Shadow of rotating regular black holes and no-horizon spacetimes

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(Dated: April 17, 2020)
Abstract

The Kerr black holes always have a photon region with its prograde radius \(r_p^-\) and retrograde radius \(r_p^+\), respectively, in the range \(M \leq r_p^- \leq 3M\) and \(3M \leq r_p^+ \leq 4M\), and always cast closed shadow silhouette for \(a \leq M\). For \(a > M\), it is a no-horizon spacetime (naked singularity) with prograde orbits spiral to end up in singularity and retrograde orbits produce an arc like shadow with dark spot at the center, i.e., no closed shadow silhouette. We compare shadows cast by Kerr black holes with those produced by three rotating regular spacetimes, viz. Bardeen, Hayward and nonsingular, which are prototype non-Kerr black hole metrics with an additional deviation parameter \(g\) related to nonlinear electrodynamics charge. It turns out that for a given \(a\), there exists a critical value of \(g\), \(g_E\) such that \(\Delta = 0\) has no zeros for \(g > g_E\), one double zero at \(r = r_E\) if \(g = g_E\), respectively, corresponding to a no-horizon regular spacetime and extremal black hole with degenerate horizon. We demonstrate, unlike the Kerr black hole, no-horizon regular spacetimes can form closed shadow silhouette when \(g_E < g \leq g_c\), e.g., for \(a = 0.10M\), Bardeen \((g_E = 0.763332M < g \leq g_c = 0.816792M)\), Hayward \((g_E = 1.05297M < g \leq g_c = 1.164846M)\) and nonsingular \((g_E = 1.2020M < g \leq g_c = 1.222461M)\) no-horizon spacetimes cause closed shadow silhouette. These results confirm that the existence of closed shadow does not prove that the compact object is necessarily a black hole. The circularity deviation parameter \(\Delta C\) for the three no-horizon rotating spacetime shadows satisfy \(\Delta C \leq 0.10\) in accordance with the observed shadow of M87\(^*\) black hole. The case of Kerr-Newman no-horizon spacetimes (naked singularities) with similar features is appended.

PACS numbers:
I. INTRODUCTION

Photons circular orbits, the extreme cases of gravitational light bending, play a crucial role in the appearance of black holes and offer a promising tool to probe the strong gravity features. A black hole embedded in a luminous background cast a shadow that is fringed by a sharp and bright emission ring, known as photon ring, whose structure solely depends upon the spacetime geometry [1]. Synge [2] and Luminet [3] led the study of a non-rotating black hole shadow, respectively, without and with a surrounding geometrically thick and optically thin accretion disk. Bardeen [4] pioneered the study of rotating black hole shadow in the luminous background, whereas Cunningham and Bardeen [5], discussed how the spectrum would be modified by gravitational redshift and lensing, and studied the shadow in presence of surrounding accretion disk. Over the past decades, the theory of black hole shadows is well-developed and the flurry of activities in analytical investigations, observational studies and numerical simulations of shadows have been elicited [6, 7]. Though the concept of the black hole shadow exists since the ’70s, the idea to image it in the galactic center using the VLBI technique was first presented by Falcke et al. in 2000 [8]. The Event Horizon Telescope (EHT) collaboration has recently captured the central compact radio emission region of nearby galaxy M87 at 1.3 mm wavelength with an unprecedented angular resolution of 20 μas [9–11]. The central flux depression in the horizon-scale emission image of residing supermassive black hole M87* infers the presence of a black hole shadow [10, 11]. Though the observed shadow is consistent with the expected general-relativistic magnetohydrodynamics simulated image of the Kerr black hole as predicted by the general relativity, alternatives to the Kerr black hole could not be completely ruled out currently [9–13]. Moreover, some attention has been paid to shadows of some exotic alternatives to black holes, namely wormholes, naked singularities, boson stars, gravastars, superspinors, etc. [14, 15], and also a probe of horizon geometry [16].

An especially attractive and intensively discussed class of black holes is sourced by the gauge-invariant Lagrangian density $\mathcal{L}(\mathcal{F})$ associated with the nonlinear electrodynamics (NED) field i.e., regular black holes. Bardeen [17], realizing the concrete idea of Sakharov [18] and Gliner [19] of replacing the central singularity by a de-Sitter core, proposed the first-ever toy model for the static spherically symmetric regular black hole. However, the NED source for the Bardeen black hole arising from the magnetic charge $g$ was obtained
much later [20]. This inspired the construction of several other regular black holes solutions involving the varieties of NED fields [21–23]. Moreover, general relativity coupled with the NED of appropriate Lagrangian density having correct weak field limits naturally leads to the regular black hole spacetimes [24, 25]. Rotating regular black holes [26–28] have also been obtained and studied extensively, which in the large–r limits retrieve the Kerr black hole solution [29].

Intriguing features of photon motions in the no-horizon non-rotating regular spacetimes, namely Bardeen and Ayon-Beato-Garcia spacetimes, have already been reported in literature yielding some engrossing optical phenomenons [30, 31]. The presence of unstable photon circular orbits in no-horizon spacetimes and of ghost images are notable among them [30], however, such possibilities in the rotating no-horizon spacetimes are still unexplored to the best of our knowledge. Further, the cosmic censorship conjecture (CCC) entails that the spacetime singularities are clothed by the black hole event horizon, thereby forbidding naked-singularities [32]. The validity of CCC is still an outstanding open problem in gravitational physics and its remains an unproven conjecture to the date. The closed shadow silhouette ensuring the CCC for the Kerr spacetimes [33] may not hold for other rotating black holes. This motivates us to investigate the photon motions in no-horizon rotating regular spacetimes and derive the analytical expression for the photon region. Seeking the astrophysical implications of no-horizon rotating spacetimes, we construct the corresponding shadows and compare them with those for the Kerr black holes and Kerr naked singularity shadows. Unlike the Kerr naked singularity, which always cast an open arc shadow, the no-horizon rotating regular spacetimes and Kerr-Newman naked singularity can cast a closed or open shadow silhouette. Indeed, over a finite parameter space \((a, g)\), the no-horizon rotating spacetimes posses unstable photon orbits and lead to the closed shadow silhouette much similar to the rotating black hole shadow. This confirms the subtlety that closed shadows silhouette may not always ensure the presence of the black hole horizon. Similar conclusions were drawn for the spherically symmetric naked singularities with or without accretion disk, which for some particular values of parameters resemble the Schwarzschild black hole shadow [15].

The paper is structured as follows: In Sect. II we give an overview of rotating regular black holes. The photon motion and formulation of the necessary technique to investigate the shadow is discussed in Sect. III. Section IV is devoted to the construction of the shadows
of three familiar rotating regular spacetimes, and we also examine how the\no-horizon spacetimes shadows can be distinguishable from the black hole's shadow. We conclude in Sect. V with a summary of the key results.

II. REGULAR BLACK HOLES

The regular black holes are solutions of general relativity minimally coupled to NED, which yielding alteration to classical black holes and near the center behave like a de-Sitter spacetime. The Einstein-Hilbert action minimally coupled with the NED field reads  

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - \mathcal{L}(\mathcal{F})),$$

(1)

where $\mathcal{F}$ is the Faraday electromagnetic field tensor for the gauge potential $A_\mu$. It describes Maxwell theory for weak fields $\mathcal{F}$: $\mathcal{L}(\mathcal{F}) \approx \mathcal{F}$ as $\mathcal{F} \to 0$. Varying action (1) leads to the following covariant equations of motion

$$G_{\mu\nu} = T_{\mu\nu} = 2 \left( \mathcal{L}_\mathcal{F} F^\sigma_{\mu} F_{\sigma\nu} - \frac{1}{4} g_{\mu\nu} \mathcal{L}(\mathcal{F}) \right),$$

(2)

$$\nabla_\mu (\mathcal{L}_\mathcal{F} F^{\mu\nu}) = 0 \text{ and } \nabla_\mu (\ast F^{\mu\nu}) = 0,$$  

(3)

where an asterisk denotes the Hodge dual, $G_{\mu\nu}$ is the Einstein tensor, and $\mathcal{L}_\mathcal{F} = \partial \mathcal{L}(\mathcal{F})/\partial F$. Bardeen [17] black hole is the first regular black hole and later several solutions based on Bardeen’s idea have been obtained [21, 22]. The most general static and spherically symmetric black hole spacetime metric can be written in the form

$$ds^2 = - \left( 1 - \frac{2m(r)}{r} \right) dt^2 + \left( 1 - \frac{2m(r)}{r} \right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

(4)

where $m(r)$ can be determined by using Eq. (4) and solving Eqs. (2) and (3). The Faraday field tensor for a spherically symmetric spacetime can only have non-zero electric field $F_{tr} = -F_{rt}$ and magnetic field $F_{\theta\phi} = -F_{\phi\theta}$, Eq. (3) leads $F_{\theta\phi} = g \sin \theta$ for the gauge field potential $A = -g \cos \theta \, d\phi$, $g$ is the magnetic charge. Equation (2) leads to only two independent equations [23]

$$-\frac{4m'(r)}{r^2} + \mathcal{L}(\mathcal{F}) = 0,$$  

(5)

$$-\frac{2m''(r)}{r} + \mathcal{L}(\mathcal{F}) - \frac{4g^2}{r^4} \mathcal{L}_F = 0.$$  

(6)
For regular black holes, the Lagrangian density has a form [23],

\[ \mathcal{L}(\mathcal{F}) = \frac{4\mu\alpha}{g^3} \frac{(2g^2\mathcal{F})^{\frac{2+\mu}{4}}}{(1 + (2g^2\mathcal{F})^{\frac{\mu}{4}})^{1+\frac{\nu}{2}}} \]  

(7)

with this choice of the Lagrangian Eq. (7), Eqs. (5) and (6) admit the solution [23]

\[ m(r) = \frac{\alpha r^\mu}{(r^\nu + g^\nu)^{\frac{\mu}{\nu}}} \]  

(8)

where, \( \mu, \nu \geq 0 \) are dimensionless constants constrained to ensure the asymptotic flatness, with \( \mu = 0 \) leads to the Schwarzschild black hole [23]. In the asymptotic limit \( r \to \infty \), the smooth mass distribution function reads \( m(r) \approx \alpha \), which can be identified as the black hole mass parameter \( M \), i.e., \( \alpha = M \). For the spacetime (4) with \( m(r) \) in Eq. (8), the highly densed central region of black hole will be a de-Sitter like region for \( \mu \geq 3 \). One can choose \( \mu \) and \( \nu \) suitably to construct exact spherically symmetric regular black hole solutions [23]. The Bardeen [17] and the Hayward [22] black holes are encompassed as special cases, respectively, for \( \nu = 2, \mu = 3 \), and \( \nu = \mu = 3 \), with the mass function in Eq. (8). If \( g \neq 0 \), these black hole solutions develop a de-Sitter core at center \( (r \to 0) \), thereby avoiding the curvature singularity. One can generate several regular black holes by appropriate choice of the Lagrangian density, e.g., if we consider [35]

\[ \mathcal{L}(\mathcal{F}) = 4\mathcal{F}e^{-s(4Mk\mathcal{F})^{1/4}} \]  

(9)

where \( s = g/2M \), and \( k = g^2/2M \). Equation (2) with Eqs. (4) and (9) leads to the exponential mass distribution function [35]

\[ m(r) = Me^{-g^2/2Mr} \]  

(10)

It encompasses Schwarzschild black hole as special case when \( g = 0 \), and can be identified as Reissner-Nordström black hole for \( r >> g^2/2M \) [27].

The rotating counterpart of these spherically symmetric matrices are also obtained [26, 27], which in the Boyer-Lindquist coordinates have the Kerr-like form

\[ ds^2 = - \left( 1 - \frac{2m(r)r}{\Sigma} \right) dt^2 - \frac{4am(r)r}{\Sigma} \sin^2 \theta \, dt \, d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma \, d\theta^2 \]

\[ + \left[ r^2 + a^2 + \frac{2m(r)ra^2}{\Sigma} \sin^2 \theta \right] \sin^2 \theta \, d\phi^2 \]  

(11)

with

\[ \Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2m(r)r, \]  

(12)
where $a$ is the black hole spin parameter. Thus, in the Eq. (11), one can replace $m(r)$ as defined in Eqs. (8) or (10) to get corresponding rotating regular black hole spacetimes, whereas for $m(r) = M$ it describes the Kerr spacetime. The rotating metric Eq. (11) is globally regular as scalar invariants like Ricci scalar and Riemann scalar are finite everywhere including $r = 0$ [26, 27]. The horizons of these spacetimes are determined by the real positive roots of

$$\Delta(r) = r^2 + a^2 - 2m(r)r = 0,$$

(13)

where $m(r)$ is defined by Eqs. (8) or (10). It turns out that there exist set of black hole parameters such that Eq. (13) admits two positive roots, viz., $r_-$ and $r_+$, respectively, corresponding to inner (Cauchy) and outer (event) horizons [26, 27]. Indeed, elementary analysis of Eq. (13) suggests a critical value $g = g_E$, corresponding to the extremal rotating regular black hole i.e., $r_- = r_+ \equiv r_E$ [27, 36]. On the other hand, $\Delta(r) = 0$ has no physical roots for $g > g_E$, and two distinct positive roots $r_\pm$ for $g < g_E$. These two cases describe, respectively, a rotating regular spacetime with no-horizon and regular non-extremal black holes. For instance, for $a = 0.30M$, the critical value of $g_E$, for rotating Bardeen spacetime reads $g_E = 0.713153M$, while the value of $g_E$ for rotating Hayward spacetimes is $g_E = 1.0046M$ and that for rotating nonsingular spacetime is $g_E = 1.12081M$. We can say that the critical parameter $g_E$ is separating the parameter space for rotating regular black holes and no-horizon rotating regular spacetimes. One can also obtain $a_E$ for a fixed $g$, such that two horizons coincides and one gets extremal black holes.

III. GEODESICS AND UNSTABLE PHOTONS ORBITS OF ROTATING SPACETIMES

The photon geodesics around compact astrophysical objects are crucially important both from phenomenological and observational perspectives, as they carry the information about the physical processes in the strong-gravity regime [8, 37]. To study the apparent shadow of the black hole we need to trace the photon’s trajectories which originates from a background source, propagate around the black hole and reach the far distant observer. We use the Hamilton-Jacobi method to obtain shadow of the rotating spacetime (11). The Hamilton-
Jacobi equation for the photon geodesics is [38]

\[ \frac{\partial S}{\partial \tau} = -\frac{1}{2} g^{\alpha\beta} \frac{\partial S}{\partial x^\alpha} \frac{\partial S}{\partial x^\beta}, \] (14)

where \( \tau \) is the affine parameter along the geodesics and \( S = S(\tau, x^\alpha) \) is the Jacobi action. Metric (11) is stationary and axisymmetric, which yield a set of corresponding Killing vectors \( \chi^\mu_{(t)} \) and \( \chi^\mu_{(\phi)} \). The projection of these Killing vectors along the photon four-momenta \( (p^\mu) \) entail the conserved quantities [39]

\[ \mathcal{E} = -\chi^\mu_{(t)} p_\mu, \quad \mathcal{L} = \chi^\mu_{(\phi)} p_\mu, \] (15)

where \( \mathcal{E} \) is specific energy and \( \mathcal{L} \) is the axial angular momentum. For the separable solution of the Eq. (14), the action must have the form [38]

\[ S = -\mathcal{E} t + \mathcal{L} \phi + S_r(r) + S_\theta(\theta). \] (16)

Using Eqs. (14) and (16), the complete equations of motion are obtained as follow [38, 39]

\[ \Sigma \frac{dt}{d\tau} = \frac{r^2 + a^2}{r^2 - 2m(r)r + a^2} (\mathcal{E}(r^2 + a^2) - a\mathcal{E} \sin^2 \theta - \mathcal{L}), \] (17)

\[ \Sigma \frac{dr}{d\tau} = \pm \sqrt{\mathcal{V}_r(r)}, \] (18)

\[ \Sigma \frac{d\theta}{d\tau} = \pm \sqrt{\mathcal{V}_\theta(\theta)}, \] (19)

\[ \Sigma \frac{d\phi}{d\tau} = \frac{a}{r^2 - 2m(r)r + a^2} (\mathcal{E}(r^2 + a^2) - a\mathcal{L}) - \left( a\mathcal{E} - \frac{\mathcal{L}}{\sin^2 \theta} \right), \] (20)

where \( \mathcal{V}_r(r) \geq 0 \) and \( \mathcal{V}_\theta(\theta) \geq 0 \) are related to the effective potentials for \( r \) and \( \theta \)-motions, and, respectively, have the following form

\[ \mathcal{V}_r(r) = \left( (r^2 + a^2)\mathcal{E} - a\mathcal{L} \right)^2 - (r^2 + a^2 - 2m(r)r)(\mathcal{K} + (a\mathcal{E} - \mathcal{L})^2), \] (21)

\[ \mathcal{V}_\theta(\theta) = \mathcal{K} - \left( \frac{\mathcal{L}^2}{\sin^2 \theta} - a^2\mathcal{E}^2 \right) \cos^2 \theta, \] (22)

where constant \( \mathcal{K} \) is the Carter integral of motion and indeed plays a crucial role in characterizing the possible latitudinal motion. For \( \mathcal{K} = 0 \), \( \theta \)-motion is suppressed and in turn, all photon orbits are restricted only to a plane \( (\theta = \pi/2) \), yielding unstable circular orbits at the equatorial plane [39]. For \( \mathcal{K} > 0 \), the photons undergo non-planer motion and form spherical orbits at a constant radial coordinate. Orbits with \( \mathcal{K} < 0 \) are irrelevant for the black hole shadow [39]. On the other hand, \( \mathcal{V}_r(r) \) determines the radial motion.
Photons having sufficient energy to cross the local maxima in radial potential $V_r(r)$ fall into the black hole, whereas those with less energy scatter to the spatial infinity. Photons with energy identically same as the height of potential barrier follow the unstable orbits, which experience continuum turning points in their trajectories, and are characterized by the condition \[ V_r\big|_{r=r_p} = \frac{\partial V_r}{\partial r}\big|_{r=r_p} = 0; \] whereas instability of orbits obeys condition \[ \frac{\partial^2 V_r}{\partial r^2}\big|_{r=r_p} \geq 0, \] where $r_p$ is the photon orbit radius. To minimize the number of constants of motion we define the two dimensionless impact parameters $\eta \equiv K/E^2$ and $\xi \equiv L/E$ [39]. Solving Eq. (23), the critical values of impact parameters for photons unstable orbits read [40]

\[ \xi_c = -\frac{a^2 - 3r_p^2m(r_p) + r_p[a^2 + r_p^2][1 + m'(r_p)]}{a[m(r_p) + r_p[-1 + m'(r_p)]]}, \]

\[ \eta_c = -\frac{r_p^3[r_p^3 + 9r_p m(r_p)^2 + 2[2a^2 + r_p^2 + r_p^2m'(r_p)]r_p m'(r_p) - 2m(r_p)[2a^2 + 3r_p^2 + 3r_p^2 m'(r_p)]]}{a^2[m(r_p) + r_p[-1 + m'(r_p)]]^2}, \] which for the Kerr black hole ($m(r) = M$) reduces to [39]

\[ \xi_c = \frac{M(r_p^2 - a^2) - r_p(r_p^2 + a^2 - 2Mr_p)}{a(r_p - M)}, \]

\[ \eta_c = -\frac{r_p^3(4Ma^2 - r_p(r_p - 3M)^2)}{a^2(r_p - M)^2}. \] (24)

The impact parameters ($\eta, \xi$) are constant along the geodesics such that depending upon their values photons may get scatter ($\xi > \xi_c$), capture ($\xi < \xi_c$), or move along the unstable orbits ($\xi = \xi_c$). On the celestial sky of a far distant observer, these scattered photons mark the bright spots, whereas captured photons account for the dark region of the shadow [5, 37]. The projections of photons circular and spherical unstable orbits on the observer’s plane define the silhouette of black hole shadow [4]. These impact parameters ($\eta_c, \xi_c$) are related with the celestial coordinates ($\alpha, \beta$) on the observer’s sky, via [39]

\[ \alpha = \lim_{r_o \rightarrow \infty} \left(-r_o^2 \sin \theta_o \frac{d\theta}{dr}\bigg|_{r_o}\right) = \xi_c, \]

\[ \beta = \lim_{r_o \rightarrow \infty} \left(r_o^2 \frac{d\theta}{dr}\bigg|_{r_o}\right) = \pm \sqrt{\eta_c}, \] (26)
where \( \beta \)–axis is chosen along the rotational axis of the black hole. The observer is considered in the far distant asymptotically flat region \((r_o \to \infty)\) at an inclination angle \(\theta_o = \pi/2\); in this plane observer witness the maximum distortion in the shadow \([41]\). The contour of \(\beta\) vs \(\alpha\) determines the black hole shadow on the observer’s sky. Though the intensity distribution in the black hole shadow certainly depends upon the detailed and complex accretion model and emission processes, the shadow silhouette shape and size are determined solely by the geometry and the photon captured region \([1, 42, 43]\). In rotating black hole spacetime, photons can have prograde or retrograde motions, whose respective orbits radii at the equatorial plane, \(r^-_p\) and \(r^+_p\), can be determined by solving \(\beta = 0\) \([39]\). Whereas generic spherical photon orbits, having radii \(r^-_p \leq r_p \leq r^+_p\), construct a photon region around the rotating black hole, which is determined by Eqs. \((22)\) and \((24)\), and given by \((V_\theta(\theta) \geq 0)\)

\[
\cot^2 \theta \left[ \left( (a^2 - 3r^2_p)m(r_p) + r_p(a^2 + r^2_p)(1 + m'(r_p)) \right)^2 - a^4 \sin^2 \theta \left( m(r_p) + r_p(-1 + m'(r_p)) \right)^2 \right] \\
\leq -a^3 r^3_p \left[ r_p \left( r_p - 3m(r_p) \right)^2 + m'(r_p) r_p \left( 2r^2_p + m'(r_p) r^2_p + 4a^2 \right) - 2m(r_p) \left( 3m'(r_p) r^2_p + 2a^2 \right) \right],
\]

which can be re-written in the simplified form as follow

\[
(4r_p \Delta - \Delta' \Sigma)^2 \leq 16a^2 r^2_p \Delta \sin^2 \theta,
\]

the gravitationally lensed image of this photon region corresponds to the black hole shadow.

Further, we consider that the circularity deviation \(\Delta C\) can be used to characterize the distortion in black hole shadow from a perfect circle \([1, 43]\)

\[
\Delta C = 2 \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \left( R(\varphi) - \bar{R} \right)^2 d\varphi},
\]

such that \(\Delta C = 0\) for non-rotating black holes having circular shadows, \(R(\varphi)\) is the shadow silhouette radial distance from the shadow center \((\alpha_C, \beta_C)\), and \(\varphi\) is the angular coordinate. The shadow average radius \(\bar{R}\) is defined as \([43]\)

\[
\bar{R} = \frac{1}{2\pi} \int_0^{2\pi} R(\varphi) d\varphi.
\]

In particular, for the observed shadow of M87\(^*\) black hole, \(\Delta C \leq 0.10\) \([9]\).
A. Photons orbits around Kerr black hole

The horizons of the Kerr black hole are located at radii \( r_+ = M \pm \sqrt{M^2 - a^2} \), and \( r_\pm \equiv r_E = M \) for the extremal case \( (a = M) \). For Kerr black hole, Eq. (27) reduces to

\[
\left[ (r_p^2(r_p - 3M) + a^2(r_p + M))^2 - a^4(r_p - M)^2 \sin^2 \theta \right] \cot^2 \theta \leq -r_p^3 \left[ r_p(r_p - 3M)^2 - 4Ma^2 \right].
\]

(31)

The prograde and retrograde orbits at the equatorial plane have radii, respectively, given by

\[
r_p^- = 2M \left[ 1 + \cos \left( \frac{2}{3} \cos^{-1} \left( -\frac{|a|}{M} \right) \right) \right],
\]

\[
r_p^+ = 2M \left[ 1 + \cos \left( \frac{2}{3} \cos^{-1} \left( \frac{|a|}{M} \right) \right) \right],
\]

(32)

and they fall in the range \( M \leq r_p^- \leq 3M \) and \( 3M \leq r_p^+ \leq 4M \). Indeed, in the limit of zero rotation \( (a = 0) \), these two orbits coincide into a single circular orbit and photon region in Eq. (31) degenerate into a photon sphere of constant radius, \( r_p = 3M \) for the Schwarzschild black hole. The \( r_p^- \leq r_p^+ \) can be attributed to the Lense-Thirring effect [43] and due to the dragging of spacetime, the particles revolve around the black hole with respect to an observer at spatial infinity. For extremal Kerr black hole prograde orbits and event horizon have the same radii, viz., \( r_p^- = r_E = M \) for \( a = M \), however, they are still separated by a finite proper distance [37]. Thus, Kerr black holes \( (a \leq M) \) always have prograde and retrograde photons unstable orbits, and form a closed shadow silhouette [37]. Indeed, in the Kerr spacetimes, the closedness of the shadow silhouette is equivalent to the CCC [33]. Furthermore, for \( a > M \), suffice to ascertain a Kerr naked singularity, prograde photons spiral in and eventually end up in the central singularity, thereby the retrograde photon orbits, least affected by the black hole frame dragging, cause an open arc shadow (cf. Fig. 1) [45–47]. In the Kerr naked singularity spacetime, orbits with \( r_p < 0 \) are also crucial for \( \theta_o \neq \pi/2 \), because they escape into the other spatial infinity \( (r \to -\infty) \) and never reaches the observer \( (r_o \to \infty) \) [45]. Indeed, these photon orbits account for a dark central spot in the naked singularity shadows (cf. Fig. 1). Figure 1 shows that the shadow of Kerr naked singularity is markedly different from that of a Kerr black hole. On the other hand, Reissner-Nordström black hole spacetime exhibits an intriguing feature that the shadow silhouette can be closed even if CCC is violated \( (Q > M) \), in fact, the silhouette is closed
for $Q < \sqrt{9/8}M$ [33]. This is because of the presence of inner (stable) and outer (unstable) photon circular orbits in the Reissner-Nordström naked singularity spacetimes.

Since the occurrence of the naked singularity leads to the violation of the CCC, however, its understanding will helps to put CCC in concrete mathematical form. Indeed, some spherically symmetric naked singularity models shadows are found to resemble the Schwarzschild black hole shadows [15]. This motivates us to investigates the other possibilities of the no-horizon cases of rotating spacetimes, which can resemble the black hole shadow.

IV. SHADOW FOR NO-HORIZON ROTATING REGULAR SPACETIME

The observations may allow non-Kerr black holes as an astrophysical black hole candidate. Hence, we can use the above technique to determine how spherical photon orbits may change because of the deviation parameter $g$. In this section, we compare shadows produced by the Kerr black hole and Kerr naked singularity with those produced by the three well-known rotating regular no-horizon spacetimes, described by viz. Bardeen, Hayward and nonsingular.
A. Bardeen spacetime

The rotating Bardeen black hole belongs to the prototype non-Kerr family and described by metric (11) with the mass function

\[ m(r) = M \left( \frac{r^2}{r^2 + g^2} \right)^{3/2}, \]

such that in the limit \( g \to 0 \), it retrieve the Kerr black hole. The rotating Bardeen black hole shadows are smaller and more distorted than the Kerr black hole shadows [40, 48, 49].

The variation of horizon radii \( r_{\pm} \) and unstable circular orbit radii \( r_p^{\pm} \) with spin parameter \( a \) for rotating Bardeen black hole are depicted in Fig. 2. The photon region around the black hole grows with \( a \) but considerably decreases with \( g \). For small \( g \) (\( g \leq 0.572M \)) and increasing \( a \), prograde orbits approach the event horizon and both radii eventually coincide for \( a = a_E \). However, for sufficiently large values of \( g \) (\( > 0.572M \)) there exists \( a = a_E \) corresponding to the extremal black hole, for which the prograde orbits \( r_p^- \) do not merge with the extremal horizon \( r_E \) (cf. Fig. 2), i.e., \( \delta = r_p^- - r_E \neq 0 \). Table I summarizes the extremal horizon radius \( r_E \) and circular orbit radii \( r_p^{\pm} \) for various extremal black hole configurations, such that \( a_E \) decreases with increasing \( g \). For non-rotating Bardeen black hole with \( g = g_E = 0.7698M \), only one unstable circular orbit for photons exist outside the event horizon at \( r_p = 2.30118M \) whereas extremal horizon radii is \( r_E = 1.0886M \). However, in the no-horizon spacetimes, two circular orbits, inner being stable and outer being unstable, can

![FIG. 2: Radii of Cauchy horizon \( r_- \) (Red dashed curve), event horizon (Black solid curve), prograde orbit \( r_p^- \) (Blue small dashed curve), and retrograde orbit \( r_p^+ \) (Magenta dotted curve) with varying \( a \) for rotating Bardeen black hole.](image-url)
TABLE I: Table summarizing the values of extremal horizon radius $r_E$, prograde and retrograde photon orbit radii $r_p^-$, $r_p^+$ for rotating Bardeen black hole. For extremal black holes photon region size deceases with increasing $g$.

| $g/M$ | $a_E/M$ | $r_E/M$ | $r_p^−/M$ | $r_p^+/M$ | $\delta = (r_p^− - r_E)/M$ |
|-------|---------|---------|-----------|-----------|--------------------------|
| 0.0   | 1       | 1       | 1         | 4         | 0                       |
| 0.10  | 0.985177 | 1.01406 | 1.01406   | 3.98135   | 0                       |
| 0.20  | 0.942439 | 1.04762 | 1.04762   | 3.92633   | 0                       |
| 0.30  | 0.875075 | 1.08604 | 1.08604   | 3.83598   | 0                       |
| 0.40  | 0.785157 | 1.11879 | 1.11879   | 3.70880   | 0                       |
| 0.50  | 0.671965 | 1.13979 | 1.13979   | 3.53828   | 0                       |
| 0.60  | 0.529499 | 1.14404 | 1.23056   | 3.30719   | 0.08651                 |
| 0.70  | 0.334007 | 1.12449 | 1.62061   | 2.96033   | 0.49612                 |
| 0.769 | 0.035112 | 1.08923 | 2.22920   | 2.37297   | 1.13997                 |
| 0.76980 | 0.0 | 1.08866 | 2.30118   | 2.30118   | 1.21252                 |

FIG. 3: Parameter plane $(a, g)$ for the rotating Bardeen spacetime. The red dashed line separates the black hole spacetimes from the no-horizon spacetimes. The no-horizon spacetimes also form closed shadow silhouette when parameters $(a, g)$ lie in the black shaded region.

exist in the interval $g_E < g < g_c = 0.858650M$, and for $g = g_c$ both coincide at $r_p = 1.717M$ and no circular orbit can exist for $g > g_c$ [30]. This is because of the fact that for $a = 0$ and $g = g_E$, photon circular orbits do not merge with the black hole horizon (cf. Table I). For some particular values of parameters $(a, g)$, unstable photon circular orbits, especially
FIG. 4: Comparison of rotating Bardeen spacetime shadows. Black solid curves correspond to black holes, whereas dashed and dotted curves are for the no-horizon spacetimes. Prograde, may exist in the no-horizon rotating Bardeen spacetimes as well (cf. Fig. 3), which is not true for the Kerr naked singularities. For a black hole having anti-clockwise rotation, the prograde and retrograde orbits, respectively, construct the left and right edges of the shadow silhouette. Therefore, the presence of prograde orbits is crucial for the closed shadow silhouette, such that in their absence retrograde orbits solely outline an open arc shadow. Shadows of rotating Bardeen black holes for different values of $a$ and $g$ are depicted in Fig. 4 and are compared with the corresponding no-horizon rotating spacetime counterparts shadows. For small values of $g$ ($\leq 0.572M$), the shadows of no-horizon rotating Bardeen spacetimes ($a > a_E$) are always open arc and distinct from that of rotating Bardeen black hole ($a < a_E$) shadows, which have closed silhouette. Interestingly, shadows of no-horizon rotating Bardeen spacetime are similar to that of Kerr naked singularity, except that now shadows do not have a central dark spot due to photon orbits from $r < 0$. However, when $g$ is large ($g > 0.572M$), the no-horizon rotating Bardeen spacetime may posses prograde orbits and thus the shadows may still have a closed silhouette (cf. Fig. 4). In Fig. 4, we have shown that shadow of no-horizon spacetime ($g = 0.70M, a = 0.35M > a_E$) is indeed a closed curve with a circularity deviation parameter $\Delta C = 0.1588$. For $a > a_E$, the prograde orbits disappear and in turns, shadow again become an open arc (cf. Figs. 3 and 4). Thus, the no-horizon rotating Bardeen spacetime, with some specific values of parameters shown as a black shaded region in Fig. 3, cast closed shadows silhouette which resembles the rotating
FIG. 5: Deviation from circularity parameter $\Delta C$ for no-horizon rotating Bardeen spacetimes closed shadows silhouettes as a function of $(a, g)$. The black solid line is for $\Delta C = 0.10$, such that region on the right-side of the black line is excluded by the measured circularity of the M87* black hole reported by the EHT, $\Delta C \leq 0.10$.

black hole shadow. Therefore, the closed shadow silhouette does not ensure the existence of the black hole horizon. Figure 5 shown the deviation from circularity parameter $\Delta C$ as a function of $a$ and $g$ for the no-horizon rotating Bardeen spacetime closed shadows, clearly the M87* bound, $\Delta C \leq 0.10$ is satisfied.

B. Charged Hayward spacetime

Frolov [50] obtained the spherically symmetric charged Hayward black hole solutions. Later, Kumar et al. [12] derived the rotating counterpart solutions, whose line element is described by metric (11) with the mass function $m(r)$

$$m(r) = \frac{M (2Mr - Q^2) r^3}{2Mr^4 + g^3 (2Mr + Q^2)},$$

(34)

where $Q$ is the electric charge. The parameter $g$ is related to the length $\ell$ associated with the region concentrating the central energy density via $g^3 = 2M\ell^2$, such that modifications in the spacetime metric appear when the curvature scalars become comparable with $\ell^{-2}$ [22]. We shall now discuss the spherical photon orbits around the charged rotating Hayward black holes. Abdujabbarov et al. [40] studied the shadows cast by rotating regular Hayward
black hole ($Q = 0$) to infer that the shadow size deceases and become more distorted with $g$. Later, Kumar et al. [12] extended this result for the charged rotating Hayward spacetimes ($Q \neq 0$).

The behaviour of horizon radii $r_\pm$ and unstable photon orbit radii $r_p^\pm$ are analyzed and depicted in Fig. 6. The retrograde photons orbits radii increase while prograde orbits radii decrease with $a$. In Table II and III, we give horizon radii $r_E^\pm$ and unstable orbit radii $r_p^\pm$, for the case of extremal charged Hayward black holes, respectively, when $Q = 0$ and $Q = 0.5M$. From Tables II and III, we notice that the radii $r_E^+$ and $r_p^-$ increase with $g$, while $a_E$ decreases with $g$ and so is $r_p^+$. It turns out that for a given $g$ ($g \leq 0.731M$ for $Q = 0$ and $g \leq 0.602M$ for $Q = 0.50M$), there exists $a = a_E$ corresponding to the extremal black hole, such that the radial coordinate of prograde orbits and extremal horizon
\[
g/M \quad a_E/M \quad r_E/M \quad r^-_p/M \quad r^+_p/M \quad \delta = (r^-_p - r_E)/M
\]

| g/M | a_E/M | r_E/M | r^-_p/M | r^+_p/M | \delta = (r^-_p - r_E)/M |
|-----|------|-------|--------|--------|--------------------------|
| 0.0 | 1    | 1     | 1      | 4      | 0                        |
| 0.10| 0.999002 | 1.010018 | 1.001018 | 3.999002 | 0.0                      |
| 0.20| 0.99215 | 1.01501 | 1.01501 | 3.99212 | 0.0                      |
| 0.30| 0.974509 | 1.04466 | 1.04466 | 3.97424 | 0.0                      |
| 0.40| 0.942997 | 1.08796 | 1.08796 | 3.9417 | 0.0                      |
| 0.50| 0.896106 | 1.13802 | 1.13802 | 3.89193 | 0.0                      |
| 0.60| 0.832765 | 1.18888 | 1.18888 | 3.82226 | 0.0                      |
| 0.70| 0.751193 | 1.23642 | 1.23642 | 3.7285 | 0.0                      |
| 0.80| 0.647497 | 1.27766 | 1.42044 | 3.60281 | 0.142784                 |
| 0.90| 0.512192 | 1.30988 | 1.70778 | 3.42781 | 0.397897                 |
| 1.05827| 0.0 | 1.33333 | 2.65236 | 2.65236 | 1.31903                  |

**TABLE II:** Table representing the values of extremal horizon radius \( r_E \), prograde and retrograde photon orbit radii, respectively, \( r^-_p \) and \( r^+_p \) for Hayward black hole with \( Q = 0.0 \).

\[
g/M \quad a_E/M \quad r_E/M \quad r^-_p/M \quad r^+_p/M \quad \delta = (r^-_p - r_E)/M
\]

| g/M | a_E/M | r_E/M | r^-_p/M | r^+_p/M | \delta = (r^-_p - r_E)/M |
|-----|------|-------|--------|--------|--------------------------|
| 0.0 | 0.866025 | 1.0 | 1.0 | 3.73205 | 0.0                      |
| 0.10| 0.864891 | 1.00192 | 1.00192 | 3.73085 | 0.0                      |
| 0.20| 0.857094 | 1.01453 | 1.01453 | 3.72258 | 0.0                      |
| 0.30| 0.836956 | 1.04316 | 1.04316 | 3.70096 | 0.0                      |
| 0.40| 0.800728 | 1.08475 | 1.08475 | 3.66125 | 0.0                      |
| 0.50| 0.746117 | 1.13234 | 1.13234 | 3.59947 | 0.0                      |
| 0.60| 0.670646 | 1.17985 | 1.17985 | 3.51044 | 0.0                      |
| 0.70| 0.569346 | 1.22287 | 1.42794 | 3.38437 | 0.205072                 |
| 0.80| 0.428932 | 1.25787 | 1.73516 | 3.19725 | 0.47729                 |
| 0.90| 0.183365 | 1.28094 | 2.21264 | 2.8352 | 0.931701                 |
| 0.921771| 0.0 | 1.28385 | 2.53615 | 2.53615 | 1.25231                  |

**TABLE III:** Table representing the values of extremal horizon radius \( r_E \), prograde and retrograde photon orbit radii, respectively, \( r^-_p \) and \( r^+_p \) for charged Hayward black hole with \( Q = 0.50M \).
radius coincides, i.e., \( \delta \equiv r_p^- - r_E = 0 \). However, when \( g > 0.731 M \) for \( Q = 0 \) and \( g > 0.602 M \) for \( Q = 0.50 M \), they do not coincide, viz. \( \delta C > 0 \) and that \( \delta \) increases with \( g \). A numerical study of \( \beta = 0 \) shows that prograde orbits may prevail for the no-horizon rotating Hayward spacetime resulting into the alteration of the shadow (cf. Fig. 7). Such as, for the non-rotating cases with \( Q = 0.0 \) and \( Q = 0.50 M \), photon circular orbits exist for \( g \leq g_c = 1.21834 M \) and \( g \leq g_c = 1.082293 M \), respectively, whereas \( g_E = 1.05827 M \) and \( g_E = 0.921771 M \). For rotating case \( a = 0.10 M \), \( g_c = 1.164846 M \) and \( g_E = 1.05297 M \) for \( Q = 0.0 \), whereas \( g_c = 1.026494 M \) and \( g_E = 0.901829 M \) for \( Q = 0.50 M \). The shadows of rotating Hayward spacetimes for different values of parameters \((a, g, Q)\) are depicted in Fig. 8. For some particular values of parameters \((a, g)\), shown as black shaded region in Fig. 7, the shadows of no-horizon rotating regular spacetimes have closed silhouette and resemble shadow of rotating black hole (cf. Fig. 8). However, for significantly large violations of extremal bound \((a > a_E)\), the shadow silhouette is again an open curve as shown in Fig. 8. The deviation from circularity parameter \( \Delta C \) of these shown no-horizon spacetime shadows \((Q = 0.0, g = 0.90 M, a = 0.52 M)\) and \((Q = 0.50 M, a = 0.43 M, g = 0.80 M)\) are, respectively, 0.19242 and 0.13816. Further, in Fig. 9, the deviation from circularity parameter \( \Delta C \) is depicted for the no-horizon rotating Hayward spacetime closed shadows, over the finite parameter space \((a, g)\) the M87* bound, \( \Delta \leq 0.10 \) is satisfied.
FIG. 8: Comparison of charged rotating Hayward spacetime shadows. Black solid curves correspond to black holes, whereas dashed and dotted curves are for the no-horizon spacetimes.

C. Nonsingular spacetime

The nonsingular black hole is a novel class of regular black hole, which in contrast to the Bardeen and Hayward regular black holes, has an asymptotically Minkowski core [51, 52]. These rotating nonsingular or regular black holes also belong to the family of non-Kerr black holes in which the metric tensor resembles with that of the Kerr black hole with mass $M$ replaced by $m(r) = Me^{-g^2/2Mr}$ [27]. Further, it may look like a Kerr black hole with a different spin [53]. It turns out that the rotating nonsingular black hole shadows get more distorted and smaller for $g \neq 0$ when compared with the Kerr black hole shadows [54].
FIG. 9: Deviation from circularity parameter $\Delta C$ for no-horizon rotating Hayward spacetimes closed shadows silhouettes as a function of $(a, g)$, $Q = 0.0$ (left panel) and $Q = 0.50M$ (right panel). The black solid line is for $\Delta C = 0.10$, such that region on the right-side of the black line is excluded by the measured circularity of the M87* black hole reported by the EHT, $\Delta C \leq 0.10$.

FIG. 10: Radii of Cauchy horizon $r_-$ (Red dashed curve), event horizon $r_+$ (Black solid curve), prograde orbit $r_p^-$ (Blue small dashed curve), retrograde orbit $r_p^+$ (Magenta dotted curve) for rotating nonsingular black holes.

The horizon radii $r_\pm$ and photon orbit radii $r_p^\pm$ are shown as a function of spin parameter $a$ in Fig. 10, and the qualitative behavior is very much similar to those for rotating Bardeen and charged rotating Hayward black holes. Table IV summarizes the values of $r_\pm$ and $r_p^\pm$ for various extremal black hole configurations, such as for given $g$ ($g \leq 1.086M$) and $a = a_E$, $\delta = r_p^- - r_E = 0$. Photon unstable circular orbits may exist in the no-horizon
\[ \delta = (r_p^- - r_E)/M \]

| \( g/M \) | \( a_E/M \) | \( r_E/M \) | \( r_p^-/M \) | \( r_p^+/M \) | \( \delta = (r_p^- - r_E)/M \) |
|---|---|---|---|---|---|
| 0.0 | 1 | 1 | 1 | 4 | 0 |
| 0.10 | 0.995 | 0.99998 | 0.99998 | 3.98999 | 0 |
| 0.20 | 0.979999 | 0.999803 | 0.999803 | 3.95985 | 0 |
| 0.30 | 0.954984 | 0.999015 | 0.999015 | 3.90921 | 0 |
| 0.40 | 0.919914 | 0.996948 | 0.996948 | 3.83741 | 0 |
| 0.50 | 0.874669 | 0.992707 | 0.992707 | 3.74339 | 0 |
| 0.60 | 0.818995 | 0.98521 | 0.98521 | 3.62551 | 0 |
| 0.70 | 0.752392 | 0.973158 | 0.973158 | 3.48125 | 0 |
| 0.80 | 0.673917 | 0.954954 | 0.954954 | 3.30659 | 0 |
| 0.90 | 0.581749 | 0.928489 | 0.928489 | 3.09472 | 0 |
| 1.00 | 0.472045 | 0.89064 | 0.89064 | 2.83264 | 0 |
| 1.10 | 0.334606 | 0.835888 | 0.884741 | 2.48879 | 0.0488538 |
| 1.20 | 0.108811 | 0.750887 | 1.34572 | 1.89459 | 0.548865 |
| 1.21306 | 0.0 | 0.73576 | 1.61053 | 1.61053 | 0.874771 |

TABLE IV: Horizon radius \( r_E \), prograde and retrograde photon orbit radii \( r_p^- \), \( r_p^+ \) for various extremal rotating nonsingular black holes. Photon region size decrease with increasing \( g \).

**FIG. 11:** Parameter plane \((a, g)\) for the rotating nonsingular spacetime. The red dashed line separates the black hole spacetimes from the no-horizon spacetimes. The no-horizon spacetimes also form closed shadow silhouette when parameters \((a, g)\) lie in the black shaded region.

nonsingular spacetime, viz., for \( a = 0 \), photon prograde orbits exist for \( g \leq 1.264523M \)
FIG. 12: Comparison of rotating nonsingular spacetime shadows. Black solid curves correspond to black holes, whereas dashed and dotted curves are for the no-horizon spacetimes.

FIG. 13: Deviation from circularity parameter $\Delta C$ for no-horizon rotating nonsingular spacetimes closed shadows silhouettes as a function of $(a, g)$. The black solid line is for $\Delta C = 0.10$, such that region on the right-side of the black line is excluded by the measured circularity of the M87* black hole reported by the EHT, $\Delta C \leq 0.10$.

whereas $g_E = 1.21306M$. Interestingly, these orbits continue to persist even in the no-horizon rotating spacetimes for suitable values of parameters $(g > 1.086M)$ shown as a black shaded region in Fig. 11. This parameter space, admitting the photon orbits, for
no-horizon rotating nonsingular spacetime, is comparatively smaller than those for Bardeen and charged Hayward spacetimes. In Fig. 12 the shadows of nonsingular spacetimes are depicted, and it is evident that the shadow of no-horizon spacetime \((a = 0.12M, g = 1.20M)\) has closed silhouette with \(\Delta C = 0.0438986\) and shows resemblance with the rotating black hole \((a = 0.10M, g = 1.20M)\) shadow. The circularity deviation parameter \(\Delta C\) for the no-horizon rotating nonsingular spacetime closed shadows is shown in Fig. 13, clearly the bound, \(\Delta C \leq 0.10\), for the observed M87* black hole shadow, is satisfied. Hence, the closed shadow silhouette may correspond to both a rotating black hole or a no-horizon rotating spacetime.

V. CONCLUSION

Black hole shadow, a projection along the photon orbits whirling around the black hole many times before reaching the observer, has imprints of the spacetime geometry and is anticipated to deduce the exact nature of the black hole. We examine shadows cast by three familiar rotating regular no-horizon spacetimes, namely Bardeen, charged Hayward and nonsingular, and compared them with the shadows produced by a Kerr black hole and Kerr naked singularity. We find that rotating regular no-horizon spacetimes (but not always), depending on the values of parameters, could cast closed shadows silhouettes that are very similar to those produced by the Kerr black holes (cf. Figs. 4, 8 and 12).

To illustrate this, we showed that the existence of photon unstable prograde and retrograde circular orbits in the no-horizon rotating spacetimes leads to the closed shadow silhouette with bounds on parameters \((a, g)\). For instance, with \(a = 0.10M\), Bardeen \((g_E = 0.763332M < g \leq g_c = 0.816792M)\), Hayward \((Q = 0.0, g_E = 1.05297M < g \leq g_c = 1.164846M\) and \(Q = 0.50M, g_E = 0.901829M < g \leq g_c = 1.026494M\) ) and nonsingular \((g_E = 1.2020M < g \leq g_c = 1.222461M)\) no-horizon spacetimes cast closed shadow silhouette. The bounds on the spin parameter for Bardeen, Hayward \((Q = 0.0\) and \(Q = 0.50M)\) and nonsingular no-horizon spacetime closed shadow silhouette \((g_E < g \leq g_c)\), respectively, are \(0 \leq a \leq 0.5730M\), \(0 \leq a \leq 0.7216M\), \(0 \leq a \leq 0.6688M\) and \(0 \leq a \leq 0.3563M\). When these values of \(a\) and \(g\) are outside the obtained bounds, the prograde orbits disappear and shadows of no-horizon rotating spacetimes turn into an open arc, but do not have a central dark disc (cf. Figs. 4, 8 and 12), thereby are different from the
Kerr naked singularity open arc shadow for $\theta_o \neq \pi/2$ (cf. Fig. 1). This result is contrary to Kerr no-horizon spacetimes (naked singularities) which never form closed shadow silhouette, but is in accordance with the Kerr-Newman no-horizon (naked singularities) which have similar features.

Our study suggesting that the no-horizon rotating spacetime shadows may resemble the Kerr black hole shadows, therefore it is of theoretical as well as of astrophysical relevance in light of ongoing shadow observations. The key indication that follows from our analysis is that the mere existence of a closed shadow does not by itself imply the existence of a black hole or event horizon.

In conclusion, we have examined shadows of rotating regular no-horizon spacetimes, compared them with those for the black holes, and saw whether the shadows corresponding to the two are clearly distinguishable. Interestingly, while black holes always cast a closed shadow silhouette, rotating regular no-horizon spacetimes may or may not. Therefore, while black holes imply the existence of shadows, the converse is not true. A shadow could be produced by rotating regular no-horizon spacetimes as well. Further, while Kerr no-horizon spacetimes never produce a closed shadow silhouette, Kerr-Newman and three rotating regular spacetimes no-horizon spacetimes, for certain parameter values, do generate a closed shadow silhouette.

Further, the circularity deviation parameter for the no-horizon rotating spacetime shadows, for certain values of parameters $(a, g)$, may satisfy the bound deduced for the observed shadow of M87* black hole, i.e., $\Delta C \leq 0.10$.

Other interesting phenomena related to the no-horizon rotating spacetimes can be expected in the presence of accretion disk, this and related study are subjects of our future investigation.

VI. ACKNOWLEDGEMENT

S.G.G. would like to thank DST INDO-SA bilateral project DST/INT/South Africa/P-06/2016, SERB-DST for the ASEAN project IMRC/AISTDF/CRD/2018/000042 and also to IUCAA, Pune for the hospitality while this work was being done. R.K. would like to thank UGC for providing SRF.
Appendix A: Kerr-Newman naked singularity

The Kerr-Newman black hole is described by metric (11) with mass function

\[ m(r) = M - \frac{Q^2}{2r}. \]  

The behavior of horizon radii \( r_\pm \) and photon orbit radii \( r_p^\pm \) with varying spin parameter \( a \) and for different values of \( Q \) is depicted in Fig. 14, which shows that prograde orbits radii \( r_p^- \) decrease and retrograde orbits radii \( r_p^+ \) increase with \( a \). Table V summarize these values for various extremal black hole configurations, such that for given \( Q \) (\( Q \leq 0.8661M \)) there exists \( a = a_E \), for which prograde orbits coincide with the extremal horizon radii, i.e., \( \delta = r_p^- - r_E = 0 \). Whereas for \( Q > 0.8661M \) and \( a = a_E, \delta = r_p^- - r_E \neq 0 \) and \( \delta \) further increases with \( Q \). Thus, for sufficiently large values of electric charge \( Q (> 0.8661M) \) and small rotation parameter, prograde photon orbits can persist for naked singularity spacetimes.

Kerr-Newman black hole spacetimes posses retrograde and prograde unstable photon orbits, moreover, these orbits also exist in the naked singularity spacetime with finite parameter space \((a, Q)\) shown as a black shaded region in Fig. 15, viz. for \( a = 0.10M, 0.99498M < Q \leq 1.0305201M \). Outside the parameter space, prograde orbits disappear. This leads to an important consequence that the loci of photon orbits in the Kerr-Newman naked singularity spacetimes may form a closed silhouette, which is strikingly different from that for the Kerr naked singularity (cf. Figs. 1, 16 and 17). The central small dark spot in Fig. 16 corresponds to the admissible photon orbits in the negative \( r \) region. It is clear that only

![FIG. 14: Radii of Cauchy horizon \( r_- \) (Red dashed curve), event horizon \( r_+ \) (Black solid curve), prograde orbit \( r_p^- \) (Blue small dashed curve), retrograde orbit \( r_p^+ \) (Magenta dotted curve) for Kerr-Newman black holes.](image)
TABLE V: Table summarizing the values of extremal horizon radius $r_E$, prograde and retrograde photon orbit radii, respectively, $r_p^-$ and $r_p^+$ of Kerr-Newman black holes. With increasing $Q$ the size of the photon region decreases.

| $Q/M$ | $aE/M$ | $rE/M$ | $r_p^-/M$ | $r_p^+/M$ | $\delta = (r_p^- - r_E)/M$ |
|-------|--------|--------|-----------|-----------|---------------------------|
| 0.0   | 1      | 1      | 1         | 4         | 0                         |
| 0.10  | 0.994987 | 1      | 1         | 3.98997   | 0                         |
| 0.20  | 0.979795 | 1      | 1         | 3.95959   | 0                         |
| 0.30  | 0.953939 | 1      | 1         | 3.90788   | 0                         |
| 0.40  | 0.916515 | 1      | 1         | 3.833033  | 0                         |
| 0.50  | 0.866025 | 1      | 1         | 3.73205   | 0                         |
| 0.60  | 0.80    | 1      | 1         | 3.60      | 0                         |
| 0.70  | 0.714143 | 1      | 1         | 3.42829   | 0                         |
| 0.80  | 0.60    | 1      | 1         | 3.20      | 0                         |
| 0.90  | 0.435890 | 1      | 1.12822   | 2.87178   | 0.12822                   |
| 0.95  | 0.312249 | 1      | 1.37550   | 2.62450   | 0.37550                   |
| 0.99  | 0.0141067| 1      | 1.71787   | 2.28213   | 0.71787                   |

FIG. 15: Parameter plane $(a, Q)$ for the Kerr-Newman spacetime. The red dashed line separates the black hole spacetimes from the naked singularity spacetimes. The naked singularity spacetimes also form closed shadow silhouette when parameters $(a, Q)$ lie in the black shaded region.

Slowly rotating no-horizon spacetimes ($0 \leq a \leq 0.4998M$) can cast closed shadow silhouette (cf. Table V and Fig. 15). In Fig. 18, shadow circularity deviation parameter $\Delta C$ is plotted.
as a function of \((a, Q)\) for the Kerr-Newman naked singularity spacetimes causing the closed shadow silhouette. It is evident that these naked singularities closed shadows can satisfy the
FIG. 18: Deviation from circularity parameter $\Delta C$ for Kerr-Newman naked singularities closed shadow silhouettes as a function of $(a, g)$. The black solid line is for $\Delta C = 0.10$, such that region on the right-side of the black line is excluded by the measured circularity of the M87* black hole reported by the EHT, $\Delta C \leq 0.10$.

bound, $\Delta C = 0.10$, inferred from the M87* black hole shadow.

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