Critical state solution of a cable made of curved thin superconducting tapes

Roberto Brambilla\textsuperscript{1} and Francesco Grilli\textsuperscript{2}

\textsuperscript{1} RSE—Ricerca sul Sistema Energetico, Via Rubattino 54, 20134 Milano, Italy
\textsuperscript{2} KIT—Karlsruhe Institute of Technology, Hermann-von-Helmholtz-Platz 1, 76344 Eggenstein-Leopoldshafen, Germany

E-mail: francesco.grilli@kit.edu

Received 4 July 2014, revised 9 September 2014
Accepted for publication 18 September 2014
Published 12 November 2014

Abstract
In this paper, we develop a method based on the critical state for calculating the current and field distributions and AC losses in a cable made of curved thin superconducting tapes. The method also includes the possibility of considering spatial variation of the critical current density, which may be the result of the manufacturing process. For example, rare-earth-based coated conductors are known to have a decrease in transport properties near the edges of the tape: this influences the way the current and field penetrate the sample and, consequently, the AC losses. We demonstrate that curved tapes arranged on a cylindrical former behave as an infinite horizontal stack of straight tapes, and we compare the AC losses in a variety of working conditions, both without and with the lateral dependence of the critical current density. This model and subsequent similar approaches can be of interest for various applications of coated conductors, including power cables and conductor-on-round-core cables.

Keywords: critical state, AC losses, thin superconductors

(Some figures may appear in colour only in the online journal)

1. Introduction
Superconducting tapes are often assembled in cable configurations to form conductors with high current capacity. The performance of such cables strongly depends on the electromagnetic interaction of the tapes and therefore it is important to have simple and fast numerical tools able to evaluate that interaction. In a recent paper [1] we considered the problem of the critical state for superconducting thin tapes forming a polygonal cable. That model is based on the Biot–Savart law for current distributions with angular periodicity and on its transformation into a Cauchy singular integral equation. In regard to the AC losses, we found that two major factors greatly influence their value: (i) the number of tapes (edges of the polygon) and (ii) the lateral gap between the tapes. In this paper, we utilize the same approach for a system of identical curved tapes, which we call, for brevity, an arc-polygonal cable. This configuration describes more accurately the curvature of the tapes in conductor-on-round-core (CORC) cables [2] and in other conductor concepts featuring curved tapes [3, 4].

2. Definition of the problem
Let us consider a set of \( n \) equidistant arches, each of length \( 2a \), positioned on a circumference of radius \( R \) and separated by a (circular) gap \( 2g \), as displayed in figure 1. Each tape carries the same sheet current density \( J(\zeta) \), where \( \zeta \) is the source point moving along the arc \( \Gamma : (-a, a) \) representing the cross section of the conductor. Applying the Biot–Savart law for current distributions with angular periodicity, one can express the magnetic field in complex form as [1]

\[
H(z) = H_y(z) + \text{i}H_x(z) = \frac{1}{2\pi} \int_{\Gamma} J(\zeta) \frac{nz^{n-1}}{z^a - \zeta^a} \text{d}\zeta, \quad (1)
\]

Defining \( L = a + g \), the radius of the supporting circle is \( R = nL/\pi \) and the half angle of the tape sector is \( \alpha = a/R \).
One can rewrite equation (1) as a function of the polar angle posing \( \zeta = Re^{i\theta} \)

\[
H(z) = \frac{n}{2\pi} \int_{-\mathcal{R}}^{\mathcal{R}} J(\theta) \frac{e^{n-1}}{z^n - R^n e^{in\theta}} R d\theta. \tag{2}
\]

If one considers, in particular, a field point on the arc, the integral must be calculated as principal value: defining \( z = R e^{i\theta} \), the magnetic field is

\[
H(\phi) = \frac{n}{2\pi} \text{p.v.} \int_{-\mathcal{R}}^{\mathcal{R}} J(\theta) \frac{e^{n-1} e^{i(n-1)\phi}}{R^n e^{in\theta} - R^n e^{in\theta}} R d\theta
\]

\[
= \frac{n}{2\pi} \text{p.v.} \int_{-\mathcal{R}}^{\mathcal{R}} J(\theta) e^{i(n-1)\phi} e^{in\theta} - e^{in\theta} d\theta. \tag{3}
\]

By using the complex form of the magnetic field, the normal to the arc \( H_n(\phi) = n(\phi) \cdot H(\phi) = \cos \phi H_x(\phi) + \sin \phi H_y(\phi) \) is given by the imaginary part of \( e^{i\phi}H(\phi) \), i.e.,

\[
H_n(\phi) = \frac{n}{2\pi} \text{p.v.} \int_{-\mathcal{R}}^{\mathcal{R}} J(\theta) \left[ \frac{1 - e^{i(n-1)\phi}}{1 - e^{i(n\theta - \phi)}} \right] d\theta
\]

\[
= \frac{n}{2\pi} \text{p.v.} \int_{-\mathcal{R}}^{\mathcal{R}} J(\theta) \frac{1}{2} \cot \left[ \frac{1}{2} n(\theta - \phi) \right] d\theta. \tag{4}
\]

Introducing the arc coordinates \( t = R\theta \) and \( s = R\phi \), by simple substitution, because \( d\theta = dt/R = \pi dt/nL \), we obtain the final expression

\[
H_n(s) = \frac{1}{2L} \text{p.v.} \int_{a}^{a} K(s, t) J(t) dt, \tag{5}
\]

where we have defined the Green function for the arc-polygonal

\[
K(s, t) = \frac{1}{2} \cot \frac{\pi(s - t)}{2L}. \tag{6}
\]

Equation (5) tells us that the field \( H_n \) is the finite Hilbert transform of the current density \( J \).

This expression is independent of \( n \), i.e., the perpendicular field on the arc \((-a, a)\) does not depend on the number of arches composing the cable. Assuming a number \( n \) approaching infinity, one also has that \( R \) approaches infinity, so that the cable becomes an infinite horizontal array (the so-called X-array [5]). Therefore, we can conclude that the distribution of the perpendicular field in a cable made of curved tapes does not depend on \( n \) and it is equal to that of the perpendicular field of an X-array made of the same tapes and gaps. The circumstance \( n = 1 \) relates to the case of a single tape with curvature of radius \( R \), where \( 2g \) is the remaining part of the circumference.

The independence of the results on the number tapes \( n \) had been already derived, although perhaps little emphasized, by Mawatari [6]. He applies a method based on conformal mapping of an exponential type that converts the curved tapes into an infinite X-array. In the transformed complex plane, the magnetic field is then obtained by applying the usual Biot–Savart formula and the results are pulled back to the original cable plane by the inverse transform. In contrast, in our approach, the magnetic field is obtained as an immediate application of the Biot–Savart formula for angular-periodic currents, see equation (1), avoiding the detour of complex transforms, which one can consider as hindered in the Hilbert kernel (6). Straightforward variable substitutions in the integral lead directly to the independence of the results on the number of tapes \( n \). In addition, as it will be shown later, the fact that \( J_t \) may not be uniform, but depend on the position (as a consequence of irregularities and difficulties of the manufacturing process), can be easily implemented in the current approach.

3. Constant \( J_c \) along the whole section of the tape

As a first application of formula (5), we consider the case of superconducting tapes in the critical state characterized by a constant \( J_c \). Because the arc-polygonal case coincides with the X-array case in this paragraph, for the sake of practicality to the reader, we simply report formulas found in the past that use different methods [1, 5, 6]. Suppose \( I_c \) is the critical current for each tape and that the magnetic field penetrates each tape only in the lateral rims \((-a, -b)\) and \((b, a)\) (figure 1). According to the critical state assumption, in those bands, the current density is constant and assumes the critical value \( J_c = L/2a \). In the central zone where the field is null (n.f.z.), the current density assumes a profile \( g_i(s) \)

\[
J_i(s) = \begin{cases} 
\frac{g_i(s)}{b} & (0 < |s| < b) \\
1 & (b < |s| < a). 
\end{cases} \tag{7}
\]

The unknown function \( g_i(s) \) satisfies the integral equation

\[
\int_{-b}^{b} K(s, t) g_i(t) dt = f_i(s), \tag{8}
\]
as a function of the transport current ratio

The AC losses (in each tape of the cable) can be computed

Inverting equation (13), the penetration

Integrating equation (7) on the arc

which is the same result obtained by Müller for an X-array [5]. The normal component of the magnetic field in the penetration band \((b, a)\) is obtained from equation (5) using equation (7)

where we have defined the auxiliary function

In this special case, the integral equation can be solved analytically and we obtain

which is the same result obtained by Müller for an X-array [5]. The normal component of the magnetic field in the penetration band \((b, a)\) is obtained from equation (5) using equation (7)

\[ H_s(x, y) = \frac{\mu_0}{L_s} \int_0^y \mathbf{H}_s \cdot \mathbf{n} \, ds. \]

\[ \mathbf{H}_s = \frac{\mathbf{H}}{\mu_0} \]

where \(\mathbf{H} = \mathbf{B}/\mu_0\) is the magnetic field and \(\mu_0\) is the magnetic permeability.

The assumption of a uniform critical current density along the cross section is mostly motivated by the necessity of simple mathematical handling. However, this is not always the case: for example, high temperature superconductor (HTS)-coated conductors have often lower \(J_c\) in the center of the tape, while the edges support a higher critical current density.

4. Lateral variation of the critical current density

4.1. Trapezoidal profile

First, we shall consider the case of a non-uniform lateral distribution of the critical current density \(J_c\), with a linear increase in the lateral bands \((-a, -c)\) and \((c, a)\)

\[ j_c(s) = \begin{cases} \frac{1}{T(s)} & (0 < |s| < c) \\ \frac{|s| - a}{c - a} & (c < |s| < a) \end{cases} \]

where

\[ T(s) = \frac{a}{c - a}, \]

so that the \(j_c(s)\) profile has a trapezoidal shape, as shown in figure 3(a). The critical current will therefore be

\[ I_c = J_c(a + c). \]

Similarly to equation (7), the current density

using Norris’s approach [7]

\[ Q = \int_0^\infty ds \int_{-a}^a J_c(s) E(s) ds \]

where \(T\) is the period of the AC cycle. This integral appears not to be expressible by means of standard functions but is easily numerically evaluated.

Figure 2 shows the AC losses as a function of the transport current ratio for a cable made of \(n\) polygonally arranged straight tapes \((n = 6, 12, 24)\) and of curved tapes. In the latter case, the losses are independent of \(n\) and are equivalent to those of a tape in an X-array configuration. The following parameters were used: tape width = 4 mm, gap = 0.25 mm. The losses are plotted as a function of the reduced current \(p = I/I_c\) and are normalized with respect to those of an isolated tape, computed with the well-known formula by Norris [7]
is

\[ J_2(s) = \begin{cases} 
J_{sT}(s) & (-a < |s| < -b) \\
g_2(s) & (-b < |s| < b) \\
J_{sT}(s) & (b < |s| < a)
\end{cases} \tag{19} \]

where \( g_2(s) \) is the unknown function to be determined. The magnetic field is still given by equation (5)

\[ H_L(s) = \frac{1}{2L} \text{p.v.} \int_{-a}^{a} K(s, t) J_2(t) \, dt. \tag{20} \]

Two cases need be considered, depending on the n.f.z. being internal or external to the interval \((-c, c)\).

Case \( b < c \): the magnetic field is null in \((-b, b)\) and one has

\[
0 = \int_{-a}^{-c} K(s, t) T(t) \, dt + \int_{-c}^{-b} K(s, t) \, dt \\
+ \text{p.v.} \int_{-b}^{b} K(s, t) g_2(t) \, dt \\
+ \int_{b}^{c} K(s, t) \, dt + \int_{c}^{a} K(s, t) T(t) \, dt
\]

or

\[
\text{p.v.} \int_{-b}^{b} K(s, t) g(t) \, dt = f_1(s)
\]

where the known term is

\[
f_1(s) = L_0 \left( \frac{s}{L} - \frac{b}{L} \right) + \frac{L^2}{a - c} \left[ s \left( \frac{s}{L} - \frac{a}{L} \right) \\
- s_0 \left( \frac{s}{L} - \frac{c}{L} \right) \right]
\]

(22)

where we have defined the auxiliary function

\[ s_1(x, y) = \frac{1}{\pi^2} \left[ \text{Li}_2(e^{i(x+y)}) - \text{Li}_2(e^{i(x-y)}) \right] \]

(\( \text{Li}_2(z) = \int_{1}^{z} \frac{\ln(1 - t)}{t} \, dt \) is the dilogarithm function and \( s_0 \) is the same as in equation (10)).

Case \( b > c \): in a completely similar way, one will have again equation (22), but with the known term given by

\[
f_2(s) = L \frac{a - b}{a - c} s_0 \left( \frac{s}{L} - \frac{b}{L} \right) + \frac{L^2}{a - c} \left[ s_0 \left( \frac{s}{L} - \frac{a}{L} \right) \\
- s_0 \left( \frac{s}{L} - \frac{b}{L} \right) \right]. \tag{24}\]

Equation (22) is singular of Cauchy type (principal value), as it can be seen developing the cotangent in series

\[
\pi = - \sum_{k=1}^{\infty} \frac{c_k(s - t)^k}{k!}.
\]

A direct solution (of the numerical type) is very problematic and it is convenient to extract the singularity from the integral. Setting \( Q(s, t) = (s - t) K(s, t) \), we can write

\[
\text{p.v.} \int_{-b}^{b} K(s, t) g_2(t) \, dt = \text{p.v.} \int_{-b}^{b} \frac{Q(s, t)}{s - t} g_2(t) \, dt = Q(s, s) \text{p.v.} \int_{-b}^{b} \frac{g_2(t)}{s - t} \, dt
\]

or

\[
\int_{-b}^{b} Q(s, t) dt = \frac{Q(s, s)}{s - t} g_2(t) \, dt. \tag{25}\]

One can immediately note that \( Q(s, s) = L/\pi \). Equation (22) therefore becomes

\[
\text{p.v.} \int_{-b}^{b} \frac{g_2(t)}{s - t} \, dt + \int_{-b}^{b} P(s, t) g_2(t) \, dt = \frac{\pi}{L} f_{23}(s) \tag{26}\]

where one has defined the non-singular kernel

\[ P(s, t) = \frac{(\pi/L) Q(s, t) - 1}{s - t}. \]

In fact, in virtue of the development of the cotangent, one obtains \( P(s, s) = 0 \). Equation (26) can now be transformed into a non-singular equivalent equation (see the appendix for details), more specifically into a Fredholm equation of the second type, which can be easily solved with the usual
numerical techniques dedicated to that purpose.
\[
g_3(s) + \int_{-b}^{b} K_0(s, t) g_2(t) dt = f_{0,1}(s)
\]
\[
K_0(s, t) = \frac{\sqrt{b^2 - s^2}}{\pi^2} \text{p.v.} \int_{-b}^{b} \frac{P(\sigma, t)}{\sqrt{b^2 - \sigma^2}} \frac{d\sigma}{\sigma - s}
\]
\[
f_{0,1}(s) = \frac{\sqrt{b^2 - s^2} \pi^2}{\pi^2} \text{p.v.} \int_{-b}^{b} \frac{f_1(\sigma)}{\sqrt{b^2 - \sigma^2}} \frac{d\sigma}{\sigma - s}
\]

Once equation (27) is solved, the distribution of current density along the whole width of the tape is known. One can therefore calculate the total current and the magnetic field. The AC losses can be obtained by adapting the Norris method to the current case, i.e.,
\[
Q_2 = 8 \int_0^T dt \int_{-a}^{a} J_2(s) E(s) ds
\]
\[
= 8 \int_0^{T/4} dt \int_{0}^{a} J_2(s) E(s) ds
\]
\[
= 8 \mu_0 \int_0^{T/4} ds \int_{-b}^{b} ds J_2(s) \int_{b}^{a} H_\perp(\sigma, t) d\sigma
\]

Finally, inverting the order of integration, one obtains
\[
Q_2 = 8 \mu_0 \int_{-b}^{b} ds H_\perp(\sigma, T/4) \int_{0}^{a} J_2(s) ds.
\]

One can easily verify that in the case of constant \( J_c \) in \((b, a)\), one obtains again equation (15).

4.2. Trapezoidal profile with base

Let us finally consider the more general case where the distribution of the critical current density \( j_c(s) \) does not drop to zero at the edges, but to a finite value \( j_{ca} \), as displayed in figure 3(b). We can treat this case as a weighted superposition of the two previous cases—the sum of a constant distribution and a trapezoidal distribution going to zero

\[
j_c(s) = j_{ca} + (1 - j_{ca}) j_{cT}(s),
\]
with \( j_{cT}(s) \) given by equation (17). This superposition is possible because the kernel of the respective integral equations is the same equation (6). For this reason, the related integral equation will be
\[
\int_{-b}^{b} K(s, t) g(t) dt = j_{ca} f_0(s) + (1 - j_{ca}) f_{1,2}(s),
\]
where \( f_0(s) \) is given by equation (9) and \( f_{1,2}(s) \) by equations (23) and (24). By the linearity of this equation, for any chosen value of the penetration \( b \), the solution will be given by the same superposition of the solutions of the previous cases
\[
g_3(s) = j_{ca} g_1(s) + (1 - j_{ca}) g_2(s)
\]
where \( g_1(s) \) is given by equation (11) and \( g_2(s) \) is the solution of equation (27). The current density will be given by
\[
J_2(s) = J_c \begin{cases} 0 & (-a < |s| < -b) \\ g_1(s) & (-b < |s| < b) \\ g_2(s) & (b < |s| < a) \end{cases}
\]

where \( J_c \) is a multiplicative constant determined by the request of the actual value of the critical current of the tape \( I_t = J_c \int_{-b}^{b} j_c(s) ds \). The transported current is \( I_t = \int_{-b}^{b} J_2(s) ds \) and finally we obtain the current ratio \( p = I_t/I_c \) as a function of \( b \). Once the current density is known, the normal magnetic field is given by equation (5) and the AC losses by an integral as in equation (28). As an application, in figure 4, we compare the current density profiles of tapes with the same geometrical properties \((a = 2 \text{ mm}, c = 1.4 \text{ mm}, \text{ and } g = 0.25 \text{ mm})\), the same critical and operational currents \((100 \text{ and } 80 \text{ A}, \text{ respectively})\), but different trapezoidal distributions of the critical current density: \( j_{ca} = 0, 0.2, 0.4, 0.6, 1 \), the latter case corresponding to a uniform current distribution. It can be clearly seen that the effect of having a non-uniform distribution is to move the
region with maximum current toward the center. Another consequence of the non-uniform distribution of \( j_c \) is the fact that the losses are higher. This is shown in figure 5, where the losses are plotted as a function of the current ratio \( p \). In all cases, the AC losses are normalized to those of a Norris strip characterized by the same given \( I_c \) and current ratio \( p \). The more pronounced the trapezoidal distribution is (i.e., the lower \( j_{ca} \)), the higher the losses. Tapes with \( j_{ca} = 0.4 \) have about three times the losses of tapes characterized by a uniform current density distribution.

5. Conclusion

In this paper, we carried out the critical state solution of curved thin tapes arranged around a cylindrical former. With our method, which avoids complex conformal mapping techniques, we immediately derived the fact that the electromagnetic behavior of this configuration is the same as that of an infinite X-array of straight tapes. As such, it does not depend on the number of tapes, but only on the tape width and on the gap between the tapes. The losses of a cable composed of curved tapes are much lower than those of a polygonal configuration, and can be seen as the lowest limit. We subsequently extended our model to include spatial variations of the critical current density, a situation that for example occurs often in HTS-coated conductors. We calculated the current profiles for several spatial distributions of the critical current, and evaluated the impact of such non-uniformity on the AC losses of the cable. Besides the specific cases considered in this paper, this modeling approach can be extended to other situations of practical interest, characterized by the presence of periodic assemblies of current-carrying tapes.

Acknowledgements

This work was supported by the Research Fund for the Italian Electrical System under the Contract Agreement between RSE and the Ministry of Economic Development (RB) and by the Helmholtz Association (FG, grant VH-NG-617).

Appendix

The Cauchy type singular equation

\[
p.v. \int_{-b}^{b} \frac{\phi(t)}{x-t} \, dt + \int_{-b}^{b} K(x, t) \phi(t) \, dt = f(x) \quad (A.1)
\]

where the kernel \( K(x, t) \) is not singular, can be rewritten as

\[
p.v. \int_{-b}^{b} \frac{\phi(t)}{x-t} \, dt = F(x) \quad (A.2)
\]

where we posed

\[
F(x) = f(x) - \int_{-b}^{b} K(x, \tau) \phi(\tau) \, d\tau. 
\]

The solution of equation (A.2) bounded at both extremes is

\[
\phi(x) = \frac{1}{\pi^2 \sqrt{b^2 - x^2}} \int_{-b}^{b} \frac{F(t)}{\sqrt{b^2 - t^2}} \, dt - \frac{1}{\pi^2 \sqrt{b^2 - x^2}} \int_{-b}^{b} \frac{1}{\sqrt{b^2 - t^2}} \, dt 
\]

with the supplementary constraint that

\[
\int_{-b}^{b} \frac{F(t)}{\sqrt{b^2 - t^2}} \, dt = 0. 
\]

By equation (A.3), this request can be satisfied; for instance, if the given term \( f(x) \) is an odd function and if the kernel \( K(x, \tau) \) transforms even functions in odd functions. If this is the case, the solution \( \phi(x) \) is an even function. Applying this solution to equation (A.2), we obtain

\[
\phi(x) = \frac{\sqrt{b^2 - x^2}}{\pi^2} \text{p.v.} \int_{-b}^{b} \frac{f(t)}{\sqrt{b^2 - t^2}} \, dt - \frac{\sqrt{b^2 - x^2}}{\pi^2} \text{p.v.} \int_{-b}^{b} \frac{1}{\sqrt{b^2 - t^2}} \, dt 
\]

\[
\times \int_{-b}^{b} K(t, \tau) \phi(\tau) \, d\tau 
\]

Changing the integration order in the second term, we have

\[
p.v. \int_{-b}^{b} \frac{1}{\sqrt{b^2 - t^2}} \, dt \int_{-b}^{b} K(t, \tau) \phi(\tau) \, d\tau = \int_{-b}^{b} \phi(\tau) \, d\tau \left( \text{p.v.} \int_{-b}^{b} \frac{K(t, \tau)}{\sqrt{b^2 - t^2}} \, dt \right) 
\]

so that, defining

\[
f_0(x) = \frac{\sqrt{b^2 - x^2}}{\pi^2} \text{p.v.} \int_{-b}^{b} \frac{f(\tau)}{\sqrt{b^2 - \tau^2}} \, d\tau \quad (A.7)
\]

\[
K_0(x, t) = \frac{\sqrt{b^2 - x^2}}{\pi^2} \text{p.v.} \int_{-b}^{b} \frac{K(\tau, t)}{\sqrt{b^2 - \tau^2}} \, d\tau, \quad (A.8)
\]

equation (A.6) becomes

\[
\phi(x) + \int_{-b}^{b} K_0(x, t) \phi(t) \, dt = f_0(x). 
\]

The singular integral equation (A.1)—with the restrictions above—has the same solution \( \phi(x) \) as the non-singular integral equation (A.9). The main advantage of this transformation is that the calculation of the principal value present in the integral equation is shifted to the calculation of a new given term and of a new kernel. A second advantage is that equation (A.9) is a Fredholm integral equation of second type, a well-posed integral equation for which many solving numerical routines exist (i.e., Nyström methods).
References

[1] Brambilla R, Grilli F and Martini L 2013 Critical state solution and AC loss computation of polygonally arranged thin superconducting tapes *Appl. Phys. Lett.* **103** 092602

[2] van der Laan D C, Lu X F and Goodrich L F 2011 Compact GdBa$_2$Cu$_3$O$_{7-δ}$ coated conductor cables for electric power transmission and magnet applications *Superconductor Sci. Tech.* **24** 042001

[3] Ma B and Balachandran U 2006 Prospects for the fabrication of low aspect ratio coated conductors by inclined substrate deposition *Superconductor Sci. Tech.* **19** 497–502

[4] Allais A, Rikel M O, Ehrenberg J and Bruzek C E 2014 Coated conductor *US Patent* US8644899 B2

[5] Müller K-H 1997 Self-field hysteresis loss in periodically arranged superconducting strips *Physica C* **289** 123–30

[6] Mawatari Y 2009 Field distributions in curved superconducting tapes conforming to a cylinder carrying transport currents *Phys. Rev. B* **80** 184508

[7] Norris W T 1970 Calculation of hysteresis losses in hard superconductors carrying ac: isolated conductors and edges of thin sheets *J. Phys. D: Appl. Phys.* **3** 489–507

[8] Amemiya N, Maruyama O, Mori M, Kashima N, Watanabe T, Nagaya S and Shiohara Y 2006 Lateral $I_c$ distribution of YBCO coated conductors fabricated by IBAD/MOCVD process *Physica C* **445–448** 712–6

[9] Grilli F, Brambilla R and Martini L 2007 Modeling High-Temperature Superconducting Tapes by Means of Edge Finite Elements *IEEE Trans. Appl. Supercond.* **17** 3155–8

[10] Gőmöry F, Šouc J, Pardo E, Seiler E, Solovyov M, Frolek L, Skarba M, Konopka P, Pekarčíková M and Janovec J 2013 AC loss in pancake coil made from 12 mm wide REBCO tape *IEEE Trans. Appl. Supercond.* **23** 5900406

[11] Solovyov M, Pardo E, Šouc J, Gőmöry F, Skarba M, Konopka P, Pekarčíková M and Janovec J 2013 Non-uniformity of coated conductor tapes *Superconductor Sci. Tech.* **26** 115013