Radiative signature of magnetic fields in internal shocks

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ABSTRACT

Common models of blazars and gamma-ray bursts assume that the plasma underlying the observed phenomenology is magnetized to some extent. Within this context, radiative signatures of dissipation of kinetic and conversion of magnetic energy in internal shocks of relativistic magnetized outflows are studied. We model internal shocks as being caused by collisions of homogeneous plasma shells. We compute the flow state after the shell interaction by solving Riemann problems at the contact surface between the colliding shells, and then compute the emission from the resulting shocks. Under the assumption of a constant flow luminosity, we find that there is a clear difference between the models where both shells are weakly magnetized (\(\sigma \lesssim 10^{-2}\)) and those where, at least, one shell has \(\sigma \gtrsim 10^{-2}\). We obtain that the radiative efficiency is largest for models in which, regardless of the ordering, one shell is weakly and the other strongly magnetized. Substantial differences between weakly and strongly magnetized shell collisions are observed in the inverse-Compton part of the spectrum, as well as in the optical, X-ray and 1-GeV light curves. We propose a way to distinguish observationally between weakly magnetized and strongly magnetized internal shocks by comparing the maximum frequency of the inverse-Compton part and synchrotron part of the spectrum to the ratio of the inverse-Compton to synchrotron fluence. Finally, our results suggest that low-frequency peaked blazars (LBL) may correspond to barely magnetized flows, while high-frequency peaked blazars (HBL) could correspond to moderately magnetized ones. Indeed, by comparing with actual blazar observations, we conclude that the magnetization of typical blazars is \(\sigma \lesssim 0.01\) for the internal shock model to be valid in these sources.

Key words: MHD – radiation mechanisms: non-thermal – radiative transfer – shock waves – gamma-ray burst: general – BL Lacertae objects: general.

1 INTRODUCTION

Highly variable radiation flux has been observed in the relativistic outflows of blazars and gamma-ray bursts (GRBs). Even though the radiation energy and time-scales are different for both classes of objects (gamma-rays on a millisecond time-scale for GRBs versus X-rays on a time-scale of hours for blazars), the underlying physics responsible for the energy dissipation might be very similar. The internal shock scenario (Rees & Meszaros 1994) has been used to explain the variability of blazars (e.g. Spada et al. 2001; Mimica et al. 2004) and GRBs (e.g. Kobayashi, Piran & Sari 1997; Daigne & Mochkovitch 1998; Spada et al. 2001; Bošnjak, Daigne & Dubus 2009). In this scenario, inhomogeneities in a relativistic outflow cause parts of the fluid to collide and produce shock waves which dissipate energy. The shell collisions are often idealized as collisions of dense shells. In recent years, 1D and 2D relativistic hydrodynamics (Kino, Mizuta & Yamada 2004; Mimica et al. 2004, 2005) and relativistic magnetohydrodynamics (Mimica, Aloy & Müller 2007) simulations of the shell collisions have been performed and have showed that the dynamics of shell interaction is much more complex than what is commonly assumed when modelling shell interactions analytically (e.g. Kobayashi et al. 1997; Daigne & Mochkovitch 1998; Spada et al. 2001; Bošnjak et al. 2009). Particularly, the influence of the magnetic field (if present) has been shown to significantly alter the dynamics (Mimica et al. 2007). In spite of these efforts, we still do not know with certainty whether the flow, whose energy is being dissipated, is significantly magnetized, or whether it is only the kinetic energy which ultimately powers the emission.

In a previous work (Mimica & Aloy 2010, hereafter MA10), we have studied the dynamic efficiency, i.e. the efficiency of conversion of kinetic to thermal and/or magnetic energy in internal shocks. We found that the dynamic efficiency is actually higher if the shells are moderately magnetized (\(\sigma \approx 0.1\), see the next section for the definition of \(\sigma\)) than if both are unmagnetized. However, we did not compute the radiative efficiency of such interactions, but instead used the dynamic efficiency as an upper bound of it. Recently, Böttcher & Dermer (2010, hereafter BD10), Joshi & Böttcher

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(2011) and Chen et al. (2011) have presented sophisticated models for the detailed computation of the emission from internal shocks. While these models assume a simple hydrodynamic evolution, they employ a time-dependent radiative transfer scheme which involves the synchrotron and synchrotron self-Compton (SSC) processes as well as the contribution Comptonized external radiation [external inverse Compton (EIC)], all while taking into account the radiative losses of the emitting non-thermal particles. We have adapted the method of BD10 and use it to perform a parametric study, addressed to infer the magnetization of the flow from the light curves and spectra of internal shocks in magnetized plasma.

The organization of this paper is as follows. Section 2 briefly summarizes the model of MA10 which is used to study the shell collision dynamics, and in Sections 3 and 4 we describe the numerical method we employ to compute the non-thermal radiation. We discuss the radiative efficiency in Section 5 and present the spectra and light curves in Section 6. A global parameter study is elaborated in Section 7. We close the paper with a discussion of our results and give our conclusions (Section 8).

2 SHELL COLLISION DYNAMICS

As was discussed in detail in the Section 2 of MA10, our aim is to model a large number of shell collisions with varying properties. Therefore, we employ a simplified model for a single-shell collision, based on the exact solution of the Riemann problem. When describing the initial states of the Riemann problem, we will use subscripts L and R to denote left (faster) and right (slower) shells, respectively.

We assume a cylindrical outflow with a radius $R$. Mimica et al. (2004) show that the jet lateral expansion in this case is negligible. For simplicity, and being consistent with previous work in the field (e.g. BD10; Joshi & Böttcher 2011), we also ignore the shell longitudinal expansion after the shock crosses the shell (see also Section 3.4). Following equation (9) of MA10, we define the luminosity as

$$L = \pi R^2 \rho c^3 \left[ \Gamma^2 (1 + \epsilon + \chi + \sigma - 1) \right] \sqrt{1 - \Gamma^{-2}},$$

(1)

where $c$ is the speed of light in vacuum, $\rho$ is the fluid rest-mass density, $\epsilon$ is the specific internal energy, $\chi := \rho/(\rho c^2)$ is the initial ratio between the thermal pressure and the rest-mass energy density, and $\sigma := B^2/(4\pi \rho \Gamma^2 c^2)$ is the magnetization parameter. Here $B$ is the strength of the large-scale magnetic field, which is perpendicular to the direction of propagation of the fluid moving with velocity $v$ and a corresponding Lorentz factor $\Gamma := 1/\sqrt{1 - (v/c)^2}$. The specific internal energy is related to the pressure and to the density through the equation of state. We use the TM analytic approximation to the Synge equation of state (de Berredo-Peixoto, Shapiro & Sobreira 2005; Mignone, Pleva & Bodo 2005) and obtain the following:

$$\epsilon := 3 \frac{P}{2 \rho c^2} + \frac{9}{4} \left( \frac{P}{\rho c^2} \right)^2 + 1 \right]^{1/2} - 1.$$

(2)

We assume that $L_L = L_R$ and $\chi_L = \chi_R$. Furthermore, as in MA10, we assume $\Gamma_L := (1 + \Delta g)\Gamma_R$. This leaves us with $R, \sigma_L, \sigma_R$ and $\Delta g$ as parameters, because all other quantities can be determined using equations (1) and (2). To these, we add an additional parameter $\Delta r$, which is the initial width of the shells in the lab frame. While it does not influence the solution of the Riemann problem, it provides the physical scale necessary for the calculation of the observed emission.

Once the initial states are constructed, we compute the exact solution of the Riemann problem using the solver of Romero et al. (2005). The initial discontinuity between left and right states decomposes into a contact discontinuity (CD), and a left-going wave and a right-going wave (in the frame in which the CD is at rest). Depending on the particulars of the initial states, these waves can either be shocks or rarefactions. We label the left-going wave with RS to denote a reverse shock and with RR in case a reverse rarefaction happens. Similarly, we label the right-going wave with FS or FR to differentiate the cases in which a forward shock or a forward rarefaction occurs, respectively. We will use a subscript S to refer to the properties of the shocked fluid in general and subscripts FS and RS when distinguishing between the front and reverse shocked fluids. Finally, we will use the subscript 0 for properties of the initial states in general, and the subscripts L and R when we need to distinguish between left and right initial states. Because we assume that the flow luminosity is the same for both initial states, using (1) we determine the number density in the shells to be

$$n_{L,R} = \frac{L}{\pi R^2 \rho c^3 \left[ \Gamma^2 \left(1 + \chi + \sigma_{L,R} \right) - \Gamma_{L,R} \right] \sqrt{1 - \Gamma_{L,R}^{-2}}}.$$

(3)

where $\Gamma_{L} = (1 + \Delta g)\Gamma_{R}$.

The Riemann solver provides us with the bulk velocity of the shocked fluid $\beta c$ and its Lorentz factor $\Gamma = (1 - \beta^2)^{-1/2}$, and velocities $\beta_{FS}$ and $\beta_{RS}$ of the FS and RS, respectively (provided they exist). The velocity of the initial (unshocked) states in the CD rest frame is

$$\beta_0 = \frac{\beta_0 - \beta}{1 - \beta \beta_0}.$$

(4)

The shock velocities in the frame of the CD can be computed as

$$\beta_0 = \frac{\beta_0 - \beta}{1 - \beta \beta_0},$$

(5)

where prime denotes quantities in the CD rest frame. In this frame, the shock crosses the shell at a time

$$t_{cross,S} = \frac{\Delta r_0}{c |\beta_0|},$$

(6)

where $\Delta r_0$ is the shell width in the CD frame,

$$\Delta r_0 = \frac{\Gamma \Delta v r}{\beta_0 - \beta}.$$

(7)

3 NON-THERMAL PARTICLES

In this section, we show the properties of non-thermal particles and their emission. We first discuss the model for the magnetic field and non-thermal particles, and then outline the method used to compute their emission.

3.1 Magnetic field

As in MA10 and BD10, we assume that there exists a stochastic magnetic field, which is created in situ by the shocks arising in the collision of the shells. We label this field by $B_{S,\text{st}}$, and by definition its strength is a fraction $\epsilon_B$ of the internal energy density of the shocked shell $\rho_0$ (obtained, in our case, by the exact Riemann solver): $B_{S,\text{st}} = \sqrt{8\epsilon_B \rho_0 s}.$

(8)

Since we allow for arbitrarily magnetized shells, there is also an ordered (macroscopic) magnetic field component $B_{S,\text{mac}}$, which is a direct output of the exact Riemann solver. The total magnetic field is then $B_S := B_{S,\text{st}} + B_{S,\text{mac}}$. © 2012 The Authors, MNRAS 421, 2635–2647 Monthly Notices of the Royal Astronomical Society © 2012 RAS
3.2 Injection spectrum of non-thermal particles

We assume that a fraction of electrons in the unshocked shell is accelerated to high energies at the shock front. Following section 3 of BD10, we assume that a fraction $\epsilon_e$ of the dissipated kinetic energy is used to accelerate electrons. We assume that some particle acceleration mechanism operates at shocks, such that a fraction of the electrons in the unshocked shell is accelerated to high energies in the vicinity of the shock front. As it is commonly done (Bykov & Meszaros 1996; Daigne & Mochkovitch 1998; Mimica et al. 2004; BD10), we assume that a fraction $\epsilon_e$ of the dissipated kinetic energy is used to accelerate electrons. The width of the acceleration zone $\Delta r_{\text{acc}}$ is parametrized as a multiple $\Delta_{\text{acc}}$ of the proton Larmor radius in the shocked fluid. Therefore, the acceleration at a given point in the shocked fluid lasts for a time $\Delta t_{\text{acc}} = \Delta r_{\text{acc}}/(\beta_p c)$. We have

$$\Delta r'_{\text{acc}} = \Delta_{\text{acc}} \frac{\Gamma_{\text{acc}}' m_p c^2}{e B_S},$$

(9)

From this expression, we compute the volume where acceleration takes place as $V'_{\text{acc}} = \pi R'^2 \Delta r_{\text{acc}}'$. The energy injection rate into the acceleration region is

$$\frac{dE_{\text{inj},0}}{dr'} = \pi R'^2 \epsilon_e u_S \frac{\Delta r_{\text{acc}}'}{\Delta_{\text{acc}}},$$

(10)

and we assume that the energy spectrum of the injected relativistic particles is a power law in the electron Lorentz factor $\gamma'$,

$$\frac{dn'_{\text{inj}}}{d\gamma'} = Q(0)\gamma'^{-\delta}H(\gamma'; \gamma_1, \gamma_2),$$

(11)

where $n'_{\text{inj}}$ is the number density of the injected electrons, $Q(0)$ is a normalization factor and $\gamma_1$ and $\gamma_2$ are the lower and upper injection cut-offs (computed below), all measured in the shocked fluid rest frame. The step function is defined as usual by $H(x; a, b) = 1$ if $a \leq x \leq b$ and by $H(x; a, b) = 0$ otherwise.

A cautionary note should be added here regarding the fact that we choose that the spectral energy distribution of the injected particles is a pure power law even in the high-$\gamma$ regime. Both theoretical arguments (e.g. Kirk & Heavens 1989) and recent particle-in-cell (PIC) simulations (e.g. Sironi & Spitkovsky 2009) have shown that particle acceleration is not very efficient in the presence of a strong magnetic field parallel to the shock front. The modifications to the particle injection spectrum might involve the presence of the thermal population. Recently, a calculation by Giannios & Spitkovsky (2009) shows how the spectrum of the GRB prompt emission might look in such a case: a bump at the spectral maximum and a lower cut-off to the injection at shock. More precisely, we use the semi-analytic solver of Mimica et al. (2004) to compute the particle distribution at any time after the start of the injection at shock. Then, the maximum Lorentz factor is obtained by equating the result to equation (10) divided by $V'_{\text{acc}}$ (in order to obtain the energy density injection rate into the acceleration region) we can compute the normalization factor for the electron injection, $Q_0 = \frac{dE_{\text{inj},0}/dr'}{V'_{\text{acc}} m_e c^2} \times \left\{ \begin{array}{ll} \frac{q - 2}{\gamma_2 - \gamma_1} & \text{if } q \neq 2 \\ 1/\ln \left( \frac{\gamma_2}{\gamma_1} \right) & \text{if } q = 2 \end{array} \right\}$.

(12)

3.3 Particle injection cut-offs

As was done in Mimica, Giannios & Aloy (2010), we assume that the upper cut-off for the electron injection is obtained by assuming that the acceleration time-scale is proportional to the gyration time-scale. Then, the maximum Lorentz factor is obtained by equating this time-scale to the cooling time-scale,

$$\gamma_2 = \left( \frac{3m_e^2 c^4}{4\pi\epsilon_{\text{acc}} e^2 B_S^2} \right)^{1/2},$$

(13)

where $\epsilon_{\text{acc}} \geq 1$ is the acceleration efficiency parameter (BD10). The lower cut-off is obtained by assuming, in complete analogy to equation (10), that the number of accelerated electrons is related to the number of electrons passing through the shock front,

$$\frac{dN_{\text{inj},0}}{dr'} = \zeta_2 \pi R'^2 n_0 \Gamma_{\text{acc}}',$$

(14)

where $\zeta_2$ is the fraction of electrons accelerated into the power-law distribution. From equations (14), (10) and (11), we get

$$\int_{\gamma_1}^{\gamma_2} d\gamma' \gamma'^{-\delta} = \frac{\epsilon_e u_S}{\zeta_2 \Gamma_{\text{acc}}' m_e c^2}.$$  

(15)

Since we are dealing with potentially highly magnetized fluids, the condition $\gamma_2 \gg \gamma_1$ cannot be assumed (see equation 13), and therefore we cannot use the equation such as equation (13) of BD10. Therefore, we compute $\gamma_1$ from equation (15) numerically using an iterative procedure.

3.4 Evolution of the particle distribution

In this work, we assume that particles cool via synchrotron and external-Compton processes. We ignore the adiabatic cooling in this work since we are primarily interested in collisions of magnetized shells, where the electrons are fast-cooling. The consequence of not accounting for the adiabatic losses of the particle distribution is that our model overestimates the emission after the shocks cross the shells. Nevertheless, most of the features that make substantively different between the dynamics triggered by magnetized and non-magnetized shells happen in the early light curve and, thus, neglecting the expansion of the shells plasma does not change the qualitative conclusions of this paper.

The radiative losses for an electron with a Lorentz factor $\gamma$ can be written as

$$\dot{\gamma} = -\frac{4}{3} \frac{u'_B + u'_\text{ext}}{m_e c^2} \gamma^2,$$

(16)

where $u'_B = B'^2/8\pi c^2$ and $u'_\text{ext}$ are the energy density of the magnetic field and the external radiation field (see Section 4.2) in the shocked fluid frame, respectively. Once the energy losses have been specified, we use the semi-analytic solver of Mimica et al. (2004) to compute the particle distribution at any time after the start of the injection at shock. More precisely, we use the solution for the continuous injection and particle cooling (equation 19 of Mimica et al. 2004) for the time $\Delta t_{\text{acc}}$ since the beginning of the shock acceleration. After that time, the shock acceleration ends and we approximate the resulting particle distribution by a piecewise power-law function. Then we employ equation (17) of Mimica et al. (2004) on each of the power-law segments to compute the subsequent evolution.

4 NON-THERMAL RADIATION

We assume that the observer’s line of sight makes an angle $\theta$ with the jet axis, which is also the direction of propagation of the fluid.
We denote by $x$ and $t$ the position and time in the observer frame, and by $x'$ and $t'$ the location and time in the CD frame. We assume that the CD is located at $x' = 0$ for all $t'$. Let $\mu = \cos \theta$, and define the time at which an observer sees the radiation emitted from $x$ at time $t$ as

$$T = t - x \mu/c.$$  

Then the time as a function of the time of observation and position can be written as

$$t' = D(T/(1 + z) + \Gamma(\mu - \beta)x'/c),$$

where $D := \Gamma(1 - \beta \mu)^{-1}$ is the Doppler factor and $z$ is the redshift. Lorentz transformations have been applied to equation (17) to obtain equation (18).

An important quantity is the time elapsed after the shock has passed a given $x'$. From such value, one can calculate the age of the electron distribution function at that position, which turns to be the time since the shock acceleration has begun. Thus, the age can be defined as

$$t'_d := t' - \frac{x'}{\beta c}.$$  

A more useful expression involves $T$. Using equations (19) and (18), we get, for the FS,

$$t'_{d,FS} = D \left[ \frac{T}{1 + z} - \frac{x'}{c \Gamma} \frac{1 - \beta_{RS}\mu}{\beta_{RS} - \beta} \right],$$

and, for the RS,

$$t'_{d,RS} = D \left[ \frac{T}{1 + z} - \frac{x'}{c \Gamma} \frac{1 - \beta_{RS}\mu}{\beta_{RS} - \beta} \right].$$

We note that equation (20) has to be used when $x' \geq 0$, while equation (21) is valid when $x' < 0$. If $t'_d \leq 0$, then the shock has not crossed that position yet and, consequently, that place does not contribute to the emission yet.

The observed luminosity in the CD rest frame is

$$L'_\nu(T) = \pi R^2 \int_{\nu_{\min}(T)}^{\nu_{\max}(T)} \dd \nu' v'_\nu j'_\nu(T, x'),$$

where the lower and upper limits depend on (1) if the shock exists and on (2) if it has crossed the shell. If the RS does not exist, then $x'_{\min} = 0$; otherwise, it is

$$x'_{\max}(T) = \max \left( \frac{\Gamma c T - \beta_{RS} - \beta}{1 + z} \frac{1 - \beta_{RS}\mu}{\beta_{RS} - \beta} - \Delta r'_L \right),$$

where $\Delta r'_L$ is the faster shell width in the CD frame. Analogously, for $x'_{\max} = 0$, the FS is non-existent; otherwise, it is

$$x'_{\max}(T) = \min \left( \frac{\Gamma c T - \beta_{RS} - \beta}{1 + z} \frac{1 - \beta_{RS}\mu}{\beta_{RS} - \beta} - \Delta r'_K \right),$$

$\Delta r'_K$ being the slower shell width in the CD frame. We point out that, to perform the integral in equation (22), $j'_\nu(t'_d)$ should be computed for the particle distribution evolved using values for the FS if

$\chi > 0$ and RS if $\chi < 0$ (i.e., in equations 12, 13, 15 and 8, the values for the corresponding shocked fluid should be used).

Considering that $v' = v(1 + z)/D$, and using equation (22), we can compute the flux in the observer frame, obtaining (Dermer 2008; BD10)

$$\nu F_\nu(T) = \frac{D^3 \pi R^2}{d_L^2} \int_{\nu_{\min}(T)}^{\nu_{\max}(T)} \dd \nu' v'_\nu j'_\nu(T', x'),$$

where $d_L$ is the luminosity distance. We perform the integration in equation (25) numerically.

The total emissivity is assumed to be the result of combining three emission processes: (1) synchrotron radiation, (2) IC with an external radiation field (EIC) and (3) the SSC upscattering. These emission processes are considered in more detail in the next sections.

### 4.1 Synchrotron emission

We compute the synchrotron emission for each power-law segment of the electron distribution (see Section 3.4) separately. In order to speed up the calculation, we use the interpolation method described in Mimica et al. (2009, section 4) and, in more detail, Mimica (2004, sections 2.1.3 and 4.3.1).

### 4.2 EIC emission

Following BD10, we assume that the external radiation field is monochromatic and isotropic in the active galactic nucleus frame. We denote the frequency and the radiation field energy density in this frame by $v_{ext}$ and $u_{ext}$, respectively. Transforming into the shocked fluid frame, we get

$$v'_{ext} = \Gamma v_{ext},$$

$$u'_{ext} = \Gamma^2 u_{ext}.$$  

Analogously to the computation of the synchrotron emission (Section 4.1), we compute the EIC emissivity for each power-law segment separately. We use equation (2.94) of Mimica (2004), but replacing $I(v_{0})/v_{0}$ by $c u_{ext}/v_{ext}$ and with an additional cut-off (approximating the Klein–Nishina decline of the Compton cross-section) such that the emissivity is zero for $h v \geq m_e^2 c^2/(\Gamma v_{ext})$ (see also Aloy & Mimica 2008). Values of $v_{ext}$ and $u_{ext}$ used in this work can be found in Table 1.

### 4.3 SSC emission

Analogously to Section 4.2, we use the equation (2.94) of Mimica (2004). However, in the case of SSC, the incoming intensity of the synchrotron radiation depends on $x'$ and $T$. For a point on the shell axis, the (angle averaged) intensity at frequency $v_0$ can be written as

$$1_{0,v_0}(T, x') = \frac{1}{2} \int_0^\pi \dd \theta' \int_0^{L(\theta')} \dd s' j'_{\nu,\text{syn}}(T) - \frac{s}{c}, x' + s \cos \theta'),$$

where $L(\theta')$ is the length of the segment in direction $\theta'$ from which synchrotron emission has had time to arrive to $x'$ at time $t'$ and $t'(T)$ is computed using equation (18). The synchrotron emissivity $j'_{\nu,\text{syn}}$ can be rewritten in terms of $T$ using equations (19)
and 20 or (21),
\[
\hat{j}_{\nu,\text{syn}} \left( \hat{r}(T) - \frac{s}{c}, x' + s \cos \theta \right) = \hat{j}_{\nu,\text{syn}} \left( \tilde{r}(T, x') - \frac{s}{c} \left( 1 + \cos \theta \frac{1 - \beta \beta_3}{\beta_3 - \beta} \right) \right).
\]
(28)
From equation (28), we can see that \( L(\theta') \) can be computed by requiring that the following condition be satisfied for each \( \theta' \):
\[
t'_i(T, x') - \frac{s}{c} \left( 1 + \cos \theta \frac{1 - \beta \beta_3}{\beta_3 - \beta} \right) > 0.
\]
If this condition is not satisfied, it means that the shock has not passed the point \( x' + s \cos \theta' \) at time \( t(T) - s/c \) yet, i.e. there is no synchrotron emission from that point to contribute to the incoming intensity. In addition, we also require that \( L(\theta') \leq R \). Finally, it should not be forgotten that when \( x' + s \cos \theta' > 0 \), the emissivity of fluid shocked by the FS should be used, and the one corresponding to the shocked fluid by the RS otherwise. Also, if either of the shocks is not present, there is no contribution from the corresponding region.

In practice, the numerical cost of a direct evaluation of double integral in equation (27) is prohibitive if we take into account that this intensity has to be evaluated for each \( x' \) in equation (25). To overcome this problem, we approximate equation (27) by discretizing the angular integral in non-uniform \( \theta' \) intervals. The choice of non-uniform intervals is motivated by the fact that most of the contribution of the incoming radiation comes from angles close to \( \mu = -\beta_3 \), so that we concentrate most of the bins close to that angle. Numerical testing shows that using 13 bins provides an acceptable trade-off between the accuracy and the computational requirements.

### 5 RADIATIVE EFFICIENCY

In this section, we compare the radiative efficiency of the internal shocks with their corresponding dynamic efficiency. We use the kinematic parameters from MA10 in the blazar regime, while the parameters used to compute emission are guided by the values from BM10 (see Table 1 for the complete list).

Table 1. Blazar model parameters used in this work. Note that \( \sigma_L \) and \( \sigma_R \) can vary continuously in the indicated range.

| Parameter | Value |
|-----------|-------|
| \( \Gamma_R \) | 10 |
| \( \Delta_R \) | 1 |
| \( \sigma_L \) | \([10^{-6}, 10^1]\) |
| \( \sigma_R \) | \([10^{-6}, 10^1]\) |
| \( \epsilon_B \) | \(10^{-3}\) |
| \( \epsilon_e \) | \(10^{-1}\) |
| \( \epsilon_{\text{acc}} \) | \(10^{-2}\) |
| \( \Delta_{\text{acc}} \) | 10 |
| \( \omega_{\text{acc}} \) | \(10^6\) |
| \( R \) | \(3 \times 10^{16} \text{ cm}\) |
| \( \Delta r \) | \(6 \times 10^{13} \text{ cm}\) |
| \( q \) | 2.6 |
| \( L \) | \(5 \times 10^{48} \text{ erg s}^{-1}\) |
| \( u_{\text{ext}} \) | \(5 \times 10^{-4} \text{ erg cm}^{-3}\) |
| \( v_{\text{ext}} \) | \(10^{14} \text{ Hz}\) |
| \( \gamma \) | 0.5 |
| \( \theta \) | 5° |

All parameters of our models are fixed except for \( \sigma_L \) and \( \sigma_R \), which can vary in the range indicated by Table 1. In the rest of the paper, we distinguish models by the value of the magnetization of each shell, e.g. a model with \( \sigma_L = 0.1 \) and \( \sigma_R = 1 \) is denoted by the pair \((0.1, 1)\).

As can be seen from equation (10), in our model only the thermal energy can be injected into the non-thermal particle population. We point out that, alternatively or simultaneously, magnetic dissipation can provide a source for emission in internal shocks (e.g. Giannios, Uzdensky & Begelman 2009; Nalewajko et al. 2011, and references therein), which we are not considering here. Thus, the radiative efficiency we compute in this paper is only a lower bound to the actual radiative efficiency of the binary collision of relativistic magnetized shells. As is shown in Appendix A, we can use the definition of the dynamic efficiency inspired by the recent work of Narayan, Kumar & Tchekhovskoy (2011). Its advantage is the Lorentz invariance, which enables us to compare it to the radiative efficiency of our model.

Following MA10, but using the definitions for the different energy components \( \dot{E}_K, \dot{E}_T \) and \( \dot{E}_M \) of Appendix A (equation A2), we denote by \( \dot{E}_0 \) the total energy in the unshocked shells,
\[
\dot{E}_0 := \dot{E}_K(\Gamma_R(1 + \Delta_g), n_{m,p} \Gamma), \quad \dot{E}_M(\Gamma_R(1 + \Delta_g), n_{m,p} \Gamma, n_{\text{L}} \Gamma), \quad \dot{E}_T(\Gamma_R(1 + \Delta_g), n_{m,p} \Gamma, n_{\text{L}} \Gamma, \frac{\omega_{\text{acc}}}{\beta_0})
\]
(29)
where \( \chi \) is the pressure-to-density ratio of the cold initial shells and is set to \(10^{-4}\). We also define the width of the shocked shells in terms of their initial width,
\[
\xi_w := \left| \frac{\beta_3 - \beta_0}{\beta_0 - \beta_3} \right|
\]
(30)
so that \(0 \leq \xi_w \leq 1\). The dynamic thermal efficiency for the faster shell is defined as
\[
\epsilon_{T, L} := \frac{1}{\dot{E}_0} \left( \frac{\dot{E}_T(\Gamma_L \gamma_s n_{L,m,p} \gamma_s n_{L,m,p} \gamma_s n_{L,m,p} \gamma_s n_{L,m,p} \gamma_s n_{L,m,p} \gamma_s n_{L,m,p})}{E_0} \right)
\]
(31)
and analogous definitions can be written for \( \epsilon_{M, L} \), \( \epsilon_{T, R} \) and \( \epsilon_{M, R} \) (see equations 13, 14, 16 and 17 of MA10). The total (Lorentz invariant) dynamic thermal and magnetic efficiency is
\[
\epsilon_T = \epsilon_{T, L} + \epsilon_{T, R}
\]
(32)
\[
\epsilon_M = \epsilon_{M, L} + \epsilon_{M, R}
\]
(33)
From these equations, it can be seen that the radiative efficiency can be at most be \( \epsilon_{T, L} + \epsilon_{T, R} \). More formally, we can write the radiative efficiency as (neglecting adiabatic cooling)
\[
\epsilon_{\text{rad}} := \epsilon_T f_{\text{rad}}(\epsilon_T + \epsilon_M).
\]
(34)
where \( f_{\text{rad}} := \epsilon_T/(\epsilon_T + \epsilon_M) \). It should be noted that equation (34) refers to the ‘bolometric’ emission, i.e. it includes all frequencies for the whole duration of the shell interaction. Since Earth-based observations have a limited spectral and temporal coverage, equation (34) is only an upper limit for radiative efficiencies inferred from actual observations.

Fig. 1 shows that the radiative efficiency is not a one-to-one map of the total dynamic efficiency. In particular, we note that \( f_{\text{rad}} \)

2 Note that comparing equations (7) and (30), we see that \( \Delta r_0 = \Gamma \xi_w \Delta r \), which is just a Lorentz transformation of the shocked shell width from lab to CD frame.
6 SPECTRA AND LIGHT CURVES OF MAGNETIZED INTERNAL SHOCKS IN BLAZARS

Our aim is to produce synthetic spectra and light curves from our numerical models of the interaction of two relativistic, magnetized shells. With this purpose, we chose three models from our parameter space, which are representative of different conditions that can be encountered in blazar jets. The first model corresponds to a regime of very low magnetization of both shells ($\sigma_L, \sigma_R > 10$). The second and third models correspond to intermediate ($10^{-6}, 10^{-2}$) and moderate/high shell magnetizations ($1, 0.1$).

6.1 Average spectra

The spectrum of the weakly magnetized model ($10^{-6}, 10^{-6}$) (Fig. 2; full lines) reproduces the typical double-peaked spectrum of blazars. The synchrotron emission (Fig. 2; solid red line) peaks at $4.6 \times 10^{13}$ Hz, while the IC emission, dominated by the SSC component, peaks at $6.7 \times 10^{21}$ Hz (Fig. 2; solid blue line). In this case, the IC spectral component is clearly dominating the overall spectrum.

At intermediate magnetizations (Fig. 2; dashed lines), the synchrotron emission peaks at higher frequencies than in the previous case, namely at $7.5 \times 10^{14}$ Hz, while the IC emission peaks at $6.0 \times 10^{20}$ Hz, as one would expect, since a larger magnetic field increases the synchrotron peak frequency (e.g. Mimica 2004). For these shell magnetizations, the SSC also dominates the high-energy emission, but now both the SSC and EIC components are significantly weaker than in the model ($10^{-6}, 10^{-6}$). Interestingly, moderate to high magnetizations (Fig. 2; double-dot–dashed lines) reduce the peak frequency ($1.9 \times 10^{14}$ Hz) and substantially flatten the synchrotron spectral component with respect to the intermediate-magnetization case. Furthermore, IC spectral components are notably weaker than the synchrotron one for large magnetization, the IC spectral peak being located at $4.2 \times 10^{18}$ Hz.

As can be seen from Fig. 2, the synchrotron emission from all three models is of comparable intensity, while the IC emission is much weaker in the strongly magnetized model ($1, 0.1$). The reason for such a large difference in the high-energy emission between the magnetized and the non-magnetized models lies in the lower number density of emitting electrons (equation 3) and in the higher magnetic field of the magnetized model. The magnetized model has a much lower number density due to its relatively high $\sigma$, in both FS- and RS-emitting regions, which means that there are less scatterers for the SSC and EIC processes. A high magnetic field also means a reduction of the upper injection cut-off (equation 13), which in turn means that the seed synchrotron photons in the SSC process are being upscattered to lower frequencies, explaining the small contribution of the SSC component to the spectrum. The EIC component’s upper cut-offs are determined by the Klein–Nishina decline (see Section 4.2) and not by the upper injection cut-off, which explains why the EIC spectral peaks of the models are in a similar frequency range.

Fig. 3 shows the contributions from the FS and RS (red and blue lines, respectively) to the total spectrum (black lines). Except close to the local minima between the two spectral peaks, the RS contribution is dominant in the average spectra of the models with low and intermediate magnetizations, ($10^{-6}, 10^{-6}$) and ($10^{-2}, 10^{-2}$), respectively. In the vicinity of the aforementioned minima (located in the X-ray range), the FS contribution tends to broaden the width of the minima and to soften the spectral slope. At the moderate to high magnetizations of the model ($1, 0.1$), the FS is dominant except in the range $10^{15} – 10^{16}$ Hz, where the FS and RS have comparable
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6.2 Light curves

The multiwavelength light curves of the models presented in the previous section are displayed in Fig. 4. We have picked up several characteristic bands to analyse the data (R band, X-ray, 0.1- and 1-GeV light curves).

Comparing the R-band light curves of the three models, we can see that models $(10^{-6}, 10^{-6})$ and $(1, 0.1)$ exhibit properties of the fast-cooling electrons emitting synchrotron radiation, while the model $(10^{-5}, 10^{-5})$ shows the opposite, slow-cooling behaviour. In the latter case, the maximum of the R-band light curve is reached when the shocks cross the shells, and afterwards the emission decays as the particles cool down (no new particles are accelerated after both shocks cross the shell). In the case of the model $(1, 0.1)$, one can clearly note two sudden drops in emission around 4 ks, which correspond to the moments when first the RS and later the FS cross their respective shells. The almost vertical drops in emission are indicative of a very efficient electron synchrotron cooling.

At intermediate magnetizations $(10^{-2}, 10^{-2})$, the first sharp drop is observable as well, though here there is a weak late-time optical emission between $10^{4}$ and $10^{5}$ s due to the SSC process. The emission in the X-ray band is a bit more involved, and to perform a proper analysis we show in Fig. 5 both the total light curve (black lines) and the individual contributions to it of the synchrotron and SSC processes (red and blue lines, respectively).

Except at very early times, the emission is dominated by the SSC process in all cases. The synchrotron emission in this band happens in an efficient fast-cooling regime, which can be inferred from the fast drop of the synchrotron components between 4 and 9 ks. The fact that increasing magnetic fields makes that particles cool faster explains that the non-magnetized model peaks much later ($\approx 60$ ks) in this band than the other two (more magnetized) models. At energies of 1 GeV, there is only emission from IC processes (Fig. 6). The model with the smaller magnetization displays a clearly dominant EIC emission at early times, while in the other two models EIC dominates the later times. In the models $(10^{-2}, 10^{-2})$ and $(1, 0.1)$, EIC, similar to the synchrotron emission in the X-ray band, sinks very quickly before 8 ks, indicating that the electrons are in a fast-cooling regime. In the latter models, because of the delays associated with the physical length of the emitting region, the SSC contribution peaks very early and decays exponentially before the sharp drop of the EIC emission (this is particularly the case of the most magnetized model, in which the SSC component does not significantly contribute to the light curve after $\approx 400$ s). In contrast, the EIC emission of the model $(10^{-6}, 10^{-6})$ shows a much more prominent peak and a shallower decay from the maximum (at $\approx 9$ ks),

Note that, since the high-latitude emission is ignored due to cylindrical geometry, the drops are too sharp and would be smoother were a conical jet geometry assumed.

We do not show the EIC light curve because its contribution is negligible at these frequencies.
both features being characteristics from electrons in a slow-cooling regime.

7 GLOBAL PARAMETER STUDY

In the following, we present the results of the global parameter study of the dependence of the emitted radiation on the shell magnetization.5 Fig. 7 shows the fluence as a function of $\sigma_L$ and $\sigma_R$. We can see that, as expected, the fluence roughly follows $f_R$ (see Section 5). The region with most luminous internal shocks (upper-left corner of Fig. 7) happens for a moderately to strongly magnetized slow shell and a weakly magnetized fast shell, whereby the FS does not exist. The emission weakens as the magnetization of the fast shell increases, with the exception of the region where the fast shell is strongly magnetized, but the slow shell is weakly magnetized (lower-right corner of Fig. 7). We conclude that, as was indicated in the section 4.4 of MA10, a large difference in the magnetization of the shells yields stronger dissipation and more luminous internal shock(s) than when both shells are weakly magnetized.

Fig. 8 shows the spectral maxima of the synchrotron, $v_{\text{max,syn}}$, and the IC emission, $v_{\text{max,IC}}$ (left- and right-hand panels, respectively). From Fig. 2, one could anticipate a trend we confirm here, with the parametric space coverage, namely that the IC emission is more sensitive to changes in the magnetization than the synchrotron emission. This statement reflects itself in Fig. 8 through the fact that the range of variation of $v_{\text{max,IC}}$ is larger than that of $v_{\text{max,syn}}$. Thus, the IC spectral peak becomes a better proxy of the magnetization of the shells than the synchrotron peak. Except at small shell magnetizations, the IC emission happens in a fast-cooling regime, and the dependence of $v_{\text{max,IC}}$ on the magnetization is similar to that of $f_{\text{rad}}$. Complementarily, at small shell magnetizations, the map of $v_{\text{max,syn}}$ resembles very much to that of $f_{\text{rad}}$ [compare the lower halves of Figs 1 and 8 (left-hand panel)].

The left-hand panel of the Fig. 9 shows the ratio of the IC to synchrotron fluence. The trend is quite similar to that of the integrated flux shown in Fig. 7. When both shells are strongly magnetized ($\sigma \gtrsim 0.1$), the IC emission drops significantly. In the region $\sigma_L \lesssim 10^{-3}$, the ratio is between a unity and $\lesssim 60$, with a similar behaviour in the region ($\sigma_L \gtrsim 0.1, \sigma_R \lesssim 0.1$). The region of low radiative efficiency around $\sigma_L \approx 0.01$ appears as a dark vertical band in the plot ($\sigma_R \lesssim 10^{-4}$), where both synchrotron and IC processes provide a similar fluence. The right-hand panel of Fig. 9 shows the ratio of the frequencies of the IC to synchrotron spectral maxima shown in Fig. 8. From an observational point of view, the fluence might be much more robust and significant than the peak IC and synchrotron frequencies, which can be difficult to measure. However, for the ratio $v_{\text{IC}}/v_{\text{syn}}$, the lower-right and upper-left corners of the plot display noticeably different values. Thus, one can use the frequency ratio together with the fluence ratio in order to break the degeneration in the fluence ratio when one of the shells is very magnetized and the other is not magnetized (see Section 8.2, for further discussion of this point).

8 DISCUSSION AND SUMMARY

We have extended the study of the dynamic efficiency performed in MA10 by computing the multwavelength, time-dependent emission from internal shocks. In this section, we discuss and summarize our findings.
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8.1 Emission mechanisms and magnetization

In Section 6, we show the average spectra and multiwavelength light curves of three typical models from the parameter space considered in this paper. Synchrotron emission dominates for $\nu \lesssim 10^{17}$ Hz, and is rather independent of the shell magnetization (Fig. 2). The RS dominates synchrotron emission for weakly magnetized shells, while in the case of strongly magnetized shells, the FS and RS have comparable contributions. $R$-band light curves (black lines on Fig. 4) show that the synchrotron emission is due to the slow-cooling electrons only for the weakly magnetized model, while for shells with $\sigma \gtrsim 0.01$ electrons are fast-cooling.

The SSC emission dominates in the X-ray band and higher frequencies (Fig. 2). However, at early times, the synchrotron emission dominates in X-rays (Fig. 5), while in gamma-rays the situation is more complex. For the weakly magnetized model (slow-cooling electrons), EIC dominates the early emission, while in the moderate to highly magnetized models, EIC dominates the late-time

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Figure 8. Contours of the frequency of the spectral maxima of the synchrotron (left-hand panel) and of the IC (right-hand panel) emission as a function of $\sigma_L$ and $\sigma_R$.

Figure 9. Left-hand panel: contours of the logarithm of the ratio of the IC to synchrotron fluence as a function of the shell magnetizations $\sigma_L$ and $\sigma_R$. Right-hand panel: same as the left-hand panel, but for the ratio of the frequency of the spectral maxima of the synchrotron to IC emission.
emission. The reason for this is that, in the magnetized models, the high-energy tail of the electron distribution disappears very quickly, so that the incoming synchrotron photons cannot be upscattered into the 1-GeV range. In the weakly magnetized models, there are enough slow-cooling electrons at sufficiently high energies for the SSC to dominate over EIC at later times.

Finally, from Fig. 2 we see that the IC emission is weaker the more magnetized the shells are. This is due to the requirement of our model that the shell luminosity (equation 1) be constant regardless of $\sigma$. From equation (3), we see that for $\sigma \gg 1$ the number density in the shells behaves as $\approx \sigma^{-1}$. Since the IC emission depends on the number of electrons (EIC linearly and SSC quadratically), it is clear that the IC emission must necessarily drop for large $\sigma$. From the analysis of the three representative models, we conclude that the shell magnetization imprints two main features on the emission properties of blazars. On the one hand, the magnetization determines whether electrons are slow-cooling (for weakly magnetized shells) or fast-cooling (moderate to high magnetization).

### 8.2 Global trends

We performed a global parameter study (Section 7) to investigate the dependence of some observational quantities on the shell magnetization. As discussed in Section 7, the integrated flux (Fig. 7) follows the trend already shown by the radiative efficiency (Fig. 1). However, the integrated flux and the spectral maxima are quantities dependent on the particular values we have taken in our model, specifically, on the physical size of the shells and their bulk Lorentz factors, as well as on the source redshift. On the other hand, in MA10 we show that the dynamic efficiency is very weakly dependent on the shell bulk Lorentz factor, i.e. it only depends on the shell magnetization for a fixed $\Delta g$. In order to eliminate the dependence on absolute quantities in Fig. 9 we show IC-to-synchrotron flux ratio, as well as the ratio of the frequency of IC to the synchrotron spectral maxima. The shape of the contours on the left-hand panel of Fig. 9 does not exactly follow the one in Fig. 1: there is a much stronger dependence on $\sigma_L$ than on $\sigma_R$ in most of the scanned parameter space. None the less, in the lower half of the plots, the behaviour of both $F_{IC}/F_{syn}$ and $f_{fr}$ is similar. For example, if we keep $\sigma_R$ constant and equal to $10^{-2}$ and vary $\sigma_L$, we note that the radiative efficiency is larger than 90 per cent for $\sigma_L < 0.01$, then it decays to a local minimum, and successively grows again to reach values in excess of 90 per cent ($\sigma_L \gtrsim 1$). Comparatively, at small values of $\sigma_R$, the ratio $F_{IC}/F_{syn}$ is close to its maximum for $0.01 < \sigma_L < 1$ and touches a minimum in the same interval as $f_{fr}$. The upper halves of Figs 9 (left-hand panel) and 1 do not show the same qualitative behaviour. The reason for this discrepancy is that RS dominates the emission, and thus the overall radiative properties are more sensitive to the magnetization of the fast shell through which it propagates.

Interestingly, there is a certain degree of degeneration in the values both of the radiative efficiency and of the $F_{IC}/F_{syn}$ ratio considering the regions where one of the two shells is very magnetized and the other is basically non-magnetized (i.e. the upper-left and lower-right corners of Figs 9 and 1). In both cases, the radiative efficiency and the fluence ratio are close to their respective maximum values. However, we can distinguish between the case of high $\sigma_L/\sigma_R$ and the case of low $\sigma_L/\sigma_R$ by looking at the ratio of peak frequencies $\nu_{IC}/\nu_{syn}$ (right-hand panel of Fig. 9). A noticeably smaller $\nu_{IC}/\nu_{syn}$ ratio corresponds to the former case than to the later.

The previous analysis suggests that with the combined information of the fluence and peak-frequency ratios, one could try to figure out, by using observational data, which is the rough magnetization of the shells of plasma whose interaction yields flares in blazars. To serve such a purpose, we display in Fig. 10 our models in a 2D parameter space whose horizontal and vertical directions are $\nu_{IC}/\nu_{syn}$ and $F_{IC}/F_{syn}$, respectively. We note that the computed models are distributed in a broad region which, nevertheless, shows

![Figure 10](https://academic.oup.com/mnras/article-abstract/421/3/2635/1746973/25732573)

Fig. 10. Left-hand panel: $F_{IC}/F_{syn}$ (ratio of the IC to the synchrotron fluence) as a function of $\nu_{IC}/\nu_{syn}$ (ratio of the maximum spectral frequencies in IC to synchrotron ranges) for the models considered in Section 7. The models have been grouped in three bins according to $\sigma_L$, and are annotated with black circles ($10^{-6} < \sigma_L < 10^{-4}$), blue triangles ($10^{-4} < \sigma_L < 10^{-2}$) and red diamonds ($10^{-2} < \sigma_L < 10^0$). Right-hand panel: same as the left-hand panel, but in this case, the models have been grouped according to $\sigma_R$, in the same bins and same colouring and symbols in as the left-hand panel. Shaded areas denote three regions of interest (see text for details). The three reference models in this paper are marked with purple crosses, and their respective magnetizations are overlaid.
a relatively tight correlation between \( F_{\text{IC}}/F_{\text{syn}} \) and \( v_{\text{IC}}/v_{\text{syn}} \). In the left-hand panel of Fig. 10, we display our models in three different colours according to the magnetization of the left shell. The same has been done in the right-hand panel, but for the right shell.

Based on the degree of variation of magnetization between the fast and the slow shells, we have divided the parameter space in three broad regions (labelled with roman numerals I, II and III in Fig. 10), where the shells have the following characteristics:

(I) moderately magnetized fast shell colliding with a weakly magnetized slow shell or weakly magnetized fast shell interacting with a strongly magnetized slow one;

(II) strongly magnetized fast and moderately magnetized slow shells;

(III) strongly magnetized fast and slow shells.

The first thing to note is that for models in the region III, both the IC emission and its frequency maximum are lower compared to the rest of the models. This leads us to the conclusion that when the flow is strongly magnetized and the magnetization does not vary substantially, the IC signature is expected to be relatively weak. Furthermore, region II shows that in the case of a larger variation in magnetization (i.e. weakly magnetized slow shell), the frequency maximum remains low, but the IC signature becomes substantially higher. Finally, in region I we see that when the variation in magnetization is more extreme (i.e. a collision of a weakly and a strongly magnetized shells), we get a very strong IC signature and its frequency maximum is shifted to much higher energies.

8.3 Radiative and dynamic efficiency

As discussed in Section 5, the radiative efficiency \( \epsilon_{\text{f,rad}}(\epsilon_{\text{T}} + \epsilon_{\text{M}}) \) does not have a one-to-one correspondence with the dynamic efficiency \( (\epsilon_{\text{T}} + \epsilon_{\text{M}}) \). While the latter peaks in the regions \( \sigma_{\text{L}} \approx 1 \) and \( \sigma_{\text{R}} \approx 0.2 \), the former reaches its maximum in the region \( \sigma_{\text{L}} \lesssim 10^{-1}, \sigma_{\text{R}} \gtrsim 10 \). The same can be concluded from the time- and frequency-integrated flux shown in Fig. 7. For purposes of the rest of this discussion, we will use \( f_{\text{rad}} \) as a proxy for the radiative efficiency.

We note that in the region of maximum \( f_{\text{rad}} \), the FS does not exist. However, we see another region of high \( f_{\text{rad}} \) in the opposite corner of Fig. 1; the efficiency is quite high as well. Consistent with the discussion in the previous subsection and with what is shown in region (I) in Fig. 10, we conclude that the radiative efficiency is maximal when the variation in magnetization between the colliding shells is large.

8.4 Conclusions and future work

Under the assumption of a constant flow luminosity, we find that there is a clear difference between the models where both shells are weakly magnetized (\( \sigma \lesssim 10^{-2} \)) and those where, at least, one shell has \( \sigma \gtrsim 10^{-2} \). We obtain that the radiative efficiency is largest in those models where, regardless of the ordering, there is a large variation in the magnetization of the interacting shells. Furthermore, substantial differences between weakly and strongly magnetized shell collisions are observed in the IC part of the spectrum, as well as in the optical, X-ray and 1-GeV light curves.

In the previous sections, we have deepened our analysis of the radiative efficiency of the process of collision of magnetized relativistic shells of plasma. We have studied this problem from a mostly theoretical point of view, where the intrinsic properties of the flow (in particular the magnetization) have been related to the properties of the resulting (synthetic) spectra and light curves. It is, however, worthwhile to provide suitable links between our theoretical results and the observed properties of blazar flares. Thus, we propose a way to distinguish observationally between weakly magnetized and strongly magnetized internal shocks by comparing the maximum frequency of the IC and synchrotron part of the spectrum to the ratio of the IC to synchrotron fluence.

For a given flare taken in isolation, our model may predict which is the range of magnetizations which have to be invoked to fit the spectrum. However, such a model fitting is not fully satisfactory, since it is strongly dependent on the details of the theoretical model. A more generic knowledge of the physical conditions in the flaring regions can be obtained by arranging the observational data in plots where the fluence ratio \( F_{\text{IC}}/F_{\text{syn}} \) is represented versus \( v_{\text{IC}}/v_{\text{syn}} \). The reason is that the fluence and frequency ratios are redshift- and source-independent, since they are mostly influenced by the variation of the bulk magnetization of the blazar jets (assuming that the viewing angle is fixed). We note that different flares of the same blazar, as well as different flares of distinct blazars, can be plotted in such graphs and compared with our theoretical predictions. In addition, an average over a number of flares of the same source might also be interpreted using our model if the magnetization ratio of different pairs of colliding shells is similar.

Our results suggest that the variability in the flow magnetization is a factor that shall be considered to explain the observed continuity of properties of the blazar sequence (e.g. Fossati et al. 1998; Ghisellini & Tavecchio 2008; Ghisellini et al. 1998). Looking at Figs 2 and 3, it is evident that if the magnetization of the shells is not too large (\( \sigma_{\text{L,R}} \lesssim 10^{-2} \)), increasing the flow magnetization shifts the synchrotron peak towards the ultraviolet band and lowers the IC peak. Leaving aside orientation effects (which we are neglecting here since we fix the viewing angle), such a behaviour suggests that LBL blazars may correspond to barely magnetized flows, while HBL blazars could correspond to moderately magnetized ones. If the magnetization is large (\( \sigma_{\text{L,R}} \gtrsim 0.1 \)), the synchrotron peak shifts towards lower frequencies, and the IC spectral peak falls three orders of magnitude below the synchrotron peak. The latter situation seems not to be observed and, thus, we conclude that this is an indication that the typical value of the magnetization in the flow of blazars is \( \sigma \lesssim 10^{-2} \). We note that this value is about one order of magnitude smaller than that suggested for the flow in GRBs (e.g. Giannios & Spruit 2006).

Our results are not in contradiction with the common view, according to which, the variation of the properties of the blazar family correspond to the changes in the bolometric luminosity of the synchrotron component \( L_{\text{bol,synch}} \) (Fossati et al. 1997). In such a case, LBLs and HBLs are extrema of a one-parameter family, where LBLs are more radio luminous than HBLs. However, \( L_{\text{bol,synch}} \) is not a ‘direct’ physical property of the plasma in a relativistic jet, since the same \( L_{\text{bol,synch}} \) may arise with an infinite number of combinations of bulk Lorentz factors, blazar orientations with respect to the line of sight, flow magnetization, etc. In this paper, we explore the role that the flow magnetization (a direct physical property of the emitting plasma) plays in shaping not only \( L_{\text{bol,synch}} \), but also the whole spectrum. We conclude that the flow magnetization alone is enough to explain the difference in \( L_{\text{bol,synch}} \) and in the high-energy part of the spectrum (i.e. in the Compton-dominated regime) found in the blazar family. We also point out that Fossati et al. (1998) arrive at a similar conclusion: by fixing the bulk Lorentz factor and assuming the validity of the SSC model for all sources, they find that the spectral differences in the blazar sequence are due to the systematic variation of the magnetic field strength; HBLs
have the highest random field intensity. We go a step further in this paper. When considering the dynamical changes induced by non-negligible macroscopic magnetic fields, the same conclusion as in Fossati et al. (1998) holds, but we remark that, in our case, the total jet luminosity is kept constant (by construction of our models); only the magnetization is varied.

A problem with the internal shock scenario is that it is apparently not able to explain the ultrafast variability of TeV blazars (e.g. Aharonian et al. 2007; Albert et al. 2007). In order to properly account for this fast variability (on the time-scales of minutes), fast ‘minijets’ (Ghisellini et al. 2009) or ‘spines/needles’ (Tavecchio & Ghisellini 2008) need to exist in a much larger, slower jet. We do not consider minijets in our current models, although they will need to be considered in the future.

In the future work, we will improve our modelling by including the resistive dissipation as a source of energy for the radiating non-thermal particles. This will make it possible to assess whether radiative and dynamical efficiencies have a one-to-one correspondence or if there is a fundamental degree of independence. In the latter case, it might be difficult to infer the flow properties except in those asymptotic cases where the dissipation either does not play a significant role or dominates the dynamics. Furthermore, we will study how changing the viewing angle and the Doppler factor reflects on the observational signature. Finally, we will include the effects of the presence of a thermal component during particle injection at very magnetized shocks (Ghisellini & Spitkovsky 2009; Sironi & Spitkovsky 2009).

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APPENDIX A: COMPARISON OF THE DYNAMIC EFFICIENCY DEFINITIONS

In this appendix, we compare two alternative definitions of the dynamic efficiency, one due to MA10 and the other inspired by Narayan et al. (2011, hereafter NK11).

MA10 define in their equation (12) three components (kinetic, thermal and magnetic, respectively) of the total energy in each shell:

\[ E_K(\Gamma, \rho, \Delta x) := \Gamma(\Gamma - 1)\rho c^2 \Delta x \]
\[ E_T(\Gamma, \rho, p, \Delta x) := (\rho c^2 + p)\Gamma^2 - p \Delta x \]
\[ E_M(\Gamma, \rho, \sigma, \Delta x) := \left( \frac{\Gamma^2 - 1}{2} \right) \rho c^2 \Delta x \]

where \( \rho, p, \sigma, \) and \( \Gamma \) are the fluid rest-mass density, thermal pressure, specific internal energy density, magnetization parameter and the Lorentz factor. \( \Delta x \) is the width of the shell. Then, the total dynamical efficiency is defined as the ratio of the sum of the magnetic and thermal energies in the post-shock state to the total initial energy (MA10).

Recently, NK11 have computed the radiative efficiency of magnetized internal shocks using a slightly different definition. The definition of NK11 has the advantage of resulting into a radiative efficiency, which is Lorentz invariant, while that of MA10 is not. Therefore, the dynamic efficiency can be computed in the lab frame and then be used in the rest frame of the CD to be directly compared with the radiative efficiency, as was done in Section 5. However, the NK11 approach does not shed any light on the problem of obtaining a true energetic efficiency in terms of the initial conditions of the shells. This is because their definition does not consider the efficiency of conversion of the initial total energy of the shells to radiation but, instead, the efficiency of conversion of the enthalpy per particle after the shell collision into radiation enthalpy.

With the aim of introducing a Lorentz invariant, energy-based definition of the radiative efficiency in terms of a quantity akin to the dynamic efficiency of MA10, we consider the following

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expressions for the kinetic, thermal and magnetic energies:

\[ \hat{E}_K(\Gamma, \rho, \Delta x) := \Gamma^2 \rho c^2 \Delta x \]
\[ \hat{E}_T(\Gamma, \rho, p, \Delta x) := \Gamma^2 (\rho \epsilon + p) \Delta x \]
\[ \hat{E}_M(\Gamma, \rho, \sigma, \Delta x) := \Gamma^2 \rho \sigma c^2 \Delta x \]  

(A2)

With these definitions, the ratio of energies shown in equation (31) becomes Lorentz invariant, and so is the total dynamic efficiency. Furthermore, when calculated in the lab frame, the exact values of the dynamical efficiency computed using any of the two sets of definitions (equations A1 or A2) differ very little, as is shown in Fig. A1.

APPENDIX B: COMPUTATIONAL REQUIREMENTS FOR REALISTIC CALCULATIONS

In this paper, we have adopted a cylindrical geometry and ignored adiabatic losses of the non-thermal electrons, as well as the high-latitude emission. However, we have included both of these effects in previous works studying afterglows of GRBs (Mimica et al. 2010; Mimica & Giannios 2011). In those calculations, which were performed using the radiative transfer code SPEV (Mimica et al. 2009), we have only taken into account the synchrotron and EIC emission processes. Even so, the calculation of a realistic multiwavelength light curve of a single model lasts anywhere between a few hours to a few days on a supercomputing cluster with several hundreds of computing cores, depending on the number of wavelengths at which the emission is computed. If the synchrotron-self absorption (SSA) is included, then the parallel scalability of the method is reduced and the calculations can become prohibitively expensive (see footnote 3 of Mimica et al. 2010).

In this paper, we have a similar problem due to the fact that we include the SSC process, which, as SSA, is non-local and difficult to parallelize in a realistic conical geometry. The use of cylindrical geometry and other simplifying assumptions has enabled us to nevertheless compute light curves of 900 models for 96 frequencies at 120 observer times. This required \(200\,000\) computing hours on the MareNostrum computer of the Barcelona Supercomputing Center. Based on our previous experience, we estimate that an order of magnitude more resources are needed if the adiabatic cooling of the electrons is included, and another order of magnitude if an accurate radiative transfer method such as SPEV is used instead of approximate method described in Section 4. We note that such a calculation requires resources in the range of 10 million computing hours on a computer such as MareNostrum.

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