Systematics of fine structure in the $\alpha$ decay of deformed even-even nuclei

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Abstract.
We present detailed systematics of the results for $\alpha$ transitions to excited $2^+$, $4^+$, and $6^+$ states in the $\alpha$ decay of deformed even-even nuclei. Both the semiclassical and coupled-channel results are analyzed in the valence nucleon scheme. This provides a good test of the reliability of the various theoretical models for the $\alpha$-decay fine structure. It is found that the coupled-channel results agree well with the experimental data in both systematic behavior and magnitude, while the semiclassical results have relatively large deviations from the experimental data. This confirms the good reliability of the coupled-channel studies.

1. Introduction
Alpha decay has been investigated for a long time in heavy and superheavy nuclei. The decay energies pose a tough test for nuclear mass models. The half-lives provide information on the stability of nuclides especially for superheavy nuclei. Since the pioneering work of Gamow [1], the experimental $\alpha$-decay half-lives have been interpreted with improved accuracy for both spherical and deformed $\alpha$-emitters [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Here we take deformed emitters for example. The semiclassical methods, based on the one-dimensional Wentzel-Kramers-Brillouin (WKB) approximation, evaluate the direction-dependent penetration probability of $\alpha$ particles and then average the probability along all directions [5, 6, 8]. The coupled-channel methods, based on the three-dimensional Schrödinger equation, use various channel wave functions with outgoing wave boundary conditions to determine partial widths and then total $\alpha$-decay half-lives [11, 12]. These two groups, in spite of different consideration, give similarly precise descriptions of total $\alpha$-decay half-lives. Nevertheless, many deformed emitters show a line spectrum of $\alpha$-groups corresponding to $\alpha$ transitions to various daughter states, which is confirmed by the fact that the energy differences between various $\alpha$-groups fit with $\gamma$-rays [13]. It was discovered by Rosenblum in 1929 [14], namely, $\alpha$-decay fine structure. Since the fine structure is a rich source of nuclear structure information, extensive theoretical studies have been devoted to pursuing a qualitative description of such an interesting phenomenon. Recently the fine structure in the $\alpha$ decay of deformed even-even nuclei has been widely investigated from both semiclassical and coupled-channel sides [15, 16, 17, 18, 19, 20, 21, 22, 23]. In addition, systematics of hindrance factors in the $\alpha$ decay of even-even trans-lead nuclei has been achieved as a function of the collectivity indicators [24].
Within the coupled-channel framework, it has been clearly shown that the $\alpha$ transitions to excited states are closely related to the structure properties of daughter nuclei such as nuclear deformation and excitation spectrum [20, 22, 23]. In other words, the fine structure observed in $\alpha$ decay is an important and sensitive probe of the structure properties of daughter nuclei. From the viewpoint of nuclear structure, it has been known that the evolution of nuclear structure is closely related to the integrated strength of the residual valence $p$-$n$ interaction, and the simple valence nucleon product $N_pN_n$ is employed to gauge this interaction [25]. Various structure quantities have been displayed to follow certain simple trends in the $N_pN_n$ scheme [26, 27, 28, 29], such as nuclear deformation, $B(E2)$ values, ground band energies of even-even nuclei, core cluster decompositions in the rare earth region, yrast energies of even-even nuclei, and so on. With this in mind, it is expected that the $\alpha$-decay fine structure, associated with nuclear structure, also follows one certain trend in the $N_pN_n$ scheme. The objective of this article is to perform systematics of various theoretical results for the $\alpha$-decay fine structure within the valence nucleon scheme. By comparison with systematics of the experimental data, one can discern the reliability of the theoretical models developed for the $\alpha$-decay fine structure.

2. Various theoretical models for fine structure observed in $\alpha$ decay

Within the semiclassical framework, the partial decay width can be easily evaluated as [5, 6, 7, 8, 9]

$$\Gamma(Q_I, \ell) = P_\alpha F \exp \left( -\frac{2}{\hbar} \int_a^b \sqrt{2\mu[V(r) + \ell(\ell + 1)h^2/(2\mu r^2) - Q_I]} dr \right), \tag{1}$$

where $P_\alpha$ represents the $\alpha$-preformation probability, $F$ is the frequency of the $\alpha$ cluster inside the barrier, $V(r)$ is the $\alpha$-nucleus potential, the decay energy $Q_I$ is determined from the decay energy for the transition to ground states and the excitation energy of the final state, $Q_I = Q_0 - E_I$, and $\ell$ is the angular momentum carried by the $\alpha$ particle. If one takes into account nuclear deformation, the expression (1) would depend on deformation and orientation due to the deformed potential $V(r)$, and the partial width is determined by averaging along all orientations [5, 6, 8]. In terms of different $\alpha$-nucleus potentials used in the calculations, there are various models such as the simple WKB barrier penetration approach [15], generalized liquid drop model (GLDM) [3, 16], unified model for $\alpha$ decay and $\alpha$ capture (UMADAC) [6, 17], Coulomb and proximity potential model for deformed nuclei (CPPMDN) [18].

The starting point of the coupled-channel study is the coupled equations for radial parts [19, 20, 21, 22, 23]

$$\left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{\ell(\ell + 1)}{r^2} - Q_I \right] u_\alpha(r) + \sum_{\alpha'} V_{\alpha,\alpha'}(r) u_{\alpha'}(r) = 0. \tag{2}$$

In this equation, $u_\alpha'$ [$\alpha \equiv (n\ell I)$ labels the channel quantum numbers] is the cluster radial function describing the relative radial motion of the $\alpha$ particle with respect to the daughter nucleus, and $V_{\alpha,\alpha'}(r)$ is the matrix element of the interaction $V(r)$ taken between channels $\alpha$ and $\alpha'$. The matrix elements could be evaluated using two different techniques, the multipole expansion of interaction potentials [19, 20, 23, 30, 31] and diagonalization of the operator matrix [21, 22, 32]. After solving the equations (2), one can express the partial width as

$$\Gamma_{Q_I, \ell}(R) = \frac{\hbar^2 k_I}{\mu} \frac{|u_{n\ell I}(R)|^2}{G_\ell(k_I R)^2 + F_\ell(k_I R)^2}, \tag{3}$$

where $R$ denotes large distances beyond the range of the nuclear potential and beyond the distance where the Coulomb potential can be regarded as spherically symmetric. It should be particularly noted that the results of $\Gamma_{Q_I, \ell}(R)$ show rather weak sensitivity to the choice of $R$. 


No matter it concerns the semiclassical or coupled-channel picture, the branching ratio (BR) for the transition from ground states to excited states $I$ is determined as

$$\text{BR}_{I\ell} = \frac{\Gamma(Q_I, \ell)}{\sum_{\ell} \Gamma(Q_I, \ell)} \times 100\%.$$  \hspace{1cm} (4)

In some cases, the hypothesis of the Boltzmann distribution (BD) for daughter states, $\rho(E_I) = \exp(-cE_I)$ is proposed [22, 23, 33, 34, 35], and hence the BR is calculated as

$$\text{BR}_{I\ell} = \frac{\rho(E_I)\Gamma(Q_I, \ell)}{\sum_{\ell} \rho(E_I)\Gamma(Q_I, \ell)} \times 100\%.$$  \hspace{1cm} (5)

To gain better insight into the $\alpha$-decay fine structure, one also define the quantity [12, 19, 20]

$$\chi_{I\ell} = \log_{10}(\text{BR}_{00}/\text{BR}_{I\ell}),$$  \hspace{1cm} (6)

which represents the relative intensity of different channels with respect to favored channels.

3. Numerical results and discussion

In the following, we will focus on the $\alpha$ transitions from ground $0^+$ states of an even-even nucleus to various members of the ground-state rotational band in the daughter nucleus (i.e., $0^+$, $2^+$, $4^+$, $6^+$ ...). The experimental data for the BRs are taken from [36]. Actually, there are other $\alpha$ transitions to highly excited states rather than the ground-state rotational band, but they usually have quite small contributions to $\alpha$ decay [36].

First, we examine the theoretical results which are obtained using the semiclassical WKB calculations such as the UMADAC [17] and CPPMDN [18]. Figure 1 displays the calculated relative intensities $\chi_{I\ell}$ for excited $2^+$, $4^+$, and $6^+$ states in the $N_pN_n/(N_p+N_n)$ scheme, compared with the systematics of the experimental data. As one can see, the results for $2^+$ states are generally smaller than the experimental values especially in the larger $N_pN_n/(N_p+N_n)$ region. Within the CPPMDN, there is also an abnormal point corresponding to the $\alpha$ decay of $^{248}$Fm where the calculated value has a sudden increasing by about two orders of magnitude with respect to the neighboring emitters [18]. This suggests that the CPPMDN calculations for $^{248}$Fm should be checked. For the case of $4^+$ states, the experimental data show a striking peak around $N_pN_n/(N_p+N_n) \approx 7.5$ and the peak value is larger than the minimum by roughly two orders of magnitude [24]. However, the WKB results do not exhibit this behavior. For the case of $6^+$ states, the experimental data show a decreasing trend in the interval of $N_pN_n/(N_p+N_n) = 4 \sim 7$, by more than one order of magnitude [24]. By contrast, the CPPMDN results decrease by more than three orders of magnitude in the same interval of $N_pN_n/(N_p+N_n)$, while the UMADAC results decrease by about one order of magnitude. Besides, one can also notice that there is a clear difference between the CPPMDN and UMADAC results for $4^+$ and $6^+$ states in the interval of $N_pN_n/(N_p+N_n) = 4.5 \sim 6.5$. In a word, the differences between the experimental data and the WKB results are considerable. This is not so surprising because some important effects associated with nuclear structure are ignored in the WKB calculations, for example, the coupling effect between decay channels resulting from nuclear deformations.

Next, we transfer our attention from the semiclassical results to the coupled-channel studies. Delion and coworkers expanded the double-folding potentials into spherical multipoles and introduced the repulsive core for the monopole component [19, 20]. They performed three-channels calculations for deformed even-even nuclei, showing that the $\alpha$-decay fine structure is closely related to the deformation parameters of daughter nuclei. We used the diagonalization technique to evaluate the interaction matrix elements and performed four-channels calculations for deformed even-even $\alpha$-emitters [21, 22]. Recently we also used the multipole expansion to deal
Figure 1. (Color online) Relative intensities $\chi_{\ell}$ for excited $2^+$, $4^+$, and $6^+$ states as a function of the quantity $N_pN_n/(N_p + N_n)$. The theoretical results are obtained using the semiclassical methods and they are taken from [17, 18].

with the interaction matrix, where the dynamic effects of core nuclei are included [23]. The five-channels analysis showed that the excitation spectrum of daughter nuclei also plays an important role in the $\alpha$-decay fine structure in addition to the deformation of daughter nuclei. In a similar fashion, the detailed numerical results of these works are showed in figure 2. One can see that the theoretical results for $2^+$ states show good agreement with the experimental data. In details, the four-channels and five-channels calculations show similarly good results and the three-channels analysis is slightly worse in the larger $N_pN_n/(N_p + N_n)$ region. For the case of $4^+$ states, the results of the three cases have a tendency to form a peak around $N_pN_n/(N_p + N_n) \approx 7.5$ like the experimental results, but this is not evident especially in the smaller $N_pN_n/(N_p + N_n)$ region (corresponding to the outset of U, Pu, and Cm isotopic chains). This can be understood since in these $\alpha$ decays members of negative-parity and excited-state rotational bands, such as $1^−$, $3^−$, $0^+_2$, and $2^+_2$, emerge with low excitation energies in the daughter nuclei [36], leading to significant effects on the $\alpha$-decay fine structure. The consideration of these states requires
suitable theoretical schemes to describe the coupling between different rotational bands. Efforts towards this goal are being made. For the case of $6^+$ states, the results of the four-channels and five-channels calculations also follow the experimental points well in both the systematic behavior and the magnitude: as mentioned above, the relative intensities decrease in the interval of $N_pN_n/(N_p + N_n) = 4 \sim 7$ and show a smooth increasing in the region $N_pN_n/(N_p + N_n) > 7$. An obvious derivation is the four-channels result for the emitter $^{244}$Pu. This is worth us further investigation. On the whole, there is good agreement between experiment and theory for the coupled-channel studies of the $\alpha$-decay fine structure, as shown in figure 2. This in itself not only gives a strong support to the applicability and reliability of the coupled-channel approach, but also gives us some guaranty for reliable predictions of the $\alpha$-decay properties of superheavy nuclei. In addition, we would like to note that both the semiclassical and coupled-channel methods can give the results for $8^+$ states. But there are quite few experimental data for the transitions to excited $8^+$ states; that is, about six data are available in experiments [36]. So in

Figure 2. (Color online) Same as in figure 1, but the theoretical results are obtained using the coupled-channel methods, containing the three-channels (3-cs) [20], four-channels (4-cs) [22], and five-channels (5-cs) [23] calculations.
the present analysis we do not perform systematics of the results for $8^+$ states.

4. Summary

In summary, the theoretical results for the $\alpha$-decay fine structure in deformed even-even nuclei, obtained from both the semiclassical and coupled-channel methods, are investigated as a function of the collectivity indicator $N_pN_n/(N_p + N_n)$. And they are compared with the systematics of the experimental data. It is found that the results obtained from the semiclassical models deviate from the experimental data evidently in both the systematic behavior and the magnitude, while the results obtained from the various coupled-channel calculations agree well with the experimental data. This confirms the good reliability of the coupled-channel studies so that coupled-channel predictions are allowed for the $\alpha$-decay fine structure in the heavier mass region. Although the systematic behaviors of the results for excited $4^+$ and $6^+$ states are clearly seen, theoretical explanations are still not available, calling for more detailed structure investigations. In future, it will be of particular interest to use reliable structure models to achieve more information on the $\alpha$-decay fine structure, which could be useful for future structure researches on superheavy nuclei.

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