Quantum-wave pattern recognition: from simulations towards implementation

Mitja Peruš * and Horst Bischof
Graz University of Technology, Institute for Computer Vision and Graphics
Inffeldgasse 16, 2.OG, A-8010 Graz, Austria
www.icg.tu-graz.ac.at/~perus & ~bischof

Abstract
A neural-net-like model, which is realizable using quantum holography, is proposed for quantum associative memory and pattern recognition. This Hopfield-based mathematical model/algorithm, translated to quantum formalism, has been successfully tested in computer simulations of concrete pattern-recognition applications. In parallel, the same mathematics governs quantum dynamics which can be harnessed for information processing by proper (de)coding manipulation. Since we are able to give quantum interpretation to all the elements (e.g., variables, couplings) of the model, and as far as we are able to show that processing, governed by that mathematics, is experimentally implementable in real quantum systems, we can expect efficient quantum computing – in our case pattern recognition based on quantum content-addressable associative memory.

Keywords: quantum, pattern recognition, Hopfield, neural net, holography, phase, associative memory

1. INTRODUCTION

Quantum neural nets [1, 2] are a branch of quantum computers needing no logic gates. It will be shown that the implementation of associative neural nets can be naturally-physical, i.e. no artificial classical-physical devices are necessary, except for encoding into and decoding from quantum systems. To demonstrate quantum implementation of certain ANN, it is best to remember a fundamental technique which has already been much experimentally tested and used – holography. Holography is a practical 3D image-storage and -reconstruction procedure [3]. Its imaging is powerful and of high resolution, although the technique is relatively simple — it uses merely reflection from the laser-illuminated object and interference of the "object"-beam with a "reference"-beam. Early associative memories were inspired by holography [4]. They were a version of digitalized amplitude-beam. Early associative memories were inspired by holography [4]. They were a version of digitalized amplitude-beam.

Since holography can be in principle realized applying any sort of coherent waves [3], it is realizable also using quantum waves or wave-packets. The latter are of the same type as the Gabor wavelets used, e.g., in computer vision [7]. Dennis Gabor got the Nobel prize for physics for his invention of holography. The fast-developing nanotechnology promises to realize it soon in quantum field, i.e. using quantum probability-distribution waves [8].

This paper introduces some relations of parallel-distributed processes (PDP) in neural and quantum systems (overview in [9]), and their relations to holography, in order to propose a quantum pattern recognition model. The mathematics of this algorithmic model to be presented has already been successfully computer-simulated, e.g. in [10]. The next step made here is to proceed from simulations to a proposal of quantum implementation, supported by considering quantum-optical / -holographic experiments.

This step was enabled by the following observation on the parallel-distributed information-encoding into waves, i.e. into \( \sum_{i=1}^{A_{1}} A_{2} \), general. It has two special cases: (I) encoding in amplitudes \( A \) only, i.e. in \( A_{1}, A_{2}, A_{N} \), and (II) encoding in oscillatory phases \( \varphi \) only, i.e. in \( e^{i\varphi_{1}}, e^{i\varphi_{2}}, ..., e^{i\varphi_{N}} \), \( i = \sqrt{-1} \). These two cases enable effectively the same information processing as far as the following variable exchange can be made in the mathematics of the model/algorithm:

\[
A \leftrightarrow e^{i\varphi}
\]

It will be shown for our wave-model (II) with \( A = 1 \): what works for real-valued coding-numbers (A) works for sinusoid encoding also. I.e., “wave-based” model (II) is equivalent to “intensity-based” model (I).

In sec. 2 we “translate” Hopfield’s model into quantum formalism [2, 9]. In sec. 3 we “transform” Hopfield’s model into wave-model (II), showing their equivalence for pattern recognition. In sec. 4 we present possible quantum implementations of the simulated model.

2. QUANTUM-WAVE HOPFIELD-BASED NET

The simplest Hopfield ANN (1982) incorporates Hebbian memory-storage “into” correlation matrix \( J \), i.e. \( J = \sum_{k=1}^{P} \vec{v} \otimes \vec{v} \) (\( \otimes \) denotes tensor/outer product), and a memory-influenced transformation of patterns \( \vec{v} \):

\[
\vec{v}_{\text{output}} = J \vec{v}_{\text{input}}
\]

Each of \( P \) patterns, simultaneously stored in the same net/\( J \), is denoted by a superscript index \( k \): \( k = 1, ..., P \). Patterns \( \vec{v}^{k} \), which become Hopfield-net’s eigenstates (attractors), can be complex-valued and
can be quantum-encoded (as will be shown). Therefore, we will henceforth use the quantum notation, \( \psi^k \), for them (i.e., \( \bar{\psi}^k = \psi^k \), instead of the standard case: \( \bar{\psi}^k = A^k \)). (So-called wave-function \( \Psi \) describes the whole state of the quantum system/net; \( \psi^k \) describes the \( k^{th} \) of its eigenstates.) Thus, patterns are assumed to be encoded into quantum eigen-wave-functions \( \psi^k \) (physical realization will be discussed later).

Turning from global description of the quantum PDP (using associative memory \( J \) and net-states \( \Psi \)) into local one (using interaction-weights \( J_{hj} \) and unit-states \( \Psi_j \); \( j, h = 1, ..., N \), for \( N \) units; \( N \) is huge), we have:

\[
J_{hj} = \sum_{k=1}^{P} \psi^k_j (\psi^k_j)^* \tag{1}
\]

where the asterisk denotes complex conjugation, and

\[
\Psi^\text{output}_h = \sum_{j=1}^{N} J_{hj} \Psi^\text{input}_j \tag{2}
\]

Inserting eq. (1) into eq. (2), we obtain:

\[
\Psi^\text{output}_h = \left( \sum_{j=1}^{N} \left( \sum_{k=1}^{P} \psi^k_j (\psi^k_j)^* \right) \Psi^\text{input}_j \right) = \sum_{k=1}^{P} e^c k \psi^k_h = \sum_{k=1}^{P} e^c k \psi^k_h \tag{3}
\]

Usually, exclusively those coefficient \( e^c \), say \( e^c_0 \), is close to 1 which belongs to the memorized pattern \( \psi^{k0} \) which is the most similar to \( \Psi^\text{input} \). Consequently, all other \( e^c \), \( k \neq k_0 \), are close to zero. In such a case of the process of eq. (3), the quantum associative net recognizes the input-pattern. See analysis of this matching process in [2] [12].

\( J \) is called Green-function propagator, \( G \), in quantum theory [9] [12]. \( G \)'s description of input–output transformations corresponds to statistical description of state-relations (relations in encoded data) by the quantum density matrix \( \rho \). We will not use \( \rho \) here; we merely wanted to emphasize \( \rho \)'s role as a "quantum archive" (of all potential input–output transformations, in contrary to \( G \)'s actual ones).

For physics-oriented readers, we write eq. (2), with kernel of eq. (1) inserted, into space-time form:

\[
\Psi(\bar{r}_2, t_2) = \int \int \left( \sum_{k=1}^{P} \psi^k(\bar{r}_2, t_2)(\psi^k(\bar{r}_1, t_1))^* \right) \Psi(\bar{r}_1, t_1) d\bar{r}_1 d\bar{t}_1 \tag{4}
\]

We replaced unit-indices \( h, j \) by \( (\bar{r}_2, t_2), (\bar{r}_1, t_1) \), and discrete summation by an integration over the whole effectively-continuous quantum system/net (if it consists of very many “units”). This is the Feynman (path-integral) version of the Schrödinger equation, the fundamental equation for quantum dynamics — in Dirac’s notation: \( | \Psi \rangle = \langle \Psi | (\Psi | \Psi) = \langle \sum_k | \psi^k \rangle (\psi^k | | \Psi) \).

The Hopfield computational model, incorporating coupled eqs. (1) & (2) with real-valued variables, has been used in very many different applications of numerous authors. Based on [4], it is a historical prototype-model, out from which so many other models, more applicable for particular problems, have been developed. Using it, the first author has computationally recognized patterns of approximated 3D structures of proteins using a huge memorized data-base (from the Brookhaven protein data bank) [10]. However, for quantum implementation of associative PDP, we should first turn to this model, eqs. (1) & (2), again using it as a “Rosetta stone”. This might then enable subsequent fantastic improvements which are promised by possibly-entangled [13] quantum field dynamics manipulated by so-called classical–quantum interactions. So, the quantum breakthrough for ANN-implementations can best be made with the prototypical associative content-addressable memory of eqs. (1) & (2), because its dynamics is relatively similar to natural processes, mainly in spin systems (i.e., spin glass) [11] and quantum fields [12].

3. SINUSOID ACTIVITIES OF NET’S ”UNITS”

Quantum wave-functions \( \psi^k \) can have many forms. For our purposes, (quantum-optical) plain-waves \( \psi^k(\bar{r}, t) = A^k(\bar{r}, t)e^{i\psi(k)(\bar{r}, t)} \) are the most appropriate. An advanced alternative, to be left for our future work, are quantum wave-packets nearly-identical to Gabor wavelets [4].

Holography shows, at least for non-quantum waves, how one can parallel-distributively encode patterns \( k \) into a web of waves \( (A^1 e^{i\psi^1(k)}, A^2 e^{i\psi^2(k)}, ..., A^N e^{i\psi^N(k)}) \). The amplitude \( A^k \) and the oscillatory phase \( \psi^k \) have the same lower index \( j \) (\( j = 1, ..., N; N \) huge), since they belong to the same "waving" point, which is our “unit” (encoding a point of the pattern).

We can use plain-waves (sinusoids) with the same constant amplitude, say \( A = 1 \); so, \( A^k = 1 \) for all \( k, j \). This is functioning for few decades, known as phase-information holography. We thus replace all \( \psi \)-variables (\( = \psi \)) in eqs. (1), (2), (3) and (4), with \( e^{i\psi} \), instead of Hopfield’s \( A \). We are allowed to do this — it’s a usual mathematical exchange of variables. The essential observation is that with this legal variable-exchange, \( A \leftrightarrow e^{i\psi} \), giving \( \psi^k = e^{i\phi^k} \) instead of \( \psi^k = A^k \), all the simulation-tested mathematics remains valid for sinusoid-encoded patterns also. Thus, we can claim that the Hopfield algorithm, i.e. eqs. (1) & (2), works with complex-valued sinusoid-inputs at least as much as with real-valued inputs! Performance of the wave-phase model (II) with eigenpatterns \( \psi^k = e^{i\phi^k} \), \( A^k = 1 \), is equal to performance of the amplitude model (I) with \( \psi^k = A^k \), \( A^k \) real number. However, when using both – different amplitudes and different phases – performance might be (much) improved, as practically proved by HNeT [3]. Much better results arise using HNeT’s preprocessing
method \[5\] where inputs \(\psi_j^k\) are sigmoidally mapped into phases \(\varphi_j^k\) to obtain a convenient symmetric (uniform) data-distribution: \(\varphi_j^k = 2\pi \left(1 + \exp \left(\frac{v_j^k - v_j^0}{\sigma(v_j^0)}\right)\right)^{-1}\).

To prove quantum-wave pattern recognition with the system of eqs. (1) & (2), it suffices to execute the exchange, \(\psi_j^k \leftrightarrow e^{i\varphi_j^k}\), first in the Hebbian eq. (1), using \((e^{i\varphi})^* = e^{-i\varphi}\):

\[
G_{hj} = \sum_{k=1}^{P} e^{i\varphi_j^k} e^{-i\varphi_j^k} = \sum_{k=1}^{P} e^{i(\varphi_j^k - \varphi_j^k)} = (5),
\]

and secondly in eq. (2). So, instead of eq. (2), when inserting now expression (5) into \(J_{hj}\) of eq. (2) and exchanging \(\psi_j^k \leftrightarrow e^{i\varphi_j^k}\) in it, we obtain the following equivalent of eq. (3):

\[
e^{i\varphi_j^\text{output}} = \sum_{j=1}^{N} \left( \sum_{k=1}^{P} e^{i\varphi_j^k} e^{-i\varphi_j^k} \right) e^{i\varphi_j^\text{input}} = \sum_{k=1}^{P} \left( \sum_{j=1}^{N} e^{i\varphi_j^\text{input}} e^{-i\varphi_j^k} \right) e^{i\varphi_j^k} = e^{i\varphi_j^k} (6).
\]

This enormous process of phase (mis)matching, producing constructive or destructive interferences, is described in detail, mathematically and informatically, in \[5\] and for quantum case in \[12\]. The right-most expression of eq. (6) really describes the output, i.e. approximately \(e^{i\varphi_j^k}\), only if the same conditions are valid as described below eq. (3): If the input wave has a similar phase to one of the memorized waves, say \(k = k_0\), then those wave will be reconstructed — the pattern it is carrying, \(k_0\), will be recognized. See \[12\] for discussion of the precise conditions for clear pattern-recall. If these conditions are not satisfied by the data correlation-structure, interferences (“cross-talk”) lead to a mixed or averaged output (details in \[10\]).

So, instead of a long series of products (correlations) of real-valued information-coding numbers, \(A\), as in the Hopfield model, we have here a long series of complex-valued exponentials (waves) with differences of information-coding phases, \(\varphi\), in each exponent. These phase-differences (peak delays) encode interferences in data. Our wave output \(e^{i\varphi_j^k}\) is the same as Hopfield’s \(A_h^k\). In sum, input–output transformations are the same in the wave case as it were in our simulated real-number (intensity) case \[10\]. All this proves the pattern recognition capabilities of the wave model (II) with phase-encoding of pattern-points \(h\). The memory is ”represented” by the hologram, i.e. wave-interference pattern, of eq. (5).

4. DISCUSSION ON IMPLEMENTATION

Mathematically and computationally \[10\], we have proved the associative memory-storage and pattern-recognition performance of Perus’s model named Quantum Associative Network \[12\]. It remains to prove it in quantum-physical experimental practice, i.e. with real quantum-pattern-encoding waves, not merely with digital simulated ones (complex sinusoids).

It is crucial that our model \[12\] is fundamental, optimized and relatively natural, i.e. almost no artificial devices are really necessary, in contrary to all other models. Laser is also not unavoidable.

Implementation of our model is most appropriate using quantum-wave holography which is within the reach of present experimental technology \[8\] \[13\]. Several applied techniques, which are at least partially holography-based, are already functioning, e.g. some sorts of tomography (MRI and PET scanning) \[14\]. Holography is a fundamental and universal procedure in the sense that, in principle, any sort of coherent waves can be applied for interference-based simultaneous recording of many objects into (and for selective reconstruction from) various hologram-media. Apart of classical optical and acoustical holography, microwave-, X-ray-, atomic- and electron-holography were realized \[3\]. There is just a step further to quantum-wave holography as described here.

This attempt is supported by the following reports: According to \[15\], universal quantum computation is realized using only projective measurement, like ours of eq. (3) or eq. (6), quantum memory, like ours of eq. (1) or eq. (5), and preparation of the initial state (the laser-wave in our case). Information-storage and -retrieval through quantum phase \[16\] and measurements of quantum relative phase \[17\] have been experimentally demonstrated. Quantum encodings in spin systems and coupled harmonic oscillators are possible \[18\], thus enabling Hopfield-like pattern-storage and -recognition in such nets, including spin-wave holographic ones.

However, if nanotechnology could not (which is unlikely) realize quantum-holographic pattern-recognition as proposed here, something like that is hypothesized to be happening in the (visual) brain \[19\]. Not only brain, the whole quantum Nature itself probably incorporates such processes, at least in interaction with our quantum-measurement devices \[19\]. In worst case, it does merely not let us to collaborate with – until tomorrow?

Recently, we found similar quantum pattern-recognition proposals \[20\], Trugenberger’s one is related to the fact that a special case of Hebbian memory-storage, eq. (1), i.e. with bipolar states (1 and -1 only), is equivalent to quantum-implementable NOT XOR gate. This makes a link between ANN-like and logic-gate-based branches of quantum pattern recognition.

The benefits of the first branch, i.e. quantum neuralnet approach, are the following: Quantum decoherence (“collapse of the wave-function”) is not devastating (as is in main-stream quantum computers), but is usefully harnessed for pattern recognition. No special mechanisms are needed for quantum error-correction, since it is done spontaneously by the net’s self-organizing process (as in ANN). Initialization problems are not as serious \[21\] as in logic-gate quantum computers, at least not when an ob-
ject is holographed. In this case, reflection from the surface determines the phases, and fluctuations do not destroy the modulation (cf., experimental quantum-phase storage and retrieval [10]). Finally, as it is characteristic for quantum computers, quantum associative net is exponentially superior to its classical counterparts in memory capacity, processing speed and in miniaturization [1]. This brings improvements in computational capacity and efficiency. Quantum ANN promise to outperform logic-gate quantum computing in associative tasks like discussed here, and in flexibility (fuzzy processing) [12], where also classical ANN outperform sequential computing. Finally, our net presented [12] is relatively inexpensive, because it is relatively natural, and is of huge (at least) theoretical importance.

ACKNOWLEDGEMENTS:
This work was enabled by the European-Union’s Marie-Curie fellowship for M.P (contract no. HPMF-CT-2002-01808). M.P. is also grateful to Professors Karl H. Pribram and Alexandr A. Ezhov for encouragement for fundamental studies.

References

[1] A.A. Ezhov, D. Ventura: Ch. 11 in N. Kasabov (ed.): Future Directions for Intelligent Systems and Information Sciences; Physica/Springer, Heidelberg, 2000. Cf.: E. Behrman et al.: arXiv:quant-ph/0202131 & Info. Sci. 128 (2000) 257-269.

[2] M. Peruš: Neural Netw. World 10 (2000) 1001-13.

[3] E.g.: H. Bjelkhagen & H. Caulfield (eds.): Selected Papers on the Fundamental Techniques in Holography; SPIE Opt. Eng. Press, Bellingham (WA), 2001. H. Caulfield (ed.): Handbook of Optical Holography; Academic Press, New York, 1979. Or see (text)books by: Hariharan; Stroke; Collier et al.; Saxby.

[4] Willshaw, Buneman, Longuet-Higgins (1969); Kohonen (1972); Hopfield (1982). Cf.: Nakano (1972); Anderson (1972); Kosko (1988). [Early holographic associative memories: P. van Heerden, 1963; Gabor, 1969, (1949). Cf., Pribram, 1963, 1969.]

[5] J. Sutherland: Int. J. Neural Sys. 1 (1990) 256-267. Applications: www.ANDcorporation.com

[6] D. Psaltis et al.: Nature 343 (1990) 325-330. Sci. Amer. 273(5) (Nov. 1995) 52-58. R. Spencer in: C. Dagli et al. (eds.): Intelligent Engineering Systems Through ANN (vol. 10); ASME Press, St. Louis, 2000; pp. 971-6.

[7] T.S. Lee: IEEE Transac. Pattern Anal. & Mach. Intell. 18(10) (1996) 1-13. C. Chui: An Introduction to Wavelets; Academic Press, Boston, 1992.

[8] A. Abouraddy et al.: "Quantum holography"; Optics Express 9(10) (2001) 498-505. B. Saleh et al. in: Proc. LEOS’96, vol. 1, Laser & El.Opt. Soc., Boston, 1996, p. 362-3. Cf.: N. Bhatchayara et al.: Phys. Rev. Lett. 88 (2002) 137901. [And: S. Takeuchi: Phys. Rev. A 62(3) (2000) 032301. R. Speeew: Phys. Rev. A 63(6) (2001) 062302.]

[9] M. Peruš: Nonlin. Phenom. in Complex Sys. 4 (2001) 157-193. Zeitschr. angew. Math. & Mech. 78, S 1 (1998) 23-26. Informatica 20 (1996) 173-183.

[10] M. Peruš: Int. J. Computing Anticip. Sys. 13 (2002) 376-391.

[11] S. Bartlett et al.: Phys. Rev. A 65 (2002) 052316. D. Lidar, O. Biham: Phys. Rev. E 56 (1997) 3661-81. [Cf.: NMR bulk / ensemble quantum computing: N. Gershenfeld, I. Chuang: Science 275 (1997) 350-6. D. Cory et al.: Proc. Nat. Acad. Sc. USA 94 (1997) 1634-9.]

[12] M. Peruš, S.K. Dey: Appl. Math. Lett. 13(8) (2000) 31-36. Orig.: M. Peruš in Proc. JC-‘IS’98, vol. 2: 3rdIC-CI&N; eds. P.P. Wang et al., N. Carolina, 1998, p. 197-200.

[13] H. Lee at al.: Phys. Rev. A 65 (2002) 030101.

[14] W. Schempp: "Quantum holography and MR tomography..."; Informatica 21 (1997) 541-562. Cf., G. D’Áriano et al.: "Quantum tomography"; Phys. Lett. A 276 (2000) 25-30.

[15] M. Nielsen: arXiv:quant-ph/0108020

[16] J. Ahn et al.: Science 287 (2000) 463-5. Y. Wu: Phys. Rev. A 63(5) (2001) 052303. Cf.: J. Denenschlag et al.: Science 287 (2000) 97-101.

[17] A. Trifonov et al.: J. Optics B 2 (2000) 105-112.

[18] M. Peruš: Informatica 25 (2001) 575-592. K.H. Pribram: Brain and Perception; LEA, Hillsdale (NJ), 1991. A.F. da Rocha et al.: Progress in Neurobiol. 64 (2001) 555-573. R. Nobili: Phys. Rev. A 32 (1985) 3518-26.

[19] Cf.: R. Cahill: arXiv:quant-ph/0111026 or in Proc. SPIE Conf. #4390: BioMEMS & Smart Nanostructures, ed. L. Kish, 2001, p. 319-328.

[20] A.Yu. Vlasov: "Quantum computations and image recognition"; arXiv:quant-ph/9703010 C.A. Trugenberger: "Quantum pattern recognition"; Phys. Rev. Lett. 86(27) (2002) 277903. Phys. Rev. Lett. 87(6) (2001) 067901. arXiv:quant-ph/0210176. A. Ezhov, A. Nifanova, D. Ventura: Info. Sci. 128 (2000) 271-293. A. Ezhov in Proc. ICAPR’01, eds. Singh et al., Rio de Jan., p. 60-71.

[21] S. Kak: Foundat. Phys. 29 (1999) 267-279.