Motion of particles in storage rings in the presence of a counterpropagating laser beam I

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Abstract

The dynamics of charged particle beams interacting with laser beams and material targets in storage rings is presented. Formulas for the radiative damping, quantum excitation and equilibrium parameters of ion and electron beams determined by the backward rayleigh and backward compton scattering are derived. General case is considered. It takes into account the possibility of the periodical displacement of the laser beam in the radial direction with varying velocity in the process of the interaction of the beams and includes the possibility of the description of dynamics of muon beams interacting with material targets in storage rings (enhanced ionization cooling). The example is considered and some applications of being cooled beams are discussed.

1 Introduction

The backward compton or rayleigh scattering of laser photons by elementary or complicated particles (electrons, protons, ions) in storage rings results simultaneously both to radiative damping and to quantum excitation of amplitudes of betatron and phase oscillations of these particles. There is an analogy of these processes with the influence of synchrotron radiation on dynamics of particles in storage rings.

The spectral-angular characteristics of radiation scattered by an ion and electron beams are identical. Therefore the equilibrium transverse emittances and damping times of amplitudes of betatron and phase oscillations of particle beams as a function of parameters of the storage ring, charge and mass of particles are identical. However there is a difference in the coefficients of proportionality of the longitudinal emittances. In the relativistic case the maximal hardness of scattered radiation according to the relativistic doppler effect is $4\gamma^2$ times higher then the hardness of the laser radiation, where $\gamma = 1/\sqrt{1-\beta^2}$ is a relativistic factor of a particle; $\beta = v/c$, relative velocity of particles. Therefore the hardness of the non-resonance radiation scattered by an electron is proportional to $4\gamma^2$. At the same time ions are resonant systems. They are excited by photons of a laser beam, and rescatter them later. The nonmonochromatic laser beam is used in order to ions of different energies scattered laser photons independently on their energy. In this case the energy of laser photons interacting with ions at resonance is inversely proportional to $2\gamma$. Therefore the hardness of radiation scattered by these ions is proportional to $2\gamma$. The difference in the dependence of hardness of radiation scattered by electron and ion beams on their energy leads to the difference $2$ times in the expressions for equilibrium energy spreads, longitudinal and transverse radial-phase dimensions of ion and electron beams.

In this paper the detailed calculations of the equilibrium dimensions, energy spread, the time of radiative damping of amplitudes of betatron and phase oscillations of the particle beams interacting with

\[1^{\text{Non-resonant crossection of scattering of laser photons by ions is 10-15 orders less then the resonance one.}}\]
the laser beams in storage rings are presented. General formulas are received for the non-stationary case, when the laser beam is displaced with some velocity in the radial direction relative to the beam of particles and such a way the degree of overlapping of beams is changed. The calculations are carried out in the assumption, that the displacement of the vertical position of the exited ions for their life-time can be neglected. The received results include the possibility of the description of muon beam dynamics when material targets are used in storage rings (ionization cooling). The example is considered to estimate the possible range of energies of electron storage rings and hardness of laser and scattered radiation proceeding from necessary life-time of the electron beam in the storage ring. Some applications of the being cooled beams are discussed.

The parameters of the particle beams for stationary conditions of interaction of laser and particle beams were derived in papers [1, 2] for ion beams and in the paper [3] for electron beams. Dynamic conditions of interaction were investigated for the case of switched off the radiofrequency system of the storage ring in the paper [4].

2 Radiative effects in laser-electron and laser-ion storage rings in stationary conditions

Below we will present in the general form a derivation of basic equations describing the dynamics of interaction of particles with a laser beam in storage rings and then we will discuss the losses of particles caused by multiple scattering of the laser beam photons and intrabeam scattering. The maximal stored current limited by coulomb repulsion of particles in the beam will be discussed as well.

2.1 The equilibrium vertical dimension of a particle beam

The dependence of a distance of a particle from the instantaneous orbit of the storage rings in the vertical direction is described by the equation

$$z(s) = A_z(s) \cos \xi(s),$$

where $A_z(s) = C_z \beta_z^{1/2}(s)$ is the amplitude of particle oscillations; $\xi(s) = \int_0^s ds/\beta_z(s) + \xi_0$, phase; $C_z$, $\xi_0$, constants determined by the initial conditions; $\beta_z(s)$, $\beta$-function of the storage ring; $s$, longitudinal coordinate $\bar{z}$ - $\bar{s}$. The derivative

$$z'(s) = A_z(s) \frac{\beta_z'(s)}{2 \beta_z(s)} \cos \xi(s) - \sin \xi(s) = \frac{\beta_z'(s)}{2 \beta_z(s)} z - \frac{A_z(s)}{\beta_z(s)} \sin \xi(s).$$

The square of the amplitude of vertical betatron oscillations, according to (1), (2), can be expressed through the initial conditions $z_0$, $z'_0 = dz/ds|_{s=s_0}$ in some point $s_0$

$$A_z^2(s) = \frac{\beta_z(s)}{\beta_z^0} A_{z_0}^2(s), \quad A_{z_0}^2 = z_0^2 + \beta_{z_0}^2 (z'_0 - \frac{\beta_{z_0}'}{2 \beta_{z_0}} z_0)^2,$$

where $\beta_{z_0}$ is the value of the vertical $\beta$-function in a point $s_0$. Further we will take into account the quantum nature of the interaction of particles with the laser beam.

At the moment of a photon scattering the coordinate of the particle is not changed ($\delta z = 0$). If a photon is emitted in a direction of a particle velocity, then the momentum and inclination angle of the particle trajectory is not changed as well ($\delta z' = 0$). Therefore in the case of vertical oscillations the particle emission of a photon leads to a change of its trajectory only in the case, when the photon is emitted under some angle to its velocity.
If photon is scattered at some angle $\theta \neq 0$ to a vector of velocity of a particle and at the scattering point $\beta_z' = 0$, then the change of the square of the amplitude of vertical betatron oscillations of the particle in a point of scattering

$$\Delta A^2_{z,sc} = \beta^2_{z,sc}[2z'_{sc}\delta z' + (\delta z')^2], \quad (4)$$

where

$$\delta z'_{1} = \frac{\hbar \omega}{\varepsilon_s} \sin \theta \cos \phi, \quad (5)$$

$\hbar \omega$ is the energy of scattered photon; $\varepsilon_s$, the equilibrium energy of the particle.

The rate of growth of a square of the amplitude of vertical betatron oscillations of particles $A_{z,1}$ caused by quantum processes of scattering of laser photons is

$$\frac{dA^2_{z,1}}{dt} = \int \frac{\partial N^2_{\gamma}}{\partial \theta \partial \phi} \Delta A^2_{z,sc} \, d\theta \, d\phi, \quad (6)$$

where $\partial^2 N_{\gamma}/\partial \theta \partial \phi = f \, dN_{\gamma}/d\theta$, is the angular distribution of the flow of scattered photons; $f$, the frequency of the revolution of a particle at the orbit of the storage ring; $do = \sin \theta \, d\theta \, d\phi$, the element of a solid angle; $\theta$, the angle between the laser beam axis and the direction of observation; $\phi$, the azimuth angle; $dN_{\gamma}/d\phi = (1/\hbar \omega)(d\varepsilon_{rad}/d\phi)$, the energy of the particle being lost at the passage of the laser beam (see section 2.4); $F(\theta, \phi)$, the normalized angular distribution of the scattered radiation ($\int F(\theta, \phi) \, d\theta \, d\phi = 1$). For circular and linearly polarized in the plane of the particle orbit laser beam radiation in the conditions of dipole radiation [8]

$$F(\theta, \phi)_{cp} = \frac{3}{8\pi \gamma^4} \left[ \frac{1}{(1 - \beta \cos \theta)^3} - \frac{\sin^2 \theta}{2\gamma^2(1 - \beta \cos \theta)^5} \right],$$

$$F(\theta, \phi)_{lp} = \frac{3}{8\pi \gamma^4} \left[ \frac{1}{(1 - \beta \cos \theta)^3} - \frac{\sin^2 \theta \cos^2 \phi}{\gamma^2(1 - \beta \cos \theta)^5} \right]. \quad (7)$$

From equations (4), (5), (6) and the dependence of the frequency of scattered radiation on the angle $\omega = \omega_L(1 + \beta)/(1 - \beta \cos \theta)$ follows a general expression for the rate of growth of a square of amplitude of vertical betatron oscillations of a particle caused by scattering of photons in an interaction region

$$\frac{dA^2_{z,1}}{dt} = \frac{\hbar \omega_L(1 + \beta)P_{rad}}{m^2 \alpha^2 \gamma^2} \beta_z(s) \beta_{z,sc} \int \frac{\sin^2 \theta \cos^2 \phi}{1 - \beta \cos \theta} F(\theta, \phi) \, d\theta \, d\phi, \quad (8)$$

where $P_{rad} = f \varepsilon_{rad}$ is the power of the radiation scattered by the particle; $\beta_{z,sc}$, the vertical $\beta$-function at the interaction region of the laser and particle beams. It is supposed, that along the length of the interaction region and along the length of a free path of ions in the exited state the change of $\beta$-function and the displacement of the ion vertical position can be neglected. At the derivation of (8) we took into account that at the averaging the first term in (4) tends to zero.

According to (7), (8) in the specific case of dipole scattering the rate of growth of the square of amplitude of particle oscillations does not depend on the kind of polarization of laser radiation and has linear time dependence

$$\frac{dA^2_{z,1}}{dt} = \frac{6\pi \hbar c P_{rad}}{5\varepsilon_s^2 \lambda_L} \beta_z(s) \beta_{z,sc}, \quad (9)$$
Within the framework of the classical electrodynamics the electromagnetic radiation is emitted in the direction of the velocity of a particle and it is not accompanied by a radiative damping of amplitudes of vertical betatron of oscillations. The damping arises at the acceleration of particles in the high-frequency resonator. In this case particle changes longitudinal component of momentum only, and consequently the angle of an inclination of a trajectory

\[ \delta z' = -z'_{sc} \frac{qU_m \cos \varphi}{\varepsilon_s} \approx -z'_{sc} \frac{\varepsilon_{rad}}{\varepsilon_s}, \]  

(10)

where \( q \) is the charge of a particle; \( U_m \) and \( \varphi \), amplitude and phase of the accelerating voltage at the moment of passing by the particle of the resonator; \( \varepsilon_{rad} \), the energy, emitted by equilibrium particle at one passing of the laser beam. We took into account, that if the equilibrium energy of particles in the storage rings is not changed, then the radiation losses of the energy by a particle are equal to its energy increment in the resonator. In this case in average \( qU_m \cos \varphi = qU_m \cos \varphi_s = \varepsilon_{rad} \), where \( \varphi_s \) is the equilibrium phase. The change of the square of amplitude of vertical betatron oscillations of a particle in region of the location of the resonator can be presented in the same form (4), if we will put \( \beta'_{z rf} = 0 \), where now \( \beta_{z rf} \) is the value of the vertical \( \beta \)-function in this region.

From (4), (10) the expression for the rate of damping of the square of the amplitude of vertical betatron oscillations of a particle caused by the change of its energy in the resonator

\[ \frac{dA_{z}^2}{dt} = f \Delta A_{z}^2 = -A_{z}^2 \frac{\varepsilon_{rad}}{\varepsilon_s}. \]  

(11)

We put \( \beta'_{z} = 0 \) at the region of location of the resonator, neglected the second term in (4), took into account, that \( z_{rf} = A_{z rf} \sin \xi_n/\beta_{z rf} \) and the average value \( < \sin^2 \xi_n > = 1/2 \), where \( \xi_n \) is the value of the phase \( \xi \) at the revolution \( n \).

The change of the square of amplitude of vertical betatron oscillations of a particle caused both quantum processes of scattering of laser photons and classical radiative damping simultaneously can be presented in the form \( dA_{z}^2/dt = dA_{z1}^2/dt + dA_{z2}^2/dt \) or

\[ \frac{dA_{z}^2}{dt} = -A_{z}^2 \frac{\varepsilon_{rad}}{\varepsilon_s} + 6\pi \hbar \frac{\varepsilon_{rad}}{5\lambda_L \varepsilon_s^2} \beta_{z}(s) \beta_{z sc}. \]  

(12)

According to (12) the square of the equilibrium amplitude of vertical betatron of oscillations of a particle is

\[ A_{z eq}^2 = \frac{dA_{z}^2}{dt} \tau_z = \frac{6\pi \Lambda_c}{5\lambda_L \gamma_s} \beta_{z}(s) \beta_{z sc}, \]  

(13)

and the time of radiative damping

\[ \tau_z = \frac{2\varepsilon_s}{\varepsilon_{rad}}, \]  

(14)

where \( \Lambda_c = h/mc \) is the compton length of a wave of a particle; \( \gamma_s = \varepsilon_s/mc^2 \), the relativistic factor of the equilibrium particles. For electron \( \Lambda_c \approx 3.86 \times 10^{-11} \) cm.

According to the central limiting theorem the density of distribution of particles of the beam in a vertical plane is described by the Gauss law

\[ \rho(z) = \frac{1}{2\pi} \sigma_z e^{-\frac{z^2}{2\sigma_z^2}}, \]  

(15)
where $\sigma_z = A_{zeq}/\sqrt{2}$ is the dispersion of the distribution of the density. Therefore the dispersion of the density distribution and the emittance of the beam of particles $\varepsilon_z = \sigma_{z eq}^2/\beta_z$ we can present in the form

$$
\sigma_z = \sqrt{\frac{3\pi\Lambda_c}{5\lambda L\gamma_s}} \sqrt{\beta_z(s)\beta_{z sc}}, \quad \varepsilon_z = \frac{3\pi\Lambda_c}{5\lambda L\gamma_s} \beta_{z sc}
$$

(16)

### 2.2 The equilibrium radial dimension of a beam determined by betatron oscillations

The radial coordinate of a particle in a storage ring is $x = x_\eta + x_b$, where $x_\eta$ is the deviation of an instantaneous orbit of this particle from equilibrium orbit and $x_b$ the deviation of the particle from the instantaneous orbit. The instantaneous orbit makes slow radial-phase oscillations relative to the equilibrium orbit, and the particle makes fast betatron oscillation relative to the instantaneous orbit.

The expression for the deviation of a particle from the instantaneous orbit in the horizontal plane of the storage ring has the form identical to the form for the vertical oscillations. It can be obtained by replacing $z$ by $x_b$ in (1) - (3). Unlike vertical, the radial betatron oscillations are connected with radial-phase oscillations. The position of the instantaneous orbit of a particle is changed intermittently after scattering of a laser photon by the value

$$
\delta x_\eta = -\frac{D_{x sc} h\omega}{\beta^2_x} \varepsilon_x,
$$

(17)

where $D_x = p(\partial x_\eta/\partial p)$ is the dispersion function of the storage ring; $p = \beta \gamma$, relative momentum of the particle. If $D_x \neq 0$, then with the intermittent change of an instantaneous orbit the amplitude betatron of oscillations will be changed intermittently as well. At that the inclination of the particle trajectory will not be changed.

The change of the square of the amplitude of horizontal betatron oscillations of particles in the scattering point under the condition that at this point $\beta_{x sc} = 0$, $\delta x_b = -\delta x_\eta$, looks like

$$
\Delta A_{x b sc}^2 = -2x_{b sc} \delta x_\eta + (\delta x_\eta)^2 + 2\beta_{x sc} \delta x_{b sc} + \beta_{x sc}^2 (\delta x_b')^2.
$$

(18)

The included in (18) values $\delta x_b'$ are similar to the corresponding values (5), (10) for vertical oscillations:

$$
\delta x_{b1} = \frac{h\omega}{mc^2\gamma} \sin \theta \cos \phi, \quad \delta x_{b2} = -x_{b0} \frac{qU_m \cos \varphi}{mc^2\gamma}.
$$

(19)

In the general case $D_x$ depends on the longitudinal coordinate and can accept zero values in some points and on the lengths of straight sections of the storage ring. In the smooth approximation $D_x \simeq aR \simeq R/\nu_x^{-2}$, where $a = \partial \ln C/\partial \ln p$, is the momentum compaction factor; $C$ the perimeter of the trajectory; $\nu_x$, the frequency of radial betatron of oscillations; $R = C/2\pi$, the average radius of the storage ring. If $D_x = 0$ then the radial betatron oscillations will be described by the expressions like (16) for the vertical one if we will replace indexes $z$ by $x_b$:

$$
\sigma_{x b} = \sqrt{\frac{3\pi\Lambda_c}{5\lambda L\gamma_s}} \sqrt{\beta_x(s)\beta_{x sc}}, \quad \varepsilon_x = \frac{3\pi\Lambda_c}{5\lambda L\gamma_s} \beta_{x sc}.
$$

(20)

If $D_x \neq 0$, then in the expression (18) for the square of amplitude of oscillations, as a rule, can be neglected the fourth item. In this case the rate of growth of the square of the amplitude of radial betatron oscillations of particles caused by quantum processes of scattering of laser photons determined, according to (17), (19) by the values $\delta x_\eta$, $\delta x_2'$, can be presented in the form
\[
\frac{dA_{x1}^2}{dt} = \int \frac{\partial N_\gamma}{\partial \omega \partial t} \Delta A_{x bs}^2 d\omega = \frac{7D_s^2 \hbar \omega_m P_s^{rad}}{10\beta_s^2 \varepsilon_s^2} + \frac{2D_{x sc} P_s^{rad}}{\beta_s^2 \varepsilon_s} \delta x_b,
\]

where \(\partial N_\gamma / \partial \omega \partial t = (f/\hbar \omega)(\partial \varepsilon_\gamma / \partial \omega), \partial \varepsilon_\gamma / \partial \omega = 3\varepsilon^{rad}[1 - 2\omega/\omega_m + 2(\omega/\omega_m)^2] \omega/\omega_m^2\) is the spectral distribution of the energy of the scattered laser radiation; \(\omega_m = (1 + \beta)^2 \gamma^2 \omega_L\), the maximal frequency in the spectrum of the scattered radiation.

The rate of damping of the square of the radial amplitude of the particle betatron oscillations caused by the change of its energy in the radiofrequency resonator, according to (18), for the value \(\delta x_{b2}^\prime\), determined by (19), can be presented in the form

\[
\frac{dA_{x2}^2}{dt} = f\Delta A_{x2}^2 = -A_{x2}^2 \frac{P_s^{rad}}{\varepsilon_s} \delta x_b.
\]

At the derivation of (22) we put \(\beta_{x rf}^\prime = 0\) in the region of location of the resonator and took into account, that \(x_{rf} = A_{x rf} \sin \xi_n/\beta_{x rf}, <\sin^2 \xi_n> = 1/2\).

The change of the square of amplitude of radial betatron oscillations of a particle caused by a quantum processes of scattering of laser photons and classical radiative damping simultaneously can be presented in the form \(dA_{x}^2/dt = dA_{x1}^2/dt + dA_{x2}^2/dt\) or

\[
\frac{dA_{x}^2}{dt} = -A_{x}^2 \frac{P_s^{rad}}{\varepsilon_s} + \frac{7\pi \hbar c (1 + \beta_s)^2 \gamma_s^2 P_s^{rad} D_{x sc}^2}{10\lambda_L \beta_s^4 \varepsilon_s^2} + \frac{2D_{x sc} P_s^{rad}}{\beta_s^2 \varepsilon_s} \delta x_b.
\]

The radiative damping time of the amplitude of radial betatron oscillations of a particle according to (23)

\[
\tau_{xb} = \tau_x = \frac{2\varepsilon_s}{P_s^{rad}},
\]

the square of equilibrium amplitude of these oscillations

\[
A_{x eq}^2 = \frac{0.7\pi \Lambda_c (1 + \beta_s)^2 \gamma_s D_{x sc}^2}{\beta_s^2 \lambda_L},
\]

the dispersion of the radial distribution of the density and emittance of the particle beam

\[
\sigma_{xb}^2 = D_{x sc} \sqrt{\frac{0.35\pi \Lambda_c (1 + \beta_s)^2 \gamma_s}{\lambda_L \beta_s^4}} \sqrt{\frac{\beta_x(s)}{\beta_{x sc}}}, \quad \varepsilon_x = \frac{0.35\pi \Lambda_c (1 + \beta_s)^2 \gamma_s D_{x sc}^2}{\lambda_L \beta_{x sc} \beta_s^4}.
\]

At the derivation of (25) we have assumed, that the laser beam has homogeneous density. Therefore \(P_s^{rad}\) does not depend on \(x_b\), on the average \(\overline{\tau_b} = 0\) and consequently the last term in (23) can be neglected. According to (20), (26) the ratio \(\sigma_{xb}^2 / \sigma_{xb}^\prime = \sqrt{7/3}(1 + \beta_s)\gamma_s D_{x sc} / \beta_{x sc} \beta_s^2\).

2.3 The equilibrium energy spread of a particle beam

The energy of a particle in the storage rings \(\varepsilon\) produce the synchrotron oscillations about the equilibrium energy \(\varepsilon_s\). The difference between the energy of a nonequilibrium particle and the equilibrium energy is connected with the derivative of the particle phase \(d\phi/dt = \dot{\phi} = h(\omega_s - \omega)\) by the dependence

\[
\Delta \varepsilon = \frac{\varepsilon_s}{hK \omega_s} \dot{\phi},
\]
where \( K = -\partial \ln \omega_r / \partial \ln \varepsilon = (\alpha \gamma_s^2 - 1)/(\gamma_s^2 - 1) \), \( \omega_r = 2\pi f \), \( h \) is the harmonic order of the radiofrequency voltage \([3]-[4]\).

The equation of phase oscillations of a particle accelerated in the resonator and losing the energy by scattering of the laser beam photons can be received from the balance of the energy received by a particle at one revolution \((d\varepsilon/dt)T = qU_m \cos \varphi - \varepsilon^\text{rad}\). Subtracting from the equation for the energy balance of a nonequilibrium particle the same equation for the equilibrium particle \((d\varepsilon_s/dt = 0, qU_m \cos \varphi_s = \varepsilon^\text{rad}_s)\) and taking into account (27) we will find

\[
\ddot{\varphi} + \frac{h\omega_s^2 K(\varepsilon^\text{rad} - \varepsilon^\text{rad}_s)}{2\pi \varepsilon_s} - \frac{hqU_m}{2\pi \varepsilon_s}(\cos \varphi - \cos \varphi_s) = 0. \tag{28}
\]

The linear equation of small phase oscillations, according to (28), has the form

\[
\dot{\psi} + \frac{h\omega_s^2 K(\varepsilon^\text{rad} - \varepsilon^\text{rad}_s)}{2\pi \varepsilon_s} + \Omega^2 \psi = 0, \tag{29}
\]

where \( \psi = \varphi - \varphi_s \ll 1, \Omega = \omega_s \sqrt{q/hKU_m \sin \phi_s / 2\pi \varepsilon_s} \).

If the laser beam is both homogeneous and motionless, its transverse dimensions are much greater than the transverse dimensions of the beam of particles, then in the relativistic case the value \( \varepsilon^\text{rad} = \varepsilon^\text{rad}_s(\gamma/\gamma_s)^2 \) for the backward compton scattering and \( \varepsilon^\text{rad} \sim D/(1 + D) \) for the backward rayleigh scattering in the field of the broadband laser beam, where \( D \sim \gamma \) is the saturation parameter \([1, 2]\). In this case \( \varepsilon^\text{rad} - \varepsilon^\text{rad}_s = (d\varepsilon^\text{rad}/d\varepsilon)(\Delta \varepsilon) = k_i \varepsilon^\text{rad}_s(\Delta \varepsilon/\varepsilon_s) \), where \( k_i = 2 \) for the backward compton scattering and \( k_i = 1/(1 + D) \) for backward rayleigh scattering. In the considered case, the equation of the small phase oscillations, according to (27), (29), has a form

\[
\ddot{\psi} + \frac{k_i P^\text{rad}_s}{\varepsilon_s} \dot{\psi} + \Omega^2 \psi = 0. \tag{30}
\]

The equation (30) has the solution of the form \( \psi = \psi_m(t) \cos \Omega'(t - t_0) \), where \( \psi_m = \psi_{m0} \exp(-t/t_s) \) is the amplitude of small phase oscillations,

\[
\tau_s = \frac{2\varepsilon_s}{k_i P^\text{rad}_s}, \tag{31}
\]

the damping time of the synchrotron oscillations of the phase and the energy of the particle; \( \psi_{m0} \), and \( t_0 \), the initial amplitude of phase oscillations and initial time; \( \Omega' = \sqrt{\Omega^2 - \tau_s^{-2}} \), the frequency of small phase oscillations of the particle.

Usually \( \Omega \tau_s \ll 1, \Omega' = \Omega \). In this approximation the square of amplitude \( \psi_{m0} \) expressed through the initial phase \( \psi_0 \) and initial derivative of a phase \( \dot{\psi}_0 = (d\psi/dt)|_{t=t_0} \) has a form \( \psi_{m0}^2 \approx \psi_0^2 + \Omega^{-2} \dot{\psi}_0^2 \). Since the phase of the particle is not changed at the scattering of the laser photons, then the change of the square of the amplitude of phase oscillations after the emission of the photon of the energy \( h\omega \) can be presented in the form

\[
\delta \psi_m^2 = [2\psi_0 \delta \dot{\psi} + (\delta \dot{\psi})^2] \cdot \Omega^{-2}, \tag{32}
\]

where \( \delta \dot{\psi} = hK\omega_s(h\omega/mc^2\gamma_s) \).

The rate of growth of the average square of the amplitude of phase oscillations of a particle caused by quantum processes of scattering of laser photons can be presented in the form

\[
\frac{d\psi_m^2}{dt} = \int \frac{\partial N_m}{\partial \omega dt} \delta \psi_m^2 d\omega = \frac{7\Lambda_c h^2 K^2 \omega_s^2 \omega_m P^\text{rad}}{10mc^3 \gamma_s^2 \Omega^2}, \tag{33}
\]
In the process of calculation of (33) we have neglected the first term in \( \delta \psi_m^2 \), since at the further integration over time it will disappear. The rate of growth of the average square of the amplitude of the energy oscillations of a particle, according to (27), \( d(\Delta \varepsilon)^2_m/dt = (\varepsilon^2 \Omega^2 / \hbar^2 K^2 \omega^2) (d \psi_m^2 / dt) \).

According to general rules (see (13)) the equilibrium values of the average squares of the amplitudes of the transverse and longitudinal oscillations of particles, and also amplitudes of oscillations of the square of their energy are determined by the product of the rate of growth of the appropriate values and the half of the time of radiative damping. Therefore, the square of the amplitude of the phase oscillations of the particle and the square of its amplitude of the energy oscillations, according to (31), (33), are equal:

\[
\overline{\psi_m^2} = \frac{11.2 \pi^2 h K \Lambda_c \gamma_s \varepsilon_s}{q U_m \sin \phi_s \lambda_L k_i}, \quad (\Delta \varepsilon)^2_m = \frac{5.6 \pi \Lambda_c \gamma_s \varepsilon_s^2}{\lambda_L k_i}.
\]

(34)

The dispersions of the particle distribution in the beam for the radial-phase oscillations \( \sigma_{x \eta} = D_x \beta_s^{-2} (\sigma_{\varepsilon}/\varepsilon_s) \), longitudinal oscillations \( \sigma_l = \psi_m R / h \sqrt{2} \), and for the energy spread \( \sigma_{\varepsilon}/\varepsilon_s \) are equal:

\[
\sigma_{x \eta} = \frac{D_x}{\beta_s^2} \sqrt{\frac{2.8 \pi \Lambda_c \gamma_s}{\lambda_L k_i}}, \quad \sigma_l = \frac{K R \omega_s}{\Omega} (\bar{\varepsilon}), \quad \sigma_{\varepsilon} = \sqrt{\frac{2.8 \pi \Lambda_c \gamma_s}{\lambda_L k_i}}.
\]

(35)

According to (26), (35) the ratio \( \sigma_{x \eta}^{\text{rad}} / \sigma_{x \eta} = (1 + \beta_s) \sqrt{k_i D_x \beta_s \gamma_s (s)} / (2 \sqrt{2} D_x \beta_s \gamma_s) \).

### 2.4 The power of radiation scattered by a particle

The energy of the laser radiation scattered by a particle at the passage of the laser beam depends on the distribution of the density of the number of photons in the beam. For Gaussian beam the density is distributed by the law

\[
n_L = \frac{N_L}{2 \pi \sigma_L^2 l_L} (\sigma_{L0}^2)^2 e^{-\sigma^2 \sigma_L^2},
\]

(36)

where \( N_L = \varepsilon_L / h \omega_L \) is the number of photons in the laser beam; \( \varepsilon_L \), the energy of the laser beam; \( l_L \), the length of the laser beam, \( \sigma_l = \sigma_{L0} \sqrt{1 + s^2 / l_R^2} \), the dispersion of the distribution of the laser beam in any point \( s \) in the longitudinal direction; \( \sigma_{L0} \), the dispersion in the point \( s = 0 \) corresponding to the waist of the laser beam; \( l_R = 4 \pi \sigma_L^2 / \lambda_L \), rayleigh length\(^4\).

If the longitudinal dimension of a particle beam \( \sigma_l \ll 4 l_R \), then the change of the transverse dimension of the laser beam for the time of its interaction with the particle beam can be neglected. In this case the energy being lost by the particle moving at distances \( r \ll \sigma_L \) from the axis of the counterpropagated laser beam

\[
\bar{\varepsilon}^{\text{rad}} = h c n_L (0) l_L \sigma_{L0}^2 \gamma_{\|} = \frac{\sigma \varepsilon_L \gamma^2}{\pi \sigma_{L0}^2},
\]

(37)

where \( n_L (0) = n_L (r = s = 0) \), \( \sigma = 8 \pi r_p^2 / 3 \) is the crosssection of scattering of laser photons; \( r_p = q^2 / mc^2 \), classical radius of the particle; \( h c = h \omega_{\max} / 2 = 2 \gamma h \omega_L \), average energy of scattered photons. In the case of compton (thompson) scattering of photons by electrons \( \sigma = \sigma_T \simeq 6.65 \cdot 10^{-25} \text{ cm}^2 \) \((r_e \simeq 2.82 \cdot 10^{-13} \text{ cm})\). For the case of the rayleigh scattering of photons by ions the crosssection is 10-15 orders higher.

If bunches of the particle beams have the gaussian distribution, then the flow of photons emitted by one bunch \( \dot{N}_c = \int n_c l_p n_L f s dxdz = N_c N_L f \sigma / S_{eff} \), where \( S_{eff} = 2 \pi (\sigma_{L0}^2 + \sigma^2) \), \( l_p \) is the length of

\(^4\)Usually in the laser technology the dimension of the laser beam \( w_L = 2 \sigma_L \) is introduced.
the bunch; \( N_p \), the number of particles in the bunch. Therefore the power of radiation scattered by one bunch

\[
P^{\text{rad}} = h \sigma N_{\gamma} \gamma \gg 1 = 2 \gamma^2 h \omega L \dot{N}_{\gamma} = \frac{2 \gamma^2 N_p P_L \sigma}{S_{\text{eff}}},
\]

where \( P_L = f \varepsilon L \) is the average reactive power of the laser beam stored in the open resonator.

### 2.5 Influence of synchrotron radiation on dynamics of particles in storage rings

Below, for a comparison, we will present the equilibrium parameters of beams in storage rings determined by the influence of synchrotron radiation on dynamics of particles in storage rings [5] - [7].

If the losses are defined by the synchrotron radiation emitted in the bending magnets of the storage ring, then the power of radiation and the time of the radiative damping of amplitudes of vertical betatron oscillations are equal accordingly

\[
\mathcal{P}_{\text{sr}} = \frac{2 c q^2 \gamma_s^4}{3 R R}, \quad \tau_{\text{sr}}^z = \frac{3 R R}{c \sigma_{\varepsilon} \gamma_s^3} \simeq 354.86 \frac{R R}{\gamma_s^3}.
\]

where \( R \) is the instantaneous radius of an orbit of a particle in bending magnets of the storage ring.

The damping times \( \tau_x, \tau_\varphi \) of the horizontal and longitudinal dimensions of a beam are approximately equal to \( \tau_z \). At that \( \tau_x^{-1} + \tau_\varphi^{-1} = 3 \tau_z^{-1} \). The equilibrium transverse emittance and the dispersion of the energy spread of the relativistic beam of particles are equal

\[
\varepsilon_{sr}^z \simeq \frac{13 \sqrt{3} \Lambda_c}{96 \nu_z}, \quad \varepsilon_{sr}^x \simeq \frac{C_s \Lambda_c \gamma_s^2}{\nu_x^3 J_x}, \quad \left( \frac{\sigma_{\varepsilon}}{\varepsilon} \right) \gamma \simeq \gamma_s \sqrt{\frac{C_s \Lambda_c}{2 R J_x}},
\]

where \( C_s = (55 \sqrt{3}/96) \simeq 0.992 \) and \( J_x, s \sim 1 \div 2 \) are constants determined by the magnetic structure of the storage rings \( (J_x + J_s = 3) \).

The dispersion of distribution of the electron beam density determined by the betatron and radial-phase oscillations in the smooth approximation are equal

\[
\sigma_{sr}^z = \frac{1}{\nu_z} \sqrt{\frac{13 \sqrt{3} \Lambda_c}{96 R \Omega}}, \quad \sigma_{sr}^x = \frac{\gamma_s R}{\nu_x^2} \sqrt{\frac{C_s \Lambda_c}{R J_x}}, \quad \sigma_{sr} = \frac{\gamma_s}{\nu_x^2} \sqrt{\frac{C_s \Lambda_c R}{J_s}}.
\]

The dispersions of the density of distribution of an electron beam over phases and in the longitudinal direction, according to (27), are connected with the dispersion of distribution of particles over the energy by the expressions

\[
\sigma_{\eta} = \frac{h K \omega_s}{\Omega} \left( \frac{\sigma_{\varepsilon}}{\varepsilon_s} \right), \quad \sigma_t = \frac{K \Omega \omega_s}{\Omega} \left( \frac{\sigma_{\varepsilon}}{\varepsilon_s} \right).
\]

### 2.6 Life-time of particles in the storage rings

The life-time of particles in the storage rings is limited by losses caused by various processes. For the case of multiple scattering of particles by photons of a laser beam

\[
\tau_{\text{loss}}^{\ell} = \frac{e^r}{2r},
\]

where \( r = A^2_{\text{lim}} / A^2_{\text{eq}}, \quad A^2_{\text{lim}} \) and \( A^2_{\text{eq}} = 2 \sigma^2 \) is the square of the limiting amplitude and the square of the equilibrium amplitude of oscillations of the particle accordingly; \( \tau \), the time of the radiative damping [3].
In the case of phase oscillations \( A_{eq}^2 = 2\sigma_{\varepsilon}^2 \), \( A_{lim}^2 = (\Delta \varepsilon)_{sep}^2 \), where \((\Delta \varepsilon)_{sep}\) is the maximal deviation of the energy of a particle moving along the separatrix from the equilibrium energy

\[
(\Delta \varepsilon)_{sep} = \sqrt{\frac{2qV\varepsilon_s(\sin \varphi_s - \varphi_s \cos \varphi_s)}{\pi h K}}.
\]  

(44)

In order to neglect the losses caused by multiple processes of quantum fluctuation of synchrotron radiation it is necessary to fulfill the condition \((\Delta \varepsilon)_{sep} \ll \sigma_{\varepsilon}\).

The intrabeam interaction of particles of a beam results in lost them from the region of stable phase oscillations (Touschek effect). The time of life of particles in storage rings in the relativistic case

\[
\tau_{\text{loss}}^T = \frac{8\pi\gamma^2\sigma_z\gamma_{\sigma_I}(\Delta \varepsilon)}{c^2 p\nu_p D(\xi) \varepsilon_s^3},
\]

(45)

where \(D(\xi) = \sqrt{\xi - (3/2) \exp(-\xi) + (\xi/2) \int_{\xi}^{\infty} \ln u/u \exp(-u) + (1/2)(3\xi - \xi \ln \xi + 2) \int \exp(-u)/u du}\),

\(\xi = [((\Delta \varepsilon)\beta_z/\gamma_{\varepsilon_s} \sigma_z]^2\).

### 2.7 Limiting current of the storage rings

The limiting number of particles in the beam of the storage ring is determined by the volume charge of the beam (coulomb repulsion). In the relativistic case \[4\]

\[
N_{\text{lim}} = \frac{\sqrt{2\pi\nu_z \Delta \nu_z} \gamma_{\sigma_I}(\sigma_z + \sigma_x)}{R r_e \sigma_z \sigma_x}.
\]

(46)

### 2.8 Example

In the Tables 1 - 3, as an example, the basic parameters of the electron storage ring, the electron and laser beam parameters and the parameters of the beam of scattered radiation are presented. It is supposed, that the interaction of the electron and circular polarized laser beams occurs in the straight section of the storage ring having zero dispersion function and small \(\beta\)-function \((\beta_{x,z,sc} = 1 \text{ cm})\). Outside of the region of interaction \(\beta_{x,z}(s) \simeq R/\nu_{x,z} = 10 \text{ cm}, D_x = R/\nu^2 = 1\text{ cm}\). The storage ring has large energy acceptance \((\Delta \varepsilon)_{A}/\varepsilon_\varepsilon = \pm 0.1\). One electron bunch regime is used.

The tables we can see, that in the conditions of small currents horizontal emittance and the horizontal dimensions of the electron beam determined by the synchrotron radiation (the laser is switched off) are much greater then the corresponding dimensions determined by backward compton scattering, if the interaction of the electron and laser beams occurs in the region with zero dispersion function. The vertical dimension of the beam determined by the synchrotron radiation is much less then the horizontal one. The vertical and horizontal dimensions of the beam determined by backward compton scattering are identical. The increase of the dispersion function of the storage ring to the value \(D_x \simeq R/\nu^2\) leads to the increase of the horizontal dimension of the beam by the backward compton scattering approximately \(\gamma_s = 10^2\) times.

The conditions of small currents in this example correspond to the currents \(\sim 0.1\ \text{mA}\). At the current \(100\ \text{mA}\) we are forced to increase the dimensions of a beam artificially so that outside of the regions of interaction they were equal to the dimensions \(\sigma_z \cdot \sigma_x \cdot \sigma_I = 75 \cdot 150 \cdot 4000\ \mu\text{m}\). At that the life-time of a beam caused by intrabeam scattering of electrons (Touschek effect) reaches the acceptable values \(\tau_{\text{loss}}^T \simeq 10^3\), and the coulomb repulsion shifts the frequencies of the betatron oscillations to the value \(\Delta \nu \simeq 0.2\), that is close to the extremely possible shift\[5\]. The increase of the horizontal dimension of a beam can

\[5\] On practice the shift \(\Delta \nu \simeq 0.4\) is achieved \[6\].

10
be achieved by a selection of a final value of the dispersion function at the interaction point (IP) or by excitation of betatron and phase oscillations of particles of the beam by a high-frequency fields. The vertical dimension of a beam can be increased by coupling of vertical and horizontal oscillations. Besides the coulomb repulsion the intrabeam scattering of the electrons leads to the increase of the transverse dimensions of the beam in this case.

The equilibrium energy spread of a beam of particles does not depend on the focusing properties and parameters of the radio-frequency system of the storage ring. It grows with the increase of the energy, decreases with increase of the laser wavelength ($\sim \sqrt{\gamma_s/\lambda_L}$) and at the energy 50 MeV has rather large value ($\sigma_{\gamma}/\varepsilon_s = 1.3 \cdot 10^{-2}$), and leads to hard requirements to the energy acceptance of the storage ring, and, hence, to the focusing forces. In this example we have limited the acceptance of the storage ring by the value $(\Delta\varepsilon)^A/\varepsilon_s = \pm 0.1$\textsuperscript{4}. Therefore with increase of the energy of particles in the storage rings it is necessary to go to the more long wavelength lasers. At the energy of particles 1 , $\lambda_L = 20 \mu m$ the maximal energy of scattered photons is increased from $\varepsilon_{\gamma m} = \hbar \omega_m = 50$ keV up to $\varepsilon_{\gamma m} = 1.0$ MeV, and the energy spread of the beam of particles is not changed ($\sim 1.3\%$).

The limiting currents in the storage ring are increased proportionally to the cube of the energy and in the inverse proportion to the electron beam crosssection in the storage ring. Therefore the increasing of the energy of the storage ring will permit to reduce the dimensions of the beams and to increase the stored current. The transverse dimensions of the laser beam must be greater then the transverse dimensions of the electron beam (in the considered example $\sigma_L = 50 \mu m$, $\sigma_{x,z} = 23.7 \mu m$ in the region of interaction). The decreasing of the electron beam current up to 10 mA and the appropriate decreasing of the transverse dimensions of the electron and laser beams the power of scattered radiation is not changed, and the brightness is increased.

The basic advantage of the storage rings compared with the linear accelerators as the sources of the hard electromagnetic radiation consists in the fact that the scattering of the laser photons by electrons do not lead to losses of these electrons, and the electrons are used repeatedly. In the considered case the losses of electrons are determined by the multiple processes of scattering in the field of the laser beam and by Touschek effect. For the life-time $\tau_{\text{loss}} \simeq 10^3$ the electron makes $N_{eff} \simeq 10^{11}$ revolutions and lose the energy $\Delta\varepsilon_{\text{loss}} = eV_s N_{eff} \simeq 10^{13}$ eV $\sim 10^7\varepsilon_s$. Therefore without losing of the efficiency the life-time of the beam can be decreased, and the frequency of the injection of electrons can be increased. Thus the requirements to vacuum in the storage ring and to the value of the energy acceptance can be reduced.

The energy of the electron is changed by the value $\varepsilon_{\gamma} \ll \varepsilon_{\text{sep}} < (\Delta\varepsilon)^A$ in the process of scattering of the laser photons. Therefore the single losses of electrons in this process are absent.

It is interesting to note, that in the considered case the average value of the jump of the electron energy caused by scattering a photon is much greater then the increase of the energy of the equilibrium electron per one revolution ($\varepsilon_{\gamma m}/2 \gg eV_s$), and the probability to the electron to scatter a photon at the passage of the laser beam is small $(2eV_s/(\Delta\varepsilon)_{\gamma m} \simeq 4.0 \cdot 10^{-2})$. It means, that the trajectory of the electron produce on average two jumps at one period of phase oscillations $(\Omega/\omega_s = 2 \cdot 10^{-2})$. At the same time the above mentioned expressions for the equilibrium amplitudes of phase oscillations remain valid, as at their derivation we are not used any restrictions on probability of the emitting of photons.

In the considered storage ring, as well as in the similar storage ring discussed in [3], the harmonic order of the radio-frequency generator is chosen equal to $h = 60$. It corresponds to the wavelength of the generator $\lambda_{rf} \simeq 10.5\ cm$. Such harmonic order is convenient at the construction of the small-dimensional radio-frequency resonator. However it is possible to reduce the harmonic order of the generator, the voltage and the quality of the resonator. So at the harmonic order $h = 4$ ($\lambda_{rf} = 1.57\ m$) the amplitude of

\textsuperscript{4}There are projects of storage rings, in which the energy acceptance is equal $(\Delta\varepsilon)^A/\varepsilon_s = \pm 0.25$ [3].
the voltage at the gap of the resonator will be decreased to the value \( V_m = 16.7 \) kV, and the dimension of the separatrix will not be changed \( (\Delta \varepsilon)_{sep}/\varepsilon_s \approx 0.073 \). At that the equilibrium transverse dimensions of the beam will not be changed, and the longitudinal dimension will be increased up to acceptable dimension \( a_{l,c}^{bs} = 7.4 \) mm \( < l_R \). The equilibrium voltage in this case remains much smaller then the amplitude of radio-frequency voltage. The analogical remark can be done relatively to the focusing force. It is important to have the zero dispersion function and low \( \beta \)-function in the straight section of the storage ring. Less important is to have very high tunes.

The use of supermirrors and storage rings with straight sections having zero dispersion function allows to lower the beam dimensions and to increase the power of the laser radiation in the resonator up to the values, when the losses of the energy of the electron in the storage ring through the backward compton scattering of the laser photons become much greater than the losses of the energy through the synchrotron radiation. Thus the radiative damping times of the amplitudes of radial and phase oscillations are decreased essentially. It allows to lower the influence of the intrabeam scattering inside the electron beam and the influence of the emission of hard backward scattered laser photons on the excitation of betatron and phase oscillations.

The considered source has the emittance \( \varepsilon_{x,z,b} = 7.3 \cdot 10^{-9} \) cm in a conditions of small currents \( (i < 1 \) mA) and the emittance \( \varepsilon_{x,z,b} = 5.4 \cdot 10^{-5} \) cm at the current \( i = 100 \) mA. It can be classified as the light source of the third generation\(^5\). The compactness, small cost, smooth tuning of the frequency, narrow range of generated waves, high average photon flux and brightness discovered in the papers [3], [12] are invaluable quality of such sources. They, without doubts, can find the wide region of applications.

The storage ring RHIC, considered in [4], [13] allows to receive ion beams \( ^{199}\text{Xe}^{47+} \) with the energy spread \( (\sigma_\varepsilon/\varepsilon)_{bs} = 1.2 \cdot 10^{-4} \) and the emittance \( \varepsilon_{x,z,b} = 8.0 \cdot 10^{-12} \) cm at the relative energy \( \gamma \sim 100 \). This is the more expensive and bulky storage ring. At the same time it allows to proceed to the light sources of the fifth generation [14].

### 2.9 Some applications of the backward compton and backward rayleigh sources of hard electromagnetic radiation

For the first time the sources of the backward compton scattering were discussed in the papers [15], [16]. Coherent scattering of electromagnetic waves by electron beams was considered. Later the incoherent scattering of photons of a laser beam by ultrarelativistic electrons was considered in the papers [17], [18] and realized in papers [19] (nonrelativistic beams) and [20], [21] (relativistic beams). The scattered \( \gamma \)-quanta were used in the fields of nuclear physics and diagnostics of electron beams.

Later the production of circular polarized backward compton \( \gamma \)-quanta and using them in the conversion systems for the production of the longitudinally polarized positron beams and in nuclear physics was discussed in papers [22] - [27]. In the paper [28] the intraresonator scheme of interaction of electron and laser beams was considered. This scheme was realized later in papers [29], [30] (in the regime of the free-electron laser). In the paper [31] the current 0.5 was stored in the storage ring NIJI-1 SR (maximum energy 270 MeV) at the energy 163 MeV. This and similar papers stimulated the interest to investigations of compact storage rings as the sources of x-ray radiation for medical and other applications [32]. In the paper [33] the use of supermirrors was suggested to increase the power of the scattered radiation.

In papers [34] - [37] the attention was paid to the rayleigh scattering sources. In this case the scattering crossection of a photon by bound electron in ion at resonance is many orders \( (\sim 10 \div 15) \) higher then the compton one and hence the power of scattered radiation is higher.

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\(^5\)According to accepted classification [10] - [11] the light sources of the third generation have emittances \( \varepsilon_{x,z,b}^{bs} < 1.0 \cdot 10^{-6} \) cm, and the light sources of the fourth generation will have the emittances \( \varepsilon_{x,z,b}^{bs} < 1.0 \cdot 10^{-9} \) cm.

\(^6\)The scheme using the helical undulator was discussed in these papers as well.
In the papers [1], [2], it was shown that the scattering of laser photons by ions in the storage rings leads to the three-dimensional radiative cooling of ion beams and, simultaneously, to the excitation of the amplitudes of betatron and phase oscillations of ions in the beam. The equilibrium parameters of ion beams according to developed theory will differ from similar parameters of beams determined by the synchrotron radiation. To reduce the influence of the quantum excitation of amplitudes of betatron and radial-phase oscillations it was suggested to use the zero dispersion function at the IP.

In the paper [3] it was shown, that the use of radiative damping in electron storage rings of the intermediate energy, supermirrors and modern lasers allows to generate the beams of scattered photons with the power exceeding the power of the synchrotron radiation from the orbit of these storage rings at higher hardness and brightness. At that the damping time of amplitudes of electron oscillations is decreased, the influence of the intrabeam scattering on the emittance of the beam is decreased. This paper stimulated a series of papers devoted to sources of backward compton scattering and their applications [38] - [40], [4]. In papers [41] - [43] the attention was paid on the opportunity of effective self-polarization of the positron beams in the storage rings with special magnetic structure in the field of the circular polarized laser beam.

3 Radiative effects in laser-electron and laser-ion storage rings in dynamics

In the previous section we have considered the dynamics of particles in storage rings in the field of a homogeneous counterpropagated laser beam in stationary regime. In this case the transverse dimensions of the laser beam were greater, than the dimensions of the particle beam and their axes coincided. At the same time we have derived the general formulas (23), (28) allowing to consider the dynamics of particle beams interacting with an arbitrary targets (undulators, laser beams, material media). The targets could be non-uniform and could be moved (oscillate) in the radial direction (or, the orbits in the storage rings could be moved in the backward direction). These formulas are

\[
\frac{dA^2}{dt} = -A^2\frac{P^{rad}}{\varepsilon_s} + \frac{7\pi\hbar c(1 + \beta_s)^2\gamma_s^2P^{rad}}{10\lambda_L\beta_s^4\varepsilon_s^2}D_{sc}^2 - \frac{2D_{sc}P^{rad}}{\beta_s^2\varepsilon_s}x_b, \tag{47}
\]

\[
\ddot{\varphi} + \frac{h\omega_s^2K(\varepsilon^{rad} - \varepsilon_s^{rad})}{2\pi\varepsilon_s} - \frac{hq\omega_s^2KU_m}{2\pi\varepsilon_s}(\cos \varphi - \cos \varphi_s) = 0. \tag{48}
\]

The values \(\varepsilon^{rad}\) and \(P^{rad} = f\varepsilon^{rad}\) in (47), (48) usually depend on the energy of particles, effective thickness of a target in a radial direction and on time by the law \(\varepsilon^{rad} = F_1(x,t)F_2(\gamma)\), where \(x = x_b + x_\eta\) is the deviation of the particle from the equilibrium orbit. The deviation \(x_\eta\) is determined by the deviation of the relative energy \(\gamma\) from the equilibrium one and by the dispersion function. For the stationary regime and non-uniform target the value

\[
\varepsilon^{rad} = \varepsilon_s^{rad} + \varepsilon_s^{rad}\frac{\partial \ln F_1}{\partial x}(x_b + x_\eta) + \varepsilon_s^{rad}\frac{\partial \ln F_2}{\partial \ln \gamma}(\gamma - \gamma_s). \tag{49}
\]

In the case of cooling of the electron and ion beams in the field of the laser beam the second term in (49) is equal \(\partial \ln F_2/\partial \ln \gamma = k_1\). In a case of ionization cooling of muon beams this term is much less then unit and has more complicated dependence on energy.

7The using of damping wigglers and undulators in storage rings, installed in the straight sections, with zero dispersion function has allowed to reduce the emittances of electron beams (three orders) 10 and to proceed to light sources of the third generation.
The second term in (49) is different from zero only in the case, when the target is non-uniform in radial
direction (axis of a beam of particles is on the slope near to caustic of the laser beam or, in case of cooling
of muon beams, wedge target is used).

The expressions (47) and (48), according to (49), can be presented in the form

$$\frac{dA_x^2}{dt} = -A_x^2 \frac{P^{rad}_x}{\varepsilon_s} \left(1 - \frac{D_{x,sc} \partial \ln F_1}{\beta_s^2 \partial x} \right) + \frac{7\pi \hbar \omega (1 + \beta_s)^2 \gamma_s^2 P^{rad}_s D_{x,sc}^2}{10 \lambda_L \beta_s^4 \varepsilon_s^3},$$

(50)

$$\ddot{\psi} + \frac{P^{rad}_s}{\varepsilon_s} (k_i + \frac{D_{x,sc} \partial \ln F_1}{\beta_s^2 \partial x}) \dot{\psi} + \Omega^2 \psi = 0.$$

(51)

According to (50), (51) the increase of damping decrement of amplitudes of betatron oscillations of
particles leads to the decrease of the damping decrement of amplitudes of phase oscillations in the same
degree. The sum of decrements is a constant (analogue of the Robinson criterion of damping). The
decrements can accept negative values.

The non-stationary regime of interaction of being cooled particle beams with targets was developed in
[4] for the case of the unbunched beams. Switching on a radio-frequency accelerating field will complicate
the problem. This problem will be considered in the separate paper.

4 Conclusion

In this paper the detailed derivation of the equations describing the dynamics and equilibrium dimensions
of the particle beams interacting with the laser beams in storage rings is presented. Brief derivation of
these equations was given in the papers [1, 2] for ion beams and in the paper [3] for the electron beams.
The general non-stationary case of interaction of particles with a target is considered. The example is
presented to illustrate the problem to estimate the possible range of the energy of electron storage rings
and the hardness of scattered radiation proceeding from life-time of the electron beam in the ring. Some
applications of the being cooled beams were discussed.

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8 A misprint is in the equilibrium energy spread ($\sigma_s = \sigma_\varepsilon/\varepsilon$) in [3] (corrected in [4]). In [2] the value ($\sigma_s = 2\sigma_\varepsilon/\varepsilon$)
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TABLE 1. Electron storage ring parameters \((J_x = 1, \ J_\varphi = 2) \ast\).

| Parameter | Value |
|-----------|-------|
| Energy \[\text{MeV}\] | \(\varepsilon = 50\) |
| Average radius \[\text{m}\] | \(R = 1\) |
| Instantaneous radius \[\text{m}\] | \(R = 0.5\) |
| Horizontal/vertical tunes | \(\nu_{x,y} = 10\) |
| \(\beta\)-function \[\text{cm}\] | \(\beta_{x,z} = 10, \ \beta_{x,z\ sc} = 1\) |
| No of electrons (aver. current \[\text{mA}\]) | \(N_e = 1.3 \cdot 10^{10} \ (i = 100)\) |
| Current in a bunch \[\text{A}\] | \(i_b = (C/2\sigma l)i = 74\) |
| Frequency of resonator \[\text{MHz}\] | 2856 \((\lambda_{rf} = 10.5 \text{ cm})\) |
| Harmonic order | \(h = 60\) |
| RF peak voltage \[\text{kV}\] | 250 |
| Damping time of amplitudes of radial betatron and phase oscillations \[\text{sec}\] | \(\tau_{sr}^{st} = 1.774, \ \tau_{x,z}^{st} = 1.28 \cdot 10^{-2}, \ \tau_{\varphi}^{bc} = 6.5 \cdot 10^{-3}\) |
| Equilibrium energy spread | \(\langle \sigma_{\varepsilon}/\varepsilon \rangle^{st} = 4.38 \cdot 10^{-5}, \ \langle \sigma_{\varepsilon}/\varepsilon \rangle^{bc} = 1.3 \cdot 10^{-2}\) |
| Equilibrium transverse emittance \[\text{cm}\] \((D_z = 0, \ \beta_{z,x\ sc} = 1 \text{ cm})\) | \(\varepsilon_{x,y}^{st} = 7.7 \cdot 10^{-11}, \ \varepsilon_{x,z\ b}^{bc} = 7.3 \cdot 10^{-9}\) |
| Transverse dimensions at IP \[\mu\text{m}\] \((D_z = 0, \ \beta_{z,x\ sc} = 1 \text{ cm})\) | \(\sigma_{x,y}^{st} = 8.75, \ \sigma_{x,z\ b}^{bc} = 0.854\) |
| Equilibrium transverse emittance \[\text{cm}\] \((D_{z\ sc} = 0.5 \text{ cm}, \ \beta_{z,x\ sc} = 1 \text{ cm})\) | \(\varepsilon_{x,y}^{st} = 7.7 \cdot 10^{-11}, \ \varepsilon_{x,z\ b}^{bc} = 5.4 \cdot 10^{-5}\) |
| Transv. beam dimensions at IP \[\mu\text{m}\] \((D_{z\ sc} = 0.5 \text{ cm}, \ \beta_{z,x\ sc} = 1 \text{ cm})\) | \(\sigma_{x,z\ b}^{bc} = 23.7\) |
| Transv. beam dimensions outside of the IP \[\mu\text{m}\] | \(\sigma_{x,y}^{bc} = 75, \ \sigma_{x,\eta}^{bc} = 65, \ \sigma_{z} \simeq 10^2\) |
| Longitudinal beam dimensions \[\text{mm}\] | \(\sigma_z^{l} = 8.49, \ \sigma_{l\ bc}^{bc} = 3.8\) |
| Relative frequency | \(\Omega/\omega_s = 2.2 \cdot 10^{-2}\) |
| Maximal energy deviation | \((\Delta\varepsilon)_{sep}/\varepsilon_s = 0.073\) |
| Energy acceptance | \((\Delta\varepsilon)_A/\varepsilon_s = \pm 0.1\) |
| Life-time \[\text{sec}\] | \(\tau_{T\ loss}^{C} = 5.6 \cdot 10^4, \ \tau_{T\ loss}^{T} = 7.1 \cdot 10^2\) |
| Accepted beam dimensions at IP \[\mu\text{m}\] | \(\sigma_{x,y}^{bcs} = 23.7, \ \sigma_{x,\eta}^{bc} = 0, \ \sigma_{l} = 4 \cdot 10^3\) |
| Accepted beam dimensions outside IP \[\mu\text{m}\] | \(\sigma_{x,y}^{bcs} = 75, \ \sigma_{x,\eta}^{bc} = 130, \ \sigma_{x\ sc}^{bc} = 150, \ \sigma_{l} = 4 \cdot 10^3\) |
| \(\sigma_{x\ sc}^{bc} = \sqrt{(\sigma_{x\ bc}^{bc})^2 + (\sigma_{x\ sc}^{bc})^2}\) | \(\sigma_{x,y}^{bcs} = 23.7, \ \sigma_{x,\eta}^{bc} = 0, \ \sigma_{l} = 4 \cdot 10^3\) |
| Limit. number of electrons for \(\sigma_z \cdot \sigma_x \cdot \sigma_l = 75 \cdot 150 \cdot 4200 \ \mu\text{m}^3; \ \Delta\nu = 0.2\) | \(N_{lim} = 1.34 \cdot 10^{10}\) |

\(\ast\) Emittances and dimensions of beams are calculated in one-particle approximation (coulomb repulsion and intrabeam scattering are neglected.)
| Parameter                                                      | Value                                      |
|---------------------------------------------------------------|--------------------------------------------|
| Wavelength [cm]                                               | $\lambda_L = 1 \cdot 10^{-4}$             |
| Photon energy [eV]                                            | $h\omega_L = 1.24$                        |
| Laser pulse length [cm]                                       | $l_L = K\lambda_L = 1.0$                  |
| Distance between wave packets [m]                            | $P_{wp} = 2\pi R = 2\pi \approx 6.3$     |
| Laser beam waist radius [cm]                                 | $\sigma_L = 5 \cdot 10^{-3}$              |
| Rayleigh length [cm]                                          | $l_R = 4\pi\sigma_L^2/\lambda_L = 3.14$  |
| Laser bunch energy in resonator [J]                           | $\varepsilon_L = 2 \ J$                   |
| Resonator quality                                             | $Q = 1/(1 - R_r) = 1.0 \cdot 10^4$        |
| Damping time of laser beam in resonator [sec]                 | $\tau_r = QL_{wp}/c = 2.1 \cdot 10^{-4}$  |
| Reactive laser pulse power in resonator [GW]                  | $P_b = 60$                                |
| Pulse intensity in the waist [(W/cm)$^2$]                     | $I = P_b/2\pi\sigma_L^2 = 3.82 \cdot 10^{14}$ |
| Magnetic field strength in the waist                          | $B_{\perp m} = \sqrt{2P_L/c\sigma_L^2} \approx 6.8 \cdot 10^5$ Gs |
| Deflecting parameter                                         | $p_{\perp m} = eB_m/2\pi mc^2 = 6.4 \cdot 10^{-3}$ |
| Electron energy losses per revolution [keV]                   | $eV_e = 1.06$                             |
| Laser pulse duration [sec]                                    | $\tau_L = 10^{-3}$                        |
| Frequency of laser pulses [Hz]                                | $f_L = 50$                                |
| Laser pulse power [kW]                                        | $P_L = \varepsilon_L \cdot f/Q = 9.5$     |
| Average laser power [W]                                       | $P_L = P_L \cdot \tau_L f_L = 475$       |

| Parameter                                                      | Value                                      |
|---------------------------------------------------------------|--------------------------------------------|
| The maximal energy of photons [keV]                           | $\varepsilon_{\gamma m} = 50$             |
| Pulse power of the beam [W]                                   | $P_{\text{rad imp}} = 95$                 |
| Average power [W]                                             | $P_{\text{rad}} = P_{\text{rad imp}} \cdot \tau_L \cdot f = 4.74$ |
| Photon flux [sec$^{-1}$]                                      | $N_{\gamma \text{imp}} = P_{\text{rad}}/h\omega = 2.37 \cdot 10^{16}$ |
| Photon flux in the range $\Delta \omega/\omega_m = 0.1$ [sec$^{-1}$] | $\Delta N_{\gamma \text{imp}} = (3P_{\text{rad}}/h\omega_m)(\Delta \omega/\omega_m)$ $= 3.6 \cdot 10^{15}$ |
| Average photon flux [sec$^{-1}$]                              | $\bar{N}_{\gamma} = N_{\gamma \text{imp}} \cdot \tau_L \cdot f = 1.19 \cdot 10^{14}$ |
| Average photon flux in a range $\Delta \omega/\omega_m = 0.1$ [sec$^{-1}$] | $\Delta N_{\gamma} = \bar{N}_{\gamma \text{imp}} \cdot \tau_L \cdot f$ $= 1.8 \cdot 10^{14}$ |
| Duration of pulses [sec]                                      | $\tau_{x-ray} = 10^{-3}$                  |
| Frequency of pulses [Hz]                                      | $f_{x-ray} = 50$                          |
| Duty cycle                                                    | $S = \tau_{x-ray} \cdot f_{x-ray} = 5 \cdot 10^{-2}$ |