Analysis on sustainable manufacturing criteria of automotive industry

Siti Aisyah Jalaludin¹ and Sumarni Abu Bakar²

¹,²Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Shah Alam, Selangor Darul Ehsan, Malaysia.
E-mail: ¹isyah91@ymail.com, ²sumarni@tmsk.uitm.edu.my

Abstract. In this paper, criteria within the most significant domain of manufacturing in automotive industry in Malaysia is analysed and prioritized. This paper aims to propose a multi criteria decision making (MCDM) procedure using concept of fuzzy theory, graph theory and Laplacian matrix approach which is embedded into Graph Theory and Matrix approach (GTMA). The manufacturing domains are ranked using Laplacian energy while the criteria within the most significant domain is prioritized using method of graph drawing whereby the criteria’s is ranked based on the average distance among other criteria in Cartesian coordinate system. The result indicates that cost, quality and service in economic domain are among significant criteria’s that practitioners should give an attention in developing a quality sustainable manufacturing framework so that the industry could sustain for a longer period of time.

1. Introduction
Sustainable manufacturing is defined as the creation of manufactured products that minimize negative environmental impacts, conserve energy and natural resources, are safe for employees, consumers, and communities, are economically sound [1]. This era, sustainable manufacturing has become a very important issue among governments and industries around the world. There is now a well-recognized need for achieving overall sustainability in industrial activities, arising due to several established and emerging causes: diminishing non-renewable resources, stricter regulations related to environment and occupational safety/health, increasing consumer preference for environmentally friendly products, etc. [2].

As for automotive industry, according to the Malaysian Automotive Association (MAA), a cooling domestic economy and uncertainty over the pace of global recovery slow the Malaysia's automotive industry, which show a decline in sales or production in year-on-year. Vehicle sales declined by 12.4% to 529,434 units in 2020 from 604,281 units of the previous year. The collapse in Malaysia's car market can be attributed to the unpopular passenger car model in the local market. In order not to make the situation worse, the government should seize the opportunity to implement sustainable manufacturing practices to improve the effectiveness, efficiency, and performance of automotive industry. Sustainable manufacturing practices have seen as an effective solution to the automotive industry whereby it supports the continued growth and expansion of the industry [3].

The practice involves development of standard framework. The framework takes into consideration of three domains namely, economic, environmental, and social. Manufacturing industry is considered as the most complex-oriented industries since it utilized many resources from other industries which may leads to practitioners to considering many criteria from each of the domain in developing a good manufacturing framework for the industry to sustain. Because of that reason, researchers come out with different method or approach in order to help practitioners in the selection of the criteria. Some of the MCDM approaches that are used to solve any problem involving decision making with many criteria’s in the automotive industry are Fuzzy Analytical Hierarchy Process (FAHP) [4,5], Analytical Hierarchy Process (AHP) [4,5,6], Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) [4], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [4],
VlseKriterijumskaOptimizacija I KomparomisooResenje (VIKOR) [5], FAHP; FTOPSIS [6], Fuzzy Analytic Network Process (FANP) [7], Analytic Network Process (ANP) [8] and Decision-Making Trial and Evaluation Laboratory (DEMATEL) [8,9]. AHP, TOPSIS and VIKOR are applied with assumption that the criteria is independent to each other even though it is likely to affect each other. Thus, it fails to capture the information of interrelationship between criteria [10]. Although the proposed ANP method cannot reflect the hierarchical interrelationship among criteria. Besides that, DEMATEL method does not provide visualization of interrelationship between criteria. Understanding of interrelationship among the criteria’s is important because changing one parameter from one state to the other state can have multiple outcomes. For example, increasing the manpower can consequently lead to increase in production cost. These dynamic changes need an advanced network analysis tool such as graph theory in order to understand the dynamics involved while analysing variations in parameter adjustment.

Therefore, this study focuses on the evaluation and selection of manufacturing criteria using Graph theory and matrix approach (GTMA) not only to provides practitioner information on significant and important criteria to be included in the framework and strategic planning but to provide a systematic visualisation on the interdependencies among criteria. Graph theory and matrix approach (GTMA) has been successfully employed to model and analyse various MCDM problems [11,12,13]. GTMA helps in identifying criteria’s and offers a better visual appraisal of the criteria’s and their interrelations [11]. The same approach is extended in our present work for selection of significant criteria for sustainable framework in automotive industry application with some modification. Due to the fact that human decision is always involving uncertainty and vague in which decision made by the decision maker may give wrong information of ranking attributes, therefore fuzzy theory is embedded into the GTMA to cater the nature of uncertainty in human decision. Here, fuzzy graph is utilised since it is better than crisp graph since interrelationship between nodes (criteria) is described using fuzzy number which gives extra information of the link. [14].

On the other hand, Laplacian matrix is used to find many useful properties of a graph. Its usefulness has been investigated in transforming non-coordinated Fuzzy Autocatalytic graph (FACS) to coordinated FACS in 2-dimensional space [15] In this work, Laplacian matrix is used in the transformation procedure of embedding the graph that picture interrelation between nodes [15] into 2D Euclidean space. Here, eigenvectors of the graph Laplacian associated with the smallest eigenvalues can be used to construct low dimensional embeddings. Subsequently, significant criteria are ranked through Euclidian distance of the node (criteria) from the origin. The larger the distance value of the node (criteria) from the origin indicates that the node is insignificant. So far, it is believed that none of existing MCDM method used graph embedding technique in determining ranking of alternatives.

2. Materials and Method

2.1 Graph theory and matrix approach (GTMA)

Graph theory and matrix approach (GTMA) is a new technique of decision making [16], which is reasonable and systematic [11,17]. The matrix is useful in analysing diagraph models in easy way which explains the system and problems in numerous science and technology [11]. GTMA methodology has several advantages but the main advantage is it can provide information of interrelationship between criteria by using pair wise comparison matrix which later the interrelationship between criteria is visualised on diagraph representation. The graph could give information of interdependencies of the criteria through visualisation while matrix representation is useful in analysing the directional graphs particularly when numbers of nodes are large, and graphs become complex to visualize.

The fact is that human perception always contains a certain degree of vagueness and ambiguity, makes the traditional GTMA fails to perceive these traits. Therefore, fuzzy set theory is used to deal with the uncertainty and imprecision associated with information on every different parameter, and the situation where only partial information is available. Fuzzy theory has been described as a problem-
solving method which can cater imprecise, vague, and uncertain information to provide final conclusions [14]. Generally, a fuzzy set is defined by its membership function, which represents the degree to which any element x of X has partial membership in M. The degree to which an element belongs to the set is defined by a value ranging from 0 to 1 [18].

One representation of fuzzy number is called triangular fuzzy number (TFN) which can be simply written as \((l, m, u)\) where \(l\) is the lowest possible value, \(m\) is the moderate possible value and \(u\) is the upper possible value describing the fuzzy event. A fuzzy number \(A = (l, m, u)\) is called triangular fuzzy number if its membership function is given by,

\[
\mu_A(x) = \begin{cases} 
0, & x < l \\
\frac{(x - l)}{(m - l)}, & l \leq x \leq m \\
\frac{(u - x)}{(u - m)}, & m \leq x \leq u \\
0, & x > u
\end{cases}
\]  

2.2 Fuzzy Graph

Fuzzy graph was another extension of fuzzy theory’s application in its relation to graph theory. This is because fuzzy theory was designed as a mathematical formalized tool in dealing with imprecise information. Thus, the emergence of fuzzy graph theory could be very beneficial in dealing with graph-theoretic problems which dealing with uncertainties. Definition of fuzzy graph given by Rosenfeld in 1975 is introduced as follows.

**Definition 1 Fuzzy Graph** [19]

Fuzzy graph \(G = (\sigma, \mu)\) is a pair of functions \(\sigma: S \rightarrow [0,1]\) and \(\mu: S \times S \rightarrow [0,1], \ \forall x, y \in S\) we have \(\mu(x,y) \leq \sigma(x) \wedge \sigma(y)\).

Rosenfeld (1975) has generalized the version of graph fuzziness in which he has considered fuzzy graph to consist both fuzzy sets of vertices as well as for the edges. Yeh and Bang (1975), on the other hand, have taken a particular case of fuzzy graph initiated by Rosenfeld (1975) into new definition as follows:

**Definition 2 Fuzzy Graph** [20]

A fuzzy graph \(G\) is a pair \((V, R)\) whereby \(V\) is a set of vertices and \(R\) is a fuzzy set of edges.

2.2.1 Combinatorial Laplacian Matrix of FACS

According to Mohar et al. (1991), Laplacian is considered as an interesting matrix since its spectrum is much more natural and important than adjacency matrix spectrum. Koren (2005) used Laplacian spectrum in graph drawing since the Laplacian spectrum is more fundamental than adjacency spectrum. Further investigation on the Laplacian matrix leads to the formation of Directed Laplacian matrix. Directed Laplacian matrix for aperiodic strongly connected graph was firstly defined by Chung in 2006. The definition is given as follows.

**Definition 3** Laplacian of a directed graph. [23]

The Laplacian of a directed graph \(G\) is defined by

\[
L = I - \frac{1}{2} \left( \phi \frac{1}{2} P \phi^{-\frac{1}{2}} + \phi^{-\frac{1}{2}} P^t \phi^{\frac{1}{2}} \right)
\]

where \(\phi\) is a diagonal matrix with entries \(\phi(v, v) = \phi(v)\) and \(P\) denotes a transition matrix while \(P^t\) denotes the conjugated transpose of \(P\).
On the other hand, Fuzzy Autocatalytic Set (FACS) is a concept that was developed in Tahir et al. (2010) by integrating graph theory, fuzzy theory and Autocatalytic Set. The graph representing FACS have characteristics of aperiodic and strongly connected graph. Since Fuzzy Autocatalytic Set (FACS) is a type of aperiodic strongly connected graph, therefore Definition 3 is used in the formation of Directed Laplacian matrix of Fuzzy Autocatalytic Set (FACS). The new definition for Directed Laplacian matrix of FACS started with the new definition of transition matrix of FACS which is given as follows.

**Definition 4** Transition Matrix of FACS [25]
Suppose $G(V, E)$ is a no loop FACS of Fuzzy Graph Type-3. The transition matrix for fuzzy graph of Type-3 of FACS is $P^*$. $P^*(u, v)$ is fuzzy value represents strength of connection from vertex $u$ to $v$ as

$$P^*(u, v) = \begin{cases} \frac{\mu(u, v)}{d_{out}(u)} & \text{if } (u, v) \in E \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (3)

Thus, the definition of combinatorial Laplacian of FACS is as below:

**Definition 5** Combinatorial Laplacian of FACS [24]
Suppose $P^*$ is the transition matrix for no loop FACS of fuzzy graph Type-3 with $P_{u,v}^*$ represent fuzzy value of moving from $u$ to $v$. Let $I$ denote the all $1$s vector then $P^*1 = 1$. If the graph is strongly connected and aperiodic, then, from Perron Frobenious Theorem, there exist a unique (row) vector, $\varphi$ for which $\varphi P^* = \varphi$ with $\varphi(v) > 0$ for all $v$ and $\sum \varphi(v) = 1$ which is called the Perron vector of $P^*$. Let $\varphi$ the diagonal matrix with $\varphi(v, v) = \varphi(v)$, then directed Laplacian of FACS is

$$L = I - \frac{1}{2} (\varphi P^* \varphi^{-1} + \varphi^{-1} P^* \varphi)$$  \hspace{1cm} (4)

where $(P^*)^*$ denotes the transpose of $P^*$. The combinatorial Laplacian of FACS is defined as:

$$L_C = \varphi - \frac{1}{2} (\varphi P^* + (P^*)^* \varphi)$$  \hspace{1cm} (5)

where $(P^*)^*$ denotes the transpose of $P^*$ and $\varphi$ is the diagonal matrix of the stationary distribution, i.e. $\varphi = \text{diag}(\pi_1, ..., \pi_n)$.

In this study, combinatorial Laplacian of FACS is adopted since the proposed graph is sharing a similar structure with FACS graph.

### 2.2.2 Graph Embedding Method
The procedure to transform the graph into 2D-Euclidean space is adopted from Carmel et al. (2002) which is based on Laplacian matrix and solving a unique one-dimensional optimization problem in determining coordinates of each node. In this study, definition of balanced vector and optimal arrangement of directed graph [23] is adopted to transform the proposed graph into coordinated graph. Definition of balance vector and optimal arrangement for the study are presented as follow.

**Definition 6** Let $G(V, W^*)$ be a digraph where $W^*$ is $n \times n$ matrix of edge weight of $G$ such that $w_{ij} \in R^+$. The balance of the $i^{th}$ node is a vector $b = (b_1, ..., b_n)^t$ such that $b_i = \sum_{j=1}^{n} w_{ij} - \sum_{j=1}^{n} w_{ji}$.

Consequently, $y = (y_1, ..., y_n)$ is determined by using concept of minimization of hierarchy energy function where it is equivalent to solving an optimal arrangement [26]. In this study, the same concept is modified and redefined as follows.
**Definition 7** Let $G(V, W^*)$ be a digraph with Laplacian, $L$ and balance, $b$. Its optimal arrangement, $y^*$, is the solution for $Ly = b$, subject to the constraint $y^T L_n = 0$.

The optimal arrangement $y^*$ is the vector for $y$-coordinates of criteria and is solved by using Conjugate Gradient method. On the other hand, $x = (x_1, ..., x_n)^T$ representing the $x$-coordinates of criteria are solved using minimization of edge squared lengths via Tutte-Hall energy function,

$$TH_E = \frac{1}{2} \sum_{j=1}^{n} w_{ij}(x_i - x_j)^2 = x^T L x$$  \hspace{1cm} (6)

According to Carmel et al., (2002), minimization of this energy function is the Fiedler vector, which is the eigenvector of the Laplacian associated with the smallest positive eigenvalue. In this case, the Fiedler vector of combinatorial directed Laplacian is obtained by solving its eigenvalue problem.

### 3. The Proposed Method and Its Application

Multi-Criteria Decision Making (MCDM) has become one of the most important and active fields of operations research or management science. MCDM represents the process of determining the best solution based on established criteria and common problems in daily life. The MCDM method has been successfully utilized in the sustainable manufacturing and solving the prioritizing problems related to enablers, issues, and indicators [27]. This study provides decision makers with a MCDM for evaluating sustainable manufacturing in the automotive industry. In order to rank automotive criteria using GTMA, a research framework is developed as shown in Figure 1. Following the research framework shown in Figure 1 the goal of ranking the criteria is determined. The criteria’s are ranked based on experts’ opinions using surveys. Experts were asked to perform pair wise comparison of the criteria based on the importance scale. The following steps show the development of an GTMA based model for sustainable manufacturing criteria evaluation in automotive industry. In this study, GTMA approach which consists of a digraph and its associated weighted matrix particularly Laplacian matrix is developed. Laplacian energy value is then calculated. The methodology which involved nine steps is presented as follows.

- Define the Goal. (Ranking of Automotive Criteria)
- Identify significant criteria (based on literature review)
- Plot diagraph for sustainable manufacturing with nodes and edges which represent interdependency
- Normalize the weight for the above matrix.
- Formulate matrix based on survey using pairwise comparison, from industrial experts for weights of interrelationship.
- Convert the diagraph into matrix structure with off-diagonal elements which represent interdependency
- Evaluation of the combinatorial Laplacian matrix
- Determine Laplacian energy based on Laplacian eigenvalues.
- Rank the domain based on Laplacian energy.
- Using Euclidean distance matrix, rank the criteria with respect to goal.
- Embeds the nodes into 2D-Euclidean space, based on Laplacian eigenvector

**Figure 1.** Proposed research framework.
Step 1: Identification the sustainable manufacturing criteria.
This study begins by listing all possible criteria that may be involved in the sustainable manufacturing automotive industry through literature review and selection of the most frequent criteria among the selected influential studies in Malaysia. The criteria must be constructed by adopting the triple bottom line of sustainability consisting of economic, environmental, and social domains. As a result, the sustainable manufacturing criteria consist of three domains divided into twenty four criteria were identified as shown in Table 1.

Table 1. Sustainable manufacturing criteria in automotive industries.

| Domain       | Criteria                      |
|--------------|-------------------------------|
| 1 Economic   | 1 Flexibility                 |
|              | 2 Delivery                    |
|              | 3 Cost                        |
|              | 4 Technology                  |
|              | 5 Production capacity         |
|              | 6 Service                     |
|              | 7 Quality                     |
|              | 8 Technical capability        |
| 2 Environmental | 9 Land Used                  |
|              | 10 Water Usage                |
|              | 11 Energy Usage               |
|              | 12 Raw Material Usage         |
|              | 13 Air Emission               |
|              | 14 Water Emission             |
|              | 15 Land Emission              |
|              | 16 Solid Waste                |
| 3 Social     | 17 Community Satisfaction    |
|              | 18 Drive by Noise             |
|              | 19 Political                  |
|              | 20 Health and Safety          |
|              | 21 Job Opportunities          |
|              | 22 Labour Relation            |
|              | 23 Turnover Rate              |
|              | 24 Training & Development     |

Step 2: Establish fuzzy pairwise comparison matrices of each criterion in every domain.
Here, the general form of the fuzzy pairwise comparison will be as follows:

\[
A = (a_{ij})_{n \times n} = \begin{bmatrix}
0 & (x_{121}, x_{12m}, x_{12u}) & \cdots & (x_{1n1}, x_{1nm}, x_{1nu}) \\
(x_{211}, x_{21m}, x_{21u}) & 0 & \cdots & (x_{2n1}, x_{2nm}, x_{2nu}) \\
\vdots & \vdots & \ddots & \vdots \\
(x_{n11}, x_{n1m}, x_{n1u}) & (x_{n21}, x_{n2m}, x_{n2u}) & \cdots & 0
\end{bmatrix}
\]

where \( a_{ij} = (x_{ij1}, x_{ijm}, x_{iju}) \) and \( a_{ji} = a_{ij}^{-1} (1/x_{ij1}, 1/x_{ijm}, 1/x_{iju}) \) for \( i, j = 1, ..., n \).

Fuzzy pairwise comparison matrices of each criteria are assigned by the expert using linguistic scale of importance. The linguistics scale and its reciprocal scales are represented by triangular fuzzy number as shown in Table 2.
Table 2. Linguistic scales of importance [28].

| Linguistic scale of importance | Triangular fuzzy scale | Triangular fuzzy reciprocal scale |
|--------------------------------|------------------------|----------------------------------|
| Equally important (EI)         | (1, 1, 1)              | (1, 1, 1)                        |
| Intermediate 1 (IM1)           | (1, 2, 3)              | (1/3, 1/2, 1)                    |
| Moderately important (MI)      | (2, 3, 4)              | (1/4, 1/3, 1/2)                  |
| Intermediate 2 (IM2)           | (3, 4, 5)              | (1/5, 1/4, 1/3)                  |
| Important (I)                  | (4, 5, 6)              | (1/6, 1/5, 1/4)                  |
| Intermediate 3 (IM3)           | (5, 6, 7)              | (1/7, 1/6, 1/5)                  |
| Very important (VI)            | (6, 7, 8)              | (1/8, 1/7, 1/6)                  |
| Intermediate 4 (IM4)           | (7, 8, 9)              | (1/9, 1/8, 1/7)                  |
| Absolutely important (AI)      | (9, 9, 9)              | (1/9, 1/9, 1/9)                  |

Step 3: Calculate the fuzzy relative importance or the fuzzy weights for each criteria in every domain.

The fuzzy relative important or the fuzzy weights for each criteria are calculated using arithmetic mean method given by

\[
x_{ijl} = \frac{1}{n} \sum_{k=1}^{n} x_{ijkl}, \quad x_{ijm} = \frac{1}{n} \sum_{k=1}^{n} x_{ijmk}, \quad x_{iju} = \frac{1}{n} \sum_{k=1}^{n} x_{ijuk}
\]  

(8)

where \(x_{ijl} = \) off-diagonal element of \(A\).

\(n = \) the number of decision maker.

\(x_{ijl}^k = \) the relative importance value given by decision maker \(k\) which based on scale in Table 2.

\(l, m\) and \(u\) is referring to left, moderate and upper value of triangular fuzzy number.

There are three fuzzy relative importance matrix for three domain in this study. The fuzzy relative importance of criteria is calculated using a modified approach which is the arithmetic mean in Equation (8) in order to obtain the off-diagonal element of matrix \(A\).

For example (Economic Domain), the off-diagonal elements of \(< x_{ijl}, x_{ijm}, x_{iju} >\) for \(i = 2\) for Delivery (DL), \(j = 1\) for Flexibility (FX)

\[
x_{21l} = \frac{1}{3} \sum_{k=1}^{3} x_{21k}^l = \frac{x_{211}^l + x_{212}^l + x_{213}^l}{3} = \frac{1 + 1/9 + 4}{3} = 1.7037
\]

\[
x_{21m} = \frac{1}{3} \sum_{k=1}^{3} x_{21k}^m = \frac{x_{212}^m + x_{212}^m + x_{212}^m}{3} = \frac{1 + 1/9 + 5}{3} = 2.0370
\]

\[
x_{21u} = \frac{1}{3} \sum_{k=1}^{3} x_{21k}^u = \frac{x_{211}^u + x_{212}^u + x_{213}^u}{3} = \frac{1 + 1/9 + 6}{3} = 2.3704
\]

The arithmetic mean aggregate is (1.7037, 2.0370, 2.3704).
Therefore, the fuzzy relative importance between criteria in the economic domain can be expressed in the matrix form as follows.

\[
A = \begin{bmatrix}
0 & (3.3889,3.4,3.4167) & (2.3333,3.3333,3.3333) & (1.375,2.0476,2.7222) \\
(1.7037,2.0370,2.3704) & 0 & (3.1389,3.1778,3.25) & (0.8148,1.2037,1.7037) \\
(0.2444,0.3333,0.5556) & (2.0370,2.7037,3.3704) & 0 & (2.7778,3.5,4.3333) \\
(2.1778,2.5833,3.1111) & (3.4167,3.7778,4.1667) & (0.4815,0.8751,1.3810) & 0 \\
(0.4556,0.4833,0.5278) & (1.4861,1.8810,2.3889) & (6.6667,7.3333,8) & (4.6667,5.6667,6.6667) \\
(1.4365,1.7809,2.15) & (0.7810,1.1389,1.5111) & (3.4,4.9333,4.7778) & (1.1204,1.4815,1.8704) \\
(7.7,3.3333,7.6667) & (3.7083,4.3810,5.0556) & (2.3704,3.0370,3.7037) & (1.4167,1.7778,2.1667) \\
(4.3333,5.3333,6.3333) & (4.1111,4.5,5.) & (4.4333,3.4,6.6667) & (2.7333,3.4167,4.1111) \\
(2.6667,3.3333,4) & (3.0556,3.7333,4.1667) & (0.1407,0.1574,0.1852) & (0.1958,0.2639,0.4476) \\
(2.3889,3.0667,3.75) & (2.75,3.4444,4.1667) & (2.0926,2.4417,2.7976) & (0.4370,0.7870,1.1481) \\
(0.1296,0.1454,0.1680) & (1.1037,1.4583,1.8254) & (3.1222,3.15,3.1944) & (0.4537,0.4814,0.5370) \\
(0.1667,0.2063,0.2778) & (3.7333,4.0833,4.4444) & (1.0667,1.4167,1.7778) & (1.1111,1.4667,1.8333) \\
0 & (1.1111,1.5,2) & (8.3333,8.6667,9) & (1.0925,1.4370,1.7870) \\
(0.751,1.1111,1.5) & 0 & (0.7333,1.0833,1.4444) & (0.1310,0.1508,0.1778) \\
(0.1111,0.1157,0.1217) & (1.4444,1.8333,2.3333) & 0 & (2.1204,2.4861,2.8810) \\
(4.4475,5.1111) & (5.6667,6.6667,7.6667) & (3.0417,3.7143,4.3889) & 0
\end{bmatrix}
\]

**Step 4:** Develop the weighted directed graph of relative importance among criteria.

The weighted directed graph is developed based on the matrix A obtained in Step 3. The weighted directed graph presents a graphical representation of interrelationship among criteria where the graph consists of nodes \( V = \{ v_i \} \) for \( i = 1,2,3,...,m \) and set of directed edges \( E = \{ e_{ij} \} \) for \( i,j = 1,2,3,...,m \). The number of criteria equal to the number of nodes in digraph model and the edges \( e_{ij} \) represent the relative importance among the criteria. The digraph representation of importance of the criteria to each other is illustrated in Figure 2.

![Figure 2. Digraph representation of interrelationship among criteria.](image)

**Step 5:** Normalized the weights.

The weights of criteria obtained in Step 3 are normalized using Definition 8 in order to ensure that the weights are effectively compared.
**Definition 8** Normalized Adjacency Matrix [29]

The normalized adjacency matrix is

\[
\bar{A}_k = D^{-1/2}AD^{-1/2}
\]

where \( A \) is the adjacency matrix of \( G \) and \( D = \text{diag}\{d_i\} \) is the degree matrix and \( D^{-1/2} \) as follow,

\[
D^{-1/2} = \begin{pmatrix}
\frac{1}{\sqrt{d(1)}} & 0 & \cdots & 0 \\
0 & \frac{1}{\sqrt{d(2)}} & \cdots & 0 \\
\vdots & \vdots & \ddots & 0 \\
0 & 0 & \cdots & \frac{1}{\sqrt{d(n)}}
\end{pmatrix}
\]

For example (Economic Domain):

i) The elements of \( x_{ijl}, x_{ijm}, x_{ijt} \) for \( i = 2 \) for Delivery (DL), \( j = 1 \) for Flexibility (FX)

\[
x_{21l} = D^{-1/2}AD^{-1/2} = \frac{1}{\sqrt{13.3259}} \times 1.7037 \times \frac{1}{\sqrt{13.3259}} = 0.1287
\]
\[
x_{21m} = D^{-1/2}AD^{-1/2} = \frac{1}{\sqrt{16.1583}} \times 2.0370 \times \frac{1}{\sqrt{16.1583}} = 0.1256
\]
\[
x_{21t} = D^{-1/2}AD^{-1/2} = \frac{1}{\sqrt{19.1865}} \times 2.3704 \times \frac{1}{\sqrt{19.1865}} = 0.1225
\]

The normalized the weights is \((0.1287, 0.1256, 0.1225)\).

Therefore, the normalized the weights \( \bar{A} \) in economic can be expressed in the matrix form as follows:

\[
\bar{A} = \begin{pmatrix}
FX & 0 & (0.2559,0.2097,0.1765) & (0.2048,0.2409,0.2623) & (0.1087,0.1337,0.1495) \\
DL & (0.1287,0.1256,0.1225) & 0 & (0.2737,0.2304,0.1984) & (0.064,0.0789,0.0944) \\
CS & (0.0215,0.0241,0.0336) & (0.1776,0.1960,0.2058) & 0 & (0.2536,0.2668,0.2811) \\
FN & (0.1722,0.1687,0.1708) & (0.2685,0.2476,0.2308) & (0.044,0.0672,0.0896) & 0 \\
PC & (0.0257,0.0231,0.0217) & (0.0834,0.0901,0.099) & (0.4349,0.4116,0.3882) & (0.2743,0.2875,0.2935) \\
SV & (0.1370,0.1347,0.1328) & (0.074,0.0861,0.0941) & (0.3745,0.3615,0.3486) & (0.1112,0.1186,0.1238) \\
QL & (0.4527,0.3971,0.3547) & (0.2383,0.238,0.2359) & (0.177,0.1933,0.2025) & (0.0953,0.1023,0.1075) \\
TC & (0.2246,0.2312,0.2348) & (0.2118,0.1957,0.1870) & (0.2394,0.2208,0.2044) & (0.1474,0.1574,0.1633) \\
n & (0.1507,0.1591,0.1643) & (0.2915,0.2812,0.2728) & (0.0091,0.0085,0.0066) & (0.0101,0.0114,0.0166) \\
(0.1341,0.1469,0.1553) & (0.2607,0.2603,0.2596) & (0.1345,0.1327,0.1306) & (0.0225,0.0342,0.0429) \\
(0.0085,0.0082,0.0082) & (0.1216,0.1291,0.1332) & (0.2332,0.2005,0.1746) & (0.0272,0.0245,0.0235) \\
(0.0098,0.0105,0.0122) & (0.3705,0.3268,0.2942) & (0.0718,0.0815,0.0882) & (0.0599,0.0676,0.0729) \\
0 & (0.0788,0.0877,0.0999) & (0.4006,0.3645,0.3339) & (0.0421,0.0464,0.0531) \\
0 & (0.0532,0.065,0.0743) & 0 & (0.0595,0.0719,0.0806) & (0.0085,0.0080,0.0079) \\
0 & (0.0553,0.0490,0.0445) & (0.1172,0.1216,0.1302) & 0 & (0.0935,0.0949,0.0965) \\
0 & (0.1695,0.1599,0.1519) & (0.3687,0.3541,0.3426) & (0.1342,0.1418,0.147) & 0
\end{pmatrix}
\]
Step 6: Develop the Fuzzy Combinatorial Laplacian matrix of the fuzzy graph.
The calculation of fuzzy combinatorial Laplacian matrix for fuzzy graph using Definition 5. There are three fuzzy combinatorial Laplacian matrix for three domain in this study. Result the Fuzzy Combinatorial Laplacian \( L_c \) in economic can be expressed in the matrix form as follows:

\[
L_c = 
\begin{bmatrix}
(0.1288, 0.1234, 0.1212) & (-0.0259, -0.0221, -0.0195) & (-0.0153, -0.0169, -0.0188) & (-0.0181, -0.0202, -0.0218) \\
(-0.025, -0.022, -0.019) & (0.1570, 0.1556, 0.1534) & (-0.0418, -0.04, -0.0378) & (-0.0226, -0.0241, -0.0251) \\
(-0.015, -0.016, -0.018) & (-0.0418, -0.04, -0.0378) & (0.1965, 0.1927, 0.1895) & (-0.0325, -0.0353, -0.0379) \\
(-0.018, -0.020, -0.021) & (-0.0226, -0.0241, -0.0251) & (-0.0325, -0.0353, -0.0379) & (0.1314, 0.141, 0.1483) \\
\end{bmatrix}
\]

Step 7: Defuzzify the fuzzy combinatorial Laplacian matrix of the graph.
Defuzzification is the process of taking the fuzzy outputs and converting them to a single or crisp output value. Let the fuzzy evaluation for criterion \( i \) be \((l_i, m_i, u_i)\) where \( l_i \) is the lowest possible value, \( m_i \) is the moderate possible value and \( u_i \) is the upper probable value. Each fuzzy evaluation of every criterion has been converted into crisp value. The Fuzzy Combinatorial Laplacian matrix of all evaluation criteria are still in the form of triangular fuzzy number and need to defuzzify using Center of Gravity method.

\[
dw_i = \frac{(u_i - l_i) + (m_i - l_i) + l_i}{3} \quad (11)
\]

For example (Economic Domain, the elements of \( <l_i, m_i, u_i> \) for \( i = 2 \) for Delivery (DL)

\[
x_2 = \frac{(u_i - l_i) + (m_i - l_i)}{3} + l_i = \frac{(-0.019 - (-0.025)) + (-0.022 - (-0.025))}{3} + (-0.025) = -0.0225
\]

Therefore, the defuzzify the Fuzzy Combinatorial Laplacian in economic can be expressed in the matrix form as follows:

### Table 3. Defuzzification of Fuzzy Combinatorial Laplacian matrix of the graph for economic domain.

| Criteria               | 1       | 2       | 3       | 4       | 5       | 6       | 7       | 8       |
|------------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| Flexibility            | 0.1244  | -0.0225 | -0.0170 | -0.0200 | -0.0100 | -0.0309 | -0.0205 | -0.0035 |
| Delivery               | -0.0225 | 0.1553  | -0.0399 | -0.0239 | -0.0133 | -0.0288 | -0.0220 | -0.0049 |
| Cost                   | -0.0170 | -0.0399 | 0.1929  | -0.0352 | -0.0107 | -0.0521 | -0.0325 | -0.0055 |
| Technology             | -0.0200 | -0.0239 | -0.0352 | 0.1402  | -0.0076 | -0.0359 | -0.0109 | -0.0067 |
| Production capacity     | -0.0100 | -0.0133 | -0.0107 | -0.0076 | 0.0623  | -0.0088 | -0.0089 | -0.0031 |
| Service                | -0.0309 | -0.0288 | -0.0521 | -0.0359 | -0.0088 | 0.1749  | -0.0134 | -0.0051 |
| Quality                | -0.0205 | -0.0220 | -0.0325 | -0.0109 | -0.0089 | 0.1145  | 0.0064  | -0.0064 |
| Technical capability   | -0.0035 | -0.0049 | -0.0055 | -0.0067 | -0.0031 | -0.0051 | -0.0064 | 0.0354  |
Step 8: Determine the Laplacian Energy.

The calculation of Laplacian energy must include eigenvalue of Fuzzy Combinatorial Laplacian using Definition 9.

Definition 9 Eigenvector and eigenvalue [30]

An eigenvector of a square matrix $A$ is a non-zero vector $v$ that, when the matrix is multiplied by $v$, yields a constant multiple of $v$, the multiplier being commonly denoted by $\lambda$. That is:

$$v = \lambda v$$  \hspace{1cm} (12)

The number $\lambda$ is called the eigenvalue of $A$ and $v$ is said to be an eigenvector corresponding to $\lambda$. It can rewrite the condition as

$$(\lambda I - A)v = 0$$  \hspace{1cm} (13)

That is, the determinant of $\lambda I - A$ must equal to 0. We call $p(\lambda) = \det(A - \lambda I)$ the characteristics polynomial of $A$. The eigenvalues of $A$ are simply the roots of the characteristic’s polynomial of $A$. The eigenvalues and eigenvector are calculated by using MATLAB Software. Let $v_{1}^{*}, ..., v_{n}^{*}$ be the eigenvectors ordered according to their eigenvalues with $v_{1}^{*}$ having the smallest eigenvalue $\lambda_{1}$ (in fact zero). Therefore, the eigenvalues for Fuzzy Combinatorial Laplacian in economic, $\mu_{Li;i}$ for $i = 1, 2, 3, 4, 5, 6, 7, 8$ as follows:

$$\bar{\mu}_{Li;1} = 0; \; \bar{\mu}_{Li;2} = 0.0402; \; \bar{\mu}_{Li;3} = 0.0725; \; \bar{\mu}_{Li;4} = 0.1292$$

$$\bar{\mu}_{Li;5} = 0.1456; \; \bar{\mu}_{Li;6} = 0.1743; \; \bar{\mu}_{Li;7} = 0.1963; \; \bar{\mu}_{Li;8} = 0.2419$$

Laplacian energy is calculated, and the domain of sustainable manufacturing is ranked. In this study, the Laplacian energy value is calculated with the help of MATLAB software. Laplacian energy is calculated based on Definition 10 as follow.

Definition 10 Laplacian Energy [31].

A fuzzy graph $\tilde{G}$ as the underlying set is pair of functions $\tilde{G} = (\sigma, \mu)$ where $\sigma: V \rightarrow [0,1]$ is a fuzzy subset and $\mu: V \times V \rightarrow [0,1]$ is a symmetric fuzzy relation on the fuzzy subset $\sigma$ where for all $u, v \in V$ such that $\mu(u, v) \leq \sigma(u)\sigma(v)$. Let $u$ be a vertex of the fuzzy graph $\tilde{G} = (\sigma, \mu)$. The degree of $u$ is defined as $d_{\mu}(u) = \sum_{v \in V} \mu(u, v)$ that the strength of interdependencies from criteria to another criteria presented vertices. Let $\tilde{G} = (\sigma, \mu)$ be a fuzzy graph with $|V| = n$ vertices and $\bar{\mu}_{1} \geq \bar{\mu}_{2} \geq \cdots \geq \bar{\mu}_{n}$ be the Laplacian eigenvalues of $\tilde{G} = (\sigma, \mu)$ is defined as

$$LE(\tilde{G}) = \left| \bar{\mu}_{i} - \frac{2}{n}(1 \leq i \leq n) \mu(\tilde{G}(\tilde{G})) \right|$$  \hspace{1cm} (14)

For example (Economic Domain), $i = 2$ for Delivery (DL)

$$LE(\tilde{G}) = \left| \bar{\mu}_{i} - \frac{2}{n}(1 \leq i \leq n) \mu(\tilde{G}) \right| = \left| 0.0402 - \frac{2(8.6512)}{8} \right| = 2.1226$$

Therefore, the result Laplacian energy in economic as follows:

$$LE(\tilde{G})_{1} = 2.1628; \; LE(\tilde{G})_{2} = 2.1226; \; LE(\tilde{G})_{3} = 2.0904; \; LE(\tilde{G})_{4} = 2.0336$$

$$LE(\tilde{G})_{5} = 2.0172; \; LE(\tilde{G})_{6} = 1.9885; \; LE(\tilde{G})_{7} = 1.9665; \; LE(\tilde{G})_{8} = 1.9209$$

Sum of Laplacian Energy, $LE(\tilde{G}) = 16.3024$.

Ranking domains are determined with the highest the Laplacian energy value, the more preferred the domain. This rank will order the highest to lowest. Table 4 shows that the economic domain is the first
rank followed by social and the last is environmental. Next, the economic domain more preferred than
analysed to determine which criteria are preferred in the economic domain.

Table 4. Ranking the Laplacian Energy for each domain.

| Domain       | Laplacian Energy | Ranking |
|--------------|------------------|---------|
| Economic     | 16.3024          | 1       |
| Environmental| 15.2164          | 3       |
| Social       | 15.3367          | 2       |

Step 9: Embeds the nodes into 2D-Euclidean space and determine the rank of criteria based on Euclidean distance.

The image of \( x_i \) embedded into \( k \) domanial space is given by \( y^* = [v_2^*, ..., v_{k+1}^*] \) [30]. Based on Definition 6 and Definition 7, y-coordinates of the nodes criteria of \( G(V,E) \) is given by \( y^* \). Herewith, solve eigenvalue problem of \( L \) to get the Fiedler vector, \( x = (x_1, ..., x_n)^T \) which represents x-coordinates of the nodes criteria [26]. It solved by using eigenvector of the Laplacian associated with the smallest positive eigenvalue. Positioned the nodes in 2D-Euclidean space using \( (x, y) \) coordinate obtained and draw its corresponding edges.

The Euclidean distance between criteria is calculated using Equation (15). The Euclidian distance matrix shows the distance between pair of criteria. By definition, criteria’s distance from itself, which is shown in the main diagonal of the matrix is zero.

**Definition 11 Euclidean Distance Matrix [30].**

A Euclidean matrix on the real \( n \) dimensional vector space is the usual distance matrix.

\[
d(x, y) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]  

(15)

For example (Economic Domain), Delivery (DL)

\[
d(x, y) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-0.1272 - (-0.1410))^2 + (0.1284 - 0.1677)^2} = 0.0417
\]

Therefore, the Euclidean distance between criteria in economic as follows:

Table 5. Euclidean distance between criteria.

| Nodes, \( v_i \) | \( v_1 \) (Flexibility) | \( v_2 \) (Delivery) | \( v_3 \) (Cost) | \( v_4 \) (Technology) | \( v_5 \) (Production capacity) | \( v_6 \) (Service) | \( v_7 \) (Quality) | \( v_8 \) (Technical capability) |
|-----------------|-------------------|-------------------|----------------|----------------------|-------------------------------|----------------|----------------|----------------------|
| \( v_1 \) (Flexibility) | 0.0000 | 0.0417 | 0.0234 | 0.0363 | 1.0821 | 0.0179 | 0.0304 | 1.0992 |
| \( v_2 \) (Delivery) | 0.0417 | 0.0000 | 0.0293 | 0.0603 | 1.0437 | 0.0440 | 0.0442 | 1.0779 |
| \( v_3 \) (Cost) | 0.0234 | 0.0293 | 0.0000 | 0.0312 | 1.0726 | 0.0158 | 0.0158 | 1.0767 |
| \( v_4 \) (Technology) | 0.0363 | 0.0603 | 0.0312 | 0.0000 | 1.1026 | 0.0197 | 0.0168 | 1.0729 |
| \( v_5 \) (Production capacity) | 1.0821 | 1.0437 | 1.0726 | 1.1026 | 0.0000 | 1.0877 | 1.0858 | 1.4131 |
| \( v_6 \) (Service) | 0.0179 | 0.0440 | 0.0158 | 0.0197 | 1.0877 | 0.0000 | 0.0133 | 1.0833 |
| \( v_7 \) (Quality) | 0.0304 | 0.0442 | 0.0158 | 0.0168 | 1.0858 | 0.0133 | 0.0000 | 1.0700 |
| \( v_8 \) (Technical capability) | 1.0992 | 1.0779 | 1.0767 | 1.0729 | 1.4131 | 1.0833 | 1.0700 | 0.0000 |
| Average distance | 0.2914 | 0.2926 | 0.2831 | 0.2925 | 0.9860 | 0.2852 | 0.2845 | 0.9866 |
4. Result and Discussion

Pairwise comparison for each criterion is established using combination of three mathematical concept that is fuzzy theory, graph theory and Laplacian matrix approach based on GTMA. The ranking domain it can be identified that economic sustainability is the first rank followed by social and the lastly is environmental. Next, the economic domain more preferred then analysed to determine which criteria are preferred in the economic domain. In this study, for each criterion can be transformed into 2D-Euclidean space as in Figure 3. Every edge of the graph in Figure 2 is associated to a membership value for fuzzy edge connectivity whereby no information on location of each node in this graph. In contrast, every edge of the graph in Figure 3 provides not only information on membership value, it also gives the information of its length. Here, every node has its own coordinate, and the location of every node is given in Table 6.

Table 6. Location of criteria (node).

| $x_i$ | Criteria              | Nodes, $v_i$ | $x$-coordinates | $y$-coordinates |
|-------|-----------------------|--------------|-----------------|----------------|
| 1     | Flexibility           | $v_1$        | -0.1410         | 0.1677         |
| 2     | Delivery              | $v_2$        | -0.1272         | 0.1284         |
| 3     | Cost                  | $v_3$        | -0.1203         | 0.1568         |
| 4     | Technology            | $v_4$        | -0.1098         | 0.1861         |
| 5     | Production capacity   | $v_5$        | -0.1998         | -0.9128        |
| 6     | Service               | $v_6$        | -0.1237         | 0.1722         |
| 7     | Quality               | $v_7$        | -0.1107         | 0.1693         |
| 8     | Technical capability  | $v_8$        | 0.9327          | -0.0676        |

![Figure 3. Location of node in 2D-Euclidean space.](image)

Table 7. Ranking criteria based on Average Euclidean distance.

| Criteria            | Nodes, $v_i$ | Average Euclidean distance | Rank Euclidean distance |
|---------------------|--------------|----------------------------|-------------------------|
| Flexibility         | $v_1$        | 0.2914                     | 4                       |
| Delivery            | $v_2$        | 0.2926                     | 6                       |
| Cost                | $v_3$        | 0.2831                     | 1                       |
| Technology          | $v_4$        | 0.2925                     | 5                       |
| Production capacity | $v_5$        | 0.9860                     | 7                       |
| Service             | $v_6$        | 0.2852                     | 3                       |
| Quality             | $v_7$        | 0.2845                     | 2                       |
| Technical capability| $v_8$        | 0.9866                     | 8                       |

Table 7 shows the lowest average Euclidean distance are 0.2831 which representing Cost criteria. This is show that Cost criteria is the most preferred followed by Quality criteria. Service is ranked in third place followed by the Flexibility, Technology, Delivery and Production capacity criteria based on the preference. The larger Euclidean distance are 0.9866 refers to technical capability. Figure 3
indicates that \((v_5)\) and \((v_8)\) which are positioned at considerable distance away from the other variables and \((v_4), (v_1), (v_3), (v_2)\) and \((v_7)\) which is closely located to each other in the graph. This information indicates that outlying variables shown in the graph can be determined from the largest value of the criteria in the Euclidean distance. Since larger value of the entries in the Euclidean distance is correspond to the least importance variable. Subsequently, the outlying variable of coordinated indicate the ‘most depleting’ variables. The sequences of ranking the criteria using Euclidean distance is Cost > Quality > Service > Flexibility > Technology > Delivery > Production capacity > Technical capability.

5. Conclusion
This paper has proposed a new multi criteria decision making using fuzzy theory, graph theory and Laplacian matrix approach based on GTMA method in the evaluation of sustainable manufacturing criteria in the automotive industry. The network relationship model is constructed using the GTMA method. The weights of the criteria sustainable manufacturing are assigned through pairwise comparison and calculated using fuzzy combinatorial Laplacian matrix approach. The combination of the three mathematical concept of fuzzy theory, graph theory and Laplacian matrix approach based on GTMA method will provide a better understanding of the interrelationships between the criteria and help solve a complex evaluation problem, so that it can enhance the quality of decision making. The model enables and assists companies to give an opportunity for the researcher and decision maker to do an analysis of criteria that related with automotive sustainability. This study also shown that cost, quality and service in economic domain are among significant criteria’s that a company should give priority in developing and formulating a comprehensive standard framework of planning, improving and thus make the company a sustainable manufacturing company in future.

References
[1] Jasiulewicz-Kaczmarek M 2013 The role and contribution of maintenance in sustainable manufacturing. In: Manufacturing Modelling, Management, and Control, 1146-1151.
[2] Jayal A D, Badurdeen F, Dillon O W, and Jawahir I S 2010 Sustainable manufacturing: modelling and optimization challenges at the product, process, and system levels. CIRP Journal of Manufacturing Science and Technology, 2(3),144–152.
[3] Habidin N F, Zubir A F M, Fuzi N M, Latip N A M, and Azman M N A 2015 Sustainable manufacturing practices in Malaysian automotive industry: confirmatory factor analysis. Journal of Global Entrepreneurship Research, 5,14.
[4] Jamil N, Besar R, and Sim H K 2013 A study of multicriteria decision making for supplier selection in automotive industry. Journal of Industrial Engineering, 2013.
[5] Nazihah A, Maznah M K, and Shanmugam K R S 2019 Comparative analysis of crisp and fuzzy multicriteria decision-making methods for supplier selection in an automotive manufacturing industry. International Journal Supply Chain Management 8(1), 951-957.
[6] Dweiri F T, Kumar S, Khan S A, Jain V 2016 Designing an integrated AHP based decision support system for supplier selection in automotive industry. Expert Systems with Applications, 62, 273-283.
[7] Dargi A, Anjomshoae A, Galankashi M R, Memari A, Tap M M D 2014 Supplier selection: A Fuzzy-ANP approach. Procedia Computer Science, 31,691 – 700.
[8] Aghae M and Fazli S 2012 An improved MCDM method for maintenance approach selection: A case study of auto industry. Management Science Letters, 2, 137–146.
[9] Alireza I, Mahdi G, Mehdi H, and Nasrin S 2012 Evaluation of the most effective criteria in green supply chain management in automotive industries using the fuzzy DEMATEL method. Journal of Basic and Applied Scientific Research, 2(9), 8952-8961.
[10] Rao R V and Padmanabhan K 2006 Selection, identification and comparison of industrial robots using digraph and matrix methods. Robotics and Computer Integrated
Manufacturing, 22(4), 373-383.

[11] Rao R V 2007 Decision making in the manufacturing environment: using graph theory and fuzzy multiple-criteria decision making methods. *Landon: Springer.*

[12] Moktadir M A, Rahman T, Rahman M H, Ali S M and Paul S K 2018 Drivers to sustainable manufacturing practices and circular economy: A perspective of leather industries in Bangladesh. *Journal of Cleaner Production, 174*, 1366-1380.

[13] Sumarni A B, Ainy N H, Kahartini A R., Agos M S N, Herniza M T and Elsie J 2019 Application of graph theory and matrix approach as decision analysis tool for smartphone selection. *ASM Sci. J., Special Issue 6*, 53-59.

[14] Dweiri F T and Kablan M M 2006 Using fuzzy decision making for the evaluation of project management internal efficiency. *Decision Support Systems, 42(2)*, 712-726.

[15] Tahir A, Sumarni A B, Sabariah B and Faisal A M B 2015 Coordinated transformation for fuzzy autocatalytic set of fuzzy graph type-3. *Journal of Mathematics and Statistics, 11(4)*, 119-127.

[16] Malik S, Kumari A and Agrawal S 2015 Selection of locations of collection centres for reverse logistics using GTMA. *Materials Today: Proceedings, 2(4-5)*, 2538-2547.

[17] Geetha N K and Sekar P 2016 Graph theory matrix approach in selecting optimal combination of operating criteria. *International Journal for Modern Trends in Science and Technology, 02(11)*, 170-175.

[18] Kahraman C, Ruan D and Dogan I 2003 Fuzzy group decision making for facility location selection. *Information Sciences, 157*, 135-153.

[19] Rosenfeld A 1975 Fuzzy graphs. In Zadeh L A, Fu K S and Shimura M Eds. Fuzzy sets and their applications to cognitive and decision processes. *New York: Academic Press.*

[20] Yeh R T and Bang S Y 1975. Fuzzy relations, fuzzy graphs and their applications to clustering analysis. In Zadeh L A, Fu K S and Shimura M Eds. Fuzzy sets and their applications to cognitive and decision processes. *New York: Academic Press.*

[21] Mohar B, Alavi Y, Chartrand G and Oellermann O R 1991 The Laplacian spectrum of graphs. *Graph theory, combinatorics, and applications, 2(871-898)*, 12.

[22] Koren Y 2005 Drawing graphs by eigenvectors: theory and practice. *Computers & Mathematics with Applications, 49(11-12)*, 1867-1888.

[23] Chung F 2006 The diameter and Laplacian eigenvalues of directed graphs. *Electronic Journal of Combinatorics, 13*, 4-6.

[24] Tahir A, Sabariah B and Sumarni A B 2010 Directed Laplacians for fuzzy autocatalytic set of fuzzy graph type-3 of an incineration process. *AIP Conference Proceedings, 1309*, 112.

[25] Sumarni A B, Tahir A and Sabariah B 2010 Transition probability matrix for fuzzy autocatalytic set of fuzzy graphs type-3. *Journal of Advances and Applications in Mathematical Sciences, MILI Publications, India, 5, 1*, 25-38.

[26] Carmel L, Harel D and Koren Y 2002 Drawing directed graphs using one-dimensional optimization, *LNCS Vol 2528*, Springer-Verlag, London.

[27] Deshmukh R A and Hiremath R 2019 Analyzing the key performance indicators of advanced sustainable manufacturing system using AHP approach. *In Techno-Societal*, 745-750.

[28] Zou X and Lu M 2012 Risk evaluation of dynamic alliance based on fuzzy analytic network process and fuzzy TOPSIS. *Journal of Service Science and Management (5)*, 3.

[29] Lau L C, Lee Y T, Gharan S O and Trevisan L 2013 Improved Cheeger’s inequality: analysis of spectral partitioning algorithms through higher order spectral gap. *Proceedings of the forty-fifth annual ACM symposium on Theory of computing*; 11-20.

[30] Anton H and Rorres C 2011 *Elementary linear algebra with supplemental applications.* 10th Edition New York. John Wiley and Sons, Inc

[31] Gutman I and Zhou B 2006 Laplacian energy of a graph. *Linear Algebra Application, 414*, 29-37.