Experimental Review of Inclusive $|V_{ub}|$ Measurements

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We review the status of inclusive $|V_{ub}|$ measurements where many new results have become available recently.

1 Introduction

Precise measurements of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ provide stringent consistency tests of the unitarity triangle. The data on the ratio $|V_{ub}|/|V_{cb}|$ form an annulus in the unitarity plane. This can be tested against the apex of the triangle, determined by the measurements of the CP violating phase $\sin 2\beta$ and the $B^0\bar{B}^0$ mixing parameter $|\Delta m_d|$. The BaBar and Belle experiments have recently measured $\sin 2\beta$ with an error of 0.055 [1] and this precision is expected to improve to 0.02 in a few years. The precision on $|V_{ub}|$ will soon limit unitarity triangle tests. Hence we aim to measure $|V_{ub}|$ to 10% or better in the near future.

2 Status and Methods

Ten Years ago, CLEO and Argus had established that $|V_{ub}|$ was non-zero [2] by observing an excess of leptons at the endpoint of the momentum spectrum (2.3 - 2.6 GeV/c). However, the accuracy of this result, $|V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$, was theoretically limited by the extrapolation into the full phase which were based on (exclusive) models. Since then huge progress has been made by both theory and experiment. As reported at this workshop [3], heavy quark effective theory (HQET) and operator production expansion (OPE) allow a calculation of the inclusive charmless semileptonic decay rate of $B$ mesons, $B \to X_u\ell\nu$. The distributions for several kinematic variables, e.g. the lepton momentum, $p_\ell$, have also been calculated up to order $a_S$ [4]. The theoretical uncertainties in the determination of $|V_{ub}|$ on the $b$-quark mass, $m_b$, and the average kinetic energy of the $b$-quark within the $B$ meson, $\lambda_1$, are now quantifiable.

The experimental challenge is to suppress the background from $B \to X_c\ell\nu$ transitions. These have a rate that is higher by two orders of magnitude. Only $\sim 10\%$ of $B \to X_u\ell\nu$ decays have a lepton momentum larger than 2.3 GeV/c and are above the endpoint of the $B \to X_c\ell\nu$ background. With the large statistics of $B$ mesons available at the $B$ factories, additional methods with lower efficiency but higher purity are now accessible.

An additional kinematic variable has been already employed by the LEP experiments, namely the mass $m_X$ of the hadronic system recoiling against the lepton pair in the rest frame of the $B$ meson. About 70% of the $B \to X_u\ell\nu$ decays have $m_X < m_D$ where the mass of the $D$ meson, $m_D$, is the minimum mass $m_X$ for $B \to X_c\ell\nu$ decays. In $\sim 25\%$ of $B \to X_u\ell\nu$ decays the invariant mass of the lepton-neutrino pair, $q^2$, exceeds the maximum possible value of background $B \to X_c\ell\nu$ decays. The energy of the hadronic recoil system could also be used to separate signal from background. The distributions of these kinematic variables have different dependencies on $m_b$ and $\lambda_1$. Thus it is important to measure all kinematic distributions.

Here, we give an overview of six new results on inclusive $|V_{ub}|$ measurements which have been made public by the CLEO, BaBar and Belle collaborations during the last year. For details on these measurements the reader is referred to other presentations at this workshop [5],[6].

Measured inclusive branching ratios $B \to X_u\ell\nu$ are converted into $|V_{ub}|$ measurements using the relation

$$|V_{ub}| = 0.00445 \sqrt{\frac{B(BXulv) 1.55ps}{0.002 \tau_B}} (1 \pm 0.020 \pm 0.052)$$

or equivalent [7],[8],[9]. The first error arises from the perturbative scale dependence and the second error is due to the $b$ quark mass for which an uncertainty of 90 MeV has been assigned. In what follows, we will refer to these two uncertainties as theoretical error.

3 Lepton Endpoint Spectrum

A new measurement on the excess of leptons in the endpoint region has been published by CLEO [10]. The full data set of 9.13 fb$^{-1}$ at the $Y(4S)$ resonance and 4.35 fb$^{-1}$ just below has been used. They have carefully designed the suppression of the background from non-$B$ decays (continuum) in order to minimise model dependence on the selection cuts. In Fig. 1 we show the resulting lepton momentum spectrum. A signal interval of 2.2 - 2.6 GeV/c was chosen as this gives the smallest total error. A clear excess of $1901 \pm 122 \pm 256$ events, attributed to $B \to X_u\ell\nu$, is visible in this region. A partial branching ratio of

$$\Delta B(B \to X_u\ell\nu) = (0.230 \pm 0.015 \pm 0.035) \times 10^{-3}$$
The CLEO measurement of $|V_{ub}|$ result and CLEO obtains

$$|V_{ub}| = (4.08 \pm 0.34 \pm 0.44 \pm 0.16 \pm 0.24) \times 10^{-3}$$

where the errors are separated into experimental, signal fraction, theoretical and shape function components, respectively.

Using the lepton endpoint method, the BaBar experiment has presented a preliminary measurement based on 20.6 fb$^{-1}$ of data at the $\Upsilon(4S)$ resonance and 2.6 fb$^{-1}$ below $\Upsilon(4S)$. An excess of 1696$\pm$133$\pm$153 events is observed in the lepton momentum range of 2.3 - 2.6 GeV/c. BaBar measures a partial branching ratio of

$$\Delta B(B \to X_u \ell \nu) = (0.152 \pm 0.014 \pm 0.014) \times 10^{-3}$$

where the experimental systematic error is dominated by the uncertainty in the continuum background subtraction. The CLEO measurement of $f_u = 0.074 \pm 0.014 \pm 0.009$ for this momentum interval is used to extrapolate to the full phase space. Due to the larger extrapolation the uncertainty is comparable to the experimental systematic error. From the $B \to X_u \ell \nu$ branching ratio BaBar measures a preliminary $|V_{ub}|$ result of

$$|V_{ub}| = (4.43 \pm 0.39 \pm 0.50 \pm 0.25 \pm 0.35) \times 10^{-3}$$

where the errors are separated into experimental, signal fraction, theoretical and shape function components, respectively.

4 Hadronic Recoil Mass

A measurement of $|V_{ub}|$ by reconstructing the mass $m_X$ of the hadronic system recoiling against the lepton pair in the $B \to X_u \ell \nu$ decay has been carried out by BaBar [5]. A data sample of about 82 fb$^{-1}$ is used. The large statistics of $B$ meson samples produced at the B-factories permits the use of new methods which have low efficiency, but allow to exploit the separation power of $m_X$. The experimental task is to separate the spatially overlapping decay products of the two $B$ mesons produced at the $\Upsilon(4S)$. The principle is to fully reconstruct one $B$ meson in many hadronic modes: $B \to D^{*+} \pi$, $D^{*0} \pi^0$, $D^{*+} \pi^0$, .... The hadronic $B$ meson tags the $b$-quark flavour, and more importantly removes the background from one of the $B$ mesons in the event. The method yields about 4000 $B$ mesons per fb$^{-1}$. All the remaining particles that are detected in a tagged event must originate from the other $B$ meson. Events are selected if a lepton with a momentum $p_\ell > 1$ GeV/c in the $B$ rest frame is identified among the particles recoiling against the tagging $B$ meson. The energy substituted mass spectrum, $m_{ES}$, of the tagging $B$ candidates is shown in Fig. 3a. The signal to background ratio is large and a fit is used to remove the continuum background. Additional criteria, e.g. charge correlation and a missing mass consistent with a neutrino are applied to increase the purity of this sample. The hadronic recoil mass, $m_X$, is reconstructed; a constraint fit imposing zero missing mass is employed to improve the $m_X$ resolution. The $B \to X_u \ell \nu$ signal region is also enriched by vetoing semileptonic $B$ candidates containing $K^0_S$ or $K^0_L$ decay products. The depleted region is used as a control sample. In Fig. 3a) we plot the measured $m_X$ spectrum. A binned fit is overlaid. The signal region is $m_X < 1.55$ GeV which gives the smallest total error. It contains one bin to minimise the model dependence. An excess of $167 \pm 21$ events is observed and attributed to $B \to X_u \ell \nu$. The signal to background ratio is about $2:1$. The background subtracted $m_X$ spectrum with finer bins in the signal region is shown in Fig. 3b). To reduce systematic errors, the ratio $R_u = \mathcal{B}(B \to X_u \ell \nu)/\mathcal{B}(B \to X \ell \nu)$ is measured. BaBar obtains a preliminary result of

$$R_u = (1.97 \pm 0.27 \pm 0.23 \pm 0.34) \times 10^{-2}$$

where the errors are statistical, systematic and extrapolation into the region above the $m_X$ cut. Using this branching
Employing a method similar to BaBar, the Belle experiment has presented a preliminary $|V_{ub}|$ result

$$|V_{ub}| = (4.52 \pm 0.31 \pm 0.27 \pm 0.30 \pm 0.26) \times 10^{-3}$$

where the errors are statistical, systematic, $m_X$ extrapolation and theoretical. The extrapolation error still dominates, but improvements in the knowledge of $\lambda_1$ and $\lambda_2$ are expected from measurements in the next few years. The potential of this method is its small systematic error, thus more statistics will allow to improve the precision on $|V_{ub}|$.

Employing a method similar to BaBar, the Belle experiment has presented a preliminary $|V_{ub}|$ result \[6\] [17]. Fully reconstructed $B^0 \rightarrow D^{*-} \ell^+\nu$ decays are used as tagging $B$ mesons. Hence, in selected events, both mesons decay semileptonically. The tag removes the decay products of one of the $B$ mesons and substantially suppresses the continuum background from non-$B$ decays. Since there are two neutrinos in the event, the $B$ momentum cannot be determined from the tagging $B$ meson. The direction of the two $B$ momenta lay on two cones which can be reconstructed up to an azimuthal angle. After mirroring one of the cones on the origin, the $B$ direction must then lay on the cross line of the two cones and can be reconstructed. As with hadronic tags, the hadronic recoil mass $m_X$ is used to separate $B \rightarrow X_u \ell \nu$ decays from $B \rightarrow X_d \ell \nu$ decays. In the signal region, defined as $m_X < 1.5$ GeV, Belle observes an excess of $172 \pm 28$ leptons with momenta $p_T > 1$ GeV/c as is shown in Fig. 4. This excess corresponds to a branching ratio

$$\mathcal{B}(B \rightarrow X_u \ell \nu) = (2.62 \pm 0.63 \pm 0.23 \pm 0.41) \times 10^{-3}$$

where the errors are statistical, systematic and extrapolation into the region above the $m_X$ cut. Belle obtains a preliminary result for $|V_{ub}|$ of

$$|V_{ub}| = (5.00 \pm 0.60 \pm 0.23 \pm 0.39 \pm 0.36) \times 10^{-3}$$

where the errors are separated into statistical, experimental, $m_X$ extrapolation and theoretical components, respectively.

5 Neutrino Reconstruction

The neutrino reconstruction technique relies on exploiting the hermeticity of the detector to infer the neutrino momentum from the missing momentum. The missing energy is
also measured and allows to require that the missing mass-
squared of the event be consistent with zero. For experi-
ments at the $\Upsilon(4S)$ resonance, this method has been pio-
nereed for exclusive $|V_{ub}|$ and $D_s^+ \to \mu^+\nu$ measurements [14],[15].

![Figure 5. Fit projection for the $q^2$ spectrum from CLEO (points). The $B \to X_u\ell\nu$ signal, $B \to X_c\ell\nu$ and other background simulations are shown as histograms.](image)

Using neutrino reconstruction, the CLEO experiment has presented a preliminary result on $|V_{ub}|$ using events with inclusive charged leptons [16]. The measurement of the neutrino momentum allows the use of kinematic variables in addition to the charged lepton energy, $E_\ell$. The hadronic recoil mass squared, $m^2_X$, the invariant mass of the lepton pair, $q^2$, and $\cos\theta_{W\ell}$, the helicity angle of the virtual $W$ are used here. Selected events are required to have a lepton with momentum $p_T > 1 \text{ GeV/c}$. A simultaneous fit to the quantities $q^2$, $m^2_X$, and $E_\ell$ is performed. The $B \to X_u\ell\nu$ signal region is restricted to the range $q^2 > 10 \text{ GeV}^2$ and $m^2_X < 2.25 \text{ GeV}^2$. In Fig. 5 we show the $q^2$ projection of the fit with the $m_X$ cut applied for a data sample of 9.4 fb$^{-1}$. From this fit CLEO extracts a preliminary result of

$$|V_{ub}| = (4.05 \pm 0.18 \pm 0.58 \pm 0.33 \pm 0.56) \times 10^{-3}$$

where the errors are statistical, experimental, $m_X$ and $q^2$ extrapolation and theoretical.

The Belle experiment has also presented a preliminary $|V_{ub}|$ result using events with charged leptons and neutrino reconstruction [16],[17]. In addition, the particles in each event are separated into a tagging and a signal $B$ meson by employing a combinatorial annealing [18]. The annealing procedure minimizes the likelihood ratio of random over the correct assignments of particles to the two

![Figure 6. Background subtracted $q^2$ spectrum (points) from Belle. The $B \to X_u\ell\nu$ simulation (histograms) is overlaid.](image)

$B$ mesons in the events. The efficiency of this method is about 0.3%. A control sample of $B^0 \to D^-\ell^+\nu$ decays is used to check the annealing and to correct for efficiency differences between data and simulation. After annealing, all kinematic variables of interest are available, for example $q^2$ and the hadronic recoil mass $m_X$ in the $B$ mesons decaying semileptonically. The signal is extracted in the region of $q^2 > 7 \text{ GeV}^2$ and $m_X < 1.5 \text{ GeV}$. Belle measures an excess of 1148 events over background is measured with a signal to background ratio of 0.27. In Fig. 6 we plot the background subtracted $q^2$ spectrum with the $m_X$ cut applied. Also shown are $B \to X_u\ell\nu$ simulations for two values of $m_b$. A preliminary branching ratio of

$$\mathcal{B}(B \to X_u\ell\nu) = (1.64 \pm 0.14 \pm 0.36 \pm 0.36) \times 10^{-3}$$

is measured. The errors are statistical, systematic and extrapolation into the region above the $m_X$ and $q^2$ cut. From this measurement, Belle obtains a preliminary $|V_{ub}|$ result of

$$|V_{ub}| = (3.96 \pm 0.17 \pm 0.44 \pm 0.43 \pm 0.29) \times 10^{-3}$$

where the errors are separated into statistical, experimental, $m_X$ and $q^2$ extrapolation and theoretical components, respectively.

6 Summary and Conclusions

The $B$-factories experiments, BaBar and Belle, and CLEO have recently produced many new results for the CKM matrix element $|V_{ub}|$. Different kinematic variables have been used in the results which are based on measurements of
the inclusive \( B \to X_d \ell \nu \) decay rate. In Fig. 7 we show a compilation of all \( |V_{ub}| \) results, carried out by the Heavy Flavour Averaging Group (HFAG) for the winter conferences 2003 [19]. A procedure to average these results is being developed. We have made a preliminary average of all inclusive \( |V_{ub}| \) measurements made by the BaBar, Belle and CLEO experiments and obtain

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|V_{ub}| = (4.32 \pm 0.57) \times 10^{-3}.
\]

Statistical and detector errors are treated uncorrelated whereas the extrapolation and theoretical errors are correlated and rescaled. An arbitrary 5\% error is assigned to the validity of the shape function being equal for \( b \to s \gamma \) and \( B \to X_d \ell \nu \). Hence we consider this a conservative average.

In this review, we have shown that substantial experimental and theoretical progress has been made over the last decade for measuring \( |V_{ub}| \), a crucial parameter in the unitarity triangle. The B-factories allow the use of new experimental methods. The single most precise measurement is the BaBar result using the hadronic recoil mass \( m_X \) spectrum. The precision of this result is 14\% which is better than the average of the LEP results [20]. Excellent neutrino reconstruction is crucial for measuring \( q^2 \) which has lower sensitivity to \( m_b \) than \( m_X \). The theoretical precision will improve as more precise measurements of the moments of the shape function moments, \( \Lambda \) and \( \lambda_1 \), will become available. The aim is to reduce the error on \( |V_{ub}| \) to substantially better than 10\% over the next five years.

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