The Dynamic and Optimized Modeling Method of Secondary Winding for Transformer

Xiudong Xu, Rui Li, Liang Zhao, Zhiyuan Shen, Jie Cheng, Bo Zeng, Yu Zhang and Pengcheng Gao
Northwest Institute of Nuclear Technology, Xi’an, Shaanxi, 710024, China
xuxiudong@nint.ac.cn

Abstract. In order to analyze dynamic characteristics of secondary winding for Transformer, one joint interface dynamic modeling and optimized methodology is provided in this paper. A layer of equivalent material between secondary winding and clamp is established, which can simulate stiffness and damp in the joint interface. The initial elastic modulus, poisson ratio and density of equivalent material are deduced from Hertz contact theory and fractal theory. The final property parameters of equivalent material are obtained by using optimized model. Simulation and experiments results show that the dynamic and optimized model can simulate experimental results exactly. The relative errors between the simulation frequencies of new model and experimental frequencies are controlled within 2%.

Key words. Secondary winding, Transformer, dynamic and optimized model, equivalent material, joint interface

1. Introduction
There are many influence factors for dynamic characteristics of complicated machine, such as structure, material, stiffness of interface, damp of interface etc. It is said that dynamic characteristics can be improved by designing the styles of structure. Besides, the characteristics of interface are the important factors, which influence on dynamics. The stiffness and damp of interface are the most important properties.

There are about three modeling methods for interface of two mechanical parts: rigid link for two parts, stiffness and damp elements modeling, non-linear modeling method. The stiffness is infinite for rigid link method, which can not simulate truly for two mechanical parts. Therefore, many researchers focus on other two methods. Multi-spring and damp method is given between two metal joint interfaces by Namazi etc, which also study dynamic characteristics for joint interface [1]. It proposes FEM joint interface model for zero thickness by Mayer etc, which analyze the non-linear response [2]. Mottershead identifies dynamic parameters through comparing input accelerations and output response[3]. Fang identifies friction damp between fixed part and movement part. Besides, he analyzes system stabilization for damp [4]. Unfortunately, these researches can not simulate accurately for non-linear dynamic problems. Therefore, one dynamic modeling field can be proposed based on Hertz contact theories [5]. GW[6] model and MB[7] model are classical research methods among Hertz contact theory and fractal theory. Tian Hongliang gives new dynamic modeling method based on above researches [8]. He converts the dynamic modeling problem to parameters identification problem, and gives calculated method for elastic modulus and poisson ratio of virtual material. Thus, this method must obtain relative surface parameters.
Transformers have been applied widely in the field of electric power. In different and complex applications of electrical Transformer, mechanical vibration may destroy the Transformer. Generally, Transformer contains about primary winding, secondary winding, inner cylinder, outer cylinder and insulator etc, which are shown in Figure 1. The secondary winding can be destroyed more easily. Therefore, we focus on dynamic modeling of secondary winding in this paper. The secondary winding contains two parts, which are basic cone and cable. The cable is enlaced around the basic cone. The dynamic characteristics of secondary winding should be analyzed and understood in order to avoid cable loosening from basic cone.

![Figure 1. Schematic diagram of a Transformer](image1.png)

![Figure 2. The diagram of equivalent material](image2.png)

Generally, the model of basic cone is regarded as important for secondary winding. Actually, the joint interface between secondary winding and clamp can impact on dynamic characteristic directly according to former researches. Therefore, one dynamic and optimized model of joint interface is presented for secondary winding in this paper. It is assumed that one equivalent material layer is existed between outer magnetic cores and secondary winding for Transformer, which is shown in Figure 2. We should get initial parameters of equivalent material firstly through calculated model. And the final parameters can be obtained by comparing simulation and experimental response of secondary winding. It is not necessary to obtain accurate surface parameters, which is the advantage for this methodology.

2. Simplified structure model of secondary winding

![Figure 3. The simulation model of the secondary winding](image3.png)

The actual model of secondary winding is shown in Figure 1. The secondary winding of Transformer contains: inner and outer cylinder, secondary and primary windings, two insulators. The inner cyliner are laid between with two insulators. The larger side of secondary winding connects to outer cylinder with six screw hole. The smaller side of secondary winding contacts inner cylinder for circle mode. The actual model can be converted to simplified structure model in Figure 3. There are secondary
winding and clamp in the simplified model, which can simulate actual situation for larger side of secondary winding. It is assumed that one layer of equivalent material exists between outer joint interface of secondary winding with inner joint interface of clamp. It is seen that this simplified model is one worse cantilever. Therefore, the dynamic characteristics of secondary winding can be checked fully in this condition.

3. Dynamic and optimized methodology for secondary winding

In the simplified model, the joint interface between secondary winding with clamp may be considered equivalent material in the same cross area. A series of analytic solutions of elastic modulus \( E \), poisson ratio \( \mu \), density \( \rho \) and thickness \( h \) could be obtained by interface dynamic model, which show in (1)–(4) respectively. The equivalent material is rigidly connected with secondary winding with clamp situated in both side of joint interface. The parameters of \( E \), \( \mu \) and \( \rho \) are the property of the equivalent material. And the \( h \) is the physical parameter of the material layer.

\[
E = f_1(E_1, E_2, \mu_1, \mu_2, R_{a1}, R_{a2}, P) \tag{1}
\]
\[
\mu = f_2(E_1, E_2, \mu_1, \mu_2, R_{a1}, R_{a2}, P) \tag{2}
\]
\[
h = f_3(h_1', h_2', R_{a1}, R_{a2}) \tag{3}
\]
\[
\rho = f_4(\rho_1, \rho_2) \tag{4}
\]

In which \( f_1 \) is the mathematics model of \( E \), \( f_2 \) is the mathematics model of \( \mu \), \( f_3 \) is the mathematics model of \( h \), \( f_4 \) is the mathematics model of \( \rho \). And the \( E_1 \) is the elastic modulus of secondary winding, \( E_2 \) is the elastic modulus of clamp, \( \mu_1 \) is the poisson ratio of secondary winding, \( \mu_2 \) is the poisson ratio of clamp, \( R_{a1} \) is the roughness of outer joint interface for secondary winding, \( R_{a2} \) is the roughness of inner joint interface for clamp, \( P \) is the pressure between secondary winding with clamp, \( \rho_1 \) is the density of secondary winding, \( \rho_2 \) is the density of clamp, \( h_1' \) is the largest height of microcontact for secondary winding, \( h_2' \) is the largest height of microcontact for clamp. The mathematics model \( f_1 \), \( f_2 \) and \( f_4 \) are provided directly respectively via Hertz contact theory and fractal theory [5], [8].

\[
E = \frac{\sqrt{2}}{6\pi\sqrt{\pi}}DE'EG^{(1-D)}\phi^{\frac{1-D}{D}}\frac{1}{a_c'^{-2}-(a_c'^{-1})^2} \tag{5}
\]
\[
\mu = \frac{(1+\mu')E}{G_x'E'} \tag{6}
\]
\[
\rho = \frac{\rho_1h_1+\rho_2h_2}{h_1+h_2} \tag{7}
\]

Where \( D \) is the fractal dimension of surface profile, \( E' \) is the equivalent elastic modulus of two surfaces, \( G \) is the fractal roughness parameter. \( \phi \) is the region expanding coefficient, which can be calculated by \( D \). \( a_c' \) is the area of largest microcontact, \( a_c' \) is the area of critical regimes. \( E' \) is the equivalent elastic modulus of material, which can be given by (8). \( \mu' \) is the equivalent poisson ratio of material, which can be given by (9), \( G_x' \) is the equivalent shear modulus of material [9].

\[
E' = \frac{E_1E_2}{E_1(1-\mu_1^2)+E_2(1-\mu_2^2)} \tag{8}
\]
\[
\mu' = \frac{E'G_x'(2-\mu_1)+E'G_x'(2-\mu_2)}{2G_x'G_2} - 1 \tag{9}
\]

Where \( G_1 \) and \( G_2 \) are the shear modulus of secondary winding and clamp, separately.
Generally, $h$ is chosen as 1mm [8]. And the density $\rho$ is obtained by (7). It is concluded that $\rho$ is less influence on dynamic characteristics in the former researches. Otherwise, the precise $E$ and $\mu$ are decided by interface parameters $D$ and $G$. In this paper, we need only imprecise $E_0$ and $\mu_0$, which can be seen as initial parameters of equivalent material.

It is assumed that $q_i^{\text{end}} (i=1,2,3,4,5)$ are experimental frequency of secondary winding, in which $i$ stands for phase index. The simulation frequency of secondary winding are $q_i (i=1,2,3,4,5)$. And the optimized model is shown as (10).

$$\min[g(x)] = \sum_{i=1,2,3,4,5} \left| q_i - q_i^{\text{end}} \right| n_i$$ (10)

Where, $x=[E \ \mu]^T$ are optimized variable, $n_i (i=1,2,3,4,5)$ are coefficients. It is seen that PATRAN software is used to optimize relative parameters, and the optimized algorithm is GA.

4. Dynamic experiment and simulations result

The material of secondary winding is PI, whose elastic modulus is 3.2Gpa, poisson ratio is 0.36, density is 1300 kg/m$^3$. The material of clamp is Q235, whose elastic modulus is 205Gpa, poisson ratio is 0.29, and density is 7850 kg/m$^3$.

![Figure 4. The diagram of vibration experimental system.](image)

![Figure 5. Comparative response of x-direction for 1g z-vibration.](image)

![Figure 6. Comparative response of y-direction for 1g z-vibration.](image)

![Figure 7. Comparative response of z-direction for 1g z-vibration.](image)
The dynamic experiment of secondary winding has been carried on, which is shown in Figure 4. We select three control points, which laid on clamp. The measurement point is fixed on the smaller side of secondary winding. The experiments of x-direction and z-direction 1g sine sweep frequency have been implemented. In order to show typical frequency, we select z-direction 1g sweep experiment, which is seen as Figure 5~Figure 7. And the final comparison between experiment and simulation are detailed by Table 1. The elastic modulus of equivalent material becomes stably from 20.21Mpa to 48.57Mpa, and poission ratio becomes steadily from 0.362 to 0.1811. The errors between simulation and experiment are smaller than the model for initial parameter value.

| Item              | The first frequency(Hz) | The second frequency(Hz) | The third frequency(Hz) | The fourth frequency(Hz) | The fifth frequency(Hz) |
|-------------------|-------------------------|--------------------------|-------------------------|--------------------------|-------------------------|
| Vibration         | 66.45                   | 102.27                   | 124.37                  | 147.35                   | 161.95                  |
| experiment        |                         |                          |                         |                          |                         |
| Final value errors| 0.45%                   | 0.22%                    | 1.83%                   | 1.32%                    | 1.79%                   |

Parameters:
Initial value: elastic modulus (20.21Mpa) poission ratio (0.362)
Final value: elastic modulus (48.57Mpa) poission ratio (0.1811)

5. Conclusions
The paper interests in the problems of joint interface modeling for secondary winding. Therefore, one joint interface modeling and optimized methodology is provided. This method can simulate the damp and stiffness between secondary winding and other part. The exact acceleration responses are obtained by using this simulation method. The simulation results show that the response curves are similar with experiment curves. The frequency errors can be controller within 2%. It can improve the simulation veracity for secondary winding.

References
[1] Namazi M, Altintas Y, Abe T, et al. 2009 Modeling and identification of tool holder-spindle interface dynamics International Journal of Machine tool and Manufacture 47 pp 1333-1341.
[2] Mayer M H, Gaul L 2007 Segment-to-Segment contact elements for modelling joint interfaces in finite element analysis Mechanical Systems and Signal Processing 21 pp 724-734.
[3] Mottershead J E, Stanway R 1986 Identification of structural vibration parameters by using a frequency domain filter Journal of Sound and Vibration 109 pp 495-506.
[4] Fang B, Devor R E 2002 Influence of friction damping on workpiece-fixture system dynamics and machining stability Journal of Manufacturing Science and Engineering 124 pp 226-233.
[5] Campbell P, Abhyankar S 1978 Fractals, forms, chance and dimension The Mathematical Intelligencer 1 pp 35-37.
[6] Fisher R 1953 Dispersion on a sphere Proceeding of the Royal Society of London. Series A. Mathematical and Physical Sciences 217 pp 295-305.
[7] Majumdar A, Bhushan B 1991 Fractal model of elastic-plastic contact between rough surfaces Journal of Tribology 113 pp 1-11.
[8] Tian Hongliang, Li Bin, Liu Hongqi, Mao Kuanmin, Peng Fangyu, Huang Xiaolei 2011 A New Method of Virtual Material Hypothesis-Based Dynamic Modeling on Fixed Joint Interface in Machine Tools International Journal of Machine Tools and Manufacture 51 pp 239-249.
[9] Ray Sudipto, Roy Chowdhury S K 2007 Prediction of Flash Temperature at the Contact Between Sliding Bodies with Nanoscale Surface Roughness ASME Journal of Tribology 129 pp 467-480.