Mathematical Model for Assessing the Limits of Crack Resistance of Structural Steels of Large-Sized Structures

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Abstract. In metal structures under operating conditions, as well as in emergency situations due to overloads, the presence of stress concentrators, etc., plastic deformations may occur. Such deformation by tensile stresses introduces some damage to the structure of steel, which adversely affects its subsequent operation under cyclic loads. Preliminary relative plastic deformation, \(\sim 1-6\%\), which occurs in structural steels under tension, unambiguously reduces plasticity. It negatively affects the endurance of the material and leads to the early formation of macrocracks. In addition, after unloading, an internal residual stress field at the meso-micro level remains in the structural elements, which also reduces the endurance limit. In causes the decrease of such an important indicator of reliability as fracture toughness, which reflects the resistance of the material to crack propagation. In this regard, designing structures, it seems important to improve the calculation methods for assessing and predicting their fatigue strength, taking into account the listed factors.

The article is devoted to the development of a mathematical model that includes dependencies for assessing the fracture toughness of metal structures according to the fracture toughness limit for all sizes of defects in the form of cracks. Analytical dependences have been obtained for calculating the relative limit of crack resistance based on the main mechanical characteristics of the state of the material. The results of the study can be used to assess the crack resistance of structural elements and welded joints pre-deformed by tension.

1. Introduction

In welded joints and bonds subjected to cyclic loads, over time, cracks, which ultimately leads to a dangerous state of structures and their destruction, often appear and develop [1]. The kinetics of formation of macrocracks and their development depend on many factors. This requires comprehensive studies of the crack resistance of welded joints and the creation of appropriate mathematical models. Macroscopic inhomogeneity of mechanical characteristics caused by thermal welding cycles leads to inhomogeneous plastic deformation. Different sections of the welded joint are characterized by a different degree of inhomogeneity of the mechanical characteristics of the material [2]. In some cases, in welded joints made of heat-hardened steels, it leads to the contact hardening of the material under an external load as a result of its plastic deformation. Researchers have also solved a number of particular problems aimed at increasing the strength and durability of welded joints with interlayers, [3, 4, 5, 6], etc. Tensile plastic deformation, as was established in [7], [8], leads to a decrease in the limit endurance and resistance to the propagation and initiation of a macrocrack of...
length L, even in a plane stress state. With preliminary tensile deformations of the steel close to the specimen elongation at break $\delta_{5(10)}$, catastrophic failure under cyclic load can be initiated by a critical crack with a much shorter length [9, 10]. In such cases, the area with a defect in the form of a crack should be assessed for the possibility of brittle fracture by the crack resistance limit under the condition of plane deformation and plastically deformed material by tension [11, 12, 13, 14].

The purpose of this study is to develop analytical dependences and a mathematical model for assessing the crack resistance of large-sized welded structures after the appearance of a crack in a previously plastically deformed material and, in particular, for plastically deformed steels as a result of overloads.

2. Microflexibility approach

Let us take as an assumption that the macrozone of plastic deformation of a structural element, in which a crack is formed, covers the area of development of a macrocrack. Moreover, its deformation is sufficiently uniform.

The endurance limit in the general case depends ambiguously on the degree of preliminary deformation [15, p. 578], and its decrease can be represented as the ratio of the endurance limit of deformed material to the undeformed one $K = \sigma_{\sigma_1}/\sigma_{\sigma_0}$ (Fig. 1). The shape of the curve in the graph depends on how the test material is processed. The empirical dependence is presented in [7, p. 68]. The endurance limit almost always reaches its minimum in a small area (1 ... 10%) of preliminary plastic deformation of the steel. After that, it increases and can reach such a value as in an undeformed material (see Fig. 1). Thus, the very fact of the appearance of a propagating crack, obviously, begins to play a predominant role in fatigue after preliminary deformation. Non-propagating cracks (even with significant preliminary plastic deformation) can only rarely lead to fracture. This is illustrated by numerous examples of damage presented in [1]. It follows from the above that, in most cases, macrocrack appears earlier in pre-deformed by tension material than in the undeformed one. Therefore, it is necessary to know its critical size $L_c$ for a pre-deformed material.

The crack resistance of a plastically deformed material up to the values $\sigma < \sigma_{\sigma_0}$ can be estimated by a two-parameter criterion of fracture - the crack resistance limit, which is calculated from the known analytical dependence [11]:

$$K_c = K_{1c} \sqrt{1 - \left(\frac{\sigma_c}{\sigma_{\sigma_0}}\right)^2},$$

where $\sigma_c$ is the critical fracture stress in the gross section; $K_{1c}$ is critical stress intensity factor (material constant).

The results of a theoretical analysis of the development of defects in the form of microcracks with a radius at the apex $\rho \gg 0$ according to equation (1) are presented in [16, p. 457]. They are aimed at leveling the gap between the approaches of the stress concentration classical theory and the mechanics of fracture of bodies with cracks.

After plastic deformation, the stress $\sigma_c$ will depend on the parameters of the defect. We assume that the main characteristic of the defect will be the critical length of the incipient crack $L_c$. Then, in the case under consideration, $\sigma_c = K_c/\sqrt{\pi L_c}$.

In order to use (1), it is necessary to know the degree of preliminary tensile deformation, since $K_c$ will primarily depend on it. The unknown value $L_c$ will be considered a function of the preliminary deformation.

The authors of [17], as a result of studying the structural fracture parameter $d$ for a disc-shaped crack, propose to use the criterion dependence of crack resistance in a one-dimensional form:
\[
\frac{K_c}{K_{1c}} = \sqrt{\frac{2\eta(1-\eta)}{\arccos(\eta)}}
\]

where \(\eta = \frac{a}{a + d}\) is a dimensionless parameter specified by the range of variation \((0 \leq \eta \leq 1)\); \(a\) is the radius of the disc-shaped crack.

Figure 1. Influence of preliminary tensile deformation on the endurance limit at normal temperatures of various steels [7]: a) -steel 45; b) -12XH3A (- - experimental data, solid line - calculation); c) - 15XCHD; d) -40X (1-grinding; 2-turning); e) an iron-based heat-resistant alloy; f) -12X18H10T.

3. Building a mathematical model

It is proposed to develop analytical dependencies in the following sequence. The remaining plastic properties of a structural element or a part of it can be estimated by deformation using relative elongation for a certain area of the material. We will assume that in some area there is a uniform plastic deformation, which is associated with the critical elongation to fracture \(\delta_{5(10)}\) by the known dependence [18]:

\[
e = \ln(1 + \delta_5).
\]
Plastic preliminary deformation leads to a decrease in the critical opening at the crack tip, which is a consequence of a decrease in the ultimate deformation of the material. According to the results of work [19], it can be noted that the dependence of the critical crack tip opening $\delta_{\text{CTO}}$ during deformation of steel by stretching is very close to the linear dependence. This fact is observed for both aluminum alloy and austenitic steel. Taking into account the fact that $\delta_{\text{CTO}} = 0$ in the limit of plastic preliminary deformation and using a linear approximation of the dependence $\delta_{\text{CTO}}(\varepsilon)$, we have the following expression:

$$
\delta(\varepsilon) = \delta_{\text{TC}} - \left( \frac{\delta_{\text{TC}}}{\varepsilon_{\text{max}}} \right) \varepsilon.
$$

CCTO (critical crack tip opening) at the crack tip is determined by the known dependence of linear fracture mechanics:

$$
\delta_{\text{TC}} = \lambda \left( \frac{K_{\text{TC}}^2}{E \sigma_t} \right),
$$

where the coefficient $\lambda$ can be calculated by the formula [20]:

$$
\lambda = \frac{(1 - 2\mu)^2 E}{1,24 \pi \sigma_t} \left( \frac{q \sigma_t}{R_{\text{mce}} D} \right)^{\frac{1}{m+1}}.
$$

Here $E$ is the modulus of elasticity of the material; $R_{\text{mce}}$ is the stress to the micro-cleavage of the deformed material; $D$ is a coefficient that takes into account the increase in the first principal component of stress for the case of a complex stress state; $q$ is a coefficient showing the correlation between the principal components of stresses; $m$ is the hardening factor. For the specified parameters and coefficients, there are quite definite calculated dependences on the main mechanical characteristics of the material.

The critical stress intensity factor in (5) is calculated by the formula:

$$
K_{\text{TC}} = \sqrt{\left( \frac{R_{\text{mce}} D}{q \sigma_t} \right)^{\frac{1}{m+1}} \sigma_t \cdot 6,18 \pi d_z},
$$

where $d_z$ is the average grain diameter of the initial material. Substituting expressions (7), (6) into (5), and (5) into (4), after simple transformations and neglecting the small component of elastic deformations, the equation in the following form is obtained:

$$
\delta_{\text{TC}}(\varepsilon) = 5(1 - 2\mu)^2 d_z \left( 1 - \frac{\varepsilon \sigma_b}{100} \right).
$$

Plastic deformation $\varepsilon$ is calculated by the formula (3). Let us find the dependence $K_{\text{TC}}(\varepsilon)$.

From (5) we have $K_{\text{TC}}(\varepsilon) = \sqrt{E \sigma_t \delta_{\text{TC}}(\varepsilon)/\lambda}$. After substituting the known expressions into this equation and simplifying, it is possible to finally obtain a simple dependence of the critical stress intensity factor $K_{\text{TC}}$ on the current value of the material elongation $\delta$ in the following form:

$$
K_{\text{TC}}(\delta) = K_{\text{c}} = K_{\text{TC}} \sqrt{1 - \frac{\sigma_b}{100} \ln (1 + \delta)},
$$

where $\delta$ is the elongation of the steel, in fractions. The ratio $K_{\text{TC}}(\delta)/K_{\text{TC}}$ shows the degree of reduction in the fracture toughness of the steel from elongation (pulling) of the elementary region of
the structural element. Since at \( \delta = \delta_5 \) the ratio \( K_c/K_{1c} \) should equal zero, we equate the radical expression (9) to zero and write the equation for \( \delta \) in the form:

\[
\delta_5 = \exp\left(\frac{100}{\sigma_n}\right) - 1.
\] (10)

Here \( \sigma_n \) is substituted in MPa.

4. Results analysis

The results of the calculated verification of the proposed dependence (10) with reference data for structural steels of the ferrite-pearlite class (steel 10, 15Г, 22К, 50, Cr3цн, 10ХСНД, 37Х3А, и 30ХГСА in two different states of delivery) of thin-sheet rolled products can be considered quite satisfactory. The relative error in calculations for these steels was no more than 7%.

In fig. 2 and fig. 3 diagrams of changes in \( K_{1c} \) are shown as a function of \( \delta \) for the indicated steels. The critical intensity factor was calculated according to (7).

After substituting (11) into (2), we obtain an invariant expression for the mean value. The comparison results show that the functions practically coincide and are invariant in relative coordinates with good convergence. Both functions well approximate the experimental data for the crack resistance limit obtained for compact specimens with characteristic dimensions \( b \) [21].

\[ \eta^* = 1 - a/(a + d). \] (11)

Comparison of the given results testifies to their good convergence. Thus, the presented computational analysis of the obtained dependences confirms the legitimate application of the proposed approach to assessing the fracture toughness of steels of large-sized structures.

5. Conclusion

A simple analytical dependence of the critical elongation to fracture (10) has been obtained. The calculation results which are in satisfactory agreement with the experimental data for most structural steels have also been obtained. The formula allows to estimate the reference characteristic \( \delta_5 \) of the
sample by one known characteristic $\sigma_b$.

An analytical dependence of a material crack resistance limit change on its preliminary plastic deformation has been proposed. The results of comparing the calculated values of the fracture toughness limit with the experimental test data of the samples and the previously carried out mathematical solutions, convincingly prove the applicability of the developed analytical approach.

It has been established that the law of decreasing the crack resistance limit in the case of preliminary plastic elongation up to fracture ($\delta_2$) and a change in the parameter ($\eta^*$) of the previously grown crack in the material up to $L_c$, becomes invariant in relative coordinates under the condition of plane deformation and the absence of the influence of the limited sample or structures.

The resulting dependencies in the system submit represent the proposed mathematical model for assessing the fracture toughness of structures.

6. References

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