The Cepheid Distance Scale: Recent Progress in Fundamental Techniques

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Abstract. This review examines progress on the Pop I, fundamental-mode Cepheid distance scale with emphasis on recent developments in geometric and quasi-geometric techniques for Cepheid distance determination. Specifically I examine the surface brightness method, interferometric pulsation method, and trigonometric measurements. The three techniques are found to be in excellent agreement for distance measures in the Galaxy. The velocity p-factor is of crucial importance in the first two of these methods. A comparison of recent determinations of the $p$-factor for Cepheids demonstrates that observational measures of $p$ and theoretical predictions agree within their uncertainties for Galactic Cepheids.

Keywords: (Stars: variables): Cepheids — Stars: distances

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INTRODUCTION

In the near-century since Henrietta Leavitt's announcement of the Cepheid period-luminosity relation (Leavitt & Pickering 1912), enormous progress has been made in our understanding of the observational properties and physical origin of Cepheid pulsation. A delightful and thorough presentation of the early history of what is now being called the Leavitt Law is given by Femie (1969). But one aspect of the relation has proved elusive—a calibration of Cepheid luminosities based on fundamental geometry. Due to the large distances to Cepheids—other than the overtone pulsator Polaris near 130 pc, the closest is $\delta$ Cep itself at 273 pc—distance determinations have depended, first upon statistical parallaxes, and later, upon the presence of Cepheids in galactic clusters. Neither of these methods can be considered fundamental in the geometric sense. Recent developments have changed that situation.

In this paper I will examine the techniques of fundamental distance measurement of Cepheids, compare the results from those techniques, and discuss the potential systematic error common to two of the techniques. I will restrict my discussion to fundamental-mode Cepheids only.

FUNDAMENTAL TECHNIQUES

Three methods qualify as geometric or quasi-geometric determinations of Cepheid distances: surface brightness pulsation distances, interferometric pulsation distances, and trigonometric parallaxes. I omit here discussion of the Cepheid distances determined by means of the maser in NGC 4258 as that result is better used as a check on the other three.

The distinction I make between geometric and quasi-geometric is the following. Trigonometric parallaxes are based on geometry. Some may quibble that the adjustment from relative to absolute parallax is not geometric, but I would argue that the distances to the reference stars may be traced back to trigonometric parallax calibration. On the other hand, the quasi-geometric methods (comparison of linear diameters to angular diameters to determine the distance) would be geometric except for complications that are not geometric. In the case of the surface brightness pulsation method, this is the $p$-factor that converts observed radial velocity into stellar pulsation velocity. In the case of interferometric pulsation distances there is the limb darkening correction to the uniform-disk angular diameter as well as the $p$-factor.

Surface Brightness Distances

The surface brightness technique is an extension of the work by Baade (1926) and by Wesselink (1946, 1969). The first two of these papers established a practical method for determining the mean radius of a Cepheid without knowing the actual surface brightness. Phases of equal color index are assumed to be phases of equal surface brightness. The difference in magnitude between the two phases is then dependent on the ratio of the radii at the two phases. The difference in radii at the two phases may be determined by integration of the radial velocity curve, appropriately converted to a pulsational velocity curve. Application to multiple phases yield the mean radius. This is the classic Baade-Wesselink
method.

Wesselink (1969) later determined actual surface brightnesses for eighteen stars of measured angular diameter. Although no Cepheid variables were among the sample, he assumed (with full acknowledgment of the risk) that the correlation between these surface brightnesses and \((B-V)\) would apply to Cepheids. From the mean \((B-V)\) of a Cepheid and his correlation he obtained the mean surface brightness; from the Baade-Wesselink method he obtained the mean stellar radius; a combination of these two provided the mean absolute magnitude. This method depends upon a reliable determination of the color excess in order to infer the Cepheid surface brightness correctly from \((B-V)\) and also to compute the distance. Later studies found that the slope of the surface brightness \(-\langle B-V\rangle\) relation determined from angular diameter stars did not agree with the slope for Cepheid variables (see for example, Thompson 1975). That notwithstanding, Wesselink obtained a distance to \(\delta\) Cep of 270 pc, in remarkable agreement with the recent trigonometric parallax distance of 273 pc.

Barnes et al. (1976, 1977) extended the Wesselink (1969) approach in two ways. Using a much larger sample of measured angular diameters, they showed that the visual surface brightness \(F_v\) correlated with \((V-R)\) much better than with \((B-V)\). The tight correlation included stars of all luminosity classes, unlike the separation of supergiants exhibited in the \((B-V)\) correlation, and thus seemed likely to be applicable to supergiant Cepheid variables. Secondly, their mathematical approach to the problem solves for the distance and radius simultaneously, unlike Wesselink’s separate solutions. Not only is this approach more appropriate mathematically, but it also, when used with \((V-R)\), renders a distance that is essentially independent of the reddening. A one magnitude error in the adopted interstellar extinction \(A_v\) causes a 4% error in distance.

Significant improvement in the surface brightness method was introduced by Welch (1994). He demonstrated that use of the infrared color index \((V-K)\) preserves the advantages of the surface brightness method (quasi-geometric, insensitivity to reddening) while reducing the uncertainties substantially. Fouquè and Gieren (1997) compared the \(V,(V-R); V,(V-K)\); and \(K,(J-K)\) versions of the method and found that the three choices yield very similar distances and radii but that the \(V,(V-K)\) combination produces percentage uncertainties nearly an order of magnitude smaller than those for \(V,(V-R)\). As a result most researchers have adopted the infrared surface brightness method. One problem with this choice is that the \(V\) and \(K\) photometric data are seldom acquired simultaneously, hence some interpolation scheme based on accurate knowledge of the period is needed to compute the colors.

Despite the shift to the infrared, the surface brightness method still suffered from two major problems. None of the mathematical solutions by various researchers to the equations for determination of distance and radius from magnitude, color and radial velocity were rigorous, and none of the angular diameters used to calibrate the surface brightness – color relation were obtained from Cepheids. These two issues were not fully addressed until very recently.

A rigorous and objective solution to the mathematics of the surface brightness equations was provided by Barnes et al. (2003) using a Bayesian Markov-Chain Monte Carlo code. This paper also provides a thorough discussion of the equations needed to solve for the distance and radius in the surface brightness method. Their analysis objectively selects a model for the radial velocity curve, correctly propagates the radial velocity uncertainties through the analysis, correctly handles the problem of uncertainties in both the inferred angular diameters and the computed linear displacements, and averages over the probabilities associated with all the models for distance and radius. The latter is demonstrated in Figure 1. The adopted parallax is the expectation value of the posterior marginal distribution and its uncertainty is given by the breadth of that distribution. In a later paper Barnes et al. (2005) showed that the linear-bisector solution to the surface brightness equations adopted by some researchers gives the same results for distance and radius as the Bayesian solution, but underestimates the uncertainties considerably and lacks the internal checks of the Bayesian approach.

The issue of calibration of the surface brightness by means of observed Cepheid angular diameters has also
been addressed recently. The first demonstration of a surface brightness relation using observed angular diameters was by Nordgren et al. (2002) using data from several interferometers. Based on 59 angular diameter measures of three Cepheids, they found an $F_\gamma - (V - K)$ relation consistent with that found for non-variable stars by Fouqué and Gieren (1997). Kervella et al. (2004a) resolved the angular pulsation curves of seven Cepheids with the VLT interferometer. Figure 2 shows the combined infrared surface brightness relation for these stars. They demonstrated that the observed surface brightness relation with the smallest scatter is the $V, (V - K)$ relation, that the slope of the surface brightness relation is independent of the period of the Cepheid to within their uncertainties, that the previously determined surface brightness relation of Fouqué and Gieren (1997), based on angular diameters of non-variable stars, matches very closely the one in Figure 2 based on Cepheid angular diameters, and that the surface brightness relations of Cepheids and main sequence stars of similar $(V - K)$ are essentially identical. It is not possible to overstate the importance of these papers to establishing the validity of surface brightness distance measures.

As to the precision of the infrared surface brightness method, the Bayesian calculations published by Barnes et al. (2005) for 38 Galactic Cepheids had a typical random uncertainty in measured distance of 4%. Gieren et al. (2005) obtained a similar uncertainty for LMC Cepheids that had periods similar to the Galactic ones studied by Barnes et al. The Gieren et al. study demonstrates that the surface brightness method makes it possible to determine with confidence quasi-geometric distances to Cepheid variables at any distance for which the requisite photometry and radial velocities can be obtained.

### Interferometric Pulsation Distances

It is obvious that a superior, if limited, application of the surface brightness method for determining Cepheid distances would use direct measurement of the angular diameters. Interferometric pulsation distances are superior in that they do not require inference of the angular diameter from a color index and because they are fully independent of reddening. They are limited because only a modest number of Cepheids are close enough to permit measurement of the angular diameter throughout the pulsation cycle. Thus there are few such measures and some of those have rather large uncertainties. The observational process of measuring stellar angular diameters is well discussed in the literature, e.g., Lane et al. (2002), and inappropriate to include here. Nonetheless, it is important to recognize that the reduction of observed fringe visibilities to angular diameters requires a model for the light distribution in the source. The standard assumption is that of a uniform intensity disk, hence the "uniform-disk angular diameter" that is usually quoted. By means of a theoretically established limb-darkening curve, the uniform-disk diameter may be converted to the (larger) limb-darkened diameter. This, together with the need for a velocity p-factor, is why I denote interferometrically determined Cepheid distances as quasi-geometric.

The first measurement of an interferometric angular diameter of a Cepheid was by Mourard et al. (1997) for δ Cep, followed soon thereafter by Nordgren et al. (1999, 2000) (Polaris η Aql, δ Cep and ζ Gem), Lane et al. (2000) (ζ Gem) and others. While some of these works showed evidence of resolved pulsation, the uncertainties were large enough to prohibit precise distance determination. That situation was soon improved. Figure 3 shows the angular pulsation curve for l Car by Davis et al. (2009) demonstrating the quality of recent measurements. Davis et al. determined a distance to l Car with random uncertainty of 3%.

Currently there are eight Cepheids with distances measured through interferometric pulsation parallaxes. They are listed in Table 5 of Fouqué et al. (2007) which is reproduced here as Table 1. I have added to Table 1 the distances and percentage uncertainty in the distance and have reordered the list by period. The mean percentage uncertainty in distance for the eight Cepheids is 12.7%, but this is dominated by three stars with large uncertainties. If those are discarded, the mean is 3.0%. While interferometric pulsation distances show promise of being superior in precision to infrared surface brightness distances, that promise is not yet fulfilled.

![Figure 2](image-url)
While a worrisome observation, we will leave it for future investigation and assume here that the interferometric pulsation distances are not affected significantly.

Merand et al. estimate the effect which she did not detect expected circumstellar material. Nevertheless, the utility of trigonometric parallaxes for Cepheids was obtained by the Hipparcos mission (Perryman et al. 1997). Hipparcos measured the parallaxes of numerous Cepheids but very few were individually useful.

The utility of trigonometric parallaxes for Cepheids took a major leap forward when Benedict and collaborators used one of the Hubble Space Telescope fine guidance sensors (FGS1a) to measure, first the parallax of δ Cep, and later nine additional Cepheid parallaxes (Benedict et al. 2002, 2007). The HST fine guidance system is intended for spacecraft control, and turns out to be extraordinarily effective for astrometry. In the papers cited above, Benedict et al. describe in detail how the relative parallaxes are measured within the FGS1a field of view, and how those relative parallaxes are converted to absolute parallaxes through ground-based observations of the reference stars.

Table 2 shows the HST parallaxes for ten Cepheids as determined by Benedict’s team. The mean uncertainty in distance is 8.0%, which is comparable to the mean uncertainty of the five best interferometric pulsation distances (7.2%).

Recently van Leeuwen et al. (2007) used the revised calibration of Hipparcos parallaxes to investigate the Leavitt Relation. It is useful to compare the Hipparcos parallax with the ones from HST. Figure 4 compares the

![Figure 3](image-url)  
**FIGURE 3.** Observed angular diameters (points) of I Car compared to scaled linear displacements (smooth curve). Data from SUSI. Figure from Davis et al. (2009).

### Table 1. Cepheids with interferometric pulsation parallaxes. Adapted from Fouqué et al. 2007.

| Star   | Log P (days) | \( \pi \) (mas) | \( \sigma(\pi) \) (mas) | Distance (pc) | \( \sigma(d) \) (%) | Source                  |
|--------|--------------|-----------------|------------------------|--------------|------------------|-------------------------|
| δ Cep  | 0.72         | 3.52            | 0.10                   | 284          | 2.8              | Mérand et al. (2005)    |
| Y Sgr  | 0.76         | 1.96            | 0.62                   | 510          | 31.6             | Mérand et al. (2009)    |
| η Aql  | 0.85         | 3.31            | 0.05                   | 302          | 1.5              | Lane et al. (2002)      |
| W Sgr  | 0.88         | 2.76            | 1.23                   | 362          | 44.6             | Kervella et al. (2004c) |
| β Dor  | 0.99         | 3.05            | 0.98                   | 328          | 3.1              | Kervella et al. (2004c), Davis et al. (2006) |
| ζ Gem  | 1.01         | 2.91            | 0.31                   | 344          | 10.6             | Lane et al. (2002)      |
| Y Oph  | 1.23         | 2.16            | 0.08                   | 463          | 3.7              | Mérand et al. (2007)    |
| I Car  | 1.55         | 1.90            | 0.07                   | 526          | 3.7              | Kervella et al. (2004b), Davis et al. (2009) |

### Table 2. Cepheids with trigonometric parallaxes from Benedict et al. 2007.

| Star   | Log P (days) | \( \pi \) (mas) | \( \sigma(\pi) \) (mas) | Distance (pc) | \( \sigma(d) \) (%) |
|--------|--------------|-----------------|------------------------|--------------|------------------|
| RT Aur | 0.57         | 2.40            | 0.19                   | 417          | 7.9              |
| T Vul  | 0.65         | 1.90            | 0.23                   | 526          | 12.1             |
| FF Aql | 0.65         | 2.81            | 0.18                   | 356          | 6.4              |
| δ Cep  | 0.73         | 3.66            | 0.15                   | 273          | 4.0              |
| Y Sgr  | 0.76         | 2.13            | 0.29                   | 499          | 13.6             |
| X Sgr  | 0.85         | 3.00            | 0.18                   | 333          | 6.0              |
| W Sgr  | 0.88         | 2.28            | 0.20                   | 483          | 8.8              |
| β Dor  | 0.99         | 3.14            | 0.16                   | 318          | 5.1              |
| ζ Gem  | 1.01         | 2.78            | 0.18                   | 360          | 6.5              |
| I Car  | 1.55         | 2.01            | 0.20                   | 497          | 9.9              |
original Hipparcos parallaxes to those from HST, and Figure 5 compares the revised Hipparcos parallaxes. The mean difference for the original Hipparcos parallaxes is $-0.229 \pm 0.534$ mas and for the revised parallaxes, $-0.005 \pm 1.093$ mas. It is remarkable that the revised Hipparcos and HST parallaxes agree almost perfectly in the mean but the scatter is much larger than was the case for the original Hipparcos parallaxes. This is largely a result of two outliers (RT Aur and Y Sgr). If those two stars are removed, the new Hipparcos and the HST parallaxes have this mean difference: $+0.123 \pm 0.405$ mas. This is a very modest improvement in consistency over the original Hipparcos catalog.

Because the two outliers in Figure 5 suggest that either the Hipparcos results or the HST results are subject to unexpectedly large errors, it is important examine the quoted uncertainties in somewhat more detail. Table 1 of van Leeuwen et al. (2007) gives the uncertainties of the Hipparcos Cepheid parallaxes in common with HST parallaxes. Setting aside RT Aur and Y Sgr for the moment, the mean quoted uncertainty of an Hipparcos Cepheid parallax is $\pm 0.31$ mas, compared to $\pm 0.20$ mas for the HST Cepheid parallaxes. Adding these values in quadrature suggests that the scatter in Figure 5 (again, ignoring the outliers) should be $\pm 0.363$ mas, which compares well with the actual $\pm 0.405$ mas and suggests that the quoted uncertainties are realistic.

On the other hand, if we include the two outliers, the mean quoted Hipparcos uncertainty is $\pm 0.38$ mas and the HST, $\pm 0.20$ mas. When added in quadrature, these yield $\pm 0.429$ mas, which is far from the actual scatter of $\pm 1.093$ mas. Which set of results causes the outliers?

In Figure 6 I show a Wesenheit Leavitt Law based on the HST parallaxes. The arrows denote the locations of RT Aur and Y Sgr. It is clear that they lie well within the scatter band of the HST relation. Had I used the Hipparcos parallaxes for the outliers, RT Aur would lie at least 2.4 mag above the relation (‘at least’ because its parallax is negative) and Y Sgr would lie 1.2 mag below it. I conclude that the anomalies lie within the Hipparcos data set. RT Aur lies 2.6 Hipparcos $\sigma$ from the HST result and Y Sgr lies 5.0 $\sigma$ away, which in a sample of ten stars suggests that some of the quoted Hipparcos uncertainties are not Gaussian.

As in the two previous sections I close this one with
FIGURE 7. A geometrically determined Leavitt Law in the Wesenheit magnitude and the K magnitude. The solid line is the OGLE slope for the LMC. Figure from Fouqué et al. 2007.

TABLE 3. Galactic Leavitt Laws from fundamental distances. Table adapted from Fouqué et al. 2007.

| Band | Slope   | Intercept | σ  | N |
|------|---------|-----------|----|---|
| B    | -2.289 ± 0.091 | -0.936 ± 0.027 | 0.207 | 58 |
| V    | -2.678 ± 0.076  | -1.275 ± 0.023 | 0.173 | 58 |
| R_c  | -2.874 ± 0.084  | -1.531 ± 0.025 | 0.180 | 54 |
| I_c  | -2.980 ± 0.074  | -1.726 ± 0.022 | 0.168 | 59 |
| J    | -3.194 ± 0.068  | -2.064 ± 0.020 | 0.155 | 59 |
| H    | -3.328 ± 0.064  | -2.215 ± 0.019 | 0.146 | 56 |
| K_s  | -3.365 ± 0.063  | -2.282 ± 0.019 | 0.144 | 58 |
| W_{ri} | -3.477 ± 0.074  | -2.414 ± 0.022 | 0.168 | 58 |
| W_{pi} | -3.600 ± 0.079  | -2.401 ± 0.023 | 0.178 | 58 |

A statement about the likelihood of adding more Cepheid distances by this technique in the near future. Each of the Cepheid parallaxes in Table 2 required eleven orbits, appropriately timed. The three Cepheids beyond 450 pc have a mean uncertainty of ±12%. Significantly more orbits per Cepheid would be needed to obtain parallaxes of Cepheids beyond 500 pc and to higher precision. To increase the sample significantly in the very near future would require considerable support within the HST TAC. More likely we will have to await the space missions GAIA and SIM.

**Fundamental Leavitt Laws**

The preceding discussion has laid the foundation for a Leavitt Law based on geometric and quasi-geometric distances to fundamental-mode Galactic Cepheids. There are seventy Cepheids with distances from the infrared surface brightness method, eight from the interferometric pulsation method and ten from HST trigonometric parallaxes. Fouqué et al. (2007) have combined these results into a single Leavitt Law in a variety of bands (Table 3). (The fourth column gives the scatter of the absolute magnitudes about the fit.) When there are multiple distances, they adopted a distance based on HST parallaxes, if available, and then chose from interferometric pulsation distance, infrared surface brightness distance, and Hipparcos distance on the basis of precision. Because of overlap in distance measures and variations in the quality of the photometry, the relations contain up to 59 distances. In Figure 7 we show their results for the Wesenheit magnitude and the Ks magnitude compared to the OGLE slope for LMC Cepheids (Udalski et al. 1999).

It is important to examine whether there are any systematic differences between these three distance indicators. Fouqué et al. (2007) quote that the infrared surface brightness W_{ri} magnitudes differ from the ones determined from HST parallaxes by 0.01 ± 0.03 mag. I have computed from their results that the interferometric pulsation distances give W_{ri} magnitudes that differ from the HST ones by 0.03 ± 0.13 mag. Clearly these fundamental methods agree with each other.

**THE P-FACTOR**

Until now I have not addressed a potential systematic error common to the infrared surface brightness distances and the interferometric pulsation distances: the velocity p-factor. Distances by both of these methods scale directly with the adopted p-factor. The other potential systematic error in interferometric pulsation distances, the limb-darkening correction, will not be discussed here.
After ruling out other possibilities, they concluded that
ness distances to Galactic and LMC Cepheids. When
using the additional HST parallaxes publish by Benedict
newegen (2007) later extended the analysis to seven stars
5
HST parallax of
Cep to invert the interferometric pul­
propriate p-factor. These efforts were both observational
method generated much interest in establishing the ap­
rightness method and the interferometric pulsation
period as a proxy) and ignores any variation with pulsa­
factures included a calculation of the p-factor. When based on
bservations of the single line Fe I \( \lambda 4896 \), they deter­
mained a dependence of \( p \) upon period to be (Nardetto
al. 2007)

\[
p = 1.39 - 0.03 \log P 
\]
where \( P \) is the period of pulsation in days. They devel­
ised this relation as a simplified fit to the theoretical cal­
culations of \( p \) by Hindsley and Bell (1986). It accounts for
the change in \( p \) in their models due to mean effective
rature and surface gravity of the Cepheid (using
period as a proxy) and ignores any variation with pulsation
phase or other factors. Hindsley and Bell’s calcula­
tions were appropriate to radial velocities determined by
cross-correlation radial velocity meters.

In recent years the success of the infrared surface
rightness method and the interferometric pulsation
method generated much interest in establishing the ap­
ropriate p-factor. These efforts were both observational
and theoretical.

The first observational determination of a p-factor for
a Cepheid was by Mérand et al. (2005). They used the
HST parallax of \( \delta \) Cep to invert the interferometric pul­
sation method. Given the distance and the observed an­
gular diameter variation, they determined that p-factor
that would match the scaled displacement curve to the
angular diameters. Their value is \( p = 1.27 \pm 0.06 \). Groe­
newegen (2007) later extended the analysis to seven stars
using the additional HST parallaxes publish by Benedict
et al. (2007). He found \( p = 1.27 \pm 0.05 \), He did not find
any period dependence.

The second observational determination was by
Gieren et al. (2005) using their infrared surface bright­
ess distances to Galactic and LMC Cepheids. When
Gieren et al. used eq. (1) in determination of LMC
Cepheid distances, they found a strong dependence of
the distance modulus upon period – an unphysical result.
After ruling out other possibilities, they concluded that
the p-factor eq. (1) was incorrect and determined a new
one that (1) yielded a zero slope in the period-distance
plane (affects the slope in eq. (1)), and (2) yielded
agreement within the Galaxy in Cepheid \( W_r \) magnitudes
between infrared surface brightness distances and
ZAMS fitting distances (affects the zero point in eq. (1)).
Their result, as modified by Gieren et al. (2009, these
proceedings) is

\[
p = 1.52(\pm 0.02) - 0.17(\pm 0.03) \log P 
\]

Finally, Benedict et al. (2007) determined \( p \) for T Vul
by inverting the \( (V - R) \) surface brightness method using
their new HST parallax. The result, while rather uncer­
tain, \( p = 1.19 \pm 0.16 \), is another independent determina­
tion.

Using very high resolution spectra, Nardetto et al.
(2004, 2006, 2007, 2009a, 2009b) have examined
Cepheid atmospheres and their motions extensively,
including a calculation of the p-factor. When based on
bservations of the single line Fe I \( \lambda 4896 \), they deter­
mained a dependence of \( p \) upon period to be (Nardetto
et al. 2007)

\[
p = 1.376(\pm 0.023) - 0.064(\pm 0.02) \log P 
\]
whereas observations based on velocities determined by
cross-correlation (Nardetto et al. 2009a) gave

\[
p = 1.31(\pm 0.06) - 0.08(\pm 0.05) \log P 
\]

They note that the cross-correlation method of measuring
velocities overestimates the velocity curve amplitude and
thus the correction factor to pulsational velocities eq.
(4) must be smaller. This indicates that researchers must
ensure that the p-factor used in their infrared surface
rightness and interferometric pulsation calculations be
the correct one for the velocities adopted.

It is interesting to put these results into context. In
Figure 8 I show the p-factor appropriate to the period
of \( \delta \) Cep according to each of the above studies. Sev­
eral conclusions may be drawn from the figure. First,
the value approximated from the work of Hindsley and
Bell (1986), source (1), is too large compared to re­
cent determinations in the Galaxy. A too large p-factor
yields too large distances in the surface brightness and
interferometric distance methods. Second, the observa­
tional results in the Galaxy and the theoretical results
for cross-correlation velocities are consistent (sources
2,4,5,7). This is very encouraging as the observational
work is based on cross-correlation velocities. Third, the
result by Gieren et al. (2005, 2009), source 3, using LMC
Cepheids is larger by somewhat more than one sigma
than the purely Galactic determinations. As the chain
or reasoning in Gieren et al. (2005) seems strong, one
immediately suspects a metallicity effect. Finally, the p-
factor for the single line Fe I \( \lambda 4896 \), source (6), is larger
Figure 8. $p$ values inferred for δ Cep from each of the recent $p$-factor studies. Sources: 1) Gieren et al. 1989; 2) Mérand et al. 2005; 3) Gieren et al. 2005, 2009; 4) Groenewegen 2007; 5) Benedict et al. 2007; 6) Nardetto et al. 2007; 7) Nardetto et al. 2009a.

than the other Galactic determinations, as expected from Nardetto et al. (2009a).

Figure 8 shows only a snapshot of $p$ at the period of δ Cep. The dependence upon pulsation period has only one observational determination, in the LMC (Gieren et al. 2005, 2009), and one recent theoretical determination, appropriate to Galactic metallicity (Nardetto et al. 2007, 2009a). These differ by a factor of two in slope of the $p$-factor with period. In order to improve this situation observationally, we need more Cepheid parallaxes of high quality to use with more resolved angular pulsation curves in an inverted interferometric pulsation calculation. It may be some time before this is possible. Improvement theoretically would be to determine if the difference between the observed LMC slope and the Galactic slope can be understood as a result of differences in the Cepheid atmospheres or whether it requires some other explanation.

In the previous section on Leavitt Laws we saw that the three fundamental methods of Cepheid distance determination agree with each other to better than 2\% in distance. The distance determinations that led to this agreement in Fouqué et al. (2007) used a dependence of $p$ upon period quite close to that in eq. (3). That relation is appropriate to velocity measures using the line Fe I $\lambda4896$, not cross-correlation velocities, which is what the data they had would require. (This is not a criticism of Fouqué et al. as the work by Nardetto et al. 2009a on the cross-correlation $p$-factor was still in the future.) The effect of using eq. (3) rather than eq. (4) at the period of δ Cep is to increase the distance by about 6 ± 4\%, or about 0.12 ± 0.08 mag in $W_p$. (There is no difference between the relations in the period dependence of $p$.) The actual difference between the $p$-dependent distances and the HST parallaxes was less than 0.03 mag. The pessimist will say that this implies a failure of our understanding of $p$; the optimist will say that it shows, to one sigma, that we actually understand the value of $p$.

### CONCLUSION

In this review I have endeavored to summarize recent work on our understanding of the distance scale of fundamental-mode Cepheids as based on geometric and quasi-geometric distance determinations. We have seen that the infrared surface brightness method has reached a level of maturity that permits it to be used as a reliable method for measurement of Cepheid distances in our Galaxy. The interferometric pulsation method has demonstrated its usefulness as a check on the infrared surface brightness distance scale and as a method to determine the $p$-factor observationally. The distances determined by both these methods are found to be in excellent agreement with the high quality parallaxes from HST.

The above agreement between $p$-dependent (quasi-geometric) and trigonometric (geometric) distances notwithstanding, there is still uncertainty in our understanding of the correction factor from observed radial velocities to pulsational velocities. The relationship between $p$ and pulsation period appears to be different in the Galaxy from that in the LMC. Until this is resolved, we must be cautious in applying the infrared surface brightness method in environments that differ greatly from the Galaxy. Within the Galaxy, observational and theoretical determinations of $p$ seem to be in agreement, and this agreement is supported by the agreement between quasi-geometric and geometric distances to Cepheids.

I am confident that Henrietta Leavitt would be both amazed and pleased at the progress made in the past century.

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