Reconciling the Predictions of Microlensing Analysis with Radial Velocity Measurements for OGLE-2011-BLG-0417

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Abstract

Microlensing is able to reveal multiple body systems located several kilo-parsec away from the Earth. Since it does not require the measurement of light from the lens, microlensing is sensitive to a range of objects, from free-floating planets to stellar black holes. But, if the lens emits enough light, the microlensing model predictions can be tested with high-resolution imaging and/or radial velocity methods. Such a follow-up was done for the microlensing event OGLE-2011-BLG-0417, which was expected to be a close by (≤1 kpc), low-mass (~0.8 M⊙) binary star with a period of P ~ 1.4 year. The spectroscopic follow-up observations conducted with the Very Large Telescope did not measure any variation in the radial velocity, which is in strong contradiction with the published microlensing model. In the present work, we model this event and find a simpler model that is in agreement with all the available measurements, including the recent Gaia Data Release 2 parallax constraints. We also present a new way to distinguish degenerate models using the Gaia Data Release 2 proper motions. This work stresses the importance of thorough microlensing modeling, especially with the horizon of the Wide Field Infrared Survey Telescope and the Euclid microlensing space missions.

Key words: binaries: general – gravitational lensing: micro – techniques: radial velocities

1. Introduction

The gravitational microlensing technique (Paczyński 1986) is a unique tool to explore stellar and planetary companions over a very wide range of masses and orbits along the line of sight toward the Galactic Center (Gould & Loeb 1992; Beaulieu et al. 2006; Cassan et al. 2012; Clanton & Gaudi 2016; Penny et al. 2016; Suzuki et al. 2016). Usually, accurate mass ratios and projected separations in units of the Einstein ring radius are extracted by modeling the microlensing light curve. One mass–distance relation for the lens is generally obtained in the case of binary lens events from the measurements of finite-source effects. Another lens mass–distance relation can be derived from the parallax effects along with the lens light detection and/or a Bayesian analysis with a galactic model (see Beaulieu et al. 2018 and references therein). This is then sufficient to estimate the physical parameters (mass, distance, and projected orbital separation) of the lens system. In rare cases where the lens is bright enough to be detected in good contrast with the source, it is possible to perform radial velocity follow-up observations to test and refine the microlensing predictions (Yee et al. 2016).

OGLE-2011-BLG-0417, originally presented by Shin et al. (2012, hereafter S12), is a microlensing event with the strong features of a binary lens. The authors conducted a deep analysis and found a full Keplerian solution for the lens system. The lens was expected to be close by (i.e., Dl ~ 0.9 kpc) and to be composed of low-mass binary stars (i.e., Mtot ~ 0.7 M⊙). Moreover, assuming partial extinction, S12 suggested that “the blended light...comes very likely from the lens itself, implying that the lens system can be directly observed.”

Based on the S12 solution, Gould et al. (2013, hereafter G13) did not repeat the modeling but derived radial velocity properties of the lens system with K = 6.31 ± 0.34 km s⁻¹ and P = 1.423 ± 0.113 year. Therefore, G13 concluded that the radial velocity signal “should be precisely measurable despite the fact that the lens primary is relatively faint (I = 16.3 mag, V = 18.2 mag).”

Following the prediction of G13, Boisse et al. (2015, hereafter B15) observed the target with the Ultraviolet and Visual Echelle Spectrograph (Dekker et al. 2000) mounted on the Very Large Telescope. From the high-precision radial velocity measurements (i.e., RMS = 94 m s⁻¹) that are based on the 10 spectra taken over a year, they found that there was no measurable modulation in the radial velocity. This result contradicted the predictions G13 and B15 concluded that the microlensing scenario of G13 was highly unlikely, given their new data (with a probability of P ~ 10⁻⁵). This raised doubts about the microlensing predictions.

In addition to the B15 investigations, Santerne et al. (2016, hereafter S16) obtained supplementary data on this target. The authors secured a near-infrared spectrum with the Astronomy Research using the Cornell InfraRed Imaging Spectrograph (ARCoIRIS) at the Cerro Tololo Inter-American Observatory (CTIO; Schlawin et al. 2014). They also obtained near-infrared high-resolution images using the Near Infra Red Camera 2 (NIRIC2) adaptive optics instrument at the Keck II telescope. Their results indicate that the source is a giant star in the Galactic Bulge and that the lens is located at a distance of Dl ~ 1 kpc. They confirm previous results from B15 and conclude that the microlensing model is probably wrong.

In this work, we reanalyze the microlensing event OGLE-2011-BLG-0417. We present the observations in Section 2. In Section 3, we first study in details the results of S12 and find clues that corroborate the conclusions of B15 and S16. We then present our modeling process and the new results. We finally conclude in Section 5.
2. Microlensing Observations

The microlensing event OGLE-2011-BLG-0417 ($\alpha = 17^{\mathrm{h}}34^{m}33^{s}12$, $\delta = -27^\circ06^\prime39^\prime3$ (J2000)) was alerted by the Optical Gravitational Lens Experiment (OGLE; Udalski et al. 2015b). Because it is a bright event (i.e., $I \lesssim 15.7$), several teams conducted follow-up observations based on the OGLE alert and obtained data during the magnification peak. In this work, we use the same data sets described in S12, which we summarize in the Table 1. OGLE, PLANET, MiNDStEp, and RoboNet data were obtained by the difference image analysis (DIA) technique. RoboNet used DanDIA (Bramich 2008; Bramich et al. 2013), OGLE used their own implementation of DIA (Udalski et al. 2015b), while PLANET and MiNDStEp used pySIS (Albrow et al. 2009). A special pipeline based on DoPHOT (Schechter et al. 1993) was used to process $\mu$FUN data.

It is common practice in microlensing to rescale the photometric uncertainties (using in mag unit)

$$\sigma' = k \sqrt{\sigma^2 + \epsilon_{\text{min}}^2},$$

where $\sigma'$ is the rescaled uncertainties, $\sigma$ is the original uncertainties, $k$ is a parameter to adjust low-magnification uncertainties, and $\epsilon_{\text{min}}$ is used to adjust uncertainties at high magnification. To compare this work with S12, we use the same rescaling parameters as S12, which we report in Table 1.

This event was also in the footprint of the Visible and Infrared Survey Telescope for Astronomy (VISTA) Variables in the Via Lactea (VVV) survey (Minniti et al. 2010). This data set was not used by S12. The $K$-band data obtained by the VVV survey were re-reduced using pySIS. We then align the pySIS reduction to an independent VVV catalog (Beaulieu et al. 2016) by adding a 0.7 mag offset to the pySIS light curve. However, the target is so bright in the near-infrared that it is close to saturation during the highest amplification. The photometry quality during the central caustic approach is, therefore, low. Moreover, the number of data points near the peak of the lensing event is small ($\lesssim 10$), and the data do not cover the caustic crossing. For these reasons and in order to make a like-for-like comparison with S12, we opted not to include the VVV data in our modeling process and instead used it afterward to obtain the source brightness in the $K$ band.

3. Revisiting the Microlensing Modeling

3.1. Review of Previous Results

As described in S12, the static binary microlensing magnitude is computed with seven parameter $t_0$ is the time of the minimum impact parameter $u_0$ (relative to origin of the system, the center of mass in this work), $t_E$ is the angular Einstein radius crossing time, $\rho$ is the normalized angular source radius, $s$ is the normalized projected separation, $q$ is the mass ratio between the two bodies lens component, and $\alpha$ is the lens/source trajectory angle relative to the binary axes. Finally, the flux measurements from each telescope are used to derive two extra linear parameters: the source flux, $f_\nu$, and the blend flux, $f_\nu$. For a more complete description of the problem, readers can refer to Mao (2012) and Gaudi (2012).

S12 modeled this event and found that the anomalies were due to source star crossing over a caustic close to the primary. This geometry might be sensitive to the close/wide degeneracy (Dominik 1999; Bozza 2000), where the central caustic shape is similar if the companion is inside (close model) or beyond (wide model) the Einstein ring. However, S12 only reported a close model ($s \lesssim 1$) in their analysis. S12 used the grid search method to find this solution, whereby they explored the $(\log(s), \log(\rho))$ parameter space on a grid, with all other parameters optimized with a downhill approach, generally using a Monte Carlo Markov Chain (MCMC) algorithm (Dong et al. 2006). Unfortunately, so far there is no method available to determine whether all possible minima have been discovered and hence no way to ensure that the optimum solutions has been identified (in a finite time). The grid search generally performs well but is sensitive to the grid sampling, and we are aware that it failed in at least two occasions. For OGLE-2008-BLG-513, Yee et al. (2011) found a model where the lens is

| Table 1: Summary of Observations |
|----------------------------------|
| Name            | Collaboration | Aperture(m) | Filter | Code       | $N_{\text{data}}$ | $k$   | $\epsilon_{\text{min}}$ |
|-----------------|---------------|------------|--------|------------|-----------------|------|------------------------|
| OGLE_I          | OGLE          | 1.3        | $I$    | Wozniak    | 1481            | 2.740| 0.0005                 |
| OGLE_V          | OGLE          | 1.3        | $I$    | Wozniak    | 32              | 1.300| 0.0005                 |
| CTOI_V          | $\mu$FUN     | 1.3        | $I$    | DoPHOT     | 18              | 1.440| 0.0005                 |
| CTOI_V          | $\mu$FUN     | 1.3        | $V$    | DoPHOT     | 5               | 0.597| 0.0005                 |
| Auckland_R      | $\mu$FUN     | 0.4        | Red    | DoPHOT     | 9               | 0.635| 0.0005                 |
| FCO_U           | $\mu$FUN     | 0.4        | $U$    | DoPHOT     | 72              | 2.470| 0.0005                 |
| Kumeu_R         | $\mu$FUN     | 0.4        | Red    | DoPHOT     | 15              | 0.773| 0.0005                 |
| ODP_1           | $\mu$FUN     | 0.6        | $I$    | DoPHOT     | 520             | 2.325| 0.0005                 |
| Canopus_I       | PLANET        | 1.0        | $I$    | pySIS      | 122             | 1.320| 0.0005                 |
| SAAO_I          | PLANET        | 1.0        | $I$    | pySIS      | 29              | 7.035| 0.0005                 |
| Danish_1        | MiNDStEp     | 1.5        | $I$    | pySIS      | 118             | 3.350| 0.0005                 |
| FTN_i           | RoboNet       | 2.0        | SDSS-$i'$ | DANDIA   | 140             | 3.860| 0.0005                 |
| LT_i            | RoboNet       | 2.0        | SDSS-$i'$ | DANDIA   | 73              | 2.250| 0.0005                 |

Note. Instrumental magnitudes of OGLE are for the Cousin $I$ filter and are close to the Johnson $V$ filter (Udalski et al. 2015b). The $\mu$FUN filters (http://www.astronomy.ohio-state.edu/~microfun/Data/disclaimer.html) are: Bessell V, Cousins $I$, and a large band $R$ and $U$ is unfiltered (modulo the CCD quantum efficiency curve). The $I$ band of PLANET is Cousin $I$ (Albrow et al. 1998), while RoboNet Filters are SDSS-$i'$. MiNDStEp $I$ Filter is Gunn i, which is close to the standard system according to Skowron et al. (2016).
composed of a super-Jupiter orbiting an M dwarf, but further modeling work by Jeong et al. (2015) found that a binary lens composed of similar mass components better described the observed data. More recently, Han et al. (2016) revisited the event OGLE-2013-BLG-0723 and concluded that it resulted from a lens composed of two low-mass stars, rather than, as originally published by Udalski et al. (2015a), a triple-body lens composed of a Venus-mass planet orbiting a brown-dwarf in a binary system.

By examining in the microlensing models of S12 (their Figure 2) in more detail, we note a bump in the residuals of their static model around HJD = 2455730. This is due to the passage of the source trajectory close to a peripheral caustic (see their Figure 3). Including the microlensing parallax (Hardy & Walker 1995; Smith et al. 2003; Gould 2004) and the orbital motion of the lens (Dominik 1998; Ioka et al. 1999; Albrow et al. 2000) in their static model allowed the necessary flexibility for the source trajectory to avoid the passage close to this peripheral caustic, dramatically improving the χ^2 since this deviation is not supported by the data. Moreover, the orbital motion effect modifies the projected separation of the lens components, inducing a modification of the caustics pattern. In the analysis of S12, the rate of binary separation change was considerable (dω_0/da = 1.314 ± 0.023 yr^{-1}). The triangular caustic is then very distant from the trajectory at the date HJD = 2455730, which again improves the χ^2 in their best model (as can be seen in their Table 3). We note that the close/wide degeneracy implies that a competitive wide model may exist, for which the source trajectory would not intersect this peripheral caustic, potentially leading to a better model.

The presence of the “bump” at HJD = 2455730 and the absence of a static wide-binary solution in the S12 analysis might indicate that their model corresponds to a local minima, raising the need for further investigation. However, we note that G13 (p. 2) favored an alternative conclusion: “Indeed, the fact that this blended-light point sits right on the “disk main sequence” or “reddening track” at roughly the position expected for the primary component of the lens is already an indication that the microlensing solution is basically correct.”

### 3.2. Description

We reanalyzed this microlensing event using the pyLIMA software (Bachelet et al. 2017). A complete description of the binary fitting with pyLIMA will be given in E. Bachelet et al. (2018, in preparation). Whereas S12 considered limb-darkening effects in their models, in this work, we consider a simpler model, the Uniform Source Binary Lens (Bozza et al. 2012). The motivation for this is that caustic crossings are covered by a small number of data points (i.e., ≤10 data points) and they were taken in similar passbands (I and SDSS-r). In this case, the limb-darkening is weakly constrained. The results of our analysis show that this assumption is verified.

Instead of using the same grid approach as S12, we run a global search on all parameters using the differential evolution method. See Storn & Price (1997) and Bachelet et al. (2017) for details. However, due to the dramatic difference in the caustic topology between the wide and close binary regimes (Erdl & Schneider 1993; Dominik 1999; Bozza 2000; Cassan 2008), we split the parameter space into the close (s < 1) and the wide (s ≥ 1) regimes. This helps to avoid the potential pitfall, described in the previous section, of missing degenerate solutions.

To obtain a more comprehensive picture, each minima is then explored using the Monte Carlo Markov Chain algorithm (Foreman-Mackey et al. 2013). To increase the performance of the modeling process, it is useful to shift the geometric origin closer to the caustics (Cassan 2008; Han 2009; Penny 2014). We found, like S12, that the deviations are induced by the central caustic, so we shifted the reference to the center of that caustic. t_0 and u_0 parameters are then replaced with t_c and u_c. Note that pyLIMA follows the convention defined in (Gould 2004) and Skowron et al. (2011): u_0 > 0 when the lens passes the source on its right.

After finding a plausible static model, second-order effects were added to test whether they significantly improved significantly the model. The annual parallax was expected to be found in the event OGLE-2017-BLG-0417 due its long duration (t_E ≥ 70 days). This can be seen in Figure 1, where the deviations in the wings of both models (after HJD ≥ 2455840) are typical for a parallax signature. S12 showed that including the microlensing parallax significantly improved the model. In principle, the orbital motion of the lens or the motion of the source, known as xallarap (Pointdexter et al. 2005; Rahvar & Dominik 2009; Miyake et al. 2012), could be considered. S12 considered the former and again found a significant improvement in the likelihood of their model. In this work, we will show that only the parallax inclusion is relevant.

#### 3.3. Results

##### 3.3.1. Static Models

As implied by the close/wide degeneracy, two competitive models exist in the wide and close binary regimes. As can be seen in Table 2, the static wide model is significantly better than the static close model (Δχ^2 ~ 400) for the reasons we discussed in Section 3.1. In the absence of a parallax signal, the close model trajectory passes over a triangular shape caustic. On the other hand, the wide model does not have such caustic anywhere close to the trajectory, as can be seen in Figure 2, which is in better agreement with the data.

##### 3.3.2. Annual Parallax

The strong asymmetric deviations observed in the rising and falling parts of the event light curve (see Figure 1) are a classic signature of the annual parallax (Gould 2004). Briefly, the orbit of the Earth around the Sun induces shifts in the source trajectory, which significantly modifies the event magnification. Considering annual parallax effects in modeling requires two extra parameters summarized in the parallax vector, which are traditionally written in two different ways: \( \pi_E = (\pi_{EN}, \pi_{EG}) = (\pi_0, \pi_\ell). \) While the former corresponds to the projection of the parallax vector into the (north, east) plane of the sky formalism (the one used in pyLIMA), the second projects the vector in a basis parallel and perpendicular to the projected Earth acceleration at the time \( t_{0,par}. \) We refer the reader to Skowron et al. (2011) for more details about the choice of \( t_{0,par}. \) In this work, we selected \( t_{0,par} = 2455820 \) HJD, which is close to the caustic crossing. On this date (2011 September 13), the Earth’s acceleration was nearly parallel to the vernal direction. The east component of the parallax is expected to be stronger, and therefore better constrained,
because \( \pi_{EE} \sim \pi_1 \). However, the long event duration (i.e., \( \geq 60 \) days) should ensure a good constraint on both components. We encounter an additional degeneracy (Skowron et al. 2011), since the models are perfectly symmetric with the transformation \( (u_0, \alpha, \pi_{EN}) \leftrightarrow -(u_0, \alpha, \pi_{EN}) \), known as the “ecliptic degeneracy” (Skowron et al. 2011, 2015). This degeneracy is especially severe when source stars are close to the ecliptic plane, which is the case for OGLE-2011-BLG-0417 (i.e., \( \lambda = 264^\circ 33 \) and \( \beta = -3^\circ 7969 \)), but this degeneracy was not investigated by S12. This degeneracy means that it is nearly impossible to distinguish the lens-source proper motion direction only using the microlensing data. Fortunately, it has only a minor impact on the mass and lens distance estimations, since they are independent of the direction (see Gould 2000, 2004 or Equation (3)).

### 3.3.3. Orbital Motion

As underlined by S12, the orbital motion of the lens can produce significant effects on the light curve, especially for long-duration events (Dominik 1998; Skowron et al. 2011). Moreover, it can mimic the parallax effect and, therefore, produce a misleading parallax measurement (Batista et al. 2011; Bachelet et al. 2012). In this work, we implement the two-dimensional orbital motion defined by the binary lens separation expansion rate \( ds/dt \) and the binary-axes rotation rate \( d\alpha/dt \) (Dong et al. 2009; Batista et al. 2011). To verify if the lens system is gravitationally bound, we used (Dong et al. 2009; Batista et al. 2011):

\[
 v_\perp = \sqrt{\left(\frac{ds}{dt}\right)^2 + s^2 \left(\frac{d\alpha}{dt}\right)^2} \frac{D_1}{\sqrt{\theta_E D_1}} \leq \frac{2GM_1}{\sqrt{\theta_E D_1}}.
\]  

We found that including the orbital motion significantly improved the close model interpretation (with \( \Delta\chi^2 \sim 600 \), see Table 2). The close model with the orbital motion reported in this work is bound since \( v_\perp = 4.4 < 7.7 \) au yr\(^{-1}\). This model is similar to that found by S12. As suspected in Section 3.1, the triangular caustic is located far from the source trajectory at HJD = 2455730 (at this date, the triangular caustic is located at \( (x, y) = (-2, 3) \)). This allows the model to tune the parameters.
### Table 2
Summary of the Various Models Presented in This Work

| Parameters       | S12 Standard | Parallax | Orbital Motion | pyLIMA close Parallax (u_c > 0) | Orbital Motion (u_c > 0) | pyLIMA wide Parallax (u_c > 0) | Orbital Motion (u_c > 0) | Orbital Motion (u_c < 0) | Orbital Motion (u_c < 0) |
|------------------|--------------|----------|----------------|---------------------------------|--------------------------|-------------------------------|--------------------------|--------------------------|--------------------------|
| t_{ref} (2450000 HJD) | 5817.302(18) | 5815.867(30) | 5813.306 (59) | 5816.387(27) | 5814.785(60) | 5812.10(17) | 5811.517(29) | 5810.92(12) | 5811.05(14) | 5811.37(52) | 5811.12(53) |
| u_{ref}          | 0.1125(1)    | -0.0971(3) | -0.0992(5)    | 0.13608(16) | 0.11659(55) | 0.1148(17) | 0.11124(63) | 0.11071(88) | -0.1128(11) | 0.1136(39) | -0.1147(37) |
| t_{E} (days)     | 60.74(8)     | 79.59(36) | 92.26(37)     | 60.79(11) | 80.97(56) | 99.5(1.6) | 98.23(52) | 105.79(62) | 104.27(70) | 103.3(2.5) | 105.7(2.7)   |
| \rho (10^{-5})   | 3.17(1)      | 2.38(2) | 2.29(2)       | 3.094(34) | 2.494(22) | 2.127(59) | 2.138(21) | 2.156(37) | 2.202(43) | 2.179(50) | 2.219(48)   |
| \delta_{L}      | 0.601(1)     | 0.574(1) | 0.577(1)      | 0.60184(39) | 0.5731(12) | 0.5681(32) | 2.6214(66) | 2.5440(69) | 2.5374(62) | 2.5378(98) | 2.5318(94) |
| \eta             | 0.402(2)     | 0.287(2) | 0.292(2)      | 0.3967(26) | 0.2829(27) | 0.2800(40) | 0.968(12) | 0.617(10) | 0.629(12) | 0.619(12) | 0.627(11)   |
| \alpha (rad)     | 1.030(2)     | -0.951(2) | -0.850(4)     | -1.0257(20) | -0.9609(30) | -0.8994(71) | -0.8422(17) | -0.8486(19) | 0.8575(88) | -0.875(35) | 0.875(33)   |
| \gamma_{E}       | *            | 0.125(4) | 0.375(15)     | *        | -0.1263(56) | -0.314(30) | *        | -0.137(19) | 0.175(25) | -0.128(21) | 0.189(25)   |
| \gamma_{E}       | *            | -0.111(5) | -0.133(3)     | *        | -0.1151(65) | -0.1500(56) | *        | -1.160(32) | -0.1495(44) | -0.1576(60) | -0.1432(66) |
| ds_{\perp}/dt (yr^{-1}) | *         | 1.314(23) | *            | 1.353(48) | *            | *          | *        | 0.06(17) | 0.20(15)   |
| ds_{\perp}/dt (rad yr^{-1}) | *          | 1.668(76) | *            | 1.26(25) | *            | *          | *        | *        | *          | *          |
| \delta_{L}       | *            | 0.467(20) | *            | *        | *            | *          | *        | *        | *          | *          |
| \chi^{2}/dof     | 4415/2627    | 2391/2625 | 1735/2621     | 4456/2627 | 2348/2625 | 1741/2623 | 4026/2627 | 1723/2625 | 1724/2625 | 1720/2621 | 1720/2621   |

**Note.** Numbers in brackets are the 1\sigma uncertainties. The columns corresponding to pyLIMA report the best model, while uncertainties are derived from the MCMC exploration (i.e., 68% intervals). Note that t_{ref} and u_{ref} refers to t_{3} and a_{0} for S12 models and t_{c} and u_{c} for pyLIMA models.
to fit the residuals remaining in the close parallax fit visible in the Figure 1. This effect also impacts the fit parameters $t_E$, $\pi_{EN}$, and $\pi_{EE}$: adding the orbital motion effect changes their values significantly.

Conversely, the orbital motion effect is not measured for the wide models. Both the likelihood improvement and orbital motion parameters values are negligible. This is not surprising, since no sign of systematics remains in the residuals of the wide parallax model in the Figure 1. Moreover, $t_E$, $\pi_{EN}$, and $\pi_{EE}$ stayed consistent with or without the inclusion of the orbital motion effect.

### 3.3.4. Model Selection

Statistical tools exist for optimized model selection, provided that their parameters are linear (see, for example, Bramich et al. 2016 for a review). However, these methods cannot be used in the present case because the model parameters are nonlinear.

Instead, we adopt the following argument by applying the Occam’s Razor. First, the wide models reproduce the data better than the close models, with or without parallax taken into account. Moreover, the parallax wide models found in this work require fewer parameters than the 3D orbital motion model with a similar likelihood. Table 2 shows that orbital motion is not detected for wide models. The most plausible models are, therefore, the wide parallax models.

For the rest of this work, we will then refer to the positive ($u_c > 0$) wide parallax model as the best model, but similar conclusions can be made using the negative ($u_c < 0$) wide parallax model. In Section 4.4, we discuss how future observations can distinguish these two models.

### 4. Reassemble the Puzzle

#### 4.1. Source and Blend Brightness

Using our MCMC exploration around the best model (i.e., the $u_c > 0$ wide parallax model), we obtain the relative brightness of the source and the blend in several bands. For the source, we found $I_0 = 16.47 \pm 0.01$ mag and $V_s = 19.04 \pm 0.01$ mag. The brightness of the blend are $I_b = 16.52 \pm 0.01$ mag and $V_b = 18.104 \pm 0.005$ mag. The $I$ and $V$ bands are in the OGLE-IV system very close to the Johnson-Cousins system (Udalski et al. 2015b), whereas S16 assumed the AB system. G13 derived a fainter source $(V, I_s) = (19.42, 16.73)$ mag and a brighter blend $(V, I_b) = (18.23, 16.29)$ mag, which is in good ($I$ band) and relative ($V$ band) agreement with our close parallax model estimations: $(V, I_s) = (19.28, 16.71)$ mag and $(V, I_b) = (18.05, 16.31)$ mag.

The Interstellar Extinction Calculator on the OGLE website, based on Nataf et al. (2013) and Gonzalez et al. (2012), returns $A_V = 2.0 \pm 0.1$ mag (we assume the error on $A_V$) and $E(V-I) = 1.58 \pm 0.08$ mag. The absorption corrected brightness of the source is then $I_{0s} = 14.5 \pm 0.1$ mag, and the intrinsic color is $(V-I)_{0s} = 1.0 \pm 0.1$ mag.

However, the OGLE-IV photometric system is slightly off-standard, especially in $V$ band (Udalski et al. 2015b). One can obtain a more accurate estimation of the intrinsic color and brightness of the source following the same method as S12. From the Figure 4 of S12, the location of the Red Giant Giant Clump (RGC) for this line of sight is $(V-I, I)_{RGC} = (2.7 \pm 0.1; 16.5 \pm 0.1)$ mag. The offset between the microlensing source and the RGC is then $\Delta(V-I) = (0.13; 0.03)$ mag. To be in the standard system, the offset in color needs to be corrected by a factor 0.94 (the microlensing event was on CCD 22 of the OGLE-IV mosaic camera), leading to an offset in the Johnson-Cousins system $\Delta(V-I); I_{0C} = (0.12; 0.03)$ mag. The absolute color and brightness of the RGC are known $(V-I) = 1.06 - 0.12 = 0.94 \pm 0.1$ mag and $I_{0C} = 14.43 - 0.03 = 14.4 \pm 0.1$ mag. Using the color-magnitude-radius relation of Kervella & Fouqué (2008), we found $\theta_e = 5.2 \pm 0.3$ mas and finally $\theta_e = 2.4 \pm 0.2$ mas and $\mu_{geo} = 8.3 \pm 0.8$ mas yr$^{-1}$ (in the geocentric frame).

Thanks to VVV observations, it is possible to estimate the source flux in the near infrared. However, the target was so bright during the peak magnification (i.e., 12 mag) that the target approached saturation. We take this into account by weighting the brightness estimations with a 0.2 mag error. Using the MCMC exploration, we compute the magnification relative to the VVV observations and estimate a source brightness of $K_s = 13.5 \pm 0.2$ mag. We can estimate the near-infrared absorption based on the extinction maps of Gonzalez et al. (2012). Using their online tool, we found (with a box size of 2’ and using the Nishiyama et al. 2009 extinction law) $A_K = 0.3 \pm 0.1$ mag and $E(J-K) = 0.6 \pm 0.1$ mag. The source brightness is, therefore, $K_{0s} = 13.2 \pm 0.2$ mag and $(I-K)_{0s} = 1.2 \pm 0.2$ mag.

Both colors are in good agreement with a G8 giant source (Bessell & Brett 1988). S16 have obtained a high-resolution image in the $K_s$ band and found the target to be $K = 12.94 \pm 0.03$ mag. They did not find any significant companion to the target star, indicating that the blend and the source are still aligned. This is consistent with our proper motion measurement. This image was taken four years after the event peak, so we would expect a separation of 30 mas, which is equivalent to 3 pixels of the narrow camera of NIRC2. So assuming the object observed is the combined image of the source and the lens, we estimate that the lens brightness is $K_b = 14.0 \pm 0.2$ mag.

#### 4.2. Is the Blend the Lens?

Similar to S12 and G13, we test the hypothesis that the blend could be the lens using several evidences. First, using the parallax norm, $\pi_{E} = \sqrt{\pi_{EN}^2 + \pi_{EE}^2}$, and the angular Einstein ring radius, we can derive the total mass of the lens (Gould 2000):

$$M_{0s} = \frac{\theta_E^2}{\kappa \pi_{E}} = \frac{\pi_{red}}{\kappa \pi_{E}} = \frac{\theta_E}{\kappa \pi_{E}}$$

with $\pi_{red} = au(D_{s}^{-1} - D_{l}^{-1})$.

We can combine the brightness measurements of the blend derived previously with isochrones from Bressan et al. (2012) and Marigo et al. (2017) (with solar metallicity and 5 Gyr isochrones) to estimate the total mass and distance of the blend under two assumptions. We assume that the blend is a binary star with the same mass ratio as the lens (i.e., $q = 0.61$) and use the partial extinction law of Bennett et al. (2015) and Beaulieu et al. (2016).

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5 http://ogle.astro.sou.edu.pl/

6 http://mill.astro.puc.cl/BEAM/calculator.php

7 http://stev.oapd.inaf.it/cgi-bin/cmd
Finally, we take advantage of the recent Gaia data release (Gaia Collaboration et al. 2016, 2018) and found the target parallax: $p = 0.8 \pm 0.2$ mas (the Gaia Data Release 2 (DR2) ID is 4061372412899533056). This leads directly to a target distance of $D_t = 1.3 \pm 0.3$ kpc. The significant blending $g = f_b/f_s \sim 1$ of this event affects the parallax measurement because Gaia has measured the displacement of the photocenter in this case. We, therefore, used a refined estimate of the distance, $D_t \in [982,1734]$ pc, which is available in the catalog of Bailer-Jones et al. (2018).

We combine this information in Figure 3. It is clear that the blend is likely to be the lens. All constraints converge to a total lens mass of $M_{\text{tot}} = 1.5 \pm 0.2 M_\odot$ and $D_t = 1.6 \pm 0.2$ kpc. According to this scenario, the lens is a binary composed of a K dwarf with $M_{l,1} = 0.9 \pm 0.1 M_\odot$ and an M dwarf with $M_{l,2} = 0.6 \pm 0.1 M_\odot$.

4.3. Wide Scenario Predictions versus B15 and S16 Measurements

Following G13, we make a prediction for the radial velocity that would be expected from follow-up observations, based on this new model. Using our best model, we find that the orbital radius of the binary is at least $a \sim 10$ au, leading to a minimum orbital period of $P \sim 25$ year. According to this new scenario, it is not surprising that B15 did not find any signal in their observations.

The wide model confirms the predictions by S16 that the source is redder ($(V-I)_s = 2.57$ mag) than the lens ($(V-I)_l = 1.58$ mag). The source brightnesses derived in this work are in agreement with the spectral energy distribution (SED) study of S16 (this work, S16): $V = (19.04 \pm 0.01, 19.4 \pm 0.58$ mag).

$I = (16.47 \pm 0.01, 16.30 \pm 0.34$ mag, and $K = (13.5 \pm 0.2, 13.55 \pm 0.17$ mag. Note that we remodel the SED presented in S16, but use a scenario with a giant source and two bright lenses. While the source and the lens primary properties stay unchanged, the mass of the companion is weakly constrained $(M_{l,2} = 0.24^{+0.04}_{-0.01} M_\odot$) but is in agreement with the estimation made in the previous section. Given the masses of the individual components $(M_{l,1} = 0.9 \pm 0.1 M_\odot$ and $M_{l,2} = 0.6 \pm 0.1 M_\odot$), one can expect the flux ratios between the binary components to be $\geq 4$ in the wavelength windows observed by S16. The SED constraints on the companion are, therefore, expected to be weak.

4.4. Predictions with Gaia DR2 Proper Motions

In addition to the parallax information, the Gaia DR2 also provides proper motion measurements in the International Celestial Reference System (ICRS) reference frame $\mu(G,N) = (-4.06 \pm 0.44, -6.39 \pm 0.35)$ mas yr$^{-1}$. Note that the components of the Gaia DR2 proper motions are $\mu_{ls} = \mu_l \cos(\delta)$ and $\mu_s$ (Luri et al. 2018). This measurement is also affected by the blending and, assuming the blend light is coming only from the lens, the reported proper motion is the proper motion of the photocenter, which is similar to the astrometric microlensing (Dominik & Sahu 2000; Nucita et al. 2017):

$$\mu_G = \frac{f_s \mu_l + f_b \mu_s}{f_s + f_b} = \frac{g \mu_l + \mu_s}{1 + g}, \quad (4)$$

with $\mu_l$ and $\mu_s$ representing the proper motion of the lens and the source, respectively. The microlensing models predict the heliocentric lens-source proper motion (Skowron et al. 2011):

$$\mu_{\text{helio}} = \mu_{\text{geo}} + \pi_{\text{rel}} V_{\odot,\perp}, \quad (5)$$

where $V_{\odot,\perp} = (-1.99, 0.58)$ km s$^{-1}$ is the projected velocity of the Earth at the time $t_{\odot,\text{par}}$ (Gould 2004). The proper motion vector is given by:

$$\mu_{\odot} = \frac{\pi_{\odot}}{\pi_{\odot}} \mu_{\odot}, \quad (6)$$

knowing that

$$\mu_{\text{helio}} = \mu_l - \mu_s, \quad (7)$$

where one can rewrite Equation (4) and derive

$$\mu_l = \mu_G + \frac{\mu_{\text{helio}}}{1 + g}, \quad (8)$$

and

$$\mu_s = \mu_G - g \frac{\mu_{\text{helio}}}{1 + g}. \quad (9)$$

Assuming that the blend light is measured and emitted by the lens, the proper motions of the source and the lens can be estimated. For the event OGLE-2011-BLG-0417, the blending of the source is $g \sim 1$. Note that it is difficult to estimate precisely the blending for the Gaia $G$ band (Jordi et al. 2010) because the filter is broad and, more importantly, there is no available light curve. As can be seen in Table 3, the projected speeds of the lens and the source can be estimated assuming (for both scenarios) a source distance of $D_s = 8.2 \pm 0.5$ kpc, a lens distance of $D_l = 1.6 \pm 0.2$ kpc, $\theta_E = 2.4 \pm 0.2$ mas,
### Table 3

Heliocentric Proper Motion and Speed Estimations for OGLE-2011-BLG-0417 (East, North)

| Model            | $\mu_{\text{heli}}$ mas yr$^{-1}$ | $\mu$ mas yr$^{-1}$ | $V_{\perp}$ km s$^{-1}$ | $\mu_{\perp}$ mas yr$^{-1}$ | $V_{\perp}$ km s$^{-1}$ |
|------------------|----------------------------------|---------------------|-------------------------|----------------------------|-------------------------|
| Wide parallax $u_0 > 0$ | $(-6.25 \pm 0.62, -5.60 \pm 0.56)$ | $(-1.44 \pm 0.40, -3.59 \pm 0.45)$ | $(-56 \pm 16, -139 \pm 19)$ | $(-7.68 \pm 0.40, -9.19 \pm 0.45)$ | $(-58.3 \pm 7.9, -69.7 \pm 9.4)$ |
| Wide parallax $u_0 < 0$ | $(-5.32 \pm 0.54, 6.08 \pm 0.62)$ | $(-1.90 \pm 0.37, -9.43 \pm 0.47)$ | $(-74 \pm 15, -367 \pm 29)$ | $(-7.22 \pm 0.37, -3.35 \pm 0.47)$ | $(-54.8 \pm 7.4, -25.4 \pm 4.8)$ |

**Note.** Proper motions are in the system $\mu = (\mu_\alpha, \mu_\delta)$. Note that the errors of $\mu_\alpha$ and $\mu_\delta$ are strictly equal since $g = 1$. 

Besides the $K$-band brightness constraint, all constraints converge to a total lens mass of $M_{\text{tot}} = 1.5 \pm 0.2 \, M_\odot$ and $D_l = 1.6 \pm 0.2 \, \text{kpc}$. According to this new scenario, the lens is a binary composed of a K dwarf with $M_{1;2} = 0.9 \pm 0.1 \, M_\odot$, and an M dwarf with $M_{1;2} = 0.6 \pm 0.1 \, M_\odot$. The lens orbital radius of the binary is at least 10 au with a minimum orbital period of $P \sim 25 \, \text{yr}$, contrary to the conclusion of G13. The absence of radial velocity modulations measured in B15 is in agreement with this new scenario. More generally, our magnitude predictions for the source (but not the blend) are in agreement with S16. We found two degenerate models in the wide-binary regime caused by the symmetry of the source trajectory with respect to the binary lens axis. We present a method using the Gaia proper motion measurements, and predict that the model $u_c > 0$ is preferred. This is verifiable with future high-resolution imaging.

Since there has not yet been developed a method to decide the optimal range and spacing in the grid search for lensing parameters, it is highly recommended to cross check results from this technique using an alternative method to explore parameters space. This especially true for models that require extreme tuning of the source trajectory (i.e., high orbital motion and/or parallax and/or xallarap). As the grid method is computationally intensive, applying it to large datasets results in excessive analysis timescales and requires costly many-CPU computing clusters. It is, therefore, beneficial and timely to explore alternatives that can handle microlensing data sets expected from the planned missions of the Wide Field Infrared Survey Telescope and Euclid.

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5. Discussion and Conclusion

From Figure 3, we notice that the $K$-band constraint is not in good agreement with the others. This may be explained by the extreme brightness of the event in this band, potentially leading to an underestimation of the source flux in $K$ (and then an overestimation of the lens brightness). The isochrones, especially for M stars, are also sensitive to the age, metallicity, and, potentially, dust models (whereas we have not considered dust in this work; Marigo et al. 2017).
Appendix

Propagation of Uncertainties in Proper Motion Transformations

Our goal is to derive the proper motion of the lens and the source in the galactic and equatorial systems. The transformation of proper motions from equatorial coordinates \( \mu_{\alpha,s} \) and \( \mu_{\delta,s} \) to galactic coordinates \( \mu_{\alpha,g} = \mu_{\alpha} \cos(b) \) and \( \mu_{\delta,g} \) is given by a rotation matrix (Johnson & Soderblom 1987; Poleski 2013; Luri et al. 2018):

\[
\begin{pmatrix} \mu_{\alpha,g} \\ \mu_{\delta,g} \end{pmatrix} = R \begin{pmatrix} \mu_{\alpha,s} \\ \mu_{\delta,s} \end{pmatrix},
\]

(10)

Differentiating the galactic coordinates defined in (Binney & Merrifield 1998), one can derive (Poleski 2013):

\[
R = \frac{1}{\cos(b)} \begin{pmatrix} R_1 & R_2 \\ -R_2 & R_1 \end{pmatrix},
\]

(11)

with

\[ R_1 = \cos(\delta) \sin(\delta b) - \sin(\delta) \cos(\alpha - \alpha b) \cos(\delta b); \]
\[ R_2 = \sin(\delta b) \sin(\alpha - \alpha b). \]

(12)

Since the rotation matrix determinant has to be unity, it implies \( \cos(b) = \sqrt{R_1^2 + R_2^2} \).

Luri et al. (2018) stress that the correlation terms can seriously impact the estimation of errors in the proper motion. Considering the general problem \( Y = f(X) \), the variance-covariance matrices relation is (Luri et al. 2018)

\[
C_Y = J C_X J^T,
\]

(13)

where \( C_Y \) and \( C_X \) are the variance-covariance matrices, and \( J \) is the Jacobian matrix of the transformation. In the following, we describe the various components of the error estimations of quantities defined in Section 4.4 in the \( \mu = (\mu_{\alpha}, \mu_{\delta}) \) system. First, the variance-covariance \( C_{\mu_{geo}} \) matrix of the vector \( \mu_{geo} \) define in Equation (6) is

\[
C_{\mu_{geo}} = J \begin{pmatrix} A & B \\ B^T & C_{\mu_{geo}} \end{pmatrix} J^T; \quad J = \frac{1}{\pi E} \begin{pmatrix} \pi_{EE} \mu_{geo}^2 \pi_{EE} & \mu_{geo} \pi_{EE} \\ \pi_{EN} \mu_{geo} \pi_{NE} & \mu_{geo} \pi_{NE} \end{pmatrix},
\]

(14)

where \( C_{\mu_{geo}} \) is the variance-covariance matrix of the normalized microlensing parallax, \( A = \sigma_{\mu_{geo}}^2 \) (i.e., the scalar error), and \( B = (0, 0) \). The former is estimated numerically from the MCMC exploration. Similarly, we can derive the \( C_{\mu_{helio}} \) matrix:

\[
C_{\mu_{helio}} = J \begin{pmatrix} C_{\mu_{geo}} & B^T \\ B & C_{\mu_{geo}} \end{pmatrix} J^T; \quad J = \begin{pmatrix} 1 & 0 & V_{E,E} \\ 0 & 1 & V_{E,N} \end{pmatrix},
\]

(15)

The errors of the source proper motion can be estimated from

\[
C_{\mu_0} = J \begin{pmatrix} C_{\mu_{geo}} & D \\ B & C_{\mu_{helio}} B^T \end{pmatrix} J^T; \quad J = \begin{pmatrix} 1 & 0 & \frac{1}{1 + g} & 0 \\ 0 & 1 & 0 & \frac{1}{1 + g} \end{pmatrix},
\]

(16)

where \( \sigma_g \) is the error on the blending ratio, \( g = f_b / f_s \), between the lens and the source, and \( C_{\mu_{helio}} \) is the variance-covariance matrix of Gaia proper motions (estimated from the Gaia DR2 archive and transformed to the \( \mu = (\mu_{\alpha}, \mu_{\delta}) \) system). The matrix \( C_{\mu_0} \) is

\[
C_{\mu_0} = J \begin{pmatrix} C_{\mu_{geo}} & D \\ B & C_{\mu_{helio}} B^T \end{pmatrix} J^T; \quad J = \begin{pmatrix} 1 & 0 & -\frac{g}{1 + g} & 0 \\ 0 & 1 & 0 & \frac{1}{1 + g^2} \end{pmatrix},
\]

(17)

Note that the errors for \( \mu_0 \) and \( \mu_s \) are, therefore, nearly similar. In Section 4.4, they are strictly identical since \( g = 1 \). The errors for the source and lens speeds can be estimated using

\[
C_{\mu_i} = J \begin{pmatrix} C_{\mu_{geo}} & D \\ B & \sigma^2_{g_i} \end{pmatrix} J^T; \quad J = \begin{pmatrix} D_i & 0 & \mu_{i,E} \\ 0 & D_i & \mu_{i,N} \end{pmatrix},
\]

(18)

and the index \( i \) indicates the source or the lens, respectively. The transformation to the system \( \mu = (\mu_{\alpha}, \mu_{\delta}) \) of any matrix \( C \) is simply

\[
\mu = J C J^T; \quad J = \begin{pmatrix} -\sin(\delta) & 0 \\ 0 & 1 \end{pmatrix},
\]

(19)

The errors from the equatorial to galactic coordinates are given by (Luri et al. 2018)

\[
C_{\mu_{geo}} = R C_{\mu} R^T,
\]

(20)

since the Jacobian of a rotation is the rotation itself. Finally, the transformation to the system \( \mu_{geo} = (\mu_{\alpha}, \mu_{\delta}) \) of the system \( \mu_{geo} = (\mu_{\alpha}, \mu_{\delta}) \) is

\[
C_{\mu_{geo}} = J C_{\mu_{geo}} J^T; \quad J = \begin{pmatrix} -\sin(b) / \cos(b)^2 & 0 \\ 0 & 1 \end{pmatrix},
\]

(21)

In this study, we use the updated coordinates of the North Galactic Pole, \( \alpha_{GP} = 192.85948 \) and \( \delta_{GP} = 27.1285 \) (Perryman 1997; Padmanabhan 2002; Poleski 2013).

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