Prediction of stress concentration at fillets using a neural network for efficient finite element analysis

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Received: 27 June 2020; Revised: 16 July 2020; Accepted: 4 August 2020

Abstract
In finite element analysis, small fillets make mesh generation difficult and accurate evaluation of stress concentration at fillets requires refined meshes. Simplified analysis is often performed using a corner model where the fillets are removed. In the analysis using a corner model, mesh division becomes easier and the number of elements is reduced, which shortens the calculation time. However, the stress concentrations cannot be evaluated, and stress singularities occur at corners. We have developed a method for predicting the stress at a fillet based on the simulation of a simplified corner model and the use of a neural network. We use the stress distribution at a corner as the neural network input such that the method can be applied to arbitrary object shapes, loading, and boundary conditions. We trained and validated the neural network using simple corner and fillet models. It was shown that stress distribution at a corner can express the difference in loading conditions. In addition, we found that the method can predict stress at fillets of models that were not used for the neural network training. These results show the possibility that the method enables efficient stress concentration evaluation in finite element analysis.

Keywords: Finite element method, Stress concentration, Stress singularity, Fillet, Neural network

1. Introduction
The parallel finite element method (FEM) and high-performance computing enable large-scale, detailed model analysis. However, in the structural analysis of a model with a small, complicated shape, it still takes time and effort to create a mesh model, and there are problems with the evaluation of stress singularities and stress concentrations. A method for efficiently evaluating stress concentrations of a huge number of structures such as bridges, which include small complicated shapes, is required as urgent challenges because the rate of deterioration of social infrastructure is increasing rapidly. In linear analysis, shapes that cause stress singularities include corners (Seweryn and Molski, 1996; Dunn et al., 1997), as shown in Fig. 1(a), and edges of a dissimilar material interface (Raju and Crews, 1981), as shown in Fig. 1(b). Stress concentration occurs in the fillet, circular hole, and notch with a curvature at the tip, as shown in Fig. 2 (Peterson, 1953). The stress increases indefinitely as the mesh size decreases in a stress singularity region, as shown in Fig. 3. The stress converges to a specific value with decreasing mesh size in a stress concentration region, as shown in Fig. 4. The both figures are the analysis results of the same model under the same conditions. Table 1 shows stresses...
at the corner and the fillet. In this study, we focus on the corners and fillets, and linear static analysis with hexahedral elements. Table 2 summarizes the features of the corners and fillets in the linear static analysis by FEM. Dividing corners into meshes is easier than dividing fillets. In addition, we can divide the corners into coarse meshes, which has the advantage of shortening the calculation time because the number of elements can be reduced. Therefore, small fillets that do not affect the overall stiffness are often removed from a CAD model before performing finite element analysis (FEA). A simplified model including corners is used for efficient analysis. Methods for automatically searching and removing fillet parts from a CAD model have also been developed (Danglade et al., 2014; Chow et al., 2015). However, because corners have stress singularities, it is necessary to analyze them using a detailed model with fillets in order to accurately evaluate the stress concentrations. The zooming method (Mao and Sun, 1991) and the overlapping mesh method (Fish et al., 1994) were developed for efficiently analyzing detailed structures composed of small parts, however it is necessary to create a detailed mesh model in the local area and to recreate the model when the fillet radius changes. Analytical solutions provide another method to evaluate stress concentrations (Peterson, 1953), although these solutions can only be used with ideal shapes and loading conditions and cannot take advantage of FEM features that can handle various shapes and boundary conditions.

For efficient FEA, we propose a method to predict the stress at a fillet by using the simulation results of a corner model without the fillet. We use a neural network (NN) (Alom et al., 2019), which is a type of machine learning, as a prediction technique. Multilayered NNs are called deep learning and have been studied and used in the field of FEA (Oishi and Yagawa, 2017; Wang et al., 2019) in recent years. The development of NNs has become easier because there exist many libraries for deep learning, such as TensorFlow (Abadi et al., 2016). Several studies on the technology of predicting stress concentrations using NNs have been conducted. In studies on the prediction of stress concentrations of gears (Shunmugam and Prasad, 2008) and T-joints (Dabiri et al., 2017) using NNs, they can only be applied to the specified shape. In order to apply a NN to various shapes, it is necessary to develop a method that focuses only on corners that do not depend on the overall shape or boundary conditions. For example, if the stress at a fillet can be predicted by only analyzing the stress distribution at a corner using a NN, it can be applied to various shapes. In this study, we developed a NN to predict the stress at a fillet from the stress distribution of a corner model. The objective of this study is to show the possibility of the proposed method using the stress distribution to predict the stress at a fillet, so we used simple stress distribution and simple configuration of multilayer perceptron NN. First, we developed the NN using simulation results of simple corner and fillet models. The effect of stress distribution on the prediction accuracy was evaluated by changing the distribution used for input. Next, we verified the prediction accuracy of the trained NN, which had a high prediction accuracy for training datasets, with a model different from that used for learning.

2. Development of a NN that predicts stress at a fillet
2.1 NN configuration

We have developed a NN that predicts stress at a fillet using the stress distribution in a corner model without a fillet and performing a linear static analysis. In this study, we used the stress distribution around the corner instead of the load

![Table 1 Comparison of stress at a corner and a fillet.](image1)

| h [mm] | Corner [MPa] | Fillet [MPa] |
|-------|--------------|--------------|
| 2.0   | 79.9         | 69.2         |
| 1.0   | 111          | 67.4         |
| 0.5   | 153          | 67.1         |

![Table 2 Points of concern for corners and fillets.](image2)

|               | Corners | Fillets |
|---------------|---------|---------|
| Mesh generation | Easy    | Difficult |
| Calculation time       | Short   | Long    |
| Difficulty     | Stress singularities | Stress concentrations |

![Fig. 3 Effect of mesh size h on stress singularity.](image3)

![Fig. 4 Effect of mesh size h on stress concentration.](image4)
and boundary conditions as the input value to the NN to enable analysis of targets with various shapes. The von Mises stress, which is often used to evaluate analysis results, was used as the stress component. As shown in Fig. 5(a), the nodal von Mises stress at a corner ($\sigma_c$) and the nodal von Mises stresses around the corner ($\sigma_1, \sigma_2, \sigma_3, \sigma_4$) were used. For the purpose of showing the possibility of the proposed method for stress prediction, we selected the von Mises stress at four nodes adjacent to $\sigma_c$. The nodal stress is the average of the values obtained by using the shape function from the stress values at the integration points of the elements that share the node. We used $\sigma_c$ to normalize $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ so that the stress prediction at a fillet using the NN could be applied to various stress analyses regardless of the stress and Young's modulus magnitude. The nodal von Mises stresses around the corner were divided by the nodal von Mises stress at the corner ($\sigma_1/\sigma_c, \sigma_2/\sigma_c, \sigma_3/\sigma_c, \sigma_4/\sigma_c$). The output layer of the NN was one node of the value obtained by dividing the Mises stress ($\sigma_f$) at the fillet portion shown in Fig. 5(b) by the Mises stress at the corner ($\sigma_c$). When $\sigma_c$ is close to zero, dividing by zero would be problematic but it is not necessary to evaluate the nodal stress from the viewpoint of strength evaluation. When $\sigma_c$ changes significantly, it is considered that the adjacent nodal stress ($\sigma_1, \sigma_2, \sigma_3, \sigma_4$) changes as well.

We evaluated the effect on the prediction accuracy of the stress distribution used as an input to the NN by comparing three types of NNs: one that inputs only the fillet radius (Type I), one that inputs the fillet radius and the nodal von Mises stress ratios ($\sigma_1/\sigma_c, \sigma_2/\sigma_c$) (Type II), and one that inputs the fillet radius and the nodal von Mises stress ratios ($\sigma_3/\sigma_c, \sigma_4/\sigma_c$) (Type III). We selected one type which didn’t use stress distribution and two types which used simple stress distribution to show the possibility of expressing the difference in loading conditions by using the stress distribution as NN input. We used a simple multilayer perceptron NN in this study. Every NN had one hidden layer that had 30 nodes. Figure 6 shows the configuration of the Type III NN. The ReLU function was used as the activation function of the hidden layer, an identity function was used as the activation function of the output layer, and the loss function was the mean square error. The Adam algorithm was used as the model optimizer. We used Python and TensorFlow to develop the NN.

![Stress at corner: $\sigma_c$](image)

(a) Evaluation points of von Mises stress at a corner.

![Stress at fillet: $\sigma_f$](image)

(b) Evaluation point of von Mises stress at a fillet.

Fig. 5 Input and output variables of the NN.

![30 nodes](image)

Fig. 6 Configuration of the NN (Type III).

### 2.2 Datasets for training the NN

We created training datasets from simple corner and fillet models with side lengths of 40 mm, as shown in Fig. 7. The fillet model shows a model with a fillet radius of 3 mm as an example. The element size of the corner model was 2 mm, that of the whole fillet model was 2 mm, and the local element size at the fillet was 20% of the fillet radius to calculate the converged stress. There were five fillet radii: 1, 2, 3, 4, and 5 mm. We analyzed both the corner model and the fillet model under the same load and boundary conditions. Figure 8 shows the boundary conditions where part of the face was fixed. Figure 9 shows the four load conditions that were used: types A, B, C, and D, where each red square has a side length of 10 mm. The distributed loads of P1 and P2 were applied. As a result of using a linear static analysis, the ratio of P1 and P2 influenced the stress distribution. 19 pressure ratios were used. A wide range of load conditions are required because the NN is applied to various shapes and load conditions, which is different from a surrogate model.
Table 3 Analysis conditions of training models.

| FEM software | Abaqus 2018 |
|--------------|-------------|
| Analysis types | Linear static analysis |
| Element type | 20-node hexahedral element |
| Young's modulus | 100 GPa |
| Poisson's ratio | 0.3 |
| Mesh size | 2 mm |
| Local mesh size at a fillet | 20% of fillet radius |
| Fillet radius | 1 mm, 2 mm, 3 mm, 4 mm, 5 mm |
| Loading radius | A, B, C, D |
| Loading conditions | A, B, C, D |
| P1[MPa] : P2[MPa] | 1:1 1:0.8 1:0.6 1:0.4 |
|                     | 1:0.2 1:0 1:-0.2 1:-0.4 |
|                     | 1:-0.6 1:-0.8 0:0.8 1:0.6 1:0.4 |
|                     | 0.4:1 0.2:1 0:1 -0.2:1 |
|                     | -0.4:-1 -0.6:1 -0.8:1 |

Fig. 7 Mesh models for learning.
Fig. 8 Displacement conditions.
Fig. 9 Loading conditions.
Fig. 10 One example of simulation results.
for the optimization problem that is applied to a specific condition. Abaqus was used as the FEM software to create the training datasets. We used a computer with an Intel® Core™ i7-7700HQ CPU and 32 GB of memory. Figure 10 shows an example of the corner and fillet models analyzed under the same conditions.

Table 3 shows a summary of the analysis conditions. We used a Young's modulus of 100 GPa in FEA, but the magnitude of Young's modulus does not affect the stress ratio as NN input values. There were 380 datasets generated from 5 fillet radii, 4 loading types, and 19 pressure ratios. Seventy percent of the datasets were allocated to training and 30% to validation. Google Colaboratory (Carneiro et al., 2018) was used for training the NN. The training set batch size was 32, and the number of training epochs was 5000.

2.3 Training results

The results of the three types of NN training are shown in Fig.11 as the loss function history over the training epochs. The loss values for both the training and validation sets decreased as the training progressed in the Type III NN. We evaluated the average prediction accuracy of each NN type. The definition of the prediction accuracy, \( p^i \), for each dataset, \( i \), is shown in Eq. (1).

\[
p^i = 1 - \frac{|\sigma_f^i - \sigma_p^i|}{\sigma_f^i}
\]  

Here, \( \sigma_f^i \) is the predicted value of the von Mises stress of dataset \( i \), and \( \sigma_f^i \) is the von Mises stress at the fillet obtained by analyzing the fillet model with a detailed mesh. \( \sigma_f^i \) has only positive values. The definition of the average prediction accuracy, \( p \), is shown in Eq. (2).

\[
p = \frac{1}{N} \sum_{i=1}^{N} p^i
\]

Here, \( N \) is the number of datasets. There were 266 training datasets and 114 validation datasets. Table 4 shows the average prediction accuracy and the standard deviation (SD) for each NN type. We calculated the average prediction accuracy when the loss function of the verification datasets was minimized during training. The average prediction accuracy of the Type III NN using \( \sigma_3/\sigma_c \) and \( \sigma_4/\sigma_c \) as input was at least 90% for both the training and validation datasets. The difference in loading conditions could be expressed by selecting an appropriate stress distribution. Relationship between \( \sigma_3/\sigma_c \) and \( \sigma_4/\sigma_c \) is shown in Fig.12 and it was shown that the relationship is nonlinear. It was found that the simple multilayer perceptron NN could learn the relationship and predict the stress at fillets with good accuracy.

![Fig. 11 Line plot of the average mean squared error loss over 3000 training epochs for each NN type.](image)

| Table 4 | Average prediction accuracy of each NN. |
|----------|---------------------------------------|
| Input values | Prediction accuracy[\%] |
| Training | Validation |
| Average | SD | Average | SD |
| Type I | Fillet radius | 68.7 | 58.2 | 68.7 | 53.8 |
| Type II | Fillet radius, \( \sigma_3/\sigma_c \), \( \sigma_4/\sigma_c \) | 72.2 | 55.2 | 71.5 | 51.7 |
| Type III | Fillet radius, \( \sigma_3/\sigma_c \), \( \sigma_4/\sigma_c \) | 92.1 | 18.5 | 91.2 | 17.7 |
3. Verification of trained NN prediction accuracy on another model that is not used for training

3.1 Datasets for verification

We verified the trained NN using shapes that were different from that used for training. If the NN that trained the results of a simple model with only a corner and a fillet could predict the stress of another shape, it would be easy to create the NN. As another shape, we targeted the 1/2 symmetric model of H-section steel (Fig.12), which is a general structural element. The end face was completely fixed, the symmetry condition was given to the symmetry plane, and a distributed load of 1 MPa was applied to the top face. We evaluated two conditions: changing the dimensions and changing the fillet radius.

3.1.1 Changing the dimensions

As shown in Table 5, the dimensions of \( a, b, \) and \( c \) in Fig. 13 were each changed to 4, 10, 20, and 30 mm, resulting in 64 models. The fillet radius was 3 mm. The steel exhibited a Young's modulus of 200 GPa and a Poisson's ratio of 0.3. We could change the Young's modulus because the Young's modulus magnitude didn’t affect the developed NN. The element sizes of the corner models were 2 mm, those of the whole fillet model were 2 mm, and the local element sizes at the fillet was 20% of the fillet radius. The von Mises stress at an evaluation point was predicted by the NN from the analysis results of a corner model and was compared with the von Mises stress obtained by the detailed analysis of a fillet model.

3.1.2 Changing the fillet radius

As shown in Table 6, the fillet radius was changed from 1 to 5 mm in 0.5 mm increments while the dimensions \( a, b, \) and \( c \) were held constant at 20, 10, and 4 mm, respectively, which resulted in 9 models. The other conditions were the same as in 3.1.1.

Fig. 12 Relationship between \( \sigma_3/\sigma_c \) and \( \sigma_2/\sigma_c \).

![Graph showing the relationship between \( \sigma_3/\sigma_c \) and \( \sigma_2/\sigma_c \) with different fillet radii.](image)

Fig. 13 Models for evaluating the trained NN were 1/2 symmetric models of H-section steel. (3.1.1)The dimensions of \( a, b, \) and \( c \) were changed to 4, 10, 20, and 30 mm, and the fillet radius was 3 mm. (3.1.2) The fillet radius was changed from 1 to 5 mm in 0.5 mm increments, where \( a \) was 20 mm, \( b \) was 10 mm, and \( c \) was 4 mm. The von Mises stress at an evaluation point predicted by the NN from the analysis results of a corner model was compared with the von Mises stress obtained by the detailed analysis of a fillet model.
Table 5 Conditions of models with r. t. dimensions.

| Fillet radius | 3 mm |
|---------------|------|
| a             | 4 mm, 10 mm, 20 mm, 30 mm |
| b             | 4 mm, 10 mm, 20 mm, 30 mm |
| c             | 4 mm, 10 mm, 20 mm, 30 mm |

Table 6 Conditions of models with r. t. fillet radius.

| Fillet radius | 1 mm, 1.5 mm, 2 mm, 2.5 mm, 3 mm, 3.5 mm, 4 mm, 4.5 mm, 5 mm |
|---------------|---------------------------------------------------------------|
| a             | 20 mm                                                          |
| b             | 10 mm                                                          |
| c             | 4 mm                                                           |

### 3.2 Verification results

#### 3.2.1 Changing the dimensions

The average prediction accuracy was 88.5% and the SD was 9.4% for 64 shapes with varying dimensions (Table 4). Table 7 shows the average number of elements, the average number of nodes, and the average calculation time of the corner models and fillet models. By using this method, the number of nodes was reduced by approximately 80%, and the calculation time was reduced by approximately 90%. It was assumed that efficient FEA could be performed by using the predictions of this method only for the stress concentration region where a detailed mesh was required since the displacement and the rough stress distribution could be calculated with a relatively coarse mesh.

Table 7 Comparison of average calculation costs.

|          | Average number of elements | Average number of nodes | Average calculation time [s] |
|----------|----------------------------|-------------------------|------------------------------|
| Corner   | 20,800                     | 93,669                  | 178                          |
| Fillet   | 107,404                    | 464,892                 | 1,949                        |

#### 3.2.2 Changing the fillet radius

The average prediction accuracy was 94.9% and the SD was 2.4% when changing the fillet radius. Table 8 shows the number of elements and nodes, the calculation time of the corner and fillet models, and von Mises stresses obtained by FEA and the NN. Figure 14 shows the corner model used for prediction and the R3 model. This method could predict the stress in the fillet of R3 in Fig. 14(b) from the simplified corner model in Fig. 14(a) with an accuracy of 95.8%.

Table 8 Calculation costs of corner and fillet models.

|          | Number of elements | Number of nodes | Calculation time [s] | FEA [MPa] | Proposed method [MPa] |
|----------|--------------------|-----------------|----------------------|-----------|-----------------------|
| Corner   | 8,000              | 40,485          | 32                   | 14.08     | -                     |
| R1.0 [mm]| 164,000            | 748,105         | 1,111                | 16.16     | 17.97                 |
| R1.5 [mm]| 99,866             | 460,266         | 712                  | 14.08     | 14.82                 |
| R2.0 [mm]| 65,000             | 304,897         | 369                  | 12.77     | 12.10                 |
| R2.5 [mm]| 50,800             | 239,279         | 292                  | 11.85     | 11.12                 |
| R3.0 [mm]| 45,484             | 212,091         | 238                  | 11.04     | 10.58                 |
| R3.5 [mm]| 37,914             | 177,138         | 202                  | 10.41     | 10.03                 |
| R4.0 [mm]| 28,224             | 135,101         | 164                  | 9.771     | 9.488                 |
| R4.5 [mm]| 25,536             | 121,981         | 133                  | 9.466     | 8.975                 |
| R5.0 [mm]| 23,100             | 110,206         | 118                  | 9.106     | 8.892                 |

Fig. 14 One example of simulation results of models for evaluation.
4. Conclusions

We have developed a NN to predict the stress of a fillet model from the stress distribution of a corner model, which was easy to analyze using a finite element method. It was found that the difference in loading conditions could be expressed by selecting an appropriate stress distribution. This method could predict the stress at a fillet with good accuracy from the result of a corner model even in a shape different from that used for training, because only the stress distribution of the corner model was used as an input. These results show the possibility that this method could be applied to finite element method stress analysis regardless of the overall shape, load conditions, and boundary conditions. In the developed NN, we used stress at a fillet shown in Fig. 5(b). It is assumed that max nodal von Mises stress at a fillet could be used as the NN output, which would be effective for strength evaluation of structures. In this study, we developed the NN for hexahedral elements with small distortion, so applying the proposed method to tetrahedral elements and element distortions is the future works. In addition, we would like to improve the prediction accuracy by increasing the nodal stress values around the corner used for the NN inputs, by increasing loading conditions, and by using deep learning, regularization methods and other machine learning methods.

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