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BORN ORDER STUDY OF $\gamma^*\gamma^* \rightarrow pp$ AT VERY HIGH ENERGY

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We calculate the cross-section for the diffractive exclusive process $\gamma^*_L(Q_1^2)\gamma^*_L(Q_2^2) \rightarrow \rho^0_L\rho^0_L$, in view of its study in the future high energy $e^+e^-\rightarrow$ linear collider. The Born order approximation of the amplitude is completely calculable in the hard region $Q_1^2, Q_2^2 \gg \Lambda_{QCD}^2$. The resulting cross-section is large enough for this process to be measurable with foreseen luminosity and energy, for $Q_1^2$ and $Q_2^2$ in the range of a few $GeV^2$.

1 Motivation

High energy QCD dynamics may be tested by the next generation of $e^+e^-\rightarrow$-colliders. Exclusive processes such as double vector meson production by two highly virtual photons give access to the kinematical regime in which the perturbative approach is justified. If additionally one selects the events with comparable photon virtualities, the perturbative Regge dynamics of QCD of the BFKL$^{[1]}$ type should dominate with respect to the conventional partonic evolution of DGLAP type. We propose$^{[2]}$ to study the electroproduction of two longitudinally polarized $\rho-$mesons,

$$\gamma^*_L(q_1) \gamma^*_L(q_2) \rightarrow \rho^0_L(k_1) \rho^0_L(k_2), \quad (1)$$
for arbitrary values of $t = (q_1 - k_1)^2$, with $s \gg -t$. The Born approximation estimate of the cross section proves the feasibility of a dedicated experiment. The extension of this study by taking into account of BFKL evolution is given in \[\text{3}\]. The case of transverse photon polarizations will be studied soon. At lower energy, some experimental data exist for $Q_2^2$ small\[\text{4}\] which may be analysed in terms of generalized distribution amplitudes.

## 2 Impact representation

The impact representation of the scattering amplitude has the form

$$
\mathcal{M} = i s \int \frac{d^2 k}{(2\pi)^4} \frac{1}{(L - k)^2} \mathcal{J}^{\gamma^*_1(q_1) \to \rho^0(k_1)}(k, L - k) \mathcal{J}^{\gamma^*_2(q_2) \to \rho^0(k_2)}(-k, -(L + k)),
$$

where $\mathcal{J}^{\gamma^*_1(q_1) \to \rho^0(k_1)}(k, L - k)$ and $\mathcal{J}^{\gamma^*_2(q_2) \to \rho^0(k_2)}(k, L - k)$ are the impact factors corresponding to the transition of $\gamma^*_1(q_1) \to \rho^0(k_1)$ and $\gamma^*_2(q_2) \to \rho^0(k_2)$ via the $t$–channel exchange of two gluons. The amplitude depends linearly on $s$, since these impact factors are $s$-independent. Calculations of the impact factors in the Born approximation are standard; they read, using the notation $\bar{z} = 1 - z$,

$$
\mathcal{J}^{\gamma^*_1(q_1) \to \rho^0(k_1)}(k, L - k) = \int_0^1 dz z \bar{z} \phi(z) 8\pi^2 \alpha_s \frac{e^{\delta_{ab}}}{\sqrt{2} N_c} Q_i f_\rho P_P(z, k, \bar{k}, \mu_i),
$$

where

$$
P_P(z, k, \bar{k}, \mu_i) = \frac{1}{z_1^2 k_1^2 + \mu_i^2} + \frac{1}{z_2^2 k_2^2 + \mu_i^2} - \frac{1}{(z_1 \bar{k}_1 - k)^2 + \mu_i^2} - \frac{1}{(z_2 \bar{k}_2 - k)^2 + \mu_i^2},
$$

originates from the impact factor of quark pair production from a longitudinally polarized photon, with two $t$-channel exchanged gluons. In formula the collinear approximation has been made, namely the relative transverse momenta of quarks with respect to the mesons momenta have been neglected in the hard part of the amplitude. We denote $\mu_i^2 = Q_i^2 z_i \bar{z}_i$ in the case of massless quarks.

In Eq.\[\text{3}\], $\phi$ is the distribution amplitude of the produced longitudinally polarized $\rho$–mesons. For simplicity, we use the asymptotic distribution amplitude

$$
\phi(z) = 6 z (1 - z).
$$

The only remaining part of the quark phase space after applying the collinear approximation are the integration with respect to quark longitudinal fractions of meson momenta $z_1$ and $z_2$.

Combining Eqs.\[\text{2} \text{ } \text{3} \text{ } \text{4}\], the amplitude can be expressed as

$$
\mathcal{M} = i s 2 \pi \frac{N_c^2 - 1}{N_c} \alpha_s^2 \alpha_{em} f_\rho^2 Q_1 Q_2 \int_0^1 dz_1 dz_2 z_1 \bar{z}_1 \phi(z_1) z_2 \bar{z}_2 \phi(z_2) M(z_1, z_2),
$$

with

$$
M(z_1, z_2) = \int \frac{d^2 k}{k^2 (L - k)^2} \left[ \frac{1}{z_1^2 k_1^2 + \mu_1^2} + \frac{1}{z_2^2 k_2^2 + \mu_1^2} - \frac{1}{(z_1 \bar{k}_1 - k)^2 + \mu_1^2} - \frac{1}{(z_2 \bar{k}_2 - k)^2 + \mu_1^2} \right] \times \left[ \frac{1}{z_1^2 k_1^2 + \mu_2^2} + \frac{1}{z_2^2 k_2^2 + \mu_2^2} - \frac{1}{(z_1 \bar{k}_1 - k)^2 + \mu_2^2} - \frac{1}{(z_2 \bar{k}_2 - k)^2 + \mu_2^2} \right].
$$

Note that at moderate values of $s$, apart from diagrams with gluon exchange in $t$–channel considered here, one should also take into account of diagrams with quark exchange. We do not consider them here since we restrict ourselves to the asymptotical region of large $s$. 

3 Forward cross section

In the simpler case \( t = t_{\text{min}} \) \( (i.e. \, \tau = 0) \), the integral over \( \vec{k} \) can readily be performed and gives

\[
M(z_1, z_2) = \frac{4\pi}{z_1 z_2 z_2 Q_1^2 Q_2^2 (z_1 z_1 Q_1^2 - z_2 z_2 Q_2^2)} \ln \frac{z_1 z_1 Q_1^2}{z_2 z_2 Q_2^2}.
\]  
(8)

The amplitude \( \mathcal{M} \) given by Eq. (6) can then be computed analytically from Eq. (8) through double integration over \( z_1 \) and \( z_2 \):

\[
\mathcal{M}_{t_{\text{min}}} = -i s \frac{N_c^2 - 1}{N_c^2} \alpha_s^2 \alpha_{\text{em}} f_{\rho}^2 \frac{9\pi^2}{2 Q_1^2 Q_2^2} \left[ 6 \left( R + \frac{1}{R} \right) \ln^2 R + 12 \left( R - \frac{1}{R} \right) \ln R \right.
\]
\[
+ 12 \left( R + \frac{1}{R} \right) + \left( 3 R^2 + 2 + \frac{3}{R^2} \right) \left( \ln(1 - R) \ln^2 R - \ln(R + 1) \ln^2 R - 2 \ln(1 - R) \ln R \right.
\]
\[
\left. + 2 \ln(1 - R) \ln R + 2 \ln(1 - R) - 2 \ln(R) \right].
\]  
(9)

where \( R = Q_1 / Q_2 \).

In the special case where \( Q = Q_1 = Q_2 \), it simplifies to

\[
\mathcal{M}_{t_{\text{min}}}(Q_1 = Q_2) \sim -i s \frac{N_c^2 - 1}{N_c^2} \alpha_s^2 \alpha_{\text{em}} f_{\rho}^2 \frac{9\pi^2}{2 Q_1^2} \left( 24 - 28 \zeta(3) \right).
\]  
(10)

![Figure 1: Left: Differential cross-section for the process \( \gamma L \gamma L \rightarrow \rho_0^L \rho_0^L \) at Born order, at the threshold \( t = t_{\text{min}} \), as a function of \( Q_2^2 / Q_1^2 \). The dots represents the value of the cross-section at the special point \( Q_1 = Q_2 \), as given by the analytical formula (10). The asymptotical curves are valid for large \( Q_2^2 / Q_1^2 \) and are given by Eq. (12). Right: The integrated cross-section for the process \( \gamma L \gamma L \rightarrow \rho_0^L \rho_0^L \) at Born order as a function of \( Q_1^2 = Q_2^2 \).](image)

In Fig.1L we display the differential cross-section

\[
\frac{d\sigma^{\gamma L \gamma L \rightarrow \rho_0^L \rho_0^L}}{dt} = \frac{|\mathcal{M}|^2}{16 \pi s^2}
\]  
(11)

for vanishing transverse \( t \)-channel momentum, i.e. \( t = t_{\text{min}} \), as a function of the ratio \( Q_2^2 / Q_1^2 \). Curves are labelled by the values of \( Q_1 \). The dots on the curves represent the values of the cross-section at the special point \( Q_1 = Q_2 \), which obviously correspond to the analytical formula (10).

The peculiar limits \( R \gg 1 \) and \( R \ll 1 \) are of special physical interest, since they correspond to the kinematics typical for deep inelastic scattering on a photon target described through collinear approximation, i.e. the usual parton model. In the limit \( R \gg 1 \), the amplitude simplifies into

\[
\mathcal{M}_{t_{\text{min}}} \sim i s \frac{N_c^2 - 1}{N_c^2} \alpha_s^2 \alpha_{\text{em}} \alpha(k_1) \beta(k_2) f_{\rho}^2 \frac{96\pi^2}{Q_1^2 Q_2^2} \left( \ln R - \frac{1}{6R} \right).
\]  
(12)

The corresponding asymptotical curves are shown on Fig.1L.
4 Integrated cross section

We get the $t-$ dependence of the cross section by computing exactly the two dimensionnal integral (7) and then performing the integration over $z_1$ and $z_2$ numerically in order to get the amplitude (6). As may be anticipated, the cross-section is strongly peaked in the forward direction. This does not mean that most particles escape detection since the virtual photons are not in the direction of the beams. The differential cross-section shows up sufficient for the $t-$dependence to be measured up to a few GeV$^2$.

Figure 1R shows the integrated over $t$ cross-section as a function of $Q^2 = Q_{1}^2 = Q_{2}^2$. The magnitude of the cross-section appears to be sufficient for a detailed study to be performed at the linear collider presently under study. Note that we did not included the virtual photon fluxes, which would amplify the dominance of smaller $Q^2$. However, triggering efficiency often increases substantially with $Q^2$\[\text{[8]}\]. At this level of calculation there is no $s$-dependence of the cross-section. It will appear after taking into account of BFKL evolution\[\text{[3]}\].

5 Conclusion

This Born order study shows that the process $\gamma^*_L(Q_{1}^2)\gamma^*_L(Q_{2}^2) \rightarrow \rho^0_L\rho^0_L$ can be measured at foreseen $e^+ - e^- -$colliders for $Q_{1}^2, Q_2^2$ up to a few GeV$^2$. Indeed, a nominal integrated luminosity of 100 fb$^{-1}$ should yield thousands of events per year, with $Q^2 \gtrsim 1$ GeV$^2$. This would open a new domain of investigation for diffractive processes in which practically all ingredients of the scattering amplitude are under control within a pertubative approach.

BFKL evolution\[\text{[8]}\] is expected to give a net and visible enhancement of the cross section.

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