The Photon Wave Function in Non-forward Diffractive Scattering with Non-vanishing Quark Masses

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Abstract

The light-cone Photon wave function in explicit helicity states, valid for massive quarks and in both momentum and configuration space, is presented by considering the leading order photon-proton hard scattering, i.e., the splitting quark pair scatters with the proton in the Regge limit. Further we apply it to the diffractive scattering at nonzero momentum transfer and reach a similar factorization as in the case of zero momentum transfer.

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I. INTRODUCTION

The hard diffractive photon-proton scattering are of particular interest in testing the perturbative QCD (pQCD) because of not only the HERA prolific experimental data but also the theoretical developments to date. Diffractive processes generally characterized by the presence of large rapidity gaps in the hadronic final state are conventionally ascribed to Pomeron exchange. Before the advent of QCD, hard diffractive processes are well described by Regge theory [1]. Although the relation between Regge asymptotics with pQCD description, the well-known BFKL equation [2], is still unclear, we have seen the possibility of gaining an understanding of the nature of diffraction, the Pomeron and its coupling to partons from the first principle. The hard diffractive processes in deep-inelastic scattering (DIS) are of particular interest toward these goals, in which people believe could reach the transition region between perturbative and non-perturbative strong interactions.

It is believed that in DIS the fast-moving photon dissociates into partons long before its interacting with the target and the dissociation of the photon into partons is described by the so-called photon wave function [3–6]. In the beginning of the scattering, the photon splits into a quark-antiquark pair, which may or may not evolve into more partons by strong interaction before it reaches the target proton [3]. The simplest process in diffractive DIS is of the $q\bar{q}$ production, which is conveniently described in terms of light-cone photon wave functions procedure in the proton rest frame. Because in DIS $Q^2$, the photon virtuality, is relatively large, the quark-antiquark pair in impact parameter space can be referred as a color dipole. The diffractive interaction of the color dipole with the proton is mediated by Pomeron, which may be described by, e.g., two gluon model [7].

The photon wave function has been introduced and used for many years (for a recent paper related to it, see e.g. [8]), however we find that in amplitude level the photon-quark-antiquark wave function with non-vanishing quark masses are not explicitly shown in the helicity basis. In [3] expressions for massless quarks and transverse photons are given explicitly in terms of helicity amplitudes. [3,9] give expressions for the squared wave functions, including longitudinal photons and also massive quarks. A derivation of the amplitudes in terms of an explicit spinor representation limited to massless quarks is given in [10]. For completeness, in this paper we will reconsider a perturbative derivation of the photon wave function, in explicit helicity basis and including fermions masses, by considering the amplitude for the diffractive process $\gamma^*p \rightarrow p + X$ both in momentum and configuration space. In practice, we consider the kinematic limit, where the $\gamma^*p$ energy $s$ is much larger than the diffractive mass $M_X^2$, the relative transverse momentum squared of the $q\bar{q}$-pair $k^2$, the virtuality of the photon $Q^2$ and the momentum transfer $t$ from the
virtual photon system to the proton. In this limit the amplitude is factorized basically into the photon wave function and the unintegrated off-diagonal gluon structure function as expected.

Instead of the space of transverse momenta one can transform this amplitude to configuration space, where the conjugate variable to the transverse momentum separation of the $q\bar{q}$ pair is its transverse size. This leads to the convenient dipole-picture in the proton rest frame, saying that the virtual photon fluctuates into the $q\bar{q}$-pair long before it undergoes the interaction with the proton and therefore this fluctuation and the interaction of the dipole with the proton are independent and hence factorize in the transverse space. In this way a description of cross sections for inclusive diffractive scattering processes, as well as of diffractive vector meson production, can be carried out completely in terms of the photon wave function and the so called dipole cross section, see e.g. [11–13].

The paper is organized as follows. In section II we will derive the photon wave function in momentum space by using the helicity method developed in [14]. Following we will transform the wave function into configuration space and to see the factorized form of photon-proton interaction in two-gluon model. In the end there will be a brief summary.

II. LIGHT-CONE WAVE FUNCTIONS IN MOMENTUM SPACE

The photon wave function is a very useful tool in the calculation of a high energy virtual photon scattering off the target. One of the nice features of it is that it can not only being used for single gluon exchange processes (e.g, in determining $F_2$), but can be extended to include multi-gluon exchange processes. Here, before considering the amplitudes for diffractive scattering, we calculate the photon wave function from a simpler process, where a single gluon is exchanged in the $t$-channel as depicted in Fig.1.

Throughout the calculation we will use the Sudakov decomposition for all momenta with respect to the light cone vectors $q' = q - (Q^2/s)p_B$ and $p' = p_B + (m_B^2/s)q'(s = 2p'\cdot q'; q$ is the momentum of the photon with virtuality $Q^2 = -q^2$ and $p_B$ is the momentum of the incoming quark $B$ with mass $m_B$). For example, in diagram (a) the momenta $k$ of internal quark line and $r$ of the exchanged gluon have the following Sudakov decomposition:

$$k = \alpha q' + \beta p' + k_\perp, \quad (1)$$
$$r = \frac{k}{s} q' + x_{p'} p' + r_\perp. \quad (2)$$

In this way the antiquark and quark carry fractions $\alpha$ and $(1 - \alpha)$ in the $q'$-direction.
Noted that in the following we often use the Euclidean form of the transverse momenta marked in boldface, i.e. $k^2_\perp = -k^2$.

From the on-shell conditions for the outgoing quarks, together with the assumptions that $Q^2 \sim |t| \sim M_X^2 \ll s$, we know $x_F$ and $\beta$ to be of the order $t/s$. In this limit we use the usual decomposition of the metric tensor

$$g_{\mu\nu} = \frac{2}{s}(p'_\mu q'_\nu + p'_\nu q'_\mu) + g^\perp_{\mu\nu}$$

for the gluon propagator and retain only the first term with an appropriate contraction, while the other terms are suppressed by powers of $t/s$. Furthermore, the denominators of the quark lines in diagrams (a) and (b) can be expressed respectively as

$$\Delta_a = k^2 - m^2 = -\frac{1}{1 - \alpha}[\alpha(1 - \alpha)Q^2 + k^2 + m^2] \equiv -\frac{1}{1 - \alpha}D(k), \quad (4)$$

$$\Delta_b = (q - k - r)^2 - m^2 = -\frac{1}{\alpha}[\alpha(1 - \alpha)Q^2 + (k + r)^2 + m^2] \equiv -\frac{1}{\alpha}D(k + r). \quad (5)$$

The mass in the upper fermion line is always denoted as $m$.

After these denotations it is quite straightforward to write down the amplitude, by virtual of the helicity method, e.g. in [14], for one gluon exchange as

$$A = -ee_f C \frac{2g^2\delta_{\lambda B,\lambda' B'}}{t} \bar{u}_\lambda(p_A') \left( \frac{\chi_a}{\Delta_a} + \frac{\chi_b}{\Delta_b} \right) v_\lambda(p_A) \quad (6)$$

with

$$\chi_a = \gamma^\mu(k + m) \gamma^\nu, \quad \chi_b = -\gamma^\mu(\gamma^\nu k - \gamma^\nu) \gamma^\nu, \quad \quad (7)$$

and $\lambda'(\lambda) = \pm$ denotes the helicity of the (anti-)quark. $C$ is the color factor and $g$ is the strong coupling constant. The photon polarization vectors in Eq. (7) can be chosen as

$$\varepsilon^0 = \frac{1}{Q}(q' + xBP'), \quad \varepsilon^{(\pm)}_\perp = \frac{1}{\sqrt{2}}(0, 1, \gamma i, 0), \quad (8)$$

denoting the longitudinal($\gamma = 0$) and transverse($\gamma = \pm$) cases, respectively.

Still using the helicity method and keeping the mass term one reaches the helicity expressions for longitudinal and transverse polarizations of the photon in diagram (a)(and a similar one for diagram (b)),

$$\chi_a(\gamma = 0) = \alpha s \varepsilon^0 + \frac{\mp}{2Q} \varepsilon_\perp - i\varepsilon(\mu, \varepsilon^0, p_{B'}, p_B)\gamma_\mu\gamma_5, \quad (9)$$

$$\chi_a(\gamma = \pm) = \alpha s \varepsilon_\perp \mp\varepsilon_\perp \cdot r \gamma^\nu - i\varepsilon(\mu, \varepsilon_\perp, p_{B'}, p_B)\gamma_\mu\gamma_5. \quad (10)$$
Here only leading terms in the high energy limit are kept. With (9) we obtain the helicity amplitude for (6), which can be written in a compact form

$$\mathcal{A} = e g^2 \frac{2 s}{t^2} \delta_{\lambda B, \lambda' B} \sqrt{\alpha(1 - \alpha)} \left( \Psi(k, \alpha) - \Psi(k + r, \alpha) \right).$$  \hfill (11)

Here, $\Psi(k, \alpha)$ is the so called light-cone photon wave function, which contains all dependence of the amplitude on the properties of the virtual photon and the $q\bar{q}$-pair in helicity basis.

$$\Psi^0_{\pm \mp}(k, \alpha) = \frac{2e^f \alpha(1 - \alpha) Q}{\alpha(1 - \alpha)Q^2 + k^2 + m^2},$$  \hfill (12)

$$\Psi^\pm_{\pm \mp}(k, \alpha) = \frac{\sqrt{2e^f \alpha k}}{\alpha(1 - \alpha)Q^2 + k^2 + m^2},$$  \hfill (13)

$$\Psi^\pm_{\mp \pm}(k, \alpha) = \frac{-\sqrt{2e^f (1 - \alpha) k}}{\alpha(1 - \alpha)Q^2 + k^2 + m^2},$$  \hfill (14)

$$\Psi^\pm_{\mp \mp}(k, \alpha) = \frac{\sqrt{2e^f i m}}{\alpha(1 - \alpha)Q^2 + k^2 + m^2},$$  \hfill (15)

$$\Psi^0_{\pm \pm}(k, \alpha) = \Psi^\pm_{\mp \mp}(k, \alpha) = 0,$$  \hfill (16)

where $\overline{k} = k_x + i \gamma k_y$. Note that in the additional term (15), appearing only for massive quarks, there is a naive helicity conservation $\gamma = (\lambda + \lambda')/2$. A similar situation was found for (12) but there the mass dependence always cancels in the difference of the wave functions. On the other hand, in the massless limit we can only have the configuration $\lambda = -\lambda'$ within the $q\bar{q}$-pair. This behavior is easily understood from the appropriate limit in $m/\sqrt{s}$: in the high energy limit the fermions’ helicities are always opposite and independent of the coupling, in contrast to the nonrelativistic limit where we have helicity conservation. Besides, in massless limit we find a agreement of the expressions (11)-(16) with references [4,11].

In the following we apply the obtained photon wave functions to the diffractive scattering in two gluon exchange model as depicted in Fig.2. In the leading log $x$ approximation the amplitude is dominated by its imaginary part, from which we will reconstruct the full amplitude later on. By virtue of the Cutkosky rules we may consider each diagram as a composition of two, where the crossed fermion lines shown in Fig.2 are on mass shell. The left part has a structure as in the one gluon exchange discussed above and the right part is just a simple one gluon exchange between two quark lines. From the cuts in the fermion lines we have the integration loop momentum

$$\ell = \alpha q' + \beta p' + \ell_\perp,$$  \hfill (17)

where from the mass-shell conditions we know $\alpha_l$ and $\beta_l$ are both of order $t/s$.

From the respective right parts of the diagrams we get a factor $2\alpha s/(r - \ell)^2$ or $2(1 - \alpha)s/(r - \ell)^2$ with helicity conservation within the fermion lines, depending on
whether the right gluon couples to the quark or the antiquark. On the other hand we get factors $1/2\alpha_s$ or $1/2(1-\alpha)s$ from the integration over the longitudinal part of the loop momentum and the on-shell conditions, therefore we are only left with the denominators from the right parts of the diagrams.

To determine the left parts, we can simply read off the transverse momentum of the virtual quark line from the diagrams and get a photon wave function with this argument, similar to the one gluon case, but now we also have the transverse part of the loop momentum as an argument of the wave functions. Furthermore we now have a different color structure. Putting everything together, we can write the full amplitude, arising from the diagrams in Fig.2, as a ‘double difference’

$$A = ieg^4 s \delta_{\lambda_B,\lambda_B'} C' \sqrt{\alpha(1-\alpha)} \int \frac{d^2 \ell}{(2\pi)^2} \frac{r^2}{\ell^2(r-\ell)^2} \times \left\{ \Psi(k,\alpha) + \Psi(k+r,\alpha) - \Psi(k+\ell,\alpha) - \Psi(k+r-\ell,\alpha) \right\},$$

with the wave function $\Psi(k,\alpha)$ defined as in Eqns. (12) – (16). This kind of ‘double difference’ was already obtained in [5] and [6] in a proper limit.

To include the non-perturbative coupling of the two gluons to the proton, one needs to go beyond the leading order and meets the unintegrated off-diagonal gluon distribution $F(x,x',\ell^2,r^2)$, a suitable generalisation of the unintegrated gluon distribution to the off-diagonal case, for more see e.g. [15] and references therein for discussions of off-diagonal gluon distributions in diffraction. Here, $x$ and $x'$ denote the longitudinal momentum fractions of the two gluons, coupling to the proton and $x_{IP} = x - x'$. Integrating over $\ell$ would give an off-diagonal gluon distribution,

$$\int d^2 \ell F(x,x',\ell^2,r^2) = G(x,x',r^2,Q^2)$$

and in the limit $x \approx x'$ we have $G(x,x,r^2 = 0,Q^2) = xg(x,Q^2)$, with $g(x,Q^2)$ being a diagonal gluon distribution. In [16,17] the deviation of $G(x,x',t,Q^2)$ from $xg(x,Q^2)$ is discussed in detail.

Now we may write the general amplitude for diffractive scattering off the proton as

$$A = i\frac{\pi}{4} eg^2 \sqrt{\alpha(1-\alpha)} s \int \frac{d^2 \ell}{\pi \ell^2} F(x,x',\ell^2,r^2) D\Psi(k,r,\ell),$$

where the shorthand notation

$$D\Psi(k,r,\ell) = \Psi(k,\alpha) + \Psi(k+r,\alpha) - \Psi(k+\ell,\alpha) - \Psi(k+r-\ell,\alpha)$$

with the wave function $\Psi(k,\alpha)$ defined as in Eqns. (12) – (16). This kind of ‘double difference’ was already obtained in [5] and [6] in a proper limit.
for the double difference of the wave functions, \( F(x, x', \ell^2, r^2) \) is normalized as in the diagonal case, i.e. \( F(x, x, \ell^2, r^2 = 0) = F(x, \ell^2) \), with \( F(x, \ell^2) \) being the unintegrated gluon structure function \([6]\). Furthermore, we left the dependence on \( t \) in the unintegrated structure function to keep the expression more general.

### III. EXPRESSIONS IN CONFIGURATION SPACE

It is more clear to consider the factorization of diffractive amplitudes in configuration space. Therefore we will proceed the Fourier transform in this section with respect to the transverse momenta \( k \) and \( r \). The variable conjugate to \( k \) is the transverse separation of the \( q\bar{q} \)-pair \( q \), or simply the called ‘dipole size’. Similarly, the variable conjugated to the momentum transfer between the diffractive system and the proton is the impact parameter \( b \).

The conjugated photon wave functions are \( \psi^{\gamma_{\lambda\lambda}}(q) \) in \( q \)-space as

\[
\psi_{\pm\pm}^{0}(q) = \frac{1}{\pi} e^{i\alpha(1-\alpha)QK_{0}(\delta q)},
\]

\[
\psi_{\pm\pm}^{\pm}(q) = \frac{i}{\pi} e^{i\alpha\frac{q \cdot \epsilon}{\ell}K_{1}(\delta q)},
\]

\[
\psi_{\pm\pm}^{-\pm}(q) = -\frac{i}{\pi} e^{i(1-\alpha)\frac{q \cdot \epsilon}{\ell}K_{1}(\delta q)},
\]

\[
\psi_{\pm\pm}^{\pm\pm}(q) = -\frac{m}{2\pi} e^{iK_{0}(\delta q)},
\]

where we used \( \delta^{2} = \alpha(1-\alpha)Q^{2} + m^{2} \) as a shorthand notation and \( K_{\nu}(z) \) is modified Bessel function. After squaring, our results agree with that of [9]. Using these wave functions, we can easily transform the diffractive amplitude (20) into configuration space,

\[
\tilde{A}^{D}(q, b) = \int \frac{d^{2}k}{(2\pi)^{2}} \int \frac{d^{2}r}{(2\pi)^{2}} e^{ik \cdot q} e^{ir \cdot b} A^{D}(k, r) \]

\[
= B \int \frac{d^{2}k}{(2\pi)^{2}} \int \frac{d^{2}r}{(2\pi)^{2}} e^{ik \cdot q} e^{ir \cdot b} \int \frac{d^{2}\ell}{\pi \ell^{2}} \alpha_{s}(\mu^{2}) F(x, x', \ell^{2}, r^{2}) D\Psi(k, r, \ell),
\]

where \( B \) contains the factors in front of the integral in (20). In doing the \( k \) integration we make use of the wave function (22)-(25) and pick up appropriate phases from a shift in the integration region, get

\[
\int \frac{d^{2}k}{(2\pi)^{2}} e^{ik \cdot q} D\Psi(k, r, \ell) = \psi(q) \left[ 1 + e^{-i\ell \cdot q} - e^{-i\ell \cdot q} - e^{-i(r-\ell) \cdot q} \right].
\]
Therefore, we may write
\[
\tilde{A}^D (\varrho, b) = B \psi (\varrho, \alpha) \int \frac{d^2 r}{(2\pi)^2} e^{i r \cdot b} \int \frac{d^2 \ell}{\pi \ell^2} \\
\times \alpha_s (\mu^2) F (x, x', \ell^2, r^2) [1 - e^{-i \ell \cdot q}] [1 - e^{-i (r - \ell) \cdot q}] \\
\equiv \psi (\varrho, \alpha) \int \frac{d^2 r}{(2\pi)^2} e^{i r \cdot b} \sigma_{q\bar{q}} (\varrho, r), \quad (28)
\]
where we have put the \( \ell \)-dependent part and the factor into the definition of the proton-dipole cross section. Carrying out the last Fourier transform from momentum transfer to impact parameter, we get the expected factorized amplitude, as in the zero momentum case in e.g. [18],
\[
\tilde{A}^D (\varrho, b) = \psi (\varrho, \alpha) \sigma_{q\bar{q}} (\varrho, b) \quad (29)
\]
in the case of nonzero momentum transfer. We see that the physical picture, where the fluctuation of the virtual photon into the \( q\bar{q} \) pair occurs long before the interaction of dipole and proton, still holds. But for now, the dipole proton cross section \( \sigma_{q\bar{q}} (\varrho, b) \) depends on not only the dipole size \( \varrho \), but also the impact parameter \( b \).

IV. SUMMARY

In the diffractive scattering of a virtual photon off a target it is technically useful to introduce the photon wave function in the calculation. In this paper we have recalculated the wave functions including massive quarks in the explicit helicity basis, which is not fully and explicitly presented in the literature by now.

On the other hand, the concept of the photon wave function turned out to arise naturally in considering the diffractive scattering, where the factorized part of the photon is expressed in terms of the wave function. We get know that in addition to the dipole size \( \varrho \), the dipole cross section also depends on the impact parameter \( b \) of virtual photon and proton in transverse configuration space. It is shown manifestly that the factorization of the diffractive amplitude into the photon wave function and the unintegrated structure function still holds in the case of non-zero momentum transfer, which is widespreadly believed.

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Figure 1: The Feynman diagrams contributing to $\gamma^* + q' \rightarrow q\bar{q} + q'$ in the high energy limit.
Figure 2: The Feynman diagrams contributing to diffractive $\gamma^* p \rightarrow q\bar{q} p$ scattering with only the transverse momenta of virtual fermion lines denoted.