Coherence and mode decomposition of weak twin beams

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Abstract. Properties of weak spatio-spectral twin beams in paraxial approximation are analyzed using the decomposition into appropriate paired modes. Numbers of paired modes as well as numbers of modes in the signal (or idler) field in the transverse wave-vector and spectral domains are analyzed as functions of pump-beam parameters. Spatial and spectral coherence of weak twin beams is described by auto- and cross-correlation functions. Relation between the numbers of modes and coherence is discussed.

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1. Introduction

Generation of photon pairs by spontaneous parametric down-conversion \[1\] belongs to the most frequently studied nonlinear processes in optics. The reason is entanglement of photons constituting a photon pair. It represents a purely quantum property. Entanglement leads to nonclassical cross-correlation functions between the signal and idler fields \[2\] that influence many physical effects. Nonclassical multiple coincidence-count rates used, e.g., for tests of the Bell inequalities \[3\] \[4\] or quantum teleportation \[5\] represent the most important manifestation of these correlations. Unusual entangled two-photon absorption with its entanglement-induced transparency \[6\] \[7\] and virtual state two-photon spectroscopy \[8\] is another example of the large impact of entanglement of photons. Also ghost imaging has to be mentioned here \[9\]. At present, there already exist several applications based upon entangled photon pairs including quantum key distribution \[10\], ultra-fast measurements \[11\] and absolute detector calibration \[12\] \[13\].

This wide use of entangled photon pairs has naturally stimulated investigations of a detailed structure of photon pairs. It has been shown that their properties are fully described by two-photon (spectral, temporal, spatial) amplitudes at the level of individual photon pairs \[14\] \[15\]. Moreover, the Schmidt decomposition of a two-photon amplitude \[16\] \[17\] has been found crucial for the quantification of entanglement by the Schmidt number. We note that no physical quantity is directly related to entanglement but entanglement influences even qualitatively the physical behavior of photon pairs. As the experimental determination of the Schmidt number \[18\] \[19\] and profiles of the modes \[20\] \[21\] \[22\] is difficult, an alternative approach based on the measurement of field width and width of the corresponding intensity cross-correlation function has been developed giving the Fedorov ratio \[23\] \[24\] as a good quantifier of entanglement.

In the last years, a greater deal of attention has been devoted to fields composed of many photon pairs arising in parametric down-conversion \[25\] \[26\] \[27\] \[28\]. Properties of these twin beams reflect those of the individual photon pairs \[29\], at least for not very intense twin beams \[30\]. For characterizing twin beams, auto-correlation functions are important as well as the cross-correlation functions \[31\]. Auto-correlation functions then give, according to statistical optics \[32\], the information about the number of modes constituting the signal (or idler) part of the twin beam. In analogy with the definition of Fedorov ratio, this number of modes can be determined by the ratio of field width and width of the corresponding auto-correlation function. Considering weak twin beams, the auto-correlation functions can even be derived from the two-photon amplitudes describing photon pairs.

In this contribution, we determine side by side the spatial and spectral auto- and cross-correlation functions as they depend on pump-field parameters. Using these functions, we obtain the number of modes in the signal (or idler) field and compare it with the Schmidt number giving the number of paired modes \[24\].

The paper is organized as follows. Theory suitable for describing weak twin beams is presented in Sec. 2. Sec. 3 is devoted to the properties of twin beams in the transverse wave-vector plane. Spectral properties of the twin beams are discussed in Sec. 4. Sec. 5 brings conclusions.

2. Theory of weak twin beams

Parametric down-conversion in a medium with tensor $d$ of the second-order nonlinear coefficients is characterized by momentum operator $G_{\text{int}}$ written as follows \[33\] \[34\]:

$$G_{\text{int}}(z) = 4\epsilon_0 \int dx dy \int_{-\infty}^{\infty} dt \left[ d : E_p^{(+)}(r,t)\hat{E}_s^{(-)}(r,t)\hat{E}_i^{(-)}(r,t) + \text{h.c.} \right];$$ \[(1)\]

where $r = (x,y,z)$. Symbol $E_p^{(+)}$ denotes the positive-frequency part of the classical pump electric-field amplitude and $\hat{E}_s^{(-)}$ \[\hat{E}_i^{(-)}\] means the negative-frequency part of the signal- \[\text{idler-}\] field operator amplitude. Permittivity of vacuum is denoted as $\epsilon_0$, symbol : is shorthand for tensor shortening with respect to its three indices and h.c. replaces the Hermitian conjugated term.

Amplitudes of all three interacting fields can be decomposed into harmonic plane waves with wave vectors $k_a$ and frequencies $\omega_a$:

$$E_a^{(+)}(r,t) = \frac{1}{\sqrt{2\pi}} \int d k_a \, E_a^{(+)}(k_a) \exp(i k_a r - i \omega_a t),$$ \[a = p, s, i.\] \[(2)\]
In the considered paraxial approximation, the plane wave with wave vector \( k_s \) is conveniently parameterized by its frequency \( \omega_s \) and transverse wave vector \( k^\perp_s \).

The quantum signal and idler spectral-field amplitudes \( \hat{E}_a^{-\perp}(k^\perp_s, \omega_a) \) can then be expressed in terms of the appropriate creation operators \( \hat{a}^{-\dagger}(k^\perp_s, \omega_a)\):

\[
\hat{E}_a^{-\perp}(k^\perp_s, \omega_a) = -i \sqrt{\frac{\hbar \omega_a^2}{2 k_a^2 \omega}} \hat{a}^{-\dagger}(k^\perp_s, \omega_a); \tag{3}
\]

\( \hbar \) is the reduced Planck constant and \( c \) is the speed of light in vacuum; \( k_a = |k_a| \).

Correlations in the transverse wave-vector planes of the signal and idler fields are described by the following function \( T_L \) depending on the pump-field transverse spectral power \( E_p^\perp(k^\perp_p) \):

\[
T_L(k^\perp_s, k^\perp_p) = E_p^\perp(k^\perp_p + k^\perp_s) \times \exp \left(-i \frac{|k^\perp_p|^2}{2k_p} - \frac{|k^\perp_p|^2}{2k_s} \right) \times \text{sinc} \left( \frac{|k^\perp_p|^2}{2k_p} - \frac{|k^\perp_p|^2}{2k_s} \right); \tag{4}
\]

\( \text{sinc}(x) \equiv \sin(x)/x \) and \( \delta \) stands for the Dirac \( \delta \)-function. In Eq. (4), \( |k^\perp_s|^2 = k_{a,x}^2 + k_{a,y}^2 \) and \( L \) denotes the crystal length.

We assume that the emitted signal and idler fields have the radial symmetry. This is a good approximation for sufficiently narrow spatial spectral profiles \( E_p^\perp \) of the pump field. The pump field in the considered type-I nonlinear interaction propagates as an extraordinary wave and so its index \( n_p \) of refraction changes linearly with the radial emission angle \( \theta_p \equiv \arcsin(|k^\perp_p|/k_p) \) in the plane containing the propagation direction and the crystal optical axis. This dependence introduces anisotropy into the generated signal and idler fields \([35][36][37]\).

Assuming the Gaussian transverse pulse profile with radius \( w_p \) \( (E_p^\perp(k^\perp_p)) = w_p^2/\sqrt{2\pi} \text{exp}[-w_p^2|k^\perp_p|^2/4]/4 \) the anisotropy is well developed for narrow transverse pump profiles with wide spatial spectra. Anisotropy considerably modifies the emitted signal and idler fields for the pump-field radii \( w_p \) comparable or smaller than \( w_p^a \) (for more details, see \([35]\)):

\[
w_p^a = \frac{1}{n_p} \left| \frac{d n_p(\omega_p^0, \theta_p)}{d \theta_p} \right| \frac{L}{x_e}; \tag{5}
\]

\( x_e \approx 2.2 \left[ \sin(x_e)/x_e = 1/e \right] \).

The assumed radial symmetry qualitatively simplifies the description as it allows to decompose the function \( T_L \) into the dual Schmidt basis in both radial and azimuthal directions in the transverse planes:

\[
T_L(k^\perp_s, k^\perp_p) = \frac{1}{2\pi \sqrt{k^\perp_p k^\perp_s}} \sum_{m=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \lambda_{ml}^s \times t_{s,ml}(k^\perp_s, \varphi_s) t_{i,ml}(k^\perp_p, \varphi_i), \tag{6}
\]

\( t_{s,ml}(k^\perp_s, \varphi_s) = u_{s,ml}(k^\perp_p) \text{exp}(im\varphi_s), \)

\( t_{i,ml}(k^\perp_p, \varphi_i) = u_{i,ml}(k^\perp_p) \text{exp}(-im\varphi_i). \)

Whereas the functions \( u_{a,ml} \) describe the radial parts of the fields, the harmonic functions \( \text{exp}(im\varphi_a)/\sqrt{2\pi} \) are appropriate for the azimuthal parts of the fields, \( a = s,i \). Symbols \( \lambda_{ml}^s \) denote the Schmidt numbers and \( t^\perp \) is the normalization constant.

Unitary transformations of the field operators,

\[
\hat{a}_{a,ml}(\omega_a, z) = \int_0^\infty dk^\perp_a \int_0^{2\pi} d\varphi_a \hat{a}_{a,ml}(k^\perp_a, \varphi_a) \times \text{exp}(im\varphi_a)\lambda_{ml}^a(\omega_a, \varphi_a, \omega, z), \quad a = s, i, \tag{7}
\]

then allow us to write the first-order perturbation solution of the Schrödinger equation as follows:

\[
|\psi\rangle_{\text{out}} = t^\perp \sum_{m,l} \lambda_{ml}^s \int_0^\infty d\omega_s \int_0^\infty d\omega_i \text{exp}(-i\omega_i) F_L(\omega_s, \omega_i) \times \hat{a}_{s,ml}(\omega_s, 0)\hat{a}_{i,ml}^\dagger(\omega_i, 0)|\text{vac}\rangle; \tag{8}
\]

\( |\text{vac}\rangle \) is the incident vacuum state. The two-photon spectral amplitude \( F_L \) introduced in Eq. (8) is determined by the formula

\[
F_L(\omega_s, \omega_i) = \frac{2id_{\text{eff}}L}{\sqrt{2\pi}} \frac{\omega_s^2}{\omega_i^2} E_p^\perp(\omega_s + \omega_i) \times \exp \left(-i|k_p(\omega_s + \omega_i) - k_s(\omega_s + \omega_i)|L/2 \right) \times \text{sinc} \left(|k_p(\omega_s + \omega_i) - k_s(\omega_s + \omega_i)|L/2 \right), \tag{9}
\]

where \( E_p^\perp \) stands for the pump-field spectrum and \( d_{\text{eff}} \) denotes an effective nonlinear coupling constant. The Schmidt decomposition of two-photon amplitude \( F_L \),

\[
F_L(\omega_s, \omega_i) = f^\perp \sum_{q=0}^{\infty} \lambda_{q}^s f_{s,q}(\omega_s) f_{i,q}(\omega_i), \tag{10}
\]

and introduction of new field operators \( \hat{a}_{a,mlq} \),

\[
\hat{a}_{a,mlq} = \int_0^\infty d\omega_s f_{a,s,q}(\omega_s) \hat{a}_{a,ml}(\omega_s, 0), \quad a = s, i, \tag{11}
\]

allows to rearrange the output state \( |\psi\rangle_{\text{out}} \) into the form:

\[
|\psi\rangle_{\text{out}} = t^\perp f^\perp \sum_{m,l,q} \lambda_{ml}^s \lambda_{mlq}^s \hat{a}_{s,mlq}^\dagger |\text{vac}\rangle. \tag{12}
\]

Formula (12) represents the output state \( |\psi\rangle_{\text{out}} \) decomposed into independent paired spatial and spectral modes. We note that there occurs degeneracy in the signal and idler mode structure for \( m = 0 \) owing to the radial and spectral symmetry of the signal and idler fields. Due to this degeneracy, we cannot distinguish a signal and an idler photons in modes with \( m = 0 \) as both photons are created in one spatio-spectral mode. This considerably modifies properties of the generated paired fields in the collinear geometry \([38]\). On the other hand, non-collinear geometries are practically unaffected by this degeneracy as the relative
The emitted signal (or idler) field is characterized by its intensity profiles along variables \( k_s^\perp \), \( \varphi_s \) and \( \omega_s \). Its internal correlations are described by amplitude correlation functions in these variables. The mutual signal and idler correlations are characterized by intensity cross-correlation functions in these variables. In detail, intensity profile \( n_{s,k} \) (expressed in photon-number density) \( [n_{s}(k_s^\perp) \approx \langle \hat{a}^\dagger_{s}(k_s^\perp)\hat{a}_{s}(k_s^\perp) \rangle] \) of the signal field in the radial wave-vector direction is derived from the function \( T_L \) given in Eq. (11):

\[
n_{s,k}(k_s^\perp) = k_s^\perp \int_0^\infty dk_s^\perp k_s^\perp \times |T_L(k_s^\perp, \varphi_s = 0, k_s^\perp, \varphi_i = \pi)|^2. \tag{13}
\]

Using Eq. (9) the signal-field intensity spectrum \( n_{s,\omega} \) given by the formula

\[
n_{s,\omega}(\omega_s) = \int_0^\infty d\omega_i |F_L(\omega_s, \omega_i)|^2. \tag{14}
\]

The amplitude signal-field correlations in their radial \( [A_{s,k}^a(k_s^\perp, k_s^\perp) \approx \langle \hat{a}^\dagger_{s}(k_s^\perp)\hat{a}_{s}(k_s^\perp) \rangle] \) and spectral \( [A_{s,\omega}^a] \) variables are described by the corresponding auto-correlation functions as given:

\[
A_{s,k}^a(k_s^\perp, k_s^\perp) = \sqrt{k_s^\perp k_s'^\perp} \int_0^\infty dk_s^\perp k_s^\perp T_L(k_s^\perp, \varphi_s^0 = 0, k_s^\perp, \varphi_i^0 = \pi)
\]

\[
A_{s,\omega}^a(\omega_s, \omega_s') = \int_0^\infty d\omega_i |F_L(\omega_s, \omega_i)|^2. \tag{15}
\]

The intensity correlations between the signal and idler fields in their radial \( [C_{s,k}, C_{s,k'}(k_s^\perp, k_s'^\perp) \approx \langle \mathcal{N} : \hat{a}^\dagger_{s}(k_s^\perp)\hat{a}_{s}(k_s^\perp)\hat{a}^\dagger_{s}(k_s'^\perp)\hat{a}_{s}(k_s'^\perp) \rangle : \mathcal{N}] \) and spectral \( [C_{s,\omega}] \) variables are characterized by the following cross-correlation functions:

\[
C_{s,k}(k_s^\perp, k_s'^\perp) = k_s^\perp k_s'^\perp |T_L(k_s^\perp, \varphi_s^0 = 0, k_s'^\perp, \varphi_i^0 = \pi)|^2, \\
C_{s,\omega}(\omega_s, \omega_s') = \int_0^\infty d\omega_i |F_L(\omega_s, \omega_i)|^2. \tag{16}
\]

Similar quantities as written in Eqs. (13–16) for the signal field can be defined also for the idler field.

The number of effectively populated paired modes in a twin beam is determined by the Schmidt number \( K \) defined as

\[
K = \frac{1}{\sum q \lambda_q^2}, \tag{17}
\]

using eigenvalues \( \lambda_q \) of the Schmidt decomposition of two-photon amplitude normalized such that \( \sum_q \lambda_q^2 = 1 \).

On the other hand, the number \( K_b^A \) of modes constituting field \( b \), \( b = s, i \), in a given variable is quantified by the ratio of appropriate intensity width \( \Delta n_b \) of field \( b \) and width \( \Delta A^b \) of the amplitude autocorrelation function introduced in Eq. (15).

\[
K_b^A = \frac{\Delta n_b}{\Delta A^b}, \quad b = s, i. \tag{18}
\]

3. Spatial properties of weak twin beams

We consider a BBO crystal 8-mm long cut for non-collinear type-I process (eoO) for the spectrally-degenerate interaction among the wavelengths \( \lambda_s^0 = 349 \text{ nm and } \lambda_i^0 = 698 \text{ nm} \) (\( \theta_{\text{BBO}} = 36.3 \text{ deg} \)). The pump field is provided by the third harmonics of the Nd:YLF laser at the wavelength 1.047 \( \mu \text{m} \). Both the Gaussian transverse profile with radius \( w_p \) and the Gaussian spectrum of the pump pulse with duration \( \tau_p \) are considered in the calculations. Assuming the pump field at normal incidence, the signal and idler fields at the central frequencies \( \omega_s^0 \) and \( \omega_i^0 \) propagate outside the crystal under the radial emission angles \( \vartheta_s = \vartheta_i = 8.45 \text{ deg} \). As this configuration is symmetric with respect to the exchange of the signal and idler fields, we restrict our discussion to only the signal field. We assume in the discussion that the conditions are such that the spectral and spatial properties of the twin beams factorize.

We first pay attention to the properties of twin beams in the wave-vector transverse plane. We assume in accord with the developed model that the twin beam has the rotational symmetry around the \( z \) axis and so it is stationary in the azimuthal angle \( \varphi \). Formula (4) suggests that the rotational symmetry is roughly observed for the pump-field radius \( w_p \) larger than \( w_s^0 \) that equals 270 \( \mu \text{m} \) for the 8-mm long crystal and the considered geometry \( |n_p = 1.568, |dn_p/d\vartheta_p| \text{ } = 1.123 \). The pump-field radius \( w_p \) is the crucial parameter that determines the properties of twin beams in the transverse plane. It gives the number \( K_{k_s} \) of independent transverse modes comprising the twin beam. In our configuration, the greater the value of radius \( w_p \), the greater the number \( K_{k_s} \) of independent modes, as documented in Fig. 1. The number \( K_{k_s} \) of overall modes in the transverse wave-vector plane can approximately be factorized into the number \( K_k \) of modes in the radial direction and the number \( K_{\varphi} \) of modes in the azimuthal direction. This factorization arises from the fact that the number \( K \) of radial modes determined for a fixed value of the azimuthal number \( m \) depends only weakly on the number \( m \). The
more strict phase-matching conditions which results in greater values of the Schmidt numbers $K$ compared to those shown in Fig. 1. Anisotropy and its role in photon-pair generation has been discussed in detail in [35].

Numbers $K^\Delta$ of modes in the signal field determined along Eq. (18) are compared with the numbers $K$ of paired modes from the Schmidt decomposition in Fig. 1. This comparison reveals that the values of $K^\Delta$ are only approx. by 40% lower than the values of $K$. The numbers $K^\Delta$ of effectively populated signal-field modes are in general smaller than the corresponding Schmidt numbers $K$ as the widths of auto-correlation functions occurring in definition (18) are not able to fully take into account the complex structure of entanglement present in a twin beam [18, 19]. In other words, numbers $K$ of paired modes and numbers $K^\Delta$ of signal-field modes are defined differently. However, their comparison done in Fig. 1 clearly reveals that both of them are suitable for quantifying the dimensionality of a twin beam.

As the range of radial angles $\theta_s$ belonging to the emitted photons practically does not change with the pump-field radius $w_p$ and the allowed azimuthal emission angles $\phi_s$ lie in interval $(0, 2\pi)$, an increase of the number $K^\Delta_{s,k_{0}}$ of signal-field modes with the radius $w_p$ is caused by the decrease of the radial and azimuthal widths of effective modes given by the extension of amplitude auto-correlation functions $A^s_{e,k}$ and $A^\phi_{e,\phi}$ defined in Eqs. (15). The dependence of widths $\Delta A^s_{e,k}$ and $\Delta A^\phi_{e,\phi}$ of their intensity counterparts on the radius $w_p$ is shown in Figs. 3(a) and 3(b), respectively, and confirms this behavior. Profiles $A^s_{e,k}$ and $A^\phi_{e,\phi}$ giving the signal-field intensity correlations for the pump field 1-mm wide are drawn in Figs. 3(b) and 3(b).

The generation of signal and idler fields by photon pairs strongly correlated in the transverse plane results in...
4. Spectral properties of weak twin beams

The behavior of twin beam in the spectral domain and under the considered conditions is more complex compared to the spatial domain. The number \( K_\omega \) of paired spectral modes attains its minimum when considered as a function of the pump-field spectral width \( \Delta \lambda_p \). The number \( K_\omega \) of paired spectral modes is larger (\( K_\omega \approx 70 \)) even in this minimum reached for the pump pulse with the spectrum approx. 0.2 nm wide, as documented in Fig. 5. It holds also in the spectral domain that the number \( K_\omega \) of signal-field modes determined from the ratio defined in Eq. (18) is smaller than the number \( K_\omega \) of paired modes arising from the Schmidt decomposition (see Fig. 3). The comparison of curves in Fig. 5 confirms that both the number \( K_\omega \) of paired modes and the number \( K_\omega \) of signal-field modes are suitable for quantifying dimensionality of the twin beam in a broad range of pump-field spectral widths.

The dependence of the number \( K_\omega \) of signal-field modes on the pump-field spectral width \( \Delta \lambda_p \) as drawn in Fig. 5 can be explained by the behavior of spectral intensity width \( \Delta n_{s,\omega} \) and width \( \Delta A_{s,\omega} \) of intensity auto-correlation function. Whereas the spectral intensity width \( \Delta n_{s,\omega} \) monotonically increases as a function of \( \Delta \lambda_p \) [see Fig. 6(a)], the width \( \Delta A_{s,\omega} \) of intensity auto-correlation function increases for lower values of \( \Delta \lambda_p \) and then it saturates [see Fig. 6(b)].

This results in an increase of the values of \( K_\omega \) for larger values of \( \Delta \lambda_p \) observed in Fig. 6. Modes of the spectral decomposition behave in the same manner as the modes in the radial direction. Thus, the intensity profile of a \( q \)-th mode has \( q \) zeros and \( q + 1 \) peaks and extends over all frequencies found in the spectrum [see Fig. 7(a)]. As a consequence of the conservation law of energy, the frequencies of the emitted signal and idler photons are correlated within the spectral interval given by the pump-field spectral width. However,
intensity auto-correlation function (plain curve) and ∆

depend on pump-field spectral width ∆

λ

of intensity cross-correlation function (dashed curve) as they

in the description of a twin beam. This behavior

A

field. (b) Signal-field spectral intensity auto-correlation

function shows that the intensity auto-correlation function

as that used in the transverse wave-vector domain

inside the individual fields. The same argumentation

signal and idler fields as well as among the intensities

determines correlations between the intensities of the

no further increase is possible [see Fig. 6(b)]. This

K

and

K

is, the lower the numbers

filtering. It suggests that the narrower the filter width

also that profiles of these correlation functions plotted

in Fig. 7(b) are similar.

The picture that the overall spectrum is composed

of adjacent independent ‘local’ modes is useful in

predicting the behavior of numbers Kω of paired modes

and K∆

ω,s,kϕ of signal-field modes with respect to spectral

filtering. It suggests that the narrower the filter width

is, the lower the numbers Kω and K∆

ω,s,kϕ of modes needed

in the description of a twin beam. This behavior

has been confirmed numerically. A sufficiently strong

filtering then allows to reach a twin beam with the

number of modes approaching one [43]. The same

conclusions are valid for the geometric filtering that

reduces the numbers Kω,ϕ and K∆

ω,s,kϕ of modes in the transverse wave-vector plane.

5. Conclusions

Weak spatio-spectral twin beams have been analyzed in

paraxial approximation using the perturbation solution of the Schrödinger equation and its decomposition into

the spatial and spectral paired modes. Applying these

modes, coherence properties of weak twin beams have

been studied using both the auto- and cross-correlation

functions. Numbers of paired modes revealed by the

Schmidt decomposition and numbers of modes

constituting the signal (or idler) field and given by the

ratio of field width and width of the appropriate

amplitude auto-correlation function have been found

mutually proportional in the broad area of the analyzed

pump-field parameters. This justifies both of them

as appropriate quantifiers of dimensionality of weak

twin beams. Whereas the number of transverse

modes increases with the increasing pump-field radius

in our configuration, the number of spectral modes

considered as a function of pump-field spectral width

has a well-formed minimum. This behavior has

been explained analyzing the widths of appropriate

correlation functions. It has been shown that the spectral

and transverse wave-vector cross-correlation functions

are broader than the corresponding auto-


correlation functions.

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