Abstract. An increasing research activity in the field of dynamic calibration of pressure transducers can be recognized at some national metrology institutes. As an example the EMPIR project “development of measurement and calibration techniques for dynamic pressures and temperatures” can be named. Despite that efforts, no national reference standard for dynamic pressure calibration is available up to now. This makes the measurement of high fluctuating pressure signals difficult and unprecise. These dynamic pressure signals appears in aerospace applications, blast test and almost every fluidic circuit which employs discontinuous discharge elements. To address that topic the authors developed a sine calibration apparatus to measure the frequency response of pressure transducers with sufficient amplitudes up to 1.2 MPa. Due to the construction of the pressure generator frequencies up to 10 kHz can be reached. Furthermore a calibration technique was developed to calculate the pressure inside a chamber primarily. The fundamental idea is to calculate the pressure based on the displacement of a piston in a pistonphone device. To do so the author had to analyses the thermodynamic conditions inside the fluid filled chamber. The paper shows that the fundamental approach was confirmed by measurements.

1 Introduction

Measuring dynamic pressure is an essential task when it comes to automotive, fluidic, aeronautic or aerospace applications. The pressure pulsation has to be measured with high accuracy in respect to the frequency and the amplitude of the pulsation. An example application for the measurement of dynamic pressure amplitudes is the compression process in an aircraft turbine. If the pressure is measured correctly this process can be analyzed and important conclusions for the design of such turbines are generated. But for a measurement of pressure in a dynamical environment, the frequency response of the transducer has to be known. Hence the transducer has to be calibrated by means of dynamic calibration methods.

As summarized by Hjelmgren [1] or the ISA dynamic pressure committee [2] some dynamic calibration methods were developed in the past. Namely the shock tube method and the fast opening devices to name a few of them. But they work mostly with pressure amplitudes of several MPa and in an aperiodic manner. If converting the aperiodic time based results to the frequency domain numerical issues and a high measurement uncertainty has to be accepted. In contrast periodic pressure calibration systems are widely used for
microphones. But this type of calibration deals with pressure amplitudes obviously lower than reached by industrial pressure sensors.

This is why the author has made an effort to develop a periodic calibration device and a primary calibration method, to investigate the frequency response of pressure transducers. The device is investigated at a frequency range of $f = 0..10 \text{ kHz}$ and pressure amplitudes higher than $\hat{p} = 1 \text{ MPa}$ can be reached.

2 Calibration device

The design is based on a pistonphone. The device consists of a piezoelectric actuator which is located between a housing and a piston. The piston compresses the enclosed liquid. Figure 1 shows the design and the mechanical model.

![Figure 1. Design of the pressure generator and the mechanical model](image)

In the left part of figure 1 it can be seen, that the device can be used for primary and secondary calibrations. When operating the device in a primary mode, the reference transducer or the DUT is calibrated with the help of the laser interferometer and the primary method which is presented in section 3. Additionally a secondary calibration can be performed, where the DUT is calibrated by the reference pressure transducer. To do so the reference transducer has to be calibrated by the primary method in advance.

From the mechanical model it is obvious that the pressure generator is not mounted to a fundament, but build up as a free two mass oscillator. The fluid and the piezoelectric drive can be modelled as a spring-damping-system. Both, the fluid and the actuator are connect to the two masses, hence they can be summed to a resultant stiffness $c = c_{\text{fluid}} + c_{\text{piezo}}$ and damping factor $d = d_{\text{fluid}} + d_{\text{piezo}}$. The system can be described by the Newtonian equation of motion.

$$\begin{pmatrix}
F_{\text{Piezo}} \\
-F_{\text{Piezo}}
\end{pmatrix} = 
\begin{pmatrix}
m_h & 0 \\
0 & m_p
\end{pmatrix}
\begin{pmatrix}
\ddot{x}_h \\
\ddot{x}_p
\end{pmatrix} + 
\begin{pmatrix}
d & -d \\
-d & d
\end{pmatrix}
\begin{pmatrix}
\dot{x}_h \\
\dot{x}_p
\end{pmatrix} + 
\begin{pmatrix}
c & -c \\
-c & c
\end{pmatrix}
\begin{pmatrix}
x_h \\
x_p
\end{pmatrix}$$  \hspace{1cm} (1)

The force $F_{\text{Piezo}}$ is generated by the piezo. The force-voltage dependency of the piezo can be evaluated with the piezo constant, since the device is operated in a low voltage mode. The displacement amplitudes of the piston and housing are limited by the dynamic motion and the power limit of the actuator. With the current setup the maximum dynamic peak pressure is approximately $\hat{p} = 1.2 \text{ MPa}$. 
3 Measurement Model

3.1. Basic Model

The basic measurement model develops from the conservation of mass in a sealed cavity, with the assumption of a uniform pressure in the whole cavity.

\[
C_H \frac{dp}{dt} = -\frac{dV}{dt} + V \gamma \frac{dT}{dt} \tag{2}
\]

Where \( p \) is the pressure, \( t \) is the time, \( C_H \) is the hydraulic capacity, \( V \) is the initial volume, \( \gamma \) is the heat expansion coefficient and \( T \) is the temperature. In a first step the temperature dependency is neglected. The equation then simplifies to

\[
C_H \frac{dp}{dt} = -\frac{dV}{dt} \tag{3}
\]

and after integration to

\[
\Delta p = -\frac{1}{C_H} \Delta V \tag{4}
\]

With the definition of the hydraulic capacity and with the volume difference applied on the actual exciter we get

\[
\Delta p = -\frac{1}{C_H} \Delta V = K \frac{\Delta V}{V} = K \frac{A \Delta x}{V} \tag{5}
\]

Where \( K \) is the bulk modulus of the fluid, \( A \) is the area of the piston and \( \Delta x \) is the difference between the movement of the piston and the housing

\[
\Delta x = x_p - x_h \tag{6}
\]

For the calculation of the dynamic pressure amplitude the three constants bulk modulus, piston area and the initial volume have to be measured. This three parameters can be regarded as constant with respect to the frequency. Hence they can be concentrated in a constant \( C_1 \) and evaluated in a static pre-measurement.

\[
\Delta p = C_1 \Delta x \tag{7}
\]

\[
C_1 = \frac{\Delta p}{\Delta x} \tag{8}
\]

To measure the displacement of the piston a laser interferometer is used. The pressure is measured by a statically calibrated pressure transducer. With the measurement of the constant \( C_1 \) all physical dependencies of the basic measurement model are evaluated and the dynamic pressure amplitude can be calculated based on the dynamic displacement of the piston and the housing. One advantage of the design of the system is, that the displacement of the housing is much smaller than the displacement of the piston, so if the relation between these two values is confirmed once, the piston displacement is may the only quantity which has to be measured later, while the motion of the housing is considered in the measurement uncertainty.
3.2. Thermodynamically Influence

3.2.1 Mathematical description

As mentioned in the previous section, the compression of the fluid due to the heat expansion of the fluid is neglected in the basic model. While do so this influence has to be quantified. This influence was evaluated by calculating the pressure to displacement change for an ideal isothermal compression by:

\[
\frac{dp}{dx} = -K \frac{A}{V} \tag{9}
\]

\[
\frac{dT}{dp} = 0 \tag{10}
\]

and afterwards for an ideal adiabatic compression by [3]:

\[
\frac{dp}{dx} = \frac{A}{V \left( \gamma \frac{dT}{dp} - \frac{1}{K} \right)} \tag{11}
\]

\[
\frac{dT}{dp} = \frac{\gamma T}{\rho c_p} \tag{12}
\]

This calculation was done for different fluids, namely water and a hydraulic oil HLP46. The pressure rise due to displacement change is shown in figure 2.

![Figure 2](https://doi.org/10.1051/metrology/201927006)

**Figure 2.** Pressure rise due to displacement change for different fluids and isothermal and adiabatic compression

The difference between the ideal isothermal and the ideal adiabatic compression can be seen in the diagram. It is obvious that the both cases differs more from each other for hydraulic oil compared to water. The difference between this to ideal cases is \( \kappa = 1.12 \) for the hydraulic oil and \( \kappa = 1.007 \) for water.
How this effect develops with respect to the frequency is qualitatively shown in figure 3.

The three regions ideal isothermal, transition and ideal adiabatic compression are marked in the upper diagram. The previous calculation analysed the difference between the adiabatic and isothermal state. The compression difference between this two cases is highlighted in the diagram. The transition range was not calculated by the employed equations and only the qualitative form of the curve is given in the diagram.

3.2.2 Measurement Results

To confirm the theoretical findings shown in the previous section, the author performed some measurements with the shown calibration apparatus. To do so two calibrations were performed where only the fluid in the cavity was changed. All other potential influence parameters like the employed sensor, the ambient conditions, the temperature or the vibration isolation, to name a few of them, were not changed.

To analyse the influence of the fluid, both measurement results were normalized to the calibration value at $f = 100 \text{ Hz}$ and afterwards the difference of both measurements was calculated (figure 4).
Figure 4. Difference between two calibrations where either water or hydraulic oil was used

In figure 4 it can be seen that there is a difference below $f = 1 \text{ Hz}$ if either water or hydraulic oil is used as a pressure transmitting fluid. As shown in the previous section this influence is caused by the heat expansion of the employed fluid. With the experiment the frequency range where the heat expansion of the fluid plays a dominant role can be identified. This influence is becoming dominant below $f = 1 \text{ Hz}$.

To verify that the experimental findings are showing the thermodynamic transition range, the experimental results were fitted to a model. The employed model has its origin in the field of primary acoustic calibration and was revisited by P. Vincent recently. The model calculates the frequency dependency of the pressure rise for a given displacement of an actuator in a closed cavity. The basics can be found in [4, 5]. The transition model was fitted on the measurement points and can be seen in figure 5.

Figure 5. Comparison of measurement results to a theoretical model from [4]

As figure 5 shows the model and the experimental curve are in good agreement. The overall difference between the isothermal compression and the adiabatic one was assumed to be $\kappa_{\text{diff}} = \kappa_{\text{oil}} - \kappa_{\text{water}} = 1.12 - 1.007 = 1.113$ or 11.3 % as it was calculated in section 3.2.1.

Besides that the frequency at which an ideal isothermal compression takes place can be evaluated. For our exciter the author estimates, that at a frequency of $f = 1 \text{ mHz}$ an ideal isothermal compression can be considered.

Nevertheless it is important to notice, that the thermodynamically influence can be neglected or considered in the measurement uncertainty if the employed fluid is water, since
the theoretical findings show that the absolute difference between the isothermal and adiabatic case is less than 1 %.

4 Conclusion

The author developed a calibration apparatus for the dynamic calibration of pressure transducers. The excitation of the dynamic pressure is done in a periodic manner over a broad frequency range and with a pressure amplitude sufficient to excite common pressure sensors.

Further the author presented the measurement model to calculate the dynamic pressure amplitude in the cavity where the pressure sensor is coupled to. The measurement model was reduced to a basic measurement model which will be used for the calibration. While doing so the uncertainty which is caused by the reduction of the model has to be evaluated. The reduction contains the thermodynamic influence of the fluid namely the compression caused by the heat expansion of the fluid. The author had shown theoretically that this effect can be reduced to under 1% if water is used as a pressure transmitting fluid. Further this thermodynamic effect was shown experimentally and the frequency range of this effect was shown. To verify the measurement data, a model for the thermodynamic transition was fitted onto the measurement points and the curves are in good agreement.

In the next steps the measurement frequency could be reduced to verify the theoretically calculated deviation between an isothermal and adiabatic compression for the chosen optimal calibration fluid water. By doing so the assumptions of the basic measurement model would be verified entirely.

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