“Stopping” of light and quantum memories for photons

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Summary. — A coherent technique for the control of photon propagation in optically thick media and its application for quantum memories is discussed. Raman adiabatic passage with an externally controlled Stokes field can be used to transfer the quantum state of a light pulse (“flying” qubit) to a collective spin-excitation (stationary qubit) and thereby slow down its propagation velocity to zero. The process is reversible and has a potential fidelity of unity without the necessity for strongly coupling resonators. A simple quasi-particle picture (dark-state polariton) of the transfer is presented. The analytic theory is supplemented with exact numerical solutions. Finally the influence of decoherence mechanisms on collective storage states, which are N-particle entangled states, is analyzed.

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1. – Introduction

Among the many challenges for the implementation of quantum information processing (QIP) is the transport of unknown quantum states between separated locations as well as the realization of quantum memories with short access times [1]. Quantum optical systems are very attractive for this, since photons provide ideal carriers of quantum information and nuclear-spin or hyperfine states of atoms are ideal storage systems. Isolation from environmental interactions on one hand and controllable coupling to the photon field on the other can most easily be realized through Raman transitions with a classical laser providing the Stokes field. Combining Raman coupling with adiabatic following provides control over the coherent absorption or emission of photons.

The application of Stimulated Raman adiabatic passage (STIRAP) [2] to single-atom cavity systems has lead to a number of important proposals for qubit transfer between atoms and photons as well as for quantum-logic gates [3]. However due to the small absorption cross section of an isolated atom, it is necessary to employ strongly coupling resonators. The realization of the strong-coupling regime for optical frequencies remains a
technically very challenging task despite the enormous experimental progress in this field [4]. Furthermore a single-atom system is by construction highly susceptible to the loss of atoms and requires a high degree of control over atomic positions. For these reasons the prospects of cavity-QED systems in qubit transfer and storage may be limited.

On the other hand a photon is absorbed with certainty, if a sufficiently large number of atoms is present. Normally such absorption is accompanied by dissipation and only a partial mapping of quantum properties of light to atomic ensembles can be achieved [5]. If dissipation is involved, it is in general not possible to store the quantum state of photons on the level of individual quanta (single qubits) in a reversible manner. Rather a quasi-stationary source is required, whose output can be considered as a train of wave-packets in identical quantum states. Similar limitations apply to techniques of classical optical-data storage in the time domain based on spin- and photon echo or Raman photon echo [6].

Recently we have proposed a method that combines the enhancement of the absorption cross section in many-atom systems with dissipation-free adiabatic passage [7]. It is based on Raman adiabatic transfer of the quantum state of photons to collective atomic excitations using electromagnetically induced transparency (EIT) [8].

In EIT a strong coherent Stokes fields renders the otherwise optically thick medium transparent for a resonant pump. Associated with the transparency is a large linear dispersion, which has been demonstrated to lead to a substantial reduction of the group velocity of light [9]. When a pump pulse propagates in such a medium, its front end gets coherently “absorbed” and the corresponding excitation is transferred to the Stokes field as well as into a spin excitation. At the back end the process is exactly reversed and all excitation returned to the field, i.e. there is no net transfer from photons to atoms or vice versa. From the point of view of the atoms slow light in an EIT systems corresponds to a complete adiabatic return. Nevertheless part of the photonic excitation is temporarily stored in the atoms. During this time interval photons and atoms form quasi-particles, called dark-state polaritons [7] that propagate with a velocity determined by the ratio of the electromagnetic to the matter component. Unfortunately EIT systems have only limited capabilities as a temporary memory, since the achievable ratio of storage time to pulse length is rather limited [10].

The limitations of EIT can be overcome, however, when the group velocity is changed in time and is adiabatically reduced to zero by dynamically decreasing the strength of the classical Stokes field. The reduction of the Stokes field in time brakes the symmetry of the process and net flow of excitation from photons to atoms or vice versa can be achieved. By adiabatically reducing the group velocity the light pulse can eventually be brought to a full stop. In this process its excitation as well as its quantum state are completely transferred to the atomic spins. The process can be reversed and the light pulse regenerated. Recent experiments [11] have already demonstrated some basic principles of this technique - the dynamic group velocity reduction to a full stop and adiabatic following in dark-state polaritons.

2. – single-atom cavity-QED

To introduce the basic idea of a controlled and reversible quantum state transfer via Raman adiabatic passage, let us first consider the case of an individual 3-level atom coupled to a single quantized mode as pump and a classical field as Stokes field. This is illustrated in Fig. 1. Both fields are assumed to be resonant with the corresponding transitions and decay from the excited state out of the system is taken into account. The
(complex) interaction “Hamiltonian” in rotating-wave approximation reads in a rotating frame

\[ H = \hbar g a \sigma_{ab} + \hbar \Omega(t) \sigma_{ac} + \hbar \text{c.c.} - i \hbar \gamma \sigma_{aa} \]  \hspace{1cm} (1)

where \( a, a^\dagger \) are the annihilation and creation operators of the quantized pump mode and \( \Omega(t) \) is the (in general time-dependent) Rabi-frequency of the classical Stokes field. \( \sigma_{ij} \equiv |i\rangle \langle j| \) is the atomic spin-flip operator from state \( j \) to state \( i \) and \( g = \varphi \sqrt{\omega/\hbar \epsilon_0 V} \) characterizes the vacuum Rabi-frequency of the quantized mode, \( \varphi \) being the dipole moment and \( V \) the mode volume. The imaginary part of the Hamiltonian effectively describes spontaneous emission from the excited state with rate \( \gamma \). The interaction couples only triplets of bare eigenstates of the combined atom-field system, viz \( |a, n+1\rangle \leftrightarrow |a, n\rangle \leftrightarrow |c, n\rangle \), where \( n = 0, 1, \ldots \) denotes the number of photons in the mode. In addition there is the total ground state \( |b, 0\rangle \) which is completely decoupled. In the basis of these states the Hamiltonian separates in \( 3 \times 3 \) block-matrices of the form

\[ H_n = \hbar \begin{bmatrix} -i\gamma & g \sqrt{n} & \Omega(t) \\ g \sqrt{n} & 0 & 0 \\ \Omega(t) & 0 & 0 \end{bmatrix} \]  \hspace{1cm} (2)

Each of these matrices has one instantaneous (adiabatic) eigenstate \( |\phi_0^{(n)}\rangle \) with eigenvalue \( \epsilon_0^{(n)} = 0 \) and two eigenstates \( |\phi_{\pm}^{(n)}\rangle \) with eigenvalues \( \epsilon_{\pm}^{(n)} = \pm \sqrt{\Omega^2(t) + g^2 n} \). The zero eigenstates read

\[ |\phi_0^{(n)}\rangle = \cos \theta_n(t) |b, n+1\rangle - \sin \theta_n(t) |c, n\rangle, \quad n = 0, 1, 2, \ldots, \]  \hspace{1cm} (3)

where the mixing angles \( \theta_n(t) \) are defined as \( \tan \theta_n(t) = g \sqrt{n}/\Omega(t) \). The important feature of the zero-eigenstates is that they do not contain the excited state and are thus immune to spontaneous emission. For this reason these states are called dark states [12]. Furthermore by changing the strength of the classical Stokes field, i.e. \( \Omega(t) \), the mixing angles \( \theta_n \) can be rotated from \( \theta_n = 0 \), where \( |\phi_0^{(n)}\rangle = |b, n+1\rangle \) to \( \theta_n = \pi/2 \), where \( |\phi_0^{(n)}\rangle = -|c, n\rangle \). If the atom-field system is initially prepared in the dark state and if \( \Omega(t) \) changes sufficiently slowly, the state vector will follow the rotation. Thus by

![Fig. 1. – a) 3-level atoms coupled to single quantized mode and classical control field of (real) Rabi-frequency \( \Omega(t) \). b) coupling of relevant bare eigenstates; \( g \)-vacuum Rabi-frequency](image-url)
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stimulated Raman adiabatic passage [2] a controlled and reversible transfer of excitation from the quantized radiation mode to the atom is possible:

$$|b, n + 1\rangle \leftrightarrow -|c, n\rangle.$$  (4)

This mechanism is the basis of several proposals for engineering of quantum states of the radiation field in resonators, for the transfer of quantum states between atoms through a resonator mode and for the transfer of quantum states between different cavities [3].

Adiabatic following requires that the rate of change of the mixing angles should be sufficiently slow. In particular the characteristic time of transfer $T$ should obey the condition

$$\frac{g^2 n}{\gamma} T \gg 1.$$  (5)

The transfer time $T$ is usually limited by the finite decoherence time of the field. In a resonator set-up the lifetime is determined for example by mirror losses and coupling to the outside. If $\kappa$ denotes the characteristic rate of photon loss, the decoherence time of a Fock-state $|n\rangle$ is $1/(\kappa n)$. Thus adiabaticity requires

$$g^2 \gg \kappa \gamma.$$  (6)

This condition is referred to as strong-coupling regime. Condition (6) is technically very difficult to satisfy. The physical origin of this strong condition is the small absorption cross section $\sigma$ of atoms in the optical frequency domain. In fact as $g^2 \sim \sigma/A \sim \lambda^2/A$, where $A$ is the cross section of the light field at the position of the atoms, tight focusing is required. As a consequence cavity-QED techniques with individual atoms in the strong-coupling regime are very sensitive to an exact positioning of the atom. Resonator systems have the further disadvantage that communication between them requires a careful timing of the control fields to avoid losses due to impedance mismatch at the cavity interfaces.

3. – Temporary storage of photons in many-atom systems: electromagnetically induced transparency and slow-light

An obvious way to overcome the limitations of single-atom systems caused by the small optical cross section is to use optically thick ensembles of atoms. This also allows to abandon the use of resonators and thus to avoid impedance matching problems. In order to identify appropriate transfer schemes in ensembles of 3-level atoms, let us first discuss pulse propagation in such media.

One of the most important phenomena associated with pulse propagation in these systems is called electromagnetically induced transparency [8]. If, as indicated in Fig. 2, a sufficiently strong coherent field of constant Rabi-frequency $\Omega$ couples the excited state $|a\rangle$ to a meta-stable state $|c\rangle$, the absorption of the probe field is exactly canceled at two-photon resonance. In Fig. 2 the imaginary part of the susceptibility

$$\chi = \eta \left[ \gamma \delta |\Omega|^2 - \delta^2 - i \gamma \delta \right] \approx \eta \left[ \frac{\gamma \delta}{|\Omega|^2} + i \left( \frac{\gamma \delta}{|\Omega|^2} \right)^2 + \mathcal{O}(\delta^3) \right].$$  (7)
is shown as a function of the detuning from resonance. Here $\eta = \frac{3}{8\pi^2}\rho\lambda^3$ is a density parameter that gives the number of atoms per cubic wavelength, $\delta$ is the probe detuning and $\gamma$ the excited-state decay rate.

![Diagram](image)

**Fig. 2.** *Left:* 3-level Λ-type medium resonantly coupled to a coherent Stokes field with Rabi-frequency $\Omega$ and a (quantum) probe field $E(z, t)$. *Right:* Typical susceptibility spectrum for probe field $E$ as function of normalized detuning for resonant and constant drive field. Real part $\chi'$ describes refractive index contribution and imaginary part $\chi''$ absorption.

At the same time the real part $\chi'$, which is responsible for the refractive index, shows a large normal linear dispersion. Associated with the linear dispersion is a reduction of the group velocity

$$v_{gr} = \frac{c}{1 + n_g(z)}, \quad \text{with} \quad n_g(z) = \frac{\omega d\chi}{d\omega} = \eta \frac{k c \gamma}{|\Omega|^2}. \quad (8)$$

Since the medium is non-absorbing, high densities can be used and rather small group velocities can be achieved. In fact values as low as few meters per second have been observed [9]. In the region of linear dispersion the propagation equation for the slowly varying complex field amplitude reads

$$\left(\frac{\partial}{\partial t} + v_{gr}(z) \frac{\partial}{\partial z}\right) E(z, t) = 0 \quad (9)$$

where we have taken into account a possible space dependence of the group velocity. The solution of this equation

$$E(z, t) = E(0, t - \int_0^z dz' \frac{1}{v_{gr}(z')}), \quad (10)$$

describes slow propagation with an invariant temporal pulse shape. Note, that the spatial shape is not conserved and depends on the spatial profile of $v_{gr}(z)$. The slow-down is a loss-less linear process and hence all properties of the slowed pulse are conserved. In particular also its quantum state. The slow-down of the group velocity is thus interesting from the point of view of quantum memories as it represents a temporary storage.

If a light pulse propagates in an absorption-free medium with linear dispersion, its total number of photons is reduced by the ratio of the group velocity $v_{gr}$ to the vacuum speed of light $n/n_0 = v_{gr}/c$. However the time-integrated number of photons crossing
a plane perpendicular to the propagation direction is constant. Thus photons must be temporarily stored in the combined system of atoms and control field.

The transfer mechanism between probe field and atomic system can be identified as STIRAP. To see this, let us consider a coherent probe pulse with all atoms initially in the ground state \( |b\rangle \). This is a natural assumption since optical pumping will prepare the atoms in this state. At the initial time the ground state \( |b\rangle \) is then identical to the dark state

\[
|d(z)\rangle = \cos \vartheta(z,t) |b\rangle - \sin \vartheta(z,t) |c\rangle \quad \longrightarrow \quad |b\rangle
\]

where the mixing angle \( \vartheta \) is now given by \( \tan \vartheta(z,t) = \Omega_{p}(z,t)/\Omega \) with \( \Omega_{p}(z,t) \) being the Rabi-frequency of the probe pulse. When the front end of the probe pulse arrives at an atom, the mixing angle makes a small excursion from zero. In this process a fraction of the state amplitude rotates from \( |b\rangle \) to \( |c\rangle \) and energy is taken out of the probe pulse. When the probe pulse reaches its maximum, the excursion of the mixing angle stops and \( \vartheta \) returns to zero. The state vector of the atoms follows and rotates back to the ground state. Thus the entire energy is put back into the probe pulse at its back end. The slow-down of light in EIT can be understood as a Raman adiabatic return of the atoms.

In order for the process to be entirely adiabatic, the condition

\[
\frac{\Omega^2}{\gamma} T_p \gg \sqrt{\eta k L}
\]

must be fulfilled, where \( T_p \) is the pulse duration, \( L \) the length of the medium, and it was assumed that \( |\Omega| \gg |\Omega_p| \). The presence of the factor \( \sqrt{\eta k L} \) instead of unity in (12), where \( \alpha = \eta k L \) is the opacity of the medium in the absence of EIT, is due to the fact that propagation effects need to be taken into account [2, 13, 14].

The inequality (12) has a very simple physical meaning. It states that the spectrum of the pulse should be much less than the spectral window of transparency in EIT given by

\[
\Delta \omega_p \ll \Delta \omega_{tr} = \frac{|\Omega|^2}{\gamma} \frac{1}{\sqrt{\eta k l}}.
\]

It is instructive to express this condition in terms of the group velocity. This yields

\[
\Delta \omega_p \ll \frac{v_{gr}}{l} \sqrt{\eta k l}.
\]

One immediately recognizes that a vanishing group velocity implies a zero spectral width of transparency. This is illustrated in Fig. 3 which shows the absorption of a homogeneously broadened EIT medium as function of the detuning from resonance and the group velocity.

Since EIT does not change the spectrum of the probe pulse, it is in this way not possible to bring a photon wave-packet to a complete stop. This is illustrated in Fig. 4, where the propagation of a pulse in an EIT medium with spatially decreasing group velocity is depicted. The left curve shows the effect of the linear dispersive part alone (\( \chi' \)). Since the front end of the pulse sees always a smaller group velocity than the back
end, the wave-packet becomes spatially compressed during the deceleration according to 
\[ \frac{\Delta l(z)}{\Delta l(0)} = \frac{v_{gr}(z)}{v_{gr}(0)}. \]

The right curve shows for comparison a numerical solution of the 1-D propagation equations. One clearly recognizes that at a particular position, where the pulse spectral width becomes comparable to the decreasing transparency width, dissipation and partial back reflection sets in.

Although EIT does not allow to bring a pulse to a complete stop, the question arises whether it can be used as a temporary memory. Expressing the transparency width in
terms of the pulse delay time \( \tau_d \equiv n_g L/c \) for the medium yields

\[
\Delta \omega_{tr} = \sqrt{\eta k L} \frac{1}{\tau_d}.
\]

(15)

Thus large delay times imply a narrow transparency window, which in turn requires a long pulse time. Hence there is an upper bound for the ratio of achievable delay (storage) time to the initial pulse duration of a photon wave-packet

\[
\frac{\tau_d}{\tau_p} \leq \sqrt{\eta k L}.
\]

(16)

\( \tau_d/\tau_p \) is the figure of merit for any memory device. In practice, the achievable opacity \( \alpha = \eta k L \) of atomic vapor systems is limited to values below \( 10^4 \) resulting in upper bounds for the ratio of time delay to pulse length of the order of 100. Thus EIT media with ultra-small group velocity are only of limited use as temporary storage devices.

The delay-time limitations are a common feature of any EIT-based scheme which does not affect the spectrum of the probe pulse and sets strong limitations e.g. to freezing of light in moving media [15] and so-called optical black holes based on EIT [16]. For instance a light pulse propagating in an EIT medium with a vortex matter flow, as suggested in [16], will be absorbed or reflected before it reaches the analog of the Schwarzschild-radius.

4. – Light stopping by adiabatic rotation of dark-state polaritons

4.1. Time-dependent group velocity. – In the last section we have seen that the essential limitation of EIT for a temporary memory and light stopping is the proportionality between spectral transmission width and group velocity. Below a certain value of \( v_{gr} \) the transmission window becomes narrower than the pulse spectrum and the pulse is absorbed unless its spectrum narrows as well. Changing the spectrum of a pulse in a linear medium is only possible if the medium properties change in time. Let us thus consider a hypothetical medium with a time-dependent group velocity. The corresponding wave equation reads

\[
\left( \frac{\partial}{\partial t} + v_{gr}(t) \frac{\partial}{\partial z} \right) E(z, t) = 0.
\]

(17)

It has the solution

\[
E(z, t) = E(z - \int_0^t d\tau v_{gr}(\tau), 0),
\]

(18)

which in contrast to the case of \( v_{gr} = v_{gr}(z) \) describes a spatially invariant propagation. At the same time the temporal profile is however not conserved and the spectrum of the probe field changes during propagation. Assuming that \( v_{gr} \) changes only slow compared to the field amplitude, one finds that the spectral width narrows (broadens) according to

\[
\Delta \omega_p(t) \approx \Delta \omega_p(0) \frac{v_{gr}(t)}{v_{gr}(0)}.
\]

(19)
In this way the bandwidth limitations of EIT can be overcome. The spectrum of the probe pulse narrows in the same way as the transparency width when the group velocity is reduced. It is thus worthwhile to consider the propagation of a probe pulse in an EIT medium with an explicitly time dependent control field.

4.2. Polariton picture of EIT and “stopping” of light. – In order to describe the propagation of a weak probe pulse in an explicitly time-dependent EIT medium, let us consider a one-dimensional model. Two fields, a strong and undepleted Stokes field of Rabi-frequency \( \Omega(t) \) and a weak probe pulse with dimensionless field amplitude \( E(z,t) \) couple resonantly the transitions in a 3-level medium as shown in Fig. 2. The propagation equation can be written in the form

\[
\frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \mathcal{E} = igN \rho_{ab}(z,t),
\]

where \( g \) is the vacuum Rabi-frequency in an interaction volume \( V \), with \( g^2N = n_kc^2\gamma \), \( N \) being the number of atoms in \( V \). \( \rho_{ab}(z,t) \) is the density matrix element of the atoms between the excited and ground states at position \( z \). Without the probe pulse, the atoms are assumed to be in state \( |b\rangle \), e.g. as a result of optical pumping by the Stokes field. I.e. in zeroth order of the probe field one has \( \rho_{ab} = 1 \) and all other elements vanish. In first order of \( E \) the following relevant density-matrix equations are obtained:

\[
\dot{\rho}_{ab} = -\gamma \rho_{ab} + igE + i\Omega \rho_{cb}, \tag{21}
\]
\[
\dot{\rho}_{cb} = i\Omega \rho_{ab}. \tag{22}
\]

It is now convenient to introduce two new dimensionless field variables corresponding to quasi-particles, called dark- and bright-state polaritons [17]

\[
\Psi(z,t) = \cos \theta(t) E(z,t) - \sin \theta(t) \sqrt{N} \rho_{cb}(z,t), \tag{23}
\]
\[
\Phi(z,t) = \sin \theta(t) E(z,t) + \cos \theta(t) \sqrt{N} \rho_{cb}(z,t), \tag{24}
\]

where \( \tan^2 \theta = g^2N/\Omega^2 = n_g \). The group velocity of slow-light propagation is thus given by \( v_{gr} = c \cos^2 \theta \). One can transform the equations of motion for the electric field and the atomic variables into the new variables. This yields

\[
\left[ \frac{\partial}{\partial t} + c \cos^2 \theta \frac{\partial}{\partial z} \right] \Psi = -\dot{\theta} \Phi - \sin \theta \cos \theta c \frac{\partial}{\partial z} \Phi, \tag{25}
\]

and

\[
\Phi = \frac{\sin \theta}{g^2N} \left( \frac{\partial}{\partial t} + \frac{\gamma}{\tan \theta} \frac{\partial}{\partial t} \right) \left( \sin \theta \Psi - \cos \theta \Phi \right). \tag{26}
\]

Introducing an adiabaticity parameter \( \varepsilon \equiv \left( g\sqrt{N}T \right)^{-1} \) with \( T \) being a characteristic time, one can expand the equations of motion in a power series in \( \varepsilon \). In lowest order i.e. in the adiabatic limit one finds \( \Phi \approx 0 \) and thus the electric field amplitude as well as the spin coherence are entirely determined by the amplitude of the dark-state polariton

\[
\mathcal{E} = \cos \theta \Psi; \quad \text{and} \quad \sqrt{N} \rho_{cb} = -\sin \theta \Psi. \tag{27}
\]
Furthermore all terms on the r.h.s. of eq.(25) vanish and one finds
\[ \left( \frac{\partial}{\partial t} + c \cos^2 \theta(t) \frac{\partial}{\partial z} \right) \Psi = 0. \] (28)

4.3. “Stopping” of light. – Eq.(28) describes a shape-preserving propagation with instantaneous velocity \( v = v_{\text{gr}}(t) = c \cos^2 \theta(t) \):
\[ \Psi(z, t) = \Psi(z - c \int_0^t d\tau \cos^2 \theta(\tau), 0). \] (29)

For \( \theta \to 0 \), i.e. for a strong external drive field \( \Omega^2 \gg g^2 N \), the polariton has purely photonic character \( \Psi = E \) and the propagation velocity is that of the vacuum speed of light. In the opposite limit of a weak drive field \( \Omega^2 \ll g^2 N \) such that \( \theta \to \pi/2 \), the polariton becomes spin-wave like \( \Psi = -\sqrt{N} \rho_{cb} \) and its propagation velocity approaches zero. Thus the following mapping can be realized
\[ E(z) \longleftrightarrow \rho_{cb}(z') \] (30)

with \( z' = z + z_0 = z + \int_0^\infty d\tau c \cos^2 \theta(\tau) \).

Eq.(30) is the essence of the transfer technique of quantum states from photon wavepackets propagating at the speed of light to stationary atomic excitations (stationary spin waves). Adiabatically rotating the mixing angle from \( \theta = 0 \) to \( \theta = \pi/2 \) decelerates the polariton to a full stop, changing its character from purely electromagnetic to purely atomic. Due to the linearity of eq.(28), the quantum state of the polariton is not changed during this process.

Likewise the polariton can be re-accelerated to the vacuum speed of light; in this process the stored quantum state is transferred back to the field. This is illustrated in Fig.5, where we have shown the coherent amplitude of a dark-state polariton which results from an initial light pulse as well as the corresponding electromagnetic and matter components obtained from a numerical solution of the 1-D Maxwell-Bloch equations.

As noted above, the spectrum of the polariton and thus of the probe pulse narrows in the same way as the EIT transmission window when the group velocity is reduced as a function of time. The adiabatic slow-down of a dark-state polariton thus avoids the bandwidth limitation of EIT and a light pulse can indeed be brought to a full stop. This is the essential difference of the present scheme to the light-freezing proposal of Ref. [15]. In the adiabatic transfer process all information carried by the pulse, i.e. also its quantum state, are transferred to a collective Raman or spin excitation of atoms.

In order to prove the validity of the simple analytic solution (29), we have compared in Fig. 6 exact numerical and analytical results for the situation of Fig. 5. As can be recognized there is an excellent agreement.

4.4. Adiabatic condition and non-adiabatic corrections. – The analysis of the conditions for Raman adiabatic passage in single-atom cavity QED in sec. 2 lead to the experimentally challenging strong-coupling requirement. Let us now investigate the corresponding requirements for the present scheme. To this end we take into account the
Fig. 5. – Numerical simulation of dark-state polariton propagation with envelope \( \exp\{-z/10\}^2 \). The mixing angle is rotated from 0 to \( \pi/2 \) and back according to \( \cot \theta(t) = 100(1 - 0.5 \tanh[0.1(t - 15)] + 0.5 \tanh[0.1(t - 125)]) \). \textit{top:} polariton amplitude, \textit{bottom-left:} electric field amplitude, \textit{bottom-right:} atomic spin coherence; all in arbitrary units. Axes are in arbitrary units with \( c = 1 \).

first-order correction to the adiabatic solution (29). Making use of the zeroth-order relation \( \dot{\Psi} = -c \cos^2 \theta \partial \Psi / \partial z \) to eliminate time derivatives in favor of spatial derivatives we find

\[
\left[ \frac{\partial}{\partial t} + c \cos^2 \theta \frac{\partial}{\partial z} \right] \Psi = -A(t)\Psi + B(t)c \frac{\partial}{\partial z} \Psi + C(t)c^2 \frac{\partial^2}{\partial z^2} \Psi - D(t)c^3 \frac{\partial^3}{\partial z^3} \Psi,
\]

where

\[
A(t) = \left( \gamma + \frac{1}{2} \frac{\partial}{\partial t} \right) \left( \frac{\dot{\theta}^2 \sin^2 \theta}{g^2 N} \right), \quad B(t) = \frac{\sin \theta}{3g^2 N} \frac{\partial^2}{\partial t^2} \sin^3 \theta,
\]

\[
C(t) = \left( \gamma + \frac{1}{2} \frac{\partial}{\partial t} \right) \frac{\sin^4 \theta \cos^2 \theta}{g^2 N}, \quad D(t) = \frac{\sin^4 \theta \cos^4 \theta}{g^2 N}.
\]

\( A(t) \) describes homogeneous losses due to non-adiabatic transitions followed by spontaneous emission. \( B(t) \) gives rise to a correction of the polariton propagation velocity, \( C(t) \) results in a pulse spreading by dissipation of high spatial frequency components.
and $D(t)$ leads to a deformation of the polariton. Since all coefficients depend only on time, eq.(31) can be solved by spatial Fourier transform and simple integration in time. In order to neglect dissipation, the terms containing $A(t)$ and $C(t)$ must be small, which results in the two conditions

\[
\frac{\gamma c^2}{L_p^2} \int_0^\infty dt \frac{\cos^2 \theta(t)}{g^2 N} = \frac{\gamma c L}{g^2 N L_p^2} \ll 1, \tag{34}
\]

and

\[
\gamma \int_0^\infty dt \frac{\dot{\theta}^2 \sin^2 \theta}{g^2 N} = \gamma \int_0^\infty dt \frac{\dot{\theta}^2}{g^2 N + \Omega^2(t)} \ll 1. \tag{35}
\]

To simplify the first expression we have used here the approximation $\sin \theta \approx 1$. $L_p$ denotes the pulse length in the medium at the initial time.

The first condition should be compared and contrasted to the strong coupling condition of sec. 2. Using $T_p = L_p/c$ one finds

\[
\frac{g^2 N'}{\gamma} T_p \gg 1 \quad \leftrightarrow \quad \frac{g^2}{\gamma} T_p \gg 1, \tag{36}
\]

where $N' \equiv N L/L_p$ is the number of atoms in a volume of length $L_p$ (rather than $L$). The large factor $N'$ is a signature of the collective interaction and strongly alleviates the requirements in the many-atom system as compared to the single-atom one. Condition (34) has again a simple physical meaning. It states that the initial pulse spectral width has to be much less than the initial transparency width, i.e. at the time when the pulse enters the medium.

\[
\Delta \omega_p(0) \ll \Delta \omega_{tr}(0). \tag{37}
\]
This is easily satisfied, if the probe pulse is not too short. Practically it does prevent however light storage of pulses shorter than a few nanoseconds.

The second condition is well-known from adiabatic passage [13, 18] and sets a limit to the rotation velocity \( \dot{\theta} \) of the mixing angle and hence to the deceleration/acceleration of the polariton. In order to stay in the adiabatic regime at all times the characteristic time scale \( T \) has to fulfill the condition

\[
T \gg \frac{l_{\text{abs}}}{c} \frac{v_{\text{gr}}}{c},
\]

where \( l_{\text{abs}} = c\gamma/g^2N \) is the absorption length in the absence of EIT. We note that for realistic experimental parameters the quantity on the right hand side is extremely small on all relevant time scales. It should be noted that a sudden and sharp switch-off of the drive field leads to a complete loss of the entire electromagnetic component of the polariton. This is illustrated by numerical simulations in Fig.7. The left figure shows the propagation of a light pulse in an EIT medium, where the cosine of the mixing angle is switched from 1 to 0 at \( t = 50 \) and back to 1 at \( t = 90 \). Apart from some small and rapidly decaying Rabi-oscillations, the light field is immediately absorbed and cannot be regenerated. On the other hand the losses are small if the cosine of the mixing angle is initially significantly less than unity. This is shown in the right picture, where \( v_{\text{gr}}/c = \cos^2 \theta(\pm \infty) = 0.1 \) and hence the output amplitude is only reduced by 10%.

![Fig. 7. – propagation of light pulse in EIT medium with sudden switching of \( \cos \theta \). left: \( \cos \theta \) is switched from 1 to zero at \( t = 50 \) and back at \( t = 90 \), right: same switch, but with initial and final values of \( \cos^2 \theta(\pm \infty) = 0.1 \), only electromagnetic component of the polariton (here 10%) gets lost; \( \gamma = 1 \).](image)

5. – Quantum memory and decoherence

An important question when discussing the application of the many-atom system to a quantum memory is the sensitivity to errors. Thus in the following the influence of decoherence processes on the fidelity of the state storage will be discussed.

The most essential properties of a quantum memory based on collective excitations can be understood looking at the case of a single mode of the radiation field as realized e.g. in
a single mode optical cavity. Consider a collection of $N$ 3-level atoms as shown in Fig. 1. The dynamics of this system is described by the complex Hamiltonian ($E_b = \hbar \omega_b = 0$): 

\begin{equation}
H = \hbar \omega a^\dagger a + \hbar (\omega_a - i\gamma) \sum_{j=1}^{N} \sigma^j_{aa} + \hbar \omega_c \sum_{j=1}^{N} \sigma^j_{cc} +
\end{equation}

\begin{equation}
+ \hbar g \sum_{i=1}^{N} a\sigma^i_{ab} + \hbar \Omega(t) e^{-i\nu t} \sum_{i=1}^{N} \sigma^i_{ac} + \text{h.c.}
\end{equation}

Here $\sigma^j_{\mu\nu} = |\mu\rangle_{ii}\langle\nu|$ is the flip operator of the $i$th atom and the vacuum Rabi-frequency is assumed to be equal for all atoms. We also have introduced an imaginary part to the Hamiltonian to take into account losses from the excited state.

When all atoms are prepared initially in level $|b\rangle$ the only states coupled by the interaction are the totally symmetric Dicke-states $|19\rangle$

\begin{equation}
|b\rangle_N = |b_1, b_2, \ldots, b_N\rangle,
\end{equation}

\begin{equation}
|a\rangle_N = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |b_1, \ldots, a_j, \ldots, b_N\rangle,
\end{equation}

\begin{equation}
|c\rangle_N = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |b_1, \ldots, c_j, \ldots, b_N\rangle,
\end{equation}

\begin{equation}
|aa\rangle_N = \frac{1}{\sqrt{2N(N-1)}} \sum_{i\neq j=1}^{N} |b_1, \ldots, a_i, \ldots, a_j, \ldots, b_N\rangle, \text{ etc.}
\end{equation}

The couplings within the sub-systems corresponding to a single and a double excitation are shown in Fig. 8.

Fig. 8. – Coupling of bare eigenstates of atom + cavity system for at most two photons

The interaction of the $N$-atom system with the quantized radiation mode has now a family of dark-states $|19\rangle$

\begin{equation}
|D, n\rangle_N = \sum_{k=0}^{n} \sqrt{\frac{n!}{k!(n-k)!}} (- \sin \theta)^k (\cos \theta)^{n-k} |e^k, n-k\rangle_N,
\end{equation}
\[
\tan \theta(t) \equiv \frac{g \sqrt{N}}{\Omega(t)}.
\]

It should be noted that although the dark states \(|D, n\rangle_N\) are degenerate they belong to exactly decoupled sub-systems. This means there is no transition between them even if non-adiabatic corrections are taken into account. Adiabatically rotating the mixing angle \(\theta\) from 0 to \(\pi/2\) leads to a complete and reversible transfer of all photonic states to a collective atomic excitation if the maximum number of photons \(n\) is less than the number of atoms \(N\). If the initial quantum state of the single-mode light field is in a mixed state described by the density matrix \(\hat{\rho}_f = \sum_{n,m} \rho_{nm} |n\rangle\langle m|\), the transfer process generates a quantum state of collective excitations according to

\[
\sum_{n,m} \rho_{nm} |n\rangle\langle m| \otimes |b\rangle_N N \langle b| \leftarrow \rightarrow |0\rangle \otimes \sum_{n,m} \rho_{nm} |c^n\rangle_N N \langle c^m|.
\]

Thus the storage state corresponding to a photon Fock state \(|n\rangle\) is an entangled many-particle state \(|c^n\rangle_N\). These states are known to be highly susceptible to decoherence. For example if any one of the \(N\) atoms undergoes a spin flip, corresponding to \(|b\rangle_j \leftrightarrow |c\rangle_j\), an orthogonal state is created. When the probability of a single-atom error is \(p\), the total probability that an orthogonal state is created is \(P = 1 - (1 - p)^N \sim Np\). Thus one might conclude that collective quantum memories display a substantially enhanced rate of decoherence, which is a substantial drawback that out-weighs the advantage of ensembles for the coupling of light and matter. Although the following discussion is far from complete, some arguments why this is indeed not the case will be presented now [20].

For this the system Hamiltonian will first be brought into a more appropriate form. Separating the oscillatory factor \(e^{-i\omega t}\) by a canonical transformation, assuming two photon resonance, i.e. \(\omega = \omega_c + \nu\), and adiabatically eliminating the excited state \(|a\rangle\) yields

\[
H = \hbar \omega \left( \Psi^\dagger \Psi + \sum_l \Phi_l^\dagger \Phi_l \right) - i\hbar \frac{\Omega^2(t)}{\gamma} \sum_l \Phi_l^\dagger \Phi_l.
\]

Here we have assumed adiabatic conditions and introduced the dark-state polariton operator

\[
\Psi = \cos \theta(t) a - \sin \theta(t) \frac{1}{\sqrt{N}} \sum_{j=1}^N \sigma_{bc}^{j},
\]

as well as the \(N\) bright-state polariton operators

\[
\Phi_0 = \sin \theta(t) a + \cos \theta(t) \frac{1}{\sqrt{N}} \sum_{j=1}^N \sigma_{bc}^{j},
\]

\[
\Phi_l = \frac{1}{\sqrt{N}} \sum_{j=1}^N \sigma_{bc}^{j} \exp \left\{ 2\pi i \frac{lj}{N} \right\} \quad \text{for} \quad l \in \{1, 2, \ldots, N - 1\}.
\]
Expression (46) shows that under adiabatic conditions all excitations of the dark-state polariton \((\Psi^\dagger)^n|b,0\rangle_N\) are conserved, while all bright-polariton excitations decay via optical pumping into the excited state and subsequent spontaneous emission.

It can also be seen from eqs.(46 - 49) that in the read-out process of the memory, where \(\theta : \pi/2 \to 0\) only dark-polariton excitations will be transferred to field excitations. Thus from the point of view of photon storage all collective states with the same number of dark-polariton excitations are equivalent. For example the symmetric dark-state

\[
|D, n\rangle_N = \frac{1}{\sqrt{n!}}(\Psi^\dagger)^n|b,0\rangle_N,
\]

which is a storage state of a radiation-mode Fock-state with \(n\) photons is equivalent to all states with arbitrary number of bright-polariton excitations as long as there are exact \(n\) excitations of \(\Psi\):

\[
|D, n\rangle_N \equiv (\Phi^\dagger_1)^k \cdots (\Phi^\dagger_2)^l (\Psi^\dagger)^n|b,0\rangle_N.
\]

The existence of these equivalence classes of storage states has an important consequence. All transitions within an equivalence class of states caused by environmental interactions leave the fidelity of the quantum memory unaffected. This leads to an exact compensation of the enhanced probability of a single-atom error to occur.

To illustrate this, let us consider the case of a Fock state of the radiation field \(|n\rangle\) stored in the symmetric dark-state \(|D, n\rangle_N\). A random spin flip of atom \(j\) from state \(|b\rangle\) to \(|c\rangle\) can be described by the operation \(|D, 1\rangle_N \to \sigma_{cb}^j|D, 1\rangle_N\). Assuming \(\theta = \pi/2\) and making use of

\[
\sigma_{cb}^j = \frac{1}{\sqrt{N}} \left( \sum_{l=1}^{N-1} \exp \left\{ -2\pi i \frac{l}{N} \right\} \Phi^\dagger_l + \Psi^\dagger \right)
\]

one finds

\[
|D, n\rangle \to \sum_{j=l}^{N-1} \frac{\exp \left\{ -2\pi i \frac{l}{N} \right\}}{\sqrt{N}} \Phi^\dagger_l |D, n\rangle + \frac{1}{\sqrt{N}} \Psi^\dagger |D, n\rangle.
\]

Only the last component belongs to a state that is not in the same equivalence class as \(|D, n\rangle_N\). Thus the probability to leave the equivalence class is only \(1/N\). This factor exactly compensates the factor \(N\) of the total probability of a spin flip error to occur in any one atom.

One can show that similar arguments hold for all possible non-cooperative decoherence mechanisms. Hence the sensitivity of the collective quantum memory does not depend on the number of particles involved. The sensitivity of many-particle quantum memories to decoherence is not enhanced as compared to the single-atom counterparts, given that there are no cooperative decay processes.

6. – Summary

A technique for a controlled and coherent transfer of the quantum state of photons to and from collective spin excitations of ensemble of atoms has been discussed. The
underlying mechanism is an adiabatic rotation of a dark-state polariton between a purely photonic and a purely atomic excitation. Due to the collectively enhanced interaction cross section, the strong-coupling requirement of single-atom cavity QED is eliminated. The mapping technique, although superficially similar looking, is quite different from conventional methods of optical data storage based on optical pumping. In the present set-up no spontaneous emission is present, making a reversible storage on the level on individual light quanta possible.

It was shown that despite the entangled character of the storage states, the collective quantum memory is not more sensitive to decoherence than a single-particle system. This is due to the existence of equivalence classes of many-particle excitations representing the same stored quantum state of light.

While many-particle excitations may have potential advantages over single-atom ones with respect to storage purposes, processing of the information in collective and thus delocalized qubits is more difficult. Recently a method was suggested, however, to implement quantum logic operations between collective excitations [21]. The underlying physical mechanism is here a blockade effect caused by resonant dipole-dipole interactions in Rydberg states which is not sensitive to the actual separation between two atoms.

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