Rugosity and hardness determination in obsidianus lapis for the design of an Yb$^{3+}$-doped fiber laser

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Abstract. Obsidianus lapis is a volcanic rock that has been worked into tools for cutting or weaponry by Teotihuacan people for hundreds of years. Currently, it is used in jewelry or for house decorative items such as elaborated sculptures. From the physico-chemical properties point of view, obsidianus lapis is considered a glass as its composition is 80% silicon dioxide. In México, there are different kinds of obsidianus lapis which are classified according to its colour: rainbow, black, brown, red, silver, golden and snowflake. The traditional grinding process for working with obsidianus lapis includes fixed grinders and sandpaper for the polishing process, where the craftsman grinds the rock manually for obtaining a variety of shapes. Laser processing of natural stones is a relatively new area. We propose the use of an Yb$^{3+}$-doped fibre laser for cutting and ablating obsidianus lapis into spherical, rectangular and oval shapes. By means of a theoretical analysis of roughness and hardness, which affect the different surfaces and final shapes, and by considering the changes in material temperature during laser interaction, this work will focus on parameter determination such as: laser fluence, incidence angle, laser average power and peak pulse energy, from the proposed Q-switched fibre laser design. Full optical, hardness and rugosity, initial and final characterization will be included in the presentation.

1. Introduction

Obsidianus lapis is classified as one of the natural volcanic glasses. It has been expelled mixed with basaltic strata and pumice rock, it is produced when molten igneous rock (magma) comes out to the earths surface as lava and cools down so quickly that their ions have not time to crystallize. The magma that forms the obsidianus lapis has its origin approximately 40 km below the earths surface and has a temperature of nearly 1200°C [1].

Obsidianus lapis is composed with 80% of silicon dioxide. The chemical composition of silicon dioxide is 46.7% silicon and 53.3% oxygen in the empirical formula SiO$_2$. The presence of hematite produces a variety of colors in obsidianus lapis. Hematite is a mineral composed of ferric oxide (Fe$_2$O$_3$). Obsidianus lapis could contain small bubbles of gas, which are aligned along the layers created in the molten rock, which flow just before cooling. These bubbles can produce interesting effects such as golden, green, gray, black, red-brown, silver, rainbow and blue shine [2].
Characterization
For obsidianus lapis characterization, we performed Vickers hardness testing in a cube with 2cm of edge. Different results of Vickers hardness were obtained, according to the indentation, by using a force of 10 Kg. The results are in a range of 110HV10 - 150HV10.

The manufacturing of obsidianus lapis into shapes is performed with a process including the 7 following steps: 4 for grinding, 2 for polishing and 1 for shining. Using a Mitutoyo surf test 301 roughness tester, the data shown on Table 1, were obtained.

Table 1. Roughness average

| Process                        | Roughness average (μm) |
|--------------------------------|------------------------|
| Diamond disc                   | 3.4                    |
| Grinding wheel grain size 35   | 3.3                    |
| Grinding wheel grain size 90   | 2.83                   |
| 120-grit sand paper            | 1.98                   |
| 180-grit sand paper            | 1.23                   |
| Cerium oxide                   | 1.02                   |

The average roughness for artistic finishing (100nm) that we need to obtain from the proposed fibre laser craftsmanship process is 1.02 μm since polishing the surface into its final values, is the most attractive feature for the osidianus lapis samples.

Absorptivity calculation
In material cutting, absorptivity calculation relies on vaporization, the focused beam heats up the surface up to the boiling point first and then generates a keyhole. The keyhole causes a sudden increase in the absorptivity due to multiple reflections, causing the hole to deepen quickly [4].

Absorptivity in obsidianus lapis can be calculated as a function of average reflectivity $R_{ave}$, as the ratio of the reflected power $P_{L,R}$ of the total laser power $P_L$. It depends on the polarization state of the incident laser radiation. Reflectivity values for $R_p$ y $R_s$, for parallel and perpendicular polarizations, are established for the Fresnel equations 1 and 2 [3], as follows:

$$R_p = \frac{(n \cos \varphi_{in} - 1)^2 + (k \cos \varphi_{in})^2}{(n \cos \varphi_{in} + 1)^2 + (k \cos \varphi_{in})^2}$$

(1)

$$R_s = \frac{(n - \cos \varphi_{in})^2 + k^2}{(n + \cos \varphi_{in})^2 + k^2}$$

(2)

Where $\varphi$ is incidence angle, $n$ is the refractive index and $k$ the extinction coefficient. In case of no polarized radiation and for circularly-polarized radiation, the reflectivity $R_{ave}$ can be estimated as the average value of the two reflectivity components, according to equation 3:
The optical parameters of refractive index “n” and the extinction coefficient are a function of permittivity, according to equations 4 and 5:

\[ n = \left( \frac{\left( \varepsilon_1^2 + \varepsilon_2^2 \right)^{\frac{1}{2}} + \varepsilon_1}{2} \right)^{\frac{1}{2}} \]  \hspace{1cm} (4) 

\[ k = \left( \frac{\left( \varepsilon_1^2 + \varepsilon_2^2 \right)^{\frac{1}{2}} - \varepsilon_1}{2} \right)^{\frac{1}{2}} \]  \hspace{1cm} (5) 

Where: \( \varepsilon_1 \) is the real and \( \varepsilon_2 \) the imaginary part of the materials permittivity. These parameters can be calculated with equations 6 and 7:

\[ \varepsilon_1 = 1 + \frac{\omega_p^2}{\omega_L^2 + V_e^2} \]  \hspace{1cm} (6) 

\[ \varepsilon_2 = \frac{V_c \cdot \omega_p^2}{\omega_L^2 \cdot \omega_c^2 + V_c^2} \]  \hspace{1cm} (7) 

The laser frequency \( \omega_L \) can be calculated using equation 8 and it is inversely proportional to the wavelength of the emitted radiation.

\[ \omega_L = \frac{2\pi \cdot c_o}{\lambda} \]  \hspace{1cm} (8) 

c_o is the speed of light in free space. The laser frequency of an Yb^{3+}-doped fibre laser corresponds to 1.762X10^{15}s^{-1} for a lambda of 1.07 \( \mu \)m. The plasma frequency is proportional to the square root of the free electron density and can be obtained with equation 9 as:

\[ \omega_p = \left( \frac{e^2 \cdot n_p(T)}{\varepsilon_0 \cdot m_e} \right)^{\frac{1}{2}} \]  \hspace{1cm} (9) 

Where the value of the electron charge is e=1.602X10^{-19}C, vacuum permittivity is 8.85419X10^{-12} F/m and electron mass is 0.91095X10^{-30} kg. Equation 10 below shows how to calculate free electron density:
\[ \eta_e(T) = N_A \cdot \frac{Z \cdot \rho(T)}{A_t} \]  

(10)

In this equation, \( N_A = 0.6022 \times 10^{24} \text{ mol}^{-1} \) is Avogadro's number. \( Z \) being the number of valence for silicon, and since obsidianus lapis is composed with 80% of this material, we use \( Z = 4 \), and the atomic mass \( A_t = 28.086 \times 10^{-3} \text{ Kg/mol} \). The density of obsidianus lapis in liquid form is a function of temperature and it is calculated with equation 11:

\[ \rho = \rho_{m.p.} - \Lambda \cdot (T - T_{m.p.}) \]  

(11)

Where \( T_{m.p.} \) is the melting point temperature and \( \rho_{m.p.} \) at the melting point, for obsidianus lapis \( \rho_{m.p.} = 1492 \text{ kg/m}^3 \), \( \Lambda = 0.88 \text{ kg/m}^3 \cdot \text{K} \) and \( T_{m.p.} = 1383 \text{ K} \), the boiling point \( T_{b.p} = 2628 \text{ K} \). For calculating \( \eta_e(T) \), one can use values from \( 1.279 \times 10^29 \text{ m}^{-3} \) up to \( 3.399 \times 10^28 \text{ m}^{-3} \). So the plasma frequency at the melting point has a value of \( \omega_p = 2.018 \times 10^{16} \text{ S}^{-1} \) and at the boiling point \( \omega_p = 1.0402 \times 10^{16} \text{ S}^{-1} \). For obtaining the collision frequency \( V_c \), we use equation 12:

\[ V_c = \frac{\rho_e(T) \cdot \eta_e(T) \cdot e^2}{m_e} \]  

(12)

Where \( \rho_e(T) \) is the electric resistivity is a function of temperature and is calculated with equation 13.

\[ \rho_e(T) = \rho_{e,0} + \Lambda_e \cdot (T - 273 \text{ K}) \]  

(13)

Where \( \rho_{e,0} = 112.3 \times 10^{-8} \text{Ωm} \) and \( \Lambda_e = 0.0154 \times 10^{-8} \text{Ωm/K} \). Therefore the electric resistance at melting point is \( \rho_e(T) = 1.294 \times 10^4 \text{Ω} \) and at boiling point it is \( \rho_e(T) = 1.485 \times 10^6 \text{Ω} \). We obtained the values for refractive index \( n \), and for extinction coefficient \( k \) at melting point, boiling point and an average, as shown in table 2.

**Table 2.** Refractive index, and extinction coefficient of obsidianus lapis, at the melting point, boiling point and at the average.

| Temperature | Wavelenght \( \lambda = 1.07 \text{ μm} \) |
|-------------|----------------------------------|
|             | \( n \)                           | \( k \)                       |
| \( T_{m.p.} \) = 1383K | 5.646                            | 3.812                         |
The average theoretical absorptivity of obsidianus lapis as a function of the angle $\phi_m$ using an Yb$^{3+}$-doped fibre laser is shown in figure 1.

![Figure 1. Behaviour of absorptivity of obsidianus lapis for an Yb$^{3+}$-doped fibre laser](image)

The angle of incidence with better absorption is 79º with absorptivity higher than 50%. After reaching this angle the absorptivity decreases, as it is observed.

For the proposed laser design we define the laser beam power as $P_{out}$ for the proposed obsidianus lapis cutting process. In the case of laser beam for cutting, we can calculate the power required for melting a volume of material per time unit, using equation 14 as:

$$P = \rho \cdot W_K \cdot T_C \cdot V_C \cdot \Delta h_m$$  

(14)

Where $\rho$ and $\Delta h_m$ are the density and the necessary increase in the specific enthalpy to cause the melting of the material. For obsidianus lapis, we consider values of $\rho = 2360 \text{ Kg/m}^3$, $W_K = 0.01 \text{m}$, $T_C = 0.001 \text{m}$, $V_C = 0.01 \text{m/s}$, $\Delta h_m = 1650\text{KJ/Kg}$, and we obtain a required average output power of $P = 389.4 \text{ W}$. 

| $T_{ave}$ | 2005K | 5.526 | 3.164 |
|----------|-------|-------|-------|
| $T_{b.p.}$ | 2628K | 4.980 | 1.689 |
Laser Design

Along with the increase in average power, the emergence of large mode area technology has made possible a number of achievements in the fundamental capabilities of fiber lasers, such as peak power and pulse energy, by means of an acousto-optic modulator. That is a valid reason for proposing a Q-switched fibre laser for obsidianus lapis processing [5].

Thus, a Q-switched fibre laser design with an Yb3+-doped for grinding and polishing obsidianus lapis is proposed. Fig. 2 shows the components for the system. It consists of a laser diode (A), a collimating lens (B), a dichroic mirror (C), a focusing lens (D), the Yb3+-doped fibre piece (E), an acousto-optic modulator (F), and high-reflectivity, broad band mirror (G).

![Figure 2. Yb3+-doped fibre laser design and components](image)

In the Q-switched fibre laser design, the pumping with a laser diode at 976 +/-10nm was made in a continuous wave output beam of 15W. After considering the losses in the system that we designated in figure 2, we calculate the average power $P_{\text{out}}$ of the system. Background losses of 0.3dB/m is employed. A quantum efficiency of Ytterbium is considered 91%, absorption in the fibre is 3dB/m @976nm, and the average power has a value of $P_{\text{avg}}=7.406$W in continuous wave.

For obtaining theoretical parameters in the fibre laser design, we use a length of 3m for the Yb3+-doped fiber, and from equation 15, the peak power of 1.33MW is obtained.

$$p_r = \frac{P_{\text{avg}}}{R \cdot \sigma_p}$$ (15)

Where $P_{\text{avg}} = 7.4$ W, repetition rate is proposed at $R_r = 500$Hz and pulse duration $\sigma_p = 11.11$ns. Pulse energy is calculated via equation 16 as:

$$P_e = \frac{P_{\text{avg}}}{R_r}$$ (16)

A value of $P_e = 0.0148$J is then obtained. After varying the repetition rate, we obtain values shown in figure 3, for $P_{\text{avg}}$ varying from 4.4 to 9.4 W.
Fluence is the ratio of the applied laser energy within the cross-sectional area of the incident laser beam. The fluence of an individual laser pulse is calculated by using equation 17\[6\].

\[
F = \frac{8E_0}{\pi\omega_0^2}
\]  
(17)

Where \(\omega_0\) is the Gaussian beam diameter; \(\omega_0 = 30\pm/2\mu m\), and \(E_0\) is the energy of the incident beam; \(E_0 = 0.0148\)J. The final determination of fluence produces a value of \(F = 4187.54\) J/cm\(^2\).

**Conclusion**

The study of the main parameters and components in a laser design has been proposed for cutting obsidianus lapis. Cutting can be made in rock and in metal even there is less evaporation and considerable melting. The analysis of crucial design parameters involved in the interaction mechanisms between the fiber laser beam and rock, allows us the investigation of novel ways of rugosity and hardenes determination along with ablation in obsidianus lapis. The designed Q-switched Yb\(^{3+}\)-doped fibre laser, with short-pulse fibre technology, produces a diffraction-limited beam at the output of the system, with which different surfaces finishes can be processed. Theoretical results can be summarized as: i) Absorptivity in obsidianus lapis with Yb\(^{3+}\)-doped fibre laser is higher than 40% with incident angles from 50\(^\circ\) to 79\(^\circ\), this range is important in beam efficiency. Calculated results show the effects on obsidianus lapis and different values for pulse energy with respect to repetition rate. ii) Obsidianus lapis structure can be melt with a Q-switched Yb\(^{3+}\) doped fibre laser with a value of pulse energy of 0.0148J that can even be modulated.

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