Mixed Convection Nanofluid Flow with Heat Source and Chemical Reaction over an Inclined Irregular Surface

Izharul Haq, Muhammad Bilal, N. Ameer Ahammad, Mohamed E. Ghoneim, Aatif Ali, and Wajaree Weera

ABSTRACT: Two-dimensional mixed convection radiative nanofluid flow along with the non-Darcy permeable medium across a wavy inclined surface are observed in the present analysis. The transformation of the plane surface from the wavy irregular surface is executed via coordinate alteration. The fluid flow has been evaluated under the outcomes of heat source, thermal radiation, and chemical reaction rate. The nonlinear system of partial differential equations is simplified into a class of dimensionless set of ordinary differential equations (ODEs) through a similarity framework, where the obtained set of ODEs are further determined by employing the computational technique parametric continuation method (PCM) via MATLAB software. The comparative assessment of the current outcomes with the earlier existing literature studies confirmed that the present findings are quite reliable, and the PCM technique is satisfactory. The effect of appropriate dimensionless flow constraints is studied versus energy, mass, and velocity profiles and listed in the form of tables and figures. It is perceived that the inclination angle and wavy surface assist to improve the flow velocity by lowering the concentration and temperature. The velocity profile enhances with the variation of the inclination angle of the wavy surface, non-Darcian term, and wavy surface term. Furthermore, the rising value of Brownian motion and thermophoresis effect diminishes the heat-transfer rate.

1. INTRODUCTION

Surfaces that are roughened or uneven transmit more thermal energy than level interfaces. In many technical and industrial operations, the exterior layer of the items is purposefully nonuniform or uneven to increase the heat conduction rate through it. The reason for this is that rough surfaces speed up the mixing of fluid particles, resulting in a higher heat-transfer rate.\(^1\) Thermal transfer is a field of heat engineering that entails the assembly, use, transformation, and exchange of heat energy in various contexts. Heat transmission is divided into many types, and energy is transferred through stage transitions.\(^2,5\) The heat transfer qualities of fuel cell technology, nuclear plants, microsystems, and transportation are all important in a variety of technical and commercial operations.\(^3\) The analysis of fluid flow induced by a continually turning expanding or contracting sheet is critical in practice. These sorts of streams can be found in a wide range of professional and scientific operations, including electrospinning, film sketching, fiber manufacture, paper production, crystal development, and the condensation process for the production of liquid films.\(^5,6\)

Zhao et al.\(^7\) experimented on the influence of varied configurations of wavy and striped fins in a multiple plate-fin compressed heat exchanger on precipitation heat conduction in the heat exchanger. Bilal et al.\(^8\) examined how a wavy oscillating whirling disc transfers energy through the nanofluid flow. In comparison to a uniform interface, the irregular rotational surface enhances energy transmission by up to 15%. Singh and Dewan\(^9\) examined the energy transmission through wavy wall-bounded jets. The heat-transfer augmentation is affected by the offset ratio’s placement and the intensity of the wavy material. Alsabery et al.\(^10\) used a two-phase water base fluid in a 3D wavy permeable medium with top and bottom wavy edges induced with a constant temperature and isothermal circumstances to investigate the Brownian diffusion as convective energy transference enhancement methods. The findings revealed that the nanofluid flow in the wavy channel improved heat transmission, particularly when the Reynolds number and the nanoparticle volume fraction increased. Some recent studies related to the heat transfer through wavy surfaces have been heavily reported by refs 11–14.

A nanofluid is a fluid that contains nanomaterials, which are nanometer size objects. These fluids have heterogeneous nanoparticulate concentrations in a base fluid.\(^15\) Metals, iron oxide, carbides, and carbon nanofibers are commonly employed as aggregates in nanofluids. Water, propylene, and...
petroleum are all frequent base fluids.\textsuperscript{16--18} Nanofluids have exclusive geographies that could make them ideal in a diversity of thermal governance domains, such as optoelectronics, fuel cells, generic drug operations, hypervisor motors, engine refrigeration, domestic refrigerator, cooler, exchangers, shredding, milling, and burner flue gas thermal minimization.\textsuperscript{19,20} When compared to the base fluid, they have higher thermal conductance and lower thermal diffusivity. It has been discovered that understanding the viscoelastic performance of nanofluids is crucial in determining their feasibility for convective energy transmission purposes.\textsuperscript{21} Bhatti et al.\textsuperscript{22} noted the fluid flow across a permeable substrate saturated with Williamson nanofluid by simultaneously moving round sheets. The upshot of Prandtl number leads to the enhancement of thermal contour; similar phenomena have been observed with greater Reynolds number. Varun Kumar et al.\textsuperscript{23} used the RK-4 strategy to quantitatively assess the outcome of a magnetic flux on the nanoliquid flow. Alharbi et al.\textsuperscript{24} described the flow of a highly conductive trihybrid nanoliquid with external magnetic impacts. It has been discovered that varying the trhybrid NP's increases the thermophysical properties of the base fluid substantially. Ikram\textsuperscript{25} devised a theoretical model for magnetized nanocrystal Darcy–Forchheimer flow through an extending disc with negligible mass flux. The temperature of the nanocrystals is improved through exothermic reactions. Waqas et al.\textsuperscript{26} documented the magneto-hydrodynamics (MHD) characteristics of nanoliquid flows across a mixed convection stretched sheet. It was discovered that for increasing scales of the buoyancy ratio constraint, the microbe's concentration profile and nanoparticle concentration enhanced. Shahid et al.\textsuperscript{27} reported the nanoliquid flow over a parabolic reflector perforated surface using MHD. The flow structure and thermal alterations of an intermittent radiation nanoliquid flow caused by the spinning disc were presented by Acharya et al.\textsuperscript{28} The energy communication increases with the effect of thermal radiation and nanolayer ratio. It was stated that the heat allocation for nanolayers improved by about 84.61 percent. Many scientists have reported substantial research on the nanofluid flow.\textsuperscript{29--32}

The thermal radiation effect is crucial for a multitude of applications, such as heat dissipation, electron microscopy, photodetectors, and energy gadgets.\textsuperscript{33} Kumar et al.\textsuperscript{34} devised a computational model for nanoliquid flow and heat transmission over an infinite diagonal plate. It has been realized that growing the value of the radiation factor enriches the thermal energy and velocity profiles. Jin et al.\textsuperscript{35} addressed the heat conduction effect of Al\textsubscript{2}O\textsubscript{3} nanoliquid flow with mixed convection and thermal radiation through a porous stretched surface. The ethylene glycol—Al\textsubscript{2}O\textsubscript{3} nanofluid is shown to be more important in the freezing phase than the water-based ferrofluid. Raza et al.\textsuperscript{36} inspected the 2D MHD stream flow across an absorbent substance with the suction/injection and heat radiation upshots. With clay nanomaterials and thermal radiation, Alrabaiah et al.\textsuperscript{37} investigated the time-fractional free laminar flow of stagnation point draining nanofluid. A quantifiable increase of 11.83% in the energy conversion of boring nanofluid versus radiation effect has been described. Bilal et al.\textsuperscript{38} designated the effect of radiation on the stretchable sheet with momentum and mass transformation. Many researchers have recently studied the thermal radiation effect on fluid flow.\textsuperscript{39--41}

The goal of this study is to present a numerical simulation for the mixed convection 2D non-Darcy model with the heat generation, thermal radiation, and chemical reaction impact on a nanofluid over a slanted wavy surface. The consequences of Brownian, buoyance ratio, and thermophoretic diffusion are also considered. According to the author's knowledge, no previous studies have been documented on this topic. The flow stream has been described in terms of partial differential equations (PDEs), which are reduced to a set of ordinary differential equations (ODEs) through a similarity framework. To propose a numerical solution, a computational approach parametric continuation method (PCM) is used. The influence of the physical variables on the dimensionless velocity, energy, and mass contours is depicted graphically.

2. MATHEMATICAL FORMULATION

We supposed steady 2D mixed convection radiative nanoliquid flow, along with the non-Darcy permeable medium across a wavy inclined surface. The schematic sketch of fluid flow over an inclined surface is portrayed in Figure 1. The following assumptions have been considered.
irregular surface, respectively. The modeled equations are formulated as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(2)

\[
\left( 1 + \frac{K}{\nu \sqrt{u'^2 + v'^2}} \right) \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \frac{K}{\nu \sqrt{u'^2 + v'^2}} \left( u'^2 \frac{\partial u}{\partial y} + v' \frac{\partial v}{\partial y} - v'^2 \frac{\partial u}{\partial x} \right) - v'^2 \frac{\partial v}{\partial x} = \left( 1 - \phi_0 K \right) \frac{\rho K g}{\mu} \left( \frac{\partial T}{\partial x} \sin A - \frac{\partial T}{\partial y} \cos A \right) - \left( \frac{\partial \phi}{\partial x} \sin A - \frac{\partial \phi}{\partial y} \cos A \right)
\]  
(3)

\[
\left( \frac{\partial T}{\partial y} \sin A - \frac{\partial T}{\partial x} \cos A \right) - \left( \frac{\partial \phi}{\partial x} \sin A - \frac{\partial \phi}{\partial y} \cos A \right)
\]  
(4)

\[
\left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \right) + \frac{\partial T}{\partial x} \frac{\partial T}{\partial y} = D_T \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \gamma D_T \left( \frac{\partial T}{\partial x} \frac{\partial T}{\partial y} \right) + D_T \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  
(5)

\[
\left( \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) = D_\psi \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right)
\]  
(6)

\[
\begin{align*}
\psi &= \frac{\psi}{U_\infty} = \left( \frac{\partial \psi}{\partial y} \sin A - \frac{\partial \psi}{\partial x} \cos A \right) &= \frac{\partial \psi}{\partial y} \sin A - \frac{\partial \psi}{\partial x} \cos A = 1/2, \\
\phi &= \frac{\phi}{\phi_\infty} = \frac{\partial \psi}{\partial y} \sin A - \frac{\partial \psi}{\partial x} \cos A = \frac{s - s_\infty}{s_\infty - s_\infty}
\end{align*}
\]  
(7)

As a result of eq 7, the dimensionless system of equations is obtained as

\[
(1 + Fc \psi) \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + Fc \frac{\partial^2 \psi}{\partial y^2} \left( \frac{\partial \psi}{\partial y} \right)^2 + 2 \frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial x^2} \left( \frac{\partial \psi}{\partial y} \right)^2 = \Delta
\]  
(8)

\[
\left( \frac{\partial \psi}{\partial y} \sin A - \frac{\partial \psi}{\partial x} \cos A \right) - \left( \frac{\partial \phi}{\partial x} \sin A - \frac{\partial \phi}{\partial y} \cos A \right)
\]  
(9)

\[
\frac{\partial \phi}{\partial x} \frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y} \frac{\partial \psi}{\partial y} = \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + Nt \left( \frac{\partial \psi}{\partial x} \sin A - \frac{\partial \psi}{\partial y} \cos A \right) + Kr \phi
\]  
(10)

where \( Ra \) is the Rayleigh number, \( Q \) is the dimensionless velocity, \( Pe \) is the Peclet number, \( Fc \) is the non-Darcian term, \( Nt \) is the thermophoresis effect, \( Nb \) is the Brownian diffusivity, \( Le \) is the Lewis number, \( R \) is the radiative factor, and \( \Delta \) is the mixed convective term defined as

\[
Ra = \frac{(1 - \phi_0 K \beta Kg(T_w - T_\infty))}{\mu \alpha}, \quad Q
\]

\[
= \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right)^{1/2} \quad Pe = \frac{U_\infty}{\alpha}, \quad Fc = \frac{K \psi}{\nu}
\]  
(11)

Eq 11 is employed to convert the wave surface impact to controlling formulas from the boundary restrictions

\[
\xi = \xi, \quad \eta = \frac{(\gamma - \delta) Pe^{1/2}}{\xi^{1/2}} \quad \psi = \sqrt{Pe \xi f(\xi, \eta)}
\]  
(12)

Replacing eq 9 into eq 8 by making \( Ra \to \infty \), we get
The local heat and nanomaterial mass transfer from the wavy surface

\[ q = kn \nabla T + n \cdot q_s, \quad q_{np} = -D_k n \nabla \phi \]  

(16)

Here, \( n = \left( -\frac{\delta}{\sqrt{1 + \delta^2}}, \frac{1}{\sqrt{1 + \delta^2}} \right) \) is a unit perpendicular vector

and \( qr = \left( -\frac{4\pi}{3k} \right) \nabla T \) is a thermal radiation heat flux.

The dimensionless Nusselt and Sherwood numbers are

\[ \frac{Nu}{\sqrt{Pe}} = \left( 1 + \frac{4R}{3} \right) \sqrt{\frac{1}{1 + \delta^2} \left( \frac{\partial \phi}{\partial \eta} \right)_{\eta=0}}, \quad \frac{Sh_k}{\sqrt{Pe}} \]  

(17)

3. NUMERICAL SOLUTION

The following are the basic steps while applying the PCM approach, and its future direction is as follows:

- **Step 1:** Reduce eqs 12−15 to first order

  \[ \delta_1 = f(\eta), \quad \delta_2 = f'(\eta), \quad \delta_3 = \theta(\eta), \quad \delta_4 = \theta'(\eta), \quad \delta_5 \]

  \[ = \phi(\eta), \quad \delta_6 = \phi'(\eta) \]  

(18)

By putting eq 18 in eqs 12−15, we get

| Step 2: Introducing parameter \( p \) |

\[ \delta_4' + 2\left( 1 + \delta^2 \right)^{-1/2} \delta_7 \delta_7' \]

\[ = \Delta(\sin A + \delta \cos A)(\delta_4 - N_r \delta_6) \]  

(19)

\[ \left( 1 + \frac{4R}{3} \right) \delta_4' + \frac{1}{2} \delta_8 \delta_8 + N_t N_b \delta_7 + N_t \delta_8 + N_t \delta_9 + N_t \delta_9' + N_t \delta_9'' + N_t \delta_9''' = 0 \]  

(20)

| The transform conditions are |

\[ \delta_4' = 0, \quad \delta_5' = 0, \quad \delta_6' = 0 \] \( \text{as } \eta \to 0 \) \]

\[ \delta_7 = \delta_7(\eta), \quad \delta_4 = \delta_4(\eta), \quad \delta_5 = \delta_5(\eta) \] \( \text{as } \eta \to 0 \) \]

(22)

Step 2: Introducing parameter \( p \)

\[ \delta_4' + 2\left( 1 + \delta^2 \right)^{-1/2} (\delta_8 - 1)p \delta_7' \]

\[ = \Delta(\sin A + \delta \cos A)(\delta_4 - N_r \delta_6) \]  

(23)

\[ \left( 1 + \frac{4R}{3} \right) \delta_4' + \frac{1}{2} \delta_8 \delta_8 + N_t N_b \delta_7 + N_t \delta_8 + N_t \delta_9 + N_t \delta_9' + N_t \delta_9'' + N_t \delta_9''' = 0 \]  

(24)
4. RESULT AND DISCUSSION

This section describes the physical process behind each figure and table. The following observations have been noted:

4.1. Velocity profile \( f'(\eta) \). Figure 2a–c explains the velocity outlines versus the inclination angle of a wavy surface \( A \), non-Darcian term \( F_c \), and the wavy surface term \( \delta \), respectively. Throughout the analysis, \( \Delta = -3 \) is used for opposing flow and \( \Delta = 5 \) for adding the flow, while the arrow represents the \((↓)\) reducing and \((↑)\) elevating behaviors of velocity, energy, and concentration profiles. Figure 2a displays that the velocity field intensifies with the difference of the inclination angle of wavy surface \( A \). Figure 2b elaborates that the effect of the non-Darcian term \( F_c \) boosts the fluid velocity within the case of \( \Delta = 5 \). Physically, the non-Darcian inertial effects improve, while the kinetic viscosity of fluid diminishes with the augmentation of \( F_c \), which results in the diminution of the momentum boundary layer. Similarly, Figure 2c demonstrates that the velocity outline increases with the variation of wavy surface term \( \delta \). Physically, due to the zigzag motion of the fluid particles, the kinetic energy of the fluid increases, which results in the advancement of the velocity field \( \Delta = 5 \), while an inverse behavior has been noticed \( \Delta = -3 \).

4.2. Temperature Profile \( \theta(\eta) \). Figure 3a–e exemplifies the energy distribution \( \theta(\eta) \) outlines versus the inclination angle of a wavy surface \( A \), non-Darcian term \( F_c \), heat source term \( hs \), radiation term \( R \), and wavy surface term \( \delta \), respectively. Figure 3a,b shows that the energy contour declines with the upshot of an inclination angle of a wavy surface \( A \), while it enhances with the non-Darcian term \( F_c \) in case of \( \Delta = 5 \). The energy field shows an opposite trend against both parameters in the case of \( \Delta = -3 \). Figure 3c,d reports that the energy profile magnifies with the effects of heat source \( hs \) and radiation term \( R \). The consequences of both
Figure 4. Outlines of mass distribution $\phi(\eta)$ vs the (a) inclination angle of a wavy surface $\alpha$, (b) non-Darcian term $F_c$, (c) chemical reaction term $K_r$, and (d) wavy surface term $\delta$.

4.3. Concentration Profile $\phi(\eta)$. Figure 4a–d describes the outlines of mass distribution $\phi(\eta)$ versus the inclination angle of a wavy surface $\alpha$, non-Darcian term $F_c$, chemical reaction term $K_r$, and wavy surface term $\delta$, respectively. Figure 4a,b reports that the mass transport rate lessens with the effect of inclination angle, while it boosts with the impact of the non-Darcian term. Physically, the fluid particles need more energy to move over an inclined surface, that is, the increment in the angle of wavy surface retards the mass-transfer rate as elaborated in Figure 4a. Figure 4c explains that the influence of chemical reaction constraints boosts the mass propagation rate because its effect improves the kinetic energy of fluid particles, which promotes the mass transition rate as shown in Figure 4c. Figure 4d discloses that the mass profile shrinks with the variation of the wavy surface term. Physically, for the fluid particles, it is difficult to move over an irregular surface; that is why the variation of $\delta$ declines the mass transmission. Figure 5 reveals the flow chart of the solution methodology.

Table 1 reveals the relative assessment of the present results with the available work to ensure the validity of the proposed method. It can be perceived that the present results are correct and reliable. Table 2 illustrates the numerical outcomes of the Nusselt number $(\theta(\eta))$ toward $(\Delta = 5)$, $-\left(1 + \frac{4K_r}{3}\right)\frac{1}{\sqrt{1 + \delta}}\theta(0)$ and opposing $(\Delta = -3)$, $-\left(1 + \frac{4K_r}{3}\right)\frac{1}{\sqrt{1 + \delta}}\theta(0)$ the flow field. It can be observed that the rising value of the Brownian motion thermophoresis effect lessens the heat-transport rate. Table 3 elaborates the numerical outcomes of the Sherwood number $(\theta(\eta))$ toward $(\Delta = 5)$, $-\left(1 + \frac{4K_r}{3}\right)\frac{1}{\sqrt{1 + \delta}}\phi(0)$ and opposing $(\Delta = -3)$, $-\left(1 + \frac{4K_r}{3}\right)\frac{1}{\sqrt{1 + \delta}}\phi(0)$ the flow field.

5. CONCLUSIONS

The aim of this work is to study the two-dimensional mixed convection radiative nanofluid flow, along with the non-Darcy permeable medium across a wavy inclined surface.
The relative evaluation of the current outcomes with employing the computational technique PCM via MATLAB where the obtained set of ODEs is further determined by dimensionless sets of ODEs through a similarity framework, nonlinear system of PDEs is simplified into a class of dimensionless sets of ODEs through a similarity framework, where the obtained set of ODEs is further determined by employing the computational technique PCM via MATLAB software. The relative evaluation of the current outcomes with the earlier existing literature studies confirmed that the present findings are quite reliable and the PCM technique is satisfactory. The core findings from the above examination have been offered below:

- The velocity profile enhances with the variation of the inclination angle of the wavy surface $A$, non-Darcian term $F_v$, and wavy surface term $d$.
- The energy profile declines with the upshot of the inclination angle of a wavy surface $A$, while it enhances with the effect of the wavy surface term $d$, non-Darcian term $F_v$, heat source $h_s$, and thermal radiation parameter in the case of $(\Delta = 5)$.
- The mass-transfer rate reduces with the effect of the wavy surface term and the inclination angle of the wavy surface, while it boosts with the impact of the non-Darcian term and chemical reaction.
- The rising value of $Nt$ and $Nb$ effect diminishes the heat-transfer rate.
- The inclination angle and wavy surface assist to improve flow by lowering the concentration and temperature.
- The present mathematical may be extended to other types of fluids. It may be modified with Das and Tiwari’s model and can be solved using other numerical techniques, analytical methods, and fractional derivatives.
- The present mathematical model may be extended to other types of fluids (i.e., non-Newtonian) by considering different sorts of physical effects over different geometries and may be solved through fractional and analytical techniques.

### Table 1. Statistical Evaluation of the Present Results with the Published Studies for Nusselt Number $-\theta'(0)$

| $\Delta$ | ref 50 | ref 51 | present work | ref 50 | ref 51 | present work |
|---------|--------|--------|--------------|--------|--------|--------------|
| 0.0     | 0.5641 | 0.5641 | 0.56415974   | -0.2   | 0.5269 | 0.526911    |
| 0.5     | 0.6473 | 0.6473 | 0.64737520   | -0.4   | 0.4865 | 0.486533    |
| 1.0     | 0.7205 | 0.7205 | 0.72055503   | -0.6   | 0.4420 | 0.442021    |
| 3.0     | 0.9574 | 0.9574 | 0.95744610   | -0.8   | 0.3916 | 0.391663    |
| 10      | 1.5160 | 1.5160 | 1.51623864   | -1.0   | 0.3320 | 0.332021    |
| 20      | 2.0660 | 2.0660 | 2.06701200   | 0.0    |        |              |

### Table 2. Numerical Outcomes of Nusselt Number $(\theta(\eta))$ toward $(\Delta = 5)$, $-\left(1 + \frac{4R}{3}\right)\frac{1}{\sqrt{1 + \delta'}}\theta'(0)$ and Opposing $(\Delta = -3)$, $-\left(1 + \frac{4R}{3}\right)\frac{1}{\sqrt{1 + \delta'}}\theta'(0)$ to the Flow Field

| $R$ | $Nb$ | $Nt$ | $(\Delta = 5)$, $-\left(1 + \frac{4R}{3}\right)\frac{1}{\sqrt{1 + \delta'}}\theta'(0)$ | $(\Delta = -3)$, $-\left(1 + \frac{4R}{3}\right)\frac{1}{\sqrt{1 + \delta'}}\theta'(0)$ |
|-----|------|------|------------------------------------------------|------------------------------------------------|
| 0.3 | 0.1  | 0.2  | 0.5653657                                          | 0.447610                                           |
| 0.5 | 0.3  | 0.4  | 0.6098267                                          | 0.482763                                           |
| 0.7 | 0.5  | 0.6  | 0.6519716                                          | 0.514105                                           |
| 3.0 | 0.1  | 0.2  | 0.6838033                                          | 0.538975                                           |
| 10  | 0.3  | 0.4  | 0.6347640                                          | 0.497362                                           |
| 20  | 0.5  | 0.6  | 0.6182457                                          | 0.487163                                           |

### Table 3. Numerical Outcomes of Sherwood Number $\phi(\eta)$ toward $(\Delta = 5)$, $-\left(1 + \frac{4R}{3}\right)\frac{1}{\sqrt{1 + \delta'}}\phi'(0)$ and Opposing $(\Delta = -3)$, $-\left(1 + \frac{4R}{3}\right)\frac{1}{\sqrt{1 + \delta'}}\phi'(0)$ to the Flow Field

| $Nb$ | $Nt$ | $Le$  | $(\Delta = 5)$, $-\frac{1}{\sqrt{1 + \delta'}}\phi'(0)$ | $(\Delta = -3)$, $-\frac{1}{\sqrt{1 + \delta'}}\phi'(0)$ |
|------|------|------|------------------------------------------------|------------------------------------------------|
| 0.1  | 0.1  | 2.0  | 0.5653687                                          | 0.448712                                           |
| 0.3  | 0.3  | 4.0  | 0.6098267                                          | 0.481864                                           |
| 0.5  | 0.5  | 6.0  | 0.6519716                                          | 0.513206                                           |
| 0.1  | 0.3  | 4.0  | 0.6838034                                          | 0.537872                                           |
| 0.5  | 0.5  | 6.0  | 0.6347631                                          | 0.499463                                           |
| 0.1  | 0.1  | 2.0  | 0.6182457                                          | 0.486262                                           |

### AUTHOR INFORMATION

**Corresponding Author**

Wajaree Weera — Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand; orcid.org/0000-0001-9595-6096; Email: wajawe@kku.ac.th
N. Ameer Ahmammad — Department of Mathematics, Faculty of Science, University of Tabuk, Tabuk 71491, Saudi Arabia
Mohamed E. Ghoneim — Department of Mathematical Sciences, Faculty of Applied Science, Umm Al Qura University, Makkah 21955, Saudi Arabia; Faculty of Computers and Artificial Intelligence Damietta University, Damietta 34511, Egypt
Aatif Ali — Department of Mathematics, Abdul Wali Khan University Mardan, Khyber Pakhtunkhwa 23200, Pakistan; Department of Mathematical Sciences, Faculty of Education at Prince Mohammad Bin Fahd University, Dhahran 34754, Saudi Arabia

Complete contact information is available at: https://pubs.acs.org/10.1021/acsomega.2c03919

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NOMENCLATURE

\( \omega, \nu \) velocity components
\( Q \), dimensionless velocity
\( K \), surface permeability
\( \beta \), volumetric thermal expansion
\( \phi \), dimensionless concentration
\( K_m \), mean absorption
\( \mu \), density Kgm\(^{-1}\)
\( \sigma \), Stefan–Boltzmann coefficient
\( (\rho c_p)_p \), nanoparticles heat capacity
\( T_{sw} \), surface temperature
\( \eta \), unit vector
\( L_e \), Lewis number
\( \Delta \), mixed convective term
\( D_b \), Brownian motion
\( \dot{F}_D \), dimensionless non-Darcian term
\( S_h \), Sherwood number
\( R \), radiative factor
\( x, y \), coordinate
\( T_w \), temperature at wall
\( K \), non-Darcian inertial effects
\( D_{th} \), thermophoretic diffusivity
\( C_p \), specific heat KJg\(^{-1}\)
\( q_0 \), heat flux
\( \alpha \), inclined angle
\( \mu_s \), dynamic viscosity Kg m\(^{-1}\) s\(^{-1}\)
\( T_\infty \), temperature at free stream [K]
(\( \theta \), dimensionless temperature
\( Ra \), Rayleigh number
\( Pe \), Peclet number
\( Nb \), Brownian diffusivity
\( Nt \), Buoyancy ratio
(\( \rho C_p \)_p), specific heat capacity
\( Nu \), Nusselt number

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