Formation of the Double White Dwarf Binary PTF J0533+0209 through Stable Mass Transfer?

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Abstract

Double white dwarf (DWD) binaries are important for studies of common-envelope (CE) evolution, Type Ia supernova progenitors and Galactic sources of low-frequency gravitational waves. PTF J0533+0209 is a DWD system with a short orbital period of $P_{\text{orb}} \sim 20$ minutes and thus a so-called LISA verification source. The formation of this system and other DWDs is still under debate. In this paper, we discuss the possible formation scenarios of this binary and argue that it is not likely to have formed through CE evolution. Applying a new magnetic-braking prescription, we use the MESA code to model the formation of this system through stable mass transfer. We find a model that can well reproduce the observed WD masses and orbital period but not the effective temperature and hydrogen abundance of the low-mass He WD component. We discuss the possibility of using H flashes to mitigate this discrepancy. Finally, we discuss the future evolution of this system into an AM CVn binary such as those that will be detected by spaceborne GW observatories like LISA, TianQin, and Taiji.

Unified Astronomy Thesaurus concepts: Close binary stars (254); Roche lobe overflow (2155); White dwarf stars (1799); Compact binary stars (283); Gravitational wave sources (677)

1. Introduction

Recently, a number of double white dwarfs (DWDs) with very short orbital periods have been discovered, such as ZTF J1539+5027 (~7 minute orbital period, Burdge et al. 2019 and PTF J0533+0209 (~20 minute orbital period, Burdge et al. 2019c). In these DWDs, the primary WD is a CO WD and the companion star is an extremely low-mass (ELM) He WD with a mass smaller than $\sim 0.2 M_\odot$. Because of their short orbital periods, they are important Galactic gravitational wave (GW) sources that will be detected by spaceborne observatories such as LISA (Amaro-Seoane et al. 2017), TianQin (Luo et al. 2016), and Taiji (Ruan et al. 2020). However, the formation of these systems is not well understood.

The formation of DWDs has been widely investigated in the past (Iben & Tutukov 1986; Lipunov & Postnov 1988; Tutukov & Yungelson 1994; Yungelson et al. 1994; Han et al. 1995; Iben et al. 1997; Han 1998; Sun & Arras 2018; Li et al. 2019). It is generally accepted that there are two possible formation scenarios for DWDs (see also Figure 2 in Li et al. 2019). In the first formation scenario, a binary system consisting of a CO WD and a low-mass donor star undergoes stable mass transfer via Roche lobe overflow (RLO) and evolves into a binary system consisting of a CO WD and a He WD. The systems produced from this scenario will follow the He WD mass–orbital-period relation (e.g., Tauris & Savonije 1999). In the second scenario, a binary system with a CO WD and a red-giant companion star is subject to unstable mass transfer and enters a common-envelope (CE) phase. If the system successfully ejects the CE, then it will evolve into a CO WD+He WD binary. Hereafter, we refer to these two scenarios as the stable RLO scenario and the CE scenario, respectively.

Sun & Arras (2018) have studied the formation of ELM WDs from binary evolution using a mass range of the progenitors of the ELM WDs between $1.0 \sim 1.50 M_\odot$. They found that ELM WDs can be produced from nonconservative mass transfer and are not likely to be formed via CE evolution.

Li et al. (2019) have comprehensively studied the formation of ELM WDs through both stable RLO and CE evolution. They found that the ELM WDs can form from both scenarios and argue that ELM WDs with a mass smaller than $\sim 0.22 M_\odot$ are formed from the stable RLO scenario, whereas systems with ELM WDs $>0.22 M_\odot$ formed via the CE scenario.

Applying so-called convection and rotation-boosted magnetic braking, Soethe & Kepler (2021) studied the formation of ELM WDs with NS companions and found that ELM WD masses of $0.15 \sim 0.27 M_\odot$ can be produced via stable RLO from low-mass X-ray binaries (LMXBs) with initial orbital periods of 1–25 days.

For the very tight-orbit DWD system with an ELM He WD component above a certain mass limit, the origin is clear. As an example, one may consider ZTF J1539+5027 with an orbital period of $\sim 7$ minutes and an ELM He WD mass of $\sim 0.21 M_\odot$. If this system were produced from stable RLO, then the orbital period at the end of mass transfer would have been around 2.4 days according to the WD mass–orbital-period relation from Tauris & Savonije (1999). It would subsequently have taken the system around $3.4 \times 10^{12}$ yr to evolve to the current orbital period of 7 minutes via GW damping. Therefore, this system cannot have been produced from stable RLO.

Regarding the system PTF J0533+0209, which is the focus of this paper (see Table 1 for parameters), Burdge et al. (2019c) suggested that it is formed through a CE scenario, but they did not discuss whether the CE can be ejected in order to produce such a system. The results of Sun & Arras (2018) and Li et al. (2019) show that this system is not likely to be formed from a CE scenario. However, it is unclear from the binary models of Li...
et al. (2019) and Soethe & Kepler (2021) if PTF J0533+0209 can be produced from stable RLO and subsequently decay its orbit sufficiently (once detaching from the WD mass–orbital-period correlation) within a Hubble time to reproduce its present orbital period of 20.6 minutes. Therefore, this system is puzzling as it is apparently also hard to explain its origin with binary orbital period correlation. As a conservative limit, we denote the binary orbital separations before and after the CE as $q_{\text{bind}}$ before entering the CE. Here $M_r$ is the mass within radial coordinate, $r$ of the donor star and $U$ is the internal energy that can be used in the ejection process. The latter includes the thermal energy of a simple perfect gas, the radiation energy, and terms related to the ionization of atoms and dissociation of molecules, as well as the Fermi energy of a degenerate electron gas (Han et al. 1994, 1995). The right-hand side expresses the release of orbital energy from in-spiral, $\Delta E_{\text{orb}}$ and $0 < \alpha_{\text{CE}} < 1$ is the efficiency of CE ejection (i.e., fraction of released orbital energy used to eject the CE). Here, $M_{\text{core}}$, $M_{\text{env}}$ and $M_{\text{CO}}$ are the masses of the core, the envelope, and that of the in-spiralling CO WD, respectively, and $a_i$ and $a_f$ denote the binary orbital separations before and after the CE phase, respectively. As a conservative limit, we first set $a_f$ to be the value of current binary separation.

Following Burdge et al. (2019c), we assume that the progenitor mass of the ELM WD is $1.20 M_\odot$. Using the stellar evolution code Modules for Experiments in Stellar Astrophysics (MESA, version: 12115, Paxton et al. 2011, 2013, 2015, 2018, 2019), we computed the evolution of a single star with a mass of $1.20 M_\odot$ and obtained a binding energy of the envelope, $E_{\text{bind}} = -1.20 \times 10^{48}$ erg, and a radius, $R_2 = 3.10 R_\odot$, when the core mass is around $0.167 M_\odot$, i.e., similar to the mass of the ELM He WD component in the PTF J0533+0209 system (Table 1). The initial binary separation before the CE can be computed from (Eggleton 1983):

$$R_i = a_i = \frac{0.49 q^{3/2}}{0.6 q^{3/2} + \ln(1 + q^{1/3})},$$

(2)

where $R_i$ is the Roche lobe radius and equal to the radius of the (sub)giant star. $q = M_{\text{donor}}/M_{\text{CO}}$ is the mass ratio, and $M_{\text{donor}}$ is the progenitor mass of the ELM WD, i.e., $M_{\text{donor}} = 1.20 M_\odot$. Solving Equations (1) and (2), we find a value of $\alpha_{\text{CE}} = 1.75$. However, this large value is not physical. Furthermore, we note that the binary separation of this system after a CE should be larger than the current orbital separation because of GW radiation since its formation until its current state, and thus the amount of released orbital energy from in-spiral would have been smaller. Therefore, the $\alpha_{\text{CE}}$ value would be larger than 1.75, making the CE even more unlikely. It should be noted that applying a larger progenitor mass of the ELM WD would only exacerbate the problem. Assuming, e.g., $M_{\text{donor}} = 1.40 M_\odot$ increases the envelope binding energy, leading to $\alpha_{\text{CE}} = 2.7$. Assuming a smaller mass, e.g., $M_{\text{donor}} = 1.0 M_\odot$ leads to longer evolution time and $\alpha_{\text{CE}} = 1.14$.

So far, the value of $\alpha_{\text{CE}}$ is not well constrained (see Ivanova et al. 2013 for a review). Davis et al. (2010) computed the detailed structure of $1-8 M_\odot$ AGB stars and computed the binding energy of the envelopes of these stars. With these

### Table 1

Comparison of Observed Data of PTF 0533+0209 (Burdge et al. 2019c) and our Simulated Binary with $M_1 = 1.40 M_\odot$, $P_{\text{orb}} = 7.00$ d and $M_{\text{CO}} = 0.55 M_\odot$.

| PTF J0533+0209 | Observations | Our Model |
|----------------|-------------|-----------|
| Orbital period (s) | 1234.0 | 1234.4 |
| CO WD mass ($M_\odot$) | 0.652 ± 0.04 | 0.669 |
| He WD mass ($M_\odot$) | 0.167 ± 0.030 | 0.155 |
| He WD radius ($R_\odot$) | 0.057 ± 0.004 | 0.060 |
| He WD log g (cgs) | 6.3 ± 0.1* | 6.1 |
| He WD log(L/L_\odot) | −0.33 ± 0.13 | −2.24 |
| He WD $T_{\text{eff}}$ (K) | 20 000 ± 800 | 6689 |
| He WD $X_{\text{surf}}$ | $4.9 \times 10^{-4}$ | 1.000 |
| Age (Gyr) | ... | 10.660 |

* Note: Our calculated value from released data is 6.2.
results, they found that it is possible to explain the population of post-CE binaries found by the SDSS with $\alpha_{\text{CE}} > 0.10$. Using a similar approach, Zorotovic et al. (2010) constrained the CE efficiency parameter to $0.20 < \alpha_{\text{CE}} < 0.30$. In addition, Nandez et al. (2015) studied the formation of DWDs from CE evolution with 3D simulations and constrained the CE efficiency parameter to $0.25 < \alpha_{\text{CE}} < 0.50$. From these studies, we expect $0.1 < \alpha_{\text{CE}} < 0.50$ for low-mass binaries. In any case, $\alpha_{\text{CE}} > 1.75$ is not realistic. Therefore, we conclude that PTF J0533+0209 did not form through a CE scenario and we will now focus on investigating the stable RLO scenario in the rest of the paper.

### 3. Stable Mass-transfer Scenario

#### 3.1. Method and Assumptions

Our binary evolution model is motivated by Chen et al. (2021), in which we modeled the formation of ELM WDs in binary MSP systems via RLO in LMXBs. Here we briefly summarize the assumptions and the method we adopt in our presented calculation of PTF J0533+0209 via RLO in a CV system.

We use the MESA code to model the binary evolution. In our calculation, the CO WD accretor is assumed to be a point mass, so we do not compute the detailed structure of this WD. The secondary star is assumed to initially be a zero-age main-sequence star with a chemical composition $X = 0.70$ (hydrogen), $Y = 0.28$ (helium), and $Z = 0.02$ (metals). We set the mixing length to be 2 times the local pressure scale height and do not consider overshooting in our calculation. The Reimer’s wind prescription is used for the secondary star (Reimers 1975):

$$M_{\text{wind}} = -4 \times 10^{-13} M_\odot \text{ yr}^{-1} \eta \left( \frac{R_2}{R_\odot} \right) \left( \frac{L_2}{L_\odot} \right) \left( \frac{M_2}{M_\odot} \right).$$

where $\eta = 1.0$ is our adopted scaling factor, $M_2$, $R_2$, $L_2$ are the mass, radius, and luminosity of the secondary star, respectively.

In our model, we have taken different mechanisms of angular momentum loss into consideration, including angular momentum loss due to GW radiation, magnetic braking, mass loss, and spin-orbital couplings. To compute the angular momentum loss due to GW radiation, magnetic braking, mass loss, and spin-orbital couplings. To compute the angular momentum loss due to spin-orbital couplings. The angular momentum loss due to GW radiation, magnetic braking, mass loss, and spin-orbital couplings.

Regarding the angular momentum loss due to magnetic braking, the standard Skumanich law (i.e., $\beta = \xi = \alpha = 0$ in Equation (5)) is usually applied to solar-type main-sequence stars. It may be not suitable for systems with donors different than the Sun since the stellar wind and the strength of the magnetic field can be very different which is important for the calculation of magnetic braking. By including a scaling of the strength of magnetic braking with convective turnover time and wind mass-loss rate, Van et al. (2019) modeled the evolution of LMXBs and found that the observed properties of persistent X-ray binaries are in better agreement with observations using a model with $\beta = 0$, $\xi = 2$, and $\alpha = 1$ (MB3), compared to models with other choices of $\beta$, $\xi$, and $\alpha$. Here, in our study of DWDs we adopt their general prescription, which can be described with the following equations:

$$\frac{dJ_{\text{mb}}}{dt} = \frac{dJ_{\text{mb, Sk}}}{dt} \left( \frac{\omega_2}{\omega_\odot} \right)^{3} \left( \frac{\tau_{\text{conv}}}{\tau_{\odot, \text{conv}}} \right) \left( \frac{M_{\text{wind}}}{M_{\odot, \text{wind}}} \right)^{\alpha},$$

where

$$\frac{dJ_{\text{mb, Sk}}}{dt} = -3.8 \times 10^{-30} M_2 R_2^5 \left( \frac{R_2}{R_\odot} \right)^{3} \omega_2^3 \text{ dyne cm.}$$

We adopt $\gamma_{\text{mb}} = 4$, $\tau_{\odot, \text{conv}} = 2.8 \times 10^6 \text{ s}$, and $M_{\odot, \text{wind}} = 2.54 \times 10^{-14} M_\odot \text{ yr}^{-1}$. $\omega$ is the orbital angular velocity and $\tau_{\text{conv}}$ is the turnover time of convective eddies. $M_\text{w}$ is the wind mass-loss rate of the secondary star.

A caveat of the applied model MB3 is that Chen et al. (2021) could not reproduce the many known wide-orbit binary MSPs using this magnetic-braking prescription (because the drain of orbital angular momentum is simply too strong if applying MB3). Nevertheless, here we explore binary evolution using MB3 as our default model.

We adopt the “isotropic re-emission model” for the mass transfer via RLO (see e.g., Tauris & van den Heuvel 2006, for a review). In this model, the material flows conservatively from the donor star to the WD. The further fate of this material will strongly depend on the accretion rate (see Chen et al. 2019). If the accretion rate is larger than the maximum rate of steady burning, the excess material will be lost in the form of an optically thick wind (Hachisu et al. 1996). Only in a narrow range of accretion, can the H or He burn stably, and the steady burning regimes of H and He do not overlap with each other (Nomoto 1982). Therefore, even in the steady burning regime of H, not all material can be retained by the WD because of unstable burning of He. If the accretion rate is lower than the minimum accretion rate of stable burning, the material will burn unstably and most of it will be ejected in violent shell flashes and nova outbursts and the WD may even reduce its mass if the accretion rate is too low (Hachisu et al. 1999). Keeping this and also the low mass of the CO WD of PTF J0533+0209 in mind, we simply assume that, as an average for the entire RLO phase spanning a wide range of mass-transfer rates ($M_2$), a fraction of $90\%$ of the transferred material is lost from the CO WD accretor and takes away with it the specific orbital angular momentum of this WD.

The mass-transfer rate, $M_2$ from the secondary star to the WD is computed with the scheme proposed by Ritter (1988). The amount of accretion onto the WD is limited by the Eddington accretion rate ($M_{\text{Edd}}$). We set the ratio of gravitational mass to rest mass of accreted material to be $0.99$ for our CO WD accretor.

In our calculation, we also take the rotation of the donor star into account. We assume that its initial rotation velocity is synchronized with the initial binary orbital period. The prescription of Paxton et al. (2015) is adopted to compute the angular momentum loss due to spin-orbital couplings. The evolution of angular velocity is computed with the following formulae:

$$\frac{d\omega_i}{dt} = \frac{\omega_i - \omega_i^*}{\tau_{\text{sync}}},$$

(7)
and
\[
\frac{1}{\tau_{\text{sync}}} = \frac{3}{q} \left( \frac{k}{T} \right) \left( \frac{R}{a} \right)^6,
\]
where \(\omega_j\) is the angular velocity at the face of cell \(j\) and \(\omega_{\text{orb}}\) is
the orbital angular velocity, \(\tau_{\text{sync}}\) is the synchronization time, \(r_g\)
is the radius of gyration, \((k/T)_c\) is the ratio of the apsidal motion constant
to the viscous dissipation timescale, \(R\) is the stellar radius, and \(a\) is the
binary separation. The synchronization time depends on the structure of stars.
For stars with convective envelopes, turbulent viscosity in the convective
region, which may operate on the equilibrium tide, is the most
efficient form of viscous dissipation. For stars with radiative
evelopes, the resonances of the free gravity modes are damped
by radiative dissipation which operates on the dynamical tide.
The value of \((k/T)_c\) is different for these two kinds of stars and
can be calculated as in Hurley et al. (2002). In our calculation,
we do not include the internal heating due to the tidal fraction
in the thermal evolution of the donors.

Chemical mixing and transport of angular momentum
induced by different kinds of instabilities resulting from
rotation (Heger et al. 2000) are also taken into account, i.e.:
chemical mixing induced by secular shear instability,
Eddington–Sweet circulation, dynamical shear instability,
and Goldreich–Schubert–Fricke instability. Following Heger et al.
(2000), the mixing efficiency factor in our calculation is
assumed to be \(f_m = 1/30\). With respect to the transport of
angular momentum, we only consider the Spruit–Tayler
dynamo (Spruit 2002; Heger et al. 2005). Following Heger
et al. (2000), we set the parameter \(f_\mu = 0.05\), which regulates
the inhibiting effect of chemical gradients on the mixing
process. Motivated by Istrate et al. (2016), we also include
element diffusion induced by gravitational settling and
chemical and thermal diffusion in our calculation.

3.2. Initial Values

In order to find a model which can reproduce the properties
of PTF J0533+0209, we compute a grid of binary evolution with
the above assumptions. Given the low mass of the currently observed
CO WD \((M_{\text{CO}} = 0.652 M_\odot)\), we assume an initial CO WD mass
of \(M_{\text{CO},i} = 0.55 M_\odot\). The donor (secondary star) mass ranges
between \(M_{2,i} = 1.0–1.4 M_\odot\) in steps of \(\Delta M_2 = 0.05 M_\odot\).
The initial orbital period ranges between \(P_{\text{orb},i} = 1.0–100\) days in steps of
\(\Delta \log(P_{\text{orb},i}/\text{days}) = 0.1\). With this grid, we evolve models
resulting in final properties relatively close to the properties of
PTF J0533+0209. Next, we decrease the steps of \(\Delta P_{\text{orb}}\) and find
improved models which can better reproduce the observed
properties of PTF J0533+0209. Based on these models, we also compute
some models with different WD or donor masses in
order to understand the influence of these parameters on our
results.

3.3. Results

In this section, we present our binary evolutionary model
with properties closest to those of PTF J0533+0209. In this
model, the initial values are \(M_{2,i} = 1.40 M_\odot\), \(P_{\text{orb},i} = 7.00\) days,
and \(M_{\text{CO},i} = 0.55 M_\odot\).

Figure 1 displays the HR diagram of the secondary star (top), the
evolution of mass-transfer rate (middle), and orbital and spin
periods (bottom), as a function of time. In the middle panel, two
phases of RLO mass transfer are identified: cataclysmic variable
(CV) and AM Canum Venaticorum (AM CVn). PTF J0533+0209
is located between these two stages.

A more qualitative description of this example is as follows:
when the secondary star evolves beyond the main sequence
(after termination of core hydrogen burning) it enters the
as the ELM He WD radius. Since the He WD mass in PTF J0533+0209 is quite small, its progenitor should start mass transfer rather early in its stellar evolution, shortly after it has produced a small core mass. The ZAMS mass of the He WD progenitor should not be too large since in that case, the RLO is no longer stable. In addition, early on the main sequence, the donor star is slightly out of synchronization. As the binary orbital period decreases significantly due to magnetic braking, the effect of tides becomes stronger and the spin of the donor star becomes synchronized with the orbit. At the very beginning of the first mass-transfer phase, because of mass loss, the donor star spins down and is out of synchronization with the orbit (see inset in bottom panel). However, due to the strong tides, it becomes synchronized again after a short time (less than 5 Myr).

At the end of the first mass-transfer phase (t = 3.23 Gyr), the secondary star detaches from its Roche lobe and slowly settles on the He WD cooling track with a mass of M_{2,i} = 0.155 M_\odot. During the RLO, the CO WD has accreted \sim 0.12 M_\odot, and now has a mass of M_{CO} = 0.669 M_\odot and the orbital period is P_{\text{orb}} = 0.22 day. The system subsequently evolves as a detached DWD system, as presently observed, with decreasing orbital separation due to GW radiation. Around t = 10.66 Gyr, the He WD fills its Roche lobe and starts to transfer material onto the CO WD and the system evolves as an AM CVn system. Slightly before that stage (t = 10.66 Gyr), when the orbital period and masses are close to those observed for PTF J0533+0209, the system has the physical parameters listed in Table 1.

During the AM CVn phase, the spin of the donor star is always out of synchronization with the orbit. This is because the donor star spin increases due to the mass loss and the effect of tides becomes weaker as the orbital period increases.

Figure 2 shows a more detailed comparison of our results with observations. Concluding from the values in Table 1 and Figure 2, we find that our model can very well reproduce the WD masses and the orbital period of PTF J0533+0209, as well as the ELM He WD radius (and log g). However, the simulated effective temperature (T_{\text{eff}} and thereby also the luminosity, \textit{L}) and the surface H abundance (X_{\text{surf}}) of the ELM He WD represent quite a mismatch. In the grid we computed, we have the surface H abundance of the ELM He WD.

For our initial binary systems, if we increase the donor star mass (M_{2,i}) or the orbital period (P_{\text{orb,i}}), we produce He WDs with too-large masses and too-small radii. If we decrease M_{2,i} or P_{\text{orb,i}}, the result is that the timescale of nuclear evolution is too long to reproduce PTF J0533+0209 or the system never detaches from RLO (producing a so-called converging system, see, e.g., Istrate et al. 2014), respectively. If we increase the initial CO WD mass (M_{CO,i}) of the accretor, the results are not very significantly different. To compensate for a different applied value of M_{CO,i}, we can simply adjust the accretion efficiency to reach the value of M_{CO} = 0.652 M_\odot for PTF J0533+0209.

4. Discussion

4.1. A Possible Way to Mitigate the Discrepancy

As we found in the previous section, our model cannot explain the high value of T_{\text{eff}} and the low H abundance of the ELM He WD in PTF J0533+0209. In the following, we will discuss the possible explanation for this discrepancy. However, we start by briefly summarizing the investigation by Burdge et al. (2019c).

The peculiar observed values of the abovementioned parameters were also discussed by Burdge et al. (2019c) in their attempt to model PFT J0533+0209. Whereas we have argued that a CE origin of PTF J0533+0209 seems unlikely based on simple energy considerations (Section 2), they disregarded this aspect and instead attempted to explain PTF J0533+0209 by modeling a post-CE system in composite steps: first, they evolved a single 1.20 M_\odot star up the red-giant branch until its core mass is 0.187 M_\odot. Then they stripped the outer layers until the star had a mass of 0.19 M_\odot and a total amount of hydrogen of 10^{-3} M_\odot. Subsequently, they placed this model star in a P_{\text{orb}} = 1 hr binary system with a 0.66 M_\odot companion and evolved it further using MESA. Despite introducing excessive rotational mixing, the surface abundance of hydrogen remained too high. They then computed a model with an extremely large ad hoc mixing diffusivity, which enforces hydrogen to mix deep enough to burn and thereby reduce the value of X_{\text{surf}}.

Burdge et al. (2019c) concluded that based on the findings of Fuller & Lai (2013), the tidal heating is unlikely to heat up the ELM He WD to the observed temperature at an orbital period of 20 minutes. Referring to the work of Burdge et al. (2019), they argued that the “tidal temperature” to which the ELM He WD can be heated is only \sim 7000 K, much below the observed temperature of T_{\text{eff}} \sim 20,000 K. Another solution discussed by Burdge et al. (2019c) is the possibility of H-shell flashes (e.g., Driebe et al. 1998; Althaus et al. 2001; Nelson et al. 2004; Althaus et al. 2013; Istrate et al. 2016). However, due to numerical reasons, they were not able to simulate flash-induced RLO of the residual H envelope. Such RLO could substantially reduce the amount of residual hydrogen (see below). Finally, Burdge et al. (2019c) attempted to model PTF J0533+0209 via the stable RLO channel as explored in this paper. However, due to the problems of reproducing T_{\text{eff}} and X_{\text{surf}}, they concluded that the CE channel is more likely.

Despite the initial issues with T_{\text{eff}} and X_{\text{surf}}, we are still optimistic with respect to the stable RLO channel. The residual H-envelope mass expected for ELM He WDs is typically between \sim 10^{-3} and 10^{-2} M_\odot depending on WD mass and evolutionary epoch.

Figure 3 displays the HR diagram of one of our grid model computations of the evolution of an ELM He WD model with a final WD mass of 0.156 M_\odot. It covers the evolution from detachment of the CV phase as a 0.163 M_\odot proto-ELM He WD (t = 3.2 Gyr and P_{\text{orb}} = 6.92 hr) until t = 13.7 Gyr (P_{\text{orb}} = 5.97 hr). The WD experiences several strong H-shell flashes during its evolution, leading to temporary episodes of additional RLO. As a consequence, the final WD mass is reduced to 0.156 M_\odot. From the plot it is seen that following each flash episode the surface H abundance is considerably reduced, although in all cases X_{\text{surf}} \lesssim 0.1. Thus, we cannot explain the observed value of X_{\text{surf}} \sim 4.9 \times 10^{-4}, which may indeed suggest that extra mixing is needed, as proposed by
Nevertheless, we speculate, that it is quite possible that PFT J0533+0209 is currently observed shortly after one of these short-lived H-shell flash episodes, thereby helping to explain the measured values of $T_{\text{eff}}$ and $L$. A caveat is that both $X_{\text{surf}}$ and $L$ ($T_{\text{eff}}$) decay back to preflash values on a timescale of order 10 Myr. Hence, the time window for observing the system in an immediate postflash evolutionary stage is rather limited.

4.2. Influence of the Accretion Disk

In our calculations, we did not consider the effect of an accretion disk during the AM CVn phase. In this phase, the binary separation is very small and the formation of an accretion disk is not obvious. In an early paper, Nelemans et al. (2001) studied this question with an analytical method. If the minimum distance at which the accretion flow passes the accretor is comparable to the radius of the accretor, it is likely that no accretion disk will be formed and the accretion flow will directly hit the surface of the accretor. Their Figure 1, suggests that no accretion disk is formed at the beginning of the AM CVn phase whereas an accretion disk will appear at later times, after the system passes through the orbital period minimum and once the donor star (He WD) mass is below $\sim 0.08 M_\odot$. Without an accretion disk, loss of angular momentum due to gravitational radiation becomes limited, and the system will remain in the AM CVn phase for a much longer time.
momentum from the orbit may destabilize the mass transfer unless the angular momentum lost to the accretor can be transferred back to the orbit (Marsh et al. 2004). Later, Kaplan et al. (2012) argued that for AM CVn systems with ELM He WD donors, the initial contraction of the ELM He WD due to mass loss allows for more stable mass transfer than that originally found for cold He WDs studied by Marsh et al. (2004). In our MESA models, we do take into account the entropy (finite temperature) of the ELM He WD, but the detailed treatment of direct impact versus disk formation is not included.

4.3. A LISA Verification Source

PTF J0533+0209 is expected to be detected by LISA (i.e., a so-called LISA verification source Kupfer et al. 2018). However, during the subsequent GW damping of the system, the more extended ELM He WD component will fill its Roche lobe and initiate RLO toward the CO WD companion. During this epoch, the system will be observable as an AM CVn system to be damped by tidal interactions (van Roestel et al. 2021). Discussions and detailed tracks of their evolution through the AM CVn phase are given in, e.g., Tauris (2018).

Figure 4 shows the evolution of characteristic strain as a function of GW frequency for the binary system presented in Figure 1 for an assumed distance of $d = 1.5$ kpc. The red, gray, and magenta solid lines represent the sensitive curves for TianQin with an observational time $T = 5$ yr, Taiji with an observational time $T = 4$ yr, and LISA with an observational time $T = 5$ yr, respectively. The color on the evolutionary track represents the chirp mass of the binary system (see color bar; the gray and dark blue colors represent chirp masses of $M_\text{chirp} > 0.30 M_\odot$ and $M_\text{chirp} < 0.05 M_\odot$, respectively). The black dot represents the end of the first mass-transfer phase and the star represents the onset of the second mass-transfer phase. The cross represents the current location of PTF J0533+0209.

with an assumed observational time $T = 4$ yr. We note that a system like PTF J0533+0209 is detected both during the detached phase and the subsequent AM CVn phase. From this plot, we find a possible signal-to-noise ratio ($S/N$) up to 10–100 for simulated systems similar to, but further evolved than, PTF J0533+0209. The chirp mass of the detectable sources is between 0.10 and 0.27 $M_\odot$ (steadily decreasing with evolutionary age; see right-hand-side color bar).
5. Conclusions

In this paper, we have discussed possible formation scenarios of the DWD binary system PTF J0533+0209. We found that explaining its origin via the CE channel (as suggested by Burdge et al. 2019c) is difficult since the low mass of the ELM He WD implies CE ejection at an early evolutionary stage (Hertzsprung gap) when the binding energy of the envelope of the progenitor star is still larger (by a factor of ~2) than the amount of orbital energy released during spiral-in.

We have instead presented a formation scenario based on stable RLO in a CV system. Using the MESA code in combination with a magnetic-braking prescription from Van et al. (2019), we have modeled the system from the ZAMS and found a solution that matches the two measured WD component masses and the present orbital period of the system. However, our model fails to reproduce the observed $T_{\text{eff}}$ and, in particular, the surface H abundance of the ELM He WD. We have discussed possible reasons for this and point to a recent H-shell flash (combined with efficient mixing) as the most plausible explanation.

The present GW signal of this system will be detected by low-frequency GW observatories such as LISA, TianQin, and Taiji. We calculated the signal of its detection and continued low-frequency GW observatories such as LISA, TianQin, and Taiji. We calculated the signal of its detection and continued

We are strongly inclined to believe that PTF J0533+0209 formed via stable RLO. However, a complete and self-consistent model that explains all observables of the binary is still missing. We consider this system a very interesting test case for future investigations of close binary star evolution and input physics and encourage the community to join our efforts.

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Software: MESA (Paxton et al. 2011, 2013, 2015, 2018, 2019).

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