Electron Density Reconstruction of Solar Coronal Mass Ejections Based on a Genetic Algorithm: Method and Application

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Abstract

We present a new method to reconstruct the three-dimensional electron density of coronal mass ejections (CMEs) based on a genetic algorithm, namely the genetic reconstruction method (GRM). GRM is first applied to model CMEs with different orientations and shapes. A set of analytic model CMEs from Gibson and Low is employed to produce synthetic CME images for GRM reconstruction. Model CMEs with longitudes of 0°, 45°, 90°, 135°, and 180° and latitudes of 0°, 15°, 30°, and 45° are used to test the performance of GRM. The model CMEs are obscured with a simulated occluder of a coronagraph to determine the influence of CME brightness incompleteness. We add random noise to some synthetic CME images to test the performance of GRM. The CME reconstructions are carried out using synthetic data from Solar Terrestrial Relations Observatory (STEREO) A and B with a separation angle of 90° and from STEREO A and the Solar and Heliospheric Observatory (SOHO) with a separation angle of 73°. The Pearson correlation coefficient and the mean relative absolute deviation are calculated to analyze the similarities in brightness and electron density between the model and reconstructed CMEs. Comparisons based on the similarity analysis under various conditions stated above give us valuable insights into the advantages and limitations of GRM reconstruction. The method is then applied to real coronagraph data from STEREO A and B, and SOHO on 2013 September 30.

Unified Astronomy Thesaurus concepts: Solar coronal mass ejections (310); Solar activity (1475); Line of sight (924); Field of view (534)

1. Introduction

Coronal mass ejections (CMEs) are usually observed by a coronagraph (Lyot 1939). The observation records the radiation from Thomson scattering (Minnaert 1930; van de Hulst 1950; Billings 1966; Vourlidas & Howard 2006; Howard & Tappin 2009; Inhester 2015), which is produced by the interaction between the radiation from the photosphere and free electrons inside CMEs. Information on the electron locations along the line of sight (LOS) is hidden from the observer after the observation (Howard et al. 2008; Temmer et al. 2009).

This hidden information can be restored by different methods of CME reconstruction (Mierla et al. 2010; Thernisien et al. 2011). We classify the various methods into three categories as follows.

1. Methods to reconstruct point features of CMEs. The three-dimensional (3D) coordinates of a CME feature can be calculated if the feature can be seen from different viewpoints, like the Solar Terrestrial Relations Observatory (STEREO) A and B (Howard et al. 2008; Kaiser et al. 2008). For example, Mierla et al. (2008) calculated the 3D position of the CME feature using height–time diagrams from COR1 on board STEREO A and B based on epipolar geometry (Inhester 2006). Liu et al. (2010a, 2010b, 2014) obtained the 3D position of the CME feature from the corona to the heliosphere using the time-elongation map from COR2, HI1, and HI2 on board STEREO A and B.

2. Methods to reconstruct the outline of CMEs. A global outline can be obtained from CME boundary measurements in the coronagraph images. For example, Pizzo & Biesecker (2004) used the CME boundaries to construct a series of quadrilaterals to approximate the three-dimensional CME outline based on synthetic STEREO images. de Koning et al. (2009) applied this method to reconstruct the CME outline using coronagraph observations from STEREO. Thernisien et al. (2006, 2009) employed a forward model of a graduated cylindrical shell to reconstruct flux-rope-like CMEs using data from either a single viewpoint or multiple viewpoints. Byrne et al. (2010) employed an elliptical tie-pointing technique to reconstruct a full CME front in 3D space. Feng et al. (2012, 2013) developed a 3D mask-fitting reconstruction method using coronograph images from three viewpoints to obtain the 3D morphology of a CME.

3. Methods to reconstruct the density inside CMEs. For example, Moran & Davila (2004) derived the 3D position of electrons along the LOS corresponding to every CME pixel through polarization analysis of single-view images. Dai et al. (2014) suggested a classification of ambiguity in this method to improve the reliability of the reconstruction. Antunes et al. (2009) reconstructed the 3D CME electron density through a combination of inversion using the PIXON method (Puetter et al. 2005) and forward modeling (Thernisien et al. 2006, 2009). Frazin et al. (2009) developed a reconstruction method based on level-set (Jacob et al. 2006) algorithm using synthetic CME images produced from a 2D slice of magnetohydrodynamic (MHD) simulation (Manchester et al. 2008). Hutton & Morgan (2017) developed automated CME triangulation (ACT) to find the most probable location where the CME passes at height of 5 R\(_\odot\). ACT is effective for different observations of three or two viewpoints over a sliding window of five hours.
The information on the CME structure obtained from 3D reconstruction gradually increases using the methods from category (1) to (3). For category (3), the reconstructed CME is close to the real CME, which is a collective of inhomogeneous magnetized plasmas. On the other hand, the ambiguities in the methods in category (3) are also larger than those in categories (1) and (2). In the current work, we develop a new method to reconstruct the 3D electron density of CMEs based on a genetic algorithm (GA; Holland 1992; He et al. 2011), namely the genetic reconstruction method (GRM). According to the classification mentioned above, GRM belongs to category (3).

The rest of this paper is organized as follows. We introduce GRM in Section 2 and present applications of the method in Sections 3.1 and 3.2 for model and observed CMEs. Finally, conclusions are given in Section 4.

2. GRM

Before describing the method in this section, we briefly explain the approach for reconstructing CME electron density using GRM. Our approach is to randomly distribute test electrons throughout a 3D reconstruction space, with their populations within voxels used to calculate the LOS integrations that create synthetic observations. These can be compared to true observations, giving a goodness of fit to the electron distribution. We can randomly distribute electrons in the reconstruction space to create many different reconstructed CMEs, and their goodness of fit sets the scoring. Based on the fitness scoring, we select a better electron distribution in the next iteration. When the iteration process meets the termination condition, we obtain the final reconstruction of the electron distribution. The reconstruction process, with five steps, is shown in Figure 1.

2.1. A General Description of CME Reconstruction

The observed brightness, \( B_{\text{obs}} \), of each CME pixel is the corresponding LOS integral of the Thomson scattering including measurement noise \( \eta \):

\[
B_{\text{obs}} = \int_{\theta_1}^{\theta_2} B_c(\theta)N_c(\theta)d\theta + \eta.
\]

Here, \( N_c(\theta) \) is the electron density on the LOS where the angle between the plane of sky and the line connecting the electron to the Sun center has the value \( \theta \). The analytic function \( B_c(\theta) \) (Quémerais & Lamy 2002) is the value of the Thomson scattering for a single electron at location \( \theta \). \( \theta_1 \) and \( \theta_2 \) are the LOS boundaries constrained by the CME shape.

CME reconstruction tries to obtain the vector \( N_c(\theta) \) when \( B_{\text{obs}} \) is observable. Methods in category (1) focus on the localized feature that contributes most of the brightness along the LOS. Methods in category (2) focus on the electron density that contributes to the brightness of the observed CME boundary. Methods in category (3) focus on the electron density along the LOS between \( \theta_1 \) and \( \theta_2 \). We summarize some of the challenges in and limitations of CME reconstruction (Frazin et al. 2009; Mierla et al. 2010; Thernisien et al. 2011) as follows:

(1) Equation (1) is highly underdetermined. There are multiple possible solutions of Equation (1) if constraints or a priori information are not enough. The possible distributions of \( N_c(\theta) \) may generate the same observed brightness \( B_{\text{obs}} \).

(2) Mixing of the CME and background corona. CME images always contain brightness from the background corona because of the optically thin nature of Equation (1), \( B_{\text{obs}} = B_{\text{cme}} + B_{\text{bg}} \). \( B_{\text{bg}} \) can be removed by subtracting the pre-CME image from the CME image (Vourlidas et al. 2010). The situation becomes complex when the background corona contains dynamic structures like streamers, jets, or other CMEs. It may be difficult to do a clean subtraction. Some advanced techniques have been developed for more effective background subtraction through more sophisticated means (Morgan et al. 2012; Morgan 2015).

(3) Measurement noise in the coronagraph images. Noise, \( \eta \), in any measurements is unavoidable. As pointed out by Thernisien et al. (2011), the presence of noise in the CME images may lead to unstable solutions of Equation (1).

(4) Incomplete observation caused by coronagraph occulter. The CME may be obscured by the coronagraph occulter especially when the CME has not propagated far away from the Sun. For different viewpoints, occulters may obscure different parts of the CME.

(5) The number of simultaneous observation viewpoints. Simultaneous observations from different viewpoints play an important role in most of the reconstruction methods classified in Section 1. Inverse methods, like tomography, that try to restore \( N_c(\theta) \) usually need data from many viewing directions. With two or three viewpoints, like Solar and Heliospheric Observatory (SOHO; Brueckner et al. 1995; Domingo et al. 1995) and STEREO, the inverse problem is extremely ill-posed. The solution of Equation (1) is not unique.

(6) Separation angle between viewing directions. For reconstructions using data from two viewpoints, the separation angle of 0° and 180° is unsatisfactory while 90° is ideal. STEREO A and B orbit the Sun with different speeds, and the angle between them varies by about 44° per year.

We should keep these limitations in mind throughout the process of CME reconstruction.

CME reconstruction using inverse methods can be stated as an optimization problem:

\[
\text{cf} = \sum_{\theta_1}^{\theta_2} |B_c(\theta)N_c(\theta) - B_{\text{obs}}|^p + \text{regularization},
\]

where \( \text{cf} \) is the cost function, \( p \) is a positive number that defines the error norm of the observations. Traditionally, \( p = 2 \) and the regularization term also depends on \( N_c^2 \) so that minimizing Equation (2) results in a least-squares problem for which many solvers exist. Because of the limitations of CME reconstruction mentioned above, the optimization is highly ill-posed unless the regularization expression is chosen properly so that it yields different penalties for the many solutions in the null space of the data term.

As shown in Equation (2), an appropriate regularization needs to be added to obtain a more stable solution to deal with the ill-posed problem. Because many solutions are possible to eliminate the data term in Equation (2), the solution that is finally returned by any procedure depends on the error norm and the regularization we choose (in that case, \( \eta = 0 \), else the data for \( B_{\text{obs}} \) are probably inconsistent and there is no exact
solution, although there are many for which the delta error is of the order of \( \eta \). We emphasize that this ambiguity lies in the heart of the CME reconstruction problem and is unavoidable because two viewpoints are not sufficient to determine a unique CME density distribution. Our goal is to test whether a certain combination of error norm \( p \) and regularization may yield more realistic solutions than others. Because conventional solvers only work for \( p = 2 \) and \( N_e^2 \)-dependent regularization, we decided to use GRM to find the minimum of Equation (2) so that we maintain maximum flexibility in choosing \( p \) and the regularization expression.

In order to show the ability of GRM to reconstruct CMEs, regularization has not been added in the current work, and we choose \( p = 1 \):

\[
\text{cf} = \left| \sum_{\tilde{\theta}} B_i(\tilde{\theta}) N_e(\tilde{\theta}) - B_{\text{obs}} \right|.
\]

The L1 norm is often favored over least-squares solutions in image processing because it attributes less weight to measurement outliers. A systematic test of different regularization operators will be made in the next paper.

2.2. Calculations of Electron Number and White-light Brightness

Optimization is initiated by calculating the electron number for each LOS and redistributing the electron density \( N_e(\theta) \) in a discrete form along each LOS. The Thomson scattering can then be calculated for the redistributed electrons, and the cost function is ready for the optimization.

In order to explain the process of initiation, a cube of plasmas is used to mimic the CME, as shown in the middle panel of Figure 2. In the first and third panels, the simulated white-light brightness is produced through a Thomson scattering mechanism in the field of view (FOV) of STEREO A and B, respectively. There is no need to subtract the background corona because we only simulate the brightness of the model CME itself. For the brightness of the real CME in Section 3.2, we rescale the CME images to 128 \( \times \) 128 pixels and make a 3 \( \times \) 3 smoothing to minimize the influence of noise which is inherent in the measurements. A pre-CME background image is subtracted to get the excess brightness of the CME (Vourlidas et al. 2010).

It is necessary to split the brightness to simplify the reconstruction. As shown in Figure 2, the brightness is equally split up into ribbons with white lines for FOV A and B. For example, the corresponding part of ribbon A in FOV A is ribbon B in FOV B, shown by the black lines. Pixels in this pair of ribbons belong to the same set of epipolar planes (Inhester 2006) between two epipolar planes indicated in the middle panel of Figure 2. The process for finding ribbon B is carried out by projecting the LOS of each pixel in ribbon A into FOV B. LOS A, which is projected onto FOV B, should intersect with the CME boundary at two cross-points at least.
Thomson scattering of a single electron, $\theta$ index the computation grid along LOS A, and equivalent to the summation of index FOV A can be indexed by along the view directions and usually nonorthogonal. Pixels in FOV A can be calculated as coordinate axes are corresponding pixel of the model CME brightness image in each other in the FOV of a given telescope. Reconstruction can be carried out for each pair of ribbons to obtain the electron density in the corresponding set of epipolar planes.

We use $i$ to index pixels along epipolar lines in FOV A, $j$ to index the computation grid along LOS A, and $k$ to index epipolar planes. In each epipolar plane, coordinate axes are along the view directions and usually nonorthogonal. Pixels in FOV A can be indexed by $(i, k)$. Integration along LOS A is equivalent to the summation of index $j$.

(1) First of all, the electron number along the LOS of the corresponding pixel of the model CME brightness image in FOV A can be calculated as

$$N_e(i, k) = \frac{B_{obs}(i, k)}{B_e(\theta(i, k))},$$

(4)

where $B_{obs}(i, k)$ is the brightness value of the pixel $(i, k)$. The Thomson scattering of a single electron, $B_e(\theta(i, k))$, can be calculated as

$$B_e(\theta(i, k)) = \frac{\pi \sigma}{2 z^2} \left[ 2((1 - u)C + uD) \right. \left. - ((1 - u)A + uB)\cos^2 \theta(i, k) \right].$$

(5)

$\theta$ is traditionally assumed to be zero (Vourlidas et al. 2010) when the electron location along the LOS is unknown. $z$ is the distance from the observer to the scattering location along the LOS. Three-dimensional reconstruction, as in Moran & Davila (2004) and Dai et al. (2015), can obtain the value of each pixel to get a more precise calculation of the electron number as done in this paper. $\sigma$ is the Thomson scattering cross section of an electron and $u$ is the limb-darkening coefficient. $A$, $B$, $C$, and $D$ are the van de Hulst coefficients (van de Hulst 1950).

In this work, the electron number is always calculated from observation 1 as shown in Figure 2. In principle, observation 1 can be any of STEREO A, B, or SOHO. Observation 2 is not restricted to STEREO B but is also possible for SOHO, as shown in Section 3. This enables us to carry out the reconstruction without data from STEREO B.

(2) The redistribution of the electrons of each pixel along the corresponding LOS. As shown in the top-right panel of Figure 1, series of LOSs are generated from brightness A, and the extent of the LOS is restricted by the length of brightness B. Then, we get the LOS segment. Each LOS segment is uniformly separated into $N_j = 64$ computational grids. The voxels on the computational grids are initially populated with a random distribution of electrons. An array of random decimals, $L(i, j, k)$, is used to represent the electron distribution.

We calculate the electron number in the $j_{th}$ voxel on the grid point along the LOS segment using the following equation:

$$N_e(i, j, k) = \frac{L(i, j, k)}{\sum_{j=1}^{N_j} L(i, j, k)} N_e(i, k).$$

(6)

(3) Calculation of the Thomson scattering for the redistributed electrons. Equation (7) converts the electron density $N_e(i, j, k)$ into white-light emission $B_{\text{av}}(i, j, k)$:

$$B_{\text{av}}(i, j, k) = B_e(\theta_{av}(i, j, k))N_e(i, j, k),$$

(7)

where $B_e(\theta_{av}(i, j, k)) = \frac{\pi \sigma}{2 z^2} \left[ 2((1 - u)c + uD) \right. \left. - ((1 - u)A - uB)\cos^2 \theta_{av}(i, j, k) \right].$

(8)

Then, we can do the summation for $j$ to get the total brightness of a CME pixel $(i, k)$ in FOV A:

$$B_{av}(i, j, k) = \sum_{j=1}^{N_j} B_{\text{av}}(i, j, k).$$

(9)

The total brightness of a CME pixel $(j, k)$ in FOV B can also be calculated using a similar summation for $i$:

$$B_{av}(j, k) = \sum_{i=1}^{N_i} B_{av}(i, j, k)$$

(10)

using Equations (7) and (8) when values of $\theta_{av}$ and the van de Hulst coefficients in FOV A are replaced by values of $\theta_{av}$ and the van de Hulst coefficients in FOV B. We emphasize that each LOS_B contains electrons from different LOS_A. The values of $\theta$ and the van de Hulst coefficients should be calculated for Equations (7) and (8) according to the electron locations in the coordinate system of FOV A and FOV B, respectively.
2.3. CME Reconstruction Using GA

Now, we can calculate the cost function for pixel \((i, k)\) in FOV A using Equation (3),

\[
\text{cf}(i, k) = |B_g(i, k) - B_{\text{obs}}(i, k)|, \quad (11)
\]

and the cost function for pixel \((j, k)\) in FOV B:

\[
\text{cf}(j, k) = |B_g(j, k) - B_{\text{obs}}(j, k)|. \quad (12)
\]

Once the cost function is ready, the process of CME reconstruction can be described in the following five steps which are typical for GA, as shown in the top-left panel of Figure 1.

1. Population initiation. The redistributed electrons along all of the LOS segments constitute one of the individuals of GA. The same process of random redistribution is carried out 200 times to produce the initial population \(L_q(i, j, k)\), \(q = 1, 2, 3, \ldots, N_q\), \(N_q = 200\). This is the beginning of GA as shown in the top-left panel of Figure 1.

2. Fitness calculation. The fitness function used in GA is inversely proportional to the cost function:

\[
\text{Fit}_q = \frac{B_{\text{obs}}}{\text{cf}_q}. \quad (13)
\]

We show an example of the fitness of \(N_q\) individuals in the top panel of Figure 3, from smallest to largest. In order to avoid a local solution, which may be produced by the steep distribution of fitness as shown in the top panel of Figure 3, we need to rescale the fitness function. First, the values of the fitness are changed to be linearly increasing:

\[
\text{Fit}_q = \frac{\text{findgen}(N_q) + 1}{N_q}, \quad (14)
\]

where \text{sort} is the ranking function implemented in IDL. The sorted array of fitness \(\text{Fit}_q\) is incremental, as shown in the top panel of Figure 3. The IDL function \text{findgen}(N_q) creates an arithmetic progression \([0, 1, 2, \ldots, N_q - 1]\). Then, the values of fitness are transformed to \([1, 2, 3, \ldots, N_q]\) by Equation (14), as plotted in the middle panel of Figure 3. Finally, the IDL function \text{exp} is used to transform the fitness values into a natural exponential distribution via

\[
\text{Fit}_q(i, k) = \exp \left( \frac{\text{Fit}_q(i, k) g}{N_q} - 0.03 \right) \quad (15)
\]

and

\[
\text{Fit}_q(j, k) = \exp \left( \frac{\text{Fit}_q(j, k) g}{N_q} - 0.03 \right), \quad (16)
\]

where \(g = 1, 2, 3, \ldots, N_q\), \(N_q = 100\) is the total number of generations in the genetic evolution. As shown in the bottom panel of Figure 3, the fitness distribution becomes steeper from generation 40 to 60. It means that the algorithm can avoid a local solution at the preliminary stage of genetic evolution and keep it more convergent at later stages.

3. A selection operator is employed to obtain the optimal electron distribution. In order to further simplify the reconstruction, we equally divide the brightness ribbon into 20 parts for FOVs A and B, respectively, as shown in Figure 4. For each part of the brightness ribbon in FOV B, a group of electron distributions \(L_q\) along the LOS segments inside a column that is marked by the purple lines, for instance, contribute to the brightness of the Thomson scattering. The mean value of the fitness is already calculated for pixels inside each part of the brightness ribbon using Equation (16) in step (2). We randomly choose two candidates from \(N_q\) candidates of the \(L_q\) group and compare their mean fitness. The candidate with the larger fitness will be chosen and passed on to the next generation. Such tournament is repeated \(N_q\) times for each part of the brightness ribbon in FOV B to produce a new population. Based on this new population, the brightness in FOV A can be updated and the mean fitness of the whole brightness ribbon in FOV A can be calculated by Equation (15) for each of the \(N_q\) individuals. Such mean fitness of the whole brightness ribbon is also calculated for FOV B based on the new population. Ten individuals with the worst fitness in FOV B are replaced by 10 individuals with the best fitness in FOV A. This replacement optimizes the population for FOV A after the tournament selection for FOV B.

4. In this work, we set the maximum number of generation to 100. If the genetic evolution meets this termination condition, the GA will stop and an optimal solution will be chosen to be the final reconstruction of the CME. If the number of generations is less than 100, the algorithm will go to step (5).

5. Crossover and mutation operators are used to update the population \(L_q\). The basic concept of these operations is presented in the bottom panel of Figure 1. The crossover operator exchanges the electron distribution between two...
randomly selected individuals while the mutation operator changes the distribution inside one individual. Equations (17) and (18) describe these operators:

\[ L_m(i, j, k) = L_n(i, j, k), \quad (17) \]

where \( L_m(i, j, k) \) ∈ CME\(_m\), \( L_n(i, j, k) \) ∈ CME\(_n\), \( m, n \in [1, N_q], m \neq n; \)

\[ L_m(i, j, k) = L_m(i, j, k) + L_m(i, j, k) \times D_r, \quad (18) \]

where \( D_r \) is a random decimal which can be positive or negative. In order to simplify the reconstruction, crossover and mutation operators are applied to all of the \( L_q \) inside one of the \( 20 \times 20 \) areas shaped like a parallelogram at each time.

After the genetic operation, we get the new CME population from parents to children. The probabilities of crossover and mutation decrease progressively to keep the optimization global and convergence.

The process from steps (2) to (5) are repeated until the number of generations is equal to \( N_g \).

In the following sections, GRM is first applied to a set of model CMEs with different directions and shapes. Then, the method is employed to reconstruct a CME observed by the coronagraphs of SOHO and STEREO.

3. Application

In order to test the GRM method in a more realistic way, we use a set of analytic GL98 (Gibson & Low 1998) model CMEs with various orientations and shapes, as well as occultation to produce synthetic coronagraph data instead of the cube of plasmas used in Figure 2 in Section 2.2. The GL98 model constructs a flux rope to create a typical three-part CME structure. The flux rope is an analytical solution of the magnetohydrostatic equation

\[ \frac{1}{4\pi} (\nabla \times B) \times B - \nabla p - \rho g = 0 \quad (19) \]

and the solenoidal condition of the magnetic field, \( \nabla \cdot B = 0 \). The explicit solution of Equation (19) can be derived from the
We change the position of the sphere center, under self-similar theory (Chandrasekhar & Prendergast 1956) to an axisymmetric sphere of magnetic flux (Chandrasekhar 1956; Chandrasekhar & Prendergast 1956) with radius $r_0$. The sphere center is located at $r_1$ from solar center. After the transformation, the magnetic flux appears to be a tear drop. Then, the magnetic field of the GL98 model can be expressed by a Bessel function and a free parameter $a_1$ (Prendergast 1956; Lites et al. 1995). As pointed out by GL98, the solution of Equation (19) can even be used to solve the MHD equation

$$\frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p = 0$$

under self-similar theory (Low 1984). The GL98 model is implemented in a space weather modeling framework (Tóth et al. 2005, 2012) and is successfully used in studies of CME simulations (Manchester et al. 2004a, 2004b, 2014a, 2014b; Lugaz et al. 2005; Jin et al. 2016, 2017a). Recently, this model was applied in a user-friendly tool named Eruptive Event Generator using the Gibson–Low configuration (Borovikov et al. 2017; Jin et al. 2017b), which has been transitioned to the Community Coordinated Modeling Center.

Finally, we obtain the distribution of plasma density frozen into the magnetic field:

$$\rho = f(B_x, B_y, B_z).$$

We change the position of the sphere center, $r_0$, $r_1$, $a$, and $a_1$ to obtain model CMEs with different orientations, sizes, shapes, and strengths of the magnetic field. Because the distribution of plasma density is known, the synthetic CME white-light images can be produced by the Thomson scattering mechanism mentioned in Section 2.2.

We apply GRM to these synthetic white-light images to reconstruct the model CMEs. Comparisons of brightness and electron density between models and reconstructed CMEs are presented to show the advantages and limitations of GRM in Section 3.1. After systematic comparisons between model and reconstructed CMEs, GRM is applied to real observations from SOHO and STEREO in Section 3.2.

### 3.1. Application to Model CMEs

We simulate CME observations using the same parameters as STEREO/COR2 and SOHO/C3 including satellite position, viewing direction, size of FOV, and spatial resolution on 2013 February 15. The positions of Earth (SOHO) and STEREO A and B in our simulation are shown in Figure 5. A noise image produced from COR2 data as shown in Figure 6 is employed to test the performance of GRM. A detailed description can be found at the end of Section 3.1.

For simulations of two viewing observations from STEREO A and B, denoted observation 1 and 2, the separation angle is set to 90° to test interferring factors in reconstruction like the longitude and latitude of the CME, whether or not to use occultation, and whether or not the CME is a halo. For STEREO A and SOHO as observations 1 and 2, the separation angle is set to 135°. We add data from COR2 on STEREO B as the third viewpoint to improve the performance of GRM.

The central longitude of the model CME is set to 0°, 45°, 90°, 135°, and 180° as shown in Figure 5. The central latitude of the model CME is routinely set to 0° and 30° for each value of the longitude. The latitudes of 15° and 45° are also set to the model CME with a longitude of 0° to test the latitude dependence of GRM more carefully. For convenience, we mark the CME with (longitude, latitude) for specific values of longitude and latitude. For example, the CME with a longitude of 0° and latitude of 10° can be called the (0, 0) CME.

In order to quantitatively evaluate the similarity between the model and reconstructed CME, we calculate the mean relative absolute deviation (MRAD),

$$MRAD = \frac{1}{N} \sum_{i=1}^{N} \frac{|x - y|}{x_i}$$

and the linear Pearson correlation coefficient (PCC; Li et al. 2018),

$$PCC = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{N} (y_i - \bar{y})^2}},$$

where $x$ and $y$ are values of brightness or electron density of the model and reconstructed CME, and $N$ is the number of CME voxels. If $y$ is similar to $x$, the value of MRAD should be close to 0 and the value of PCC should be close to 1. The upper limit for MRAD is set to 1.0.

We show the similarities between the model and reconstructed CMEs qualitatively with images and quantitatively with PCC in Figure 7 and the following figures. For convenience, we call these figures “Figures of Similarity” (FOS). Figure 7 and 8 show the results of CMEs with a longitude of 0 degrees. Figure 9 shows the results of (45, 0) CME. Figures 10–13 show the results of CMEs with a longitude of 135 degrees. Figures 14 and 15 show the results of (135, 30) CME using SOHO as observation 2. We run GRM three times for each model CME to prove consistency of the method as shown in the FOS, which is labeled with “1st,” “2nd,” and “3rd.” Because of the ill-posed property of CME
specific CME model on the $xy$, $xz$, and $yz$ planes. The average density in these planes enables us to make more comprehensive comparisons between the model and reconstructed CMEs.

To compare brightness, we display the original brightness of the GL98 model CME in the first column of the first and second rows of the FOS. The brightness reconstructed by GRM at generation 100 is labeled “1st,” “2nd,” and “3rd” for different runs of GRM. We plot the PCC of the brightness from generations 0 to 100 in the third row to show the process of genetic evolution. The plots of PCC at the “1st,” “2nd,” and “3rd” runs for reconstruction 1 are shown in the first column in red, green, and blue, respectively. Plots of PCC for reconstruction 2 are shown in the second column. PCC becomes larger from generation 0 to 100, which means that the optimization is convergent.

The electron density in 3D space as shown in the FOS is the average inside cubes with width 2.0 $R_{\odot}$. To compare electron density, the original density of the GL98 model CME in 3D space and the $xy$, $xz$, and $yz$ planes is first presented in the fourth row of the FOS. The reconstructed density of the “1st,” “2nd” and “3rd” runs of GRM are displayed in the last three rows of FOS. The Heliocentric Earth Equatorial (Thompson 2006) coordinate system is used to illustrate the CME density in the FOS. The corresponding heliographic coordinates are Stonyhurst. In Figure 7, the longitude and latitude of the model CME are both equal to 0°. It means that the direction of the (0, 0) CME is along $z$-axis. In order to simulate the brightness of the (135, 30) CME, the longitude and latitude of the model CME are changed to 135° and 30° respectively. Based on the simulated brightness, we reconstruct the electron density of the CME using GRM. Then, we transform the model and reconstructed CME by $-135^\circ$ and $-30^\circ$ for the longitude and latitude, respectively. After this kind of transformation, the density of the model and reconstructed CME in Figure 11 can be viewed under the same perspective as in Figure 7. Similar transformations are applied to the rest of the FOS. The PCCs of the electron density distribution between the model and reconstructed CME labeled in the FOS are summarized in Table 1. In order to give a more complete picture of the GRM performance, values of MRAD along with those of the PCC are shown in columns 2–5 of Table 1.

We further discuss the interfering factors of GRM based on the FOS and Table 1 as follows.

1. As shown in Table 1 and FOS, the values of PCC in the $xz$ plane are usually lower than those in the $xy$ and $yz$ planes when the latitude is equal to 0°. The value of PCC in the $xz$ plane is even less than 0.6 at the second run for the (90, 0) CME. An obvious improvement of PCC in the $xz$ plane can be seen when the latitude increases from 0° to 30° for CMEs with all values of longitude. A comparison between the (0, 0) CME in Figure 7 and the (0, 30) CME in Figure 8 validates such improvement as an example. For latitudes from 30° to 45°, the improvement in the PCC stops as shown for the 0° longitude CME.

The PCC of the (135, 30) CME is the best among all of the reconstructed CMEs. For this CME, the PCC in 3D space is larger than 0.8 while the PCCs in the $xy$, $xz$, and $yz$ plane are even larger than 0.9.

2. The (45, 0) CME appears as a full halo in FOV B as a backside event and the (135, 0) CME appears as a full halo in FOV A as a frontside event. The (45, 30) and

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**Figure 6.** Noise level of the COR2 image at 09:08 on 2019 July 10 after subtracting the background image, which is observed at 08:08. Top panel: noise image. Center of the Sun is marked with a plus. Rings with heights of 5, 10, and 15 $R_{\odot}$ are labeled with a number. Second panel: plot of average noise in the rings, $B_n$. Third panel: plot of average total brightness, $B_t$. Bottom panel: plot of ratio of $B_t$ to $B_n$. Reconstruction and the inherent randomness of GRM, reconstruction results including the Thomson scattering brightness and electron density of the same model CME should be slightly different between each run.

For each run of GRM, we not only show the 3D distribution of the electron density in the first column of the last three rows of FOS but also the 2D distribution of the density averaged along the directions of the $x$-, $y$-, and $z$-axes as shown in the last three columns of these rows. For example, the density averaged along the $z$ direction forms the 2D distribution in the $xy$ plane as shown in the second column which is labeled “$xy$” in the lower-left corner. The performance of GRM is different for a
Figure 7. FOS of the (0, 0) CME. Top two rows: brightness of model CME (observations 1 and 2) and brightness of reconstructed CME (reconstructions 1 and 2 of the “1st,” “2nd,” and “3rd” GRM runs) at generation 100. Third row: PCC of brightness between model and reconstructed CMEs for observations 1 (left panel) and 2 (right panel) from generation 0 to 100 of the “1st” (red), “2nd” (green), and “3rd” (blue) GRM runs. Fourth row, from left to right: distribution of the electron density of the GL98 model CME in 3D space, xy plane, xz plane, and yz plane. Bottom three rows: distribution of the electron density of the reconstructed CME of the “1st,” “2nd,” and “3rd” GRM runs. The unit of the electron density color bar is $10^4$ electron cm$^{-3}$. Return to Table 1.
Figure 8. FOS of the (0, 30) CME. Return to Table 1.
Figure 9. FOS of the (45, 0) CME with the occulter. Return to Table 1.
Figure 10. FOS of the (135, 0) CME with the occulter. Return to Table 1.
Figure 11. FOS of the (135, 30) CME. Return to Table 1.
Figure 12. FOS of the (135, 30) CME with the occulter. Return to Table 1.
Figure 13. FOS of the (135, 30) CME with normal noise. Return to Table 1.
Figure 14. FOS of the (135, 30) CME using STEREO A and SOHO as observations 1 and 2. Return to Table 1.
Figure 15. FOS of the (135, 30) CME using STEREO A, SOHO, and STEREO B as observations 1, 2, and 3. Return to Table 1.
(135, 30) CMEs become partial halos when the latitude is changed to 30°.

The halo CMEs are obscured by the occulter of the coronagraph as shown in Figures 9, 10, and 12. For halo CMEs, the influence of the occulter is more obvious than for limb CMEs. The central part of the halo CME is hidden behind the occulter. For GRM reconstruction, the situations in Figures 9 and 10 are different, although these two CMEs are both full halos. The (45, 0) CME is a full halo for observation 2 as shown in Figure 9. On the other hand, the CME is almost not affected by the occulter as it is a limb event for observation 1. We obtain a relatively complete initial electron number of the CME for GRM because the electron number is calculated from the brightness in observation 1. In contrast, the initial electron number is obviously incomplete in Figure 10 for the (135, 0) CME.

As a result, the brightness of reconstruction 2 is obviously larger than that of observation 2 for the (45, 0) CME as shown in Figure 9 because of the completeness of the CME in observation 1 and its incompleteness in observation 2. The situation is opposite for the (135, 0) CME as shown in Figure 10.

(3) As shown in Figure 14, the (135, 30) CME is reconstructed using simulation data from the view from SOHO/C3 as observation 2. The separation angle between STEREO A and SOHO is 135°. Comparing to the orthogonal coordinates with a 90° separation angle, a 135° separation angle is not ideal for GRM reconstruction. In order to improve the performance of GRM under a non-orthogonal coordinate system, the view from STEREO B is added into the reconstruction as observation 3 in Figure 15. The outline of the CME in observation 3 constrains the 3D electron distribution as shown in Figure 15 compared to Figure 14. In the current work, we just use the outline of CME in observation 3 as a constraint. Such a constraint makes the boundary of the LOS more accurate. In the future, we may calculate the fitness function of the brightness in observation 3

### Table 1

| Modeled CME position (longitude, latitude) | MRAD of $xy$ space 1st, 2nd, 3rd | PCC of $xy$ plane 1st, 2nd, 3rd | PCC of $yz$ plane 1st, 2nd, 3rd | PCC of $xz$ plane 1st, 2nd, 3rd |
|------------------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 0, 0                                     | 0.612, 0.620, 0.592              | 0.784, 0.782, 0.780              | 0.848, 0.883, 0.899              | 0.634, 0.642, 0.666              |
| 0, 15                                    | 0.615, 0.608, 0.622              | 0.799, 0.798, 0.816              | 0.905, 0.891, 0.886              | 0.750, 0.751, 0.779              |
| 0, 30                                    | 0.652, 0.651, 0.666              | 0.808, 0.795, 0.800              | 0.892, 0.883, 0.864              | 0.853, 0.851, 0.851              |
| 0, 45                                    | 0.744, 0.741, 0.732              | 0.796, 0.800, 0.793              | 0.841, 0.849, 0.817              | 0.864, 0.871, 0.863              |
| 45, 0                                    | 0.569, 0.577, 0.569              | 0.845, 0.870, 0.862              | 0.923, 0.915, 0.925              | 0.762, 0.805, 0.796              |
| 45, 0°                                   | 0.640, 0.635, 0.632              | 0.737, 0.772, 0.759              | 0.703, 0.786, 0.739              | 0.502, 0.586, 0.562              |
| 45, 30                                   | 0.631, 0.630, 0.620              | 0.840, 0.823, 0.809              | 0.857, 0.850, 0.831              | 0.846, 0.803, 0.798              |
| 90, 0                                    | 0.608, 0.599, 0.612              | 0.810, 0.789, 0.799              | 0.938, 0.944, 0.924              | 0.665, 0.580, 0.652              |
| 90, 30                                   | 0.651, 0.668, 0.656              | 0.782, 0.798, 0.787              | 0.893, 0.890, 0.908              | 0.843, 0.802, 0.860              |
| 135, 0                                   | 0.592, 0.597, 0.581              | 0.838, 0.868, 0.855              | 0.980, 0.975, 0.977              | 0.685, 0.679, 0.726              |
| 135, 0°                                  | 0.578, 0.589, 0.579              | 0.821, 0.811, 0.817              | 0.790, 0.779, 0.782              | 0.649, 0.654, 0.653              |
| 135, 30                                  | 0.621, 0.618, 0.611              | 0.822, 0.821, 0.847              | 0.957, 0.953, 0.955              | 0.913, 0.899, 0.910              |
| 135, 30°                                 | 0.630, 0.643, 0.620              | 0.813, 0.835, 0.840              | 0.945, 0.953, 0.955              | 0.888, 0.872, 0.899              |
| 135, 30°                                 | 0.740, 0.727, 0.711              | 0.795, 0.801, 0.814              | 0.820, 0.881, 0.862              | 0.843, 0.838, 0.873              |
| 135, 30°                                 | 0.894, 0.897, 0.898              | 0.206, 0.398, 0.408              | 0.507, 0.284, 0.414              | 0.252, 0.499, 0.433              |
| 135, 30°                                 | 0.999, 0.999, 0.999              | 0.139, 0.184, 0.164              | 0.231, 0.139, 0.132              | 0.157, 0.218, 0.230              |
| 135, 30°                                 | 0.719, 0.710, 0.705              | 0.688, 0.731, 0.709              | 0.851, 0.857, 0.868              | 0.762, 0.773, 0.740              |
| 135, 30°                                 | 0.750, 0.761, 0.755              | 0.728, 0.716, 0.704              | 0.871, 0.869, 0.868              | 0.786, 0.822, 0.833              |

### Notes

Please click on the blue values of the central longitude and latitude in the first column to go to the corresponding FOS.

- The CME is obscured by the modeled occulter of the coronagraph.
- Randomly distributed noise are added to the synthetic CME observations.
- Randomly distributed noise ×10 are added to the synthetic CME observations.
- The CME is reconstructed from modeled coronagraph images of STEREO A and SOHO as observations 1 and 2.
- The CME is reconstructed from modeled coronagraph images of STEREO A, SOHO, and STEREO B as observations 1, 2, and 3.
- The CME is reconstructed from real coronagraph images of STEREO A and B, SOHO as observations 1, 2, and 3.
- The CME is reconstructed from real coronagraph images of STEREO A, SOHO, and STEREO B as observations 1, 2, and 3.
together with those of observations 1 and 2 to optimize
the reconstruction of the electron distribution.

(4) For the original synthetic observations of the model
CME, there is no noise. On the contrary, noise is
inevitable in real observations. We add noise to the
synthetic images of the (135, 30) CME as shown in
Figure 13. This is a critical test to evaluate the
performance of GRM when the method is applied to real
observations from coronagraphs. The noise image is
produced by subtracting the background for a
coronagraph image which is clear of CME. In a perfect
subtraction of a noiseless image, pixel values should be
zero because there is no CME brightness. However, the
brightness values usually deviate from zero for real
observations as shown in the top panel of Figure 6.
This deviation can approximate the normal noise level in real
observations. In the second and third panels of Figure 6,
the average values of noise, \(B_n\), and total brightness, \(B_t\),
are calculated within rings of 0.5 \(R_\odot\) width. It is obvious
that both \(B_n\) and \(B_t\) decrease rapidly outward from the
Sun. However, the relative noise \(B_n/B_t\) increases with
height.

This normal noise level is added to the synthetic CME
brightness as shown in Figure 13 and Table 1. Results with
the noise level multiplied by 10 and 100 are also shown in Table 1.
We can see that the difference in PCC values between
Figure 13 with normal noise and Figure 12 without noise is
not obvious. In the case of normal noise level, values of PCC
are still larger than 0.8 and values of MRAD are still less than
0.8. The difference becomes larger when the noise is multiplied
by 10 and 100. An obvious decline of GRM performance can
be seen in the case of noise multiplied by 100. In this case,
values of PCC are even less than 0.3 and values of MRAD are
close to 1.0. It means that the correlation between model and
reconstructed CME is very low and GRM is no longer
effective. On the other hand, GRM is still feasible under normal
noise levels common in real CME observations.

### 3.2. Application to Observed CMEs

In this section, we show the application of GRM to a real
CME event observed by STEREO and SOHO on 2013
September 30. The CME is observed by COR2 on STEREO
A and B at 00:08 and by C3 on SOHO at 00:04. The difference
in observation time is four minutes between COR2 and C3. The
general case is that differences in observation timings usually
exist between COR2 and C3. What we can do is choose the
nearest observation time of C3 to match the COR2 observation.
The coordinate system of this GRM reconstruction is
nonorthogonal because the separation angle between STEREO
A and B is 73°.5 while the separation angle between STEREO
A and SOHO is 147°.1.

In Figure 16, the CME is reconstructed from data of
STEREO A and B, and SOHO as observations 1, 2, and 3. In
Figure 17, we reconstruct the CME using STEREO A, SOHO,
and STEREO B as observations 1, 2, and 3. For this real CME,
the PCC of the brightness can still be calculated between the
real observation and the reconstruction. Plots of the brightness
PCC from generation 0 to 100 still converges as shown in the
FOS. However, the PCC of the electron distribution cannot be
calculated between the real and reconstructed CME because we
do not know the 3D electron distribution of the real CME.

Instead, the PCC of the electron distribution can be calculated for the “2nd” and “3rd” reconstructions by comparing to the
“1st” reconstruction. These PCC values are also summarized in
Table 1, marked as “ABS” and “ASB” for observation 2 using
data from STEREO B and SOHO, respectively. Because we
use the density distribution in the “1st” reconstruction as the
reference (“model CME”), values of PCC are equal to 1.0 and
values of MRAD are equal to 0.0 for the “1st” reconstruction as
shown in Table 1. For the “2nd” and “3rd” reconstructions,
PCC values are larger than 0.78 and MRAD values are less
than 0.4. These results show us the stability of reconstructions
in different runs of GRM.

GRM reconstructs this real CME using data from STEREO
B and SOHO as observation 2. The CME is a partial halo in
these two FOVs. Thus, we cannot detect the complete
brightness of the CME core which is obscured by the occulter
of the coronagraph. On the other hand, the CME brightness is
relatively complete in FOV A as observation 1 because the
CME is a limb event in this FOV. A similar situation can be
seen in Figure 9 for the reconstruction of the model (45, 0)
CME with the occulter. This model CME is a limb event in
observation 1 and is a halo event in observation 2. Although the
CME brightness is complete in observation 1, the reconstruc-
tion is still not correct because of the incomplete brightness in
observation 2. The structure of the reconstructed CME core is
evidently destroyed as shown in Figure 9, comparing to the
model CME. This may also be the reason why we cannot find a
typical dense core in the reconstruction of the real CME as
shown in Figures 16 and 17. Besides the influence of the
occulter, the CME itself may not be a typical three-part event.
The dense core may become indistinct during the outward
displacement.

As pointed out by Morgan (2015), the accuracy of the
reconstructed density depends on the reliability of the real input
data besides the reconstruction method. Calibration and
background subtraction play important roles to obtain the real
CME brightness without contamination from vignetting, stray-
light, F corona, and other static structures in K-coronas, such as
streamers, coronal holes, etc.

In this work, we download the level 0.5 fits files of the
coronagraph images and process them to level 1.0 using standard
calibration routines from the Solarsoft library. For the total
brightness \(B_t\) observed by coronagraph C3 on board the SOHO/
Large Angle and Spectromeric Coronagraph Experiment, we
apply the Solarsoft routine reduce_level_1.pro to make calibrations
for dark current, flat field, stray light, distortion, vignetting,
photometry, corrected time and position, etc. For \(B_t\) observed by
coronagraph COR2 on board STEREO/SECCHI, we apply
secchi_prep.pro to make calibrations for subtracting the CCD bias,
multiplying by the calibration factor and vignetting function,
dividing by the exposure time, etc.

After the standard calibration, \(B_t\) contains a combination of
K corona brightness \(B_K\) and F corona brightness \(B_F\). Morgan
(2015) developed a method to separate \(B_K\) and \(B_F\) for C2
observations with FOV between 2.2 and 6.0 \(R_\odot\). In this
method, an in-flight calibration using star brightness is
employed to improve the standard calibration for the polarized
brightness \(B_P\) of C2, which is usually obtained once a day. A
series of calibrated \(B_P\) during half period of solar rotation is
subtracted from the corresponding \(B_t\) to produce a \(B_F\)
approximation. Finally, \(B_K = B_t - B_F\). For C3 observations
with FOV between 3.7 and 30 \(R_\odot\), we may use this method to
Figure 16. FOS of the CME using real data from STEREO A and B and SOHO as observations 1, 2, and 3. Return to Table 1.
Figure 17. FOS of the CME using real data from STEREO A, SOHO, and STEREO B as observations 1, 2, and 3. Return to Table 1.
obtain cleaner corona brightness by removing \( B_f \) from \( B_i \) in the future.

\( B_i \) contains the brightness from the Thomson scattering in coronal structure like streamers, coronal holes, and CME, etc., among other. For CME reconstruction, we need to remove the background including the relatively static components like streamers and coronal holes. Subtraction of a pre-CME image observed before the CME is a standard approach, as we have done in this work. After such subtraction, the excess brightness of the CME is available. Morgan et al. (2012) and Morgan (2015) developed a useful Dynamic Separation Technique (DST) to make the subtraction more effective. DST is valid under the assumption that quiescent structures like streamers are basically smooth in the radial direction and slowly evolve in time, while dynamic structures like CMEs are not radially smooth and evolve faster. Then, DST is able to separate \( B_i \) into quiescent and dynamic components using iterative deconvolution in the radial and time dimensions. In a future improvement, we will employ DST to subtract the background corona for CME reconstruction. Morgan (2015) presented a method to cross-calibrate between C2 and COR2 which is necessary for electron density reconstruction using observations from different coronagraphs (Morgan 2019). Such method will also be considered in a future work.

4. Conclusions

In this study, GRM is used to reconstruct the 3D distribution of CME electrons. We first gave a general description of CME reconstruction. An ill-posed property of CME reconstruction is pointed out. A set of analytic GL98 model CMEs with different orientations and shapes is employed to produce synthetic CME images without noise for genetic reconstruction. Random noise is artificially added to some synthetic CME images to imitate the measurement noise that is unavoidable in the real CME observations. Because the electron distribution is known for the model CMEs, we compare the PCCs of both the electron distribution and its corresponding Thomson scattering brightness between the model and reconstructed CMEs. The PCC of the brightness from generation 0 to 100 shows the convergence of GRM for all of the model CMEs. The PCC of the electron distribution in 3D space and 2D planes shows the ability of GRM to obtain a stable and reasonable CME reconstruction. MRAD values of the electron distribution in 3D space validate the results of PCC.

Based on the comparison between model and reconstructed CMEs presented in the FOS and Table 1, we understand in more depth the advantages and limitations of GRM. The performance of GRM depends on the longitude and latitude of the model CME as well as the completeness of the observation and the separation angle between points of view. A more reliable reconstruction can be obtained if (1) the model CME in the coronagraph is not that obscured by the occulter, (2) the central latitude of the CME is about 30°, (3) the separation angle is about 90°, and (4) data from the third observation can be added into the reconstruction.

The method is then applied to real coronagraph data from STEREO A and B, and SOHO. We compare the reconstructed brightness with observation to show the convergence of GRM. Comparisons of electron distribution between reconstructions from different runs tell us that the results of GRM reconstruction are stable. As pointed out in Section 3.2, we should employ more effective techniques to calibrate the coronagraph data and for background subtraction in the future work.

The purpose of this paper is to demonstrate how GRM could be used to find a solution. Because of the ill-posed nature of CME reconstruction using data from only two or three viewpoints and the random nature of GRM, the results of GRM for the same model CME are different among three runs. This just illustrates the range of solutions that are possible for the unregularized problem. Regularization as in Equation (2) is helpful to further stabilize GRM reconstruction and may also mitigate its ill-posed characteristic toward a unique solution. How realistic the solution is then depends on how reasonable the regularization operator is. The GRM may allow the least restrictions in the choice of regularization. In a future work, we may test different forms of regularization to find suitable constraints for the CME reconstruction.

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