On Flux Compactification and Moduli Stabilization

A. Awad\textsuperscript{1,2*}, N.Chamoun\textsuperscript{1,3†}, and S.Khalil\textsuperscript{2,4‡}

\textsuperscript{1} The Abdus Salam ICTP, P.O. Box 586, 34100 Trieste, Italy.
\textsuperscript{2} Ain Shams University, Faculty of Science, Cairo 11566, Egypt.
\textsuperscript{3} Physics Department, HIAST, P.O.Box 31983, Damascus, Syria.
\textsuperscript{4} Department of Mathematics, German University in Cairo, New Cairo city, El Tagamo\textasciitilde El Khames, Egypt.

Abstract

We study the effect of adding charged matter fields to both $D3$ and $D7$ branes in type IIB string theory compactification with fluxes. Generically, charged matter fields induce additional terms to the Kähler form, the superpotential and the D-terms. These terms allow for minima with positive or zero cosmological constants, even in the absence of non-perturbative effects. We show this result first by decoupling the dilaton field along the lines of the KKLT, and second by reincorporating it in the action with the Kähler moduli.
1 Introduction

In flux compactifications of type IIB supergravity, all the complex structure moduli and the dilaton are generically fixed by the non-trivial superpotential induced by the 3-form field strengths[1]. However the Kähler moduli are not fixed by the fluxes, and the resulting 4D model is of no-scale type.

Kachru, Kallosh, Linde and Trivedi (KKLT)’s approach [2] is the first explicit realization of 4D de Sitter space as a solution to the low-energy equations of string theory where all the moduli are stabilized. Their scenario consists of three main stages. First, geometrical fluxes due to RR and NS-NS 3-form field strengths are introduced to stabilize the dilaton $S$ and the complex structure moduli $Z_i$ (CSM). Second, non-perturbative effects due to gaugino condensation in the gauge theory on $D7$ branes [3, 4], or $D3$ instantons [5] are used to stabilize the Kähler moduli. The resulting potential has an AdS–SUSY minimum. In the final uplifting stage, adding $\bar{D}3$ antibranes breaks SUSY explicitly and allows a fine tuning of the cosmological constant to a small positive value (de Sitter space).

Since the KKLT set up was proposed, it has been thoroughly discussed and studied by many authors. It has been noticed [6, 7] that the potential of the anti-branes $\bar{D}3$ should be added to the ten dimensional theory and not to the effective four dimensional action as was adopted in the KKLT set up. In this case, supersymmetry is explicitly broken and the resulting effective theory is not in a supergravity form, which leaves the theory uncontrollable. This problem was the motivation for many authors to consider alternative mechanisms in order to uplift the AdS vacuum to a Minkowski or dS vacuum within the supergravity framework. In Ref. [6], fluxes of gauge fields that reside on $D7$ branes have been used to induce a positive D-term to the scalar potential allowing, thus, to obtain dS vacua. However, it has been emphasized in Ref. [8] that one can not use D-terms to uplift the AdS SUSY vacuum to a dS non-SUSY one.

In this letter, we study the effect of charged matter and gauge fluxes that live on $D3$ and/or $D7$ branes. The matter fields induce additional terms to the Kähler function, the superpotential and the D-terms. We show that, by incorporating these electric fluxes with geometric fluxes in the presence of matter fields, one can stabilize the complex structure moduli $Z_i$, the dilaton $S$, and the real part of the Kähler modulus $T_R$, without invoking any non-perturbative mechanism. We will not be concerned with the imaginary part of $T$ which is analogous to the QCD $\theta$-term and may be fixed by the non-perturbative axionic effect that breaks Peccei-Quinn symmetry, see for example [9]. We also find that the matter field contributions to the D-terms play a crucial role in obtaining non-SUSY Minkowski or dS vacua. We show these results in the two most common scenarios: $i)$ The KKLT-like case where $S$ and $Z_i$ are integrated out at a higher scale. $ii)$ The case in which both $S$ and $T$ are kept light in the effective theory.

It is worth mentioning that in the later case ($ii$) the KKLT set up fails to obtain any local minimum to the potential [10]. In contrast, we show that the new corrections due to
matter fields and electric fluxes allow the existence of a local AdS non-SUSY minimum in both \( (T, S) \) directions. In this analysis we assume that some of the matter fields acquire non-vanishing vevs which are higher than the moduli mass scale.

This letter is organized as follows. In section 2 we review briefly the KKLT set up and its variations including the dilaton in the effective action. Section 3 is devoted to analyzing the impact of adding charged matter in \( D3 \) and/or \( D7 \) branes. We show that these matter fields modify the Kähler potential and induce D-terms that lead to a non-SUSY Minkowski or de Sitter vacua, even in the absence of non-perturbative effects and also regardless of whether the dilaton is decoupled or not from the low energy regime. Our conclusions are given in section 4.

## 2 KKLT and its variants

In this section we briefly review flux compactifications and the KKLT approach and its variations for stabilizing all moduli. In type IIB theory, strings can have RR and NS-NS antisymmetric 3-form field strengths \( (H_3 \text{ and } F_3 \text{ respectively}) \) which can wrap 3-cycles of the compactification manifold labeled by \( P \) and \( Q \), leading to the following background fluxes

\[
\frac{1}{4\pi^2\alpha'} \int_P F_3 = L, \quad \frac{1}{4\pi^2\alpha'} \int_Q H_3 = -K,
\]

where \( K \) and \( L \) are integers. In the effective 4D supergravity, these geometric fluxes generate a superpotential for the Calabi-Yau (CY) moduli, which is of the form [11]

\[
W = \int_M G_3 \wedge \Omega
\]

(2)

where \( \Omega \) is the holomorphic 3-form which depends on the CSM moduli \( Z_i \) and \( G_3 = F_3 - iSH_3 \). The axion-dilaton field \( S \) and the overall scale modulus \( T \) are defined in type IIB by:

\[
S = \frac{e^{-\phi}}{2\pi} + ic_0, \quad T = \frac{e^{-\phi}}{2\pi} (M_{st} R)^4 + ic_4,
\]

(3)

where \( c_0 \) and \( c_4 \) are the axions from the RR 0-form and 4-form, respectively, \( e^{-\phi} = g_{st} \) denotes the string coupling, \( 1/M_{st}^2 \) is the string tension, and \( R \) is the compactification radius of the CY volume \( V_{CY} \equiv (2\pi R)^6 \).

Giddings, Kachru and Polchinski (GKP) showed that these 3-form fluxes can generically stabilize the dilaton \( S \) and all the CMS moduli \( Z_i \) [12]. However, since the Kähler modulus \( T \) does not appear in the potential, it can not be fixed by the geometric fluxes and the potential is of no-scale type. This partial fixing of moduli in GKP framework can be understood by considering the following tree-level Kähler potential:

\[
K = -3 \log(T + \overline{T}) - \log(S + \overline{S}) - \log[-i \int_M \Omega \wedge \overline{\Omega}].
\]

(4)
The superpotential (2) and the Kähler potential (4) lead to the following F-term potential:

$$V_F = e^K (G^{IJ} D_I W \overline{D_J W} - 3|W|^2) = e^K (G^{ij} D_i W \overline{D_j W})$$

where $I$ and $J$ run over all moduli while $i$ and $j$ run over dilaton and CSM moduli only. The covariant derivative is defined as $D_I W = \partial_I W + (\partial_I K) W$. Here $G^{IJ} = G^{-1}_{IJ}$ and $G_{ij}$ is given by $G_{ij} = K_{ij} = \partial_i \partial_j K$. Note that $K^T |D_T W|^2$ cancels with $3|W|^2$, leaving the potential $V_F$ independent of $T$. As can be seen from Eq.(5), the potential $V_F$ is positive definite, so that its global minimum is at zero and hence the dilaton and CSM moduli are fixed by the condition $D_i W = 0$. This minimum is non-supersymmetric due to the fact: $F_T \propto D_T W \propto W \neq 0$, i.e., SUSY is broken by the $T$ field.

In order to fix $T$, KKLT considered a nonperturbative superpotential, either generated by D3-brane instantons or by gaugino condensation within a hidden non-abelian gauge sector on the D7-branes. Since the dilaton and the CSM have been fixed at a high scale, their contribution to the superpotential is a constant $W_0$ and the total effective superpotential is given by

$$W = W_0 + Ae^{-\alpha T},$$

where $A$ and $a$ are constants ($1/a$ is proportional to the beta function coefficient of the gauge group in which the condensate occurs). The Kähler potential after integrating out $S$ and $Z_i$ is reduced to

$$K = -3 \log(T + \overline{T}).$$

The new potential can now fix the field $T$ and one gets a supersymmetric AdS vacuum. To uplift this AdS minimum to a Minkowski or dS one, they added antibranes $\overline{D3}$. The $\overline{D3}$ effect amounts to an additional term in the scalar potential that is proportional to $\frac{1}{(T + \overline{T})^2}$, and for reasonable choices of parameters it yields de Sitter vacua.

As mentioned in the introduction, the $\overline{D3}$ explicitly breaks supersymmetry and therefore the scalar potential is no longer in its supergravity form. This complicates the analysis of the low energy theory. A possible solution to overcome this problem has been proposed in Ref.[6] where the authors used the $D$-term induced by the gauge fluxes on $D7$ branes. It turns out that if the matter fields charged under the gauge group on $D7$ branes are minimized at zero vevs, then the $D$-term gives the same contribution to the scalar potential as $\overline{D3}$. However, it is important to note that after including the non-perturbative gaugino condensation in the second step of the KKLT approach, supersymmetry is restored and the effective theory is fully supersymmetric. Therefore, the potential $V_D$ is always minimized at zero, as pointed out by several authors [8], and the $D$-term can not be used for uplifting the AdS SUSY vacua.

Another point of concern in the KKLT approach is the assumption that the Kähler moduli are the only light moduli. As discussed in [10,13], there are situations where
fluxes would keep the dilaton light while the CSM have string scale masses. In such cases, one can integrate out the CSM but should leave the dilaton in the Kähler form and superpotential:

\[ K = -3 \log(T + \overline{T}) - \log(S + \overline{S}) \quad (8) \]

\[ W = A + BS + Ce^{-aT} \quad (9) \]

where \(A, B, C\) and \(a\) are constants. In [10] it was shown that the AdS supersymmetric stationary point is in fact a saddle point with instabilities along the moduli and axion directions, and if one keeps \(S\) as well as \(T\) after including the gaugino condensate then there are no local minima even in the presence of a lifting potential of the form \(\Delta V = \frac{D}{(T + \overline{T})^{n_t}(S + \overline{S})^{n_s}}\) where \(n_t\) and \(n_s\) are positive or zero integers.

### 3 Charged matter fields and D-terms

As advocated in the introduction, the problem of the no scale type potential that is encountered in the KKLT setup can be overcome by considering the effect of the charged chiral fields living on \(D3\) and \(D7\) branes in type IIB string. These fields generate a new \(T\)-dependence in the Kähler potential which helps to stabilize the \(T\) moduli without assuming any non-perturbative mechanism. Generically, in type IIB there are two types of massless \(N = 1\) chiral fields: closed string chiral fields (which include the dilaton and moduli) and open string chiral fields which are charged under the D-branes’ gauge groups.

In this setup we will consider two types of branes, namely: \(D3\) and \(D7_i\) branes (which are dual to \(D9\) and \(D5_i\) branes respectively). The index \(i = 1, 2, 3\), denotes the complex compact coordinate transverse to the \(D7\)-brane world-volume. In this respect, we may have the following charged matter fields: \(C_{3}^{i}\) which arise from open strings starting and ending on a \(D3\) brane and \(C_{7}^{i} \) which come from open strings starting and ending on the same \(7_i\). The lowest order expansion of the Kähler potential in the matter field is given by [14]

\[ K = -\sum_{i=1}^{3} \log(T_i + \overline{T_i}) - \log(S + \overline{S}) + \sum_{i=1}^{3} \frac{|C_{3}^{i}|^2}{T_i + \overline{T_i}} + \sum_{i=1}^{3} \frac{|C_{7}^{i}|^2}{S + \overline{S}} + \sum_{i,j,k=1}^{3} d_{ijk} \frac{|C_{7}^{k}|^2}{T_i + \overline{T_i}}, \quad (10) \]

where \(d_{ijk} = 0\) if \(i \neq j \neq k\), otherwise \(d_{ijk} = 1\). One can also extract the renormalizable contributions of the charged matter (i.e., the mass terms and tri-linear couplings for the charged chiral superfields) to the superpotential [14]. Therefore, combining the charged matter and geometric fluxes contributions leads to the following superpotential

\[ W = A + BS + g_3 C_{1}^{3} C_{2}^{3} C_{3}^{3} + \sum_{i=1}^{3} g_{7} C_{1}^{7_i} C_{2}^{7_i} C_{3}^{7_i}. \quad (11) \]
The Yukawa coupling constants are given by the gauge couplings \( g_2^2 = 4\pi/\text{Re}S \) and \( g_7^2 = 4\pi/\text{Re}T \). For simplicity, we assume that one field, at most, on each type of \( C_i^3 \) and \( C_i^7 \) gets a vev. Furthermore, the vevs of Kähler moduli acquire equal values and the fields \( C_j^{3i} \) for \( i \neq j \) are assumed to be zero. Here, two comments are in order: i) The scale of the vevs of the charged fields \( C_i^3, C_i^7 \) is assumed to be an intermediate, i.e., below the scale of the CSM \( Z_i \) and well above the scale of the modulus \( T \). ii) These vevs can be explicitly determined from the full potential of the charged fields. It is interesting to find an explicit example which leads to these desired vevs in type IIB, however this beyond the scope of this letter and will be considered elsewhere. In this case, the Kähler potential and the superpotential take the form:

\[
K = -3 \log(T + \bar{T}) - \log(S + \bar{S}) + \frac{|\langle C_3 \rangle|^2}{(T + \bar{T})} + \frac{|\langle C_7 \rangle|^2}{(S + \bar{S})} \tag{12}
\]

\[
W = A + BS \tag{13}
\]

In addition to their contributions in the Kähler potential and the superpotential, charged matter together with gauge fluxes give rise to two different contributions to D-term. The first contribution is coming from gauge fluxes which are known to induce terms in the 4D \( \mathcal{N} = 1 \) supersymmetric effective action identified as Fayet-Iliopoulos (FI) D-terms [6]. Such terms have the following form in the D7 brane case

\[
T_7 \int_{\Gamma} F \wedge \bar{F} = 2\pi E^2 \left( \frac{T + \bar{T}}{(T + \bar{T})^3} \right) \tag{14}
\]

where \( T_7 \) is the D7 brane tension, \( \Gamma \) is the 4-cycle around which the D7 branes are wrapped and \( E \) is the flux strength. The other contribution is coming from matter fields. Matter fields trigger spontaneous gauge symmetry breaking through D-terms if they have non-vanishing vevs. The general form of D-terms for the D7 and D3 branes can be expressed as [15]

\[
V_D = \frac{g_2^2}{2} \left( \sum_{i, 7} q_i \Phi_i K_i + \xi_7 \right)^2 + \frac{g_7^2}{2} \left( \sum_{i, 3} q_i \Phi_i K_i + \xi_3 \right)^2 \tag{15}
\]

where \( K_i \) is the derivative of the Kähler potential \( K \) with respect to the matter fields \( \Phi_i \) (a subset of \( C_i \)'s) which has charge \( q_i \) under the FI \( U(1) \) group. The FI terms \( \xi_i \), where \( i = 3, 7 \) denotes the brane type, are given by \( \xi_3 = E_3/\text{Re}S \) and \( \xi_7 = E_7/\text{Re}T \) [16].

It is plausible to fine-tune the parameters: \( \langle T \rangle, \langle S \rangle, E \) and the vevs of the charged matter fields \( \langle \Phi_{3,7} \rangle \equiv v_{3,7} \) so that gauge symmetry is broken while SUSY remains exact. In Ref. [6], it is assumed that charged matter fields acquire a vanishing vev as a result of having only one type of \( U(1) \) charge either positive or negative. Here, we will not assume either of these situations, but, on the contrary, we will consider the two D-term contributions mentioned above. Writing D-terms as a function of the moduli fields, one gets the following expression

\[
V_D = \frac{1}{T + \bar{T}} \left( \frac{E_7}{T + \bar{T}} + \frac{F_7}{S + \bar{S}} \right)^2 + \frac{1}{S + \bar{S}} \left( \frac{E_3}{S + \bar{S}} + \frac{F_3}{T + \bar{T}} \right)^2 \tag{16}
\]
where $E_7(E_3)$ is a measure of the strength of the flux on D7 (D3) brane, and $F_7(F_3)$ denotes a ‘charged’ vev, i.e.:

$$F_{3,7} \propto q_{3,7}|v_{3,7}|^2.$$ \hfill (17)

If the matter fields are minimized at $v_i = 0$, then the D-term, $V_D$, mimics the effect of adding $\overline{D3}$ anti-branes, as pointed out in Ref.[6]. However, as we will show, having non-vanishing vevs for these fields, i.e., $F_i \neq 0$ and also assuming the non-cancellation of the D-terms can play a crucial role in obtaining a de Sitter vacuum.

Now we present our results. In the generic case where the dilaton field can be integrated out, we have the following scalar potential

$$V = e^K \left( K^T D_T W D_T W - 3|W|^2 \right) + V_D$$

$$= e^K W_0 \left( \frac{|v_3|^4}{T + \overline{T}} \right) \left( \frac{v_3}{T + \overline{T}} + 2|v_3|^2 \right) + V_D \hfill (18)$$

where

$$K = -3 \log(T + \overline{T}) + \frac{|v_3|^2}{T + \overline{T}} + |v_7|^2$$

$$W = W_0$$

$$V_D = \frac{1}{T + \overline{T}} \left( \frac{E_7}{T + \overline{T}} + F_7 \right)^2 + \left( E_3 + \frac{F_3}{T + \overline{T}} \right)^2 \hfill (19)$$

where the dilaton $S$ was integrated out.

As one can observe, the parameter space ($E_3, v_3, E_7,$ and $v_7$) in this case is large, and for simplicity we choose that matter and gauge flux are present on the D7 branes but no matter are present on the D3 branes. Thus, the minimum of the above potential occurs at $t_{min} = -\frac{E_7}{2F_7}$ with $E_7F_7 < 0$ (the other stationary point at $t = -\frac{3F_7}{2E_7}$ would give a non-SUSY maximum). This is a non-SUSY Minkowski or de Sitter vacuum, according to whether or not we switch on gauge flux on the D3 branes, since $V(t = t_{min}) = E_3^2$. In this simple example the existence of non-vanishing charged vev $v_7$ is crucial in order to obtain the above non-SUSY dS minimum. Here, the D-terms alone (with both contributions from the FI term and the vev) is capable of stabilizing the volume modulus and producing a dS vacuum without the need of non-perturbative effects.

In the case of non-vanishing $v_3$, one can show numerically the existence of a non-SUSY dS minimum. Taking the values of the parameters to be:

$$F_3 = F_7 = 0.1, \quad E_7 = 0, \quad E_3 = -1.0, \quad v_3 = v_7 = 0.1, \quad W_0 = -0.01,$$ \hfill (20)

one can check that the minimum of the potential occurs at $t_{min} = 0.05267$ and $V(t)|_{min} = 0.0977$. The minimum is again a de Sitter vacuum.
Now turning to the case where $T$ and $S$ are both left light. The analytical expressions are difficult to obtain for the full parameter space, but by setting $v_3 = 0$, and $E_3 = 0$, one can show the existence of a non-SUSY Minkowski vacua. For instance, choosing

$$v_3 = E_3 = 0, \quad q = -1, \quad E_7 = v_7^2, \quad B = \frac{3A}{v_7^2},$$

(21)

one finds a non-SUSY minimum at $t_{min} = E_7$, $s_{min} = v_7^2$, which is indeed a Minkowski vacuum. It is important to notice that for the cases we have discussed where SUSY is broken, one might expect a relevant one-loop corrections to the potentials. Therefore one should not rule out the possibility of having Minkowski or AdS vacua since these corrections can uplift them to dS vacua with small cosmological constant as has been discussed in Ref.[17].

4 Conclusions

In this letter we have argued that it is possible to obtain de Sitter or Minkowski non-SUSY vacua in type IIB string theory compactification with fluxes without using neither non-perturbative effects nor adding anti-branes $\bar{D}3$. We have accomplished this by adding charged matter and gauge fluxes on both $D7$ and $D3$ branes. We found that the matter field contributions to the D-terms, the Kähler form and the superpotential are crucial to stabilize the complex structure moduli $Z_i$, the dilaton $S$, and the real part of the Kähler modulus $T_R$ with dS or Minkowski non-SUSY vacua. We showed that these results are valid in scenarios where either $T$ or $(T, S)$ are the only light moduli in the low energy effective theory.

Acknowledgements

It is always a pleasure to thank B. Acharya, P. Argyres, K. Choi, N. Mahajan and A. Shapere for enlightening discussions and useful comments. Major part of this work was done within the Associate Scheme of ICTP. S.K would like to thank CERN theory group for their hospitality, where part of this work took place.

References

[1] J. Polchinski and A. Strominger, Phys. Lett. B388 (1996) 736.

[2] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D68, 046005 (2003).

[3] J. P. Derendinger, L. E. Ibanez and H. P. Nilles, Phys. Lett. B 155, 65 (1985); Nucl. Phys. B267, 365 (1986).
[4] M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B156, 55 (1985).

[5] E. Witten, Nucl. Phys. B474, 343 (1996).

[6] C. P. Burgess, R. Kallosh and F. Quevedo, JHEP 0310, 056 (2003).

[7] R. Brustein and S. P. de Alwis, Phys. Rev. D69 (2004) 126006.

[8] K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B718, 113 (2005).

[9] L. E. Ibanez and D. Lust, Phys. Lett. B267, 51 (1991).

[10] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, JHEP 0411, 076 (2004).

[11] S. Gukov, C. Vafa and E. Witten, Nucl. Phys. B584, 69 (2000).

[12] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D66, 106006 (2002).

[13] S. P. de Alwis, hep-th/0506266.

[14] L. E. Ibáñez, C. Muñoz and S. Rigolin, Nucl. Phys. B553, 43 (1999).

[15] L. E. Ibanez and F. Quevedo, JHEP 9910 (1999) 001.

[16] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987) 585; J. Atick, L. Dixon and A. Sen, Nucl. Phys. B292 (1987) 109; M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. B293 (1987) 253.

[17] H. P. Nilles, Phys. Rept. 110, 1 (1984).