Controlling the parametric roll of a container ship model by changing the forward velocity

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Abstract. Parametric roll on ships is a resonance phenomenon whose onset causes heavy roll oscillations leading to dangerous situations for the ship, the cargo and the crew. It affects both small vessels with marginal stability and large container ships if four conditions are met simultaneously: the wave length and ship length are approximately equal, the frequency of encounter wave is twice the roll natural frequency, the ship’s roll damping is low enough and the wave height exceeds a limit value characteristic to each vessel. A good start to capture the basic features of the parametric roll is to cancel the couplings between roll and the other motions of the ship and to analyse the influence of parameters on the ship stability and the appearance of large roll amplitudes by using one-degree-of-freedom model. By doing this, one realizes that, besides increasing the damping, the roll motion can be stabilized by changing the wave encounter frequency in such a way that the resonance condition is no longer fulfilled. Two parameters mainly affect the encounter frequency: the ship forward velocity and the heading angle. In the paper, we have numerically investigated the effect of increasing or decreasing the forward speed on the roll amplitudes. We followed two strategies. In the first, we allowed the speed to change with a constant acceleration (positive or negative) so that the encounter frequency has left the dangerous region of the resonance. In the second strategy, we changed the forward speed up and down around a value that generates large roll amplitudes, by using different periodic profiles such as sinusoidal, saw tooth or trapezoidal. We noticed that both control techniques generally managed to significantly reduce the roll amplitudes (with a few exceptions). At the same time, we found that very important in how the amplitudes diminish are parameters like the moment of time when the control is activated, the speed rate of change or the period and amplitude of the speed change around the average value (in the second strategy). The data used in the paper have been obtained from experiments with a container ship model in a towing tank followed by expansion to a full scale ship.

1. Introduction

Parametric roll resonance is a nonlinear phenomenon that results in heavy roll motions, with amplitudes of 30 – 40 degrees for ships sailing in following and head seas, which may put the ship in a very dangerous situation that can even capsize it. Fishing vessels, container carriers and passenger vessels are known to be prone to parametric roll in both regular and irregular waves. Several incidents have been reported with significant damage to cargo and ship for millions of euro’s, including APL China and Maersk Carolina, and then carefully analysed [1, 2].

Parametric resonance is related to the periodic changes of those coefficients in the roll equation which describe the restoring arm variation in longitudinal waves (connected to the ship stability). These changes are most accentuated in waves of length close to that of the ship and involve an
increase in stability when a wave trough is amidships or a decrease in stability when a wave crest is in the same position. Other conditions for parametric roll may also be considered such as the wave encounter period is approximately half the ship roll natural period, the ship damping in roll is insufficient to avoid the onset of the resonance condition and the wave height exceeds a ship-dependent threshold value [3, 4].

For the modern containerships, factors as the exaggerated bow flare and the pronounced overhang stern make the vessel more susceptible to parametric roll [5, 6].

Apart from the varying restoring arm, other causes for parametric resonance are the coupling between roll, heave and pitch motions. In the last decades, several mathematical models including these couplings have been proposed by scientists [7 – 10].

Once the parametric roll has been installed, the stringent problem to be solved by the shipmaster is to find solutions to quickly decrease the roll amplitudes. A good idea is to significantly change the wave encounter frequency, which would mean that the resonance condition would no longer be satisfied. This can be done by changing at least one of the following two human dependent parameters: the ship forward speed and the heading angle.

In the paper, we focused on the first strategy, so we conducted a comprehensive numerical investigation on the effect of changing (up or down) the ship forward speed. In our approach we used a simplified mathematical model, in which the only degree of freedom was the roll angle. Even if it can be criticized for simplicity, this model has proven to be able to capture the basic features of the parametric roll resonance [11].

2. The parametric roll equation
The equation used throughout the paper for describing the parametric roll motion is written as

\[ (I_x + \delta I_x) \dddot{\phi} + d_1 \dddot{\phi} + d_2 \dot{\phi} + d_3 \ddot{\phi} + \rho g \nabla (GM_m + GM_d \cos \omega_c t) \phi + k_3 \phi^3 = 0 \]  

(1)

with \( I_x \) and \( \delta I_x \) denoting the ship and added mass inertia in roll, \( d_1 \) and \( d_2 \) the linear and quadratic roll damping coefficients, \( \rho \) the water density, \( g \) the gravitational acceleration, \( \nabla \) the water displacement, \( k_3 \) the cubic coefficient of the roll restoring moment, \( GM_m \) and \( GM_d \) the constant and variable parts of the metacentric height, and \( \omega_c \) the wave encounter frequency.

The last parameter results from

\[ \omega_c = \omega_0 - \frac{\omega_0^2 U \cos \beta}{g} \]  

(2)

where \( \omega_0 \) stands for the wave frequency, \( U \) for the ship forward velocity and \( \beta \) for the ship heading angle. The parameters’ values have been taken from an experimental research with a container ship model in a towing tank followed by an expansion to a full scale ship [12, 13]. They are presented in table 1. The roll natural frequency, \( \omega_\phi \), is provided by

\[ \omega_\phi = \sqrt{\frac{\rho g \nabla GM_m}{I_x + \delta I_x}} \]  

(3)

For the values given in table 1 and \( U = 7.4 \) m/s, one obtains \( \omega_\phi = 0.2975 \text{ rad/s} \) and \( \omega_c = 0.6239 \text{ rad/s} \), such as \( \omega_c/\omega_\phi = 2.0971 \).
Table 1: The ship parameters’ values requested in equation (1).

| Parameter | Value | Unit |
|-----------|-------|------|
| $I_x$     | $1.4014 \cdot 10^{10}$ | kg $\cdot$ m$^2$ |
| $\delta I_x$ | $2.17 \cdot 10^9$ | kg $\cdot$ m$^2$ |
| $d_1$    | $3.5393 \cdot 10^8$ | kg $\cdot$ m$^2$/s |
| $d_2$    | $1.8826 \cdot 10^8$ | kg $\cdot$ m$^2$/rad |
| $k_3$    | $2.974 \cdot 10^9$ | kg $\cdot$ m$^2$/s$^2$ rad$^2$ |
| $\rho$   | 1000 | kg/m$^3$ |
| $\nabla$ | 76,468 | m$^3$ |
| $GM_m$   | 1.91 | m |
| $GM_\phi$ | 0.84 | m |
| $\omega_0$ | 0.464 | rad/s |
| $\beta$ | 180 | degrees |
| $g$      | 9.81 | m/s$^2$ |

3. Numerical simulations

In this section, we performed an extensive analysis of the ship response in the event that the ship forward speed is changed so that the ratio $\omega/c/\omega_\phi$ is removed in the long/short term from the resonant value 2.

![Figure 1](image1.png)

Figure 1. (a) The time evolution of roll angle $\phi$ and of its derivative for the parameters displayed in Table 1; (b) The amplitudes and periods of each oscillation cycle.

The initial conditions were $\phi(0) = 0.1, \dot{\phi}(0) = 0$.

It is well known that in achieving the resonance conditions, a mechanical system greatly amplifies any initial perturbation, regardless of its size [14]. The motion described by equation (1) is not an exception. Thus, figure 1(a) shows the time evolution of roll angle $\phi$ and its derivative for the parameters’ values given in Table 1. It is noticeable that the initial perturbation of only 0.1 degrees is amplified in about 400 seconds to values that raise serious problems for, at least, the transported goods. To get a better picture of the convergence process to the steady state, figure 1(b) presents the amplitude, respectively the period, of each oscillation cycle.
From the figure above, it is apparent that the stationary state is reached in a few hundred seconds. The maximum amplitude is associated either with the stationary state or with a pre-run time interval. In the following we chose as parametric roll amplitude the average value of the last ten periods of oscillation for a study time of 800 seconds. In these conditions, to determine how far the ship’s forward speed (or the heading angle) should be changed so the roll amplitude becomes acceptable, we computed these amplitudes for 200 x 200 pairs \((U, \beta) \in [3, 10] \times [90, 270]\). Our findings are reported in figure 2.

Keeping the course of the ship unchanged \((\beta \text{ constant})\), it is obvious that the ship speed will have to be slowed to about 3 m/s or increased to at least 8.5 m/s. Two variables can be involved in this process: the time when the speed control is activated and the acceleration of motion. We will analyse the effect of their change in the following.

The black point in figure 2 stands for the data included in table 1.

![Figure 2. Steady state roll amplitudes for \((U, \beta) \in [3, 10] \times [90, 270]\).](image)

The speed started to grow after 300 s. The black colour is associated with the time intervals in which the speed is constant \((U = 7.4 \text{ m/s or } U_{\text{max}})\) and the red colour for the period when the speed is changed with constant acceleration.

![Figure 3. The time evolution of roll angle for various increases of ship’s speed: (a) \(a = 0.005 \text{ m/s}^2\); (b) \(a = 0.02 \text{ m/s}^2\).](image)
Case 1: The ship’s speed is increased (positive acceleration)

Let assume that the speed control is switched on at 300 s after the parametric roll was installed. The initial excitation of only 0.1° increased during this time to 9.5°. An insufficient speed increase (for example up to $U = 8 \text{ m/s}$) will create an even more drastic situation for the ship (amplitudes of 30° - 35°). Even if the ship’s speed reaches 9 - 10 m/s, but the acceleration is small, the desired effect does not occur. The roll amplitudes return to values of 1° – 2° only if the speed rate of increase exceeds a threshold value, as illustrated in figures 3 and 5(a).

A delay in activating the speed control only worsens the situation of the ship. Thus, if the speed control is initiated after 400 s, when the roll angle is 26.5°, both acceleration and higher speeds are required to remove the large amplitudes, as shown in figures 4 and 5(b).

Case 2: The ship’s speed is decreased (negative acceleration)

If the deceleration solution is chosen, the only condition that must be fulfilled for attenuating the roll oscillations is reaching a speed of approximately 3 m/s. The acceleration’s magnitude only affects the time interval when the ship is subjected to severe oscillations and the value of the amplitudes in the deceleration stage, as shown in figures 6 and 7. The effect of slowing down the ship is beneficial in all
cases. Thus, even if the speed is reduced to only 4.5 m/s, the roll oscillations almost disappear, before starting to develop again with the speed’s stabilization (according to figure 2).

Figure 6. The time evolution of roll angle for various decreases of ship’s speed: (a) \(a = -0.005 \text{ m/s}^2\); (b) \(a = -0.02 \text{ m/s}^2\). The speed started to reduce after 300 s.

Figure 7. The time evolution of roll angle for various decreases of ship’s speed: (a) \(a = -0.005 \text{ m/s}^2\); (b) \(a = -0.02 \text{ m/s}^2\). The speed started to reduce after 400 s.

The fact that a change of speed initially produces a decrease of the roll angle can be used to imagine another strategy for controlling the parametric roll [15]. It consists in imposing a periodic ship speed regime, having as an average the reference value \(U = 7.4 \text{ m/s}\). Different options for this profile can be thought of, such as triangular, trapezoidal or sinusoidal.

a) Triangular profile

We will consider a triangular profile of period \(T\), consisting of three right segments corresponding to a decrease in speed from \(U\) to \(U - \Delta U\) (for the first quarter of the period), followed by an increase of speed to \(U + \Delta U\) (in a half period) and a new reduction to the reference value \(U\) (in the last quarter of the period). Another possible choice is that of a profile symmetrical to the one previously considered.

Figure 8 demonstrates the efficiency of this idea in combating the parametric roll. The values \(T = 400 \text{ s}\) and \(\Delta U = 0.5 \text{ m/s}\) were used and the speed control activation took place after 400 s (when the parametric roll was already well developed). For both profiles the roll oscillations vanished in less than two periods \(T\) but they are extremely dangerous in the first phase.
Figure 8. Triangular speed profiles and the time evolution of roll angle. The speed profile’s period is $T = 400 \text{ s}$ and $U \in [6.9, 7.9] \text{ m/s}$. Firstly the speed is decreased (a), respectively increased (b).

A better situation is obtained with the decrease of the $T$ period, as shown in figure 9. At 400 - 600 seconds after the speed control was activated, the danger of large amplitudes completely disappeared. This state is preserved for other amplitudes $\Delta U$ of speed profile, as illustrated in figure 10. A variation of only 0.15 - 0.25 m/s of ship forward speed is sufficient to eliminate the parametric roll. It should be noted, however, that the triangular profile requires a step shaped traction force, which is not possible for a real ship. A “softening” of the corners of this profile is desirable.

Figure 9. Triangular speed profiles and the time evolution of roll angle. The speed profile’s period is $T = 100 \text{ s}$ and $U \in [6.9, 7.9] \text{ m/s}$. Firstly the speed is decreased (a), respectively increased (b).

b) Trapezoidal profile

The second speed profile we tested was a trapezoidal one. In the first $T/8$ seconds, the speed decreases from the reference value $U$ at $U - \Delta U$ , then remains constant for a quarter of the period and, after, it grows to $U + \Delta U$ in another $T/4$ seconds. A last quarter of period follows, when the speed does not change, after which the speed profile ends with a decreasing segment towards the reference value $U$. A symmetric profile with respect to horizontal axis is also possible (see figure 11).

This speed profile is only able to attenuate partially the roll oscillations. Whenever the speed remains constant, there is a tendency to return to large amplitudes (according to figure 2). At the same time, if the speed changes, up or down, the oscillations decrease in amplitude.
Figure 10. The roll amplitudes at 800 s after the triangular speed profile were initiated. Firstly the speed is decreased (a), respectively increased (b).

Figure 11. Trapezoidal speed profiles and the time evolution of roll angle. The speed profile’s period is $T = 400 \text{s}$ and $U \in [6.9, 7.9]$ m/s. Firstly the speed is decreased (a), respectively increased (b).

c) Sinusoidal profile

The last speed profile used for the parametric roll’s control was a sinusoidal one, of period $T$ and amplitude $\Delta U$. Like the triangular profile, it has been shown to be effective in attenuating the roll oscillations, regardless of the size of the parameters $T$ and $\Delta U$, as well as the moment the control was switched on. An example is given in figure 12. It can be observed that before the oscillations decrease in amplitude as a consequence of control initiation, they were even more dangerous than in the absence of an action to combat the parametric roll. This has been observed for many combinations ($T$, $\Delta U$).

One would think that a faster action to control the parametric roll by changing the speed would give much better results. It is not necessarily true. In figure 13 is presented, as an evidence, the case where the control was activated after 250 s from the beginning of the parametric roll (the amplitude was only 5°). In the first period of the sinusoidal profile, the oscillations were extremely hardened (up to 40 degrees!), and then they were slowly smothered by a series of oscillations reminiscent of the beating phenomenon.
Figure 12. Sinusoidal speed profiles and the time evolution of roll angle. The speed profile’s period is $T = 200$ s and $U \in [6.9, 7.9]$ m/s. Firstly the speed is decreased (a), respectively increased (b).

Figure 13. Sinusoidal speed profiles and the time evolution of roll angle. The speed profile’s period is $T = 400$ s and $U \in [6.9, 7.9]$ m/s. The control was switched on after 250 s.

The period $T$ and the amplitude $\Delta U$, as well as the number of repetitions of the sinusoidal profile, play an important role for combating the parametric roll, as shown in figures 14 and 15. The control by sinusoidal speed profile is always successful for a $\Delta U$ greater than a threshold value and a number of 5 to 10 repetitions of the profile.

Figure 14. The roll amplitudes at 800 s after the sinusoidal speed profile were initiated. Firstly the speed is decreased (a), respectively increased (b).
Figure 15. The roll amplitudes at 1400 s after the sinusoidal speed profile was initiated. Firstly the speed is decreased (a), respectively increased (b).

4. Conclusions
In the paper, a control strategy of a ship’s parametric roll was analyzed, namely the change of surge speed. This will affect the encounter wave frequency and, implicitly, one of the necessary conditions for parametric roll. The ship forward speed has been modified in two ways:

a) Using a constant acceleration, it was increased or decreased to an ending value that allowed the stabilization of the roll angles to zero. We noticed that the absolute value of the acceleration must be higher than a threshold, depending on the moment of time when the speed control is turned on.

b) Using a triangular, trapezoidal or sinusoidal periodic speed profile. This approach has the advantage of maintaining the average ship speed equal to the reference one, so the ship’s schedule remains unaffected. Except for the trapezoidal profile, the use of the other two profiles produced the desired effect, namely the cancelling of the large roll amplitudes. Two conditions for the success are required: the number of repetitions of the speed profile shall be around 5 to 10 and the speed profile’s amplitude at least 0.2 – 0.4 m/s. Interesting is that in this case the resonance condition is still valid.

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