We analyze the new threshold data for neutral pion photoproduction off protons in the framework of heavy baryon chiral perturbation theory. We show that large loop corrections are needed to understand the S–wave multipole $E_{0+}$ and that all pertinent low–energy constants can be understood within the framework of resonance exchange saturation. Previous inconsistencies in the description of this reaction in the threshold region are resolved.
Neutral pion photoproduction off protons has been a hot topic ever since the Saclay \cite{1} and Mainz \cite{2} groups claimed a sizeable deviation from a so-called low–energy theorem (LET) for the electric dipole amplitude $E_{0+}$ derived in 1970 \cite{3} \cite{4}. However, reexaminations of these data seemed to bring the empirical value in agreement with the theoretical prediction, see e.g. \cite{5} \cite{6}. On the theoretical side, it was shown that the low–energy theorem of Refs. \cite{3} \cite{4} is indeed incomplete \cite{7} and that the expansion of the electric dipole amplitude in powers of $\mu = M_\pi/m$ (with $M_\pi$ and $m$ the pion and the nucleon mass, respectively) is slowly converging and therefore hard to pin down accurately. The numerical closeness of the empirical value of $E_{0+}$ at threshold with the one based on the incomplete LET has led to a flurry of proposals to reinterpret or resurrect the latter (for a detailed discussion, see e.g. \cite{8}). Two new developments, however, allow us to show in this letter that indeed there is no mystery about the threshold data for $\gamma p \to \pi^0 p$ if one performs a sufficiently accurate calculation in chiral perturbation theory. First, the theoretical framework to do just that was laid out in Ref. \cite{9}, as discussed very briefly below. Second, the new data from the TAPS collaboration have now been released \cite{10} and they show some discrepancies to the previously considered best data of Beck et al. \cite{2} \cite{11}.

Let us briefly review the pertinent results of Ref. \cite{9}. In that paper, heavy baryon chiral perturbation theory \cite{13} was used to calculate the S–wave multipole $E_{0+}$ to order $q^4$ (where $q$ denotes a small momentum) and the P–wave multipoles $P_{1,2,3}$ to order $q^3$ \cite{12}. The pertinent effective Lagrangian takes the form

\begin{equation}
\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_{\pi \pi}^{(2)}
\end{equation}

where the superscript $(i)$ refers to the number of derivatives or meson mass insertions. The structure of $\mathcal{L}_{\pi N}$ is discussed in detail in the review \cite{14} and a pedagogical introduction can be found in \cite{15}. Besides the loop and pseudovector (pv) Born contributions, there are to this order two counter terms in the S–wave and one in the P–wave $P_3$,

\begin{equation}
E_{0+}(\omega) = E_{0+}^{\text{Born}}(\omega) + E_{0+}^{\text{loop}}(\omega) + e a_1 \omega M_\pi^2 + e a_2 \omega^3,
\end{equation}

\begin{equation}
P_i(\omega) = P_i^{\text{Born}}(\omega) + P_i^{\text{loop}}(\omega) \quad i = 1, 2,
\end{equation}

\begin{equation}
P_3(\omega) = P_3^{\text{Born}}(\omega) + e b_P \omega |\vec{q}| \quad (4)
\end{equation}

with $\omega$ the pion energy in the cms system, $\vec{q}$ the pion momentum and $e^2/4\pi = 1/137.036$. We have not made explicit the scale dependence of the low–energy constants $a_1$ and $a_2$. In what follows, we use $\lambda = m$. At threshold, we have $\omega_0 = M_{\pi^0} = 134.97 \text{ MeV}$. The pv Born contributions include the coupling proportional to the anomalous magnetic moment of the proton, $\kappa_p$. In the chiral counting, these stem from the dimension two Lagrangian $\mathcal{L}_{\pi N}^{(2)}$. Based on the data of Ref. \cite{2}, the three low–energy constants $a_{1,2}$ and $b_P$ could be determined by a best fit. First, the numerical value for $b_P$ can be estimated from resonance exchange \cite{17}, in this case from the $\Delta$ (in the static isobar model) and the vector mesons, $V = \rho^0 + \omega$,

\begin{equation}
b_P^{\text{reso}} = b_P^\Delta + b_P^V = (9.7 + 3.1) \text{ GeV}^{-3}
\end{equation}

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which shows that the vector meson contribution can not be neglected \[16\]. The fitted value for \(b_p\) is close to the number given in Eq.(3). However, letting the two S–wave constants \(a_1\) and \(a_2\) completely free, they turn out to be very large in magnitude but of different sign. If one restricts these coefficients again by resonance exchange, one can only vary the \(\Delta\) off-shell parameters within some bounds \(18\) and finds much smaller values for \(a_1\) and \(a_2\) (typically a factor 20 smaller than in the free fit). This signals that there are either enormous higher order corrections or that the strong energy dependence of \(E_{0^+}(\omega)\) as suggested by the data of \[2\] is incorrect. It is important to note, however, that the sum \(a_1 + a_2\) is roughly the same in both procedures. In effect, if no anomalously large coefficients appear, only this sum plays a role (in the threshold region). We also remark that the form of Eqs.(3) has lead to novel P–wave LETs for \(P_1\) and \(P_2\). These will be tested directly in polarization measurements at MAMI soon (for a somewhat model–dependent analysis, see \[19\]).

We can now use this formalism to analyze the new TAPS data of Fuchs et al. \[10\]. In Fig.1, we show the fit constrained by resonance exchange for the differential cross sections and in Fig.2 for the total cross section. For the differential cross sections, all data up to \(E_\gamma = 160\) MeV were used in the fit but only the ones up to \(E_\gamma = 152\) MeV are shown in Fig.1. Before discussing our fit parameters, we note that the differential cross sections above the \(\pi^+ n\) threshold show much less of the pronounced bell shape as inferred from the older data and that the total cross section is somewhat decreased, which is of particular relevance for the extraction of \(E_{0^+}\). Let us now discuss in more detail the fits based on the theoretical framework of Ref. \[1\]. For the \(\Delta\), we keep the two \(\gamma N \Delta\) couplings fixed, \(g_1 = g_2 = 5\). The off–shell parameter \(Y\) is severely constrained by the \(\Delta\) contribution to the magnetic polarizability of the proton, we set \(0.1 \leq Y \leq 0.14\) , i.e. \(6.4 \leq \delta \beta^p_\Delta \leq 7.5\) (in units of \(10^{-4}\) fm\(^3\)) \[21\]. Also, \(Z\) is bounded by the \(\Delta\) contribution to the \(\pi N\) scattering volume \(a_{33}, -0.4 \leq Z \leq -0.2\) \[22\]. \(X\) is varied within the range given in \[18\]. We find \(X = 2.75\) \[23\], \(Y = 0.10\) and \(Z = -0.21\) which translates into

\[
(a_1 + a_2)^{V+\Delta} = (2.67 + 3.92)\text{ GeV}^{-4} = 6.59\text{ GeV}^{-4}, \quad b_p^{RESO} = 13.0\text{ GeV}^{-3}. \tag{6}
\]

The \(\chi^2/\text{dof}\) is 2.21. We remark that the value for \(b_p\) does indeed nicely agree with the resonance saturation estimate, Eq.(4). Note that \(b_p^{RESO}\) does not depend on \(X\) and its possible values are strongly constrained by the ranges of \(Y\) and \(Z\) discussed above. It is thus gratifying that one can obtain such a consistent description of this low–energy constant. The sum \((a_1 + a_2)\) is consistent with the free fit value of 6.60 GeV\(^{-4}\) (we do not show the free fit since it is essentially the same as the resonance one). We also note that the resonance fit of Ref. \[1\] already had \(a_1 + a_2 = 6.67\text{ GeV}^{-4}\). The apparent mismatch between the free and the resonance fit discussed in \[1\] has turned out to be an artefact related to the old data.

In Fig. 3, we show the electric dipole amplitude \(E_{0^+}\). Its values at the \(\pi^0 p\) and the \(\pi^+ n\) threshold are

\[
E_{0^+}(\pi^0 p) = -1.16 \cdot 10^{-3}/M_{\pi^+}, \quad E_{0^+}(\pi^+ n) = -0.44 \cdot 10^{-3}/M_{\pi^+}, \tag{7}
\]

to be compared with \(E_{0^+}^{EXP}(\pi^0 p) = -1.31 \pm 0.08 \cdot 10^{-3}/M_{\pi^+} \[10\] and \(E_{0^+}^{EXP}(\pi^+ n) \simeq -0.4 \cdot 10^{-3}/M_{\pi^+}\) (as read off from Fig.4 of \[10\]). The value of \(a_1 + a_2\) in Eq.(4) amounts to an \(E_{0^+}\)-contribution of +0.3 from vector mesons and +0.4 from the \(\Delta\) (in units of \(10^{-3}/M_{\pi^+}\)). Almost the same number for the sum of vector meson and nucleon resonance contributions
to $E_{0+}$ at threshold is reported in Ref. [24] (see also [25]). In the threshold region, the shape for $E_{0+}(\omega)$ shown in Fig. 3 can be well represented by a two–parameter fit of the form (as discussed in some detail in Ref. [9])

$$E_{0+}(\omega) = -a - b \sqrt{1 - \omega^2/\omega_c^2},$$

with $\omega_c = 140.11$ MeV the pion energy at the $\pi^+n$ threshold. We find $a = 0.44 \cdot 10^{-3}/M_{\pi^+}$, $b = 2.9 \cdot 10^{-3}/M_{\pi^+}$ and $b_P = 12.9$ GeV$^{-3}$. The $\chi^2$/dof is 2.22, i.e. almost identical to the one of the resonance fit. This indicates that the mild slope of $\text{Re}E_{0+}(\omega)$ behind the $\pi^+n$ threshold is not significant. The value for $b$ is somewhat below the one estimated from the Fermi–Watson theorem, $b_{FW} = 3.7 \cdot 10^{-3}/M_{\pi^+}$. Note, however, that this is based on the assumption of exact isospin symmetry, whereas the clearly visible cusp effect in $E_{0+}(\omega)$ is due to the pion mass difference, i.e. an isospin–violating effect. A more consistent treatment of such effects is certainly needed. For a study of the cusp effect in $E_{0+}$ in terms of a multi–channel S–matrix, see Ref. [26].

For larger energies, however, the new SAL data agree with the older Mainz data [2]. This does not affect the threshold value of $E_{0+}$ but rather leads to a larger value of $b_P$. The experimental discrepancy remains to be clarified.

For the respective slopes of the P–wave multipoles, the LETs together with the best resonance fit give $P_1/|\vec{q}| = 0.480$ GeV$^{-2}$, $P_2/|\vec{q}| = -0.512$ GeV$^{-2}$ and $P_3/|\vec{q}| = 0.544$ GeV$^{-2}$, i.e. all P–waves $P_{1,2,3}$ are of the same magnitude close to threshold. Consequently, the photon asymmetry $\Sigma(\theta)$ to be measured at MAMI is expected to be small in the threshold region since $\Sigma \sim (|P_3|^2 - |P_2|^2)$.

To summarize, we have shown that within the framework of chiral perturbation theory, one is able to consistently understand the new threshold data for the reaction $\gamma p \rightarrow \pi^0p$. Loop effects are clearly visible in the S–wave. Furthermore, the three low–energy constants are fully understood within the framework of resonance saturation. In contrast to common folklore, these resonances pose no problem and do not have to be treated as dynamical degrees of freedom (as long as one stays in the threshold region). In the next step, this formalism should be extended to electroproduction to discuss the new data from NIKHEF [28] and MAMI [29].

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\[ P_1 = 3E_{1+} + M_{1+} - M_{1-} , \quad P_2 = 3E_{1+} - M_{1+} + M_{1-} \quad \text{and} \quad P_3 = 2M_{1+} + M_{1-} . \]

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FIGURES

Fig.1 Differential cross sections in the threshold region (in nb/sr) for lowest 9 values of the photon lab energy $E_\gamma$ versus the cm scattering angle $\theta$. The solid line is the best resonance fit, the data are from \[10\].

Fig.2 Total cross section in the threshold region (in $\mu$b) versus $E_\gamma$. For notations, see Fig.1.

Fig.3 The real part of the electric dipole amplitude in the threshold region.
Figure 1
Figure 2

Figure 3