In his Comment [1], Wójcik proposes two simple schemes for performing eavesdropping attacks to the Quantum Dense Key Distribution protocol (QDKD) in Ref. [2]. These schemes enable Eve to achieve a larger mutual information than Alice's and Bob's one, but maintaining inviolated the security condition (Eq. (6) in Ref. [2]).

Eve's attack in Wójcik's schemes relies on the possibility of subtraction of photons from the quantum channel without being disclosed by the Anticorrelation Check. In fact the security proof given in [2] is incomplete because it includes only the effect that Eve's presence induces a bit flip on the travelling qubit.

In this reply we complete the security proof of QDKD even against individual eavesdropping attack with injection or subtraction of photons in the quantum channels. Furthermore we discuss the security of the experimental realization of the QDKD protocol performed in [2], according to the arguments raised by the modified proof.

A dedicated formalism is introduced to account for these attacks. Let \( |\psi^+_A\rangle \) be the state with \( n \) photons on the quantum channel \( X \), where \( m \) photons have horizontal polarization and \( (n-m) \) vertical polarization. The orthonormal base \( B_X : \{|m\rangle\} \) spans the Hilbert space \( \mathcal{H}_X \) of photons in channel \( X \); \( |0\rangle \) is the vacuum state.

In the actual formalism, referring to Fig. 1 in Ref. [2], Alice produces pairs of photons in the singlet state \( |\psi^-_{AB}\rangle = \frac{1}{\sqrt{2}}(|0^+_A 1^-_B\rangle - |1^+_A 0^-_B\rangle) \). Photon A is stored in her laboratory while on photon B she performs either the or subtraction of photons in the quantum channels.

In this reply we complete the security proof of QDKD modified by means of general unitary operators \( \tilde{K}_{BE} \) and \( \bar{K}_{BE} \) before and after Bob's operations, respectively. The final state belongs to the widened Hilbert space \( \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_E \). This approach is general because any physical non-unitary interaction is equivalent to a unitary one with a higher dimensional ancilla space \( \mathcal{H}_E \).

The final state after Bob's and Eve's operations is described by the trace preserving quantum operation \( \mathcal{E} \)

\[
\mathcal{E}(\tilde{\rho}_{AB}) = \frac{1}{2} \mathcal{E}_{1a}(\tilde{\rho}_{AB}) + \frac{1}{2} \mathcal{E}_{2a}(\tilde{\rho}_{AB}),
\]

with \( |\psi^+_{AB}\rangle = \frac{1}{\sqrt{2}}(|0^+_A 1^-_B\rangle + |1^+_A 0^-_B\rangle) \).

Bob randomly switches photon B towards either the Anticorrelation Check or his encoding apparatus.

The Anticorrelation Check is performed by Bob projecting photon B on the states \( |0^-_B\rangle \) and \( |1^-_B\rangle \) and Alice projecting photon A on the states \( |1^-_A\rangle \), and \( |0^-_A\rangle \), respectively. The non-local measurement guarantees the security of the transmission.

Bob's encoding apparatus is identical to Alice's. The communication takes place sending back photon B to Alice, who performs the complete Bell's state analysis.

Specifically, \( |\psi^+_{AB}\rangle \) corresponds to Alice and Bob encoding 0 and 1 or 1 and 0 respectively, while \( |\psi^-_{AB}\rangle \) corresponds to Alice and Bob both encoding 0 or 1. In other words the measurement of \( |\psi^+_{AB}\rangle \), \( |\psi^-_{AB}\rangle \) corresponds to the sum mod 2 of the bits encoded by Alice and Bob.

We model the general individual Eve's attack by coupling photon B with an ancilla system of Hilbert space \( \mathcal{H}_E \) in the initial state \( |\psi_E\rangle \) by means of general unitary operators \( \tilde{J}_{BE} \) and \( \bar{J}_{BE} \) before and after Bob's operations, respectively. The final state belongs to the widened Hilbert space \( \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_E \). This approach is general because any physical non-unitary interaction is equivalent to a unitary one with a higher dimensional ancilla space \( \mathcal{H}_E \).

The final state after Bob's and Eve's operations is described by the trace preserving quantum operation \( \mathcal{E} \)

\[
\mathcal{E}(\tilde{\rho}_{AB}) = \frac{1}{2} \mathcal{E}_{1a}(\tilde{\rho}_{AB}) + \frac{1}{2} \mathcal{E}_{2a}(\tilde{\rho}_{AB}),
\]

where \( \mathcal{E}_{2a} \) and \( \mathcal{E}_{1a} \) are quantum operations describing the evolution of the initial state \( \tilde{\rho}_{AB} \) prepared by Alice and modified by Bob's and Eve's actions

\[
\mathcal{E}_{1a}(\tilde{\rho}_{AB}) = \frac{1}{2} \tilde{K}_{BE}\tilde{J}_{BE}\tilde{\rho}_{AB} \otimes |\psi_E\rangle\langle\psi_E| \tilde{J}_{BE}\tilde{J}_{BE}^\dagger, \\
\mathcal{E}_{2a}(\tilde{\rho}_{AB}) = \frac{1}{2} \bar{K}_{BE}\bar{J}_{BE}\bar{\rho}_{AB} \otimes |\psi_E\rangle\langle\psi_E| \bar{J}_{BE}\bar{J}_{BE}^\dagger
\]

It is assumed that Bob encodes bit 0 or 1 with probability 1/2.

Our aim is to quantify the maximum information achievable by Eve in terms of the quantities measured.
by Alice and Bob in the Anticorrelation Check. We define the quantities $P_{01}$ and $P_{10}$ as the probabilities of anticorrelated results according to

$$
P_{01} = \text{tr}[\hat{J}_{BE}\hat{\rho}_{AB} \otimes |e_e\rangle \langle e_e| \hat{J}_{BE}\hat{\Pi}_{01}],
$$
$$
P_{10} = \text{tr}[\hat{J}_{BE}\hat{\rho}_{AB} \otimes |e_e\rangle \langle e_e| \hat{J}_{BE}\hat{\Pi}_{10}],
$$
(4)

where $\hat{\Pi}_{01} = |1^1_A1^1_B\rangle\langle 1^1_A1^1_B|$, $\hat{\Pi}_{10} = |1^1_A0^1_B\rangle\langle 1^1_A0^1_B|$ are projection operators. We assume $\hat{\rho}_{AB} = 1/2|\psi_{AB}\rangle\langle \psi_{AB}| + 1/2|\psi_{AB}\rangle\langle \psi_{AB}|$ (Alice prepares only states $|\psi_{AB}\rangle$ with probability $\frac{1}{2}$), in the absence of Eve’s attack $P_{01} = P_{10} = 0.5$ (perfect anticorrelation). Eve’s actions lower the value of $P_{01}$ and $P_{10}$ and this is basically the signature of her presence.

The maximum of the mutual information between Bob and Eve $I_{BE}$, i.e. Eve’s ability to distinguish Bob’s operations, can be evaluated exploiting the Holevo bound [4]: $I_{BE} \leq I_{BE}$. Consider the following scenario: Alice prepares the state $\hat{\rho}_{AB}$, Bob encodes his key, and Eve couples her system to photon B. The maximum mutual information between Bob and Eve $I_{BE}$ is bounded by

$$
I_{BE} = S[\mathcal{E}(\hat{\rho}_{AB})] - \frac{1}{2} S[\mathcal{E}_A(\hat{\rho}_{AB})] - \frac{1}{2} S[\mathcal{E}_B(\hat{\rho}_{AB})],
$$
(5)

where $S(\hat{\rho})$ is the Von Neumann entropy [5] of the generic state $\hat{\rho}$.

Analogously, also the maximum of mutual information between Alice and Eve $I_{AE}$, i.e. Eve’s ability to distinguish the states prepared by Alice, is calculated exploiting the Holevo bound ($I_{AE} \leq I_{AE}$). In this case

$$
I_{AE} = S[\mathcal{E}(\hat{\rho}_{AB})] - \frac{1}{2} S[\mathcal{E}(\hat{\psi}_{AB})] - \frac{1}{2} S[\mathcal{E}(\hat{\psi}_{AB})],
$$
(6)

To evaluate $I_{AE}$ and $I_{BE}$ in terms of $P_{01}$ and $P_{10}$ we define the final states in the coupled space $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_E$ after Bob’s and Eve’s operations

$$
\hat{K}_{BE} |1^1_B\rangle \otimes |e_e\rangle = |\mu_{ABE}\rangle, 
\hat{\hat{K}}_{BE} |0^1_B\rangle \otimes |e_e\rangle = |\nu_{ABE}\rangle, 
\hat{K}_{BE} |1^1_B\rangle \otimes |e_e\rangle = |\nu_{ABE}\rangle, 
\hat{\hat{K}}_{BE} |0^1_B\rangle \otimes |e_e\rangle = |\nu_{ABE}\rangle,
$$
(7)

where we observe that $\langle \mu_{ABE}|\mu_{ABE}\rangle = 0$, $\langle \nu_{ABE}|\nu_{ABE}\rangle = 0$.

Before the Anticorrelation Check (or the Bob’s operation) the evolution of the system can be completely described by Eve’s operation of coupling photon B with her ancilla system

$$
\hat{J}_{BE}|1^1_B\rangle \otimes |e_e\rangle = a|1^1_B\rangle \otimes |\alpha_e\rangle + \gamma |\Gamma_{BE}\rangle, 
\hat{J}_{BE}|0^1_B\rangle \otimes |e_e\rangle = \beta|0^1_B\rangle \otimes |\beta_e\rangle + |\Delta_{BE}\rangle,
$$
(8)

with $|1^1_B\rangle |\Gamma_{BE}\rangle = 0$ and $|0^1_B\rangle |\Delta_{BE}\rangle = 0$. $|\alpha_e\rangle$, $|\beta_e\rangle$, $|\Gamma_{BE}\rangle$, $|\Delta_{BE}\rangle$ are normalized to one and the operator $\hat{J}_{BE}$ is unitary thus $|\alpha|^2 + |\gamma|^2 = |\beta|^2 + |\delta|^2 = 1$. The states $|\Gamma_{BE}\rangle$ and $|\Delta_{BE}\rangle$ represent situations in which the Anticorrelation Check produces unexpected (“wrong”) results due to e.g. bit-flip, vacuum state or state with more than one photon in the channel B. This is the main difference with respect to the security proof proposed in [2] where Alice and Bob considered that only the bit-flip was the signature of Eve’s presence. In this respect the probabilities of anticorrelated results are $P_{01} = |\alpha|^2/2$ and $P_{10} = |\beta|^2/2$.

Thus, we can obtain the relation between Bob and P10, and the final states of the coupled system, by inserting Eqs. 9 in the left hand side of Eqs. 4. Observing that $\langle \mu_{ABE}|\nu_{ABE}\rangle = \langle \mu_{ABE}|\nu_{ABE}\rangle \equiv p$, $\langle \mu_{ABE}|\nu_{ABE}\rangle \equiv q$ we obtain

$$
p = \frac{c - d}{2} - P_{01}(1 + c) - P_{10}(1 - d),
$$
$$
q = \frac{c + d}{2} - P_{01}(1 + c) + P_{10}(1 - d),
$$
(9)

where $c = \langle\Gamma_{BE}|\hat{Z}_B|\Gamma_{BE}\rangle$ and $d = \langle\Delta_{BE}|\hat{Z}_B|\Delta_{BE}\rangle$ are two real parameters ($c$ and $d$ in $[-1,1]$) under Eve’s control that cannot be evaluated by Alice and Bob [3].

According to Eq. 9, we evaluate $I_{BE}$. From Eqs. 8 and Eqs. 7 we obtain $S[\mathcal{E}_{10}(\hat{\rho}_{AB})] = 1$, $S[\mathcal{E}_{10}(\hat{\rho}_{AB})] = 1$. The calculation of $S[\mathcal{E}(\hat{\rho}_{AB})]$ is not trivial. In order to obtain the diagonal representation of the state $\mathcal{E}(\hat{\rho}_{AB})$ we introduce the orthonormal base $S = \{|\mu_{ABE}\rangle, |\mu_{ABE}\rangle, |\xi_{ABE}^{(1)}\rangle, |\xi_{ABE}^{(2)}\rangle\}$. $S$ spans the generic subspace of the Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_E$ support of $\mathcal{E}(\hat{\rho}_{AB})$. According to Eqs. 8, the states $|\mu_{ABE}\rangle$ and $|\nu_{ABE}\rangle$ can be rewritten as

$$
|\mu_{ABE}\rangle = p|\mu_{ABE}\rangle + q|\nu_{ABE}\rangle + r|\xi_{ABE}^{(1)}\rangle + t|\xi_{ABE}^{(2)}\rangle,
$$
$$
|\nu_{ABE}\rangle = q|\mu_{ABE}\rangle + p|\nu_{ABE}\rangle + r|\xi_{ABE}^{(1)}\rangle,
$$

where $r$, $s$ and $t$ are complex, and $p$ and $q$, according with Eq. 9, are real. From the normalization and orthogonality conditions on $|\nu_{ABE}\rangle$ and $|\tilde{\nu}_{ABE}\rangle$ we obtain

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Plot of the maximum of $I_{BE}$ (a) and $I_{AE}$ (b) versus $P_{01}$ and $P_{10}$}
\end{figure}
the Von Neumann entropy of $\mathcal{E}(\hat{\rho}_{AB})$ as
\[
S[\mathcal{E}(\hat{\rho}_{AB})] = -\sum_{i=1}^{4} \lambda_i \log \lambda_i.
\] (10)
where $\lambda_1 = \frac{1}{4}(1-p-q)$, $\lambda_2 = \frac{1}{4}(1+p+q)$, $\lambda_3 = \frac{1}{4}(1-p+q)$ and $\lambda_4 = \frac{1}{4}(1+p-q)$. Thus,
\[
\mathcal{I}_{B:E} = -\sum_{i=1}^{4} \lambda_i \log \lambda_i - 1.
\] (11)

As Alice and Bob have only access to the results of the Anticorrelation Check, for any fixed pair of values $P_{01}$ and $P_{10}$ the maximum information achievable by Eve, $\max_{c,d} \{\mathcal{I}_{B:E}\}$, corresponds to the maximum value of $\mathcal{I}_{B:E}$ in the range of values allowed for $c$ and $d$. As shown in Fig. 1 (a) the behavior of $\max_{c,d} \{\mathcal{I}_{B:E}\}$ versus $P_{01}$ and $P_{10}$ can be analyzed by considering four regions. Specifically

- for $P_{01} \geq 0.25$ and $P_{10} \geq 0.25$
  $\max_{c,d} \{\mathcal{I}_{B:E}\} = \mathcal{I}_{B:E}(P_{01}, P_{10}, c = 1, d = -1)$
- for $P_{01} < 0.25$ and $P_{10} \geq 0.25$
  $\max_{c,d} \{\mathcal{I}_{B:E}\} = \mathcal{I}_{B:E}(P_{01}, P_{10}, c = \frac{2P_{01}}{1-2P_{10}}, d = -1)$
- for $P_{01} \geq 0.25$ and $P_{10} < 0.25$
  $\max_{c,d} \{\mathcal{I}_{B:E}\} = \mathcal{I}_{B:E}(P_{01}, P_{10}, c = 1, d = \frac{2P_{10}}{2P_{10}-1})$
- for $P_{01} < 0.25$ and $P_{10} < 0.25$
  $\max_{c,d} \{\mathcal{I}_{B:E}\} = 1$.

Thus, for $P_{01} \geq 0.25$ or $P_{10} \geq 0.25$ the maximum information achievable by Eve is upper-bounded, and for $P_{01} = P_{10} = 0.5$ (perfect anticorrelation) Eve can get no information at all.

According to Eq. (9), we evaluate $\mathcal{I}_{A:E}$. Observing that the eigenvalues of both $\mathcal{E}(\langle \psi_{AB}^- | \psi_{AB}^- \rangle)$ and $\mathcal{E}(\langle \psi_{AB}^+ | \psi_{AB}^+ \rangle)$ are $\lambda_1 = \frac{1}{2}(1-q)$ and $\lambda_2 = \frac{1}{2}(1+q)$, we obtain
\[
\mathcal{I}_{A:E} = \sum_{i=1}^{2} \lambda_i \log \lambda_i - \sum_{j=1}^{4} \lambda_j \log \lambda_j.
\] (12)

As previously, Alice and Bob have only access to $P_{01}$ and $P_{10}$, thus we evaluate $\max_{c,d} \{\mathcal{I}_{A:E}\}$, the maximum value of $\mathcal{I}_{A:E}$ in the range of $c$ and $d$ allowed values.

In Fig. 1 (b) $\max_{c,d} \{\mathcal{I}_{A:E}\}$ is plotted versus $P_{01}$ and $P_{10}$ where only two regions are identified.

- For $P_{01} + P_{10} \geq 0.5$
  $\max_{c,d} \{\mathcal{I}_{A:E}\} = \mathcal{I}_{A:E}(P_{01}, P_{10}, c = 1, d = -1)$
- for $P_{01} + P_{10} < 0.5$
  $\max_{c,d} \{\mathcal{I}_{A:E}\} = 1$.

Note that the maximum information achievable by Eve is upper-bounded only if $P_{01} + P_{10} \geq 0.5$, but also in this case Eve cannot gain any information for $P_{01} = P_{10} = 0.5$. It is straightforward to demonstrate that $\max_{c,d} \{\mathcal{I}_{A:B;E}\}(P_{01} = P_{10} = \mathcal{P}) \geq \max_{c,d} \{\mathcal{I}_{A:B;E}\}(P_{01} = P_{10} = \mathcal{P}) \geq \mathcal{H}(1-2\mathcal{P})$ for $0.25 < \mathcal{P} \leq 0.5$, while $\max_{c,d} \{\mathcal{I}_{A:B;E}\}(P_{01} = P_{10} = \mathcal{P}) \geq \mathcal{H}(1-2\mathcal{P})$ for $0 \leq \mathcal{P} < 0.25$ where $\mathcal{H}$ is the Shannon entropy of a binary channel [4].

In summary, the maximum information achievable by Eve is upper-bounded in some region, even when Eve can modify the number of photons in the quantum channel. Moreover these upper bounds have been demonstrated to be strictly related to the Anticorrelation Check outcomes. Thus measurement of $P_{01}$, $P_{10}$ would allow Alice and Bob to determine the security level of the communication. In addition we point out that, as in [2], Eve’s resources have been heavily overestimated in deriving Eqs. (11) and (12). In order to extract information on Alice and Bob operations, we assumed that Eve should perform any POVM on the final state of the whole space $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_E$ as clearly stated in Eqs. (11) and (12). This is obviously not the case. Even if Eve can perform any POVM on the final state of her ancilla system, about the Alice-Bob system, she can only know the results disclosed during the public discussion. This induces to think that information achievable by Eve should be in some cases well below these limits.

As already pointed out in [2], Alice’s and Bob’s are able to recover secure cryptographic keys in spite of Eve’s attack if the condition $I_{A:B} > I_{A:E}$ and $I_{A:B} > I_{B:E}$ is satisfied [4], where $I_{A:B}$ is the mutual information between Alice and Bob (Ref. [7] demonstrates that this condition is, in some cases, too restrictive). $I_{A:B}$ can be simply calculated considering the capacity of a noisy channel of quantum bit error rate $Q$, as $I_{A:B} = 1 - H(Q)$ [4]. To ensure the security of the two generated keys, we replace $I_{A:E}$ and $I_{B:E}$ with $\max_{c,d} \{\mathcal{I}_{A:E}\}$ and $\max_{c,d} \{\mathcal{I}_{B:E}\}$ calculated for $P_{01} = P_{10} = \mathcal{P}$. This means that Alice and Bob can distill common secret keys when $0.25 < \mathcal{P} \leq 0.5$ and
\[
H(Q) + H(1-2\mathcal{P}) < 1.
\] (13)

Despite the fact that Eq. (13) appears to be formally analogous to Eq. (6) in [2], we underline that in Eq. (13) the term $\mathcal{P}$ should be carefully evaluated from the experimental data as the ratio between the anticorrelated results and all possible results (anticorrelated and "wrong") of the Anticorrelation Check. The results of the Anticorrelation Check should be considered "wrong" not only in the case of correlated results, but also if more than two photons are detected in coincidence by the Alice and Bob apparatuses, or if only one of Alice’s detector fires. This last case makes the QDKD protocol not practical for today technology as all the transmission losses and detection inefficiencies would be considered due to eavesdropping attack.
Let us consider, as an example, the experimental realization of QDKD protocol performed in [2]. Losses due to the detection apparatuses (which is the main contribution) as well as to the source and to the encoder apparatuses strongly affect the estimation of the parameter $\mathcal{P}$. In fact, the probability of losing one photon of the pair by the generation-detection apparatuses was estimated to be $P_{\text{loss}} \simeq 0.77$, and we observed a probability of correlated results $P_{\text{corr}} < 0.05$ due to optics imperfections, misalignment and dark counts. The probability of measuring a three-fold or four-fold coincidences due to the presence of more than a photon pair in the quantum channels or to dark counts was completely negligible. Also losses in the quantum channels are completely negligible (propagation in air for less than one meter). Thus the anticorrelation parameter $\mathcal{P}$ is evaluated as $\mathcal{P} = (1 - P_{\text{loss}} - P_{\text{corr}})/2 \simeq 0.09$, and, according to the considerations related to Eq. (13), common secret keys cannot be distilled.

But if we assume that Eve cannot modify Alice and Bob detection apparatuses as she has not access to their keys cannot be distilled. To the considerations related to Eq. (13), common secret keys cannot be distilled.

In Refs. [8] and [9] two protocols are proposed for unidirectional secure direct communication (message from Bob to Alice) based on local operations on one photon of an EPR pair. We observe that the QDKD scheme can be used to implement bidirectional secure deterministic communication (either message from Alice to Bob or from Bob to Alice).

When a deterministic message is sent from Bob to Alice, Alice encodes a random sequence of bits and Bob encodes the deterministic message. As Alice is aware of her operations ($1_B$ or $\tilde{Z}_B$), she can extract the message-bit encoded by Bob from the measurement result $|\psi_A^+\rangle$ or $|\psi_A^-\rangle$. During the communication Bob discloses the results obtained from his Anticorrelation Check apparatus on a non-jammable public channel. According to these results as well as to her measurements, Alice estimates the security level of the communication. If communication is secure, Alice discloses the results of her measurements corresponding to the message encrypted by Bob’s key. Since Bob is aware of his key, he can extract the message encoded by Alice. Only if the security of the communication is ascertained the encrypted message is publicly disclosed by Alice, thus this protocol is not affected by the security drawback present in the communication from Bob to Alice and in protocols of Refs. [8] and [9].

Furthermore in both protocols of Refs. [8] and [9] the security proofs consider that Eve’s presence only induces bit-flips and they do not consider eavesdropping strategies based on injection-subtraction of photons in the quantum channel. In fact a successful eavesdropping attack against protocol in Ref. [8] based on subtraction of photons has already been proposed [10], while valid eavesdropping strategies have not yet been found against protocol of Ref. [9]. We observe that for both these protocols a security proof analogous to the one here proposed is necessary in order to guarantee the security against also the attacks based on injection-subtraction of photons.

In conclusion, this Reply proposes an improved security proof of the QDKD protocol which is able to detect any individual eavesdropping attack and to provide an upper bound to the information achievable by Eve, even in the case of attacks exploiting the possibility of injecting-subtracting photons in the quantum channel as the ones proposed in Wójcik’s Comment [1].

The work was supported by MIUR (Project 67679) and by Elsag S.p.A.

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