Casimir effect: a novel experimental approach at large separation

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Abstract. A novel experimental set-up for the measurement of the Casimir effect at separations larger than a few microns is presented. The apparatus is based on a mechanical resonator and will use a homodyne detection technique to sense the Casimir force in the plane–parallel configuration.

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1. Introduction and motivation

In quantum electrodynamics, the description of the vacuum is modified by the presence of boundary conditions: the introduction of conducting surface changes the energy associated with the nonzero ground state of the electromagnetic field. This corresponds to a net force acting on the surface, and is known as the Casimir effect [1]–[3]. The attractive force between two parallel and perfectly conducting metal plates is

\[ F = \frac{\pi^2 \hbar c S}{240 d^4}, \]  

(1)

where \( c \) is the speed of light, \( \hbar \) the reduced Planck constant, \( S \) the surface of the plates, and \( d \) their separation.

The experimental study of the Casimir effect has had great impetus in the last decade, following the first precision measurement conducted by Lamoreaux [4]. Several experiments followed this measurement, all but one using a plane–sphere geometry: Mohideen and Roy [5], Decca et al [6] and Ederth [7]. The group of Bressi et al [8] was able to perform a measurement in the plane–parallel geometry originally proposed by Casimir. With this geometry the experimental difficulties are bigger, and this explains why the precision is worse compared to other configurations.

The Casimir formula (1) is valid only in an ideal situation, and has to be corrected to take into account for example the finite conductivity of the metal plates and their roughness. The experimental study of these corrections, that are not negligible only at very small distances (\( \leq 1 \mu m \)), was possible up to now only with the plane–sphere configuration due to its better precision. These measurements were performed for separations smaller than 1 \( \mu \)m, while for larger distances the Casimir force was below the sensitivity of the experiment.

An important correction to the Casimir formula is due to the presence of thermal photons: the original calculation has been made at zero temperature but experiments are normally performed at room temperature.

Due to the form of the black body spectrum, thermal corrections start to be significative only for separations larger then a few microns, and become the dominant part for distances larger than

\[ \lambda_T = \frac{2\pi c}{\omega_t} = \frac{\hbar c}{k_B T}, \]

(2)

corresponding to 7 \( \mu \)m at 300 K [9]. Since the separation has to be rather large, the most promising geometry for observing the thermal contributions to the Casimir force is the plane–parallel one, where the possibility to use relatively wide surfaces results in large enough forces. A plane–cylinder geometry has also been recently proposed [10, 11]. In this paper, we present a set-up based on the plane–parallel geometry where we aim to be able to measure the Casimir force for distances of several microns, thus entering in the regime of strong thermal corrections. Our apparatus is based on an oscillating plate whose motion is detected using a Michelson type interferometer. This mechanical resonator is made of aluminized silicon and its motion is excited at low frequency by means of a movable source plate. A homodyne detection scheme is employed, allowing for the very long integration times necessary to measure the very small motions of the resonator induced by the weak force. Care has been taken in the design of the
apparatus to reduce background noise and systematic effects. Vibrations are in fact driving the resonator and thus limiting the sensitivity and increasing the integration time. The most important systematic comes from the presence of voltage biases between the plates even when they are short cut. These voltages must be counter-biased with high precision otherwise they will hide the Casimir force. A detailed study of their properties has been done and will be carefully described.

The exact calculation of the finite temperature corrections to the Casimir force is still an open question: there are at least two models, both are predicting a sizeable change in the force for separations above \(1–2 \mu m\). Without the necessity of entering into this dispute, we quote the results of Bordag et al [3]: in the plane–parallel configuration, at a temperature \(T = 300 \text{ K}\) and a separation of \(5 \mu m\), the correction to the standard force is about 50\%, thus a precision of the order of 10–20\% in the measurement of the force would then be sufficient to separate this contribution.

2. Experimental set-up

2.1. The apparatus

The plane–parallel configuration is the most convenient geometry for the measurement of the force at large separations, providing a possibly measurable signal even at distances of the order of a few microns. In our set-up, two flat parallel surfaces are fronting each other. One of the two is called the source, its distance from the second surface can be changed acting on a piezoelectric actuator (PZT1), the second one is called the resonator. The source can exert a force on the resonator, through an electrostatic field or the Casimir effect. The resonator is free to move around its equilibrium position, and by monitoring its position using a laser interferometer, it is possible to gather information on the forces acting on it.

The resonator is made of silicon crystal and is essentially a plate \(1 \text{ cm} \times 1 \text{ cm} \times 77 \mu m\) connected to a frame through four parallel wires \(1 \text{ cm}\) long and \(77 \mu m \times 77 \mu m\) in section. Plate, wires and frame are carved from the same crystal, which was initially coated with 0.5 \(\mu m\) of aluminum. A front view of the resonator can be seen in figure 1. We measured its resonance frequency to be \(\nu_r = 172 \text{ Hz}\) with a quality factor around 2000.

The source is a \(1 \text{ cm} \times 1 \text{ cm}, 500 \mu m\) thick silicon crystal coated on both sides with 0.8 \(\mu m\) of aluminium. The source is glued to an aluminium block, which is in turn glued to PZT1 providing a controllable translational motion of the source itself.

Aluminium in air quickly oxidizes, forming a thin film over the surfaces of the two plates. The film is very thin, of the order of two atomic layers, since the aluminium oxide is chemically rather stable and thus prevents the diffusion of the oxygen into the bulk. It has been shown [12] that the effect of a thin film on the Casimir force is relevant only at very short ranges, of the order of 100–200 nm, much shorter than the range of interest of this experimental set-up.

The flatness of the two surfaces were measured interferometrically. An interferometric profile of the source is shown in figure 2, and shows that the planarity is 419 nm peak—valley over it whole surfaces. Similar measurements on the resonators gave as results planarity of the order of 200 nm. Using equation (5.95) of [3], we estimate a relative error less than \(10^{-4}\), using \(\alpha L/d \approx 0.1\), setting \(\alpha \approx \delta/L \) (\(\delta\) is the deviation from flatness, 0.5 \(\mu m\), \(L\) the length of the sides of the surfaces (1 cm) and \(d\) the separation between the surfaces, 5 \(\mu m\).

PZT1 is clamped to a 6-axis stepper-motor-controlled positioning stage (Thorlabs, 6-Axis NanoMax Nanopositioner), which is used to control the parallelism between the two.
**Figure 1.** A picture of the resonator, before cleaning. The square in the centre is the oscillating plate, connected to the supporting frame by four wires. A small square (1 mm × 1 mm) appears in the centre of the resonators. It is a hole that was intended for interferometric applications. New resonators will be made without the hole.

**Figure 2.** An interferometric measurement of the surface of the source shows its planarity of 419 nm peak—valley over its whole surface.
Figure 3. Side view showing the resonator, the source and the positioning system used for the parallelization.

Figure 4. Top view of the experimental set-up.

plates. This arrangement is shown in figure 3, while the complete apparatus scheme is shown in figure 4.

The source, the resonator and the positioning stage are in a vacuum chamber, at a pressure of about \(3 \times 10^{-6}\) mbar. The vacuum is provided by a vibrations-free ion pump. The whole experiment lies inside a clean room (class 1000), with thermal stabilization at the level of \(\Delta T = 0.1^\circ \text{C}\). The motion of the resonator is detected by means of a Michelson type interferometer set-up with a 1 mW amplitude stabilized He–Ne laser at 633 nm. One arm of the interferometer enters the vacuum region through a glass window and is reflected from the back side of the resonator. The second arm is directed on to a mirror (M2) controlled by a piezoelectric transducer (PZT2). The interference between the beam impinging on the resonator and the reference-crossed beam is detected by a photodiode, whose current is proportional to the intensity of the combined beams:

\[
i = i_0 \left[ 1 - v \cos \left( \frac{4\pi l}{\lambda} \right) \right],
\]

\(i_0\)
where \( v \) is the fringe visibility, \( i_0 \) the average intensity corresponding to an incoherent sum of the beams, \( \lambda \) the wavelength of the laser and \( l \) the path difference between the two beams. The sensitivity of the interferometer is maximum when the phases of the two beams are in quadrature, i.e. when \( l = (2n + 1)\lambda/8 \). Around these points, the photodiode is sensitive to change in \( l \) and in the wavelength of the laser:

\[
\frac{\Delta l}{i_0} = 4\pi v \left( \frac{\delta l}{\lambda} + \frac{\delta \lambda}{\lambda^2} \right),
\]

where \( \delta l \) is the gap change and \( \delta \lambda \) the wavelength change, which is of the order of \( 10^{-8} \) over 8 h for a stabilized He–Ne laser. A proportional-integral feedback loop acts on the piezoelectric actuator PZT2 of mirror M2 (cf figure 4) to keep the interferometer around the maximum of the sensitivity. The photodiode signal is amplified, the voltage corresponding to a fringe is \( V_{fr} = 1.5 \text{ V} \). In the point of maximum sensitivity of the interferometer, the measured amplitude \( V_M \) corresponds to a distance

\[
\delta x = \frac{\lambda}{2\pi} \frac{V_M}{V_{fr}}.
\]

In order to achieve maximum sensitivity of the resonator, particular care has been taken to minimize its free motion. The whole apparatus is mounted on a passive low frequency damper and on a floating optical table. As explained next, we want to minimize the low frequency noise, below a few Hz: the passive damper (Minus-k Technologies 250 BM 6) is a mechanical low pass filter with a cut-off frequency at 0.5 Hz in the transmissibility curve. The clean room also represents a quiet environment to avoid acoustical noise: all the electronics lie in a separate control room.

2.2. Detection technique

The set-up presented in this paper is based on an amplitude homodyne detection technique, already described in [13].

A static detection scheme is not sensitive enough, because it is affected by low frequency drifts due to thermal expansion, electronic instabilities, drift of the laser frequency. For this reason, we use the dynamical homodyne detection technique: we move the source with a periodical movement in the direction of the resonator. The Casimir force acting between the two surfaces makes the resonator moving, and its motion is detected using the interferometer. If the chosen frequency is well below the resonator proper frequency, a fixed phase difference will be present between source motion and resonator motion, thus allowing for vector averaging of the interferometer signal.

If between the source and the resonator there is a spatially dependent force \( F(d) \), then the equation of motion of the equivalent harmonic oscillator is

\[
m\ddot{x}_r(t) = -m\omega_r^2 x_r(t) + F(d),
\]

where \( x_r(t) \) is the position of the resonator, \( \omega_r = 2\pi v_r \) is its angular proper frequency and \( m \) its mass.

We can decompose the separation \( d \) between source and resonator into two components, namely a fixed distance \( d_0 \) and a time-dependent component \( x_s(t) \), due to the movement of the source, that is \( d = d_0 + x_s(t) \). The free motion of the resonator is considered to be much smaller.
and is neglected. In the case of a generic force acting between the source and the resonator, of the form \( F(d) = C/d^n \), we obtain a dynamical force, at the second order in the expansion parameter \( |x_s/d_0| \ll 1 \):

\[
F(x_s, d_0) \simeq \frac{C}{d_0^n} \left[ 1 - n \frac{x_s}{d_0} + \frac{n(n + 1)}{2} \left( \frac{x_s}{d_0} \right)^2 \right].
\]  

(7)

These components of the force modify the amplitude, the frequency and the phase of the free oscillations of the resonator.

A periodic modulation of the position of the source \( x_s = x_s^0 \cos \omega_s t \) gives a force between source and resonator in the form (at first order):

\[
F(t) = \frac{C}{d_0^n} \left( 1 - n \frac{x_s^0}{d_0} \cos \omega_s t \right).
\]  

(8)

The oscillation of the resonator will be at the same frequency, and its amplitude together with the value of the spring constant of the resonator allows a calculation of the force parameters. In the Fourier spectrum of the solution \( x_r(t) \) of equation (6), there is a peak at the frequency \( \nu_s = \omega_s / 2\pi \), whose amplitude is:

\[
A_s^0 = n \frac{C x_s^0}{m \omega_r^2 d_0^{n+1}}.
\]  

(9)

An overall calibration of the system is obtainable using controllable electrostatic forces. In fact, a constant voltage \( V \) between the resonator and the source will induce a force

\[
F_V(d) = -\frac{1}{2} \varepsilon_0 S \left( \frac{V}{d} \right)^2,
\]  

(10)

It is then possible to compare the amplitude induced by the Casimir force \( A_C \) and the one obtained by the voltage calibration at fixed bias \( A_V \):

\[
A_C = \frac{1}{60} \frac{\pi^2 \hbar c S}{m \omega_r^2 d_0^5} x_s^0,
\]  

(11)

\[
A_V = \frac{\varepsilon_0 SV^2}{m \omega_r^2 d_0^5} x_s^0.
\]  

(12)

The calibrations are useful to infer the parameters common to both expressions, without relying on their direct determination.

2.3. Stray effects

In order to measure the tiny Casimir force, one must be able to eliminate or compensate other forces acting between the two plates. In particular electrostatic attraction between the two plates can hide the Casimir force. It is not enough to put both plates to the same potential (e.g. at ground). In fact if different metals connect the two plates, the different work functions of the metals cause a built-in potential. This potential can be measured and counter-biased. This
technique is valid only if this built-in potential remains constant. Variations of temperature in the experimental set-up cause variations of the built-in potential. To minimize the built-in potential all connections between the plates and the devices used for the measurements has to be made of the same material. For this reason the two plates were connected to the measuring devices or to the voltage supply used for the counter-bias using only aluminium, with aluminium bars connected to the two plates by aluminium strips, as can be seen in figure 5.

Figure 5. Electric contacts to the plates.

Figure 6 shows the built-in voltage that would build a force between the two surfaces of the same amount of the Casimir force. The measurement can be made only if the built-in voltage is controlled within a fraction of this.

Another source of systematic uncertainty is due to the patch effect [14, 15]. Patch forces can mimic the Casimir force, spoiling the accuracy of the measurement of a few percentage, that must be taken into account.
In precision mechanical instruments, the thermal noise excites the mechanical resonance with a root-mean-square level that corresponds to an energy of $k_B T$. At frequencies that are lower than the resonance frequency seismic noise will dominate the noise budget [16].

3. Experimental results

A search for the best signal to noise ratio has been conducted in order to optimize the homodyne detection technique. As explained above, the frequency at which the source is moved has to be well below the proper frequency of the resonator. Reducing this pumping frequency also has the advantage of reducing the amount of energy introduced into the system, which can cause a spurious effect due to a mechanical coupling between source and resonator via their basements. We therefore study the spectrum of the interferometer noise coupled with the free resonator, having the source far away from the resonator itself. A low noise region has been found in the frequency range 6 to 8 Hz. We decided to pump the source at the frequency $\nu_s = 7$ Hz, with an amplitude $x_s^0 = 150$ nm. To be sure that there was no mechanical pick-up a very long run has been performed with a large separation, around 50 $\mu$m, between the source and the resonator: no peak appeared at the expected frequency.

The precise measurement of the separation distance between source and resonator is a key parameter of the experiment. Relative distances are controlled using the PZT1 transducer glued to the source and the translational stage of the 6-axis mount. The PZT transducer has been calibrated using the interferometer: it has an actuation parameter of 0.15 $\mu$m V$^{-1}$. The 6-axis mount is used for coarse motion, with a resolution of 50 nm. To obtain the absolute distance of the gap, one has to obtain the actual separation corresponding to the zero value in the relative displacement. We have employed two different schemes to extract this information, both of them also allow to check the parallelism between the surfaces.

In one method one measures the capacitance $C$ between the opposing surfaces with a precision LCR meter, varying the gap separation. In the case of perfect parallelism, the expected behaviour is

$$C = \frac{\epsilon_0 S}{d - d_{ref}} \tag{13}$$

where $d_{ref}$ is the reference distance that can be found fitting the data. The parasitic capacitance of the electrical circuit was measured with a measure of capacity with a very large distance between the plates (more than 4 mm), and subtracted from the measures of capacity. This method also gives an indication of the parallelism between the two plates, in fact any deviation from the $1/d$ dependence of the capacity reveals a non parallelism of the plates, that could thus be minimized [17].

In the second method, a fixed bias voltage is applied in the gap and the force acting on the resonator is measured using the homodyne detection previously described: the voltage is chosen of the order of a few volts and results in a large signal. Figure 7 shows a typical spectrum obtained with 1.5 V in the gap at a separation about 15 $\mu$m.

Precisely varying the distance using the PZT transducer a $1/d^3$ behaviour is expected for the amplitude $A_V$ of equation (12) (see figure 8). The displacement source-resonator was parameterized as function of the DC voltage $V_{PZT}$ applied to the calibrated PZT1. The measured
Figure 7. A typical spectrum of the interferometer signal, used for the measurement of $A_V$ as in equation (12). Here, the bias voltage was of 1 V, at a gap separation around 18 $\mu$m. The relevant peak is at 7 Hz.

points of the displacement as a function of the DC voltage actuator can be fitted with the function

$$P = \frac{a}{(b - 0.15V_{\text{PZT}})^3}$$

(14)

where $a$ and $b$ are two parameters, 0.15 corresponds to the calibration of the actuator in micrometres/volt. The parameter $b$ provides the distance between source and resonator when no voltage was applied to the actuator. The minimum distance source-resonator was then calculated from the maximum voltage ($V_{\text{PZT}}^{\text{max}}$) applied without contact between the plates: $d_{\text{min}} = b - 0.15V_{\text{PZT}}^{\text{max}}$. In a preliminary set-up, we obtained a value of about 10 $\mu$m. A deviation from the $1/d^3$ law also would reveal a non parallelization of the plates.
Another method is used to improve the parallelization between the two plates: using the beam of the interferometer reflected by the source and the back side of the resonator, it is possible to get a parallelization of the order of $10^{-3}$ rad. Since this is not enough for the scope of the experiment, a more sophisticated technique is under construction, using an auxiliary laser beam. This method should provide a real-time measurement of the parallelization of the order of $10^{-4}$ rad.

The parallelization of the two surfaces is of great importance. As shown in [3], a deviation from perfect parallelization would cause deviations from the Casimir force between parallel plates $F(d, \alpha)$:

$$F(d, \alpha) = F_C \left[ 1 + \frac{10}{3} \left( \frac{\alpha L}{d} \right)^2 + 7 \left( \frac{\alpha L}{d} \right)^4 \right].$$ (15)

The resulting force $F(d, \alpha)$ is plotted in figure, and shows that for angles of a few $10 \mu$rad the deviation is of the order of a few percent.

In order to measure and study the properties of the built-in voltage $V_{\text{bi}}$, several calibrations have been performed. In these measurements, a varying bias voltage is applied in the gap, keeping the distance fixed. The amplitude of the resonator motion is plotted versus the applied voltage: figure 10 shows one of the resulting curves. The voltage corresponding to the minimum of this curve corresponds to the inverse built-in voltage. An accuracy of about 1 mV is obtained using about ten different bias voltages and a total measurement time about an hour. Typical values for $V_{\text{bi}}$ are about 100 mV. Since this is rather large, we studied its stability with time. We have found that over the timescale of one day the built-in voltage is constant within the measurement accuracy. The duration of a Casimir force measurement will be of about half a day, short enough to consider the built-in voltage constant over the whole measurement. At any rate, improvements to reduce the changes of the potential are planned.

Finally it is possible to calculate the total sensitivity of the apparatus. We obtained a limit sensitivity on the resonator displacement of $x_{\text{limit}} = 7 \times 10^{-13}$ m over an integration time of $\sim 5000$ s. Apparently for longer integration times the noise figure remains constant, a study of this noise behaviour is under way. The maximum distance at which the Casimir force can be
measured with this set-up can be obtained by inverting formula (11):

\[ d_{\text{max}} = \left[ \frac{\hbar c S x_0^0}{240 m \nu^2 x_{\text{limit}}} \right]^{1/5} \]  (16)

that results \( d_{\text{max}} = 5.5 \, \mu m \), distance at which the Casimir force can be measured with a SNR = 1, corresponding to a dynamical force \( \Delta F = 4.8 \times 10^{-11} \, N \). We are testing new geometries for the resonator in order to improve the force sensitivity.

4. Conclusions

We have described an experimental apparatus and the measurement technique aimed to measure the Casimir effect in the parallel plates configuration in the 3–6 \( \mu m \) range.

Calibrations show that a force of \( 5 \times 10^{-11} \, N \) can be measured with this set-up. This corresponds to the Casimir force between the two 1 cm \( \times \) 1 cm aluminium parallel plates at a distance of 5.5 \( \mu m \). It is possible to measure the built-in voltage and to control it with an accuracy of 1 mV, that opens up the possibility to measure the Casimir force at large distances. The minimum distance between the surfaces so far obtained is 10 \( \mu m \), limited by imperfect parallelization, at level of \( 10^{-3} \) rad. Improvement are in progress to increase the parallelization at a level of \( 10^{-4} \) rad, and to decrease other sources of systematic errors.

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