An EOQ model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments

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A finite time horizon inventory problem for a deteriorating item is developed with constant demand and time-varying deterioration rate under inflation and time value of money. Some of the items may deteriorate in the course of time. In this regard, the authors develop an EOQ model for time-varying deterioration rate. Shortages are allowed in each cycle and backlogged them completely. In this model, the fixed credit period is offered by the supplier to the retailer. During the credit period, the retailers are allowed a trade-credit offer by the suppliers to buy more items and earn more by selling their products. The interest on purchasing cost is charged for the delay of payment by the retailers. The objective of the study is to find optimal decision variables and order quantities of the products so that the net present value of total system cost over a finite planning horizon is minimized. Also, the profit function of the model is maximized. Finally, numerical results are presented to analyse the sensitivity of the optimal policies with respect to changes in some parameters of the system.

Keywords: inventory; order quantity; deteriorating items; shortages; credit periods; inflation

1. Introduction

In recent years, many researchers have investigated on inventory models for deteriorating items. The deterioration of items becomes a common factor in daily life. In reality, many products such as fruits, vegetables, medicines, volatile liquids, blood banks, high-tech products and others deteriorate continuously due to evaporation, spoilage, obsolescence, etc.

The retailer must pay off as soon as the items are received. It is tacitly assumed in the classical economic order quantity inventory model. In the actual business world, that would not be true always in today’s competitive business environment. A supplier frequently offers his retailers a delay of payment for settling the amount owed to him. The permissible delay in payments is an effective method of attracting new customers and increasing sales. It may be applied as an alternative to price discount additionally, because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions. Hu and Liu (2010) analysed an optimal replenishment policy for the EPQ model with permissible delay in payments and allowable shortages. Huang (2007) developed economic order quantity model under conditionally permissible delay in payments. Ouyang, Wu, and Yang (2006) developed a study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Uthayakumar and Parvathi (2006) developed a deterministic inventory model for deteriorating items with time-dependent demand, backlogged partially when delayed in payments is permissible. Chung and Huang (2009) analysed an ordering policy with allowable shortage and permissible delay in payments. Chang, Wu, and Chen (2009) studied optimal payment time with deteriorating items under inflation and permissible delay in payments.

Inflation also plays an important role for the optimal order policy and influences the demand of certain products. The value of money goes down and erodes the future worth of saving and forces one for more current spending as inflation increases. These spending are on peripherals and luxury items that give rise to demand of these items usually. As a result, the effect of inflation and time value of the money cannot be ignored for determining the optimal inventory policy.

Hou (2006) studied an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. Hou and Lin (2006) developed an EOQ model for deteriorating items with price- and stock-dependent selling rates under inflation and time value of money. Mirzazadeh, Seyyed-Esfahani, and Fatemi-Ghomi (2009) analysed an inventory model under uncertain inflationary conditions, finite production rate and inflation-dependent demand rate for deteriorating items with shortages. Roy, Pal, and Maiti (2009) analysed a production inventory model with

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stock-dependent demand incorporating learning and inflationary effect in a random planning horizon. Sarkar and Moon (2011) developed an EPQ model with inflation in an imperfect production system. Sarkar, Sana, and Chaudhuri (2011) analysed an imperfect production process for time-varying demand with inflation and time value of money — an EMQ model. Taheri-Tolgari, Mirzazadeh, and Jolai (2012) developed an inventory model for imperfect items under inflationary conditions with considering inspection errors. Guria, Das, Mondal, and Maiti (2013) considered the inventory policy for an item with inflation-induced purchasing price, selling price and demand with immediate partial payment. Guan and Liang (2014) studied optimal reinsurance and investment strategies for insurer under interest rate and inflation risks. Sarkar, Mandal, and Sarkar (2014) analysed an EMQ model with price- and time-dependent demand under the effect of reliability and inflation. Fiti and Hichri (2014) considered price stability under an inflation targeting regime: an analysis with a new intermediate approach. Gilding (2014) studied inflation and the optimal inventory replenishment schedule within a finite planning horizon.

Combining the above arguments, so for not all the EOQ models considered all situations such as shortages, deterioration, delayed payments, inflation and time value of money. In the proposed paper, we develop an inventory model for time-varying deteriorating items with constant demand, shortages, delayed payment, inflation and time value of money. The total cost per inventory cycle depends on two decision variables \( N \) (number of replenishment) and \( F \) (fraction of replenishment cycle). An algorithm is proposed to derive optimal decision variables \( N^*, F^* \) so as to minimize the total cost and maximize the profit.

The rest of the paper is organized as follows. In the next section, assumptions, notations and mathematical formulation are given. In Section 3, a numerical example and sensitivity analysis are given in detail to illustrate the models. Finally, conclusion and summary are presented.

2. Model formulation

The following notations and assumptions are used to develop the model.

2.1. Notations

- \( D \) the constant demand rate per unit time
- \( L \) the length of the finite planning horizon
- \( \theta \) the deterioration rate per unit time
- \( r \) the fixed ordering cost, $ per order
- \( h \) the fixed holding cost, $ per order
- \( p_0 \) the unit purchase cost at time zero, $ per order
- \( V \) the unit selling price at time zero, $/item
- \( p_t \) the unit purchase cost at time \( t, p_t = p_0 e^{-Rt} \)
- \( s \) shortage cost, per unit time $ per order
- \( s_t \) shortage cost, per unit per unit time at time \( t, s_t = s e^{-Rt} \)
- \( d \) the discount rate, representing the time value of money
- \( R \) \( d - i \), representing the net constant discount rate of inflation
- \( M \) permissible delay in settling the accounts and \( 0 < M < 1 \)
- \( Q \) the order quantity per replenishment
- \( I(t) \) the inventory level at time \( t \)
- \( T \) the length of replenishment cycle
- \( t_j \) the total time that is elapsed up to and including the \( j \)th replenishment cycle where \( t_0 = 0, \ t_1 = T, \ t_N = L \)
- \( T_j \) the time at which the inventory level in the \( j \)th replenishment cycle drops to zeros

**Dependent variables**

- \( I_b \) the maximum shortage quantity
- \( I_m \) the maximum inventory level
- \( Q_m \) the maximum order quantity
- \( T_{C_r} \) the total ordering cost during \((0, L)\)
- \( T_{C_h} \) the total holding cost during \((0, L)\)
- \( T_{C_p} \) the total purchasing cost during \((0, L)\)
- \( T_{C_s} \) the total shortage cost during \((0, L)\)
- \( \phi(N, F) \) the total relevant cost during \((0, L)\)

**Decision variables**

- \( F \) the fraction of replenishment cycle where the net stock is positive
- \( N \) the number of replenishment during the planning horizon \( N = (L/T) \)

2.2. Assumptions

1. The inventory system considers a single item and the demand rate is known and constant.
2. Deterioration rate is depending on time and there is no replacement or repair of deteriorated units.
3. Shortages are allowed and completely backlogged.
4. The time horizon of the inventory system is finite.
5. The fixed credit period is offered by the supplier to the retailer.
6. Inflation rate is constant and time value of money is considered.

2.3. Mathematical formulation

\[
\frac{dI_b(t)}{dt} + \theta t I_b(t) = -D, \quad 0 \leq t \leq T_1, \quad (1)
\]
\[
\frac{dI_2(t)}{dt} = -D, \quad T_1 \leq t \leq T, \quad (2)
\]

with the boundary condition \( I(T_1) = 0 \). The solutions of Equations (1) and (2) are

\[
I_1(t) = D e^{-\theta t/2} [(T_1 - t) + \frac{\theta}{6} (T_1^3 - t^3)], \quad 0 \leq t \leq T_1 \\
I_2(t) = -D(t - T_1), \quad T_1 \leq t \leq T. \quad (3)
\]

According to Equations (3) and (4), the maximum inventory quantity at the beginning of each period and the maximum shortage quantity at the end of each period are

\[
I_m = I_1(0) = D \left[ T_1 + \frac{\theta}{6} T_1^3 \right], \quad T_1 = \frac{FL}{N} \\
I_b = D(T - T_1) = D \left( T - \frac{FL}{N} \right). \quad (5)
\]

The total minimum cost can include the following elements:

(i) Fixed ordering cost

Since the number of replenishment or period is \( N \), the fixed ordering cost over the planning horizon under net present value and inflation is

\[
TC_f = \sum_{j=0}^{N} r e^{-\theta RT} = r \left[ \frac{e^{-((N+1)RT)/N} - 1}{e^{-RT} - 1} \right], \quad (7)
\]

\[
= r \left[ \frac{e^{-((N+1)/N)RL} - 1}{e^{-RL} - 1} \right] \text{ since } T = L/N. \quad (8)
\]

(ii) Holding cost

In order to determine the holding cost, firstly the average inventory quantity should be determined. Using Equation (3), the average inventory is

\[
\bar{I} = \int_0^{T_1} I_1(t) \, dt = \int_0^{T_1} D e^{-\theta t/2} \left[ (T_1 - t) + \frac{\theta}{6} (T_1^3 - t^3) \right] \, dt \\
= D \left[ \frac{T_1^2}{2} + \frac{\theta}{12} T_1^4 - \frac{\theta^2}{72} T_1^6 \right]. \quad (9)
\]

Then, using Equation (9), the holding cost over the planning horizon under net present value and inflation is

\[
TC_h = \sum_{j=0}^{N-1} hp_0 e^{-\theta RT} \bar{I} = h D p_0 \left[ \frac{T_1^2}{2} + \frac{\theta}{12} \frac{T_1^4}{T} - \frac{\theta^2}{72} \frac{T_1^6}{T^2} \right] \left[ e^{-NRT} - 1 \right]. \quad (10)
\]

Put \( T = L/N \) in Equation (10)

\[
TC_h = h D p_0 \left[ \frac{(FL/N)^2}{2} + \frac{\theta}{12} \left( \frac{FL/N}{N} \right)^4 - \frac{\theta^2}{72} \left( \frac{FL/N}{N} \right)^6 \right] \left[ e^{-RL} - 1 \right]. \quad (11)
\]

(iii) Shortage cost

\[
\bar{B} = \int_0^T I_2(t) \, dt = \int_0^T D(t - T_1) \, dt \\
= D \left[ T - T_1 \right]^2 = D \frac{L}{N} - \frac{FL}{N} \gamma^2. \quad (12)
\]

Therefore, the shortage cost over the planning horizon under net present value and inflation is

\[
TC_s = \sum_{j=0}^{N-1} s e^{-\theta RT} \bar{B} = s \left[ \frac{e^{-NRT} - 1}{e^{-RT} - 1} \right] \bar{B} \\
= s D \left[ \frac{L}{N} - \frac{FL}{N} \right] \gamma^2 \left[ e^{-RL} - 1 \right]. \quad (13)
\]

(iv) Purchasing cost

Purchasing cost of the \( j \)th cycle is

\[
c_p(j) = c_{ij} I_m + c_{i(j+1)} I_b \\
= c_{ij} D \left[ \frac{FL}{N} + \frac{\theta}{6} \left( \frac{FL}{N} \right)^3 \right] + c_{j+1} D \left( \frac{L}{N} - \frac{FL}{N} \right). \quad (14)
\]

Therefore, the total purchasing cost over the planning horizon with \( T = L/N \) is

\[
TC_p = \sum_{j=0}^{N-1} c_p(j) \\
= p_0 D \left[ \frac{FL}{N} + \frac{\theta}{6} \left( \frac{FL}{N} \right)^3 \right] \left[ e^{-RL} - 1 \right] \left[ \frac{e^{-RL/N} - 1}{e^{-RL/N} - 1} \right] \left[ e^{-RL/N} - 1 \right]. \quad (15)
\]

(v) Interest charged and earned
Regarding interests charged and earned, we have the following two possible cases based on the value of $M \leq T_1$ and $M > T_1$ which are described as follows.

**Case i: $M \leq T_1$**

**Interest earned**

As items are sold and before the replenishment account is settled, the sales revenue is used to earn interest. At the beginning of the time interval, the backordered quantity which is $I_0$, should be replenished first and the maximum accumulated is sold until $M$ is equal to $\int_0^T M \, dt$.

Therefore, the interest earned in the first cycle is

$$IE_1 = I_0 V(0) \left[ I_0 M + \int_0^M D t \, dt \right]$$

$$= I_0 V(0) \left[ MD \left( \frac{L}{N} - \frac{FL}{N} \right) + \frac{DM^2}{2} \right].$$

And total interest earned over the horizon planning using $T = L/N$ will be

$$TIE_1(N, F) = \sum_{j=0}^{N-1} IE_1(j) = \sum_{j=0}^{N-1} I_1(e^{-jRT})$$

$$= I_0 V \left[ MD \left( \frac{L}{N} - \frac{FL}{N} \right) + \frac{DM^2}{2} \right] \times \left[ \frac{e^{-RL}}{e^{-RL/N}} - 1 \right].$$

**Interest charged**

When the replenishment account is settled, the situation is reversed and effectively the items still in stock, which is equal to $\int_0^{T_1} I(t) \, dt$, have to be financed at interest rate $I_0$.

Therefore, the interest payable in the first cycle is

$$I_p = p_0 I_0 \int_M^{T_1} I(t) \, dt = p_0 I_0 \int_M^{T_1} D e^{-\theta t/2}$$

$$= p_0 I_0 D \left[ \frac{T_1^2}{2} - M \left( T_1 - \frac{M}{2} \right) \right]$$

$$= \theta \left[ \frac{MT_1}{6} (T_1^2 - 2M^2 + \frac{T_1^4 + M^4}{12}) \right]$$

$$- \theta^2 \left[ \frac{T_1^6 + M^6}{72} - \frac{T_1^3 M^3}{36} \right].$$

And total interest payable over the horizon planning using $T = L/N$ will be

$$TIP(N, F) = \sum_{j=0}^{N-1} I_p(j) = \sum_{j=0}^{N-1} I_p e^{-jRT}$$

$$= p_0 I_0 D \left( \frac{(FL)^2}{2N^2} - \frac{M}{2} \left( \frac{FL}{N} - \frac{M}{2} \right) \right)$$

$$- \theta \left[ \frac{MT_1}{6} \left( \frac{(FL)^2}{N^2} - 2M^2 \right) + \left( \frac{(FL)^4}{N^4} + \frac{M^4}{12} \right) \right]$$

$$- \theta^2 \left[ \frac{(FL)^6}{72} + \frac{M^6}{36N^3} - \frac{(FL)^3 M^3}{36N^3} \right] \times \left[ \frac{e^{-RL}}{e^{-RL/N}} - 1 \right].$$

$$\varphi_1(N, F) = \text{ordering cost + holding cost}$$

$$+ \text{shortage cost}$$

$$+ \text{purchasing cost + interest payable}$$

$$- \text{interest earned}$$

$$\frac{\partial \varphi_1}{\partial N} = \text{RL} \left( 1 - e^{-RL/N} \right) \frac{e^{-RL/N} + \frac{e^{-RL/N} - 1}{RL}}{N} \times$$

$$\left[ \frac{hDp_0}{N} (FL)^2 \left( \frac{(FL)^4}{12N^3} - \frac{(FL)^2}{3N^2} - 1 \right) \right]$$

$$- \frac{D_0}{N} \left( FL + \frac{(FL)^3}{2N^2} + L \left( 1 - F \right) e^{-RL/N} \left( 1 - \frac{1}{N} \right) \right)$$

$$\times \left[ \frac{hDp_0}{N} \left( \frac{(FL)^2}{2N^2} + \frac{(FL)^4}{12N^4} - \frac{(FL)^6}{72N^6} \right) \right]$$

$$+ D_0 \left( FL + \frac{(FL)^3}{6N^3} + \frac{e^{-RL/N} L \left( 1 - F \right)}{N} \right).$$
\[ + sDL^2(1 - F)^2 2N^2 + I_p_0 D \left[ \frac{(FL)^2}{2N^2} - M \left( \frac{FL}{N} - \frac{M}{2} \right) \right] \]
\[ - \theta \left( \frac{M(FL)^3}{6N^3} - M^3 \frac{FL}{3N} + \frac{(FL)^4}{12N^2} \right) + \theta^2 \left( \frac{M^6}{72} + \frac{(FL)^6}{72N^6} - \frac{M^3(FL)^3}{36N^3} \right) \]
\[- I_v V \left( \frac{ML}{N} (1 - F) - \frac{M^2}{2} \right) \right] \right] = 0. \quad (21) \]
\[ \frac{\partial \varphi_1}{\partial F} = hD_p_0 \left[ \frac{FL^2}{N^2} + \frac{\theta}{3} \left( \frac{L}{N} \right)^4 F^3 - \frac{\theta^2}{12} \left( \frac{L}{N} \right)^6 F^5 \right] \]
\[ + D_p_0 \left[ \frac{L}{N} + \frac{\theta}{2} \left( \frac{L}{N} \right)^3 F^2 - e^{-RL/N} \left( \frac{L}{N} \right) \right] \]
\[- sD \left( \frac{L}{N} \right)^2 \left( 1 - F \right) + I_e V D \frac{ML}{N} = 0. \quad (22) \]

Case ii: \( M > T_1 \)

Interest earned

At the beginning of the time interval, the backordered quantity, \( I_b \), should be replenished first and interest earned for the first cycle will be \( I_e V(0)(M - I_b) \). Then, the maximum accumulated is sold until \( M \) is equal to \( I_e V(0)(M - T_1)D_T_1 + (D_T_1^2/2) \), while the interest earned will be \( I_e V(0)((M - T_1)D_T_1 + (D_T_1^2/2)) \).

Therefore, the interest earned in the first cycle is

\[ I_e V(0) \left[ I_b M + (M - T_1)D_T_1 + \int_0^m D_T_1 dt \right] \]
\[ = I_e V(0) \left[ MD \left( \frac{L}{N} - \frac{FL}{N} \right) + \left( M - \frac{FL}{N} \right) D \frac{FL}{N} \right] \]
\[ + \left( \frac{FL^2}{2N^2} \right) \right]. \quad (23) \]

And total interest earned over the planning horizon using \( T = L/N \) will be

\[ TIE_2 (N,F) = \sum_{j=0}^{N-1} I_e V(0) \left[ I_b M + (M - T_1)D_T_1 + \int_0^m D_T_1 dt \right] \]
\[ = I_e V(0) \left[ MD \left( \frac{L}{N} - \frac{FL}{N} \right) + \left( M - \frac{FL}{N} \right) D \frac{FL}{N} \right] \]
\[ + \left( \frac{FL^2}{2N^2} \right) \right]. \quad (24) \]

Interest charged

In this case when the replenishment accounts is settled, the number of items which are in stock is zero, so the interest payable will be zero.

Therefore, the total inventory cost function is

\[ \varphi_2(N,F) = \text{ordering cost} + \text{holding cost} + \text{shortage cost} + \text{purchasing cost} - \text{interest earned}, \]
\[ \varphi_2(N,F) = r \left[ \frac{e^{-((N+1)/N)RL-1}}{e^{-RL/N} - 1} \right] \]
\[ + \left\{ hD_p_0 \left[ \frac{(FL/N)^2}{2} + \frac{\theta(FL/N)^4}{12} - \frac{\theta^2(FL/N)^6}{72} \right] \right\} \]
\[ + sD \left[ \frac{L}{N} - \frac{FL^2}{N^2} \right] + p_0 D \left[ \frac{FL}{N} + \frac{\theta}{6} \left( \frac{FL}{N} \right)^3 \right] \]
\[ + p_0 D e^{-RL/N} \left( \frac{L}{N} - \frac{FL}{N} \right) - I_e V \left[ MD \left( \frac{L}{N} - \frac{FL}{N} \right) \right] \]
\[ + \left( M - \frac{FL}{N} \right) D \frac{FL}{N} + \left( \frac{FL^2}{2N^2} \right) \right] \left[ \frac{e^{-RL/N} - 1}{e^{-RL/N} - 1} \right]. \quad (25) \]

For optimality, \( \frac{\partial \varphi_2}{\partial N} = 0 \) and \( \frac{\partial \varphi_2}{\partial F} = 0 \)

\[ \frac{\partial \varphi_2}{\partial N} = RL(1 - e^{-RL/N}) \left[ e^{-RL/N} + \frac{e^{-RL/N} - 1}{RL} \right] \]
\[ \frac{\partial \varphi_2}{\partial N} = RL(1 - e^{-RL/N}) \left[ e^{-RL/N} + \frac{e^{-RL/N} - 1}{RL} \right] \]
\[ hD_p_0 \left[ \frac{(FL)^2}{2N^2} + \frac{\theta(FL)^4}{12N^2} - \frac{\theta^2(FL)^6}{72N^6} \right] \]
\[ + D_p_0 \left[ \frac{FL}{N} + \frac{\theta(FL)^3}{6N^3} + e^{-RL/N} \left( \frac{L}{N} - \frac{FL}{N} \right) \right] \]
\[ + \left( \frac{FL^2}{2N^2} - I_e V D \left( \frac{ML}{N} - \frac{(FL)^2}{2N^2} \right) \right] = 0. \quad (26) \]

\[ \frac{\partial \varphi_2}{\partial F} = hD_p_0 \left[ \frac{FL^2}{2N^2} + \frac{\theta}{3} \left( \frac{L}{N} \right)^4 F^3 - \frac{\theta^2}{12} \left( \frac{L}{N} \right)^6 F^5 \right] \]
\[ + D_p_0 \left[ \frac{L}{N} + \frac{\theta}{2} \left( \frac{L}{N} \right)^3 F^2 - e^{-RL/N} \left( \frac{L}{N} \right) \right] \]
\[ + \left( \frac{FL^2}{2N^2} - I_e V D \left( \frac{ML}{N} - \frac{(FL)^2}{2N^2} \right) \right] = 0. \quad (27) \]

Calculation of profit

Revenue = price \times \text{order quantity} = p_0 (I_m + I_b),

where \( I_m = D[FL/N + (\theta/6)(FL/N)^3] \) and \( I_b = D(L/N - FL/N) \).
The total maximum order quantity
\[ Q_m = I_m + I_b = D \left[ \frac{L}{N} + \frac{\theta}{6} \left( \frac{FL}{N} \right)^3 \right]. \]

Therefore, the revenue cost over the planning horizon under net present value and inflation consideration is
\[ TR_e = \sum_{j=0}^{N-1} r_{p_0} Q_m = r_{p_0} D \left[ \frac{L}{N} + \frac{\theta}{6} \left( \frac{FL}{N} \right)^3 \right] \left[ \frac{e^{-RL} - 1}{e^{-RL} - 1} \right], \]

\[ \phi(N^*, F^*) = TR_e - \psi(N^*, F^*). \]

By using the following algorithm, we have to find optimal values of \( N^* \) and \( F^* \) to minimize the total cost \( \psi(N^*, F^*) \) and maximize the profit \( \phi(N^*, F^*) \).

**Algorithm 1**
**Step 1 Perform (i)–(v)**

(i) Input the values
(ii) Substituting the values into Equation (21) and find \( N_1(1) \)
(iii) Using \( N_1(1) \) determine \( F_1(1) \) from Equation (22)
(iv) Using Equation (20) determine \( \varphi_1(N_1(1), F_1(1)) \)
(v) Repeat (ii) and (v) until \( \varphi_1(N_1(n-1), F_1(n-1)) \leq \varphi_1(N_1(n), F_1(n)) \). Consider \( N_1 = N_1(n-1), F_1 = F_1(n-1) \) and go to step 2

**Step 2 Perform (i)–(v)**

(i) Input the values
(ii) Substituting the values into Equation (26) and find \( N_1(1) \)
(iii) Using \( N_1(1) \) determine \( F_1(1) \) from Equation (27)
(iv) Using Equation (25) determine \( \varphi_2(N_1(1), F_1(1)) \)
(v) Repeat (ii) and (v) until \( \varphi_2(N_1(n-1), F_1(n-1)) \leq \varphi_2(N_1(n), F_1(n)) \). Consider \( N_2 = N_1(n-1), F_2 = F_1(n-1) \) and go to step 3

**Step 3 Case (i)** \( M \leq F_1L/N_1 \) and \( M > F_2L/N_2 \)

(i) If \( \varphi_1(N_1, F_1) \leq \varphi_2(N_2, F_2) \) then \( N^* = N_1, F^* = F_1 \) and \( \phi(N^*, F^*) = \varphi_1(N_1, F_1) \)
(ii) If \( \varphi_1(N_1, F_1) > \varphi_2(N_2, F_2) \) then \( N^* = N_2, F^* = F_2 \) and \( \phi(N^*, F^*) = \varphi_2(N_2, F_2) \)
(iii) Go to step 4

**Case (ii)** \( M > F_1L/N_1 \) and \( M > F_2L/N_2 \)

(i) Input the values
(ii) Substituting the values into Equation (21) and find \( N_1(1) \)
(iii) Using \( N_1(1) \) determine \( F_1(1) \) by using \( F_1(1) = N_1(1)M/L \)
(iv) Using Equation (20) determine \( \varphi_1(N_1(1), F_1(1)) \)
(v) Repeat (ii) and (v) until \( \varphi_1(N_1(n-1), F_1(n-1)) \leq \varphi_1(N_1(n), F_1(n)) \). Consider \( N_1 = N_1(n-1), F_1 = F_1(n-1) \)
(vi) If \( \varphi_1(N_1, F_1) \leq \varphi_2(N_2, F_2) \) then \( N^* = N_1, F^* = F_1 \) and \( \phi(N^*, F^*) = \varphi_1(N_1, F_1) \)
(vii) If \( \varphi_1(N_1, F_1) > \varphi_2(N_2, F_2) \) then \( N^* = N_2, F^* = F_2 \) and \( \phi(N^*, F^*) = \varphi_2(N_2, F_2) \)
(viii) Go to step 4

**Case (iii)** \( M \leq F_1L/N_1 \) and \( M \leq F_2L/N_2 \)

(i) Input the values
(ii) Substituting the values into Equation (26) and find \( N_1(1) \)
(iii) Using \( N_1(1) \) determine \( F_1(1) \) by using \( F_1(1) = N_1(1)M/L \)
(iv) Using Equation (25) determine \( \varphi_2(N_1(1), F_1(1)) \)
(v) Repeat (ii) and (v) until \( \varphi_2(N_1(n-1), F_1(n-1)) \leq \varphi_2(N_1(n), F_1(n)) \). Consider \( N_1 = N_1(n-1), F_1 = F_1(n-1) \)
(vi) If \( \varphi_1(N_1, F_1) \leq \varphi_2(N_2, F_2) \) then \( N^* = N_1, F^* = F_1 \) and \( \phi(N^*, F^*) = \varphi_1(N_1, F_1) \)
(vii) If \( \varphi_1(N_1, F_1) > \varphi_2(N_2, F_2) \) then \( N^* = N_2, F^* = F_2 \) and \( \phi(N^*, F^*) = \varphi_2(N_2, F_2) \)
(viii) Go to step 4

**Case (iv)** \( M > F_1L/N_1 \) and \( M \leq F_2L/N_2 \)

(i) If \( \varphi_1(N_1, F_1) \leq \varphi_2(N_2, F_2) \) then \( N^* = N_2, F^* = F_1 \) and \( \phi(N^*, F^*) = \varphi_1(N_1, F_1) \)
(ii) If \( \varphi_1(N_1, F_1) > \varphi_2(N_2, F_2) \) then \( N^* = N_2, F^* = F_2 \) and \( \phi(N^*, F^*) = \varphi_2(N_2, F_2) \)
(iii) Go to step 4

**Step 4 Determination of \( I_m, I_b \) and \( \phi(N^*, F^*) \)**

(i) Determine \( I_m \) by using \( I_m = D(F^*/L)^8 + \theta/f(F^*/L)^8 \)
(ii) Determine \( I_b \) by using \( I_b = D(L/N)^8 - (F^*/L)^8 \)
(iii) Determine order quantity \( Q_m = I_m + I_b \)
(iv) Determine maximum profit \( \phi(N^*, F^*) \) by using Equation (28)

3. Numerical examples

**Example 1**
Let \( r = 100, D = 10,000, R = 0.05, L = 30/365, F = 0.001, \theta = 0.01, h = 0.5, p_0 = 8, s = 2, I_t = 0.06, I_e = 0.05, N = 1, V = 10, M = 5/365. \) The computational result shows the following optimal values by using the above algorithm:

\[ N^* = 1.8309, F^* = 0.2413, \epsilon = 0.00449, Q_m' = 449, \]
\[ \psi(N^*, F^*) = 112.3436, \phi(N^*, F^*) = 2214.7. \]

**Example 2**
Let \( r = 500, D = 10,000, R = 0.05, L = 60/365, F = 0.001, \theta = 0.01, h = 0.8, p_0 = 8, s = 4, I_t = 0.06, I_e = 0.05, N = 1, V = 10, M = 10/365. \) The computational result shows the following optimal values by using above algorithm:

\[ N^* = 1.6577, F^* = 0.2763, T^* = 0.0992, Q_m' = 992, \]
\[ \psi(N^*, F^*) = 560.9096, \phi(N^*, F^*) = 50.657. \]

3.1. Sensitivity analysis

We now study the effects of changes in the value of system parameters \( R, L, \theta, D, r, I_t, I_e, s, p_0, M, V \) on the optimal length of order cycle \( T^* \), the optimal number of replenishment \( N^* \), the optimal order quantity per cycle \( Q_m' \), the minimum total relevant cost per unit time \( \psi(N^*, F^*) \) and the total profit \( \phi(N^*, F^*) \) of Example 1. The analysis is carried out by changing the value of only one parameter at a time keeping the rest of the parameters at their initial values. The results are shown in Table 1.

From Table 1, the following inferences can be observed:

(1) An increase in the value of demand \( D \), the relevant total costs \( \psi(N^*, F^*) \), optimum order size \( Q_m' \) and profit \( \phi(N^*, F^*) \) will increase, that is, if the retailer’s demand
### Table 1. Sensitivity of the optimal solution with respect to change in values of the model parameters.

| Parameters | $N^*$ | $F^*$ | $Q_m^*$ | $\phi(N^*, F^*)$ | $\psi(N^*, F^*)$ |
|------------|-------|-------|---------|------------------|------------------|
| $r$        | 100   | 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $R$        | 0.05  | 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $D$        | 10,000| 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $L$        | 30/365| 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $P_0$      | 6     | 1.8781| .3088   | 438              | 109.7437         | 1591.7           |
| $\theta$   | 0.01  | 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $I_r$      | 0.06  | 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $I_e$      | 0.05  | 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $s$        | 2     | 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $V$        | 10    | 1.8309| .2413   | 449              | 112.3436         | 2214.7           |
| $M$        | 5/365 | 1.8309| .2413   | 449              | 112.3436         | 2214.7           |

increases, then the retailer orders a high amount of quantity with the supplier, so that the costs automatically increase.

(2) An increase in the value of deterioration rate $\theta$, the relevant total costs $\psi(N^*, F^*)$ and profit $\phi(N^*, F^*)$ increases without affecting the optimum order size $Q_m^*$, that is, $\theta$ is independent of optimum order size $Q_m^*$.

(3) An increase in the value of ordering cost $r$, the relevant total costs $\psi(N^*, F^*)$, profit $\phi(N^*, F^*)$ will increase and the optimum order size $Q_m^*$ will decrease.

(4) An increase in the value of the parameters $R, L, s, P_0$ the optimum order size $Q_m^*$, the relevant total costs $\psi(N^*, F^*)$ and profit $\phi(N^*, F^*)$ will increase.

(5) An increase in the value of the parameters $I_r, V, h$ and $M$ the optimum order size $Q_m^*$, the profit $\phi(N^*, F^*)$ will increase and the relevant total costs $\psi(N^*, F^*)$ will decrease.

(6) An increase in value of the parameter $I_r$, the relevant total costs $\psi(N^*, F^*)$ will decrease, the profit $\phi(N^*, F^*)$ will increase and the optimum order size $Q_m^*$ will remain same.

## 4. Conclusion

This paper deals with the EOQ model for deteriorating items with inflation and time value of money over the finite horizon. The model assumes constant demand, time-varying deteriorating rate and finite planning horizon. Shortages are allowed in the inventory system and are fully backlogged and also supplier offers the delayed payment strategy. Our aim is to find the optimal decision
variables to maximize the net present value of the profit and minimize the net present value of the total inventory system cost over a finite horizon. Numerical examples are also provided to illustrate the proposed model. Moreover, sensitivity analysis of the optimal solutions with respect to major parameters is carried out. The model can be extended in several ways. For instance, we may generalize the models by allowing price- or stock-dependent demand, fuzzy demand, two-level trade-credit policy, temporary discounts, quantity discounts, etc.

Disclosure statement
No potential conflict of interest was reported by the authors.

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