Quantum Noise in Gravitation and Cosmology

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Abstract

We begin by enumerating the many processes in gravitation and cosmology where quantum noise and fluctuations play an active role such as particle creation, galaxy formation and entropy generation. Using the influence functional we first explain the origin and nature of noise in quantum systems interacting with an environment at a finite temperature. With linear coupling to nonohmic baths or at low temperatures, colored noise and nonlocal dissipation would appear and for nonlinear coupling multiplicative noise is generally expected. We derive a generalized fluctuation-dissipation relation for these systems. Then using a model of quantum Brownian motion in a bath of parametric oscillators, we show how noise and dissipation can be related to the Bogolubov coefficients of parametric amplification, which in the second-quantized sense, depicts cosmological particle creation in a dynamic background. We then calculate the influence functional and study the noise characteristics of quantum fields as probed by a particle detector. As examples, we show that an uniformly-accelerated observer in flat space or an inertial observer in an exponentially expanding (de Sitter) universe would see a thermal particle spectrum, recovering the well-known results of Unruh and Gibbons and Hawking, as inspired by the Hawking effect in black holes. We show how this method can be effectively used for treating the backreaction of particle creation and other quantum field processes on the dynamics of the early universe and black holes. We also discuss the advantage of adopting the viewpoint of quantum open systems in addressing some basic issues of semiclassical gravity and quantum cosmology.

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1 Introduction

It is no secret that the message of this conference is ‘NOISE IS GOOD’. In this talk I (BLH) want to show that not only is noise good, it is absolutely essential. This is because, as one school of cosmology believes and I think what every cosmologist would have to confront ultimately when the questions of why and how are insisted upon, noise, as fluctuations of the vacuum, is possibly the only thing there was in the beginning which, when ingeniously construed can explain the birth of the universe and the germination of everything in it, including spacetime and matter in their multifarious forms and structures.

Noise from quantum fields play an active and sometimes decisive role in many fundamental processes in cosmology and gravitation, especially near the Planck time (10^{-43} sec from the Big Bang). This is the time when many of us believe that the familiar concept of spacetime depicted by Einstein’s theory of general relativity emerged from an as-yet-unknown quantum theory of gravity. Below this scale the world can be adequately described by a semiclassical theory of gravity, where quantized matter fields coexist with a classical spacetime. Many qualitative changes are believed to have taken place at this energy scale, amongst them the formation of spacetime depictable as a manifold, the emergence of time, the creation of particle pairs from the vacuum, the growth of fluctuations as seeds for galaxies, and possible phase transitions and the ensuing entropy generation. It is also the cross-over point of quantum to classical transition. It is for this reason that I think one can indeed view semiclassical gravity as a branch of mesoscopic physics, the topic of the session to which this talk belongs. The only difference is that instead of dealing with the quantum to classical and micro- to macro- transition in the state of matter we are dealing with the corresponding issues for spacetime and fields.

In two recent conference papers I have described how the concepts, viewpoints and techniques of non-equilibrium quantum statistical mechanics can benefit the studies of some basic issues in gravitation and cosmology, including quantum processes in black holes and the early universe. Some examples are:
1. Particle creation as parametric amplification of vacuum fluctuations
2. Thermal radiance from accelerated observers and black holes as fluctuation-dissipation phenomena
3. Entropy generation from quantum stochastic and kinetic processes
4. Phase transitions in the early universe as noise-induced processes
5. Galaxy formation from primordial quantum fluctuations
6. Anisotropy dissipation from particle creation as backreaction processes
7. Dissipation in quantum cosmology and the issue of the initial state

1This general belief of the speaker (which may or may not be shared by his collaborators) may echo the more expressed views of the ‘birth of the universe from nothing’ scenario of Tryon, Brout, Englert, Gunzig, Spindel, Vilenkin, Linde and others, the ‘no-boundary’ proposal of Hartle and Hawking, the ‘free-lunch’ advertisement of Guth’s inflationary cosmology, or the philosophical underpinnings of the ‘austerity principle’ and ‘it from bit’ quests of Wheeler, or the ‘gravity as metric elasticity’ materialist worldview of Sakharov, but the differences are probably greater than similarities when it comes to comparing one’s view on the universe.
Recently there is an increasing effort to understand these processes in terms of statistical field theory (for a review, see [9]). Topics 5-8 are discussed in the references given at the end of this introduction. Some of these processes are not as well-understood as others. Indeed Topics 9-12 may not even be well-defined or posed. But there is hope that if one can construct a more rigorous theory of noise in quantum fields in curved spacetimes one can begin to formulate these problems in a meaningful and solvable way.

Since this workshop is about noise and not about cosmology, we will not discuss any cosmological issue here, but focus on the theory of noise in quantum fields. We will however for illustration purpose draw in two basic processes in gravitation and cosmology where noise manifests in a simple and direct way. The two processes are particle creation and detection (Topics 1,2 above). For background material on noise in open systems see, e.g., [10, 11, 12].

The starting point for the study of quantum fields is the vacuum, that for statistical mechanics the fluctuations. If the ultimate task for the cosmologist is to explain (better yet, to reconstruct) everything from nothing, it is prudent to first understand the nature and behavior of noise in fields and spacetime. Therefore, in this talk we will aim at addressing the following problems:

1) **How to define the characteristics of noise from a coarse-grained environment.** To prepare the stage for particle-field interaction and field-spacetime coupling we study the quantum Brownian model (QBM) of a particle interacting with a coupled oscillator bath. (See e.g.,[10, 11] for references to earlier work on this old topic). To see the more general features of noise we consider the cases of nonlinear coupling between the particle and bath. We use the Feynman-Vernon influence functional formalism [13, 14, 15] to show how to calculate the averaged effect of the environment on the particle, here viewed as the system. In Sec. 2 we show how all the statistical information of the bath (field) is contained in the two kernels, the noise kernel and the dissipation kernel, and we explain the physical meaning of these two kernels following [17]. The nonlocal kernels which appear at low temperature and for nonohmic baths (which are already present for the bilinear coupling) lead to colored noise. The nonlinear coupling leads to multiplicative noise. One of us had earlier suggested that colored noise, rather than the familiar white noise, is expected to appear commonly in cosmological and gravitational problems. This was shown systematically in [18, 19, 20]. Multiplicative noise in cosmology based on a nonlinear Langevin equation of classical oscillator model is discussed in [21].

2) **How to deduce a generalized fluctuation-dissipation relation** for quantum field systems. In the nonlinear QBM studied by Hu, Paz and Zhang [19] the coupling of the particle to the bath is via a form general in the system variable but of polynomial power in
the bath variable. This generalizes the bilinear coupling case studied by most researchers previously. They found a general fluctuation-dissipation relation (FDR) for this class of models with nonlinear coupling [Eq. (3.8)]. The FD kernel has a temperature-dependent factor which varies with the polynomial order \((k)\) in the bath variables in the coupling. But at high and zero temperatures this kernel becomes the same for linear and nonlinear cases and the relation becomes insensitive to the coupling. It is the belief of these authors that the FDR is a categorical relation because it reflects on the self-consistency in the interaction between the stochastic stimulation (noise and fluctuation) from the environment and the averaged response of the system (relaxation-dissipation) (cf. [10]). Later we will remark on FDR for quantum fields in curved spacetime. (See point 5 below.)

3) **How to model the particle-field coupling with a parametric oscillator bath** for the study of non-equilibrium quantum field processes in a time-dependent background spacetime. We can extract the statistical information of a quantum field (like quantum noise) by coupling a particle detector to it [23] and studying the detector’s response to the fluctuations of the field. We model the detector as a Brownian particle and the quantum field as a bath of coupled oscillators with time dependent frequencies [24]. We assume in Sec. 4 and 5 a monopole detector coupled bilinearly to the field. The normal modes of a quantum field obey an oscillator equation with time-dependent natural frequencies. For flat space (Minkowski spacetime) the time dependence of the modes is a simple sinusoidal, whereas for dynamic spacetimes it has a more complex behavior which leads to parametric amplification of vacuum fluctuations and backscattering of waves. In the second-quantized formulation this corresponds to particle creation [25, 26]. Cosmological particle creation is very strong at the Planck time and its effect on the dynamics of the universe can be very powerful [27]. The backreaction of these quantum field processes manifests as dissipation effect, which is described by the dissipation kernel [28]. We show how the influence functional can be expressed in terms of the Bogolubov coefficients which appear in the unitary transformation between the Fock space operators defined at different times. Since these coefficients determine the amount of particles produced, *one can identify the origin of noise in this system to particle creation* [29, 30, 31]. On a related point, the transition of the system from quantum to classical requires the diminishing of coherence in the wave function. The noise kernel is found to be primarily responsible for this decoherence process [22, 23]. Decoherence can be studied by analyzing the magnitude of the diffusion coefficients in the master equation. We write down without derivation (see [23] for details) the master equation for a QBM in a parametric oscillator bath (POB) and indicate how it is different from the case studied before with time-independent frequencies. This new result is useful for the analysis of decoherence where parametric excitation is present in the environment. Here we aim not at the decoherence or the dissipation process, but to focus on the very definition and nature of noise associated with quantum fields in gravitation and cosmology.

4) **How do the characteristics of quantum noise vary with the nature of the field**, the type of coupling between the field and the background spacetime, and the time-dependence of the scale factor of the universe. As an example, in Sec. 5 we illustrate how a uniformly accelerating detector observes a thermal spectrum. This way of understanding the Unruh effect
is recently discussed by Anglin [34]. As another example, we show that in a cosmology with an exponentially expanding scale factor (the de Sitter universe), a thermal spectrum is observed for a comoving observer (in the vacuum defined with respect to the proper time). This effect was first proposed by Gibbons and Hawking [35] after the Hawking effect for black holes was discovered [36]. The viewpoint of quantum open systems and the method of influence functionals can, in our opinion, lead to a deeper understanding of black hole thermodynamics and quantum processes in the early universe [8]. A fluctuation-dissipation relation for quantum fields in black hole spacetimes was first suggested by Sciana [37], and later derived for de Sitter spacetime via linear response theory by Mottola [38]. These familiar cases all deal with spacetimes with event horizons and thermal particle creation. From earlier backreaction studies in semiclassical gravity a general FDR was conjectured by one of us [16] for quantum fields in spacetimes without event horizon. This corresponds to a non-equilibrium generalization of the black hole case which we believe should capture the essence of the particle creation backreaction processes in curved spacetime [39].

5) The backreaction of particle creation has been studied in detail before [27, 28]. Indeed it was with the aim of understanding the statistical meaning of dissipation in this backreaction which led one of us to adopt the influence functional (or the equivalent coarse-grained Schwinger-Keldysh, or closed-time-path effective action [40]) method to problems in semiclassical gravity and cosmology [11, 14, 8]. In Sec. 6 we outline a program for studying the backreaction of particle creation in semiclassical cosmology. We use a model where the quantum Brownian particle and the oscillator bath are coupled parametrically. The field parameters change in time through the time-dependence of the scale factor of the universe, which is governed by the semiclassical Einstein equation. We give an expression for the influence functional in terms of the Bogolubov coefficients as a function of the scale factor. We indicate how one can obtain a new, extended theory of semiclassical gravity which, in our opinion, is necessary for furthering the investigation of quantum and statistical processes in curved spacetimes [30, 31].

This talk aims only as an introduction to the theory of noise in quantum systems. Its content is based mainly on two recent papers: Sec. 2, 3 on [19] and Sec. 4, 5 on [29]. The main theme of this talk, on the origin and nature of quantum noise in gravitation and cosmology, is also discussed in the following related work from which the reader can find a wider exposition of topics:

1) On the galaxy formation problem in inflationary cosmology [20, 41, 42];
2) On noise and fluctuations in semiclassical gravity [30, 31, 43];
3) On dissipation and initial conditions in quantum cosmology [14, 44, 46];
4) On a fluctuation-dissipation theorem in cosmology, geometrodynamic noise and gravitational entropy [8, 39].
2 Quantum Noise from the Influence Functional

Consider a Brownian particle interacting with a set of harmonic oscillators. The classical action of the Brownian particle is given by

$$S[x] = \int_0^t ds \left\{ \frac{1}{2} M \dot{x}^2 - V(x) \right\}. \quad (2.1)$$

The action for the bath is given by

$$S_b[q_n] = \int_0^t ds \sum_n \left\{ \frac{1}{2} m_n \dot{q}_n^2 - \frac{1}{2} m_n \omega_n^2 q_n^2 \right\}. \quad (2.2)$$

We will assume in this and the next section that the action for the system-environment interaction has the following form

$$S_{int}[x, \{q_n\}] = \int_0^t ds \sum_n v_n(x) q_n^k \quad (2.3)$$

where $v_n(x) = -\lambda c_n f(x)$ and $\lambda$ is a dimensionless coupling constant. If one is interested only in the averaged effect of the environment on the system the appropriate object to study is the reduced density matrix of the system $\rho_r$, which is related to the full density matrix $\rho$ as follows

$$\rho_r(x, x') = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dq' \rho(x, q; x', q') \delta(q - q'). \quad (2.4)$$

It is propagated in time by the propagator $J_r$

$$\rho_r(x, x', t) = \int_{-\infty}^{+\infty} dx_i \int_{-\infty}^{+\infty} dx'_i J_r(x, x', t | x_i, x'_i, 0) \rho_r(x_i, x'_i, 0). \quad (2.5)$$

If we assume that at a given time $t = 0$ the system and the environment are uncorrelated

$$\hat{\rho}(0) = \hat{\rho}_s(0) \times \hat{\rho}_b(0), \quad (2.6)$$

then $J_r$ does not depend on the initial state of the system and can be written as

$$J_r(x_f, x'_f, t | x_i, x'_i, 0) = \int_{x_i}^{x_f} dx \int_{x'_i}^{x'_f} dx' \exp \frac{i}{\hbar} \left\{ S[x] - S[x'] \right\} F[x, x'] = \int_{x_i}^{x_f} dx \int_{x'_i}^{x'_f} dx' \exp \frac{i}{\hbar} A[x, x'] \quad (2.7)$$
where the subscripts \( i, f \) denote initial and final variables, and \( \mathcal{A}[x, x'] \) is the effective action for the open quantum system. The influence functional \( \mathcal{F}[x, x'] \) is defined as

\[
\mathcal{F}[x, x'] = \int_{-\infty}^{+\infty} dq_i \int_{-\infty}^{+\infty} dq'_i \int_{-\infty}^{+\infty} dq_i \int_{-\infty}^{+\infty} dq'_i \left\{ S_b[q] + S_{int}[x, q] - S_b[q'] - S_{int}[x', q'] \right\} \rho_b(q_i, q'_i, 0) 
\]

where \( \delta \mathcal{A}[x, x'] \) is the influence action. Thus \( \mathcal{A}[x, x'] = S[x] - S[x'] + \delta \mathcal{A}[x, x'] \).

From its definition it is obvious that if the interaction term is zero, the influence functional is equal to unity and the influence action is zero. In general, the influence functional is a highly non-local object. Not only does it depend on the time history, but—and this is the more important property—it also irreducibly mixes the two sets of histories in the path integral of (2.7). Note that the histories \( x \) and \( x' \) could be interpreted as moving forward and backward in time respectively. Viewed in this way, one can see the similarity of the influence functional and the generating functional in the closed-time-path, or Schwinger-Keldysh integral formalism.

It can be shown \[13\] that the influence action for the model given by the interaction in (2.3) to second order in \( \lambda \) is given by

\[
\delta \mathcal{A}[x, x'] = \left\{ \int_0^t ds \left[ -\delta V(x) \right] - \int_0^t ds \left[ -\delta V(x') \right] \right\} 
- \int_0^t ds_1 \int_0^{s_2} \xi^2 \left[ f(x(s_1)) - f(x'(s_1)) \right] \mu^{(k)}(s_1 - s_2) \left[ f(x(s_2)) + f(x'(s_2)) \right] 
+ i \int_0^t ds_1 \int_0^{s_2} \xi^2 \left[ f(x(s_1)) - f(x'(s_1)) \right] \nu^{(k)}(s_1 - s_2) \left[ f(x(s_2)) - f(x'(s_2)) \right] \tag{2.9}
\]

where \( \delta V(x) \) is a renormalization of the potential that arises from the contribution of the bath. It appears only for even \( k \) couplings. For the case \( k = 1 \) the above result is exact. This is a generalization of the result obtained in \[13\] where it was shown that the non-local kernel \( \mu^{(k)}(s_1 - s_2) \) is associated with dissipation or the generalized viscosity function that appears in the corresponding Langevin equation and \( \nu^{(k)}(s_1 - s_2) \) is associated with the time correlation function of the stochastic noise term. The dissipation part has been studied in detail by Calzetta, Hu, Paz, Sinha and others \[28, 44, 45, 46, 30\] in cosmological backreaction problems. We shall only discuss the noise part of the problem. In general \( \nu \) is nonlocal, which gives rise to colored noises. Only at high temperatures would the noise kernel become a delta function, which corresponds to a white noise source. Let us see the meaning of the noise kernel.
The influence functional can then be rewritten as
\[ \nu \]
so that the reduced density matrix can be rewritten as
\[ \rho_r(x, x') = \int D\xi^{(k)}(s) P[\xi^{(k)}] \rho_r(x, x', [\xi^{(k)}]). \] (2.16)

From equation (2.15) we can view \( \xi^{(k)}(s) \) as a nonlinear external stochastic force and the reduced density matrix is calculated by taking a stochastic average over the distribution \( P[\xi^{(k)}] \) of this source.
This is a Gaussian type noise since from (2.12), we can see that the distribution functional is Gaussian. The noise is therefore completely characterized by

\[
\langle \xi^{(k)}(s) \rangle_{\xi^{(k)}} = 0 \]
\[
\langle \xi^{(k)}(s_1) \xi^{(k)}(s_2) \rangle = \hbar \nu^{(k)}(s_1 - s_2).
\] (2.17)

We see that the non-local kernel \( \nu^{(k)}(s_1 - s_2) \) is just the two point time correlation function of the external stochastic source \( \xi^{(k)}(s) \) multiplied by \( \hbar \).

In this framework, the expectation value of any quantum mechanical variable \( Q(x) \) is given by [17]

\[
\langle Q(x) \rangle = \int D\xi^{(k)}(s) \mathcal{P}[\xi^{(k)}] \int d\rho_r(x, x, [\xi^{(k)}]) Q(x)
\]
\[
= \left\langle \langle Q(x) \rangle_{\text{quantum}} \right\rangle_{\text{noise}}.
\] (2.18)

This summarizes the interpretation of \( \nu^{(k)}(s_1 - s_2) \) as a noise or fluctuation kernel.

We will now derive the semiclassical equation of motion generated by the influence action (2.9). This will allow us to see why the kernel \( \mu^{(k)}(s_1 - s_2) \) should be associated with dissipation. Define a “center-of-mass” coordinate \( \Sigma \) and a “relative” coordinate \( \Delta \) as follows

\[
\Sigma(s) = \frac{1}{2} [x(s) + x'(s)]
\]
\[
\Delta(s) = x'(s) - x(s).
\] (2.19)

The semiclassical equation of motion for \( \Sigma \) is derived by demanding (cf. [28])

\[
\frac{\delta}{\delta \Delta} \left[ S[x] - S[x'] + \delta \mathcal{A}[x, x'] \right]_{\Delta=0} = 0.
\] (2.20)

Using the sum and difference coordinates (2.19) and the influence action (2.9) we find that (2.20) leads to

\[
\frac{\partial L_r}{\partial \Sigma} - \frac{d}{dt} \frac{\partial L_r}{\partial \dot{\Sigma}} - 2 \frac{\partial f(\Sigma)}{\partial \dot{\Sigma}} \int_0^t ds \gamma^{(k)}(t - s) \frac{\partial f(\Sigma)}{\partial \Sigma} \dot{\Sigma} = F_{\xi^{(k)}}(t)
\] (2.21)

where \( \frac{d}{ds} \gamma^{(k)}(t - s) = \mu^{(k)}(t - s) \). We see that this is in the form of a classical Langevin equation with a nonlinear stochastic force \( F_{\xi^{(k)}}(s) = -\xi^{(k)}(s) \frac{\partial f(\Sigma)}{\partial \Sigma} \). This corresponds to multiplicative noise unless \( f(\Sigma) = \Sigma \) in which case it is additive. \( L_r \) denotes a renormalised system Lagrangian. This is obtained by absorbing a surface term and the potential renormalisation in the influence action into the system action. The nonlocal kernel \( \gamma^{(k)}(t - s) \) is responsible for non-local dissipation. In special cases like a high temperature ohmic environment, this kernel becomes a delta function and hence the dissipation is local.
3 Fluctuation-Dissipation Relation for Systems with Colored and Multiplicative Noise

Recall that the label $k$ is the order of the bath variable to which the system variable is coupled to. $\gamma^{(k)}(s)$ can be written as a sum of various contributions

$$\gamma^{(k)}(s) = \sum_l \gamma_l^{(k)}(s)$$  \hspace{1cm} (3.1)

where the sum is over even (odd) values of $l$ when $k$ is even (odd). To derive the explicit forms of each dissipation kernel, it is useful to define first the spectral density functions

$$I^{(k)}(\omega) = \sum_n \delta(\omega - \omega_n) k_0^{-1} \omega^k \frac{\frac{\lambda^2 C_n^2(\omega_n)}{2m_n \omega_n^k}}.$$  \hspace{1cm} (3.2)

It contains the information about the environmental mode density and coupling strength as a function of frequency. Different environments are classified according to the functional form of the spectral density $I(\omega)$. [On physical grounds, one expects the spectral density to go to zero for very high frequencies, and thus a certain cutoff frequency $\Lambda$ (which is a property of the environment) is often introduced such that $I(\omega) \rightarrow 0$ for $\omega > \Lambda$.] The environment is classified as ohmic \[17, 15, 24, 18\] if in the physical range of frequencies ($\omega < \Lambda$) the spectral density is such that $I(\omega) \sim \omega^n$, as supra-ohmic if $I(\omega) \sim \omega^n$, $n > 1$ or as sub-ohmic if $n < 1$. The most studied ohmic case corresponds to an environment which induces a dissipative force linear in the velocity of the system.

In terms of these functions, the dissipation kernels can be written as

$$\gamma_l^{(k)}(s) = \int_0^{+\infty} \frac{d\omega}{\pi} \frac{1}{\omega} I^{(k)}(\omega) M_l^{(k)}(z) \cos l\omega s$$  \hspace{1cm} (3.3)

where $M_l^{(k)}(z)$ are temperature dependent factors derived in \[19\]. Analogously, the noise kernels $\nu_l^{(k)}(s)$ can also be written as a sum of various contributions

$$\nu^{(k)}(s) = \sum_l \nu_l^{(k)}(s)$$  \hspace{1cm} (3.4)

where the sum runs again over even (odd) values of $l$ for $k$ even (odd). The kernels $\nu_l^{(k)}(s)$ can be written as

$$\nu_l^{(k)} = \hbar \int_0^{+\infty} \frac{d\omega}{\pi} I^{(k)}(\omega) N_l^{(k)}(z) \cos l\omega s$$  \hspace{1cm} (3.5)

where $N_l^{(k)}(z)$ is another set of temperature-dependent factors given by \[19\].

To understand the physical meaning of the noise kernels of different orders, we can think of them as being associated with $l$ independent stochastic sources that are coupled to the
Brownian particle through interaction terms of the form [Eq.(2.15)]

\[ \int_{0}^{t} ds \sum_{l} \xi_{l}^{(k)}(s) f(x). \]  

(3.6)

This type of coupling generates a stochastic force in the associated Langevin equation

\[ F_{\xi_{l}^{(k)}}(s) = -\xi_{l}^{(k)}(s) \frac{\partial f(x)}{\partial x} \]  

(3.7)

which corresponds to multiplicative noise. The stochastic sources \( \xi_{l}^{(k)} \) have a probability distribution given by (2.12) which generates the correlation functions (2.17) for each \( k \) and \( l \).

To every stochastic source we can associate a dissipative term that is present in the real part of the influence action. The dissipative and the noise kernels are related by generalized fluctuation–dissipation relations of the following form

\[ \nu_{l}^{(k)}(t) = \int_{-\infty}^{+\infty} ds K_{l}^{(k)}(t - s) \gamma_{l}^{(k)}(s) \]  

(3.8)

where the kernel \( K_{l}^{(k)}(s) \) is

\[ K_{l}^{(k)}(s) = \int_{0}^{+\infty} \frac{d\omega}{\pi} L_{l}^{(k)}(z) l \omega \cos l\omega s \]  

(3.9)

and the temperature-dependent factor \( L_{l}^{(k)}(z) = N_{l}^{(k)}(z)/M_{l}^{(k)}(z) \).

A fluctuation dissipation relation of the form (3.8) exists for the linear case where the temperature dependent factor appearing in (3.9) is simply \( L^{(1)} = z \). The fluctuation-dissipation kernels \( K_{l}^{(k)} \) have rather complicated forms except in some special cases. In the high temperature limit, which is characterized by the condition \( k_{B}T \gg \hbar \Lambda \), where \( \Lambda \) is the cutoff frequency of the environment, \( z = \coth \beta \hbar \omega / 2 \rightarrow 2/\beta \hbar \omega \) we obtain

\[ L_{l}^{(k)}(z) \rightarrow \frac{2k_{B}T}{\hbar \omega}. \]  

(3.10)

In the limit \( \Lambda \rightarrow +\infty \), we get the general result

\[ K_{l}^{(k)}(s) = \frac{2k_{B}T}{\hbar} \delta(s) \]  

(3.11)

which tells us that at high temperature there is only one form of fluctuation-dissipation relation, the Green-Kubo relation [22]

\[ \nu_{l}^{(k)}(s) = \frac{2k_{B}T}{\hbar} \gamma_{l}^{(k)}(s). \]  

(3.12)
In the zero temperature limit, characterized by $z \to 1$, we have
\[ L_l^{(k)}(z) \to l. \] (3.13)

The fluctuation-dissipation kernel becomes $k$-independent and hence identical to the one for the linearly-coupled case
\[ K(s) = \int_0^{+\infty} d\omega \frac{\omega}{\pi} \omega \cos\omega s. \] (3.14)

It is interesting to note that the fluctuation-dissipation relations for the linear and the nonlinear dissipation models are exactly identical both in the high temperature and in the zero temperature limits. In other words, they are not very sensitive to the different system-bath couplings at both high and zero temperature limits. The fluctuation-dissipation relation reflects a categorical relation (backreaction) between the stochastic stimulation (fluctuation-noise) of the environment and the averaged response of a system (dissipation) which has a much deeper and universal meaning than that manifested in specific cases or under special conditions.

## 4 Brownian Particle in a Bath of Parametric Oscillators

The previous two sections showed how noise and dissipation are generated using general system environment couplings within the quantum Brownian motion paradigm. This extension of the quantum Brownian motion paradigm to non-linear couplings and nonlocal noise and dissipation is essential if we are to address the issues in cosmology and gravity outlined in the Introduction. Since the early universe is rapidly expanding we need a formalism which allows us to study the non-equilibrium quantum statistical processes in time-dependent backgrounds. In the following two sections we will discuss the Brownian motion of a quantum particle in a bath of parametric oscillators and show how this model can be used to treat particle creation and detection processes in the early universe and black holes. To lessen the complexity of the problem we will consider only linear coupling between the system and the bath.

Consider now the system being a parametric oscillator with mass $M(s)$, cross term $B(s)$ and natural (bare) frequency $\Omega(s)$. The environment is also modelled by a set of parametric

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A given environment is characterized by the spectral densities $I^{(k)}(\omega)$ and it is clear that if these functions are appropriately chosen, the form of the noise and dissipation kernels can be simplified considerably. For example, if the spectral density is $I^{(k)}(\omega) \sim \omega^k$, the noise and the dissipation kernels become local kernels in the high temperature limit. In that case we have $\gamma_l^{(k)}(s) \sim (k_B T)^{k-1} \delta(s)$, $\nu_l^{(k)}(s) \sim (k_B T)^k \delta(s)$. Note that $\gamma_l^{(k)}$ depends upon the temperature and will produce a temperature dependent friction term in the effective equations of motion. On the other hand if the spectral density is the same linear function for all $(k)$, i.e., $I^{(k)}(\omega) \sim \omega$, the dissipation kernel will become local in the low temperature limit: $\gamma_l^{(k)} \sim \delta(s)$, but the noise remains colored due to the nontrivial fluctuation dissipation relation (3.8). However, as we will show below, for quantum fields the spectral density are fixed by their own character and cannot be adjusted arbitrarily.
oscillators with mass $m_n(s)$, cross term $b_n(s)$ and natural frequency $\omega_n(s)$. We assume that the system-environment coupling is linear in the environment coordinates with strength $c_n(s)$, but general in the system coordinate. The action of the combined system + environment is

$$S[x, q] = S[x] + S_E[q] + S_{int}[x, q]$$

$$= \int_0^t ds \left[ \frac{1}{2} M(s) \left( \dot{x}^2 + B(s)x\dot{x} - \Omega^2(s)x^2 \right) \right.$$ 

$$+ \sum_n \left\{ \frac{1}{2} m_n(s) \left( \dot{q}_n^2 + b_n(s)q_n\dot{q}_n - \omega_n^2(s)q_n^2 \right) \right\} + \sum_n \left( -c_n(s)f(x)q_n \right) \right]$$

(4.1)

where $x$ and $q_n$ are the coordinates of the particle and the oscillators respectively.

### 4.1 Bogolubov Transformation and Particle Creation

All the information about the quantum dynamics of the bath parametric oscillators are contained in the two complex numbers, $\alpha$ and $\beta$, known as the Bogolubov coefficients. They obey two coupled first order equations [25, 26]

$$\dot{\alpha} = -ig^*\beta - ih\alpha$$

$$\dot{\beta} = ih\beta + ig\alpha$$

(4.2)

and are related by the Wronskian condition $|\alpha|^2 - |\beta|^2 = 1$. The time-dependent coefficients are given by

$$g(t) = \frac{1}{2} \left( \frac{m(t)\omega^2(t)}{\kappa} + \frac{m(t)b^2(t)}{4\kappa} - \frac{\kappa}{m(t)} + ib(t) \right)$$

(4.3)

$$h(t) = \frac{1}{2} \left( \frac{\kappa}{m(t)} + \frac{m(t)\omega^2(t)}{\kappa} + \frac{m(t)b^2(t)}{4\kappa} \right).$$

(4.4)

where $\kappa$ is an arbitrary positive real constant that is usually chosen so that $g = 0$ at $t_i$. Thus if $b_n = 0$ we will usually have $\kappa = m(t_i)\omega(t_i)$. Given the initial condition for the propagator, Eq.(4.2) must satisfy the initial conditions $\alpha(t_i) = 1, \beta(t_i) = 0$. In a cosmological background, the time dependence of $g$ and $h$ are parametric in nature, i.e., it comes from the time-dependent scale factor $a$.

One can use the squeeze state language to depict particle creation [48, 49]. The unitary evolution operator $\hat{U}$ for this time-dependent system can be expressed as a product of the squeeze and rotation operators $\hat{S}, \hat{R}$

$$\hat{U}(t, t_i) = \hat{S}(r, \phi)\hat{R}(\theta)$$

(4.5)

where

$$\hat{R}(\theta) = e^{-i\theta \hat{B}}, \quad \hat{S}(r, \phi) = \exp[r(\hat{A}e^{-2i\phi} - \hat{A}^\dagger e^{2i\phi})].$$

(4.6)

Here

$$\hat{A} = \hat{a}^2/2, \quad \hat{B} = \hat{a}^\dagger\hat{a} + 1/2$$

(4.7)
and $a, a^\dagger$ are the annihilation and creation operators of the second-quantized modes. The
Bogolubov coefficients $\alpha$ and $\beta$ are related to the three real parameters, $r$, the squeeze
parameter, $\phi$, the squeeze angle, and $\theta$, the rotation angle by

$$\alpha = e^{-i\theta} \cosh r, \quad \beta = -e^{-i(2\phi+\theta)} \sinh r.$$ (4.8)

The exact influence action for the model (4.1) takes the form (2.9) with $\delta V(x) = 0$ and
$k = \lambda = 1$. For an initial thermal state \[3\], the dissipation and noise kernels are calculated to
be\[29\]

$$\mu(s, s') = \frac{i}{2} \int_0^\infty d\omega I(\omega, s, s')[X^* X' - XX'^*]$$ (4.9)

$$\nu(s, s') = \frac{1}{2} \int_0^\infty d\omega I(\omega, s, s') \coth \left( \frac{\hbar \omega(t_i)}{2kB T} \right) [X^* X' + XX'^*]$$ (4.10)

where $X \equiv X_\omega(s) \equiv \alpha_\omega(s) + \beta_\omega(s)$ and $X' \equiv X_\omega(s')$.\[4\] The spectral density, $I(\omega, s, s')$ defined by

$$I(\omega, s, s') = \sum_n \delta(\omega - \omega_n) \frac{c_n(s)c_n(s')}{2\kappa}$$ (4.11)

is obtained in the continuum limit. It contains information about the environmental mode
density and coupling strength as a function of frequency. We see from (4.9-10) that the effect
of parametric amplification in the bath affects both the noise and dissipation kernels.

From (2.20) with the influence action (2.9) (with $\delta V(x) = 0$ and $k = \lambda = 1$), we find
that the semiclassical equation is given by

$$\frac{\partial L_r}{\partial x} - \frac{d}{dt} \frac{\partial L_r}{\partial \dot{x}} - 2 \frac{\partial f(x)}{\partial x} \int_0^t ds \gamma(t, s) \frac{\partial f(x)}{\partial x} \dot{x} = -\frac{\partial f(x)}{\partial x} \xi(t)$$ (4.12)

where $\xi$ is a zero mean gaussian stochastic force with the correlator $\langle \xi(t) \xi(t') \rangle = h\nu(t, t')$
and $L_r$ is an arbitrary system Lagrangian renormalised by a surface term from the nonlocal
kernel. The noise and dissipation kernels are no longer stationary due to the time dependence
of the bath.

4.2 Noise and Decoherence

Finding how the classical features arise from a quantum system is a fundamental issue for all
physical systems, including the description of the universe itself \[21\]. In many cosmological
processes it is essential to be able to say when some degree of freedom has become effectively

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\[3\] In Ref. \[29\] we consider a squeezed thermal initial state which is a generalisation of the initial state used
here. This form of initial state is of interest in quantum optics.

\[4\] If we assume $b = 0$ and $m = 1$ we can show using (4.2) that $X$ is a solution of $\ddot{X} + \omega_n^2(t) X = 0$
subject to the boundary condition $X(t_i) = 1$. From this one can show that with a thermal initial state in the high
temperature limit our quantum theory gives the correct classical result \[21\].
classical. Only when this happens will the semiclassical description will appropriate. \(^5\) To understand the quantum to classical transition an essential fact or is the suppression of the interference terms in the system of interest. This can be achieved by letting the system interact with a coarse-grained environment \(^5\). (The problem is better formulated in terms of decoherent or consistent histories \(^5\).) Noise in the environment here plays two important roles: one in decohering the system and causing it to assume a classical character, the other in imparting a dissipative behavior in the system dynamics. These two processes, decoherence and dissipation, usually occur at very different time scales. Decoherence is effectively achieved when thermal fluctuations overtake the vacuum fluctuations \(^5\). For macroscopic objects at high temperatures, decoherence time is much faster than relaxation time. This can be studied by analyzing the relative importance of the respective terms in the master equation for the quantum open system.

A full quantum mechanical description of the dynamics of the open system is given by the propagator \(J_r\) of the reduced density matrix (2.7), which can be, and has been derived exactly for the bilinear coupling case \(^2\). Using this we can derive the master equation for the reduced density matrix. The master equation is useful because it separates out the different non-equilibrium quantum processes generated by the bath on the system.

The exact master equation for a system interacting with a bath described by a general time-dependent quadratic Hamiltonian in a squeezed thermal initial state is derived to be \(^2\)

\[
\begin{align*}
\hat{H}_{\text{ren}}(t) &= \frac{i}{\hbar} \partial_t \hat{\rho}_r(x, x', t) \\
&= \left\{ -\frac{\hbar^2}{2M(t)} \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x'^2} \right) + \frac{M(t)}{2} \Omega_{\text{ren}}^2(t, t_i)(x^2 - x'^2) \right\} \rho_r(x, x', t) \\
&\quad - i\hbar \Gamma(t, t_i)(x - x')(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}) + iD_{pp}(t, t_i)(x - x')^2 \\
&\quad - \hbar \left( D_{xp}(t, t_i) + D_{px}(t, t_i) \right)(x - x')(\frac{\partial}{\partial x} + \frac{\partial}{\partial x'}) - i\hbar^2 D_{xx}(t, t_i) \left( \frac{\partial^2}{\partial x^2 + \partial x'^2} \right) \rho_r(x, x', t)
\end{align*}
\]

where \(\Omega_{\text{ren}}\) is the renormalized frequency, \(\Gamma\) is the dissipation coefficient and the \(D's\) are the diffusion coefficients. \(^5\) In operator form, it reads

\[
\begin{align*}
\hat{H}_{\text{ren}, \hat{\rho}}(t) &= \left[ \hat{H}_{\text{ren}}, \hat{\rho} \right] + iD_{pp}(t)[\hat{x}, [\hat{x}, \hat{\rho}]] + iD_{xx}(t)[\hat{p}, [\hat{p}, \hat{\rho}]] \\
&\quad + iD_{xp}(t)[\hat{x}, [\hat{p}, \hat{\rho}]] + iD_{px}(t)[\hat{p}, [\hat{x}, \hat{\rho}]] + \Gamma(t)[\hat{x}, \{\hat{p}, \hat{\rho}\}]
\end{align*}
\]

\(^5\)For example, in the inflationary universe it seems plausible that the amplification of quantum fluctuations of a scalar field can act as seeds for primordial density perturbations. In this case the transition from quantum to classical fluctuations is critical since it is responsible for breaking the spatial translational invariance of the vacuum, which is necessary in order to generate primordial density perturbations \(^5\). The quantum origin and nature of noise in the generation of structure in the primordial universe is discussed in \(^2\).

\(^6\) The master equation differs from that of the time-independent oscillator bath \(^5\) by more than changing the kernels. There the dissipation kernel is stationary (i.e a function of \((s - s')\)) and \(D_{xx}\) is absent. A non-stationary factor enters in all the diffusion coefficients and \(D_{xx}\) depends solely on it.
where
\[ \hat{H}_{\text{ren}} = \frac{\hat{p}^2}{2M} + \frac{M}{2} \Omega_{\text{ren}}(t) \hat{x}^2. \] (4.14)

The first term is the Liouville or streaming term describing the free unitary dynamics of the system but with a renormalised frequency. The last term proportional to \( \Gamma \) is responsible for dissipation. The renormalised frequency and the dissipation coefficient \( \Gamma \) do not depend on the noise kernel (as they depend on the environment only via the functions \( u_i \) defined in [18, 19, 28]). The terms proportional to \( D_{xx}, D_{pp}, D_{xp}, \) and \( D_{px} \) generate diffusion in the variables \( p^2, x^2 \), and \( xp + px \) respectively. This can be seen by going from the master equation to the Fokker-Planck equation for the Wigner function [55]. The diffusion coefficients are affected by both noise and dissipation kernels (as they depend both on \( u_i \) and \( a_{ij} \)). From the master equation we know that \( D_{xx} \) and \( D_{pp} \) generate decoherence in \( p \) and \( x \) respectively. Decoherence is a critical process for the quantum to classical transition. It is also responsible for entropy generation and other quantum statistical effects.

5 Particle-Field Interaction

Let us now examine how to define the noise of a quantum field, both in flat and curved spacetimes. It is easy to show that a field can be represented as a parametric bath of oscillators [4]. To study the noise properties of a quantum field, we introduce a particle detector and assume some interaction, the simplest being a monopole linear coupling. We consider two cases: an accelerated detector in flat space, and an inertial detector in an expanding universe. It is seen that if the acceleration is uniform or if the scale factor undergoes an exponential expansion then the noise observed in the detector’s proper time is thermal. These two examples were first given in 1976 and 1977 by Unruh [23] and Gibbons and Hawking [35] respectively. We use these well-known examples to illustrate the physics of the problem and to demonstrate the power of the influence functional formalism in extracting the statistical information of the system and bath.

5.1 Accelerated Observer

We consider a massive scalar field \( \Phi \) in a two-dimensional flat space with mode decomposition
\[ \Phi(x) = \sqrt{\frac{2}{L}} \sum_k [q_k^+ \cos kx + q_k^- \sin kx]. \] (5.1)

The Lagrangian for the field can be expressed as a sum of oscillators with amplitudes \( q_k^\pm \) for each mode
\[ L(s) = \frac{1}{2} \sum_\sigma \sum_k \left[ (q_k^\sigma)^2 - \omega_k^2 q_k^\sigma^2 \right]. \] (5.2)

This corresponds to the case in (4.1) with \( m_n = 1, b_n = 0 \). Since \( \omega_k^2 = (k^2 + m^2) \) in flat space is time-independent, \( \alpha = e^{-i\omega t}, \ \beta = 0, \) where \( \alpha = 1 \) initially \( t = 0 \). Substituting these into
(4.9-10) one obtains for an inertial detector in a thermal bath
\[
\mu(s, s') = -\int_0^\infty dk \, I(k) \sin \omega(s - s')
\] (5.3)
and
\[
\nu(s, s') = \int_0^\infty dk \, \coth \left( \frac{\hbar \omega}{2k_B T} \right) I(k) \cos \omega(s - s').
\] (5.4)

Now consider an observer undergoing constant acceleration \(a\) in this field with trajectory
\[
x(\tau) = \frac{1}{a} \cosh a \tau, \quad s(\tau) = \frac{1}{a} \sinh a \tau.
\] (5.5)

We want to show that the observer detects thermal radiation. Let us first find the spectral density. For a monopole detector the particle-field interaction is
\[
L_{\text{int}}(x) = -\epsilon r \Phi(x) \delta(x(\tau)).
\] (5.6)

where they are coupled at the spatial point \(x(\tau)\) with coupling strength \(\epsilon\) and \(r\) is the detector’s internal coordinate. Integrating out the spatial variables we find that
\[
L_{\text{int}}(\tau) = \int L_{\text{int}}(x) dx = -\epsilon r \Phi(x(\tau)).
\] (5.7)

Using (5.1-2) we see that the accelerated observer is coupled to the field with effective coupling constants
\[
c_n^+(s) = \epsilon \sqrt{\frac{2}{L}} \cos kx(\tau), \quad c_n^-(s) = \epsilon \sqrt{\frac{2}{L}} \sin kx(\tau).
\] (5.8)

With this we find from (4.11) that in the continuum limit (using \(\sum_k \to \frac{L}{2\pi} \int dk; \, \kappa = \omega\))
\[
I(k) = \frac{\epsilon^2}{2\pi \omega} \cos k[x(\tau) - x(\tau')].
\] (5.9)

The expressions for the noise and dissipation kernels in a zero-temperature field (assumed in an initial vacuum state) can be obtained from the above result for finite temperature bath by setting \(z \equiv \coth \beta \hbar \omega/2 \to 1\). We have
\[
\zeta(s(\tau), s(\tau')) = \nu(s, s') + i \mu(s, s') = \int_0^\infty dk \, I(k)e^{-i\omega[s(\tau) - s(\tau')]}.
\] (5.10)

Now putting in the spectral density function \(I(k)\) we get
\[
\zeta(s(\tau), s(\tau')) = \frac{\epsilon^2}{4\pi} \int_{-\infty}^\infty \frac{dk}{\omega} e^{-ik[x(\tau) - x(\tau')]} e^{-i\omega[s(\tau) - s(\tau')]}.
\] (5.11)
We can write this as
\[ \zeta(\tau, \tau') = \int \frac{d\omega}{\omega} G(a, \omega) \left[ \coth(\pi \omega / a) \cos(\omega (\tau - \tau')) - i \sin(\omega (\tau - \tau')) \right] \]

(5.12)

where
\[ G(a, \omega) = \frac{\epsilon^2 \omega}{a^2} \sinh(\pi \omega / a) [K_{i\omega/a}(m/a)]^2 \]

(5.13)

and \( K \) is the Bessel function. Comparing this with (5.4) we see that a thermal spectrum is detected by a uniformly-accelerating observer at temperature
\[ k_B T = \frac{a}{2\pi}. \]

(5.14)

This was first found by Unruh and presented in this form recently by Anglin [34]. Note that there is a trade-off between the thermal effect of the bath as detected by an inertial observer and the kinematic effect of the detector undergoing uniform acceleration in a vacuum. Although it is well-known that a uniformly accelerating observer sees an exact thermal radiance, the case of arbitrary motion is perhaps lesser known. It is certainly more difficult to analyze if one interprets the Unruh effect (or Hawking effect for black holes) in the geometric sense, i.e., via the event horizon, which does not exist for all times in this more general case. In the statistical mechanical viewpoint we are espousing, it is easier to understand that noise is always present no matter how the detector moves. Indeed this formalism tells one how to calculate the form of noise for an arbitrary particle trajectory and field. It also separates the kinematic and the thermal effects so one can interpret the physics clearly.

### 5.2 Thermal Radiance in de Sitter Space

Consider now the 4-dimensional Robertson-Walker (RW) spacetime with line element
\[ ds^2 = dt^2 - \sum_i a^2(t) dx_i^2. \]

(5.15)

For this metric the Lagrangian density of a massless conformally coupled scalar field is
\[ \mathcal{L}(x) = \frac{a^3}{2} \left[ (\dot{\Phi})^2 - \frac{1}{a^2} \sum_i (\Phi,_{i})^2 - \left( \frac{\dot{a}}{a} - \frac{\ddot{a}}{a} \right) \Phi^2 \right] \]

(5.16)

where a dot denotes a derivative with respect to \( t \). Decomposing \( \Phi \) in normal modes we find (after adding a surface term)
\[ L(t) = \sum_{\sigma} \sum_{\vec{k}} \frac{a^3}{2} \left[ (\dot{q}_k^\sigma)^2 + 2 \frac{\dot{a}}{a} \dot{q}_k^\sigma q_k^\sigma - \left( \frac{k^2}{a^2} - \frac{\ddot{a}}{a} \right) q_k^\sigma \right] \]

(5.17)

where \( k = |\vec{k}| \) and \( L(t) = \int \mathcal{L}(x) d^3\vec{x} \). If the detector-field interaction is of the same form as (5.6), but at fixed \( x \), we see that in a four-dimensional field the spectral density is given by
\[ I(k) = \left( \frac{\epsilon}{2\pi} \right)^2 k. \]

(5.18)
Using the Lagrangian (5.17) we find from (4.2) that the Bogolubov coefficients are
\[
\alpha = \frac{(1 + a^2)}{2a} e^{-ik\eta}, \quad \beta = \frac{(1 - a^2)}{2a} e^{-ik\eta}
\] (5.19)
where \( \eta = \int_{t_i}^{t} dt / a(t) \) with \( a(t_i) = 1 \). Using these we find that the noise and dissipation kernels (4.9-10) are
\[
\zeta(t, t') = \nu(t, t') + i\mu(t, t') = \frac{1}{a(t)a(t')} \int_{0}^{\infty} dk \ I(k) e^{-ik(\eta - \eta')}.
\] (5.20)

We will now specialise to the de Sitter space where, in the spatially-flat RW coordinatization, the scale factor has the form
\[ a(t) = e^{Ht} \] (5.21)
In this case \( \eta = -\frac{1}{H} e^{-Ht} \) with \( t_i = 0 \). If we define \( \Delta = t - t' \), \( \Sigma = t + t' \) we find that (5.20) becomes
\[
\zeta(t, t') = e^{-H\Sigma} \int_{0}^{\infty} dk \ I(k) \exp \left[ -\frac{2ik}{H} e^{-H\Sigma/2} \sinh(H\Delta/2) \right].
\] (5.22)
Making use of [34]
\[
e^{-i\alpha \sinh(x/2)} = \frac{4}{\pi} \int_{0}^{\infty} d\nu K_{2\nu}(\alpha) \left[ \cosh(\pi\nu) \cos(\nu x) - i \sinh(\pi\nu) \sin(\nu x) \right]
\] (5.23)
we find that
\[
\zeta(t, t') = \int_{0}^{\infty} dk \ G(k) \left[ \coth \left( \frac{\pi k}{H} \right) \cos k(t - t') - i \sin k(t - t') \right]
\] (5.24)
where
\[
G(k) = \frac{4 \sinh(\pi k/H)}{\pi H e^{H\Sigma}} \int_{0}^{\infty} dk' \ I(k') K_{2i k/H}(2k' e^{-H\Sigma/2}/H)
\nonumber
= \left( \frac{e}{2\pi} \right)^2 k = I(k).
\] (5.25)

We have used the integral identity
\[
\int_{0}^{\infty} dx \ x^\mu K_\nu(ax) = 2^{\nu - 1} a^{-\mu - 1} \Gamma \left( \frac{1 + \mu + \nu}{2} \right) \Gamma \left( \frac{1 + \mu - \nu}{2} \right)
\] (5.26)
and the properties of gamma functions. Comparing (5.24) with (5.4) we see that a thermal spectrum is detected by an inertial observer in de Sitter space at temperature
\[
k_B T = \frac{H}{2\pi}.
\] (5.27)
6 Field-Spacetime Coupling: backreaction in semiclassical cosmology

The QBM paradigm can be applied effectively to treat problems in semiclassical gravity, i.e., quantum matter fields in a classical spacetime and its backreaction on the dynamics of spacetime. A well-studied example is the dissipation of anisotropy due to particle creation in the early universe [28]. (For a description of how a study of this problem led to the discovery of the relevance of the closed-time-path effective action method [40] and the influence functional [13] formalism, see e.g., [8].) Here spacetime is regarded as the system and the quantum field as the environment. The effect of the quantum field on the dynamics of spacetime is captured in the influence functional in the same way as the oscillator bath on the Brownian particle. The motivation is explained in [18, 19, 20] and the details can be found in [30, 31].

Consider now another model with the action

\[ S[a,q] = \int ds \left[ L_g(a, \dot{a}, s) + \sum_n \left( \frac{1}{2} m_n(a, \dot{a}) \left( \dot{q}_n^2 + b_n(a, \dot{a}) q_n \dot{q}_n - \omega_n^2(a, \dot{a}) q_n^2 \right) \right] \]  

(6.1)

where \( L_g \) is the classical gravitational action. This action can be used to describe a free quantized scalar field propagating in a spatially flat Friedmann-Robertson-Walker (FRW) universe with scale factor \( a(s) \). The field and spacetime are coupled parametrically: The time dependence of the mass and frequencies of the oscillators comes from the scale factor (system variable) whose dynamics is determined by the Einstein equations. The detector is left out of the picture, replacing it formally is the scale factor of the universe. By tracing out the scalar field we can obtain an influence functional from which an equation of motion for the scale factor in the semiclassical regime can be derived. We find that for an initial vacuum state the exact influence functional for this model is [31, 30]

\[ F[a,a'] = \prod_n \frac{1}{\sqrt{\alpha_n[a]\alpha_n^*[a] - \beta_n[a]\beta_n^*[a]}}. \]  

(6.2)

The Bogolubov coefficients are derived as before via equations (4.2), where now \( g \) and \( h \) are functions of the system variable \( a \). The Bogolubov coefficients describe particle creation in each normal mode \( n \) of the scalar field generated by the expansion of the universe.

The influence functional tells us how particle creation reacts back on the dynamics of the universe and determines the time-dependence of the scale factor \( a \). Using the influence functional method we have studied the backreaction in detail [30, 31] and obtained in the semiclassical limit an equation of motion for \( a \) which includes non-local dissipation and colored noise. The vacuum fluctuations of the scalar field is the source of the noise. This approach to semiclassical gravity goes beyond the conventional semiclassical Einstein equation, which considers only the average value of the matter energy momentum tensor. By approaching semiclassical gravity from the statistical mechanical viewpoint we see how
all statistical information in the quantum matter source, in particular the noise and fluctuations of quantum fields, can be systematically included in the backreaction. Recognition of the stochastic nature of semiclassical gravity is important because it represents a qualitative change in the theory which can lead to a deeper understanding of all semiclassical gravity phenomena.

7 Discussion

This talk aims at addressing the origin and nature of noise in quantum systems. The methodology and concepts we introduced can be used to tackle a wide range of problems, such as those listed in the Introduction.

On a practical level there are at least two simple advantages in discussing noise in gravitation and cosmology:
1) The frequent occurrence and ubiquitous nature of colored noise in cosmology enrich the repertoire and enhance the function of noise in physical systems as we have learned in this meeting.
2) The theory of noise developed in ordinary physical systems can be deepened to probe into the statistical properties of the vacuum in curved spacetime. From this one can perform more in-depth analysis of the quantum statistical processes in the early universe and black holes.

Although we have used examples from quantum cosmology to illustrate the physical relevance of the quantum Brownian model with a parametric oscillator bath, the range of applicability of this model goes beyond. An important area where parametric amplification in coupled oscillators plays a central role is in quantum optics [56]. There it is believed that squeezed baths can serve to retain certain quantum behaviour of the system in its interaction with the environment. The model studied in this work is exact and its general results will therefore prove useful for addressing similar issues in this context.

On a theoretical level, the theory of noise and fluctuations lie at the foundations of statistical mechanics and quantum field theory. Some basic issues in gravitation and cosmology require the understanding of both of these aspects. These two branches of theoretical physics are however almost entirely disjoint until the discovery of Hawking effect for black holes [30] and Unruh’s discovery of thermal radiance in a uniformly accelerated observers [23]. Sciama [37], in celebration of Einstein’s other major accomplishment, was the first to introduce a fluctuation theory viewpoint in explaining these effects. This very insightful viewpoint has unfortunately received little attention. From the work of one of us over the years in trying to understand the nature of dissipation in semiclassical gravity [16] we are led to focusing more on the effect of noise and fluctuations in quantum fields as the source of decoherence in the emergence of classical spacetime and dissipation in the spacetime dynamics [7]. Noise and fluctuation could also be instrumental in inducing a phase transition at the Planck scale between the phases described by a theory of quantum gravity and the classical theory of general relativity [1]. We think this viewpoint can stimulate new ideas and open new channels of fruitful investigations in gravitation and cosmology research.
From this discussion you can perhaps understand why we said in the beginning of the talk that noise in the cosmos is essential. From it particles are created, galaxies are formed and entropy is generated, not to mention their role in ‘creating’ the quantum universe and bringing about the comfortable classical world we live in. Actually considering noise as good or bad is rather irrelevant in this regard, as we owe our existence to it in the final analysis. Paraphrasing Voltaire: they simply exist and thrive in spite of us.

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