Charge Tempered Cosmological Model

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Abstract

The main purpose of this work is to obtain the metric of a Charge Tempered Cosmological Model, a slightly modified Standard Cosmological Model by a small excess of charge density, distributed uniformly in accordance with the Cosmological Principle, the global Coulomb interaction incorporated in this metric. The particularity of this model is that the comoving observer referential where the metric belongs is non inertial, which consequence is that clocks at different position cannot be synchronized. The new metric is constrained to $k = 0$, with dependence on a charge parameter, and related to a modified Friedmann equation, but it is constrained to a positive deceleration parameter and hyperbolic solution. Nevertheless, there are corrections to do, valid just for a long range distances. For example, the red shift has, now, dependences on the gravitational potential together the recessional motion. In any way, this model accepts as well the cosmological constant and its physical counterpart, the dark energy.
I. INTRODUCTION

It is a common sense that, among the two long range interaction we know, gravitational and electromagnetic, only the first, being universal and cumulative, can be responsible for the great scale cosmic dynamic. So, the Standard Friedmann and Robertson-Walker Cosmological Model \([1−3]\) uses as a recipe an universe with an uniform distribution of matter and energy, which the dynamic given solely by the gravitational interaction obeying Einstein equations, except at the primordial Big Bang inflation, where all the interaction is believed to be unified and obeying the Quantum Mechanics laws \([4]\). As a result, we have an evolutionary, actually expanding universe, which must be decelerated due to the gravitational attractive interaction.

However, systematic observations of the recession velocities of type Ia supernova indicated a positive acceleration (or negative deceleration parameter) of the universe expansion\([5−9]\). To take account to these observations, the cosmological constant was reintroduced in the Einstein equations, which gives us then the positively accelerated solutions \([10−12]\). Physically, addition of the cosmological constant is equivalent to add an uniform and constant energy distribution with negative pressure responsible by the positive acceleration. It is known as dark energy, a counterpart of the dark matter used to solve the problem of stelar rotation in galaxies. The nature of the dark energy, as well as the dark matter, is not well understood, while the dark energy must be of the order of 70 per cent of the whole energy content of the universe, the dark matter 25 per cent and the normal matter just 5 per cent.

A natural way to introduce a repulsive acceleration is the Coulomb interaction between charged particles, and there are many attempts to deal with, but actual physics theories are strongly based upon symmetries and conservation laws, and one of the most stringent is the charge conservation. It is generally accepted that the universe as a whole have to be neutral because there is no mechanism for charge production in any actual theories like the Standard Model and other unified theories \([13−15]\). The electromagnetic interaction \([16]\), due to the presence of opposite charges, positive and negative, in perfect balanced amount, as it is believed, shouldn’t be relevant at the great cosmological scale. There are many works to show how much charge asymmetry is admissible to be in agreement with present data \([17−26]\).

Actually, we are not going to be stressed by such considerations because it is reasonable
to think that they depend on the geometrical environment, which is considered as given by the Robertson-Walker metric

$$ds^2 = -d\tau^2 - dt^2 + R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right], \quad (1)$$

the building base of the Standard Cosmological Model. We are using the metric $g^{\mu\nu}$ compatible with the Minkowskian metric $\eta^{\alpha\beta}$ with $\text{diag}(\eta^{\alpha\beta}) = (-1, 1, 1, 1)$.

The first thing we have to do is to conciliate the geometrical environment with the physics content. If the physics content includes charge excess, the geometrical environment should be changed because we are adding global non gravitational interaction in such a way that the comoving referential frame is not a free fall anymore.

Charged gravitational systems, in the context of General Relativity, has been objects of recent studies, specially for isotropic and with rotational symmetric systems as stars and black holes. In such studies, the electromagnetic energy is incorporated to the total energy and momentum tensor, and it contributes to the metric tensor to take account this additional gravitational interaction source. However, Coulomb interaction is not considered at all but for its global effects that cause instability of the system until the great part of the charge excess had got away, as in the charged black hole formation $^{[27-31]}$.

The region exterior to a spherically symmetric system with mass $M$ and charge $Q$ is described by the Reissner-Nordstrom metric,

$$d\tau^2 = \left[ 1 - \frac{2MG}{r} + \frac{Q^2G}{r^2} \right] dt^2 - \left[ 1 - \frac{2MG}{r} + \frac{Q^2G}{r^2} \right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2. \quad (2)$$

In the absence of charge, $Q = 0$, and obviously it remits to the well known Schwarzschild metric,

$$d\tau^2 = \left[ 1 - \frac{2MG}{r} \right] dt^2 - \left[ 1 - \frac{2MG}{r} \right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2. \quad (3)$$

A more generalized, taking into account a rotational symmetry, is the Kerr-Newman metric, for systems with angular momentum.

To make clear the role of the charge content of the Reissner-Nordstrom metric (2), let us go to write the equation of motion of a particle of mass $m$ and charge $q$ placed in a region of influence of this metric at radial distance $r$,

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = f^i_{\text{ext}}, \quad (4)$$
where $f_{ext}^i$ is an external non gravitational force, actually the electrostatic force due to the charge $Q$. In Newtonian approximation, we have

$$m \frac{d^2 x^i}{dt^2} + m \Gamma^i_{00} = F_{ext}^i \quad (5)$$

for

$$\Gamma^i_{00} = -\frac{1}{2} \eta^{ij} \frac{\partial g_{00}}{\partial x^j} = -\frac{1}{2} \frac{\partial g_{00}}{\partial x^i}, \quad (6)$$

where

$$g_{00} = -\left[ 1 - \frac{2MG}{r} + \frac{Q^2G}{r^2} \right] \quad (7)$$

and $\eta^{ij}$ are the spatial components of the Minkowskian metric. As the system is isotropic, the equation of motion reduces to its radial component

$$m \frac{d^2 r}{dt^2} = \frac{m}{2} \frac{\partial g_{00}}{\partial r} + F_{ext}^r. \quad (8)$$

From (2),

$$\frac{1}{2} \frac{\partial g_{00}}{\partial r} = -\frac{MG}{r^2} + \frac{Q^2G}{r^3}, \quad (9)$$

and the equation of motion can be written as

$$m \frac{d^2 r}{dt^2} = -m [M - Q\phi(r)] \frac{G}{r^2} + F_{ext}^r. \quad (10)$$

It is clear the role of the charge component of the metric tensor as an additional gravitational source due to the electrostatic self energy $Q\phi(r)$, where

$$\phi(r) = -\frac{Q}{r} \quad (11)$$

is the electrostatic potential, and the Coulomb interaction force between charges $q$ and $Q$ has to be put by hand. It is so because the charge term in the metric (2) is just related to the additional electromagnetic energy and momentum tensor put together with the matter source term of the Einstein equations.

II. COSMOLOGICAL MODEL

Let us go to consider the Standard Cosmological Model slightly modified by an uniform distribution of a small excess of electric charge, no matter where it is from. In cosmological scale, there is no chance the charges scape to get away. Instead, due to the electrostatic
repulsion, any excess of charge will be distributed uniformly in the whole universe and the Coulomb interaction will act equally at all portion of the universe and certainly it is too much stronger than the gravitational force due to its electromagnetic energy. At this cosmological scale, matter and charge are constrained to move together due to a combined gravitational and electrostatic forces, and therefore, as the universe evolution is given by a scale factor, it is convenient to incorporate this global electrostatic interaction in the structure of the metric. It is the basic idea to build what one are going to name as the Charge Tempered Cosmological Model.

Of course, the charge distribution depends on the nature of a possible charge asymmetry, which can result in a charge distribution proportional to or independent of the matter distribution. For example, if this eventual excess of charges, positive or negative, is due to a proton-electron charge asymmetry, the charge distribution is likely to be proportional to the matter distribution. In the other hand, if it is from an asymmetry between the number of positive and negative charged particles like the matter-antimatter asymmetry, with a final excess of electrons in relation to protons or vice-versa, due to the electrostatic repulsion and the extreme mobility of the particles such as electrons, any excess of charge, positive or negative, will be distributed uniformly, surpassing any matter inhomogeneity and getting a complete annihilation of the electromagnetic field, a scenario we are seeking for.

In an uniform universe, with matter, charge and anything else distributed uniformly, there is no vector field, and scalar fields such as the electrostatic potential must be spatially uniform. In such sense, Maxwell equations are insensitive to an uniform charge distribution. Actually, it is possible to think that the sources of Maxwell equations are related to the fluctuations of the positive or negative charge distribution around a sea of an uniform charge distribution that should be not necessarily neutral. From the electromagnetic point of view, the assumption that the total charge distribution of the universe is null may be so arbitrary as to assume that it is not so.

A. Commoving Referential

The Standard Cosmological Model is built on the basis of the Cosmological Principle, which postulates the homogeneity and isotropy of the universe, geometrically traduced by the Robertson-Walker metric (1), where the coordinates are defined on a commoving referential
frame. It is locally inertial and the time coordinate can be defined to coincide with the proper time,

$$d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu = -g_{\mu\nu}dt^2 = dt^2$$, \hspace{1cm} (12)

which permits the synchronization of all clocks at rest in this commoving referential, and any object at rest will satisfy the free fall equation of motion,

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = \Gamma^i_{tt} = 0$$ . \hspace{1cm} (13)

So, it implies

$$\Gamma^i_{00} = \frac{1}{2}g^{i\mu} \left( \frac{\partial g_{0\mu}}{\partial x^0} + \frac{\partial g_{0\mu}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^\mu} \right) = g^{ij} \frac{\partial g_{j0}}{\partial x^0} = 0$$ \hspace{1cm} (14)

and, since $g^{ij}$ are components of a non singular matrix,

$$\frac{\partial g_{j0}}{\partial x^0} = 0$$, \hspace{1cm} (15)

which is identically satisfied by the Robertson-Walker metric.

The absence of electrostatic field does not mean absence of electrostatic interaction between parts of the system. Actually, each part of the system acts, with a repulsive electrostatic force, on all the rest of the system, and while the total force on it is null, it implies that the system as a whole is under a nonzero pressure favor to increase the whole volume. Our present challenge is to incorporate the global Coulomb interaction into the metric, and we have to be ready to pay a just price for it. In a general case, it is not possible to make the geometrization of the electromagnetic interaction as it is done in gravitational interaction because the Equivalence Principle is valid just for gravitation. However, in an uniform distribution of the charge density in the whole universe, electromagnetic and gravitational forces, in a large cosmological scale, must work together because there is just only one degree of freedom to keep the homogeneity and isotropy of the Universe, and the commoving referential is not any more a free fall locally inertial system. As it acts globally, the Coulomb interaction can be incorporated into the metric as the responsible for the referential acceleration.

Let us consider the equation of motion of a electrically charged particle with charge $q$ and proper mass $m_0$,

$$m_0 \frac{d^2x^\lambda}{d\tau^2} + m_0 \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = qF^\lambda_{\mu} \frac{dx^\mu}{d\tau}$$, \hspace{1cm} (16)

which spatial component is
\[ m_0 \frac{d^2 x^i}{d\tau^2} + m_0 \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = f^i_{\text{ext}}, \tag{17} \]

where \( f^i_{\text{ext}} \) is the Coulomb force that, in the commoving referential, is
\[ f^r_{\text{ext}} = qF^r_\mu U^\mu = qF_0^r U^0. \]

For the relativistic invariants \( p^\mu p_\mu = -m_0^2 c^4 \) and \( g_{\mu\nu} U^\mu U^\nu = -1 \), where \( p^\mu = m_0 U^\mu \), we have, in the commoving referential, \( g_{00} U^0 U^0 = -1 \), so that
\[ U^0 = \frac{1}{\sqrt{-g_{00}}} \quad \text{and} \quad U_0 = \sqrt{-g_{00}} \tag{18} \]
and therefore
\[ p^\mu = m_0 \left( U^0, U^i \right) = \frac{m_0}{\sqrt{-g_{00}}} \left( c, v^i \right) = m \left( c, v \right) = \left( \frac{E}{c}, \mathbf{p} \right). \tag{19} \]

The radial equation of motion of a rest particle turn to be
\[ \Gamma^r_{\mu\nu} \frac{d^2 x^r}{d\tau^2} = \frac{f^r_{\text{ext}}}{m_0} = -\frac{q}{m\sqrt{-g_{00}}} E^r g_{00} U^0 = -\frac{q}{m\sqrt{-g_{00}}} E^r g_{00} \frac{1}{\sqrt{-g_{00}}} = \frac{q}{m} E^r, \tag{20} \]
where we are using the relativistic mass \( m = m_0/\sqrt{-g_{00}} \) and we are considering the isotropy.

So, in a commoving referential, any particle at rest must satisfy the condition
\[ \Gamma^0_{00} U^0 - \frac{f^0_{\text{ext}}}{m_0} = 0 \tag{21} \]
or, considering radial symmetry,
\[ -\frac{\Gamma^r_0}{g_{00}} - \frac{f^r_{\text{ext}}}{m_0} = 0 \tag{22} \]
as well as
\[ -\frac{\Gamma^0_0}{g_{00}} - \frac{f^0_{\text{ext}}}{m_0} = 0. \tag{23} \]

For electrostatic force, due to the asymmetry of the electromagnetic tensor \( F^{\mu\nu} \),
\[ f^0_{\text{ext}} = qF^0_0 \frac{dx^0}{d\tau} = 0 \tag{24} \]
such that we will have \( \Gamma^0_{00} = 0 \).

From the affine connection
\[ \Gamma^\rho_\lambda^\mu = \frac{1}{2} g^{\rho\sigma} \left\{ \frac{\partial g_{\mu\nu}}{\partial x^\lambda} + \frac{\partial g_{\lambda\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\lambda}}{\partial x^\nu} \right\}, \tag{25} \]
considering the isotropy, we obtain, from equations (23) and (24),
\[ \Gamma^0_{00} = g^{r0} \left\{ \frac{\partial g_{0r}}{\partial x^0} - \frac{1}{2} \frac{\partial g_{00}}{\partial r} \right\} + \frac{1}{2} g^{00} \frac{\partial g_{00}}{\partial x^0} = 0. \tag{26} \]
Because it is possible, always, to impose $g_{00}$ be time independent, it results

$$\Gamma_{00}^{0} = g^{r0} \left\{ \frac{\partial g_{0r}}{\partial x^{0}} - \frac{1}{2} \frac{\partial g_{00}}{\partial r} \right\} = 0 . \quad (27)$$

From the spatial equation (22), we have

$$\Gamma_{00}^{i} = g^{r0} \left( \frac{\partial g_{0r}}{\partial x^{0}} - \frac{1}{2} \frac{\partial g_{00}}{\partial r} \right) \neq 0 . \quad (28)$$

Equations (27) and (28) are compatible only if $g^{r0} = 0$. Also, in a symmetric space, non-diagonal components can be eliminated with a redefinition of the coordinates. After these considerations, we can go to construct the metric of our charge tempered universe.

**B. Charge tempered metric**

Let us consider the general form of the metric of an uniform space,

$$ds^2 = -d\tau^2 = g_{00}(r)dt^2 + R^2(t)\tilde{g}_{rr}(r)dr^2 + R^2(t)r^2 \left[ d\theta^2 + \sin^2 \theta d\phi^2 \right] , \quad (29)$$

with the usual construction

$$g_{rr}(r,t) = R^2(t)\tilde{g}_{rr}(r) , \quad (30)$$

where $R(t)$ is the space scale factor that carries the global time dependence of the radial coordinate,

$$r(t) = R(t)r . \quad (31)$$

Now, substitution of the component

$$\Gamma_{00}^{r} = -\frac{1}{2} g^{rr} \frac{\partial g_{00}}{\partial r} \quad (32)$$

of the affine connection in equation (22) leads to the condition

$$\frac{1}{2} g^{rr} \frac{\partial g_{00}}{\partial r} = \frac{f_{ext}}{m_0} = \frac{F}{m} , \quad (33)$$

where $F$ is the radial Coulomb force that acts on any region of the universe.

Let us consider a spherical region with radius $r$ and charge $Q$, where

$$Q = \frac{4\pi}{3} \rho_Q r^3 \quad (34)$$
for an uniform charge density $\rho_Q$. Now, let us take a small volume $\Delta v$ with charge and mass contents, $q = \rho_Q \Delta v$ and $m = \rho \Delta v$, respectively, where $\rho$ is the uniform mass density and the small volume $\Delta v$ is placed on the surface of the spherical region. The Coulomb force acting on $\Delta v$ due to its charge $q$ due to the charge $Q$ is

$$F = \frac{qQ}{r^2} = \frac{4\pi}{3} \rho_Q^2 r \Delta v , \quad (35)$$

corresponding to the force per mass unit

$$\frac{F}{m} = \frac{F}{\rho \Delta v} = \frac{4\pi \rho_Q^2}{3} \rho r . \quad (36)$$

Substituted in (33), it results

$$\frac{1}{2} g_{rr} \frac{\partial g_{00}}{\partial r} = \frac{4\pi \rho_Q^2}{3} \rho r . \quad (37)$$

For a diagonal metric tensor, the orthogonality condition $g^{\mu\nu} g_{\nu\sigma} = g^{\mu}_{\sigma} = \delta_{\mu\sigma}$ implies $g^{\mu\nu} = (g_{\mu\nu})^{-1}$ and, therefore $g^{rr}(r,t) = R^{-2}(t) \tilde{g}^{rr}(r)$, so equation (37) turn to be

$$\frac{1}{R^2 g_{00}} \frac{\partial g_{00}}{\partial r} = \frac{8\pi \rho_Q^2}{3} \rho r . \quad (38)$$

After separation of temporal and spatial dependences, we have

$$\frac{1}{g_{00}} \frac{\partial g_{00}}{\partial r} = \frac{8\pi \rho_Q^2 R^3}{3} \rho = \lambda_Q , \quad (39)$$

where $\lambda_Q$ is the constant of separation of variables. The spatial part, using $\tilde{g}^{rr}(r) = (\tilde{g}_{rr}(r))^{-1}$, becomes

$$\frac{1}{g_{00}} \frac{\partial g_{00}}{\partial r} = \lambda_Q \tilde{g}_{rr} , \quad (40)$$

and the temporal part defines the charge parameter

$$\lambda_Q = \frac{8\pi \rho_Q^2 R^3}{3} \rho . \quad (41)$$

The metric functions $g_{00}(r)$ and $g_{rr}(r,t)$ must be obtained solving the Einstein equations

$$R_{\mu\nu} = -8\pi G S_{\mu\nu} \quad (42)$$

together the constraint equation (40).

In principle, we must to add, explicitly, the electromagnetic energy contribution to the source term of the Einstein equations. But the electromagnetic energy must be uniformly
distributed in such a way that it can be incorporated to the matter density $\rho$, so the energy momentum tensor will be the same perfect fluid of the Standard Model,

$$ T_{\mu\nu} = pg_{\mu\nu} + (p + \rho)U_\mu U_\nu. \quad (43) $$

The source term

$$ S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^\lambda_\lambda = \frac{1}{2}g_{\mu\nu}(\rho - p) + (p + \rho)U_\mu U_\nu \quad (44) $$

has the time-time

$$ S_{00} = -\frac{1}{2}(3p + \rho)g_{00} \quad (45) $$

and the three diagonal space-space

$$ S_{ij} = \frac{1}{2}g_{ij}(\rho - p) \quad (46) $$
as the non zero components.

Spatial homogeneity and isotropy impose the metric in the form (29), its unknown components $g_{00}(r)$ and $g_{rr}(r,t)$ being connected by (40), the energy and matter distribution given by (43) and the constraint (41) for charged matter distribution, all of them then related by Einstein equations (42).

The left side term of the Einstein equations (42) is the second order Ricci tensor

$$ R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}, \quad (47) $$
contraction of the Riemann curvature tensor

$$ R^\lambda_{\mu\nu\kappa} = \frac{\partial \Gamma^\lambda_{\mu\nu}}{\partial x^\kappa} - \frac{\partial \Gamma^\lambda_{\mu\kappa}}{\partial x^\nu} + \Gamma^\eta_{\mu\nu} \Gamma^\lambda_{\kappa\eta} - \Gamma^\eta_{\mu\kappa} \Gamma^\lambda_{\nu\eta}, \quad (48) $$

fully dependent of the space-time geometry and its metric via affine connection (25),

$$ R_{\mu\nu} = \frac{\partial \Gamma^\lambda_{\mu\lambda}}{\partial x^\nu} - \frac{\partial \Gamma^\lambda_{\nu\lambda}}{\partial x^\mu} + \Gamma^\eta_{\mu\lambda} \Gamma^\lambda_{\nu\eta} - \Gamma^\eta_{\nu\lambda} \Gamma^\lambda_{\mu\eta} \quad (49) $$

In particular, the non zero components of the Ricci tensor are

$$ R_{00} = \frac{\partial \Gamma^\lambda_{0\lambda}}{\partial x^0} - \frac{\partial \Gamma^\lambda_{0\lambda}}{\partial x^0} + \Gamma^\eta_{0\lambda} \Gamma^\lambda_{0\eta} - \Gamma^\eta_{0\lambda} \Gamma^\lambda_{0\eta} \quad (50) $$

and

$$ R_{ij} = \frac{\partial \Gamma^\lambda_{i\lambda}}{\partial x^j} - \frac{\partial \Gamma^\lambda_{i\lambda}}{\partial x^j} + \Gamma^\eta_{i\lambda} \Gamma^\lambda_{j\eta} - \Gamma^\eta_{i\lambda} \Gamma^\lambda_{j\eta} \quad (51) $$
Exhaustive and systematic calculations are necessary to obtain, first, all components of the affine connection, and then these Ricci tensor components. Components of the affine connection are given in the appendix, from which we can obtain the Ricci tensor. The time-time component (50) become

\[ R_{00} = \frac{3}{R} \frac{\ddot{R}}{R} + \frac{3}{2R^2} \lambda Q g_{00} + \frac{1}{4R^2} \left( \lambda Q r - \frac{\partial \tilde{g}^{rr}}{\partial r} \right) \lambda Q r \tilde{g}_{rr} g_{00} \]  (52)

which, using equation (?), can be rewritten as

\[ R_{00} = \frac{3}{R} \frac{\ddot{R}}{R} + \frac{3}{2R^2} \lambda Q g_{00} + \frac{1}{4R^2} \frac{1}{\tilde{g}_{rr}} \frac{\partial g_{00}}{\partial r} \frac{\partial}{\partial r} \left[ \ln \left( g_{00} \times \tilde{g}_{rr} \right) \right], \]  (53)

where we are using the auxiliary notation

\[ \frac{\partial R}{\partial t} = \frac{dR}{dt} = \dot{R} \]  (54)

with large upper dot to indicate time derivative to distinguish it from the proper time derivative with normal upper dot

\[ \frac{\partial R}{\partial \tau} = \frac{dR}{d\tau} = \dot{R}. \]  (55)

The space-space component of the Ricci tensor, equation (51), can be decomposed as

\[ R_{ij} = \frac{\partial \Gamma^0_{0i}}{\partial x^j} - \frac{\partial \Gamma^0_{ij}}{\partial x^0} + (\Gamma^0_{i0} \Gamma^0_{j0} + \Gamma^0_{ik} \Gamma^0_{jk} + \Gamma^0_{i0} \Gamma^0_{jk}) - (\Gamma^0_{ij} \Gamma^0_{k0} + \Gamma^0_{ij} \Gamma^0_{0k}) + \tilde{R}_{ij} \]  (56)

where

\[ \tilde{R}_{ij} = \frac{\partial \Gamma^k_{ik}}{\partial x^j} - \frac{\partial \Gamma^k_{ik}}{\partial x^k} + \Gamma^m_{im} \Gamma^m_{jk} - \Gamma^k_{ij} \Gamma^m_{mk}. \]  (57)

The general expression valid for the space-space components is

\[ R_{ij} = \frac{\partial}{\partial r} \left( \frac{1}{2} \lambda Q r \tilde{g}_{rr} \right) \delta_i \delta_j + \frac{\partial}{\partial t} \left( \frac{\ddot{R}}{g_{00}} \right) \tilde{g}_{ij} + \left( \frac{1}{2} \lambda Q r \tilde{g}_{rr}(r) \right)^2 \delta_i \delta_j + \frac{\ddot{R}}{g_{00}} \tilde{g}_{ij} + \]  

\[ + \left( \frac{1}{2} \frac{\partial \tilde{g}^{rr}}{\partial r} \delta_i \delta_j + \frac{\tilde{g}^{rr}}{r} \delta_i \delta_j + \frac{\tilde{g}^{rr}}{r} \delta_i \delta_j \right) \tilde{g}_{ij} \left( \frac{1}{2} \lambda Q r \tilde{g}_{rr} \right) + \tilde{R}_{ij} . \]  (58)

The radial component can be written as

\[ R_{rr} = \left( \frac{\ddot{R} R + 2 \frac{\ddot{R}}{R}^{\ddot{R}}}{g_{00}} \right) \tilde{g}_{rr} + \frac{1}{2} \lambda Q \tilde{g}_{rr} + \frac{1}{4} \lambda Q r \tilde{g}_{rr} \left( \frac{\partial \tilde{g}^{rr}}{\partial r} - \lambda Q r \right) \tilde{g}_{rr} + \tilde{R}_{rr} \]  (59)
or the alternative form

\[ R_{rr} = \left( \frac{RR + 2R^2}{g_{00}} \right) \tilde{g}_{rr} + \frac{1}{2} \lambda_Q \tilde{g}_{rr} - \frac{1}{4} \lambda_Q \frac{\partial}{\partial r} \ln (\tilde{g}_{rr} \times g_{00}) + \tilde{R}_{rr}, \]  

(60)

which contains additional compared with the two angular components. As the three diagonal space-space components of the Einstein equation must have the same form, the additional term of the radial equation can be eliminated by imposing the condition

\[ \lambda_Q r \tilde{g}_{rr} + \tilde{g}_{rr} \frac{\partial \tilde{g}_{rr}}{\partial r} = 0 \iff \frac{\partial}{\partial r} \ln (\tilde{g}_{rr} \times g_{00}) = 0. \]  

(61)

This condition, applied to the time-time component given by equation (52) or (53), leads it to

\[ R_{00} = \frac{3}{\lambda_Q} + \frac{3}{2R^2} \lambda_Q g_{00}, \]  

(62)

and the space-space components, equations (58), assume the general form

\[ R_{ij} = \left( \frac{RR + 2R^2}{g_{00}} \right) \tilde{g}_{ij} + \frac{1}{2} \lambda_Q \tilde{g}_{ij} + \tilde{R}_{ij}. \]  

(63)

The condition (61), left equation, can be simplified,

\[ \lambda_Q r \tilde{g}_{rr} + \tilde{g}_{rr} \frac{\partial \tilde{g}_{rr}}{\partial r} = 0 \Rightarrow \lambda_Q r \tilde{g}_{rr} - \tilde{g}_{rr} \frac{\partial \tilde{g}_{rr}}{\partial r} = 0, \]  

(64)

and rewritten in the simple form

\[ \frac{\partial \tilde{g}_{rr}}{\partial r} - \lambda_Q r = 0, \]  

(65)

its solution given by

\[ \tilde{g}_{rr} = A + \frac{1}{2} \lambda_Q r^2. \]  

(66)

The same condition (61), right equation,

\[ \frac{\partial}{\partial r} \ln (\tilde{g}_{rr} \times g_{00}) = 0 \]  

(67)

implies

\[ g_{00} = \frac{C}{\tilde{g}_{rr}} = C \tilde{g}_{rr} = C \left( A + \frac{1}{2} \lambda_Q r^2 \right). \]  

(68)

We are going to impose \( A = 1 \) and \( C = -1 \) such that, for \( \lambda_Q = 0 \), we will have \( \tilde{g}_{rr} = 1 \) and \( g_{00} = -1 \), as in the Robertson-Walker metric, equation (1), for the case \( k = 0 \). From such conditions, we get

\[ \tilde{g}_{rr} = \frac{1}{1 + \frac{1}{2} \lambda_Q r^2} \]  

(69)
and
\[ g_{00} = \left( 1 + \frac{1}{2} \lambda Q r^2 \right), \quad (70) \]

the metric (29) taking the final form
\[ ds^2 = -d\tau^2 = - \left( 1 + \frac{1}{2} \lambda Q r^2 \right) dt^2 + R^2(t) \left[ \frac{dr^2}{(1 + \frac{1}{2} \lambda Q r^2)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]. \quad (71) \]

The scale factor \( R(t) \) defines the time evolution of the universe, and must be obtained solving the Einstein equations (42). Remember that, to define completely the space-space components of the Ricci tensor (63), we need to obtain the terms \( \tilde{R}_{ij} \) defined in equation (57), which reduces to
\[ \tilde{R}_{ij} = \frac{1}{r} \frac{\partial \tilde{g}^{rr}}{\partial r} \tilde{g}_{ij} = \lambda Q \tilde{g}_{ij}, \quad (72) \]

using the components of affine connection given in the appendix. So, equation (63) becomes
\[ R_{ij} = \left( \frac{RR + 2 \dot{R}}{g_{00}} \right) \tilde{g}_{ij} + \frac{3}{2} \lambda Q \tilde{g}_{ij}. \quad (73) \]

The metric (71) was built imposing just space homogeneity and isotropy as in the case of the Robertson-Walker metric, but it has a very important difference between them. Both are for commoving referential, but the Robertson-Walker was built such that the cosmological time coincides with the proper time of each observer referential and any observer at rest in any place will measure the same time. It is possible because all of them are local free fall referential. On the other hand, in the charge tempered model, the commoving referential is not a free fall referential, but instead it is an accelerated referential, the acceleration due to the Coulomb force, and the metric (71) does not permit the synchronization of clocks at different place. The relation between the time in the commoving referential and the proper time is given by
\[ -d\tau^2 = g_{00} dt^2 \Rightarrow \frac{dt}{d\tau} = \frac{1}{\sqrt{-g_{00}}}. \quad (74) \]

Relations between the time derivatives in relation to these two times are given by
\[ \ddot{R} = \frac{\partial R}{\partial x^0} \frac{dR}{d\tau} = \frac{dR}{d\tau} \frac{d\tau}{dt} = \sqrt{-g_{00}} \frac{dR}{d\tau} = \sqrt{-g_{00}} \dot{R}, \quad (75) \]

and, as the \( g_{00} \) is time independent,
\[ \dddot{R} = \sqrt{-g_{00}} \frac{\ddot{R}}{d\tau} = -g_{00} \frac{d^2 R}{d\tau^2} = -g_{00} \ddot{R}, \quad (76) \]
where
\[ \dot{R} = \frac{dR}{d\tau}, \quad \ddot{R} = \frac{d^2R}{d\tau^2}, \text{ etc..} \] (77)
define proper time derivatives. Using such relations, the time-time and space-space components, (62) and (73) become
\[ R_{00} = -3 \frac{\ddot{R}}{R} g_{00} + \frac{3}{2} \frac{\lambda_Q}{R^2} g_{00} \] (78)
and
\[ R_{ij} = -\left( R\ddot{R} + 2\dot{R}^2 \right) \tilde{g}_{ij} + \frac{3}{2} \lambda_Q \tilde{g}_{ij}, \] (79)
respectively.
From the time-time component of the Einstein equations (42),
\[ R_{00} = -8\pi G S_{00}, \] (80)
the Ricci tensor component (78) and the source term (45), we obtain
\[ \dot{R} \dddot{R} = \frac{1}{2} \lambda_Q - \frac{4\pi}{3} G R^2 (\rho + 3p). \] (81)
From the space-space components of the Einstein equations,
\[ R_{ij} = -8\pi G S_{ij}, \] (82)
which the Ricci tensor and the source components given by (79) and (46), respectively, results
\[ \dot{R} \dddot{R} + 2\dot{R}^2 = \frac{3}{2} \lambda_Q + 4\pi G R^2 (\rho - p) \] (83)
which, combined with the acceleration equation (81) results the velocity equation
\[ H^2 = \frac{\dot{R}^2}{R^2} = \frac{1}{2} \frac{\lambda_Q}{R^2} + \frac{8\pi}{3} G \rho, \] (84)
the charge tempered model version of the Friedmann equation. Apart from the algebraic relations of the equations (81), (83) and (84), there is a differential constraint, also, in such a way that the Friedmann equation (84) derived once in relation to the time variable must leads back to equation (81). It implies the differential relation
\[ \frac{\partial}{\partial R} \left( \rho R^3 \right) = -3pR^2 + \frac{3\lambda_Q}{8\pi G} \] (85)
or, equivalently,
\[
\frac{\partial (\rho R^2)}{\partial R} = -(3p + \rho) R + \frac{3\lambda Q}{8\pi G R}.
\]  
(86)

Considering that the charge parameter (41) is consistent with \( \rho \propto R^{-3} \) dependence, that is,
\[
\frac{\partial}{\partial R} (\rho R^3) = 0 \iff \rho = \frac{\rho_0}{R^3} \]  
(87)

one has the pressure equation
\[
3pR^2 = \frac{3\lambda Q}{8\pi G} \iff p = \frac{\lambda Q}{8\pi G R^2}.
\]  
(88)

This last equality, substituted in (81), implies a negative defined acceleration,
\[
\ddot{R} = -\frac{4\pi}{3} G R^2 \rho < 0,
\]  
(89)

and a positive defined deceleration parameter,
\[
q = -\frac{\ddot{R}}{\dot{R}^2} = \frac{4\pi}{2\lambda Q} + \frac{8\pi}{3 G \rho R^2} > 0.
\]  
(90)

To have a positive acceleration (negative deceleration parameter) it is necessary positive energy \((k = -1)\), not allowed for a charged model, which demands \( k = 0 \).

C. Solution of Friedmann Equation

Let us consider the Friedmann equation (84) as differential equation,
\[
\left( \frac{dR}{d\tau} \right)^2 = \frac{8\pi G \rho_0}{3R} + \frac{\lambda Q}{2},
\]  
(91)

where the charge parameter
\[
\lambda_Q = 2H_0^2 (1 - \Omega_0)
\]  
(92)

is given in terms of the Hubble constant and the density parameter \( \Omega = \rho/\rho_c \) evaluated at present time, \( H_0 \) and \( \Omega_0 = \rho_0/\rho_{c,0} \), respectively, where the mass density critical value is defined as usual,
\[
H^2 = \frac{8\pi G}{3} \rho_c.
\]

Using auxiliary variables
\[
D = \frac{16\pi G \rho_0}{3\lambda_Q} = \frac{\Omega_0}{(1 - \Omega_0)}
\]  
(93)
and
\[ dt' = \sqrt{\frac{\lambda Q}{2}} d\tau = H_0 \sqrt{(1 - \Omega_0)} d\tau , \]
(94)

Friedmann equation assumes the simple form
\[ \left( \frac{dR}{dt'} \right)^2 = \frac{D}{R} + 1 , \]
(95)

which parametric solution given by
\[ R = \frac{\Omega_0}{(1 - \Omega_0)} \sinh^2 \psi = \frac{\Omega_0}{2 (1 - \Omega_0)} (\cosh 2\psi - 1) \]
(96)

and
\[ t = \frac{1}{H_0} \frac{\Omega_0}{2 (1 - \Omega_0)^{3/2}} (\sinh 2\psi - 2\psi) . \]
(97)

Notice that in this charged environment, while \( k = 0 \), \( \Omega_0 \neq 1 \), with a charge dependence on the mass density related by
\[ \rho_0 = \Omega_0 \rho_{c,0} = \rho_{c,0} - \frac{3}{16 \pi G} \lambda Q . \]
(98)

For a present matter density (forgetting for a moment the dark energy and dark matter densities) \( \rho_0 \approx 0.03 \rho_{c,0} \), using \( G = 6.67259 \times 10^{-8} \text{cm}^3/(\text{g.s}^2) \), \( \rho_{c,0} \approx 1.88 \times 10^{-29} \text{g/cm}^3 \) and \( e = 4.8032 \times 10^{-10} \text{esu} = 4.8032 \times 10^{-10} \sqrt{\text{gcm}^3}/\text{s} \), one obtain
\[ \lambda Q = \frac{16 \pi G}{3} (\rho_{c,0} - \rho_0) = 2.04 \times 10^{-35} \times 1/s^2 \]
(99)

and
\[ \rho_{Q,0} = 2.5 \times 10^{-24} \frac{e}{\text{cm}^3} \approx \frac{e}{(1000 \text{km})^3} . \]
(100)

An interesting relation can be obtained equations (41) and (92),
\[ \rho_{Q,0}^2 = 2G \rho_{c,0}^2 \Omega_0 (1 - \Omega_0) , \]
imposing an upper limit to the charge density,
\[ \rho_{Q,0} \lesssim \rho_{c,0} \sqrt{G/2} \approx \frac{7 \times e}{(1000 \text{km})^3} , \]
where \( e \) is the absolute value of the electron charge.

Equations (96) and (97) give us the age of the universe
\[ t_0 = \frac{1}{H_0} \frac{\Omega_0}{2 (1 - \Omega_0)^{3/2}} (\sinh 2\psi_0 - 2\psi_0) . \]
(101)
where the parameter $\psi_0$ is obtained imposing the condition $R(\psi_0) = 1$,

$$\sinh \psi_0 = \sqrt{\frac{(1 - \Omega_0)}{\Omega_0}}. \quad (102)$$

It is to note that as $\Omega_0 \to \infty$, the charge distribution is going to vanish, $\lambda Q \to 0$, the equation (97) going to

$$\psi = (1 - \Omega_0)^{1/2} \left(\frac{3}{2}H_0t\right)^{1/3} \quad (103)$$
in such a way that (96) is going to

$$R(t) = \frac{1}{(1 - \Omega_0)^2} \left(\frac{3}{2}H_0t\right)^{2/3}, \quad (104)$$

the usual solution of the Friedmann-Robertson-Walker standard model for the case $k = 0$.

D. Red Shift

A non relativistic Doppler effect due to the recessional kinetic motion is given by

$$\lambda_0 = \lambda_1 \left(1 + \beta_1\right),$$

where $\lambda_0$ and $\lambda_1$ are the observer (at position $r_0$) and the source (at position $r_1$) wave length, respectively, and $\beta_1$ is the source recession velocity. Now, there is a gravitational red shift due to the time dilatation, equation (74), which implies

$$\lambda_0 = \frac{\lambda_1}{\sqrt{-g_{00}(r_1)}},$$

and, to the first order approximation, it is to be combined as

$$\frac{\lambda_0}{\lambda_1} = \frac{1 + \beta_1}{\sqrt{-g_{00}(r_1)}}. \quad (105)$$

On the other hand, from (31), the radial velocity is given by

$$v(t) = \frac{dr}{dt} = R \frac{dr}{d\tau} = \sqrt{-g_{00}(r)} H(t) r(t), \quad (106)$$

which at the present time is going to be

$$v(t_0) = \sqrt{-g_{00}(r)} H_0 \ r(t_0), \quad (107)$$
the Hubble law, now modified by the \( g_{00}(r) \) potential term. This potential term correction, for a charge parameter value given by (99), turn to be relevant only at a distance near about Gpc.

Also, for a light signal travelling from the source at distance \( r_1 \) to the observer, the first wave front emitted at time \( t_1 \) and the next at the time \( t_1 + T_1 \), arriving to the observer at time \( t_0 \) and \( t_0 + T_0 \), respectively, one obtain the relation

\[
\frac{T_1}{T_0} = \frac{\lambda_1}{\lambda_0} = \frac{R(t_1)}{R(t_0)} = R(t_1)
\]  

(108)

such that the red shift parameter is

\[
z_1 = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{1}{R(t_1)} - 1 = \frac{1 + \beta_1}{\sqrt{-g_{00}(r_1)}} - 1,
\]  

(109)

with the inverse relation

\[
R(z_1) = \frac{1}{z_1 + 1} = \frac{\sqrt{-g_{00}(r_1)}}{1 + \beta_1}
\]  

(110)

and the velocity as a function of the parameter of red shift,

\[
\beta_1 = (z_1 + 1) \sqrt{-g_{00}(r_1)} - 1.
\]  

(111a)

This last equation can be combined with equation (106), resulting

\[
z_1 = H_0 \frac{r_1(t_0)}{\sqrt{-g_{00}(r_1)}} - 1 \approx H_0 \frac{r_1(t_0)}{r_1}. 
\]  

(112)

III. CONCLUSION

A charge tempered cosmological model is proposed to describe an universe with a small charge asymmetry, which excess is distributed uniformly in accordance to the Cosmological Principle. A very characteristic of this charged model is a non inertial observer commoving frame of reference, which implies a metric with a potential term carried by the time-time component of the metric tensor. Another important feature is that an unique possibility is for a metric with the curvature parameter \( k = 0 \), in such a way that the amount of any excess of charge is strongly constrained to the Coulomb force does not surpass the gravitational one (which can occur just for \( k = -1 \)). As a consequence, there is not allowed to be responsible of the positive acceleration of the recessional motion of the universe, as shown by the positive defined deceleration parameter.
Reduction of the Einstein equations to a modified Friedmann equation is done, as well as its solution obtained. While $k = 0$, it does not mean that the matter density is equal to the critical one; instead, the small charge parameter can simulate a $k = -1$ condition, as suggested by the hyperbolic solution of the Friedmann equation. As a final remark, positive acceleration can be obtained as usual, introducing the cosmological constant and its physical counterpart, the dark energy. It remains to be necessary, also, to correct the problem of the age of the universe, which seems not affected by the charge asymmetry.

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Appendix

It contains all of the 40 independent components of the affine connection

\[ \Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\sigma\lambda} \left\{ \frac{\partial g_{\nu\sigma}}{\partial x^\mu} + \frac{\partial g_{\mu\sigma}}{\partial x^\nu} - \frac{\partial g_{\nu\mu}}{\partial x^\sigma} \right\} : \]

Components \( \Gamma^0_{\lambda\mu} \):

\[ \Gamma^0_{00} = \frac{1}{2} g^{00} \frac{\partial g_{00}}{\partial x^0} = 0 \]
\[ \Gamma^0_{r0} = \frac{1}{2} g^{00} \frac{\partial g_{00}}{\partial r} = \frac{1}{2} \lambda Q \cdot r \tilde{g}_{rr}(r) \]
\[ \Gamma^0_{r0} = \Gamma^0_{\varphi0} = 0 \]
\[ \Gamma^0_{rr} = -\frac{1}{2} g^{00} \frac{\partial g_{rr}}{\partial x^0} = - \frac{\hat{R} \circ \hat{R}}{g_{00}} g_{00} \]
\[ \Gamma^0_{\theta0} = \Gamma^0_{r\theta} = 0 \]
\[ \Gamma^0_{\varphi0} = -\frac{1}{2} g^{00} \frac{\partial g_{\varphi\varphi}}{\partial x^0} = - \frac{\hat{R} \circ \hat{R}}{g_{00}} g_{\varphi\varphi} \]

Components \( \Gamma^r_{\lambda\mu} \):

\[ \Gamma^r_{00} = -\frac{1}{2} g^{rr} \frac{\partial g_{00}}{\partial r} = - \frac{1}{2 R^2} \tilde{g}^{rr} \lambda Q \cdot r \tilde{g}_{rr} g_{00} = - \frac{1}{2 R^2} \lambda Q \cdot r g_{00} \]
\[ \Gamma^r_{r0} = \frac{1}{2} g^{rr} \frac{\partial g_{rr}}{\partial x^0} = \frac{1}{2 R^2} \tilde{g}^{rr} \frac{\hat{R}}{\hat{R}} \]
\[ \Gamma^r_{r0} = \Gamma^r_{\varphi0} = 0 \]
\[ \Gamma^r_{rr} = \frac{1}{2} g^{rr} \frac{\partial g_{rr}}{\partial r} = -\frac{1}{2} \frac{\partial \tilde{g}^{rr}}{\partial r} \tilde{g}_{rr} \]
\[ \Gamma^r_{\theta0} = \Gamma^r_{r\theta} = 0 \]
\[ \Gamma^r_{\theta\theta} = \frac{1}{2} g^{rr} \frac{\partial g_{\theta\theta}}{\partial r} = -\tilde{g}^{rr} \frac{\hat{g}_{\theta\theta}}{r} \]
\[ \Gamma^r_{\varphi\theta} = 0 \]
\[ \Gamma^r_{\varphi\varphi} = \frac{1}{2} \tilde{g}^{rr} \frac{\partial \hat{g}_{\varphi\varphi}}{\partial r} = - r \tilde{g}^{rr} \sin^2 \theta = - \frac{\tilde{g}^{rr} \tilde{g}_{\varphi\varphi}}{r} \]
Components $\Gamma^\theta_{\lambda\mu}$:

$$
\Gamma^\theta_{00} = -\frac{1}{2} g^{\theta\theta} \frac{\partial g_{00}}{\partial \theta} = 0
$$

$$
\Gamma^\theta_{\theta0} = \frac{1}{2} g^{\theta\theta} \frac{\partial g_{\theta\theta}}{\partial x^0} = \frac{\tilde{g}^{\theta\theta}}{2} \frac{\partial \tilde{g}_{\theta\theta}}{\partial \theta} = \frac{\tilde{R}}{R}
$$

$$
\Gamma^\theta_{r0} = \Gamma^\theta_{\varphi0} = 0
$$

$$
\Gamma^\theta_{rr} = -\frac{1}{2} g^{\theta\theta} \frac{\partial g_{rr}}{\partial \theta} = 0
$$

$$
\Gamma^\theta_{\theta\varphi} = \frac{1}{2} g^{\theta\theta} \left\{ \frac{\partial g_{\theta\varphi}}{\partial \varphi} + \frac{\partial g_{\varphi\theta}}{\partial r} - \frac{\partial g_{r\varphi}}{\partial \theta} = 0 \right\}
$$

$$
\Gamma^\theta_{\theta\theta} = \frac{1}{2} g^{\theta\theta} \frac{\partial g_{\theta\theta}}{\partial \theta} = 0
$$

$$
\Gamma^\theta_{\varphi\varphi} = \frac{1}{2} g^{\theta\theta} \frac{\partial g_{\varphi\varphi}}{\partial \theta} = \frac{\cos \theta \tilde{g}^{\theta\theta} \tilde{g}_{\varphi\varphi}}{\sin \theta} = -\sin \theta \cos \theta
$$

Components $\Gamma^\varphi_{jk}$:

$$
\Gamma^\varphi_{00} = -\frac{1}{2} g^{\varphi\varphi} \frac{\partial g_{00}}{\partial \varphi} = 0
$$

$$
\Gamma^\varphi_{r0} = \Gamma^\varphi_{\theta0} = 0
$$

$$
\Gamma^\varphi_{\varphi0} = \frac{1}{2} g^{\varphi\varphi} \frac{\partial g_{\varphi\varphi}}{\partial x^0} = \frac{\tilde{R}}{R}
$$

$$
\Gamma^\varphi_{rr} = \Gamma^\varphi_{\theta\varphi} = 0
$$

$$
\Gamma^\varphi_{\varphi\varphi} = \frac{1}{2} g^{\varphi\varphi} \frac{\partial g_{\varphi\varphi}}{\partial \varphi} = \frac{\tilde{g}^{\varphi\varphi}}{r} \sin^2 \theta = \frac{\tilde{g}^{\varphi\varphi} \tilde{g}_{\varphi\varphi}}{r}
$$

$$
\Gamma^\varphi_{r\theta} = \Gamma^\varphi_{\theta\varphi} = 0
$$

$$
\Gamma^\varphi_{\theta\varphi} = \frac{1}{2} g^{\varphi\varphi} \frac{\partial g_{\theta\varphi}}{\partial \theta} = \frac{\cos \theta \tilde{g}^{\varphi\varphi} \tilde{g}_{\theta\varphi}}{\sin \theta} = \frac{\cos \theta \tilde{g}^{\varphi\varphi} \tilde{g}_{\theta\varphi}}{\sin \theta}
$$

$$
\Gamma^\varphi_{\varphi\varphi} = \frac{1}{2} g^{\varphi\varphi} \frac{\partial g_{\varphi\varphi}}{\partial \varphi} = 0
$$