Development of two-photon event generators for the KEDR experiment

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Abstract. The KEDR experiment, which started in 2002, is dedicated to a study of c- and b-quarks and the two-photon physics at the $e^+e^-$ collider VEPP-4M in the Budker INP. To analyze the $2\gamma$ data and estimate contribution of two-photon background events in the $1\gamma$ data samples, the event generators $e^+e^-\rightarrow e^+e^-+\text{hadrons}$, $e^+e^-\rightarrow e^+e^-+\pi^+\pi^-$, and $e^+e^-\rightarrow e^+e^-+\text{PS}$ (PS – pseudoscalar meson) have been developed.

1. Introduction  
Since the beginning of the 2000s at the $e^+e^-$ collider VEPP-4M (energy $2E_b = 2 - 11$ GeV) experiments with the detector KEDR [1] are carried out at energies $2E_b \lesssim 4$ GeV. To the present time the KEDR performed a series of precision measurements of masses of elementary particles [2], $R$ [3], and some others.

For the $\gamma\gamma$ experiments the KEDR detector is equipped with a special system of scattered electron ($e^\pm$) tagging (TS), consisting of two identical subsystems, each of 4 blocks located on both sides of the interaction point (see figure 1). The TS detects electrons (positrons), scattered in interaction, in the angle range 0-10 mrad with energies 45\% – 98\% of the beam energy.

![Figure 1. Drawing of the TS of the KEDR detector.](image)

The accuracy of measurement of the scattered $e^\pm$ energy is about 0.1\% of the beam energy and resolution in the invariant mass of $\gamma\gamma$ system $\Delta W/W \sim 1\%$. When scattered electrons are
detected in the TS, corresponding $Q_i$ of photons are small: $Q_i \ll m_\rho$, mass of $\rho$-meson.

$$Q_i^2 = |k_i^2| \approx E_b E'_i (\theta_i^2) \lesssim 10^{-2} \text{ GeV}^2.$$  

(Variables are shown in figure 2).

Using TS in the double tag mode, one can define $W$ independent of the central detector:

$$W^2 \approx 4\omega_1\omega_2 = 4(E_b - E'_1)(E_b - E'_2).$$

The main goal of experiments with the KEDR in the $\gamma\gamma$ physics – precision measurement of the total cross section of two-photon hadron production at energies $W \lesssim 4$ GeV and the cross sections of production of pairs $\pi^+\pi^-$ and $K^+K^-$ at $W \lesssim 1.5$ GeV. For this purpose collection of luminosity integral of 100 pb$^{-1}$ or more is planed. There are two measurements of the total cross section $\sigma(\gamma\gamma \rightarrow \text{hadrons})$ at low $\gamma\gamma$ energy $W$ and $Q^2 \approx 0$: by the MD-1 detector and by the TPC/2$\gamma$ detector [4]. In figure 3 the measurement of the MD-1 is shown. For comparison, the TPC/2$\gamma$ obtained $\sigma(\gamma\gamma \rightarrow \text{hadrons})=471\pm12$ and $479\pm16$ nb at $W=2$-3 and 3-4 GeV.

![Figure 2. Variables of process $e^+e^- \rightarrow e^+e^- + \text{hadrons.}$](image1.png)

![Figure 3. Cross section $\gamma\gamma \rightarrow \text{hadrons}$ measured by the MD-1 detector [4].](image2.png)

2. Lowest-order cross section

For a description of the $\gamma\gamma$ process $e^+e^- \rightarrow e^+e^- + f$, variables shown in figure 2, and invariants: $t_1 = -Q_2^2 = k_1^2$, $t_2 = -Q_2^2 = k_2^2$, $s_1 = (p_1^2 + k)^2$, $s_2 = (p_2^2 + k)^2$, $s = (p_1 + p_2)^2$, $W^2 = k^2 = (k_1 + k_2)^2$ are used.

The differential cross section for the unpolarized beams in the lowest order of QED is [5]:

$$d\sigma = \frac{\alpha^2}{16\pi^4 t_1 t_2} \sqrt{(k_1 k_2)^2 - t_1 t_2} \frac{d^3q_1}{E_1} \frac{d^3q_2}{E_2},$$  \hspace{1cm} (1)

where the function $\sum$ contains sum of 6 hadron $\gamma\gamma$ cross sections with calculated in QED factors.

The simulation can be divided into two stages: (i) $e^+e^- \rightarrow e^+e^- + f$ (reaction 2 $\rightarrow$ 3) and (ii) $f \rightarrow n$ particles. The phase space of $n + 2$ particles can be represented as [6]:

$$R_{n+2} = \int dW^2 R_3 R_n, \quad dR_n = \prod_i \frac{d^3q_i}{2E_i} \delta^4(k - \sum_{j=1}^{n} q_j),$$  \hspace{1cm} (2)

where $R_3$ – the phase space of the final state of the $2 \rightarrow 3$ reaction [6], $R_n$ – phase space of $n$ particles with 4-moments $q_i$ from decay of the system $f$. The 3-particle phase volume $R_3$ as a function of variables $s, s_1, s_2, t_1, t_2, W$ after integration over the azimuthal angle $\varphi$ [6].

The energy-momentum conservation is fulfilled exactly in the generators described below.
3. Event generator $e^+e^- \to e^+e^+ + \text{hadrons}$

For small $Q^2 \to 0$ all hadron cross sections in $\sum$ (equation (1)), except $\sigma_{TT}$ – cross section for transverse photons, tend to 0, and $\sigma_{TT} \to \sigma_{\gamma\gamma}$, where $\sigma_{\gamma\gamma}$ – cross section for real transverse photons [5]. From formulas (1) and (2) one obtains

$$d\sigma = \frac{\alpha^2 \sqrt{X_K_{TT}} \cdot \sigma_{TT}}{32\pi^3 s(s - 4m^2) t_1 t_2 \sqrt{-\Delta_4}} dW^2 d\tau_1 \tau_2 ds_1 ds_2.$$  \hspace{1cm} \text{(3)}

Here $\sigma_{TT} = [F(t_1, t_2)/F(0, 0)]^2 \sigma_{\gamma\gamma}$, transition form factors are included in $\sigma_{TT}$. $\Delta_4$ – Gram determinant [6]. The formulas for the $X$ and $K_{TT}$ can be found in [5]. For the $F(t_1, t_2)$ two options can be used: $|F(t_1, t_2)|^2 = |F(0, 0)|^2$ and the vector dominance model (VDM): $|F|^2 = (1 - t_1^2/m_P^2)^{-2}(1 - t_2^2/m_P^2)^{-2}$.

In the generator the hadron system consists of pions with uniform distribution in phase space [7]. This follows from the $e^+e^-$ data at energy of several GeV. Simulation $e^+e^- \to e^+e^+ + \text{hadrons}$ includes simulation of invariant $W$ in the range of $W_{\text{min}} - W_{\text{max}}$, invariants $t_1$, $t_2$, $s_1$, $s_2$ in the kinematics limits of the problem [7], [8], as well as rotation angle $\varphi$ of the whole system. From these values one obtains laboratory 4-moments of scattered $e^\pm$ and of the $\gamma\gamma$ system. 4-moments of pions from decay of the system $f$ are simulated according to [9].

In figure 4 the distribution in $n_\pi$ – number of $\pi^\pm$, obtained in this model at $W=3$ GeV, is compared with one measured in the $e^+e^- \to \text{hadrons}$ experiment at $\sqrt{s} = 3$ GeV [10].

![Figure 4. Distribution on $n_\pi$ at 3 GeV.](image)

4. Event generator $e^+e^- \to e^+e^- + \pi^+\pi^-$

This generator is based on the formula (3) and the simple model [11], which includes interference of the Born amplitude for helicity $\lambda=2$ in the continuum with the amplitude of the resonance $f_2(1270)$ with spin 2. The cross section $\sigma_{\gamma\gamma}$ in (3) equals $\sigma_{\gamma\gamma} = \int (d\sigma/d\varOmega) d\varOmega$, where, in the $\gamma\gamma$ system,

$$d\sigma = \left( \left( \frac{d\sigma}{d\varOmega} \right)_B \right)^{\lambda=0} 2 + \left( \left( \frac{d\sigma}{d\varOmega} \right)_R \right)^{\lambda=2} 2.$$ \hspace{1cm} \text{(4)}

The calculated Born, resonance and total $\gamma\gamma$ cross sections are shown in figure 5 (left panel). The cross section $\sigma(e^+e^- \to e^+e^- + \pi^+\pi^-)$ (integral of (4) for $W = 2m_{\pi^-} - 1.5$ GeV) as a function of beam energy is shown in figure 5 (right panel). Integral of (4) at $W = 0.8 - 1.5$ GeV for $|\cos\theta^*| < 0.6$ is consistent with a factor of 2 with the measurement of the Belle detector [12].

5. Event generator $e^+e^- \to e^+e^- + \text{PS} \text{ (PS – pseudoscalar meson)}$

This generator for simulation two photon production of pseudoscalar mesons is described in detail in [13]. In the generator an option for account radiative corrections in the single tag mode is included.

The differential production cross section of narrow pseudoscalar meson with mass $M_R$ and $\gamma\gamma$ width $\Gamma_{\gamma\gamma}$ can be written as [13]

$$d\sigma = \frac{4\alpha^2 \Gamma_{\gamma\gamma}}{\pi s^2 t_1^2 t_2^2 M_R^3} \left| \frac{F(t_1, t_2)}{F(0, 0)} \right|^2 \frac{d\tau_1 \tau_2 ds_1 ds_2}{\sqrt{-\Delta_4}}.$$ \hspace{1cm} \text{(5)}
Figure 5. Model [11]: cross sections $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$ as a function of $\pi^+\pi^-$ mass (left panel) and cross section $\sigma(e^+e^- \rightarrow e^+e^-\pi^+\pi^-)$ as a function of beam energy (right panel).

Function $B$ was calculated in [14] and is given by

$$B = 0.25t_1t_2B_1 - 4B_2^2 + m_e^2B_3,$$

where

$$B_1 = (4p_1p_2 - 2p_1k_2 - 2p_2k_1 + k_1k_2)^2 + (k_1k_2)^2 - 16t_1t_2 - 16m_e^4,$$

$$B_2 = (p_1p_2)(k_1k_2) - (p_1k_2)(p_2k_1),$$

$$B_3 = t_1(2p_1k_2 - k_1k_2)^2 + t_2(2p_2k_1 - k_1k_2)^2 + 4m_e^2(k_1k_2)^2.$$  \hspace{1cm} (6)

Algorithm of simulation of events $e^+e^- \rightarrow e^+e^- + \text{PS}$ is the same as for the modeling of $e^+e^- \rightarrow e^+e^- + \text{hadrons}$, only it is not necessary to simulate the energy $W$, since $W = M_R$.

Measurement of the cross section $\sigma(e^+e^- \rightarrow e^+e^+ + \eta)$ at $\sqrt{s} = 1$ GeV by the KLOE detector [15] is consistent with the MC calculation with and without VDM within about two errors:

$$\sigma_{\text{exp}} = 41.7 \pm 4 \text{ pb [KLOE]}; \sigma_{\text{MC}} = 32.4 \text{ pb [with VDM]} \text{ and } \sigma_{\text{MC}} = 35.5 \text{ pb [without VDM]}.$$  

For experiments on two-photon physics with the KEDR detector at small $Q^2$, radiative corrections are negligible (see [13]).

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