Systematic study of two-proton radioactivity half-lives within the two-potential approach with Skyrme-Hartree-Fock

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Abstract: In this work, we systematically study the two-proton (2p) radioactivity half-lives using the two-potential approach while the nuclear potential is obtained by using Skyrme-Hartree-Fock approach with the Skyrme effective interaction of SLy8. For true 2p radioactivity (Q_{2p}>0 and Q_p<0), where the Q_p and Q_{2p} are the released energy of the one-proton and two-proton radioactivity), the standard deviation between the experimental half-lives and our theoretical calculations is 0.701. In addition, we extend this model to predict the half-lives of 15 possible 2p radioactivity candidates with Q_{2p}>0 taken from the evaluated atomic mass table AME2016. The calculated results indicate that a clear linear relationship between the logarithmic 2p radioactivity half-lives log_{10}T_{1/2} and coulomb parameters [ (Z_p^2+Z_{2p}^2+Q_{2p}^2/2) ] considered the effect of orbital angular momentum proposed by Liu et al [Chin. Phys. C 45, 024108 (2021)] is also existed. For comparison, the generalized liquid drop model(GLDM), the effective liquid drop model(ELDM) and Gamow-like model are also used. Our predicted results are consistent with the ones obtained by the other models.

Key words: two-proton radioactivity, Skyrme-Hartree-Fock, two-potential approach

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1 Introduction

In recent years, the study of exotic nuclei far from the β-stability line has become an interesting topic in nuclear physics with the developments of radioactive beam facilities[1,2]. Two-proton (2p) radioactivity, as an important exotic decay mode, provides a new way to obtain the nuclear structure information of rich-proton nuclei[3,10]. In the 1960s, this decay mode was firstly predicted by Zel’dovich[11] and Goldansky[12,13], independently. However due to the limitations in experimental, until 2002, the true 2p radioactivity(Q_{2p}>0 and Q_p<0), where the Q_p and Q_{2p} are the released energy of the proton and two-proton radioactivity) from 46Fe ground state was observed at Grand accélérateur national d’ions lourds (GANIL) [14] and Gesellschaft für Schwerionenforschung (GSI) [15], respectively. Later, the 2p radioactivity of 54Zn, 48Ni, 19Mg and 67Kr were consecutively identified at different radioactive beam facilities [16–19].

The 2p radioactivity process was treated as the isotropic emission with no angular correlation or a correlated emission forming 4He-like cluster with strongly correlation from the even-Z nuclei either in the vicinity or beyond the proton drip line [14, 20, 22]. Based on the above physical mechanisms, many theoretical models have been proposed to study the 2p radioactivity, such as the direct decay model [23, 29], the simultaneous...
versus sequential decay model[30], the diproton model[31,32], three-body model[33,34] and so on. Moreover, some empirical formulas can also successfully reproduce the half-lives of 2p radioactive nuclei including a four-parameter empirical formula proposed by Sreeja et al.[37] and New Geiger-Nuttall law for two-proton radioactivity proposed by Liu et al.[38]. The two-potential approach (TPA)[39,40] proposed by Gurvitz was initially used to deal with quasi-stationary problems and has been extended to study α decay, cluster radioactivity and proton radioactivity[41,52]. In our previous works, we systematically study the proton radioactivity within the TPA while the nuclear potential is calculated by SHF approach[53,54] denoted as TPA-SHF. The calculated results can reproduce the experimental data well. Since 2p radioactivity process may be share the similar theory of barrier penetration with proton radioactivity[58–61], whether TPA-SHF can be extended to study the 2p radioactivity or not is an interesting question. To this end, in this work, considering the spectroscopic factor $S_{2p}$, we extend TPA-SHF to systematically study the 2p radioactivity half-lives of nuclei with $4<Z<36$. For comparison, the generalized liquid drop model(GLDM)[62], the effective liquid drop model(ELDM)[20] and Gamow-like models[63] are also used.

This article is organized as follows. In the next section, the theoretical framework for the TPA-SHF is described in detail. The calculated results and discussion are given in Section 3. In Section 4, a brief summary is given.

2 Theoretical framework

The 2p radioactivity half-life $T_{1/2}$ as an important indicator of nuclear stability, can be calculated by

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln n_{2}}{\Gamma}. \quad (1)$$

Here $\lambda$, $\Gamma$ and $\hbar$ are the two-proton radioactivity constant, decay width and reduced Planck constant, respectively. In the framework of TPA[39,40], $\Gamma$ can be represented by the normalized factor $F$ and the penetration probability $P$. It is expressed as

$$\Gamma = \frac{\hbar^2 S_{2p} F P}{4 \mu}. \quad (2)$$

where $S_{2p} = G^2[A/(A-2)]^2 \chi^2$ denotes the spectroscopic factor of the 2p radioactivity. It can be obtained by the cluster overlap approximation[64]. Here $G^2 = (2n)!/[2^{2n}(n!)^2]$ with $n \approx (3Z)^{1/3}$ is the average principal proton oscillator quantum number[63]. $\chi^2$ is chosen as 0.0143 according to Cui et al. work[62]. $A$ and $Z$ are the mass and proton number of parent nucleus, respectively. $F$ is the normalized factor. It can be calculated by

$$F \int_{r_1}^{r_2} \frac{1}{2k(r)} dr = 1, \quad (3)$$

where $k(r) = -\mu \frac{dV(r)}{dr} + \frac{\mu}{k} |Q_{2p} - V(r)|$ is the wave number. $\mu = m_2m_d/(m_{2p}+m_d)$ is the reduced mass with $m_{2p}$ and $m_d$ being the mass of the emitted two protons and the residual daughter nucleus, respectively. $Q_{2p}$ is the released energy of the two-proton radioactivity. $V(r)$ is the total interaction potential between the emitted two protons and daughter nucleus which will be given more in detail in the following. $r_1$, $r_2$ and the following $r_3$ are the classical turning points. They satisfy the conditions $V(r_1) = V(r_2) = V(r_3) = Q_{2p}$. The penetration probability $P$ can be obtained by

$$P = \exp\left[-2\int_{r_2}^{r_3} k(r) dr\right]. \quad (4)$$

The total interaction potential $V(r)$ is composed by the nuclear potential $V_N(r)$, Coulomb potential $V_C(r)$ and the centrifugal potential $V_l(r)$. It can be written as

$$V(r) = V_N(r) + V_C(r) + V_l(r). \quad (5)$$

In this work, based on the assumption that the two protons spontaneously emitted from parent nuclear share momentum $p$ on average and the nuclear interaction potential of the emitted two protons-daughter nucleus is twice of the one between the emitted proton and daughter nucleus, we can obtained the nuclear potential of the emitted two protons $V_N(r) = 2U_0(\rho, \rho_p, \frac{p}{2})$ with SHF. In this model, the nuclear effective interaction is expressed as the standard Skyrme form. It is written as[67]

$$V_{12}(r_1, r_2) = t_0(1+x_0P_0)[\delta(r_1-r_2)$$

$$+ \frac{1}{2}t_1(1+x_1P_0)[P^2 \delta(r_1-r_2)+\delta(r_1-r_2)P^2]$$

$$+t_2(1+x_2P_0)P' \cdot \delta(r_1-r_2)P$$

$$+ \frac{1}{2}t_3(1+x_3P_0)[\rho(\delta(r_1-r_2))\alpha\delta(r_1-r_2)$$

$$+iW_0\sigma \cdot P' \times \delta(r_1-r_2)P], \quad (6)$$

where $t_0$, $t_3$, $x_0$, $x_3$, $W_0$ and $\alpha$ are the Skyrme parameters. $r_i$ (i=1, 2) is the coordinate vector of $i$-th nucleon. $P'$ and $P$ are the relative momentum operator acting on the left and right. $P_0$ and $\sigma$ are the spin exchange operator and the Pauli spin operator. In the SHF model, single-nucleon potential depended on the momentum of nucleon $p$ can be calculated by[68]

$$U_0(\rho, \rho_0, \frac{p}{2}) = a(\frac{p}{2})^2 + b, \quad (7)$$

where the subscript $q$ stands for proton/neutron ($q = p/n$). Total nucleonic density $\rho$ is sum of the proton density $\rho_p$ and neutron density $\rho_n$. The coefficients $a$
and \( b \) can be written as

\[
a = \frac{4}{5}\left[t_1(x_1 + 2) + t_2(x_2 + 2)\right] \rho + \frac{4}{5}\left[-t_1(2x_1 + 1) + t_2(2x_2 + 1)\right] \rho_q, (8)
\]

\[
b = \frac{4}{5}\left[t_1(x_1 + 2) + t_2(x_2 + 2)\right] k_1 \rho + \frac{4}{5} k_2 \rho_q + \frac{4}{5} t_0(x_0 + 2) \rho - \frac{4}{5} t_0(2x_0 + 1) \rho_q + \frac{4}{5} t_3(x_3 + 2) (\alpha + 2) \rho^{(\alpha + 1)} - \frac{1}{15} t_3(2x_3 + 1) \alpha \rho^{(\alpha - 1)} (\rho^2 q + \rho^2 q^2) - \frac{1}{15} t_3(x_3 + 1) \rho^2 q. (9)
\]

Here \( k_{f,q} = (3\pi \rho_q)^{1/3} \) represents the Fermi momentum. The relationship among total energy \( E \) of 2p emission in nuclear medium, nuclear potential and Coulomb potential can be written as

\[
E = 2U_q(\rho, \rho_q, \frac{b}{\rho}) + \frac{E^2}{2m_{2p}} + V_C(r). (10)
\]

In this work, \( E \) is obtained by the corresponding \( Q_{2p} \) with \( E = [(A-2)/A]Q_{2p} \). Based on the premise that the total energy keeps constant when 2p emit from parent nuclei, using Eq. (7) and Eq. (10) we obtained the momentum of two emitted protons \( |p| \) written as

\[
|p| = \sqrt{\frac{2(E - 2b - V_C(r))}{a + \frac{1}{m_{2p}}}}. (11)
\]

The Coulomb potential \( V_C(r) \) can be obtained from a uniformly charged sphere with radius \( R \). It is written as

\[
V_C(r) = \begin{cases} \frac{Z_i Z_2 e^2}{2R} [3 - \left(\frac{r}{R}\right)^2], & r < R, \\ \frac{Z_i Z_2 e^2}{r}, & r > R, \end{cases} (12)
\]

where \( Z_2 = 2 \) is the proton number of the two emitted protons in 2p radioactivity. The radius \( R \) is given by

\[
R = 1.28A^{1/3} - 0.76 + 0.8A^{-1/3}. (13)
\]

For the last part of Eq. (3), centrifugal potential \( V_l(r) \), we choose the Langer modified form since \( l(l+1) \rightarrow (l+1/2)^2 \) is necessary in one-dimensional problems. It can be expressed as

\[
V_l(r) = \frac{\hbar^2 (l + \frac{1}{2})^2}{2\mu r^2}, (14)
\]

where \( l \) is the orbital angular momentum taken away by the two emitted protons in 2p radioactivity.

### 3 Results and discussion

In this work, we firstly calculate the 2p radioactivity half-lives of nuclei with \( 4 < Z < 36 \) using the TPA while the nuclear potential is obtained by SHF and compare our calculated results with the experimental data and theoretical results calculated by GLDM, ELDM, Gamow-like models. For Skyrme effective interaction, there are about 120 sets current Skyrme parameters. The SLy series parameters are widely used to describe the different nuclear reactions in various studies and the \( \alpha \) decay since spin-gradient term or a more refined two-body cent of mass correction is considered.

These parameters are listed in Table 1. As an example, we choose the Skyrme parameters of SLy8 in this work. The detailed calculation results are listed in Table 2. In this table, the first two columns represent the two-proton emitter and the experimental released energy of 2p radioactivity \( Q_{2p} \). The experimental data of 2p radioactivity half-lives, the theoretical ones obtained by GLDM, ELDM, Gamow-like and our model in logarithmic form are shown in 3–7 columns, respectively. From Table 2, we can see that the theoretical 2p radioactivity half-lives calculated by our work can reproduce experimental data well. In order to intuitively survey their deviations, we plot the difference of 2p radioactivity logarithmic half-lives between the experimental data and the ones calculated by these four models (our model, GLDM, ELDM and Gamow-like) in Fig. 1. From this figure, we can clearly see that all the points representing the difference are basically within \( \pm 1 \). Especially for \( ^{48}\text{Ni} \), \( Q_{2p} = 1.350 \text{ MeV} \) and \( ^{54}\text{Zn} \), \( Q_{2p} = 1.280 \text{ MeV} \), our calculated results can better reproduce the experimental data than the other models.

To obtain further insight into the well of agreement and the systematics of results, the standard deviation \( \sigma \) between the theoretical values and experimental ones is used to quantify the calculated capabilities of the above four models for 2p radioactivity half-lives. In this work, it is defined as follows:

\[
\sigma = \left[ \sum_{i=1}^{n} \left( \log_{10} T_{1/2}^i (\text{expt.}) - \log_{10} T_{1/2}^i (\text{cal.}) \right)^2 / n \right]^{1/2}. (15)
\]

Here \( \log_{10} T_{1/2}^i (\text{expt.}) \) and \( \log_{10} T_{1/2}^i (\text{cal.}) \) denote the logarithmic forms of experimental and calculated 2p radioactivity half-lives for the \( i-th \) nucleus, respectively. For comparison, the \( \sigma \) values of these four models are listed in Table 4. From this table, we can clearly see that the \( \sigma = 0.701 \) for this work is better than GLDM, Gamow-like with the same data. It indicates our work is suitable to study 2p radioactivity half-lives.

In addition, as an application, we extend our model...
to predict the half-lives of 15 possible $2\,p$ radioactivity candidates with $Q_{2p}>0$ taken from the evaluated atomic mass table AME2016[79, 80]. For comparison, the GLDM, ELDM and Gamow-like models are also used. The detailed results are given in Table 3. In this table, the first three columns represent the $2\,p$ radioactivity candidates, the experimental $2\,p$ radioactivity released energy $Q_{2p}$ and orbital angular momentum $l$, respectively. The last four columns are the theoretical values of $2\,p$ radioactivity half-lives calculated by GLDM, ELDM, Gamow-like and our model in logarithmic form, respectively. From this table, it is clearly seen that for short-lived $2\,p$ radioactivity nuclei, the order of magnitude of most predicted results calculated by our work are consistent with the ones obtained by the other three models. However, for long-lived $2\,p$ radioactivity nuclei, such as $^{49}$Ni and $^{60}$Ge, the magnitude of our work are less than 2-3 order to the other three models. In order to further clearly compare the evaluation capabilities of those four models, the relationship between the predicted results of those four models listed in Table 3 and coulomb parameters considering orbital angular momentum $(Z_d^2 s + l^{0.25}) Q_{2p}^{-1/2}$ i.e. New Geiger-Nuttall law for two-proton radioactivity proposed by Liu et al[38] was plot in Fig.2. From this figure, we can see that the predicted results of those four models are all linearly dependent on $(Z_d^2 s + l^{0.25}) Q_{2p}^{-1/2}$ and our work can better conform to the linear relationship.

Table 4. The standard deviation $\sigma$ between the experimental data and theoretical ones calculated by our model, GLDM, ELDM and Gamow-like model .

| model       | our model | GLDM | ELDM | Gamow-like |
|------------|-----------|------|------|------------|
| $\sigma$   | 0.701(10) | 0.852(10) | 0.531(4) | 0.844(10)  |

Fig. 1. (color online) The difference between the experimental data of $2\,p$ radioactivity half-lives and theoretical ones calculated by GLDM, ELDM, Gamow-like and our model in logarithmic form.
Table 1. The Skyrme parameters of SLy series.

| model  | $t_0$  | $t_1$  | $t_2$  | $t_3$  | $x_0$  | $x_1$  | $x_2$  | $x_3$  | $W_0$  | $\alpha$ |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| Sly0   | -2486.40 | 485.20 | -440.50 | 13783.0 | 0.790  | -0.500 | -0.930 | 1.290  | 123.0  | 1/6      |
| Sly1   | -2487.60 | 488.30 | -568.90 | 13791.0 | 0.800  | -0.310 | -1.000 | 1.290  | 125.0  | 1/6      |
| Sly2   | -2484.20 | 482.20 | -290.00 | 13763.0 | 0.790  | -0.730 | -0.780 | 1.280  | 125.0  | 1/6      |
| Sly3   | -2481.10 | 481.00 | -540.80 | 13731.0 | 0.840  | -0.340 | -1.000 | 1.360  | 125.0  | 1/6      |
| Sly4   | -2488.91 | 486.82 | -546.39 | 13777.0 | 0.834  | -0.344 | -1.000 | 1.354  | 123.0  | 1/6      |
| Sly5   | -2484.88 | 483.13 | -549.40 | 13763.0 | 0.778  | -0.328 | -1.000 | 1.267  | 126.0  | 1/6      |
| Sly6   | -2479.50 | 462.18 | -448.61 | 13673.0 | 0.825  | -0.465 | -1.000 | 1.355  | 122.0  | 1/6      |
| Sly7   | -2482.41 | 457.97 | -419.85 | 13677.0 | 0.846  | -0.511 | -1.000 | 1.391  | 126.0  | 1/6      |
| Sly8   | -2481.40 | 480.80 | -538.30 | 13731.0 | 0.800  | -0.340 | -1.000 | 1.310  | 125.0  | 1/6      |
| Sly9   | -2511.10 | 510.60 | -429.80 | 13716.0 | 0.800  | -0.620 | -1.000 | 1.370  | 125.0  | 1/6      |

Table 2. The experimental data and theoretical ones of $2p$ radioactivity half-lives calculated by GLDM, ELDM, Gamow-like and our model.

| Nucleus | $Q_{2p}$ (MeV) | $\log_{10} T_e^{1/2}$ (s) | $\log_{10} T_{GLDM}^{1/2}$ (s) | $\log_{10} T_{ELDM}^{1/2}$ (s) | $\log_{10} T_{Gamow-like}$ (s) | $\log_{10} T_{our~model}$ (s) |
|---------|----------------|----------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $^{19}$Mg | 0.750[18] | -11.40[18] | -11.79 | -11.72 | -11.46 | -11.00 |
| $^{45}$Fe | 1.100[15] | -2.40[15] | -2.23 | - | -2.09 | -2.31 |
|          | 1.140[14] | -2.07[14] | -2.71 | - | -2.58 | -2.87 |
|          | 1.210[76] | -2.42[76] | -3.50 | - | -3.37 | -3.53 |
|          | 1.154[17] | -2.55[17] | -2.87 | -2.43 | -2.74 | -2.88 |
| $^{48}$Ni | 1.350[17] | -2.08[17] | -3.24 | - | -3.21 | -2.27 |
|          | 1.290[77] | -2.52[77] | -2.62 | - | -2.59 | -2.23 |
| $^{54}$Zn | 1.480[16] | -2.43 [16] | -2.95 | -1.32 | -3.01 | -2.08 |
|          | 1.280[78] | -2.76[78] | -0.87 | - | -0.93 | -1.32 |
| $^{67}$Kr | 1.690[19] | -1.70[19] | -1.25 | -0.06 | -0.76 | -1.05 |
4 Summary

In the present work, based on the two-potential approach while the nuclear potential is calculated by Skyrme-Hartree-Fock with the Skyrme effective interaction of SLy8, we systematically study the 2p radioactivity half-lives of nuclei with 4<Z<36. The calculated results can reproduce the experimental ones well. In addition, we extend our model to predict the half-lives of 15 possible 2p radioactivity candidates with \(Q_{2p}>0\) taken from the evaluated atomic mass table AME2016 and compared our calculated results with the theoretical one calculated by GLDM, ELDM and Gamow-like models. The predicted results of these four models are all linearly dependent on \((Z_0^{0.8}+l_0^{0.25})Q_{2p}^{-1/2}\) i.e. New Geiger-Nuttall law for two-proton radioactivity proposed by Liu et al.

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Table 3. Comparison of the predicted $2p$ radioactivity half-lives using GLDM, ELDM, Gamow-like and our model. The $2p$ radioactivity released energy $Q_{2p}$ and orbital angular momentum $l$ taken away by the two emitted protons are taken from Ref. [20].

| Nucleus | $Q_{2p}$ (MeV) | $l$ | $\log_{10} T_{1/2}^{\text{GLDM}}$ (s) \cite{62} | $\log_{10} T_{1/2}^{\text{ELDM}}$ (s) \cite{20} | $\log_{10} T_{1/2}^{\text{Gamow-like}}$ (s) \cite{63} | $\log_{10} T_{1/2}^{\text{our model}}$ (s) |
|---------|----------------|-----|---------------------------------------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|
| $^{22}\text{Si}$ | 1.283 | 0 | $-13.30$ | $-13.32$ | $-13.25$ | $-11.78$ |
| $^{26}\text{S}$ | 1.755 | 0 | $-14.59$ | $-13.86$ | $-13.92$ | $-12.93$ |
| $^{34}\text{Ca}$ | 1.474 | 0 | $-10.71$ | $-9.91$ | $-10.10$ | $-9.51$ |
| $^{36}\text{Sc}$ | 1.993 | 0 | $-11.74$ | $-12.00$ | $-11.12$ | $-11.12$ |
| $^{38}\text{Ti}$ | 2.743 | 0 | $-14.27$ | $-13.56$ | $-13.84$ | $-11.77$ |
| $^{39}\text{Ti}$ | 0.758 | 0 | $-1.34$ | $-0.81$ | $-0.91$ | $-1.62$ |
| $^{40}\text{V}$ | 1.842 | 0 | $-9.85$ | $-10.15$ | $-9.34$ | $-9.34$ |
| $^{42}\text{Cr}$ | 1.002 | 0 | $-2.88$ | $-2.43$ | $-2.65$ | $-2.83$ |
| $^{47}\text{Co}$ | 1.042 | 0 | $-0.11$ | $-0.42$ | $-0.97$ | $-0.97$ |
| $^{49}\text{Ni}$ | 0.492 | 0 | $14.46$ | $14.64$ | $14.54$ | $11.05$ |
| $^{56}\text{Ga}$ | 2.443 | 0 | $-8.00$ | $-8.57$ | $-7.51$ | $-7.51$ |
| $^{58}\text{Ge}$ | 3.732 | 0 | $-13.10$ | $-11.74$ | $-12.32$ | $-11.06$ |
| $^{59}\text{Ge}$ | 2.102 | 0 | $-6.97$ | $-5.71$ | $-6.31$ | $-5.88$ |
| $^{60}\text{Ge}$ | 0.631 | 0 | $13.55$ | $14.62$ | $14.24$ | $12.09$ |
| $^{61}\text{As}$ | 2.282 | 0 | $-6.12$ | $-6.76$ | $-6.07$ | $-6.07$ |

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