Enhance Convolutional Neural Networks with Noise Incentive Block

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Abstract
As a generic modeling tool, Convolutional Neural Networks (CNNs) have been widely employed in image generation and translation tasks. However, when fed with a flat input, current CNN models may fail to generate vivid results due to the spatially shared convolution kernels. We call it the flatness degradation of CNNs. Unfortunately, such degradation is the greatest obstacles to generate a spatially-variant output from a flat input, which has been barely discussed in the previous literature. To tackle this problem, we propose a model agnostic solution, i.e. Noise Incentive Block (NIB), which serves as a generic plug-in for any CNN generation model. The key idea is to break the flat input condition while keeping the intactness of the original information. Specifically, the NIB perturbs the input data symmetrically with a noise map and reassembles them in the feature domain as driven by the objective function. Extensive experiments show that existing CNN models equipped with NIB survive from the flatness degradation and are able to generate visually better results with richer details in some specific image generation tasks given flat inputs, e.g. semantic image synthesis, data-hidden image generation, and deep neural dithering.

1 Introduction
Recently, Convolutional Neural Networks (CNNs) have demonstrated great success in various image processing and computer vision applications [Simonyan and Zisserman, ; Park et al., 2019]. They can be roughly categorized into two main directions, image understanding and image generation. Image understanding usually summarizes massive information into a more abstract form, like semantic segmentation. Just the opposite, Image generation aims to create massive information from an abstract image, such as semantic image synthesis. However, existing image generation models usually face a vital but easily overlooked problem. When CNNs are fed with flat inputs, they may fail to generate vivid results.

It is the nature inherited from the convolution operation with spatially shared kernels, i.e. given a constant function $f(x) \equiv c$ and an arbitrary local kernel function $k(x)$, the convolution of $f(x)$ and $k(x)$ can only produce another constant function $g(x) \equiv c \cdot \mu(k(y))$, where $\mu(\cdot)$ computes the mean value. Regarding this, a standard CNN model, integrating cascaded convolutional layers with bias terms and activation functions, retains this property as well. That is, given a flat input $X$, the transformation by CNN degrades to a scaling operation $Y = \alpha X$, where the scalar $\alpha$ is determined by the CNN parameters and $X$, and hence the output $Y$ is flat for sure. We call this phenomenon flatness degradation throughout this paper. Theoretically, it will always happen when the receptive field of a CNN (or its submodules) is filled with flat data, which is inevitable with sparse input. Figure 1 demonstrates how flatness degradation affects several typical applications: (a) Semantic image synthesis that synthesizes...
photo-realistic textures from piece-wise flat semantic layout; (b) Data-hidden image generation that encodes extra information through imperceptible intensity variation; (c) Neural dithering that reproduces tone through discrete halftone patterns.

To circumvent the flatness degradation of CNNs, we propose a generic plug-in block, Noise Incentive Block (NIB). Basically, it is based on a straightforward insight that by reforming the input data to be locally non-flat, the CNN would never work on flat signals. More importantly, our proposed reform never compromise the input information fidelity. In particular, the NIB perturbs the input data symmetrically with a noise map and then reassembles the two complementary noisy variants in the feature domain (see Figure 2). The design rationale is that the noisy perturbation breaks flat input condition while the symmetric perturbation strategy generates two complementary components that reserves the information intactness together. In other words, the NIB enables CNNs to generate adaptive output free of flatness degradation, and meanwhile guarantees the information intactness for further task-oriented usage. This two important functionalities of NIB are verified and studied through various experiments. Results tell that equipping CNN models with NIB avoids flatness degradation completely, which even comes with performance gain in general cases. Such advantages are further confirmed by the evaluations on related image generation applications.

Note that, our proposed NIB is a generic solution to tackle flatness degradation issue of CNNs, however, it is possible to have more sophisticated alternatives in specific application scenarios. In some sense, this work serves as a discussion opener and expects to inspire further studies on this problem. To summarize, this work makes the contributions:

• We raise the concept of flatness degradation for the first time, which is demonstrated to degrade the performance of CNN-based generative models.
• We propose a simple but effective convolutional plug-in, Noise Incentive Block that resolves flatness degradation for CNNs in a generic manner.
• Our proposed method consistently improves the performance of multiple existing state-of-the-art models in relevant applications.

2 Related Works

We briefly review existing noise utilization in deep neural networks (DNNs), and then discuss with recent works on spatially variant convolution designs.

Promoting training efficiency. It is a non-convex optimization problem to train DNNs and there are usually many saddle points and local optima that are associated with bad generalization [Barrera and Jara, 2015]. Anyhow, despite the highly complex landscape, Stochastic Gradient Descent (SGD) [Bottou, 2010] and its variants are practically successful in training DNNs. Empirical evidences [Zhang et al., 2017; Keskar et al., 2017] imply that the noise (or say randomness) in Stochastic Gradient Descent (SGD) is very crucial and enables SGD to achieve good optima. The theoretic insights behind these phenomena are discussed in [Zhou et al., 2019]. For data augmentation, injecting noise to input data is leveraged to improve the model robustness to adversarial noise [Jin et al., 2015]. Also, [Neelakantan et al., 2015] finds that adding noise to gradient potentially benefits the network training. Noh et al. [Noh et al., 2017] propose that the noise injection to the DNN neurons actually optimize the lower bound of the true objective, and a tighter lower bound can be achieved by carefully constructing multiple samples per training examples. Different from the above methods that take dynamic noises to introduce randomness to the training, our model just utilizes the local non-flatness nature of the noise map to break flatness, so even a stationary noise map is feasible in our case.

Modeling generative space. Generative models generally construct the generation space by exploiting random noises under prior distributions (e.g. normal distribution), so that the target domain could be sampled by controlling the input variables. As the most popular generative framework, Generative adversarial networks (GAN) [Goodfellow et al., 2014] and Variable Autoencoder (VAE) [Kingma and Welling, 2019] model the generation space from the input variables and from the latent variables respectively. To allow user control of the generation, conditionally generative models [Zhu et al., 2017; Isola et al., 2017] synthesize the target domain samples from a preset random distribution and the conditional input. In particular, the conditional input can be either a visually perceptible image [Park et al., 2019] or a latent variable from another distribution function [Karras et al., 2019]. Seemingly, conditional GANs taking as input a conditional image and a noise map, is similar to our noise incentive block. In essence, there is a significant difference. Conditional GANs require the noise map to be dynamically sampled under a well-defined distribution function, by which the noise map carries distribution information to the GAN models for output attributes modeling. In contrast, the noise map equipped in our NIB just serves as a medium of local non-flatness, which offers no information and never affects the property of the output.

Content-adaptive convolution. Some recent researches [Verma et al., 2016; Dai et al., 2017; Su et al., 2019] raise concerns about the convolutional flexibility under spatial sharing scheme. Particularly, it may be sub-optimal to have the same convolutional filter banks applied to all the images and all the pixels irrespective of their content. Dynamic filters [Verma et al., 2016; Dai et al., 2017; Hu et al., 2019] and pixel-adaptive convolution [Su et al., 2019] are two well-known techniques to alleviate this restriction, both of which share the same spirit of dynamically generating spatially varying convolution kernels or weighting coefficients from the input features. Indeed, these content-adaptive convolution modules boost performance clearly in mainstream tasks. However, they still suffer from flatness degradation since their spatially varying kernels are generated from the input features by typical convolutions. In this regard, our proposed noise incentive block can be applied to enhance these techniques.
3 Noise Incentive Block

To address the flatness degradation for CNNs, we propose a generic plug-in, Noise Incentive Block (NIB), which can be used along with standard convolution layers. The conceptual diagram is illustrated in Figure 2.

3.1 Main Idea

Since flatness degradation happens upon flat input, an intuitive idea is to represent the input data in a spatially variant fashion. To this end, we propose to construct a convolutional block that perturbs the input data in a spatially varying manner and preserves the input information from contamination. Below, we discuss how the two characteristics are achieved by our proposed NIB.

Spatially variant proxy. We propose to perturbs the input data with a spatially variant proxy. Formally, this proxy is required to satisfy three conditions: (i) non-zero gradient, to avoid flat patch for any cross-pixel convolutional kernels. (ii) non-periodic repeating unit, to avoid the restriction of generating regularly repeated values (see "Regular grid" in Fig 4(b)). (iii) independence on the input data, to avoid information contamination. Naturally, the random noise map, with independent and identically distributed elements, is a qualified choice. Firstly, since each element is sampled as an independent random variable, the noise map has non-zero gradient almost everywhere and is free of periodic repeating values. Besides, the noise map is generated in a purely random fashion, which definitely has no correlation with the input data. Therefore, the random noise map is a good match to our requirements. According to Figure 4, the distribution function used for sampling the noise map makes little difference. As default, we take a stationary white noise map N sampled from $\mathcal{N}(0.0, 0.3)$.

Introduce mechanism. With the spatially variant proxy, e.g. the noise map N, we propose to perturbs the input data I in a symmetric manner, i.e. $I + N$ and $I - N$, and then resemble them in an adaptively learned feature domain to obtain the potentially non-flat feature representation $\tilde{I} = f_1(I + N) + f_2(I - N)$. As a default setting, $f_1$ and $f_2$ are implemented with two convolutional layers. These operations constitute our proposed Noise Incentive Block (NIB), as shown in Figure 2. By this way, the input data I is represented by $\tilde{I}$ that is endowed with the potential of offering the intact information of I and getting rid of flatness degradation. It worth noting that the two functionality of NIB are not hard properties but potentials to be realized by the adaptively learned $f_1$ and $f_2$ under task-oriented loss functions.

Despite flatness degradation risking every convolution layer, it is unnecessary to substitute all convolutional layers with our NIBs. In fact, the degradation-free potential of NIBs can be transferred to its successor layers. Specifically, supposing the precursor layer is free of flatness degradation, it could generate a non-flat feature maps such that its successor layer inherit this potential accordingly. In other words, adopting a NIB as the first layer could potentially secure the whole CNN model free of flatness degradation.

3.2 Model Analysis

We equip typical CNNs with the proposed NIB to verify its functionalities and explore the working mechanisms. In particular, the questions below are investigated sequentially.

How does NIB work? For clear visualization purpose, we explore this problem by applying CNNs for image halftoning [Ulichney, 1987]. Formally, given a grayscale image $I_g$, the CNN model is required to generate a halftone image $I_h$ that reproduces the tone of $I_g$, with binary pixels. For simplicity, we only consider the tone similarity for quality assessment, though other properties (e.g. structure similarity, blue noise profile, etc.) are also necessary to quality halftones. The loss function and training details are provided in the supplementary materials. We take a ResNet consisting of 10 residual blocks [He et al., 2016] as the baseline model, and the NIB equipped model has a NIB plugged in the first layer. Figure 3 compares the results on two typical example. Due
to flatness degradation, the standard model can only generate halftone pattern in the surroundings of structure edges, where the radial neighborhood gets larger as the layers get deeper. This indicates that enlarging the receptive field of the CNN can only alleviate the convolution degradation for those deeper layers while the shallower layer still get stuck in those locally flat regions. In contrast, our proposed NIB-equipped model is free of this restriction and can synthesize non-flat feature maps adaptively at all convolution layers.

To demonstrate the advantages in general cases, we further quantitatively evaluate the models on comic pictures and natural images. Specifically, 93 comic frames of resolution 994 × 1500 (about 6 pictures per frame) are collected from the Internet, named COMIC; 3367 images are collected from VOC2012 dataset [Everingham et al., 2012] and resized to 256 × 256, named VOC. Two kinds of CNN architecture with different receptive fields: ResNet (41 × 41) and U-Net (183 × 183), are employed for comparison. Table 1 tabulates the results. For both CNN architectures, our NIB-equipped model outperform the standard model on COMIC clearly since comic pictures almost are filled with flat patches. Interestingly, such superiority is even maintained on the natural image dataset VOC when natural images barely contains constant patches. It is probably because that the CNNs’ shallower layers gets stuck in those locally flat spot scattered in smooth regions and thus hinders the model’s capability to some extent. Moreover, as the flat area increases, more layers get affected by flatness degradation. Anyhow, NIB-equipped model has no such issue.

What role is the noise map? As discussed in Section 3.1, the noise map only serves as a spatially variant proxy that is passively exploited by the task-driven NIB to build non-flat feature maps. In some sense, it is a kind of incentive that enables convolutions to work fluently even the input data is a singular constant map. In this regard, the noise map used in NIB could be either dynamic or stationary and generated from whatever distribution. To verify these hypotheses, we compare the NIB variants that adopt different noise injection modes. Specifically, we take the Stationary noise generation from Normal distribution \( \mathcal{N}(0,0.3) \) as the reference group (S-N-0.3), and further construct four control groups: dynamic noise from \( \mathcal{N}(0.0,0.3) \) (D-N-0.3), stationary noise from \( \mathcal{N}(0,0.03) \) (S-N-0.03), and stationary noise from uniform distribution \( \mathcal{U}(-0.3,0.3) \) (S-U-0.3). Here, dynamic noise means freshly generating a noise map for each input and stationary noise means using a fixed noise map for all input data. In addition, a regular grid with alternative black and white lines, is compared as an baseline of disobeying the principle of non-periodic repeating unit. In this study, the ResNet is employed and the quantitative evaluation is performed on COMIC dataset. Figure 4 compares the performance of the NIB variants that take different noise injection modes. Except for the regular grid case, all variants achieve comparable performance and address flatness degradation effectively (see the constant-valued “sky” and “tree” regions). It means that the functionality of NIB is insensitive to the noise type and injection mode. This justifies our hypothesis that the noise map offers no information but passively serves as certain incentive medium. Notably, the periodicity of the regular grid is absorbed by the model during training, as the halftone patterns illustrates. This extra constraint inevitably impedes the model flexibility and thus degrades the performance.

In practice, our proposed NIB, formulated as \( \tilde{I} = f_2(I\cdot N) + f_2(I\cdot (1 - B)) \), can be naturally extended to other paradigms under the same concept. For instance, we may introduce the perturbation by an impulse noise map \( B \) with each element sampled from Bernoulli distribution, and then it turns into \( \tilde{I} = f_2(I\cdot B) + f_1(I\cdot (1 - B)) \). Also, the perturbation even could be performed in the feature domain only, i.e. \( \tilde{I} = f_1(I) + f_2(N) \), which is especially suitable to the case of semantic input. Accordingly, they are evaluated in Figure 4 as two variants of new paradigms (D-B-0.5 and F-D-N-0.3), and present equally decent performance.

Does NIB contaminate data? As one of the required features, the NIB is expected to reserve the intactness of the input information. To verify this, we employ NIB into an autoencoder model that reconstructs the input from a learned latent representation. For comparison, we build a naive baseline that adds white noise to the input data. The rationale is that if the NIB contaminates the input data, the reconstruction accuracy will be affected consequently. The detailed experiment setting is provided in the supplementary material. Figure 5 illustrates an comparative example. Mea-
Figure 5: Reconstruction accuracy of: Autoencoder (a), additive noise imposed Autoencoder (b), and NIB equipped autoencoder (c). The color-coded error maps are attached respectively. For reference, the noisy image is shown in (d). Thanks to the symmetric noise mechanism, NIB makes no harm to information reconstruction, while additive noise ruins the fine structures irrevocably.

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4 Application Evaluation

We evaluate our proposed Noise Incentive Block (NIB) on three applications, i.e., semantic image synthesis, data-hidden image generation and deep neural dithering, all of which are required to generate spatial variation from potential flat input. Typically, semantic image synthesis utilizes CNNs with global receptive field for context interpretation, while data-hidden image generation and deep neural dithering adopt CNNs with local receptive field for feature translation.

4.1 Semantic Image Synthesis

Semantic image synthesis aims to synthesize photorealistic images from semantic layout. In general, the input semantic layout consists of constant-valued segments. According to our analysis, such flat input may hinder the CNN performance with flatness degradation, which motivates us to use NIB in this task. Particularly, we investigate the benefits by equipping NIBs into two typical semantic synthesis models, Pix2Pix [Isola et al., 2017] and SPADE [Park et al., 2019]. Figure 6 compares the results on several typical examples. Comparing to the original variants, the NIB-equipped models exhibit advantages in synthesizing more realistic textures with sharp details. Although the CNN has a receptive field covering the whole input image, the shallow convolution layers still degrades in local flatness areas, which affects the model flexibility in some degree. In contrast, the NIB-equipped model is free of flatness degradation and enables the CNN to work with full capability.

We perform quantitative evaluation on the testing set of Cityscapes [Cords et al., 2016]. A pretrained FCN model [Yu et al., 2017] is utilized to perform segmentation on the synthesized images. The segmentation performance is measured by mean Intersection-over-Union (mIOU) and pixel accuracy, which reflect how the synthesized images are aligned with the corresponding input semantic layouts. Besides, we further calculate the distribution distances between the generated images and real images by the Fréchet Inception Distance (FID) [Martin Heusel and Hochreiter, 2017]. The results tabulated in Table 2 show that our NIB makes notably improvement in FID while no gain in the indirect segmentation accuracy. These results are consistent with the properties of NIB as it benefits spatially variant generation while having negligible influences on pixel alignment.

4.2 Data-Hidden Image Generation

Invertible grayscale (IG) [Xia et al., 2018] refers to a kind of grayscale image that has the original color information represented as certain imperceptible texture patterns. Based on the learned encoding scheme via CNN model, it works very well in all kinds of cases except for constant patches. Figure 7(b)(c) shows two examples, where the original colors

| Method       | mIOU ↑ | Accu ↑ | FID ↓ |
|--------------|--------|--------|-------|
| Pix2Pix [2017] | 12.05% | 53.6%  | 92.9  |
| Pix2Pix+NIB   | 12.27% | 53.6%  | 88.1  |
| SPADE [2019]  | 62.3%  | 81.9%  | 71.8  |
| SPADE+NIB     | 61.9%  | 81.8%  | 54.6  |

Figure 6: Semantic image synthesis by state-of-the-art models and their NIB-equipped variants. The blowup regions highlight the performance differences.
fail to be recovered from the invertible grayscales. To spot the cause, readers are recommended to zoom in the grayscale images and carefully inspect the weak texture patterns. It shows that no color-encoded textures are generated in those flat or smooth regions, due to flatness degradation. In contrast, with the NIB plugged to [Xia et al., 2018], these failure cases are addressed successfully, as the results compared in Figure 7. Table 3 tabulates the quantitative evaluation on the testing set of DIV2K dataset [Agustsson and Timofte, 2017] (resized to \(1024 \times 1024\)). The statistic result shows less significant advantages because natural images seldom cover pure flatness. So, the advantages of the NIB-equipped models can be summarized two-fold: it can handle extreme cases (e.g. comic pictures) well; it also benefit general cases to some extent. More qualitative results are available in the supplementary material.

### 4.3 Deep Neural Dithering

Early dithering methods mainly focus on tone simulation and blue-noise profile [Ostromoukhov, 2001], and generally suffer from blurry structures. Pang et al. [Pang et al., 2008] propose an optimization based method that balances clear structures and good blue-noise profile. However, this method is time-consuming and sensitive to parameter tuning, which makes it less practical in applications. To tackle these problems, we make the first attempt to apply neural networks for image dithering. By taking the same loss function of [Pang et al., 2008], the CNN model can generate halftone images with good tone similarity (measured by PSNR) but fails to achieve desired blue-noise profile (see those visually annoying strip patterns in the orange blowup in Figure 8). Since blue noise depicts a kind of point distribution without high-frequency component, we formulate an additional loss term to encourage this target: 

\[
\mathcal{L}_B = \| \text{DCT}(\mathbf{H}) \odot \mathbf{M} \|_1,
\]

where $\mathcal{L}_B$ denotes the low-frequency assumption, this constraint can only be imposed on the dithered image $\mathbf{H}$ of constant-grayness, which challenges standard CNNs but is well enabled by NIB-equipped CNNs. Figure 8 shows the comparative results, which evidences the effectiveness of our proposed deep neural dithering method. Also, the quantitative comparison on the classic Structure dataset [Pang et al., 2008] is tabulated in Table 4. Note that, besides of the better quality, our proposed neural dithering is parameter-free and as efficient as a CNN forward process.

### 5 Conclusion

We illustrate the flatness degradation concept of CNNs through typical applications, and proposed a generic solution to address it. It is a simple but effective convolutional plug-in, Noise Incentive Block (NIB), which adaptively turns input data into non-flat features while reserves high fidelity. The rationale and effectiveness of this design are studied in-depth. Extensive experiments show that NIB-equipped CNN models are free of flatness degradation, and thus can handle previously intractable cases. As micro-flatness commonly exists, statistical results evidence the advantage of NIB in general cases. All these results justify the importance of our proposed NIB, as a model-agnostic plug-in for CNNs. Additionally, the concept presented in this paper is expected to inspire further studies in this direction.
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