Missing Mass, Dark Energy and the Acceleration of the Universe.

Is Acceleration Here to Stay?

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Abstract

Recent measurements of the temperature fluctuations in the cosmic microwave background radiation indicate that we live in an open universe. Size of these fluctuations also indicate that the universe is almost flat. In terms Friedmann models this implies a mass density within 10% of the critical density. However, the dynamical mass measurements can only account for around 30% of this mass. Recently, a series of outstanding observations revealed that the cosmos is accelerating. This motivated some astronomers to explain the missing 70% as some exotic dark energy called the quintessence or as the cosmological constant. In this paper we present an alternative explanation to these cosmological issues in terms of the Friedmann Thermodynamics. This model has the capability of making definite predictions in-line with the current observations of the universe. According to this model, cosmos was expanding slower at the beginning. During the galaxy formation era; \( z_c \in [0.54, 0.91] \), due to a change in the global equation of state, it accelerates for a brief period of time. We expect to see this as a discontinuity in the Hubble diagram. Recent data about the galaxies with redshifts \( 0.5 < z < 0.9 \) displays this discontinuity clearly. We expect the deceleration to re-appear as more data with redshifts \( z \gtrsim 1 \) is gathered. These galaxies will be among the very first galaxies formed in the universe, thus still showing the kinematics of the pre-galaxy formation era. This point is now clearly evidenced in the recent data by Riess et al. on Type Ia supernovae with redshifts \( z > 1.25 \) (2004 astro-ph/0402512). In our model, galaxies with redshifts \( 0 \lesssim z \lesssim 0.5 \) should reflect the kinematics of the universe after the transition is completed. These galaxies are now receding from each other faster. However, for \( z \) values towards the upper end of this range we still expect to see deceleration. This is in contrast with the predictions of the dark energy models.
I. INTRODUCTION

One of the first applications of the Einstein’s theory of gravitation was given by Einstein to cosmology. Which later developed into what is now known as the standard model, and has been extremely successful in explaining the overall features of the universe to times as early as $10^{-2}$ sec\[1-3]. One of the basic features of the standard model (also called the Friedmann models) is the large scale homogeneity and isotropy of the universe. This is evidenced in the uniformity of the temperature distribution of the CMBR (cosmic microwave background radiation). Which is verified to one part in $10^5$ with antennas separated by angles ranging from 10 arcsec to 180 degrees. Among its other successful predictions; we could name the age of the universe, existence of CMBR, expansion of the universe, abundances of light elements, and the existence of structure. When the standard model is extrapolated to the first microseconds of the universe, problems about the horizon and flatness appear\[1-3]. To solve these problems inflationary models have been proposed. To explain the details of the universe up to the Planck time of the origin may indeed be a tall order, even for a successful theory like the Einstein’s theory of gravity. However, there are also serious problems regarding the relatively recent eras of the universe [1-3].

1. Missing Mass Problem

Recombination starts when the universe was about 300,000 years old. Prior to this time, light created with the big bang was constantly being scattered by the free electrons in a plasma of primordial hydrogen and helium atoms. At about this time, universe has sufficiently cooled for the electrons and protons to form atoms hence, scattering has stopped. For this reason photons that we see today as the CMBR at 2.73 $K$, carries information about the state of the universe when it was only 300,000 years old. However, CMBR is not completely uniform. The theory predicts that there should exist temperature fluctuations at the order of $10^{-5}$ in order to seed structure formation. Indeed, such fluctuations have
been observed by various groups [4-9]. Since curvature acts like a lens, from the size of these fluctuations one could obtain valuable information about the geometry of the universe. Compared to the critically open (flat) universe, these fluctuations should appear smaller for a closed (spherical), and larger for an open (hyperbolic) universe. Most recent observations [4] indicate that the size of these fluctuations has not changed much since they were formed. Thus indicating that the geometry of the universe is very close to flat. Einstein’s theory relates the matter content of the universe to its geometry. Observations about the geometry of the universe, considered with the Einstein’s field equations and the Hubble constant ($H_0$) measurements, imply that the present density of mass should be within 10% of the critical density defined as [4-11]

$$\rho_c = \frac{3H_0^2}{8\pi G}.$$  

(1)

This result should naturally be confirmed by the dynamical mass measurements in the universe. This is a challenging task. Various methods indicate that only around 4% of the matter is ordinary matter i.e. baryonic. From the orbital speeds of galaxies within a cluster we also know that there is approximately 6 times as much dark matter as baryonic matter. Dark matter is basically composed of particles like neutrinos and other weakly interacting massive particles (WIMPS). These particles interact weakly with other matter and do not cluster. However, their presence could be detected through their gravitational effects [4,12-14]. In other words, the total amount of all kinds of matter -both dark and ordinary- in the universe only accounts for 30% of the critical density. The remaining 70% is far too large to be missed by the current methods of detection, and yet it is still unaccounted for. This is the so called ‘missing mass’ problem.

2. Accelerating Universe?

In the standard models, Einstein’s theory presents three alternatives for the geometry of the universe which depends on the density of the universe. For densities above the critical
value (1), universe is closed and the geometry is spherical (Gaussian). For densities equal to the critical density, the geometry is flat (Euclidean), and for densities less than the critical density, universe is open and the geometry is hyperbolic (Lobachevskian). In all these cases universe starts with a big bang, and the geometry of the universe depends on the strength of the explosion i.e. the total amount of energy (matter) in the universe.

Another property of the standard models is that for all conventional equation of states which satisfy the inequality $(\rho + 3P) \geq 0$, due to the mutual gravitational attraction of different parts of the universe, expansion decelerates [15a]. The specific amount of the deceleration depends on the geometry, as well as the equation of state. In order to see this deceleration, extending the Hubble diagram (velocity vs. distance or redshift plot of galaxies) to higher and higher redshifts has been another challenge for observational astronomy, and another means for determining the geometry of the universe.

For nearby galaxies with redshifts 0.01-0.05, this relation is linear and gives the value of the Hubble constant $H_0$ roughly as 66km/sec/Mpc [8-10]. For higher redshifts we expect the graph to deviate from linearity thus showing the effects of deceleration. In fact, just when we were beginning to think that we are seeing the effects of deceleration [15b], new observations with exciting and equally shocking results came in [16-18a,b]. For galaxies with redshifts 0.1 - 1, this data indicates that the universe is accelerating? This is particularly surprising since acceleration starts roughly at a time when the galaxies began forming: This is an era where we have confidence in our theories of matter and Einstein’s field equations.

3. Inflation and Quintessence in a Nutshell

Currently, popular solutions offered to the problems of the early universe, as well as the missing mass and the accelerating universe problems are related. They are all based on the revival of the cosmological constant term, in one form or another. This term was originally introduced by Einstein into his field equations to salvage his static universe model. After the discovery of the expansion of the universe, he decided that it is no longer needed and wanted
to abandon it. However, since than this term has become a center of attention in cosmology and kept appearing in Einstein’s equations for various reasons. Einstein expressed his dislike for this term by saying ”the biggest blunder of my life” and ”I introduced it, now I can not get rid of it”. Einstein’s dislike for this term could hardly be dismissed as purely emotional. If you consider it as a part of the left hand side of the Einstein’s field equations, it destroys the purely geometric nature of this side. Besides, it adds a new arbitrary parameter to be determined by observations, thus reducing the predictive power of the theory.

The other alternative is to consider it on the right hand side as a part of the energy-momentum tensor. Now, the Lorentz covariance of this term makes it convenient to interpret it as the energy-momentum tensor of the quantum vacuum. When the cosmological term is compared with a perfect fluid, it has the equation of state given as

$$P = -\rho = -\Lambda.$$  \hfill (2)

Where, $\Lambda$ is the cosmological constant. This equation of state has the unusual property of adding a repulsive force into the dynamics of the universe. It is this property that is used in the inflationary models to inflate the scale factor exponentially [1-3]. Thus, solving the horizon and the flatness problems.

It is again this feature that is used for the missing mass and the acceleration of the universe problems. However, if $\Lambda$ is taken as a constant, it runs into the problem of fine tuning. To avoid this, it is taken as a function of time, which is now interpreted as the density of a new form of matter called ’quintessence’ or dark energy [12,19]. This new form of matter has positive energy and yet have the unusual property of responding to gravity by repulsion, thus causing the acceleration. However, not only its physical nature at the classical and the quantum levels is not clear, it also requires rather special initial conditions to work.

In this paper, we present an alternative explanation to these cosmological issues in terms of the Friedmann Thermodynamics. This model has the capability of making definite pre-
dictions about the geometry of the universe, the missing mass problem, and the acceleration of the universe, all in-line with the current observations [20-26]. For future observations, we also predict where this model will start differing from the dark energy or the quintessence models. Models with the cosmological constant are also referred to as dark energy (or quintessence) with constant density.

II. THERMODYNAMICS AND GEOMETRY

Left hand side of the Einstein’s field equations is a purely geometric term constructed entirely from the metric tensor and its derivatives. However, the right hand side i.e. the energy-momentum distribution of the universe, which should describe the source of this geometry also contains the metric tensor. Thus indicating that matter and geometry are interrelated in an intricate way. For a given geometry, using the metric tensor Einstein’s equations could be used to obtain the total energy-momentum distribution of the source. However, many different sources could be associated with a given energy-momentum distribution. This indeterminacy about the details of the source, which should be related to the information content of a given geometry, immediately reminds us the entropy concept. In standard statistical mechanics entropy is defined as proportional to the log of the number of microstates which leads to the same macrostate. However, due to the non-extensive nature of the self gravitating systems, even if we could find a way to count the internal states of a given geometry, we could not expect the corresponding ‘curvature entropy’ to be proportional to the log of this number. A potential candidate may be the Tsallis’ definition of entropy [27].

Another approach to search the connection between the geometry and thermodynamics could be through the use of the second law, which states that the total entropy of the universe can not decrease. However, due to the fact that it is the total entropy that the second law is talking about, looking for a geometric quantity that is an ever increasing function of time and identifying it as the curvature entropy is not a reliable method. Besides, for non-
extensive systems the total entropy will not be a simple sum of its components [27]. Thus, making the contribution coming from the geometry even harder to identify. Considering these difficulties, we have recently concentrated on the thermodynamic side of this problem and argued that a system with finite 'curvature entropy' should also be endowed with a finite 'curvature temperature'. Being homogeneous and isotropic, Friedmann geometries are ideal for searching this connection, where the methods of equilibrium thermodynamics could still be used [20-26].

Starting point of the Friedman thermodynamics was the definition of the 'curvature temperature' as

$$T = \alpha_0 \mid \frac{k_0}{R(t)} \mid^2$$

(3)

\(\alpha_0\) is a dimensional constant to be determined later and \(R(t)\) is the scale factor. \(k_0^2/R(t)^2\) is proportional to the curvature scalar of the constant time slices of the Friedman geometry (\(k_0^2 = 0\) for critically open i.e. flat, \(k_0^2 = 1\) for closed, and \(k_0^2 = -1\) for the open universes.). As expected from a temperature like property, (1) is uniform throughout the system and also a three-scalar. With this new information (equation) added to the Einstein’s field equations, and for a 'local' (flat spacetime) equation of state taken as

$$P = \alpha \rho,$$

(4)

we were able to extract a 'global' equation of state, which now incorporates the effects of curvature (temperature) as

$$\rho_{\text{open}}(T, P) = -\frac{c_0^2}{8\pi}(3 + \frac{1}{\alpha})T^2 + \frac{P}{\alpha},$$
$$\rho_{\text{closed}}(T, P) = \frac{c_0^2}{8\pi}(3 + \frac{1}{\alpha})T^2 + \frac{P}{\alpha},$$

(5)

\(c_0^2 = \frac{4\pi^2 k^2 c^2}{G \hbar^2}\)

for the open and closed models, respectively. These expressions, once identified as the Gibbs energy densities, could be used to derive all the required thermodynamic properties of the system. Note that \(\rho\) and \(P\) in (5) are no longer the same with their local values given in (4). They reduce to their local values only in the ideal case where the geometry is 'exactly' flat. [20,21].
One remarkable consequence of this model is that one could now determine the geometry of the universe by thermodynamic arguments. When we compare the two geometries, we see that $\rho_{\text{open}}$ is always less than $\rho_{\text{closed}}$ i.e.

$$\rho_{\text{open}}(T, P) - \rho_{\text{closed}}(T, P) = -\frac{2c_s^2}{8\pi}(3 + \frac{1}{\alpha})T^2 \langle 0$$

Thus, making $\rho_{\text{open}}$ the more stable phase [20,21]. It is interesting that recent observations on the inhomogeneities of the universal background radiation, considered together with the dynamical mass measurements also indicate that the universe is open. Even though it is very close to the critically open i.e. flat case [4-9]. For the universe to be 'exactly' flat today, density has to be tuned to the critical density (1) with infinite precession. This will make the flatness problem in the early phases of the universe even more acute. Dynamical mass measurements can only account for 30% of the critical value [4].

In search for a justification of our definition of the curvature temperature, we have studied Casimir effect in closed Friedmann models. By taking the effective temperature of the Casimir energy as the curvature temperature, we have identified the dimensional constant $\alpha_0$ as $\frac{1}{2\pi} \frac{\hbar}{k}$. Later, by using the concept of local thermodynamic equilibrium, we have extended our definition of curvature temperature to the sufficiently slowly varying but otherwise arbitrary spacetimes [22,23]. When this definition was used for spherically symmetric stars, we have shown that in the black hole limit, the curvature temperature at the surface of the star reduces to the Hawking temperature, precisely.

III. CHANGES IN THE LOCAL EQUATION OF STATE AND THE FRIEDMANN THERMODYNAMICS

A large class of phase changes in the local matter distribution, including the transition from the radiation to the matter era could be described as

$$P = \alpha_1 \rho \rightarrow P = \alpha_2 \rho .$$

(7)
Aside from a change in the amount of deceleration, these transitions do not lead to anything interesting within the context of standard Friedmann models. However, considered in the light of Friedmann thermodynamics, they offer new insights into some of the basic issues of cosmology.

We now concentrate on the beginning of the galaxy formation era, where the $\alpha$ value of the universe is expected to decrease. This follows from the fact that at the onset of the galaxy formation, some of the gas in the universe will be immobilized. Thus, giving less pressure for the same mean density. For $P = \alpha \rho$, and an open universe the Gibbs energy density was given in (3). For the transition $\alpha_1 \rightarrow \alpha_2$, the difference between them could be written as

$$\rho_{\text{open}, \alpha_2}(T, P) - \rho_{\text{open}, \alpha_1}(T, P) = \frac{c_s^2(\alpha_2 - \alpha_1)}{8\pi \alpha_1 \alpha_2}[T^2 - \frac{8\pi}{c_0^2}P].$$

(8)

The two surfaces intersect along the curve

$$T_c^2 = \frac{8\pi}{c_0^2}P_c.$$ (9)

We could use the curvature temperature at the onset of the galaxy formation era as the critical temperature $T_c$, and obtain $P_c$ from the above relation. In ordinary phase transitions critical temperature is usually defined with respect to the constant atmospheric pressure. In our case, at the critical point both phases are expected to coexist, thus it is natural to expect $P_c$ to lie somewhere in between the pressures just before the transition has started, and after it has completed. In this regard, due to a reduction in the local pressure, we expect $T^2 - \frac{8\pi}{c_0^2}P < 0$ before the critical point is reached, and $T^2 - \frac{8\pi}{c_0^2}P > 0$ after the transition is completed. Considering that $\alpha_2 - \alpha_1 < 0$, we could conclude that $\rho_{\text{open}, \alpha_1}(T, P)$, and $\rho_{\text{open}, \alpha_2}(T, P)$ are the stable phases before and after the critical temperature, respectively.

**IV. DARK ENERGY OR THE MISSING MASS**

Now let us now see what new insights that this model contribute to cosmology. Enthalpy density corresponding to the local equation of state $P = \alpha \rho$, could be written as
where $s$ is the entropy density. During the phase transition ($\alpha_1 \to \alpha_2$) change in the enthalpy density could be written as

\[
\Delta h(s, P) = \frac{8\pi}{2c_0^2} (3 + \frac{1}{\alpha})^{-1} s\Delta s + \frac{1}{\alpha} \Delta P, \quad \text{and} \quad \Delta h(s, P) = T\Delta s + \frac{1}{\alpha} \Delta P.
\]

At constant pressure $\Delta h(s, P)$ gives us the energy density needed for this phase transition. In ordinary phase transitions this energy would be absorbed from a heat bath at constant temperature. In our case, since the universe is a closed system, it could only come from within the system. Calling this energy density $q_c$, we obtain it as

\[
q_c = \frac{2c_0^2 T_c^2}{8\pi} \frac{(\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2}, \quad \text{where}
\]

\[
T_c = \frac{1}{2\pi} \frac{\hbar c}{k R_c}.
\]

$R_c$ is the scale factor of the universe at the time of the transition. $q_c$ is the energy spent (used) by the system (universe) to perform the above phase transition, which is required by the entropy criteria. In the energy budget of the universe, this energy would show up as missing with respect to the critically open (flat) case. To find how this energy would be observed today, we use the scaling property of $q_c$, to obtain

\[
q_{\text{now}} = \frac{2}{8\pi G} \frac{c^4 (\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2} \frac{1}{R_{\text{now}}^2}.
\]

During the matter era baryons constitute the main source of pressure, whose equation of state could be taken as the ideal gas law;

\[
P = \left[\frac{kT}{3\mu H c^2}\right] \rho.
\]

$\rho$ is the energy density of baryons, $\mu$ is the mean molecular weight and $H$ is the atomic mass unit. During the galaxy formation period, which is expected to be short compared to
the age of the universe, change in temperature due to the expansion of the universe could be ignored. In other words, we could treat $\alpha$ as a sufficiently slow varying parameter and use equation (16) to assign an average value for it. Starting with the recombination era, appearance of the first galaxies spans a temperature range from several thousands to tens of K $[1 - 3]$. Taking 700K as an average temperature and the mean molecular weight as 1, we obtain 2/3 as a useful mean value for $\alpha$. However, even though the baryonic matter is the main source of pressure, it is not the main source of matter. In terms of the total energy density let us calculate an effective value for $\alpha$. We write the total local pressure as

$$P_{total} = \alpha_1 \rho_b + \alpha_2 \rho_{nb}.$$  

First term represents the baryonic component, while the second represents the nonbaryonic contribution. We take

$$\alpha_1 = \frac{2}{3},$$

and for weakly interacting particles we consider $\alpha_2$ as a number very close to zero. Expressing (17) as

$$P_{total} = \frac{[\alpha_1 \rho_b + \alpha_2 \rho_{nb}]}{\rho_b + \rho_{nb}} \rho_{total},$$

$$P_{total} = \left[\alpha_1 \left(\frac{\rho_b}{\rho_b + \rho_{nb}}\right) + \alpha_2 \left(\frac{\rho_{nb}}{\rho_b + \rho_{nb}}\right)\right] \rho_{total},$$

using the fact that $\alpha_2$ is a very small number, we could introduce an effective $\alpha_{eff}$ value for the universe as

$$P_{total} = \alpha_{eff} \rho_{total},$$

$$\alpha_{eff} = \alpha_1 \left(\frac{\rho_b}{\rho_b + \rho_{nb}}\right),$$

where $\rho_{total} = \rho_b + \rho_{nb}$. Current observations $[3,4,12]$ indicate that the present density of matter is roughly distributed as:
Radiation (photons) 0.005%
Ordinary visible dark matter (baryons) 0.5%
Ordinary nonluminous dark matter (baryons) 3.5%
Dark matter (WIMPS) 26%

Percentages are given with respect to the $\rho_{\text{critical}}$ (1). Considering that roughly 10% of the baryonic matter condenses in the form of luminous matter thus decoupling from the expansion of the universe, we could take the ranges of $\alpha_1$ and $\alpha_2$ as [3]

$$\alpha_1 \in \left( \frac{4}{100}, \frac{5}{100} \right) \rightarrow \alpha_2 \in \left( \frac{3.5}{100}, \frac{4.5}{100} \right), \text{ or}$$

$$\alpha_1 \in (0.02667, 0.03333) \rightarrow \alpha_2 \in (0.02333, 0.03000).$$

These percentages are consistent with the recent results from the cosmic background imager (CBI) observations [4], and leads to

$$\frac{(\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2} \in (3.33, 5.36).$$

To calculate the range of $q_{\text{now}}$, we take the value of $R_{\text{now}}$ as the radius of the observable universe which could be taken as

$$R_{\text{now}} \simeq 2 \times 10^{10} \text{ly} = 2 \times 10^{28} \text{cm}.$$

This gives $q_{\text{now}}$ in the range

$$q_{\text{now}} \in (8.89 \times 10^{-30}, 1.43 \times 10^{-29}) \text{gm/cc.}$$

Cosmic microwave background radiation data is sometimes used to claim that the geometry of the universe is flat (critically open). However, all it actually says is that the geometry is open but very close to flat [4,11]. The 'huge' difference between the two cases and their potential consequences is usually overlooked [28]. For the universe to be exactly flat, its density must be tuned to the critical density (1) with infinite precession. From the recent data of VSA and CBI we could conclude that the density of the universe is only within 10% of the critical value [4,5 also see 11]. Considering that the observed matter density of the
universe only adds up to 30% of the critical density, the rest is declared either as 'missing', or as exotic dark energy (quintessence) [12,19,28]. Recent data indicates that the Hubble constant could be taken in the range [8-10].

\[ H_0 \in (55, 75) \text{ km/ sec/Mpc}. \]

This gives the range of the critical density \( \left( \frac{3H_0^2}{8\pi G} \right) \) as

\[ \rho_{\text{critical}} \in [6.05 \times 10^{-30}, 1.44 \times 10^{-29}] \text{ gm/cc}, \quad (27) \]

Taking the missing mass as the 70% of the critical density we find

\[ \rho_{\text{missing}} \in (4.23 \times 10^{-30}, 1.01 \times 10^{-29}) \text{ gm/cc}. \quad (28) \]

\( q_{\text{now}} \) is now well in the range given in (28). Certainly this energy does not disappear from the universe, but it is needed for the phase transition, and it is used for it.

This phase transition takes place during the formation of structure for the first time in the universe. These are the first clusters, galaxies, quasars, and superstars etc. Modern galaxies appear only during the last 10-15 billion years. During this era equation of state changes roughly from \( P = \frac{1}{3} \rho \) to \( P = 0 \) in a relatively short time compared to the age of the universe. Thus, even a very crude approach like taking the arithmetic mean of the \( \alpha \) values of these equation of states for the average \( \alpha_2 \) value i.e. Taking

\[ \alpha_1 = 1/3 \rightarrow \alpha_2 = 1/6 \]

gives \( \frac{(\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2} = 3 \), which already leads to numbers that are very close to what we have obtained before (22-28).

V. WHERE DID THE MISSING MASS GO?

To see how \( q_{\text{now}} \) is spent, we write the free energy density and its change as
\[ f(T, v) = -\frac{c_0^2}{8\pi}(3 + v)T^2, \quad (29) \]
\[ \Delta f(T, v) = -\frac{c_0^2}{8\pi}2T(3 + v)\Delta T - \frac{T^2c_0^2}{8\pi}\Delta v, \quad (30) \]
\[ \Delta f(T, v) = -s\Delta T - P\Delta v. \quad (31) \]

For constant temperature processes, \( \Delta f(T, v) \) would usually give the work done by the system on the environment through the action of a boundary. Since we have a closed system, this work is done by those parts of the system expanding under its own internal pressure:

\[ w_c = P_c\Delta v = \frac{c_0^2T_c^2}{8\pi} (\alpha_1 - \alpha_2) \frac{\alpha_1}{\alpha_2}, \quad (32) \]

where \( w_c \) denotes the work done by the system at the time of the transition. Now, at the critical point, \( q_c \) amount of energy is used from the system, while \( w_c \) amount of it is used to do work to increase the specific volume. In a closed system, we expect these two terms to cancel each other. However, as opposed to the usually studied systems in thermodynamics, where the changes take place infinitesimally slowly, our system is dynamic i.e. The universe does not stop and go through this phase transition infinitesimally slowly. As a result, we should also take into account the change in the kinetic energy of the expansion. Hence, the energy balance should be written as

\[ -q_c + w_c + \Delta(K.E.)_c = 0. \quad (33) \]

This implies

\[ \Delta(K.E.)_c = \frac{c^4}{8\pi G R_c^2} (\alpha_1 - \alpha_2) \frac{\alpha_1}{\alpha_2} > 0 \quad \text{(in ergs/cc)}, \quad (34) \]

which is the amount of energy that has gone into increasing the kinetic energy of the expansion. In this model, \( q_c \) amount of energy (density) has been used (or converted) within the system. Part of it has gone into work to increase the specific volume, while the rest is used to increase the kinetic energy of the expansion. Why should the universe go through all this trouble? Basically, for the same reason that water starts boiling when the critical temperature is reached i.e. to increase its entropy.
VI. LOCATION OF THE CRITICAL POINT

To find the location of the critical point we first remember that in our model the scale factor $R(t)$ (not the geometry) could be determined exactly by using the local equation of state $P = \alpha \rho$ as [19, 20]

$$R(t) = \sqrt{C_1} \left[ \frac{3}{4} (\alpha + 1) t + C_0 \right]^{2/(3(\alpha + 1))},$$

where

$$C_0 = \frac{1}{2H_0} - \frac{3}{4} (\alpha + 1) t_0,$$ (35)

$$\sqrt{C_1} = R_0 (2H_0)^{2/(3(\alpha + 1))}.$$

Using these we could write the ratio of the present value of the scale factor to its value at the time of the transition as,

$$\left( \frac{R_0}{R_c} \right)^2 = \frac{1}{\left[ \frac{3}{2} (\alpha_2 + 1) H_0 (t_c - t_0) + 1 \right]^{1/(3(\alpha_2 + 1))}}.$$ (36)

Here, $t_0$ and $t_c$ represent the ages of the universe now and at the time of the transition, respectively. Since redshift at the critical point is given as

$$z_c = \frac{R_0}{R_c} - 1.$$ (37)

we could write

$$z_c = \frac{1}{\left[ \frac{3}{4} (\alpha_2 + 1) 2H_0 (t_c - t_0) + 1 \right]^{2/(3(\alpha_2 + 1))}} - 1.$$ (38)

In the light of the recent observations we could take the age $t_0$ and the Hubble constant $H_0$ as

$$t_0 = 15 \times 10^9 \text{yrs},$$ (39)

$$H_0 = 60 \text{km/sec/Mpc}.$$ (40)

These are among the most probable values for these parameters [9,10]. We have also determined the range of $\alpha_2$ in equation(23)as
\[
\alpha_2 \in (0.0233, 0.03) . 
\]

For the age at the critical point we consider that first galaxies began forming around

\[
t = 1 - 2 \times 10^9 \text{yrs} .
\]

Naturally this transformation takes some time to be completed. Age of the globular clusters is given around \(\sim 13 \times 10^9 \text{yrs}\). Thus we think that it is reasonable to take \([1]\)

\[
t_c \in (2 - 5) \times 10^9 \text{yrs} 
\]

as the effective time of the transition. With these numbers we now obtain the critical point in the range

\[
z_c \in (0.54, 0.91) .
\]

This is consistent with the value \(z \approx 0.73\) given by Perlmutter et al. as the location of the cross-over point between deceleration and acceleration \([18a]\). In our model cosmos was expanding slower at the beginning. When the galaxy formation started at \(z_c\), due to a change in the global equation of state, it accelerates for a brief period of time. We expect to see this as a discontinuity in the Hubble diagram, which is usually plotted as relative intensity vs. redshift \([16-18a,b,29]\). Recent data indicates that galaxies with redshifts \(0.5 < z < 0.9\) just began to display the change in the Hubble parameter as our model predicts\([16-18a,b]\).

We have mentioned that the deceleration should reappear as more data with redshifts \(z \gtrsim 1\) is gathered \([26]\). It is interesting to see that the recent data obtained by Riess et al. clearly demonstrates this point \([29]\). These galaxies will be among the very first galaxies formed in the universe, thus still showing the kinematics of the pre-galaxy formation era. Galaxies with redshifts \(0 \lesssim z \lesssim 0.5\) should reflect the kinematics of the universe after the transition. These galaxies are receding from each other faster now, however for \(z\) values towards the upper end of this range we still expect to see deceleration. This is in contrast with the predictions of the dark energy models, where the acceleration is forever once the quintessence overtakes ordinary matter.
VII. CHANGE IN THE HUBBLE CONSTANT AT THE CRITICAL POINT

Let us finally estimate the fractional change in the Hubble parameter at $z_c$. In Friedmann thermodynamics, local mass density calculated for a region sufficiently small so that the effects of curvature could be neglected was given as [20-25]

$$\frac{8\pi G}{3c^2} \rho = \frac{R^2}{R^2 c^2}.$$  \hspace{1cm} (45)

In terms of the Hubble parameter $H$ this could be written as

$$\frac{8\pi G}{3c^2} \rho = \frac{H^2}{c^2},$$  \hspace{1cm} (46)

and the fractional change in $H$ could now be obtained as

$$\frac{\delta H}{H} = \frac{8\pi G}{6H^2} \delta \rho.$$  \hspace{1cm} (47)

$\delta \rho$ is the energy used to increase the Hubble parameter (i.e. for acceleration). At the critical point this was obtained as (34) thus,

$$\delta \rho = \frac{c^2}{8\pi G R_c^2} \frac{(\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2},$$ \hspace{1cm} (48)

and we obtain

$$\left(\frac{\delta H}{H}\right)_c = \frac{c^2}{6R_c H_c^2 R_c} \frac{1}{\alpha_1 \alpha_2}.$$ \hspace{1cm} (49)

Using

$$R_c = \frac{R_0}{1 + z_c},$$ \hspace{1cm} (50)

and taking $R_0$ as $2 \times 10^{28} cm$ this could be written as

$$\left(\frac{\delta H}{H}\right)_c = \left[7.5 \times 10^{-9} (1 + z_c)\right] \frac{1}{H_c^2 R_c} \frac{(\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2}. \hspace{1cm} (51)$$

Before we go any further we now follow a rather crude approach and obtain another result for the fractional change in $H$. We take the average local kinetic energy density of the expansion as
\[ K.E. = \frac{1}{2} \rho \ R^2, \quad (52) \]

we write its change \( \Delta(K.E.) \) as

\[ \Delta(K.E.) = \rho R^2 H^2 \frac{\Delta H}{H}. \quad (53) \]

We have assumed \( \Delta R \ll \Delta \dot{R} \) during the transition. At the time of the transition (actually after it has been completed) this is equal to (34) thus, we could write

\[ \left( \frac{\Delta H}{H} \right)_c = 4.83 \times 10^{47} \frac{1}{\rho_c R_c^4 H_c^2} \frac{\alpha_1 - \alpha_2}{\alpha_1 \alpha_2}. \quad (54) \]

In this equation subscript \( c \) indicates the value of that parameter at \( z_c \). Using the scaling property of \( \rho \) as

\[ \frac{\rho_o}{\rho_c} = \left( \frac{R_o}{R_c} \right)^3, \quad (55) \]

we could write (54) as

\[ \left( \frac{\Delta H}{H} \right)_c = 4.83 \times 10^{47} \frac{1}{\rho_o R_c R_o^3 H_c^2} \frac{\alpha_1 - \alpha_2}{\alpha_1 \alpha_2}. \quad (56) \]

Using the values

\[ \rho_o = 7.16 \times 10^{-30} \text{gm/cc}, \quad \text{and} \]
\[ R_o = 2 \times 10^{28} \text{cm} \]

(56) becomes

\[ \left( \frac{\Delta H}{H} \right)_c = 8.43 \times 10^{-9} \frac{1}{R_c H_c^2} \frac{\alpha_1 - \alpha_2}{\alpha_1 \alpha_2}. \quad (58) \]

We could also use the relation (50), and

\[ H_c = H_o (1 + z)^{\frac{3(\alpha_2 + 1)}{2}} \quad (59) \]

to write expression (54) entirely in terms of the present day values of \( R \) and \( H \). Thus using the ranges.
\[ \alpha_1 \in (0.027, 0.033), \alpha_2 \in (0.023, 0.030), \]
\[ z_c \in (0.4516, 0.54057), \text{ and taking} \]
\[ H_o = 60 \text{km/sec/Mpc}, \]
we obtain
\[ \left( \frac{\Delta H}{H} \right)_c \in (14, 26)\%. \] (61)

Observationally we expect \( \left( \frac{\Delta H}{H} \right)_c \) to be among the difficult parameters to determine. Since it gives a broader range for \( \left( \frac{\Delta H}{H} \right)_c \), we have used (60) for \( z_c \), which is obtained by taking,
\[ t_c \in (5 - 6 \times 10^9) \text{yrs} \]
in (38).

In this model, effect of this phase transition will show up as a discontinuity in the slope of the Hubble diagram roughly given by the amount in (61). Considering the range of \( z_c \) given in (60), this result is comparable to what (51) would give. Using (44) one obtains
\[ \left( \frac{\Delta H}{H} \right)_c \approx (7.39 - 15)\%. \]
VIII. CONCLUSIONS

Predictions of our model is in contrast with the predictions of the quintessence models, where the acceleration starts around the galaxy formation era but continues forever at an ever increasing pace [12, 19]. This is due to the unusual nature of quintessence or dark energy which responds to gravity by repulsion. Quintessence models, where the density of dark energy remains constant are the models with the cosmological constant. Recently, Wang and Tegmark have claimed that the CMBR data actually favors the quintessence models with constant density i.e. the cosmological constant models [28].

Due to the Lorentz covariant nature of its equation of state, cosmological constant is usually interpreted as the quantum vacuum energy density. In this case, as the universe expands, the amount of 'vacuum' (volume) and thus the vacuum energy increases, while its density remains constant. In the mean time, ordinary matter continues to thin out, thus increasing the effect of repulsive force and the acceleration. Galaxy formation era is around where the vacuum energy is expected to overtake ordinary matter. However, it is not clear why the quantum vacuum energy should be Lorentz covariant. A proper derivation of the renormalized quantum vacuum energy in curved background geometries as the Casimir effect, gives a different result [21–23]. Actually, It is even difficult to philosophize about what 'pure' vacuum- classical or quantum- should be. As soon as one considers the presence of matter and/or observers, nature of the quantum vacuum energy changes. Calculating the renormalized energy of the massless conformal scalar field with a thermal spectrum at temperature T, in background closed Friedmann geometry via the mode sum method, one sees that the quantum vacuum energy gets completely washed out in the high temperature limit ($\frac{\hbar c}{kRT} \ll 1$), and is modified in the low temperature limit ($\frac{\hbar c}{kRT} \gg 1$). Considering that quintessence coexists with other matter and the high temperature limit is the limit to be considered, interpretation of quintessence as the quantum vacuum energy is bound to be problematic. Indeed, a recent article by Ford discusses this point [30].

All three Friedman models are homogeneous and isotropic, and start with a big bang.
Critically open and the open universes are infinite in extent. Hence, they start with an
infinite amount of matter distributed uniformly over an infinite space. However, due to the
existence of particle horizon, one could only observe a finite part of it. Thus, assertions about
the global topology of the universe are essentially very difficult to justify. In this regard,
all our arguments appearing in this paper are local and independent of the global topology
of the universe. In [31], Gomero et al. discusses problems regarding the observability
of the global topology of the universe. In our approach, we view geometry like different
crystal structures of matter i.e. matter distributed over different spaces (lattices) with
distinct symmetry properties. Thus, changes in symmetry are allowed and interpreted as
phase transitions [20 − 26]. However, It should also be emphasized that in Friedmann
thermodynamics topology does not have to change. Indeed, for local equation of states
given as $P = \alpha \rho$, which covers a wide range of physically interesting cases, global topology
is always open (6). It would be interesting to see what kind of physically acceptable local
equation of states would induce such topology changes, if at all possible.

Like the quintessence, Friedmann thermodynamics is also a suggestive model. However,
despite the missing pieces in its theoretical foundations, its predictive power is incredibly
high and not only it offers some very interesting potential answers to the existing cosmo-
logical problems, but also makes definite predictions for future observations. Other models
suggested for the accelerating universe and the dark energy problems could be found in
[32,33].
IX. REFERENCES

[1] P. Coles, and F. Lucchin, Cosmology, John Wiley & Sons Ltd. (2002).

[2] A.D. Dolgov, M.V. Sazhin, Ya.B. Zeldovich, Modern Cosmology, Editions Frontiers (1990).

[3] E.W. Wolb, and M.S. Turner, The Early Universe, Addison-Wesley Publishing Co. (1990).

[4] J.L. Sievers et al. arXiv:astro-ph/0205387 (2002).

[5] J. Silk, Physics World, 15, no.8, pg.21 (2002).

[6] J. Silk, Physics World, 13, no.6, pg.23 (2000).

[7] E. Cartlidge, Physics World, 14, no.6, pg.5 (2001).

[8] R. Ellis, Physics World, 12, no7, pg.19 (1999).

[9] G.A. Tammann et al., arXiv:astro-ph/0112489 (2001).

[10] W.L. Freedman, arXiv:astro-ph/0202006 (2002).

[11] M. Tegmark et al., arXiv:astro-ph/0310723 (2004).

[12] J. P. Ostriker, and P.J. Steinhardt, Sci. Am., Jan., pg.37 (2001).

[13] A. Taylor, and J. Peacock, Physics World, 14, no.3, pg.37 (2001).

[14] N. Smith, and N. Spooner, Physics World, 13, no.1, pg.23 (2000).

[15a] M.R. Robinson, The Cosmological Distance Ladder, W.H. Freemann and Company NY (1985).

[15b] D. Goldsmith, Science, 276, 37 (1997).

[16] C.J. Hogan, R.P. Kirchner, and N.B. Suntzeff, Sci. Am. Jan. pg.28 (1999).

[17] A.G. Riess, et al., Astronomical Journal, 116, 1009 (1998).

[18a] S. Permutter, et al. arXiv:astro-ph/9812133 (1998).

[18b] R.A. Knop, Ap. J. 598, 102 (2003).

[19] R. R. Caldwell, and P. J. Steinhardt, Physics World, 13, no.11, pg.31 (2000).

[20] S. Bayin, Ap. J., 301, 517 (1986).

[21] S. Bayin, Gen.Rel.Grav., 19, 899 (1987).
[22] S. Bayin, Gen. Rel. Grav., 22, 179 (1990).

[23] S. Bayin, Gen. Rel. Grav., 26, 951 (1994).

[24] S. Bayin, Proceedings of ECOS-95, pg. 3, July 11-15, Istanbul. Editors; Y.A. Gogus, A. Ozturk, G. Tsatsaronis. Also available at http://www.physics.metu.edu/~bayin/.

[25] S. Bayin, Workshop on Second Law of Thermodynamics- Proceedings, Erciyes Univ. - T.I.B.T.D. 27-30/8/90 Kayseri(1990). Also available at http://www.physics.metu.edu/~bayin/.

[26] S. Bayin, IJMPD 11, 1523: arXiv:astro-ph/0211097 (2002).

[27] R. Slazar, and R. Toral, Physica, A290, 159 (2001).

[28] Y. Wang and M. Tegmark, arXiv:astro-ph/0403292 (2004).

[29] A.G. Riess, et al., arXiv:astro-ph/0402512 (2004).

[30] L.H. Ford, arXiv:gr-qc/0210096v1 (2002).

[31] G.I. Gomero, M.J. Rebuças, R. Tvakal, arXiv:gr-qc/0210016v1 (2002).

[32] J. Ponce de Leon, arXiv:gr-qc/0401026v2 (2004).

[33] R.G. Vishwakarma and P. Singh, Class. Quan. Grav. 20, 2033 (2003): arXiv:astro-ph/0211285v3 (2003).