Programmable quantum motherboard for logical qubits

N S Perminov\textsuperscript{1,2}, D Y Tarankova\textsuperscript{3} and S A Moiseev\textsuperscript{1,2}

\textsuperscript{1} Kazan Quantum Center, Kazan National Research Technical University n.a. A.N.Tupolev-KAI, 10 K. Marx, Kazan 420111, Russia
\textsuperscript{2} Zavoisky Physical-Technical Institute, Kazan Scientific Center of the Russian Academy of Sciences, 10/7 Sibirsky Tract, Kazan 420209, Russia
\textsuperscript{3} Department of Radio-Electronics and Information-Measuring Technique, Kazan National Research Technical University n.a. A.N.Tupolev-KAI, 10 K. Marx, Kazan 420111, Russia

E-mail: s.a.moiseev@kazanqc.org

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Abstract

We propose a scheme for a programmable quantum motherboard based on a system of three interacting high-Q resonators coupled with two-level atoms. By using algebraic methods, we found that the investigated atomic-resonator platform can possess an equidistant spectrum of eigenfrequencies at which the simple reversible dynamics of single photon excitation becomes possible. It was shown that such a multiresonator quantum motherboard scheme allows us to achieve an efficient and programmable quantum state transfer between distributed atoms and generate logical qubits and qutrits. The use of the studied circuit in quantum processing is proposed.

Keywords: quantum motherboard, logical qubits and qutrits, high-Q resonators, quantum interface, periodic frequency comb, spectrum optimization

(Some figures may appear in colour only in the online journal)

1. Introduction

The creation of an effective quantum motherboard (QMB) with long-lived subsystems is of critical importance for quantum information technologies \cite{1, 2}. In the practical implementation of multiqubit QMBs, a sufficiently strong and reversible interaction of light/microwave qubits is required for many long-lived quantum systems \cite{3}, in particular with NV-centers in diamond \cite{4}, rare-earth ions in inorganic crystals \cite{5} and quantum dots \cite{6}. In this approach, the best realization of the controllable qubit transfer between distant nodes provides quantum efficiency up to 92\% \cite{7, 8}, while practical quantum computers require at least 99.9\%.

A robust solution to high efficiency transfer could be possible with the proper use of the rich features of many-particle dynamics \cite{9–11}. Herein, the basic problem is the construction of multi-particle systems \cite{12} providing the convenient control of time-reversible dynamics. Considerable progress in the construction of high-Q resonators \cite{13–17} with convenient spatial control opens up rich spectral opportunities for the solution of this problem in a system of coupled resonators \cite{18–23}. In particular, a system of resonators with periodic frequencies coupled to the common waveguide \cite{24–26} can extend the dynamical capabilities of linear chains of resonators with the same frequencies \cite{27}. Moreover, such multiresonator schemes demonstrate a significant increase in the operating spectral range of the quantum interface (QI). In these systems, it is possible to considerably enhance the coupling constant of light signals with resonant atoms, and herewith a broadband system of high-Q resonators allows the effects of relaxation and decoherence to be reduced due to the transition to faster storage processes. These properties promise the possibility of obtaining higher QI efficiency and using these systems \cite{28, 29} in decoherence-free quantum processing \cite{2, 30}.

A multiresonator system with atoms and controlled coupling between resonators seems useful as an efficient tool for entanglement generators, quantum memory and interfaces \cite{21, 26}, and can also be applicable for constructing photon–photon multi-qubit gates \cite{31, 32}. In this work, we show that combining a system of high-Q resonators with atoms...
two-level atoms geometrically located inside the resonators. The dashed line is the geometric border of the board.

The principle scheme of a programmable quantum motherboard: large circles—resonators connected by a dynamically controlled coupling $g$, small circles—two-level atoms geometrically located inside the resonators. The dashed line is the geometric border of the board.

The proposed platform consists of three high-Q resonators forming a controllable frequency comb structure and interacting with long-lived resonant atoms (figure 1). A small number of coupled resonators greatly facilitates the spectral-topological optimization of parameters of the QMB [29]. We analytically and numerically study the optimization criterion of the platform to obtain reversible and controlled atomic dynamics. By using the optimization method, we found the optimal values of the freely used QMB scheme parameters and show that super highly efficient interatomic quantum state transport takes place at the equidistant eigenfrequencies of the studied platform. Finally we discuss the obtained results, their possible experimental implementation and potential application.

### 2. Theoretical model

The basic idea of the scheme under consideration is based on the photon echo quantum memory approach, demonstrating reversible dynamics in inhomogeneously broadened quantum systems [33, 34], especially on its variant of the periodic spectral structure of inhomogeneous broadening known as the AFC protocol [35, 36], and its realization in the optimal resonator [37–39]. Moreover, this approach has also been recently extended to the systems of several coupled resonators [24, 25, 28].

We analyse the dynamics of three single mode resonators and resonant two-level systems by using the quantum optics approach [40]. For the case of single-photon excitation considered in this paper, the total quantum state is

$$\Psi(t) = e^{-i\omega_0 t} \left\{ \sum_s s_n(t) |n^{\text{atom}}; 0^{\text{res}}\rangle + \sum_n a_n(t) |0^{\text{atom}}; n^{\text{res}}\rangle \right\}$$

(where $|0^{\text{atom}}\rangle$ and $|0^{\text{res}}\rangle$ are the ground states of atoms and cavity modes, $|n^{\text{atom}}\rangle$ and $|m^{\text{res}}\rangle$ are exited states of the $n$th atom and $m$th cavity mode with the ground state of other quantum systems). In the framework of this approach, we get a system of linear equations for the slowly varied atomic coherences $s_n(t)$ and field mode amplitudes $a_n(t)$:

$$\begin{align*}
[\partial_t - i\Delta] s_1(t) + if_2 s_2(t) &= 0, \\
[\partial_t + i\Delta] s_2(t) + if_1 a_2(t) &= 0, \\
[\partial_t - i\Delta] a_1(t) + if_3 s_1(t) + ig a_2(t) &= 0, \\
[\partial_t + i\Delta] a_2(t) + if_1 s_2(t) + ig a_1(t) + ig a_3(t) &= 0, \\
[\partial_t + i\Delta] a_3(t) + if_3 s_3(t) + ig a_2(t) &= 0,
\end{align*}$$

(1)

where full atomic and field modes are obtained by multiplying slow modes by the factor $e^{-i\omega_0 t}$, $\omega_0$ is a central frequency, $\Delta$ is the frequency detuning of the first and third resonators relative to the second one, $g$ is the interaction constant between the resonators, $\{f_1, f_2\}$ is the interaction constant between atoms and field modes and the resonant frequencies of the atoms are equal to the resonant frequencies of the resonators in which they are located. We also ignored the relaxation terms [40, 41] in the equation (1), focusing only on the search for fast quantum transfer in the QMB scheme with weak decoherence processes in the resonators and atomic qubits.

### 3. Laplace solution and spectra

By assuming that the second atom is initially in the excited state $\langle s_2(t=0) = 1 \rangle$ and applying the Laplace transform of (1), we obtain a system of algebraic equations:

$$\begin{align*}
[p - i\Delta] s_1 &= if_2 a_1, \\
p s_2 &= -f_1 a_2, \\
p + i\Delta] s_3 &= if_3 a_3, \\
p - i\Delta] s_1 + if_2 s_1 + ig a_2 &= 0, \\
p a_2 + if_1 s_2 + ig a_1 + ig a_3 &= 0, \\
p + i\Delta] a_3 + if_3 s_3 + ig a_2 &= 0,
\end{align*}$$

(2)

where for all the field modes the Laplace transform is defined as $u(p) = \int_0^\infty dp \ e^{-\omega_0 t} u(t)$, where $\omega = ip$ is the frequency counted from the central frequency of the radiation $\omega_0$.

We find the solution of (2) for the amplitude $s_2(p)$:

$$s_2 = Det^{-1}(p^4 + 2(\Delta^2 + g^2 + f_1^2) p^2 + \Delta^4 + 2\Delta^2 g^2 - 2\Delta^2 f_2^2 + 2g^2 f_1^2 + f_2^2)),$$

$$Det = p^6 + (\Delta^2 + 2g^2 + f_1^2 + f_2^2) p^4 + (\Delta^4 + 2(g^2 + f_1^2 - f_2^2) \Delta^2 + 2(g^2 + f_1^2) f_2^2 + f_2^4) p^2 + f_1^2 (\Delta - f_2^2)^2 (\Delta + f_2^2)^2,$$

(3)

where Det is the determinant of the linear algebraic equation (2), which determines the eigenvalues $p_n = -i\omega_n$ according to the standard rule $Det(p = p_n) = 0$.

In figure 2 we see the typical dependence of the eigenfrequency distribution of our system versus $g$ for the case when the initial eigenfrequencies (in the absence of interaction) and
the coupling constants of all resonators and atoms are the same ($f_1 = f_2 = 1, \Delta = 0$). We see that the initial spectrum (without interaction), consisting of only one frequency, is split into exactly six different frequencies (three independent frequency pairs) due to the interaction between atoms and resonator modes. For such a situation, all three independent frequencies are controlled by only one free parameter $g$, but this requires at least three free parameters to be used to get full control at three frequencies.

4. Equidistance criterion for eigenfrequencies

The appearance of reversible/periodic dynamics in a closed multiparticle system is only possible if the multiplicity of the eigenfrequencies is determined by the determinant of system (3). One of the cases for implementing the condition of frequency multiplicity is the condition of the presence of an equidistant frequency comb, which has previously been used to implement a highly efficient QI and quantum memory [26, 29]. The equidistance criterion for a six-particle system with eigenfrequencies $\omega_{k,1}, \omega_{k,2}, \omega_{k,3}$ ($\omega_1 \leq \omega_2 \leq \omega_3, \omega_{-n} = -\omega_n$) without degenerate levels is $\omega_1 : \omega_2 : \omega_3 = 1 : 3 : 5$ and corresponds to the fulfilment of the condition $\delta = 0$, where the non-equidistance error

$$\delta = |\omega_2/\omega_1 - 3| + |\omega_3/\omega_1 - 5|. \quad (4)$$

Firstly we analysed the case of resonant interaction: $f_1 = f_2 = f = 1, \Delta = 0$. Using the algebraic solution (3), we did not find any efficient transfer of the initial second atomic excitation to the first and third atoms for the arbitrary value $g$ at any moment of time. We get the spectra of the atom-resonator systems (also called the atomic-photon molecule) depicted in figure 2 and its non-equidistance error (4) shown in figure 3.

5. Algebraic optimization for QMB

To obtain an equidistant frequency comb without degenerate frequencies and well-controlled dynamics, control over all the system parameters is required. Herein we assume the initial difference between the frequencies of the resonators and the atoms located in them. Below, we focus on the particular case of an equidistant frequency comb with a single frequency degeneracy in the middle of it (similar to works [25, 29]). The presence of spectral degeneracy not only simplifies the analysis, but also provides amazing new possibilities for programming the motherboard.

To obtain the equidistant frequency comb for our QMB, it is necessary to use $\Delta \neq 0$ and $f_1 \neq f_2$. At the beginning, we impose the condition for the presence of degeneracy, which is determined from (3) as the equality $\Delta = f_2$ (see the structure of the factors of the last term in the expression for the $Det$). Next, we impose the condition for the presence of an equidistant frequency comb in increments of 1, i.e. $\omega_k = \{-2, -1, 0, 1, 2\}$, which definitely gives us the determinant $Det$ structure in the form:

$$Det_0 = p^2(p^2 + 2^2). \quad (5)$$

Equating the terms with equal powers of $p$ in the expressions for the determinant from (3) and (5) (condition $Det(p) = Det_0(p)$ for all $p$), we obtain a system of algebraic equations for the parameters $g, \Delta, f_1, f_2$, the solution of which is given by the following optimal parameters:

$$\Delta = f_2,$$

$$f_1 = 2^{-\frac{1}{2}} \sqrt{5 - 3g^2 - \sqrt{g^4 - 10g^2 + 9}},$$

$$f_2 = 2^{-\frac{1}{2}} \sqrt{5 - g^2 - \sqrt{g^4 - 10g^2 + 9}}. \quad (6)$$

For such a set of parameters, a complete reversal of the time dynamics will occur at the time moment $t = 2\pi$. Then after time $t = \pi$ at $g \cong 0.755 614 2107$ there will be a complete transfer of energy from the first atom to atoms two and three. For this interesting case ($g \cong 0.755 614 2107$ and the
basis of rapidly computable algebraic feedback. This method gives a significant advantage for the rapidly reconfigurable quantum computing platform, which requires a highly accurate method for physically adjusting the internal parameters according to the observed spectroscopic data. We also note that the possibility of algebraic control for ultra-high-dimensional multiparticle quantum systems was opened along this path [51–53], which will allow the creation of a distributed quantum computer scheme with truly great power in the future.

6. Programmable quantum dynamics

A unique observation for our platform is the fact that the presence of a comb with frequency degeneration in the central zone corresponding to (5) does not fix one degree of freedom \( g \), while maintaining the relationship between \( f_3 \) and \( g \) (see figure 5). This allows the energy dynamics of a QMB to be programmed according to the formula for energy in the second atom \( E(x_2) \):

\[
E_{\pi}(x_2) = \frac{(g^4 - 2g^2 + (1 - g^2)\sqrt{g^4 - 10g^2 + 9})^2}{9},
\]

\[
g(E(x_2)) = 0 \approx 0.7556142107,
\]

\[
g(E(x_2)) = 1/3 \approx 0.4531870484,
\]

which can be used to generate a spatially separated logical qubit \( E(x_2) = 0, E(x_1) = E(x_3) = 1/2 \) and qutrit \( E(x_1) = E(x_2) = E(x_3) = 1/3 \) in one compact platform. When the coupling constant \( g \) is quickly disconnected at the time \( t = \pi \), these logical qubits can be fixed for further use. Fast dynamic control of parameter \( g \) in such systems is possible due to Josephson junctions and is used to generate entangled states and quantum storage [21, 54]. For the important case of complete transfer of energy from the second atom to atoms one and three (case \( E(x_2) = 0 \) in the formula (7)) quantum dynamics near \( t = \pi \) is characterized by a broad plateau (an almost rectangular shape). Due to this temporal behaviour, we can perform efficient noise-free operations in subsystems where there is no energy, and we can also perform efficient operations to transform the generated logical qubits. If we do not dynamically change the parameters of the platform over a period of time \([0; 2\pi]\), we get the quantum state storage regime: \( E_{\pi}(x_2) = 1 \rightarrow E_{\pi}(x_2) = 0 \rightarrow E_{2\pi}(x_2) = 1 \).

It is emphasized that such an effect for coupled many-particle quantum systems is only possible due to the presence of exceptional points determined by zeros by the higher discriminants [48]; without special points, programming a highly efficient reversible transfer (while maintaining the structure of the spectrum) is impossible.

7. Conclusion

In this paper, we show that a multiresonator quantum motherboard scheme allows us to achieve an efficient and programmable quantum state transfer between the spatially distributed subsystems of a multiparticle quantum system. Controlled transfer is possible for quantum systems with equidistant
eigenfrequencies created at a special choice of coupling constants and initial atomic and resonator frequencies. The studied scheme can be implemented in the studied systems of high-quality whispering gallery mode microresonators [13, 55] coupled to quantum systems, NV-centers in diamond [4] and rare-earth ions [5]. Herein, the coherent control of optical atomic coherence can be carried out by additional lasers tuned to the other eigenfrequencies of microresonators.

The predicted dynamics extends the algorithmic capabilities of quantum computation with logical qubits and qudits. The optimization of all the parameters in QMB with three resonators is possible for atomic parameters covering a wide frequency band, providing a fast operation rate. We note that the optimization of multipartite dynamics can be performed analytically in a more general way on the basis of applied methods of algebraic geometry [48–50], in particular, for the application of our QMB scheme in large-scale quantum computation with many spatial channels. The proposed QMB can be used to combine several quantum devices into a single broadband multi-qubit pre-processing block with highly controlled dynamic properties [29]. In the proposed way, it is possible to create a scalable hybrid QMB for a universal computation with many spatial channels. The proposed QMB can be used to combine several quantum devices into a single broadband multi-qubit pre-processing block with highly controlled dynamic properties.

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