Stagnation temperature effect on the supersonic flow around pointed airfoils with application for air

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Received: 3 February 2017 / Accepted: 2 January 2018

Abstract. The aim of this work is to develop a new numerical calculation program to determine the effect of the stagnation temperature on the calculation of the supersonic flow around a pointed airfoils using the equations for oblique shock wave and the Prandtl Meyer expansion, under the model at high temperature, calorically imperfect and thermally perfect gas, lower than the dissociation threshold of the molecules. The specific heat at constant pressure does not remain constant and varies with the temperature. The new model allows making corrections to the perfect gas model designed for low stagnation temperature, low Mach number, low incidence angle and low airfoil thickness. The stagnation temperature is an important parameter in our model. The airfoil should be pointed at the leading edge to allow an attached shock solution to be seen. The airfoil is discretized into several panels on the extrados and the intrados, placed one adjacent to the other. The distribution of the flow on the panel in question gives a compression or an expansion according to the deviation of the flow with respect to the old adjacent panel. The program determines all the aerodynamic characteristics of the flow and in particular the aerodynamic coefficients. The calculation accuracy depends on the number of panels considered on the airfoil. The application is made for high values of stagnation temperature, Mach number and airfoil thickness. A comparison between our high temperature model and the perfect gas model is presented, in order to determine an application limit of the latter. The application is for air.

Keywords: Supersonic flow / pointed airfoil / oblique shock / high temperature / aerodynamic coefficients / Prandtl Meyer function / calorically imperfect gas / thermally perfect gas / specific heat at constant pressure / error of computation

1 Introduction

The aerodynamics study problems on a numerical way is a relatively new research area. Most previous work either theoretical [1–6], or numerical [3,5,7–16] or even experimental in wind tunnel [9] on supersonic flows around airfoils are devoted to rounded airfoils at the leading edge, that is to say a development of a detached shock wave at the leading edge. These studies are generally based on the numerical solution of the Euler equations [1–6] or the equation of potential speed [1,4,5].

The pointed shape of the airfoil at the leading edge gives the possibility of having an attached shock wave, where a numerical technique can be used to evaluate the aerodynamic parameters of the flow. Since the flow is supersonic in the open air, far from the presence of any other obstacle, this technique makes it possible to progressively follow the flow on the airfoil surface as a function of the parameters of the upstream flow.

Given the complexity of the methods used in the supersonic aerodynamics [1–6], and in particular that presented in this study, called a relaxation shock method, the authors use an analytical technique named by thin-airfoil theory [3–6] to evaluate approximately the flow parameters, and in particular the calculation of the aerodynamic coefficients. This method gives acceptable results for very small airfoil thicknesses and upstream Mach number.

The first study on the use of the expansion shock method is presented in reference [17]. This method is used for the calorically and thermally perfect gas. They assume in this case that the specific heat at constant pressure $c_p$ is constant and does not depend on the temperature. This approach gives acceptable results only if the three parameters $M_0$, $T_0$ and $t/C$ are very small. It is our opinion that the PG model does not depend on the stagnation temperature $T_0$.

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The aim of this work is to develop a new mathematical model and to develop in this context a new numerical calculation program to determine the stagnation temperature effect on the supersonic flow around a pointed airfoil, using the equations of an oblique shock and the Prandtl Meyer expansion in the case at high temperature, calorically imperfect and thermally perfect gas, lower than the dissociation threshold of the molecules, in order to correct the PG model, and to determine the aerodynamic characteristics presented by the lift, drag and pitching moment coefficients as a function of upstream Mach number, incidence angle, airfoil shape and airfoil thickness and especially of the stagnation temperature, when they are high. In this case, the specific heat at constant pressure does not remain constant and varies with the increase in temperature, which will be taken into account in our HT model. The stagnation temperature $T_0$ is an important parameter in our model, which will allow for considerable corrections to the results given by the PG model. The latter gives good results only if the values of $M_0$, $T_0$, $\alpha$, $t/C$ are very small. Then, given the current and future applications in supersonic aerodynamics requiring high values of $M_0$, $T_0$ and $t/C$ with increasing respectively $2.00$, $1000$ K and $10.0$, and can arrive at $5.00$, $3500$ K respectively. The PG model falls failing, and the results given by this model becomes very far from the reality, which requires to make corrections to this model, hence the interest of the application of our HT model. We can consider that the HT model becomes a generalization of the PG model. In the other word, the PG model becomes a particular case of our HT model. The PG model falls failing, and the results given by this model becomes very far from the reality, which requires to make corrections to this model, hence the interest of the application of our HT model. We can consider that the HT model becomes a generalization of the PG model, were the application of the HT model is extended to high value of $T_0$, $M_0$, $\alpha$ and $t/C$. The application is for air. In this case and for the $C_P$ function, one finds in references [5,8,9,18,19] a series of a tabulated values for the variation of $C_P$ according to the temperature, between 55 K and 3550 K (limit not to have a dissociation of the molecules). In this temperature margin and according to references [15,16,18,19], only the translational, rotational and chemical vibrational energies are present and are included in the total evaluation of the specific heat $C_P(T)$ of air. Other sources of energy, such as the molecular dissociation energy and the ionization energy of the atoms, are absent since the temperature margin does not exceed 3500 K. A polynomial interpolation is made to these values, after several tests, in order to find an analytic function with the variation of $C_P(T)$ with the temperature. A choice on a 9th degree polynomial is made, giving a maximum error less than 0.01%. More details are found in references [8,9].

2 Mathematical model at high temperature

A flow deviation on the airfoil surface may result in a compression or a Prandtl Meyer expansion. If the compression is produced in the flow, a shock wave develops at the beginning of the deviation of the obstacle as shown in Figure 1. Otherwise, expansion waves develop at the beginning of the deviation as Figure 2 shown it. In both cases, there is a perturbation of the flow which results in a change of all the flow parameters through this deviation. It can be shown that, through the shock, only the total temperature [5,8,9,15,16] is conserved. Then $T_0 = T_{01} = T_{02}$. But the total pressure and density change values. Hence $P_{02}/P_{01} = \rho_{02}/\rho_{01} \neq 1.0$.

For the Prandtl Meyer expansion, the temperature and the total pressure will be preserved. We set the conditions at infinity upstream by $T_0$, $\rho_0$ and $P_0$. We assume that the state equation of a perfect gas ($P = rRT$) remains valid, with $R = 287.102$ J/(kg K). For PG model $\gamma = 1.402$ is taken [8,9,14].

2.1 Oblique shock wave

Figure 1 shows a general diagram of the development of an oblique shock wave at the beginning of the deflection of an obstacle by an angle $\psi = |\theta_2 - \theta_1|$ and the envisaged parameters.

For the determination of the parameters ($M_2$, $\beta$, $T_2$, $T_1$, $P_2/P_1$, $P_{02}/P_{01}$, $\Delta S_{21}$) through the oblique shock, the HT model presented in references [10–13] is used after making a correction to the relation between $\beta$, $\psi$ and $M_1$, since the authors used the equation designed for the PG model to constant $C_P$ [10], given the difficulty of finding an analytic form, which gives results far enough of reality, and that does not meet the need for HT assumptions This equation is the most interesting in the calculation of the shock parameters, since all the other parameters depend on $\beta$, $\psi$ and $M_1$. Then, in the quality of the results, corrections will be found to the results presented in the said references. Another modification made at the $C_P(T)$ level used in these references. It has been observed that $C_P(T)$ used exhibits a slight discontinuity in the passage of $T = 1000$ K with a 27% error between the function used and the tabulated values [18,19].

The relationship can be summarized as follows:

$$\frac{\rho_2}{\rho_1} = \frac{tg(\beta)}{tg(\beta - \psi)}, \quad (1)$$

$$\frac{T_2}{T_1} = \frac{\rho_2}{\rho_1} = \frac{M_1^2 \sin^2(\beta) \times \gamma(T_1) \times \left(1 - \frac{\rho_2}{\rho_1}\right)}{\left(\frac{\rho_2}{\rho_1}\right)^2}, \quad (2)$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \left[\frac{\rho_2}{\rho_1}\right]. \quad (3)$$
analytic relation between presented in this work. Since the development of an shock that occurs in nature.

\[ M_2 = \frac{1 - \frac{P_2}{P_1}}{\sin^2(\beta - \psi) \times \gamma(T_2)} \times \left[ 1 - \frac{\frac{P_2}{P_1}}{\frac{\rho_2}{\rho_{02}}} \right] \]  

\[ \frac{\rho_2}{\rho_{02}} = \text{Exp} \left( -\int_{T_0}^{T_2} \frac{C_p(T)}{a^2(T)} \ dT \right) \]  

\[ \frac{P_2}{P_{02}} = \frac{P_2}{P_{01}} \left( \frac{\rho_2}{\rho_{02}} \right) \]  

\[ \frac{P_{02}}{P_{01}} = \frac{\frac{P_2}{P_{02}}}{\frac{P_2}{P_{01}}} \]  

\[ \frac{\Delta S_{21}}{R} = -\text{Log} \left( \frac{P_{02}}{P_{01}} \right) \]  

\[ \gamma(T) = \frac{C_p(T)}{C_p(T) - R} \]  

Two solutions can be found depending on the value of \( M_2 \), implying that all physical parameters will admit two solutions. If \( M_2 > 1.00 \), a weak shock is obtained. If \( M_2 < 1.00 \), a strong shock is obtained. In general, the weak shock that occurs in nature.

Prior to the determination of the flow parameters by (1–8), we must determine the angle \( \beta \) corresponding to \( \psi \) and \( M_1 \). Then in this work, we will determine the deviation \( \beta \) with high precision according to the real \( HT \) model presented in this work. Since the development of an analytic relation between \( \beta, \psi \) and \( M_1 \) is quite complicated, we will use the relations of a normal shock wave to \( HT \) model [14].

2.2 Expansion of Prandtl – Meyer

The situation of the presence of a Prandtl Meyer expansion is presented in Figure 2. In this case we will have a deviation of an angle \( \psi = |\theta_2 - \theta_1| \). The flow becomes parallel to the wall after the deviation, and the calculation of the parameters after the expansion takes place after the calculation of the new value of the Prandtl Meyer function by the following relation:

\[ v(T_2) = v(T_1) + \psi \]  

The function \( PM \) at \( HT \) can be calculated by the following equation [20,21]:

\[ v(T) = \int_1^T \frac{C_p(T)}{2H(T)} \sqrt{M^2(T) - 1} \ dT, \]  

where [8,9]:

\[ M(T) = \frac{\sqrt{2H(T)}}{a(T)}, \]  

\[ a^2(T) = \gamma(T)RT, \]  

\[ H(T) = \int_1^T \frac{C_p(T)}{RT} \ dT. \]  

First, it is necessary to calculate the critical temperature \( T^* \) corresponding to the Mach number \( M = 1.00 \). This temperature depends only on \( T_0 \). It can be determined numerically by solving the nonlinear equation obtained from relation (12) by replacing \( M = 1.00 \) and \( T = T^* \) using the bipartition algorithm [22–24]. We obtain \( T^* < T_0 \). It is calculated once in the problem.

In equation (10), \( \psi \) and \( T_1 \) are known. Then \( v(T_1) \) can be calculated from equation (11) by replacing \( T = T_1 \). The evaluation of the obtained integral is done by the use of the Simpson quadrature with condensation of the nodes [20] or using the Gauss Legendre quadrature of a function having a weight term to accelerate the numerical process [21]. Let us replace the obtained result in (10) to determine the new value \( v(T_2) \). Let us replace again this value in (11). In this case we fall into an inverse problem. That is to say, it is necessary to determine the temperature \( T_2 \) which gives the integral (11) equal to the value given by (10). To determine \( T_2 \) from (11), a combination of the bipartition algorithm with the Gauss Legendre quadrature was used. It should be noted that \( T_2 < T^* \). The precision chosen in the calculation is \( \epsilon = 10^{-8} \).

In this case, the bipartition algorithm is used 27 times.

Once \( T_2 \) is determined, it is possible to obtain the corresponding ratio \( T_2/T_0 \) and the Mach number \( M_2 \) by relation (12) and \( \rho_2/\rho_0 \) and \( P_2/P_0 \) respectively by relations (5) and (6). We will have an increase in Mach number, i.e., \( M_2 > M_1 \). Integration (5) is done by using the Simpson method with nodes condensation [8,9].

3 Numerical procedure

The aim is to determine the aerodynamic characteristics summarized by the variation of \( M, T/T_0, \rho/\rho_0, P/P_0 \) along the airfoil surface and consequently the determination of the aerodynamic coefficients \( C_D, C_L \) and \( C_m \) for different airfoils.

Figures 1 and 2 shows the case of the flow deflection on the extrados. For the intrados, the opposite occurs. To group the problem in a single relation, we used the absolute value for the evaluation of the angle. In the developed program, we first compute the angle \( \psi = \theta_2 - \theta_1 \). Then, if we are on the extrados, the expansion occurs if \( \psi < 0 \) and the compression occurs if \( \psi > 0 \). The opposite is taken into account on the intrados.

Subdividing the selected airfoil into \( k \) nodes on the upper surface and into \( l \) nodes in the lower surface as shown in Figure 3. The total number of the nodes on the chosen
The airfoil surface is equal to \( n = k + l - 2 \). The number of regions on the upper surface is equal to \( k - 1 \) and it is equal to \( l - 1 \) in the lower surface.

A streamline coming from infinity upstream is divided into two parts at the leading edge of the airfoil (point 1). One is through the upper surface and the other through the lower surface, so that the two lines meet again at the trailing edges. We can consider the flow calculation according to the upper part and the lower surface after the other.

Every two juxtaposed panels are connected by a node. At each node, the flow undergoes a deviation by generating either a compression or expansion, or similar continuous across it, Figure 3.

To expansion either on the upper or lower surfaces, it will be an increase in Mach number, that is to say that \( M_2 > M_1 \). As against a compression will decrease the Mach number \( M_2 < M_1 \). Since it was considered that the shock is weak (real case).

Note that these parameters are constant along the segment for the shock and expansion. The flow properties in a region on the right are function of the flow parameters in the region of the left. Since these are known. There may be a detached shock if the angle \( \psi \) is minimum. The jump in entropy is zero for an expansion [1–6]. For a compression, the entropy is different to zero. The relation (8) gives the variation of the entropy between the passages across it, Figure 3.

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The jump in entropy is zero for an expansion [1–6]. For a compression, the entropy is different to zero. The relation (8) gives the variation of the entropy between the passages from one adjacent segment to another. To determine the total flow leap around the airfoil, all the local entropy leaps must be summed.

We may encounter a cases where the flow on the upper and lower surfaces has no deviation, i.e., \( \psi = (\theta_2 - \theta_1) = 0 \), \( \theta_2 = \theta_1 \). In this case, the flow properties remain unchanged.

To determine the position \((x, y)\) of a node number \( i \) on the upper surface of coordinates \((x_E, y_E)\), or the lower surface of coordinates \((x_I, y_I)\) we choose a reference coordinate and a small step. The step can be constant or variable. Typically the reference coordinates is selected at the leading edge of the airfoil. By replacing the value of \( x_E \) in the airfoil equation, is easily to determine the value of its ordinate \( y_E \). Similarly for the lower airfoil surface, there will be \( y_I \) by the following relations:

\[
x_E = \frac{(i - 1)}{(k - 1)} \times C, \quad y_E = f_E(x_E) \quad i = 1, 2, \ldots, k \tag{15}
\]

\[
x_I = \frac{(j - 1)}{(l - 1)} \times C, \quad y_I = f_I(x_I) \quad j = 1, 2, \ldots, l \tag{16}
\]

For the determination of the angle deviation \( \theta_\alpha \) made by a line segment connected between the points \( i \) and \( i+1 \) with the horizontal, in order to determine the flow angle deflection \( \psi \), the following relationship is used:

\[
\theta_\alpha = \arctan \left( \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \right), \tag{17}
\]

with: \( i = 1, 2, \ldots, k \) for extrados, and \( i = 1, 2, \ldots, l \) for intrados.

The region number 0 is the upstream infinity provided free data.

The flow in the region number 1 of the upper and the lower surface is given by the region number 0 of the upstream infinity.

The region number \( i \) of the upper or lower surface is limited by the nodes number \( i \) on the left and the node number \( i + 1 \) on the right.

The pressures \( P_E \) and \( P_I \) are broken down into two components each one, as shown in Figure 4. The vertical component that represents the lift force is \( L_E \) on the upper, and \( L_I \) on the lower surface. Similarly, the horizontal components that represent the drag are \( D_E \) on the upper and \( D_I \) on the lower surface. The pitching moment of the two forces exerted on the extrados is named by \( M_E \) and the moment of the two forces exerted on the intrados is \( M_I \). The pitching moment is calculated with respect to the point \( p \), Figure 4. For the applications, \( p = 0 \) is taken. Then, on the upper surface and per unit of depth we have

\[
L_E = -P_E \times (x_{E,i+1} - x_E),
\]

\[
D_E = P_E \times (y_{E,i+1} - y_E),
\]

\[
m_E = -L_E \frac{x_{E,i+1} + x_E}{2} - D_E \frac{y_{E,i+1} + y_E}{2},
\]

with \( i = 1, 2, \ldots, k - 1 \)

On the lower surface and per unit of depth we have

\[
L_I = P_I \times (x_{I,j+1} - x_I),
\]

\[
D_I = -P_I \times (y_{I,j+1} - y_I),
\]

\[
m_I = L_I \frac{x_{I,j+1} + x_I}{2} + D_I \frac{y_{I,j+1} + y_I}{2},
\]

with \( j = 1, 2, \ldots, l - 1 \)

The number of nodes \( k \) on the upper surface is not necessarily equal to the number of the nodes \( l \) on the upper surface. For applications, the leading edge is placed at the point \( O \) of the reference of calculation.
The total lift and drag across the airfoil are respectively considered as the sum of the forces on all segments (regions) of the upper and lower surfaces.

\[
L = (L_{E1} + L_{E2} + \cdots + L_{Ek}) + (L_{I1} + L_{I2} + \cdots + L_{Ik}),
\]

\[
D = (D_{E1} + D_{E2} + \cdots + D_{Ek}) + (D_{I1} + D_{I2} + \cdots + D_{Ik}),
\]

\[
m = (m_{E1} + m_{E2} + \cdots + m_{Ek}) + (m_{I1} + m_{I2} + \cdots + m_{Ik}).
\]

The pitching moment is considered positive when it rotates counter clockwise, Figure 4.

The aerodynamic coefficients are obtained as follows:

\[
C_L = \frac{L}{q_0 \times S},
\]

\[
C_D = \frac{D}{q_0 \times S},
\]

\[
C_m = \frac{m}{q_0 \times S \times C},
\]

where

\[
q_0 = \frac{1}{2} \gamma(T_0)P_0 M_0^2.
\]

The surface of reference \(S\) is considered to be the airfoil chord per unit of depth.

By varying the parameters \(M_0\), \(t/C\), \(\alpha\) and \(T_0\) as well as the airfoil shape, it is possible to find all the possible parameters and in particular the effect of \(T_0\) when it is high. A comparison between the \(PG\) model and the \(HT\) model is performed.

4 Applications

Our applications are limited by three types of airfoils which are symmetrical lozenge, Symmetrical curved airfoil and unsymmetrical curved airfoil. The calculation of the pitching moment is made to the leading edge.

4.1 Symmetrical lozenge

The lozenge is shown in Figure 5. We can meet this type of airfoil for building applications wings of supersonic aircraft. The geometry is given by the maximum thickness \(t\). It is chosen in the middle of the chord. With \(0 \leq x/C \leq 1\).

We discretize the airfoil into \(k = l - 3\) nodes. Two regions in upper and lower surface will be sufficient to describe the flow and to have the exact solution.

4.2 Symmetrical curved airfoil

The form of this type of airfoil is shown in Figure 6. The equation of the extrados and the intrados is chosen as a polynomial of 3rd degree. Its equation is given by:

\[
f_E(x) = -f_l(x) = \frac{27}{8} t \frac{x}{C} \left(1 - \frac{x}{C}\right)^2,
\]

where \(0 \leq x/C \leq 1\). The maximum thickness \(t\) of this airfoil is at a distance \(x/C = 1/3\) from the leading edge.

4.3 Curved non symmetrical airfoil

The shape of the upwards curved airfoil is shown in Figure 7. The upper and lower surface are selected from the parabolic equation with respect 3 chosen conditions. We encountered this type of airfoil applications for blade of compressor. The equation of the extrados and the intrados are respectively chosen by:

\[
\frac{f_E(x)}{t_E} = \frac{f_l(x)}{t_E} = 4 \frac{x}{C} \left(1 - \frac{x}{C}\right).
\]

With \(0 \leq x/C \leq 1\). In this case, the maximum thickness \(t\) of this airfoil lies in the middle of the chord. This type of airfoil is referred to as a skeletal airfoil. The value of \(t_E\) is chosen arbitrarily greater than 0. While \(t/C = 0.03\) for the applications.

5 Error of \(PG\) model compared to \(HT\) model

For each parameter, the error given by the \(PG\) model compared to our \(HT\) model can be calculated by the following relation, for aim to compare the two models:

\[
\varepsilon_{\text{Parameter}}(\%) = \left|1 - \frac{\text{Parameter}_{PG}}{\text{Parameter}_{HT}}\right| \times 100.
\]

6 Results and comments

The results were divided into eight parts. In order to obtain graphical results, we have used a discretization of \(k = l = 1000\) points on the extrados and on the intrados. For the tabulated results, we have used a discretization of \(k = l = 8000\) points. This discretization is chosen in such a way that there will be 5 exact digits of the solution. Figures 9–21 contain four curves. Curves 1–4 respectively represent the variation of the parameter at \(HT\) for \(T_0 = 3000\, \text{K}, \ 2000\, \text{K}, \ 1000\, \text{K}\)
and the case PG for $\gamma = 1.402$. Figure 9–12 contain 8 curves. The 4 curves in continuous line are to represent the variation of the selected parameter on the extrados of the airfoil with HT. The 4 dashed curves represent the variation of the selected parameter on the airfoil intrados.

In this study three airfoils have been considered as Figures 5–7 shown them. The third airfoil (Fig. 7) is characterized by two parameters. While the 1st and 2nd airfoils are characterized by one parameter. One can even consider airfoils with several parameters.

The results for the PG model can be found in references [17]. They are presented for comparison with the HT model.

6.1 Typical example

In this example, we chose three very interesting airfoils in aerodynamics. The aim is to present the effect of $T_0$ on $C_D$, $C_L$, $C_m$ and $\Delta S_{21}$ in a numerical way as well as the calculation of the error committed by the PG model with respect to the HT model for each value of $T_0$.

Table 1 shows the effect of $T_0$ on $C_D$, $C_L$, $C_m$ and $\Delta S_{21}$ for the symmetrical lozenges of Figure 5 when $\alpha = 2.0^\circ$, $M_0 = 4.00$ and $t/C = 0.1$, followed by the given errors of PG model on these coefficients with respect to the HT model as represented in Table 2.

Table 3 shows the effect of $T_0$ on $C_D$, $C_L$, $C_m$ and $\Delta S_{21}$ for the symmetric curved airfoil of Figure 6 when $\alpha = 2.0^\circ$, $M_0 = 4.00$ and $t/C = 0.1$, followed by the given errors of PG model on these coefficients with respect to HT model as represented by Table 4.

Table 5 shows the effect of $T_0$ on $C_D$, $C_L$, $C_m$ and $\Delta S_{21}$ for the non-symmetric curved airfoil of Figure 7 when $\alpha = 2.0^\circ$, $M_0 = 4.00$ and $t/C = 0.2$, followed by the given errors of PG Model on these coefficients with respect to HT model as represented by Table 6. For this third airfoil, $t_E/C = 0.03$ was taken for the application.

We clearly notice the $T_0$ effect on all aerodynamic parameters for the selected three airfoils. The difference between the PG and HT models increases with increasing of $T_0$. The maximum error is noticed on the coefficient $C_L$ which can arrive at 35% when $T_0 = 3000$ K and which can arrive at 39.38% when $T_0 = 3500$ K for $M_0 = 4.00$ and $\alpha = 2.00$. It is noted that the shape of the airfoil also influences the difference between the two models, although the parameters $\alpha$, $M_0$ and $t/C$ are the same for the three airfoils. The PG model determines the aerodynamic parameters with excess. It is designed to solve low $T_0$ problems. Then, if $T_0$ increases, the performance of the flow will be degraded with respect to the PG model.

6.2 Variation of the flow deviation along airfoil surface

Figure 8 shows the variation in the deviation of the airfoil wall which also represents the variation of the flow deviation along the surface of the selected three airfoils. This deviation enters into the calculation of the value of $\psi$ determining the type of the flow, whether compression or expansion, given by this deviation.

6.3 Variation of the parameters along the airfoil surface

Figures 9–12 show the effect of $T_0$ on the variation of $M$, $T/T_0$, $\rho/\rho_0$ and $P/P_0$ along the surface of the extrados and the intrados of the three selected airfoils. In Figures 11 and 12 the ratios $\rho/\rho_0$ and $P/P_0$ are calculated with respect to the total local conditions of the segments. We clearly notice the effect of $T_0$ on all thermodynamic parameters with a degradation of $M$ and $T/T_0$ when $T_0$ decrease, and increase...
of $\rho/\rho_0$ and $P/P_0$ when $T_0$ increases gradually. This influence necessarily gives the influence of $T_0$ on all the aerodynamic coefficients $C_D$, $C_L$ and $C_m$.

The variation of these parameters is related to the deviation of the wall, shown in Figure 8, along the airfoil surface. As the wall deviation takes two values on the extrados and on the intrados of lozenge, one will consequently have 2 values of these parameters on the extrados and on the intrados. While for the other two airfoils there is a continuous variation along the surface of the airfoils. Thus the airfoil shape, in addition to $T_0$, affects the variation of the parameters on the airfoil surface.

Fig. 9. Effect of $T_0$ on the variation of $M$ along the surface of the three selected airfoils when $\alpha = 2.0^\circ$ and $M_0 = 4.00$.

Fig. 10. Effect of $T_0$ on the variation of $T/T_0$ along the surface of the three selected airfoils when $\alpha = 2.0^\circ$ and $M_0 = 4.00$. 
6.4 Effect of $\alpha$ to $HT$ on the aerodynamic coefficients for fixed $M_0$ and $t/C$

Figures 13–15 represent the effect of $T_0$ respectively on the variation of $C_D$, $C_L$, and $C_m$ as a function of $\alpha$ for the three airfoils when $M_0 = 4.00$. For the symmetrical airfoils, we have found a symmetry of the variation of $C_D$, $C_L$ and $C_m$. While for the third airfoil, this symmetry is not present, since the airfoil is not symmetrical. We clearly notice the effect of $T_0$ on the three parameters. To determine the actual value of $C_D$, $C_L$ and $C_m$, the value obtained from Figures 13–15 must be divided by $10^3$. 

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In Figures 13–15, the range of variation of the incidence angle \( \alpha \) is taken \([-10^\circ, 10^\circ]\). Generally beyond this interval, we will have the phenomenon of stall.

Table 1. Effect of \( T_0 \) on \( C_D \), \( C_L \), \( C_m \) and \( \Delta S_{21}/R \) for the lozenge when \( \alpha = 2.0^\circ \), \( M_0 = 4.00 \) and \( t/C = 0.1 \).

| \( PG \)  | \( HT \) | \( T_0 \) (K) |
|---------|---------|---------|
| \( C_D (10^3) \) | \( 1.402 \) | \( 1000 \) | \( 2000 \) | \( 3000 \) |
| \( 0.07047 \) | \( 0.06961 \) | \( 0.06015 \) | \( 0.05489 \) |
| \( C_L (10^3) \) | \( 0.25847 \) | \( 0.25451 \) | \( 0.21908 \) | \( 0.19981 \) |
| \( C_m (10^3) \) | \( 0.10140 \) | \( 0.09980 \) | \( 0.08620 \) | \( 0.07895 \) |
| \( \Delta S_{21}/R \) | \( 0.04326 \) | \( 0.03848 \) | \( 0.03459 \) | \( 0.03541 \) |

Table 2. Error of \( PG \) model compared to \( HT \) model according to Table 1.

| \( \epsilon(\%) \) | \( T_0 = 1000 \) K | \( T_0 = 2000 \) K | \( T_0 = 3000 \) K |
|-----------------|-----------------|-----------------|-----------------|
| \( \epsilon(C_D) \) | 1.23 | 17.15 | 28.38 |
| \( \epsilon(C_L) \) | 1.55 | 17.97 | 29.35 |
| \( \epsilon(C_m) \) | 1.60 | 17.63 | 28.43 |

Table 3. Effect of \( T_0 \) on \( C_D \), \( C_L \), \( C_m \) and \( \Delta S_{21}/R \) for the symmetrical curved airfoil when \( \alpha = 2.0^\circ \), \( M_0 = 4.00 \) and \( t/C = 0.1 \).

| \( PG \)  | \( HT \) | \( T_0 \) (K) |
|---------|---------|---------|
| \( C_D (10^3) \) | \( 1.402 \) | \( 1000 \) | \( 2000 \) | \( 3000 \) |
| \( 0.14376 \) | \( 0.14138 \) | \( 0.12087 \) | \( 0.10980 \) |
| \( C_L (10^3) \) | \( 0.27933 \) | \( 0.27055 \) | \( 0.22723 \) | \( 0.20630 \) |
| \( C_m (10^3) \) | \( 0.10603 \) | \( 0.10236 \) | \( 0.08608 \) | \( 0.07844 \) |
| \( \Delta S_{21}/R \) | \( 0.73606 \) | \( 0.68655 \) | \( 0.64513 \) | \( 0.66500 \) |

Table 4. Error of \( PG \) model compared to \( HT \) model according to Table 3.

| \( \epsilon(\%) \) | \( T_0 = 1000 \) K | \( T_0 = 2000 \) K | \( T_0 = 3000 \) K |
|-----------------|-----------------|-----------------|-----------------|
| \( \epsilon(C_D) \) | 1.68 | 18.93 | 30.92 |
| \( \epsilon(C_L) \) | 3.24 | 22.92 | 35.40 |
| \( \epsilon(C_m) \) | 3.58 | 23.17 | 35.17 |

Table 5. Effect of \( T_0 \) on \( C_D \), \( C_L \), \( C_m \) and \( \Delta S_{21}/R \) for the non-symmetrical curved airfoil when \( \alpha = 2.0^\circ \), \( M_0 = 4.00 \) and \( t/C = 0.03 \) and \( t_E/C = 0.2 \).

| \( PG \)  | \( HT \) | \( T_0 \) (K) |
|---------|---------|---------|
| \( C_D (10^3) \) | \( 1.402 \) | \( 1000 \) | \( 2000 \) | \( 3000 \) |
| \( 1.85497 \) | \( 1.81198 \) | \( 1.52967 \) | \( 1.38558 \) |
| \( C_L (10^3) \) | \( 0.20469 \) | \( 0.23986 \) | \( 0.26185 \) | \( 0.25586 \) |
| \( C_m (10^3) \) | \( 1.23532 \) | \( 1.22749 \) | \( 1.06738 \) | \( 0.97667 \) |
| \( \Delta S_{21}/R \) | \( 1.41865 \) | \( 1.37919 \) | \( 1.36966 \) | \( 1.39133 \) |

It is very important to determine an angle of incidence which makes it possible to find \( C_L = 0 \), called angle of zero lift, and the incidence angle making it possible to find \( C_m = 0 \), called zero moment angle. For symmetrical airfoils, this angle is equal to 0.0. Whereas for the non-symmetrical airfoils, force will have a value other than zero and which depends on \( T_0 \). Table 7 shows the effect of \( T_0 \) on the zero lift angle for the third non-symmetric airfoil and Table 8 shows the effect of \( T_0 \) on the zero moment angle for the third airfoil for some values of \( M_0 \) when \( t/C = 0.03 \) and \( t_E/C = 0.1 \). We notice the effect of \( T_0 \), \( M_0 \), \( t/C \), the shape of airfoil and \( t_E/C \) on these two remarkable angles.

It is clear that for an incidence angle \( \alpha \) in the vicinity of the angle of zero lift, or the angle of zero moment, of one degree, there is no influence of \( T_0 \) on these parameters.
6.5 Effect of $M_0$ at HT on the aerodynamic coefficients for fixed $a$ and $t/C$

Figures 16–18 represent the effect of $T_0$ respectively on the variation of the aerodynamic coefficients $C_D$, $C_L$ and $C_m$ as a function of $M_0$ for the three selected airfoils when

$\alpha = 2.0^\circ$. The presentation is done on a Logarithmic scale because of the fact that the small values are grouped in a Figure with the large values of the aerodynamic coefficients. We clearly notice the effect of $T_0$ on these

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### Table 6. Error of $PG$ model compared to HT model according to Table 5.

| $T_0$ (K) | $\varepsilon(C_D)$ | $\varepsilon(C_L)$ | $\varepsilon(C_m)$ |
|----------|-------------------|-------------------|-------------------|
| 1000     | 2.31              | 17.18             | 0.63              |
| 2000     | 17.53             | 27.92             | 13.59             |
| 3000     | 25.30             | 33.56             | 22.93             |

### Table 7. Effect of $T_0$ the zero lift angle for the third non-symmetric airfoil in function of $M_0$.

| $M_0$ | $\gamma = 1.402$ | $T_0$ (K) | $\alpha$ (degree) |
|-------|-----------------|----------|------------------|
| 2.00  | 1.12621         | 1.09631  | 1.00951          |
| 3.00  | 1.22095         | 1.16998  | 1.10840          |
| 4.00  | 1.37949         | 1.30565  | 1.21869          |
| 5.00  | 1.44868         | 1.38326  | 1.26308          |
coefficients. This effect becomes important as $M_0$ increases gradually. Then for small values of $M_0$ up to about $M_0 < 2.00$, the difference between the $PG$ and $HT$ models is not significant, which shows that the results given by the $PG$ model are acceptable. But if $M_0$ increases, independently of $T_0$, the corrections made by the $HT$ model are necessary, which shows the interest of the $HT$ model for the large values of $M_0 > 2.00$. Generally this limit depends on the error considered in the calculation. It should also be noted that if $M_0$ decreases, there will be the appearance of a detached shock wave which develops at the leading edge of the airfoil. Commentaries remain valid for any values of $t/C$ and $t_{E}/C$.

The variation of $M_0$ in Figures 16–18 starts from $M_{0\text{min}}$ up to 5.0. We note the effect of $T_0$ on the minimum value of the Mach number upstream $M_{0\text{min}}$, that can have the flow to limit the attached shock with the detached shock, respectively for the three selected airfoils. Forcing this limit depends on $T_0, \alpha, t/C$ and $t_{E}/C$. The limit given by the $PG$ model $M_{0\text{min}}(PG) > M_{0\text{min}}(HT)$ is considered as an attached shock for the $HT$ model, since, that is to say one can have a Mach

### Table 8. Effect of $T_0$ the zero moment angle for the non-symmetric airfoil in function of $M_0$.

| $M_0$ | $PG$ | $HT$ | $T_0$ (K) |
|------|------|------|----------|
| 2.00 | /    | /    | 1000     |
| 3.00 | -4.62572 | -4.72499 | 2000     |
| 4.00 | -4.37190 | -4.47564 | 3000     |
| 5.00 | -4.12962 | -4.22671 | /        |

Fig. 16. Effect of $T_0$ on the variation of $C_D$ as a function of $M_0$.

Fig. 17. Effect of $T_0$ on the variation of $C_L$ as a function of $M_0$. 

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number $M_0 < M_{\text{th}}(PG)$ for HT with the possibility of finding an attached shock. It is noted that the margin of finding a detached shock is greater for the third non-symmetric airfoil than the two others. Then $M_{\text{th}}$ not only depends on $\alpha$, $t/C$, but also depends on the shape of the airfoil.

6.6 Effect of $t/C$ at HT on the aerodynamic coefficients for fixed $M_0$ and $\alpha$.

Figures 19–21 represent the effect of $T_0$ respectively on the variation of $C_D$, $Cl$, and $C_m$ as a function of the thickness $t/C$ of the three airfoils when $M_0 = 4.00$ and $\alpha = 2.00$. For the non-symmetrical airfoil, the curvature of the extrados was varied without touching the airfoil thickness taken at $t/C = 0.03$. We note the effect of $T_0$ on these parameters, despite the small thickness, which requires the use of the HT model for corrections despite at low $t/C$. It is also noted that if $T_0$ decreases, the HT model becomes non-comparable with the PG model. Then when $T_0 < 240$ K, the PG model gives acceptable results. It should also be noted that the shape of the airfoil will still influence the variation of these coefficients.

It is noted that if $t/C$ increases, it will be possible to find a detached shock. If $t/C \leq (t/C)_{\text{max}}$, there will necessarily be an attached shock, hence a solution can be given by the developed program. If $t/C > (t/C)_{\text{max}}$, we will have the appearance of a detached shock wave.

6.7 Variation of the aerodynamic coefficients as a function of $T_0$ for fixed $\alpha$, $M_0$ and $t/C$.

Figures 22–24 represent the variation of the aerodynamic coefficients of the three selected airfoils as a function of $T_0$.
and the comparison with the PG model. The latter does not depend on $T_0$, where it is represented by a horizontal straight line along the entire interval. The $C_D$, $C_L$, and $C_m$ variation is chosen for $M_0 = 4.00$ and $\alpha = 2.00^\circ$. We can note the effect of $T_0$ on these parameters. Hence the need to use the HT model for possible corrections. One notices again when $T_0$ is small, the HT model becomes confounded with the PG model. We can go to about 240 K. Then when $T_0 < 240$ K, the PG model gives acceptable results. It is also noted that the shape of the airfoil still influences the variation of these coefficients since the variation of the aerodynamic coefficients of the three airfoils is not the same. We note that the HT model degrades the $C_D$, $C_L$, and $C_m$ values due to their decrease with increasing of $T_0$, which does not work with the physical and real behavior of the flow.

6.8 Variation of the error caused by the PG model compared to HT model as a function of $M_0$

Figures 25–27 represent the variation of the relative error caused by the use of the PG model with respect to the HT model on the aerodynamic coefficients $C_D$, $C_L$, and $C_m$ of the three selected airfoils. The application is made for the temperatures $T_0 = 1000$ K, 2000 K and 3000 K and for $\alpha = 2.00^\circ$ and $t/C = 0.1$. It can be seen that the error increases considerably and can reach 55% when $T_0 = 3000$ K and $M_0 = 5.00$. This error is noticed for the coefficients $C_L$ and $C_m$ for the symmetrical curved airfoil. This value also increases with the airfoil shape, $t/C$ and $\alpha$. This value of the maximum error give the obligation to use
the HT model for possible corrections to the results given by the PG model when $T_0$ is high and begins to exceed the 240 K approximately.

**7 Conclusion**

The conclusions that can be deduced are:

A detached shock occurs for lower $M_0$.

Detached shock still occurs when the angle $c$ exceeds a certain maximum angle.

The computational accuracy for the PG and HT models depends on the discretization, which results in the choice of $k$ and $l$ for the curved shapes. More $k$ and $l$ will be high, we will have a good accuracy.

The PG model gives acceptable results for small values of $M_0 < 2.00$, $T_0 < 240$ K and $t/C < 1.0$ approximately. On the other hand, when $M_0$ or $T_0$ or $t/C$ increases, the PG model gives results which are different from the real case, hence the need for the HT model.

The developed numerical program can process any gas found in nature. In this case, we must add the variation of the specific heat $C_p(T)$ and the constant $R$ of the gas with the calculation of $H(T)$.

The convergence of the results requires an additional calculation time for the HT model compared to the PG model for the same accuracy.

A condensation of the nodes in the Simpson quadrature is used to integrate the function with high precision in a reduced time.
For $M_0$ given, there is a maximum deflection of the airfoil to avoid the detached shock. This limit also depends on $T_0$.

For each airfoil geometry, there is a minimum value of the upstream Mach number to avoid the detached shock. This limit depends on $T_0$, $\alpha$ and the gas used.

The $T_0$ is an essential parameter of our HT model. The PG model results do not depend on $T_0$.

The stagnation temperature $T_0$ degrades the $C_D$, $C_L$ and $C_m$ parameters compared to the PG models. This difference increases with the increase in $T_0$.

The maximum error is noticed on the coefficient $C_L$ which can arrive at 39.38% when $T_0 = 3500$ K for the selected airfoils when $M_0 = 4.00$ and $\alpha = 2.00$ and can reach 59% when $T_0 = 3500$ K for $M_0 = 5.00$ and $\alpha = 2.00$.

For $t/C < 1.0$ or $M_0 < 2.00$ or $T_0 < 240$ K, the PG model can be used to evaluate the flow independently of the values of $M_0$ and $T_0$. But if $t/C > 1.0$ or $M_0 > 2.00$ or $T_0 > 240$ K, the corrections given by the HT model become necessary to evaluate the flow parameters accurately.

The PG model use limit in terms of maximum values of $M_0$, $t/C$, and $T_0$ is set to the required accuracy.

The presentation of the results on the choice of three airfoils differs. The developed program can do the calculation for any airfoil shape.

Not only do the parameters $\alpha$, $T_0$, $M_0$, $t/C$ influence the aerodynamic coefficients, the airfoil geometry also influences.
The flow around an airfoil is characterized by the generation of a shock wave at the leading edge in addition to the possibility of having a progressive shock through the surface of a certain airfoil, like the third airfoil.

The flow around an airfoil is characterized by an entropy jump due to developed shock on the airfoil.

It can be considered that the work carried out can be considered as a numerical wind tunnel. It allows to numerically validate the new HT model with the old existing PG model that is experimentally validated.

As a perspective, this problem can be studied for the airfoils having a slat and ailerons to have a more maneuverability for the variation of the aerodynamic coefficients.

Fig. 26. Variation of the relative error caused by the PG model compared to HT model on the coefficient $C_L$ versus $M_0$.

Fig. 27. Variation of the relative error caused by the PG model compared to HT model on the coefficient $C_m$ versus $M_0$.

Nomenclature

- $\theta$: Deviation angle of an airfoil segment
- $\psi$: Flow angle deviation
- $M$: Mach number
- $\beta$: Shock wave deviation
- $\mu$: Mach angle
- $\gamma$: Specific heats ratio
- $R$: Thermodynamic constant of air
- $C_p$: Specific heat at constant pressure
- $v$: Prandtl-Meyer function
- $\alpha$: Angle of incidence of the airfoil
- $D$: Drag force
- $L$: Lift force
- $m$: Pitching moment
Dynamic pressure at upstream infinity
Reference surface
Pressure
Temperature
Density
Airfoil chord
Pitching moment coefficient
Drag coefficient
Lift coefficient
Maximum thickness of the airfoil
Position of the point on the airfoil.
Error of computation
Equation of the airfoil extrados
Equation of the airfoil intrados
Nodes number on the extrados
Nodes number on the intrados
Total number of node on the airfoil
Perfect gas
High temperature
Prandtl Meyer
Total variation of entropy

Subscripts
0 Upstream condition
1 Upstream state at the panel under consideration
2 Considered panel
E Extrados
I Intrados
p Point for the calculation of the pitching moment

Acknowledgments. The authors acknowledges Khaoula, Abdel-Ghani Amine, Ritadj and Assil Zebbiche and Monza Ouahiba for granting time to prepare this manuscript.

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Cite this article as: R. Takhnouni, T. Zebbiche, A. Allali, Stagnation temperature effect on the supersonic flow around pointed airfoils with application for air, Mechanics & Industry 19, 312 (2018)