Research Article

Extended Yearly LMDI Approaches: A Case Study of Energy Consumption

Jiandong Chen,1 Ming Gao,1 Ding Li,2 Malin Song3, Qianjiao Xie,3 and Jixian Zhou4

1School of Public Administration, Southwestern University of Finance and Economics, Chengdu, China
2Institute of Development Studies, Southwestern University of Finance and Economics, Chengdu, China
3School of Statistics and Applied Mathematics, Anhui University of Finance and Economics, Bengbu, China
4School of Economic Information Engineering, Southwestern University of Finance and Economics, Chengdu, China

Correspondence should be addressed to Malin Song; songmartin@126.com

Received 12 September 2019; Accepted 26 December 2019; Published 30 January 2020

Abstract

Although the logarithmic mean Divisia index (LMDI) approach has been widely used in the field of energy and environmental research, it has a shortcoming. Since the LMDI approach only focuses on the base year and reporting year, in situations in which the research period is long, the annual changes during the research period may be difficult to capture. In particular, if there were huge fluctuations in the indicators (such as the energy consumption and carbon emissions) or their drivers during the middle of the research period, a substantial amount of information about the fluctuations will be ignored. Therefore, we propose four extended yearly LMDI approaches, including pure LMDI, weighted LMDI, comprehensive LMDI, and scenario LMDI approaches to better capture fluctuations and compensate for the original LMDI approach’s shortcomings. Additionally, we found that there are mathematical relationships among the four extended LMDI approaches. We further compare these four approaches’ advantages, disadvantages, and applicable situations and analyze a case study on China’s energy consumption based on the four proposed approaches.

1. Introduction

The index decomposition analysis (IDA) approach has been widely used in the field of energy and environmental economics [1, 2]. Among the different types of IDA approaches, the logarithmic mean Divisia index (LMDI) approach is one of the most popular methods; it was proposed by Ang and Liu [3] and further developed by Ang [4–6] and helps to analyze the effects of driving factors on carbon emissions or energy consumption [7–9]. Since its numerical properties and economic implications are optimal [5, 10], the LMDI approach has been adopted in different fields of research, including the manufacturing, textile, and power industries [11–14]. Additionally, even though the LMDI approach cannot handle zero-value data, Ang [5] overcame this problem by using an analytical limit technique where zero values were replaced by arbitrarily small numbers that do not affect the results.

Although the LMDI approach originally proposed by Ang [4] has many advantages, it also has a shortcoming. Since the decomposition created by the original LMDI approach only relies on the base year and reporting year, substantial information during the middle of the research period will be missed if the research period is long. Simultaneously, if there are huge fluctuations in the indicators (such as energy consumption) or their decomposed drivers (such as energy intensity) during the research period, annual changes will be difficult to capture, leading to inaccurate results. For example, if the base and reporting years’ energy consumption (or its drivers) are the same, research based on the original LMDI approach will conclude that energy consumption (or its drivers) does not change (or influence anything) even if there are fluctuations in the middle of the research period.
research period, which is neither comprehensive nor reasonable. Hence, our proposal of an extended yearly LMDI approach is necessary to account for these limitations.

Although many scholars have used the original LMDI approach to measure the impact of various incentives on energy consumption (or other indicators, such as carbon emissions) during consecutive two-year periods [15, 16], which may not ignore the yearly information, few studies further discuss and explore the applicable scenarios of this type of decomposition. Actually, this is also one of the extended yearly LMDI methods we intend to introduce and call it as the weighted LMDI approach. It makes the settings of the weights depend on consecutive two-year periods and change yearly, which is helpful to capture the indicator’s short-term fluctuations but fail to exclude the influences of changes in the weights and reflect the changes caused by drivers themselves. Simultaneously, few scholars have systematically derived a formula for extended yearly LMDI approaches and discuss their mathematical relationships or applicable situations. Hence, there leaves room for further explorations.

Therefore, in this study, we propose four extended yearly LMDI approaches, which are, respectively, called pure, weighted, comprehensive, and scenario LMDI approaches to better capture fluctuations of the indicator (or its decomposed drivers) and make up for the original LMDI approach’s shortcomings. Moreover, we further analyze the mathematical relationships among the four extended yearly LMDI approaches and compare their advantages, disadvantages, and applicable situations. Finally, we provide a case study of China’s energy consumption based on the extended yearly LMDI approaches and discuss their mathematical relationships or applicable situations.

As a result, this paper makes three marginal contributions to the literature: (1) we proposed four extended yearly LMDI approaches to make up for the original LMDI approach’s limitations and better capture different fluctuations during the middle of the research period; (2) we derived and proved the mathematical relationships among the four approaches; and (3) we further compared these four approaches’ advantages and disadvantages and discussed their applicable situations.

This paper is organized as follows. The first part presents the introduction; the second part proposes the four extended yearly LMDI approaches; the third part provides a case study of China’s energy consumption based on the extended yearly LMDI approach; and the fourth part presents conclusions.

2. Methodology

If the interval between the base year (b) and the reporting year (t) is long, the yearly changes during the middle of the research period may be difficult to capture. In particular, if there were huge fluctuations in the indicators (such as energy consumption) or their decomposed drivers (such as energy intensity) during the middle of the research period, substantial information about the fluctuations would be ignored. Therefore, this paper derives and proposes four extended yearly LMDI approaches to capture the fluctuations in the middle of the research period.

2.1. Four Extended Yearly LMDI Approaches. In line with the guideline and analysis of the LMDI approach originally proposed by Ang [6], we adopt a quantity indicator \( D^f \) and we assume that there are three driving factors—\( A^t, B^t, \) and \( C^t \)—which, respectively, represent the well-known activity, structure, and intensity effects. Additionally, \( D^f \) can refer to any indicator, such as energy consumption, electric power consumption, or carbon emissions.

Considering a study in which a change in the quantity indicator \( D^f \) is to be decomposed, we begin with the following IDA identity:

\[
D^f = A^t \times B^t \times C^t. \tag{1}
\]

The change in \( D^f \) from the base year (b) to the reporting year (t) can be, respectively, decomposed into \( D^{f,b} \), \( D^{f,t} \), and \( D^{f,t} \) based on the original LMDI approach. Furthermore, we conduct a yearly decomposition and obtain the following equation:

\[
\Delta D^{f,b} = \Delta D^{h,b+1} + \Delta D^{b+1,b+2} + \cdots + \Delta D^{t-1,t} = (\Delta D_{A}^{h,b+1} + \Delta D_{B}^{b+1,b+2} + \Delta D_{C}^{b+2,b+3}) + \cdots (\Delta D_{A}^{t-1,t} + \Delta D_{B}^{t-1,t} + \Delta D_{C}^{t-1,t})
\]

\[
\Delta D^{f,t} = (\Delta D_{A}^{h,b+1} + \Delta D_{B}^{b+1,b+2} + \Delta D_{C}^{b+2,b+3}) + \cdots (\Delta D_{A}^{t-1,t} + \Delta D_{B}^{t-1,t} + \Delta D_{C}^{t-1,t}). \tag{2}
\]

The yearly decomposition presented in equation (2) can be realized by the following derivation.

In accordance with the equation \( D^f = A^t \times B^t \times C^t \), we take the logarithm on both sides of the equation and get another equation—\( \ln(D^f) = \ln(A^t) + \ln(B^t) + \ln(C^t) \). We can then obtain the following set of equations as follows:

\[
\begin{align*}
\ln(D^f) &= \ln(A^t) + \ln(B^t) + \ln(C^t) + \cdots + (1) \\
\ln(D^{f,t}) &= \ln(A^{t-1}) + \ln(B^{t-1}) + \ln(C^{t-1}) + \cdots + (2) \\
& \vdots \\
& \vdots \\
\ln(D^{f,b}) &= \ln(A^b) + \ln(B^b) + \ln(C^b) + \cdots + (t - b + 1) \\
\end{align*}
\]

Then, we subtract the two adjacent equations in equation set (3) and obtain the following set of equations:
Given equation set (4), we have four approaches to decompose \((D^t - D^b)\) and realize the yearly decomposition presented in equation (2): the pure LMDI, weighted LMDI, comprehensive LMDI, and scenario LMDI approaches.

2.1.1. Pure LMDI Approach. The pure LMDI approach is one of the four extended yearly LMDI approaches, which leaves the weights unchanged annually and the weights depend on the base and reporting years. Therefore, a pure LMDI approach helps compare the effects purely caused by driving factors themselves and considers long-term indicator trends. The derivation of the pure LMDI approach is as follows.

We stack equation (1) through \((t - b)\) in equation set (4) and obtain the following equation:

\[
\ln\left(\frac{D_t}{D_b}\right) = \ln\left(\frac{A_t}{A_{t-1}}\right) + \cdots + \ln\left(\frac{A_{b+1}}{A_b}\right)
\]

\[
+ \left(\ln\left(\frac{B_t}{B_{t-1}}\right) + \cdots + \ln\left(\frac{B_{b+1}}{B_b}\right)\right) + \ln\left(\frac{C_t}{C_{t-1}}\right)
\]

\[
+ \cdots + \ln\left(\frac{C_{b+1}}{C_b}\right).
\]

(5)

Then, we multiply both sides of equation (5) by \((D^t - D^b)\ln(D^t/D^b)\) and obtain \(\Delta D^{b+1}\) on the left side of equation (5) and the yearly decomposition on the right side of equation (5). The detailed formula for this type of the extended yearly LMDI approach is proposed in Table 1. Actually, this kind of yearly decomposition is similar to the method provided by Chen et al. [17] which also remains the weight unchanged every year. But they failed to discuss and explain the advantages and disadvantages of this type of yearly decomposition. This method can only be applied in one kind of situation when scholars only hope to analyze fluctuations of drivers themselves, but excludes the influence of changes in weights, which can put changes in external situations into consideration.

2.1.2. Weighted LMDI Approach. The weighted LMDI approach is another kind of extended yearly LMDI approach. Its weights change every year and rely on consecutive two-year periods, which help to reflect the short-term trends in the indicator and reflects changes in the external situation. The derivation is as follows.

For equation (1) through \((t - b)\) in equation set (4), we can first multiply both sides of the equation by \((D^{n+1} - D^n)/\ln(D^{n+1}/D^n)\) \((n = b, b + 1, \ldots, t - 1, t)\) and obtain the following equations:

\[
\begin{align*}
\ln\left(\frac{D_t}{D_{t-1}}\right) &= \ln\left(\frac{A_t}{A_{t-1}}\right) + \ln\left(\frac{B_t}{B_{t-1}}\right) + \ln\left(\frac{C_t}{C_{t-1}}\right) + \cdots + (1) \\
\ln\left(\frac{D^{t-1}}{D^{t-2}}\right) &= \ln\left(\frac{A^{t-1}}{A^{t-2}}\right) + \ln\left(\frac{B^{t-1}}{B^{t-2}}\right) + \ln\left(\frac{C^{t-1}}{C^{t-2}}\right) + \cdots + (2) \\
\vdots & \ddots \\
\ln\left(\frac{D^{b+1}}{D^b}\right) &= \ln\left(\frac{A^{b+1}}{A^b}\right) + \ln\left(\frac{B^{b+1}}{B^b}\right) + \ln\left(\frac{C^{b+1}}{C^b}\right) + \cdots + (t - b)
\end{align*}
\]

(6)
Table 1: The pure LMDI approach’s decomposition formula for the three driving factors.

| Additive decomposition formula | Weight |
|-------------------------------|--------|
| $\Delta D^\beta_A$ | $\Delta D^\beta_A = \sum_{i=1}^{k} w_i \ln (A_i^{\beta_i}/A_{i-1}^{\beta_i})$ | $w_i = ((D^i - D^0)/(\ln(D^i) - \ln(D^0)))$ |
| $\Delta D^\beta_B$ | $\Delta D^\beta_B = \sum_{i=1}^{k} w_j \ln (B_i^{\beta_j}/B_{i-1}^{\beta_j})$ | $w_j = (D^j - D^0)/(\ln(D^j) - \ln(D^0))$ |
| $\Delta D^\beta_C$ | $\Delta D^\beta_C = \sum_{i=1}^{k} w_j \ln (C_i^{\beta_j}/C_{i-1}^{\beta_j})$ | $w_j = ((D^j - D^0)/(\ln(D^j) - \ln(D^0)))$ |

Then, we stack the equations above together. Hence, we obtain $\Delta D^\beta_k$ on the left side of the equation and the yearly decomposition on the right side of the equation. The detailed formula for this type of the extended yearly LMDI approach is proposed in Table 2.

2.1.3. Scenario LMDI Approach. In the long run, when certain major external events occur, such as an economic crisis, government regulation, trade wars, and taxation policies, they may have an important impact on the external situation and development trends of the indicator (such as energy consumption or electric power consumption). Therefore, we should divide the long research period into several short periods in light of major external shocks and, respectively, set corresponding weights in different divided periods. For these reasons, we propose the scenario LMDI approach as another extended yearly LMDI approach.

We assume that there was an important event that occurred in year $(k)$ ($b < k < t$), which had an important impact on the development trends of indicator $D^t$. Therefore, the research period should be divided into two periods—year $(b)$ to year $(k)$ and year $(k)$ to year $(t)$. The interval between year $(b)$ and year $(k)$ should have the same weight, while the period between year $(k)$ and year $(t)$ should be set at another weight. The derivation of the scenario LMDI approach is as follows.

First, we, respectively, stack equation (1) through $(t-k)$ and equation $(t-k+1)$ through $(t-b)$ in equation set (4) and obtain the following two equations:

$$\ln\left(\frac{D^t}{D^b}\right) = \ln\left(\frac{A^t}{A^{t-1}}\right) + \ln\left(\frac{B^t}{B^{t-1}}\right) + \ln\left(\frac{C^t}{C^{t-1}}\right) + \cdots$$

$$+ \left(\ln\left(\frac{A^{k+1}}{A^k}\right) + \ln\left(\frac{B^{k+1}}{B^k}\right) + \ln\left(\frac{C^{k+1}}{C^k}\right)\right).$$

(7)

$$\ln\left(\frac{D^k}{D^b}\right) = \ln\left(\frac{A^k}{A^{k-1}}\right) + \ln\left(\frac{B^k}{B^{k-1}}\right) + \ln\left(\frac{C^k}{C^{k-1}}\right) + \cdots$$

$$+ \left(\ln\left(\frac{A^{b+1}}{A^b}\right) + \ln\left(\frac{B^{b+1}}{B^b}\right) + \ln\left(\frac{C^{b+1}}{C^b}\right)\right).$$

(8)

Then, we, respectively, multiply both sides of equation (7) by $(D^k - D^b)/\ln(D^k/D^b)$ and multiply both sides of equation (8) by $(D^t - D^b)/\ln(D^t/D^b)$. Then, we obtain the following two equations:

$$D^t - D^b = \frac{D^t - D^b}{\ln(D^t/D^b)} \left[\ln\left(\frac{A^t}{A^{t-1}}\right) + \ln\left(\frac{B^t}{B^{t-1}}\right) + \ln\left(\frac{C^t}{C^{t-1}}\right) + \cdots + \left(\ln\left(\frac{A^{k+1}}{A^k}\right) + \ln\left(\frac{B^{k+1}}{B^k}\right) + \ln\left(\frac{C^{k+1}}{C^k}\right)\right)\right],$$

(9)

$$D^k - D^b = \frac{D^k - D^b}{\ln(D^k/D^b)} \left[\ln\left(\frac{A^k}{A^{k-1}}\right) + \ln\left(\frac{B^k}{B^{k-1}}\right) + \ln\left(\frac{C^k}{C^{k-1}}\right) + \cdots + \left(\ln\left(\frac{A^{b+1}}{A^b}\right) + \ln\left(\frac{B^{b+1}}{B^b}\right) + \ln\left(\frac{C^{b+1}}{C^b}\right)\right)\right].$$

(10)

Secondly, we stack equations (9) and (10) and obtain $\Delta D^{k,b}$ on the left side of the equation and the yearly decomposition on the right side. Since this kind of yearly LMDI approach considers great changes in the external situation, we call it a scenario LMDI approach. The detailed formula for this type of extended yearly LMDI approach is proposed in Table 3. Additionally, it is obvious that if $k = b$ or $k = t$, the decomposition approach is the same as that for the pure LMDI approach.

Although the scenario LMDI approach considers great changes in the external situation, it is not able to provide the point in time at which the structural mutation happened, so it is necessary for scholars to presuppose the point in time at which the external shock caused the structural mutation. Such a deficiency may lead to subjectivity in the process.

2.1.4. Comprehensive LMDI Approach. Although the scenario LMDI approach can better reflect large changes in the external situation, it cannot provide the point in time at which the structural mutation happened. Hence, the comprehensive LMDI approach is proposed to make up for the shortcomings of the previous approach.

As for equation (1) through $(t-b)$ in equation set (4), we can first, respectively, multiply both sides of the equation by
obtaining the following equation:

\[
\frac{1}{\ln(D^{n+1}/D^b)} (n = b, b+1, \ldots, t-1, t) \text{ and add them up,}
\] obtained the following equation:

\[
(t - b) = \frac{1}{\ln(D^d/D^{d-1})} \left( \ln\left( \frac{A^t}{A^{t-1}} \right) + \ln\left( \frac{B^t}{B^{t-1}} \right) + \ln\left( \frac{C^t}{C^{t-1}} \right) \right)
\]

\[
+ \cdots + \frac{1}{\ln(D^{n+1}/D^b)} \left( \ln\left( \frac{A^{b+1}}{A^b} \right) + \ln\left( \frac{B^{b+1}}{B^b} \right) \right)
\]

\[
+ \ln\left( \frac{C^{b+1}}{C^b} \right) \right). \]

(11)

Then, we multiply both sides of equation (11) by \((D^d/D^b)/(t - b)\). Hence, we can obtain another yearly decomposition approach, and the detailed formula for this type of extended yearly LMDI approach is presented in Table 4.

Evidently, the comprehensive approach considers both short-term and long-term trends (i.e., it includes changes during consecutive two-year periods and changes during the base and reporting years). If there are no external shocks (i.e., the short-term and long-term trends remain relatively stable), the weights should remain stable annually.

However, once there is an external shock that influences the external situation, the short-term trend will be very inconsistent with the long-term trend, resulting in a drastic change in weight, and the calculated results will also fluctuate drastically. Hence, it is useful to identify the point in time at which the structural mutation happened, which also helps compensate for the defects in the scenario LMDI approach.

### 2.2. Relationships among the Four Extended LMDI Approaches

The four extended yearly LMDI approaches we proposed are clearly different and are derived from differences in the weight settings. These different weights have certain mathematical relationships and different applicable situations based on the features of each approach. Simultaneously, since the four approaches have their own advantages and disadvantages, we also provide the applicable situation of the four approaches based on their features.

#### 2.2.1. Mathematical Relationships among the Four Extended LMDI Approaches

We regard the weights used by the pure LMDI approach as a reference and compare the weights of the other approaches accordingly. We set the weights of the four methods separately as \(\theta, \lambda^b, \delta^b, \alpha, \) and \(\beta:\)

\[
\begin{align*}
\frac{(D^d - D^b)}{\ln(D^d) - \ln(D^b)} &= \theta, \\
\frac{(D^{n+1} - D^{n+2})}{\ln(D^{n+1}) - \ln(D^{n+2})} &= \lambda^n, \quad (n = 1, 2, \ldots, (t - b)) \\
\frac{(D^d - D^b)}{(t - b)\ln(D^{d/b}/D^{d-1/b})} &= \delta^{d/b}, \quad (n = 1, 2, \ldots, (t - b)) \\
\frac{(D^d - D^k)}{\ln(D^d) - \ln(D^k)} &= \alpha, \quad (b < k < t) \\
\frac{(D^d - D^b)}{\ln(D^d) - \ln(D^b)} &= \beta, \quad (b < k < t)
\end{align*}
\]

(12)

We first derive the relationship between weights of the pure LMDI approach and the weighted LMDI approach:
Table 4: The comprehensive LMDI approach’s decomposition formula for the three driving factors.

| Additive decomposition formula | Weight |
|-------------------------------|--------|
| $\Delta D_{A}^{\theta}$      | $\Delta D_{A}^{\theta} = \sum_{n=1}^{b} w_{n} \ln \left( A_{n+b}^{\theta} / A_{n+b-1}^{\theta} \right)$ | $w_{\theta} = \left( \frac{D^{\theta} - D^{b}}{(t-b) \ln (D^{n+b} / D^{n+b-1})} \right)$ |
| $\Delta D_{B}^{\theta}$      | $\Delta D_{B}^{\theta} = \sum_{n=1}^{b} w_{n} \ln \left( B_{n+b}^{\theta} / B_{n+b-1}^{\theta} \right)$ | $w_{\theta} = \left( \frac{D^{\theta} - D^{b}}{(t-b) \ln (D^{n+b} / D^{n+b-1})} \right)$ |
| $\Delta D_{C}^{\theta}$      | $\Delta D_{C}^{\theta} = \sum_{n=1}^{b} w_{n} \ln \left( C_{n+b}^{\theta} / C_{n+b-1}^{\theta} \right)$ | $w_{\theta} = \left( \frac{D^{\theta} - D^{b}}{(t-b) \ln (D^{n+b} / D^{n+b-1})} \right)$ |

\[ \begin{align*}
\therefore \frac{(D' - D^b)}{\ln (D') - \ln (D^b)} &= \theta, \\
\therefore \frac{(D^{n+b} - D^{n+b-1})}{\ln (D^{n+b}) - \ln (D^{n+b-1})} &= \lambda^{n+b}, \quad (n = 1, 2, \ldots, (t-b)), \\
\therefore \theta \ln \left( \frac{D'}{D^b} \right) &= \lambda^{t} \ln \left( \frac{D'}{D^{t-1}} \right) + \lambda^{t-1} \ln \left( \frac{D'}{D^{t-2}} \right) + \ldots + \lambda^{b+1} \ln \left( \frac{D'}{D^b} \right), \\
\therefore \theta \ln \left( \frac{D'}{D^b} \right) &= \theta \ln \left( \frac{D'}{D^{t-1}} \right) + \theta \ln \left( \frac{D'}{D^{t-2}} \right) + \ldots + \theta \ln \left( \frac{D'}{D^b} \right), \\
\therefore (\theta - \lambda^{t}) \ln \left( \frac{D'}{D^{t-1}} \right) + (\theta - \lambda^{t-1}) \ln \left( \frac{D'}{D^{t-2}} \right) + \ldots + (\theta - \lambda^{b+1}) \ln \left( \frac{D'}{D^b} \right) &= 0, \\
\therefore (\theta - \lambda^{t}) \frac{(D' - D^{t-1})}{\lambda^{t}} + (\theta - \lambda^{t-1}) \frac{(D' - D^{t-2})}{\lambda^{t-1}} + \ldots + (\theta - \lambda^{b+1}) \frac{(D' - D^b)}{\lambda^{b+1}} &= 0, \\
\therefore \frac{\theta (D' - D^{t-1})}{\lambda^{t}} + \ldots + \frac{\theta (D' - D^b)}{\lambda^{b+1}} &= (D' - D^b), \\
\therefore \sum_{n=b+1}^{t} \frac{\theta}{\lambda^{n}} \times \frac{(D^{n} - D^{n-1})}{(D' - D^b)} &= 1.
\end{align*} \]

Therefore, equation (13) reflects the relationship between the pure LMDI approach and the weighted LMDI approach.

As for the relationship between the pure LMDI approach and the comprehensive LMDI approach, in line with the derivation process provided above and the weights set by the comprehensive LMDI approach—$(D' - D^b)/(t-b) \ln (D^{n+b} / D^{n+b-1}) = \theta^{n+b}$—we can obtain the following derivation process:

\[ \begin{align*}
\therefore \frac{\theta (D' - D^{t-1})}{\lambda^{t}} + \ldots + \frac{\theta (D' - D^{t})}{\lambda^{b+1}} &= (D' - D^b), \\
\therefore \frac{\theta}{(t-b)\delta^{t}} + \frac{\theta}{(t-b)\delta^{t-1}} + \ldots + \frac{\theta}{(t-b)\delta^{b+1}} &= 1, \\
\therefore \sum_{n=b+1}^{t} \frac{\theta}{(t-b)\delta^{n}} &= 1.
\end{align*} \]

Evidently, the derivation of the relationship between the pure LMDI approach and the comprehensive LMDI approach comes from the derivation of the relationship between the pure LMDI approach and the weighted LMDI approach. Hence, there are $(\theta/\lambda^{t}) \times ((D^{n} - D^{n-1})/(D' - D^b)) = \theta/((t-b)\delta^{n})$ and $\sum_{n=b+1}^{t} (\theta/\lambda^{n}) \times ((D^{n} - D^{n-1})/(D' - D^b)) = \sum_{n=b+1}^{t} \theta/((t-b)\delta^{n}) = 1.

2.2.2. Applicable Situation of the Four Extended Yearly LMDI Approaches. Compared with the other three extended yearly LMDI approaches, the pure LMDI approach has the advantage of identifying and comparing the yearly fluctuations
of a driver’s changes, excluding the influences of different weights since the weights remain unchanged every year. Additionally, since the weight adopted by the pure LMDI approach relies only on the base and reporting years, the effects of the drivers calculated may only reflect the long-term trends of the indicator (such as energy consumption), ignoring the changes in the short-term trends and external situation. Hence, the pure LMDI approach may be adopted in several situations: (1) identifying and comparing the yearly fluctuations of the drivers themselves, not considering changes in the short-term trends and external situation of the indicator; (2) a research period that is not very long; and (3) no major external events occurred or events that occurred did not greatly impact the external situation during the study period.

As for the weighted LMDI approach, the settings of the weights depend on consecutive two-year periods and change yearly. Hence, the effects of the drivers decomposed by this method reflect changes in the driving factors and short-term trends of the indicator (such as energy consumption). Compared with the pure LMDI approach, the weights set by the weighted LMDI approach change every year. Hence, it can better reflect any changes in short-term trends in detail no matter how big or small the external shocks occurred during the period; however, since both the weights and driving factors change every year, it is difficult to identify the fluctuations caused by the drivers’ changes. At the same time, although the weighted LMDI approach can capture short-term changes, it cannot identify the point in time at which great short-term changes happened. And the weighted LMDI approach may ignore the long-term trends of the indicator because its weights cannot consider the base and reporting year at the same time. Therefore, the weighted LMDI approach may be adopted in several situations: (1) analyzing the yearly effects, which also considers the short-term trends of the indicator; (2) long research periods in which it is uncertain whether major external factors will occur during the research period; and (3) as a supplement to the pure LMDI approach.

As for the comprehensive LMDI approach and the scenario LMDI approach, these two methods can complement each other and make up for each other’s shortcomings. A comprehensive LMDI approach cannot be used alone to analyze the effects of the drivers because its weight will fluctuate drastically if the short-term trends of the indicator (such as energy consumption) are not consistent with the long-term trends (i.e., there were major external shocks during this time that influenced the development trends of the indicator). However, it can be adopted to better analyze the time at which major external shocks happened and affect the development trend.

Based on the point in time provided by the comprehensive LMDI approach, the scenario LMDI approach can more reasonably divide the long period of time into multiple short periods instead of presupposing the time at which the external shock influences the external situation and causes a structural mutation. This approach can more reasonably analyze the yearly effects of drivers, considering the substantial changes in the external situation. Simultaneously, it helps to partially compare and analyze fluctuations purely caused by changes in the drivers themselves during the divided period since the weights are identical during a specific divided period.

Therefore, the comprehensive LMDI approach and scenario LMDI approach may be combined and adopted in several situations: (1) analyzing the yearly effects of drivers, which considers changes in the external situation of the indicator (such as energy consumption and carbon emissions); (2) if the interval between the base and reporting years is too long, and there are major external shocks during this time that could influence the external situation; and (3) partially comparing fluctuations purely caused by changes in the drivers themselves during the divided period.

In conclusion, the advantages and disadvantages of each approach are summarized in Table 5, which is helpful to compare these four more intuitively.

### 3. Case Study Results and Discussion

In this section, we use China’s energy consumption as a case study and first adopt the original LMDI approach to explore the effects of driving factors on China’s energy consumption from 1980 to 2016, thereby considering only the base and reporting years. Then, we further analyze them based on the four extended LMDI approaches to analyze the yearly effects of the drivers and make comparisons between the original LMDI approach and the extended yearly LMDI approaches.

To make our case study easy to understand, we adopted the IPAT model, which has been widely used in previous studies [18–20]:

$$E_t = \frac{E_t}{Y_t} \times \frac{Y_t}{P_t} \times P_t. \tag{16}$$

According to the definition of the IPAT model [21], $E_t/Y_t$ represents energy intensity, reflecting technological progress; $Y_t/P_t$ represents the gross domestic product (GDP) per capita, reflecting the level of economic development; and $P_t$ represents China’s population. For simplicity, we use $E_t$ and $P_t$, respectively, representing $E_t/Y_t$ and $Y_t/P_t$.

#### 3.1. Data

The data include energy consumption, economic output, and population. All data concerning total energy consumption were obtained from the China Energy Statistics Yearbook [22]. Economic output was expressed as the GDP, and relevant data were obtained from the China Statistical Yearbook [23]. We used constant prices for the GDP (with a base year of 1978). These data have been widely used in previous studies (e.g., [24, 25]).

#### 3.2. Empirical Results Based on the LMDI Approach

We first adopt the original LMDI approach to study the long-term effects of energy intensity, GDP per capita, and population on China’s energy consumption (we set 1980 as the base year, and then set each year in the period 1981–2016 as the reporting year). The results are represented in Figure 1.

Evidently, the effects of energy intensity were continuously negative. The energy-saving effect of the energy intensity becomes increasingly stronger, and both the GDP per capita and population continuously result in an increase in energy consumption.
In accordance with the shortcomings of the original LMDI approach, if there were huge fluctuations in energy consumption or its decomposed drivers and only the base and reporting years were considered, changes during the middle of the period would be difficult to capture. We therefore developed Figure 2 to demonstrate the changes in energy consumption and the three drivers. Obviously, the energy intensity was not continuously decreasing from 1980 to 2016 and there were evident fluctuations that happened during 2001 to 2004. Hence, the energy intensity may fail to continuously lead to a reduction in the energy consumption. We should further adopt the extended yearly LMDI approaches to estimate the yearly effects of the three drivers on China’s energy consumption, which will allow us to capture more detailed information about the fluctuations.

3.3. Empirical Results Based on Four Extended Yearly LMDI Approaches

3.3.1. Effects of Driving Factors Based on the Pure LMDI Approach. We first adopt the pure LMDI approach to estimate the yearly effects of driving factors on China’s energy consumption during 1980–2016, and the result is represented in the first panel of Figure 3. Among them, YEI, YPY, and YP, respectively, represent the effects of energy intensity, GDP per capita, and population on China’s energy consumption calculated via the pure LMDI approach.

Evidently, if we only pay attention to the base and reporting years, we may ignore the abnormal yearly effects of drivers and miss substantial information. Even though the GDP per capita constantly resulted in an increase in China’s energy consumption, its influences on energy consumption became very small in the period of 1988–1990, which cannot be seen in Figure 1. This further confirms the shortcomings of the original LMDI approach when the research period is long, which ignores fluctuations in the middle of the research period, making the extended approaches necessary.

Since the pure LMDI approach maintains the same weights every year, the fluctuations in the drivers themselves are easy to identify. We found that the GDP per capita and energy intensity fluctuated during 1980–2016, while the population remained relatively stable.

3.3.2. Effects of Driving Factors Based on the Weighted LMDI Approach. The second panel of Figure 3 represents the empirical results calculated by the weighted LMDI approach. Among them, WEI, WPY, and WP, respectively, represent the effects of energy intensity, GDP per capita, and...
population on China’s energy consumption calculated by the weighted LMDI approach.

Obviously, the empirical results estimated by the weighted LMDI approach were also very different from those provided in Figure 1. In a similar manner as the pure LMDI approach, the yearly effects of the driving factors on energy consumption calculated by the weighted LMDI approach also fluctuated during 1980–2016. However, the magnitude of the effects’ fluctuations measured by the two methods and the oscillation points have significant differences.

Since the weights adopted by the pure LMDI approach and the weighted LMDI approach, respectively, reflect changes in the long-term and short-term trends of energy consumption, the different empirical results may indicate that the short-term trends were not consistent with the long-term trends and that there were changes in the external situation that influenced the development trends of energy consumption during 1980–2016; however, we cannot clearly identify when the external situation changed if we only adopt the pure or weighted LMDI approach.

3.3.3. Effects of Driving Factors Based on the Comprehensive LMDI Approach. The third panel of Figure 3 represents the empirical results estimated by the comprehensive LMDI approach. Among them, CEI, CPY, and CP, respectively, represent the effects of energy intensity, GDP per capita, and population on China’s energy consumption calculated by the comprehensive LMDI approach. In accordance with the weight set established by the comprehensive LMDI approach, it is evident that the weight reflects both changes in the short-term and long-term trends of energy consumption. Normally, if the short-term and long-term trends of energy consumption are consistent (i.e., there were no structural mutations caused by external shocks), the weights calculated will be stable; however, if the short-term trends in energy consumption are significantly inconsistent with the long-term trends, the results of the calculation will oscillate substantially.

According to the third panel of Figure 3, we found that the effects of the driving factors on energy consumption were stable in most years, while they intensely fluctuated in ~1998 and 2015, indicating that there were structural mutations that happened in the development trend of energy consumption.

Notably, on November 1, 1997, the 28th meeting of the Standing Committee of the Eighth National People’s Congress passed the “Energy Conservation Law of the People’s Republic of China.” The establishment of the “Energy Conservation Law of the People’s Republic of China” indicated that China officially adopted the goal of saving energy and improving energy efficiency throughout the society from a legal perspective. This may explain why there was a fluctuation that occurred in 1998. Simultaneously, since the Chinese President Jinping Xi came to power in 2012, he has continuously emphasized the importance of the ecological environment and proposed the idea of green development. In 2015, the Fifth Plenary Session of the 18th Central Committee put forward the concept of green development.
development and regarded green development as a long-term development concept. This may explain the other fluctuation that happened in 2015.

In conclusion, a comprehensive LMDI approach helps to identify the time at which structural mutations (i.e., significant changes in the external situation) happened, compared with the pure and weighted LMDI approaches.

3.3.4. Effects of Driving Factors Based on the Scenario LMDI Approach. In line with the empirical results provided by the comprehensive LMDI approach, we found that there were two large structural mutations that happened in 1998 and 2015 and the mutations may have come from significant external shocks.

Hence, we set three different weights in the periods of 1980–1998, 1998–2015, and 2015–2016 and we adopted the scenario LMDI approach to estimate the yearly effects of drivers on China’s energy consumption. The results are presented in the fourth panel of Figure 3. Among them, SEI, SPY, and SP, respectively, represent the effects of energy intensity, GDP per capita, and population on China’s energy consumption calculated by the scenario LMDI approach.

Evidently, the yearly effects of energy intensity and GDP per capita on energy consumption were similar to the effects calculated by the weighted LMDI approach. This indicates that the scenario LMDI approach is also helpful for considering notable changes in the external situation.

3.4. Relationships among the Four Extended LMDI Approaches. Considering that the energy intensity trend showed huge fluctuations during 1980–2016, we use energy intensity as an example for comparison among the four approaches. Additionally, in order to facilitate comparison with the other three methods, we remove the mutation points estimated by the comprehensive LMDI approach since their fluctuation amplitudes were far greater than those of others; the results are shown in Figure 4.

As expected, we found that the results estimated by the pure LMDI approach were very different from those calculated by weighted and scenario LMDI approaches (since comprehensive LMDI is mainly used to identify the point in time at which the external shock caused the structural mutation and is not suitable for analyzing the effects of driving factors, we do not compare it directly with the other three methods). This may be because both weighted and scenario LMDI approaches put the short-term fluctuations caused by the external shocks into consideration, while the pure LMDI approach only focused on long-term trends. We found that the trends measured by the weighted and scenario LMDI approaches were close; this may be because both approaches help reflect changes in the external situation.
In addition, even though the results estimated by the pure LMDI approach were very different from those of the others, there are also some mathematical relationships to consider between the pure approach and the proposed alternatives.

According to the methodology provided in Section 2.1, the pure LMDI approach has the following respective mathematical relationships with the weighted LMDI approach and comprehensive LMDI approach:

\[ \sum_{i=1981}^{1980} (\theta/\lambda^n) \times ((E^n - E^{n-1}))/((E^{n-1} - E^n)) = \sum_{i=1981}^{2016} (\theta/366^n) = 1. \]

In order to confirm the results, we set \( R_n = \sum_{i=1981}^{1980} (\theta/\lambda^n) \times ((E^n - E^{n-1}))/((E^{n-1} - E^n)) = \sum_{i=1981}^{2016} (\theta/366^n) (n = 1981, \ldots, 2016) \) and draw a picture about \( R_n \) in Figure 5.

As can be seen in Figure 5, it is obvious that the last column is equal to 1. Hence, the equation \( \sum_{i=1981}^{1980} (\theta/\lambda^n) \times ((E^n - E^{n-1}))/((E^{n-1} - E^n)) = \sum_{i=1981}^{2016} (\theta/366^n) = 1 \) will be established when \( n \) equals 2016, which is consistent with our derived equations (13) and (14).

And in accordance with the mathematical relationship between the pure LMDI approach and scenario LMDI approach derived in methodology, we can obtain the following equation:

\[ (\theta(E^{1980} - E^{1988}))/((E^{1986} - E^{1988}))) + (\theta(E^{1981} - E^{1988}))/((E^{1986} - E^{1988}))) + (\theta(E^{1980} - E^{1988}))/((E^{1986} - E^{1988}))) = 1. \]

Among them, \( \theta \) represents the weights estimated by the pure LMDI approach, while \( \alpha, \beta, \text{and} \gamma \) represent the weights, respectively, adopted in the periods 1980–1988, 1988–2014, and 2014–2016, in line with the scenario LMDI approach.

To confirm the result, we set \( X_i (i = 1, 2, 3, 4) \), which sequentially represents \( (\theta(E^{1980} - E^{1988}))/((E^{1986} - E^{1988}))) + (\theta(E^{1981} - E^{1988}))/((E^{1986} - E^{1988}))) + (\theta(E^{1980} - E^{1988}))/((E^{1986} - E^{1988}))) \), and \( (\theta(E^{1981} - E^{1988}))/((E^{1986} - E^{1988}))) + (\theta(E^{1980} - E^{1988}))/((E^{1986} - E^{1988}))) \). Then, we draw a picture about \( X_i (i = 1, 2, 3, 4) \), which is demonstrated in Figure 6.

It is obvious that the last column is equal to 1. Therefore, it confirms that \( X_4 \) is equal to 1 (i.e., \( (\theta(E^{1980} - E^{1988}))/((E^{1986} - E^{1988}))) + (\theta(E^{1981} - E^{1988}))/((E^{1986} - E^{1988}))) + (\theta(E^{1980} - E^{1988}))/((E^{1986} - E^{1988}))) = 1 \), which is consistent with our derived equation (15).
4. Conclusions

Although the original LMDI approach has been widely adopted, it has many shortcomings. Since the original LMDI approach only considers the base and reporting years, if the research period is long, substantial information in the middle of the research period will be ignored. If there are huge fluctuations in the indicators (such as energy consumption or carbon emissions) or their drivers (such as energy intensity or carbon intensity) during the research period, it will be difficult to capture them and studies will not be able to obtain comprehensive conclusions. For example, if the indicator (or its driver) has a U-shaped change and studies use the original LMDI approach, which only focuses on the base and reporting years, then the researchers will conclude that the indicator (or drivers) has no change (or no effect). Obviously, such a conclusion is not comprehensive and is inaccurate.

Thus, we introduced four yearly extended LMDI approaches to better capture different fluctuations in the indicator or its drivers. Additionally, we adopted a case study of China’s energy consumption as a convincing example of the applicability of these approaches. Based on the original LMDI approach, we cannot capture the fluctuation of the effects of energy intensity and may draw an inaccurate conclusion that the energy intensity led to continuously reduced energy consumption during 1980–2016. However, the extended yearly LMDI approach will capture detailed information that the energy intensity failed to reduce energy consumption during 2001–2004.

The four extended yearly LMDI approaches have their own advantages and disadvantages. Overall, the pure LMDI approach focuses more attention on long-term trends of the indicator and can better reflect fluctuations of the driving factors themselves, excluding influences caused by changes in weights, but it cannot reflect changes in the external situation of the indicator (such as new taxes on energy consumption or carbon emissions). The weighted LMDI focuses more attention on any large or small changes in short-term trends and the external situation, but it fails to identify and compare fluctuations caused by changes in the driving factors themselves. The comprehensive LMDI approach helps to identify the point in time at which changes in the external situation happened, and the scenario LMDI approach helps to consider great changes in external situation and partially compare fluctuations in the driving factors themselves during a specific divided period. Hence, the comprehensive LMDI approach and scenario LMDI approach can be combined together to make up for each other’s shortcomings and can better consider substantial changes in the external situation.

Additionally, although the different weight settings of the four approaches have a certain mathematical relationship, they have very different applicable meanings, which rely on research goals. If scholars hope to focus more on fluctuations purely caused by changes in driving factors themselves, they may adopt the pure LMDI approach; if they focus more on the short-term trends of the indicator and hope to capture any detailed changes in the external situation of the indicator, the weighted LMDI approach may be adopted; if they consider significant changes in the external situation and hope to partially analyze the fluctuations of drivers in a specific divided period, the comprehensive and scenario LMDI approaches should be adopted simultaneously.

Data Availability

All data concerning total energy consumption were obtained from the China Energy Statistics Yearbook (http://tongji.cnki.net/kns55/Navi/HomePage.aspx?id=N2014030143&name=YCXM_E&floor=1) and relevant data of economic output were obtained from the China Statistical Yearbook (http://www.stats.gov.cn/tjsj/ndsj).

Conflicts of Interest

The authors declare that they have no conflicts of interest.
Acknowledgments

This work was supported by the National Natural Science Foundation of China (grant nos. 71471001, 41771568, 71533004, and 71503001) and the National Key Research and Development Program of China (grant no. 2016YFA0602500). Additionally, the authors would like to thank Ms. Chen Boyang for her important and valuable guidance on the methodology section of this article.

References

[1] G. P. Hammond and J. B. Norman, “Decomposition analysis of energy-related carbon emissions from UK manufacturing,” *Energy*, vol. 41, no. 1, pp. 220–227, 2012.

[2] G. Kumbaroglu, “A sectoral decomposition analysis of Turkish CO₂ emissions over 1990–2007,” *Energy*, vol. 36, no. 5, pp. 2419–2433, 2011.

[3] B. W. Ang and F. L. Liu, “A new energy decomposition method: perfect in decomposition and consistent in aggregation,” *Energy*, vol. 26, no. 6, pp. 537–548, 2001.

[4] B. W. Ang, “Decomposition analysis for policymaking in energy: which is the preferred method?,” *Energy Policy*, vol. 32, no. 9, pp. 1131–1139, 2004.

[5] B. W. Ang, “The LMDI approach to decomposition analysis: a practical guide,” *Energy Policy*, vol. 33, no. 7, pp. 867–871, 2005.

[6] B. W. Ang, “LMDI decomposition approach: a guide for implementation,” *Energy Policy*, vol. 86, pp. 233–238, 2015.

[7] W. Li, H. Li, H. Zhang, and S. Sun, “The analysis of CO₂ emissions and reduction potential in China’s transport sector,” *Mathematical Problems in Engineering*, vol. 2016, Article ID 1043717, 12 pages, 2016.

[8] B. Lin and M. Wang, “Dynamic analysis of carbon dioxide emissions in China’s petroleum refining and coking industry,” *Science of The Total Environment*, vol. 671, pp. 937–947, 2019.

[9] J. Wang, Y. Shi, X. Zhao, and X. Zhang, “Factors affecting energy-related carbon emissions in Beijing-Tianjin-Hebei region,” *Mathematical Problems in Engineering*, vol. 2017, Article ID 1524023, 17 pages, 2017.

[10] J. Chen, S. Cheng, M. Song, and Y. Wu, “A carbon emissions reduction index: integrating the volume and allocation of regional emissions,” *Applied Energy*, vol. 184, pp. 1154–1164, 2016.

[11] B. Zhang and H. Long, “How to promote energy conservation in China’s chemical industry,” *Energy Policy*, vol. 73, pp. 93–102, 2014.

[12] B. Lin and X. Ouyang, “Analysis of energy-related CO₂ (carbon dioxide) emissions and reduction potential in the Chinese non-metallic mineral products industry,” *Energy*, vol. 68, pp. 688–697, 2014.

[13] B. Lin and Z. Zhang, “Carbon emissions in China’s cement industry: a sector and policy analysis,” *Renewable and Sustainable Energy Reviews*, vol. 58, pp. 1387–1394, 2016.

[14] X. Yan and Y.-P. Fang, “CO₂ emissions and mitigation potential of the Chinese manufacturing industry,” *Journal of Cleaner Production*, vol. 103, pp. 759–773, 2015.

[15] C. Liu and B. Lin, “Analysis of the changes in the scale of natural gas subsidy in China and its decomposition factors,” *Energy Economics*, vol. 70, pp. 37–44, 2018.

[16] R. Tan and B. Lin, “What factors lead to the decline of energy intensity in China’s energy intensive industries?,” *Energy Economics*, vol. 71, pp. 213–221, 2018.