ANOMALOUS EVOLUTION OF
NON-SINGLET NUCLEON STRUCTURE FUNCTIONS

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Abstract

We calculate the scale dependence of non-singlet nucleon structure functions. Due to anomalous axial symmetry breaking a large flavour asymmetry of the quark–antiquark sea is generated nonperturbatively. This produces a strong scale dependence of the non-singlet structure function in an intermediate range of $Q^2$. Evolving nonperturbatively a pure valence distribution from an infrared scale we can thus compute $F_2^p - F_2^n$ as measured by the NMC, and give detailed predictions for its $Q^2$ dependence at fixed $x$. We also compare our results with Drell–Yan data.

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The detailed description of logarithmic scaling violations is perhaps the most successful result of perturbative QCD, particularly in the context of deep inelastic scattering, where the scale dependence of the nucleon structure functions can be accurately computed perturbatively either through the operator product expansion or equivalently by solving the Altarelli–Parisi equations. Recent improvements in experimental accuracy are now opening up the possibility of measuring subleading perturbative corrections, as well as corrections to perturbation theory as expressed by higher twist effects. However, it has been recently suggested [1] that in an intermediate region of $Q^2 \sim 1 \text{ GeV}^2$ the scale dependence of the nonsinglet nucleon structure function may be qualitatively rather different from that predicted by purely perturbative QCD, due to nonperturbative effects related to the anomalous breaking of axial U(1) symmetry.

In ref.[1] it was shown that recent data on the first moment of the nonsinglet nucleon structure function [2,3], which seem to contradict standard assumptions on the symmetry of the light quark sea in the nucleon, can be understood in a natural way if nonperturbative scaling violations are taken into account by suitable modification of the Altarelli–Parisi equations. These scaling violations are relatively large in the nonsinglet channel due to the fact that the $U(N_f)$ symmetry of the spectrum of light pseudoscalar bound states (the Goldstone bosons of spontaneous chiral symmetry breakdown) is broken explicitly down to $SU(N_f)$ by the axial anomaly and fluctuations of topological charge in the QCD vacuum. If this (anomalous) symmetry breaking effect, which is essentially nonperturbative [4], is taken into account in the evolution equations, then the first moments of nonsinglet charge conjugation even structure functions, which perturbatively are almost scale independent, acquire a strong scale dependence for $Q^2$ up to a few GeV$^2$, due to the generation of a flavour asymmetric component of the quark–antiquark sea.

Here we will investigate the consequences of such a modification of the evolution equations on the $Q^2$ dependence of the full $x$-dependent nonsinglet, charge conjugation even structure function $[F_2^p(x, Q^2) − F_2^n(x, Q^2)]$. We will show that the shape of this structure function is also significantly affected by the production of the asymmetric sea induced by the nonperturbative effects mentioned above, in a way which is experimentally verifiable independently of the effect on the first moment. We will show further that if one attempts to compute the shape of the nonsinglet structure function when $Q^2$ is of the order of several GeV$^2$ by evolving some valence quark distribution from an infrared scale, then the inclusion of our nonperturbative effects leads to a qualitative improvement in the agreement with the NMC data, which is independent of the details of the initial valence quark distribution.
Finally, we will show that our results are also in agreement with available data from the Drell–Yan process on the $x$-dependence of the ratio of $\bar{u}$ and $\bar{d}$ sea quark distributions.

Let us first summarize briefly the formalism developed in ref. [1] to include anomalous symmetry breaking in the nonsinglet Altarelli–Parisi evolution equations. We will concentrate on the charge conjugation even sector, i.e. on the combination

$$q_i^+(x) \equiv q_i(x) + \bar{q}_i(x)$$

of the quark and antiquark distributions of flavour $i$. These are directly measurable in deep inelastic scattering, since in the parton model the structure function for unpolarized electroproduction $F_2$ is given (to all orders, in the parton scheme [5]) by

$$F_2(x) \equiv x \sum_i e_i^2 q_i^+(x),$$

where $e_i$ are the electric charges of the quarks.

Now, in perturbative QCD the evolution equation for any nonsinglet combination $q^+(x)$ of the distributions $q_i^+(x)$ (for example $q^+ = u^+(x) - d^+(x)$) is given by [5]

$$\frac{d}{dt} q^+ = (Q_{qq} + Q_{q\bar{q}}) \otimes q^+, \quad (3)$$

where

$$Q_{qq} \equiv \mathcal{P}_{qq}^D - \mathcal{P}_{qq}^{ND}, \quad (4)$$

and $\mathcal{P}^D$ ($\mathcal{P}^{ND}$) is any quark–quark splitting function $\mathcal{P}_{qs,q}$ which is diagonal (nondiagonal) in flavour, i.e., such that $i = j$ ($i \neq j$), and analogously for $Q_{q\bar{q}}$. At one loop, all the splitting functions $\mathcal{P}^{ND}$ and $\mathcal{P}_{q\bar{q}}$ vanish, hence the evolution of $q^+$ is driven entirely by the quark–quark splitting function $\mathcal{P}_{qq}^D$: the quarks lose momentum by radiation of gluons. This leaves the first moment of $q^+$ unchanged (by charge conservation), whereas all higher moments necessarily decrease. Thus at one loop a nonsinglet quark-antiquark sea cannot be generated radiatively: if the sea vanishes at some scale, it will vanish at all scales, and only the valence will evolve. At two loops all the splitting functions are nonzero, but the two loop contribution to the evolution of $q^+$ eqn.(3) is extremely small, because $\mathcal{P}^D$ and $\mathcal{P}^{ND}$ only differ due to antisymmetrization of the sea quark pairs in the final state [8]. Indeed, the $U(N_f)$ flavour symmetry of the QCD Lagrangian ensures that perturbatively no nonsinglet component of the sea can be generated, modulo the tiny final state antisymmetrization effect.
The nonperturbative breaking of the $U(N_f)$ symmetry due to the axial anomaly can be incorporated into the evolution equations by considering the contribution to $Q^2$ evolution from diagrams where the quark couples directly to a $q\bar{q}$ bound state. These can be viewed as being generated dynamically from the diagram which provides the two loop contribution to the usual splitting function $P_{q_iq_j}$, but where the emitted (and unobserved) quark–antiquark pair is allowed to bind nonperturbatively into a meson state. A full, self-consistent set of coupled evolution equations for quarks and mesonic bound states in the nonsinglet channel can then be written down \[1\]; it turns out that the evolution equation for $q^+$ retains the form of eqn.(1), but now the splitting functions $P$ receive contributions due to (one loop) bound state emission. The splitting functions can thus be computed explicitly from the respective particles’ emission cross section, according to

$$\frac{d}{dt}P_{q_iq_j}(x,t) = \frac{d}{dt}\sigma_{q^+}^\gamma X_q(x,t),$$

(5)

where $\sigma_{q^+}^\gamma X_q(x,t)$ is the total cross section for absorption of a virtual photon $\gamma^*$ and emission of the state $X_q$, expressed in terms of the usual scaling variables and integrated over all $k_{\perp}$.

Because the bound states carry flavour, $P^{ND}$ is nonvanishing already at leading order in the quark–meson coupling. Furthermore, the combination of spontaneous chiral symmetry breaking $SU(N_f) \times SU(N_f) \rightarrow SU(N_f)$ and the explicit anomalous breaking of the axial $U(1)$, resulting in the relatively large π-$\eta'$ mass splitting, leads kinematically to a substantial difference between the splitting functions $P^D$ and $P^{ND}$, and thus to the generation of a sizable nonsinglet component in the quark–antiquark sea. As the breaking of $U(N_f)$ chiral symmetry is only manifested in the pseudoscalar meson channel, these are actually the only bound states which contribute significantly to the nonsinglet evolution; for other multiplets the splitting functions $P^D$ and $P^{ND}$ are almost equal, and are in any case suppressed kinematically (by at least an order of magnitude) due to the relatively large masses of the emitted bound states.

The full splitting function (4) is thus determined according to eqn.(5) in terms of the set of pseudoscalar octet meson emission cross sections, which, due to the extended nature of the bound states, depend on a quark–meson vertex function. This may be decomposed as a sum of four independent couplings (a pseudoscalar, two axial, and a tensor), yielding four independent (scalar) vertex functions. However, once the cross section is computed\[1\], it turns out to depend only on a pseudoscalar vertex function $\varphi$ and an axial vertex function $\tilde{\varphi}$; the former dominates for intermediate values of $Q^2$ (up to a few GeV$^2$), while the latter
controls the large $Q^2$ tail. The pseudoscalar vertex function, which, consistent with known asymptotic behaviour, can be taken to have the simple dipole form

$$\varphi(p^2) = \frac{m_d \Lambda^2 + m_d^2}{f_\pi \Lambda^2 + p^2}, \quad (6)$$

can be further constrained by using the chiral Ward identity to express the constituent quark mass $m_d$ in terms of the parameter $\Lambda$; this restricts the allowed range of values of $\Lambda$ to be $0.4 \text{ GeV} \lesssim \Lambda \lesssim 0.8 \text{ GeV}$. The axial vertex function may be assumed to have the similar form

$$\tilde{\varphi}(p^2) = \frac{g_\pi \tilde{\Lambda}^2 + m_d^2}{f_\pi \tilde{\Lambda}^2 + p^2}, \quad (7)$$

with $\tilde{\Lambda} \sim \Lambda$ while the parameter $g_\pi$ is likely to be rather small, say $g_\pi \lesssim \frac{1}{2}$.

An explicit computation of the various cross sections [1] shows that even though the vertex function (6) is soft (i.e. $\Lambda$ is rather small) the contribution to the anomalous dimensions calculated from eqn.(5) remains sizable for values of $Q^2$ up to 5–10 GeV$^2$ because there exists a region of phase space (at small $x$ in the $t$-channel and at large $x$ in the $s$-channel) where the vertex function is pointlike for arbitrarily large values of $Q^2$. Furthermore it is precisely in this dominant region that the emitted bound state and quark are quasi–collinear, which justifies the use of a parton picture, and the Altarelli–Parisi evolution equations. However, at both very small $Q^2 \sim 0.05 \text{ GeV}^2$, and at large $Q^2 > 10 \text{ GeV}^2$, all the anomalous dimensions flatten, ensuring both a smooth connection to a valence quark picture in the infrared and to the usual perturbative behaviour in the ultraviolet.

We wish now to solve the evolution equation (3) for the nonsinglet combination of structure functions

$$\frac{1}{x} (F_2^p(x) - F_2^n(x)) \equiv \frac{1}{3} q^+(x) \equiv \frac{1}{3} [q^v(x) + 2\bar{q}(x)], \quad (8)$$

using the nonperturbative splitting function due to bound state emission determined from eqn.(5) (and given explicitly in the appendix of ref.[1]). If in eqn.(5) we assume exact isospin symmetry, $q^+ = u^+ - d^+$ is just the nonsinglet combination of quark distributions eqn.(1) with isospin one; in the last step we have introduced for convenience valence quark distributions defined as $q^v(x) = q(x) - \bar{q}(x)$. Whereas the computation of the evolution of the first moment of $q^+$ (as performed in ref.[1]) required only the nonperturbative splitting function which governs the generation of an asymmetric sea component due to bound state
emission (the perturbative evolution of the first moment being negligible), the determination of the evolution of the full structure function requires in addition the contribution to the splitting function which determines the evolution of the valence distribution due to gluon emission. Furthermore, whereas the initial condition for the evolution of the first moment is very simple — it is just the flavour asymmetry of the quark sea at the ‘constituent quark’ scale, which can be taken to be zero — that for the full distribution is nontrivial: even assuming that there is initially no asymmetry in the sea one still needs to know the initial shape of the nonsinglet valence distribution, eqn.(8).

In order to actually perform the computation, we will assume that bound state emission contributes significantly only to the generation of an asymmetric sea. This is in keeping with the observation that the anomalous dimensions for bound state emission are generally much smaller than the usual perturbative ones, and are indeed only significant at all because of the nonperturbative breaking of the axial symmetry. The gluon emission responsible for evolution of the valence distribution, and the bound state emission responsible for the nonsinglet sea evolution, are then to be regarded as truly distinct mechanisms. The full splitting functions to be used in eqns (3) and (4) are thus given by

\[ P^{ND}_{qq} = \left[ P^{\Pi}_{qq} \right]^{ND}, \quad P^{D}_{qq} = \left[ P^{\Pi}_{qq} \right]^{D} + \left[ P^{g}_{qq} \right]^{D}, \]  

where \( P^{\Pi} \) are the nonperturbative splitting functions due to meson emission computed from eqn.(5), while \( P^{g} \) is the splitting function due to gluon emission.

Here, we are interested in studying the effects of the generation of an asymmetric sea due to the nonperturbative splitting function \( P^{\Pi} \); however, we need a model of the gluon emission mechanism in order to be able to obtain predictions which may be compared with the data. The perturbative gluon splitting function is itself inadequate because we wish to begin our evolution at a low scale \( Q^2 \sim 0.04 \text{ GeV}^2 \) where the nonperturbative evolution due to meson emission flattens \( \bar{1} \) and the sea and gluon distributions may be reasonably assumed to vanish; at such low scales perturbation theory is clearly useless. Indeed, early studies \( \bar{1} \) show that at large \( Q^2 \gtrsim 1 \text{ GeV}^2 \) the perturbatively generated sea and gluon distributions rise far too steeply at small \( x \). More reasonable distributions may be obtained \( \bar{8} \) by suitably suppressing the emission of very soft gluons, and this is the approach we will follow here.\( \bar{3} \)

\[ \bar{1} \] An alternative possibility \( \bar{1} \) is to assume a large gluon component with a valence–like distribution at very low scales (“valence gluons”). We will not consider this option as it would require us to introduce for consistency also an ansatz for the starting sea asymmetry; the connection with the constituent quark model would then be lost, and the evolved distributions would become almost entirely model dependent.
We will thus model the evolution of the valence distribution by using the general-
ized splitting function of ref.[8]. This is computed by freezing the strong coupling below
$Q = 0.5$ GeV, and then assuming instead that the gluons have a ‘mass’ $\mu_G$ of several
hundreds of MeV; this ensures that the evolved gluon distributions will be harder than
the corresponding perturbative ones. The flux of gluons generated by the target quark
is then determined by solving an appropriate evolution equation which factorizes into an
Altarelli–Parisi–like equation only at scales rather larger that the gluon mass. Gluon and
sea quark distributions are thus generated dynamically by evolving a valence quark dis-
tribution. The starting valence distribution can be obtained from a three–quark wave
function defined in the limit of small $Q^2$; it has been shown [8] that a simple gaussian
wave function, whose width is fixed in terms of the r.m.s. radius of the proton, yields both
a correct shape for the gluon distribution and a correct balance between valence and sea
components. The gaussian ansatz is sufficient for our purposes, since we are only really
interested in the scale dependence of the sea asymmetry, which is due almost entirely to
the nonperturbative evolution induced by meson emission; we will check that our results
do not depend on the detailed form of the starting valence evolution by varying the shape
of this ansatz.

We have therefore solved the evolution equation (3),(4) with the splitting functions
eqn.(9), $P^{\Pi}$ from ref.[1], and $P^g$ from ref.[8]. Because the nonperturbative coupling to
bound states generates a contribution to the nonsinglet sea $\bar{u} - \bar{d}$, it will also contribute
to the singlet structure function: clearly $|\bar{u} + \bar{d}| \geq |\bar{u} - \bar{d}|$. But then the parameters which
control the valence evolution according to the splitting function $P^g$ must be adjusted
in order to insure that the balance of the fractions of nucleon momentum carried by
valence, sea and gluons is kept in agreement with experiment; the unobserved fraction of
the total momentum (namely, the momentum which is not carried by quarks or antiquarks)
should be around 45% at 4 GeV$^2$. Due to the nonperturbative mechanism, an unobserved
momentum fraction carried by bound states is generated, and the momentum carried by
the gluons must be accordingly decreased. This is done by computing the second moment
of the nonperturbatively generated nonsinglet quark–antiquark sea, using it to estimate
the momentum that goes into the bound states, and then readjusting $\mu_G$ in order to reduce
the momentum fraction carried by the gluon sea by the same amount.

With typical values of the parameters (see below) the momentum fraction carried by
the nonperturbatively generated sea at $Q^2 = 0.5$ GeV$^2$ is around 15–20% of the total
momentum, the momentum carried by the glue is 20–25%, and the valence and perturbatively generated sea quarks carry the rest. As $Q^2$ grows further, the nonperturbative sea generation flattens rapidly and eventually disappears above $Q^2 \sim 5 \text{ GeV}^2$; standard perturbative evolution, and the usual balance of momentum between quarks and gluons, are then recovered.

Firstly, we consider the scale dependence of the full nonperturbative structure function, displayed in fig. 1. The evolution starts at a very low scale $Q^2 = 0.04 \text{ GeV}^2$, where the nonperturbative evolution flattens and it can thus be assumed that there is no sea asymmetry. The valence distribution is constructed from a gaussian nucleon wave function, whose radius is determined with an uncertainty of the order of 20%. The values of the two parameters which control the nonperturbative evolution are fixed by requiring the value of the Gottfried sum rule measured experimentally at $Q^2 = 4 \text{ GeV}^2$ to be correctly reproduced. Combined with the theoretical expectation that $g_\pi \lesssim \frac{1}{2}$, this fixes $\Lambda = 0.4 \text{ GeV}$ with an uncertainty of order 20%. As may be seen from the figure, the scale dependence of the nonsinglet structure function turns out to be rather strong in the medium–large $x$ region, $0.15 < x < 0.6$.

The nonsinglet structure function at $Q^2 = 4 \text{ GeV}^2$ is then compared in fig. 2 with the most recent data from the NMC. The figure also shows the nonsinglet structure function obtained from the model of ref., i.e., with the evolution due to meson emission switched off ($P_\Pi \equiv 0$), and thus with a flavour symmetric sea. Comparing this curve with the data demonstrates a problem common to most dynamical evolution mechanisms: because initially the valence quarks carry all of the nucleon’s momentum, the nonsinglet distribution is typically peaked at $x \approx \frac{1}{3}$; if the evolution of this distribution is entirely due to the emission of flavour singlet objects (for example gluons) then since the momentum of

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2 Notice that the value published originally was recently reevaluated to give $S_G = 0.258 \pm 0.017$. However, the data which lead to this reevaluation also show evidence for shadowing effects in deuterium, which were neglected in both analyses. Taking current theoretical estimates for these effects and correcting the new data leads back to a value $S_G \approx 0.20 - 0.24$.

3 The dependence of the results on $g_\pi$ is actually rather weak, because $g_\pi$ only affects the large $Q^2$ tail of the evolution of the first moment (which is weakly evolving), hence the asymptotic value of $S_G$, rather than the measured one.

4 An alternative determination of the nonsinglet distribution, which is peaked at lower $x$ ($x \sim 0.35$), has been offered in ref., but never published. Our computation is in fair agreement with this analysis too.
the valence quarks is necessarily degraded, at large scales the maximum of the distribution occurs at a lower value, $x \sim 0.1 - 0.2$. This disagrees with the data [3],[11], which are instead peaked at $x \approx 0.35 - 0.45$.

By contrast, the nonperturbative evolution generated by emission of bound states leads to a net increase in the number of sea quark pairs; the charge conjugation even first moment is no longer conserved (even though the charge conjugation odd is, of course, in agreement with the Adler sum rule). It is then possible for the sea distributions to become harder, rather than softer, and indeed it turns out that this is the case: if the nonperturbative mechanism were the only one responsible for evolution, the maximum of the distribution would shift to larger, rather than smaller values of $x$ (compare the dash-dot curve in fig. 2). Because the flavour off–diagonal nonperturbative emission is favoured with respect to the flavour diagonal one [1], the $\bar{d}$ distribution in eqn.(8) increases more rapidly than the $\bar{u}$ one, and the net effect of the nonperturbative evolution is to make the full nonsinglet distribution eqn.(8) decrease more rapidly than its valence component. This effect is sufficient to bring the calculated distribution in line with the data [3], because it counteracts the degrading effect of the gluon emission. The nonsinglet nonperturbative evolution mechanism is thus probably the key ingredient in the determination of the shape of the nonsinglet structure function.

The sensitivity of this result to the choice of parameters of the nonperturbative evolution is tested in fig. 3, where the variation of the structure function at $Q^2 = 4$ GeV$^2$ is shown as $\Lambda$ and $g_\pi$ are varied within their admissible ranges discussed above. It appears that the sensitivity to $g_\pi$ (which cannot be fixed accurately from independent data) is minimal, and concentrated in the small $x$ region; the sensitivity to $\Lambda$ is rather more significant. The dependence of the results on the precise form of the ansatz for the starting valence distribution is shown in fig. 4, which compares to the previous results those obtained choosing a power like form for the starting valence,

$$u^v(x) - d^v(x) = C x^{0.5} (1 - x)^2 \quad \text{at} \ 0.04 \ \text{GeV}^2,$$

where the values of the exponents are determined from the observed valence distribution. It appears that indeed the $Q^2$ evolution of the nonsinglet structure function is largely independent of the precise form of the valence distribution which is chosen.

Indeed the valence distributions extracted from fits to structure function data [12] appear to have a maximum located at $x \sim 0.2$.  

It follows from this discussion that a measurement of the $Q^2$ dependence of the nonsinglet nucleon structure function in the range $1 \text{ GeV}^2 \lesssim Q^2 \lesssim 5 \text{ GeV}^2$ allows an experimental test of the nonperturbative sea generation mechanism. A detailed set of predictions is thus provided in fig. 5 where the $Q^2$ dependence of the structure function over a suitable range of $Q^2$ is shown and compared with the scale dependence due to gluon emission alone (applied to the same purely valence starting distribution at $Q^2 = 0.04 \text{ GeV}^2$). It is apparent that the nonperturbative nonsinglet evolution is maximal (and differs most strongly from the purely gluonic evolution) in the central region of $Q^2$ indicated above, and in the region $0.1 \lesssim x \lesssim 0.35$ where the maxima of the two evolved distributions are located. For larger values of $Q^2$ the perturbative evolution is regained; since the evolution is multiplicative, this means that, as a function of $Q^2$, the structure function generated through nonsinglet evolution becomes eventually parallel (rather than equal) to that evolved by gluon emission, as displayed in fig. 5 for $Q^2 \gtrsim 5 \text{ GeV}^2$.

Figs. 2 and 5 also show that the effect which is responsible for violation of the Gottfried sum rule [1], rather than being concentrated at small $x$, is relevant in the intermediate $x$ region as well. Specifically, computing the difference $\Delta S_G$ between the value of the Gottfried sum $S_G$ (i.e., the first moment of the nonsinglet structure function eqn. (8)) which we get at $Q^2 = 4 \text{ GeV}^2$, and the naive parton model prediction $S_G = 1/3$, we find that only about 55% of $\Delta S_G$ comes from the region $x < 0.05$; 20% comes from the region $0.05 \leq x \leq 0.1$, and 25% comes from the region $0.1 \leq x \leq 0.4$. Therefore, the nonperturbative effect studied here significantly affects the scale dependence of higher moments of the nonsinglet structure function, and not only the first moment.

A flavour asymmetry in the quark–antiquark sea may also be measured directly in Drell–Yan production, which allows [13] a determination of the asymmetry ratio

$$R \equiv \frac{\bar{d}(x) - \bar{u}(x)}{\bar{d}(x) + \bar{u}(x)}$$

from the comparison of the cross sections for production on isoscalar targets with that on targets with a proton or neutron excess. We can provide a prediction for this quantity by extracting the sea asymmetry in the numerator from the nonsinglet structure function according to the decomposition eqn. (8), where the valence distribution $q^v$ at any scale is determined as the nonsinglet structure function obtained by setting the nonperturbative splitting function $\mathcal{P} \equiv 0$ in the evolution equations. The denominator is then found by computing the symmetric portion of the sea according to the procedure of ref. [8] (but with
the parameters of the gluon distribution suitably adjusted to ensure that the momentum sum rule is correctly satisfied in the presence of an asymmetric sea, as discussed above) and adding to it the asymmetric sea determined as above. We use this self-consistent procedure, rather than, for example, computing the numerator and taking the denominator from available fits to the data, in order to minimize the error due to the very poor experimental knowledge of the sea distributions for \( x > 0.15 \), where the sea is tiny and drops very rapidly with increasing \( x \).

Our prediction for the ratio \( R \), eqn.(11), is shown in fig. 6. At present, the most precise determination of \( R \) (although limited to one point in the \((x, Q^2)\) plane, namely, \( x = 0.18, Q^2 \approx 25 \text{ GeV}^2 \)) comes from the NA51 experiment on Drell–Yan production on deuteron [14]; there are also upper limits on \( R \) in a broad \( x \) range, obtained from Drell–Yan production on tungsten [15]. The data are in good agreement with our calculation. Of course one should always keep in mind that there is an intrinsic theoretical uncertainty in our determination of \( R \), due to the need to estimate the singlet structure function which contributes to the denominator of eqn.(11). However, this does not affect the overall trend of our prediction (specifically, the fact that as a function of \( x \) \( R \) should flatten for \( x > 0.2 \)) and data in a broader \( x \)-range should provide a test of it.

Finally, it is interesting to see how a nonperturbative contribution to the scale dependence of the nonsinglet structure function would affect the scale dependence of the ratio \( F_{n/p} \equiv F_{2}^{n}/F_{2}^{p} \). Indeed, there exists an indication that the observed scale dependence of \( F_{n/p} \) disagrees with the prediction of perturbative QCD [18]. Even though the experimental situation is inconclusive, due to the large errors involved in the determination of the scale dependence, it would appear that \( \gamma_{n/p}(x) = \frac{d}{dt}F_{n/p}(x) \) is negative at small \( x \approx 0.3 \) and positive at large \( x \approx 0.4 \), whereas numerical solution of the (perturbative) Altarelli–Parisi equations gives [19] negative values of \( \gamma_{n/p} \) up to \( x \approx 0.6 \); furthermore at small \( x \) the measured value of \(|\gamma_{n/p}| \) appears to be larger by perhaps one order of magnitude than that expected perturbatively.

A precise determination of \( \gamma_{n/p} \) requires a computation of the scale dependence of the singlet structure functions, and thus goes beyond the scope of the present work, where we concentrate on nonsinglet evolution. However, using the fact that the nonperturbatively generated asymmetry satisfies \( |\frac{d}{dt}(\bar{u} - \bar{d})^{NP}| \leq |\frac{d}{dt}(\bar{u} + \bar{d})^{NP}| \) it is easy to see that the nonperturbative contribution to \( \gamma_{n/p} \) has the opposite sign to \( \frac{d}{dt}(F_{2}^{p} - F_{2}^{n}) \). This means that (compare fig. 3) the effect discussed here will lead to a negative contribution to \( \gamma_{n/p} \) for \( x \approx 0.1 \) and a positive one for \( x \approx 0.15 \) growing quite large around \( x \approx 0.25 \).
in qualitative agreement with the experimental trend. A more conclusive study of this issue will require an improvement in the accuracy of the experiments, as well as a better theoretical understanding of the singlet evolution.

In conclusion, we have discussed the consequences for the scale dependence of the nonsinglet nucleon structure function of the nonperturbative generation of a sea quark flavour asymmetry, due essentially to the anomalous breaking of the U(1) symmetry in the spectrum of pseudoscalar mesons. These contribute to the generalized Altarelli–Parisi equations constructed in ref.\[1\], through splitting functions for the emission of bound states from quarks. The nonperturbative evolution, besides explaining the observed violation of the Gottfried sum rule as a consequence of strong evolution of the first nonsinglet moment in the region $0.5 \lesssim Q^2 \lesssim 5 \text{ GeV}^2$, predicts a stronger scale dependence of the full structure function in the same $Q^2$ range for medium–large values of $x$, $0.1 \lesssim x \lesssim 0.6$. This is due to the fact that the nonperturbative symmetry breaking allows for the generation of a relatively hard sea component, peaked around $x \approx 0.35$, which, due to its flavour asymmetry, contributes to the nonsinglet structure function. Perturbative evolution is regained at large $Q^2 \gtrsim 5 \text{ GeV}^2$.

The nonperturbatively generated sea asymmetry leads to a considerable improvement in the agreement between the computed nonsinglet structure function and available data, quite independently of the way the initial valence distribution is modeled and evolved, thus suggesting that indeed the model presently under discussion is responsible for the bulk of the generation of the nonsinglet sea. The model can be tested by measuring the scale dependence of the nonsinglet structure function, for which we have presented a detailed set of predictions. We have also provided a prediction for the sea asymmetry as measured in Drell–Yan production experiments; our prediction is compatible with available data which however do not allow any definite conclusion. If confirmed, the effects discussed here and in ref.\[1\] would provide the first conclusive evidence for scale dependence which cannot be obtained through purely perturbative techniques.

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References

[1] R. D. Ball and S. Forte, Oxford preprint OUTP-93-18P and Torino preprint DFTT 9/93 (1993).
[2] P. Amaudruz et al., Phys. Rev. Lett. 66 (1991) 560.
[3] P. Amaudruz et al., CERN preprint CERN-PPE/93-117 (1993).
[4] G. ’t Hooft, Phys. Rep. 142 (1986) 357 and ref. therein.
[5] G. Altarelli, Phys. Rep. 81 (1982) 1.
[6] D. A. Ross and C. T. Sachrajda, Nucl. Phys. B149 (1979) 497.
[7] G. Parisi and R. Petronzio, Phys. Lett. B62 (1976) 331; V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Ann. Phys. 105 (1977) 276; M. Glück and E. Reya, Nucl. Phys. B130 (1977) 76; M. Glück, R.M. Godbole and E. Reya, Zeit. Phys. C41 (1989) 667.
[8] V. Barone et al., Zeit. Phys. C58 (1993) 541; Int. Jour. Mod. Phys. A8 (1993) 2779.
[9] M. Glück, E. Reya and A. Vogt, Zeit. Phys. C48 (1990) 471; C53 (1992) 127.
[10] N.N. Nikolaev and B.G. Zakharov, Zeit. Phys. C49 (1991) 607; Phys. Lett. B260 (1991) 414; V. R. Zoller, Phys. Lett. B279 (1992) 145; B. Badelek and J. Kwieciński, Nucl. Phys. B370 (1992) 278; V. Barone et al., Zeit. Phys. C58 (1993) 541; Phys. Lett. B321 (1994) 137; W. Melnitchouk and A.W. Thomas, Phys. Rev. D47 (1993) 3783.
[11] E.M. Kabuß (NMC), Nucl. Phys. B (Proc. Suppl.) 29A (1992) 1; A. Brüll (NMC), Ph.D. Thesis, Freiburg University, 1993.
[12] A. D. Martin, W. J. Stirling and R. G. Roberts, J. Phys. G19 (1993) 1429.
[13] S.D. Ellis and W.J. Stirling, Phys. Lett. B256 (1991) 258.
[14] NA51 collaboration; P. Sonderegger et al., proposal CERN SPSLC/92-15/P267 (1992); and work in preparation, B. Alessandro, and M. Monteno (NA51), private communication.
[15] P. L. McGaughey et al. (E772), Phys. Rev. Lett. 69 (1992) 1726.
[16] S. Kumano and J. T. Londergan, Phys. Rev. D44 (1991) 717.
[17] E. J. Eichten, I. Hinchliffe and C. Quigg, Phys. Rev. D45 (1992) 2269.
[18] P. Amaudruz et al. (NMC), Nucl. Phys. B371 (1992) 3.
[19] M. Virchaux and A. Milsztajn, Phys. Lett. B274 (1992) 221.
Figure Captions

Fig. 1. The structure function $F_2^p - F_2^n$ for various values of $Q^2$.

Fig. 2. Comparison of the calculated nonsinglet structure function at $Q^2 = 4$ GeV$^2$ (solid curve of fig. 1) with the data of ref. [3]. The dotted curve displays the result obtained with $P^{II} = 0$ in eqn. (9) and the dot-dashed curve the result obtained setting $P^g = 0$.

Fig. 3. Dependence of the nonsinglet structure function on the parameters $A$ and $g_{\pi}$. The NMC data are also shown.

Fig. 4. Comparison of the nonsinglet structure functions obtained from two different initial valence distributions: the gaussian wave function valence distribution of ref. [8] (dot-dashed curve at the input scale, solid curve at $Q^2 = 4$ GeV$^2$) and the power–like valence eqn. (11) (dotted curve at the input scale, dashed curve at $Q^2 = 4$ GeV$^2$). The NMC data are also shown.

Fig. 5. The $Q^2$ dependence of the structure function at various values of $x$ (solid curves) compared with the standard $Q^2$ dependence obtained omitting the nonperturbative evolution mechanism (dashed lines). Note the change of vertical scale between frames.

Fig. 6. Our computation of the asymmetry ratio $R$ eqn. (11) compared with the available experimental information from the Drell–Yan process. The curve is given at $Q^2 = 10$ GeV$^2$, though it is essentially $Q^2$–independent for $Q^2 \gtrsim 5$ GeV$^2$. The square is the NA51 determination [14] ($Q^2 = 25$ GeV$^2$). The region below the finely dotted curve is the area allowed by the E772 data ($25 \sim Q^2 \sim 170$ GeV$^2$) at a $2\sigma$ statistical error level [15]. The broadly dotted, dot-dashed, and dashed curves are predictions from the models of refs. [16] and [17], and the fit of ref. [13], respectively (see ref. [15]).
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