Irreversibility field and anisotropic $\delta l$-pinning in type II superconductors

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Abstract. The article presents a brief generalization of our recent experiments confirming that “irreversibility field” in Nb-Ti superconducting tapes is determined by the inhomogeneity of superconducting properties. It is shown that the effect is a direct consequence of the so-called $\delta l$-pinning, i.e. pinning caused by local reduction of the electrons mean free path. Based on the understanding of the irreversibility field nature, an assessment of the individual pinning force is performed. Good agreement is obtained with the estimations performed in the framework of the Larkin-Ovchinnikov pinning model (LO), which is valid for magnetic field less than the irreversibility field. The competing model is considered which is also valid for field range below the irreversibility field - the anisotropic pinning model (APM). The comparison of the fundamental length scales introduced in the LO and APM models is given.

1. Introduction. Irreversibility field

It is known that pinning in superconducting single-phase niobium-base alloys is realized at grain boundaries, in places of dislocation accumulation [1]. At the microscopic level, this occurs due to a local decrease in the mean free path of electrons $l$, which implies a local decrease in the coherence length $\xi$, and thus the grain boundaries become energy-efficient for vortices pinning [2]. This is the so-called $\delta l$-pinning [3]. At the same time, the upper critical field $H_{c2}$ is in inverse proportion to the square of the coherence length $\xi$ [4]:

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2}$$

(1)

where $\Phi_0$ - magnetic flux quantum. Thus, any $\delta l$ - pinning through the local decrease in $\xi$ causes a local increase in $H_{c2}$. In practice, this leads to the fact that in a single-phase $\beta$-Nb-Ti cold-rolled tape the grain boundaries (GB) and internal grain volumes (bulk) have different upper critical fields and therefore can be considered as two separate “superconducting phases” [5]. As a result of cold deformation, the grains are elongated in the tape rolling direction and flattened in the normal direction. In sufficiently high magnetic field in range $H_{c2}^{\text{bulk}} < H < H_{c2}^{\text{GB}}$ there are only grain boundaries in the superconducting state. The upper critical field manifests itself as an “apparent” anisotropy of $H_{c2}$ [6, 7]. So the vast majority of grain boundaries acquire a highly anisotropic geometry of superconducting “thin-film stacks” [6]. The thickness of these thin films is of the order of $\xi$, therefore, when the magnetic field is in the plane of the tape, the grain boundaries are too small to accommodate the...
vortex. In perpendicular geometry a two-dimensional vortex lattice is expected, but due to the small
distance between the vortices it seems that individual vortices cannot be distinguished and only weak
modulations of the order parameter exist. This leads to the fact that in the field range $H_{2}^{bulk} < H <
H_{2}^{GB}$ the picture of classical vortices and their pinning is not applicable [8]. An interesting feature of
this unusual state is the abnormal hysteresis of the current – voltage characteristics, which can be
explained using the superconducting glass model [9] under assumption that there are weak links
between the stacks of thin films [10].

When the external magnetic field decreases below $H_{2}^{bulk}$, the internal volume of the grains passes
from the normal state to the superconducting state. Vortices appear in the material and grain
boundaries start to work as the $\delta l$-pinning centers. Macroscopically, this manifests itself as a transition
from the reversible to irreversible behavior of the magnetization [11], marking the so-called
“irreversibility field” $H^*$. From practical point of view the irreversibility field $H^*$ and its temperature
dependence $H^*(T)$ are of paramount importance since it actually determines the limit of material
applicability [12]. This explains the great attention of the scientific community to the phenomenon and
many attempts have been made to give a theoretical justification (see for example [9, 13-16]). The
number of models suggest that the vortex state keeps above the irreversibility field but their pinning
for one or another reason becomes “ineffective”. An experimental refutation of this hypothesis for Nb-
Ti tapes using the method of two-dimensional current-voltage characteristics (2D-CVC) is given in
[5]. Note that the measurement of 2D-CVC is a good tool for solving the long-standing problem of
distinguishing between the state of superconducting glass, which is determined by inhomogeneity and
the state of vortex glass in which electrodynamics are described by vortices [17]. Without the 2D-CVC
method the crossover between states can be recognized only indirectly by the different temperature
dependence of $H^*(T)$ [18].

Thus, in contrast to the common belief, we argue that the irreversibility field $H^*$ in low-
temperature superconductors with $\delta l$-pinning is determined by the inhomogeneity of the
superconducting properties and in many cases it is the natural limit of the vortex state models. In this
paper, we demonstrate a reasonable quantitative agreement of this model with Larkin-Ovchinnikov
pinning theory (LO), developed for $H < H^*$. We also consider the competing to LO-theory
phenomenological model of anisotropic pinning (APM) and give a comparison between the
fundamental length scales introduced in these theories.

2. Theoretical models in vortex state

2.1. Anisotropic pinning model (APM)

In the phenomenological model proposed in [19], an ensemble of a sufficiently large number of
vortices is considered as a whole, located in a cooperative potential well caused by the entire ensemble
of pinning centers. This ensemble of vortices occupies the most favorable place at the bottom of the
potential well. This condition does not imply that each of the vortices captured by individual pinning
centers and stands at the bottom of its own individual potential well. Under the influence of the
transport current, the vortices are jostled along the well slope, and if the Lorentz force is greater than
the maximum slope angle of the well, the vortices start to move with constant velocity. The maximum
pinning force, which counteracts the Lorentz force, is defined as:

$$ F_p = -\max(\frac{\partial U}{\partial l}) = -e_i \frac{U_0(B)}{I_0(j,B)} $$

(2)

where $U$ is the cooperative potential well depth, $e_i$ is the unit vector in the direction of the Lorentz
force, $L_0(j, B) = U_0(B)/\max(\partial U/\partial l)$ is the effective size of the cooperative potential well, $U_0(B)$ - the
effective depth of the cooperative potential well.

An essential assumption of the model is that the effective depth of the cooperative potential well
$U_0(B)$ is assumed to depend only on the magnitude and direction of the magnetic induction (and not
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potential well \( L_0(\mathbf{j}, \mathbf{B}) \) is assumed to depend on the absolute value of the magnetic induction (density of the vortices) and the direction of the Lorentz force.

According to this model all pinning features in a particular material are determined by the specific angular dependences of the depth \( U_0 \) and the size \( L_0 \). In the case of cold-rolled Nb-Ti tape, the extreme values of \( U_0 \) and \( L_0 \) are achieved with orientations of magnetic induction and Lorentz force along the main orthogonal directions in the material: normal direction (ND), rolling direction (RD) and direction perpendicular to rolling in the plane of the tape (PD). For other directions, \( U_0 \) and \( L_0 \) smoothly vary from minimum to maximum. Thus, the depth \( U_0 \) and the width \( L_0 \) of the cooperative potential well are described by ellipsoids:

\[
(\frac{\cos \alpha''}{U_{RD}})^2 + (\frac{\cos \beta''}{U_{PD}})^2 + (\frac{\cos \gamma''}{U_{ND}})^2 = \frac{1}{U_0^2}
\]

(3a)

\[
(\frac{\cos \alpha'}{L_{RD}})^2 + (\frac{\cos \beta'}{L_{PD}})^2 + (\frac{\cos \gamma'}{L_{ND}})^2 = \frac{1}{L_0^2}
\]

(3b)

where \( \cos \alpha'' \), \( \cos \beta'' \) and \( \cos \gamma'' \) are the direction cosines of induction \( \mathbf{B} \); \( \cos \alpha' \), \( \cos \beta' \) and \( \cos \gamma' \) are the direction cosines of the Lorentz force vector; \( U_{RD} \), \( U_{PD} \), \( U_{ND} \), \( L_{RD} \), \( L_{PD} \) and \( L_{ND} \) – depth and width of the cooperative potential well in the main orthogonal directions.

The anisotropic pinning model well describes a large set of experimental data [19], including guided vortices motion phenomenon and the so-called peak effect in the field dependence of the volume pinning force \( F_p \) in a magnetic field perpendicular to the plane of the Nb-Ti tape, the \( F_p(H) \) dependence has a complex two-humped shape (see below in Fig. 1). The phenomenological APM model does not provide a microscopic explanation for this effect; however, based on an analysis of all experimental data, it was concluded that the effect is associated not so much with an increase in the binding energy \( U_0(B) \), but with a decrease in the width of the cooperative well \( L_0(B) \). Since at the experiment only the ratio of the model parameters is measured (see (2)), it was not possible to determine the absolute values of \( U_{RD} \), \( U_{PD} \), \( U_{ND} \), \( L_{RD} \), \( L_{PD} \) and \( L_{ND} \) and ways to measure them were not given. This greatly complicates their comparison with the structural features and it is the main deficiency of the model.

Recently, the predictions of the AMP model were also confirmed by magneto-optical studies [20] in low field range in perpendicular geometry.

2.2. Larkin-Ovchinnikov model (LO)

Following the original work [21], we present here the formulas in the CGS system of units, with the translation of the calculation results into the International System (SI) of units. Initially, the work [21] was developed to explain the peak effect. Amplification of macroscopic pinning \( F_p \) with magnetic field is realized in the competition between the elastic forces of the vortex lattice and the forces of the individual interaction of the vortices with the pinning centers \( f \).

In the case of “weak” pinning the rigidity of the vortex lattice exceeds the individual forces \( f \). In this case the volume pinning force \( F_p \) is formed due to collective effects: small displacements of the vortices from the lattice sites, caused by various defects, accumulate and the long-range order disappears. The volume of the vortex lattice, in which the short-range order is preserved, is called the correlation volume \( V_c = L_c R_c^2 \), where \( L_c \) is the correlation length in the field direction, \( R_c \) – in the direction of the unit cell vector of the vortices. It is assuming that these correlated regions are elastically independent. Thus the Larkin-Ovchinnikov theory introduces the length scales \( L_c \) and \( R_c \) which depends on the magnetic field.

In the opposite case, when \( f \)-forces dominate over the lattice rigidity, the “strong” pinning is realized. In this case, the value of \( R_c \) becomes smaller than spacing between the vortices and the vortex lattice is amorphized. The change in free energy during the transition from one metastable state to another determines the value of the critical current. The correlation lengths in this case cannot be
computed from the LO expressions, so that the correlated volume must be considered as an experimental parameter [22].

Further, we write out the necessary formulas for the case of $\delta l$-pinning at grain boundaries [21]. The strength of individual pinning $f$ is estimated as:

$$f = \nu g_1 \Delta^2 \xi l_1 (1 + \frac{l_1}{\Delta})$$

where $\nu$ is the density of states on the Fermi surface, $\Delta$ is the superconducting order parameter, $l_1$ and $\Delta$ are the sizes of the pinning center in the direction of the unit cell vector of the vortex lattice $a$, and the applied magnetic field, respectively, $g_1$ – the deviation of the electron interaction from the average, $\xi$ – coherence length.

The case of the strong limit can be realized in low fields $H \ll H_{c2}$ and near $H_{c2}$ if the grain size $l$ satisfies the condition:

$$\xi < l < \kappa \xi \frac{\tau}{g_1}$$

where $\tau = l - T/T_c$, $T_c$ – critical temperature of superconductor, $\kappa$ – Ginzburg-Landau parameter.

The positions of the maxima of the volume pinning force $F_p$ are given by following:

1. Near $H_{c2}$, the value of the reduced magnetic field ($b = H/H^*$, $H^*$ - irreversibility field) is $b_p^{max}$:

$$1 - b_p^{max} = \frac{g_1}{\tau} \left[1 + \left(\frac{\xi^2}{l^2} - \frac{1}{g_1}\right)^{1/2}\ln\left(\frac{\pi^2 l^2}{g_1^2}\right)\right]^{-1}$$

(6)

2. The position of the low field maximum $b_p^{min}$:

$$b_p^{min} = \frac{g_1}{\tau} \left(1 + \frac{l}{\kappa \xi}\right) \left[1 + \frac{r}{\kappa \xi^2} \frac{\pi^2 l^2}{g_1^2} \ln\left(\frac{\pi^2 l^2}{g_1^2}\right)\right]^{-1}$$

(7)

3. Results and discussion

The transport method [19] allows one to obtain quantitative data on the current carrying capacity; however, this method is rather laborious, which makes it difficult to accurately determine the position of the maxima on the $F_p(H)$ dependence. In this work, the position of the peaks was determined by measuring the field dependence of the magnetic moment $M(H)$ (Fig. 2). The critical current was estimated from the width of the hysteresis loop $\Delta M(H)$. Since shielding currents are closed loops and flow throughout the sample during such measurements, the obtained data reflect the current carrying capacity of the tape averaged over all angles to the rolling direction (compare with Fig. 1). The positions of the pinning force peaks $\mu_0 H_{p_{min}} = 0.96$ T and $\mu_0 H_{p_{max}} = 9.23$ T. Using the irreversibility field value $\mu_0 H^* = 10.6$ T [10] according to formulas (6) and (7), we have determined the parameters $g_1/\tau = 0.19$ and the size of the pinning centers $l = 40.5$ nm, which were considered as fitting parameters in this procedure. The obtained $l$ value agrees well with the most probable size of the Nb-Ti grain in the direction normal to the tape — 38 nm, which we determined earlier by statistical processing of a large number of images from an electron microscope. We also note that the obtained quantities $l$ and $g_1/\tau$ satisfy condition (5).

Substituting into (4) the values for the density of states on the Fermi surface $\nu = 2 \times 10^{14}$ erg$^{-1}$cm$^{-1}$ [23], $g_1 = 0.19(1 - T/T_c) = 0.1$, $\Delta = 1.76T_c = 2.2 \times 10^{15}$ erg (BCS ratio), $a = 2 \times 10^{-8}$ cm (vortex spacing), $\xi = 5.4 \times 10^{-7}$ cm, and setting $I_p \sim l_1 \sim l = 4.05 \times 10^6$ cm, we obtain an estimate for the individual pinning force $f = 2 \times 10^9$ dyn or, in the SI system:

$$f = 2 \times 10^{13} \text{N}$$

(8)
On the other hand, it is also possible to evaluate the individual pinning force $f$ from the ideas about the nature of the irreversibility field described in the introduction. Indeed, if the irreversibility field $H^*$ is determined by the value of the upper critical field of the grain volume $H_{c2}^{\text{bulk}}$, and the transition to the normal state $H_{c2}$ by $H_{c2}^{\text{GB}}$, then using the data on $H^*$ [11] and $H_{c2}$ [6], we can estimate the decrease in the coherence length at the grain boundary:

\begin{align}
\delta \xi &= \left( \frac{\Phi_0}{2\pi H_{c2}^{\text{bulk}}} \right)^{1/2} - \left( \frac{\Phi_0}{2\pi H_{c2}^{\text{GB}}} \right)^{1/2} = 0.26 \text{ nm} \\
\end{align}

The gain in energy due to a decrease in the volume of the vortex core at the grain boundary is:

\begin{align}
\delta \mathcal{E} &= \frac{\mu_0 H_{c2}^2}{2} \pi \xi \delta \xi
\end{align}

where $H_{c2}=H_{c2}/\kappa\sqrt{2}$ – thermodynamic critical field. The individual pinning force, per unit of vortex length:

\begin{align}
f_p &= \frac{\delta \mathcal{E}}{\xi} = \frac{\pi \mu_0 H_{c2}^2}{2\kappa^2} \delta \xi
\end{align}

Substituting the corresponding values in (11), we find $f_p = 1.16 \times 10^5$ N/m. With the pinning center length $l = 38$ nm, the value of the individual pinning force $f = f_p l = 4.5 \times 10^{-13}$ N. This value has a reasonable agreement with the estimate (8) obtained in the Larkin-Ovchinnikov model.

The successes of the LO and APM models entice their integration: the introduction of $R_c$ as a scaling parameter for the width of the potential well (3b). Indeed, in both models the peak effect is well explained by a decrease in the length parameter. However, it should be noted that the LO model operates with a static state of vortex matter, while the APM model with a dynamic one. So, for example, it is much easier to shift the vortex matter in the rolling direction (sample with $\xi = 90^\circ$ in Fig. 1) than in the perpendicular direction (sample with $\xi = 0^\circ$ in Fig. 1), which is reflected in various values of $L^{RD}$ and $L^{PD}$, and it is completely not taken into account in the theory of LO. In physical sense, $R_c$ describes the size of the vortex matter part in which the short-range order is preserved, while

**Figure 1.** Field dependence of the volume pinning force $F_p = \mu_0 H\xi$ for the series of samples cut from cold-rolled Nb-Ti tape with a magnetic field oriented perpendicular to the tape. Here $\xi$ is the angle between sample axial axis and the tape rolling direction. Figure is taken from paper [19].

**Figure 2.** The field dependence of the volume pinning force (in arbitrary units) $F_p = \mu_0 H\Delta M$ for the cold-rolled Nb-Ti tape with the magnetic field oriented perpendicular to the tape, determined from the width of the hysteresis loop of the magnetic moment.
$L_0$ of the APM model is the distance over which this part must be moved so that the vortices can irreversibly leave the cooperative potential well.

4. Conclusion
In this work, the force $f$ with which the pinning center attracts a vortex is estimated. For the cold-rolled $\beta$–Nb-Ti tape in perpendicular geometry (magnetic field is directed along the normal) this force is of the order of $10^{13}$ N. This estimate is the same both in the Larkin Ovchinnikov model and in the model of the reversibility field of superconductors with $\delta l$-pinning. This coincidence is especially noteworthy because in the first case (Larkin-Ovchinnikov model) the magnetic field range $0 < H < H^*$ was used for estimation, while in the second case the difference between the irreversibility field $H^*$ and the upper critical field $H_{c2}$ (which is higher than $H^*$).

The Larkin-Ovchinnikov model gives a qualitative explanation of the peak effect in the field dependence of the volume force of pinning but does not take into account the pinning force anisotropy. This problem is solved using the anisotropic pinning model (APM). However, in APM model a cooperative potential well arises with parameters whose absolute values are not determined. It is remarkable that both in the Larkin-Ovchinnikov model and in the anisotropic pinning model the peak effect is associated with a decrease in the parameter having a length dimension. However, the above reasoning allows us to conclude that, apparently, the correlation length of the Larkin-Ovchinnikov model cannot give the scale of the cooperative potential well.

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