A Minimum Capacity Optimization Scheme for Airport Terminals During Peak Periods

GUANGMIN XIE1, ZHONGYUAN JIANG2, MINGXING HE1, XIAOLIANG CHEN1, AND JIAN MAO3

1School of Computer and Software Engineering, Xihua University, Chengdu 610039, China
2College of Big Data Statistics, Guizhou University of Finance and Economics, Guiyang 550025, China
3Department of Research and Development, Chengdu Civil Aviation Information Technology Company Limited, Chengdu 611435, China

Corresponding author: Zhongyuan Jiang (jiangzyj@163.com)

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ABSTRACT Air transportation as a public transport method is preferentially considered by passengers. However, this method will not only bring congestion problems but also affect the passenger experiences in airport terminals during peak periods. Considering the fact that the capacities of airport terminals are proportional to their building costs, this paper proposes a scheme to calculate the minimum capacity for each airport terminal. The main purpose is to avoid the saturated congestion problems and reduce the waste of transition bus resources. For each airport terminal, its capacity is closely related with its inputs and outputs. The proposed scheme proves a relational expression for each airport terminal according to its inputs and outputs. Moreover, this work formally models the limited flow structures of flight process guidance systems by using finite capacity Petri nets and then verifies the correctness of the proposed relational expression. Finally, an example is presented to illustrate the proposed scheme.

INDEX TERMS Petri net, airport terminal, minimum capacity, saturated congestion.

I. INTRODUCTION

With the rapid development of social economy globalization, the demands of air transportation have increased sharply. Unfortunately, they may lead to a series of problems, such as the congestion at airport terminals during peak periods, the flight delays, and the serious declines in service quality. A core reason of these problems is that the current capacities of airport terminals are unreasonable for satisfying the demands of air transportation. These problems will greatly affect the developments of air transportation. In order to make sure the normal operations of air transportation and relieve passenger congestion in airport terminals, two urgent problems, i.e., how to improve the service quality and how to properly optimize the capacities of airport terminals, should be resolved more rationally.

As a long-term strategy, an excessively ideal approach is to build high capacity airports to resolve these urgent problems. However, it not only requires a lot of construction costs but also cannot be implemented in short periods. This approach has no advantage in alleviating the pressure of air transportation demands. Another practical approach is to optimize and expand the existing resources, such as airport terminals, runways, aprons and so on. The main purpose is to appropriately improve the capacities of these existing resources to improve the utilization rate and passenger satisfaction degree [1]. The prominent advantages of this approach are simple construction, rapid implementation, and relatively low cost. In recent years, various airport capacity optimization schemes are proposed to resolve these urgent problems, which can be roughly
summarized into two categories, i.e., queuing theory [2]–[4] and genetic algorithms [5]–[7].

Queuing theory as an excellent mathematical tool that can be used to simulate and analyze airport behavior [8], [9]. The authors of [10] observe the arrival time, queuing time, and queuing behavior of passengers in the terminals of Kerala airport. They establish an M/M/C queuing system to reduce passenger dissatisfaction and increase terminal capacities. The authors of [11] present a queuing model to simulate the departure processes of an airport. The model simulates airport departure processes with a current gate assignment and a robust gate assignment to assess the impact of gate assignments on departure metering. As a result, it improves the operations of crowded airports with limited gate resources and increases airport capacities. In [12]–[15], they focus on the resource allocation of airport security scanning channels. Some models are built based on the queuing theory to describe passenger flows in airport security processes. Finally, the optimization of airport security processes is solved and the utilization of airport resources is improved.

In general, the queue theory can be used to establish the corresponding mathematical models and optimize the existing resources. It is easy to build a real model to improve the capacities of these existing resources based on queue theory. However, such a model requires a large amount of real-time aviation data that is affected by numerous random factors.

Genetic algorithms are a class of search algorithms based on evolutionary theory [16], which greatly improve the adaptability of each individual and finally converge to the optimal solutions [17]. They are often used for scheduling airport resources to achieve the optimal utilization of airport resources. In [18], a genetic algorithm is provided to achieve flight scheduling optimization. This scheme uses genetic algorithms to solve a two-runway flight scheduling problem, which can not only efficiently determine the runways of aircraft and landing sequences, but also achieve a runway capacity optimization scheme. The authors of [19] present a model based on genetic algorithms. This model can reasonably solve the problem of airport capacity management. A simulation study shows that this scheme can improve runway capacities. In [20], an improved genetic algorithm is proposed to avoid the premature of genetic algorithms by considering structural characteristics. The experimental results show that the gate capacities are optimized by using genetic algorithms. The authors of [21] establish a multi-flight multi-service parallel timing optimization model based on genetic algorithms. The model can ensure the normal operations of flights, realize the coordinated task scheduling scheme among various businesses, and optimize the capacities of airport terminals.

We know that genetic algorithms are based on probability rules, which make the search algorithms more flexible and their parameters have as little impact as possible. Meanwhile, genetic algorithms can be well combined with other related algorithms. They can be used to optimize the capacities of the existing airport resources. However, it may involve a lot of complex calculations by using genetic algorithms to optimize complex systems. Unfortunately, airport systems coincidentally are complex systems and it is difficult to achieve a global optimal solution for a complex airport layout with genetic algorithms. Furthermore, the speeds of many search algorithms based on genetic algorithms are relatively slow if the feedback information of the networks cannot be used in time. Moreover, the efficiency of genetic algorithms depends on the selections of initial populations.

Petri nets [22] as a graphical tool are widely used to describe and analyze discrete event systems [23]. They have established applications to discrete event systems from modeling [24], optimization control [25], and deadlock analysis [26]. It is possible to create mathematical models, state equations, and algebraic equations to analyze and verify the behavior of discrete event systems by using Petri nets [27]. With the help of formal modeling and analysis methods of Petri nets, researchers can analyze the static models and the operational dynamic behavior of discrete event systems. The obtained results are more similar to the exact results for the real systems. For example, a Petri net model of an automated manufacturing system is established in [28]. In this model, genetic algorithms are used to solve multi-objective scheduling problems of automated manufacturing systems with limited resource capacities and high deadlocks. The studies in [29] use Petri nets to establish a model and propose a method to avoid deadlock in flexible manufacturing systems. In [30], the fault diagnosis of power systems is realized by building a model with Petri nets. The authors of [31] propose a scheme to ease the traffic congestion problems based on Petri nets in traffic management systems. The authors of [32] present two heuristic algorithms to solve a marking optimization problem and a cycle time optimization problem based on a subclass of timed Petri nets called deterministic timed weighted marked graphs. The simulation studies show that their presented algorithms are significantly more efficient. Moreover, they also establish a formal model for resource allocation systems by deterministic timed weighted marked graphs in [33]. The performance optimization problems are considered with the aim of maximizing their throughput for resource allocation systems under a given budget for acquiring resources.

Considering the above characteristics of Petri nets, many researchers also use Petri nets to optimize the capacities for airport resources. It can not only avoid the inputs of a large amount of real-time aviation data, but also establish formal models with intuitive graphics to represent complex airport systems. The authors in [34] use Petri nets to establish a situational awareness method. This method introduces the queuing theory and perceptual parameters into the existing Petri nets. Moreover, the perceptual Petri net model of a general service system is constructed, which can quickly establish models by different scene service systems. In [35], the authors study the model and simulation of baggage handling systems based on colored Petri nets. The simulation results show that the model can not only realize fast baggage sorting, but also improve baggage efficiency and airport capacities. The authors in [36]
propose a new Petri net model to dynamically analyze aircraft movements on the runways with given input and predetermined exit parameters. Their main purpose is to improve the runway capacities. In [37], a stochastic Petri net model is used to optimize the airport security processes and finally the security processes of terminals are optimized. In [38], a dynamic modeling method is proposed for airport aprons based on Petri nets, which realizes the optimization of apron capacities.

Airport terminals as a critical part of airports, their role is to provide waiting services for passengers. The capacity of an airport terminal largely determines the congestion levels during peak periods. For example, Beijing Capital International Airport as one of the three complex airport hubs in China, it has three main airport terminals, i.e., T₁, T₂ and T₃ [39], where terminal T₃ is the largest single terminal in Asia and it is composed of three terminals, i.e., T₃-C, T₃-D, and T₃-E [40]. Terminal T₃-C is a main building of T₃. Its role is to perform ticket purchasing, security checking, and waiting for boarding. Passengers should take transition buses from T₃-C to T₃-D or T₃-E, where they are waiting for boarding if T₃-C is extremely congestion during peak periods. There are many runways around terminals T₃-D and T₃-E, which can improve airplane utilization and reduce passenger congestion during peak periods. Similarly, passengers should take transition buses from T₃-D or T₃-E to T₃-C to leave the airports. Therefore, the capacity of T₃-D will affect the congestion levels in Beijing Capital International Airport. A whole process is shown in Fig. 1.

![FIGURE 1. The passenger flows of terminal T₃ in Beijing Capital International Airport during peak periods.](image)

In Beijing Capital International Airport, passengers should go through a series of complex passenger handing processes from entering the airport to returning and leaving the airport [41]. It is difficult to completely analyze the whole processes. In order to simplify these processes, this paper divides the passenger handling processes into six core steps, as shown in Fig. 2. The six core steps form a flight process guidance system that is a typical discrete event system. Therefore, a flight process guidance system can be modeled and analyzed by using Petri nets.

In a flight process guidance system, it usually suffers from the saturated congestion problems but also exists the waste of transition bus resources if many passengers are waiting in terminals and the capacities of the terminals are unsuitable, which are illustrated by the following examples.

To facilitate the dispatch and control of flights and transition buses and enhance the statistical and analysis of passengers, all passengers who are entering and exiting a terminal will be divided into different groups. Let \( x = 200 \) and \( y = 150 \) be the input and output of T₃-D, respectively, \( C = 250 \) be the capacity of T₃-D, and there be 250 passengers who are waiting to leave T₃-D, where \( x \) and \( y \) represent the numbers of passengers who are entering and exiting T₃-D, respectively. Specially, \( x \) is closely controlled by the arrived flights and \( y \) is closely related with the number of transition bus seats at T₃-D. If the flight process guidance system starts to run during the peak period, it may suffer a failure state after a few system running periods. At the failure state, there are 100 passengers who still are waiting to leave terminal T₃-D by transition buses. Certainly, they can leave T₃-D by a transition bus at this state. However, this transition bus cannot work with a full load and 50 seats of this transition bus are wasted since \( y = 150 > 100 \). Moreover, a more noteworthy problem is that the other passengers cannot enter T₃-D at this failure state since \( x + 100 = 300 > C = 250 \). Therefore, the flight process guidance system suffers a saturated congestion problem in T₃-D at this failure state. To avoid this problem, three ways can be considered as follows.

1) Decreasing the value of \( x \), such as \( x = 150 \). This means that the number of passengers that are entering T₃-D will be reduced to 150 and the remaining 50 passengers cannot enter T₃-D timely. It will further aggravate the congestion in the entrance of T₃-D.
2) Decreasing the value of \( y \), such as \( y = 100 \). This means that the number of passengers that are exiting T₃-D will be reduced to 100. In other words, the number of passengers that will leave T₃-D by transition buses will be reduced, where the seats of transition buses are fixed. Therefore, the transition bus resources will be wasted unless more passengers are exiting T₃-D rapidly.
3) Expanding the value of \( C \), such as \( C = 400 \). The saturated congestion problems can be avoided if the flights and transition bus resources are difficult to be controlled during the peak periods.

The first way tries to decrease the value of \( x \), but the value of \( x \) is determined by the number of arrived flights during the peak periods. It is obviously inadvisable to restrict the arrived flights. Conversely, it will further aggravate the congestion in the entrance of T₃-D. The second way tries to decrease the value of \( y \), but the value of \( y \) is determined by the number of
transition bus seats that are usually constant. Comparing with the first two ways, we know that the third way is a reasonable method to solve the saturated congestion problems. But it also exists the waste of transition bus resources for different capacities. For example, assume that the capacity of $T_3$-$D$ is expanded to $C = 400$ and there are 400 passengers who are waiting to leave $T_3$-$D$. If the flight process guidance system starts to run during the peak periods, it may also suffer another failure state after a few system running periods. At the failure state, 100 passengers also are waiting to leave terminal $T_3$-$D$ by transition buses. Similarly, the transition bus cannot work with a full load and 50 seats of the transition bus will be wasted if the 100 passengers leave $T_3$-$D$ by a transition bus. There exists the waste of transition bus resources in the flight process guidance system. Fortunately, the other passengers can enter $T_3$-$D$ at this failure state. If we assume that the capacity of $T_3$-$D$ is expanded to $C = 450$ and 450 passengers are waiting to leave $T_3$-$D$ at the initial state, the above two problems can be avoided. Therefore, a suitable capacity for $T_3$-$D$ is very important to avoid the saturated congestion problem and reduce the waste of transition bus resources. The existing studies ignore these problems.

In this paper, a scheme that is used to calculate the minimum capacity for each terminal is proposed to solve the above two problems. For each terminal, its capacity is closely related to its inputs and outputs. Therefore, a Petri net model is created for the terminal. Moreover, a relational expression among the capacity, inputs, and outputs of the terminal is derived based on the mathematical methods of Petri nets to relieve the congestion in the entrances of the terminal and reduce the waste of transition bus resources in its exits. The main contributions of this paper are concluded as follows.

1) The formal model of each terminal is proposed based on finite capacity Petri nets. The main purpose is to facilitate the analysis and verification for the behavior of terminals in a flight process guidance system by using the mathematical methods of Petri nets.

2) A relational expression is derived to calculate the minimum capacity for each terminal by analyzing the properties of the proposed Petri net model. It not only can avoid the saturated congestion problems in the entrances of terminals but also can reduce the waste of transition bus resources during the peak periods. Moreover, it also can reduce the building costs by minimizing the capacities of the terminals.

3) The proposed relational expression can also be expanded to control the dispatch of passengers among multiple infrastructures in aviation systems if the capacities of these infrastructures are difficult to be extended. The main purpose is also to avoid the congestion and reduce the waste of transition bus resources by controlling the inputs and outputs of the infrastructures.

This paper is organized as follows. Section II introduces the basics of Petri nets. Section III introduces a saturated congestion problem in a terminal. Section IV presents a capacity optimization scheme to solve the saturated congestion problem for each terminal. Moreover, a simulated study is given in Section V and Section VI concludes this paper.

II. BASICS OF PETRI NETS

This paper only provides some fundamental concepts of finite capacity Petri nets. More details about Petri nets can be found in [42]–[44]. A finite capacity Petri net [22] is a five-tuple $N = (P, T, F, W, C)$, where $P$ and $T$ are finite, $P \neq \emptyset$ and $T \neq \emptyset$. $P$ is a set of places and $T$ is a set of transitions. $F \subseteq (P \times T) \cup (T \times P)$ is called a flow relationship from places to transitions or transitions to places. $W : F \rightarrow N$ is a weight function that gives weights to arcs. $C : P \rightarrow N$ is a capacity function that gives capacities to places. Besides, a finite capacity Petri net can also be expressed as an input matrix $N^+(p, t) = W(t, p)$ and an output matrix $N^-(p, t) = W(p, t)$, where $p \in P$ and $t \in T$.

Let $M : P \rightarrow N$ be a marking of $N$. $M(p)$ represents the number of tokens in place $p$ at marking $M$. Place $p$ is marked if $M(p) > 0$. $(N, M_0)$ is called a network system, where $M_0$ is an initial marking of $N$.

Let $x \in P \cup U$ be a node of $N$. The preset of $x$ is defined as $*x = \{y \in P \cup U \mid (y, x) \in E\}$. The post set of $x$ is defined as $x^* = \{y \in P \cup U \mid (x, y) \in E\}$. For a set of nodes $X \subseteq P \cup U$, the preset of $*X = \cup_{x \in X} x^*$ and the post set of $X^* = \cup_{x \in X} x^*$. The number of elements in the set $X$ is defined as $|X|$.

In a finite capacity Petri net, $t \in T$ is enabled at marking $M$ if $\forall p \in *t \ s.t. M(p) \geq W(t, p)$ and $\forall p' \in t^* \ s.t. M(p') \leq C(p') - W(t, p')$, which is denoted as $M[t]$. If a fire is at $M$, a new marking $M'$ is obtained such that $\forall p'' \in P,$

$$M'(p'') = M(p'') - W(p'', t) + W(t, p''),$$

which is denoted as $M[t]M'$. Marking $M''$ is called a reachable marking if there exists a transition sequence $\sigma = t_1t_2 \ldots t_n$ such that $M[t_1]M[t_2]M[t_3] \ldots M[t_{n-1}]M[t_n]$ is denoted as $M[\sigma]M''$. Marking $M''$ can be calculated by

$$M'' = M + N^+\sigma - N^-\sigma,$$

where $\sigma : T \rightarrow N$ is the Parikh vector of $\sigma$. The $\sigma(t)$ represents the sum of all occurrences of $t$ in $\sigma$. The set of reachable markings from $M$ in $N$ is denoted as $R(N, M)$.

To clearly indicate the location of the components in a marking or a Parikh vector, the marking or the Parikh vector can be described using a special multiset. Let $N = (P, T, F, W, C)$ be a finite capacity Petri net with $P = \{p_1, p_2, p_3, \ldots, p_m\}$ and $T = \{t_1, t_2, t_3, \ldots, t_n\}$. We use

$$(M(p_1)p_2, M(p_2)p_3, \ldots, M(p_m)p_m)$$

to denote marking $M$. Similarly, we use

$$(\sigma(t_1)t_1, \sigma(t_2)t_2, \sigma(t_3)t_3, \ldots, \sigma(t_n)t_n)$$

to denote Parikh vector $\sigma$.

Let $p \in P$ be a place in $N$. All transitions in $*p \cup p^*$ are enabled at marking $M$ if

$$1) \forall t_i \in *p : \forall p' \in *t_i, M(p') \geq W(p', t_i);$$
2) \( M(p) \geq \sum_{t \in p} W(p, t) \);  

3) \( C(p) - M(p) \geq \sum_{t \in p'} W(t, p) \);  

4) \( \forall t \in p^* : \forall p'' \in t^* \), \( C(p'' - M(p'') \geq W(t, p'').) \)

According to (1) and (2), if all transitions in \( p \cup p^* \) are enabled, it can obtain

\[
C(p) \geq \sum_{t \in p^*} W(t, p) + \sum_{t \in p} W(p, t).
\]

Let \( t_1 \) and \( t_2 \) be two transitions, \( M \) be a marking, and \( \sigma \) be a transition sequence. If \( t_1 \) and \( t_2 \) fire at \( M \), it is denoted as

\[
\sigma = \begin{cases} 
  t_1 \ t_2 & \text{if } t_1 \text{ fire before } t_2, \\
  t_2 \ t_1 & \text{if } t_2 \text{ fire before } t_1, \\
  \{t_1 \ t_2\} & \text{if } t_1 \text{ and } t_2 \text{ fire at the same time.}
\end{cases}
\]

To facilitate the description of synchronous discrete events in this paper, we have a following assumption.

**Assumption 1:** Let \( N = (P, T, F, W, C) \) be a finite capacity Petri net, \( M \) be a marking, and \( t_1 t_2 \ldots t_n \) be \( n \) transitions. The \( n \) transitions \( t_1 t_2 \ldots t_n \) should fire simultaneously at marking \( M \) if they can fire at marking \( M \), denoted as \( M(\sigma) \), where \( \sigma = \{t_1 t_2 \ldots t_n\} \).

**FIGURE 3.** An example of finite capacity Petri nets.

Fig. 3 shows a finite capacity Petri net, where \( \bullet t_1 = \{p_1\}, \bullet t_2 = \{p_2\}, \bullet t_3 = \{p_3\}, C(p_1) = 11, C(p_2) = 12, C(p_3) = 19, W(p_1, t_1) = W(t_1, p_2) = 7, \) and \( W(p_2, t_2) = W(t_2, p_3) = 5 \). The input matrix \( N^+ \) and output matrix \( N^- \) are expressed as follows.

\[
N^+ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad \text{and} \quad N^- = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}.
\]

The initial marking is \( M_0 = (9, 5, 7)^T \).\( ^T \) Therefore, transitions \( t_1 \) and \( t_2 \) can both fire simultaneously since \( M_0(p_1) \geq W(p_1, t_1) \) and \( M_0(p_2) \geq W(p_2, t_2) \).\( ^T \) \( C(p_1) - M_0(p_2) \geq W(t_1, p_2) \), and \( C(p_3) - M_0(p_3) \geq W(t_2, p_3) \), denoted as \( M_0(\sigma_1) \), where \( \sigma_1 = \{t_1 t_2\} \).

If transitions \( t_1 \) and \( t_2 \) simultaneously fire at marking \( M_0 \), a new marking \( M_1 \) can be obtained by

\[
M_1 = (M_0 + \sigma_1 - \sigma_2) = (2, 7, 12)^T,
\]

where \( \sigma_1 = (1, 1)^T \). In the same way, only the transition \( t_2 \) can fire at marking \( M_1 \). Let \( \sigma_2 = t_2 \). We have \( \sigma_2 = (0, 1)^T \). A new marking \( M_2 \) can be obtained by

\[
M_2 = (M_1 + \sigma_2 - \sigma_1) = (2, 2, 17)^T
\]

if transition \( t_2 \) fires at marking \( M_1 \). Finally, neither \( t_1 \) nor \( t_2 \) can fire at marking \( M_2 \). The whole process can be denoted as \( M_0(\sigma) M_2 \), where \( \sigma = \sigma_1 + \sigma_2 = (1, 2)^T \).

**III. THE SATURATED CONGESTION PROBLEMS**

In a flight process guidance system, as shown in Fig. 2, its all processes can be classified into two major steps as follows.

Step 1: During the passenger boarding process, passengers first need to purchase tickets in terminal \( T_3-C \). They should take the transition buses to terminal \( T_3-D \) to wait for the boarding announcements. Next, they begin to check-in and wait in line for boarding after they hear the boarding announcements.

Step 2: During the passenger landing process, passengers begin to disembark from the airplane and enter the terminal \( T_3-D \) in an orderly manner to take transition buses to the terminal \( T_3-C \) after the airplane arrived at the destination. Finally, they leave terminal \( T_3-D \) by transition buses in an orderly manner. To facilitate a formal description, we assume that passengers who enter terminal \( T_3-D \) are formally divided into several groups with equal numbers. Each group should queue up to enter the terminal \( T_3-D \) to avoid unordered congestion.

**FIGURE 4.** The Petri net model of a flight process guidance system.

Fig 4 shows the Petri net model for the flight process guidance system that is shown in Fig. 2. It contains 12 places and 16 transitions. Each place has a capacity that represents the capacity of the terminal \( T_3-D \), terminal \( T_3-C \), and airplanes. Table 1 shows their descriptions.

In order to make the Petri net model of a flight process guidance system more clearer in Fig. 4, we simplify the Petri net model, as shown in Fig. 5, where path \( p_{3c} \rightarrow t_{cd} \rightarrow p_{3d} \rightarrow t_{cdi} \rightarrow p_{ai} \) is corresponding to step 1 of Fig. 2, denoted as \( p_{3c} p_{d} p_{d} p_{ai} \). The path \( p_{ai} \rightarrow t_{cd} \rightarrow p_{3d} \rightarrow t_{cdi} \rightarrow p_{3c} \) is similar with step 2, denoted as \( p_{ai} p_{id} p_{3c} \). In path \( p_{ai} p_{id} p_{3c} \), the capacity of \( p_{3d} \) is limited because its building costs are proportional to its capacity. Therefore, a limited flow structure is defined based on finite capacity Petri nets.

**Definition 1:** A limited flow structure is defined as a finite capacity Petri net \( N_e = (P, T, F, W, C) \), where
1) \( P = \{p_1, p_2, p_0\} \), where \( p_1 \), \( p_2 \) and \( p_0 \) are input capacity place, limited capacity place, and output capacity place, respectively, such that \( p_1^\bullet = \{p_1\} \) and \( p_2^\bullet = \{p_0\} \).
2) \( T = \{t_1, t_o\} \), where \( t_1 \) and \( t_o \) are the processes that passengers are entering and leaving \( p_1 \), respectively.
3) \( F = \{(p_1 \times t_1) \cup (t_1 \times p_1) \cup (p_1 \times t_o) \cup (t_o \times p_0)\} \) is the set of passenger flow direction arcs.
4) \( p_1^* = \{t_1\}, t_1^* = \{p_1\}, t_o^* = \{p_1\}, t_o^* = \{t_o\}, p_2^* = \{p_0\}, p_0^* = \{t_o\} \).
5) \( W : F \rightarrow N \) is a mapping that assigns a number of passengers to a passenger flow arc.
6) \( C : F \rightarrow N \) is a mapping that assigns a capacity to a limited capacity place \( p_1 \).

In a limited flow structure \( N_s = (P, T, F, W, C) \) with \( P = \{p_1, p_2, p_0\} \) and \( T = \{t_1, t_o\} \) \( p_1 \) has an input flow that represents the number of passengers who are entering \( p_1 \), denoted as \( p_1^I \), and an output flow that represents the number of passengers who are leaving \( p_1 \) by transition buses, denoted as \( p_1^O \), such that
\[
p_1^I = W(t_1, p_1) \quad \text{and} \quad p_1^O = W(p_1, t_o).
\]

Generally, \( p_1^I < p_1^O \), it is a normal state. However, we have \( p_1^I < p_1^O \) during the peak periods.

In a limited flow structure, it may not only suffer from the saturated congestion problems but also exist the waste of transition bus resources if many passengers are waiting in terminals and the capacities of the terminals are unsuitable. For example, Fig. 6(a) shows a limited flow structure \( N_s = \{(p_3, p_4, p_5), (t_3, t_4), F, W, C\} \), which is the part of the flight process guidance system in Fig. 5. Let \( p_4^I = 640 \), \( p_4^O = 600 \), and \( M_0(p_4) = C(p_4) = 1000 \) during the peak periods. It will suffer from the saturated congestion problems. The scenario begins as follows.

At marking \( M_0 \), transition \( t_3 \) cannot fire and only \( t_4 \) can fire since \( C(p_4) - M_0(p_4) = 0 < p_4^I \) and \( M_0(p_4) \geq p_4^O \). If transition \( t_4 \) fires, a new marking \( M_1 \) is obtained by
\[
M_1(p_4) = M_0(p_4) - p_4^O = 1000 - 600 = 400.
\]

At marking \( M_1 \), neither \( t_3 \) nor \( t_4 \) can fire since \( C(p_4) - M_1(p_4) = 600 < p_4^I \) and \( M_1(p_4) < p_4^O \), as shown in Fig. 6(b). This means that there exists a saturated congestion problem in the limited flow structure \( N_s \).

If the capacity of \( p_4 \) is expanded to \( C(p_4) = 1500 \), as shown in Fig. 7(a), where \( M_0(p_4) = C(p_4) \), the similar scenario begins as follows.

At marking \( M_0 \), transition \( t_3 \) cannot fire and only \( t_4 \) can fire since \( C(p_4) - M_0(p_4) = 0 < p_4^I \) and \( M_0(p_4) \geq p_4^O \). If transition \( t_4 \) fires, a new marking \( M_1 \) is obtained by
\[
M_1(p_4) = M_0(p_4) - p_4^O = 1500 - 600 = 900.
\]

At marking \( M_1 \), transition \( t_3 \) cannot fire and only \( t_4 \) can fire since \( C(p_4) - M_1(p_4) = 600 < p_4^I \) and \( M_1(p_4) \geq p_4^O \). If transition \( t_4 \) fires, a new marking \( M_2 \) is obtained by
\[
2p_4 = M_1(p_4) - p_4^O = 900 - 600 = 300.
\]

At marking \( M_2 \), transition \( t_4 \) cannot fire and only \( t_3 \) can fire since \( C(p_4) - M_2(p_4) = 1200 \geq p_4^I \) and \( M_2(p_4) < p_4^O \), as shown in Fig. 7(b). This means that there exists the waste of transition bus resources in the limited flow structure \( N_s \).

For a limited flow structure \( N_s = (P, T, F, W, C) \) with the initial marking \( M_0(p_1) = C(p_1) \). We can conclude that \( N_s \)
may exist a marking \( M \in R(N_s, M_0) \) such that \( p^I_1 + M(p_1) > C(p_1) \) and \( M(p_1) < p^O_1 \). It may suffer a saturated congestion problem, as shown in Fig. 6(b). In addition, \( N_t \) may also exist another marking \( M' \in R(N_t, M_0) \) such that \( M'(p_1) < p^I_1 \). It may exist the waste of transition bus resources, as shown in Fig. 7(b). This means that the inconsistent among \( p^I_1, p^O_1 \) and \( C(p_1) \) will cause the above two urgent problems during peak periods.

IV. CAPACITY OPTIMIZATION FOR TERMINALS

In order to resolve the two urgent problems during peak periods, it is necessary to calculate appropriate capacities for terminals in limited flow structures. Therefore, a calculation scheme is proposed to calculate the minimum capacities for terminals by also considering their building costs. The proposed scheme not only can avoid the saturated congestion problems but also reduce the waste of transition bus resources during peak periods. The minimized capacities can also economize the costs for building or expanding the terminals.

**Definition 2:** Let \( N_s = (P, T, F, W, C) \) be a limited flow structure and \( M_0 \) be an initial marking of \( N_s \), \( \forall M \in R(N_s, M_0) \), marking \( M \) is called an error marking if \( M(p_1) < p^O_1 \) or marking \( M \) is called a legal marking if \( M(p_1) \geq p^O_1 \).

Let \( N_t = (P, T, F, W, C) \) be a limited flow structure with \( P = \{p_i, p_1, p_o\} \) and \( T = \{t_i, t_o\} \). We have
1) \( \forall M \in R(N_s, M_0) \), both \( t_i \) and \( t_o \) can fire at \( M \) if
   \[
   M(p_1) \in [p^I_1, C(p_1) - p^I_1].
   \]
   \[\exists M_1 \in R(N_s, M) \text{ such that } M([t_i, t_o])M_1, \text{ it can obtain } M_1(p_1) = M(p_1) + p^I_1 - p^O_1 \geq M(p_1) \]
   since \( p^I_1 \geq p^O_1 > 0 \). It means that the number of passengers in \( p_1 \) is increased gradually during the peak periods.
2) \( \forall M \in R(N_s, M_0) \), transition \( t_i \) cannot fire and only transition \( t_o \) can fire at \( M \) if
   \[
   M(p_1) \in [C(p_1) - p^I_1 + 1, C(p_1)].
   \]
   \[\exists M_1 \in R(N_s, M) \text{ such that } M([t_o])M_1, \text{ it can obtain } M_1(p_1) = M(p_1) - p^O_1 < M(p_1) \]
   since \( p^O_1 > 0 \). It means that the number of passengers in \( p_1 \) is reduced gradually.

**Definition 3:** Let \( N_s = (P, T, F, W, C) \) be a limited flow structure and \( p_1 \in P \) be a terminal. Interval \( [p^O_1, C(p_1) - p^I_1] \) represents the passenger increasing region of \( p_1 \), denote as \( p^I_1 \). Interval \( [C(p_1) - p^I_1 + 1, C(p_1)] \) represents the passenger decreasing region of \( p_1 \), denote as \( p^O_1 \).

Fig. 8 shows the partition of error marking region, legal marking region, passenger increasing region, and passenger decreasing region for \( p_1 \).

**Property 1:** Let \( N_s = (P, T, F, W, C) \) be a limited flow structure, \( p_1 \in P \) be a terminal, \( M_0 \) be an initial marking such that \( M_0(p_1) = C(p_1), M \in R(N_s, M_0) \) be a reachable marking, \( p^I_1 \) and \( p^O_1 \) be two prime numbers. Then,
   \[
   M(p_1) = C(p_1) - p^I_1 - p^O_1 + 1
   \]
   is the minimum number of passengers in \( p_1 \).

**Proof:** Let \( M_0(p_1) \in p^I_1 \) since \( M_0(p_1) = C(p_1) \). This means that the passenger numbers in \( p_1 \) will be reduced gradually. Let \( M_1 \in R(N_s, M_0) \) be a reachable marking such that \( M_1(p_1) = C(p_1) - p^I_1 + 1 \) is the minimum number in the passenger decreasing region \( p^O_1 \). At marking \( M_1 \), a new marking \( M \) can be obtained after transition \( t \in p^I_1 \) fires by
   \[
   M(p_1) = M_1(p_1) - p^I_1 = C(p_1) - p^I_1 - p^O_1 + 1.
   \]
   At marking \( M \), we have \( M(p_1) \in p^O_1 \). It means that the number of passengers in \( p_1 \) will be increased gradually. Therefore,
   \[
   M(p_1) = C(p_1) - p^I_1 - p^O_1 + 1
   \]
   is the minimum number in \( p_1 \). For example, we scale down the data to quickly visualize the results, as shown in Fig. 9. Assume that \( M_0(p_4) = C(p_4) = 10, p^I_4 = 5, \) and \( p^O_4 = 3 \) in a limited flow structure \( N_s \). At marking \( M_0 \), only transition \( t_4 \) can fire. If transition \( t_4 \) fires, a new marking \( M_1 \) is obtained by
   \[
   M_1(p_4) = M_0(p_4) - p^O_4 = 7.
   \]
   At marking \( M_1 \), only transition \( t_4 \) can fire. If transition \( t_4 \) fires, a new marking \( M_2 \) is obtained by
   \[
   M_2(p_4) = M_1(p_4) - p^O_4 = 4.
   \]
   At marking \( M_2 \), transitions \( t_3 \) and \( t_4 \) can fire simultaneously. If transitions \( t_3 \) and \( t_4 \) fire, a new marking \( M_3 \) is obtained by
   \[
   M_3(p_4) = M_2(p_4) - p^O_4 = 6.
   \]
   At marking \( M_3 \), only transition \( t_4 \) can fire. If transition \( t_4 \) fires, a new marking \( M_4 \) is obtained by
   \[
   M_4(p_4) = M_3(p_4) - p^O_4 = 3.
   \]
At marking $M_4$, transitions $t_3$ and $t_4$ can fire simultaneously. If transitions $t_3$ and $t_4$ fire, a new marking $M_5$ is obtained by

$$M_5(p_4) = M_4(p_4) + p_4^1 - p_4^0 = 5.$$ 

At marking $M_5$, transitions $t_3$ and $t_4$ can fire simultaneously. If transitions $t_3$ and $t_4$ fire, a new marking $M_6$ is obtained by

$$M_6(p_4) = M_5(p_4) + p_4^1 - p_4^0 = M_1(p_4) = 7.$$ 

It forms a loop from marking $M_1$ to marking $M_6$ for $p_4$ since $M_6(p_4) = M_1(p_4)$. Therefore, it can obtain that

$$M(p_4) = C(p_4) - p_4^1 - p_4^0 + 1 = 3$$

is the minimum number according to Property 1.

According to Property 1, we can obtain that $C(p_1) - p_1^1 - p_1^0 + 1$ is the minimum number in $p_1$ if $M_0(p_1) = C(p_1)$. Therefore, we still need to prove that $\exists M \in R(N_s, M_0)$ such that

$$M(p_1) = C(p_1) - p_1^1 - p_1^0 + 1.$$ 

**Lemma 1:** Let $x$ and $y$ be two prime numbers and $a_1, a_2, \ldots, a_l$ be a complete system of residues for a module $y$. Therefore, $x_1, x_2, \ldots, x_y$ are also a complete system of residues for a module $y$. This means that $x_1, x_2, \ldots, x_y$ can traverse $a_1, a_2, \ldots, a_l [45]$.

**Property 2:** Let $N_s = (P, T, F, W, C)$ be a limited flow structure, $p_1 \in P$ be a terminal, $M_0$ be an initial marking such that $M_0(p_1) = C(p_1)$. Then, $\exists M \in R(N_s, M_0)$ such that

$$M(p_1) = C(p_1) - p_1^1 - p_1^0 + 1.$$ 

**Proof:** It is known that $p_1^1$ and $p_1^0$ are two prime numbers, 0, 1, 2, $\ldots$, $p_1^1 - 1$ are a typical complete system of residues for a module $p_1^1$ and $M_0(p_1) = C(p_1)$. It is true that the number of passengers is changed between the interval $[C(p_1) - p_1^1 - p_1^0 + 1, C(p_1)]$ according to Property 1. Therefore, it is true that $\forall M \in R(N_s, M_0)$ such that

$$M(p_1) = M_0(p_1) + mp_1^1 - np_1^0 = C(p_1) + mp_1^1 - np_1^0,$$

where $m$ and $n$ are two positive integers and the increment of $n$ is one. It can obtain that

$$C(p_1) - M(p_1) = (C(p_1) + mp_1^1 - np_1^0) - mp_1^1 - mp_1^0.$$ 

Therefore, we have

$$(C(p_1) - M(p_1)) \mod p_1^1 = ((np_1^0 - mp_1^0) \mod p_1^1.$$ 

It means that $C(p_1) - M(p_1)$ will traverse the complete system of residues 0, 1, 2, $\ldots$, $p_1^1 - 1$ for a module $p_1^1$ according to Lemma 1. We can obtain that $\exists M' \in R(N_s, M_0)$ such that $C(p_1) - M'(p_1) = p_1^1 - 1$. At marking $M'$, transition $t \in p_1^*$ can fire since $M'(p_1) \in p_1^1$. This means that transitions are enabled only in $p_1^*$ at marking $M'$. Therefore, $\exists M \in R(N, M') \subset M'(p_1^*)M$, that is

$$M(p_1) = M'(p_1) - p_1^0 = C(p_1) - p_1^1 - p_1^0 + 1.$$ 

**Property 3:** Let $N_s = (P, T, F, W, C)$ be a limited flow structure, $p_1 \in P$ be a terminal, $M_0$ be an initial marking such that $M_0(p_1) = C(p_1)$ and $p_1^1$ and $p_1^0$ be two prime numbers. Then, $\exists M \in R(N_s, M_0)$ such that $M(p_1) = C(p_1) - p_1^1 - p_1^0 + 1$ and $M' \in R(N_s, M_0)$ such that $M'(p_1) \geq M(p_1)$.

**Proof:** Let $N_s = (P, T, F, W, C)$ be a limited flow structure, $p_1 \in P$ be a terminal and $M_0$ be an initial marking such that $M_0(p_1) = C(p_1)$. It is impossible that $\exists M \in R(N_s, M_0)$ such that $M(p_1) < p_1^0$ ($M$ is an error marking) if

$$C(p_1) \geq k(x + 2y - 1),$$

where $p_1^1 = kx$, $p_1^0 = ky$, $k$ is the greatest common divisor of $p_1^1$ and $p_1^0$, $x$ and $y$ are two prime numbers. Moreover, $k$, $x$, and $y$ are positive integers and greater than or equal to one.

**Proof:** For $p_1$, if $p_1^1$ and $p_1^0$ are two prime numbers, it obtains that $k = 1$, $x = p_1^1$, and $y = p_1^0$. $\exists M \in R(N_s, M_0)$ such that $M(p_1) = C(p_1) - p_1^1 - p_1^0 + 1$ and $M' \in R(N_s, M_0)$ such that $M'(p_1) \geq M(p_1)$ according to Property 3. $\forall M \in R(N_s, M_0)$, marking $M$ is a legal marking if and only if $M(p_1) \geq p_1^0$. We have

$$C(p_1) - p_1^1 - p_1^0 + 1 \geq p_1^0 \Rightarrow C(p_1) \geq p_1^1 + 2p_1^0 - 1 = k(x + 2y - 1).$$ 

In addition, if $p_1^1$ and $p_1^0$ are not prime numbers, it obtains that $k \neq 1$, $p_1^1 = kx$, and $p_1^0 = ky$. It can be considered as that $x$ and $y$ are the new inputs and outputs. Similarly, it can obtain that

$$C(p_1) \geq k(x + 2y - 1).$$

Let $N_s = (P, T, F, W, C)$ be a limited flow structure, $p_1 \in P$ be a terminal and $M_0$ be an initial marking such that $M_0(p_1) = C(p_1)$. $\forall M \in R(N_s, M_0)$, it is impossible to suffer a saturated congestion problem at marking $M$ in the terminal $p_1$ if $(p_1)$ satisfies the conditions of Property 4. Moreover, it is also impossible to exist the waste of transition bus resources if $C(p_1) \geq k(x + 2y - 1)$. Therefore, the proposed scheme not only can calculate the minimum capacities for terminals but also can reduce the waste of transition bus resources.
V. EXAMPLE ANALYSIS

As one of the three complex airport hubs in China, Beijing Capital International Airport includes three airlines, i.e., China Southern Airlines, Air China Airlines, and China Eastern Airlines. Among the three airlines, the mainstream model of China Eastern Airlines is the Air bus A320 that is a typical seating 2-class with a capacity range of 150-180. The mainstream model of Air China Airlines is the Boeing B737-800 that is also a typical seating 2-class with a capacity range of 162-178 and the maximum capacity is 210. In this example, we assume that the average capacity of the mainstream models in Beijing Capital International Airport is 163 and the average capacity of the transition buses is 102 by analyzing their aviation resources.

The statistical results show that approximately five airplanes were arrived at the airport during fifteen minutes and passengers should enter terminal T3-D as quickly as possible during the peak periods. At the same time, passengers should leave the terminal T3-D by five transition buses. The Petri net model for the above scenario is shown in Fig. 9 and Table 2 shows its descriptions. This Petri net model contains seven places \( p_{1} \), \( p_{5} \), and ten transitions \( t_{s1}, t_{s2}, t_{s5}, t_{r1}, t_{r5} \). The inputs of \( p_{4} \) is \( 163 \times 5 = 815 \), i.e., \( p_{4}^I = 815 \), and the outputs of \( p_{4} \) is \( 102 \times 5 = 510 \), i.e., \( p_{4}^O = 510 \).

![Table 2. The descriptions for the Petri net model of Fig. 10.](image)

### FIGURE 10. The Petri net model of a real scenario in Beijing Capital International Airport.

In order to make the Petri net model more clearer in Fig. 10, it can be simplified to a flight process guidance system that is a limited flow structure \( N_f = (p_{a}, C) \), as shown in Fig. 11, where place \( p_{a} \) is associated with places \( p_{1}, p_{5} \); transition \( t_{a} \) is associated with transitions \( t_{s1}, t_{s2}, t_{s5}, t_{r1}, t_{r5} \). Let \( M_0 \) be the initial marking of \( N_f \), it is necessary to ensure \( M_\{t_{a} \} \). The main purpose is to reduce the waste of transition bus resources. Meanwhile, it is also necessary to ensure \( M_\{t_{r1} \} \) or \( M_\{t_{r5} \} \). The purpose is to avoid the saturated congestion problem. Both the necessary requirements require an appropriate capacity for \( p_{4} \). We know that \( p_{4}^I = 815 \) and \( p_{4}^O = 510 \). According to Property 4, we have \( p_{4}^I = xk \) and \( p_{4}^O = ky \), where \( k = 5 \), \( x = 163 \), \( y = 102 \) since \( p_{4}^I \) and \( p_{4}^O \) are not prime numbers and \( x \) and \( y \) are two prime numbers. Therefore, it can obtain

\[
C(p_{4}) \geq k(x + 2y - 1)
\]

\[
\geq 5 \times (163 + 2 \times 102 - 1)
\]

\[
\geq 1830.
\]

Assume \( M_0(p_{4}) = C(p_{4}) = 1830 \), as shown in Fig. 11. We have \( t_{a} = \{p_{a}\}, t_{s1} = \{p_{s}\}, t_{s2} = \{p_{s}\}, t_{s5} = \{p_{s}\}, C(p_{a}) = 815, C(p_{s}) = 1830, C(p_{s}) = 2000, W(t_{a}, t_{s1}) = p_{4}^I = 815, \) and \( p_{4}^O = W(t_{r1}, t_{r5}) = 510. \)

The input matrix \( N^+ \) and output matrix \( N^- \) are expressed as follows

\[
N^+ = \begin{bmatrix} 0 & 0 \\ 815 & 0 \end{bmatrix}
\]

\[
N^- = \begin{bmatrix} 0 & 510 \\ 0 & 0 \end{bmatrix}
\]

The initial marking is \( M_0 = (815, 1830, 350)^T \). At marking \( M_0 \), we have \( M_1(t_{a}) \) since \( M_0(p_{4}) \geq p_{4}^O \) and \( W(t_{a}, p_{s}) + M_0(p_{s}) \leq C(p_{s}) \). Transition \( t_{s1} \) is disenable since \( M_0(p_{s}) + p_{4}^O > C(p_{s}) \). If transition \( t_{s1} \) fires, a new marking \( M_1 \) is obtained by

\[
M_1(p_{4}) = (M_0(p_{4}) + N^+ \cdot \sigma_1 - N^- \cdot \sigma_1)
\]

\[
= (815, 1320, 350)^T,
\]

where \( \sigma_1 = (0, 1)^T \). At marking \( M_1 \), we have \( M_1(t_{r1}) \) since \( M_1(p_{4}) \geq p_{4}^O \) and \( W(t_{a}, p_{s}) + M_1(p_{s}) \leq C(p_{s}) \). Transition \( t_{r1} \) is disenable since \( M_1(p_{s}) + p_{4}^O > C(p_{s}) \). If transition \( t_{r1} \) fires, a new marking \( M_2 \) is obtained by

\[
M_2(p_{4}) = (M_1(p_{4}) + N^+ \cdot \sigma_2 - N^- \cdot \sigma_2)
\]

\[
= (815, 810, 350)^T,
\]

where \( \sigma_2 = (0, 1)^T \). At marking \( M_2 \), we have \( M_2(t_{a}) \) since \( M_2(p_{4}) \geq p_{4}^O \) and \( W(t_{a}, p_{s}) + M_2(p_{s}) \leq C(p_{s}) \). If transitions \( t_{a} \) and \( t_{r1} \) fire simultaneously, a new marking \( M_3 \) is obtained by

\[
M_3(p_{4}) = (M_2(p_{4}) + N^+ \cdot \sigma_3 - N^- \cdot \sigma_3)
\]

\[
= (815, 1115, 350)^T,
\]

where \( \sigma_3 = (1, 1)^T \). At marking \( M_3 \), we have \( M_3(t_{r1}) \) since \( M_3(p_{4}) \geq p_{4}^O \) and \( W(t_{a}, p_{s}) + M_3(p_{s}) \leq C(p_{s}) \). Transition \( t_{a} \) is disenable since \( M_3(p_{s}) + p_{4}^O > C(p_{s}) \). If transition \( t_{r1} \) fires, a new marking \( M_4 \) is obtained by

\[
M_4(p_{4}) = (M_3(p_{4}) + N^+ \cdot \sigma_4 - N^- \cdot \sigma_4)
\]

\[
= (815, 605, 350)^T,
\]
where \( \sigma_4 = (0, 1)^T \). At marking \( M_4 \), we have \( M_4[t_{a_4}, t_{t_4}] \) since \( M_4(p_{a_4}) > p_4^0, M_4(p_{a_4}) + p_4^1 \leq C(p_{a_4}) \), and \( W(t_{t_1}, p_{s_8}) + M_4(p_{s_8}) \leq C(p_{s_8}) \). If transitions \( t_{a_4} \) and \( t_{t_4} \) fire simultaneously, a new marking \( M_5 \) is obtained by

\[
M_5(p_{a_4}) = (M_4(p_{a_4}) + N^+ \cdot \sigma_5 - N^- \cdot \sigma_5) = (815, 910, 350)^T,
\]

where \( \sigma_5 = (1, 1)^T \). From marking \( M_5 \), we can obtain a new marking \( M_{164} \) by

\[
M_{164}(p_{a_4}) = (M_5(p_{a_4}) + 100x - 159y) = (815, 1320, 350)^T = M_1(p_{a_4})
\]

after a series of transitions among \( t_{a_4}, t_{t_4} \), or \( t_{a_4}, t_{t_4} \) fire, where

\[
\sigma = t_{t_1}t_{a_1}t_{a_4} \cdots t_{a_4}t_{t_4}t_{a_4}t_{t_4}t_{a_4}t_{t_4}
\]

\[
= \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \cdots + \sigma_{162} + \sigma_{163} + \sigma_{164}
\]

\[
= \sigma_5 + 100t_{a_4} + 159t_{t_4}.
\]

The firing processes are modeled by a tool that is developed from marking \( M_1 \) to marking \( M_{164} \). Therefore, it can effectively avoid the saturated congestion problem.

By carefully verifying the results, we know that transition \( t_{t_4} \) can fire at marking \( M_0 \). Moreover, \( VM \in \{M_1, M_2, M_4, M_5, \cdots, M_{164} \} \), transition \( t_{t_4} \) can always fire at marking \( M \). This means that passengers in \( p_{a_4} \) are continuously leaving terminal \( p_{t_4} \) by transition buses and the transition buses are fully loaded. It can reduce the waste of transition bus resources.

It knows from the example above that can avoid the saturated congestion problem if \( C(p_{t_4}) \) satisfies the conditions of \( C(p_{t_4}) \geq k(x + 2y - 1) \) in Property 4. If \( C(p_{a_4}) < k(x + 2y - 1) \), that exists the waste of transition bus resources, the examples are as follows.

\[
\begin{array}{cc}
C(p_{a_4}) &= 815 \\
C(p_{t_4}) &= 1250 \\
C(p_{s_8}) &= 2000
\end{array}
\]

According to Property 4, we have \( p_{t_4}^1 = kx \) and \( p_{t_4}^0 = ky \), where \( k = 1, x = p_{t_4}^1 = 743, \) and \( y = p_{t_4}^0 = 401 \) since \( p_{t_4}^1 \) and \( p_{t_4}^0 \) are prime numbers. Therefore, we have

\[
C(p_{t_4}) \geq k(x + 2y - 1) \geq 1 \times (743 + 2 \times 401 - 1) \geq 1544.
\]

If \( C(p_{t_4}) < k(x + 2y - 1) \), terminal \( T_3-E \) may exists the waste of transition bus resources, such as \( C(p_{t_4}) = 1533 \), \( C(p_{t_4}) = 1532 \), or \( C(p_{t_4}) = 1250 \). For example, we assume \( C(p_{t_4}) = 1250 \) in order to quickly visualize the results, as shown in Fig. 12. We have \( t_{a_1} = [p_{a_1}, t_{a_1}, t_{t_1} = [p_{t_1}], t_{t_1}^* = [p_{t_1}], C(p_{a_1}) = 815, C(p_{t_1}) = 1250, C(p_{t_1}) = 2000, W(p_{a_1}, t_{a_1}) = p_{t_1}^1 = 743, \) and \( p_{t_1}^0 = W(t_{a_1}, p_{t_1}) = 401 \). The input matrix \( N^+ \) and output matrix \( N^- \) are

\[
N^+ = \begin{bmatrix}
0 & 0 \\
743 & 0 \\
401 & 0
\end{bmatrix} \quad \text{and} \quad N^- = \begin{bmatrix}
743 & 0 \\
0 & 401 \\
0 & 0
\end{bmatrix}.
\]

Let the initial marking be \( M_0 = (815, 1250, 350)^T \). At marking \( M_0 \), we have \( M_0(t_{a_1}) \) since \( M_0(p_{t_1}) > p_{t_1}^0 \) and \( W(t_{a_1}, p_{a_1}) + M_0(p_{a_1}) \leq C(p_{a_1}) \). Transition \( t_{a_1} \) is disenable since \( M_0(p_{a_1}) + p_{t_1}^1 > C(p_{t_1}) \). If transition \( t_{a_1} \) fires, a new marking \( M_1 \) is obtained by

\[
m_1(p_{t_1}) = (M_0(p_{t_1}) + N^+ \cdot \sigma_1 - N^- \cdot \sigma_1) = (815, 849, 350)^T,
\]

where \( \sigma_1 = (0, 1)^T \). At marking \( M_1 \), we have \( M_1(t_{a_1}) \) since \( M_1(p_{t_1}) > p_{t_1}^0 \) and \( W(t_{a_1}, p_{a_1}) + M_1(p_{a_1}) \leq C(p_{a_1}) \). Transition \( t_{a_1} \) is disenable since \( M_1(p_{t_1}) + p_{t_1}^1 > C(p_{t_1}) \). If transition \( t_{a_1} \) fires, a new marking \( M_2 \) is obtained by

\[
m_2(p_{t_1}) = (M_1(p_{t_1}) + N^+ \cdot \sigma_2 - N^- \cdot \sigma_2) = (815, 448, 350)^T,
\]

where \( \sigma_2 = (0, 1)^T \). At marking \( M_2 \), we have \( M_2[t_{a_1}t_{a_1}] \) since \( M_2(p_{t_1}) > p_{t_1}^0 \) and \( M_2(p_{a_1}) + M_2(p_{a_1}) \leq C(p_{a_1}) \). If transition \( t_{a_1} \) fires, a new marking \( M_3 \) is obtained by

\[
m_3(p_{t_1}) = (M_2(p_{t_1}) + N^+ \cdot \sigma_3 - N^- \cdot \sigma_3) = (815, 790, 350)^T,
\]

where \( \sigma_3 = (1, 1)^T \). At marking \( M_3 \), we have \( M_3[t_{a_1}t_{a_1}] \) since \( M_3(p_{t_1}) > p_{t_1}^0 \) and \( M_3(p_{a_1}) + M_3(p_{a_1}) \leq C(p_{a_1}) \). Transition \( t_{a_1} \) is disenable since \( M_3(p_{t_1}) + p_{t_1}^1 > C(p_{t_1}) \). If transition \( t_{a_1} \) fires, a new marking \( M_4 \) is obtained by

\[
m_4(p_{t_1}) = (M_3(p_{t_1}) + N^+ \cdot \sigma_4 - N^- \cdot \sigma_4) = (823, 389, 350)^T,
\]

where \( \sigma_4 = (0, 1)^T \). At marking \( M_4 \), transitions \( t_{a_1} \) cannot fire since \( M_4(p_{a_1}) < p_{t_1}^1 \). This means that the limited flow structure \( N_f \) exists the waste of transition bus resources.

According to Property 4, we have \( C(p_{t_1}) \geq k(x + 2y - 1) = 1544 \). Let \( M_0(p_{t_1}) = C(p_{t_1}) = 1544 \). We have \( M_0 = (815, 1544, 350)^T \). At marking \( M_0 \), we have \( M_0[t_{a_1}t_{a_1}] \)

![FIGURE 12. A limited flow structure in terminal T3-E.](image-url)
since $M_0(p_7) = p_7^0$ and $W(t_{11}, p_8) + M_0(p_8) \leq C(p_8)$. Transition $t_{11}$ is enabled since $M_0(p_7) + p_7^1 > C(p_7)$.

If transition $t_{11}$ fires, a new marking $M_1$ is obtained by

$$M_1(p_7) = (M_0(p_7) + N^+ \cdot \sigma_1 - N^- \cdot \sigma_1)$$

$$= (815, 1143, 350)^T,$$

where $\sigma_1 = (0, 1)^T$. At marking $M_1$, we have $M_1[t_{11}]$ since $M_1(p_7) = p_7^0$ and $W(t_{11}, p_8) + M_1(p_8) \leq C(p_8)$.

Transition $t_{11}$ is enabled since $M_1(p_7) + p_7^1 > C(p_7)$.

If transition $t_{11}$ fires, a new marking $M_2$ is obtained by

$$M_2(p_7) = (M_1(p_7) + N^+ \cdot \sigma_2 - N^- \cdot \sigma_2)$$

$$= (815, 742, 350)^T,$$

where $\sigma_2 = (0, 1)^T$. At marking $M_2$, we have $M_2[t_{11}]$ since $M_2(p_7) = p_7^0$, $M_2(p_7) + p_7^1 \leq C(p_7)$, and $W(t_{11}, p_8) + M_2(p_8) \leq C(p_8)$. If transitions $t_{11}$ and $t_{11}$ fire simultaneously, a new marking $M_3$ is obtained by

$$M_3(p_7) = (M_2(p_7) + N^+ \cdot \sigma_3 - N^- \cdot \sigma_3)$$

$$= (815, 1084, 350)^T,$$

where $\sigma_3 = (1, 1)^T$. At marking $M_3$, we have $M_3[t_{11}]$ since $M_3(p_7) = p_7^0$ and $W(t_{11}, p_8) + M_3(p_8) \leq C(p_8)$.

Transition $t_{11}$ is enabled since $M_3(p_7) + p_7^1 > C(p_7)$.

If transition $t_{11}$ fires, a new marking $M_4$ is obtained by

$$M_4(p_7) = (M_3(p_7) + N^+ \cdot \sigma_4 - N^- \cdot \sigma_4)$$

$$= (823, 683, 350)^T,$$

where $\sigma_4 = (0, 1)^T$. At marking $M_4$, we have $M_4[t_{11}]$ since $M_4(p_7) = p_7^0$, $M_4(p_7) + p_7^1 \leq C(p_7)$, and $W(t_{11}, p_8) + M_4(p_8) \leq C(p_8)$. If transitions $t_{11}$ and $t_{11}$ fire simultaneously, a new marking $M_5$ is obtained by

$$M_5(p_7) = (M_4(p_7) + N^+ \cdot \sigma_5 - N^- \cdot \sigma_5)$$

$$= (823, 1025, 350)^T,$$

where $\sigma_5 = (1, 1)^T$. From marking $M_5$, we can obtain a new marking $M_{744}$ by

$$M_{744}(p_7) = M_5(p_7) + 399 p_7^1 - 739 p_7^0$$

$$= (815, 1143, 350)^T$$

$$= M_1(p_7)$$

after a series of transitions among $t_{a1}$, $t_{11}$, or $t_{a1}, t_{11}$ fire, where

$$\sigma = t_{a1}t_{11}[t_{a1}t_{11}]t_{11} \cdots [t_{a1}t_{11}]t_{11}[t_{a1}t_{11}]t_{11}$$

$$= \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \cdots + \sigma_{742} + \sigma_{743} + \sigma_{744}$$

$$= \sigma_3 + 399\sigma_{a1} + 739\sigma_{t1}.$$}

It forms a loop from marking $M_1$ to marking $M_{744}$. Therefore, it can effectively avoid the waste of transition bus resources and the saturated congestion problems during the peak periods if $C(p_7) = 1544$.

The above examples verify that the terminals can avoid the waste of transition bus resources and the saturated congestion problems during the peak periods if their capacities such that the condition in Property 4.

**VI. CONCLUSION**

This paper proposes a formal model to show the limited flow structure of flight process guidance systems based on finite capacity Petri nets. Compared with the previous studies on capacity optimization for terminals, the proposed scheme not only can calculate the minimum capacities for terminals, but also can avoid the saturated congestion problems and reduce the waste of transition bus resources. Moreover, the proposed scheme shows the whole processes of passenger guidance systems by using the graphical tool of Petri nets. The derivation processes of passengers entering and leaving the terminals are verified by using the mathematical tool of Petri nets. By analyzing an example, the results show that the proposed relational expression can calculate the minimum capacity for each terminal during the peak periods. It reduces the building costs, avoids the saturated congestion problems, and reduces the waste of transition bus resources.

However, this paper only considers the minimum capacity for terminals, the complex coordination control among multiple terminals are ignored during the peak periods. In the future, the complex models for multiple terminals will be proposed by Petri nets. Moreover, the average time passengers spend in terminals is also important for evaluating the throughput of terminals during peak periods. In the future work, the complex models for multiple terminals will be proposed by a class of timed Petri nets and the average time that passengers spend in the terminals will be considered with the passenger flow control protocols to optimize the congestion problems among multiple terminals.

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**GUANGMIN XIE** received the B.S. degree from Xihua University, Chengdu, China, in 2017, where she is currently pursuing the M.S. degree in computer technology. Her current research interests include cryptography, Petri nets theory, and information security.

**ZHONGYUAN JIANG** received the B.S. and M.S. degrees from Xihua University, Chengdu, China, in 2004 and 2008, respectively, and the Ph.D. degree from the School of Electro-Mechanical Engineering, Xiidian University, Xi’an, China, in 2014. From 2015 to 2019, he was with an Associate Professor with the School of Computer and Software Engineering, Xihua University. Since 2019, he has been an Associate Professor with the College of Big Data Statistics, Guizhou University of Finance and Economics. His research interests include public opinion analysis, supervisor control theory, and fault diagnosis of discrete event systems.
MINGXING HE received the M.S. degree from Chongqing University, in 1990, and the Ph.D. degree from Southwest Jiaotong University in 2003. He is currently a Full Professor with the School of Computer and Software Engineering, Xihua University, Chengdu, China. He has coauthored five books and has published more than 100 articles in refereed professional journals and international conferences. His current research interests include cryptography and information security. He received the DAAD Scholarship Reward of Germany, in 2002, the Excellent Ph.D. Dissertation Award in Southwest Jiaotong University, in 2003, and the Grant of National Science Foundation of China (NSFC), in 2004, 2007, and 2015. He is a Senior Member of CACR and a member of the ACM.

XIAOLIANG CHEN received the B.S. and M.S. degrees in computer science from Xihua University, Chengdu, China, in 2007 and 2010, respectively, and the Ph.D. degree in mechatronic engineering from Electro-Mechanical Engineering, Xidian University, Xi’an, China in 2014. He joined Xihua University, in 2014, where he is currently an Associate Professor with the School of Computer and Software Engineering and the Deputy Director of the Intelligent Information Systems Laboratory. From October 2017 to October 2018, he was a Visiting Professor with the Department of Computer Science and Operations Research, University of Montreal, Canada. His research interests include social networks, natural language processing, and supervisor control theory.

JIAN MAO received the B.D. and M.S. degrees from Sichuan University, in 2004 and 2008, respectively. He joined Chengdu Civil Aviation Information Technology Company Limited, in 2014, where he is currently the Manager of the Department of Research and Development and a Senior Engineer of airport engineering. His research interests include airport engineering, airport operation, and airport control.