Fermion-Higgs model with strong Wilson-Yukawa coupling in two dimensions

Wolfgang Bock\(^1\), Asit K. De\(^2,3,\#\), Erich Focht\(^2,3\)& and Jan Smit\(^1,\ast\)

\(^1\)Institute of Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands
\(^2\)Institute of Theoretical Physics E, RWTH Aachen, Sommerfeldstr., 5100 Aachen, Germany
\(^3\)HLRZ c/o KFA Jülich, P.O.Box 1913, 5170 Jülich, Germany

Abstract

The fermion mass spectrum is studied in the quenched approximation in the strong coupling vortex phase (VXS) of a globally $U(1)_L \otimes U(1)_R$ symmetric scalar-fermion model in two dimensions. In this phase fermion doublers can be completely removed from the physical spectrum by means of a strong Wilson-Yukawa coupling. The lowest lying fermion spectrum in this phase consists most probably only of a massive Dirac fermion which has charge zero with respect to the $U(1)_L$ group. We give evidence that the fermion which is charged with respect to that subgroup is absent in the VXS phase. When the $U(1)_L$ gauge fields are turned on, the neutral fermion may couple chirally to the massive vector boson state in the confinement phase. The outcome is very similar to our findings in the strong coupling symmetric phase (PMS) of fermion-Higgs models with Wilson-Yukawa coupling in four dimensions, with the exception that in four dimensions the neutral fermion does most probably decouple from the bosonic bound states.

\$\text{e-mail: bock@phys.uva.nl}$
\# e-mail: hkf212@djukfa11.bitnet
\& e-mail: hkf247@djukfa11.bitnet
\ast e-mail: jsmit@phys.uva.nl
1 Introduction

A non-perturbative formulation of a chiral gauge theory on the lattice has proved to be a difficult issue. In a chiral gauge theory naively transcribed to the lattice, each fermion is accompanied by fifteen ‘doubler fermions’ where eight of these couple as mirror fermions and spoil the chiral couplings. One way to deal with this problem is to decouple the unwanted extra species by rendering them very heavy.

The standard Wilson mass term, which is known to remove the doublers in the case of vector-like theories on the lattice, obviously breaks gauge invariance of the chiral gauge theory. A proposal to overcome this difficulty is the so-called Wilson-Yukawa approach [1, 2] which has recently received a lot of attention. Instead of a standard Wilson mass term one uses a so-called Wilson-Yukawa term which contains the Higgs fields in a way that it is manifestly invariant under the chiral gauge transformation. The goal of prime importance is now to try to decouple the unwanted species doublers by means of a strong Wilson-Yukawa coupling and to give them a mass of the order of the cutoff. It is well known that such a decoupling is a non-trivial and non-perturbative issue. For recent overview articles on the Wilson-Yukawa approach see refs. [3-9].

Recently extensive investigations of globally $G_L \otimes G_R$ symmetric (with $G_{L,R} = SU(2), U(1)$) fermion-Higgs models with Wilson-Yukawa coupling in four space-time dimensions have shown that it is rather unlikely that this method leads to the desired non-perturbative formulation of the standard model on the lattice. The reasons for this may be summarized as follows: The phase diagram contains, apart from weak coupling symmetric (PMW) and broken (FM(W)) phases where fermion masses behave according to perturbation theory, also strong coupling symmetric (PMS) and broken (FM(S)) phases where these masses exhibit a non-perturbative behavior. The Wilson-Yukawa approach fails in the weak coupling phases because there the masses of the doubler fermions are restricted by the triviality of Yukawa couplings and cannot be made sufficiently heavy [10]. They remain as additional particles in the spectrum.

On the other hand, in the strong coupling phases PMS and FM(S) the doubler fermion states can be removed completely from the particle spectrum by making them as heavy as the cut-off. In contrast to the weak coupling region, fermions become massive also in the PMS phase, a situation which differs already from that in the fermion-Higgs sector of the perturbative standard model where the fermion mass vanishes in the symmetric phase. Symmetry considerations show that the particle spectrum in the PMS phase can a priori contain a fermion which is neutral with respect to the $G_L$ group and a fermion which is charged with respect to that group [11-19]. The existence of the neutral fermion was confirmed by the good agreement of analytic and numerical calculations [14, 17, 19]. On the other hand a $1/w$ expansion ($w$ is the Wilson-Yukawa coupling) of the charged fermion propagator [20] and a numerical investigation of the fermion spectrum [13] gave evidence that the charged fermion does not exist as a particle in the spectrum, though this is not generally accepted [21]. Under the assumption that the charged fermion is absent one can show that the coupling of the neutral fermion to Higgs and gauge fields vanishes as a power of the lattice spacing $a$ and the neutral fermion becomes non-interacting in the scaling region [13]. It was also argued that the renormalized Yukawa coupling in the FM(S) phase vanishes as a power of $a$ rather than logarithm of $a$ [13, 14].

In this paper we extend the investigations to a $U(1)_L \otimes U(1)_R$ symmetric fermion-Higgs...
model with Wilson-Yukawa coupling in two dimensions. The important advantage of an investigation in two dimensions is that the simulations can be carried out on lattices of large linear dimension, enabling one to achieve large correlation lengths for the scalar fields with better control over finite size effects. The numerical data are of much superior quality than in the four dimensional case.

In two dimensions spontaneous breakdown of a continuous symmetry cannot occur because of the Mermin-Wagner-Coleman theorem [22]. In spite of that, fermions are observed to acquire a mass also in two dimensional fermion-scalar models. For example a $1/N$ expansion in the Gross Neveu models with a continuous chiral symmetry (which can be also viewed as fermion-scalar models after the introduction of an auxiliary scalar field) shows that the fermion mass does not vanish [23]. At a first glance this appears to be contradictory. The contradiction could be resolved by expressing the action in terms of new fermionic fields which can have a mass term without destroying the original chiral symmetry [24]. Interestingly, the existence of massive fermions in the strong coupling symmetric phase PMS in the four dimensional models with Wilson-Yukawa coupling may be viewed in a similar way [11, 12]. The new fermionic variables are here the above mentioned neutral and charged fermion fields which transform vectorially under the original chiral symmetry group and allow therefore for the construction of invariant mass terms.

The phase structure of the two dimensional model in the quenched approximation is similar to the one found before in the four dimensional models. The analogues of the weak coupling phases FM(W) and PMW are respectively a weak coupling spin-wave SW(W) and vortex VXW phase. The strong coupling phases FM(S) and PMS get replaced by strong coupling spin-wave SW(S) and vortex VXS phases. We find that fermions are massive in the strong coupling phases VXS and SW(S). The existence of the strong coupling phases is recently confirmed also by an investigation of a two dimensional U(1) fermion-Higgs model with dynamical naive fermions. There are, however, some indications favoring the absence of the VXW phase [25]. The weak coupling spin-wave SW(W) would in this case extend down to zero Yukawa coupling. If this is correct, the VXW phase has to be regarded as an artefact of the quenched approximation.

In this paper we study the fermion spectrum in the quenched approximation in the strong coupling vortex phase VXS whose existence is guaranteed in the full model with dynamical fermions. In this phase the species doublers can be completely removed from the spectrum and the generation of the fermion masses is based on the same mechanism as in the PMS phase of the four dimensional models. Also here the fermion spectrum can a priori consist of a neutral and a charged fermion. Similar to the four dimensional models we find strong indications for the absence of the charged fermion. The spectrum consists then only of the neutral fermion and the scalar particles. We shall show in sect. 7 that the neutral fermion may exhibit a chiral coupling to the vector boson state in the confinement phase after the gauge interactions are turned on.

The outline of the paper is as follows: In sect. 2 we introduce the model and describe its phase structure in the quenched approximation. Sect. 3 deals with the fermion spectrum in the strong coupling phases SW(S) and VXS. In sect. 4 we present the results of a leading order hopping expansion for the neutral and charged fermion propagators. After giving a brief report on the technical details of the numerical simulations in sect. 5, we compare in sect. 6 the numerical results for the rest energies of the charged and neutral fermion with
those obtained from the hopping expansion. Based on the results of sect. 6 we discuss in sect. 7 different outcomes for the physics in the VXS phase. A brief conclusion is given in sect. 8.

2 The model and its phase diagram

The model of interest is given by the following gauge invariant euclidean lattice action in two dimensions:

\[ S = \sum_x \mathcal{L} , \quad \mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} \]  

(2.1)

with

\[
\mathcal{L}_{\text{gauge}} = -\frac{1}{g^2} \text{Re} \, U_{12x},
\]

(2.2)

\[
\mathcal{L}_{\text{scalar}} = -\kappa \sum_{\mu=1}^{2} \left[ \Phi_{x}^* U_{x \mu} \Phi_{x+\hat{\mu}} + \Phi_{x+\hat{\mu}}^* U_{x \mu}^* \Phi_{x} \right] + \Phi_{x}^* \Phi_{x} + \lambda (\Phi_{x}^* \Phi_{x} - 1)^2 ,
\]

(2.3)

\[
\mathcal{L}_{\text{fermion}} = \frac{1}{2} \sum_{\mu=1}^{2} \overline{\psi} \gamma_{\mu} \left[ (D_{\mu}^+ + D_{\mu}^-) P_{L} + (\partial_{\mu}^+ + \partial_{\mu}^-) P_{R} \right] \psi + g \overline{\psi} (\Phi P_{R} + \Phi^* P_{L}) \psi
\]

\[ - w \left[ \overline{\psi} \Phi \right] P_{R} \sum_{\mu=1}^{2} \partial_{\mu}^+ \partial_{\mu}^- \psi + \overline{\psi} P_{L} \sum_{\mu=1}^{2} \partial_{\mu}^+ \partial_{\mu}^- (\Phi^* \Phi) \right] ,
\]

\[ \]  

(2.4)

where in the second line of eq. (2.4) we have included the Wilson-Yukawa coupling with strength \( w \). Besides the Wilson-Yukawa term we also included a usual Yukawa term. The symbols \( D_{\mu}^+ \), \( D_{\mu}^- \), \( \partial_{\mu}^+ \) and \( \partial_{\mu}^- \) denote the covariant and normal lattice derivatives which are defined by the relations \( D_{\mu}^+ \psi_x = U_{\mu x} \psi_{x+\hat{\mu}} - \psi_x, \) \( D_{\mu}^- \psi_x = \psi_{x-\hat{\mu}} - U_{\mu x}^* \psi_x, \) \( \partial_{\mu}^+ = D_{\mu}^+ |_{U=1} \) and \( \partial_{\mu}^- = D_{\mu}^- |_{U=1} \). The symbols \( \gamma_{\mu}, \mu = 1,2 \) denote the two dimensional \( \gamma \) matrices and \( P_{R,L} = \frac{1}{2} (\mathbb{1} \pm \gamma_5) \) with \( \gamma_5 = -i \gamma_1 \gamma_2 \) are the right and left-handed chiral projectors. The field \( U_{12x} \equiv U_{x,x+1} U_{x+1,x+2} U_{x+2,x} \) is the usual plaquette variable in the Wilson action, \( g \) is the gauge coupling, \( \kappa \) is the hopping parameter for the scalar field and \( \lambda \) the quartic coupling. If we succeed in removing the species doublers from the particle spectrum by means of a strong Wilson-Yukawa coupling \( w \), one would expect that the lattice lagrangian defined by eq. (2.1) reproduces in the continuum limit the target model given by the lagrangian

\[ \mathcal{L}_0 = \frac{1}{4} F_{\mu \nu} F_{\mu \nu} + D_{\mu} \phi^* D^\mu \phi + m_0^2 \phi^* \phi + \lambda_0 (\phi^* \phi)^2 \]

\[ + \bar{\psi} (\vec{\partial} P_{L} + \vec{\partial} P_{R}) \psi + y_0 \bar{\psi} (\phi P_{R} + \phi^* P_{L}) \psi , \]  

(2.5)

where \( F_{\mu \nu}(x) = \partial_{\mu} A_{\nu}(x) - \partial_{\nu} A_{\mu}(x) \) and \( D_{\mu} = \partial_{\mu} - ig_0 A_{\mu}(x) \). We will demonstrate in this paper that this naive expectation is not correct and that the effective lagrangian which describes the physics in the VXS phase differs substantially from eq. (2.5). The continuum fields \( \psi(x), \phi(x), A_{\mu}(x) \) in eq. (2.3) are related to the corresponding lattice fields in (2.1) by the transformations

\[ \Psi_x = a^{1/2} \psi(x) , \quad \Phi_x = \frac{1}{\sqrt{\kappa}} \phi(x) , \quad U_{\mu x} = \exp(-i g_0 A_{\mu x}) . \]  

(2.6)

The coupling parameters in eq. (2.3) can be expressed in terms of the lattice couplings by the relations

\[ m_0^2 = \frac{1 - 2 \lambda - 4 \kappa}{a^2 \kappa} , \quad y_0 = \frac{y}{a \sqrt{\kappa}} , \quad g_0 = \frac{g}{a} , \quad \lambda_0 = \frac{\lambda}{(a \kappa)^2} . \]  

(2.7)
We note that in the continuum formulation the Yukawa coupling, the gauge coupling and the quartic coupling carry a dimension, whereas the gauge and scalar fields are dimensionless. Throughout this paper we will study the model in the limit \( \lambda \to \infty \) which implies that the scalar fields become radially frozen, i.e. \( \Phi^* \Phi = 1 \).

The lattice lagrangian in eq. (2.1) is invariant under the local gauge transformations of the form \( \Psi_{L,x} \to \Omega_{L,x} \Psi_{L,x}, \Psi_{R,x} \to \Omega_{R,x} \Psi_{R,x}, \Phi_x \to \Omega_{L,x} \Phi_x, U_{\mu x} \to \Omega_{L,x} U_{\mu x} \Omega_{L,x + \hat{\mu}}^*, \) with \( \Omega_{L,x} \in U(1)_L \). The model is furthermore also invariant under the global gauge transformations \( \Psi_{R,x} \to \Omega_R \Psi_{R,x}, \Psi_{R,x} \to \Omega_R \Psi_{R,x}, \Phi_x \to \Phi_x \Omega_R^* \) with \( \Omega_R \in U(1)_R \). At \( y = 0 \) the action (2.1) possesses a shift symmetry for the right-handed fermion fields,

\[
\Psi_{R,x} \to \Psi_{R,x} + \epsilon_R, \quad \Psi_{R,x} \to \Psi_{R,x} + \bar{\epsilon}_R,
\]

where \( \epsilon_R \) and \( \bar{\epsilon}_R \) are the constant shifts in the right-handed Weyl spinors. This symmetry guarantees that the mass of the fermion with the quantum number \( s \) of the \( \Psi_R \) fermion field vanishes at \( y = 0 \) for all values of \( \kappa \) and \( g \) [26].

Although we have included for later convenience in (2.1) also the interactions to the \( U(1)_L \) gauge fields, in our numerical work, however, we will restrict ourselves to the global limit \( g = 0 \) where \( U_{\mu x} = 1 \) and the local \( U(1)_L \) gauge symmetry turns into a global one. In this case eq. (2.3) (with \( \lambda = \infty \)) reduces to the lagrangian of the XY model in two dimensions. We shall furthermore study the model (2.1) in the quenched approximation where the effects of the fermion determinant are neglected. The use of the quenched approximation for the investigation of the fermion spectrum in strong coupling phase VXS is justified since this phase was established also in the full model with dynamical fermions [25].

Next we turn to the phase structure of the model in the quenched approximation. The XY model is known to have a phase transition at \( \kappa = \kappa_c \approx 0.56 \) which separates a vortex (VX) phase \( (\kappa < \kappa_c) \) with finite scalar correlation length from a spin-wave (SW) phase where the scalar correlation length is infinite [27]. We note that the spectrum in the VX phase consists of two scalar particles which have the same mass. In the quenched approximation fermions do not have a feedback on the scalar sector and \( \kappa_c \) is independent of \( y \) and \( w \). A phase transition may, however, occur in the fermionic sector. Such a phase transition or ‘crossover’ was discovered before in four dimensional models with Wilson-Yukawa coupling at \( y + 4w \approx \sqrt{2} \) [28, 29]. It separates the weak coupling phases PMW and FM(W) \( (y + 4w \approx \sqrt{2}) \) from the strong coupling phases PMS and FM(S) \( (y + 4w \approx \sqrt{2}) \). The existence of this ‘crossover’ has manifested itself in a different behavior of the fermion mass as a function of \( \kappa \) in the weak and strong coupling regimes.

In order to monitor the fermionic phase structure of the quenched model in two dimensions we have computed the mass \( m_F \) of the \( \Psi_R \) fermion by fitting the \( \langle \Psi_R \overline{\Psi_R} \rangle \) propagator to a free Wilson fermion propagator ansatz (for more information about the technical details see sect. 5). In fig. 1 we have displayed this mass as a function of \( \kappa \) for several values of the Yukawa coupling \( y \) and for the special case of naive fermions \( (w = 0) \). As in four dimensions the fermion mass shows a qualitatively different behavior at small and large values of \( y \). It decreases when approaching the SW-VX phase transition in the SW phase for \( y < y^* \approx 1 \).

On the contrary the mass is observed to increase when the value of \( \kappa \) is lowered for \( y > y^* \). This increase is seen to continue into the VX phase and fermions become massive in that phase. A similar behavior of the physical and doubler fermion mass as a function of \( \kappa \) is observed also for \( w > 0 \). From the different behavior of the fermion mass we can localize
the position of a ‘crossover’. This position is within the precision of our resolution independent of $\kappa$ and is approximately given by the relation $y + 2w = 1$. As in four dimensions the ‘crossover’ sheet splits the VX and SW phases into strong (S) and weak (W) coupling regions, which we will denote in the following by VXS, SW(S) ($y + 2w \gtrsim 1$) and VXW, SW(W) ($y + 2w \lesssim 1$). As we mentioned already in the introduction, the VXW phase does not seem to be present in the full model with dynamical fermions and could be an artefact of the quenched approximation \cite{25}. However, this does not affect our investigations in the VXS phase.

Our numerical results on the $48^2$ lattice show that the fermion mass stays non-zero everywhere in the SW phase and also in the VXS phase though the chiral symmetry cannot be broken according to the Mermin-Wagner-Coleman theorem. Studies on different lattices show that the finite size dependence of the fermion mass is extremely small and it appears very unlikely that the fermion mass could vanish in the infinite volume limit. The existence of a massive neutral fermion in the strong coupling phases SW(S) and VXS is indeed strongly supported by the good agreement between analytic calculations which are based on strong coupling expansions and the results of the numerical simulation. We will report more on these results in the following sections of this paper.
The fact that fermions may be massive within the SW phase although the chiral symmetry is unbroken in that phase was to our knowledge first explained in ref. [24]. The basic idea is as follows: The original action in terms of the $\Psi$ fields does not provide a correct description of the physics in the SW phase. It can, however, be rewritten in terms of new fermionic field variables, which transform vectorially under the original symmetry transformations thereby allowing for the construction of a chirally invariant mass term. This new form of the action may give a more appropriate description of the physics in this phase (in the sense of a weak coupling expansion) if the fermions are indeed observed to be massive.

3 The fermion spectrum in the strong coupling phases

Since the $U(1)_L \otimes U(1)_R$ symmetry is unbroken everywhere, the states excited by the fields $\Psi_L$ and $\Psi_R$ need not be the same since the corresponding fields carry different quantum numbers under the unbroken symmetry group. We will refer to the $\Psi_R = \Psi^{(n)}_R$ field as the neutral ($n$) fermion field since it has charge zero under the $U(1)_L$ group ($qL = 0$). The $\Psi_L = \Psi^{(c)}_L$ field will be called charged ($c$) fermion field since it has charge one under the local $U(1)_L$ gauge group ($qL = 1$). When discussing the phase diagram in the previous section we have already mentioned that the numerical results give strong evidence that the $\Psi^{(n)}_R$ fermion becomes massive both in the SW(S) and VXS phases. This will be substantiated later by the good agreement of the numerical results with a hopping expansion. In order to describe this situation the $\Psi^{(n)}_R$ field may be regarded as the right-handed component of a massive Dirac fermion field $\Psi^{(n)}$. A possible choice for $\Psi^{(n)}_L$ is the composite field $\Phi^* \Psi_L$ which transforms in the same manner as $\Psi^{(n)}_R$. The neutral Dirac fermion field may then be written as

$$\Psi^{(n)} = (\Phi^* P_L + P_R) \Psi, \quad \Psi^{(n)} = (\Phi P_R + P_L) \Psi.$$  (3.1)

Along the same lines we may also introduce a charged Dirac fermion field

$$\Psi^{(c)} = (P_L + \Phi P_R) \Psi, \quad \Psi^{(c)} = (P_R + \Phi^* P_L) \Psi.$$  (3.2)

The fields $\Psi^{(c)}$ and $\Psi^{(n)}$ transform vectorially under $U(1)_L$ and $U(1)_R$ respectively.

On a finite lattice the long range fluctuations in the SW phase cause a non-vanishing value of the magnetization $M$ which may be defined by the relation

$$M = \langle \frac{1}{V} \sum_{\mathbf{x}} \Phi_{\mathbf{x}} \rangle_{\text{rot}},$$  (3.3)

where $V$ is the lattice volume. The index “rot” means that each configuration is rotated such that $\frac{1}{V} \sum_{\mathbf{x}} \Phi_{\mathbf{x}}$ points into a given direction. This rotation is necessary since on a finite lattice the magnetization $M$ is drifting through the group space and when averaging over many configurations one would get zero. A measure for the magnetization in the SW phase may then be defined by $v = |M|$. Since there is no spontaneous symmetry breaking in two dimensional systems, this quantity has to vanish in the infinite volume limit $V \to \infty$. Some typical values for $v$ on a $48 \times 48$ lattice are given by $v(\kappa) = 0.3545(44), 0.5565(22), 0.6734(12)$ for $\kappa = 0.48, 0.52, 0.60$. Even on large lattices (e.g. $400^2$) $v$ is clearly non-zero in the SW phase and increases when raising the values of $\kappa$. Therefore, on a finite lattice the situation in the SW phase is very similar to the broken phase in the four dimensional model where the $U(1)_L \otimes U(1)_R$ symmetry is broken to the diagonal subgroup $U(1)_{L=R}$. As a consequence the
fields $\Psi^{(n)}$ and $\Psi^{(c)}$ appear to behave almost as equivalent interpolating fields. Indeed, the numerically found rest energies obtained from the neutral and charged fermion propagators coincide in that phase within the statistical errors. In the infinite volume limit, however, the two rest energies are expected in general to be different for $w > 0$.

## 4 Hopping expansion for the neutral and charged fermion propagators

An appropriate method to try to find analytic expressions for the neutral and charged fermion propagators in the strong coupling phases is the hopping expansion. The hopping expansion deals only with the fermionic integration in the path integral, the bosonic integration has to be performed e.g. by a $1/d$ expansion \[15\] or by numerical simulations. When starting from the lagrangian (2.4) with the single-site Yukawa-coupling and expanding the Boltzmann factor in the path integral in powers of the hopping parameter $\alpha = 1/(y + 2w)$ one comes upon cancellations of the type $\Phi^* \Phi \rightarrow 1$, emerging from the single-site terms and the one-link terms. The hopping expansion becomes more transparent after removing the $\Phi$ fields from the single-site Yukawa term by means of a unitary transformation to the one link terms. Two transformations of this type are given by the inverses of eqs. (3.1) and (3.2) which express the neutral and charged fermion fields in terms of the original $\Psi$ fields. The associated jacobian for these transformations is in both cases equal to one since the scalar field is radially frozen. Replacing $\Psi$ and $\overline{\Psi}$ in (2.4) by $\Psi^{(n)}$ and $\overline{\Psi}^{(n)}$ gives

\[
\mathcal{L}_F = \frac{1}{2} \sum_{\mu=1}^{2} \left[ \overline{\Psi}^{(n)} \Phi^*(D^+_\mu + D^-_{\mu})(\Phi \Psi^{(n)}) + \overline{\Psi}^{(n)} \Phi^*(D^+_\mu + D^-_{\mu})\Psi^{(n)} \right] \\
+ y \overline{\Psi}^{(n)} \Psi^{(n)} - \frac{w}{2} \sum_{\mu=1}^{2} \partial^+_{\mu} \partial^-_{\mu} \Psi^{(n)}. \tag{4.1}
\]

This substitution transforms the Yukawa term into a bare mass and the Wilson–Yukawa term into a free Wilson term. Expressing the $\Psi$ and $\overline{\Psi}$ fields in terms of the $\Psi^{(c)}$ and $\overline{\Psi}^{(c)}$ fields leads to

\[
\mathcal{L}_F = \frac{1}{2} \sum_{\mu=1}^{2} \left[ \overline{\Psi}^{(c)} \Phi^*(D^+_\mu + D^-_{\mu})\Psi^{(c)} + (\overline{\Psi}^{(c)} \Phi)(\partial^+_{\mu} + \partial^-_{\mu})\Phi^*(\Psi^{(c)}) \right] \\
+ y \overline{\Psi}^{(c)} \Psi^{(c)} - \frac{w}{2} (\overline{\Psi}^{(c)} \Phi) \sum_{\mu=1}^{2} \partial^+_{\mu} \partial^-_{\mu} \Phi^*(\Psi^{(c)}) \tag{4.2}. \]

The lagrangian (4.1) has a shift symmetry (2.8) in terms of the neutral field because $\Psi^{(n)}_R = \Psi_R$. Such a symmetry, however, is absent for the action (4.2) in terms of the $\Psi^{(c)}$ fields. This different behavior under the shift symmetry holds also if the local U(1)$_L$ gauge interactions are turned off. It makes therefore sense to distinguish between the charged and the neutral fermion also in this globally symmetric U(1)$_L \otimes$U(1)$_R$ theory.

Using the lagrangians (4.1) and (4.2) one finds to lowest order in $\alpha$ the following expressions for the charged and neutral fermion propagators in momentum space,

$$S^{(n)}(k) \equiv \left\langle \frac{1}{V} \sum_{x,y} \Psi^{(n)}_x \overline{\Psi}^{(n)}_y e^{ik(x-y)} \right\rangle$$
\[ z^2 = \langle \text{Re}(\Phi_x^* U_{\mu x} \Phi_{x+\mu}) \rangle \] (4.5)

is the scalar field link expectation value. This quantity has a non-vanishing value in the VX as well as SW phase.

From the expressions (4.3) and (4.4) one can read off expressions for the fermion masses. For the masses of the neutral fermion and its species doublers we obtain

\[ m_F^{(n)} \approx yz^{-1}, \quad m_D^{(n)} \approx m_F^{(n)} + 2wlz^{-1}, \quad l = 1, 2, \] (4.6)

where \( l \) is the number of momentum components equal to \( \pi \) in the two dimensional Brillouin zone. From eq. (4.6) we can read off an effective Wilson \( r \)-parameter:

\[ r^{(n)} \approx wz^{-1}. \] (4.7)

As we shall see later, these expressions are in good agreement with the numerical simulations in the VXS phase. This was also found to be the case in the four dimensional models [14, 17, 19].

In agreement with the shift symmetry [26] the mass \( m_F^{(n)} \) of the physical fermion is seen to vanish in the limit \( y \to 0 \). The doubler fermions, however, are non-zero within the strong coupling phases, since \( 2wlz^{-1} \neq 0 \) everywhere in this region. This means that in the continuum limit the species doublers for the neutral fermion decouple from the particle spectrum within this phase.

Similar formulas can be obtained from eq. (4.4) for the masses of the charged fermion (assuming it exists for the moment) and its species doublers

\[ m_F^{(c)} \approx (y + 4w)z^{-1} - 4wz, \quad m_D^{(c)} \approx m_F^{(c)} + 2wzl, \quad l = 1, 2. \] (4.8)

The effective Wilson parameter is now given by

\[ r^{(c)} \approx wz. \] (4.9)

The discussion in the following section will show that the formulas (4.8) and (4.9) for the charged fermion are in disagreement with the numerical results. A calculation of higher order terms in four dimensional models showed that they appear to be small for the neutral propagator \( S^{(n)} \), but not for the charged propagator \( S^{(c)} \) [18, 19]. On the basis of our numerical results we will give in sect. 7 an argument why the hopping expansion to lowest orders leads to a wrong result for charged fermion propagator. In sect. 6 we will compare the analytic formulas that have been derived in this section with the results of the numerical simulation.
5 Details of the numerical simulation

The neutral and charged fermion propagators were determined by inverting the corresponding fermion matrices on a set of uncorrelated scalar field configurations which were generated by means of the reflection cluster algorithm [30] for the XY model. We have computed the fermion propagators in coordinate space. As an example we give the expression for the RR component of neutral fermion propagator

$$S^{(n)}_{RR}(t) = \left\langle \frac{1}{L} \sum_{x_1} \Psi^{(n)}_{x_1,x_2} \overline{\Psi^{(n)}_{y_1,y_2}} e^{ip_1(x_1-y_1)} \right\rangle, \quad t = |x_2 - y_2|,$$

where $t = 1, \ldots, T$. The symbols $L$ and $T$ denote here and in the following the spatial and time extent of a rectangular lattice. The physical fermion propagator is obtained for $p_1 = 0$ and the propagator of the lowest lying doubler fermion for $p_1 = \pi$. The fermion and the doubler fermion propagators were computed for all four $L$-$R$ combinations. We have used for the fermion fields periodic boundary conditions in the spatial direction and anti-periodic boundary conditions in the time direction. The scalar fields had periodic boundary conditions in all directions.

For the neutral fermion propagator we have inverted the fermion matrix on typically 1000-2000 scalar field configurations. A problem which we were confronted with in the four dimensional models was the large number of matrix inversions which was required to obtain a stable signal for the charged fermion propagator in the PMS phase. In the two dimensional models it is possible to enlarge the statistics in the VXS phase on relatively large lattices (e. g. $32^2$) by an order of magnitude. For the determination of the charged fermion propagator we have inverted the fermion matrix on typically 1000-5000 and deeper in the VXS phase on 20000 scalar field configurations.

Most of the results were obtained on a $32 \times 32$ lattice. In order to estimate the finite size effects we varied $L$ at a particular point in the VXS phase from 16 to 64 while keeping $T$ fixed at 64.

We have fitted the neutral and charged propagators at zero spatial momentum to the free Wilson fermion ansatz

$$S^{(n,c)}(t) \rightarrow \frac{Z}{2\sqrt{1 + 2r_l m_l + m_l^2}} \times \left[ \frac{\exp(-E_l t) + \zeta \exp(-E_l(T - t))}{1 + \exp(-E_l T)} - \zeta(-1)^t \frac{\exp(-E'_l t) + \zeta \exp(-E'_l(T - t))}{1 + \exp(-E'_l T)} \right],$$

where $\zeta = 1(-1)$ for the RR and LL (RL and LR) components. This ansatz holds only for $r_l < 1$, for $r_l > 1$ the oscillating factor $(-1)^t$ has to be omitted.

From this fit we obtain the numerical values for the rest energies $E_l$ and $E'_l$ of the fermion and its ‘time doubler’ and for the wave-function renormalization constant $Z$. The subscript $l$ is 0 for the physical fermion propagator ($p_1 = 0$ in eq. (5.1)) and 1 for the doubler fermion propagator ($p_1 = \pi$). The masses $m_l$ and the Wilson parameters $r_l$ for which we obtained expressions in the previous section are related to the rest energies $E_l$ and $E'_l$ by the lattice
The physical charged fermion propagator $S^{(c)}_{RL}(t)$ ($p_1 = 0$) as a function of the time coordinate $t$ for $(\kappa, y, w) = (0.45, 0.3, 2.0)$. The lattice size is $32 \times 64$. The solid line was obtained by fitting $S^{(c)}_{RL}(t)$ to the free Wilson fermion ansatz in eq. (5.2).

The effective Wilson parameter $r_l$ could be in principle a function of $l = 1, 2$. The numerical results, however, show that $r_l$ is independent of $l$ (therefore we will subsequently use the notation $r = r_l$), which supports the interpretation of the numerical results in terms of the free fermion formula.

We find that the rest energies obtained from the four chiral components $S^{(n)}_{LL}(t)$, $S^{(n)}_{RR}(t)$, $S^{(c)}_{RL}(t)$ and $S^{(c)}_{LR}(t)$ for $p_1 = 0, \pi$ agreed always within the statistical errors. The same holds also for the rest energies determined from the four chiral components of the charged fermion propagator. In the following we will use the notation $E^{(n)}_F$ and $E^{(c)}_F$ for the rest energies of the neutral and charged physical fermion and similarly $E^{(n)}_D$ and $E^{(c)}_D$ for the rest energies of the lowest lying doubler fermions.

In fig. 2 we have displayed as an example the charged fermion propagator $S^{(c)}_{RL}(t)$ for $p_1 = 0$ as a function of $t$. In this example we have chosen $L = 32$ and $T = 64$. The solid curve was obtained by fitting the numerical data to the free Wilson fermion ansatz (5.2). The high quality of the numerical results for the propagators allowed for an accurate determination of the rest energies $E^{(n)}_F$, $E^{(n)}_D$, $E^{(c)}_F$ and $E^{(c)}_D$. 

The dispersion relations

$$e^{E_{l}} = \frac{\sqrt{1 + 2r_l m_l + m_l^2 + r_l + m_l}}{1 + r_l}, \quad e^{E'_{l}} = \frac{\sqrt{1 + 2r_l m_l + m_l^2 + r_l + m_l}}{1 - r_l}. \quad (5.3)$$
For the considerations in sect. 7 we have to know also the numerical values for the rest energy $E_\Phi$ of the scalar particles in the VX phase. This rest energy was determined numerically by fitting the scalar propagator in momentum space defined by

$$G_\Phi(p) = \left\langle \frac{1}{2V} \sum_{x,y} \Phi^*_x \Phi_y e^{ip(x-y)} \right\rangle$$

(5.4)

to a free scalar propagator ansatz

$$G_\Phi(p) \rightarrow \frac{Z_\Phi}{p^2 + m_\Phi^2},$$

(5.5)

where $p^2 = 2 \sum_{\mu=1}^2 (1 - \cos p_\mu)$ is a lattice equivalent of the momentum squared in the continuum. The rest energy $E_\Phi$ of the scalar particles in the VX phase is obtained from the lattice dispersion relation at zero momentum, $\cosh E_\Phi = 1 + \frac{m_\Phi^2}{2}$.

6 Comparison of the numerical results with the hopping expansion

In this section we are going to compare the numerically found values for the rest energies $E_F^{(n)}$ and $E_F^{(c)}$ of the neutral and the charged fermion and the corresponding rest energies $E_D^{(n)}$ and $E_D^{(c)}$ for the lowest lying doubler fermions with the analytic expressions which result from the formula (5.3) after inserting eqs. (4.6), (4.7), (4.8) and (4.9) from the lowest order hopping expansion. For $z^2$ we use the numerical value measured on the same lattice which we are using for the determination of the fermion propagators. In fig. 3 we have displayed the numerical values for $E_F^{(n)}$, $E_D^{(n)}$, $E_F^{(c)}$ and $E_D^{(c)}$ as a function of $y$ for $\kappa = 0.4$ and $w = 2.0$. The coupling parameter values we have chosen here lie well inside the VXS phase. The results from the hopping expansion are represented by the curves. The dashed, solid, dash-dotted and dotted lines correspond respectively to the rest energies $E_F^{(n)}$, $E_D^{(n)}$, $E_F^{(c)}$ and $E_D^{(c)}$. The figure shows that the agreement between the numerical result and the analytic prediction is quite impressive for the rest energies $E_F^{(n)}$ and $E_D^{(n)}$ while the curves for $E_F^{(c)}$ and $E_D^{(c)}$ exhibit a strong deviation from the numerical results. In the case of $E_F^{(c)}$ the deviation is larger than a factor two. The figure shows furthermore that $E_F^{(n)}$ appears to vanish in the limit $y \rightarrow 0$, in agreement with the shift symmetry mentioned before, whereas $E_D^{(n)}$ stays non-zero for all different values of $y$ which implies the decoupling of the species doublers of the neutral fermion in the continuum limit. Also the numerical values for $E_D^{(c)}$ are larger than one for all values of $y$ with no indication of dropping in the limit $y \rightarrow 0$. Thereby also the species doublers of the charged fermion can be completely removed from the physical spectrum. Provided it exists at all as a particle in the spectrum, the charged fermion is massive in the VXS phase.

In fig. 4 the rest energies $E_F^{(n)}$ and $E_F^{(c)}$ are plotted as a function of $\kappa$ where the coupling parameters $y$ and $w$ were fixed to 0.4 and 2.0. The analytic results for $E_F^{(n)}$ and $E_F^{(c)}$ are represented also in this figure by the dashed and dash-dotted lines. We find again that analytic expressions from the hopping expansion provide a good description of the rest energy of the neutral, but not of the charged fermion. The other details in this figure will be explained in the next section where we will develop, on the basis of the results of this section, two different scenarios for the physics in the strong coupling region.
Figure 3: The rest energies $E_F^{(n)}$, $E_D^{(n)}$, $E_F^{(c)}$ and $E_D^{(c)}$ as a function of $y$ for $\kappa = 0.4$ and $w = 2.0$. The dashed, solid, dash-dotted and dotted curves represent respectively the analytic results for $E_F^{(n)}$, $E_D^{(n)}$, $E_F^{(c)}$ and $E_D^{(c)}$ obtained from the hopping expansion.

7 Scenarios for an effective field theory in the strong coupling regime

The results of the previous section showed that the analytic results deduced from the lagrangian (4.1) are in good agreement with the numerical results for the rest energies $E_F^{(n)}$ and $E_D^{(n)}$, whereas the lagrangian (4.2) led to expressions which were in a strong disagreement with the numerical data. This suggests that the physics in the strong coupling region is well described by the lagrangian (4.1) in terms of the neutral fermion fields. The charged fermion fields $\Psi^{(c)} = \Phi\Psi^{(n)}$ and $\overline{\Psi}^{(c)} = \overline{\Psi}^{(n)}\Phi^*$ can then be regarded as composite fields and the charged fermion, provided it exists at all in the particle spectrum, can be considered as a bound state of the scalar particle and the neutral fermion. The question arises now whether the interactions in eq. (4.1) can produce such a $\Phi-\Psi^{(n)}$ bound state. In the four dimensional model the scalar fields $\Phi$ carry a dimension of a mass and as a consequence of this the four-point coupling

$$\frac{1}{2} \sum_{\mu=1}^{d} (\overline{\Psi}_L^{(n)}\Phi^*)\gamma_\mu (D^\mu_+ + D^-_\mu)(\Phi\Psi_L^{(n)})$$

(7.1)

with $d = 4$ vanishes in the classical continuum limit like $a^2$ which makes the formation of a $\Phi-\Psi^{(n)}$ bound state already very unlikely. Indeed a $1/w$ expansion [21] and the numerical
Figure 4: The rest energies $E^{(n)}_F$ and $E^{(c)}_F$ plotted against $\kappa$ for $y = 0.4$ and $w = 2.0$. The dashed and dash-dotted curves represent the analytic results from the hopping expansion for $E^{(n)}_F$ and $E^{(c)}_F$. The solid line gives the result for the sum $E^{(n)}_F + E_\Phi$. The error bars for $E^{(n)}_F + E_\Phi$ are much smaller than the symbol sizes for $E^{(c)}_F$.

Simulations [14] gave strong evidence for the absence of the charged fermion in the particle spectrum of the PMS phase. In the case of the two dimensional model the scalar fields are dimensionless and for $d = 2$ the coupling (7.1) does not vanish as a power of the lattice spacing $a$. Therefore the formation of a $\Phi-\Psi^{(n)}_L$ bound state appears at the first glance to be more favored than in the four dimensional model. If the four-point interaction (4.1) is strong enough to produce a $\Phi-\Psi^{(n)}_L$ bound state we expect to find the following relation among the rest energies of the neutral and the charged fermion and of the scalar particles in the VXS phase

$$E^{(c)}_F = E^{(n)}_F + E_\Phi + \epsilon_B, \quad \epsilon_B < 0,$$

(7.2)

where the quantity $\epsilon_B$ denotes the binding energy of the $\Phi-\Psi^{(n)}_L$ bound state. This relation implies that the charged fermion could only exist as a particle at a point in the coupling parameter space where the rest energies $E^{(c)}_F$, $E^{(n)}_F$ and $\epsilon_B$ scale simultaneously to zero. This can happen only at the point $\kappa = \kappa_c$, $y = 0$, since only there $E^{(n)}_F$ and $E_\Phi$ can vanish simultaneously.

**Scenario A:** Let us assume now for the moment that $\epsilon_B$ scales to zero like $E^{(n)}_F$ and $E_\Phi$ in the limit $\kappa \not\to \kappa_c$, $y \to 0$ and the charged fermion exists together with the neutral fermion as a Dirac fermion in the particle spectrum. The coupling of the $\Psi^{(c)}_L$ field to the gauge fields is necessarily vectorial because the charged fermion is massive ($m^{(c)} > 0$). The model we end
Table 1: The rest energies $E_\Phi$, $E_F^{(n)}$, $E_F^{(c)}$ and $\epsilon_B$ as a function of the spatial extent $L$ of a lattice with volume $L \times 64$ obtained at the point $(\kappa, y, w) = (0.45, 0.3, 2.0)$. The error bars for $E_F^{(n)}$ were omitted since they are smaller than 0.001.

| $L$  | $E_\Phi$  | $E_F^{(n)}$ | $E_F^{(c)}$ | $\epsilon_B$ |
|------|------------|-------------|-------------|-------------|
| 16   | 0.126(3)   | 0.275       | 0.368(13)   | -0.033(16)  |
| 32   | 0.115(3)   | 0.277       | 0.374(9)    | -0.018(12)  |
| 48   | 0.120(5)   | 0.276       | 0.379(12)   | -0.017(17)  |
| 64   | 0.119(5)   | 0.277       | 0.389(11)   | -0.007(16)  |

Scenario B: In order to figure out whether the above requirements for the binding energy are fulfilled we have determined $\epsilon_B$ numerically in a wide range of the bare parameters in the VXS phase. The details about the numerical determination of the rest energy $E_\Phi$ were given in sect. 5. In the figs. 4 and 5 we compare the rest energy $E_F^{(c)}$ with the sum $E_F^{(n)} + E_\Phi$ which in these graphs is represented by the solid lines. For $E_F^{(n)}$ we have used the results from the hopping expansion which are in perfect agreement with the actual data, as we reported in sect. 6. The error bars for the sum $E_F^{(n)} + E_\Phi$ are always much smaller than the symbol sizes for $E_F^{(c)}$. Both plots indicate that $\epsilon_B = 0$ for all values of $\kappa$ and $y$ in the VXS phase. This suggests that the formation of a bound state is very unlikely. Fig. 5 shows that also for the lowest lying doubler fermion the numerical results for the rest energy $E_F^{(c)}$ are nicely represented by the sum $E_F^{(n)} + E_\Phi$ (upper solid line). In order to get an impression about the finite size dependence of the energies $E_F^{(n)}$, $E_F^{(c)}$, $E_\Phi$ and $\epsilon_B$ we have computed the $S^{(n)}$, $S^{(c)}$ and $G_\Phi$ propagators at a fixed point in VXS phase on a sequence of lattices with size $L \times 64$, where spatial extent $L$ was varied in a range from 16 to 64. The results for $E_F^{(n)}$, $E_F^{(c)}$, $E_\Phi$ and $\epsilon_B$ are summarized in table 1. It can be seen also here that the binding energies are very small and even on the smallest lattices almost compatible with

up with in the strong coupling VX phase is clearly different from the original target model given in eq. (2.7), although we succeeded in removing the species doublers from the spectrum. One possible form of an effective action in the VXS phase is given by the expression

$$
L^{eff}_F = \bar{\psi}^{(n)}(\phi_\Psi^{(n)} + \phi_\Phi^{(c)}) + m^{(n)}\bar{\psi}^{(n)}\psi^{(n)} + m^{(c)}\bar{\psi}^{(c)}\psi^{(c)} + y_R[\bar{\psi}_R^{(n)}\phi^*_L + \bar{\psi}_L^{(c)}\phi_R^{(n)}],
$$

where we used the concise continuum notation of eq. (2.5) and all fields and coupling parameters are considered to be effective. The symbol $y_R$ denotes the renormalized Yukawa coupling. The failure of the Wilson-Yukawa approach in giving a chiral gauge theory is related to the fact that the original fermion fields $\Psi_R$ and $\Psi_L$ combine with composite ‘mirror’ fermion fields $\chi_R \equiv \phi_\Psi R$ and $\chi_L \equiv \phi_\Psi L$ and form the two massive Dirac fields $\Phi^{(n)}$ and $\Phi^{(c)}$ which would couple vectorially to “external” gauge fields. The model (7.3) is a special case of the mirror fermion model [31], when transcribed to the case of two dimensions. It has, however, less flexibility in tuning coupling parameters.

Eq. (7.3) is, however, not the only possible form of an effective action in the strong coupling phase. For example it allowed by the symmetries to add a term, which couples the neutral fermion chirally to the massive vector bosons in the confinement phase. We will come to this in the last part of this section.
Figure 5: The rest energies $E_F^{(c)}$ and $E_D^{(c)}$ as a function of $y$ for $\kappa = 0.4$ and $w = 2.0$. The rest energies are compared with the sums $E_F^{(n)} + E_\Phi$ and $E_D^{(n)} + E_\Phi$ which are represented by the lines. The error bars of $E_F^{(n)} + E_\Phi$ and $E_D^{(n)} + E_\Phi$ are much smaller than the symbol sizes for $E_F^{(c)}$.

zero within the quoted error bars. Furthermore one recognizes a systematic trend of $|\epsilon_B|$ to decrease when the spatial extent of the lattice is enlarged. These results strongly suggest that the signal which we detected in the charged fermion propagators is simply caused by a two particle state of the neutral fermion and the scalar particle. We now can understand also the fact why the hopping expansion to lowest order for the charged fermion propagator is very misleading. To reproduce the inverse propagator of a two particle state an infinite number of hopping terms would be needed. Only on the basis of our numerical results we can, of course, not completely exclude the possibility that in principle a very small and non-vanishing binding energy might be left over in the continuum limit. We could also not find a good field theoretical argument which would rule out the existence of a bound state with zero binding energy. Both cases would bring us back to scenario A which we drew up first in this section.

Let us now proceed under the assumption that the charged fermion does not exist as a particle in the spectrum. In order to find an effective lagrangian for the model in the VXS phase we first rewrite the lattice lagrangian (1.1) in the following way

$$L_F = \frac{1}{2} \sum_{\mu=1}^{2} \left[ (\overline{\Psi}_{L,x+\mu} \gamma_\mu U_{\mu x} \Psi_{L,x+\mu} - \overline{\Psi}_{L,x+\mu} \gamma_\mu U^{\dagger}_{\mu x} \Psi^{(n)}_{L,x}) + (\overline{\Psi}_{R,x} \gamma_\mu \Psi^{(n)}_{R,x+\mu} - \overline{\Psi}_{R,x} \gamma_\mu \Psi_{R,x}) \right]$$
where we introduced the effective gauge field
\[
U'_{\mu x} = \Phi_\mu^* x U_{\mu x} \Phi_x + \hat{\mu}.
\]

Although the interaction in the first term of eq. (7.4) appears to be too weak for a formation of a \(\Phi-\Psi\) bound state, the form (7.4) leaves still the possibility of a chiral coupling between the neutral fermion and the effective gauge field \(U'_{\mu x}\) since this field has dimension one according to a naive power counting analysis. This outcome would be very interesting since one would have found at least one example for a lattice regularized theory where fermions exhibit a chiral coupling to an “external” gauge field. In contrast in four dimensional models the naive power counting analysis suggests that the coupling to the effective gauge field \(U'_{\mu x}\) vanishes like \(a^2\) in the continuum limit.

The \(U(1)\) pure gauge model (2.2) is confining for all values of the gauge coupling \(g\). The confinement phase is expected to be present also in the two dimensional \(U(1)\) Higgs model at small values of \(\kappa\) and to turn in the limit \(g \to 0\) into the vortex phase. For \(g > 0\) the scalar particles get confined into massive bosonic particles. The effective gauge field in eq. (7.5) can be written in the form
\[
U'_{\mu x} = z^2 + H_{\mu x} + iW_{\mu x}
\]
where \(z^2\) is given in eq. (4.5) and \(H_{\mu x}\) and \(W_{\mu x}\) are interpolating fields for bosonic bound states in the confinement phase with quantum numbers \(J^{PC} = 0^{++}\) and \(1^{--}\) in lowest spin state [32]. The field \(H_{\mu x}\) couples primarily to the Higgs-like scalar particle according to
\[
H_{\mu x} \to m_H H_x
\]
where \(m_H\) is some mass scale which has to be introduced since the scalar field \(H_x\) is dimensionless \((H_{\mu x} \text{ is not a vector under lattice rotations, which forbids a relation of the form } H_{\mu x} \to \partial_\mu H(x))\). The field \(W_{\mu x}\) couples to the vector boson.

After inserting (7.6) into (7.4) and a trivial rescaling of the fields \(\Psi^{(n)}_L\) and \(\overline{\Psi}^{(n)}_L\) we obtain for \(\mathcal{L}_F\) the form
\[
\mathcal{L}_F = \frac{1}{2} \sum_{\mu=1}^2 \left[ \overline{\Psi}_x^{(n)} \gamma_\mu \Psi^{(n)}_{x+\hat{\mu}} - \overline{\Psi}_{x+\hat{\mu}} \gamma_\mu \Psi^{(n)}_x \right] + \frac{y}{z} \overline{\Psi}^{(n)} \Psi^{(n)} - \frac{w}{2z} \overline{\Psi}^{(n)} \sum_{\mu=1}^2 \partial_\mu \partial_{\mu} \Psi^{(n)}
\]
\[
+ \frac{m_H}{z^2} \sum_{\mu=1}^2 \left[ \overline{\Psi}^{(n)} \gamma_\mu \Psi^{(n)}_{x+\hat{\mu}} - \overline{\Psi}^{(n)}_{x+\hat{\mu}} \gamma_\mu \Psi^{(n)}_x \right] + \frac{1}{2} \sum_{\mu=1}^2 iW_{\mu x} \left[ \overline{\Psi}^{(n)} L_x \gamma_\mu \Psi^{(n)}_{L,x+\hat{\mu}} + \overline{\Psi}^{(n)}_{L,x+\hat{\mu}} \gamma_\mu \Psi^{(n)}_{L,x} \right]
\]

The term (7.8) describes a free neutral fermion with mass \(m_F^{(n)} = y/z\). The expression (7.9) suggests that the coupling of the neutral fermion to the Higgs-like bound state vanishes like \(a\). However, the neutral fermion couples chirally in (7.10) to the vector boson field \(W_{\mu x}\) if its dimension is one, as suggested by the naive dimensional analysis.

In order to find out whether the fields \(W_{\mu x}\) and \(H_{\mu x}\) are indeed dimension one operators we have computed the scale dependence of the corresponding wave-function renormalization
constants $Z_H$ and $Z_W$. From the naive dimensional analysis these wave-function renormalization constants $Z_H$ and $Z_W$ are expected to vanish like $a^2$. To see whether this expectation is correct we have computed the momentum space propagators

$$
G_H(p_2) = \left\langle \frac{1}{V} \sum_{x,y} H_{1x} H_{1y} e^{i p_2 (x_2 - y_2)} \right\rangle, \quad G_W(p_2) = \left\langle \frac{1}{V} \sum_{x,y} W_{1x} W_{1y} e^{i p_2 (x_2 - y_2)} \right\rangle
$$

(7.11)

in the confinement phase of the U(1) gauge-Higgs model for several values of $g$ and $\kappa$ and fitted the results for sufficiently small $p_2$ to the free boson propagator ansatz given in eq. (5.5), which for $G_W$ is considered as a special case ($p_1 = 0$) of a free massive vector boson propagator, $(\delta_{\mu\nu} + p_\mu p_\nu/m^2)(m^2 + p^2)^{-1}$. In fig. 6 we have displayed the resulting wave-function renormalization constant $Z_W$ (squares) and $Z_H$ (circles) respectively as a function of $m^2 = a^2 m_{W,\text{phys}}^2$ and $m^2 = a^2 m_{H,\text{phys}}^2$ for the fixed ratio $m_H/m_W = 1.14$. In both cases the wave-function renormalization constants obey nicely a linear dependence, supporting our expectation from the dimensional analysis.

This result suggests that the neutral fermion exhibits in two dimensions indeed a non-vanishing chiral coupling to the massive vector bosons. This coupling is universal in the quenched approximation since the field $W_{\mu x}$ is proportional to a current which is conserved in the two dimensional U(1) gauge-Higgs model. The properties of this coupling in the full model with dynamical fermions are, however, not yet clear to us.
The fermion couplings in the VXS phase may then be summarized qualitatively by the following effective lagrangian

\[ \mathcal{L}^{\text{eff}}_F = \bar{\psi}^{(n)} \frac{\partial}{\partial \psi^{(n)}} + m_F^{(n)} \bar{\psi}^{(n)} + g_R \bar{\psi}^{(n)} \gamma_\mu \psi_L^{(n)} W^{(c)}_\mu , \]  

(7.12)

where \( g_R = \sqrt{Z_W/z^2} \) and \( W^{(c)}_\mu \) is the vector field in the continuum with standard normalization. This expression for the effective lagrangian gives a satisfactory description of the fermion couplings at distances which are large in comparison with the typical length scale of the vector boson bound state. When lowering the value of the gauge couplings \( g \) the string tension becomes smaller and the bound states extends over larger distances. At small distances the scalar particles are then almost free and a more appropriate form of the action is given then by

\[ \mathcal{L}^{\text{eff}}_F \rightarrow \bar{\psi}^{(n)} \frac{\partial}{\partial \psi^{(n)}} + m_F^{(n)} \bar{\psi}^{(n)} + \frac{1}{z^2} \bar{\psi}^{(n)} \gamma_\mu \psi_L^{(n)} (\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) . \]  

(7.13)

This form is expected to describe the fermion couplings in particular in the global limit, i.e. \( g = 0 \), where the bosonic spectrum consists of unbound scalar particles. The massive neutral fermion in eq. (7.13) is coupled to a two particle current. This interaction seems, however, to be quite weak since our numerical results for the neutral fermion propagator are in good agreement with the analytic prediction from the lowest order hopping expansion, which leads to a free fermion propagator. Furthermore our numerical results for the binding energy \( \epsilon_B \) suggest that the interaction in (7.13) is too weak as to give rise to the formation of a \( \Phi-\Psi^{(n)} \) bound state. Also the expressions (7.12) and (7.13) are certainly different from the target model in eq. (2.5) which we had originally in mind.

8 Conclusion

We started out our investigations from the lattice lagrangian given in eq. (2.1) in the hope to obtain in the continuum limit the target model in eq. (2.3). In the global limit of the model \( g \rightarrow 0 \) the unwanted species doublers can be removed completely from the spectrum within the strong coupling vortex phase (VXS). The physics in this phase differs, however, substantially from the target action which we had originally in mind. In the previous chapter we have developed two different scenarios (A and B) for the effective theory in the VXS phase which were summarized by the continuum lagrangians (7.3) and (7.12), (7.13). Our numerical results are in favor of scenario B: In this case the fermionic spectrum in VXS contains only a neutral fermion which has zero charge with respect to the U(1) group and which in the global limit \( g \rightarrow 0 \) exhibits a left-handed coupling to a two particle current. This coupling, however, appears to be weak since the neutral fermion propagator data are in nice agreement with the results from the lowest order hopping expansion which implies a free fermion behavior. Furthermore this coupling seems to be too weak as to give rise to the formation of a \( \Phi-\Psi^{(n)} \) bound state. When the gauge coupling is turned on, we argue that the neutral fermion couples chirally to the massive vector boson state in the confinement phase. If this scenario is correct, it would have been the first time that a chirally coupled fermion has been detected on the lattice. This result is also different from the previous findings in the strong coupling symmetric phase (PMS) of the fermion-Higgs models with Wilson-Yukawa coupling in four dimensions where the coupling of the neutral fermion to the bosonic bounds states vanishes presumably as a power of the lattice spacing [13].
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