Research Article

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Mathematical and Coding Lessons Based on Creative Origami Activities

https://doi.org/10.1515/edu-2019-0016
received April 29, 2019; accepted October 28, 2019.

Abstract: This paper considers how creativity and creative activities can be encouraged in regular mathematical classes by combining different teaching approaches and academic disciplines. We combined origami and paper folding with fractals and their mathematical properties as well as with coding in Scratch in order to facilitate learning mathematics and computer science. We conducted a case study experiment in a Serbian school with 15 high school students and applied different strategies for learning profound mathematical and coding concepts such as fractals dimension and recursion. The goal of the study was to employ creative activities and examine students’ activities during this process in regular classrooms and during extracurricular activities. We used Scratch as a programming language, since it is simple enough for students and it focuses on the concept rather than on the content. Real-life situation of folding Dragon curve was used to highlight points that could cause difficulties in the coding process. Classroom observations and interviews revealed that different approaches guided students through their learning processes and gradually made the introduced concepts meaningful and applicable. With the introduction of this approach, students acquired understanding of the concept of coding recursion through paper folding and applied it in the higher-level programming. In addition, our teaching approach made students enthusiastic, motivated and engaged with the learning of usually difficult subjects.

Keywords: origami; fractals; coding; Scratch

1 Introduction

Technological development has influenced many aspects of 21st century life. Besides everyday changes, adjustments are also required in the field of education. Modern education needs to develop students’ skills and knowledge necessary for their current, but also future lives. According to research and current trends, such education needs to be engaging and enquiry-based with the valuable and relevant context to the individuals (Baron & Darling-Hammond, 2008), where teaching should be based on transformed pedagogies and redesigned learning tasks promoting students’ autonomy and creativity supported by technology (Leadbeater, 2008). Some of the general 21st century skills, outlined in the research literature, are critical thinking, problem solving, creativity, innovations, communication and collaboration (CCR, 2015), but also skills that emerge from the development of technology and their effects on everyday life, such as computational thinking.

Technological development has influenced numerous aspects of life in the 21st century. It has made our lives more convenient and easier. Every time we press a button, e.g. on a phone, a dishwasher, a door, a computer behind it is executing a program. Progress is made in many areas, such as communication, health care, entertainment and finance. Technology is basically created to meet our various everyday needs. Besides everyday changes, adjustments are also needed to be made in the field of education. That is why many countries worldwide are introducing coding as a compulsory school activity. Learning coding gives an opportunity to develop digital skills, as a chance to communicate solutions in a logical and structured programming language. Even though it is recognized as a 21st century skill and literacy, coding knowledge is not enough to become successful in contemporary, complex and dynamic world and to confront challenges with confidence. Besides coding, students need to develop other skills such as collaboration, critical thinking, communication, creativity and problem solving. Teaching coding in a 21st century school requires different teaching
strategies. Many factors constrain the changes from traditional practice (OECD, 2004; OECD, 2018) and we often face the problem of traditional content-based teaching applied in the classroom rather than active learning.

2 Unusual Educational Combination of Mathematics, Origami and Technology

In this paper, we investigate a teaching approach that combines mathematics, arts and coding in order to introduce computational thinking and creativity into the learning process and focus mainly on learning concepts rather than content. We will work with the concept of orientation as it is recognized to be one of the three attributes that enhance learning tasks, beside observation-grounded and value-beyond-school learning (Donham, 2010). Through such concepts students can obtain a broad and abstract view on the topic and it may help them to transfer knowledge across different teaching and learning settings. Emphasizing core concepts in mathematical teaching and learning beyond specific contents can facilitate students’ acquisition of meaningful and transferable knowledge (Erickson, 2008; Erickson, Lanning & French, 2017). For example, utilising origami activities to illustrate concepts in mathematics could assist students in learning a variety of topics and approaches in other subjects too.

Mathematical lessons based on origami used to be organised in workshop formats, starting with folding instructions and completion of objects, followed by basic mathematical concepts connected with the object and finally with formal mathematical definitions (e.g. symmetry, self-symmetry, or dimension) and proofs. Since origami is based on axiomatic theory, origami as a mathematical principle of paper folding provided solutions for numerous geometrical constructions, that were impossible to construct with a straight edge and compass (Auckly & Cleveland, 1995). Since it is established as a mathematical discipline by axiomatizing, it could be seriously concerned as a concept that could be more involved in classrooms and mathematical education. Playful origami activities of folding paper are underlined with substantial mathematical scientific theory. Exploring seven Huzita-Hatori axioms (Huzita, 1989) or Kawasaki’s and Maekawa’s theorems (Justin, 1986; Hull, 1994) of paper folding and relationships between creases and angles in flat origami models could be an excellent geometrical exercise. Also, it can be an introduction to the deeper concepts, such as computational origami (Demaine et al., 2000) which is dealing with efficient algorithms for solutions of paper folding problems. Nevertheless, even the simple origami activities, not based directly on axiomatic theory could be useful in developing understanding of spatial procedures and relationships. Creating origami models requires the ability to follow procedures in order to solve open-ended problems (Meyer & Meyer, 1999).

Based on literature and experience, origami could be used at least in two ways in mathematical lessons. Firstly, it could be used to develop basic mathematical ideas building on its theories, secondly, it could be employed to motive students to develop their logical thinking, spatial reasoning, or connecting mathematics with artistic aesthetics. Certainly, the combination of these two approaches is utilised most of the time in teaching and learning with origami. In order to utilise potentials of origami, we have been experimenting with origami instructions in mathematics for several year mainly in Petro Kuzmjak school in Serbia, but offered teaching and workshops at many different parts of the world. As an example, we used simple origami folds to explore basic mathematical notions, such as polygons, segments or angles (Budinski et al., 2019), while we noticed that more complicated folds assist in spatial visualisation or in other project we introduced concepts of Platonic solids and fractals with origami (Budinski, 2016; Budinski & Novta, 2017).

During the past decades, besides our work, numerous educational researchers noticed the educational potentials of origami (Rovichaux & Rodrigue, 2003; Cippoletti & Wilson, 2004; Boakes, 2009; Fiol et al., 2011; Fenyesi et al., 2014; Gur & Kobak-Demir, 2017), making origami a useful and popular tool in mathematics education. The most prevalent connection is between origami and geometry, where students can easily examine 2- and 3-dimensional forms (Weinsstein, 2015). Even simple folds can be useful for teaching substantial mathematics concepts, e.g. folding square form paper by connecting opposite vertices of the square and creating a diagonal. Naturally, more advanced folds enable the examination of deeper mathematical concepts.

Recently, we started to research another possibility for origami using in the classroom and connecting it to technology and different software in order to develop digital and coding skills besides mathematical ideas. After mainly using the free educational software GeoGebra (www.geogebra.org) to support origami activities (Budinski et al., 2018; Budinski et al., 2019), for this study we chose
The data gathering and exploration of the case of applying creative paper folding activities in teaching mathematical and coding concepts and examining students’ activities in the process of application. The two main tasks were defined as follows. The first consisted in describing the settings of this type of lessons, so other teachers and educators could follow and apply the format in their classrooms and the second was to observe students’ learning activities in this kind of approach and note its benefits. In addition, our goal with the paper was to introduce opportunities to combine several subjects and topics into regular classroom teaching. The strength of the paper, we believe, is offering a concept and opportunity for transdisciplinarity and multiple teaching approaches.

In this paper we followed the growing trend in educational research (Silver, 2004) and use qualitative approach of case study. Case studies are known for allowing a deep and detailed description of situation, interactions and observed behaviors (Labuschange, 2003). Even though qualitative research has many challenges, such as reliability, generality and validity, our motivation for this choice was to collect data that could not be collected by surveys or large-scale research. The case study format provided us with details of close observation of students’ activities in a real classroom settings. Also, we applied a novel method of combining creative activities in teaching mathematics and coding, which are not a formal part of a typical school practice, so that narrowed our possibilities. The innovation in our case was only possible to be applied with the small group of students, so we could follow every interaction and step of the implementation in details. It was students’ activities that were the predominant source of qualitative data that helped in understanding the concept of combining creative activities in teaching mathematics and coding.

We also opted for the qualitative study, because they mostly do not aim to quantify results, but to understand the dynamics in the classroom situation and during the activities. The data from interviews and observations were analyzed and we used qualitative data to gauge students’ interest and engagement. The main findings were collected through interviews and observations.

In our qualitative research we relied on validity and reliability in the process of research. Making work available for public examination supports the improvements of the research practice and increases the chance that the work becomes useful to other mathematics educators. Since it was a case study in real classroom settings, we focused our research more on usefulness, meaningfulness and shareability as validity criteria (Sharma, 2013).

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mathematics and coding concepts in a typical context were done during regular school activities in the Petro Kuzmjak school in Serbia. Fifteen students (8 males and 7 females, aged 17) took part in the case study in the second term of the 2017/2018 school year. The case study had several phases, which took place on a weekly basis. It started with the regular curriculum requirements and logarithms' properties lessons (two school lessons). In order to illustrate application of logarithms, the next phase was introduction to fractals, whose dimension calculations are based on logarithms (one school lesson). This phase was followed by paper folding activities (one school lesson and extracurricular activities). During those activities, beside learning about fractal properties students had to make Dragon curve fractal by folding paper strips. The final activity was dedicated to exploration of recursion concept in coding on the example of Dragon curve fractal. It was an extracurricular activity that last for two hours.

4 Description of the Study

Results to be presented here were obtained from a study conducted in Petro Kuzmjak school in Serbia with 15 students (17 years old) during the second term of the school year. The aim of the study was to implement activities to connect mathematics, arts and coding during regular in-class and extracurricular activities. We created a framework to combine, at first glance unconnected disciplines, in order to convey complex scientific concepts to students and influence their creativity and computational thinking. During the study we planned activities, conducted interviews and collected observational data.

In our case, we combined mathematical concepts of fractals with the ancient paper folding skill of origami and basic coding in Scratch to teach students the concept of recursion appearing in different disciplines. In our teaching approach, we directed the teaching process not only to teach facts, but also to emphasize concepts important for both mathematics and coding. For instance, this process was organized to highlight the concept of recursion not only to develop understanding and to be helpful in deriving more general rules, but also to support knowledge applied in new situations. We were working on developing creative and computational thinking, making the teaching process diverse in each step. We emphasized the “big ideas” and helped students to be involved in the class design process and their interests with the subjects.

5 Fractals as a Connection Between Mathematics, Arts and Coding

In our research, the combination of origami and coding activities were connected through the topic of fractals in order to help students to better understand the concept of recursion. Fractals are often barely taught in the curricula across the world (Fraboni & Moller, 2008; Chen, 2015), even though there is substantial educational potential of bringing fractals into mathematics classrooms (Budinski & Novta, 2017; Frame & Mandelbrot, 2002). Mathematical properties of fractals such as shapes, symmetries, structures, recursive and iterative forms, could be useful in the visualization of topics covered in mathematics and computer science lessons. The mathematical root of the idea of fractals started with the notion of recursion and was further developed in 20th and 21st centuries greatly assisted with the development of computer-based modeling. Besides being a mathematical concept, fractals are also widely used as artistic inspirations.

In order to clarify our approach and highlight the concept of recursion, we selected a fractal called the Dragon curve. The Dragon curve is also known as the Heighway dragon, named after John Heighway, a NASA physicist, who was among the first to research its properties (Tabachnikov, 2014). The idea of the Dragon curve fractal can be illustrated by folding a paper strip in half to the left (or right), repetitively, which, after finishing the process at some point and unfolding, turns the Dragon curve into a fractal. In Figure 1, we can see Dragon curve fractal obtained by folding a paper strip in half four times.

When we started with the introduction of fractals to students, the first step was to present fractals as geometrical curves that consist of infinitely repeating identical forms that are decreasing in range with each iteration (Debnath, 2016). We presented the concept of fractals using the Dragon curve example during regular lessons dedicated to logarithms in order to fulfill curricular requirements. We used the connection between fractal dimension and logarithmic equations as the ground for explaining complex mathematical concept of fractals and their dimensions. Since the fractal dimension can be mathematically explained with the formula \( d = \log n / \log s \), where \( d \) is dimension, \( s \) is the decreasing scale, and \( n \) is the number of self-similar copies, the determination of the dimension required solving a logarithmic equation. After exploring the dimension, we continued with other properties such as self-similarity by exploring the parts of objects that were exactly or approximately similar to the
A theoretical presentation was insufficient to explain the complexity of fractals, so that we used origami and paper folding to assist students to acquire fractals in a more explicable way. Students’ task was to fold the Dragon curve using paper strips, which required understanding of the fractal’s properties in order to make as many iterations of the Dragon curve as possible. Result of the students’ folding is shown in Figure 3.

The activity was performed during two school lessons, and students who did not complete the task took it as a homework activity. Also, after fully understanding the concept of fractals, students participated in extracurricular activities such as Science fairs, workshops for mathematics promotion and presentations.

After theoretical analyses and paper-folding activities, we used students’ knowledge of fractals to help them acquire one more concept interwoven in fractals and that was recursion and its application in writing computer programs. We were following the recommendation that coding lessons should not only focus on coding syntaxes, but also on creativities involved and on supporting students’ development of computational thinking. The coding solutions should be understood on a deeper level and not only explored on the surface (Grover, 2013). In our case, the task was to analyze a program written in Scratch that generated the Dragon curve and assisted in learning about functions that were emerging from the program generating the Dragon curve. We chose Scratch since we worked with students who were familiar with the basics of programming in Scratch and as explained before it has a strong online community where users can share their designs and redesign them for their own purposes. For the lessons, we took already existing programs that generated the Dragon curve, such as https://scratch.mit.edu/projects/11149112/#editor, which is shown in Figure 4, so that students could explore commands that were used to generate the fractal. At that point students were familiar with the Dragon curve through hands-on activities, so we used a completed program to discuss important and advanced coding concepts such as recursion, which is widely used in programming solutions. The idea was to highlight that recursion is based on the fact that overall solution of a problem relies on the solution of smaller instances of the same problem. Fractals and the Dragon curve, which are predominantly generated by iterations of more complex forms were used to clarify that concept and helped to use them in writing computer programs.

Figure 1: The Dragon curve received by folding paper strip four times.

Figure 2: Self-similarities of the Dragon curve.

Figure 3: Paper-folded Dragon curve.

Figure 4: Completed program generated the Dragon curve.
Dragon curve is a fractal made of segments connecting at right angles, where iterations can be presented as strings of left and right turns. The focus of the task was on analyzing the iterations in the Dragon curve fractal and its representations in Scratch functions. The process was repeated backwards, as students already had the solutions, but it was used to clarify the concept of recursion.

Simulation and altering different input parameters were also helpful in understanding how the Dragon curve was drawn in Scratch. It started with searching for analogies with the same process derived from paper folding of the Dragon curve from one strip which is shown in Figure 5 (Budinski & Novta, 2017).

The analysis of the computer program that drew the Dragon curve was in the form of discussion, where we analyzed the folded papers and the virtual solutions in parallel. Students concluded that side length was not changing during the simulation and it was defining the length of a single line part. During each call to Left and Right dragon blocks, the level was decreasing by one. The Left dragon and the Right dragon blocks recalled themselves and each of the other blocks, which was an example of recursion that we wanted to show to students. The process was repeated until the level became zero, when each function draw a line with its side length. The only difference between the Left and the Right dragon was the left and right 90-degree rotation between these two calls. At that point of the discussion, students noticed that when we were folding paper strips, each folding was applied in the middle of the paper strip. That lead students to the observation where after unfolding a paper strip that was folded three times, it become clear that defining left and right folding and writing down the result would be useful. One series of correct folding was L, L, R, L, L, R, R for a three-fold Dragon curve. While the middle folding was the third L, its halves were L, L, R and L, R, R which was centrally symmetrical and opposite to the middle L. Opposite in terms that L was opposite to R, and vice versa. This result was also investigated by folding paper strips two and four times and comparing them to the virtual solution.
6 Results

We interviewed the students at the end of the all phases and collected their answers. In the interviews students were asked to highlight one of the benefits and one of the obstacles of this kind of learning. After analyzing the answers we noticed some generalities. Regarding the benefits, all students highlighted that they learned a lot about application of logarithms. Application of mathematical knowledge is, in their opinion not presented well enough in regular classes, so this kind of activities gave them an opportunity to contextualize their knowledge. Students expressed their satisfaction with the unusual combination, since every student could find an activity where he or she could express their potential. For example, student A said that he liked to code and that motivated him to be active in lessons, while student B said that she was not that much into mathematics, but art activities attracted her to be more involved in mathematical tasks. As obstacles, students highlighted the need for more time devoted to the activities, which made it necessary for them to take part in extracurricular activities. In their opinion it would be better if all activities were part of regular lessons.

7 Conclusions

Based on the interviews and observation analyses, students stated that using the combination of origami and Scratch for creating fractals helped them and motivated them in the process of developing mathematical and coding concepts through visualizations in physical and virtual settings. They also expressed that this process encouraged their creativity and contributed to their better understanding of computational thinking methods. In addition, during the interviews, it became evident that students developed fundamental understanding of both mathematical and computer science concepts and drew their knowledge from both physical and virtual experiences. We deliberately focused on conceptual approaches in the activities and hoped to make connections between students’ experiences with paper folding and coding. Activities, concept and learning environments were new to all students, they were learning from basics, which made them more open to “big” ideas and encouraged their curiosity. Students constructed the concept of recursion when they analyzed fractals by folding paper strips and then looked for the same structure in the lines of code.

Unlike in traditional mathematical or programming lessons, origami in combination with mathematics and Scratch helped students to focus on the concept which could be used in other, much more advanced mathematical and coding problems. Concept and the experiences in both environments could be transferred to other solutions of similar problems. The teaching approach of combining origami, mathematics, and coding could be a promising platform for developing creative computational thinking. It could be a potential learning framework where students could gain knowledge within an authentic context by participating in projects and solving problems, and using creative expressions, all of which contribute to their learning experiences, creativities, and motivation for learning.

Even though those answers could not be generalized since they are based on students’ experience, they gave us an insight into the nature of problems of educational approach that combine mathematics, coding and art. This study offered insights into concepts and understanding activities of new educational methods. This could be the base for further, more extensive research, including larger scale studies. However, in further studies we may include quantitative research elements. Applying this experimental design in the real classroom settings and its making its description available for public examination could support the improvements of the educational research and teaching practice. Rearranging and supporting teaching mathematics and coding with creative and artistic activities and examples could broaden the picture of innovation in education in a desired way: by developing students’ creative and problem solving abilities.

Acknowledgments: The paper was written during the exchange program of one of the authors, Natalija Budinski, and supported by a CEEPUS scholarship and the OEAD organization at the Johannes Kepler University in Linz during the winter semester of the 2018/2019 academic year.

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