Adjoint Chiral Supermultiplets and Their Phenomenology

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Abstract

Matter fields in the MSSM are chiral supermultiplets in fundamental (or singlet) representations of the standard model gauge group. In this paper we introduce chiral superfields in the adjoint representation of $SU(3)_C$ and study the effective field theory and phenomenology of them. These states are well motivated by intersecting D-brane models in which additional massless adjoint chiral supermultiplets appear generically in the low energy spectrum. Although it has been pointed out that the existence of these additional fields may make it difficult to obtain asymptotic freedom, we demonstrate that this consideration does not rule out the existence of adjoints. The QCD gauge coupling can be perturbative up to a sufficiently high scale, and therefore a perturbative description for a D-brane model is valid. The full supersymmetric and soft SUSY breaking Lagrangians and the resulting renormalization group equations are given. Phenomenological aspects of the adjoint matter are also studied, including the decay and production processes. The similarity in gauge interaction between the adjoint fermion and gluino facilitates our study on these aspects. It is found that these adjoint multiplets can give detectable signals at colliders and satisfy the constraints from cosmology.

1 Introduction

It is well known that in both of the two benchmark models of particle physics — the Standard Model (SM) and its supersymmetric extension, the Minimal Supersymmetric Standard Model (MSSM) — all the matter fields are in the fundamental (or singlet) representations under the
SM group $SU(3)_C \times SU(2)_L \times U(1)_Y$, and all the gauge fields are in the adjoint representation under the corresponding gauge group. However, it is interesting to ask whether it is possible to consistently include matter fields in higher representations under the gauge groups. This is not an unimportant question to ask since this possibility is also strongly motivated by intersecting D-brane models. Intersecting D-brane models are a class of phenomenologically interesting string models where the gauge fields of the SM are localized on D-branes wrapping certain cycles on an underlying compact geometry. Chiral matter emerges at the intersections of these D-branes and the family number is determined by the intersection number of the branes. However, in addition to the ordinary chiral matter localized at intersections, these models typically possess non-chiral open string states resulting from the deformation and Wilson line moduli of the D-branes\cite{1,2}. These states appear as adjoint matter in the four dimensional low-energy effective field theory. Although they have not been observed yet, they could be phenomenologically interesting at future colliders, such as the LHC, and bring new knowledge to our understanding of elementary physics. Especially, an interesting point is: if the adjoint matter originating from D-brane moduli is observed, it is possible to be an indirect signal and probe of extra dimensions. As we mentioned, the adjoints in D-brane models geometrically originate from the transverse fluctuations of D-branes in higher dimensions. On the other hand, if we look at the T-dual picture, these geometric moduli become the higher dimensional components of the relevant gauge field, which appear as scalars in the four-dimensional theory. Taking into account that these adjoints have gauge couplings to SM fields, once observed, they might produce stronger signal at colliders, compared to the KK modes of the graviton as a probe of extra dimensions. Of course, adjoint matter fields in four-dimensional effective theory can have other sources and may not be relevant to extra dimensions in these cases. In any case now we can expect some nontrivial phenomenological implications of the adjoint matter. However, it was pointed out that these exotic multiplets should be got rid of in some way since they give additional positive contributions to the gauge coupling $\beta$-function. The main motivation for this argument was to obtain an asymptotically free gauge theory (for $SU(3)_C$) which facilitates perturbative gauge coupling unification enjoyed by the MSSM. This consideration has been taken seriously and D-brane models without additional adjoint fields have been developed\cite{1,3}. In these specific models, the D-branes wrap rigid cycles so that the open string moduli are frozen.

However, the possible trouble with asymptotic freedom brought in by the adjoint fields does not actually impose a no-go theorem on the existence of new fields in the low energy spectrum. First of all, unlike in heterotic string models, gauge coupling unification, as the main motivation for asymptotic freedom, is not a prerequisite in intersecting D-brane models. Instead, all that is required is that the gauge coupling be perturbative up to the string scale. After deriving the RG equations of the gauge couplings in our model in section 3, we will demonstrate in detail that generically these additional adjoint fields do respect this requirement. Furthermore, it is possible to preserve gauge coupling unification in an intersecting D-brane model with these new fields. This issue has been discussed in detail in ref.\cite{7}, in which the authors demonstrate that under a few reasonable assumptions, gauge coupling unification can be realized in realistic three generation supersymmetric intersecting brane models and the result is still valid even when the
contributions from additional non-chiral matter are included. An earlier study of unification with adjoints in SUSY scenario can be found in [4, 5].

Based on the discussions above, we see that adjoint matter fields are worthy of serious study despite some possible challenges of phenomenology. In the following sections of this paper, we will study the effective field theory of these adjoint matter fields by embedding them into the MSSM.\(^1\)

## 2 Lagrangian of Chiral Adjoint Matter

In order to derive the relevant Lagrangian, we need to briefly review the transformation properties of adjoint matter fields under gauge and supersymmetry transformations. In the intersecting D-brane models which reproduce the SM gauge group \(SU(3)_C \times SU(2)_L \times U(1)_Y\), there is an adjoint field for each of the three subgroups. These fields are in the adjoint representation of the corresponding subgroup and are in the singlet representation under the other two subgroups. For each gauge group, the multiplicity \(n\) of the adjoint (an analogy to the family number of ordinary matter) is determined by the specific configuration of D-branes. Under supersymmetry, all of the adjoints are in chiral supermultiplets, as in the case for ordinary matter. Considering the theory above the electroweak symmetry breaking (EWSB) scale, we require the Lagrangian to be invariant under the SM gauge group. We will focus on \(SU(3)_C\) adjoints. It can be expected that compared to ordinary matter, the distinct and interesting features of adjoint matter come from its distinct representation under the gauge group.

The complete renormalizable supersymmetric and gauge invariant Lagrangian is:

\[
\mathcal{L}_1 = \mathcal{L}_{g_0} + \mathcal{L}_g + \mathcal{L}_{\bar{\mu}} + \mathcal{L}_Y
\]

\(\mathcal{L}_{g_0}\) is the kinetic term for free gauge fields and has the usual form as we see in the MSSM. \(\mathcal{L}_g\) is the connective gauge interaction between the adjoints and the gauge fields (\(SU(3)_C\) in this paper). \(\mathcal{L}_{\bar{\mu}}\) is the mass term of the adjoints. The non-chiral property of the adjoints allows this gauge invariant mass term. The fermionic component of the adjoint superfield is Majorana which is an important point for our discussion on phenomenological issues later. Although these adjoints typically get mass only after supersymmetry breaking, SUSY breaking effects in the Kähler potential in supergravity can result in such a supersymmetric mass term from \(F^\dagger\) insertions, which is similar to the Giudice-Masiero mechanism in the MSSM [15]. \(\mathcal{L}_Y\) is the Yukawa interaction between the adjoints.

Let us now address each term of the Lagrangian in turn in terms of the F-term and D-term of the relevant chiral superfield and vector superfield. We will mainly use the notation and

\(^1\)Although inspired by D-brane models, we do not dwell on string model building in this paper. The considerations will be based on effective field theory. However, it is plausible to realize this MSSM+ \(SU(3)_C\) adjoints model from a real intersecting D-brane model, without auxiliary matter fields, e.g. \(SU(2)\) adjoints, symmetric or antisymmetric matter [1, 2-6].
conventions of refs. [13] [14].

\[ L_1 = \left( \int d^2 \theta \frac{1}{16k g^2} \overline{T} R W^\alpha W_\alpha + h.c. \right) + \int d^4 \theta \frac{\overline{T} R (\Phi^{a \mu a} e^{2 g V^c T^c} \Phi^{b \nu b})}{l} \]

\[ + \left[ \int d^2 \theta \left( \frac{1}{2} \overline{T} R (\Phi^{a \mu a} \Phi^{b \nu b}) \right) + \frac{\overline{T} R (\Phi^{a \mu a} \Phi^{b \nu b} \Phi^{c \rho c})}{l^{3/2}} \right] + h.c. \]

where \( \Phi^{a \mu a} \) is the adjoint chiral superfield with color index \( a \) and can be expanded in component fields as

\[ \Phi = A + \sqrt{2} \psi \theta + F \theta \theta \quad (3) \]

\( t^a \) is \( N \times N \) matrix for \( SU(N) \) group where \( N = 3 \) in this case. \( V \) is the usual vector superfield, whose matrix \( T^c \) in this case is \( 8 \times 8 \) when acting on adjoint matter. \( \tilde{\mu} \) is the supersymmetric Majorana mass of the adjoint field. \( y \) is the Yukawa coupling constant. \( k \) is the Dynkin index of the representation of the gauge field according to the matter field it acts on. \( l \) is defined by

\[ \overline{T} R (t^a t^b) = l \delta^{ab} \quad (4) \]

For the fundamental representation, \( l = \frac{1}{2} \).

The combinations of color indices \((a, b)\) and \((a, b, c)\) in eq.(2) run over all the possible combinations. The notation of \( \overline{T} R \) means the symmetrized trace of the involved matrices which gives the invariant tensor of certain representation. This notation is no different from the usual \( Tr \) if two matrices are included. When three matrices are involved, this new notation sums over all the possible invariants. \( \overline{T} R (t^a t^b t^c) \) is actually exactly the form of triangle anomaly which we are familiar with. An interesting upshot here is that there is no gauge invariant Yukawa interaction term for \( SU(2) \) adjoint since the triangle anomaly for an \( SU(N) \) group is non-vanishing only for \( N > 2 \). The following results are useful for our later discussion in this paper [16]:

\[ Tr([t_a, t_b] t_c) = \frac{1}{2} d_{abc} \]

\[ \sum_{b,c} d_{abc} d_{bcd} = \frac{N^2 - 4}{N} \delta_{ad} \quad (6) \]

\[ \sum_{a,b,c} d_{abc} d_{abe} = \frac{(N^2 - 4)(N^2 - 1)}{N} \quad (7) \]

\[ Tr([t_a, t_b] t_c) = i \frac{1}{2} f_{abc} \]

\[ \sum_{c,d} f_{acd} f_{bcd} = N \delta_{ab} \quad (9) \]

We would like to point out some important properties of the adjoint fields. From eq.(2), we see that in the gauge invariant renormalizable Lagrangian, they only couple to \( SU(3)_C \) gauge
fields (including gluons and gluinos) of the MSSM. They can only directly couple to Higgs and quarks through non-renormalizable interactions which we will consider in section (2.2). For a complete MSSM augmented by chiral adjoint matter, it is fair to include the connective gauge terms for ordinary matter and the Higgs Yukawa terms. But they are simply in the usual form we know well. These terms will contribute to the RG equation of the gauge coupling, but not to the 1-loop RG running of adjoint masses.

In order to obtain RG equations and discuss phenomenological issues, we derive the explicit form of the Lagrangian by component fields:

$$\mathcal{L}_{\mu} = \bar{\psi}(A^a F^a - \psi^a \psi^a) + h.c.$$  \hspace{1cm} (10)

$$\mathcal{L}_{\phi^0} = \frac{1}{4} v_{\mu} v_{\mu} + i \bar{\lambda} \sigma^\mu D^a \lambda^a + \frac{1}{2} D^a D^a$$ \hspace{1cm} (11)

$$\mathcal{L}_Y = \sqrt{2} g \eta^{abc} (F^a A^b A^c - \psi^a \psi^b \psi^c) + h.c.$$ \hspace{1cm} (12)

$$\mathcal{L}_g = \frac{Tr(t^a t^a)}{l} [D_\mu A^a D^a F^a + i \bar{\psi} \sigma^\mu D^a \psi^a + F^a \bar{A}^a - \sqrt{2} g (\bar{\lambda} \psi^a T^a A^a + h.c.) \hspace{1cm} (13)

\quad + g A^a D^a D^a A^a]$$

\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (\partial_\mu A^a D^a A^a + g^2 f^{aef} v^e \psi^f A^a A^d f - g f^{aef} (v^c A^d \sigma^\mu \bar{A}^a + v^c D^a \sigma^\mu A^d) + i \bar{\psi} \sigma^\mu \partial_\mu \psi^a - ig f^{aef} (\bar{A}^d \lambda^a \psi^a + \bar{\psi} \lambda^a A^d) - ig f^{aef} A^a D^a A^d$$

where $v_\mu$ is the gauge boson, $\lambda$ is gaugino, $D$ is the auxiliary field

$$v_\mu^a = \partial_\mu v_\nu^a - \partial_\nu v_\mu^a - g f^{abc} v_\mu^b v_\nu^c \hspace{1cm} (14)$$

$$D_\mu \lambda^a = \partial_\mu \lambda^a - g f^{abc} v_\mu^a \lambda^c \hspace{1cm} (15)$$

Eq. (15) is the generic expression for the covariant derivative of the adjoint representation which applies for both gaugino and the adjoint fields $\psi, A$ which we used in getting eq. (13)).

We can obtain the explicit expression of the auxiliary fields from the equation of motion:

$$F^a = -\sqrt{2} y^a A^a A^a - \mu^a A^a \hspace{1cm} (16)$$

$$F^{\dagger a} = -\sqrt{2} y A^{\dagger a} A^a - \mu^a A^a \hspace{1cm} (17)$$

$$D^c = ig f^{cad} A^d A^a \hspace{1cm} (18)$$

In order to study the low energy phenomenology, in addition to the manifestly supersymmetric Lagrangian $\mathcal{L}_1$, we must include soft breaking terms, which we collectively denote as $\mathcal{L}_2$. This is analogous to the usual case $\mathcal{L}_2$:

$$\mathcal{L}_2 = -\frac{1}{2} (M \lambda^a \lambda^a + c.c.) - m^2 A^a A^a - \left( \frac{1}{2} b A^a A^a + \sqrt{2} d^{abc} A^a A^b A^c + c.c. \right) \hspace{1cm} (19)$$
Since the adjoint fields do not couple to ordinary matter directly through renormalizable interactions, we need to include dim-5 non-renormalizable terms suppressed by some high scale \( M^* \) in order to study the phenomenology at colliders. In order to simplify, we assume that flavor symmetry breaks at a scale above \( M^* \) and that the Yukawa matrix \( (Y') \) for such dim-5 interactions has the similar hierarchical structure as the Yukawa in the SM, since they share the same flavor physics at high scale. Concretely, this means that all the mixings are negligible and the only non-vanishing entry of the Yukawa matrices is \((3,3)\), with the value \( y'_t \) or \( y'_b \).

If we further assume that the hierarchy between the top quark mass and bottom quark mass mainly comes from their difference in Yukawa coupling, \( y'_b \) is also negligible. Under the above assumptions, only \( y'_t \sim 1 \) enters our discussion. (Of course, if we need, it is not hard to write down the full dim-5 Lagrangian without those assumptions. However, the expressions will be more complicated.) Here we use \( T_a \) instead of \( t^a \) to represent the matrix representation of the adjoint field, in order not to mix up with the notation \( t^c \) for the conjugate of right-handed top quark field. Now we get:

\[
L'_{Y} = \int d^2 \theta \frac{1}{M_s} Q_3 \Phi_a T_a t^c H_u + h.c. \tag{20}
\]

\[
= \int d^2 \theta \frac{1}{M_s} \Phi_a T_a (t^c H_u^0 - t^c b H_u^+) + h.c.
\]

\[
= \frac{1}{M_s} T_a [A_a (F_t H_u^0 t^c + F_b H_u^0 b^c + F_c H_u^0 c^c - t^c t H_u^0 - t^c \tilde{H}_u^0 t^c - t^c H_u^0)] + h.c.
\]

\[
+ 2 \psi_a (t^c H_u^0 t^c + \tilde{t}^c \tilde{H}_u^0 t^c + t^c H_u^0 t^c + F_a \tilde{c} H_u^0 t^c - (t \rightarrow b, H_u^0 \rightarrow H_u^+)] + h.c.
\]

3 Renormalization Group Equations (RGEs) and Asymptotic Freedom Problem

3.1 RGES

In order to study phenomenology at the low scale (electroweak scale, typically), we need to derive RGEs to see how the parameters evolve. Our results are obtained from explicit diagrammatic techniques, based on conventional Feynman diagrams. According to the Lagrangian obtained in section 2, using DR, we can obtain RGEs for all the relevant parameters of the theory.

First let’s look at the 1-loop RGE of scalar mass squared \( \tilde{m}^2 = m^2 + |\tilde{\mu}|^2 \). Note that in this model this quantity is the sum over soft SUSY breaking \( \tilde{m}^2 \) and the ‘supersymmetric’ \( |\tilde{\mu}|^2 \) which results from the non-chiral property of the adjoints. For simplicity, we can assume \( \tilde{\mu} \) is real so that it is rightly the physical mass of \( \Psi \). \( M \) is the gaugino mass. \( t = \ln(p/p_0) \), where \( p \) is the renormalization scale, \( p_0 \) is an arbitrary reference scale.

\[
\frac{d\tilde{m}^2}{dt} = \frac{2}{(4\pi)^2} \left\{ -g^2 [12(M^2 + \tilde{\mu}^2) - 9\tilde{m}^2] + |\tilde{y}|^2 (10\tilde{m}^2 - \frac{40}{3} \tilde{\mu}^2) + \frac{10}{3} |a|^2 \right\} \tag{21}
\]
1-loop RGE of b-term is
\[
\frac{db}{dt} = \frac{2}{(4\pi)^2} \left[ \tilde{\mu} \left( 24g^2M + \frac{20}{3} \tilde{y}^* a \right) + b \left( 9g^2 + \frac{10}{3} |\tilde{y}|^2 \right) \right]
\] (22)

The physical squared masses \( x^2 \) of the scalar adjoint are
\[
x^2 = \tilde{m}^2 \pm |b|
\] (23)

The fermion mass does not get enhanced from soft SUSY breaking terms. The 1-loop RGE of the fermion mass \( \tilde{\mu} \) is
\[
\frac{d\tilde{\mu}}{dt} = \frac{2}{(4\pi)^2} \left( \frac{3}{4} g^2 + \frac{5}{3} |\tilde{y}|^2 \right) \tilde{\mu}
\] (24)

Based on these RGEs, we are ready to analyze the evolution of these mass parameters. When running towards lower scale, the fermion mass \( \tilde{\mu} \) is strictly reduced by gauge coupling and Yukawa coupling. The situation is not so definite for \( \tilde{m}^2 \) and \( b \), depending on the evolutions and relative sizes of different soft parameters. Nevertheless we can draw the conclusion that the physical squared mass \( x^2 \) does not go tachyonic in a large region of parameter space, and it is reasonable to assume that at the EW scale, the mass of the scalar adjoint \( A \) is larger than that of the fermionic adjoint \( \psi \). This case is very similar to that of the MSSM: all the scalar fields are heavier than their fermionic partners because of the enhancement from soft SUSY breaking terms. In section 4, we will take this case as a typical example to study the decay process of adjoint fields at colliders.

Now let’s examine the evolution of the \( SU(3)_C \) gauge coupling \( g \). Since we already know the matter content of the model, we can directly apply the general RGEs [19]. We give the results up to 2-loop contribution. Our important discussion about asymptotic freedom problem will be based on these.

\[
\frac{dg}{dt} = \frac{1}{16\pi^2} \beta_g^{(1)} + \frac{1}{(16\pi^2)^2} \beta_g^{(2)}
\] (25)

\[
\beta_g = \beta_{g_{MSSM}} + \beta_{g_{\Phi}}
\]

\[
\beta_{g_{MSSM}}^{(1)} = -3g^3
\] (26)

\[
\beta_{g_{MSSM}}^{(2)} = g^3 \left( \frac{11}{5} g_1^2 + 9g_2^2 + 14g^2 - 4|y_t|^2 \right)
\] (27)

\[
\beta_{g_{\Phi}}^{(1)} = 3ng^3
\] (28)

\[
\beta_{g_{\Phi}}^{(2)} = 54ng^5 - \frac{9}{4} ng^3 |\tilde{y}|^2
\] (29)

Summing over eq.(26)-(29), taking them into eq.(25), we get the full RGE for \( g \) up to 2-loop corrections:
\[
\frac{dg}{dt} = \frac{1}{16\pi^2} g^3 (-3 + 3n) + \frac{1}{(16\pi^2)^2} g^3 \left[ \frac{11}{5} g_1^2 + 9g_2^2 + (14 + 54n)g^2 - 4|y_t|^2 - \frac{9}{4} n |\tilde{y}|^2 \right]
\] (30)
In the 2-loop contribution from the MSSM eq.(27), for simplicity we assume that the only Yukawa that is considerable is \( y_t \sim 1 \). Eq.(28) and (29) are additional contributions from the adjoint fields. \( n \) is the family number of the adjoint fields which is determined by the D-brane configuration.

Before going to the discussion of asymptotic freedom using eq.(30), we write down the 1-loop RGE of gaugino mass\textsuperscript{[19]}:

\[
\frac{dM}{dt} = \frac{1}{8\pi^2} \beta_g M \tag{31}
\]

### 3.2 Adjoint Matter Fields and Asymptotic Freedom

As we know, asymptotic freedom behavior of the strong coupling enables gauge coupling unification, or at least helps preserve the perturbative property of the gauge coupling at high scale. We also know that adding any new matter fields to the MSSM will make it more difficult to preserve the asymptotic freedom property. This is why many suggest getting rid of the adjoints emerging from intersecting D-brane models, although these states are too generic to dismiss immediately. As we mentioned in the introduction: gauge coupling unification is not a prerequisite for intersecting D-brane models and the corresponding requirement in these models should be that the gauge coupling does not diverge below the string scale, which can comfortably be the intermediate scale. This means that for D-brane models, as long as the theory is still perturbative at high scale, asymptotic freedom is unnecessary.

First let’s see if it is still possible to realize asymptotic freedom with these additional adjoint fields. From eq.(30), we see that when \( n > 1 \), the 1-loop \( \beta_g \) is positive. From the dominance of the 1-loop correction over the 2-loop correction, we can draw the conclusion that asymptotic freedom cannot be realized for \( n > 1 \). When \( n = 1 \), it is interesting that the 1-loop \( \beta \) function is 0. So we need to look into the 2-loop correction in this case. Unfortunately, it is easy to see that 2-loop contribution is positive unless the Yukawa coupling \( y \) is unnaturally large. However, the divergence scale in this case can hopefully be above string scale (preferably some intermediate scale), especially when threshold corrections are taken into account. We will demonstrate how threshold correction can lift the divergence scale up to string scale for \( n > 1 \) cases. Similarly, we can expect the divergence scale to be as high as the string scale for the case of \( n = 1 \) since the divergence rate of 2-loop correction is \( \frac{1}{16\pi^2} \) suppressed, compared to that of 1-loop correction.

Now we see it is truly hard to preserve asymptotic freedom for MSSM+chiral adjoint model. We need to analyze the divergence scale of the gauge coupling, which is preferred to be above the string scale in order to preserve the validity of perturbation theory. The solution of the 1-loop RGE for \( g \) is:

\[
g^2(p) = \frac{g^2(p_0)}{1 + \frac{g^2(p_0)}{(4\pi)^2} (-3n + 3) \ln\left(\frac{p^2}{p_0^2}\right)} \tag{32}
\]

where \( p_0 \) is the reference scale.
We can solve for the divergence scale $\Lambda$ from eq.(32):

$$\Lambda = p_0 \exp \left[ \frac{2\pi}{\alpha_3(p_0) 3n - 3} \right]$$

where as usual, $\alpha_3 = \frac{g^2}{4\pi}$.

First we assume the threshold where all the MSSM particles and the $SU(3)_C$ adjoints enter the loop diagrams is just above the scale $m_Z$ (we actually assume a degenerate SSM spectrum for simplicity). Then $p_0 = m_Z = 91$ GeV. According to experimental result, $\alpha_3(m_Z) = 0.119 \pm 0.002$. Applying this to eq.(33), we find:

for $n = 1$, $\Lambda = \infty$
for $n = 2$, $\Lambda \sim 4 \times 10^9$ GeV
for $n = 3$, $\Lambda \sim 6 \times 10^5$ GeV

Therefore, to 1-loop order when $n = 1$, the strong coupling in our new model is always perturbative. When $n = 2$, $g$ diverges a bit lower than the intermediate scale which seems not so good if we assume the string scale to be there. However, if we assume the string scale is at the TeV scale as is the case in large extra dimension scenario, even the $n = 2$ and $n = 3$ cases are satisfactory.

Furthermore, it is too restrictive to assume that all beyond-the-SM particles run in the loops just above $m_Z$. Let’s assume the SM RGE dominates (where $\beta^{(1)} = -7g^3$) until the threshold at 1 TeV where MSSM particles and the adjoints run in. In this case, we get more satisfactory results:

for $n = 1$, $\Lambda = \infty$
for $n = 2$, $\Lambda \sim 1.2 \times 10^{13}$ GeV
for $n = 3$, $\Lambda \sim 1.1 \times 10^8$ GeV

Now for $n = 2$, $g$ diverges well above intermediate scale.

Thus, we find that although it is hard to preserve asymptotic freedom in our model, when $n$ is small it is still possible to keep the theory perturbative up to the string scale (either intermediate or low scale) in the intersecting D-brane scenario and to do so becomes even easier when that scale is low. It is also possible that threshold corrections can further lift the divergence scale. Especially, when $n = 1$, $g$ is always perturbative at any high scale (to 1-loop order). These solidify the motivation of this effective field theory model with chiral adjoint fields. Furthermore, it should be kept in mind that in more general intersecting D-brane models which contain other beyond-the-MSSM particles in addition to the adjoint matter addressed in this work, it may be possible to further increase the divergence scale or even to restore gauge coupling unification\(^2\) (however, see\(^1\))

\(^2\)In the relevant D-brane models, the family number of adjoints can be up to 3. That is why we consider the $n = 1, 2, 3$ cases.
4 Phenomenology of Chiral Adjoint Fields

We are now ready to study the phenomenology of these $SU(3)_C$ chiral adjoint states: their behavior at colliders and the role they play in cosmology. As mentioned in section 3.1, in the following discussion, we will assume the mass of the scalar component $A$ is larger than that of the fermionic component $\psi$. Based on this assumption, whether $\psi$ is the LSP or not (the lightest particle in the MSSM+adjoints model) will result in different stories, which we will discuss respectively.

4.1 Decays

The lifetime or decay process of a particle is an important issue to study in phenomenology since it is relevant to both its cosmological ramifications (whether satisfies the constraint from BBN) and collider physics (whether decays promptly enough to be detected). Now let’s study this issue for $A(A^\dagger)$ and $\psi$ in turn.

The possible decay of $A(A^\dagger)$ comes from the gauge interaction

$$L_{\text{dec}} = i\sqrt{2}gf^{\text{f} \text{c} \text{d}}(-A^d \psi^c + \bar{\psi}^a \bar{\lambda} A^d)$$

(34)

We also assume that $m_A > m_\psi + m_\lambda$ to allow an on-shell two-body decay. We will calculate the decay rate of the process $A^\dagger \rightarrow \Psi + \Lambda$ as an example. Here we have constructed the 4-component Majorana particles $\Psi$ and $\Lambda$ from the original Weyl fermions.

If $m_{A^\dagger} \gg m_\Psi, m_\Lambda$ the integration over phase space simplifies, and the decay rate is [20]:

$$\Gamma_1 = \frac{3}{8\pi}g^2m_{A^\dagger}$$

(35)

We know that for the $SU(3)$ gauge coupling, $\frac{g^2}{4\pi} \approx 0.1$. To make a numerical estimation, we assume that $m_\Psi, m_\Lambda \sim 100$ GeV, $m_{A^\dagger} \sim 1$ TeV. Then taking the numbers in eq.(35):

$$\Gamma_1 \approx 150\text{ GeV} \approx 2 \times 10^{26}\text{ sec}^{-1}$$

(36)

We see that this decay is a prompt process and therefore can be detected at a collider. Meanwhile, because the lifetime is much shorter than 1 sec, $A^\dagger$ ($A$) will not violate the BBN constraints from cosmology.

Now we come to study decay processes of $\Psi$. If $\Psi$ is the LSP or very long-lived, we need to see if it respects the cosmological constraints, which we will discuss in the third subsection. If $\Psi$ is not the LSP, three-body on-shell decays are possible according to the nonrenormalizable term in the Lagrangian:

$$L' = \frac{1}{M^c}T_a[2\psi_a(t^c H^0_t + \tilde{t}^c \tilde{H}^0_t + t^c H^0_u) - (t \rightarrow b, H^0_u \rightarrow H^+_u)] + h.c.$$  

(37)
To give an example of the decay rate, we will focus on the second term, which results in a $4t + \not{E}$ signature (Fig.1) at colliders. With little background from SM, and with sufficient signal cross section, this signature can be very interesting evidence for new physics. We will find that this is quite plausible at the end of section 4.3 after we make an estimation of the pair production cross section of $\Psi$. Here we focus on giving the decay rate of the typical process: $\Psi \rightarrow \chi_1 + \tilde{t}_1 + \tilde{t}_c$, where we transformed $\psi$ and $\tilde{H}_u$ from Weyl basis to Majorana basis and rotated from $SU(2)$ eigenstates to mass eigenstates. If we assume $\chi_1$ is the LSP, then the outgoing $\chi_1$ results in pure missing energy signature. The rotation from gauge eigenstates to mass eigenstates results in the factors $\frac{1}{2} \sin 2\theta_{\tilde{t}}$ and $N_{31}$. Again, to simplify the integration over the phase space of this 3-body decay, we assume that $m_\Psi \gg m_{\chi_1}, m_{\tilde{t}_1}, m_{\tilde{t}_c}$. We get the decay rate:

$$\Gamma' = \frac{1}{4(4\pi)^3} \left( \sin 2\theta_{\tilde{t}} \right)^2 N_{31}^2 \frac{m_\Psi^3}{M_*^2}$$  \hspace{1cm} (38)

Figure 1: $4t + \not{E}$ signal from our model: $g$ is gluon, $\chi_i$ is neutralino which result in the signal of missing energy, $i=1,2,3,4$. For mass eigenstates of stops, $j,k=1,2$

To make a numerical estimate, we assume the mixing factors to be of order 1, and $m_\Psi \sim 1$TeV. For the three typical suppression scales, the results are:

| Suppression Scale $M_*$ | Decay rate/Lifetime of $\Psi$ |
|-------------------------|--------------------------------|
| 1 TeV (low scale)       | $10^{-1}$ GeV$/10^{-23}$ sec  |
| $10^{11}$ GeV (intermediate scale) | $10^{-17}$ GeV$/10^{-7}$ sec  |
| $10^{18}$ GeV (Planck scale) | $10^{-31}$ GeV$/10^{7}$ sec  |

We see that for TeV or intermediate scale suppression, the decay of $\Psi$ is prompt and detectable at colliders, and thus will not violate constraints from BBN. However, for higher suppression scales, some additional mechanism may need to be invoked for $\Psi$ decays so as to
prevent late $\Psi_1$ decays from violating the BBN constraints.

Again, we will discuss more about the $4t + E_1$ signal at the end of the next subsection, which should be detectable at colliders if the suppression scale is not too high.

### 4.2 Pair Production of $\Psi_1$ at Hadron Colliders

Another interesting aspect of phenomenology is production. Since $A$ typically decays promptly once produced and is heavier, we will focus on $\Psi_1$. As we see in the derived renormalizable Lagrangians for the adjoints, the only direct coupling between the adjoints and SM fields is through the connective term of the gauge interaction. For $\Psi_1$, we see this is

$$\mathcal{L}_{\Psi-g} = i\bar{\psi}_a \bar{\sigma}^\mu D_\mu \psi^a$$  \hspace{1cm} (39)

Here a striking equivalence resulted from gauge invariance is that this is exactly the same form with the gluino contribution in the kinetic term of gauge interaction:

$$\mathcal{L}_{\lambda-g-1} = i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a$$  \hspace{1cm} (40)

Since both the gluino and $\psi$ are in the adjoint representation of $SU(3)$, their covariant derivatives $D_\mu$ are in the same form as in eq.(15). Meanwhile, both the gluino and $\psi$ are Majorana, eq.(39) and (40) will still take the same form after transforming to the Majorana basis. Therefore, the diagrams will have exactly the same Feynman rules in this form (they can have quite different mass parameters). Or to say it more accurately, formally, gluino and $\Psi_1$ have exactly the same interactions with gluons. Since there have been much study and existing checks of the phenomenology of gluinos, we can refer to these references for $\Psi_1$ because of their close similarity\cite{21-34}. We will discuss some details below. However, here it is also important to notice that the major difference between them is that the gluino can couple directly to quarks and squarks through a connective term in the gauge interaction:

$$\mathcal{L}_{\lambda-g-2} = -\sqrt{2}g(\bar{q}^t a \bar{q} \lambda^a + \bar{\lambda}^a t^a \bar{q} q)$$  \hspace{1cm} (41)

On the other hand, as the component of a chiral superfield, $\Psi_1$ does not have this direct renormalizable coupling to quarks and squarks. This is an important point we need to keep in mind when considering the $\Psi_1$ collider phenomenology.

The important pair production processes at leading order for gluino\cite{22} and $\Psi_1$ are shown in Fig.2 and Fig.3 respectively. For production mechanism (1), i.e. produced by quark-antiquark pair, the gluino has three channels. However, for $\Psi_1$, only the first channel of mechanism (1) is allowed by the Lagrangian (no direct coupling to $q, \bar{q}$). On the other hand, they all have the same three channels in the production mechanism (2), i.e. produced by gluon pair. For the gluino, the amplitude of the total production cross-section is already calculated for both the Tevatron and the LHC\cite{22}. Therefore, we can apply these known results to $\Psi_1$ to calculate the total production cross-section of $\Psi_1$. Numerically, according to their result, at LHC where $\sqrt{s}=14$TeV, the total cross-section for gluino pair production ranges from $10^3$pb to $10^{-1}$pb.
when \( m_{\tilde{g}} \) ranges from 200GeV to 1TeV. If we reasonably assume that the amplitudes of the two channels involving squarks are negligible (if the squarks are very heavy) or of the same order with the other channels, we can expect a very similar evaluation on production cross-section for our adjoint fermion. In a word, we can expect the pair production cross-section of \( \Psi \) to be large at the LHC. Now we see that the similarity in gauge interaction between gluino and \( \Psi \) facilitates our study on the collider physics of \( \Psi \).

We are also interested in the detection of \( \Psi \) once it is produced. If stable, \( \Psi \) will also form R-hadrons, as what happens to long-lived gluinos. This case has been discussed in refs. [30, 31, 32] for gluinos. If \( \Psi \) is not stable, we need to return to the discussion of its non-renormalizable decay in section 4.1.2. The \( 4t \) signature arises here, as was mentioned earlier. The first three terms in eq.(37) can result in \( 4t + X \) signature at colliders, where \( X \) can be neutralinos (missing energy, if the neutralino is the LSP) or gluinos. Since it may be hard to discern \( X \), we can make the inclusive signature of \( 4t + X \) as a signature (or rather, the many energetic states from the \( t \) decays). Such a signature has very low background from the SM and therefore can be good evidence for beyond the SM physics if it has a reasonable (i.e., many fb) cross-section. The two largest sources of \( 4t \) background from the SM are: \( gg \rightarrow t\bar{t}t\bar{t} \) and QCD background. Their total cross-section is less than 10fb (after all the cuts, about 1fb) [25]-[29]. In order to estimate its cross-section in our model, we need to know the pair production cross-section of \( \Psi \) and the branching ratio of the \( 4t + X \) signature. As we mentioned, utilizing analogous calculations of gluino production[22], the pair production cross-section of \( \Psi \) can be of the order of pb. Taking into account all the terms in eq.(37), we know that all the possible decay products of the \( \Psi \) pair are: \( 4t + X_1, 4b + X_2, 2b + 2t + X_3 \). If we assume that the mass of \( \Psi \) is much larger...
than the masses of its decay products, different decay channels have negligible difference in the integration over phase space and therefore have almost the same branching-ratio. In conclusion, the branching ratio of $4t + X$ is about $\frac{1}{4}$, and therefore its cross-section can be around pb as long as $m_\Psi \lesssim 2$ TeV. In this way, compared with the fb background from the SM, $4t + X$ can be a notable signature at hadron colliders.

4.3 Cosmology of $\Psi$

Based on the discussion of the non-renormalizable decay of $\Psi$, we see that the lifetime of this adjoint fermion can span a wide range, depending on the suppression scale $M_*$. For a TeV scale $\Psi$, it is safe cosmologically if $M_*$ is lower or not much higher than the intermediate scale. Otherwise we need to check if it respects the constraints from BBN. If it is very long-lived or is the LSP, it can be a cold dark matter (CDM) candidate and we need to check if its relic abundance does not overclose the universe. An even stronger constraint, which applies to long lived particles charged under $SU(3)_C$ (including stable gluinos) comes from the fact that any such particles will bind into nuclei to produce anomalously heavy elements. However, current experiments put severe limits on the abundance of heavy elements on the earth today \cite{26}. The limits range from $\eta < 3.5 \times 10^{-30}$ for $M = 100$ GeV to $\eta < 8 \times 10^{-28}$ for $M = 1.2$ TeV, where $M$ is the mass of the anomalously heavy hydrogen, composing the anomalous water molecules. Here, $\eta$ is the ‘anomalous concentration’, namely the number of anomalous water molecules per usual water molecule in the oceans. With such a small relic density, it appears unlikely for $\Psi$ to be a dark matter candidate. On the other hand, this severe bound can be weakened if we assume that the heavy elements sink towards the center of the earth so that they have not been discovered yet in the experiments on the ground\cite{33}. Furthermore, we should notice the fact that in these known experiments that give severe bound on the existence of stable charged particles, the samples are taken from terrestrial water, which may not reflect the true
abundance of heavy elements in the whole universe\cite{27}. If these are true, we can still speculate that $\Psi$ is a CDM candidate if it is stable. Now that we have outlined the constraints on $\Psi$ from cosmology, we would like to see what they imply for our model.

For a $\Psi$ of intermediate lifetime, the constraints from BBN are relevant. Once again, we can take advantage of the analogy to gluino physics. There has been careful study about the gluinos on this aspect. A typical result \cite{33} is that in split supersymmetry, a TeV mass gluino must have a lifetime shorter than 100 seconds in order not to change the abundances of D and $^6$Li. However, the corresponding study of $\Psi$ can be somewhat different from that of the gluino since they have different decay processes, which are relevant for the BBN constraint. Of course, if the relic density of $\Psi$ is already small enough during the BBN era, its influence on BBN is negligible.

For a very long-lived $\Psi$, we demand it to have a very small relic density. As we know, the relic density of a thermal relic, e.g. neutralino LSP, is largely determined by the cross-section of its annihilation. For $\Psi$, or any strongly interacting particle like the gluino, there are two main regimes of annihilation: perturbative era before QCD phase transtion, and non-perturbative era after QCD phase transition. For the perturbative stage, the pair annihilation is obviously the inverse process of pair production, which we discussed in detail in section 4.2. So again, we can take advantage of the similarity between the gluino and $\Psi$ and refer to the results for gluino for an estimation. The upshot is, for a TeV $\Psi$, if we naively only take the perturbative annihilation into consideration, the relic abundance $\Omega_p h^2$ is around $10^{-3}$ \cite{30,33,34}, which is obviously too large if we take seriously the constraints given in ref.\cite{26}. For the non-perturbative stage, we can expect that the difference between the $\Psi$ and the gluinos is even more blurred since the physics is taken over by hadronic dynamics when these microscopic particles have become confined in color-singlet R-hadrons. So it still makes sense to refer to the corresponding analysis for the gluinos. Since the picture of the non-perturbative annihilation during the QCD era is not clear, most of the related study has large theoretical uncertainties and is quite model-dependent. In most of the known models, it is still hard to obtain a small enough relic density to satisfy the constraints from anomalous nuclei in seawater \cite{30,33,34}. Perhaps the most optimistic result is in ref.\cite{30}, where $\Omega_{np} h^2 \sim 10^{-11}$ for a TeV gluino or $\Psi$, which is still larger than the seawater constraint by a factor of $10^{15}$. However, there is still hope in that the non-perturbative physics issue is not clarified yet, and more importantly, the connection between the relic density calculation and the existing limits is unclear as discussed above. On the other hand, the resulting relic density can be small enough to satisfy the seawater constraints by virtue of some other reasonable mechanism such as late time second inflation \cite{30} or a low reheating temperature \cite{34}.

5 Conclusion

The existence of matter fields in higher representations is an interesting issue to consider in our search for new physics beyond the SM. In this paper we extend the MSSM by chiral adjoint
matter which is inspired by intersecting D-brane models. We construct the Lagrangian of this effective field theory model which is independent of the details of the original string model. The renormalization group equations are given for the study of low energy physics. Based on the RGEs, we carefully discuss the asymptotic freedom problem and perturbativity of strong coupling in this model. We find that although asymptotic freedom is hard to be realized here, perturbativity can be preserved up to the string scale. In this way, the existence of chiral adjoint matter in low energy physics does not undermine the motivation of the model, since the bottom-line of calculable intersecting D-brane models is the validity of perturbativity at the string scale. We also study the phenomenology of this model. Under natural assumptions, the scalar adjoint decays promptly and therefore can be detected at colliders and is safe cosmologically. The story of the fermion adjoint can be a bit complex because of the uncertainty in its lifetime. With a short lifetime, it can be detected at colliders through a significant beyond the SM signature: $4t + X$. If it has an intermediate lifetime, we need to check if it respects the limit from BBN, which we argue is plausible. If it is long-lived or even the stable LSP, there are many mechanisms to make it satisfy the constraints from cosmology.

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References

[1] R. Blumenhagen, M. Cvetic, F. Marchesano and G. Shiu, JHEP 0503, 050 (2005) [arXiv:hep-th/0502095].

[2] R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, Ann. Rev. Nucl. Part. Sci. 55, 71 (2005) [arXiv:hep-th/0502005].

[3] E. Dudas and C. Timirgaziu, Nucl. Phys. B 716, 65 (2005) [arXiv:hep-th/0502085].

[4] J. E. Bjorkman and D. R. T. Jones, Nucl. Phys. B 259, 533 (1985).

[5] P. M. Ferreira, I. Jack and D. R. T. Jones, Phys. Lett. B 387, 80 (1996) [arXiv:hep-ph/9605440].

[6] L. E. Ibanez, F. Marchesano and R. Rabadan, JHEP 0111, 002 (2001) [arXiv:hep-th/0105155].

[7] R. Blumenhagen, D. Lust and S. Stieberger, JHEP 0307, 036 (2003) [arXiv:hep-th/0305146].
[8] R. Blumenhagen, B. Kors, D. Lust and T. Ott, Nucl. Phys. B 616, 3 (2001) arXiv:hep-th/0107138.

[9] D. Cremades, L. E. Ibanez and F. Marchesano, JHEP 0207, 022 (2002) arXiv:hep-th/0203160.

[10] C. Kokorelis, JHEP 0209, 029 (2002) arXiv:hep-th/0205147.

[11] C. Kokorelis, JHEP 0208, 036 (2002) arXiv:hep-th/0206108.

[12] D. Berenstein, arXiv:hep-th/0603103.

[13] J. Wess and J. Bagger, “Supersymmetry and supergravity”, (second edition), 1992

[14] M. Drees, R. Godbole and P. Roy, “Theory and phenomenology of sparticles: An account of four-dimensional N=1 supersymmetry in high energy physics”, 2004

[15] G. F. Giudice and A. Masiero, Phys. Lett. B 206, 480 (1988).

[16] J. A. de Azcarraga, A. J. Macfarlane, A. J. Mountain and J. C. Perez Bueno, Nucl. Phys. B 510, 657 (1998) arXiv:physics/9706006.

[17] S. P. Martin, arXiv:hep-ph/9709356.

[18] A. J. Buras, arXiv:hep-ph/9806471.

[19] S. P. Martin and M. T. Vaughn, Phys. Rev. D 50, 2282 (1994) arXiv:hep-ph/9311340.

[20] H. E. Haber and G. L. Kane, Phys. Rept. 117, 75 (1985).

[21] W. Beenakker, R. Hopker, M. Spira and P. M. Zerwas, Z. Phys. C 69, 163 (1995) arXiv:hep-ph/9505416.

[22] W. Beenakker, R. Hopker, M. Spira and P. M. Zerwas, Nucl. Phys. B 492, 51 (1997) arXiv:hep-ph/9610490.

[23] A. Mafi and S. Raby, Phys. Rev. D 62, 035003 (2000) arXiv:hep-ph/9912436.

[24] F. Mahmoudi and A. Deandrea, arXiv:hep-ph/0503286.

[25] M. Spira and J. D. Wells, Nucl. Phys. B 523, 3 (1998) arXiv:hep-ph/9711410.

[26] P. F. Smith, J. R. J. Bennett, G. J. Homer, J. D. Lewin, H. E. Walford and W. A. Smith, Nucl. Phys. B 206, 333 (1982).

[27] A. De Rujula, S. L. Glashow and U. Sarid, Nucl. Phys. B 333, 173 (1990).

[28] K. m. Cheung, arXiv:hep-ph/9507411
[29] V. D. Barger, A. L. Stange and R. J. N. Phillips, Phys. Rev. D 44, 1987 (1991); Phys. Rev. D 45, 1484 (1992).

[30] H. Baer, K. m. Cheung and J. F. Gunion, Phys. Rev. D 59, 075002 (1999) [arXiv:hep-ph/9806361].

[31] S. Raby and K. Tobe, Nucl. Phys. B 539, 3 (1999) [arXiv:hep-ph/9807281].

[32] J. L. Hewett, B. Lillie, M. Masip and T. G. Rizzo, JHEP 0409, 070 (2004) [arXiv:hep-ph/0408248].

[33] A. Arvanitaki, C. Davis, P. W. Graham, A. Pierce and J. G. Wacker, Phys. Rev. D 72, 075011 (2005) [arXiv:hep-ph/0504210].

[34] N. Arkani-Hamed and S. Dimopoulos, JHEP 0506, 073 (2005) [arXiv:hep-th/0405159].