Photon flux and distance from the source: consequences for quantum foundations and technologies

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Abstract

The paper explores the fundamental physical principles of quantum mechanics (in fact, quantum field theory) which limit the bit rate for long distances. Propagation of photons in optical fibers is modeled using methods of quantum electrodynamics. We define photon “duration” as the standard deviation of the photon arrival time; we find its asymptotics for long distances and then obtain the main result of the paper: the linear dependence of photon duration on the distance. This effect puts the limit to joint increasing of the photon flux and the distance from the source and it has important consequences both for quantum information technologies and quantum foundations. Once quantum communication develops into a real technology, it would be appealing to the engineers to increase both the photon flux and the distance. And here our “photon flux/distance effect” has to be taken into account (at least if successively emitted photons are considered
as independent). This effect also has to be taken into account in a loophole free test of Bell’s type – to close jointly the detection and locality loopholes.

Keywords: Photon propagation, optical fiber, photon duration, linear dependence of photon duration on the distance, loophole free Bell test, photon flux/distance effect

1 Introduction

Last years were characterized by a tremendous development of quantum information, in both theory and experiment. Quantum information technologies, especially quantum cryptography, approached the level of market products. Transmission of quantum information for long distances is one of the most important problems of theoretical and experimental research [1]. This problem has also a foundational dimension as playing a crucial role in performance of a loophole-free test for Bell’s type [2] inequalities, see [3]–[8]. Such a test should finally close all possibilities to interpret quantum mechanics as emergent from a local realistic model (although, see, e.g., [9]–[18] for discussions, cf., e.g., [19]). It is clear that without a test which is free from every loophole, the present foundational grounds of quantum mechanics can be questioned. And it is not only the foundations that can be questioned, but even the most successful quantum technologies such as quantum cryptography and quantum random generators.

As was already mentioned, recently two world leading experimental groups working in quantum foundations, first in Vienna [6] and then in Urbana-Champaign [8], performed the Bell-type tests closing the detection loophole. Both groups have approached sufficiently high levels of detection efficiency (which includes efficiency of detectors and all optical losses in the pathways from the source to the detectors) to close the detection loophole, i.e., to discard the fair sampling assumption [11]. However, the locality loophole has

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1 The present situation both in quantum foundations and quantum technologies (especially quantum cryptography and quantum random generators) is highly unsatisfying from the scientific viewpoint. There have been performed Bell’s type tests closing the locality and (recently) detection loopholes, separately, see [4], [6], [8] (in this paper we discuss only experiments with photons). The tests of both types demonstrated statistically significant deviations from the predictions of the local realistic model proposed by Bell [2], see [17] for a discussion on generality of this model. Often the results of these tests are interpreted as sufficient to reject local realism completely.

2 For us it is important to point to the basic role of losses in the optical pathways, because the modern detectors which have been used both in the Vienna-experiment and the Urbana-Champaign experiment have the efficiency near 100%.
not yet been closed jointly with the detection loophole and nowadays the leading experimental groups work to improve the experimental setups from [6] and [8] (by increasing the distance between detectors) in order to solve this problem. The main difficulty is the essential increase in losses of photons with distance. Hence, it will be very difficult (if at all possible) to approach sufficiently high (for proceeding without the fair sampling assumption) arm-efficiency for long distances.

In this paper we study spatial and temporal dependencies of detection probabilities for photons propagating in optical fibers. We define photon “duration” as the standard deviation of the photon arrival time; we find its asymptotics for long distances and then obtain the main result of the paper: the linear dependence of photon duration on the distance. This effect puts the limit to joint increasing of the photon flux and the distance from the source and it has important consequences both for quantum information technologies and quantum foundations. In quantum technologies it is appealing to increase both the photon flux and the distance. And here our “photon flux/distance effect” has to be taken into account (at least if successively emitted photons or pairs of entangled photons are treated as independent). This effect also has to be taken into account in a loophole free test of Bell’s type – to close jointly the detection and locality loopholes. As the result of increasing of losses in optical pathways, to collect sufficient statistical data in this test, experimenters have to increase duration of runs (for the fixed pairs of the orientations of polarization beam splitters). However, runs’ durations cannot be too large, since for very long runs it would be impossible to neglect the drift effect, see [7] for details. Thus the photon flux has to be sufficiently high. To exclude correlations between successively emitted pairs of entangled photons, experimenters have to select the photon flux compatible with photon duration which (as we shall show) increases linearly with the distance.

Now we outline the structure of the paper and its main result in details. As was already pointed out, the main result of the paper is the derivation of the spatial asymptotics of the “photon duration”, the standard deviation of instances of detection of photons, see (10).

In general, investigations of quantum correlations at long distances imply the use of optical fibers. Nowadays, quantum communication links can be established over considerable distances using the art fiber technology [22, 23, 24]. Indeed, up to 100 km entanglement distributions have been created in optical fibers [24, 25, 26, 27, 28].

In [20] two authors of this paper formulated (on the heuristic grounds) a kind of complementarity principle for closing the detection and locality loopholes.
The confinements of the photons to a fiber has certain drawbacks. Firstly, there is a loss of photons dependent mainly on material absorption, through internal resonances or impurities, and Rayleigh scattering from local fluctuations in material density\cite{29}. Secondly, the fiber is dispersive making the photon velocity frequency dependent. The photon loss can be reduced by a high degree of purification of the fiber material in combination with an optimal selection of the frequency band, whereas the dispersion can be reduced by dispersion shift \cite{30}. Even with such measures, the dispersion effect must usually be taken into account for the design and evaluation of experiments on quantum correlations. However, the loss of photons due to the mechanisms mentioned above is often not critical and will be neglected in what follows.

One way of describing the effect of dispersion in the fiber is to concentrate on scattering in the photon arrival time for an ensemble of photons having the same initial state. In this paper, the standard deviation of photon arrival time is denoted the ”photon duration” time. This implies that in vacuum, where dispersion is absent, the photon duration time vanishes. As an example Zhang et al. \cite{25} report that the photon duration time increases from 4 ps to 25 ps in a 100 km long fiber, despite that both dispersion shift and a narrow frequency band have been employed.

Expressions for the photon duration time based on quantum field theory are the goal of the current paper. Their asymptotic form, valid for large distances, should be useful for the design and evaluation of quantum correlation experiments in optical fibers. A photon initial state serves as a model for the generation of photons that usually is performed by down-conversion using a laser and a non-linear crystal; see e.g. \cite{31} for calculation of the spatial modes produced in a non-linear waveguide.

A quantum field model of the fiber was given by Khrennikov et al. first for a hollow waveguide \cite{21, 32} and then for a single core optical fiber \cite{33} with the aim of calculation of quantum correlations in a single discrete mode. As a continuation of \cite{33}, the quantization of a single discrete mode of the single core optical fiber was re-examined, giving it a solid foundation by constructing the Hamilton function in terms of cylindrical vector wave functions \cite{34}. This recent paper \cite{34} is the foundation of the current paper.

The paper is organized so that it strictly defines (in section 2) the photon duration time relating it to the probability density of finding a photon. This probability density is then constructed in section 3 using the quantization of a discrete mode of the single core fiber \cite{34} and then specialized to the HE_{11} mode that is usually used in single core fiber communication. The expressions for the photon duration time given in sections 2-3 are simplified in section 4 for the large distance regime. In the final section, a discussion of the results is given.
2 The photon arrival time and its standard deviation

Consider a straight circularly symmetric (of radius $a > 0$) fiber with the core and exterior domain with material properties given by parameters $\mu_1, \varepsilon_1$ and $\mu_2, \varepsilon_2$, respectively. A cylindrical coordinate system $(\rho, \varphi, z)$ with the $z$–axis in the middle of the core is attached to the fiber and the coordinates are chosen so that the photons originate inside the fiber near the axial coordinate $z = 0$ and time $t = 0$. The photon detection takes place in the core at fixed axial coordinate: $z$ for one photon; $z_1$ and $z_2$ for a photon pair. The arrival time is denoted $t(z)$ for a single photon and $t_1(z_1, z_2), t_2(z_1, z_2)$ for a photon pair. This paper deals primarily with the mean value $\bar{t}(z)$ and the standard deviation $\sigma(z)$ for a single photon arrival time.

$\sigma(z)$ is a measure on the scattering of the photon arrival time. A natural name for $\sigma(z)$ is therefore [25, 30] the photon duration time. This notation should not be confused with the detection time, the time it takes to detect the photon, which is neglected in this study. From experimental values $\{t_n(z)\}_{n=1}^N$ of the photon arrival time, the photon duration time $\sigma(z)$ can be estimated with

$$\sigma(z) = \sqrt{\frac{1}{N-1} \left[ \sum_{n=1}^{N} t_n^2(z) - \frac{1}{N} \left( \sum_{n=1}^{N} t_n(z) \right)^2 \right]}.$$  \hspace{1cm} (1)

A quantum electrodynamic field description of the fiber gives $P(\rho, z, t)$, via Glaubert’s formulæ [35], so that the probability of detecting a photon with electrical polarization $\nu$ in a specific discrete mode at time $t$ in the volume element $dV$ at $(\rho, \varphi, z)$ is proportional to $P(\rho, z, t)dV$. In this section, formulæ for $\bar{t}(z)$ and $\sigma(z)$ are derived in terms of $P(\rho, z, t)$.

The probability density $p(z, t)$ with respect to $t$ for locating a photon anywhere in the core of the fiber at the cross-section located at $z$ is

$$p(z, t) = \frac{P(z, t)}{P_{\nu} \int_{0}^{\infty} P(z, t)dt},$$  \hspace{1cm} (2)

where $P_{\nu}$ is the probability of finding a photon in polarization $\nu$ at any time, and

$$P(z, t) = 2\pi \int_{0}^{a} P(\rho, z, t)\rho d\rho.$$  \hspace{1cm} (3)

$P_{\nu}$, which depends on the experimental set-up, is assumed to be a constant, independent of the axial distance $z$. This means that the probability for
locating a photon anywhere in the core at the cross-section at $z$ during the time interval $(t_1, t_2)$ is

$$\int_{t_1}^{t_2} p(z, t) dt. \quad (4)$$

Note that the joint probability density with respect to $t'$ and $z'$ is $p(z, t')\delta(z - z')$.

Let

$$\mathcal{E}\{f(t)\}(z) = \int_0^\infty f(t)p(z, t) dt \quad (5)$$

denote the expectation value of $f(t)$ with respect to the probability density $p(t, z)$. With this notation, we get that the expectation value $\bar{t}(z)$ of the arrival time of the photon is given by

$$\bar{t}(z) = \mathcal{E}\{t\}(z) \quad (6)$$

and the corresponding standard deviation

$$\sigma(z) = \sqrt{\mathcal{E}\{t^2\}(z) - [\bar{t}(z)]^2}. \quad (7)$$

It is convenient to express the results in terms of the moments

$$\tau_n(z) = \int_0^\infty t^n P(z, t) dt \quad (8)$$

with respect to the un-normalized probability density $P(t, z)$, rather than with respect to $p(z, t)$, giving

$$\left\{ \begin{array}{l}
\bar{t}(z) = \tau_1(z) \\
\sigma(z) = \sqrt{\tau_2(z) - \left(\frac{\tau_1(z)}{\gamma_0(z)}\right)^2}.
\end{array} \right. \quad (9)$$

The explicit dependence on the polarization $\nu$ and mode properties for $\bar{t}(z)$ and $\sigma(z)$ are suppressed in what follows.

A major result of this paper is that

$$\left\{ \begin{array}{l}
\bar{t}(z) \sim Az \\
\sigma(z) \sim Bz, \quad z \to \infty,
\end{array} \right. \quad (10)$$

where $A$ and $B$ are the explicit expressions independent of $z$. This means that asymptotically for large $z$, the photon duration time $\sigma(z)$ increases linearly with $z$. As a consequence, the measurement time like the time required to
determine probabilities also increases with $z$. Conversely, $\sigma(z)$ can be used to estimate the measurement time at a given distance $z$. An efficient method to determine $\sigma(z)$ is provided by (10): first, the proportionality constant is determined for sufficiently large $z = z_0$, then $\sigma(z) \sim Bz$ is calculated for $z > z_0$.

3 Quantization and the probability density

In this section, the probability density for detecting a photon is constructed. The basis for this is a quantization of a discrete mode in the single core fiber \[34\]; all other discrete and the non-discrete modes can be neglected for long distance interaction; see e.g. \[36\].

Consider one mode with azimuthal index $m$ and let $|0\rangle$ be the corresponding Fock vacuum with the creation operator $a^{(+)}_{mk}$; the annihilation operator is $a^{(-)}_{mk}$. With the initial state

$$|\Psi_1\rangle = \int dE g_{mk} a^{(+)}_{mk} |0\rangle,$$

(11)

the probability density for finding a photon in polarization $\nu$ is

$$P_{\nu m}(\rho, z, t) = \langle \Psi_1 | E^{(-)}_{\nu m}(\rho, \varphi, z, t) E^{(+)}_{\nu m}(\rho, \varphi, z, t) | \Psi_1 \rangle,$$

(12)

using Glauber's formula \[35\]. The dependence on the polarization, which was suppressed in the notation in section 2, is made explicit with a sub-index $\nu$. According to \[34\], the electrical operator projected in direction $\nu$ is

$$E^{(\pm)}_{\nu m}(\rho, \varphi, z, t) = \int \frac{dk}{2\pi} \sqrt{\frac{i\omega_{mk}}{2\epsilon_0}} a^{(\pm)}_{mk} \nu \cdot \psi_{mk}(\rho) e^{i(kz - \omega_{mk}t + m\varphi)}.$$

Here, $E^{(-)}_{\nu m}(\rho, \varphi, z, t)$ is the Hermitian conjugate of $E^{(+)}_{\nu m}(\rho, \varphi, z, t)$. $\omega_{mk}$ solves the dispersion relation

$$G_m(\omega, k) = \frac{a^2 \kappa^2 q^2 \mu_1 \mu_2 J_m^2(\kappa a) K_m^2(qa)}{k^2 \mu_1 \mu_2} \left[ -\frac{m^2 k^2}{k_0^2} \left( \frac{1}{(qa)^2} + \frac{1}{(\kappa a)^2} \right)^2 + \left( \frac{\mu_1 J_m(\kappa a)}{\kappa a J_m(\kappa a)} + \frac{\mu_2 K_m(qa)}{qa K_m(qa)} \right) \left( \frac{\epsilon_1 J_m(\kappa a)}{\kappa a J_m(\kappa a)} + \frac{\epsilon_2 K_m(qa)}{qa K_m(qa)} \right) \right].$$

(14)

where the $\omega$-dependence is introduced via $k_0 = \omega \mu_0 \epsilon_0$, $\kappa = \sqrt{k_0^2 \mu_1 \epsilon_1 - k^2}$ and $q = \sqrt{k^2 - k_0^2 \mu_2 \epsilon_2}$. It is assumed that $\mu_1 \epsilon_1 > \mu_2 \epsilon_2$. Here, $\mu_0$ and $\epsilon_0$ are the permeability and permittivity in vacuum, $\mu_j$ and $\epsilon_j$ the relative permeability and permittivity in the medium.
and permittivity in material $j$ with $j = 1$ referring to the core and $j = 2$ to the exterior region, see Fig. 1. Explicit procedures for calculating $\nu \cdot \psi_{mk}(\rho)$ are given in [34]. From symmetry considerations follows that $P_{\nu m}(\rho, z, t)$ is independent of the azimuthal angle $\varphi$ as shown in the notation. Only propagating modes are considered and it is sufficient to consider non-negative $m$ so that $\omega_{mk} = \omega_{m-k}$ is real and non-negative.

A regularization of the theory developed below for calculating $\tau_0(z)$ and $\sigma(z)$ will be required since $\omega_{mk}$ may vanish for $k = 0$. This is the case for the $\text{HE}_{11}$ mode, with the asymptotic behaviour $\omega_{mk} \sim |k|/c_0(\varepsilon_2 \mu_2)^{-1/2}, k \to 0$. The regularization is done by replacing $k^2$ with $k^2 + \varepsilon^2$ and letting $\varepsilon$ tend to zero at the end of the analysis. Then $\omega_{mk\varepsilon}$ is positive and satisfies $G_m(\omega_{mk\varepsilon}, \sqrt{k^2 + \varepsilon^2}) = 0$ and $\omega_{mk\varepsilon} = \lim_{\varepsilon \to 0} \omega_{mk\varepsilon}$. To assure that this limit exists an initial state is selected so that $g_{mk\varepsilon}$ is vanishing sufficiently fast when $k$ tends to zero.

The calculation of the moments $\tau_n(z)$ starts with standard procedures with the commutation relations $c_{mk\varepsilon}^{(-)} a_{mk\varepsilon}^{(+)} - a_{mk\varepsilon}^{(-)} c_{mk\varepsilon}^{(+)} = 2\pi \delta(k - k')I[m\varepsilon]$, with $I$ being the identity operator. Assuming a one photon state the result is

$$P_{\nu m}(\rho, z, t) = |A_{\nu m}(\rho, z, t)|^2, \quad (15)$$

where

$$A_{\nu m}(\rho, z, t) = \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} dk f_{\nu m\varepsilon}(\rho) e^{ikz - \omega_{mk\varepsilon}t}$$

$$f_{\nu m\varepsilon}(\rho) = g_{mk\varepsilon} \sqrt{\frac{\hbar \omega_{mk\varepsilon}}{2\epsilon_0}} \nu \cdot \psi_{mk\varepsilon}(\rho). \quad (16)$$

The reality condition $f_{\nu m-k\varepsilon}(\rho) = f_{\nu m\varepsilon}^*(\rho)$ follows from the definitions of $g_{mk}$ and $\psi_{mk\varepsilon}(\rho)$.

4 The photon duration time for large distances

For a given initial state $|\Psi_1\rangle$, defined by $g_{mk\varepsilon}$, the mean value of the arrival time $\bar{t}(z)$ and the photon duration time $\sigma(z)$ are given by (9), (8) and (3) with $P(\rho, z, t) = P_{\nu m}(\rho, z, t)$ according to (15)-(16).

It is assumed that

$$\omega_{mk\varepsilon} - \omega_{mk\varepsilon} = (k' - k)(k' + k)F_{me}(k', k), \varepsilon > 0 \quad (17)$$

with $F_{me}(k', k) > 0$, has zeros only at $k' = \pm k$ and that these zeros are simple. This means that $\omega_{mk\varepsilon}$ is increasing with $|k|$ with a minimum positive value at $k = 0$. From $\omega_{mk\varepsilon} = \omega_{m-k\varepsilon}$ and (17) follows that $F_{me}(k', k) =$
\( F_{me}(k, k') = F_{me}(-k', k) \). The assumption (17) and \( F_{me}(k', k) > 0 \) holds for the HE_{11} mode.

As a starting point for deriving asymptotic results for the \( n \)th moment \( \tau_n(z) \) when \( z \) is large, (15-16) are inserted into (8) and the order of integration is changed to get

\[
\tau_n(z) = \lim_{\varepsilon \to 0} \frac{2\pi}{\varepsilon} \int_0^a \rho \, d\rho \int_{\mathbb{R}^2} dk \, dk' \, f_{\nu mk\varepsilon}(\rho) f_{\nu mk'\varepsilon}(\rho) e^{i(k-k')z} I_{nm\varepsilon}(k', k). \tag{18}
\]

Here,

\[
I_{nm\varepsilon}(k', k) = \frac{n!}{[-i(\omega_{mk\varepsilon} - \omega_{mk\varepsilon})]^{n+1}} + \pi (-i)^n \delta^{(n)}(\omega_{mk\varepsilon} - \omega_{mk\varepsilon}) \tag{19}
\]

is a distribution meaning that the first term on the right hand side of (20) requires that the integration in \( k \) and \( k' \) be interpreted as Cauchy principle value integrals. Now, (17) is introduced into (20). The integration in (18) related to the second term in (20) can be performed using the definition of the \( \delta^{(n)} \) distribution whereas for the first term an asymptotic analysis for large \( z \), based on the standard Laplace transform

\[
\int_0^\infty \ln t \, e^{-st} \, dt = - \frac{1}{s} \left( \gamma + \ln s \right), \tag{21}
\]

is appropriate, \( \gamma = 0.5772 \ldots \) being Euler’s constant. Let us define

\[
|f_{\nu mk\varepsilon}|^2 = 2\pi \int_0^a \rho \, d\rho \, |f_{\nu mk\varepsilon}(\rho)|^2, \tag{22}
\]

which can be expressed in terms of Bessel functions using the results of [34]. Then, the result is

\[
\tau_n(z) = \tau_n^\sim z^n [1 + o(1)], \quad z \to \infty, \quad n = 0, 1, 2, \tag{23}
\]

where the asymptotic constants

\[
\begin{cases}
\tau_0^\sim = \lim_{\varepsilon \to 0} \frac{\pi}{2} \int_{\mathbb{R}} \, dk \frac{|f_{\nu mk\varepsilon}|^2}{|k| F_{me}(k, k)} \\
\tau_1^\sim = - \lim_{\varepsilon \to 0} \frac{\pi}{4} \int_{\mathbb{R}} \, dk \ln |2k| \frac{d^2}{dk^2} \frac{|f_{\nu mk\varepsilon}|^2}{[F_{me}(k, k)]^2} \\
\tau_2^\sim = \lim_{\varepsilon \to 0} \frac{\pi}{8} \int_{\mathbb{R}} \, dk \frac{|f_{\nu mk\varepsilon}|^2}{|k|^3 [F_{me}(k, k)]^3}
\end{cases} \tag{24}
\]

9
With (6), (7) and (23) we finally have

\[
\begin{align*}
\tau(z) &= \frac{1}{P_\nu} \tau_0^z + o(z), z \to \infty \\
\sigma(z) &= \sqrt{\frac{\tau_2^z}{P_\nu \tau_0^z} - \left(\frac{\tau_1^z}{P_\nu \tau_0^z}\right)^2} + o(z), z \to \infty.
\end{align*}
\] (25)

Provided that the moments \(\tau_n(z), n = 0, 1, 2\) exist, the variance \(\sigma^2(z)\) is non-negative that shows that the asymptotic expression (25) for \(\sigma(z)\) is well defined.

Asymptotic photon duration times for entangled states can also be derived straightforwardly, although the calculations are technically more involved. Only the results for the biphoton case are presented omitting all details in the derivation. An ensemble of identical initial states consisting of 2 photons is considered. The two detectors are located at \(z_1\) and \(z_2\), respectively. For the duration time \(\sigma_j(z_1, z_2)\) of photon \(j\), defined as the standard deviation of the arrival time, we have

\[
\sigma_j(z_1, z_2) \sim B_j |z_j|, j = 1, 2 \text{ when } |z_1|, |z_2| \to \infty.
\] (26)

This means that the asymptotic one photon results (10) is transferred to the two photon case provided that both observation points tends to \(\infty\). Note that the interference terms found in the asymptotic formulae for the probability density, see (33) in [33] for details, are not present in the lowest order asymptotic result (26) for the duration time.

5 Discussion

Our statistical theory is based on the assumption that there is an ensemble of photons prepared in the same way. To test our predictions in an experiment, there must be sufficiently long time interval between the emitted photons, so that they can be considered independent.

In a real experiment one cannot wait an infinite time before emitting the next photon. Still, the time interval between the successive emissions of photons must be (in average) much larger than the other time scales of the problem. One such time scale is the duration time of the photon. If the time interval is not much larger than the duration time of each photon then the two successively emitted photons (propagating in the same direction) can no more be considered as independent systems with their own intrinsic properties; they should be considered as an entangled photon pair. Thus, for measurements in an optic fiber, the distance between the source and the
detector comes into play if the photons may interact; the longer the distance the more interaction there is for a given initial time interval. Hence, two photons may be considered as two independent systems on short distances, but we cannot assume that they are independent for sufficiently large distances (the photons may be entangled).

This inter-relation between the photon flux and the distance between the source and the detectors is the main experimental consequence of our study. It has important implications for quantum technologies and quantum foundations.

Once quantum communication develops into a real technology, it would be appealing to the engineers to increase both the photon flux and the distance. Our paper explores the fundamental physical principles of quantum mechanics (in fact, quantum field theory) that limits the bit rate for long distances.

In coming experimental realizations of the loophole free Bell’s test, see Introduction, the same problem arises. On one hand, to close the locality loophole the distance has to be essentially increased comparing to the Vienna and Urbana-Champaign experiments, [6], [8]. On the other hand, the photon flux has to be sufficiently high: otherwise the duration of each run would become too large to neglect the drift effect, see [7] for details. Our paper says that one has to take into account the photon flux/distance effect.

In a real Bell’s type experiment a laser beam is sent to a nonlinear crystal and beams of photons go out from the crystal. A beam has a conical structure. One captures the photons in the beam with the end of the core of the optic fiber. To do this, the beam must have sufficiently many photons in a small solid angle. Then, it is natural to define the brightness of the source as power/solid angle, while in quantum optics it may be convenient to use the number of photons emitted each second (rather than power). It is also of importance to have nearly the same energy or frequency of the photons in the beam. Of course, it is impossible that all photons have precisely the same frequency. Therefore, it is of interest to define the spectral brightness which is given as brightness $\times$ frequency/width of frequency band. Thus, a source with a high spectral brightness at a particular angle and frequency has its photons concentrated to this direction and this frequency. Both the photon flux and brightness are proportional to the number of photons emitted per second (but the two notions are not identical). To get a closer connection to experiments testing Bell inequalities, in addition to the number of photons emitted each second, the spectral brightness is also useful. Therefore, it may be convenient to speak not about the photon flux/distance effect, but the spectral brightness/distance effect.
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