Precision Calculation of Dark Radiation from Spinning Primordial Black Holes and Early Matter Dominated Eras

Barmak Shams
U. Utah

with: Alexandre Arbey, Jeremy Auffinger, Pearl Sandick, and Kuver Sinha

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Outline

- Formation and evaporation of spinning primordial black holes (PBHs) in an early matter dominated era (EMDE)

- Precision calculation of dark radiation (graviton) emitted by spinning PBHs

- Constraints on PBHs from BBN, CMB, and the projected sensitivity of CMB Stage 4
PBHs: motivated by many different scenarios

BHe can be fully characterized by their mass, spin, and charge

image credit: https://slideplayer.com/slide/7773485/
mass:

\[ M_i \sim \frac{4}{3} \pi H(t_i)^{-3} \rho(t_i) \]

mass distribution:

1) monochromatic, e.g., if the PBH production occurs at a precise time.

2) extended, e.g., if the power spectrum of primordial inhomogeneities embeds a wide peak around some spatial scale

spin (more involved):

Radiation-dominated: small angular momentum

Matter-dominated: large angular momentum

can also gain spin either through early accretion or hierarchical merger
Hawking evaporation of Schwarzschild BHs:

\[
T_{\text{BH}} = \frac{M_{\text{Pl}}^2}{8\pi M_{\text{BH}}}
\]

\[
\frac{d^2 u_i(E, t)}{dt dE} = \frac{g_i}{8\pi^2} \frac{E^3}{e^{E/T_{\text{BH}}} + 1}
\]

\[
M(t) = M_i \left(1 - \frac{(t - t_i)}{\tau}\right)^{1/3}, \quad \tau = \frac{10240\pi}{g_*(T_{\text{BH}})} \frac{M_i^3}{M_{\text{Pl}}^4}
\]

\[
N_i = \frac{120 \zeta(3)}{\pi^3} \frac{g_i}{g_*(T_{\text{BH}})} \frac{M_{\text{BH}}^2}{M_{\text{Pl}}^2}, \quad T_{\text{BH}} > m_i
\]

\[
N_i = \frac{15 \zeta(3)}{8\pi^5} \frac{g_i}{g_*(T_{\text{BH}})} \frac{M_{\text{Pl}}^2}{m_i^2}, \quad T_{\text{BH}} < m_i
\]
Hawking evaporation of Kerr BHs:

\[ T_K = \frac{1}{2\pi} \left( \frac{r_+ - M_{BH}}{r_+^2 + a^* M_{BH}^2} \right) \]

\[ a^* \equiv \frac{L}{M_{BH}^2} \quad \quad r_+ \equiv M_{BH} (1 + \sqrt{1 - a^*^2}) \]

\[ \frac{d^2 N_i}{dt \, dE} = \frac{1}{2\pi} \frac{\Gamma_{s_i}^{l,m}}{e^{E'/T_K} - (-1)^{2s_i}} \]

\[ E' \equiv E - m\Omega = E - ma^*/2r_+ \]

\[ \frac{dM_{BH}}{dt} = - \frac{f(M_{BH}, a^*)}{M_{BH}^2} \]

\[ \frac{dM_{BH}}{dt} = \frac{a^* [2f(M_{BH}, a^*) - g(M_{BH}, a^*)]}{M_{BH}^3} \]

\[ f(M_{BH}, a^*) \equiv M_{BH}^2 \int_0^{+\infty} \sum_i \sum_{dof} \frac{E}{2\pi} \frac{\Gamma_{s_i}^{l,m}(E, M_{BH}, a^*)}{e^{E'/T_K} - (-1)^{2s_i}} dE, \]

\[ g(M_{BH}, a^*) \equiv \frac{M_{BH}}{a^*} \int_0^{+\infty} \sum_i \sum_{dof} \frac{m}{2\pi} \frac{\Gamma_{s_i}^{l,m}(E, M_{BH}, a^*)}{e^{E'/T_K} - (-1)^{2s_i}} dE \]

\[ BHawk \quad \quad Arbey, Auffinger1905.04268 \]

Kerr BHs have enhanced emission of particles with higher spin.

Gravitons: impact the effective number of relativistic degrees of freedom \[ \Delta N_{\text{eff}} \]
assuming PBHs are abundant enough to initiate an EMDE and reheat the universe via Hawking evaporation:

\[
\Delta N_{\text{eff}} = \frac{\rho_{\text{DR}}(t_{\text{EQ}})}{\rho_{\text{R}}(t_{\text{EQ}})} \left[ N_\nu + \frac{8}{7} \left( \frac{11}{4} \right)^{4/3} \right] \\
\frac{\rho_{\text{DR}}(t_{\text{EQ}})}{\rho_{\text{R}}(t_{\text{EQ}})} = \frac{\rho_{\text{DR}}(t_{\text{RH}})}{\rho_{\text{R}}(t_{\text{RH}})} \left( \frac{g_*(T_{\text{RH}})}{g_*(T_{\text{EQ}})} \right) \left( \frac{g_*,S(T_{\text{EQ}})}{g_*,S(T_{\text{RH}})} \right)^{4/3} \\
N_\nu = 3.046
\]

Hooper, Krnjaic, McDermott 1905.01301

current experimental limits and projected sensitivities of future experiments: CMB(two are taken from the Planck)

CMB\(^1\) (TT+low E, conservative)
CMB\(^2\) (TT,TE,EE+low E, more stringent)

**BBN: AlterBBN**
the sensitivity of the future CMB Stage 4 (CMB-S4) experiment
precision calculation:

- extended spin distribution

\[ \int_{0}^{1} \frac{dn}{da^*} da^* = 1 \]

\[ \rho_{\text{DR}/\text{SM}}(t_{\text{RH}}) = \int_{0}^{1} da^* \frac{dn}{da^*} \int_{0}^{t_{\text{RH}}} dt \int_{0}^{+\infty} dE E \frac{d^2 N_{\text{DR}/\text{SM}}}{dt dE}(M, a^*) \]

- reheating temperature

1) RH time = time at which last PBH (with lowest spin evaporates)
2) RH time = the average PBH lifetime, weighted by the spin distribution

- degrees of freedom

\[ g_*(T), g_*, s(T) \]  

SuperIso Relic
benchmark spin distributions:

1) Early matter domination era (EMDE):

spin source:
- the first-order contribution: from deviation of the boundary of the volume from a sphere.
- the second-order contribution: density fluctuations in the comoving region.

The first-order effect usually dominates (when the initial deviation of the boundary of collapsing region from a sphere is large)

In both cases, spin distribution is a function of the mean variance of the density perturbations at horizon entry

\[ \sigma_H = \langle \delta_s(t_H)^2 \rangle^{1/2} \]

Harada, Yoo, Kohri, Nakao 1707.03595
2) distribution from inspirals:

Fishbach, Holz, Farr 1703.06869
PBH spins compares to the monochromatic approximation

\[ \langle a^* \rangle \simeq 0.7 \]

\[ a^* = 0.7 \]

\[ \Delta N_{\text{eff}} \]

\[ M_{\text{BH}} (\text{g}) \]

CMB – S4

EMDE: 2nd order
Early Matter domination:

**EMDE:**
1st order

**EMDE:**
2nd order

\[ f_{\text{EMDE}}(a) \]

\[
\begin{align*}
\Delta N_{\text{eff}} & \quad \text{BBN} \\
& \quad \text{CMB\textsuperscript{1}} \quad \text{CMB\textsuperscript{2}} \\
\end{align*}
\]

\[
\begin{align*}
\Delta N_{\text{eff}} & \quad \text{CMB\textsuperscript{1}} \quad \text{CMB\textsuperscript{2}} \\
& \quad \text{BBN} \\
\end{align*}
\]
near-extremal PBH spins (monochromatic spin distribution):

- Large PBH masses already excluded.

- CMB-S4 will probe all near-extremal monochromatic spin distributions.

**Low and high cut-off for CMB -S4:**

- The smallest monochromatic spin for which CMB-S4 will be sensitive to the entire mass range:
  \[ a_{\text{min, all}}^* = 0.81 \]

- The largest monochromatic spin for which CMB-S4 will not be sensitive to any part of the mass range:
  \[ a_{\text{max, no}}^* = 0.69 \]
Early matter domination era leading to a PBH-dominated era:
a gravitationally coupled modulus field
Conclusions:

- The first precision study of dark radiation emission by Kerr PBHs

- Precision study includes: extended spin distributions, careful prescription for reheating time, precise evaluation of degrees of freedom.

- Precision results for PBHs formed during an EMDE.

- Provided PBHs dominate the energy density of the Universe before evaporation, CMB-S4 will be sensitive to nearly all EMDE distributions and monochromatic spins greater than 0.69.