I. INTRODUCTION

The light mesons sector has been studied over several decades and many properties of the mesons were found. However, there are still uncertainties about the properties of some of these mesons. In particular, their structure are not yet fully established. For this reason, this sector still arouses interest from both a theoretical and an experimental point of view. The possible existence of exotic hadrons is also subject of several studies. Most of these exotic mesons are not yet experimentally detected, but several theoretical and phenomenological works predicts their existence and properties. The glueball is an exotic state predicted theoretically but not yet experimentally observed. Glueballs and $q\bar{q}$ mesons states can mixing strongly which brings numerous experimental difficulties to detect glueball states due to the interference of $q\bar{q}$ states.

The radiative decay of mesons can be used to establish their internal structure. For this reason, the meson in the final state must have a well established internal structure. This idea was used to predict the radiative decay rates calculated in the context of a non-relativistic model as a function of the mixing angles of glueball and meson states. In their work Close el al have considered the $f_1(1285)$ as a member of the axial vector meson nonet.

The $f_1(1285)$, with quantum numbers $J^{PC} = 1^{++}$, mass $1281.9 \pm 0.5$ MeV and total width $22.7 \pm 1.1$ MeV, is usually considered as a member of the axial vector meson nonet. Recently, the photoproduction of $f_1(1285)$ was studied by the CLAS Collaboration, where the photoproduction differential cross section and the branching ratio $\Gamma(\gamma^*\rho^0)/\Gamma(\eta^\prime\pi^0) = 0.047 \pm 0.018$ were measured. Then they combine this data with PDG results and found $\Gamma[f_1(1285) \rightarrow \gamma\rho^0] = 453 \pm 177$ keV which is in poor agreement with the PDG estimation of $1203 \pm 280$ keV. The PDG Collaboration also shown the results for the branching ratio of $\Gamma[f_1(1285) \rightarrow \gamma\phi] = 17.0 \pm 6.3$ keV.

From the point of view of its nature, the $f_1(1285)$ was mostly considered as a $q\bar{q}$ state, more precisely composed by $u$ and $d$ quarks. However, it was also predicted in the literature a mixture of gluons in $f_1$ wave function. This mixture was described in a form similar to the mixing in the scalar sector. On this way the resonances $f_1(1285)$, $f_1(1420)$ and $f_1(1510)$ were considered as a mixing of $1/\sqrt{2} | uu + dd \rangle$, $| ss \rangle$ and $| G \rangle$ states. This possibility was also mentioned by Kochelev et al in their paper about $f_1(1285)$ photoproduction.

In this work we calculate the radiative decay rates for $f_1(1285) \rightarrow \gamma\rho$ and $f_1(1285) \rightarrow \gamma\phi$. The choice of $\rho$ and $\phi$ is due to the $\rho$ is a state of $u$ and $d$ quarks and $\phi$ is a state of $s$ quarks. Then we can use these results and the experimental data available to estimate the quarks content of $f_1$ and consequently the glue content of this resonance. In the next section we show the meson confining potential and the relation of this potential with the parameters of the meson wave function. In section III the non-relativistic radiative decay model is presented. Our results are shown in section IV. The section V is about the Summary and Conclusions.

II. THE CONFINING POTENTIAL

In a non-relativistic constituent quark model, the meson is described as a quark-antiquark system with masses $m_q$ and $m_{\bar{q}}$ respectively. The potential used here is the Cornell potential which describe the behavior of the color interaction in the two asymptotic limits.

$$V(r) =Kr - \frac{4\alpha_s}{3r} + C. \quad (1)$$

where $K = 0.18 \text{ GeV}^2$ and $\alpha_s = 0.39$. This potential allow us to solve variationally the Schrödinger equation and connect the meson masses and the parameters which are needed in the next section.

$$\left( m_q + m_{\bar{q}} + \frac{p^2}{2\mu} + V(r) \right) | \Psi_M \rangle = M | \Psi_M \rangle. \quad (2)$$

The wave functions are taken to be Gaussian multiplied by a polynomial. For $L = 0$ the normalized wave function

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is given by
\[ \Psi_M = \frac{2\beta_M^{3/2}}{\pi^{1/4}} e^{-\frac{s^2 \beta_M^2}{4}} Y_{00}(\Omega), \] (3)
and for \( L = 1 \) we have
\[ \Psi_M = 2 \sqrt{\frac{3}{2}} \frac{\beta_M^{5/2}}{\pi^{1/4}} r e^{-\frac{s^2 \beta_M^2}{4}} Y_{10}(\Omega). \] (4)
where \( \beta_M \) is the parameter which will be obtained from the mesons masses.

In the present work we consider two possibilities for the mesons states. The first one is that only quarks \( u \) and \( d \) are part of the meson. On this way we consider \( m_u = m_d = 0.33 \) GeV. With these assumptions we obtain the mesons masses as a function of the \( \beta_M \) parameters for \( \rho(770) \) and \( f_1(1285) \). The results obtained are presented in Table I. A similar procedure is used for \( s \) quarks composite mesons. In this case the quark mass is \( m_s = m_s = 0.54 \) GeV [11]. The results for \( \beta_M \) parameters and the corresponding masses are shown in Table I. In the next section the parameters found here will be

**III. RADIATIVE DECAY MODEL**

In this section we describe the formalism for the radiative decay of a meson \( A \) into a meson \( B \) [9, 12],
\[ A \rightarrow \gamma B. \] (5)
In a non-relativistic quark model the transition amplitude, for a meson \( A \) which decays at rest to a meson \( B \) plus a photon with momentum \( p \), is given by the sum of two contributions
\[ \mathcal{M}_{A \rightarrow \gamma B} = \mathcal{M}_{A \rightarrow \gamma B}^{(a)} + \mathcal{M}_{A \rightarrow \gamma B}^{(b)}, \] (6)
The first contribution is related to the probability of emission of a photon by the quark
\[ \mathcal{M}_{A \rightarrow \gamma B}^{(a)} = \frac{\langle e \rangle}{2m_1} \int d^3p \Phi_B^* \left( \vec{p} - \frac{m_2}{m_1 + m_2} \vec{k} \right) \times \left[ 2\vec{p} - i\sigma_1 \times \vec{k} \right] \Phi_A(\vec{p}) \] (7)
and the second one is related to the probability of emission of a photon by the antiquark
\[ \mathcal{M}_{A \rightarrow \gamma B}^{(b)} = \frac{\langle e \rangle}{2m_2} \int d^3p \Phi_B^* \left( \vec{p} + \frac{m_1}{m_1 + m_2} \vec{k} \right) \times \left[ -2\vec{p} - i\sigma_2 \times \vec{k} \right] \Phi_A(\vec{p}) \] (8)
The two contributions for the amplitude are represented by the diagrams in Fig. 1.

**TABLE I. Parameters for mesons composed by \( u \) and \( d \) quarks.**

| Meson | Mass (GeV) | \( \beta_M \) (GeV) |
|-------|------------|---------------------|
| \( 1^- \) | 0.758 | 0.305 |
| \( 1^+ \) | 1.298 | 0.270 |

**TABLE II. Parameters for mesons composed by \( s \) quarks.**

| Meson | Mass (GeV) | \( \beta \) (GeV) |
|-------|------------|------------------|
| \( 1^- \) | 0.981 | 0.371 |
| \( 1^+ \) | 1.463 | 0.323 |

used as an input for the radiative decay model.

**FIG. 1. Diagrams representing the two possibilities for the radiative decay of \( f_1(1285) \).**

The wave function for a spin 1 meson can be written in the following form
\[ \Phi_M = \frac{1}{\sqrt{2}} \mathcal{Y}_{lm}(\vec{p}) \sigma R_M(\vec{p}), \] (9)
where \( \sigma \) is the Pauli matrix and \( R_M(p) \) is the meson radial wave function
\[ R_M(q) = \exp \left( -\frac{p}{2\beta_M} \right), \] (10)
The decay width of \( f_1(1285) \to \gamma V \) is given by
\[ \Gamma(f_1 \to \gamma V) = \frac{8\alpha E_V \beta^2 F^2 \langle e^2 \rangle^2}{3m_{f_1} m_q^2} \left( 1 + \lambda x + \frac{\lambda^2}{2} x^2 \right), \] (11)
where \( x = p^2 / \beta^2 \), the photon momentum is \( p = m_{f_1} - m_V \), the vector meson energy is \( E_V \approx m_V \),
\[ \beta = \sqrt{\frac{2\beta_{f_1}^2 + \beta_V^2}{\beta_{f_1}^2 + \beta_V^2}}, \] (12)
\[ \lambda = \frac{\beta_{f_1}^2}{2(\beta_{f_1}^2 + \beta_V^2)}, \] (13)
and
\[ F = \frac{\beta^4}{\beta_{f_1}^{3/2} \beta_V^{3/2}} \exp \left( -\frac{p^2}{8(\beta_{f_1}^2 + \beta_V^2)} \right). \] (14)
The isospin factors for the radiative decay were shown by Close et al in Ref. [5]. Here we use the following values for these factors

$$\langle e^2 \rangle = \frac{1}{4}$$  (15)

for \( n\bar{n} \rightarrow n\bar{n} \) with different isospin and

$$\langle e^2 \rangle = \frac{1}{9}$$  (16)

for \( s\bar{s} \rightarrow s\bar{s} \). For more details about the calculation of this factors see Ref. [13].

IV. RESULTS

The decay of \( f_1(1285) \rightarrow \gamma \rho^0 \) can be related to the content of quarks \( u \) and \( d \) to the \( \rho^0 \) is composed exclusively of these quarks. Our result for the decay rate is

$$\Gamma (f_1(1285) \rightarrow \gamma \rho^0) = 1608 \text{ keV}.$$  (17)

When we compare this result with PDG data for the same process (1203 ± 280 keV) [7] we can see that the \( u \) and \( d \) content is about 75% of \( f_1(1285) \). Which means that the \( f_1(1285) \) is mostly composed by \( u \) and \( d \) quarks. It is important to note that our results are also in agreement with the results of Close et al [6]. However, when we compare our result with the CLAS data (453 ± 177 keV) the content of \( u \) and \( d \) on \( f_1(1285) \) decreases to about of 28%. Then we need to investigate the quarks content of \( f_1(1285) \), on this way we consider the decay \( f_1(1285) \rightarrow \gamma \phi \). This decay channel give us information about the \( s \) component because the \( \phi \) meson is constituted basically of \( s\bar{s} \) quarks. The result obtained is

$$\Gamma (f_1(1285) \rightarrow \gamma \phi) = 214 \text{ keV}.$$  (18)

When we compare this result with PDG data for the same process (17.0 ± 6.3 keV) the content of \( s \) quarks can be estimated as 8% of total constituents. The CLAS collaboration do not have data about this channel. These results for \( u \), \( d \) and \( s \) quarks content of \( f_1(1285) \) add up to 83% which imply in a glue content of 17% in the \( f_1(1285) \) when the PDG data was considered. When we consider the CLAS data the total contribution of \( u \), \( d \) and \( s \) is about 36%, which imply in a large glue content of about 64%. Our results, when compared with PDG data indicates a glue content compatible with the results obtained by Birkel and Fritzsch [8] where the estimated glue content is about 14%.

V. SUMMARY AND CONCLUSIONS

In this paper we have calculated the radiative decay rates of \( f_1(1285) \) to \( \rho \) and \( \phi \) vector mesons. The choice of the \( \rho \) and \( \phi \) mesons is due to the fact that they have a well-known internal structure and because there are experimental data for these decay channel. The \( \rho \) is exclusively composed by \( u \) and \( d \) quarks and the \( \phi \) meson is exclusively composed by \( s \) quarks. The results were compared to experimental data and this comparison allowed us to estimate the contribution of \( u \), \( d \) and \( s \) quarks for \( f_1(1285) \) and consequently the glue content too. When compared to the PDG data our results indicated that \( f_1(1285) \) has a small glue content which is compatible with other theoretical results. However, when our results were compared to the CLAS data the \( u \), \( d \) and \( s \) quarks content decreased and consequently we found a large glue contribution to the \( f_1(1285) \) wave function. Even if the glue content of \( f_1(1285) \) is small (about 17%) in a more precise calculation it must be taken into account.

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