Model for Economic Optimization of Iron Production in the Blast Furnace

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A method for economic optimization of ironmaking in the blast furnace has been developed. Values for some of the most central operational variables of the furnace were generated by a factorial plan, and were used as inputs to a first-principles model that estimated important performance variables of the process. On the basis of these data a linear process model was developed and subsequently used for minimizing the hot metal production costs. The optimization task was formulated as a mixed integer linear programming problem and was solved by a spreadsheet program. The performance of the model is illustrated by examples, where the specific hot metal production cost is minimized under realistic process constraints. The sensitivity of the model to changes in the constraints or in the price structure is also demonstrated.

KEY WORDS: blast furnace; pig iron production optimization; MILP.

1. Introduction

In spite of its ancient origin, the blast furnace has remained competitive against other alternative processes for production of iron for primary steelmaking. Several inherent bottlenecks have been removed gradually through ingenious modifications of the process, e.g., by introducing metallurgical coke in the 18th century, pre-heating of the blast in hot stoves in the 19th century, agglomeration of iron oxides, the injection of auxiliary reductants in the 20th century, and computer-based analysis and control during the last few decades. However, along with the emergence of new competing ironmaking technologies (HIsmelt, Midrex, Corex, etc.), and the dramatic changes in the prices of raw materials and steel experienced during the last few years, it has become increasingly important to use an optimal mix of both iron-bearing components and reductants in order to operate the process economically.

Since the development of linear programming techniques about half a century ago, production planning and optimization methods in the steel industry have been actively studied starting with principal material and energy flow optimization in the late 1950’s. In spite of a rather simplified treatment of unit operations and constraints in such models, the problem of finding optimal performance of the system is extremely complex. Investigators have therefore often focused on separate subtasks, such as inventory optimization or optimal scheduling. In all these models, however, the blast furnace operation has been described by strongly simplified equations, and the “internal” operation constraints have often been neglected or considered as equality constraints (e.g., operation under constant flame temperature). In summary, very few attempts have been made to rigorously optimize the economy of ironmaking in the blast furnace under realistic operation conditions and constraints.

This paper presents a method by which the economy of ironmaking in a blast furnace can be optimized. The method is based on a two-step approach. First, a first-principles simulation model of the blast furnace is used to generate data points for the development of a linear model of central performance (“output”) variables of the process. To minimize and systematize this effort and to create a model with good interpolation properties, a factorial plan was designed for some of the most central operational variables and the model was executed on these inputs. The inputs and the resulting outputs formed the data on which the linear model was developed. Next, the linear model was used for optimization of the economy of the process by minimizing the specific costs of hot metal. To accomplish this, binary variables have to be introduced, which yields a mixed integer linear programming (MILP) problem. The development of the linear model is described in the next section of the paper, followed by a presentation in Sec. 3 of the formulation of the optimization problem. Sec. 4 presents some illustrative results, while the last section concludes the paper with a discussion of possible improvements and refinements of the method, as well as on a possible future use of the optimization tool.

2. Development of a Linear Process Model

As the goal of the work was to find the most profitable way to operate the blast furnace, an important step in the analysis was the choice of model to describe the performance of the process. Due to the complexity of the problem,
the optimization was estimated to require a substantial number of iterations, so it was decided to base the analysis on a linear model. In lack of a good linear blast furnace model, an existing first-principles model was used to generate points to which an approximate linear model was fitted. The first-principles model is static and one-dimensional, but the conditions approach chemical and thermal equilibrium. The model, which also includes an overall description of the state of the raceways, can, e.g., be used for the calculation of chemical and thermodynamic values ranging from 93.9 to 99.9%. Note that since the representations of the Calcium (expressing the volumetric flow rates of air and oxygen, the specific oil rate, the blast temperature, and the specific pelleted and limestone rates) as well as the thirteen outputs, \( Y_{1}, \ldots, Y_{13} \) (cf. Table 3). The outputs are the production rate, the specific coke rate, the flame temperature, the top gas temperature, the bosh gas volume, the residence time of the burden, the (simple) slag basicity, the slag rate, the top gas volume, the (dry) top gas composition, and the calorific heating value. The resulting regression equations are thus written as

\[
Y_i = K_{1,1} + K_{1,2} \left( \frac{\dot{V}_{\text{air}}}{\text{kNm}^3/\text{h}} \right) + K_{1,3} \left( \frac{\dot{V}_{O_2}}{\text{kNm}^3/\text{h}} \right) + K_{1,4} \left( \frac{m_{\text{oil}}}{\text{kg/t hm}} \right) + K_{1,5} \left( \frac{T_{bl}}{\text{C}} \right) + K_{1,6} \left( \frac{m_{\text{pel}}}{\text{kg/t hm}} \right) + K_{1,7} \left( \frac{m_{\text{line}}}{\text{kg/t hm}} \right) \quad i = 1, \ldots, 13
\]

or in matrix notation

\[
Y = K \hat{X}
\]

where \( Y = (Y_1, \ldots, Y_{13})^T \) and \( \hat{X} = [1, X_1, \ldots, X_{13}]^T \). The elements of the matrix \( K \) were determined by linear regression, yielding accurate models of the thirteen outputs (with \( R^2 \) values ranging from 93.9 to 99.9%). Note that since the sinter, pellet and hot metal compositions are fixed, the specific mass flow of sinter can be solved from the mass balance of iron (here neglecting possible contributions from coke)

\[
1000 \frac{\text{kg}}{\text{t hm}} \times x_{\text{Fe,hm}} = m_{\text{air}} + m_{\text{pel}} \left( x_{\text{Fe,pel}} \right)
\]

### Table 1. Number of levels and minimum and maximum values in the trial plan.

| Variable       | Levels | Minimum | Maximum |
|----------------|--------|---------|---------|
| Blast volume   | 3      | 100     | 140     |
| Blast oxygen   | 5      | 21      | 49      |
| Oil rate (kg/t hm) | 6   | 0       | 200     |
| Blast temperature (°C) | 5   | 950     | 1100    |
| Pellets (%)    | 5      | 20      | 40      |
| Limestone (kg/t hm) | 4 | 0       | 60      |

### Table 2. Oxygen content (%) in blast for different oil rates.

| Oil rate (kg/t hm) | Level | 1 | 2 | 3 | 4 | 5 |
|--------------------|-------|---|---|---|---|---|
| 0                  |       | 21.00 | 22.00 | 23.00 | 24.00 | 25.00 |
| 40                 |       | 21.00 | 22.50 | 24.00 | 25.50 | 27.00 |
| 80                 |       | 21.00 | 23.50 | 26.00 | 28.50 | 31.00 |
| 120                |       | 21.00 | 24.75 | 28.50 | 32.25 | 36.00 |
| 160                |       | 21.00 | 26.25 | 31.50 | 36.75 | 42.00 |
| 200                |       | 21.00 | 28.00 | 35.00 | 42.00 | 49.00 |

### Table 3. Variables predicted by linear model, Eq. (1).

| Variable      | Symbol | Unit | Range |
|---------------|--------|------|-------|
| Production rate | \( n_m \) | t hm/h | 130-200 |
| Specific coke rate | \( m_{cok} \) | kg/t hm | - |
| Flame temperature | \( T_{bl} \) | °C | 2000-2300 |
| Top gas temperature | \( T_{t} \) | °C | 100-250 |
| Bosh gas volume | \( V_{bl} \) | kNm³/h | 150-195 |
| Residence time of solids | \( \tau \) | h | 6.5-9.5 |
| Slag basicity, (CaO)/(SiO₂) | \( B \) | - | 1.05-1.2 |
| Slag rate | \( m_{slag} \) | kg/t hm | - |
| Top gas volume | \( V_{t} \) | kNm³/h | - |
| Top gas CO content | \( y_{CO} \) | % | - |
| Top gas CO₂ content | \( y_{CO_2} \) | % | - |
| Top gas H₂ content | \( y_{H_2} \) | % | - |
| Heating value of top gas | \( Q_b \) | MJ/Nm³ | - |

### 3. Objective Function and Constraints

#### 3.1. Objective Function

The economy of ironmaking in the blast furnace can be expressed in terms of specific costs of hot metal by considering the costs of the raw materials, including their prepara-
tion with a compensation ("income") for the produced gas, \( i.e. \)

\[
\frac{C}{\text{€/t km}} = \left( \frac{\dot{m}_{\text{cok,int}}}{\text{t/h}} + \frac{\dot{m}_{\text{cok,ext}}}{\text{t/h}} \right) \frac{C_{\text{cok,ext}}}{\text{€/t}} + \frac{\dot{m}_{\text{cok,ext}}}{\text{t/h}} \frac{C_{\text{cok,ext}}}{\text{€/t}} + \frac{\dot{m}_{\text{oil}}}{\text{t/h}} \frac{C_{\text{oil}}}{\text{€/t}} + \frac{\dot{m}_{\text{ini}}}{\text{t/h}} \frac{C_{\text{ini}}}{\text{€/t}} + \frac{\dot{m}_{\text{pet}}}{\text{t/h}} \frac{C_{\text{pet}}}{\text{€/t}} + \frac{\dot{V}_{\text{O}_2}}{\text{kNm}^3/\text{h}} \frac{C_{\text{O}_2}}{\text{€/kNm}^3/\text{h}} + \frac{\dot{E}_{\text{bl}}}{\text{MW}} \frac{C_{\text{bl}}}{\text{€/MW}} - \frac{\dot{E}_{\text{tg}}}{\text{MW}} \frac{C_{\text{tg}}}{\text{€/MW}} \right) / \frac{\dot{m}_{\text{ini}}}{\text{t/h}} \text{.........(4)}
\]

where \( \dot{m}_{i} \) and \( \dot{V}_{i} \) are the main mass and volume flow rates respectively, and \( C_{i} \) are corresponding specific or volumetric cost terms, and subscripts "int" and "ext" denote internal (own) and external ("bought") coke. Here investment and other fixed costs have been neglected, and an assumption has been made that variation of the variables within admissible ranges does not cause additional (\( e.g. \), in-works transportation) costs.

The fact that specific hot metal costs are considered instead of over-all (\( e.g. \), hourly or daily) costs eliminates the problem of bilinear terms (\( e.g. \), of the type \( Y_{1}, Y_{2} \) for the costs of coke). In order to eliminate the need for division by a variable production rate, as indicated in Eq. (4), for the other contributions, the hot metal production rate was kept fixed at different levels within the interval of interest, \( (\dot{m}_{\text{ini}}^{\text{min}}, \dot{m}_{\text{ini}}^{\text{max}}) \), in successive solutions of the optimization problem.

The variables in the last two terms of the numerator of Eq. (4), the energy flow required for heating the blast and the outflow of (useful) energy with the top gas, correspond to bilinear terms, since both the flows and the temperature or composition are variable. The blast is heated in the hot stoves, where a part (normally 40–50\%) of the top gas is burned. The required energy is given by the blast volume multiplied by the increase of the specific enthalpy of the blast. To remove the bilinear term, the change in the molar enthalpy of the blast was fixed at the upper limit, \( \Delta H_{\text{bl}}^{\text{max}} \), not to underestimate the costs, yielding \( E_{\text{bl}} = K \dot{V}_{\text{bl}} \Delta H_{\text{bl}}^{\text{max}} \), where \( K (\geq 1) \) is a factor introduced to roughly consider the losses in hot stoves. The cost of compression of the blast in the turbo blowers has not been considered in this study. As for the energy flow with the top gas, the calorific heating value, \( Q_{\text{g}} \), strongly depends on the top gas composition, which varies considerably with the rate of injected reductants and with the oxygen enrichment. However, if the bosh gas volume is limited to a certain operation interval, the top gas volume is rather constant. Therefore, as an approximation, a constant top gas volume was used in estimating the flow of useful top gas energy, \( \dot{E}_{\text{tg}} \). In order not to overestimate the profit, the lowest possible value of the top gas volume, obtained at the lower limit of the bosh gas volume, was used in the approximation, \( i.e. \), \( \dot{E}_{\text{tg}} = \dot{V}_{\text{tg}} Q_{\text{tg}} \).

In summary, the minimization of the hot metal production costs can be described as

\[
\min C \quad s.t \quad AZ \leq b \quad .......................(5)
\]

where the process constraints are considered in the inequalities, where \( A \) is a matrix of coefficients and \( Z \) is a vector of variables holding both inputs and outputs (\( X \) and \( Y \)) of the linear model and \( b \) is a vector of constants.

### 3.2. Constraints

#### 3.2.1. Box Constraints

In order to carry out a meaningful optimization, practical operational constraints\(^{[9]} \) have to be considered. Naturally, the maximum flows of available burden material (coke, sinter and pellets) as well as injected reductants and oxygen should be considered. In addition to such external constraints, internal limits should be imposed on variables such as blast and flame temperature, top gas temperature, bosh gas volume, residence time for solids, and slag basicity (see, \( e.g. \) Ref. 11). The motivation for constraining the blast temperature is, obviously, the limits imposed by the operation of the hot stoves. The flame temperature, in turn, has a lower constraint set by the condition that the hot metal and slag be sufficiently superheated at the tuyere level, and an upper constraint to prevent excessive bosh gas flow rates and undesired gasification of components in the raceways. Also the bosh gas volume should be limited to prevent fluidization or flooding in the lower furnace.\(^{11} \) As for the top gas temperature, limits are set by thermal aspects of the gas transportation and handling system (upper limit) and by a sufficient capacity to carry out moisture (lower limit). The latter limit is also analogous to an upper limit of the thermal (or heat) flow ratio between the descending solid burden (s) and the ascending gas (\( g \)), often expresses as \( \gamma = \dot{m}_{\text{g}} C_{p,g}/\dot{m}_{\text{s}} C_{p,s} \), in the shaft, which controls the heating of the burden.\(^{12} \)

It is also reasonable to introduce limits on the residence time of the burden; a too short time may prevent the burden from approaching the desired equilibrium conditions in the reserve zone, while a too long time can lead to, \( e.g. \), sinter degradation. Finally, it is reasonable to set an upper limit for the productivity imposed by the volume and drainage of the hearth.

#### 3.2.2. Special Constraints

As both the air and the oxygen volume flow rates are unknown variables in the optimization, the flow of blast is calculated from the condition

\[
\dot{V}_{bl} = \dot{V}_{air} + \dot{V}_{O_2} \quad .......................(6)
\]

It has been reported that the oxygen content of the blast should not exceed a critical limit, \( (e.g., \) 32\%) in order to avoid the risk of explosion in the hot stoves, so at higher blast oxygen contents the excess (cold) oxygen has to be injected through lances. This simultaneously decreases the maximum blast temperature. The material balance equations and an energy balance equation for the final "mixing" of blast from the hot stoves (subscript "st") and the lance oxygen (subscript "lan") give rise to an expression for the maximum blast temperature

\[
T_{bl}^{\text{max}} = T_{bl,\text{st}} - ((1 + a) y_{O_2,\text{bl}} - a) (T_{bl,\text{st}} - T_{lan}) \quad ..........(7)
\]

where \( a \) is a constant that can be determined analytically.
(see Appendix A). However, use of this equation would require the blast oxygen content, $y_{O_2,bl}$, to be determined. To avoid this non-linear term, the oxygen content is not used as a variable, but is instead densely discretized between $y_{O_2,bl}^{min}$ and $y_{O_2,bl}^{max}$ using an increment of $\Delta y_{O_2,bl}$. To model this, a binary variable, $I_i$, is introduced for each discretized level $i$, where $I_i = 1$ implies that $y_{O_2,bl} > y_{O_2,bl}^{min} + \Delta y_{O_2,bl} \sum_{j=1}^{i} I_j$.

4. Results

The model described in the previous sections is next illustrated by optimizing the iron production costs in a medium-sized blast furnace, using typical conditions of a Finnish blast furnace as a reference point. This furnace, which is charged with sinter, pellets and coke and uses heavy oil as auxiliary reductant, has a production rate of about 3 500 t/hm/d and a slag ratio of 200 kg/thm. Upper limits applied for the sinter and coke mass flow rates were 160 and 45 t/h respectively. Constraints should also be imposed on, e.g., the maximum specific oil, pellet and limestone rates (kg/thm) and blast and oxygen rates (kNm³/h). Furthermore, it is reasonable to constrain the top gas and flame temperatures, bosh gas volume, slag basicity, and the residence time of the burden as well as the production rate.

4.1. Reference Case

As a reference case, the optimization was run under the constraints reported in Tables 1–3 using the (rather arbitrary) cost structure expressed in Table 4. The admissible hot metal production rate range was 130 to 160 t/h, which was considered appropriate for the reference furnace, and the maximum oxygen supply rate was set to 40 kNm³/h. Figure 1 depicts how the specific hot metal cost $C$ varies with the production rate: it shows a shallow minimum at about 3 500 t/hm/d, beyond which the cost increases by about 5 €/t hm till the highest production rate is reached. Figures 2 and 3 show some central process variables at these points, where the latter figure mainly illustrates the variables with constraints imposed. They show that at the optimum, the maximum admissible oil injection rate (200 kg/thm) is reached. Also note that the residence time (upper) constraint is active up to the optimum point, where it is relaxed but “replaced” by the bosh gas volume (upper) constraint. After the point with the lowest cost, the oxygen content of the blast is gradually increased, while the blast volume and blast temperature decrease. This makes the bosh gas volume stay constant at its upper limit. A deeper understanding of the role of the constraints can be gained by studying the contour plots (of some variables) presented in Appendix B.

4.2. Fixed Blast Temperature

In order to evaluate the effect of an added equality constraint, the blast temperature was fixed at $T_{bl}=1100°C$, and

| Variable       | Unit | $C$ / € |
|----------------|------|---------|
| Sinter         | t    | 35      |
| Pellets        | t    | 50      |
| Coke, own      | t    | 200     |
| Coke, bought   | t    | 30      |
| Oil            | t    | 150     |
| Limestone      | t    | 30      |
| Oxygen         | kNm³ | 30      |
| Blast heating  | MWh  | 15      |
| Tog gas energy | MWh  | 10      |

Fig. 1. Optimal specific costs as a function of the production rate for the reference case.

Fig. 2. Specific oil rate, blast oxygen content, volume and temperature for the reference case.

Fig. 3. Flame and top gas temperatures, bosh gas volume and residence time for the reference case.
the costs were minimized, keeping all other parameters as in the reference run. This setup gives the results depicted by circles in Figs. 4–6 and is seen to initially yield the same optimum performance as in the reference case, but the economy strongly suffers at higher productivity. This is initially due to the lost possibilities to use massive oxygen enrichment to restrict the bosh gas volume at the maximum oil injection rate. Instead, to achieve the high productivity required, the blast volume must be increased, but simultaneously the oil injection rate is reduced. This, in turn, must be compensated for by the use of (own) coke. At a productivity of about 3 650 t/hm/d, a need arises for external (bought) coke, which is more expensive, causing the dramatic increase in costs. In the reference case (cf. Figs. 1–3) there is a slight (2 t/h) surplus of own coke at the highest productivity (3 800 t/hm/d).

It is also interesting to study the optimal operation strategy at a lower blast temperature, which could correspond to operation when only two hot stoves are available, e.g., during a modernization of the third stove. With the blast temperature fixed at $T_{bl}/H=950^\circ C$, the results illustrated by asterisks in Figs. 4–6 were obtained. The lower blast temperature is seen to yield a clearly higher cost that, furthermore, increases with the productivity. At the lower productivities illustrated, the reduced blast temperature gives rise to an increase of 15–17 kg/t hm in the fuel rate, corresponding to about 3 €/t hm with the present cost structure. Beyond 3 600 t/d there are no feasible solutions. Note that the flame temperature is initially at its lower boundary, which is relaxed at a productivity of 3 400 t/hm/d, where the bosh gas volume reaches its upper limit. In comparison with the reference case, the lower energy input with the blast is seen to be compensated for by an increased blast volume over the whole (feasible) range of productivities.

### 4.3. Restricted Resources

The impact of a restriction in the oil injection rate or in the oxygen flow rate is illustrated in Figs. 7–9. The circles in the figures correspond to the case where the maximum specific oil rate is set to $m_{oil}/H=120$ kg/t hm; this could also be taken to correspond to a situation where operation beyond this point is not feasible, e.g., due to oil combustion problems, excessive soot formation, etc. Naturally, the restricted oil injection rate leads to an increase in the hot metal production cost due to the large price difference between oil and coke, and the cost increases dramatically with

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**Fig. 4.** Optimal specific hot metal costs as a function of the production rate for fixed blast temperatures: (○) $T_{bl}/H=1 100^\circ C$, (●) $T_{bl}/H=950^\circ C$.

**Fig. 5.** Specific oil rate, blast oxygen content, volume and temperature for fixed blast temperatures: (○) $T_{bl}/H=1 100^\circ C$, (●) $T_{bl}/H=950^\circ C$.

**Fig. 6.** Flame and top gas temperatures, bosh gas volume and residence time for fixed blast temperatures: (○) $T_{bl}/H=1 100^\circ C$, (●) $T_{bl}/H=950^\circ C$.

**Fig. 7.** Specific costs at a lower maximum oil rate (○) $m_{oil}/H=120$ kg/t hm, and a restricted oxygen supply rate (●) $V_{O2}/H=20$ kNm$^3$/h.
productivity. The blast temperature and oxygen enrichment are seen to be constant—and the former at its upper limit except for the cases where the productivity comes close to 3 800 t hm/d.

If the available oxygen flow is restricted to 20 kNm³/h, the results (asterisks in Figs. 7–9) initially follow the ones of the reference case (cf. Figs. 1–3), but when the oxygen limit is reached (at a productivity of 3 600 t hm/d) the cost increases dramatically. For higher productivities it is not possible (like in the reference case) to keep the oil injection rate high, and this has a large impact on the operational state and the production economy. It is interesting to note that the variables (depicted in Figs. 8, 9) exhibit rather complex behaviour, and, furthermore, that the end points of the two cases studied in this paragraph practically coincide.

4.4. Reduced Costs for Oxygen

Finally, the impact of a cost term on the optimal state of operation of the furnace is illustrated. Here, the price of oxygen was halved even though this is admittedly an exaggerated change. However, it was introduced to cause a sufficient change in the state of operation of the furnace for the purpose of illustration.

As seen in Fig. 10, where the new case (•) is compared with the reference case (○) of subsection 4.1, the reduced costs of oxygen give rise to a reduction of the specific hot metal costs by about 3 €/t hm. At lower productivity the optimal state is found at higher oil injection and oxygen rates and a lower blast volume, keeping the blast temperature constant $T_{bl}$≈1 000°C (see Fig. 11). When the upper boundary of the oil injection rate is reached (at 3 400 t hm/d) the oxygen content is gradually taken down, simultaneously increasing the blast temperature and volume, and for productivities exceeding 3 500 t hm/d the two cases depicted in the figures coincide. Figure 12 illustrates the evolution of the other variables.

5. Conclusions and Discussion

A model has been developed for economic optimization of ironmaking in the blast furnace. Values for some of the most central operational variables of the furnace were first generated by a factorial plan, and these were used as inputs for an existing first-principles model that estimates impor-
tand performance variables of the blast furnace. A linear model was next developed on the resulting data set and was used for minimizing the hot metal production costs. Since some of the terms in the objective function include bilinear expressions, binary variables were introduced to maintain convexity of the optimization problem. The resulting problem is a mixed integer linear programming (MILP) problem that can be efficiently and robustly solved, e.g., by a spreadsheet program such as Excel.13)

The performance of the model has been illustrated by examples, where the specific hot metal production cost is minimized under realistic process constraints. With reference to a base case, the effect of added constraints was evaluated and it was demonstrated that, e.g., allowing for a variable blast temperature leads to novel economically advantageous states of operation characterized by a complex balance between blast volume, oxygen enrichment and oil injection. The effects of a restricted supply of oil and oxygen have also been studied, and for a case where the price of oxygen was assumed to be halved the optimal states exhibited intricate transition between the constraints. Clearly, such states would not be possible to find by manual trial-and-error with a simulation model.

The present model formulation does not include the possibility to inject steam to control the heat level of the lower part of the process, and this limitation will be relaxed in forthcoming work. One reason for neglecting this possibility is that steam injection is known to increase the reductant rate and therefore also the costs, while another motivation is that steam injection is known to increase the reductant allows CO2 emissions.

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Appendix A.

Consider the system outlined in Fig. A1, where oxygen can be added both before and after the regenerative heat exchanges (hot stoves) that are used for heating of the combustion air (blast). This arrangement is used since the oxygen content of the blast should not exceed a critical limit, $y_{O_2,at}^{max}$ in the hot stoves.

With reference to the figure we have

\[ \dot{V}_{bl,at} = \dot{V}_{Ar} + \dot{V}_{O_2,at} \] (A1)

and

\[ \dot{y}_{O_2,at} = y_{O_2,at} + \dot{V}_{O_2,at} \dot{V}_{Ar} \leq y_{O_2,at}^{max} \] (A2)

\[ \dot{V}_{bl} = \dot{V}_{bl,at} + \dot{V}_{O_2,lan} \] (A3)

\[ \dot{V}_{O_2} = \dot{V}_{O_2,at} + \dot{V}_{O_2,lan} \] (A4)
An overall oxygen balance yields
\[ \dot{V}_{m}y_{O_{2},bl} = \dot{V}_{m}y_{O_{2},air} + \dot{V}_{O_{2}} \] ..........................(A5)
Furthermore, an energy balance equation for the final mixing of the blast from the hot stoves and the lances (assuming constant specific heat capacity of the gases) gives
\[ \dot{V}_{bl}T_{bl} = \dot{V}_{bl}T_{bl,at} + \dot{V}_{O_{2},lan}T_{lan} \] ..........................(A6)
If the temperature of the gas from the hot stoves, \( T_{bl,at} \) is assumed to be fixed, the maximum blast temperature can be written as
\[ T_{bl} = \frac{\dot{V}_{bl}}{\dot{V}_{bl}}T_{bl,at} + \frac{\dot{V}_{O_{2},lan}}{\dot{V}_{bl}}T_{lan} \]
\[ = T_{bl,at} - \frac{\dot{V}_{O_{2},lan}}{\dot{V}_{bl}}(T_{bl,at} - T_{lan}) \] ..........................(A7)
Equation (A5) gives
\[ \frac{\dot{V}_{O_{2}}}{\dot{V}_{bl}} = y_{O_{2},bl} + \frac{\dot{V}_{air}}{\dot{V}_{bl}}y_{O_{2},air} \] ..........................(A8)
and, if lance oxygen is used, the constraint of Eq. (A2) is active, so it can be replaced by an equality, yielding
\[ \dot{V}_{O_{2},at} = \frac{\dot{V}_{max}}{1 - \frac{\dot{V}_{max}}{\dot{V}_{air}}} \] ..........................(A9)
Inserting Eqs. (A8) and (A9) into Eq. (A7) gives after some algebra
\[ T_{bl} = T_{bl,at} - \left[ y_{O_{2},bl} - \frac{\dot{V}_{air}}{\dot{V}_{bl}} \left( y_{O_{2},air} + \frac{\dot{V}_{max}}{1 - \frac{\dot{V}_{max}}{\dot{V}_{air}}} - y_{O_{2},air} \right) \right] \times (T_{bl,at} - T_{lan}) \] ..........................(A10)
Noting that \( \dot{V}_{air}/\dot{V}_{bl} \) can be written as \((\dot{V}_{air} - \dot{V}_{bl})/\dot{V}_{bl} \), the former term will appear recursively in the expression. After introducing the sum of the infinite series, one finally obtains
\[ T_{bl} = T_{bl,at} - \left[ \left( 1 + \frac{\psi}{1 - y_{O_{2},air}} \right) y_{O_{2},bl} - \frac{\psi}{1 - y_{O_{2},air}} \right] \times (T_{bl,at} - T_{lan}) \] ..........................(A11)

**Appendix B.**

This appendix briefly illustrates the feasible region and the constraints of the optimization problem in a two-dimensional space with the specific oil rate on the abscissa and the oxygen content of the blast on the ordinate, using the parameter settings of the reference case (Subsec. 4.1). **Figure B1** illustrates how the feasible region at different production rates (130, 140 and 150 t hm/h, corresponding to 3 120, 3 320 and 3 500 t hm/d) shrinks and moves towards the upper right of the figure, i.e., towards higher oil injection rates and oxygen enrichment. The feasible regions have been detected by optimizing with different fixed discretized values of the specific oil rate and the blast oxygen content.

Combinations for which the solution violates any of the constraints were excluded from the feasible region. For instance, studying the top gas temperature in more detail (Fig. B2) at a production rate of 140 t hm/h, one may conclude that the left boundary of the region is set by the lower constraint (100°C) of the top gas temperature, while the right part of the region is partially constrained by the upper temperature limit (250°C). Correspondingly (cf. Fig. B3), the lower boundary of the region is seen to be formed by the upper limit of the bosh gas volume (195 kNm3/h). A summary of the constraints active at the boundary of the feasible region is given in **Fig. B4**. Starting from the top, and moving clockwise along the boundary, the following can be concluded: The oxygen injection rate is limited by the lower bound of the blast temperature, while the oil rate is limited by its own upper bound (200 kg/t hm) but first by the maximum residence time of the burden. Moving downwards, at lower oxygen enrichment, the lower bound of the flame temperature also becomes an active constraint. At low oxygen rates the feasible boundary is set by the upper limit of the bosh gas volume (and of the blast temperature). At the low oil rates encountered in the lower left corner of the feasible region, the active constraint is the minimum...
residence time, while the lowest allowable top gas temperature becomes the active constraint when moving up and right, and finally also the lower limit of the bosh gas volume. The path is completed when the lower limit of the blast temperature is encountered again.

**Fig. B3.** Feasible region at a production rate of 140 t h⁻¹, and contour plot of the bosh gas volume (165–190 kNm⁻³/h).

**Fig. B4.** Constraints active at the boundary of the feasible region at a production rate of 140 t h⁻¹.