Constraints on Quartic Vector-Boson Interactions

from $Z$ Physics

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(March 26, 2022)

Abstract

We obtain the constraints on possible anomalous quartic vector-boson vertices arising from the precision measurements at the $Z$ pole. In the framework of $SU(2)_L \otimes U(1)_Y$ chiral Lagrangians, we examine all effective operators of order $D = 4$ that lead to four-gauge-boson interactions but do not induce anomalous trilinear vertices. We constrain the anomalous quartic interactions by evaluating their one-loop corrections to the $Z$ pole physics. Our analysis is performed in a generic $R_\xi$ gauge and it shows that only the operators that break the $SU(2)_C$ custodial symmetry get limits close to the theoretical expectations. Our results also indicate that these anomalous couplings are already out of reach of the Next Linear $e^+e^-$ Collider, while the Large Hadron Collider could be able to further extend the bounds on some of these couplings.

CERN-TH/96-22

February 1996
I. INTRODUCTION

The standard model (SM) of electroweak interactions has been the subject of an intense experimental research that confirmed its predictions for the interactions between fermions and vector bosons [1]. However, some elements of the SM, such as the symmetry breaking mechanism and the interaction among the gauge bosons, have not been the object of direct experimental observation yet. In particular, the structure of the triple and quartic vector-boson couplings is completely determined by the $SU(2)_L \otimes U(1)_Y$ gauge structure of the model, and a detailed study of these interactions can either further confirm the local gauge invariance of the theory or indicate the existence of new physics beyond the SM.

Presently only hadronic colliders have directly studied the triple vertex $W^+W^-\gamma$ [2]; however, their constraints on this coupling are very loose. One of the main goals of LEP II at CERN will be the investigation of the reaction $e^+e^- \rightarrow W^+W^-$, which can furnish direct bounds on anomalous $W^+W^-\gamma$ and $W^+W^-Z$ interactions [3]. Future hadron [4] and $e^+e^-$, $e\gamma$, and $\gamma\gamma$ [5] colliders will also provide information on these couplings and improve significantly our knowledge of possible anomalous gauge-boson interactions.

Direct studies of quartic vector-boson interactions cannot be performed at the colliders in operation since the available centre-of-mass energy is not sufficient for multiple vector-boson production. This crucial test of the gauge structure of the SM will be possible only at the CERN Large Hadron Collider (LHC) through the reaction $pp \rightarrow V_LV_LX$ [6–8] or at the next linear collider (NLC) through the processes $e^+e^- \rightarrow VVV$ [9,10], $e^-e^- \rightarrow FFVV$ [11], $e\gamma \rightarrow VVF$ [12], $\gamma\gamma \rightarrow VV$ [13], and $\gamma\gamma \rightarrow VVV$ [14], where $V = Z$, $W^\pm$ or $\gamma$ and $F = e$ or $\nu_e$. However, these machines will not operate in the near future, and consequently we will have to rely on indirect information on the quartic vertices for quite some time.

Valuable information on anomalous interactions can also be gathered from the low energy data [15] and the results of the $Z$ physics [16–19], which can also constrain substantially the possible deviations of the gauge boson self-interactions from the SM predictions through their contributions to the electroweak radiative corrections. So far all the analyses have
concentrated on operators which generate tree-level modifications to the gauge-boson two- 
point or three-point functions.

In this work, we obtain the constraints on quartic vector-boson self-interactions arising 
from the precision measurements at LEP and the SLC. We focus our attention on genuinely 
anomalous quartic operators, \textit{i.e.} operators that do not modify the trilinear vertices. These 
anomalous interactions cannot be constrained by the direct LEP II analysis of the vertices 
$W^+W^-\gamma$ and $W^+W^-Z$. In general, anomalous quartic couplings arise as the low-energy 
limit of heavy state exchange, whereas trilinear couplings are modified by integrating out 
heavy fields. Therefore, deviations on the triple gauge-vector couplings should be harder to 
observe than the ones on the quartic couplings, since the former are suppressed by factors 
of $1/16\pi^2$ \cite{20}. Furthermore, it is even possible to conceive extensions of the SM where the 
trilinear couplings remain unchanged, while the quartic vertices receive new contributions. 
For instance, the introduction of a new heavy scalar singlet, which interacts strongly with 
the Higgs sector of the SM, enhances the quartic vector-boson interaction without affecting 
either the triple vector-boson couplings or the SM predictions for the $\rho$ parameter \cite{21}.

At the one-loop level, anomalous quartic vector-boson interactions contribute to the $Z$ 
pole physics through universal corrections to the gauge-boson self-energies. In general, the 
oblique radiative corrections can be parametrized in terms of three observables $S$, $T$, and 
$U$ \cite{22}, or equivalently $\epsilon^1$, $\epsilon^2$, and $\epsilon^3$ \cite{23}. We shall obtain the constraints on anomalous 
quartic vertices by imposing that their one-loop contributions are compatible with the $Z$ 
pole data \cite{24,25}. Since the SLC and LEP I achieved a precision of the order of a per mille 
in some observables, the $Z$ pole physics is the best available source of information on quartic 
vector-boson interactions.

\textbf{II. THEORETICAL FRAMEWORK}

If the electroweak symmetry breaking is due to a heavy (strongly interacting) Higgs 
boson, which can be effectively removed from the physical low-energy spectrum, or to no
fundamental Higgs scalar at all, one is led to consider the most general effective Lagrangian which employs a nonlinear representation of the spontaneously broken $SU(2)_L \otimes U(1)_Y$ gauge symmetry \cite{26}. The resulting chiral Lagrangian is a non-renormalizable non-linear $\sigma$ model coupled in a gauge-invariant way to the Yang-Mills theory. This model independent approach incorporates by construction the low-energy theorems \cite{27}, that predict the general behavior of Goldstone boson amplitudes irrespective of the details of the symmetry breaking mechanism. Notwithstanding, unitarity implies that this low-energy effective theory should be valid up to some energy scale smaller than $4\pi v \simeq 3$ TeV, where new physics would come into play.

To specify the effective Lagrangian one must first fix the symmetry breaking pattern. We consider that the system presents a global $SU(2)_L \otimes SU(2)_R$ symmetry that is broken to $SU(2)_C$. With this choice, the building block of the chiral Lagrangian is the dimensionless unimodular matrix field $\Sigma(x)$, which transforms under $SU(2)_L \otimes SU(2)_R$ as $(2,2)$:

$$\Sigma(x) = \exp\left(i \frac{\varphi^a(x) \tau^a}{v}\right), \quad (1)$$

where the $\varphi^a$ fields are the would-be Goldstone fields and $\tau^a$ ($a = 1, 2, 3$) are the Pauli matrices. The $SU(2)_L \otimes U(1)_Y$ covariant derivative of $\Sigma$ is defined as

$$D_\mu \Sigma \equiv \partial_\mu \Sigma + ig \frac{\tau^a}{2} W^a_\mu \Sigma - ig' \Sigma \frac{\tau^3}{2} B_\mu . \quad (2)$$

At this point, it is convenient to introduce the following auxiliary quantities

$$T \equiv \Sigma \tau^3 \Sigma^\dagger , \quad (3)$$

$$V_\mu \equiv (D_\mu \Sigma) \Sigma^\dagger , \quad (4)$$

which are $SU(2)_L$-covariant and $U(1)_Y$-invariant. Notice that $T$ is not invariant under $SU(2)_C$ custodial due to the appearance of $\tau^3$ in its expression.

The lowest-order terms in the derivative expansion of the effective Lagrangian are

\footnote{We follow the notation of Ref. \cite{26}.}
The first term of the above equation is responsible for giving mass to the gauge bosons $W^\pm$ and $Z$ for $v = (\sqrt{2} G_F)^{-1}$. The second term violates the custodial $SU(2)_C$ symmetry and contributes to $\Delta \rho$ at the tree level, being strongly constrained by the low-energy data. This term can be understood as the low-energy remnant of the high-energy custodial symmetry breaking physics, which has been integrated out above a certain scale $\Lambda$. Moreover, at the one-loop order, it is also required in order to cancel the divergences in $\Delta \rho$, arising from diagrams containing a hypercharge boson in the loop [26]. This subtraction renders $\Delta \rho$ finite, although dependent on the renormalization scale.

At the next order in the derivative expansion $D = 4$, there are many operators that can be written down [26]. We shall restrict ourselves to the ones that exhibit genuine quartic vector-boson interactions. These operators are

$$L_4^{(4)} = \alpha_4 [\text{Tr} (V_{\mu} V_{\nu})] \frac{v^2}{4} \left[ (D_{\mu} \Sigma)^\dagger (D^{\nu} \Sigma) \right],$$

$$L_5^{(4)} = \alpha_5 [\text{Tr} (V_{\mu} V^{\mu})] \frac{v^2}{4} \left[ (\text{Tr} (TV_{\mu}) \right)^2,$$

$$L_6^{(4)} = \alpha_6 \text{Tr} (V_{\mu} V_{\nu}) \text{Tr} (TV^{\mu}) \text{Tr} (TV^{\nu}) ,$$

$$L_7^{(4)} = \alpha_7 \text{Tr} (V_{\mu} V^{\mu}) [\text{Tr} (TV^{\nu})]^2,$$

$$L_{10}^{(4)} = \alpha_{10} [\text{Tr} (TV_{\mu}) \text{Tr} (TV_{\nu})]^2 .$$

In an arbitrary gauge, these Lagrangian densities lead to quartic vertices involving gauge bosons and/or Goldstone bosons. In the unitary gauge, these effective operators give rise to anomalous $ZZZZ$ (all operators), $W^+W^-ZZ$ (all operators except $L_{10}^{(4)}$), and $W^+W^-W^+W^-$ ($L_4^{(4)}$ and $L_5^{(4)}$) interactions. Moreover, the interaction Lagrangians $L_6^{(4)}$, $L_7^{(4)}$, and $L_{10}^{(4)}$ violate the $SU(2)_C$ custodial symmetry. Notice that quartic couplings involving photons remain untouched by the genuinely quartic anomalous interactions at the order $D = 4$. The Feynman rules for the quartic couplings generated by these operators can be found in the last article of Ref. [26].

In our calculations, we adopted an arbitrary $R_\xi$ gauge, whose gauge-fixing Lagrangian is
\[
\mathcal{L}_{GF} = -\frac{1}{2\xi_B} f_0^2 - \frac{1}{2\xi_W} \left( \sum_{i=1}^{3} f_i^2 \right),
\]

(11)

where

\[
f_0 = \partial_{\mu} B^\mu - \frac{i}{4} g' v \xi_B \text{Tr}(\tau^3 \Sigma),
\]

(12)

\[
f_i = \partial_{\mu} W_{i}^\mu + \frac{i}{4} g v \xi_W \text{Tr}(\tau^i \Sigma),
\]

(13)

with \( g \) (\( g' \)) being the \( SU(2)_L \) (\( U(1)_Y \)) coupling constant.

At the one-loop level, the effective interactions (6) – (10) contribute to the \( Z \) physics only through corrections to the gauge boson propagators (\( \Sigma_{\text{new}} \)). The anomalous oblique corrections can be efficiently summarized in terms of the parameters \( S_{\text{new}}, T_{\text{new}}, \) and \( U_{\text{new}} \) \([22]\), or the equivalent set \( \epsilon^1_{\text{new}}, \epsilon^2_{\text{new}}, \) and \( \epsilon^3_{\text{new}} \) \([23]\), whose expressions as functions of the unrenormalized gauge boson self-energies are

\[
\frac{\alpha S_{\text{new}}}{4 s_w^2} \equiv \frac{1}{M_Z^2} \left\{ c_w^2 \left[ \Sigma_{\text{new}}^\gamma(M_Z^2) + \Sigma_{\text{new}}^Z(0) - \Sigma_{\text{new}}(M_Z^2) \right] \right. \\
- s_w c_w \left( \frac{c_w}{s_w} - 1 \right) \left[ \Sigma_{\text{new}}^Z(M_Z^2) - \Sigma_{\text{new}}^Z(0) \right] \left\} = \epsilon^3_{\text{new}}, \right.
\]

(14)

\[
\alpha T_{\text{new}} \equiv \frac{\Sigma_{\text{new}}^Z(0)}{M_Z^2} - \frac{\Sigma_{\text{new}}^W(0)}{M_W^2} - 2 s_w^2 c_w \Sigma_{\text{new}}^Z(0) = \epsilon^1_{\text{new}}; \]

(15)

\[
\alpha U_{\text{new}} \equiv \left\{ \frac{\Sigma_{\text{new}}^W(0) - \Sigma_{\text{new}}^W(M_W^2)}{M_W^2} + 2 s_w^2 \Sigma_{\text{new}}^Z(M_Z^2) \right. \\
- 2 s_w c_w \left. \left[ \Sigma_{\text{new}}^Z(M_Z^2) - \Sigma_{\text{new}}^Z(0) \right] + c_w^2 \frac{\Sigma_{\text{new}}^Z(M_Z^2) - \Sigma_{\text{new}}^Z(0)}{M_Z^2} \right\} = -\epsilon^2_{\text{new}}, \right.
\]

(16)

where \( \alpha \) is the fine structure constant and \( s_w \) (\( c_w \)) is the sine (cosine) of the weak mixing angle. These expressions for \( S_{\text{new}}, T_{\text{new}}, \) and \( U_{\text{new}} \) are valid for an arbitrary momentum dependence of the vacuum polarization diagrams \([28]\); they recover the original definitions of Ref. \([22]\), when we consider only the first two terms in the momentum expansion of the self-energies. This is the case of the present work. Since the contribution from the new operators to the \( Z \) observables occurs only through the gauge-boson vacuum polarization diagrams, which are momentum-independent, we can also express the new contributions in terms of the \( \epsilon \) parameters from Ref. \([23]\).
Recent global analyses of the LEP, SLD, and low-energy data yield the following values for the oblique parameters [24]:

\[
\begin{align*}
\epsilon^1 &= \epsilon_{SM}^1 + \epsilon_{new}^1 = (5.1 \pm 2.2) \times 10^{-3}, \\
\epsilon^2 &= \epsilon_{SM}^2 + \epsilon_{new}^2 = (-4.1 \pm 4.8) \times 10^{-3}, \\
\epsilon^3 &= \epsilon_{SM}^3 + \epsilon_{new}^3 = (5.1 \pm 2.0) \times 10^{-3}.
\end{align*}
\]

In Ref. [25], the results are obtained in terms of \( S_{new}, T_{new}, \) and \( U_{new} \), consistent with Eqs. (17). In order to extract the value of the oblique parameters due to new physics, we must subtract the SM contribution, which depends upon the SM parameters, in particular, on the top quark mass \( m_{top} \).

**III. RESULTS AND CONCLUSIONS**

We used dimensional regularization [23] to evaluate the one-loop contributions from the effective interactions (6) – (10), in order to preserve gauge invariance and to keep the ordering of the different contributions simple. In analogy to what happens for chiral Lagrangians applied to low-energy QCD [30], the electroweak chiral Lagrangian leads to an expansion in powers of the momentum \( p \) and the weak coupling constant \( g \). This \( g \) dependence is due to the introduction of new degrees of freedom associated to the gauge bosons. At a given order in \( g \) and \( p \), there are just a finite number of operators and loop diagrams that contribute to a process. Therefore, the effective theory renormalization can be carried out by renormalizing the coupling constants of the operators that appear in the process at the order that the analysis is being done. In this work, we evaluate the one-loop contributions from \( D = 4 \) interactions, where \( D \) counts the number of derivative plus the numbers of gauge bosons in the operator, which lead to \( g^4 p^2 \) corrections. In a complete calculation, we should also include the effects of two-loop graphs of \( D = 2 \) operators and tree-level contributions from \( D = 6 \) operators. At the end of the day, the experimental results for the oblique parameters would constrain combinations of the coupling constants appearing in the \( D = 2, D = 4, \) and \( D = 6 \) operators [31]. However, this next-to-next-to-leading order calculation contains a
large number of free parameters that reduces its usefulness. Notwithstanding, we can bound the anomalous quartic interactions from their oblique corrections under the naturalness assumption that no cancelation takes place amongst the $L^{(2)}$, $L^{(4)}$, and $L^{(6)}$ contributions that appear at the same order in the expansion. The motivation for this assumption is that the effects of the new physics and states from higher-energy scales must manifest themselves in a very clear way, otherwise they are very hard to observe.

Our procedure to bound the operators (6) – (10) is the following: first we evaluate their oblique corrections using dimensional regularization. Then, we use the leading non-analytic contributions from the loop diagrams to constrain the quartic interactions – that is, we keep only the terms proportional to $\log(\mu^2)$, dropping all others. The contributions that are relevant for our analysis are easily obtained by the substitution

$$\frac{2}{4 - d} \rightarrow \log \frac{\Lambda^2}{M_Z^2},$$

where $\Lambda$ is the energy scale which characterizes the appearance of new physics.

Using the above procedure, we obtained that $S_{new} = U_{new} = 0$ and that only $T_{new}$ is non-vanishing for all the quartic anomalous interactions, being given by

$$\alpha T_{new} = \epsilon^1_{new} = -\frac{15\alpha_4}{64\pi^2} g^4 (1 + c_W^2) \frac{s_W^2}{c_W^2} \log \frac{\Lambda^2}{M_Z^2},$$

$$\alpha T_{new} = \epsilon^1_{new} = -\frac{3\alpha_5}{32\pi^2} g^4 (1 + c_W^2) \frac{s_W^2}{c_W^2} \log \frac{\Lambda^2}{M_Z^2},$$

$$\alpha T_{new} = \epsilon^1_{new} = -\frac{3\alpha_6}{64\pi^2} g^4 \left(2 + \frac{11}{c_W^4}\right) \log \frac{\Lambda^2}{M_Z^2},$$

$$\alpha T_{new} = \epsilon^1_{new} = -\frac{3\alpha_7}{64\pi^2} g^4 \left(\frac{1}{c_W^4}\right) \log \frac{\Lambda^2}{M_Z^2},$$

$$\alpha T_{new} = \epsilon^1_{new} = -\frac{9\alpha_{10}}{8\pi^2} g^4 \left(\frac{1}{c_W^4}\right) \log \frac{\Lambda^2}{M_Z^2},$$

for $L^{(4)}_4$, $L^{(4)}_5$, $L^{(4)}_6$, $L^{(4)}_7$ and $L^{(4)}_{10}$, respectively.

Our calculation has been done in a general $R_\xi$ gauge and we have explicitly verified the cancellation of the $\xi$-dependent terms, indicating that our result is gauge-invariant. Ward identities relate the two- and three-point functions, and consequently the anomalous
contributions to the two-point functions are gauge-independent since there is no one-loop three-point contribution due to the effective quartic interactions.

Our first step towards obtaining the bounds on the anomalous quartic vertices is to determine the SM contribution to $\epsilon^1$. As discussed above, the gauge-boson contribution to this parameter is infinite as a consequence of the absence of the elementary Higgs. On the other hand, we must also include the tree level effect due to the $\beta_1$ operator in Eq. (3), which absorbs this infinity through the renormalization of the $\beta_1$ constant. If the renormalization condition is imposed at a scale $\Lambda$, we are left with the contribution due to the running of $\beta_1$ from the scale $\Lambda$ to $M_Z$. Therefore, the SM contribution without the Higgs boson will be the same as that of the SM with an elementary Higgs, with the substitution $\ln(M_H) \rightarrow \ln(\Lambda)$.

We show in Table I the 90% CL constraints on the quartic anomalous vector-boson interactions which are obtained from Eq. (17) assuming that $\Lambda = 2 \text{ TeV}$. In this case the SM contribution to $\epsilon^1$ is in the range $(2.68 - 7.58) \times 10^{-3}$ for $m_{t\text{top}} = 170 - 220 \text{ GeV}$. Our bounds for $\alpha_4$ and $\alpha_5$ agree with the ones in Ref. [31] which were obtained in the unitary gauge.

It is interesting to notice that our analysis does not show any indication of new physics beyond the SM since all the anomalous couplings are compatible with zero at 90% CL. A natural order of magnitude of the anomalous couplings $\alpha_i$ in a fundamental gauge theory is $g^2 v^2 / \Lambda^2$ [20], since the quartic anomalous interactions can be generated by tree diagrams. Thus, we might expect that the size of the $\alpha$’s should be of the order of $M_Z^2 / \Lambda^2 \simeq 2 \times 10^{-3}$. From our results we see that only the operators that break the custodial $SU(2)_C$ symmetry, $(\mathcal{L}^{(4)}_{6,7,10})$ get limits close to this expectation[4].

Future colliders will be able to search for anomalous quartic interactions through multiple gauge-boson production. Assuming, as in Ref. [7], that an anomalous coupling is observable at the LHC if it induces a 50% change in the integrated cross section for the production of

\footnote{It is interesting to notice that models containing spin-0 and spin-1 resonances also lead to couplings of this order [7].}
pairs $V_L V_L$ ($V = W^\pm, Z$), it will be possible to detect the couplings $\alpha_4$ and $\alpha_5$ provided they satisfy $|\alpha_4, \alpha_5| \sim \mathcal{O}(0.005)$. These constraints are stronger than the limits obtained from the $Z$ physics, as also concluded in Ref. [31]. In Ref. [3], the capabilities of the NLC to study quartic anomalous couplings via the production mechanisms $e^+e^- \rightarrow W^+W^-Z$ and $e^+e^- \rightarrow ZZ Z$ are analysed, this last mechanism being the one yielding stronger constraints. Using their values for the $ZZZ$ cross section associated with the couplings $\alpha_{4,5}$, we translated their results into 90% CL limits $|\alpha_4, \alpha_5| \lesssim 0.2$, $|\alpha_6, \alpha_7| \lesssim 0.1$, and $|\alpha_{10}| \lesssim 0.05$. These future direct bounds are weaker than the limits already imposed by the LEP data for most of the quartic anomalous couplings. Therefore, we can see that our results show that only the LHC can improve what we have learned from the radiative corrections at the $Z$ pole.

Summarizing, we have analyzed the effects of possible anomalous quartic vector-boson interactions that appear in a scenario where there is no particle associated to the symmetry-breaking sector in the low-energy spectrum. Using a chiral Lagrangian at the order $D = 4$ and an arbitrary gauge $R_\xi$, we draw the limits on the anomalous interactions $ZZZZ$, $W^+W^-ZZ$, and $W^+W^-W^+W^-$ arising from the precision measurements at the $Z$ pole. We extended previous results by considering all possible anomalous couplings that appear at order $D = 4$ and by working in an arbitrary $R_\xi$ gauge. Our analysis shows that, with the present limits, these anomalous couplings are already out of reach of a 500 GeV $e^+e^-$ collider for most of the values of $m_{t_{top}}$. However, the LHC will be able to further extend the bounds on some of these couplings.

Acknowledgements

This work was partially supported by the U.S. Department of Energy under Grants Nos. DE-FG02-95ER40896 and DE-FG02-91ER40661, by the University of Wisconsin Research

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3 These limits should be taken with a grain of salt since the analysis in Ref. [3] does not take into account the efficiency to detect the gauge bosons. This will probably make the limits be a factor of 10 weaker.
Committee with funds granted by the Wisconsin Alumni Research Foundation, by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), and by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP).
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TABLE I. Limits on the anomalous quartic vector boson couplings at 90% CL for $\Lambda = 2$ TeV.

| $m_{\text{top}}$ | $m_{\text{top}} = 170$ GeV | $m_{\text{top}} = 200$ GeV | $m_{\text{top}} = 220$ GeV |
|-----------------|----------------------------|----------------------------|----------------------------|
| $-0.060 \leq \alpha_4 \leq 0.30$ | $-0.20 \leq \alpha_4 \leq 0.16$ | $-0.30 \leq \alpha_4 \leq 0.056$ |
| $-0.15 \leq \alpha_5 \leq 0.76$ | $-0.50 \leq \alpha_5 \leq 0.40$ | $-0.77 \leq \alpha_5 \leq 0.14$ |
| $-0.010 \leq \alpha_6 \leq 0.053$ | $-0.035 \leq \alpha_6 \leq 0.028$ | $-0.054 \leq \alpha_6 \leq 0.0099$ |
| $-0.077 \leq \alpha_7 \leq 0.39$ | $-0.26 \leq \alpha_7 \leq 0.21$ | $-0.39 \leq \alpha_7 \leq 0.072$ |
| $-0.0051 \leq \alpha_{10} \leq 0.026$ | $-0.017 \leq \alpha_{10} \leq 0.014$ | $-0.026 \leq \alpha_{10} \leq 0.0048$ |
FIG. 1. Feynman diagrams that contain contributions from anomalous quartic vertices.