1 Abstract

The article gives the explicit interpretation of the nonlinear locally anisotropic $\sigma$-model of Boze string\footnote{The author is indebted to Dr. Sergiu Vacaru for some conceptual discussions.}. The equations of motions and tensor of energy-momentum, the background field method applied to get order extensions of the effective action consequently accommodated to the geometry of locally anisotropic spaces, additional contributions-corrections resulted from the richer geometric character of the model are calculated and interpreted from the possible physical point of view. It is shown that the strings theory considered on spaces with local anisotropy and nonlinear connections are renormalizable taken the similar nature of the explicitly calculated anisotropic corrections with the Riemannian one.

Author explains shortly necessary geometrical background and conventions of locally anisotropic geometry as well as the effects of this geometry on strings. Mention is given particularly to the interpretation of the results and "new" aspects that features the locally anisotropic strings form their Riemannian counterparts.
2 Introduction

The concept of locally anisotropic strings and of the nonlinear $\sigma$—model had already been advanced in some works [32, 33, 27]. There has been given the substantial body of conceptual motivation and necessary geometrical language adopted to develop the locally anisotropic (super)strings [15, 7, 6], and their extensions to superstrings [10, 14, 30].

The diversity of results formulated and obtained in locally anisotropic physics [31, 37, 35, 36, 34] provoked and justified a special interest for modelling of nonlinear $\sigma$—model on spaces with local anisotropy.

As the nonlinear geometry considered sometimes inadequate or rather inappropriate for the easily accepted standard mathematical tools to explain physical phenomena, physicists always remain "unhappy" that the diverse nature of the phenomenon considered is somewhat straiten out by isotropic and/or linear approximations. A physicist always wants that linear isotropic mathematical tool applied to describe a phenomena keep the model flexible enough to consider possible intrusions deviations, interferences, interactions and corrections. It has been a predominant practice that intrinsically desired flexibility is introduced or accounted for with the help of some external to the initial mathematical tool of model assumptions or approximations. It does though make happy the physicist, however demonstrating after all, how innovative the human intelligence is, yet leaves one with the sense of self-inconsistency of the theory itself.

The proposed example demonstrates how the nonlinear geometry-spaces with nonlinear connection or spaces with local anisotropy, spaces that evidently will make happy any physicist to modulate a phenomena, once he/she has an appropriate mathematical tool to work with, allows the generalization of the non-linear $\sigma$—model. The explicitly constructed model on locally anisotropic spaces gives not just necessarily locally anisotropic corrections, but is manifestly renormalizable and consistent with the standard previously obtained results of nonlinear $\sigma$—model and what is important essentially widens the horizon of modelling string theories.

The rich nature of the fiber-base structures allows, apart from construction of strict physically friendly results, hypothetical interpretation of the fiber-base objects, as for instance the interaction gauge like fields. This objects serve also simultaneously as some additional terms-calculated explicitly in this work-for the standard nonlinear $\sigma$—model. The fiber-base objects,
reasonably judged, may lead to some further developments of string theory as for, instance the consistent explication of the string interactions. The above suggested ideas may justifiably present certain interest for physicists.

The article is structured as follows: first goes a brief introduction into locally anisotropic geometry and objects; then the equations of motion and tensor of energy-momentum of locally anisotropic strings is considered; follows the accommodation of the standard background field method; later the effective action order extension and locally anisotropic counterterms discussed widely; the article finalizes conclusions and discussions of the obtained results.

3 An Outline of Locally Anisotropic Geometry.

We consider $\epsilon = (E, \pi, F, G, M)$ to be a locally trivial vector bundle, $v$-bundle, where $F$ is a vector space with $\dim F = m$, $G$ is a group of automorphism of $F$, $\pi = E \to M$ is a surjective map and a differentiable manifold $F, \dim F = m + n$ is called as the total space of $v$-bundle $\epsilon$. We can locally parameterize $\epsilon$ by coordinates $u^\alpha = (x^i, y^a)$, where $i, j, k, l, m, n, u, \ldots$ take value $0, 1, \ldots n - 1$ and Greek indices $a, b, c, d, e, f, \ldots$ take value $0, 1, \ldots m - 1$.

Coordinate transforms $(x^i, y^a) \to (x'^i, y'^a)$ on differentiable manifold $\epsilon$ are given by formulas $x'^i = x^i(x^i), \text{rank}(\partial x'^i / \partial x^i) = n$, and $y'^a = M_a^i(x)y^a, M_a^i(x) \in G$.

We provide $\epsilon$ with the structure of nonlinear connection that splits $v$-bundle into horizontal, $HE$ and vertical $VE$ subbundles of the tangent bundle $TE$

$$TE = HE \oplus VE$$

(1)

For a $N$-connection on $\epsilon$ one can associate the covariant derivation operator

$$\nabla_X A = y^i \left\{ \frac{\partial A^a}{\partial x^i} + N_i^a(x, A) \right\} s_a$$

(2)

where $s_a$ are local linearly independent sections of $\epsilon, A = A^a s_a$ and $Y = Y^i s_i$ is a vector field decomposition on local bases $s_i$ on $M$. $N_i^a(x, y)$ are called as coefficients of $N$-connection.
The transformation law for $N$-connection under coordinate transforms 

$$
N^a_i \frac{\partial x^i}{\partial x^i} = M^a_i(x)N^a_i \frac{\partial M_i^a}{\partial x^i}(x)y^a
$$

(3)

$N$-connection $N_i^a(x, y)$ is characterized by its curvature 

$$
\Omega = \frac{1}{2} \Omega^a_{ij} dx^i \wedge dx^j \otimes \frac{\partial}{\partial y^a}
$$

(4)

with coefficients 

$$
\Omega^a_{ij} = \frac{\partial N^a_j}{\partial x^i} - \frac{\partial N^a_i}{\partial x^j} + N^b_j \frac{\partial N^a_i}{\partial y^b} - N^b_i \frac{\partial N^a_j}{\partial y^b}
$$

(5)

For further needs we define a locally adopted (to $N$-connection) reaper basis as 

$$
u_\alpha = \frac{\delta}{\delta u_\alpha} = (x_i = \frac{\delta}{\delta x_i} = \frac{\partial}{\partial x_i} - N^a_i(x, y) \frac{\partial}{\partial y^a}; y_a = \frac{\delta}{\delta y^a} = \frac{\partial}{\partial y^a})
$$

(6)

The dual basis to $x_\alpha$ is as 

$$
u^\alpha = \delta \nu^\alpha = (x^i = dx^i; x^a = \delta y^a = dy^a + N^a_i(x, y) dx^i)
$$

(7)

The algebra of tensors distinguished fields on $\epsilon$ can be introduced by using bases 

$$
t = t_{i_1 \ldots i_p a_1 \ldots a_r} \left(x, y\right) \frac{\delta}{\delta x^{i_1}} \otimes \cdots \otimes \frac{\delta}{\delta x^{i_p}} \otimes dx^{j_1} \otimes \cdots \otimes dx^{j_q} \otimes \cdots \frac{\partial}{\partial y^{a_1}} \otimes \cdots \otimes \frac{\partial}{\partial y^{a_r}} \otimes \delta y^{b_1} \otimes \cdots \otimes \delta y^{b_s}
$$

(8)

Along with nonlinear $N$-connection one can define a distinguished linear connection $(d$-connection) $\Gamma^a_{\beta \gamma}$ associated to a fixed $N$-connection structure on $\epsilon$ 

$$
D_{\gamma} \frac{\delta}{\delta u_\beta} = D_{\gamma} \frac{\delta}{\delta u_\beta} = \Gamma^a_{\beta \gamma} \frac{\delta}{\delta u_\alpha}
$$

(9)
Torsion \( T^\alpha_{\beta\gamma} \) and curvature \( R^\alpha_{\beta\gamma\delta} \) of \( d\)-connection \( \Gamma^\alpha_{\beta\gamma} \) are defined respectively as:

\[
T \left( \frac{\delta}{\delta u^\gamma}, \frac{\delta}{\delta u^\beta} \right) = T^\alpha_{\beta\gamma} \frac{\delta}{\delta u^\alpha}
\]

(10)

where \( T^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta} + \omega^\alpha_{\beta\gamma} \) and

\[
R \left( \frac{\delta}{\delta u^\beta}, \frac{\delta}{\delta u^\gamma}, \frac{\delta}{\delta u^\delta} \right) = R^\alpha_{\beta\gamma\delta} \frac{\delta}{\delta u^\alpha}
\]

(11)

where \( R^\alpha_{\beta\gamma\delta} = \frac{\delta}{\delta u^\delta} \Gamma^\alpha_{\beta\gamma} - \frac{\delta}{\delta u^\gamma} \Gamma^\alpha_{\beta\delta} + \Gamma^\alpha_{\beta\gamma} \phi^\delta - \Gamma^\alpha_{\beta\delta} \Gamma^\phi_{\gamma\delta} + \Gamma^\alpha_{\beta\gamma} \omega^\phi_{\gamma\delta} \). Throughout the formulas used below \( \omega^\alpha_{\beta\gamma} \) are nonholonomic coefficients of locally adopted reapers.

Global decomposition of bundle \( \epsilon \) into horizontal and vertical parts by nonlinear connection structure splits components of \( d\)-connection and \( d\)-tensor fields into horizontal and vertical ones. Locally they appear for horizontal components:

\[
D^h_{\delta x} \left( \frac{\delta}{\delta x^i} \right) = L^i_{jk}(x, y) \frac{\delta}{\delta x^j}; \quad D^h_{\delta y} \left( \frac{\delta}{\delta y^a} \right) = L^a_{bk}(x, y) \frac{\delta}{\delta y^b}
\]

and \( D^h_{\delta x} f = \frac{\delta f}{\delta x} = \frac{\partial f}{\partial x} - N^a_{k}(x, y) \frac{\partial f}{\partial y^a} \) where \( f(x, y) \) is a scalar function on \( \epsilon \) and for vertical components

\[
D^v_{\delta \Sigma} \left( \frac{\delta}{\delta x^j} \right) = C^i_{jc}(x, y) \frac{\delta}{\delta x^i}; \quad D^v_{\delta y} \left( \frac{\delta}{\delta y^b} \right) = C^a_{bc}(x, y) \frac{\partial}{\partial y^a}; \quad D^v_{\delta y} f = \frac{\partial f}{\partial x^j}.
\]

For components of torsion an explicit calculation gives

\[
h T \left( \frac{\delta}{\delta x^i}, \frac{\delta}{\delta x^j} \right) = T^i_{jk} \frac{\delta}{\delta x^k}; \quad v T \left( \frac{\delta}{\delta x^i}, \frac{\delta}{\delta x^j} \right) = T^a_{jk} \frac{\partial}{\partial y^a},
\]

\[
h T \left( \frac{\partial}{\partial y^a}, \frac{\partial}{\partial x^i} \right) = P^i_{jb} \frac{\partial}{\partial y^b}; \quad v T \left( \frac{\partial}{\partial y^b}, \frac{\partial}{\partial x^i} \right) = P^a_{jb} \frac{\partial}{\partial y^a},
\]

\[
v T \left( \frac{\partial}{\partial y^a}, \frac{\partial}{\partial y^b} \right) = S^a_{bc} \frac{\partial}{\partial y^c}.
\]
the corresponding components of torsion are

\[ T^i_{jk} = L^i_{jk} - L^i_{kj}, T^a_{jk} = R^a_{jk} = \frac{\delta N^a}{\delta x_j}, P^i_{jb} = C^i_{jb}, \]  

(12)

For the components of curvature an explicit calculation gives

\[ R \left( \frac{\delta}{\delta x^i}, \frac{\delta}{\delta x^j} \right) = R^i_{jk}, R \left( \frac{\delta}{\delta x^i}, \frac{\delta}{\delta x^j}, \frac{\delta}{\delta y^k} \right) = R^i_{jkl}, \]

\[ P^i_{jkl} = \frac{\partial N^i}{\partial y^l} - L^i_{bj}, S^a_{bc} = C^a_{bc} - C^a_{eb} \]

(13)

the corresponding components of curvature are

\[ R^i_{kl} = \frac{\delta L^i_{hl}}{\delta x^l} + L^i_{jk}L^h_{kl} - L^h_{jl}L^j_{kl} + C^i_{jl}R^a_{kl}, \]

\[ R^a_{kl} = \frac{\delta L^a_{hl}}{\delta x^l} + L^a_{jk}L^h_{kl} - L^h_{jl}L^j_{kl} + C^a_{jl}R^c_{kl}, \]

\[ P^i_{jkl} = \frac{\partial L^i_{kl}}{\partial y^l} - C^i_{jl}P^b_{kl}, \]

\[ P^i_{jkl} = \frac{\partial L^i_{kh}}{\partial y^k} - C^a_{be}P^c_{kl}, \]

\[ S^a_{bc} = \frac{\partial C^a_{bc}}{\partial y^c} - C^h_{jb}C^c_{hc} - C^h_{jc}C^c_{hb}, \]

\[ S^a_{cd} = \frac{\partial C^a_{cd}}{\partial y^d} - C^f_{bc}C^c_{fd} - C^f_{bd}C^c_{fc}, \]

(14)

In addition to d- connection structure, metric structure on v- bundle \( \epsilon \) being associated to a map \( G(u) : T_u \epsilon \otimes T_u \epsilon \to R \). Choosing a concordance between \( N \)- connection and metric \( G \) on \( \epsilon \) when condition \( G \left( \frac{\delta}{\delta x^i}, \frac{\partial}{\partial y^j} \right) = 0 \)

or equivalently \( N^a_{ij}(x, y) = G_{ib}(x, y)G^{ba}(x, y) \) are held. A metric \( G \) on \( \epsilon \) is defined by two independent d-tensors \( g_{ij}(x, y) \) of type \( \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \) and \( h_{ab}(x, y) \)

of type \( \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \) and with respect to the local adapted bases can be written

as \( G = g_{ij}(x, y)dx^i \otimes dx^j + h_{ab}(x, y)dy^a dy^b \). d- connection \( \Gamma^a_{\beta \gamma} \) is compatible with metric structure \( G \):

\[ D_\alpha G^{\beta \gamma} = \frac{\delta}{\delta u^\alpha} G^{\beta \gamma} = \Gamma^\alpha_{\beta \gamma} \Gamma^\beta_\gamma - \Gamma^\alpha_{\beta \gamma} \Gamma^\beta_\gamma = 0 \]  

(15)

In v- bundle \( \epsilon \) we can consider the canonical (metric) d- connection \( \Gamma(N) \) with components \( \Gamma^a_{\beta \gamma} = (L^i_{jk}, L^a_{bi}, C^i_{jc}, C^a_{bc}) \) determined by metric \( G \). Below we write out the components of d- connections:
\[
L^i_{jk} = \frac{1}{2} g^{ip} \left( \frac{\delta g_{jk}}{\delta x^p} + \frac{\delta g_{kp}}{\delta x^j} - \frac{\delta g_{jk}}{\delta x^p} \right), \\
L^a_{bi} = \frac{\partial N^a_{\gamma}}{\partial y^i} + \frac{1}{2} h^{ac} \left( \frac{\partial h_{bc}}{\partial y^a} - \frac{\partial N^c_{\gamma}}{\partial y^b} h_{ac} - \frac{\partial N^a_{\gamma}}{\partial y^b} h_{ab} \right), \\
C_{ijb} = \frac{1}{2} g^{ik} \frac{\partial g_{jk}}{\partial y^b}, \\
C_{abc} = \frac{1}{2} h^{ad} \left( \frac{\partial h_{bd}}{\partial y^c} + \frac{\partial h_{dc}}{\partial y^b} - \frac{\partial h_{bc}}{\partial y^d} \right) 
\]

The Ricci tensor \( R_{\beta \gamma} = R_{\beta \alpha \gamma}^{\alpha} \) defines the \( cd \)-connection and has the following components with respect to adapted reaper basis

\[
R_{ij} = R_{i \; jk}^{k}, \quad R_{ia} = P_{i \; ak}^{k} = -P_{ia}, \\
R_{ai} = P_{a \; ib}^{b} = P_{ai}, \quad R_{ab} = S_{a \; bc}^{c} = S_{ab} 
\]

In general tensor Ricci is not symmetric. The scalar curvature of the \( cd \)-connection \( R = g^{ij} R_{ij} \) and \( S = h^{ab} S_{ab} \) is given by

\[
4 \quad \text{Equation of Motion and Energy-Momentum Tensor of Locally Anisotropic Strings.}
\]

We are intending to get strings equation of motion considering the movement of the two-dimensional surface imbed in \( (n + m) \) dimensional locally anisotropic space-time geometry and expansion of the tensor energy-momentum to Locally Anisotropic Geometry (LAG). We discuss some particular consequences of the Locally Anisotropic Strings (LAS).

The \( \sigma \)-model which corresponds to the Bose string action with Witten-Zumino term in locally anisotropic case is

\[
I_0 = \frac{1}{4 \pi \alpha'} \int dz^\alpha \left( \delta^A u^\alpha \delta_A u^\beta G_{\alpha \beta}(u) + i \epsilon^{AB} \delta_A u^\alpha \delta_B u^\beta H_{\alpha \beta}(u) \right) 
\]

Varying with respect to \( \delta G_{\alpha \beta}(u) \) in order to get string’s equation of motion we note that due to the locally antisymmetric nature of the second term in (13) it does not contribute to string’s equation of motion and taking into account the compatibility condition

\[
D_{\gamma} G_{\alpha \beta}(u) = \delta_{u^\gamma} G_{\alpha \beta} - \Gamma_{\alpha \sigma}^{\gamma} G_{\gamma \beta} - \Gamma_{\beta \sigma}^{\gamma} G_{\alpha \gamma} = 0 
\]
Lagrangian is given by

\[ L(u, G) = \delta^A u^\alpha \delta_A u^\beta G_{\alpha\beta}(u) \]  

we get

\[ \frac{\delta L}{\delta u^\gamma} = \frac{\delta}{\delta u^\gamma} \left( \delta^A u^\alpha \delta_A u^\beta G_{\alpha\beta}(u) \right) = \delta^A u^\alpha \delta_A u^\beta \frac{\delta}{\delta u^\gamma} G_{\alpha\beta}(u) = \delta^A u^\alpha \delta_A u^\beta \frac{\delta}{\delta u^\gamma} G_{\alpha\beta}(u) = \delta^A u^\alpha \delta_A u^\beta \frac{\delta}{\delta u^\gamma} G_{\alpha\beta}(u) = \left( \partial_k - N^a_k \partial_a \right) \left( \frac{g_{ij}(u)}{0} \right) = \delta^\alpha_\gamma \delta^A u^\beta + \left( \delta_k g_{ij} \right) = 0 \]

multiplying on \( G^{\alpha\beta} \) and bearing in mind that \( G^{\alpha\beta} G_{\alpha\beta} = (m+n-1) \) we have the string equation of motion of the locally anisotropic string

\[ (m+n-1) \partial_A \partial_B u^\beta + \Gamma^\alpha_\beta_\gamma(u) \partial_A u^\alpha \partial_B u^\beta = 0 \]  

where \( \Gamma^\alpha_\beta_\gamma \) as was expected is a symmetric part of the distinguished linear connection in the adapted bases and the last becomes possible owing to (19).

One can split \( \Gamma^\alpha_\beta_\gamma(u) \) into symmetric part \( s\Gamma^\alpha_\beta_\gamma(u) \) and nonsymmetric \( n\Gamma^\alpha_\beta_\gamma \) parts on the different bases.

Let us consider one of them minding one more that will be taken as essential in next section.

Symmetric part of \( \Gamma^\alpha_\beta_\gamma(u) \) can be obtained by simple substraction of nonsymmetric part \( n\Gamma^\alpha_\beta_\gamma(u) \) of \( \Gamma^\alpha_\beta_\gamma(u) \):

\[ s\Gamma^\alpha_\beta_\gamma(u) = \Gamma^\alpha_\beta_\gamma(u) - \begin{pmatrix} 0 & L^{i}_{j\gamma} \\ C^{a}_{bi} & 0 \end{pmatrix} = \begin{pmatrix} L^{i}_{j\gamma} & 0 \\ 0 & C^{a}_{bc} \end{pmatrix} \]  

\( s\Gamma^\alpha_\beta_\gamma(u) \) Levi-Civita connection or any other. Than equation (21) take the form of a system of two equations

\[ \begin{cases} (m - 1) \partial_A \partial_B x^j + L^{i}_{j\gamma}(x, y) \partial_A x^i \partial_B x^j = 0 \\ (n - 1) \partial_A \partial_B y^b + C^{a}_{bc}(x, y) \partial_A y^a \partial_B y^b = 0 \end{cases} \]  

the equation (23) gives rise to the idea that in locally anisotropic case we have two way directions to move each being in corresponding subspaces (fiber and base). However, they are not independent due to independence of \( L^{i}_{j\gamma}(x, y) \) and \( C^{a}_{bc}(x, y) \) on \( u(x, y) \) so that motion in fiber bears an impact from the
point of view of physical applications consequences. We intend to discuss it in the next paper.

Our next step is to have energy-momentum tensor extended to the case of the locally anisotropic geometry. We proceed by varying with respect to $G_{\alpha\beta}(u)$:

$$
\frac{\delta I_0}{\delta G_{\alpha\beta}(u)} = \frac{1}{2} \int d^2z \sqrt{G} T_{\alpha\beta}(u) = 0
$$

where

$$
T_{\alpha\beta} = \delta_A u_{\alpha} \delta^A u_{\beta} - \frac{1}{2} G_{\alpha\beta} G^{\gamma\delta} \delta_A u_{\gamma} \delta^A u_{\delta} = 0. \quad (24)
$$

The later result perfectly fits with the standard one in Riemannian case[28], as is to expect.

5 Locally Anisotropic Geometry Background

Field Method.

This section deals with formulating of the Riemannian coordinates to the case of the locally anisotropic geometry. We consider also the possibility to obtain symmetric part of $\Gamma_{\alpha\beta\gamma}(u)$ on the basis of having certain restrictions fixed on linear connection in the adapted basis.

One embraces here a different approach adapted in [33, 32] rather developing the standard normal (Riemannian) coordinates [29], generalizing them to the case of locally anisotropic geometry [27]. Author follows the general logic of the order extension contained in [3, 25]. As far as $u^\alpha = (x^1, y^a)$, $\xi^\alpha = (\theta^i, \lambda^a)$ and making shift $u^\alpha \rightarrow u^\alpha + \xi^\alpha$ and expanding it as a power series in $\xi^\alpha$ we get

$$
u^\alpha = u^\alpha_0 + \xi^\alpha s - \frac{1}{2} \left( \frac{s^\alpha}{\Gamma_{\beta\gamma}} \right)_0 \xi^\beta \xi^\gamma - \frac{1}{3!} (\Gamma_{\beta\gamma\phi})_0 \xi^\beta \xi^\gamma \xi^\phi - \cdots \quad (25)
$$

and accommodating the well-known relations to the case

$$
\Gamma_{(\alpha_1, \alpha_2, ..., \alpha_n)}(u) = \frac{1}{N} \left( \frac{\delta \Gamma_{\alpha_1, \alpha_2, ..., \alpha_n}}{\delta u^{\alpha_n}} \right) - \Gamma_{\gamma, \alpha_2, ..., \alpha_n-1} \Gamma_{\alpha_1 \alpha_2} - \Gamma_{\alpha_1 \gamma \alpha_3, ..., \alpha_{n-1}} \Gamma_{\alpha_2 \alpha_n} - \Gamma_{\alpha_1 \alpha_2 \gamma} \Gamma_{\alpha_n \alpha_{n-1}}
$$

and
\[
\Gamma^\delta_{(\alpha_1,\alpha_2,..,\alpha_n)}(u) = \frac{1}{N} \left( \delta^\delta_{\alpha_1,\alpha_2,..,\alpha_{n-1}} - (N-1)\Gamma^\delta_{\gamma,\alpha_2,..,\alpha_n} \Gamma^\gamma_{\alpha_1 \alpha_n} \right)
\]

where

\[
\frac{\delta^2 u^\alpha}{\delta s^2} + \Gamma^\alpha_{\beta\gamma} \frac{\delta u^\beta}{\delta s} \frac{\delta u^\gamma}{\delta s} = 0, \quad u^\alpha = \xi^\alpha s
\]

we finally obtain decomposition

\[
u^\alpha = u_0^\alpha + v^\alpha - \frac{1}{2} \left( \Gamma^\alpha_{\beta\gamma} \right)_0 v^\beta v^\gamma - \frac{1}{3!} \left( \Gamma^\alpha_{\beta\gamma\delta} \right)_0 v^\beta v^\gamma v^\delta - \ldots
\]

we referred to \( \Gamma^\alpha_{\beta\gamma} \) as a symmetric part of the distinguished linear connection in adapted bases and bearing in mind (19) and (21) takes form

\[
u^\alpha = u_0^\alpha + \xi^\alpha s - \frac{1}{2} \left( \Gamma^\alpha_{\beta\gamma} \right)_0 \xi^\beta \xi^\gamma s^2 - \frac{1}{3!} \left( \Gamma^\alpha_{\beta\gamma\delta} \right)_0 \xi^\beta \xi^\gamma \xi^\delta s^3 - \ldots
\]

and as \[29\]

\[
\begin{align*}
\{ \partial(\beta \Gamma^\nu_{\alpha})_\mu \}_0 &= \frac{1}{3} \delta^\nu_{(\alpha \beta)}_\mu, \\
\{ \partial(\gamma \Gamma^\nu_{\alpha})_\mu \}_0 &= -\frac{1}{2} \delta^\nu_{\mu(\gamma,\alpha)}, \\
\{ \partial(\delta \Gamma^\nu_{\alpha})_\mu \}_0 &= -3/5(2/9 \delta^\nu_{(\alpha \beta)} \delta^\beta_{\delta \gamma} \delta^\gamma_{\mu \omega} + R_{\mu(\delta,\gamma,\alpha)\beta});
\end{align*}
\]

a locally anisotropic tensor field \( W_{\alpha_1..\alpha_p}(u) \), can be order expanded

\[
W_{\alpha_1..\alpha_p} = 0_{W_{\alpha_1..\alpha_p}} + \left( \frac{\partial W_{\alpha_1..\alpha_p}}{\partial \xi^\nu} \right)_0 \xi^\nu + \frac{1}{2!} \left( \frac{\partial^2 W_{\alpha_1..\alpha_p}}{\partial \xi^\mu \partial \xi^\nu} \right)_0 \xi^\mu \xi^\nu + \ldots
\]

one can express \( W_{\alpha_1..\alpha_p} \) by means of the (30) and (31)

\[
W_{\alpha_1..\alpha_p} = 0_{W_{\alpha_1..\alpha_p}} + 0_{W_{\alpha_1..\alpha_p}} \xi^\mu + 1/2! \{ W_{\alpha_1..\alpha_p} \xi^\mu, \xi^\omega \} + \ldots
\]

10
\[- \sum_{k=1}^{p} \hat{R}_{\mu k \omega} W^{\alpha_1 \ldots \alpha_{k-1} \nu \alpha_{k+1} \ldots \alpha_p \sigma} \]

\[-1/2 \sum_{k=1}^{p} \hat{R}_{\mu \omega, \sigma} W^{\alpha_1 \ldots \alpha_{k-1} \nu \alpha_{k+1} \ldots \alpha_p} \xi^\mu \xi^\omega \xi^\sigma + \ldots \]

The general form is the same as in Riemannian space-time but the essence consists in the fact that \(\xi\) splits in fiber \(\lambda^a\) and \(\theta^i\) components and as we see later contractions are possible only for \((\lambda^a \lambda^b)\) or \((\theta^i \theta^j)\) as far as \(\dim M \neq \dim F\) (dimension of the fiber is not equal to the dimension of the base in general case. Yet, \((\lambda \theta)\) contractions present special interest in vector bundle space).

Symmetric part of \(\hat{\Gamma}^{a}_{\beta \gamma}\) can be also achieved by reducing local anisotropic space-time i.e. restricting the mixed components of the \(d\)-connection. Proceeding this way we get that torsion vanishes and we can define locally and along with a curve \(\Gamma^{\alpha}_{\beta \gamma}\) and summing up over the fiber-fiber and base-base indices in \((16)\) we get that \((10)\),

\[
L^a_{ai}(u) = \frac{\delta}{\delta x^i} \ln \left( \sqrt{-h} \right) \\
C^{i a}(u) = \frac{\partial}{\partial y^a} \ln \left( \sqrt{-g} \right)
\]

(32)

The geometrical sense of the considered restrictions is evident from the definition of connection components

\[
D^b_{\delta \delta} \left( \frac{\partial}{\partial x^a} \right) = L^b_{ak}(x, y) \frac{\partial}{\partial y^a} \\
D^c_{\delta \delta} \left( \frac{\delta}{\delta x^a} \right) = C^c_{ic}(x, y) \frac{\delta}{\delta x^a}
\]

(33)

bases vectors \(\frac{\partial}{\partial y^a}, \frac{\delta}{\delta x^a}\) lost their flexibility in \(\frac{\delta}{\delta x^a}, \frac{\beta}{\partial y^a}\) directions respectively but did not in \(\frac{\beta}{\partial y^a}, \frac{\partial}{\partial x^a}\) directions. Our assumption makes locally anisotropic geometry to be rather awkward, but even it allows us to test the simplest extension of Riemannian bosonic strings to the locally anisotropic bosonic strings.
The components of the curvature take form:

\[
R_{ijkl} = \frac{\delta L_{ij}}{\delta x^k} - \frac{\delta L_{ik}}{\delta x^j} + L^h_{jk}L^i_{hl} - L^h_{jl}L^i_{hk},
\]

\[
R_{abkl} = \frac{\delta L_{ab}}{\delta x^k} - \frac{\delta L_{ak}}{\delta x^b} + L^c_{bk}L^a_{cl} - L^c_{bl}L^a_{ck},
\]

\[
P_{jikl} = \frac{\partial L_{ijk}}{\partial y^c} - C_{jck}^i,
\]

\[
P_{jikc} = \frac{\partial L_{abk}}{\partial y^c} - C_{abc}^i,
\]

\[
S_{jibc} = \frac{\partial C_{ijb}}{\partial y^c} - \frac{\partial C_{ijc}}{\partial y^b} + C_{hjb}^iC_{ihc}^k - C_{hjc}^iC_{chb}^k,
\]

\[
S_{bacd} = \frac{\partial C_{bda}}{\partial y^c} - \frac{\partial C_{abd}}{\partial y^c} + C_{fbc}^aC_{afd}^c - C_{fbd}^aC_{afc}^c.
\]

The number of curvature components are just the same as in the local anisotropic case that permits us to make a conclusion that reduction will give a quantitative appropriate result to the original locally anisotropic case.

The reduced \(d\)-connection in components is

\[
\Gamma_{\alpha \beta \gamma}^s = (L_{ijk}, L^a_{bi}, C^i_{jc}, C^a_{bc}).
\]

The reduced components of Ricci tensor in adapted bases take form

\[
R_{ij} = R^k_{i \, jk}, R_{ia} = -P^k_{i \, ka} = -\frac{2}{\alpha'} P_{ia}, R_{ai} = P^b_{a \, ib} = \frac{1}{\alpha'} P_{ai}, R_{ab} = S_{a \, bc} = S_{ab}.
\]

6 Effective Action Order Extension and Locally Anisotropic Counterterms

The effective action of a locally anisotropic \(\sigma\)-model of Boze string is given by

\[
I = \frac{1}{4\pi\alpha'} \int d^2z (\sqrt{\eta^{AB}} \delta^A u^\alpha \delta^B u^\beta G_{\alpha\beta}(u) + i\epsilon^{AB} \delta^A u^\alpha \delta_B u^\beta H_{\alpha\beta}(u) + \alpha' \sqrt{\eta_{<\sigma>}} R_{(2)}^2 \phi(u))
\]

where the second term is a nonsymmetric one \(H_{\alpha\beta} = -H_{\beta\alpha}\), It’s been used the following notations herein:
peculiarities of the locally anisotropic space gives \[25\].

In what follows one explicitly calculates exclusively the contributions of the contributions of WZW term solely given the

\[D_A \xi^\alpha = \delta A \xi^\alpha + \langle i_\ge \Gamma^{\alpha}_{\beta\gamma} \delta A u^\beta \xi^\beta, \quad (37)\]

\[<i_\ge \Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} G^{\alpha\beta} (\delta_\gamma G_{\beta\delta} + \delta_\delta G_{\gamma\beta} - \delta_\beta G_{\gamma\delta}),\]

\[T_{\alpha\beta\gamma} = \frac{1}{4} (D_\gamma H_{\alpha\beta} + D_\beta H_{\gamma\alpha} + D_\alpha H_{\beta\gamma}),\]

\[D_\gamma H_{\alpha\beta} = \delta_\gamma H_{\alpha\beta} - H_{\alpha\epsilon} \langle i_\ge \Gamma^{\alpha}_{\beta\epsilon} - H_{\epsilon\beta} \langle i_\ge \Gamma^{\alpha}_{\epsilon\gamma},\]

\[\delta_r = \frac{\delta}{\delta u^\alpha}.\]

A standard order extension with respect to \(\xi\) and considering also the peculiarities of the locally anisotropic space gives \[25\].

\[I[u] = I[\pi] + \frac{1}{4\pi\alpha'} \left\{ \left[ \frac{1}{2} G_{\alpha\beta}(u) \delta A u^\alpha D^A u^\beta + G_{\alpha\beta}(u) D_A \xi^\alpha D^A \xi^\beta \right. \right.\]

\[+ \langle i_\ge \rangle R_{\alpha\beta\gamma\delta} \delta A u^\beta \delta A u^\delta \xi^\gamma \xi^\delta + \frac{1}{3} \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon} \delta A u^\beta \delta B u^\epsilon \xi^\gamma \xi^\delta \]

\[+ \frac{4}{3} \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon} \delta A u^\epsilon \delta B \xi^\alpha \xi^\beta \xi^\gamma \]

\[+ \frac{1}{12} (D_\alpha D_\beta \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon} + 4 \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon}) \delta A u^\alpha \delta B u^\delta \xi^\gamma \xi^\delta \]

\[+ \frac{1}{2} D_\beta \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon} \delta B u^\delta \xi^\alpha \xi^\beta \xi^\gamma \]

\[+ \frac{1}{3} \langle i_\ge \rangle R_{\alpha\beta\gamma\delta} \delta A \xi^\alpha \xi^\beta \xi^\gamma \xi^\delta \|

\[+ i \varepsilon^{AB} \left( \frac{1}{2} D_\beta T_{\alpha\gamma\delta} \delta A u^\alpha \delta A u^\delta \xi^\gamma \xi^\beta + T_{\alpha\beta\gamma} \delta A u^\alpha D^A \xi^\beta \xi^\gamma \right)\]

\[+ \frac{1}{6} (D_A D^A T_{\alpha\delta} + 2 T_{\alpha\delta} \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon}) \delta A u^\alpha \delta A \xi^\beta \xi^\gamma \xi^\delta \]

\[+ \frac{1}{2} D_A T_{\alpha\beta\gamma} \delta A u^\beta D_B \xi^\gamma \xi^\delta \xi^\alpha \xi^\epsilon + \frac{1}{3} T_{\alpha\beta\gamma} D_A \xi^\alpha D^B \xi^\beta \xi^\gamma \]

\[+ \frac{1}{24} (D_\alpha D_\beta D_\gamma T_{\beta\alpha\epsilon} + 6 D_\delta T_{\alpha\theta} \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon} + 2 T_{\alpha\theta} \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon})\]

\[\times \delta A u^\beta \delta B u^\epsilon \xi^\alpha \xi^\gamma \xi^\delta \xi^\epsilon + \frac{1}{4} \langle D_\beta D_\gamma T_{\beta\alpha\epsilon} + T_{\alpha\beta\epsilon} \rangle R_{\beta\gamma\delta\epsilon}\]

\[- \frac{1}{3} T_{\beta\alpha} \langle i_\ge \rangle R_{\beta\gamma\delta\epsilon} \times \delta A u^\beta D_B \xi^\alpha \xi^\gamma \xi^\delta \xi^\epsilon \|

\[+ \frac{1}{4} \langle D_A T_{\beta\gamma\delta} D_A \xi^\alpha D^A \xi^\beta \xi^\gamma \xi^\delta \rangle + O(\xi^5).\]

In what follows one explicitly calculates exclusively the contributions given by the richer aspects of the locally anisotropic space time only. Also, we omit the contributions of the contributions of WZW term solely given the
intention to avoid overwhelming of the calculations. The obtained results will be added to the standard one cited here from \[24\].

So, the additional action $I_{la}[u]$ to be added to the standard action $I[u]$ comes explicitly from the properties of the locally anisotropic strings. The general structure of the locally anisotropic effective action of non-linear $\sigma$-model is:

$$I_{la}^G[u] = I_{la}[u] + I_{la}^h(x) + I_{la}^{g-h}(x, y);$$

where $I_{la}^h(x)$ string effective action on base coinciding to standard Riemannian one. The term $I_{la}^{g-h}(x, y)$ has influence of both fiber and base. The contributions of $I_{la}^{g-h}(x, y)$ projected on base, so that to the standard model, add nonlinear anisotropic corrections we pursue to calculate. The contribution of $I_{la}^{g-h}(x, y)$ can be virtually interpreted\(^2\) as the effective action of interaction between effective action of, say base residing string $I_{la}^g(x)$ and similarly fiber string $I_{la}^h(y)$.

$$I_{la}^{g-h}(x, y) = (4\pi\alpha')^{-1} \{ D_A \theta^i D^A \lambda^a + R_{abkl} \delta_{A}^a \delta_{A}^a \lambda^b \delta^l \lambda^k + P_{jikc} \delta_{A}^a \delta_{A}^c \lambda^b \delta^l \lambda^k + P_{abkc} \delta_{A}^a \delta_{A}^c \lambda^b \delta^l \lambda^k + S_{jikc} \delta_{A}^a \delta_{A}^c \lambda^b \delta^l \lambda^k + \frac{1}{3} D_{i} R_{abkl} \delta_{A}^a \delta_{A}^a \lambda^b \delta^l \lambda^k \}
\[3\]

$$+ \frac{1}{3} D_{j} P_{jikl} \delta_{A}^a \delta_{A}^j \delta_{B}^l \delta_{B}^k \delta^l \lambda^d \lambda^k
+ \frac{1}{3} D_{k} P_{abkl} \delta_{A}^a \delta_{A}^b \delta_{B}^l \delta_{B}^k \delta^l \lambda^d \lambda^k
+ \frac{1}{3} D_{l} S_{jikl} \delta_{A}^a \delta_{A}^j \delta_{B}^l \delta_{B}^k \delta^l \lambda^d \lambda^k
$$

$\[4\]

$$+ \frac{4}{3} D_{i} R_{abkl} \delta_{A}^a \delta_{A}^a \lambda^b \delta^l \lambda^k
+ \frac{4}{3} P_{jikc} \delta_{A}^a \delta_{A}^c \lambda^b \delta^l \lambda^k
+ \frac{4}{3} P_{abkc} \delta_{A}^a \delta_{A}^c \lambda^b \delta^l \lambda^k
+ \frac{4}{3} S_{jikc} \delta_{A}^a \delta_{A}^c \lambda^b \delta^l \lambda^k
+ \frac{1}{12} (D_{i} D_{j} R_{abkl} + 4 R_{abkl}^{c} R_{cijb}) \delta_{A}^a \delta_{A}^b \delta_{A}^c \lambda^a \theta^b \theta^c \theta^l \lambda^k \}
\[5\]

\(2\)The nature of the interaction, to be interpreted from the structure of the additional terms, will be considered as we go along with presentation.
\[ + \frac{1}{12} (D_c D_d R_{ijkl} + 4 R_{ijkl} S_{cdef}) \delta A^b \delta B^c \lambda^a \theta^d \lambda^e \]
\[ + \frac{1}{12} (D_m D_n P_{ijkl} + 4 P_{ijkl} R_{ijkl}) \delta A^b \delta B^c \theta^d \theta^e \theta^f \theta^g \]
\[ + \frac{1}{12} (D_c D_d P_{ijkl} + 4 P_{ijkl} R_{ijkl}) \delta A^b \delta B^c \lambda^d \lambda^e \lambda^f \]
\[ + \frac{1}{12} (D_c D_d P_{ijkl} + 4 P_{ijkl} R_{ijkl}) \delta A^b \delta B^c \lambda^d \lambda^e \]
\[ + \frac{1}{12} (D_c D_d P_{ijkl} + 4 P_{ijkl} R_{ijkl}) \delta A^b \delta B^c \lambda^d \lambda^e \]
\[ + \frac{1}{12} (D_k D_l S_{jibe} + 4 S_{ijbe} R_{ijkl}) \delta A^b \delta B^c \lambda^d \lambda^e \lambda^f \]
\[ + \frac{1}{12} (D_k D_l S_{jibe} + 4 S_{ijbe} R_{ijkl}) \delta A^b \delta B^c \lambda^d \lambda^e \lambda^f \]
\[ + \frac{1}{2} D_t R_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{2} D_R R_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{2} D_t R_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{2} D_R R_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{2} D_t \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{2} D_R \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{3} R_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{3} P_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{3} P_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]
\[ + \frac{1}{3} P_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \]

where \( I_{la}^b (y) \) the fiber effective action gets quite already familiar expression

\[ I_{la}^b (y) = \frac{1}{4 \pi \alpha^e} \{ D_A \lambda^a D^A \lambda^b + h_{ab} D_A \lambda^a D^A \lambda^b + S_{abcd} \delta A^b \delta A^c \delta B^d \theta^e \theta^f + \frac{1}{3} D_{ijkl} \delta B^c \lambda^d \lambda^e \lambda^f \lambda^g \lambda^h \]
\[ + \frac{1}{12} (D_e D_u S_{baced} + 4 S_{acbe} s_{abeu}) \delta A^b \delta B^c \lambda^d \lambda^e \lambda^u \]
\[ + \frac{1}{2} D_e S_{baced} \delta B^c \lambda^d \lambda^e \lambda^a + \frac{1}{3} S_{baced} \delta A^b \lambda^a D^A \lambda^a \lambda^c \} \]
The contribution and effects of $I_{\text{la}}^{g-h}(y)$ are absolutely symmetric to $I_{\text{la}}^{b}(x)$ and to Riemannian string. Therefore, we intentionally omit consideration in these respect also for the diagrams and counterterms resulted from $I_{\text{la}}^{h}(y)$ do not interfere with those of base component as well as, it has been intended to see the effects upon the base evolutions.

Here one can envisage at least two possible ways to count the effects of $I_{\text{la}}^{g-h}(x, y)$. First is to project the additional terms on base, so that they lost vehemently the fiber aspect whatsoever. Second, is to interpret the fiber-base nature of the corrections as they have significance from the point of view of fiber-base interactions. There are both conveniently approached, whereas the first one is of primarily interest in the framework of exclusive calculation of locally anisotropic corrections, while the second one appealed to justifying the physically charged nature of the additional terms.

The explicit expression for fiber-base locally anisotropic effective action $I_{\text{la}}^{g-h}(x, y)$ is given by:

\[
I_{\text{la}}^{g-h}(x, y) = [P_{ijkc}\delta A x^j \delta A y^c + \frac{1}{3} D_t R_{abkl} \delta A y^a \delta B x^l \lambda^b + \frac{1}{3} D_d P_{jkl} \delta A x^j \delta B x^l \lambda^d + \frac{1}{3} D_t P_{bakc} \delta A y^b \delta B y^c \lambda^a + \frac{1}{3} D_k S_{jibc} \delta A x^j \delta B y^c \lambda^b + \frac{4}{3} P_{jikc} \delta A x^j D_B \lambda^c + \frac{1}{12}(D_t D_d P_{iklc} + 4 P_{ijkR}R_{fde}) \delta A y^b \delta B x^l \lambda^c \lambda^d] \times \theta^i \theta^k + \frac{4}{3} R_{bakd} \delta A y^b \lambda^a + \frac{1}{2} D_d R_{bakc} \delta B y^c \lambda^a + \frac{1}{2} D_t P_{bakd} \delta B y^b \lambda^d + \frac{1}{3} S_{ikbc} D^A \lambda^c \lambda^b + \frac{1}{2} D_t S_{jibc} \delta B x^j \lambda^b \lambda^c + \frac{1}{3} R_{bacl} D_A \lambda^b \lambda^a] \times D^A \theta^i \theta^k + \frac{1}{3} P_{jikd} \lambda^d D_A \theta^i D^A \theta^k
\]

\[
+ \frac{1}{3} D_t P_{jikl} \delta A x^j \delta B x^l + \frac{1}{12}(D_t D_t R_{bacl} + 4 R_{akl} R_{dlic}) \delta A y^b \delta B x^l \lambda^a + \frac{1}{12}(D_t D_t P_{abkc} + 4 P_{bka} R_{dlic}) \delta A y^a \delta B y^c \lambda^b + \frac{1}{12}(D_k D_t S_{jibc} + 4 S_{ijkc} R_{dlic}) \delta A x^j \delta B y^c \lambda^b \times \theta^i \theta^k \theta^j
\]

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the standard procedure for doing calculations of divergencies \[8\] implies the following notations:

\[ A_{\gamma\zeta} \times (\xi\xi\xi) = -\frac{1}{6} (\eta^{AB} + \varepsilon^{AB}) (D_{\xi<1>}R^{\alpha\gamma\beta\varsigma} - 2<_{1>}R^{\alpha\gamma\beta\lambda}T_{\beta\varsigma\lambda}) D_A u^\alpha \partial_B u^\beta \times (\xi^\gamma \xi^\zeta), \]

\[ B_{\gamma\varsigma}^{AB} \times (D\xi\xi\xi) = -\frac{2}{3} (\eta^{AB} \frac{<_{1>}R_{\gamma(\alpha\beta)}}{<_{1>}R_{\gamma(\alpha\beta)\varsigma}} + \varepsilon^{AB} <_{1>}R_{\gamma(\alpha\beta)\varsigma}) \partial_A u^\alpha \times (D\xi^\beta \xi^\varsigma), \]

\[ C_{\alpha\gamma}^{AB} \times (D\xi D\xi\xi) = -\frac{1}{3} \varepsilon^{AB} T_{\alpha\beta\gamma} \times (\xi^\alpha D_A \xi^\beta D_B \xi^\gamma), \]

\[ L_{AB}^{\alpha\beta\gamma\delta} \times (D\xi D\xi\xi\xi) = -\frac{1}{4} \left[ -\eta^{AB} (\frac{<_{1>}R_{\alpha\beta}}{<_{1>}R_{\alpha\beta\gamma}} + T_{\alpha\beta} T_{\gamma\lambda}) + \varepsilon^{AB} \frac{<_{1>}R_{\alpha\beta\gamma}}{<_{1>}R_{\alpha\beta\gamma\delta}} \right] \times (D\xi^\alpha D\xi^\beta D\xi^\gamma \xi^\delta), \]

\[ H^{\delta\epsilon\lambda} \times (\xi\xi\xi\xi) = -\frac{1}{24} (\eta^{AB} + \varepsilon^{AB}) D_{\epsilon} D_{\gamma} \frac{<_{1>}R_{\alpha\gamma\delta\epsilon\lambda}}{<_{1>}R_{\alpha\gamma\delta\epsilon\lambda}} + \frac{3<_{1>}R_{\alpha\gamma\delta<1>}R_{\epsilon\lambda}}{<_{1>}R_{\alpha\gamma\delta<1>}R_{\epsilon\lambda}} + 4D_{\delta} \frac{<_{1>}R_{\alpha\gamma\delta\epsilon\lambda}}{<_{1>}R_{\alpha\gamma\delta\epsilon\lambda}} T_{\beta\lambda} + 4<_{1>}R_{\alpha\gamma\delta<1>}T_{\epsilon\lambda} T_{\beta\lambda} \delta_A u^\alpha \delta_B u^\beta \times (\xi^\delta \xi^\epsilon \xi^\lambda), \]

\[ X^{\gamma\delta} \times (\xi\xi) = -\frac{1}{2} (\eta^{AB} + \varepsilon^{AB}) \frac{<_{1>}R_{\alpha\beta\gamma\delta}}{<_{1>}R_{\alpha\beta\gamma\delta}} \delta_A u^\alpha \delta_B u^\beta \times (\xi^\gamma \xi^\delta). \]

The counterterms coming out of the one loop diagrams are proportional to

\[ X_{\alpha\beta} = \sum_{\gamma\delta} \frac{1}{2} \left( <_{1>}R_{\alpha\beta\gamma\delta} \delta_A u^\alpha \delta_B u^\beta \xi^\gamma \xi^\delta - \frac{1}{2} \varepsilon^{AB} D_{\delta} T_{\alpha\beta\gamma} \delta_A u^\alpha \delta_B u^\beta \xi^\delta \xi^\gamma \right) + \frac{1}{2} \frac{m^2_{<1>} R_{\alpha\beta} u^\alpha u^\beta}{m^2_{<1>} R_{\alpha\beta} u^\alpha u^\beta} \] (43)

where there has been considered the contributions of the relations from base \[24\] and of corrections fiber-base projected on base \[12\].

\[ I^{\text{count,1}}_{(a)} = \Gamma^{(1)}_{\infty,1} = (4\pi\epsilon)^{-1} \int d^4 z (\delta^4 - \frac{1}{2} \frac{m^2_{<1>} R_{\alpha\beta} u^\alpha u^\beta}{m^2_{<1>} R_{\alpha\beta} u^\alpha u^\beta}) \]

\[ + \varepsilon^{AB} D_{\delta} T_{\alpha\beta\gamma} \delta_A u^\alpha \delta_B u^\beta \left( \frac{1}{2} \frac{m^2_{<1>} R_{\alpha\beta} u^\alpha u^\beta}{m^2_{<1>} R_{\alpha\beta} u^\alpha u^\beta} \right) \] (44)
In the explicit form the results projected on base (42) are given:

\[
I^{(la)}_{\text{count},1} = (4\pi\epsilon)^{-1} \int d^n z (\delta^A x^i \delta_A x^j (R_{ij} - 1/4 T_{ij}^4)) \\
+ V_{jB} \delta_A x^j + K_{ij} \delta_A x^i \delta_B x^j + W_{iA} \delta_B x^i \\
+ \epsilon^{AB} \delta_A x^i \delta_B x^j (-\frac{1}{2} D_i D_j T_{ij}) + \frac{1}{3} m^2 R_{ij} x^i x^j \tag{45}
\]

Expression (45) gives counterterms with one loop contributions resulted from (39), where one introduced the notations

\[
F_{lA} = 1/3 D_k R_{kabl} \delta_A y^a \lambda^b, M_{jB} = 1/3 D_k S_{kjbc} \delta_B y^c \lambda^b, K_{jl} = 4/3 D_d P_{jl} \lambda^d \tag{46}
\]

\[
C_{lA} = \frac{1}{12} (D_c D_d P_{lc} + 4 P_f R_{fdeb}) \delta_A y^b \lambda^c \lambda^d, O_{jB} = P_{jc} D_B \lambda^c, U_{jA} = P_{jc} \delta_A y^c, W_{jA} = U_{jA} + C_{jA}, V_{jB} = M_{jB} + O_{jB}.
\]

Expression (45) differs from the standard effective action by the component

\[
p I^{(g-h)}_{\text{count},1} = (4\pi\epsilon)^{-1} \int d^n z (V_{jB} \delta_A x^j + K_{ij} \delta_A x^i \delta_B x^j + W_{iA} \delta_B x^i) \tag{47}
\]

where fields \( V_{j}^{AB} = \epsilon^{AB} V_{jB} \delta_A x^j \), \( W_{i}^{AB} = W_{iA} \delta_B x^i \) are transformed as gauge like fields, so that the only possible invariant of which has four dimension and cannot contribute to UV divergencies of two dimensional space. Therefore, the only additional UV divergent contribution remains

\[
p I^{(g-h)}_{\text{count},1} = (4\pi\epsilon)^{-1} \int d^n z (K_{ij} \delta_A x^i \delta_B x^j) \tag{48}
\]

The last produces a set of new diagrams, presented below. The structure of \( p I^{(g-h)}_{\text{count},1} \) has not been essentially different from the rest of the standard counterterms that thus allows to presuppose, within the first mentioned above
approach, simple locally anisotropic correction.

The contributions of some additional locally anisotropic one loop diagrams are calculated making use the standard procedure [1]:

\begin{align*}
(1a) &= -\frac{1}{4\varepsilon} K_{ij}, \\
(1b) &= \frac{1}{4\varepsilon^2} (1 + \varepsilon) K_{il} D_k H^{kl}_j, \\
(1c) &= -\frac{1}{24\varepsilon} R^{mk} K_{ml} K^{kl}_{ij}, \\
(1d) &= -\frac{1}{4\varepsilon^2} K_{il} R^l_{ij}, \ldots
\end{align*}

(49)

If not projected on base the diagrams resulted from (48), will bear tensor sings of fiber. Also the one loop base-fiber diagrams will not be transformed in two loop diagrams, one of which will be base and another one fiber loop.

\footnote{See also relevant discussions in S.Vacaru, Locally Anisotropic Interactions: I. Non-linear Connections in Higher Order Anisotropic Superspaces, E-print: \texttt{hep-th/9607194}.
Locally Anisotropic Interactions: II. Torsions and Curvatures of Higher Order Anisotropic Superspaces, E-print: \texttt{hep-th/9607194}.
Locally Anisotropic Interactions: III. Higher Order Anisotropic Supergravity, E-print: \texttt{hep-th/9607196}.}
This is explicable by the fact that the fiber tensor order is low enough to prevent order expansion in those one loop counterterms that contribute to base standard counterterms. The fiber terms allowing fiber order expansion and forming up fiber one loop do not result in base loops.

At the same time the tensor structure of the terms $V_{jB} \delta_A x^j$, $W_iA \delta_B x^i$ suggests their gauge like character. Their role is not interpreted and clear yet.

the action the two loop diagrams to be read from is

$$I_{\text{count}, 2} = I_{la}^g(x) + pI_{la}^{g-h}(x, y) + pI_{\text{count}, 2}^{(g-h)} + 1/3 m^2 R_{ij} \theta^i \theta^j + H − \text{dependent part}$$  \hspace{1cm} (50)

where

$$pI_{\text{count}, 2}^{(g-h)} = +(4\pi \epsilon)^{-1} \int d^4 z (R_{ij} - 1/4 T_{ij}^2 + K_{ij}) D_A \theta^i D^A \theta^j$$

$$+ [1/2 D_i D_j (R_{ik} - 1/4 T_{ik}^2 + K_{ik})] \times \delta_A x^i \delta_B x^j \theta^i \theta^j$$

$$+ 2 D_j (R_{ij} - 1/4 T_{ij}^2 + K_{ij}) \delta_A x^j D_B \theta^j$$

$$+ D_j V_{Bi} D_A \theta^j + D_j W_{Ai} D_B \theta^j$$

$$+ [1/3 R_{ijkl} (V_{Bl} \delta_A x^k + W_{Al} \delta_B x^k)] \times \theta^i \theta^j$$  \hspace{1cm} (51)

Below we bring only some of the two loop diagrams resulted from (50),

$$I_{\text{count}, 2} = I_{la}^g(x) + pI_{la}^{g-h}(x, y) + pI_{\text{count}, 2}^{(g-h)} + 1/3 m^2 R_{ij} \theta^i \theta^j + H − \text{dependent part}$$  \hspace{1cm} (50)

where

$$pI_{\text{count}, 2}^{(g-h)} = +(4\pi \epsilon)^{-1} \int d^4 z (R_{ij} - 1/4 T_{ij}^2 + K_{ij}) D_A \theta^i D^A \theta^j$$

$$+ [1/2 D_i D_j (R_{ik} - 1/4 T_{ik}^2 + K_{ik})] \times \delta_A x^i \delta_B x^j \theta^i \theta^j$$

$$+ 2 D_j (R_{ij} - 1/4 T_{ij}^2 + K_{ij}) \delta_A x^j D_B \theta^j$$

$$+ D_j V_{Bi} D_A \theta^j + D_j W_{Ai} D_B \theta^j$$

$$+ [1/3 R_{ijkl} (V_{Bl} \delta_A x^k + W_{Al} \delta_B x^k)] \times \theta^i \theta^j$$  \hspace{1cm} (51)

Below we bring only some of the two loop diagrams resulted from (50),
which are being projected to base again:

\[ (2a) = \frac{1}{16\pi m^2\varepsilon} K^{ij}_{klm} K^{k\ell m}_i, \]
\[ (2b) = -\frac{9}{16} \left( \frac{1}{4\pi^2\varepsilon^2} + \frac{1}{2\pi\varepsilon} \right) K^{ijkl} T_{km} T_{lm}, \]
\[ (2c) = \frac{1}{16\pi m^2\varepsilon} K^{iklm}_{klm} K_{ij} \] \hspace{1cm} (55)

The calculations of these two loop locally anisotropic diagrams gives the result:

Adding up easily the contributions of one and two loop locally anisotropic counterterms of two loop locally anisotropic \( \beta^{(g-h)}_{(2)ij} \) function, one gets the complete two loop locally anisotropic \( \beta^{(G)la}_{(2)\alpha\beta} \) function including all terms of base fiber-base:

\[ \beta^{(G)la}_{(2)\alpha\beta} = \beta^{(g)}_{(2)ij} + \beta^{(g-h)}_{(2)ij} \] \hspace{1cm} (56)

where \( \beta^{(g-h)}_{(2)ij} \) accounts for the contributions of the diagrams (49) si (53) and other possible diagrams to be resulted from locally anisotropic dependence nature of the locally anisotropic string model. The result (56) taken the already well known, see for instance [24], \( \beta^{(g)}_{(2)ij} \) functions render the explicit contribution projected on base of the effects from base-fiber, otherwise essentially anisotropic nature of the geometrical background of the model.
7 Conclusions and Interpretation of Results

It is clear that the locally anisotropic spaces, spaces provided with the structure of nonlinear connection allows conceptual and consistent generalization of the (super)strings models, as they are modelled on a explicitly anisotropic geometric background.

The above presented results show that the basic techniques developed in (super)strings theory after some accommodations to the features of locally anisotropic spaces are successfully applicable. The locally anisotropic nonlinear $\sigma$–model is apparently renormalizable, the locally anisotropic corrections having manifestly similar structure to those of standard nonlinear $\sigma$–model. The extension of the explicit locally anisotropic strings to locally anisotropic superstrings is far only a matter of technical aspects.

An interesting feature of the locally anisotropic strings comes from the role of the $V^i_{jB}\delta_A x^j, W_iA\delta_B x^i$ gauge like fields and their fiber symmetric counterparts. The explicit form of these fields are given by (46). These terms on one hand can contribute to two loop base diagrams and are gauge like fields in base one loop approximation. At the same time they are manifestly gauge like fields in fiber space. This twofold nature of the terms: a contributing term on base and simultaneously gauge like field in fiber may virtually inspire to physically interesting interpretations, as for instance can be string interaction $^4$.

In the number of above cited works the geometry of locally anisotropic spaces had been groundly considered physically natural and quite appropriate bases to modeling and investigating fundamental interactions. The explicitly presented example is considered by author to be one of the convincing argument for raising the validity and trustworthy of the locally anisotropic spaces in physicists’ perception.

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