Spectrum of positive and negative parity pentaquarks, including $SU(3)_F$ breaking.

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Abstract

We present the spectrum of the lightest pentaquark states of both parities and compare it with the present experimental evidence for these states. We have assumed that the main role for their mass splittings is played by the chromo-magnetic interaction. We have also kept into account the $SU(3)_F$ breaking for their contribution and for the spin orbit term. The resulting pattern is in good agreement with experiment.

Keywords: Pentaquarks, Chromo-magnetic interaction

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1 Introduction

Exotic baryonic resonances in $KN$ scattering have been found by phase shift analysis \([1, 2, 3]\). Evidence has also been claimed at CERN SPS for the existence of a narrow $\Xi^−\pi^−$ baryon resonance with mass $1.862 \pm 0.002$ GeV at $4.0\sigma$ \([4]\). This state would be an exotic baryon $\Xi^{−−}$ with isospin $I = 3/2$, hypercharge $Y = −1$ and a quark content $d\bar{s}s\bar{u}$. The original observation of a narrow exotic baryon resonance $\Theta^+$ (with $I = 0$ and $Y = 2$) in two independent experiments \([5]\) was confirmed \([6]\). Such discovery has motivated several attempts to study it as a $uudd\bar{s}$ state \([7, 8, 9]\).

The circumstance that the previous evidence in photoproduction for the $\Theta^+$ \([6]\) has been recently disproved \([10]\) does not seem to be the last word. Recently, new results have become available, partly based on new data, confirming seeing the $\Theta^+$ \([11]\); the results of a new run, which should increase the statistics by 10, are expected at LEPS.

The $N\pi\frac{1}{2}^+, I = 1/2, Y = 1$ resonance with mass 1358 MeV discovered at BES \([12]\) in the decay $J/\psi \to p\bar{p}\pi^0$ and the $P_{11}(1860)$ and $P_{13}(1900)$ states found in the photoproduction on proton of $K\Lambda$ and $K\Sigma$ \([13]\) are natural candidates for a pentaquark interpretation.

Exotic baryons, consisting of $4q$ and a $\bar{q}$ have been studied \([14]\), at the times of bubble chambers, the best device to detect these particles.

In this paper, we evaluate their spectrum with the assumption that the mass splittings between the different states are due to the effect of the chromo-magnetic interaction; we also include the effect of $SU(3)_F$ flavour symmetry breaking. Such simple model has been proved remarkably successfully at describing the spectrum of the standard baryons \([15]\), which transform as the 56 representation of flavour-spin $SU(6)_{FS}$ \([16]\). The same approach has already been exploited to evaluate the spectrum of the positive and negative parity $Y = 2$ baryons \([17]\) and the spectrum of the scalar mesons \([18]\).

Here, we extend the analysis to the pentaquarks with one or more strange constituents, that is to $Y < 2$. As in \([17]\), we consider $4q$ in $S$ and $P$ wave, which give rise, together with the $\bar{q}$ in $S$-wave with respect to them, to negative and positive parity states, in the last case with an extension of the proposal of Jaffe and Wilczek in \([7]\). We shall consider states exclusively of the type $(4q)\bar{q}$. We shall call $p$ this state and $t$ the corresponding $(4q)$ subsystem.
2 The chromo-magnetic interaction

The hyperfine interaction arising from one gluon exchange between constituents leads to a simple Hamiltonian involving the colour and spin degrees of freedom:

\[
H_{CM} = \sum_i m_i - \frac{1}{4} \sum_{i<j} \frac{K_{ij}}{m_i m_j} \sum_{a=1}^{8} \sum_{k=1}^{3} (\lambda_a \otimes \sigma_k)^{(i)} (\lambda_a \otimes \sigma_k)^{(j)}
\]

where the index \(i\) \((j)\) refers to the \(i\)th \((j)\)th quark, \(\lambda_a\) are the 8 Gell-Mann matrices, \(\sigma_k\) the Pauli matrices, \(m_i\) the mass of the \(i\)th \((anti)\)quark and \(K_{ij}\) appropriate coupling constants; the sum above depends on the spatial relative configuration of quarks \(i\) and \(j\), since one has to include only pairs which effectively interact with each other via the short range QCD interaction. It’s natural to define the operators \(O_{CM}^{(ij)}\) for the 2-body chromo-magnetic operators by

\[
O_{CM}^{(ij)} = \frac{1}{4} \sum_{a=1}^{8} \sum_{k=1}^{3} (\lambda_a \otimes \sigma_k)^{(i)} (\lambda_a \otimes \sigma_k)^{(j)}.
\]

Quarks belong to the fundamental representation of \(SU(6)_{CS} \times SU(6)_{CS}\) and transform as \((3C, 2S)\) with respect to \(SU(3)_C \times SU(2)_S\), they are represented by a wave function \(\psi_{\alpha p}\), while the antiquark \(\chi^{\alpha p}\) trasmforms in the conjugate representation \((\bar{3}C, 2S)\), with \(\alpha\) the color index \((\alpha = 1, 2, 3)\) and \(p\) the spin index \((p = 1, 2)\). Since the Hamiltonian involves only two body forces, we have just to consider the action of the chromo-magnetic operator on a specific quark pair \(\psi_{\beta q} \chi^{\nu l}\) or a quark-antiquark pair \(\psi_{\beta q} \chi^{\mu m}\) that is displayed below:

\[
\begin{align*}
\left(O_{CM}^{(12)} \psi^{(1)}_1 \psi^{(2)}_2\right)_{\alpha p, \mu m} &= \psi'^{(1)}_{\alpha p} \psi'^{(2)}_{\mu m} \\
&= \frac{1}{4} \sum_{\beta, \nu=1,2,3} \sum_{q,l=1,2} \sum_{a=1,\ldots,8} \sum_{k=1,2,3} (\lambda_a)^{\beta} (\sigma_k)^{q}_{\nu} \left((\lambda_a)^{\nu} (\sigma_k)^{l}_{\mu}\right) \psi'^{(1)}_{\beta q} \psi_{\nu l}
\end{align*}
\]

\[
\begin{align*}
\left(O_{CM}^{(15)} \psi^{(1)}_1 \chi^{(5)}_{\nu l}\right)_{\alpha p} &= \psi'^{(1)}_{\alpha p} \chi'^{\nu l}_{\nu l} \\
&= \frac{1}{4} \sum_{\beta, \mu=1,2,3} \sum_{q,m=1,2} \sum_{a=1,\ldots,8} (\lambda_a)^{\beta} (\sigma_k)^{q}_{\mu} \left((\lambda_a)^{\nu} (\sigma_k)^{l}_{\mu}\right) \psi'^{(1)}_{\beta q} \\
&= \frac{1}{6} \psi'^{(1)}_{\alpha p} \chi'^{\nu l}_{\nu l} - \frac{1}{3} \delta^{\nu}_{\rho \rho} \sum_{n=1}^{2} \psi'^{(1)}_{\alpha n} \chi'^{\nu n}_{\nu n} - \frac{1}{2} \delta^{\nu}_{\rho \rho} \sum_{n=1}^{3} \psi'^{(1)}_{\rho n} \chi'^{\nu l}_{\nu l} + \delta^{\nu}_{\rho \rho} \sum_{n=1}^{3} \psi'^{(1)}_{\rho n} \chi'^{\nu n}_{\nu n}.
\end{align*}
\]
The action of $H_{CM}$ on the pentaquark states $|\Phi_A\rangle$ (a complete set of states for assigned flavour and spin of the pentaquark) is readily obtained as follows. Since the $|\Phi_A\rangle$’s can be written as $|\Phi_A\rangle = \Theta^{(12)}_{\alpha p, \mu m} \psi^{(1)}_{\alpha p} \psi^{(2)}_{\mu m}$ where $\Theta^{(12)}_{\alpha p, \mu m}$ is trilinear in $\psi^{(3)}$, $\psi^{(4)}$ and the antiquark $\chi$ define

$$O^{(12)}_{CM} |\Phi_A\rangle = |\Phi_A\rangle' = \Theta^{(12)}_{\alpha p, \mu m} \psi'^{(1)}_{\alpha p} \psi'^{(2)}_{\mu m}$$

replacing $\psi'^{(1)}_{\alpha p}$ and $\psi'^{(2)}_{\mu m}$ according to Eq.2. We get the new states $|\Phi_A\rangle'$ as

$$|\Phi_A\rangle' = \sum C^B_A |\Phi_B\rangle$$

where $C^B_A = \langle \Phi_B | \Phi_A' \rangle = \langle B | O^{(12)} | A \rangle$ are the matrix elements of the operator $O^{(12)}$ between pentaquark states. The same reasoning applies to the operator $O^{(i5)}$ for $q\bar{q}$ interaction.

In the flavour symmetry limit (i.e. $m_i = m$), the hyperfine interaction reduces to a term proportional to

$$\sum_{i<j} \sum_{a=1}^{8} \sum_{k=1}^{3} (\lambda_a \otimes \sigma_k)^{(i)} (\lambda_a \otimes \sigma_k)^{(j)}$$

which can be expressed in terms of the Casimir operators of $SU(6)_{CS}$, $SU(3)_{F}$ and $SU(2)_{S}$ [14] denoted in the following by $C_6$, $C_3$ and $C_2$, respectively.

### 2.1 Pauli Principle and Flavour Content of Pentaquarks

The Pauli principle imposes the complete antisymmetry of the wave function of the quarks in the tetraquark. On the other hand, the requirement that the pentaquark is a colour singlet enforces the tetraquark wave function to transform as a $3^F_C$. The only representations occurring in the direct product of four $6_{CS}$’s, which contain a $3^F_C$, are the $210_{CS}$, $105'_{CS}$, $105_{CS}$ and $\bar{105}_{CS}$ of $SU(6)_{CS}$. For all these representations there is a $3^F_C$ with $S = 1$ while a $3^F_C$ with $S = 0$ is present in $210_{CS}$ and $105'$, which contains also a $3^F_C$ with $S = 2$.

For a symmetric spatial wave function, as for the tetraquark in S-wave, the corresponding flavour wave functions must transform congruently in order to fulfill the Pauli principle and one gets straightforwardly the correspondence between the colour-spin and $SU(3)_{F}$ flavour contents

$$210_{CS} \leftrightarrow 3^F_C \quad 105_{CS} \leftrightarrow \bar{6}^F_C \quad 105'_{CS} \leftrightarrow 15^F_C \quad \bar{105}_{CS} \leftrightarrow 15'_{F}$$

In the case the tetraquark subsystem be in P wave, we need to take into account, besides the flavour, also the spatial degrees of freedom. In that case we get the following correspondances [17]:

$$210^{(1)}_{CS} \leftrightarrow \bar{6}^F_C \quad 105^{(1)}_{CS} \leftrightarrow 3^F_C \quad 210^{(2)}_{CS} + 105'^{(1)}_{CS} \leftrightarrow 15^F_C + 3^F_C$$

$$105'^{(2)}_{CS} \leftrightarrow 15'_{F} + \bar{6}^F_C \quad 105^{(2)}_{CS} + \bar{105}_{CS} \leftrightarrow 15^F_C$$
With the exception of the $\overline{15}_{CS}$, all the representations appear twice, since there are two inequivalent ways of obtaining them, namely:

\[
\begin{align*}
21_{CS} \otimes 21_{CS} & = 126_{CS} + 210^{(1)}_{CS} + 105^{(1)}_{CS} \quad (12) \\
21_{CS} \otimes 15_{CS} & = 210^{(2)}_{CS} + 105^{(1)}_{CS} \quad (13) \\
15_{CS} \otimes 15_{CS} & = 105^{(2)}_{CS} + 105^{(2)}_{CS} + \overline{15}_{CS} \quad (14)
\end{align*}
\]

### 2.2 Negative parity pentaquarks

By composing the S-wave $t$'s with the $\bar{q}$, one gets the following $1_C$ flavour spin multiplets:

\[
\begin{align*}
8_F + 1_F, S & = 1/2 + 1/2 + 3/2 \quad (15) \\
\overline{10}_F + 8_F, S & = 1/2 + 3/2 \quad (16) \\
27_F + 10_F, +8_F, S & = 1/2 + 1/2 + 3/2 + 3/2 + 5/2 \quad (17) \\
35_F + 10_F, S & = 1/2 + 3/2 \quad (18)
\end{align*}
\]

Let us construct explicitly the pentaquark states relevant for the calculation of the spectrum. Since we have at disposal only 3 flavours, at least 2 quarks must have identical flavour, say $uu$. The more general state would then correspond to the case the remaining pair differ in flavour from each other and from $u$, so we can denote it by $ds$. The $uu$ pair must be symmetric, so it is a $6_F$ to be combined with a $ds$ pair that can be a $6_F$ (symmetric under the exchange $d \leftrightarrow s$) or a $3_F$ (antisymmetric under the exchange $d \leftrightarrow s$). As $6_F \otimes 6_F = \overline{6}_F + 15'_F + 15_{SF}$ and $6_F \otimes 3_F = 3_F + 15_{AF}$, we call the $15_{SF}$ the representation appearing in the state $uu(ds)_S$ and $15_{AF}$ that appearing in $uu(ds)_A$. The other representations $3_F, \overline{6}_F, 15'_F$ appear unambiguously.

So the states in this case can be classified according to the spin and flavour of the tetraquark. The more transparent way of getting the classification of the states is based on a well known argument concerning the tranformation properties of the tetraquark wave function under the group $SU(6)_FS \otimes SU(3)_C$. Since the only occurring $3_C$ state is a $210_{FS}$, whose decomposition under $SU(3)_F \otimes SU(2)_S$ is given by:

\[
(3_F, 1_S) + (3_F, 3_S) + (\overline{6}_F, 3_S) + (15_F, 1_S) + (15_F, 3_S) + (15_F, 5_S) + (15'_F, 3_S),
\]

we readily get the 17 states below:\footnote{When the two quarks in the second pair are equal, the $\bar{3}$ and the $15_{AF}$ are absent. In the case one of the}
As a matter of fact, in order to operate with the chromo-magnetic Hamiltonian Eq.’s. (3,4) we need the explicit expression of the wave functions, which for the sake of completeness are given in Appendix A. Let’s write explicitly the expression of \( m^{(S)} \) for our conventional state \( uuds \overline{q} \):

\[
m^{(S)} = 2m_u + m_d + m_s + m_{\overline{q}} + K^S S
\]

where \( S \) is a 17 \( \times \) 17 matrix, which splits into 8 \( \times \) 8, 7 \( \times \) 7 and 2 \( \times \) 2 matrices corresponding to spin 1/2, 3/2 and 5/2, respectively. The matrix elements of \( S \) between the states in Appendix A may be computed through the use of Eq.’s. (3,4) and their values are reported in Appendix B. By an appropriate choice of \( \overline{q} \) and also with suitable change of the set \( uuds \), one can apply Eq.(19) to any negative parity pentaquark. For instance, the \( I_3 = 1/2, \ Y = -3, \ J = 1/2 \) and 3/2 states have the quark content \( ssss \) and \( \overline{q} \).

### 2.3 The Flavour Symmetry Limit

As mentioned before, in the case we can disregard the breaking of \( SU(3)_F \), the hyperfine interaction Eq.(8) can be expressed in terms of a purely grupal expression involving the quadratic Casimir operators.

A weaker, and more useful, limit is when all the quarks have the same constituent mass (we assume isospin invariance \( m_u = m_d \)), while the antiquark may be a light or strange one, corresponding to the \( Y = +2 \) baryons (and some cases with \( Y \leq 1 \)). In that limit the mass of a negative parity pentaquark state is

\[
m^{(S)} = 4m_q + m_{\overline{q}} + \frac{K^S}{m_q m_{\overline{q}}} \left[ C_6(p) - C_6(t) - \frac{1}{3}C_2(p) + \frac{1}{3}C_2(t) - \frac{4}{3}\right] \\
- \frac{K^S}{m_q^2} \left[ C_6(t) - \frac{1}{3}C_2(t) - \frac{26}{3}\right]
\]

(20)

two quarks, in the second pair, is equal to the ones in the first pair, no 3 and \( \overline{6}_F \) occur. If all quarks are the same, then only the \( 15'_F \) is present.

\(^2\) Our normalization for the Casimir operators, at difference with [14], is the one, which takes the value \( n \) for the adjoint representation of \( SU(n) \).
$K^S$ being the chromo-magnetic coupling constant for $qq$ and $q\bar{q}$ (all in $S$-wave), $m_q$ the common quark mass and $m_{q\bar{q}}$ the antiquark mass. The above expression in (20) shows that the lightest states have large $SU(6)_{CS}$ Casimir for the $4q$ and as small as possible for the pentaquark.

Hypercharge $Y = +2$ baryon resonances (notice that in this case $q = \bar{s}$), called $Z^*$, have been revealed in $KN$ interactions. The $Z^*$ resonances $D_{03}$ and $D_{15}$ (the two lower indexes stand for the isospin and twice the spin, respectively) have negative parity and have possibly been revealed within mass ranges $m_{D_{03}} = 1788 - 1865$ and $m_{D_{15}} = 2074 - 2160$.

From the spin content of the $3C$ tetraquarks given before and from the tensor products:

$$105_{CS} \otimes \bar{6}_{CS} = 560_{CS} \oplus 70_{CS} \quad (21)$$
$$105'_{CS} \otimes \bar{6}_{CS} = 540_{CS} \oplus 70_{CS} \oplus 20_{CS} \quad (22)$$

the spin $S = 5/2$ and the isospin $I = 1$ of the $D_{15}$ state imply that the pentaquark is in the $540_{CS}$ with the respective tetraquark being in the $105'_{CS}$ of $SU(6)_{CS}$. By inserting in Eq. (20) the Casimir values: $C_2(t) = C_2(5) = 6$, $C_6(t) = C_6(105) = 26/3$, $C_6(p) = C_6(540) = 49/4$ and $C_2(p) = C_2(6) = 35/4$, the chromomagnetic contribution to the $D_{15}$ mass turns out to be

$$K^S \left( \frac{4}{3m_q m_s} + \frac{2}{m_q^2} \right).$$

Similar reasonings hold for $D_{03}$, the pentaquark state being in the $560_{CS}$ and the tetraquark in the $105_{CS}$. The Casimir values involved in the calculation are $C_2(t) = 2$, $C_6(t) = C_6(105) = 32/3$, $C_6(p) = C_6(560) = 57/4$ and $C_2(p) = C_2(4) = 15/4$. The observed mass difference

$$m_{D_{15}} - m_{D_{03}} = -\frac{1}{3} \frac{K^S}{m_q m_s} + \frac{10}{3} \frac{K^S}{m_q^2} \quad (23)$$

implies a larger value for the mass of the $D_{15}$ in agreement with experiment.

### 3 “Open door” channels for pentaquarks

It has been observed for the first time by Jaffe [19] that some $qq\bar{q}\bar{q}$ mesons may decay into two ordinary mesons (PP, PV, VV) by simple separation of the constituents: he called these channels “open door”.

Many years later a group theoretical criterium has been found [20] to give a necessary condition for a PP and PV channels to be ”open door”, according to $SU(6)_{CS}$ symmetry. Since a pseudoscalar and vector meson transform as the singlet $1_{CS}$ or the adjoint $35_{CS}$ representation

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3By P we mean a pseudoscalar meson, by V a vector meson.
of $SU(6)_{CS}$, respectively, only states, which transform as $1_{CS}$ (or $35_{CS}$) of $SU(6)_{CS}$ may have "open door" amplitudes into PP (or VP) final states.

The contributions of the chromomagnetic interaction in the flavour symmetry limit are proportional to a combination of quadratic Casimir operators \[14\] and depend mostly on the $SU(6)_{CS}$ Casimir operator. Therefore the eigenstates of the mass spectrum belong to almost irreducible representations of $SU(6)_{CS}$. This property is weakly affected by the breaking of $SU(3)_F$. In particular, the lighter tetraquark meson scalar (or axial) states, which transform approximately as a singlet (or $35_{CS}$), have large "open door" amplitudes into PP (or VP) channels \[18\].

These considerations can be extended to pentaquarks, as a consequence of the $SU(6)_{CS}$ transformation properties of the baryon $1/2^+$ octet and of the $3/2^+$ decuplet, respectively in the $70_{CS}$ and the $20_{CS}$ representations. Since the pseudoscalars are colour-spin singlets only pentaquarks with the same $SU(6)_{CS}$ transformation properties have "open door" amplitudes into a channel consisting of one of these baryons and a pseudoscalar meson \[20\]. This selection rule often coincide with the one proposed in \[21\] in analogy with the $SU(6)_{FS}$ selection rule found in \[8\], but is more restrictive.

As seen in Eq.(9), for the negative parity pentaquarks there is a relation among $SU(3)_F$ and $SU(6)_{CS}$ transformation properties of the $4q$. This relation implies, according to Eq.(20), larger masses for higher dimension $SU(3)_F$ representations, since they correspond to smaller $SU(6)_{CS}$ representations (more precisely with smaller quadratic Casimir) for the $4q$, as a consequence of the sign of the chromo-magnetic contribution proportional to $K^S$. This implies that the lightest $J = S = 1/2$ or $3/2$ states will be those transforming as the $70_{CS}, J = S = 1/2$ or the $20_{CS}, J = S = 3/2$ representations. Therefore there is a correlation between smaller mass and large couplings to the final channels consisting of a baryon of the 56 of $SU(6)_{FS}$ and a pseudoscalar meson. For these negative parity pentaquarks we expect the "open door" channels above threshold to be difficult to detect for their broad width, as the long controversy about the $f_0$ has shown.

Instead we expect the more likely detectable positive parity pentaquarks to be those with large couplings to the final states. In conclusion we expect P and D-wave resonances to have been already found.

As long as for the positive parity pentaquarks with the $\bar{q}$ in P-wave with respect to the $4q$, there are is no "open door" channel, since the $\bar{q}$ has no quark in S-wave to build a meson \[14\]. So we expect these states to be difficult to detect and for this reason we will not discuss them here.
3.1 Positive parity pentaquarks

Let us consider the pentaquark with positive parity with $t$ in $P$-wave and $\bar{q}$ in $S$-wave with respect to $t$.

In this case, the mass of the $Y = 2$ pentaquark state, that we indicate with $m_P$, can be calculated considering the system as composed of a pair of diquarks with total orbital momentum $L = 1$ and the $\bar{s}$ \cite{7}, whose chromo-magnetic interaction (with flavour independent coupling constant $\overline{K}^{(P)}$) with the quarks can be expressed in terms of Casimirs \cite{17}. The spin orbit term, proportional to $\sum_{i=1}^{4} 1/m_i \vec{L}_i \cdot \vec{S}_i$, in the limit of equal masses depends only on the spin of the tetraquark $\vec{S}_t$ and on the colour of the two diquarks. Besides the nude masses and kinetic energy ($E_{kin}$) contributions, we must add the mass defects for the diquark clusters $\Delta m_{qq}^{(12)}$ and $\Delta m_{qq}^{(34)}$ \cite{17}, so obtaining:

$$m^{(P)} = 4m_q + m_s + \Delta m_{qq}^{(12)} + \Delta m_{qq}^{(34)} - \frac{a}{4} \lambda_{b}^{(12)} \lambda_{b}^{(34)} \vec{L}_i \cdot \vec{S}_t + E_{kin}$$

$$+ \frac{\overline{K}^{(P)}}{m_q m_s} \left[ C_6(p) - C_6(t) - \frac{1}{3} C_2(p) + \frac{1}{3} C_2(t) - \frac{4}{3} \right]$$

where the upper indices (12) and (34) refer to the two diquarks. The mass defects $\Delta m_{qq}^{(12)}$ and $\Delta m_{qq}^{(34)}$ can be equally calculated in terms of Casimirs according to the relation below:

$$\frac{1}{4} \sum_{b=1}^{8} \sum_{k=1}^{3} (\lambda_b \otimes \sigma_k)^{(1)} (\lambda_b \otimes \sigma_k)^{(2)} \Rightarrow \left( C_6(q_1 q_2) - \frac{1}{2} C_3(q_1 q_2) - \frac{1}{3} C_2(q_1 q_2) - 4 \right).$$

This contribution depends on the colour and spin of the pair of quarks $q_1 q_2$ and it is reported in Table 1. It is assumed that the chromo-magnetic interaction concerns the quarks in $S$-wave

| $SU(3)_C \times SU(2)_S$ | $\frac{2 \Delta m_{qq}}{C_{qq}}$ |
|-------------------------|-----------------|
| $\overline{3}, 1$       | $-2$            |
| $(6, 3)$                | $-\frac{1}{3}$ |
| $\overline{3}, 3$       | $\frac{2}{3}$  |
| $(6, 1)$                | $+1$            |

Table 1: Chromomagnetic splittings for 2q states

in the same pair \cite{7} and the $\bar{q}$ with both pairs. The interaction among components not in $S$-wave is neglected; this is why the interaction among the two diquark pairs does not contribute
to Eq.(24). We shall take for the qq interaction in the pair the same coupling as in S-wave, i.e. $K^S$ and at difference from [17], but as in [21], for the quark antiquark coupling half of that of S-wave. Indeed the factor 1/2 is a consequence of the total antisymmetrization of the tetraquark wave function, which implies that the $\bar{q}$ has probability 1/2 to be in S-wave with either pair:

$$\bar{K}^P = \frac{1}{2} K^S$$

(26)

In our treatment we shall consider the tetraquark state as two diquark clusters, namely of quarks $(q_1q_2)$ and $(q_3q_4)$ of masses $m_{12}$ and $m_{34}$ respectively, orbiting about each other with $L = 1$, and interacting chromo-magnetically only with the antiquark, denoted with index 5. The spin-orbit term arises, as in electrodynamics, from the interaction of the quarks with the coloured current. It is proportional to the giro-chromomagnetic factor of the quarks in P wave as well to the product of their colour matrices: more precisely, if the representation $3_C$ of the 4q state is originated by the $\bar{3}_C \otimes \bar{3}_C$, or the $6_C \otimes \bar{3}_C$ representation of the two diquark pairs, the coefficients will be in the ratio 2 : 5. Since the colour and spatial degrees of freedom are independent, the interaction should be typically proportional to $\vec{L} \cdot S^{(\pm)}_i$, being $\vec{S}^{(\pm)}_i$ combinations of quarks spins, $\vec{S}^{(\pm)}_i = \vec{S}_i \pm \vec{S}_j$. We include also the short range chromo-magnetic interaction between quarks in the same cluster and neglect the mass defects in the kinetic energy and the spin orbit Hamiltonian as a higher order effect. So $m^{(P)} = H_0 + H_{CM} + H_{SO}$, with

$$H_0 = \sum_{i=1}^{4} m_i + m_5 + \frac{1}{2} \left( \frac{1}{m_{12}} + \frac{1}{m_{34}} \right) \frac{P^2}{2}$$

(27)

$$\approx \sum_{i=1}^{4} m_i + m_5 + \left( \frac{1}{m_1 + m_2} + \frac{1}{m_3 + m_4} \right) \frac{P^2}{2}$$

(28)

$$H_{CM} = K^S P$$

(29)

$$H_{SO} = a_{12} \vec{L} \cdot \vec{S}^{(+)}_{12} + b_{12} \vec{L} \cdot \vec{S}^{(-)}_{12} + a_{34} \vec{L} \cdot \vec{S}^{(+)}_{34} + b_{34} \vec{L} \cdot \vec{S}^{(-)}_{34}$$

(30)

where $P$ is a 30$\times$30 matrix, which splits into 15$\times$15, 12$\times$12 and 3$\times$3 matrices corresponding to spin 1/2, 3/2 and 5/2, respectively. The matrix elements of $P$ between the states in Appendix A may be computed through the use of Eq.'s [31] and their values are reported in Appendix B. As long for the spin orbit interaction, $a_{12}$, $a_{34}$, $b_{12}$, $b_{34}$ are the appropriate kinematic factors

$$a_{ij} = -\frac{a}{4} \lambda_b^{(12)} \lambda_b^{(34)} \frac{m_i^2}{2} \left( \frac{1}{m_1 + m_2} + \frac{1}{m_3 + m_4} \right) \left( \frac{1}{m_i} - \frac{1}{m_j} \right)$$

(31)

$$b_{ij} = -\frac{a}{4} \lambda_b^{(12)} \lambda_b^{(34)} \frac{m_i^2}{2} \left( \frac{1}{m_1 + m_2} + \frac{1}{m_3 + m_4} \right) \left( \frac{1}{m_i} + \frac{1}{m_j} \right)$$

(32)
The total antisymmetry with respect to the exchange of the quarks, which are in the two S-wave pairs, and of the two pairs (which are in P-wave), fixes the \( SU(3)_F \) quantum numbers of the pentaquarks. The \( SU(3)_F \) breaking in the chromomagnetic interaction and in the spin-orbit term implies the mixing between different representations of \( SU(3)_F \).

As it was the case of the negative parity states, the qualitative form of the spectrum are shown in the symmetry limit: the lightest states will be the ones, where both the two diquarks transform as a \( 21_{CS} \) and the pentaquark as the smallest possible representation of \( SU(6)_{CS} \). From Eq.’s(10,21,22,24) and the tensor products:

\[
\begin{align*}
210_{CS} \otimes 6_{CS} &= 1134_{CS} \oplus 70_{CS} \oplus 56_{CS} \quad (33) \\
\overline{15}_{CS} \otimes 6_{CS} &= \overline{70}_{CS} \oplus 20_{CS}. \quad (34)
\end{align*}
\]

we deduce that the lightest \( Y = 2 \) state has \( J^P = 1/2^+ \) and \( I = 0 \) and may be identified with the \( \Theta^+ \). The corresponding state with a light \( \bar{q} \) can be identified with the \( 1/2^+ \) \( Y = 1 \) \( I = 1/2 \) seen by BES \[12\] at 1358 MeV. At higher mass there are three \( J^P = 1/2^+ \) \( Y = 2 \) \( I = 1 \) states, one of which may be identified with the \( P_{11} \) resonance seen in \[1\] at 1720 MeV; the \( P_{13} \) (1780) with the same internal quantum numbers, seen in the same experiment, may be identified with the corresponding \( 3/2^+ \) state. Finally the \( \Xi^{--} \) state seen at CERN \[4\] can be identified with his partner in the \( 27_F \). In the next section we shall fix the parameters to reproduce the values of the masses of the five states just quoted consistently with the ranges found for the \( Y = 2 \) \( D_{03} \) and \( D_{15} \) previously mentioned states. Besides the positive parity states chosen to fix the parameters, we shall plot only the other ”open door” states, which, according to Eq.’s.(10-14,21,22,33,34) will be the flavour \( J \) multiplets with positive parity

\[
\begin{align*}
[10 + 8, \ 8 + 1, \ twice \ (27 + 10 + 8 + 8 + 1), \ 35 + 10 + \bar{10} + 8 \ and \ 27 + 10 + 8, \ 1/2 + 3/2] \\
[27 + 10 + 8 + 8 + 1, \ 35 + 10 + \bar{10} + 8 \ and \ 27 + 10 + 8, \ 1/2 + 3/2 + 5/2]
\end{align*}
\]

with ”open door” decay into a pseudoscalar meson and a baryon of the octet or the decuplet, respectively.

### 4 Comparison with data

To reproduce the masses of the four states \( 1/2^+ \) mentioned at the end of the previous section, of the \( (P_{13}, Y = 2, 1780) \) and of the \( D_{03} \) and \( D_{15} \) resonances, we find the following values for
the parameters:

\[
\frac{K^S}{m_u^2} = 74.5 \text{ MeV} \quad (35)
\]
\[
a = 42 \text{ MeV} \quad (36)
\]
\[
< p^2 > = (276 \text{ MeV}/c^2) \quad (37)
\]
\[
m_u = 346.8 \text{ MeV} \quad (38)
\]
\[
m_s = 480 \text{ MeV} \quad (39)
\]

With these values, we get for the states mentioned at the end of the previous section the masses as a function of \( J^P, Y, I \):

\[
m(1/2^+, 1, 1/2) = 1356 \text{ MeV} \quad (40)
\]
\[
m(1/2^+, 2, 0) = 1545 \text{ MeV} \quad (41)
\]
\[
m(1/2^+, 2, 1) = 1732 \text{ MeV} \quad (42)
\]
\[
m(3/2^+, 2, 1) = 1789 \text{ MeV} \quad (43)
\]
\[
m(1/2^+, -1, 3/2) = 1851 \text{ MeV} \quad (44)
\]
\[
m(3/2^-, 2, 0) = 1858 \text{ MeV} \quad (45)
\]
\[
m(5/2^-, 2, 1) = 2088 \text{ MeV} \quad (46)
\]

With the same values of the parameters one may identify the resonances seen in the photo-production of \( \Sigma K \) and \( \Lambda K \) resonances [13] with hidden strangeness partners of the \((1/2^+, Y = 2, I = 1, 1734)\) and the \((3/2^+, Y = 2, I = 1, 1789)\) with masses 1862 and 1908, respectively. The masses of all the negative parity and of the ”open door” positive parity states corresponding to the parameters just written are reported in Appendix C, where we write a lower index \( s \) for the states with hidden strangeness and the isospin for the values impossible for \( qqq \) states. We put a * for the negative parity multiplets with ”open door” decays and for the positive parity states with ”open door” decays into a \( MB^* \) channel.

Instead of grouping the multiplets according to their \( SU(3)_F \) transformation properties, we group the different \( I, Y \) multiplets in a hybrid way following the same principle used for the vector \((\omega, \phi)\) states and related to the fact that states with or without hidden strangeness, which are components of the same \( SU(3)_F \) multiplet, differ in mass by about 270 Mev: we combine 4\( q \) in \( SU(3)_F \) multiplets either with \( \bar{s} \) or with \( \bar{u} \) and \( \bar{d} \).

For each \( SU(3)_F \) reducible representation we report in Figures (1-8) the spectrum of at least one \( J^P \) multiplet. More precisely:

1) For \( 1/2^+ \): \( 8 + 1, \overline{10} + 8, 27 + 10 + 8 \) and \( 27 + 10 + 8 + 8 + 1 \)
2) For $3/2^+ : 27 + 10 + 8 + 8 + 1$

3) For $3/2^- : 10 + 8$, and $35 + 10$

4) For $5/2^- : 27 + 10 + 8$

There is evidence of two partners for the $Z_1 (1/2^+, 1734)$ (see Fig.4 in Appendix D) and for one partner of the $Z_0 (1/2^+, 1545)$ (see Fig.2) and of the $Z_1 (3/2^+, 1789)$ (see Fig.6). The interpretation of the Roper resonance as a pentaquark was proposed in [7].

As long as for $\Delta K$ states, it is not easy to find them in $KN$ reactions, since they have no common "open door" channel. The best way to find them should be in deep-inelastic reactions on strange partons, where the remaining $\bar{s}$ with the three valence quarks and another light quark may form a $Y = 2$ state.

We conclude that the actual knowledge about the spectrum of the pentaquarks is well consistent with the hypothesis that the chromo-magnetic interaction plays the main role in describing their mass splittings.

5 Conclusion

The experimental situation is up to now controversial, as shown from the disparition from $PDG$ of the $KN$ resonances in [1], [2] and [3], the oscillating evidence for $\Theta^+$ and the $\Xi^{-\bar{\Xi}^+}$ found at CERN.

Indeed a recent report [22] is rather negative on the existence of the $\Theta^+$ and of the $\Xi^{-\bar{\Xi}^+}$, as well as on the $C = -1$ pentaquark claimed in [23]. We show, anyway, that the spectrum of these states can be described in the framework of QCD, as it happened for ordinary hadrons [15].

Also the recent discoveries of the $(1/2^+, Y = 1, I = 1/2)(1356)$ at BES [12] and of the $\Lambda K$ and $\Sigma K$ $P_{11}$ and $P_{13}$ resonances in photoproduction on proton [13] support the existence of pentaquarks with the spectrum well described in a constituent model, where the chromo-magnetic interaction and the spin orbit term, both expected within QCD, play the main role.

There is also an excess of $I = 1/2, Y = 1 \ N\pi$ positive parity states beyond the $56, L = 2$ in the partial wave analysis performed at BES [24] in $J/\psi \rightarrow p\bar{p}\pi^0$, which may interpreted as pentaquarks, in particular the $P_{11}(1710)$ and the $P_{13}(1900)$. There are many states up to now escaped to observation, but the evidence shown here encourages further experimental research, for which this work can be a useful source of suggestions where to look for pentaquarks.
Acknowledgments

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### A States

| State | tetra Flavour & Spin | Wave function |
|-------|----------------------|---------------|
| 1, $S = \frac{1}{2}$ | $3_F S_t = 0$ | $+\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{ij \kappa \ell} \sqrt{82} \left( + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (s) \right. \left. + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (d) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (s) \psi_{\gamma j}^C (u) \psi_{\ell j}^D (d) \right)/96$ |
| 2, $S = \frac{1}{2}$ | $15_A S_t = 0$ | $+\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{ij \kappa \ell} \sqrt{82} \left( + \psi_{\alpha h}^A (s) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (s) \right. \left. - \psi_{\alpha h}^A (d) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d) \right) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (d) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d))/96/\sqrt{3}$ |
| 3, $S = \frac{1}{2}$ | $15_S S_t = 0$ | $+\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{ij \kappa \ell} \sqrt{82} \left( + \psi_{\alpha h}^A (s) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (s) \right. \left. + \psi_{\alpha h}^A (d) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d) \right)/48/\sqrt{3}$ |
| 4, $S = \frac{1}{2}$ | $3_F S_t = 1$ | $+\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{ij \kappa \ell} \sqrt{82} \left( + \psi_{\alpha h}^A (s) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (s) \right. \left. - \psi_{\alpha h}^A (d) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d) \right) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (d) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d))/96/\sqrt{3}$ |
| 5, $S = \frac{1}{2}$ | $15_A S_t = 1$ | $+\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{ij \kappa \ell} \sqrt{82} \left( + \psi_{\alpha h}^A (s) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (s) \right. \left. - \psi_{\alpha h}^A (d) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d) \right) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (d) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d))/96/\sqrt{3}$ |
| 6, $S = \frac{1}{2}$ | $6_F S_t = 1$ | $+\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{ij \kappa \ell} \sqrt{82} \left( + \psi_{\alpha h}^A (s) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (s) \right. \left. - \psi_{\alpha h}^A (d) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d) \right) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (d) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d))/144/\sqrt{3}$ |
| 7, $S = \frac{1}{2}$ | $15_S S_t = 1$ | $+\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{ij \kappa \ell} \sqrt{82} \left( + \psi_{\alpha h}^A (s) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (s) \right. \left. - \psi_{\alpha h}^A (d) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d) \right) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (d) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d))/48/\sqrt{3}$ |
| 8, $S = \frac{1}{2}$ | $15_F S_t = 1$ | $+\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{ij \kappa \ell} \sqrt{82} \left( + \psi_{\alpha h}^A (d) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (u) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (d) \psi_{\ell j}^D (s) \right. \left. - \psi_{\alpha h}^A (d) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d) \right) + \psi_{\alpha h}^A (u) \psi_{\beta h}^B (d) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (u) - \psi_{\alpha h}^A (u) \psi_{\beta h}^B (u) \psi_{\gamma j}^C (s) \psi_{\ell j}^D (d))/576$ |

Table 2: Pentaquark states with $J^P = \frac{1}{2}^-$
| State | \( t F & S \) | Wave function |
|---|---|---|
| 1, \( S = \frac{3}{2} \) | 3F \( S_t = 1 \) | \( +\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \chi^{\alpha \beta \gamma \delta} + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (d) \psi_{\alpha \beta}^{D} (s) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (d) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (u) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \) |
| 2, \( S = \frac{3}{2} \) | 15A \( S_t = 1 \) | \( +\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \chi^{\alpha \beta \gamma \delta} + \psi_{\alpha \beta}^{A} (s) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (d) \psi_{\alpha \beta}^{D} (u) - \psi_{\alpha \beta}^{A} (d) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (s) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (u) - \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (d) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (u) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \) |
| 3, \( S = \frac{3}{2} \) | 15A \( S_t = 2 \) | \( +\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \chi^{\alpha \beta \gamma \delta} + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (d) \psi_{\alpha \beta}^{D} (u) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) - \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) - \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) \) |
| 4, \( S = \frac{3}{2} \) | 6F \( S_t = 1 \) | \( +\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \chi^{\alpha \beta \gamma \delta} + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (d) \psi_{\alpha \beta}^{D} (d) - \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) - \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) \) |
| 5, \( S = \frac{3}{2} \) | 15S \( S_t = 1 \) | \( +\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \chi^{\alpha \beta \gamma \delta} + \psi_{\alpha \beta}^{A} (s) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (d) \psi_{\alpha \beta}^{D} (u) + \psi_{\alpha \beta}^{A} (d) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (d) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (u) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (d) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (u) \) |
| 6, \( S = \frac{3}{2} \) | 15S \( S_t = 2 \) | \( +\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \chi^{\alpha \beta \gamma \delta} + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (d) \psi_{\alpha \beta}^{D} (d) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) + \psi_{\alpha \beta}^{A} (u) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) \) |
| 7, \( S = \frac{3}{2} \) | 15F \( S_t = 1 \) | \( +\epsilon^{ABCD} \epsilon^{\alpha \beta \gamma \delta} \chi^{\alpha \beta \gamma \delta} + \psi_{\alpha \beta}^{A} (d) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (d) \psi_{\alpha \beta}^{D} (u) + \psi_{\alpha \beta}^{A} (d) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) + \psi_{\alpha \beta}^{A} (d) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) + \psi_{\alpha \beta}^{A} (d) \psi_{\alpha \beta}^{B} (u) \psi_{\alpha \beta}^{C} (s) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) \psi_{\alpha \beta}^{D} (d) \) |

Table 3: Pentaquark states with \( J^P = \frac{3}{2}^- \)
| State       | $tF$ & $S$ | Wave function                                                                 |
|-------------|------------|-------------------------------------------------------------------------------|
| $|1, S = \frac{7}{2}\rangle$ | 15$_S$ $S_t = 2$ | $\pm\epsilon^{ABCD} \epsilon^{\alpha\beta\gamma}\chi^{52}$ $
(\pm\psi_{\alpha_1}^A (u) \psi_{\beta_1}^B (u) \psi_{\gamma_1}^C (d) \psi_{\delta_1}^D (s) + \psi_{\alpha_1}^A (u) \psi_{\beta_1}^B (u) \psi_{\gamma_1}^C (s) \psi_{\delta_1}^D (d))/24/\sqrt{2}$ |
| $|2, S = \frac{7}{2}\rangle$ | 15$_A$ $S_t = 2$ | $\pm\epsilon^{ABCD} \epsilon^{\alpha\beta\gamma}\chi^{52}$ $\n(\pm\psi_{\alpha_1}^A (u) \psi_{\beta_1}^B (u) \psi_{\gamma_1}^C (d) \psi_{\delta_1}^D (s) - \psi_{\alpha_1}^A (u) \psi_{\beta_1}^B (u) \psi_{\gamma_1}^C (s) \psi_{\delta_1}^D (d))/48$ |

Table 4: Pentaquark states with $J^P = \frac{5}{2}^-$
Table 5: Pentaquark states. The first two quarks are indicated by \( \psi \), the other two quarks with \( \phi \), the antiquark with \( \chi \); the Greek indexes refer to colour and the other ones to spin. In the first column, the group representations of \( SU(3)_C \) and \( SU(2)_S \), named after their dimension \( d \), are represented, for the first and the second pair of quarks, respectively. The total spin \( S_q \) of the 4 quark state is also indicated. In the second column the pentaquark states are listed. The *kets* \( |i> \) have total spin \( S = S_z = 1/2 \) for \( i \leq 15 \), \( S = S_z = 3/2 \) for \( 16 \leq i \leq 27 \), \( S = S_z = 5/2 \) for \( i \geq 28 \)

| \((d_C,d_S)_{qq}(d_C,d_S)_{qq}\) | States |
|-----------------------------|---------------------|
| \((6,3),(3,1)\) \(|S_q=1\) | \( | \phi_{ai} \psi_{bj} + \psi_{ai} \phi_{bj} + \psi_{ai} \phi_{bj} + \phi_{ai} \psi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,1),(6,3)\) \(|S_q=0\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,1),(3,1)\) \(|S_q=0\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((6,1),(3,1)\) \(|S_q=0\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,3),(6,3)\) \(|S_q=1\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((6,3),(3,3)\) \(|S_q=0\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,1),(3,3)\) \(|S_q=0\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((6,1),(3,3)\) \(|S_q=0\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,3),(3,3)\) \(|S_q=2\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((6,3),(3,3)\) \(|S_q=2\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,3),(3,1)\) \(|S_q=1\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((6,3),(3,1)\) \(|S_q=2\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,3),(3,1)\) \(|S_q=2\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,1),(3,3)\) \(|S_q=2\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((6,3),(3,1)\) \(|S_q=2\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |
| \((3,3),(3,3)\) \(|S_q=2\) | \( | \phi_{ai} \psi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} + \phi_{ai} \phi_{bj} \rangle \phi_{ai} \phi_{bj} \chi_{3}^{\alpha\beta\gamma} \) |


B Matrix elements

The non vanishing matrix elements of the chromo-magnetic interaction for negative parity pentaquarks with $J = 1/2, 3/2$ and $5/2$ where $C_{qi,qj} = \frac{1}{m_{q_i} m_{q_j}}$ and $C_{qi} = \frac{1}{m_{q_i} m_q}$.

4 quarks in S-wave

$J=1/2$

\[ S_{1/2}[1,1] = -\frac{3C_{ds}}{4} - 2C_{ud} - 2C_{us} + \frac{3C_{uu}}{4} \]

\[ S_{1/2}[1,2] = \frac{5C_{ds}}{4\sqrt{3}} - \frac{C_{ud}}{2\sqrt{3}} - \frac{C_{us}}{2\sqrt{3}} - \frac{C_{uu}}{4\sqrt{3}} \]

\[ S_{1/2}[1,3] = \frac{1}{2} \sqrt{\frac{3}{2}} C_{us} - \frac{1}{2} \sqrt{\frac{3}{2}} C_{ud} \]

\[ S_{1/2}[1,4] = -\frac{2C_{d}}{\sqrt{3}} - \frac{2C_{s}}{\sqrt{3}} - \frac{2C_{u}}{\sqrt{3}} \]

\[ S_{1/2}[1,5] = -\frac{C_{d}}{2\sqrt{3}} - \frac{C_{s}}{2\sqrt{3}} + \frac{C_{u}}{\sqrt{3}} \]

\[ S_{1/2}[1,6] = \frac{3C_{d}}{2\sqrt{2}} - \frac{3C_{s}}{2\sqrt{2}} \]

\[ S_{1/2}[1,7] = \frac{C_{d}}{\sqrt{6}} - \frac{C_{s}}{\sqrt{6}} \]

\[ S_{1/2}[2,2] = -\frac{19C_{ds}}{12} + \frac{C_{ud}}{3} + \frac{C_{us}}{3} + \frac{11C_{uu}}{12} \]

\[ S_{1/2}[2,3] = \frac{5C_{us}}{2\sqrt{2}} - \frac{5C_{ud}}{2\sqrt{2}} \]

\[ S_{1/2}[2,4] = -\frac{2C_{d}}{3} - \frac{2C_{s}}{3} + \frac{4C_{u}}{3} \]

\[ S_{1/2}[2,5] = -\frac{C_{d}}{6} - \frac{C_{s}}{6} - \frac{5C_{u}}{4} \]

\[ S_{1/2}[2,6] = \frac{7C_{d}}{6\sqrt{6}} - \frac{7C_{s}}{6\sqrt{6}} \]

\[ S_{1/2}[2,7] = \frac{\sqrt{2}C_{d}}{3} - \frac{\sqrt{2}C_{s}}{3} \]

\[ S_{1/2}[2,8] = \frac{C_{d}}{3\sqrt{3}} - \frac{C_{s}}{3\sqrt{3}} \]

\[ S_{1/2}[3,3] = \frac{2C_{ds}}{3} - \frac{2C_{ud}}{3} - \frac{2C_{us}}{3} + \frac{2C_{uu}}{3} \]

\[ S_{1/2}[3,4] = \frac{2\sqrt{7}C_{d}}{3} - \frac{2\sqrt{7}C_{s}}{3} \]

\[ S_{1/2}[3,5] = \frac{\sqrt{2}C_{d}}{3} - \frac{\sqrt{2}C_{s}}{3} \]

\[ S_{1/2}[3,6] = -\frac{7C_{d}}{6\sqrt{3}} - \frac{7C_{s}}{6\sqrt{3}} + \frac{7C_{u}}{3\sqrt{3}} \]

\[ S_{1/2}[3,7] = -\frac{C_{d}}{2} - \frac{C_{s}}{2} - C_{u} \]
\[ S_{1/2} [3, 8] = \frac{Cd}{3\sqrt{6}} + \frac{Cs}{3\sqrt{6}} - \frac{1}{3} \sqrt{3} \text{Cu} \]
\[ S_{1/2} [4, 4] = -\frac{17Cd}{12} - \frac{7Cds}{6} - \frac{17Cs}{12} - \frac{Cu}{2} - \frac{17Cud}{12} - \frac{17Cus}{12} + \frac{2Cu}{3} \]
\[ S_{1/2} [4, 5] = -\frac{5Cd}{12} + \frac{5Cds}{6} - \frac{5Cs}{12} + \frac{5Cu}{6} - \frac{5Cud}{12} - \frac{5Cus}{12} \]
\[ S_{1/2} [4, 6] = \frac{5Cd}{2\sqrt{6}} - \frac{5Cs}{2\sqrt{6}} - \frac{5Cud}{2\sqrt{6}} + \frac{5Cus}{2\sqrt{6}} \]
\[ S_{1/2} [4, 7] = \frac{5Cd}{6\sqrt{2}} - \frac{5Cs}{6\sqrt{2}} - \frac{5Cud}{6\sqrt{2}} + \frac{5Cus}{6\sqrt{2}} \]
\[ S_{1/2} [5, 5] = \frac{7Cd}{12} - \frac{7Cds}{6} + \frac{7Cs}{12} - \frac{Cu}{2} + \frac{7Cud}{12} + \frac{7Cus}{12} + \frac{2Cu}{3} \]
\[ S_{1/2} [5, 6] = \frac{7Cd}{6\sqrt{6}} - \frac{7Cds}{6\sqrt{6}} + \frac{Cu}{6\sqrt{6}} + \frac{Cus}{6\sqrt{6}} \]
\[ S_{1/2} [5, 7] = \frac{5Cd}{6\sqrt{2}} - \frac{5Cs}{6\sqrt{2}} + \frac{11Cud}{6\sqrt{2}} + \frac{11Cus}{6\sqrt{2}} \]
\[ S_{1/2} [5, 8] = -\frac{2Cd}{3\sqrt{3}} + \frac{2Cs}{3\sqrt{3}} - \frac{Cud}{3\sqrt{3}} + \frac{Cus}{3\sqrt{3}} \]
\[ S_{1/2} [6, 6] = \frac{13Cds}{18} - \frac{5Cs}{6} - \frac{5Cu}{3} - \frac{18}{25Cud} - \frac{25Cus}{18} + \frac{13Cu}{18} \]
\[ S_{1/2} [6, 7] = \frac{7Cd}{6\sqrt{3}} + \frac{Cds}{6\sqrt{3}} - \frac{7Cs}{6\sqrt{3}} + \frac{7Cu}{3\sqrt{3}} - \frac{Cu}{6\sqrt{3}} \]
\[ S_{1/2} [6, 8] = -\frac{Cd}{9\sqrt{2}} + \frac{Cud}{9\sqrt{2}} + \frac{Cus}{9\sqrt{2}} - \frac{Cu}{9\sqrt{2}} \]
\[ S_{1/2} [7, 7] = \frac{Cd}{6} + \frac{5Cds}{6} + \frac{Cs}{6} + \frac{Cu}{3} - \frac{Cud}{2} - \frac{Cus}{2} + \frac{5Cu}{6} \]
\[ S_{1/2} [7, 8] = -\frac{1}{3} \sqrt{3} \frac{Cd}{3\sqrt{6}} - \frac{1}{3} \sqrt{3} \frac{Cds}{3\sqrt{6}} - \frac{1}{3} \sqrt{3} \frac{Cs}{3\sqrt{6}} + \frac{2}{3} \sqrt{3} \frac{Cu}{3\sqrt{6}} + \frac{Cus}{3\sqrt{6}} \]
\[ S_{1/2} [8, 8] = \frac{2Cd}{3} + \frac{7Cds}{9} + \frac{2Cs}{3} + \frac{4Cu}{3} + \frac{14Cud}{9} + \frac{14Cus}{9} + \frac{7Cu}{9} \]

4 quarks in S-wave

\[ J = 3/2 \]

\[ S_{3/2} [1, 1] = \frac{17Cd}{24} - \frac{7Cds}{6} + \frac{17Cs}{24} + \frac{Cu}{4} - \frac{17Cud}{12} - \frac{17Cus}{12} + \frac{2Cu}{3} \]
\[ S_{3/2} [1, 2] = \frac{5Cd}{24} + \frac{5Cds}{6} + \frac{5Cs}{12} - \frac{5Cu}{2} + \frac{5Cud}{12} - \frac{5Cus}{12} \]
\[ S_{3/2} [1, 3] = \frac{1}{12} \sqrt{2} Cd + \frac{1}{12} \sqrt{2} Cds - \frac{1}{6} \sqrt{2} Cs - \frac{1}{6} \sqrt{2} Cu \]
\[ S_{3/2} [1, 4] = -\frac{5Cd}{4\sqrt{6}} - \frac{5Cds}{4\sqrt{6}} - \frac{5Cs}{2\sqrt{6}} + \frac{5Cu}{2\sqrt{6}} \]
\[ S_{3/2} [1, 5] = -\frac{5Cd}{12\sqrt{2}} - \frac{5Cds}{12\sqrt{2}} - \frac{5Cs}{6\sqrt{2}} + \frac{5Cu}{6\sqrt{2}} \]
\[ S_{3/2} [1, 6] = \frac{5Cd}{12} - \frac{5Cs}{12} \]
\[
S_{3/2} [2, 2] = -\frac{7\text{Cd}}{12} - \frac{7\text{Cs}}{6} - \frac{7\text{Cu}}{4} + \frac{7\text{Cus}}{12} + \frac{7\text{Cus}}{12} + \frac{2\text{Cuu}}{3}
\]
\[
S_{3/2} [2, 3] = -\frac{11\sqrt{2} \text{Cd}}{12} - \frac{11\sqrt{2} \text{Cs}}{12} - \frac{5\text{Cs}}{3} + \frac{5\text{Cu}}{2} + \frac{5\text{Cu}}{6} + \frac{5\text{Cu}}{6} + \frac{2\text{Cuu}}{3}
\]
\[
S_{3/2} [2, 4] = -\frac{7\text{Cd}}{12\sqrt{6}} + \frac{7\text{Cs}}{12\sqrt{6}} + \frac{\text{Cud}}{6\sqrt{3}} + \frac{\text{Cus}}{6\sqrt{3}}
\]
\[
S_{3/2} [2, 5] = -\frac{5\text{Cd}}{12\sqrt{2}} + \frac{5\text{Cs}}{12\sqrt{2}} - \frac{11\text{Cud}}{6\sqrt{2}} + \frac{11\text{Cus}}{6\sqrt{2}}
\]
\[
S_{3/2} [2, 6] = \frac{5\sqrt{2} \text{Cd}}{12} - \frac{5\sqrt{2} \text{Cs}}{12}
\]
\[
S_{3/2} [2, 7] = \frac{\text{Cud}}{3\sqrt{3}} - \frac{\text{Cus}}{3\sqrt{3}} + \frac{\text{Cud}}{3\sqrt{3}} + \frac{\text{Cus}}{3\sqrt{3}}
\]
\[
S_{3/2} [3, 3] = -\frac{5\text{Cd}}{4} - \frac{5\text{Cs}}{3} - \frac{5\text{Cu}}{2} + \frac{5\text{Cu}}{6} + \frac{5\text{Cu}}{6} + \frac{2\text{Cuu}}{3}
\]
\[
S_{3/2} [3, 4] = \frac{1}{12} \sqrt{\frac{5}{3} \text{Cd}} - \frac{1}{12} \sqrt{\frac{5}{3} \text{Cs}}
\]
\[
S_{3/2} [3, 5] = \frac{5\sqrt{3} \text{Cs}}{12} - \frac{5\sqrt{3} \text{Cd}}{12}
\]
\[
S_{3/2} [3, 6] = \frac{3\text{Cd}}{2\sqrt{2}} + \frac{3\text{Cs}}{2\sqrt{2}} + \frac{\text{Cud}}{\sqrt{2}} + \frac{\text{Cus}}{\sqrt{2}}
\]
\[
S_{3/2} [3, 7] = \frac{2}{3} \sqrt{\frac{10}{3} \text{Cd}} - \frac{2}{3} \sqrt{\frac{10}{3} \text{Cs}}
\]
\[
S_{3/2} [4, 4] = \frac{5\text{Cd}}{12} + \frac{13\text{Cs}}{18} + \frac{5\text{Cs}}{12} + \frac{5\text{Cu}}{6} - \frac{25\text{Cu}}{18} - \frac{25\text{Cu}}{18} + \frac{13\text{Cuu}}{18}
\]
\[
S_{3/2} [4, 5] = \frac{7\text{Cd}}{12\sqrt{3}} + \frac{7\text{Cs}}{12\sqrt{3}} + \frac{7\text{Cus}}{6\sqrt{3}} - \frac{7\text{Cus}}{6\sqrt{3}} - \frac{7\text{Cu}}{6\sqrt{3}} - \frac{7\text{Cu}}{6\sqrt{3}}
\]
\[
S_{3/2} [4, 6] = \frac{1}{6} \sqrt{\frac{5}{3} \text{Cd}} + \frac{1}{6} \sqrt{\frac{5}{3} \text{Cs}} - \frac{1}{6} \sqrt{\frac{5}{3} \text{Cu}}
\]
\[
S_{3/2} [4, 7] = -\frac{\text{Cud}}{9\sqrt{2}} + \frac{\text{Cud}}{9\sqrt{2}} + \frac{\text{Cus}}{9\sqrt{2}} - \frac{\text{Cuu}}{9\sqrt{2}}
\]
\[
S_{3/2} [5, 5] = -\frac{\text{Cd}}{12} + \frac{5\text{Cd}}{6} - \frac{5\text{Cs}}{12} - \frac{5\text{Cu}}{6} - \frac{5\text{Cu}}{6} - \frac{5\text{Cuu}}{6}
\]
\[
S_{3/2} [5, 6] = \frac{1}{2} \sqrt{\frac{5}{2} \text{Cd}} - \frac{1}{2} \sqrt{\frac{5}{2} \text{Cs}} + \frac{1}{2} \sqrt{\frac{5}{2} \text{Cus}} + \frac{1}{2} \sqrt{\frac{5}{2} \text{Cuu}}
\]
\[
S_{3/2} [5, 7] = \frac{\text{Cd}}{3\sqrt{6}} + \frac{\text{Cd}}{3\sqrt{6}} - \frac{\text{Cus}}{3\sqrt{6}} - \frac{1}{3} \sqrt{\frac{2}{3} \text{Cu}} + \frac{\text{Cuu}}{3\sqrt{6}}
\]
\[
S_{3/2} [6, 6] = -\frac{\text{Cd}}{2} + \frac{2\text{Cs}}{3} - \frac{\text{Cd}}{2} - \text{Cu} + \frac{\text{Cd}}{3} + \frac{\text{Cus}}{3} + \frac{2\text{Cuu}}{3}
\]
\[
S_{3/2} [6, 7] = -\frac{2}{3} \sqrt{\frac{2}{3} \text{Cd}} - \frac{2}{3} \sqrt{\frac{2}{3} \text{Cs}} + \frac{4}{3} \sqrt{\frac{5}{3} \text{Cu}}
\]
\[
S_{3/2} [7, 7] = -\frac{\text{Cd}}{3} + \frac{7\text{Cs}}{9} - \frac{\text{Cus}}{3} - \frac{2\text{Cuu}}{3} + \frac{14\text{Cud}}{9} + \frac{14\text{Cuu}}{9} + \frac{7\text{Cuu}}{9}
\]
4 quarks in S-wave

\( J = \frac{5}{2} \)

\[
S_{5/2} [1,1] = \frac{Cd}{3} + \frac{2Cds}{3} + \frac{Cs}{3} + \frac{2Cu}{3} + \frac{Cud}{3} + \frac{Cus}{3} + \frac{2Cuu}{3}
\]

\[
S_{5/2} [1,2] = -\frac{Cd}{\sqrt{2}} + \frac{Cs}{\sqrt{2}} + \frac{Cud}{\sqrt{2}} - \frac{Cus}{\sqrt{2}}
\]

\[
S_{5/2} [2,2] = \frac{5Cd}{6} - \frac{Cds}{3} + \frac{5Cs}{6} - \frac{Cu}{3} + \frac{5Cud}{6} + \frac{5Cus}{6} + \frac{2Cuu}{3}
\]

In the following we report the non vanishing matrix elements of the chromo-magnetic interaction for positive parity pentaquarks with Spin \( \frac{1}{2}, \frac{3}{2} \) and \( \frac{5}{2} \) where \( c_{12} = \frac{1}{m_1 m_2}, c_{34} = \frac{1}{m_3 m_4} \) and \( c_{i5} = \frac{1}{m_i m_q} \).

4 quarks in P-wave

Block from \([1,1]\) to \([15,15]\) with \( S = \frac{1}{2} \)

\[
P[1,1] = -\frac{c_{12}}{3} - \frac{5c_{15}}{3} - \frac{5c_{25}}{3} - 2c_{34}
\]

\[
P[1,3] = -\sqrt{\frac{3}{2}}c_{15} - \sqrt{\frac{3}{2}}c_{25}
\]

\[
P[1,4] = \frac{5c_{15}}{2\sqrt{3}} - \frac{5c_{25}}{2\sqrt{3}}
\]

\[
P[1,5] = \sqrt{2}c_{15} - \sqrt{2}c_{25}
\]

\[
P[1,8] = \frac{c_{35}}{3\sqrt{2}} - \frac{c_{45}}{3\sqrt{2}}
\]

\[
P[1,9] = \frac{c_{35}}{6} - \frac{c_{45}}{6}
\]

\[
P[2,2] = -2c_{12} - \frac{c_{34}}{3} - \frac{5c_{35}}{3} - \frac{5c_{45}}{3}
\]

\[
P[2,3] = \sqrt{\frac{3}{2}}c_{35} + \sqrt{\frac{3}{2}}c_{45}
\]

\[
P[2,6] = \frac{c_{25}}{3\sqrt{2}} - \frac{c_{15}}{3\sqrt{2}}
\]

\[
P[2,7] = \frac{c_{15}}{6} - \frac{c_{25}}{6}
\]

\[
P[2,10] = \frac{5c_{35}}{2\sqrt{3}} - \frac{5c_{45}}{2\sqrt{3}}
\]

\[
P[2,11] = \sqrt{2}c_{35} - \sqrt{2}c_{45}
\]

\[
P[3,3] = -2c_{12} - 2c_{34}
\]

\[
P[3,5] = \frac{c_{15}}{\sqrt{3}} - \frac{c_{25}}{\sqrt{3}}
\]
\[ P[3, 11] = \frac{c_{45}}{\sqrt{3}} - \frac{c_{35}}{\sqrt{3}} \]
\[ P[4, 4] = c_{12} - 2c_{34} \]
\[ P[4, 5] = -\sqrt{\frac{3}{2}}c_{15} - \sqrt{\frac{3}{2}}c_{25} \]
\[ P[4, 12] = \frac{c_{45}}{2\sqrt{3}} - \frac{c_{35}}{2\sqrt{3}} \]
\[ P[5, 5] = \frac{2c_{12}}{3} - \frac{2c_{15}}{3} - \frac{2c_{25}}{3} - 2c_{34} \]
\[ P[5, 6] = -c_{35} - c_{45} \]
\[ P[5, 7] = -\frac{c_{35}}{\sqrt{2}} - \frac{c_{45}}{\sqrt{2}} \]
\[ P[5, 13] = \frac{c_{45}}{3} - \frac{c_{35}}{\sqrt{2}} \]
\[ P[5, 14] = \sqrt{\frac{3}{2}}c_{45} - \sqrt{\frac{3}{2}}c_{35} \]
\[ P[6, 6] = \frac{2c_{12}}{3} + \frac{c_{15}}{6} + \frac{c_{25}}{6} - \frac{c_{34}}{3} - \frac{5c_{35}}{6} - \frac{5c_{45}}{6} \]
\[ P[6, 7] = -\frac{c_{15}}{3\sqrt{2}} - \frac{c_{25}}{3\sqrt{2}} - \frac{5c_{35}}{3\sqrt{2}} - \frac{5c_{45}}{3\sqrt{2}} \]
\[ P[6, 13] = c_{45} - c_{35} \]
\[ P[6, 14] = \frac{c_{45}}{\sqrt{2}} - \frac{c_{35}}{\sqrt{2}} \]
\[ P[6, 15] = \frac{5c_{45}}{3\sqrt{2}} - \frac{5c_{35}}{3\sqrt{2}} \]
\[ P[7, 7] = \frac{2c_{12}}{3} - \frac{c_{34}}{3} \]
\[ P[7, 14] = c_{45} - c_{35} \]
\[ P[7, 15] = \frac{5c_{45}}{6} - \frac{5c_{35}}{6} \]
\[ P[8, 8] = -\frac{c_{12}}{3} - \frac{5c_{15}}{6} - \frac{5c_{25}}{6} + \frac{2c_{34}}{3} + \frac{c_{35}}{6} + \frac{c_{45}}{6} \]
\[ P[8, 9] = \frac{5c_{15}}{3\sqrt{2}} + \frac{5c_{25}}{3\sqrt{2}} + \frac{c_{35}}{3\sqrt{2}} + \frac{c_{45}}{3\sqrt{2}} \]
\[ P[8, 11] = c_{15} + c_{25} \]
\[ P[8, 12] = \frac{5c_{15}}{3\sqrt{2}} - \frac{5c_{25}}{3\sqrt{2}} \]
\[ P[8, 13] = c_{25} - c_{15} \]
\[ P[8, 14] = \frac{c_{15}}{\sqrt{2}} - \frac{c_{25}}{\sqrt{2}} \]
\[ P[9, 9] = \frac{2c_{34}}{3} - \frac{c_{12}}{3} \]
\[ P[9, 11] = -\frac{c_{15}}{\sqrt{2}} - \frac{c_{25}}{\sqrt{2}} \]
\[ P[9, 12] = \frac{5c_{25}}{6} - \frac{5c_{15}}{6} \]
\[ P[9, 14] = c_{25} - c_{15} \]
\[ P[10, 10] = c_{34} - 2c_{12} \]
\[ P[10, 11] = -\sqrt{\frac{2}{3}} c_{35} - \sqrt{\frac{2}{3}} c_{45} \]
\[ P[10, 15] = \frac{c_{25}}{2\sqrt{3}} - \frac{c_{15}}{2\sqrt{3}} \]
\[ P[11, 11] = -2c_{12} + \frac{2c_{34}}{3} - \frac{2c_{35}}{3} - \frac{2c_{45}}{3} \]
\[ P[11, 13] = \frac{c_{15}}{3} - \frac{c_{25}}{3} \]
\[ P[11, 14] = \frac{\sqrt{2}c_{25}}{3} - \frac{\sqrt{2}c_{15}}{3} \]
\[ P[12, 12] = c_{12} + \frac{2c_{34}}{3} + \frac{c_{35}}{3} + \frac{c_{45}}{3} \]
\[ P[12, 13] = \frac{c_{15}}{\sqrt{2}} + \frac{c_{25}}{\sqrt{2}} \]
\[ P[12, 14] = -c_{15} - c_{25} \]
\[ P[13, 13] = \frac{2c_{12}}{3} + \frac{2c_{34}}{3} \]
\[ P[13, 14] = \frac{\sqrt{2}c_{15}}{3} + \frac{\sqrt{2}c_{25}}{3} - \frac{\sqrt{2}c_{35}}{3} - \frac{\sqrt{2}c_{45}}{3} \]
\[ P[13, 15] = -\frac{c_{35}}{\sqrt{2}} - \frac{c_{45}}{\sqrt{2}} \]
\[ P[14, 14] = \frac{2c_{12}}{3} - \frac{c_{15}}{3} - \frac{c_{25}}{3} + \frac{2c_{34}}{3} - \frac{c_{35}}{3} - \frac{c_{45}}{3} \]
\[ P[14, 15] = -c_{35} - c_{45} \]
\[ P[15, 15] = \frac{2c_{12}}{3} + \frac{c_{15}}{3} + \frac{c_{25}}{3} + c_{34} \]

4 quarks in P-wave
Block from \([16,16]\) to \([27,27]\) with \(S = 3/2\)

\[ P[16, 16] = \frac{2c_{12}}{3} - \frac{c_{15}}{2} - \frac{c_{25}}{2} + \frac{2c_{34}}{3} - \frac{c_{35}}{2} - \frac{c_{45}}{2} \]
\[ P[16, 17] = \frac{\sqrt{6}c_{15}}{6} + \frac{\sqrt{6}c_{25}}{6} - \frac{\sqrt{6}c_{35}}{6} - \frac{\sqrt{6}c_{45}}{6} \]
\[ P[16, 18] = \frac{\sqrt{6}c_{25}}{2} + \frac{\sqrt{6}c_{45}}{2} \]
\[ P[16, 19] = -\frac{\sqrt{6}c_{15}}{2} - \frac{\sqrt{6}c_{25}}{2} \]
\[ P[16, 20] = \frac{1}{3}\sqrt{\frac{3}{2}c_{15}} - \frac{1}{3}\sqrt{\frac{3}{2}c_{25}} \]
$P_{[16, 21]} = \frac{3c_{15}}{2\sqrt{2}} - \frac{3c_{25}}{2\sqrt{2}}$

$P_{[16, 22]} = \frac{1}{2} \sqrt{\frac{5}{2}} c_{25} - \frac{1}{2} \sqrt{\frac{5}{2}} c_{15}$

$P_{[16, 23]} = \frac{1}{3} \sqrt{\frac{5}{2}} c_{45} - \frac{1}{3} \sqrt{\frac{5}{2}} c_{35}$

$P_{[16, 24]} = \frac{3c_{45}}{2\sqrt{2}} - \frac{3c_{35}}{2\sqrt{2}}$

$P_{[16, 25]} = \frac{1}{2} \sqrt{\frac{5}{2}} c_{45} - \frac{1}{2} \sqrt{\frac{5}{2}} c_{35}$

$P_{[17, 17]} = \frac{2c_{12}}{3} + \frac{c_{15}}{6} + \frac{c_{25}}{6} + \frac{2c_{34}}{3} + \frac{c_{35}}{6} + \frac{c_{45}}{6}$

$P_{[17, 18]} = \frac{c_{35}}{2} + \frac{c_{45}}{2}$

$P_{[17, 19]} = \frac{c_{15}}{2} + \frac{c_{25}}{2}$

$P_{[17, 20]} = \frac{c_{35}}{3\sqrt{2}} - \frac{c_{15}}{3\sqrt{2}}$

$P_{[17, 21]} = \frac{1}{2} \sqrt{\frac{5}{2}} c_{25} - \frac{1}{2} \sqrt{\frac{5}{2}} c_{15}$

$P_{[17, 22]} = \frac{c_{25}}{2\sqrt{2}} - \frac{c_{15}}{2\sqrt{2}}$

$P_{[17, 23]} = \frac{c_{45}}{3\sqrt{2}} - \frac{c_{35}}{3\sqrt{2}}$

$P_{[17, 24]} = \frac{1}{2} \sqrt{\frac{5}{2}} c_{45} - \frac{1}{2} \sqrt{\frac{5}{2}} c_{35}$

$P_{[17, 25]} = \frac{c_{35}}{2\sqrt{2}} - \frac{c_{45}}{2\sqrt{2}}$

$P_{[18, 18]} = \frac{2c_{12}}{3} - \frac{c_{15}}{6} - \frac{c_{25}}{6} + c_{34}$

$P_{[18, 24]} = \frac{5}{6} \sqrt{\frac{5}{2}} c_{35} - \frac{5}{6} \sqrt{\frac{5}{2}} c_{45}$

$P_{[18, 25]} = \frac{5c_{35}}{6\sqrt{2}} - \frac{5c_{45}}{6\sqrt{2}}$

$P_{[19, 19]} = c_{12} + \frac{2c_{34}}{3} - \frac{c_{35}}{6} - \frac{c_{45}}{6}$

$P_{[19, 21]} = \frac{5}{6} \sqrt{\frac{5}{2}} c_{15} - \frac{5}{6} \sqrt{\frac{5}{2}} c_{25}$

$P_{[19, 22]} = \frac{5c_{25}}{6\sqrt{2}} - \frac{5c_{15}}{6\sqrt{2}}$

$P_{[20, 20]} = -2c_{12} + \frac{2c_{34}}{3} + \frac{c_{35}}{3} + \frac{c_{45}}{3}$

$P_{[20, 21]} = -\sqrt{\frac{5}{2}} c_{15} - \sqrt{\frac{5}{2}} c_{25}$

$P_{[20, 22]} = \frac{c_{15}}{2} + \frac{c_{25}}{2}$

$P_{[20, 27]} = \frac{c_{35}}{\sqrt{2}} - \frac{c_{45}}{\sqrt{2}}$
\[ P[21, 21] = -\frac{c_{12}}{3} - \frac{5c_{15}}{4} - \frac{5c_{25}}{4} + \frac{2c_{34}}{3} + \frac{c_{35}}{4} + \frac{c_{45}}{4} \]
\[ P[21, 22] = \frac{5\sqrt{5}c_{15}}{12} + \frac{5\sqrt{5}c_{25}}{12} + \frac{\sqrt{5}c_{35}}{12} + \frac{\sqrt{5}c_{45}}{12} \]
\[ P[21, 26] = \frac{1}{6} \sqrt{\frac{5}{2}}c_{45} - \frac{1}{6} \sqrt{\frac{5}{2}}c_{35} \]
\[ P[22, 22] = -\frac{c_{12}}{3} + \frac{5c_{15}}{4} + \frac{5c_{25}}{4} + \frac{2c_{34}}{3} - \frac{c_{35}}{12} - \frac{c_{45}}{12} \]
\[ P[22, 26] = \frac{c_{45}}{6\sqrt{2}} - \frac{c_{35}}{6\sqrt{2}} \]
\[ P[23, 23] = \frac{2c_{12}}{3} + \frac{c_{15}}{3} + \frac{c_{25}}{3} - 2c_{34} \]
\[ P[23, 24] = -\frac{\sqrt{5}c_{35}}{2} - \frac{\sqrt{5}c_{45}}{2} \]
\[ P[23, 25] = -\frac{c_{35}}{2} - \frac{c_{45}}{2} \]
\[ P[23, 26] = \frac{c_{15}}{\sqrt{2}} - \frac{c_{25}}{\sqrt{2}} \]
\[ P[24, 24] = \frac{2c_{12}}{3} + \frac{c_{15}}{4} + \frac{c_{25}}{4} - \frac{c_{34}}{3} - \frac{5c_{35}}{4} - \frac{5c_{45}}{4} \]
\[ P[24, 25] = -\frac{\sqrt{5}c_{15}}{12} - \frac{\sqrt{5}c_{25}}{12} - \frac{\sqrt{5}c_{35}}{12} - \frac{\sqrt{5}c_{45}}{12} \]
\[ P[24, 27] = \frac{1}{6} \sqrt{\frac{5}{2}}c_{25} - \frac{1}{6} \sqrt{\frac{5}{2}}c_{15} \]
\[ P[25, 25] = \frac{2c_{12}}{3} - \frac{c_{15}}{12} - \frac{c_{25}}{12} - \frac{c_{34}}{3} + \frac{5c_{35}}{12} + \frac{5c_{45}}{12} \]
\[ P[25, 27] = \frac{c_{15}}{6\sqrt{2}} - \frac{c_{25}}{6\sqrt{2}} \]
\[ P[26, 26] = -\frac{c_{12}}{3} + \frac{5c_{15}}{6} + \frac{5c_{25}}{6} - 2c_{34} \]
\[ P[27, 27] = -2c_{12} - \frac{c_{34}}{3} + \frac{5c_{35}}{6} + \frac{5c_{45}}{6} \]

4 quarks in P-wave
Block from [28,28] to [30,30] with \( S = 5/2 \)

\[ P[28, 28] = \frac{2c_{12}}{3} + \frac{c_{15}}{3} + \frac{c_{25}}{3} + \frac{2c_{34}}{3} + \frac{c_{35}}{3} + \frac{c_{45}}{3} \]
\[ P[28, 29] = \frac{c_{35}}{\sqrt{2}} - \frac{c_{45}}{\sqrt{2}} \]
\[ P[28, 30] = \frac{c_{25}}{\sqrt{2}} - \frac{c_{15}}{\sqrt{2}} \]
\[ P[29, 29] = \frac{2c_{12}}{3} - \frac{c_{15}}{6} - \frac{c_{25}}{6} - \frac{c_{34}}{3} + \frac{5c_{35}}{6} + \frac{5c_{45}}{6} \]
\[ P[30, 30] = -\frac{c_{12}}{3} + \frac{5c_{15}}{6} + \frac{5c_{25}}{6} + \frac{2c_{34}}{3} - \frac{c_{35}}{6} - \frac{c_{45}}{6} \]
The spectrum of the \( SU(3)_F \) reducible multiplets of negative parity and “open door” positive parity. We call \( O \) the \( s = -4 \) states with quark content \( ssqs \). A star has been put for the negative parity states with “open door” decays and for the positive parity states with “open door” decays into a pseudoscalar meson and bayon of the decuplet.

\[
\begin{array}{|c|c|c|c|c|}
\hline
3 \times \bar{3}_F & N_s & \Lambda_s & \Lambda + \Sigma & \Xi \\
\hline
1/2^- & 1525 & 1636 & 1289 & 1397 \\
1/2^- & 1878 & 1995 & 1794 & 1918 \\
3/2^- & 1884 & 2011 & 1775 & 1907 \\
1/2^+ & 1775 & 1944 & 1611 & 1780 \\
3/2^+ & 1870 & 2025 & 1707 & 1865 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
6 \times \bar{3} & Z^0 & N_s & \Sigma_s & N & \Lambda + \Sigma & \Xi + \Xi_{3/2} \\
\hline
1/2^- & 1588 & 1744 & 1907 & 1386 & 1547 & 1712 \\
3/2^- & 1858 & 2005 & 2142 & 1759 & 1905 & 2038 \\
1/2^+ & 1545 & 1719 & 1893 & 1356 & 1534 & 1711 \\
3/2^+ & 1614 & 1779 & 1994 & 1434 & 1608 & 1761 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
15 \times \bar{3} & Z^1 | N_s | \Delta_s | \Lambda_s | \Sigma_s | \Xi_s \\
\hline
1/2^- & 1794 & 1920 & 1964 & 2045 & 2077 & 2184 \\
1/2^- & 2026 & 2126 & 2189 & 2229 & 2269 & 2355 \\
3/2^- & 1733 & 1831 & 2175 & 1939 & 2008 & 2088 \\
3/2^- & 2074 & 2175 & 2201 & 2278 & 2293 & 2391 \\
5/2^- & 2088 & 2201 & 2193 & 2307 & 2301 & 2416 \\
1/2^+ & 1922 & 2052 & 2052 & 2180 & 2180 & 2306 \\
3/2^+ & 2030 & 2120 & 2120 & 2236 & 2236 & 2358 \\
1/2^{++} & 1959 & 2082 & 2082 & 2204 & 2204 & 2336 \\
3/2^{++} & 1999 & 2132 & 2132 & 2246 & 2246 & 2367 \\
5/2^{++} & 2101 & 2208 & 2208 & 2323 & 2323 & 2426 \\
\hline
\end{array}
\]
| 15 × 3 | $N + \Delta$ | $\Lambda + \Sigma$ | $\Sigma + \Sigma_2$ | $\Xi$ | $\Xi + \Xi_{3/2}$ | $\Omega + \Omega_1$ |
|---------|---------------|-------------------|-------------------|-----|-------------------|-------------------|
| $1/2^-$ | 1627          | 1749              | 1793              | 1871| 1916              | 2030              |
| $1/2^-$ | 1941          | 2032              | 2102              | 2137| 2177              | 2259              |
| $3/2^-$ | 1511          | 1612              | 1727              | 1719| 1796              | 1869              |
| $3/2^-$ | 1943          | 2082              | 2123              | 2184| 2219              | 2311              |
| $5/2^-$ | 1982          | 2088              | 2088              | 2197| 2197              | 2311              |
| $1/2^+$ | 1761          | 1892              | 1892              | 2023| 2021              | 2151              |
| $3/2^+$ | 1841          | 1960              | 1960              | 2086| 2076              | 2203              |
| $1/2^{++}$ | 1810       | 1932              | 1932              | 2059| 2057              | 2187              |
| $3/2^{++}$ | 1869       | 1984              | 1984              | 2104| 2101              | 2225              |
| $5/2^{++}$ | 1958       | 2068              | 2068              | 2173| 2179              | 2285              |

| $15' \times 3$ | $Z_2$ | $\Delta_s$ | $\Sigma_s$ | $\Xi_s$ | $\Omega_s$ |
|-----------------|------|-----------|-----------|--------|----------|
| $1/2^-$ | 2358 | 2434      | 2513      | 2597   | 2685     |
| $3/2^-$ | 2143 | 2242      | 2338      | 2434   | 2529     |

| $15' \times 3$ | $\Delta + \Delta_{5/2}$ | $\Sigma + \Sigma_2$ | $\Xi + \Xi_{3/2}$ | $\Omega + \Omega_1$ | $O$ |
|-----------------|---------------------|-------------------|-------------------|-------------------|-----|
| $1/2^-$ | 2280              | 2352              | 2428              | 2508              | 2591 |
| $3/2^-$ | 1943              | 2066              | 2161              | 2263              | 2376 |

| $15 + 3 \times 3$ | $Z^1$ | $N_s$ | $N_s + \Delta_s$ | $\Lambda_s$ | $\Lambda_s + \Sigma_s$ | $\Xi_s$ |
|-------------------|------|------|------------------|-------------|----------------------|------|
| $1/2^+$ | 1732          | 1862 | 1901              | 1994        | 2030                 | 2161 |
| $1/2^+$ | 1851          | 1979 | 2001              | 2106        | 2126                 | 2256 |
| $3/2^+$ | 1789          | 1908 | 1957              | 2027        | 2077-2069            | 2194 |
| $3/2^+$ | 1908          | 2014 | 2063              | 2123        | 2172-2168            | 2281 |
| $1/2^{++}$ | 1683          | 1862 | 1901              | 1958        | 1992-1996            | 2133 |
| $3/2^{++}$ | 1767          | 1892 | 1934              | 2016        | 2061-2058            | 2185 |
| $5/2^{++}$ | 1888          | 1989 | 2049              | 2098        | 2199-2201            | 2259 |
| $15 + 3 \times 3$ | $N + \Delta$ | $\Lambda + \Sigma$ | $\Sigma + \Sigma_2$ | $\Xi$ | $\Xi + \Xi_{3/2}$ | $\Omega + \Omega_1$ |
|-------------------|--------------|-----------------|------------------|------|----------------|------------------|
| $1/2^+$           | 1547         | 1681            | 1719             | 1814-1852 | 1851          | 1984             |
| $1/2^+$           | 1705         | 1833            | 1851             | 1960-1980 | 1p81          | 2110             |
| $3/2^+$           | 1607         | 1736            | 1779             | 1853-1902 | 1897          | 2019             |
| $3/2^+$           | 1761         | 1870            | 1912             | 1980-2020 | 2026          | 2134             |
| $1/2^{++}$        | 1506         | 1644            | 1688             | 1779-1826 | 1823          | 1960             |
| $3/2^{++}$        | 1597         | 1722            | 1766             | 1839-1892 | 1889          | 2013             |
| $5/2^{++}$        | 1729         | 1828            | 1888             | 1935-1991 | 1986          | 2096             |

| $15' + 6 \times 3$ | $Z_0 + Z_2$ | $N_s + \Delta_s$ | $\Sigma_s$ | $\Xi_s$ | $\Omega_s$ |
|-------------------|-------------|-------------------|-------------|---------|---------|
| $1/2^+$           | 1983        | 2107              | 2229-2235   | 2354    | 2476    |
| $3/2^+$           | 2048        | 2165              | 2275-2280   | 2395    | 2508    |
| $1/2^{++}$        | 1863        | 1994              | 2125-2130   | 2262    | 2390    |
| $3/2^{++}$        | 1920        | 2045              | 2163-2173   | 2298    | 2426    |
| $5/2^{++}$        | 2018        | 2129              | 2242-2243   | 2360    | 2546    |

| $15' + 6 \times 3$ | $N + \Delta + \Delta_{5/2}$ | $\Lambda + \Sigma + \Sigma + \Sigma_2$ | $\Xi + \Xi_{3/2}$ | $\Omega + \Omega_1$ | $O$ |
|-------------------|-------------------------------|-----------------------------------------|-------------------|-------------------|-----|
| $1/2^+$           | 1838                          | 1964                                    | 2086-2091         | 2210              | 2332 |
| $3/2^+$           | 1899                          | 2017                                    | 2130-2149         | 2245              | 2366 |
| $1/2^{++}$        | 1685                          | 1821                                    | 1957-1959         | 2094              | 2230 |
| $3/2^{++}$        | 1742                          | 1872                                    | 2001-2001         | 2130              | 2260 |
| $5/2^{++}$        | 1843                          | 1958                                    | 2075-2075         | 2192              | 2313 |

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Figure 1: States with $J^P = 1/2^+$ obtained with the tetraquark in $3_F$ representation. The upper diagram (1A) correspond to $\bar{s}$ and the lower one (1B) to $\bar{u}$ and $\bar{d}$. Small circles denote a weight degeneracy.
Figure 2: States with $J^P = 1/2^+$ obtained with the tetraquark in $\bar{6}_F$ representation. The upper diagram (2A) correspond to $\bar{s}$ and the lower one (2B) to $\bar{u}$ and $\bar{d}$. Small circles denote a weight degeneracy.
Figure 3: States with $J^P = 1/2^+$ obtained with the tetraquark in $15_F$ representation. The upper diagram (3A) correspond to $\bar{s}$ and the lower one (3B) to $\bar{u}$ and $\bar{d}$. Small circles denote a weight degeneracy.
Figure 4: States with $J^P = 1/2^+$ obtained with the tetraquark in mixed representation $15_F + 3_F$. The upper diagram (4A) correspond to $\bar{s}$ and the lower one (4B) to $\bar{u}$ and $\bar{d}$. Small circles denote a weight degeneracy.
Figure 5: States with $J^P = 3/2^-$ obtained with the tetraquark in a $15'_F$ representation. The upper diagram (5A) correspond to $\bar{s}$ and the lower one (5B) to $\bar{u}$ and $\bar{d}$. Small circles denote a weight degeneracy.
Figure 6: States with $J^P = 3/2^+$ obtained with the tetraquark in a mixed $15_F + \bar{3}_F$ representation. The upper diagram (6A) correspond to $\bar{s}$ and the lower one (6B) to $\bar{u}$ and $\bar{d}$. Small circles denote a weight degeneracy.
Figure 7: States with $J^P = 3/2^-$ obtained with the tetraquark in a $6_F$ representation. The upper diagram (7A) correspond to $\bar{s}$ and the lower one (7B) to $\bar{u}$ and $\bar{d}$. Small circles denote a weight degeneracy.
Figure 8: States with $J^P = 5/2^-$ obtained with the tetraquark in a $15_F$ representation. The upper diagram (8A) correspond to $\bar{s}$ and the lower one (8B) to $\bar{u}$ and $\bar{d}$. Small circles denote a weight degeneracy.
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