Multiscale Features of Cross Correlation of Price and Trading Volume

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Abstract

Price without transaction makes no sense. Trading volume authenticates its corresponding price, so there is mutual information and entanglement between price and volume. On the other hand, we are curious about scaling features of this entanglement and need to know how structures in different scales translate information. So, markets are faced with a variety of dimensions of price and trading volume. Investment size (volume), price-wise expectations (gain/loss), and time-wise expectations (time-scale) differ from one investor to another. This study, by applying the MF-DXA method, demonstrates that price and trading volume and their coupling contain power law information and are multifractal in the markets we investigated. Also, the resultant correlation coefficients present scaling behaviors which are totally significant in the investigated time-scales and they decrease with increasing time-scales. Meanwhile, considering developed markets, the price-volume coupling is more dominated by trading volume rather than price. This domination increases price validity. We can confirm that in a developed market, traders, with a certain price, are more rational and show their enthusiasm to price by applying trading volume. This approach for emerging
markets is weak. As a whole, in emerging markets, market behavior is guided by a phenomenon other than volume.

Keywords: Price-Volume Cross Correlation, Stock Market, Multifractal Behavior, MF-DFA

1. Introduction

*Price-Volume entanglement.* The phenomenon which leads to price discovery in an auction, is translation of trading volume. Nevertheless, higher trading volume may help one to distinguish how much tendency there is to a certain price. Hence, if a certain price corresponds to a relatively higher trading volume, it can be an indication of more price reliability [1], at that time, in the market. It may lead to smaller spreads and also smaller liquidation volatilities [2] within that time-scale. Ausloos and Ivanova [3], combined classical technical analysis with thermodynamic proxies. Considering that from a physics point of view, it is doubtful to investigate price movements without price-volume dependencies, they obtained some fruitful measures which can model market behavior in terms of trends and amplitude of volatilities by price and trading volume time series. Since trading volume is a touchstone of how traders work in the market and translate information via their transactions, this phenomenon can shape movements in a market. As a result, Ahmad [4] constructed a price-volume joint distribution and investigated price-volume relations by comparing with a bivariate normal distribution and the related marginal distributions and conditional distributions. Podobnik et al. [5] obtained a power-law in price-volume cross correlations. They investigated logarithmic changes of price and corresponding volume and showed that just the cross correlation of absolute values of volatilities are statistically significant. It is noteworthy to state that in more developed markets, the entanglement of price return fluctuations and higher volumes [1], leads to a situation whereby an increase in volume shall (to some extent (turning point)) increase price fluctuations. But then a further increase of trading volume leads to a lowering of price fluctuation. Meanwhile,
Guo et al. [6] investigated the price-volume cross-correlation of commodities in futures markets and compared their multifractal behaviors. Also, some other researchers such as Campbell et al. [7]; Wang [8]; Copeland [9]; Saatcioglu and Starks [10]; Chen et al. [11]; Chuang et al. [12] and Osborne [13], showed the existence of an information translation between price and trading volumes by using econometric and statistical methods. In reference to the theme of this current paper, Podobnik et al. [5]; Nasiri et al. [1] and Guo et al. [6], found that price-volume cross volatilities, may have scaling behavior at the levels of time and magnitude-scales. Since it makes no sense to study price volatilities without considering the reliability which a certain price owes to trading volume, we investigate the scaling behavior of the cross correlation of price-volume volatilities.

Some interactions in some scales may present uncorrelated behavior, but there is this possibility that in some scales, these interactions show significant correlation. Different scaling behavior toward events of different magnitudes, causes multifractal behavior [14]. In this study, we apply a multifractal detrended cross correlation analysis to investigate the multifractal behavior of price, volume and their coupling in some stock markets (DJIA, S&P500, TOPIX, TSE, SSEC). It is shown that, in addition to time-wise multiscale patterns, price and trading volume and their cross correlations have magnitude-wise multifractal behaviors. Moreover, the correlation coefficients of their volatilities decrease with an increase of the time-scale.

In order to reveal scaling behavior, we need to differentiate large-scale and small-scale patterns in a stock market as complex system [15, 16, 17, 18, 19]. Since large-scale patterns (main trends) are somehow evident they do not present much latent information [14]. Hence, initially it is necessary to apply ‘detrended fluctuation analysis’ (DFA) for nonstationary signals. See Peng et al. [20] and also [15, 20, 21, 22, 23, 24].

Because of psychological biases and irrational decisions of investors [19, 25, 26, 27, 28, 29]; out-of-market effects [19, 30]; uncertainty about the transparency of fundamental analysis [25, 31]; the coexistence of collective effects and noise
and the lagging diffusion of internal and external information between several dynamics, a situation occurs where some assumptions of the EMH (Efficient Market Hypothesis) become doubtful. In spite of the complexity aspects in financial markets such as self-organizing (in order to increase adaptability) and scaling patterns of nonlinear dynamics, we observe power-law (scale invariance) behavior in some scales \([19, 33]\). Industrial, economic and political cycles which are dynamic and ever changing, may cause a variability of statistical properties of time series in several scales. This is what we call ‘nonstationarity’. These behaviors cause multiscale correlations between time series. Systems which contain persistent nonlinear interactions as inputs and outputs of the constituents, may cause the emergence of some simultaneous and some lagging effects. In the case of persistent information, these effects cause long-range autocorrelation (for one constituent) and long-range cross correlation (between several constituents) \([34]\).

Since our time series may be nonstationary and limited in length, we need a method which considers limited-length effects and nonstationarity effects \([35]\). The mentioned methodology has been applied successfully in finance \([5, 36, 37, 38, 39, 40, 41, 42, 43]\) and developed to Fourier-DFA and MF-DFA \([44]\). As a heuristic method, Caraiani and Haven \([45]\) by applying the EMD (Empirical Mode Decomposition) method on the detrending process of MF-DFA, investigated multifractality in currency data against Euro and nonlinearity of the market.

Podobnik and Stanley \([46]\) introduced detrended cross correlation analysis (DXA) for studying two power-law nonstationary time series. Then, Zhou \([47]\) developed the DXA method to multifractal DXA (MF-DXA) to investigate multifractal behaviors of two power law nonstationary time series. In the case of combining the MF-DXA within magnitude-wise scales, Shadkhoo and Jafari \([48]\), Cottet et al. \([49]\), Podobnik et al. \([5]\), Campillo and Paul \([50]\), Hajian and Movahed \([51]\) applied some studies on detrended covariance. Moreover, Hedayatifar et al. \([14]\) introduced the Coupling-DXA method to investigate coupling behavior among more than two nonstationary power law time series. They be-
lieved that studying more than two signals in complex systems leads researchers to better understanding the FOREX market structure, and they showed that several (more than two) constituents have a scaling coupling behavior. It also needs mentioning that, for the separation of large-scale and small-scale hierarchical sinusoidal modes, Fourier-DFA is suggested [52, 53].

In this paper, we apply multifractal detrended cross correlation analysis to investigate the scaling behavior of volatilities of price-volume coupling in markets such as DJIA, S&P500, TOPIX, TSE and SSEC. In section 2, a review on methodology is presented. Then in section 3, we analyze and describe the input data used in the methodology. In section 4, the empirical results are discussed and then in section 5, we provide for the conclusion.

2. Methodology

*MF-DXA*. The general steps of MF-DXA methodology [15, 20, 21, 22, 23, 24] are as follows. Initially we convert the time series to standardized logarithmic changes. After demeaning each data-point, we execute a cumulative summation which is called profile series (Eq 1).

\[
X(i) = \sum_{k=1}^{i} [x_k - <x>]
\]

\[
Y(i) = \sum_{k=1}^{i} [y_k - <y>]
\]

\[i = 1, 2, \ldots, N.\]  (1)

where \(<...>\) is the average value of the time series; \(N\) is the total length; and \(X(i)\) and \(Y(i)\) are profile series. To investigate time-scale effects on series, we consider several time-scale windows with the length of \(s\) which are iterated on time series. Variable \(N_s\) is obtained by \(int(N/s)\) which provides the maximum number of segments (with length of \(s\)) in each time series. Since the length \(N\) may not be an integer multiple of scale length \(s\), we repeat the same iteration from ending data point to starting one. It means that each non-overlapping
scale window, is allocated to sequential time locations without ignoring any data points (Eq 3). Consequently, the total number of time-wise scales would be \(2N_s\).

In order to convert profile series to non-stationary type series, we should eliminate the main trends (large-scale behavior) from time series. Among lots of detrending methods (such as polynomial detrending, Fourier detrending), it is better to start from the simplest one to avoid over-fitting and also to avoid eliminating excessive amount of information. So in this study, one-degree linear detrending is applied. As follows, detrending covariance for each time-wise scale \((s)\) and windows location \((\nu)\) is obtained by subtracting local trend of each window. After detrending, local deviation in each non-overlapping window is obtained.

\[
F^2_{(s,\nu)} = \frac{1}{s} \sum_{j=1}^{s} |X^{(j)}_{(\nu-1)s+j} - \tilde{X}^{(j)}_{\nu}||Y^{(j)}_{(\nu-1)s+j} - \tilde{Y}^{(j)}_{\nu}|
\]

\[
\nu = 1, 2, ..., N_s
\]

\[
F^2_{(s,\nu)} = \frac{1}{s} \sum_{j=1}^{s} |X^{(j)}_{N-(\nu-N_s)s+j} - \tilde{X}^{(j)}_{\nu}||Y^{(j)}_{N-(\nu-N_s)s+j} - \tilde{Y}^{(j)}_{\nu}|
\]

\[
\nu = N_s + 1, N_s + 2, ..., 2N_s;
\]

where \(\tilde{X}_\nu\) and \(\tilde{Y}_\nu\) are local trends which are fitted polynomials in each window \(\nu\) with the local length of \(s\). The fitting style may be linear or nonlinear (quadratic, cubic, DFAm) with the order of \(m\). Trends with the order \(m\) in profile series, and \(m - 1\) in the original time series are eliminated by fitting \(m\)th-order polynomials \([15, 20, 35, 54]\) which are best to be chosen based on data type and in comparison with each other \([22, 23]\). For a special case of triangular trends, when a certain detrending can not indicate the power-law of fluctuation function in the corresponding scale range, Fourier-detrending is a suitable method.

The less (more) the length of scale windows, the more detrended covariance is
affected by small and rapid (large and slow) fluctuations. Because of the shorter (longer) scales, on average, there will be an increase (decrease) of local effects in the detrended covariance function. Accordingly, small-scale (large-scale) behaviors are observed significantly in small (large) windows. We can further explain this by saying that monofractals are normally distributed and the volatilities can be explained just by the second statistical moment, i.e. the ‘variance’. On the contrary, when it comes to multifractals, in large (small) scales, local variations are excessively large (small). As a result, to consider different behaviors of fluctuations, we magnify our concentration from small and frequent, to large and rare fluctuations by weighting them. Hence, for considering effects of events on magnitude scales, the parameter \(q\) is applied. The bigger \(q\), the more weight is applied to the tails (rare and enormous events) of the logarithmic changes histogram. To be unbiased toward small or large variations, \(q = 0\) is applied.

\[
F_q(s) = \left( \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} F^2(s, \nu)^{q/2} \right)^{\frac{1}{q}} \quad q \neq 0.
\]

\[
F_q(s) = \exp\left( \frac{1}{4N_s} \sum_{\nu=1}^{2N_s} \ln[F^2(s, \nu)] \right) \quad q = 0.
\]

In the case of the second moment, Eq. 4 leads to \(\sigma = \sqrt{\sigma_1 \sigma_2}\) [14, 46].

If the time series are scale-invariant and have long-range correlation, the MF-DXA approach would contain a scaling behavior as follows:

\[
F_q(s) \sim s^{h_{sv}(q)},
\]

where \(F_q(s)\) is a fluctuation function in order of \(q\) and scale length of \(s\).

The slopes of \(\log F - \log s\) are called a generalized Hurst exponent \(h(q)\). For each moment \(q\), there is an \(F(s)\). Whilst within a range of time-scales, the slope of \(\log F - \log s\) for various moment \(q\) is constant, the time series is scale-invariant.

On the contrary, if there is a change in the value of slope \(h(q)\) for a single \(q\), it is called a ‘cross-over’ and the time series is now scale variant. For a range of \(q\), a spectrum of \(h(q)\) is obtained. The degree of multifractality [55] as a risk measure [56] is as follows:

\[
\Delta h = h_{\max}(q) - h_{\min}(q).
\]
If $\Delta h = 0$, the system is monofractal so the time series does not have segments with extreme small and extreme large fluctuations. Hence, its detrended covariance within a same time-scale window length, when powered by $q$th-order in different windows location $\nu$, will yield no peak. On the contrary, for $\Delta h \rightarrow 0$, the system is multifractal. In this case, the average value for residuals of local trends for different $s$ and $\nu$, are not similar. So, large variations dominate the results for larger $q$, and small variations dominate the results for smaller $q$.

The higher the multifractality degree, the less efficient and the more predictable is the market. It is worthy to state, if the Hurst exponent $h(q = 2) = 0.5$, the time series is a random walk. For a Hurst exponent $h(q = 2) < 0.5$ ($h(q = 2) > 0.5$), it tends to be anti-persistent (persistent) and is negatively-correlated (positively-correlated). The Hurst exponent for developed markets is less than 0.5 and for emerging markets is larger than 0.5. Rényi’s exponent (scaling exponent function) is as follows [48]:

$$\tau_{xy}(q) = q h_{xy}(q) - 1.$$ 

If $\tau_{xy}(q)$ (which is a derivative of $h_{xy}(q)$) is a linear function of $q$, the time series is monofractal. To examine the singularity content of time series, the singularity spectrum is as follows:

$$\alpha = h_{xy}(q) + q h'_{xy}(q)$$

$$f_{xy}(\alpha) = q(\alpha - h_{xy}(q)) + 1; \quad (4)$$

where $\alpha$ is the singularity of a time series and $f_{xy}(\alpha)$ is the multifractality spectrum. In addition, the multifractality strength is observed by the singularity width $\Delta \alpha$, as below:

$$\Delta \alpha = \alpha_{max} - \alpha_{min}; \quad (5)$$

where $\alpha_{max}$ relates to $q_{min}$ and $\alpha_{min}$ relates to $q_{max}$. If $\Delta \alpha = 0$, the time series is monofractal and the response of cross correlation toward different events of $q$, in large and small lengths of the time-scale is identical, and the multifractality spectrum is just a point.
Until this section, the fluctuation function $F(s)$ for each $q$, is obtained for the coupling of time series.

**MF-DFA.** When applying the detrended covariance function for just a single time series, the detrended variance function is presented. The rest of the methodology is similar to the MF-DXA. This process needs to be applied for price and trading volume separately.

**Correlation.** The absence of any correlation leads to $\rho_{DXA} = 0$. Based on Podobnik et al. [34] this claim is just valid for an unlimited-length of time series. In the case of limited length, the situation of $\rho_{DXA} \neq 0$, but no correlation is probable. If the investigated time series have power law -as we will show in the rest of this research- with the help of cross correlation statistic [34, 56], by the equation below we will show the scales which the cross correlation coefficient is statistically significant:

$$Q_{cc}(m) = N^2 \sum_{i=1}^{m} \frac{X_i^2}{N-i},$$

where $X_i$ is cross correlation function as follows:

$$X_i = \frac{\sum_{k=i+1}^{N} x_k y_{k-i}}{\sqrt{\sum_{k=1}^{N} x_k^2} \sqrt{\sum_{k=1}^{N} y_k^2}};$$

where $x_k$ and $y_k$ are detrended logarithmic changes of time series.

Since the cross correlation statistic $Q_{cc}(m)$ is somehow similar to the $\chi^2(m)$ (Chi-squared) distribution, the critical value is measured by $\chi^2(m)$. So, when $Q_{cc}(m)$ is less than the critical value (null hypothesis) it means there is no significant cross correlation. On the contrary, if $Q_{cc}(m)$ is more than the critical value (alternative hypothesis), then there is no reason to reject the existence of cross correlation.

In order to reveal cross-correlation scaling behavior, we calculate the correlation by DXA fluctuation as follows:

$$\rho_{DXA} = \frac{F_{DXA}^2(n)}{F_{DFA_1}(n) F_{DFA_2}(n)};$$
where $\rho_{DXA}$ is cross correlation behavior and $n$ is the time-scale value. $F_{DF}$ is the fluctuation function of each time series and $F_{DXA}$ is the fluctuation function of cross correlation.

3. Empirical Data

In this study, we gathered the price index and trading volume of the DJIA, S&P500, TOPIX, TSE and SSEC markets during March 21st, 2013 until March 20th, 2018. This includes around 1300 trading days from which we can investigate the multifractal behavior of time series volatilities and their related cross correlations.

In Fig. 1 top panel, the descriptive statistics for one of the markets is shown. Also, as illustrated in Fig. 1 bottom panel, none of the time series are normally distributed so they are not monofractal. Initially, after calculating the logarithmic changes of time series, they were standardized. Then

$$
    r_{\text{price}} = \ln P_t - \ln P_{t-1},
$$

$$
    r_{\text{volume}} = \ln V_t - \ln V_{t-1},
$$

where $t$ is daily data point, and also, $P$ and $V$ refer to price and trading volume, respectively. The above mentioned results are used as inputs to the methodology.

4. Empirical Results

MF-DXA. After detrending in each nonoverlapping segment $\nu$ with length of $s$ for price, volume, and price-volume cross series, we shall in order to estimate the power-law relation, compute $\log F - \log s$. This is illustrated in Fig. 2 for the markets we investigated. For instance, the results for two markets are shown. In Fig. 4, the multifractality features of price-volume coupling for the investigated markets is presented.
Figure 1: Descriptive statistics of investigated markets are demonstrated.

| Log. changes of | TSE     | DAX     | S&P500  | TOPIX   | SSEC   |
|-----------------|---------|---------|---------|---------|--------|
| $\mu$           | 0.001, 0.001 | 0.000, 0.001 | 0.000, 0.001 | 0.000, 0.001 | 0.000, 0.000 |
| $\sigma$        | 0.007, 0.320 | 0.008, 0.284 | 0.008, 0.240 | 0.011, 0.165 | 0.014, 0.180 |
| skewness         | 0.144, -0.120 | -0.619, -0.315 | -0.592, 0.070 | -0.399, 0.306 | -3.327, 0.408 |
| kurtosis         | 0.144, 1.639 | 4.051, 6.225 | 3.631, 7.594 | 5.474, 1.822 | 8.277, 4.834 |
| Jarqu-Bra P-value| 0.000, 0.000 | 0.000, 0.000 | 0.000, 0.000 | 0.000, 0.000 | 0.000, 0.000 |
Altering a power law relation in different scales, means there is scale variance and is an indication of structural transformation. On the other hand, a system is scale invariant when it generates its structure in different intervals. To obtain valid power laws, proper scales are extracted which are shown in Fig. 4. It is worthy to say that for window lengths near $N$, the local trend of the segments become more similar to the whole time series, and in this process $F(s)$ becomes more independent of iterations on $\nu$.

Another reason for this phenomenon is the effect size of variations. Large segments (small segments) contain large-scale (small-scale) behaviors of the time series. Hence, applying the $q$th-order effect on the fluctuation function, leads to a divergence of $\log F - \log s$ for smaller scale sizes. Nevertheless, for $s \ll N$ (local deviation of detrended segments are smaller on smaller scales), the fitting polynomial is better fitted on the segments and results in a magnification of the divergence of $F(s)$ for different $q$ values.
Figure 3: Top: Multifractality features such as Hurst exponent spectrum (left), singularity spectrum (middle) and scaling exponent (right) of price, volume, and their coupling for TSE (as an emerging market) are shown. Bottom: Multifractality features such as Hurst exponent spectrum (left), singularity spectrum (middle) and scaling exponent (right) of price, volume, and their coupling for DJIA (as a developing market) are shown.

Figure 4: Top: Multifractality features of price-volume coupling: Hurst exponent spectrum (left), singularity spectrum (middle) and scaling exponent (right) are shown. Bottom: Multifractality features of price timeseries, volume timeseries, and price-volume coupling are presented.
The generalized Hurst exponent presents multifractality of correlations. Based on Podobnik and Stanley [46], the Binomial measure from the $p$-model for the cross-correlation exponent of the fractionally autoregressive integrated moving average (FARIMA) with identical stochastic noises, is equal to their arithmetic mean at $q = 2$. However, Zhou [47] illustrated that the same relation for $q$ values other than 2, is valid [56] which is presented in red colour in the left panels in 3 for an emerging market (TSE) and for a developed market (DJIA).

Considering the left columns of Fig. 3, if the arithmetic mean of the generalized Hurst exponent of time series is larger/smaller than the cross-correlation generalized Hurst exponent, in those $q$ moments, then the cross correlation multifractal behaviors are more affected by the effects of trading volume/price time series.

Also, when it comes to the middle column in Fig. 3, price multifractalities of all investigated markets are larger than volume multifractalities of the corresponding markets. If the time series is multifractal and $q$ is extremely large for the system, it may yield to $f(\alpha) < 0$ [57]. On the other hand, $f(\alpha) < 0$, may occur on extremely large or small singularity spectra. [58]. Since a negative dimension is an unreal solution, this may warn us to increase the total length of the time series, so that events near tails of the histogram increase and eliminate limited-length effects. In addition, the right panels of Fig. 4, which relate to the nonlinearity of scaling exponents, prove that price multifractalities occur more often than volume multifractalities of the markets we investigated. As a result, the multifractality of price-volume coupling can be found to be between the corresponding price and trading volume multifractalities.

As shown in Fig. 4, among the investigated markets, the TSE has the least multifractality degree of price-volume coupling. Conversely, for more developed markets, the multifractality degrees of price-volume coupling are larger. However, considering the singularity spectrum, lower efficiency of the markets contributes to higher singularity strength in price-volume couplings. Furthermore, less efficiency in the markets coincides with more linear scaling exponents of price-volume coupling. It is notable that, price-volume coupling of
markets which are more developed such as DJIA, S&P500 and TOPIX (with $H_{\text{price}} < 0.5$), are more affected by trading volume. Cross correlations are dominated by volume, and also, their $h_{cc}(q)$ show a more descending pattern. What is noteworthy, is that the singularity spectrum for an emerging market such as the TSE (with $H_{\text{price}} > 0.5$) is more left-hooked. The price-volume cross correlation of an emerging market is more led by some attributes other than volume of transactions (such as the manipulation of the supply and demand at certain prices which causes markets to be led more by price rather than trading volume!). This is proven by the left panel of Fig. 3.

**Cross Correlation Coefficient.** Firstly, by applying eq. 8 the scales with significant cross correlation coefficients are distinguished, as shown in Fig. 5. After a statistical confirmation of correlation significance, by applying eq. 9, we evaluate the scaling behavior of correlation coefficients of the investigated markets throughout different time-scales, Fig. 6. We filtered the area with significant cross correlations in order to investigate the cross correlation coefficients in the next step. Fig. 5 contains cross correlation statistics versus critical value, and Fig. 6 shows the corresponding correlation coefficients in a multiscale pattern, all for $h(q = 2)$, of price-volume coupling for investigated markets. As shown, the cross correlation statistics are significant for the studied range of

![Figure 5: Cross correlation statistics of the investigated market versus critical value are shown.](image)
Figure 6: Scaling behavior of correlation coefficients of the investigated markets are demonstrated.

time-scales.

As illustrated in Fig. 6, the larger temporal segments size, the smaller the cross-correlation coefficient and the larger fluctuation functions are (such as $DFA_{\text{price}}, DFA_{\text{volume}}, DXA_{\text{price-volume}}$). When it comes to larger scales, the multifractal behavior of fluctuation functions corresponding to price and volume, does not present an extreme correlation and the scaling behavior of price and trading volumes are totally different in larger scales. Hence, in larger scales, it makes no sense to describe price volatilities by trading volume volatilities.

It is noteworthy that cross correlations of domains maintaining a power law in both price time-series and volume time-series of each market, will show that the decrease of the correlation coefficient co-occurs faster by an increase of time-scales in less developed markets such as the TSE and SSEC. Consequently, as shown in Fig. 5 and Fig. 6, by increasing time-scale, the behavior of emerging markets deviate from the behavior of developed markets. The segregation between the correlation behaviors occurs after the emerging power law. In the period of our study, it is obvious that among all investigated markets, the scaling correlation coefficients of the developed markets maintain to 0.4 to around 0.6. On the other hand, the scaling correlation coefficients of the less developed markets maintain around less than 1 and ultimately tend to somehow 0.2.
to 0.35 in larger time-scales. It means that the decrease in scaling correlation coefficients of less developed markets is more sensitive rather than is the case with more developed markets toward time-scale size.

5. Conclusion

In multifractal time series, we need to consider several statistical moments to describe the behavior of the system. Therefore, investors need to execute a multifractal analysis to be more familiar with the system’s behavior. Nevertheless, in the aforementioned markets, the generalized Hurst exponent is not distributed stochastically. By illustrating the singularity spectrum, this study shows nonlinear behavior of price, volume, and price-volume structure. As a consequence, studying a single variable without considering simultaneous collective effects and their cross effects, may be biased.

Since the Hurst exponent of price is larger than 0.5 ($h_{price} > 0.5$) for the TSE, it is an emerging market with persistent behavior of price volatilities. Conversely, because of $h_{price}(q = 2) < 0.5$, the DJIA, S&P500 and TOPIX, are classified as developed markets with short range correlation and anti-persistent behavior. The investigated markets are affected by their memory and models which analyze these markets based on the Efficient Market Hypothesis may no longer estimate the market behavior accurately. In an inefficient market, the long-range correlation and short-range correlation phenomena exist in price. Long-ranged correlation leads to more predictability for price. Long-range correlations cause more persistent information effects and lessen the speed of fading-out of information from past dynamics. On the other hand, the trading volume volatilities present negatively correlated behavior. Furthermore, the multifractal behavior of volume DFA is less than the multifractal behavior of price DFA. Moreover, the cross correlation coefficients show scaling behaviors which are totally significant in the investigated time-scales and decreases by increasing time-scales.
As investigated, when considering developed markets, the price-volume coupling is more dominated by the trading volume rather than price. This domination increases price validity. We can confirm that in a developed market, traders, in a certain price, rationally show their enthusiasm to price by applying trading volume. This approach for emerging markets is weak. As a whole, in emerging markets, market behavior is guided more by a phenomenon other than trading volume.

Multiscale volatilities in financial markets have become more important in risk management. Efficient risk management requires better understanding of information translation between couplings of stock markets structures and internal and external dynamics in multiscale patterns. One of the most practical measures of this study is applying the cross correlation matrix based on Laloux et al. [59] and Plerou et al. [60] for several fluctuation functions to measure market risk.

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