Soft electroweak breaking from hard
supersymmetry breaking

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Abstract
We present a class of four-dimensional models, with a non-supersymmetric spectrum, in which the radiative corrections to the Higgs mass are not sensitive, at least at one-loop, to the UV completion of the theory. At one loop, Yukawa interactions of the top quark contribute to a finite and negative Higgs squared mass which triggers the electroweak symmetry breaking, as in softly broken supersymmetric theories, while gauge interactions lead to a logarithmic cutoff dependent correction that can remain subdominant. Our construction relies on a hard supersymmetry breaking localized in the theory space of deconstruction models and predicts, within a renormalizable setup, analogous physics as five-dimensional scenarios of Scherk–Schwarz supersymmetry breaking. The electroweak symmetry breaking can be calculated in terms of the deconstruction scale, replication number, top-quark mass and electroweak gauge couplings. For $m_{\text{top}} \sim 170$ Gev, the Higgs mass varies from 158 GeV for $N = 2$ to 178 GeV for $N = 10$. 
The weak scale is unlikely to be a fundamental scale of physics. Its calculation in terms of more fundamental scales is one of the central problems in particle physics. The problem is aggravated by the fact that in the Standard Model (SM) the Higgs field mass parameter gets radiative corrections that grow quadratically with the scale \( \Lambda \) of new physics:

\[
m_h^2 \sim \Lambda^2.
\]

Thus, any attempt to calculate \( m_h \) in terms of a more fundamental scale \( \Lambda \) and to make it stable against radiative corrections needs a mechanism of suppression of the quadratic dependence on \( \Lambda \); the higher the scale \( \Lambda \) is, the stronger suppression is required.

An attractive solution to the stability aspect of the hierarchy problem is provided by softly broken supersymmetry. Quantum corrections to the weak scale depend quadratically on \( M_{\text{SUSY}} \) and only logarithmically on the cutoff scale \( \Lambda \):

\[
\delta m^2 \sim M_{\text{SUSY}}^2 \ln \Lambda \quad \text{(therefore } \Lambda \text{ can be taken as high as the Planck scale).}
\]

However, the generation of the weak scale \( v_h \ll \Lambda \) is overshadowed by the \( \mu \)-problem, i.e., by the question why the supersymmetric parameter \( \mu \) is of the order of the weak scale. Also, some other aspects of softly broken supersymmetric theories are sufficiently troublesome to justify the quest for alternative routes of solving the hierarchy problem.

An important new element in attempts to solve the hierarchy problem is the idea of large (TeV\(^{-1}\) size) extra dimensions [1], realized by low scale string theories or at the level of effective theories [2]. One possibility, which does not require supersymmetry, is to consider the Higgs boson to be a component of the gauge field propagating in extra dimensions. The Higgs potential, by higher-dimensional gauge invariance, does not depend on the cutoff scale and is calculable in terms of the compactification scale \( M_c \sim 1/R \) [3]. Another possibility [1, 2, 3] is to break supersymmetry via the Scherk–Schwarz mechanism [7]. The non-local character of this mechanism ensures that at least one-loop corrections to the Higgs mass are finite [5, 8]. In the effective theory supersymmetry is broken in a hard-way and it is conceivable that divergences re-appear at higher-loop level. However, large extra dimensions and related low value of the cutoff scale \( \Lambda \) change qualitatively the hierarchy problem in the sense that calculating \( m_h \) in terms of \( R \) and \( \Lambda \) does not require as strong suppression of quadratic divergences as for the canonical case with \( \Lambda = M_{\text{PL}} \). From a phenomenological point of view, with the cutoff scale close to the compactification scale, one-loop finiteness of the leading corrections to the Higgs potential is sufficient for the cutoff dependence to be very weak. This point of view is taken in the model of Ref. [5]. The electroweak symmetry breaking (EWSB) is triggered by the top/stop loops. Although the gauge interactions contribute to a quadratically divergent result [5], the finiteness of the leading corrections still allows to make predictions about the Higgs boson mass [10]. The obvious advantage of this scenario is that the full Higgs potential and the superpartner masses are calculable to a good precision in terms of one dimensionful parameter — the compactification radius, and the soft breaking masses are not necessary.

Recently, a new idea called deconstruction appeared [11, 12] which allows to realize the physics of extra dimensions in a strictly four-dimensional set-up. Soon after, the 4D analogue of the mechanism [3] was constructed [13] where the Higgs boson mass is protected from receiving divergent radiative corrections by the pseudo-Goldstone mechanism (see also [4] for another deconstruction model of electroweak symmetry breaking and [15] for a deconstruction model where the radiative corrections are highly suppressed as a result of the topological nature of the supersymmetry breaking). Although the deconstruction models yield no unambiguous predictions about the fundamental scale, the low-scale unification [16] suggests that the fundamental scale could be much lower than the Planck scale. Thus, similarly as in the large extra
dimensions models, less suppression of the quadratic divergence is required to alleviate the hierarchy problem. In this paper we investigate the four-dimensional analogue of the Scherk–Schwarz mechanism and take the model proposed by Barbieri, Hall and Nomura (BHN) [5] as our reference point. We do not aim at constructing a complete and phenomenologically viable model, which would give the Standard Model as its low-energy approximation. We rather aim at analyzing the general situation, when divergences in non-supersymmetric theories are considerably softened.

More precisely, we start with $\mathcal{N} = 1$ supersymmetric models consisting of a chain of $N$ gauge groups which communicate to each other through $N - 1$ bifundamental link-Higgs fields $\Phi_i$ [17]. The matter and Higgs fields are also replicated and represented by a set of $N$ chiral superfields transforming in fundamental representation of the corresponding gauge group. When the link-Higgs fields acquire vacuum expectation values (vev) the mass pattern of the gauge, Higgs and matter fields is similar as in the theories with extra dimensions. We are mainly interested in the models in which the low-energy spectrum (zero-modes of the mass matrix) shows no sign of supersymmetry but still the radiative corrections to the mass parameters are weakly dependent on the cutoff scale.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{quiver.png}
\caption{The quiver diagram of the model. Each circle (site) represents an $SU(2)$ $\mathcal{N} = 1$ Yang-Mills multiplet. Each line pointing outwards a circle represents a chiral multiplet in the fundamental representation of the group while a line pointing towards a circle stands for a chiral multiplet in the anti-fundamental representation of the group.}
\end{figure}

First, recall how the 5D supersymmetric Yang-Mills theories on $S_1/Z_2$ are realized in the 4D set-up [17]. We write a supersymmetric Lagrangian for a chain of $N$ gauge multiplets ($A_i^a, \chi_i^a$) (with common gauge coupling $g_0$) and $N$ chiral, link-Higgs multiplets ($\Phi_i, \Psi_i$). In this paper all the gauge groups are $SU(2)$ and the diagonal subgroup is identified with the SM weak hypercharge group. The link fields are $2 \times 2$ complex matrices transforming in the fundamental representation of the $i$-th and antifundamental of the $(i+1)$-th gauge group. The orbifolding procedure is accounted for by the fact that the first and the last gauge groups are not linked, thus the quiver diagram has 'topology' of the line segment (see Fig. [1]). The product group is spontaneously broken by the link-Higgs fields which acquire a common expectation value $\langle \Phi_i \rangle = v1$. Diagonalizing the mass matrix, one finds that the spectrum in the large $N$ limit is the same as in the 5D super-YM theory compactified on $S_1/Z_2$. In particular, the link-Higgs degrees of freedom account for completing $\mathcal{N} = 1$ gauge multiplets up to $\mathcal{N} = 2$ at every massive level.
To realize the SM matter and Higgs fields in the bulk we need to deconstruct 5D hypermultiplets. To this end, to every gauge group we attach a set of chiral multiplets: 'Higgs doublets' \( H_i = (h_i, \psi_{H_i}) \) and 'quark doublets' \( Q_i = (q_i, \psi_{Q_i}) \), in the fundamental of the \( i \)-th group and their mirror partners with opposite quantum numbers \( \tilde{H}_i = (\tilde{h}_i, \tilde{\psi}_{H_i}) \), \( \tilde{Q}_i = (\tilde{q}_i, \tilde{\psi}_{Q_i}) \) which complete the spectrum to \( N = 2 \) hypermultiplets. The superpotential is chosen as:

\[
W = \left( \sum_{i=1}^{N-1} y_i^b \tilde{H}_i \Phi_1 H_{i+1} + \sum_{i=1}^{N} m_i^b \tilde{H}_i H_i + \sum_{i=1}^{N-1} y_i^t \tilde{Q}_i \Phi_1 Q_{i+1} - \sum_{i=1}^{N} m_i^t \tilde{Q}_i Q_i \right) 
\]

(1)

To complete the Standard Model quark spectrum we need to add right-handed quark multiplets \( U_i \) and \( D_i \) and their mirrors. Since no color or hypercharge group is present in our toy-model these fields are singlets. The superpotential is chosen as

\[
W = \left( \sum_{i=1}^{N-1} y_i^u \tilde{U}_i U_{i+1} - \sum_{i=1}^{N} m_i^u \tilde{U}_i U_i \right) \quad \text{and analogously for } D_i.
\]

In order that the hypermultiplet mass towers match that of the gauge multiplet one has to fine-tune the parameters of the superpotential \( y_i = g_0, m_i = g_0 v \). For hypermultiplets implementing the \( S_1/Z_2 \) orbifold would consist in removing the mirror multiplets \( \tilde{H}_N, \tilde{Q}_N, \tilde{U}_N, \tilde{D}_N \) at the \( N \)-th site. In general, one removes those fields at the \( N \)-th site which in the 5D picture have negative \( Z_2 \) parity.

In order to get the Yukawa interactions of the Higgs boson with the up-quarks it is sufficient to add to the superpotential the Yukawa term which involves only the superfields from the first site (we omit in our notations the \( SU(2) \) antisymmetric tensor used to build an invariant from two fundamental representations):

\[
W = \lambda Q_1 H_1 U_1
\]

(2)

In the 5D picture this choice corresponds to \textit{brane} Yukawa interactions at \( x_5 = 0 \). In the 4D set-up such a choice is stable in the large \( N \) limit when \( N = 2 \) supersymmetry is recovered. For finite \( N \), when supersymmetry is broken, Yukawa interactions at other sites will be generated at higher-loop level. At the moment we do not have the Yukawa interactions of the Higgs boson with the down-quarks; we comment on this issue at the end of the paper.

At this point one could proceed towards the phenomenological models in the standard way, that is add soft terms to obtain splittings of the multiplets and to trigger the electroweak symmetry breaking. In this paper we investigate an alternative road.

As a first step we investigate loop corrections to the Higgs boson mass from the Yukawa couplings. Generically, the dominant contribution to the one-loop Higgs mass is due to Yukawa interactions with the top quarks. In SM this contribution is quadratically divergent, while in the MSSM the quadratic divergence is canceled by the top squark loops. Here, we analyze the set-up where such boson-fermion cancellation occurs when we break supersymmetry in a \textit{hard way} by removing some of the degrees of freedom in a non-supersymmetric way. For the discussion of divergences it is irrelevant what is the precise pattern of the breaking; the only important thing is that the part of the Lagrangian involving the fields of the first site maintains the supersymmetric form. In particular, we assume that all supertraces at the first site are vanishing.

If the link-Higgs vevs are absent it is clear that at one-loop Yukawa interactions do not feel the supersymmetry breaking on the other end of the chain. Thus, the one-loop radiative correction to the \( h_1 \) squared mass proportional to \( \lambda \) are absent. As soon as we switch on the
link-Higgs vevs, the fields living at different sites are allowed to mix and we have to perform an orthogonal transformation to diagonalize the mass matrix. Since supersymmetry is broken, generically the spectrum is completely non-supersymmetric (boson and fermion masses will be different and there can be a different number of bosonic and fermionic degrees of freedom). However, the $\lambda$-proportional corrections to the Higgs mass are still controlled by the first site and, as a consequence, they are finite! To see this we need to perform an orthogonal transformation to express the original fields in terms of the mass eigenstates: $q_i = \sum_n a^{q}_{i n} q(n)$, $\psi_{Q_i} = \sum_n b^{Q}_{i n} \psi_{Q(n)}$. The zero mode Higgs mass receives one-loop radiative corrections proportional to the Yukawa coupling through the diagrams depicted in Fig. 2, which results in:

\[
-i \delta m^2 = 2 \lambda^2 \int \frac{d^4k}{(2\pi)^4} \left( \sum_n \frac{|a^{q}_{1 n}|^2}{k^2 - m^{q}_{n}} + \sum_{m,n} \frac{|a^{q}_{1 m}|^2}{k^2 - m^{q}_{n}} + \frac{|a^{q}_{1 n}|^2}{k^2 - m^{q}_{m}} \right)
\]

\[
-k^2 \sum_{m,n} \frac{|b^{Q}_{1 n}|^2}{k^2 - m^{Q}_{n}} + \frac{|b^{Q}_{1 m}|^2}{k^2 - m^{Q}_{m}} \right)
\]

(3)

The divergences in this expression are proportional to:

\[
\delta m^2 \sim \Lambda^2 \left( \sum_n |a^{q}_{1 n}|^2 - \left( \sum_n |b^{Q}_{1 n}|^2 \right)^2 \right)
\]

\[
+ \ln \Lambda^2 \left( - \sum_n m^{q^2}_{(n)} |a^{q}_{1 n}|^2 - m^{Q^2}_{(n)} \left( \sum_n |a^{q}_{1 n}|^2 \right)^2 + 2 \sum_{m,n} m^{Q 2}_{(m)} |b^{Q}_{1 n}|^2 |b^{Q}_{1 m}|^2 \right)
\]

(4)

The coefficient of the quadratic divergence vanishes by the fact, that $a^{q}_{1 n}$, $b^{Q}_{1 n}$ are coefficients of the orthogonal transformation diagonalizing the squark and quark squared mass matrices, respectively. Furthermore, orthogonality identities from the diagonalization of the mass matrices also give: $\sum_n m^{q^2}_{(n)} |a^{q}_{1 n}|^2 = (m^{q^2})_{11}$, $\sum_n m^{Q^2}_{(n)} |b^{Q}_{1 n}|^2 = (m^{Q^2} m^{Q^t})_{11}$. This leads to the conclusion

1From now, $x_i$ and $\psi_{X_i}$ will denote respectively the scalar and the chiral fermionic components of the chiral superfield $X_i$. The mass eigenstates will be denoted with parenthesized subscripts: $x_{(m)}$ and $\psi_{X_{(m)}}$ with masses $m^2_{(m)}$ and $m^X_{(m)}$. The Lagrangians involving fermions will be written in two component notation.
that the Higgs mass gets logarithmically divergent contribution proportional to the supertrace in the quark sector at the first site, which we assumed to vanish. Thus, in spite of the fact that the theory is non-supersymmetric, the Higgs mass (in fact the same holds for the squarks) gets, from the Yukawa interactions, only a finite one-loop correction to its mass. These conclusions hold even if the model has a different number of bosonic and fermionic degrees of freedom!

To illustrate this discussion we present a construction inspired by the five-dimensional of the BHN model [5]. Arriving at the spectrum of [5] involves some tunings of the parameters. But we stress that these tunings are by no means important for the cancellation of divergences; they serve only to obtain simple mass matrices, so that formulae for the Higgs boson mass can be evaluated explicitly. So we tune the parameters in the superpotential (1) as:

$$y_i^h = y_i^q = y_i^d = y_i^u = g_0$$

and

$$m_i^h = m_i^q = m_i^d = m_i^u = g_0 v$$

We also add $\Phi_N$ link-Higgs, as in Fig. (3), which we need to avoid a massless gaugino mode.

Figure 3: The magnifying glass view of the $N$-th site of the model. The fields $\phi_N$, $\tilde{h}_N$ and $\psi_{\tilde{Q}_N}$ have been removed in order to break supersymmetry and induce a mass splitting in the low-energy theory. This specific construction mimics the spectrum of the 5D supersymmetric model on $S_1/Z_2 \times Z'_2$. Similarly as in the deconstruction of $S_1/Z_2$ models, the orbifolding procedure amounts to removing those fields from the mirror multiplets at the $N$-th site which in the 5D picture have both $Z_2$ quantum numbers negative, the $(- -)$ states of [5].

In the second step we break supersymmetry by setting $\phi_N$, $\tilde{h}_N$ and $\psi_{\tilde{Q}_N}$ to zero in the Lagrangian (see Fig. 3). This is of course hard breaking of supersymmetry, as some of the fields at the last site lose their superpartners (a similar supersymmetry breaking has also been proposed in Ref. [15]). Moreover, in order for the Higgs mass matrix to have a zero mode we add a soft breaking negative squared mass term, $-g_0^2 v^2 |h_N|^2$. Diagonalizing the mass matrices yields a spectrum as in Table 1. At the massless level we have only the gauge field, quarks and the Higgs boson. Their lightest superpartners have masses $\tilde{m}_{(1)} \sim g_0 v/(2N + 1)$ and include a Dirac gaugino, two squarks for every quark and a Dirac Higgsino. The complete mass matrices and their eigenvector can be found in Appendix A. Within this particular model, we can now get a close expression of the finite radiative Yukawa correction. Let us evaluate it.

2The last step is not necessary as the radiative corrections will drive the lowest level higgs mass negative, even if its tree-level mass is not precisely zero.
Table 1: Spectrum of the model after the diagonal symmetry breaking. The bifundamental link fields are decomposed into irreducible representations of the diagonal \( SU(2) \): \( \psi_{\phi^a} = \sqrt{2} \text{tr}(T^a \psi_\phi) \) transforms as an adjoint, while \( \psi_{\phi^a} = \text{tr}(T^a \psi_\phi) / \sqrt{2} \) transforms as a singlet – similar definitions for the bosonic components hold. Note that the singlet bosonic as well as fermionic fields remain massless, fortunately they will also remain uncharged under the full SM gauge group.

The top-quark Yukawa term \( (2) \) in the superpotential leads to the following interactions:

\[
\mathcal{L} = -\lambda^2 h_1^2 (|u_1|^2 + |q_1|^2) + \lambda g_0 v (h_1 u_1 \tilde{q}_1^\dagger + h_1 q_1 \tilde{u}_1^\dagger + \text{h.c.}) - \lambda (h_1 \psi_{U1} \psi_{Q1} + \text{h.c.})
\]  

(6)

The orthogonal transformations into the mass eigenstates are given in Appendix A. Plugging the results in the mode decomposition we find the following interactions involving the zero mode Higgs doublet \( h(0) \equiv H \):

\[
\mathcal{L} = -\frac{4\lambda^2}{N(2N + 1)} |H|^2 \sum_{n,m=1}^N \left( u(n) u^\dagger_{(m)} + q(n) q^\dagger_{(m)} \right) \cos \left( \frac{(2n - 1)\pi}{4N + 2} \right) \cos \left( \frac{(2m - 1)\pi}{4N + 2} \right) + \frac{4\lambda}{\sqrt{N(2N + 1)}} \sum_{n,m=1}^N \left( H u(n) \tilde{q}^\dagger_{(m)} + H q(n) \tilde{u}^\dagger_{(m)} + \text{h.c.} \right) m_n \cos \left( \frac{(2n - 1)\pi}{4N + 2} \right) \cos \left( \frac{(2m - 1)\pi}{4N + 2} \right) - \frac{2\lambda}{\sqrt{N}} \sum_{n,m=0}^{N-1} \left( H \psi_{U(n)} \psi_{Q(m)} + \text{h.c.} \right) \eta_n \eta_m \cos \left( \frac{n\pi}{2N} \right) \cos \left( \frac{m\pi}{2N} \right)
\]  

(7)

The one-loop radiative correction to the Higgs mass can now be computed explicitly and after some algebra, we arrive at:

\[
\delta m^2 = -\lambda_T^2 (gv)^2 F(\Lambda, N)
\]  

(8)

where \( F(\Lambda, N) \) is a purely numerical factor given by:

\[
F(\Lambda, N) = \pi^{-2} N^3 \int_0^X dx \cosh x \sinh^3 x \frac{(\cosh 2x + 1)(\cosh 2x + 2 \cosh 4Nx - 1)}{\sinh^2 2Nx \cosh^2(2N + 1)x}
\]

(9)

the cutoff, \( X \), being related to the cutoff scale of the theory by \( \Lambda = 2g_0v \sinh X \). The normalized coupling, \( \lambda_T = \lambda / N^{3/2} \), is the Yukawa coupling of the zero mode Higgs to the zero mode quarks,
i.e., the Yukawa coupling of the effective SM; similarly, \( g = \frac{g_0}{\sqrt{N}} \) is the zero mode \( SU(2) \) gauge coupling. Notice that according to our general discussion, \( F(\Lambda, N) \) is finite when \( \Lambda \) goes to infinity. The Fig. 4 illustrates the insensitivity of the Higgs mass to the high energy physics.

\[
F(\Lambda, N)
\]

Figure 4: UV insensitivity of the one-loop Yukawa correction to the Higgs mass. We have plotted for different replication number, \( N \), the numerical factor that enters in the one-loop correction to the Higgs mass as a function of the cutoff scale. We conclude that this factor is completely determined by the IR physics.

Note that in the 4D model, the sums over the KK modes are finite and one can freely exchange the sum with the integral. Thus, contrary to the 5D models, one does not rely on the procedure of KK-regularization, which is sometimes questioned \([18]\) or at least requires a careful treatment of the symmetries of the theory \([19]\). In a sense, the 4D model can be considered a regularization of the construction \([5]\), which truncates the KK-tower at a finite value, but still preserves the finiteness properties.

We have shown in general that one-loop corrections in certain non-supersymmetric theories can be surprisingly softened. What about two- and higher-loop corrections? The one-loop cancellation of quadratic divergences depends crucially on the tree-level equality of the Yukawa and 4-scalar couplings of the Higgs field on the first site. However, due to the mass splitting between quarks and squarks these couplings are renormalized differently.\(^3\) Thus we expect

\(^3\)More precisely, at one-loop the infinite part of renormalization is equal, thus one-loop running of both coupling is the same. The difference is in the finite part of the renormalization.
quadratic divergences to reappear at the two-loop level.

We come back to the general analysis of the model defined by the superpotential (1). Except for the top-Yukawa couplings there are other sources of quadratic divergences which are proportional to the gauge coupling or to the Yukawa couplings to the link-Higgs. Following the discussion of the top-Yukawa contributions we analyze the general conditions to avoid any quadratic divergences. The first potential source originates from the couplings of the Higgs field to the gauge multiplet (and to itself in the D-term scalar potential) and the relevant coupling leading to quadratic divergences are:

\[
\mathcal{L} = \sum_i \left( g_0^2 h_i^T A_{\mu i} A_{\mu i} - \frac{1}{2} g_0^2 \left( h_i^T h_i \right)^2 + \left( i \sqrt{2} g_0 h_i^T \phi_i \right) \right)
\]

(10)

The second source comes from the F-term of the superpotential (1) and the dangerous terms which yield quadratic divergences are:

\[
\mathcal{L} = - \sum_i \left( y_i^2 |h_i|^2 |h_{i-1}|^2 - y_{i-1}^2 h_i^T \phi_{i-1} \phi_{i-1} h_i - \left( y_i \psi_i \tilde{H}_{i-1} \psi_{i-1} h_i + \text{h.c.} \right) \right)
\]

(11)

Here, the situation is qualitatively different than in the case of top-Yukawa contribution, as interactions occur at all sites. To avoid quadratic divergences proportional to \( g_0 \) we have to ensure that at every site the full Higgs multiplet interacts with the full gauge multiplet. In particular we need \( N \) gauginos. When the link-Higgs fields get vevs, the gauginos get Dirac masses through bilinear coupling to the \( N \) Dirac mass term. In our specific model we chose to add the Dirac mass terms for gauginos or in a hard way — coupling gauginos and link-Higgsinos to add supersymmetry breaking terms. This can be done either in a soft way — by adding the particular we need \( N \) to ensure that at every site the full Higgs multiplet interacts with the full gauge multiplet. In as interactions occur at all sites. To avoid quadratic divergences proportional to \( \text{links-Higgs and mirror degrees of freedom at the } N \)-th site has no consequence for the divergence which yield quadratic divergences are:

\[
\mathcal{L} = \sum_i \left( y_i^2 |h_i|^2 |h_{i-1}|^2 - y_{i-1}^2 h_i^T \phi_{i-1} \phi_{i-1} h_i - \left( y_i \psi_i \tilde{H}_{i-1} \psi_{i-1} h_i + \text{h.c.} \right) \right)
\]

This gives us, in the large \( N \) limit, the same mass spectrum of gauginos as in [5].

Similarly, for quadratic divergences proportional to \( y_i \) to be absent, we need full link and mirror multiplets to be present at the \( i \)-th site. Note that, since \( y_N \equiv 0 \), adding or removing links-Higgs and mirror degrees of freedom at the \( N \)-th site has no consequence for the divergence of the Higgs mass. We used this fact in our model and placed the hard supersymmetry breaking sector at the \( N \)-th site.

In our specific model these interaction are all proportional to the gauge coupling (as we tuned the link-Yukawa couplings with the gauge coupling). The diagrams that contribute to the one-loop zero mode Higgs mass are depicted in Fig 5. They give:

\[
\delta m^2 = -g^4 v^2 G(\Lambda, N)
\]

(12)

where \( g \) is the zero mode \( SU(2) \) gauge coupling and \( v \) is the deconstruction scale. The numerical factor \( G(\Lambda, N) \) is given in terms of a complicated integral:

\[
G(\Lambda, N) = \frac{N}{\pi^2} \int_0^{\Lambda/(2g_0v)} dx \ x^3 \left( \frac{7}{2} \sum_{n=1}^{N} \frac{1}{x^2 + \sin^2 \frac{(2n-1)\pi}{4N+2}} - \frac{1}{2} \sum_{n=1}^{N-1} \frac{1}{x^2 + \sin^2 \frac{\pi n}{2N} - 3 + \frac{1}{2} \frac{1}{x^2}} \right)
\]

\[
- \frac{14}{2N+1} \sum_{n=1}^{N} \frac{\cos^2 \frac{(2n-1)\pi}{4N+2}}{x^2 + \sin^2 \frac{(2n-1)\pi}{4N+2}} + \frac{24}{(2N+1)^2} \sum_{m,n=1}^{N} \frac{x^2 \cos^2 \frac{(2m-1)\pi}{4N+2} + \cos^2 \frac{(2m-1)\pi}{4N+2}}{(x^2 + \sin^2 \frac{(2m-1)\pi}{4N+2})(x^2 + \sin^2 \frac{(2m-1)\pi}{4N+2})}
\]

(13)
\[ \delta m^2 = \frac{6}{8\pi^2} g^2 v^2 N \ln \left( \frac{\Lambda}{2gv} \right)^2 \]  

(14)

The cutoff dependence is similar as in the softly broken supersymmetry, but the \( M_{\text{SUSY}} \) scale is replaced here by the deconstruction scale \( v \). If \( v \) is close to the weak scale (which is the case as long as \( N \) is not too large) then the one-loop sensitivity to the cutoff is weak.

The previous evaluation of the Yukawa and gauge radiative corrections to the zero mode Higgs mass suggests that they will trigger the electroweak symmetry breaking. To study in full details this breaking, we need now to compute the one-loop effective potential given by:

\[ V = \frac{1}{2} \text{Tr} \int \frac{d^4 k_E}{(2\pi)^4} \ln \frac{k_E^2 + m_b^2(v_H)}{k_E^2 + m_f^2(v_H)} \]  

(15)

where \( m_b \) and \( m_f \) are respectively the bosonic and fermionic mass matrices as functions of the vev of the Higgs field, \( v_H \). We consider only the top-multiplet contribution and the dependence on the Higgs vev is coming from the Yukawa interactions localized on the first site only.

For the stop sector, in the basis \((u_{(m)} R, \tilde{u}_{(n)} L)\) and \((u_{(m)} L, \tilde{u}_{(n)} R)\), we obtain the following \(2N \times 2N\) squared mass matrix \((m, n, p, q = 1 \ldots N)\):

\[
\begin{pmatrix}
\tilde{m}^2_{(m)} \delta_{mp} + a_b c(m) c(p) & b_b \tilde{m}_{(m)} c(m) c(q) \\
 b_b \tilde{m}_{(p)} c(m) c(p) & \tilde{m}^2_{(n)} \delta_{nq}
\end{pmatrix}
\]  

(16)

where we have defined: \( c(m) = \cos \frac{(2m-1)\pi}{4N+2} \), and the two coefficients \( a_b \) and \( b_b \) are related to the Yukawa coupling as:

\[
a_b = \frac{4\lambda^2 v_H^2}{(2N+1)N} \quad \text{and} \quad b_b = -\frac{4\lambda v_H}{(2N+1)\sqrt{N}}
\]  

(17)

\[4\] In this paper, our convention is to define \( v_H \) as the vev of the complex Higgs field, i.e., \( v_H \sim 174 \text{ GeV} \).
Note that squarks mix with mirror squarks once the electroweak symmetry is broken.

Similarly in the \( \textbf{top sector} \), in the basis \((\psi_\bar{\psi} U_L, m), (\psi \psi U_R, n)\), the \((2N - 1) \times (2N - 1)\) squared mass matrix reads \((m, p = 0 \ldots N - 1, n, q = 1 \ldots N - 1)\):

\[
\begin{pmatrix}
 m_{(m)}^2 \delta_{mp} + a_f \eta_m \eta_p d(m) d(p) \\
 b_f \eta_p m_{(m)} d(m) d(p) \\
 b_f \eta_p m_{(n)} d(m) d(p) \\
 m_{(n)}^2 \delta_{nq}
\end{pmatrix}
\]

where we have now defined \(d(m) = \cos \frac{m \pi}{2N}\), and the two coefficients \(a_f\) and \(b_f\) are given by:

\[
a_b = \frac{2 \lambda^2 v_H^2}{N^2} \quad \text{and} \quad b_b = -\frac{2 \lambda v_H}{N \sqrt{N}}
\]

We will discuss in a moment the diagonalization of these matrices. But first, we would like to show that two important supertraces are independent of the vev of the Higgs. Indeed:

\[
\begin{align*}
\text{STr}_\text{top} M^2 &= 2 \left( \sum_{m=1}^{N} \tilde{m}_m^2 - \sum_{m=1}^{N-1} m_m^2 \right) + \frac{2N + 1}{4} a_b - \frac{N}{2} a_f = 2 g_0^2 v^2 \\
\text{STr}_\text{top} M^4 &= 2 \left( \sum_{m=1}^{N} \tilde{m}_m^4 - \sum_{m=1}^{N-1} m_m^4 \right) + \frac{(2N + 1)^2}{16} a_b^2 - \frac{N^2}{4} a_f^2 \\
&\quad + 2 \left( a_b + \frac{2N + 1}{4} b_b^2 \right) \left( \sum_{m=1}^{N} \tilde{m}_m^2 \cos^2 \left( \frac{(2m - 1)\pi}{4N + 2} \right) \right) \\
&\quad - 2 \left( a_f + \frac{N}{2} b_f^2 \right) \left( \sum_{m=1}^{N} m_m^2 \cos^2 \frac{m \pi}{2N} \right)
= 6 g_0^4 v^4
\end{align*}
\]

This ensures that the one-loop potential for \( v_H \) has no divergent dependence on the cutoff of the theory: the EWSB is triggered by the low energy physics and is not dominated by unknown physics that will be revealed at or above the cutoff scale. Adding the tree-level Higgs self-coupling originating from the D-terms, we get:

\[
V(v_H) = \frac{1}{8} g^2 v_H^4 + \frac{3}{16 \pi^2} \text{STr}_\text{top} m^4 \ln \left( \frac{m^2}{2g_0 v} \right)
\]

where the supertrace is over the \(2N\) bosonic and \(2N - 1\) fermionic eigenvalues of the matrices \((16)-(18)\).

Let us now turn to the determination of the spectrum. Using the general formula for the determinants, the secular equation of the stop squared mass matrix is given by:

\[
1 - \frac{16 \lambda^2 v_H^2}{N(2N + 1)^2} \rho^2 \left( \sum_{m=1}^{N} \frac{\cos^2 \left( \frac{(2m - 1)\pi}{4N + 2} \right)}{m_m^2} \right)^2 = 0
\]

\(^5\)Once the \(U(1)\) hypercharge interactions are included along the line discussed later, the gauge coupling, \(g\), in the tree level term is replaced by \(\sqrt{g^2 + g'^2}\), where \(g'\) is the \(U(1)\) gauge coupling. The numerical simulations reported at the end of the paper include of course the \(U(1)\) interactions.
\[
\text{Table 2: Spectrum of the model after the EWSB for different values of the replication number, } N. \\
\begin{array}{|c|c|c|c|c|}
\hline
N = 2 & 158 & 142 & 502 & 437 \\
N = 3 & 166 & 158 & 532 & 565 \\
N = 4 & 170 & 161 & 533 & 664 \\
N = 5 & 172 & 167 & 537 & 745 \\
N = 10 & 178 & 164 & 517 & 1051 \\
\hline
\end{array}
\]

which, using some remarkable identities, can be rewritten as a polynomial equation of degree 2\(N\):

\[
RT_N(1 - x^2) = \tau x RU_{N-1}(1 - x^2)
\]

where \(x\) is the dimensionless eigenvalue \(x = \rho/(2g_0v)\), \(\tau = \lambda_T v_H N/(g_0 v)\) characterizes the Higgs vev in units of \(g_0 v\), and \(RT_N\) and \(RU_{N-1}\) are the reduced Chebyshev polynomials:

\[
RT_N(X) = \frac{T_{2N+1}(\sqrt{X})}{\sqrt{X}} \quad \text{and} \quad RU_{N-1}(X) = \frac{U_{2N-1}(\sqrt{X})}{\sqrt{X}}
\]

Similarly, the fermionic secular equation is

\[
1 - \frac{4\lambda^2 v_H^2}{N^3} \rho^2 \left( \sum_{m=0}^{N-1} \eta_m^2 \frac{\cos^2 \frac{m\pi}{2N}}{m^2 - \rho^2} \right)^2 = 0
\]

and it can be written in the form of a polynomial equation of degree \(2N - 1\):

\[
RT_{N-1}(1 - x^2) = \tau^{-1} x RU_{N-1}(1 - x^2)
\]

All the pieces are now set to numerically evaluate the potential \(V(v_H)\) and find its minimum. The results are plotted in Fig. 6 for different values of the replication number \(N\). The Higgs mass after EWSB becomes a function of low energy parameters only: the top Yukawa coupling, the Higgs vev, the electroweak gauge coupling and the replication number. The deconstruction scale, \(v\) is indeed fixed in terms of the low energy parameters once the Higgs vev is fitted to its phenomenological value. Note that the relation between the top mass and the top Yukawa coupling is modified compare to the Standard Model since, here, the top mass corresponds to the lowest eigenvalue of the fermionic mass matrix (18). Some numerical results are given in Table 2 for the Higgs mass, the stop mass, the first KK excitation of the top and the deconstruction scale, \(v\). Interestingly enough, after EWSB, the stop is lighter than the top.

Finally, we comment on how the toy-model presented here can be extended to match the Standard Model phenomenology. The obvious missing pieces are:

- Leptons
  One can replicate the lepton superfields, \(SU(2)\) doublets \(L_i\) and singlets \(E_i\), and write superpotential exactly in the same way as for the quarks. Analogously, one introduces Yukawa interactions of the leptons and the Higgs at the first site.
Figure 6: One-loop effective potential for the Higgs scalar field for different values of the replication number $N$. As in softly broken supersymmetric theories, radiative corrections trigger EWSB. The Higgs vev is a function of the top Yukawa coupling, the SM gauge coupling constants and the deconstruction scale. Fitting this Higgs vev to its phenomenological value, we get the prediction for the Higgs mass. The UV insensitivity of the Higgs one-loop effective potential ensures that the EWSB is reliably computed in our effective theory.

- **SU(3) color group**
  In the real world quarks transform in the $3$ or $\bar{3}$ representation of the color group. Replicating $SU(3)$ gauge group so as to obtain only one octet of massless gluon and superpartners separated by a mass gap does not bring any complication. One introduces a set of link-Higgs superfields $\Gamma_i$ transforming as $(3_i, \bar{3}_{i+1})$ and the rest of the construction is analogous to the $SU(2)$ case. The problem appears when we want to obtain the KK-tower for the quark doublets; the gauge invariant superpotential must now have the form $W = g_0 \sum (\pm \bar{Q}_i \Phi_i \Gamma_i Q_{i+1} - v \bar{Q}_i Q_i)$ and leads to non-renormalizable interactions. A more satisfactory alternative which allows to maintain renormalizability is not to replicate the color gauge group and assume that all quark superfields $Q_i$, $U_i$ and $D_i$ are charged under a single $SU(3)$. Then the superpotential for these superfields has the same form as in the pure $SU(2)$ case. Of course, then the model has no extra-dimensional interpretation.
but this does not change any conclusions about softness of the radiative corrections. It is nothing but a deconstructed version of a brane-world scenario where QCD interactions are localized on the brane while weak interactions are free to propagate in the bulk.

- \(U(1)\) hypercharge group
  Similar problems as in the \(SU(3)\) case arise when we replicate the hypercharge group: writing an invariant superpotential so as to get the KK-tower of quarks and leptons implies non-renormalizable interactions. In addition, one must worry about anomalies, which must be compensated, \(e.g.\), by deconstructed Wess–Zumino terms \([20]\). Therefore, the more plausible alternative is not to replicate the hypercharge group. One avoids non-renormalizable interactions and as a byproduct the anomalies automatically cancel. Indeed, the fermion spectrum at all sites but the last one is vector-like: every fermion is accompanied by the mirror fermion with opposite quantum numbers. At the \(N\)-th site the fermion spectrum is the same as in the MSSM. Note that in the 5D model of Ref. \([5]\), the issue of the \(U(1)\) anomaly is more subtle \([9, 21, 22]\).

- Down-quark masses
  In the supersymmetric models with only one light Higgs doublet there is the well-known problem of coupling the neutral Higgs boson to the down quarks, as holomorphicity forbids the \(H^1QD\) term in the superpotential. A novel way to circumvent this problem was worked out in Ref. \([5]\). It was noted that at the \(Z'_2\) fixed point one can write Yukawa interactions between the multiplets of the second supersymmetry, which is not projected out at that point. In the models of deconstruction this solution has no direct analogue, as the second supersymmetry appears only in the large \(N \to \infty\) limit. Nevertheless, at the \(N\)-th site one can still construct multiplets \(H^N = (h^+_N, \psi_{H_N}), Q'_N = (\tilde{q}^+_N, \psi_{Q_N}), D'_N = (\tilde{d}^+_N, \psi_{D_N})\) and write the Yukawa interactions as if resulting from the superpotential term \(W = \lambda_D H^N Q'_N D'_N:\)

\[
\mathcal{L} = -\lambda_D^2 \left( \bar{d}^+_N \tilde{q}^+_N \tilde{d} + \bar{h}^+_N h_N \bar{d}^+_N \tilde{d} + \bar{q}^+_N \tilde{h}^+_N h_N \bar{q} \right) \lambda_D g_0 v \left( \bar{h}^+_N \tilde{q}^+_N d + \bar{h}^+_N \tilde{d}^+_N q + \bar{q}^+_N \tilde{d}^+_N \tilde{h} + \text{h.c.} \right) - \lambda_D \left( h_N \tilde{q} \psi_{D_N} + \tilde{q} \psi_{H_N} \psi_{D_N} + \bar{d} \psi_{H_N} \psi_{Q_N} + \text{h.c.} \right)
\]

It is easy to check that, similarly as the Yukawa interactions \(6\) at the first site, the quark and squark loops resulting from \(28\) do not produce any divergences in the Higgs mass.

So far supersymmetry is the best ingredient to protect the Higgs mass from high energy physics. Unfortunately, a new sector plugged by unknowns and even some fine-tunings is called for to break supersymmetry and to allow for a phenomenologically viable spectrum. Recently, Barbieri, Hall and Nomura proposed to circumvent this supersymmetry breaking sector; the supersymmetry breaking would rather have a geometrical origin as resulting from a compactification on a non-trivial orbifold. We propose a four-dimensional set-up, relying on a fully renormalizable theory, that shares similar properties. Electroweak symmetry breaking is triggered by radiative corrections protected by some remnants of supersymmetry, yet completely broken. The Higgs mass, at one loop, is insensitive to UV physics and can be computed in
terms of low energy parameters only. In the past few years, several non supersymmetric four-dimensional models have been proposed \cite{23, 24, 25} in which the one-loop Higgs potential is not destabilized by radiative corrections, however higher loop corrections are much more dangerous if not vexatious \cite{26}. Localization of supersymmetry breaking in the theory space of deconstruction models might be here the key to protect the predictability of the theory at higher loops.

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Appendix A  Mass matrices and mode decomposition

In this appendix we give the explicit formulae for the mass matrices and mode decomposition into mass eigenstates in the model considered in the paper. As it does not introduce any complication, we can be more general here and consider a chain of $SU(K)$ groups with arbitrary $K$.

Let us begin with the gauge sector. We consider $N$ super-Yang-Mills $SU(K)$ multiplets $(A^a_i, \chi^a_i)$ communicating to each other through $N$ link-Higgs chiral multiplets $(\phi_i, \psi_i)$ transforming as $(K_i, \bar{K}_{i+1})$; in other words links are $K \times K$ complex matrices transforming as $\Phi_i \rightarrow U_i \Phi_i U_{i+1}^\dagger$. The scalar $\phi_N$ is removed from the Lagrangian while the $N$-th link-Higgsino transforms as $(K_N, \bar{K}_N)$. The D-term potential has vacua for $\langle \Phi_i \rangle = v_i 1$. If we take the special point on the moduli space where all vevs are equal (and real) $v_i = v$, then the product gauge group is broken down to its diagonal subgroup. The mass term for the gauge bosons are \cite{12}:

$$\mathcal{L} = \frac{1}{2} g_0^2 v^2 \sum_{i=1}^{N-1} (A^\mu_i - A^\mu_{i+1})^2$$  \hspace{1cm} (A.1)

This mass term is diagonalized by the following orthogonal transformation:

$$A_j = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \eta_n \cos \left( \frac{(2j - 1)n\pi}{2N} \right) A_{(n)}$$ \hspace{1cm} (A.2)

where $\eta_0 = 1/\sqrt{2}$ and otherwise $\eta_n = 1$. The spectrum of gauge boson contains a massless mode $A_{(0)}$ corresponding to the unbroken diagonal $SU(K)$ and a tower of gauge bosons $A_{(n)}$ with masses:

$$m_{(n)} = 2g_0 v \sin \left( \frac{n\pi}{2N} \right) \quad n = 1 \ldots N - 1$$ \hspace{1cm} (A.3)
In the large $N$ limit the gauge boson masses are $m_{(n)} \approx \frac{g_0 v \pi}{2N} 2n$. This matches the gauge boson spectrum of the BHN model upon identification:

$$\frac{1}{R} \sim \frac{g_0 v \pi}{2N}$$  \hspace{1cm} (A.4)$$

This differs from the aliphatic case by a factor of one half, which can be easily understood as resulting from orbifolding the circle twice.

The scalar link-Higgs degrees of freedom split into:

- $N - 1$ real scalars $G_i^a = i Tr [T^a (\phi_i - \phi_i^\dagger)]$ in adjoint representation of the diagonal group. These are massless (in the Landau gauge) Goldstone boson which become the longitudinal components of the massive gauge fields. Their counterpart in the 5D models are KK-modes of the fifth component of the gauge field.

- $N - 1$ real scalars $\phi_i^a = Tr [T^a (\phi_i + \phi_i^\dagger)]$ in adjoint representation of the diagonal group. When the link Higgs fields get vev, the D-term potential generates mass terms:

$$L = \frac{1}{2} g_0^2 v^2 \sum_{i=1}^{N} (\phi_{i-1}^a - \phi_i^a)^2 \hspace{1cm} \phi_0^a \equiv \phi_N^a \equiv 0$$  \hspace{1cm} (A.5)$$

The mass matrix is diagonalized by the orthogonal transformation:

$$\phi_j^a = \sqrt{\frac{2}{N}} \sum_{n=1}^{N-1} \sin \left( \frac{j n \pi}{N} \right) \phi_{(n)}^a$$  \hspace{1cm} (A.6)$$

The $n$-th mode $\phi_{(n)}^a$ has mass equal to $m_n$, matching at every massive level that of the gauge boson. Thus, $\phi_{(n)}^a$ corresponds to an adjoint scalar of 5d $\mathcal{N} = 2$ gauge multiplets.

- $N - 1$ complex scalars $\phi_i^a = \frac{1}{\sqrt{K}} Tr [\phi_i - v]$, singlets under the diagonal subgroup. They correspond to flat directions of the D-term potential and have no counterpart in the 5D supersymmetric $SU(K)$ model. One can get rid of them [17] but in these paper we kept them in the physical spectrum. However, if we add a product $U(1)$ hypercharge group to our model (this implies that the link Higgs also must also have the $U(1)$ charge $(1/2, -1/2)$) then the real part of $\phi_i^a$ turns into a tower of massive scalars with masses $m_n^2 \Phi_0^a$ ($g_0'$ being the gauge coupling of the $U(1)$ interactions), while the imaginary part is eaten by the massive modes of the $U(1)$ gauge fields.

We now turn to the fermionic part of the gauge sector. Similarly as for the link-scalars, we split the link-Higgsino degrees of freedom into adjoints, $\psi_i^a = \sqrt{2} Tr [T^a \Psi_{\phi_i}]$, and singlets, $\psi_i^a = \frac{1}{\sqrt{K}} Tr [\Psi_{\phi_i}]$, with respect to the diagonal group. When the link-Higgs field get vev, the adjoints $\psi_i^a$ mix with gauginos. In addition, we need to add by hand a mass term $i g_0 v \psi_{\phi_N}^a \chi_N^a$. The full mass matrix becomes:

$$L = i g_0 v \sum_{i=1}^{N} (\psi_i^a - \psi_{i-1}^a) \chi_i^a + h.c$$  \hspace{1cm} (A.7)$$
which is diagonalized by:

\[ \chi^a_j = \sqrt{\frac{2}{N+1/2}} \sum_{n=1}^{N} \cos \left( \frac{(2j-1)(2n-1)\pi}{2(N+1)} \right) \chi^a_n \]

\[ \psi^a_{\phi_j} = \sqrt{\frac{2}{N+1/2}} \sum_{n=1}^{N} \sin \left( \frac{j(2n-1)\pi}{2(N+1)} \right) \psi^a_n \]  

(A.8)

The Dirac fermion \( \lambda_{(n)} = \begin{pmatrix} \psi^a_{(n)} \\ -\sigma_2 \chi^a_{(n)} \end{pmatrix} \) acquires mass:

\[ \tilde{m}_{(n)} = 2g_0v \sin \left( \frac{(2n-1)\pi}{2(N+1)} \right) \quad n = 1 \ldots N \]  

(A.9)

The singlets \( \psi^a_{\phi} \) remain massless, as long as links do not have any \( U(1) \) charges. Apart of the singlets, in the large \( N \) limit we recover the mass tower of the BHN model.

The Higgs sector is represented by the set of chiral multiplets \( H_i = (h_i, \psi^a_{H_i}) \) in the fundamental representation of the \( i \)-th group, and their antifundamental mirrors \( \tilde{H}_i = (\tilde{h}_i, \psi^a_{\tilde{H}_i}) \). These degrees of freedom acquire their masses from the F-term potential. In order to obtain the same mass tower as in the gauge sector we choose the superpotential of the form:

\[ W = g_0 \left( \sum_{i=1}^{N-1} \bar{H}_i \Phi_i H_{i+1} - \sum_{i=1}^{N} v \bar{H}_i H_i \right) \]  

(A.10)

At the \( N \)-th site we remove the mirror Higgs \( \tilde{h}_N \) and add the scalar mass term \( \Delta \mathcal{L} = g_0^2v^2h_N^2 \) (otherwise the Higgs field would not have a zero mode). After the diagonal symmetry breaking the scalar mass matrix is:

\[ \mathcal{L} = -g_0^2v^2 \sum_{i=1}^{N-1} |h_{i+1} - h_i|^2 - g_0^2v^2 \sum_{i=1}^{N} |\bar{h}_{i-1} - \bar{h}_i|^2 \quad \bar{h}_0 \equiv \bar{h}_N \equiv 0 \]  

(A.11)

which is diagonalized by:

\[ h_j = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \eta_n \cos \left( \frac{(2j-1)n\pi}{2N} \right) h_{(n)} \]

\[ \bar{h}_j = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \sin \left( \frac{jn\pi}{N} \right) \bar{h}_{(n)} \]  

(A.12)

Both Higgs towers have masses \( m_{(n)} \) but only \( h \) has the zero mode.

The mass matrix of the Higgsini is:

\[ \mathcal{L} = -g_0v \sum_{i=1}^{N} \psi^a_{\bar{H}_i} (\psi_{H_{i+1}} - \psi_{H_i}) + h.c \]  

(A.13)

and is diagonalized by:

\[ \psi_{H_j} = \sqrt{\frac{2}{N+1/2}} \sum_{n=1}^{N} \cos \left( \frac{(2j-1)(2n-1)\pi}{2(N+1)} \right) \psi_{H_{(n)}} \]

\[ \psi_{\bar{H}_j} = \sqrt{\frac{2}{N+1/2}} \sum_{n=1}^{N} \sin \left( \frac{j(2n-1)\pi}{2N+1} \right) \psi_{\bar{H}_{(n)}} \]  

(A.14)
The Dirac fermion $\Psi_{H^{(n)}} = \left( \begin{array}{c} \psi_{H^{(n)}} \\ -i \sigma_2 \psi_{H^{(n)}}^* \end{array} \right)$ acquires mass $\tilde{m}_{(n)}$.

As for the quark doublet degrees of freedom, we again start with a set of fundamental chiral multiplets $Q_i = (q_i, \psi_{Q_i})$ and their antifundamental mirrors $\tilde{Q}_i = (\tilde{q}_i, \psi_{\tilde{Q}_i})$. This time we remove the mirror quark $\psi_{\tilde{Q}_N}$ at the $N$-th site. The squark mass matrix is:

$$L = -g_0^2 v^2 \sum_{i=1}^{N} |q_{i+1} - q_{i}|^2 - g_0^2 v^2 \sum_{i=1}^{N} |\tilde{q}_{i-1} - \tilde{q}_{i}|^2 \quad q_{N+1} \equiv \tilde{q}_{0} \equiv 0$$

(A.15)

which is diagonalized by:

$$q_j = \sqrt{\frac{2}{N+1/2}} \sum_{n=1}^{N} \eta_n \cos \left( \frac{(2j-1)(2n-1)\pi}{4N+2} \right) q_{(n)}$$

$$\tilde{q}_j = \sqrt{\frac{2}{N+1/2}} \sum_{n=1}^{N} \sin \left( \frac{j(2n-1)\pi}{4N+2} \right) \tilde{q}_{(n)}$$

(A.16)

Both squark towers have masses $\tilde{m}_{(n)}$ and there is no zero mode. The quark mass matrix:

$$L = -g_0 v \sum_{i=1}^{N} \psi_{\tilde{Q}_i} (\psi_{Q_{i+1}} - \psi_{Q_i}) + \text{h.c} \quad \psi_{\tilde{Q}_N} \equiv 0$$

(A.17)

is diagonalized by:

$$\psi_{Q_j} = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \eta_n \cos \left( \frac{(2j-1)n\pi}{2N} \right) \psi_{Q_{(n)}}$$

$$\psi_{\tilde{Q}_j} = \sqrt{\frac{2}{N}} \sum_{n=1}^{N-1} \sin \left( \frac{jn\pi}{N} \right) \psi_{\tilde{Q}_{(n)}}$$

(A.18)

The Dirac fermion $\Psi_{Q^{(n)}} = \left( \begin{array}{c} \psi_{Q^{(n)}} \\ -i \sigma_2 \psi_{Q^{(n)}}^* \end{array} \right)$ acquires mass $m_{(n)}$ and there is single chiral zero mode $\psi_{Q^{(0)}}$.

**Appendix B  Trigonometric sums and determinant**

We collect in this appendix the results of different trigonometric sums that appear throughout the paper. First, the normalizations of the mass eigenstates modes of the different fields follow from the identities:

$$\sum_{k=0}^{N-1} \eta_k^2 \cos \left( \frac{(2i-1)k\pi}{2N} \right) \cos \left( \frac{(2j-1)k\pi}{2N} \right) = \frac{N}{2} \delta_{ij} \quad i, j = 1 \ldots N$$

(B.1)

$$\sum_{k=1}^{N-1} \sin \left( \frac{ik\pi}{N} \right) \sin \left( \frac{jk\pi}{N} \right) = \frac{N}{2} \delta_{ij} \quad i, j = 1 \ldots N - 1$$

(B.2)

$$\sum_{k=1}^{N} \cos \left( \frac{(2i-1)(2k-1)\pi}{4N+2} \right) \cos \left( \frac{(2j-1)(2k-1)\pi}{4N+2} \right) = \frac{2N+1}{4} \delta_{ij} \quad i, j = 1 \ldots N$$

(B.3)

$$\sum_{k=1}^{N} \sin \left( \frac{i(2k-1)\pi}{2N+1} \right) \sin \left( \frac{j(2k-1)\pi}{2N+1} \right) = \frac{2N+1}{4} \delta_{ij} \quad i, j = 1 \ldots N$$

(B.4)
The computation of the numerical loop factor in the Yukawa correction to the Higgs squared mass uses the two relations:

\[
\sum_{k=0}^{N-1} \eta_k^2 \frac{\cos^2 \frac{k\pi}{2N} \sinh^2 x + \sin^2 \frac{k\pi}{2N}}{\sinh 2N x \sinh x} = N \frac{\cosh(2N-1)x}{\sinh 2Nx} \sinh x
\]

(B.5)

\[
\sum_{k=1}^{N} \frac{\cos^2 \frac{(2k-1)\pi}{4N+2} \sin^2 \frac{(2k-1)\pi}{4N+2}}{\sinh^2 x + \sin^2 \frac{(2k-1)\pi}{4N+2}} = \frac{2N+1}{2} \frac{\sinh 2Nx}{\cosh(2N+1)x \sinh x}
\]

(B.6)

And finally to compute the logarithmic gauge correction to the Higgs squared mass, we need to know:

\[
\sum_{k=1}^{N} \sin \frac{(2k-1)\pi}{4N+2} = \frac{2N-1}{4}
\]

(B.7)

\[
\sum_{k=1}^{N} \sin \frac{k\pi}{2N} = \frac{N-1}{2}
\]

(B.8)

\[
\sum_{k=1}^{N} \cos \frac{(2k-1)\pi}{4N+2} \sin \frac{(2k-1)\pi}{4N+2} = \frac{2N+1}{8}
\]

(B.9)

The computation of the two supertraces (20)–(21) requires the evaluation of the two sums:

\[
\sum_{k=1}^{N-1} \sin^4 \frac{k\pi}{2N} = \frac{3N-4}{8}
\]

(B.10)

\[
\sum_{k=1}^{N} \sin^4 \frac{(2k-1)\pi}{4N+2} = \frac{6N-5}{16}
\]

(B.11)

We finish this appendix by a formula for the determinant leading to the mass spectrum of bosons and fermions after EWSB \((m, p = 1 \ldots M, n, q = 1 \ldots N)\):

\[
\begin{align*}
\det & \left( (m_m^2 - \rho^2)\delta_{mp} + f_m g_p f_m c_q, d_m g_p (\tilde{m}_n^2 - \rho^2)\delta_{nq} \right) \\
& = \prod_{m=1}^{M} (m_m^2 - \rho^2) \prod_{n=1}^{N} (\tilde{m}_n^2 - \rho^2) \left( 1 + \sum_{m=1}^{M} \frac{f_m g_m}{m_m^2 - \rho^2} \right) \left( 1 - \sum_{n=1}^{N} \frac{c_n d_n}{\tilde{m}_n^2 - \rho^2} \right)
\end{align*}
\]

(B.12)

References

[1] I. Antoniadis, Phys. Lett. B246 (1990) 377.

[2] I. Antoniadis, C. Muñoz and M. Quirós, Nucl. Phys. B397 (1993) 515 [hep-ph/9211309]; I. Antoniadis and K. Benakli, Phys. Lett. B326 (1994) 69 [hep-th/9310151].
[3] Y. Hosotani, *Phys. Lett.* **B126** (1983) 309; P. Fayet, *Nucl. Phys.* **B237** (1984) 367. H. Hatanaka, T. Inami and C. S. Lim, *Mod. Phys. Lett.* **A13** (1998) 2601 [hep-th/9805067]; H. Hatanaka, *Prog. Theor. Phys.* **102** (1999) 407 [hep-th/9905100]; G. R. Dvali, S. Randjbar-Daemi and R. Tabbash, [hep-ph/0102307]; I. Antoniadis, K. Benakli and M. Quiros, [hep-th/0108003].

[4] I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quirós, *Nucl. Phys.* **B544** (1999) 503 [hep-ph/9810410]; A. Delgado, A. Pomarol and M. Quirós, *Phys. Rev.* **D60** (1999) 095008 [hep-ph/9812489]. I. Antoniadis, K. Benakli and M. Quirós, *Nucl. Phys.* **B583** (2000) 35 [hep-ph/0004091].

[5] R. Barbieri, L. J. Hall and Y. Nomura, *Phys. Rev.* **D63** (2001) 105007 [hep-ph/0011311].

[6] N. Arkani-Hamed, L. J. Hall, Y. Nomura, D. R. Smith and N. Weiner, *Nucl. Phys.* **B605** (2001) 81 [hep-ph/0102090].

[7] J. Scherk and J. H. Schwarz, *Phys. Lett.* **B82** (1979) 60; *Nucl. Phys.* **B153** (1979) 61; P. Fayet, *Nucl. Phys.* **B263** (1986) 649; C. Kounnas and M. Porrati, *Nucl. Phys.* **B310** (1988) 355; S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, *Nucl. Phys.* **B318** (1989) 75; C. Kounnas and B. Rostand, *Nucl. Phys.* **B341** (1990) 641; I. Antoniadis and M. Quirós, *Nucl. Phys.* **B505** (1997) 109 [th/9705037]; E. Dudas and C. Grojean, *Nucl. Phys.* **B507** (1997) 553 [hep-th/9704177]; I. Antoniadis, E. Dudas and A. Sagnotti, *Nucl. Phys.* **B544** (1999) 469 [hep-th/9807011].

[8] A. Delgado and M. Quirós, *Nucl. Phys.* **B607** (2001) 99 [hep-ph/0103055]; A. Delgado, G. von Gersdorff, P. John and M. Quirós, *Phys. Lett.* **B517** (2001) 445 [hep-ph/0104112]; A. Masiero, C. A. Scrutti, M. Serone and L. Silvestrini, *Phys. Rev. Lett.* **87** (2001) 251601 [hep-ph/0107201].

[9] D. M. Ghilencea, S. Groot Nibbelink and H. P. Nilles, *Nucl. Phys.* **B619** (2001) 385 [hep-th/0108184].

[10] R. Barbieri, L. J. Hall and Y. Nomura, [hep-ph/0110102].

[11] N. Arkani-Hamed, A. G. Cohen and H. Georgi, *Phys. Rev. Lett.* **86** (2001) 4757 [hep-th/0104003].

[12] C. T. Hill, S. Pokorski and J. Wang, *Phys. Rev.* **D64** (2001) 105005 [hep-th/0104035].

[13] N. Arkani-Hamed, A. G. Cohen and H. Georgi, *Phys. Lett.* **B513** (2001) 232 [hep-ph/0105239]; N. Arkani-Hamed, A. G. Cohen, T. Gregoire and J. G. Wacker, [hep-ph/0202093]; K. Lane, [hep-ph/0202093].

[14] H. C. Cheng, C. T. Hill and J. Wang, *Phys. Rev.* **D64** (2001) 095003 [hep-ph/0105323].

[15] N. Arkani-Hamed, A. G. Cohen and H. Georgi, [hep-th/0109082].

[16] N. Arkani-Hamed, A. G. Cohen and H. Georgi, [hep-th/0108089]; P. H. Chankowski, A. Falkowski and S. Pokorski, [hep-ph/0109272]; C. Csáki, G. D. Kribs and J. Terning, *Phys. Rev.* **D65** (2002) 015004 [hep-ph/0107266]; H. C. Cheng, K. T. Matchev and J. Wang, *Phys. Lett.* **B521** (2001) 308 [hep-ph/0107268].

[17] C. Csáki, J. Erlich, C. Grojean and G. D. Kribs, *Phys. Rev.* **D65** (2002) 015003 [hep-ph/0106043].

[18] D. M. Ghilencea and H. P. Nilles, *Phys. Lett.* **B507** (2001) 327 [hep-ph/0103151]; H. D. Kim, [hep-ph/0106072].
[19] A. Delgado, G. von Gersdorff, P. John and M. Quirós, Phys. Lett. B517 (2001) 445 [hep-ph/0104112]; R. Contino and L. Pilo, Phys. Lett. B523 (2001) 347 [hep-ph/0104130]; V. Di Clemente, S. F. King and D. A. Rayner, Nucl. Phys. B617 (2001) 71 [hep-ph/0107290]; T. Kobayashi and H. Terao, hep-ph/0108072; V. Di Clemente and Y. A. Kubyshin, hep-th/0108117; D. M. Ghilencea, H. P. Nilles and S. Stieberger, hep-th/0108183; S. Groot Nibbelink, Nucl. Phys. B619 (2001) 373 [hep-th/0108185].

[20] W. Skiba and D. Smith, hep-ph/0201056.

[21] C. A. Scrucca, M. Serone, L. Silvestrini and F. Zwirner, Phys. Lett. B525 (2002) 169 [hep-th/0110073].

[22] L. Pilo and A. Riotto, hep-th/0202144.

[23] P. H. Frampton and C. Vafa, hep-th/9903220.

[24] K. R. Dienes, Nucl. Phys. B611 (2001) 146 [hep-ph/0104274].

[25] D. Cremades, L. E. Ibáñez and F. Marchesano, hep-th/0201205.

[26] C. Csáki, W. Skiba and J. Terning, Phys. Rev. D61 (2000) 025019 [hep-th/9906057].