Incomplete vs. Complete Fusion at E/A ≈ 4-7 MeV

Pushpendra P. Singh1, Abhishek Yadav2, Vijay R. Sharma2, Devendra P. Singh2, Unnati Gupta2, Manoj K. Sharma3, R. Kumar4, K. S. Golda4, R. P. Singh1, S. Muralithar4, B. P. Singh2, R. K. Bhowmik4, R. Prasad2

1INFN - Laboratori Nazionali di Legnaro, I-35020 Legnaro, Italy
2Accelerator Laboratory, Department of Physics, A. M. University, Aligarh-202 002, India
3Physics Department, S. V. College, Aligarh-202 001, India
4NP-Group, Inter-University Accelerator Center, New Delhi-110 067, India

E-mail: pushpendrapsingh@gmail.com

Abstract. With a view to study onset and strength of incomplete fusion at low projectile energies (i.e., ≈ 4-7 MeV/nucleon) three sets of experiments have been performed in 12C, 16O+169Tm systems. In first set of experiments, spin-distributions and feeding intensity profiles for xn,αxn/2αxn-channels have been measured to figure out associated ℓ-values. The spin-distributions for direct-α-emitting channels (associated with incomplete fusion) have been found to be distinctly different than that observed for fusion-evaporation (complete fusion) channels. The mean value of driving input angular momenta associated with direct-α-emitting-channels have been found to be higher than that observed for fusion-evaporation xn/α-emitting-channels, and increases with direct-α-multiplicity in forward cone. The second set of experiments has been performed to understand influence of incomplete fusion on complete fusion at these energies. Incomplete fusion strength function has been deduced from the analysis of experimental excitation functions. The third set of experiments deals with the validation of data reduction procedure used to deduce incomplete fusion fraction, and to confirm the fusion incompleteness at slightly above barrier energies. Forward-recoil-ranges of heavy reaction products have been measured and analysed on the basis of break-up fusion model. More than one linear-momentum-transfer components associated with full- and/or partial-fusion of projectile with target nucleus have been observed. Experimental ranges of forward-recoils are found to be in good agreement with that estimated using range-energy formulation. The relative strengths of complete and incomplete fusion components deduced from the analysis of forward-recoil-ranges and excitation functions complement each other. Result presented in this paper conclusively demonstrate substantial incomplete fusion contribution at energy as low as 7% above the barrier.

1. Introduction

Dynamics of incomplete fusion has been intensively investigated in recent years using light heavy ion (A≤16) beams at low projectile energies (i.e., ≈ 4-7 MeV/nucleon), where complete fusion (CF) supposed to be the sole contributor [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. At these energies, the interaction trajectories lying within the target dimensions prominently lead to the reaction modes namely: (i) CF: defined as the capture of entire projectile by target nucleus, and (ii) IF: where only a part of projectile fuses with target nucleus. In a qualitative way, both reaction modes can be disentangled on the basis of driving angular momenta at different interaction trajectories (ℓ-bins) [15, 16, 17]. Central and/or near-central trajectories (0 ≤ ℓ ≤
lead to CF, where an excited composite system forms after intimate contact and transient amalgamation of projectile and target nucleus. In this case, the attractive nuclear potential influences the sum of repulsive Coulomb and centrifugal potentials. Eventually, the projectile’s kinetic energy and driving angular momenta are equally distributed among all the accessible internal degrees of freedom of composite system to form an equilibrated compound nucleus (CN). However, at relatively higher \( \ell \)-values (\( \geq \ell_{\text{crit}} \)) imparted in the entrance channel due to non-central interactions) centrifugal potential overwhelms attractive nuclear potential, therefore, the pocket in the entrance channel potential disappears. As a consequence, fusion of entire projectile (CF) hinders and gives way to fusion incompleteness. In this case, a part of projectile emits as a spectator (P\(^s\)) to release excess driving angular momenta. After such an emission, the remnant (participant: P\(^p\)) supposed to have effective driving angular momenta less than or equal to its own critical limit (\( \ell_{\text{eff}} \leq \ell_{\text{crit}}^{\text{pr}+T} \)) for fusion to occur, which leads to an incompletely fused composite (IFC), and direct projectile-like-fragments (PLFs) centred in forward cone. The CN formed via CF is expected to have pre-determined mass/charge, excitation energy and angular-momenta. However, the IFC system (reduced CN) forms with relatively less mass/charge and excitation energy (due to partial fusion of projectile), but at high angular-momenta (imparted due to non-central interactions) as compared to the CN formed via CF.

After first experimental observation of direct PLFs (in massive transfer reactions) by Britt and Quinton [18] and Galin et al.[19], several dynamical models have been proposed to explain ICF dynamics. In SUMRULE model, Wilczyński et al.[16, 17], suggested that ICF is mainly confined to the \( \ell \)-space above \( \ell_{\text{crit}} \) for CF, and originates from peripheral interactions or non-central collisions. The non-central nature of ICF has been emphasized by Trautmann et al.[20], and Inamura et al.[21]. The Break-up Fusion (BF)-model [22] of Udagawa and Tamura is based on the Distorted Wave Born Approximation (DWBA), in which the projectile is assumed to break-up into constituent \( \alpha \)-clusters (e.g., \(^{16}\text{O}\) may break-up into \(^{12}\text{C}+\alpha\) and/or \(^{8}\text{Be}+^{8}\text{Be}\)) within the nuclear field of target nucleus. One of the fragments may get fuse with target nucleus and remnant behaves like a spectator (ejected in the forward cone). According to Promptly Emitted Particles (PEP)-model [23], the nucleons transferred from projectile to the target nucleus may get accelerated in the nuclear field of target nucleus and consequently acquire extra velocity to escape before equilibration. Further, Morgenstern et al. [24], correlated the onset of ICF with relative velocity (\( v_{\text{rel}} \)) where ICF found to be contributed significantly above \( v_{\text{rel}} \approx 0.06 \) (6 \% speed of light). In a review, Gerschel[25] inferred that the localization of \( \ell \)-window also depends on the target deformation. Despite a variety of existing studies (see ref.[9, 10, 14] for detail), the dynamics of ICF at these energies is still not fairly understood, thus continues to be an active area of investigations. The most debated issues related to ICF are; (i) the localization of \( \ell \)-values, and (ii) the onset and influence on complete fusion at low projectile energies. In order to have an insight into the aforementioned issues and to better understand ICF dynamics, three sets of experiments have been performed [6, 7, 8, 9, 10, 11]. The experimental details, obtained results and their interpretations are discussed in section-2. The summary and conclusions of the present work are given in the last section of this paper.

2. Experimental details, results and interpretations

Experiments have been carried out at the Inter-University Accelerator Center (IUAC), New Delhi, using different setups and techniques. Some of the interesting results along with a short account of experimental conditions are given in the following sub-sections. In first set of experiments, particle-\( \gamma \)-coincidence technique has been employed for spin-distribution measurement [6, 7, 8]. A brief account of experimental conditions related to spin-distribution measurement are given in section-2.1, where the localization of \( \ell \)-values in different reaction products is discussed. For second and third sets of experiments, the novel activation technique has been used where ICF strength function [9, 11], and forward-recoil-ranges (FRRs) [10] of
heavy recoils have been measured. An approach to deduce ICF strength function from the analysis of experimental excitation functions is given in section 2.2. While, the validation of this approach through the analysis of experimental FRRs is discussed in section 2.3. Detailed discussion on the results obtained from these experiments can be found in refs. [6, 7, 8, 9, 10, 11].

2.1. spin-distributions: a sensitive probe for incomplete fusion dynamics

Aiming to probe the role of high $\ell$-values in the onset of ICF at low projectile energies, the spin-distributions (SDs) of xn/pxn/\alpha n/2\alpha n-channels have been measured in $^{12}$C,$^{16}$O+$^{169}$Tm systems at energies $\approx 4$-7 MeV/nucleon [6, 7, 8]. In this work, particle-\gamma coincidence technique has been employed for reaction channel selection. The particle-\gamma-coincidences have been recorded using Gamma Detector Array (GDA) and Charged Particle Detector Array (CPDA) setup. The GDA is an assembly of 12 Compton suppressed, high resolution HPGe-detectors installed at angles 45$^\circ$, 99$^\circ$, 153$^\circ$ with respect to the beam axis and there are 4 detectors at each of these angles. The CPDA is a set of 14-phoswich detectors housed in a 14-cm diameter scattering chamber, covering nearly 90% of the total solid angle. All 14 detectors of CPDA have been divided into the angular rings; (i) Forward angle (F) 10$^\circ$ – 60$^\circ$, (ii) Sideways (S) 60$^\circ$ – 120$^\circ$ and (iii) Backward angle (B) 120$^\circ$ – 170$^\circ$. Depending on the fast and slow components of the CPDA, proton and \alpha-particles in each angular ring can be identified. Isotopically pure, self-supporting $^{169}$Tm (100%) target of thickness $\approx 1$ mg/cm$^2$ (prepared by rolling technique) has been bombarded with $^{12}$C$^+$ ($E$=54-90 MeV) and $^{16}$O$^+$ ($E$=90 MeV) beams delivered from 15UD-Pelletron Accelerator. CPD’s were covered by Al-absorbers of appropriate thicknesses to remove the scattered beam. At forward angles (F) 10$^\circ$ – 60$^\circ$, the detectors are supposed to detect both; (i) fusion-evaporation (CF) \alpha-component of average energy i.e., $E_{\alpha}$ (evaporation) $\approx$ 18 MeV, and (ii) ICF ‘fast’ \alpha-component belonging to the same velocity as that of incident projectile, e.g., $E_{\alpha}$ = 22.5 MeV in case of 90 MeV $^{16}$O beam. In order to cut-off fusion-evaporation \alpha-particles from forward cone, an additional Al-absorber of appropriate thickness has been kept on forward angle (F) 10$^\circ$ – 60$^\circ$ CPD’s so that only ICF ‘fast’ \alpha-component ($E$ $\geq$ 18 MeV) can be detected in forward cone. In-beam prompt \gamma-ray spectra have been recorded in event-by-event multi parameter mode, which includes different coincidences like; \alpha and 2\alpha detected in backward (B), forward (F) and 90$^\circ$ (S)-angles. Singles data have also been collected to identify xn-channels (produced via CF). Data analysis has been performed in two steps. In first step, spin distributions of xn-channels have been measured by looking into singles spectra. For the identification of pxn-channels, backward(B)-\alpha-gated spectra has been subtracted from backward(B)-\alpha-particles (Z=1,2)-gated spectra to achieve proton-gated spectra. However, \alpha n/2\alpha n(CN-\alpha)-channels produced via CF have been identified from the backward(B)-\alpha-gated spectra. Further, as per the definition of ICF, the fast-\alpha-particles (particles of the order of projectile velocity) are expected to be emitted only in forward cone (F). As such, \alpha n/2\alpha n(Direct-\alpha)-channels produced via ICF have been identified from forward(F)-\alpha-gated spectra. The intensity and area under the photo-peak (efficiency corrected) of the characteristic prompt \gamma-transitions assigned to the particular reaction products were used to determine the relative production yield.

As a matter of fact, the relative number of statistical and ‘yrast’-like transitions depend on entry state angular momenta and available excitation energy (E*). The CF reaction products are formed at high E* and low angular momenta leading to more statistical transitions, where ‘yrast’ states are expected to fed by statistical \gamma-transitions. However, the ICF reaction products achieve low E* (due to the involvement of partial degrees of excitations) and high angular momenta (relatively higher values of impact parameters contribute to the high spin states) at a given projectile energy. In such a case, number of ‘yrast’-like transitions are much larger than that of statistical ones, where less or no feeding is expected. Therefore, the spin-distribution of CF and ICF products are expected to be entirely different in nature and can be used as a sensitive tool to probe reaction dynamics by looking into entry state spin population. As such,
in order to generate experimental spin-distributions of various CF and ICF reaction products, relative production yields have been plotted as a function of experimentally observed spin \( J_{\text{exp}} \) and corresponding to prompt \( \gamma \)-transitions. For better comparison of different reaction channels (\( xn, \alpha xn \) and \( 2\alpha xn \)) in a panel, relative production yields have been normalised with their own highest experimentally measured values \( Y_{\alpha, \text{obs}}^{\text{max}} \) at lowest \( J_{\text{obs}, \alpha}^{\text{min}} \).

Further, as an analytical representation of data, the experimentally measured spin-distributions obtained as mentioned above have been fitted by a function of following type:

\[
Y = Y_0/[1 + \exp(J - J_0)/\Delta]
\]

Where; \( \Delta \) is related to the width of mean input angular momenta \( (J_0) \) and \( Y_0 \) is the normalisation constant. Here, \( J_0 \) is a sensitive parameter, which provides the qualitative information about the driving input angular momenta associated with various reaction channels.

As a representative case, experimental spin-distributions of \( xn/\alpha xn/2\alpha xn \)-channels populated via CF and ICF in \( ^{12}C, ^{16}O+^{169}Tm \) systems are plotted in Fig.1. The errors have not been shown in the figures as they have been estimated to be \( \leq 10\% \), and the inclusion of these errors is not likely to modify the present analysis. The nomenclature used in this figure indicate the involved reaction dynamics i.e., ‘B’ and ‘F’ respectively indicate CF-channel: identified from backward(B)-\( \alpha \)-gated spectra, and ICF-channel: identified from forward(F)-\( \alpha \)-gated spectra. As can be observed from this figure, in general, there is a striking difference in the spin-distribution for different channels which indicates the involvement of entirely different reaction dynamics in the production of these reaction products. The intensity of \( xn \)-(singles)/\( \alpha xn \)-B-channels (only populated via CF) falls off rather quickly with observed spin \( (J_{\text{obs}}) \), indicating strong feeding and/or broad spin population during the de-excitation of CN. However, for \( \alpha xn \)-F-channels (expected to be populated via ICF), the intensity appears to be almost constant up to a certain value of \( J_{\text{obs}} \), and then decreases towards entry side. This indicates the absence of feeding to the lowest members of the ‘yrast’ band and/or the population of low spin states are strongly hindered in ICF channels. Further, the feeding intensity profiles (FIPs) for different reaction products have been generated from experimentally measured spin-distributions, and are plotted in Fig.1(c,f,h). As indicated in Fig.1c, the feeding intensity for CF-channels (\( xn/pxn-B \)) is showing sharp exponential rise towards low spin states, which indicates regular population with strong feeding contribution for each \( \gamma \)-transition up to \( J_{\text{obs}}^{\text{min}} \). For better comparison of direct-\( \alpha \)-emitting channels (ICF) and fusion-evaporation \( \alpha \)-emitting channels (CF), the feeding intensity profiles for CF and ICF products have been plotted in a single panel (see Figs.1f). As can be observed from this figure, the feeding intensity of \( \alpha xn \)-channels \( ^{174}Ta-CF-B \) identified from backward(B)-\( \alpha \)-gated spectra shows similar trend as has been observed for \( xn/pxn-B \)-channels, where the band is fed over a broad spin range. While, the feeding intensity for forward(F)-\( \alpha xn/2\alpha xn \)-channels (identified from forward \( \alpha \)-gated spectra) is found to be increasing up to a certain values of \( J_{\text{obs}} \) and then decreases gradually towards band head, see Fig.1(f,h). This trend indicates that the high spin states are strongly fed even in case of ICF channels. However, as the residual nucleus de-excites, the feeding intensity decreases gradually with available excitation energy and/or angular momenta, which indicates the absence of feeding to the lowest members of the ‘yrast’ band or the low spin states are less populated in ICF-\( \alpha xn/2\alpha xn \)-F channels. Such feeding intensity pattern is expected to arise from narrow \( \ell \)-window, localized near and/or above to the critical angular momentum for CF. Similar trends have also been observed in other \( xn/\alpha xn/2\alpha xn \)-channels (see ref.[6, 7, 8] for detail).

In order to understand the multitude of mean driving angular momenta \( (\ell \text{-values}) \), and to examine the possibility to populate high spin states via ICF, mean driving angular momenta involved in CF and ICF channels have been deduced from the analysis of spin-distributions. The \( \ell \)-values involved in various modes of reactions are plotted in Fig.1(i). As indicated in this figure, the value of \( < \ell > \) involved in the production of CF-\( xn/pxn-B/\alpha xn-B \), and ICF-\( \alpha xn-F \),
and ICF-2oxn-F channels are found to be $\approx 7.5 \hbar$, $\approx 10 \hbar$ and $\approx 13.5 \hbar$, respectively, at projectile energy $\approx 5.6$ AMeV. However, at projectile energy $\approx 6.5$ AMeV, the value of $<\ell>$ for CF-xn/pxn-B/oxn-B, and ICF-oxn-F, and ICF-2oxn-F channels are found to be $\approx 10 \hbar$, $\approx 14 \hbar$ and $\approx 17 \hbar$, respectively. The enhancement in the value of $<\ell>$ in case of direct-$\alpha$-emitting channels (ICF products) indicates their origin from high $\ell$-values as compared to CF-channels.
A very useful correlation between the value of \(< \ell >\) and the successively opened ICF channels can be obtained from the data presented in Fig.8. Following the approach presented in ref.[7], the value of \(< \ell >\) associated with ICF in contrast with CF can be represented as:

\[(i)\] for \(^{12}\text{C}+^{169}\text{Tm}\) system at \(E_{\text{lab}} \approx 5.6\) AMeV;
\[
\ell (\text{ICF} - \alpha xn) \approx 1.33 \ell (\text{CF} - \alpha xn/pxn/\alpha xn),
\]
\[
\ell (\text{ICF} - 2\alpha xn) \approx 1.35 \ell (\text{ICF} - \alpha xn) \approx 1.8 \ell (\text{CF} - \alpha xn/\alpha xn),
\]

\[(ii)\] at \(E_{\text{lab}} \approx 6.5\) AMeV;
\[
\ell (\text{ICF} - \alpha xn) \approx 1.4 \ell (\text{CF} - \alpha xn/pxn/\alpha xn),
\]
\[
\ell (\text{ICF} - 2\alpha xn) \approx 1.2 \ell (\text{ICF} - \alpha xn) \approx 1.7 \ell (\text{CF} - \alpha xn/\alpha xn).
\]

From the correlations presented above, it is interesting to note that the values of \(< \ell >\) involved in the production of various ICF-\(\alpha xn/2\alpha xn\)-channels are found to be \(\approx 30\) to \(70\) \% higher as compared to the CF-\(\alpha xn/pxn/\alpha xn\)-channels at both the energies. This clearly indicates the involvement of high \(\ell\)-values in the production of ICF products at a constant projectile energy, essentially due to non-central interactions, where a significant amount of orbital angular momentum between projectile and target nucleus transformed into high spin states of final reaction products. It can also be observed from Fig.1(i), the involved \(\ell\)-values in different reaction channels are found to increase linearly with the projectile energy, and almost same (within \(\leq 0.5\) \(h\)) for each set of reaction channels at a given projectile energy. It is also clear from Fig.1(i), the CF is not able to approach the value of angular momenta even at \(\approx 6.5\) AMeV which has been populated via ICF for the same residue at relatively low projectile energy. The above striking features strongly support the possibility to populate high spin states via ICF in final reaction products even at low projectile energy. The smallness of \(J_o\) indicates the involvement of less input angular momenta in CF reactions. It may also be seen from the deduced values of \(J_o\) that the multiplicity of \(\alpha\)-particles associated with ICF increases with the driving input angular momenta, indicating the variation of different \(\ell\)-values (impact parameter dependent) at a given projectile energy. This indicates that the partial waves of lower \(\ell\)-values do not contribute to the ICF reaction dynamics significantly. Therefore, it may not be out of order to state that ICF reactions occur in the peripheral interactions, probably at finite values of impact parameters, where the centrifugal force field overtake the sum of nuclear and Coulomb potentials.

2.2. Incomplete fusion strength function: measure of strength and onset

The measurements presented in previous section are relative and don’t give information on how ICF show up with various entrance channel parameters. As such, in order to understand influence of ICF on CF at these energies, absolute cross-sections for individual reaction residues have been measured for for \(^{12}\text{C},^{16}\text{O}+^{169}\text{Tm}\) systems at energies near and above the fusion barrier [9, 11, 14]. Experiments have been performed using GPSC-setup at IUAC. Experimental methodology and setup have been detailed in refs.[9, 11, 14]. However, a brief account of experimental conditions is given here for ready reference. Isotopically pure (abundance = 100\%) \(^{169}\text{Tm}\) Targets of thickness \(t_m \approx 1-1.8\) mg-cm\(^{-2}\) were prepared by uniform pressure rolling technique. Uniformity of each target foil was verified by multi-point thickness measurement using \(\alpha\)-transmission method. The error in thickness was estimated to be \(\leq 1\)\%. For \(^{16}\text{O}+^{169}\text{Tm}\) system, energy degradation technique has been employed to cover a wide energy range in single run, and corrections were made for energy spread and beam intensity variations. while, in case of \(^{12}\text{C}+^{169}\text{Tm}\) system, the energy spread and beam intensity variations have been avoided by adopting single target irradiation methodology for each projectile energy. Irradiations were carried out using \(^{12}\text{C},^{16}\text{O}\)-beams (\(E_{\text{beam}} = \approx 4-7\) MeV/nucleon with beam current \(\approx 20-30\) nA) delivered from 15UD Pelletron accelerator. The targets were backed by Al-catchers of
The two data sets show good agreement. (d) Experimentally measured and systematically deduced (see text for deduction procedure) EFs of ICF products along with the sum of all ICF-channels ($\Sigma_\text{ICF}$), (e) Total fusion cross-section ($\sigma_\text{TF} = \Sigma_\text{CF} + \Sigma_\text{ICF}$) along with the sum of all CF-channels ($\Sigma_\text{CF}$) and ICF-channels ($\Sigma_\text{ICF}$) as a function of projectile energy, and (f) the zoom of $\Sigma_\text{CF}$ and $\sigma_\text{TF}$ comparison on linear scale for an easy visualization of increasing ICF strength with projectile energy. Solid curves represent best fit to the data points.

Figure 2. (a) – (c) The experimental excitation functions of $x_n/\alpha x_n/2\alpha x_n$-channels populated via CF and/or ICF in $^{12}$C+$^{169}$Tm system. Lines and symbols are used for self explanatory notations. In case of $^{173,174}$Ta($\alpha 4n$,\,$\alpha 3n$) and $^{171,172}$Lu(2\,$\alpha 2n$,\,$2\alpha n$) residues, the red dotted lines through the data points represent best fit to the both experimental data sets (I run and II run). The two data sets show good agreement. (d) Experimentally measured and systematically deduced (see text for deduction procedure) EFs of ICF products along with the sum of all ICF-channels ($\Sigma_\text{ICF}$). (e) Total fusion cross-section ($\sigma_\text{TF} = \Sigma_\text{CF} + \Sigma_\text{ICF}$) along with the sum of all CF-channels ($\Sigma_\text{CF}$) and ICF-channels ($\Sigma_\text{ICF}$) as a function of projectile energy, and (f) the zoom of $\Sigma_\text{CF}$ and $\sigma_\text{TF}$ comparison on linear scale for an easy visualization of increasing ICF strength with projectile energy. Solid curves represent best fit to the data points.

Appropriate thicknesses, so that the recoiling nuclei can be trapped in the catcher foil thickness. Proper care was taken to maintain the constant beam current during the irradiations so that the uncertainty in the production cross-sections due to beam current fluctuations can be minimised. In order to detect the residues of small half-lives ($t_{1/2} \approx 5$-10 minutes), in-situ measurements of $\gamma$-activities produced during the irradiations were performed off-line using two pre-calibrated HPGe detectors. The uncertainty in geometry dependent efficiency of detectors is estimated to be $\leq 2\%$. Prompt-$\gamma$-rays spectra have been recorded in singles mode for an additional degree of freedom in residue identification. Reaction products have been identified by their characteristic prompt and decay $\gamma$-lines. In order to gain high confidence in reaction products identification, the decay-curve analysis for each residue has been performed using decay $\gamma$-lines. The most intense decay $\gamma$-lines have been used for decay-curve analysis and for the production cross-
section \((\sigma_{ER})\) measurement [7]. The projectile energy dependent reaction cross-sections \((\sigma_r(E))\) for different radio-nuclides have been determined using standard formulation \([?]\). The overall error in \(\sigma_{ER}\) is estimated to be \(\leq 13\%\), excluding the uncertainty in branching ratio, decay constant etc. Details of error estimation and causing factors are given in ref.[14].

![Graph](image)

**Figure 3.** (a) the ICF strength function for \(^{12}\text{C}+^{169}\text{Tm}\) system (see text for description), (b) the value of \(F_{ICF}\) as a function of relative velocity \((v_{rel})\) for \(^{12}\text{C},^{16}\text{O}+^{169}\text{Tm}\) systems.

In order to deduce ICF fraction \((F_{ICF})\), experimental EFs have been analysed in the framework of theoretical model code PACE4 based on equilibrated CN-decay. Details of this code can be found elsewhere [9, 14]. In this code, level density parameter \((a = A/K)\) is an important input parameter which affects the equilibrium component. In order to know the suitable value of ‘a’ for studied systems at given energy range, different values of ‘a’ from \(A/10\) - \(A/8\) MeV\(^{-1}\) have been tested. As a representative case, the predictions of PACE4 for different ‘a’ values from \(a = A/10\) to \(a = A/8\) MeV\(^{-1}\) are plotted in Figs.2(a). Experimental EFs of \(^{176,177}\text{Re}(5n,4n)\) residues are reasonably well reproduced for \(a = A/8\) MeV\(^{-1}\). This confirms the production of these residues \((^{176,177}\text{Re})\) through the equilibrated CN-decay via emission of \(x\)-neutrons from the excited \(^{185}\text{Ir}^*\) formed via CF. As such, the value of \(a = A/8\) MeV\(^{-1}\) can be used as default parameter for further analysis within the tested energy range. As shown in Fig.2(b), the EFs of \(\alpha\)-emitting channels are significantly enhanced than theoretical predictions. The enhanced cross-section in case of \(\alpha xn\)-channels points towards some physical effect which is not included in this code. It may be pointed out that PACE4 do not take ICF into account.
Therefore, the enhancement in case of α-emitting channels (α3n,α4n) may be attributed to the onset of ICF. Assuming the enhancement as ICF fraction in case of α3n,α4n-channels, the ICF contribution has been accounted using data reduction procedure suggested by Gomes et al.[3]. For some α-emitting channels (as shown in Fig.2c), PACE4 gives negligible cross-section for 2α2n and α2n-channels. This indicates the population of these residues only via CF. Experimental EFs of α-emitting channels are subtracted from the theoretical ones (σ_{ICF} = σ_{exp} - σ_{pace}) to get ICF contribution, and are plotted in Fig.1(d). To show how does ICF contribute to the total fusion cross-section (σ_{TF} = σ_{CF} + σ_{ICF})? the overall ICF cross-section (σ_{ICF}) is plotted with the sum CF-channels (σ_{CF}) and σ_{TF} in Fig.2(e). For an easy visualisation of increasing ICF contribution with energy, the zoom of σ_{CF} and σ_{TF} comparison is plotted on linear scale in Fig.2(f). As shown in this figure, the increasing separation between σ_{CF} and σ_{TF} with projectile energy indicates strong energy dependence on ICF strength. For better insight into the projectile energy effect on ICF strength, the percentage fraction of ICF (F_{ICF}) has been deduced and plotted as a function of projectile energy in Fig.3(a), i.e., termed as ICF strength function. The ICF strength function defines empirical probability of ICF at different projectile energies. As shown in this figure, the value of F_{ICF} is found to be ≈7 % at ≈ 59 MeV, i.e., 1.075Vb (7.5 % above the barrier), and increases smoothly up to ≈18 % at highest measured energy i.e., 1.64 Vb. The existence of ICF at such a low energy has been justified as a consequence of high input ℓ-values imparted into the system in non-central interactions [6, 7, 8, 16, 17].

Further, the values of F_{ICF} obtained in ¹²C,¹⁶O+¹⁶⁹Tm systems are plotted as a function of \(v_{rel}\) in Fig.3(b). According to Morgenstern’s systematics [24], ICF contributes significantly above \(v_{rel} ≈ 0.06\) (6 % speed of light). As indicated in this figure, the values of \(v_{rel}\) are in the range from \(≈ 0.027\) (2.7 % of c) to \(≈ 0.084\) (8.4 % of c) for ¹²C, and from \(≈ 0.014\) (1.4 % of c) to \(≈ 0.053\) (5.3 % of c) for ¹⁶O. Therefore, no significant ICF contribution is expected at the given values of \(v_{rel}\). However, the results presented in Fig.3(b) clearly demonstrate the onset of ICF at relatively lower value of \(v_{rel}\) i.e., \(≈ 0.027\) (F_{ICF} ≈ 7 %) in ¹²C+¹⁶⁹Tm system, and at \(≈ 0.014\) (F_{ICF} ≈ 10 %) in ¹⁶O+¹⁶⁹Tm system. In both cases, the observed value of F_{ICF} is significant at well below the proposed onset value of \(v_{rel}\) (i.e., 6 % of c). As such, it can be inferred that the ICF starts competing with CF even at slightly above barrier energies.

### 2.3. Forward-recoil-ranges: proof of fusion incompleteness

In order to validate data reduction procedure to deduce F_{ICF}, and to proof fusion-incompleteness at slightly above barrier energies, forward-recoil-ranges (FRRs) of heavy reaction residues produced in ¹²C,¹⁶O+¹⁶⁹Tm systems have been measured at different energies [10, 13]. Kinematically, the FRRs depend upon degree of linear momentum transfer (\(\rho_{LMT}\)) from projectile to target nucleus. For ICF, where only a part of projectile fuses with target nucleus, the \(\rho_{LMT}\) may be given as:

\[
\rho_{LMT} = \frac{P_{frac}}{P_{proj}}
\]

where; \(P_{frac}\) is the linear momentum of fused fraction of projectile, and \(P_{proj}\) is the full linear momentum of projectile. As per kinematical range-energy calculations, entire LMT (i.e., \(\rho_{LMT} = 1\)) from projectile to target nucleus (in case of CF) supposed to give maximum recoil velocity to the composite (\(A_{target}+A_{proj}\)) system, and relatively less recoil velocity to the incompletely fused composite (ICF: \(A_{T}+A_{proj-frac}\)). Consequently, the radio-nuclides populated via small \(\rho_{LMT}\) are expected to show relatively smaller penetration depth in the stopping medium as compared to the entire LMT populations. On the basis of aforementioned kinematics, CF and ICF products can be disentangled from the analysis of FRRs of heavy reaction residues.
For the measurement of FRRs, Natural Thulium (\(^{169}\)Tm) targets of thickness \(\approx 38 - 56\ \mu g/cm^2\), and Aluminium catcher foils (of thickness \(\approx 10 - 55\ \mu g/cm^2\)) were prepared by UHV-vacuum deposition technique. The irradiations have been carried out in GPSC-setup and followed by off-line gamma spectroscopy. In order to stop the recoiling reaction products, a stack of thin Al-catcher foils of the thicknesses varying from \(\approx 10 - 55\ \mu g/cm^2\) (sufficient to stop entire LMT events) has been placed just after the target. The recoiling products populated via CF and/or ICF have been stopped at various cumulative depths in the Al-catcher foils. Keeping in mind the half-lives of interest, the irradiations were carried out for \(\approx 10\) hrs duration for each incident energy. After the irradiation, the Al-catcher foils along with the target foil were taken out from the scattering chamber for post-irradiation analysis. The radio-activities produced in catcher foils were counted separately using two pre-calibrated, high resolution, HPGe-detector of 100 c.c. active volume. The HPGe-detectors were pre-calibrated for energy using \(^{60}\)Co, \(^{133}\)Ba and \(^{152}\)Eu \(\gamma\)-sources. The resolution of \(\gamma\)-detector was found to be \(\approx 2.4\) keV, for 1.33 MeV \(\gamma\)-ray of \(^{60}\)Co. The residues were identified by their characteristic \(\gamma\)-lines and confirmed by the decay-curve analysis. The induced activities can be used to measure the production probability of the reaction products. The measured intensities of the characteristic \(\gamma\)-lines have been used to determine the production cross-sections of the residues in each catcher foil using standard formulation. The overall errors from all possible factors (for detail see ref. [10, 13]) including the statistical error are estimated to be \(\leq 14\%\).

Figure 4. Experimentally measured profile of FRRs for; (a) \(^{182}\)Os(p2n), (b) \(^{179}\)Re(\(\alpha\)2n), and (c) \(^{178}\)Ta(\(\alpha\)2pn) residues populated via CF and/or ICF of \(^{16}\)O with \(^{169}\)Tm at energies \(\approx 87\) MeV. Relative strengths of different LMT transfer components are indicated in percentage.

In order to generate experimental FRRs, the normalised yield of reaction products have been plotted as a function of cumulative catcher foil thicknesses. The production yield of different reaction products have been deduced by the normalisation of experimentally measured production cross-sections with the respective catcher foil thicknesses. The FRRs have been fitted with Gaussian distribution for better analytical representation of data, and to deduce the value of most probable recoil ranges \((R_{p(exp)})\) in the stopping medium, similar to that given in ref.[10, 13].

Fig.4(a) shows the profile of FRRs for \(^{182}\)Os(p2n), populated in \(^{16}\)O+\(^{169}\)Tm system at \(\approx 87\) MeV. As can be seen from this figures, the data can be fitted by a single Gaussian peak indicating
Figure 5. (a) Experimentally measured profile of FRRs for $^{181}\text{Re}(\alpha/2p2n)$, (b) Relative strengths of CF and ICF of $^{16}\text{O}$ with $^{169}\text{Tm}$ at projectile different energies for $^{181}\text{Re}$ ($\alpha$ or 2p2n - channel). (c) The value of $F_{ICF}$ as a function of reduced projectile energy. The values of $F_{ICF}$ given in different colors have been obtained from the analysis of three different sets of data. As indicated, the value of $F_{ICF}$ deduced from three different techniques at is in good agreement within the experimental uncertainties. Lines through the data points are drawn to guide the eyes.

single LMT-component involved in the production of this residue. However, Fig.4(b-c) show the FRRs for $^{179}\text{Re}(\alpha 2n)$ and $^{178}\text{Ta}(\alpha 2pn)$ radio-isotopes. As shown in these figures, the FRRs can be resolved into two Gaussian peaks, revealing the presence of more than one LMT-components associated with the fusion of $^{16}\text{O}$, $^{12}\text{C}$ and/or $^8\text{Be}$ with the target nucleus. As an example, the FRR of residue $^{179}\text{Re}(\alpha 2n)$ is showing two LMT components at $\approx 383 \mu g/cm^2$ (due to $^{16}\text{O}$-fusion) and at $\approx 212 \mu g/cm^2$ ($^{12}\text{C}$-fusion, and an $\alpha$ as spectator) at projectile energy $\approx 87$ MeV. As such, on the basis of observed $R_{P(Exp)}$, it can be inferred that the residues $^{179}\text{Re}(\alpha 2n)$ is populated via both CF and ICF processes. This implies fractional momentum transfer in ICF. The value of $R_{P(Exp)}$ for different residues have also been compared with the theoretically estimated most probable FRRs have been estimated by adopting break-up-fusion description of massive transfer reactions given by Udagawa and Tamura [22], and are found to be in good agreement with theoretical ones (see ref.[10, 13]).

Further, an attempt has been made to find out the relative strengths of CF and ICF contributions to understand the energy dependence of full (CF products) and partial (ICF products) linear-momentum-transfer components. Fig.5(a) shows the FRRs for $^{181}\text{Re}(\alpha$ or 2p2n) which can be resolved into two Gaussian peaks. Peak at $\approx 260 \mu g/cm^2$ indicating contribution from ICF of $^{12}\text{C}$-fusion. For this case, the relative contributions of CF and ICF obtained from the analysis of FRRs at different energies are plotted in Fig.5(b). As indicated in this figure, the CF (fusion of $^{16}\text{O}$) contribution decreases at higher projectile energies, while the ICF (fusion of $^{12}\text{C}$) contribution is found to increase with projectile energy. The observed trend reveals that the influence of ICF over CF increases with projectile energy, as expected.

In order to validate data reduction procedure given in section-2.2, the value of $F_{ICF}$ has been deduced from the analysis of FRRs using following relation,

$$F_{ICF} = \frac{\Sigma \sigma_{ICF}}{\Sigma \sigma_{CF} + \Sigma \sigma_{ICF}} \times 100$$
Where, $\Sigma \sigma_{CF}$ and $\Sigma \sigma_{ICF}$ are the sum of cross-sections (obtained from the analysis of FRRs) of CF and ICF processes, respectively. The $F_{ICF}$ has been deduced as a function of normalized projectile energy ($E_{\text{beam}}/CB$), where $E_{\text{beam}}$ is the beam energy and $CB$ is the Coulomb barrier for the $^{16}$O+$^{169}$Tm system. For better comparison and to have confidence in the measured values, the values of $F_{ICF}$ deduced from three different techniques, namely: (i) from the analysis of FRRs, (ii) angular distributions, and (iii) from the analysis of EFs, have been plotted as a function of $E_{\text{beam}}/CB$ in Fig.5(c). As shown in this figure, the value of $F_{ICF}$ deduced from three different methods at a constant normalized projectile energy ($E_{\text{beam}}/CB = 1.2$) is found to be in good mutual agreement within the experimental uncertainties, which puts faith in our measurements and data reduction procedure. Lines through the data points are drawn to guide the eyes. The value of $F_{ICF}$ deduced from the analysis of FRRs is found to be $\approx 15 \%$ at energy $\approx 76$ MeV ($\approx 10 \%$ above the CB), while, at energy $\approx 81$ MeV ($\approx 17 \%$ above the CB), the value of $F_{ICF}$ approaches up to $\approx 29 \%$.

3. Summary and Conclusions

This paper deals with recent experimental results from three set of experiments performed to study ICF at energies $\approx 4$-$7$ MeV/nucleon. In first set of experiments, the spin-distributions and feeding intensity profiles have been measured at near and above barrier energies. The spin-distributions of ICF-xn/2xn-channels are found to be distinctly different than that observed for CF-xn/pxn/alpha-channels, which indicates entirely different de-excitation patterns in CF and ICF products. In case of CF products, strong feeding through broad range of spin population has been observed towards the band head. However, the spin-distribution(s) associated with ICF are found to be arrived from the narrow spin population, localised near and/or above to the critical angular momentum for CF, where a given PLF is emitted to release excess driving angular momenta. This indicates the competition from successively opened ICF channels for each value of $\ell$-value above $\ell_{\text{crit}}$ for normal fusion (CF). The population of low spin states are hindered and/or less fed in case of ICF. This reveals the occurrence of ICF due to the influence of centrifugal potential in peripheral interactions, where driving angular momentum limit do not allow CF. It is also shown that the direct alpha-multiplicity increases in forward cone with the value of $<\ell>$ at a particular projectile energy. For an example, at $\approx 5.6$ AMeV the value of $<\ell>$ for CF-xn/pxn-B/alpha-B, and ICF-alpha-F, and ICF-2alpha-F channels are found to be $\approx 7.5 \ h$, $\approx 10 \ h$ and $\approx 13.5 \ h$, respectively. On the basis of above results, it can be concluded that the high $\ell$-values associated with non-central interactions essentially contribute to open up direct-alpha-emitting channels. It may, further, be pointed out that the value of $<\ell>$ associated with 2alpha-emitting channel likely to be originated from higher impact parameters as that associated with the production of single direct-alpha-emitting channel. Further, from the comparison of $\ell$-values involved in the production of direct-alpha-emitting (ICF products) and normal-alpha-emitting channels (CF-products), we presented direct evidence that ICF can populate high spin states in final reaction products which is not possible to achieve via CF at a given projectile energy and/or even higher energy.

In second set of experiments, ICF strength function has been measured using some systematics. Existence of ICF at energy as low as $\approx 7.5 \%$ above the barrier is conclusively demonstrated. The value of $F_{ICF}$ increases from $\approx 7 \%$ to $\approx 18 \%$ with in the studied energy range (i.e., $1.02V_b$ to $1.64V_b$). The results presented in section-2.2 clearly demonstrate the onset of ICF at relatively lower value of $v_{rel}$ i.e., $\approx 0.027$ ($F_{ICF} \approx 7 \%$) in $^{12}$C+$^{169}$Tm system, and at $\approx 0.014$ ($F_{ICF} \approx 10 \%$) in $^{16}$O+$^{169}$Tm system. In both cases, the observed value of $F_{ICF}$ is significant at well below the proposed onset value of $v_{rel}$ (i.e., $6 \%$ of c). Further, a comparison of $F_{ICF}$ for $^{12}$C+$^{169}$Tm and $^{16}$O+$^{169}$Tm systems displays higher ICF probability for $^{16}$O+$^{169}$Tm system for the entire measured energy range. This indicates strong projectile dependence on $F_{ICF}$. Presence of significant ICF fraction at well below the proposed onset value of $v_{rel}$ (i.e., $6$
% of c) supplements Morgenstern’s systematics.

Moreover, the results presented in section-2.3 are from the third set of experiments where FRRs of heavy reaction products have been measured to confirm the findings of first two set of experiments. From the analysis of FRRs, it has been observed that the α-emitting channels have significant contribution from both CF and ICF processes. Different LMT components attributed to the full (fusion of $^{16}$O) and partial fusion ($^{12}$C and/or $^{8}$Be fusion from $^{16}$O) of projectile with target nucleus have been observed. The $R_{P}^{(exp)}$ are found to consistent with the theoretical predictions, within the experimental errors. The value of $F_{ICF}$ obtained from the analysis of FRRs is found to be in good agreement with that estimated by the measurement and analysis of EFs. The results presented in section-2.3 support BF-model description of massive transfer reaction given by Udagawa and Tamura [22]. As such, it can be concluded that the ICF processes contribute significantly to the reaction cross-section at these energies. The extension of this work using $^{13}$C, $^{14}$N and $^{18}$O beams with $^{169}$Tm target would be interesting, and will be helpful for further refinement of present systematics.

Acknowledgments
We thank Prof. Amit Roy, Director of IUAC for extending experimental facilities to perform these experiments. One of the authors (P.P.S.) thanks to INFN-Laboratori Nazionali di Legnaro (LNL), and R. P. & B. P. S. thank to DST and UGC for financial support.

References
[1] A. Diaz-Torres, J. Phys. G: Nucl. Part. Phys. 37, 075109 (2010), Phys. Rev. Lett. 98, 152701 (2007), and the references therein.
[2] D. J. Hinde and M. Dasgupta, Phys. Rev. C 81, 064611 (2010), and the references therein.
[3] P. R. S. Gomes et al., Phys. Lett. B601, 20 (2004); Phys. Rev. C73, 064606 (2006); Phys. Rev. Lett. B53, 1630 (1984).
[4] E. Z. Buthelezi et al., Nucl. Phys. A734, 553 (2004).
[5] L. R. Gasques et al., Phys. Rev. C79, 034605 (2009); Phys. Rev. C74, 064615 (2006).
[6] Pushpendra P. Singh et al., Phys. Lett. B671, 20 (2009), and the references therein.
[7] Pushpendra P. Singh et al., Phys. Rev. C80, 064603 (2009)
[8] Pushpendra P. Singh et al., Phys. Rev. C78, 017602 (2008), and the references therein.
[9] Pushpendra P. Singh et al., Phys. Rev. C77, 014607 (2008).
[10] Pushpendra P. Singh et al., Eur. Phys. J. A 34, 29 (2007), and the references there in.
[11] Pushpendra P. Singh et al., Phys. Rev. C (submitted)
[12] Devendra P. Singh et al., Phys. Rev. C81, 054607 (2010).
[13] Unnati Gupta et al., Phys. Rev. C80, 024613 (2009).
[14] Unnati Gupta et al., Nucl. Phys. A811, 77 (2008).
[15] P. E. Hodgson, E. Gadioli and E. Gadioli Erba, Introductory Nuclear Physics, Chapter 23, Clarendon Press, Oxford (1997).
[16] J. Wilczyński, et al., Phys. Rev. Lett. 45, 606 (1980); Nucl. Phys. A 373, 109 (1982).
[17] K. Siwek-Wilczyńska et al., Phys. Rev. Lett. 42, 1599 (1979).
[18] H. C. Britt and A. R. Quinton, Phys. Rev. 124, 877 (1961).
[19] J. Galin et. al., Phys. Rev. C 9, 1126 (1974).
[20] W. Trautmann, et al., Phys. Rev. Lett. B 53, 1630 (1984).
[21] T. Inamura, et al., Phys. Lett. 68B, 51 (1977), Phys. Lett. 84B, 71 (1982); Phys. Rev. C32, 1539 (1985).
[22] T. Udagawa and T. Tamura, Phys. Rev. Lett. 45, 1311 (1980).
[23] J. P. Bondorf, J. N. De, G. Fai, A. O. T. Karvinen, and J. Randrup, Nucl. Phys. A 333, 285 (1980).
[24] H. Morgenstern et al., Z. Phys. A313, 39 (1983); Phys. Rev. Lett. 52, 1104 (1984); Z. Phys. A324, 443 (1986).
[25] C. Gerschel, Nucl. Phys. A 387, 297 (1982), and references there in.