Large-scale magnetic fields from inflation due to Chern–Simons-like effective interaction

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Abstract. We discuss the generation of large-scale magnetic fields due to the breaking of the conformal invariance in an electromagnetic field through the $CPT$-even dimension-6 Chern–Simons-like effective interaction with a fermion current in inflationary cosmology. It is shown that the magnetic fields on the 1 Mpc scale with the field strength of $\sim 10^{-9}$ G at the present time can be generated even for the scale of the effective interaction being the Planck scale.

Keywords: magnetic fields, inflation, cosmology of theories beyond the SM

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1. Introduction

It is observationally known that there exist magnetic fields with the field strength $10^{-7}$–$10^{-6}$ G on 10 kpc–1 Mpc scales in clusters of galaxies as well as $\sim10^{-6}$ G on 1–10 kpc scales in galaxies of all types and at cosmological distances (for reviews, see [1]). However, the origin of the cosmic magnetic fields, in particular the large-scale magnetic fields in clusters of galaxies, is not well understood yet. Although the dynamo amplification mechanism [2] amplifies very weak seed magnetic fields up to $\sim10^{-6}$ G, its effectiveness in galaxies at high redshifts and clusters of galaxies is still unsatisfactory. Furthermore, it is difficult for astrophysical processes [3, 4], cosmological phase transitions [5] and primordial density perturbations before or at the epoch of recombination [6] to generate the magnetic fields on megaparsec scales with sufficient field strengths to account for the magnetic fields observed in galaxies and clusters of galaxies without a dynamo amplification mechanism.

The most natural origin of the large-scale magnetic fields is from electromagnetic quantum fluctuations that existed at the inflationary stage [7]. This is because inflation naturally produces effects on very large scales, larger than the Hubble horizon, starting from microphysical processes operating on a causally connected volume. In the Friedmann–Robertson–Walker (FRW) spacetime, the metric is conformally flat, while the ordinary Maxwell theory is conformally invariant. It is clear that the conformal invariance must have been broken at the inflationary stage in order that electromagnetic quantum fluctuations can be induced during inflation [8]. Note that this does not apply when the FRW background has non-zero spatial curvature [9]. Various breaking mechanisms of the conformal invariance in the electromagnetic field, such as non-minimal gravitational coupling [7], coupling to a scalar field [10–18], the conformal anomaly induced by quantum effects [19], spontaneous breaking of the Lorentz invariance [20], non-commutative field theories [21], a preferred minimal length [22], and cosmic defects [23], have been proposed.

Recently, an $CPT$-even dimension-6 Chern–Simons-like effective interaction between a fermion current and the electromagnetic field in inflationary cosmology was proposed for inducing the cosmological birefringence [24, 25] as well as baryon number asymmetry [26].
The electromagnetic quantum fluctuations can be generated during inflation as the interaction breaks not only the Lorentz invariance but also the conformal invariance of the electromagnetic field. In the present paper, we study the generation of large-scale magnetic fields due to this effective interaction.

This paper is organized as follows. In section 2 we describe our model and derive equations of motion for the $U(1)$ gauge field. In section 3 we consider the evolution of the $U(1)$ gauge field and estimate the present strength of the large-scale magnetic fields. Finally, section 4 is devoted to a conclusion. We use units where $k_B = c = \hbar = 1$ and adopt Heaviside–Lorentz units of electromagnetism.

2. Model

The action for the Maxwell theory with the CPT-even dimension-6 Chern–Simons-like effective interaction between a fermion current ($j_\mu$) and the electromagnetic field ($A_\mu$) [24,25] is given by

$$S = \int d^4x \sqrt{-g} [\mathcal{L}_M + \mathcal{L}_{CS}], \quad \mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \text{and} \quad \mathcal{L}_{CS} = -\frac{\beta}{M^2} j_\mu A_\nu \tilde{F}^{\mu\nu},$$

(2.1)

where $g$ is the determinant of the metric tensor $g_{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, $\tilde{F}^{\mu\nu} = [1/(2\sqrt{-g})]\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ is the dual of $F_{\mu\nu}$ with $\epsilon^{\mu\nu\rho\sigma}$ being the Levi-Civita tensor normalized by $\epsilon_{0123} = +1$, $\beta$ is a dimensionless coupling parameter, and $M = \Lambda/4\pi$ with $\Lambda$ being the scale of the effective interaction. The Chern–Simons-like effective interaction might originate from the low energy effective theory in superstring theory [24].

From the action in equation (2.1), the equation of motion for the electromagnetic field can be derived as

$$\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} F^{\mu\nu} + \frac{\beta}{M^2} j_\rho A_\sigma \epsilon^{\rho\sigma\mu\nu} \right) - \frac{\beta}{M^2} j_\mu \tilde{F}^{\mu\nu} = 0.$$

(2.2)

We take the flat FRW spacetime with the metric $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) dx^2$, where $a(t)$ is the scale factor. The fermion current $j_\mu$ has the following form [24,25] for a comoving observer:

$$j_\mu = \bar{\psi} \gamma_\mu \psi = (n, 0), \quad n \equiv n_\psi - n_{\bar{\psi}},$$

(2.3)

where $n_\psi$ and $n_{\bar{\psi}}$ are the number densities of the fermion $\psi$ and antifermion $\bar{\psi}$, respectively. In the FRW background, for the Coulomb gauge of $A_0(t, x) = 0$ and the case of $\partial_j A^j(t, x) = 0$, equation (2.2) becomes

$$\ddot{A}_i(t, x) + H \dot{A}_i(t, x) - \frac{1}{a^2} \partial_j \partial_j A_i(t, x) + 2 \frac{\beta}{M^2} na^{-1} \epsilon_{ijk} \partial_j A_k(t, x) = 0,$$

(2.4)

where a dot denotes a time derivative, $H = \dot{a}/a$ is the Hubble parameter, and $\epsilon_{ijk}$ is the totally antisymmetric tensor ($\epsilon_{123} = +1$).
3. Large-scale magnetic fields

3.1. Evolution of the $U(1)$ gauge field

We now consider the case in which a slow-roll exponential inflation is realized with the scale factor $a(t)$ given by $a(t) = a_1 \exp[H_{\text{inf}}(t - t_1)]$, where $a_1$ is the scale factor at the time $t_1$ when a comoving wavelength $2 \pi/k$ of the $U(1)$ gauge field first crosses outside the horizon during inflation, $k/(a_1 H_{\text{inf}}) = 1$, and $H_{\text{inf}}$ is the Hubble constant at the inflationary stage. From the quantization of the $U(1)$ gauge field $A_\mu(t, x)$, we obtain the expression for $A_i(t, x)$ as

$$A_i(t, x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ \hat{b}(k) A_i(t, k) e^{i k \cdot x} + \hat{b}^\dagger(k) A_i^*(t, k) e^{-i k \cdot x} \right],$$

(3.1)

where $k$ is the comoving wavenumber, $k$ denotes its amplitude $|k|$, and $\hat{b}(k)$ and $\hat{b}^\dagger(k)$ are the annihilation and creation operators which satisfy $[\hat{b}(k), \hat{b}^\dagger(k')] = \delta^3(k - k')$ and others $= 0$. In what follows, we choose the $x^3$ axis to lie along the spatial momentum direction $k$ and denote the transverse directions $x^I$ with $I = 1, 2$. We use circular polarizations expressed by the combination of linear polarizations as $A_\pm(k, t) \equiv A_1(k, t) \pm i A_2(k, t)$. From equation (2.4), we find that

$$\ddot{A}_\pm(k, t) + H_{\text{inf}} \dot{A}_\pm(k, t) + \left[ \frac{\beta}{M^2} n \left( \frac{k}{a} \right) \right] A_\pm(k, t) = 0.$$ 

(3.2)

Unfortunately, it is impossible to obtain the analytic solution of equation (3.2) for a generic evolution of the fermion number density $n$ at the inflationary stage. If $n$ evolves as $n \propto a^{-\eta}$, where $\eta = \int dt/a(t)$ is conformal time, or $n \propto a^{-1}$, analytic solutions for equation (3.2) can be derived. We will investigate these cases in the next subsection. Here we will numerically solve equation (3.2) at the inflationary stage. We assume that the initial amplitudes of $A_+(k, t)$ and $A_-(k, t)$ are the same for the time $t_1$ as for the initial time. During inflation ($t_1 \leq t \leq t_R$, where $t_R$ is the end of inflation), the amplitudes of $A_\pm(k, t)$ are expressed as $A_\pm(k, t) = C_\pm(k, t_1) A_\pm(k, t_1)$, where $C_\pm(k, t)$ are obtained by numerical calculations with $C_\pm(k, t_1) = 1$. We note that $C_\pm(k, t)$ is $k$ dependent. Hence, in numerical calculations we chose a comoving scale $L = 2\pi/k$. On requiring that the vacuum should reduce to the one in Minkowski spacetime in the short-wavelength limit, we have $|A_\pm(k, t_1)| = 1/\sqrt{2k}$ and $|A_\pm(k, t_1)| = H_{\text{inf}}/\sqrt{2k}$.

For $n = \bar{n} a^{-3}$, the numerical results for the evolutions of $C_+(k, t)$ and $C_-(k, t)$ are shown in figures 1 and 2, respectively, where $\bar{n}$ is a constant, $\xi_n \equiv (\bar{n}/[\text{cm}^{-3}])^{1/2}/(M/[\text{GeV}]) = 4.53 \times 10^{-44}$, $H_{\text{inf}} = 10^{10}$ GeV, $\beta = 1.0$ and a comoving scale $L = 2\pi/k = 1$ Mpc. By using the five-year Wilkinson Microwave Anisotropy Probe (WMAP) data on the anisotropy of the cosmic microwave background (CMB) radiation [27], one gets that $H_{\text{inf}} < 6.0 \times 10^{14}$ GeV from tensor perturbations [28]. As shown later, the large-scale magnetic fields with a sufficient amplitude at the present time can be generated. Here, we have used $k/a = \exp[-H_{\text{inf}}(t - t_1)]H_{\text{inf}}$ and $a_1 = k/H_{\text{inf}}$, and calculated equation (3.2) numerically at $t = t_1 = H_{\text{inf}}^{-1}$ with $C_\pm(k, t_1) = 1.0$ and $\dot{C}_\pm(k, t_1) = H_{\text{inf}}$. 

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Figure 1. Evolution of $C_+(k,t)$ for $n = \bar{n}a^{-3}$, $\xi_n = (\bar{n}/[\text{cm}^{-3}])^{1/2}/(M/[\text{GeV}]) = 4.53 \times 10^{-44}$, $H_{\text{inf}} = 10^{10}$ GeV, $\beta = 1.0$ and a comoving scale $L = 2\pi/k = 1$ Mpc.

The numerical results in figures 1 and 2 are understood as follows. From equation (3.2) for $n = \bar{n}a^{-m}$ with $m$ being an integer, the equation of $C_{\pm}(k,t)$ is given by

$$C_{\pm}''(k,\tilde{t}) + C_{\pm}'(k,\tilde{t}) + \exp \left[ -2 \left( \tilde{t} - \tilde{t}_1 \right) \right] \left\{ 1 \mp J \exp \left[ -(m-1) \left( \tilde{t} - \tilde{t}_1 \right) \right] \right\} C_{\pm}(k,\tilde{t}) = 0,$$

where

$$J = 2 \frac{\beta}{M^2} \frac{1}{H_{\text{inf}}} \bar{n}a_1^{-m}, \quad \tilde{t} \equiv H_{\text{inf}} t, \quad \tilde{t}_1 = H_{\text{inf}} t_1,$$

and the prime denotes the derivative with respect to $\tilde{t}$. We now consider the case of $m = 3$ shown in figures 1 and 2. At the early stage of inflation, the second term in the braces $\{}$ on the left-hand side of equation (3.3) is much larger than the first one. This means that the conformal invariance of the electromagnetic field is broken, and hence
the amplification of $C_{\pm}(k, \tilde{t})$ can be realized. However, as $\tilde{t}$ becomes much larger than $\tilde{t}_1$, the second term becomes much smaller than the first one due to the existence of the exponential suppression term, $\exp[-2(\tilde{t} - \tilde{t}_1)]$. After about ten Hubble expansion times, the second term become negligible comparing with the first one, and the above equation becomes almost equal to the equation of the ordinary Maxwell theory, which can be rewritten to the following form by replacing the independent variable $t$ with the conformal time $\eta$:

$$\frac{d^2 A_{\pm}(k, \eta)}{d\eta^2} + k^2 A_{\pm}(k, \eta) = 0. \quad (3.5)$$

The solution of equation (3.5) is given by $A_{\pm}(k, \eta) = 1/\sqrt{2k} e^{-ik\eta}$. Thus, the absolute value of the amplitude is constant, as $|A_{\pm}(k, \eta)| = 1/\sqrt{2k}$. Consequently, the solutions of $C_{\pm}(k, t)$ become asymptotically constant and the behaviors of $C_{\pm}(k, t)$ in figures 1 and 2 are reasonable.

### 3.2. Strength of the large-scale magnetic fields

Next, we estimate the present strength of the large-scale magnetic fields. The proper magnetic fields are given by [10]

$$B_i^{\text{proper}}(t, \mathbf{x}) = a^{-1} B_i(t, \mathbf{x}) = a^{-2} \epsilon_{ijk} \partial_j A_k(t, \mathbf{x}), \quad (3.6)$$

where $B_i(t, \mathbf{x})$ is the comoving magnetic field. From equation (3.6), the energy density in Fourier space is

$$\rho_B(k, t) = \frac{1}{2} \left[ |B_1^{\text{proper}}(k, t)|^2 + |B_2^{\text{proper}}(k, t)|^2 \right], \quad (3.7)$$

$$|B_1^{\text{proper}}(k, t)|^2 = \frac{1}{a^2} \left( \frac{k}{a} \right)^2 |A_{\pm}(k, t)|^2, \quad (3.8)$$

where $B_1^{\text{proper}}(k, t) \equiv B_1^{\text{proper}}(k, t) \pm i B_2^{\text{proper}}(k, t)$. Multiplying $\rho_B(k, t)$ by the phase-space density of $4\pi k^3/(2\pi)^3$, we get the energy density of the proper magnetic field as

$$\rho_B(L, t) = \frac{k^3}{4\pi^2} \left[ |B_1^{\text{proper}}(k, t)|^2 + |B_2^{\text{proper}}(k, t)|^2 \right] \quad (3.9)$$

in the position space on $L$. Using equations (3.7)–(3.9), we find

$$\rho_B(L, t) = \frac{1}{8\pi^2} \left( \frac{k}{a} \right)^4 \mathcal{I}(k, t), \quad \mathcal{I}(k, t) \equiv |C_+(k, t)|^2 + |C_-(k, t)|^2, \quad (3.10)$$

where $\mathcal{I}(k, t)$ corresponds to the amplification factor at the inflationary stage. Here, we concentrate on the case in which after inflation the universe is reheated immediately at $t = t_R$. The conductivity of the universe $\sigma_c$ is negligibly small during inflation because there are few charged particles at that time. After reheating, a number of charged particles are produced and so the conductivity immediately jumps to a large value: $\sigma_c \gg \dot{H}$. For a large enough $\sigma_c$, magnetic fields evolve in proportion to $a^{-2}(t)$ [10]. From $B(L, t) = \sqrt{2\rho_B(L, t)}$ and equation (3.10), we find that the present strength of the
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magnetic fields is

\[ B(L, t_0) = 8.2 \times 10^{18} \exp(-2N) \left( \frac{H_{\text{inf}}}{\text{GeV}} \right)^2 \left( \frac{a_R}{a_0} \right)^2 \sqrt{I(k, t_0)} \quad [\text{G}], \]

(3.11)

where \( a_R/a_0 = (g_R/3.91)^{-1/3}T_{\text{end}}/T_R \) with \( T_R \) being the reheating temperature and \( T_{\text{end}} = 2.73 \) [K]) are the present temperature of the CMB radiation [29], \( a_R \) and \( a_0 = 1 \) are the values of \( a \) at \( t = t_0 \) and the present time \( t_0 \), and \( N \) is the number of e-folds between the time \( t_1 \) and \( t_R \), given by \( N = 45 + \ln(L/\text{[Mpc]}) + \ln J \), where \( J = [30/(\pi^2 g_R)]^{1/12} \rho_R^{1/4}/(10^{38/3} \text{[GeV]}) \), \( g_R \approx 100 \) is the total number of degrees of freedom for relativistic particles at the reheating epoch, and \( \rho_R = (\pi^2/30)g_R T_R^4 \) is the energy density of radiation at the reheating stage.

Using equation (3.11) and \( H_{\text{inf}}^2 = (8\pi/3)\rho_R/M_{\text{Pl}}^2 \), where \( M_{\text{Pl}} \) is the Planck mass, we find that for \( n = \tilde{n}_a^{-3} \), \( \xi_n = 4.53 \times 10^{-44} \), \( H_{\text{inf}} = 10^{10} \text{ GeV} \) and \( \beta = 1.0 \), \( C_+(k, t_R) = 1.9 \times 10^{-48} \) and \( C_-(k, t_R) = -4.6 \), and consequently the field strength of the magnetic fields generated on the 1 Mpc scale at the present time is \( B_0(L = 1 \text{ Mpc}, t_0) = 1.1 \times 10^{-9} \text{ G} \).

We account for the relation of the amplitude of the magnetic fields and its scale dependence to the evolution of the fermion number density via \( n = \tilde{n}_a^{-m} \). It follows from equation (3.3) that when the value of \( J \) is large, the breaking magnitude of the conformal invariance of the electromagnetic field becomes large and hence strong magnetic fields can be generated. From equation (3.4) with \( a_1 = k/H_{\text{inf}} \), we find

\[ J = 2\beta(\tilde{n}/M^2)(1/H_{\text{inf}})(k/H_{\text{inf}})^{-m} \propto \xi_n^2 H_{\text{inf}}^{m-1}k^{-m}. \]

When \( m > 0 \), the spectrum of the resultant magnetic fields is a red one, and the magnetic fields on some given large scale, i.e., small \( k \), become stronger as \( m \) becomes larger.

For the strength of the primordial magnetic fields, there are constraints from big bang nucleosynthesis (BBN) on smaller scales. The limit on the present strength of the magnetic fields around the BBN horizon size \( \sim 9.8 \times 10^{-5}h^{-1} \text{ Mpc} \) is less than \( 10^{-6} \text{ G} \) [30]. Here, \( h \) is the related quantity to the present Hubble parameter as \( H_0 = 2.13h \times 10^{-42} \text{ GeV} \) [29]. Note that throughout this paper, we use \( h = 0.7 \) [31]. In the case in figures 1 and 2, the present strength on the BBN horizon scale is \( 5.9 \times 10^{-50} \text{ G} \). Clearly, the constraints from BBN are satisfied. On the other hand, there exist constraints from the CMB anisotropy measurements on larger scales. The result \( \sim 10^{-9} \text{ G} \) in figures 1 and 2 on the 1 Mpc scale is consistent with the recent observational upper bound derived by using the WMAP five-year data [32]. Similar bounds have been studied in [33]. According to [34], the limit on the current strength on scales larger than the present horizon is less than \( 4.8 \times 10^{-9} \text{ G} \). For \( \xi_n = 1.87 \times 10^{-49} \), \( H_{\text{inf}} = 10^{10} \text{ GeV} \) and \( \beta = 1.0 \), the present strength of the magnetic fields on the horizon scale is \( 2.4 \times 10^{-9} \text{ G} \), which is consistent with the above upper limit. In this case, the field strength for 1 Mpc is \( 1.2 \times 10^{-57} \text{ G} \).

In addition, we study the case in which \( n \) evolves as \( n = \tilde{n}a \). In this case, we can obtain the following analytic solution of equation (3.2):

\[ A_\pm(k, a) = \sqrt{\frac{\pi}{4k}} \left( \frac{k}{aH_{\text{inf}}} \right) H'^{(1)}(\nu) \left( \frac{k}{aH_{\text{inf}}} \right) \exp([2\nu+1]\pi/4), \]

(3.12)

\[ \nu = \sqrt{\frac{1}{4} \pm 2 \frac{\beta}{M^2} \tilde{n}_a \frac{1}{H_{\text{inf}}^2}}, \]

(3.13)
where $H^{(1)}_{
u}$ is a $\nu$th-order Hankel function of type 1, and we have taken $\nu > 0$ and chosen the integral constant so that the vacuum reduces to the one in Minkowski spacetime at the short-wavelength limit. Being interested in large-scale magnetic fields, we investigate the behavior of this solution in the large-wavelength limit. Expanding the Hankel function in equation (3.13) and taking first the leading order in $k/(aH_{\text{inf}})$, from equations (3.7)–(3.9) we find that the present strength of the large-scale magnetic fields is given by

$$B(L, t_0) = 5.1 \times 10^{19} \frac{2^{\nu-1}}{\pi^{3/2}} \Gamma(\nu) \left( \frac{H_{\text{inf}}}{\text{GeV}} \right)^2 \left( \frac{a_R}{a_0} \right)^2 \left( \frac{k}{aH_{\text{inf}}} \right)^{5/2-\nu} \text{[G]}.$$

If $\nu = 2.57$ and $H_{\text{inf}} = 10^{14}$ GeV, the present strength of the large-scale magnetic fields on 1 Mpc scale is $B(L = 1 \text{ Mpc}, t_0) = 1.4 \times 10^{-9}$ G. In this case, $\xi_n = 3.27 \times 10^{53}$.

It is interesting to note that for $n = \tilde{n}a^{-1}$, we can also obtain an analytic solution of equation (3.2). In this case, equation (3.2) is rewritten to the following form by replacing the independent variable $t$ with the conformal time $\eta$: $d^2A_{\pm}(\tilde{k}, \eta)/(d\eta^2) + \tilde{k}^2A_{\pm}(\tilde{k}, \eta) = 0$, where $\tilde{k}^2 = k^2 \mp 2(\beta/M^2)\tilde{n}k$. The solution of this equation is given by $A_{\pm}(\tilde{k}, \eta) \propto \exp(-i\tilde{k}\eta)$. This is an oscillating solution and its absolute value is constant. Hence $A_{\pm}(k, t)$ cannot be amplified.

We remark that if $\psi$ is an unknown particle and $n = \tilde{n}a^{-3}$, the present number density $\tilde{n}$ should be smaller than that of the neutrino $1.1 \times 10^2$ cm$^{-3}$ [29]. For $\tilde{n}/(\text{cm}^{-3}) = 3.1 \times 10^{-49}$, which satisfies the above constraint, it follows from $\xi_n = (\tilde{n}/(\text{cm}^{-3}))^{1/2}/(M/[\text{GeV}]) = 4.53 \times 10^{-44}$ that $M$ can be the Planck scale, i.e., $M = M_{\text{Pl}} = 1.2 \times 10^{19}$ GeV.

In this paper, we treat the fermion $\psi$ as being relativistic during inflation. The energy density of the inflaton is given by $\rho_{\text{inf}} = [3/(8\pi)]H_{\text{inf}}^2M_{\text{Pl}}^2$. The energy density of radiation at the reheating stage is given by $\rho_R = (\pi^2/30)g_RT_R^4$. Since we consider instantaneous reheating in our study, $\rho_{\text{inf}} \approx \rho_R$. The energy density of the relativistic fermion is given by $\rho_{\text{fermion}} = (7/8)(\pi^2/30)g_RT_R^4$ [29]. Thus, $\rho_{\text{fermion}} = (7/8)\rho_{\text{inf}}$, namely, the energy density of the fermion $\psi$ is smaller than that of the inflaton. This is reasonable because in the standard inflationary cosmology the potential energy of the inflaton is mainly responsible for inflation. Moreover, in this paper we do not study the production mechanism of the fermion $\psi$. We assume that it is produced by some other mechanism.

In addition, we note that the present value of the ratio of the Chern–Simons interaction term to the Maxwell one is given by $|\mathcal{L}_{CS}/\mathcal{L}_M| \approx 10\beta \xi_n^2$, where we have used $|\partial_\mu A_\mu/A_\mu| \approx H_0$. Hence, for $n = \tilde{n}a^{-3}$, $\xi_n = 4.53 \times 10^{-44}$ and $\beta = 1.0$, the above ratio is much smaller than unity. Thus, the relative contribution of the Chern–Simons interaction term at the present time is very small.

4. Summary

In summary, we have studied the generation of the large-scale magnetic fields due to the breaking of the conformal invariance in the electromagnetic field through the CPT-even dimension-6 Chern–Simons-like effective interaction with a fermion current in inflationary cosmology. We have found that the magnetic fields on the 1 Mpc scale with the present amplitude of $\sim 10^{-9}$ G can be generated. This strength is enough to explain the magnetic fields observed in galaxies and clusters of galaxies through only adiabatic compression without requiring any dynamo amplification [7]. If the number density of the fermion $\psi$...
interacting with the electromagnetic field evolves in proportion to $a^{-3}(t)$ during inflation, the scale of the effective interaction can be the Planck scale.

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Note added. After this work was completed, we became aware of a related work [35] considering the generation of the primordial magnetic fields during inflation in a Lorentz violating theory of electrodynamics due to a Chern–Simons term coupling the $U(1)$ gauge field to an external 4-vector, proposed by Carroll, Field and Jackiw [36]. In the scenario of [35], during inflation the induced magnetic fields are peaked on much smaller scales than the Hubble horizon at that time. To extend the scales to the galactic one at the time of protogalactic collapse, the inverse cascade mechanism has to work. On the other hand, in our scenario, during inflation the magnetic fields can be generated on scales much larger than the Hubble horizon at that time, so magnetic fields with sufficient strength on the scale of cluster of galaxies at the present time are produced without any secondary extension mechanism such as the inverse cascade. This is the advantageous feature of our scenario.

References

[1] Kronberg P P, 1994 Rep. Prog. Phys. 57 325
Grasso D and Rubinstein H R, 2001 Phys. Rep. 348 163 [SPIRES]
Carilli C L and Taylor G B, 2002 Ann. Rev. Astron. Astrophys. 40 319 [SPIRES]

[2] Parker E N, 1971 Astrophys. J. 163 255 [SPIRES]
Parker E N, 1979 Cosmical Magnetic Fields (Oxford: Clarendon)
Zel’dovich Ya B, Ruzmaikin A A and Sokoloff D D, 1983 Magnetic Fields in Astrophysics
(New York: Gordon and Breach)

[3] Biermann L and Schlüter A, 1951 Phys. Rev. 82 863 [SPIRES]

[4] Weibel E S, 1959 Phys. Rev. Lett. 2 83 [SPIRES]

[5] Quashnock J M, Loeb A and Spergel D N, 1989 Astrophys. J. 344 L49 [SPIRES]
Baym G, Bodeker D and McLerran L D, 1996 Phys. Rev. D 53 662 [SPIRES]

[6] Berezhiani Z and Dolgov A D, 2004 Astropart. Phys. 21 59 [SPIRES]

[7] Bamba K and Yokoyama J, 2004 Phys. Rev. D 70 023502 [SPIRES]

[8] Gasperini M, Giovannini M and Veneziano G, 1995 Phys. Rev. Lett. 75 3796 [SPIRES]

[9] Bamba K and Yokoyama J, 2004 Phys. Rev. D 69 043507 [SPIRES]

[10] Martin J and Yokoyama J, 2008 J. Cosmol. Astropart. Phys. JCAP01(2008)025 [SPIRES]

[11] Giovannini M, 2001 Phys. Rev. D 64 061301 [SPIRES]
Large-scale magnetic fields from inflation due to Chern–Simons-like effective interaction

Bertolami O and Monteiro R, 2005 Phys. Rev. D 71 123525 [SPIRES]

Giovannini M, 2008 Phys. Lett. B 659 661 [SPIRES]

[13] Garretson W D, Field G B and Carroll S M, 1992 Phys. Rev. D 46 5346 [SPIRES]

Field G B and Carroll S M, 2000 Phys. Rev. D 62 103008 [SPIRES]

Anber M M and Sorbo L, 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)018 [SPIRES]

Campanelli L, 2008 arXiv:0805.0575 [astro-ph]

[14] Calzetta E A, Kandus A and Mazzitelli F D, 1998 Phys. Rev. D 57 7139 [SPIRES]

Kandus A, Calzetta E A, Mazzitelli F D and Wagner C E M, 2000 Phys. Lett. B 472 287 [SPIRES]

Giovannini M and Shaposhnikov M E, 2000 Phys. Rev. D 62 103512 [SPIRES]

Field G B and Carroll S M, 2000 Phys. Rev. D 62 103008 [SPIRES]

Davis A C, Dimopoulos K, Prokopec T and Tornkvist O, 2001 Phys. Lett. B 501 165 [SPIRES]

Davis A C, Dimopoulos K, Prokopec T and Tornkvist O, 2002 Phys. Rev. D 65 063505 [SPIRES]

Giovannini M and Kunze K E, 2008 arXiv:0804.2238 [astro-ph]

Barrow J D, Ferreira P G and Silk J, 1997 Phys. Rev. Lett. 78 3610 [SPIRES]

Campanelli L, Cea P and Fogli G L, 2008 arXiv:0805.1851 [astro-ph]

Carroll S M, Field G B and Jackiw R, 1990 Phys. Rev. D 41 1231 [SPIRES]