HIGHLIGHTS

- This paper examines two Russian stock market volatility indices
- These indices are the RTSVX and the new RVI that has replaced it
- Daily data over the period 2010-2018 are used
- The two series are found to be mean-reverting
- This is true regardless of whether the errors are white noise or autocorrelated
- On the whole, it seems shocks do not have permanent effects on volatility in the Russian stock market.
Abstract

This paper applies a fractional integration framework to analyse the stochastic behaviour of two Russian stock market volatility indices (namely the originally created RTSVX and the new RVI that has replaced it) using daily data over the period 2010-2018. The empirical findings are consistent and imply in all cases that the two series are mean-reverting, i.e. they are not highly persistent and the effects of shocks disappear over time. This is true regardless of whether the errors are assumed to follow a white noise or autocorrelated process; this is confirmed by the rolling window estimation, and it holds for both subsamples, before and after the detected break. On the whole, it seems shocks do not have permanent effects on volatility in the Russian stock market.

Keywords: RTSVX, RVI, volatility, persistence, fractional integration, long memory

JEL Classification: C22, G12

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1. Introduction

Financial market instabilities have become more frequent and acute in the era of
globalisation (Bordo et al., 2001), and have raised concerns about the benefits of
traditional portfolio diversification strategies. Those involving instruments based on the
VIX volatility index (which is negatively correlated to equity returns) are thought to be
particularly effective during periods of market turmoil for tail risk hedging (Whaley,
1993). The VIX is especially attractive to investors with high skewness preferences
(Barberis and Huang, 2008). Unlike credit derivative instruments, the liquidity of VIX
derivatives improves during periods of markets turmoil, when investors are in search of
hedging instruments (Bahaji and Aberkane, 2016). The existing literature also shows the
diversification benefits of VIX exposures in institutional investment portfolios (Szado,
2009). In particular, a VIX short future exposure in a benchmark portfolio triggers a
positive expansion of the efficient frontier (Chen et al., 2011); moreover, the addition of
VIX futures to pension fund equity portfolios can significantly improve their in-sample
performance, whilst incorporating VIX instruments into long-only equity portfolios
significantly enhances Value-at-Risk optimisation (Briere et al., 2010).

A number of empirical papers have examined the features of the VIX,
specifically its information content (Canina and Figlewski, 1993; Fleming, 1998;
Christensen and Prabhala, 1998; Koopman et al, 2005; Becker, et al., 2009, Smales,
2014), importance and effectiveness (Whaley, 1993; Barberis and Huang, 2008; Bahaji
and Aberkane, 2016; Szado, 2009; Briere et al, 2010), statistical properties (Lee and
Ree, 2005), dynamic association and regime-switching behaviour (Baba and Sakurai,
2011), as well as the presence of a day-of-the-week effect (Qadan, 2013), and its
usefulness as a measure of investor sentiment (Brown and Cliff, 2004; Bandopadhyaya
and Jones, 2008) and/or risk aversion and market fear (Bekaert et al., 2013; Caporale et
al., 2018), and as a stock market indicator/barometer (Iso-Markku, 2009; Fernandes et al., 2014).

Most of the studies mentioned above focus on the developed economies. By contrast, the present paper provides new evidence for the VIX in an emerging economy such as Russia. Moreover, it considers both the old and the new VIX constructed for the Russian stock market and analyses in depth the statistical properties of both (long-range dependence, non-linearities and breaks) in a fractional integration framework.

Understanding the behaviour of the VIX is important because this index can be used as a predictor of stock returns and volatility, economic activity and financial instability. Further, it can be the basis of portfolio diversification strategies designed by domestic and international institutional investors. Specifically, the choice of the hedging effectiveness measure aimed at capturing the tail risk in the portfolio depends on the stochastic properties of the VIX. This is the motivation for the present study, which examines two different VIX measures (RTSVX and RVI) in a comparative framework in the case of Russia, a country for which very little evidence is available at present. The newly constructed RVI has replaced the originally created RTSVX in order to comply with the latest international financial industry standards and take into account feedback from market participants (see Section 2 for more details).

The layout of the paper is as follows. Section 2 provides background information on the Russian VIX, Section 3 outlines the empirical methodology, Section 4 describes the data and the empirical findings. Section 5 offers some concluding remarks.

2. The VIX in the Russian Stock Market

The idea of constructing a volatility index using option prices was first formulated at the time of the introduction of exchange trade index options in 1973. In subsequent years,
the original methodology of Gastineau (1977), Cox and Rubinstein (1985) and others was considerably developed. The first implied volatility index, the VIX, was introduced by the Chicago Board Options Exchange (CBOE) in 1993 and was based on the S&P 100 index. It aimed to measure market expectations of the short-term volatility implied by stock index option prices. Subsequently, similar indices have been constructed for many developed and emerging markets.

Russia, one of the most important emerging economies, first introduced a volatility index, named RTSVX (Russian Trading System Volatility Index) on 7 December 2010. It is an aggregate indicator of the performance of futures and options in the Russian market based on the volatility of the nearby and next option series for the RTS (Russian Trading System) Index futures (for further details see the Moscow Exchange website, https://www.moex.com/en/index/RVI). However, in late 2013, the Moscow Exchange decided to replace the RTSVX with a new Russian Volatility Index (RVI) to catch up with developments in international financial industry standards and in response to feedback from market agents; this was launched on 16 April 2014. The Moscow Exchange also decided to keep calculating the RTSVX until futures contracts on the index expired and to discontinue it from 12 December 2016 (RTSVX futures are not traded anymore, with RVI futures having being available instead to trade from June 2014).

The new RVI measures market expectation of the 30-day volatility, calculated from real prices of nearby and next RTS Index option series. In the previous RTSVX volatility index, a parameterised volatility smile was used to calculate continuous, theoretical Black-Scholes prices of the nearby and next RTS Index option series. The RVI is calculated in real time during both day and evening sessions (first values 19:00 –
23:50 MSK\(^1\) and then 10:00 – 18:45 MSK), and differs from the RTSVX in three main respects, i.e. it is discrete, it uses actual option prices over 15 strikes, and calculates the 30-day volatility. Specifically, it is defined as follows:

\[
RVI = \left\{ 100 \times \sqrt{\frac{T_{30}}{T_{365}}} \left[ T_1 \times \sigma_1^2 + \frac{T_2 - T_{30}}{T_2 - T_1} \right] + T_2 \times \sigma_2^2 + \frac{T_{30} - T_1}{T_2 - T_1} \right\},
\]

(1)

where \(T_1\) and \(T_2\) are the time to expiration expressed as a fraction of a year consisting of 365 days for the nearby and far option series respectively; \(T_{30}\) and \(T_{365}\) stand for 30 and 365 days respectively, expressed as a fraction of a year; \(\sigma_1^2\) and \(\sigma_2^2\) are the variance of the nearby and next option series respectively.

There is only a limited number of studies on the Russian stock market, possibly because of the lack of long series of reliable data. As Mirkin and Lebedeva (2006) point out, Russian companies are more dependent on debt financing than equity financing since only about 6 percent of listed companies are traded in the largest Russian exchange; ownership in the equity market is highly concentrated; the Russian bond and equity markets are easily accessible to international investors and the corporate bond market has proven to be highly profitable without any defaults. Russian financial markets are rather stable and integrated in terms of international capital flows (Peresetsky and Ivanter, 2000); the degree of financial liberalisation in Russia determines the strength of its international integration (Hayo and Kutan, 2005); since the Russian stock market is not cointegrated with the US one investors should focus on the Russian VIX for predicting Russian stock market returns (Mariničevaitė & Ražauskaitė, 2015); in general, they have become more knowledgeable about the effects

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\(^1\) Moscow Standard Time
of the VIX on stock price indices for developed and emerging economies (Natarajan et al., 2014).1

3. Methodology

The concept of long memory was originally introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981), and allows the differencing parameter required to make a series stationary I(0) to take a fractional value. Assuming that \( u_t \) is a covariance-stationary I(0) process (denoted as \( u_t \approx I(0) \)) with a spectral density function that is positive and bounded at all frequencies, \( x_t \) is said to be integrated of order \( d \) (and denoted as \( x_t \approx I(d) \)), if it can be represented as

\[
(1 - L)^d x_t = u_t, \quad A = 0, \pm 1, \ldots, \tag{2}
\]

with \( x_t = 0 \) for \( t \leq 0 \), and where \( L \) is the lag operator \( (Lx_t = x_{t-1}) \) and \( d \) can be any real value. Then, \( u_t \) is I(0) and \( x_t \) is I(d), and \( d \) measures the persistence of the series. In such a case, one can use the following Binomial expansion for the polynomial on the left-hand side of (2) for all real \( d \):

\[
(1 - L)^d = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{d} \left(-1\right)^j \frac{d!}{j!(d-j)!} L^j = 1 - d \ L + \frac{d(d-1)}{2} L^2 - \ldots ,
\]

and thus, noting that \( L^2 x_t = x_{t-2} \),

\[
(1 - L)^d x_t = x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \ldots .
\]

The main advantage of this model, which became popular in the late 1990s and early 2000s (see Baillie, 1996; Gil-Alana and Robinson, 1997; Michelacci and Zaffaroni, 2000; Gil-Alana and Moreno, 2004; Abbritti et al., 2016; etc.), is that it is more general than standard models based on integer differentiation: it includes the stationary I(0) and

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1 Other papers studying the Russian stock market and its volatility include Goriaev and Zabotkin (2006), Luukka et al. (2016) and Korhonen and Peresetsky (2016).
nonstationary I(1) series as particular cases of interest when \( d = 0 \) and \( 1 \) respectively, but also nonstationary though mean-reverting processes if the differencing parameter is in the range \([0.5, 1)\).

We estimate the fractional differencing parameter \( d \) along with the rest of the parameters in the model by using the Whittle function in the frequency domain (Dahlhaus, 1989; Robinson, 1994) under the assumption that the estimated errors are uncorrelated and autocorrelated in turn. In particular, we adopt a parametric method that involves imposing a structure on the error term. Robinson’s (1994) test is most suitable in this case very convenient in this context since it is valid for any range of values of \( d \) and therefore it does not require preliminary differencing; moreover, it allows the inclusion of deterministic terms such as an intercept and a time trend, and its limit distribution is standard normal.\(^2\)

4. **Data and Empirical Results**

We analyse daily transaction level data for both the old (RTSVX) and new (RVI) volatility indices obtained from the Moscow exchange web database; the sample period goes from 7 December 2010 to 12 December 2014 and 6 January 2014 to 9 February 2018 respectively. Appendix 1 provides some descriptive statistics. RTSVX has a slightly higher mean but is less volatile than RVI; further, it has a lower kurtosis coefficient, but a higher skewness one.

4a. **The RTSVX index**

As a first step we estimate the following model:

\(^2\) See Gil-Alana and Robinson (1997) for a description of the functional form of the version of the tests of Robinson (1994) used in this paper.
where $y_t$ is the series of interest, in this case the original volatility index and the log-transformed data. Three specifications are considered, namely i) without deterministic terms (i.e. $\alpha = \beta = 0$ a priori in (3)); (ii) with an intercept ($\alpha$ is estimated and $\beta = 0$ a priori), and iii) with an intercept and a linear time trend (as in equation (3)), and assuming that the errors are uncorrelated (white noise) and autocorrelated (Bloomfield, 1973) in turn.

[Insert Table 1 about here]

Table 1 shows the estimated values of $d$ with their 95% confidence intervals. The t-stats imply that the time trend is not a significant regressor; therefore the selected model includes a constant only in all cases; the estimates of $d$ are slightly higher in the case of uncorrelated errors, and in all cases favour fractional integration over the I(0) stationarity and the I(1) nonstationary hypotheses; since they are below 1, they imply mean reversion, with the effects of shocks disappearing in the long run.

Next, we check if the differencing parameter has remained constant over the sample period, and for this purpose we compute rolling estimates of $d$ with a window of size 10 moving along a subsample of 500 observations. The results are displayed in Figure 1. Under the white noise assumption, the estimates of $d$ (the degree of persistence) start around 0.9, then they decline in the subsample [301-800] and till the subsample [621-1120]; then they increase again till the subsample [931-1430] and only start decreasing again in the final two subsamples, when the unit root null cannot be rejected.

[Insert Figure 1 about here]
Under the assumption of autocorrelation, the estimates of \( d \) are initially around 0.8, and then decrease from the subsample [381-880] till the end of the sample; all of them are below 1, implying mean-reverting behaviour.

Next, we test for breaks using the approach suggested by Bai and Perron (2003) and then its extension to the fractional case by Gil-Alana (2008). The results (not reported) suggest in both cases that there is a single break occurring on 5 August 2011. Around this date, some of the main stock markets including those in the US, the Middle East, Europe and Asia plunged owing to the fear of contagion effects of the sovereign debt crisis in Spain and Italy, credit rating worries in France and slow economic growth in the US.

We then split the sample in two subsamples accordingly. The results for the two cases of uncorrelated and autocorrelated errors are presented, respectively, in Tables 2 and 3. The estimates of \( d \) are significantly below 1 in both subsamples, with both white noise and autocorrelated errors, and for both the original and the logged data.

[Insert Tables 2 and 3 about here]

4b. **The RVI index**

Table 4 has the same structure as Table 1 (i.e., it displays the estimates of \( d \) for the three cases of no regressors, an intercept, and intercept with a linear trend, for both white noise and autocorrelated errors, and for both the original and the logged data) for the new RVI index. The results are fairly similar to the previous ones, with the estimates of \( d \) in all cases in the interval \((0.5, 1)\) and the unit root null hypothesis being rejected in all cases in favour of mean reversion \((d < 1)\).

[Insert Table 4 and Figure 2 about here]
As in Figure 1, Figure 2 displays rolling estimates of $d$ using a window with size 10 moving along a subsample of 500 observations. A clear break is found around the 25th subsample; the Bai and Perron (2003) and Gil-Alana (2008) tests detect a single break on 20 July 2016. Two major rulings of the Bank of Russia (Regulation No. 550-P dated 19-20, July 2016 and Ordinance No. 4077-U dated July 20, 2016) could have been the reasons for such a break in the VIX series. The first ruling concerned the procedure for computing the capital of the professional securities market participants, (ii) the procedure for applicants for the professional securities market participant’s Licence and (iii) the procedure for communicating to credit organisations and non-credit financial institutions information on cases of refusal to fulfil a customer’s instruction for a transaction, and refusal to terminate a bank account (deposit) agreement with a customer. The second ruling concerned the procedure for submission by credit institutions to an authorised body of information on cases of refusal to terminate a bank account (deposit) agreement with a customer on the initiative of a credit institution, and on cases of refusal to fulfil a customer’s instruction for a transaction.

[Insert Tables 5 and 6 about here]

Tables 5 and 6 display the estimates of $d$ for each subsample under the assumption of white noise and autocorrelated errors respectively. As for the other index, the estimates of $d$ are all statistically smaller than 1 (which implies mean reversion) and decline in the second subsample. Specifically, with uncorrelated errors, they are 0.91 (original series) and 0.89 (logged data) for the first subsample, and 0.60 and 0.63 for the second one; with autocorrelated errors, they shift from 0.72 and 0.85 in the first subsample to 0.55 and 0.61 in the second one. It should be noted that a direct comparison of the rolling-window results obtained for the two indices is not appropriate.
since the sample period are different and the evolution of the parameters reflects different economic and stock markets developments.

5. Conclusions

This paper has applied a fractional integration framework to analyse the stochastic behaviour of two Russian stock market volatility indices, namely the originally created RTSVX and the new RVI that has replaced it (for both of which very limited evidence was previously available), using daily data over the period 2010-2018. The chosen approach is more general than those based on the I(0) v. I(1) dichotomy and provides useful information on the long-memory properties and degree of persistence of the series being analysed.

The empirical findings are consistent and imply in all cases that the two series are mean-reverting, i.e. their degree of persistence is limited and the effects of shocks disappear over time. This is consistent with the results reported in Cont and Fonseca (2002) and others on volatility in stock markets, it is true regardless of whether the errors are assumed to follow a white noise or autocorrelated process, and it holds for both subsamples, before and after the detected break. The rolling window estimation reveals the presence of some degree of time variation, but does not affect the general conclusion about the behaviour of the two series under examination.

This type of volatility index can also be seen as a measure of market fear, which therefore does not seem to be permanently affected by shocks in the case of the Russian stock market. Moreover, given the fact that the effects of shocks are not long-lived there does not seem to be any need of strong policy measures to push the series back to their original trends. Finally, our findings represent useful information for investors aiming to design appropriate portfolio diversification strategies.
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Table 1: Estimated coefficients of d and 95% confidence bands, RTSVX

|                          | No terms       | An intercept   | A linear time trend |
|--------------------------|----------------|----------------|--------------------|
| **i) Original data (RTSVX)** |                |                |                    |
| White noise              | 0.89 (0.85, 0.93) | **0.86 (0.82, 0.90)** | 0.86 (0.82, 0.90) |
| Bloomfield               | 0.80 (0.74, 0.85) | **0.76 (0.71, 0.82)** | 0.76 (0.72, 0.82) |
| **ii) Log-transformed data (Log RTSVX)** |                |                |                    |
| White noise              | 0.97 (0.93, 1.01) | **0.88 (0.84, 0.92)** | 0.88 (0.84, 0.92) |
| Bloomfield               | 0.96 (0.90, 1.01) | **0.81 (0.76, 0.87)** | 0.81 (0.76, 0.87) |

**Note:** This table displays the estimated values of the differencing parameter, d, (and their 95% confidence bands) using three different models: a) with no deterministic terms (2nd column); b) with a constant (3rd column) and c) with an intercept and a time trend (4th column). The data are the original old volatility index (RTSVX) (in panel i) and its log transformation (in panel ii). The sample period goes from 7 Dec. 2010 to 12 Dec. 2014. In bold, the selected model according to the deterministic terms.
**Figure 1: Rolling window estimates of d and 95% confidence bands, RTSVX**

i) Uncorrelated errors

![Uncorrelated errors graph]

ii) Autocorrelated errors

![Autocorrelated errors graph]

**Note:** The first value in the figure is the estimate of d for the first subsample with the first 500 observations, i.e. [1 – 500]; the second one corresponds to [11-510] and so on. For the white noise case, the first decrease takes place around the 30th subsample, corresponding to [301-800]; the first jump occurs at the 65th subsample ([621-1120]) and another jump occurs at the 95th ([931-1430]). In the case of autocorrelation, the only noticeable change takes place at the 40th subsample corresponding to [381-880].
Table 2: Results for the two subsamples using white noise errors, RTSVX

|                  | No terms | An intercept | A linear time trend |
|------------------|----------|--------------|---------------------|
| **First subsample** | 1.06 (0.97, 1.17) | **0.87 (0.79, 0.98)** | 0.88 (0.80, 0.98) |
| **Second subsample** | 0.89 (0.85, 0.95) | **0.82 (0.78, 0.86)** | 0.82 (0.78, 0.86) |

**Note:** This table displays the estimated values of the differencing parameter, d, (and their 95% confidence bands) using three different models: a) with no deterministic terms (2nd column); b) with a constant (3rd column) and c) with an intercept and a time trend (4th column). The data are the original old volatility index (RTSVX) (in panel i) and its log transformation (in panel ii). The errors are assumed to be white noise, and the sample period is separated in two subsamples: from 7 Dec. 2010 to 5 Aug. 2011, and from 8 Aug. 2011 to 12 Dec. 2014. In bold, the selected model according to the deterministic terms.

Table 3: Results for two subsamples with autocorrelated errors, RTSVX

|                  | No terms | An intercept | A linear time trend |
|------------------|----------|--------------|---------------------|
| **First subsample** | 0.98 (0.83, 1.21) | 0.80 (0.67, 1.00) | **0.82 (0.71, 1.00)** |
| **Second subsample** | 0.78 (0.72, 0.82) | **0.74 (0.69, 0.81)** | 0.74 (0.70, 0.81) |

**Note:** This table displays the estimated values of the differencing parameter, d, (and their 95% confidence bands) using three different models: a) with no deterministic terms (2nd column); b) with a constant (3rd column) and c) with an intercept and a time trend (4th column). The data are the original old volatility index (RTSVX) (in panel i) and its log transformation (in panel ii). The errors are assumed to be autocorrelated, and the sample period is separated in two subsamples: from 7 Dec. 2010 to 5 Aug. 2011, and from 8 Aug. 2011 to 12 Dec. 2014. In bold, the selected model according to the deterministic terms.
Table 4: Estimated coefficients of $d$ and 95% confidence bands, RVI

|                  | i) Original data (RVI) | ii) Log-transformed data (Log RVI) |
|------------------|------------------------|------------------------------------|
|                  | No terms               | An intercept                       | A linear time trend               |
| White noise      | 0.90 (0.86, 0.96)      | **0.89 (0.84, 0.95)**              | 0.89 (0.84, 0.95)                |
| Bloomfield       | 0.80 (0.73, 0.86)      | **0.74 (0.68, 0.81)**              | 0.74 (0.68, 0.81)                |
|                  |                        |                                    |                                   |
|                  | No terms               | An intercept                       | A linear time trend               |
| White noise      | 0.97 (0.93, 1.01)      | **0.84 (0.80, 0.88)**              | 0.84 (0.80, 0.88)                |
| Bloomfield       | 0.99 (0.93, 1.06)      | **0.82 (0.77, 0.89)**              | 0.82 (0.77, 0.89)                |

Note: This table displays the estimated values of the differencing parameter, $d$, (and their 95% confidence bands) using three different models: a) with no deterministic terms (2nd column); b) with a constant (3rd column) and c) with an intercept and a time trend (4th column). The data are the original new volatility index (RVI) (in panel i) and its log transformation (in panel ii). The sample period goes from 6 Jan 2014 to 9 Feb. 2018. In bold, the selected model according to the deterministic terms.
Figure 2: Rolling window estimates of $d$ and 95% confidence band, RVI

i) Uncorrelated errors

ii) Autocorrelated errors

Note: The first value in the figure is the estimate of $d$ for the first subsample with the first 500 observations, i.e., [1–500]; the second one corresponds to [11-510] and so on. The most noticeable change takes place around the 25th subsample [241-740].
Table 5: Results for the two subsamples using white noise errors, RVI

|                  | No terms | An intercept | A linear time trend |
|------------------|----------|--------------|---------------------|
| **First subsample** | 0.90 (0.84, 0.98) | **0.91 (0.84, 0.99)** | 0.91 (0.84, 0.99) |
| **Second subsample** | 0.90 (0.84, 0.97) | 0.62 (0.57, 0.68) | **0.60 (0.54, 0.68)** |

**ii)** Logged data

|                  | No terms | An intercept | A linear time trend |
|------------------|----------|--------------|---------------------|
| **First subsample** | 0.94 (0.90, 1.00) | **0.89 (0.84, 0.95)** | 0.89 (0.84, 0.95) |
| **Second subsample** | 0.98 (0.92, 1.06) | 0.64 (0.59, 0.71) | **0.63 (0.57, 0.70)** |

**Note:** This table displays the estimated values of the differencing parameter, \( d \), (and their 95% confidence bands) using three different models: a) with no deterministic terms (2\(^{nd}\) column); b) with a constant (3\(^{rd}\) column) and c) with an intercept and a time trend (4\(^{th}\) column). The data are the original old volatility index (RVI) (in panel i) and its log transformation (in panel ii). The errors are assumed to be white noise, and the sample period is separated in two subsamples: from 6 Jan. 2014 to 20 Jul. 2016, and from 21 Jul. 2016 to 9 Feb. 2018. In bold, the selected model according to the deterministic terms.

Table 6: Results for two subsamples with autocorrelated errors, RVI

|                  | No terms | An intercept | A linear time trend |
|------------------|----------|--------------|---------------------|
| **First subsample** | 0.77 (0.70, 0.85) | **0.72 (0.65, 0.82)** | 0.72 (0.65, 0.82) |
| **Second subsample** | 0.92 (0.84, 1.04) | 0.61 (0.54, 0.70) | **0.55 (0.46, 0.68)** |

**ii)** Logged data

|                  | No terms | An intercept | A linear time trend |
|------------------|----------|--------------|---------------------|
| **First subsample** | 0.97 (0.90, 1.06) | **0.85 (0.77, 0.96)** | 0.85 (0.77, 0.96) |
| **Second subsample** | 0.98 (0.88, 1.10) | 0.63 (0.57, 0.73) | **0.61 (0.52, 0.72)** |

**Note:** This table displays the estimated values of the differencing parameter, \( d \), (and their 95% confidence bands) using three different models: a) with no deterministic terms (2\(^{nd}\) column); b) with a constant (3\(^{rd}\) column) and c) with an intercept and a time trend (4\(^{th}\) column). The data are the original old volatility index (RVI) (in panel i) and its log transformation (in panel ii). The errors are assumed to be autocorrelated, and the sample period is separated in two subsamples: from 6 Jan. 2014 to 20 Jul. 2016, and from 21 Jul. 2016 to 9 Feb. 2018. In bold, the selected model according to the deterministic terms.
### APPENDIX 1

Descriptive statistics

|                   | RTSVX   | RVI     |
|-------------------|---------|---------|
| **Mean**          | 3.440813| 3.417787|
| **Standard Error**| 0.006984| 0.010149|
| **Median**        | 3.438814| 3.460566|
| **Mode**          | 3.302113| 3.766997|
| **Standard Deviation** | 0.28239 | 0.326046|
| **Sample Variance** | 0.079744| 0.106306|
| **Kurtosis**      | -0.03088| 0.000875|
| **Skewness**      | 0.480844| 0.162611|
| **Range**         | 1.875387| 2.040248|
| **Minimum**       | 2.735665| 2.678965|
| **Maximum**       | 4.611053| 4.719213|
| **Sum**           | 5625.729| 3527.156|
| **Count**         | 1635    | 1032    |