Super reflector and invisible object: analytical investigation of photonic cluster reflectivity

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Abstract

The reflectivity of photonic cluster is important property for nanophotonic applications especially when one needs to hide the cluster or to make it extremely visible. Currently, where are no methods clearly describing how to create the photonic cluster with the predefined reflectivity. In this paper several analytical methods are proposed to create the photonic cluster with minimal or maximal reflectivity. The proposed methods are applicable for the clusters made of small particles.

1 Introduction

Invisible men, time traveling, and teleportation are the very known and fascinating subjects from ancient tales and modern fiction books. For centuries these subjects were something easy imaginable but "not of this world". While time traveling and teleportation are still far from a realization (for living organisms at least) the idea of invisibility slowly began to translate into reality [1]-[5]. The super reflection can be considered as antipode to the invisibility and it has many practical applications already (in Bragg gratings and road

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signs, for example). In some sense the invisibility and the super reflection are closely related but inverted phenomena and that is why they can be studied in parallel.

The discussion about the invisible objects and nonradiating sources was started in the scientific literature many years ago (see for example the works [6]-[7] and the informative review [8]). Recently it was suggested that metamaterials can be used as building blocks for invisible objects and the key component of these materials is actually nanostructured photonic crystal [4]-[5], [9]-[10]. While it was demonstrated that some devices show relatively low reflectivity in some directions and at some wavelengths [3], [5], the transparent algorithm for design of invisible photonic clusters is not proposed yet. It is worth to mention several methods using supernatural conditions. One of them is the transformation optics method where the coordinates are transformed in such a way that the object becomes invisible [11]-[13]. The catch of the method is that transformation of the coordinates is equivalent to the redistribution of the permittivity inside the object and as the result of such transformation the object becomes invisible. It is not clear however, how such method can be applied to the photonic crystals formed by many discrete particles. Another method based on so-called negative refractive index metamaterials requires supernatural conditions (see, for example, the review [14] and the refreshing work [15]) and it is hardly feasible [2]. Inverse scattering methods [16]-[17] can be used, in principle, for construction of invisible objects, however a robust algorithm is not developed yet.

In this paper I will study visibility of the cluster made of small dielectric particles by using the local perturbation method (LPM) approach [18]-[20]. I will present and discuss several methods to construct invisible and extremely visible photonic cluster made of independent and interacting scatterers.

2 The field scattered by the photonic cluster in the LPM approximation

The theoretical framework I use is presented in many works (see for example [18]-[20] and references wherein) and it will be only briefly presented here for
convenience and consistency. Consider the photonic cluster made of particles which characteristic sizes are small compared to the incident wavelength $\lambda$. For definiteness it is assumed that the cluster is positioned at the origin of the coordinates. The electric field $E$ propagating in the host medium filled with $N$ small particles is described by the following equation \[20\]

\[
(\nabla^2 - \nabla \otimes \nabla + k^2) E(r) + \frac{k^2}{\varepsilon_0} \sum_{n=0}^{N-1} E(r_n)(\varepsilon_{sc,n} - \varepsilon_0)f_n(r - r_n) = S(r), \tag{1}
\]

where

\[
k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \sqrt{\varepsilon_0}, \quad f_n(r - r_n) = \begin{cases} 
1, & \text{inside particle} \\
0, & \text{outside particle} \end{cases} \tag{2}
\]

Here $\nabla$ and $\nabla$ are the Laplacian and nabla operators respectively, $\otimes$ defines tensor product, $k \equiv |k|$ is a wave number in the host medium (the $|...|$ brackets denote an absolute value), $\omega$ is the angular frequency, $c$ is the light velocity in vacuum, $\varepsilon_{sc,n}$ and $\varepsilon_0$ are the permittivity of the $n$-th particle and the medium respectively, $f_n$ is the function describing the shape of the $n$-th scatterer positioned at $r_n$, and $S$ is the field source. The characteristic size of $n$-th scatterer is denoted as $L_n$.

It should be noted that the equation (1) is an approximate one and it is valid when the small scatterers ($kL_n \ll 1$) are considered. The solution of the equation (1) is presented in the Appendix A and I will use in the following discussion the final result presented by the Eq. (A6)

\[
E_{sc}(r) = \frac{k^2}{4\pi\varepsilon_0} \left( \hat{I} + \frac{\nabla \otimes \nabla}{k^2} \right) \sum_{n=0}^{N-1} E(r_n)V_n(\varepsilon_{sc,n} - \varepsilon_0) \frac{e^{ikR_n}}{R_n}, \tag{3}
\]

where

\[
R_n \equiv |r - r_n| \gg L_n. \tag{4}
\]

Here $R_n$ is the distance between the observer positioned at $r$ and the $n$-th scatterer placed at $r_n$, $V_n$ is the volume of the $n$-th scatterer.

In many practical cases, the distance between the cluster and the observer is much more larger than the size of the cluster and the inequality $|r| \gg
max(|r_n|) is fulfilled. In this case the scattered field (3) can be simplified and it can be rewritten in the far zone (k |r| \gg 1) in the following form

\[ E_{sc}(r) = \frac{k^2 e^{ikr}}{4\pi r \varepsilon_0} \left( \hat{I} - \mathbf{l} \otimes \mathbf{l} \right) \sum_{n=0}^{N-1} E(r_n)V_n(\varepsilon_{sc,n} - \varepsilon_0)e^{-ik\mathbf{l} \cdot \mathbf{r}_n}, \]  

(5)

where

\[ \mathbf{l} = \mathbf{r}/r, \quad r \equiv |\mathbf{r}| \gg \max(|r_n|) \text{ and } kr \gg 1. \]

The formula (5) is the main result of this section and it will be used extensively in the following discussion. The formula shows that the field scattered by the cluster is mainly defined by the weighted fields \( E(r_n)V_n(\varepsilon_{sc,n} - \varepsilon_0) \) inside the small scatterers and by the phase \( kl \cdot r \) in the scattering direction \( \mathbf{l} \). Note, that the interactions between the particles in the cluster are present in the fields \( E(r_n) \).

3 The visibility of the photonic cluster

3.1 Criteria of the invisibility and the super reflectivity

The visibility of any photonic cluster depends on the intensity \( I_{sc}(r) \) of the field scattered by the cluster (and incident on the detector positioned at \( r \)) and on the sensitivity level \( \mathcal{L} \) of the detector. The sensitivity level of the detector is defined here as minimal intensity at which a signal is detected. A photonic cluster will be invisible (in the frequency span \( \Delta \omega \) and at the cone of the directions \( \Delta \mathbf{l} \)) when \( I_{sc}(r) \) is smaller or equal to \( \mathcal{L} \). It is convenient to write this statement for the invisible cluster in the following form

\[ I_{sc}(r) \equiv |E_{sc}(r)|^2 \leq \mathcal{L}, \quad (\omega \in \Delta \omega \text{ and } \mathbf{l} \in \Delta \mathbf{l}), \]  

(6)

where \( \mathcal{L} \) is the sensitivity level of the receiver. The sign \( \leq \) in the expression (6) can be used due to the presence of the noise masking the detecting signal when \( I_{sc}(r) = \mathcal{L} \).

To define the super reflectivity one should use the criterium based on the cluster’s reflectivity rather than on the detector sensitivity. The cluster will
be qualified as super reflective when the following relation is true

\[ I_{sc}(r) \sim \Im, \quad (\omega \in \Delta \omega \text{ and } l \in \Delta l), \tag{7} \]

where \( \Im \) is the maximal possible intensity of the field scattered by the cluster. For the cluster made of small particles the maximal scattered intensity can be estimated with the help of the Eq. (5) in the approximation of the independent particles (when \( E(r_n) = E_{in}(r_n)/(1 + (\varepsilon_{sc,n} - \varepsilon_0)\gamma_n/3\varepsilon_0) \)) and without depolarization (when \( (\hat{I} - l \otimes l) E = \hat{I}E \)) and it has the form

\[ \Im = \frac{|E_{in}(r)|^2 k^4}{16\pi^2\varepsilon_0^2 r^2} \left| \sum_{n=0}^{N-1} V_n \varepsilon_n \right|^2. \tag{8} \]

Here

\[ \varepsilon_n = \frac{\varepsilon_{sc,n} - \varepsilon_0}{1 + (\varepsilon_{sc,n} - \varepsilon_0)\gamma_n/3\varepsilon_0} \tag{9} \]

where \( \gamma_n \) is the parameter defined by the shape of the particles and for spheres, for example, it is

\[ \gamma_n = 1 - k^2L_n^2 \left( 1 + i\frac{2}{3}kL_n \right), \quad (kL_n \ll 1). \tag{10} \]

In the most ideal case one can modify the number of the particles \( N \), their volumes \( V_n \), positions \( r_n \), and the permittivities \( \varepsilon_n \) to make the cluster invisible or extremely visible in accordance with the conditions (6) or (7) respectively.

### 3.2 The ways to achieve the invisibility or super reflectivity

The scattering process involves the inhomogeneous medium (photonic cluster in our case) and the incident light. That is why the visibility of the cluster can be maximized or minimized by the changing the cluster or the incident light. The second possibility is discussed in the section 7 where it is shown how the given cluster can be hidden or made super reflective by using artificial illumination. In the next section I will discuss approaches to achieve the invisibility or the super reflectivity of the cluster by modifying its structure.
The photonic clusters can be subdivided in two main categories: composed of independent particles and made of interacting particles. The clusters made of the independent particles will be discussed in the section 5 while the clusters composed of the interacting particles will be discussed in the section 6.

There are several possibilities to satisfy the conditions (6) or (7) and to achieve the invisibility or the super reflectance of the photonic cluster. One of the ways is to solve the inequations (6) and (7) with respect to the positions of the particles \( r_n \) when the fields \( E(r_n) \), the number of the particles \( N \), and the properties of the particles (\( \varepsilon_{sc,n} \) and \( L_n \)) are known. (Note that the fields \( E(r_n) \) can be found analytically by solving the system of \( 3N \) linear equations of type (A5).) This way, though clearly formulated, will involve solution of the system of nonlinear equations that is not feasible. Alternatively, the positions of the particles in the cluster can be modified till \( I_{sc} \) will be smaller than \( \mathcal{L} \) or of the order of \( \mathcal{L} \). This way was investigated in the work [21] where encouraging results were obtained. The method closely related to the linear sampling method can be also used. It will be described in the section 6 in more detail.

Consider the cluster positioned far from the observer. In this case the formula (5) is applicable and the invisibility condition (6) can be presented in the following form

\[
\left| \left( \hat{I} - 1 \otimes 1 \right) K \right| \leq \sqrt{\mathcal{L}} \frac{4\pi\varepsilon_0}{k^2}
\]

(11)

and the condition (7) for the super reflectivity is

\[
\left| \left( \hat{I} - 1 \otimes 1 \right) K \right| \sim \left| E_{in}(0) \right| \left| \sum_{n=0}^{N-1} V_n \varepsilon_n \right|.
\]

(12)

Here the vector \( K \) is defined as

\[
K \equiv \sum_{n=0}^{N-1} E(r_n) V_n (\varepsilon_{sc,n} - \varepsilon_0) e^{-ik1 \cdot r_n}.
\]

(13)

The expressions (11) and (12) can be considered as the equations with respect to the positions \( r_n, V_n, \) and \( \varepsilon_{sc,n} \). The solution of these equations should give
the properties of the particles \((V_n, \varepsilon_{sc,n})\) and their coordinates \(r_n\) in the invisible cluster or the super reflector when the observer is positioned at the distance \(r\) from the cluster in the direction \(l\). The equations (11) and (12) should be complemented by the additional equations for the fields \(E(r_n)\) and they should contain at least \(N\) equations formulated for \(N\) observation directions \(l\). It should be emphasized that the equations are nonlinear with respect to the unknowns and they are extremely complicated. While the equations (11) and (12) can be solved numerically in principle, a number of serious drawbacks and limitations exist reducing the value of the numerical solution. The most serious limitations are the complex values of the solutions and the excessive number of the solutions.

Below, the conditions of invisibility and super reflectance will be discussed for the photonic cluster made of small particles.

### 4 The zero and the maximal scattering by the cluster of independent particles

Consider now the cluster made of \(N\) scatterers placed in such a way that interactions between the particles are negligible. The interaction between a central particle and \(M\) particles surrounding it is negligible when the cube of the distance between the particles is much larger than the total volume of the particles, i.e. when the following condition holds

\[
R_{mn}^3 \gg MV_n/4.
\]

For more information one can see the formula (B4) in the Appendix B and the required conditions. In this case the fields \(E(r_n)\) inside the scatterers can be calculated explicitly by using the Eq. \(A5\)

\[
E(r_n) = \frac{E_{in}(r_n)}{1 + (\varepsilon_{sc,n} - \varepsilon_0)\gamma_n/3\varepsilon_0},
\]

where \(\gamma_n\) are the parameters taking into account the characteristic size \(L_n\) and the shape of the particles. For example, for spheres and for cubes \(\gamma_n\)

\[
\gamma_n = 7.
\]
respectively are
\[
\gamma_n = 1 - k^2 L_n^2 \left( 1 + \frac{2}{3} k L_n \right)
\]
(16)
\[
\gamma_n = 1 - k^2 L_n^2 \left( 1.52 + i \frac{4}{\pi} k L_n \right),
\]
where \( L_n \) is characteristic size of the particle (radius of the sphere or half of the cube size). Substituting the fields (15) into the equations (11) and (12) we have
\[
\left| \left( \hat{I} - 1 \otimes 1 \right) \sum_{n=0}^{N-1} E_{in}(r_n) V_n \epsilon_n e^{-ik_0 \cdot r_n} \right| \leq \sqrt{N} \frac{4 \pi \epsilon_0 R}{k^2}. \]
(17)
\[
\left| \left( \hat{I} - 1 \otimes 1 \right) \sum_{n=0}^{N-1} E_{in}(r_n) V_n \epsilon_n e^{-ik_0 \cdot r_n} \right| \sim |E_{in}(0)| \sum_{n=0}^{N-1} V_n \epsilon_n. \]
(18)
The equations (17) and (18) show that even for the independent particles the cluster’s visibility is extremely complex phenomenon depending on all particles (via the fields \( E(r_n) \)) and it is extremely sensitive to the parameters of each particle \( (V_n, \epsilon_{sc,n}, \gamma_n, \text{and } L_n) \). The equations (17) and (18) are difficult to solve and that is why some simplifications are required. Below I will use two important simplifications: the scatterers with identical permittivity \( (\epsilon_{sc,n} = \epsilon_{sc}) \) and the long wavelength approximation.

4.1 The cluster made of independent particles with identical permittivity

Consider the case when the particles in the cluster have the same permittivity \( \epsilon_{sc} \) and the incident field is the plane wave, i.e. \( E_{in}(r) \equiv A e^{ik \cdot r} \). In this case the expressions (17) and (18) can be essentially simplified and they can be rewritten in the following ultimate form
\[
\left| \sum_{n=0}^{N-1} V_n e^{i k_{sc} \cdot r_n} \right| = \left\{ \begin{array}{ll} 0 & \text{if } \sum_{n=0}^{N-1} V_n \epsilon_n = 0, \end{array} \right. \]
(19)
where \( k_{sc} = k - k l \) is the scattering vector. The formula (19) was obtained in the assumption that the sensitivity level is \( \mathcal{E} \) and that depolarization is not
essential in this case. The Eq. (19) shows that the visibility of the cluster made of the independent scatterers is solely defined by the phases $k_{sc} \cdot r_n$ and by the volumes $V_n$ playing the role of weight factors.

The expression (19) is actually system of nonlinear equations for the positions $r_n$. When the positions $r_n$ are known one can produce invisible or super reflective cluster. The downside is that even this simplified system (19) is difficult to solve analytically.

However, for some specific systems the solutions are clearly visible. Consider, for example, the cluster with central symmetry. In this case the expression (19) transforms into the following one

$$\left|\sum_{n=0}^{(N-1)/2} V_n \cos(k_{sc} \cdot r_n)\right| = \left\{ \begin{array}{ll} 0 & \sum_{n=0}^{(N-1)/2} V_n \\ \sum_{n=0}^{(N-1)/2} V_n & \end{array} \right..$$

(20)

One set of solutions of the Eq. (20) is clearly visible and for the invisibility it is

$$k_{sc} \cdot r_n = \pi(1/2 + n)$$

(21)

and for the maximal reflectivity it is

$$k_{sc} \cdot r_n = \left\{ \begin{array}{ll} 2m\pi & \\ (2m + 1)\pi & \end{array} \right.,$$

(22)

where $m$ is an integer.

It is interesting to note that the maximal scattering happens only when the conditions (22) are fulfilled and the scattering is not affected by the “weight factors” $V_n$. On the other hand, for the minimal scattering, the weight factors $V_n$ are extremely important such that invisibility can happen in multiple ways (i. e. other solutions are possible where the weight factors $V_n$ play active role).

4.2 The cluster of independent particles in the long wavelength approximation

It is interesting to note that despite the independency (defined by the condition (14) the particles in the cluster can be placed such that the distance
between the adjacent scatterers will be much smaller than the incident wavelength such that the following condition will be satisfied

\[ |k_{sc} \cdot (r_n - r_m)| \ll 1, \]  

(23)

where the particles positioned at \( r_n \) and \( r_m \) are the adjacent ones. By using the condition (23) the sum (17) and (18) can be replaced by the following integral

\[ \Gamma \equiv \frac{1}{d^3} \left| \int_{V_{cl}} \epsilon(r) e^{ik_{sc} \cdot r} dr \right|, \]  

(24)

where

\[ d \equiv \langle |r_n - r_m| \rangle, \quad \epsilon(r) = \frac{(\varepsilon_{sc}(r) - \varepsilon_0)}{1 + (\varepsilon_{sc}(r) - \varepsilon_0)/3\varepsilon_0}. \]  

(25)

Here \( d \) is the average period of the cluster. Note that the integral \( \Gamma \) depends on the shape of the cluster which is not known beforehand and the shape should be defined from other considerations (practical or guess, for example).

Since \( \epsilon(r) \) vanishes outside of the cluster, the integration in (24) can be extended till infinity and \( \Gamma \) is actually the Fourier transform of the \( \epsilon(r) \). In this case we can present \( \Gamma \) as

\[ \Gamma = \frac{8\pi^3}{d^3} |\tilde{\epsilon}(-k_{sc})|, \]  

(26)

where

\[ \tilde{\epsilon}(k_{sc}) \equiv \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \epsilon(r) e^{-ik_{sc} \cdot r} dr \]  

(27)

is the Fourier transform of the contrast function \( \epsilon(r) \).

The formula (26) shows that when the long wavelength approximation is valid, the visibility of the cluster made of the independent scatterers is defined by the Fourier transform of the contrast function \( \epsilon(r) \). This property allows to construct the permittivity \( \varepsilon_{sc}(r) \) of the cluster by using the following formula

\[ \varepsilon_{sc}(r) = \varepsilon_0 \left( 1 + \frac{\epsilon(r)}{\varepsilon_0 - \epsilon(r)/3} \right), \]  

(28)

It is worth to note that this result resembles the one presented in [8] for the invisible scatterer in the first Born approximation.
5 The visibility of the cluster of interacting particles: the linear sampling method approach

Consider the cluster of arbitrary form made of interacting particles. Surely, the cluster can be surrounded by the sphere of the radius $\rho$ and we can subdivide the sphere into small cells positioned at the points $r_n (|r_n| \leq \rho)$. Suppose that we know the field scattered by the cluster at the observation points $r_m$ such that $|r_m| = r$. In this case one can write the following system of equations in respect to the fields $E(r_n)$ and the weighted contrasts $\mu_n$

$$E_{sc}(r_m) = \frac{k^2 e^{ik|r_m|}}{4\pi \varepsilon_0 r} \left( \hat{I} - l_m \otimes l_m \right) \sum_{n=0}^{N-1} E(r_n)\mu_n e^{-ikl_m \cdot r_n},$$

(29)

where $m = 1...M \ (M \geq N)$ and

$$\mu_n \equiv V_n(\varepsilon_{sc,n} - \varepsilon_0), \ l_m \equiv r_m/r, \ |r_m| = r.$$

(30)

The equations (29) can be resolved in respect to the multiplication product $E(r_n)\mu_n$. The next step is to find the fields $E(r_j)$ by using the following equations

$$E(r_j) = E_{in}(r_j) + E(r_j)\mu_j \gamma_j +$$

$$\frac{k^2}{4\pi \varepsilon_0} \left( a \hat{I} - bl_{nj} \otimes l_{nj} \right) \sum_{n \neq j} E(r_n)\mu_{n} e^{ik|r_n-r_j|/|r_n-r_j|},$$

(31)

$$a = 1 + i/kR - 1/k^2R^2,$$

(32)

$$b = -1 - 3i/kR + 3/k^2R^2,$$

(33)

where $\gamma_j$ takes into account the shape of the $j$-th particle and it is described by the formulae above.

When the field $E(r_j)$ is known, the weighed contrast $\mu_j$ is found from the multiplication product $E(r_n)\mu_n$. The number of the cells ($N$) will be larger than the number of the particles in the cluster, however the contrast of some cells should be close to zero such that shape of the cluster (defined by the non zero contrast) will be not spherical but similar to the real one.
The only open question is the values of the scattered fields $E_{sc}(r_m)$ since we need to have definitive values of them to construct the invisible or the super reflective cluster. We note that the amplitudes of the fields $E_{sc}(r_m)$ are defined in some sense by the Eqs. (11) and (12) while the phases are not clearly defined. This can lead to ambiguity which can be reduced by imposing additional restrictions (from design or engineering point of view, for example).

6 The visibility of the cluster in artificially created incident field

Some applications require to hide a given cluster or make it clearly visible (super reflective, for example). One of the ways to do it is to synthesize the incident field $E_{in}$ in such a way that the cluster will change its reflectance (at the wavelength $\lambda$ in the direction $l$). To do this, the fields $E(r_n)$ inside the particles should be found from the following system of equations

$$\frac{k^2 e^{ikr}}{4\pi\varepsilon_0 r} \left( \hat{I} - l_m \otimes l_m \right) \sum_{n=0}^{N-1} E(r_n) V_n (\varepsilon_{sc,n} - \varepsilon_0) e^{-ikl_m r_n} = E_{sc}(r_m), \quad (34)$$

where $l_m$ and $r_m$ are the directions and the points respectively in which the cluster should be invisible. The system (34) consists of linear equations with respect to the unknown fields $E(r_n)$ and it can be easily resolved. When the fields $E(r_n)$ are known, the scattered field $E_{sc}(r_n)$ can be found by using the formula (5). The incident fields $E_{in}(r_n)$ are found from the following formula (see Eq. (A2) for the reference)

$$E_{in}(r_n) = E(r_n) - E_{sc}(r_n). \quad (35)$$

When the cluster is illuminated by the incident field $E_{in}$ synthesized in accordance with the expression (35), the cluster will be invisible or super reflective for the observers positioned in the directions $l_k$ at the wavelength $\lambda$. 

12
7 Conclusions

The visibility criteria have been discussed for the photonic cluster made of small particles. The conditions of the zero and of the maximal scattering have been studied for the clusters made of independent and interacting particles. It has been shown that in the long wavelength approximation the visibility of the cluster made of the independent particles is governed by the Fourier transform of the optical contrast.

The clear algorithm to construct the invisible or the super reflective photonic cluster has been proposed for the clusters made of interacting particles.

The new method to hide a given photonic cluster or to make it extremely visible by creating artificial incident field has been presented.

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8 Appendix A: The LPM formalism

The equation (1)

\[ (\Delta - \nabla \otimes \nabla + k^2) E(\mathbf{r}) + \frac{k^2}{\varepsilon_0} \sum_{n=0}^{N-1} E(\mathbf{r}_n)(\varepsilon_{sc,n} - \varepsilon_0) f_n(\mathbf{r} - \mathbf{r}_n) = S(\mathbf{r}), \quad (A1) \]

is easily solvable with respect to the fields \( E(\mathbf{r}_n) \) when the positions \( \mathbf{r}_n \) of the particles are known (it will invoke solution of \( 3N \) linear equations for each frequency \( \omega \)). The solution of the equation \((A1)\) can be presented in the following form

\[ E(\mathbf{r}) \equiv E_m(\mathbf{r}) + E_{sc}(\mathbf{r}), \quad (A2) \]

where the scattered field \( E_{sc} \) is

\[ E_{sc}(\mathbf{r}) = \frac{k^2}{\varepsilon_0} \left( \hat{I} + \frac{\nabla \otimes \nabla}{k^2} \right) \sum_{n=0}^{N-1} E(\mathbf{r}_n)(\varepsilon_{sc,n} - \varepsilon_0) \Phi_n(\mathbf{r}) \quad (A3) \]
\[ \Phi_n(r) \equiv \int_{-\infty}^{\infty} \widetilde{f}_n(q) e^{i q \cdot (r - r_n)} dq, \quad \widetilde{f}_n(q) \equiv \frac{1}{8 \pi^3} \int_{-\infty}^{\infty} f_n(u) e^{-i q \cdot u} du. \]  

(A4)

Here the incident field \( \mathbf{E}_{in} \) is created by the source \( S \) in the host medium and it is not important for our consideration (see for example [22] for more details). The tensor \( \hat{I} \) is the 3 \times 3 unitary tensor in polarization space and \( r_n \) is the radius vector of the \( n \)-th particle. The field \( \mathbf{E}(r_n) \) is the field inside the \( n \)-th particle, \( \widetilde{f}_n \) is the Fourier transform of the function \( f_n \), and \( \cdot \) defines scalar product. Note that the integration in Eq. (A4) is over infinite three dimensional spaces.

The equations for the fields \( \mathbf{E}(r_n) \) are found by substituting \( r = r_n \) into Eq. (A2)

\[ \mathbf{E}(r_n) = \mathbf{E}_{in}(r_n) + \frac{k^2}{\varepsilon_0} \sum_{n=0}^{N-1} (\varepsilon_{sc,n} - \varepsilon_0) \int_{-\infty}^{\infty} \left( \frac{\hat{I} - \frac{\partial \hat{I}}{\partial q^2}}{q^2 - k^2} \right) \widetilde{f}_n(q) \left( q^2 - k^2 \right) dq \mathbf{E}(r_n). \]  

(A5)

It should be emphasized that the formula (A2) is rather general one and it describes the field in the medium with photonic cluster of arbitrary form made of small particles of arbitrary form. The formula for the scattered field (A3) can be simplified when the distance between the observer and the \( n \)-th scatterer (\( R_n \)) is large, i.e. when \( R_n \gg L_n \). In this case the integral \( \Phi_n \) can be calculated approximately. Note also that the integral \( \Phi_n \) can be calculated exactly at least for the spherical particles. When \( R_n \gg L_n \) the integration in (A3) gives

\[ \mathbf{E}_{sc}(r) = \frac{k^2}{4\pi \varepsilon_0} \left( \hat{I} + \frac{\nabla \otimes \nabla}{k^2} \right) \sum_{n=0}^{N-1} \mathbf{E}(r_n) V_n (\varepsilon_{sc,n} - \varepsilon_0) \frac{e^{ikR_n}}{R_n}, \]  

(A6)

where

\[ R_n \equiv |r - r_n| \gg L_n. \]  

(A7)

Here \( R_n \) is the distance between the observer positioned at \( r \) and the \( n \)-th scatterer placed at \( r_n \), \( V_n \) is the volume of the \( n \)-th scatterer.

\[ \mathbf{E}_{sc}(r) = \frac{k^2}{4\pi \varepsilon_0} \left( \hat{I} + \frac{\nabla \otimes \nabla}{k^2} \right) \sum_{n=0}^{N-1} \mathbf{E}(r_n) V_n (\varepsilon_{sc,n} - \varepsilon_0) \frac{e^{ikR_n}}{R_n}, \]  

(A6)
where
\[ R_n \equiv |r - r_n| \gg L_n. \] (A7)
Here \( R_n \) is the distance between the observer positioned at \( r \) and the \( n \)-th scatterer placed at \( r_n \), \( V_n \) is the volume of the \( n \)-th scatterer.

9 Appendix B: The condition of the independence of the particles in the cluster

Consider the cluster made of identical small scatterers positioned at \( r_n \). The cluster consists of the central particle (positioned at \( r_0 \)) and the particles surrounding it (\( M \) particles are placed at the distance \( R \) from the central particle). The field inside the central particle positioned at \( r_0 \) has the following form (see Eq. (A5))

\[
E(r_0) = \left[ \frac{k^2 e^{ikR}}{4\pi \varepsilon_0 R} V(\varepsilon_{sc} - \varepsilon_0) \sum_{n=1}^{M} \left( a\mathbf{l} + b l_{0n} \otimes l_{0n} \right) E(r_n) + E_{in}(r_0) \right] / (1 + (\varepsilon_{sc} - \varepsilon_0)/3\varepsilon_0),
\] (B1)

where
\[
a = 1 + i/kR - 1/k^2 R^2, \quad (B2)
\]
\[
b = -1 - 3i/kR + 3/k^2 R^2,
\]
and
\[
l_{0n} = (r_0 - r_n) / R, \quad R = |r_0 - r_n|. \quad (B3)
\]
Here \( V \) and \( \varepsilon_{sc} \) are the volume and the permittivity of the scatterers respectively.

The importance of the formula (B1) is that it allows us to estimate the distance at which the particles are independent, i.e. when the field inside the particle located at \( r_0 \) will be independent from the particles located at \( r_n \). This will happen when the term containing \( E(r_n) \) in Eq. (B1) will be much smaller than the incident field \( E_{in}(r_0) \). The analysis of the Eq. (B1) shows
that for long wavelengths (when $kR \ll 1$) the particles are independent when the following condition is satisfied

$$R^3 \gg MV/4,$$

where $M$ is the number of the particles surrounding the central one. The condition (B4) is used in the paper to discriminate the clusters with independent and interacting particles. We note that this is over estimated condition since it was supposed that all the particles interfere in a constructive way. The condition (B4) shows, for example, that for the cluster with $M = 6$ surrounding spheres (case of simple cubic lattice) the particles will be independent when $R \geq 3.3L$.

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