REVISITING SVD TO GENERATE POWERFUL NODE EMBEDDINGS FOR RECOMMENDATION SYSTEMS

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ABSTRACT
Learning node representations is at the heart of Graph Representation Learning (GRL), and has been proven successful in establishing the state-of-the-art in multiple domains including recommendation systems. In this paper, we benchmark node embeddings generated through Singular Value Decomposition (SVD) of adjacency matrix for embedding generation of users and items and use a two-layer neural network on top of these embeddings to learn relevance between user-item pairs. Inspired by the success of higher-order learning in GRL, we further propose an extension of this method to include two-hop neighbors for SVD through the second order of the adjacency matrix and demonstrate improved performance compared with the simple SVD method which only uses one-hop neighbors. Empirical validation on three publicly available datasets of recommendation system demonstrates that the proposed methods, despite being simple, beat many state-of-the-art methods and for two of three datasets beats all of them up to a margin of 10%. Through our research, we want to shed light on the effectiveness of matrix factorization approaches, specifically SVD, in the deep learning era and show that these methods still contribute as important baselines in recommendation systems.

Index Terms— Graph Neural Networks, Singular Value Decomposition, Node Embeddings, Representation Learning

1. INTRODUCTION
Graph Representation Learning (GRL) presents a very promising direction in terms of machine learning on graphs [1] [2] [3] [4]. A central idea in GRL is to represent each node in the graph as a vector of floating-point numbers to capture some desired properties of a node with respect to the graph. For node2vec [1], this property is the neighborhood of the node; for GraphSage [5], it is about capturing the feature neighborhood of a node based on a selected neighbor set; for Graph Convolutional Network (GCN) [2], the purpose of a node embedding is to capture both feature and neighborhood similarity. These methods have been profoundly useful in several domains such as bio-inspired machine learning and genomics [6] [7] [8], spam detection [9] [10], natural language processing [11] [12] [13] [14] and recommendation systems [15] [16] [17] [18] [19].

In recommendation systems, GRL has been applied to further advance collaborative filtering algorithms by considering multi-hop relationships between users and items [15]. The authors in [15] further proposed the notions of message dropout and node dropout to reduce overfitting in GCN like methods. In a follow-up study [17], it was demonstrated that simplifying GCN network by reducing non-linearity from the network can give a boost to the performance of these higher-order methods. Their work also corresponded with a similar study done in [20] where the authors argued that for GCN, even after removing non-linearity and collapsing weight matrices into a single one, the performance does not degrade in downstream tasks. The research carried out in the above papers compares several state-of-the-art methods to the proposed methods and shows that the simplicity of models is leading to higher performance, credited to better generalization of the models. Motivated by these studies, we set out to benchmark a simple SVD based approach in this paper on the recommendation systems problem to understand if further simplicity of the modelling approach can improve the performance metrics. In the proposed method, we first generate user and item embeddings using SVD of the adjacency matrix of the user-item interaction graph and then employ a two-layer neural network with these embeddings as inputs to estimate the relevance of an item to a user.

Given the success of multi-hop graph neural network models in previous studies, we augment the simple SVD method to consider a two-hop adjacency matrix for generating the embeddings and found that this method outperforms the simpler one-hop SVD method as well. Empirical results on three public datasets demonstrate that the performance of the proposed methods is indeed comparable to state-of-art approaches, and these methods beat many of them despite their simplicity. For two out of three datasets, the methods even outperform all compared approaches and effectively establish new state-of-the-art performance with the margin of improvement as much as 10%.

The rest of the paper is divided into three sections: Section 2 describes the proposed methods, Section 3 contains the empirical experiments, and Section 4 provides the conclusion and future work.
2. PROPOSED METHODS

In this section, we elaborate on the proposed methods. We first discuss the SVD based baseline followed by an extension of the same by using two-hop matrices. In the last part, we describe the loss function and model training. Before discussing the proposed methods, we list our notations in Table I.

2.1. Simple SVD Baseline

Matrix factorization is a well-studied problem in linear algebra and has been extensively applied to recommendation systems [21][22][23][24], typically in the form of collaborative filtering. In this paper, we propose a simple approach to generate user and item embeddings using Single Value Decomposition (SVD) [25] of the adjacency matrix between users and items. Using a two-layer perceptron model, we transform these embeddings in a supervised fashion to learn the relevance between the user and item pairs. We call this method Simple SVD Baseline (SSB).

To compute the SVD embeddings, we consider the adjacency matrix of the user-item interaction graph, $A$. We first convert the asymmetric matrix to a symmetric matrix as follows:

$$A' = \begin{pmatrix} 0 & A^T \\ A & 0 \end{pmatrix}.$$  

We then compute a Laplacian Normalization of $A'$ as discussed in [2]: $\tilde{A} = D^{-\frac{1}{2}} A D^{-\frac{1}{2}}$, where $A$ is the Laplacian Normalized of adjacency matrix, $A'$, and $D$ is the degree matrix derived from $A'$.

We perform matrix factorization on $\tilde{A}$ using Truncated SVD on this normalized matrix to generate user embeddings ($e_u^{SV D}$) and item embeddings ($e_i^{SV D}$), where the number of components in Truncated SVD correspond to the embedding dimension. We use Truncated SVD [26] since it has shown to be scalable on large matrices.

After generating these embeddings, we transform them through a two-layer perceptron model ($f(x) = x$, as the activation function) and concatenate the output of both the layers of the perceptron model along with original SVD embedding to generate a user embedding ($e_u$) or an item embedding ($e_i$). The intuition behind using the perceptron model is to allow supervised transformation of $e_u^{SV D}$ and $e_i^{SV D}$ to learn the relevance between user and item. Fig. 1 shows the model architecture. The dot product between $e_u$ and $e_i$ acts as the relevance score for the user-item pair and is optimized by the model through tuning of the weights of the two-layer perceptron model via back-propagation.

2.2. Two-Hop SVD Approach

Motivated by the success of multi-hop graph neural networks and the performance of the SSB approach on recommendation tasks, we attempt at joining both of these into a single method to capture higher-order relationships between users and items, similar to graph neural networks like GCN [2].

The overall model architecture remains the same as SSB, except for the change in how the $e_u^{SV D}$ and $e_i^{SV D}$ embeddings are computed. To compute an embedding that can capture the two-hop signals, we compute the second power of the Laplacian Normalized adjacency matrix, $\tilde{A}$, and then compute its Truncated SVD. We finally concatenate the embeddings from SVD of $\tilde{A}$ (corresponding to one-hop neighborhood) and SVD of $\tilde{A}^2$ (corresponding to two-hop neighborhood) to generate $e_u^{SV D}$ and $e_i^{SV D}$ embeddings for this approach. The embedding size of TSA is the size of the vector after this concatenation. Since this approach contains two-hop signals from the graph, we denote this method as Two-Hop SVD Approach (TSA).

2.3. Model Training

The learnable parameters in the proposed methods are only the weights of the multi-layer perceptron model. To optimize the user-item relevance, we employ the Bayesian Personalized Ranked (BPR) loss [27] similar to [15]. It is a pair-wise loss that encourages correct predictions on observed instances than on unobserved instances. We use the Adam optimizer [28] in a mini-batch setting, where the batch size is a hyper-
### Table 1: Symbols and their meaning used in the paper.

| Symbol | Definition |
|--------|------------|
| $\mathcal{A}$ | Adjacency matrix between Users and Items |
| $\mathcal{A}'$ | Symmetric adjacency matrix between Users and Items |
| $\tilde{\mathcal{A}}$ | Laplacian Normalization of matrix $\mathcal{A}'$ |
| $D$ | Degree Matrix of $\mathcal{A}'$ |
| $u$ | User |
| $i$ | Item |
| $e_{SVD}^u$ | Embedding of user generated from SVD of $\mathcal{A}$ |
| $e_{SVD}^i$ | Embedding of item generated from SVD of $\mathcal{A}$ |
| $e_{mj}^u$ | Embedding of user outputted from the $j^{th}$ layer of perceptron model |
| $e_{mj}^i$ | Embedding of item outputted from the $j^{th}$ layer of perceptron model |
| $e_u$ | Concatenation of : $e_{SVD}^u$ and $e_{mj}^u$ ($j = \{1, 2\}$) |
| $e_i$ | Concatenation of : $e_{SVD}^i$ and $e_{mj}^i$ ($j = \{1, 2\}$) |

### 3. EXPERIMENTS

#### 3.1. Datasets and Performance Metrics

We use the same three datasets (Gowalla, Yelp2018 and Amazon-Book) as [15, 17] with the same train and test split in order to make a fair comparison with the already reported results. Table 2 summarizes the dataset statistics. We refer the reader to [15] for more details of the datasets. We evaluate the performance on mean NDCG@K and mean Recall@K per user for K=20. We keep K=20 to enable fair comparison with previous studies which use the same metrics [15, 17]. For the rest of the paper, we denote Recall@20 as Recall and NDCG@20 as NDCG. It should be noted that for both Recall and NDCG, the items retrieved for top-20 are solely from the test partition of the dataset.

#### 3.2. Hyperparameter Tuning

For the proposed methods, there are four key hyperparameters - SVD embedding size ($|e_{SVD}^u|$ and $|e_{SVD}^i|$), batch size, learning rate and size of the multi-layer perceptron. For this study, we fix the size of the multi-layer perceptron to be 512 neurons each and keep the learning rate as $10^{-3}$ motivated by the experiments in [17]. We tune the SVD embedding size over the following set: $\{2^6, 2^7, 2^8, 2^9, 2^{10}\}$. We keep the batch size as 1024 for Gowalla and Yelp2018, and 2048 for Amazon-Book as used in the study of NGCF [15] and LightGCN [17].

| Dataset       | #Users | #Items | #Interactions | Density |
|---------------|--------|--------|---------------|---------|
| Gowalla       | 29,858 | 40,981 | 1,027,370     | 0.00084 |
| Yelp2018      | 31,688 | 38,048 | 1,561,406     | 0.00130 |
| Amazon-Book   | 52,643 | 91,599 | 2,984,108     | 0.00062 |

### 3.3. Empirical Results

#### 3.3.1. Comparison with state-of-the-art methods

In this section, we report the performance metrics for the proposed methods - Simple SVD Baseline (SSB) and Two-Hop SVD Approach (TSA). We benchmark the approach against NGCF [15], Mult-V AE [29], GRMF [30], LightGCN [17], MF [27], and NeuMF [31]. Although MF [27], and NeuMF [31] methods are relatively older methods to compare. However, we report their performance here as these are closely related to matrix factorization in the context of recommendation systems. LightGCN [17] is the state-of-the-art method showing the best performance compared to all related approaches as shown in their paper. Table 3 shows the performance metrics for the proposed methods and the compared methods. We replicate the results of these approaches from the original papers of NGCF [15] and LightGCN [17]. We follow the same experimental methodology as stated in the papers and followed in the code and datasets made available by the authors of these studies to make a fair comparison.

We can observe that TSA performs considerably better for Amazon-Book dataset than all the compared state-of-art methods, including LightGCN and NGCF. The relative gain of TSA over LightGCN, which performs the best among the compared methods, is approximately 9.86% in Recall@20 and 13.4% in NDCG@20. The performance of SSB is also...
Table 3: Comparison of the proposed methods - SBB and TSA with related methods. It can be observed that the proposed methods, despite being very simple, beat all the compared methods for the Yelp2018 and Amazon-Book datasets. For the Gowalla dataset, the proposed methods prove to be a strong baseline and was able to beat all but one state-of-the-art methods on this dataset. \( \psi \) Results reused from [15]; \( \phi \) Results reused from [17].

Table 4: Comparison of the proposed methods with each with respect to training metrics on Yelp-2018 dataset. It should be noticed that SVD is only performed once at the start of training the model.
3.3. Impact of Embedding Size on Performance on TSA

Out of the four hyperparameters discussed, we only optimize the SVD based embedding size for users and items, which become the input to the multi-layer perceptron. In this section, we will describe the observations on how the performance and training loss changes as we change the embedding dimensions for TSA, which performs better than SSB across datasets. We vary the embedding dimension as follows: \{2^6, 2^7, 2^8, 2^9, 2^{10}\}. Fig. 2(a) shows the training loss for different embedding sizes for the TSA approach. As expected, it can be seen that as the embedding size increases, the loss decreases faster and also to a lower value. We observe a similar trend in test performance metrics, and Fig. 2(b) shows Recall@20 for the test set for different embedding sizes, and it can be seen that higher embedding sizes lead to better performance.

4. CONCLUSIONS AND FUTURE WORK

In this paper, we started out to benchmark SVD based methods against the state-of-the-art GRL methods. We propose two approaches for the same, and experiments on three real-world open datasets demonstrate that these methods are powerful enough to beat many GRL methods and even come out as state-of-the-art themselves in two out three datasets. We observed the most significant relative gain of over 10% against the state-of-the-art methods.

This particular work raises many research questions, and we envision the following future work. We plan on investigating how to generalize the approach from two-hop to n-hop since we saw in earlier research articles that with a higher order of neighborhood, we could expect better performance. There is also a need to propose an inductive version of these methods since transductive versions (as proposed in this paper), do not work with new nodes or new edges in the graph and would require frequent retraining in the current form.

We also plan to investigate the aspects around the better implementation of SVD in big data frameworks such as Spark [32] [33]. This would help us further understand the time taken for the proposed methods for the training of models. We also want to explore how matrix factorization or SVD can be integrated with GRL to improve empirical performance and is it possible to extract the goodness from both methods and merge them. We also plan to work on data profiling for the proposed methods and existing literature to understand why one approach performs well on some datasets but does not perform equally well on other datasets. On the empirical investigation front, we also intend to benchmark these approaches on the Open Graph Benchmark [34]. We also plan to test out these approaches in tasks beyond recommendation systems such as graph-based formulations in NLP, social network modelling and graph applications in biology.

Through our work, we want to highlight that matrix factorization based methods still contribute as important baselines and should not be ignored in empirical benchmarking while making more advances in GRL or recommendation systems.

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