The Universe out of an Elementary Particle?

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Abstract

We consider a model of an elementary particle as a $2+1$ dimensional brane evolving in a $3+1$ dimensional space. Introducing gauge fields that live in the brane as well as normal surface tension can lead to a stable "elementary particle" configuration. Considering the possibility of non vanishing vacuum energy inside the bubble leads, when gravitational effects are considered, to the possibility of a quantum decay of such "elementary particle" into an infinite universe. Some remarkable features of the quantum mechanics of this process are discussed, in particular the relation between possible boundary conditions and the question of instability towards Universe formation is analyzed.

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1. Introduction

In recent years it has become evident that the interplay between elementary particle theory and cosmology is a subject of great relevance for the understanding of the fundamental features of the Early Universe. In this paper we propose yet another possible connection between elementary particles and cosmology, which is that the whole universe may have had as its origin in the decay of a single elementary particle. This can be regarded as a modern realization of the vision Lemaître [1] had regarding the origin of the universe in the decay of a ”primeval atom”. In recent years a number of authors [2] have studied the possibility that a small bubble of false vacuum could evolve into a whole universe, including the quantum mechanical possibility of tunneling towards an infinite universe. Such false vacuum bubbles do not qualify as ”elementary particles ” since they are not static-classical configurations. From a physical point of view it is hard to see how for a state which is not at least quasi-stationary, tunneling could be physically relevant since we do not have then a situation, like the well known α decay process, where the particle can be available for decay for a very long time and although the decay rate can be very small, the very long time of existence of the quasi stationary state renders the decay relevant from a physical point of view. Here we build just such an scenario. Furthermore, we shall also find some remarkable features associated with the quantum mechanics of the tunneling process that can lead to the formation of a universe in this way. In particular, we discuss the relation between possible boundary conditions on the wave function and the question of instability towards universe formation.
2. Stable particle-like membrane solutions in flat space-time

We want to view an elementary particle as a $2 + 1$-dimensional membrane evolving in a $3 + 1$-dimensional embedding space-time. We associate to this membrane, following the standard approach to the theory of extended objects \[8\], a surface tension. If this was the end of the story, we could not achieve a stable configurations since the surface tension alone wants to make the surface of the membrane as small as possible and left without anything to balance it, it will lead to a collapse of the membrane. In order to compensate the effect of the surface tension we consider the effect of matter fields that live in the membrane itself, for example a $2 + 1$-dimensional gauge theory defined on the brane surface. The simplest form for the membrane action that incorporates the above requirements is \[3\]:

$$S = \sigma_0 \int \sqrt{-h} d^3 y + \lambda \int \sqrt{-h} F^{\alpha\beta} F_{\alpha\beta} d^3 y$$

where $h = \text{det}(h_{\alpha\beta}) \, \alpha, \beta = 0, 1, 2$ and $h_{\alpha\beta}$ is the induced metric on the surface as given by

$$h_{\alpha\beta} = \frac{\partial x^\mu(y)}{\partial y^\alpha} \frac{\partial x^\nu(y)}{\partial y^\beta} g_{\mu\nu}[x(y)]$$

where $g_{\mu\nu}(\mu, \nu = 0, 1, 2, 3)$ is the embedding $3 + 1$-dimensional space time and $x^\mu$ is the position of the membrane is this embedding space-time, and $F_{\alpha\beta} F^{\alpha\beta} \equiv F_{\alpha\beta} F^\alpha_{\delta\epsilon} h^{\delta\epsilon}$. Considering a spherically symmetric bubble and using spherical coordinates $\theta, \phi$, the simplest non-trivial potential that respects spherical symmetry (up to a gauge transformation) is the magnetic monopole configuration, given by

$$A_\phi = f(1 - \cos \theta)$$
which means that
\[ F_{\theta\phi} = f \sin \theta \quad (4) \]

Considering the most general spherically symmetric metric for the 2 + 1-surface (c=1)
\[ ds^2 = h_{\alpha\beta}dy^\alpha dy^\beta = -d\tau^2 + r^2(\tau)(d\theta^2 + \sin^2 \theta d\phi^2) \quad (5) \]

In this case the form (4) leads to \( F_{\alpha\beta}F^{\alpha\beta} = \frac{2f^2}{r^4} \), which leads to an action of the form
\[ S = \lambda \int 8\pi \frac{f^2}{r^2(\tau)}d\tau + 4\pi \int \sigma_0 d\tau \]
we can therefore regard the gauge field contribution as an r-dependent contribution to the surface tension
\[ \sigma = \sigma_0 + \frac{\sigma_1}{r^4} \quad (7) \]
\( \sigma_1 \) being \( 2\lambda f^2 \). The energy of the static wall being \( 4\pi r^2 \sigma \) has a non trivial minimum for any \( \lambda > 0 \) which permit a stable configuration. In addition to allowing for the stabilization of the bubble, the 2 + 1-internal gauge fields can also play a role in defining the electromagnetic currents that compete to external 3 + 1 gauge fields [3].

3. The effect of gravity and of an internal false vacuum

Generically, thin layers define boundaries between different phases and unless there is a reason, those different phases have different values for their energy densities. In what follows, we shall consider the case when the internal phase, defined by the inside of the membrane, has a non vanishing positive vacuum energy. The resulting system, as we shall see in section 4, display the necessary features which we are after in
In this work, that is, the possibility of defining a metastable, long lived configuration which can be identified as an elementary particle, and which has the possibility, when gravitational and quantum mechanical effects are taken into account, of decaying into an infinite universe. In this case, we can also solve for the dynamics of the membrane by the W. Israel method [4], which determine the discontinuity of the extrinsic curvatures along the thin shell. The internal solution, which corresponds to the existence of a non-vanishing vacuum energy density with a $T^{\mu \nu} = g^{\mu \nu} \rho$, is the well known de Sitter space defined by

$$ds^2 = -(1 - \chi^2 r^2)dt^2 + \frac{dr^2}{(1 - \chi^2 r^2)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

(8)

where $\chi^2 = \frac{8\pi \rho G}{3}$, $\rho$ is the energy density. Outside, in the empty space, we can have only a Schwarzschild space time according to Birkhoff’s theorem

$$ds^2 = -(1 - \frac{2GM}{r})dt^2 + \frac{dr^2}{(1 - \frac{2GM}{r})} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

(9)

In the boundary, we have a singular energy momentum tensor. In order to obtain the Israel conditions, the simplest way is to use the Gaussian normal coordinates. This is done as follows: denoting the 2 + 1 dimensional hypersurface of the wall by $\Sigma$, we begin by introducing a coordinate system in $\Sigma$. For definiteness, two of the coordinates can be taken to be the angular variables $\theta$ and $\phi$, which are always well defined up to an overall rotation for a spherical symmetric configuration. For the other coordinate in the wall, one can use the proper time variable $\tau$ that would be measured by an observer moving along with the wall. Now consider geodesics normal to $\Sigma$, $|\eta|$ is then defined as the proper length along one such geodesics, starting from
the surface $\Sigma$ to a given point outside $\Sigma$. We adopt the convention that $\eta$ is taken to be positive in the Schwarzschild regime and negative in the de Sitter regime. $\eta = 0$ is of course the position of the wall. At least in a neighborhood of the wall, any point is going to be intersected by one and only one such geodesics. Specifying then the associated value of $\eta$ and the coordinates of the point in $\Sigma$ where the geodesics originated, we can define in this way a coordinate system in a neighborhood of $\Sigma$. Thus the full set of coordinates is given by $x^\mu = (x^i, \eta), x^i = (\tau, \theta, \phi)$. In these coordinates $g^\eta\eta = g_{\eta\eta} = 1$ and $g^\eta\mu = g_{\eta\mu} = 0$. Also, we define $\xi_\mu$ to be the normal to an $\eta =$ constant hypersurface, which in Gaussian normal coordinates has the simple form $\xi^\mu = \xi_\mu = (0,0,0,1)$. We then define intrinsic curvature to an $\eta =$ constant surface as

$$K_{ij} = \xi_{i;j} = \frac{\partial \xi_i}{\partial x^j} - \Gamma^\eta_{ij} = -\frac{1}{2} \partial_\eta g_{ij}$$

(10)

In terms of these variables, Einstein’s equations take the form (as usual $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$)

$$G^\eta_{\eta} = -\frac{1}{2} (3)R + \frac{1}{2} [(K^2)_i^i - K_i^i K_j^j] = 8\pi G T^\eta_{\eta}$$

(11)

$$G_i^\eta = K_{ij}^j - K_j^j = 8\pi G T^\eta_i$$

(12)

$$G_j^i = (3)G_j^i + K_i^i K_j^j - \frac{1}{2} \delta_j^i [ (K^2)_i^i + (K_i^i)^2 ] + \partial_\eta [K_i^i - \delta_j^i K_j^i] = 8\pi G T_j^i$$

(13)

Where $|$ means covariant derivative in a three dimensional sense (in the 2+1 dimensional space of coordinates ($\tau, \theta, \phi$), which we denote with Latin indices $i,j,k,l,m$ ...).

Also quantities $(3)R$, $(3)G_j^i$, etc. are to be evaluated as if they concerned to a purely 3-dimensional metric $g_{ij}$, without any reference as to how it is embedded in the higher four dimensional space. By definition, for a thin wall, the energy moment tensor
\( T^{\mu\nu} \) has a delta function singularity at the wall, so one can define a surface energy momentum \( S^{\mu\nu} \), by writing

\[
T^{\mu\nu} = S^{\mu\nu}\delta(\eta) + \text{regular terms} \tag{14}
\]

When the energy momentum tensor (14) is inserted into the field equations (11-13), we obtain that (11) and (12) are satisfied automatically, provided they are satisfied for \( \eta \neq 0 \) (so that \( K_{ij} \) does not acquire a delta function singularity). Eq.(13) however, when integrated from \( \eta = -\epsilon \) to \( \eta = +\epsilon \) (\( \epsilon \to 0 \) and \( \epsilon > 0 \)), leads to the discontinuity conditions

\[
\gamma^i_j - \delta^i_j Tr\gamma = 8\pi G S^i_j \tag{15}
\]

where \( \gamma_{ij} = \lim_{\epsilon \to 0} (K_{ij}(\eta = +\epsilon) - K_{ij}(\eta = -\epsilon)) \). Solving from (15) for the trace of \( \gamma^i_j \) and substituting back into (15), we get

\[
\gamma^i_j = 8\pi G [S^i_j - \frac{1}{2}\delta^i_j TrS] \tag{16}
\]

It is easy to see that the local conservation of \( T_{\mu\nu} \), when it is of the form (14) implies that

\[
S^{\eta\eta} = S^{\eta\iota} = 0 \tag{17}
\]

If we combine (17) with the demand of spherical symmetry, we arrive at the form

\[
S^{\mu\nu} = \sigma(\tau) u^\mu u^\nu - \omega(\tau)[h^{\mu\nu} + u^\mu u^\nu] \tag{18}
\]

where

\[
h^{\mu\nu} = g^{\mu\nu} - \xi^\mu \xi^\nu \tag{19}
\]
is the metric projected onto the hypersurface of the wall, and

\[ u^\nu = (1, 0, 0, 0) = \text{four velocity of the wall} \quad (20) \]

In (18), \( \sigma \) has the interpretation of energy per unit surface, as detected by an observer at rest with respect to the wall, and \( \omega(\tau) \) has the interpretation of surface tension.

The form (18) is also obtained directly from the variation of the matter Lagrangian (1,4). In this case we obtain \( \sigma = \sigma_0 + \frac{\sigma_1}{r(\tau)}, \omega = \sigma_0 - \frac{\sigma_1}{r(\tau)} \) in agreement with (7). We now consider the matching through a thin spherical wall of the asymptotically flat (as \( r \to \infty \)) Schwarzschild solution (9), to the de Sitter solution (8). First consider the discontinuity of \( K_{\theta \theta} \). Using equation (10), we get for \( K_{\theta \theta} \)

\[ K_{\theta \theta} = \frac{1}{2} \partial_{\eta} g_{\theta \theta} = \frac{1}{2} \xi^\mu \partial_\mu g_{\theta \theta} \quad (21) \]

In the last step, in (21) we have expressed the derivative in the direction of the normal \( \partial_{\eta} \), in an arbitrary coordinate system using the normal vector \( \xi_\mu \). Adopting the convention where the normal points from the de Sitter space towards the Schwarzschild space, we have, denoting by - quantities referred to the de Sitter space and + to the related to the Schwarzschild space, that according to (16), (18), and (19),

\[ K_{\theta \theta}^- - K_{\theta \theta}^+ = 4\pi G \sigma r^2 \quad (22) \]

Both de Sitter and Schwarzschild can be expressed in the form

\[ ds^2_{\pm} = -A_{\pm}(r)dt^2_{\pm} + A_{\pm}^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (23) \]

In these coordinates \( U^\mu_{\pm} = (\dot{t}_{\pm}, \dot{r}, 0, 0) \) (for the velocity of the membrane) and the normal to the membrane \( \xi_{\pm}^\mu = (A_{\pm}^{-1} \dot{r}, \beta_{\pm}, 0, 0) \), \( \beta_\pm = \sqrt{A_{\pm} + \dot{r}^2} \). There is the possibility
of + or - sign for the \( \sqrt{A_\pm + \dot{r}^2} \). Full information concerning signs is obtained when going to a globally good coordinate system. Then \( K_{\theta\theta}^\pm \) is given by

\[
K_{\theta\theta}^\pm = \frac{1}{2} \xi^\mu \partial_\mu r^2 = r \beta^\pm \tag{24}
\]

4. The effective potential for the membrane: stable "particle-like" solutions and their possibility of tunneling to an infinite universe

Using equations (24), (22) and the specific values of \( A_\pm \) in our case, we obtain,

\[
\sqrt{1 - \chi^2 r^2 + \dot{r}^2} - \sqrt{1 - \frac{2GM}{r}} + \dot{r}^2 = 4\pi G \sigma r \tag{25}
\]

where

\[
\sigma = \sigma_0 + \frac{\sigma_1}{r^4} \tag{26}
\]

After solving for \( \sqrt{1 - \chi^2 r^2 + \dot{r}^2} \) and squaring both sides and afterwards solving for \( \sqrt{1 - \frac{2GM}{r}} + \dot{r}^2 \) and squaring again, we obtain the following equation

\[
\dot{r}^2 + V_{eff}(M, r) = -1 , \quad V_{eff} = \frac{2GM}{r} \left( 2\frac{\alpha^2}{\beta^2} - 1 \right) - \frac{4G^2M^2}{\beta^2 r^4} - \frac{\alpha^4 r^2}{\beta^2} \tag{27}
\]

being

\[
\alpha^2(r) = \chi^2 + \left( 4\pi \sigma(r) \right)^2 \tag{28}
\]

and

\[
\beta(r) = 8\pi G \sigma(r) \tag{29}
\]

which resembles the equation of a particle in a potential. \( V_{eff} \) with the form (7) for \( \sigma(r) \) is depicted in Fig 1. (in Fig. 1 the behavior as \( r \to 0 \) is not depicted. One
finds $V_{eff} \to -\infty$ as $r \to 0$). As we can see from Fig 1., $V_{eff}$ allows for a classically stable solution at $r=r_{min}$ (defined by $\frac{\partial V_{eff}}{\partial r} = 0$ and $\frac{\partial^2 V_{eff}}{\partial r^2} > 0$). Since $V_{eff} \to -\infty$ as $r \to \infty$, such a solution can be only classically stable but quantum mechanically can decay to a membrane containing an infinite universe inside (since $r \to \infty$) is expected. In the next chapter we investigate some interesting features of the quantum mechanics of such a process.

5. Some quantum mechanical aspects of the dynamics of the membrane

For the form (23) of the internal (-) and external (+) metrics it is guaranteed that the 2 + 1-dimensional energy momentum tensor of the membrane is conserved in a 2 + 1-dimensional sense [7]. This makes us expect that a Hamiltonian formalism which makes reference only to the 2 + 1 dynamical variables and not to the embedding space makes sense. Such is the ”proper time” quantization developed in ref.[6]. Following (4), we take as the Hamiltonian of the system the mass of the system. Using eq. (25), we obtain for the mass

$$M = \frac{\chi^2 r^3}{2G} - \frac{(4\pi G)^2 \sigma^2 r^3}{2G} + 4\pi \sigma r^2 (1 - \chi^2 r^2 + \dot{r}^2)^{1/2}$$

(30)

This mass $M$ corresponds also to the conserved zero-component of the 4-momentum as defined for example in [3] page 168. In an arbitrary Lorentz frame the standard particle energy momentum relation $P^0 = \sqrt{\vec{P}^2 + M^2}$ holds. Having obtained the Hamiltonian, all the other classical dynamical variables can be obtained as was done in [3]. The conjugate momentum $p$ will be equal to

$$p = \frac{\partial L}{\partial \dot{r}}$$

(31)
The Lagrangian will be equal to

\[ L = \dot{r} \int \frac{H}{\dot{r}^2} \]  

(32)

This give for the conjugate momentum

\[ p = \int \frac{\partial H}{\partial \dot{r}} \frac{\dot{r}}{r} \]  

(33)

Using H as before we arrive at the value of p, which is equal to:

\[ p = 4\pi \sigma r^2 \arcsinh\left( \frac{\dot{r}}{\sqrt{1 - \chi^2 r^2}} \right) \]  

(34)

for the membrane inside horizon and

\[ p = 4\pi \sigma r^2 \arccosh\left( \frac{\dot{r}}{\sqrt{\chi^2 r^2 - 1}} \right) \]  

(35)

outside. An arbitrary function of r can be added in the definition of p. Classically it corresponds to an additional total derivative of a function of r in the Lagrangian, while in Quantum Mechanics it corresponds to a redefinition of the wavefunction \( \Psi' = e^{i\phi(r)}\Psi \). That means that the Hamiltonian can be taken as

\[ H = \frac{\chi^2 r^3}{2G} - \frac{(4\pi G)^2 \sigma^2 r^3}{2G} + 4\pi \sigma r^2 \sqrt{1 - \chi^2 r^2 \cosh\left( \frac{p}{4\pi \sigma r^2} \right)} \]  

(36)

inside the horizon and

\[ H = \frac{\chi^2 r^3}{2G} - \frac{(4\pi G)^2 \sigma^2 r^3}{2G} + 4\pi \sigma r^2 \sqrt{\chi^2 r^2 - 1 \sinh\left( \frac{p}{4\pi \sigma r^2} \right)} \]  

(37)

outside it. In order to achieve a quantum mechanical approach we shall assume that

\[ p = -i \frac{\partial}{\partial r} \]  

(38)
and from this
\[ e^{-ia \frac{\partial}{\partial r}} \Psi(r) = \Psi(r - ia) \] (39)

and from this assumption, the Schroedinger equation is
\[ H \Psi = m \Psi \] (40)

being m the mass parameter of the external Schwarzschild solution. Defining the
dimensionless variable (in units where \( \hbar = c = 1 \))
\[ x = \frac{4 \pi r^3 \sigma_0}{3} - 4 \pi \frac{\sigma_1}{r} \] (41)

we receive the following difference equation for \( \Psi \), interpreting the order of operators
in \( \frac{p}{4 \pi \sigma r^2} \) as \( \frac{1}{4 \pi \sigma r^2} p \)
\[ f(x) \Psi(x) + g(x) [\Psi(x + i) + \Psi(x - i)] = 0 \] (42)
f and g being real functions of x, inside the horizon, and
\[ f(x) \Psi(x) + g(x) [\Psi(x + i) - \Psi(x - i)] = 0 \] (43)

outside. Expanding the equation for \( \Psi \) outside the horizon, taking \( x >> 1 \) (setting
\( r \sim \frac{1}{\chi} \) and \( \chi \sim \mathcal{O}^{1/2} \rho_0^{1/2} \), we see that \( x >> 1 \) is satisfied if the typical energy scales
determining \( \sigma_0, \sigma_1 \) and \( \rho_0 \) are \( << \) Planck scale) and keeping the first non vanishing
contribution only, we receive the equation:
\[ -\frac{f}{2g} \Psi(x) = i \frac{\partial \Psi}{\partial x} \] (44)

This has the form of a Schroedinger equation (x is time-like outside the horizon). The
solution is:
\[ \Psi = Ce^{i \int \frac{f}{2g} dx} \] (45)
where \( C = \text{constant} \). That means that once a bubble passes the horizon it will expand indefinitely, since from (45) we obtain that \( |\Psi|^2 = \text{constant} \) and therefore the modulus of the amplitude for the bubble being at \( r = \frac{1}{\chi} + \epsilon \) (\( \epsilon > 0 \) is very small) is the same as the amplitude for the membrane being at \( r \to \infty \) with probability equal 1. Notice that we could in principle avoid the outflow outside \( r = \frac{1}{\chi} \) by taking \( \Psi = 0 \) at the point \( r = \frac{1}{\chi} \). In this case, since \( \frac{\partial |\Psi|}{\partial r} = 0 \) for \( r > \frac{1}{\chi} \), this will imply that \( \Psi = 0 \) for all \( r > \frac{1}{\chi} \) as well. Such particles would be stable against decay towards a \( r \to \infty \) state at this level of approximation. If we were to expose the particle so defined to an external field which can change the vanishing value of the wave function towards a non zero value at \( r = \frac{1}{\chi} \), we would then find of course that an irreversible flow towards \( r \to \infty \) will be generated this way. It is important to notice that the decay of the particle is an entirely gravitational effect. Indeed when \( \mathcal{G} \to 0 \) the form of the effective potential contains a minimum at \( r = r_{\text{min}} \) and as \( r \to 0 \) and \( r \to \infty \), \( V_{\text{eff}} \to \infty \) leaving no other possibility of a totally stable discrete spectrum on very general grounds.

6. Discussion and Prospects for Future Research

We want to enlarge the research in the future by studying the following items:

1. In the above discussion, we took the mass as the Hamiltonian. By ADM theorems this must be. But there is always a possibility that the Hamiltonian will be equal to the mass plus other terms that are equal to zero, if the constraints of the theory are used. These terms will not affect the dynamics of the problem, but will change the
picture we are working with, in particular, the phase of the wave function depends on the definition of the canonically conjugate momentum which is not clear. We want to arrive at the Hamiltonian by the canonical formalism, where the meaning of the phase of the wave function could be better understood. Also, we want to show how to obtain our results with an arbitrary choice of time and not only the proper time that has been used here. It should be noticed that the proper time method and the canonical formalism give the same results in other examples that have been investigated [11].

2. We also saw that when $r > \frac{1}{\chi}$ the equation of motion change from a second order equation to an equation with first derivative. We want to study this more in detail. In particular, the issues related to the boundary conditions inside and outside the horizon. Also issues related to the question of information loss (the external first order equation requires less boundary conditions than the internal one). It is very interesting to notice that as we cross the value $r = \frac{1}{\chi}$ the equation becomes like the time independent Schroedinger equation with $r$ playing the role of time and therefore the spontaneous appearance of a time variable takes place. Further implications of this research on the question of time in quantum cosmology [8] should be of considerable interest.

3. The problem of the boundary conditions. Boundary conditions like $\Psi(r = 0) = 0$ and $\Psi(r = \frac{1}{\chi}) \neq 0$ will give us a particle that can decay to a universe and $\Psi(r = \frac{1}{\chi}) = 0$ a totally stable particle. We want to study this in greater details.
4. The possibility of a solution generalized with spin, as in Davidson and Paz [10],
but now including gravitational effects and the instabilities discovered here.

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**FIGURE CAPTIONS**

Fig1. The potential of the membrane $V(r)$ vs. $r$
