Coupled elasto-electromagnetic waves in bounded piezoelectric structures

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Abstract. The work studies theoretically the effect of electromagnetic wave generation on the acoustic wave reflection/transmission in anisotropic materials possessing piezoelectric properties. We are concerned with quasi-normal incidence at angles $\theta_i \leq v_a/v_{el} \sim 10^{-3} \div 10^{-5}$, where $v_a$ and $v_{el}$ are the typical velocities of the acoustic and electromagnetic waves. It is shown that electromagnetic and acoustic waves are able to interact strongly despite a huge difference in velocities so that the wave behavior of time-dependent electric fields can drastically change the coefficients of mode conversion. In particular, examples exist of the situations where the acoustic wave must be totally reflected but quasi-electrostatic calculations predict almost total transmission.

1. Introduction

The theory of acoustic wave propagation in piezoelectric materials commonly rests on the quasi-electrostatic (QE) description of electric fields accompanying elastic deformations [1, 2]. This approximation uses the fact that the sound velocity $v_a$ is several order of magnitude smaller than the velocity $v_{el}$ of electromagnetic waves and the electric field is characterized by means of time-dependent electric potential. The electromagnetic (EM) correction to the acoustic wave velocity appears to be of the order $(v_a/v_{el})^2$.

Considering the reflection-transmission problems in piezoelectric media within the frame of QE approximation, we find that the incident plane wave usually converts in each medium into three homogeneous or inhomogeneous "acoustic" modes and a fourth inhomogeneous mode often referred to as the electrostatic, or Coulomb, mode. Generally speaking, the QE approximation is absolutely adequate for solving the reflection-transmission problems. The exception is the incidence under small angles $\theta_i$ of the order of $v_a/v_{el}$ to the normal to the interface. In this case, when striking the interface, the incident elastic wave can be converted into EM waves. The reason is that the synchronism condition requiring the equality of the tangential component $k$ of the wave vectors can be fulfilled at $\theta_i \sim v_a/v_{el}$. Besides, the QE approximation can be used provided that the typical distance of electric field variations in space is far smaller than the length of EM waves. If $\theta_i \gg v_a/v_{el}$, then the penetration depth of the Coulomb mode has the order of the acoustic wave-length and is smaller by a factor $v_a/v_{el}$ than the EM wave-length. But the penetration depth of electric fields tends to infinity when the angle of incidence decreases [3]. Thus, the QE approximation is not totally applicable to the reflection-transmission problems at
quasi-normal incidence. It does not describe electric fields generated by the incident wave on the surface.

EM effects in the reflection-transmission phenomena has been discussed in Ref. [4]. The authors have investigated theoretically the reflection of the SH acoustic wave from the hexagonal crystal - vacuum interface. They have found that the acoustic wave can be totally converted into EM waves provided that the velocities of EM waves in the crystal and the vacuum coincide. In the present paper we consider more thoroughly the role of the coupling between EM and acoustic waves. Our analysis is based on the matrix approach put forward in Refs. [5]. It generalizes the method developed in Refs. [6] as applied to the QE approximation. In section 2 we briefly review the derivation of the $10 \times 10$ matrix introduced in [5]. Afterwards, we consider how the EM effects influence the coefficients of mode conversion.

2. Eigenvalue problem
Consider a half-infinite medium. The coordinate axes $x$ and $y$ lie in the surface. The axis $z$ is directed along the unit internal normal $n$ to the surface. The unit vector $m$ is directed along the axis $x$. Linear acoustic fields in non-magnetic piezoelectric materials are described by the elastodynamic equation and Maxwell’s equations in conjunction with constitutive equations. We are interested in the partial solutions of the form $f(z) \exp[i(kz - \omega t)]$, where the functions $f(z)$ specify the $z$-dependence of the plane mode characteristics.

The unknown functions $f(z)$ can be found from a system of ten first-order ODEs [5]. For our purposes it is convenient to construct the unknown ten-component column vector $\xi(z)$ as follows: $\xi = (U, V)^t$, where $U = (A, \mathcal{E}_x, \mathcal{E}_y)^t$ and $V = (\mathcal{F}, \mathcal{H}_y, \mathcal{H}_x)^t$ are five-component vector columns involving three components of the polarization vector $A = (A_x, A_y, A_z)^t$ of mechanical displacement, three components of the traction $\mathcal{F} = i\hat{\sigma} \cdot n = (\mathcal{F}_x, \mathcal{F}_y, \mathcal{F}_z)^t$, and four quantities related to the $x$- and $y$-components of the electric $E$ and magnetic $H$ fields : $\mathcal{E}_{x,y} = iE_{x,y}/\omega$, $\mathcal{H}_y = -iH_y$, and $\mathcal{H}_x = iH_x$. The symbol $\hat{\sigma}$ stands for the mechanical stress tensor and $t$ means transposition. By expressing the $z$-components of $E$ and $H$ as well as the tangential projection of stress tensor $\hat{\sigma} \cdot m$ in terms of $A, \mathcal{F}, \mathcal{E}_{x,y},$ and $\mathcal{H}_{x,y}$, and combining the relations thus obtained, we eventually arrive at the equation

$$\frac{1}{i} \frac{d \xi}{dz} = \hat{N} \xi,$$

where $\hat{N}$ is a $10 \times 10$ real matrix.

The solutions of Eq. (1) are sought in the form $\xi(z) = \xi_\alpha \exp[ip_\alpha z]$, where $\xi_\alpha$ and $p_\alpha$ are eigenvectors and eigenvalues of $\hat{N}$, respectively: $\hat{N} \xi_\alpha = p_\alpha \xi_\alpha, \quad \alpha = 1, \ldots, 10$. The subscript $\alpha$ labels the partial modes. The eigenvalues are the normal components of the wave vectors of partial modes. Thus, given $\omega$ and the tangential projection $k$ of the wave vectors, Eq. (1) has ten partial solutions describing coupled elasto-electromagnetic fields. With piezoelectric effect excluded, this set of modes is split into six elastic modes and four electromagnetic modes. Omitting the explicit expression of $\hat{N}$ we note that the connection between the exact description of ac electric fields and the QE approximation is discussed in Ref [7].

3. Reflection and transmission at quasi-normal incidence
The EM effects generally yield small corrections to the results obtained on the basis of QE approximation. However, as it has been noted in Introduction, the QE approximation is not valid in the case of quasi-normal incidence so that we pass on to discuss how EM effects influence the reflection and transmission of plane waves in the vicinity of the normal to the surface.

Consider a half-infinite piezoelectric material of symmetry $6mm$ characterized by the elastic constants $c_{ij}$, the piezoelectric coefficients $e_{ij}$, the dielectric permittivity $\varepsilon_i$, and the density $\rho$. The axis 6 is parallel to the coordinate axis $y$. We are concerned with pure shear horizontal
(SH) modes propagating in the coordinate plane $xz$. Due to the simplifications entailed by symmetry the elements of the matrix $N$ involved in the problem form only a $4 \times 4$ matrix and the corresponding eigenvectors contain only four components: $\xi = (A_y, \xi_x, F_y, H_y)^t$.

We find two "acoustic" ($\alpha = 1, 3$) and two "EM" ($\alpha = 2, 4$) solutions with the normal components of the wave vector $p_1 = \sqrt{\omega^2/v_a^2 - k^2}$, $p_2 = \sqrt{\omega^2/v_{a1}^2 - k^2}$ and $p_{3,4} = -p_{1,2}$, where $v_a = (c_{55}^D/\rho)^{1/2}$, $v_{a1} = (\epsilon_1\mu_0)^{-1/2}$, $c_{55}^D = c_{55} + \epsilon_{15}^2/\epsilon_1$, and $\mu_0$ is the magnetic constant. In what follows, we use $\xi_\alpha$ in the form

$$\xi_1 = i||\xi_1||(1, k e_{15}/\omega \epsilon, -p_1 c_{55}^D, 0)^t, \quad \xi_2 = (0, -p_2/\omega^2 \epsilon, -k e_{15}/\omega \epsilon, 1)^t,$$

where $||\xi_1|| = (p_1 c_{55}^D)^{-1/2}$. To obtain $\xi_{3,4}$, we should substitute $-p_{1,2}$ for $p_{1,2}$ and omit the imaginary unit before $||\xi_1||$.

The modes $\alpha = 1, 2$ are reflected and $\alpha = 3, 4$ are incident for the half-space $z > 0$. It is seen that the $y$-component of the displacement vanishes in the "EM" modes, like in the Coulomb modes when the quasi-static approximation is used. The $y$-component of the magnetic field is zero for "the acoustic" modes. Thus, the EM description does not modify anyhow SH acoustic modes in our case.

Let the SH "acoustic" wave $\alpha = 3$ be incident on the perfect rigid contact of hexagonal crystals 1 and 2 from crystal 1. It is reasonable to expect the contribution of EM effects to be more pronounced if the reflection occurs exclusively because of the piezoelectric properties of the structure. In this connection, we assume that crystals 1 and 2 are identical and oriented such that their axes 6 are anti parallel ("180°-domain" structure). Hence, the bi-crystal is characterized by the material constants $c_{55}^{(1)D} = c_{55}^{(2)D} = c^D$, $\epsilon_1^{(1)} = \epsilon_1^{(2)} = \epsilon$, $\rho^{(1)} = \rho^{(2)} = \rho$, and $\epsilon_{15}^{(1)} = -\epsilon_{15}^{(2)} = \epsilon$ with respect to the common coordinate frame (the superscript (1) and (2) will label the parameters of crystals 1 and 2, respectively). Accordingly, $v_{a1}^{(1)} = v_{a1}^{(2)} = v_a$, and $v_{el}^{(1)} = v_{el}^{(2)} = v_{el}$, and also $p_0^{(1)} = p_0^{(2)} = p_0$. We evaluate the reflection coefficient:

$$R_a = -ik^2 \tau^2/(k^2 \tau^2 + p_2 p_1),$$

where $\tau = \epsilon/\sqrt{\epsilon c^D}$. The QE yields

$$R_a^{(qs)} = -ik \tau^2/(\tau^2 + ip_1).$$

It is seen that $R_a$ and $R_a^{(qs)}$ behave markedly differently in the whole interval of small $k$ (Fig. 1). However, the most considerable difference shows up in the vicinity of $k_L = \omega/v_{el}$. As $k$ tends
to \(k_L\), \(p_2\) tends to zero and the magnitude of \(R_a\) approaches unity (not shown in Fig. 1). The incident plane wave is reflected totally from the interface at \(k_L\), while the QE approximation predicts weak reflection: \(|R_a^{qs}| \approx v_a \tau^2/v_{el} \ll 1\).

By making use of (2) we obtain the coefficient of conversion into the reflected EM wave:

\[
\tilde{R}_e = -\omega \sqrt{\varepsilon_0} k \tau / (k^2 \tau^2 + p_2 p_1) .
\]

This coefficient determines the component \(H_y\) of the magnetic field when the displacement of the incident acoustic wave is equal to \(||\xi_l|||\). In particular, it follows from Eq. (5) that \(|\tilde{R}_e|\) sharply increases in the vicinity of \(k_L\) and reaches the maximum \(|\tilde{R}_e|_{\text{max}} = (\omega e v_a)^{1/2} v_{el} / \tau\) at \(k = k_L\). The QE approximation would yield the value \(|\tilde{R}_e^{qs}| = (\omega e v_a)^{1/2} \tau\) smaller by a factor \(v_{el}/v_a \tau^2\) than \(|\tilde{R}_e|_{\text{max}}\). Note that the peaks of \(|R_a|\) and \(|\tilde{R}_e|\) in the vicinity of \(k_L\) are very narrow. Their relative width has the order of \(10^{-2} v_{el}/v_a^2\).

Usually the anomalies of the above-discussed type in the behavior of the coefficients of mode conversion are due to the excitation of a leaky wave, see [8] and references therein. However, no leaky wave appears in our case. The dispersion equation does not have a complex root of the type \(k = k' + ik''\), \(k'' > 0\), in the vicinity of \(k_L\). Nevertheless, a situation is met where the EM bulk wave nearly fulfills the boundary conditions. Accordingly, the peculiarities of the coefficients of conversion can be attributed to the fact that one of the excited modes is nearly intrinsic for the structure.

4. Conclusion

EM and acoustic waves are able to interact strongly in piezoelectric materials despite a huge difference in velocities. However, it appears that all these significant deviations from the behavior of the coefficients of conversion predicted by the QE approximation are practically unobservable. In particular, the total reflection of the acoustic wave that could occur in the "180°-domain" structure requires an extremely weakly divergent acoustic bounded beam. The angle of divergence must be at least of the order of \((k_L/v_a)(v_a/v_{el})^2 \sim (v_a/v_{el})^{3}\). As it has been already noted, the discrepancy between the reflection coefficients calculated on the basis of the EM theory and the QE approximation is strong in the angle interval of the order of \(v_a/v_{el}\) (Fig. 1). Accordingly, the angle of divergence of the beam can be \(\sim v_a/v_{el}\). But in this case the coefficients has the order of smallness \(v_a/v_{el}\). Thus it can be thought that regardless of the angle of incidence the QE approximation provides the acceptable precision of estimations.

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