Research on Equivalent Factor Boundary of Equivalent Consumption Minimization Strategy for PHEVs

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Abstract—The equivalent fuel consumption minimum strategy (ECMS) based on the Pontryagin’s minimum principle (PMP) enables real-time energy management optimization of plug-in hybrid electric vehicles (PHEVs). However, it remains challenging to accurately determine the equivalent factor (EF). In this study, an analytical expression of the optimal EF boundary is addressed to facilitate more efficient search of the optimal EF. The deterministic expression of the optimal EF boundary is derived from the Hamilton equation of PMP. By combining the optimal EF boundary and differential evolution algorithm, a novel fusion adaptive ECMS (A-ECMS) is proposed to demonstrate the application of the proposed boundary in the framework of model predictive control (MPC). Two different simulations with and without prior knowledge of driving cycle were respectively conducted, and the simulation results verify the feasibility of optimal EF boundary and manifest that the energy savings by the proposed A-ECMS can reach more than 97% of dynamic programing, highlighting the superior performance of the proposed strategy.

Index Terms—Plug-in hybrid electric vehicles (PHEVs), equivalent fuel consumption minimum strategy (ECMS), Hamilton equation, equivalent factor (EF) boundary, differential evolution algorithm.

I. INTRODUCTION

Plug-in hybrid electric vehicles (PHEVs) are usually equipped with larger capacity lithium-ion batteries and more powerful electric motors, compared to conventional hybrid electric vehicles (HEVs) [1], thus leading to more energy-saving potential than HEVs [2]. Effective energy distribution between different power sources, achieved by the so-called energy management strategies (EMS), is critical to exploit energy saving potential and optimize performance of the vehicle powertrain [3], [4]. Currently, there have been a variety of researches in terms of EMSs, which can be classified into two categories, rules-based and optimization-based [5], [6]. Rule-based EMSs mainly include deterministic rules [7] and fuzzy rules [8]. A representative of deterministic rule-based EMSs is the charge depletion/charge sustaining (CD/CS) strategy [9]. In the CD mode, the battery mainly supplies the driving power, and the engine is engaged when the battery cannot meet the power demand; while in the CS mode, the main target is to maintain battery state of charge (SOC) in the vicinity of setting boundary by regulating the engine power and battery power. Fuzzy rule-based EMSs can distribute energy of different sources according to the preset rules with the inputs of power demand, vehicle speed and battery SOC. In [10], a multi-input fuzzy logic controller is developed to manage the energy allocation of power-split HEVs, and showcases better energy-saving effects than conventional rule-based controllers. In [11], an EMS is proposed based on the traffic condition recognition and genetic-fuzzy logic controller. The fuzzy rule controller is adaptively participated in energy management by predicting and identifying the traffic condition. Usually, the rule-based EMS is simple and easy to implement. However, the energy saving and controlling effect is highly dependent on developer’s expertise and engineering experience. It is time saving, reliable, robust, and yet usually non-optimal.

Optimization-based EMSs can be divided into two categories, global optimization algorithms and instantaneous optimization algorithms [12]. The global optimization EMS stems from power allocation of a hybrid electric truck achieved by dynamic programming (DP) [13]. Theoretically, DP can attain global optimal solution [14], whereas it needs to acquire global transportation information a priori and then conducts backward optimization for searching proper controlling sequences from the beginning [15]. Obviously, its online application is restricted; whereas its optimal solution is often considered as a benchmark for evaluating performance of other EMSs.

In contrast, instantaneous optimization EMSs, just as the name implies, can conduct energy management optimization immediately or in a short duration [16]. A typical instantaneous optimization based EMS is the equivalent consumption minimum
strategy (ECMS) built based upon the Pontryagin’s minimum principle (PMP) [17]. The key principle of ECMS is to translate the global optimization into a local optimization problem that minimizes the real-time equivalent fuel consumption at each step [18]. A key process of conducting ECMS is to properly estimate the equivalent factor (EF). In the premise of acquiring the whole driving condition, the offline algorithm can be employed to determine the optimal EF, thereby enabling fuel consumption based on ECMS to be close or equal to the global optimal solution [19]. Actually, the ECMS is a typical open-loop control strategy that is not qualified for real-time control. For adapting to complex and time-varying traffic environment, a body of more in-depth studies have been spurred, such as adaptive ECMS (A-ECMS) and predictive ECMS [20]. The main challenge when applying A-ECMS lies in the real-time regulation and optimization of EF. In [21], [22], a novel A-ECMS is introduced for energy management of a plug-in hybrid electric bus. Firstly, the optimal EF of different driving routes is calculated offline, and then the EF is determined online in the light of vehicle states and driving conditions. However, it is only applicable to the vehicles with relatively fixed driving routes. In [23], [24], adaptive adjustment of the optimal EF based on the proportional-integral (PI) controller is leveraged with regard to the error of reference SOC and its feedback. Nonetheless, it is difficult to find a deterministic rule for regulating the parameters of PI controller. Moreover, the reference SOC trajectory will directly influence the saving effect of this closed-loop control strategy. Particularly for PHEVs, to attain the satisfactory energy saving effect, planning of SOC curves should sufficiently consider the global driving information [25].

In addition, model predictive control (MPC) can realize online instantaneous optimization, and consequently has been widely applied in energy management of PHEVs [26]–[28]. The advantage of MPC lies in the supplied extra prediction information in the receding horizon, thereby reducing excessive dependence on the global driving information [29]. Since MPC requires iterative calculations, the PMP with lower calculation intensity will be preferred for real-time control [30]. In [31], the optimal co-state is assumed to be a constant. Then, the dichotomic algorithm is applied to obtain it in the receding horizon. In [32], the MPC is combined with the numerical method to give the explicit solution of optimal torque distribution and shifting rules, thereby achieving fast energy management of PHEV.

The shortcomings of A-ECMS and the advantage of MPC discussed above motivate the in-depth investigation of instantaneous energy management of PHEVs. To achieve fast search of the optimal EF, the analysis of optimal EF boundary is conducted. The proposed boundary correlates to only the real-time operating efficiency of vehicle powertrain. Additionally, determination of the optimal EF boundary is simple and easy to be implemented online. On this basis, the differential evolution (DE) algorithm, to the best of authors’ knowledge, is firstly applied to search the optimal EF [33]. Then, a novel fusion A-ECMS is proposed with the combination of metaheuristic algorithm and classical MPC algorithm. It can comprehensively consider the variation of SOC and expected speed information to adjust the optimal EF in a receding horizon. The simulation results highlight the application prospect of proposed EF boundary theory, including high efficiency of searching the optimal EF and satisfactory fuel consumption savings. Compared with the PI based A-ECMS and conventional CD/CS algorithm, the proposed algorithm can reduce the dependence on the reference SOC trajectory and feature better adaptability and robustness.

The main contribution of this study is summarized as follows: Firstly, the optimal EF boundary is determined by analyzing and deducing the Hamiltonian equation of PMP. Then, a novel fusion A-ECMS is proposed on the basis of MPC. The optimal EF can be determined in real time based on the reference SOC and short-term prediction velocity in a receding horizon according to the refined boundary and DE algorithm. Finally, the feasibility of proposed EF boundary is validated by simulation, and the proposed A-ECMS can attain more than 97% fuel savings provided by DP, manifesting the preferable performance of improved A-ECMS.

The remainder of this paper is organized as follows: Section II describes the powertrain of parallel PHEV and the conventional ECMS. Section III details the deduction process of EF boundary and the proposed fusion A-ECMS. Next, simulations are conducted and discussed in Section IV. Finally, Section V summarizes the main conclusions of this study.

II. STATEMENT OF SYSTEM MODEL AND ECMS

A. Powertrain Description and Modeling

![Powertrain structure of a parallel PHEV.](image)

The inner powertrain structure of studied parallel PHEV is shown in Fig. 1. As can be seen, the powertrain includes three energy transfer paths: the engine driving, the electric driving, and the mechanical transmission path. When the vehicle is accelerated uphill with the speed \( v \) and acceleration \( a \), the free body diagram is shown in Fig. 2.

According to Fig. 2, the driving resistance of vehicle can be determined as:

\[
\begin{align*}
\sum F &= (F_f + F_r) + F_i + F_j + F_w \\
&= \left( m g f \cos \alpha \right) + F_i + F_j + F_w
\end{align*}
\]

where \( F_f + F_r = m g f \cos \alpha \), \( F_i = m g \sin \alpha \), \( F_j = \delta \alpha a \), and \( F_w = 0.5 C_D \rho D v^2 \). \( f \) is the rolling resistance coefficient, \( \delta \) is the rotating mass coefficient, \( C_D \) is the air resistance coefficient, \( D \) is the windward area of vehicle, and \( \rho \) is the air density. Generally, when \( \alpha \) is small, the rolling resistance can be simplified to \( m g f \). Therefore, the required driving power.
of vehicle at the wheel $P_{\text{wheel}}$ can be calculated as:

$$P_{\text{wheel}} = \sum F \cdot v$$  \hspace{1cm} (2)

As shown in Fig. 1, from the coupling mechanism to the wheel, the second half of powertrain mainly belongs to the mechanical energy transfer path. Assuming that the total efficiency of mechanical transmission path is $\eta_m$, the relationship between the motor power, engine power and driving power can be calculated as:

$$P_{\text{wheel}} = (P_{\text{mot}} + P_{\text{eng}})\eta_m$$  \hspace{1cm} (3)

where $P_{\text{mot}}$ and $P_{\text{eng}}$ denote the motor power and engine power, respectively. Note that the efficiency of clutch is assumed to be 1 when it is engaged. On the basis of our previous research [34], a two-dimensional efficiency map of engine and motor can be constructed based on the experimental data. Therefore, the real-time efficiency of engine and motor can be determined by means of interpolation with the inputs of torque and speed, as:

$$\begin{cases}
\eta_{\text{eng}}(t) = f(T_{\text{eng}}(t), n_{\text{eng}}(t)) \\
\eta_{\text{mot}}(t) = f(T_{\text{mot}}(t), n_{\text{mot}}(t))
\end{cases}$$  \hspace{1cm} (4)

where $\eta_{\text{eng}}$ and $\eta_{\text{mot}}$ denote the efficiency of engine and motor; and $T_{\text{eng}}, T_{\text{mot}}, n_{\text{eng}}$ and $n_{\text{mot}}$ represent their torque and speed, respectively. Consequently, the instantaneous fuel consumption $\dot{m}_{\text{fuel}}$ can be calculated as:

$$\dot{m}_{\text{fuel}} = P_{\text{eng}}/(H_{\text{LHV}}\eta_{\text{eng}})$$  \hspace{1cm} (5)

where $H_{\text{LHV}}$ is the low heating value of fuel. In this study, the battery is modeled in a manner of an internal resistance and an open circuit voltage (OCV) source connected in series, similar as introduced in [35]. According to the definition of SOC [36], [37], its variation rate $\dot{\text{SOC}}$ can be formulated as:

$$\dot{\text{SOC}} = -\frac{E_{\text{OCV}}(\text{SOC})}{2R(\text{SOC})C_{\text{bat}}}$$  \hspace{1cm} (6)

where $C_{\text{bat}}$ means the battery capacity, $E_{\text{OCV}}(\text{SOC})$ is the OCV, $R(\text{SOC})$ is the internal resistance, and both are related to SOC. $P(t) = P_{\text{motor}}/(\eta_{\text{motor}}\eta_{\text{inv}})$ denotes the output power of battery, and $\eta_{\text{inv}}$ expresses the efficiency of inverter.

B. Equivalent Consumption Minimum Strategy

The energy management of PHEV is essentially a sequential optimal problem with the target of minimizing the fuel consumption. The battery SOC is often selected as the state variable $x(t)$, and $T_{\text{eng}}$ is considered as the control variable $u(t)$. Therefore, the objective function, state equation, initial condition and terminal constraint of the optimal control problem can be formulated sequentially, as:

$$J(u, x) = \int_{t_0}^{t_f} \dot{m}_f(u(t), x(t), t)\, dt$$  \hspace{1cm} (7)

$$\dot{x}(t) = f(x(t), u(t), t) = -\frac{I(u(t), E(x), R(x))}{C_{\text{bat}}}$$  \hspace{1cm} (8)

$$x(t_0) = \text{SOC}_{\text{int}}$$  \hspace{1cm} (9)

$$G[x(t_f), t_f] = \text{SOC}(t_f) - \text{SOC}_{\text{end}} = 0$$  \hspace{1cm} (10)

where $\dot{x}(t)$ denotes $\dot{\text{SOC}}$, $t_0$ and $t_f$ represent the starting and ending time of trip, $\text{SOC}_{\text{int}}$ and $\text{SOC}_{\text{end}}$ express the initial and ending values of SOC in a certain trip. In addition, SOC and $T_{\text{eng}}$ should be subject to:

$$\begin{cases}
\text{SOC}_{\text{min}} \leq \text{SOC} \leq \text{SOC}_{\text{max}} \\
T_{\text{eng, min}} \leq T_{\text{eng}} \leq T_{\text{eng, max}}
\end{cases}$$  \hspace{1cm} (11)

Intuitively, it is difficult to directly solve out the optimal control trajectory $u^*$ that satisfies the aforementioned constraints and meanwhile minimizes $J(u, x)$. Yes, this extremum problem of objective function is solved by PMP. To attain it, the co-state $\lambda(t)$ is introduced to construct the Hamilton function $H$:

$$H(x(t), \lambda(t), u(t), t) = \lambda(t)f(x(t), u(t), t) + \dot{m}_f(u(t), t)$$

$$= \lambda(t)\dot{x}(t) + \dot{m}_f(u(t), t)$$  \hspace{1cm} (12)

Additionally, $\lambda(t)$ should satisfy the co-state equation and terminal constraint, as:

$$\dot{\lambda} = -\frac{\partial H(x(t), \lambda(t), u(t), t)}{\partial x(t)} = -\lambda(t)\frac{\partial I(u(t), E(x), R(x))}{\partial x(t)}$$  \hspace{1cm} (13)

$$\lambda(t_f) = \partial G(t)/\partial x(t_f)$$  \hspace{1cm} (14)

From (13), it can be observed that $\dot{\lambda}$ is directly related to the OCV and internal resistance. However, since the PHEV is equipped with a lithium-ion battery with relatively large capacity, the SOC deviation is quite small, and even can be neglected, over a short period. Therefore, the OCV and internal resistance can be regarded unchanged in a short duration, i.e., $\dot{\lambda} = 0$, and thus $\lambda(t)$ will be a constant. In this study, $s(t) = -\lambda(t)H_{\text{LHV}}/(E_{\text{OCV}}(\text{SOC})C_{\text{bat}})$ is introduced to replace $\lambda(t)$ in Hamilton equation, and now (12) can be rewritten into:

$$H(x, s, u, t) = s(t)P_{\text{bat}}(t)/H_{\text{LHV}} + \dot{m}_f(t)$$

$$= \dot{m}_{\text{equ}}(x, s, u, t) = \dot{m}_{\text{bat}}(x, s, t) + \dot{m}_f(u, t)$$  \hspace{1cm} (15)

where $\dot{m}_{\text{equ}}(t)$ denotes the instantaneous equivalent fuel rate, $P_{\text{bat}}(t)$ denotes the battery power, and $s(t)P_{\text{bat}}(t)/H_{\text{LHV}}$ represents the equivalent fuel rate consumed corresponding to
the battery power. Actually, equation (15) describes the key principle of ECMS. Based on PMP, a necessary condition of \( u^*(t) = \arg \min [H(x(t), \lambda(t), u(t), t)] \) should be satisfied for minimization of \( J(u, x) \). In this premise, the control sequence \( u^* \) exists that can achieve the optimal energy saving if and only if (8) and the following condition are obeyed.

\[
u^*(t) = \arg \min \{\dot{m}_{equ} (x(t), s(t), u(t), t)\} \tag{16}\]

In the light of ECMS, different EFs will directly affect the equivalent fuel rate of battery, as presented in (15). When EF increases, the equivalent fuel rate will increase, and the control strategy tends to increase the engine power and decrease the motor power. On the contrary, when EF decreases, the control strategy will endeavor to release the energy of battery to power the vehicle and simultaneously reduce the engine power. Nonetheless, it is difficult to find a deterministic analytical solution for the optimal EF according to (9), (13) and (14), as this is obviously an optimal searching problem with a two-point boundary.

### III. Derivation and Application of Optimal Equivalent Factor Boundary

As discussed above, it is difficult to find EF in a deterministic manner within a two-point boundary; however, if the wide boundary of EF can be narrowed in advance, it will become easier and more efficient to search its optimal value.

#### A. Derivation of Optimal Equivalent Factor Boundary

As presented previously, the variation of EF can directly affect the optimal engine torque. At time \( t \), the relationship between the optimal engine power and EF is shown in Fig. 3.

In this study, the maximum and minimum power of engine is assumed to be \( P_{eng, max} \) and 0, respectively. According to the engine power, we can define three operating modes of PHEV: pure electrical driving mode \( P_{eng} = 0 \), blended driving mode, i.e., \( P_{eng} \in \{0, P_{eng, max}\} \) and maximum engine power mode \( P_{eng} = P_{eng, max} \). As can be found in Fig. 3, when \( P_{eng} \) equals \( P_{eng, max} \), the current EF is defined as the upper limit \( s_{chg} \); and when \( P_{eng} = 0 \), the EF is defined as the lower limit \( s_{dis} \). Thus, a closed interval \([s_{dis}, s_{chg}]\) with respect to EF can be generated. When the EF locates beyond \([s_{dis}, s_{chg}]\), the vehicle will operate in either the pure electrical driving mode or maximum engine power mode. In this case, no matter how EF changes, the engine power cannot be further changed. Therefore, only EF within the closed interval will affect the power distribution between the engine and motor. Based on (5) and (15), the instantaneous equivalent fuel consumption rate \( \dot{m}_{equ}(t) \) can be obtained by \( P_{eng} \) and \( P_{bat} \), as:

\[
\dot{m}_{equ}(t) = P_{eng}(t)/(H_{LHV} \eta_f) + s(t)P_{bat}(t)/H_{LHV} \tag{17}
\]

where \( \eta_f = \eta_{eng} \) denotes the efficiency of engine driving path, as shown in Fig. 1 (Note that the fuel loss of oil circuit is not considered). Here we suppose that the efficiency of electric driving path, as detailed in Fig. 1, is \( \eta_{ele} = \eta_{mot_{lin}}/\eta_{bat} \), where \( \eta_{bat} \) is the efficiency of battery system (A similar assumption is made that the energy loss of conductor is not taken into account).

Now, equation (17) can be transformed into:

\[
\dot{m}_{equ}(t) = P_{eng}(t)/(H_{LHV} \eta_f) + s(t)P_{mot}(t)\eta_{ele}^{-\text{sgn}(P_{mot}(t))}/H_{LHV} \tag{18}
\]

According to (3), \( P_{mot} = P_{wheel}(t)/\eta_m - P_{eng}(t) \), and we can get:

\[
\dot{m}_{equ}(t) = \frac{1}{H_{LHV}} \left[ s(t)P_{wheel}(t)\eta_{ele}^{-\text{sgn}(P_{mot}(t))}/\eta_m + \left(\eta_f^{-1} - s(t)\eta_{ele}^{-\text{sgn}(P_{mot}(t))}\right) P_{eng}(t) \right] \tag{19}
\]

where \( P_{eng}(t) \in [0, P_{eng, max}] \). Actually, equation (19) is a monotonic function with \( P_{eng}(t) \), and the situations of \( P_{eng}(t) > 0 \) and \( P_{eng}(t) < 0 \) will be respectively discussed.

1) **Situation A:** when \( \eta_f^{-1} - s(t)\eta_{ele}^{-\text{sgn}(P_{mot}(t))} > 0 \), equation (19) increases monotonically. Now, \( P_{eng}(t) \) should be minimized to reduce \( \dot{m}_{equ}(t) \) and the vehicle will operate in the pure electrical driving mode.

a) If \( P_{mot} > 0 \), then:

\[
1/\eta_f - s(t)\eta_{ele} > 0 \Rightarrow s(t) < \eta_{ele}/\eta_f \tag{20}
\]

b) If \( P_{mot} < 0 \), then:

\[
1/\eta_f - s(t)\eta_{ele} > 0 \Rightarrow s(t) < 1/(\eta_f\eta_{ele}) \tag{21}
\]

where \( \eta_{ele}/\eta_f < 1/(\eta_f\eta_{ele}) \). Based on (20) and (21), when EF satisfies \( s(t) < \eta_{ele}/\eta_f \), the engine will not be turned on, regardless of the demanded power \( P_{wheel}(t) \). As such, \( \eta_{ele}/\eta_f \) should be the lower limit of EF.

2) **Situation B:** when \( \eta_f^{-1} - s(t)\eta_{ele}^{-\text{sgn}(P_{mot}(t))} < 0 \), equation (19) decreases monotonically. Then, \( \dot{m}_{equ}(t) \) should be maximized to minimize \( \dot{m}_{equ}(t) \). Now, the vehicle should operate in the maximum engine power mode.

a) If \( P_{mot} > 0 \), then:

\[
1/\eta_f - s(t)\eta_{ele} < 0 \Rightarrow s(t) > \eta_{ele}/\eta_f \tag{22}
\]

b) If \( P_{mot} < 0 \), then:

\[
1/\eta_f - s(t)\eta_{ele} < 0 \Rightarrow s(t) > 1/(\eta_f\eta_{ele}) \tag{23}
\]

Based on (22) and (23), when EF satisfies the condition of \( s(t) > 1/(\eta_f\eta_{ele}) \), the engine will work with the maximum power output in spite of \( P_{wheel}(t) \). Therefore, \( 1/(\eta_f\eta_{ele}) \) can be regarded as the upper limit of EF. From the aforementioned discussion, we can know that the optimal EF constraint should

![Fig. 3. Qualitative relationship between optimal engine power and EF.](image-url)
be $s^*(t) \in [s_{\text{dis}}, s_{\text{chg}}]$, i.e.,

$$\eta_{\text{ele}}/\eta_f \leq s^*(t) \leq 1/\eta_f \eta_{\text{ele}}$$

(24)

In this manner, a pair of limit values for the EF is determined. Next, by incorporating the optimal EF boundary and DE algorithm, a novel fusion A-ECMS is proposed to achieve the effective energy management of PHEV with acceptable calculation intensity and preferable fuel savings.

B. Application of EF Boundary: A Novel Adaptive ECMS

The overall structure of A-ECMS proposed in this study is shown in Fig. 4, which mainly includes three modules: the SOC trajectory planning, vehicle speed prediction and rolling optimization. The SOC trajectory planning module can rapidly design the global reference trajectory based only on the expected driving distance of spatial domain, and it can be considered as the reference for the SOC variation within the prediction horizon. The vehicle speed prediction module can predict the future short-term speed information based on the genetic algorithm-back propagation neural network (GA-BPNN) predictor. In each time step, the rolling optimization module combines the EF boundary and DE algorithm to regulate the EF online by incorporating the variation of SOC and expected speed information in the prediction horizon, and the optimal EF can be applied for real-time control of ECMS. The rapid planning algorithm of SOC trajectory and vehicle speed prediction method have been widely researched in our previous work [35], and the research emphasis of this study is the rolling optimization based on the built EF boundary.

1) Calculation of EF Boundary in Rolling Optimization: At time $t$, the predicted speed $V_{\text{pre}}$ can be expressed as:

$$V_{\text{pre}} = [V(t+1), V(t+2), \ldots, V(t+p)]$$

(25)

where $p$ denotes the length of predicted velocity sequence. The expected driving distance with $p$ steps can be calculated, as:

$$S_{\text{dis}}(t+p) = S_{\text{dis}}(t) + \sum_{i=1}^{p} (V(t+i))$$

(26)

where $S_{\text{dis}}(t)$ denotes the driving distance at time $t$, $S_{\text{dis}}(t+p)$ is the expected travel distance at time $t+p$. The reference $SOC_{\text{ref}}$ at the end of prediction horizon can be attained by interpolating the reference trajectory with $S_{\text{dis}}(t+p)$. Based on $V_{\text{pre}}$, the demanded power within the prediction horizon can be calculated according to (2). If the PHEV operates in the maximum engine power mode, the ending SOC should be the maximum value, i.e., $SOC_{\text{max}}$. Based on (24), the variation range of EF should be $[s_{\text{dis}} \cdot \text{eng}_\text{max}, s_{\text{chg}} \cdot \text{eng}_\text{max}]$. As discussed previously, the EF in this mode should be maximum and equal to the upper limit, and hence the optimal EF is certainly less than the upper limit. As such, within the prediction horizon, the maximum value of $s_{\text{chg}} \cdot \text{eng}_\text{max}$, which is highly conservative and rigorous, can be regarded as the upper limit of EF, as:

$$s_{\text{chg}, \text{pre}} = \max \{s_{\text{chg}} \cdot \text{eng}_\text{max}\}$$

(27)

where $s_{\text{chg}, \text{pre}}$ is the upper limit of EF, and the subscript $\text{eng}_\text{max}$ denotes that the variable is calculated in the maximum engine power mode. Similarly, when the vehicle operates in the pure electrical driving mode, the ending SOC should be minimum, and we define it as $SOC_{\text{min}}$. Now, the lower limit $s_{\text{dis}, \text{pre}}$ of EF within the prediction period can be calculated as:

$$s_{\text{dis}, \text{pre}} = \min \{S_{\text{dis}} \cdot \text{eng}_0\} = \min \left\{\frac{s_{\text{dis}} \cdot \text{eng}_0(t+1)}{s_{\text{dis}} \cdot \text{eng}_0(t+2)}, \ldots, \frac{s_{\text{dis}} \cdot \text{eng}_0(t+p)}{s_{\text{dis}} \cdot \text{eng}_0(t+1)}\right\}$$

(28)

where the subscript $\text{eng}_0$ denotes that the variable is calculated in the pure electrical driving mode. In this manner, the optimal EF boundary $[s_{\text{dis}, \text{pre}}, s_{\text{chg}, \text{pre}}]$ can be constructed, and the reachable range of SOC in the end of prediction horizon can be generated, i.e., $[SOC_{\text{min}}, SOC_{\text{max}}]$.  

2) Implementation of DE Algorithm in Rolling Optimization: The DE algorithm, firstly proposed by Storn and Price [38], [39], features simple structure, fast convergence and strong robustness [40], [41]. In this study, the optimization target of DE is EF, i.e., $s = [s_1, s_2, \ldots, s_p]$, and the optimal EF should enable $SOC(t+p)$ to satisfy the terminal constraint presented in (10). As such, the objective function of the DE can be expressed as:

$$\min(f) = \min((SOC(t+p) - SOC_{\text{ref}}(t+p)))$$

s.t. $s_{\min} \leq s_j \leq s_{\max}, j = 1, 2, \ldots, p$ (29)

where $[s_{\min}, s_{\max}]$ denotes the variation range of EF. In the rolling optimization module, $[s_{\min}, s_{\max}]$ can be updated in real-time based on the built EF boundary, i.e., $s_{\min} = s_{\text{dis}, \text{pre}}$ and $s_{\max} = s_{\text{chg}, \text{pre}}$. The calculation process of the optimal EF based on DE is illustrated in Algorithm I and detailed as follows.

1) Initialize the population: $g = 0, 1, 2, \ldots, Gm$ is the current generation ($Gm$ denotes the maximum generation). In it, the $i$th individual of $g$th generation can be expressed as:

$$s_i(g) = [s_{i,1}(g), s_{i,2}(g), \ldots, s_{i,p}(g)]$$

(30)

Based on (13), EF will be a constant within the prediction horizon. Hence, the dimension of optimization parameters...
can be reduced from \( p \) to one, thereby greatly reducing the computation intensity. The elements of initial population after simplification can be expressed as:

\[
s_i(0) = s_{\text{dis-pre}} + \text{rand}(0, 1) \cdot (s_{\text{chg-pre}} - s_{\text{dis-pre}})
\]

where \( i = 1, 2, \ldots, N \), in which \( N \) denotes the population size, and \( \text{rand}(0, 1) \) denotes a random value confined within \([0, 1]\).

2) Apply the differential strategy to conduct mutation for generating the intermediates \( v_i(g) \), as:

\[
v_i(g) = s_{r1}(g) + F \cdot (s_{r2}(g) - s_{r3}(g))
\]

where \( F \) is the scaling factor. \( s_{r1}(g) \), \( s_{r2}(g) \) and \( s_{r3}(g) \) are randomly selected from the current population, and the range of \( r1, r2 \) and \( r3 \) is equal with \( i \).

3) Determine \( s_i(g) \) of the \( g \)th population and \( v_i(g) \) based on the crossover operation.

\[
u_i(g) = \begin{cases} v_i(g), & \text{if } \text{rand}(0, 1) \leq CR \\ s_i(g), & \text{otherwise} \end{cases}
\]

where \( CR \) denotes the crossover rate.

4) Apply the greedy algorithm to select the individuals which are survived into the next generation.

\[
s_i(g + 1) = \begin{cases} u_i(g), & \text{if } f(u_i(g)) \leq f(s_i(g)) \\ s_i(g), & \text{otherwise} \end{cases}
\]

In this study, \( N = 30, Gm = 10, F = 0.5, CR = 0.9 \), and \( \varepsilon = 1 \times 10^{-4} \). In the application of Algorithm I, three scenarios can be defined according to the relationship between the reference SOC \( SOC_{\text{ref}}(t + p) \) and the reachable range of SOC in the end of prediction horizon\([SOC_{\text{min-pre}}, SOC_{\text{max-pre}}]\), as shown in Fig. 5. As can be seen, when \( SOC(t + p) \) is greater than \( SOC_{\text{max-pre}} \) or less than \( SOC_{\text{min-pre}} \), the optimal condition \( \min(|SOC(t + p) - SOC_{\text{ref}}(t + p)|) = 0 \) does not exist within the boundary of EF. Only when \( SOC_{\text{ref}}(t + p) \) is located within \([SOC_{\text{min-pre}}, SOC_{\text{max-pre}}]\), the optimal EF can be found within the boundary of EF to meet the optimal target \( \min(|SOC(t + p) - SOC_{\text{ref}}(t + p)|) = 0 \). This article adopts a simple but effective method to solve the problem. The improved calculation process of the optimal EF is shown in Algorithm II. When \( SOC(t + p) \) is greater than \( SOC_{\text{max-pre}} \) or less than \( SOC_{\text{min-pre}} \), the optimal EF can be directly set to \( SOC_{\text{chp-pre}} \) or \( SOC_{\text{dis-pre}} \) without calculation provided by the Algorithm I. In addition, this method can also avoids unnecessary searches and reduces the calculation intensity. In the next step, the simulation is conducted to verify the effectiveness of the proposed strategy.

IV. SIMULATION AND DISCUSSION

In this section, we conduct the simulations from two aspects to evaluate the proposed A-ECMS with the incorporation of built boundary theory. 1) The proposed strategy is simulated and analyzed under ideal situation to verify the feasibility of proposed EF boundary and the performance of rolling optimization module. 2) In actual situation without prior knowledge of driving conditions, the performance of proposed strategy is analyzed by comparing with DP, conventional ECMS, A-ECMS and CD/CS strategy. All the simulations were conducted on Matlab by a
TABLE I

| Vehicle Type   | Parallel PHEV |
|---------------|---------------|
| Vehicle       | Mass 1680 kg  |
| Engine        | Maximum power 73 kW |
|               | Maximum torque 154 Nm |
| Motor         | Maximum power 52 kW |
|               | Maximum torque 400 Nm |
| Transmission  | 5 speed AMT |
| Lithium-ion Battery | Maximum power 60 kW |
|               | Rated capacity 37 Ah |

desktop computer with a processor of i5 core. The target vehicle is a parallel PHEV, as shown in Fig. 1, and the basic parameters are list in Table I. The engine/motor efficiency map and battery parameters are shown in Figs. 6–7.

The efficiency of engine driving path \( \eta_f \) is equal to the engine efficiency \( \eta_{eng} \), and can be calculated according to the interpolation shown in Fig. 6(a). The efficiency of electric driving path \( \eta_{ele} \) can be calculated as \( \eta_{mot} \times (\eta_{inv} \eta_{bat}) \), where \( \eta_{mot} \) can be calculated based on Fig. 6(b), and \( \eta_{inv} \eta_{bat} \) represents the efficiency of battery system. In this paper, the inverter efficiency \( \eta_{inv} \) is set as a constant value, and the battery efficiency \( \eta_{bat} \) can be expressed as:

\[
\eta_{bat} = \left( 1 + \sqrt{1 - \left(4R(SOC)P(t)/E_{OCV}(SOC)\right)^2} \right)^{\text{sgn}(P(t))}
\]

where \( R(SOC) \) and \( E_{OCV}(SOC) \) can be interpolated with respect to the SOC shown in Fig. 7. As such, the efficiency map of battery system can be determined, as shown in Fig. 8, and the efficiency of electric driving path \( \eta_{ele} \) can be determined based on Figs. 6(b) and 8.

A. Performance Analysis of the Rolling Optimization Module

To verify the feasibility of proposed EF boundary and the performance of proposed rolling optimization module, it is necessary to avoid influence incurred by the SOC trajectory planning and vehicle speed prediction modules. As such, under the ideal situation of knowing the full driving condition, the optimal global reference SOC trajectory for the rolling optimization module is calculated by the conventional ECMS, and the future short-term vehicle speed for the rolling optimization module is replaced by the prior knowledge of driving cycle. Here, the length of predicted velocity sequence is set to 6 s. Four typical driving cycles are selected including urban dynamometer driving schedule (UDDS), 2 consecutive new European driving cycles (NEDC), worldwide harmonized light vehicles test cycle (WLTC) and 8 consecutive Manhattan cycles. After calculation, the optimal EF is \( s_{UDDS} = 2.485 \), \( s_{2 \times NEDC} = 2.569 \), \( s_{WLTC} = 2.592 \) and \( s_{8 \times Manhattan} = 2.417 \). In addition, to verify the benefits of proposed boundary for optimal EF calculation, a comparison is conducted with a loose range of EF, i.e., [0, 30]. The simulation results of the conventional ECMS, proposed strategy and loose boundary based ECMS under four typical driving cycles are compared in Table II. The number of iterations...
TABLE II
SIMULATION RESULTS OF THE THREE DIFFERENT STRATEGIES UNDER FOUR DRIVING CYCLES

|                | Initial SOC | Final SOC | Fuel (g) | Time (s) |
|----------------|-------------|-----------|----------|----------|
| UDDS           | 0.4         | 0.302     | 105.24   | /        |
| Conventional ECMS | 0.4         | 0.302     | 105.25   | 94.90    |
| Proposed Algorithm | 0.4         | 0.302     | 106.01   | 569.19   |
| Loose boundary ECMS | 0.4         | 0.316     | 302.71   | /        |
| Conventional ECMS | 0.45        | 0.316     | 302.71   | 170.63   |
| Proposed Algorithm | 0.45        | 0.316     | 303.52   | 875.22   |
| Loose boundary ECMS | 0.4         | 0.311     | 538.51   | /        |
| Conventional ECMS | 0.4         | 0.311     | 538.51   | 254.06   |
| Proposed Algorithm | 0.4         | 0.311     | 540.59   | 1103.06  |
| Loose boundary ECMS | 0.4         | 0.302     | 177.21   | /        |
| Conventional ECMS | 0.6         | 0.302     | 177.35   | 369.38   |
| Proposed Algorithm | 0.6         | 0.302     | 179.47   | 1338.29  |

Note: fuel denotes the fuel consumption, time represents the total simulation calculation time of the proposed algorithm under the ideal situation with full knowledge of the driving cycle.

From Table II, we can find that the proposed strategy can achieve the same fuel consumption, compared with that of the conventional ECMS, under ideal situation with prior knowledge of driving conditions, and the calculation time of proposed strategy is much less than that of the loose boundary based ECMS. This is because the initial searching range of EF can be narrowed by the proposed boundary theory, thus reducing the calculation effort of DE algorithm. As shown in Fig. 9, the probability that the proposed strategy satisfies the termination criteria in one iteration is more than 80%; and in contrast, most of the time the DE with the loose boundary needs to iterate two times before reaching the termination conditions. As compared in Fig. 10 (b), the SOC trajectory of proposed algorithm is almost identical with the conventional ECMS, highlighting that the proposed strategy can accurately track the global reference SOC trajectory. From Fig. 10(c) and (d), we can find that the torques of engine and motor calculated by the two algorithms are almost the same. This indicates that the proposed rolling optimization module based on the EF boundary and DE algorithm can accurately calculate the optimal EF. To conclude, the proposed algorithm enables that the powertrain of PHEV can achieve the same performance as the conventional ECMS.

From Table II, we can find that the fuel consumption is relatively low under the UDDS and Manhattan cycle and this indicates that most of the time, the engine remains off. Even the
engine is turned on, its power is still low. Therefore, the EFs in Fig. 11(a) and (d) are very close to $s_{dis}$. From Fig. 10(d) and Fig. 11(a), we can find that when the real-time optimal EF is equal to $s_{dis}$, the engine torque is zero, and the PHEV will operate in the pure electrical driving mode; and when the engine is turned on, the real-time EF will vary within the limitations. For example, when $t$ equals 100 s, $s(t) = s_{dis} = 2.263$, and the engine output torque is 0 Nm; when $t$ equals 200 s, $s_{dis} = 2.432$, $s_{chg} = 3.405$, the optimal EF $s(t)$ searched by the DE algorithm in the receding horizon is 2.494, and the engine output torque is 134 Nm. In this manner, the effectiveness of proposed EF boundary theory is manifested.

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length increases, the RSME will increase, which means that the accuracy of GA-BPNN predictor will be deteriorated. In other words, the EF cannot be estimated with higher precision, and thus the fuel economy cannot be further improved. In addition, longer prediction horizon will incur heavier computation burden. By trading off computation time and fuel consumption savings, we chose 12 s as the final prediction length in this study. In addition, the true speed, and two different velocity predictors, i.e., the exponentially varying predictor and Markov-chain predictor [45], are applied to compare the influences of different prediction accuracy on the energy management effects. Here, the true speed represents the ideal case, of which the prediction accuracy is 100%. The exponentially varying predictor and Markov-chain based predictor are introduced in the previous study [45]. The predicted velocity sequences of each algorithm under the UDDS are shown in Fig. 13. Table III provides the simulation results and predicting performance of different algorithms.

From Fig. 13, the exponentially varying predictor and Markov-chain predictor roughly predict the vehicle speed. In contrast, the predicted velocity sequences of the GA-BPNN predictor can captures the trend of the actual vehicle speed better than the others. In Table III, $T_{com}$ is the online computation time of the velocity predictor at each time step (Note that the time step in this paper is 1 s). As the online computation time of GA-BPNN predictor is only 0.0714 ms at each time step, the delay effects in detection of the prior data is not considered in the simulation. As presented in Table III, more accurate velocity prediction can lead to lower fuel consumption. The results show that improving the accuracy of velocity prediction can contribute to the energy saving of proposed algorithm. By in-depth analysis of $\Delta$Fuel and $\Delta$RSME, we can find that as the forecast accuracy increases, the energy consumption savings raised by the prediction will become less. That said when the prediction is high enough, the further improvement of fuel savings is tiny and even neglectable.

2) Energy-Saving Performance of the Proposed Strategy Under Combined Driving Cycle: To furthermore evaluate the proposed strategy, two combined driving cycles are designed and simulated. Their speed profiles are shown in Fig. 14, and we can find that the cycle I is mainly focused on the urban conditions with frequent acceleration and deceleration. The cycle I, of which the duration is 3550 s and the length is 18.64 Km, consists of two Manhattan cycles and one UDDS. The cycle II is comprised of one WLTC, one Japanese 10-15 mode (JN1015) cycle and one UDDS cycle. According to our previous work [35], the SOC reference trajectory can be quickly determined as:

$$SOC(t) = SOC_{int} - \frac{S(t)}{S_{cyc}}(SOC_{int} - SOC_{min}) \quad (37)$$

where $S(t)$ is the driving distance at time $t$, and $S_{cyc}$ is the entire travel distance. The SOC reference trajectory is shown in Fig. 15. As the speed profile of cycle I meets the simplified assumption in [35] more than the cycle II, the reference SOC trajectory of cycle I is closer to the ideal SOC trajectory calculated by DP.

| Predictor          | Initial SOC | Final SOC | $T_{com}$ (ms) | Fuel (g) | RSME (m/s) | $\Delta$RSME (m/s) | $\Delta$Fuel (g) |
|--------------------|-------------|-----------|----------------|---------|------------|-------------------|-----------------|
| Exponentially      | 0.4         | 0.302     | 0.0018         | 111.584 | 3.64       | 0.38              | 1.83            |
| Markov-chain       | 0.4         | 0.302     | 1.2142         | 109.715 | 3.26       | 0.53              | 1.407           |
| GA-BPNN            | 0.4         | 0.302     | 0.0714         | 108.308 | 2.73       | 2.73              | 1.227           |
| True speed         | 0.4         | 0.302     | /              | 107.081 | 0          |                   |                 |

**TABLE III**

**SIMULATION RESULTS AND PREDICTOR PERFORMANCE METRICS OF THE TESTED PREDICTORS**

![Fig. 13. Predicted velocity sequences of UDDS. (a) Exponentially varying predictor; (b) Markov-chain predictor; (c) GA-BPNN predictor. The black curve presents the actual speed of UDDS, and the red curve denotes the predicted velocity sequences across the prediction horizon at each time step.](image)

![Fig. 14. Speed profiles of the two combined driving cycles. (a) Combined driving cycle I; (b) Combined driving cycle II.](image)

![Fig. 15. SOC reference trajectory of the two combined driving cycles. (a) Combined driving cycle I; (b) Combined driving cycle II.](image)
TABLE IV
SIMULATION RESULTS OF DIFFERENT STRATEGIES UNDER COMBINED DRIVING CYCLES

| Initial SOC | Final SOC | Fuel (g) | Situation | Type       | Fuel saving | Time (s) |
|-------------|-----------|---------|-----------|------------|-------------|----------|
| Combined driving cycle I
| DP         | 0.4       | 0.301   | 196.68    | Ideal      | Offline     | /        | 725.96   |
| ECMS        | 0.4       | 0.302   | 197.37    | Ideal      | Offline     | /        | /        |
| CD/CS       | 0.4       | 0.302   | 233.82    | Actual     | Online     | 81.12% (DP) | 2.47     |
|             |           |         |           |            |             | 81.53% (ECMS) |         |
| A-ECMS      | 0.4       | 0.301   | 205.68    | Actual     | Online     | 95.42% (DP) | 2.73     |
|             |           |         |           |            |             | 95.79% (ECMS) |         |
| Proposed    | 0.4       | 0.302   | 202.18    | Actual     | Online     | 97.20% (DP) | 390.01   |
|             |           |         |           |            |             | 97.56% (ECMS) |         |
| Combined driving cycle II
| DP         | 0.4       | 0.301   | 796.28    | Ideal      | Offline     | /        | 904.38   |
| ECMS        | 0.4       | 0.302   | 797.47    | Ideal      | Offline     | /        | /        |
| CD/CS       | 0.4       | 0.304   | 880.14    | Actual     | Online     | 89.47% (DP) | 3.03     |
|             |           |         |           |            |             | 89.63% (ECMS) |         |
| A-ECMS      | 0.4       | 0.301   | 843.84    | Actual     | Online     | 94.03% (DP) | 3.24     |
|             |           |         |           |            |             | 94.16% (ECMS) |         |
| Proposed    | 0.4       | 0.302   | 820.22    | Actual     | Online     | 97.14% (ECMS) | 479.70   |

To analyze the performance of proposed strategy in depth, we compare the simulation results based on the CD/CS strategy, A-ECMS and proposed strategy without prior knowledge of driving condition. In addition, the simulation results of DP and conventional ECMS under ideal situation are considered as the benchmarks for comparison. The optimal EF of conventional ECMS under two cycles are \(s_{I} = 2.5040\) and \(s_{II} = 2.6175\), respectively. Here, we designed the A-ECMS based on the PI controller. According to [46], the real-time EF update law can be expressed as:

\[
s(t) = s_0 + K_p \Delta SOC + K_i \int_0^t (\Delta SOC) d\tau \tag{38}
\]

where \(s_0 = 2.55\) denotes the initial value of EF, and \(\Delta SOC = SOC_{ref}(t) - SOC(t)\) is the difference between the reference SOC and actual value. \(K_p = 800\) and \(K_i = 300\) are the proportional and integral parameters of PI controller, and \(SOC_{ref}\) is the SOC reference trajectory, as shown in Fig. 15. For ease of comparing the energy saving effect incurred by different strategies, an evaluation parameter is introduced, as:

\[
Y = \left( 1 - \frac{\text{consumption} - \text{benchmark}}{\text{benchmark}} \right) \times 100\% \tag{39}
\]

where \(\text{consumption}\) is the fuel consumption of CD/CS strategy, A-ECMS and proposed strategy, and \(\text{benchmark}\) denotes the fuel consumption of DP and conventional ECMS. Table IV provides quantitative simulation results based on these five strategies, and Fig. 16 shows the SOC trajectories in terms of DP, CD/CS, A-ECMS and proposed strategy.

As can be found in Table IV, the global optimization EMS based on DP and the conventional ECMS with optimal EF lead to the optimal fuel consumption. Nonetheless, both need to acquire the driving condition in advance, and obviously it is impossible to apply online. The proposed strategy in this study only needs to obtain the expected driving distance before departure, and thus becomes more applicable than DP and ECMS. Although the CD/CS strategy can be applied online, it yet leads to poor energy savings, which is only 81.12% of that by DP and 81.53% of that by conventional ECMS.

The proposed strategy and PI based A-ECMS can both obtain the preferable fuel savings, i.e., more than 95% of DP savings, in the cycle I. The reasons can be attributed to the following two aspects. Firstly, as can be found in Fig. 15(a), only slight difference exists between the reference SOC trajectory calculated based on the driving distance and the ideal SOC curved solved by DP. Secondly, the A-ECMS can regulate the EF in real time according to the error between the feedback SOC and reference...
value, thus the real-time SOC can follow the SOC reference trajectory and the A-ECMS can attain 95.42% savings of DP. The proposed algorithm can integrate the reference SOC trajectory and predicted velocity, of which the latter can compensate the inaccurate reference SOC trajectory. Thus, the energy saving effect of proposed strategy is 2% higher than that of the PI based A-ECMS, and it attains more than 97% effect of DP solution.

In terms of the cycle II that is mainly dominated by moderate and high speed conditions, there exists obvious differences between the reference SOC trajectories calculated according to the driving distance and that solved by DP. The energy saving of A-ECMS decreases by 1% due to the influence raised by inaccurate reference SOC trajectory. In contrast, the proposed algorithm can consider the reference SOC trajectory and predicted velocity, of which the latter can compensate the drawback of inaccurate reference SOC trajectory, thereby still attaining 97% savings. As can be seen from the enlarged subgraphs in Fig. 16, the SOC variation solved by A-ECMS is closer to the referred one; while the SOC based on the proposed algorithm remains the similar declining trend, compared with the referred trajectory, and is closer to the ideal SOC trajectory calculated by DP.

Table IV compares the total calculation time of all algorithms. As the conventional ECMS needs to adjust and find the optimal EF offline by iterative regulation manually, which is difficult to get the exact time cost. Therefore, it is not taken into account. As DP is a typical offline EMS, the global transportation information should be acquired in advance, and then the optimal policy matrix can be inversely solved. As well known, it is not qualified for real-time application. The total time cost of proposed algorithm is 390 s when it is applied in the combined cycle I with the duration of 3550 s, and the average time cost per time step is 0.11 s. When it is applied in the cycle II with the duration of 3831 s, the total calculation time 480 s and the average time per time step is 0.125 s. Although it is slightly longer than those of the CD/CS strategy and A-ECMS, it can still satisfy the demand of online application. Additionally, as illustrated in Fig. 16, the proposed strategy can accurately track the global reference SOC trajectory and meanwhile satisfy the SOC terminal constraints, manifesting the effectiveness of the proposed algorithm. To sum up, the proposed algorithm features better self-adaptivity and robustness as well as better fuel saving potential, compared with the CD/CS strategy and A-ECMS.

V. CONCLUSION

This paper derives the optimal EF boundary of ECMS by analyzing only efficiency characteristics of the vehicle powertrain. A conclusion can be made that in the blended driving mode of PHEV, the optimal EF must be between the proposed boundaries. In other two special operating modes, the optimal EF can be directly derived from the proposed boundary principle. Then, the optimal EF boundary is applied in a framework of MPC. By combining the optimal EF boundary and DE algorithm, a novel predictive A-ECMS is introduced to highlight the application prospect of the proposed optimal boundary theory. On the account of knowing global driving information, the proposed A-ECMS can achieve the near-optimal energy savings, and in the premise of only acquire the anticipated driving distance, the proposed strategy can attain more than 97% fuel savings solved by DP, suggesting effectiveness of the proposed boundary and the novel energy management strategy.

In the next step, how to incorporate identification of the driving condition and online optimal EF search to faster the operating efficiency will be our research focus. In addition, the hardware-in-the-loop validation and real vehicle test will also be conducted in the future.

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