Refining the Proof of Planar Equivalence

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Abstract

We outline a full non-perturbative proof of planar (large-$N$) equivalence between bosonic correlators in a theory with Majorana fermions in the adjoint representation and one with Dirac fermions in the two-index (anti)symmetric representation. In a particular case (one flavor), this reduces to our previous result — planar equivalence between super-Yang–Mills theory and a non-supersymmetric “orientifold field theory.” The latter theory becomes one-flavor massless QCD at $N = 3$. 
Recently, we have argued [1] that a bosonic sector of $\mathcal{N} = 1$ super-Yang–Mills (SYM) theory is equivalent, in the large-$N$ planar limit, to a corresponding sector of a non-supersymmetric Yang–Mills theory with a Dirac fermion in the two–index antisymmetric or symmetric representation. We will refer to these as the parent and daughter theories, respectively, all being endowed with the same gauge group, $\text{SU}(N)$. The daughter theories represent orientifold projections of the parent one, as first discussed in Ref.[2]. As we shall see, all our results apply equally well to the antisymmetric (orienti-A), and to the symmetric (orienti-S) case. For a detailed review see Ref. [3].

For the orienti-A case the daughter theory reduces, at $N = 3$, to one-flavor massless QCD. Thus, as an intriguing consequence of planar equivalence, one can copy, within an $O(1/N)$ error, non-perturbative quantities from SYM theory to the corresponding ones in one-flavor massless QCD [4]. In particular, in [5] we have obtained a very encouraging value for the quark condensate. Orientifold planar equivalence has further possible applications, both in phenomenology [6] and in string theory [7].

In Refs. [1, 3] we provided a perturbative proof of the planar equivalence and outlined a non-perturbative extension of it. In this paper we present a detailed analysis of non-perturbative planar equivalence (including theories with $N_f$ flavors, $N_f > 1$), with emphasis on the assumptions made. In our view, this completes the non-perturbative proof, under very mild assumptions.

The basic idea behind our proof is the comparison of generating functionals of appropriate gauge-invariant correlators in the parent and daughter theories by, first, integrating out their respective fermions in a fixed gauge background — a feature which could not be implemented for the orbifold projection $^1$ — and, then, averaging over the gauge field itself. In Refs. [1, 3] the main emphasis was on the first step. Here we mainly focus on the second.

Let us define, for a generic Dirac fermion in the representation $r$, the generating functional,

$$e^{-W_r(J_{\text{YM}}, J_{\Psi})} = \int DA_\mu D\Psi D\bar{\Psi} e^{-S_{\text{YM}}[A,J_{\text{YM}}]} \exp \left\{ \bar{\Psi} (i \not\partial + A^a T^a_r + J_{\Psi}) \Psi \right\},$$

where $S_{\text{YM}}$ is the Yang–Mills action, $J_{\text{YM}}$ is a source which can couple to any

$^1$Planar equivalence for “orbifold field theories” was conjectured by M. Strassler in Ref. [8]. Orbifold theories always contain a product of gauge factors.
gauge-invariant operator built from gauge fields and covariant derivatives, and the quark (color-singlet) source $J_\Psi$ can contain Lorentz $\gamma$ matrices. A mass term is a particular case of such quark source. We will always assume that a small fermion mass term is introduced for infrared regularization. It can be set to zero at the very end. The generating functional $W_r(J_{YM}, J_\Psi)$ is written in (1), for definiteness, in Euclidean space. This is not crucial: one can carry out all our derivations in Minkowski space as well.

After the fermions are integrated out we arrive at

$$e^{-W_r(J_{YM}, J_\Psi)} = \int DA_\mu e^{-S_{YM}[A,J_{YM}]+\Gamma_r[A,J_\Psi]} ,$$

where

$$\Gamma_r[A, J_\Psi] = \log \det (i \not \partial + A_a T_r^a + J_\Psi) .$$

For what follows it is convenient to write the effective action $\Gamma_r[A, J_\Psi]$ in the world-line formalism, see [9, 10, 11, 12], as an integral over (super-)Wilson loops, namely\(^2\)

$$\Gamma_r[A, J_\Psi] = -\frac{1}{2} \int_0^\infty \frac{dT}{T}$$

$$\times \int Dx D\psi \exp \left\{ -\int_0^T d\tau \left( \frac{1}{2} \dot{x}^\mu \dot{x}^\mu + \frac{1}{2} \dot{\psi}^\mu \dot{\psi}^\mu - \frac{1}{2} J_\Psi^2 \right) \right\}$$

$$\times \text{Tr} \mathcal{P} \exp \left\{ i \int_0^T d\tau \left( A_\mu^a \dot{x}^\mu - \frac{1}{2} \psi^\mu F^a_{\mu\nu} \psi_\nu^a \right) T_r^a \right\} ,$$

where the functional integral runs over all closed paths $x_\mu(\tau)$,

$$x_\mu(0) = x_\mu(T) ,$$

$A_\mu(x)$ is a fixed gauge background, and

$$T_r^a = T_{\text{adjoint}}^a , \quad T_{AS}^a , \quad T_S^a .$$

\(^2\)Strictly speaking Eq.(4) is only valid for space-time independent currents proportional to 1 or $\gamma_5$. The extension to non-constant currents can be found in [12] for those $\gamma$-matrix structures and do not affect our considerations below. As discussed below and at the end of the paper, we also expect the same to be true for other $\gamma$-matrix structures (see [13]) provided suitable identifications are made for the currents in the various theories we consider.
are the generators for the adjoint, two–index antisymmetric and two–index symmetric representations, respectively. Moreover, $\psi^\mu(\tau)$ are superpartners to $x^\mu(\tau)$; they occur due to the fact that we are dealing with spin 1/2 matter.

Eq. (4) can be written symbolically as:

$$\Gamma_r[A, J_\Psi] = \sum_\alpha C_\alpha(J_\Psi)W_\alpha^r(A_\mu), \quad (5)$$

where the summation symbol also stands for the functional integrals appearing in (4). The expansion coefficients $C_\alpha(J_\Psi)$ depend, in general, on the representation $r$ through the sources $J_\Psi$. However, for the case at hand, the sources $J_\Psi$ can be matched in the three theories in such a way that, to leading order in $1/N$, the $C_\alpha(J_\Psi)$ become representation–independent. Examples of such a matching will be given at the end of this paper, also for the case of more than one flavor. With this in mind, we shall assume hereafter that representation dependence resides entirely in the (super) Wilson factors $W_\alpha^r(A_\mu)$. Inserting the above result in (2) we arrive at

$$e^{-W_r(J_{YM}, J_\Psi)} = \langle e^{\Sigma_\alpha C_\alpha(J_\Psi)W_\alpha^r(A_\mu)} \rangle_c, \quad (6)$$

where the angle brackets stand for the remaining functional integral (average) over the gauge field in the presence of a generic gluon source $J_{YM}$.

As usual, taking the logarithm of both sides of (6) picks up the connected contributions from the expansion of the right-hand side,

$$-W_r(J_{YM}, J_\Psi) = \sum_n \sum_{\alpha_1, \alpha_2, \ldots, \alpha_n} \frac{C_{\alpha_1} \cdots C_{\alpha_n}}{n!}$$

$$\times \langle W_{r_1}^{\alpha_1}(A_\mu) W_{r_2}^{\alpha_2}(A_\mu) \cdots W_{r_n}^{\alpha_n}(A_\mu) \rangle_c, \quad (7)$$

where the subscript $c$ stands for connected. In fact a subtlety, representing the main thrust of this paper, is related to the issue of “connectedness” which, in turn, is related to the process of averaging over the gluon field the multi-Wilson-loop operators appearing in Eq. (7). In Ref. [1, 3] we dealt with a single loop (Fig. 1), now we will carefully treat multiloop averaging (see Fig. 2 which displays, as a particular example, five loops).

We will compare two cases: $r =$adjoint and $r =$two–index antisymmetric. The dimensions of the corresponding representations are

$$N^2 - 1 \quad \text{and} \quad N(N - 1)/2,$$
Figure 1: One fermion loop \( (i \phi + A^a T^a + J_\Psi) \) in the gluon field background (shown as shaded areas). The gluon fields “inside” and “outside” the loop do not communicate with each other at \( N \to \infty \). This is indicated by distinct shadings. Averaging over the gluon field inside the loop is independent of averaging outside. Topologically, of course, the distinction between inside and outside is immaterial respectively. Note, however, that the adjoint fermions are taken to be Majorana, while the two-index antisymmetric ones are Dirac. As a consequence, for \( r = \text{adjoint} \), Eq. (4) has to be multiplied by \( \frac{1}{2} \). Let us also note, in passing, that the dimension of the two-index symmetric representation is \( N(N+1)/2 \).

Our statement now is as follows: As \( N \to \infty \) each term in Eq. (7) for \( r = \text{two-index antisymmetric} \) has a corresponding term, with exactly the same value, for \( r = \text{adjoint} \). The proof is based on well-known trace identities.

Since \( W^r_\alpha (A_\mu) \), for a given loop and given \( A_\mu \), is just the trace of one concrete \( SU(N) \) group element, written in the representation \( r \), the following relations hold (see, for example, [14]):

\[
W_S = \frac{1}{2} \left( (\text{Tr } U^2 + \text{Tr } U^2) + (U \to U^\dagger) \right), \tag{8}
\]

\[
W_{AS} = \frac{1}{2} \left( (\text{Tr } U^2 - \text{Tr } U^2) + (U \to U^\dagger) \right), \tag{9}
\]

\[
W_{\text{adjoint}} = \text{Tr } U \text{Tr } U^\dagger - 1 + (U \to U^\dagger) = 2 \left( \text{Tr } U \text{Tr } U^\dagger - 1 \right), \tag{10}
\]

where \( U \) (resp. \( U^\dagger \)) represents the same group element in the fundamental (resp. antifundamental) representation of \( SU(N) \). An important point
Here is the occurrence of the \((U \rightarrow U^\dagger)\) terms in Eqs. (8), (9). The origin of these terms, whose presence is very natural since, for Dirac fermions, a representation and its complex conjugate are equivalent, can be explained as follows. For each given oriented contour \(x^\mu(\tau)\) in (4) there exists also the same contour with the opposite orientation, see Fig. (3). We can therefore group Wilson loops pairwise (In QED this contour “pairing” is responsible for the Furry theorem.) and thus obtain the additional complex conjugate terms. For the real representations, such as the adjoint, this gives simply a factor 2, as on the right-hand side of Eq. (10), which cancels however in our case against the factor \(\frac{1}{2}\) due to the Majorana condition.

Consider now, as the simplest example, the term with \(n = 1\) in (7). At large \(N\), after integrating over the gauge field, the terms \(\text{Tr} (U^2)\) and \(1\) are subleading in \(1/N\) with respect to the terms of \(O(N^2)(\text{Tr} U)^2\) or \(\text{Tr} U \text{Tr} U^\dagger\). Furthermore, \(\langle \text{Tr} U\rangle = \langle \text{Tr} U^\dagger\rangle\). It then follows immediately that, at \(N \rightarrow \infty\):

\[
\frac{1}{2} \langle W_{\text{adjoint}} \rangle \rightarrow \langle W_S \rangle \rightarrow \langle W_{AS} \rangle \rightarrow \langle \text{Tr} U\rangle^2.
\]  

(11)
Figure 3: Two opposite-orientation contours in the sum (4).

Figure 4: The 't Hooft double-line representation for $\langle W_{\text{adjoint}} \rangle$. On the right we display a convenient graphic shorthand notation that we suggest to use in this problem. The black circle corresponds to $\langle \text{Tr} U \rangle$, the white circle to $\langle \text{Tr} U^\dagger \rangle$, while the segment connecting them indicates that both circles originate from one and the same fermion loop, see Fig. 1.

Note that this common leading contribution is connected, in spite of the fact that, when written in terms of Wilson loops with $r =$fundamental (the last step in Eq. (11)), it looks disconnected. It is instructive to graphically illustrate Eq. (11). To this end we redraw Fig. 1 using the 't Hooft double-line notation, as shown in Fig. 4.

It is easy to show that, also for higher-order terms in Eq. (7), we can drop the subleading contributions in Eqs. (8)–(10), namely, Tr $(U^2)$ and 1, so that, hereafter, we will deal with the large-$N$ limit,

$$W_S = W_{\text{AS}} = \frac{1}{2} \left((\text{Tr} U)^2 + (\text{Tr} U^\dagger)^2\right),$$
\[
\frac{1}{2} W_{\text{adjoint}} = \text{Tr} U \text{Tr} U^\dagger. \tag{12}
\]

Equations (12) suggest a convenient graphic notation (see again Figs. 1 and 4). Associate with every \( W_{\text{adjoint}} \) a black and a white circle (related to \( \text{Tr} U \) and to \( \text{Tr} U^\dagger \), respectively) connected by a short segment (just to show that it represents a single Wilson loop) and to either \( W_S \) or \( W_{\text{AS}} \) a similar drawing with two whites or two black circles (with a factor \( \frac{1}{2} \) each). It is easy to see that, as \( N \to \infty \), the leading diagrams for a generic contribution of the form \( \langle W_1 W_2...W_p \rangle_c \) in Eq. (7) are given, in the above notation, by a connected tree where \( p \) segments are joined through “\( i \)-vertices”, i.e. vertices that couple any number \( i \) \((i = 1, 2, \ldots)\) of dots. By using trivial properties of tree diagrams, and the fact that an \( i \)-vertex gives a contribution \( \mathcal{O}(N^{2-i}) \), we arrive immediately to the conclusion that all tree diagrams are \( \mathcal{O}(N^2) \) while all loops are suppressed. It is amusing to notice that this large-\( N \) counting resembles closely the one of closed-string amplitudes if one associates with each \( \text{Tr} U \) or \( \text{Tr} U^\dagger \) a closed string and with each shaded region a tree-level vertex among the closed-strings that define that region’s boundary.

We recall once more that the subscript \( c \) stands for connected. In order to ease understanding of this point we show, in Fig. 5, one of contributions in the parent theory (five fermion loops, see Fig. 2) in the shorthand notation introduced in Fig. 4. This figure represents a certain large-\( N \) connected correlator of five Wilson loops in the parent theory, namely

\[
\langle W_1 W_2 W_3 W_4 W_5 \rangle_c \longrightarrow \langle \text{Tr} U_2 \rangle \langle \text{Tr} U_3^\dagger \rangle \langle \text{Tr} U_2^\dagger \text{Tr} U_3 \text{Tr} U_1 \rangle
\times \langle \text{Tr} U_1^\dagger \text{Tr} U_4 \rangle \langle \text{Tr} U_4^\dagger \text{Tr} U_5^\dagger \rangle \langle \text{Tr} U_5 \rangle. \tag{13}
\]

A similar contribution in the daughter theory (see Fig. 6) is

\[
\langle W_1 W_2 W_3 W_4 W_5 \rangle_c \longrightarrow \langle \text{Tr} U_2^\dagger \rangle \langle \text{Tr} U_3 \rangle \langle \text{Tr} U_2^\dagger \text{Tr} U_3 \text{Tr} U_1 \rangle
\times \langle \text{Tr} U_1 \text{Tr} U_4^\dagger \rangle \langle \text{Tr} U_4^\dagger \text{Tr} U_5^\dagger \rangle \langle \text{Tr} U_5^\dagger \rangle. \tag{14}
\]

To complete the proof of the parent-daughter planar equivalence we now show that, to every such tree diagram in the adjoint theory, one can associate a corresponding tree shorthand diagram of the S or AS theory, having exactly the same value. To see that this is the case one can interchange white and black circles at every other vertex along the tree as shown in Fig. 6. After
Figure 5: A particular connected contribution to an expectation value $\langle W_1W_2W_3W_4W_5 \rangle_c$ in $\mathcal{N} = 1$ SYM theory. Dashed circles indicate averaging over a connected background field, for instance, the external lines of loop 1 and loop 4 are averaged over one and the same gluon field.

Figure 6: The same as in Fig. 5 after interchange “black circle ↔ white circle” in vertices 5, (1, 4), 2 and 3.
doing so we arrive at a five fermion loop contribution in the daughter (A or S) theory. This operation obviously transforms a generic graph of the adjoint theory into a corresponding graph of the S or AS theory. The fact that one can perform the interchange “black circle ↔ white circle” at any vertex separately, is rather obvious. Take, for instance, the vertex (1,4) in Figs. 5 and 6. The above interchange is nothing but the use of an obvious equality
\[ \langle \text{Tr} U^\dagger_1 \text{Tr} U_4 \rangle = \langle \text{Tr} U_1 \text{Tr} U^\dagger_4 \rangle, \]
which is a straightforward generalization of the equality \( \langle \text{Tr} U \rangle = \langle \text{Tr} U^\dagger \rangle \).

It is easy to show that the above procedure is biunivocal i.e. it associates to every graph of the parent theory a graph of the daughter theory, and vice versa.

Other one-to-one transformations of the graphs of the parent theory into those of the daughter theories are also possible. For instance, one can use the fact that \( W_{\text{adjoint}} \) is real even before averaging over the gluon field, see the second line in Eq. (12). This means, in essence, that the loops of the parent theory are unoriented (the consequence of the reality of the adjoint representation). In addition, it is not necessary to isolate and “pair” together, from the very beginning, contours of the opposite orientation, as shown in Fig. 3. One can let the sum run over all contours independently. This will lead to untangling the terms \( (\text{Tr} U)^2 \) and \( (\text{Tr} U^\dagger)^2 \) in the first line in Eq. (12). They will appear as separate contributions. And, nevertheless, each of these separate contributions will have an equal counterpart in the parent theory.

To conclude, we proved a non-perturbative equivalence between the partition function of \( \mathcal{N} = 1 \) SYM theory and “orientifold field theory”. The equivalence holds also in the presence of certain external currents. While we do not provide an exact detailed dictionary of the “common sector” of the two theories, it is clear from our proof that correlation functions that involve powers of \( \text{Tr} F^2 \) as well as \( \text{Tr} FF^\dagger \) match at large \( N \). The bifermion operators \( \Psi \Psi \) and \( \bar{\Psi} \gamma_5 \Psi \) are also in the common sector, and so is the axial current \( \bar{\Psi} \gamma_\mu \gamma_5 \Psi \). The vector current \( \bar{\Psi} \gamma_\mu \Psi \) and the tensor operator \( \bar{\Psi} \sigma_{\mu\nu} \Psi \) do not belong to the common sector, however. From our proof it follows that the bosonic hadron spectra as well as the domain wall spectra (mass and charge) are the same in the two theories. In general, every operator in the parent theory that survives the orientifold projection belongs to the common
Finally, we would like to briefly discuss a rather obvious generalization which had been called \cite{15} flavor proliferation. Our proof of planar equivalence can be readily generalized to the case of many flavors:

\textit{SU}(N) Yang–Mills theory with }\textit{N_f} \textit{Majorana fermions in the adjoint representation (non-supersymmetric if }\textit{N_f} > 1\text{) is equivalent, in the common sector, to }\textit{SU}(N) \textit{Yang–Mills theory with }\textit{N_f} \textit{Dirac fermions in the two-index antisymmetric representation} \footnote{In general, the }\textit{m} \rightarrow 0 \text{ and }\textit{1/N} \rightarrow 0\text{ limits need not (and do not) commute. We consider here planar equivalence, namely we take the limit }\textit{1/N} \rightarrow 0\text{ first.} \textit{The common sector includes all operators built of gluon fields and covariant derivatives, and a subset of bifermion operators.}

A few explanatory remarks are in order here regarding the determination of the common sector in the multiflavor case. In selecting bifermion operators that belong to the common sector (i.e. the set of sources }J_\Psi\text{) one should exercise caution. The pattern of flavor symmetry in these two theories are drastically different. In the parent theory with the Majorana fermions, the global flavor symmetry is }\textit{SU}(\textit{N_f})\text{; it is believed to be spontaneously broken down to }\textit{SO}(\textit{N_f}), \text{ see e.g. Ref. [16], while in the daughter theory the pattern is the same as in QCD, namely}

\[ \textit{SU}(\textit{N_f})_L \times \textit{SU}(\textit{N_f})_R \rightarrow \textit{SU}(\textit{N_f})_V. \] \tag{16}

\textit{This is the reason why many fermion bilinears do not belong to the common sector.}

Operationally, it can be defined as follows. Start from the parent theory with }\textit{N_f} \textit{Majorana flavors in the adjoint. Write all possible bilinears which do not vanish by symmetry. Perform orientifoldization and find the projection of the above set to the daughter. Call this “common class” }C\text{. Alternatively, one can also start from the daughter theory. Write all possible bilinears. Examine which ones of them can be elevated to the parent theory. These should define the same class }C\text{.}

There is a large number of fermion bilinears which do not lie in }C\text{ in the parent theory, and the same is true for the daughter theory. These do not belong to the common sector.}
For fermion bilinears with no derivatives one can readily present a complete catalogue. With respect to the Lorentz symmetry they form the following representations:

\[ (0, 0), \quad \left( \frac{1}{2}, \frac{1}{2} \right), \quad (0, 1) + (1, 0). \]

The \((0, 0)\) operators in the parent theory are of the type

\[ \text{Tr} \lambda^f_\alpha \lambda^{\alpha g} \]

where \(f, g\) are the flavor indices. To have a non-vanishing operator, we must symmetrize with respect to \(f, g\). Altogether we get \(N_f(N_f + 1)/2\) operators of the type (17) plus \(N_f(N_f + 1)/2\) complex conjugate operators. Let us denote the orientifold projection of \(\lambda^f_\alpha\) as

\[ \lambda^f_\alpha \rightarrow \{ \chi^f_{[ij]}, \eta^{[ij]}_f \} \]

where \(i, j\) are the color indices and \(\eta, \chi\) are chiral (left-handed) spinors of the daughter theory. Each pair \(\chi, \bar{\eta}\) forms one Dirac flavor. It is clear that (17) projects onto \(\chi^f \eta_g + \chi^g \eta_f\). In Dirac notation the projection is onto

\[ \bar{\Psi}_f \frac{1 + \gamma_5}{2} \Psi^g, \quad f, g\text{-symmetrized}, \]

and similarly for the complex-conjugate bilinears. The \((0, 1)+\,(1, 0)\) operators in the parent theory are of the type

\[ \text{Tr} \lambda^{[f}_{\{\alpha} \lambda^g_{\beta]} \]

with symmetrized \(\alpha, \beta\) and antysymetrized \(f, g\). There are \(N_f(N_f - 1)/2\) operators (19) and the same amount of complex conjugate operators. They project onto

\[ \bar{\Psi}_f \sigma_{\mu\nu} \Psi^g, \quad f, g\text{-antisymmetrized}, \]

in the daughter theory.

Finally, the operators \((1/2, 1/2)\) are currents. Their classification is discussed in sufficient detail in Ref. [16]. The total number of currents in the parent theory is \(N_f^2\) (including one anomalous current), while the total number of currents in the daugther theory is \(2N_f^2\) (namely, \(N_f^2\) vector and \(N_f^2\)
axial currents). It is clear that a half of the daughter theory currents have no projection onto the parent one.

What currents can be projected? To answer this question we can use, again, the basic projection (18). The $N_f^2$ currents of the parent theory are

$$\bar{\chi}_{\dot{\alpha}, g} \chi_{\beta}^f, \quad f, g = 1, 2, ..., N_f.$$  \hspace{1cm} (20)

They are projected as

$$\bar{\chi}_{\dot{\alpha}, g} \chi_{\beta}^f + \bar{\eta}_{\alpha}^g \eta_{\beta, f}.$$  \hspace{1cm} (21)

The daughter theory has $2N_f^2$ currents,

$$\bar{\chi}_{\dot{\alpha}, g} \chi_{\beta}^f, \quad \text{and} \quad \bar{\eta}_{\alpha}^g \eta_{\beta, f}.$$  \hspace{1cm} (22)

Comparing Eqs. (21) and (22) we conclude that the minus combination of the currents in (22) does not make it to the common sector.

The analysis becomes even easier if we use the Majorana rather than Weyl’s representation of the adjoint spinors in the parent theory. In this case the nonvanishing $(1/2, 1/2)$ operators in the parent theory are

$$\bar{\chi}[\gamma_\mu \lambda^g], \quad \bar{\lambda}\{\gamma_\mu \gamma_5 \lambda^{g}\}.$$  \hspace{1cm} (23)

where curly and square brackets denote symmetrization and antisymmetrization, respectively. The total number of the currents (23) is $N_f^2$. Performing orientifoldization (i.e. replacing $\lambda \rightarrow \Psi$ and $\bar{\lambda} \rightarrow \bar{\Psi}$) we get $N_f^2$ currents of the daughter theory which belong to the common sector.\footnote{For instance, in this way it is easy to check that “extra” Goldstone bosons that exist in the daughter theory but are absent from the parent one \cite{15} do not belong to the common sector. They can be produced only in pairs. This contribution is subleading in $1/N$.}

Their charges generate an unconventional SU($N_f$) subgroup of SU($N_f$)$_L \times$ SU($N_f$)$_R$, containing both vector and axial transformations.

If we allow for no-derivative bifermion operators with gluon fields included, the set of allowed operators expands dramatically. We will make no attempt at a complete classification in this case. Let us give just one example. With a single insertion of the gluon field, one can build combinations

$$\text{Tr} \lambda_p^f F_{\alpha\beta} \lambda^g \varepsilon^{\rho\alpha}$$

with all possible symmetry patterns for $\gamma$, $\beta$ and $f, g$.  

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Inclusion of derivatives leads to a further enlargement of the common sector. The issue of a complete classification in this case is left for future work.

Acknowledgments We would like to thank L. Alvarez-Gaumé, R. Casalbuoni, L. Del-Debbio, P. Di Vecchia, M. Lüscher, R. Musto and A. Ritz for very useful discussions. The work of M.S. was supported in part by DOE grant DE-FG02-94ER408. A.A. is supported by the PPARC advanced fellowship.

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