Mathematical Fallacies and Applications of Graph Theory in Electrical Engineering

Z Zahedi 1, Herman Mawengkang 2, Mahrizal Masri 3, Herdi Ramon 4, Yasmine Maharani Putri 5

1,3,4 Institut Teknologi Medan, Indonesia
2 Universitas Sumatera Utara, Medan - Indonesia
5 Sutomo High School, Indonesia

* hmawengkang@yahoo.com

Abstract. A graph is a pair of two sets. A graph is a representation of a system that uses two basic elements of vertices and edges, a node represented by a circle and an edge represented by a line connecting two nodes. Graph theory is widely used in every field of engineering. In this paper, an application description of the graph is presented to find the shortest path and circuit network. The purpose of this paper is to connect graphs with engineering and electrical subjects. But beforehand, some fundamental mistakes of thinking in understanding mathematics, also called as fallacies are also introduced.

1. Introducing
Graphs are often used to represent an object and its relationship with other objects. This theory began when Leonhard Euler solved an old puzzle about the possibility of finding a trail on seven bridges that stretched along a branched river that passed through an island as long as it was not allowed to cross the bridge twice. The problem which became known as the Konisberg bridge problem, according to Euler, was that there would be no way like this. The proof only refers to the physical arrangement of bridges, but essentially Euler proves the first theorem in graph theory.

As used in graph theory, the graph terms here do not refer to graphs such as line graphs or bar graphs. Instead, this term refers to a set of nodes (i.e. points or vertices) and edges (or lines) that connect each node. When two vertices are combined in more than one edge, the graph is called multi graph. No loop and at most one edge between two vertices is called a simple graph. Unless otherwise stated, the graph is assumed to refer as a simple graph. When each node is connected at the end of each other’s point, the graph is called a complete graph. If suitable, directions can be given to each end to produce what is known as a directed graph, or digraph.

The graph basically has components in the form of nodes and sides and in the graph so that it forms an open graph, and the graph is closed so as to form a number of trajectories and circuits. Before describing some examples of the use of graph theory in the field of engineering (electric), a number of examples of fundamental thinking errors are explained in understanding mathematics.

2. Methodology
2.1. Plus Minus Dillemma
When people first learn mathematics, lots of them misunderstood and it can become bad habits in the next learning process. For example, when we talk about the form of $x^2 = a^2$. Some people will immediately conclude that the solution to the above equation is $x = a$, where the answer should be $x = \pm a$ (we have to choose $+a$ or $-a$). This fallacy can be fatal in the process of further calculations, as exemplified below.

We will prove that $-1 = 1$ with the concept of the Pythagorean theorem.

Previously, consider the following right triangle

![Figure 1. Triangle](image)

In summary, the formula of the Pythagorean theorem states that the sum of its squared sides is equal to the square of the hypotenuse or written with:

$$a^2 = b^2 + c^2$$
$$a = \sqrt{b^2 + c^2}$$

At first glance, we will see that this problem is so simple and easy. We often forget that simple problems can actually cause problems if they are not properly understood.

Now, let's look at the problem below, equated with the following functions of trigonometry identity:

$$\sin^2 x + \cos^2 x = 1$$

The equation above can be manipulated as

$$\sqrt{\cos^2 x} = \sqrt{1 - \sin^2 x}$$
$$\cos x = (1 - \sin^2 x)^{1/2}$$

$$x = \pi$$
$$\cos \pi = (1 - \sin^2 x)^{1/2}$$ Assume, we get

$$-1 = (1 - 0)^{1/2}$$
$$-1 = 1$$

The answer to this problem is a thinking error in concluding the answer of the root withdrawal problem above. Some people will then answer that because it is the side or length of a line, then it will not be possible to have a negative sign.

For this reason, let's look at an example of further thinking errors.

2.2. The Dangerous Games of Algebra
Other thinking errors can also occur in algebraic equations. Ignoring the concept of conjunction will confuse the results obtained. We shall prove that $3=0$ by using the concept of equation and algebra manipulation. Given a square equation $x^2 + x + 1 = 0$. Multiply by $x$ to get

\[
x^3 + x^2 + x = 0.
\]

\[
x^3 + x^2 + x = 1 - 1
\]

\[
x^3 + x^2 + x + 1 = 1
\]

Substitute equation $x^2 + x + 1 = 0$ to this equation, and then we get

\[
x^3 + 0 = 1
\]

\[
x^3 = 1
\]

\[
x = \sqrt[3]{1} = 1
\]

Now, substitute $x = 1$ to the first equation, and we will get $1^2 + 1 + 1 = 0$, which means $3 = 0$.

Next, given two examples of graph theory application in the major of engineering that is adopted from Narsingh and Vasudev’s book.

2.3. Traveling-Salesman Problem

The traveling-salesman problem, stated as follows:

"A salesman is required to visit a number of locations during his trip. Given the distances between the locations, what sequence should be done so that he can visit each location only once and return to the initial location with the minimum distance?" The location will be represented by vertices and distances or paths between nodes will be represented by edges. A graph will be obtained. In this graph, each edge $ei$ is associated with a real number (for example, the distance in kilometers, say), $w(ei)$. The graph formed is called a weighted graph; $w(ei)$ becomes the weight of the edge $ei$. (meaning, a traveling salesman wants to visit each of the location exactly once and return to the starting point) if each of the locations has a road to another location, it will have a complete weighted graph.

For example, assume that the salesman will visit five locations, namely, P, Q, R, S, and T (see Figure). In what order does it have to visit these locations so that the minimum total distance is obtained? To solve this problem it is assumed that the salesman will start his journey from point P (because this must be part of the circuit) and examine all possible ways for him to visit the other four locations and then return to the starting point (point P) (starting from another place will produce the same circuit). There are a total of 24 such circuits, but because he travels the same distance when traveling a circuit in reverse order, it is only necessary to consider 12 different circuits to find the minimum total distance that must be traveled. There is a list of 12 different circuits and the total distance traveled for each circuit.

From the list, the minimum total distance must be found using the P - Q - T - S - R - P circuit (or vice versa)

| No. | Route     | Total Distance |
|-----|-----------|----------------|
| 1.  | P - Q - S - R - T - P | $a + f + g + f + d$ |
| 2.  | P - Q - S - T - R - P | $a + f + h + f + b$ |
| 3.  | P - Q - T - R - S - P | $a + i + f + f + c$ |
| 4.  | P - Q - T - S - R - P | $a + i + h + f + b$ |
| 5.  | P - Q - R - T - S - P | $a + e + f + h + c$ |
| 6.  | P - Q - R - T - P    | $a + e + f + h + d$ |
| 7.  | P - R - Q - T - S - P | $b + e + f + i + h + d$ |
| 8.  | P - R - Q - T - S - P | $b + e + i + h + c$ |
| 9.  | P - R - T - Q - S - P | $b + f + i + j + c$ |
| 10. | P - R - S - Q - T - P | $b + g + j + i + d$ |
| 11. | P - S - R - Q - T - P | $c + g + e + i + d$ |
| 12. | P - S - Q - R - T - P | $c + f + e + f + d$ |

The graph showing the distance between five cities (P, Q, R, S, T)
The problem of traveling salesman is to look for a minimum total weight circuit in a weighted, complete, undirected graph that visits each point (or vertex) exactly once and returns to its starting point. This is the same as finding a Hamilton circuit with a minimum total weight in the complete graph, because at the circuit each point is visited.

The simplest way to solve a traveling salesman problem is to check all possible Hamilton circuits and choose one from the minimum total length.

The question is how many circuits should be checked to solve this problem if there are \( n \) vertices in the graph? After the starting point is selected, there are \((n-1)!\) different Hamilton circuits to check, because there are choices of \(n-1\) for the second vertex, \(n-2\) choices for the third point or vertex, and so on. Because Hamilton circuits can be used in reverse order, we only need to check the \(\frac{(n-1)!}{2}\) circuit to find a satisfying answer. Note that \(\frac{(n-1)!}{2}\) increases very quickly. Trying to solve the problem of salesman traveling this way when there are only a few vertices are not practical.

For example, with 25 vertices, a total of \(\frac{24!}{2}\) (approximately \(3.1 \times 10^{23}\)) different Hamilton circuits must be considered. If it takes only one second to examine each Hamilton circuit, it's hard to imagine if you want to calculate the total time needed to get the minimum-length of the Hamilton circuit in this graph with a complete searching technique.

### 3. Results and Discussion

Considering to convert any current source into an equivalent voltage source, we will obtain a set of \(\mu = e - n + 1\) simultaneous loop equations,

\[
B_f Z(s) B_f^T I_e(s) = E(s)
\]  

(7)

Where \(B_f\) is a series circuit matrix of the network associate with several spanning tree, where the transpose are \(B_f^T\). The \(e \times e\) matrix \(Z(s)\) is the edge impedance matrix that describes the electrical property of each of \(e\) edges in the network; written,

\[
\begin{bmatrix}
V_1(s) \\
V_2(s) \\
\vdots \\
V_e(s)
\end{bmatrix} =
\begin{bmatrix}
Z_1(s) & 0 & \cdots & 0 \\
0 & Z_2(s) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & Z_e(s)
\end{bmatrix}
\begin{bmatrix}
I_1(s) \\
I_2(s) \\
\vdots \\
I_e(s)
\end{bmatrix}
\]

See that for an RLC Network the edge impedance matrix \(Z(s)\) is the inverse of its edge admittance matrix \(Y(s)\). \(I_e(s)\) is the Laplace of the loop current vector \(i_L(t)\), and \(E(s)\) is the Laplace transform of the voltage sources (or equivalent voltage sources) applied externally in the \(\mu\) fundamental circuits.

This is the same as the previous equation if we derivatate equation (7) gradually.

The \(\mu \times \mu\) matrix \(B_f Z(s) B_f^T\) in equation (7) is called the loop impedance matrix and is usually denoted by \(Z_I(s)\). thus equation (7) is written as

\[
Z_I(s) I_e(s) = E(s)
\]  

(8)
Figure 2. Network of figure(a) for a loop analyses

Now, let’s look at the electrical network from the figure below. By replacing the current source $x(t)$ with an equivalent voltage source, we get the network as shown in the figure above (a) and its graph shown in figure b.

$$Z_L(s) = \begin{bmatrix} \frac{L_1(s)}{1+C_1L_1s^2} + \frac{L_2(s)}{1+C_2L_2s^2} + \frac{1}{C_3s} & \left( -\frac{1}{C_3s} \right) \\ \left( -\frac{1}{C_3s} \right) & \left( \frac{1}{C_3s} \right) + L_3s + R \end{bmatrix}.$$  

$$I_L = \begin{bmatrix} I_{L1}(s) \\ I_{L2}(s) \end{bmatrix}, \text{ and } E(s) = \begin{bmatrix} X(s)L_2(s) \\ 1+L_2C_2s^2 \\ -V(s) \end{bmatrix}.$$  

To get a solution from the equation (8), the first step is to get the determinant and cofactors of $Z_L(s)$. The expression for $\Delta_L$, the determinant of $Z_L(s)$, according to the Binet-Cauchy theorem is given by

$$\Delta_L = \det Z_L(s) = \det (B_f Z(s)B_f^T).$$  

$$= \text{sum of products of all pairs of corresponding majors of } [B_fZ(s)] \text{ and } B_f^T.$$  

Since a major of $B_f$ is nonzero if and only if it corresponds to a chord set, equation (8) becomes

$$\Delta_L = \text{sum of chord impedance products for all spanning trees of the network.}$$

Equation (9) was originally given by Kirchoff for a purely resistive network. For the network of figure (b), all possible chord sets are $ce$, $cd$, $be$, $d$, $bc$, $ae$, $ac$, and $ad$. Therefore,

$$\Delta_L = Z_c(s)Z_e(s) + Z_c(s)Z_d(s) + Z_b(s)Z_e(s) + Z_b(s)Z_d(s)$$
The expressions for the cofactors of $Z_L(s)$ both symmetrical and asymmetrical can be obtained in a fashion similar to those for $Y_N(s)$.

4. Conclusion
Deep understanding of basic mathematics material is needed. Inaccuracy in seeing fundamental problems can cause errors in the process of subsequent calculations. This fallacy or misguided thought can also affect the use of mathematics in the engineering area. Furthermore, the application of graph theory to the technical field has also been introduced.

References
[1]. Carlson, Stephan C. “Graph Theory.” Encyclopædia Britannica, Encyclopædia Britannica, Inc., www.britannica.com/topic/graph-theory, 2017.
[2]. Choudum, S. A., Graph Theory, A NPTEL Course. Department of Mathematics, IIT Madras, Chennai, India.
[3]. Duffy, D.G., Advanced Engineering Mathematics. Boca raton: CRC Press, 1998.
[4]. James, G., Advanced Modern Engineering Mathematics, Fourth Edition. Essex: Pearson Educational Ltd., 2011
[5]. Kelly, S.G., Advanced Engineering Mathematics with Modelling Aplications. Boca raton: CRC Press, 2009.
[6]. Kreyzig, E., Advanced Engineering Mathematics, Tenth Edition. New Jersey: John Wiley & Sons, Inc., 2011.
[7]. Ramdhani, M., Rangkaian Listrik (Revisi). Sekolah Tinggi Teknologi Telkom, Bandung. 2005.
[8]. J. A. Bondy and U. S. R. Murty., Graph Theory with Applications. Elsevier Science Publishing Co., Inc. USA. Fifth Printing, 1982.
[9]. A. Wanto, M. Zarlis, Sawaluddin, and D. Hartama, “Analysis of Artificial Neural Network Backpropagation Using Conjugate Gradient Fletcher Reeves in the Predicting Process,” Journal of Physics: Conference Series, vol. 930, no. 1, pp. 1–7, 2017.
[10]. J. A. Bondy and U. S. R. Murty., Graph Theory. Springer, 2008.
[11]. Narsingh Deo., Graph Theory with Applications to Engineering and Computer Sciences. Prentice-Hall, Inc. Englewood Cliffs, N. J. 1974.
[12]. Vasudev, C., Graph Theory with Applications. New Age International (P) Limited, Publishers. Daryaganj, New Delhi, 2006.
[13]. Wai-Kai Chen., Some Application of Graph Teory. Report R-211. University Science Laboratory, University of Illinois, Urbana Illinois, 1964.
[14]. Zill, D.G. and Cullen, M.R., Advanced Engineering Mathematics, Fifth Edition. Ontario: Jones and Bartlett Publisher, Inc., 2012.