The three-boson bound state in the massive Schwinger model

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Abstract

We derive the (matrix-valued) Feynman rules of mass perturbation theory of the massive Schwinger model for non-zero vacuum angle $\theta$. Further, we discuss the properties of the three-boson bound state and compute – by a partial resummation of the mass perturbation series – its mass and its partial decay widths.

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1 Introduction

The massive Schwinger model is two-dimensional QED with one massive fermion. In this model some nontrivial field-theoretic features occur. E.g. instanton-like gauge field configurations are present in the model and, therefore, a $\theta$-vacuum has to be introduced as a new, physical vacuum ([1, 2]). Further, confinement is realized in this model and there are no fermions in the physical spectrum ([3, 4, 5]). The fundamental physical particle of the theory is a massive, scalar boson (the Schwinger boson with mass $\mu$). This boson is interacting and may form bound states ([2, 6, 7]) and undergo scattering ([8]). A proper way to discuss the model is mass perturbation theory (i.e. perturbation in the fermion mass term), as it preserves the nontrivial structure of the model (like e.g. $\theta$-vacuum) and is performed in terms of physical (bosonic) variables only ([2, 9, 10, 11]). This mass perturbation theory uses the known exact solution of the massless Schwinger model as a starting point (in the massless model the Schwinger boson is a free scalar particle with mass $\mu_0 = e\sqrt{\pi}$; [3], [12] – [18]).

In this paper we first derive the Feynman rules of the mass perturbation theory for general vacuum angle $\theta$ (which slightly complicates them as they acquire a matrix structure). We then discuss the three-boson bound state and show how, by a resummation of the perturbation series, we may compute the mass $M_3$ of the three-boson bound state and its partial decay widths for the decays into two Schwinger bosons and into one Schwinger boson and one two-boson bound state.

The mass perturbation theory that is used throughout this paper is based on the Euclidean path integral formalism and, therefore, we have to take into account our specific Euclidean conventions (see e.g. [17]).

2 Mass perturbation theory

First let us shortly review the mass perturbation theory. By simply expanding the mass term, the vacuum functional and VEVs of the massive model may be traced back to space-time integrations of VEVs of the massless model. E.g. the vacuum functional is

$$Z(m, \theta) = \sum_{k=-\infty}^{\infty} e^{ik\theta}N \int D\bar{\Psi}D\Psi DA_k^k,$$

$$\cdot \sum_{n=0}^{\infty} \frac{m^n}{n!} \prod_{i=1}^{n} \int dx_i \bar{\Psi}(x_i)\Psi(x_i) \exp \int dx \left[ \bar{\Psi}(i\not{\partial} - eA)\Psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \right]$$

($k$ ... instanton number). Therefore, one needs scalar VEVs $\langle S(x_1)\ldots S(x_n) \rangle_0$ of the massless model, where $S = \bar{\Psi}\Psi$, $S_\pm = \frac{1}{2} \bar{\Psi}(1 \pm \gamma_5)\Psi$. Chiral VEVs $\langle S_{H_1}\ldots S_{H_n} \rangle_0$, $H_i = \pm$, are especially easily computed, as only a definite instanton sector contributes (see e.g. [10, 7, 13]),

$$\langle S_{H_1}(x_1)\ldots S_{H_n}(x_n) \rangle_0 = e^{ik\theta} \left( \frac{\Sigma}{2} \right)^n \exp \left[ \sum_{i<j} (-)^{\sigma_i,\sigma_j} 4\pi D_{\mu\nu}(x_i - x_j) \right]$$
\[ k = \sum_{i=1}^{n} \sigma_i = n_+ - n_- \]

where \( \sigma_i = \pm 1 \) for \( H_i = \pm \), \( D_{\mu_0} \) is the massive scalar propagator of the Schwinger boson \( (\mu_0^2 = \frac{\nu}{\pi}) \) and \( \Sigma \) is the fermion condensate of the massless model.

The Schwinger boson \( \Phi \) is related to the vector current, \( J_\mu = \frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu \Phi \), and, therefore, the \( S \) and \( \Phi \) VEVs, which we need for the perturbative calculation of massive VEVs, are related to the vector and scalar current VEVs of the massless model. Explicitly the \( S \) and \( \Phi \) VEVs may be computed from the generating functional (which is at the same time a VEV for \( n \) chiral currents)

\[
\langle S_{H_1}(x_1) \cdots S_{H_n}(x_n) \rangle_0[\lambda] = e^{ik\theta} \left( \frac{\Sigma}{2} \right)^n \exp \left[ \sum_{i<j} (-)^{\sigma_i \sigma_j} 4\pi D_{\mu_0}(x_i - x_j) \right] \cdot \exp \left[ -\int dy_1 dy_2 \lambda(y_1) D_{\mu_0}(y_1 - y_2) \lambda(y_2) + 2i\sqrt{\pi} \sum_{l=1}^{n} (-)^{\sigma_l} \int dy \lambda(y) D_{\mu_0}(y - x_l) \right]
\]

(see [20] for an explicit computation), where \( \lambda \) is the external source for the Schwinger boson \( \Phi \). Observe the \((-)^n\) in the last term of the exponent. As a consequence, whenever an odd number of external \( \Phi \) lines meets at a point \( x_i \), the corresponding \( S_- (x_i) \) acquires a \(-\), i.e. instead of a \( S = S_+ + S_- \) vertex there is a \( P = S_+ - S_- \) vertex.

From equs. (2), (3) one finds that exponentials \( \exp \pm 4\pi D_{\mu_0}(x_i - x_j) \) are running from any vertex \( x_i \) to any other vertex \( x_j \); however, in order to get an IR finite perturbation theory, one has to expand these exponentials into the functions

\[
E_{\pm}(x) = e^{\pm 4\pi D_{\mu_0}(x)} - 1,
\]

(or their Fourier transforms \( \widetilde{E}_{\pm}(p) \) for momentum space Feynman rules). This expansion is analogous to the cluster expansion of statistical physics.

The mass perturbation theory may be summarized by the following Feynman rules

- \( D_{\mu_0}(x_i - x_j) \)
- \( E_{\pm}(x_i - x_j) \)
- \( m\Sigma \)
- \( g_0 \)
- \( 1 \)

Fig. 1
where $m \Sigma$ is the bare coupling constant of mass perturbation theory and $g$ the renormalized one (including all tadpole-like corrections, see e.g. [7]). Strictly speaking, we added the $S_+$ and $S_-$ contributions of each vertex, therefore the rules, as they stand, are true only for the special case $\theta = 0$. The general $\theta$ case we will discuss in a subsequent section, where we will find that the graphical notation of Fig. 1 may be used, but the meaning of the individual graphs is slightly changed.

Using this mass perturbation theory the following features could be shown to hold in [6, 7]:

1. The mass corrections for the Schwinger boson (computable from $\langle \Phi(x_1)\Phi(x_2)\rangle_m$, [9]) also occur as corrections to the internal boson lines of the exponentials $E_\pm(x)$. Therefore, the bare Schwinger mass $\mu_0$ may be replaced by the renormalized one, $\mu$, in all internal propagators.

2. In the perturbation series of Fig. 1 there occur, among other terms, strings of $E_\pm$, $E_\pm(x_1 - x_2) \cdot E_\pm(x_2 - x_3) \ldots$ When the two-boson part $\frac{16\pi^2}{2} D_\mu^2(x)$ is separated in the expanded exponential $E_\pm(x)$, (4), these two-boson blobs may be resummed in momentum space, via a geometric series formula and give the leading contribution to the two-boson bound state mass pole (with mass $M_2$). Higher $n$-boson blobs give only higher order corrections to the mass, because the two-boson blob is near its threshold singularity at $M_2$ (see [8]).

Explicitly one finds the mass pole ($s = -p^2 > 0$, remember our Euclidean conventions)

$$1 - \frac{16\pi^2}{2} m \Sigma \cos \theta \frac{1}{\pi s} \frac{1}{\sqrt{4\mu^2 - 1}} \arctan \frac{1}{\sqrt{4\mu^2 - 1}}$$

with the pole mass

$$M_2^2 = 4\mu^2 - \Delta_2 \quad , \quad \Delta_2 = \frac{4\pi^4 (m \Sigma \cos \theta)^2}{\mu^2}$$

and residue at the pole

$$R_2 = \frac{2\Delta_2}{m \Sigma \cos \theta} = \frac{8\pi^4 m \Sigma \cos \theta}{\mu^2}.$$  

Analogously, we want to show in this paper that
1. there is a resummation that leads to a $\mu-M_2$-blob

\[ \text{Fig. 3} \]

(this is quite obvious from the Feynman rules, Fig. 1 and from the fact that the propagators $E_{\pm}$ may connect all vertices);

2. two- and three- boson blobs and the above-mentioned $\mu-M_2$-blob (Fig. 3) occur in such combinations that they may again be resummed via the geometric series formula to result in a propagator

\[ \frac{1}{1 - m\Sigma \cos \theta f(p)} \]  

(8)

and $f(p)$ is the sum of the three above-mentioned blobs,

\[ \text{Fig. 4} \]

where the three-boson blob will give rise to the three-boson bound-state mass pole $M_3$, whereas the $2\mu$- and $\mu-M_2$-blobs have imaginary parts that will give the partial decay widths for the decays of the three-boson bound state $M_3$ into $2\mu$ and $\mu + M_2$, respectively. All other types of blobs that we ignored are unimportant for these features.

Actually, things are a little bit more complicated. As mentioned above, up to now the Feynman rules of Fig. 1 are true only for the special case $\theta = 0$. But this restriction is too strong. E.g. the decay $M_3 \to 2\mu$ is not possible in that case because of parity conservation. So let us discuss the general $\theta$ Feynman rules as a next step.

## 3 Feynman rules for general $\theta$

In the mass perturbation theory each vertex is $mS = m(S_+ + S_-)$. For general $\theta$ the two chiral components $S_+$ and $S_-$ couple differently, with (renormalized) coupling $g_\theta = m^2 e^{i\theta} + o(m^2)$ for $S_+$ and $g_\theta^* = m^2 e^{-i\theta} + o(m^2)$ for $S_-$. As a consequence, the Feynman rules of Fig. 1 acquire a matrix structure. More precisely, the wavy line (the $E_{\pm}$ propagator) turns into a matrix,

\[ \mathcal{E}(p) = \begin{pmatrix} \bar{E}_+(p) & \bar{E}_-(p) \\ \bar{E}_-(p) & \bar{E}_+(p) \end{pmatrix} \]  

(9)
whereas the vertex $g$ turns into an $n$-th rank tensor $G$ when $n$ wavy lines $\mathcal{E}$ meet at this vertex. Only two components of this tensor are nonzero, namely $G_{++...+} = g_\theta$ and $G_{----...} = g_\theta^*$. E.g. when two lines meet at one vertex, then $G$ is the matrix

$$G = \begin{pmatrix} g_\theta & 0 \\ 0 & g_\theta^* \end{pmatrix}. \quad (10)$$

External bosons $\Phi$ that meet at a vertex are represented by $P$ ($S$), when an odd (even) number of bosons meets at the vertex, where

$$P = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad S = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (11)$$

Among other Feynman graphs, there exist the following ones (that may be resummed), where we amputate external bosons,

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\end{array} \quad \text{Fig. 5}$$

or

$$G\Pi(p) := G + G\mathcal{E}(p)G + G\mathcal{E}(p)G\mathcal{E}(p)G + \ldots = G(1 + \mathcal{E}(p)G\Pi(p)) \quad (12)$$

with the solution

$$\Pi(p) = \frac{1}{N(p)} \begin{pmatrix} 1 - g_\theta^* E_+(p) & g_\theta^* E_-(p) \\ g_\theta E_-(p) & 1 - g_\theta E_+(p) \end{pmatrix} \quad (13)$$

where

$$N(p) = \det(1 - G\mathcal{E}(p)) = 1 - (g_\theta + g_\theta^*) E_+(p) + g_\theta g_\theta^* (\bar{E}_+^2(p) - \bar{E}_-^2(p)). \quad (14)$$

So we succeeded in inverting the propagator $\Pi(p)$, analogously to our above discussion.

For the determination of the mass pole and decay widths only the denominator $N(p)$ of (13) will be important. In the denominator $N(p)$ we find all the $n$-boson propagators

$$d_n(p) := \frac{(4\pi)^n}{n!} \bar{D}^n(p). \quad (15)$$

But we want to include the $\mu, M_2$ two-boson loop

$$d_{1,1}(p) := \int \frac{d^2q}{(2\pi)^2} \frac{8\pi^4 m \Sigma \cos \theta}{\mu^2 (q^2 + M_2^2)} \frac{4\pi}{(p-q)^2 + \mu^2}, \quad (16)$$

too. In principle, its existence is obvious from the Feynman rules of Fig. 1. But how does it fit into our matrix scheme? The answer is that it is parity odd and may be treated just like any odd $n$-boson propagator (e.g. like $d_3$), as may be checked by expanding
the perturbation series into the individual $n$-boson propagators and including the $\mu-M_2$ two-boson propagator term by term. So we just include it into $E_{\pm}$ with a $\pm$ sign as a higher order contribution that we need.

Inserting the $n$-boson propagators $d_n$, we may rewrite the denominator $N(p)$ like

\[
N(p) \simeq 1 - (g_\theta + g_\theta^*) (d_1 + d_2 + d_{1,1} + d_3 + \ldots) + 4g_\theta g_\theta^* (d_1 (d_2 + d_4 + \ldots) + d_{1,1} (d_2 + d_4 + \ldots) + d_3 (d_2 + d_4 + \ldots) + \ldots)
\]  

(17)

where we included $d_{1,1}$, as stated above.

Now suppose we are at the three-boson bound-state mass $M_3$. Then the real part of (17) vanishes by definition, and the leading order contribution (of order $\frac{1}{g_\theta + g_\theta^*}$) stems from the $d_3(s = -p^2)$ function that is near its threshold singularity,

\[
(g_\theta + g_\theta^*) d_3(s = M_3^2) = 1 + o(m^2).
\]

(18)

In addition, all functions $d_i$ that are above their respective mass thresholds will have imaginary parts at $M_3^2$, and result thereby in partial decay widths. This is obviously so for $d_2$, and it is also true for $d_{1,1}$ provided $M_3 > \mu + M_2$ (actually this is the case as we will see in the computations). Therefore, at $M_3$ (17) may be rewritten like

\[
-i(g_\theta + g_\theta^*) \left( \text{Im} d_2(M_3^2) + \text{Im} d_{1,1}(M_3^2) \right) + 4g_\theta g_\theta^* d_3(M_3^2) \text{Im} d_2(M_3^2) + o(m^2)
\]

\[
= -im\Sigma \cos \theta \left( \text{Im} d_2(M_3^2) + \text{Im} d_{1,1}(M_3^2) \right) + i\frac{m\Sigma}{\cos \theta} \text{Im} d_2(M_3^2) + o(m^2)
\]

(19)

and we find that parity forbidden decay widths acquire a factor $(\cos \theta - \frac{1}{\cos \theta})$, whereas parity allowed decays acquire a $\cos \theta$ factor.

So we have, indeed, succeeded in rewriting the mass pole equation, as indicated in Fig. 4, and may start the actual computations.

4  Bound state mass and decay width computations

For the computation of the three-boson bound-state mass $M_3$ we need the three-boson propagator $d_3$ and find, in lowest order (see (18))

\[
1 = \frac{1}{3!} m\Sigma \cos \theta \cdot 64\pi^3 \overline{D}_3^\mu(p)
\]

(20)

or, after a rescaling $p \rightarrow \frac{p}{\mu}$ to dimensionless momenta

\[
1 = \frac{64\pi^3}{6} m\Sigma \mu^2 \cos \theta \overline{D}_3^\mu(p).
\]

(21)

$\overline{D}_3^\mu(p)$ is given by the graph (where we introduce positive squared momentum $s = -p^2 > 0$)
\[ \bar{D}_\mu^3(p) = -\int \frac{d^2q_1d^2q_2}{(2\pi)^4} \frac{1}{(p + q_1 + q_2)^2 + 1} \frac{1}{q_1^2 + 1} \frac{1}{q_2^2 + 1} = \]

\[ -2 \int_0^1 dx \int_0^x dy \int \frac{d^2q_1d^2q_2}{(2\pi)^4} \frac{1}{[q_1^2 + 1 + (q_2^2 - q_1^2)x + ((p + q_1 + q_2)^2 - q_2^2)y]^3} = \]

\[ \int \frac{dx}{(4\pi)^2} \int_0^x dy \int \frac{d^2q_1d^2q_2}{(2\pi)^4} \frac{1}{s(xy - x^2y - y^2 + xy^2) - x + x^2 - xy + y^2} = \]

\[ \int \frac{dx}{8\pi^2(1 - s(1 - x))} \int_0^1 \frac{dz}{z^2 + T^2(s, x)} = \]

\[ \int_0^1 \frac{dx}{8\pi^2(1 - s(1 - x))} \frac{1}{T(s, x)} \arctan \frac{x}{2T(s, x)}, \quad (22) \]

\[ T^2(s, x) = \frac{x^2 - sx^2(1 - x) + 4x(1 - x)}{4(s(1 - x) - 1)}, \quad (23) \]

where, as usual, we introduced Feynman parameter integrals and performed the momentum integrations. Further, the first Feynman parameter integral could be done analytically. The numerator of \( T^2 \) has a double zero at \( s = 9 \):

\[ 9x(x - \frac{2}{3})^2. \quad (24) \]

This double zero is in the integration range of \( x \) and is precisely the threshold singularity. Setting

\[ s = 9 - \Delta_3 \quad (25) \]

in the numerator of \( T^2 \) in the factor \( \frac{1}{x} \), and \( s = 9 \) everywhere else, where it is safe, one arrives at:

\[ \frac{1}{12\pi^2} \int_0^1 \frac{dx}{\sqrt{|9x - 8|}} \frac{\arctan \sqrt{\frac{|9x - 8|}{3x - \frac{9}{4}}}}{\sqrt{\left((x - \frac{2}{3})^2x + \frac{\Delta_3}{3}x^2(1 - x)\right)}} =: I(\Delta_3). \quad (26) \]

The mass-pole equation reads

\[ 1 = \frac{64\pi^3}{6} m \Sigma \cos \theta I(\Delta_3) \quad (27) \]

and must be evaluated numerically. It gives rise to an extremely tiny mass correction \( \Delta_3 \).

For sufficiently small \( m \) it is very well saturated by

\[ \Delta_3(m \Sigma \cos \theta) \simeq 6.993 \exp\left(-\frac{0.263}{m \Sigma \cos \theta}\right) \quad (28) \]
and is therefore smaller than polynomial in \( m \). (I checked the numerical formula (28) for \( 30 < \frac{1}{m \Sigma \cos \theta} < 1000 \), corresponding to \( 10^{-2} < \Delta_3 < 10^{-100} \), but I am convinced that it remains true for even larger \( \frac{1}{m \Sigma \cos \theta} \); however, there the numerical integration is quite difficult because of the pole in (26).)

We conclude that the three-boson bound state mass is nearly entirely given by three times the Schwinger boson mass (we change back to dimensionful quantities now),

\[
M_3^2 = 9 \mu^2 - \Delta_3, \quad \Delta_3 = 6.993 \mu^2 \exp(-0.263 \frac{\mu^2}{m \Sigma \cos \theta})
\]  

(29)
or, differently stated, that the binding of three bosons is extremely weak.

Therefore it holds that \( M_3 > \mu + M_2 \), as indicated above, and, consequently, a decay of \( M_3 \) into \( \mu + M_2 \) is possible. This has the consequence that the three-boson bound state is unstable even for \( \theta = 0 \), contrary to some earlier conjecture ([2]).

So let us turn to the decay width computation. For this purpose we need the first Taylor coefficient of the denominator \( 1 - m \Sigma \cos \theta d_3(s) \) (see (14)) around the mass pole. Observe that because of formulae (25), (28) \( m \Sigma \cos \theta d_3(s) \) may be written, in the vicinity of \( s = M_3^2 \), like

\[
m \Sigma \cos \theta d_3(s) \sim \frac{m \Sigma \cos \theta}{0.263 \Delta_3} \ln \frac{6.993 \mu^2}{9 \mu^2 - s}.
\]  

(30)

Therefore, we find the Taylor coefficient

\[
c_3 = \frac{m \Sigma \cos \theta}{0.263 \Delta_3}.
\]  

(31)

Generally, a decay width may be inferred from the imaginary part of a propagator,

\[
G(p) \sim \frac{\text{const.}}{s - M^2 - i\Gamma},
\]  

(32)

where \( \Gamma \) is the decay width.

In our case we have for \( \frac{1}{N(p)} \), equ. (19),

\[
\frac{1}{N(p)} \sim \frac{1}{c_3(s - M_3^2)} - \frac{1}{im \Sigma (\cos \theta - \frac{1}{\cos \theta}) \text{Im} d_2(M_3^2) - im \Sigma \cos \theta \text{Im} d_{1,1}(M_3^2)}
\]

\[
\approx \frac{\text{const.}}{s - M_3^2 - i\frac{m \Sigma}{c_3} \left[(\cos \theta - \frac{1}{\cos \theta}) \text{Im} d_2(M_3^2) + \cos \theta \text{Im} d_{1,1}(M_3^2)\right]}.
\]  

(33)

Next we need the imaginary parts \( \text{Im} d_2 \), \( \text{Im} d_{1,1} \). Both of them stem from a two-boson blob, so let us write down the general result (which is standard) \( (s = -p^2) \)

\[
\text{Im} \left( D_{M_1} D_{M_2} \right)(s) = \text{Im} \int \frac{d^2q}{(2\pi)^2} \frac{-1}{q^2 + M_1^2} \frac{-1}{(p-q)^2 + M_2^2} = \frac{1}{2w(s, M_1^2, M_2^2)}.
\]  

(34)
\[ w(x, y, z) = (x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)^{\frac{1}{2}}. \]  

Therefore we can write for (34) (see (7) and (16) for the normalization factors of \( d_2 \) and \( d_{1,1} \))

\[
\frac{\text{const.}}{s - M_3^2 - i\frac{m\Sigma}{c_3} (\cos \theta - \frac{1}{\cos \theta}) w(M_3^2, \mu^2, \mu^2) - i\frac{(m\Sigma \cos \theta)^2}{c_3} \frac{16\pi^5}{\mu^2 w(M_3^2, \mu^2, M_2^2)}}.
\]

There seems to be something wrong with the sign of the parity forbidden partial decay width \( \Gamma_{M_3 \rightarrow 2\mu} \) (the \( d_2 \) term). Actually the sign is o.k. and the problem is a remnant of the Euclidean conventions that are implicit in the whole computation (see e.g. [17, 18]). In these conventions \( \theta \) is imaginary and therefore \( (\cos \theta - \frac{1}{\cos \theta}) \geq 0 \). Of course, this is not a reasonable convention for a final result. When performing the whole computation in Minkowski space and for real \( \theta \), roughly speaking, the roles of \( E_+ \) and \( E_- \) are exchanged in (14). This gives an additional relative sign between parity even and odd \( n \)-boson propagators and, therefore, changes the factor of \( d_2 \) to \( \frac{1}{\cos \theta - \cos \theta} \), which is \( \geq 0 \) for real \( \theta \).

With this remark in mind, and expressing the final results for real \( \theta \), we find, by using the approximations

\[
w(M_3^2, \mu^2, \mu^2) \simeq w(9\mu^2, \mu^2, \mu^2) = 3\sqrt{5}\mu^2
\]

\[
w(M_3^2, M_2^2, \mu^2) \simeq w(9\mu^2, M_2^2, \mu^2) = 2\sqrt{3}\mu\sqrt{\Delta_3} + o(m^2),
\]

the following results:

\[
\Gamma_{M_3 \rightarrow 2\mu} = 0.263 \frac{4\pi^2 \Delta_3}{9\sqrt{5}\mu} \left( \frac{1}{\cos^2 \theta} - 1 \right)
\]

\[
\simeq 3.608\mu \left( \frac{1}{\cos^2 \theta} - 1 \right) \exp(-0.929 \frac{\mu}{m\cos \theta})
\]

and

\[
\Gamma_{M_3 \rightarrow \mu + M_2} = 0.263 \frac{4\pi^2 \Delta_3}{\sqrt{3}\mu}
\]

\[
\simeq 43.9\mu \exp(-0.929 \frac{\mu}{m\cos \theta})
\]

where we inserted the numerical value \( \Sigma = \frac{e^2\mu}{2\pi} = 0.283\mu \).

The ratio of the two partial decay widths does not depend on the approximations that were used for the \( M_3 \) computation,

\[
\frac{\Gamma_{M_3 \rightarrow 2\mu}}{\Gamma_{M_3 \rightarrow \mu + M_2}} = \frac{1}{\cos^2 \theta} - 1
\]

Contrary to a naive expectation it holds that \( \Gamma_{M_3 \rightarrow \mu + M_2} > \Gamma_{M_3 \rightarrow 2\mu} \), although \( \mu + M_2 \sim M_3 \). This is not surprizing in two dimensions, because there the phase space “volume” does not grow with increasing momentum.
5 Summary

So we have succeeded in computing the three-boson bound state mass and partial decay widths via (resummed) mass perturbation theory. The computations may be generalized and lead to the following physical picture: there are two stable particles in the theory, namely the fundamental Schwinger boson $\mu$ and the two-boson bound state $M_2$. Higher bound states are unstable (resonances) and may decay into all combinations of $\mu$ and $M_2$ final particles that are allowed kinematically.

There exists also another way for the computation of partial decay widths, namely the usual perturbative method of putting all the initial and final states on their mass shells, inserting the squared transition matrix element and summing over all kinematically possible final states. In our theory this needs the construction of the exact higher $n$-point functions (here the three-point function) as a prerequisite, but once this is done it is easy to show that the decay widths (40), (41) just correspond to a first order perturbative computation (there the nontrivial numerical factors in (40), (41) are caused by the proper normalizations of the initial and final states, which in our computation correspond to the residues of the mass poles).

Observe that the decay widths (40), (41) are related to the $M_3$ binding energy, $\Gamma_{M_3} \sim \Delta_3$. This is a very reasonable result that enables us to interpret the $M_3$ bound state (and, a fortiori, higher bound states) as a resonance. Indeed, suppose that the propagator (13) belongs to the matrix element of a scattering process (to be discussed elsewhere [8]). The denominator of (13) has zero real part at $s = M_3^2$ and infinite real part at the real production threshold $s = 9\mu_2^2 = M_3^2 + \Delta_3$, therefore it will give a local maximum of the scattering cross section (resonance) near $M_3^2$, and a local minimum at $9\mu_2^2$, and the resonance width must be related to the binding energy, $\Gamma_{M_3} \sim \Delta_3$.

Acknowledgement

The author thanks the members of the Institute of Theoretical Physics of the Friedrich-Schiller-Universität Jena, where this work was done, for their hospitality. Further thanks are due to A. Wipf and J. Pawlowski for helpful discussions.

This work was supported by a research stipendium of the Vienna University.

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