Kinematics analysis of a 3-DOF hydraulic driven parallel mechanism

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Abstract: In order to provide the basis about the selection of parts for the mechanical design of a 3-DOF parallel mechanism, such as hydraulic cylinders, valves and so on. Based on coordinate transformation and vector guidance method, the kinematics of the hydraulic drive 3-DOF parallel mechanism is analyzed at first. Secondly, deducing the displacement of each hydraulic cylinder according to the position of the upper platform on parallel mechanism. Then, The ADAMS is used to build virtual prototype of 3-DOF parallel mechanism, the validity about the kinematics of the parallel mechanism is verified by the co-simulation of MATLAB / SIMULINK and ADAMS, which provide the theoretical support for dynamics analysis.

1 Introduction

Parallel mechanism is widely used in testing equipment, machine tools, sensors and entertainment equipment because of its large stiffness, fast response, high precision and high load carrying capacity. In view of the drive system, it can be divided into hydraulic drive, pneumatic drive, motor drive and so on. Hydraulic drive is suitable for the testing equipment of heavy equipment because of its high power density, large output characteristics, such as vehicles, tanks and other heavy engineering machineries. Kinematics analysis is the basis to analyze the dynamics, singularity and workspace of parallel mechanisms, mainly including the kinematics forward and inverse solution, speed and acceleration analysis and so on. Kinematic inverse solution gives the pose of platform to solve the position parameter of driving system, kinematic forward solution is the opposite. Velocity and acceleration analysis refers to the relationship between the velocity and acceleration of the platform with the velocity and acceleration of driving mechanism, and they can be linked by the Jacobi matrix.

Recently, few degree of freedom parallel mechanism has become research hotspot, which has broad prospects in the field of industrial applications because of its low cost. At present, 3-DOF parallel mechanisms mostly evolve from Delta mechanism, while 3-RPS spatial parallel mechanism is proposed by foreign scholars. Generally speaking, 3-DOF parallel mechanisms are fewer models and inadequate research depth. The schematic diagram of mechanism about the novel 3-DOF parallel mechanism researched in this paper is shown in Fig. 1.

As can be seen from figure 1, the compositions of 3-DOF parallel mechanism researched in this paper are upper platform, lower platform, three sets of drive hydraulic cylinders (1°, 2° and 3°) and three sets of folding combinations (upper folding are a1d1, a2d2 and a3d3; the corresponding lower folding are d1b4, d2b5, d3b6).
Three degrees of freedom movements that the 3-DOF can achieve are: when the hydraulic cylinder 1\(^*\) stays in the middle position, the cylinder 2\(^*\) and 3\(^*\) do differential motion, achieving the roll motion. When the three hydraulic cylinders run synchronously in middle position, the platform achieves the heave movement. When the hydraulic cylinder 1\(^*\) does reciprocate motion in the middle position, and the other two hydraulic cylinders run synchronously with differential motion of cylinder 1\(^*\) to achieve pitch movement at the same time.

2 Kinematic analysis of parallel mechanism

In order to express spatial motion of parallel mechanism clearly, two coordinates systems are needed. They are body coordinate system\{M\} and static coordinate system\{G\} that respectively shown in Fig. 1. Spatial motion pose parameter of parallel mechanism upper platform is defined as \(q=[q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6]^{T}\); Let \(q_r=[q_1 \ q_2 \ q_3]^{T}\) be the angle of body coordinate system relative to static coordinate system; \(c=[q_4 \ q_5 \ q_6]^{T}\) is the position of upper platform center in static coordinate system. Meanwhile, let \(q_0=[q_1 \ q_2 \ q_6]^{T}\) be the generalized coordinate of 3-DOF parallel mechanism that is researched, respectively represent roll, pitch and heave motion of parallel mechanism.

The system rotation transformation matrix can be obtained after three axes rotation transformation as follows

\[
R = \begin{bmatrix}
  cq_1cq_2 & cq_1sq_2sq_4 + sq_1cq_4 & sq_1sq_4 + cq_1sq_2sq_4 \\
  sq_1cq_2 & sq_1cq_4 - sq_2sq_4 & sq_1sq_4 - sq_2sq_4 \\
  -sq_1sq_2 & cq_2sq_4 & cq_2sq_4
\end{bmatrix}
\]

(1)

where, \(cq_1 = \cos q_1\), \(sq_1 = \sin q_1\), and the others are same defined. Therefore, the transformation matrix from body coordinate system to static coordinate system can be given as

\[
T = \begin{bmatrix}
  R & c \\
  0 & 1
\end{bmatrix}
\]

(2)

According to the triangle cosine theorem, the length \(b1c1\) and total length of hydraulic cylinder 1\(^*\) can be derived, and the hydraulic cylinders 2\(^*\) and 3\(^*\) are calculated Similarly. The geometric relationship about 1\(^*\) compounds of driving chain of 3-DOF parallel mechanism is shown in figure 2.

(a) Side view of 3-DOF parallel mechanism  (b) Top view of 3-DOF parallel mechanism

Figure 2. Schematic of kinematic inverse solution of parallel mechanism

If homogeneous coordinate matrix \(A\) in body coordinate system of upper platform hinge point \(a_i\)\((i=1,2,3)\) , homogeneous coordinate matrix \(B_1\) and \(B_2\) in static coordinate system of lower platform
hinge point  \( b_i(i=1,2,3) \), \( b_{i,3}(i=1,2,3) \) are
\[
A = \begin{bmatrix} a_1 & a_2 & a_3 \\ 1 & 1 & 1 \end{bmatrix}, \quad B_i = \begin{bmatrix} b_1 & b_2 & b_3 \\ 1 & 1 & 1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} b_4 & b_5 & b_6 \\ 1 & 1 & 1 \end{bmatrix}
\]
(3)

Through coordinate transformation, the coordinate matrix \( G \) in the static coordinate system of upper platform hinge points can be derived as
\[
G = TA = \begin{bmatrix} a_i^e & a_i^e & a_i^e \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} R & c \end{bmatrix} \begin{bmatrix} a_i & a_i & a_i \\ 0 & 1 & 1 \end{bmatrix}
\]
(4)

where, \( a_i^e \) is coordinate matrix of upper hinge point \( a_i \) in static system, \( a_i^f = [x_i^f, y_i^f, z_i^f]^T \) \((i=1,2,3)\); \( a_i \) is coordinate matrix of upper hinge point \( a_i \) in body system, \( a_i = [x_i, y_i, z_i]^T \) \((i=1,2,3)\); \( R \) is coordinate rotation matrix; \( c \) is coordinate origin translation matrix of body coordinate system.

The upper hinge can only move in one plane due to the limitation of the three folds, for example, point \( a_i \) can only move in the plane determined by point \( b_1, b_4 \) and \( d_i \). Thus three constraint equations can be described as
\[
y_i^f = 0, y_i^e = \sqrt{3}x_i^e, y_i^e = -\sqrt{3}x_i^e
\]
(5)

Combining formula (3) and (4). \( q_3, q_4, q_5 \) can be derived as
\[
q_3 = \arctan \frac{s q_3 s q_2}{c q_1 + q_2}, q_4 = -\frac{r}{2} (c q_3 q_4 + s q_3 s q_2 - c q_4 q_2), q_5 = -c q_3 q_4 \cdot r
\]
(6)

At this time, the six parameters \( q_1, q_2, q_3, q_4, q_5, q_6 \) of motion platform pose have already been solved, so transform matrix \( T \) is known. The coordinates of the upper hinge point \( a_i(i=1,2,3) \) in the static coordinate system can be derived by formula (4). The expression of the position vector between the upper hinge point \( a_i \) and the lower hinge point \( b_i \) in the static coordinate system can be given by
\[
L_{a_h} = a_i^e - b_i = Ra_i + c - b_i \quad (i=1,2,3), \quad L_{a_{h,3}} = a_i^e - b_{i,3} = Ra_i + c - b_{i,3} \quad (i=1,2,3)
\]
(7)

Then the distance between the upper and lower hinge points can be expressed as
\[
l_{a_h} = \|L_{a_h}\|, l_{a_{h,3}} = \|L_{a_{h,3}}\| \quad (i=1,2,3)
\]
(8)

So the angle \( \theta_i (i=1,2,3,4,5,6) \) in Fig. 2(a) can be derived as
\[
\theta_i = \arccos \left( \frac{l_{a_h}^2 + l_{a_{h,3}}^2 - l_{b_h}^2}{2l_{a_h}l_{a_{h,3}}} \right) \quad (i=1,2,3), \quad \theta_{i,3} = \arccos \left( \frac{l_{a_{h,3}}^2 + l_{a_{h,3}}^2 - l_{b_{h,3}}^2}{2l_{a_{h,3}}l_{a_{h,3}}} \right) \quad (i=1,2,3)
\]
(9)

And \( \alpha_i(i=1,2,3) \) can be defined as
\[
\alpha_i = \theta_i + \theta_{i,3}(i=1,2,3)
\]
(10)

According to the cosine theorem, the length of hydraulic cylinder in \( \Delta a_i b_i c_i (i=1,2,3) \) can be derived as
\[
l_i = l_{a_i c_i} = \sqrt{l_{a_h}^2 + l_{a_{h,3}}^2 - 2l_{a_h}l_{a_{h,3}} \cos \alpha_i} \quad (i=1,2,3)
\]
(11)

If the initial length of hydraulic cylinder is \( l_{i,0} (i=1,2,3) \), then the piston rod movement of hydraulic cylinder can be described by
\[
\Delta l_i = l_i - l_{i,0} (i=1,2,3)
\]
(12)

3 Co-simulation verification of parallel mechanism system

It is a low cost, high precision and high efficiency method to verify the mathematical model of system by using virtual prototype instead of physical prototype. In this paper, the kinematics model, dynamic model and control strategy of 3-DOF parallel mechanism will be established in MATLAB/SIMULINK; The virtual prototype of parallel mechanism is established in ADAMS
software. To verify the feasibility and effectiveness of the kinematics model, dynamic model and control algorithm of 3-DOF parallel mechanism built above by co-simulation.

The parallel mechanism is in initial state while three cylinders are in middle position, and three groups of driving and supporting structures are evenly distribute, the structural parameters are shown in table 1 when the parallel mechanism is in middle position.

**Table 1 Table of the 3-DOF parallel robot's structural parameters (m)**

| Radius of upper hinge circle \( r \) | Radius of lower hinge circle \( R_1 \) | Radius of lower hinge circle \( R_2 \) | Fold vertical distance between upper and lower point \( L_{aad} \) | Length of upper fold \( L_{ad} \) | Length of lower fold \( L_{db} \) | Distance between fold and hinge point of cylinder \( L_{ac} \) |
|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| 2.000                               | 1.600                                | 2.000                                | 2.000                                | 1.154                                | 1.154                                | 0.450                                |

The process of kinematic model verification is shown as: the displacement of hydraulic cylinder is obtained by using inverse kinematic with input pose of parallel mechanism, and the output pose can be obtained by using motion virtual prototype of displacement driven platform. The validity of kinematic model is evaluated by the error between input and output pose, its principle is shown in figure 3.

![Figure 3. The verification principle of kinematics](image)

The desired input pose signals are \( q_1 = 8^\circ \), \( q_2 = 8^\circ \), \( q_6 = 0.28m \). So the results of system simulation are shown in figure 4.

![Figure 4. Validation error of kinematic model](image)

(a) Pose error of roll motion  
(b) Pose error of pitch motion  
(c) Pose error of heave motion

**Figure 4. Validation error of kinematic model**

According to the simulation results, the roll motion error is \( 1.48 \times 10^{-4} \), that is \( 0.00185\% \); The input of pitch motion and heave motion is zero, and its errors are \( 9.85 \times 10^{-5} \) and \( 7.22 \times 10^{-5} \) respectively; The pitch motion error is \( 4.42 \times 10^{-4} \), that is \( 0.0055\% \); The errors of roll motion and the heave motion at zero input are respectively \( 9.82 \times 10^{-9} \) and \( 7.65 \times 10^{-5} \); The heave motion error is \( 9.21 \times 10^{-3} \text{m} \), that is \( 0.0033\% \); The errors of roll motion and the pitch motion are respectively \( 7.65 \times 10^{-10} \) and \( 8.97 \times 10^{-7} \). From the simulation results, it can be concluded that the kinematic model established to analyze the parallel mechanism is accurate.
Cylinder length range during the three degrees of freedom motion

| Cylinder | Roll | Pitch | Heavy |
|----------|------|-------|-------|
| 1\textsuperscript{st} | $-4.32 \times 10^{-3}$ m to $-1.78 \times 10^{-3}$ m | $-0.180$ m to $-0.162$ m | $-0.175$ m to $-0.168$ m |
| 2\textsuperscript{nd} | $-0.155$ m to $-0.144$ m | $-0.088$ m to $-0.087$ m | $-0.175$ m to $-0.168$ m |
| 3\textsuperscript{rd} | $-0.155$ m to $-0.144$ m | $-0.088$ m to $-0.087$ m | $-0.175$ m to $-0.168$ m |

According to Table 2, the final motion range of hydraulic cylinder can be obtained as follows: Cylinder 1\textsuperscript{st}: $-180$ mm to $-168$ mm; Cylinder 2\textsuperscript{nd} and 3\textsuperscript{rd}: $-175$ mm to $-168$ mm, which can be ready for the selection of hydraulic power mechanism.

4 Conclusion

The kinematics of a novel 3-DOF parallel mechanism was established using the derivative methods, and the high precision of model was verified by co-simulation in using MATLAB/SIMULINK and ADAMS software. The length of each hydraulic cylinder was gained by calculating the kinematics model under given work condition, which guides the choice of power mechanism. And the inverse kinematic model supplies the fundament for the joint control of parallel mechanism. There is a slight translational movement along the axis $x$ in roll and pitch movements because of the existence of implicated movement.

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