Observation of quantum state collapse and revival due to the single-photon Kerr effect

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To create and manipulate non-classical states of light for quantum information protocols, a strong, nonlinear interaction at the single-photon level is required. One approach to the generation of suitable interactions is to couple photons to atoms, as in the strong coupling regime of cavity quantum electrodynamics systems1,2. In these systems, however, the quantum state of the light is only indirectly controlled by manipulating the atoms3. A direct photon–photon interaction occurs in so-called Kerr media, which typically induce only weak nonlinearity at the cost of significant loss. So far, it has not been possible to reach the single-photon Kerr regime, in which the interaction strength between individual photons exceeds the loss rate. Here, using a three-dimensional circuit quantum electrodynamic architecture4, we engineer an artificial Kerr medium that enters this regime and allows the observation of new quantum effects. We realize a gedanken experiment5 in which the collapse and revival of a coherent state can be observed. This time evolution is a consequence of the quantization of the light field in the cavity and the nonlinear interaction between individual photons. During the evolution, non-classical superpositions of coherent states (that is, multi-component Schrödinger cat states) are formed. We visualize this evolution by measuring the Husimi Q function and confirm the non-classical properties of these transient states by cavity state tomography. The ability to create and manipulate superpositions of coherent states in such a high-quality-factor photon mode opens perspectives for combining the physics of continuous variables6 with superconducting circuits. The single-photon Kerr effect could be used in quantum non-demolition measurement of photons7, single-photon generation8, autonomous quantum feedback schemes9 and quantum logic operations10.

A material whose refractive index depends on the intensity of the light field is called a Kerr medium. A light beam travelling through such a material acquires a phase shift \( \phi_{\text{Kerr}} = KtI \) (ref. 11), where \( K \) is the Kerr constant, \( t \) is the interaction time of the light field with the material, and \( I \) is the intensity of the beam. The Kerr effect is a widely used phenomenon in nonlinear quantum optics, and has been successfully used to generate quadrature and amplitude squeezed states12, used in quantum state manipulation13, and create ultra-fast pulses14. The Kerr effect for a quantized mode of light with frequency \( \omega_L \) can be described by the normal ordered Hamiltonian, 

\[
H_{\text{Kerr}} = \hbar \omega_L a^\dagger a - \hbar K \frac{1}{2} a^\dagger a^\dagger a a,
\]

with \( K \) the Kerr frequency shift per photon and \( a, a^\dagger \) the ladder operators. The average phase shift per photon is again given by \( \phi_{\text{Kerr}} = KtI \), with \( K \) the decay rate of the photon mode. Typical Kerr effects are so small that they are not visible on the single-photon level because \( K > K \). Applications which require \( K \) much bigger than the cavity-kerr frequency shift per photon can be used to generate coherent states with a mean photon number \( \langle n \rangle \) much higher than the single-photon level because

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\langle n \rangle = \frac{1}{4} (KtI)^2.
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The Kerr nonlinearity of a Josephson junction is naturally created by the nonlinear inductance \( I \) due to the single-photon Kerr effect. Observation of quantum state collapse and revival

LETTER

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factor of about one million, limited by internal losses, corresponding to a single-photon decay rate $\kappa/2\pi = 10$ kHz. The vertical transmon consists of a single Josephson junction embedded in a transmission line structure, which couples the junction to both cavities. The observed transition frequency of the qubit is $\omega_0/2\pi = 8.2564$ GHz and its anharmonicity is $K/2\pi = \{\omega_0 - \omega_0\} / 2\pi = 7.34$ MHz using the standard convention for labelling from lowest to highest energy level in the qubit as $|g,e,f,h,...\rangle$ (see Supplementary Information). The energy relaxation time of the qubit is $T_1 = 10$ $\mu$s with a Ramsey time $T_2 = 8$ $\mu$s. The qubit is used to interrogate the state of the storage cavity, which acts as a Kerr medium. The other cavity is used to read out the state of the qubit after the interrogation, similarly to ref. 24.

The analysis of the distributed stripline elements and the cavity electrodynamics can be performed using finite-element calculations for the actual geometry. Combined with ‘black-box’ circuit quantization, one can derive dressed frequencies, couplings and anharmonicities with good relative accuracy (see Supplementary Information). For the purposes of the experiments discussed here, the coupling of the qubit to the storage resonator, in the strong dispersive limit of circuit QED, is well described by the Hamiltonian

$$\frac{H}{\hbar} = \frac{\omega_0}{2} a^\dagger a + \left(\frac{\omega_0 - K}{2}\right) a^\dagger a + \frac{K}{2} a^3 a$$

(1)

taking into account only the lowest two energy levels of the qubit. The operators $a^\dagger a$ are the usual raising/lowering operators for the harmonic oscillator and $a$ is the Pauli operator. In this description, we completely omit the measurement cavity because it is only used for reading out the state of the qubit and otherwise stays in its ground state. The energy level diagram described by the Hamiltonian given in equation (1) can be seen in Fig. 1b. The second term on the right-hand side of equation (1) is the state-dependent shift per photon $\sqrt{2}\gamma/2\pi = 9.4$ MHz of the qubit transition frequency. The last two terms on the right-hand side of equation (1) describe the cavity as an anharmonic oscillator with a dressed resonance frequency $\omega_c$ and a nonlinearity $K/2\pi = 325$ kHz which is given by $K = \gamma^2/4K_\text{eff}$ (ref. 15). All interaction strengths in the above Hamiltonian are at least one order of magnitude bigger than any decoherence rate in the system.

To visualize and understand the evolution of the resonator state, we measure the Husimi $Q$ function $Q_\theta$ in a space spanned by the expectation value of the dimensionless field quadratures $Re(\chi)$ and $Im(\chi)$. $Q_\theta$ is defined as the modulus squared of the overlap of the resonator state $|\Psi\rangle$ with a coherent state $|\beta\rangle$ by $Q_\theta(\chi) = |\langle \beta | \Psi \rangle|^2$. Alternatively, we can write $Q_\theta$ using the displacement operator $D_\beta = e^{\beta a^\dagger - \beta^* a}$ (note that $D_\beta^2 = D_{-\beta}$), as $Q_\theta(\chi) = \frac{1}{\pi} \left| \langle 0 | D_{-\chi} | \Psi \rangle \right|^2$, which describes the actual measurement procedure used in the experiment. The sequence to measure $Q_\theta$ can be seen in Fig. 2a. The initial displacement, $D_\beta$, creates a coherent state $|\Psi\rangle = \beta |0\rangle$ in the cavity, whose $Q_\theta$ is given by a Gaussian, $\frac{1}{\sqrt{2\pi}} e^{-|\beta|^2}$. After a variable waiting time $t$, we measure $Q_\theta(\chi)$ by displacing the cavity state by $-\chi$ and determine the overlap of the resulting wavefunction with the cavity ground state. The population of the cavity ground state can be measured by applying a photon number state selective pulse, $X_{\beta n} = e^{\beta a^\dagger n a - \beta^* a^\dagger n a}$, to the qubit (see Supplementary Information), similarly to ref. 25. The qubit is excited if and only if the cavity is in the $n = 0$ Fock state, $|n\rangle$ being the photon number, after the analysis displacement. This scheme allows us to determine $Q_\theta(\chi)$ of the resonator up to experimental imperfections (see Supplementary Information for details). Applying $\sigma$ pulses to the qubit conditioned on other photon numbers, $X_{\beta n}$ allows us to measure the overlap of the displaced state with any Fock state $|n\rangle$, which we will call the generalized $Q$ functions $Q_\theta(\chi) = \frac{1}{\pi} |\langle n | D_{-\chi} | \Psi \rangle|^2$. In essence, we can ask the question ‘are there $n$ photons in the resonator?’, using photon number state selective pulses. To test the analysis protocol, we measured $Q_\theta$ and $Q_\chi$ of the cavity in the ground state, Fig. 2b–e, by omitting the first displacement pulse of the sequence given in Fig. 2a.

Using this method, we can follow the time evolution of a coherent state in the presence of the Kerr effect. In the experiment, we prepare a coherent state with an average photon number $|\beta|^2 = \bar{n} = 4$ using a microwave pulse to displace the cavity state. We then measure $Q_\theta$ for different delays between the preparation and analysis pulses. A comparison of the theoretical evolution of the coherent state and the measured evolution can be seen in Fig. 3. The time evolution of the state is described by considering the action of the Kerr Hamiltonian $H_{\text{Kerr}}$ on a coherent state $|\beta\rangle$ in the cavity. In the rotating frame of the harmonic oscillator, with the qubit in the ground state, we can write:

$$|\Psi(t)\rangle = e^{i\chi a^\dagger a} |\beta\rangle = e^{-|\beta|^2/2} \sum \frac{\beta^n}{\sqrt{n!}} e^{i\chi a^\dagger a} |n\rangle$$

(2)

For short times, the nonlinear phase evolution of the Fock states $|n\rangle$ is closely approximated by a rotation of the state with an angle $\phi_{\text{Kerr}} = KT(|\beta|^2 + 1/2)$ with respect to the frame rotating at $\omega_c$. The onset of this rotation can be seen in Fig. 3a, which is taken at the minimal waiting time of 15 ns between the two displacement pulses. Because of this waiting time, the state rotates under the influence of the Kerr effect from $\beta = 2$ to $|\beta|^2 = 2e^{0.13}$. For longer times we can see how the state rotates further and spreads out on a circle (Fig. 3b, c). This spreading can be simply understood in a semi-classical picture, in which the amplitude components in the coherent state further away from the origin evolve with a higher angular velocity given by the $n^2$ dependence of the Kerr effect. Complete phase collapse is reached at a time when the phase dispersion across the width of the photon number distribution corresponds to $\pi$, which can be estimated as $T_{\text{col}} = \frac{\pi}{2\sqrt{nK}}$ (ref. 2). For our system, the complete phase collapse

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**Figure 1** | Device layout and energy level diagram of the two-cavity, one-qubit device. a. Photograph of one half of two aluminium (alloy 6061) waveguide cavities coupled to a vertical transmon qubit. Right, a magnified view of the indicated area of the photograph, showing a detail of the qubit, fabricated on a $c$-plane sapphire substrate 1.4 mm wide, 15 mm long and 430 $\mu$m thick. The coupling strength of the qubit is determined by the length of the stripline coupling antenna which extends into each cavity. The upper cavity, with a resonant frequency of $\omega_{01}/2\pi = 8.2564$ GHz, is used for qubit readout, and the lower cavity, with a frequency of $\omega_{02}/2\pi = 9.2747$ GHz, is used to store and manipulate quantum states. b. Combined energy level diagram of the qubit coupled to the storage cavity. The qubit states are denoted as $|g\rangle$ and $|e\rangle$, respectively, while the cavity states are labelled as $|o\rangle$, with $n$ the number of photons in the cavity. Each photon in the cavity reduces the qubit transition frequency by $\chi$. Equivalently, exciting the qubit reduces the cavity transition frequency by $\chi$. The energy levels of the cavity are not evenly spaced owing to the induced Kerr anharmonicity, $K/2\pi = 325$ kHz.
complete state revival to a coherent state with opposite phase. For the oscillator as a superposition of coherent states, the coherent states have to be separated by more than twice their width on a circle with a radius given by the initial displacement. In other words, the coherent states have to be quasi-orthogonal. This means that for a displacement of $|\beta| = 2$, the maximum number of coherent states that can be distinguished is 4.

In Fig. 3g, we can see how the state again completely dephases shortly before the coherent revival in Fig. 3h after $t = 3.065$ ns. After this time, we get a state with amplitude $|\beta| = 1.78(2)$, which fits to the expected decay of the resonator state. The theoretical plots for Fig. 3i were simulated by solving a master equation using the decay rate $\kappa/2\pi = 10$ kHz of the resonator and introducing a small detuning of 5 kHz of our drive from the resonator frequency $\omega_c$. The time evolution of the state in the experiment agrees well with the theory. The hazy ring that can be seen in theory and experiment in Fig. 3h is produced by cavity decay. The evolution of the state from 0 to 6.05 μs over 50 frames, including two revivals, can be seen in the Supplementary Video.

To get a more quantitative comparison of experiment and theory, we need to determine the quantum state of the resonator. Although in principle one can reconstruct the cavity wavefunction from the measured Husimi Q function, in practice there is important information, such as the interference fringes between coherent state superpositions, which is exponentially suppressed as the separation of the coherent states increases. This makes it hard to distinguish a mixture of coherent states from a coherent superposition in an experiment due to a finite signal to noise ratio. An experimentally more practical way to determine the quantum state of a resonator is, for example, to reconstruct its Wigner function, as this emphasizes the interference fringes.

The Wigner function of a cavity has been determined by measuring the statistical density of states from a coherent superposition in an experiment due to a Poisson distribution $|x|^n e^{-|x|^2}$. For $q = 2$, we get the two-component Schrödinger cat state, similar to the cat states created in refs 22 and 27. To distinguish the $q$ components of a cat state, the coherent states have to be separated by more than twice their width on a circle with a radius given by the initial displacement. In other words, the coherent states have to be quasi-orthogonal. This means that for a displacement of $|\beta| = 2$, the maximum number of coherent states that can be distinguished is 4.

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The decay of the resonator state is also responsible for a reduction in the fidelity is due to the spurious excited state population well described by the wavefunction given in equation (2). The main fringes, which demonstrates that the evolution is indeed coherent and cat states, respectively. The Wigner functions show clear interference actions in a cavity, entering the single-photon Kerr regime where $K \gg k$. We are able to observe the collapse and revival of a coherent state due to the intensity-dependent dispersion between Fock states in the cavity. This opens the possibility of using such a Kerr medium for error correction schemes where a nonlinear cavity is used to realize the necessary components. The good agreement between the theory and the experiment demonstrates the accurate understanding of this system. It also confirms our ability to predict higher-order couplings, which is a necessary ingredient for understanding the behaviour of large circuit QED systems. Furthermore, we have measured the evolution of a coherent state in a Kerr medium at the single-photon level, and shown a new experimental way for creating and measuring multi-component Schrödinger cat states. This demonstrates the ability to create, manipulate and visualize coherent states in a larger Hilbert space, and opens up new directions for continuous variable quantum computations.

Figure 3 | Time evolution of $Q_0$ for a coherent state in the nonlinear cavity. Shown are experiment (upper row) and theory (lower row) for a coherent state $\beta = 2$; the time $t$ for the frames shown in a–h is given above each panel. We measure $Q_0$ at 441 different analysis displacements $x$. The resolution of the pictures was doubled by interpolation. Initially the phase of the state spreads rapidly, a–c, leading to a complete phase collapse after a characteristic time $T_{\text{coll}} = \frac{\pi}{2\sqrt{2K}} = 385$ ns. For short times, the Kerr interaction leads to a quadrature squeezed state along the Re($\alpha$) axis, which can be seen in b. After the complete phase collapse, structure emerges again, d–h, at times which are integer submultiples of the complete revival time $T_{\text{rev}} = \frac{2\pi}{K}$. At these times, one obtains coherent state superpositions which are multi-component cat states, up to a maximum number of resolvable components set by the average photon number in the initial coherent state displacement. The colour scale for $Q_0$ of each pair of plots is individually rescaled, with the scaling parameter given for a–h respectively by $A = 1.0, 0.93, 0.32, 0.24, 0.3, 0.46, 0.24, 0.89$. The negative amplitudes in plot a are due to a 10% excited state population of the qubit which evolves in a different rotating frame (see Supplementary Information). This excited state population is not visible in the other frames as it disperses quickly and vanishes in the large positive amplitudes.

Figure 4 | Wigner function of the multi-component cat states emerging during the Kerr interaction. The top row shows the Wigner functions, reconstructed by cavity state tomography, of a coherent state subject to a Kerr interaction for a time $t$. The lower row shows the theoretically expected Wigner functions for the same interaction time obtained by a simulation including the decay of the cavity. The Wigner functions are reconstructed from measurements of the quasi-probability distributions $Q_n(x)$ for $n = 0–7$. Each $Q_n(x)$ was measured at the same displacements as the corresponding data in Fig. 3. Comparing the experimentally obtained state to an ideal cat state, we find a fidelity $F_2 = 0.71, F_3 = 0.70, F_4 = 0.71$ for the two-, three- and four-component cat states respectively. The experimental data show excellent correspondence with theoretical predictions, including the interference fringes and regions of negative quasi-probability distribution, confirming that highly non-classical states are produced by the Kerr evolution.

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