On the Stabilization of the Size of Extra Dimensions

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Abstract

We derive a general formalism for constructing a four-dimensional effective action for the radion field in 5 dimensions, taking into account the possible dependence of the scale factor on the internal coordinate. First and second variations of the action with respect to the size of the extra dimension reveals two types of algebraic constraints on the combination of brane and bulk Lagrangians that extremize the action, and provide an effective mass for the radion field. We derive a compact formula for the radion mass squared in terms of brane and bulk potentials for the case of a bulk scalar field. We further specialize these constraints for several, five-dimensional realizations of brane world models.
1 Introduction

Theoretical models with internal compactified space-like dimensions play a special role in string theory [1], particle physics [2], and cosmology [3, 4, 6]. To be consistent with phenomenological requirements, the volumes of internal dimensions have to be stabilized, i.e. the corresponding moduli, or radions, have to acquire a regular mass term. Depending on the model at hand, a mass for a radion field may vary in a wild range from $10^{-3}$ eV to the Planck scale. The stabilization of extra dimensions is a phenomenological necessity both in models with a smooth distribution of matter in the extra dimensions and in models with branes, where the metric may acquire a strong dependence on the coordinates of the extra dimensions, $a = a(y, z, ..)$. One of the most interesting recent developments for model-building is the idea that the strong dependence of the scale factor on the coordinates in extra dimensions may have profound consequences for the gauge hierarchy problem in particle physics [4, 5].

The presence of matter in these models, either placed on the brane or smoothly distributed in the bulk, usually has a linear coupling to the radion field, displacing the radion from its vacuum value, and creating the $T^n_n$ pressure-like components of the energy-momentum tensor, where $n$ labels the internal dimensions. The presence of this extra-dimensional pressure enables one to recover the usual 4D cosmological expansion [7] and cures the crisis in “brane cosmology” [6] that consists of a drastically different cosmological evolution at late times. The firm connection between the stabilization mechanism and extra-dimensional pressure was further exemplified in Refs. [8, 9].

The static solution for the radion field equations ($\dot{b} = 0$) implies that the energy density, “normal” 4D pressure of matter and the extra-dimensional pressure $T^n_n$ must be related by an integral condition. The first of a series of conditions was found in Ref. [10], in the context of the Randall-Sundrum model, and presented as a topological constraint necessary for the consistency of the particular solution. A similar integral condition, suitable for a non-warped case, $a(y) = \text{const}$, was found in Ref. [2] and it was shown to be associated with the stabilization of the size of extra dimension. Finally, a whole family of different sum rules was presented in Ref. [11], applicable to the case of matter that is uniformly distributed ($x^\mu$-independent) in “our” dimensions. One of these sum rules was shown to provide a consistency check for solutions with a constant radion field ($\dot{b} = 0$). The same sum rules were generalized in 6 dimensions in [13] and in d dimensions in [14].

This paper extends the analysis given in Ref. [9] and includes the case where the scale factor depend on the extra-dimensional coordinate of a 5D spacetime. This results in a constraint on the components of the higher-dimensional energy momentum tensor, the extremization constraint, that generalizes the simplest constraint obtained in [9]. Using the fact that the variation over $b$ with branes at fixed positions, $y_1$ and $y_2$, is equivalent in fact to the variation over the length of the extra-dimensional circle, we reduce integral constraints to a set of algebraic relations between brane Lagrangians and the bulk Lagrangian, calculated at the position of the branes.

The presence of a stabilization mechanism leads to the radion relaxation to its minimum
at \( b = b_0 \), with \( \dot{b} = 0 \). However, the inverse is not true. The existence of a solution with \( \dot{b} = 0 \) might be a result of a fine-tuning that does not lead to a physically viable “brane model”. Indeed, small perturbations of the solution around the extremal point \( b = b_0 \) may prove to be unstable. Such an instability can manifest itself as a tachyonic mass and/or a ghost-like signature for the kinetic term of the radion. To decide whether a particular brane world scenario is stable against radial perturbations or not, one has to find a close family of solutions around \( b = b_0 \) and expand in \( b - b_0 \). In many existing brane-world scenarios such an analysis is possible, whereas in some models that specify \( a(y) \) and \( T_{mn} \) only at a given point \( b = b_0 \), the stability cannot be analyzed. In cases when it is possible, the analysis of the stability \( (\ddot{b}/b < 0) \) can be done with the use of Einstein’s equations \([15]\) or directly in the action \([16, 17]\). We choose the latter method, and obtain a rather compact algebraic formula for the radion mass squared, again in terms of brane parameters and bulk quantities calculated at the position of the branes. The stabilization constraint can be regarded as the condition on the positivity of the radion mass squared.

After obtaining the generic forms of the \( \dot{b} = 0 \) and \( \ddot{b}/b < 0 \) constraints, we apply them for some particular realizations of brane-world models. We exemplify how the extremization constraint \( (\dot{b} = 0) \) is satisfied in a number of static, five-dimensional models, namely, in the Randall-Sundrum model \([4]\), in the two-brane model with a Casimir force \([18, 19]\), and in the two-brane model with a classical massless bulk scalar \([20]\). In the above cases, we also analyze the stabilization constraint and determine whether, or under which assumptions, the radion has a positive or negative mass squared. The results of this paper can be used to check the consistency of various brane model solutions and present a useful formalism that helps to analyze the stability of a particular extra-dimensional solution against small perturbations of the radius of the internal spacetime.

This paper is organized as follows. Section 2 presents the complete study of the 5D case addressing the generic form of the stabilization and extremization constraints. Section 3 applies the derived constraints to particular brane-world models. Section 4 presents our conclusions.

## 2 The radion potential in 5-dimensions

Here, we assume that the brane self-energies, \( V_i \), do not depend on the four-dimensional coordinates either explicitly or implicitly. Therefore, \( V_i \)’s can be, at most, functions of the extra coordinate \( y \). In this case, we may write down the following ansatz for the five-dimensional spacetime

\[
ds^2 = a(t, y)^2(-dt^2 + \delta_{ij} dx^i dx^j) + b(t)^2 dy^2
\]

where \( a(t, y) \) stands for the conformal factor multiplying a four-dimensional, flat line element and \( b(t) \) denotes the scale factor of the extra dimension.
We first focus on the gravitational part of the theory. For the above ansatz, the 5D scalar curvature takes the form

\[ \hat{R} = R^{(4)}(t, y) - \frac{2}{b} D_\mu D^\mu b + \hat{R}^{(y)} = \frac{6\ddot{a}}{a^3} - \frac{2}{b} D_\mu D^\mu b + \left( -\frac{12a'^2}{a^2b^2} - \frac{8a''}{ab^2} \right). \]  

(2.2)

The last two terms inside brackets, denoted as \( \hat{R}^{(y)} \), are the part of the five-dimensional scalar curvature involving solely derivatives with respect to the extra coordinate \( y \). The covariant derivative of the scale factor \( b(t) \) will turn out to be a total derivative in 4D spacetime, and hence drops out. The scalar curvature \( R^{(4)}(t, y) \) is still \( y \)-dependent and, therefore, the purely time-dependent, four-dimensional scalar curvature \( R^{(4)} \) needs to be extracted. For this reason, we define \( a(t, y = 0) \equiv a_0(t) \) and write the conformal factor as

\[ a(t, y) = a_0(t) \tilde{a}(t, y) \]  

(2.3)

where \( \tilde{a}(t, y = 0) \equiv 1 \), by definition\(^1\). \( a_0(t) \) will play the role of the 4D metric component that we need for the effective theory and so we write

\[ R^{(4)}(t, y) = \frac{1}{\tilde{a}^4} \mathcal{R}^{(4)} - \frac{6}{\tilde{a}^3a_0^4} \partial_\mu [a_0^4 \partial^\mu \tilde{a}], \]  

(2.4)

with \( \mathcal{R}^{(4)} = 6\ddot{a}_0/a_0^3 \).

We now turn to the action of the five-dimensional gravitational theory, which can be written as

\[ S = -\int d^4 x \oint dy \sqrt{-G} \left\{ -\frac{\hat{R}}{2\kappa_5^2} + \mathcal{L} \right\} \]  

(2.5)

where \( \kappa_5^2 \) stands for the five-dimensional Newton’s constant and \( \mathcal{L} \) is a generic Lagrangian that may include bulk cosmological constants, brane-self energies and various bulk and brane matter fields. The integral over \( y \) is performed over a compact dimension. We can recover finite volume, but infinite extra dimension type models by sending one of the branes off to infinity. In case when two infinitely thin branes are located at \( y_1 \) and \( y_2 \), the domain of integration over \( y \) can be decomposed into three separate pieces that represent the integral over the bulk and the integrals across the branes:

\[ \oint = \int_{y_1}^{y_2} + \int_{y_1}^{y_1 + \epsilon} + \int_{y_1 - \epsilon}^{y_2 - \epsilon} + \int_{y_2 + \epsilon}. \]  

(2.6)

The part of the above action consisting of the five-dimensional scalar curvature, \( \hat{R} \), will provide the kinetic part of the four-dimensional effective action describing the dynamics of the graviton and the radion field. By using the expressions (2.2) and (2.4) and integrating by parts, the gravitational part takes the form

\[ S_G = \int d^4 x \oint dy \sqrt{-g_4} \frac{\hat{R}}{2\kappa_5^2} \]

\[ = \int d^4 x \sqrt{-g_4} \left\{ \left( \oint dy \ddot{a}^2 b \right) \mathcal{R}^{(4)} - \frac{3}{\kappa_5^2} \oint dy \partial_\mu (\tilde{a} b) \partial^\mu \tilde{a} + \oint dy \ddot{a}^4 b \right\} \].  

(2.7)

\(^1\)Note that, in the case of static branes, this definition is not necessary as \( a_0(t) \) reduces to a constant.
with $\sqrt{-G} = a_0^4 \tilde{a} b$ and $\sqrt{-g_4} = a_0^4$. In order to obtain standard 4D Einstein gravity we perform a conformal transformation of the four-dimensional metric tensor in order to eliminate the coupling between the radion field and the scalar curvature $R^{(4)}$. We set:

$$\sqrt{-g_4} = \frac{1}{A^2(b)} \sqrt{-g_4},$$

where

$$A(b) = \frac{\kappa_5^2 b}{\kappa_5^2} \int dy \tilde{a}^2(t, y),$$

which leads to the result

$$R^{(4)} = A(b) \left[ \hat{R}^{(4)} + 3D_\mu D^\mu \ln A(b) - \frac{3}{2} \partial_\mu \ln A \partial^\mu \ln A \right].$$

Then, the entire action can be written as,

$$S = -\int d^4x \sqrt{-g_4} \left\{ -\frac{\hat{R}^{(4)}}{2\kappa_5^2} + B(b) \partial_\mu b \partial^\mu b + \bar{V}_{eff}(b) \right\},$$

with

$$B(b) = -\frac{3}{\kappa_5^2} \frac{1}{A(b)} \left( \int dy \partial_\mu (\tilde{a} b) \partial^\mu \tilde{a} \right) - \frac{3}{2\kappa_5^2} \left( \frac{\partial \ln A}{\partial b} \right)^2,$$

and

$$\bar{V}_{eff}(b) = \frac{1}{A^2(b)} \int dy \tilde{a}^2 b \left\{ -\frac{\hat{R}^{(y)}}{2\kappa_5^2} + L \right\}.$$

Note that, having restored the four-dimensional gravitational term, all remaining terms in the action will constitute the building blocks of the effective action for the radion field. Therefore, terms involving $x^\mu$-derivatives of the auxiliary functions $\tilde{a}$ and $A(b)$ are bound to contribute to the kinetic term of the only remaining degree of freedom in the theory, that is, of the radion field, while any other terms will contribute to its effective potential.

At this point, we are able to obtain the constraints on the combination of the scale factors and components of the stress-energy that were previously derived in [9] and [11]. These constraints were derived under the assumption that $\delta \tilde{a}/\delta b = 0$, that is, that the perturbation with respect to the value of the radion field does not affect the value of the scale factor. In this case, $A(b)$ is simply proportional to $b$ and the variation of the action (2.3) takes the following simple form:

$$\delta S = -\int d^4x dy \left[ \frac{\delta(\sqrt{-G} \hat{R})}{\delta G_{MN}} \frac{\delta G_{MN}}{2\kappa_5^2} - \frac{\sqrt{-G}}{2} T^{MN} \delta G_{MN} \right]$$

$$= -\int d^4x dy \left[ \frac{\delta(\sqrt{-g_4 \tilde{a}^4})}{2\kappa_5^2 b A^2} \left[ 8a'' + \frac{12a'' a}{a^2} \right] - \frac{1}{2} T_5^5 a^4 \delta b^2 + \frac{1}{2} T_5^5 a^4 b \delta (a^2) \right]$$

$$= -\int d^4x dy \sqrt{-G} \left[ -\frac{12a''}{\kappa_5^2 b^2 a} + T_5^\mu - 2T_5^5 \right] \delta b^2.$$
Note that the variation in the quantity $b^2 \hat{R}$ vanishes with these assumptions. In the above expression we performed an integration by parts and used the standard definition for the stress energy tensor. It is easy to see that the square brackets in the second line of Eq. (2.15) simply contains a combination of the $ii$ and 55 components of Einstein’s equations,

$$R^M_N - \frac{1}{2} G^M_N R^K_R - \kappa_5^2 T^M_N = 0.$$ 

Obviously, Eq. (2.15) must vanish identically when $a(y)$ is the solution. Integrating by parts the term with $a''$ we obtain the following constraint:

$$\oint d^4x dy \sqrt{-G} \left[ \frac{36a'^2}{\kappa_5^2 a^2} + T^\mu_\mu - 2T^5_5 \right] = 0.$$ 

This constraint generalizes a formula from [9] and includes the dependence of $a$ on $y$ (i.e. warping). The constraint derived in [9] is valid only in the case of small $T^\mu_\mu$. Indeed, a non-trivial dependence on $y$-coordinate usually arises due to the presence of branes. In this case, $a'/a$ is proportional to the brane energy density $\rho$. In this case, the first term in (2.15) is just an $O(\rho^2)$ correction and can be dropped, leading exactly to the expression found in [9]. For 5-dimensional spacetimes with significant warping, the constraint (2.15) is valid instead. A similar, but not identical, constraint was derived in Ref. [11] for solutions with large warping by rearranging the components of Einstein’s equations. The same combination $T^\mu_\mu - 2T^5_5$ appears in both constraints, however Eq. (2.15) has an extra term, proportional to $a'^2/a^2$, and a different coefficient in front coming from $\sqrt{-G}$. The apparent differences are due to the fact that the two constraints were derived with different methods, however, they are both valid under the same assumptions and, are therefore, equivalent.

In the above derivation we assumed that $\tilde{a}$ and $b$ are totally independent, and this is why the form of the constraint comes out to be identical with the combination suggested by Einstein’s equations. However, in order to study the stability of the physical size of the extra dimension, one can no longer assume that $\tilde{a}$ and $b$ are independent and that $A(b)$ is simply linear in $b$. Instead, we would have to consider $\tilde{a}$ as a function of $by$, $\tilde{a} = \tilde{a}(by)$ and the variation of action over $b$ might be a more complicated procedure. It is important to note that there is always a residual symmetry under a simultaneous rescaling of $y$ and $b$. All physical parameters must remain invariant under $y \rightarrow Cy$ and $b \rightarrow b/C$, where $C$ is an arbitrary constant. Below, we will consider a two-brane model with branes positioned at $y_1$ and $y_2$. Then, the extremal size of the extra dimension is given by certain fixed values of $y_1 b_0$ and $y_2 b_0$, which are invariant under the above symmetry. The variation over $b$ around $b_0$ with fixed $y_1$ and $y_2$ is equivalent to a variation over the length of the fifth dimension. Moreover, using this rescaling symmetry, we can always choose $b_0 = 1$, i.e. absorb $b_0$ into the definition of $y_1$ and $y_2$.

A canonically normalized radion field $r(t)$ is obtained by demanding that the kinetic term $2B(b) \partial_\mu b \partial^\mu b$ can be written as $\partial_\mu r \partial^\mu r$. For small deviations of the scale factor $b(t)$ from a stable minimum, $b_0$, of its potential, the normalized radion field may be written as:

$$r(t) = \sqrt{2B(b_0)} [b(t) - b_0],$$ 

if the radion is not a ghost-like degree of freedom, i.e. $B > 0$. Note that, in the pursuit of deriving extremization and stabilization constraints, we can
continue working with the metric function $b(t)$ instead of the radion field $r(t)$ since:

$$\frac{\partial V_{\text{eff}}}{\partial r} \bigg|_{r_0} = \left( \frac{\partial b}{\partial r} \right) \frac{\partial V_{\text{eff}}}{\partial b} \bigg|_{b_0}, \quad \frac{\partial^2 V_{\text{eff}}}{\partial r^2} \bigg|_{r_0} = \left( \frac{\partial b}{\partial r} \right) \frac{1}{2} \frac{\partial^2 V_{\text{eff}}}{\partial b^2} \bigg|_{b_0}. \quad (2.16)$$

Therefore, for a well defined relation between $r(t)$ and $b(t)$, the transition from one field to the other makes no difference in the vanishing of the first derivative of the effective potential. In the same way, stability of the extra dimension in terms of $b(t)$, that is $\partial^2_b V_{\text{eff}} > 0$, leads to stability in terms of $r(t)$, $\partial^2_r V_{\text{eff}} > 0$, and vice versa. We note that in particular models, the value of $B(b_0)$ may also be very important because it leads to a rescaling of the mass term, and a very large value of $B(b_0)$ may lead to a phenomenologically unacceptably low value of the radion mass.

Before we go into explicit calculation of the variation of the action over $b$ with $b$-dependent $\tilde{a}$, we would like to specify some components of the matter Lagrangian that we consider here. We allow $L$ to include the bulk Lagrangian and brane self-energies that may depend on the value of bulk fields:

$$\int dy \, \tilde{a}^4 b \, L = \int_{y_1}^{y_2} dy \, \tilde{a}^4 b \, L_B + \int_{y_2}^{y_1} dy \, \tilde{a}^4 b \, L_B + \sum_{i=1,2} \tilde{a}^4(b_i) V_i + U(by_1, by_2). \quad (2.17)$$

In this expression, the bulk Lagrangian may include a variety of fields, however, for the sake of simplicity we will restrict to the case of a single scalar field $\phi$

$$L_B = \frac{1}{2b^2} \left( \frac{d\phi}{dy} \right)^2 + V_B(\phi). \quad (2.18)$$

Note that the bulk cosmological constant $\Lambda_B$ has been absorbed into the definition of $V_B$. We also allow the brane self-energies to depend on the value of the same field $\phi$. A constant part of $V_i$ at the extremal value $\phi = \phi(b_0y_i)$ is a usual brane self-energy, $V_i = V_i(b_0y_i)$. Like $\tilde{a}$, $\phi$ is always a function of $by$, $L_B = L_B(by)$ and $V_i = V_i(by_i)$.

Finally, $U(by_1, by_2)$ in (2.17) represents an effective interaction that may arise due to quantum effects in the final volume and which often cannot be formalized in the language of the Lagrange density in $y$-space. Sometimes this piece of the potential by itself may lead to the stabilization of the extra dimension. For the remainder of the calculations we would like to keep a generic form for this potential, specifying its form only in particular applications.

We now turn to the explicit calculation of the variation of action with respect to $b$. In order to do that, we perform a change of variables, $\xi = by$. This change brings about a significant simplification, as $b$ now only enters in the calculation through the limits of integration, i.e. at specific points $y_1$ and $y_2$

$$A^2(by_1, by_2) V_{\text{eff}} = 2 \int_{by_1}^{by_2} \tilde{a}^4(\xi) \left[ -\frac{1}{2\kappa^2_5} \frac{12 \tilde{a}^2(\xi)}{\tilde{a}^2(\xi)} + \mathcal{L}_B(\xi) \right] d\xi + \sum_{i=1,2} \tilde{a}^4(by_i) V_i + U(by_1, by_2). \quad (2.19)$$
where we have now assumed equally spaced branes on the circle. In the above expression, and henceforth, primes denote differentiation with respect to $\xi$. Note, that we used the integration by part to remove $\tilde{a}''$ from the $R^y$-part of the effective potential. This allows us to remove the so-called Gibbons-Hawking boundary terms, i.e. singular terms in $\hat{R}^y$ when $a''$ is taken at the positions of the branes. When $b$ is taken at its extremum, $\bar{V}_{eff}$ can be interpreted as the effective four-dimensional cosmological constant.

The first derivative over $b$ is computed trivially, and the result takes the following form:

$$A^2(by_1, by_2) \frac{d\bar{V}_{eff}}{db} = \left(2.20\right)$$

$$= - \frac{4\kappa^2}{\kappa_5^2} A y_i \bar{a}^2(by_i) \bar{V}_{eff} \left[ \sum_{i=1}^{i=2} y_i \tilde{a}_i^4 \left[ -\frac{12\tilde{a}_i'^2}{\kappa_5^2 \tilde{a}_i^2} + 2\mathcal{L}_B(by_i) \right] \right] + \sum_{i=1,2} y_i \frac{\partial}{\partial(by_i)} \left[ V_i \tilde{a}_i^4 + U \right],$$

where $a_i \equiv a(by_i)$. The terms with $V_i$'s can be further expanded as

$$\frac{\partial}{\partial(by_i)} V_i \tilde{a}_i^4 = \tilde{a}_i^4 \left[ \frac{4}{a_i} V_i + \frac{dV_i}{d\phi} \phi_i' \right]. \quad \left(2.21\right)$$

In this expression, the signs of $\tilde{a}_i'$ and $\phi_i'$ are defined according to the following rule:

$$\tilde{a}_1' = \frac{d\tilde{a}}{d\xi} \bigg|_{(by_1+0)}, \quad \left(2.22\right)$$

$$\tilde{a}_2' = \frac{d\tilde{a}}{d\xi} \bigg|_{(by_2-0)}, \quad \left(2.23\right)$$

with similar definitions for $\phi_i'$. In other words, the derivatives are taken on the right side of brane 1, and on the left side of brane 2 since we assumed $y_1 < y_2$.

To derive the extremization constraint, we put $b = b_0 = 1$, and use equations of motion for the scale factor and $\phi$, as well as junction conditions on the branes, both for the scale factor and the scalar field:

$$\frac{\tilde{a}'}{\tilde{a}}(y_1) = -\frac{\kappa_5^2}{6} V_1, \quad \frac{\tilde{a}'}{\tilde{a}}(y_2) = \frac{\kappa_5^2}{6} V_2, \quad \left(2.24\right)$$

$$\phi'(y_1) = \frac{1}{2} \frac{dV_1}{d\phi}, \quad \phi'(y_2) = -\frac{1}{2} \frac{dV_2}{d\phi}.$$

The final result takes the following form:

$$2\tilde{a}_i^4 \left[ \mathcal{L}_B(y_i) + \frac{\kappa_5^2}{6} V_i^2 - \frac{1}{4} \left( \frac{dV_i}{d\phi} \right)^2 \right] \bigg|^{i=2}_{i=1} + \sum_{i=1,2} y_i \frac{\partial}{\partial(by_i)} U(by_1, by_2) \bigg|_{b=b_0=1} = 0. \quad \left(2.25\right)$$

Here we used the smallness of the four-dimensional effective cosmological constant and put the first term in (2.20) to zero. This algebraic constraint (as opposed to an integral
condition (2.13) can be used to check the consistency of particular solutions. We note that in the particular cases that we study later, the separate cancellations occur for $y_1$ and $y_2$ proportional terms.

It is easy to see that the extremization constraint is not uniquely defined. Indeed, one can add to (2.23) linear combinations of $T_5^i(y_i) - \kappa_5^2 V_i^2/6$ and $\mathcal{L}_B(y_i) - V_B(y_i) - (dV_i/d\phi)^2/8$, since these combinations are equal to zero due to the 55-component of Einstein’s equations and the boundary conditions for the scale factor and the scalar field:

$$
\frac{1}{\kappa_5^2} \tilde{a}_i^2 - \frac{\kappa_5^2}{6} V_i^2 = T_5^i(y_i),
$$

$$
\mathcal{L}_B(y_i) = \frac{1}{8} \left( \frac{dV_i}{d\phi} \right)^2 + V_B(y_i).
$$

(2.26)

Using the above relations, we would like to give another legitimate form of (2.25):

$$
2\tilde{a}_i^4 y_i \left[ V_B(y_i) + \frac{\kappa_5^2}{6} V_i^2 - \frac{1}{8} \left( \frac{dV_i}{d\phi} \right)^2 \right] \bigg|_{i=2}^{i=1} \sum_{i=1,2} y_i \frac{\partial}{\partial (by_i)} U(b y_1, b y_2) \bigg|_{b=b_0=1} = 0.
$$

(2.27)

In order to obtain a radion mass term and impose a stabilization constraint, we differentiate $d\tilde{V}_{eff}(b)/db$ over $b$:

$$
A^2 \frac{d^2\tilde{V}_{eff}}{db^2} =
$$

$$
y_i^2 \left[ -\frac{12}{\kappa_5^2} (\tilde{a}_i^2 \tilde{a}_i')' + 2(\mathcal{L}_B(by_i) a_i^4)' \right] \bigg|_{i=2}^{i=1} \sum_{i=1,2} y_i^2 (\tilde{a}_i^4 V_i)'' + \sum_{i=1,2} y_i y_j \frac{\partial^2}{\partial (by_i) \partial (by_j)} U.
$$

(2.28)

Not shown here are the terms proportional to the first and second derivatives of $A(b)$. These terms exist in principle, but vanish when $b = b_0$ because of $\Lambda_{eff} = 0$ and because of the extremization constraint (2.23), and thus have no relevance for the calculation of the radion mass term. It is easy to see that the above expression contains $\tilde{a}_i''$. This may look worrisome as $\tilde{a}_i''$ has a singularity right at the position of the branes. However, in view of the definitions (2.22) the double derivatives in (2.28) should be understood as the limit of the continuous part of $\tilde{a}_i''(y)$ at $y \to y_i$.

As in the case of the extremization constraint, the expression (2.28) can be significantly simplified at $b = b_0 = 1$, because of the use of the boundary conditions and the equations of motion for the scale factor and the scalar field. The extensive use of these relations allows to reduce (2.28) to the following rather compact expression:

$$
A^2 \frac{d^2\tilde{V}_{eff}}{db^2} \bigg|_{b=b_0=1} = \frac{4\kappa_5^2}{3} \sum_{i=1,2} y_i^2 \tilde{a}_i^4 V_i \left[ T_5^i(y_i) + \mathcal{L}_B(y_i) - \frac{1}{2} \left( \frac{dV_i}{d\phi} \right)^2 \right]
$$

$$
+ \sum_{i=1,2} y_i^2 \tilde{a}_i^4 \left[ \frac{1}{4} \frac{d^2V_i}{d\phi^2} \left( \frac{dV_i}{d\phi} \right)^2 - \frac{dV_i}{d\phi} \frac{dV_B}{d\phi}(y_i) \right] + \sum_{i=1,2} y_i y_j \frac{\partial^2}{\partial (by_i) \partial (by_j)} U \bigg|_{b=1}.
$$

(2.29)
The stabilization constraint is the requirement on the positivity of \((2.29)\). This result can be further modified to an equivalent form, using relations \((2.26)\) and the extremization constraints \((2.25)\) and \((2.27)\). One of these forms contains only bulk and brane potentials and their derivatives over \(\phi\):

\[
A^2 d^2 V_{\text{eff}} \bigg|_{b=b_0=1} = \frac{4\kappa_5^2}{3} \sum_{i=1,2} y_i^2 \tilde{a}_i^4 V_i \left[ V_B(y_i) + \frac{\kappa_5^2}{6} V_i^2 - \frac{3}{8} \left( \frac{dV_i}{d\phi} \right)^2 \right] + \sum_{i=1,2} y_i^2 \tilde{a}_i^4 \left[ \frac{1}{4} \frac{d^2 V_i}{d\phi^2} \left( \frac{dV_i}{d\phi} \right)^2 - \frac{dV_i}{d\phi} \frac{dV_B}{d\phi}(y_i) \right] + \sum_{i=1,2} y_i y_j \frac{\partial^2}{\partial (b y_i) \partial (b y_j)} U \bigg|_{b=1} .
\]

The generalization of \((2.25)\), \((2.27)\), \((2.29)\) and \((2.30)\) to the case of multiple scalar fields is straightforward.

### 3 Applications to brane-world models

1. **No branes, trivial scalar field profiles.**

   In this case we take \(V_i \equiv 0\), \(\phi = \text{const}\), \(V_B \equiv \Lambda_B\) and \(\tilde{a}_i = 1\). In the absence of branes, the individual positions \(y_i\) lose their meaning and translational invariance in \(y\)-direction requires that all quantities depend only on the difference \(y_2 - y_1\), which is simply related to the size of the compact dimension. Therefore, \(\tilde{V}_{\text{eff}}(by_1, by_2) = \tilde{V}_{\text{eff}}(b(y_2 - y_1))\). Furthermore, since \(V_i = 0\), we also have \(T_5^5 = 0\) (in the absence of branes this is true for all \(y\)). Looking at the extremization constraints, we immediately discover that

\[
2\Lambda_B (y_2 - y_1) + \frac{dU}{db} \bigg|_{b=1} = 0 .
\]

The requirement \(\Lambda_{\text{eff}} = 0\) is equivalent to the condition \(2\Lambda_B (y_2 - y_1) + U = 0\) that ensures the vanishing of the other components of \(T_M^N\). The stabilization constraint reduces to

\[
A^2 d^2 \tilde{V}_{\text{eff}} \bigg|_{b=b_0=1} = \frac{d^2 U}{db^2} \bigg|_{b=1} > 0 .
\]

Particular examples of such a stabilization due to the Casimir force generated by multiple massive and massless scalar fields while classical bulk profiles of these fields are trivial were considered recently in Ref. [21].

2. **Randall-Sundrum model: empty bulk, empty branes, \(U = 0\).**

   In this case, the extremization constraint takes the form

\[
2\tilde{a}_i^4 y_i \left[ \Lambda_B + \frac{\kappa_5^2}{6} \Lambda_i^2 \right]_{i=1}^{i=2} = 0 .
\]
and leads to the usual relation between the bulk cosmological constant and the brane tension in the RS model: \(-\Lambda_B = \kappa_5^2 \Lambda_i^2 / 6\). The radion mass term, Eq. (2.29) is also zero because \(T_5^0 + \mathcal{L}_B = T_5^0 + \Lambda_B = 0\). It can actually be shown that every derivative of the radion effective potential is trivially zero for this particular solution. Going back to the expression of the potential, Eq. (2.13), substituting the relevant quantities inside the integral and performing the integration over the extra coordinate, we are led to the result

\[
A^2 \tilde{V}_{\text{eff}} = -\sqrt{\frac{6}{\kappa_5^2 |\Lambda_B|}} \tilde{a}^i \left[ \Lambda_B + \frac{\kappa_5^2}{6} \Lambda_i^2 \right]_{i=2}^{i=1}
\] (3.4)

We therefore conclude that every derivative of the effective potential with respect to \(b\), once evaluated for the specific solution, will be identically zero. Therefore, this particular solution is an absolute saddle point (a flat direction) of the radion effective potential.

3. Empty branes, trivial scalar field profile, \(U \neq 0\).

Next, we turn to a particular two-brane-world solution in which the Casimir force generated by a conformally coupled scalar field was taken into account [18, 19]. For the case of an attractive Casimir force, the \(y\)-profile of the scale factor takes the following form:

\[
\tilde{a}(t, y) = \cosh^{2/5}(\omega by), \quad \omega^2 = \frac{25}{24} \kappa_5^2 |\Lambda_B|.
\] (3.5)

For a repulsive Casimir force, the above cosh-like solution is substituted by a sinh-like one and the analysis follows along the same lines. The solution assumed a vanishing 4D cosmological constant, \(\Lambda_{\text{eff}} = 0\), and no bulk scalar fields contributed classically to the five-dimensional action. For both cases, the presence of the Casimir force is accounted for by the \(U\) potential in the action:

\[
U(by_1, by_2) = -\frac{2\alpha}{L^5},
\] (3.6)

where \(\alpha\) is a dimensionless constant and \(L\) the inter-brane distance in 5D conformally-flat coordinates, defined as

\[
L = \frac{1}{\omega} I(\omega b) = \frac{1}{\omega} \int_{\omega y_1}^{\omega y_2} \frac{d\xi}{\cosh^{2/5}(\xi)}.
\] (3.7)

The extremization constraint (2.27) takes the following simple form:

\[
2\tilde{a}^i y_i \left[ \Lambda_B + \frac{\kappa_5^2}{6} \Lambda_i^2 + \frac{4\alpha \omega^5}{\tilde{a}^i I^5(\omega)} \right]_{i=2}^{i=1} = 0.
\] (3.8)

Indeed, after applying the following relations from [19],

\[
\Lambda_i^2 = \frac{6|\Lambda_B|}{\kappa_5^2} \tanh^2(\omega y_i), \quad |\Lambda_B| = \frac{4\alpha \omega^5}{I^5(\omega)},
\] (3.9)
Eq. (3.8) is satisfied, separately for $y_1$- and $y_2$-proportional terms.

Using the above relations, we can calculate the radion mass term from Eq. (2.30):

$$A^2 \frac{d^2 V_{eff}}{db^2} \bigg|_{b=1} = -10|\Lambda_B| \omega \left\{ \frac{1}{I(\omega)} \left[ \frac{y_2}{a_2} - \frac{y_1}{a_1} \right]^2 + \frac{\kappa_5^2}{6} \left[ \frac{y_2^2 \Lambda_2}{a_2} + \frac{y_1^2 \Lambda_1}{a_1} \right] \right\} < 0 ,$$

(3.10)

which coincides with the result of [19] upon a trivial overall rescaling of $\tilde{a}$. The solution with the attractive Casimir force turns out to have a tachyonic radion, and therefore is unstable.

Before concluding this subsection, we would like to briefly discuss another solution with non-zero $U$ and with two positive tension branes (with a negative bulk cosmological constant) which was derived in [20]. The conformal factor, for this solution, had the form

$$\tilde{a}(t, y) = \cosh^{1/2}(\omega by) , \quad \omega^2 = \frac{2}{3} \frac{\kappa_5^2}{|\Lambda_B|} .$$

(3.11)

The individual brane tensions (located at $y = y_1$ and $y = y_2$) were given by

$$\Lambda_i = \frac{3\omega}{\kappa_5^2} \tanh(\omega by_i) ,$$

(3.12)

In this case, the extremization constraint (2.27) becomes,

$$2\tilde{a}_i^4 y_i (\Lambda_B + \frac{\kappa_5^2}{6} \Lambda_i) \bigg|_{i=1,2} + \sum_{i=1,2} y_i \frac{\partial}{\partial (by_i)} U(by_1, by_2) \bigg|_{b=b_0=1} = 0 ,$$

(3.13)

and easily reduces to

$$2\Lambda_B(y_2 - y_1) + \sum_{i=1,2} y_i \frac{\partial}{\partial (by_i)} U(by_1, by_2) \bigg|_{b=b_0=1} = 0 .$$

(3.14)

As one can plainly see, the existence of this solution requires $dU/db \neq 0$. In [20], this term was linked with $T_5^3$.

Finally, we apply the stabilization constraint (2.30), which becomes

$$-2\omega^2 (y_1^2 \Lambda_1 + y_2^2 \Lambda_2) + \sum_{i=1,2} y_i y_j \frac{\partial^2}{\partial (by_i) \partial (by_j)} U \bigg|_{b=1} .$$

(3.15)

Thus provided that $d^2U/db^2$ is positive and sufficiently large, this solution is stable.

4. Non-trivial $V_i(\phi)$, trivial bulk potential $V_B = \Lambda_B$, $U = 0$.

Exact solutions of this type, a massless scalar field in the bulk with non-trivial interactions on the branes $V_i(\phi)$, were found in Refs. [22, 20]. Originally, it was hoped that such
a solution had some relevance for the cosmological constant problem, until more careful considerations [23] exemplified the necessity of fine-tuning in this model. For $y_1 < y_2 < 0$, the solution was found [20] to be

$$\tilde{a}^4(by) = \sinh[\omega b (-y)], \quad \frac{1}{b} \frac{d\phi}{dy} = \pm \frac{\sqrt{2|\Lambda_{B}|}}{\sinh(\omega b |y|)},$$

(3.16)

where $\omega^2 = 8\kappa_5^2|\Lambda_{B}|/3$, and the sign ambiguity is resolved only when the scalar potential on either of the two branes is specified.

The extremization constraint (2.27) takes the form

$$2\tilde{a}_i^4 y_i \left[ \Lambda_B + \frac{\kappa_5^2}{6} V_i^2 - \frac{1}{8} \left( \frac{dV_i}{d\phi} \right)^2 \right]_{i=1}^{i=2} = 0,$$

(3.17)

and is trivially satisfied (separately for $y_1$ and $y_2$) upon the following substitutions:

$$V_i^2 = \frac{6|\Lambda_B|}{\kappa_5^2} \coth^2(\omega |y_i|), \quad \frac{1}{4} \left( \frac{dV_i}{d\phi} \right)^2 = (\phi_i')^2 = \frac{2|\Lambda_B|}{\sinh^2(\omega |y_i|)}.$$

(3.18)

These relations allow one to express $dV_i/d\phi$ in terms of $V_i$ and the bulk cosmological constant, which obviously requires fine-tuning:

$$\left( \frac{dV_i}{d\phi} \right)^2 = \frac{8}{\Lambda_B + \frac{\kappa_5^2}{6} V_i^2}. \quad (3.19)$$

Armed with these relations, we easily obtain the radion mass term from (2.30), if we make one further assumption for $d^2V_i/d\phi^2$. In the case of linear dependence, $V_i = \alpha_i \phi_i$, the second line in (2.30) is zero and

$$A^2 \frac{d^2 V_{eff}}{d\phi^2} \bigg|_{b=1} = \frac{8\kappa_5^2}{3} \sum_{i=1,2} y_i^2 \tilde{a}_i^4 V_i \left[ |\Lambda_B| - \frac{\kappa_5^2}{6} V_i^2 \right].$$

(3.20)

By using the boundary conditions (B.18) twice, the radion mass term may be written as

$$A^2 \frac{d^2 V_{eff}}{d\phi^2} \bigg|_{b=1} = -4\omega |\Lambda_B| \left[ y_1^2 \frac{\coth(\omega |y_1|)}{\sinh(\omega |y_1|)} - y_2^2 \frac{\coth(\omega |y_2|)}{\sinh(\omega |y_2|)} \right].$$

(3.21)

The above expression is not sign definite. A simple numerical analysis reveals that the sign of the radion mass squared depends on the inter-brane distance (large inter-brane distances add a positive contribution to the radion mass term), and the distance of the brane system from the singularity located at $y_0 = 0$ (small separation between the branes and the singularity adds a negative contribution). When $|y_1 - y_0| = |y_1| < |y_1|_{crit}$, the stability of the system strongly depends on the location of the second brane: if it is located closer to the first brane rather than the singularity, the destabilizing force is minimal and
the solution is stable; if, however, the second brane is moved closer to the singularity, the radion field is inevitably tachyonic. On the other hand, for $|y_1| > |y_1|_{\text{crit}}$, the stabilizing force coming from the large inter-brane distance is dominant, which guarantees the overall stability of the system even when the second brane is placed close to the singularity.

For non-linear interaction terms $V_i$, there is an extra term in the expression for the radion mass term proportional to $d^2V_i/d\phi^2$. If the sign of this term is chosen to be positive then, for large inter-brane distances, this term tends to stabilize the solution even more, while, for small brane separations, it may reduce, or even overcome, the destabilizing force due to the singularity.

As a final comment, let us stress at this point that the two-brane solution with a bulk scalar field, studied above, turned out to be stable, for much of the parameter space, in the absence of a bulk potential $V_B$ for the scalar field. This result differs from that in [16], where the mass of the radion is found to vanish in the limit of the vanishing bulk mass for the $\phi$ field. Our study shows that the non-trivial interaction terms of the scalar field with the branes are sufficient for both fixing the inter-brane distance [20] and stabilizing the brane-system under small time-dependent perturbations.

4 Conclusions

In this paper, we have focused on the study of the stabilization of the size of extra dimensions. A variety of solutions in extra-dimensional brane-world models have appeared in the literature that are characterized by a constant radion field ($\dot{b} = 0$). These solutions correspond to extrema of the corresponding radion effective potential and constraints involving the components of the energy-momentum tensor, for the vanishing of the first derivative of $V_{\text{eff}}$, have been written down for particular classes of models.

In the context of our analysis, the derivation of those two types of constraints took place in the framework of the four-dimensional, effective theory for the graviton and radion field. We demonstrated that the variation over the size of the extra dimension reduces the integral constraints to a set of algebraic conditions formulated at the positions of the branes. Clearly, the extremization constraints give the same information as Einstein’s equations. However, the stabilization constraint is not trivial. We were able to derive it in terms of the bulk and brane potentials, which may be used for a direct calculation of the radion mass term in many brane-world models.

The resulting constraints were then applied for a number of static, five-dimensional solutions that had previously appeared in the literature. All of them were shown to satisfy the extremization constraint as expected, and furthermore their stability behaviour was studied by making use of the stabilization constraint. The two-brane Randall-Sundrum model was shown to be an absolute saddle point (a flat direction) of the radion potential while the implementation of an attractive Casimir force between the two branes led to an unstable configuration, in agreement with a previous work. Finally, the two-brane model with a bulk scalar field was shown to be stable for large inter-brane distances, i.e. when the
branes lie in the exponential regime of the sinh-like solution, while a potential instability arises for small brane separations. Of particular importance was the fact that the bulk potential is not a necessary ingredient for stabilizing an extra dimension with the scalar field. Indeed, the interaction terms between the bulk scalar field and the branes lead to a sufficient condition for stabilization.

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