Current induced transition of anisotropic quantum Hall states

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(Dated: March 23, 2022)

We compare the energies of the striped Hall state and the anisotropic charge density wave (ACDW) state at half-filled third and higher Landau levels in the system with injected currents. With no injected current, the ACDW state has a lower energy. We find that the striped Hall state becomes the lower energy state when the injected current exceeds a critical value. The critical value is estimated as about 0.04-0.05 nA.

PACS numbers: 73.43.-f

One of the most fascinating states in the quantum Hall system is a highly anisotropic state. The highly anisotropic state has been found around half-filled third and higher Landau levels (LLs) in ultra-high mobility samples at low temperature [1, 2]. The state reveals highly anisotropy in the longitudinal resistivity $\rho_{xx}$ and $\rho_{yy}$. In the experiments at the filling factor $\nu = 9/2$, $\rho_{xx}$ is about 1000Ω while $\rho_{yy}$ is about several Ω at low temperatures of order tens of mK. In this state, no quantized plateaus in the Hall resistivity $\rho_{xy}$ have been found. While several states have been proposed to explain the experimental results so far [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], it is still an open problem which state is realized in the experiments.

We focus on two Hartree-Fock (HF) states among them. One is a striped Hall state and another is an anisotropic charge density wave (ACDW) state. The striped Hall state is a unidirectional charge density wave state [3] and is a gapless state with an anisotropic Fermi surface [15, 16] (Fig. 1, Fig. 3). The ACDW state [9] is a state which has the similar CDW order perpendicular to the stripe direction as the striped Hall state, but in addition has the density-wave modulation along stripes (Fig. 2). The density modulation along stripes results in an energy gap. These properties suggest that the anisotropic state is the striped Hall state since the anisotropic longitudinal resistivity and the un-quantized Hall resistivity are naturally explained by the anisotropic Fermi surface [15, 16], while it is difficult to explain these experimental features with the ACDW state because of the energy gap. However it has been pointed out that the striped Hall state is unstable within the HF approximation to formation of modulations along stripes so that the ACDW state is the lower energy state [9]. This has been an enigma as to the anisotropic states.

In this paper, we study energy corrections due to injected currents flowing in the stripe direction for the two HF states. The effect of the injected current has not been taken into account in the previous studies. The density modulations along stripes shown in Fig. 2 suggest that the effect of the injected current becomes larger in the ACDW state. When the current is injected, charges accumulate around both edges perpendicular to the current flow with the opposite sign as in the case of the classical Hall effect and the accumulated charges cause the energy enhancement via the Coulomb interaction. We calculate this type of energy corrections for the present HF states and find that the energy correction for the ACDW state is larger than that for the striped Hall state.

Let us consider the two-dimensional (2D) electron system in the $x$-$y$ plane with an external magnetic field $B$.
where \( l \) denotes the LL index, \( b_{l}(p) \) is the anti-commuting annihilation operator with momentum \( p \) defined in the BZ \( (|p_{\nu}| < \pi) \), and \( \langle x|f_{l} \otimes \beta_{p} \rangle \) is given in Ref. \[18\]. The momentum state is the Fourier transform of the Wannier function.

The striped Hall state is obtained by assuming \[19\] which gives the anisotropic Fermi surface (Fig. \[3\] \[15, 16\]). Since the anisotropic Fermi surface has the inter-LL energy gap in the \( p_x \)-direction and is gapless in the \( p_y \)-direction, the striped Hall state would have the anisotropic longitudinal resistivity. The total energy depends on \( r_s \) and the minimum energy per particle is given by \(-0.3074 \ (r_s = 2.474) \) for \( l = 2 \) and \(-0.2800 \ (r_s = 2.875) \) for \( l = 3 \) in units of \( \sqrt{2}/l_B \), where \( l_B = \sqrt{1/eB} \) is the magnetic length.

**ACDW state.**— The ACDW state is obtained by assuming \[19\]

\[
\langle \tilde{\rho}_{l}(k) \rangle_{\text{ACDW}} = \sum_{\mathbf{Q}} \Delta_{l}(\mathbf{Q}) (2\pi)^2 \delta^{2}(k - \mathbf{Q}),
\]

where \( \mathbf{Q} = (2\pi N_x/r_{0x}, 2\pi N_y/r_{0y}) \), \( r_{0x} \) and \( r_{0y} \) are the periods of the density in the \( x \)-direction and the \( y \)-direction respectively, \( \Delta_{l}(\mathbf{Q}) \) is an order parameter determined self-consistently, and \( \Delta_{l}(0) = \nu^{*} \). The density modulations in both directions generate the energy bands and the BZ is reduced to a smaller BZ as shown in Fig. \[2\]. The direction with a shorter period of the density modulation is referred to as the stripe direction of the ACDW state in this paper. We concentrate on the case of \( \nu^{*} = 1/2 \). The HF Hamiltonian is diagonalized under the assumption \[19\] taking \( r_s = r_0 \). Since the number of ACDW unit cells is equal to the number of electrons within one LL, the area of the ACDW unit cell is just twice as large as the vNL unit cell. Hence the self-consistent solution gives two energy bands and the BZ is reduced to the half size of the original BZ, \( |p_{x}| < \pi \) and \( |p_{y}| < \pi \). The total energy depends on \( r_s \) and the minimum energy per particle is given by \(-0.3097 \ (r_s = 0.82) \) for \( l = 2 \) and \(-0.2814 \ (r_s = 0.70) \) for \( l = 3 \) in units of \( \sqrt{2}/l_B \), where there are two values of \( r_s \) at each LL due to the \( \pi/2 \)-rotational symmetry. The magnitudes of energy gaps are 0.2470 for \( l = 2 \) and 0.1967 for \( l = 3 \) in units of \( \sqrt{2}/l_B \). These values are estimated as of order 10K for a few tesla in terms of temperature while experiments for the anisotropic states have shown the anisotropic longi-

\[
\psi(x) = \int_{BZ} \frac{d^2p}{(2\pi)^2} \sum_{l=0}^{\infty} b_{l}(p) \langle x|f_{l} \otimes \beta_{p} \rangle,
\]
ductance) and quantum Hall state, finite for the ACDW state \[20\]. In the case of the integer quantum Hall state, \(\nu = 0\) for the ACDW state are 0 respectively. We consider the small static current flowing in the stripe direction which is set to the \(y\)-direction and depending only on \(x\). The injected current generates the deviation of the electron density \(\delta \rho(x)\) from the original value in the HF state which also depends only on \(x\) and gives the energy correction via the 2D Coulomb interaction between the deviated charges. This energy correction is given by

\[
\delta E = \frac{2e^2}{L_x} \int_{-L_x/2}^{L_x/2} dx dx' \delta \rho(x) \ln |x - x'| \delta \rho(x'). \tag{6}
\]

The deviated charges generates a potential \(a_0(x)\). The current distribution \((j_y(x))\) and \(\delta \rho(x)\) are related with \(a_0(x)\) as \((j_y(x)) = -\sigma_{xy}^{(v)} \partial_x a_0(x)\) and \((-e)\delta \rho(x) = 2\pi \gamma \partial_x^2 a_0(x)\) in the long wave length limit. Here \(\gamma = (1 + \beta \sqrt{B/\nu}) \sigma_{xy}^{(v)} / 2\pi \omega_c\) (\(\sigma_{xy}^{(v)} = e^2/\nu/2\pi\) is a Hall conductance) and \(\beta\) is zero for the striped Hall state and finite for the ACDW state \[20\]. In the case of the integer quantum Hall state, \(\beta\) becomes zero. The values of \(\beta\) for the ACDW state are 0.472 for \(l = 2\) and 0.581 for \(l = 3\) in units of \((\text{tesla})^{-1/2}\). The origin of the finite \(\beta\) for the ACDW state is the density modulation along stripes. When the current flows in the stripe direction, the deviation of the electron density occurs perpendicularly to the stripe direction. The Fermi surface of the striped Hall state has the inter-LL energy gap in the perpendicular direction to the stripes so that \(\beta\) becomes zero as in the case of the integer quantum Hall state. On the other hand, the ACDW state has the intra-LL energy gap in addition to the inter-LL energy gap. This intra-LL effect gives additional corrections. Our study has shown that \(\beta\) becomes larger as the magnitude of the ACDW energy gap becomes smaller. Since the energy gap of the ACDW state is caused by the density modulation along stripes, the finite \(\beta\) is the result of the density modulation along stripes. \(a_0(x)\) is approximately given by

\[
a_0(x) = \alpha \ln \left| \frac{x - L_x/2}{x + L_x/2} \right| \quad \text{for} \quad |x| \leq \frac{L_x}{2} - \gamma, \tag{7}
\]

with a linear extrapolation of \(a_0\) to \(\pm IR_H/2\) in the interval within \(\gamma\) from the edge, where \(\alpha = IR_H/2(1 + \ln(L_x/\gamma))\), and \(R_H = 1/\sigma_{xy}^{(v)}\) is the Hall resistivity \[20\]. Note that \(\gamma\) has the dimension of length and is very small for the magnetic fields of order several tesla, e.g., if \(\epsilon = 13\mu_0\) and \(m = 0.067m_e\) (these are parameters in GaAs), then \(\gamma\) is of order 10\(^{-5}\) m. For the integer quantum Hall states with \(\beta = 0\), the potential distribution (7) becomes the same form as that obtained by MacDonald et al. \[21\] and other authors \[22, 23, 24\]. The charge distribution obtained from Eq. (7) reveals the accumulation of charges around the both edges with the opposite sign. Substituting Eq. (7) into Eq. (6) and performing the \(x\)-integration, we obtain the final result as

\[
\delta E[I] = C \times I^2 \left( \frac{q^2}{l_B} \right), \quad C = \frac{\pi \epsilon}{L_x(\sigma_{xy}^{(v)})^2} \times \ln(2/b) - 1 \left( \ln(2/b) + 1 \right)^2. \tag{8}
\]

where \(b\) is a dimensionless constant given by \(b = \gamma/((L_x/2)(\ll 1))\).

The energy correction \(C\) depends on the magnitude of the total current, the filling factor, and experimental parameters. Since the actual filling factor includes the spin degree of freedom, we use \(\nu_{ex} = 2l + \nu^*\) for lower spin bands and \(\nu_{ex} = (2l + 1) + \nu^*\) for upper spin bands instead of \(\nu\). We use \(\epsilon = 13\mu_0\), \(m = 0.067m_e\), \(v_x = 2.67 \times 10^{15} \text{ m/s}\), and \(L_x = 5 \times 10^{-3} \text{ m}\) (referred to as Lilly’s parameters). These parameters have been used in the experiment by M. P. Lilly et al. \[9\]. Substituting Lilly’s parameters into Eq. (8), the values of the coefficient \(C\) are given as shown in Table I. Including the energy correction due to current, the energy difference between the striped Hall state and the ACDW state is given by (Fig. 4) \(\delta E[I] = -\Delta E_0 + (C_{\text{ACDW}} - C_{\text{Stripe}})(q^2/l_B) \times (I/|nA|)^2\). The signs of \(\Delta E[I]\) change at the critical values of current \(I_c\). The critical values are about 0.04-0.05 nA (Table I). The current used in the experiments \[1, 3\] is above 1 nA and is much larger than the critical value. Hence the striped Hall state becomes the lower energy state and should be realized in the experiments.

In summary, we have investigated the effect of the injected current on the striped Hall state and the ACDW state. The injected current flowing in the stripe direction has been considered. It is found that the charge accumulation occurs around the both edges with the opposite sign and the accumulated charges give energy corrections via the Coulomb interaction. The energies of the two HF states including the energy corrections due to current have been compared. In the system with no

| \(\nu_{ex}\) | \(C_{\text{Stripe}}\) | \(C_{\text{ACDW}}\) | \(I_c\) |
|---|---|---|---|
| 9/2 | 144.7 | 146.1 | 0.040 |
| 11/2 | 109.9 | 110.7 | 0.053 |
| 13/2 | 87.44 | 88.08 | 0.047 |

TABLE I: Values of the coefficient \(C\) in units of nA\(^{-2}\) and the critical current \(I_c\) in units of nA.
injected current, the ACDW state has slightly lower energy than the striped Hall state. We found that the energy of the ACDW state increases faster than that of the striped Hall state as the injected current increases. The naive expectation suggested from the density modulation along stripes has been verified. Hence the striped Hall state becomes the lower energy state when the current exceeds a critical value. The critical value is estimated as about 0.04-0.05 nA, which is much smaller than the current used in the experiments. Our results suggest that the striped Hall state is realized in the experiment rather than the ACDW state and the ACDW state is realized if the experiment is done with a current smaller than the critical value.

This work was partially supported by the special Grant-in-Aid for Promotion of Education and Science in Hokkaido University, a Grant-in-Aid for Scientific Research on Priority Area (Dynamics of Superstrings and Field Theories, Grant No. 13135201) and (Progress in Elementary Particle Physics of the 21st Century through Discoveries of Higgs Boson and Supersymmetry, Grant No. 16081201), provided by the Ministry of Education, Culture, Sports, Science, and Technology, Japan.

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