1. Introduction

Our aims in this discussion are to summarize briefly on the main features of YFS/CEEX exponentiation [1] in QED and to present examples of recent theoretical results [2,3] relevant for the LEP2 and LC physics programs.

In the next section, we review the older EEX exclusive realization and summarize the new CEEX exclusive realization of the YFS [4] theory in QED. In this way we illustrate the latter’s advantages over the former, which is also very successful. We also stress the key common aspects of our MC implementations of the two approaches to exponentiation, such as the exact treatment of phase space in both cases, the strict realization of the factorization theorem, etc. In Sect. 3, we illustrate recent improvements in the KK MC realization of CEEX for the $\nu\bar{\nu}$ channel. In Sect. 4 we illustrate recent exact results on the single hard bremsstrahlung in 2f processes which quantify the size of the missing sub-leading $O(\alpha^2)\mathrm{L}$ terms in the KK MC. Sect. 5 contains our summary.

2. Standard Model calculations for LEP with YFS exponentiation

There are currently many successful applications [5] of the YFS theory of exponentiation for LEP and LC physics: (1), for $e^+e^-\rightarrow f\bar{f}+n\gamma$, $f = \tau, \mu, d, u, s, c$ there are YFS1 (1987-1989) $O(\alpha^1)_{\exp}$ ISR, YFS2 $\in$ KORALZ (1989-1990), $O(\alpha^1 + h.o.LL)_{\exp}$ ISR, YFS3 $\in$ KORALZ (1990-1998), $O(\alpha^1 + h.o.LL)_{\exp}$ ISR+FSR, and KK MC (98-02) $O(\alpha^2 + h.o.LL)_{\exp}$ ISR+FSR+IFI with $d\sigma/\sigma = 0.2\%$; (2), for $e^+e^-\rightarrow e^+e^-+n\gamma$ for $\theta < 6^\circ$ there are BHLUMI 1.x, (1987-1990), $O(\alpha^1)_{\exp}$ and BHLUMI 2.x,4.x, (1990-1996), $O(\alpha^1 + h.o.LL)_{\exp}$ with $d\sigma/\sigma = 0.07\%$; (3), for $e^+e^-\rightarrow e^+e^-+n\gamma$ for $\theta > 6^\circ$ there is BH-WIDE (1994-1998), $O(\alpha^1 + h.o.LL)_{\exp}$ with $d\sigma/\sigma = 0.2(0.5)\%$ at the Z peak (just off the Z peak); (4), for $e^+e^-\rightarrow W^+W^-+n\gamma$, $W^\pm \rightarrow f\bar{f}$ there is KORALW (1994-2001); and, (5), for $e^+e^-\rightarrow W^+W^-+n\gamma$, $W^\pm \rightarrow f\bar{f}$ there is YFS-WWW3 (1995-2001), YFS exponentiation + Leading Pole Approximation with $d\sigma/\sigma = 0.4\%$ at LEP2 energies above the WW threshold. The typical MC realization we effect is in the form of “matrix element $\times$ exact phase space” principle, as we illustrate in the following diagram:
In practice it means:

- The universal exact Phase-space MC simulator is a separate module producing "raw events" (with importance sampling).
- The library of several types of SM/QED matrix elements which provides the "model weight" is another independent module (the KKMC example is shown).
- Tau decays and hadronization come afterwards of course.

The main steps in YFS exponentiation are the reorganization of the perturbative complete $\mathcal{O}(\alpha^\infty)$ series such that IR-finite $\bar{\beta}$ components are isolated (factorization theorem) and the truncation of the IR-finite $\bar{\beta}$s to finite $\mathcal{O}(\alpha^n)$ with the attendant calculation of them from Feynman diagrams recursively. We illustrate here the respective factorization for overlapping IR divergences for the $2\gamma$ case – $R_{12} \in R_1$ and $R_{12} \in R_2$ as they are shown in the following picture:

\[
D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) = \bar{\beta}_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4});
\]

\[
p_{f_1} + p_{f_2} = p_{f_3} + p_{f_4}
\]

\[
D_1(p_{f_1}; k_1) = \bar{\beta}_0(p_{f_1}) \bar{S}(k_1) + \bar{\beta}_1(p_{f_1}; k_1);
\]

\[
p_{f_1} + p_{f_2} \neq p_{f_3} + p_{f_4}
\]

\[
D_2(k_1, k_2) = \bar{\beta}_0(k_1) \bar{S}(k_2) + \bar{\beta}_1(k_1) \hat{S}(k_2) + \bar{\beta}_2(k_1, k_2).
\]

Note: $\bar{\beta}_0$ and $\bar{\beta}_1$ are used beyond their usual (Born and $1\gamma$) phase space. A kind of smooth "extrapolation" or "projection" is always necessary. We see that a recursive order-by-order calculation of the IR-finite $\bar{\beta}$s to a given fixed $\mathcal{O}(\alpha^n)$ is possible:

Specifically,

\[
\bar{\beta}_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) = D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}),
\]

\[
\bar{\beta}_1(p_{f_1}; k_1) = D_1(p_{f_1}; k_1) - \bar{\beta}_0(p_{f_1}) \bar{S}(k_1),
\]

\[
\bar{\beta}_2(k_1, k_2) = D_2(k_1, k_2) - \bar{\beta}_0(k_1) \bar{S}(k_2) - \bar{\beta}_1(k_1) \hat{S}(k_2) - \bar{\beta}_1(k_1, k_2) \bar{S}(k_2), \ldots,
\]

allow such a truncation.

In the classic EEX/YFS schematically the $\beta$’s are truncated to $\mathcal{O}(\alpha^1)$, in the ISR example. For $e^-_{(p_1, \lambda_1)} + e^+_{(p_2, \lambda_2)} \rightarrow f_1(q_1, \lambda'_1) + f_2(q_2, \lambda'_2) + \gamma(k_1, \sigma_1) + \ldots + \gamma(k_n, \sigma_n)$, we have

\[
\sigma = \sum_{m=0}^{\infty} \int d\Phi_{n+2} e^{\chi(m)} D_n(q_1, q_2, k_1, \ldots, k_n)
\]

with

\[
D_0 = \bar{\beta}_0, \quad D_1(k_1) = \bar{\beta}_0 \bar{S}(k_1) + \bar{\beta}_1(k_1),
\]

\[
D_2(k_1, k_2) = \bar{\beta}_0 \bar{S}(k_1) \bar{S}(k_2) + \bar{\beta}_1(k_1) \hat{S}(k_2) + \bar{\beta}_2(k_2) \bar{S}(k_1)
\]

\[
D_n(k_1, k_2, \ldots, k_n) = \bar{\beta}_0 \bar{S}(k_1) \bar{S}(k_2) \ldots \hat{S}(k_n)
\]

\[
+ \bar{\beta}_1(k_1) \bar{S}(k_2) \bar{S}(k_3) \ldots \hat{S}(k_n)
\]

\[
+ \bar{\beta}_2(k_1, k_2) \bar{S}(k_3) \ldots \hat{S}(k_n)
\]

\[
+ \ldots + \bar{\beta}_n(k_1, k_2, \ldots, k_n).
\]

The real soft factors are $\hat{S}(k) = \sum_{\sigma} |\hat{\sigma}(k)|^2 = \ldots$
\[ |s_+(k)|^2 + |s_-(k)|^2 = \frac{2 |\hat{\beta}_0^\lambda|^2 - 2 |\hat{\beta}_0^\lambda|^2}{2 \text{Re} \{ \hat{\beta}_0^\lambda \}} \]

and the IR-finite building blocks are

\[ \hat{\beta}_0 = (e^{-i \Phi \lambda} \sum_\lambda |M_{\beta_0}^{\text{Born+Virt}}(\sigma)|^2) \hat{\sigma} + \text{fermion hel.}, \]

\[ \hat{\sigma} = \text{photon hel.}, \text{ and} \]

\[ \hat{\beta}_1(k) = - \sum_\lambda |M_{\beta_1}^{\text{Born+Virt}}(\sigma)|^2 - \sum_\sigma |s_\sigma(k)|^2 \sum_\lambda |M_{\lambda}^{\text{Born}}(\sigma)|^2. \]

Distributions are \( \leq 0 \) by construction!

In KKMC the above is done up to \( O(\alpha^2) \) for ISR and FSR.

The full scale CEEX \( O(\alpha^2) \), \( r=1,2 \), master formula for the polarized total cross section reads as

\[ \sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\Phi_{n+2} \sum_{\lambda,\sigma_1,\ldots,\sigma_n} |\hat{\sigma}^{(r)}(\lambda_{\sigma_1},\ldots,\sigma_n)|^2 \]

\[ \times \left\{ \beta^{(2)}(\lambda_{\sigma_1},\ldots,\sigma_n) \Phi_{n+2} + \sum_{j=1}^{n} \beta^{(1)}(\lambda_{\sigma_1},\ldots,\sigma_n) \Phi_{n+2} \left[ \Phi_{n+2}^{(j)} \Phi_{n+2}^{(j) \dagger} \right] \right\} \]

\[ \times \sum_{i,j} \frac{\beta^{(2)}(\lambda_{\sigma_1},\ldots,\sigma_n)}{s_{ij}} \left[ \Phi_{n+2}^{(i)} \Phi_{n+2}^{(i) \dagger} \right] \sum_{i,j} \frac{\beta^{(2)}(\lambda_{\sigma_1},\ldots,\sigma_n)}{s_{ij}} \left[ \Phi_{n+2}^{(j)} \Phi_{n+2}^{(j) \dagger} \right]. \]

For the details see ref. [1].

The precision tags of the KKMC are determined by comparisons with our own semi-analytical and independent MC results and by comparison with the semi-analytical results of the program ZFITTER [6]. In Fig. 1 we illustrate such comparisons, which lead to the KKMC precision tag \( \sigma/\sigma = 0.2\% \) for example. The ISR of ZFITTER is based on the \( O(\alpha^2) \) result of ref. [7], while KKMC is totally independent! See ref. [1] for a more complete discussion.

3. Extension of CEEX in KKMC to the \( e^+e^- \rightarrow \nu\bar{\nu}\gamma \) process

The respective tree level process is given by the Feynman diagrams in Fig. 2. As described
in ref. [2], the KK MC with CEEX matrix element is now extended to the neutrino mode. It is a replacement for the older KORALZ program. This new mode of the KKMC is useful for LEP final data analysis and for the first steps toward the LC. We note the following properties and improvements due to this new KKMC CEEX treatment of the $e^+e^-\rightarrow\nu\bar{\nu}\gamma$ process: (1), the systematic error is now estimated to be 1.3% for $\nu_e\bar{\nu}_e\gamma$ and 0.8% for $\nu_\mu\bar{\nu}_\mu\gamma$ and $\nu_\tau\bar{\nu}_\tau\gamma$; (2), for observables with two observed photons we estimate the uncertainty to be about 5%; (3), these new improved results were obtained thanks to the inclusion of the non-photonic electroweak corrections of the ZFITTER package and due to newly constructed, exact, single and double photon emission amplitudes in the KK MC for the contribution with the $t$-channel $W$ exchange; and, (4), the virtual corrections for the $W$ exchange are at present introduced in an approximated form but the CEEX exponentiation scheme is the same as in the original KK MC program.

4. Exact Differential $O(\alpha^2)$ Results for Hard Bremsstrahlung in $e^+e^-\rightarrow 2f$

The respective $O(\alpha^2)$ process is illustrated in Fig. 3. In ref. [3], we have presented fully differen-

5. Conclusions

YFS inspired EEX and CEEX MC schemes are successful examples of Monte Carlos based directly on the factorization theorem (albeit for the IR soft case for Abelian QED only). These
schemes work well in practice: KORALZ, BH-LUMI, YWSWW3, BHWIDE and KKMC are examples. The extension of such schemes (as far as possible) to all collinear singularities would be very desirable and practically important! Work on this is in progress.

Here, we have illustrated that the KKMC program is extended to the neutrino channel. Moreover, we have shown that the missing fully differential $2f + 1\gamma_{\text{virt}} + 1\gamma_{\text{real}}$ distributions for $\mathcal{O}(\alpha^2)$ CEEX are now available. Applications to final LEP data analysis and to LC studies are in progress.

The authors thank Profs. G. Altarelli and Wolf-Dieter Schlatter for the support and kind hospitality of the CERN Theory Division and the CERN LEP Collaborations while this work was in progress. One of the authors (B.F.L.W.) also thanks Profs. S. Bethke and L. Stodolsky for the support and kind hospitality of the Werner-Heisenberg-Institut, Max-Planck-Institut, Munich, while this work was in progress.

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