QCD sum rules study of the meson $Z^+(4430)$

Su Houng Le†
Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea

Antonio Mihara‡
Instituto de Cincias Exatas e Tecnologia, Universidade Federal do Amazonas, R. Nossa Senhora do Rosario 3863, 69100-000 - Itacoatiara, AM, Brazil

Fernando S. Navarra† and Marina Nielsen§
Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil

We use QCD sum rules to study the recently observed meson $Z^+(4430)$, considered as a $D^*D_1$ molecule with $J^P = 0^-$. We consider the contributions of condensates up to dimension eight and work at leading order in $\alpha_s$. We get $m_Z = (4.40 \pm 0.10) \text{ GeV}$ in a very good agreement with the experimental value. We also make predictions for the analogous mesons $Z_0$ and $Z_0^*$, considered as $D^*_1D^*_1$ and $B^*B_1$ molecules respectively. For $Z_0$, we predict $m_{Z_0} = (4.70 \pm 0.06) \text{ GeV}$, which is above the $D^*_1D^*_1$ threshold, indicating that it is probably a very broad state and, therefore, difficult to observe experimentally. For $Z'_0$, we predict $m_{Z'_0} = (10.74 \pm 0.12) \text{ GeV}$, in agreement with quark model predictions.

PACS numbers: 11.55.Hx, 12.38.Lg, 12.39.-x

In the last years many new mesons have been observed by the BaBar, BELLE, CLEO, D0 and FOCUS collaborations. Among these new mesons, some have been considered as good candidates for tetraquark states like the $D_{sJ}(2317)$, the $D_{sJ}(2460)$, the $X(3872)$ and more recently the $Z^+(4430)$.

While there are many indications that the charmed mesons, $D_{sJ}(2317)$ and $D_{sJ}(2460)$, are not four-quark states, this is not the case for the charmonium like states, $X(3872)$ and $Z^+(4430)$. The $X(3872)$, with quantum numbers $J^{PC} = 1^{++}$, does not fit in the charmonium spectrum and presents a strong isospin violating decay, disfavoring a $c\bar{c}$ assignment. The $Z^+(4430)$, recently observed in the $Z^+ \to J/\psi \pi^+$ decay mode, is the most interesting one since, being a charged state, it can not be a pure $c\bar{c}$ state.

There are already many theoretical interpretations for the structure of the $Z^+(4430)$ meson: molecular $D^*D_1$ state, tetraquark state or a cusp in the $D^*D_1$ channel. In ref. [8], the authors have interpreted the $Z^+(4430)$ meson as the first radial excitation of the diquark-antidiquark $[cu][\bar{c}\bar{d}]$ state, with $J^{PC} = 1^{++}$. The low lying tetraquark state, $[cu][\bar{c}\bar{d}]$, is interpreted as the charged partner of the $X(3872)$ meson. Supposing that the mass of the low lying tetraquark state, $[cu][\bar{c}\bar{d}]$ is compatible with the mass of the $X(3872)$ meson, the mass difference between the $Z^+(4430)$ and the $X(3872)$ would be close to the mass difference between the $\psi'$ and $J/\psi$: $m_{\psi'} - m_{J/\psi} = 590 \text{ MeV}$. Therefore, they arrive at $m_Z \sim 3872 + 590 \sim 4460 \text{ MeV}$, which is compatible with the observed mass. In ref. [7], the closeness of the $Z^+(4430)$ mass to the threshold of $D^*(2010)\bar{D}_1(2420)$ lead the authors to consider the $Z^+(4430)$ as a $D^*D_1$ molecule. In this case, $J^P = 0^-$, it would be $0^-$ or $2^-$, although the $2^-$ assignment is probably suppressed in the $B \to Z(4430)K$ decay, by the small phase space. Among the remaining possible $0^-$ and $1^-$ states, the former will be more stable as the later can also decay to $DD_1$ in s-wave. Moreover, one expects a bigger mass for a $J^P = 1^-$ state as compared to a $J^P = 0^-$ state. Therefore, in this work we use QCD sum rules (QCDSR) to study the two-point function of the state $Z^+(4430)$ considered as a $D^*D_1$ molecule with $J^P = 0^-$. In a previous calculation, the QCDSR approach was used to study the $X(3872)$ meson, considered as a diquark-antidiquark state, and a good agreement with the experimental mass was obtained. If we suppose, as in ref. [8], that the $Z^+(4430)$ is related to the first radial excitation of the $X(3872)$, in the QCDSR approach its mass would be given by $\sqrt{s_0}$, where $s_0$ is the continuum threshold. In ref. [8] it was found that $\sqrt{s_0} = (4.3 \pm 0.1) \text{ GeV}$ also in a very good agreement with the experimental mass of

*Electronic address: suhoung@phya.yonsei.ac.kr
†Electronic address: mihara74@gmail.com
‡Electronic address: navarra@if.usp.br
§Electronic address: mnielsen@if.usp.br
\( Z^+(4430) \). However, this is not a precise determination of the mass of the first excited state, since the continuum threshold gives only a lower bound for the mass of the first excited states.

Considering \( Z^+(4430) \) as a \( D^* D_1 \) molecule with \( J^P = 0^- \), a possible current describing such state is given by:

\[
j = \frac{1}{\sqrt{2}} \left[ (\bar{d} \gamma_\mu c a)(\bar{c} \gamma^\mu \gamma_5 u_b) + (\bar{d} \gamma_\mu c \gamma_5 a)(\bar{c} \gamma_5 \gamma^\mu u_b) \right],
\]

where \( a \) and \( b \) are color indices. We have considered the symmetrical state \( D^{*+} D_1^0 + \bar{D}^{*0} D_1^+ \) because it has positive \( G \)-parity, which is consistent with the observed decay \( Z^+(4430) \to \psi' \pi^+ \).

The two-point correlation function is given by:

\[
\Pi(q) = i \int d^4x \ e^{iq.x} \langle 0| T[j(x)j^\dagger(0)]|0\rangle.
\]

On the OPE side, we work at leading order in \( \alpha_s \) and consider the contributions of condensates up to dimension eight. We calculate the light quark part of the correlation function in the coordinate-space and at \( D = 4 \) in Eq. (3).

The calculation of the phenomenological side at the hadron level proceeds by writing a dispersion relation to the correlation function in Eq. (2):

\[
\Pi^{\text{phen}}(q^2) = \int ds \frac{\rho^{\text{phen}}(s)}{s - q^2} + \cdots,
\]
where \( \rho^{\text{phen}}(s) \) is the spectral density and the dots represent subtraction terms. The spectral density is described, as usual, as a single sharp pole representing the lowest resonance plus a smooth continuum representing higher mass states:

\[
\rho^{\text{phen}}(s) = f_Z^2 \delta(s - m_Z^2) + \rho^{\text{cont}}(s),
\]

where \( f_Z \) gives the coupling of the current to the meson \( Z^+ \):

\[
\langle 0 | j | Z^+ \rangle = f_Z.
\]

For simplicity, it is assumed that the continuum contribution to the spectral density, \( \rho^{\text{cont}}(s) \) in Eq. (7), vanishes below a certain continuum threshold \( s_0 \). Above this threshold, it is assumed to be given by the result obtained with the OPE. Therefore, one uses the ansatz [17]

\[
\rho^{\text{cont}}(s) = \rho^{\text{OPE}}(s) \Theta(s - s_0),
\]

After making a Borel transform to both sides of the sum rule, and transferring the continuum contribution to the OPE side, the sum rules for the pseudoscalar meson \( Z^+ \), up to dimension-eight condensates, can be written as:

\[
f_Z^2 e^{-m_Z^2/M^2} = \int_{4m_c^2}^{s_0} ds \ e^{-s/M^2} \rho^{\text{OPE}}(s) + \Pi^{\text{mix} \langle \bar{q}q \rangle}(M^2),
\]

where

\[
\Pi^{\text{mix} \langle \bar{q}q \rangle}(M^2) = \frac{m_c^2 \langle \bar{q}q \sigma G \bar{q}q \rangle}{2\pi^2} \int_0^1 d\alpha \exp\left[-\frac{m_c^2}{\alpha(1-\alpha)M^2}\right]\left[1 + \frac{m_c^2}{\alpha(1-\alpha)M^2}\right].
\]

To extract the mass \( m_Z \) we take the derivative of Eq. (10) with respect to \( 1/M^2 \), and divide the result by Eq. (10).

The values used for the quark masses and condensates are [13, 18]: \( m_c(m_c) = (1.23 \pm 0.05) \text{ GeV} \), \( \langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3 \), \( \langle \bar{q}q \sigma G \bar{q}q \rangle = m_0^2 \langle \bar{q}q \rangle \) with \( m_0^2 = 0.8 \text{ GeV}^2 \), \( \langle g^2 G^2 \rangle = 0.88 \text{ GeV}^4 \).

We evaluate the sum rules in the Borel range \( 2.2 \leq M^2 \leq 3.5 \text{ GeV}^2 \), and in the \( s_0 \) range \( 4.8 \leq \sqrt{s_0} \leq 5.0 \text{ GeV} \).

From Fig. 1 we see that we obtain a quite good OPE convergence for \( M^2 \geq 2.5 \text{ GeV}^2 \). Therefore, we fix the lower value of \( M^2 \) in the sum rule window as \( M^2_{\text{min}} = 2.5 \text{ GeV}^2 \). This figure also shows that, although there is a change of sign between dimension-six and dimension-eight condensate contributions,

\[\text{FIG. 1: The OPE convergence in the region } 2.2 \leq M^2 \leq 3.5 \text{ GeV}^2 \text{ for } \sqrt{s_0} = 4.9 \text{ GeV}. \text{ Perturbative contribution (long-dashed line), } \langle g^2 G^2 \rangle \text{ contribution (dashed line), } \langle \bar{q}q \rangle^2 \text{ contribution (dotted-line), } \langle \bar{q}q \sigma G \bar{q}q \rangle \langle \bar{q}q \rangle \text{ (dot-dashed line) and the total contribution (solid line).}\]
as noticed in [19], both contributions are very small and, therefore, they do not spoil the convergence of the OPE. It is also important to mention that the OPE convergence in this case is much better than the OPE convergence for the $X(3872)$ meson [15], and is comparable with the OPE convergence for heavy baryons [20].

To get an upper limit constraint for $M^2$ we impose that the QCD continuum contribution should be smaller than the pole contribution. The comparison between pole and continuum contributions for $\sqrt{s_0} = 4.9$ GeV is shown in Fig. 2. From this figure we see that the pole contribution is bigger than the continuum for $M^2 \leq 3.05$ GeV$^2$. The maximum value of $M^2$ for which this constraint is satisfied depends on the value of $s_0$. The same analysis for the other values of the continuum threshold gives $M^2 \leq 2.85$ GeV$^2$ for $\sqrt{s_0} = 4.8$ GeV and $M^2 \leq 3.25$ GeV$^2$ for $\sqrt{s_0} = 5.0$ GeV. In our numerical analysis, we shall then consider the range of $M^2$ values from 2.5 GeV$^2$ until the one allowed by the pole dominance criteria given above.

In Fig. 3 we show the $Z^+$ meson mass, for $\sqrt{s_0} = 4.9$ GeV, in the relevant sum rule window, with the
upper and lower validity limits indicated. From this figure we see that the results are reasonably stable as a function of $M^2$.

Using the Borel window, for each value of $s_0$, to evaluate the mass of the $Z^+$ meson and then varying the value of the continuum threshold in the range $\sqrt{s_0} = (4.9 \pm 0.1)$ GeV, we arrive at

$$m_Z = (4.40 \pm 0.10) \text{ GeV},$$

in a very good agreement with the experimental value $[4]$.

To check the dependence of our results with the experimental value $[4]$.

We find that the results for the quartic quark condensate, gluon condensate, perturbative contributions. The solid line gives the total OPE contribution to the sum rule.

We can extend our results to the bottom analogous state $Z_{bb}$, considered as a pseudoscalar $B^*B_1$ molecule, by exchanging the charm quark in Eqs. (1) to (11), by the bottom quark. Therefore, in the case of the pseudoscalar meson $Z_{bb}$, using consistently the perturbative $MS$-mass $m_b(m_b) = (4.24 \pm 0.6)$ GeV, and the continuum threshold in the range $11.2 \leq \sqrt{s_0} \leq 11.6$ GeV, we find a good OPE convergence for $M^2 \geq 8.0$ GeV$^2$. The OPE convergence in this case is even better than the one presented in Fig. 1. We also find that the pole contribution is bigger than the continuum contribution for $M^2 \leq 8.25$ GeV$^2$ for $\sqrt{s_0} < 11.2$ GeV, and for $M^2 \leq 9.9$ GeV$^2$ for $\sqrt{s_0} < 11.6$ GeV. For $\sqrt{s_0} < 11.2$ GeV we found no Borel window, since $M^2_{\max} < 8.0$ GeV$^2$.

We find that the results for the $Z_{bb}$ meson mass, in the allowed sum rule window, are very stable as a function of $M^2$. Taking into account the variation of $M^2$ and varying $s_0$ and $m_b$ in the regions indicated we get:

$$m_{Z_{bb}} = (10.74 \pm 0.12) \text{ GeV},$$

in a very good agreement with the prediction in ref. $[21]$.

For completeness, we also predict the mass of the strange analogous meson $Z_s^+$ considered as a pseudoscalar $D^*_sD_1$ molecule. The current is obtained by exchanging the $d$ quark in Eq. (1) by the $s$ quark.

![FIG. 4: The OPE convergence for the sum rule for $Z_s$, using $\sqrt{s_0} = 5.0$ GeV. The solid with triangles, long-dashed, dashed, solid with circles, dot-dashed and dotted lines give, respectively, $m_s$ times the quark condensate, four-quark condensate, gluon condensate, $m_s$ times the mixed condensate, dimension eight condensate and the perturbative contributions. The solid line gives the total OPE contribution to the sum rule.](image)

The expressions obtained in Eqs. (12) for $\rho^{(\bar{q}q)^2}(s)$ and $\Pi^{mix(q\bar{q})}(q^2)$ should be changed to:

$$\rho^{(\bar{q}q)^2}(s) = -\frac{m_s^2(m_b^2 + 4\pi^2)\langle \bar{q}q \rangle B}{4\pi^2} \sqrt{1 - 4m_c^2/s},$$

$$\Pi^{mix(q\bar{q})}(q^2) = \frac{m_s^2 m_b^2 B \langle \bar{q}q \rangle}{2^9 \pi^2} \int_0^1 d\alpha \frac{\alpha(1 - \alpha)}{m_c^2 - \alpha(1 - \alpha)q^2} \left[ 1 + \frac{m_s^2}{m_c^2 - \alpha(1 - \alpha)q^2} \right].$$
We get also two new contributions due to the strange quark mass:

\[ \rho_{\text{ms}}(\bar{q}q)_{(s)} = \frac{3m_s}{2^4\pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \frac{\tilde{B}}{4} \left( \frac{m_s^2 - s\alpha(1-\alpha)^2}{1-\alpha} - \frac{m_s^2(\bar{q}q)}{\beta_{\text{min}}} \right)^{1-\alpha} \int_{\beta_{\text{min}}}^{\beta} d\beta \frac{\tilde{B}(\alpha + \beta)m_s^2 - \alpha\beta s}{\beta} \right], \]

\[ \rho_{\text{ms}}(\bar{q}g\sigma Gq)_{(s)} = \frac{m_s m_0^2}{2^6\pi^6} \sqrt{1 - 4m_c^2/s} \left( \frac{\tilde{B}}{2}(2m_c^2 - s) - 3m_c^2(\bar{q}q) \right). \quad (15) \]

Using \( m_s = (0.13 \pm 0.03) \text{ GeV} \), and the continuum threshold in the range \( \sqrt{s_0} = (5.1 \pm 0.1) \text{ GeV} \) we see, from Fig. 4, that there is a good OPE convergence for \( M^2 \geq 2.5 \text{ GeV}^2 \). From Fig. 4 we also see that, although proportional to \( m_s \), the dimension four condensate \( m_s(\bar{q}q) \) (the solid line with triangles) is the most important condensate contribution.

The upper limits for \( M^2 \) for each value of \( \sqrt{s_0} \) are given in Table I.

| \( \sqrt{s_0} \) (GeV) | \( M^2_{\text{max}} \) (GeV\(^2\)) |
|-------------------------|---------------------|
| 5.0                     | 2.80                |
| 5.1                     | 3.14                |
| 5.2                     | 3.43                |

Table I: Upper limits in the Borel window for \( Z_s \).

In Fig. 5 we show the relative continuum (solid line) versus pole (dashed line) contribution, for \( Z_s \) using \( \sqrt{s_0} = 5.0 \text{ GeV} \), from where we clearly see that the pole contribution is bigger than the continuum contribution for \( M^2 < 2.80 \text{ GeV}^2 \).

\[ \text{FIG. 5: Same as Fig. 2 for } Z_s \text{ using } \sqrt{s_0} = 5.0 \text{ GeV.} \]

In the case of \( Z_s \) we get a remarkable stability for the \( Z_s \) mass, in the allowed sum rule window, as a function of \( M^2 \) as can be seen by Fig. 6.

Taking into account the variations on \( M^2, s_0, m_s \) and \( m_c \) in the regions indicated above we get:

\[ m_{Z_s} = (4.70 \pm 0.06) \text{ GeV}, \quad (16) \]

which is bigger than the \( D_s^*D_1 \) threshold \( \sim 4.5 \text{ GeV} \), indicating that this state is probably a very broad one and, therefore, it might be very difficult to be seen experimentally.

In conclusion, we have presented a QCDSR analysis of the two-point functions of the recently observed \( Z^+(4430) \) meson. Due to the closeness of the \( Z^+(4430) \) mass to the threshold of \( D_s^*D_1(2010)D_s(2420) \), we have followed ref. [7], and have considered the \( Z^+(4430) \) meson as a \( D_s^*D_1 \) molecule. We have also presented a QCDSR study for the analogous mesons \( Z_{bb} \) and \( Z_s \) considered as \( B^*B_1 \) molecule and \( D_s^*D_1 \) molecule respectively. We find very good OPE convergence for these three four-quark mesons, although
FIG. 6: The $Z_s$ meson mass as a function of the sum rule parameter ($M^2$) for $\sqrt{s_0} = 5.0$ GeV. The crosses delimit the region allowed for the sum rule.

this is not in general the case for tetraquark states [6]. We got for $Z^+$ a mass in a very good agreement with the experimental result.

In the case of $Z_s$ we have obtained a mass bigger than the $D^*_sD_1$ threshold. Therefore, our results indicate that the $Z_s$ meson is probably very broad.

Acknowledgements

This work has been partly supported by FAPESP and CNPq-Brazil, and by the Korea Research Foundation KRF-2006-C00011.

[1] BaBar Coll., B. Auber et al., Phys. Rev. Lett. 90, 242001 (2003).
[2] CLEO Coll., D. Besson et al., Phys. Rev. D68, 032002 (2003).
[3] BELLE Coll., S.-K. Choi et al., Phys. Rev. Lett. 91, 262001 (2003).
[4] BELLE Coll., K. Abe et al., arXiv:0708.1700 [hep-ex].
[5] E. S. Swanson, Phys. Rept. 429, 243 (2006).
[6] R.D. Matheus et al., Phys. Rev. D76, 056005 (2007).
[7] C. Meng, K.-T. Chao, arXiv:0708.4222 [hep-ph].
[8] L. Maiani, A.D. Polosa, V. Riquer, arXiv:0708.3997 [hep-ph].
[9] J.L. Rosner, arXiv:0708.4496 [hep-ph].
[10] D.V. Bugg, arXiv:0709.1254 [hep-ph].
[11] M.A. Shifman, A.I. and Vainshtein and V.I. Zakharov, Nucl. Phys. B147, 385 (1979).
[12] L.J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127, 1 (1985).
[13] For a review and references to original works, see e.g., S. Narison, QCD as a theory of hadrons, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. 17, 1 (2002) [hep-ph/0205006]: QCD spectral sum rules, World Sci. Lect. Notes Phys. 26, 1 (1989); Acta Phys. Pol. B26, 687 (1995); Riv. Nuov. Cim. 10N2, 1 (1987); Phys. Rept. 84, 263 (1982).
[14] K.-C. Yang, W.-Y.P. Hwang, E.M. Henley and L.S. Kisslinger, Phys. Rev. D47, 3001 (1993).
[15] R.D. Matheus, S. Narison, M. Nielsen and J.-M. Richard, Phys. Rev. D75, 014005 (2007).
[16] F.S. Navarra, M. Nielsen and S.H. Lee, Phys. Lett. B649, 166 (2007).
[17] B. L. Ioffe, Nucl. Phys. B188, 317 (1981); B191, 591(E) (1981).
[18] S. Narison, Phys. Lett. B466, 345 (1999); S. Narison, Phys. Lett. B361, 121 (1995); S. Narison, Phys. Lett. B387, 162 (1996). S. Narison, Phys. Lett. B624, 223 (2005).
[19] A.G. Oganesian, hep-ph/0510327.
[20] F.O. Durães and M. Nielsen, Phys. Lett. B658, 40 (2007), arXiv:0708.3030 [hep-ph].
[21] K. Cheung, W.-Y. Keung, T.-C. Yuan, arXiv:0709.1312 [hep-ph].
[22] X.-M. Jin, M. Nielsen, Phys. Rev. C51, 347 (1995).