Solution of the Skyrme-Hartree-Fock equations in the Cartesian deformed harmonic oscillator basis. (II) The program HFODD.

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Abstract

We describe the code HFODD which solves the nuclear Skyrme-Hartree-Fock problem by using the deformed Cartesian harmonic oscillator basis. The user has a possibility of choosing among various symmetries of the nuclear HF problem for rotating or non-rotating nuclei; they vary from the non-axial parity-invariant nuclear shapes, through those also breaking the intrinsic parity, towards the least-restrictive case corresponding to only one symmetry plane. The code provides a solution for a complete superdeformed rotational band in an $A\sim150$ nucleus within one CPU hour of the CRAY C-90 supercomputer or within two-three CPU hours of a fast workstation.

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PROGRAM SUMMARY

Title of the program: HFODD (v1.60n)

Catalogue number:

Program obtainable from: CPC Program Library, Queen’s University of Belfast, N. Ireland (see application form in this issue)

Licensing provisions: none

Computer for which the program is designed and others on which it has been tested: CRAY C-90, SG Power Challenge L, IBM RS/6000

Operating systems: UNIX, UNICOS, IRIX, AIX

Programming language used: FORTRAN-77

Memory required to execute with typical data: 10 Mwords

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**No. of bits in a word:** 64

**Has the code been vectorised?** Yes

**No. of lines in distributed program:** 19 438 (of which 8 354 are comments and separators)

**Keywords:** Hartree-Fock; Skyrme interaction; Self-consistent mean-field; Nuclear many-body problem; Superdeformation; Quadrupole deformation; Octupole deformation; Pairing; Nuclear radii; Single-particle spectra; Nuclear rotation; High-spin states; Moments of inertia; Level crossings; Harmonic oscillator; Coulomb field; Point symmetries

**Nature of physical problem**
The nuclear mean-field and an analysis of its symmetries in realistic cases are the main ingredients of a description of nuclear states. Within the Local Density Approximation, or for a zero-range velocity-dependent Skyrme interaction, the nuclear mean-field is local and velocity dependent. This allows an effective and fast solution of the self-consistent Hartree-Fock equations even for heavy nuclei and for different configurations, deformations, excitation energies, or angular momenta.

**Method of solution**
The program uses the Cartesian harmonic oscillator basis to expand single-particle wave functions of neutrons and protons interacting by means of the Skyrme effective interaction. The expansion coefficients are determined by the iterative diagonalization of the mean field Hamiltonians or Routhians which depend nonlinearly on the local neutron and proton densities. Suitable constraints are used to obtain states corresponding to a given configuration, deformation or angular momentum.

**Restrictions on the complexity of the problem**
The main restriction is the CPU time required for calculations of heavy deformed nuclei and for a given precision required. One symmetry plane is assumed. Pairing correlations are only included in the BCS limit and for the conserved time-reversal symmetry.

**Typical running time**
One Hartree-Fock iteration for the superdeformed, rotating, parity conserving state of $^{152}_{66}$Dy$_{86}$ takes about nine seconds on the CRAY C-90 computer. Starting from the Woods-Saxon wave functions, about fifty iterations are required to obtain the energy converged within the precision of about 0.1 keV. In case when every value of the angular velocity is converged separately, the complete superdeformed band with precisely determined dynamical moments $J^{(2)}$ can be obtained within one hour of CPU on the CRAY C-90, or within two to three hours of CPU on the SG Power Challenge L or IBM RS/6000 computers. This time can be often reduced by a factor of three when a self-consistent solution for a given rotational frequency is used as a starting point for a neighboring rotational frequency.

**Unusual features of the program**
The user must have an access to the NAGLIB subroutine f02axe or to the ESSL subroutine zhpev which diagonalize complex hermitian matrices, or provide another subroutine which can perform such a task. The code is written in single-precision for the use on a 64-bit processor. The compiler option -r8 or +autodblpad (or equivalent) has to be used to promote all real and complex single-precision floating-point items to double precision when the code is used on a 32-bit machine.

LONG WRITE UP

1 Introduction

The method of solving the Skyrme-Hartree-Fock (HF) equations in the Cartesian harmonic oscillator (HO) basis was described in the previous publication [1] which is referred to as I. The present paper is a long write up of the code hfodd which implements this method and solves the three-dimensional HF equations. A one-dimensional Skyrme-HF code restricted to the spherical symmetry has been published by P.-G. Reinhard [2]. Although several HF codes allowing triaxial deformations exist [3, 4, 5, 6], the code hfodd is the first one of that kind which is made available in the public domain. Several earlier beta versions of the code have already been distributed and used by various groups. The present version (v1.60n) is published for the first time and replaces all previous versions.

In Sec. 2 we present the numerical tests of the code, and in Secs. 3, 4, and 5 we describe its input, output, and source files, respectively.

2 Numerical tests

Accuracy of the solution of the HF equations with the wave functions expanded onto the Cartesian HO basis, Eq. (I-70), depends on the three parameters $\hbar \omega_x$, $\hbar \omega_y$, $\hbar \omega_z$ defining the HO frequencies in three Cartesian directions, and on the number $M$ of the HO states included in the basis. In the code hfodd we use the standard prescription [7, 8] to chose the HO states included in the basis, namely, the $M$ states with the lowest HO single-particle energies,

$$\epsilon_{n_x n_y n_z} = \hbar \omega_x (n_x + \frac{1}{2}) + \hbar \omega_y (n_y + \frac{1}{2}) + \hbar \omega_z (n_z + \frac{1}{2}),$$

(1)

are selected among those which have $n_x \leq N_0$, $n_y \leq N_0$, and $n_z \leq N_0$, where $N_0$ is the fixed maximum number of HO quanta. It should be noted that in general both $M$ and $N_0$ have to be specified to define the basis. Only for large $N_0$, the basis is defined solely by $M$ and does not depend on $N_0$. In this case, the grid of points $(n_x, n_y, n_z)$ defining the states included in the basis forms a pyramid in three dimensions, with the inclined face delimited by the condition $\epsilon_{n_x n_y n_z} \leq \text{const}$. On the other hand, only for small values of $N_0$ the basis is defined solely by $N_0$ and does not depend on the energy cut-off. In this case the corresponding grid of points $n_x n_y n_z$ forms a cube of the size $N_0$. In all intermediate

\footnote{Symbol (I-70) refers to Eq. (70) of I.}
cases the shape of the basis corresponds to a pyramid with the corners cut off, or to a cube with the corners cut off. Usually \( N_0 \) is chosen large enough so that all the states allowed by the energy cut-off are included in the basis. The shape of the basis is printed in form of three projections by the code \texttt{HFODD} on the output file, see the TEST RUN OUTPUT below.

In what follows we discuss only the results obtained with axially deformed bases for \( \omega_\perp = \omega_x = \omega_y \), and present the results in terms of the standard parameters

\[
\omega_0 = (\omega^2_\perp \omega_z)^{1/3} \quad \text{and} \quad q = \omega_\perp / \omega_z.
\]  

(2)

(The code accepts of course in general a triaxially deformed HO basis what offers the user some additional flexibility in approaching the actual physical problem). In Ref. [8] the dependence of results on the basis parameters and on the basis size has been tested for \( N_0 \leq 10 \), which at that time was at the limit of the numerical feasibility. In the present study we discuss similar tests up to \( N_0 = 20 \) (spherical case) and \( N_0 = 26 \) (deformed case).

From the tests based on low values of \( N_0 \), the authors of Ref. [8] concluded that the basis parameters \( \omega_0 \) and \( q \) have to be optimized for a given basis size in such a way that the HF energy is minimized. This procedure is consistent with the variational character of the HF theory in which the best approximation to the energy is obtained by a minimization of the energy. Although the variational principle does not ensure that the best approximation to other observables is obtained for the wave functions corresponding to the minimum of energy, this turned out to be the case for low values of \( N_0 \). The necessity to optimize the basis parameters was the real bottleneck of the method, because it requires many HF calculations to be performed before one physical solution can be obtained.

The picture described above does not apply to calculations performed with relatively large values of \( N_0 \). Of course, the best energy is still obtained with optimized values of the basis parameters, but the energy depends now very weakly on the basis parameters and the error generated by using a non-optimal basis is small. Moreover, the best results for other observables are not necessarily obtained with the optimal basis parameters. This can be well understood by noticing that the minimum obtained from a very weak dependence of the energy on the basis parameters may be ill-conditioned, and may occur at rather accidental value. Therefore, for large values of \( N_0 \), the basis optimization does not bring any substantial improvement of precision. Instead of the optimization, suitable “physical” values of the basis parameters can be determined based on simple geometrical considerations. This is in line with results of Ref. [8], where it was shown that a partial optimization followed by the liquid-drop-model estimates reproduces the optimal basis parameters even for small values of \( N_0 \).

In the code \texttt{HFODD} the physical basis parameters are determined as follows (cf. also Ref. [8]). First we define \( \omega_0 \) by the standard value [9] multiplied by some phenomenological factor \( f \) close to unity,

\[
\hbar \omega_0 = f \times 41 \text{ MeV} / A^{1/3}.
\]  

(3)

Based on the experience gained from the diagonalization of the Woods-Saxon Hamiltonian on the HO basis [10, 11], the recommended value is \( f = 1.2 \). This particular scaling of the oscillator frequencies turns out to help in stabilizing the results of the diagonalization
with respect to adding the new basis states. Second, suppose we wish to perform the HF calculations for a given set of multipole deformations \( \alpha_{\lambda \mu} \), i.e., for a nuclear shape defined by the surface \( \Sigma \)

\[
\Sigma : \quad R(\theta, \phi) = c(\alpha) \left[ 1 + \sum_{\lambda=0}^{\lambda_{\text{max}}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda \mu} Y_{\lambda \mu}(\theta, \phi) \right],
\]

where \( c(\alpha) \) is a function of \( \alpha_{\lambda \mu} \) such that the volume enclosed by the surface \( \Sigma \) does not depend on \( \alpha \). Due to the assumed \( y \)-simplex symmetry, see Sec. 3.1 of I, all multipole deformations \( \alpha_{\lambda \mu} \) are real, and only those with \( \mu \geq 0 \) are used in the code HFODD because then \( \alpha_{\lambda,-\mu}=(-1)^{\mu}\alpha_{\lambda \mu} \).

The lengths of principal axes of the volume enclosed in the surface \( \Sigma \) can be defined as

\[
R_x = R\left(\frac{\pi}{2}, 0\right), \quad R_y = R\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \quad \text{and} \quad R_z = R(0, 0).
\]

For complicated shapes they do not necessarily correspond to the principal axes of the moment of inertia or quadrupole moment. However, when the quadrupole deformation dominates, this will roughly be so if the quadrupole deformation is considered in its intrinsic frame of reference, i.e., if one uses \( \alpha_{21}=0 \). The relative values of the HO frequencies are then defined by the condition

\[
\omega_x R_x = \omega_y R_y = \omega_z R_z,
\]

while the common proportionality constant is given by the condition \( (\omega_x \omega_y \omega_z)^{1/3}=\omega_0 \).

The above prescription is very helpful if the values of physical deformations are (at least roughly) known for a given nucleus. This is true in particular for the superdeformed shapes which have very similar deformations in a given region of nuclei. For example, the superdeformed nuclei in the \( A\sim150 \) region can be very well described in the basis corresponding to the \( ^{152}\text{Dy} \) nucleus with deformations \( \alpha_{20}=0.61 \) and \( \alpha_{40}=0.10 \) [12]. This gives the physical basis parameters, \( \hbar\omega_x=\hbar\omega_y=6.246 \text{ MeV} \) and \( \hbar\omega_z=11.200 \text{ MeV} \), i.e., \( \hbar\omega_0=9.219 \text{ MeV} \) and \( q=1.793 \).

We may now present results of tests performed for different basis parameters and compare them with those obtained for the physical basis parameters. All calculations below have been performed for the Skyrme parametrization SkM* with the coupling constants \( C_t^J=C_t^T=C_t^\Delta s=0 \) in the energy functional (I-12). The latter conditions correspond to simplifications of the Skyrme functional usually assumed in studies made within the spatial coordinates [13, 14], and allow a comparison of our results with those obtained by an independent method and an independent HF code.

From now on we are going to use the two adjectives introduced above and associated with the basis parameters: 

- **optimal**, i.e., adjusted to a given basis size so as to minimize the HF energy, and
- **physical**, i.e., corresponding to the actual information about the deformations of nuclei studied, and calculated through Eqs. (3), (4), and (5) based on the experimental result or earlier Strutinsky-type calculations.

### 2.1 Spherical nuclei

We begin with the results obtained for a spherical nucleus \(^{208}\text{Pb} \) for which the spherical basis is used, \( q=1 \), and the results are studied in function of \( \hbar\omega_0 \). The physical value for this nucleus is \( \hbar\omega_0=8.304 \text{ MeV} \), and is represented in Figs. 3 and 4 by the vertical dashed
lines. The horizontal dashed lines represent very precise solutions obtained by the simple one-dimensional spherical code in spatial coordinates, and are considered as the exact values. The solid lines represent the results calculated by the code HFODD for \( N_0 \) from 10 to 18, and the asterisk represents the calculation with \( N_0 = 20 \). For the spherical bases the number of the HO states is given by \( M = (N_0+1)(N_0+2)(N_0+3)/6 \), and hence in Figs. 1 and 2 it increases from 286 to 1771.

The energies of \(^{208}\text{Pb}\), shown in Fig. 1, converge rather slowly to the exact value of \(-1635.956\) MeV. For \( N_0 = 10 \) the error is about 6 MeV and only for \( N_0 \geq 16 \) becomes smaller than 1 MeV. For every value of \( N_0 \), the dot denotes the minimum of energy which defines the optimal value of \( \bar{\hbar}\omega_0 \). For \( N_0 = 10 \) and 12 the optimal values are rather remote from the physical value and, moreover, two minima appear for \( N_0 = 12 \). For \( N_0 = 14 \) the minimum occurs at the physical value of \( \bar{\hbar}\omega_0 \), but for larger bases, \( N_0 = 16 \) and 18, the minimum becomes poorly defined and deviates from the physical value of \( \bar{\hbar}\omega_0 \).

Fig. 2 shows the corresponding results of calculations for the matter rms radii of \(^{208}\text{Pb}\). For every curve the dots represent the minima of the energies shown in Fig. 1. One should note that the vertical scale of the Figure is very much expanded and shows a narrow region of radii within about 0.3% around the exact value of \( R_{\text{rms}} = 5.5546 \) fm. For \( N_0 = 10 \) the radii significantly depend on \( \bar{\hbar}\omega_0 \) but the value of \( R_{\text{rms}} \) at the physical basis parameter is very close to the exact result, while the value at the optimal basis parameter is much worse. For \( N_0 = 12 \) the value at the physical \( \bar{\hbar}\omega_0 \) is again much better than those at any of the two optimal bases. For \( N_0 = 14 \) the physical and optimal bases coincide and give the exact value with the error of 0.05%, while for larger bases the dependence of \( R_{\text{rms}} \) on \( \bar{\hbar}\omega_0 \) becomes weak, and the results for the optimal bases are only slightly better than those for the physical basis. Altogether, for any value of \( N_0 \geq 10 \) the physical basis gives the radii precise up to \( \pm 0.003 \) fm, or up to \( \pm 0.05\% \).

### 2.2 Deformed nuclei

In order to study the properties of the optimization of the deformation of the basis, we have performed a series of calculations for the non-rotating superdeformed state in \(^{152}\text{Dy}\) \((\bar{\hbar}\omega = 0)\) with \( \bar{\hbar}\omega_0 \) fixed at the physical value, and for several different values of \( q \) and \( M \). These results can be compared to those obtained in spatial coordinates for the same nucleus and force, and with the three-dimensional cubic grid of points spaced by \( \Delta x = 1.0, 0.8, \) or 0.7 fm. In this case smaller spacing gives more accurate results, and one obtains \([15]\), respectively, the total energies \( \mathcal{E} = -1234.611, -1230.769, \) or \(-1230.104\) MeV and the proton quadrupole moments \( Q_p = 18.316, 18.369, \) or 18.376 b.

These values of energies were obtained by using the finite difference expressions for derivatives, which are less precise than the Fourier expressions \([10]\). If the HF equations are solved using the first type of approximation but nevertheless at the end of the convergence the energies are calculated with the second method, one obtains a good and cost-effective estimate of the energy \([16, 17]\). Such a procedure yields \([15]\) for \( \Delta x = 0.7 \) fm the value of \( \mathcal{E} = -1229.365 \) which we may consider for the purpose of the discussion below as the exact result. This energy is shown by the horizontal dashed line in Fig. 3. In practice, the three-dimensional calculations in the spatial coordinates are usually done with \( \Delta x = 1 \) fm and employ the finite difference method. For these standard conditions the binding energy
of $^{152}\text{Dy}$ is overestimated by about 5 MeV.

Fig. 3 shows the total energies in $^{152}\text{Dy}$ calculated by the code HFODD for $M=300$, 600, 900, or 1200. When $q$ varies from 1.3 to 2.3 the maximum numbers of the HO quanta in the directions perpendicular/parallel to the elongation axis vary for $M=300$ from 9/13 to 8/18 quanta. Similarly, for $M=1200$ and $q=1.793$ the HO basis contains 14/26 quanta. Here and in the following, the results for $M=300$ are presented as broken lines with visible jumps corresponding to individual states entering and leaving the HO basis. Indeed, keeping a fixed number $M$ of states with varying $q$ means that a given HO orbital may cross the boundary $\epsilon_{n_x n_y n_z} \leq \text{const}$. For relatively small values of $M$ this may create some effects on various quantities (visible in the expanded scales of presented Figures), while for larger values of $M$ such effects disappear.

The convergence of energy in function of $M$ is again rather slow; for $M=300$ the error is around 5 MeV and it decreases to below 1 MeV only at $M>600$. For $M=300$ and 600 the optimal basis deformations are rather different from the physical value. However, the energy gain from optimizing the value of $q$ is very small. For $M=900$ the optimal and physical values of $q$ are close to one another, but the minimum of energy is hardly visible. For $M=1200$ we obtain $\mathcal{E}=-1229.383$ in a very good agreement with the exact result. I should be noted that both estimates compared here are variational, and hence both may still slightly decrease with further improvement of the numerical parameters.

Fig. 4 shows the values of proton quadrupole moments calculated by the code HFODD and compared with the value of 18.376 b obtained for $\Delta x=0.7 \text{ fm}$ (see above). The latter value is for the purpose of the present test considered as the exact result. Again the region shown on the ordinate is very narrow; within 0.5% of the value of $Q_p$. On the other hand, the region of $q$ presented on the abscissa is fairly large; it corresponds to the standard quadrupole deformations $\beta \equiv \alpha_{20}$ of the nuclear surface (II) ranging from 0.3 to 1.0.

For $M=300$ one obtains much a better value of $Q_p$ for the physical basis than for the optimal basis. This situation is reversed for $M=600$. For $M=900$ the dependence on basis parameters is very weak and the results are very close to the physical-basis value of $Q_p=18.393$ obtained for $M=1200$. The values of $Q_p$ calculated by the code HFODD tend to be slightly larger than the exact value obtained within the spatial-coordinate method. We have checked that the optimization of $\hbar \omega_0$ does not remove this residual difference.

Similarly as for tests performed in spherical nuclei, one can conclude that the optimization of the basis does not, in general, improve the results as compared to those obtained for the physical basis. Within a very limited space of $M=300$ HO states one may obtain a very reasonable estimate of $Q_p$ when the physical basis is used. Taking into account the fact that the discrepancy is larger for $M=600$, we may attribute a conservative error of 0.05 b, or 0.3% to the calculated values of the quadrupole moments. A precision of this order is perfectly sufficient for all practical purposes, and can of course be improved by using larger bases, instead of optimizing the basis parameters. In fact, the obtained precision is comparable to that of the spatial-coordinate method when it is used with the standard value of $\Delta x=1 \text{ fm}$ (see above).
2.3 Rotating nuclei

In Fig. 5 we show the values of the angular momentum $I$ calculated for $^{152}$Dy at $\hbar \omega = 0.5$ MeV. For all values of $q$ and $M$ considered in the Figure one obtains results which are within $0.1 \hbar$ of the value calculated for $M=1200$. Again, there is no advantage in using the optimal bases as compared to the physical one. The optimal values of $q$ were obtained by minimizing the Routhian (I-21) at constant $\hbar \omega$. We do not show here the plot of the Routhian in function of $q$ because such a plot is very similar to that of the energy at zero spin, Fig. 3, except for an almost constant shift by $-\omega I$.

The properties of superdeformed bands are very often described in terms of moments of inertia, which can be calculated as the static or dynamic (first or second) moments

$$J^{(1)} = \frac{I}{\omega}, \quad (6a)$$

$$J^{(2)} = \frac{dI}{d\omega}, \quad (6b)$$

respectively. The precision of determining $J^{(1)}$ is therefore governed by that with which the total angular momentum $I$ is calculated, and for $\hbar \omega=0.5$ MeV it is equal to about $0.2 \hbar^2$/MeV. In practical calculations, the dynamic moment $J^{(2)}$ has to be calculated by the finite-difference expression

$$J^{(2)} = \frac{I(\omega_2) - I(\omega_1)}{\omega_2 - \omega_1} \quad (7)$$

which is precise up to

$$\delta J^{(2)} \sim \frac{1}{\omega_2 - \omega_1} 2 (\omega_2 - \omega_1)^2 \frac{d^2 J^{(2)}}{d\omega^2}. \quad (8)$$

This expression for $\delta J^{(2)}$ corresponds to the lowest-order term neglected in the Taylor expansion leading to the finite-difference formula (7). The curvature of $J^{(2)}$, which determines its precision $\delta J^{(2)}$, is usually fairly small ($200\text{–}300 \hbar^4$/MeV$^3$ at most), except at the crossing frequencies where the precise values of $J^{(2)}$ are physically not important anyhow. Therefore, the finite difference expression (8) can usually be used already at $\omega_2 - \omega_1 \approx 0.1$ MeV/\hbar.

In Fig. 6 we present results for $J^{(2)}$ calculated at $\hbar \omega=0.55$ MeV by using the values of $I$ obtained at $\omega_1=0.5$ and $\omega_2=0.6$ MeV/\hbar. In view of the precision of the total angular momentum $I$, estimated above to be equal to $0.1 \hbar$, one could have expected that the values of $J^{(2)}$ obtained in this way will be precise only up to about $1 \hbar^2$/MeV. In fact, the real precision of $J^{(2)}$ turns out to be an order of magnitude better. This is so because most of the errors related to the use of the finite HO basis cancel out, and the difference of angular momenta turns out to be much more precise than the total value of $I$ itself. As a result, the dynamic moments $J^{(2)}$ shown in Fig. 6 are precise up to $0.1 \hbar^2$/MeV. This observation concerns also many other observables, and will be discussed in the next section in more detail.

2.4 Relative energies, moments, and alignments

As already pointed out in Ref. 8, the differences of energies (the relative energies) depend much less on the basis parameters than the total (absolute) energies. This is illustrated
in Fig. 7, where we show the proton separation energies $S_p$ in the SD $^{152}$Dy nucleus, calculated at spin zero for different values of $q$ and $M$. The values of $S_p$ have been calculated by subtracting the total energy of $^{152}$Dy from the total energy of $^{151}$Tb after both have been determined self-consistently by using the same HO basis. The SD state in $^{151}$Tb has been constructed by creating a hole in the $\pi[651]3/2(+)i$ orbital, cf. Ref. [12]. As seen in the Figure, the separation energies are precise up to about 50 keV, which is two orders of magnitude better than the total energies in either of $^{151}$Tb or $^{152}$Dy nucleus. Similar precision is also expected for the excitation energies in a given nucleus, i.e., for the differences of energies corresponding to different configurations or angular momenta.

The same principle also works for the quadrupole polarizations [18] which are obtained by subtracting the quadrupole moments of the SD states in $^{151}$Tb and $^{152}$Dy nuclei. This is shown in Fig. 8, where the obtained quadrupole polarizations are precise up to 0.01 b, compared to the 0.05 b precision obtained for the values of proton quadrupole moments $Q_p$ discussed above. Similarly, the relative alignments between these two nuclei, i.e., the differences of angular momenta at a given angular frequency, $\hbar \omega = 0.5$ MeV here, are presented in Fig. 9. These values characterize the quality of the identical bands [19], and have been discussed in Ref. [12]. Here we see that using the truncated HO basis they are precise up to 0.03 $\hbar$, which is a factor three better than the values of the angular momenta discussed in Sec. 2.3.

2.5 Summary

Table 1 summarizes the results discussed in Sections 2.1–2.4. The column denoted “absolute values” refers to the errors of observables calculated in $^{208}$Pb or $^{152}$Dy for $M=300$ and for the physical basis parameters. The errors are deduced by comparing these results with the exact ones, where available, or with the results obtained with $M=1200$. The percentage errors are also given. The column denoted “relative values” refers to the errors of differences of observables calculated in $^{151}$Tb and $^{152}$Dy. Due to the fact that most of the errors are systematic (they originate from the truncated HO basis), they tend to cancel out, and the errors of relative observables are smaller than those of the absolute observables. Since the relative values are much smaller than the absolute values, the percentage errors are larger in this case, but still only of the order of several per cent. In all cases the precisions of relative values are sufficient for a meaningful comparison with data or with other theories. Whenever more precise results are required, they can be easily obtained by increasing the size of the basis. The basis optimization is not a competing procedure which would lead to a better precision.

3 Input data file

Different aspects of the organization of the code HFODD are presented in this section together with the description of the input data file. The code reads the input data from the standard FORTRAN input file, prints results on the standard FORTRAN output file, and also writes two auxiliary files which below are called the review file and the record file. If required, the code may also start the iteration from the Woods-Saxon wave functions, which have to be provided in the specialized file called the Woods-Saxon file, or from the
previously recorded HF potentials provided in the file called the replay file.

The general structure of the input data file is defined by the following rules:

1. Input data file is an ASCII file composed of independent items.

2. An item is composed of the keyword line, and of the data line which follows immediately the keyword line. Two items contain no data lines, see point 6 below.

3. The keyword line contains in columns 1 through 10 a keyword which is a specific text defining the item. If the keyword has less than 10 characters it has to be obligatorily padded with appropriate number of trailing spaces. Columns beyond 10 are ignored and can be used to place arbitrary comments or texts.

4. The data line contains a prescribed number of data values which are read in the free FORTRAN FORMAT. The type of data (REAL, INTEGER, or CHARACTER) should match the definition of the given data line. The CHARACTER data should be provided starting from the 13-th column of the data line.

5. Items can be separated by an arbitrary numbers of comment lines. A line is treated as a comment and ignored provided it does not contain in columns 1 through 10 any of the valid keywords.

6. Two items contain no data lines. The first one is defined by the keyword EXECUTE, and starts a calculation for the currently defined input parameters. The second one is defined by the keyword ALLDONE, and terminates the program. The latter item is not required if the compiler is able to properly recognize the end of input data file. When the end of input data file is encountered, the program terminates as if the keyword ALLDONE was found.

7. A given item may occur any number of times in different places of the input data file. Only the last one of the same items occurring before a given EXECUTE keyword is taken into account.

8. Between two consecutive items EXECUTE, the order of other items is arbitrary. These other items define the data set, i.e, the set of input parameters, for which the calculation will start at the moment when the next item EXECUTE is found.

9. All input parameters have the default values defined in the code (subroutine PREDEF). Therefore, if the input data file contains only the item EXECUTE, and no other lines, a calculation for the default values will be performed. In the present version, this will result in performing 50 iterations for the superdeformed state of $^{152}_{66}$Dy$_{86}$ at the angular frequency of $\hbar \omega = 0.5 \text{ MeV}$.

10. Only those items which define the values of input parameters which are different than the default values have to be included in the input data file. The values of input parameters defined by any item will stay in effect till another occurrence of the same item, or till the code terminates. This rule is valid irrespectively of how many EXECUTE items follow the given item.
Together with the FORTRAN source code in the file `hfodd.f`, two input data files are provided, `dy152-a.dat` and `dy152-b.dat`. The first one contains only one line with the keyword **EXECUTE**. The second one, which is reproduced in the section **TEST RUN INPUT** below, contains all valid keywords and the input parameters identical to the default values. Therefore, the results of executing the program for the both provided input data files are strictly identical. The second input data file serves only as a suitable pattern to modify the input parameters. However, it should be a good practice to include in the input data file only those items which modify the input parameters with respect to the default values. In this way, a danger of accidentally modifying some intricate numerical parameters of the code is minimized.

In the following subsections we discuss all the valid keywords and their corresponding data lines. The default values are given in the data lines following the keyword lines. In the data lines we also show names of the corresponding variables which are referred to in the text. The meaning of several parameters that are introduced below in the context of their function in the code is explained in detail in Chapter I.

### 3.1 General data

**Keyword: NUCLID**

\[ 86, \ 66 = \text{IN\_FIX, IZ\_FIX} \]

Numbers of neutrons (IN\_FIX) and protons (IZ\_FIX) in the nucleus under consideration. Calculations for odd and odd-odd nuclei require IROTAT=1.

**Keyword: ITERATIONS**

\[ 50 = \text{NOITER} \]

For NOITER>0, the specified number of iterations is performed. The code starts with the iteration number 0 by constructing the initial potentials. Specifying NOITER=0 requests only this initial phase. If the iteration is restarted from the previously recorded potentials (ICONTI=1), then the counting of iterations continues from the previous value on.

### 3.2 Interaction

**Keyword: SKYRME-SET**

\[ \text{SKM* = SKYRME} \]

Character*4 acronym of the Skyrme force parameter set. Must start at the 13-th column of the data line. At present, valid acronyms are SKM*, SIII, SKP, and SKII. Other sets of parameters can easily be included in the subroutine `params`.

**Keyword: EVE_SCA_TS**

\[ 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1. \]

SRHO\_T, SRHO\_S, SRHODT, SRHODS, SLPR\_T, SLPR\_S,
STAU\_T, STAU\_S,
SSCU\_T, SCCU\_S,
SDIV\_T, SDIV\_S

By using this item in the data set the coupling constants corresponding to a given Skyrme parameter set can be arbitrarily scaled. This allows calculations with modified Skyrme
functionals [13]. The time-even coupling constants in the total-sum representation (I-14) are multiplied by the 12 numbers from \( \text{SRHO}_T \) to \( \text{SDIV}_S \). The variables with names ending with \( _T \) and \( _S \) multiply the “total” and “sum” coupling constants, respectively. The variables with names containing the acronyms \( \text{RHO}, \text{LPR}, \text{TAU}, \text{SCU}, \) and \( \text{DIV} \), multiply the coupling constants with superscripts \( \rho, \Delta \rho, \tau, J, \) and \( \nabla J \), respectively, and those with \( \text{RHOD} \) multiply the density-dependent part of \( C^\rho \). Similar name convention is used for many other variables in the code hfodd.

**Keyword:** ODD_SCA_TS

\[
1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.
\]

\( \text{SSPI}_T, \text{SSPI}_S, \text{SSPIDT}, \text{SSPIDS}, \text{SLPS}_T, \text{SLPS}_S, \text{SCUR}_T, \text{SCUR}_S, \text{SKIS}_T, \text{SKIS}_S, \text{SROT}_T, \text{SROT}_S \)

Same as above but for the time-odd coupling constants. Acronyms \( \text{SPI}, \text{LPS}, \text{CUR}, \text{KIS}, \) and \( \text{ROT} \) correspond to coupling constants with superscripts \( s, \Delta s, T, j, \) and \( \nabla j \), respectively, and those with \( \text{SPID} \) correspond to the density-dependent part of \( C^s \).

**Keyword:** EVE_SCA_PM

\[
1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.
\]

\( \text{SRHO}_P, \text{SRHO}_M, \text{SRHODP}, \text{SRHODM}, \text{SLPR}_P, \text{SLPR}_M, \text{STAU}_P, \text{STAU}_M, \text{SSCU}_P, \text{SSCU}_M, \text{SDIV}_P, \text{SDIV}_M \)

Same as above but for the time-even coupling constants in the isoscalar-isovector representation, Eq. (I-15). The variables with names ending with \( _P \) and \( _M \) multiply the isoscalar and isovector coupling constants, respectively. The total-sum scaling factors are used first, and the isoscalar-isovector scaling factors are used afterwards.

**Keyword:** ODD_SCA_PM

\[
1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.
\]

\( \text{SSPI}_P, \text{SSPI}_M, \text{SSPIDP}, \text{SSPIDM}, \text{SLPS}_P, \text{SLPS}_M, \text{SCUR}_P, \text{SCUR}_M, \text{SKIS}_P, \text{SKIS}_M, \text{SROT}_P, \text{SROT}_M \)

Same as above but for the time-odd coupling constants in the isoscalar-isovector representation, Eq. (I-15).

**Keyword:** G_SCALING

\[
1.0, 1.0 = \text{FACTGN, FACTGP}
\]

For \( \text{IPAIRI}=1 \) the code hfodd solves the BCS equations with the neutron and proton pairing strengths defined in Ref. [20]. These values can be modified by defining here the multiplicative factors \( \text{FACTGN} \) and \( \text{FACTGP} \) for neutrons and protons, respectively.

### 3.3 Symmetries

**Keyword:** ROTATION

\[
1 = \text{IROTAT}
\]
Calculation with time-reversal breaking will be performed for $\text{IROTAT}=1$, while the time-reversal symmetry will be conserved for $\text{IROTAT}=0$. In the latter case the calculations will be performed only for one value of the simplex, $s=+i$, which gives almost twice shorter execution times. $\text{IROTAT}=0$ is incompatible with providing a non-zero value of the angular frequency or with attempting a calculation for an odd or odd-odd nucleus.

**Keyword**: $\text{SIGNATUREY}$

1 = $\text{ISIGNY}$

Calculation with parity/signature symmetry conserved are performed for $\text{ISIGNY}=1$, while the parity (and signature) will be broken for $\text{ISIGNY}=0$. In the former case the parity blocks will be separately diagonalized and the densities will be summed only in the 1/8 of all the Gauss-Hermite nodes. This may give almost twice shorter execution times. $\text{ISIGNY}=1$ requires $\text{ISIMTX}=\text{ISIMTZ}=1$, and $\text{ISIGNY}=0$ requires $\text{ISIMTX}=0$, or $\text{ISIMTZ}=0$, or $\text{ISIMTX}=\text{ISIMTZ}=0$, see Sec. 3.4 of I.

**Keyword**: $\text{TSIMPLEXES}$

1, 1 = $\text{ISIMTX}, \text{ISIMTZ}$

Calculation with conserved symmetries given by $x$-simplex$^T$ and/or $z$-simplex$^T$, see Sec. 3.4 of I, will be performed for $\text{ISIMTX}=1$ and/or $\text{ISIMTZ}=1$, respectively. These symmetries will be broken for $\text{ISIMTX}=0$ and $\text{ISIMTZ}=0$. Values of $\text{ISIMTX}$ and $\text{ISIMTZ}$ should be compatible with that of $\text{ISIGNY}$ in such a way that $\text{ISIGNY}=\text{ISIMTX}\times\text{ISIMTZ}$.

**Keyword**: $\text{PAIRING}$

0 = $\text{IPAIRI}$

Calculation without pairing correlations (pure HF) will be performed for $\text{IPAIRI}=0$ while the BCS pairing correlations will be included for $\text{IPAIRI}=1$. The latter case is incompatible with $\text{IROTAT}=1$.

### 3.4 Configurations

**Keyword**: $\text{VACSIM\_NEU}$

43, 43 = $\text{KVASIM}(0), \text{KVASIM}(1)$

Numbers of lowest neutron states occupied in the two blocks, denoted by $(+)$ and $(-)$, of given simplexxes, $s=+i$ and $s=-i$, respectively. These numbers define the simplex reference configuration from which the particle-hole excitations are counted. The definitions of simplex reference configuration and excitations are ignored unless $\text{ISIGNY}=0$ and $\text{IPAIRI}=0$.

**Keyword**: $\text{VACSIM\_PRO}$

33, 33 = $\text{KVASIM}(0), \text{KVASIM}(1)$

Same as above but for the numbers of proton states.

**Keyword**: $\text{PHSIM\_NEU}$

1, 00, 00, 00, 00 = $\text{NUPAHO}, \text{KPSIMP}, \text{KPSIMM}, \text{KHSIMP}, \text{KHSIMM}$

Neutron particle-hole excitations in the simplex blocks. $\text{NUPAHO}$ is the consecutive number from 1 to 5 (up to five sets of excitations can be specified in separate items). Particles are removed from the $\text{KHSIMP}$-th state in the $(+)$ block and from the $\text{KHSIMM}$-th state in the $(-)$ block, and put in the $\text{KPSIMP}$-th state in the $(+)$ block and in the $\text{KPSIMM}$-th
state in the $(-)$ block. At every stage of constructing excitations the Pauli exclusion principle has to be respected (particle removed from an occupied state and put in an empty state). Values equal zero have no effect. In practice, reasonable excitations can only be constructed by consulting the printed lists of single-particle states with their consecutive numbers in given blocks, see the TEST RUN OUTPUT below.

**Keyword:** PHSIMP_PRO

$1, 00, 00, 00, 00 = NUPAHO, KPSIMP, KPSIMM, KHSIMP, KHSIMM$

Same as above but for the proton particle-hole excitations.

**Keyword:** VACSIG_NEU

$22, 22, 21, 21 = KVASIG(0,0), KVASIG(0,1), KVASIG(1,0), KVASIG(1,1)$

Numbers of lowest neutron states occupied in the four blocks, denoted by $(+,+)$, $(+,-)$, $(-,+)$, and $(-,-)$, of given (parity,signature) combinations, i.e., $(\pi, r) = (+1,+i)$, $(+1,-i)$, $(-1,+i)$, and $(-1,-i)$, respectively. These numbers define the parity/signature reference configuration from which the particle-hole excitations are counted. The definitions of parity/signature reference configuration and excitations are ignored unless ISIGNY=1 and IPAIRI=0.

**Keyword:** VACSIG_PRO

$16, 16, 17, 17 = KVASIG(0,0), KVASIG(0,1), KVASIG(1,0), KVASIG(1,1)$

Same as above but for the numbers of proton states.

**Keyword:** PHSIGN_NEU

$1, 00, 00, 00, 00, 00, 00, 00, 00 = NUPAHO, KPPPSP, KPPPSM, KPPMSP, KPPMSM, KHPPSP, KHPPSM, KHPMSP, KHPMSM$

Neutron particle-hole excitations in the parity/signature blocks. Basic principles are the same as for the excitations in the simplex blocks. Particles are removed from the KHPPSP-th state in the $(+,+)$ block, from the KHPPSM-th state in the $(+,-)$ block, from the KHPMSP-th state in the $(-,+)$ block, and from the KHPMSM-th state in the $(-,-)$ block, and put in the KPPPSP-th state in the $(+,+)$ block, in the KPPPSM-th state in the $(+,-)$ block, in the KPPMSP-th state in the $(-,+)$ block, and in the KPPMSM-th state in the $(-,-)$ block.

**Keyword:** PHSIGN_PRO

$1, 00, 00, 00, 00, 00, 00, 00, 00 = NUPAHO, KPPPSP, KPPPSM, KPPMSP, KPPMSM, KHPPSP, KHPPSM, KHPMSP, KHPMSM$

Same as above but for the proton particle-hole excitations.

### 3.5 Ensemble of specific parameters referred to as “Numerical data”

**Keyword:** MAX_MULTIP

$2, 4, 4 = NMUCON, NMUCOU, NMUPRI$

Maximum multipolarities $\lambda$ of multipole moments used in the code for the constraints, Eq. (I-22), surface term of the Coulomb field, Eq. (I-101), and printed on the output,
respectively. Values not larger than $\lambda=9$ are currently allowed. In case of the conserved parity/signature, only even multipoles are used in the Coulomb field.

**Keyword: COULOMB**

$$80, 79, 0.25 = \text{NUMCOU}, \text{NUMETA}, \text{FURMAX}$$

$\text{NUMCOU}$ gives the number of points $N_{\text{Coul}}$, Eq. (I-100), used when summing up the Coulomb Green function. The dimensionless parameter $d$ defining the size of the Coulomb parallelepiped, Eqs. (I-102) and (I-96), is given by $\text{NUMCOU} \times \text{FURMAX}$. $\text{NUMETA}$ gives the order of the Simpson integration of the solid harmonics on the faces of the parallelepiped, see Sec. 5 of I.

**Keyword: SLOW-DOWN**

$$0.5, 0.5 = \text{SLOWEV}, \text{SLOWOD}$$

The standard prescription to calculate the HF potential in the next iteration is to mix a given fraction $\epsilon$ of the HF potentials from the previous iteration, with the fraction $1-\epsilon$ of potentials given by expressions (I-27). $\text{SLOWEV}$ and $\text{SLOWOD}$ give the values of $\epsilon$ separately for the time-even and time-odd potentials.

**Keyword: EPS_HERMIT**

$$1.0 \times 10^{-14} = \text{EPSHER}$$

Numerical precision requested for determining values of Hermite polynomials.

**Keyword: OPTI_GAUSS**

$$1 = \text{IOPTGS}$$

For $\text{IOPTGS}=1$ and $\text{IREAWS}=0$, expression (I-94) is used to calculate the order of the Gauss-Hermite integration, and the input parameters $\text{NXHERM}, \text{NYHERM},$ and $\text{NZHERM}$ are ignored.

**Keyword: GAUSHERMIT**

$$18, 18, 32 = \text{NXHERM}, \text{NYHERM}, \text{NZHERM}$$

Orders $L_x$, $L_y$, and $L_z$ of the Gauss-Hermite integration in three Cartesian directions. Must be even. Ignored if $\text{IOPTGS}=0$ or $\text{IREAWS}=1$.

### 3.6 Parameters of the HO basis

**Keyword: BASIS_SIZE**

$$15, 301, 800.0 = \text{NOSCIL}, \text{NLIMIT}, \text{ENECUT}$$

The HO basis is composed of states having not more than $N_0=\text{NOSCIL}$ quanta in either of the Cartesian directions, and not more than $M=\text{NLIMIT}$ states in total, see Sec. 3. The states are added to the basis according to the increasing energy of the deformed harmonic oscillator (1). In case of degenerate HO states (e.g., for an axially deformed HO) the complete multiplets are included, so the actual number of states $\text{LDBASE}$ can be slightly larger than $\text{NLIMIT}$. If $\text{NLIMIT}>0$, the cut-off energy $\text{ENECUT}$ is ignored; otherwise all states having HO energy smaller then $\text{ENECUT}$ are included in the basis, and $\text{NLIMIT}$ is ignored.

**Keyword: HOMEGAZERO**

$$1.2 = \text{FCHOM0}$$

The code uses the standard value of the spherical HO frequency $\hbar \omega_0=41 \text{ MeV}/A^{1/3}$ multiplied by the scaling factor $f=\text{FCHOM0}$, see Eq. (3).
Keyword: **SURFAC_PAR**

86, 66, 1.23 = **INNUMB, IZNUMB, ROPARM**

The code HFODD calculates parameters of the HO basis, and the zero-iteration Nilsson potential, by defining the standard classical surface $\Sigma$, Eq. (4), corresponding to the volume $\frac{4}{3} \pi R^3 \left( \text{INNUMB} + \text{IZNUMB} \right)$.

**Keyword: SURFAC_DEF**

2, 0, 0.61 = **LAMBDA, MIU, ALPHAR**

The code defines frequencies of the deformed HO in three directions by using relation (5), $\omega_x R_x = \omega_y R_y = \omega_z R_z$, where $R_\mu$ are the lengths of principal axes of the nuclear surface $\Sigma$ defined in the standard way (4) by real deformation parameters $\alpha_{\lambda \mu} = \text{ALPHAR} (\text{LAMBDA}, \text{MIU})$. The overall factor is defined by $(\omega_x \omega_y \omega_z)^{1/3} = \omega_0$. For details see Sec. 4.

### 3.7 Constraints

**Keyword: OMEGAY**

0.5 = **OMEGAY**

Value of the cranking rotational frequency $\omega_J = \text{OMEGAY}$, Eqs. (I-23) and (I-24). Non-zero value requires IROTTAT=1. Pure linear constraint on the value of spin, $\omega_y = \omega_J$ is used for IFLAGI=0.

**Keyword: MULTCONSTR**

2, 0, 0.01, 42.0, 1 = **LAMBDA, MIU, STIFFQ, QASKED, IFLAGQ**

For IFLAGQ=1, the mass multipole moment of the given multipolarity $\lambda$ and $\mu$ is constrained. Values of LAMBDA, MIU, STIFFQ, and QASKED correspond respectively to $\lambda$, $\mu$, $C_{\lambda \mu}$, and $\bar{Q}_{\lambda \mu}$ in Eq. (I-22). For IFLAGQ=0, there is no constraint in the given multipolarity.

**Keyword: SPINCONSTR**

0.0, 0.0, 0 = **STIFFI, ASKEDI, IFLAGI**

For IFLAGI=1, the quadratic constraint on spin is used together with the linear constraint. Values of STIFFI and ASKEDI correspond respectively to $C_J$ and $\bar{J}_y$ in Eq. (I-23). For IFLAGI=0, there is no quadratic constraint on spin, (but there still can be the standard cranking linear constraint defined by OMEGAY).

### 3.8 Output-file parameters

**Keyword: PRINT-ITER**

1, 0, 1 = **IPRSTA, IPRMID, IPRSTO**

The code prints results for the first, middle, and/or last iteration if the corresponding parameters IPRSTA, IPRMID, and IPRSTO equal 1.

**Keyword: EALLMINMAX**

-12.0, 0.0 = **EMINAL, EMAXAL**

The code prints tables of single-particle properties for states with values of the single-particle Routhians between EMINAL and EMAXAL. No table is printed unless EMINAL $\leq$ EMAXAL.
3.9 Files

Keyword: REVIEWFILE
HFODD.REV = FILREV

CHARACTER*68 file name of the review file. Must start at the 13-th column of the data line. The ASCII review file is written after calculating every data set (i.e., once per every EXECUTE item), provided IREVIE>0. The file is not rewound, so the results for several data sets can be accumulated in a single file. This is so provided the filename FILREV is not changed between the EXECUTE items. The file contains all relevant parameters and results of calculation in a form suitable for reading by another program. It is meant as an interface to programs which analyze and/or plot the results. The file contains sections defined by keywords (different than keywords used in the input data file described here). The detailed structure of the review file is not documented in the present write up, and can be inferred from inspecting the specimen produced by the sample run, and from the subroutine REVIEW.

Keyword: REVIEW
2 = IREVIE

The review file will not be written if IREVIE=0. For IREVIE=2, the table of single-particle properties is included in the review file in addition to other results which are written for IREVIE=1.

Keyword: RECORDFILE
HFODD.REC = FILREC

CHARACTER*68 file name of the record file. Must start at the 13-th column of the data line. The binary record file is written after each HF iteration. It contains complete information which allows restarting the iteration in another run of the code. To restart, one has to specify ICONTI=1 and provide the name of the file by defining FILREP. In case of the computer crush, the record file contains the results of the last performed iteration. Upon a successful completion of the given input data set it contains the results of the last performed iteration. The file is always rewound before it is written, so the results for consecutive iterations do not pile up.

Keyword: REPLAYFILE
HFODD.REP = FILREP

CHARACTER*68 file name of the replay file. Must start at the 13-th column of the data line. The binary replay file with the name defined in FILREP must exist if ICONTI=1, and will be read. If the filenames FILREP and FILREC are identical, the replay file will be subsequently overwritten as a new record file. These feature is implemented to facilitate chaining of jobs which follow one another.

Keyword: WOODSAFILE
WOODS.WFN = FILWOO

CHARACTER*68 file name of the Woods-Saxon file. Must start at the 13-th column of the data line. This file is read provided IREAWS=1. The binary Woods-Saxon file constitutes an interface between the Woods-Saxon code and the code HFODD. It contains the Woods-Saxon wave functions and numerous other parameters which define the current calculation.
The parameters read from the Woods-Saxon file overwrite the values provided in the input data file. Since the current version of the Woods-Saxon code will be published separately [21], the feature of starting the iteration from the Woods-Saxon results is not documented in the present write up.

3.10 Starting the iteration

**Keyword: RESTART**

0 = ICONTI

For ICONTI=1, results stored in the replay file (written as a record file in a previous run) will be used to start the iteration. The replay file name should be provided by defining FILREP. If the previous run was done with IREAWS=1, the current run must also use IREAWS=1, and the same Woods-Saxon file must be provided. This so because the Woods-Saxon file contains not only the information about the starting potential (which is ignored for ICONTI=1) but also defines the HO basis.

**Keyword: READ_WOODS**

0 = IREAWS

For IREAWS=1, the results stored by the previously performed Woods-Saxon calculation will be used to start the iteration and to define the HO basis. For ICONTI=1 and IREAWS=1 the Woods-Saxon file must also be provided, and will be used only to define the HO basis.

**Keyword: NILSSONPAR**

0, -1.175, -0.247, -1.175, -0.352, 11.17, 11.17, 6.28 = NILDAT, CNILSN, DNILSN, CNILSP, DNILSP, HBANIX, HBANIY, HBANIZ

For IREAWS=0 and ICONTI=0, the code starts the calculation from the Nilsson potential. If NILDAT=1, the Nilsson parameters C and D [Ref. [22], Eq. (2.89)] are for neutrons given by CNILSN and DNILSN and for protons by CNILSP and DNILSP, while the HO deformation is defined by HBANIX, HBANIY, and HBANIZ. If NILDAT=0, the Nilsson parameters C and D are defined by Eq. (2.91) and Table 2.3 of Ref. [22], while the HO deformation is the same as that of the HO basis defined in Sec. 3.6. In the latter case the Nilsson parameters read from the input file are ignored.

4 Output file

Together with the FORTRAN source code in the file hfodd.f, an example of the output file is provided in dy152.out. Selected lines from this file are presented in the section TEST RUN OUTPUT below. This output file corresponds to the input file dy152-b.dat reproduced in the section TEST RUN INPUT below. Most of the information printed on the output file is self-explanatory. Here we only give some details which are not explicit in the output file.

The output file begins with the information pertaining to the general parameters of the calculation, then gives information about the starting point of the iteration, provides the convergence report, and finally contains the results calculated at the last iteration.
Section CLASSICAL NUCLEAR SURFACE lists the deformation parameters used to define the nuclear surface, Eq. (4), from which the basis parameters are derived, as described in Sec. 2. It also gives the basis parameters such as the OSCILLATOR FREQUENCIES: HBAROX, HBARQ, and HBARZ corresponding to $\hbar \omega_x$, $\hbar \omega_y$, and $\hbar \omega_z$.

Section PHYSICAL CONSTANTS gives the values of $\hbar c$ in MeV fm (H_BARC), of $\hbar c/e^2$ (H_COE2), of the neutron and proton masses in MeV/c$^2$ (X_MASSN and X_MASSP), of the kinetic-energy coefficient $\hbar^2/2m$ before (H_BMASS) and after the center-of-mass correction (H_BMRPA), Eq. (I-10), of the elementary charge squared $e^2$ in MeV fm (E_CHAR2), and of the coefficient preceding the integral in the Coulomb exchange energy (COULEX), Eq. (I-19). For details see the comments in the SETBAS subroutine.

Section OSCILLATOR LENGTHS, CONSTANTS, and FREQUENCIES gives the values of $1/b_\mu$, $b_\mu$, and $\hbar \omega_\mu$, respectively, which characterize the HO basis in three Cartesian directions, $\mu=x, y, z$.

Section BASIS CUT-OFF CONTROL PARAMETERS gives maximum numbers of the HO quanta in three directions NXMAXX, NYMAXX, and NZMAXX corresponding to $N_x$, $N_y$, and $N_z$, Eq. (I-70), as well as the orders of the Gauss-Hermite quadratures NXHERM, NYHERM, and NZHERM, corresponding to $L_x$, $L_y$, and $L_z$, Eq. (I-92). It also gives the number $M$ of the HO states included in the basis as requested in the input file (NLIMIT) and as used in the calculation (LDBASE).

Section SHAPE OF THE OSCILLATOR-BASIS DIAMOND gives the numbers of the HO quanta in a given direction for fixed numbers of the HO quanta in both remaining directions. The output is arranged in such a way that the shape of the grid of points $n_xn_yn_z$ is clearly visualized by projections in every of the three directions.

Section PARAMETER SET gives the name and the values of the Skyrme force parameters, while the following section COEFFICIENTS DEFINING THE SKYRME FUNCTIONAL gives the corresponding values of the coupling constants in the Skyrme functional (I-12). These values take into account the scaling factors, Sec. 3.2, which are printed in the section SCALING FACTORS unless all are equal 1.

Sections PARITY/SIGNATURE CONFIGURATIONS or SIMPLEX CONFIGURATIONS give the vacuum and particle-hole configurations requested in the input file for the case of the parity symmetry conserved or not conserved, respectively.

Section CONVERGENCE REPORT gives the list of performed iterations. For each iteration one line is printed with the energy (I-9), stability (I-37), average values of the quadrupole moments $Q_{20}$ and $Q_{22}$, total angular momentum, angular frequency (I-24), and the ratio of energies $\mathcal{E}/\tilde{\mathcal{E}}$, see Sec. 2.5 of I. The line corresponding to the last iteration is not printed because a more detailed information is given for the final state below this section.

Section SINGLE-PARTICLE PROPERTIES lists the single-particle states calculated for the Nilsson, Woods-Saxon, or Hartree-Fock Routhian operators. For every state one line is printed which gives the value of the single-particle Routhian, cf. Eq. (31), the consecutive numbers in the parity/signature or simplex blocks, the quantum numbers $[N, n_z, \Lambda|\Omega$ of the asymptotic Nilsson state which has the largest component in the given state, the average value of the parity operator (in %), the average values of projections of the intrinsic and total angular momenta (in $\hbar$), and their ratio called the $g$-factor.

Sections MULTIPOLE MOMENTS give the values of neutron, proton, or mass multipole moments in units of $(10 \text{ fm})^\lambda$. Traditional normalization factors are used as explained in
the comments in the subroutine DEFUNI. In particular, the \( \lambda=0 \) moment corresponds to the number of particles.

Sections **ROOT-MEAN-SQUARE AND GEOMETRIC SIZES** gives the rms average values of the radius and of the \( x, y, \) and \( z \) coordinates. In order to better visualize the size of the nucleus, the geometric sizes are also calculated by multiplying the rms radius by \( \sqrt{5}/3 \) and the rms coordinates by \( \sqrt{5} \).

Section **DENSITY INTEGRALS IN THE SKYRME FUNCTIONAL** gives the integrals of products of densities, which appear in the Skyrme functional (I-12). The terms in the functional are identified by the acronyms described in Sec. 3.2.

Section **CONTRIBUTIONS TO ENERGY IN THE SKYRME FUNCTIONAL** gives the values of various terms which appear in the Skyrme functional (I-12). These contributions are the products of the coupling constants and of the density integrals described above. The sums of time-even and time-odd contributions are also printed.

Section **ANGULAR MOMENTA** gives the average values of the total and intrinsic neutron, proton, and total angular momentum (in \( \hbar \)). It also gives the corresponding values and contributions to the first moment of inertia \( J^{(1)} = I/\omega \) (in \( \hbar^2/\text{MeV} \)).

Sections **NEUTRON CONFIGURATIONS** and **PROTON CONFIGURATIONS** give a visual representation of states occupied in the parity/signature or simplex blocks. The lines denoted by **CONF**: give the configurations requested in the input data set, while those denoted by **VACC**: give the configurations characterizing the given HF state. By comparing the two sets one can verify whether the requested configuration has been obtained, and eventually devise a new configuration to be calculated. The consecutive numbers printed in the horizontal direction correspond to the consecutive numbers in blocks printed in the section **SINGLE-PARTICLE PROPERTIES**. By comparing the two sections one can effectively associate the Nilsson labels with the calculated configurations and also prepare the configuration input data described in Sec. 3.4.

Section **ENERGIES** gives a summary of the energies calculated for the HF state. The kinetic (I-10), single-particle (I-32), and pairing (I-20) energies are printed for neutrons (NEU), protons (PRO), and all particles (TOT). The direct (DIR) and exchange (EXC) Coulomb energies, Eqs. (I-17a) and (I-19), are printed together with their sum (TOT). The multipole (MULT) and cranking (SPIN) constraint energies, Eqs. (I-22) and (I-23), are printed together with the corresponding corrections (CORR.) given by Eqs. (I-35) and (I-36), respectively. Then the rearrangement energy (I-34) is printed followed by the value of the Routhian (I-21) and the spin-orbit and Skyrme energies, the latter two split in the time-even (EVE) and time-odd (ODD) contributions. Finally, the total energies \( \mathcal{E} \) (I-33) and \( \bar{\mathcal{E}} \) (I-9) are printed as (SP) and (FUN), respectively, while their difference (I-37) is printed as the stability (STAB).

5 **FORTRAN source file**

The FORTRAN source code in is provided in the file hfodd.f and can be modified in several places which are described in this section.
5.1 Dimensions of arrays

The code HFODD uses the arrays’ dimensions declared through the PARAMETER statements. This allows changing the dimensions and adapting the size of the reserved memory to the problem being solved. Whenever too small a dimension is defined the code aborts with a message indicating the dimension which should be increased. Substantial amount of memory is required only for arrays depending on the following PARAMETER values:

PARAMETER (NDMAIN=16)
Should be larger or equal to the input parameter NOSCIL defined in Sec. 3.0.

PARAMETER (NDBASE=307)
Should be larger or equal to the input parameter NLIMIT defined in Sec. 3.0. It should also be larger or equal to the actual size of the HO basis LDBASE, which can be larger than NLIMIT in case of degenerate HO states.

PARAMETER (NDXHRM=19,NDYHRM=19,NDZHRM=33)
Should be respectively larger or equal to the input parameters NXHERM, NYHERM, and NZHERM defined in Sec. 3.3.

On vector machines, parameters NDXHRM, NDYHRM, NDZHRM, and NDBASE should be odd in order to minimize the risk of bank memory conflicts. By the same token, parameter NDMAIN should be even, because it defines the matrix dimensions beginning with 0.

5.2 Vectorization properties

As discussed in Sec 4.3 of I, the code HFODD has to operate by using seven-fold nested short loops, and this part does not perform well in a vector processor. However, it turns out that the loops can be artificially made longer in such a way that the final CPU time in a vector processor actually becomes much shorter. All the places where this trick has been applied can be identified in the source file by finding the lines beginning with CVVECTOR and CSCALAR, for example:

CVVECTOR
DO KZ=0,LAZOXY(NX,NY)+LAZOXY(MX,MY)
CSCALAR DO KZ=NZ+MZ,0,-2
RESULT=RESULT+COEF00(KZ,NZ,MZ,3)*T_AUXI(KZ)
END DO

The line beginning with CSCALAR should be made active on a scalar or superscalar machine. The line immediately below the line which begins with CVVECTOR should be active on a vector machine. The results of calculation do not depend on which version of loops is activated.

5.3 Library subroutines

The code HFODD requires an external subroutine which diagonalizes complex hermitian matrices. In the present version, the code calls the NAGLIB subroutine F02AXE. The call to this subroutine is performed through the interface subroutine ZHPEV, which can be replaced by the ESSL subroutine of the same name and the same calling parameters.
In the present version, the code also uses an external subroutine CGEMM from the BLAS CRAY library, which multiplies three complex matrices. By using the value of \texttt{PARAMETER (I\_CRAY=0)} the call is diverted to the functionally equivalent subroutine ZGEMUL, which is provided in the source file. However, the latter one can also be replaced by the ESSL subroutine of the same name and the same calling parameters.

6 Acknowledgments

Useful comments by G. Hackman, P.-H. Heenen, and W. Satula are gratefully acknowledged. We thank P.-H. Heenen for providing us with results of test calculations performed with his HF code, and K. Burzyński, W.D. Luo, H. Molique, and T.R. Werner for help in coding the subroutines SKFILD, SOLHAR, PHASES, and CYLXYZ. We would like to express our thanks to the Institut du Développement et de Ressources en Informatique Scientifique (IDRIS) of CNRS, France, which provided us with the computing facilities under Project No. 960333. This research was supported in part by the Polish Committee for Scientific Research under Contract No. 2 P03B 034 08, and by the computational grant from the Interdisciplinary Centre for Mathematical and Computational Modeling (ICM) of the Warsaw University.

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## TEST RUN INPUT

**NUCLID**

```
  86 66
```

**ITERATIONS**

```
  50
```

**SKYRME-SET**

```
| EVE_SCA_TS | RHOD | RHOD | LPR | TAU | SCU | DIV |
|------------|------|------|-----|-----|-----|-----|
| ODD_SCA_TS | SPI  | SPI  | LPS | CUR | KIS | ROT |
| EVE_SCA_PM | RHOD | RHOD | LPR | TAU | SCU | DIV |
| ODD_SCA_PM | SPI  | SPI  | LPS | CUR | KIS | ROT |
| G_SCALING  | 1.1  | 1.1  | 1.1 | 1.1 | 1.1 | 1.1 |
```

**G_SYMMETRIES**

```
  1.0 1.0
```

**Symmetry**

```
  1
```

**TSIMPLEXES**

```
  1
```

**PAIRING**

```
  0
```

**VACSIM_NEU**

```
  SIMP SIMM
```

**VACSIM_PRO**

```
  SIMP SIMM
```

**PHSIMP_NEU**

```
  PART HOLE
```

**PHSIMP_PRO**

```
  PART HOLE
```

**VACSIG_NEU**

```
  PPSM PMSP PMSP PMSP
```

**VACSIG_PRO**

```
  PPSM PMSP PMSP PMSP
```

**PHSIGN_NEU**

```
  PARTICLES HOLES
```

**PHSIGN_PRO**

```
  PARTICLES HOLES
```

**MAX_MULTIP**

```
  2 4 4
```

**COULOMB**

```
  80 79 0.25
```

**SLOW-DOWN**

```
  0.5 0.5
```

**EPS_HERMIT**

```
  1.00E-14
```

**OPTI_GAUSS**

```
  1
```

**GAUSHERMIT**

```
  18 18 32
```

**BASIS_SIZE**

```
  15 301 800.
```

**HOMEZZERO**

```
  1.2
```

**SURFAC_PAR**

```
  86 66 1.23
```

**SURFAC_DEF**

```
  2 0 0.61
```

**OMEGAY**

```
  0.50
```

**MULTCONST**

```
  2 0 0.01 42. 1
```

**SPINCONST**

```
  0.00 0. 0
```

**PRINT-ITER**

```
  1 0 1
```

**EALLMINMAX**

```
  -12. 0.
```

**REVIEWFILE**

```
  HFODD.REV
```

**REVIEW**

```
  2
```

**RECORDFILE**

```
  HFODD.REC
```

**REPLAYFILE**

```
  HFODD.REP
```

**WOODSAFILE**

```
  WOODS.WFN
```

**RESTART**

```
  0
```

**READ_WOODS**

```
  0
```

**NILSSONPAR**

```
  0 -1.175 -0.247 -1.175 -0.352 11.170 11.170 6.280
```

**EXECUTE**

```
  Calculate
```

**ALL_DONE**

```
  Terminate
```
TEST RUN OUTPUT

SKYRME-HARTREE-FOCK CODE VERSION 1.60N
ONE SYMMETRY-PLANE AND NO TIME-REVERSAL SYMMETRY
DEFORMED CARTESIAN HARMONIC-OSCILLATOR BASIS

JACEK DOBACZEWSKI AND JERZY DUDEK
CENTRE DE RECHERCHES NUCLEAIRES, STRASBOURG, 1993-96

CLASSICAL NUCLEAR SURFACE DEFINED FOR: N = 86  Z = 66

AL10 = ZERO AL11 = ZERO ...................... ......................
AL20 = 0.610 AL21 = ZERO AL22 = ZERO ......................
AL30 = ZERO AL31 = ZERO AL32 = ZERO AL33 = ZERO ......................
AL40 = 0.100 AL41 = ZERO AL42 = ZERO AL43 = ZERO AL44 = ZERO

HOMEGA= 9.2190 FCHOM0= 1.2000

OSCILLATOR FREQUENCIES: HBAROX= 11.1998 HBARY= 11.1998 HBAROZ= 6.2464
MOMENTS OF INERTIA: XMOMFC= 90.2596 YMOMFC= 90.2596 ZMOMFC= 42.8300
CENTRES OF MASS: CMSXFC= 0.0000 CMSYFC= 0.0000 CMSZFC= 0.0000

PHYSICAL CONSTANTS: H_BARC=197.32891000 HBCDE2=137.03602000
XMASSN=938.90590000 XMASSP=938.27231000
HBMASS= 20.73620941 HBMRF= 20.59978698
ECHAR2= 1.43997841 COULEX= -1.06350868

OSCILLATOR LENGTHS: X= 1.9243099 Y= 1.9243099 Z= 2.5766958
OSCILLATOR CONSTANTS: X= 0.5196668 Y= 0.5196668 Z= 0.3880939
OSCILLATOR FREQUENCIES: X=11.1997759 Y=11.1997759 Z= 6.2464470

BASIS CUT-OFF CONTROL PARAMETERS: NMAXX= 8 NYMAXX= 8 NZMAXX= 15
OPTIMUM NUMBERS OF GAUSS POINTS: NXHERM= 18 NYHERM= 18 NZHERM= 32
NLIMIT= 301 LDATABASE= 306 MCDV= 4096
ENEVET= 800.0000 ELIMIT= 112.7539

SHAPE OF THE OSCILLATOR-BASIS DIAMOND

WZ >>>>> 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
MAX.NX => 8 8 7 7 6 6 5 4 4 3 3 2 2 1 0 0

NX= 0 (15) | 8 8 7 7 6 6 5 4 4 3 3 2 2 1 0 0
NX= 1 (13) | 7 7 6 6 5 5 4 4 3 3 2 2 1 1 0 0
NX= 2 (12) | 6 6 5 5 4 4 3 3 2 2 1 1 0 0 0
NX= 3 (10) | 5 5 4 4 3 3 2 2 1 1 0 0 0
NX= 4 ( 8) | 4 4 3 3 2 2 1 0 0 0
NX= 5 ( 6) | 3 3 2 2 1 0 0 0
NX= 6 ( 5) | 2 2 1 0 0
NX= 7 ( 3) | 1 0 0
NX= 8 ( 1) | 0 0

PARAMETER SET SKM*: T0= -2645.00 T1= 410.00 T2= -135.00 T3= 15595.00
POWER=0.1667 W=130 X0= 0.09000 X1= 0.00000 X2= 0.00000 X3= 0.00000
### Coefficients Defining the Skyrme Functional

| TOTAL(T) | SUM(S) | ISOSCALAR(P) | ISOVECTOR(M) |
|----------|--------|--------------|--------------|
| CRHO_ =  | -1382.012500 | 780.275000 | -991.875000 | 390.137500 |
| CRHOD =  | 1299.583333 | -649.791667 | 974.687500 | -324.895833 |
| CLPR_ =  | -85.312500 | 34.218750 | -68.203125 | 17.109375 |
| CTAU_ =  | 68.750000 | -68.125000 | 34.687500 | -34.062500 |
| CSCU_ =  | 0.000000 | 68.125000 | 34.062500 | 34.062500 |
| CDIV_ =  | -65.000000 | -65.000000 | -97.500000 | -32.500000 |
| CSPI_ =  | -59.512500 | 661.250000 | 271.112500 | 330.625000 |
| CSPID =  | 0.000000 | -649.791667 | -324.895833 | -324.895833 |
| CLPS_ =  | 0.000000 | 34.218750 | 17.109375 | 17.109375 |
| CCUR_ =  | -68.750000 | 68.125000 | -34.687500 | 34.062500 |
| CKIS_ =  | 0.000000 | -68.125000 | -34.062500 | -34.062500 |
| CROT_ =  | -65.000000 | -65.000000 | -97.500000 | -32.500000 |

---

### Parity/Signature Configurations:

| VACUUM | PARTICLES | HOLES |
|--------|-----------|-------|
| (+++) | (+-) | (-+) | (--) | (++) | (+-) | (-+) | (--) | (++) | (+-) | (-+) | (--) |

**Neutrons:**

| 22 | 22 | 21 | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Protons:**

| 16 | 16 | 17 | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

---

### Convergence Report

| ITER | ENERGY | STABILITY | Q20 | Q22 | SPIN | OMEGA | HOW | NICE |
|------|--------|-----------|-----|-----|------|-------|-----|------|
| 0    | -559.936804 | -759.775974 | 54.178 | -0.027 | 98.777 | 0.500 | 0.42 | 4287 |
| 1    | -1163.385944 | 88.001030 | 48.812 | 0.025 | 53.243 | 0.500 | ... |

---

### Single-Particle Properties: Hartree-Fock

| NO | ENERGY | (+,+,+-,-+,--) | N,nz,\,\,\,O\,\,MEG\,\,\,<P> | JY | SY | GFACT |
|----|--------|----------------|---------------------------|----|----|------|
| 76 | -11.743 | 0, 0, 21, 0 | 5, 3, 2, 3/2 | -100 | 0.103 | -0.092 | -0.896 |
| 77 | -11.728 | 0, 0, 0, 20 | 5, 3, 2, 3/2 | -100 | 0.209 | -0.101 | -0.485 |
| 79 | -11.334 | 19, 0, 0, 0 | 4, 1, 1, 1/2 | 100 | 0.007 | -0.182 | -3.661 |
| 80 | -11.241 | 0, 19, 0, 0 | 4, 1, 3, 5/2 | 100 | 0.119 | -0.176 | -0.157 |
| 81 | -11.069 | 0, 20, 0, 0 | 6, 5, 1, 1/2 | 100 | 0.984 | 0.084 | 0.086 |
| 82 | -10.881 | 22, 0, 0, 0 | 6, 4, 2, 5/2 | 100 | 0.156 | -0.064 | 0.414 |
| 83 | -10.385 | 0, 22, 0, 0 | 6, 4, 2, 5/2 | 100 | 0.007 | -0.024 | -3.300 |
| 84 | -9.557 | 0, 0, 21, 0 | 7, 6, 1, 3/2 | -100 | 2.517 | 0.041 | 0.016 |
| 85 | -7.880 | 0, 0, 22, 0 | 5, 2, 1, 3/2 | -100 | 0.903 | 0.201 | 0.222 |
| 86 | -7.738 | 23, 0, 0, 0 | 4, 0, 2, 5/2 | -100 | 0.247 | 0.161 | 0.650 |
| 87 | -7.732 | 23, 0, 0, 0 | 4, 0, 2, 5/2 | -100 | 0.235 | 0.162 | 0.690 |
| 88 | -7.562 | 0, 0, 23, 0 | 5, 2, 1, 3/2 | -100 | 1.088 | 0.152 | 0.144 |
| 89 | -7.221 | 0, 0, 0, 23 | 5, 1, 4, 9/2 | -100 | 0.355 | 0.023 | -0.083 |
| 90 | -7.096 | 0, 0, 0, 24 | 5, 1, 4, 9/2 | -100 | 0.337 | 0.028 | -0.038 |
| 91 | -7.076 | 0, 24, 0, 0 | 6, 4, 0, 1/2 | 100 | 1.232 | 0.328 | 0.266 |
| 92 | -6.350 | 0, 25, 0, 0 | 6, 4, 0, 1/2 | 100 | 0.078 | 0.453 | 5.825 |
| 93 | -6.284 | 24, 0, 0, 0 | 6, 3, 3, 7/2 | 100 | 0.140 | -0.044 | -0.313 |

---

### Density Integrals in the Skyrme Functional

| TOTAL(T) | SUM(S) | ISOSCALAR(P) | ISOVECTOR(M) |
|----------|--------|--------------|--------------|
| DRHO_ =  | 1.783618 | 8.912767 | 17.566618 | 0.258917 |
| DRHOD =  | 12.388132 | 6.334617 | 12.488132 | 0.181102 |
| DLPR_ =  | -3.719401 | -1.886188 | -3.719401 | -0.052976 |
| DTAU_ =  | 15.598603 | 7.983209 | 15.598603 | 0.367815 |
| DSCU_ =  | 0.119929 | 0.419204 | 0.119929 | 0.014912 |
| DDIV_ =  | 0.823496 | 0.10522 | 0.823496 | 0.014912 |
| DSPI_ =  | 0.018303 | 0.010522 | 0.018303 | 0.002740 |
| DSPID =  | -0.037372 | -0.023155 | -0.037372 | -0.008937 |
| DLPS_ =  | 0.007844 | 0.004322 | 0.007844 | 0.001941 |
### Contributions to Energy in the Skyrme Functional

|       | TOTAL(T) | SUM(S) | ISOSCALAR(P) | ISOVECTOR(M) |
|-------|----------|--------|--------------|--------------|
| ERHO_ | -24277.285107 | 6954.409417 | -17423.888833 | 101.013143   |
| ERHOD | 16229.367796  | -4116.181326 | 12172.025847 | -58.83977    |
| ELPR_ | 317.311377    | -64.543010  | 253.874755   | -0.963888    |
| ETAU_ | 1072.403924   | -543.856103 | 541.076525   | -12.528704   |
| ESCU_ | 0.000000      | 4.365047    | 4.085077     | 0.279970     |
| EDIV_ | -53.527218    | -27.248236  | -80.290827   | -0.484627    |

#### Sum Even:

-6711.729227  2206.945789 -4533.317455  28.534017

#### Sum Odd:

-5.773917  2.093065 -3.823078  0.142226

|       | Q_{00} | Q_{21} | Q_{22} | Q_{30} | Q_{31} | Q_{32} | Q_{33} |
|-------|--------|--------|--------|--------|--------|--------|--------|
|       | 152.0000 | ZERO   | ZERO   | ZERO   | ZERO   | ZERO   | ZERO   |
|       | 41.8059  | Q_{21}  | 0.0675 |        |        |        |        |
|       | 4.7902   | Q_{41}  | 0.0058 | Q_{43} | 0.0011 |        |        |
Table 1: Precision of results obtained for various observables calculated by the code HFODD with the physical basis parameters and $M=300$. Absolute values correspond to the results for $^{152}$Dy and $^{208}$Pb, while the relative values correspond to the differences between $^{151}$Tb and $^{152}$Dy. The values denoted as “precision” represent also typical orders of magnitude of the accuracy obtained in other cases for the same numerical conditions.

| Observable               | Absolute values | Relative values |
|--------------------------|-----------------|-----------------|
|                          | Precision | %       | Precision | %       |
| Energy $\mathcal{E}$     | 5 MeV     | 0.4%    | 0.05 MeV  | 1%      |
| Radius $R_{\text{rms}}$  | 0.003 fm   | 0.05%   | —         | —       |
| Quadrupole moment $Q_p$  | 0.05 b     | 0.3%    | 0.01 b    | 1%      |
| Angular momentum $I$     | $0.1\,\hbar$ | 0.2%    | $0.03\,\hbar$ | 2%    |
| Static moment $\mathcal{J}^{(1)}$ | $0.2\,\hbar^2$/MeV | 0.2% | $0.06\,\hbar^2$/MeV | 2% |
| Dynamic moment $\mathcal{J}^{(2)}$ | $0.1\,\hbar^2$/MeV | 0.1% | $0.1\,\hbar^2$/MeV | 2-10% |
Figure 1: Ground-state energies of $^{208}$Pb (solid lines) calculated as functions of $\hbar \omega_0$ for several values of the number $N_0$ of HO shells included in the basis. Dots denote the minima, the asterisk gives the value calculated at $N_0=20$ for the physical value of $\hbar \omega_0$ shown by the vertical dashed line. The horizontal dashed line represents the exact result.

Figure 2: Same as in Fig. 1 but for the rms radii of $^{208}$Pb.

Figure 3: Energies of superdeformed state in $^{152}$Dy (solid lines) calculated at $\hbar \omega=0$ as functions of the basis deformation parameter $q$ for several values of the number $M$ of states included in the HO basis. Dots denote the minima, the asterisk gives the value calculated at $M=1200$ for the physical value of $q$ shown by the vertical dashed line. The horizontal dashed line represents the exact result, see text.

Figure 4: Same as in Fig. 3 but for the proton quadrupole moments $Q_p$ of superdeformed $^{152}$Dy at $\hbar \omega=0$.

Figure 5: Angular momenta of the rotating superdeformed nucleus $^{152}$Dy (solid lines) calculated at $\hbar \omega=0.5$ MeV as functions of the basis deformation parameter $q$ for several values of the number $M$ of states included in the HO basis. Dots denote the minima of the Routhian (I-21), the asterisk and the horizontal dashed line gives the value calculated at $M=1200$ for the physical value of $q$ shown by the vertical dashed line.

Figure 6: Same as in Fig. 5 but for the dynamic moment $J^{(2)}$ of superdeformed $^{152}$Dy at $\hbar \omega=0.55$ MeV.

Figure 7: Proton separation energies in SD $^{152}$Dy (solid lines) relative to the SD state in $^{151}$Tb, i.e., the differences of energies $E^{SD}(^{151}$Tb$)-E^{SD}(^{152}$Dy$)$, obtained at $\hbar \omega=0$. Solid lines denote values calculated as functions of the basis deformation parameter $q$ for several values of the number $M$ of states included in the HO basis. Dots denote the minima of the energy in $^{152}$Dy (Fig. 3), the asterisk and the horizontal dashed line gives the value calculated at $M=1200$ for the physical value of $q$ shown by the vertical dashed line.
Figure 8: Same as in Fig. [4] but for the quadrupole polarizations induced in the $^{152}$Dy core by the $\pi[651](3/2)(+i)$ hole, i.e., the differences of proton quadrupole moments $Q_{p}^{SD}(^{151}\text{Tb})-Q_{p}^{SD}(^{152}\text{Dy})$.

Figure 9: Same as in Fig. [4] but for the relative alignments between the $^{151}\text{Tb}$ and $^{152}\text{Dy}$ nuclei, i.e., the differences of angular momenta $I(^{151}\text{Tb})-I(^{152}\text{Dy})$ at $\hbar\omega=0.5\text{MeV}$. 
Figure 2.
Figure 3.

![Diagram of energy vs. q=(\omega_x/\omega_z) = (\omega_y/\omega_z)]
Figure 4.

Quadrupole Moment $Q_p$ (b)

$q = \omega_x / \omega_z = \omega_y / \omega_z$

$^{152}\text{Dy SD}$
Figure 5.
Figure 6.
Figure 7.
Figure 8.
Figure 9.