Isospin Matter in AdS/QCD

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Abstract

We study strange and isospin asymmetric matter in a bottom-up AdS/QCD model. We first consider isospin matter, which has served as a good testing ground for nonperturbative QCD. We calculate the isospin chemical potential dependence of hadronic observables such as the masses and the decay constants of the pseudo-scalar, vector, and axial-vector mesons. We discuss a possibility of the charged pion condensation in the matter within the bottom-up AdS/QCD model. Then, we study the properties of the hadronic observables in strange matter. We calculate the deconfinement temperature in strange and isospin asymmetric matter. One of the interesting results of our study is that the critical temperature at a fixed baryon number density increases when the strangeness chemical potential is introduced. This suggests that if matter undergoes a first-order transition to strange matter, the critical temperature shows a sudden jump at the transition point.

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I. INTRODUCTION

Dense matter is one of the most challenging problems of modern physics. Understanding the properties of such matter is important for the physics of relativistic heavy-ion collisions and dense stellar objects such as neutron stars. Recent developments based on Gauge/Gravity correspondence about Anti de Sitter space (AdS) and Conformal field theory (CFT) [1] have opened a new way to study such matter in the framework of a holographic model of Quantum chromodynamics (QCD), or AdS/QCD [2, 3, 4], see Ref. [5] for a review. Also, there have been many examples of such studies on dense nuclear matter [6] and isospin matter [7, 8, 9].

Isospin matter, where \( \mu_I \) (isospin chemical potential) is finite and \( \mu_B \) (baryon chemical potential) is zero, was proposed by Son and Stephanov [10] as a useful setting to improve our understanding of cold dense QCD. Although a QCD system with finite \( \mu_I \) and zero \( \mu_B \) hardly exists in nature, it has many interesting and useful features. The standard lattice QCD technique is applicable to isospin matter, unlike to the QCD at finite baryon chemical potential. Furthermore, we can analytically study the system at very low \( \mu_I \) by using chiral perturbation theory and at very high \( \mu_I \) by using perturbative QCD. Therefore, it seems quite natural that one adopts the system to test a theoretical tool for its suitability to tackle the nonperturbative physics of QCD.

At the same time, strange matter has been of great interest ever since the original discussions that it could be stable [11] and form strangelets [12]. Although no proof for its existence has been found yet, heavy ion collisions at the Large Hadron Collider (LHC) could lead to its discovery as strange quarks will be amply produced if a quark-gluon plasma is formed [13]. Matter with strangeness chemical potentials is also of interest in relation to neutron stars and the phases of QCD at high density.

In this work, we first consider isospin matter, which has been well-studied by using lattice QCD and effective theories of QCD, to test our tool [3, 4]. Some part of the present work is briefly reported in Ref. [8]. We calculate the pseudo-scalar, vector and axial-vector meson masses in isospin matter and discuss the validity region of the model. We also evaluate the \( \mu_I \)-dependence of the meson decay constants. Then, we study the strangeness chemical potential dependence of the hadronic observables and investigate a possible onset of strange matter or hyperonization in the AdS/QCD model. Finally, we study the effect of the isospin...
and the strangeness number densities on the deconfinement transition temperature.

II. ADS/QCD MODEL WITH VARIOUS CHEMICAL POTENTIALS

The action of the model given in Ref.[3, 4] is

\[ S_5 = \int d^4x \int dz L_5 = \int d^4x \int dz \sqrt{g} M_5 Tr[-\frac{1}{4} (L_{MN} L^{MN} + R_{MN} R^{MN})] + \frac{1}{2} |D_{M}\Phi|^2 - \frac{1}{2} M_\Phi^2 |\Phi|^2, \]

(1)

where \( M_5 = \frac{N_c}{12\pi^2} \) and the field strength tensor is defined by \( L_{MN} = \partial_M L_N - \partial_N L_M + i[L_M, L_N] \) with gauge fields \( L_M = L_M^a t^a \) (and similar definition on \( R_M \)). Here, \( Tr(t^a t^b) = \delta^{ab} \).

Also, the covariant derivative has the form \( D_M \Phi = \partial_M \Phi + iL_M \Phi - i\Phi R_M \). The background is given by

\[ ds^2 = a^2(z) \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right), \]

(2)

where \( a(z) = 1/z \) and \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \). Note that we don’t consider the back-reaction due to the chiral condensate to the AdS background. In the model [3, 4], which is sometimes called the hard-wall model, the AdS_5 space is compactified such that \( z_0 \leq z \leq z_m \). Here, \( z_0 \) is the UV cutoff and \( z_m \) is for the IR cutoff. In Ref.[3, 4], the IR cutoff, \( z_m \), is fixed by the rho-meson mass. The vector- and axial-vector mesons are defined by

\[ V_M = \frac{1}{\sqrt{2}} (L_M + R_M), \]

\[ A_M = \frac{1}{\sqrt{2}} (L_M - R_M). \]

(3)

The bulk scalar field is defined by \( \Phi = Se^{iP/v(z)} \), where \( S \) and \( P \) correspond to a real scalar and a pseudoscalar, respectively. Here, the vacuum expectation value of \( S \), \( v(z) \), depends on the quark mass matrix \( M \) and the chiral condensate \( \Sigma \),

\[ v(z) = M z + \Sigma z^3. \]

(4)

In Ref.[3], it is assumed that \( \Sigma = \sigma E \) and \( M = m_q E \), where \( E \) is the two-by-two identity matrix for \( N_f = 2 \). For \( N_f = 3 \) [14, 15, 16, 17], \( M \) and \( \Sigma \) are assumed to be \( M = \text{diag}(\hat{m}, \hat{m}, m_s) \) and \( \Sigma = \text{diag}(\sigma, \sigma, \sigma) \) or \( \Sigma = \text{diag}(\sigma, \sigma, \sigma_s) \), where \( \hat{m} = (m_u + m_d)/2 \). In
Ref. [15], with the choice of \( m_u = m_d \neq m_s \) and \( \langle \bar{u}u \rangle = \langle \bar{d}d \rangle \neq \langle \bar{s}s \rangle \), the strange quark sector of the hard-wall model is extensively studied, and the result shows good agreement with the experiment and the lattice QCD results, except for the strange pseudoscalar mass. In Ref. [15, 16], a deformed AdS background due to the meson action is investigated with \( N_f = 3 \).

Now, we discuss how to introduce various chemical potentials into our model. For this, we first consider the QCD side, where the chemical potentials are introduced as follows:

\[
\mathcal{L}_{\text{QCD}}^\mu = \mu_u u^\dagger u + \mu_d d^\dagger d + \mu_s s^\dagger s \\
= (\mu_q + \mu_I) u^\dagger u + (\mu_q - \mu_I) d^\dagger d + \mu_s s^\dagger s ,
\]

(5)

where \( \mu_q = \frac{1}{2}(\mu_u + \mu_d) \) and \( \mu_I \equiv \frac{1}{2}(\mu_u - \mu_d) \). According to an AdS/CFT dictionary, a chemical potential in 4D QCD is encoded in the boundary value of the time component of the 5D bulk U(1) gauge field. To this end, we generalize the 5D gauge symmetry of the model [3, 4] from \( \text{SU}(3)_L \times \text{SU}(3)_R \) to \( \text{U}(3)_L \times \text{U}(3)_R \) [18]. As shown in Ref. [18], the time component of the bulk U(1) vector field has the following profile:

\[
V_0 = c_1 + c_2 z^2 .
\]

(6)

Then, \( c_1 \) is a chemical potential \( \mu \), and \( c_2 \) is a conjugate number density \( \sim \rho \). If \( c_1 \) is the quark chemical potential, then \( c_2 \) must be the quark number density, \( c_2 = 12\pi^2 n_q / N_c \) [18].

We remark here that although the AdS/CFT dictionary states that the integration constants of the vector profile must be interpreted as a chemical potential and its conjugate density, it does not fix the values or the relation of the chemical potential and the corresponding density, which is a limitation of the present approach. In this work, we consider the baryon (or quark) chemical potential, the isospin chemical potential, and the strangeness chemical potential. To include such chemical potentials in the model, we rewrite the time component of the 5D bulk U(1) field as

\[
V_0 = \hat{V}_0 \hat{t} + V_0^a t^a \\
= \hat{V}_0 \hat{t} + V_0^8 t^8 + V_0^3 t^3 + \ldots \\
= 2\hat{V}_0 \hat{t} + \sqrt{2}V_0^8 + V_0^3 t^3 + \ldots ,
\]

(7)

where \( t^a \) are the generators of \( \text{SU}(3) \), \( \hat{t} \) is the generator of the U(1) subalgebra of U(3),
\[ \hat{t} = \text{diag}(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}) \], and

\[ \tilde{t} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \bar{t} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad t^3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}. \quad (8) \]

From the form of redefined generators, we see that \( \tilde{V}_0 \leftrightarrow (\tilde{\mu}_q, \rho_q), V_0^3 \leftrightarrow (\bar{\mu}_I, \rho_I) \) and \( \bar{V}_0 \leftrightarrow (\bar{\mu}_s, \rho_s) \). More explicitly, we write

\[ \tilde{V}_0 = \tilde{\mu}_q + c_q z^2, \quad \bar{V}_0 = \bar{\mu}_s + c_s z^2, \quad V_0^3 = \bar{\mu}_I + c_I z^2. \quad (9) \]

Among the three chemical potentials, the most interesting one might be the baryon (quark) chemical potential. To look at this, we put \( \tilde{V}_0 \) into the action in Eq. (1). However, \( \tilde{V}_0 \) does not couple to the bulk fields in the action, simply because the action contains only mesons. An exceptional case is to consider the Chern-Simons couplings at finite baryon density. Therefore, we consider the isospin matter to test our model in the medium because the matter has been well-studied by using various approaches.

A. Isospin Matter

To study isospin matter, we turn on only \( V_0^3 \) and \( \tilde{\mu}_I \) and turn off the others in Eq. (7). Then, the profile of the isospin-triplet bulk vector field is given by

\[ V_0^3 t^3 = (\tilde{\mu}_I + c_I z^2) t^3. \quad (10) \]

Since we cannot determine the relation between \( \tilde{\mu}_I \) and \( c_I \) within our framework, we treat both of them as external parameters. For simplicity, we may borrow the relation from QCD. For example, in Ref. [10], it is given by

\[ c_I = 0: \text{normal phase with no pion condensation} \]

\[ c_I = f_\pi^2 \tilde{\mu}_I (1 - m_\pi^4 / f_\pi^4): \text{pion condensed phase.} \]

Now, we calculate the pion masses at small \( |\tilde{\mu}_I| \) and study if pion condensation occurs as we increases \( |\tilde{\mu}_I| \). For a small isospin chemical potential, we have

\[ V_0^3 t^3 = \tilde{\mu}_I t^3. \quad (11) \]

We use \( \tilde{\mu} \) instead of \( \mu \) because our chemical potential may be different from the one in Eq. (5) by an overall normalization constant.
Although we are not able to describe the physics in the pion condensed phase with Eq. (11), we should be able to observe the onset of condensation, if it occurs. Note that if $\mu_I$ is negative, pion condensation means $\pi^{-}$ condensation. From the action in Eq. (1), we obtain the equation of motion of the pion field in isospin matter as

$$D \left( m_{\pm}^2 + (2v^2 a^2 D) \pm 2\tilde{\mu}_I m_{\pm} \right) \pi^\pm = 0, \quad (12)$$

$$D \left( m_0^2 + (2v^2 a^2 D) \right) \pi^0 = 0, \quad (13)$$

where

$$D \equiv 1 - \partial_5 \left( \frac{1}{2a^3 v^2 \partial_5 a} \right).$$

Here $\pi^{\pm}$ are linear combinations of $\pi^1$ and $\pi^2$, and $\pi^0$ is for the neutral pion, $\pi^3$. Note that the mass of the neutral pion is independent of $\tilde{\mu}_I$, as it should be, because a neutral pion has a zero third component of the isospin. We can immediately read off the $\tilde{\mu}_I$ dependence of the charged pion masses from Eq. (12):

$$m_{\pm} = \mp \tilde{\mu}_I + \sqrt{\tilde{\mu}_I^2 + m_0^2}, \quad (14)$$

We find that the mass of $\pi^+$ increases with $|\tilde{\mu}_I|$ while that of $\pi^-$ decreases as we raise $|\tilde{\mu}_I|$. At small $\tilde{\mu}_I$, we have $m_{\pm} \approx \mp \tilde{\mu}_I + m_0$, which is consistent with the observation made in Ref. [10].

Before we move on to the pion decay constant, we present some remarks here on the pion condensation. In Ref. [10], the critical isospin chemical potential $\tilde{\mu}_I^c$ for the condensation is determined by the condition that the mass of $\pi^-$ be zero. In the chiral limit, $m_0 = 0$, we always have the pion condensation because $m_-$ is zero for any value of $\tilde{\mu}_I$, which is consistent with the observation made in a top-down approach [9]. With nonzero $m_0$, however, the mass of $\pi^-$ obtained in the present work is always non-zero, unless $\tilde{\mu}_I$ goes to infinity. This means that in the present set-up, we have no pion condensation, which is quite different from what was observed in Ref. [10], in the Nambu-Jona-Lasinio(NJL) model [20], in Lattice [21] and in other model calculations [22]. This may be understood as follows: At small isospin chemical potential, we may ignore the back-reaction due to $V^3_0$, or the isospin chemical potential, so the AdS metric in Eq. (11) is good for describing the isospin matter. As we increase the value of the isospin chemical potential, the effect of the back-reaction of the isospin chemical potential on the metric is no longer negligible, so our expectation is that if we consider
the back-reaction from the flavor sector (chemical potential or density) and work with a deformed AdS$_5$, as for instance in Ref. [23], then we are able to observe pion condensation in AdS/QCD. For pion condensation in dense matter other than isospin matter, we refer to Ref. [24].

Now, we calculate the $\tilde{\mu}_I$-dependence of the pion decay constant. In matter, due to the lack of Lorentz invariance, we may define the pion decay constant as

$$ (f_{\pi}^{t,s})^2 = -\frac{1}{g^2} \left. \frac{\partial_z A_{0,i}}{z} \right|_{z=\epsilon}, $$

(15)

where $\epsilon \rightarrow 0$ and $A_{0,i}$ is the solution of the axial-vector equation of motion at zero 4D energy-momentum,

$$ [a^{-1} \partial_5 a \partial_5 - 2v^2 a^2 + \tilde{\mu}_I^2] A_i = 0. $$

(16)

Here, $A_i$ is the space component of the charged axial-vector ($A_1, A_2$), and the equation of motion of the charge-neutral axial-vector field does not depend on $\tilde{\mu}_I$. The $\tilde{\mu}_I$-dependent $f_{\pi}^s$ is plotted in Fig. 1.

![FIG. 1: The spatial component of the pion decay constant in isospin matter. Here, $R \equiv f_{\pi}^s(\tilde{\mu}_I)/f_\pi(0)$.](image)
Finally, we consider the vector and the axial-vector mesons, especially the $\rho$ and the $a_1$ mesons that are the lowest Kaluza-Klein (KK) states of the bulk vector field and the axial-vector field respectively. The KK decomposition of the bulk vector field is defined by

$$V_\mu(x, z) = \frac{1}{\sqrt{M_5 L}} \sum_n f_n^V(z) V^{(n)}_\mu(x).$$

After integration by parts in the action, we obtain quadratic terms for the vector field:

$$L_\mu I V V = \frac{M_5 a}{2} \{ V^\pm_i \partial^2 - a^{-1} \partial_5 a \partial_5 \mp 2 \tilde{\mu}_I i \partial_0 - \tilde{\mu}_I^2 \} V^\pm_i + V^0_i \partial^2 - a^{-1} \partial_5 a \partial_5 V^0_i \}, \quad (17)$$

where $V_0^3 T^3 = \tilde{\mu}_I T^3$ is used. Again, the charge-neutral vector field is independent of $\mu_I$.

From the quadratic action, we find

$$m_{\rho^\pm} = m_{\rho^0} \mp \tilde{\mu}_I. \quad (18)$$

As expected, the negatively-charged $\rho$-meson mass decreases with increasing $|\tilde{\mu}_I|$. Here we observe an interesting possibility, namely, the vector meson condensation, because the mass of $\rho^-$ is zero at $|\tilde{\mu}_I| = m_{\rho^0}$ (See Ref. [26] for a review on vector meson condensation.) As discussed previously, to discuss the physics of condensation in the present set-up, we may have to consider the back-reaction from the isospin chemical potential to the background, so any definite statement on the possibility of vector condensation has to be postponed until we successfully obtain the back-reacted (deformed) background. We refer to Ref. [9] for a discussion on vector condensation in the chiral limit in a top-down model. We also evaluate the $\rho$-meson decay constant in the isospin matter and find it to be independent of $\tilde{\mu}_I$. This is obvious from Eq. (17). The corrections due to finite $\tilde{\mu}_I$ to the equation of motion for the vector field, $f_n(z)$, are $z$-independent; consequently, the KK wave-function for the vector field is blind to the isospin chemical potential. For the axial-vector meson, $a_1$, we obtain the same results for the $\rho$-meson:

$$m_{a_1^\pm} = m_{a_1^0} \mp \tilde{\mu}_I, \quad (19)$$

and the $a_1$ decay constant is independent of $\tilde{\mu}_I$. To complete the analysis, we need to calculate the isospin-chemical-potential dependence of scalar mesons. The scalar sector of the present model is, however, very sensitive to the details of added scalar potentials, which is to support nonzero chiral condensate in $v(z)$, even in free space with no chemical potentials [17]. Therefore, we don’t consider the scalar sector in this work.
B. Strange Matter

We first study the strangeness-chemical-potential dependence of hadronic observables. The mass of the kaon is given by

\[ m_{K^\pm} = \mp \tilde{\mu}_s + \sqrt{\tilde{\mu}_s^2 + m_0^2}, \]  

where \( m_0 \) is the mass with no strangeness chemical potential. When \( \tilde{\mu}_s \) increases, the masses of particles that contains \( \bar{s} \) quarks, \( K^+ \) and \( K^0 \), decrease while the masses of \( K^- \) and \( \bar{K}^0 \), which have \( s \) quarks, increase with \( \tilde{\mu}_s \).

Similarly, we study the vector meson case and find that the masses of \( K^{*+} \) and \( \bar{K}^{*0} \) increase and that those of \( K^{*-} \) and \( K^{*0} \) decrease with increasing \( \tilde{\mu}_s \):

\[ m_{K^{*-}} = \mp \tilde{\mu}_s + m_0. \]  

Now, we study a transition from nuclear matter to strange matter in a simple way. The boundary action, which is nothing but the 4D grand canonical potential \([18]\), is

\[ S_b = \frac{1}{2} M_5 \text{Tr} \int d^4x \left( \frac{1}{z} V_0 \partial_z V_0 \right)_{z=\epsilon} \]
\[ = \frac{1}{2} M_5 \int d^4x \left( \frac{1}{z} \left( 4V_0 \partial_z V_0 + 2V_0 \partial_z V_0 + V_0^3 \partial_z V_0^3 \right) \right)_{z=\epsilon} \]
\[ = M_5 \left( 4\tilde{\mu}_q c_q + 2\tilde{\mu}_s c_s + \tilde{\mu}_I c_I \right) \int d^4x, \]  

where \( c_i = 12\pi^2 \rho_i / N_c \) with \( i = q, s, I \), and \( \rho_i \) is a conjugate number density of \( \tilde{\mu}_i \). Here, we turn on the isospin chemical potential and study the transition from symmetric nuclear matter to strange matter. To this end, we introduce

\[ \rho_Q = 2\rho_q + \rho_s, \]
\[ \rho_q = \frac{1}{2} (1 - x) \rho_Q, \quad \rho_s = x \rho_Q. \]  

As previously mentioned, we cannot determine the relation between a chemical potential and the corresponding number density in a self-consistent way. Therefore, we have to borrow the relation from other studies using lattice QCD or models of QCD. In this work, we simply adopt the relation from a free quark gas:

\[ \tilde{\mu}_q = k_F^q = (\pi^2 \rho_q)^{1/3}, \quad \tilde{\mu}_s = \sqrt{m_s^2 + k_F^{s2}} = \sqrt{m_s^2 + (\pi^2 \rho_s)^{2/3}}. \]
The relevant part of Eq. (22) to study the transition is
\[
f(x) = 2\tilde{\mu}_q\rho_q + \tilde{\mu}_s\rho_s \\
= (1 - x)\rho_Q\left(\pi^2 \frac{1}{2} (1 - x)\rho_Q\right)^{1/3} + x\rho_Q\left(m_s^2 + (\pi^2 x\rho_Q)^{2/3}\right)^{1/2}.
\]
If \(f(x)\) has an absolute minimum at a nonzero and positive \(x\), then we have a transition at that \(x\). We calculate the transition density at different strange quark masses to see how the transition density changes with \(m_s\).

The value of the fixed strange quark mass shows some discrepancy in AdS/QCD studies [14, 15, 16, 17]. The discrepancy is mainly due to differences in the choices for the values of the parameters and the backgrounds in the studies. For instance, one finds \(m_s = 90\) MeV and \(\langle \bar{s}s \rangle = (457.53\) MeV\(^3\) in the pure AdS [15] model, \(m_s = 138.5\) MeV and \(\langle \bar{s}s \rangle = (176\) MeV\(^3\) in a deformed AdS [16], and \(m_s = 40\) MeV and \(\langle \bar{s}s \rangle = (333\) MeV\(^3\) in pure AdS background [17]. Note that in Ref. [17], \(\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle\) is assumed, so we choose values of \(m_s\) in the range of 50 - 150 MeV, which are introduced in other AdS Models. Then, the transition densities are
\[
\begin{align*}
m_s = 50\text{ MeV}, & \quad \tilde{c}_q = 0.004, \\
m_s = 100\text{ MeV}, & \quad \tilde{c}_q = 0.035, \\
m_s = 150\text{ MeV}, & \quad \tilde{c}_q = 0.116.
\end{align*}
\]
Here, we have introduced the dimensionless quark number density \(\tilde{c}_q = c_q z_m^3\). As expected, the transition density becomes larger with heavier strange quark mass. We remark, however, that the results in Eq. (25) should be taken as being very schematic because free quark gas has been assumed.

### III. DECONFINEMENT TRANSITION IN DENSE MATTER

In this section, we generalize the analysis done in Ref. [27, 28] to a system with various chemical potentials. We note here that, unlike isospin matter, we turn on the baryon chemical potential, as well as the others. Therefore, even with no pion condensation, we can have \(\rho_t \neq 0\).

We first summarize [27, 28]. The Euclidean gravitational action is given by
\[
S_{\text{grav}} = -\frac{1}{2\kappa^2} \int d^5x \sqrt{g} (R + 12),
\]
where $\kappa^2$ is the 5D Newton constant. The cut-off thermal AdS (tAdS) is given by

$$ds^2 = \frac{1}{z^2} (d\tau^2 + dz^2 + d\vec{x}_3^2),$$  \hspace{1cm} (27)$$

and the cut-off AdS black hole (AdSBH) reads

$$ds^2 = \frac{1}{z^2} \left( f(z) d\tau^2 + \frac{dz^2}{f(z)} + d\vec{x}_3^2 \right),$$  \hspace{1cm} (28)$$

where $f(z) = 1 - (z/z_h)^4$, and $z_h$ is the horizon of the black hole. The Hawking temperature is defined by $T = 1/(\pi z_h)$. The periodicity of the compactified Euclidean time direction of the tAdS is given by

$$\beta' = \pi z_h \sqrt{f(\epsilon)}. \hspace{1cm} (29)$$

Now, we calculate the action density $V$, which is defined by the action divided by the volume $d\vec{x}_3^2$. For the tAdS, we obtain

$$V_1 = \frac{4}{\kappa^2} \int_0^{\beta'} d\tau \int_{z_0}^{2m} \frac{dz}{z^5}, \hspace{1cm} (30)$$

and for the AdSBH, we have

$$V_2 = \frac{4}{\kappa^2} \int_0^{\pi z_h} d\tau \int_{z_0}^{\min(z_m, z_h)} \frac{dz}{z^5}. \hspace{1cm} (31)$$

Then, we arrive at

$$\Delta V_g = V_2 - V_1 = \begin{cases} \frac{\pi z_h}{\kappa^2 (1 - \frac{1}{2z_h})} & z_m < z_h \\ \frac{\pi z_h}{\kappa^2 (1 - \frac{1}{2z_h})} (\frac{1}{z_m} - \frac{1}{z_h}) & z_m > z_h. \end{cases} \hspace{1cm} (32)$$

When $\Delta V_g$ is positive (negative), the thermal AdS (the AdS black hole) is stable [27]. Thus, at $\Delta V_g = 0$, there exists a Hawking-Page transition. In the second case, $z_m > z_h$, the Hawking-Page transition occurs at

$$T_0 = 2^{1/4}/(\pi z_m). \hspace{1cm} (33)$$

This is the result in Ref. [27] for the hard wall model.

To study the Hawking-Page transition at finite density [28], we consider the meson action given in Eq. (1). The Euclidean action for mesons is given by

$$S_{\text{matter}} = M_5 \int d^5x \sqrt{g} \text{Tr} \left[ \frac{1}{2} |D_{\mu} \Phi|^2 + \frac{1}{2} M_\Phi^2 |\Phi|^2 + \frac{1}{4} (L_{MN} L^{MN} + R_{MN} R^{MN}) \right]. \hspace{1cm} (34)$$
The solution of the equation of motion for the (Euclidean) time component of the U(1) bulk vector field is given by

\[ V_\tau = c_1 + c_2 z^2. \]  

(35)

In Ref. [28], \(c_1\) is identified as the baryon (quark) chemical potential, and \(c_2\) is nothing but the baryon number density. Following the same procedure as for the gravity action, we obtain the regularized action density as follows:

\[ V_{v1} = \pi z_h M_5 N_f c_2^2 z_m^2 \]  

(36)

for the tAdS and

\[
V_{v2} = \begin{cases} 
\pi z_h M_5 N_f c_2^2 z_h^2 & z_h < z_m \\
\pi z_h M_5 N_f c_2^2 z_m^2 & z_h > z_m 
\end{cases}
\]

(37)

for the AdSBH. Then, the difference of the actions reads

\[
\Delta V = V_{v2} - V_{v1} = \begin{cases} 
-\pi z_h M_5 N_f c_2^2 (z_m^2 - z_h^2) & z_h < z_m \\
0 & z_h > z_m
\end{cases}
\]

(38)

Combining Eq. (32) and Eq. (38), we obtain [28], for \(z_h < z_m\),

\[
\Delta V = \frac{\pi z_h}{\kappa^2} \left[ \frac{1}{z_m^4} - \frac{1}{2z_h^4} - \frac{N_f c_2^2}{48 N_c} (z_m^2 - z_h^2) \right]
\]

(39)

where we have used

\[
\frac{1}{\kappa^2} = \frac{1}{8 \pi G_5}, \quad \frac{1}{G_5} = \frac{32 N_f^2}{\pi L^5}, \quad \text{and} \quad M_5 = \frac{N_c}{12 \pi^2 L}.
\]

(40)

Here, the second relation is taken from Ref. [29]. From Eq. (39), it is obvious that \(T_c \equiv 1/(\pi z_h^0)\), where \(z_h^0\) is a solution of \(\Delta V = 0\), will depend on \(c_2\) or the baryon number density.

Now, we extend the analysis done in Ref. [28] with the isospin and the strangeness chemical potentials. For this, we generalize Eq. (39) to obtain

\[
\Delta V = \frac{\pi z_h}{\kappa^2} \left[ \frac{1}{z_m^4} - \frac{1}{2z_h^4} - \frac{1}{48 N_c} \left( 4c_q^2 + 2c_s^2 + c_I^2 \right) (z_m^2 - z_h^2) \right]
\]

(41)

We consider a few situations that may be relevant to the physics of relativistic heavy-ion collisions and neutron stars (or quark stars [30]).
First we consider the case where $c_I = 0$ with $c_q$ and $c_s$ being nonzero. Here, we introduce $c_b = 2c_q + c_s$ and $c_s = \text{constant}$. We assume that the weak interaction time scale is long enough to be ignored and that the strangeness is conserved. This situation could correspond to the initial stages of a heavy-ion collision, where the initially produced strangeness is conserved. As can be seen in Fig. 2, we observe that the deconfinement temperature with finite strangeness density is higher than it is with zero strangeness density.

Second, we consider the case where $c_I = 0$ with $c_q$ and $c_s$ being nonzero and where the weak processes and charge conservation have to be considered. In studies based on QCD effective theories [31], the transition to strange matter takes place at a density around $2 - 3$ times that of normal nuclear matter, and the transition is first order. To simulate such conditions, we introduce $c_s = yc_b$, where $0 \leq y \leq 1$. We further assume that the transition occurs at some density $c_b$ and that at this density, $y$ acquires a finite value. For example, we take $y = 0.1$, which is close to the value in Ref. [31], to obtain Fig. 3. An interesting feature observed in Fig. 3 is that the first-order nature of the transition to strange matter leads to a cusp in $T_c$.

Finally, we consider the case where $c_s = 0$ and $c_I \neq 0$. In this case, the critical temperature decreases as compared to the case with $c_I = 0$, as shown in Fig. 4. Similar results are
FIG. 3: The critical temperature at the phase transition to strange matter. The solid line is for the case where the phase transition occurs at $\tilde{c}_h$ while the dashed line is for the case with no transition. We assume $\tilde{c}_h = 12$.

observed in a model calculation based on a hadron resonance gas [32].

FIG. 4: The critical temperature at finite isospin. The dashed line is for $\tilde{c}_I = 0$ while the solid line is for $\tilde{c}_I \neq 0 (\tilde{c}_I = c_I z_m^3)$. 
Closing this section, we discuss possible corrections, other than the one studied in this work, to the Hawking-Page transition temperature $T_0$. Note that $T_0$ is the transition temperature determined with the gravity action only. As is well-known, the matter action is $1/N_c$ sub-leading compared to the leading gravity action, so the correction to $T_0$ from the matter action is suppressed by $1/N_c$, as shown in Eq. (41). There are, however, other sources of the $1/N_c$ corrections to $T_0$. First of all, we need to consider the effect of the back-reaction due to the matter, such as the chemical potentials and corresponding densities on the background. So far, however, a full back-reacted background in confined phase due to the chemical potentials and corresponding densities has not been found. We refer to Ref. [23] for back-reacted backgrounds in some other situations. Another source of a correction could be from 5D loops, which are completely ignored in the present work. This defect, however, seems common to studies based on AdS/CFT or AdS/QCD. Although we may not be able to calculate the corrections from the back-reaction and 5D loops in a simple manner, we have to include them to obtain complete corrections to $T_0$ due to the matters. Therefore, we may conclude that in the present work, we have calculated some part of the leading $1/N_c$ corrections due to the matters.

IV. SUMMARY

Using the AdS/QCD approach, we studied isospin and strange matters, their effects on the hadronic observables, and the critical temperature for the deconfinement transition. The isospin matter can serve as a good testing ground for the AdS/QCD approach because it has been well-studied by using lattice QCD, by using chiral perturbation theory, and by using effective models of QCD, including the NJL model. Therefore, we first considered the isospin matter and found that the mass of $\pi^+$ increases with $|\tilde{\mu}_I|$ while that of $\pi^-$ decreases, which is in agreement with the results from lattice QCD and from other studies at small isospin chemical potential. When the isospin chemical potential is high enough, one may expect the pion condensation observed in the previous studies [10, 20, 21, 22]. In the chiral limit, $m_0 = 0$, we showed that the pion condensation occurs in the present study, which is consistent with the observations made in various studies [10, 20, 21, 22] and in a top-down approach [9]. With nonzero $m_0$, however, the mass of $\pi^-$ obtained in the present work is always non-zero unless $\tilde{\mu}_I$ goes to infinity. Therefore, pion condensation can exist only if
$\tilde{\mu}_I = \infty$, which is not the case in Ref. [10, 20, 21, 22]. We may attribute this discrepancy to the fact that we don’t take into account the back-reaction due to the isospin chemical potential on the metric, especially when the chemical potential is large. We observed that the space component of the pion decay constant and the masses of $\rho^-$ and $\alpha_1$ decrease with increasing $\tilde{\mu}_I$ while the $\rho$ and $\alpha_1$ decay constants are independent of $\tilde{\mu}_I$.

Then, we calculated the dependence of the hadronic observables and found that the dependence is similar to that of $\tilde{\mu}_I$. We also investigated the transition from nuclear matter to strange matter in our model. The relation between the chemical potential and the density, however, could not be obtained within the present approach. For simplicity, we adopted the relation from a free quark gas and estimated the critical density for the transition. We found that the critical density was very sensitive to the mass of the strange quark, as expected.

Finally, we estimated the critical temperature of the deconfinement transition for different isospin and strangeness number densities. For the strangeness, we considered two cases that might be relevant to relativistic heavy-ion collisions and neutron stars, which are summarized in Figs. 2 and 3. At a fixed baryon number density with a finite isospin chemical potential, we found that $T_c$ in asymmetric nuclear matter, $\tilde{c}_I \neq 0$, is smaller than that in symmetric nuclear matter, $\tilde{c}_I = 0$.

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