Simulating the Collapse of Rotating Primordial Gas Clouds to Study the Possibility of the Survival of Population III Protostars

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Abstract

It has been argued that the low-mass primordial stars ($m_{\text{pop III}} \lesssim 0.8 M_\odot$) are likely to enter the main sequence and hence may possibly be found in present-day galaxies. However, due to limitations in existing numerical capabilities, current three-dimensional (3D) simulations of disk fragmentation are only capable of following a few thousand years of evolution after the formation of the first protostar. In this work, we use a modified version of the GADGET-2 smoothed particle hydrodynamics code to present the results of the nonlinear collapse of the gas clouds associated with various degrees of initial solid body rotation (parameterized by $\beta$) using a piecewise polytropic equation of state. The 3D simulations are followed until the epoch that occurs when 50$M_\odot$ of mass has been accreted in protostellar objects, which is adequate enough to investigate the dynamics of the protostars with the surrounding gaseous medium and to determine the mass function, accretion rate, and possibility of the survival of these protostellar objects to the present epoch. We found that evolving protostars that stay within slow-rotating parent clouds can become massive enough to survive, due to accretion in the absence of radiative feedback, whereas 10%–12% of those formed within fast-rotating clouds ($\beta \gtrsim 0.1$) could possibly be ejected from the gravitational bound cluster as low-mass stars.

Unified Astronomy Thesaurus concepts: Population III stars (1285); Cosmology (343); Stellar accretion (1578); Hydrodynamical simulations (767); Protostars (1302); Initial mass function (796); Low mass stars (2050)

1. Introduction

The age of the universe and the expected time at which the very first stars formed makes direct observations a difficult prospect (see recent surveys by, e.g., Frebel et al. 2019; Schauer et al. 2020; Suda et al. 2021; Finkelstein et al. 2022; Hartwig et al. 2022). Theoretical prediction of the Lambda cold dark matter model shows that the entire process is led by the gravitational collapse of dark matter halos as a consequence of hierarchical structure formation (see the latest results in, e.g., Bohr et al. 2020; Springel et al. 2020; Wang et al. 2020; May & Springel 2021; Latif et al. 2022). At the time of collapse, the primordial gas in halos is very hot and remains spread out due to its high pressure (Barrow et al. 2017; Barkana 2018; Chon et al. 2018). Gas cools by radiating away energy and collapses to form a thin rotating circumstellar disk that grows over time and fragments due to gravitational and spiral-arm instability (Inoue & Yoshida 2020; Wollenberg et al. 2020; Chiaki & Yoshida 2022). Some of the fragments that go on to become stars are not isolated and continue to interact with the surrounding gas. This interaction leads to an increase in the mass of fragments as well as changes in their orbits. This leads to the very basic question of what is the fate of these evolving fragments in the cluster. Do they merge with the central star (Klessen 2019; Kulkarni et al. 2019), or do they move away from the cluster after their dynamical interaction with each other and with the surrounding gas (Sharda et al. 2019; Sugimura et al. 2020)?

It may be that a fraction of them can either become massive due to rapid accretion (Umeda et al. 2016; Woods et al. 2017; Fukushima et al. 2020) such that the resulting stars explode as (pair-instability) supernovae (Whalen et al. 2014; Welsh et al. 2019; Jeon et al. 2021) or collapse into black holes (Madau & Rees 2001; Matsumoto et al. 2015) or may lead to the formation of a supermassive black hole (SMBH; Alister Seguel et al. 2020; Herrington et al. 2022). There might also exist a fraction of fragments that remain low-mass protostars and hence are able to survive to the present epoch provided their mass remains as low as 0.8 $M_\odot$ (Marigo et al. 2001; Ishiyama et al. 2016; Susa 2019; Dutta et al. 2020a). Thus, the mass function of these fragments remains unclear and requires further investigation (see review by Haemmerlé et al. 2020). While it is possible to run detailed simulations of a few systems, it is difficult to explore a wide range of parameter space with this approach. When the numerical integration over the density regime is computed beyond the formation of the first protostellar core, the collapse tends to be chaotic and highly nonlinear, and it becomes difficult to follow the dynamical system for a long time. As a consequence, current simulations lack the ability to follow the evolution of fragments over a sufficient number of orbital revolutions within the disk.

In this paper, we aim to develop a model, building upon our earlier work (Dutta 2016a) on the fragmentation of the unstable disk centered within the rotating collapsing gas clouds, using the modified version of the Gadget-2 SPH simulations and piecewise polytropic equation of state, in order to place some upper bounds on the final mass of the protostars after the long time evolution of the gas. In Section 2, we describe in detail the initial conditions, implementation of a polytropic index profile in the mathematical model, and the modified numerical scheme. The details of the dynamics are outlined in Section 3 with an emphasis on fragments that stay below the critical mass required to survive to the present epoch. We summarize our work in Section 4.
2. Numerical Methodology

We start our discussion by considering the uniform density of spheres of gas with number density \( n = 10^4 \text{ cm}^{-3} \) and temperature \( T = 250 \text{ K} \), with initial solid body rotation. The gas density is represented by SPH particles. The clouds are numerically designed to model the local thermodynamic equilibrium (LTE) conditions for primordial gas and to study the effects of rotation on the timescales associated with the collapse and subsequent fragmentation.

2.1. Initial Condition

The gas clouds are modeled with approximately 2 million SPH particles, each with mass \( m_{\text{gas}} = 4.639 \times 10^{-4} \text{ M}_{\odot} \), uniformly distributed inside a sphere of radius equal to the Jeans radius at LTE, i.e., \( R = R_J \approx 0.857 \text{ pc} \) with total mass \( M = M_J \approx 940 \text{ M}_{\odot} \). This numerical setup allows us to follow the collapse accurately for about 10 orders of magnitude in density up to the number density \( 5 \times 10^{14} \text{ cm}^{-3} \) until the formation of the first central sink (i.e., central hydrostatic core), and about 4 orders of magnitude in size up to about 10 au. The mass resolution for \( N_{\text{gb}} = 100 \text{ SPH neighbors} \) is about 0.04639M\(_{\odot}\). This implies that the rotating gas clouds are numerically well resolved up to a critical number density \( n_{\text{crit}} = 2.02 \times 10^{15} \text{ cm}^{-3} \), given by

\[
n_{\text{crit}} = \left( \frac{3}{4\pi} \right) \left( \frac{5k_B T}{G} \right) \left( \frac{1}{\mu m_p} \right)^{4/3} \left( \frac{1}{m_{\text{gas}}N_{\text{gb}}} \right)^{2/3} \tag{1}
\]

for temperature \( T = 1300 \text{ K} \). Here \( \mu \) is the hydrogen mass fraction of the gas, \( m_p \) is the mass of the proton, and all other symbols have their usual meaning. The freefall time over sound-crossing time \( t_{\text{ff}}/t_{\text{sc}} \sim 0.1 \) for the clouds confirms the validity of the initial conditions for some degree of gravitational collapse.

Initial velocities are assigned to the SPH particles depending upon the angular velocity (\( \Omega \)) of the clouds in addition to the thermal distribution of velocities and no internal turbulent motion. In the absence of internal turbulent motions, the degree of rotation (i.e., the strength of the centrifugal support) of clouds is modeled by estimating the rotational energy over the total gravitational potential energy (Sterzik et al. 2003 quantified it using the parameter \( \beta = R^2 \Omega^2 / 3GM \)). We also model the distribution of angular momentum that originates from the distortion of the clouds or from their nonaxisymmetric nature due to differential rotation between the high- and low-density regimes (Larson 1984; Meynet & Maeder 2002). The gravitational forces from the dark matter are negligible compared to the self-gravity of the gas on the length scales of our simulation; therefore, for the sake of simplicity, we do not consider the dark matter or the expansion of space itself in our simulations.

Although our calculation is based on the initial condition that assumes a spherical cloud with uniform density distribution, it is to be noted that molecular clouds are in general very irregular in shape. This is because, in reality, the formation of molecular clouds due to the continual accumulation of baryonic matter within a dark matter halo encounters different types of forces occurring simultaneously (Larson 1972; Shu 1977; Bodenheimer & Boss 1981). For example, the interplay between the self-gravity of the cloud and the internal pressure of the infalling gas can introduce asymmetries or inhomogeneities in the clouds, which will be amplified during the collapse at a later time. That is why we see in the cosmological simulation that the initial density distribution of molecular clouds before it reaches LTE is nonhomogeneous in nature and rather closer to the non-singular isothermal sphere (Suto & Silk 1988; Omukai & Nishi 1998).

In addition, as the gas cloud possesses small angular momentum, it soon experiences a differential rotation between the layers of the gas due to which the cloud becomes slightly centrally condensed. As a consequence, nonaxisymmetric features are likely to appear at the initial phase of the formation of molecular clouds and even tend to wind up forming trailing spiral patterns at later stages of collapse. Larson (1984) has elaborated on the fact that these tiny irregularities during the formation of the molecular cloud can certainly develop non-radial gravitational forces in a nonaxisymmetric mass distribution. The associated gravitational/tidal torques that transfer angular momentum outward on an orbital timescale are another reason for the inhomogeneities, which we see in cosmological simulations. Besides turbulent motion viscosity, even magnetic fields can also play a role in shaping the clouds during their initial dynamical phase (Truelove 1998; McKee 2002; Krumholz & McKee 2005).

2.2. Modeling the Polytropic Equation of State

In order to account for various heating and cooling processes occurring simultaneously, which become important at certain number densities during the collapse, we model the thermal behavior of the gas clouds (following the discussion in Jappsen et al. 2005) with a piecewise polytropic equation of state,

\[
T_i(n_i) = a_i n_i^{-\gamma_i}, \quad i = 1, 2, 3. \tag{2}
\]

Here the polytropic index \( \gamma_i \) changes values in a piecewise constant manner as a function of the number density \( n \). The constant of proportionality \( a_i \), which is initially determined from the thermal conditions of the gas, is also rescaled in order to maintain the continuity of the temperature across the certain intermediate densities \( n_{\text{int}} \),

\[
T_i(n_{\text{int}}) = T_{i+1}(n_{\text{int}}), \tag{3}
\]

according to the following equation:

\[
a_{i+1} = a_i n_{\text{int}}^{-\gamma_i}. \tag{4}
\]

The intermediate densities and values for the polytropic index in different density intervals are chosen carefully in order to reproduce the temperature-density profile resulting from the primordial chemistry (Dutta et al. 2015; Pallottini et al. 2017; Bovino et al. 2019). Furthermore, as the fast-rotating clouds have larger timescales associated with the collapse and tend to have lower rates of compressional heating, they are significantly colder than their slow-rotating counterparts (Dutta 2016a). Therefore, all the values of the polytropic index also depend on the degree of rotation of the clouds. Table 1 summarizes the chosen values for the polytropic indices for all the clouds, where the piecewise polytropic index profile is divided into three regions separated by intermediate densities \( n_{\text{int}} = 10^9 \) and \( 10^{12} \text{ cm}^{-3} \).

In order to implement a general polytropic process in the publicly available GADGET-2, we added a polytropic index variable that controls the rate of change of the entropic function.
to be the same as the adiabatic index in the original code. In addition, we identified the original adiabatic index variable in the code with the quantity $1 + 1/C_V$, where $C_V$ is the specific heat at constant volume for the gas. The implementation is explained in detail in Appendix A. This is necessary in order to accurately model the thermal and chemical evolution of gas. This also reduces the computational cost and allows the simulations to be followed for a long period of time.

2.3. Simulation Details

Once the central density reaches the critical value given by Equation (1), the total mass enclosed in a single kernel volume, $(m_{\text{gas}}N_{\text{ ngh}})$, becomes greater than the local Jeans mass, which limits the density resolution for SPH simulations. Furthermore, the adaptive time steps for the integration near the critical value become of the order of 0.01 yr, which is too small to be able to follow the simulations for any reasonable amount of time after the formation of the central core, and hence, no fragmentation can be seen.

To circumvent this problem, we searched among all the processors for the highest density particle every 10 time steps after the number density $n$ reached $5 \times 10^{13}$ cm$^{-3}$ for the first time. Since the particles are distributed over a number of processors according to the domain decomposition, we broadcast the information of this highest-density particle to all the processors to check for neighbors in their own domain. We then dynamically replace the entire region centralized at the highest density particle with $n > 5 \times 10^{14}$ cm$^{-3}$ and $T > 1300$ K by nongaseous sink particles upon satisfaction of the sink formation criteria as given in Bate & Bonnell (1997), i.e., the particle is on the current time step and the divergence of both the velocity and acceleration are negative in the vicinity of this particle. The total potential energy within two smoothing lengths is greater than the sum of thermal and rotational kinetic energies and this region is also virially unstable.

The sink particles are formed from about 50 neighboring gas particles within one smoothing length and thereafter interact with the rest of the gas gravitationally. A sink can accrete the gas particles falling into an accretion radius $r_{\text{acc}}$ that we fix to be about 8 au. This is done provided the particle is on the current time step, and the total energy of a gas particle relative to the sink is negative, i.e., the particle is gravitationally bound to the candidate sink. In addition, the specific angular momentum of the gas particle around the sink is less than what is required to form a circular orbit.

When a gas particle is accreted, its mass and linear momentum are added to the sink particle and the location of the sink particle is shifted to occupy the center of mass of the two. The accreted gas particles are removed from the simulation and their effect is taken into account using appropriate boundary conditions near the accretion region. In addition to the accretion radius, we also define an outer accretion radius $r_{\text{outeracc}} = 1.25r_{\text{acc}}$ such that the gas particles falling into this outer accretion radius are only evolved gravitationally until they reach the accretion radius and are possibly accreted by the candidate sink. The gas particles may also leave the outer accretion region during the course of their motion. We prevent sink particles from being formed within $2r_{\text{outeracc}}$ of each other in order to restrain the spurious formation of sink particles from the gas, which eventually would have been accreted by the candidate sink.

As a check, we also keep track of the global quantities of the gaseous system such as total energy, angular momentum, and entropy throughout the simulation. The sink particles are usually created at protostellar density and temperature, and are subsequently identified as being growing protostars. The gravitational softening for the sink particles is set to be equal to $r_{\text{acc}}$, while for the gas particles, we use a variable gravitational softening length, which is proportional to their SPH smoothing length. This greatly improves the time it takes to run the simulations. Besides, following the discussion in Clark et al. (2011), we also implement a constant external pressure boundary in addition to vacuum and periodic boundary conditions in GADGET-2. To this end, we modify the original SPH momentum equation

\[
\frac{dv_i}{dt} = - \sum_j m_j \left[ \frac{p_i}{p_j} \nabla W_0(h_i) + \frac{p_j}{p_i} \nabla W_0(h_j) \right]
\]

by subtracting the contribution of the external pressure, $P_{\text{ext}} = 2.5 \times 10^6 K_\odot K \text{ cm}^{-3}$ from both $P_i$ and $P_j$. All the other symbols have their usual meaning.

2.4. Check for Self-similarity Solution

In this section, we quickly check the gas distributions at different epochs of time associated with the runaway phase of the collapse of the clouds for various degrees of initial solid body rotation. For the nonrotating ($\beta = 0$) clouds, the density remains spherically symmetric throughout and follows a well-known power-law profile $r^{-2.2}$ while the central density increases monotonically with time as $\propto 1/(t - t_0)^2$. The initial phase of the collapse is likely to remain self-similar at different epochs of freefall time. This means that the collapsing gas distribution is invariant, i.e., looks similar on every scale of the density regime. This can be seen from Figure 1, consistent with the conventional studies (Shu 1977; Suto & Silk 1988; Omukai & Nishi 1998) in which the gas distribution is self-similar corresponding to $\gamma_{\text{eff}} \sim 1.09$. Clouds with various rotational support also roughly follow the same power-law density profile. However, as collapsed gas gets redistributed and accumulates near the center of the mass of the cloud, the degree of rotational support also increases. This causes the density and its gradient to be slightly lower and gas temperatures to be a little lower near the center of clouds with a higher degree of rotation (Saigo et al. 2008; Bromm & Yoshida 2011; Meynet et al. 2013; Dutta 2015).

3. Results

The transport of angular momentum to smaller scales results in the formation of a rotationally supported spiral arm, the
so-called circumstellar disk or disk-like structures around the central hydrostatic core. Until this point, the collapse has been studied rigorously (Greif et al. 2012; Hirano et al. 2014; Dutta 2016b; Riaz et al. 2018). See Appendix B for a discussion of the runaway collapse phase. Here we follow the simulation further until the epoch of time when 50M_☉ of mass is accreted in total onto the dynamically created sink particles as a consequence of instabilities within the spiral arm and its fragmentation. We study the dynamics associated with multiple sinks and their interaction with ambient gas.

3.1. Gas Distribution during Disk Fragmentation

Figures 2 and 3 show snapshots of logarithmically scaled densities and the temperature distribution of the gas in the equatorial plane of the circumstellar disk for the 16 values of rotation parameter β = 0.0–0.15. All the images understandably reflect the fact that the collapsed gas in the spiral arm becomes unstable by accreting mass from the surroundings and hence is prone to fragmentation. This may also be a consequence of the interplay between the gravitational torque and pressure gradient throughout the layers of the spiral arms. In all these snapshots, the white dots represent the sink particles in the simulation and the circle shows the scale in astronomical units at which the fragmentation takes place. The size of the circle generally continues extending with increasing β parameter. For example, the central region for β = 0.02 is approximately 600 au, whereas for β = 0.1 it is ∼3000 au. The snapshots also reflect the fact that all the clouds with nonzero rotation form a small N-body protostellar system immersed in the ambient medium near the center of the clouds. As expected, the fast-rotating clouds are likely to develop noticeable dense spiral arms in which protostellar mass evolves substantially, and in all probability to fragment reasonably more, say, N ∼8–11 for β > 0.05, as seen in Figures 2 and 3. This satisfies the justification for the conservation of angular momentum during the gas evolution (Hirano & Bromm 2018a). In contrast, the slow-rotating clouds contain a relatively small number of protostars, N ∼4–7, indicating the possibility of having a high accretion rate, as predicted in theoretical calculations (Machida & Doi 2013; Liu et al. 2021). We have limited our calculations up to this epoch because it is extremely difficult to follow simulations for the fast-rotating clouds above β > 0.15 beyond this stage of evolution.

Another feature to be noted in Figure 2 is that the gas distribution is highly complicated and nonlinear in nature, which can be thought of as a supersonic and compressible flow coupled to the accreting sinks in the gravitationally bound protostellar system. Due to interaction with the ambient medium, the evolving sinks experience strong friction, also known as drag forces (Dutta et al. 2020b), which can change the movement and orbit of the sinks (Bobrick et al. 2017 similar to what is seen in the X-ray binary system).

3.2. Evolution of the Sinks

The fragmentation of the rotating unstable spiral arms within the circumstellar disk has significant implications for the final mass of the evolving sinks. Figures 4 and 5 show the mass of the sinks in our simulation as a function of time. The first feature to be noted is that all clouds are inclined to fragment on an estimated mass scale that evolves up to ∼0.001–20 M_☉, depending on the strength of rotational support of the parent clouds. Because the fragmentation takes place as a consequence of the gravitational instability, the characteristic mass scale may be substantially smaller. Second, as the gas continues to collapse to higher densities, the spiral arms continue developing instabilities that result in the successive formation of secondary sinks within the circumstellar disk. We see that most of the fragmentation takes place within ∼100–200 yr from the formation of the central core for clouds with β ≤ 0.05. The higher the degree of rotation is, the longer the time t_{frag} takes via the gas to become gravitationally unstable to fragmentation. This is expected as the sinks within slow-rotating clouds begin to be Jeans unstable much earlier due to the very strong accretion rate, ∼1M_☉ yr⁻¹ (as can be seen in Figure 5). In addition, we see that sinks moving with lower radial velocity within dense ambience are likely to have a high accretion rate. Thus, even if the sinks had low mass at the time of formation, their mass can be increased approximately by an order of magnitude relative to their initial mass. As a consequence, sinks now face more gravitational drag due to the increase in mass, and therefore they are likely to change their orbits. This also triggers the sinks to be more centrally condensed and continue to accrete gas ending up as massive protostars within a few thousand years of evolution. We also see that sinks for low β-values are quite strongly bound gravitationally. However, clouds with a higher degree of rotation tend to fragment more vigorously due to spiral-arm instabilities on larger scales and contain both low and high-mass sinks (some of which even have quite a high radial velocity as compared to the escape velocity of the cloud). For example, a number of protostars with m_* < 1 M_☉ are formed for the clouds with β > 0.1. This is consistent with the probability of the existence of the smallest fragmentation scale ∼0.03 au with ∼0.01 M_☉ (Becerra et al. 2015; Hirano et al. 2017). We conclude that the formation of the sinks and their dynamical interaction with the ambience depend on the history of collapse (i.e., evolution history of chemical/thermal changes, turbulence, and angular momentum conservation).
3.3. Histograms of Mass Function and Radial Velocity

Here we try to understand the basic properties of the sinks, such as mass, radial velocity, and rotational velocity, as reflected in Figure 6. On the other hand, Figure 7 depicts a histogram of the mass function (top) and the ratio of the radial velocity of the sink particles to the local escape velocity of the cloud (bottom) at the end of the simulations, i.e., when 50 M_\odot has been accreted in total. The newly formed sinks in their parent clouds tend to have a wide range of velocities, the typical value of both the radial (v_{rad}) and rotational (v_{rot}) components lies within a span of roughly ~0.01–25 km s\(^{-1}\). This is consistent with previous studies (e.g., Greif et al. 2011; Dutta 2016a). In addition, we see that in the absence of radiative feedback, relatively high-mass sinks are likely to form irrespective of the rotation of the clouds. Another interesting aspect of the simulations is that a tiny fraction of low-mass sinks for \( \beta \equiv 0.1–0.15 \) move with relatively high radial velocity compared to others. They can therefore directly travel at the periphery at a later stage of evolution, and can even move away from the potential well of the gaseous system with their radial velocity exceeding the escape velocity (v_{escape} \equiv 10–12 km s\(^{-1}\), as seen in Dutta et al. 2020a). Due to the high velocity, the ejected sinks can accrete negligible mass from the surroundings. There are subtle deviations between the rotationally supported clouds. From the analysis, it is clear that the mass function for fast-rotating clouds peaks at a lower mass value, and also the central sink can get quite massive for fast-rotating clouds due to the larger timescale for fragmentation. The radial velocities for these sinks can also get 2–3 times

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Figure 2. Logarithmically scaled distributions of the number density of the gas clouds in the equatorial plane (plane of rotation of the circumstellar disk) for 16 realizations \( \beta \equiv 0.0–0.15 \) at an epoch of time when 50 M_\odot have been accreted into the sinks in total. The white dots are the evolving sink particles in the simulation and the circle shows the size of the central region in astronomical units. As shown in the images, the fast-rotating clouds are likely to develop a small N-body system that has a noticeable dense spiral arm-like structure in which protostellar mass evolves substantially on larger length scales as compared to their lower rotating counterparts.
their escape velocity, depending on the strength of the cloud’s rotation.

3.4. Possibility of Survival

A very important issue to address is the possibility of the survival of such evolving sinks that may be identified as primordial protostars. Note that the lifetime of a star is inversely proportional to the mass that it contains, say a combination of hydrogen, helium, and other higher metallicity gas (Binney & Tremaine 2008). In this scenario, a star can survive for billions of years provided its accretion rate remains very minimal so that the estimated final mass would be as low as \( \sim 0.8 M_\odot \) (Komiya et al. 2009; Kirihara et al. 2019). In Figure 8, we plot the mass of the sinks that get ejected from the multi-scaled N-body cluster for rotating clouds parameterized by \( \beta \). As expected, the ejections are likely to happen for the higher rotating gas clouds (e.g., \( \beta > 0.1 \), as shown in Figure 6, the overall mass distribution of the protostars). These realizations show that due to the conservation of angular momentum, a fraction of protostars are spread out to the outer periphery of the clouds out of which only a tiny fraction (10%–12%) are ejected as low-mass stars. As they continue to have a negligible accretion rate for a long period of time, their final mass remains as low as 0.8\( M_\odot \), and hence are likely to survive to the present epoch. The dotted horizontal line in Figure 8 reflects the threshold mass (0.8\( M_\odot \)) for the ejected protostars.
Figure 4. Mass evolution of the sinks in the simulations is shown as a function of time since the formation of the first central hydrostatic core all the way up to the epoch when 50 $M_e$ has been accreted by the sinks in total, for 15 instances of the clouds parameterized by $\beta \equiv 0.008$–0.15. Depending on the rotation of the clouds, most of the sinks are formed within a few hundred to thousands of years of evolution from the formation of the central hydrostatic core. Note that fast-rotating clouds tend to fragment more to form sinks over a span of mass 0.001–20 $M_e$, depending on the strength of rotational support.
Figure 5. Different panels show the time evolution of the mass accretion rate by the sinks within the rotating gas clouds quantified by the parameter $\beta$, similar to Figure 4, i.e., once roughly $\sum m_{\text{sink}} \sim 50 M_\odot$ has been accreted by the sinks in total. Sinks formed out of the slow-rotating clouds are expected to accrete the ambient gas at a comparatively higher rate (say, $dm/dt \sim 10^{-1} - 10^{-1} \text{ yr}^{-1}$), whereas accretion for those formed in fast-rotating clouds span over roughly four orders of magnitude, and could be as low as $dm/dt \sim 10^{-4} \text{ yr}^{-1}$.
The others can be the massive ones. There is hence a high possibility that a fraction of the sinks within the fast-rotating clouds can escape the cluster as low-mass sinks. There is a considerable chance that they can evolve into main-sequence stars before entering ZAMS to survive for billions of years to the present epoch. This also confirms the theoretical prediction of the probability of the existence of the first generation of stars (it may be either Population III/Population II stars or extremely metal-poor stars) if they would have contained very low mass ($\lesssim 1 \, M_\odot$) before entering as a ZAMS (Andersen et al. 2009). Following the recent study by Dutta et al. (2020a) of the Bondi–Hoyle accretion flow, one can estimate the mass-velocity relations for the protostars in which an initial high speed ensures that the mass accretion is relatively smaller. There is hence a good possibility of their existence even in our Galaxy (either in a bulge or in a halo). Recent state-of-the-art observational tools and wide-range surveys have suggested the existence of extremely metal-poor stars that could be possible candidates for low-mass Population III/Population II stars (e.g., Komiya et al. 2010; de Bennassuti et al. 2017; Husain et al. 2021). Therefore, searching for more such low-mass stars and where they are located in our Galaxy has become one of the primary interests in present-day observations (Johnson 2015; El-Badry et al. 2018; Griffen et al. 2018; Susa 2019; Liu & Bromm 2020).

4. Summary and Discussion

In this work, we have performed a suite of 3D simulations using our modified version of the GADGET-2 SPH code to follow the gravothermal evolution of a number of primordial gas clouds associated with a degree of rotation that spans over roughly two orders of magnitude. The heating and cooling phenomenon that arises during the chemical and thermal evolution of the collapsing gas is approximated with a piecewise polytropic equation of state appropriate for the primordial chemistry. Below we outline the main points related to the modification of the SPH simulations and a summary of the results along with open issues.

4.1. Modification of the Code

1. In addition to the “ADIABATIC” and “ISOTHERMAL” modes available in the publicly available version of GADGET-2, we added a “POLYTROPIC” mode in the code, enabling of which in the makefile evolves the gas system with a general polytropic equation of state. This is done by introducing a polytropic exponent, and appropriately modifying the formulas for internal energy, entropy, and rate of change of entropy.

2. We have carried out the sink particle technique in the original GADGET-2, following the discussion in Bate & Bonnell (1997) along with the sink boundary conditions near the accretion surface. We have also defined an outer accretion radius of $r_{\text{outer}} \approx 1.25 r_{\text{acc}}$ for the sinks, such that the gas particles falling into this outer accretion radius are only evolved gravitationally. Sink particles can be activated in the code by enabling the SINK mode in the makefile. In this mode, the simulations run until 50 units of mass (e.g., here we use $50 \, M_\odot$) have been accreted into the sinks in total, and writes the data related to all the sinks (e.g., mass, position, velocity, internal spin) in their individual text files.

4.2. Summary of Results

1. Irrespective of the cloud’s rotation, most of the sinks start to accrete from the ambient gaseous medium while orbiting the central region. In the absence of a radiation mechanism, the continued accretion results in the increase of the mass of the evolving sinks, which are likely to turn...
Figure 7. Panels showing histograms for the mass of the sink particles (top) and the ratio of the magnitude of velocities of the sink particle to the escape velocity of the cloud (bottom) at end of the simulations, i.e., at the epoch when the multi-scaled $N$-body protostellar system attains a total mass of $\sum m_{\text{sink}} \sim 50 M_\odot$. It is clear that the slow-rotating clouds (e.g., $\beta = 0.008, 0.01$) are inclined to fragment on a mass scale of $\sim 0.001–20 M_\odot$ within the central regime $\sim 20–100$ au. On the contrary, fast-rotating clouds are likely to contain a few low-mass sinks that move faster toward the outer end and accrete a negligible amount of mass from the surrounding gaseous medium.
obtain an estimation of the initial mass function of Population III protostars. However, one needs to perform a more rigorous investigation along with the inclusion of more sophisticated simulations such as magnetic field, radiative feedback, and primordial chemistry.

4.3. Effect of Magnetic Field

A number of studies have confirmed the impact of primordial magnetic fields on the thermodynamics of Population III star formation (Sur et al. 2010) and on the 21 cm emission line of atomic hydrogen (Schleicher et al. 2009). Other magnetohydrodynamic simulations (e.g., Price & Bate 2007; Machida & Matsumoto 2008a; Schleicher et al. 2010) show the contribution from so-called magnetic flux (Maki & Susa 2007) and magnetic braking (Meynet et al. 2011). Besides, magnetic fields can also be amplified by orders of magnitudes over their initial cosmological strengths by a combination of small-scale dynamo action and field compression (Doi & Susa 2011; Sur et al. 2012; Turk et al. 2012; Machida & Doi 2013). Magnetic fields may also provide support against fragmentation (Peters et al. 2014) and outflow (Hirano & Machida 2019). Hence, it is important to note that the inclusion of magnetic fields may substantially influence the gas dynamics, especially the disk evolution and fragmentation phenomenon (see, e.g., McKee et al. 2020; Stacy et al. 2022).

4.4. Effect of Radiative Feedback

The radiation-hydrodynamic simulations on the other hand show that the radiation from protostars can significantly change the accretion phenomenon (Whalen et al. 2004, 2008; Wise & Abel 2008; Susa et al. 2009; Wise et al. 2012) and can even evaporate the disk (Hosokawa et al. 2011; Hirano & Yoshida 2013; Johnson et al. 2013). In general, we expect feedback to lower the accretion of mass and metals onto protostars (Suzuki 2018) and hence our estimates can be thought of as upper bounds on these masses (Bar- kana 2016; Barrow et al. 2017; Chon et al. 2018). However, it is noted that the radiative feedback becomes important approximately after ~10^4 yr have elapsed from the time of formation of the first protostar. Here, the results from our calculations clearly demonstrate that protostars with v_r ≥ v_esc are able to escape clusters within a few tens of thousands of years. Hence, they can accrete a negligible bulk of mass, which implies that radiation emitted from the surface of these stars is unlikely to have an impact on their mass accretion process. On the other hand, it is obvious that feedback effects need to be included for those protostars roaming around with v_r < v_esc. However, analyzing the fate of these protostars is beyond the scope of the present study.

4.5. Effect of the Primordial Chemical Network

In a realistic collapse scenario, various chemical species go through numerous reactions that happen concurrently. This results in the formation of a number of other molecules depending on the local thermodynamic state of the gas (see the excellent reviews by Loeb & Barkana 2001; Ciardi & Ferrara 2005). The thermodynamic balance among these primordial chemical species then determines the net rate of compressional heating and radiative cooling in the gas. Therefore, a detailed knowledge of several possible chemical reactions and mass fractions of all the chemical species is

![Figure 8. Mass of the sinks ejected from the multi-scaled N-body protostellar system is plotted as a function of the rotation parameter \( \beta \) of the clouds. The dotted horizontal line indicates the threshold mass (0.8M_\odot) for the ejected protostars. Only a few sinks are capable of overcoming the strong gravitational drag of the dense gaseous medium and can move away from the potential well of the cluster with radial velocities greater than the escape speed. However, it is noted that only a tiny fraction of them (~10%-12%) continue to have a negligible accretion rate for a long period of time to end up as low-mass sinks (<0.8 M_\odot). One aspect is that they are likely to remain in the main sequence for billions of years to survive to the present epoch.](image-url)
required in order to specify the overall chemo-thermal state of the gas (Omukai & Yoshii 2003; Ripamonti 2007; Dutta 2015). Hence, one has to follow the entire network in detail in order to determine the thermodynamic evolution of the gas, which seems to be fairly complicated to follow in the simulations even for primordial gas with a relatively simpler chemical network of hydrogen and helium.

4.6. Combined 3D Large-scale Simulation with a Bondi–Hoyle Semi-numerical Approach

As discussed above, the protostellar system can numerically be considered a classical N-body problem in which the evolving protostars accrete ambient gas and are coupled with gravity while orbiting around the central gas cloud. However, in reality, this complex phenomenon is indeed difficult to simulate in full 3D simulations (Bagla & Khandai 2009; Stacy et al. 2010) as there is a huge difference in density gradient between the layers of spiral arms. This results in a substantial computational cost. However, with some approximation, the protostellar system can be modeled using semi-analytical calculations with the help of Bondi–Hoyle accretion (Bondi & Hoyle 1944; Bondi 1952) that may provide a satisfactory outcome for the study of stellar dynamics (see, e.g., Edgar 2004; Lee et al. 2014; Bobrick et al. 2017; Xu & Stone 2019). It is therefore important to focus on combining the 3D simulation along with the Bondi–Hoyle semi-numerical approach (Dutta et al. 2020a; Park et al. 2021; Keto & Kuiper 2022) to study the long-term evolution of gas, especially the instabilities that grow during the buildup of the circumstellar disk and complex fragmentation process of its spiral arms (in preparation).

4.7. Evidence for Metal-poor Stars

In the recent past, a number of studies have predicted that there is substantial feasibility of finding the first generation of stars that could be possible candidates for Population III stars or extremely metal-poor (EMP) stars, in the present-day local universe (White & Springel 2000; Tumlinson 2010; Schneider et al. 2012; Gibson et al. 2013; Bland-Hawthorn et al. 2015; Ishiyama et al. 2016; Komiya et al. 2016). Today, the search for EMP stars or very metal-poor stars involves cutting-edge research, and with the advent of the new ultramodern telescope and futuristic surveys, there have been a number of pioneering observational studies that provide crucial information about the early universe (Dawson et al. 2004, 2013; Eisenstein et al. 2005; Furlanetto et al. 2006; Tegmark et al. 2006; Bagla et al. 2009; Cooke et al. 2009; Caffau et al. 2011, 2012; Ahn et al. 2012; MacDonald et al. 2013; Rydberg et al. 2013; Friel et al. 2014; Mirocha et al. 2017). There have been numerous observational analyses of the evolution of baryonic matter both from a cosmological viewpoint (Wells & Norman 2022) and a statistical interpretation of how the universe has evolved into its present state (please see the recent studies by Planck Collaboration et al. 2011, 2014, 2016, 2020). Besides, Population III stars can also be major origins of both merging binary black holes and EMP stars, as shown in Tanikawa et al. (2022). Recent studies have also shown the possibility of detecting primeval galaxies at higher redshift $z \gtrsim 10$ through JWST observations (Jeon & Bromm 2019; Riaz et al. 2022). This may constrain the mass function of Population III stars. Interestingly, Hubble has just detected a very old magnified star of mass $\sim 50–100 M_\odot$ around redshift 6.2 (Welch et al. 2022). With the help of state-of-the-art observations from the Atacama Large Millimeter/submillimeter Array, Tokuoka et al. (2022) has also confirmed the possible systematic rotation in the mature stellar population of a $z \approx 9.1$ Galaxy.

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Appendix A

Implementation of the Polytropic Equation of State in GADGET-2

The standard model of the thermodynamic behavior of the primordial gas clouds has been well understood (see, e.g., Abel et al. 2002; Yoshida et al. 2006; Glover & Savin 2009; Turk et al. 2009). The primordial chemical network primarily contains numerous concurrent reactions between hydrogen and helium. For example, at low densities ($n_H \approx 1–10^3 \text{ cm}^{-3}$), the hydrogen atoms combine with the free electrons to produce hydrogen molecule ion $H^+$, which in turn combines with the hydrogen atoms to form a small abundance of $H_2$. The gas is then cooled through $H_2$ rotational and vibrational line emission up to a temperature of 200 K. However, the small abundance of $H_2$ is not sufficient to cool the gas further and the gas begins to heat up with an increase in density up to a number density of about $10^8 \text{ cm}^{-3}$. At this stage of collapse, the hydrogen atoms are converted to molecules via three-body reactions, which again cool the gas through line emissions. The cloud becomes optically thick to the strongest of $H_2$ emission lines beyond number density $10^{11} \text{ cm}^{-3}$ (Ripamonti & Abel 2004; Clark et al. 2011; Dutta et al. 2015). However, if we are only interested in the thermodynamic evolution of the gas and not the detailed balance between abundant chemical species, then we can make substantial simplifications and use a general polytropic equation of state for all the chemical processes that involve the transfer of heat (Jappsen et al. 2005):

$$T = a \rho^{\eta - 1}, \quad (A1)$$

where $\eta$ is the polytropic index. Therefore, following the above discussion, we use a polytropic equation of state with a piecewise constant profile for a polytropic index appropriate for the thermodynamic evolution of primordial gas clouds. Where the polytropic index $\eta$ changes its values at certain critical number densities, according to the following relations. The publicly available version of GADGET-2 can be used to examine the numerically evolved ideal gas system with the adiabatic equation of state

$$P = A \rho^\gamma, \quad (A2)$$

where $A$ is constant and $\gamma = C_P/C_V$ is the adiabatic index. The internal energy per unit of mass, $u$, of the gas as calculated in the code is

$$u = \left(\frac{A}{\gamma - 1}\right) \rho^{\gamma - 1}. \quad (A3)$$

In order to model the general polytropic process that can involve heat transfer as well, we write the following identity
from the first law of thermodynamics:
\[ NK_B C \Delta T = P \Delta V + NK_B C_V \Delta T, \]
where \( C \) is the rate of heat added to the system (for adiabatic process \( C = 0 \)), \( N \) is the total number of gas particles, and the other symbols have their usual meaning. From the above equation and ideal gas law \( PV = NK_BT \) we can derive the polytropic equation of state as
\[ P = B \rho^\gamma, \]
where \( B \) is constant and \( \eta = (C - C_p)/(C - C_V) \) is the polytropic index. Unlike the adiabatic index \( (\gamma) \), the polytropic index \( (\eta) \) can be greater than, smaller than, or equal to 1, and the two are related by
\[ \gamma = \eta + \frac{C}{C_V}(1 - \eta). \]

Therefore, in the code we choose \( \gamma = 1 + 1/C_V = 7/5 \) for diatomic gas and replace \( \gamma \) with \( \eta \) at appropriate places; for example, the internal energy Formula in A3 is modified to be
\[ u = \left( \frac{B}{\gamma - 1} \right) \rho^{\gamma - 1}. \]

With these modifications, the code can handle general polytropic processes where the temperature can increase \((\eta > 1)\) or decrease \((\eta < 1)\) with density and does not require special treatment for the isothermal \( (\eta = 1) \) case.

Appendix B
Runaway Collapse Phase

In this appendix, we describe the gas distributions, velocity profile, and timescales associated with the initial phase of collapse for the clouds for various degrees of initial solid body rotation until the formation of the central core in our simulations, as illustrated in Figure 9. All the physical quantities are radially averaged within logarithmic binned as calculated from our simulations.

The estimated freefall time over sound-crossing time justifies the gravitational collapse of primordial gas, for which the density distribution obeys the power-law profile with \( n \sim r^{-2.2} \) irrespective of the rotational strength of the clouds. This also confirms that collapse is a self-similar (Susa et al. 1998) process. Therefore, the density profile of the collapsed clouds at the outer part is nearly the same as that of the inner regime until the core is formed at the center of the clouds (see, e.g., Meynet et al. 2013; Stacy et al. 2013; Latif & Schleicher 2015; Dutta 2016b). Due to the high strength of rotation, the radial velocities are considerably lower, as expected for clouds with \( \beta \geq 0.05 \). So the radial component of the velocity is less dominant and gradually becomes comparable to the rotational component near the center of the mass until about 100 au. This is due to the fact that the infalling gas loses angular momentum near the center as it is accreted by the central core. The loss of angular momentum near the center is compensated by the transport of angular momentum farther from the center. Hence, the transport of angular momentum is more noticeable for the clouds with a higher degree of rotational support. This is evident from the distribution of the rotational component of the velocities. This also confirms the formation of larger rotationally supported spiral arms in the clouds with fast-rotating clouds. The flow outside the spiral arms remains sub-Keplerian, i.e., \( v_{\text{rot}}(r) \leq v_{\text{Kep}}(r) \), as can be seen in the middle panel of Figure 9.

In order to quantify the effects of rotation on the accretion phenomenon and associated timescales, we estimate the mass accretion rate, \( M(r) = 4\pi r^2 \rho(r) v_{\text{rad}}(r) \), and the accretion time, \( t_{\text{acc}} = M_{\text{enc}}(r)/4\pi \rho v_{\text{rad}}(r) r^2 \), and plot them as a function of radial distance for different values of the \( \beta \) parameter. As can be seen from Figure 9, the mass accretion rate reaches a maximum value of about 0.1 \( M_\odot \text{ yr}^{-1} \) at about 20 au from the center of the mass. For distances smaller than this the sound-crossing time tends to become comparable to the freefall time, which decreases the mass accretion rate near the center. Because the accretion phenomenon is directly related to the instability within the gas, we also check the degree of Jeans instability in our clouds by measuring the two quantities, \( t_{\text{acc}}(r)/t_{\text{ff}}(r) \) and \( t_{\text{frag}}(r)/t_{\text{ff}}(r) \), as a function of the radial distance. Here, the fragmentation time, \( t_{\text{frag}}(r) \), has been estimated by the ratio of the Jeans mass and the mass accretion rate of the core (Dutta et al. 2015). This can provide an approximation of the fragmentation of the gas for different strengths of the rotation of the clouds. As the infalling gas is redistributed near the center of the mass of the cloud, the rotational support near the center also increases, resulting in the formation of a spiral arm or disk-like structure. These spiral arms are likely to become unstable and prone to fragmentation by accreting mass near their boundaries. The fragmentation timescales and sizes of the disks are proportional to their degree of rotation and are considerably larger for fast-rotating clouds \( \beta \geq 0.05 \) than their slowly rotating counterparts for the fixed polytropic index profile.
Figure 9. The initial (runway) collapse phase is shown for the gas clouds associated with various degrees of initial solid body rotation just before the formation of the central core. The physical properties (estimated as logarithmically scaled radially averaged) of gravitational collapse, such as gas distribution (top panel), velocity structure (middle panel), and accretion phenomenon (bottom panel) are plotted as a function of the radius. The collapse remains self-similar at different scales and satisfies the power-law density profile $n \sim r^{-2.2}$ irrespective of the rotation of the clouds (in agreement with previous studies, e.g., Bromm & Larson 2004; Glover 2005; Yoshida et al. 2006). It also shows that the slow-rotating clouds have a higher degree of compressional heating and are therefore hotter compared to their fast-rotating counterparts. Once the primordial gas is redistributed near the center of mass of the cloud, the ratio $v_{\text{rad}}(r)/v_{\text{kep}}(r)$, where $v_{\text{kep}} = \sqrt{G M_{\text{enc}}(r)/r}$ and $M_{\text{enc}}(r)$ is the mass enclosed inside the sphere of the region of radius $r$, indicates that the clouds with a higher degree of rotational support go through a more efficient phase of angular momentum transport (as shown in Greif et al. 2012; Stacy & Bromm 2014; Dutta 2016b; Hirano & Bromm 2018b).
