Nonminimal Inflation in Supersymmetric GUTs with $U(1)_R \times Z_n$ Symmetry

Muhammad Atif Masoud$^a$, Mansoor Ur Rehman$^a$, Mian Muhammad Azeem Abid$^a$

$^a$Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan

Abstract

A supersymmetric hybrid inflation framework is employed to realize a class of non-minimal inflation models with $U(1)_R \times Z_n$ global symmetry. This framework naturally incorporates models based on grand unified theories by avoiding the most commonly faced monopole problem. The predictions of inflationary observables, the scalar spectral index $n_s = 0.960 - 0.966$ and the tensor to scalar ratio $r = 0.0031 - 0.0045$, are in perfect agreement with the Planck 2018 data. For sub-Planckian values of the field the $Z_n$ symmetry is only allowed for $n \leq 4$.

1 Introduction

One of the most favored inflationary model according to Planck 2018 results $^1$ is the Starobinsky model $^2$. The scalar field version of this model is equivalent to an inflationary model which exploits a strong non-minimal coupling of the scalar field with gravity. See for example $^3$ $^4$ for a few of the non-supersymmetric models of non-minimal Higgs inflation. In order to realize non-minimal inflation in supersymmetric framework a special form of Kähler potential is employed. For the feasibility of realizing inflation with standard model like Higgs boson in the minimal supersymmetric standard model see $^5$ $^6$ $^7$. Further this idea has also been applied to Higgs fields in grand unified theories (GUTs) $^8$ $^9$.

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$^1$E-Mail: atifmasood23@gmail.com
$^2$ E-Mail: mansoor@qau.edu.pk
$^3$ E-Mail: azeem_92@live.com
The supersymmetric hybrid inflation model provides an elegant framework to incorporate GUTs [10][11][12]. However, the standard version of supersymmetric hybrid inflation is plagued with the monopole problem which is a generic prediction of GUTs based on a simple gauge group. In this paper we effectively consider a model of non-minimal GUT Higgs inflation with a special form of Kähler potential that is usually employed in no-scale supergravity models [13]. In this model monopoles are produced during inflation and are inflated away. The viability of non-minimal inflation is explored in a broader context with an additional $Z_n$ symmetry. The predictions of various inflationary parameters are obtained in a generic GUT framework and are consistent with the Planck 2018 results.

2 Superpotential with $U(1)_R \times Z_n$ Symmetry

In a typical supersymmetric hybrid inflation framework based on a given GUT gauge group, $G$, we usually consider a gauge singlet superfield, $S$, along with a gauge non-singlet conjugate pair of Higgs superfields $H$ and $\bar{H}$. Some of the examples of the GUT gauge groups are $SO(10)$, $SU(5) \times U(1)$ and $SU(4)_c \times SU(2)_L \times SU(2)_R$ with Higgs superfields residing in the 16, 10 and $(4,1,2)$ dimensional representations of the respective gauge groups [14]. With this minimal content of superfields and the $U(1)_R \times Z_n$ global symmetry we obtain the following simple form of the superpotential [15, 16],

$$W = \kappa S \left( -\mu^2 + \frac{(H\bar{H})^m}{\Lambda^{2m-2}} \right). \tag{1}$$

Here, $\kappa$ is a dimensionless coupling, $\mu$ is some superheavy mass and $\Lambda$ is the cut-off scale. Under $Z_n$ symmetry the superfield, $S$, carries zero charge whereas Higgs superfields carry unit charges. This makes the integer $m = n$ for odd values of $n$ and $m = n/2$ for even values of $n$ [16]. For example, values of $m = 1, 2, 3$ correspond to $n = 2, 4, 3$ respectively. However, for the special case of $m = 1$ we do not need to impose any $Z_2$ symmetry as GUT gauge symmetry alone is sufficient to restrict the form of the superpotential. Further, the superfield $S$ and the superpotential $W$ carry one unit of $R$ charge whereas $H\bar{H}$ is neutral under $U(1)_R$ symmetry. This $R$ charge assignment ensures a linear relationship of $W$ in terms of $S$ which is necessary to realize a consistent model of inflation [10].

The global supersymmetric minimum occurs at

$$\langle S \rangle = 0, \quad \langle (H\bar{H})^m \rangle = M^{2m} \equiv \mu^2 \Lambda^{2m-2}, \tag{2}$$
where the Higgs vacuum expectation value (VEV) is described by $\mathcal{M}$. This gauge symmetry breaking scale is taken to be the GUT scale, $M_{\text{GUT}} \equiv 2 \times 10^{16}$ GeV, in our numerical calculations. Further we set $\Lambda = m_P$ where $m_P = 2 \times 10^{18}$ GeV is the reduced Planck mass.

The form of the superpotential considered above has been used before mostly in the context of new inflation in a supersymmetric framework. Once the field $S$ is stabilized we obtain an effective Higgs potential which, for values of fields below $M$, can be used for new inflation. For example, see [15] where it is used to realize pre-inflation in order to justify the initial conditions of new inflation. In ref. [16], it was used to realize a model of new inflation itself. In ref. [17], flavon inflation is discussed using a similar form of the superpotential. For $SU(5)$ and flipped $SU(5)$ based GUT realization of new inflation see [18]. In this paper, however, we consider the other side of the Higgs potential where field values lie above $M$ and the potential is steep. A special form of the Kähler potential, which is usually employed in the no-scale gravity models, helps to reduce the slope of the potential and makes it suitable for the slow-roll conditions to apply. This setup gives rise to non-minimal Higgs inflation which is discussed below in detail with additional $Z_n$ symmetry.

3 Non-minimal Higgs Inflation with $Z_n$ Symmetry

To achieve non-minimal inflation we consider the following special form of the Kähler potential

$$K = -3m_P^2 \log \left( 1 - \frac{|S|^2 + |H|^2 + |\bar{H}|^2}{3m_P^2} \right) + \frac{\chi}{2m_P^2} \left( \frac{(H\bar{H})^n}{\Lambda^{2m-2}} + \text{h.c} \right) + \frac{\gamma |S|^4}{3m_P^2},$$

(3)

where $\chi$ and $\gamma$ are dimensionless parameters. This is a variant of the Kähler potential usually employed in the no-scale supergravity models where moduli fields are assumed to be stabilized [13]. The addition of last term is necessary for the stabilization of $S$ field [7]. The scalar potential and the metric in Jordan and Einstein frames are related via the conformal rescaling factor $\Omega^2 = e^{-K/3m_P^2}$ as,

$$V_J = \Omega^4 V_E, \quad g_{\mu\nu}^J = \Omega^2 g_{\mu\nu}^E.$$  

(4)

This defines the Einstein-frame scalar potential $V_E$ in terms of $W$ and $K$ as

$$V_E = e^{K/m_P^2} \left( K_{ij}^{-1} D_iW D_jW^* - 3m_P^{-2} |W|^2 \right) + V_E^D,$$  

(5)
where

\[ D_z W = \frac{\partial W}{\partial z_i} + \frac{1}{m_P^2} \frac{\partial K}{\partial z_i} W, \quad K_{ij} = \frac{\partial^2 K}{\partial z_i \partial z_j}, \quad D_z W^* = (D_z W)^*, \quad (6) \]

with \( z_i \in \{ S, H, \overline{H} \} \). Here, same notation has been used for the superfields and their scalar components. The Einstein-frame D-term potential is given by

\[ V_E^D \propto g^2 (|H|^2 - |\overline{H}|^2). \quad (7) \]

Writing complex Higgs fields in terms of real scalar fields,

\[ H = \frac{\phi}{\sqrt{2}} e^{i\alpha} \cos \beta, \quad \overline{H} = \frac{\phi}{\sqrt{2}} e^{i\alpha} \sin \beta, \quad (8) \]

the stabilized D-flat direction is obtained for \( \beta = \pi/4, \alpha = \overline{\alpha} = 0 \) and this implies that

\[ H = \overline{H} = \frac{\phi}{2^i}, \quad (9) \]

where \( \phi \) is the canonically normalized real scalar field in the Jordan frame. Finally the scalar potential in the Einstein frame takes the following form

\[ V_E = \frac{\kappa^2 \mu^4 \left( 1 - \left( \frac{\phi}{2M} \right)^{2m} \right)^2}{\left( 1 - \frac{2}{3} \left( \frac{\phi}{2m_P} \right)^2 + \chi \left( \frac{\phi}{2m_P} \right)^{2m} \right)^2}. \quad (10) \]

After conformal rescaling the canonically normalized inflaton field \( \hat{\phi}(\phi) \) in the Einstein frame becomes a function of field \( \phi \) as

\[ J(\phi) \equiv \left( \frac{d \hat{\phi}}{d \phi} \right) = \sqrt{\frac{1}{\Omega^2(\phi)} + \frac{3}{2} m_P^2 \left( \frac{d \ln \Omega^2(\phi)}{d \phi} \right)^2}. \quad (11) \]

The slow-roll parameters can now be expressed in terms of \( \phi \) as

\[ \epsilon(\phi) = \frac{1}{2} m_P^2 \left( \frac{V_E'}{JV_E} \right)^2, \quad \eta(\phi) = m_P^2 \left( \frac{V_E''}{J^2 V_E} - \frac{J' V_E'}{J^3 V_E} \right), \quad (12) \]

where a prime denotes a derivative with respect to \( \phi \). The scalar spectral index \( n_s \) and the tensor to scalar ratio \( r \) to the first order in slow-roll approximation are given by

\[ n_s \simeq 1 - 6\epsilon(\phi_0) + 2\eta(\phi_0), \quad r \simeq 16 \epsilon(\phi_0), \quad (13) \]
Figure 1: The variation of field value $\phi_0$ versus $\kappa$ for $m = 1, 2, 3$ and $N_0 = 50$(left panel), 60(right panel). We set the gauge symmetry breaking scale $M = 2 \times 10^{16}$ GeV.

where the field value, $\phi_0$, corresponds to the number of e-folds,

$$N_0 = \frac{1}{\sqrt{2m_P}} \int_{\phi_e}^{\phi_0} \frac{J(\phi)}{\sqrt{\epsilon(\phi)}} d\phi,$$

before the end of inflation at $\phi = \phi_e$ defined by the condition $\epsilon(\phi_e) = 1$. Also, $\phi_0$ corresponds to the pivot scale where the amplitude of the scalar power spectrum is normalized by Planck to be,

$$A_s(k_0) = \frac{1}{24 \pi^2 \epsilon(\phi)} \left| V_E(\phi) \right|_{\phi(k_0) = \phi_0} = 2.137 \times 10^{-9},$$

at $k_0 = 0.05$ Mpc$^{-1}$. For non-minimal inflation with sub-Planckian values of the field we need to consider the large $\chi$ limit such that $\chi \left(\frac{\phi}{m_P}\right)^{2m} \gg 1$. Therefore, in the non-minimal limit with $\phi_0 \gg M$, above relation can be used to eliminate $\kappa$ in terms of $\phi_0$ as

$$\kappa \simeq \chi \sqrt{24\pi^2 A_s(k_0) \epsilon(\phi_0)} \simeq \sqrt{32\pi^2 A_s(k_0)} \left(\frac{2m_P}{\phi_0}\right)^{2m}. \quad (16)$$

Now we look for a relation of $\phi_0$ in terms of $N_0$. Using Eq. (14) and $\epsilon(\phi_e) = 1$, the field values $\phi_0$ and $\phi_e$ can be written in terms of $N_0$ as

$$\frac{\phi_0}{2m_P} \simeq \left(\frac{4N_0}{3\chi}\right)^{1/2m}, \quad \frac{\phi_e}{2m_P} \simeq \left(\frac{4}{3\chi^2}\right)^{1/4m}. \quad (17)$$

Therefore, field values are expected to change with $m$ or $Z_n$ symmetry. This is confirmed by the exact numerical results shown in the Fig. (1) for the variation of $\phi_0$ with respect to $\kappa$. With sub-Planckian field values $\phi_0 \lesssim m_P$. 

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we obtain $\chi \gtrsim \frac{2^{2m+2}}{3} N_0 \gg 1$ which provides a cross-check for using the large $\chi$ limit at first place. Using again above value of $\phi_0$ in Eq. (16) we obtain a constant value for the ratio $\kappa/\chi$ written in terms of $N_0$ as

$$
\frac{\kappa}{\chi} \simeq \frac{3\sqrt{2\pi^2 A_s(k_0)}}{N_0} \simeq \begin{cases} 
1.23 \times 10^{-5} & \text{for } N_0 = 50, \\
1.03 \times 10^{-5} & \text{for } N_0 = 60.
\end{cases}
$$

This ratio turns out to be of order $10^{-5}$ showing a weak dependence on $m$ or $Z_n$ symmetry in the non-minimal limit and this can also be seen in our numerical results displayed in the Fig. [2]. We can express field value $\phi_0$ in terms of $\kappa$ using Eq. (18) in Eq. (17) as

$$
\frac{\phi_0}{m_P} \simeq 2 \left( \frac{4N_0/\kappa}{3 \times 10^5} \right)^{1/2m}.
$$

For a given value of $\kappa$, this expression explains the observed increasing trend of field values with respect to $m$ as shown in Fig. [1] This trend leads to fine tuning in the solutions with large values of $m$ as soon as $\phi_0$ becomes tran-Planckian. Therefore, we allow $m \leq 3$ or $n \leq 4$ for $\phi_0 \lesssim m_P$ with perturbative values of $\kappa \lesssim 0.1$. For $SU(5)$ GUT with Higgs field in the adjoint representation we expect to obtain two more solutions for $m = 3/2$ and $m = 5/2$ effectively.

Finally, the expression of $\phi_0$ is used to obtain the scalar spectral index $n_s$ and the tensor to scalar ratio $r$ in terms of $N_0$,

$$
n_s \simeq 1 - \frac{2}{N_0} \simeq \begin{cases} 
0.960 & \text{for } N_0 = 50, \\
0.967 & \text{for } N_0 = 60,
\end{cases} \quad r \simeq \frac{12}{N_0^2} \simeq \begin{cases} 
0.0048 & \text{for } N_0 = 50, \\
0.0033 & \text{for } N_0 = 60.
\end{cases}
$$
For \( N_0 = 50 \)

| \( m \) | \( r \) | \( n_s \) | \( \phi_0 \) | \( \phi_e \) | \( \chi \) |
|------|------|------|------|------|------|
| 1    | 0.0045 | 0.960 | \( 5.5 \times 10^{17} \) | \( 7.4 \times 10^{17} \) | 8194 |
| 2    | 0.0051 | 0.957 | \( 1.4 \times 10^{18} \) | \( 4.0 \times 10^{17} \) | 7718 |
| 3    | 0.0051 | 0.957 | \( 2.1 \times 10^{18} \) | \( 8.6 \times 10^{17} \) | 7693 |

For \( N_0 = 60 \)

| \( m \) | \( r \) | \( n_s \) | \( \phi_0 \) | \( \phi_e \) | \( \chi \) |
|------|------|------|------|------|------|
| 1    | 0.0031 | 0.966 | \( 5.7 \times 10^{17} \) | \( 7.2 \times 10^{16} \) | 9848 |
| 2    | 0.0035 | 0.965 | \( 1.4 \times 10^{18} \) | \( 3.8 \times 10^{17} \) | 9337 |
| 3    | 0.0035 | 0.964 | \( 2.2 \times 10^{18} \) | \( 8.3 \times 10^{17} \) | 9331 |

Table 1: The predicted values of inflationary parameters with gauge symmetry breaking scale \( M = 2 \times 10^{16} \text{GeV} \) and \( \kappa = 0.1 \).

where, \( \epsilon(\phi_0) = \frac{3}{4N_0^2} \) and \( \eta(\phi_0) = -\frac{1}{N_0} \). This result holds in the leading order approximation and also explains the weak dependence of \( n_s \) and \( r \) on \( m \) or \( Z_n \) symmetry as confirmed by our numerical estimates displayed in Table-I. We show the predictions of various inflationary parameters in Table-I, using first order slow-roll approximation, for \( \kappa = 0.1, M = 2 \times 10^{16} \text{ GeV} \) and \( N_0 = (50, 60) \). We obtain \( n_s \approx 0.96 (0.966) \) and \( r \approx 0.0045 (0.0031) \) for \( N_0 = 50 (60) \) respectively, independent of \( m \) and \( \kappa \) values. The non-minimal coupling parameter is large \( \chi \sim 10^5 \) and this is a common feature of these models. An order of magnitude estimate of error expectancy in inflationary parameters can be calculated from the second order slow-roll contribution. This can be described as a fractional change in the corresponding quantity, e.g., \( \Delta n_s/n_s \approx 0.01\%, \Delta r/r \approx 1.5\%, \Delta \phi/\phi \approx 0.4\%, \Delta \chi/\chi \approx 1\% \).

4 Conclusion

We have studied a class of models based on the realization of non-minimal inflation in \( R \)-symmetric supersymmetric hybrid inflation framework with an additional \( Z_n \) symmetry. The requirement of sub-Planckian field values is satisfied in the large \( \chi \) limit. This also restricts the possible values of \( n \leq 4 \) with \( \kappa \lesssim 0.1 \). We have calculated the predictions of \( n_s \) and \( r \) numerically and also provided the analytic justification of these results. Finally, we conclude that the results of non-minimal inflation hold in a rather broad class of supersymmetric GUT models.
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