperatures to observe the topological contribution from the lowest energy magnon band.

Here, we demonstrate a topological thermal Hall effect of magnons realized by magnetic skyrmions in the polar magnet GaV$_4$Se$_8$. The polar structure stabilizes cycloidal spin order with the magnetic modulation vectors perpendicular to the polar axis, realizing a Néel-type skyrmion phase down to the lowest temperature by applying a magnetic field parallel to the polar axis$^{3,4}$. We find that a distinct thermal Hall conductivity emerges only in the skyrmion lattice phase but not in the forced-ferromagnetic nor in the cycloidal phase. Remarkably, $\kappa_{xy}$ does not depend on the field strength in the skyrmion phase, suggesting the topological nature of the thermal Hall effect. By theoretical calculations of the topological thermal Hall effect of the magnons in the triangular Néel-type skyrmion lattice, we indeed confirm that $\kappa_{xy}$ is determined by the Chern number of the lowest energy band. We further find the difference in $\kappa_{xy}$ at low temperatures by entering the magnetic skyrmion phase from the low-field cycloidal and from the forced-ferromagnetic phase, indicating a high sensitivity of $\kappa_{xy}$ to the skyrmion dynamics.

The lacunar spinel compound GaV$_4$Se$_8$, a magnetic insulator in which each (V$_4$Se$_4$)$_{5+}$ cluster carries $S = 1/2$ spin, undergoes a structural transition to a polar phase below $T_S = 41$ K by elongating one of the $<111>$ axes$^3$. This first-order transition is clearly seen as a jump in the temperature dependence of the longitudinal thermal conductivity $\kappa_{xx}$ (Fig. 1e). A ferromagnetic interaction in the polar phase induces magnetic order below $T_C = 18$ K (Fig. 1d). This ferromagnetic transition only slightly increases $\kappa_{xx}$ just below $T_C$ (Fig. 1e), showing a dominant phonon contribution in $\kappa_{xx}$. A competition between the ferromagnetic interaction and the DM interaction results in multiple non-collinear magnetic structures in GaV$_4$Se$_8$ below $T_C$. Previous magnetization$^3$ and elastic neutron scattering$^4$ studies have shown that a cycloidal phase at zero field turns into a Néel-type skyrmion phase in a magnetic field of $B_{c1} \sim 0.1$ T applied along the [111] axis, which is followed by a transition to the forced-ferromagnetic phase above $B_{sat} \sim 0.4$ T. One should note that the structural transition results in four crystallographic polar domains with different elongating axis. The magnetic skyrmion phase appears in one of them in which the elongating axis is parallel to the magnetic field. In the other domains, the cycloidal phase changes to a conical phase above $B_{c2} \sim 0.15$ T.

The field dependence of the magnetization $M$, $[\kappa_{xx}(B) - \kappa_{xx}(0)]/\kappa_{xx}(0)$, and $\kappa_{xy}/T$ at different temperatures is shown in Fig. 2 (see Methods and S1 in Supplementary
Information (SI)). In the paramagnetic phase above $T_C$ (Fig. 2a), $\kappa_{xx}$ monotonously increases with $B$, showing a typical field effect on phonons by suppressing spin fluctuations. Although a finite $\kappa_{xy}$ is observed at high fields (Fig. S7), $\kappa_{xy}$ is virtually absent below 0.5 T (Fig. 2a). Below $T_C$, the magnetic transitions between the multiple magnetic phases are observed by the features in $\partial M/\partial B$ at $B_{c1}$, $B_{c2}$, and $B_{sat}$ (top panels in Fig. 2) as reported in previous study\(^3\). Although $\kappa_{xy}/T$ is absent just below $T_C$ (Fig. 2b), distinct $\kappa_{xy}/T$ with an almost flat field dependence is observed at 10 K in the magnetic skyrmion phase (Fig. 2c). In contrast, $\kappa_{xy}/T$ is clearly absent both in the cycloidal and forced-ferromagnetic phases. Moreover, the effect of the other domains at $B_{c2}$ is absent in $\kappa_{xy}/T$, which is in sharp contrast to the field dependence of $\kappa_{xx}$ changing intricately at all magnetic transitions. These results demonstrate the high sensitivity of $\kappa_{xy}$ to the emergence of the magnetic skyrmion phase. As shown in Figs. 1f and 1g, the color plot of $\kappa_{xy}/T$ in the $B$–$T$ phase diagram clearly indicates the stable region of the magnetic skyrmion phase. The reproducibility of $\kappa_{xy}$ in the magnetic skyrmion phase is confirmed in the measurements of another sample (see S2 in SI). In addition, we confirm the domain volume effect on $\kappa_{xy}$ by cooling the sample in a magnetic field across $T_S$ (see S3 in SI).

We find a large hysteresis in the field dependence of $\kappa_{xy}/T$ at lower temperatures (see Figs. 2d and 2e). When entering the magnetic skyrmion phase from the forced-ferromagnetic phase (“$|B|$ down” process), $\kappa_{xy}/T$ becomes smaller below 5 K and is almost absent below 2 K. On the other hand, when entering the magnetic skyrmion phase from the cycloidal phase (“$|B|$ up” process), the field dependence $\kappa_{xy}/T$ turns to be a shoulder shape with a peak near the boundary to the forced-ferromagnetic phase. The magnitude of the peak in $\kappa_{xy}/T$ increases down to 2 K, which is followed by a rapid decrease to zero at lower temperatures (Fig. 3e). This hysteresis effect is not seen in the field dependence of $M$ and $\kappa_{xx}$ except that caused by the different phase boundary of the forced-ferromagnetic phase owing to the first-order transition nature.

Here, we discuss the origin of the thermal Hall effect observed in GaV\(_4\)Se\(_8\). In a magnetic insulator, there are four possible origins for the thermal Hall effect; (1) a conventional magnon thermal Hall effect\(^{13}\) ($\kappa_{xy}^{mag}$) (2) a conventional phonon thermal Hall effect\(^{26\text{-}30}\) ($\kappa_{xy}^{ph}$) (3) a topological magnon thermal Hall effect ($\kappa_{xy}^{top}$), and (4) skyrmion Hall effect ($\kappa_{xy}^{skr}$). In $\kappa_{xy}^{mag}$ and $\kappa_{xy}^{ph}$, the semi-classical trajectories of magnons and phonons are bent by the applied magnetic fields, due to the momentum-space topology of the bands.
By contrast, in $\kappa_{xy}^{\text{topo}}$, magnons are deflected by the emergent field produced by the magnetic skyrmions, which survives as long as magnetic skyrmions exist, regardless of the presence or absence of an external magnetic field. In $\kappa_{xy}^{\text{skr}}$, the skyrmions are transferred by the magnons as the back-action of $\kappa_{xy}^{\text{topo}}$ (Ref. 31). The magnitudes of $\kappa_{xy}^{\text{mag}}$ and $\kappa_{xy}^{\text{ph}}$ can be estimated by the thermal Hall measurements above the saturating field, showing that both $\kappa_{xy}^{\text{mag}}$ and $\kappa_{xy}^{\text{ph}}$ are negligible below $B_{\text{sat}}$ (see S4 in SI). A skyrmion Hall effect appears when the thermal current $Q$ exceeds the depinning threshold of the magnetic skyrmions. This skyrmion Hall effect works reversely to $\kappa_{xy}^{\text{topo}}$ because the depinning weakens the deflecting force on the magnons as the skyrmions are set in motion. We find that $\kappa_{xy}$ does not depend on $Q$ within the $Q$-range where we measured, showing the absence of $\kappa_{xy}^{\text{skr}}$ (see S5 in SI). From these results, we conclude that the thermal Hall effect observed in GaV$_4$Se$_8$ is dominated by $\kappa_{xy}^{\text{topo}}$ induced by the magnetic skyrmions.

The conclusion is strongly supported by our theoretical investigation of the temperature dependence of $\kappa_{xy}^{\text{topo}}$ due to the magnon-skyrmion interaction. We consider the Néel-type skyrmions (Fig. 3a) forming a triangular lattice in a two-dimensional layer and solve the bosonic tight-binding problem

$$H = -S J \sum_{\langle ij \rangle} \left( t_{ij} b_i^\dagger b_j + h.c. \right)$$

$[S = \text{size of magnetic moment}, J = \text{spin interaction energy}, b_i = \text{boson amplitude at lattice site } i ]$ where the hopping phase term $t_{ij}$ is affected by the local spin texture.

Specifically, one can write $t_{ij} = e^{2i \hat{m}(i) \cdot \hat{n}_0}$ when the local magnetic configuration varies as $\hat{n}_i = (\sin \theta_i \cos \phi_i, \sin \theta_i \sin \phi_i, \cos \theta_i)$, while $\hat{m}_i$ is given locally by $\left( \sin \frac{\theta_i}{2} \cos \phi_i, \sin \frac{\theta_i}{2} \sin \phi_i, \cos \frac{\theta_i}{2} \right)$, and $\hat{n}_0 = (0,0,1)$. One chooses $\hat{n}_i$ to be that of the triangular lattice of the Néel-type skyrmions and diagonalize the bosonic Hamiltonian (see S6 in SI).

The lowest ten magnon bands are presented in Fig. 3d. The Berry curvature of the $n$-th band $\Omega_n(\vec{k})$ can be evaluated using the standard formula. Integrating the Berry curvature over the Brillouin zone gives the Chern number $C_n = \frac{1}{2\pi} \int_{BZ} \Omega_n(\vec{k})$ of the $n$-
th band. In our calculation, the lowest band is topologically non-trivial with a Chern number $C_1 = +1$ as specified in Fig. 3d, while the second and third bands are trivial with zero Chern number (A full list of Chern numbers for all the bands is given in S6 in SI). Now, we compute $\kappa_{xy}^{\text{topo}}$ using Eq. (1), and the result is presented in Fig. 3e as a function of temperature. We adopt the mean-field-like temperature dependence of the magnetization, $S = \sqrt{1 - T/T_c}$, and set the critical temperature $T_c \sim J$ (the entire magnon band width is about 10J). To compare the calculation with the experimental data, we also estimate $\kappa_{xy}^{\text{topo}}$ of the two-dimensional layer of the magnetic skyrmions ($\kappa_{xy}^{2D}$) from the experimental data at a fixed field by $\kappa_{xy}^{2D} = \kappa_{xy} d$, where $d = 0.5854$ nm is the interlayer distance (Fig. 1a).

As the temperature increases, the magnon bandwidth effectively decreases by $\sqrt{1 - T/T_c}$, whereas the thermal factor $k_B T$ begins to weigh the higher-band contributions more and more. The sum of all Chern numbers is zero, $\Sigma_n C_n = 0$, and consequently $\kappa_{xy}^{\text{topo}}$ must vanish at a sufficiently high temperature. Surprisingly, we find a nearly complete suppression of $\kappa_{xy}^{\text{topo}}$ at temperatures above $0.7T_c$ and a peak at about $0.2T_c$, showing a good agreement with the experimental result (Fig. 3e). Moreover, notice that $\kappa_{xy}^{\text{topo}}$ peaks around the temperature comparable to the energy of the lowest magnon band, proving that topological magnons from the lowest band are responsible for the temperature dependence of $\kappa_{xy}^{\text{topo}}$ up to the peak temperature. To be more precise, we present the Berry curvature of the lowest and second-lowest bands in Fig. 3b and Fig. 3c, respectively. It is always positive and peaked at $K$ point in the lowest band (Fig. 3b), while negative peaks appear at $\Gamma$ and $K$ points in the second band (Fig. 3c). Considering the proximity of the magnon energies at the $K$ point for the two bands as shown in Fig. 3d, we conclude that as $k_B T$ begins to exceed the peak temperature, the negative Berry curvature contribution from the second magnon band begins to undermine the lowest-band contribution to $\kappa_{xy}^{\text{topo}}$.

As shown in Fig. 3e, whereas $\kappa_{xy}/T$ observed in the demagnetizing process agrees well to the theoretical simulation, that in the magnetizing process shows a peak at a clearly lower temperature, implying a lower energy of the lowest magnon band. Meanwhile, as shown in the top panels in Fig. 2, $M$ does not show a difference in the two processes, showing no difference in the number of the magnetic skyrmions. Therefore, the difference in $\kappa_{xy}$ suggests that the lattice structure of the magnetic skyrmions depends on the magnetizing process of forming them from the forced-ferromagnetic phase or from the cycloidal phase. For example, if the distribution of the skyrmions is non-uniform, the
energy of the lowest magnon band may become lower owing to a different size of the Brillouin zone. In general, a more regular skyrmion lattice is expected in the magnetizing process in which the cycloidal phase changes to the magnetic skyrmion phase, because the magnetic skyrmions are formed by cutting the down-spin stripes that are regularly arranged in the helix. On the other hand, the skyrmion lattice is presumed to be irregular in the demagnetizing process because magnetic skyrmions would be more randomly formed from the forced-ferromagnetic phase. However, our data implies that the skyrmion lattice formed in the demagnetizing process is well approximated to the triangular lattice than that by the magnetizing process. This inconsistency requires further experimental and theoretical studies to study the detailed mechanism about the formation of the skyrmion lattice.

Our thermal Hall measurements demonstrate a distinct example of a topological thermal transport of bosons in a magnetic insulator. These results will pave the way to realize potential applications of nondissipative magnonics and microspintronics, as well as further investigations to find a topological transport of other charge-neutral bosons. Moreover, we find that $\kappa_{xy}^{\text{topo}}$ reflects a difference of the lattice structure of the magnetic skyrmions in insulators, prompting new studies for the magnetic skyrmions in insulators.
Figures and legends:

Fig. 1 Magnetic skyrmion phase in GaV₄Se₈. **a, b**, Crystal structure of GaV₄Se₈ viewed parallel (**a**) and perpendicular (**b**) to the plane of (V₄Se₄)⁵⁺ clusters. The arrow in (**a**) shows the inter-layer distance $d$. **c**, An illustration of the experimental setup. One heater and three thermometers ($T_H$, $T_{L1}$, and $T_{L2}$) are attached to the sample fixed on the LiF heat bath (see Methods). **d, e**, The temperature dependence of the magnetization ($M$) and the longitudinal thermal conductivity ($\kappa_{xx}$). The structural transition temperature ($T_S$) and the magnetic ordering temperature ($T_C$) are shown. **f, g**, Color plots of the thermal Hall conductivity ($\kappa_{xy}/T$) measured in the magnetizing ($|B|$ up, **f**) and demagnetizing ($|B|$ down, **g**) procedure. Four magnetic phases – paramagnetic (Para), cycloidal (Cyc), magnetic skyrmion (SkX), and forced-ferromagnetic (FFM) phases – are indicated. The red solid lines and black circles show the magnetic phase boundary determined by the previous measurements^{3} and by the magnetization measurements (top panels of Fig. 2), respectively.
Fig. 2 Thermal Hall effect by magnetic skyrmions. Top, middle, and bottom panels show the field dependence of the magnetization ($M$), the field dependence of the longitudinal thermal conductivity ($\kappa_{xx}(B)$) normalized by the zero-field value ($\kappa_{xx}(0)$), and the thermal Hall conductivity divided by the temperature ($\kappa_{xy}/T$), respectively. The field differential of the magnetization $\partial M/\partial B$ is shown in arb. unit in the top panels (solid lines). The data obtained in the magnetizing ("$|B|$ up") and that in the demagnetizing ("$|B|$ down") process are shown in red and blue, respectively. For clarity, the data in the middle panels is multiplied by a constant indicated in the panel.
Fig. 3 Theoretical calculations of topological magnon Hall effects. a, a schematic illustration of a spin texture of a Néel-type skyrmion. b, c, Color plots of the Berry curvature of the lowest (b) and the second lowest (c) energy bands of the magnons in a triangular lattice of the Néel-type magnetic skyrmions. See S6 in SI for details. d, The energy bands along the high-symmetry points. The color denotes the Berry curvature that is normalized in each band. e, The temperature dependence of $\kappa_{xy}^{2D}/T$ (left) and $M$ (right) obtained by the calculation (solid lines) and the experiments (symbols). The experimental data obtained at 0.36 T in the magnetizing (“$|B|$ up”) and that at 0.2 T in the demagnetizing (“$|B|$ down”) process are shown in red and blue circles, respectively. The data of $\kappa_{xy}^{2D}/T$ in the DR measurements are multiplied by 0.5 to include the domain volume effect (see S3 in SI). Then, the experimental values of $\kappa_{xy}^{2D}/T$ are multiplied by 10 and the horizontal axis is normalized by $T_C = 20$ K to fit with the theoretical result.
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Materials and Methods

Single crystals of GaV₄Se₈ were synthesized by the chemical vapor transport method as described in the previous paper³.

Magnetization was measured by using a magnetic property measurement system (MPMS, Quantum Design) under a magnetic field applied along the [111] axis.

The thermal-transport measurements were performed by the steady-state method as described in Refs. 14–17 by using a variable temperature insert (VTI, 2–60 K, 0–15 T) and in a dilution refrigerator (DR, 0.15–4 K, 0–14 T). One heater and three thermometers were attached to the sample by using a silver paste, and then the temperature difference ΔTₓ (ΔTₓ = Tₓ − Tₓ₋₁) and ΔTᵧ (ΔTᵧ = Tᵧ₁ − Tᵧ₂) were measured as a function of the heat current Q (Fig. 1(c)). To cancel the longitudinal component in ΔTᵧ by the misalignment effect, ΔTᵧ is asymmetrized with respect to the field direction as ΔTᵧᵦᵦ = ΔTᵧ(+B) − ΔTᵧ(−B). The thermal (Hall) conductivity κₓₓ (κₓᵧ) is derived by

\[
\left( \frac{Q}{wt} \right) = \begin{pmatrix} \kappaₓₓ & \kappaₓᵧ \\ -\kappaₓᵧ & \kappaₓₓ \end{pmatrix} \begin{pmatrix} \frac{ΔTₓ / L}{ΔTᵧᵦᵦ / w'} \end{pmatrix},
\]

where t is the thickness of the sample, L is the length between Tₓ and Tₓ₋₁, and w is the sample width between Tₓ₋₁ and Tₓ₋₂, and w' is the length between Tᵧ₁ and Tᵧ₂. A typical field dependence of ΔTᵧ at a fixed temperature is shown in Fig. S1. To investigate a difference of κₓᵧ by entering the skyrmion phase from the low-field cycloidal phase and from the high-field forced-ferromagnetic phase, we measured the field dependence of ΔTᵧ both in the magnetization and demagnetization process. We then antisymmetrized ΔTᵧ for each magnetizing and demagnetizing process (see Fig. S1b and Fig. S1c) to obtain the field dependence of κₓᵧ/T shown in Fig. 2.

We confirmed the reproducibility of the data by performing the magnetization and the thermal-transport measurements of two samples (sample 1 and 2). All the data shown in the main text are obtained from sample 1. The data of sample 2 is described in S2 in SI.

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Author Contributions:
T.A. and M.Y. conceived the project. Y.F., Y.T., and T.A. prepared the samples. M.A., H.T. and M.Y. performed the thermal-transport measurements. M.A., H.T. Y.T., and T.A. performed magnetization measurements. H.-Y.L. and J.H.H. performed the theoretical calculations. All authors discussed the experimental and theoretical results. M.Y. prepared manuscripts based on the discussions with all coauthors.
Supporting Online Material

S1. Raw data of the transverse temperature difference $\Delta T_y$

Here we explain the procedure of the thermal Hall measurement. To investigate a difference of the thermal Hall effect by entering the skyrmion phase from the low-field cycloidal phase and from the high-field forced-ferromagnetic phase, we measured the field dependence of the transverse temperature difference, $\Delta T_y$, both in the magnetization and demagnetization process under the positive and the negative fields (Fig. S1a). We then antisymmetrized the data with respect to the field direction for each the magnetizing (red and orange data in Fig. S1b) and the demagnetizing (blue and cyan data in Fig. S1b) process to obtain $\Delta T_y^{asym}$ for both procedures (Fig. S1c).

Fig. S1 The field dependence of the transverse temperature difference $\Delta T_y = T_{L1} - T_{L2}$.

a, The field procedure of the thermal Hall measurement. b, The field dependence of $\Delta T_y / Q$ of sample 1 observed at 10 K. c, The field dependence of $\Delta T_y^{asym} / Q$ obtained for magnetizing (red) and demagnetizing (blue) process.

S2. Sample dependence of the thermal Hall effect

Figures S2 to S4 show the data of sample 2. We confirmed the reproducibility of the thermal Hall measurements of sample 1 whereas the magnitude of $\kappa_{xy}$ of sample 2 is smaller than that of sample 1 (Fig. S4). This small thermal Hall signal is attributed to a smaller fraction of the domain volume of the magnetic skyrmion phase. This smaller fraction of the skyrmion domain volume can be seen in the smaller increase of $M$ at $B_{c1}$ of sample 2 (top panels in Fig. S2) than that in sample 1 (see Fig. 2 in the main text). The relation between the skyrmion domain volume and $\kappa_{xy}$ is confirmed by field cooling, as discussed in the next section.
**Fig. S2 a-e.** The data of sample 2. The field dependence of the magnetization ($M$, top panels), the field dependence of the longitudinal thermal conductivity [$\kappa_{xx}(B)$] normalized by the zero-field value [$\kappa_{xx}(0)$] (mid panels), and the thermal Hall conductivity divided by the temperature ($\kappa_{xy}/T$, bottom panels) at different temperatures. The field differential of the magnetization $\partial M/\partial B$ is shown in arb. unit in the top panels (solid lines). The data obtained in the magnetizing (“$|B|$ up”) and that in the demagnetizing (“$|B|$ down”) process are shown in red and blue, respectively. For clarity, the data in the middle panels is multiplied by a constant indicated in the panel.

**Fig. S3 a,b** Color plots of the thermal Hall conductivity divided by the temperature
(\(\kappa_{xy}/T\)) of sample 2 measured in the magnetizing (\(|B|\) up, a) and demagnetizing (\(|B|\) down, b) procedure. The red solid lines show the magnetic phase boundary determined by the previous measurement\(^3\). The black circles show the phase boundary of sample 2 determined by the magnetization measurement (top panels of Fig. S2).

![Graph showing temperature dependence of \(\kappa_{xy}/T\) for samples 1 and 2.](image)

**Fig. S4** The temperature dependence of \(\kappa_{xy}/T\) at 0.36 T (\(|B|\) up) and that at 0.2 T (\(|B|\) down, blue circles) of sample 1 and sample 2. The DR data of sample 1 is multiplied by 0.5 as described in the next section. The data of sample 2 is multiplied by 5 for a comparison.

**S3. Domain volume effect on the magnitude of the thermal Hall conductivity**

In GaV\(_4\)Se\(_8\), four crystallographic polar domains with a different elongating axis are formed below the structural transition temperature \(T_s = 41\) K. The magnetic skyrmion phase appears only in one of them for \(B_{c1} < B < B_{sat}\) in which the elongating axis is parallel to the applied field (denoted as “skyrmion domain”) whereas the cycloidal phase in the other domains changes to a conical phase at \(B_{c2}\). Therefore, the magnitude of \(\kappa_{xy}\) is expected to depend on the fraction of the skyrmion domain volume.

In the thermal-transport measurements of sample 1, we find that the magnitude of \(\kappa_{xy}\) is increased in the DR measurements (triangles in Fig. S5) performed after the VTI measurements. As shown in Fig. S5a, the field dependence of \(\kappa_{xy}/T\) at 2 K is almost twice as large as that in the VTI at the same temperature, showing that the fraction of the skyrmion domain volume is doubled in this cooling process. To take into account this domain size effect, all the DR data of sample 1 shown in the main text is multiplied by
0.5 (see also Fig. S6c). We then multiply a constant to the experimental data shown in Fig. 3e, to fit with the theoretical result. Note that, data of the DR measurements in the demagnetization process are not shown in Fig. 3e, because the appearance of the skyrmion signal is no longer discernible in the field dependence of $\kappa_{xy}$ below 2 K as shown in Fig. S5.

**Fig. S5 a-d**, The field dependence of $\kappa_{xy}/T$ of sample 1 in the DR measurements (triangles). The magnitude of $\kappa_{xy}/T$ is almost twice as large as that observed in the previous VTI measurements (circles).

To investigate the domain volume effect on $\kappa_{xy}$, we compare the field dependence of $M$ and $\kappa_{xy}$ of sample 2 by cooling through $T_S$ under the zero field (ZFC) and a constant field (FC). In the field dependence of $M$, the skyrmion domain shows a step-like increase of $M$ both at $B_{c1}$ and $B_{sat}$. On the other hand, $M$ in the other domains increases in proportional to $B$ with a decrease of the slope at $B_{c2}$. As shown in Fig. S6a, the field dependence of $M$ in the FC process shows a smaller slope of $M$ below $B_{c1}$. In addition, increases of $M$ both at $B_{c1}$ and $B_{sat}$ become sharper by the field cooling. These results indicate a larger fraction of the skyrmion domain volume formed in the FC process. We then checked this field-cooling effect on $\kappa_{xy}$ of sample 2 in the 2nd run of the VTI measurements (Fig. S6b). As shown in Fig. S6b, whereas $\kappa_{xy}/T$ in the ZFC process is similar both in 1st and 2nd run, $\kappa_{xy}/T$ in FC process becomes about three times larger than that in ZFC process. These enhancements of $M$ and $\kappa_{xy}/T$ by FC process demonstrate that the skyrmion domain volume can be increased by field cooling through $T_S$, and that the magnitude of $\kappa_{xy}/T$ reflects the skyrmion domain volume.
Fig. S6 a,b, The comparison of the field dependence of $M$ (a) and $\kappa_{xy}/T$ (b) of sample 2 in the zero-field cooling (ZFC) and field cooling (FC) at $5\,K$. The field differential of the magnetization $\partial M/\partial B$ is shown in arb. unit in a. In the FC process, the constant magnetic field of 7 T and 15 T is applied when cooling through $T_s$ for $M$ and $\kappa_{xy}/T$ measurements, respectively. c, The temperature dependence of $\kappa_{xy}/T$ at 0.36 T ($|B|$ up) and that at 0.2 T ($|B|$ down) measured in the VTI measurements (filled circles) and the DR measurements (filled triangles) of sample 1. The data of sample 2 (multiplied by 2 for a clarity) are obtained in the 1st ZFC, 2nd ZFC and FC process (diamonds).

S4. Estimation of the conventional thermal Hall effects of phonons and magnons

Here we describe the thermal Hall measurements up to high fields and the estimation of the conventional thermal Hall effect by phonons ($\kappa_{xy}^{ph}$) and magnons ($\kappa_{xy}^{mag}$) in the magnetic skyrmion phase.

The field dependence of $\kappa_{xy}/T$ of sample 1 up to 15 T ($\gg B_{sat}$) is shown in Fig. S7. In the forced ferromagnetic phase above $B_{sat}$, both $\kappa_{xy}^{ph}$ and $\kappa_{xy}^{mag}$ are expected to appear. As discussed in Ref. 13, the field dependence of $\kappa_{xy}^{mag}$ is mainly given by a polylogarithmic function of $e^{-g\mu_B H/kT}$, where $g$ is the g-factor and $\mu_B$ the Bohr magneton. Therefore, $\kappa_{xy}^{mag}$ shows a peak near $B = 0$ which is followed by a monotonic decrease at higher fields. Therefore, as shown in Fig. 2 in the main text, $\kappa_{xy}$ is virtually absent or very small both in the cycloidal phase ($0 < B < B_{c1}$) and in the forced-ferromagnetic phase just above $B_{sat}$, showing that $\kappa_{xy}^{mag}$ is negligible in the magnetic skyrmion phase of GaV$_4$Se$_8$.

On the other hand, given the large energy scale of the Debye energy, $\kappa_{xy}^{ph}$ is expected to
be linear in field. In fact, as shown in Fig. S7, $\kappa_{xy}$ at high fields show an almost linear field dependence. However, extrapolating $\kappa_{xy}^{ph}$ at high fields to the field region of the magnetic skyrmion phase results in a negligible $\kappa_{xy}^{ph}$ in the magnetic skyrmion phase. We can thus safely conclude that both $\kappa_{xy}^{ph}$ and $\kappa_{xy}^{mag}$ are negligible in the magnetic skyrmion phase.

**Fig. S7 a-d.** The field dependence of $\kappa_{xy}/T$ of sample 1 up to 15 T at 2 K (a), 8 K (b), 12 K (c), and 20 K (d).

**S5. The thermal current dependence of the thermal Hall effect**

Here, we show the thermal current ($Q$) dependence of the thermal Hall effect in the magnetic skyrmion phase. As discussed in the main text, when the magnetic skyrmions are depinned by applying a large $Q$, the thermal Hall effect is suppressed by the skyrmion Hall effect ($\kappa_{xy}^{sky}$). To observe the skyrmion Hall effect, we applied $Q$ as large as possible
at several temperatures in the magnetic skyrmion phase. However, as shown in Fig. S8, only the linear $Q$ dependence of $\Delta T_{y}^{asym}$ is observed. We conclude that the skyrmion Hall effect is absent in GaV$_4$Se$_8$ within the $Q$ range that we used, owing to a strong pinning of the magnetic skyrmions.

The absence of $\kappa_{xy}^{skr}$ is also supported by the field dependence of $\kappa_{xx}$. The itinerant magnetic skyrmions are expected to move in the opposite direction to the thermal current$^{23}$. Therefore, $\kappa_{xx}$ would decrease when the magnetic skyrmions are depinned. However, $\kappa_{xx}$ increases in the magnetic skyrmion phase as shown in Fig. 2 in the main text.

![Fig. S8](image.jpg)

**Fig. S8** The thermal current dependence of the thermal Hall effect of sample 1 at 10 K.

**S6. Details of the theoretical calculation.**

In this section, we discuss the effective magnon Hamiltonian and provide detailed information for simulation. For convenience, we rotate the spin quantization axis spatially such that the local spin configuration, $\vec{n}_i = (\sin \theta_i \cos \phi_i, \sin \theta_i \sin \phi_i, \cos \theta_i)$, becomes aligned with $\vec{n}_0 = (0,0,1)$. The rotation of the frame gives rise to an additional vector potential or emergent gauge field to the spin-wave excitation leading to the topological magnon modes$^2$. Taking into account the emergent potential, a minimal magnon Hamiltonian reads as $H = -S J \sum_{ij} (t_{ij} b_i^\dagger b_j + h.c.)$ where $S$ is the size of spin, $\langle ij \rangle$ denotes the pair of nearest-neighbor sites $i$ and $j$ on the triangular lattice. Since the magnons sense the skyrmion as a two-flux quanta effectively, a phase $2(\vec{m}_i \times \vec{m}_j) \cdot \vec{n}_0$ is accumulated while it hops on neighboring sites.$^2$ The skyrmion
deflects the magnon and electron in the same direction\textsuperscript{31}, and thus leads to positive overall sign of the phase. After all, it is reasonable to define the hopping phase term as \( t_{ij} = e^{2i(\vec{m}_i \times \vec{m}_j) \cdot \vec{n}_0} \) with \( \vec{m}_i = \left( \sin \frac{\theta_i}{2} \cos \phi_i, \sin \frac{\theta_i}{2} \sin \phi_i, \cos \frac{\theta_i}{2} \right) \).\textsuperscript{2} After the Fourier transformation into the momentum space, the Hamiltonian is recast as

\[
H = -SJ \sum_{\vec{k} \in \text{BZ}} \sum_{n=1}^{N_u} \sum_{m=1}^{N_u} \left[ t_{nm} e^{i\vec{k} \cdot (\vec{r}_n - \vec{r}_m)} + c.c. \right] b_{kn}^\dagger b_{km},
\]

where \( N_u \) stands for the number of sites in the unit-cell and \( \vec{r}_n \) for the position of site \( n \) in the magnetic unit-cell. By diagonalization of the \( N_u \times N_u \)-matrix, one acquires the magnon spectrum \( \varepsilon_{nk} \) for a given momentum \( \vec{k} \). In our simulation, we employ a 64-site unit-cell of the Néel-type skyrmion lattices, and thus 64 magnon bands appear in the Brillouin zone. Note that the Hamiltonian above does not need to be positive-definite while it should be with a full consideration of the spin interactions and magnetic field. We therefore add a constant to the spectrum to guarantee the positive-definite spectrum with a tiny gap. To be more concrete, the gap is set to be 0.1% of the entire band.

The Berry curvature of the \( n \)-th band can be evaluated by the following formula\textsuperscript{35}:

\[
\Omega_n(\vec{k}) = -2 \sum_{m \neq n} \frac{\text{Im}[\langle n, \vec{k} | v_x | m, \vec{k} \rangle \langle m, \vec{k} | v_y | n, \vec{k} \rangle]}{(\varepsilon_{nk} - \varepsilon_{mk})^2},
\]

where \( v_y = \partial H / \partial k_y \). The Berry curvature distribution of the lowest and second-lowest bands in the momentum space are shown in Fig. 3b and Fig. 3c in the main text, respectively. It should be noted that our color plot of the Berry curvature has the slight distortion in the momentum space owing to a small and local variance in the skyrmion we employed. Integrating the Berry curvature over the Brillouin zone, one obtains the Chern number of the \( n \)-th band, \( C_n = \frac{1}{2\pi} \int_{\text{BZ}} \Omega_n(\vec{k}) \). The full list of \( C_n \) from the lowest to top band is shown in Table 1. We then calculate \( \kappa_{xy} \) by using Eq. (1) in the main text with

\[
f(E) = -c_2 \left[ n_B \left( \frac{E}{k_B T} \right) \right],
\]

where \( c_2(x) = (1 + x) \left( \ln \frac{1+x}{x} \right)^2 - (\ln x)^2 - 2 \text{Li}_2(-x) \), \( n_B(x) = (e^x - 1)^{-1} \) the Bose-Einstein distribution function, and \( \text{Li}_2 \) is the polylogarithm function\footnote{11}.
We note that, although other models for magnon bands in the presence of skyrmion texture exist and give different Chern number distributions of the magnon bands\textsuperscript{25}, they do not give as good a description of our experimental data as the model adopted in this paper.

**Table 1** Full list of the Chern number $C_n$ of the magnon bands in the lattice of the magnetic skyrmions.

| $n$ | $C_n$ | $n$ | $C_n$ | $n$ | $C_n$ | $n$ | $C_n$ |
|-----|-------|-----|-------|-----|-------|-----|-------|
| 1   | 1     | 17  | 10    | 33  | 1     | 49  | -4    |
| 2   | 0     | 18  | 1     | 34  | 3     | 50  | 2     |
| 3   | 0     | 19  | -4    | 35  | -1    | 51  | 3     |
| 4   | -1    | 20  | 0     | 36  | -6    | 52  | 3     |
| 5   | 3     | 21  | -2    | 37  | 2     | 53  | 0     |
| 6   | -3    | 22  | 5     | 38  | 1     | 54  | 5     |
| 7   | 3     | 23  | 7     | 39  | -1    | 55  | 2     |
| 8   | -1    | 24  | -8    | 40  | -1    | 56  | 2     |
| 9   | 1     | 25  | 3     | 41  | 6     | 57  | -4    |
| 10  | 0     | 26  | -1    | 42  | -4    | 58  | 2     |
| 11  | -3    | 27  | 0     | 43  | -2    | 59  | 2     |
| 12  | 9     | 28  | 3     | 44  | 2     | 60  | 0     |
| 13  | -5    | 29  | -6    | 45  | 0     | 61  | 1     |
| 14  | -4    | 30  | 11    | 46  | -4    | 62  | -1    |
| 15  | 0     | 31  | -7    | 47  | -2    | 63  | 0     |
| 16  | -3    | 32  | -2    | 48  | -2    | 64  | -2    |