Control of Charging of Electric Vehicles through Menu-Based Pricing under Uncertainty

Arnob Ghosh and Vaneet Aggarwal

Abstract—We propose an online pricing mechanism for electric vehicle (EV) charging. A charging station decides prices for each arriving EV depending on the energy and the time within which the EV will be served (i.e., deadline). The user selects either one of the contracts by paying the prescribed price or rejects all depending on their utilities. The charging station has to select a price without knowing the future arrival times of the EVs and the utilities of the EV users. We show that there exists a social welfare pricing strategy, however, the above may not maximize the expected profit of the charging station and even the profit may be 0. We propose a fixed profit pricing strategy which provides a guaranteed fixed profit to the charging station. Numerically, we show that how the charging station can select a profit margin to trade-off between profit and the users’ surpluses. We also show empirically that since our proposed mechanism also controls the deadline of the vehicles compared to the existing pricing mechanisms, hence, the number of charging spots required can be lower.

I. INTRODUCTION

Electric Vehicles (EVs) have several advantages over the traditional gasoline powered vehicles. For example, EVs are more environment friendly and more energy efficient. Thus, regulators (e.g., Federal Energy Regulator Commission (FERC)) are providing incentives to the consumers to switch to electric vehicles. However, the successful deployment of charging stations crucially depends on the profit of the charging stations and how efficiently the resources are used for charging the electric vehicles. Without profitable charging stations, the wide deployment of the EVs will remain a distant dream. It is also important for the regulators to increase the user (or, consumer) surplus to provide an incentive for the users. Hence, selecting a price is an imperative issue for the charging stations. The charging station may have limited charging spots or renewable energy harvesting devices. Hence, intelligent allocation of the resources among the EVs through pricing is a key component for fulfilling the potential of EVs’ deployment.

We propose a menu-based pricing scheme for charging an EV. Whenever an EV arrives, the charging station offers a variety of contracts \((l, t_{\text{dead}})\) at price \(p(l, t_{\text{dead}})\) to the user where the user will be able to use up to \(l\) units of energy within the deadline \(t_{\text{dead}}\) for completion. We assume that the EV users are equipped with smart devices (e.g., smart phone) which can communicate to the charging station over a wireless link. The EV user either accepts one of the contracts by paying the specified price or rejects all of those based on its payoff. We assume that the user gets an utility for consuming \(l\) amount of energy with the deadline \(t_{\text{dead}}\). The payoff of the user (or, user’s surplus) for a contract is the difference between the utility and the price paid for one contract. The user will select the option which fetches the highest payoff. The various advantages of the above pricing scheme should be noted. First, it is an online pricing scheme. It can be adapted for each arriving user. Second, since the charging station offers prices for different levels of energy and the deadline, the charging station can prioritize one contract over the others depending on the energy resources available. Favorable prices for shorter deadlines can attract users to vacate the charging stations early and only use it when it is necessary. Third, the user’s decision is much simplified. She only needs to select one of the contracts (or, reject all) and will receive the prescribed amount within the prescribed deadline.

We consider that the charging station is equipped with renewable energy harvesting devices and a storage device for storing energies. The charging station may also buy conventional energy from the market to fulfill the contract of the user if required. Hence, if a new user accepts the contract \((l, t_{\text{dead}})\), a cost is incurred to the charging station. This cost may also depend on the existing EVs and their resource requirements. Hence, the charging station needs to find the optimal cost for fulfilling each contract. We show that obtaining that cost is equivalent to solve a linear programming problem.

We consider two optimization problems—i) social welfare maximization, and ii) the EV charging station’s profit maximization. We first propose a pricing scheme which maximizes the social welfare irrespective of whether the charging station is aware of the utilities of the users or not. The pricing scheme is simple to compute, as the charging station selects a price which is equal to the marginal cost for fulfilling a certain contract for a new user (Theorem 1). However, the above pricing scheme only provides zero profit to the charging stations. Thus, such a pricing scheme may not be useful to the charging station.

We show that there may not exist a pricing strategy which simultaneously maximizes the ex-post social welfare and the expected profit. One has to give away the ex-post social welfare maximization in order to achieve expected profit maximization. However, whether a contract will be selected by the user does not depend on the price of the contract, but also the prices of other contracts. Thus, achieving a pricing scheme which maximizes the expected profit is difficult because of the discontinuous nature of the profits. We propose a pricing strategy which yields a fixed (say, \(\beta\)) amount of profit to the charging station. Lower values of \(\beta\) may reduce the profit; however, higher values of \(\beta\) may also distract the users for charging. We show that a suitable choice of \(\beta\) can maximize the profit of the charging station for a class of utility functions (Theorem 3).

Finally, we, empirically provide insights how a trade-off between the profit of the charging station and the social welfare can be achieved for various pricing schemes (Section VI). We also show that how our pricing scheme can increase greater utilization of the resources and result in a lower number of charging spots compared to the existing ones.

The proofs are deferred to the technical report [1] owing to the space constraint.

Related Literature: To the best our knowledge this is the first attempt to consider contract based online pricing for controlling

\(^1\)Social welfare is the sum of the profit of the charging station and the user surplus.
both the energy and deadline of the EVs. In [2]–[5] online scheduling algorithms have been proposed for charging EVs. The main focus of these papers was scheduling, they did not consider the optimal pricing approach for the charging station which we did. Further, unlike in [2]–[5], in our menu-based pricing scheme, the charging station can control the energy requirement and the deadline of the users by selecting the prices to the users. Hence, a greater flexibility can be achieved. Additionally, [2], [5] did not guarantee that the energy demand will be fulfilled. In contrast, in our model once a user opts for an option, the EV charging station always fulfills the request of users.

[6]–[9] considered a deadline differentiated pricing scheme where users select amounts of energies they will consume at different times depending on the prices. The above papers considered an offline or day-ahead market. However, in practice, the users’ utilities only depend on the total energy they receive over the entire time horizon, not specifically for each time period. Hence, it is difficult to bid for energy at each deadline. In contrast, in our proposed mechanism, the user selects one of the contracts and receives the prescribed energy within the prescribed deadline. Thus, user’s decision burden and the communication overhead are greatly reduced. In contrast to all the above papers, in our proposed pricing mechanism, the user can choose a deadline by selecting one of the menu-based prices. Choosing suitable prices the charging stations can facilitate the efficient use of the limited charging spots and resources.

II. MODEL

We consider a charging station which wants to select a pricing strategy in order to maximize its payoff over a certain period of time \( T \) (e.g. one day). Suppose that user \( k = 1, \ldots, K \) arrives at the charging station at time \( t_k \). The charging station decides a price menu or a contract \( p_{k,l,t} \) to user \( k \) for different energy levels \( l \in \{1, \ldots, L\} \) and deadline \( t \in \{t_k + 1, \ldots, T\} \) (Fig. 1). It is needless to say that we can discretize the time and energy at any level that one may want\(^2\), however, the computational cost will increase. User \( k \) has to decide \( l \) and \( t \) based on the menu; if she decided to accept any option on the menu, she has to pay the prescribed price \( p_{k,l,t} \). The user can decide not to accept any price too (Fig 1). The EV may not be charged continuously i.e. preemption is allowed. A preempted battery of the EV can be resumed charging from the previous battery level upon preemption.

A. User’s utilities

If user \( k \) selects the price option \( p_{k,l,t} \), it will get an utility \( u_{k,l,t} \). Hence, its surplus or payoff will be \( u_{k,l,t} - p_{k,l,t} \). If the user rejects all the options then its utility is 0 (Fig. 1). We assume that the realized value \( u_{k,l,t} \) is drawn from a distribution function of the random variable \( U_{k,l,t} \). The random variables \( U_{k,l,t} \) need not be independent, in fact, they can be generated from a joint distribution. In practice, there is a correlation of the utilities among different deadlines and charging amount. For example, \( U_{k,l,t} \geq U_{k,l,t} \) if \( l_1 > l \) as a higher amount of energy for a fixed deadline should induce higher utility to a user. Similarly, \( U_{k,l,t} \leq U_{k,l,t} \) if \( t_1 > t_2 \) since for a similar level of charge, the user will prefer the smaller deadline menu as it will give the user more flexibility. On the other hand, a user who wants to park a long time may not mind a longer deadline. Thus, we do not make any a priori assumptions on the utility functions since they can be different for different users. We assume that the car vacates the charging spot once it exceeds its prescribed deadline\(^3\).

B. The charging Station

1) Hybrid Energy Source: We assume that the charging station can obtain energies for fulfilling the charging request both from the renewable sources and conventional sources (Fig. 2). The charging station can buy a conventional energy \( q_t \) at a price \( c_t \) for usage during the interval \([t, t+1]\). We do not assume any specific type of pricing schemes for buying conventional energy, however, we assume that \( c_t \) is known. If the real time pricing is used, then we consider \( c_t \) as the expected real-time price at time \( t \).

The charging station is also equipped with an energy harvesting device and a storage capacity of \( B_{\text{max}} \) (Fig. 2). The harvesting device harvests \( E^d_t \) amount of energy between \([t, t+1]\). We assume that the marginal cost to harvest renewable energy is 0. The amount of energy that the charging station uses from the storage for the time \([t, t+1]\) be \( r_t \).

2) System Constraints: The charging station must procure the \( l \) amount of energy for user \( k \) by time \( d_k \) if the user accepts the price menu \( p_{k,l,d_k} \). Let \( q_{k,t} \) be the conventional energy and \( r_{k,t} \) be the energy from the storage device used by the charging station to charge user \( k \) for time interval \([t, t+1]\). Then, we must have the following constraint

\[
\sum_{t=t_k}^{d_k-1} (r_{k,t} + q_{k,t}) \geq l
\]  

Suppose that initially, there is a set of users \( K_0 \) already present in the charging station at time \( t_k \). Now, the user \( i \in K_0 \) has a deadline of \( w_i \) and additional demand \( N_i \). The charging station must have to satisfy the demand of those users. Thus, the charging station must also satisfy

\[
\sum_{t=t_k}^{w_i-1} (r_{i,t} + q_{i,t}) \geq N_i, \forall i \in K_0
\]  

We also assume that the charging station has one kind of charging equipment (either slow charging or fast charging) and there is a maximum rate constraint \( (R_{\text{max}}) \). Hence,

\[
r_{k,t} + q_{k,t} \leq R_{\text{max}}, \quad r_{k,t} + q_{i,t} \leq R_{\text{max}}, \forall i \in K_0 \quad \forall t.
\]  

Since the total energy \( r_t \) used for charging from the storage device and the amount of conventional energy, \( q_t \) bought from the market, thus,

\[
r_{k,t} + \sum_{i \in K_0} r_{i,t} = r_t, \quad q_{k,t} + \sum_{i \in K_0} q_{i,t} \leq q_t \quad \forall t.
\]

Note that the charging station may store the unused conventional energy bought from the market i.e. \( q_t - q_{k,t} - \sum_{i \in K_0} q_{i,t} \) is stored in the storage device. The charging station may buy an additional amount of conventional energy at time \( t \), if the future prices are higher.

Let the battery level at time \( t + 1 \) be \( B^{t+1} \) and \( B_0 \) be the initial battery level. The charging station also wants to keep the battery level at the end of the day as \( B_0 \). If the final battery level need not match the initial level, our pricing approach can be easily extended to that scenario. If the battery can not hold the excessive energy, then it is wasted. Let use denote the wasted energy for time \([t, t+1]\) be \( D_t \geq 0 \), then

\(^3\)It the user can not take away its car, the charging spot will be automatically downgraded to a mere parking spot i.e. without any charging facility.

\(^4\)It is also straightforward to extend our setting when the charging station can sell the excess energy to the grid, i.e. it will sell \( D_t \) to the grid.
that user can get is 0 process of the EVs and their demand. Thus, the charging station does not need to know the arrival times for the future vehicles. We consider that the charging station is myopic or near-sighted i.e., it selects its price for user \( k \) without considering the future arrival process of the vehicles. However, it will consider the cost incurred to charge the existing EVs. Thus, the charging station does not need to know the arrival process of the EVs and their demand.

**III. PROBLEM FORMULATION**

The profit of the charging station inherently depends on whether the user will accept that menu or not. Hence, before how the charging station will select \( p_{k,l,t} \) for user \( k \), we delve into the decision process of the users.

**A. User's decision**

A user selects at most one of the price menus in order to maximize its payoff or surplus. We assume that the user is a price taker. Thus, for a menu of prices \( p_{k,l,t} \), the user \( k \) selects \( A_{k,l,t} \in \{0,1\} \) such that it maximizes the following

\[
\text{maximize} \sum_{l=1}^{L} \sum_{t=t_{k}+1}^{T} A_{k,l,t} (u_{k,l,t} - p_{k,l,t}) \\
\text{subject to} \sum_{l=1}^{L} \sum_{t=t_{k}+1}^{T} A_{k,l,t} \leq 1 \]

Note from the formulation in (6) the maximum is achieved when \( A_{k,l,t} = 1 \) for the contract which maximizes the user \( k \)'s payoff (i.e., \( \max_{i,j} \{u_{k,i,j} - p_{k,i,j}\} = u_{k,l,t} - p_{k,l,t} \) and is 0 otherwise. If such a solution is not unique, any convex combination of these solutions is also optimal since a user can select any of the maximum payoff contracts. Note that if the maximum payoff that user gets among all the price menus (or, contracts) is negative, then the user will not charge i.e. \( A_{k,l,t} = 0 \) for all \( l \) and \( t \). We also assume that if there is a tie between charging and not charging, then the user will decide to charge i.e. if the maximum payoff that user can get is 0, then the user will decide to charge.\(^5\)

**B. Myopic Charging Station**

Since the users arrive for the charging request at any time throughout the day, the charging station does not know the exact arrival times for the future vehicles. We consider that the charging station is myopic or near-sighted i.e., it selects its price for user \( k \) without considering the future arrival process of the vehicles. However, it will consider the cost incurred to charge the existing EVs. Thus, the charging station does not need to know the arrival process of the EVs and their demand.

\(^5\)Our result can be readily extended to the other options, in that case the price strategies given in this paper have to be decreased by an \( \epsilon > 0 \) amount.

**C. Charging Station’s Decisions and cost**

Note that if the user \( k \) accepts the menu \( (l,d_k) \), then the charging station needs to allocate resources among the EVs in order to minimize the total cost of fulfilling the contract. First, we introduce some notations which we use throughout.

**Definition 1.** The charging station has to incur the cost \( v_{l,d_k} \) for fulfilling the contracts of existing customers and the contract \((l,d_k)\) of the new user \( k \), where \( v_{l,d_k} \) is the value of the following linear optimization problem:

\[
\mathcal{P}_{l,d_k} : \min \sum_{t=1}^{T-1} c_i q_t \\
\text{subject to (1), (2), (3), (4), (5)} \\
\text{var:} \ r_{k,t}, q_{k,t}, q_t, r_t, D_t \geq 0
\]

Note that our model can also incorporate time varying, strictly increasing convex costs \( C_i(t) \). \(^6\) Since \( \mathcal{P}_{l,d_k} \) is a linear optimization problem, it is easy to compute \( v_{l,d_k} \). Also, note that if the above problem is infeasible for some \( l \) and \( t \), then we consider \( v_{l,t} \) as \( \infty \). We assume that the prediction \( E^t \) is perfect for all future times and known to the charging station.\(^7\)

**Definition 2.** Let \( v_{-k} \) be the amount that the charging station has to incur to satisfy the requirements of the existing EVs if the new user does not opt for any of the price menus.

If user \( k \) does not accept any price menu, then the charging station still needs to satisfy the demand of existing users i.e. the charging station must solve the problem \( \mathcal{P}_{l,t} \) with \( q_{k,t} = r_{k,t} = 0 \) and \( v_{-k} \) is the value of that optimization problem. Thus, from Definitions 1 and 2 we can visualize \( v_{l,d_k} - v_{-k} \) as the additional cost or marginal cost to the charging station when the user \( k \) accepts the price menu \( p_{k,l,d_k} \). It is easy to discern that \( v_{l,d_k} - v_{-k} \) is non-negative for any \( d_k \) and \( l \).

**D. Profit of the charging station**

Now, we discuss the profit of the charging station based on its pricing strategies. Note that if all the spots are occupied then, the charging station can not accommodate a new user. Thus, we consider the scenario where a charging spot is available.

Let \( R_{k,l,t} \) be the event that the price menu \( p_{k,l,t} \) is selected, hence, the profit of the charging station is
The expected profit maximization problem for the charging station is to maximize the above objective over \( p_{k,l,t} \). We assume that the utilities are distributed according to some continuous distribution. Hence, \( \Pr(R_{l,t}) \) is given by

\[
\Pr(R_{l,t}) = \Pr(U_{k,l,t} - p_{k,l,t} \geq \max_{i,j} \{U_{k,i,j} - p_{k,i,j}\})^+
\]

Thus, \( \Pr(R_{l,t}) \) not only depends on \( p_{k,l,t} \), but also prices for other menus.

**E. Objectives**

We consider that the charging station decides the price menus in order to fulfill one of the two objectives (or, both)--i) Social Welfare Maximization and ii) its profit maximization.

1) **Social Welfare**: The social welfare is the sum of user surplus and the profit of the charging station. As discussed in Section III-A for a certain realized values \( u_{k,l,t} \) if the user \( k \) selects the price menu \( p_{k,l,t} \), then its surplus is \( u_{k,l,t} - p_{k,l,t} \), otherwise, it is 0.

As discussed in Section III-D the profit of the charging station is \( p_{k,l,t} = v_{l,t} + v_k \) for a given price \( p_{k,l,t} \) if the user selects the menu, otherwise it is 0. Hence, the ex-post social welfare maximization problem is to select the price menu \( p_{k,l,t} \) which will maximize the following

\[
P_{\text{social welfare}} \triangleq \max \sum_{l=1}^{L} \sum_{t=t_{l}+1}^{T} (u_{k,l,t} - v_{l,t} + v_k) A_{k,l,t}(p_k)
\]

\[
\text{var} : p_{k,l,t} \geq 0.
\]

where \( A_{k,l,t}(p_k) \) denotes the decision of the user \( k \) for the price vector \( p_k \) (Section III-A). Recall that in order to find \( v_{l,t} \) we have to solve \( P_{l,t} \) (cf. (7)) which is a constrained optimization problem. Since EV is expected to increase the social value such as providing a cleaner environment, and higher energy efficiency, hence, it is important for the regulator (e.g. FERC) whether there exists a pricing strategy which maximizes the social welfare of the system. If the charging station is operated by the regulator or some government agency, then the main objective is indeed maximizing the social welfare or user surplus is maintained.

Note that we consider maximizing the ex-post social welfare. When the charging station is unaware of the utilities of the users, then two options are considered--i) decides a price and hopes that it will maximize the social welfare for the realized values of utilities (ex-post maximization), or ii) decides a price and hopes that it will maximize the social welfare in an expected sense (ex-ante maximization). Thus, the ex-ante maximization does not guarantee that the social welfare will be maximized for every realization of the random variables \( U_{k,l,t} \). However, in the ex-post maximization, the social welfare is maximized for each realization of the random variables. Thus, ex-post maximization is a stronger concept of maximization (and thus, more desirable) and it is not necessary that there exist pricing strategies which maximize the ex-post social welfare.

2) **Profit Maximization**: Social welfare maximization does not guarantee that the charging station may get a positive profit. It is important for the wide-scale deployment of the charging stations, the charging station must have some profit. Further, if the charging station is operated by a private entity its objective is indeed to maximize the profit. When the charging station is clairvoyant, then the charging station wants to maximize the profit given in (8) by selecting \( p_{k,l,t} \).

3) **Separation Problem**: Note that in order to select optimal \( p_{k,l,t} \), the charging station has to obtain \( v_{l,t} \) and \( v_k \) (Definitions 1 & 2). However, \( v_{l,t} \) and \( v_k \) do not depend on \( p_{k,l,t} \). Hence, we can separate the problem--first the charging station finds \( v_{l,t} \) and \( v_k \), and then it will select \( p_{k,l,t} \) to fulfill the objective. We now focus on finding optimal \( p_{k,l,t} \).

**IV. RESULTS: SOCIAL WELFARE MAXIMIZATION**

The social welfare maximizing pricing strategy is not unique. However, in the next we state one of the simplest pricing strategies--

**Theorem 1.** The pricing strategy \( p_{k,l,t} = v_{l,t} - v_k \) maximizes the ex-post social welfare.

Though the pricing strategy maximizes the social welfare, the above pricing strategy does not provide any positive profit to the charging station. The charging station also does not need to know the distribution of the utilities, such a pricing strategy is known as prior free.

Also note that the pricing strategy also maximizes the social welfare in the long run when the additional cost of fulfilling a contract (i.e. \( v_{l,t} - v_k \)) does not depend on the existing users in the charging station. The condition that \( v_{l,t} - v_k \) is independent of the existing EVs in the charging station is satisfied if either all demand can be fulfilled using renewable energy or there is no renewable energy generation and the conventional energy is bought at a flat rate. Hence, in the two above extreme cases, the myopic pricing strategy is also optimal in the long run.

**V. PROFIT MAXIMIZATION OF THE CHARGING STATION**

We have already seen (Theorem 1) that a pricing strategy which can maximize the ex-post social welfare, however, it does not give any positive profit. We now consider a pricing strategy which gives a guaranteed profit to the charging station. First, we introduce a notation which we use throughout.

**Definition 3.** Let \( L_{k,l,t} \) be the lowest end-point of the marginal distribution of the utility \( U_{k,l,t} \).

Consider the following pricing strategy--

\[
p_{k,l,t} = v_{l,t} - v_k - (\max_{i,j} \{L_{k,i,j} - v_{i,j} + v_k\})^+ + \beta
\]

where \( \beta \geq 0 \). The expected profit of the charging station for the above pricing strategy can be readily obtained–

**Theorem 2.** Suppose that \( p_{k,l,t} \) is set according to (10), then the expected profit of the charging station is (\( \max_{i,j} \{L_{k,i,j} - v_{i,j} + v_k\}^+ + \beta \)) \( \max_{l,t} \{\Pr(U_{k,l,t} \geq v_{l,t} - v_k + \beta + L_{k,l,t})\} \).

The ex-post social welfare is maximized only when \( \beta = 0 \).

\( \beta = 0 \) in (10) gives the maximum possible profit that the charging station can have under the condition that it maximizes the ex-post social welfare with probability 1. However, it may not maximize the expected profit of the charging station or in other words. Thus, the above theorem shows that a pricing strategy may not simultaneously maximize the ex-post social welfare and the expected profit of the charging station.

Note that if \( \max_{i,j} \{L_{k,i,j} - v_{i,j} + v_k\} > 0 \), then the charging station’s profit increases. If the charging station has large storage or large renewable energy harvesting devices, then, the cost \( v_{l,t} - v_k \) will be lower and thus, the charging station can get a higher profit. It also increases the user surplus, as the price set by the charging station decreases. Thus, the impact of higher degrees of
renewable energy integration for the charging station increases both the profit of the charging station and the user surplus. The above illustrates the importance of the storage and harvesting energy devices in the charging station. The regulator (e.g. FERC) can also provide incentives to the charging station to set up those devices as the pricing strategy increases profit to the charging station as well as the ex-post social welfare.

In the extreme, when \(v_{l,t} = 0\) for all \(l\) and \(t\), then the profit of the charging station becomes maximum. However, further decreasing \(v_{l,t}\) will not have any effect on the profit of the charging station as well as the user surplus; hence, it also shows the investment that the charging station needs to make for storage and renewable energy harvesting devices.

The charging station needs to know the lowest end-points of the support set of the utilities unlike in Theorem 1. For example, \(L_{k,l,t}\) may be computed by the lowest possible price that the user accepts for the energy level \(l\) and the deadline \(t\).

Now, we provide an example where the above pricing strategy can also be a profit maximizing for a suitable choice of \(\beta\). First, we introduce a notation

**Definition 4.** Let \(\zeta = \max\{\gamma\} = \arg\max_{\beta \ge 0}(\max_{i,j}(L_{k,i,j} - v_{i,j} + v_{-k}))^+ + \beta \{\max_{i,j}\Pr(U_{k,i,j} - L_{k,i,j} > \beta + v_{i,j} - v_{-k})\} \).

Note that since \(U_{k,l,t}\) is bounded and the probability distribution is continuous, thus, \(\zeta\) exists. Note from Theorem 2 that \(\zeta\) corresponds to the \(\beta\) for which the charging station can get the maximum possible profit when the prices are of the form \(p_{k,l,t} = v_{l,t} - v_{-k} + \beta\).

Now consider a class of widely seen utility functions

**Assumption 1.** Suppose that the utility function \(U_{k,l,t} = (Y_{k,l,t} + X_k)^+\) for all \(l\) and \(t\); \(Y_{k,l,t}\) is a constant and known to the charging station, however, \(X_k\) is random variable and whose realized value is not known to the charging station.

In the above class of utility function, the uncertainty is only regarding the realized value of the random variable \(X_k\). Note that \(X_k\) is independent of \(l\) and \(t\), hence, \(X_k\) is considered to be an additive white noise.

It is important to note that we do not put any assumption whether \(X_k\) should be drawn from a continuous or distribution. However, if the distribution is discrete, we need the condition that \(\zeta\) must exist.

**Theorem 3.** Consider the pricing strategy \(p_{k,l,t} = v_{l,t} - v_{-k} + \zeta\); where \(\zeta\) is defined in Definition 4.

The pricing strategy maximizes the expected profit of the charging station (given in (8)) when the utility functions are of the form given in Assumption 1.

**Remark:** The above result is surprising. It shows that a simple pricing mechanism such as fixed profit can maximize the expected payoff for a large class of utility functions. However, if the utilities do not satisfy Assumption 1 then, the above pricing strategy may not be optimal.

### VI. Simulation Results

We numerically study and compare various pricing strategies presented in this paper. We evaluate the profit of the charging station and the user’s surplus achieved in those pricing strategies. We also show that our mechanism requires less charging spots compared to our nearest pricing model.

**A. Simulation Setup**

Similar to [10], the user’s utility for energy \(x\) is taken to be of the form

\[
\min\{-ax^2 + bx, \frac{b^2}{4a}\}
\]

Thus, the user’s utility is a strictly increasing and concave function in the amount energy consumed \(x\). The quadratic utility functions for EV charging have also been considered in [11]. Note that the user’s desired level of charging is \(b/(2a)\). We assume that \(b/(2a)\) is a random variable. [12] shows that in a commercial charging station, the average amount of energy consumed per EV is 6.9 kWh with standard deviation 4.9 kWh. We thus consider that \(b/(2a)\) is a truncated Gaussian random variable with mean 6.9 kWh and standard deviation 4.9 kWh in the interval [2, 20]. We assume \(a\) is a uniform random variable in the interval [1/20, 1/8].

From [12], the deadline or the time spent by an electric vehicle in a commercial charging is distributed with an exponential distribution with mean 2.5 hours. Thus, we also consider the preferred deadline \((T_{pref})\) of the user to be an exponentially distributed random variable with mean 2.5. The user strictly prefers a lower deadline. Hence, we assume that the utility is a convex decreasing function of the deadline [2]. The utility of the user after the preferred deadline is considered to be 0. Hence, the user’s utility is

\[
U_{k,l,t} = \min\{-al^2 + lb, \frac{b^2}{4a}\} \times \left(\exp(T_{pref} - t - t_k) - 1\right) \times \left(\exp(T_{pref} - t_k) - 1\right)
\]

The arrival process of electric vehicles is considered to be a Poisson arrival process. However, the arrival rates vary over time. For example, during the peak-hours (8 am to 5 pm) the arrival rate is higher compared to the off-peak hours. We, thus, consider a Poisson arrival process. However, the arrival rates vary over time. For example, during the peak-hours (8 am to 5 pm) the arrival rate is higher compared to the off-peak hours. We, thus, consider a non-homogeneous Poisson process with the arrival rate is 15 (5, resp.) vehicles per hour during the peak period (off-peak period, resp.). We also assume that the maximum charging rate \(R_{max}\) is 3.3 kW.

We assume that the renewable energy is harvested according to a truncated Gaussian distribution with mean 2 and variance 2 per hour. The storage unit is assumed to be of capacity 20 kWh. Initial battery level is assumed to be 0 i.e. it is fully discharged. The prices for the conventional energy is assumed to be governed by Time-of-Use (ToU) time scale.

**B. Results**

We consider the scenario where the charging station is unaware of the exact utilities of the users, however, it knows the lower end-points. We consider the pricing strategy stated in (10).

1. **Effect of \(\beta\) on User’s Surplus and Profit of the charging station:** The total surpluses of the users decreases with increase in \(\beta\) (Fig. 3) as the user pays larger price when \(\beta\) increases. The user surplus is maximum at \(\beta = 0\). The decrement of total users’ surplus is not significant with \(\beta\) for \(\beta < 1.6\). However, as \(\beta > 1.6\), it decreases rapidly. For \(\beta < 1.6\), the number of users served does not decrease much with \(\beta\). Hence, the total users’ surpluses decrease slowly.

As \(\beta\) increases the profit increases initially (Fig. 4). However, as \(\beta > 3\), the number of users served decreases rapidly, hence, the profit also drops.

At high values of \(\beta\) both users’ surpluses and the profit decrease significantly. Low values of \(\beta\) give high users’ surpluses, however, the profit is low. \(\beta \in [0.8, 1.6]\) is the best candidate for the balance between profit and users’ surpluses.

2. **Advantages of our proposed mechanism:** Fig. 5 shows that our pricing algorithm requires less charging spots compared to the differentiated pricing mechanism [6], [7] closest to our proposed approach. Similar to [6], [7] the users select the amount of energy to be consumed for each time period based on the price set by the
The energy used from the battery of the charging station is also β = 0 the peak energy consumption from the grid is very high. Even applied i.e., the EVs are charged as soon as they arrive, then, the charging station can also limit the peak energy consumption from β accepted users decreases with β bought from the grid decreases. This is because the number of users is large, serving additional user can be significant and thus, higher as β the admitted users is higher, hence the price variation is also selecting higher prices during the peak time to flatten the demand pricing mechanism is consistent with the FERC’s objective of.

In our proposed mechanism, the charging station controls the time spent by an EV through pricing and results into lower charging spots.

3) Effect on the price selected by the charging station: The price is higher during the peak period when the arrival rates is higher and the time-of-use price is high (Fig. 6). Hence, the pricing mechanism is consistent with the FERC’s objective of selecting higher prices during the peak time to flatten the demand curve. A new price is selected when an EV arrives. As β decreases the admitted users is higher, hence the price variation is also higher as β decreases. Also note that when the number of active users is large, serving additional user can be significant and thus, the price is also high.

4) Impact on the energy drawn from the grid and the storage of the charging station: Fig. 7 shows that as β increases the energy bought from the grid decreases. This is because the number of accepted users decreases with β. The energy used from the battery also decreases as β increases. Note that by selecting a β the charging station can also limit the peak energy consumption from the grid. Fig. 7 also shows that when no menu based pricing is applied i.e., the EVs are charged as soon as they arrive, then, the peak energy consumption from the grid is very high. Even β = 0 lowers the energy consumption from the grid significantly. The energy used from the battery of the charging station is also

low when there is no menu-based pricing. The above shows the usefulness of the menu-based pricing in reducing the peak-energy consumption and efficient use of the renewable energy.

VII. FUTURE WORKS

We considered that the EV charging station is myopic which does not consider the future arrival process while selecting an optimal price for an incoming EV. In future we consider the case where the charging station knows the statistics of the future arrival process of the EVs and selects price accordingly. Considering stochastic pattern of energy harvesting is an important next step. Finally, the consideration of the multiple charging stations which set prices in a competitive manner also constitutes a future research direction.

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An EV is removed when it is fully charged.

We assume that the EV is removed after its reported deadline. When the deadline is over, the EV is moved to a parking spot without any charging facility.