An Alternative Tuning Scheme for Simple Adaptive Flight Control System

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Abstract. The occurrence of system failure or of external disturbances are problem that must be considered in the design of safe flight control systems. Adaptive control schemes show the ability to handle such issues without any a priori knowledge about the fault or disturbance. The main condition for obtaining a stable adaptive controller is the passivity of the plant but most real system do not satisfy this condition. The use of properly defined Parallel Feedforward Compensator (PFC) can allow the augmented system to meet the passivity requirements. An alternative design approach for tuning the PFC and the invariant gains of the Simple Adaptive Controller (SAC) is presented in this work. The tuning procedures is a modification of the PSO and is called Population Decline Swarm Optimization (PDSO) since it takes into account a decline demographic model to speed up the tuning steps. The method is applied to adaptively control the pitch response of an aeroplane during level flight. Tuning results are presented along with flight mechanics simulation taking into account different command laws.

1. Introduction
Modern control systems, as the fly by wire, have become a standard for most aircrafts leading to an increase in term of safety and performance. With the increase of flight control systems complexity the requirement for safety and reliability has become more and more stringent [1, 2]. It has stemmed the need to design control systems capable of tolerating potential faults and disturbances as well. For such reason, robust, fault-tolerant or adaptive flight control systems are usually investigated [3, 4]. Different Adaptive Control techniques have been developed and applied to manage the changes that the aircraft may undergo during its flight. Among these, the following may be cited: Viswanathan and Lakshmi [5] that redesign an autopilot system using Sliding Mode Control; Lee et al. [6] that study an L1 adaptive control capable of recovering nominal performance in the presence of failures and disturbances; Nivison and Khargonekar [7] that present a Model Reference Adaptive Control system based on Neural network to take into account uncertainties; and Nishiyama et al. [8] that propose the use of Simple Adaptive Control to realize fault tolerant flight systems.

The use of the adaptive control requests for the passivity of the system which, for the SAC problem, can be identified by the Almost Strictly Positive Realness condition for the transfer functions that model the plant [9]. However, real systems generally do not satisfy such condition. For this reason Barkana [9] has proposed the use of a Parallel Feedforward Compensator to render the augmented plant ASPR.

An alternative design procedure for synthesizing the PFC and tuning the SAC invariant gains is studied in this work. The procedure, called PDSO - Population Decline Swarm Optimizer, is a modification of the particle swarm optimization (PSO) [10] that takes into account a population decline model [11] allowing the use of a large swarm for the exploration steps and then reducing it during the exploitation. The tuned adaptive scheme is then applied to control aircraft pitch motion,
whose model is taken from literature [12]. Outline on the simple adaptive flight control system is given in section 2; the population decline swarm optimizer is briefly described in section 3 while tuning and simulation results are commented in section 4.

2. Simple Adaptive Flight Control System

Let us assume that the aircraft longitudinal behaviour can be represented by a simplified controllable and observable SISO linear model through the following transfer function [12] that links the elevator deflection \( \delta \) to the aircraft pitch angle \( \theta \) as

\[
G_s(s) = \frac{4.22s^2 + 4.31s + 0.212}{s^4 + 3.01s^3 + 6.96s^2 + 0.232s + 0.224}
\]

Such model is obtained by linearizing the aircraft longitudinal dynamic about the steady level flight trim conditions given by true airspeed 66.5 m/s and angle of attack of 4.98 deg at an altitude of 1524 m with constant thrust. Let now assume that the plant has to follow a stable reference model that can be written as a first order transfer function whose output \( y_m \) gives the state \( x_m \) while the bounded input scalar signal is \( u_m \).

The SAC scheme is similar to the MRAC (Model Reference Adaptive Control) but it requests for an output feedback term that allows to build the input control signal as a linear combination of the model reference input, of the model reference state vector and of the output tracking error \( e(t) = y(t) - y_m(t) \) and writes as

\[
u(t) = K_e(t)e(t) + K_u(t)u_m(t) + K_x(t)x_m(t)
\]

The terms \( K_i(t) \), with \( i = \{e, um, xm\} \) are the adaptive gains of the SAC, obtained by summing a proportional and an integrative terms, and each is properly defined as

\[
K_i(t) = K_{i,prop} + K_{i,int}
\]

where \( \Gamma_i \) are the invariant gains of the SAC algorithm. The condition requested for stability of the adaptive controller is the passivity of the plant. This means that the plant transfer function must be at least Almost Strictly Positive Real (ASPR) which, for a SISO transfer function, implies that all zeroes are negative, the relative degree is 1 and that the leading coefficient is positive. It is to be said, however, that almost all real systems do not satisfy the ASPR. Such problem is tackled by adding to the plant a PFC such that the augmented system meets the passivity requirements. Moreover the PFC transfer function must be such that the output of the augmented plant \( G_a = G_0 + G_{PFC} \) is almost equal to the original plant output, namely \( y_a = y + y_{PFC} \approx y \). For more detail about the SAC algorithm, the ASPR and PFC and for the proof of stability, the interested reader is referred to the literature [9]. The block diagram representation of the plant augmented by the parallel feedforward compensator and controlled by SAC to follow the reference model is shown in Figure 1.

![Block diagram representation of the controlled plant.](image-url)
3. Population Decline Swarm Optimizer

In order to synthesize the parallel feedforward compensator and to properly select the invariant parameters of the SAC scheme the population decline swarm optimizer is used. The PD\textsubscript{SO} is a modification of the particle swarm optimization and it relies upon the simplified social model that describes a swarm of birds (the particles) looking for food (minimum/maximum value of the objective function) while flying on field (the search space). More in detail, each particle \( p \) represents a position vector in the \( n \)-dimensional search space and, after a random initialization, it moves according to the following law

\[
p_{i+1} = p_i + v_{i+1}
\]

where \( \lambda = \{1, 2, \ldots, \Lambda \} \) is the iteration step being \( \Lambda \) the max number of iteration while \( i = \{1, 2, \ldots, P_\lambda \} \) labels the \( i \)-th particle of the swarm and \( P_\lambda \) is the number of particle present in the swarm at the iteration \( \lambda \). The updated particle velocity in equation (4) writes as

\[
v_{i+1} = \rho \left[ \mu A v_A + c_s r_1 (p_b^i - p_i^i) + c_r r_2 (p_b^i - p_g^i) \right]
\]

being \( c_i \) the cognitive acceleration constant, \( c_s \) the social constant, \( p_b^i \) is the current best position attained by any of the particle of the swarm while \( p_b^i \) is the best position of all the swarm; \( r_1 \) and \( r_2 \) are random numbers in the interval \([0; 1]\); \( \mu \) and \( \chi \) are the linear decreasing inertia weight and the constriction factor [11], respectively. Last, the decline population model introduced to modify the standard PSO read as

\[
P_\lambda = \begin{cases} P_{\text{max}} & \text{if } \frac{\lambda}{\Delta} = \left\lfloor \frac{\lambda}{\Delta} \right\rfloor \\ P_{\lambda-1} & \text{otherwise} \end{cases}
\]

being \( \Delta \) the number of iterations during which the swarm size is constant, \( \xi \) is the percentage reduction of the population and \( \lfloor x \rfloor \) is the integer part of the variable \( x \) and \( \lceil -x \rceil = -\lfloor -x \rfloor \). It is worth noting that the use of the decline population model allows to use a large size swarm during the initialization to increase the exploration capability of the swarm and allows to continuously reduce the number of particles in the swarm during the iterations to speed up the procedure with the objective of minimizing an \textit{a priori} chosen fitness function by reducing the overall computational cost.

More in detail, with reference to the problem described in the previous Section 2, the PD\textsubscript{SO} is applied twice to design both the parallel feedforward compensator transfer function \( G_{\text{PFC}} \) and to select the SAC invariant parameters \( \Gamma_k \). The fitness function to be minimized to synthesize the PFC transfer function is a modified norm of the discrepancy between the augmented \( y_a \) and the actual \( y \) systems outputs undergoing a unitary step input excitation, \( u \). The PFC is written as a first order transfer function having DC gain \( \kappa_{\text{PFC}} \) and time constant \( \tau_{\text{PFC}} \), that are the element of the particle vector \( p \) while the fitness function is

\[
\mathcal{J}_{\text{PFC}} (p) = \phi(p) \left[ \int_{t_0}^{t_\text{a}} [y_a(p, t) - y(t)]^2 dt \right]^{\frac{1}{2}}
\]

being \( \phi(p) \) a penalty function that equals 1 if \( G_p \) meets the ASPR condition and becomes numerically infinite otherwise. This approach allows to constraint the optimization problem, that specifies as in equation. (8), in such a way to look for ASPR augmented plant only.

\[
\min_{p} \mathcal{J}_{\text{PFC}} (p)
\]

\[
\text{S.T. } p \in [p_{\text{PFC,min}}, p_{\text{PFC,max}}]
\]

\[(8)\]
On the other hand, in order to define the SAC invariant parameters, the particle position vector is defined as \( p = [\Gamma_s, \Gamma_m, \Gamma_{sm}] \) and the minimization problem write as

\[
\min_{s.t. p \in [p_{SAC,min}, p_{SAC,max}]} \mathcal{I}_{SAC}(p)
\]

where the objective function specifies as

\[
\mathcal{I}_{SAC}(p) = \int_{t_0}^{t_f} \left[ a e^2(p,t) + b \delta^2(p,t) \right] dt
\]

In equation 10 \( \alpha \) and \( \beta \) are weight parameters while \( T_w \) is the integration time window.

4. Results

Simulation results are summarized in this section. At first PFC and SAC tuning results by mean of the Population Decline Swarm Optimizer are presented and commented, successively flight mechanics simulation results are given.

4.1. PFC tuning results

In order to synthesize the PFC by using the PDPSO, the particle is defined as \( p = [\kappa_{PFC}, \tau_{PFC}] \), the minimum and maximum value of the search space are set to 0.01 and 1 for both particle elements (in accord with [13]) while the time window extent to compute the index to be minimized, namely equation 7, is set to 600 s. The parameters of the optimization algorithm are: number of swarm = 1; cognitive constant \( c_c = 2.05 \); social constant \( c_s = 2.05 \); minimum and maximum of inertia weight are 0.4 and 0.9, respectively; maximum number of iterations \( \Lambda = 20 \); time for population decline \( \Delta = 3 \); percentage reduction of the population \( \xi = 0.5 \); initial population size \( P_{max} = 10 \). To account for the stochastic nature of the PDPSO, 50 optimizations have been run. Figure 2 shows convergence history of the fitness function equation 7. It can be appreciated the random initialization and that after the fifth iteration the convergence is met.

![Figure 2. PFC tuning convergence (five representative curves).](image)

Table 1 collects the results of the minimization procedure carried out using the proposed population decline swarm optimizer and the standard PSO using the same optimizer parameters previously given. It can be noted that the proposed optimizer, that takes into account a decline model for the population of the swarm, manages to find the same results of the standard (constant population) particle swarm optimizer in terms of minimum value of the objective function equation 7 and values of the best position particle.
Table 1. PFC tuning results.

|                | PdSO | PSO |
|----------------|------|-----|
| $\min \mathcal{J}_{PFC}$ | 0.495 | 0.495 |
| $p_{PFC}$      | [0.01,1] | [0.01,1] |
| $t_{PFC}/t_{PSO}$ | 0.27  | 1   |

On the contrary it is worth noting that, in the present case, the PdSO requests for a computational time corresponding to the 27% of the one needed by the standard PSO and resulting in an increased computational efficiency. The comparison of step responses of original $G_0$ and PFC augmented $G_a=G_0+G_{PFC}$ plants is shown in Figure 3 evidencing that the parallel feedforward compensator does not modify the plant response. This is true at low frequencies, below 100 rad/s where phugoid and short period dynamics lie. However the PFC makes the plant ASPR allowing for the application of the simple adaptive control technique.

Figure 3. Unitary step response: comparison of original and PFC augmented plant.

4.2. SAC tuning results

To synthesize the SAC controller, a smoothed square wave defined in equation 11 is passed as command $u_m$ to the reference model. The amplitude of the square wave is $A_{sw}=2$ deg, the circular frequency is $\omega_{sw}=0.1$ rad/s while smoothing parameter $\rho_{sw}$ is set to 50.

$$\delta_c = \frac{2A_{sw}}{\pi} \tan^{-1}\left[\rho_{sw} \sin(\omega_{sw} t)\right]$$

The reference model transfer function is defined as $G_{ref}=(0.05s+1)^{-1}$ while the computation time window is $T_0=10$ s. The PdSO parameters are set as: number of swarm $= 1$; cognitive constant $c_s=2.05$; social constant $c_c=2.05$; minimum and maximum of inertia weight are 0.4 and 0.9, respectively; maximum number of iterations $\Lambda = 20$; time for population decline $\Delta = 5$; percentage reduction of the population $\xi=0.8$; initial population size $P_{max} = 20$. To account for the stochastic nature of the PdSO, 50 optimizations have been run. The minimum and maximum values of the search domain are set to 0.001 and 100, respectively, for all SAC parameters while the weights of the objective function equation 10 are set as $\alpha=1$ and $\beta=0.04$.

The convergence history of the objective function equation 10 is given in figure 4 showing the random initialization and the convergence after about five iteration. The optimization results are collected in Table 2 and, even in this case, the PdSO results match well with the standard PSO solution, showing a speed up ratio $t_{PdSO}/t_{PSO}=0.71$ that confirms a reduced computational effort of the proposed decline population scheme with respect to the standard procedure.
Figure 4. SAC tuning convergence (five representative curves).

Table 2. SAC tuning results

|       | P_{SO} | PSO       |
|-------|--------|-----------|
| min \( \zeta_{PFC} \) | 0.447  | 0.447     |
| \( p_{SAC} \) | [24.7,0.001,0.277] | [25.0,0.001,0.272] |
| \( t/t_{PSO} \) | 0.71  | 1         |

Time history results for the ten second analyzed are given in figure 5 in terms of the aircraft pitch angle variation \( \theta \)commanded by the elevator deflection \( \delta \). It can be seen that the pitch angle never exceed the value of 2 deg and, after a transient response, reaches the reference model at about \( t=4 \) sec.

Figure 5. Time history of pitch angle (solid: plant response; dashed: reference).

4.3. Flight simulation results

Once the controller parameters have been set by using the proposed alternative optimization procedure that has allowed to minimize the chosen objective functions, the pitch dynamic behaviour of the controlled aircraft is investigated. Again, the reference model transfer function that the plant is request to follow is \( G_{ref}=(0.05s+1)^{-1} \), the smoothed square wave equation 11 is used as input while the computation time window is set to \( T_W=300 \) s.

The effects of the input command frequency and of the smoothing parameters on the aeroplane pitch and elevator deflection angle are investigated. In such study it is taken into account that plant transfer function models the behaviour of the experimental aircraft called MuPAL-\( \alpha \) [12]. The phugoid and short period frequencies are 0.0287 Hz and 0.343 Hz, respectively, while the elevator actuator (modelled as a first order transfer function) has a cut-off circular frequency of 1.59 Hz [14], maximum
deflection angle of 25 deg and minimum deflection angle of -30 deg [15]. The system transient response in terms of pitch angle $\theta$ and elevator deflection $\delta$ is plotted in figure 6 for the case $\omega_{ssw}=0.1$ rad/s and $\rho_{ssw}=75$.

![Figure 6. Time history of pitch angle and elevator deflection.](image1)

Looking at the figure 6 it can be concluded that the simple adaptive controller tuned by means of the PdSO is capable of leading the plant in such a way to closely follow the reference. The maximum pitch value in absolute value is 2.28 deg at about 31 seconds while the frequency of oscillation of pitch variable is about 1.51 Hz which is below the plant cut-off frequency of 1.59 Hz; on the other hand the maximum deflection reached by the elevator is about 20 deg and the transient behaviour is characterized by a frequency of 1.58 Hz that is close but below the cut-off one. A closeup representation of the transient behaviour of the system is given in Figure 7.

![Figure 7. Particular of transient response.](image2)

Last, the elevator command parameters $\omega_{ssw}$ and $\rho_{ssw}$ are let vary to study the influence of the command signal on the plant response. Results collected in Table 3 and 4 highlights that as the smoothing parameter $\rho_{ssw}$ increases, which means that the square wave becomes sharper, maximum value of the pitch angle increases over 2 deg. The maximum elevator deflection increases as well with the smoothing parameter and the same stands for the transient frequency of both the input $\delta$ and output $\theta$ variables of the plant. The effect of the actuation frequency $\omega_{ssw}$ is similar since all the investigated variables increase with it. In addition, it can be noted that the transient frequency of both $\delta$ and $\theta$
move towards the cut-off frequency in both cases. Moreover, looking at the maximum elevator deflection results it can be noted that when $\omega_{ssw}$=0.1 rad/s and $\rho_{ssw}$=100 the elevator deflection exceeds the limit value of 25 deg suggesting the use of a lower smoothing parameter when an actuation frequency higher or equal to 0.1 rad/s is requested.

| $\rho_{ssw}$ | $|\theta|_{max}$ [deg] | $f_\theta$ [Hz] | $|\delta|_{max}$ [deg] | $f_\delta$ [Hz] |
|------------|-----------------|-------------|-----------------|-------------|
| 25         | 1.99            | 0.04        | 2.6             | 1.1         |
| 50         | 2.08            | 0.08        | 5.7             | 0.94        |
| 75         | 2.12            | 1.17        | 8.7             | 1.15        |

Table 3. Influence of command parameters $\rho_{ssw}$ for $\omega_{ssw}$=0.05 rad/s

| $\rho_{ssw}$ | $|\theta|_{max}$ [deg] | $f_\theta$ [Hz] | $|\delta|_{max}$ [deg] | $f_\delta$ [Hz] |
|------------|-----------------|-------------|-----------------|-------------|
| 25         | 2.05            | 1.07        | 5.7             | 1.07        |
| 50         | 2.11            | 1.25        | 11.5            | 1.28        |
| 75         | 2.28            | 1.51        | 20.8            | 1.58        |

Table 4. Influence of command parameters $\rho_{ssw}$ for $\omega_{ssw}$=0.1 rad/s

5. Conclusions
An alternative method based on a modification of the particle swarm optimization has been presented in this work with the aim of tuning the PFC and the invariant gains of the SAC scheme. The simple adaptive controller has been employed to control the pitch dynamic of an aircraft during horizontal flight. Tuning results have shown that the search capability of the Population Decline Swarm Optimizer – PDSO is comparable with the standard PSO but have also evidenced a reduced computational effort of the proposed method. Numerical simulation have also been performed to investigate the influence of the input to the reference model on the aircraft response taking into account physics limitation of the system. Next steps of present work will deal with the analysis of the fault tolerant and disturbance rejection capabilities of the proposed controller taking into account the deflection limits of the elevator surface.

6. References
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