Quantification of a concentrated point mass by Haar wavelets and machine learning

Ljubov Jaanuska, Helle Hein
University of Tartu, Institute of Computer Science, 51009 Narva mnt 18 Tartu, Estonia
ljubov.jaanuska@ut.ee

Abstract. The inverse problem of determining location and mass ratio of a concentrated point mass attached to the homogeneous Euler–Bernoulli beam was considered in this article. Under the assumption that the size of the point mass was small compared to the total mass of the beam, it was shown that the problem could be solved in terms of point-mass-induced changes in the natural frequencies or mode shapes. Predictions of the point mass location and its mass ratio were made by the artificial neural networks or the random forests. The dimensionless natural frequency parameters or the first mode shape transformed into the Haar wavelet coefficients were used at the inputs of the machine learning methods. The simulation studies indicated that the combined approach of the natural frequencies, Haar wavelets and neural networks produced accurate predictions. The results presented in this article could help in understanding the behaviour of more complex structures under similar conditions and provide apparent influence on design of beams.

1. Introduction
A large number of modern engineering problems require identification of a mass loading on the basis of dynamic responses (an inverse problem). The solution uniqueness of such problems is not guaranteed; therefore, approximation methods are needed. Hosseini and Abbas [1] studied deflection of clamped beams struck transversely by a mass using the linear regression (LR) and various artificial neural networks (ANNs). Material properties and geometry of the beams were selected as independent variables of the model to predict the deflection of the beam. It was found out that a simple feed-forward back propagation ANN was either as good as or even slightly better than the LR or other sophisticated networks such as the cascade-forward back propagation ANN or the radial basis function network. The analysis on sensitivity of different variables showed that raw input variables performed better than mathematical combinations of the variables. Other studies on the natural frequency parameters (FPs) of beams and rods carrying masses can be found in [2-5]. Yet, none research has been found on the concentrated point mass quantification using the mode shape transformation into a set of the Haar wavelet transform coefficients (HWTCs). The wavelet transform has been recently successfully used in many fields including vibration-based damage detection in beams or plates [6-8].

This article demonstrates that the ANNs and the random forests (RFs) trained on the NFs or the HWTCs are capable of accurately quantifying the attached concentrated point mass.
2. Dynamic response of a beam with a concentrated mass

The differential equation associated with the present eigenvalue problem is [10]:

$$\frac{d^4 V}{dx^4} - k^4 V = 0, \quad k^4 = \frac{\rho A \omega^2}{EI}$$  \hspace{1cm} (1)

where $k^4$ is the natural frequency parameter, $\rho$ denotes the beam’s density, $A$ is the cross-sectional area, $E$ is Young’s modulus and $I$ is the moment of inertia. The general solutions of the ordinary differential equation (1) can be presented as

$$V_i(x) = C_i \sinh(k_i x) + C_j \cosh(k_i x) + C_k \cosh(k_i x), \quad x_i \in [0, a]$$

$$V_j(x) = C_i \sinh(k_i x) + C_j \cos(k_i x) + C_k \sinh(k_i x), \quad x_j \in [0, b]$$  \hspace{1cm} (2)

where $V_i$ and $V_j$ are the left and right mode shapes with respect to the concentrated mass $M$, and $C_i$ are the constants to be determined from the boundary and continuity conditions [10].

3. Datasets

In this simulation study, a homogeneous Euler-Bernoulli type cantilever carrying a concentrated point mass in different locations was investigated. The beam length was scaled to one. A concentrated point mass of an arbitrary mass ratio ($M$) was modelled to occur at a random point of the beam ($L$). The point mass ratios were chosen in the range from one to 100; the locations were chosen in the scaled range from 0.001 to 0.991. In total, the dataset contained 860 random cases. For each case, the first eight FPs $k_i$ and 16 HWTCs $h_j$ were calculated numerically using the procedure described in [10]. The calculated values of the first FPs and the mode shapes had good agreement with the results obtained by Low [11]. Here it should be noted that the calculation of 16 HWTCs was 22 times faster than the calculation of eight FPs.

The sensitivity tests were conducted to determine a relative significance of each of the independent variable. For this, a feed-forward back propagation ANN with one hidden layer, ten hidden nodes, Levenberg-Marquardt training algorithm and Elliot sigmoid transfer function was created. In each test, the weights of the ANNs were set to the same initial values. In the sensitivity analysis, each input neuron was eliminated in turns and its influence on the prediction was evaluated using the mean square error (MSE). The results are shown in Tables 1 and 2. In general, if the first two FPs were removed from the feature vector, the prediction error increased. In the case of the HWTCs, no single metrics had significant influence on the point mass quantification. The observation was confirmed by the correlation analysis (Fig. 1). The mass ratio hardly correlated to any feature. The location of the point mass closely correlated to the first FP, or the first, fourth, fifth, eighth, ninth and sixteenth HWTCs ($R > |0.7|$).

### Table 1. Significance of each FP on the point mass quantification.

|   | $k_1$  | $k_2$  | $k_3$  | $k_4$  | $k_5$  | $k_6$  | $k_7$  | $k_8$  |
|---|--------|--------|--------|--------|--------|--------|--------|--------|
| $M$ | 7.19e-2 | 1.28e-2 | 6.30e-3 | 4.00e-3 | 4.80e-3 | 9.20e-3 | 5.00e-3 | 6.40e-3 |
| $L$ | 8.60e-3 | 1.00e-3 | 1.00e-4 | 4.00e-4 | 7.00e-4 | 6.00e-4 | 5.00e-4 | 1.00e-4 |

### Table 2. Significance of each HWTC on the point mass quantification.

|   | $h_1$  | $h_2$  | $h_3$  | $h_4$  | $h_5$  | $h_6$  | $h_7$  | $h_8$  |
|---|--------|--------|--------|--------|--------|--------|--------|--------|
| $M$ | 7.00e-2 | 7.39e-2 | 7.42e-2 | 7.62e-2 | 6.35e-2 | 9.45e-2 | 7.38e-2 | 7.42e-2 |
| $L$ | 2.10e-5 | 2.01e-6 | 4.22e-5 | 8.83e-6 | 4.94e-5 | 5.18e-5 | 4.09e-5 | 4.46e-5 |

|   | $h_9$  | $h_{10}$  | $h_{11}$  | $h_{12}$  | $h_{13}$  | $h_{14}$  | $h_{15}$  | $h_{16}$ |
|---|--------|------------|------------|------------|------------|------------|------------|----------|
| $M$ | 6.97e-2 | 7.97e-2 | 7.86e-2 | 7.60e-2 | 8.18e-2 | 7.03e-2 | 7.69e-2 | 6.48e-2 |
| $L$ | 2.00e-5 | 3.94e-5 | 2.62e-5 | 8.07e-6 | 3.83e-5 | 3.57e-6 | 1.89e-5 | 3.12e-5 |
Taking into account the results of the analyses, the machine learning methods were examined on several datasets (Tables 3 and 4).

**Table 3. Datasets to quantify mass ratio.**

| Set nr | Feature nature | Feature coefficients |
|--------|----------------|----------------------|
| 1      | $k_i^j$        | $i = 1, 2, 6$        |
| 2      | $k_i^j$        | $i = 1, 2, 3, 4, 8$  |
| 3      | $k_i^j$        | $i = 1, \ldots, 8$  |
| 4      | $h_j$          | $j = 1\text{-}4, 6\text{-}8, 10\text{-}15$ |
| 5      | $h_j$          | $j = 1, \ldots, 16$ |
| 6      | $k_i^j, h_j$   | $i = 1, j = 1, \ldots, 16$ |

**Table 4. Datasets to localise point mass.**

| Set nr | Feature nature | Feature coefficients |
|--------|----------------|----------------------|
| 7      | $k_i^2$        | $i = 1, 3, 2, 4$     |
| 8      | $k_i^2$        | $i = 1, \ldots, 8$  |
| 9      | $h_j$          | $j = 1, 5, 9, 3, 4, 8$ |
| 10     | $h_j$          | $j = 1, \ldots, 16$ |
| 11     | $k_i^2, h_j$   | $i = 1, j = 1, \ldots, 16$ |

In the following machine learning tests, each initial dataset of 860 patterns was divided into two sets: 774 patterns were used for training and 86 patterns were held back for the testing and method evaluation. Before applying the data into the training and testing of the machine learning methods, the values were scaled within the range of zero and one.

**4. Quantification of the point mass by neural**

In the simulation study, a feed-forward back propagation ANN with one hidden layer was used. The ANN was trained using one of the following training functions: resilient back propagation (RP), scaled conjugate gradient (SCG), Broyden-Fletcher-Goldfarb-Shanno (BFGS), Levenberg-Marquardt (LM) or Bayesian regularization (BR). Taking into account the results published in [1, 10], the point mass parameters were predicted one-by-one (one output per ANN). The number of hidden neurons was increased gradually from 10 to 50 with an increment of ten neurons in order to select the most promising network structure. The differentiable transfer function in the hidden layer was Elliot sigmoid; the linear transfer function was used in the output layer. The training of the ANN was stopped when an acceptable level of error was achieved (MSE = 1e-4), or when the number of the iterations exceeded 1e+3 epochs or the number of the validation checks was six. Table 5 summarises the results of different trainings. Each row describes the ANN configuration, which produces the
lowest MSE of the five-fold cross-validation using a particular training function, the number of hidden neurons and the dataset.

### Table 5. The lowest MSE of five-fold cross-validation by ANN.

| Training function | $M$  | $L$  |
|-------------------|------|------|
|                   | MSE  | Hidden neurons | Dataset | MSE  | Hidden neurons | Dataset |
| LM                | 2.6e-3 | 10     | 3       | 3e-6 | 10     | 11 |
| SCG               | 2.17e-2 | 50     | 2       | 2e-4 | 40     | 10 |
| RP                | 1.64e-2 | 30     | 3       | 1.64e-2 | 50     | 10 |
| BFGS              | 1.82e-2 | 50     | 3       | 6e-4 | 50     | 10 |
| BR                | 2.1e-3 | 10     | 3       | 3e-7 | 10     | 10 |

According to the results, the mass ratio is harder to predict than the location of the point mass. It is noted that the dataset with eight FPs performs better on the prediction of the mass ratios (MSE = 2.1e-3); however, the dataset of 16 HWTCs produces more accurate results on the location (MSE = 3e-7). The corresponding ANN has ten hidden neurons and the Bayesian regularisation function.

Next, two ANNs with ten hidden neurons and the Bayesian regularisation training function were examined on the set of independent data. One ANN was trained on the first eight FPs to predict the mass ratios, and the second ANN was trained on 16 HWTCs to predict the locations of the point masses. The predicted values against the target (observed) values are plotted in Figures 2 and 3. For clarity and descriptive reasons, the mass ratios are shown in the unscaled format. The MSE of the point mass localisation was 2e-7; the MSE of the mass ratio quantification was 1.6e-3. The R-values were 1.0 and 9.61e-1, respectively. The absolute prediction error of the point mass localisation was less than 0.0015 in 85 cases out of 86; the absolute prediction error of the mass ratio quantification was less than 0.9 in 85 cases.

5. Quantification of the point mass by random forests

Next, the point masses were quantified by the RFs using the following configurations:
- predictors without replacement in samples p (1/6, 1/2 and 2/3 or all);
- the number of trees (25, 50, 75, 100, 150, 300, 500, 750, 1000).

Table 6 shows the lowest MSE and the configurations of the corresponding RF. As in the case of the ANNs, the RFs produced accurate results if they were provided either with the FPs (dataset 3) or the HWTCs (dataset 10); the combination of the FPs and HWTCs did not produce promising results. In the case of training on the FPs, the lowest MSE of the point mass localisation was 2e-4. The lowest MSE to estimate the mass ratio was 3.9e-3. In the case of training on the HWTCs, the lowest MSE to
localise the point mass was 6e-6. The lowest MSE to estimate the mass ratio of the point masses was 8.67e-2.

Table 6. The lowest MSE of five-fold cross-validation by RF.

| Predictors in a pattern | p | Number of trees | Average MSE |
|------------------------|---|----------------|-------------|
| M                      | 8 FPs | 8   | 75 | 3.9e-3 |
| 16 HWTCs               | 8 FPs | 8   | 50 | 2e-4  |
| L                      | 8 FPs | 16  | 150| 6e-6  |

Next, two RFs were merged to form a combined approach of the FPs and the HWTCs. Namely, the locations were predicted by the ensemble of 150 trees provided with the HWTCs, and the mass ratios were predicted by the ensemble of 75 trees provided with the FPs. The combined approach was tested on the testing set with 86 records. The results are visualised in Figure 3. The MSE of the point mass localisation was 6e-6, the R-value was 9.998e-1. The absolute prediction error was less than 0.007 in 84 cases. The MSE of the mass ratio estimation was 3.7e-3, the R-value was 9.781e-1. The absolute prediction error was less than 0.5 in 34 cases. On the whole, the RFs provided with the dataset of FPs produced more accurate predictions of the mass ratios; meanwhile, the RFs provided with the dataset of HWTCs produced precise predictions of the locations of the concentrated point masses.

6. Conclusion
The development of effective methods for timely identification of a concentrated point mass on vibrating beams is an issue of increasing interest in several fields. Motivated by the need for a fast and accurate tool for the quantification of the concentrated point masses, an Euler-Bernoulli type cantilever was investigated in the present article. The solution to the point mass quantification was based on the FPs and the first mode shape decomposition into 16 HWTCs. The mass ratio and location of the point masses were predicted by the ANNs and the RFs. According to the results, the ANN trained by the Bayesian regularisation function was either as good as or even slightly better than other ANNs or RFs. The results also indicated that the use of the HWTCs at inputs might be more beneficial for the localisation of the point masses than the FPs. On the other hand, the FP based methods predicted more precisely the mass ratio of the concentrated point masses.

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