Prediction of Astrometric-microlensing Events from Gaia eDR3 Proper Motions*†

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Abstract

Astrometric microlensing is a unique tool to measure stellar masses. It allows us to determine the mass of the lensing star with an accuracy of a few percent. In this paper, we update, extend, and refine our predictions of astrometric-microlensing events based on Gaia’s early Data release 3 (eDR3). We selected about 500,000 high-proper-motion stars from Gaia eDR3 with \( \mu_{\text{BGS}} > 100 \) mas yr\(^{-1} \) and searched for background sources close to their paths. We applied various selection criteria and cuts in order to exclude spurious sources and co-moving stars. By forecasting the future positions of lens and source, we determined epoch of and angular separation at closest approach, and determined an expected positional shift and magnification. Using Gaia eDR3, we predict 1758 new microlensing events with expected shifts larger than 0.1 mas between the epochs J2010.5 and mid J2066.0. Further, we provide more precise information on the angular separation at closest approach for 3084 previously predicted events. This helps to select better targets for observations, especially for events that occur within the next decade. Our search lead to the new prediction of an interesting astrometric-microlensing event by the white dwarf Gaia eDR3-4053455379420641152. In 2025 it will pass by a \( G = 20.25 \) mag star, which will lead to a positional shift of the major image of \( \Delta \theta_e = 1.2^{+0.5}_{-0.5} \) mas. Since the background source is only \( \Delta G = 2.45 \) mag fainter than the lens, also the shift of the combined center of light will be measurable, especially using a near-infrared filter, where the background star is brighter than the lens (\( \Delta K_S = -1.1 \) mag).

Unified Astronomy Thesaurus concepts: Astrometry (80); Proper motions (1295); Catalogs (205); Astrometric microlensing effect (2140); Stellar masses (1614); White dwarf stars (1799)

Supporting material: machine-readable table

1. Introduction

The mass of a star is one of its most important parameters. It defines its luminosity, temperature, surface gravity, appearance, and evolutionary path. Testing evolutionary and stellar models requires accurate and direct measurements of fundamental stellar parameters. Direct masses are usually derived from double-lined spectroscopic and eclipsing binaries. However, for most of the isolated stars, masses can only be derived indirectly, typically by using the mass–luminosity or mass–radius relations. For the determination of such relations, a set of accurately known masses is required. These are mainly derived from binary stars (Andersen 1991; Torres et al. 2010). However, binary stars and isolated stars may evolve differently. Therefore it is not known how well these empirical relations describe the masses of single stars. For a better understanding of the mass–luminosity relations, direct mass measurements of single stars are important. Besides asteroseismology, which itself is strongly model dependent, gravitational microlensing is the only available tool. Further, the direct determined mass of white dwarfs (WDs) provides a unique test sample for comparison with theoretical mass–radius relations and evolutionary cooling tracks of WDs; a first such measurement was achieved by Sahu et al. (2017).

As a subarea of gravitational lensing, microlensing describes the time-dependent positional deflection (astrometric microlensing) and magnification (photometric microlensing) of a background source (BGS) due to an intervening star (“lens”; Paczyński 1986a, 1986b). Using microlensing, it is possible to determine the mass of the lens star with uncertainties on the order of a few percent, either by detecting finite-source effects in photometric-microlensing events and measuring the microlens parallaxes (Gould 1992) or by observing the positional deflection of astrometric-microlensing events (Paczyński 1991, 1995). While the usage of photometric microlensing is slightly dependent on empirical relations to determine the radius of the BGS, astrometric microlensing is completely model independent and depends directly on the mass of the lens and its distance. In the last few years, this was used by Sahu et al. (2017) and Zurlo et al. (2018) to determine the masses of Stein 2051b and Proxima Centauri, respectively. In comparison to photometric microlensing, astrometric microlensing has two additional advantages. First, it can be observed at larger angular separations between the BGS and the foreground lens, which results furthermore in a longer timescale for the event of the order of months to years (Dominik & Sahu 2000). The second advantage is the possibility to confidently predict astrometric-microlensing events for stars with known proper motions (Refsdal 1964; Paczyński 1995). Further, these precise predictions are also needed for the mass determination to derive the unlensed separation.

Salim & Gould (2000) systematically searched for astrometric-microlensing events for the first time, and were followed...
by several studies based on non-Gaia proper motions (Proft et al. 2011; Lepine & DeStefano 2012; Harding et al. 2018). Presently the most precise predictions make use of astrometric data from the Gaia satellite (Gaia Collaboration et al. 2016). Using Gaia DR1, the first data release, McGill et al. (2018) predicted one event caused by a WD in 2019. With the second Gaia data release (Gaia DR2; Gaia Collaboration et al. 2018) about 5700 events were predicted, either by using solely Gaia DR2 (Bramich 2018; Bramich & Nielsen 2018; Klüter et al. 2018a, 2018b; McGill et al. 2019b) or by combining it with additional catalogs (Nielsen & Bramich 2018; McGill et al. 2019a), such as Pan-STARRS (Chambers et al. 2016),4 or VVV (Minniti et al. 2010),5 respectively. Additionally, Mustill et al. (2018) searched for astrometric-microlensing events with impact parameters of the order of one Einstein radius. Such events will lead to a potentially measurable photometric magnification of the source star. They found 30 such events that will occur until the year 2032 that have a >10% probability of having an impact parameter smaller than the Einstein radius. However, the astrometric precision of Gaia DR2 (and also of eDR3) is not sufficient to securely predict photometric-microlensing events.

In 2020 December, Gaia published its early Data Release 3 (Gaia eDR3; Gaia Collaboration et al. 2020a), which contains updated and more precise astrometric data of about two billion stars. McGill et al. (2020) showed that events predicted from Gaia DR2 that occur in the coming few years are contaminated by spurious background stars and by pairs of co-moving stars. Gaia eDR3 includes fewer spurious sources, and a better exclusion of co-moving stars is possible due to the larger fraction of stars with available proper motions. Further, the increase in the astrometric precision allows for more accurate predictions.

In this paper we update our previous search for astrometric-microlensing events in Klüter et al. (2018b, hereafter K18b) by using Gaia eDR3. First, in Section 2, we shortly explain astrometric and photometric microlensing. In Section 3 we explain our search for astrometric-microlensing events, starting with the selection of potential lens and source stars in Sections 3.1 and 3.2, respectively, and continuing with the determination of approximate masses and Einstein radii in Section 3.3, the forecast of their paths and the detection of close encounters in Section 3.4, and the determination of the expected effects and selection of observable events in Section 3.5. In Section 4, we present the predicted astrometric-microlensing events and compare this list with previous studies. Finally, we draw conclusions in Section 5.

2. Basics of Microlensing

Microlensing describes the time-dependent photometric magnification and positional change of a BGS by a point-like foreground mass (“lens”) passing by. While the lens is passing the source, two images of the source are created: a bright major image (+) close to the unlensed source position, and a faint minor image (−) close to the lens position. In case of perfect alignment, the BGS appears as a so-called Einstein ring. Its radius is given by Chwolson (1924), Einstein (1936), and Paczyński (1986a) as

\[
\theta_E = \sqrt{\frac{4GM_{L}D_{ES}D_{LS}}{c^2D_{LS}D_{ES}}},
\]

(1)

where \(G\) is the gravitational constant, \(c\) is the speed of light, \(M_{L}\) is the mass of the lens, \(D_{E}\) and \(D_{S}\) are the distances of lens and source, respectively, from the observer, and \(\theta_{EL}\) and \(\theta_{ES}\) are the parallaxes of lens and source, respectively. This Einstein radius is often used as an angular scale for the microlensing event, e.g., to define the (unlensed) scaled angular separation on the sky \(u = \theta/\theta_E\), where \(\theta\) is the two-dimensional unlensed angular separation.

2.1. Photometric Microlensing

The photometric magnification only depends on this unitless angular separation \(u\). The total magnification of the two images is given by Paczyński (1986a):

\[
A = A_+ + A_- = \frac{u^2 + 2}{u\sqrt{u^2 + 4}},
\]

(2)

where \(u = |u|\). The apparent magnification of the BGS can be reduced due to blending with light from additional unresolved light sources. For the here-predicted events, the most important additional light source is the lens itself. With a flux ratio \(f_{LS}\) between lens and source, it can be expressed as (Dominik & Sahu 2000):

\[
A_{\text{lam}} = \frac{f_{LS} + A}{f_{LS} + 1},
\]

(3)

and in units of magnitude, it is given by

\[
\Delta m = 2.5 \cdot \log_{10}\left(\frac{f_{LS} + A}{f_{LS} + 1}\right).
\]

(4)

Due to a strong decline (\(A \approx 1 + 2/u^2\); Paczyński 1996), a magnification is only observable when the impact parameter is on the order of the Einstein radius. This also means that a photometric-microlensing event usually has a short timescale on the order of days to weeks. This can be expressed by the Einstein time (Gould 1992):

\[
f_E = \frac{\theta_E}{\mu_{\text{rel}}},
\]

(5)

where \(\mu_{\text{rel}}\) is the absolute value of the relative proper motion between lens and source.

2.2. Astrometric Microlensing

In astrometric microlensing, the signal of interest is the change of the positions of the two images. The positional shift of the major image with respect to the unlensed position of the source is given by:

\[
\delta\theta_L = \frac{\sqrt{u^2 + 4} - u}{2} \cdot \frac{u}{\mu_{\text{rel}}} \cdot \theta_E.
\]

(6)

In the unresolved case, this deflection is reduced due to blending. First, due to the minor image, which leads to a shift of the center of light (with respect to the unlensed position of
the source),
\[ \delta \theta_s = \frac{u}{u^2 + 2} \cdot \theta_s, \]
and second due to a likely blending with a luminous lens (Hog et al. 1995; Miyamoto & Yoshii 1995; Walker 1995):
\[ \delta \theta_{c,\text{lum}} = \frac{u \cdot \theta_s}{1 + f_{LS}} \frac{1 + f_{LS} (u^2 - 3 - u \sqrt{u^2 + 4})}{u^2 + 2 + f_{LS} \sqrt{u^2 + 4}}. \]
This also changes the reference point to the center of light of the lens and the unlensed source.
For large impact parameters \( u \gg \sqrt{2} \), this simplifies to (Dominik & Sahu 2000):
\[ \delta \theta_{c,\text{lum}} \simeq \frac{\delta \theta_s}{1 + f_{LS}}. \]
Please note that \( \delta \theta_s \) and \( \delta \theta_{c,\text{lum}} \) have different reference points, so only the amount of the positional shifts is connected via Equation (9).
For \( u \gg 5 \), the center of light is dominated by the major image. Therefore, the shift of the major images can be approximated by the shift of the center of light:
\[ \delta \theta_s \simeq \delta \theta_s = \frac{\theta_s}{u} \theta \sim \frac{M_L (\varpi_L - \varpi_S)}{\theta}, \]
where \( \theta = |\theta| \). This direct dependency on the mass shows the strength of astrometric microlensing. Further, due to the weaker dependence on the impact parameter, which is \( 1/u \) rather than \( 1/u^2 \) for photometric microlensing, astrometric microlensing can be observed at larger impact parameters, which also results in longer timescales (Miralda-Escude 1996; Paczyński 1996). It can be described by (Honma 2001)
\[ t_{\text{land}} = 2 \cdot t_e \left( \frac{\theta_s}{\theta_{\text{min}}} \right)^2 - u_{\text{min}}^2, \]
where \( \theta_{\text{min}} \) is the astrometric precision threshold of the used instrument. This assumes a rectilinear lens-source trajectory, and \( \theta_s / \theta_{\text{min}} \gg 5 \), which means we can observe the effect in a regime where Equation (10) is valid. With high-precision instruments, this can be on the order of \( \theta_{\text{min}} = 0.1 \) mas, which may lead to timescales of many months or even a few years. Further, the weaker \( u \) dependence also allows for more confident predictions of astrometric-microlensing events.
On the other hand, the long timescales can lead to small variation within a given time, which can be hard to measure. To indicate how fast the position changes due to microlensing, we estimate two timescales, \( t_{0.1\text{mas}} \) and \( t_{50\%} \). These indicate how long it takes until \( \delta \theta \) change by \( 0.1 \) mas (i.e., \( |\delta \theta| \cdot (\text{CA}) - \delta \theta_s (\text{CA} - t_{0.1\text{mas}}) = 0.1 \) mas, where \( \text{CA} \) is the epoch of the closest approach) and \( 0.5 \cdot \delta \theta_s \) from the maximum deflection, respectively. We note that \( t_{50\%} \) is longer than \( t_{0.1\text{mas}} \) for events with a shift larger than \( 0.2 \) mas and vice versa if the maximum shift is smaller than \( 0.2 \) mas. Figure 1 shows the positional shift due to astrometric microlensing. \( t_{\text{land}} \) corresponds to the duration outside of the dashed circle \( (\delta \theta_s > 0.1 \) mas, orange and green part) and \( t_{0.1\text{mas}} \) corresponds to the duration between entering the solid circle \( (0.1 \) mas around the point of the maximum positional shift) and reaching the maximum positional shift (green part).
We estimates those timescales numerically using a 1 week grid. If we encounter a duration below 2 weeks, we switch to a 1 day grid. Later, this typically effects events with very high proper motions compared to their impact parameters squared \( (\mu_{\text{el}}/u_{\text{min}}^2) \gg 5 \) mas yr\(^{-1} \).

3. Prediction of Microlensing Events

In principle, our search follows the method we described in K18b or in Proft et al. (2011). However, we made major adjustments in the selection of potential BGS and high-proper-motion stars (HPMSs). The selection process is shown in Figure 2.

3.1. List of High-proper-motion Stars

We started with the selection of HPMSs as potential lenses. Compared with our previous study, we extended our search to HPMSs with absolute proper motions \( \mu_{\text{tot}} = \sqrt{\mu_\alpha^2 + \mu_\delta^2} > 100 \) mas yr\(^{-1} \). The limit of \( 100 \) mas yr\(^{-1} \) still allows a clear separation between background stars and co-moving stars (see Figure 3).
In Gaia eDR3, about 500,000 sources fulfill this criterion. As mentioned by Fabricius et al. (2020) and Lindegren et al. (2020), the Gaia eDR3 contains a small fraction of erroneous data. For example, about 1000 HPMSs show a significantly negative parallax. To exclude such sources, and to ensure a good astrometric solution in Gaia eDR3, we applied the following quality cuts. We first excluded all HPMSs with a renormalized unit weight error \( (\Sigma_{\text{ruwe}}) \) above 2. This deleted about 33,000 HPMSs. Further, we excluded HPMSs with a significance of the parallax \( (\varpi/\sigma_{\varpi}) \) less than 5. About 30,000 do not pass this criterion. While the \( \Sigma_{\text{ruwe}} \) limit excludes mostly bright HPMSs \((G < 18 \) mag\)), faint HPMSs \((G > 18 \) mag\)) are mostly excluded by the parallax criterion. Finally, we excluded 413 sources, since Gaia eDR3 does not provide a \( G \) magnitude for them.
In order to validate our selection, we cross-matched our list of HPMSs with the Gaia Catalogue of Nearby Stars (GCNS; Gaia Collaboration et al. 2020b). We found \( \sim 144,000 \) common stars in the GCNS proper and \( \sim 50,000 \) common stars in the GCNS rejected catalog. The latter are almost exclusively rejected due to parallaxes between 8 and 10 mas. Only 95 common sources are rejected due to a low GCNS probability.
In K18b we also found a suspicious data cluster in the number of photometric observations \( (n_{\text{obs}}) \)–\( G \)-flux significance \( (G_{\text{flux}}/G_{\text{gau}}) \) space. This is also true for most of the 95 HPMSs rejected by the GCNS (see Figure 4). Consequently, we excluded \( \sim 9500 \) HPMSs with
\[ G_{\text{flux}}/G_{\text{gau}} \cdot n_{\text{obs}}^{1.5} < 3 \times 10^5. \]
This limit is indicated by the line in Figure 4.
Using Gaia DR2, in K18b we had further found two populations of erroneous data with typical tangential velocities of \( (v_{\text{tan}} \approx 4 \) km s\(^{-1} \)) and \( (v_{\text{tan}} \approx 11 \) km s\(^{-1} \)), respectively. In Gaia eDR3, we could not detect these, nor any other unexpected population (see Figure 3). Consequently, we did not apply a filter on the tangential velocity. About 440,000 HPMSs pass all five criteria. For these, we searched for BGSs close to their expected paths on the sky.

\[6\] The source code for this analysis is publicly available at https://github.com/klutter/amilensing.
Figure 1. Geometric illustration for the different timescales. The wobbling line shows the position of the major image with respect to the unlensed position of the source at the same epoch. During the event, the major images move along this line in a clockwise or counterclockwise direction (clockwise for the shown event). The wobbling motion is caused by the parallactic motion of the lens. The black dashed and solid circles are centered on the origin, and the position of the major image at the epoch of the closest approach. Both have a radius of 0.1 mas. \( t_{\text{max}} \) describes the duration where the major image is outside of the dashed circle (orange + green, part of the line). \( t_{0.1 \text{mas}} \) describes the duration between entering the solid circle and reaching the center (green part of the line). Similar to \( t_{0.1 \text{mas}} \), \( t_{0.5 \text{mas}} \) describes how long it would take if the circle had a radius of \( 0.5\theta_{\text{e}} \).

Figure 2. Illustration of the selection process. The search for astrometric-microlensing events is divided into four major parts. The selection of good HPMSs (blue), the selection of good background stars (green), the exclusion of co-moving stars (red), and the determination of the closest approach and estimation of the expected microlensing effect (black). The search for close-by BGSs is performed on the sample of \( \sim 500,000 \) HPMSs, and the selections of good HPMSs and BGSs are performed independently from each other. The illustration lists the different selection criteria, showing the number of excluded HPMSs, BGSs, candidates, or events. In each process, stars can be excluded due to multiple criteria. The small rectangular boxes indicate locations where the major computation is performed.
Rybicki et al. (2022) determined a classifier in order to indicate sources with spurious astrometric data in Gaia eDR3. Using a machine-learning algorithm, they determined a fidelity value between 0 and 1 for each source with a five-parameter solution in Gaia eDR3. A value of 1.0 means a perfectly trustable solution, and the lowest possible value of 0.0 indicates a lot of issues in the astrometric solution. Further, they proposed a cut at 0.5 to differentiate between “good” and

Figure 3. Proper motions ($\mu_{\text{tot}}$) and parallaxes ($\pi$) for all used HPMSs (blue), excluded HPMSs (red), and BGSs (green). The high number of HPMSs excluded with small parallaxes is due to the selection of HPMSs with $\pi/\sigma_\pi > 5$. The BGSs and HPMSs show a clear separation, the few BGSs $\mu_{\text{tot}} > 100$ mas yr$^{-1}$ are passed by HPMSs with even higher proper motions.

Figure 4. Number of photometric observations by Gaia ($n_{\text{obs}}$) and significance of the $G$ flux ($G_{\text{flux}}/\sigma_{G_{\text{flux}}}$) for all potential HPMSs. The black squares indicate the HPMSs rejected in the GCNS. The HPMSs with low fidelity values also cluster in the bottom-left region. The excluded HPMSs are plotted in red, and the HPMSs that passes this criterion are plotted in blue.
spurious sources. Only 126 of all HPMSs that passed our five criteria have a fidelity value below 0.75, and none of those result in a predicted event, even without any cuts on the fidelity value. On the other hand, a large fraction of the excluded HPMSs (97%) have a fidelity value above 0.75. However, to ensure a good prediction, we kept the above-described criteria.

3.2. Potential Background Stars

As in K18b, for each HPMS, we defined a rectangular box on the sky, using the J2010.5 and J2066.0 positions and a half-width of $w = 7$ mas (see Figure 5) to search for suitable BGSs. In the following, the pair of a foreground HPMS and BGS is called a “candidate,” where the BGS parameters are labeled with the prefix “Sou_.”

Among all stars in the boxes, we first excluded BGSs with significantly negative parallaxes ($\text{Sou}_\varpi + 3 \cdot \text{Sou}_{\sigma_{\varpi}} < 0$ mas), $G$ magnitudes fainter than $G = 21.5$, and without $G$ magnitudes in Gaia eDR3. Second, we excluded all BGSs with a ruwe above 2. This criterion can only be applied for BGSs with a five-parameter solution in Gaia eDR3. However, the ruwe strongly correlates with the astrometric goodness-of-fit over the square root of the number of good astrometric microlensing along-scan observations ($\text{Sou}_{\text{GoF}}/\sqrt{\text{Sou}_N}$; see Figure 6). Hence, if a five-parameter solution does not exist in Gaia eDR3, we only considered sources with $\text{Sou}_{\text{GoF}}/\sqrt{\text{Sou}_N} < 1.24$, which is equivalent to $\text{ruwe} < 2$.

In order to avoid physical binary stars as well as co-moving pairs of stars, we apply various filters on the parallax and proper motion. We first consider only candidates with the parallax of the source smaller than 0.9 times the parallax of the lens, i.e.,

$$\text{Sou}_\varpi/\varpi < 0.9. \quad (13)$$

The main purpose of this cut is to avoid imaginary Einstein radii (i.e., the “source star” being closer than the “lens star”), since the proper motion is more effective to distinguish between background and co-moving stars, especially for distant stars ($\varpi < 10$ mas). For the difference in proper motion, we applied the following two criteria:

$$|\text{Sou}_\mu - \mu| > 0.7 \cdot \mu_{\text{tot}} \quad (14)$$

$$\text{Sou}_{\mu_{\text{tot}}} < 0.8 \cdot \mu_{\text{tot}}. \quad (15)$$

The difference in proper motion between HPMSs and BGSs is shown in the left panel of Figure 7.

3.2.1. Displacement between DR2 and eDR3 as Proxy for Missing Proper Motions

These criteria can only be used when a proper motion is given in Gaia eDR3. However, upon visual inspection (originally required due to reasons mentioned in Section 3.2.2), most bright BGSs without a five-parameter solution are within a small angular separation to an even brighter star, which results in a limited performance of Gaia. An example is shown in panel (a) of Figure 8. For many of these supposed BGSs, we observed that the difference between the Gaia DR2 and Gaia eDR3 positions is similar to the difference between the positions of the HPMSs, and that they are therefore co-moving stars. To distinguish between co-moving stars and true candidates, we used this positional offset to determine an approximate proper motion.
Using the DR2-neighborhood catalog (Torra et al. 2020), we searched for matched sources. For $\sim 185,000$ of our BGS, we found at least one entry in this catalog. To ensure a correct match, we applied the following steps. Some sources have multiple entries in this DR2-neighborhood catalog. For those, we only considered the eDR3-DR2 pair with the smallest angular distance. We then selected only matches where the angular distance ($\Delta \phi$) is $\Delta \phi < 400$ mas and the magnitude difference is $|\Delta G| < 0.3$ mag ($\Delta \phi / 1$ mas)$^{0.2}$ (see Figure 9). We also excluded 1061 sources where we found a Gaia DR2 source within an angular distance less than 400 mas, but with a magnitude difference larger than the above limit. Typically, these are brighter in Gaia DR2. We found that for several candidates, a star can be identified at the listed position, but is much fainter than the listed $G$ magnitude in Gaia DR2. Hence the reason for the disagreement might be an incorrect $G$ magnitude in Gaia DR2.

We calculated an estimate of the proper motion from the positional displacement between Gaia eDR3 and Gaia DR2; that is,

$$\text{Sou}_\Delta \phi_{x2} = \left( \frac{\text{Sou}_{-\alpha_{\text{DR3}}}}{\text{Sou}_{\delta_{\text{DR3}}}} - \frac{\text{Sou}_{-\alpha_{\text{DR2}}}}{\text{Sou}_{\delta_{\text{DR2}}}} \right) \cdot \frac{\cos \text{Sou}_{\delta_{\text{DR3}}}}{1} / \Delta t$$

where $\Delta t$ is the difference between the catalog epochs of Gaia eDR3 and Gaia DR2 (0.5 yr; see Figure 10). We also computed

![Figure 6. Goodness-of-fit for Gaia’s astrometry vs. number of good astrometric observations. BGSs with an RUWE less than 2 are plotted in green. These show a sharp edge, which we used to determine a limit for stars that do not have a five-parameter solution. Such sources that pass this criterion are shown in blue, and the excluded BGSs are plotted in red.](image)

![Figure 7. Difference in the proper motion between HPMSs and BGSs. The left panel shows the stars with a proper motion listed in Gaia eDR3, while in the right panel, we used the positional displacement between DR2 and DR3 as an estimate for the proper motion. The excluded candidates are plotted in red. The sparsely scattered candidates with large differences in the right panel are excluded, since the absolute value of the proper motions is similar.](image)
a rough estimate for the parallax as follows:

$$\text{Sou}_{\pi_{\text{approx}}} = 4.74 \text{ km s}^{-1} \text{ mas yr}^{-1} \cdot \frac{|\text{Sou}_{\Delta \phi_{x \gamma}}/1 \text{ yr}|}{v_{\text{tan}}} \text{ mas},$$

(17)

where we use a typical tangential velocity of $v_{\text{tan}} = 75 \text{ km s}^{-1}$.

If the proper motion of the BGS was not given in Gaia eDR3, we used Sou$_{\Delta \phi_{x \gamma}}$ and Sou$_{\pi_{\text{approx}}}$ as estimates for the proper motion and parallax of the BGS, with assumed standard errors of Sou$_{\pi_{\text{approx}}}$ = 5 mas and Sou$_{\pi_{\text{approx}}}$ = 3 mas, respectively. These standard errors roughly reflect the half-width of the distributions of all BGSs with a five-parameter solution, for the corresponding parameter. Finally we applied the same criteria as those we had for BGSs with five-parameter solutions (i.e., Equations (13)–(15)).

For candidates where neither Gaia eDR3 lists a proper motion nor a partner could be found in Gaia DR2, we set the
parallax and proper motion to zero. These cases were also visually inspected. We excluded 52 out of the 279 inspected candidates since we could not identify the BGSs in any Pan-STARRS, DSS2,7 2MASS (Skrutskie et al. 2006),8 or VVV Image. We note that the large majority of candidates were only excluded because the source is blended in all of the considered surveys. We draw this conclusion from the fact that two-thirds are located at a decl. less than $-30^\circ$ and outside of the Pan-STARRS footprint, while only two-fifths of all inspected candidates are below $-30^\circ$, and most of the inspected candidates are too faint to be identified in a DSS2 or 2MASS image (compare the images in the right column of Figure 8). Further, we note that we only inspected candidates that passed all criteria and would lead to a measurable effect.

3.2.2. Selection on the $\Psi$-value

For sources brighter than $G = 18$ mag, McGill et al. (2020) proposed to use

$$\Psi = \frac{\sigma_{5d_{\text{max}}}}{1.2 \text{ mas} \cdot \gamma(G)} < 1$$

(18)

\[\text{Figure 9. Absolute value of the magnitude difference ($\Delta G$) vs. angular separation ($\Delta \phi$) for the cross-match between Gaia eDR3 and DR2, as listed in the DR2-neighborhood catalog. The gray dots in the background show the distribution for a random sample. The green dots ($\Delta \phi > 400$ mas) are interpreted as false matches (these are ignored), while the good matches are shown in blue. The population of stars with $\Delta \phi$ around 100 mas is interpreted as comprising co-moving stars, without a five-parameter solution in Gaia DR2. The red dots indicate sources excluded due to a mismatch in the magnitudes. The orange lines indicate the limits for our classification. We note that including more false matches leads to a stricter selection of candidates.}\]

\[\text{Figure 10. Approximate proper motion from the position difference Gaia DR2 minus eDR3 vs. the actual proper motion given in Gaia eDR3. For most BGSs, these two are in good agreement, especially if a five-parameter solution exists in Gaia DR2 (green dots). The few dozen candidates with large discrepancy are most likely mismatches.}\]
as an additional criterion, to exclude spurious data, where 

\[ \sigma_5 \Delta d_{\text{max}} = \text{the square root of the largest singular value of} \]

the 5 × 5 covariance matrix of the astrometric parameters, and 

\[ \gamma = \max [1, 10^{9.2(G - 18)}]. \]  

(19)

The distribution of \( \Psi \)-values as function of \( G \) magnitude is shown in Figure 11. In our raw sample (i.e., if none of our exclusion criteria are applied), 2071 candidates would not fulfill this criterion. Out of those, 1545 violate either our criterion on the Sou_GoF/\( \sqrt{\text{Sou}_N} \) (1450) or show a bad match between DR2 and DR3 (1468). The comparison with Gaia DR2 has shown that a large fraction of those are most likely physical binary stars, which leads to the selection described above. Further, 98 candidate pairs are excluded due to the displacement between Gaia eDR3 and DR2.

Instead of a strict criterion, we visually inspected the Pan-STARRS, 2MASS, and DSS2 images for 562 of the candidates where the background stars show a \( \Psi \) larger than the 90th percentile of the stars in a random sample, with similar magnitudes (shown as the orange line in the top panel of Figure 11). This leads to the exclusion of 210 candidates. Examples of the visual inspection are shown in Figure 8. The bottom panel of Figure 11 shows the \( \Psi \)-value for our predicted events in blue, and the excluded candidates are plotted as red or magenta squares. We note that for most of the events, a clear falsification was not possible due to the same reasons as mentioned in Section 3.2.1.

### 3.3. Approximate Masses and Einstein Radii

For each of the potential astrometric-microlensing events, we determined the Einstein radii based on an assumed lens mass. We used this to determine an expected displacement between Gaia eDR3 and DR2.

For MSs, we assumed an uncertainty of 10%. Stars with \( G_{\text{abs}} > 15 \) lie in the brown dwarf (BD) regime, where the mass–luminosity function is not valid. For these, we assumed a mass of

\[ M_{\text{BD}} = (0.07 \pm 0.03) M_\odot. \]  

(24)

We note that this is only a rough estimate. For example, it treats subdwarfs like MSs. However, measuring the mass is aim of the observations of the astrometric-microlensing event.

Using Equation (1), we then determined the Einstein radii, using the masses determined above and the Gaia eDR3 parallaxes of lens and source. For the source parallaxes, we also used the prior derived from the DR2-eDR3 displacement, if only a two-parameter solution was provided by Gaia eDR3.

### 3.4. Forecasting the Motion

For about 136,000 candidates, we searched for the closest approach between source and lens. Using elementary geometry, we first determined an approximate minimum angular separation (\( d_{\text{CA,approx}} \)) and approximate epoch of the closest approach (\( T_{\text{CA,approx}} \)).

\[ d_{\text{CA,approx}} = \left[ \frac{\Delta \alpha^* \cdot \Delta \mu_x - \Delta \delta \cdot \Delta \mu_y^*}{\sqrt{\Delta \mu_x^2 + \Delta \mu_y^2}} \right] \]  

(25)

\[ T_{\text{CA,approx}} = \frac{\Delta \alpha^* \cdot \Delta \mu_x + \Delta \delta \cdot \Delta \mu_y^*}{\Delta \mu_x^2 + \Delta \mu_y^2} + 2016.0 \]  

(26)

where \( \Delta \alpha^*, \Delta \delta, \Delta \mu_x^*, \) and \( \Delta \mu_y \) are the differences in positions and proper motions between lens and source in R.A. and decl. Note that this approximation neglects the periodic motion caused by the parallax difference.

For the next step, we only considered the ~16,000 candidates where the predicted shift of the major image was larger than 0.03 mas or where the approximate minimum angular separation was less than two times the parallax difference. For those, we searched for the exact closest approach. We determined the position of lens and source in equatorial Cartesian coordinates as a function of time \( t \) after the Gaia reference epoch \( t_0 = 2016.0 \),

\[ x(t) = \begin{pmatrix} \cos(\delta + \mu_y t) \cdot \cos(\alpha + \mu_x t) \\ \cos(\delta + \mu_y t) \cdot \sin(\alpha + \mu_x t) \\ \sin(\delta + \mu_y t) \end{pmatrix} \times \frac{1000 \text{ mas pc}}{c} + E(t) \]  

(27)

where \( E(t) \) is the geocentric location of the Sun at \( t_0 + t \), in equatorial Cartesian coordinates. We note that \( \mu_y = \mu_x^* / \cos(\delta) \). This is retrieved from the astropy packages (The Astropy Collaboration et al. 2013). Further, we determined the angular separation between lens and source via:

\[ d(t) = 2 \cdot \arcsin \left[ \frac{x(t)}{|x(t)|} - \frac{\text{Sou}_x x(t)}{|\text{Sou}_x x(t)|} \cdot 0.5 \right]. \]  

(28)

We then evaluated this function on a 1 week grid within 2 yr around the previously determined approximate epoch. By comparing consecutive grid points, we detected local minima and then applied a nested-intervals algorithm to determine the precise epoch and minimum distance for each local minimum. For edges \( (t_0, t_3) \) of the starting interval, we use the two adjacent neighbors of the previously found minimum, and split it according to the golden ration at \( t_1 = t_0 + 38.2\% \cdot (t_3 - t_0) \),
Figure 11. $\Psi$ vs. $G$ magnitude. Top: the distribution for all potential BGSs. BGSs that would not pass the criterion defined by McGill et al. (2020) are shown in red. BGSs that had passed all of our selection criteria are plotted in yellow. Only a few dozen of those would not have passed the criterion of McGill et al. (2020). A sample of randomly selected stars is shown in gray, and 90% of those are below the orange line. We note that our sample has slightly larger $\Psi$ values compared to those of the random sample. Bottom: in blue, the BGSs of all predicted events are shown. The ones excluded after the visual inspection are plotted as red ($\delta > -30^\circ$) and magenta ($\delta < -30^\circ$) squares. The excluded candidates below the orange line were inspected since Gaia eDR3 does not provide proper motions, and they were not found in DR2. The gray dots indicate the random sample as for the top panel, with the 90th percentile shown as the orange line.
\[ t_2 = 61.8\% \cdot (t_3 - t_0) \] of the interval length. We then shrink the interval such that, in each step, the epoch for the minimum of \( d(t_1), d(t_2) \) stays in the center of our interval (the new intervals is \( t_0 - t_2 \) or \( t_1 - t_3 \), respectively), until the width is less than \( 3 \times 10^{-10} \text{yr} \approx 10 \text{ms} \), i.e., much smaller than the expected error. Finally, by comparing all detected local minima, we selected the global minimum.

### 3.5. Computing the Expected Astrometric Effects

Using Equations (6)–(8) and Equation (4), we computed the expected astrometric shifts for the major image (\( \delta \theta_{\text{maj}} \)), for the center of light (\( \delta \theta_{\text{c}} \)), and for the center of light including luminous-lens effects (\( \delta \theta_{\text{c,lm}} \)), as well as the expected photometric magnification. We determined the uncertainties of these predictions using a Monte Carlo approach, where we picked 5000 realizations. The results correspond to the median and to the upper and lower errors of the 15.87th–50th percentiles and 84.13th–50th percentiles, respectively. We do not include any co-variances between the different input parameters.

Finally, we only selected predicted events with an expected shift of the major image larger than \( \delta \theta_{\text{maj}} = 0.1 \text{ mas} \) or with an expected magnification larger than \( \Delta m = 1 \text{ mmag} \).

Since several high-precision astrometric instruments like Gaia or James Webb Space Telescope (JWST) are, or will be, located at the Sun–Earth Lagrange point L2, we repeated our calculations with a 1% larger parallax. These results are labeled in the online table with “L2_” as a prefix. However, the epochs and expected separations differ only very little.

### 4. Results: Predicted Astrometric-microlensing Events

We report the prediction of 4842 astrometric-microlensing events based on Gaia eDR3, caused by 3791 distinct HPMSs. The relevant dates on the individual events are listed in the machine-readable version of the table. This can be accessed through the GAVO Data Center,\(^9\) and is also available through the TAP service\(^10\) or through the Virtual Observatory (look for “Astrometric-Microlensing Events Predicted from Gaia eDR3”). A description of all of the columns can be find in Tables 1 and 2.

The table contains updated parameters for 3084 events already predicted from Gaia DR2 (see Section 4.1). For those, we provide updated and more precise information on the separation and epoch of the closest approach. The 1758 newly predicted events are mainly due to the reduced lower limit for the proper motion of the HPMSs (\( \mu_{\text{tot}} > 100 \text{ mas yr}^{-1} \) instead of \( \mu_{\text{tot}} > 150 \text{ mas yr}^{-1} \) in K18b; see Section 4.2). A total of 869 events predicted in K18b could not be rediscovered with Gaia eDR3, mainly due to the stricter quality requirements. For about 200 of those events, Gaia eDR3 led to strong evidence that lens and source are co-moving stars, based either on the proper motion given in Gaia eDR3 or on the positional displacement between Gaia eDR3 and Gaia DR2.

#### 4.1. Updated Results from Gaia DR2

In Klüter et al. (2018a) and K18b, we predicted 3908 astrometric-microlensing events, and Bramich & Nielsen (2018) and Bramich (2018) predicted 79 and 2509 further events, respectively. These partly overlap, so that a total of 5766 events have been predicted from Gaia DR2. In this section we investigate which of the events are found within our new results, and why certain events do not show up within our

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9 German Astrophysical Virtual Observatory, https://dc.g-vo.org/amlensing/q3/q/form.
10 https://dc.g-vo.org/tap
new sample. In the rest of this section, we provide numbers for the sample of the 3908 events that we predicted with Gaia DR2, followed in parentheses by the number for the full sample of 5766 Gaia DR2 predictions.

4.1.1. Match between Gaia DR2 and eDR3

In order to match the source IDs between Gaia eDR3 and Gaia DR2, we again used the DR2-neighborhood catalog provided with Gaia eDR3. If in Gaia eDR3 multiple sources are provided for a single Gaia DR2 source, we only considered those with the smaller angular separation as a genuine match. Source pairs with angular separations below 450 mas and with magnitude differences below 0.5 mag are labeled as good matches, and the others are labeled as bad matches. For each of the HPMSs, we were able to detect a counterpart in Gaia eDR3. Among those, 45 (45) were labeled as bad matches due to large angular separations and three (three) due to a missing $G$ magnitude in Gaia eDR3. For five (six) of the BGSs, the DR2-neighborhood catalog does not list any entry. For three out of the six, the BGS is clearly visible in the Pan-STARRS images, and for one of them, a slight distortion of the point-spread function can be observed in VVV. For the other two, the images were strongly blended by the HPMSs. Additionally, 114 (140) BGSs show a bad match between Gaia DR2 and Gaia eDR3 ($d > 450$ mas; 26 (39), $ΔG > 0.5$ mag; 90 (103); $G_{DR3} = \text{None}$: 18 (31)). In total, for 166 (193) events, it was not possible to find a good match for the BGSs and HPMSs. Further, for 21 (33) of those, the BGSs and HPMSs are matched to the same Gaia eDR3 source. This is also true for two events where both matches seem to be good. Additionally, two BGSs are matched to the same Gaia eDR3 source before the quality cuts. We note that an event could be excluded due to multiple reasons.

4.1.2. Excluded Events

This subsection gives the statistics of the various reasons for excluding some of the predicted events. We note that an event could be excluded due to multiple reasons.

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### Table 1

Description of the Columns in the Machine-readable Table

| Name                  | Unit       | Description                                                                 |
|-----------------------|------------|-----------------------------------------------------------------------------|
| event_id              |            | Unique identifier of this event. This is the decimal representation of the Gaia eDR3 source_id of the lens, and a disambiguator for the lensed object |
| lens_id               |            | Gaia eDR3 source_id of the lens                                              |
| lens_ra               | deg        | Gaia eDR3 R.A. of the lens                                                  |
| lens_dec              | deg        | Gaia eDR3 decl. of the lens                                                  |
| lens_err_ra           | mas        | Error of lens__ra from Gaia eDR3                                             |
| lens_err_dec          | mas        | Error of lens__dec from Gaia eDR3                                            |
| lens_pmra             | mas yr<sup>-1</sup> | Proper motion in R.A. of the lens from Gaia eDR3                            |
| lens_pmdec            | mas yr<sup>-1</sup> | Proper motion in decl. of the lens from Gaia eDR3                           |
| lens_err_pmra         | mas yr<sup>-1</sup> | Error of lens_pmra from Gaia eDR3                                           |
| lens_err_pmdec        | mas yr<sup>-1</sup> | Error of lens_pmdec from Gaia eDR3                                          |
| lens_parallax         | mas        | Parallax of the lens from Gaia eDR3                                         |
| lens_err_parallax     | mas        | Standard error of the parallax the lens from Gaia eDR3                      |
| lens_phot_g_mean_mag  | mag        | Mean magnitude of the lens in the integrated $G$ band from Gaia eDR3        |
| lens_phot_rp_mean_mag | mag        | Mean magnitude of the lens in the integrated RP band from Gaia eDR3         |
| lens_phot_bp_mean_mag | mag        | Mean magnitude of the lens in the integrated BP band from Gaia eDR3         |
| ob_id                 |            | Gaia eDR3 source_id of the lens                                              |
| ob_ra                 | deg        | Gaia eDR3 R.A. of the lens                                                  |
| ob_dec                | deg        | Gaia eDR3 decl. of the lens                                                  |
| ob_err_ra             | mas        | Error of ob__ra from Gaia eDR3                                              |
| ob_err_dec            | mas        | Error of ob__dec from Gaia eDR3                                             |
| ob_pmra               | mas yr<sup>-1</sup> | Proper motion in R.A. of the lens from Gaia eDR3                            |
| ob_pmdec              | mas yr<sup>-1</sup> | Proper motion in decl. of the lens from Gaia eDR3                           |
| ob_err_pmra           | mas yr<sup>-1</sup> | Error of ob_pmra from Gaia eDR3                                           |
| ob_err_pmdec          | mas yr<sup>-1</sup> | Error of ob_pmdec from Gaia eDR3                                          |
| ob_parallax           | mas        | Parallax of the lensed object from Gaia eDR3                               |
| ob_err_parallax       | mas        | Standard error of the parallax the lensed object from Gaia eDR3             |
| ob_phot_g_mean_mag    | mag        | Mean magnitude of the lensed object in the integrated $G$ band from Gaia eDR3 |
| ob_phot_rp_mean_mag   | mag        | Mean magnitude of the lensed object in the integrated RP band from Gaia eDR3 |
| ob_phot_bp_mean_mag   | mag        | Mean magnitude of the lensed object in the integrated BP band from Gaia eDR3 |
| ob_displacement_ra_doubled | mas       | Doubled displacement in R.A. between DR2 and DR3, $\cos(θ)$ applied           |
| ob_displacement_dec_doubled | mas | Doubled displacement in decl. between DR2 and DR3                          |

Note. This part lists all of the important parameters directly taken from Gaia eDR3. It is continued in Table 2, where all of the “from us” derived columns are described. (This table is available in its entirety in machine-readable form.)
Table 2  
Continuation of Table 1

| Name               | Unit        | Description                                                                 |
|--------------------|-------------|-----------------------------------------------------------------------------|
| 32 star_type       |             | Type of the lensing star: WD = White Dwarf, MS = Main Sequence, RG = Red Giant, BD = Brown Dwarf |
| 33 mass             | solMass     | Estimated mass of the lens from Gaia eDR3                                    |
| 34 err_mass         | solMass     | Error in the mass of the lens from Gaia eDR3                                 |
| 35 theta_e          | mas         | Einstein radius of the event                                                 |
| 36 err_theta_e      | mas         | Error in the Einstein radius of the event                                    |
| 37 t_aml            | yr          | Approximate duration of the event (i.e., shift \( \geq 0.1 \) mas)            |
| 38 t_0_1mas         | yr          | Approximate duration on which shift_plus changes by 0.1 mas from its maximum abs(shift_plus_vector(tca)-shift_plus_vector(tca-t_0_1mas)) = 0.1 mas |
| 39 t_50pc           | yr          | Approximate duration on which shift_plus changes by 50% from its maximum abs(shift_plus_vector(tca)-shift_plus_vector(tca-t_50pc)) = 50% * shift_plus |
| 40 tca              | yr          | Estimated time of the closest approach                                        |
| 41 err_tca          | yr          | Error in tca                                                                 |
| 42 dist             | mas         | Estimated distance at closest approach                                        |
| 43 err_dist         | mas         | Error d_min                                                                  |
| 44 u                |             | Estimated distance at closest approach in Einstein radii                     |
| 45 u_error_m        |             | Left 67% confidence interval of u                                             |
| 46 u_error_p        |             | Right 67% confidence interval of u                                            |
| 47 shift             | mas         | Maximal astrometric shift of the center of light                             |
| 48 shift_error_m    | mas         | Left 67% confidence interval of shift                                         |
| 49 shift_error_p    | mas         | Right 67% confidence interval of shift                                        |
| 50 shift_lum        | mas         | Maximal astrometric shift including lens-luminosity effects                  |
| 51 shift_lum_error_p| mas         | Left 67% confidence interval of shift_lum                                    |
| 52 shift_lum_error_m| mas         | Left 67% confidence interval of shift_lum                                    |
| 53 shift_plus        | mas         | Maximal astrometric shift of brighter image                                  |
| 54 shift_plus_error_m| mas         | Left 67% confidence interval of shift_plus                                   |
| 55 shift_plus_error_p| mas         | Right 67% confidence interval of shift_plus                                  |
| 56 magnification    | mag         | Maximal magnification                                                        |
| 57 magnification_error_m| mag   | Left 67% confidence interval of magnification                               |
| 58 magnification_error_p| mag   | Right 67% confidence interval of magnification                              |
| 59 l2_tca           | yr          | Estimated time of the closest approaches seen from Earth–Sun L2 point       |
| 60 l2_err_tca       | yr          | Error in l2_tca                                                             |
| 61 l2_dist          | mas         | Estimated distance at closest approaches seen from Earth–Sun L2 point        |
| 62 l2_err_dist      | mas         | Error l2_d_min                                                              |
| 63 l2_u             |             | Estimated distance at closest approach in Einstein radii as seen from Earth–Sun L2 point |
| 64 l2_u_error_m     |             | Left 67% confidence interval of l2_u                                         |
| 65 l2_u_error_p     |             | Right 67% confidence interval of l2_u                                        |
| 66 l2_shift          | mas         | Maximal astrometric shift of the center of light as seen from Earth–Sun L2 point |
| 67 l2_shift_error_m |             | Left 67% confidence interval of l2_shift                                      |
| 68 l2_shift_error_p |             | Right 67% confidence interval of l2_shift                                    |
| 69 l2_shift_lum      | mas         | Maximal astrometric shift including lens-luminosity effects as seen from Earth–Sun L2 point |
| 70 l2_shift_lum_error_m| mas         | Left 67% confidence interval of l2_shift_lum                                |
| 71 l2_shift_lum_error_p| mas         | Left 67% confidence interval of l2_shift_lum                                |
| 72 l2_shift_plus     | mas         | Maximal astrometric shift of brighter image as seen from Earth–Sun L2 point |
| 73 l2_shift_plus_error_m| mas         | Left 67% confidence interval of l2_shift_plus                               |
| 74 l2_shift_plus_error_p| mas         | Right 67% confidence interval of l2_shift_plus                              |
| 75 l2_magnification  | mag         | Maximal magnifications seen from Earth–Sun L2 point                          |
| 76 l2_magnification_error_m| mag   | Left 67% confidence interval of l2_magnification                            |
| 77 l2_magnification_error_p| mag   | Right 67% confidence interval of l2_magnification                           |

Note. Description of the columns estimated in this analysis.

Out of the 3755 (3990) events described in the previous subsection, 402 (404) events were excluded because the HPMS did not pass all quality criteria. This was mainly caused by an \( \text{ruwe} > 2 \) (396 (398) events). For seven (seven) events, the HPMSs did not pass the \( G_{\text{flux}}/G_{\text{flux}} - n_{\text{obs}} > 3 \times 10^5 \) criterion. We note that Rybizki et al. (2022) determined a fidelity value less than 0.5 only for two of the excluded events. Except for one, which has a value of 0.87, they have a value larger than 0.99 (the fidelity value lies between 1.0 = Good and 0.0 = Bad). Hence the \( \text{ruwe} = 2 \) might be too strict of a limit. Consequently, events that are only excluded due to a high \( \text{ruwe} \) might be real events.

Based on the information of the BGSs, we excluded 283 (285) events. These are mainly due to a high GoF/\( n \) > 1.24 (197 (197) events), which is equivalent to \( \text{ruwe} > 2 \), or due to a strange magnitude difference between DR2 and DR3 (207 (209) events). Additionally, 18 (19) events were excluded due to a significantly negative parallax, and 34 (35) events were excluded due to a standard error in the position larger than 10 mas. Furthermore we found 198 (200) events where the...
motion of the BGSs in Gaia eDR3, or the positional change between Gaia eDR3 and Gaia DR2, indicates that the supposed lens and source star are co-moving stars. Finally, 26 (29) events were excluded after visual inspection of the BGSs with an atypically high $\Psi$-value. In addition, it is possible that some of the events found in Gaia eDR3 but with an expected shift less than 0.1 mas would not have passed the visual inspection.

In the end, 3039 (3264) astrometric-microlensing events passed all of our criteria (and 3352 (3579) when using the fidelity value instead of the $\kappa$ and parallax error).

### 4.1.3. Redetected Events

For 150 (180) of these 3039 (3264) redetected events, the expected effects are below our selection limit of $\delta \theta > 0.1$ mas or $\delta m > 1$ mmag. For eight (20) events, this is caused by a significant difference in the mass of the lens. For the eight events in K18b, this is due to a classification as MS instead of WD. For the other 142 (150) events, the differences between the values determined from Gaia DR2 and eDR3 are not significant ($< 3 \sigma$).

Finally, our result contains 2888 (3083) events previously predicted. The majority of the events, 2818 (2997), have comparable angular separations. Only for 70 (86) events did we find a significant difference ($> 3 \sigma$) between Gaia DR2 and eDR3. These are caused by significant changes within the Gaia data. For the epoch of the closest approach, we also found good agreement for most of the events. However, 181 (193) show offsets between 0.5 and 1.0 yr. Other than the significant changes within the Gaia data, this is due to two local minima with roughly the same minimum separation, resulting in a switch from one to the other as global minimum. This effect is not covered in our error estimation. However, for most of those events, a measurable shift is also expected during the epoch predicted from Gaia DR2. By using Gaia eDR3, the uncertainties for both epoch and position are smaller by roughly a factor of two. Except for 17 (78) events, the expected effects are in a good agreement, too. Significant differences are mainly caused by large differences in the assumed masses—this concerns 10 (56) events. For seven (22) events, this is caused by significant differences in the impact parameters. We do not observe a reduction of the mean errors, since these are dominated by the uncertainties of the mass.

### 4.2. New Events from Gaia eDR3

In addition to the 2888 (3083) previous events, we predict 1756 new astrometric-microlensing events. This is mainly due to the reduction of the lower limit in the HPMS proper motion. However, we also found 563 new events with a proper motion larger than 150 mas yr$^{-1}$. In most of these cases (487 events), Gaia DR2 did not contain the BGSs, and for 76 events, Gaia eDR3 indicates the lens as a binary star, while Gaia DR2 only lists one component.

### 4.3. Properties of the Full Sample

Our full sample of predicted astrometric-microlensing events based on Gaia eDR3 contains 4842 events caused by 3791 distinguished HPMSs. The number of events for the different criteria are summarized in Table 3. As in K18b, these are mainly located toward dense areas in the galactic disk, or toward the Magellanic Clouds. Since astrometric microlensing favors close-by massive stars as lenses, roughly half (2160) of the events have a magnitude difference larger than six. For 2682 events, the magnitude difference is smaller than six, and for 1246 events, the lens star is at most three magnitudes brighter. In the following, we give the corresponding numbers for each of these two subsamples in parentheses, i.e., the total number of events is followed by events with $\Delta G < 6$, and events with $\Delta G < 3$.

Finally, we found 285 events where the BGS is brighter than the lens. We note that bright lenses tend to have either large masses or small distances; both lead to large (angular) Einstein radii. Hence, a measurable shift is also expected at larger angular separations, where the source might be detectable next to a bright star.

Out of the 4842 (2682, 1246) events, 473 (260, 111) have an expected shift larger than 1 mas, and for 532 (301, 135), the expected shift is between 0.5 mas and 1 mas. The 260 events with $\Delta G < 6$ and $\delta \theta > 1$ mas are especially promising targets. However, the impact parameter and observability should also be considered in order to select targets for observation, which can differ between telescopes and instruments.

In the unresolved case, i.e., when only the shift of the center of light can be observed, we found 393 (393, 380) events with an expected effect larger than $\delta \theta_{\text{lim}} > 0.1$ mas. This can only be observed for events with low contrast between HPMSs and BGSs, since the blending by the HPMSs strongly reduces the expected shift. All of the 50 events with an expected shift of the center of light above 0.5 mas have a magnitude difference below $\Delta G = 3$. About 40% of those (147 out of 393) are caused by a WD. We note that the contrast between HPMSs and BGSs, and therefore the expected shift of the center of light, depends on the flux ratio in the selected filters.

The expected shifts and predicted epochs of minimal separation are shown in the top panel of Figure 13, and the number of events per year are shown in the bottom panel. The number of events per year is 94 (52, 24). Until the year $\sim 2030$, the number of predicted events is low. This is caused by Gaia’s limited spatial resolution and performance for close pairs. For the next decade (2021–2031), we found 685 (443, 218) events; these and the events during the Gaia mission are shown in Figure 14. The latter will become especially interesting when Gaia publishes its individual astrometric measurements, planned for the Data Release 4 (expected in the year 2024) and for the final data release after the extended mission (expected about 2029). Within the next decade, the number of events per year will strongly increase with time, where events, which will be caused by faster HPMSs, tend to happen earlier. Within the next decade, we expect 76 (46, 17) events with an expected shift larger than 1 mas.

### 4.3.1. Timescales

Typically, a shift larger than 0.1 mas from the unlensed position is expected over a time range between 1.1 yr (15.87th percentile) and 7.7 yr (84.13th percentile), with a median of 3.2 yr. And it takes typically between 0.037 yr (27 weeks, 15.87th percentile) and 3.6 yr (84.13th percentile) to observe a change in the positional shift by 0.1 mas, with a median of 0.53 yr. Both distributions are shown in Figure 15. For 303 events, $t_{0.1 \text{ mas}}$ is between 5 and 10 yr, and for 209 events, $t_{0.1 \text{ mas}}$ is longer than 10 yr. The expected shifts for those events are below 0.2 mas and 0.15 mas, respectively. Additionally, 39 events do not list a $t_{0.1 \text{ mas}}$ since they show large uncertainties, and the expected shift is below 0.1 mas when the uncertainties
Figure 13. Top: expected maximum shifts for all astrometric-microlensing events with a magnitude difference $\Delta G < 6$ mag. The gray dots indicate the events where the proper motion and parallax of the source are unknown. The blue dots show the events with a five-parameter solution for the BGS, as well as the determined standard errors. On the $x$-axis, the epoch of closest approach $T_{CA}$ is displayed. The apparent paucity of events during the Gaia mission time is due to the angular resolution limit of Gaia eDR3. Bottom: number of predicted events per year. The red bars show the events with $\Delta G < 3$ mag, the red + blue bars show the events with $\Delta G < 6$ mag, and the red + blue + gray bars show all predicted events.

Table 3

| Event Type                          | $\Delta G \leq 3$ | $3 < \Delta G \leq 6$ | $\Delta G > 6$ | Total |
|------------------------------------|-------------------|------------------------|----------------|-------|
| all events                          |                   |                        |                | 4842  |
| $\delta \theta_{+} > 0.1$ mas       | 1246              | 1436                   | 2160           |       |
| $0.5$ mas $> \delta \theta_{+} > 0.1$ mas | 1000              | 1121                   | 1716           |       |
| $1$ mas $> \delta \theta_{+} > 0.5$ mas | 135               | 166                    | 231            |       |
| $\delta \theta_{+} > 1$ mas         | 111               | 149                    | 213            |       |
| luminous lens                       |                   |                        |                | 473   |
| $\delta \theta_{\text{lum}} > 0.1$ mas | 380               | 13                     | 0              |       |
| $0.5$ mas $> \delta \theta_{\text{lum}} > 0.1$ mas | 330               | 13                     | 0              | 343   |
| $1$ mas $> \delta \theta_{\text{lum}} > 0.5$ mas | 0                 | 0                      | 35             |       |
| $\delta \theta_{\text{lum}} > 1$ mas | 15                | 0                      | 0              | 15    |
| events by white dwarfs              |                   |                        |                |       |
| $\delta \theta_{+} > 0.1$ mas       | 409               | 154                    | 62             | 625   |
| $0.5$ mas $> \delta \theta_{+} > 0.1$ mas | 333               | 129                    | 43             | 505   |
| $1$ mas $> \delta \theta_{+} > 0.5$ mas | 41                | 11                     | 8              | 60    |
| $\delta \theta_{+} > 1$ mas         | 35                | 14                     | 11             | 60    |
| photometric events                  |                   |                        |                |       |
| $\delta m > 1$ mmag                | 99                | 37                     | 3              | 139   |
| $10$ mmag $> \delta m > 1$ mmag     | 59                | 28                     | 3              | 90    |
| $100$ mmag $> \delta m > 10$ mmag   | 31                | 9                      | 0              | 40    |
| $\delta m > 100$ mmag              | 9                 | 0                      | 0              | 9     |

Note. For each subset, three different ranges for the expected effects are given, and the first line is the sum of the three lines below. The numbers are also given for different magnitude differences between lens and source, where the last column gives the total number of events.
are not included in the analysis. Since all of the events are included in the Gaia eDR3, events with large $t_{0.1,\text{mas}}$ can benefit from the existence of a precise J2016.0 position. However, those are hard to observe with an accuracy of 0.1 mas anyway. For 263 of the 512 “slow” events ($t_{0.1,\text{mas}} > 5$ yr), a change in the position by 50% · $\delta \theta$ ($\approx 0.05$–0.8 mas) is between 2 yr and 5 yr, and for 221, is between shorter than 2 yr. We found a strong correlation between $\log(t_{0.1,\text{mas}})$ and $\log(\mu^2/\mu_{\text{rel}})$ with a correlation coefficient of 0.98, leading to:

$$t_{0.1,\text{mas}} \simeq 0.11 \text{ yr} \cdot \left( \frac{\mu_{\text{min}}^2}{\mu_{\text{rel}}} \right)^{1.16}. \quad (29)$$

4.3.2. Events Caused by White Dwarfs

For 625 (563, 409) events, the HPMS is classified as WD. In the subsequent decade, 112 (95, 75) events occur, out of which nine (five, four) have an expected angular shift larger than 1 mas. As a median, we find an occurrence rate of 12 (10, 8) events per year with expected shifts larger than 0.1 mas, and about 1 event per year with expected shifts larger than 1 mas, respectively (see Figure 16). These events are of special interest, for two reasons. First, WDs are relatively faint for a given mass, i.e., the magnitude differences between lens and source are much smaller than for MS star-lenses. Hence, they provide ideal targets with large effects and only little blending. This is also shown by the larger fraction of events with smaller magnitude differences ($66\%$ events with $\Delta G < 3$, compared to 25% for the full sample). Further, relations concerning the masses of WDs are currently only poorly known, due to the lack of WDs with directly determined masses. This increases the importance of direct mass measurements for WDs.

Harding et al. (2018) estimates the event rate of astrometric-microlensing events caused by stellar remnants. For Gaia DR2 sources and WDs as lenses, they estimates $1.52 \times 10^{-3}$ events per WD per decade. With about 3800 WDs in our sample of good HPMSs, we would expect about 20 microlensing events.
between 2030 and 2065.5 (where our sample shows a constant behavior). However, for the same time range, we found 146 events caused by WDs, with an expected shift larger than 0.3 mas (the detection limit in Harding et al. 2018). This would lead to an event rate of $1.1 \times 10^{-2}$. The higher event rate is due to our selection of HPMSs only, which are more likely to cause astrometric-microlensing events than the average population of WDs.
4.3.3. Events Caused by Binary Stars

Our sample also contains pairs of events with identical BGSs, but different HPMSs. In total, we found 24 such pairs, caused by 15 binary systems. A visual inspection showed that all of these are true events. For 15 of the 48 events, the magnitude difference is below six. Additionally, for 421 events, the HPMS is part of a binary system, but a measurable effect is not expected for the second component. We note that the orbital motion of the binary system is not included in our analysis, nor accounted for in the error estimation. It is possible that additional events are caused by binary stars that have angular separations larger than 30″ (which we used as the radius for a cross-match), or by binaries that are not resolved in Gaia eDR3, or where the other component does not satisfy our quality criteria. Due to the orbital motion of the binary components, the true epoch and the separation of the closest approach might differ from our predictions. Corresponding corrections can be included in our analysis, once the binary motion is known. For some of the events, this will come with the full Gaia Data Release 3 (i.e., in 2022).

4.3.4. Astrometric-microlensing Events with Photometric Signature

In K18b, we predicted 127 astrometric-microlensing events that might lead to a noticeable photometric signal. Using Gaia eDR3, we redetect 51 of those events. Further, 44 events were redetected, but with an expected magnification below 0.1 mmag. Additionally, we found 88 events with an expected magnification larger than 1 mmag. Thirty-two of those were predicted in K18b but with an expected magnification below 1 mmag. The other newly predicted events are caused by stars with proper motions between 100 and 150 mas yr\(^{-1}\). In total, we found 139 astrometric-microlensing events with potential photometric signatures. However, all predicted magnifications have large uncertainties. These are shown in Figure 17. For 49 (49, 40) events, we expected a magnification larger than 10 mmag, and for nine (nine, nine) events, we expected a magnification even larger than 100 mmag. We do not find any photometric events with high contrast (ΔG > 6), due to the blending of the lens. We note that our estimation only considers blending caused by the lens star, and neglects any blends from additional BGSs.

4.3.5. Astrometric-microlensing Event by Gaia eDR3-4053455379420641152

As an example, we want to show a promising newly predicted astrometric-microlensing event caused by Gaia eDR3-4053455379420641152 (VVV 176144893) in 2025 (hereafter G-2025). G-2025 is located toward the galactic center (R. A. J2016 = 17h38m37.04721; decl. J2016 = -34°27′35″.4946) with a proper motion of (μ\(_a\), μ\(_δ\)) = (-316.1 mas yr\(^{-1}\), -388.6 mas yr\(^{-1}\)), and a parallax of ϖ = 25.5 mas. Gaia provides magnitudes of G = 17.8 mag, G\(_{\text{BP}}\) = 18.2 mag, and G\(_{\text{RP}}\) = 16.9 mag, respectively. Therefore, we classify the star as a WD. For those, we determined an approximated mass of m = 0.65 M\(_{\odot}\). G-2025 is located within the VVV footprint, and VVV lists a brightness of KS = 15.5 mag. A stamp from VVV is shown in Figure 18, where G-2025 is marked by the green square. G-2025 and Gaia eDR3-4053455379465036800 (2MASS J17383723-3427304; VVV 176850871, marked by the black cross in Figure 18). This is also true for the proper motion provided by VIRAC (Smith et al. 2018). The motion can also be seen by comparing VVV images from different epochs. Hence, G-2025 and Gaia eDR3-4053455379465036800 are forming a binary (or multiple stars) system.
In 2025.17 $\pm$ 0.12, it will pass by a G $= 20.25$ mag (Gaia eDR3-405345379420641152; VVV 1765850869) background star with an impact parameter of $d_{\text{min}} = (111 \pm 69)$ mas. The large uncertainties are due to the missing five-parameter solution for the background star in Gaia eDR3. Hence we used the displacement between Gaia eDR3 and DR2 for the proper motion. We note that the so-derived proper motion does not agree with the proper motion provided by VVV. Using the VVV proper motion will lead to a $\sim 10\%$ smaller minimal angular separation and thus to a $\sim 10\%$ larger shift, but with an epoch of the closest approach about 6 months earlier. (We can also find large discrepancies between Gaia and VVV proper motion for other close-by stars.) The fourth Gaia data release might provide proper motions and parallaxes for this source, which will help in the analysis after a successful observation of the event. In Figure 18, the background star is indicated by the red circle. For the background star, VVV lists $J$, $H$, and $K_s$ magnitudes of $J = 15.7$ mag, $H = 14.8$ mag, and $K_s = 14.4$ mag, respectively.

We determined an Einstein radius of $\theta_E = (11.6 \pm 1.5)$ mas. This results in an expected maximum shift of the major image of $\delta \theta_{\pi} = 1.2^{+0.5}_{-0.3}$ mas. If it is not possible to resolve lens and source, only the combined center of light can be observed. For the $G$-band observation, we then expect a shift of $\delta \theta_{\text{sum}} = 0.11^{+0.05}_{-0.03}$ mas. Since in the near-infrared, the source is brighter than the lens, a larger shift of the center of light can be observed. Using the $K_s$ filter, a maximum shift of $\delta \theta_{\text{sum},K_s} \simeq 0.87$ mas is expected (red dotted line in Figure 19). JWST\(^{11}\) will be able to measure this small shift. Both the shift of the major image and the shift of the center of light, as well as the angular separation between lens and source star, are shown in Figure 19 as a function of time. The expected shift for the major image will be above 0.5 mas over a period of about 11 months (about 5.5 yr for $\delta \theta_{\pi} > 0.1$ mas).

We found that G-2025 will pass by further BGSs, including one in early 2028, with an expected impact parameter of $d_{\text{min}} = (1300 \pm 60)$ mas. For this event, we expect a shift of the major image of $\delta \theta_{\pi} = (0.10 \pm 0.03)$ mas, which is only slightly above our selection criterion. Another will occur in 2036, with an expected impact parameter of $d_{\text{min}} = (260 \pm 3)$ mas. This will lead to a shift of the major image of $\delta \theta_{\pi} = 0.52 \pm 0.13$ mas. The BGS for the later event is marked by a cyan hexagon in Figure 18.

5. Summary and Conclusion

From Gaia eDR3, we found 136,000 close passages of background stars by high proper-motion (foreground) stars with proper motions larger than 100 mas per year. We defined a set of quality criteria to avoid spurious entries. Furthermore, we sorted out potentially co-moving star pairs using the cross-match between Gaia DR2 and eDR3. By forecasting the motion of HPMSs and BGSs, we derived the angular separations and epochs of the closest approach for each case. Using approximate masses for the lens stars, we estimated the expected microlensing effects, both astrometric and photometric.

We give these predictions for 4842 events by 3791 distinct HPMSs, where we expect a shift larger than 0.1 mas. Most, 3084, of these events had already been predicted from Gaia DR2. For those, we give updated parameters based on Gaia eDR3. For a majority of the events, we find good agreement for the angular separation and epoch of the closest approach.

\footnote{https://www.jwst.nasa.gov}
Significant differences are mainly due to a significant changes of the proper-motion values between Gaia DR2 and eDR3.

Compared to K18b, we improved the exclusion of spurious sources and the detection of co-moving star pairs, especially for cases in which Gaia eDR3 does not provide a proper motion for the BGS. Due to the higher precision in the proper motions, the uncertainties for the expected epochs and minimum separations decrease roughly by a factor two. However, the uncertainties of the expected shifts stay roughly constant since they are dominated by the errors of the assumed masses. This should be improved for future studies. However, measuring the masses is the ultimate aim of observing the astrometric-microlensing events. For that purpose, the unlensed angular separation is of special interest, since it cannot directly be observed but is essential to determine the deflection. And this parameter does not depend on the assumed mass.

The observation of the predicted events and the subsequent mass determination will lead to a better understanding of stellar masses for isolated stars. Based on the prediction from Gaia DR2, several observing programs started. With the present work, it is possible to further improve the selection of promising events. The 625 events caused by 473 different WDs might be of particular interest. On the one hand, these are easier to observe, due to the lower magnitude difference (caused by the high mass-to-luminosity ratio). On the other hand, such observations will lead to a better understanding of WDs, and thus of the final phase of the evolution of low- to medium-mass stars. It might also be valuable to observe multiple events caused by the same lens star. This can strongly reduce the uncertainties of the mass determination for such cases.

Using the upcoming Gaia data releases will allow to further improve the analysis. Also, the future Gaia releases will provide orbital parameters of binaries. These can strongly improve the selection of co-moving stars and might also be included in the forecast of the paths. Further improvements from future releases will be achieved due to the more precise astrometric parameters and due to the expected increase in the effective angular resolution. Finally, with the upcoming data releases, more detailed knowledge of the spurious sources is expected, which will lead to better selections of BGSs. New event predictions will be mainly added with epochs of closest approach during the Gaia mission or in the near future. These will be of special interest once the individual Gaia measurements are published.

Finally, we showed that the WD Gaia eDR3-4053455379420641152 will cause a promising astrometric-microlensing event in 2025. This event is ideal for a JWST observation, since the background star is brighter in the near-infrared wavelength regime, and a measurable shift of the center of light is expected even if it is not possible to resolve the lens and source stars.

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Software: astropy (The Astropy collaboration et al. 2013), astroquery, (Ginsburg et al. 2019), matplotlib (Hunter 2007), numpy (Harris et al. 2020), TOPCAT (Taylor 2005).

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