Abstract

Gamma ray burst (GRB) objects are now widely thought to be at cosmological distances, and thus represent enormous energy emission. Gamma ray spectra extending to GeV energies suggest the possibility of accompanying neutrino emission, and there are several models proposed suggesting the potential detectability of such coincident neutrino bursts. With this in view, we examine possible measurements that might be conducted to give experimental data useful for astronomy, for cosmology and also neutrino properties.

Of interest to astronomy and cosmology, we show how measurement of neutrino flavor ratios yields information on the nature and relative distance of the source. We point out that cosmological time dilation might be measured for these sources using neutrinos, as has been done for photons, and that neutrino oscillation lengths in the range of $1$ to $10^5$ Mpc can be probed with GRB neutrinos. We thus note that these sources may make possible the first non-electromagnetic measurements of the scale size of the universe. We discuss tests of the weak equivalence principle, tests for flavor dependent gravitational couplings, and tests for long time scale variation of physical
constants. We also show that a number of new bounds on neutrino properties (charge, mass, speed, lifetime) could be facilitated to levels well beyond those already inferred from the neutrino observation of SN1987A.

We also examine the implications of these physics opportunities for designers of neutrino telescopes. We conclude that detection may be possible in planned instruments if the spectra are power law extending to the $\text{TeV}$ energy region, and if the neutrino fluxes are equal to or greater than the gamma ray fluxes. We emphasize the importance of low energy detection in future experiments for the tests described above.

1 Introduction

Recent observations of gamma ray bursts (GRBs) supports the idea that these objects may be distributed over cosmological ranges\[1\]. The GRBs may be among the most luminous objects in the universe, with peak gamma ray luminosities $L \sim 10^{51} \text{ergs/sec}$. There is evidence that the range of intrinsic luminosities is very narrow, which might allow distance determinations out to red–shifts of 2 or more\[2\]. It is plausible that when such large amounts of energy are released in objects which are evidently quite compact, pions will be produced in hadronic collisions, and that the GRBs may serve as standard candles in bursts of neutrinos as well. The ($\sim 1/E^2$) power law spectrum of gamma rays which is observed to extend at least to several GeV, suggests the possibility of particle acceleration and radiation from other than electromagnetic interactions of electrons. Paczynski and Xu \[3\] have recently proposed a specific GRB model in which the gamma rays and neutrinos arise from decay of pions produced in shock front collisions. In a second class of models (e.g. Plaga \[4\]) gamma rays are hypothesized to come from superconducting cosmic strings (SCSs); the luminosities are very high and one expects neutrino emission as well.

Models of the first kind\[3\] will have the following generic features for the neutrinos emitted: the neutrino energies range over MeV to (a few) GeV, or perhaps much higher, and since the source is $\pi$-decay, the flavor content has the proportions $\nu_e : \nu_\mu : \nu_\tau = 2 : 1 : 0$. We also note that due to gamma ray absorption within the source, the luminosity in neutrinos can, in principle,
exceed the observed luminosity in gamma rays, potentially by a large factor. In models of the second kind, the neutrino flavor content is expected to be $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$ and the energies can reach up to $10 \text{ TeV}$.

In this paper we suppose that the GRBs are indeed cosmological standard candles (at least in an ensemble average sense) for neutrinos and gammas, in order to maximize the physical inferences to be drawn from coincident neutrino and photon detection. We investigate the opportunities for neutrino astronomy, for cosmology, and for particle physics. Some of this analysis may also be applicable to other astrophysical neutrino sources, such as AGNs which are expected to emit neutrinos over a very large energy range; AGNs will probably have observational data exceeding what is possible from GRBs, but are unlikely to have such short time pulse emission. We hope that our analysis will be useful to designers of neutrino telescopes and inspire them to consider GRB neutrino detection as an important goal for future instruments.

2 Neutrino Mass Mixing and Oscillations

One of the unique properties of three generations of neutrinos, not shared with photons, is that they may carry with them two additional scale lengths associated with flavor oscillations. The oscillation lengths are related to the neutrino mass differences according to

$$l_{ij}(\text{km}) \approx 2.5\left[\frac{p(\text{GeV})}{\delta m_{ij}^2 (\text{eV}^2)}\right],$$

where $i$ and $j$ are flavors and $\delta m_{ij}$ is the, currently unknown, mass difference between neutrinos of the relevant flavors. The probability of observing neutrinos of a given flavor at any distance from their source is a well known function of the distance, the oscillation length and mixing parameters, as well as the initial conditions. Therefore, if the neutrinos are produced in the same

---

1 Recent emphasis has centered on neutron star models, motivated in part by the observation that the gamma-ray energy released by GRBs at cosmological distances is of order of a per cent of the binding energy of a neutron star if the emission is isotropic, and less if the emission is beamed. In analogy to a supernova one might expect 99 to 99.9% of the binding energy is expected to emerge as neutrinos, yielding a neutrino–to–photon energy emission ratio of $10^2$ to $10^3$.

2 However, Markarian 421 has been seen by the Whipple Observatory very–high–energy $\gamma$-ray telescope to have a flux increase of a factor of ten on a time scale of one day.
flavor ratio in each GRB, then the measured flavor ratios can, in principle, determine the relative distances between various sources, at least for the case of small mass differences and consequent long oscillation lengths. Absolute distances will not be given directly by the measured flavor ratios, because the oscillation length itself will be difficult to measure independently.

However, if GRBs are standard candles whose distances can be calibrated (e.g., by the time dilation of their spectra, or by gravity waveforms of inspiraling objects), then the measured flavor ratios may provide a direct determination of very small neutrino mass differences. Competing models place the GRBs (i) within the solar neighborhood \((D \sim \text{pc})\), (ii) in the galactic halo \((10^{-2} \text{kpc} \sim D \sim 10^{2} \text{kpc})\), and (iii) at cosmic distances \((D \sim \text{Mpc})\) as we assume herein. Assuming detection of a correlated neutrino with GeV energy, the \(\delta m^2\)'s which are probed at each distance scale are \(\sim 10^{-13} \text{eV}^2\), \(\sim 10^{-14}\) to \(\sim 10^{-18} \text{eV}^2\), and \(\sim 10^{-19} \text{eV}^2\), respectively.

If the neutrino oscillation indications from the atmospheric neutrino observations are correct \((\delta m^2_{\mu\tau} \sim 0.01 \text{eV}^2\), and \(\theta_{\mu\tau}\) large, \(\sim 20^\circ - 40^\circ\)\), then the observed flavor content of the neutrinos from pion–producing GRBs should show the same effect, viz. a \(\nu_{\mu}:\nu_e\) flavor content of 1.2:1 rather than 2:1. This applies to models of the Paczynski-Xu kind. Furthermore, because the atmospheric solution has a short oscillation length \(l_{\mu\tau} \sim 250\text{p(GeV) km}\), this flavor ratio should be universal for all GRBs independent of their distance; individual GRBs might have hot source spots smaller in size than the oscillation length \(l\), but individual oscillations will sum to zero in an ensemble average. For the models like that of Plaga, based on SCS’s, the democratic flavor mixture is unchanged by oscillations and remains \(\nu_{\mu}:\nu_e: \nu_{\tau} = 1:1:1\). It may be thus possible to distinguish between these two classes of source models.

If the atmospheric neutrino anomaly is not due to neutrino oscillations, then there is a more interesting possibility for GRB neutrinos. According to the see-saw mechanism,

\[
m_\nu \sim m_D^2/M
\]

where \(m_D\) is the generation–dependent Dirac mass of either the up quarks, down quarks, or charged leptons \(m_l\). If we use \(m_D \sim m_l\) and \(M \sim \text{Planck Mass}\), the neutrino masses\(^3\) are \(m_{\nu_e} \sim 2 \cdot 10^{-17} \text{eV}\), \(m_{\nu_\mu} \sim 10^{-12} \text{eV}\), \(m_{\nu_\tau} \sim \)

\(^3\)Neutrino mass differences induced by scattering on the inter-galactic medium are
3 \cdot 10^{-10} \text{eV}, \text{ and hence } \delta m^2_{ee} \sim 10^{-24} \text{eV}^2 \text{ and } \delta m^2_{\mu\tau} \sim 10^{-19} \text{eV}^2.

In this see–saw case, the oscillation lengths are in the cosmic range 1 to 10^5 \text{ Mpc} for neutrino energies in the GeV range\(^4\). There is a maximum distance over which a mixture of mass eigenstates can be expected to remain coherent\(^1\). However, for the tiny mass differences here, the coherence length appears to exceed 10^5 \text{ Mpc}.

The longer oscillation length possibilities exceed the size of the visible universe \(\sim H_0^{-1} = 3000h^{-1} \text{ Mpc}\), where \(H_0 = 100h \text{ km/sec/Mpc}\) is the present value of the Hubble parameter. If \(l\) exceeds \(H_0^{-1}\), then the neutrinos do not oscillate, and the flavor ratios observed at earth are just those established at emission. On the other hand, for \(l\) less than \(\sim 10^4 \text{ Mpc}\), the flavor content of the neutrinos from pion–producing GRBs will vary strongly with distance; and again, for Plaga-like models the flux will remain universal with no dependence on distance.

Alternatively, if mass differences are large (of order 0.01 \text{eV}^2 or larger), then the separation of the mass eigenstates in time offers another handle on \(\delta m^2\). At a fixed energy \(E\), the arrival times of the \(\nu\)'s are separated by

\[
\delta t = 5 \times 10^{-3} \left(\frac{L}{100 \text{ Mpc}}\right) \left(\frac{\delta m^2}{10^{-2} \text{eV}^2}\right) \left(\frac{E}{100 \text{ MeV}}\right)^2 \text{ sec},
\]

assuming much smaller time difference at the source. For low energies and large \(\delta m^2\), \(\delta t\) can range from \(10^{-3} \text{ sec}\) to several seconds. With a spectrum of energies, the neutrino burst would be spread more than the accompanying gamma ray burst.

## 3 Other Neutrino Properties

As with the observations of Supernova 1987A, the distance, short emission time and trajectory through varying gravitational fields, leads to the potential for some fundamental tests of neutrino properties, which are not possible in terrestrial laboratories\(^{12}\). In the following we enumerate some of the possibilities.

\(^{10}\) expected to be smaller than these numbers\(^{10}\).

\(^{12}\) For most purposes in this paper, it is sufficient to equate time and distance as if the universe were static. The generalization to an expanding cosmology is reserved for §4.
3.1 Neutrino Lifetime

From the detection of neutrinos arriving form the 50kpc distant SN1987A it was possible to place a limit on the (laboratory frame) lifetime of the $\bar{\nu}_e$ of $\tau(\bar{\nu}_e) > 5 \cdot 10^{12}$ sec for $E_\nu$ of the order of 10 MeV. In the same way, observation of $\nu$'s from GRBs would place bounds on $\tau(\nu) > 10^{17}(D/Gpc)$ sec, which for a 10 Mpc distant source is some 200 times stronger than the bounds on $\bar{\nu}_e$ from SN1987A. Of course this is really a bound on the lifetime of the dominant mass eigenstate.

3.2 Neutrino Electric Charge

If neutrinos carry even a tiny electric charge (as favored in some theoretical models\textsuperscript{[13]}), then the passage through magnetic field regions enroute to the earth from a distant source creates an additional dispersion in arrival time, $\delta t$. From the observed upper bound on any additional dispersion beyond the gamma pulse time distribution one can then derive a limit on the neutrino charge from the Barbillieni-Cocconi formula\textsuperscript{[14]}:

$$\frac{Q_\nu}{|e|} < \frac{\delta t}{D} \sqrt{\frac{<E>}{0.6DB}} (\delta E/E)^{-1/2},$$

where $\delta t$ is the burst duration, $D$ is the flight distance, $B$ is the root–mean–squared average magnetic field experienced by the neutrinos enroute, and $\delta E/E$ is the relative spread in energies. For $\delta t \sim 10^{-3}$ sec, $D \sim 10$ Mpc, $<E> \sim 1$ GeV, $B \sim 10^{-12}$ Tesla and $\delta E/E \sim 1$, one has $Q_\nu/|e| < 10^{-27}$. Hence it may be possible to improve considerably on the SN1987A limit of $Q_{\nu_e} < 10^{-14}|e|$, and the somewhat better laboratory limit of $Q_{\nu_e} < 10^{-19}|e|$. Moreover, one could place the first limit on $\nu_\mu$, and possibly on $\nu_\tau$.

3.3 Neutrino Speed

To the extent that the $\gamma$-ray and neutrino pulses coincide, new limits on neutrino speed relative to the speed of light may be placed. If $\delta t = t_\nu - t_\gamma$, then

$$1 - v_\nu/c \leq \delta t/D.$$
For $\delta t$ as large as 1 second this can place upper bounds of the order of $10^{-15}(10\text{Mpc}/D)$ on $1 - \beta_\nu$, to be compared with $10^{-9}$ (for $\nu_e$) from SN1987A [15].

### 3.4 Equivalence principle

The neutrinos from SN1987A were used to establish limits on parameters in various theories of gravitation and to test whether the Weak Equivalence Principle (WEP) is symmetric with respect to bosons and fermions, and with respect to matter and anti-matter. Specifically, the Shapiro delays for gammas, neutrinos, and anti-neutrinos passing the Galactic nucleus were compared and found to be the same, within errors [16, 17]. Also, SN1987A provided tests for the presence of proposed new forces in nature and tests of the equality of parameters when applied to photons, neutrinos, or anti-neutrinos [17, 18]. The same methods would apply to neutrinos from GRBs. However, under our assumptions, the distances would be greater and the impact parameter with the Galactic nucleus would vary from event to event, offering much improved sensitivity and a new distance scale, as well as a large increase in statistics.

To test the possibility that the neutrino and antineutrino gravity couplings differ requires tagging of neutrino and antineutrino events separately. Distinguishing between $\nu_e$ and $\bar{\nu}_e$ interactions at low energy may be possible because of the charged current capture by protons. At higher energies distinguishing between $\nu_\mu$ and $\bar{\nu}_\mu$ interactions is in principle possible if one employed a magnetic field, or if one could adequately detect the muon capture by nuclei; but in practice it cannot be done in instruments proposed at this time. A $\nu - \bar{\nu}$ separation would also be useful for studying possible CP violation in neutrino oscillations [19].

A flavor violating gravitational coupling has been proposed as a possible mechanism for accounting for atmospheric neutrino as well as solar anomalies [20]. It is remarkable that one choice of parameters ($\sin^2 2\theta_G \sim 1$, $\delta f \sim O(10^{-15})$) can account for both problems. The transition probability for $\nu_\mu \leftrightarrow \nu_e$ is $\sin^2(2\theta_G)\sin^2(\frac{1}{2}LE\phi(L)\delta f)$, where $\phi$ is the gravitational potential and $\delta f$ is a measure of the degree of violation of the WEP. Without a knowledge of $\phi$ along the path it is not possible to calculate the net effect on the mixing, but it is very likely similar to the expectation in the oscillation case (i.e., $\nu_e : \nu_\mu$ becomes 1.2 : 1 from 2 : 1).
4 Cosmology

Norris, et al. [21], have analyzed the experimental data assuming that the GRBs are standard candles in gamma rays and have found evidence for a cosmological time dilation. If the GRBs are neutrino sources, then the same analysis can provide, for the first time, non-electromagnetic evidence for the expansion of the universe.

The prevailing view, that the expansion of the universe is of a universal nature, leads to the expectation that the gamma ray and neutrino time dilations will be identical. Unfortunately, a nearly identical dilation may occur if the cause is evolutionary effects. This is because the charged and neutral pions of the pion–production model, and the emitted photons and neutrinos of the SCS model arise from a common mechanism. Still, it is possible that the neutrino and photon opacities evolve differently, in which case studies of the differing dilations may yield information on cosmic evolution.

An important probe of the universe at an early time is the oscillation phase itself, $\phi_{ij}$. In Minkowski space, the quantum mechanical phase is just $Et$. However, it is a bit more complicated in the expanding universe. In the adiabatic approximation, the phase is

$$\phi = R_0 \int_0^\tau \frac{d\tau'}{R(\tau')} E. \quad (2)$$

We have assumed a time–dependent Robertson–Walker metric, with scale factor $R(\tau)$; $\tau$ is the lookback time to the source emission, and $R(0) \equiv R_0$. The red–shift factor $R_0/R(\tau)$ accounts for the time dilation of our clocks compared to early universe clocks. Taking the difference of two mass–eigenstate phases, expanding $E_i \simeq p + m_i^2/2p$, and red–shifting the momentum to its present–time observed value via $p(\tau) = (R_0/R(\tau))p_0$, one obtains from Eq. (3) the very simple result for the oscillation phase:

$$\phi_{ij} = \frac{\tau \delta m_{ij}^2}{2p_0}. \quad (3)$$

The blue–shift of the inverse momentum exactly cancels the red–shift from time dilation. Thus, the correct generalization of the oscillation phase

---

5 We note that such an analysis would exclude any cosmological model which attempts to explain the observed red–shifts by “tired photons”; earlier arguments against the tired photon hypothesis were given by Zeldovich, et al. [22].
from Minkowski space to an expanding cosmology is obtained by replacing laboratory time with cosmic lookback time, or equivalently, replacing $l_{ij}$ in Eq. (1) with $c\tau_{ij}$.

This deceptively simple result is rich in cosmological information. In particular, if $\delta m_2^2$ were a priori known, then a measurement of the flavor ratio and $E_\nu$ would directly yield $\tau$. This is analogous to obtaining the cosmic red–shift $z$ directly from a measurement of a photon’s energy, when the unshifted spectral line of the photon source is a priori known. Furthermore, $\tau$ contains as much information as is conveyed by $z$. In fact, $\tau$, $z$, and the distance $D$ are linearly related for small $z$, $\tau$, $D$, and nonlinearly related for large $z$, $\tau$, $D$. For example, to first nonlinear order, some Taylor series expansions relating these three variables are

\[ z(\tau) = H_0\tau + (1 + q_0^2)(H_0\tau)^2 + \ldots \]

and its inverse relation

\[ \tau(z) = H_0^{-1}z - (1 + q_0^2)z^2 + \ldots \]

and

\[ D_L = H_0^{-1}[z + \frac{1}{2}(1 - q_0)z^2 + \ldots] \]

Here, $D_L$ is the “luminosity” distance defined as $D_L^2 = \mathcal{L}/4\pi F$, where $\mathcal{L}$ is the absolute luminosity at the source (energy/time), and $F$ is the fluence measured at earth (energy/time/area). Results for other distance definitions are similar; e.g. the “proper” distance ($R_0 \times$ comoving coordinate distance), is given as

\[ D_P = H_0^{-1}[z - \frac{1}{2}(1 + q_0)z^2 + \ldots] \]

In all of these relations, $q_0$ is the present value of the deceleration parameter.

The value of $q_0$ is unknown; since the GRBs seem to exist at cosmic distances, it is possible that oscillation measurements with GRB neutrinos may shed some “neutrino light” on this important parameter. An independent measurement of even two of the three variables $z$, $\tau$, and $D$ would potentially determine $q_0$ and test cosmological models, because of the nonlinear relations. The series expansions make it clear that the linear Hubble relation fails by a fractional amount $z$ in red–shift, $H_0\tau$ in lookback time, and $D/H_0^{-1}$ in distance. Interpreting the burst dilation of fainter GRBs as due to time dilation leads to the estimate $z \sim 1$ to 2 for these GRBs, so there is indeed hope that oscillation measurements may yield a value for $q_0$. We have seen that for small neutrino masses, it may be possible to measure the phase of oscillations from pion–producing GRBs. Of course, one would require a much better understanding of the structure and mechanisms of GRBs than we have today, in order to draw useful conclusions.

Let us assume for the moment that such measurements can be made with enough precision to yield values for $\tau \delta m_2^2$. If an independent measurement of $z$ or $D$ is available, then a single GRB would suffice to fix $\delta m_2^2$, and a
second GRB measurement would yield $q_0$. In this idealized situation, we may also inquire about the nature of higher order terms in the nonlinear expansions relating $z$, $\tau$, and $D$. In fact, given a cosmological model, the nonlinear relations among $z$, $\tau$, and $D$ are exactly calculable. For example, $\tau = \int_{R(\tau)}^{R_0} dR/\dot{R} = \int_{R(\tau)}^{R_0} d\ln R/H$ will yield $\tau(1 + z = R_0/R(\tau))$ once $\dot{R}(\tau)$ or equivalently, $H \equiv \dot{R}/R$ are determined as a function of $R$ (or $z$) from the Friedmann equation. Ignoring the radiation energy density of the recent universe compared to the matter density, the Friedmann equation reads $H(z)^2 = H_0^2 \left[ (1 + z)^2 (1 + z\Omega_0) - z(2 + z)\Omega_\Lambda \right]$, where $\Omega_0$ is the present matter density compared to the critical value $\rho_c = 3H_0^2/8\pi G$, and $\Omega_\Lambda = \Lambda/3H_0^2$; $\Lambda$ is the cosmological constant. (The Friedmann universe is open or closed according to whether $\Omega_0 + \Omega_\Lambda$ is less than, or greater than, unity.) The Friedmann equation may be manipulated to yield

$$\tau = H_0^{-1} \int_{1}^{1+z} \frac{d\omega}{\omega} \left( \Omega_0 \omega^3 + \Omega_\Lambda - \Omega_k \omega^2 \right)^{-\frac{1}{2}}, \quad (4)$$

where the curvature term $\Omega_k = \Omega_0 + \Omega_\Lambda - 1$ is a constrained by the Friedmann equation itself. A thorough set of numerical solutions to this equation may be found in ref.[24]. With $\Omega_\Lambda$ omitted, the integral is easily solved analytically. The form of the solution for $\tau(z)$ depends on whether the universe is closed ($\Omega_0 > 1$), critical ($\Omega_0 = 1$) or open ($\Omega_0 < 1$). For the critical case, motivated by inflation, the result is

$$\tau = \frac{2}{3} H_0^{-1} \left[ 1 - (1 + z)^{-3/2} \right]. \quad (5)$$

For small $z$, the integral is just $z$, and the linear Hubble relation $H_0\tau \simeq z$ results. A related calculation yields the simple and useful relation between the redshift and the luminosity distance[25], valid for $\Omega_\Lambda \ll \Omega_0$: $D_L = (2H_0^{-1}/\Omega_0^2) \left[ z\Omega_0 + (\Omega_0 - 2)(\sqrt{z\Omega_0 + 1} - 1) \right]$. Then for $\Omega_0 = 1$ as suggested in inflationary cosmologies, $D_L = 2H_0^{-1} \left[ z - 1 - \sqrt{z + 1} \right]$. Given a cosmological model, measurements of a second GRB would potentially validate or invalidate the model, by fitting or not fitting the nonlinear $\tau(z)$ or $\tau(D)$ relation.

In static Minkowski space, a neutrino source is most useful for oscillation studies if its distance is comparable to the oscillation length; a shorter distance does not provide sufficient path length for oscillations to develop,
and in a longer distance the information is effectively averaged over many oscillations. However, in an expanding metric, distance must be carefully defined, and the situation is different. We may calculate the lookback times and luminosity distances to typical GRBs using the $\tau(z)$ or $D_L(z)$ formulae just given. The results are $\tau(z = 1) = 0.43H_0^{-1}$ and $\tau(z = 2) = 0.54H_0^{-1}$, and $D_L(z = 1) = 1.18H_0^{-1}$ and $D_L(z = 2) = 2.54H_0^{-1}$. Note that for this matter-dominated, critical-density example, $D_L(z)$ is unbounded as $z$ increases, whereas the lookback time $\tau(z)$ has approached $\frac{2}{3}H_0^{-1}$ for large $z$. The lookback time is the relevant variable for neutrino oscillation, since it is directly proportional to the oscillation phase, according to Eq. (3). The fact that $\tau$ has become asymptotic for the red-shift values typical of GRBs has an important implication: the value of $\tau$ in the oscillation phase is nearly $\tau(z = \infty)$, which in any cosmological model is a known fraction of $H_0^{-1}$. This fact means that the uncertainty in $\tau$ is dominated by the uncertainty in $H_0$, which is only a factor of two. Thus, a single measurement of the oscillation phase and neutrino energy may permit a determination of $\delta m^2$ to a factor of two!

Comparing an exact cosmological solution for $z, \tau, \text{or } D$ to the appropriate Taylor series approximation relates each term in the series to more fundamental quantities. The simplest relation is between the parameter $q_0$ multiplying the quadratic term, and the parameters $\Omega_0$ and $\Omega_\Lambda$ in the Friedmann equation. The relation is $q_0 = \Omega_0/2 - \Omega_\Lambda$. Thus, a neutrino oscillation determination of $q_0$ would establish a fundamental constraint between the two parameters of the Friedmann universe.

Studying cosmology by measuring neutrino flavor ratios has one tremendous advantage over other methods, namely that flavor ratios should be independent of any evolutionary effects in the ensemble of sources. Absolute neutrino luminosities may evolve, but the initial neutrino flavor ratios are fixed by microphysics; it is hard to imagine that these ratios will change with cosmic history. This is in sharp contrast to the use of distant “candle” luminosities to infer deviations from the linear Hubble Law, which may be due to the deceleration of the universe or to evolutionary effects in the candles.
5 Physical Constants - Time Dependence

There is a long tradition in physics of asking whether various constants are, in fact, constant over cosmological time. The best known of these questions was raised by Dirac. In 1981, Barrow [26] reviewed these ideas. Among his conclusions is the idea that only dimensionless constants can have a meaningful time dependence. In fact there are some new strong limits upon the time variation of the fine structure constant and the electron to proton mass ratio, and upon possible variations across causally disconnected regions of space [27].

However, any possible time variation of the dimensionless parameters upon which neutrino oscillations depend (mass ratios and mixing angles, or equivalently, the Yukawa couplings at the origin of fermion mass generation) are without constraint. If the distances to the GRBs can be measured by independent means, and the neutrino mass differences turn out to be very small, then a time dependence of the mixing angles over cosmological times may be detectable as deviations from the expected flavor ratios.

It has been speculated that dimensionful cosmological parameters may have a time–dependence. After all, the Hubble parameter itself has a complicated dependence on time, varying inversely during power law expansion, and exponentially during inflationary expansion. The cosmological constant \( \Lambda(t) \) (or equivalently, the energy density of the vacuum), has been hypothesized to relax to zero asymptotically with time [28]. And the Hubble parameter and Newton’s gravitational constant have been hypothesized to consist of two terms each, one standard and one oscillating periodically in time [29]. Motivation for the relaxation of \( \Lambda \) is that the present value of \( \Lambda/8\pi G \) is known to be less than \( 10^{-47} \text{ GeV}^4 \) [28], whereas there is no good theoretical understanding of why this should be so small. Motivation for the periodic oscillation in \( H \) and \( G \) is that periodic modulation offers an explanation for the controversial observation [30] of a 128/\( h \) Mpc quasi–period in deep “pencil beam” surveys of galaxy positions. If \( \Lambda(\tau), G(\tau) \), and/or \( H(\tau) \) are time–dependent, then the Friedmann equation is modified. As discussed in the previous section, the oscillation phases of neutrinos emitted at large lookback time \( \tau \) are sensitive to the parameters in the (now modified) Friedmann equation. Thus, measured flavor ratios can offer information on the time dependences of these cosmic parameters.
6 Implications for Neutrino Telescopes

We discuss some implications of our analysis for designers of neutrino telescopes. The spectrum of gammas from GRBs has been seen out to energies of a few GeV, so lacking a detailed GRB model one would do well to focus upon neutrino detection in a similar energy range. There are already weak limits on the neutrino flux associated with GRBs from the IMB experiment[31] and others. The largest deep mine experiment yet planned, the SuperKamiokande detector with a 50 kiloton sensitive volume (scheduled for operation in 1996) will have about ten times the sensitivity for events with energies between 5 MeV and a few GeV as had previous instruments.

Much further progress for sensitivity down to the few MeV region is not presently on the horizon. One possibility would be parasitic use of a megaton size detector contructed for observation of supernova neutrinos out to a few Mpc. The capability to sense neutrino versus antineutrino interactions via interaction characteristics, muon absorption, or magnetic fields to distinguish charge, would provide a powerful tool for many of the tests described above.

However, if the GRB neutrino spectrum extends to energies of many GeV or even TeV, then there is more hope for the near future. This is because the detectability of signals rises strongly with energy. For example, the AMANDA, Baikal, DUMAND and NESTOR ice/water instruments now under construction have effective volumes for 100 GeV neutrinos of order $10^6$ tonnes, up by two orders of magnitude from underground instruments[32]. Observation of the ratio of muon charged current events to muonless events would potentially allow for discrimination between the putative neutrino–flavor democratic source and the pion source with $\pi \rightarrow \mu \rightarrow e$ decay and $\nu_\mu/\nu_e = 2$.

We can make a rough estimate of counting rates for neutrinos by assuming a spectral shape which we take to be $1/E^2$, and a gamma flux which we take to be $1 \gamma/cm^2/burst$ with energy greater than 1 MeV. The ratio of neutrinos to gamma rays, labeled as $\eta$, could be zero in the case of a purely electromagnetic origin of the gammas, in which case most of the foregoing is irrelevant except for the important constraint upon the source model. For the situation of particle acceleration with power law spectra, as discussed by Paczynski, $\eta \sim 1$. $\eta \simeq 1$ holds if the source is not heavily shielded. Even better for our purposes, in situations similar to that expected near AGN[33] the attenuation of the gamma rays can be severe, while the neutrinos flow
freely and so \( \eta > 1 \), possibly even as large as the \( 10^3 \) energy emission ratio expected in supernovae.

We take a simplistic model of a standard 10,000 \( m^2 \) effective area (for muon detection) instrument (e.g., AMANDA, Baikal, DUMAND or NESTOR). In underwater (or ice) experiments the area grows with energy somewhat, but for simplicity we make the conservative assumption that this area independent of energy (as in mine detectors). We take the effective detector area for neutrinos then as the the muon range, times the effective area for muons, times the density of the medium, times Avogadro’s Number, times the neutrino–nucleon cross section. This turns out to be 90 cm\(^2\) for the nominal detector at 1 TeV, and it scales roughly as \( E_\nu^2 \) from cross section and range, for energies from 1 GeV to 10 TeV.

A given detector will have some threshold detection energy, which in practice is not a step function, though for simplicity we take it to be so, at 20 GeV. We also take an arbitrary maximum neutrino energy of 1 TeV. With these assumptions we find for the expected number of neutrino interactions per burst which are below the neutrino detector horizon, the value \( 9 \cdot 10^{-5} \eta/\text{burst} \) in muon neutrinos. If we take the rate of GRB (as now detected) as about 1/day, then the total expected number of correlated muons for this standard neutrino detector is \( 1.5 \cdot 10^{-2} \eta/\text{year} \). Since the input assumptions are certainly imprecise to a factor of ten, this could easily be well detectable or beyond experimental reach.

If \( \eta = 1 \), it is easy to see why existing underground instruments have not seen such correlations as yet, despite lower thresholds. The IMB detector had 400 \( m^2 \) area, 25 times less than our assumed instrument. In fact, one can turn this around and ask what limit on \( \eta \) is implied by the non-detections in present underground instruments, under our assumptions of spectrum. In Figure 4, we show the combinations of maximum GRB neutrino energy and \( \eta \) which would lead to one event detected per year in IMB (400 \( m^2 \)), a next generation instrument (10\(^4 \) \( m^2 \)), and a hypothetical \( km^3 \) detector (KM3). One sees that for a TeV GRB neutrino cutoff energy, the IMB limit on the neutrino to gamma ratio is of the order of a few thousand.

Another approach is to consider the brightest GRBs, and ask whether or not multiple events from a single source are possible. If we assume that the distribution of GRB gamma fluxes are dominated by spatial distribution, then 0.1% of the GRBs will be at 0.1 of the distance of the typical burst, and offer 100 times the neutrino flux at earth; this is the famous “3/2 law”. Thus
in one year of operation during which there will be about 1000 GRBs, one might find a GRB with $9 \cdot 10^{-3}$ $\eta$ muons. One can conclude that detection of multiple muons per GRB is likely in the standard next generation instrument only if $\eta \gg 1$. This is illustrated in Figure 2, where the diagonal lines indicate the $\eta$ and maximum neutrino energy combinations needed to see an event with ten muons once per two years in the various classes of detectors (and roughly one in three such GRBs would have a coincident GRO observation).

Better possibilities exist with a $km^3$ scale instrument. Even though current design discussions center around optimization for detection of $TeV$ to $PeV$ neutrinos from AGN’s, neutrinos from GRB’s (as well as other opportunities such as dark matter searching and atmospheric neutrino oscillation measurement) argue in favor of reducing the threshold to as low a value as feasible, say of the order of $10 GeV$. Given the lack of understanding of the process which generates the GRB’s we cannot do much in terms of $a$ priori optimization of detector design. Good timing is obviously important, but high angular resolution is not as demanding as for neutrino point source searches.

If we take the energy threshold to again be $20 GeV$ for the ($km^3$) neutrino detector with $10^6 m^2$ effective muon area, then under the same assumptions we employed for the standard next generation instrument we get an expected event number of $9 \cdot 10^{-3} \eta/burst$, or about $1.5 \eta/year$ GRO coincident detections. More encouraging yet is the rate of multiples: the brightest 0.1% of GRBs might produce the spectacular signature of 90 $\mu$’s once in two years if $\eta$ should be 100 and the maximum energy 1 $TeV$, in which case one could begin the studies outlined above.

7 Conclusion

Any detection of GRBs in neutrinos would have great significance in understanding these enigmatic objects. What we advertise herein is that moreover such a detection can lead to fundamental exploration of neutrino physics, astrophysics, and maybe even cosmology. This exploration is not possible by any other means we know.

The implications for the telescope designers then are fairly obvious: make instruments with as low an energy threshold as practical, and allow for upgrades (in terms of energy sensitivity and capability to resolve neutrino fla-
vors) to follow the path of discovery.

Acknowledgements

We want to thank Xerxes Tata for many useful discussions. We also acknowledge Jack VanderVelde for a useful suggestion about calculating rates. We also want to thank Andy Szentgyorgyi for help with the BATSE data. This work was supported in part by the U.S. Department of Energy grants no. DE-FG05-85ER40226 and no. DE-FG03-94ER40833.

References

[1] See “Gamma Ray Bursts”, Huntsville, 1993, AIP Conference Proceedings 307, Ed., G. J. Fishman, J. J. Brainead, K. Hurley, AIP, N.Y. 1994.

[2] J. M. Horack, A. G. Emslie and C. A. Meegan, ApJ 426, L5 (1994).

[3] B. Paczynski and G. Xu, Astrophys. J. 427, 708 (1994).

[4] R. Plaga, Astrophys. J. Lett. 424, L9 (1994).

[5] Over a hundred papers modeling gamm–ray bursters are put into a historical context by R. J. Nemiroff, Comments Astrophs. 17, 189 (1994).

[6] A. D. Kerrick, et al., Ap. J. Lett., in press (1/95).

[7] B. F. Schutz, Nature 323, 310 (1986); Class. Quantum. Grav. 6, 1761 (1989); C. Cutler et al., Phys. Rev. Lett. 70, 2984 (1993).

[8] Y. Fukuda, et al., KAMIOKANDE Collaboration, Phys. Lett. B72, 333 (1994), presents the most recent data sample (higher energy muon neutrinos showing a zenith–angle dependent depletion); prior analyses are referenced in this work. The first interpretation of the atmospheric anomaly as neutrino oscillations can be found in J. G. Learned, S. Pakvasa, and T. J. Weiler, Phys. Lett. B207, 79 (1988); the significance of the zenith angle measurement is also outlined in this work.
[9] T. Yanagida, Proc. of Workshop on “The Unified Theory and the Baryon Number in the Universe”, KEK, Feb. 13-14, 1979, Ed. by O. Sawada and A. Sugamoto, KEK-79-18, p.95; M. Gell-Mann, P. Ramond and R. Slansky, “Supergravity”, ed. by D. Z. Freedman and P. von Nieuwenhuizen, N. Holland (1979).

[10] J. G. Learned, S. Pakvasa, W. A. Simmons and X. Tata, Q.J.R. astr. Soc. 35, 321 (1994).

[11] The coherence condition in Minkowski space is given in S. Nussinov, Phys. Lett. B63, 201 (1976).

[12] For a summary of neutrino properties deduced from SN1987A, see S. Pakvasa, Prosiding Pertemuan Ilmiah VI HFIY, Salatiga, Indonesia, Aug. 1990, eds. Muslim, A. Pramudita and Sumaji, p. 15.

[13] A. Yu. Ignatiev and G. C. Joshi, UM-P-94-73 (hep-ph 9407346); Mod. Phys. Lett., A9, 1979 (1994).

[14] R. Barbillieni and G. Cocconi, Nature 329, 21 (1987).

[15] L. Stodolsky, Phys. Lett. 201B, 353 (1988).

[16] M. J. Longo, Phys. Rev. Lett. 60, 173 (1988); L. M. Krauss and S. Tremaine, Phys. Rev. Lett. 60, 176 (1988).

[17] S. Pakvasa, W. Simmons, and T. J. Weiler, Phys. Rev. D39, 1761 (1989); J. LoSecco, Phys. Rev. D 38, 3313 (1988).

[18] J. A. Grifols, E. Masso and S. Peris, Phys. Lett. B207, 493 (1988); Astropart. Phys. 2, 161 (1994).

[19] N. Cabibbo, Phys. Lett. B72, 333 (1978); S. Pakvasa, Proc. of XXth Int. Conf. High Energy Phys., Madison, July 1980, ed. L. Durand & L. Pomed, A.I.P. 1981, p. 1164; V. Barger, K. Whisnant and R. J. N. Phillips, Phys. Rev. Lett. 45, 2084 (1980).

[20] J. Pantaleone, A. Halprin and C. N. Leung, Phys. Rev. D47, 4199 (1993).
[21] J. P. Norris, et al., Astroph. J. Lett., in press (1994); R. J. Nemiroff et al., Astroph. J. Lett., in press; R. A. M. J. Wijers and B. Paczynski, Princeton preprint POP–567 (1994).

[22] Ya. B. Zeldovich, Usp. Fiz. Nauk. 80, 357 (1963); M. Geller and P. J. E. Peebles, Ap. J. 174, 1 (1972).

[23] E. Kolb and M. S. Turner, The Early Universe, Addison–Wesley, (1990); a thorough discussion of the cosmological constant is given by S. Weinberg, in Rev. Mod. Phys. 61, 1 (1989).

[24] J. E. Felten and R. Isaacman, Rev. Mod. Phys. 58, 689 (1986).

[25] S. Weinberg, Gravitation and Cosmology, Wiley, N.Y. (1972).

[26] J. D. Barrow, Q. J. R. as. Soc., 22, 388 (1981).

[27] L. Cowie., et al., IfA preprint in preparation 11/94.

[28] I. Antoniadis and N. C. Tsamis, Phys. Lett. B144, 55 (1984); T. Banks, Nucl. Phys. B249, 332 (1985); L. Abbott, Phys. Lett. B150, 427 (1985); M. Gasperini, Phys. Lett. 194, 347 (1987); M. Reuter and C. Wetterich, Phys. Lett. B188, 38 (1987); M. Ozer and M. O. Taha, Nucl. Phys. B287, 776 (1987); R. D. Peccei, J. Sola, and C. Wetterich, Phys. Lett. B188, 38 (1987); P. J. E. Peebles and B. Ratra, Ap. J. 325, L17 (1988); Phys. Rev. D37, 3406 (1988); C. Wetterich, Nucl. Phys. B302, 645 and 668 (1988).

[29] C. Hill, P. J. Steinhardt, and M. S. Turner, Phys. Lett. 252, 343 (1990).

[30] T. J. Broadhurst, R. S. Ellis, D. C. Koo, and A. S. Szalay, Nature 343, 726 (1990).

[31] R. Becker-Szendy, et al., Ap. J. in press (1994), IMB limits on neutrinos correlated with GRBs.

[32] For a discussion of future of large neutrino detectors, see, e. g., J. G. Learned, Plenary Talk at the XVI International Conference on Neutrino Physics and Astrophysics, May 29–June 3, 1994, Eilat, Israel, eds. A. Dar and M. Gronau, Nucl. Phys. B. Proceed. Suppl., in press (1994).
[33] HENA Proceedings, ed. V. Stenger, et al., (1992).
Figure 1: One event per year in coincidence with GRO for detectors of 400 m$^2$ (IMB), 10,000 m$^2$ (BAND), and 10$^6$ m$^2$ (KM3), for various hypothetical neutrino to gamma ratios ($\eta$) and GRB maximum neutrino energy. The upper right hand corner region is already ruled out by IMB data.
Figure 2: 10 muons in one burst for brightest GRBs, seen in detectors of 400 $m^2$ (IMB), 10,000 $m^2$ (BAND), and $10^6$ $m^2$ (KM3), for various hypothetical neutrino to gamma ratios ($\eta$) and GRB maximum neutrino energy. The upper right hand corner region is already ruled out by IMB data. The rate may be 0.5/year.