THE $\pi\gamma$ TRANSITION FORM FACTOR AND ITS IMPACT ON CHARMONIUM DECAYS INTO TWO PIONS *

P. Kroll

Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany

*Invited talk presented at the Third Workshop on Diquarks, Torino (October 1996)
THE $\pi\gamma$ TRANSITION FORM FACTOR
AND ITS IMPACT ON CHARMONIUM DECAYS INTO TWO PIONS

PETER KROLL †
Fachbereich Physik, Universität Wuppertal,
D-42097 Wuppertal, Germany

ABSTRACT

The analysis of the $\pi\gamma$ transition form factor provides severe constraints on the pion’s wave function. This information is used to examine charmonium decays into two pions critically. It will be argued that the standard perturbative QCD analysis of these reactions fails, i.e. the need for additional contributions can convincingly be demonstrated. Colour-octet admixtures to the charmonium states are proposed as a possible dynamical mechanisms to solve the puzzle. Consequences of the $\pi\gamma$ analysis for the electromagnetic form factor of the pion are also discussed.

1. Introduction

At large momentum transfer the hard scattering approach (HSA) [1] provides a scheme to calculate exclusive processes. Observables are described as convolutions of hadronic wave functions which embody soft non-perturbative physics, and hard scattering amplitudes $T_H$ to be calculated from perturbative QCD. In most cases only the contribution from the lowest-order pQCD approach in the collinear approximation using valence Fock states only (termed the standard HSA) has been worked out. Applications of the standard HSA to space-like exclusive reactions, as for instance the magnetic form factor of the nucleon, the pion form factor or Compton scattering off protons revealed that the results are only in fair agreement with experiment if hadronic wave functions are used that are strongly concentrated in the end-point regions where one of the quark momentum fractions, $x$, tends to zero. As has been pointed out by several authors (e.g. [2,3]), the results obtained from such wave functions are dominated by contributions from the end-point regions where perturbative QCD cannot readily be applied. Hence, despite the agreement with experiment, the predictions of the standard HSA are theoretically inconsistent for such wave functions. It should also be stressed that the large momentum transfer behaviour of the helicity-flip controlled Pauli form factor of the proton remains unexplained within the standard HSA.

Applications of the HSA to time-like exclusive processes fail in most cases (e.g. $G_M$, $F_T$, $\gamma\gamma \to p\bar{p}$). The predictions for the integrated $\gamma\gamma \to \pi\pi$ cross-section ($|\cos \theta| \leq 0.6$) are in fair agreement with the data whereas the predictions for the angular distribution fails. Exclusive charmonium decays constitute another class of time-like reactions. If the end-point region concentrated wave functions are employed

† e-mail: kroll@theorie.physik.uni-wuppertal.de
Supported in part by the TMR Network ERB 4061 PL 95 0115.
again, the standard HSA provides results in fair agreement with the data in many cases. It should be noted that in most calculations of exclusive charmonium decays \[4\] values of the order of \(0.2 - 0.3\) are employed. Such values do not match with \(\alpha_s\) evaluated at the charm quark mass, the characteristic scale for these decays \((\alpha_s(m_c = 1.5\text{ GeV}) = 0.37\) in one-loop approximation with \(\Lambda_{QCD} = 200\text{ MeV})\). Since high powers of \(\alpha_s\) are involved in charmonium decays a large factor of uncertainty is hidden in the predictions.

Constraining the pion wave function \[5, 6\] from the recent precise data on the \(\pi\gamma\) transition form factor \[7\], one observes an order-of-magnitude discrepancy between data and HSA predictions for charmonium decays into two pions. In \[8\] contributions from the \(c\bar{c}g\) Fock state are suggested as the solution of this puzzle. I am going to discuss these topics in my talk. Also I shall discuss the large momentum transfer behaviour of the pion form factor in the light of the new information on the pion’s wave function.

2. The \(\pi-\gamma\) transition form factor

The apparent success of the end-point concentrated wave functions, in spite of the theoretical inconsistencies, prevented progress in understanding hard exclusive reactions for some time. Recently, with the advent of the CLEO data on the \(\pi\gamma\) transition form factor \[8\], the situation has changed. The leading twist result for that form factor\(^\dagger\) including \(\alpha_s\)-corrections, reads \[8\]

\[
F_{\pi\gamma}(Q^2) = \frac{\sqrt{2}}{3} \frac{f_\pi}{Q^2} \left[ 1 + \frac{\alpha_s(\mu_R)}{2\pi} K_{\pi\gamma}(Q^2, \mu_R) + \mathcal{O}(\alpha_s^2) \right].
\]

\(f_\pi\) is the usual pion decay constant (130.7 MeV) and \(\mu_R\) represents the renormalization scale. The function \(K_{\pi\gamma}\) has been calculated by Braaten \[9\] in the \(\overline{MS}\) scheme. \(\langle x^{-1} \rangle\) is the \(1/x\) moment of the pion distribution amplitude, \(\phi\), which represents the light-cone wave function of the pion integrated over transverse quark momenta, \(k_\perp\), up to a factorization scale, \(\mu_F\), of order \(Q\). The distribution amplitude can be expanded upon Gegenbauer polynomials, \(C_n^{3/2}\), the eigenfunctions of the evolution kernel for mesons \[8\]

\[
\phi_{\pi}(x, \mu_F) = \phi_{AS}(x) \left[ 1 + \sum_{n=2,4,...} \frac{1}{B_n(\mu_0)} \left( \frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^n C_n^{3/2}(2x - 1) \right]
\]

where the asymptotic distribution amplitude is \(\phi_{AS}(x) = 6x(1-x)\). The \(1/x\) moment of the distribution amplitude reads

\[
\langle x^{-1} \rangle = 3 \left[ 1 + \sum_{n=2,4,...} \frac{1}{B_n(\mu_0)} \left( \frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^n \right] = 3 \left[ 1 + \sum_{n=2,4,...} B_n(\mu_F) \right].
\]

\(^\dagger\) The pion mass as well as the light current quark masses are neglected throughout.
Figure 1: The scaled $\pi\gamma$ transition form factor vs. $Q^2$. The solid (dashed) line represents the results obtained with the modified HSA using the asymptotic (Chernyak-Zhitnitsky) wave function. The evolution of the Chernyak-Zhitnitsky wave function is taken into account. The dotted line represents the limiting behaviour $\sqrt{2}f_\pi$. Data are taken from [7, 12].

The process-independent expansion coefficients $B_n$ embody the soft physics; they are not calculable at present. $\mu_0$ is a typical hadronic scale, actually $\mu_0 = 0.5$ GeV. Since the anomalous dimensions, $\gamma_n$, are positive fractional numbers increasing with $n$ (e.g. $\gamma_2 = 50/81$) any distribution amplitude evolves into the asymptotic distribution amplitude for $\ln Q^2 \to \infty$; higher order terms are gradually suppressed. Hence, the limiting behaviour of the transition form factor is

$$F_{\pi\gamma} \to \sqrt{2}f_\pi/Q^2$$

which is a parameter-free QCD prediction [11]. As comparison with the CLEO data [7] reveals, the limiting behaviour is approached from below. At 8 GeV$^2$ the data only deviate by about 15% from (4) (see Fig.1). In order to give a quantitative estimate of the allowed deviations from the asymptotic distribution amplitude one may assume that $B_2$ is the only non-zero expansion coefficient in (2). The truncated series suffices to parametrize small deviations. Moreover, it has the advantage of interpolating smoothly between the asymptotic distribution amplitude and the frequently used Chernyak-Zhitnitsky distribution amplitude [13] ($B_2 = 2/3$; $C_2^{3/2}(\xi) = 3/2(5\xi^2 - 1)$). For large momentum transfer the assumption on the expansion coefficients is justified by the properties of the anomalous dimensions $\gamma_n$.

In [6] it is shown that the leading twist, lowest order pQCD result [4] nicely fits the CLEO data for $B_2^{LO}(\mu_0) = -0.39 \pm 0.05$. Using Braaten’s result for $K_{\pi\gamma}$ [4] that, choosing $\mu_F = \mu_R = Q$, reads

$$K_{\pi\gamma} = \frac{-10}{3} \frac{1 - 59/72 B_2(Q^2)}{1 + B_2(Q^2)},$$

(5)
one finds $B_{2}^{NLO}(\mu_{0}) = -0.17 \pm 0.05$ from a fit to the CLEO data. Braaten’s analysis is however incomplete in so far as only the $\alpha_{s}$ corrections to the hard scattering amplitude have been considered but the corresponding corrections to the kernel of the evolution equation for the pion’s distribution amplitude were ignored. As has been shown by Müller [11] recently next-to-leading order evolution provides logarithmic modifications in the end-point regions for any distribution amplitude, i.e. for the asymptotic one too. An estimate however reveals that the modifications of the evolution behaviour in next-to-leading order are very small and can safely be neglected.

To summarize the $F_{\pi\gamma}$ form factor requires a distribution amplitude in a leading twist analysis that is narrower than the asymptotic one in the momentum transfer region of a few GeV$^{2}$. The Chernyak-Zhitnitsky distribution amplitude is in clear conflict with the data and should, therefore, be discarded.

Recently a modified HSA has been proposed by Botts, Li and Sterman [16] in which transverse degrees of freedom as well as Sudakov suppressions are taken into account. This approach has the advantage of strongly suppressed end-point regions. Hence, the perturbative contributions can be calculated self-consistently in the sense that the bulk of the perturbative contribution is accumulated in regions of reasonably small values of the strong coupling constant. It is to be stressed that the effects of the transverse degrees of freedom taken into account in the modified HSA represent soft contributions of higher-twist type. Still, modified HSA calculations are restricted to the dominant (valence) Fock state. Another advantage of the modified HSA is that the renormalization scale can be chosen in such a way that large logs from higher order perturbation theory are eliminated. Such a choice of the renormalization scale are accompanied by $\alpha_{s}$ singularities in the end-point regions which are, however, compensated by the Sudakov factor in the modified HSA. Singularities produced by the evolution of the wave function are also cancelled by the Sudakov factor.

Adapting the modified HSA to the case of $\pi\gamma$ transitions, one can write the corresponding form factor as

$$F_{\pi\gamma}(Q^{2}) = \int \frac{d^{2}b}{4\pi} \hat{\Psi}_{\pi}(x, -b, \mu_{F}) \hat{T}_{H}(x, b, Q) \exp [-S(x, b, Q)]$$

up to $O(\alpha_{s}, k_{\perp}^{2}/Q^{2})$ corrections. $b$ is the quark-antiquark separation and is canonically conjugated to the usual transverse momentum $k_{\perp}$. The use of the transverse configuration space is mandatory because the Sudakov exponent $S$ is only known in that space [13]. The Sudakov exponent comprises those gluonic radiative corrections not taken into account in the evolution of the wave function. $\hat{T}_{H}$ is the Fourier

§ In [14, 15] a modification of the pion wave function is proposed where the distribution amplitude is multiplied by the exponential $\exp \left[-m_{q}^{2}g_{a}^{2}/x(1-x)\right]$. The parameter $m_{q}$ represents a constituent quark mass of, say, 330 MeV. Since the exponential substantially deviates from unity only in the end-point regions it leads to a strong additional suppression in the case of the Chernyak-Zhitnitsky distribution amplitude ($\langle x^{-1} \rangle$ changes from a value of 5 to 3.71 at the scale $\mu_{0}$). For narrow distribution amplitudes ($B_{2} \leq 0$), on the other hand, the exponential has only a minor bearing on the results for $F_{\pi\gamma}$. 

\[\]
transform of the lowest order momentum space hard scattering amplitude. It reads

$$\hat{T}_H(x, b, Q) = \frac{2}{\sqrt{3\pi}} K_0 \left( \sqrt{1-x} Q b \right)$$

(7)

where $K_0$ is the modified Bessel function of order zero. Due to the properties of the Sudakov exponent any contribution is damped asymptotically, i.e. for $\ln(Q^2/\mu_0^2) \to \infty$, except those from configurations with small quark-antiquark separations and, as can be shown, the limiting behaviour (7) emerges. $b$ plays the role of an infrared cut-off; it sets up the interface between non-perturbative soft gluon contributions - still contained in the hadronic wave function - and perturbative soft gluon contributions accounted for by the Sudakov factor. Hence, the factorization scale $\mu_F$ is to be taken as $1/b$.

Finally, $\hat{\Psi}_\pi$ is the Fourier transform of the momentum space (light-cone) wave function of the pion for which a Gaussian $k_\perp$-dependence is employed

$$\Psi_\pi(x, k_\perp; \mu_F) = \frac{f_\pi}{2\sqrt{6}} \phi_\pi(x, \mu_F) N \exp \left( -a_\pi^2(\mu_F) \frac{k_\perp^2}{x(1-x)} \right),$$

(8)

Here $N = 16\pi^2 a_\pi^2/(x(1-x))$ and, for a distribution amplitude with $B_n = 0$ for $n \geq 4$, $a_\pi = 1/(\pi f_\pi \sqrt{8(1+B_2)}$. The $\pi^0 \to \gamma\gamma$ constraint [14] is automatically satisfied for that choice of the transverse size parameter $a_\pi$. $\Psi_\pi$ represents a soft wave function, i.e. a full wave function with its pertubative tail removed from it. For $B_2 = 0$ the wave function (8) leads to a valence Fock state probability of 0.25 and a r.m.s. radius of 0.42 fm. Using the wave function (8) in a modified HSA calculation, one finds excellent agreement with the CLEO [7] and CELLO [12] data above $Q^2 \simeq 1$ GeV$^2$ for $B_2(\mu_0) = -0.006 \pm 0.014$ [3, 8] (see Fig.4). Hence, the asymptotic wave function, i.e. the asymptotic distribution amplitude combined with the Gaussian $k_\perp$-dependence, works very well if the modified HSA is used.

A similar analysis of the $\eta\gamma$ and the $\eta'\gamma$ transition form factors has been carried through in [1]. It is important thereby to take into account mass corrections and the $\eta-\eta'$ mixing. The results of that analysis are in excellent agreement with the available data including the recent CLEO data [7]. The values of the $\eta-\eta'$ mixing angle and the decay constants are calculated in [1] to be $\theta_P = -18^\circ \pm 2^\circ$, $f_\eta = 175 \pm 10$ MeV and $f_{\eta'} = 95 \pm 6$ MeV, respectively. The $\eta\gamma$ transition form factor can be analyzed in the same manner. Estimates of that form factor can be found in [17].

With the advent of the CLEO data the $\pi\gamma$ transition form factor attracted much interest and, besides [1, 8], many papers have been devoted to its analysis elucidating various aspects of it [13, 18, 19, 20, 21]. Particularly interesting is the generalization of (1) to the case of two virtual photons. In the standard HSA and again with $B_n = 0$ for $n \geq 4$, the $\pi\gamma^*$ transition form factor reads [13]

$$F_{\pi\gamma^*}(Q^2, \omega) = \sqrt{2} \frac{f_\pi}{Q^2 (1-\omega)^3} \left\{ [1 - \omega^2 + 2\omega \ln \omega] \times [1 + \frac{1 + 28\omega + \omega^2}{(1-\omega)^2} B_2(\mu_F)] + 10\omega \ln \omega B_2(\mu_F) \right\}$$

(9)
where \( \omega = Q'^2/Q^2 \). The larger one of the two photon virtualities is denoted by \( Q^2 \), the smaller one by \( Q'^2 \). The factorization scale may be chosen as \( \mu_F = Q\sqrt{1+\omega} \). \( \alpha_s \)-corrections to \( F_{\pi\gamma^*} \) can be found in \[9\] and an estimate of power corrections in \[22\]. The treatment of \( \pi\gamma^* \) transitions within the modified HSA is straightforward generalization of (6) \[18\].

Interestingly, the \( F_{\pi\gamma^*} \) form factor still behaves as \( Q^{-2} \) at large \( Q^2 \). This is to be contrasted with the \( Q^{-2}Q'^{-2} \) behaviour of the vector meson dominance model \[23\]. In the limes \( \omega \to 1 \) (9) simplifies to

\[
F_{\pi\gamma^*} = \frac{\sqrt{2}}{3} f_\pi \left[ 1 + \frac{1}{2}(1-\omega)(1-12B_2(\mu_F)) \right].
\]

(10)

The limiting behaviour of the form factor for \( \omega = 1 \) which is strictly independent on the form of the distribution amplitude, has also been derived from QCD sum rules \[24\]. In \[21\] the triangle diagram is analyzed with the most general form of the \( \pi q\bar{q} \) vertex. The result obtained for \( F_{\pi\gamma^*} \) in that paper is similar to (9) provided \( B_2 \) is put to zero in (9). The differences between the two results are strongest at \( \omega = 0 \) (about 9%) while both the results coincide at \( \omega = 1 \).

3. Pionic decays of charmonium

In view of the results for \( F_{\pi\gamma} \) a fresh analysis of the decays \( \chi_{cJ} \to \pi\pi \) is in order. Using the information on the \( \pi \) wave function obtained from the analysis of \( F_{\pi\gamma} \), one finds the following values for the partial widths

\[
\Gamma(\chi_{c0(2)} \to \pi^+\pi^-) = 0.872 (0.011) \text{ keV}
\]

(11)

within the standard HSA \[8\]. As usual the renormalization and the factorization scales are identified in that calculation and put equal to the \( c \)-quark mass. The parameter describing the \( \chi_{cJ} \) state is the derivative \( R'_p(0) \) of the non-relativistic \( cc\bar{c} \) wave function at the origin (in coordinate space) appropriate for the dominant Fock state of the \( \chi_{cJ} \), a \( cc\bar{c} \) pair in a colour-singlet state with quantum numbers \( ^{2S+1}L_J = ^3P_J \). \( m_c = 1.5 \text{ GeV} \) and, of course, the leading order standard HSA value -0.39 for \( B_2(\mu_0) \) are chosen as well as \( R'_p(0) = 0.22 \text{ GeV}^{5/2} \) which is consistent with a global fit of charmonium parameters \[25\] as well as with results for charmonium radii from potential models \[26\].

In \[8\] the modified HSA is also used to calculate the \( \chi_{cJ} \to \pi\pi \) decay widths. Taking \( B_2 = 0 \) and the other parameters as quoted above, one finds

\[
\Gamma(\chi_{c0(2)} \to \pi^+\pi^-) = 8.22 (0.41) \text{ keV}.
\]

(12)

For comparison the experimental data as quoted in \[27\] and reported in a recent paper of the BES collaboration \[28\] are

\[
\begin{align*}
\Gamma(\chi_{c0} \to \pi^+\pi^-) & = 1.05 \pm 0.30 \text{ keV (PDG),} \\
& = 62.3 \pm 17.3 \text{ keV (BES),}
\end{align*}
\]

\[
\begin{align*}
\Gamma(\chi_{c2} \to \pi^+\pi^-) & = 3.8 \pm 2.0 \text{ keV (PDG),} \\
& = 3.04 \pm 0.73 \text{ keV (BES).}
\end{align*}
\]

(13)
One notes that both the theoretical results, (11) and (12), fail by at least an order of magnitude. To assess the uncertainties of the theoretical results one may vary the parameters, $m_c$, $B_2$ and $\Lambda_{QCD}$. However, even if the parameters are pushed to their extreme values the predicted rates are well below data. Thus, one has to conclude that calculations based on the assumption that the $\chi_{cJ}$ is a pure $c\bar{c}$ state, are not sufficient to explain the observed rates. The necessary corrections would have to be larger than the leading terms. A new mechanism is therefore called for.

Recently, the importance of higher Fock states in understanding the production and the inclusive decays of charmonium has been pointed out [29]. It is therefore tempting to assume the inclusion of contributions from the $|c\bar{s}(3S_1)g\rangle$ Fock state to exclusive $\chi_{cJ}$ decays as the solution to the failure of the HSA. The usual higher Fock state suppression by powers of $1/Q^2$ [30] where $Q = m_c$ in the present case, does not appear as a simple dimensional argument reveals: the colour-singlet and octet contributions to the decay amplitude behave as

$$M_{fJ}^{(c)} \sim f_2^{(c)} f_8^{(e)} m_c^{-n_c}.$$  

The singlet decay constant, $f_{J}^{(1)}$, represents the derivative of a two-particle coordinate space wave function at the origin. Hence it is of dimension GeV$^2$. The octet decay constant, $f_{J}^{(8)}$, as a three-particle coordinate space wave function at the origin, is also of dimension GeV$^2$. Since $M_{fJ}^{(c)}$ is of dimension GeV, $n_c = 3$ in both cases. Note that the $\chi_{cJ}$ decay constants may also depend on $m_c$. Obviously, the colour-octet contribution will also play an important role in the case of the $\chi_{bJ}$ decays.

In [8] the colour-octet contributions to the exclusive $\chi_{cJ}$ decays are estimated by calculating the hard scattering amplitude from the set of Feynman graphs shown in Fig. 2 and convoluting it with the asymptotic pion wave function. The colour-octet and singlet contributions are to be added coherently. The $\chi_{cJ} \rightarrow \pi\pi$ decay widths are given in terms of a single non-perturbative parameter $\kappa$ which approximately accounts for the soft physics in the colour-octet contribution. A fit to the data [27, 28] yields $\kappa = 0.16$ GeV$^2$ (with $m_c = 1.5$ GeV; $\Lambda_{QCD} = 0.2$ GeV) and the widths

$$\Gamma(\chi_{c0(2)} \rightarrow \pi^+\pi^-) = 49.85 (3.54) \text{ keV}. \quad (15)$$

Comparison with (13) reveals that the inclusion of the colour-octet mechanism brings predictions and data in generally good agreement. The value found for the parameter $\kappa$ has a reasonable interpretation in terms of charmonium properties and the mean transverse momentum of the quarks inside the pions. Results for the decays into pairs of uncharged pions are presented in Table 2. The quoted results for the pionic charmonium decays refer to a calculation within the standard HSA but similar good results are found when the modified HSA is used [31]. The only soft parameter appearing in the latter calculation is the octet-decay constant $f_{J}^{(8)}$ of the charmonium state.

Thus it seems that the colour-octet mechanism leads to a satisfactory explanation of the decay rates of the $\chi_{cJ}$ into two pions. Of course, that mechanism has to pass more tests in exclusive reactions before this issue can be considered as being settled.
4. The $\pi$ form factor

Let us now turn to the case of the $\pi$ form factor and discuss the implications of the constraints on the pion’s wave function obtained from the $\pi\gamma$ analysis. The leading twist result for the pion form factor can be brought into a form similar to (14):

$$F_\pi(Q^2) = \frac{8\pi}{9} (x^{-1})^2 \frac{F_\pi^2}{Q^2} \alpha_s(\mu_R) \left[ 1 + \frac{\alpha_s(\mu_R)}{2\pi} K_\pi(Q^2, \mu_R) + \mathcal{O}(\alpha_s^2) \right].$$

(16)

Choosing $\mu_F = \mu_R$ and using the value of $B_2^{LO}(\mu_0)$ determined in the leading twist analysis of the $F_{\pi\gamma}$ as well as a value of, say, 0.4 for $\alpha_s$, one obtains the result $0.097 \text{ GeV}^2/Q^2$ for $F_\pi$ to lowest order pQCD. That result is much smaller than the admittedly poor experimental result [33]: $F_\pi = 0.35 \pm 0.10 \text{ GeV}^2/Q^2$. The $\alpha_s$-corrections are too small to account for that discrepancy [32]. The modified HSA likewise pro-
vides too small a perturbative contribution \[34\]. It is important to remember at this point that, formally, the perturbative contribution to the pion form factor represents the overlap of the large momentum tails of the initial and final state wave functions $\Psi_\pi$. That contribution, frequently termed the Feynman contribution, is customarily assumed to be negligible already at momentum transfers as low as a few GeV$^2$. Examining the validity of that presumption by estimating the Feynman contribution from the asymptotic wave function (8), one finds results of appropriate magnitude to fill in the gap between the perturbative contribution and the data of \[33\]. The results exhibit a broad flat maximum which, for momentum transfers between 3 and about 15 GeV$^2$, simulates the dimensional counting behaviour. For a wave function based on the Chernyak-Zhitnitsky distribution amplitude, on the other hand, the Feynman contribution exceeds the data significantly \[3, 34\]. Large Feynman contributions have also been found by other authors \[2, 35\]. Thus, the small size of the perturbative contribution to the elastic form factor finds a comforting although model-dependent explanation, a fact which has been pointed out by Isgur and Llewellyn Smith \[2\] long time ago. Of course this line of reasoning is based on the assumption that the data of \[33\] are essentially correct.

A comment concerning the pion form factor in the time-like region is in order. In the standard HSA the predictions for the form factor in both the time-like and the space-like region, are identical. The experimental information on the time-like form factor comes from two sources, $e^+ e^- \rightarrow \pi^+ \pi^-$ and $J/\Psi \rightarrow \pi^+ \pi^-$, which provides, to a very good approximation\[1\] the form factor at $s = M_{J/\Psi}^2$. Although the $e^+ e^-$ annihilation data of Bollini et al. \[36\] suffer from low statistics, they agree very well with the result obtained from the $J/\Psi$ decay. Combining both the data, one finds $|F_\pi| = 0.93 \pm 0.08$ GeV$^2$/s in the momentum transfer range between 2 and 10 GeV$^2$, a value which is roughly a factor of 3 larger than the space-like data. The modified HSA can account for that large ratio of the time-like over space-like form factors \[38\] although the perturbative contributions to the form factor in both the regions are too small.

The structure function of the pion offers another possibility to test the wave function against data. As has been pointed out in \[14\] the parton distribution functions are determined by the Fock state wave functions. Since each Fock state contributes through the modulus squared of its wave function integrated over transverse momenta up to $Q$ and over all fractions $x$ except those pertaining to the type of parton considered, the contribution from the valence Fock state should not exceed the data of the valence quark structure function. As discussed in \[5, 39\] the asymptotic wave function respects this inequality while the Chernyak-Zhitnitsky one again fails dramatically.

\[1\] At large $Q^2$ the Feynman contribution is suppressed by $1/Q^2$ as compared to the perturbative contribution.

\[1\] The contribution from $c\bar{c}$ transitions into the light quarks via three gluons cancel to zero if quark masses are neglected \[37\].
5. Summary

The study of hard exclusive reactions is an interesting and challenging subject. The standard HSA, i.e. the valence Fock state contribution in collinear approximation to lowest order perturbative QCD, while asymptotically correct (at least for form factors), does not lead to a consistent description of the data. In many cases the predicted perturbative contribution to particular exclusive reactions are much smaller than the data. The observed spin effects do not find a comforting explanation. In some reactions agreement between prediction and experiment is found although at the expense of dominant contributions from the soft end-point regions rendering the perturbative analysis inconsistent.

From a detailed analysis of the $\pi\gamma$ transition form factor it turned out that the pion distribution amplitude is close to the asymptotic form. Strongly end-point concentrated distribution amplitudes are obsolete and the ostensible successes in describing the large momentum transfer behaviour of the pion form factor in the space-like region and charmonium decays into two pions with such distribution amplitudes must therefore be dismissed.

In view of these observations it seems that higher Fock state and/or higher twist contributions have to be included in the analysis of exclusive reactions. However, not much is known about them as yet. We are lacking systematic investigations of such contributions to exclusive reactions. The colour octet model for exclusive charmonium decays is discussed in this talk as an example of such contributions.

1. G.P. Lepage and S.J. Brodsky, *Phys. Rev.* D 22, 2157 (1980).
2. N. Isgur and C.H. Llewellyn Smith, *Nucl. Phys.* B 317, 526 (1989).
3. A.V. Radyushkin, *Nucl. Phys.* A 532, 141c (1991).
4. A. Duncan and A.H. Mueller, *Phys. Lett.* B 93, 119 (1980); V.L. Chernyak et al., *Z. Phys.* C 42, 583 (1989); N.G. Stefanis and M. Bergmann, *Phys. Rev.* D 47, R3685 (1993).
5. R. Jakob, P. Kroll and M. Raulfs, *J. Phys.* G 22, 45 (1996); P. Kroll, Proceedings of the PHOTON95 Workshop, Sheffield (1995), eds. D.J. Miller et al., World Scientific.
6. P. Kroll and M. Raulfs, *Phys. Lett.* B 387, 1996 (848).
7. V. Savinov et al., CLEO collaboration, Proceedings of the PHOTON95 Workshop, Sheffield (1995), eds. D.J. Miller et al., World Scientific.
8. J. Bolz, P. Kroll and G.A. Schuler, preprint CERN-TH/96-266 (1996), [hep-ph/9610263](http://arxiv.org/abs/hep-ph/9610263), to be published in Phys. Lett. B.
9. E. Braaten, *Phys. Rev.* D 28, 524 (1983).
10. D. M"uller, *Phys. Rev.* D 51, 3855 (1995).
11. T.F. Walsh and P. Zerwas, *Nucl. Phys.* B 41, 551 (1972).
12. CELLO coll., H.-J. Behrend et al., *Z. Phys.* C 49, 401 (1991).
13. V.L. Chernyak and A.R. Zhitnitsky, *Nucl. Phys.* B 201, 492 (1982).
14. S.J. Brodsky, T. Huang and G.P. Lepage, *Banff Summer Institute, Particles and Fields* 2, p. 143, A.Z. Capri and A.N. Kamal (eds.), 1983.
15. F.-G. Cao, T. Huang and B.-Q. Ma, *Phys. Rev.* D **53**, 6582 (1996).
16. J. Botts and G. Sterman, *Nucl. Phys.* B **325**, 62 (1989);
   H.N. Li and G. Sterman, *Nucl. Phys.* B **381**, 129 (1992).
17. P. Aurenche et al., γγ Physics, [hep-ph/9601317](hep-ph/9601317).
18. S. Ong, *Phys. Rev.* D **52**, 3111 (1995).
19. A.V. Radyushkin and R.T. Ruskov, preprint [hep-ph/9511270](hep-ph/9511270) (1995).
20. V.V. Anisovich, D.I. Melikhov and V.A. Nikonov, preprint [hep-ph/9607215](hep-ph/9607215).
21. A. Anselm, A. Johansen, E. Leader and L. Lukaszuk, preprint hep-ph/9603444.
22. A.S. Gorskiï, *Sov. J. Nucl. Phys.* **50**, 498 (1989).
23. P. Kessler and S. Ong, *Phys. Rev.* D **48**, R2974 (1993).
24. V.A. Novikov, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I.
   Zakharov, *Nucl. Phys.* B **237**, 525 (1984); V.A. Nesterenko and A.V.
   Radyushkin, *Yad. Fiz.* **38**, 476 (1983) (*Soviet J. Nucl. Phys.* **38**, 284 (1983).
25. M.L. Mangano and A. Petrelli, *Phys. Lett.* B **352**, 445 (1995).
26. W. Buchmüller and S.-H. Tye, *Phys. Rev.* D **24**, 132 (1981).
27. Particle Data Group: Review of Particle Properties,
   *Phys. Rev.* D **54**, 1 (1996).
28. Y. Zhu for the BES coll., talk presented at the XXVIII Int. Conf. on High
   Energy Physics, 25-31 July 1996, Warsaw, Poland.
29. G.T. Bodwin, E. Braaten and G.P. Lepage, *Phys. Rev.* D **51**, 1125 (1995).
30. S.J. Brodsky and G.R. Farrar, *Phys. Rev. Lett.* **31**, 1153 (1973);
    V.A. Matveev, R.M. Murradyan and A.V. Tavkheldize,
    *Lett. Nuovo Cim.* **7**, 719 (1973).
31. J. Bolz, P. Kroll and G.A. Schuler, in preparation.
32. R.D. Field et al., *Nucl. Phys.* B **186**, 429 (1981);
    F.M. Dittes and A.V. Radyushkin, *Sov. J. Nucl. Phys.* **34**, 293 (1981);
    E. Braaten and S.-M. Tse, *Phys. Rev.* D **35**, 2255 (1987).
33. C.J. Bebek et al., *Phys. Rev.* D **13**, 25 (1976) and D **17**, 1693 (1978).
34. R. Jakob and P. Kroll, *Phys. Lett.* B **315**, 463 (1993); B **319**, 545(E) (1993).
35. P.L. Chung, F. Coester and W.N. Polyzou, *Phys. Lett.* B **205**, 545 (1988);
    L.S. Kisslinger and S.W. Wang, *Nucl. Phys.* B **399**, 63 (1993);
    V. Braun and I. Halperin, *Phys. Lett.* B **328**, 457 (1994);
    A.E. Dorokhov, *Nuovo Cimento A* **109**, 391 (1996).
36. D. Bollini et al., *Lett. Nuovo Cim.* **14**, 418 (1975).
37. S.J. Brodsky and G.P. Lepage, *Phys. Rev.* D **24**, 2848 (1981).
38. T. Gousset and B. Pire, *Phys. Rev.* D **51**, 15 (1995).
39. T. Huang, B.-Q. Ma and Q.-X. Sheng, *Phys. Rev.* D **49**, 1490 (1994).