Research Article

Neurons with Hidden Attractor via Only One Controller

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Compared with the integer-order neuron model, the fractional-order neuron model can depict richer electrical activities of the neuron system and becomes a hot topic. To better understand the complex phenomenon of neuron, according to Caputo’s fractional derivative operator, the fractional-order Hindmarsh-Rose neuron model is introduced, and dynamics of it are investigated. Firstly, the hidden attractor of the proposed model is discussed via theoretical analysis and numerical simulation. Secondly, synchronization between fractional-order Hindmarsh-Rose neurons is realized by designing one controller whether the order is the same or different. Simultaneously, the impact of the order on the synchronization speed of considered systems with the same order is explored, and it is found that lower order is beneficial for speeding up synchronization. Theoretical results are confirmed via numerical simulations. Research results contribute to reveal some complex phenomena of neuron systems and control the complex dynamics effectively.

1. Introduction

As an important component of the nervous system, neuron plays a pivotal role during signal transmission and exchange. Research on the firing rhythm of the neuron is beneficial for understanding and revealing the neurophysiological phenomenon of the neuron system. Therefore, various kinds of neuron models were addressed from aspects of the structure and function of the neuron. According to electrophysiological experiments, Hodgkin-Huxley (HH) neuron model [1] was established for revealing the electrophysiological mechanism of neurophysiological activity. And then on, some other neuron models were raised for discussing the rich electrical activity of the neuron, such as FitzHugh-Nagumo (FHN) model [2], Morris-Lecar (ML) model [3], Chay model [4], and Hindmarsh-Rose (HR) model [5]. Recently, Wang et al. established a novel neuron model [6] depicting a new working mechanism of neuron activity from an energy perspective and verified its equivalence to HH model based on nerve energy calculation [7].

With the establishment of various neuron models, much work has been done about neuron dynamics, such as Hopf bifurcation analysis of FHN neuron model [8], dynamical behaviors dependent on noise variance of HH neuron model [9], chaotic resonance of Izhikevich neuron model [10], nonsmooth dynamics of integrate-and-fire (IF) model, Hopf bifurcation in ML neuron controlled by constructing special controller [11]. Considering the effect of the electric field, memristor was introduced into neuron models, and dynamical behaviors of the improved models were discussed, including the electrical activity of neurons under electromagnetic induction [12], electric activities of time-delay memristive neuron disturbed by Gaussian white noise [13], different electrical activity modes of the neuron under magnetic field [14], and finite-time synchronization of improved HR neuron model under electric field effect [15]. Existing results suggest that integer-order neurons are provided with rich dynamics, which can reveal the complex phenomenon of the neuron system.

Recently, the dynamics of fractional-order systems attracted researchers’ interest, and some results have been obtained, such as chaotic behaviors in fractional order unified system [16], synchronization of fractional-order hyperchaotic systems [17], and chaotic maps [18]. In neuroscience, it was found that multiple time scale adaptation of neuron was consistent with fractional-order differentiation.
incorporating them and also increases the degree. The neuron model can improve the classical neuron model by being noted, applying fractional-order derivative in the system and its application have been imposed much attention, such as dynamical characteristics of the FHN model differing from that of integer-order model [24, 25], stability and bifurcation of fractional-order HR neuron model relying on the order and different firing frequency [26], firing patterns of fractional-order ML model [27], bursting patterns of a modified fractional-order ML neuron which were not obtained in corresponding integer-order model. As has been noted, applying fractional-order derivative in the neuron model can improve the classical neuron model by incorporating the memory effect and also increase the degree of freedom [28].

Via research on dynamical systems, it was believed that chaotic systems mainly involved two general categories, self-excited attractor and hidden attractor [29]. According to the Shilnikov theorem [30], a system can produce a chaotic attractor if and only if the equilibrium point is unstable. But, researchers found a class of chaotic systems that may have only stable equilibrium points or have no equilibrium points, but chaotic oscillations may still occur. This phenomenon is called hidden attractor differing from self-excited attractor induced by unstable equilibrium point. Further study showed that self-excited periodic and chaotic oscillations did not give exhaustive information about the possible types of oscillations. In the middle of the 20th century, a hidden attractor was found [31]. As a challenging problem, hidden attractor received much attention in the past decades because of its effect in revealing new phenomena in the nonlinear system different from the self-excited attractor. Especially, hidden attractor may lead to devastating consequences. For example, at the end of the last century, in the simulation of aircraft's control systems (antiwindup scheme), it was found that hidden oscillations could cause aircraft crash [32]. Hidden attractor is not just one of the phenomena seen in the nonlinear system described by the differential equations. It may suggest some specific physical meaning in the natural world. In the real nervous system, a hidden attractor corresponds to certain electrical activity of neuron in a particular situation. Therefore, research on hidden dynamics in neuron system is helpful for explaining the pathogenesis of neuronal diseases. It is of great significance for preventing and controlling some neuronal diseases.

It was found that fractional-order neuron system can produce biophysical variability for certain parameters which were not presented in the classical model [27]. This result suggests that fractional-order neuron model is provided with complex dynamics. Therefore, in this paper, hidden attractor of fractional-order HR neuron system with only a stable equilibrium point is to be investigated, and the synchronization of addressed neurons with the same or different orders will be realized by a single controller.

2. Preliminaries and System Description

In this section, to study the hidden attractor of fractional-order HR neuron and achieve the synchronization between fractional-order HR neurons with same or different orders, some relative preliminaries about fractional-order system are introduced, and fractional-order HR neuron model is depicted.

2.1. Preliminaries

Definition 1 (see [33]). Suppose \( f(t): [0, +\infty) \rightarrow \mathbb{R} \) be continuous and differentiable, and then, Caputo’s fractional differential operator of \( q \) for \( f(t) \) can be defined as
\[
D^q f(t) = \frac{1}{\Gamma(n-q)} \int_0^t \frac{f^n(r)}{(t-r)^{q-n+1}} dr,
\]
where \( n-1 < q < n \) with \( n \in \mathbb{N}^+ \), \( \Gamma(\cdot) \) means gamma function defined as \( \Gamma(\tau) = \int_0^\infty e^{-t} \tau^t dt \) satisfying \( \Gamma(\tau+1) = \tau \Gamma(\tau) \). Especially, when \( n = 1 \), that is, \( 0 < q < 1 \), we have
\[
D^q_0 f(t) = \frac{1}{\Gamma(1-q)} \int_0^t \frac{f(r)}{(t-r)^q} dr.
\]
This definition is widely used in control theory because the Laplace transformation formula of the Caputo fractional differential operator has the same form as that of an integer-order derivative. For simplicity, in the following discussions, the Caputo operator \( D^q \) is denoted as \( D^q f(t) \).

Lemma 1 (see [34]). Considering system
\[
\frac{d^n X}{dt^n} = A(X)X,
\]
where \( X = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \) is state variable and \( 0 < q < 1 \). If each eigenvalue \( \lambda \) of the coefficient matrix \( A(X) \) in (3) satisfies \( \arg(\lambda) \geq q\pi/2 \), then system (3) is said to be stable.

Remark 1. According to Lemma 1, for equilibrium point \( X^* \), if all eigenvalues of \( A(X^*) \) are provided with negative real part, then system (3) is stable at \( X^* \).

Lemma 2 (see [35]). For fractional-order nonlinear system
\[
\frac{d^n X}{dt^n} = A(X)X,
\]
where \( X \in R^n \) and \( 0 < q < 1 \), if there exists a positive definite matrix \( P \), for any variable \( X \), \( X^T P \frac{dX}{dt} \leq 0 \) holds, then fractional-order system (4) will be stable.

Lemma 3 (see [36]). Suppose \( f(t) \in C^2_a([a, b]) \), \( D^a_t f(t) \in C^\beta([a, b]) \), \( 0 < \alpha, \beta < 1 \), then

\[
D^\beta(D^a_t f(t)) = D^{a+\beta}_t f(t),
\]

holds.

2.2. System Description. The well-known HR neuron model [37] is an integer-order model, about which there has been much attention because of its superior computational speed. Its dynamics relies on the immediate previous response, while the fractional-order derivative depends on all the previous responses. That is to say, the fractional-order derivative has a memory effect [38] and can provide a wide range in understanding neuron dynamics [39]. The result in [19] suggested that the firing frequency of neocortical pyramidal neurons disturbed by the sinusoidal current can be well depicted with fractional-order derivative. In the early investigation of human memory, it was found that the accuracy of memory is relative to the power-law model; [40] suggested that the firing frequency of neocortical pyramidal neurons disturbed by the sinusoidal current can be well depicted with fractional-order derivative.

\[
\begin{align*}
\frac{dx}{dt} &= bx^2 - ax^3 + y - z + I_{\text{ext}}, \\
\frac{dy}{dt} &= c - dx^2 - y, \\
\frac{dz}{dt} &= r(S(x + k) - z),
\end{align*}
\]

where \( x \) denotes the membrane potential, \( y \) means the recovery variable for slow current, and \( z \) is adaption current. \( I_{\text{ext}} \) represents external forcing current. \( \frac{d^q}{dt^q} \) is a differential operator in terms of Caputo’s definition. The order \( q \) means the change rate of current satisfying \( 0 < q < 1 \).

3.1. Existence of Hidden Attractor. The equilibrium point of system (6) can be determined as

\[
A = (\delta, c - d\delta^2, S(\delta + k)),
\]

where \( \delta \) can be regarded as the solution of

\[
a\delta^3 + (d - b)\delta^2 + S\delta + Sk - c - I_{\text{ext}} = 0.
\]

By solving equation (8), \( \delta \) can be gained. And then the equilibrium point of (6) can be obtained from (7). Owing to \( a \neq 0 \), equation (8) always has a real solution; namely, there exists an equilibrium point of the system (6). The stability of the equilibrium point can be analyzed using the Jacobian matrix.

Suppose \((x^*, y^*, z^*)\) be the equilibrium point of HR system (6), corresponding Jacobian matrix of system (6) at \((x^*, y^*, z^*)\) can be given as

\[
J = \begin{bmatrix}
2bx^* - 3ax^*^2 & 1 & -1 \\
-2dx^* & -1 & 0 \\
rS & 0 & -r
\end{bmatrix}.
\]

The stability of the equilibrium point \((x^*, y^*, z^*)\) is closely dependent on the eigenvalues of (9).

According to Lemma 1, if eigenvalues \( \lambda_i (i = 1, 2, 3) \) of (9) satisfy \( |\arg(\lambda_i)| \geq \pi/2 \), then equilibrium point \((x^*, y^*, z^*)\) of system (6) is said to be stable. As we all know, by controlling the system parameters, system (6) can appear various firing modes. Especially, for fixed system parameters \( a, b, c, d, r, S, k, \) and external stimulus \( I_{\text{ext}} \), appropriate order \( q \) can be achieved using \( |\arg(\lambda_i)| \geq \pi/2 \). Namely, there exist system parameters, which can make the equilibrium point \((x^*, y^*, z^*)\) of system (6) stable. In this situation, if system (6) has attractor, it should be a hidden attractor.

3.2. Numerical Simulation Verification. Via theoretical analysis in Section 3.1, it can be seen that, for some system parameters, the fractional-order HR neuron model (6) may only have stable equilibrium point, which cannot induce self-excited attractor, while attractor also can be detected. That is to say, the fractional-order HR neuron model (6) may be provided with hidden attractor. To verify this result, select \( a = 1.0, b = 3.0, c = 2.0, d = 5.0, r = 0.006, S = 4.0, k = 1.6, I_{\text{ext}} = 4.5 \), equilibrium point of (6) can be solved as \( x^*_c = (0.0247, 1.9970, 6.4988) \). By means of the Jacobian matrix (9), corresponding eigenvalues to the equilibrium point \( x^*_c \) are gained as \( \lambda_1 = -0.7279, \lambda_2 = -0.0659 + 0.1716i \), and \( \lambda_3 = -0.0659 - 0.1716i \). The eigenvalues at equilibrium point \( x^*_c \) all have negative real part such that \( |\arg(\lambda_i)| > \pi/2 \geq \pi/2 (i = 1, 2, 3) \) hold for \( 0 < q < 1 \). In terms of Lemma 1, \( x^*_c \) is a stable equilibrium point. For the initial value in the basin of attraction of \( x^*_c \), time series of system (6) will stabilize to the equilibrium point \( x^*_c \). But for the initial value not in the basin of attraction of \( x^*_c \), time series of system (6) can converge to a periodic orbit. For example, choose initial value \( (x_0, y_0, z_0) = (1, 0.9, 0.1) \), time series and phase portrait of system (6) are calculated via Adams-
Bashforth-Moulton predictor-corrector algorithm [41] and depicted in Figure 1, which suggests that the dynamical behavior of system (6) converges to a periodic orbit over time \( t \). Namely, fractional-order Hindmarsh-Rose neuron has a periodic orbit, which is hidden attractor. Further investigation via large amount of numerical simulation indicates that, when system parameters change in the neighborhood of \( a = 1.0, b = 3.0, c = 2.0, d = 5.0, r = 0.006, S = 4.0, k = 1.6 \), system (6) mainly appear hidden periodic orbit.

As has been noted, fractional-order HR neural model (6) can demonstrate hidden periodic attractor for suitable parameters and external forcing current. Theoretical analysis and numerical simulations all verified this result.

4. Synchronization of Fractional-Order HR Neuron Model via Only One Controller

In this section, synchronization between fractional-order HR neurons is to be investigated via only one controller from two aspects. And the numerical simulations are also carried out via the Adams-Bashforth-Moulton algorithm.

4.1. Synchronization between Fractional-Order HR Neurons with Same Order. In this section, synchronization between fractional-order HR neurons with the same order is discussed. For this end, system (6) is rewritten as

\[
\begin{align*}
\frac{d^q x_1}{dt^q} &= bx_1^2 - ax_1^3 + x_2 - x_3 + I_{\text{ext}}, \\
\frac{d^q x_2}{dt^q} &= c - dx_1^2 - x_2, \\
\frac{d^q x_3}{dt^q} &= r(S(x_1 + k) - x_3),
\end{align*}
\]

(10)

which is taken as the master system, and the slave system is assigned as

\[
\begin{align*}
\frac{d^q y_1}{dt^q} &= by_1^2 - ay_1^3 + y_2 - y_3 + I_{\text{ext}} + u(t), \\
\frac{d^q y_2}{dt^q} &= c - dy_1^2 - y_2, \\
\frac{d^q y_3}{dt^q} &= r(S(y_1 + k) - y_3),
\end{align*}
\]

(11)

where \( u(t) \) is the controller to be designed.

By constructing controller \( u(t) \), synchronization between fractional-order HR neurons can be achieved. The main result is given as follows.

**Theorem 1.** If the controller is chosen as

\[
u(t) = - (y_1 - x_1) + a(y_1 - x_1)(y_1^2 + y_1x_1 + x_1^2) \\
\quad - b(y_1 - x_1)(y_1 + x_1) - (1 - d)(y_1 - dx_1)(y_2 - x_2) \\
\quad - (rS - 1)(y_3 - x_3),
\]

(12)

then system (11) can synchronize system (10).

**Proof.** Let \( e_1 = y_1 - x_1, e_2 = y_2 - x_2, e_3 = y_3 - x_3 \), and then error system between systems (10) and (11) can be gained as

\[
\begin{align*}
\frac{d^q e_1}{dt^q} &= e_2 - ae_1(y_1^2 + y_1x_1 + x_1^2) + be_1(y_1 + x_1) - e_1 + u(t), \\
\frac{d^q e_2}{dt^q} &= -de_1(y_1 + x_1) - e_2, \\
\frac{d^q e_3}{dt^q} &= rse_1 - re_3.
\end{align*}
\]

(13)
Substitute (12) into (13), and we can get

\[
\begin{align*}
\dot{e}_1 & = e_2 - ae_1(y_1^2 + y_1 x_1 + x_1^2) + be_1(y_1 + x_1) - e_3 + u(t) \\
& \quad + e_2(-de_1(y_1 + x_1) - e_2) + e_3(rSe_1 - re_3) \\
& = e_1(e_2 - ae_1(y_1^2 + y_1 x_1 + x_1^2) + be_1(y_1 + x_1) - e_3 \\
& \quad + (-y_1 - x_1) + a(y_1 - x_1)(y_1^2 + y_1 x_1 + x_1^2) - b(y_1 - x_1)(y_1 + x_1) \\
& \quad -(1 - dy_1 - dx_1)(y_2 - x_2) - (rS - 1)(y_3 - x_3)) \\
& \quad + e_2(-de_1(y_1 + x_1) - e_2) + e_3(rSe_1 - re_3) \\
& = -e_1^2 - e_2^2 - re_3^2 \leq 0.
\end{align*}
\]

According to Lemma 2, the error system (13) will be stable at zero under controller (12). It means that the synchronization between systems (10) and (11) can be realized with only one controller. Theorem 1 is proved.

MATLAB program and Adams-Bashforth-Moulton predictor-corrector algorithm are utilized to implement numerical simulations for illustrating the theoretical result of Theorem 1. In the numerical simulations, system parameters of (10) and (11) are selected as \( a = 1.0, b = 3.0, c = 2.0, d = 5.0, r = 0.006, S = 4.0, k = 1.6, \) and \( I_{ext} = 4.5. \)

The initial values are chosen as \( x_1(0) = 1.0, x_2(0) = 0.9, x_3(0) = 0.1 \) and \( y_1(0) = 0.1, y_2(0) = 0.3, y_3(0) = 0.8, \) respectively. The order is taken as \( q = 0.5. \) Controller \( u \) is taken as the expression in (12). The time series of the error system (13) is computed and drawn in Figure 2, which suggests that the error system converges to zero over time \( t. \) It means that the synchronization between fractional-order neuron systems can be achieved by designing one controller.

To discuss the effect of order \( q \) on the convergence speed of the error system, a kind of total error is defined as

\[
E = \sum_{i=1}^{3} e_i^2,
\]

where \( e_1, e_2, e_3 \) are the errors between systems (10) and (11). When the controller is regarded as (12), the evolution curves of \( E \) for different values of \( q \) are calculated and depicted in Figure 3, from which we know that the total error \( E \) converges slower for larger value of \( q. \) Then, we know that, for fractional-order HR neuron, the higher the order is, the time required to obtain the synchronization is longer.
To further explain this phenomenon, the energy consumed by the controller with different fractional orders for reaching synchronization is to be discussed. In our work, the average integral value is used to estimate the energy consumed by controller. For simplicity, let $H = \int |u(t)| dt / T$, the integral is presented within $[0, T]$, and $T$ is the calculating period, which is the time to reach synchronization. Therefore, fractional order is selected as different values, and the energy the controller consumed for reaching synchronization can be estimated and depicted in Table 1, which suggests that the controller $u$ can consume lower energy for reaching synchronization when the fractional order $q$ is selected with smaller value.

4.2. Synchronization between Fractional-Order HR Neurons with Different Orders. As we all know, fractional-order derivative can depict the synaptic plasticity of the neuron. Generally speaking, there always exist some neurons with different synaptic plasticity. Therefore, research on the synchronization between different fractional-order neurons is helpful for understanding the collective behavior of these neurons. By designing a controller, namely, adjusting the synapse, the dynamical behavior of neuron can be controlled.

In this section, the synchronization between fractional-order HR neurons with different orders is studied. For this purpose, the master system is rewritten as

$$u(t) = -L(y_1 - x_1) - (bx_1^2 - ax_1^3 + x_2 - x_3 + I_{ext})$$

$$+ (bx_1^2 - ax_1^3 + x_2 - x_3 + I_{ext})^{(\beta-a)}$$

$$+ \left[ (c - dx_1^2 - x_2) \right]^{(\beta-a)} - (c - dx_1^2 - x_2) (y_2 - x_2)/(y_1 - x_1)$$

$$+ \left[ (rSx_1 + rSk - rx_3) \right]^{(\beta-a)} - (rSx_1 + rSk - rx_3) (y_3 - x_3)/(y_1 - x_1),$$

where $L$ is a positive real number.

**Proof.** Since $\alpha, \beta$ are two real numbers, we suppose $0 < \alpha < \beta < 1$. Considering Lemma 3, the drive system (16) can be depicted as

$$\frac{d^\alpha x_1}{dt^\alpha} = (bx_1^2 - ax_1^3 + x_2 - x_3 + I_{ext})^{(\beta-a)},$$

$$\frac{d^\alpha x_2}{dt^\alpha} = (c - dx_1^2 - x_2)^{(\beta-a)},$$

$$\frac{d^\alpha x_3}{dt^\alpha} = (r(S(x_1 + k) - x_3))^{(\beta-a)},$$

and the controlled slave system is depicted as

$$\frac{d^\beta y_1}{dt^\beta} = by_1^2 - ay_1^3 + y_2 - y_3 + I_{ext} + u(t),$$

$$\frac{d^\beta y_2}{dt^\beta} = c - dy_1^2 - y_2,$$

$$\frac{d^\beta y_3}{dt^\beta} = r(S(y_1 + k) - y_3),$$

where $0 < \alpha, \beta < 1$.

**Theorem 2.** For different $\alpha$ and $\beta$, the synchronization between systems (16) and (17) can be realized via controller $u$.

Let $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$ and utilize (18), error system between systems (16) and (17) can be gained as

$$\frac{d^\beta e_1}{dt^\beta} = (bx_1^2 - ax_1^3 + x_2 - x_3 + I_{ext})^{(\beta-a)},$$

$$\frac{d^\beta e_2}{dt^\beta} = (c - dx_1^2 - x_2)^{(\beta-a)},$$

$$\frac{d^\beta e_3}{dt^\beta} = (rSx_1 + rSk - rx_3)^{(\beta-a)}.$$
Sinesystem (6) has bounded trajectories, there exists a positive constant $M$, such that $|x_i| < M$, $|y_i| < M$, and $|z_i| < M$ for $i = 1, 2, 3$. Then, it can be gained that

$$e_1 \frac{d^6 e_1}{dt^6} + e_2 \frac{d^6 e_2}{dt^6} + e_3 \frac{d^6 e_3}{dt^6} \leq \left(2bM + 3aM^2 - L\right)e_1^2 + e_1 e_2 - e_1 e_3$$

Substitute (18) into (23), and we have

$$e_1 \frac{d^6 e_1}{dt^6} + e_2 \frac{d^6 e_2}{dt^6} + e_3 \frac{d^6 e_3}{dt^6} \leq \left(2bM + 3aM^2 - L\right)e_1^2 + e_1 e_2 - e_1 e_3 + 2 dM e_1 e_2 - e_2^2 + rSe_1 e_3 - re_3^2$$

$$\leq \left(2bM + 3aM^2 - L\right)e_1^2 + (1 + 2 dM)e_1 e_2 + (2S - 1)e_1 e_3 - e_2^2 - re_3^2$$

$$= -(e_1, e_2, e_3)Q(e_1, e_2, e_3)^T,$$
In this paper, the hidden attractor of fractional-order Hindmarsh-Rose neuron is investigated. Synchronization between fractional-order HR neurons is discussed. The main results are depicted as follows.

Firstly, by selecting suitable system parameters and external forcing current, hidden attractor of fractional-order Hindmarsh-Rose neuron is detected through theoretical analysis and numerical simulation, which has not been found in the previous literature about integer-order Hindmarsh-Rose neuron up to now and can be regarded as a new phenomenon of the HR neuron model.

Secondly, the synchronization between fractional-order neurons can be achieved by designing one controller whether the order is the same or different. For synchronization of fractional-order HR neurons with the same order, the lower the order is, the less time is required to obtain the synchronization. Meanwhile, it is found that, with order increasing, energy consumed by the controller becomes more for reaching synchronization.

Research results suggest that, for certain system parameters, the fractional-order HR neuron system is provided with hidden attractor, which is beneficial for revealing some complex phenomena of the neuron system and can enrich the functional neuronal mechanisms. On the one hand, fractional-order HR neuron can be regarded as an extension of HR neuron. On the other hand, hidden attractor of neuron can make people recognize the cognitive activity of the brain from another aspect.

Data Availability

There are no underlying data in our study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] A.L. Hodgkin and A.F. Huxley, "A quantitative description of membrane current and its application to conduction and excitation in nerve," *The Journal of Physiology*, vol. 117, no. 4, pp. 500–544, 1952.

[2] R. FitzHugh, "Mathematical models of threshold phenomena in the nerve membrane," *Bulletin of Mathematical Biophysics*, vol. 17, no. 4, pp. 257–278, 1955.

[3] C. Morris and H. Lecar, "Voltage oscillations in the barnacle giant muscle fiber," *Biophysical Journal*, vol. 35, no. 1, pp. 193–213, 1981.

[4] T. R. Chay, "Chaos in a three-variable model of an excitable cell," *Physica D: Nonlinear Phenomena*, vol. 16, no. 2, pp. 233–242, 1985.
[5] J. L. Hindmarsh and R. M. Rose, “A model of the nerve impulse using two first-order differential equations,” *Nature*, vol. 296, no. 5853, pp. 162–164, 1982.

[6] R. Wang, I. Tsuda, and Z. Zhang, “A new work mechanization on neuronal activity,” *International Journal of Neural Systems*, vol. 25, no. 3, Article ID 1450037, 2015.

[7] R. Wang, Z. Wang, and Z. Zhu, “The essence of neuronal activity from the consistency of two different neuron models,” *Nonlinear Dynamics*, vol. 92, no. 3, pp. 973–982, 2018.

[8] M.-H. Chou, “Computer-aided experiments on the Hopf bifurcation of the FitzHugh-Nagumo nerve model,” *Computers & Mathematics with Applications*, vol. 29, no. 10, pp. 19–33, 1995.

[9] Y. Yu and T. Sing Lee, “Adaptation of the transfer function of the Hodgkin-Huxley (HH) neuronal model,” *Neurocomputing*, vol. 52–54, pp. 441–445, 2003.

[10] S. Nobukawa, H. Nishimura, and T. Yamanishi, “Chaotic resonance in typical routes to chaos in the Izhikevich neuron model,” *Scientific Reports*, vol. 7, no. 1, pp. 1331–1924, 2017.

[11] S. Coombes, R. Thul, and K. C. A. Wedgwood, “Nonsmooth dynamics in spiking neuron models,” *Physica D: Nonlinear Phenomena*, vol. 241, no. 22, pp. 2042–2057, 2012.

[12] Y. Wang, J. Ma, Y. Xu, F. Wu, and P. Zhou, “The electrical activity of neurons subject to electromagnetic induction and Gaussian white noise,” *International Journal of Bifurcation and Chaos*, vol. 27, no. 2, Article ID 1750030, 2017.

[13] Z. Wang and X. Shi, “Electric activities of time-delay memristive neuron disturbed by Gaussian white noise,” *Cognitive Neurodynamics*, vol. 14, no. 1, pp. 115–124, 2020.

[14] M. Lv, C. Wang, G. Ren, J. Ma, and X. Song, “Model of electrical activity in a neuron under magnetic flow effect,” *Nonlinear Dynamics*, vol. 85, no. 3, pp. 1479–1490, 2016.

[15] K. M. Wouapi, B. H. Fotsin, F. P. Louodop, K. F. Feudjo, Z. T. Njitacke, and T. H. Djeudjo, “Various firing activities and finite-time synchronization of an improved Hindmarsh-Rose neuron model under electric field effect,” *Cognitive Neurodynamics*, vol. 14, no. 3, pp. 375–397, 2020.

[16] X. Wu, J. Li, and G. Chen, “Chaos in the fractional order unified system and its synchronization,” *Journal of the Franklin Institute*, vol. 345, no. 4, pp. 392–401, 2008.

[17] S. He, K. Sun, H. Wang, X. Mei, and Y. Sun, “Generalized synchronization of fractional-order hyperchaotic systems and its DSP implementation,” *Nonlinear Dynamics*, vol. 92, no. 1, pp. 85–96, 2018.

[18] Y. Peng, K. Sun, and S. He, “Synchronization for the integer-order and fractional-order chaotic maps based on parameter estimation with JAYA-IPSO algorithm,” *The European Physical Journal Plus*, vol. 135, no. 3, pp. 331, 2020.

[19] B. N. Lundstrom, M. H. Higgs, W. J. Spain, and A. L. Fairhall, “Fractional differentiation by neocortical pyramidal neurons,” *Nature Neuroscience*, vol. 11, no. 11, pp. 1335–1342, 2008.

[20] T. J. Anastasio, “The fractional-order dynamics of brainstem vestibulo-ocular motor neurons,” *Biological Cybernetics*, vol. 72, no. 1, pp. 69–79, 1994.

[21] H. G. Sun, W. Chen, H. Wei, and Y. Q. Chen, “A comparative study of constant-order and variable-order fractional models in characterizing memory property of systems,” *The European Physical Journal—Special Topics*, vol. 193, no. 1, pp. 185–192, 2011.

[22] J. Ma, Z.-Q. Yang, L.-J. Yang, and J. Tang, “A physical view of computational neurodynamics,” *Journal of Zhejiang University—Science*, vol. 20, no. 9, pp. 639–659, 2019.

[23] P. Zhou, J. Ma, and J. Tang, “Clarify the physical process for fractional dynamical systems,” *Nonlinear Dynamics*, vol. 100, no. 3, pp. 2353–2364, 2020.

[24] Y. Liu Tong and Y. Xie Yong, “Dynamical characteristics of the fractional-order FitzHugh-Nagumo model neuron and its synchronization,” *Acta Physica Sinica*, vol. 59, no. 3, pp. 2147–2155, 2010.

[25] Y. Xie, Y. Kang, Y. Liu, and Y. Wu, “Firing properties and synchronization rate in fractional-order Hindmarsh-Rose model neurons,” *Science China Technological Sciences*, vol. 57, no. 5, pp. 914–922, 2014.

[26] A. Mondal, S. K. Sharma, R. K. Upadhayay, and A. Mondal, “Firing activities of a fractional-order FitzHugh-Rinzel bursting neuron model and its coupled dynamics,” *Scientific Reports*, vol. 9, no. 1, Article ID 13721, 2019.

[27] M. Shi and Z. Wang, “Abundant bursting patterns of a fractional-order Morris-Lecar neuron model,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 6, pp. 1956–1969, 2014.

[28] Z. Wang, X. Wang, Y. Li, and X. Huang, “Stability and hopf bifurcation of fractional-order complex-valued single neuron model with time delay,” *International Journal of Bifurcation and Chaos*, vol. 27, no. 13, Article ID 1750209, 2017.

[29] Z. Wei, I. Moroz, J. C. Sprott, A. Akgul, and W. Zhang, “Hidden hyperchaos and electronic circuit application in a 5d self-exciting homopolar disc dynamo,” *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 27, no. 3, Article ID 033101, 2017.

[30] G. Y. Wang Guan-Yi, S. S. Qiu Shui-Sheng, and Z. Y. Xu Zhi-Yi, “A new three-dimensional quadratic chaotic system and its circuitry implementation,” *Acta Physica Sinica*, vol. 55, no. 7, pp. 3295–3301, 2006.

[31] G. A. Leonov, N. V. Kuznetsov, and V. I. Vagaitsev, “Localization of hidden Chua’s attractors,” *Physics Letters A*, vol. 375, no. 23, pp. 2230–2233, 2011.

[32] T. Lauvdal, R. Murray, and T. Fossen, “Stabilization of integrator chains in the presence of magnitude and rate saturations: a gain scheduling approach,” *Proc. IEEE Control and Decision Conf.*, vol. 1-5, pp. 4004–4005, 1997.

[33] I. Podlubny, *Fractional Differential Equations*, Academic Press, New York, NY, USA, 1999.

[34] J. B. Hu Jian-Bing, Y. Han Yan, and L. D. Zhao Ling-Dong, “A stability theorem about fractional systems and synchronizing fractional unified chaotic systems based on the theorem,” *Acta Physica Sinica*, vol. 58, no. 7, pp. 4402–4405, 2009.

[35] J. B. Hu Jian-Bing, Y. Han Yan, and L. D. Zhao Ling-Dong, “A novel stability theorem for fractional systems and its applying in synchronizing fractional chaotic system based on backstepping approach,” *Acta Physica Sinica*, vol. 58, no. 4, pp. 2235–2239, 2009.

[36] J. B. Hu Jian-Bing, J. Xiao Jian, and L. D. Zhao Ling-Dong, “Synchronizing fractional chaotic systems with different orders,” *Acta Physica Sinica*, vol. 60, no. 11, Article ID 110515, 2011.

[37] J. L. Hindmarsh and R. M. Rose, “A model of neuronal bursting using three coupled first order differential equations,” *Proceedings of the Royal Society of London—Series B: Biological Sciences*, vol. 221, no. 1222, pp. 87–102, 1984.

[38] W. W. Teka, R. K. Upadhayay, and A. Mondal, “Spiking and bursting patterns of fractional-order Izhikevich model,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 56, pp. 161–176, 2018.
[39] I. Podlubny, “Geometric and physical interpretation of fractional integration and fractional differentiation,” Fract. Calc. Appl. Anal. vol. 5, pp. 367–386, 2002.

[40] C. Donkin and R. M. Nosofsky, “A power-law model of psychological memory strength in short- and long-term recognition,” Psychological Science, vol. 23, no. 6, pp. 625–634, 2012.

[41] K. Diethelm, N. J. Ford, and A. D. Freed. “A predictor-corrector approach for the numerical solution of fractional differential equations,” Nonlinear Dynamics, vol. 29, no. 1/4, pp. 3–22, 2002.