Semi-direct Gauge Mediation in Conformal Windows of Vector-like Gauge Theories

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Abstract

Direct gauge mediation models using the Intriligator-Seiberg-Shih (ISS) metastable vacua suffer from the Landau pole problem of the standard model gauge couplings and the existence of R symmetry forbidding gaugino masses. These problems may be solved by using the recently proposed SUSY breaking models in a conformal window of the vector-like $SU(N_C)$ gauge theory with gauge singlets. In this paper we propose a model of gauge mediation based on the SUSY-breaking model in the conformal window, and study the dynamics for the SUSY breaking. In the model, there are massive vector-like bifundamental fields charged under both $SU(N_C)$ and the standard model gauge group, and our model can be regarded as a semi-direct gauge mediation model. The color number $N_C$ can be small to avoid the Landau pole problem, and the R symmetry is also broken under a reasonable assumption on the strong dynamics of the model. The model possesses only one free parameter, and the gaugino and sfermion masses are naturally of the same order.
1 Introduction

It is well known \[1\] that supersymmetric (SUSY) \(SU(N_C)\) gauge theories with \(N_F\) pairs of massive quarks and antiquarks, \(Q\) and \(\tilde{Q}\), have SUSY-breaking metastable vacua for \(N_C + 1 \leq N_F < \frac{3}{2} N_C\). In the SUSY-breaking vacua, the direct gauge mediation takes place naturally if some pairs of quarks and antiquarks carry the standard-model (SM) gauge charges by embedding the \(SU(5)_{\text{GUT}}\) gauge group into a subgroup of the flavor \(SU(N_F)\) (see Ref. \[2\] and references therein). From the point of view of the electric description of the \(SU(N_C)\) theory, the fields charged under \(SU(5)_{\text{GUT}}\) have a mass term, \(W = mQ\tilde{Q}\), in the superpotential, and the model resembles a semi-direct gauge mediation \[3, 4\] in this point. However, the flavor symmetry \(SU(N_F)\) is broken down to \(SU(N_F - N_C) \times SU(N_C)\) in the SUSY breaking vacua and hence we have a constraint, \(N_F - N_C \geq 5\) or \(N_C \geq 5\) for keeping the SM gauge symmetries unbroken. From the constraint, \(N_C + 1 \leq N_F < \frac{3}{2} N_C\), we obtain \(N_C > 10\) or \(N_C \geq 5\), respectively. Because of this large number of \(N_C\), we lose the success of the SM gauge coupling unification at the GUT scale \[5\] for a low-scale gauge mediation\[1\].

The above problem is originated from the constraint on \(N_F < \frac{3}{2} N_C\) in the model of Intriligator-Seiberg-Shih (ISS) \[1\], as discussed in Ref. \[7\]. However, it has been shown recently \[8\] that the SUSY is dynamically broken even in the conformal windows of the vector-like theories, that is, for \(\frac{3}{2} N_C \leq N_F < 3 N_C\), if we introduce gauge singlet multiplets.

In this paper, we show that a gauge mediation consistent with the GUT unification is easily constructed in the above conformal windows. In fact, the gauge mediation dynamics is similar to the one in a semi-direct mediation and the model possesses the merit of the conformal gauge mediation \[9\], where all the scales of the model are determined only by one parameter, that is, the mass of messengers. We also discuss a new mechanism for generating SUSY-breaking masses of the gauginos in the SUSY standard model (SSM). The generation of the gaugino masses requires some deformation of the model in the ISS model, due to the existence of an R symmetry. However, we do not need such a deformation in our model. The SUSY breaking field has a fractional R charge, whose \(F\)-
term breaks the R symmetry as well as the SUSY. We show that the gauginos acquire the SUSY breaking masses through instanton effects by picking up the $F$-term R breaking.

## 2 SUSY breaking in conformal windows of vector-like gauge theories

In this section we briefly review the SUSY breaking in conformal windows of vector-like gauge theories [8]. We use a slightly different approach from the one in Ref. [8]. The argument of this section is less rigorous but may give a more intuitive physical picture for the dynamics of the model.

The model is based on a SUSY $SU(N_C)$ gauge theory with $N_Q$ flavors of quarks $Q^i$, $\tilde{Q}^i_i$ ($i = 1, \cdots, N_Q$) in the fundamental and anti-fundamental representations of $SU(N_C)$, $N_P$ flavors of massive quarks $P^a$, $\tilde{P}^a_a$ ($a = 1, \cdots, N_P$) in the same representation as $Q^i$, $\tilde{Q}^i_i$, and gauge singlet fields $S^i_j$. We omit gauge indices for simplicity. We consider the dynamics of the model for $\frac{3}{2}N_C < N_Q + N_P < 3N_C$ throughout this paper. The tree level superpotential of the model is given by

$$W = \lambda \text{tr}(S Q \tilde{Q}) + m P \tilde{P},$$

where $\text{tr}(S Q \tilde{Q}) = S^i_j Q^i \tilde{Q}^i_j$ and $P \tilde{P} = P^a \tilde{P}_a$. In a regime where the mass $m$ can be neglected, this theory has an infrared conformal fixed point [10] if $\frac{3}{2}N_C < N_Q + N_P < 3N_C$ is satisfied. We also assume $N_Q < N_C$ and $N_P > N_C$, as discussed in Ref. [8].

Consider the region where the vacuum expectation value (vev) of $S$ is large. Then $Q$, $\tilde{Q}$ become massive and we can integrate out them. After integrating out all quarks, a low-energy gaugino condensation induces an effective superpotential, (see Ref. [11] for a review)

$$W_{\text{eff}} = N_C \left( m^{N_P} \Lambda^{3N_C - N_F} \det(\Lambda S) \right)^{\frac{1}{N_C}},$$

where $\Lambda$ is the (holomorphic) dynamical scale of the model, and $N_F \equiv N_Q + N_P$. One can easily see that the superpotential Eq. (2) is of runaway type. Naively, there seems to be no stable vacuum in the theory. However, as emphasized in Ref. [8], we must consider the quantum corrections to the Kähler potential to determine the behavior of the potential.
To obtain the effective Kähler potential, we follow the Wilsonian approach of Ref. [12]. Let us consider the effective Kähler potential of the fluctuation $\tilde{S}_{ij} = S_{ij} - (S_0)_{ij}$ around the background $(S_0)_{ij} = \text{const}$. We consider the case $(S_0)_{ij} = S_0 \delta_{ij}$ for simplicity. At energies much higher than the mass of the quarks $Q, \tilde{Q}$ and $P, \tilde{P}$, the Kähler potential of $S$ is given by

$$K_{\text{eff}} = Z_S(M) \text{tr}(S^\dagger S) + \cdots,$$

(3)

where $Z_S(M)$ is the wave function renormalization of $S$ at the Wilson cutoff scale $M$, and dots denote higher dimensional operators. Below the effective mass of $Q, \tilde{Q}, m_Q \propto S_0$, the quarks $Q, \tilde{Q}$ decouple from the dynamics at the scale $M$. The effective mass $m_Q$ depends on $S_0$, and we consider the region of $S_0$ in which $m_Q$ is much larger than the mass of $P, \tilde{P}$. Then, $S$ has no (relevant or marginal) interactions below the scale $m_Q$. The Kähler potential becomes

$$K_{\text{eff}} = Z_S(M) \left[ (1 + \delta_1) \text{tr}(\tilde{S}^\dagger \tilde{S}) + \delta_2 \text{tr}(\tilde{S}^\dagger) \text{tr}(\tilde{S}) \right] + \cdots,$$

(4)

where $\delta_1$ and $\delta_2$ are $\mathcal{O}(1)$ corrections which appear at the threshold $m_Q$. We neglect $\delta_1$ and $\delta_2$ in the following discussions since they are not important. (For the reader who are interested in more rigorous definition of the effective Kähler potential, see Ref. [8].)

By using Eqs. (2) and (4), and setting the fluctuation to be zero, $\tilde{S}_{ij} = 0$, the effective potential for $\tilde{S}_{ij} = S_0 \delta_{ij}$ is given by

$$V_{\text{eff}}(S_0) = Z_S(M)^{-1} N_Q \left| m^{N_F} \Lambda^{3N_C-N_F} \lambda^{N_Q} \frac{N_C}{N_F} \right| S_0^{1 + \frac{N_Q}{N_F}}.$$

(5)

We should take $M \to 0$ to integrate out all momentum modes. The factor $Z_S(M)^{-1}$ represents the effect of the quantum corrections.

Now we determine $Z_S(M)^{-1}$ for $M \to 0$ as a function of $S_0$. First, let us determine $m_Q$. $m_Q$ is determined by the equation

$$m_Q = Z_Q(m_Q)^{-1} |\lambda S_0|,$$

(6)

where $Z_Q(M)$ is the wave function renormalization of $Q, \tilde{Q}$. $Z_S$ and $Z_Q$ are given by

$$Z_S(M) = \exp \left( - \int_{M_s}^M \gamma_S(M') d \log M' \right),$$

(7)

$$Z_Q(M) = \exp \left( - \int_{M_s}^M \gamma_Q(M') d \log M' \right),$$

(8)
where $\gamma_S$ and $\gamma_Q$ are the anomalous dimensions of $S$ and $Q$, $\bar{Q}$ respectively, and $M_*$ is the scale (taken to be larger than all the other scales) at which the fields are normalized canonically.

Notice that the theory is assumed to be on the conformal fixed point above the scale $m_Q$, i.e. $M > m_Q$. Then, $\gamma_S$ and $\gamma_Q$ are constant at the conformal fixed point, which we denote $\gamma_{S*}$ and $\gamma_{Q*}$. They satisfy the relation $\gamma_{S*} + 2\gamma_{Q*} = 0$ which is required by the renormalization group equation of the Yukawa coupling $\lambda$ at the fixed point. Then, we can do the integrations in Eqs. (7) and (8) to obtain for $M > m_Q$,

$$Z_S(M) = \left( \frac{M}{M_*} \right)^{-\gamma_{S*}}, \quad Z_Q(M) = \left( \frac{M}{M_*} \right)^{\gamma_{S*}/2}. \quad (9)$$

Using Eqs. (6) and (9), we obtain

$$m_Q = M_* \left( \frac{|\lambda S_0|}{M_*} \right)^{1/(1+\gamma_{S*}/2)}. \quad (10)$$

Now let us determine $Z_S(M)$ for $M < m_Q$. Below the effective mass $m_Q$, $\hat{S}$ has no relevant or marginal interaction and hence $\gamma_S(M < m_Q) \simeq 0$. Therefore, $Z_S(M) = Z_S(m_Q)$ for $M < m_Q$ and we obtain

$$Z_S(M < m_Q)^{-1} = Z_S(m_Q)^{-1} = \left( \frac{|\lambda S_0|}{M_*} \right)^{\gamma_{S*}/(1+\gamma_{S*}/2)}. \quad (11)$$

Thus, the power of $|S_0|$ in the potential is given by,

$$V_{\text{eff}} \propto |S_0|^4, \quad (12)$$

where

$$A = \frac{\gamma_{S*}/2}{1+\gamma_{S*}/2} - \left( 1 - \frac{N_Q}{N_C} \right). \quad (13)$$

If $A$ is positive, the runaway of the potential is stabilized. Numerical values of $\gamma_{S*}$ are listed in Table I. There exist many sets of $N_C$, $N_Q$, $N_P$ satisfying the condition $A > 0$, so the runaway can be stopped and the theory has well defined vacua.

The above argument breaks down when $S_0$ becomes small and the effective mass $m_Q$ becomes smaller than the mass of $P, \bar{P}$. Therefore, we expect that the potential minimum
Table 1: The anomalous dimension of $S$ at the conformal fixed point for several values of $N_C$, $N_Q$, and $N_P$. This table is taken from Ref. [8].

| $(N_C, N_Q N_P)$ | $\gamma_{S^*}/2$ |
|------------------|------------------|
| $(3, 2, 3)$      | 0.70             |
| $(3, 2, 4)$      | 0.36             |
| $(4, 3, 3)$      | 1.00             |
| $(4, 3, 4)$      | 0.59             |
| $(4, 3, 5)$      | 0.35             |
| $(5, 3, 5)$      | 0.81             |

of the theory exists in the region of small $S_0$. In Ref. [8], the SUSY was shown to be broken by an indirect argument using the Witten index [13]. In the next section, we will show by using Seiberg duality [10] that the SUSY is indeed broken at the tree level in the dual magnetic theory. We will also show explicitly that the potential minimum exists at $\langle S \rangle = 0$ if the theory is very strongly coupled in the electric theory.

3 Magnetic dual of the theory and SUSY breaking

In the SUSY breaking model reviewed above, the couplings become very strong after the decoupling of $P^a$, $\tilde{P}_a$. Then, it is convenient to use the Seiberg duality to study the low energy dynamics of the model. We can see explicitly that the SUSY is broken in the dual theory. Furthermore, as we will see below, if the couplings are too strong at the conformal fixed point in the electric theory (so that the dual magnetic theory is quite weakly coupled), the flavor $SU(N_P)$ symmetry breaks down spontaneously, implying the $SU(5)_{GUT}$ breaking in the gauge mediation.

3.1 Seiberg duality

Before considering the dual of our theory, let us review the original work of Ref. [10], which motivates the duality of the present model with the singlet $S^i_j$.

Consider an $SU(N_C)$ SUSY QCD with $N_F$ flavors of quarks $Q^i$, $\tilde{Q}^i$. If $\frac{3}{2}N_C < N_F < 3N_C$, this theory flows to a conformal fixed point at low energies. Seiberg argued that this theory is dual to an $SU(N_F - N_C)$ gauge theory with $N_F$ flavors of quarks $q_i$, $\tilde{q}^i$ and $N_F \times N_F$ singlets $M^i_j$, with a superpotential,  

$$W = \frac{1}{\mu} M^i_j q_i \tilde{q}^j,$$  

where $\mu$ is related to the electric and magnetic holomorphic dynamical scales $\Lambda$ and $\tilde{\Lambda}$.
by \[11\]

\[
\Lambda^{3N_C-N_F} \bar{\Lambda}^{3(N_F-N_C)-N_F} = (-1)^{N_F-N_C} \mu^{N_F},
\]

and the singlets \(M_j^i\) are the mesons \(M_j^i = Q^i \bar{Q}_j^i\) of the electric theory.

Let us take a dual of the dual. The dual of the dual theory is an \(SU(N_C)\) gauge theory with quarks \(Q^i', \bar{Q}_j'^i\), and singlets \(M_j^i, N_i^j \equiv q_i \bar{q}_j^3\) with a superpotential

\[
W = \frac{1}{\mu} M_j^i q_i \bar{q}_j^3 + \frac{1}{\mu} N_i^j Q^i \bar{Q}_j^i = \frac{1}{\mu} (M_j^i - Q^i \bar{Q}_j^i) N_i^j,
\]

where \(\tilde{\mu} = -\mu\). From this superpotential, \(M\) and \(N\) become massive and we can integrate out these fields. Then we obtain \(N_i^j = 0\) and \(M_j^i = Q^i \bar{Q}_j^i\), recovering the original electric theory, provided with \(Q^i' = Q^i\) and \(\bar{Q}_j'^i = \bar{Q}_j^i\).

Now let us apply the above considerations to our model with \(m = 0\). First, we define mesons \(K^a_b = P^a \bar{P}_b, L^a_i = P^a \bar{Q}_i, \bar{L}^i_{\bar{a}} = Q^i \bar{P}_{\bar{a}}\) and \(N_{i,j} \equiv q_i \bar{q}_j^3\). Second, consider an \(SU(N_F-N_C)\) gauge theory \(\langle N_F = N_Q + N_P \rangle\) with quarks \(q_i, \bar{q}_j^3, p_a, \bar{p}_b\) in the fundamental and anti-fundamental representation of \(SU(N_F-N_C)\). The superpotential is,

\[
W = \lambda \text{tr}(SN) + \frac{1}{\mu} \left\{ \text{tr}(N \bar{q} q) + \text{tr}(L \bar{q} p) + \text{tr}(\bar{L} \bar{q} p) + \text{tr}(K \bar{p} p) \right\},
\]

where contractions of flavor indices are represented by trace as before. The first term in this superpotential is the one present in the original electric theory, and other terms appear because of taking duality. From this superpotential, we can see that \(S\) and \(N\) become massive, as in the dual of the dual theory considered above. Thus, we can integrate them out, and obtain \(N = 0, \lambda S = -\frac{1}{\mu} \bar{q} q\). Then we finally obtain a theory with mesons \(K, L, \bar{L}\), quarks \(q, \bar{q}, p, \bar{p}\) and a superpotential,

\[
W = \frac{1}{\mu} \left\{ \text{tr}(L \bar{q} p) + \text{tr}(\bar{L} \bar{q} p) + \text{tr}(K \bar{p} p) \right\}.
\]

This is the dual of our theory with \(m = 0\). We believe that this duality is correct because it is a straightforward extension of the original Seiberg’s duality.

\(^2\) When \(m = 0\), there are \(SU(N_P) \times SU(N_P)\) symmetry acting on \(P\) and \(\bar{P}\) separately. Thus we use different indices \(a\) and \(\bar{a}\) for them.
As a check, let us consider the 't Hooft anomaly matching condition. We see, in the following argument, that the anomaly matching condition is indeed satisfied. Without the singlet $S$, the duality is the original one considered in Ref. [10], and the anomaly matching condition is satisfied. Then, introducing $S$ in both the electric theory and the magnetic theory, the global symmetry of the theory reduces to a subgroup of the one in the original theory. The anomaly is still matched, since we have only added the same singlet $S$ to both the electric and magnetic theory. Finally, let us integrate out $S$ and $N$ in the magnetic theory. Massive fields in general do not contribute to anomalies, so it has no effect on the anomaly matching to integrate out the massive fields $N$ and $S$ in the magnetic theory. Thus, we can conclude that the anomaly matching condition is satisfied in our duality.

### 3.2 Low energy dynamics

In this subsection, we analyze the dual magnetic theory of the present model with $m \neq 0$. We take a more general mass term $\text{tr}(m P \tilde{P}) = m_{a}^{b} P^{a} \tilde{P}_{b}$. Then we have an additional term in the superpotential, $\text{tr}(m K)$, in the dual theory (see Eq. (19)).

Although the couplings of the model are uniquely determined by $N_{C}$, $N_{Q}$ and $N_{P}$ at the fixed point, we can make the dual theory very weakly coupled as follows. Consider a parameter region where masses have a hierarchy; $m = \text{diag}(m_{1}, m_{2}, \ldots, m_{N_{P}})$ with $|m_{N_{P}}| \gg \cdots \gg |m_{1}|$. In this case, we can integrate out massive quarks $P, \tilde{P}$ step by step in the electric theory, and eventually the electric theory enters into confining phase. In the magnetic theory, the mass $m_{N_{P}}$ induces a vacuum expectation value for $\tilde{p}_{N_{P}} P^{N_{P}}$, and hence gauge symmetry is broken down to $SU(N_{F} - N_{C} - 1)$. Then, some fields become massive and we obtain the theory with $N_{F} - N_{C} \rightarrow N_{F} - N_{C} - 1$ and $N_{P} \rightarrow N_{P} - 1$. This process can be continued and eventually we obtain an asymptotic non-free theory in the magnetic description. In this way, we reach an asymptotic non-free theory where a weak coupling analysis becomes reliable. In the following analyses, we assume that the dual magnetic theory is weakly coupled (i.e. we assume the mass hierarchy discussed above or a very weak coupling at the fixed point in the magnetic theory).
The tree level superpotential is now given by

\[ W_{\text{tree}} = \frac{1}{\mu} \left\{ \text{tr}(L\bar{q}p) + \text{tr}(\bar{L}\bar{p}q) + \text{tr}(K\bar{p}p) \right\} + \text{tr}(mK). \] (19)

From this superpotential, we can see the SUSY breaking very easily. \( F \)-term of \( K \) is

\[ -F^\dagger_K \propto \frac{\partial W_{\text{tree}}}{\partial K} = \frac{1}{\mu} \bar{p}p + m. \] (20)

This equation is an \( N_P \times N_P \) matrix equation, and \( \bar{p} \) and \( p \) are \( N_P \times (N_F - N_C) \) and \( (N_F - N_C) \times N_P \) matrices, respectively. Because we have assumed \( N_Q < N_C \) to obtain a runaway superpotential in the electric theory, we have \( (N_F - N_C) - N_P = N_Q - N_C < 0 \), so \( \text{rank}(p) < N_P \). Thus we can conclude that Eq. (20) cannot be zero and the SUSY is broken, since \( \text{rank}(|\bar{p}p|) < N_P \) and \( \text{rank}(m) = N_P \). This is the “rank condition SUSY breaking” as in the Intriligator-Seiberg-Shih (ISS) model \[1\]. If \( N_Q \geq N_C \), Eq. (20) can be zero and the SUSY is not broken. This is consistent because we know that in this case \( (N_Q \geq N_C) \) there are no runaway superpotentials and SUSY vacua exist in the electric theory. It is remarkable that the rank condition for the SUSY breaking in the magnetic theory coincides with the runaway condition in the electric theory.

What happens if we take into account non-perturbative effects? It is known that a dynamically generated superpotential restores the SUSY in the ISS model \[1\]. However, in the present model, the SUSY is broken even if non-perturbative effects are taken into account. Suppose that mesons \( K, L, \) and \( \bar{L} \) have vevs and all the quarks become massive. (This is possible only if \( N_Q \leq N_P \). If \( N_Q > N_P \), some quarks have to be massless. Note that we have assumed \( N_Q < N_C < N_P \) in the previous section.) Then the following superpotential is generated by gaugino condensation,

\[ W_{\text{dyn}} = (N_F - N_C) \left( \Lambda^{3(N_F-N_C)-N_F} \det(M/\mu) \right)^{\frac{1}{N_F-N_C}} \]

\[ = -(N_F - N_C) \left( \Lambda^{-(3N_C-N_F)} \det M \right)^{\frac{1}{N_F-N_C}}, \] (21)

where we have used Eq. (15), and \( M \) is defined by

\[ M = \begin{pmatrix} K & L \\ \bar{L} & 0 \end{pmatrix}. \] (22)

Notice that \( M^a_b = K^a_b, \) \( M^a_i = L^a_i, \) \( M^i_a = \bar{L}^i_a \) and \( M^i_j = 0. \)
Integrating out the massive quarks, the superpotential is
\[
W_{\text{eff}} = \text{tr}(mK) - (N_F - N_C) \left( \Lambda^{-(3N_C - N_F)} \det M \right)^{-1}_{N_F - N_C}.
\] (23)

The F-term of K is
\[
- \left( F_K^a \right)_b \propto \frac{\partial W}{\partial K^b} = \left( \Lambda^{-(3N_C - N_F)} \det M \right)^{-1}_{N_F - N_C} (M^{-1})^a_b.
\] (24)

For this F-term to vanish, the inverse matrix \( M^{-1} \) must be of the form
\[
M^{-1} = \left( \begin{array}{cc} \alpha m & A \\ B & C \end{array} \right),
\] (25)
where \( \alpha \) is some non-zero constant and \( A, B \) and \( C \) are some matrices. However, \( M^{-1} \) of the above form does not exist. The product of the above \( M^{-1} \) and \( M \) is
\[
\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) = M^{-1} M = \left( \begin{array}{cc} \alpha mK + A\tilde{L} & \alpha mL \\ BK + C\tilde{L} & BL \end{array} \right).
\] (26)

The Equation \( \alpha mL = 0 \) implies \( L = 0 \) because \( \alpha \) is non-zero and \( m \) is an invertible matrix, but this contradicts the equation \( BL = 1 \). Thus the F-term of \( K \) cannot vanish. This shows that even if the non-perturbative effect Eq. (21) is taken into account, SUSY cannot be restored. We see that \( M^i_j = 0 \) plays a crucial role in the above proof which comes from the integration of the singlet \( \tilde{S}^i_j \).

Finally, we show another evidence supporting the duality in the present model. In the above consideration, we have investigated the direction in the classical moduli space where \( K, L \) and \( \tilde{L} \) have vevs and all the quarks become massive, so that the quarks have vanishing vevs. Now we consider another direction in the classical moduli space. In the case \( m = 0 \), the vevs of \( \tilde{p}p, \tilde{q}p \) and \( \tilde{p}q \) are constrained to be zero by the equations of motion of \( K, L \) and \( \tilde{L} \), respectively, but the vevs of \( \tilde{q}q \) are not. Let us consider the case that \( K \) and \( \tilde{q}q \) have very large vevs. These vevs give masses to \( p, \tilde{p}, L \) and \( \tilde{L} \), and the Affleck-Dine-Seiberg superpotential [14] is generated at low energies with a dynamical scale \( \Lambda^3_{L}(N_F - N_C - N_Q) = \Lambda^3_{L}(N_F - N_C - N_F \det(K/\mu)) \) as
\[
W_{\text{dyn}} = (N_F - N_C - N_Q) \left( \frac{\Lambda^3_{L}(N_F - N_C - N_Q)}{\det \tilde{q}q} \right)^{-1}_{N_F - N_C - N_Q}
\] (27)
Adding the mass term $\text{tr}(mK)$ to this superpotential, solving the equation of motion of $K$, and using Eq. (15), we obtain

$$K = m^{-1} \left( \Lambda^{3N_C-N_F} \det m \det \left( \frac{\tilde{q}q}{-\mu} \right) \right)^{\frac{1}{N_C}},$$

(28)

and an effective superpotential,

$$W_{\text{eff}} = N_C \left( \Lambda^{3N_C-N_F} \det m \det \left( \frac{\tilde{q}q}{-\mu} \right) \right)^{\frac{1}{N_C}}.$$  

(29)

In the process of taking duality, we have found that the electric variable $S$ and the magnetic variables $q, \tilde{q}$ are related by $\lambda S = -\frac{1}{\mu} \tilde{q}q$. Using this relation, Eq. (29) is just the superpotential Eq. (2) derived in the electric theory. This is another evidence for the correctness of our duality.

### 3.3 GUT breaking and R symmetry breaking

For a gauge mediation model to be phenomenologically viable, following conditions must be satisfied:

1. The standard model gauge group $SU(3) \times SU(2) \times U(1)$ should not be broken spontaneously by the SUSY breaking dynamics.

2. R symmetry (if exist) should be broken to generate the gaugino masses.

Unfortunately, both of the conditions may not be satisfied if the dual theory is weakly coupled. To see this, consider Eq. (20) with $m^a b = m \delta^a b$. From Eq. (20), we see that $p$ develops a vev of the form

$$p = \left( \sqrt{-\mu m} \cdot 1_{(N_F-N_C) \times (N_F-N_C)} , 0_{(N_F-N_C) \times (N_C-N_Q)} \right),$$

(30)

and similarly for $\tilde{p}$. This vev breaks the flavor $SU(N_P)$ symmetry. Because we want to identify a (sub)group of $SU(N_P)$ with the GUT gauge group $SU(5)_{\text{GUT}}$, the SM gauge group may be broken down. Of course it is possible that the SM gauge group is in a residual symmetry group after the breaking of $SU(N_P)$ as in the case of direct mediation models in the ISS model, but in that case the Landau pole problem of the SM gauge
coulplings is unavoidable for the low-scale gauge mediation. Furthermore, there is an R
symmetry with the charge assignment,
\[ Q, \bar{Q} : 1 - \frac{N_C}{N_Q}, \quad P, \bar{P} : 1, \quad S : \frac{2N_C}{N_Q}, \]  
(31)
or equivalently,
\[ q, \bar{q} : \frac{N_C}{N_Q}, \quad p, \bar{p} : 0, \quad K : 2, \quad L, \bar{L} : 2 - \frac{N_C}{N_Q}. \]  
(32)
The vev Eq. (30) does not break this R symmetry, so the gaugino masses in the SSM are not generated.

The breaking of \( SU(5)_{\text{GUT}} \) and the non-breaking of \( U(1)_R \) should be regarded as a consequence of weak couplings in the dual magnetic theory. Consider the other limit, i.e. weak couplings at the fixed point in the electric theory. Then, it is convenient to use the electric description of the dynamics. In this case, the gauge and Yukawa couplings above the mass threshold of \( P, \bar{P} \) are weak and hence the low energy dynamical scale (at which SUSY is broken) is much lower than the mass of \( P, \bar{P} \). Then \( P, \bar{P} \) are decoupled at the low energies much before the couplings become strong, so it is highly unlikely that those fields, \( P \) and \( \bar{P} \), develop vevs. Therefore, it is very natural to assume that the \( SU(N_P) \) symmetry remains unbroken. Furthermore, \( F_K = F_{P\bar{P}} \) is likely to be zero, so the SUSY breaking must be developed by other fields. Here recall that the SUSY is broken in the present model as shown in Ref. [8]. After the decoupling of \( P, \bar{P} \), the gauge invariant chiral fields at the low energies are \( S \) and \( Q \bar{Q} \). Then, it is reasonable to consider that at least one of those fields develop \( F \) terms.\(^3\) Now, it is important that the \( F \) terms of those fields carry nonzero \( U(1)_R \) charges, so the R symmetry is also broken by the \( F \) terms. For example, the \( F \) term of \( S \), \( F_S \), has \( R \) charge \(-2(1 - N_C/N_Q)\) as seen from Eq. (31), which may be useful for a gauge mediation as we will see in the next section.

The above consideration suggests that a phase transition occurs as the strengths of couplings are changed. We have seen that \( F_S \sim F_{q\bar{q}} = 0 \) in the tree level analyses in the

\(^3\) If the equation of motion of the chiral field \( S \), \( \frac{1}{4} \partial^2 S^\dagger = \lambda Q \bar{Q} \), is correct as an operator equation, then by taking the components we obtain \(-\langle F_S^\dagger \rangle = \lambda \langle Q \bar{Q} \rangle \) and \(-\partial^\mu \partial_\mu \langle S \rangle^\dagger = \lambda \langle F_{Q\bar{Q}} \rangle \), where the lowest components of the chiral fields are denoted by the same symbol as the chiral fields themselves. \( \partial^\mu \partial_\mu \langle S \rangle^\dagger \) vanishes because Lorentz invariance is not broken, so we have \( \langle F_{Q\bar{Q}} \rangle = 0 \). Thus the SUSY breaking is perhaps induced by \( \langle F_S \rangle \). However, we cannot exclude a possibility that \( \langle F_S \rangle \) is also zero, and the SUSY breaking is induced by a vev of some other vector superfield operator.
dual magnetic description when the coupling is too strong in the electric theory. However, when the coupling is weak at the fixed point in the electric theory so that the dynamical scale is much smaller than the physical mass of $P, \tilde{P}$, the $S$ and $Q$ most likely have non-vanishing $F$-terms. We assume that $F_S \neq 0$ in the next section. See Appendix A for an explicit toy model where a similar phase transition from $F_S = 0$ to $F_S \neq 0$ occurs.

4 Gauge mediation model

Now let us consider a candidate of the semi-direct gauge mediation model with the above SUSY breaking mechanism. We identify a subgroup of the flavor $SU(N_P)$ symmetry as the $SU(5)_{\text{GUT}}$ gauge group. Our model is an explicit example of the strongly coupled semi-direct gauge mediation of Ref. [15]. We impose the following conditions on $(N_C, N_Q, N_P)$.

1. $N_C \leq 4$. From the point of view of $SU(5)_{\text{GUT}}$ gauge group, the color number $N_C$ of the hidden gauge group becomes the messenger number in gauge mediation. For the perturbative GUT unification to be maintained, the messenger number must be small. In particular, for the low-scale gauge mediation, the constraint on the messenger number is rather severe, $N_C \leq 5$ with $N_C = 5$ marginal [5]. In our case, the messenger fields have large negative anomalous dimension, which effectively increase the messenger number in the SM $\beta$ functions [6]. Thus we impose $N_C \leq 4$ in this paper. However, see Ref. [6] for a mechanism which allows $N_C \geq 5$ without spoiling the perturbative GUT unification.

2. $N_P = 5$. To identify a subgroup of the flavor $SU(N_P)$ symmetry with $SU(5)_{\text{GUT}}$, we only need $N_P \geq 5$ as a necessary condition. However, it is more appealing to take $N_P = 5$, because when $N_P > 5$, there seems to be no reason for the mass of messenger quarks and the other $N_P - 5$ flavors of quarks to be the same. Thus, to achieve a one parameter model of SUSY breaking and gauge mediation, that is, the conformal gauge mediation [9], it is more desirable to take $N_P = 5$.

3. $N_Q < N_C$ and $A > 0$ in Eq. (13). This is the requirement for the present SUSY

\footnote{In fact, a large negative anomalous dimension of $P, \tilde{P}$ makes the messenger contribution to the SM $\beta$ function effectively larger than 5 even for $N_C = 4$. So we should assume that the theory is off the conformal fixed point and the couplings are small at high energies.}
breaking dynamics to work.

Imposition of those conditions uniquely leads to the model \((N_C, N_Q, N_P) = (4, 3, 5)\).
We call this model the \(SU(4)\) model from now on.

In the \(SU(4)\) model, there is a relation \(N_F = 2N_C\), i.e. in the middle of conformal window. This relation means that the model is strongly coupled at the fixed point in both the electric description and the magnetic description. The strong coupling in the electric description means that the dynamical scale \(\Lambda_{\text{phys}}\) and the mass of \(P, \tilde{P}, m_{\text{phys}}\) are of the same order, \(\Lambda_{\text{phys}} \sim m_{\text{phys}}\). This is desirable [9] because we may obtain the same order of gaugino and sfermion masses, and also obtain a light gravitino mass \(m_{3/2} \ll O(10)\) eV, which is free from the cosmological gravitino problems [16]. The strong coupling in the dual magnetic description means that \(SU(5)_{\text{GUT}}\) breaking and \(U(1)_R\) non-breaking argument discussed in the previous section is not applicable to this model. Because the electric theory is also strongly coupled, we can say nothing about the spontaneous breaking of those symmetries. In the following, we assume that \(SU(5)_{\text{GUT}}\) is not broken down, and also assume \(F_S \neq 0\) as discussed in the previous section. Notice that the SUSY is broken whether the theory is strongly coupled or not, as shown in Ref. [8].

Let us investigate the dynamics of the model at the fixed point at the 1-loop level. The 1-loop anomalous dimensions of \(S, Q, \) and \(P\) are given by

\[
\gamma_{1-\text{loop}}^S = N_C \frac{\lambda^2}{8\pi^2}, \quad \gamma_{1-\text{loop}}^Q = N_Q \frac{\lambda^2}{8\pi^2} - \frac{N_C^2 - 1}{N_C} \frac{g^2}{8\pi^2}, \quad \gamma_{1-\text{loop}}^P = -\frac{N_C^2 - 1}{N_C} \frac{g^2}{8\pi^2}.
\]

The RG running equations of the gauge and Yukawa couplings are given by

\[
M \frac{d}{dM} |\lambda|^2 = (\gamma_S + 2\gamma_Q)|\lambda|^2, \quad (34)
\]

\[
M \frac{d}{dM} g^2 = -\frac{g^4}{8\pi^2} \frac{3N_C - (1 - \gamma_Q)N_Q - (1 - \gamma_P)N_P}{1 - N_C g^2 / 8\pi^2}, \quad (35)
\]

where we have adopted the NSVZ \(\beta\) function [17] for the gauge coupling \(\beta\) function. Requiring these \(\beta\) functions to vanish and substituting \((N_C, N_Q, N_P) = (4, 3, 5)\), we obtain the fixed point values of the coupling constants as

\[
\left. \frac{|\lambda|^2}{8\pi^2} \right|_{1-\text{loop}} = \frac{4}{31} \simeq 0.13, \quad \left. \frac{g^2}{8\pi^2} \right|_{1-\text{loop}} = \frac{16}{93} \simeq 0.17.
\]
and
\[ \gamma^{1-\text{loop}}_S = \frac{16}{31} \approx 0.52, \quad \gamma^{1-\text{loop}}_Q = -\frac{8}{31} \approx -0.26, \quad \gamma^{1-\text{loop}}_P = -\frac{20}{31} \approx -0.65. \] (37)

The exact values of the anomalous dimensions determined by the \( \alpha \)-maximization technique \[18\] are given \[8\] by
\[ \gamma_S \approx 0.70, \quad \gamma_Q \approx -0.35, \quad \gamma_P \approx -0.59. \] (38)

The difference between the 1-loop and exact values indicates that the fixed point theory is strongly coupled. Although the 1-loop approximation is not a good one, if we evaluate the dynamical scale \( \Lambda_{\text{phys}} \) from a simple matching at the 1-loop level, we obtain
\[ \Lambda_{\text{phys}} \sim \exp \left( -\frac{8\pi^2}{(3N_C - N_Q)g_*^2} \right) m_{\text{phys}} \sim 0.52 \times m_{\text{phys}}. \] (39)

Thus we can expect that \( \Lambda_{\text{phys}} \) and \( m_{\text{phys}} \) are almost the same order.

Examples of operators generating the sfermion and gaugino masses are as follows \[9\]. The lowest dimensional operator which generates the sfermion masses are given by
\[ \int d^4\theta \frac{g_{\text{SM}}^2}{16\pi^2} \frac{c_1}{m_{\text{phys}}^2} \text{tr}(S\dagger S)\phi\dagger\phi, \] (40)

where \( \phi \) is an SSM field, \( g_{\text{SM}} \) the SM couplings, and \( c_1 \) an \( \mathcal{O}(1) \) constant. We assume that \( c_1 \) is positive\(^5\) as in Ref. \[9\]. At the tree level, this term generates the sfermion masses of order
\[ m^2_{\text{sfermion}} \sim c_1 \left( \frac{g_{\text{SM}}^2}{16\pi^2} \right)^2 \frac{|F_S|^2}{m_{\text{phys}}^2}, \] (41)

with \( F_S \) of order \( \Lambda_{\text{phys}} \sim m_{\text{phys}} \).

The lowest dimensional operator generating the SSM gaugino masses (which respect the R symmetry) is given by
\[ \int d^4\theta \frac{c_2}{m_{\text{phys}}^6} \left( \frac{1}{16\pi^2} \right) \text{tr}(S\dagger SS\dagger D^2S)W_{\text{SM}}^2, \] (42)

\(^5\) If \( c_1 \) is negative, we may gauge the \( SU(N_P) \) symmetry by another gauge group (not by \( SU(5)_{\text{GUT}} \)), and also introduce a vector-like massive pair of quarks in the bifundamental representation of \( SU(N_P) \times SU(5)_{\text{GUT}} \). Then, we may have positive sfermion masses due to the property of the semi-direct gauge mediation discussed in Ref. \[19\].
where $W_{\text{SM}}$ is the field strength chiral field of the SSM gauge field with the kinetic term normalized as $\int d^2 \theta (1/4g_{\text{SM}}^2) W_{\text{SM}}^2 + \text{h.c.}$ This term generates the gaugino masses of order

$$m_{\text{gaugino}} \sim c_2 \left( \frac{g_{\text{SM}}^2}{16\pi^2} \right) \left( \frac{\left| F_S \right|^2 \langle S \rangle \langle S \rangle \dagger}{m_{\text{phys}}^6} \right).$$

(43)

This is nonzero only if the vev of $S$, $\langle S \rangle$, is nonzero. However, there may be no convincing argument to show $\langle S \rangle \neq 0$ and hence it is desirable to find another mechanism for the gaugino mass generation.

Fortunately, there is an operator which generates the gaugino masses even if $\langle S \rangle = 0$. Let us consider an $SU(N_C)$ theory with the flavor number of $Q, \tilde{Q}$ given by $N_Q = N_C - 1$ (in the $SU(4)$ model, $N_C = 4$ and $N_Q = 3$). Then, the R charge of $S$ is given by $2N_C/N_Q = 2 + 2/N_Q$, and the R charge of $F_S$ is $2/N_Q$. Thus, the nonzero vev for $F_S$ breaks the R symmetry spontaneously. In fact, there is an operator which respect the R symmetry,

$$\int d^4\theta c_3 \left( \frac{1}{16\pi^2} \right) \frac{(\Lambda_L^-)^{2N_C+1}}{m_{\text{phys}}^{4N_C+2}} \text{tr}(S^\dagger S) \det(D^2S^\dagger) W_{\text{SM}}^2,$$

(44)

where $\Lambda_L$ is the holomorphic dynamical scale below the threshold of $P, \tilde{P}$ (see the discussion below). Notice that the operator $D^2S^\dagger = -4F^\dagger_S + \cdots$ has R charge $-2/N_Q$, so the operator $\det(D^2S^\dagger)$ has R charge $-2$. This operator gives the gaugino masses of order

$$m_{\text{gaugino}} \sim c_3 \left( \frac{g_{\text{SM}}^2}{16\pi^2} \right) \frac{(\Lambda_L^-)^{2N_C+1}}{m_{\text{phys}}^{4N_C+2}} \left| F_S \right|^2 \langle F_S \rangle \langle S \rangle N_Q.$$

(45)

One can see that Eq. (44) may be generated by one-anti-instanton effect, by considering an anomalous $U(1)$ symmetry which we call $U(1)_A$. Assign the $U(1)_A$ charge for the fields as $Q, \tilde{Q}: +1$, $S: -2$ and $P, \tilde{P}: 0$. Then, this is a symmetry at the classical level. However, the $U(1)_A$ transformation $Q \to e^{i\alpha}Q$ and $\tilde{Q} \to e^{i\alpha}\tilde{Q}$ have the $SU(N_C)$ anomaly, inducing the shift of Lagrangian as

$$\delta \mathcal{L} = \frac{2N_Q \alpha}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu},$$

(46)

This is a notable difference from the case of the $Sp(2)$ model considered in Ref. [9]. In that model, the R charge of the SUSY breaking field $S$ is 2, so it is necessary to have $\langle S \rangle \neq 0$ as well as $F_S \neq 0$ for the generation of the gaugino masses.
where $F_{\mu\nu}$ is the field strength of $SU(N_C)$ and $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$. This anomaly can be cancelled by the sift of the vacuum angle of $SU(N_C)$, $\theta \rightarrow \theta + 2N_Q \alpha$, where the topological term in the Lagrangian is given by

$$\mathcal{L}_\theta = -\frac{\theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (47)$$

Therefore, $U(1)_A$ becomes a symmetry if we assign $U(1)_A$ charge $2N_Q$ to $\exp(i\theta)$. Because $\det(\tilde{D}^2 S^\dagger)$ has $U(1)_A$ charge $2N_Q$, the $U(1)_A$ charge of $\exp(i\theta)$ implies that the operator Eq. (44) must be accompanied with the anti-instanton factor,

$$(\Lambda_L^1)^{2N_C+1} \equiv M^{2N_C+1} \exp(-S_{\text{anti-inst}}) = M^{2N_C+1} \exp \left(-\frac{8\pi^2}{g^2(M)} - i\theta \right), \quad (48)$$

where $M$ is a renormalization scale below the threshold of $P, \tilde{P}$, and $S_{\text{anti-inst}} = \frac{8\pi^2}{g^2} + i\theta$ is the anti-instanton classical action. $(\Lambda_L^1)^{2N_C+1}$ has $U(1)_A$ charge $-2N_Q$. This is the reason for the appearance of $(\Lambda_L^1)^{2N_C+1}$ in Eq. (44).

In Fig. 1, we show an example of an anti-instanton diagram which may generate the operator Eq. (44). We have not done the computation of diagrams explicitly, and hence Fig. 1 should be taken only as a schematic picture. The existence of a diagram does not always mean the existence of a non-vanishing operator in SUSY theory, because there is a possibility that cancellation may occur among various diagrams. But the operator Eq. (44) respects all the symmetry of the theory and it is not protected by holomorphy, so there seems to be no mechanism which forbids the existence of the operator. It seems, at the first glance, that Eq. (44) is quite small since it is generated by the anti-instanton effect. However, the contribution Eq. (45) may give not-so-small gaugino masses compared to the sfermion masses, since the gauge coupling is large.

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\[\text{footnote 6 in the first paper of Ref. [9]}\]
Figure 1: An anti-instanton diagram which may generate the operator (44). We take $N_C = N_Q + 1 = 2$, for simplicity. $\lambda$ and $\lambda_{SM}$ are the gauginos in the SU($N_C$) and the SM gauge theory, respectively. $\psi_Q, \psi_{\tilde{Q}}, \psi_P$ and $\psi_{\tilde{P}}$ are the fermionic components of the chiral fields $Q, \tilde{Q}, P$, and $\tilde{P}$, respectively. The scalar components of the chiral fields are denoted by the same symbol as the chiral fields itself. There are $2N_C$ zero modes for $\lambda^\dagger$, and one zero mode for each $\psi_Q^\dagger, \psi_{\tilde{Q}}^\dagger, \psi_P^\dagger$, and $\psi_{\tilde{P}}^\dagger$ in the anti-instanton background ($\psi$'s are right-handed and $\psi^\dagger$'s are left-handed). The zero modes of $\psi_P, \psi_{\tilde{P}}$ are contracted by their mass term $m \psi_P \psi_{\tilde{P}}$, and are not written in the diagram. The appearance of $\Lambda_L^{2N_C+1} = m^N_P \Lambda^{2N_C+1-N_P}$ in Eq. (44) instead of $\Lambda^{2N_C+1-N_P}$ (where $\Lambda^{2N_C+1-N_P}$ is the dynamical scale defined above the threshold of $P, \tilde{P}$) is due to this contraction of zero modes of $\psi_P, \psi_{\tilde{P}}$ by the mass term. The diagram may generate an effective Lagrangian $\mathcal{L} \propto |F_S|^2 \lambda_{SM} \lambda_{SM}$. 

18
Appendix A  Example of phase transition

To see an example of the phase transition discussed in Section 3.3, let us consider a toy model which have a similarity to our model.

Consider an $SU(2)$ IYIT model of SUSY breaking [20, 21], with one extra massive flavor. The matter chiral fields of the model are quarks $Q^i$, $P^a$ ($i = 1, 2, 3, 4$ $a = 1, 2$) in the fundamental representation of the $SU(2)$ gauge group, and six singlets $S_{ij} = -S_{ji}$. We take the tree level superpotential to be

$$W_{\text{tree}} = \frac{1}{2} \lambda S_{ij} Q^i Q^j + m P^1 P^2. \quad (A.1)$$

One can easily see an analogy between this toy model and our model, although there is neither a runaway superpotential nor a conformal fixed point in this toy model.

Let us consider two limits of this toy model; $m \gg \Lambda$ and $m \ll \Lambda$, where $\Lambda$ is the dynamical scale of the gauge theory. In the following analysis, we neglect any perturbative effects and RG evolution of parameters, and only consider the strong gauge dynamics.

First, consider the limit $m \gg \Lambda$. In this limit, $P^a$ are massive and they can be integrated out. The low energy theory is the usual IYIT model with the dynamical scale $\Lambda_L^4 = m \Lambda^3$. Confinement occurs and the effective superpotential is given by

$$W_{\text{eff}} = \frac{1}{2} \lambda S_{ij} N^{ij} + X \{ \text{Pf}(N) - \Lambda_L^4 \}, \quad (A.2)$$

where $N^{ij} = Q^i Q^j$ are low energy mesons, and $X$ is a lagrange multiplier. The equation of motion of $X$ gives $\langle N \rangle \sim \Lambda_L^2$, then the $F$-term of $S$ is $F_S^i \sim \langle N \rangle \sim \Lambda_L^2$. Thus the SUSY is broken by the $F$-term of $S$.

Next consider the limit $m \ll \Lambda$. In this case, the low energy theory can be described by mesons $N^{ij} = Q^i Q^j$, $K = P^1 P^2$, and $L^a = Q^i P^a$. The superpotential is,

$$W_{\text{eff}} = \frac{1}{2} \lambda S_{ij} N^{ij} + m K - \frac{1}{\Lambda^3} (K \text{Pf}(N) + \cdots), \quad (A.3)$$

where dots denote terms containing $L^a$, which are unimportant for the discussion below. The leading terms in the Kähler potential of the low energy theory may be of the form

$$K_{\text{eff}} \simeq \sum_{i < j} \left\{ |S_{ij}|^2 + \frac{1}{c^2 |\Lambda|^2} |N^{ij}|^2 \right\} + \frac{1}{c^2 |\Lambda|^2} |K|^2 + \cdots, \quad (A.4)$$

Footnote 8 This toy model is a simplified version of the model studied by E. Nakamura [22]. We thank him for explanation of his result.
where $c$ is a positive numerical constant. Then the potential is
\[
V \simeq \sum_{i<j} \left( |\lambda N^{ij}|^2 + c^2|\Lambda|^2 \left| \lambda S_{ij} - \frac{1}{2\Lambda^3} K \epsilon_{ijkl} N^{kl} \right|^2 \right) + c^2|\Lambda|^2 \left| m - \frac{1}{\Lambda^3} \text{Pf}(N) \right|^2,
\]
where $\epsilon_{ijkl}$ is the totally anti-symmetric tensor with $\epsilon_{1234} = 1$. Using this potential, we can see that the minima of the potential are at $S = N = 0$ with $-F^\dagger_K = c^2|\Lambda|^2 m$ and $F^\dagger_S = 0$, if the condition $|m/\Lambda| \ll |\lambda|^2/c^2$ is satisfied. Around $|m/\Lambda| \sim |\lambda|^2/c^2$, one can see that a phase transition occurs.

Thus we can conclude that in the limit $m \ll \Lambda$, the $F$-term of $S$ is zero, while in the other limit $m \gg \Lambda$, the $F$-term of $S$ is non-zero. Such a transition may also occur in our model.

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