We propose an integrability setup for the computation of correlation functions of gauge-invariant operators at any value of the 't Hooft coupling and at any order in the large $N$ 't Hooft expansion in $\mathcal{N} = 4$ SYM theory. In this multi-step proposal, one polygonizes the string worldsheet in all possible ways, hexagonalizes all resulting polygons, and sprinkles mirror particles over all hexagon junctions to obtain the full correlator. We test our integrability-based conjecture against a non-planar four-point correlator of large half-BPS operators at one and two loops.

**INTRODUCTION**

Integrable theories are rather special 2d QFTs where the scattering of fundamental excitations factorizes into a sequence of two-body scattering events. This simplification often translates into solvability. The worldsheet theory describing superstrings in $AdS_5 \times S^5$ is integrable \[1, 2\]. Exploiting integrability machinery, the full finite-size spectrum has been obtained at any value of the coupling \[3–5\], yielding the energy spectra of single strings in this curved background or – equivalently – the spectra of anomalous dimensions of single-trace operators in $\mathcal{N} = 4$ SYM theory in the planar limit.

Beyond the planar limit we are dealing with world-sheets with handles. These induce non-local interactions in the two-dimensional theory, wormholes of sorts, which also appear in the gauge theory spin-chain description. One would guess that such non-local interactions could ruin integrability. Indeed, known degeneracies in the spectrum of the weakly coupled gauge theory – related to the hidden higher charges of the integrable theory – are lifted as one takes nonplanar corrections into account \[6\], and fermionic T-duality – responsible for dual conformal symmetry, which in turn is closely related to integrability in the usual sense – is not a symmetry of string theory at higher genus \[7, 8\]. Because of all this, it has been common lore that integrability would not be useful beyond the planar limit \[9\]. See \[10\] for a very nice summary.

On the other hand, numerous other planar quantities have been explored at finite coupling using integrability, from scattering amplitudes / Wilson loops \[11\] to OPE structure constants \[12\], higher point correlation functions \[13–15\], and even mixed quantities involving correlation functions in the presence of Wilson loops \[16, 17\]. Underlying all these computations is the idea of taming complicated string topologies by cutting the string into smaller and simpler patches (hexagonal or pentagonal), which are then glued back together. This is implemented by so-called branch-point twist field operators \[18, 19\], whose expectation values can be bootstrapped.

All these works strongly suggest that, instead of thinking about the nonplanar effects as non-local corrections to the planar world-sheet, we should from the get-go consider the theory in more general topologies, treat handles using the twist operators mentioned above, and keep everything else as local as possible. Following this philosophy, in this note we propose a framework for computing correlation functions at any higher genus order and any value of the 't Hooft coupling using integrability.

**THE DATA**

Our experimental data – against which we will test our integrability predictions – are the four-point corre-
Putting the above ingredients together, and keeping only operators with \(k\) and similar expressions for all other color factors, one finds \(k\) using the data points at finite \(k\) by their combinatorial nature – the various color factors where \(k\) ary cases at extremal values of \(l\) and various values of \(k\). The quantum corrections dressing the propagator structures depend on the conformally invariant cross ratios \(|\varepsilon|^2 = x_1^2 x_2^2 x_3^2 x_4^2\) and \(1 - |\varepsilon|^2 = x_1^2 x_2^2 - x_3^2 x_4^2\). The one- and two-loop contributions were computed in [20, 21].

A key ingredient are the conformal box and double-box functions.

\[
F^{(1)}(z, \bar{z}) = \frac{x_1^2 x_2^2}{\pi^2} \int \frac{d^4 x_5}{x_1^2 x_2^2 x_3^2 x_4^2} = \frac{1}{2} \delta_{12},
\]

\[
F^{(2)} = \frac{x_1^2 x_2^2}{x_4^2} \int \frac{d^4 x_5}{x_1^2 x_2^2 x_3^2 x_4^2} = \frac{1}{2} \delta_{12}.
\]

Other key players are the so-called color factors, which consist of color contractions of four symmetrized traces from the four operators, dressed with insertions of gauge group structure constants. For instance [22],

\[
C_m = \frac{f_{abc} f_{def} f_{pqr} f_{st}}{2m!} \times \left( \begin{array}{c} \text{tr}(d_1 \ldots d_k a_1 \ldots a_m b_1 \ldots b_k \alpha r) \\
\times \text{tr}(d_1 \ldots d_k c_1 \ldots c_m \alpha p)
\end{array} \right),
\]

where \(k' = k - m - 2\). We explicitly performed the contractions with Mathematica, for up to \(k = 8\) or 9 and various values of \(m\). Then, we used the fact that – by their combinatorial nature – the various color factors should be quartic polynomials in \(k\) and \(m\) (up to boundary cases at extremal values of \(k\) or \(m\)), which we can fit using the data points at finite \(k\) and \(m\) at the end of the day, one finds \(C_m = N^2 k^4 = 2k^4 + \mathcal{O}(N^4)\), where \(\mathcal{O}(N^4)\) is the leading large \(k\) result at each genus order, we finally obtain our much desired experimental data

\[
F^{(1)}_{k,m} = \frac{-2k^2}{N^2 \left( \begin{array}{c} m \end{array} \right)} \left( 1 + \frac{k^4 (\frac{17}{6} - \frac{7}{4} + \frac{11}{32})}{N^2} \right) |z - 1|^2 F^{(1)},
\]

The outermost sum runs over all graphs with \(n\) vertices, including all topologies, planar and nonplanar. Each edge (bridge) stands for a collection of one or more (planar, non-crossing) propagators connecting two operators (hence parallel edges must be identified). Next, we sum over all vertex labelings (distribution of operators on the vertices), and over all (nonzero) bridge fillings (numbers of propagators on each edge) compatible with the charges of the operators. All this combinatorial process is what we call **polygonization**. Next, we have what we call **hexagonization**: After inserting the operators, graphs may consist of hexagons and higher polygons. For the latter we pick a subdivision into hexagons by inserting zero-length bridges (ZLBs). Each hexagon gives rise to one hexagon form factor. Finally, at the end, we sum over mirror excitations (complete basis of states) on each edge.
FIG. 3. Bridge configurations on the torus that contribute to the leading term in $1/k$ for correlators of the type $\mathbb{I}$.

(including the ZLBs). This last step we denote as sprinkling.

These three main processes are represented in figure 2 and discussed in detail below. For illustration and simplicity, in this first note, we restrict ourselves to $n = 4$ large BPS operators computed up to the first subleading correction in $1/N_c^2$ (i.e. genus 0 and genus 1).

There is one last step represented by the seemingly innocuous $\mathcal{S}$ in [5] which stands for subtractions or stratification. The point is that the sum over polygonizations discretizes the integration over the moduli space of the Riemann surface, whose boundary contains degeneration points: At its boundary, a torus degenerates into a sphere for instance. $\mathcal{S}$ stands for the appropriate subtractions which remove these boundary contributions, see e.g. [23]. In this paper, we will consider four large BPS operators on the torus, which are controlled by configurations where all cycles of the torus will be populated by many propagators. The relevant worldsheets are thus very far from the boundary of the moduli space, and we can ignore $\mathcal{S}$ altogether. We will come back to it in [24].

(LARGE $k$) POLYGONIZATION

As indicated by the first line of (5), the polygonization proceeds in three steps: (A) construct all inequivalent graphs with $n$ vertices on the given topology, (B) sum over all inequivalent labelings of the vertices, and (C) for each labeled graph, sum over all possible distributions of propagators on the edges (bridges) of the graph, such that each edge carries at least one propagator.

In a generic graph on the torus, any two operators will be connected by one or more bridges. In this work, we are interested in the leading contribution for large operator weights $k \gg 1$ with $m/k$ finite. In this limit, graphs with a non-maximal number of bridges will be suppressed by powers of $1/k$. At the same time, we consider operator polarizations that do not admit propagator structures of the type $Z \equiv (\alpha_1 \cdot \alpha_4)(\alpha_2 \cdot \alpha_3)/x_{14}^2 x_{23}^2$, see [1]. Hence only graphs where the four operators are connected cyclically, as in $1-2-3-4-1$, will contribute. We have classified all graphs of this type (with a maximal number of bridges), and have found the six cases shown in figure 3.

For these six graphs, we have to consider all possible inequivalent operator labelings. In addition, each labeled graph comes with a combinatorial factor from the distribution of propagators on the various bridges via

$$
\sum_{n_0 \leq n_1 \leq \ldots \leq n_j \leq n \atop \sum_j n_j = n} 1 = \frac{n^{j-1}}{(j-1)!} + O(n^{-2}).
$$

We list all inequivalent labelings for the relevant graphs as well as their combinatorial factors in table 1 [24].

(LARGE $k$) HEXAGONALIZATION

Next, we further decompose all polygons in figure 3 – which are bounded by the finite bridges – into hexagons by adding ZLBs. There are typically various ways of adding these ZLBs, and they are all equivalent. We can easily see that all graphs in figure 3 are made out of four octagons, hence we simply need to split each of those octagons into two hexagons. A hexagonalization of case A is illustrated in figure 2. The physical operators correspond to the thick solid lines, the thin grey lines are the large bridges, and the dashed lines are the ZLBs.

(LARGE $k$) SPRINKLING

Finally, we have to sprinkle mirror particles on the hexagonalizations of the previous section. Our large $k$ result is given by a set of octagons separated by large bridges. Putting particles on those bridges is very costly in perturbation theory, so the only thing we can do is to put particles on each ZLB for each octagon. Furthermore, two particles on the same bridge are also very

TABLE I. All inequivalent operator labelings for the graphs that contribute to leading order in $1/k$, together with their combinatorial factors according to [6]. The order of the labels runs from top to bottom, left to right in the graphs of figure 3.

| Case | Inequivalent Labelings | Combinatorial Factor |
|------|------------------------|----------------------|
| A    | 1234, 3412             | $m^3/24$             |
| A    | 1324, 2413             | $(k - m)^3/24$       |
| B    | 1234, 2143, 3412, 4321 | $m^3(k - m)/6$       |
| B    | 1324, 3142, 2413, 4321 | $m(k - m)^3/6$       |
| C    | 1234, 3412, 2143, 4321 | $m^2/2 \cdot (k - m)^2/2$ |
| D    | 1234, 2143, 3, 1324, 3142 | $m^2(k - m)^2/2$     |
| E    | 1234                    | $m^2(k - m)^2$       |
| F    | 1234                    | $m^2(k - m)^2$       |
costly (appearing at four loops only), so up to two loops only two contributions will matter: A single particle placed on a ZLB, and two particles placed simultaneously on two distinct ZLBs. This latter contribution is essentially the square of the former one. The single-particle contribution has been studied in \cite{13} and yields

\[ \mathcal{M}^{(1)} = \left[ z + \bar{z} - (\alpha + \bar{\alpha}) \frac{\alpha \bar{\alpha} + z \bar{z}}{2 \alpha \bar{\alpha}} \right] (g^2 F^{(1)} - 2g^4 F^{(2)}) , \]  

where for the correlators considered here, the R-charge cross-ratios \( \alpha \) and \( \bar{\alpha} \) are given by

\[ \alpha = z \bar{z} X/Y , \quad \text{and} \quad \bar{\alpha} = 1 . \]  

(To get the \( g^4 \) term in \((7)\), we simply expanded the integrand in \((13)\) to one more loop in perturbation theory.)

The above factors of \( X \) and \( Y \) are contained in the factors in the green parentheses in the second line of \((5)\), and combine with the propagator factors in the first line of that formula. Hence to read off particular coefficients of monomials in \( X \) and \( Y \) to compare with perturbation theory predictions such as \((4)\), we often need to consider the contribution of a few “neighboring” graphs.

Consider for illustration the particular case A in figure 2. There are four octagons to be considered, as shown in figure 4. The first two contain pairs of physical edges associated to the same external operator and thus give a vanishing contribution, as can be easily seen by taking an OPE limit of the generic octagon. For each of the labelings in table I the resulting expressions for the remaining two octagons are summarized in table II. Accounting for the labeling and combinatorial factors listed in table I we can then read off the coefficient of \( X^m Y^{k-m} \) as

\[ \text{case A: } X^m Y^{k-m} \text{ coeff.} = k^4 \left[ \frac{(r + 1/2)^4 + (r - 1/2)^4}{24} \right] \times \]

\[ (4M + 2M^2) \sum_{a=2}^{2} X^{m+a} Y^{k-m-a} \]  

where we have used that \( \mathcal{M}^{(1)}(1/z) = \mathcal{M}^{(1)}(z) \equiv \mathcal{M} \). As explained above, this coefficient receives contributions from a few neighboring polygonizations, accounted for by the sum in the second line. The remaining cases follow in complete analogy. When we sum them all we obtain a perfect match with \((4)\).

\[ TABLE II. Contributions for the one-particle octagons for each distinct operator labeling of case A. \]

| Labeling | Octagon (c) | Octagon (d) |
|----------|-------------|-------------|
| 1234     | \( \mathcal{M}^{(1)}(z) \) | \( \mathcal{M}^{(1)}(z) \) |
| 1324     | \( \mathcal{M}^{(1)}(1/z) \) | \( \mathcal{M}^{(1)}(1/z) \) |
| 2413     | \( \mathcal{M}^{(1)}(1/z) \) | \( \mathcal{M}^{(1)}(1/z) \) |
| 3412     | \( \mathcal{M}^{(1)}(z) \) | \( \mathcal{M}^{(1)}(z) \) |

**CONCLUSIONS AND OVERLOOK**

We proposed here a novel formalism for computing correlation functions of local gauge-invariant operators in \( N = 4 \) SYM theory at any genus and any order in the coupling in the large \( N_c \) \( 't \) Hooft expansion.

In this paper we already performed one very non-trivial check of our conjecture. We reproduced the first non-planar correction to the correlation function of four large BPS operators at one loop and two loops from integrability. At the end of the day, this computation is rather simple, and only uses formulae for a single mirror particle already worked out in \cite{13}. In an upcoming paper \cite{24}, we perform numerous other checks that probe all steps in our proposal in great detail: the polygonization, the hexagonalization, the sprinkling, and the stratification. These include finite-size corrections to the computation above, correlators at strict finite size, higher-genus examples, and subtleties related to the choice of the gauge group. Through the OPE of the obtained correlators we can read off conformal data of non-BPS operators beyond the planar limit.

One of the advantages of dealing with BPS external operators (as considered in this note) is avoiding the subtlety of double-trace mixing. It would be interesting to study the mixing effects. (See \cite{26} for very interesting first explorations in this direction.) It would also be important to better understand the integrand one obtains after sprinkling the hexagons with a few mirror particles. As we increase the number of mirror particles, it quickly becomes monstrous. How to tame it efficiently? Another interesting problem – which can only be realistically addressed once we progress with the former – concerns going to strong coupling and making contact with the recent exciting developments on the bootstrap approach to loop corrections in AdS \cite{27,33}. One can then explore various interesting questions such as the emergence of bulk locality \cite{34,35}. Will we find higher genus subtleties in our integrability-based formalism akin to the complications with supermoduli integrations recently observed in the RNS formalism \cite{36,38}?

Finally, a fun project would be to re-sum the \( 't \) Hooft expansion – perhaps starting with some simplifying kinematical limits. What awaits us there, and what can we learn about string (field) theory?
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