I. INTRODUCTION

Inflation has now become part of the standard model of the cosmology \(^1\). With new data coming soon, and in particular with the Planck satellite mission, the different inflationary models will need to pass a strong test. Inflation set up the initial perturbations from which gravity forms the large scale structures LSS in the universe. At the same time, a huge amount of evidence is gathering in structure formation via the large galaxy maps that are currently under way \(^2\). These surveys have detected not long ago the Baryon Acoustic Oscillation BAO \(^2\), which have the same origin as the CMB \(^3\) but are measured via galaxy distribution. The formation structure requires the existence of dark matter and therefore the nature of inflation and of dark matter are then essential building blocks to understand the observations.

Inflation is associated with a scalar field, the "inflaton", and the energy scale at which inflation occurs is typically of the order of \(E_I = 10^{16}\text{GeV} \(^1\) but it is possible to have consistent inflationary models with \(E_I\) as low as \(O(100)\text{MeV} \(^4\). On the other hand dark matter is described by an energy density which redshifts as \(\rho_{\text{dm}} \sim a(t)^{-3}\) and is described by particles where its mass \(m \gg T\) with \(T\) its temperature. These particles can be either fermions or scalar fields. In the case of scalar fields the scalar potential must be \(V(\phi) = m^2_\phi \phi^2 + 2\) and independently on the value of the mass its classical equation of motion ensures that \(\rho_\phi \sim V \sim a(t)^{-3}\).

Here, since we want to unify inflation with dark matter we will assume that DM is made out of the same field, a scalar field \(\phi\). \(^5\). A scalar field can easily give inflation and DM if the potential is flat at high energies and at low energies the potential approaches the limit \(V(\phi) = m^2_\phi \phi^2/2\). However, most of the time our universe was dominated by radiation. Therefore, any realistic model must not only explain the two stages of inflation and dark matter but must also allow for a long period of radiation domination. Typically the inflaton decays while it oscillates around the minimum of its quadratic potential \(^6\). If the inflaton decay is not complete then the remaining energy density of the inflaton after reheating must be fine tuned to give the correct amount dark matter.

In order to avoid a fine tuning problem we follow the quantum generation work presented in \(^8\). In \(^8\) the quantum re-generation process was used to unify inflation with dark energy but here we use the same idea to unify inflation with dark matter. To reheat the universe we couple \(\phi\) to a relativistic field \(\varphi\) via an interaction term \(L_{\text{int}}\). This field \(\varphi\) may be a standard model "SM" particle, as for example neutrinos, but it could also be an extra relativistic particle not contained in the SM. An extra relativistic degree of freedom is fine with the cosmological data \(^7\). After inflation the \(\phi\) decays into this field \(\varphi\) at \(E_{\text{Din}}\) and to reheat the universe with "SM" particles we couple them to \(\varphi\) at high energies, e.g. the same energy as the decay of \(\phi\). Thermal equilibrium "TE" between \(\varphi\) and SM particles will be maintained as long as \(\varphi\) and the SM particles remain relativistic. At low energies we show that we can re-generate the dark matter field \(\phi\) at \(E_{\text{gen}}\) using the same interaction term \(L_{\text{int}}\) as for the high energy inflaton \(\phi\) decay. The appearance of the dark matter field \(\phi\) at late times is then via a quantum transition and not due to a classical evolution. A requirement on the mass \(m^2_\phi = V''(0)\) of the inflaton-dark matter field \(\phi\) is that \(m_\phi(E_I) > E_{\text{Din}} \gg E_{\text{gen}} > m_\phi(E_{\text{gen}})\) so the simplest potential \(V = m^2_\phi \phi^2\) will not work.

The unification scheme presented here has three parameters, the mass of the dark matter particle \(m_\omega\), the...
parameter in the inflation potential $\lambda$ and the coupling $g$ for the inflaton decay. Density perturbations normalized to COBE fixes the value for $\lambda$ and the correct amount of dark matter determines the cross section of dark matter at decoupling (wimp particle) gives a constraint between $g$ and $m_o$, leaving only the mass of the dark matter particle $m_o$ as a free parameter. We will show that the same coupling strength that gives the inflaton decay gives the dark matter re-generation at low scales and sets in combination with $m_o$ the wimp decoupling cross section. These same three parameters are present in models with inflation and a dark matter wimp particle \[8\] but without unification. This implies that our unification scheme does not increase the number of parameters and it accomplishes the desired unification between inflaton and dark matter for free.

II. GENERAL FRAMEWORK

Our starting point is a flat FRW universe with the inflaton-dark matter field $\phi$ coupled to a relativistic scalar $\varphi$. This field $\varphi$ can be either a fermion field or scalar field and we only require that it is relativistic at least for energies larger than the mass of the dark matter particle $m_o$. For presentation purposes we take $\varphi$ as a scalar field, but generalizing this work to a fermion field is straightforward and the fermion field could well be part of the standard model SM as for example a neutrino. We take the lagrangian $L = L_{SM} + \bar{L}$, where $L_{SM}$ is the standard model SM lagrangian and

\[
\bar{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + \frac{1}{2} \partial_{\mu} \varphi \partial^{\mu} \varphi - V(\phi) - B(\varphi) + L_{int}(\phi, \varphi, SM),
\]

$V(\phi), B(\varphi)$ are the scalar potentials for $\phi, \varphi$ and $V_{int} = -L_{int}$ is the interaction potential. The classical evolution of $\phi$ and $\varphi$ are given by the equations of motion,

\[
\ddot{\phi} + 3H \dot{\phi} + V' + V_{int} = 0 \quad (2)
\]

\[
\ddot{\varphi} + 3H \dot{\varphi} + B_{\varphi} + V_{int, \varphi} = 0 \quad (3)
\]

with $H^2 = (\dot{a}/a)^2 = \rho/3$, a prime denotes derivative w.r.t. $\phi$, $B_{\varphi} \equiv \partial B/\partial \varphi$ and we take natural units $m_{pl}^2 = 1/8\pi G \equiv 1$. The mass of $\phi$ is given by

\[
m_\phi^2(\phi) \equiv V''(\phi)
\]

and it is in general a field and time dependent quantity.

A. Inflaton-Dark Matter Potential

1. Inflaton Potential

There are many potentials $V(\phi)$ that lead to an inflation epoch. Inflation with a single scalar field can be classified in small or large field models. Small fields are potentials that inflate for values of the inflaton $\phi \ll m_{pl}$ as in new inflation models, e.g. $V = V_o(\phi^2 - \mu^2)^2$, while large field models are when inflation occurs for $\phi > m_{pl}$ as in chaotic models, e.g. $V = V_o \phi^n$. Here we will assume the simplest inflaton potential because we are more interesting in showing how the inflaton-dark matter unification scheme takes place in the context of quantum re-generation process. However, it is simple to work with other inflaton potentials.

The potential $V(\phi)$ must satisfy at the inflation scale the slows roll conditions $|V'/V| < 1, |V''/V| < 1$ and the constraint on the energy density perturbation normalized to COBE \[3\]

\[
\frac{\delta \rho}{\rho} = \frac{1}{\sqrt{75\pi^2}} \frac{V^{3/2}}{V'} = 1.9 \times 10^{-5}.
\]

For example for chaotic potentials $V_2 \equiv m_o^2 \phi^2/2$ or $V_4 \equiv \lambda \phi^4$ inflation occurs for $\phi > \phi_c$ and the value of the mass $m_o$ and the dimensionless parameter $\lambda$ and the scale of inflation $E_I \equiv V(\phi_c)^{1/4}$ are given by

\[
V_2 \equiv \frac{1}{7}m_o^2 \phi_c^2, \quad |\phi_c| = 2m_{pl}, \quad m_o = 8.7 \times 10^{14}GeV, \quad E_I = 5 \times 10^{16}GeV (7)
\]

and

\[
V_4 \equiv \lambda \phi^4, \quad |\phi_c| = 4m_{pl}, \quad \lambda = 10^{-9}, \quad E_I = 5 \times 10^{16}GeV. \quad (9)
\]

2. Dark Matter Potential

To obtain Dark Matter at low energies the scalar potential $V(\phi)$ must have the limit

\[
V(\phi) \rightarrow V_2 \equiv m_o^2 \phi_c^2
\]

with $m_o$ a constant mass term. A scalar field with potential $V(\phi) = V_o \phi^n$ redshifts as $a^{-3(1+w)}$ with $w = (n-2)/(n+2)$ (for n-even) and only for $n = 2$ do we have a energy density redshifting as matter, $V_2 \propto \phi^2 \propto a^{-3}$ while for $V_4 \propto \phi^4 \propto a^{-3}$.\[4\]. The constraint on $V_2$ is that

\[
\rho_\phi(t_o) = \frac{1}{2} \dot{\phi}_c^2 + V_2(t_o) = 2V_2(t_o) = m_o^2 \phi_c^2 = \rho_{dm.o}
\]

where we used that the pressure vanishes $p = \frac{\dot{\phi}_c^2}{2} - V_2 = 0$ and $\rho_{dm.o}$ is the present time dark matter density (from here on the subscript $o$ gives present time quantities).

B. Interaction Term

The interaction term $L_{int}(\phi, \varphi)$ between $\phi$ and $\varphi$ should allow the following processes:

i) at high energies $E_{Dim}f$ the inflaton $\phi$ must decay into $\varphi$ efficiently,

ii) at much lower energies $E_{gen}$ it should re-generate $\phi$
(with \(E_{\text{gen}} > m_\phi\)),

iii) it must inhibit the decay of dark matter particle \(\phi\).

The point (iii) is needed because dark matter must last for a long period of time, however, depending on the ratio of its decay it may be relax and we could have for example a dark matter dark energy interaction. However, we will not follow this interesting path here and will be presented elsewhere \([12]\). There are many possible interaction terms between \(\phi\) and \(\varphi\) and generalization of our work presented here is straightforward. If we take only renormalizable terms with dimensionless constant we have \(L_{\text{int}} = g \phi \varphi^2\) with \(a + b = 4\). We show in the Appendix that the terms \(L_{13} = g \phi \varphi^3\) and \(L_{31} = g \varphi^3\), i.e. \(a = 1, b = 3\) and \(a = 3, b = 1\) do not satisfy the conditions (i-iii). The term \(L_{31} = g \varphi^3\) does not allow for \(\phi\) to decay into \(\varphi\) if \(m_\phi \neq m_\varphi\) and our working hypothesis has a massive inflaton field \(\phi\) and a relativistic field \(\varphi\), i.e. \(m_\phi \neq m_\varphi\). On the other hand the term \(L_{13} = g \phi \varphi^3\) does not work because it gives a decaying dark matter particle. Therefore, we are left with the interaction term

\[
L_{22}(\phi, \varphi) = g \phi^2 \varphi^2. \tag{12}
\]

To produce all SM particle the field \(\varphi\) must be either part of the SM or it must couple to at least one field of the SM. It is standard to assume the interaction term as \([1]\)

\[
L_{\text{int}}(\varphi, SM) = h \varphi^2 \chi^2 + \sqrt{h} \varphi \bar{\psi} \psi \tag{13}
\]

where \(\chi, \psi\) are SM particles. We will choose \(h\) such that the transition \(\varphi \leftrightarrow SM\) takes place as soon as the \(\varphi\) are produced. If the fields \(\chi, \psi\) acquire a large mass then they will decouple from \(\varphi\) at \(T < m_\chi, m_\psi\) below this temperature \(n_\chi, n_\psi\) are exponentially suppressed and \(\Gamma/H\) will be smaller than one. However, as long as \(\varphi\) remains relativistic the \(\varphi\) temperature redshifts as \(T \propto 1/a(t)\) and since it was in thermal equilibrium with the SM we can determine \(T_\phi(t) = q T_{\chi\psi}(t)\) with \(q = (g_{\text{SM}}(t)/g_{\text{dec}})^{1/4}\) and \(g_{\text{dec}}\) the number of relativistic degrees of freedom of the SM at decoupling.

1. Interaction rates

The differential transition rate is given by \([10]\)

\[
d\Gamma = V_t (2\pi)^4 |M_{ab}|^2 \delta^4 (P_I - P_F) \Pi_a \frac{1}{2 E_a V_{1b}} \frac{d^3 p_b}{2 E_b (2\pi)^3} \tag{14}
\]

where \(P_I (P_F)\) is the initial (final) momentum, \(V_t\) is the volume (normalized to one particle per volume) and \(M_{ab} \equiv \langle b|M|a\rangle\) is the transition amplitude. The conservation of energy-momentum requires that initial and final energies are equal, \(E_I = E_F\) and \(p_i = p_f\), and this ensures that the amount of homogeneity of the universe is preserved by the interaction. In a process of \(a\) initial particles with the same energy \(E_a\) and a final state consisting of \(b\) particles with the same energy \(E_b\) and one has \(E_i = a E_a = b E_b = E_f\) and the differential transition rate is given by

\[
\Gamma_{ab} = c_{ab} |M_{ab}|^2 n_a^{-1} p_b^{-1} E_a^{b-a-3} \tag{15}
\]

with \(c_{ab} = (2\pi)^3 2^{-(a+b)} (a/b)^{3(b-2)}\).

2. Interaction rates for \(L_{\text{int}} = g \phi^2 \varphi^2\)

The decay rate for \(\phi \rightarrow \varphi + \varphi\) using eq.(15) with \(a = 1, b = 2\) is given by

\[
\Gamma_{12} = \frac{c_{12}|M_{12}|^2}{m_\phi(\phi)} = \frac{c_{12} g^2 \phi^2}{m_\phi(\phi)} \tag{16}
\]

with \(c_{12} = 1/16\pi\) and the interaction term in eq.(12) and we used in the last equality in eq.(10). The \(2 \leftrightarrow 2\) process is given by

\[
\Gamma_{22} = \frac{\bar{c}_{22} |M_{ab}|^2 n_a}{E_a^a} = \langle \sigma v \rangle n_a. \tag{17}
\]

and the in last equality in eq.(13) we used eq.(12). If one of the initial particles becomes non-relativistic then \(n_a\) in eq.(17) is exponentially suppressed by \(m/T\) with \(n_a = g_a(2\pi/\mu)^{3/2} \exp [-(m - \mu)/T]\). However, if the two initial particles are relativistic than the number density is given by \(n_a = g_a(\zeta(3)/\pi^2 T^3/30) = c_n E^4\) with \(E = r T\) and \(c_n = g_a(\zeta(3)/\pi^2 T^3), \ r \equiv \rho/n T = \pi^4/300\zeta(3) \approx 2.7\) and eq.(17) becomes

\[
\Gamma_{22} = c_{22} g^2 E_a \tag{19}
\]

with \(c_{22} = \bar{c}_{22} c_n = \zeta(3)/(32\pi^3 T^3)\), and \(g_a = 1\) for a real scalar field.

3. Decay Rate for \(g \phi^2 \varphi^2\)

The decay process takes place as long as \(m_\phi \gg m_\varphi\) and the decay rate \(\Gamma_{12}\) for \(\phi \rightarrow \varphi + \varphi\) is given by

\[
\Gamma_{12} = \frac{g^2 \phi^2}{m_\phi(\phi)} \tag{20}
\]

with \(c_{12} = 1/16\pi\) and it is field and time dependent through the terms \(\phi, m_\phi(\phi)\). The field \(\phi\) in eq.(20) may evolve with time or it may be constant if the scalar potential \(V(\phi)\) gives a nonvanishing v.e.v. \(\langle \phi \rangle \neq 0\), as in new inflation models. In order to have a \(\phi\) decay we require that

\[
\frac{\Gamma_{12}}{H} = \frac{c_{12} \sqrt{g^2 \phi^2}}{m_\phi(\phi) \sqrt{V}} \tag{21}
\]
is larger than one, $\Gamma_{12}/H > 1$. If $\langle \phi \rangle \neq 0$ and $m_\phi$ is constant then the decay in eq. (22) will be efficient since $\Gamma$ is constant and $H \propto 1/t$. If however, the scalar potential is $V = m_\phi^2 \phi^2$, as for dark matter, than the decay is not efficient since $\sqrt{2V} = m_\phi \propto 1/t$ and $\Gamma_{12}/H \propto \phi^2/(m_\phi \sqrt{V}) = \phi/m_\phi^2 \propto 1/\sqrt{t} \to 0$. In general, the evolution of $\phi$, $m_\phi$ and $\Gamma_{12}/H$ will depend on the choice of scalar potential $V$.

C. Decay Efficiency

The interaction or decay process depends on the transition rate $\Gamma$ and $H$ and it takes place if the ratio $\Gamma/H > 1$. The functional form of $\Gamma$ depends on the interaction term $L_{int}(\phi, \varphi)$ and it may be field and time dependent. The classical evolution of the fields depends on the scalar potentials $V(\phi), B(\varphi)$ and it is then the combination of $L_{int}(\phi, \varphi)$ and $V(\phi), B(\varphi)$ which determines the transition process.

The inflaton decay can be efficient or not efficient. The process is not efficient when $\Gamma(t)/H(t) \leq 1$ for $t > t_{dec}$, where $t_{dec}$ is the decoupling time when $\Gamma(t_{dec})/H(t_{dec}) = 1$. In this case we will have a remnant energy density $\rho_\phi(t_{dec})$. The energy density $\rho_\phi(t)$ will evolve classically depending on the form of the potential $V(\phi)$. If the potential evolves as matter then the amount of energy density $\rho_\phi$ at decoupling is easily determined and it is given by

$$\frac{\Omega_{dm}}{\Omega_{\Gamma}} = \Omega_{\phi}(E_d) = \frac{\rho_\phi}{\rho_r} = \frac{\rho_\phi}{\rho_\phi} \frac{a_d}{a_0} = \frac{\Omega_{\phi}}{\Omega_{\phi}} \left( \frac{E_d}{E_d} \right) = 3 \times 10^{-23} \left( \frac{10^{14} GeV}{E_d} \right)^2$$

with $\Omega_{\phi} = \Omega_{dm_{\phi}} = 0.22$ and $\Omega_{\phi} h^2 = 4.15 \times 10^{-5}$ the present time relativistic energy density. Clearly one requires a huge amount of fine tuning in the value of $\rho_\phi$ at a high decoupling energy since our universe had a large radiation domination epoch. We conclude that if we only take into account a classical evolution of $\phi$ after reheating the inflaton-dark matter unification requires a large fine tuning of initial conditions.

On the other hand the transition process is efficient when the inflaton decays completely and this requires that $\Gamma(t)/H(t) > 1$ for $t > t_{dec}$, where $t_{dec}$ is the time when $\Gamma(t_{dec})/H(t_{dec}) = 1$. In this case the $\phi$ particles decay completely and disappear from the spectrum and $\Omega_{\phi} \to 0$. A simple example is when $\Gamma$ is constant since $H \propto 1/t$ and $\Gamma/H \propto t \to \infty$. However if $\Gamma(t)/H(t)$ becomes smaller than one for $t > t_{dec}$ then we will also say that the decay is efficient if the residual energy density $\rho_\phi(t)$ is subdominant. For example if $\rho_\phi(t)$ redshifts as matter then that the decay is efficient if $\rho_\phi(t) \ll \rho_{dm}(t)$ for $t > t_{dec}$ with $\rho_{dm} = \rho_{dm_{\phi}}(a_0/a)^3$ the dark matter energy density. An efficient decay would clearly not allow $\rho_\phi$ to account for dark matter.

III. INFLATON-DARK MATTER UNIFICATION WITH QUANTUM GENERATION

A. Generic Quantum Transitions

Another possibility to achieve inflaton-dark matter unification is if the $\phi$ particles are re-generated via a quantum transition $|\phi\rangle$ at some late time but before $a_{eq}$, the matter-radiation equivalence scale factor. In this case, the decay $\phi \to \varphi$ at high energies $E_{Dinf}$, i.e. below inflation, must be efficient and $\phi$ disappears from the spectrum. At much lower scales $E_{gen}$, with $E_{gen} \ll E_{Dinf}$, the $\phi$ can be re-generated by $\varphi$.

After inflation the energy of the $\phi$ particles is $E_1^2 = \rho_1^2 + m_\phi(E_1)^2 \simeq m_\phi(E_1)^2$ since the momentum $p_1$ redshifted with the expansion of the universe, i.e. $p_1 = e^{-N} p_{fi}$ with $N$ the number of e-folds of inflation. If $\phi$ decays into $\varphi$ at $E_{Dinf}$ we will produce relativistic $\varphi$ particles with energy $E_{Dinf} < m_\phi(E_1)$. On the other hand at low energies $E_{gen}$ when we have the inverse process of $\phi$ production from $\varphi$ we must have that the energy of the $\varphi$ particles $E_{gen}$ must satisfy $E_{gen} > m_\phi(E_{gen})$ and therefore we must have

$$m_\phi(E_1) > E_{Dinf} \gg E_{gen} > m_\phi(E_{gen}),$$

which implies that the mass of $\phi$ after inflation must be much larger than at generation scale $E_{gen}$. Since $m_\phi(E_1) = V''(E_1) \gg m_\phi(E_{gen}) \equiv V''(E_{gen})$ clearly the simplest inflationary potential $V = m_\phi^2/2$, with $m_\phi$ constant, will not work. However, potentials such as $V = m_\phi^2/2 + \lambda \phi^4$ or of the new inflation type, e.g. $V = V_0 (\phi^2 - \mu^2)^2$, where the $\phi$ field rolls down a flat region for small $\phi$ and then oscillates around the minimum of the potential at $\langle \phi \rangle = \mu$ may work in this scenario.

IV. INFLATON-DARK MATTER UNIFICATION MODEL

We will work out the inflation-dark matter unification through a simple example. Of course, the whole scheme is much more general and other inflaton-dark matter $V(\phi)$ potentials or interaction terms may be used, however in all cases $V(\phi) = m_\phi^2 \phi^2$ for dark matter. We take the following potential

$$V(\phi) = V_2 + V_4 = \phi^2 m_\phi^2 + \lambda \phi^4$$

with a mass

$$m_\phi^2 = m_\phi^4 + 12 \lambda \phi^2$$

and we coupled $\phi$ with $\varphi$ via the interaction term $L_{22} = g \phi^2 \varphi^2$. For

$$\phi \geq \phi_{24} \equiv \frac{m_\phi}{\sqrt{2\lambda}}$$

(26)
The interaction term in eq. (28) will produce relativistic particles and that the evolution of $\rho_\phi$ goes from relativistic to dark matter type at $N_{\text{dec}}$.

$$E_{24} \equiv V(\phi_{24}) = 2 \left( \frac{m_\phi^2 \sigma_{24}}{2} \right)^{1/4} = \left( \frac{4}{3} \right)^{1/4} m \tag{27}$$

So, if $\lambda \neq 0, m_\phi \neq 0$ the potential $V_4$ dominates at high energies, during inflation, while $V_2$ at low energies, when dark matter prevails. If either potential $V_4$ or $V_2$ dominates then its classical evolution is $V_4 \propto \phi^4/\alpha^4$ with $\phi \propto 1/\alpha$ and $V_2 \propto \phi^2 1/\alpha^2$ with $\phi \propto 1/\alpha^{3/2}$. From eq. (19) we know that inflation requires $\lambda = 10^{-9}$ and inflation ends at $\phi = 2.6$ an energy $E_I \equiv V_4^{1/4}(\phi_I) = 5 \times 10^{16} \text{ GeV}$ and a mass $m_\phi = V_4^{1/2} = \sqrt{12}\lambda\phi$ and $m_\phi(E_I) \simeq 10^{14} \text{ GeV}$. At low energies the mass of the dark matter particle is given by $m_\phi$, and for CDM the mass of $\phi$ must be $m_\phi \geq O(\text{GeV})$ while warm DM requires a smaller mass with $m_\phi > O(10 - 100) \text{ keV}$.

We will use in this model the interaction term

$$L_{22} = g\phi^2 \varphi^2. \tag{28}$$

This term will allow the inflaton to decay efficiently after inflation at high energies $E_{\text{Inf}}$ into the relativistic $\varphi$ particles and $\phi$ will disappear from the spectrum (at most $\Omega_\phi$ is negligible). At a much later time the same interaction term in eq. (28) will produce relativistic particles $\phi$ at energies below $E_{\text{gen}}$ with $E_{\text{gen}} \gg m_\phi$. Eventually the $\phi$ particles become non-relativistic and they will decouple from $\varphi$ as a WIMP particle. The constraint to give the correct amount of dark matter today fixes the cross section $\sigma = g^2/(32\pi m_\phi^2)$. Finally, since $\phi$ becomes massive we also show that $\phi$ at low energies does not decay into $\varphi$ and allows for dark matter to dominate a long period as in a standard cosmological scenario. We show in fig. 1 the evolution of the inflaton-dark matter and relativistic energy densities. We see that for a long period of time the field $\phi$ disappears from the spectrum from the inflaton decay to the re-generation scale. Since the $\phi$ is re-generated while relativistic $\rho_\phi$ redshifts as radiation first and then when it becomes non-relativistic it redshifts as matter and $\phi$ decouples from $\varphi$ as any WIMP particle.

\section{Quantum Transitions}

The inflaton-dark matter field $\phi$ should decay efficiently at high energies, after inflation, to reheat the universe but at low energies when dark matter dominates we do no longer want the $\phi$ to decay since the period of dark matter domination must last from $a_{eq}$ to $a_{dec} \simeq a_o/2$ when dark energy starts to dominate.

\subsection{1. $\phi$ Decay at Inflation: $E_{\text{Inf}}$}

The interaction term in eq. (28) gives a decay $\phi \rightarrow \varphi + \varphi$ with a decay rate given by eq. (16) and at high energies when $V_4$ dominates $V$ one has $m_\phi = 12 \lambda \phi^2$ and

$$\Gamma_{\text{Inf}} = c_{12} g_3^2 \frac{\phi^2}{m_\phi} = c_{12} g_3^2 \frac{\phi}{\sqrt{12}\lambda} \tag{29}$$

with $c_{12} = 1/16\pi$. Using $H = \sqrt{V_4}/3 = \phi^2 \sqrt{\lambda}/3$ we have

$$\Gamma_{\text{Inf}} \frac{H}{E} = c_{12} g_3^2 \frac{2\lambda\phi}{E} \equiv \frac{E_{\text{Inf}}}{E_4} \tag{30}$$

where $E_4 \equiv V_4^{1/4} = \lambda^{1/4}\phi$ is an energy scale which depends on the field $\phi$ and

$$E_{\text{Inf}} = \frac{c_{12} g_3^2 \lambda^{-3/4}}{2} \tag{31}$$

is a constant quantity with energy dimensions and set the scale of the decay. We have a decay for energies $E_4 < E_{\text{Inf}}$. The evolution of $E_4 \propto \phi \propto 1/\alpha$ and $\Gamma_{\text{Inf}}/H \propto 1/\phi \propto \alpha$ grows with time giving an efficient decay. Once $\Gamma_{\text{Inf}}/H > 1$ the decay of $\phi$ takes place and does not stop (as long as $V_4$ dominates $V$).

\subsection{2. Quantum Re-generation: $E_{\text{gen}}$}

If the field $\phi$ decays completely after inflation than there will no $\rho_\phi$ left to account for dark matter. In order to re-generate the field $\phi$ we follow [6] and we use the same interaction term $L_{22}$ as for the inflaton decay but now the universe contains $\varphi$ relativistic particles and no $\phi$ particles. As long as the energy $E_\varphi$ of the relativistic particles is larger than the mass of $\phi$, i.e. $E_\varphi > m_\phi$, we can produce $\phi$ particles. If $E_\varphi \gg m_\phi$ then both fields $\varphi, \phi$ are relativistic and the transition rate for the $2 \leftrightarrow 2$ process is given by eq. (19) with

$$\Gamma_{\text{gen}} = c_{22} g_2^2 E, \quad H = \sqrt{\rho_\varphi / 3\Omega_r} = c_H E^2, \quad \tag{32}$$
\[
\Gamma_{\text{gen}} \over H = \frac{c_{\text{gen}} g^2}{E} = \frac{E_{\text{gen}}}{E}
\]  
(33)

with \(c_{22} = \zeta(3)/(32\pi^2g^3)\), \(c_H^2 \equiv g_{\text{rel}}\pi^2/(900\Omega_{\text{rel}}\Omega^2)\) and \(c_{\text{gen}} \equiv c_{22}/c_{H} = (\zeta(3)/32\pi^2\pi^2)(90\Omega_{\text{rel}}/g_{\text{rel}})^{1/2} \simeq 10^{-4}\) with \(g_{\text{rel}} \simeq 106, \Omega_{\text{rel}} \simeq 1\). We have taken in eq. (32) that \(n_\phi\) is proportional to \(T^3\) since \(\phi\) is relativistic. The process takes place for energies \(E \propto T \propto 1/a\) of the relativistic particles below the constant scale \(E_{\text{gen}}\) with \(\Gamma_{\text{gen}}/H > 1\) or

\[
E \leq E_{\text{gen}} \equiv c_{\text{gen}}g^2.
\]  
(34)

Since the particles \(\varphi\) are relativistic and they are in thermal equilibrium with SM particles we have \(T_\varphi = T_{\text{sm}} \simeq T_{\text{dec}}\). As long as \(T \gg m_\varphi\) we will \(\Omega_\varphi = \Omega_{\text{dec}}\), but once we reach the region with \(T \gtrsim m_\varphi\) the two fields will decouple since \(n_\phi\) will be exponentially suppressed.

3. Non-relativistic \(\phi\) Decoupling: \(E_{\text{dec}}\)

If two relativistic particles are in thermal equilibrium and one (in our case \(\phi\)) becomes non-relativistic then the density number \(n_\phi\) is exponentially suppressed and \(\varphi\) and \(\phi\) decouple. This is just the standard WIMP particle decoupling. The transition rate for a \(2 \leftrightarrow 2\) process is given by eq. (17)

\[
\Gamma_{\text{dec}} = \langle \sigma v \rangle n_\phi.
\]  
(35)

In order to have the correct amount of dark matter a WIMP must decouple at \(\Omega_{\phi} h^2 = \frac{3 \times 10^{-27}}{\langle \sigma v \rangle} \text{cm}^3\text{s}^{-1}\)

\[
(36)
\]

For the \(2 \leftrightarrow 2\) transition this implies a cross section

\[
\langle \sigma \rangle = \frac{g^2}{32\pi m_\phi^2} = 0.1 \text{pb}
\]  
(37)

with \(v \simeq c\). Eq. (37) gives a constraint for \(g\) in terms of the mass \(m_\phi\). The freeze out takes place at \(x_F = m_\phi/T_F \simeq 10\) giving a decoupling constant energy \(E_{\text{dec}}\) which is a function of the dark matter mass

\[
E_{\text{dec}} = c_{\text{dec}} T_F = c_{\text{dec}} m_\phi x^{-1}_F \simeq 0.12 m_\phi
\]  
(38)

with \(c_{\text{dec}} = (\pi^2g_{\text{rel}}/30)^{1/4} \simeq 1.2\) with \(g_{\text{rel}} \simeq 5.5+1 = 6.6\) at \(E < O(MeV)\). For energies \(E < E_{\text{dec}}\) the fields \(\phi\) and \(\varphi\) are no longer coupled and \(\phi\) evolves classically as matter with \(V \sim V_2 \propto \phi^2 \propto 1/a^3\). If \(m_\phi \gg T_q \simeq eV\), where \(T_q\) is the radiation-matter equality, the decoupling takes place while the universe is radiation dominated and the constraint on warm dark matter sets a lower scale \(m_\phi > 10-100keV\).

4. Dark Matter Decay?: \(E_{Ddm}\)

We have seen that at \(E_{\text{dec}}\) the field \(\phi\) ceases to maintain thermal equilibrium with \(\varphi\) through the \(2 \leftrightarrow 2\) process. However, the field \(\phi\) may decay into \(\varphi\) since \(m_\phi \gg m_\varphi\). Of course, we do not want \(\phi\) to decay since it must account for dark matter. In this case the decay rate is the same in eq. (16)

\[
\Gamma_{Ddm} = \frac{c_{12}g^2\varphi^2}{m_\phi}\]

(39)

but the mass \(m_\phi\) is now constant and \(\phi\) evolves as \(\phi \propto a^{-3/2}\) since now the potential that dominates is \(V_2 \gg V_4\) and \(\rho_{\phi} = 2V_2 = m_\phi \varphi^2 \propto 1/a^3\). For radiation dominated epoch we have \(H = \sqrt{\frac{\rho_{\text{dec}}}{4\pi}} \equiv c_H E^2\) and

\[
\Gamma_{Ddm} = \frac{c_{Ddm} g^2 \varphi^2 E}{m_\phi E^2} = \frac{E}{E_{Ddm}}.
\]  
(40)

We will not have a \(\phi\) decay for \(\Gamma_{Ddm}/H < 1\), i.e. energies \(E\) below the constant energy \(E_{Ddm}\),

\[
E < E_{\text{Ddm}} \equiv \frac{m_\phi E_{\text{Ddm}}}{c_{\text{Ddm}} g^2 \varphi^2} \simeq 10 m_\phi
\]  
(42)

where we have used \(\varphi = \varphi_{\text{eq}}(a_{\text{eq}}/a)\), \(E = E_{\text{eq}}(a_{\text{eq}}/a)\), \(\varphi_{\text{eq}}^2 E_{\text{eq}}^2 = (\varphi_{\text{eq}}^2/E_{\text{eq}}^2)(a_{\text{eq}}/a) = \varphi_{\text{eq}}^2 E_{\text{eq}}^2(H/E)\) we can write

\[
\Gamma_{Ddm} = \frac{c_{Ddm} g^2 \varphi_{\text{eq}}^2 E}{m_\phi E_{\text{eq}}^2} = \frac{E}{E_{Ddm}}.
\]  
(41)

so that after decoupling at \(E_{\text{dec}}\) there is no \(\phi\) decay.

In the region when dark matter dominates the universe we can easily estimate \(\Gamma_{Ddm}/H\) as

\[
\frac{\Gamma_{Ddm}(t_r)}{H(t_m)} \frac{H(t_m)}{\Gamma_{Ddm}(t_m)} = \frac{\varphi^2(t_r)}{\varphi^2(t_m)} \frac{H(t_m)}{H(t_r)} = \frac{a(t_m)}{a(t_r)} \left(\frac{a(t_m)}{a(t_{eq})}\right) > 1
\]  
(44)

since \(a(t_r) < a(t_{eq}) < a(t_m)\) where \(t_r, t_{eq}, t_m\) are times in radiation, equality and matter domination and \(H(t_r) \propto a^{-2}, H(t_m) \propto a^{-3/2}\). From eq. (41) and (44) we have \(1 > \Gamma_{Ddm}(t_r)/H(t_r) > \Gamma_{Ddm}(t_m)/H(t_m)\) for energies \(E < E_{\text{Ddm}}\), so we conclude that neither in radiation nor in matter domination can \(\phi\) decay into \(\varphi\).

B. Universe Reheating

The reheating of the universe takes place via a process \(\varphi + \varphi \leftrightarrow \chi + \chi\) (or \(\varphi + \varphi \leftrightarrow \psi + \psi\)) with a cross section
for relativistic particles $\sigma = h^2/32\pi E^2$ (we take the same strength for the $\chi$ and $\psi$) and an interaction rate

$$\Gamma_R = c_2 h^2E, \quad H = \sqrt{\frac{\rho_r}{3\Omega_r}} \equiv c_H E^2, \quad \Gamma_R \equiv c_h h^2$$

(45)

with $c_2 = \zeta(3)/(32\pi^3 r^3)$, $c_H^2 \equiv g_\chi \pi^2/(90\Omega_r r^4)$ and $c_r \equiv c_2/c_H$. For $E > 10^3 GeV$ we have $g_r \simeq 100, \Omega_r \simeq 1$ and $c_r \simeq 10^{-3}$. Clearly eq. (45) maintains a TE for $E \leq E_R$. A good choice of $h$ is such that reheating takes as soon as the $\phi$ particles are produced at $E_{Dinf}$ with at $E_R = c_h h^2 = E_{Dinf}$ but if can take any values as in the range $E_{Dinf} > E_R > O(10^{-100}) MeV$ which is the lower limit for reheating [4].

1. Summary of Energies

We present the different energy scales relevant in the process of our inflation-dark matter unification scheme. From high energy to low energy we have the following energy scales: Inflation occurs at $E_I$, then $\phi$ decays efficiently into $\varphi$ at $E_{Dinf}$ via the interaction term $L_{22}$ and disappears from the spectrum. Using the same interaction term the field $\phi$ is re-generated at a much lower scale $E_{gen}$. The field $\phi$ becomes none relativistic while in thermal equilibrium with $\varphi$ and decouples at $E_{dec}$. We also show that below $E_{Ddm}$ the field $\phi$ does not decay again into $\varphi$ with $E_{Ddm}/E_{dec} = 8m_o/T_F > 1$, which ensures that after thermal decoupling the field $\phi$ does not further decay. We have then the following order of energies

$$E_I > E_{Dinf} > E_{gen} > E_{24} > E_{Ddm} > E_{dec} > E_{eq}$$

(47)

SM particles are produced at $E_R$ via the coupling with $\varphi$ and the constraint is $E_{Dinf} > E_R > O(10 - 10^4 MeV)$ [4].

Concerning the inflaton-dark matter unification scheme we have 3 different parameters $\lambda, m_o$ in the potential $V$ and a coupling $g$ between $\varphi$ and $\phi$. The seven energies in eqs. (49)-(56) are given in terms of these three parameters. Inflation fixes one parameter, $\lambda$, and the amount of dark matter today gives a constraint between $g$ and $m_o$. We are left with one single free parameter which we take it to be the mass $m_o$. We show in fig. (4) the dependence of the $E’$s on $m_o$ and in table 1 we give the values for $10^{-4} GeV < m_o < 10^6 GeV$. We see that $E_{dec} > E_{eq} = O(10^{-9} GeV)$ and that the values of $g$ and all other energies are phenomenologically viable. This implies that it is feasible to implement the inflation-dark matter unification. We would like to point out when $E_{Dinf} > E_I$, or even larger than $m_{pl}$ this only means that as soon as $\phi$ ends inflation at $E_I$, $\phi$ decays immediately since the condition for its decay is satisfied. We summarize the definitions and values of these energies

$$g = (32\pi \langle \sigma \rangle)^{1/2} m_o = 1.6 \times 10^{-4} \left( \frac{m_o}{GeV} \right) \quad (48)$$

$$E_I = \lambda^{1/4} \phi e = 5.4 \times 10^{16} GeV \quad (49)$$

$$E_{Dinf} = \frac{c_2 g^2 \lambda^{-3/4}}{2} = \frac{16 \pi c_2 \langle \sigma \rangle m_o^2}{\lambda^{3/4}} \quad (50)$$

$$E_R = c_h h^2 \quad (51)$$

$$E_{gen} = c_{gen} g^2 = 32 \pi m_{gen} \langle \sigma \rangle m_o^2 \quad (52)$$

$$= 2.6 \times 10^6 \left( \frac{m_o}{GeV} \right)^2 GeV$$

$$E_{Ddm} = \frac{E_o^3 m_o}{c_{Ddm} g^2 \sigma} = \frac{32 \pi c_{Ddm} \rho_{dm} \langle \sigma \rangle}{(rT_o)^3 m_o} \quad (53)$$

$$= 10 \left( \frac{m_o}{GeV} \right) GeV$$

$$E_{dec} = c_{dec} T_F = \left( \frac{\pi^2 g_{rel}}{30} \right)^{1/4} \frac{m_o}{x_F} \quad (54)$$

$$= 0.1 \left( \frac{10}{x_F} \right) \left( \frac{m_o}{GeV} \right) GeV$$

$$E_{24} = \left( \frac{m_o^2 c_{24}}{2} \right)^{1/4} = \left( \frac{4}{\lambda} \right)^{1/4} m_o \quad (55)$$

V. SUMMARY AND CONCLUSIONS

We have presented a model where inflation and dark matter take place via a single scalar field $\phi$. This unification is realized via a quantum re-generation of the $\phi$ field at low energies. The late time appearance of $\phi$ solves the fine tuning problem of the initial condition of dark
matter density \( \Omega_{\text{dm}} \) at high energies in models without quantum re-generation, and allows for having a long lasting radiation dominated universe after reheating.

The unification scheme presented here has three parameters, the mass of the dark matter particle \( m_o \), the inflation parameter \( \lambda \) and the coupling \( g \) for the inflaton decay. Phenomenology sets the values for \( \lambda \) in terms of the mass of the dark matter particle \( m_o \), and the coupling \( g \) for the inflaton decay. We have shown that the same coupling strength that gives the inflaton decay gives the dark matter re-generation at low scales and sets in combination with \( m_o \) the wimp decoupling cross section. These same three parameters are present in models with inflation and a dark matter wimp particle but without unification. This implies that our unification scheme does not increase the number of parameters and it accomplishes the desired unification between inflaton and dark matter for free.

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**APPENDIX A: OTHER INTERACTION TERMS**

We will nos show that the terms \( L_{13} = g \phi \phi^3 \) and \( L_{31} = g \phi \phi^3 \) do not allow to implement the inflaton-dark matter unification scheme. The term \( L_{31} = g \phi \phi^3 \) does not allow \( \phi \) to decay into \( \phi \) if \( m_\phi \neq m_o \) and our working hypothesis has a massive inflaton field \( \phi \) and a relativistic field \( \varphi \), so that \( m_\phi \neq m_\varphi \). This can be easily seen since the decay process is \( \phi \rightarrow \varphi \) and energy momentum conservation implies that \( E_\phi = E_\varphi \) and \( E_3 = p_\varphi \) which requires the masses to be equal, \( m_\phi^2 = E_\phi^2 - p_\varphi^2 = E_\varphi^2 - p_\varphi^2 = m_\varphi^2 \).

On the other hand the term \( L_{13} = g \phi \phi^3 \) will not work because it gives a decaying dark matter particle. The decay process \( \phi \rightarrow \varphi + \phi + \varphi \) has a decay rate

\[
\Gamma_{13} = c_{13} g^2 m_\phi(\phi), \quad H = \sqrt{\frac{\rho_\phi}{3}} = \sqrt{\frac{V}{3}},
\]

with \( c_{13} = 1/((2\pi)^3144) \simeq 10^{-5} \) and we have used that the universe is dominated by \( \phi \) and \( \rho_\phi \sim V \), valid after inflation and matter domination. We have

\[
\frac{\Gamma_{13}}{H} = c_{13} g^2 \sqrt{\frac{3m_\phi^2}{V}}
\]

and \( \phi \) decays into \( \varphi \) if \( \Gamma/H > 1 \), i.e. as long as \( 3c_{13} g^2 m_\phi^2(t) > V(t) \). If \( m_o \) is constant, as for dark matter, and since \( H \propto \sqrt{V(t)} \propto 1/t \rightarrow 0 \) we have \( \Gamma/H \propto m_o/\sqrt{V} \propto t \rightarrow \infty \), i.e. we have an efficient decay. This is fine for reheating the universe at high energies but at low energies, when dark matter dominates, the same process takes place. This is clearly a problem since we know that the dark matter potential has a constant mass \( m_o \).

If we impose that \( \phi \) should not decay until present time than we must impose that \( \Gamma(t_o)/H(t_o) < 1 \) for \( t > t_o \). Taking a dark matter particle mass \( m_o > 10^{-18} \), i.e. \( m_o > 1 GeV \), and \( V_o = m_o^2 \phi_o^2/2 \simeq 10^{-120} \) we have \( \sqrt{m_\phi^2/V(\phi_o)} > 10^{42} \). Therefore, \( \Gamma/H < 1 \) gives a small coupling \( g^2 < 10^{-42}/c_{13} = 10^{-37} \). On the other hand, for the inflaton \( \phi \) to decay with such a small coupling we need \( \Gamma/H > 1 \) and an inflation potential \( V \lesssim c_{13} g^2 m_\phi^2 < 10^{-84} m_\phi^2 \) which for \( m_\phi < m_{pl} \) gives an inflation scale \( V < M eV^4 \). We can conclude that the term \( L_{13} \) does not allow for inflation-dark matter unification since it cannot give a high enough reheating scale without having a low energy dark matter decay.

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