Effects of the Electromagnetic Field on Five-dimensional Gravitational Collapse

M. Sharif * and G. Abbas †
Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore-54590, Pakistan.

This paper investigates the five-dimensional (5D) spherically symmetric gravitational collapse with positive cosmological constant in the presence of an electromagnetic field. The junction conditions between the 5D non-static interior and the static exterior spacetimes are derived using the Israel criteria modified by Santos. We use the energy conditions to discuss solution to the field equations of the interior spacetime with a charged perfect fluid for the marginally bound and the non-marginally bound cases. We found that the range of apparent horizon was larger than that for 4D gravitational collapse with an electromagnetic field. This analysis gives the irreducible and the reducible extensions of 4D perfect fluid collapse with an electromagnetic field and 5D perfect fluid collapse, respectively. Moreover, for the later case, the results can be recovered under some restrictions.

**Keywords:** Electromagnetic field, Gravitational collapse, Cosmological constant.

**PACS number:** 04.20.Cv; 04.20.Dw

* msharif@math.pu.edu.pk
† abbasg91@yahoo.com
I. INTRODUCTION

In general Relativity (GR), the spacetime singularity forms due to gravitational collapse of a massive astrophysical object. According to singularity theorems [1], the singularities exist in the form of either future or past incomplete spacelike geodesics. In other words, trajectories of material particles (photons) will come to a sudden end (disappear from the spacetime). The singularity theorems fail to provide practical information on the nature of the singularity.

It has been an interesting open problem to know the final fate of the gravitational collapse. Penrose [2] suggested that the spacetime singularity is always covered by the event horizon. This is known as the Cosmic Censorship Hypothesis (CCH), which has no mathematical or theoretical proof. The spacetime singularity would be a black hole or a naked singularity, depending on the initial data and the equation of state. Many efforts have been made to prove or disprove the CCH. For this purpose, Virbhadra et al. [3] introduced a new theoretical tool using gravitational lensing. Virbhadra [4] used gravitational lensing to find an improved form of the CCH.

Oppenheimer and Snyder [5] are pioneers who investigated dust collapse by taking the static Schwarzschild spacetime as an exterior spacetime and a Friedmann like solution as an interior spacetime. They concluded that a black hole is the final fate of gravitational collapse. Markovic and Shapiro [6] generalized this work by taking a positive cosmological constant. Lake [7] extended it for both positive and negative cosmological constants. Sharif and Ahmad [8] extended spherically-symmetric dust gravitational collapse with a positive cosmological constant to a perfect fluid.

Recent developments in string theory and other field theories indicate that gravity is a higher-dimensional interaction. It would be worthwhile to study gravitational collapse and singularity formation in higher dimensions. Ghosh and Banerjee [9] have studied dust collapse with five-dimensional Tolman-Bondi spacetime. In a recent paper, Sharif and Ahmad [10] studied the gravitational collapse of a perfect fluid in 5D for spherically symmetric spacetimes.

The behavior of the electromagnetic field in a gravitational field has been the subject of interest for researchers over the past decades. The inclusion of an electromagnetic field in gravitational collapse predicts that the gravitational attraction is counterbalanced by the Coulomb repulsive force along with the pressure gradient [11]. Sharma et al. [12] concluded that the electromagnetic field affects the values of the red-shifts, the luminosity, and the
masses of relativistic compact objects. Nath et al. [13] demonstrated that
the electromagnetic field reduces the pressure during gravitational collapse.
Recently, we [14] found that the electromagnetic field, along with the matter
field, in gravitational collapse increases the rate of collapse.

It would be interesting to study the gravitational collapse of a perfect
fluid in 5D with an electromagnetic field. The junction conditions between
the spherically-symmetric spacetimes are discussed using the Israel criteria
modified by Santos. A closed form of the exact solution of the field equations
in 5D exists for marginally bound \((F(r) = 1)\) and non-marginally \((F(r) \neq 1)\)
bound cases. This distinguishes the 5D case from the 4D, in which only a
marginally bound solution is possible. The main objectives of this work are
to study the effects of an electromagnetic field on the rate of collapse in 5D
and to see whether or not the CCH is valid in this framework.

The plan of the paper is as follows: In the next section, the junction
conditions are given. In Section III, we find the 5D spherically symmetric
perfect fluid solution of the Einstein field equations with a positive cosmo-
logical constant in the presence of an electromagnetic field for the marginally
bound \((F(r) = 1)\) and the non-marginally bound \((F(r) \neq 1)\) cases. The
apparent horizons and their physical significance are presented in section IV.
We conclude our discussion in the last section. The geometrized units (i.e.,
the gravitational constant \(G=1\) and speed of light in vacuum \(c=1\) so that
\(M \equiv \frac{M G}{c^2}\) and \(\kappa \equiv \frac{8 
\pi G}{c^4} = 8 \pi\)) are used. All the Latin and Greek indices vary
from 0 to 4; otherwise, the indices will be given.

**II. JUNCTION CONDITIONS**

A timelike 4D hypersurface \(\Sigma\) is taken such that it divides a 5D space-
time into two 5D manifolds, \(V^-\) and \(V^+\), respectively. The 5D spherically
symmetric spacetime is taken as an interior manifold \(V^-\) [15]
\[
ds_2^2 = dt^2 - X^2 dr^2 - Y^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \sin^2 \theta \sin^2 \phi d\psi^2),
\]
where \(X = X(t, r)\) and \(Y = Y(r, t)\). For the exterior manifold \(V^+\), we take
the 5D Reissner-Nordström de-Sitter spacetime
\[
ds_+^2 = ZdT^2 - \frac{1}{Z} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \sin^2 \theta \sin^2 \phi d\psi^2),
\]
where
\[
Z(R) = 1 - \frac{M}{R^2} + \frac{Q^2}{2R^3} - \frac{\Lambda}{6} R^2,
\]
$M$ and $\Lambda$ are constants and $Q$ is the charge. The Israel junction conditions modified by Santos [16, 17] are

1. The continuity of the first fundamental form over $\Sigma$ gives

$$(ds^2)_\Sigma = (ds^2_\Sigma) = ds^2_\Sigma.$$  

(4)

2. The continuity of the second fundamental form over $\Sigma$ gives

$$[K_{ij}] = K^+_{ij} - K^-_{ij} = 0, \quad (i, j = 0, 2, 3, 4),$$

(5)

where $K_{ij}$ is the extrinsic curvature defined as

$$K^{\pm}_{ij} = -n^\pm_\sigma \left( \frac{\partial^2 x^\sigma_{\pm}}{\partial \xi^i \partial \xi^j} + \Gamma^\sigma_{\mu\nu} \frac{\partial x^\mu_{\pm}}{\partial \xi^i} \frac{\partial x^\nu_{\pm}}{\partial \xi^j} \right).$$

(6)

$\xi^i$ correspond to the coordinates on $\Sigma$ and $x^\sigma_{\pm}$ are the coordinates for $V^\pm$. The Christoffel symbols $\Gamma^\sigma_{\mu\nu}$ are found from the interior or the exterior spacetime and $n^\pm_\sigma$ are components of the outward unit normals to $\Sigma$ in the coordinates $x^\sigma_{\pm}$.

The equations of the hypersurface $\Sigma$ in terms of the coordinates of the interior and the exterior spacetimes are given as [18]

$$h_-(r, t) = r - r_\Sigma = 0,$$

(7)

$$h_+(R, T) = R - R_\Sigma(T) = 0,$$

(8)

where $r_\Sigma$ is a constant. Using Eqs. (7) and (8) in Eqs. (1) and (2) respectively, it follows that

$$(ds^2)_\Sigma = dt^2 - [Y(r_\Sigma, t)]^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \sin^2 \theta \sin^2 \phi d\psi^2),$$

(9)

$$(ds^2_\Sigma) = [Z(R_\Sigma) - \frac{1}{Z(R_\Sigma)} (\frac{dR_\Sigma}{dT})^2] dT^2 - R_\Sigma^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \sin^2 \theta \sin^2 \phi d\psi^2).$$

(10)

For $T$ to be a timelike coordinate, we assume that

$$Z(R_\Sigma) - \frac{1}{Z(R_\Sigma)} (\frac{dR_\Sigma}{dT})^2 > 0.$$

(11)

From Eqs. (11), (9) and (10), it follows that

$$R_\Sigma = Y(r_\Sigma, t),$$

(12)

$$[Z(R_\Sigma) - \frac{1}{Z(R_\Sigma)} (\frac{dR_\Sigma}{dT})^2] \frac{1}{2} dT = dt.$$
The outward unit normals to $\Sigma$ in $M^-$ and $M^+$ are
\begin{align}
n^-_\mu &= (0, X(r_\Sigma, t), 0, 0, 0), \quad (14) \\
n^+_\mu &= (-R_\Sigma, \ddot{T}, 0, 0, 0). \quad (15)
\end{align}

The components of the extrinsic curvature $K^\pm_{ij}$ turn out to be
\begin{align}
K^-_{00} &= 0, \quad (16) \\
K^-_{22} &= \csc^2 \theta K^-_{33} = \csc^2 \theta \csc^2 \phi K^-_{44} = \left(\frac{YY'}{X}\right)_\Sigma, \quad (17) \\
K^+_{00} &= (\dddot{R}\dddot{T} - \ddot{T}\dddot{R} - \frac{Z}{2} \frac{dZ}{dR} \dddot{T}^3 + \frac{3}{2Z} \frac{dZ}{dR} T \dot{T} \dddot{T})_\Sigma, \quad (18) \\
K^+_{22} &= \csc^2 \theta K^+_{33} = \csc^2 \theta \csc^2 \phi K^+_{44} = (Z \dddot{R})_\Sigma, \quad (19)
\end{align}

where dot and prime indicate differentiations with respect to $t$ and $r$, respectively. Continuity of the extrinsic curvature gives
\begin{equation}
K^+_{00} = 0, \quad K^+_{22} = K^-_{22}. \quad (20)
\end{equation}

When we make use of Eqs. (16)-(20) along with Eqs. (3), (12) and (13), the junction conditions turn out to be
\begin{align}
(X\dot{Y}' - \dot{X}Y')_\Sigma &= 0, \quad (21) \\
M &= (Y^2 + \frac{Q^2}{2Y} - \frac{\Lambda}{6} Y^4 + \dot{Y}^2 Y^2 - Y^2 \frac{Y'^2}{X^2})_\Sigma. \quad (22)
\end{align}

Equations. (12), (13) and Eqs. (21), (22) are the necessary and the sufficient conditions for matching of the interior and the exterior regions.

**III. SOLUTION OF THE EINSTEIN FIELD EQUATIONS**

The Einstein field equations with a cosmological constant are
\begin{equation}
G^\mu_\nu - \Lambda \delta^\mu_\nu = \kappa (T^\mu_\nu + E^\mu_\nu). \quad (23)
\end{equation}

The energy-momentum tensor for a perfect fluid is
\begin{equation}
T^\mu_\nu = (\rho + p)u^\mu u_\nu - p \delta^\mu_\nu, \quad (24)
\end{equation}
where $\rho$ is the energy density, $p$ is the pressure, and $u_\mu = (1, 0, 0, 0, 0)$ is the 5-vector co-moving velocity. $E^\mu_\nu$ is the energy-momentum tensor for the electromagnetic field given by

$$E^\mu_\nu = \frac{1}{4\pi} (-F^{\mu\omega}F_{\nu\omega} + \frac{1}{4} \delta^\mu_\nu F_{\delta\omega} F^{\delta\omega}).$$  \hfill (25)

First, we solve the Maxwell field equations

$$F_{\mu\nu} = \phi_{\nu,\mu} - \phi_{\mu,\nu}, \quad \hfill (26)$$
$$F^{\mu\nu} = -4\pi J^\mu, \quad \hfill (27)$$

where $\phi_\mu$ is the five potential and $J^\mu$ is the five current. Because we have taken the charged fluid in a co-moving coordinate system, the magnetic field will be zero in this case. Consequently, the five potential and the five current can be taken as

$$\phi_\mu = (\phi(t, r), 0, 0, 0, 0), \quad \hfill (28)$$
$$J^\mu = \sigma \delta^\mu_0, \quad \hfill (29)$$

where $\sigma$ is the charge density.

We treat $\mu$ and $\nu$ as local coordinates for the solution of Maxwell field equation, Eq. (27). Thus, the non-zero components of the field tensor are evaluated by using Eqs. (26) and (28) as

$$F_{tr} = -F_{rt} = -\frac{\partial \phi}{\partial r}. \quad \hfill (30)$$

Also, Eqs. (27) and (29) yield

$$\frac{1}{X} \frac{\partial^2 \phi}{\partial r^2} - \frac{\partial \phi}{\partial r} \frac{X'}{X^2} = -4\pi \sigma X, \quad \hfill (31)$$

$$\frac{1}{X} \frac{\partial^2 \phi}{\partial r \partial t} - \frac{X}{X^2} \frac{\partial \phi}{\partial r} = 0. \quad \hfill (32)$$

The last equation implies that

$$\left( \frac{1}{X} \frac{\partial \phi}{\partial r} \right) = K(r), \quad \hfill (33)$$

where $K$ is an integrating function and gives the electromagnetic field contribution. Equations (31) and (33) yield

$$K'(r) = -4\pi \sigma X. \quad \hfill (34)$$
The field equations in Eq. (23) for the interior spacetime are written as

\[
G^i_t = -\frac{3}{X^2} \left( \frac{Y''}{Y} - \frac{X'Y'}{XY} + \frac{Y'^2}{Y^2} \right) + 3 \left( \frac{\dot{Y}^2}{Y^2} + \frac{\dot{X} \dot{Y}}{XY} \right) + \frac{3}{Y^2} = \Lambda + K^2 + 8\pi \rho,
\]

(35)

\[
G^r_r = -\frac{3Y'^2}{X^2Y^2} + 3 \left( \frac{\ddot{Y}}{Y} + \frac{\dot{Y}^2}{Y^2} \right) + \frac{3}{Y^2} = \Lambda - 8\pi p + K^2,
\]

(36)

\[
G^\theta_\theta = -\frac{2}{X^2} \left( \frac{Y''}{Y} - \frac{X'Y'}{XY} - \frac{Y''}{2Y^2} \right) + 2 \left( \frac{\ddot{Y}}{Y} + \frac{\dot{X} \dot{Y}}{XY} + \frac{\dot{Y}^2}{2Y^2} + \frac{\dot{X}}{2X} \right) + \frac{1}{Y^2} = \Lambda - 8\pi p - K^2,
\]

(37)

\[
G^\phi_\phi = G^\psi_\psi = G^\theta_\theta = \Lambda - 8\pi p - K^2,
\]

(38)

\[
G^r_t = -3 \frac{\ddot{Y}}{Y} + 3 \frac{\dot{X} Y'}{XY} = 0.
\]

(39)

To solve these equations, we first integrate Eq. (39) with respect to \( t \) so that

\[
X = \frac{Y'}{F},
\]

(40)

where \( F = F(r) \) represents the energy inside the hypersurface \( \Sigma \). Making use of Eqs. (36) and (40), we get

\[
\frac{\ddot{Y}}{Y} + (\frac{\dot{Y}}{Y})^2 + \frac{1 - F^2}{Y^2} = \frac{\Lambda - 8\pi p + K^2}{3}.
\]

(41)

The energy-momentum conservation equation for matter, \( T^\nu_{\mu\nu} = 0 \), shows that the pressure is a function of \( t \) only, i.e., \( p = p(t) \). Using this value of \( p \) in the above equation, it follows that

\[
\frac{\ddot{Y}}{Y} + (\frac{\dot{Y}}{Y})^2 + \frac{1 - F^2}{Y^2} = \frac{\Lambda - 8\pi p(t) + K^2}{3}.
\]

(42)

Here, we consider \( p \) as a polynomial in \( t \) given by [8]

\[
p(t) = p_0 (\frac{t}{T})^{-q},
\]

(43)

where \( T \) is a constant time introduced in the problem by a re-scaling of \( t \) for physical reasons and \( p_0 \) and \( q \) are constants. Further, for simplicity, we take \( q = 0 \) so that \( p(t) = p_0 \).
Inserting this value of $p(t)$ in Eq. (42) and then integrating with respect to $t$, we have

$$\dot{Y}^2 = F^2 - 1 + \frac{2m}{Y^2} + (\Lambda - 8\pi p_0 + K^2) \frac{Y^2}{6},$$

(44)

where $m$ is an arbitrary function of $r$ and is related to the mass of the collapsing system. Using Eqs. (44) and (40) in Eq. (35), we obtain

$$m' = \frac{8\pi}{3} (\rho + p_0) Y^3 Y' - \frac{KK'}{6} Y^4.$$

(45)

For physical reasons, we assume the following energy conditions [1] are satisfied:

$$\rho \geq 0, \quad \rho + p \geq 0,$$

(46)

$$\rho \geq 0, \quad -\rho \leq p \leq \rho.$$  

(47)

Integrating Eq. (45) with respect to $r$, we obtain

$$m(r) = \frac{8\pi}{3} \int_0^r (\rho + p_0) Y^3 Y' - \frac{1}{6} \int_0^r KK' Y^4,$$

(48)

where $m_0 = 0$ due to the finite distribution of the mass at the origin ($r = 0$). The mass function $m(r) > 0$ because $m(r) < 0$ is not meaningful. Using Eqs. (40) and (44) in the junction condition, Eq. (22), it follows that

$$M = \frac{Q^2}{2Y} + 2m + \frac{1}{6} (K^2 - 8\pi p_c) Y^4.$$

(49)

This mass is located at the origin of the spherical symmetry and produces a gravitational field in the exterior region of the sphere.

The total energy $\tilde{M}(r, t)$ up to a radius $r$ at time $t$ inside the hypersurface $\Sigma$ can be evaluated by using the definition of Misner-Sharp mass [18] given by

$$\tilde{M}(r, t) = \frac{1}{2} Y(1 + g^{\mu\nu} Y_{,\mu} Y_{,\nu}).$$

(50)

For the interior metric, it takes the form

$$\tilde{M}(r, t) = \frac{1}{2} Y(1 + \dot{Y}^2 - \frac{Y'^2}{X^2}).$$

(51)
Putting Eqs. (40) and (44) in Eq. (51), we obtain

$$\tilde{M}(r,t) = \frac{m(r)}{Y} + (\Lambda + K^2 - 8\pi p_0)\frac{Y^3}{12}. \quad (52)$$

Now we solve Eq. (44) for the following two cases:

$$F(r) = 1, \quad F(r) \neq 1.$$

1. **Solution with** $F(r) = 1$

First, we discuss the solution with a positive pressure. For $\Lambda - 8\pi p_0 + K^2 > 0$, the analytic solutions in closed form can be obtained from Eqs. (40) and (44) as follows:

$$Y(r,t) = \left( \frac{12m}{\Lambda - 8\pi p_0 + K^2} \right)^{\frac{1}{2}} \sinh^{\frac{1}{2}} \alpha(r,t), \quad (53)$$

$$X = \left( \frac{12m}{\Lambda + K^2 - 8\pi p_0} \right)^{\frac{1}{2}} \left[ \left\{ \frac{4m'}{m} - \frac{2KK'}{\sqrt{\Lambda + K^2 - 8\pi p_0}} \right\} \sinh \alpha(r,t) \right. + \left. \{2(t_0(r) - t)KK'\sqrt{\frac{3}{2(\Lambda + K^2 - 8\pi p_0)}} + t_0'(r)\sqrt{\frac{(\Lambda + K^2 - 8\pi p_0)}{6}} \right] \cosh \alpha(r,t) \right] \sinh^{-\frac{1}{2}} \alpha(r,t), \quad (54)$$

where

$$\alpha(r,t) = \sqrt{\frac{2(\Lambda - 8\pi p_0 + K^2)}{3}}[t_0(r) - t]. \quad (55)$$

Here, $t_0(r)$ is an arbitrary function of $r$ and is related to the time of the formation of the singularity. In the limit $(8\pi p_0 - K^2) \rightarrow \Lambda$, the above solution corresponds to the 5D Tolman-Bondi solution [19]

$$\lim_{(8\pi p_0 - K^2) \rightarrow \Lambda} Y(r,t) = \left[ 8m(t_0 - t)^2 \right] \frac{1}{4}, \quad (56)$$

$$\lim_{(8\pi p_0 - K^2) \rightarrow \Lambda} X(r,t) = \frac{m'(t_0 - t) + 2mt_0'}{32m^3(t_0 - t)^2} \frac{1}{4}. \quad (57)$$
2. Solution with $F(r) \neq 1$

Integrating Eq. (44) with the conditions $\Lambda - 8\pi p_0 + k^2 > 0$ and $F(r) \neq 1$, it follows that

\[
Y(r, t) = \left\{ \frac{12m}{\Lambda - 8\pi p_0 + K^2} - \frac{9(F^2 - 1)^2}{(\Lambda - 8\pi p_0 + K^2)^2} \right\}^{\frac{1}{2}} \sinh \alpha(r, t)
\]

\[
- \frac{3(F^2 - 1)}{\Lambda - 8\pi p_0 + K^2} \left\{ \frac{12m}{\Lambda - 8\pi p_0 + K^2} - \frac{9(F^2 - 1)^2}{(\Lambda - 8\pi p_0 + K^2)^2} \right\}^{\frac{1}{2}}.
\]

(58)

Using Eq. (58) in Eq. (40), we obtain

\[
X = \frac{1}{2F} \left\{ \frac{12m}{\Lambda - 8\pi p_0 + K^2} - \frac{9(F^2 - 1)^2}{(\Lambda - 8\pi p_0 + K^2)^2} \right\}^{\frac{1}{2}} \sinh \alpha(r, t)
\]

\[
- \frac{3(F^2 - 1)}{\Lambda - 8\pi p_0 + K^2} \left\{ \frac{12m}{\Lambda - 8\pi p_0 + K^2} - \frac{9(F^2 - 1)^2}{(\Lambda - 8\pi p_0 + K^2)^2} \right\}^{\frac{1}{2}}
\]

\[
\times \frac{12m'}{\Lambda - 8\pi p_0 + K^2} - \frac{9(F^2 - 1)^2}{(\Lambda - 8\pi p_0 + K^2)^2}
\]

\[
+ \frac{36KK'(F^2 - 1)^2}{(\Lambda - 8\pi p_0 + K^2)^3} \sinh \alpha(r, t) + \left\{ \frac{12m}{\Lambda - 8\pi p_0 + K^2} - \frac{9(F^2 - 1)^2}{(\Lambda - 8\pi p_0 + K^2)^2} \right\}^{\frac{1}{2}}
\]

\[
\times \left\{ t_0(r) - t \right\} \cosh \alpha(r, t) + \left\{ \frac{2(\Lambda - 8\pi p_0 + K^2)}{6(\Lambda - 8\pi p_0 + K^2)^2} \right\}^{\frac{1}{2}} \left\{ \frac{2K'}{6(K^2 - 1)KK'} \right\}^{\frac{1}{2}}
\]

\[
- \left\{ \frac{6FF'}{\Lambda - 8\pi p_0 + K^2} \right\}^{\frac{1}{2}}.
\]

(59)

where $\alpha(r, t)$ is given by Eq. (55). If the dominant energy condition holds (i.e., negative pressure), the restriction $\Lambda - 8\pi p_0 + K^2 > 0$ is removed, and a solution is possible for all values of $\Lambda$, $p_0$, and $K$. Equations (58) and (59) represent the non-marginally bound solution corresponding to $F(r) \neq 1$.

One can easily verify the marginally bound solution given by Eqs. (53) and (54) by substituting $F(r) = 1$ into Eqs. (58) and (59). The non-marginally bound solution is impossible in a 4D gravitational collapse with an electromagnetic field [14]. For $K = 0$, Eqs. (53), (54), (58), and (59) reduce to the marginally bound and the non-marginally bound solutions of the 5D perfect fluid collapse case [10].
IV. APPARENT HORIZONS

In this section, we discuss the apparent horizons and their physical significance by using the solution to the field equations. The apparent horizons can be found by using the boundary of the three trapped spheres whose outward normals are null. For the interior metric, this is given as follows:

\[ g^{\mu\nu}Y_{,\mu}Y_{,\nu} = \dot{Y}^2 - \left( \frac{Y'}{X} \right)^2 = 0. \]  

(60)

Inserting Eqs. (40) and (44) in the above equation, it follows that

\[ (\Lambda + K^2 - 8\pi p_c)Y^4 - 6Y^2 + 12m = 0. \]  

(61)

In particular, when we take \( \Lambda = 8\pi p_c - K^2 \), \( Y = \sqrt{2m} \). This is called the Schwarzschild horizon. For \( m = 0 \), \( p_c = 0 \), and \( K = 0 \), we have \( Y = \sqrt{\frac{2}{\Lambda}} \), which is called the de-Sitter horizon. The following positive roots are found from Eq. (61).

**Case (i):** For \( 4m < \frac{3}{\Lambda - 8\pi p_c + K^2} \), we obtain two horizons:

\[ Y_1 = \sqrt{\frac{3}{\Lambda - 8\pi p_c + K^2} + \frac{\sqrt{9 - 12m(\Lambda - 8\pi p_c + K^2)}}{\Lambda - 8\pi p_c + K^2}}, \]  

(62)

\[ Y_2 = \sqrt{\frac{3}{\Lambda - 8\pi p_c + K^2} - \frac{\sqrt{9 - 12m(\Lambda - 8\pi p_c + K^2)}}{\Lambda - 8\pi p_c + K^2}}. \]  

(63)

When \( m = 0 \), these reduce to \( Y_1 = \sqrt{\frac{6}{\Lambda + K^2 - 8\pi p_c}} \) and \( Y_2 = 0 \). \( Y_1 \) and \( Y_2 \) are called the cosmological horizon and the black hole horizon, respectively. For \( m \neq 0 \) and \( \Lambda \neq 8\pi p_c - K^2 \), \( Y_1 \) and \( Y_2 \) can be generalized [20], respectively.

**Case (ii):** For \( 4m = \frac{3}{\sqrt{(\Lambda + K^2 - 8\pi p_c)}} \), we have repeated roots:

\[ Y_1 = Y_2 = \frac{3}{\sqrt{(\Lambda + K^2 - 8\pi p_c)}} = Y, \]  

(64)

which shows that both horizons coincide. The ranges for the cosmological and the black hole horizons are

\[ 0 \leq Y_2 \leq \sqrt{\frac{3}{\Lambda - 8\pi p_c + K^2}} \leq Y_1 \leq \sqrt{\frac{6}{\Lambda - 8\pi p_c + K^2}}. \]  

(65)
The black hole horizon has its largest proper area \(4\pi Y^2 = \frac{12\pi}{(\Lambda + K^2 - 8\pi p_0)}\), and the cosmological horizon has its area between \(\frac{24\pi}{(\Lambda + K^2 - 8\pi p_0)}\) and \(\frac{12\pi}{(\Lambda + K^2 - 8\pi p_0)}\).

**Case (iii):** For \(4m > \frac{3}{\sqrt{(\Lambda + K^2 - 8\pi p_0)}}\), there are no positive roots; consequently, there are no apparent horizons.

Now we find the formation time for the apparent horizon with the help of Eqs. (53) and (61) given by

\[
 t_n = t_0 - \sqrt{\frac{3}{2(\Lambda - 8\pi p_0 + K^2)}} \sinh^{-1}\left(\frac{Y_n^2}{2m} - 1\right)^{\frac{1}{2}}, \quad (n = 1, 2).
\]  

(66)

In the limit \((8\pi p_0 - K^2) \to \Lambda\), we obtain the result corresponding to the 5D Tolman-Bondi solution [19],

\[
 t_{ah} = t_0 - \sqrt{\frac{m}{2}}.
\]  

(67)

Equations (65) and (66) imply that \(Y_1 \geq Y_2\) and \(t_2 \geq t_1\), respectively. The inequality \(t_2 \geq t_1\) indicates that the cosmological horizon forms earlier than the black hole horizon.

**V. OUTLOOK**

This paper provides an extension of our previous analysis for an electromagnetic field in perfect fluid gravitational collapse with a cosmological constant [14] from the 4D to the 5D case. The exact solution for the interior spacetime with a charged perfect fluid in the presence of a positive cosmological constant is derived. The effects of an electromagnetic field on the 5D gravitational collapse are as follows:

The relation for the Newtonian potential is \(\phi = \frac{1}{2}(1 - g_{00})\). Using Eqs. (12) and (49), for the exterior spacetime, the Newtonian potential turns out to be

\[
 \phi(R) = \frac{m}{R^2} + (\Lambda - 8\pi p_0 + K^2)\frac{R^2}{12}.
\]  

(68)

The corresponding Newtonian force is

\[
 F = -\frac{2m}{R^3} + (\Lambda - 8\pi p_0 + K^2)\frac{R}{6}.
\]  

(69)

This force will be zero for
\[ R = \frac{1}{(\Lambda - 8\pi p_0 + K^2)} \] and \[ m = \frac{1}{12(\Lambda - 8\pi p_0 + K^2)} \] while it becomes repulsive (attractive) for larger (smaller) values of \( m \) and \( R \). For the repulsive case, one must have \( \Lambda > (8\pi p_0 - K^2) \) such that \( 8\pi p_0 > K^2 \) over the entire range of the collapsing sphere. Notice that \( K = K(r) \) gives the electromagnetic field contribution. From Eq. (44), the rate of collapse turns out be

\[ \ddot{Y} = -\frac{2m}{Y^3} + (\Lambda - 8\pi p_0 + K^2)\frac{Y}{6}. \] (70)

Here, we have re-formulated the Newtonian model in terms of the acceleration of the collapsing process. If \( 8\pi p_0 - K^2 > \Lambda \) over the entire range of the collapsing sphere, then the force becomes attractive, and the cosmological constant would favor the collapsing process.

Further, we have found two apparent horizons (cosmological and black hole horizons), whose areas are larger due to the extra dimension as compared to the values obtained from our 4D analysis [14]. The solution for the non-marginally bound case \( (F(r) \neq 1) \) is also possible in 5D, which is not possible in the 4D case. Equation (48) represents the total amount of the collapsing mass contained in a sphere of radius 0 to \( r \). This amount of mass is larger than that to the 4D case; hence, collapsing process is faster in this case.

We would like to mention here that all our results match to 5D perfect fluid case [10] if we take \( K = 0 \). However, the results for a 4D gravitational collapse with an electromagnetic field [14] cannot be recovered directly. The reason is that Eq. (61) is quartic polynomial while in the 4D case, the corresponding equation is a cubic polynomial. In the 5D case, the roots under the possible conditions are different than the roots in 4D. Thus, there is only a difference of one degree in both cases, but the solutions are much different, depending on the nature of the polynomials.

The cosmological horizon was found to form earlier than the black hole horizon. Also, Eq. (66) shows that the apparent horizon forms earlier than the singularity; hence, the end state of the gravitational collapse is a black hole. These evidences agree with the 4D case. There was a possibility of a locally naked singularity in the 4D case due to the presence of an electromagnetic field, but such a possibility may be avoided due to the extra dimension. Thus, we conclude that 5D favors the formation of a black hole. It is interesting to mention here that our study supports the CCH.
ACKNOWLEDGMENT

We would like to thank the Higher Education Commission, Islamabad, Pakistan, for its financial support through the Indigenous Ph.D. 5000 Fellowship Program Batch-IV.

REFERENCES

[1] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Spacetime* (Cambridge University Press, Cambridge, 1979).

[2] R. Penrose, Riv. Nuovo Cimento **1**, 252(1969).

[3] K. S. Virbhadra, D. Narasimha and S. M. Chitre, Astron. Astrophys. **337**, 1(1998).

[4] K. S. Virbhadra, Phys. Rev. **D79**, 083004(2009).

[5] J. R. Oppenheimer and H. Snyder, Phys. Rev. **56**, 455(1939).

[6] D. Markovic and S. L. Shapiro, Phys. Rev. **D61**, 084029(2000).

[7] K. Lake, Phys. Rev. **D62**, 027301(2000).

[8] M. Sharif and Z. Ahmad, Mod. Phys. Lett. **A22**, 1493(2007); ibid. 2947.

[9] S. G. Ghosh and G. Baneerje, Int. J. Mod. Phys. **D12**, 639(2003).

[10] M. Sharif, and Z. Ahmad, J. Korean Phys. Society **52**, 980(2008).

[11] S. Thirukkanesh and S. D. Maharaj, Math. Meth. Appl. Sci. **32**, 684(2009).

[12] R. Sharma, S. Mukharjee and S. D. Maharaj, Gen. Relativ. Grav. **33**, 999(2001).

[13] S. Nath, U. Debnath, and S. Chakraborty, Astrophys Space Sci. **313**, 431(2008).

[14] M. Sharif and G. Abbas, Mod. Phys. Lett. **A24**, 2551(2009).

[15] S. G. Ghosh and A. Beesham, Phys. Rev. **D64**, 124005(2001).
[16] N. O. Santos, Phys. Lett. A106, 296(1984).

[17] W. Israel, Nuovo Cimento B44, 1(1966).

[18] C. W. Misner and D. Sharp, Phys. Rev. 136, b571(1964).

[19] D. M. Eardley and L. Smarr, Phys. Rev. D19, 2239(1979).

[20] S. A. Hayward, T. Shiromizu and K. Nakao, Phys. Rev. D49, 5080(1994).