Gauge Threshold Corrections for Local String Models

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Local vs Global

There are many different proposals to realise Standard Model in string theory:

- Weakly coupled heterotic string / heterotic M-theory
- M-theory on G2 manifolds
- Intersecting/magnetised brane worlds in IIA/IIB string theory
- Branes at singularities
- F-theory GUTs
Local vs Global

These approaches are usefully classified as either local or global.

Global models:
- Canonical example is weakly coupled heterotic string.
- Model specification requires global consistency conditions.
- Relies on geometry of entire compact space
- Limit $\mathcal{V} \to \infty$ also gives $\alpha_{SM} \to 0$: cannot decouple string
  and Planck scales.
- Other examples: IIA/IIB intersecting brane worlds, M-theory
  on G2 manifolds
Local vs Global

Local models:

- Canonical example: branes at singularity
- Model specification only requires knowledge of local geometry and local tadpole cancellation.
- Full consistency depends on existence of a compact embedding of the local geometry.
- Standard Model gauge and Yukawa couplings remain finite in the limit $\mathcal{V} \to \infty$.

It is possible to have $M_P \gg M_s$ by taking $\mathcal{V} \to \infty$.

- Examples: branes at singularities, local F-theory GUTs.
Local models have various promising features:

- Easier to construct than fully global models.
- Typically have small numbers of families.
- Combine easily with moduli stabilisation, supersymmetry breaking and hierarchy generation (LARGE volume construction)
- Promising recent constructions of local stringy GUTs.
Local vs Global

One of the most phenomenologically important quantities in local models is the bulk volume. This determines

- String scale $M_s = \frac{M_P}{\sqrt{V}}$
- Gravitino mass through the flux superpotential
  \[ m_{3/2} \sim \frac{\langle \int G_3 \wedge \Omega \rangle}{V} \]
- The unification scale in models where gauge couplings naturally unify.

The purpose of this talk is to study this question precisely.
Threshold Corrections

- If gauge coupling unification is non-accidental, it is important to understand the significance of $M_{GUT} \sim 3 \times 10^{16}\text{GeV}$.
- In particular, we want to understand the relationship of $M_{GUT}$ to the string scale $M_s$ and the Planck scale $M_P = 2.4 \times 10^{18}\text{GeV}$.
- Is $M_{GUT}$ an actual scale or a mirage scale?
- I will discuss this first using supergravity arguments and subsequently directly in string theory.
In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

\[ g_{\text{phys}}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \frac{b_a}{16\pi^2} \ln \left( \frac{M_P^2}{\mu^2} \right) + \frac{T(G)}{8\pi^2} \ln g_{\text{phys}}^{-2}(\Phi, \bar{\Phi}, \mu) + \left( \sum_r n_r T_a(r) - T(G) \right) \hat{K}(\Phi, \bar{\Phi}) - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu). \]

(Holomorphic coupling)  
(\(\beta\)-function running)  
(NSVZ term)  
(Kähler-Weyl anomaly)  
(Konishi anomaly)

Relates *measurable* couplings and *holomorphic* couplings.
For local models in IIB

- Kähler potential $\hat{K}$ is given by
  
  $$\hat{K} = -2 \ln V + \ldots$$

- Matter kinetic terms $\hat{Z}$ are given by
  
  $$\hat{Z} = \frac{f(\tau_s)}{V^{2/3}}$$

Why? When we decouple gravity the physical couplings

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\hat{Z}_\alpha \hat{Z}_\beta \hat{Z}_\gamma}}$$

should remain finite and be $V$-independent.
\[
\hat{\mathcal{K}} = -2 \ln \mathcal{V}, \quad \hat{Z} = \frac{f(\tau_s)}{\mathcal{V}^{2/3}}
\]

- Local models require a LARGE bulk volume ($\mathcal{V} \sim 10^4$ for $M_s \sim M_{GUT}$, $\mathcal{V} \sim 10^{15}$ for $M_s \sim 10^{11}$ GeV).

- Kähler and Konishi anomalies are formally one-loop suppressed. However if volume is LARGE, both anomalies are enhanced by $\ln \mathcal{V}$ factors.

- This implies the existence of large anomalous contributions to physical gauge couplings!
Plug in \( \hat{K} = -2 \ln \mathcal{V} \) and \( \hat{Z} = \frac{1}{\mathcal{V}^{2/3}} \) into Kaplunovsky-Louis formula.

We restrict to terms enhanced by \( \ln \mathcal{V} \) and obtain:

\[
g_{phys}^{-2} (\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \frac{\left( \sum_r n_r T_a(r) - 3 T_a(G) \right)}{8\pi^2} \ln \left( \frac{M_P}{\mathcal{V}^{1/3} \mu} \right) \\
= \text{Re}(f_a(\Phi)) + \beta_a \ln \left( \frac{(RM_s)^2}{\mu^2} \right).
\]

- Gauge couplings start running from an effective scale \( RM_s \) rather than \( M_s \).
- Universal \( \text{Re}(f_a(\Phi)) \) implies unification occurs at a super-stringy scale \( RM_s \) rather than \( M_s \).
Argument implies inferred low-energy unification scale is systematically above the string scale.

Argument has only relied on model-independent $\mathcal{V}$ factors - result should hold for any local model (D3 at singularities, IIB GUTs, F-theory GUTs, local M-theory models).

Unification scale is a mirage scale - new string states already occur at $M_s = M_{GUT}/R \ll M_{GUT}$. 

We now want to investigate this directly in string theory.

In string theory gauge couplings are

\[
\frac{1}{g_a^2(\mu)} = \frac{1}{g_{0,a}^2} + \frac{b_a}{16\pi^2} \ln \left( \frac{M_s^2}{\mu^2} \right) + \Delta_a(M, \bar{M})
\]

\(\Delta_a(M, \bar{M})\) are the threshold corrections induced by massive string/KK states.

Study of threshold corrections pioneered by Kaplunovsky and Louis for weakly coupled heterotic string.

For our calculations we use the background field method.
Running gauge couplings are the 1-loop coefficient of

$$\frac{1}{4g^2} \int d^4x \sqrt{g} F^a_{\mu\nu} F^{a,\mu\nu}$$

- Turn on background magnetic field $F_{23} = B$.
- Compute the quantised string spectrum.
- Use the string partition function to compute the 1-loop vacuum energy

$$\Lambda = \Lambda_0 + \frac{1}{2} \left( \frac{B}{2\pi^2} \right)^2 \Lambda_2 + \frac{1}{4!} \left( \frac{B}{2\pi^2} \right)^4 \Lambda_4 + \ldots$$

- From $\Lambda_2$ term we can extract beta function running and threshold corrections.
String theory 1-loop vacuum function given by partition function

\[ \Lambda_{1-loop} = \frac{1}{2} (T + KB + A(B) + MS(B)). \]

- Require \( O(B^2) \) term of this expansion.
- Background magnetic field only shifts moding of open string states.
- Torus and Klein Bottle amplitudes do not couple to open strings.
- Only annulus and Möbius strip amplitudes contribute at \( O(B^2) \).
We want examples of calculable local models with non-zero beta functions.

- The simplest such examples are (fractional) D3 branes at orbifold singularities.
- String can be exactly quantised and all calculations can be performed explicitly.
- Orbifold singularities only involve annulus amplitude further simplifying the computations.
- Have studied D3 branes on $\mathbb{C}^3/\mathbb{Z}_4$, $\mathbb{C}^3/\mathbb{Z}_6$, $\mathbb{C}^3/\mathbb{Z}_6'$, $\mathbb{C}^3/\Delta_{27}$.
- Will focus here on D-branes at $\mathbb{C}^3/\mathbb{Z}_4$ (results all generalise).
The quiver for $\mathbb{C}^3/\mathbb{Z}_4$ is:

Diagram showing the quiver with vertices labeled $n_0$, $n_1$, $n_2$, and $n_3$ connected by arrows indicating the directions and types of quiver relations.

Anomaly cancellation requires $n_0 = n_2$, $n_1 = n_3$. 
Orbifold action generated by $z_i \rightarrow \exp(2\pi i \theta_i)$ with $
abla = (1/4, 1/4, -1/2)$.

We only need to compute the annulus diagram

$$A(B) = \int_0^\infty \frac{dt}{2t} \mathrm{STr} \left( \frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} q^{(p_\mu p_\mu + m^2)/2} \right)$$

Here

$$q = e^{-\pi t}, \quad \mathrm{STr} = \sum_{\text{bosons}} - \sum_{\text{fermions}} = \sum_{\text{NS}} - \sum_{\text{R}}, \quad \alpha' = 1/2$$

$\beta$-function running and threshold corrections are encoded in the $O(B^2)$ term.
We separately evaluate each amplitude in the $\theta^N$ sector.

$$A(B) = \int_0^\infty \frac{dt}{2t} \text{STr} \left( \frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} q^{(p^\mu p_\mu + m^2)/2} \right)$$

- $\theta^0 = (1, 1, 1)$ is an ‘$\mathcal{N} = 4$’ sector.
- $\theta^1 = (1/4, 1/4, -1/2)$ and $\theta^3 = (-1/4, -1/4, 1/2)$ are ‘$\mathcal{N} = 1$’ sectors.
- $\theta^2 = (1/2, 1/2, 0)$ is an ‘$\mathcal{N} = 2$’ sector.

The amplitudes reduce to products of Jacobi $\vartheta$-functions with different prefactors.
\[ A_{untwisted} = \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times 0 = 0. \quad (\mathcal{N} = 4 \text{ susy}) \]

- The untwisted sector has effective $\mathcal{N} = 4$ supersymmetry and cannot contribute to the running gauge coupling.

\[ A_\theta = A_{\theta^3} = \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times \frac{(n_0 - n_2)}{2} (\vartheta - \text{functions}) \]

- The contribution of $\mathcal{N} = 1$ sectors to gauge coupling running has a prefactor $(n_0 - n_2)$.
- This necessarily vanishes once non-abelian anomaly cancellation is imposed.
\[ A_{\theta^2} = \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times (-3n_0 + n_1 + n_2 + n_3) \left( \vartheta - \text{function} \right). \]

Here \( \left( \vartheta - \text{function} \right) \) is

\[
\frac{-1}{4\pi^2} \sum \eta_{\alpha\beta} (-1)^{2\alpha} \vartheta'' \left[ \begin{array}{c} \alpha \\ \beta \end{array} \right] \vartheta \left[ \begin{array}{c} \alpha \\ \beta \end{array} \right] \frac{\vartheta \left[ \begin{array}{c} \alpha \\ \beta + \theta_1 \end{array} \right]}{\eta^3} \frac{\vartheta \left[ \begin{array}{c} \alpha \\ \beta + \theta_2 \end{array} \right]}{\eta^3} = 1.
\]

We obtain

\[ A_{\theta^2} = \int \frac{dt}{2t} \frac{1}{2} \left( \frac{B}{2\pi^2} \right)^2 \times (-3n_0 + n_1 + n_2 + n_3). \]
\[
A_{\theta^2} = \int \frac{dt}{2t^2} \left( \frac{B}{2\pi^2} \right)^2 \times \frac{(-3n_0 + n_1 + n_2 + n_3)}{b_0}.
\]

- Reduction of \( \vartheta \)-functions to a constant is a consequence of \( N = 2 \) supersymmetry.
- Only BPS multiplets can affect gauge coupling running and excited string states are non-BPS.
- Resultant amplitude is non-zero and gives field theory \( \beta \)-function running in both IR and UV limits.
Summary:

- Untwisted sector has $\mathcal{N} = 4$ susy and gives no contribution to running of gauge couplings.

- $\theta$ and $\theta^3$ twisted sectors have $\mathcal{N} = 1$ susy. Contributions vanish when anomaly cancellation is imposed.

- $\mathcal{N} = 2 \theta^2$ sectors gives non-vanishing contribution

$$\left[ \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \right] \times \int_{1/\mu^2}^{1/\infty} \frac{dt}{2t} b_a$$

- How should we interpret this?
\[ \int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a \]

- Divergence in the $t \to \infty$ limit is physical: this is the IR limit and we recover ordinary $\beta$-function running.
- Divergence in $t \to 0$ limit is unphysical: this is the open string UV limit and this amplitude must be finite in a consistent string theory.
- Physical understanding of the divergence is best understood from closed string channel.
Annulus amplitude:

Annulus amplitude in $t \rightarrow 0$ limit:
- $t \to 0$ divergence corresponds to a source for a partially twisted RR 2-form.
- In the local model this propagates into the bulk of the Calabi-Yau.
- Logarithmic divergence is divergence for a 2-dimensional source.
- In a compact model this becomes a physical divergence and tadpole must be cancelled by bulk sink branes/O-planes.
- Tadpole is sourced locally but must be cancelled globally.
The purely local computation omits the following worldsheets:
The purely local string computation includes all open string states for $t > 1/(RM_s)^2$, i.e. $M < RM_s$.

However for $t < 1/(RM_s)^2$ we must include new winding states in the partition function.

These are essential for global consistency but are omitted by a purely local computation.

These enter the computation for $t < 1/(RM_s)^2$ and enforce finiteness (RR tadpole cancellation).
The bulk worldsheets enforce global RR tadpole cancellation and effectively cut off the integral at $t = \frac{1}{(RM_s)^2}$.

Threshold corrections become finite

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a \rightarrow \int_{1/(RM_s)^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a$$

Effective UV cutoff is actually $RM_s$ and not $M_s$.
Result Summary

- For all cases studied string computation reproduces result of supergravity analysis.
- Effective unification scale is $RM_s \gg M_s$.
- In string theory, presence of radius arises from an RR tadpole sourced by the local model but which is cancelled by the bulk.
- In open string channel, model does not ‘know’ its self-consistency until an energy scale $RM_s$. 
Result Summary

- Main result: for local models, both supergravity and string theory imply gauge couplings start running from $RM_s$ and not $M_s$.

- This should hold for all local models: D3 branes at singularities, F-theory GUTs, IIB GUTs...

Note the hypercharge flux in F-theory/IIB GUTs has necessary properties for relevant physics to apply.

- Large effect: for $M_s \sim 10^{12}$ GeV changes $\Lambda_{UV}$ by a factor of 100 and for $M_s \sim 10^{15}$ GeV changes $\Lambda_{UV}$ by a factor of 10.
What should the string scale be?

- $M_s = 10^{11} - 10^{12}\text{GeV}$ is good for moduli stabilisation, the hierarchy problem, TeV supersymmetry and axions. Threshold corrections shift the unification scale to $10^{13} \rightarrow 10^{14}\text{GeV}$.

- If we want unification, then threshold corrections shift the required string scale from $10^{16}\text{GeV}$ to $10^{15}\text{GeV}$. 