Experimental Verification of a Vehicle Localization based on Moving Horizon Estimation Integrating LRS and Odometry

Kuniyuki Sakaeta¹, Kenichiro Nonaka², Kazuma Sekiguchi²

¹ Graduate school of Engineering, Mechanical Systems Engineering, Tokyo City University, 1-28-1, Tamazutsumi, Setagaya, Tokyo, 158-8587, Japan
² Faculty of Engineering, Department of Mechanical Systems Engineering, Tokyo City University, 1-28-1, Tamazutsumi, Setagaya, Tokyo, 158-8587, Japan
E-mail: g1581211@tcu.ac.jp

Abstract. Localization is an important function for the robots to complete various tasks. For localization, both internal and external sensors are used generally. The odometry is widely used as the method based on the internal sensors, but it suffers from cumulative errors. In the method using the laser range sensor (LRS) which is a kind of external sensor, the estimation accuracy is affected by the number of available measurement data. In our previous study, we applied moving horizon estimation (MHE) to the vehicle localization for integrating the LRS measurement data and the odometry information where the weightings of them are balanced relatively adapting to the number of the available LRS measurement data. In this paper, the effectiveness of the proposed localization method is verified through both numerical simulations and experiments using a 1/10 scale vehicle. The verification is conducted in the situations where the vehicle position cannot be localized uniquely on a certain direction using the LRS measurement data only. We achieve accurate localization even in such a situation by integrating the odometry and LRS based on MHE. We also show the superiority of the method through comparisons with a method using extended Kalman filter (EKF).

1. Introduction

In recent years, autonomous mobile robots have been researched actively to improve working efficiencies. When mobile robots work autonomously, they have to know the self-pose composed of position and heading angle; many studies have been conducted to estimate it with high accuracy [1]. In self-localizations, internal sensors such as rotary encoders and acceleration sensors, and external sensors such as cameras and laser range sensors (LRS) are used. The odometry is widely used as the method based on the internal sensors [2], but the estimation error is caused by measurement noises and slippages of tire; their accumulation results in cumulative errors. On the other hand, using external sensors, the robot’s pose is inferred from the correspondence of the sensor data with the surrounding environment which inherently avoids the influence of cumulative errors. Cameras are used for the localization based on landmarks [3] but detections and associations of the landmarks are required. Using LRS measurement data, map-matching methods are proposed [4, 5]: the localization is achieved by comparing and matching the known map-data with the measured distance data for each orientation. However, the decrease of LRS measurement data deteriorates the estimation accuracy. To deal with these
problems, extended Kalman filter (EKF) and particle filter (PF) are applied to improve the estimation accuracy and suppress the effect of both cumulative errors and observation-noises by integrating multiple sensors \([6, 7, 8]\).

Moving horizon estimation (MHE) is an alternative method to estimate the state of a dynamical system contaminated by noises. MHE is usually formulated as a real-time optimization problem which becomes feasible by the advances of computational power of CPU and optimization algorithms. The states are estimated using the multiple measurement data from finite time past through current time and the evaluation interval recedes every sampling time. In \([9]\), the superiority of MHE is indicated by comparisons between the estimation accuracies of MHE and EKF. The distinct advantage over EKF or PF is that MHE can naturally include the physical constraints into optimization framework. MHE is applied to localization using a camera system in \([10]\), where the effects of the outliers are suppressed by introduction of constraints. In the estimation methods based on EKF and PF, outliers detection and countermeasures are generally required to maintain the estimation accuracy. In our past study, localization methods integrating the LRS and odometry based on MHE are proposed \([11, 12]\). In \([11]\), motion update equations of the robot are used as constraints to suppress the effects of outliers and missing data of LRS, however, it receives large influence of the error of the odometry. To solve this problem, the equations are considered in the index function: the evaluation weightings for the LRS and odometry are varied adapting to the number of the LRS measurement data \([12]\). As a result, the localization is achieved even in the absence of the LRS measurement data by rating the odometry information highly.

However, in \([12]\), the effectiveness of the method is verified through the numerical simulations only. Moreover, the verifications are not conducted in singular environments where the self-pose cannot be determined uniquely using LRS measurement data only. Since several environments like corridors can be singular environments, the feasibility of the localization method in the singular environments has to be verified. So in this study, we verify the effectiveness of the localization method based on MHE through the experiments conducted in a corridor as a singular environment. In addition, superiority of the method is indicated through comparisons with EKF.

2. Localization based on odometry [11]

In this paper, we treat the front steering vehicle as a single track model illustrated in Fig. 1. The vehicle model is represented by

\[
\frac{dx}{dt} = V \cos(\theta), \quad (1)
\]

\[
\frac{dy}{dt} = V \sin(\theta), \quad (2)
\]

\[
\frac{d\theta}{dt} = \frac{V}{L} \tan(\delta), \quad (3)
\]

where the state vector \(\mathbf{x} := [x, y, \theta]^T\) are the rear wheel position and heading angle, \(V\) is the vehicle velocity, \(\delta\) is the front steering angle and \(L\) is the wheel base length. Using discretized equations of (1)(2)(3), the vehicle state \(\mathbf{x}_k\) at the current time \(k\) is estimated based on odometry as follows:

\[\mathbf{x}_k = f(\mathbf{x}_{k-1}, \delta_{k-1}, V_{k-1}), \quad (4)\]

where

\[f(\mathbf{x}_{k-1}, \delta_{k-1}, V_{k-1}) = \mathbf{x}_{k-1} + \begin{pmatrix} V_{k-1} \cos(\theta_{k-1}) \\ V_{k-1} \sin(\theta_{k-1}) \\ \frac{V_{k-1}}{L} \tan(\delta_{k-1}) \end{pmatrix} \Delta t, \quad (5)\]
and $\Delta t$ is the time interval. It is noted that repetitive integration of this motion model results in cumulative errors.

![Figure 1. Front steering vehicle as a single track model.](image)

3. Localization based on LRS [6]
In this section, we express a localization method based on map-matching using 2-D LRS measurement data, where this method is based on the assumptions that the rough initial pose of the robot and the map-data of the environment are known. Figure 2 depicts the outline of the map-matching method using the LRS measurement data: the self-pose is estimated by minimizing errors between the measured distance $r_j$ of each bearing $\varphi_j$ ($j=0,1,\cdots,n$) and theoretical distance $\hat{r}_j(x_k, y_k, \theta_k, \varphi_j)$ calculated using the map-data. The optimization problem of the localization method by the map-matching is as follows:

**Localization based on LRF:**

Minimize

$$J(x_k) = \frac{1}{N_k} \sum_{j \in C_k} \left( \hat{r}_j(x_k, y_k, \theta_k, \varphi_j) - r_j \right)^2,$$

with

$$N_k = n(C_k),$$

$$C_k = \{ j \mid |\hat{r}_j(x_k, y_k, \theta_k, \varphi_j) - r_j| < \epsilon \},$$

for $x_k = (x_k, y_k, \theta_k)$,

where $N_k$ is the number of available measurement data reflected from known objects included in the map-data. To estimate the accurate pose, unavailable data resulting from outliers or unknown obstacles are rejected using threshold $\epsilon$.

It is noted that the map-matching method is not feasible when the self-pose cannot be determined uniquely. A trivial example is when there is no available data. Practical problem is that one straight wall or two parallel straight walls are only detected as illustrated in Fig. 3.

4. Localization method based on MHE

4.1. Localization method based on MHE [12]
In this section, we explain a proposed localization method integrating the LRS and odometry based on moving horizon estimation (MHE). Figure 4 depicts the outline of MHE where $T$
Figure 2. Localization based on map-matching using LRS measurement data.

Figure 3. The singular environment in which the map-matching fails such as long corridors.

is current time and $H$ is the number of horizon: the self-pose is estimated using the multiple sampling measurement data from $T - H$ through $T$. The optimization problem of the localization method based on MHE is represented as the following problem.

**Localization based on MHE:**

Minimize

$$J(\hat{x}_k) = \sum_{k=T-H}^{T} \frac{1}{N_{typ}} \sum_{j \in C_k} \left( \frac{\hat{r}_{k,j}(\hat{x}_k, \varphi_{k,j}) - r_{k,j}}{\sigma^2} \right)^2 + \sum_{k=T-H}^{T-1} \| \hat{x}_{k+1} - f(\hat{x}_k, \delta_k, V_k) \|_Q^2 + \| \hat{x}_{T-H} - \hat{x}_{T-H} \|_P^2,$$

for $\hat{x}_k = (\hat{x}_k, \hat{y}_k, \hat{\theta}_k)$ \hspace{0.5cm} $(k = T - H, \ldots, T)$,

where $\sigma^2$ is typical value of the variance of LRS measurement distance, $Q$ and $P$ are the weighting matrices for the covariances of system-noise and error between estimated values at current and last sampling time, $\hat{x}_{T-H}$ is estimated value at $k = T - H + 1$ of the last sampling time, $N_{typ}$ is a typical number of available LRS measurement data. It is also noted that the evaluations of each term are normalized by their variances or covariances. It is noted that $N_{typ}$ is introduced to normalize the relative weightings for the map-matching and odometry depending on the number of available data: the map-matching is rated high when the number of available data is more than $N_{typ}$. When there is no available LRS measurement data, the self-pose can be estimated from the odometry information. Thus, localization suppressing the effects of noises and missing LRS measurement data is achieved using the multiple sampling measurement data while considering the number of available LRS data.

In the second term of the index function, the motion update equations (5) are approximated to first-order through a Taylor expansion around the each estimated values at last sampling time $\hat{x}_k$ ($k = T - H, \ldots, T - 1$) as follows:

$$f(x_k, \delta_k, V_k) \approx f(\hat{x}_k, \delta_k, V_k) + \frac{\partial f}{\partial x_k}_{x_k = \hat{x}_k} (x_k - \hat{x}_k).$$

(9)
Moreover, the index function (8) is minimized using sequential quadratic programming (SQP), where the second-order Taylor expansion is used for the first term of the index function around the each estimated values at last sampling time. In this study, CVXGEN [13] is used for solving the optimization problem of MHE, which generates a custom code computing the optimal solution fast.

4.2. Localization based on MHE in singular environments
In this part, we discuss the challenge of the localization in singular environments where the state of the robot cannot be determined uniquely by the map-matching. Estimating the pose by only the odometry information is one way for achieving the localization in such environments, however, the influence of the cumulative errors largely affect the estimation accuracies to all of \( x, y \) and \( \theta \). On the other hand, in the localization method based on MHE, by simultaneously optimizing errors of both the equations of motion (5) and map-matching (6) within an index function (8), the information for estimating the state which cannot be inferred by the map-matching is complemented by the odometry information. As a typical situation indoor, for the case illustrated in Fig. 3, \( y \) and \( \theta \) are estimated using information for both upper and lower walls, while \( x \) can be estimated based on the odometry which cause cumulative errors for \( x \) direction. Moreover, since the motion update equation of \( x \) is a function with respect to \( \theta \), the influence of the estimation error \( \theta \) on \( x \) is suppressed. In the following sections, the advantage of the localization method based on MHE is proved through the simulation and experiment.

5. Numerical simulations
5.1. Simulation set-up
In this section, we verify the effectiveness of MHE by comparing with EKF [6] through numerical simulations. The simulation environment is shown in Fig. 5: a corridor inside the building is used for a singular environment. In this environment, LRS measurement data for estimating \( x \) ...
or y are measured from walls indicated as $W_x$ or $W_y$, respectively. To conduct the simulations, we prepare the data of LRS measurement, vehicle velocity and input steering angle obtained from the path-tracking simulation result, the map-data illustrated in Fig. 5 and the specification of LRS shown in Fig. 6 and Table 1. White Gaussian noise with the mean value 0 and the variance $\sigma^2 = 1.92 \times 10^{-5}$, respectively, is appended to the prepared LRS measurement data as the observation-noise and the value $\sigma^2$ is also used in the index function (8), where the variance $\sigma^2$ is calculated from the actual measured value. As the system-noise, white Gaussian noise that the mean and the covariance are 0 and diag$(3.0 \times 10^{-6}, 3.0 \times 10^{-6}, 7.8 \times 10^{-7})$ is added to the motion update equation (4): the covariance is decided based on the variance of the measured vehicle velocity. The simulation parameters are shown in Table 2: the weighting matrices $Q$ and $P$ are the tuning parameters, $N_{\text{typ}}$ is decided based on the average value of the number of LRS measurement data for estimating $x$. In this paper, we compare the proposed method with a localization method integrating the LRS and the odometry based on the EKF as a conventional method. In this method, the localization is conducted based on the odometry when neither $W_x$ nor $W_y$, or neither of them can be observed, since the result of the map-matching is used as the observation value of the EKF [6].

### Table 1. Specifications of UBG-04LX-F01.

| Element                  | Parameter                                      |
|--------------------------|------------------------------------------------|
| Manufacture              | HOKUYO AUTOMATIC CO., LTD.                     |
| Model                    | UBG-04LX-F01                                   |
| Measuring distance       | 20 to 5600 mm                                  |
| Measurement accuracy     | 0.06 to 1 m : $\pm$ 10 mm, 1 to 5.6 m : 1 % of distance |
| Scanning angle           | 240 deg                                       |
| Angular resolution       | 0.36 deg(360 deg/1024)                         |
| Scanning time            | 28 ms/scan                                    |

### Table 2. Simulation condition.

| Element                  | Parameter                                      |
|--------------------------|------------------------------------------------|
| Initial state $(x, y, \theta)$ | (0.55, 1.0, 0.0)                              |
| Vehicle Velocity $V$     | 0.3 m/s                                       |
| Wheel base $L$           | 0.256 m                                       |
| Threshold value $\epsilon$ | 0.056 m                                    |
| Horizon $H$              | 3                                              |
| Weight $\sigma^2$       | $1.92 \times 10^{-5}$                         |
| Weighting matrix $Q$     | diag$(3.2 \times 10^{-4}, 3.2 \times 10^{-4}, 8.0 \times 10^{-5})$ |
| Weighting matrix $P$     | diag$(1.3 \times 10^{-4}, 1.3 \times 10^{-4}, 7.1 \times 10^{-5})$ |
| Typical measurement number $N_{\text{typ}}$ | 5                                             |
Figure 7 and 8 depict the simulation results of both methods of proposed and conventional, respectively. Figure 7(a) and 8(a) depict the number of available LRS measurement data, and they indicate that the map-matching fails in 10.1–12.1 s and 13.6–20.0 s. The estimated vehicle trajectories are depicted in Fig. 7(b) and 8(b), which indicate that the localizations are achieved by both methods even in the singular environment as confirmed from Fig. 7(c) and 8(c). Figure 7(d) and 8(d) depicts the estimation errors of MHE and EKF, respectively: in MHE, the error of only location $x$ is accumulated during the failure of map-matching. On the other hand, in
the conventional method, since the localization conducted using only the odometry information, the estimation accuracies of all of vehicle states \( x, y \) and \( \theta \) are deteriorated in those intervals. Figure 9 depicts the root mean square errors of both methods of the first half \( 0.0 \sim 10.0 \) s and the latter half \( 10.0 \sim 20.0 \) s: in Fig. 9(a), the errors of \( x \) and \( y \) are in the order of \( 10^{-3} \) m and the error of \( \theta \) is in the order of \( 10^{-4} \) rad. In Fig. 9(b), they are in the order of \( 10^{-2} \) m and \( 10^{-3} \) rad, respectively. From Figure 9(a), both estimation accuracies are in the same range in the first half because the LRS available measurement data for the map-matching are measured sufficiently. However, in the later half depicted in Fig. 9(b), since the proposed method can estimate \( y \) and \( \theta \) using both the odometry and LRS measurement, the accuracy of the proposed method is high comparing with the conventional method for \( y \) and \( \theta \). As shown above, in the proposed method, by using both information of the odometry and LRS measurement data effectively, the effect of the cumulative errors are suppressed even in the singular environment where the self-pose cannot be determined uniquely on a certain direction from the LRS measurement data.

![Figure 9](image.png)

**Figure 9.** Root mean square errors of localization.

6. Experiments
6.1. Experimental set-up
In this paper, we use the RoboCar of ZMP Inc. as an experimental vehicle depicted in Fig. 10 with its specification shown in Table 3. It is a one-tenth scale and front steering/rear driving vehicle equipped with the UBG-04LX-F01 of HOKUYO Inc. as a LRS and the vehicle velocity \( V \) is measured by rotary encoders. Figure 11 depicts the block diagram of the experimental system: for the localization based on MHE, the multiple samplings of LRS measurement data, vehicle velocity \( V \) measured by the rotary encoders and input steering angle \( \delta \) are used.

To verify the effectiveness of the proposed localization method, we conduct the path-tracking control in a singular environment depicted in Fig. 12 which is same one used in the simulations. The map-data of the experimental environment is illustrated in Fig. 13 where the black and green lines indicate the known and unknown walls, respectively. The localization is conducted using the known walls only and the unknown walls are not used for it. It is noted that this environment becomes singular again for \( x \) direction. The initial state \((x, y, \theta)\) is \((1.0, 1.0, 0.0)\) and the parameters are same as the simulation. To confirm the superiority of the proposed method, we compare the results of MHE with that of EKF [6] at off-line.
Table 3. Specifications of ZMP RoboCar.

| Name/Number      | RoboCar/ZMP RC-Z |
|------------------|------------------|
| Size/Weight      | 429.0 × 195.0 × 212.2 mm, 3kg |
| Wheel base L     | 256.0 mm         |
| Main controller  | CPU: AMD Geode LX800 (Processor: 500MHz), OS: Linux(Fedora10) |
| Internal sensor  | Rotary encoders (Wheel×4, DC motor×1) |
| External sensor  | UBG-04LX-F01     |

Figure 10. RoboCar of ZMP Inc.

Figure 11. Block diagram of experimental system.

Figure 12. Experimental environment: corridor.

Figure 13. Map data of experimental environment.

6.2. Experimental results

Figure 14(a) depicts the number of available LRS measurement data where it indicates the environment is singular since the LRS measurement data for estimating \( x \) cannot be measured in \( t = 8.84 \sim 10.87\) s and \( t = 12.5 \sim 17.7\) s. However, from Fig. 14(b), we can confirm that the path-tracking control is achieved even in the environment while estimating self-pose by the proposed method. It can also be depicted in Fig. 14(c) that the estimated vehicle state \((\hat{x}, \hat{y}, \hat{\theta})\) follow the reference values. Figure 14(d) depicts the computational time of the real-time task and localization including optimization of MHE. It indicates that the real-time localization and control are achieved. The measured surrounding shapes in \( x-y \) plane at \( t = 0.0, 4.0, 8.0, 12.0 \) and \( 17.7\) s and enlarged view at \( t = 17.7\) s are depicted in Fig. 17: red x-marks are the measurement points of LRS calculated from the orientation and distance data of LRS and the estimated vehicle states. From these figures, since the configurations shaped by the marks correspond to the actual map-data, we can confirm the localization is achieved. As shown above, the feasibility of the proposed localization method based on MHE is verified through the experiment which is conducted in the singular environment.
Available data number of LRS.

(a) Available data number of LRS.

(b) Trajectory.

(c) States.

(d) Computational time.

**Figure 14.** Experimental result of MHE.

Trajectory.

States.

**Figure 15.** Off-line localization result of EKF.

Trajectory.

States.

**Figure 16.** Off-line localization result of EKF (enlarged view around $t = 10.87\text{s}$).
Figure 17. LRS measurements of MHE.

Figure 18. LRS measurements of EKF.
Next, Fig. 15 depicts the off-line localization results of the conventional method [6], where (a) and (b) depict the estimated vehicle trajectory and state, respectively, and its enlarged views at \( t = 10.87 \) are shown in Fig. 16. From the results, the localization is achieved even in the case of the conventional method; however, from Fig. 16(a) and (b), the estimated states change discontinuously around \( t = 10.87 \):s. This reason is the correction of the cumulative errors caused by the observation of wall \( W_{x3} \) for estimating \( x \) at that time as shown in Fig. 14(a). Figure 18 shows the measured surrounding shapes at \( t = 0.0, 4.0, 8.0, 12.0 \) and \( 17.7 \)s and enlarged view at \( t = 17.7 \)s: red x-marks are calculated using the vehicle states estimated by the conventional method. In the first half \( (a) \sim (c) \), the shapes are almost the same with map-data and results of MHE depicted in Fig. 17. However, in the later half \( (d) \sim (f) \), particularly in the \( (f) \), the shapes do not correspond to the map-data which indicates that larger errors appear in the EKF localization. Since the all of vehicle state are estimated by odometry, not only \( x \) which cannot be determined uniquely but also \( y \) and \( \theta \) are affected by the cumulative errors as with the simulation. From above results, the superiority of MHE is proved comparing with EKF through the experiment.

7. Conclusion
In this paper, we conduct the experimental verification of a localization method integrating LRS and odometry based on MHE. In addition, we verify the effectiveness of the proposed method through experiments conducted in a corridor as a singular environment. As a result, by evaluating the errors of map-matching and motion update equations simultaneously, the real-time localization is achieved suppressing the cumulative error of the odometry even in the singular environment. Moreover, the accurate localization is achieved comparing with EKF by using both information of odometry and LRS, effectively.

References
[1] S. Thrun, W. Burgard and D. Fox, Probabilistic Robotics, 672, The MIT Press (2005)
[2] K. S. Chong and L. Kleeman, Accurate odometry and error modelling for a mobile robot, Proc. of IEEE Robotics and Automation, 2783-2788 (1997).
[3] M. Betke and L. Gurvits, Mobile Robot Localization Using Landmarks, Trans. on Robotics and Automation, 13-2, 251-263 (1997).
[4] J.-S. Gutmann and C. Schlegel: AMOS, Comparison of Scan Matching Approaches for Self-Localization in Indoor Environments, Proc. of Euromicro Workshop on Advanced Mobile Robots, 61-67 (1996).
[5] L. Teslic, I.Skrjanc and G. Klancar, Using a LRF sensor in the Kalman-filtering-based localization of a mobile robot, ISA Transactions, 49-1, 145-153 (2010).
[6] Y. Hiromachi, K. Nonaka and K. Sekiguchi, EKF Localization with Variable Covariance for LRS and Odometry: Experimental Verification, Advanced Motion Control 2014, 231-236 (2014).
[7] S. Jain, S. Nandy, R. Ray and S. N. Shome, Application of Particle Filtering Technique for sensor fusion in mobile robotics, Proc. of IEEE Mechatronics and Automation, 2285-2290 (2011).
[8] F. Chausse, S. M. Bake, S. Bonnet, R. Chapuis and J. P. Derutin, Experimental comparison of EKF and Constraint Manifold Particle Filter for robot localization, Proc. of IEEE Multisensor Fusion and Integration for Intelligent Systems, 399-404 (2008).
[9] E. L. Haseltine and J. B. Rawlings, Critical Evaluation of Extended Kalman Filtering and Moving-Horizon Estimation, Proc. of Industrial and Engineering Chemistry Research, 44-8, 2451-2460 (2005).
[10] M. Takahashi, K. Nonaka and K. Sekiguchi, Vehicle State Estimation by Moving Horizon Estimation Considering Occlusion and Outlier on 3D Static Cameras, Multi-Conference on Systems and Control 2015, 1211-1216 (2015).
[11] K. Kimura, Y. Hiromachi, K. Nonaka and K. Sekiguchi, Vehicle Localization by Sensor Fusion of LRS Measurement and Odometry Information based on Moving Horizon Estimation, Multi-Conference on Systems and Control 2014, 1306-1311 (2014).
[12] K. Kimura, K. Nonaka and K. Sekiguchi, MHE Localization Using LRF Measurement and Nonlinear Vehicle Dynamics, Proc. of Multi-symposium on Control Systems 2015, 662-4 (2015) (In Japanese)
[13] J. Mattingley and S. Boyd, CVXGEN: A Code Generator for Embedded Convex Optimization, Optimization and Engineering, 13-1, 1-27 (2012).