OUTER-GAP VERSUS SLOT-GAP MODELS FOR PULSAR HIGH-ENERGY EMISSIONS:
THE CASE OF THE CRAB PULSAR
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ABSTRACT

We analytically examine the capabilities of rotation-powered pulsars as the sources of gamma rays and show that their phase-averaged gamma-ray flux is proportional to the product of the spin-down flux and the gap transfield thickness cubed irrespective of the emission models. Applying the scheme to the Crab pulsar, we demonstrate that the outer-gap model reproduces the observed GeV fluxes and that the slot-gap model reproduces at most 20% of the observed fluxes because of the small transfield thickness. An implication for the relationship between the gamma-ray and the spin-down fluxes is discussed.

Subject headings: gamma rays: observations — gamma rays: theory — magnetic fields — methods: analytical — pulsars: individual (Crab)

Online material: color figures

1. INTRODUCTION

The launch of the Fermi Gamma-Ray Space Telescope will soon open a new era for the studies of rotation-powered pulsars. The unprecedented sensitivity and spectral resolution of the Large Area Telescope (LAT) aboard Fermi will allow for detailed studies of particle acceleration and radiation in rotating neutron star (NS) magnetospheres. To make the best use of the power of LAT observations, we need the most sophisticated model with minimum assumptions.

In all the pulsar emission models, $e^-$’s and/or $e^+$’s are accelerated by the magnetic-field-aligned electric field, $E_b$, to radiate photons in the open zone (Fig. 1 in Hirotani 2008) mainly via the synchro-curvature process. In polar cap (PC) models, emission takes place within several NS radii above a PC surface (Arons & Scharlemann 1979; Daugherty & Harding 1982, 1996). However, such a low-altitude emission predicts too small beam size to produce the observed wide pulse profiles. Therefore, extending the original idea by Arons (1983), Muslimov & Harding (2003, 2004a, 2004b) and Dyks et al. (2004) sought the possibility of a wide hollow cone of high-energy radiation due to the flaring of field lines. They proposed the slot-gap (SG) model, in which emission takes place along the last-open field lines. Recently, Harding et al. (2008, hereafter HSDF08) demonstrated that the SG model reasonably reproduces the Crab pulsar phase-resolved spectrum (Fierro et al. 1998; Kuiper et al. 2001; Nolan et al. 1993).

The outer-gap (OG) model gives an alternative possibility (Cheng et al. 1986a, 1986b; Romani 1996; Cheng & Zhang 1996; Hirotani 2006a, 2006b). It differs from the SG model in the following ways: (1) An OG extends between the null surface with $v_{\text{crit}}$, where the upper boundary and lower boundary of the outer gap are designated with $v_{\text{min}}$ and $v_{\text{max}}$, respectively. We further introduce the dimensionless gap transfield thickness, $h_g \equiv (\theta_{\text{max}} - \theta_{\text{min}})/\theta_{\text{max}}$. In this Letter, we assume that $h_g$ is constant for both $\phi$ and $s$ (distance along the field line) for simplicity.

The purpose here is to explore further the two models. We examine their common emission properties in § 2, and separately consider the OG and SG models in §§ 3 and 4. Section 5 is for discussion.

2. GAMMA-RAY FLUX

In this Letter, we concentrate on the phase-averaged spectrum of pulsar magnetospheric emissions, sacrificing the examinations of light curves and phase-resolved spectra. In this context, we can neglect the aberration of photon propagation directions and the time-of-flight delay due to different emission altitudes from the NS, because these two relativistic effects do not change the total number of photons to be detected. Although this thought experiment does not describe any realistic pulsar emissions, it significantly reduces the calculation of photon propagations and gives the correct phase-averaged spectrum.

To investigate the upper limit of photon fluxes, we neglect photon absorptions. Then the radiative transfer equation gives the specific intensity $I_r \approx 2b\gamma j_\gamma$, where $b \approx \gamma^{-1}$ denotes the emission beaming angle, $\gamma$ the Lorentz factor of $e^-$’s, $g_\gamma$ the local curvature radius of the magnetic field line, $j_\gamma$ the emission coefficient, and $2b\gamma g_\gamma$ the distance interval from which the photons are detected by the observer (Fig. 6.2 in Rybicki & Lightman 1979). Giving the emission coefficient as $j_\gamma \approx N(dP/d\nu)/(\pi b^2)$, we obtain

$$I_r \approx \frac{2}{\pi} \frac{\rho_n}{b} \frac{dP}{d\nu},$$

where $N$ denotes the spatial density of $e^-$’s or $e^+$’s, and $dP/d\nu$ the radiation power per particle.

At each magnetic azimuth $\phi$, on the PC surface, we parameterize the field-line footpoint with their magnetic colatitude $\theta$, measured from the magnetic axis. We define that the primary photons are emitted only along the field lines threading the PC surface with $\theta_{\text{min}} < \theta < \theta_{\text{max}}$, where the upper boundary and the last-open field lines are designated with $\theta_{\text{min}}$ and $\theta_{\text{max}}$, respectively. We further introduce the dimensionless gap transfield thickness, $h_g \equiv (\theta_{\text{max}} - \theta_{\text{min}})/\theta_{\text{max}}$. In this Letter, we assume that $h_g$ is constant for both $\phi$ and $s$ (distance along the field line) for simplicity.
Let us introduce the gap meridional thickness $\Delta z$ that represents the region between the last-open field line and the upper boundary measured perpendicularly to the field line. Then, the observer detects emissions from the magnetic-flux cross section $\Delta A \approx \Delta z \times 2br \sin \theta$, where $r \sin \theta$ refers to the distance from the magnetic axis, and $2b$ is the azimuthal full opening angle of the points from which the photons propagate toward the observer. For an aligned rotator, the azimuthal length from which the photons propagate toward the observer is given by $2br \sin \theta$; thus, $\Delta A = \Delta z \times 2br \sin \theta$ holds. For an oblique rotator, in the outer magnetosphere, toroidal expansion of the field line flux tubes is similar to an aligned case; thus, $\Delta A \approx \Delta z \times 2br \sin \theta$ approximately holds for a dipole field, where $\Delta z$ is the distance from the magnetic axis, and the quantity expresses approximately $\Delta z \times 2br \sin \theta$ for $\Delta A \approx \Delta z \times 2br \sin \theta$ holds for an oblique rotator. In the outer magnetosphere, toroidal expansion of the field line flux tubes is similar to an aligned case; thus, $\Delta A \approx \Delta z \times 2br \sin \theta$ holds. For an oblique rotator, in the outer magnetosphere, toroidal expansion of the field line flux tubes is similar to an aligned case; thus, $\Delta A \approx \Delta z \times 2br \sin \theta$ holds for an oblique rotator.

$$\Delta A \approx 2b_{\text{mc}}(r_{/\sigma_{\text{LC}}})^{3}$$ (2)

where $r$ denotes the distance from the NS center, $\theta$ the magnetic colatitude, $\sigma_{\text{mc}} = c/\Omega$ the light cylinder radius, $\Omega$ the NS angular frequency. The magnetic field strength $B$ is evaluated at the emission point and related with its surface value $B_{s}$ by $B_{s} / B_{r} = (r_{/r})^{3}(1 + 3 \cos^{2} \theta)^{1/2}$. The photon energy flux can be computed by

$$\nu F_{\nu} = \nu I_{\nu}(\Delta A/d^{2})$$ (3)

where $d$ is the distance to the pulsar. Substituting equations (1) and (2) into equation (3), we obtain

$$\nu F_{\nu} \approx \frac{4}{\pi} h_{\nu} \frac{\Omega B_{s} / (2\pi e c)}{n_{0} c^{2} \nu} \frac{d}{d} \left( \frac{r_{/r}}{\sigma_{\text{LC}}} \right)^{2} r_{/r}$$ (4)

where $e$ is the charge on the positron; the dimensionless particle density per magnetic flux tube, $n \equiv (2\pi e c/\Omega B_{s})N$, becomes approximately $\cos \alpha_{s}$, where $\alpha_{s}$ is the inclination with respect to the rotation axis. The quantity $\Omega B_{s} / (2\pi e c)$ expresses the typical Goldreich-Julian (GJ) particle number density at the PC surface (Goldreich & Julian 1969). It follows from equation (4) that the photon flux $\nu F_{\nu}$ does not depend on $b$.

For saturated $e^{-}$'s or $e^{+}$'s, electrostatic force balance, $eE_{i} = 2e^{2}\gamma^{4}(3g_{s})$, gives the terminal Lorentz factor

$$\gamma = 2.20 \times 10^{5} \Omega_{2}^{1/2} \left( \frac{\rho_{/0.5\sigma_{\text{LC}}}}{0.5\sigma_{\text{LC}}} \right)^{1/2} \left( \frac{E_{i}}{\sqrt{\text{V m}^{-1}}} \right)^{1/4}$$ (5)

where $\Omega_{2} \equiv \Omega/(10^{2} \text{rad s}^{-1})$ and $E_{i} \equiv -(B/B_{r}) \cdot \nabla \Psi$. The non-corotational potential $\Psi$ is given by the inhomogeneous part of the Maxwell equations,

$$-\nabla^{2} \Psi = 4\pi (\rho - \rho_{G}) = (2\Omega B_{s}/c) \kappa$$ (6)

where $\rho$ and $\rho_{G}$ denote the real and GJ charge densities, respectively, and $B_{s}$ the magnetic field component projected along the rotation axis. The effective charge density, $\rho_{\text{eff}} = \rho$, is parameterized by $\kappa$, which is a function of position. In the Newtonian limit, we would have $\kappa = 1$ for a vacuum gap while $\kappa < 1$ for a non-vacuum gap.

Since $e^{-}$'s are ultrarelativistic in the gap, the primary emission is dominated by the pure-curvature component. Thus,

$$\nu F_{\nu} = \frac{3\sqrt{3}}{4\pi} \frac{e^{2} \gamma^{4} \nu^{2}}{\rho^{2}} \int_{s}^{\infty} K_{0.5}(\xi) d\xi$$ (7)

peaks at $x = \nu / \nu_{e} = 1.318$ with the maximum value $x^{2} \int_{0}^{\infty} K_{0.5}(\xi) d\xi = 0.6826$, where $K_{0.5}$ is the modified Bessel function of $5/3$ order, and $\nu_{e} = (3/4\pi)(c/\gamma) / (\text{GJ})$ the characteristic frequency of the emission. The electrostatic force balance, $(2/3)e^{2}\gamma^{4}g_{s}^{2} = eE_{i}$, gives

$$\nu F_{\nu} = \frac{9\sqrt{3}}{8\pi} \nu E_{i} x^{2} \int_{s}^{\infty} K_{0.5}(\xi) d\xi$$ (8)

For a thin gap ($h_{\alpha} \ll 1$), equation (8) in Hirotani (2006a) gives

$$E_{i} \approx \frac{h_{\alpha}^{2}}{4} \left( \frac{r_{/r}}{\sigma_{\text{LC}}} \right)^{3} B_{s} \frac{\partial(-sB_{/B})}{\partial(s/\sigma_{\text{LC}})}$$ (9)

where $s$ is the distance along the field line. For the field lines curving away (or toward) the rotation axis, $1.0 < -\partial(sB_{/B}) / \partial(s/\sigma_{\text{LC}})$, $0.7 < \partial(sB_{/B}) / \partial(s/\sigma_{\text{LC}}) < 2.2$ is the typical range.

Substituting $B_{s} = 2\mu_{0} r_{/r}$ and $x^{2} \int_{0}^{\infty} K_{0.5}(\xi) d\xi = 0.68$, and combining equations (4), (8), and (9), we finally obtain the peak flux,

$$\left(\nu F_{\nu}\right)_{\text{peak}} \approx 0.0450 h_{\alpha}^{2} \kappa \frac{\mu^{2} \Omega^{4}}{c^{3}}$$ (10)

where $\mu$ denotes the magnetic dipole moment and

$$f = \frac{n_{0}}{0.7} \frac{\rho_{/0.5\sigma_{\text{LC}}}}{1.5k} \frac{1}{\gamma x^{2}} \frac{\partial(-sB_{/B})}{\partial(s/\sigma_{\text{LC}})}$$ (11)

is close to unity (see also Zhang & Cheng 2003). If we apply $\nu F_{\nu} = 2.36 \left(\nu F_{\nu}\right)_{\text{peak}}$, Note that the factor $\mu^{2} \Omega^{4} / (c^{3})$ is proportional to the spin-down flux ($\xi$ 5).

It should be emphasized that equation (10) estimates the upper limit of the phase-averaged flux. For example, for an axisymmetric gap in an aligned rotator ($\alpha_{s} = 0^{0}$), the phase-averaged flux equals equation (10), which is constant during the whole NS rotation. Since equation (10) holds particularly well for an aligned rotator, and since the phase-averaged flux decreases with increasing $\alpha_{s}$ (e.g., Muslimov & Harding 2003; Dyks et al. 2004), equation (10) gives the upper limit for general $\alpha_{s}$. The actual flux will be a few times less than equation (10), because only the field lines in a limited azimuthal range are active, and because the $\rho_{\text{eff}}$ at the PC surface decreases with increasing $\alpha_{s}$. In § 4, we confirm this by comparing with numerical results.

The difference between the OG and the SG models comes into equation (10) through $h_{\alpha}$, $\kappa$, and the assumed $\mu$. In §§ 3 and 4, we apply this equation to these two models, considering the brightest spin-down-flux pulsar, the Crab pulsar, assuming $d = 2$ kpc.
First, let us apply equation (10) to the OG models. Using the vacuum ($\kappa = 1.0$) OG models of Cheng et al. (2000), Takata et al. (2008), and Tang et al. (2008), which proposed $h_n \approx 0.11$ (i.e., $f \approx 0.11$ in their notation), we obtain

$$ (\nu F_n)_\text{peak} \approx 6.58 \times 10^{-4} f \frac{\mu_{30}}{3.8} \text{MeV s}^{-1} \text{cm}^{-2}, $$

with $E_i \approx 2.55 \times 10^8 \text{ V m}^{-1}$, $\gamma \approx 2.0 \times 10^7$, where $\mu_{30} \equiv \mu/(10^{30} \text{ G cm}^3)$. The $(\nu F_n)_\text{peak}$ flux peaks at $1.3 h_n \approx 4.1 \text{ GeV}$. These results are consistent with their vacuum OG models and with the observed phase-averaged spectrum, where the photon flux around GeV is dominated by the primary component rather than the reprocessed synchrotron-self-Compton one (Takata & Chang 2007). Specifically, the vacuum OG model is not self-consistent electrodynamicistically, because it assumes a vanishing charge density while it adopts the GJ flux for the actual flux in a (nearly) vacuum OG model of Cheng et al. (2000), Takata et al. (2008), and Tang et al. (2008), which proposed $h_n \approx 0.11$ (i.e., $f \approx 0.11$ in their notation), we obtain

$$ (\nu F_n)_\text{peak} \approx 6.02 \times 10^{-5} f \frac{\kappa}{0.22} \frac{\mu_{30}}{8} \text{MeV s}^{-1} \text{cm}^{-2}, $$

with $|E_i| \approx 2.4 \times 10^7 \text{ V m}^{-1}$ and $\gamma \approx 1.1 \times 10^7$.

If we adopt the same parameter set $h_n = 0.04$ and $\mu_{30} = 11$ as HSDF08, and if we adopt $|E_i| \approx 7 \times 10^6 \text{ V m}^{-1}$, which is derived from the potential drop $0.5 \times 1.3 \times 10^{13} \text{ V}$ in the higher altitudes SG (HSDF08), equations (4) and (8) give

$$ (\nu F_n)_\text{peak} \approx 1.9 \times 10^{-8} \text{MeV s}^{-1} \text{cm}^{-2}. $$

This analytical prediction can be confirmed by a numerical computation of the 3D SG model (Hirotani 2008). If we adopt the same parameter set as HSDF08 and if we adopt the same $E_i$ as their equation (4) for $\alpha_i = 45^\circ$, we obtain the photon map as Figure 1, which shows a caustic emission from the higher altitudes by virtue of the exclusion of the strong lower altitude emission. Specifying the observer’s viewing angle with respect to the rotation axis, and integrating over the entire NS rotation, we obtain the phase-averaged spectrum (Fig. 2), which lies much below the observed value. Since $|E_i| \approx 7 \times 10^6 \text{ V m}^{-1}$ gives $\gamma \approx 8.3 \times 10^6$ (eq. [5]), we interpret that HSDF08, who adopted $\gamma \approx 2 \times 10^7$, overestimated $(\nu F_n)_\text{peak}$ by the factor $(2/0.83)^4 \approx 33$.

Equation (14) could give the observed flux if $\mu_{30} > 17$. However, such a large $\mu_{30}$ is not allowed in recent analyses of force-free electrodynamics. For example, Spitkovsky (2006) derived $\dot{E} \approx 4.5 \dot{Q}_\Omega \approx (\mu^2 \Omega/c^3)(1 + \sin^2 \alpha_i)$, where $I$ denotes the NS moment of inertia and $\Omega$ the temporal derivative of $\Omega$. Imposing $I_{\text{fs}} = I(10^{45} \text{ g cm}^2) < 5$ (with $r_i = 16 \text{ km}$ and $M < 2.5 M_\odot$; e.g., Lattimer & Prakash 2000), we obtain $\mu_{30} < 3.10 \times I_{\text{fs}} < 7.07$ for the Crab. Thus, even HSDF08’s value, $\mu_{30} = 11$, may be a little too large. In short, equation (14) gives the conservative upper limit and the SG model can explain at most 20% of the observed flux.
5. DISCUSSION

If we assume that the spin down follows the dipole radiation formula, $E = 2\mu^2 \Omega^4 \sin^2 \alpha/(3c^3)$, equation (10) becomes

$$\dot{E} = \frac{\nu F_{\text{peak}}}{0.707 h_m^3 k} \frac{\dot{E}}{d^2 \sin^2 \alpha},$$  \hspace{1cm} (15)

where $\dot{E} d^2$ denotes the spin-down flux at the Sun. It was, therefore, natural that the largest spin-down-flux pulsars were preferentially detected with the Energetic Gamma Ray Experiment Telescope (EGRET). The same tendency can be predicted for Fermi.

For young pulsars, both $h_m$ and $k$ are less than unity (e.g., $h_m \approx 0.1$ and $k \approx 0.3$ for the Crab) owing to the copious pair production in the magnetosphere. As a result, $\int_0^\infty F_d d\nu$ becomes much less than $\dot{E} d^2$ as demonstrated by Hirotani (2008). For middle-aged pulsars, on the contrary, $h_m > 0.5$ and $k \approx 1$ holds (Hirotani et al. 2003), leading to an increasing ratio of $\int_0^\infty F_d d\nu/\dot{E} d^2$ with age.

To explain the observed relationship $\int_0^\infty F_d d\nu \propto (\dot{E} d^2)^{0.5}$ of pulsed $\gamma$-ray emissions (Thompson et al. 1994; Nel et al. 1996), it is essential to examine the evolution of $h_m$ with age. From equation (10), we can at least state that the index $\alpha$ defined by $\int_0^\infty F_d d\nu \propto (\dot{E} d^2)^{\alpha}$ becomes less than unity, because $h_m$ increases with decreasing $E$, as discussed just above. To examine the evolution of $h_m$, we must solve the screening of due to the discharge of the produced pairs in 3D pulsar magnetospheres. In subsequent papers, we shall look more carefully into this issue, by simultaneously solving equation (6), the Boltzmann equations for $e^\pm$'s, and the radiative transfer equation under minimum assumptions.

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