Symbol-Based Over-the-Air Digital Predistortion Using Reinforcement Learning

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Abstract—We propose an over-the-air digital predistortion optimization algorithm using reinforcement learning. Based on a symbol-based criterion, the algorithm minimizes the errors between downsampled messages at the receiver side. The algorithm does not require any knowledge about the underlying hardware or channel. For a generalized memory polynomial power amplifier and additive white Gaussian noise channel, we show that the proposed algorithm achieves performance improvements in terms of symbol error rate compared with an indirect learning architecture even when the latter is coupled with a full sampling rate ADC in the feedback path. Furthermore, it maintains a satisfactory adjacent channel power ratio.

I. INTRODUCTION

Digital predistortion (DPD) is a technique to linearize the nonlinear power amplifier (PA) in a radio frequency (RF) chain to achieve the best energy efficiency while maintaining the spectral mask requirements [1]. It is customary to implement DPD using parametric models. For simplicity, DPD parameters are mostly optimized at the transmitter side, which requires a feedback data acquisition path to collect the PA output signal [2]. To capture the full-band behavior of the PA for DPD optimization, high sampling rate feedback analog-to-digital converters (ADCs) in the feedback path are needed, which is challenging for wideband signals. High sampling rate ADCs add a huge cost, which increases linearly with the number of RF chains in massive multiple-input multiple-output (MIMO) [3], where each RF chain may require a separate feedback path [2].

To tackle the high cost problem of the feedback path, recent works have shifted toward low sampling rate ADC methods [4]–[7], where the feedback path is coupled with a low sampling rate ADC. These works focus on recovering the full-rate PA output signal from the undersampled output signal of the low-rate ADC. The recovered signal is then used for DPD optimization. To reduce the cost of the feedback path further, over-the-air (OTA) methods have become a promising solution for the DPD optimization [8]–[10]. Instead of using a feedback path, OTA methods utilize an observation receiver to acquire the PA output signal over the channel for DPD optimization. Such methods achieve promising linearization performance and cost savings compared with methods using the feedback path.

Several parametric models for DPD have been utilized, e.g., Volterra-series-based [4]–[11] or neural network (NN)-based models [12]–[15]. To optimize the models, the works [8]–[15] consider a sample-based criterion that minimizes the sample error between oversampled signals. This is optimal when the aim is to minimize the ACPR and normalized mean squared error (NMSE). When the aim is to minimize the symbol error rate (SER), however, this approach is not optimal, and a symbol-based criterion is more appropriate.

A symbol-based criterion has been used to optimize the constellation mapper and demapper in [16] and the pulse-shaping/matched filters in [17], [18] in an end-to-end manner via supervised learning (SL), but SL requires differential models for all hardware and channel, which limits the usage in real communication systems. Instead, reinforcement learning (RL)-based optimization of the constellation mapper and demapper has been presented in [19], [20], which does not require any models for hardware and channel. However, in [19], [20], the (de)mapper operates at the same symbol level as the optimization criterion, and no memory effects of hardware are considered. It requires a generalization for RL-based DPD optimization with a symbol-based criterion due to the data rate difference between DPD parameters and criterion, and memory effects in the PA. To the best of our knowledge, OTA DPD optimization using RL with a symbol-based criterion has not yet been addressed.

In this work, we propose a symbol-based DPD optimization algorithm using OTA observations. Instead of using a sample-based criterion that minimizes the error between oversampled signals, the proposed algorithm minimizes the cross-entropy between transmitted and received symbols. Using the policy gradient theorem [21], we generalize the work [19] by connecting the symbol-based criterion with the sample-based policy optimization, which allows to optimize DPD parameters at the sample level. For a generalized memory polynomial (GMP) PA and additive white Gaussian noise (AWGN) channel, we show that the proposed symbol-based DPD optimization algorithm achieves symbol error rate (SER) gains over the indirect learning architecture (ILA)-based DPD optimization algorithm [22] even with full-rate ADCs. Error spectrum results show that, although the DPD optimized by the symbol-based criterion focuses more on reducing the in-band errors (i.e., symbol errors), out-of-band errors are still at a

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The predistorted signal \( \hat{m} \) is obtained from the probability matrix \( \hat{P} = [\hat{p}_1, ..., \hat{p}_N] \in \mathbb{R}_+^{M \times N} \). The demapper can be implemented by an NN as in [16], [19], [20], which can be pretrained to have similar decoding performance as the maximum likelihood demapper.

Given a parametric DPD model \( f_{\text{g}_0} \) with parameters \( \theta_{\text{g}_0} \), our objective is to optimize \( \theta_{\text{DPD}} \) with respect to a given loss function \( L \),

\[
\theta_{\text{DPD}} = \arg\min_{\theta_{\text{g}_0}} L(\theta_{\text{DPD}}).
\]

### B. Indirect Learning Architecture

The ILA [22] indirectly optimizes the DPD parameters by estimating an inverse behavior of the PA as shown in Fig. 2. The optimization of the parameters of the distorser is conducted after the PA, and hence the distorser is referred to as the postdistorter. After the parameter optimization, the postdistorter is used as the predistorter, placed before the PA. To optimize the postdistorter, a feedback path is required for signal acquisition of the PA. Here we consider an ADC in the feedback path with a sampling rate \( F_{\text{ADC}} \). We denote the output of the ADC by \( \hat{x}^{\text{PA}} \). The parameters of the postdistorter are usually optimized by minimizing the mean squared error (MSE) between the postdistorter output signal \( \hat{x} \) and the PA input signal \( x \). In this case, the loss function \( L \) can be expressed as

\[
L(\theta_{\text{DPD}}) = \mathbb{E}_x \{||x - \hat{x}||^2\} = \mathbb{E}_x \{||x - f_{\text{g}_0}(\hat{x}^{n_{\text{PA}}})||^2\},
\]

where \( \hat{x}^{n_{\text{PA}} - K_1} = [\hat{x}_{n_{\text{PA}}}^{n_{\text{PA}}}, ..., \hat{x}_{n_{\text{PA}}}^{n_{\text{PA}} - K_1}] \) is the input of the postdistorter with memory \( K_1 \). Substituting (4) into (3), \( \theta_{\text{DPD}} \) is optimized by minimizing the sample difference between the postdistorter output \( \hat{x} \) and the PA output \( \hat{x}^{\text{PA}} \) collected by the feedback path, i.e., the optimization is based on a sample-based criterion.

### III. Proposed Symbol-Based DPD Optimization Algorithm

We propose to optimize the DPD based on a symbol-based criterion using OTA measurements. Specifically, the symbol-based criterion minimizes the cross-entropy between the transmitted and received messages \( m \) and \( \hat{m} \).
A. Symbol-Based Criterion

To implement the symbol-based criterion for DPD optimization, we consider a symbol-based cross-entropy loss function, which defines the transmitted message sequence \( m \) and received message probabilities \( P \) as

\[
L_\text{DPD}(\theta) = -E_m \left\{ \log (\hat{P}_m|m_n) \right\},
\]

where the loss function between the transmitted message \( m_n \) and the corresponding vector of probabilities \( \hat{P} \), denoted by \( l_n \), referred to as the cross-entropy per-example loss.

B. Supervised Learning

Assuming that all blocks in the communication system are differentiable, the derivatives of the loss function at the receiver side with respect to any trainable parameter in the system can be analytically calculated using the chain rule. Thus, \( \theta_\text{DPD} \) can be updated through back-propagation via a mini-batch stochastic gradient descent (SGD) algorithm as

\[
\theta_{j+1} = \theta_j - \eta \nabla_{\theta} L(\theta_j),
\]

where \( \eta > 0 \) denotes the learning rate and \( \nabla_{\theta} L(\theta_j) \) is the derivative of \( L \) with respect to \( \theta_j \) at step \( j \).

However, in a real communication system, most of the blocks are non-differentiable, and thus it is infeasible to apply the chain rule to calculate \( \nabla_{\theta} L \). Although we can circumvent this problem using surrogate parametric models of the hardware components, e.g., pretrained PA model in [23], it is cumbersome to pretrain such models, and the performance highly depends on the model accuracy.

C. Reinforcement Learning

RL is defined as a learning process of how an agent takes actions in an environment to minimize a given loss [21]. The optimization of the DPD parameters can be viewed through the lens of a RL problem. The DPD acts as an agent that takes actions following a policy, which is optimized to minimize the loss \( L \). RL has already been used in the context of communications, e.g., to optimize the constellation mapper and demapper [19, 20]. Here, we consider a different scenario with more components, which raises the problem of how to optimize the DPD parameters \( \theta_\text{DPD} \) at the sample-based using a loss function \( L \) at the symbol-based.

As shown in Fig. 3, we consider a Gaussian policy \( \pi(\tilde{x}|x) \) that converts the deterministic actions \( x \) to stochastic actions \( \tilde{x} \), which enables the exploration of possible actions. For an arbitrary action \( x_n \), we consider an independent Gaussian policy \( \pi(\tilde{x}_n|x_n) \), which generates output \( \tilde{x}_n \) by adding a Gaussian perturbation \( w \sim \mathcal{CN}(0, \sigma_n^2) \) as

\[
\tilde{x}_n = \sqrt{1 - \sigma_n^2} x_n + w,
\]

where \( \sigma_n^2 \) is the variance of the perturbation, which is a hyper-parameter that is fixed during the training. Thus, the policy \( \pi(\tilde{x}_n|x_n) \) is the probability density function (PDF) of a complex Gaussian variable with mean \( \sqrt{1 - \sigma_n^2} x_n \) and variance \( \sigma_n^2 \).

\[
\pi(\tilde{x}_n|x_n) \propto \exp \left( -\frac{\left| \tilde{x}_n - \sqrt{1 - \sigma_n^2} f_{\theta_\text{DPD}}(u_{n:n-K_1}) \right|^2}{\sigma_n^2} \right).
\]

Based on the received observation \( y_n \), the receiver can compute the corresponding per-examiple loss \( l_n \) by \( l_n \). The loss \( l_n \) is related to a subset of the entire sequence \( x \) because of the memory effects of the PA and convolution operation in the matched filtering. Denote this subset by \( x(G) = \{x_{n-R-G}, \ldots, x_{n+R+G}\} \), where \( G \) denotes the number of signals being related. Because of the convolution operation in the pulse shaping, the subset \( x(G) \) depends on a subset of the messages in \( m \), denoted by \( m(G) = \{m_{n-R-G}, \ldots, m_{n+R+G}\} \). Similarly, we can define \( \tilde{x}(G) = \{\tilde{x}_{n-R-G}, \ldots, \tilde{x}_{n+R+G}\} \) and \( y(G) = \{y_{n-R-G}, \ldots, y_{n+R+G}\} \).

The objective of the DPD agent is to minimize the loss function \( L(\theta_\text{DPD}) \), defined as

\[
L(\theta_\text{DPD}) = \mathbb{E}_{m,x,y} \{ l_n \},
\]

where \( l_n \) is \( l_n \) (see (5)). In order to minimize the loss function, we compute the gradient with respect to \( \theta_\text{DPD} \) according to the following proposition.

Proposition 1: The gradient of \( L(\theta_\text{DPD}) \) with respect to \( \theta_\text{DPD} \) is approximated by

\[
\nabla_{\theta_\text{DPD}} L(\theta_\text{DPD}) \approx \frac{2}{N} \left[ \frac{\sum_{n=1}^{N} l_n \sum_{g=-G}^{G} \left( \tilde{x}_n - \sqrt{1 - \sigma_n^2} f_{\theta_\text{DPD}}(u_{n:n-K_1}) \right) \nabla_{\theta_\text{DPD}} f_{\theta_\text{DPD}}(u_{n:n-K_1})}{\sigma_n^2} \right] \times \nabla_{\theta_\text{DPD}} f_{\theta_\text{DPD}}(u_{n:n-K_1}.
\]
Algorithm 1: Symbol-based optimization for $\theta_{DPD}$.

Input: $N, N_b, \sigma_n^2, R, G$

1: for A number of iterations $N_b$ do
2: Messages to symbols: $m \rightarrow u^{ym}$
3: Upsampled and pulse-shaped with an upsampling rate $R$: $u^{ym} \rightarrow u^{up} \rightarrow u$
4: DPD: $f_{\theta_{DPD}}(u) \rightarrow x$ via (1)
5: Policy: $\pi([x]) \rightarrow \hat{x}$ via (7)
6: PA: $f_{PA}(\hat{x}) \rightarrow x_{PA}$ via (2)
7: Channel: $P(x_{PA}) \rightarrow y$
8: Matched filtered and downsampled with a downsampling rate $R$: $y \rightarrow \hat{u}^{up} \rightarrow \hat{u}^{ym}$
9: Symbols to messages: $\hat{u}^{ym} \rightarrow m$
10: Per-example losses: $l_n$
11: Update $\theta_{DPD}$: SGD($\theta_{DPD}, L$) $\rightarrow \theta_{DPD}$ via (6) and (10)
12: end for
13: Remove policy $\pi(\cdot)$

Proof: Exploiting the policy gradient theorem [21] and using the fact that $\nabla \log(\pi) = (\nabla \pi) / \pi$, we can write

$$\nabla_{\theta_{DPD}} \mathcal{L}(\theta_{DPD}) = E_{m, x, y} \{l_n \nabla_{\theta_{DPD}} \log(\pi(\hat{x}|x))\}. \quad (11)$$

The loss $l_n$ is related to a fraction of $x$ consisting of $G$ signals. Restricting (11) to this subset of signals, we can approximate it as

$$E_{m, \hat{x}, y} \{l_n \nabla_{\theta_{DPD}} \log(\pi(\hat{x}|x))\} = E_{m, \hat{x}, y} \{L_n \nabla_{\theta_{DPD}} \log(\prod_{g=-G}^{G} \pi(\hat{x}_G|x_{G}))\}, \quad (12)$$

where (12) follows as the conditional probability $\pi(\hat{x}|x)$ reduces to the product of conditional probabilities $\pi(\hat{x}_G|x_G)$ due to the independence of each action. Now, using the fact that the product of logarithms can be written as the logarithm of a sum and approximating the expectation by the average of $N$ (correlated) samples from the underlying distribution $p(m, \hat{x}, y)$, we obtain

$$\nabla_{\theta_{DPD}} \mathcal{L}(\theta_{DPD}) \approx \frac{1}{N} \sum_{n=1}^{N} l_n \sum_{g=-G}^{G} \nabla_{\theta_{DPD}} \log(\pi(\hat{x}_G|x_G)). \quad (13)$$

From (8), it follows that

$$\nabla_{\theta_{DPD}} \log(\pi(\hat{x}_G|x_G)) = \frac{2\sqrt{1-\sigma_n^2}}{\sigma_n^2} \frac{1}{\sigma_n^2} f_{\theta_{DPD}}(u_{n:n-K_c}) \nabla_{\theta_{DPD}} f_{\theta_{DPD}}(\cdot). \quad (14)$$

Substituting (14) into (13) completes the proof.

The approximation of the gradient of the loss function $\mathcal{L}$ with respect to $\theta_{DPD}$ in Proposition 1 can be used to optimize $\theta_{DPD}$ via any SGD-based algorithm as in (6).

The details of the symbol-based DPD optimization procedure of $\theta_{DPD}$ are given in Algorithm 1. First, each batch of $N$ messages, $m$, is generated and transformed to a sequence of symbols $u^{ym}$, then upsampled and pulse-shaped to $u$ with an upsampling rate $R$. After the DPD model $f_{\theta_{DPD}}(\cdot)$, the Gaussian policy is applied by adding a perturbation $w$ to generate exploration samples $\hat{x}$ as in (1). These samples are sent through the PA and channel. The output of the channel, $y$, is transformed to $\hat{u}^{ym}$ by the matched filter, downsampled with a downsampling rate $R$, and eventually decoded to $\hat{m}$. Then, the cross-entropy per-example losses, $l_n$, are calculated, which are assumed to be known at the transmitter via a reliable feedback channel [19], [20], [24]. Finally, $\theta_{DPD}$ is updated by an SGD-based algorithm as in (6), where $\nabla_{\theta_{DPD}} \mathcal{L}(\theta_{DPD})$ is computed using (10). The whole procedure is iterated for a number of iterations $N_b$, and the policy $\pi(\cdot)$ is removed.

IV. NUMERICAL RESULTS

A. Setup

1) Parameters: We consider a GMP model as the PA with nonlinear orders $K_a = K_b = K_c = 7$, memory lengths $L_a = L_b = L_c = 3$, and cross-term lengths $M_b = M_c = 1$ [11] Eq. (23)]. The corresponding parameters are estimated from the measurements of the RF WebLab using the ILA and a 55 MHz signal. The measured saturation point and measurement noise standard deviation of the PA are 20.9 V ($\approx 36.4$ dBm) and 0.053 V. We consider a 50 $\Omega$ load impedance. The remaining parameters are given in Table I. The number of training messages in $m$ for each batch is $N = 1024$. The optimizer for gradient descent is Adam [25] with a learning rate of 0.001.

TABLE I: Parameter setup

| $M$ | $R$ | $N$ | $G$ | $\sigma_n^2$ | $\sigma_{\text{th}}$ [V] |
|-----|-----|-----|-----|---------------|------------------|
| 64  | 4   | 1024| 3   | 0.08          | 0.3              |

Fig. 4: Structure of the DPD model $f_{\theta_{DPD}}$ using R2TDNN [15], which is placed before the policy block. The normalization block aligns the average output power of DPD to be the same as its input.

Fig. 5: Structure of the NN-based 64 QAM decoder, whose parameters are pretrained and frozen during the training of DPD.
We consider a simulated ADC in the feedback path of the work \[15\] and shows to outperform many other NNs and GMP for DPD in terms of complexity versus performance. We consider the same parameter settings (i.e., nonlinear order, memory length, and cross-term length) for the GMP DPD. The specific structure of R2TDNN along with policy \(\pi(\cdot)\) is shown in Fig. 4. The block \(\mathbb{C}_2\mathbb{R}\) transforms the complex-valued signal, \(u\), to a real-valued signal. We consider two hidden layers with 12 neurons each, and 3 input memory length as the same as the PA model. The output of the linear layer is added with the input via the residual connection and then normalized by the normalization layer, which ensures that the average output power of the DPD is the same as its average input power.

We consider an NN-based \(M = 64\) QAM decoder as shown in Fig. 5 which are trained to have similar performance as the maximum likelihood decoder. The learned detector is frozen during the training of the DPD. Specifically, the softmax layer outputs a probability vector over \(M\) messages, where the largest probability represents the predicted message.

### B. Simulation results

#### 1) SER versus average PA output power:

Fig. 6 shows the testing SER results versus the average PA output power for the cases of no DPD, ILA-optimized NN DPD \([15]\) with under-sampling (\(F_{\text{ADC}}^{\text{ILA}} = 110\) and 220 MHz) and full-rate (\(F_{\text{ADC}}^{\text{ILA}} = 550\) MHz) ADCs in the feedback path, ILA-optimized GMP DPD with full-rate ADC \([11]\), the proposed RL-optimized DPD, the case with linear-clipping PA \([26]\) and the theoretical SER bound of 64 QAM, where the average energy per bit is converted to the average PA output power considering a distortion-free PA. The RL-optimized DPD is optimized using the cross-entropy loss function. Note that the policy \(\pi(\cdot)\) is removed once the training ends, and the input of the PA becomes \(x\). The number of QAM symbols used to calculate the SER is \(10^3\).

We observe that the SER curves exhibit two different behaviors. When the average PA output power is below 29 dBm, most of the PA input signal \(x\) is in the linear region of the PA, and the SER of all scenarios with DPD improve quickly with increasing average PA output power \(P_{\text{out}}\). The proposed RL-optimized NN DPD achieves similar SER as the ILA-optimized NN and GMP DPDs with full-rate ADCs, and has substantial SER gain over the ILA-optimized NN DPDs with undersampling ADCs. As the average PA output power increases above 29 dBm, \(x\) starts to be clipped by the saturation region of the PA. While the clipping effect makes the SER of all DPD cases degrade rapidly, we note that the RL-optimized DPD exhibits considerable SER gains over other ILA-optimized DPDs at the highly nonlinear region of the PA, i.e., \(30\) dBm < \(P_{\text{out}} < 32\) dBm. This indicates the advantage of the symbol-based criterion for DPD optimization over the sample-based criterion in terms of SER. Overall, the results prove the effectiveness of the sample-based DPD optimization using a symbol-based criterion without knowing the hardware and channel for a PA with memory and the AWGN channel. The SER gap between all the DPD cases and the linear-clipping case may come from some residual distortions due to irreversible nonlinearity.

#### 2) In-band and Out-of-band Errors:

Fig. 7 shows the error spectrum (i.e., the power spectral density (PSD) of the difference between the real and ideal PA output signals) of schemes in Fig. 6 at the average PA output power \(P_{\text{out}} = 30.2\) dBm. The corresponding NMSE and ACPR results are presented in Table 11.

Due to the aliasing and band-limiting effects of the undersampling ADC, the linearization performance of the ILA-NN DPD with \(F_{\text{ADC}}^{\text{ILA}} = 110\) MHz is affected severely, exhibiting large in-band and out-of-band errors compared with DPDs of full-rate ADCs and even larger out-of-band
which is an attractive feature in practice. Exploiting the policy of DPD achieves SER gains over the ILA-based DPD even the real PA over the AWGN channel. The proposed RL-NN-based the sample-based policy optimization. The proposed algorithm optimizes DPD parameters using a symbol-based criterion at the receiver side instead of a sample-based criterion at the ADC. As expected, the in-band spectral error of the RL-NN-based DPD still maintains a satisfactory ACPR (−41.3 dBc) considering that the lower bound is −41.3 dBc.

V. CONCLUSION

We proposed a novel DPD optimization algorithm that optimizes DPD parameters using a symbol-based criterion at the receiver side instead of a sample-based criterion at the transmitter side, which avoids the cumbersome feedback path in the transmitter. The proposed optimization algorithm, based on RL, does not require a model for the hardware or channel, which is an attractive feature in practice. Exploiting the policy gradient theorem, we connect the symbol-based criterion with the sample-based policy optimization. The proposed algorithm is verified by simulation results for a GMP PA modeled from a real PA over the AWGN channel. The proposed RL-NN-based DPD achieves SER gains over the ILA-based DPD even the latter uses a full-rate ADC in the feedback path, while it also maintains satisfactory out-of-band errors. It is expected to see a performance improvement for a real PA as the performance of ILA is limited by the nature of its identification process. Some limitations of this work are the AWGN channel and hardware models, which can be generalized to a more realistic scenario considering more realistic channels and more hardware impairments (e.g., quadrature imbalance).

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