Computing experiment for unloading dumper truck at a sloping pad

Yu N Baryshnikov
Bauman Moscow State Technical University, Moscow, Russia
E-mail: mhts@list.ru

Abstract. The results of a computational experiment on unloading a dump truck are described. The experiment was carried out on the mathematical model of a car built on the basis of Lagrange's equations. Before the movement (unloading) of cargo, the lifting of the platform is considered as a quasi-static process. From the equations of the equilibrium of the car and geometric relationships, analytical expressions are derived for calculating the force interaction between the platform and the frame. When the cargo moves as a monolithic block, the mathematical model is represented in the form of a system of differential equations. After the adoption of a number of assumptions, its solution was substantially simplified. An algorithm for solving the problem is proposed that implements the method of incremental change of parameters. The results of calculation of forces in hydraulic cylinders during unloading of BelAZ mining dump truck are given.

1. Introduction
Determination of design loads is an important part of the strength analysis of any design. In many fields of technology, much attention has been paid to this issue. So, for example, for aircraft, ships, railway cars and subway cars, the main load regimes have been established and methods for calculating loads have been developed, which are approved by the relevant regulatory documents. In the automotive industry, there are no such standards yet. This situation can be explained both by the variety of loading regimes and by the complexity of calculating the operating loads. One of these modes is hitting cars on uneven roads. The studies of loads in this case are devoted to the work of a number of authors [1] - [4]. Another calculation case, in particular for dump trucks, is the unloading of the cargo platform. Experience shows that within a day dump trucks can perform more than twenty loading and unloading operations, and over the entire period of operation - more than ten thousand. This can serve as one of the reasons for the destruction of their supporting structure [5]. That is why the calculation of the force interaction between the platform and the frame during unloading is of particular interest.

An attempt to numerically simulate the dump truck unloading process was done in [6]. However, it considers a simplified model obtained on the basis of the d'Alembert principle. In this connection, further study of the dump truck unloading process requires the creation of more sophisticated mathematical models.
2. A mathematical model of the unloading of the damper truck

Usually the unloading process consists of two stages. At the first stage, the cargo remains stationary. In the second stage, the movement of the cargo begins when the platform is raised to the angle of the natural slope and ends with its full unloading.

Consider lifting the platform with a fixed cargo. The normal rise time of the platform is from 20 to 30 seconds, with the angular velocity of the platform turning small, constant and does not exceed 0.03-0.05 sec\(^{-1}\). Therefore, lifting the platform with a fixed weight can be considered as a quasi-static process. In Figure 1 shows the calculation scheme for unloading a dump truck.

We denote by \(m_1\) and \(m_2\) - the mass of the platform 1 and the cargo 2; \(\varphi\) - platform inclination angle; \(\alpha\) - angle of inclination of the hydraulic cylinder; \(s\) - movement of cargo; \((X_K, Y_K)\) and \((x_K, y_K)\) - the coordinates of the center of mass of the platform and the cargo in the stationary \(OXYZ\) and in the mobile \(Oxyz\) coordinate system, respectively.

![Figure 1. Calculation scheme for unloading the truck.](image)

Before the movement of the cargo, as well as in the case of its adhesion (example), the force \(F\) of the hydraulic cylinder is found from the condition that the sum of the moments of all forces relative to the point \(O\) be zero.

\[
F = \frac{m_1gX_1 + m_2gX_2}{a \sin \alpha - b \cos \alpha}.
\]

(1)

Here the \(X_1\) and \(X_2\) coordinates centers of mass of the platform and cargo in a fixed coordinate system \(OXYZ\).

Using the equations of coupling of a moving \(Oxyz\) and a fixed \(OXYZ\) coordinate system, we obtain:

\[
X_K = x_K \cos \varphi - y_K \sin \varphi, \quad (k = 1, 2)
\]

(2)

In turn, the angle of inclination of the hydraulic cylinder is determined from the geometric relationships:

\[
\alpha = \arctg \left( \frac{b + d \sin \varphi + h \cos \varphi}{a - d \cos \varphi + h \sin \varphi} \right).
\]

(3)
Thus, using equations (1) - (3), you can calculate the force $F$ of the hydraulic cylinder before the movement of the cargo at any angle $\varphi$ of inclination of the platform. The total reaction $R$ in the hinged support $O$ is also found from the equilibrium condition of the platform:

$$
R_X = F \cos \alpha; \quad R_Y = m_1 g + m_2 g - F \sin \alpha; \quad R = \sqrt{R_X^2 + R_Y^2} \tag{4}
$$

When the platform is raised to the angle of a natural slope ($\varphi \approx 30^\circ$), the cargo comes into motion. Let us consider the case when the cargo moves as a monolithic block symmetric with respect to the symmetry plane of the car (Figure 1). To derive the differential equations of motion of such a system, we use the Lagrange equations of the second kind

$$
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} = Q_i. \tag{5}
$$

The platform-cargo system has two degrees of freedom, which correspond to two generalized coordinates: the angle of the platform $q_1 = \varphi$ and the movement of the cargo $q_2 = s$. The kinetic energy of the system is found as the sum of the kinetic energies of the platform and the cargo:

$$
T = \frac{1}{2} J_{OZ}^{(1)} \dot{\varphi}^2 + \frac{1}{2} m_2 \dot{v}_c^2 + \frac{1}{2} J_{cz}^{(2)} \varphi^2, \tag{6}
$$

where $J_{OZ}^{(1)}$ - moment of inertia of the platform relative to the axis $OZ$; $J_{cz}^{(2)}$ - moment of inertia of the cargo relative to the main central axis $CZ$.

The speed $v_C$ of the center of mass $C$ of the cargo is found from the velocity vector diagram (Figure 2)

$$
\mathbf{v}_C^2 = \mathbf{v}_r^2 + \mathbf{v}_e^2 + 2 \mathbf{v}_r \mathbf{v}_e \cos(\mathbf{v}_r \cdot \mathbf{v}_e) \tag{7}
$$

Here $\mathbf{v}_r = \dot{s}$ is the relative velocity of the center of mass of the cargo; $\mathbf{v}_e = \mathbf{OC} \cdot \mathbf{\dot{\varphi}}$ - the portable speed of movement of the center of mass of the cargo. After the substitution $\mathbf{v}_r$ and $\mathbf{v}_e$ formula (7) takes the form
\[ v_c^2 = \dot{s}^2 + \left( x_2^2 + y_2^2 \right) \cdot \dot{\phi}^2 + 2 y_2 \dot{s} \dot{\phi}, \]  

Finally, the kinetic energy of the platform-cargo system is  
\[ T = \frac{1}{2} \left[ J_{ac}^{(1)} + m_2 \left( x_2^2 + y_2^2 \right) + J_{cz}^{(2)} \right] \cdot \ddot{\phi}^2 + m_2 y_2 \ddot{s} \dot{\phi} + \frac{1}{2} m_2 \dot{s}^2 \]  

Generalized forces entering the right-hand sides of equations (5), we obtain in the form  
\[ Q_1 = -m_1 g \cdot X_1 - m_2 g \cdot X_2 + F \sin \alpha - F b \cos \alpha; \]  
\[ Q_2 = m_2 g \sin \phi - f (m_2 g \cos \phi - 2m_2 \dot{\phi} \cdot \dot{s}), \]  

where \( f \) is the sliding friction coefficient of the cargo along the platform. Substituting these expressions into Lagrange's equations (5), we write the differential equations of motion of the platform-cargo system:  
\[ \left[ J_{ac}^{(1)} + m_2 \left( x_2^2 + y_2^2 \right) + J_{cz}^{(2)} \right] \cdot \ddot{\phi} + m_2 y_2 \ddot{s} = -m_1 g \dot{X}_1 - m_2 g \dot{X}_2 + F (a \sin \alpha - b \cos \alpha); \]  
\[ m_2 y_2 \ddot{\phi} + m_2 \ddot{s} + m_2 x_2 \dot{s}^2 = m_2 g (\sin \phi - f \cos \phi) - 2m_2 f \dot{\phi} \cdot \dot{s}. \]  

To solve the resulting system of differential equations, a numerical integration procedure is required, for example, based on the Runge-Kutta method. However, these equations can be simplified. Indeed, since the angular velocity of rotation of the platform is small and constant, then \( \dot{\phi} = 0 \) and \( \dot{\phi}^2 \approx 0 \). In addition, the last term in Eq. (13) has a second order of smallness and can also be neglected. Then the differential equations (12) and (13) can be transformed to the form  
\[ m_2 y_2 \ddot{s} = -m_1 g X_1 - m_2 g X_2 + F (a \sin \alpha - b \cos \alpha); \]  
\[ \ddot{s} = g (\sin \phi - f \cos \phi). \]  

Eliminating the acceleration \( \ddot{s} \) of the cargo from equation (14), we obtain the force of the hydraulic cylinder  
\[ F = \frac{m_1 g X_1 + m_2 g X_2 + m_2 g \cdot y_2 (\sin \phi - f \cos \phi)}{a \sin \alpha - b \cos \alpha}. \]  

To calculate the force of the hydraulic cylinder, we apply a step-by-step change of parameters. Starting with the angle of a natural slope, with a step \( \Delta \phi = 1^\circ \) from equation (15) we find the speed and the movement of the cargo. Then calculate the coordinate of the center of mass of the moving cargo \( x_2 = x_{02} - \dot{s} \) and from the communication equations (2) determine the coordinates \( X_1 \) and \( X_2 \) centers of mass of the platform and the cargo. From the equation (3) we find the angle of inclination of the hydraulic cylinder. We substitute all the obtained values into the formula (16). The process will be repeated until the angle of inclination of the platform reaches the maximum. We find the total reactions in the hinged supports using the d'Alembert principle  
\[ R_X = F \cos \alpha - m_2 \dot{s} \cos \phi; \quad R_Y = m_1 g + m_2 g - F \sin \alpha - m_2 \dot{s} \sin \phi \]  
\[ R = \sqrt{R_X^2 + R_Y^2} \]  

(17)
3. The results of the calculation of the loads of dump trucks BelAZ

Based on the proposed mathematical model, the forces of the hydro cylinders of the tilting mechanism of the BelAZ dump truck platform are calculated. The cases of lifting the platform with a fixed cargo and unloading the cargo as a monolithic block were investigated. The results of the calculation are shown in Figure 3.

![Figure 3. Efforts of the hydraulic cylinder during the lifting of the BelAZ platform with a fixed cargo (1) and with unloading of a monolithic block (2).](image)

Graphs of effort $F$ in hydraulic cylinders (curves 1 and 2) correspond to the two specified extreme cases that arise when unloading the platform. The analysis of the obtained results showed that in regular cases, when the cargo is bulk material, the force $F$ change schedules are located in the area selected by hatching. Maximum effort $F_{\text{max}}$ in the hydraulic cylinders is observed at the beginning of the platform lift ($\varphi = -15^\circ$). When the platform is raised more than $40^\circ$ an angle in the hydraulic cylinders, tensile forces may occur which can lead to the vehicle tipping over.

4. Unloading dumper truck at a sloping pad

Now we will examine the process of unloading the truck on an inclined platform. The calculation scheme of the car in this case is shown in Figure 4.
Figure 4. Calculation scheme for unloading the truck on an inclined platform.

Here is the $m_1$ - mass of the platform 1; $m_2$ - mass of cargo 2; $m_3$ - mass of other sprung aggregates 3; $m_4$ and $m_5$ - the mass of the front 4 and rear 5 bridges of the car; $\phi$ - platform elevation angle; $\theta$ - angle of inclination of the unloading site; $s$ - movement of cargo; $R_1$ and $R_2$ - the total reaction on the wheels of the front and rear axles of the car.

To estimate the longitudinal stability of the truck, we use the d'Alembert principle:

$$
\sum M_Z(m_i \vec{g}) + \sum M_Z(R_k) + \sum M_Z(R_w^\theta) = 0
$$

In our case, Eq. (18) takes the form:

$$
g \cos \theta \sum_{k=1}^4 m_k X_k + g \sin \theta \sum_{k=1}^4 m_k Y_k + R_1 X_4 - R_w^\theta \cos \phi \cdot Y_2 + R_w^\theta \sin \phi \cdot X_2 = 0,
$$

where $X_k$ and $Y_k$ are the coordinates of the centers of mass of the platform and cargo ($k = 1, 2$); $\vec{R}_w^\theta = -m_2 \vec{a}_c$ - the main vector of inertia forces of the cargo ($\vec{a}_c = \vec{s}$).

Note that the main moment of forces of inertia of the load is equal to zero, since angular velocity of the platform is constant.

If the angle of inclination of the platform exceeds the critical one, the car will lose its stability and overturn. At the same time, the front wheels lose contact with the platform and the reaction on the wheels of the front axle $R_1$ turns to zero. Then we rewrite equation (19) in the following form:

$$
g \cos \theta \sum_{k=1}^4 m_k X_k - g \sin \theta \sum_{k=1}^4 m_k Y_k + m_2 \vec{s} \cdot (Y_2 \cos \phi - X_2 \sin \phi) = 0.
$$

To determine the critical angle $\theta^*$ of the slope of the site, we apply a step-by-step change of parameters. By incrementally increasing the angle $\theta$, we simulate the rise of the platform at each step.
of changing the angle of inclination of the platform. When the sum of the angles $(\phi + \theta)$ becomes equal to the angle of the natural slope, the movement of the cargo will begin. We calculate the acceleration of the center of mass of the cargo. To this end, we use equation (15), where the slope of the platform is replaced by $(\phi + \theta)$:

$$\ddot{s} = g\left[\sin(\phi + \theta) - f \cos(\phi + \theta)\right].$$

In turn, the coordinates of the centers of mass of the platform and the cargo at each step of the platform lifting can be found by means of the rotation matrix

$$X_k = x_k \cos \phi - y_k \sin \phi - X_O;$$

$$Y_k = x_k \sin \phi + y_k \cos \phi + Y_O,$$

where $X_O$ and $Y_O$ are the coordinates of the center of the hinged support (point $O$).

Substituting these values into equation (20), we check the condition for the detachment of the front wheels. If the sum of the moments entering into the specified equation will be less or equal to zero, the angle $\theta^*$ of an inclination of a platform has reached critical value and the dump truck will overturn. Otherwise, increase the angle $\theta$ of the platform by one step and repeat the platform unloading procedure.

5. Conclusion

As a result of the computational experiment, the force interaction of the platform and the frame of the dumper truck during the unloading of the monolithic block was investigated. The results of calculations of these loads during unloading of the BelAZ mining dump truck are presented. A mathematical model and an algorithm for calculating the maximum permissible inclination angle of the platform, excluding the overturning of the dump truck, is developed. The obtained results can be used in calculating the strength of the carrier system of dump trucks. The proposed approach allows carrying out multivariate calculations of the car with the purpose of choosing the optimal constructive solution.

References

[1] Erz K 1957 Uber die durch Unebenheiten der Fahrbauhn hervorgerufene Verdrehung von Strassenfahrzeugen Z. Automobiltechnik Bd 59 4 pp 89-96
[2] Pavlovsky Ya 1977 Automobile bodies: trans. with Polish. (Moscow: Mashinostroenie) p 544
[3] Teser E 1979 The body of heavy vehicles: trans. with Polish. (Moscow: Mashinostroenie) p 232
[4] Baryshnikov Yu N 2014 Express analysis of loads when driving a car on uneven roads Electronic J. Science and Education (Moscow: BMSTU) 8 pp 224-236, DOI: 10.7463/0814.0724703
[5] Belokurov V N, Gladkov O V, Zakharov A A and Melik-Sarkisyants A S 1987 Dump trucks ed. Melik-Sarkisyants (Moscow: Mashinostroenie) p 216
[6] Baryshnikov Yu N 2015 Numerical modeling of the process of unloading dump trucks J. Natural and technical sciences 11 pp 57-59