Modelling corners flow in rectangular microchannel

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Abstract. Rectangular microchannels are most common configuration in microfluidics. They can be used in many industries, for example in lab-on-chip devices. Despite standard fluid dynamics, microfluidics has a significant impact of wall boundary conditions on fluid flow. And in microfluidics, we cannot simply set no-slip boundary conditions if our goal is accurate modeling results. In rectangular microchannels, there is another important moment in modeling that is not present in circular pipes. The velocity profile of the fluid depends on the shear stress at the edges and the velocities at the walls of the microchannel change at different points of the cross-sectional wall of the microchannel. The fluid velocity is lower at the corners of a rectangular microchannel. In this paper, a solution is proposed to find a more accurate way to model the fluid flow in a rectangular microchannel by knowing the friction factor without shear stress distribution.

1. Introduction

Numerical computational methods have been intensively developed in the last decades. There are thousands of applications of numerical methods and microfluidics is one of them. The aim of the application in microfluidics is to provide an accurate way to predict the fluid, mass and heat transfer for various devices like Lab on Chip. Laminar fluid flow is a widely explored, but there is still much room for engineering improvement, especially in microfluidics, where new crunch points can be found in modelling due to boundary conditions.

In our previous work \cite{1}, we found the dependence of friction factor on Reynolds number for the rectangular microchannel and calculated the flow numerically by setting a constant shear stress on the rectangular microchannel walls (Figure 1). On the microchannel walls, we used a boundary condition for the shear stress instead of a no-slip condition, as we would have done for a macrochannel, since in a microchannel a no-slip condition does not give a suitable result. This method of modelling laminar flow in a rectangular microchannel, where the wall boundary conditions are important, shows good agreement with theory and good accuracy for variables such as pressure drop or volume flow rate. But this method is not accurate in modelling the corners of the microchannel. Figure 2 shows the velocity profile of the rectangular 108x242 μm\textsuperscript{2} microchannel for a Reynolds number of 500. It is obvious that the constant shear stress distribution does not give an accurate velocity profile, especially in the corners due to the high shear stress, despite accurate macroscopic quantities. It leads to the fact that the velocity of the fluid changes direction. This is the consequence of the dependence between shear stress and velocity:

\[ \tau_w = \frac{\partial u}{\partial \xi} \] (1)
where $\mu$ is the dynamic viscosity, $u$ is the flow velocity along the boundary, $\xi$ is the coordinate normal to the wall.

We have found a simple technical solution how to solve this problem without additional study of the shear stress distribution on the walls, using only the friction factor.

**Figure 1.** Scheme with rectangular microchannel a) constant shear stress on walls b) real shear stress distribution on walls.

**Figure 2.** Scheme with boundary conditions where the shear stress $\tau_w$ has a constant distribution on the walls. The results for $Re=500$ show artifacts in the corners and on the right and left walls, where the flow velocity should not be reversed.

2. **Calculation**

One of the common quantities that can determine the pipe or channel throughput is the friction factor. The article [4] contains a review of friction factors from the literature for different Reynolds numbers. The friction factor can be estimated from dimensional analysis [3]:

$$f = \frac{\Delta p}{\rho \cdot \frac{D}{4} \cdot \frac{V^2}{2}}$$

where $D$ - the hydraulic diameter, $l$ - the length of the channel, $\Delta p$ – the pressure drop, $\rho$ – the density and $V$ – the mean velocity.
For pipes, there is a simple dependence between friction factor and shear stress, which can be obtained from Newton’s law [2]:

\[
\tau = \frac{1}{8} \rho f |V| \tag{3}
\]

where \(\rho\) is the density, \(V\) is the mean flow velocity, \(f\) is the friction factor.

However, for rectangular channels, due to equation (1), shear stress \(\tau\) cannot be constant and equation (3) must be more complicated.

In this work, the laminar 3D flow in 108x242 \(\mu\)m\(^2\) rectangular microchannel was calculated using CFD SIMPLE method with a residual parameter of 10\(^{-4}\). The grid of the microchannel was made of hexagonal elements with a side size of 5\(\mu\)m. The fluid density, viscosity and pressure drop in the microchannel were set. The shear stress determined in the paper [1] was used as the mean shear stress.

The fluid flow with Re=500 was calculated with no-slip boundary condition, constant shear stress boundary condition and with non-constant shear stress. For the non-constant shear stress boundary condition, the shear stress distribution was given by the results of the solution with slip-free wall boundary condition. The diagram of the shear stress distribution for the no-slip wall boundary condition is shown in Figure 3 for the y-direction, and in Figure 4 for the z-direction.

**Figure 3.** The dependence of the wall shear stress on the y-coordinate direction.

Using the results shown in Figures 3 and 4, a function approximation was made and used in the next calculation with the same shear stress distribution but a different mean value (Equation (4)).

\[
\tau = \frac{\tau_w}{\langle f(y,z) \rangle} \cdot f(y,z) \tag{4}
\]

where \(f(y,z)\) – the function from Figures 3 and 4, \(\langle f(y,z) \rangle\) - the mean value of this function, \(\tau_w\) – the mean shear stress estimated using the friction factor and equation (3).
3. Results
The comparison between constant shear stress and non-constant shear stress is shown on Figures 5 and 6. As can be seen, this method provides a simple and effective way to increase the accuracy of the fluid flow calculation when a specific shear stress is to be set on the walls.

4. Conclusion
This article is devoted to CFD modeling of a fluid flow in a rectangular microchannel, where a certain shear stress on the walls is set. The presented method is simple and can be used also for microchannels with other cross-sectional shapes. The average shear stress can be estimated using the friction factor, and the distribution of shear stress can be obtained from calculation with no-slip condition at the wall. The results show that this method gives more accurate results in the numerical flow calculation.
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References
[1] Gluzdov D S, Gatapova E Y 2021 Physics of Fluids 33(6) 062003.
[2] Chaudhry M H 2014 Applied Hydraulic Transients (New York: Springer) p 45
[3] Munson B R, Okiishi T H, Huebsch W W, Rothmayer A P 2013 Fundamentals of fluid mechanics (Jefferson City: John Wiley & Sons) p 413
[4] Kohl M J, Abdel-Khalik S I, Jeter S M, Sadowski D L 2005 Int. J. Heat Mass Transfer 48 1518-1533