Performance Assessment for Energy Efficient NOMA over Nakagami Fading Channel

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Abstract. Nowadays, the Non-Orthogonal Multiple Access (NOMA) has become the most candidate as the multiple access technique for the Fifth Generation (5G). Although, the massive increase in traffic usage is necessary, it must not come on the account of Energy Efficiency (EE). Therefore, Green Radio (GR) which one of its main points is energy efficiency (EE) gained a lot of attention in last few years in both academic and industrial fields. In this paper, the Energy-Efficiency in the NOMA downlink system under the effect of the Nakagami-m fading channel is studied. Moreover, the Nakagami-m fading channel can tackle several fading cases according to the parameter m. The system has been solved via an iterative algorithm to investigate the required power level for maximum energy efficiency. Different channel models will be investigated such as: one-sided Gaussian faded channel, Rayleigh faded channel, Raician (Nakagami-n) faded channel, and LOS.

1. Introduction
Recently, the traffic usage in wireless networks has massively increased triggering the energy consumption problem. The transmission of data and communication infrastructure consumes about 3% of energy worldwide\cite{1}. Nonetheless, the massive amount of traffic must not come on behalf of the energy consumption. Green Radio (GR) which one of its main points is Energy Efficiency (EE) gained a lot of attention recently in both academic and industrial fields. Moreover, research project on Green Radio, such as Green touch consortium, which has a main goal to improve the EE by a factor of 1000x by 2020 as compared to the 2010 level\cite{2}. The massive increase of traffic and necessity of higher data rate and spectral efficiency (SE), the NOMA has been introduced as a promising radio access technique for the upcoming IMT2020\cite{3} or fifth generation (5G). In NOMA superposition coding (SC) is applied at the transmitter side and successive interference cancellation is applied at the receiver side, this allows multiplexing several users on the same sub channel simultaneously \cite{4}, \cite{5}. System level performance for NOMA downlink systems are studied in \cite{6}. Power allocation strategies in NOMA systems was studied in \cite{7}, \cite{8} in which the system throughput is maximized. In \cite{9}, a cooperative transmission scheme has been studied which shows that some users in NOMA systems have prior information about other’s messages. Energy Efficiency problem was studied in NOMA downlink systems in \cite{10}, \cite{11}.

The aim of the presented work is to study the trade-off transmitted power and Energy-Efficiency in the NOMA downlink system under the effect of the Nakagami-m fading channel. The equivalent
channel gain is considered a general model based on Nakagami-m fading channel. Moreover, the Nakagami-m faded channel can tackle several fading occasions according to the different values of parameter $m$. The following scenarios can be considered: the minimum value of $m$ is 0.5, which describes a one-sided Gaussian fading channel [12]. At $m = 1$, Nakagami behaves as a Rayleigh faded channel associated with path-loss as discussed in [13]. Higher values of $m$ shows the presence of a strong line of sight (LOS) component that describes less severe scenarios such as Rician faded channel [12]. As the value of $m$ approaches infinity, the Nakagami-m fading channel describes the least severe cases.

The rest of paper is organized as follows: Section 2 includes the Nakagami-m fading channel and its different variations. Section 3 describes the system model. Results and Analysis will be illustrated and discussed in Section 4. Finally, Section 5 provides the main conclusions of the presented work.

2. Nakagami-m Fading Channel and Its Variations

2.1. Nakagami-m Fading Channel

The Nakagami-m fading Probability Density Function (PDF) of the envelope is as [14]

$$P(N) = 2 \left(\frac{m}{\Omega}\right)^m N^{2m-1} \frac{\Gamma(m)}{\Gamma(m)} \exp\left(-\frac{m}{\Omega}N^2\right); \; m > 0.5$$

(1)

where, $N$ is the Nakagami-m random variable $\Omega = E[R^2]$ is the average power, $m = (E[R^2])^2/\text{Var}[R^2]$ is the shaping factor which describes the severity of the fading, $\Gamma(.)$ is the gamma function. According to [14] the Nakagami-m faded channel is considered to be the most generalized model of different faded channels; such as, Rayleigh faded channel, one-sided Gaussian faded channel, and Rician faded channel depending on the value of $m$. Furthermore, depending on the value that $m$ obtains the Nakagami-m faded channel will be estimated to a certain channel. For instance, when $m = 0.5$ the Nakagami-m faded channel will be approximated to a one-sided Gaussian faded channel [12]. Moreover, a very special case is when $m = 1$ the Nakagami-m faded channel will be approximated to a Rayleigh faded channel [15]. Therefore, if $m < 1$ this represents the most severe fading case, and when $m$ approaches $\infty$ this means that there is no effect of small scale fading on the channel (AWGN channel). Nevertheless, when $m > 1$ the Nakagami-m faded channel is modelled as a Rician faded channel and the effect of fading is less severe [15]. Table 1 summarizes the relationship between Nakagami-m fading channel and its different variations.

| Nakagami-m fading channels | Related fading channels | Substituted Parameter |
|---------------------------|-------------------------|----------------------|
| $N(m, \Omega)$            | Nakagami-m              | $m > 0.5, \Omega > 0$|
| $N(1, \Omega)$            | Rayleigh                | $m = 1, \Omega = 2\sigma^2$|
| $N(0.5, \Omega)$          | One-sided Gaussian      | $m = 0.5, \Omega = \sigma^2$|

2.2. One-sided Gaussian Fading Channel

When $m = 0.5$ and $\Omega = \sigma^2$ [16] which is the minimum value $m$ can take; therefore, the Nakagami-m PDF can be modelled as a one-sided Gaussian faded channel which represents the most severe case of fading.

$$P(N) = \sqrt{\frac{2}{\pi}} \frac{n^2}{\sigma^2} e^{-\frac{n^2}{2\sigma^2}}$$

(2)

where, $\sigma^2$ is the scale parameter or variance.
2.3. Rayleigh Fading Channel

The Rayleigh fading channel is considered to be the simplest fading model. For Rayleigh model, it is assumed many non-line of sight and scattered multi-paths of the signal received by the receiver. When $m = 1$, and $\Omega = 2\sigma^2$ the Nakagami-m PDF is shown as a Rayleigh fading channel

$$P(N) = \frac{N}{\sigma^2} e^{-\frac{N^2}{2\sigma^2}}$$

(3)

2.4. Rician Fading Channel

In this case the larger the $m$ less severe fading will be presence on the channel; this indicates the existence of the line of sight (LOS) component. Therefore, the Rician model, assumes the presence of a LOS component in addition to the scattered multipaths of the signal. For any value greater than unity ($m > 1$) the Nakagami-m fading channel describes a Rician fading channel. In addition, the Rician fading channel can also be called Nakagami-n fading channel [14], [17]. The PDF of Rician fading channel is

$$P(N) = \frac{N}{\sigma^2} e^{-\frac{(N^2+v^2)}{2\sigma^2}} I_0 \left( \frac{Nv}{\sigma^2} \right)$$

(4)

Where, $I_0(.)$ is the Bessel function of the first order kind with zero order, $v$ is the direct (LOS) component.

3. System Model

The system is considered as a downlink NOMA system with a single cell. A single base station (BS) serves two users simultaneously. Perfect knowledge of channel state information (CSI) is assumed at the BS (transmitter) and the users (receiver). The channel coefficient between the user and the BS is $h_i$, where $h_i$ is the Nakagami-m fading coefficient. The $i^{th}$ users is denoted to by $i$ ($i = 1, 2$). The transmitted signal from BS is $s_i$ with $E[|s_i|^2] = 1$. Figure 1 illustrates the proposed NOMA model with SIC applied at the user. For NOMA the superposition coding of $s_1$ and $s_2$ are represented by $x$ as given by [5]

$$x = \sqrt{\alpha Ps_1} + \sqrt{(1-\alpha)Ps_2}$$

(5)

where $\alpha$ is the power allocation ($0 < \alpha < 0.5$) constant and $P$ is the total transmitted power from the BS.

The received signal is modelled as

$$y = h_ix + w_i$$

(6)

where, $w_i$ is the AWGN, which follows the Gaussian distribution with zero mean and variance $\sigma_w^2$. The channel gain $H_i$ will be modelled as [13]

$$H_i = \frac{h_i}{\rho L}$$

(7)

According to the NOMA principle, the successive interference cancellation (SIC) is implemented at the receiver (user). Therefore, without loss of generality, it is assumed that the optimal order of decoding is in the order of increasing of the equivalent channel gain $H_i$ ($H_1 > H_2$). The transmit power will be divided between the two users. The transmit power of the weak user $U_2$ is greater that the transmit power of the strong user $U_1$, and that is governed by the power allocation coefficient $\alpha$, i.e., $0 < \alpha < 0.5$. The SIC at the receiver cancels the inter-cell interference. The strong user ($U_1$) will firstly decode the weak user’s signal then subtract it from the received signal (cancelling interference), obtaining its own signal.

According to Shannon capacity equation for system bandwidth ($W$), the data rate of the two users can be formulated as follows. The data rate of the strong user $U_1$ is $R_1$:

$$R_1 = W \log_2 \left( 1 + \frac{\alpha H_1 P}{\sigma_w^2} \right)$$

(8)

The data rate of the weak user $U_2$ is $R_2$:

$$R_2 = W \log_2 \left( 1 + \frac{(1-\alpha)H_2 P}{\alpha H_2 P + \sigma_w^2} \right)$$

(9)
Therefore, the total rate $R$ of the NOMA system is represented as follows:

$$R = W \log_2(\sigma_w^2 + H_2P) + W \log_2 \frac{\sigma_w^2 + aH_1P}{aH_2P\sigma_w^2 + \sigma_w^4}$$

(10)

Actually, a Nakagami random variable can be obtained from the gamma random variable by taking the square root of gamma random variable [16]. For simulation purposes, the generation of the Nakagami-m fading coefficient will be generated using the `gamrnd()` in MATLAB [18].

3.1. Energy efficiency and Spectral efficiency

The Energy Efficiency (EE) is defined as a number of bits that can be transmitted per Joule. The Spectral Efficiency (SE) efficiency is defined as the number of bits transmitted per bandwidth. The EE and SE is defined as follows [13]

$$EE = \frac{R}{W(\xi P + P_c)}$$

(11)

$$SE = \frac{R}{W}$$

(12)

where, $\xi$ is the drain efficiency of the BS power amplifier and $P_c$ is the power consumption of the circuits.

4. Results and discussion

In this section, the results are presented to confirm the numerical solution proposed in this paper. The radius of the cell is considered to be 500 meters. The total bandwidth of the system is 1MHz. The minimum distance between the user and the base station is 50 meters. The AWGN power spectral density is -174dBm/Hz. The pathloss exponent ($n$) is 3. To maintain the QoS the minimum data rate of each user is set to be 1Mbps. These parameter are in consistence with [13].

The system has been solved via an iterative algorithm based on many number of iterations had been run. The obtained results show a convergence around the values that will be illustrated in the following figures. These values are tested for several iterations starting at 10 iterations up to $10^5$ iterations. Therefore, the presented results showed a marginal convergence with confidence of $\pm 7\%$, and the average value of all these iterations are shown in the following figures. The following subsections illustrate the different operation scenarios.
4.1. Rayleigh fading channel \((m = 1)\) and system validation

A very special case of Nakagami is the Rayleigh fading channel when \(m = 1\). The results published in [13] compared to the proposed iterative algorithm are shown in Figure 2.

![Figure 2. Energy Efficiency versus total transmitted Power \(m=1\) [13]](image)

The comparison presents that both of the presented work and published work [13] are monotonically the same. On the other hand, the presented work out performs the work of [13] by investigating the general channel models not only Rayleigh fading channel model.

4.2. One-Sided Gaussian fading channel \((m = 0.5)\)

In this case, \(m = 0.5\) which means that the line of sight component is not available. According to Nakagami-m fading channel the minimum value that \(m\) can take is 0.5; therefore this represents the most severe fading case. Figure 3, represents the EE versus the total transmitted power for \(m = 0.5\).

![Figure 3. Energy Efficiency versus total transmitted Power \(m=0.5\)](image)
According to Figure 3, the maximum EE is about 18 bits/Joule/Hz at 25dBm. Moreover, when compared to the case of the Rayleigh fading in Figure 2 the maximum EE achieved is about 19.5 bits/Joule/Hz at 25dBm. Therefore, this explains that as \( m \) increases the severity of fading decrease. Furthermore, from Figure 3 if the power is greater than 25dBm then the total system EE will start to decrease.

### 4.3. Rician fading channel and LOS (\( m > 1 \))

In this case, \( m > 1 \) indicates the presence of the dominant (LOS) component. Figure 4 and Figure 5 show the effect of fading on the system EE for two cases, at \( m = 2 \) (Rician Fading channel) and \( m \geq 10 \) (non-faded channel) respectively.

![Figure 4](image.jpg)  
**Figure 4.** Energy Efficiency versus total transmitted Power \( m=2 \)

![Figure 5](image.jpg)  
**Figure 5.** Energy Efficiency versus total transmitted Power \( m \geq 10 \)

In accordance to the figures above, as the value of \( m \) increases from 2 to 10 the EE also increases and reaches its maximum at the power level of 25dBm. From Figure 5., (at \( m \geq 10 \)) higher value of EE is achieved at maximum power level 25dBm; unlike, Figure 4 lower value of EE is achieved at maximum
power level 25dBm. This can be explained due to the fact that $m \geq 10$ represents the case of non-faded channel (best coverage condition); therefore, higher value of EE is achieved. Therefore, increasing the power beyond the 25dBm will degrade the system EE due to the inherent system interference.

4.4. Comparison of Energy Efficiency for different channel conditions

Figure 6 shows that the EE has been improved for different $m$. According to the increasing value of $m$ (LOS component increases) the obtained EE is improved. Moreover, Figure 6 includes the comparison between One Sided Gaussian channel (at $m = 0.5$), Rayleigh channel (at $m = 1$), Rician channel (at $m = 2$) and Line of Sight (LOS) or non-faded channel (at $m \geq 10$).

4.5. EE relation to the shaping parameter ($m$)

By obtaining the maximum EE from previous results and applying Vandermonde matrix (\texttt{vander()}) an approximate relation between $m$ and $\Delta EE$ can be provided. Therefore, Figure 7. proposes an asymptotic relation for the estimated $\Delta EE$ versus $m$ the shaping parameter. From Figure 7. the estimated value of $\Delta EE$ can be obtained using the following equation:

$$\Delta EE = EE(\text{@}m = 10) - EE(\text{@} \text{any} \ m) \quad (13)$$

where $\Delta EE$ is the difference between EE (at $m=10$) and EE (at any $m$). Therefore, form Figure 7. a polynomial equation of the 11th degree can be obtained by using the function \texttt{vander()} that describes the relation between $\Delta EE$ and $m$.

$$\Delta EE(m) = 6.3037 \times 10^{-6} m^{11} - 3.3202 \times 10^{-4} m^{10} + 0.0076 m^9 - 0.1004 m^8 - 0.8388 m^7 - 4.645 m^6 + 17.287 m^5 - 42.8549 m^4 + 68.339 m^3 - 64.79 m^2 + 30.31 m - 0.151 \quad (14)$$

So that, Equation (14) may be used by system design to estimate the difference between best coverage condition (non-faded channel at $m \geq 10$) and any channel model (different $m$).
5. Conclusion
In this paper, the Energy-Efficiency in the NOMA downlink system under the effect of the Nakagami-\(m\) faded channels is studied. According to the presented results, these results are monotonically the same, but it is shown that at 25dBm the EE will be maximized. The maximum of the maximum of each figure will be reached by 25dBm. Therefore an asymptotic relation had been provided for the estimated \(\Delta EE\) for each \(m\). From these results, it is recommended that to use 25dBm as transmission power. Otherwise, if the power is less than 25dBm, then the system will not be fully power supported so it will not utilize the EE. On the other hand, by increasing the power to more than 25dBm the inherent interference will be enlarged; therefore, it will damage and degrade the system EE. Furthermore, an asymptotic relation had been provided for the estimated \(\Delta EE\) for each \(m\).

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Figure 7. \(\Delta EE\) versus \(m\)
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