Noise reduction and hyperfine level coherence in spontaneous noise spectroscopy of atomic vapor

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We develop a system for measurements of power spectra of transmitted light intensity fluctuations, in which the extraneous noise, including shot noise, is reduced. In essence, we just apply light, measure the power of the transmitted light and derive its power spectrum. We use this to observe the spontaneous noise spectra of photon atom interactions. Applying light with frequency modulation, we can also observe the spontaneous noise reflecting the coherence between the hyperfine levels in the excited state. There are two main novel components in the measurement system, the noise reduction scheme and the stabilization of the laser system. The noise reduction mechanism can be used to reduce the shot noise contribution to arbitrarily low levels through averaging, in principle. This is combined with differential detection to keep unwanted noise at low levels. The laser system is stabilized to obtain spectral width below 1 kHz without high frequency (≥ 10 MHz) noise. These methods are described systematically and the performance of the measurement system is examined through experimental results.

I. INTRODUCTION

To study the nature of photon atom interactions, a simple experiment would be to just shine light on atoms and measure the fluctuations in the transmitted light. We call these fluctuations “spontaneous noise” since they arise solely from photon atom interactions, without any other external perturbations. By measuring its spectra over a wide frequency range and analyzing its structure, we may obtain a comprehensive picture of photon atom interactions. Transmitted light has a large intensity when compared to fluorescence, making it possible to measure details of the spontaneous noise spectra.

There is a well known obstacle that needs to be overcome in achieving such measurements; namely, the shot noise that inevitably arises in the photon flux. The shot noise appears as white noise in the current fluctuation spectrum, through photoconversion, as

\[ \langle (\Delta I)^2 \rangle = 2eI\Delta f \]  

(1)

Here, \( e \) is the elementary charge, \( I \) the electric current and \( f \), the frequency. As explained below, we reduce this noise statistically, while retaining the spontaneous noise, to obtain its spectra below shot noise levels. In practice, there will be other unwanted noise that occur and we shall also need to eliminate them. The two main components of the measurement system are the noise reduction scheme and the laser stabilization. This measurement system, which combines the averagings of correlations and differential detection was first used in [4], leading to high contrast observations of spontaneous noise spectra. A main objective in this work is to describe the principles underlying this system and to investigate its efficacy through the systematic analysis of experimental results. The measurement system is relatively simple and easy to set up, so that we believe that there is some merit in explaining this system for future use. Another main objective here is to report on the observations of spontaneous noise arising from hyperfine levels with coherence, which has not been seen previously.

To reduce shot noise in measurements, several approaches are conceivable: If one is trying to measure a signal with a known frequency, one can make measurements at this frequency to reduce the shot noise, by effectively making \( \Delta f \rightarrow 0 \) in Eq. (1). However, such method precludes obtaining spectra over a wide frequency range, which is our objective. Another approach is to use a squeezed light source which has smaller shot noise from the start. While this approach is elegant and has yielded shot noise reduction by a factor of two, it seems so far to be difficult to use this to reduce the shot noise by orders of magnitude. Yet another approach to reduce shot noise, relatively, is to increase the intensity of the light source, since the intensity power spectrum behaves as the square of the intensity and the shot noise behaves as a single power, as seen in Eq. (1). Such approach is not applicable to the spontaneous noise spectra measurements, since changing the light intensity changes the properties of the spontaneous noise itself, which is what we wish to study.

In this work, we use a coherent light source and reduce shot noise through averagings by a factor of \( 10^{-3} \). This noise reduction principle is simple and has a wide range of applicability; it has proven effective in obtaining surface thermal fluctuation spectra of liquids, complex fluids and biological materials.

In Sect. II, the noise reduction scheme we employ is systematically explained and the methods used to stabilize the laser system are described in Sect. III. The performance of these methods are examined in the measurements of spontaneous noise spectra in Sect. IV. In Sect. V, spontaneous noise arising from Rabi flopping between hyperfine levels coupled through the frequency modulation of the applied light is investigated. Sect. VI contains conclusions and discussion.
Fig. 1(a). A laser light is shone on a cell containing spontaneous noise intact, we partition the photons in each transmitted light beam to allow for correlation measurements. These measurements contain extraneous noise, both correlated and uncorrelated, in addition to the spontaneous noise, which we want to measure. We shall remove this extraneous noise by combining statistical methods and differential detection. The experimental setup with varying degrees of noise reduction are shown in Fig. 1(a)–(d) and are correspondingly explained in the following subsections a–d. They will subsequently be referred to as systems a–d.

a. The basic concept of the measurement: Conceptually, the measurement of the power spectrum of spontaneous noise is straightforward and is indicated in Fig. 1(a). A laser light is shone on a cell containing atomic vapor. The transmitted light intensity is converted to an electric current by a photodetector (PD), which is then converted to a digital signal through an analog-to-digital converter (ADC) and Fourier transformed to obtain the spectrum.

The shortcoming of this setup is that unwanted noise can not be reduced enough to obtain the spectrum of the spontaneous noise, $S$. Two main kinds of such noise exist. First is the shot noise, $N$, which is a quantum fluctuation of the photon number that leads to the noise in the photocurrent as Eq. (1). Second is the noise $L$ induced by the laser noise. Some amplitude modulation (AM) and frequency modulation (FM) noise exist in the laser system, even when care is taken to minimize them, as described below. In particular, the effects of FM noise are not negligible even when the spectral width is narrowed using an external cavity.

Denoting the measured photocurrent at time $t$ as $\langle \hat{D}_A(t) \rangle = \langle \hat{S}(\omega) \rangle^2 + \langle \hat{L}(\omega) \rangle^2 + \langle \hat{N}(\omega) \rangle^2$.

Here, $\langle \rangle$ denotes averages, tildes denote Fourier transforms and $\omega$ is the (angular) frequency. We used the property that $\langle \hat{S}(t) \rangle$, $\langle \hat{L}(t) \rangle$, $\langle \hat{N}(t) \rangle$ arise from different physics and hence are uncorrelated in this setup. In the measurements, the magnitudes of these three noises are comparable and there are no distinct criteria for separating them. So it is not possible to extract the spontaneous noise spectrum $\langle \hat{S}(\omega) \rangle^2$ with this method.

b. Differential detection: In Fig. 1(b), a measurement system using two independent vapor cells is shown. Since $S$ is generated by atoms spontaneously, signals arising in the two separate atomic systems (cells A, B) are statistically uncorrelated, even if they are set up identically. On the other hand, $L$, caused by a single laser light source is identical in these two measurements. This allows us to eliminate laser noise effects using differential detection. The measured photocurrent using differential detection is $\langle \hat{D}_B(t) \rangle = \langle \hat{S}_A(t) \rangle - \langle \hat{S}_B(t) \rangle + \langle \hat{N}_A(t) \rangle - \langle \hat{N}_B(t) \rangle$, where the suffixes A, B refer to the two separate vapor cells. Since $S_{A,B}, N_{A,B}$ are all uncorrelated, the power spectrum is

$$\frac{1}{2} \langle |\hat{D}_B(\omega)|^2 \rangle = \frac{1}{2} \left( \langle |\hat{S}_A(\omega)|^2 \rangle + \langle |\hat{S}_B(\omega)|^2 \rangle \right)$$

$$+ \langle |\hat{N}_A(\omega)|^2 \rangle + \langle |\hat{N}_B(\omega)|^2 \rangle$$

$$= \langle |\hat{S}(\omega)|^2 \rangle + \langle |\hat{N}(\omega)|^2 \rangle$$.

Since cells A, B contain atomic vapor with identical properties, $\langle |\hat{S}_A(\omega)|^2 \rangle = \langle |\hat{S}_B(\omega)|^2 \rangle = \langle |\hat{S}(\omega)|^2 \rangle$. Shot noise spectra for A, B are also equal. While it is clear that the laser noise induced fluctuations $L$ can be eliminated with this method, effects due to shot noise $N$ still remain, since they are uncorrelated in PD A, B. Therefore, another method is required to extract the spontaneous noise spectrum.
c. Shot noise reduction: While shot noise is inherent in any light source, it is random and its properties are universal. Therefore, we can use correlation measurements to remove it\cite{12}. In the setup of Fig. 1(c), the laser light passing through the vapor cell is partitioned into two and measured by the two photodetectors, PD1, PD2. By Fourier transforming the current signals from PD1, PD2, $D_{ci}(t) = S(t) + L(t) + N_j(t)$ ($j = 1, 2$), and taking their correlation, we obtain

$$\langle D_{c1}(\omega)D_{c2}(\omega) \rangle \rightarrow \langle |\tilde{S}(\omega)|^2 \rangle + \langle |\tilde{L}(\omega)|^2 \rangle,$$

in the limit of infinite number of averagings. In the formula, bar denotes complex conjugation. Here, we used the property that $\tilde{S}(\omega)$, $\tilde{L}(\omega)$, $N_1(\omega)$, $N_2(\omega)$ are uncorrelated in the measurements. Denoting the number of $N$, the experimental uncertainty in Eq. $\ref{eq:5}$ is statistical and its relative size is $1/\sqrt{N}$. The crucial element here is the quantum property that the shot noise $N_{1,2}$ measured in the two independent photodetectors are statistically uncorrelated, even if they occur in the partitions of a single laser light source. It should be noted that this property holds for a coherent light source, but not necessarily for a source with different photon statistics\cite{5}. This correlation measurement allows us to achieve shot noise reduction to arbitrary levels, yet $\langle |\tilde{L}(\omega)|^2 \rangle$ still remains since the laser noise originates from a single light source and are correlated. Since it is impossible to measure just $\langle |\tilde{L}(\omega)|^2 \rangle$, which is induced in the atoms by the laser noise, further improvements are necessary to obtain $\langle |\tilde{S}(\omega)|^2 \rangle$. It should be noted that this correlation measurement removes any noise uncorrelated in $D_{c1}, D_{c2}$, such as instrumental noise, in addition to shot noise.

d. Differential detection and shot noise reduction: To remove both $L$ and $N$, we combine the above two noise reduction schemes effectively. Shown in Fig. 1(d) is the experimental setup used in this work, which incorporates both the differential detection and the shot noise reduction. The differentially detected photocurrents sent to ADC1,2 are $D_{di}(t) = S_A(t) - S_B(t) + N_{IA} - N_{IB}$ ($j = 1, 2$) and the correlation of their Fourier transforms is

$$\frac{1}{2} \langle D_{d1}(\omega)D_{d2}(\omega) \rangle = \frac{1}{2} \left( \langle |\tilde{S}_A(\omega)|^2 \rangle + \langle |\tilde{S}_B(\omega)|^2 \rangle \right) = \langle |\tilde{S}(\omega)|^2 \rangle.$$

It can be seen that the spontaneous noise spectrum can be extracted, without the shot noise nor the fluctuations induced by the laser noise.

### III. STABILIZATION OF THE LASER SYSTEM

A necessary part of the measurement system is a highly stabilized light source with low levels of both AM and FM noise. We explain in this section what is required and how to accomplish these requirements. In this experiment, we used a semiconductor laser, DFB-0780-080, Sacher lasertechnik, on a vapor of rubidium (Rb) atoms. The laser has a wavelength of 780 nm and a spectral width of about 3 MHz, which is of the same order as the spectral width of Rb-D2 transitions, about 6 MHz. This FM noise induces fluctuations in the photon absorption by Rb atoms\cite{11}, whose magnitude is much larger than that of the spontaneous noise we aim to extract. The most effective method for stabilizing a semiconductor laser and obtaining a narrow spectral width is to optically couple a confocal cavity to it. We have been able to stabilize a distributed feedback (DFB) laser diode (LD) to narrow down the spectral width below 1 kHz, using a previously proposed method\cite{13}. However, with this approach, laser instability induced noise arises at the same levels as the shot noise, so that it was impossible to extract the spontaneous noise spectrum over a wide frequency range. One reason for this is that in this method, the confocal cavity is operated off axis, so that spurious modes tend to arise in the emitted light. Another reason is that the method is ineffective in suppressing FM noise at higher frequencies ($f \gtrsim 10$ MHz). Since semiconductor lasers contain FM noise up to few GHz frequencies\cite{14}, another method is needed to achieve a stabilized laser system.

The stabilized laser system used in this experiment is shown in Fig. 2. While the optical system is somewhat complex, in essence, the light transmitted on axis through the confocal cavity (SA–300, Technical Optics) is fed back into the laser diode, which does not give rise to spurious modes and facilitates adjustments. However, similarly to the previously proposed method\cite{13}, the suppression of FM noise at high frequencies is insufficient in the traditional output from the system (output 2 in Fig. 2). Therefore, we used light transmitted through the confocal cavity (output 1) to obtain the spontaneous noise spectrum in this experiment. The FM noise, however, can be used to generate interesting physics effects, which we shall explore in Sect. V. With our experimen-
eral setup, laser noise induced effects at frequencies above 10 MHz were undetectable when using output 1. The use of light passed through the confocal cavity as in output 1 and the coupling of the confocal cavity to the light source on axis are new, we believe, and can be instrumental in achieving precision measurements at low noise levels. Light power of DFB LD was 60 mW, while that of output 1 (Fig. 2) was 2 mW. So, one aspect of this method is that we have traded in some of the power for achieving a source with lower noise levels. Since output 1 is an output from a confocal cavity, FM noise for frequencies below 10 MHz is converted to AM noise. Therefore, AM noise is increased at lower frequencies in output 1. So, output 2 (Fig. 2) can be useful for spectral measurements at lower frequencies and it also has a higher power than output 1.

IV. EXPERIMENTAL RESULTS AND THE PERFORMANCE OF THE MEASUREMENT SYSTEM

We now examine, using experimental results, the performance of the measurement system that includes noise reduction scheme and the stabilized laser system explained above. In the experiment, Rb atoms sealed in vacuum Pyrex glass cells were used as samples and were heated as necessary. We transmitted circularly polarized light in resonance with the $^{85}$Rb-D$_2$ transition from the hyperfine level $5^2S_{1/2}(F = 3)$ to $5^3P_{3/2}$ through the cells. Light source was circularly polarized using a quarter-wave plate.

In Fig. 3 measurements of the spontaneous noise spectrum using the four measurement systems a–d in the previous section, are displayed for beam waist 0.64 mm, cell temperature 47.4°C and light powers $P = 28, 546 \mu W$. Let us briefly explain the physics underlying these spectra. The main components of this spontaneous noise spectrum are the transit noise and Rabi noise. Broad structure at lower frequencies is the transit noise, caused by the effect of atoms transiting the beam. The atoms travel at thermal velocities and the average transit time for crossing the beam is $\sim 5\mu s$, corresponding to a frequency, $f = 2 \times 10^5$ Hz, consistent with the frequency this noise drops off. The higher frequency structure is caused by the Rabi flopping of the atoms, in which the atom is excited and de-excited by the incoming photons. Including the effects of the spontaneous decay from the excited level, the gaussian nature of the electric field strength within the light beam and Doppler shifts due to atomic motion gives rise to the observed structure, which is not a simple peak. While not seen here, Zeeman noise due to Larmor precession of the atoms can also be observed when a static magnetic field is additionally applied.

In the observed spectra, it should be noted that the unwanted noise in the laser and the detector system has been reduced to a level such that differential detection has very little effect when the power is weak (see Fig. 3(a), $P = 28 \mu W$ case, a vs. b, c vs. d). On the other hand, we can clearly see a reduction by a factor of $10^3$ in the shot noise, achieved by using an averaged correlation of the measurements (Eqs. 5, 6, a vs. c, b vs d), in both cases, $P = 28, 546 \mu W$. For the case $P = 546 \mu W$ (Fig. 3(b)), laser noise induces substantial changes to the spectrum, which is clearly eliminated using differential detection (c vs. d). Optically coupling an external cavity to the semiconductor laser destabilizes it weakly, resulting in the contributions to the spectrum at frequencies around 1 MHz. The excess signal below 10 kHz observed without differential detection (a, c) also originates from the laser noise. Looking closely, even for the case $P = 28 \mu W$, small effects of differential detection are visible below 10 kHz and around 1 MHz, due to the reduction in the laser noise induced signals mentioned above.

FIG. 3. Spectra measured using the measurement systems a–d, for $P = 28$ (a), 564 $\mu W$ (b). The notations are the same for both figures. a: Straightforward measurement with no noise reduction, Eq. (2), Fig. 1(a) (red). b: Measurement with differential detection, Eq. (3), Fig. 1(b) (green). c: Measurement with shot noise reduction, Eq. (4), Fig. 1(c) (blue). d: Measurement with both differential detection and shot noise reduction, Eq. (5), Fig. 1(d) (magenta). The shot noise level (black, dashed).
The number of averagings is $N = T \Delta f$, where $T$ is the total measurement time and $\Delta f$ is the frequency resolution. In the examples shown in Fig. 3, $N$ is not uniform across the spectrum. We used $\Delta f \geq 100 \text{kHz}$ for $f \geq 10 \text{MHz}$ and $T \simeq 10 \text{s}$, so that $N \geq 10^6$, leading to a noise reduction factor of $1/\sqrt{N} \leq 10^{-3}$ for higher frequencies, where the spectrum is the smallest. To obtain noise reduction by a factor $10^{-4}$, $T \simeq 1000 \text{s}$ is necessary for $\Delta f = 100 \text{kHz}$. At lower frequencies, $\Delta f$ needs to be smaller, so that $N$ is also smaller and the shot noise is not reduced as much. However, the spectrum has a larger value so that a small noise reduction factor is also unnecessary. At lowest frequencies, the small number of samples is reflected in the slightly jittery form of the observed spectrum. Analog to digital conversion was performed by PicoScope5203 (ADC, 8 bit, sampling rate 500 MHz, PicoTechnology). Fourier transforms and averagings were computed on a personal computer. In practice, the actual time needed for the experiment can be longer than $T$ due to two factors, the data transfer rate of the ADC and the computation speed of the computer.

V. HYPERFINE LEVEL COHERENCE EFFECTS

![Power spectrum](image)

**FIG. 4.** Spontaneous noise spectrum of $^{85}\text{Rb-D}_2$ transitions when a light source with FM noise is used. $P = 250$ (red), 552 (blue) and 1240 $\mu$W (cyan). The vertical black line indicates $F = 2, 4$ hyperfine splitting in the excited state, 184.0 MHz. A clear peak structure at this frequency, irrespective of the light power, is seen for $P = 552, 1240 \mu$W cases, when the spontaneous noise is also appreciable at this frequency. The beam waist is 0.14 mm. The spontaneous noise spectrum obtained using a light source with approximately the same parameters, without the FM noise are also shown (black). Their overall magnitude of the spectrum has been rescaled to make the comparisons clearer.

When the spectral narrowing is insufficient, the applied laser light contains FM noise. Such is the property of the stabilized laser output 2, which is not the output solely from the external cavity but includes direct light from the laser diode. The frequency modulation in the light couples the hyperfine levels of the atoms in the vapor. Since the hyperfine levels are essentially mixed, the states evolve with the time dependence dictated by the hyperfine splittings. This coherence of the hyperfine levels by itself is not visible in the fluctuation spectrum, since the frequency modulation of the laser is common to all atoms and is canceled in the differential measurement, system $b$ in Sect. III. However, this coherence affects Rabi noise and when it does so, it appears in the spontaneous noise spectrum. In Fig. 4, we can clearly see the effects induced by the laser FM noise and its distinct structure at a hyperfine level splitting frequency of the excited state, regardless of the light power. While not shown here, the structure can be seen at the same frequency also when linearly polarized light is applied. The effect is small when the spontaneous noise is small, since this is necessary for the effect to appear in the differential measurement, in addition to the coherence of the hyperfine levels. This interesting interplay of hyperfine level mixing and spontaneous noise has not been seen previously, to our knowledge.

VI. CONCLUSIONS AND DISCUSSION

We have developed a noise reduction system for measuring the fluctuation spectra of transmitted light power, which allows us to measure spectra at sub-shot noise levels. In the process, we also devised a method for stabilizing the laser to achieve low levels of both AM and FM noise. With these methods, we measured the spontaneous noise spectra down to levels $10^{-3}$ times the shot noise level in this work and $10^{-4}$ times the level elsewhere. We further used the measurement system to observe a new phenomenon, the spontaneous noise spectra for atomic level transitions with coherence between hyperfine levels. The shot noise reduction is statistical, so that given enough measurement time, the shot noise contribution can be reduced to arbitrarily low levels. This reduction method relies on the uncorrelated nature of the photon flux fluctuations in multiple measurements. The existence of cross talk between detected signals, which can arise at low levels, is one possible limitation in this regard. Naturally, to achieve noise reduction, all extraneous noise needs to be reduced to the desired level. Another practical limitation in this regard is the instability in the laser system.

Our measurement system with noise reduction is not much more difficult nor expensive to set up than the simplest measurement scheme in Fig. 1(a) for obtaining the intensity fluctuation spectrum. We expect such measurements of spontaneous noise to bring about a deeper understanding of photon atom interactions, which underlie many ongoing developments in physics, such as the physics of cold atoms, quantum computing and atomic physics.
clocks. Our spectral measurements were performed using atomic vapor cells, which are easy to handle and allows us also to examine properties of buffered gas. By comparing the unbuffered case to this, the physics with time scales longer than the buffered gas collision time scale is delineated, including transit noise and Rabi noise[4]. Our measurement system can also be applied spectral measurements of atomic beams.

The spectral measurements of transmitted light power fluctuations complement those of atomic fluorescence, where Rabi flopping was first observed[16]. In comparison, transmitted light spectroscopy has some advantages in that it has a higher power and is naturally a heterodyne measurement so that the spectrum can be studied with a larger dynamic range. Furthermore, the measurements can be more easily set up, take less time and phase information can also be extracted due to their heterodyne nature.

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