New Result for the Pion–Nucleon Sigma Term from an Updated VPI/GW 
πN Partial–Wave and Dispersion Relation Analysis

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Abstract

A new result for the πN sigma term from an updated πN partial–wave and dispersion relation analysis of the Virginia Polytechnic Institute (now George Washington University) group is discussed. Using a method similar to that of Gasser, Leutwyler, Locher, and Sainio, we obtain Σ = 90 ± 8 MeV (preliminary), in disagreement with the canonical result 64 ± 8 MeV, but consistent with expectations based on new information on the πNN coupling constant, pionic atoms, and the ∆ resonance width.

Introduction

The pion nucleon sigma term (Σ) continues to be a puzzle some thirty years after initial attempts to determine it. The keen interest in Σ comes from the fact that it vanishes in the massless quark (chiral) limit of QCD, and becomes non-zero only for a non-zero light (up or down) quark mass, so it is a crucial parameter in the understanding of chiral symmetry breaking (see e.g. Refs.[1,2]). The nucleon’s strange quark content can be inferred from Σ (see e.g. Ref.3), so Σ is also relevant to quark confinement, not yet fully understood, since one must understand the mechanism for accommodating strange quarks in an ostensibly light quark object 3. Thus Σ is a parameter of fundamental significance to low energy QCD, making it crucial to obtain its value as precisely as possible. The canonical result for Σ ≃ 64 MeV [4,5] implies a large nucleon strangeness content [2], and much effort has been spent trying to understand that. This article outlines recent work of the (former) Virginia Polytechnic Institute (VPI), (now George Washington University (GWU)) group to extract the “experimental” value of the sigma term (Σ) from the πN scattering data as part of ongoing πN partial-wave (PWA) and dispersion relation (DR) analyses.

Experimental Σ Term

The “experimental” sigma term Σ is related to the πN isoscalar amplitude $\bar{D}^+(\nu = 0, t = 2m^2_\pi)$ at the “Cheng-Dashen point” [6]:

$$\Sigma = F^2_\pi \bar{D}^+(\nu = 0, t = 2m^2_\pi)$$  (1)

where $F_\pi = 92.4$ MeV is the pion decay constant, $\nu$ is the crossing energy variable, and $t$ is the four-momentum transfer. Since the Cheng-Dashen point lies outside the physical πN scattering region, the experimental πN amplitudes must be extrapolated in order to obtain Σ. The most theoretically well-founded extrapolation approach is based on dispersion relation (DR) analyses of the scattering amplitudes[6]. In the early 80s, the Karlsruhe-Helsinki group performed extensive investigations into obtaining Σ from πN dispersion relations [6]. The canonical result Σ = 64 ± 8 MeV was based on hyperbolic dispersion relation [6] calculations using the groups’ πN [6] and ππ [6] phase shifts.

The only recent dispersion theoretic determinations have been by Sainio [6], based on the method of Gasser, Leutwyler, Locher, and Sainio (GLLS)[6]. The method exploits the fact that $\bar{D}^+(t)$ can be expressed as a power series in $t$[6], the coefficients determined from dispersion relation subtraction constants. The coefficients up to $O(t)$, $d^{+}_{00}$ and $d^{+}_{01}$, are determined from the forward $\bar{C}^+$ and “derivative” $\bar{E}^+$ DRs, respectively. The smaller

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$E^{+} + \frac{g^2}{4\pi} = 13.73$

$\Sigma = F_{\pi}^{2} \cdot (\bar{d}_{00}^{+} + 2m_{\pi}^{2} \cdot \bar{d}_{01}^{+}) + \Delta_{D} \equiv \Sigma_{d} + \Delta_{D}$

Figure 1. Left: Subtraction constant (“scattering length + pole”) and Born term in the forward $C^{+}$ dispersion relation as a function of energy from our $\pi N$ analysis SM99; Right: same, for the Karlsruhe KA84 analysis [9]. This DR yields the coefficient $\bar{d}_{00}^{+}$ in Eqn. 2.

Figure 2. Left: Subtraction constant in the forward $E^{+}$ dispersion relation as a function of energy from our $\pi N$ analysis SM99; Right: same, for the Karlsruhe KA84 analysis [9]. This DR yields the coefficient $\bar{d}_{01}^{+}$ in Eqn. 2.

$O(t^{2})$ correction $\Delta_{D} \approx 12 \text{ MeV}$ is determined employing $\pi\pi N\bar{N}$ phase shifts (15 MeV), and $\Delta$ isobar exchange (-3 MeV) [2]. $\Sigma$ is then expressed as:

$\Sigma = F_{\pi}^{2} \cdot (\bar{d}_{00}^{+} + 2m_{\pi}^{2} \cdot \bar{d}_{01}^{+}) + \Delta_{D} \equiv \Sigma_{d} + \Delta_{D}$

In the GLLS approach, the Karlsruhe KH80 [8] or KA84 [9] $\pi N$ phases shifts are used as fixed input above about $T_{\pi}=70 \text{ MeV}$, and the $D$ and higher phases are used below the cutoff as well in six forward dispersion relations ($B^{\pm}, C^{\pm}, E^{\pm}$). By fitting the low energy data, $\bar{d}_{00}^{+}$ and $\bar{d}_{01}^{+}$ can be determined. Their result [2] was $\Sigma_{d} \approx 50 \text{ MeV}$, and $\Delta_{D}=12 \text{ MeV}$, leading to $\Sigma \approx 62 \text{ MeV}$, in agreement with the Karlsruhe results [5,4]. However, since the dispersion relations were constrained to be satisfied, the subtraction constants, which are energy independent, must be the same at low energies where the data were fit as at high energies where they were fixed input. Therefore, $\Sigma_{d}$ could not have come out significantly different than the Karlsruhe result. Nonetheless, this analysis provided a very useful validation of the method. The technique has been criticized [10] since the $E$ DR is more sensitive to the higher partial waves than the other DRs, so it could be rather uncertain due to uncertainty in the higher phases. What the GLLS analyses showed was that this is in fact not the case, and the method can be used reliably to extract $\Sigma_{d}$.

Since the GLLS analyses simply demonstrated another method to get $\Sigma_{d}$ from the KH80 $\pi N$ analysis, there have been no recent DR–based Sigma term analyses independent of the results of the Karlsruhe group [5,4]. Consequently, our group has developed a version of the GLLS technique as part of our own $\pi N$ partial-wave and dispersion relation analysis. The method will be outlined in the following sections.

**VPI/GW $\Sigma$ Term Analysis Method**

The VPI/GW $\pi N$ partial-wave and dispersion relation analysis is an ongoing project, where new solutions are released when changes to the database and analysis method...
warrant \[1\]. Analysis details can be found in Ref.\[2,13,14\]. Presently, our partial-wave analysis is constrained by the forward \(C^\pm(\omega)\) and “derivative” \(E^\pm(\omega)\) dispersion relations, as well as the fixed-\(t\) \(B_{\pm}(\nu, t)\) (“Hüper” \[3\]) and \(C_{\pm}(\nu, t)\) dispersion relations. These DRs are constrained to be satisfied to within \(\sim 1\%\) up to \(\sim 800\) MeV. As our analysis extends up to \(2\) GeV, the KHz80 \[5\] phases are used from \(2\) to \(4\) GeV in the dispersion integrals. A \(4\) GeV cutoff is sufficient for adequate convergence in the fixed-\(t\) \(B_{\pm}\) and \(C_{\pm}\) DR integrals, however the \(E_{\pm}\) and \(C_{\pm}\) DR integrals require a parameterization for the high energy parts. After the report at MENU97 \[13\], we included the high energy parts of the latter DRs using formulas from Ref.\[5\], resulting in much more satisfactory results.

Pion-nucleon dispersion relations depend on \textit{a priori} unknown constants including the \(\pi NN\) coupling constant \(f_2\) and the subtraction constants (usually chosen to be scattering lengths). Our analysis treats these constants as unknown parameters to be determined by a best fit to data. In practice, for our work-in-progress “SM99” \[11\], the coupling \(f_2\) and the p-wave scattering volume \(a_{1+}^+\) were searched, while the s-wave scattering lengths were taken from the P.S.I. pionic hydrogen results\[15\]. We also insisted that the GMO sum rule \[16\] be satisfied.

For every solution, the subthreshold coefficient \(\tilde{d}_{00}^+\) is calculated using the chosen parameter set and \(\pi N\) phases from:

\[
\tilde{d}_{00}^+ = K_1 \cdot a_{0+}^+ + K_2 \cdot f^2 + \int d\nu' K_3(\nu') \text{Im} D^+(\nu')
\]

where \(K_i\) are kinematical factors, and \(a_{0+}^+\) is the isoscalar s-wave scattering length. The expression for \(d_{01}^+\) is analogous, involving instead the isoscalar p-wave volume \(a_{1+}^+\) and the amplitudes \(E^+, B^+,\) and \(C^+\). By noting how \(\Sigma_d\) varies for solutions away from the optimum, and fluctuations of the extracted constants with respect to energy, one obtains an indication of the uncertainty. To determine the experimental sigma term \(\Sigma\), we use \(\Delta_D=12\) MeV (see e.g. Ref.\[3\]), which is insensitive to the \(\pi N\) partial wave input \[2,17\].

The fixed-\(t\) \(C^+\) DR subtraction constants \(C^+(\nu = 0, t)\) are equivalent to \(D^+(0, t)\). Thus the slope of these constants as a function of \(t\) at \(t=0\) is \(d_{00}^+ + t \cdot d_{01}^+\), so we have another method to determine \(\Sigma_d\). Note that these subtraction constants are not fixed \textit{a priori} in the DR parameter search procedure (unlike e.g. \(f^2\)), so this method of obtaining \(\Sigma_d\) is independent to the GLLS approach and a valuable consistency check.

\textbf{Results and Discussion}

Our solution “SM99” satisfies fixed-\(t\) and forward dispersion relations well (up to our \(\sim 800\) MeV constraint limit), and the data (up to \(2\) GeV) are fit with \(\chi^2/\text{data point} = (2, 2, 2.5)\) for \((\pi^+,\pi^-,\text{CEX})\). Compared to the Karlsruhe KA84 solution\[9\], these same dispersion relations are better satisfied (see Figs.\[1,2,3\]), and the data much better fit

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.jpg}
\caption{Left: Subtraction constants in the fixed-\(t\) \(C^+\) dispersion relation from our \(\pi N\) analysis SM99 as a function of energy at three values of momentum transfer \(t\); Right: same, for the Karlsruhe KA84 analysis \[9\]. This DR yields the data points shown in Fig.\[4\] used to extract the coefficients \(\bar{d}_{00}^+\) (Eqn.\[3\]).}
\end{figure}
(χ^2/point = (4, 5, 3.5) for KA84). The PWA and DR solutions clearly favour a πNN coupling constant f^2 = 0.0759 ± 0.0004 (χ^2/ν = 13.72 ± 0.07) [1], consistent with our recently published solutions [12]. This value is compatible with most recent determinations [15] and ~5% below the canonical value 0.079 used in the KH80 and KA84 solutions.

For the subthreshold coefficients from the GLLS method, we obtain d_0^+= -1.15±0.03 and d_1^+ = 1.27±0.03 m_π^{-1}, where the uncertainty is from the energy fluctuations only (see Figs. 1 and 2). This implies Σ_d ≃ 78 MeV (Eqn. 2), which is ~55% larger than the canonical result ≃ 50 MeV [2,6,5]. As a check of our dispersion relation machinery, we inputed the Karlsruhe KA84 [9] phases and reproduced their f^2 and Σ_d results exactly. Table 1 shows a term by term comparison between SM99 and KA84 to analyze the differences.

Though the difference between the SM99 and KA84 Σ_d values is surprisingly large, one expects about 21 MeV of the difference from new pionic atom data, a lower coupling constant, and a narrower Δ resonance width. The isoscalar scattering length a_{0+}^+ ≃ -0.008 m_π^{-1} for KA84 (and KH80), but analyses of recent PSI pionic hydrogen and deuterium results yields |a_{0+}^+| ≃ 0.0015 [8] or ≃ +0.0002 [15]. Our analysis used the latter, while the “expectation” in Table 1 assumes 0.000±0.003. A lower coupling constant around f^2 = 0.0755 ± 0.0010 is favoured by most analyses [15] and this “expectation” contributes +7 MeV in Table 1 from the E^+ Born term. The C^+ Born term does not change due to a well known insensitivity to f^2. And it is well known that the Δ resonance width is too wide in KA84 (overshoots the total cross sections on the left wing), so since ImD^+ is proportional to the sum of the π^+p and π^-p total cross sections via the optical theorem, one expects the D^+ integral contribution to decrease. Due to Δ region dominance of the C DR, the Δ width and f^2 are correlated, and a ~5% decrease in f^2 roughly corresponds to a same decrease in the integrals, and this expectation is reflected in Table 1. A narrower Δ also would reduce the E^+ DR integral, but possible changes in higher partial waves make predictions less clear. So from rather general considerations, one expects a significant increase from the canonical value for Σ_d based on new experimental information.

The result from the tangent of the C^+{(0, t)} subtraction constants at t=0 yields d_0^+= -1.15±0.03 and d_1^+ = 1.23±0.03, where the uncertainties reflect only the energy fluctuations of the constants (see Fig. 3). This yields Σ_d=80 MeV, consistent with our other determination. Figure 4 shows this result along with the tangent inferred from the forward C^+ and E^+ DR analysis. The consistency is not perfect, and the slight differences in the d_0^+ values, which are believed to be understood, are being studied further.

In summary, we have performed a new πN partial wave and dispersion relation analysis, from which we obtain Σ = 91±8 MeV using two different methods, about 27 MeV larger than the canonical result 64±8 MeV from Ref. [1]. At first glance the result is indeed surprising, but a large upward change is in fact expected based on new information on a_{0+}^+ ≃ 0.000 from pionic hydrogen and deuterium [15,18], a lower πNN coupling constant f^2 ≃ 0.0755 [13], and a narrower Δ resonance width. Further study is planned to explore systematic uncertainties and to resolve small inconsistencies. A new analysis based on the the Karlsruhe methods [3,4] applied to the modern data is urged to check these findings.

*See our companion article on our f^2 determination in these proceedings for details [14].
Figure. 4. Tangent at $t = 0$ (dashed line) of the SM99 $\bar{C}(0,t)$ subtraction constants (solid squares, which include r.m.s. errors), with tangent inferred from our forward $C+$ and $E^+$ DR analysis overlayed (solid line). The slight discrepancy is understood and under investigation. Nonetheless, both yield $\Sigma_d \simeq 79$ MeV, and clearly inconsistent with the KA84 result $\simeq 50$ MeV.

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