Gravitational collapse to a Kerr–Newman black hole

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1 INTRODUCTION

The rotating black hole (Kerr) solution of the Einstein equations in vacuum and axisymmetric space–times is a fundamental block in relativistic astrophysics and has been studied in an enormously vast literature for its mathematical and astrophysical properties. On the other hand, the rotating and electrically charged black hole (Kerr–Newman, KN hereafter) solution of the Einstein–Maxwell equations in axisymmetric space–times, while equally well studied for its mathematical properties, is also normally disregarded as astrophysically relevant. The rationale is that if such an object was indeed created in an astrophysical scenario, then the abundant free charges that accompany astrophysical plasmas would neutralize it very rapidly, yielding therefore a standard Kerr solution.

Yet, KN black holes continue to be considered within astrophysical scenarios to explain, for instance, potential electromagnetic counterparts to merging stellar-mass binary black hole systems (Liebling & Palenzuela 2016; Liu et al. 2016; Zhang 2016). We here take a different view and do not explore the phenomenology of KN black holes when these are taken to be long-lived astrophysical solutions. Rather, we are interested to determine how such black holes are produced in the first place as, for instance, in the collapse of rotating and magnetized stars. We note that even if these solutions are short-lived astrophysically (Contopoulos, Nathanail & Pugliese 2014; Punsly & Bini 2016), the study of their genesis can provide useful information and shed light on some of the most puzzling astronomical phenomena of the last decade: fast radio bursts (FRBs; Lorimer et al. 2007; Thornton et al. 2013). FRBs are bright, highly dispersed millisecond radio single pulses that do not normally repeat and are not associated with a known pulsar or gamma-ray burst. Their high dispersion suggests that they are produced by sources at cosmological distances and thus with an extremely high radio luminosity, far larger than the power of single pulses from a pulsar. The event rate is also estimated to be very high and of a few per cent that of supernovae explosions, making them very common. Several theoretical models have been proposed over the last few years, but the ‘blazar’ model (Falcke & Rezzolla 2014) is particularly relevant for our exploration of the formation of KN black holes.

We recall that if a neutron star exceeds a certain limit in mass and angular momentum, it will reach a state in which it cannot support itself against gravitational collapse to a black hole. It is also widely accepted that rotating magnetized neutron stars emitting pulsed radio emission, i.e. pulsars, spin down because of electromagnetic energy losses and could therefore reach the stability line against collapse to a black hole. During the collapse of such a pulsar, an apparent horizon is formed, which will cover all the stellar matter, while the magnetic-field lines will snap violently launching an intense electromagnetic wave moving at the speed of light. Free electrons will be accelerated by the travelling magnetic shock, thus dissipating a significant fraction of the magnetosphere energy into coherent electromagnetic emission and hence produce a massive radio burst that could be observable out to cosmological distances (Falcke & Rezzolla 2014).

One aspect of this scenario that has not yet been fully clarified is the following: does the gravitational collapse of a rotating magnetized neutron star lead to a KN black hole? The purpose of this Letter is to provide an answer to this question and to determine, through numerical-relativity simulations, the conditions under which a collapsing pulsar will lead to the formation of a Kerr or a KN black

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hole. In particular, we show that when using self-consistent initial
data representing an unstable rotating and magnetized neutron star
in general relativity, the consequent collapse yields a black hole that
has all the features expected from a KN black hole. In particular, the
space–time undergoes a transition from being magnetically domi-
nated before the collapse to being electrically dominated after black
hole formation, which is indeed a key feature of a KN black hole.
We further provide evidence by carefully analysing the Weyl scalar
ψ2 and by showing that the black hole space–time possesses a net
electric charge and a behaviour, which is the one expected for a KN
black hole. These results will be contrasted with those coming from
the gravitational collapse of a non-rotating magnetized neutron star,
where the outcome is an uncharged non-rotating (Schwarzschild)
black hole.

The plan of the Letter is the following one. In Section 2, we briefly
review the numerical setup and how the initial data are computed,
leaving the analysis of the numerical results in Section 3. Finally,
the discussion of the astrophysical impact of the results and our
conclusions are presented in Section 4.

2 NUMERICAL SETUP AND INITIAL DATA

All simulations presented here have been performed employing the
general-relativistic resistive magnetohydrodynamics (MHD) code
WHISKYRMHD (Dionysopoulou et al. 2013; Dionysopoulou, Alic
& Rezzolla 2015), which uses high-resolution shock capturing meth-
ods like the Harten-Lax-van Leer-Einfeldt approximate Riemann
solver coupled with effectively second-order piece-wise parabolic
reconstruction. Differently from the implementation reported in the
references (Dionysopoulou et al. 2013, 2015), we reconstruct our
primitive variables at the cell interfaces using the enhanced piece-
wise parabolic reconstruction (Colella & Sekora 2008; Reisswig
et al. 2013), which does not reduce to first order at local maxima.
Also, we opt for reconstructing the quantity Wνi, where W is the
Lorentz factor, instead of the three-velocity νi; this choice enforces
subluminal velocities at the cell interface. Regarding the electric-
field evolution, we choose not to evolve the electrical charge q di-
rectly through an evolution equation, and instead compute q = ∇·Ei
at every time-step, as it has been done by Dionysopoulou et al.
(2013) and Bucciantini & Del Zanna (2013). The WHISKYRMHD code
exploits the Einstein Toolkit, with the evolution of the space–time
obtained using the MCLACHLAN code (Löffler et al. 2012), while
the adaptive mesh refinement is provided by CARPET (Schnetter, Hawley
& Hawke 2004).

The use of a resistive-MHD framework has the important ad-

cantage that it allows us to model the exterior of the neutron star
as an electrovacuum, where the electrical conductivity is set to be
negligibly small, so that electromagnetic fields essentially evolve
according to the Maxwell equations in vacuum. These are the phys-

ic conditions that are expected for a pulsar that has passed its
‘death line’, that is, one for which either the slow rotation or a
comparatively weak magnetic field are such that it is not possible
to trigger pair creation and its magnetosphere can be well approx-
imated as an electrovacuum (Chen & Ruderman 1993).\footnote{We recall
that the voltage drop ΔV along magnetic field lines needed for
the creation of pairs scales with the magnetic field B and rotation frequency
Ω simply as ΔV ~ BΩ (Ruderman & Sutherland 1975).} At the
same time, the resistive framework also enables us to model the
interior of the star as highly conducting fluid, so that our equations
recover the ideal-MHD limit (i.e. infinite conductivity) in such re-

gions. WHISKYRMHD achieves this by including a current that is valid
both in the electrovacuum and in the ideal-MHD limit, where it
becomes stiff, however. To accurately treat such a current, the code
uses an implicit–explicit Runge–Kutta time stepping (RKIMEX
(Pareschi & Russo 2005). For more details on the numerical setup,
we refer the interested reader to Dionysopoulou et al. (2013, 2015).

Our initial data are produced using the MAGSTAR code of the
LORENE library, which can compute self-consistent solutions of the
Einstein–Maxwell equations relative to uniformly rotating stars
with either purely poloidal (Bocquet et al. 1995) or toroidal mag-
netic fields (Frieben & Rezzolla 2012); hereafter, we will consider
only poloidal magnetic fields of dipolar type. We have consid-
ered a number of different possible configurations for the electric
field with the aim of minimizing the amount of external electric
charges. In practice, the smallest external charge has been achieved
when prescribing a corotating interior electric field matched to a
divergence-free electric field produced by a rotating magnetized
sphere. More precisely, we set the electric field in the stellar interior
using the ideal-MHD condition, i.e. Ei = −√(εij/ε)|v|/c Bj, where
v|/c is the corotation velocity, γ the three-metric determinant and
εijk the totally anti-symmetric permutation symbol. This field is
then matched to an exterior electrovacuum solution for a magne-
tized and rotating uncharged sphere in general relativity (Rezzolla,
Ahmedov & Miller 2001, 2003). Note that because the analytic solu-
tion is obtained in the slow-rotation approximation, which assumes
a spherical star, a small mismatch in the electric field is present near
the pole. Furthermore, monopolar and quadrupolar terms are added
to the solution so as to match the charge produced by the corotating
interior electric field following Ruffini & Treves (1973).

Also, as customary in this type of simulations, the stellar exter-

ior is filled with a very low-density fluid, or ‘atmosphere’, whose
velocity is set to be zero (Dionysopoulou et al. 2013); at the same
time, and from an electrodynamical point of view, we treat such
a region as an electrovacuum, so that the electrical conductivity is
set to zero. This has the important consequence that the magnetic
fields are no longer frozen in the atmosphere and are therefore free
to rotate following the stellar rotation if one is present.

Our reference rotating model is represented by a neutron star
with gravitational mass of M = 2.104 M⊙, a period of P = 1.25 ms
(or 800 Hz), and a central (and maximum) magnetic field of 1015 G;
for such a model, the light cylinder is at about 60 km from the
origin. The corresponding reference non-rotating model has instead
a gravitational mass of M = 2.100 M⊙ and the same magnetic field
of 1015 G. Finally, we will also consider a model with the same
properties as the rotating one, but with zero magnetic field.
All models are constructed from a single polytrope p = Kρ7 with
Γ = 2. The polytropic constant K = 164.708 has been adjusted so
that the maximum mass of a non-rotating star is limited to about
2.1 M⊙. The evolution is however performed using an ideal-fluid
equation of state p = ρε(Γ − 1), where ε is the specific internal
energy. In spite of using a very simplified equation of state, we
do not expect this to have any effect on the results of this Letter
since we are merely interested in a prompt collapse to a black
hole.

An important issue to discuss at this point is whether or not the star
possesses initially a net electrical charge. As it happens, the standard
solution provided by MAGSTAR does have a net charge, although we
decided not to use such a solution as it is not the one leading to
the smallest external charge. At the same time, it is reasonable to
expect that the strong electromagnetic fields in a pulsar will not only
generate a charge separation, but they will also lead naturally to a

\footnote{We recall that the voltage drop ΔV along magnetic field lines needed for
the creation of pairs scales with the magnetic field B and rotation frequency
Ω simply as ΔV ~ BΩ (Ruderman & Sutherland 1975).}
net charge. Assuming that the rotating neutron star is endowed with a dipolar magnetic field aligned with the rotation axis and that it is surrounded by an ionized medium, it will induce a radial electric field (Cohen, Kegeles & Rosenblum 1975; Michel & Li 1999).

\[ E' = B \frac{\Omega R \sin \theta}{c} \approx B \frac{\Omega R}{c} \sin^2 \theta, \]

where \( B \) is the equatorial value of the dipole magnetic field as measured by a non-rotating observer, while \( \Omega \) and \( R \) are the angular velocity and the radius of the star, respectively. As a result, the net electric charge can be computed as

\[ Q = \int_0^{\Omega \pi} 2\pi R^2 \sin \theta E' \, d\theta \approx \frac{8\pi}{3c} R^3 \Omega B. \]

In the stellar interior, this charge is distributed so as to satisfy the infinite-conductivity condition \( \mathbf{E} \cdot \mathbf{B} = 0 \) everywhere. Stated differently, having a net charge is not necessarily unrealistic, at least in this simplified model (see also Péri 2012 and Péri 2016); for our choice of initial stellar model, equation (2) would yield \( Q \approx 2.6 \times 10^{17} \) C.

We should also remark that the values we have chosen above for the magnetic field and spin frequency are untypically high for a pulsar past the death line. However, they are chosen to maximize the initial charge in order to stabilize the numerical evolution and aid the final determination of the charge from numerical noise. It is also simple to check that a neutron star with such magnetic field and rapid rotation is far from electrovacuum in its magnetosphere. However, we believe that this does not affect the general outcome of our simulations, which should be viewed as a proof of concept. Finally, our numerical grid consists of seven refinement levels extending to about 1075 km, with a finest resolution of 147 m. Additional runs with resolutions of 184, 220 m have been performed to test the consistency of the results, but we here discuss only the results of the high-resolution runs.

3 NUMERICAL RESULTS AND ANALYSIS

Overall, the gravitational collapse of our stellar models follows the dynamics already discussed in detail by Dionysopoulou et al. (2013) (see also Baumgarte & Shapiro 2003 and Lehner et al. 2012 for different but similar approaches), and the corresponding electromagnetic emission under a variety of conditions will be presented by Most, Nathanail & Rezzolla (in preparation). We here focus our attention on comparing and contrasting the collapse of the magnetized rotating and non-rotating models. Both stars are magnetized, but only the rotating model possesses also an electric field induced by the rotation; as a consequence of the presence/absence of this initial electric field, the rotating/non-rotating star is initially charged/uncharged.

Since the dynamics is rather similar in the two cases (the magnetic fields and rotation speeds are still a small portion of the binding energy), the differences between the two collapses are best tracked by the electromagnetic energy invariant

\[ F_{\mu\nu} F^{\mu\nu} = 2(B^2 - E^2), \]

where \( F^{\mu\nu} \) is the Faraday tensor, while \( B \) and \( E \) are, respectively, the magnetic and electric field three-vectors measured by a normal observer. Being an invariant, the quantity (3) is coordinate independent and can provide a sharp signature of the properties of the resulting black hole. We recall that equation (3) is identically zero for a Schwarzschild or a Kerr black hole, while it is negative in the case of a KN black hole (Misner, Thorne & Wheeler 1973).

Fig. 1 summarizes the dynamics and outcome of the gravitational collapse by showing as colour code the values of the energy invariant \( F_{\mu\nu} F^{\mu\nu} \) at three representative times (the initial one, the final one and an intermediate stage). The top row, in particular, refers to the non-rotating (but magnetized) stellar model, while the bottom row shows the evolution in the case of the model rotating at 800 Hz; also shown are the magnetic field lines.

What is simple to recognize in Fig. 1 is that both the rotating and non-rotating stars start being magnetically dominated, i.e. with \( F_{\mu\nu} F^{\mu\nu} > 0 \) (top and bottom left panels). However, while the collapse of the magnetized non-rotating star leads to a Schwarzschild black hole for which \( F_{\mu\nu} F^{\mu\nu} \approx 0 \) (top right panel), the collapse of the magnetized rotating star yields an electrically dominated black hole, i.e. with \( F_{\mu\nu} F^{\mu\nu} < 0 \) (bottom right panel). Furthermore, while electrically dominated regions are produced in both collapses, these are radiated away in the case of a non-rotating star (see also Dionysopoulou et al. 2013), in contrast with what happens for the rotating star (cf. blue regions in Fig. 1). Also worth remarking in Fig. 1 is that the magnetic field at the end of the simulation becomes essentially uniform and extremely weak (not shown, but see Most et al., in preparation) in the case of the non-rotating model, while it asymptotes to a dipolar magnetic-field configuration in the rotating case. Note that this field does not seem to have a neutral point. This is consistent with the magnetic-field geometry of a KN black hole (Pekeris & Frankowski 1987), where it can be imagined that the dipolar field is generated by a ring-like current at the location of the ring singularity of the corresponding Kerr black hole; our time and spatial gauges prevent the appearance of such singularity and push it...
to the origin of the coordinates. As discussed in the previous section, the initial data contain a charge density also in the stellar exterior, so that the overall charge in the computational domain is given by the sum of the stellar charge and of the exterior one; hereafter, we refer to this charge as to $Q_{\text{tot}}$. Shown in the two panels of Fig. 2 is the electrical charge distribution at the initial time (left-hand panel) and at the end of the simulation (right-hand panel), together with the magnetic-field lines, the location of the stellar surface (white solid and dashed lines in the left- and right-hand panels, respectively) and of the apparent horizon (red solid line in the right-hand panel). Note that the initial charge density falls off very rapidly with distance from the stellar surface and that after a black hole has been formed, the charge is mostly dominated by very small values with alternating signs; this behaviour is very similar to the one observed when collapsing a non-rotating (uncharged) star and hence indicates that the charge distribution in the right-hand panel is very close to the discretization error.

The presence of an external charge complicates the calculation of the charge of the final black hole. In fact, when computing the total electric charge as a surface integral of the normal electric field, $Q_{\text{tot}}$, which is well conserved.

At this point it is not difficult to estimate the ‘external’ charge $Q_{\text{out}}$ by subtracting the electrical charge trapped inside the event horizon from the total one, i.e. $Q_{\text{out}} := Q_{\text{tot}} - Q_{\text{BH}} \approx -0.46 \times 10^{-4} M_{\odot} \sim -5.31 \times 10^{15} \text{C}$. Note that $Q_{\text{out}}$ is smaller than $Q_{\text{in}}$, but not much smaller and while $Q_{\text{in}}$ is mostly positive, $Q_{\text{out}}$ is mostly negative and present across the computational domain. While the precise value we obtain for $Q_{\text{out}}$ depends sensitively on the initial electric field, the overall order of magnitude of the charge is robust, as we discuss below. We can in fact validate that the space–time produced by the collapse of the rotating and magnetized star is indeed a KN space–time by considering a completely different gauge-invariant quantity that is not directly related to electromagnetic fields, but is instead a pure measure of curvature. More specifically, for a KN black hole of mass $M_{\text{BH}}$, the only non-vanishing Weyl scalar $\psi_2(r, \theta)$ is given by (Newman & Adamo 2014)

$$\psi_2 = -\frac{M_{\text{BH}}}{(r - ia \cos \theta)^3} + \frac{Q_{\text{BH}}^2}{(r + ia \cos \theta)(r - ia \cos \theta)^3}. \tag{4}$$

This expression simplifies considerably on the equatorial plane (i.e. for $\theta = 0$), where it becomes purely real and is

$$r^4 \psi_2 = -r M_{\text{BH}} + Q_{\text{BH}}^2. \tag{5}$$

Because expression (4) holds true only in a pure KN space–time, which is not our case since our space–time also contains a small but non-negligible external charge, we expect equation (5) to be more a consistency check than an accurate measurement.

In practice, to distinguish the contribution in $\psi_2$ due to the mass term from one due to the black hole charge, we compare the Weyl scalar (5) in two black holes produced, respectively, by a rotating magnetized star and by a rotating non-magnetized star. Bearing in mind that the magnetic field provides only a small contribution to the energy budget, so that $M_{\text{BH}} |_{\theta=0} \approx M_{\text{BH}} |_{\theta=0}$ to a very good precision, we then obtain

$$Q_{\text{BH}}^2 = r^4 \left( \psi_2 |_{\theta=0} - \psi_2 |_{\theta=0} \right). \tag{6}$$

In principle, this quantity should be a constant in a pure KN solution, despite $\psi_2$ being a function of position. In practice, in our calculations, this quantity has an oscillatory behaviour around a constant value in a region with $20 \lesssim r \lesssim 90 \text{km}$, while higher deviations appear near the apparent horizon (where the spatial gauge conditions are very different from those considered by Newman & Adamo 2014) and at very large distances (where the imperfect Sommerfeld boundary conditions spoil the solution locally). Averaging...
around the constant value, we read off an estimate of the space-
time charge from equation (6), which is $Q_{\text{BH}} \approx 10^{-4} M_\odot$. Given the uncertainties in the measurement, this estimate is to be taken mostly as an order-of-magnitude validation of the charge of the black hole $Q_{\text{BH}}$ measured as a surface integral. We expect the precision of the geometrical measurement of the black hole charge to improve when increasing the resolution for the outer regions of the computational domain and considering longer evolutions that would lead to a more stationary solution. In summary, by using a rather different measurement based on curvature rather than on electromagnetic fields, we converge on the conclusion that the collapse of the rotating magnetized star leads to a KN black hole with a charge that is of a few parts in $10^5$ of its mass for the initial data considered here.

4 CONCLUSION

We have carried out a systematic analysis of the gravitational col-
lapse of rotating and non-rotating magnetized neutron stars as a way to model the fate of pulsars that have passed their death line but that are too massive to be in stable equilibrium. The initial magnetized models are the self-consistent solution of the Einstein–Maxwell equations and when a rotation is present, they possess a magneto-
sphere and an initial electrical net charge, as expected in the case of ordinary pulsars. By using a resistive-MHD framework, we can model the exterior of the neutron star as an electrovacuum, so that electromagnetic fields essentially evolve according to the Maxwell equations in vacuum. This is not a fully consistent description of the magnetosphere, but it has the advantage of simplicity and we expect it to be reasonable if the charge is sufficiently small as for a pulsar that has crossed the death line.

The gravitational collapse, which is smoothly triggered by a pro-
gressive reduction of the pressure, will lead to a burst of electro-
magnetic radiation as explored in a number of works (Baumgarte
& Shapiro 2003; Lehner et al. 2012; Dionysopoulou et al. 2013;
Palenzuela 2013) and could serve as the basic mechanism to ex-
plain the phenomenology of FRBs (Falcke & Rezzolla 2014). The
end product of the collapse is either a Schwarzschild black hole,
if no rotation is present, or a KN black hole if the star is initially
rotating. For this latter case, we have provided multiple evidences
that the solution found is of KN type either by considering elec-
 tromagnetic and curvature invariants, or by measuring the charge
contained inside the apparent horizon. Hence, we conclude that
the production of a KN black hole from the collapse of a rotating and
magnetized neutron star is a robust process unless the star has zero
initial charge.

At the same time, a number of caveats should be made about our
approach. Our simulations have a simplistic treatment of the stellar
exterior and no microphysical description is attempted. It is ex-
pected, however, that a distribution of electrons and positrons could
be produced during the collapse through pair production, leading to
a different evolution (Lyutikov & McKinney 2011). These charges
could reduce the charge of the black hole and even discharge it
completely, possibly leading to a radio signal that could be associ-
ated with FRBs (Punsly & Bini 2016; Liu et al. 2016). Furthermore,
in the case of a force-free magnetosphere filled with charges, the
outcome of the collapse will likely be different, although still yield-
ing a KN black hole. Also, if present magnetic reconnection in the
exterior could change the evolution of the electromagnetic fields
and have an impact on the evolution of the charge density. As a
final remark, we note that the dynamical production of a KN black
hole should not be taken as evidence for the astrophysical existence
of such objects. We still hold the expectation that stray charges
will rapidly neutralize the black hole charge, so that a KN solution
should only be regarded as an intermediate and temporary stage
between the collapse of a rotating and magnetized star, e.g. a pulsar
that has crossed the death line, and the final Kerr solution.

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