A Minimal Model of Gravitino Dark Matter

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Motivated by the absence of signals of new physics in both searches for new particles at LHC and for a Weakly Interacting Massive Particle (WIMP) dark matter candidate, we consider a scenario where supersymmetry is broken at a scale above the reheating temperature. The low energy particle content consists then only in Standard Model states and a gravitino. We investigate the possibility that the latter provides the main component of dark matter through the annihilation of thermalized Standard Model particles. We focus on the case where its production through scattering in the thermal plasma is well approximated by the non-linear supersymmetric effective Lagrangian of the associated goldstino and identify the parameter space allowed by the cosmological constraints, allowing the possibility of large reheating temperature compatible with leptogenesis scenarios, alleviating the so called "gravitino problem".

I. INTRODUCTION

Among the possible hidden symmetries of Nature, supersymmetry is of the most appealing. It endows a candidate for a fundamental theory of Nature with a better ultraviolet behavior. If it is realized at low energy, it allows to address the hierarchy problem of the electroweak symmetry breaking sector, provides a dark matter candidate when R-parity is conserved and allows unification of the gauge couplings. However, the benefits of this aesthetically attractive scenario need to be questioned in the light of increasing tensions with experimental data. On one hand, there is no sign for new physics in the searches at LHC implying strong constraints on the parameter spaces of the models. On the other hand, the negative results in direct and indirect searches for WIMPs are closing the window of parameters corresponding to a neutralino dark matter. This motivates to push further up the scale of supersymmetry breaking extending the energy range of validity of the Standard Model. We consider here this possibility within a peculiar cosmological scenario.

Weakly Interacting Massive Particle (WIMP)\textsuperscript{1} and Freeze-In Massive Particle (FIMP)\textsuperscript{2} are different theoretical frameworks that have been postulated for the production mechanisms of dark matter. Whereas WIMP dark matter are in equilibrium with other particles in the early universe, when the temperature drops below the dark matter mass, they freeze-out and then form the present relic abundance. In supersymmetric frameworks, the lightest neutralino with a mass around the weak scale is a natural WIMP candidate\textsuperscript{1}. The recent analysis on dark matter detections, from the indirect methods by its annihilation products (FERMI\textsuperscript{3}, HESS\textsuperscript{4}, AMS\textsuperscript{5}) or its direct detection processes through the measurements of nuclear recoils (LUX\textsuperscript{6}, PANDAX\textsuperscript{7}, or XENON100\textsuperscript{8}) still did not find any significative signals. However, the sensitivity reached by the different groups begins to exclude large parts of the parameter space predicted by simple WIMPy extensions of the Standard Model (Higgs-portal\textsuperscript{9}, Z-portal\textsuperscript{10}, even Z’ portal\textsuperscript{11}). At the supersymmetric level, the anithors of\textsuperscript{12} showed that the well tempered neutralino, which was the most robust supersymmetric candidates during the last years in now severely constrained by the last results of LUX\textsuperscript{6}. Wino-like neutralino is also severely constrained by the latest indirect detection searches released by the FERMI collaboration\textsuperscript{3}. A nice review of the status of dark matter SUSY searches in different supersymmetric scenario with neutralino dark matter can be found in\textsuperscript{13}. In any case, the recent prospects exposed by the LUX\textsuperscript{6} and FERMI\textsuperscript{15} collaboration showed that the WIMP paradigm should be excluded (or discovered), for dark matter masse below 10 TeV in the present generation of detectors.

The lack of experimental signals motivates investigations of other production mechanisms with weaker couplings. FIMP dark matter is an alternative using couplings between the dark matter and Standard Model particles suppressed by a much higher scale than the weak scale. Thus they are not in equilibrium with other particles in the early universe and never reach equilibrium among themselves, their yields keeping growing as temperature drops. One can distinguish three cases of FIMPs: (i) Decay products of some heavier particles in equilibrium; (ii) Final products of infrared (IR) $|M_{DM}|$ dominated processes; (iii) Final products of ultraviolet (UV) $|T_{RH}|$ dominated ones. Gravitinos produced by

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the decay of Next to Lightest Supersymmetric Particles (NLSP) \cite{16} belongs to the first class. Because the production is dominated at low temperatures, the final yields are largely independent of the reheating temperature. This scenario has a constraint from Big Bang nucleosynthesis (BBN) since the late decay of NLSP influences the nucleosynthesis mechanisms strongly constrained by the observed abundance of D and $^4$He in the Universe \cite{17}. The IR dominated productions usually correspond to renormalizable operators or $2 \rightarrow 1$ processes \cite{2}. These are most efficient when the temperature is near the FIMP mass. Thus the yields are not dependent on the reheating temperature. On the other hand, non-renormalizable operators usually lead to UV dominated production and the final results are highly dependent on the reheating temperature.

Gravitino is an universal prediction of local supersymmetry models. Its role in cosmology depends on its abundance and its lifetime. Even if in some non-minimal scenario it can be non-thermally produced at the end of inflation during preheating due to fine-tuned coupling to the inflaton, the amount expected is model dependent and can be small \cite{20} \cite{21}. We will however not consider this possibility in this work and will instead focus on the case of thermal production. The gravitinos are produced by scattering of Standard Model states in the thermal plasma after reheating or through the decay of the NLSP. Within the assumption that the reheating temperature is lower than the mass of all the supersymmetric particles, we are left with only the former possibility. However, the standard scenario of gravitino dark matter suffers from different cosmological difficulties which are referred to collectively as the "cosmological gravitino problems": i) the late decaying superpartners can strongly affect the Big Bang Nucleosynthesis \cite{17}, and ii) if thermalized, the relic gravitinos produced overclose the Universe for $m_{3/2} \gtrsim 1$ keV \cite{18}, making it difficult to be a warm dark matter candidate if one also takes in consideration large scale structure formations, Tremaine-Gunn bound or Lyman $\alpha$ constraints \cite{19}. We will show that in the high scale Supersymmetry framework we propose, where the gravitino is directly produced from the thermal bath scattering, these two issues do not hold anymore. As a consequence, our analysis leads naturally to a prediction of possible large reheating temperature $T_{RH}$, usually favored by inflationary or leptogenesis scenarios.

The paper is organized as follows: we settle the framework of our model in section II, insisting on the fundamental mass scales entering in the analysis, before building the effective Lagrangian and computing the cosmological observables in section III. We then conclude in section IV.

## II. SUPERSYMMETRY BREAKING AND THE REHEATING TEMPERATURE

We review in this section the different scales relevant for our analysis, and their origins: the SUSY breaking scale, the soft mass terms, the messenger scale and the gravitino mass.

### 1. The supersymmetry breaking parameters:

We denote by $F$ the order parameter for supersymmetry breaking, a generic combination of auxiliary $F$ or $D$ terms vacuum expectation values . It corresponds to a spontaneous breaking, thereof it implies the existence of a Goldstone fermion, the goldstino $G$. The super-Higgs mechanism at work leads to a mass for the gravitino which value at present time reads \cite{22}:

$$m_{3/2} = \frac{F}{\sqrt{3}M_{Pl}},$$

in which $M_{Pl}$ is the reduced Planck mass. The breaking is mediated to the visible sector through messengers lying at a scale $\Lambda_{mess}$. This leads to soft-terms of order $M_{SUSY}$:

$$M_{SUSY} = \frac{F}{\Lambda_{mess}}.$$  \hspace{1cm} (2)

We shall assume for simplicity that all the masses of sparticles, squarks, sleptons, gauginos and higgsinos as well as all the new scalars in the extended Higgs sector are at least of the order of the scale of supersymmetry breaking $M_{SUSY}$. These particles are thus decoupled at reheating time $T_{RH}$. Below $M_{SUSY}$, the particle content is the Standard Model (SM) (with possibly right-handed neutrinos) and the goldstino. How realistic is this assumption in explicit supersymmetry breaking models is a model-dependent question. In O’Rafartaigh models of supersymmetry breaking, the partner of the goldstino, the sgoldstino $\tilde{G}$, is usually massless at tree-level. Quantum corrections are however expected to fix its mass to be one-loop suppressed with respect to the supersymmetry breaking scale $m_G^2 \sim \frac{g^2}{16\pi^2} F$, that has to be above the reheating temperature for our model to be self-consistent. In string effective supergravities it is also often the case for the sgoldstino to be light, with mass of the order of the gravitino mass \cite{23}. This is also however a model-dependent statement; this can be avoided in models with a large Riemann curvature in the Kahler space \cite{24}. On the other hand, asking for $m_{3/2} \ll M_{SUSY}$ implies

$$\Lambda_{mess} \ll M_{Pl} \hspace{1cm} (3)$$

and in the energy range under consideration the Renormalisation Group Equations (RGEs) are
those of the SM. In particular for a Higgs boson of 126 GeV, it leads to a vanishing of the quartic coupling at scales of order $2 \times 10^{10}$ GeV to $3 \times 10^{11}$ GeV depending on the assumption on the degeneracy of superparticles soft masses, the exact value of the top mass and the strong interaction gauge coupling (see for instance \[26\]). We then considered

$$M_{SUSY} \lesssim \{10^{-10} - 10^{11}\} \text{GeV}. \quad (4)$$

Supersymmetry breaking scales above this value can be achieved by a modification of the RGEs through introduction of new light particles. We shall not discuss these cases in details here, the generalisation being straightforward.

2. The cosmological parameters:

The cosmological history described here starts after the Universe is reheated. Some assumptions are made for this epoch: (i) The reheating temperature $T_{RH}$ is small enough to not produce superpartners of the Standard Model particles, thereof $T_{RH} \lesssim M_{SUSY}$ (ii) in the reheating process gravitinos are scarcely produced. This second condition is a constraint of the nature of the inflaton, for instance in \[25\]. We then considered the top mass and the strong interaction gauge coupling at the end of inflation can be found for example in \[27\].

We consider that the dark matter gravitino interactions are well approximated by the helicity $\pm 1/2$ components. This is true in virtue of the equivalence theorem if the energy $E$ of the gravitinos is much bigger than their mass. The enhancement of the interactions of the (longitudinal component of the) light gravitino is a direct consequence of the interactions of the (longitudinal component) light gravitino is a direct consequence of the minimal couplings expected from the low energy theorem if the energy $E$ of the gravitinos is much bigger than their mass. The enhancement theorem if the energy $E$ of the gravitinos is much bigger than their mass.

$$\mathcal{L}_{2G} = \frac{i}{2F^2} (G^\mu \partial^\nu G - \partial^\mu G^\nu G) T_{\mu \nu}, \quad (8)$$

where $G$ is the gravitino field and $T_{\mu \nu}$ is the energy momentum tensor of the SM matter fields. The energy momentum tensor is given by:

$$T_{\mu \nu} = +\eta_{\mu \nu} \tilde{L} + \sum_f \left( -\frac{i}{4} D_{\mu} \bar{\psi}_f \gamma^\mu \gamma^5 \psi_f + \frac{i}{4} \bar{\psi}_f \gamma^\mu \gamma^5 D_{\mu} \psi_f \right) - D_{\mu} H D^\nu H^\dagger + \sum_{SM_{group}} \frac{1}{2} F^a_{\mu \nu} F^{a}_{\mu \nu} + (\mu \leftrightarrow \nu). \quad (9)$$

1 Reheating temperature below superpartner masses was proposed and investigated in particular in \[29\] and \[30\]. The novelty in our case is that we consider high-scale supersymmetry, so our reheating temperature is much higher compared to these references.

2 As we will see, our result will not depend drastically on this hypothesis.

III. GOLDSTINO DARK MATTER

A. Effective goldstino interactions

Under the assumption $m_{3/2} \ll E \sim T$ discussed above, the gravitino interactions with SM fields are dominated by the helicity $\pm 1/2$ components. Moreover, for $E \sim T \lesssim T_{RH} \lesssim M_{SUSY}$, these are described by a non-linear realization of supersymmetry in all the observable SM sector, since we will consider all superpartners to be heavy and therefore not accessible in the thermal bath after reheating. The leading order goldstino-matter interactions can be divided into two types of contributions: universal \[31\] and non-universal ones \[32\]. We will restrict our analysis to the former, which corresponds to the minimal couplings expected from the low energy theorem \[3\]. Their construction starts by defining a ”vierbein” \[34\].

$$e^a_m = \frac{\delta^a_m - i \frac{i}{2 F^2} D_{m} G^a G + \frac{i}{2 F^2} G^a \partial^m G}{\partial^m G}, \quad (6)$$

that under a supersymmetry transformation of parameter $\epsilon$ transform as a diffeomorphism in general relativity

$$\delta e^a_m = \partial^m \xi^a e^a_n + \xi^a \partial^m e^a_n, \quad (7)$$

where $\xi^a = \frac{1}{2} e^a_m (\epsilon^m \bar{G} - G^a \bar{\epsilon})$. The couplings to matter in this original geometrical prescription follows therefore the standard coupling to matter of a metric tensor built out from the vierbein $g_{mn} = \eta_{abc} e^a_m e^b_n$. The corresponding goldstino-matter effective operators are consequently of dimension eight and take the form:

$$L_{2G} = \frac{i}{2 F^2} (G^a \partial^\mu \partial^\nu G^a - \partial^\mu G^a \partial^\nu G^a)/T_{\mu \nu} \theta^a_{\mu \nu} \psi_{\bar{\psi}}.$$
The scalar potential and mass terms for scalar and fermions appear in the first term. After the contraction between $\eta_{\mu\nu}$ and $G^a\sigma^\mu \partial^\nu \tilde G$, the on-shell production of two goldstinos give a cross section proportional to $m_{3/2}^2$. As $m_{3/2}$ is much smaller than $T_{RH}$, these contributions can be neglected, as we will see later. Then the $2 \rightarrow 2$ scatterings for the goldstino production is dominated by the following operator:

$$i \frac{\eta_{\mu\nu}}{2 F^2} (G^a\sigma^\mu \partial^\nu \tilde G - \partial^\mu G^a \sigma^\nu \tilde G) \left( \partial_\mu H \partial_\nu H^\dagger + \partial_\mu H \partial_\nu H^\dagger \right),$$

$$i \frac{\eta_{\mu\nu}}{8 F^2} (G^a\sigma^\mu \partial^\nu \tilde G - \partial^\mu G^a \sigma^\nu \tilde G) \times$$

$$\left( \tilde \psi \sigma_\nu \partial_\mu \psi + \psi \sigma_\mu \partial_\nu \psi - \partial_\mu \psi \tilde \psi \sigma_\nu \psi - \partial_\nu \psi \tilde \psi \sigma_\mu \psi \right),$$

$$\sum_a \frac{i}{2 F^2} (G^a \partial_\mu \tilde G - \partial_\mu G^a \tilde G) F^{a\nu a} F_{\nu \xi}^a, \quad (10)$$

where $h$, $\psi$ and $F^{a\xi}_{\nu}$ stand for a complex scalar (Higgs doublet), gauge bosons and two-component fermions (quarks and leptons), respectively. Another way to describe the two goldstinos interactions to matter is to replace the superpartner soft mass terms by couplings between the goldstino superfield and the matter superfield multiplets. One can integrate out the heavy (superpartner) components and eliminate them as a function of the light degrees of freedom: the SM fields and goldstino. This leads to an effective low-energy theory where the incomplete multiplets are described in terms of constrained superfields. The kinetic terms of the sparticles will then lead to dimension-eight operators containing two goldstinos and two SM fields that generically differ from the ones computed from the low-energy theory couplings. For the gauge and the SM fermion sectors, the resulting cross sections only differ in the angular distribution and numerical constants, whereas the energy dependence is the same as for the low-energy theory couplings.

Since the masses of the superpartners are of order $M_{SUSY} < \sqrt{F}$, one can worry about effective operators generated after decoupling heavy superpartners, with larger coefficients. In particular, there can be dimension-eight operators proportional to $1/M_{SUSY}^2$ and $1/M_{SUSY} F$, that would be dominant over the universal couplings we use in our paper. This issue was investigated in the first reference in [35], where it was shown that starting from MSSM only dimension-eight R-parity violating couplings of this type are generated. The reason for this is the following: Integrating out heavy superpartners (without R-parity violation) leads to factors of $1/M_{SUSY}$ from the propagators (the square of them for scalar superpartners). The leading interactions of goldstino to matter, on the other hand, through the soft terms, are proportional to $M_{SUSY}/F$ (the square for the scalar superpartners). As a result, the factors of $M_{SUSY}$ cancel out leaving generically dimension-eight operators suppressed by $1/F^2$. The effect of the R-parity violating couplings on the gravitino production was investigated more recently in [30].

### B. Computation of the gravitino relic density

#### 1. The framework

Contrarily to the weakly interacting neutralino, the gravitino falls in the category of feebly interacting dark matter. Its interactions at high energies are governed by the helicity-1/2 component whose couplings are naturally suppressed by the supersymmetry breaking scale. In gravity mediated supersymmetry breaking the gravitino is often heavier than the supersymmetric spectrum that it generates. As a consequence, if the gravitino is not sufficiently heavy (ie below 30 TeV) it is a long-lived particle which usually decay around the BBN epoch. This gives rise to the famous "gravitino problem" [37, 38]. In that case, in order to minimize the observable effects, the gravitino density has to be small enough at the cost of an upper bound on the reheating temperature of the Universe (see eg [39]). On the other hand, if gravitino is the LSP, it can be a very good dark matter candidate, either as stable or metastable particle, with lifetime much longer than the age of the Universe.

The gravitino was in fact the first supersymmetric dark matter candidate ever proposed by Pagels and Primack [18] and Khlopov and Linde [41]. However, they showed that if thermalized, its mass is restricted to a window $m_{3/2} \lesssim 1$ keV which place it in the hot scenario, nowadays in strong tensions by large scale formation constraints [42] or Tremaine Gunn bound ($m_{3/2} \gtrsim 400$ eV) [19]. Then, the authors of [39] famously showed that the overabundance problem can be avoided if, instead of thermalizing, the gravitino is produced through scattering of gaugino with a reheating temperature below a critical value, depending on the gaugino spectrum. They obtained

$$\Omega_{3/2} h^2 \sim 0.3 \left( \frac{1 \text{ GeV}}{m_{3/2}} \right) \left( \frac{\text{RH}}{10^{10} \text{ GeV}} \right) \sum_i c_i \left( \frac{M_i}{100 \text{ GeV}} \right)^2, \quad (11)$$

where $c_i$ are coefficient of order one, and $M_i$ are the three gaugino masses. We clearly see from Eq. (11) that the density is settled by the reheating temperature. Lower
limits on $M_3$ obtained by the non-observation of gluino at LHC set (for a given gravitino mass) an upper limit on reheating temperature to avoid overclosure of the Universe. These constraints are usually in tension with baryogenesis mechanisms [27], even if some interesting scenario with low reheating temperature ($T_{RH} \lesssim 100$ TeV) can be found in [28].

Moreover, later on in [16] it was shown that another contribution, called ”gravitino freeze in” is playing an important role. It corresponds to the decay of the superpartners while they are still in thermal equilibrium. Indeed, for a sufficiently large supersymmetric spectrum (as it seems to be observed at the LHC) the short lifetime of squarks or sleptons induced this process. The only way to circumvent the overabundance, is to lower the reheating temperature below the supersymmetric spectrum to deal with the queue of the distribution. However, the origin of the gravitino is still the supersymmetric partners, through their decay. A nice summary can be found in [43]. Adding the BBN constraints give an upper bound on the gravitino mass of about 10 GeV [44–46].

All the scenario discussed above made the hypothesis of thermal production of gravitino, through supersymmetric partners in thermal equilibrium with the primordial plasma. And, this thermalisation hypothesis is the deep source of tension between the cosmological observables (density of dark matter, structure formation, BBN or leptogenesis) and the data. However, if for some reasons the supersymmetric breaking scale is above the reheating temperature while still keeping it at high scale, the SM superpartners will be too heavy to reach the thermal equilibrium. That solves naturally the preceding tension, but the issue of the gravitino production remains.

Note also that for $T_{RH} \lesssim M_{SUSY}$ thermal corrections to the effective potential may become small enough to lead to only negligible displacements of the scalars vacuum expectation values from their late time values. This kind of high scale SUSY scenario can be originated easily and naturally in string inspired constructions (see [17] and references therein for instance). A way to populate the Universe with gravitinos is through a direct freeze-in from the thermal bath itself. In this new scenario, the gravitinos are produced at a rate smaller than the one corresponding to the expansion of the Universe, therefore they do not have time to reach the thermal equilibrium. It ”freezes in” in the process to reach it as the strong suppression of the scattering cross sections by the scale $F^2$ in Eq.(10) prevents the gravitinos to be in thermal equilibrium with the Standard Model bath. That is this scenario we propose to confront with cosmological data.

2. Gravitino production through freeze in

From the interaction generated through the Lagrangian Eq.(10), one can compute the production rate $R = \langle \sigma v \rangle n_{eq}$ of the gravitino $\tilde{G}$, generated by the annihilation of the standard model bath of density $n_{eq}$. The detail of the computation is developed in the appendix Eq.(27), and we obtain

$$R = \sum_i n_{eq}^2 (\sigma v)_i \simeq 21.65 \times \frac{T_{RH}^{12}}{F^4}$$  \hspace{1cm} (12)

The Boltzmann equation for the gravitino density $n_{3/2}$ can be written

$$\frac{dY_{3/2}}{dx} = \frac{45}{g_\pi} \frac{1}{4\pi^2} \frac{1}{m_{3/2}^3} x^4 R,$$ \hspace{1cm} (13)

with $x = m_{3/2}/T$, $Y_{3/2} = n_{3/2}/s$, $s$ the density of entropy and $g_\pi$ is the effective number of degrees of freedom thermalized at the time of gravitino decoupling (106.75 for the Standard Model). Here, we use the Planck mass $M_P = 1.2 \times 10^{19}$ GeV. We then obtain after integration

$$Y_{3/2} = \frac{21.65 M_P T_{RH}^7}{28\pi^2 F^4} \left( \frac{45}{g_\pi} \right) \frac{3}{2} \simeq 3.85 \times 10^{-3} \frac{M_P T_{RH}^7}{F^4}$$ \hspace{1cm} (14)

The relic abundance

$$\Omega h^2 = \frac{\rho_{3/2}}{\rho_c} = \frac{Y_{3/2}}{s_0} \frac{m_{3/2}}{\rho_c} \simeq 5.84 \times 10^8 \frac{m_{3/2}}{1 \text{ GeV}} Y_{3/2}$$ \hspace{1cm} (15)

is then

$$\Omega_{3/2} h^2 \simeq 0.11 \left( \frac{100 \text{ GeV}}{m_{3/2}} \right)^3 \left( \frac{T_{RH}}{5.4 \times 10^7 \text{ GeV}} \right)^7$$ \hspace{1cm} (16)

As we notice, the dependence on the reheating temperature is completely different from the case where the gravitino is produced through the scattering of the gaugino in Eq.(11). A similar behavior can be observed in SO(10) framework [48] or in extended neutrino sectors [29]. All these models have in common that the production process appears at the beginning of the thermal history, and is then very mildly dependent on the hypothesis or the physics appearing after reheating. The reheating temperature is then a prediction of the model (for a given gravitino mass) once one applies the experimental constraints of WMAP [49] and PLANCK [50]. Another interesting point, is that a look at Eqs.(14) and (16) shows that even the dependance on the particle content is very mild. Indeed, due to the large power $T_{RH}^7$, the total number of degrees of freedom, or even channels does not influence that much the final reheating temperature, which is predicted to be around $10^8$ GeV for a gravitino with electroweak scale. Even the hypothesis of universal couplings [31] or non-universal ones [32–33] will not affect drastically our Eq.(16).
Our result is plotted in Fig. 1 where we represent the parameter space allowed by the relic abundance constraints $\Omega_{3/2} h^2 \simeq 0.12$ [49, 50]. As we notice, there exist a large part of the parameter space allowed by cosmology, giving reasonable values of $T_{RH} \sim 10^5 - 10^{10}$ GeV for a large range of gravitino masses MeV-PeV. The region below the orange (dashed) line is excluded as the gravitino would be too heavy to be produced by freeze-in mechanism, whereas the region above the green (dotted) line corresponds to a freeze out scenario. In the latter region, the production cross section $\langle \sigma v \rangle$ is sufficiently high to reach the thermal equilibrium. This occurs when $n(\sigma v) \gtrsim H(T_{RH}) \simeq T_{RH}^2 / M_{Pl}$. A quick look at Eq. (12) shows that such large cross section is obtained for high reheating temperature or small values of $F$ (and thus light gravitino), explaining the shape of the green region in Fig. 1. However, once the gravitino is in thermal equilibrium, its density is given by the classical Freeze Out (FO) mechanism

$$\Omega_{3/2}^{FO} = \frac{n_{3/2} m_{3/2}}{\rho_c^o} = \frac{m_{3/2}^2}{180 \text{ eV}} \Rightarrow 0.1 \left( \frac{m_{3/2}}{180 \text{ eV}} \right)$$

which corresponds obviously to the intersecting point in Fig. 1.

There exists potentially another non-thermal source of gravitino production: the decay of the NLSP. Indeed, this contribution also exist in standard supersymmetric framework, through the relic abundance produced by the decay of the NLSP (usually a sfermion $f$) into $f \rightarrow G \tilde{f}$. This process being proportional to $n_{3/2}^f$, is highly Boltzmann suppressed in our scenario where $T_{RH} \ll M_{NLSP}$. But there still exists some parameter space when NLSP are in equilibrium. Then the production of goldstinos is a combination of the decay of NLSP, QCD process and SM freeze in. An analysis in scenario with very low $T_{RH}$ ($\lesssim$ GeV) can be found in [50].

**C. Comments on the R-parity violation operators**

R-parity violation operators can also be introduced in the high-scale supersymmetry scenario discussed in this work. The corresponding operators involving goldstino fields include dimension-five operators as

$$\frac{\mu_i}{F} \epsilon_{ijk} d_j \epsilon G^{\mu} h_j + h.c.$$

dimension-six ones as [51]

$$\frac{iC^I}{F} \epsilon_{ij} (l_j^i \partial_{\mu} G) D^{\mu} h_j + h.c.,$$

and dimension-eight operators of the form

$$\frac{\lambda_{ijk}}{m_i^2 F} u_i d_j \epsilon (d_k G) + \frac{\lambda_{ijk}}{m_i^2 F} q_i l_j \epsilon (d_k G) + \frac{\lambda_{ijk}}{m_i^2 F} l_i l_j \epsilon (e_k G),$$

plus permutations. Here $\mu_i$ and $C$ are dimensionful and dimensionless coefficients respectively, $l_i$ are the three lepton doublets in the SM and $m_i^2$ are soft terms of the heavy superpartners that were integrated out. The $2 \rightarrow 1$ gravitino production through these operators will be suppressed at temperatures higher than the gravitino mass and only become important at late times, therefore do not need to be considered for the production of gravitino dark matter.

When R-parity is violated, the gravitinos are no more stable but can decay, giving rise to observable signatures. The latter are independent of the production mechanisms and the previous analysis in the literature apply to our case. The relevant operators can be derived from the above but should be written using the gravitino field. Since the heavy supersymmetric particles decouple in our case, the coefficients of the R-parity violating operators are not necessarily constrained from preserving baryon asymmetry as in previous studies [53].

However, one characteristic of our construction is that it allows for very heavy gravitino (above the PeV scale).
Smoking gun signals like tilde $G \rightarrow h\nu$ and $G \rightarrow \gamma\nu$ can be observable in telescope like Icecube for neutrino [54], or the future Cerenkov Telescope Array (CTA) for the photon [55]. In both case, a monochromatic high energy signal should be the signature of the gravitino decay, the spatial morphology distinguishing decaying dark matter (proportional to its density $\rho$) to annihilating one (proportional to $\rho^2$).

IV. CONCLUSION

We considered the framework of high scale supersymmetry, where the scale of superpartners $M_{SUSY}$ lies above the reheating temperature whereas the gravitino mass $m_{3/2}$ stays below. In this case, there still exist processes which produce thermally gravitinos through scattering of the Standard Model particles at the earliest time of reheating. Our result is well summarized by Fig.(1) and Eq.(16) where one can observe and understand the strong dependence of the relic abundance on the reheating temperature. Our result predicts large reheating temperature ($\sim 10^8$ GeV for a $\sim 100$ GeV gravitino). This scale pattern $m_{3/2} \ll T_{RH} \ll M_{SUSY}$ is common in some string models with high-scale supersymmetry breaking [47] and opens new possibilities in model building.

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APPENDIX

Computing the gravitino production rate $R$

We provide in this appendix the detail of the computation of the annihilation rate $n_{\chi\chi}^2 \langle \sigma v \rangle$. Indeed, after symmetrization and by switching to four-component fermionic notation, one can extract from Eqs.(10) the effective Lagrangian

$$\mathcal{L} = -\frac{i}{2F^2} \left( \partial_\mu \bar{G} \gamma^\mu \gamma^5 G - \bar{G} \gamma^\rho G \frac{1}{2} \frac{1}{2} \partial_\rho G \right)$$

with $G$ being the goldstino in a four-component Dirac fermion notation and $H$, $\Psi$ and $F_{\mu\nu}$ are the Higgs field, Standard Model fermions and gauge field strength respectively. It becomes then straightforward to compute the averaged production rate $R$ for the process $1 + 2 \rightarrow 3 + 4$ in the case of early decoupling, when all the particles $i$ in the thermal bath, of temperature $T$, are relativistic ($m_i \ll T \Rightarrow E_i = p_i$):

$$R_i = n_{\chi\chi}^2 \langle \sigma v \rangle_i = \int f_1 f_2 d\cos \beta \frac{E_1 E_2 dE_1 dE_2}{1024 \pi^6} \int |\mathcal{M}|^2 d\Omega$$

with $f_i = \frac{1}{e_{i\chi\chi}^{1/4}}$ for a fermionic (bosonic) distribution, $\beta$ is the angle between the colliding particles 1 and 2 of energies $E_1$ and $E_2$ respectively in the laboratory frame, and $\Omega$ is the solid angle between the incoming particle 1 and outgoing particle 3 in the center of mass frame [52]. From Eq.(21) one can easily deduce

5 See refs [52] and [2] for details.
\[ |\mathcal{M}_h|^2 = \frac{s^4}{16F_4^4} (\cos^2 \theta - \cos^4 \theta) \]  \hspace{1cm} (22)

\[ |\mathcal{M}_f|^2 = \frac{s^4}{256F_4^4} (1 + \cos^2 \theta)(1 - 2\cos^2 \theta)^2 \]  \hspace{1cm} (23)

\[ |\mathcal{M}_V|^2 = \frac{s^4}{128F_4^4} (2 - \cos^2 \theta - \cos^4 \theta) \]  \hspace{1cm} (24)

for the scalar, fermionic and vectorial contribution respectively. The integration on the phase space should be treated with care, noticing that the lorentz invariant \[ \sum_{i} n_{\mathcal{Q}}^2(\sigma v)_i = 4n_{\mathcal{Q}}^2(\sigma v)_h + 45n_{\mathcal{Q}}^2(\sigma v)_f + 12n_{\mathcal{Q}}^2(\sigma v)_V \]

implying

\[ R = \frac{6400 [\zeta(6)]^2}{\pi^5 F_4^4} T^{12} \approx 21.65 \times \frac{T^{12}}{F_4^4} \]  \hspace{1cm} (27)

\[ n_{\mathcal{Q}}^2(\sigma v)_h = \frac{48\zeta(6)^2}{\pi^5 F_4^4} T^{12} = \frac{48\pi^7}{(945)^2 F_4^4} T^{12} \]

\[ n_{\mathcal{Q}}^2(\sigma v)_f = \frac{72\zeta(6)^2}{\pi^5 F_4^4} \left(\frac{31}{32}\right)^2 T^{12} = \frac{72\pi^7}{(945)^2 F_4^4} \left(\frac{31}{32}\right)^2 T^{12} \]

\[ n_{\mathcal{Q}}^2(\sigma v)_V = \frac{264\zeta(6)^2}{\pi^5 F_4^4} T^{12} = \frac{264\pi^7}{(945)^2 F_4^4} T^{12} \]  \hspace{1cm} (26)

\[ R = \sum_{i} n_{\mathcal{Q}}^2(\sigma v)_i = 4n_{\mathcal{Q}}^2(\sigma v)_h + 45n_{\mathcal{Q}}^2(\sigma v)_f + 12n_{\mathcal{Q}}^2(\sigma v)_V \]  \hspace{1cm} (25)

The integration on the phase space should be treated with care, noticing that the lorentz invariant \[ s = (P_1 + P_2)^2 = 2P_1P_2 = 2E_1E_2(1 - \cos \beta) \] in the laboratory frame and \[ \int \frac{d^3p}{\pi^3} = n(\zeta)(n + 1). \]
