Emergence of rotational modes in nuclear fission

Bency John
Nuclear Physics Division,
Bhabha Atomic Research Centre, Mumbai-400085, India
(Dated: May 1, 2023)

In the present work, the dinucleus at pre-scission stage is treated as an overall closed thermodynamic system that encloses an open subsystem of rotational degrees of freedom. This subsystem can exchange matter and energy with its surrounding heat bath that comprising of all remaining degrees of freedom. Energy budget for the exchanges is accounted for by the pre-scission potential energy release. For the intrinsic rotational modes, the coherent degrees from among multitude of other intrinsic excitations. Thermodynamic sub-system concept that explain the emergence of intrinsic rotational degrees of freedom in excited dinuclei is discussed in the present work, some probably for first time in fission theory. In the following, Sect. II describes the method for calculations. Physical environments of sliding, rolling, and sticking in the early phase of heavy ion collisions have similarity to the environment of dinuclear rotations and hence the former is described in Sect. II as a supporting argument. The results are presented in Sect. III and a discussion is given in Sect. IV. Finally the summary and conclusions are given in Sect. V.

I. INTRODUCTION

Spontaneous generation of intrinsic rotational modes that cause ample gamma ray emissions from fission fragments is a barely understood aspect in nuclear fission theory. In general, the nuclear rotation is a complicated interplay between the collective and intrinsic degrees of freedom. This situation is further enriched in the fission process since spontaneous generation of intrinsic rotational modes occurs near the scission stage where the nucleus is already transformed into an excited dinucleus under non-equilibrium conditions. While these modes carry no net angular momentum, they impart angular momenta to the individual fragments, and they occur relative to rigid rotation, if the latter is present in the dinucleus. In induced fission reaction, very strong damping is present in the compound nuclear evolution just after the initial formation stage, and this strong damping phase continues towards the stages of saddle and at times even beyond, as long as the collective kinetic motion in elongation is slow. Thereafter, as the collective kinetic motions pick up speed, the nature of damping changes to weak damping and this phase continues till final scission. The weak damping is characterized by couplings between collective and intrinsic degrees and it allows elasto-plastic property for the nuclear matter. The weak damping phase is also known in literature as non-adiabatic phase or as diabatic phase. In fission, the collective pre-scission kinetic energy is largely accumulated during the weak damping phase. This phase also allows generation of other collective degrees such as rotational and vibrational modes that carry no net angular momentum. Thus, overall, this is a very important phase of fission, however, diabatic descriptions of the same are not as well developed as the adiabatic descriptions of the earlier phases. For this reason the nuclear scission process and the inherent dinuclear rotations are not well understood till this time.

As a many body quantum system far from equilibrium, the dinucleus can manifest quite different macroscopic degrees of freedom apart from deformations. Coherent and incoherent excitation modes can co-exist in a dinucleus with their characteristic growth governed by the weak damping phase. The free energy difference contributed by the potential energy release while moving from saddle to scission can set stage for a diabatic environment that aids emergence of ordered intrinsic rotational degrees from among multitude of other intrinsic excitations. Thermodynamic sub-system concept that explain the emergence of intrinsic rotational degrees of freedom in excited dinuclei is discussed in the present work, some probably for first time in fission theory. In the following, Sect. II describes the method for calculations. Physical environments of sliding, rolling, and sticking in the early phase of heavy ion collisions have similarity to the environment of dinuclear rotations and hence the former is described in Sect. II as a supporting argument. The results are presented in Sect. III and a discussion is given in Sect. IV. Finally the summary and conclusions are given in Sect. V.

II. METHOD

A. Intrinsic rotational modes in fission

In the present work, the dinucleus at pre-scission stage is treated as an overall closed thermodynamic system that encloses an open subsystem of rotational degrees of freedom. This subsystem can exchange matter and energy with its surrounding heat bath that comprising of all remaining degrees of freedom. Energy budget for the exchanges is accounted for by the pre-scission potential energy release. For the intrinsic rotational modes, the
model assumes their origination by nucleon pair transfers from the dinuclear neck region aided by their characteristic couplings (dynamical pairs). This assumption may be justified since in the final phase, the volume from which balance nucleons retreat to either fragments is spatially confined to the neck volume. The nucleon retreats occur mostly pair wise, one nucleon each to each pre-fragment, and preferentially to the surface of pre-fragment (surface-peaked nature [9]). The vector-sum of nucleon pair’s linear momenta will be approximately zero, and the center of mass of the whole will be at rest. Although it is not necessary that these twin transfers occur simultaneously from the same location and they are of exactly opposite linear momentum, these qualities are to be maintained on an average, over a short time period, that satisfy the Heisenberg uncertainty relations. This mode of retreat of the nucleons accomplishes a progressive necking-in and an eventual split that occurs when no nucleons are left in the neck. The average velocities with which these nucleons recede from the neck region to either side are close to Fermi velocity of the nucleons. Grossly, the angular momenta recede from the neck region to either side are close to Fermi velocity of the nucleons. Grossly, the angular momentum imparted on a prefragment of radius vector to Fermi velocity of the nucleons. Angular momenta of intrinsic modes generated are identified. Further, from a macroscopic and leptodermous idealization, the initiation of sliding and rolling like motions and the sticking configuration in a dinucleus were studied in the seminal work by Tsang [12], where, instead of outgoing fission fragments as in our case, a pair of incoming colliding nuclei of mid-mass that experience frictional forces was considered. The initial phase of the nuclear collision has weak damping characteristics wherein the sliding and rolling motions arise. In the center of mass frame, once the projectile and target approach each other sufficiently closely with a finite impact parameter, they initially slide on each other and experience sliding friction. Frictional force will be maximum when the relative velocity at each point in the overlapping region is the highest. As a result of friction, the relative velocities at these points decrease and when the relative velocity at the contact point is zero, the sliding motion would stop. Rolling motion will continue if the relative angular velocity is non-zero. By the action of rolling friction, which is lower in magnitude, the relative angular velocity also will decrease to zero. The latter condition causes the dinuclear system to get stuck completely and rotate as a rigid rotor. The sliding frictional force transfers the initial orbital angular momentum into colliding partner individual angular momenta \( I_1 \) and \( I_2 \) which lead the two partners to roll on each other. By the action of rolling friction, the \( L \) transfer gets completed and the partners get stuck and become a rigid rotor. Equations of motion for the radial direction, individual angular momenta \( I_1 \) and \( I_2 \), and the orbital angular momentum \( L \) have been obtained with a single frictional coefficient (K) as the only parameter in [12, 13]. Assuming that the mass asymmetry degree is frozen, and with \( \rho_1 \) and \( \rho_2 \) as densities of first and second nuclei in the overlapping region, the equations of motion (except the radial) are given by

\[
\dot{I}_1 = K \int \rho_1 \rho_2 d^3 \tau [(\theta R - \theta_1 R_1' - \theta_2 R_2') R_1' - g^2 (\theta_1 - \theta_2)],
\tag{1}
\]

\[
\dot{I}_2 = K \int \rho_1 \rho_2 d^3 \tau [(\theta R - \theta_1 R_1' - \theta_2 R_2') R_2' + g^2 (\theta_1 - \theta_2)],
\tag{2}
\]

and

\[
\dot{L} = -(\dot{I}_1 + \dot{I}_2). \tag{3}
\]

As dynamical variables, the polar coordinates of nuclear centers \( R \) and \( \theta \), and the angles \( \theta_1 \) and \( \theta_2 \) that characterize the space orientation of both nuclei, are used in the above equations. Dots over the symbols denote time derivatives. The arm lengths \( R_1' \) and \( R_2' \) are the distances from the nuclear centers to the centroid of the overlap region and they are also measures of the respective nuclear deformations. Parameter \( g \) is the effective radius of gyration of the overlap region around its centroid.

Equation (3) implies that if \( I_1 \) and \( I_2 \) are increased during the collision process due to friction, then \( L \) is decreased and vice versa, and it ensures conservation of the total angular momentum. The equations of motion for \( \dot{I}_1 \) and \( \dot{I}_2 \) (Eq.(1) and Eq.(2)), each has two terms in the square bracket, of which the first term is the major one. This major term has in the parenthesis the relative
C. Similarities of rotational mode environments in fission and heavy ion collisions

As mentioned above in Sect. II B in the entrance phase of heavy ion collisions, the partners first undergo sliding motion and then undergo rolling motion. Whereas the fission reaction, while it proceeds towards the scission stage, has similarities to the entrance phase of heavy ion collisions but in a time reversed manner. The prefragments, in their journey to scission, first undergo rolling motion and then reach the stage of nucleon pair transfers from the neck. The ‘pair transfers from neck’ are like the sliding motion; they cause ‘local’ high relative tangential velocity for the participating nucleons.

The rolling frictional motion causes $\dot{I}_1$ and $\dot{I}_2$ of same magnitude but opposite in sign. This is a characteristic of occurrence of the bending mode. The frictional motion generated due to the relative tangential velocity (arising due to pair transfers) will have $\dot{I}_1$ and $\dot{I}_2$ of same sign but possibly of different magnitudes in the two prefragments due to their different arm lengths. The pattern of motion generated due to pair transfers therefore has characteristic of the wriggling mode. The wriggling mode occurs much more strongly than the bending mode [12,14]. The net result of these actions can lead to probable unequal values for $\dot{I}_1$ and $\dot{I}_2$, at same time satisfy the condition given by Eq.(3). Recent advanced experimental and theoretical works have renewed interest in this topic and they forwarded competing explanations [15–17], however we maintain that, as also mentioned earlier, relevant average behaviors are explained by the macroscopic dynamical model given originally in [12].

The pair transfers can also have off-axis displacements due to which the twisting mode is generated. The twisting mode populates angular momentum components along the dinuclear symmetry axis but in opposite directions in the two prefragments so that no net angular momentum is generated in the dinucleus as a whole. The axial rotation (tilting) mode is not directly excitable by the pair transfers, for which the activation is due to the orbital rigid-rotation present in the dinucleus [3]. Note that the tilting confers angular momentum component along the symmetry axis, and only if the total angular momentum $I$ of the dinucleus is non-zero. The relaxation time for the tilting mode is relatively long compared to other intrinsic rotational modes owing to its different origin.

D. Nonequilibrium relaxation

The mechanism of the intrinsic rotational modes is described above. How the available free energy fuel these modes, from among multitude of many other degrees of freedom? It depends on how relevant flows and forces couple among themselves and exchange the energy available. For such processes, the formulation of nonequilibrium sub-system thermodynamics in terms of the free energy dissipation is very useful. This is particularly so when detailed knowledge of the flows and forces are non-existent. The formulation we used relates firmly the forces and fluxes to a free energy difference in terms of the relative entropy [18].

Relatively slow relaxation of rotational modes as compared to the nucleonic degrees and still slower relaxation of elongation (fission) degree allows one to calculate temperature $T$ of the heat bath at various instances of elongation. As the nucleon transfers ceases at scission elongation, $T$ reaches its saturation value equal to $T_s$, thereby marking the end of the transient phase of thermal generation of intrinsic rotational modes. Till the transient phase ends the dissipative part of the dynamics cause the system to experience a monotonic change of the relative entropy [18] as shown in Sect. III below.

E. Statistical model for density of states at a given $T$

The wriggling, bending, twisting and tilting mode populations are assumed to obey statistical distributions with respect to their population probabilities and relaxation times as they are embedded in a heat bath of temperature $T$. The total angular momentum $j$ in a prefragment is the sum of contributions from individual particles, conferred by the intrinsic rotational modes as well as the rigid rotational or single particle degrees. In thermal occupation condition, the density of states of a given total angular momentum $j$ in a prefragment has been obtained as

$$P(j) = \frac{(2j+1)}{2\sigma^2} \exp\left(-\frac{j(j+1)}{2\sigma^2}T\right)$$

(4)

where $\sigma^2$ is the ‘spin-cutoff’ parameter given by $\sigma^2 = I/I_n$ where $I$ is the moment of inertia and $T$ is the temperature [20]. This form for fragment angular momentum distribution has been experimentally validated
in numerous studies, see for instance\textsuperscript{[15]}. How the axi-
al alignment of the prefragments modify the density of states? Thermal excitation of the twisting mode results in a certain width for the intrinsic spin states that occur parallel to the dinuclear symmetry axis. Summation over these intrinsic spin states leads to same above form for $P(j)$ even when the prefragments are axially aligned\textsuperscript{[21]}.

Time dependent values of $P(j, t)$ and steady state saturation value of $P_{ss}(j)$ are calculated using relevant values of the temperature and moment of inertia. The distribution $P_{ss}(j)$ has to be the same with and without the knowledge of the damping rate the nucleus has taken till scission, whereas $P(j, t)$, on contrary, depends on the time scales of excitation energy and matter buildup in the prefragment. (The latter aspect is discussed in Sect. II G.)

**F. Generalized non-equilibrium free energy and entropy balance equation**

The transient nonequilibrium deviations of the subsystem can be described by a generalized free energy difference ($F(t)$) associated with this process\textsuperscript{[15]}. In this exploratory work using $F(t)$, it is assumed that the prefragments are of symmetric mass and the shell structure effects are not taken into consideration. $F(t)$ in one of the forming prefragments is given by

$$F(t) = k_B T(t) \sum_j P(j, t) \ln \frac{P(j, t)}{P_{ss}(j)}$$

where $P(j, t)$ is the probability distribution for total angular momentum $j$ at an instant of time $t$ and $P_{ss}(j)$ is the equilibrium or steady probability distribution for $j$ reached at the end of the transient phase. $T(t)$ is the absolute temperature and $k_B$ is the Boltzmann constant. How an arbitrary distribution $P(j, t) \neq P_{ss}(j)$ is changing with time and approaching $P_{ss}(j)$ is determined by $F(t)$.

$F(t)$, which is also known as relative entropy, is the free energy difference between a distribution at a point of time $t$ on way to equilibrium, and the distribution at equilibrium \textsuperscript{[15]}. It is well known that fluctuations at the equilibrium returns to its mean value driven by a thermodynamic force. The same thermodynamic force drives the non-equilibrium relaxation towards the equilibrium distribution. The equilibrium fluctuations and non-equilibrium relaxations are thus connected \textsuperscript{[15, 22]}. The connection is given formally by $F(t)$. It has many important mathematical properties, for example, it is positive and it always decreases until it reaches its minimum at equilibrium.

The free energy dissipation rate $f_d$ is given by

$$f_d = -\frac{dF}{dt} = T e_p$$

where $e_p$ is the rate of entropy production and $T$ is the absolute temperature. In literature $e_p$ is referred also as non-adiabatic entropy production rate and it is purely determined by the damping mechanism\textsuperscript{[23]}. The entropy of the subsystem of rotational modes increases due to the entropy generated in spontaneous irreversible processes and decreases when heat is expelled into the surrounding heat bath. It is has been shown\textsuperscript{[23–25]} that for such a system, the net rate of change of entropy as a function of time is given by the well known entropy balance equation as

$$\frac{dS(t)}{dt} = e_p(t) - e_e(t)$$

where $e_p$ is the rate of entropy production in the subsystem and $e_e$ is the rate of entropy expulsion to the surrounding heat bath and both are functions of time. Using the methods of non-equilibrium thermodynamics the rate of entropy production $e_p$ is calculated from Eq.$(6)$. The rate of entropy production $e_e$ is given by $e_e = \frac{h_d}{T}$, where $h_d$ is the heat dissipation.

**G. Fragment excitation buildup during diabatic phase**

In non-equilibrium thermodynamics for nuclear fission, dissipation and fluctuation are the key ingredients. The transient phenomenon of excitation energy buildup in the dinucleus (pre$TXE$) leads to the heat dissipation in prefragments given by $h_d$. A simple formalism has been used for calculation of pre$TXE$. We presently adopt simplified approaches such as this with the expectation that it will be be sufficient for a first orientation towards the main aim of this work, i.e., the first-time application of entropy balance equation in depiction of the intrinsic rotational modes as dissipative structures.

It is known that an excited nucleus de-excites itself and become relatively cold early in the fission process by emission of particles. Therefore it is assumed that a net longitudinal motion in the elongation coordinate $R$ gets initiated from a relatively cold nucleus. At an early stage like the saddle point, the deformation forces and consequential net longitudinal motions are practically zero, and the motion picks up only during the saddle to scission period. A reduced elongation parameter $\bar{R} = R - R_s$ is defined, where, $R_s$ is the value of elongation parameter at the saddle point. Equation of motion for the reduced elongation parameter $\bar{R}$ is given by

$$M_{eff} \frac{d^2 \bar{R}}{dt^2} = k \bar{R}$$

where $\bar{R}$ vary within the bounds of saddle point and scission point values and $k$ is the constant of proportionality with positive sign\textsuperscript{[26]}. The effective mass $M_{eff}$ is approximated to be a constant value given by the reduced mass of the binary nascent fission fragments. This is a good approximation within the limits of weak to non-damping
The solution for the equation of motion with initial condition $R(t=0) = 0$ is given by

$$R = c \sinh \lambda t$$

which leads to

$$\dot{R} = c \lambda \cosh \lambda t$$

with constants $c$ and $\lambda$ determined from empirical considerations and the constant of proportionality is given by

$$k = \lambda^2 M_{\text{eff}}.$$ 

The kinetic energy from the above one dimensional model of motion is given by $E_K = \frac{1}{2} M_{\text{eff}} \dot{R}^2$. In this model of saddle to scission motion, the transverse motions are not taken into account. There are degrees of freedom with transverse components in the dinucleus; for example, the thermally generated rotational degrees and such degrees have to be taken into account. Once the restriction to just one dimension is lifted, numerous exchanges occur that result in coherent excitations and non-coherent intrinsic excitations. The non-coherent intrinsic excitations together constitute a heat bath of energy content $preTXE$. An approximation is made in this work that the heat bath acquires half of the quantity of the kinetic energy gain from the one-dimensional model and the rest half is retained by the collective motions, i.e., $preTXE = \frac{1}{2} E_K$. Sharing of liberated energy between intrinsic and collective motions is one of the main characteristic of the weak damping phase. On reaching scission, the accumulation of $preTXE$ comes to an end.

The reduced elongation parameter $\bar{R}$ and the $preTXE$, both as functions of time, obtained for fission of $^{236}\text{U}$ nucleus using the above model, are given in Fig.1. Values of the empirical constants used are $c = 0.007$ fm and $\lambda = 1.0 \times 10^{-21}$ s$^{-1}$ $^{26}$. The results presented in Fig.1 are consistent with the microscopic calculations and empirical observations cited in $^{32}$, $^{33}$. It is to be noted that the collective kinetic motions, especially when they are near the originating stage and of small amplitudes, are themselves result of a gather of guided intrinsic motions arising from high dissipation. When collective kinetic motion increases there will be more coherency built and the dinucleus reach the weak damping phase.

For the present model for thermal excitation of rotational modes, the above estimation of $preTXE$ and its time dependence provide useful inputs. For symmetric fission, each prefragment will have half of the $preTXE$ as its excitation energy at a given instant. When collective kinetic motion increases there will be more coherency built and the dinucleus reach the weak damping phase.

For the present model for thermal excitation of rotational modes, the above estimation of $preTXE$ and its time dependence provide useful inputs. For symmetric fission, each prefragment will have half of the $preTXE$ as its excitation energy at a given instant. When collective kinetic motion increases there will be more coherency built and the dinucleus reach the weak damping phase.

Reaching this temperature has been used as the criterion that the system has reached the final scission point. The value of $j_{\text{rms}}$ calculated using $T = 1$ MeV in Eq.(4) is $8.6 \hbar$ for the symmetric split. Experimental systematics of $j_{\text{rms}}$ in $n_{th}+^{235}\text{U}$ fission given in $^{32}$ compare well with this estimate. For general applications, a better definition of scission point is surely desirable.

### III. RESULTS

#### A. Free energy and entropy rates

The generalized non-equilibrium free energy difference $F$ and the entropy production rate $e_p$ are calculated using Eq.(5) and Eq.(6), respectively. The free energy differ-
The irreversible processes in the dinuclear rotation are described by Eq.(5) to Eq.(7) in terms of the non-equilibrium thermodynamics. As the nucleon pairs transfer their kinetic energy to the prefragments, a part of the energy, with a spread from fluctuations in its values, is utilized to build up the subsystem of dinuclear rotations. Initially, as many associated degrees of freedom with the above spread in energy get opened up, the entropy production rate $e_p$ would increase. From the perspective of information entropy, the increase in $e_p$ comes about by discarding information due to the spreading out and from information loss associated with the couplings. As the rotations are limited by an energy budget due to the spin-cutoff values, the initial increase of $e_p$ is followed by a decrease but it remains a positive quantity ($e_p \geq 0$). The positivity of entropy production rate $e_p$ is a general criterion of irreversibility of the process. The part of the nucleon kinetic energy that is unutilized for rotations is assumed to cross the boundary of the ‘open subsystem of rotation’ and it gets dissipated in the heat bath. Therefore the term $e_r$ carry negative sign in the entropy balance equation. The resultant rate of change of entropy $dS/dt$ (Fig.3 continuous line) confirms that an efficient free energy dissipation and associated transfer of a higher amount of lower-grade heat across the boundary can change its sign from positive to negative. This change results in spontaneous high degree of organizations in space, time, and function, and it ultimately lead to the emergence of intrinsic rotational modes. This is a novel description for the intrinsic rotational modes. Under appropriate non-equilibrium conditions, a small fraction of the microscopic motions is organised into a well defined macroscopic motion such as a rotational mode exemplifying a dissipative structure.

The entropy rates have the form of power per Kelvin. While the free energy $F$ drops gradually, two regimes are displayed; first an ‘incoherent’ power transfer followed by a more coherent power transfer. During this process of power transfer, a fraction of the power may get reflected from the dinucleus in the form of radiation. Tracking the radiated power using advanced radiation detector set-ups may provide experimental confirmations for these important quantities.

**IV. DISCUSSION**

The importance of infusing concepts associated with diabaticity, open subsystems, free energy, and spontaneous emergence of order in dinuclear systems as discussed in this work lies in obtaining a better understanding of dissipative systems like the fissioning nucleus. For instance, the free energy dissipation rate $f_d$ (Eq.(6)) signifies an instantaneous current of the thermodynamic process. The dissipative systems export entropy to its environment to maintain a low internal entropy for itself. As the above mechanism evolves, the system’s ‘metabolic’ paths become more efficient. The mechanism of scission itself, a random rupture or a less or more violent process, will be guided by the system’s dissipative paths. Decades-old gray area such as the scission process description needs to be looked at from newer perspectives and the present work brings in one. Moreover, the literature on thermally excited intrinsic rotational modes in fission often cites an elusive and abstract nature for them. This is partially due to lack of information on how these modes are initiated by the nucleon motions and their growth in magnitude as collective modes. Using a simple formalism, the present work brings a clearer
Out of two types of non-equilibrium processes widely studied, namely transient and stationary, the emergence of rotational modes is of transient nature. Whereas, the second type, the stationary non-equilibrium process, is a driven process and this type requires sustained energy supply. For all such non-equilibrium systems, the free energy function assumes a leading role, and the entropy production in a non-equilibrium system is regarded as a matter of primary importance. The interest is not only focused on why the entropy change as the system evolves, but also on how the entropy is produced. The reason being that the entropy production rate is determined purely by the damping mechanism and hence the irreversibility. The present work probes some of these important aspects in context of nuclear fission, where a dramatic rearrangement of nuclear matter occur with accompanying large energy release. Far from equilibrium nature is a characteristic of many other finite many-particle quantum systems as well, such as atoms and atomic clusters, and similar systems in quantum chemistry and biology. In comparison to these systems, the transition energies in nuclear fission are far larger and this gives advantages to dedicated experimental investigations where results obtained can be connected to the free energy functional and the entropy production rate.

V. SUMMARY AND CONCLUSIONS

In summary, we considered the diabatic evolution of pre-scission nucleus in a duration when the dinuclear intrinsic rotational degrees of freedom emerge to existence, by using certain thermodynamic concepts that normally characterise macroscopic systems. As is well known, many properties of nuclei are indeed successfully explained using macroscopic formalisms and nuclei are ‘macroscopic objects’ owing to that they contain many particles and a very large number of states. Rotational modes in fission nuclei that cause ample gamma ray emissions remained barely understood even by the macroscopic formalisms. We analysed the issue of intrinsic rotational modes from a new perspective in terms of thermodynamics of irreversible processes. A reasonable explanation has been given by identifying the intrinsic rotational modes as an open subsystem that exchange matter and energy by means of coupling to a heat bath that comprises of the balance degrees of freedom of the system. Essentially, the available energy has to be appropriately coupled to the work needed for emergence of self-ordering rotational modes. As the initiator of intrinsic rotational modes, the pair transfer of nucleons from neck region play the role of such a coupling. Justification for this identification is provided in part by the equations of motion derived under certain macroscopic idealizations [12]. A simple formalism is presented such that the magnitude of relevant energy flows through the subsystem are obtained in terms of entropy production rate and entropy expulsion rate in sub-units of MeV zs⁻¹ K⁻¹. The rate of change of entropy dS/dt which is the difference of the above rates show a transition from positive to negative values as the dinucleus reached the scission point. This signifies emergence of a new order at a higher level of organisation for the subsystem, i.e., the emergence of intrinsic rotational modes as dissipative structures.

[1] L. G. Moretto, G. F. Peaslee, G. J. Wozniak, Nucl. Phys. A 502 (1989) 453c.
[2] T. Dossing, J. Randrup, Nucl. Phys. A 433 (1985) 215.
[3] J. Randrup, Nucl. Phys. A 447 (1985) 133c-148c.
[4] W. Norenberg, Nucl. Phys. A 428 (1984) 177c-188c.
[5] C. Simenel, A. S. Umar, Prog. Part. Nucl. Phys. 103 (2018) 19.
[6] R. P. Schmitt, D. R. Haenni, L. Cooke, H. Dejbakhsh, G. Mouchaty, T. Shutt, H. Utsunomiya, Nucl. Phys. A 487 (1988) 370.
[7] J. R. Nix, W. J. Swiatecki, Nucl. Phys. 71 (1962) 1.
[8] B. John, Pramana- J. Phys. 85 (2015) 267.
[9] W. J. Swiatecki, Prog. Part. Nucl. Phys. 4 (1980) 383.
[10] J. Randrup, Nucl. Phys. A 383 (1982) 468.
[11] E. Karsenti, Nat.Rev. Mol. Cell. Biol. 9 (2008) 255.
[12] C. F. Tsang, Physica Scripta 10A (1974) 90.
[13] V. V. Volkov, Phys. Rep. 44 (1978) 93.
[14] T. Dossing, J. Randrup, Phys. Lett. B 155 (1985) 333.
[15] J. N. Wilson et al., Nature 590 (2021) 566.
[16] J. Randrup, R. Vogt, Phys. Rev. Lett. 127 (2021) 062502.
[17] A. Bulgac, I. Abdurrahman, K. Godbey, I. Stetcu, Phys. Rev. Lett. 128 (2022) 022501.
[18] H. Qian, Phys. Rev. E 63 (2001) 042103.
[19] P. Ao, Commun. Theor. Phys. 49 (2008) 1073.
[20] T. Ericson, Advances in Physics 9:36 (1960) 425.
[21] B. John, S. K. Kataria, Phys. Rev. C 57 (1998) 1337.
[22] P. Mazur, Physica A 274 (1999) 491.
[23] H. Qian, Phys. Lett. A 378 (2014) 609.
[24] I. Prigogine, G. Nicolis, J. Chem. Phys. 46 (1967) 3542.
[25] H. Ge, H. Qian, Phys. Rev. E 81 (2010) 051133.
[26] H. Walliser, K. Wildermuth, F. Gonnenwein, Z. Phys. A-Atomic Nuclei 329 (1988) 209.
[27] W. J. Swiatecki, S. Bjornholm, Phys. Reports 4 (1972) 325.
[28] W. Younes, D. Gogny, Phys. Rev. Lett. 107 (2011) 132501.
[29] C. Simenel, A. S. Umar, Phys. Rev. C 89 (2014) 031601.
[30] Yu Qiang, J. C. Pei, Phys. Rev. C 104 (2021) 054604.
[31] J. F. Lemaitre, S. Panebianco, J. L. Sida, S. Hilaire, S. Heinrich, Phys. Rev. C 92 (2015) 034617.
[32] B. Becker, P. Talou, T. Kawano, Y. Danon, I. Stetcu, Phys. Rev. C 87 (2013) 014617.
[33] J. Randrup, Phys. Lett. B 110 (1982) 25.