Collapse and revival of ultracold atoms in a microwave cavity and of photons in parametric down-conversion

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We present a new theoretical method to study a trapped gas of bosonic two-level atoms interacting with a single mode of a microwave cavity. This interaction is described by a trilinear Hamiltonian which is formally completely equivalent to the one describing parametric down-conversion in quantum optics. A system of differential equations describing the evolution, including the long-time behaviour, of not only the mean value but also the variance of the number of excited atoms is derived and solved analytically. For different initial states the mean number of excited atoms exhibits periodically reappearing dips, with an accompanying peak in the variance, or fractional collapses and revivals. Closed expressions for the period and the revival time are obtained.

I. THE PHYSICAL SYSTEMS

Since the experimental realization of weakly interacting atomic Bose-Einstein condensates confined in magnetic traps \[1\] much work was done to study the interaction of such a system with light. Examples include the scattering of light by a condensate \[2\], its refraction index \[3\], nonlinear atom optics \[4,5\], the optical detection of the relative phase between two condensates \[6\], and the spontaneous emission in the presence of two condensates \[7\].

In this paper we are concerned with the question of how a condensate of two-level atoms interacts with a single resonant quantized cavity mode of the electromagnetic field. The system can be realized by placing the trap containing the condensate in a microwave cavity. We consider the limit of vanishing temperature so that essentially all atoms are in the condensate. The two states \(|g\rangle\) and \(|e\rangle\) are both members of the internal ground-state manifold of the condensed atoms but have a different hyperfine magnetic quantum number \(m_F\). For both states \(m_F\) should be chosen in such a way that the atoms remain trapped after the transition from \(|g\rangle\) to \(|e\rangle\). The internal energies corresponding to these states are denoted by \(E_g\) and \(E_e\), respectively. Although for the existing condensates the trapping potential is different for states having a different magnetic quantum number we assume for simplicity that the trapping potential is the same for \(|g\rangle\) and \(|e\rangle\). This situation may be realized by using non-magnetic traps or by exposing the atoms to a highly detuned laser beam. If the shape of the beam is appropriately chosen the induced dipole-potential can compensate for the difference between the trap potentials.

Under these conditions the Hamiltonian in its second quantized form is given by

\[
H_0 = \int d^3x \sum_{i=e,g} \left\{ H_{c.m.} + E_i \right\} \Psi_i^\dagger(\vec{x}) \Psi_i(\vec{x}) \right\} 
\]

\[+ \hbar \omega_c a^\dagger a + H_{int} + H_{n.l.}\] (1)

where \(\Psi_i, i = e, g\) is the field operator for atoms in the state \(|g\rangle\) and \(|e\rangle\), respectively. \(a\) is the annihilation operator for photons in the microwave cavity. The center-of-mass Hamiltonian

\[
H_{c.m.} := \frac{-\hbar^2 \Delta}{2M} + \frac{\hbar^2}{2} \omega_z^2 z^2 + \frac{\hbar^2}{2} \omega_y^2 (x^2 + y^2) \] (2)

is the same for both internal states. \(M\) denotes the atomic mass, and \(\omega_z\) and \(\omega_y\) are the trap frequencies in the \(x - y\) plane and in the \(z\)-direction, respectively. The nonlinear Hamiltonian

\[
H_{n.l.} = \frac{1}{2} \int d^3x \sum_{i,j = e,g} g_{ij} \Psi_i^\dagger(\vec{x}) \Psi_j^\dagger(\vec{x}) \Psi_j(\vec{x}) \Psi_i(\vec{x}) \] . (3)

describes the interaction between the atoms. The quantities \(g_{ij}\) are connected to the scattering lengths for the scattering between atoms in the internal states \(|i\rangle\) and \(|j\rangle\). We will make the Hartree approximation and consider the case that all atoms are in the same state \(\varphi_0(\vec{x})\) regarding the center-of-mass motion. This state fulfills the stationary nonlinear Schrödinger equation (see, e.g., Ref. \[8\]) with energy eigenvalue \(\mu\) and is the same for both internal states.

We model the interaction of the atoms with the microwave photons by the usual magnetic dipole coupling

\[
H_{int} = -[a\vec{B} + a^\dagger \vec{B}^*] \cdot \int d^3x \left\{ \vec{m}_{ge} \Psi_e^\dagger(\vec{x}) \Psi_g(\vec{x}) + H.c. \right\} \] (4)

where \(\vec{m}_{ge}\) is the magnetic dipole moment of the atoms and \(\vec{B}\) the magnetic field of the cavity mode at the position of the trap. As we consider the case that the trap size is much smaller than the wavelength of the
microwave photons the interaction does not affect the center-of-mass motion of the condensed atoms. This justifies the assumption that the atoms will always remain in the spatial mode $\varphi_0(x)$.

Under these assumptions we can neglect all spatial modes beside $\varphi_0(x)$ and can replace the atomic field operators $\Psi_i(x)$ by $\varphi_0(x)b_i(x)$, $i = e, g$, where the operators $b_i := \int d^3x \varphi_i^\dagger(x)\Psi_i(x)$ are the annihilation operators for atoms in the internal state $|i\rangle$. Substituting this in the Hamiltonian (1) the latter can be simplified to

$$H = (\mu + E_e)b_i^\dagger b_e + (\mu + E_g)b_i^\dagger b_g + \hbar\omega_e a^\dagger a$$

$$- [a\vec{B} + a^\dagger \vec{B}^\ast] \cdot \{b_g b_e \vec{m}_{ge} + b_g^\dagger b_e^\dagger \vec{m}_{ge}^\ast\}. \quad (5)$$

In the interaction picture and after the rotating wave approximation the Hamiltonian reduces in resonance ($\hbar\omega_e = E_e - E_g$) to

$$\tilde{H} = -\hbar\Omega \{b_e^\dagger b_e a^\dagger + b_g b_g^\dagger a\} . \quad (6)$$

The Rabi frequency $\Omega$ is given by $|\vec{m}_{ge}| \cdot \vec{B}^\ast|/\hbar$.

The Hamiltonian (6) is formally equivalent to that used to describe parametric down conversion in nonlinear optics (see, e.g., Ref. [9]). In this system $b_e$ destroys a photon of the pumping beam and $a^\dagger$ and $b_g^\dagger$ create a signal and an idler photon, respectively. The interaction between the different modes is caused by a nonlinear medium and describes the process that a pumping photon is converted into two other photons under conservation of energy. Theoretically the pumping beam is often considered as a classical field. To be short we will focus in the following on the interpretation of the Hamiltonian (6) as an interaction between two-level atoms in a cavity.

Though the physical interpretation is different our results can immediately be applied to parametric down conversion if one considers $\tilde{N}_e = b_e^\dagger b_e$ as the number operator for pumping photons.

A trilinear Hamiltonian has been examined numerically by Walls and Barakat [10], Kumar and Mehta [11] derived analytical expressions for the time evolution under the condition that the number of one of the three particle species involved in Eq. (6) remains strongly populated. That the degenerate parametric oscillator may show collapse and revival has been predicted numerically by Jyotsna and Agarwal [12]. These results are similar to the numerical results of Drobný and Jex [13]. An exact solution for a fixed number of atoms was given in a non-closed form by Tavis and Cummings [14].

II. COUPLED DIFFERENTIAL EQUATIONS FOR THE MEAN OCCUPATION NUMBER AND ITS VARIANCE

Our aim is to present a new analytical method for solving the underlying operator equations. In particular this will then enable us to describe for the system given above the long-time behaviour of the mean occupation $\bar{N}_e(\tau)$ of the excited atomic mode for the case that initially all atoms are excited and there are zero photons in the cavity and to confront it with the time dependence of its variance. To do so we want to give closed-form approximate analytical solutions for $\bar{N}_e(\tau)$ for different initial states of the condensate like atomic number state, coherent state, or mixture. This will allow us to demonstrate that $\bar{N}_e(\tau)$ shows a periodic behaviour or the existence of collapse and revival, respectively.

The Hamiltonian (1) leads to two conserved quantities: the total number of atoms $S_A := b_e^\dagger b_e + b_g^\dagger b_g$ and, because of the rotating-wave approximation, the number of excitations $S_E := b_e^\dagger b_e + a^\dagger a$. Because any two of the number operators $N_e, N_g, N_a$ can be expressed by the third and the two conserved quantities, it suffices to solve the Heisenberg equation for only one of the three number operators. We will consider the number operator $N_e$ of excited atoms.

Introducing the new time variable $\tau := \Omega t$ and considering the Heisenberg equation for $N_e$ as well as for

$$\dot{\bar{N}}_e = -ib_e^\dagger b_e a^\dagger + ib_g b_g^\dagger a , \quad (7)$$

where the derivative is taken with respect to $\tau$, one easily finds (see, e.g., Ref. [11])

$$\dot{\bar{N}}_e = 6N_e^2 - 2AN_e + 2B . \quad (8)$$

For notational convenience we have introduced the operators $A := 2S_E + 2S_A + 1$ and $B := S_S + S_E$. Note that while $S_A$ and $S_E$ commute with every operator occurring in the problem, $N_e$ does not commute with $N_e$. This fact seems to prevent the integration of the second order differential equation to an equation of first order as it is usually done with differential equations for ordinary functions. A common way to circumvent this difficulty is the vanishing variance approximation, where a differential equation for $\bar{N}_e$ is derived under the assumption that the variance $\Delta_e$ defined by $\Delta_e^2 := (\langle N_e - \bar{N}_e \rangle^2)$ is negligible. Here and in the following the mean value of an operator $O$ will be represented by a bar, $\bar{O} := \langle \psi | O | \psi \rangle$. The complete state of the system is denoted by $| \psi \rangle$. For the system under consideration this approximation was studied by Kumar and Mehta [11] starting from Eq. (6). They showed that it is valid for short times only and can be applied only to a very limited class of physical problems.

Our objective is to improve this scheme: We want to go a step further and derive differential equations for the mean value $\bar{N}_e$ as well as for the variance $\Delta_e$ and study the respective long time behaviour for different states of the condensate. Because of $\partial^2 \langle N_e^2 \rangle = \langle \bar{N}_e N_e + N_e \bar{N}_e + 2\bar{N}_e^2 \rangle$, the discussion of $\Delta_e (\tau)$ needs the knowledge of $\bar{N}_e^2$ and therefore the exact integration of the operator equation (6). This can indeed be done by observing that the commutator between $N_e$ and $\bar{N}_e$ is given by $[N_e, \bar{N}_e] = \bar{H}/(i\hbar\Omega)$ so that $\bar{N}_e^2 N_e + N_e \bar{N}_e^2 = (2/3)(dN_e^2/d\tau) + (1/3)\bar{N}_e$ follows, what can be used to integrate Eq. (8) to
\[ (\hat{N}_e)^2 = 4N_e^3 - 2AN_e^2 + 2N_e(2B + 1) + C. \]  
(9)

Here \( C \) is a constant of motion which after some algebra can be written as 
\[ C = -\hbar^2/(\hbar^2\Omega^2) + 2S_A S_E. \]  
It is therefore related to the conservation of energy and particle numbers. Note that no approximation has been made up to this point.

Although Eq. (9) cannot be exactly integrated because \( C \) does not commute with \( N_e \), we are now able to treat \( N_e \) together with \( \Delta_e \) if we make the two following assumptions of a vanishing asymmetry approximation: (i) the conserved particle numbers are always uncorrelated to the number of excited atoms so that \( \langle S_i N_e^i \rangle = \langle S_i \rangle \langle N_e^i \rangle \) holds for \( i = A, E \) and any integer \( l \). This equality is exactly fulfilled for all times if the initial state is an eigenstate of \( S_i \), e.g., a number state. (ii) we can always neglect \( \langle N_e - N_e^3 \rangle^3 \) so that \( \langle N_e^5 \rangle \) can be approximated by \( \hat{N}_e^5 + 3\hat{N}_e^2\Delta_e^2 \). Accordingly, in contrast to the vanishing variance approximation, we only assume the vanishing of the higher moment \( \langle (N_e - N_e^3)^3 \rangle \) in the hierarchy of moments \( \langle (N_e - N_e^m)^m \rangle \), \( m = 2, 3, \ldots \) of the probability distribution. The case \( m = 3 \) describes its degree of asymmetry.

Based on this we derive from Eqs. (3) and (4) the system of ordinary differential equations

\[
\begin{align*}
\partial_t^2 \hat{N}_e &= 6(\Delta_e^2 + \hat{N}_e^2) - 2A\hat{N}_e + 2\hat{B}, \\
\partial_t^2 \Delta_e^2 &= 20\hat{N}_e + 60\Delta_e^2\hat{N}_e - 8A(\Delta_e^2 + \hat{N}_e^2) + 4(1 + 3\hat{B})\hat{N}_e + 2\hat{C} - \partial_t^2 \hat{N}_e^2,
\end{align*}
\]

Eq. (10) may be considered as an algebraic equation for \( \Delta_e^2 \) which can be inserted into Eq. (11) to decouple the system. This results finally in

\[
\partial_t^4 \hat{N}_e = -10A\partial_t^2 \hat{N}_e + 60\hat{N}_e\partial_t^2 \hat{N}_e - 240\hat{N}_e^3 + 120A\hat{N}_e^2 + \hat{N}_e(24 - 48\hat{B} - 16\hat{A}^2) + 12\hat{C} + 16\hat{A}\hat{B} \tag{12}
\]

\( \Delta_e \) can then be obtained from Eq. (11).

### III. ANALYTICAL SOLUTION FOR A NUMBER STATE

We are now able to treat the envisaged physical situation: all atoms are initially for \( \tau = 0 \) excited and no photons are present. It is desirable to obtain the development in time of \( \hat{N}_e(\tau) \) for the case of the atomic state being a number state with \( n \) atoms, the corresponding complete state of the system being denoted by \( |\psi_n\rangle \). We will demonstrate below that \( \hat{N}_e(\tau) \) for other physical situations can be reduced to this. The operator equations (8) and (4) show that the initial conditions for \( \hat{N}_e(\tau) \) which are to be fulfilled for a number state are: \( \hat{N}_e(0) = n \), \( \partial_t \hat{N}_e(0) = \partial_t^2 \hat{N}_e(0) = 0 \), and \( \partial_t^2 \hat{N}_e(0) = -2n \).

We have in fact been able to find an exact solution of Eq. (12) fulfilling the correct number state initial conditions for \( \hat{N}_e(0), \partial_t \hat{N}_e(0), \partial_t^2 \hat{N}_e(0) \), and, to leading order in \( n \), for \( \partial_t^2 \hat{N}_e(0) \). It is given by

\[ \hat{N}_e^{\text{num}}(\tau) = \hat{N}_e^{\text{num}}(0) - \frac{1}{2}m\omega^2\text{cn}^2(\omega\tau - K(m)|m), \tag{13} \]

where the parameters \( m \) and \( \omega \) are solutions of the algebraic equations

\[
\begin{align*}
0 &= \omega^4m(m - 1)(5[6\hat{N}_e^{\text{num}}(0) - \hat{A}] - 2\omega^2(2m - 1)) - 120(\hat{N}_e^{\text{num}}(0))^3 + 60\hat{A}(\hat{N}_e^{\text{num}}(0))^2 + 8\hat{A}\hat{B} + 6\hat{C} + 4\hat{N}_e^{\text{num}}(0)(3 - 2\hat{A}^2 - 6\hat{B}) \\
0 &= \omega^4[4 + 19m(m - 1)] + 10\omega^2(2m - 1)[\hat{A} - 6\hat{N}_e^{\text{num}}(0)] + 180(\hat{N}_e^{\text{num}}(0))^2 - 60\hat{A}\hat{N}_e^{\text{num}}(0) - 6 + 4\hat{A}^2 + 12\hat{B}.
\end{align*}
\]

The function \( \text{cn}(z|m) \) is one of the Jacobian elliptic functions, and \( K(m) \) is the complete elliptic integral \( [13] \). If \( m \) is close to one \( \text{cn}(z|m) \) becomes almost identical to the expression \( 1/\cosh(z) \) except that \( \text{cn}^2(z|m) \) is periodic in \( z \) with period \( 2K(m) \approx \ln[16/(1 - m)] \). For a number state \( |\psi_n\rangle \) we have \( \hat{A} = 4n + 1, \hat{B} = n^2, \) and \( \hat{C} = 2n^2 - n \). The corresponding solution [13] is, to the first two leading orders in \( n \gg 1 \), characterized by \( \omega \approx \sqrt{n + 2} \) and \( m \approx 1 - 2/n \). A second exact solution of Eq. (12) is structurally similar to that of Eq. (13) if \( \text{cn}(z|m) \) is replaced by the elliptic function \( \text{dn}(z|m) \). In this case the parameters are determined by different algebraic equations which we will omit in this paper.

![FIG. 1. Mean value \( \hat{N}_e^{\text{num}} \) (solid curve) and variance \( \Delta_e \) (dashed curve) of the number of excited atoms as a function of time for 100 initially excited atoms in an atomic number state. The dotted-dashed curve is obtained in the vanishing variance approximation. \( \Omega \) is the generalized Rabi frequency.](image-url)

Our improved mean value solution \( \hat{N}_e^{\text{num}}(\tau) \) of Eq. (13) is plotted in Fig. 1 together with the ordinary mean value solution obtained in the vanishing variance approximation \( (\Delta_e = 0) \). As one can see, the fluctuations become very large when the dip in the number of excited atoms occurs. As compared with the vanishing-variance approximation the dip is reduced by a factor of 1/2. This is in qualitative agreement with the results of Ref. [14] where a similar physical system was examined under the assumption that the photons escape quickly so that the
leading to 

\[ \langle N \rangle = \langle N \rangle_0 + 2K(1 - 2/n) \]

leading to \( T_p = \ln(8n)/[\sqrt{n + 2\Omega}] \). It diminishes with increasing number of atoms.

IV. COLLAPSE AND REVIVAL

Based on the solution \( \psi_\text{coh} \) we are now able to calculate the mean number of excited atoms if initially no photons and only excited atoms are present which form a coherent state with mean number \( \bar{n} \) of atoms. We expand this state in the number states \( |\psi_i\rangle \). Since each of the states \( \sqrt{N_e(0)} \exp[-i\hat{H}t/\hbar]|\psi_i\rangle \) is an eigenstate of \( S_A \) with eigenvalue \( l \) they are orthogonal for different \( l \). This reduces the mean value of \( N_e \) to the expression

\[
N_e^{\text{coh}}(\tau) = e^{-\bar{n}} \sum_{l=0}^{\infty} \frac{\bar{n}^l}{l!} \langle \psi_i | N_e(\tau) | \psi_i \rangle
\]

(14)

Because the r.h.s. is only a function of the mean values \( N_e^{\text{num}}(\tau) \) of the number state case, it may directly be evaluated with the help of Eq. (13). The physical reason for this simplification is that the interaction conserves the total number of atoms. Thus, no interferences between number states of different total atom number can occur. Note that the expression (14) is identical to the mean value for a statistical mixture of number states with Poissonian statistics.

![Fig. 2](image)

**FIG. 2.** For a coherent state of 100 initially excited atoms or a Poissonian mixture of number states the time evolution of the mean number of excited atoms exhibits collapses and revivals.

Whereas \( N_e^{\text{num}}(\tau) \) for a number state shows a periodic behaviour with period \( T_p \) it is an important consequence of Eq. (14) that a coherent atomic state as initial state leads to collapse and revival in the number \( N_e^{\text{coh}}(\tau) \) of excited atoms (see Fig. 2). This is also the case for a statistical mixture. Leaving open the question if a condensate can at all be described by a coherent state (compare, e.g., Ref. \[17\]) an experimental verification of collapse and revival cannot be used to discriminate between coherent state and mixture.

We turn to the details of the time development of \( N_e^{\text{coh}}(\tau) \). The time \( T_R \) when the first (large) revival occurs can be determined by the criterion given in Ref. \[18\]: the expression \( \langle \psi_\text{coh} | N_e(\tau) | \psi_\text{coh} \rangle \) must simultaneously have a maximum for the two neighboring values \( l = \bar{n} \) and \( l = \bar{n} + 1 \). This guarantees that many terms in the sum in Eq. (14) become simultaneously large. Since \( \text{cn}(z|m) \) is periodic this condition becomes a condition on the argument of the Jacobian elliptic function, namely \( \sqrt{n^2(0) \exp(-i\hat{H}t/\hbar)}|\psi_i\rangle \) is an eigenstate of \( S_A \) with eigenvalue \( l \) they are orthogonal for different \( l \). This can be solved for \( r \) and \( T_R \) and results to leading order in \( \bar{n} \) in

\[
T_R \approx \frac{2\sqrt{n^2(\bar{n})}}{\Omega} \ln\frac{\bar{n}}{\ln(n^2(\bar{n}) - 2)}.
\]

According to its derivation the revival time \( T_R \) is independent of the probability weights of the mixture or state when it is expanded in \( |\psi_i\rangle \). It is therefore the same for all mixtures with the same initial mean number of excited atoms.

In addition to the large revival at time \( T_R \) there are many smaller revivals at earlier times which arise at fractions of \( T_R \) (see Fig. 3). These fractional revivals have been predicted numerically for the degenerate parametrical oscillator in Refs. \[12,13\]. The fractional revivals arise when only a part of the sum terms in Eq. (14) have simultaneously a maximum. Their revival times can be calculated by assuming that the terms for \( l = \bar{n} \) and \( l = \bar{n} + r, r = 2,3, \ldots \) are simultaneously maximal. This leads to \( T_R(r) = T_R(1)/r \). Since the revivals reappear periodically one generally finds them at fractions of small integers of the time \( T_R \).

![Fig. 3](image)

**FIG. 3.** Between the collapse and the main revival fractional revivals do appear.
An estimation of the almost constant value $\bar{N}_{\text{const}}$ taken by $N^\text{coh}(\tau)$ between the collapse and the first revival can be derived under the assumption that the arguments of $c_n(z|m)$ of Eq. (13) that appear in the sum in Eq. (14) are statistically distributed over the whole period of $c_n(z|m)$ (i.e., $z \in [0, 2K(m)]$). $N_{\text{const}}$ then is simply the average of Eq. (13) over one period $T_p$. The corresponding integration $(2K)^{-1} \int_0^{2K} c_n^2(z|m)dz$ can be performed if $c_n^2(z|m) \approx 1$ is approximated by $1/\cosh^2(z)$ and results to leading order in $\bar{n}$ in

$$\bar{N}_{\text{const}} \approx \bar{n} \left\{ 1 - \frac{1}{\ln(8\bar{n})} \right\}.$$  \hspace{1cm} (16)

The two analytical expressions for $T_R$ and $N_{\text{const}}$ are in good agreement with the numerical evaluation of Eq. (14).

In conclusion we have presented analytical results for the collapse and revival of the mean occupation number and the variance of excited atoms in a microwave cavity. The phenomenon of a revival in the context of Bose-Einstein condensation was also examined for different physical systems. Zhang and Walls [19] discovered it numerically for atoms passing through a standing light wave. Wright et al. [20] showed that it occurs if the Bose condensate is described by a superposition of states with different total number of atoms. Our results differ from these approaches in that a third mode (the microwave photons) is fully incorporated and that the new theoretical method applied here allows to derive closed expressions for the interesting physical quantities.

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