Effect of kinematic hardening on the yield surface evolution after strain-path change

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Abstract. Abrupt strain path changes without elastic unloading have been used to investigate the yield surface of sheet metals, both experimentally and theoretically. These investigations emphasized an apparent non-normality phenomenon, following such a strain-path change. Subsequently, these results inspired the development of plasticity models including non-associated flow rules and non-normality theories. In this study, this type of abrupt strain-path changes is investigated using kinematic hardening models in the context of associated plasticity. The aim is to emphasize the contribution of kinematic hardening ingredient to the apparent violation of the normality condition. The results show that kinematic hardening slope and curvature of the material have a strong influence to contribute to this apparent non-normality.

1. Introduction
Abrupt strain path changes without elastic unloading have been applied to investigate the shape of the subsequent yield surface evolution after an initial biaxial loading. Plasticity theoretical calculations and experimental validations have shown the existence of an apparent non-normality of plastic flow, with a vertex formed on the yield surface [1-3]. Based on such pioneering studies, numerous phenomenological and physical plasticity models including non-associated flow rules were developed, e.g., in [4-9].

According to these research results, some reasons have been pointed out to explain the presence of a vertex and of the loss of normality, like the role of rate sensitivity, elasticity. However, a majority of the former studies consider only focus on isotropic hardening, thus the influence of kinematic hardening contributes to non-normality were not investigated in detail, which need to be further clarified.

This paper analyzes the effect on the apparent non-normality on subsequent yield surface from kinematic hardening factor after abrupt strain-path change, which can be quantified for typical sheet steel subject to biaxial-to-shear strain path change. The apparent non-normality is simply quantified by the angle between the plastic strain rate tensor and the trace of the subsequent loading path, which is supposed to stay close to the yield surface, as illustrated by angle $\beta$ in Figure 1. The constitutive model used for this research is described in Section 2, while Section 3 describes the numerical simulations and the results, leading to the final conclusions.
2. Elaso-visco-plastic model

In computational plasticity, the total strain rate tensor $\dot{\varepsilon}$ is additively decomposed in elastic strain rate $\dot{\varepsilon}^e$ and visco-plastic strain rate $\dot{\varepsilon}^{vp}$. A hypo-elastic law linearly relating the Cauchy stress rate is expressed as

$$\dot{\sigma} = C : (\dot{\varepsilon} - \dot{\varepsilon}^{vp})$$

(1)

where $C$ is the fourth-order elasticity tensor.

The plastic flow rule defines the direction of the visco-plastic strain rate as a function of the stress tensor components. The equivalent stress $\sigma_{eq}$ employed here is von Mises for simplicity

$$\sigma_{eq} = \sqrt{\frac{3}{2}(\sigma' - X) : (\sigma' - X)}$$

(2)

where $\sigma'$ denotes the deviatoric part of the Cauchy stress tensor and $X$ is a second-order tensor which describes kinematic hardening. The plastic flow rule can be written as

$$\dot{\varepsilon}^{vp} = \dot{\varepsilon}_{eq} V, V = \frac{\partial \sigma_{eq}}{\partial \sigma} = \frac{3}{2} \frac{(\sigma' - X)}{\sigma_{eq}}$$

(3)

where $\dot{\varepsilon}_{eq}$ is the equivalent visco-plastic strain rate which is defined by the following relationship

$$\dot{\varepsilon}_{eq}^{vp} = \dot{\varepsilon}^{'} sinh\left(\frac{\sigma^*}{k^*}\right)$$

(4)

where $k^*$ and $\dot{\varepsilon}^*$ are material parameters. The scalar “overstress” $\sigma^*$ denotes the increase of stress intensity due to visco-plastic strain rate so that

$$\sigma_{eq} - \sigma_0 - R - \sigma^* \leq 0$$

(5)

In this equation, $\sigma_0$ is a material parameter designating the initial yield stress, and $R$ is the isotropic hardening of the material.
The classical framework of combined isotropic-kinematic hardening is used and its internal variables are a scalar \( R \), describing the isotropic hardening, and a second-order variable \( X \), describing the kinematic hardening

\[
X = C_x (X_{sat} N - X) \varepsilon_{eq}^{vp}
\]  
(6)

where \( C_x \) and \( X_{sat} \) are material parameters and \( N \) is the unit tensor parallel to the visco-plastic flow direction.

The isotropic hardening evolution gives the variation of the yield surface size \( Y \) by

\[
Y = Y_0 + R
\]  
(7)

where \( Y_0 \) is the initial yield stress. In the particular case of pure isotropic hardening, the variable \( Y \) describes the evolution of the tensile stress \( \sigma_T \)

\[
\sigma_T = Y
\]  
(8)

where \( \sigma_T \) is the tensile stress component in uniaxial tensile condition, seen in Figure 2a.

\[ \text{Figure 2. Uniaxial tensile stress components (a) Only isotropic hardening (b) Combined isotropic with kinematic hardening (c) Combined hardening with larger amount of kinematic hardening.} \]

However, when kinematic hardening is taken into account, the formal equivalence in equation (8) is no longer valid. In order to avoid this undesired interference, instead of using \( Y \) or \( R \), explicitly model \( \sigma_T \) is applied. Here, for example, Swift’s power law was used:

\[
\dot{\sigma}_T = H_{\sigma_T} \cdot \dot{\varepsilon}_{eq}^{vp}, \quad H_{\sigma_T} = n \cdot K \left( \frac{\sigma_T}{\sigma_0} \right)^{\frac{n-1}{n}}, \quad \text{with } \sigma_T(t = 0) = Y_0 = K \varepsilon_0^n
\]  
(9)

where \( K \), \( n \), \( \varepsilon_0 \) are material parameters. The size \( Y \) of the yield surface is determined by

\[
Y = \sigma_T - X
\]  
(10)

where \( X \) is a scalar variable corresponding to the tensile component of \( X \) under monotonic tensile loading:

\[
H_X = C_x \left( \frac{3}{2} \cdot X_{sat} - X \right)
\]  
(11)

Figure 2b and Figure 2c illustrate this case. The rate form of equation (10) writes:
3. Numerical model and simulation results

According to the experimental study in [2], abrupt strain path change simulations were performed by a specifically developed program. Among possible two-step loading processes, the first step of loading considered in the simulations was a biaxial loading prescribed by \( \dot{\varepsilon}_{11} = \dot{\varepsilon}_{22} > 0 \). When the nominal strains reach the values \( \varepsilon_{11} = \varepsilon_{22} = 0.01 \), the applied total strain rate is abruptly converted to \( \dot{\varepsilon}_{22} = -\dot{\varepsilon}_{11} \) with \( \dot{\varepsilon}_{11} > 0 \).

**Table 1.** Material parameters applied in kinematic hardening investigated simulations

| \( E \) (GPa) | \( K \) (MPa) | \( \varepsilon_0 \) | \( n \) | \( \dot{\varepsilon}^* \) | \( k^* \) |
|---|---|---|---|---|---|
| 70 | 600 | 0.02 | 0.4 | 0.1 | 20 |

In Table 1, the corresponding material parameters used in these investigated simulations is listed. For kinematic hardening parameters \( C_x \) and \( X_{sat} \), they are selected by choosing the minimum value of size of yield surface \( Y \) at different accumulated equivalent plastic strain levels. Finally, 16 simulation groups of \( C_x \) and \( X_{sat} \) parameters shown in Table 2 were simulated and analysed.

**Table 2.** Kinematic hardening parameters \((C_x, X_{sat})\) values at minimum value of size of yield surface \( Y_{min} \) with different accumulated equivalent plastic strain \( \varepsilon_{min} \) levels

| \( \varepsilon_{min} \) (MPa) | 1 | 5 | 15 | 50 |
|---|---|---|---|---|
| 0.005 | (850,112.5) | (850,109) | (850,101) | (850,72) |
| 0.01 | (300,126) | (300,122) | (300,113.5) | (250,85) |
| 0.02 | (130,145) | (130,141) | (130,132.5) | (105,107) |
| 0.03 | (80,161.5) | (75,160) | (75,151.5) | (65,124) |

The stress path for abrupt strain path change following equibiaxial pre-strain loading and the normality situation are shown in Figures 3-6 for different values of the accumulated equivalent plastic and size of the yield surface. It appears that kinematic hardening has an influence on the apparent non-normality of plastic strain-rate direction with respect to the stress path. It is also obvious that kinematic hardening plays a significant role in the shape of the subsequent yield surface and the loss of normality. In Figures 3a-6a, the second strain path is accompanied by an increase of the plastic strain. In order to reach \( \varphi = 45^\circ \) (\( \sigma_{22} = 0 \)), this amount of strain varied between 0.002 and 0.005, dependably on the kinematic hardening parameters. This is consistent with the experiment values in reference [5], which were considered negligible. However, for the combination of kinematic hardening is significant, this small strain induces very large stress variations, are illustrated by Figures 3-6.

4. Conclusion

An elasto-visco-plastic model including kinematic hardening combined modified isotropic hardening was used to simulate a typical abrupt strain-path change as described in [4, 5]. The apparent loss of normality observed in this configuration was shown to rely on the constitutive response of the material according to classical plasticity modeling. Kinematic hardening parameters \((C_x,X_{sat})\) have a strong influence to contribute to this apparent non-normality. In Figure 7, compared with experimental results
from reference [5], the simulation results using kinematic parameters which are calculated using $\varepsilon_{\text{min}} = 0.005$ for the equivalent plastic strain at which the isotropic hardening contribution has a minimum value are closer to experiments investigation.

\[ \varepsilon_{\text{min}} = 0.005 \]

Figure 3. Yield surface and non-normality evolution predicted by kinematic hardening after abrupt strain-path change at minimum value of size of yield surface $Y_{\text{min}} = 1\text{MPa}$.

Figure 4. Yield surface and non-normality evolution predicted by kinematic hardening after abrupt strain-path change at minimum value of size of yield surface $Y_{\text{min}} = 5\text{MPa}$.
Figure 5. Yield surface and non-normality evolution predicted by kinematic hardening after abrupt strain-path change at minimum value of size of yield surface $Y_{\min} = 15$MPa.

Figure 6. Yield surface and non-normality evolution predicted by kinematic hardening after abrupt strain-path change at minimum value of size of yield surface $Y_{\min} = 50$MPa.
Figure 7. Yield surface and non-normality evolution predicted by kinematic hardening after abrupt strain-path change at $\varepsilon_{\text{min}} = 0.005$, compared with experimental results from reference [5].

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