Effects of Radial Distances on Small-scale Magnetic Flux Ropes in the Solar Wind

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Abstract

Small-scale magnetic flux ropes (SFRs) in the solar wind have been studied for decades. Statistical analysis utilizing various in situ spacecraft measurements is the main observational approach to investigating the generation and evolution of these small-scale structures. Based on the Grad–Shafranov reconstruction technique, we use the automated detection algorithm to build the databases of these small-scale structures via various spacecraft measurements at different heliocentric distances. We present the SFR properties, including the magnetic field and plasma parameters at different radial distances from the Sun near the ecliptic plane. It is found that the event occurrence rate is still of the order of a few hundreds per month, the duration and scale-size distributions follow power laws, and the flux-rope axis orientations are approximately centered around the local Parker spiral directions. In general, most SFR properties exhibit radial decays. In addition, with various databases established, we derive scaling laws for the changes in average field magnitude, event counts, and SFR scale sizes, with respect to the radial distances, ranging from ~0.3 au for Helios to ~7 au for the Voyager spacecraft. The implications of our results for comparisons with the relevant theoretical works and for applications to the Parker Solar Probe mission are discussed.

Unified Astronomy Thesaurus concepts: Solar wind (1534); Interplanetary turbulence (830); Magnetohydrodynamics (1964); Astronomy data analysis (1858); Astronomy databases (83)

1. Introduction

In the past few decades, there have been a number of observational studies of small-scale magnetic flux ropes (hereafter, SFRs) in the solar wind. These relatively small-scale structures, first identified from Ulysses spacecraft measurements at ~5 au by Moldwin et al. (1995), generally are believed to have the same helical magnetic field configuration as their large-scale counterparts, i.e., magnetic clouds (MCs). Employing various in situ spacecraft mission data sets, such as those from IMP-8, WIND, and STEREO at about 1 au, a limited number (usually of the order of hundreds in total at most) of SFRs were identified for multiple years via mostly visual inspection or semi-automated methods. For example, Cartwright & Moldwin (2010) used Helios 1 and 2, IMP-8, WIND, ACE, and Ulysses spacecraft data sets with the corresponding heliocentric distances ranging from ~0.3 to 5.5 au to investigate the occurrence and the evolution of hundreds of SFRs. This represents the first comprehensive study of SFRs and the generation of SFR databases from multiple spacecraft missions, although the event counts are very limited. The corresponding statistical studies of these limited-size samples suggested that these small structures can last from tens of minutes up to a few hours, and thus have smaller-scale sizes over their cross sections, compared with MCs (Moldwin et al. 2000).

The distinctions between the two populations were sought out, and two possible sources, i.e., small coronal mass ejection from solar eruption and magnetic reconnection across the heliospheric current sheet in the solar wind, were proposed as origins of SFRs (Feng et al. 2007, 2008; Cartwright & Moldwin 2008). In addition, Yu et al. (2016, 2014) performed careful and detailed analysis of in situ observations of small transients including SFRs in the solar wind around 1 au. They examined their overall magnetic and plasma properties, and concluded that these relatively small-scale structures may originate from both the solar corona and the interplanetary medium.

One step forward in the study of SFRs is the implementation of an automated and computerized algorithm, based on the Grad–Shafranov (GS) reconstruction technique, for identifying these structures from in situ spacecraft measurements. This has yielded significantly more events, which is useful for statistical investigations. The GS method (see Hu 2017, for a comprehensive review) is a unique data analysis technique capable of recovering the two-dimensional (2D) structure from one-dimensional (1D) time-series data (Sonnerup & Guo 1996; Hau & Sonnerup 1999; Hu & Sonnerup 2001, 2002). Zheng & Hu (2018) created the computer-based program to identify SFRs automatically. This automated detection has succeeded in finding 74,241 SFRs using the WIND spacecraft in situ measurements from 1996 to 2016 (Hu et al. 2018). By this abundant event count number (of the order of a few hundreds per month on average), they compared the monthly counts of SFRs with the corresponding sunspot numbers and indicated that the occurrence of SFRs has an obvious solar cycle dependency with a short lag. Later, this automated detection was applied to ACE and Ulysses measurements (Chen et al. 2019). The bulk properties of identified SFRs, including the magnetic field strength and plasma parameters, are presented in terms of their variations with time, heliographic latitudes, and radial distances. It is found that the solar cycle dependency or the temporal variation of SFRs appears to be affected by both latitudinal and radial-distance changes owing to the unique orbit of Ulysses. An earlier study on par with the number of events identified by the GS-based approach was performed by Borovsky (2008) using ACE measurements to identify boundaries of about 65,860 flux tubes for seven years worth of data. That study suggested that these flux tubes are tangled...
along the Parker spiral direction and form “spaghetti-like” structures originating from the Sun.

Other relevant studies have hinted of local generation of these flux tube/rope structures from 2D magnetohydrodynamic (MHD) turbulence. This 2D turbulence is characterized by quasi-2D coherent structures manifest as current sheets and flux ropes of variable sizes, corresponding to spatial scales in the inertia range (Matthaeus et al. 2007; Servidio et al. 2008; Wan et al. 2013; Zank et al. 2017). Various studies using in situ spacecraft measurements (e.g., Greco et al. 2008, 2009, 2018; Osman et al. 2014) were carried out for identifying and characterizing discontinuities, i.e., current sheets, which may be considered proxies to boundaries (or so-called “walls”) of flux ropes. In particular, the partial variance of increments (PVI) method is commonly used to identify discontinuities from in situ magnetic field measurements (see the review by Greco et al. 2018). For instance, the correspondence between distributions of the wall-to-wall times of SFRs and the waiting times of current sheets yielded consistent results in determining the correlation length scale for 2D MHD turbulence (Greco et al. 2009; Zheng & Hu 2018). Recently, Pecora et al. (2019) examined WIND in situ data and related the SFRs and current sheets by combing the GS reconstruction and the PVI methods. They showed the correspondence between the flux-rope boundaries and current sheets, where each type of structure was identified from the same data set but with different and independent approaches. They certified again that these small-scale structures can be generated self-consistently from quasi-2D MHD turbulence.

To further extend and complete our SFR event databases for the existing and past spacecraft missions, here we apply our automated detection algorithm to Helios 1 and 2 and Voyager 1 and 2 data sets. Considering that they offer observations at additional heliocentric distances complementary to the ACE and Ulysses missions, we will also perform a comprehensive analysis of the possible evolution of SFR properties with radial distances between ~0.3 and 8 au, especially for the uniquely controlled subset of events to be described below.

This paper is organized as follows. The automated detection method is described briefly together with the data selection for this study in Section 2. In Section 3, the SFRs identified from the full Helios mission are categorized into three groups by their corresponding heliocentric distances. One specific year is selected for identification via the Voyager mission. The SFR properties include the axis orientation angles, the magnetic field and plasma parameters, and the duration and scale size are for each event set. In Section 4, the radial distributions of SFRs via these two missions, as well as ACE and Ulysses, are presented. The radial effects associated with the possible flux-rope merger scenario are discussed. Our findings and additional discussions, particularly regarding the applications to the Parker Solar Probe (PSP) mission, are summarized in the last section.

2. Method and Data Selection

The data analysis method we employ is the recently developed automated flux-rope detection algorithm based on the GS reconstruction technique. The GS reconstruction method is an advanced data analysis tool based on the 2D GS equation describing space plasma structures in approximate 2D quasi-static equilibrium and employing in situ spacecraft measurements. It has been widely applied to various space plasma regimes by a number of research groups worldwide for over 20 years (see Hu 2017, for a comprehensive review). The latest development has been the application of the basic GS reconstruction procedures to identifying relatively SFRs in the solar wind in a completely computerized and automated manner (Hu et al. 2018; Zheng & Hu 2018; Chen et al. 2019). This has enabled the generation of exhaustive event lists for the flagship NASA solar-terrestrial spacecraft missions. The event occurrence rate is of the order of a few hundreds per month, which had not been achieved by other means before. A detailed documentation of the approach, including an algorithm flowchart, was provided in Hu et al. (2018), and should enable the independent implementation of the algorithm by interested users. We provide below a brief description of the basic concepts underlying the automated detection algorithm, which is also utilized in the current study.

The basic quantity characterizing the 2D flux-rope configuration is defined through a magnetic flux function, \(A(x, y)\), which fully characterizes the transverse magnetic field on the cross-section plane \((x, y)\), perpendicular to the flux-rope axis, \(z\). In other words, the isosurfaces of \(A\), i.e., where \(A = \text{const}\), represent flux surfaces on which the magnetic field lines are winding along the central axis \(z\) with \(B_z = 0\) and \(\partial B_z/\partial z \approx 0\). Therefore, a solution of the scalar function \(A\) governed by the GS equation, as well as a non-vanishing axial field component \(B_z\), fully characterizes a cylindrical flux-rope configuration with nested flux surfaces surrounding one central \(z\)-axis. Thus, one important property associated with such a configuration is the single-valued behavior of the so-called field-line invariants, i.e., a few quantities as single-variable functions of \(A\) only, i.e., also being constant on each flux surface. They include the axial field component \(B_x\), the plasma pressure \(p\), and the transverse pressure \(P_t = p + B_y^2/2\mu_0\). These quantities, together with the \(A\) values, can be directly evaluated along a single spacecraft path once a \(z\)-axis is chosen. For a cylindrical flux-rope configuration and a spacecraft path across multiple nested flux surfaces, these quantities, as single-variable functions of \(A\), all exhibit a discernible double-folding behavior when displayed against \(A\) values along the path. This is because along the spacecraft path, each flux surface is crossed twice by the spacecraft, once along the inbound (“first” half of the path), and the other along the outbound (or the “second” half). Therefore, the corresponding pairs of \(A\) values are the same because each is on the same flux surface. So is each pair of the corresponding invariant quantity, as a single-variable function of \(A\), thus leading to the “second” half of the data points folding and overlapping with the “first” half, becoming so-called double-folding.

The \(A\) values along the spacecraft path at \(y = 0\) generally exhibit a monotonically increasing or decreasing pattern along the “first” half, then the trend reverses for the “second” half, after an extremum is reached. These values \(A(x, 0)\) are calculated from the “rotating” component of the magnetic field via \(A(x, 0) = -\int_0^x B_y d\xi\), where the spatial increment \(d\xi = -V_F \cdot \hat{x} dt\) is related to a frame velocity \(V_F\) (commonly the average solar wind velocity) and the time increment \(dt\) of the time-series data. The point at which the value \(A(x, 0)\) reaches an extremum is called the turning point. It is also where the component \(B_y\) changes sign, separating the “first” and the “second” halves. Therefore, the resulting double-folding behavior in the field-line invariants, as dictated by the GS equation, constitutes the key feature we utilize to devise the algorithm for detecting magnetic flux-rope intervals from
in situ spacecraft measurements. The quality of the “double-folding” pattern is assessed quantitatively by several metrics to result in the identification of flux-rope candidates. These metrics include the definition of two residues evaluating primarily the goodness of the satisfaction for these quantities, in particular the transverse pressure \( P_{ts} \) being single-valued and double-folded. A Wälén slope threshold is also used to exclude mostly Alfvénic fluctuations. In addition, an optional threshold condition on the average magnetic field magnitude over a candidate event interval can also be applied to reduce contamination of small-amplitude fluctuations whose flux-rope characteristics are less certain. The detailed descriptions of the procedures were provided in Hu et al. (2018). We refer interested readers to that report for further details.

The automated detection algorithm has been applied to a number of in situ spacecraft missions, including ACE, WIND, and Ulysses (Hu et al. 2018; Chen et al. 2019). A designated website containing the event databases is available at http://fluxrope.info. Due to the implementation of the highly computerized algorithm and the usage of cluster machines, the analysis of a whole mission data set becomes feasible. We continue our analysis in this study for additional in situ spacecraft data sets in the heliosphere, specifically the Helios and Voyager missions. The specific time periods and the resulting event counts for each mission are listed in Table 1. Since we focus on studying the flux-rope properties at different heliocentric distances and attempt to interrelate these properties considered to be radially distributed in the solar wind, we select as many as periods as possible when data are available for the whole Helios mission, but only include year 1980 for the Voyagers, when they were at large radial distances, but still in low helio-latitudes near the ecliptic plane. These selections are also largely affected by data integrity issues, i.e., the existence of data gaps. When they are prevalent, especially for the plasma parameters that are required in our analysis, and for relatively historical missions, the data integrity is significantly reduced and negatively impacts the search results. Nonetheless, on average, the event occurrence rate is still of the order of a few thousands a year, owing to the fairly exhaustive detection approach, using 1 minute cadence data throughout (Hu et al. 2018).

### 3. Analysis Results

The analysis of the heliospheric missions listed in Table 1 was carried out systematically for all available data, except for the Voyager mission. Due to data integrity issues involving low-quality or missing data and insufficient resolution of the time series, the analysis results for Voyagers are limited to year 1980 only. At that time, the spacecraft were at a heliocentric distance \( \sim 6.05-9.57 \) au near the ecliptic plane. The event counts from the Helios mission are also relatively lower, mostly due to significant numbers of data gaps. In Section 3.1, we present the properties of identified SFRs for the Helios and Voyager missions, respectively. The detailed descriptions of Ulysses, ACE, and WIND results were reported in Chen et al. (2019) and Hu et al. (2018). In Section 3.2 we present results from a subset of events for all four missions under similar conditions of over exactly a one year time period near the ecliptic plane, but at different heliocentric distances.

#### 3.1. Distributions of Selected Flux-rope Properties at Different Radial Distances

We have previously reported a comprehensive analysis of SFR databases generated for the WIND/ACE and Ulysses spacecraft missions. Here we have applied the automated detection algorithm to the Helios and Voyager missions for the first time. The Helios mission, consisting of two identical probes, took an elliptical orbit deep into the inner heliosphere around the Sun with a perihelion around 0.3 au. Thus, it provided in situ measurements of solar wind parameters in the range of heliocentric distances between \(~0.3 \) and \( 0.99 \) au over the whole mission spanning years 1975–1984. We found a total number of 15,041 SFR intervals for Helios 1 and 7981 for Helios 2, respectively, with the search window size (or duration) ranging from 9 to 2255 minutes. Figure 1 shows the distributions in terms of the probability density functions (PDFs) of the duration and the corresponding cross-section scale sizes of identified SFRs, taking into account the orientation of each identified cylindrical flux rope, relative to the spacecraft path. They are divided into three groups according to the ranges of radial distances in au, as indicated by the legend. The maximum duration rarely exceeds 1000 minutes, due to the relatively frequent occurrence of data gaps. The distributions of duration for smaller values, i.e., \(<100\) minutes, appear to exhibit power laws. Such trends seem to only persist for scale sizes within a narrow range \(~[0.002, 0.01]\) au.

Figure 2 shows the distributions of orientation angles of the flux-rope cylindrical axis \( z \) for the three groups, separately. Figure 2(a) shows the histograms of the polar angle distributions overlaid for the three groups, as indicated by the legend, corresponding to events identified at different radial distances. They all tend to have increasing counts toward \( \theta \approx 90^\circ \), i.e., increasingly more events with smaller inclination angles with respect to the ecliptic plane. Figure 2(b) shows the corresponding distributions of the azimuthal angles, \( \phi \), measured with respect to the positive \( R \) axis for the \( z \)-axis projected onto the \( RT \) plane. The distributions are folded into the range \([0^\circ, 180^\circ]\) such that this axis measures the smaller angle between the projected \( z \)-axis onto the \( RT \) plane and the positive \( R \) axis. Unlike the clear tendency in the polar angle distributions, the azimuthal angle \( \phi \) has a less clear tendency and is more broadly distributed, although it still tends to peak near one end at about \( 170^\circ \) and at the other around \( 30-50^\circ \). This behavior is likely owing to the wide range of radial distances where these events were identified. A strict Parker spiral angle distribution would correspond to azimuthal angles changing from near \( 0^\circ \) or \( 180^\circ \) near the Sun to about \( 45^\circ \) or \( 135^\circ \) near 1 au. Such a trend is somewhat embedded in Figure 2(b).
Figure 3 shows the distributions of various flux-rope properties averaged over each flux-rope interval for all Helios events, again separated into three groups. The radial decay in the average field magnitude $\mathbf{B}$ and in proton number density are readily seen. On the other hand, the changes in proton temperature and the resulting proton $\beta$ are less pronounced, especially for the two groups with radial distances in the range $<0.6$ au, where the distributions are nearly identical. For all three groups, the mean proton $\beta$ values are around 0.5.

For Voyagers, due to the data integrity issue and the constraint that we mainly focus on low-latitude observations in this report, only the data in year 1980 for Voyager 1 and 2 are analyzed. The total number of events is very limited compared with other missions. Therefore, the statistical significance in the distributions of relevant flux-rope properties is also much reduced. Nonetheless, for completeness we present the same set of parameters representative of flux-rope properties for the first time at the heliocentric distance around and beyond 6 au near the ecliptic plane. Figure 4 shows the distributions of duration and scale size. The power-law trend is not clear due to the relatively large scattering of the data points and low counts.

Figure 5 shows the corresponding distributions of selected parameters averaged over flux-rope intervals, similar to Figure 3. The decreases in the magnitudes of $\langle \mathbf{B} \rangle$, $\langle T_p \rangle$, and $\langle N_p \rangle$ are evident at such a large radial distance away from the Sun. However, the resulting proton $\langle \beta \rangle$ remains modest, with an order of magnitude $\sim 0.1$ on average, even at this radial distance. The flux-rope axis orientation angle distributions, given in Figure 6, indicate a trend with more events lying on the ecliptic plane along the nominal Parker spiral direction. The counts for the $z$-axis polar angle gradually increase toward 90°, while the corresponding azimuthal angle distribution has a broad peak around 90° (the direction perpendicular to the radial direction).

It is worth noting that we were not able to present the statistical results of waiting time and wall-to-wall time distributions (see, e.g., Zheng & Hu 2018) for the Helios and Voyager missions because of widespread data gaps. These gaps will interfere with the distributions of these two quantities.
Figure 3. Distributions of flux-rope properties derived from each flux-rope interval identified from both Helios 1 and 2: (a) the average field magnitude, (b) the average proton temperature, (c) the average proton number density, and (d) the average proton $\beta$. The different symbols represent the corresponding subgroups of SFR events identified with the radial-distance range, as indicated by the legend. The statistical quantities for each distribution are also denoted in each panel.

Figure 4. Distributions of SFR duration and scale size for the Voyager event set. The format follows that of Figure 1. See the legend for different groups.
3.2. Radial Distributions of Flux-rope Properties under Similar Conditions

To further investigate the variation of SFR properties at different radial distances, we select a subset of events from each spacecraft mission. Each spans a time period of the same length, i.e., one year, to facilitate comparison among selected flux-rope properties under similar conditions, except for being radially distributed. For example, Figure 7 shows the average solar wind velocity distributions in those flux-rope intervals. The distributions are similar if we exclude the counts in single-digit numbers. They
mostly lie in the range between 300 and 600 km s\(^{-1}\), with a notable exception for Helios, whose counts extend well into the low-speed wind regime.

The selection criteria and the resulting event counts are listed in Table 2. In a previous study by Hu et al. (2018), the possible effect of radial distances on SFRs via ACE and Ulysses measurements was reported. Now, with more available databases we plan to further extend that study by selecting specific years for four in situ data sets. Specifically, in Table 2, we present basic information and special criteria applied to four sets of measurements. (1) For Helios, we select SFR records detected in less than 0.4 au for both spacecraft and set a lower limit of magnetic field magnitude of 25 nT to remove small fluctuations. (2) For ACE and Ulysses, the year 2004 is selected since it is one of the years when Ulysses was at low latitudes (less than 30\(^\circ\)) and at far radial distances. (3) For Voyagers, 1980 is selected. Specifically, we select a 3 month period when Voyager 1 traveled from 6.9 to 7.58 au, and a 9 month period when Voyager 2 traveled from 6.5 to 8.08 au. Then we combine these two time periods together to yield detection results for one full year. All the detection results are obtained using the duration range from 9 to 2255 minutes under the scenario with thermal pressure included in the calculation except for Ulysses (Chen et al. 2019). Due to the 4–8 minute resolution of the plasma data and the 1 minute magnetic field data, we switch off the thermal pressure for Ulysses in order to have a consistent comparison with other missions.

For the Helios and Voyager missions, the events are selected from both probes and from multiple years in order to account for the significant number of data gaps while maintaining a narrow range of radial distances. The events from ACE and Ulysses are from the same and continuous one year period in 2004 when they were radially aligned near the ecliptic plane (Chen et al. 2019). Figure 8 shows the distributions of duration and scale sizes for these selected events, respectively. Because of reduced event counts, the power-law distribution in each event set is not as pronounced as those in each individual mission, e.g., as seen in Figure 1. They still exhibit power laws, especially in scale-size distributions shown in Figure 8(b), with different power-law slopes. The mean values seem to increase with radial distances, which ceases at about 7 au for the Voyagers. The disruption of such an increase for the Voyagers is likely due to the interruption in the continuous coverage of time-series data, which prohibits the detection of longer/larger event intervals, although these events are much rarer.

Figure 9 again shows the collective corresponding SFR parameters. They generally exhibit variations largely owing to the varying radial distances. Their values span wide ranges, and are well separated for different radial distances. One exception is the proton \((\beta)\), which exhibits similar distributions among all the event sets, with the mean values of the same order of magnitude, \(\sim 0.1\), despite the wide variation in radial distances.

Finally, to examine and quantify the trend in radial variation for a few selected flux-rope parameters, we show radial variations of the average field magnitude \(\langle B \rangle\), the average transverse field \(\langle B_t \rangle\), and the axial field \(\langle B_z \rangle\) over all flux-rope intervals with associated uncertainties (standard deviations) for each event set at the corresponding radial distance \(r\). A power-law fit of the form \(\propto r^{-\alpha}\) is also obtained for each quantity and the corresponding power-law indices are denoted, as given in Figure 10. They show a consistent trend of decaying with increasing radial distances, with the power-law indices \(\alpha \approx -1.4\). This value falls between \(-1\) and \(-2\). The former number corresponds to the radially decaying power-law index for the azimuthal component of the nominal Parker spiral field, while the latter corresponds to that for the radial field.

### 4. Radial Evolution of Magnetic Flux Ropes

One scenario for the radial evolution of SFRs is flux-rope merging associated with inverse magnetic energy transfer under the assumption that the poloidal magnetic flux \(A_m\) remains conserved. We choose a typical range of poloidal magnetic flux \(A_m \in [0.5, 1.5]\) T \cdot m and examine the corresponding radial evolution of selected properties. A preliminary analysis of the radial evolution of selected SFR properties (Chen et al. 2018) hinted at power-law decaying relations with respect to radial distance, with power-law indices ranging from \(-1.5\) to \(-0.5\) for a wide range of \(A_m\) values. This quantity, \(A_m\), represents the amount of poloidal magnetic flux per unit length (per meter) for a cylindrical flux rope, which is an output from our GS-based search algorithm and is directly derived from in situ spacecraft measurements. It is simply the difference between two flux function values, one at the center and the other at the boundary of a flux rope.

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**Table 2**

| Spacecraft | Year | Radial Distance (au) | Wan Slope Threshold | \(|B|\) (nT) | Counts |
|------------|------|----------------------|---------------------|-------------|--------|
| Helios 1 and 2 | ...\(a\) | 0.3–0.4 | 0.3 | >25 | 2491 |
| ACE | 2004 | ~1.0 | 0.3 | >5 | 3049 |
| Ulysses | 2004 | 5.3–5.4 | 0.5 | ≥0.2 | 2620 |
| Voyager 1 and 2 | 1980 | 6.5–8.1 | 0.5 | ... | 1967 |

**Note.**

\(a\) Multiple years from 1975 to 1981.
In order to compare with the relevant theoretical work (e.g., Zhou et al. 2019), we consider a scenario of consecutive flux-rope merging, leading to an increase in scale size but a decrease in magnetic field magnitude, while maintaining the poloidal magnetic flux. We further scrutinize our event sets by imposing the criterion that the unit poloidal magnetic flux, $\Phi_m = [0.5, 1.5]$ T $\cdot$ m, i.e., approximately a constant $\sim 1$ T $\cdot$ m with uncertainty. Such a value around 1 T $\cdot$ m is typical for an
SFR in the solar wind. A quantitative case study of flux-rope merging in interplanetary space was reported in Hu et al. (2019). It was found that given a dynamic evolution time on the order of $10^3$ s for the two adjacent flux ropes to fully merge into one via magnetic reconnection, the reconnected magnetic flux during the process would be equal to the amount of the poloidal flux of each flux rope. If we take the typical value 1 T m, then the reconnection rate can be estimated to be approximately $1 \times 10^{21}$ V m$^{-1}$ or 0.1 mV m$^{-1}$, which is not unreasonable for the solar wind, given that a value on the order of $\lesssim 1$ mV m$^{-1}$ was typically found for the reconnection events at the Earth’s magnetopause (e.g., Hasegawa et al. 2010). Such a range also ensures a more certain flux-rope configuration for the corresponding events by excluding the ones with small values of $A_m$ (i.e., < 0.5 T m), which are sometimes caused by a small rotating field component ($B_z$), indicating a less certain flux-rope configuration. Additionally, for the large-scale counterparts of SFRs, i.e., the MCs, the poloidal flux can amount to $\sim 100$ T m, corresponding to total poloidal magnetic flux of the order of magnitude $\sim 10^{12-13}$ Mx that can be related to the flux contents in solar source regions (Qiu et al. 2007; Hu et al. 2014).

Figure 11 shows the corresponding solar wind speed distributions for these subsets of events. Again, there are significant numbers of events from Helios with relatively low speeds around 300 km s$^{-1}$. Figure 12 shows distributions of the poloidal magnetic flux $A_m$ for the four spacecraft missions at different radial distances. They generally span a similar range, with maximum values of the order of tens of T m, except for the Voyagers. The event counts generally decrease with increasing radial distances, especially for $A_m \gtrsim 0.5$ T m. In Zhou et al. (2019), it was predicted that the time evolution of consecutive flux-rope merging yields the following power laws in time, $t$:

$$N \sim t^{-1.0}, \quad \langle B \rangle \sim t^{-0.5}, \quad k \sim t^{-0.5}. \quad (1)$$

These quantities are the flux-rope count $N$ over a 2D domain, the average magnetic field magnitude $\langle B \rangle$, and the inverse scale size $k$, under the condition that poloidal magnetic flux is conserved after merging. In other words, the amount of poloidal magnetic flux contained in each individual flux rope is not varying in time, although two neighboring flux ropes will merge into one consecutively. We intend to carry out an analysis from observations and the GS-based search result, following these conditions, with the additional assumption that the evolution in time as envisaged by Zhou et al. (2019) can be translated into the variation in radial distance, $r$, as we demonstrate below, by considering a constant radial solar wind flow. For instance, in Figure 11, the mean values among the average solar wind speed speed distributions for these events are similar. One obvious caveat that is also inevitable in this type of analysis of spacecraft data is the intrinsic radial change of the background field, i.e., the Parker interplanetary field in the solar wind, which was not incorporated into the theory of Zhou et al. (2019).

Table 3 summarizes the findings from the analysis of our unique event sets, with values of $A_m$ strictly confined within the range [0.5, 1.5] T m. The list of parameters includes event counts $N$, $\langle B \rangle$, $\langle B_z \rangle$, and $\langle B_\perp \rangle$ with associated uncertainties at corresponding radial distances, $r$. These results are further illustrated in Figure 13 with the corresponding power-law fittings, $\alpha r^\beta$. The count $N$ is obtained only along the radial dimension. In order to compare with the count predicted by Zhou et al. (2019) from their 2D simulations over a square area, we may compute $N^2$ to approximate such a count in 2D, i.e., $N'^{2} \approx N$. This yields a power-law index $\alpha \approx -1.5$. Therefore, similar to what Zhou et al. (2019) obtained as given in Equation (1), the radial evolutions of these quantities seem to obey the following power laws from our data analysis,

$$N^2 \sim r^{-1.5}, \quad \langle B \rangle \sim r^{-1.3}, \quad k \sim r^{-1.1}. \quad (2)$$

This set of power laws still differs significantly from those presented by Zhou et al. (2019), when assuming the equivalence between the radial distance $r$ and time $t$. 

Figure 10. Radial dependence with the heliocentric distance $r$ for (a) the average magnetic field magnitude and (b) the corresponding transverse and axial component. The symbols with error bars represent the averages over all selected flux-rope intervals and associated standard deviations, while the solid lines represent the linear fits, $\alpha r^\beta$. The corresponding slopes ($\alpha$) on the log-log scales are given.
5. Summary and Discussion

In summary, we have carried out a quantitative analysis of interplanetary spacecraft mission data, in addition to the ACE/WIND and Ulysses missions, following the automated detection approach for SFRs based on the GS reconstruction technique. The new results reported here yield the following total numbers of SFR event counts: 15,041 for Helios 1 and 7981 for Helios 2 throughout their whole mission periods; 1480 (1991) for Voyager 1 (2) in year 1980 only. The SFR properties derived from each event set are summarized and presented via statistical analysis. Targeted studies using subsets of events are performed, especially for the purpose of examining the radial evolution of selected flux-rope properties relevant to other works. Such a study is made feasible by including all derived event sets distributed over the range of heliocentric distances, namely, \( r \in [0.3, 7] \) au, and the unique approach of characterizing SFRs by the amount of poloidal magnetic flux obtained through the GS reconstruction method. The main findings are summarized as follows:

1. The event occurrence rate is still on the order of a few hundreds per month, for the range of radial distances between 0.3 and 7–8 au near the ecliptic plane.
2. The duration and scale-size distributions of SFRs again exhibit power laws. They possess different power-law slopes at different radial distances.
3. The axis orientations of the identified cylindrical SFRs have broad distribution peaks grossly centered around the nominal Parker spiral field directions at different radial distances. The trend is more pronounced for the polar angles than for the azimuthal ones.
4. The bulk properties of SFRs, such as the average magnetic field magnitude, proton number density, and temperature, generally exhibit clear decay in magnitudes with increasing radial distances, while the proton (\( \beta \)) remains largely unchanged.
5. The radial changes in magnetic field magnitudes, separately for the total field, the axial and the transverse components, seem to follow the general rule for a Parker magnetic field model, all with a power-law index close to \(-1.5\).
6. For a uniquely controlled subset of SFR events with corresponding unit poloidal magnetic flux \( A_m \in [0.5, 1.5] \) T \cdot m. The radial decaying in \( r \) for the quantities event counts \( N \), average field magnitude \( \langle B \rangle \), and inverse scale size \( k \), yield the corresponding power laws: \( N^2 \sim r^{-3} \), \( \langle B \rangle \sim r^{-1.3} \), and \( k \sim r^{-1.1} \).

The radial change in magnetic field seems to be consistent with the theoretical and observational analysis of 2D MHD turbulence throughout the heliosphere. For instance, theoretical work as well as the analysis of turbulence properties based on in situ spacecraft observations by Zank et al. (2017), Adhikari et al. (2017), and Zhao et al. (2017) showed the radial change of fluctuating magnetic power, following largely a power law \( \sim r^{\gamma} \) with the value \( \gamma \approx 2\alpha \) lying between \(-2 \) and \(-3 \), consistent with \( \alpha \approx -1.3 \) to \(-1.4 \) for the magnetic field variations from our analysis results. In particular, the radial change of the correlation length scale in the ranges \( r \in [0.3, 1] \) and \( >1 \) au seems to follow mostly power laws with different power-law indices (Adhikari et al. 2017;Zhao et al. 2017). According to Zhao et al. (2017), the correlation length scale changes from about the order of \( \lesssim 0.01 \) au for \( r \in [0.3, 1] \) au to about \( \gtrsim 0.01 \), approaching 0.1 au for \( r \in [1, 10] \) au, which is consistent with the change of the characteristic (average inverse) scale size of the SFRs from our analysis (see Figure 13) over these radial distances. It is also worth noting that the radial dependence of \( k \sim r^{-1.1} \) should not be interpreted as an indication of self-similar expansion, because this scale represents dimensions in the radial direction only, not the lateral (i.e., longitudinal or latitudinal) dimension, which has an \( r^{-1} \) dependence simply due to the

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**Figure 11.** Distribution of the average solar wind speed, \( v_{sw} \), for the events listed in Table 3. The format follows that of Figure 7.

**Figure 12.** Distributions of the poloidal magnetic flux per unit length, \( A_m \), for all the event sets listed in Table 2.

**Table 3**

| \( r \) (au) | \( N \) | \( \langle B \rangle \pm \Delta B \) (nT) | \( k \pm \Delta k \) (au\(^{-1}\)) | \( \langle B \rangle \pm \Delta B \) (nT) | \( \langle B \rangle \pm \Delta B \) (nT) |
|------------|------|----------------|-----------------|----------------|----------------|
| 0.34       | 866  | 37 ± 0.91      | 391 ± 77        | 22 ± 1.7       | 26 ± 2.0       |
| 1.0        | 599  | 8.4 ± 0.59     | 141 ± 32        | 4.3 ± 0.17     | 6.6 ± 0.72     |
| 5.4        | 157  | 1.1 ± 0.20     | 14 ± 3.2        | 0.58 ± 0.092   | 0.85 ± 0.22    |
| 7.2        | 72   | 0.91 ± 0.21    | 18 ± 5.3        | 0.54 ± 0.16    | 0.66 ± 0.15    |
radially outward flow. Therefore, such a dependence for the radial dimension may hint at certain intrinsic processes subject to local conditions when examined separately at different radial distances. Generally speaking, the apparent effect due to expansion in the radial dimension is controlled by setting the Walén slope threshold in our approach, with which events with significant remaining flows, an indication of expansion, are excluded from our event sets.

Although there appears to be more events from Helios occurring in slow solar winds, there is also a trend of increasing Alfvénicity at closer radial distances to the Sun, as indicated by the reduction in event counts (see Table 2) for Helios. A number of newly published studies have indicated this trend using PSP data but with different approaches (see, e.g., Kasper et al. 2019; Zhao et al. 2020). It is shown that highly Alfvénic activity persisted at rather low-speed solar wind streams. How the nature of solar wind fluctuations changes with radial distance or different streams can be further elucidated by checking the newly acquired PSP data. The application of the GS-based automated SFR detection approach to the publicly available PSP data is currently ongoing.

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Figure 13. Radial dependence of the parameters listed in Table 3 (read in order from left to right): the SFR event counts, the field magnitude, and the transverse and axial field components, respectively, and the inverse scale size. The format follows that of Figure 10.

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