Abstract

Recent large scale structure observations, including COBE, have prompted many authors to discuss modifications of the standard Cold Dark Matter model. Two of these, a tilted spectrum and a gravitational wave contribution to COBE, are at some level demanded by theory under the usual assumption that inflation generates the primeval perturbations. The third, whose motivation comes by contrast from observation, is the introduction of a component of hot dark matter to give the Mixed Dark Matter model. We discuss the implication of taking these modifications together. Should Mixed Dark Matter prove necessary, very strong constraints on inflationary models will ensue.

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1 Introduction: New Parameters for CDM

Recent observations of large scale structure in the universe, and particularly that of the Cosmic Microwave Explorer (COBE) satellite DMR experiment (Smoot et al 1992), have been widely interpreted as indicating that the standard Cold Dark Matter (CDM) model gives a qualitatively correct picture of structure formation, but requires quantitative modifications. Relying on a gaussian, Harrison–Zel’dovich initial spectrum, the standard CDM model is specified by a single parameter, the amplitude of the power spectrum, and the success of this model on confrontation with observation is truly remarkable (Efstathiou 1990; Liddle & Lyth 1993). Nevertheless, the required amplitude as inferred on small scales by pairwise velocities or cluster abundances appears to differ by a factor of around two from that required by COBE, and the pattern of clustering in the galaxy distribution on intermediate scales appears to indicate that the standard CDM spectrum has an incorrect shape on these scales.

The post-COBE rush of papers has introduced, among other things, three prominent new parameters into the CDM model. These are\(^1\) (Liddle, Lyth & Sutherland 1992; Wright et al 1992; Krauss & White 1992; Schaefer & Shafi 1992; Davis, Summers & Schlegel 1992; Taylor & Rowan-Robinson 1992; Cen et al 1992; Salopek 1992; Liddle & Lyth 1992; Adams et al 1993)

- A gravitational wave component contributing a fraction \(R\) of the large angle microwave anisotropies.
- A ‘tilt’ \(n\) of the primeval spectrum away from a flat \((n = 1)\) spectrum.
- An admixture of hot dark matter (HDM), contributing a fraction \(\Omega_\nu\) to the critical density

To our knowledge, all three have not been considered together before this paper. A hot dark matter component has most often been discussed without either of the others, though Schaefer and Shafi (1993) have included tilt in their studies but not gravitational waves. Particularly in \(N\)-body studies, tilt is typically discussed on its own, but the present authors have offered a study including both tilt and gravitational waves (Liddle & Lyth 1993). One aim of this paper is to examine the rationale behind these new parameters, both from a theoretical and observational viewpoint.

The motivation for these new parameters differs. A component of hot dark matter is clearly an optional extra. In order to have a third component of nonrelativistic matter (along with the baryonic and cold dark matter components) with density of the order of the critical density, it appears that some form of tuning of the parameters is needed. The relative abundances of particle species is a complicated function of masses and couplings, and can in principle take on a wide range of values. To have CDM and baryons with similar densities is already a modest coincidence; to have the HDM and baryon density similar too exacerbates this. Nevertheless there may well exist particle physics models which do exhibit these appropriate tunings. The point to make here is that this extra parameter is at best

\(^1\)We are not considering here the possibility of a fourth parameter, namely the introduction of a cosmological constant, since it does not have very strong motivation from either theory or observation (though see Kofman, Gnedin & Bahcall 1992).
poorly motivated by presently understood particle physics; the reason why the so-called mixed dark matter (MDM) model\(^\text{2}\), where \(\Omega_\nu \simeq 25\text{–}30\%\), is so popular is because of its strong phenomenology, as we shall discuss.

The other new parameters are not just well founded theoretically — provided one takes the step of believing inflation as the cause of the primeval inhomogeneities they are at some level inevitable. The inflationary prediction has long been advertised as a Harrison–Zel’doovich spectrum with a gravitational wave spectrum of negligible amplitude (Kolb & Turner 1990; Linde 1990). In the past, when observations were restricted to a limited range of scales, this was a very reasonable approximation. However, with COBE probing length scales vastly in excess of those studied previously, it seems that this approximation is no longer good enough (Davis et al 1992; Salopek 1992; Liddle & Lyth 1992; Adams et al 1993). The generic prediction from inflation is a primeval density perturbation spectrum that can be well approximated by a power-law \(P(k) \propto k^n\), but where \(n\) is ‘tilted’ from the flat \(n = 1\) case. Almost generically, the tilt is to \(n < 1\), removing short-scale power from a COBE normalised spectrum\(^\text{3}\). Gravitational waves are also generically created, and though their contribution to COBE is normally less than that of the density perturbations, it can easily be tens of percent, which can in no way be regarded as negligible.

A case in point is provided by the simplest model of inflation, chaotic inflation with a free massive scalar field (Linde 1990). This model produces the smallest distortive effects amongst the more popular inflationary models. The deviation from the flat spectrum is technically logarithmic, but in practice excellently described by a power-law over scales of interest. The combined effects of tilt and gravitational waves in this model reduce \(\sigma_8\), the dispersion of the density field at \(8h^{-1}\) Mpc, by 13\% (when normalised to the COBE 1\(^0\) result). This is not startling, but it is the minimum deviation expected from inflation and certainly large enough to convert an unlikely looking 2-sigma result into a comfortable 1-sigma result at present observational accuracy. In a self-coupled chaotic inflation model, this leaps to 20\%, and in others yet more.

While the complete details of predicting the perturbation spectra from inflation are quite involved, the outcome is very simple. In present versions, an inflationary model consists of no more than a scalar field \(\phi\) evolving in a potential \(V(\phi)\) and a mechanism to end inflation. The form of the potential is largely up for grabs, but barring pathologies the following parameters, the slow-roll parameters, must be small compared with unity,

\[
\epsilon = \frac{m^2_{Pl}}{16\pi} \left(\frac{V'}{V}\right)^2
\]

\[
\eta = \frac{m^2_{Pl}}{8\pi} \frac{V''}{V}
\]

where primes are derivatives with respect to \(\phi\). In general these depend on the scalar field value \(\phi\), but usually the scales of interest for large scale structure leave the horizon over a short interval and they can be treated as constant. The exception is ‘designer’ models of inflation, where one contrives dramatic features in the potential just at the points appropriate for large scale structure. These simple parameters are vitally important, because they

\(^{2}\)Often denoted C+HDM (or CPHDM), which is more descriptive but more cumbersome.

\(^{3}\)It is possible to have \(n > 1\), but this is not achieved by any of the well-accepted inflationary models and with present understanding should be regarded as unnatural.
alone determine to high accuracy\footnote{It has recently been verified by explicit calculation that the corrections to the following formulae are indeed small (Stewart & Lyth 1993).} the degrees of tilt of both the density perturbation and the gravitational waves, as well as their relative normalisations at the COBE scale (Davis et al 1992; Liddle & Lyth 1992).

Scales of cosmological interest leave the horizon about 60 $e$-foldings from the end of inflation (that is, when the scale factor was smaller by a factor $e^{60}$ than at the end), and it is the value of the slow-roll parameters then that is required. Usually, inflation ends when the field approaches a minimum, and $\epsilon$ exceeds unity\footnote{There are exceptions, where more involved means of ending inflation are introduced. The key example is power-law inflation, which requires an exponential potential. Conveniently, the slow-roll parameters are exactly constant in this case, so the prediction for the spectra is independent of the precise means of ending inflation. Other exceptions are discussed by Liddle & Lyth (1993).}. The scalar field value $N$ $e$-foldings from the end of inflation is easily obtained via

$$N = \frac{8\pi}{m_P^2} \int_{\phi_e}^{\phi} \frac{V}{V'} \, d\phi$$

where $\phi_e$ is the value at the end of inflation. Given a potential, it is thus easy to calculate the appropriate scalar field value, and hence the parameters $\epsilon$ and $\eta$.

The degree of tilt of the density perturbation, defined as the departure of its spectral index from the scale invariant value $n = 1$, is

$$1 - n = 6\epsilon - 2\eta$$

The degree of tilt of the gravitational wave amplitude, defined as the departure of its spectral index from the scale invariant value $n_g = 0$, is

$$- n_g = 2\epsilon$$

Finally, the ratio $R$ of the gravitational wave and density perturbation contributions to the expected mean square microwave background anisotropy measured by COBE is

$$R \simeq 12\epsilon$$

The tilt of the gravitational wave amplitude will be hard to measure because it can be probed only through the microwave background, but the actual magnitude could be crucial. In plain language, if gravitational waves are significant the $rms$ density perturbation $\sigma$ when normalised to COBE is only a fraction $F = 1/\sqrt{1 + R}$ of what you thought it would be.

For example, consider the potential $V(\phi) \propto \phi^\alpha$, proposed in the context of chaotic inflation (Linde 1990). From above, $\phi_{60}/m_P \simeq \sqrt{60\alpha/4\pi}$, so

$$\epsilon = \frac{\alpha}{240}; \quad \eta = \frac{\alpha - 1}{120}$$

So we immediately know that $n = 1 - (2 + \alpha)/120$, and that the gravitational waves will reduce the dispersion $\sigma_8$ by a factor $1/\sqrt{1 + \alpha/20}$. For the smallest conceivable power $\alpha = 2$ (corresponding to a free field), the tilt can be shown to normalise $\sigma_8$ down by 8% relative to CDM, and the gravitational waves by a further 5%, as advertised above.
In this polynomial model the degree of tilt $1 - n$ and the relative contribution of the gravitational waves $R$ are related by $6(1 - n) = 0.1 + R$. A similar relation, $6(1 - n) = R$, holds in the power-law inflation models mentioned earlier, but such a relation is not generic and in particular one can have significant tilt without significant gravitational waves (Liddle & Lyth 1992; Adams et al 1993). The essential point, though, is that barring fine-tuned ‘designer’ models the whole gamut of possible inflationary models introduces only two additional parameters into the standard CDM model.

2 Observations

The two parameters associated with generic inflation models, plus the third one invoked by the MDM model, have different effects. Let us assume for the time being that the theory is normalised to the COBE observations. The gravitational waves simply normalise down the amplitude of the whole spectrum. Tilt removes short-scale power from the spectrum, progressively across the whole range of scales. The hot dark matter component, on the other hand, removes power from the spectrum only up to the scale on which free-streaming of the HDM can occur, typically tens of megaparsecs, while leaving the large scales the same as in CDM. The effect of the HDM is somewhat subtle, in that usually only one free parameter, $\Omega_\nu$, is allowed. It is then assumed that the HDM has standard properties, effectively those of a neutrino, which relate its abundance to its mass. The mass then finally provides the free-streaming length. The one parameter thus determines both the extent to which free-streaming removes short-scale power, and also the scale up to which the free-streaming is effective. The success of the MDM model is that with a choice of the parameter as 25–30%, these two features are respectively of the size, and at the scale, at which one would wish them, allowing one free parameter to simultaneously fit several pieces of data.

One should note that, barring unusual inflation, all the new parameters serve to progressively subtract power relative to the CDM spectrum as one progresses to shorter scales. Observations on a given scale are commonly interpreted by giving the amplitude a CDM spectrum would require to explain them; this is normally specified by the $\sigma_8$ this amplitude would give, regardless of the scale on which the observations apply. As long as observations towards progressively smaller scales lead to progressively smaller predictions for the CDM amplitude, then one can reasonably expect these new parameters to be useful. It so happens that the available observational data is precisely of this sort.

There are a variety of observations giving the amplitude of the mass fluctuations across a range of scales. However, it appears possible to take a very crude view and conclude that there are effectively only about four different measurements which one must satisfy, as different measurements on the same scales appear pretty much in agreement. We list such a set below, noting in each case the value of $\sigma_8$ which would be required in the standard CDM model, denoted by $\sigma_8^{\text{CDM}}$.

- Scales $10^2 h^{-1}$ to $10^4 h^{-1}$ Mpc: COBE provides a measurement of the spectrum here,

$^6$Should things go badly for MDM, there is a rather unpalatable opportunity to add an extra parameter here, measuring in some way the ‘nonstandardness’ of the connection between the relic abundance and mass of the HDM particles.

$^7$As usual $h$ is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and in making predictions is taken to be equal to 0.5.
allowing a one-sigma range from about \( \sigma_{8\text{CDM}} = 1 \) to 1.3. The top of this range is however disfavoured by subsequent analysis and other experiments, and the most likely true value in the light of these is perhaps the COBE 1-sigma lower limit.

- Scales \( 20h^{-1} \) to \( 40h^{-1} \) Mpc: Velocity flows appear the most unambiguous measure here. For example, QDOT (Kaiser et al 1991) provides \( \sigma_{8\text{CDM}} \) ranging from between about 1 and 0.7. A comparison of POTENT with the 1.2 Jansky survey (Dekel et al 1992) yields similar results, as does a direct comparison with POTENT bulk flows. Following Efstathiou, Bond and White (1992), we utilise the QDOT results on the IRAS bias and number count variance in \( 30h^{-1} \) Mpc cubes, converted to spheres of equal volume, ie radius \( 19h^{-1} \) Mpc. Dealing directly with the variance on this scale, this gives a 1-sigma range \( \sigma_{19} = 0.37 \pm 0.07 \). [The prediction of standard CDM at COBE normalisation is \( \sigma_{19} = 0.45 \), which can be translated into the \( \sigma_{8\text{CDM}} \) limits above.] Primarily we are interested in the lower limit, and it is worth noting that POTENT/IRAS gives a much stronger version, as they obtain a 95% confidence upper limit on the IRAS bias which is less than the QDOT 1-sigma upper limit.

- The scale of order \( 10h^{-1} \) Mpc: A direct measure of the amplitude at \( 8h^{-1} \) Mpc is provided by the abundance of galaxy clusters. A recent analysis by White, Efstathiou & Frenk (1993) gives \( \sigma_{8\text{CDM}} \) in the range 0.30–0.63 (the lower limit takes into account the possibility that current cluster mass estimates could be a factor of 3 or so too big). Recently, it has been claimed that through non-linear effects the pairwise galaxy velocity dispersion for galaxy separations of of order 1 Mpc also measures the primeval amplitude on the scale of order \( 10h^{-1} \) Mpc (Gelb, Gradwohl & Frieman 1993), requiring roughly \( .3 < \sigma_{8\text{MDM}} < .5 \). We shall use the galaxy cluster range in what follows.

- The scale of order \( 1h^{-1} \) Mpc: This comoving scale encloses (before gravitational collapse) a mass comparable to that of a large galaxy or quasar. At high redshift such objects are rare because only a small fraction of the matter has had time to collapse, but lower limits on their abundances can be estimated which translate into lower limits on \( \sigma \) on this scale (Efstathiou & Rees 1988; Adams et al 1993; Cen et al 1993; Haehnelt 1993). Haehnelt uses the observed quasar luminosity function (Irwin, McMahon & Hazard 1991; Boyle et al 1991), together with reasonable assumptions about quasar astrophysics, to deduce that at redshift 4 the fraction \( f(> M) \) of mass bound into objects with \( M > 10^{13} M_{\odot} \) is at least \( 1 \times 10^{-7} \). He compares this result with the Press–Schechter formula

\[
f(> M) = 1 - \text{erfc} \left( \frac{\delta_c}{\sqrt{2\sigma(M, z)}} \right)
\]

where \( \delta_c \) is the value in linear theory of the density contrast at which gravitational collapse is assumed to take place. Comparisons with numerical simulations have suggested values of \( \delta_c \) in the range 1.3 to 1.7, and taking the lower value one finds a bound on \( \sigma(10^{13} M_{\odot}, 4) \) which is equivalent to \( \sigma_{8\text{CDM}} > .40 \). (Haehnelt obtains a somewhat stronger constraint by taking the higher value.) Note that when comparing this \( z = 4 \) result with the MDM model one has to allow for the \( z \)-dependence of the MDM transfer function, reflecting the slower growth of the perturbation relative to the CDM model.
In our view, a model which manages to satisfy all of these limits has a good chance of agreeing with all other available observations. In particular, though we have not mentioned it explicitly, it should satisfy all the clustering data such as the APM survey (Maddox et al 1990) and a host of later surveys. When first produced, these were seen, rightly, as a major problem for CDM, indicating that the clustering strength falls off less rapidly than expected with increasing scale. However, if one fits the data above there is clearly going to be an excess in amplitude as one goes from $8h^{-1}$ to the scales around $20h^{-1}$ Mpc on which the velocity flow data operates. Conveniently, the excess clustering data has already been reinterpreted by Wright et al (1992), using a quantity they call the ‘excess power’ $E$, defined simply as

$$E = 3.4 \frac{\sigma(25h^{-1}\text{Mpc})}{\sigma(8h^{-1}\text{Mpc})}$$

The prefactor is chosen to make the CDM value unity, and they suggest that values of $E$ in the range 1.15 to 1.45 will fit the clustering data. We see from the figures above that if we fit the cluster abundance and velocity flow data, we can hardly fail to satisfy this excess power criterion.

\section{Confrontation with observations}

The figures illustrate the parameter space regions which satisfy these data points. This is intended only to be illustrative of trends, for two reasons. Firstly, the observational data are not particularly strict, and most people would accept a reasonable amount of freedom to manipulate the figures above. Secondly, the theoretical calculations are not as accurate as one would like. We have utilised transfer functions from van Dalen and Schaefer (1992), who supply parametrised forms for a set of $\Omega_{\nu}$, to make our calculations. However, these are only accurate to perhaps ten percent or worse across the full range of scales, which in many places is comparable to the observational uncertainties. Klypin et al (1992) have provided what appears a more accurate transfer function for $\Omega_{\nu} = 0.30$; when normalised to COBE it gives values for the dispersion of between 10% and 15% lower across the scales where we compare with data.

Equally, one should not be expecting any dramatic conclusions. After all, we are allowing four free parameters (amplitude, tilt, gravitational waves, HDM fraction) and have only four data points to fit. So what are we looking for? The important points appear to be the following

1. The three parameters naturally motivated by inflation are amplitude, tilt and gravitational waves. Given such freedom, it is perhaps surprising that they appear insufficient to allow one to fit the data (see Liddle & Lyth (1993) for a more extensive discussion). Because of its progression across the entire range of scales, tilt seems incapable of providing the sharp drop in power between the bulk flow and cluster abundance.

\footnote{We have specifically mentioned APM. However, other surveys indicating excess power appear consistent with APM — this is normally indicated by authors quoting viable power spectra in terms of the $\Gamma = \Omega h$ parametrisation of Efstathiou, Bond & White (1992). Usually $\Gamma = 0.2-0.3$ is required (Kofman et al 1992), which can be translated into the same excess power criterion.}
scales. As a further symptom of the same shortcoming, it fails to fit the APM data if it fits QDOT. A modification, such as MDM, seems essential.

2. As advertised, satisfying the cluster abundance and bulk flow data effectively guarantees a fit to clustering data such as APM.

3. MDM without tilt or gravitational waves does rather well. Unfortunately, one cannot motivate the complete absence of tilt and gravitational waves by appealing to inflation. So it appears that one really ought to allow all three new parameters.

4. MDM only works well provided that the tilt and gravitational waves are very small. Thus, if one believes the MDM model one has to accept very strong constraints on models of inflation.

The last point is worthy of additional comment. Our present understanding of the fundamental interactions, which is embodied in the Standard Model, does not lead to inflation. However, the Standard Model has been tested only on energy scales $\lesssim 100$ GeV, and for reasons that have nothing to do with cosmology one anticipates an extension of the Standard Model at higher energy scales. The subject of particle cosmology came into being when it was realised that the early universe constitutes a ‘cosmic accelerator’, allowing one to probe energy scales far beyond the reach of laboratory physics, and the last point is a specific example of this remarkable fact. In another publication (Liddle & Lyth 1992) we have already ruled out a class of otherwise attractive inflationary models (if they are to generate the density perturbation required to form structure), and we are here pointing to the possibility of drawing stronger conclusions from better data.

As an illustration of the sort of thing that might become possible, let us suppose that the observational bounds all turn out to be correct, and that MDM turns out to be necessary. Then with COBE normalisation, the degree of tilt $1 - n$ is constrained to the range 0.07 to 0.10 in models with negligible gravitational waves, and to the range 0.03 to 0.05 for chaotic and power-law models. For the former case, the exponent $\alpha$ in the potential is constrained to the range 1.6 to 4.0, which would for example rule out the otherwise attractive inflationary model recently proposed by Lazarides and Shafi (1993) within the context of superstrings. While it should not be taken seriously at the present time, this example serves as a reminder that every improvement in observational cosmology has potential implications regarding the search for a viable model of the fundamental interactions.

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**Figure Captions**

*Figure 1*

The constraints in the $n$–$\Omega_\nu$ parameter space, for (a), inflationary models with no gravitational waves, COBE normalised; (b), chaotic (or power-law) inflationary models, incorporating gravitational waves, COBE normalised; and (c), as (b), but normalised to the COBE 1-sigma lower limit. The lines shown are solid, quasar abundance; dashed, cluster abundance; dot-dashed, bulk flows from QDOT; dotted, clustering data from APM. The shaded region indicates the region satisfying all data (and can be used in each case to see which side of the line is the allowed side). We strongly urge the reader to treat the details with skepticism, following the caveats in the text, but to pay attention to the trends and possibilities. Finally, the notches on the top axis of (b) and (c) indicate the location of chaotic inflation models with exponent $\alpha=2, 4, 6$ and 8.