Dependability breakeven point mathematical model for production - quality strategy support

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Abstract: This paper connects the field of dependability system with the production-quality strategies through a new mathematical model based on breakeven points. The novelties consist in the identification of the parameters of dependability system which, in safety control, represent the degree to which an item is capable of performing its required function at any randomly chosen time during its specified operating period disregarding non-operation related influences, as well as the analysis of the production-quality strategies, defining a mathematical model based on a new concept – dependability breakeven points, model validation on datasets and shows the practical applicability of this new approach.

1. Introduction

The existing maintenance system is based on a down-top type statistic analysis of the system/process data. The novelty of this work consists in a top/down type approach based on a mathematical model, for the analysis of the influence of Production (P)/Quality (Q) strategies on the parameters of the safety in operation, namely reliability and maintainability. This model is validated and then brought to canonical form which, through the geometric interpretation, gives the possibility to analyze the evolution of the optimum dependent parameters (identification of the directions of maximum influence of the P/Q parameters on the reliability (R)/ maintainability (M) parameters).

The issue is topical in maintenance management typology [4], [5], [7], [8], [14] but these approaches have been made on each of the four parameters of operational safety individually, missing an integrated approach as a single mathematical model.

The importance of this new approach derives from maintenance management context to quickly find good solutions in case of permanent change production-quality policies.

2. Material and method

Safety in operation (SO) represents the global aptitude of a technical system to perform the mission it was conceived for, and is characterized through 4 components [1], [2], [5].

Reliability: expresses the safety in operation of a system under specified conditions at established parameters during a time interval t. Reliability is practically the probability of good operation, with values ranging within the interval (0, 1). The Mean Time Between Failures (MTBF) represents a technico-economical index that can characterize dependability.
**Maintainability**: represents the characteristic of a system to be easy to maintain and repair. The technico-economical index that characterizes the maintainability is the Mean Time To Repair (MTTR)

**Availability**: the availability of a technical system to perform all the functions it was conceived for, within a specified time interval. The coefficient that characterizes this function is determined from the following relation: \( A = \frac{MTBF}{MTBF + MTTR} \)

**Safety**: is the aptitude of a system to perform its function under safe conditions for goods, persons and environment.

![Figure 1. The influence of Production-Quality strategies on the Operation Dependability.](image)

Figure 1 presents the influence of Production-Quality strategies on the Safety in Operation. Depending on the elaboration of its own maintenance strategies that supports, in a certain manner, the production-quality strategies, one imposes values for the four parameters to ensure a certain Safety in Operation value [3] [4].

Therefore, for Management of Operation Dependability (MOD), one needs an easy-to-use simple, efficient, flexible method.

One defines the Management of Operation Dependability Through Breakeven Points (MODBP) as the management method that proposes a technical and economical validation of the equipment operation under certain conditions imposed by breakeven points of the four parameters (safety, dependability, availability and maintainability) [6], [7] [8], [9]. The method can be represented by the following flowchart (figure 2).

![Figure 2. Structure of MODBP method.](image)
In figure 2, the safety component is shown off at the beginning of the diagram, given its importance in relation with the other SF components. After reaching the breakeven, one defines a function of the other SO components [10], [11], [12], [13]. The parameters of this function are variable in terms of P-Q strategies policy.

The definition and optimization of the function \( F(B_a, B_m, B_r) \) constitutes the subject of this work.

The present applicative research was performed on industrial equipment presented in [14].

A first stage in the processes/systems optimization process is the determination of the mathematical model. It will serve to determine the extreme values of the parameters to be optimized in the bi-dimensional factorial space determined by the two independent parameters P and Q.

The chosen model is of second order non-linear type with the following mathematical expression:

\[
y = b_0 + \sum_{i=1}^{k} b_i \cdot x_i + \sum_{i=1}^{k} \sum_{j=i+1}^{k} b_{ij} \cdot x_i \cdot x_j + \sum_{i=1}^{k} b_{ij} \cdot x_i^2
\]

(1)

where \( y \) is the resulting function or the mathematical response which approximates the dependent variable (R or M respectively), and \( x_i \) are the values of the independent variable. Model definition includes the following stages:

- define the matrix for experimentation and codification of the values measured in theoretical points adequate to the model;
- determine the model coefficients using the least-squares method, and validate them statistically;
- model statistic validation according to theoretic statistic distributions;
- canonic transformation of the function for a geometric interpretation of the dependent parameter evolution within the independent parameters evolution.

3. Mathematical models MODBP through the analysis of reliability and maintainability parameters

3.1. Mathematical model for reliability definition in terms of the P/Q plane

3.1.1. Definition of experimental matrix

The independent parameters of the considered technical process are:

- \( x_1 \) – production, whose values are rated within the interval (0,1)
- \( x_2 \) – quality, whose values are rated within the interval (0,1).

Table 1 synthetically presents the real and coded values of the independent parameters.

| Independent parameter | Code \(-1.414\) | -1 | 0 | +1 | +1.414 |
|-----------------------|----------------|----|---|----|--------|
| \( x_1 \) – Production (P) | 0.1 | 0.25 | 0.5 | 0.75 | 0.9 |
| \( x_2 \) – Quality (Q) | 0.1 | 0.25 | 0.5 | 0.75 | 0.9 |

Table 2 presents the experimental matrix based on the variation level of the two independent parameters, with 13 experimental variants [13].

| Experimental variants | Independent variables |
|-----------------------|-----------------------|
| \( x_1(P) \) | \( x_2(Q) \) |
| Code | Real | Code | Real |
3.1.2. Value and significance of the coefficients of the regression equation

Table 3 presents the coefficients of the regression equation determined through the least squares method.

| Coefficient | Statistics tc | Statistics \( t_{ab} (0.05;4) \) | Significance |
|-------------|---------------|-------------------------------|--------------|
| \( b_0 \)   | 0.522123      | 78.713                        | significant  |
| \( b_1 \)   | 0.238422      | 45.4654                       | significant  |
| \( b_2 \)   | 0.0398725     | 7.60339                       | significant  |
| \( b_{11} \)| -0.00234936   | 0.417694                      | insignificant |
| \( b_{22} \)| 0.00264913    | 0.470991                      | insignificant |
| \( b_{12} \)| -0.03         | 4.0452                        | significant  |

After eliminating the insignificant factors, the final form of the equation of regression becomes:

\[
y = 0.522123 + 0.238422x_1 + 0.0398725x_2 - 0.03x_1x_2
\]  

(2)

3.1.3. Statistic validation of the model

Table 4 presents the measured reliability values (\( R_m \)), computed reliability values (\( R_c \)) and amplitude (\( A= \text{abs}(F_m-F_c) \)).

| \( R_m \) | \( R_c \) | \( A \) |
|-----------|-----------|--------|
| 0.21      | 0.213828  | 0.0038 |
| 0.75      | 0.750673  | 0.0007 |
| 0.38      | 0.353573  | 0.0264 |
| 0.80      | 0.770418  | 0.0296 |
| 0.17      | 0.184994  | 0.0150 |
| 0.84      | 0.859253  | 0.0193 |
| 0.48      | 0.465744  | 0.0143 |
| 0.55      | 0.578503  | 0.0285 |
| 0.50      | 0.522123  | 0.0221 |
| 0.52      | 0.522123  | 0.0021 |
| 0.54      | 0.522123  | 0.0179 |
| 0.52      | 0.522123  | 0.0021 |
| 0.53      | 0.522123  | 0.0079 |

The mathematic model is statistically adequate, as it satisfies the three conditions:

a) \( F_c = 4.84816 \); \( F_{tab(0.05;4;4)} = 6.39 \); condition \( F_c < F_{tab} \) is satisfied.

b) \( F_c = 116.484 \); \( F_{tab(0.05;12;12)} = 2.69 \); condition \( F_c > F_{tab} \) is satisfied.

c) all the percentage deviations are under 10% (table 4).

The absence of the quadratic terms in the mathematic model means that reliability depends quasi-linearly on the production and quality, their weights being given by the ratio of the coefficients \( x_1 \) and \( x_2 \); he coefficient \( b_1 \) is 6 times higher than \( b_2 \), which implies a 6 times stronger influence of the production on reliability as compared to that of the quality. The presence of the term \( x_1x_2 \) leads to the non-linearity of the response \( F \) as function of the plane \( P/Q \).
The coordinates of the new center used to convert the regression equation into the standard form are: \( x_{10} = 1.32908 \) and \( x_{20} = 7.94742 \), with a computed value of 0.839007. The rotation angle of the axes is \( \alpha = -45^\circ \), with the new system of coordinates:

\[
X_1 = 0.707107*(x_1-1.32908) + -0.707107*(x_2-7.94742) \quad (3)
\]

\[
X_2 = 0.707107*(x_1-1.32908) + 0.707107*(x_2-7.94742) \quad (4)
\]

The canonic equation for the analysed parameter in the new system of coordinates \((X_1, X_2)\) has the form:

\[
Y-0.839007=0.015*Z_1^2+-0.015*Z_2^2 \quad (5)
\]

The coefficients B11 and B22 are equal, but of opposite signs; accordingly, the surface shape is a saddle, symmetrically turned at 45° in the plane \((x_1, x_2)\). Since B11 = 0.015 > 0, and B22 = -0.015 < 0, the response surface described by the canonic equation is a hyperbolic paraboloid (figure 3). The lines of equal value of the parameter Y represent a family of hyperbolas in the plane \((x_1, x_2)\); its value increases when moving along \(X_1\) (P) direction.

\[\text{Figure 3. The response surface } R=f(P/Q).\]

Figure 4 presents reliability evolution in terms of production \((x_1)\) for constant values of the quality \((x_2)\). One can notice that, regardless of the quality level, one obtains the maximum point for reliability for maximum values of the parameter \(x_1\) (P). The quality influence on reliability is noticed for small production values. Figure 4 also reveals the direct proportionality between reliability and production.

3.2. Mathematic model for maintainability definition in terms of \( P/Q \) plane

3.2.1. The experimental matrix - definition preserves the hypotheses given in Section III.1.1. Table 5 presents the measured values of maintainability normalized within the interval \((0, 1)\); \( x_1 \) codifies the independent parameter “Production”; \( x_2 \) codifies the independent parameter “Quality”, and Mm represents the measured values of the parameter “Maintainability” for the considered technical process.

| \( x_1 \) | -1 | 1 | -1 | 1 | -1.414 | 1.414 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( x_2 \) | -1 | -1 | 1 | 1 | 0 | 0 | 0 | -1.414 | 1.414 | 0 | 0 | 0 | 0 | 0 |
| Mm | 0.18 | 0.39 | 0.75 | 0.79 | 0.48 | 0.61 | 0.12 | 0.91 | 0.5 | 0.51 | 0.54 | 0.49 | 0.55 |

\[\text{Table 5. The measured values of the parameter Maintainability.}\]
3.2.2. *Value and significance of the coefficients of the regression equation*

The coefficients of the model whose mathematical expression is given by the relation (1), determined through the least squares method and their significance are presented in Table 6.

**Table 6. Values and significance of the equation coefficients.**

| Coefficient | Statistics tc | Statistics t_{tab(0.05;4)} | Significance |
|-------------|---------------|-----------------------------|--------------|
| b0          | 0.518128      | 44.7594                     | Significant  |
| b1          | 0.0542275     | 5.92553                     | Significant  |
| b2          | 0.260883      | 28.5071                     | Significant  |
| b11         | 0.0127687     | 1.30286                     | Insignificant|
| b22         | -0.00222677   | 0.226861                    | Insignificant|
| b12         | -0.0425       | 3.28384                     | Significant  |

After eliminating insignificant coefficients, the final form of the regression equation becomes:

\[ y = 0.518128 + 0.0542275x_1 + 0.260883x_2 - 0.0425x_1x_2 \] (6)

3.2.3. *Statistic validation of the model*

**Table 7. The maintainability values (measured and computed).**

| Mm   | Mc   | A   |
|------|------|-----|
| 0.18 | 0.160518 | 0.019 |
| 0.39 | 0.353973 | 0.036 |
| 0.75 | 0.767283 | 0.017 |
| 0.79 | 0.790738 | 0.001 |
| 0.48 | 0.44145  | 0.039 |
| 0.61 | 0.594806 | 0.015 |
| 0.12 | 0.14924  | 0.029 |
| 0.91 | 0.887016 | 0.023 |
| 0.5  | 0.518128 | 0.018 |
| 0.51 | 0.518128 | 0.008 |
| 0.54 | 0.518128 | 0.022 |
| 0.49 | 0.518128 | 0.028 |
| 0.55 | 0.518128 | 0.032 |

Table 7 presents the measured values of maintainability (Mm), calculated values of maintainability (Mc) and the amplitude (A=\text{abs}(Mm- Mc))

a) \( F_c=2.52588, \ F_{\text{tab}(0.05;4;4)}= 6.39, \ \text{condition } F_c<F_{\text{tab}} \) is satisfied.

b) \( F_c'=75.0963; \ F_{\text{tab}(0.05;12;12)}=2.69; \ \text{condition } F_c'>F_{\text{tab}} \) is satisfied.

c) all the percentage deviations are under 10% (table 7).

The absence of the quadratic terms from the mathematic model means that maintainability depends quasi-linearly on production and quality. The scalar that weights the “quality” parameter is about 5 times bigger than the scalar that weights the parameter “production”, defining their importance/influence on maintainability. The presence of the term \( x_1x_2 \) leads to a non-linearity of the response \( M \) function of the plane \( P/Q \).

The coordinates of the new center used to convert the regression equation into the standard forms are: \( x_{1n}=6.13841 \) and \( x_{2n}=1.27594 \), with a computed value of 0.850999. The rotation angle of the axes is \( \alpha = -45^\circ \), with the new coordinate system:

\[ X_1=0.707107*(x_1-6.13841) + 0.707107*(x_2+1.27594) \] (7)

\[ X_2=0.707107*(x_1-6.13841) + 0.707107*(x_2-1.27594) \] (8)

The canonic equation for the analysed parameter in the new coordinate system \( (X_1, X_2) \) has the form:

\[ Y-0.850999=0.02125*Z_1^2+0.02125*Z_2^2 \] (9)

The response surface has the same aspect as that for reliability, with the difference that the parameter \( Y \) increases when moving along the direction \( X_2(Q) \).
4. Conclusions and future research directions
The present work proposes a new approach of top-down type in the elaboration of maintenance strategies for supporting the production/quality policies. This method defines a function that validates technically and economically the operation of an industrial equipment, by determining certain values of the breakeven points of maintainability and reliability. From figure 7, one can determine geometrically, from the intersection of the two curves R= f₁(P,Q) and M=f₂(P,Q) in the same system of axes, the values for reliability and maintainability according to the policies of production-quality of the technological process/system.

The obtained results agree with the theoretical formulas of reliability and maintainability parameters (specified in Section 2).

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