The medium modification of dihadron fragmentation functions in the deeply inelastic scattering (DIS) off a large nucleus is studied within the framework of the higher-twist expansion in the collinear factorization formalism. It is demonstrated that the modification due to multiple parton scattering in the nuclear medium is similar to that of the single hadron fragmentation function. However, the conditional distribution of the associated hadron given by the ratio of dihadron to single hadron fragmentation function shows only slight modification. The final results depend modestly on the nuclear density distribution. Comparisons with the experimental results on two hadron correlations as obtained by the HERMES collaboration at DESY are presented.

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I. INTRODUCTION

The modification of the properties of jets as they pass through dense matter has emerged as a new diagnostic tool in the study of the partonic structure of such an environment. Such modification goes beyond a mere suppression of the leading hadron’s multiplicity and could in principle be extended to include many particle observables such as dihadron fragmentation functions. Defined as the two-hadron expectation values of partonic operators, these functions may be factorized from the hard collisions and their evolution with the energy scale may be systematically studied in perturbative Quantum Chromodynamics (pQCD). As the single hadron fragmentation, dihadron fragmentation functions in vacuum can be defined independently of the jet production processes. Once measured experimentally as semi-inclusive distributions of di-hadrons from one hard process, they can be used to predict dihadron distributions in other hard processes. The medium modification of the dihadron fragmentation functions in the multiple parton scattering picture will be dictated by jet-medium interaction as in the single hadron modification and therefore will provide an independent constraint on the jet transport parameters which in turn can provide important information on the properties of the dense matter.

In both high-energy heavy-ion collisions and DIS off nuclei, two-hadron correlations have been measured in addition to the single hadron spectra. Two-hadron correlations are measured as the conditional probability distributions of a secondary hadron associated with a triggered hadron with given momentum. Therefore, they are given by the ratio of dihadron to single hadron fragmentation functions, while the single inclusive spectra are related to single hadron fragmentation function. The study of the medium modification of single and dihadron fragmentation functions in heavy-ion collisions involves a somewhat different kinematics than that in the DIS experiments and will be discussed separately. In this paper, the theoretical description of the dihadron correlations observed in DIS experiments off large nuclei will be presented.

In conformity with the conventional picture of DIS on nucleons, the general process may be sketched as in Fig. 2. One of the quarks in the incoming nucleus receives a hard momentum transfer from the virtual photon. The struck quark then travels the remaining length of the nucleus while undergoing multiple scattering off soft gluons from various nucleons in its path. It escapes the nuclear environment and after some time, characterized by the confinement scale and dilated by its boost, undergoes the fragmentation process into a shower of hadrons. The presence of the hard scale ($Q^2$), provided by the momentum transfer between the lepton and the nucleus, allows for a factorized approach to the process of DIS off a nucleus, where the hard partonic parts can be calculated reliably within perturbative Quantum Chromodynamics (pQCD). At leading order, the differential hadronic cross section is estimated as a convolution of the quark distribution function in a nucleus with a leading order partonic cross section and the requisite fragmentation functions. The presence of the medium will be felt by the propagating quark through multiple final state scattering at next-to-leading twist level and the effects are in general suppressed by the hard scale $Q^2$. While the final state interaction may be ignored in DIS off a nucleon, they play an important role in the DIS off a large nucleus where they are enhanced by the nuclear size [$A^{1/3}$]. This is due to the fact that the subsequent scattering with the gluon field (confined in nucleons) may occur at any of the nucleons which lie along the path of the struck quark. Such medium effects can be factorised from the leading order hard cross section and manifest themselves as medium modified structure and fragmentation functions with the leading order (LO) hard partonic part remains unchanged.
In this paper, we will focus our attention on the medium modification of the fragmentation functions. The nuclear modification of the quark distribution functions will be included parametrically. We will work within the formalism of higher-twist expansion in Ref. [23] to calculate the nuclear enhanced power corrections to the semi-inclusive cross section of DIS off a large nucleus. The modification of the single fragmentation functions within this formalism has been studied in Ref. [4]. In Sec. 2 we give a brief review of the formalism and isolate the contributions generic to the modification of the dihadron fragmentation functions. The equations governing the medium modification of the dihadron fragmentation functions will be demonstrated to assume a form similar to that of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [24, 25, 26] of the dihadron fragmentation functions in the vacuum [11, 12]. This is quite similar to the case of the modification of the single fragmentation functions. The additional term as compared to the DGLAP evolution equation will be a twist-four matrix element identical to the case of the single fragmentation functions. This matrix element will have to be evaluated in a nucleus of size \( A \), and depends on the nuclear density distribution. In Sec. 3 the general methodology of the evaluation of this matrix element is outlined and its dependence on the nuclear density distribution is discussed. Numerical results for the medium modification will be presented in Sec. 4 for two different nuclear density distributions and are compared with the experimental data from the HERMES experiment. We summarize and present our conclusions in Sec. 5.

II. FORMALISM

The focus of this article is restricted to the semi-inclusive process of DIS off a nucleus where hadrons are detected in the final state. In particular, we will consider the following process,

\[
e(L_1) + A(p) \rightarrow e(L_2) + h_1(p_1) + h_2(p_2) + X,
\]

when at least two hadrons are detected in the final state, where \( L_1 \) and \( L_2 \) represent the momentum of the incoming and outgoing leptons. The incoming nucleus of atomic mass \( A \) is endowed with a momentum \( Ap \). In the final state, two hadrons \( (h_1, h_2) \) with momenta \( p_1, p_2 \) are detected.

The kinematics is defined in a frame as sketched in Fig. 1. In such a frame, the virtual photon \( \gamma^* \) and the nucleus have momentum four vectors \( q, P_A \) given as,

\[
q = L_2 - L_1 = \left[ -\frac{Q^2}{2q^+}, q^-, 0, 0 \right], \quad P_A = A[p^+, 0, 0, 0],
\]

where we continue with the notation of Ref. [11] of denoting four vectors in bold face. In this frame, the Bjorken variable is defined as \( x_B = Q^2/2p^+q^- \).

\[\text{FIG. 1: The lorentz frame of the process where a nucleon in a large nucleus is struck by a hard space-like photon.}\]

The differential cross section of the semi-inclusive process with two detected hadrons may be expressed as

\[
\frac{E_{L_2}E_{p_1}E_{p_2}d\sigma_{h_1,h_2}}{d^3L_2d^3p_1d^3p_2} = \frac{\alpha^2_{EM}}{2\pi s} \frac{E_{p_1}E_{p_2}}{4Q^4} L_{\mu\nu} dW^{\mu\nu},
\]

where \( s = (p + L_1)^2 \) is the total invariant mass of the lepton nucleon system and the leptonic tensor is

\[
L_{\mu\nu} = \text{Tr}[L_1\gamma_\mu L_2\gamma_\nu].
\]
In the notation used in this paper, \(|A,p⟩\) represents the initial state of an incoming nucleus with \(A\) nucleons each carrying momentum \(p\). The hadronic part of the final state with two detected hadrons is represented as \(|X,h_1,h_2⟩\). As a result The semi-inclusive hadronic tensor may be defined as

\[
E_{p_1}E_{p_2}\frac{dW^{\mu\nu}}{d^3p_1d^3p_2} = \sum_X 2\pi \delta(q + P_A - P_X - p_1 - p_2) \\
\times \langle A,p | J^\mu(0) | X,h_1,h_2⟩ \langle X,h_1,h_2 | J^\nu(0) | A,p⟩,
\]

where the sum \((\sum_X)\) runs over all possible hadronic states and \(J^\mu\) is the hadronic electromagnetic current \((J^\mu = e_q\bar{\psi}_qγ^\mu\psi_q)\). It is understood that the quark wavefunctions are written in the Heisenberg picture. The leptonic tensor will not be discussed further. The focus in the remaining of this paper will be exclusively on the hadronic tensor. This tensor will be expanded order by order at the leading log. The leading and next-to-leading twist contributions that are nuclear enhanced will be isolated.

A. Leading twist

We start by noting that within the kinematics chosen there are two sets of final states in the opposite directions: one set \(|S_1⟩\) is in the direction of the nuclear beam and consists mostly of its remnants, while the other \(|S_2⟩\) is in the direction of the hard photon and consists of a jet of hadrons. At leading twist, there is minimal overlap between these two sets and this leads to a simplified expression for the integrated hadronic tensor,

\[
W^{\mu\nu} = \frac{1}{2} \sum_{\lambda_p} \sum_{h_1,h_2} \int \frac{d^3p_1}{(2\pi)^32E_1} \frac{d^3p_2}{(2\pi)^32E_2} \sum_{S_1,S_2} (2\pi)^4\delta^4(q + P_A - p_{S_1} - p_{S_2} - p_1 - p_2) \\
\times \langle p\lambda_p | \bar{\psi}(0) | S_1⟩ γ^\mu(0) | ψ(0)⟩ \langle S_2p_1p_2 | \bar{\psi}(0) | 0⟩ γ^\nu ⟨S_1 | ψ(0) | p\lambda_p⟩.
\]

In the above equation, we include an average over the initial spin states \(\lambda_p\) of the incoming nucleus, in the following this will be implied. We have also simplified \(|A,p⟩\) as simply \(|p⟩\); the presence of \(A\) should always be understood. The above equation is restricted to the case of a quark inside the target undergoing a hard collision with the virtual photon. The case where an antiquark is struck is analogous and will not be discussed. The above being the leading order term in the usual expansion of the quark wavefunction operators in the interaction picture. In the rest of this section, the factorization of the dihadron fragmentation function from the hard part and the structure function will be discussed. It will be demonstrated that no new assumptions need to be invoked. The short hand notation

\[
\sum_{h_1,h_2}^f = \sum_{h_1,h_2} \int \frac{d^3p_1}{(2\pi)^32E_1} \frac{d^3p_2}{(2\pi)^32E_2}
\]

will be used henceforth. As in Ref. \[11\] we use the notation that \(p_1 + p_2 = p_0\).

The \(δ\)-function can be espresed as an integral over four space-time, where phase factors involving the proton’s momentum and that of its remnants may be absorbed into the matrix elements of the quark wavefunction operator on the proton state. Under a reorganization of the spinor matrix elements and introducing the momentum fractions \(ζ\), one obtains the hadronic tensor as

\[
W^{\mu\nu} = \sum_{h_1,h_2}^f \int d^4xe^{i\zeta(\mathbf{a} - p_{S_2} - p_n)} \int d^4k dζ δ^4(p_{S_2} + p_n - k - q) δ \left(ζ - \frac{k^+}{p^+}\right) \\
\times \text{Tr} \left[ \sum_{S_1} ⟨S_1 | ψ(0) | p⟩ ⟨p | \bar{ψ}(x) | S_1⟩ γ^\mu(0) | ψ(0)⟩ \langle S_2p_1p_2 | \bar{ψ}(0) | 0⟩ γ^\nu ⟨S_1 | ψ(0) | p⟩ \right]
\]

In the above, the four-vector \(k\) has been introduced which represents the momentum of the struck quark inside the nucleus. Using the four \(δ\)-function, one may re-express \(\exp[i\zeta \cdot (\mathbf{q} - p_{S_2} - p_n)]\) as simply \(\exp[-i\zeta \cdot \mathbf{k}]\). Taking the Fourier transform of the \(δ\)-function over \(k\), leads to the expression,

\[
W^{\mu\nu} = \sum_{h_1,h_2}^f \int dζ \int \frac{d^4k}{(2\pi)^4} d^4xe^{i\zeta\cdot\mathbf{k}} δ \left(ζ - \frac{k^+}{p^+}\right) \\
\times \text{Tr} \left[ ⟨S_1 | ψ(0) | p⟩ ⟨p | \bar{ψ}(x) | S_1⟩ γ^\mu \int d^4ye^{-iy·ζ(\mathbf{p}+\mathbf{q})} ⟨0 | ψ(0)⟩ \langle S_2p_1p_2 | \bar{ψ}(0) | 0⟩ γ^\nu⟩ \rangle \right]
\]
Invoking the collinear approximation that the (+)-components of the target momenta $p^+, k^+$ are much larger than their ($\perp$)-components and (−)-components, one may approximate the leading twist contribution of the $x$ dependent matrix element as

$$
\hat{T}(\zeta, p) = \int \frac{d^4k}{(2\pi)^4} d^4x e^{-i k \cdot x} \delta(\zeta - \frac{k^+}{p^+}) \sum_{S_i} \langle S_i | \bar{\psi}(0) | p \rangle \langle p | \bar{\psi}(x) | S_i \rangle \simeq \frac{\not{p}}{2} f(\zeta).
$$

(8)

Where, $f(\zeta)$ is identified as the unpolarized quark distribution function in a nucleus, which now has the obvious expression,

$$
f(\zeta) = \int \frac{d^4k}{(2\pi)^4} d^4x e^{-i k \cdot x} \delta(\zeta - \frac{k^+}{p^+}) |p | \bar{\psi}(x) \gamma^+ \psi(0) | p \rangle.
$$

(9)

Thus, the initial quark distribution function may be factorized from the hard cross section and the final fragmentation into hadrons under the collinear approximation. Factorization of the dihadron fragmentation function from the hard part requires the use of the collinear approximation in the opposite direction of the produced hadronic jet. One introduces the quark momentum in the direction of the hadronic jet as

$$
\int d^4l \delta^4(1 - (\zeta p + q)),
$$

as well as the momentum fractions of the two detected hadrons as

$$
\int dz_1 dz_2 \delta (z_1 - \frac{p_1^+}{l^+}) \delta (z_2 - \frac{p_2^+}{l^+})
$$

We continue with the notation of Ref. [11] where $z = z_1 + z_2$. As a result, the simplified expression for the differential hadronic tensor for the production of two hadrons with momentum fractions $z_1, z_2$ in the direction of the quark jet is obtained as

$$
\frac{W^{\mu\nu}}{dz_1 dz_2} = \int \frac{d\zeta f(\zeta)}{2\zeta} \sum_{h_1, h_2} (2\pi)^4 \delta^4(\frac{p_{h_1}}{z} - q - \zeta p) \int \frac{d^4l}{(2\pi)^4} d^4y e^{-iy \cdot l} \delta (z_1 - \frac{p_1^+}{l^+}) (z_2 - \frac{p_2^+}{l^+}) \times \text{Tr} \left[ \gamma^\mu \left( \frac{p_{h_1}}{z} - \gamma^5 \right) \gamma^\nu (0) | \bar{\psi}(0) \rangle \langle S_2 p_1 p_2 | S_2 p_1 p_2 | \bar{\psi}(y) | 0 \right].
$$

(10)

**FIG. 2:** The Lowest order contribution to $W^{\mu\nu}$.

The double integral over the momenta of the detected hadrons may be re-expressed as an integral over the sum of the two momenta and the difference. The functions $\delta(p^+ - q^+ - \zeta p^+) \delta^2(\not{p}_{h,\perp})$ may be invoked to restrict the sum of the momenta of the detected hadrons. While in the limit afforded by the collinear approximation, the $\delta$-function over the (+)-components of the momenta yields,

$$
\delta \left( \frac{p_{h_1}^+}{z} - q^+ - \zeta p^+ \right) \simeq \delta \left( \frac{Q^2}{2q^-} - \zeta p^+ \right) \simeq \frac{\delta(\zeta - x_B)}{p^+}.
$$


In the above expression, $x_B$ is the Bjorken scaling variable. The two $\delta$-functions over the two momentum fractions $z_1, z_2$ may be readjusted as demonstrated in Ref. [11] which leads to the final factorized expression for the differential hadronic tensor as,

$$\frac{W^{\mu\nu}_{12}}{dz_1dz_2} = \int dx_B f(x_B) H^{(0)\mu\nu}_q D_{q_1, q_2}(z_1, z_2), \quad (11)$$

where, $H^{(0)\mu\nu}$ is the hard part of the quark-photon scattering $i.e.$,

$$H^{(0)\mu\nu} = \pi\delta[(q + x_Bp)^2] \text{Tr} \{ \gamma^\nu \not p \gamma^\mu (\not q - x_B \not p) \}, \quad (12)$$

and $D_{q_1, q_2}(z_1, z_2)$ is the dihadron fragmentation function,

$$D_{q_1, q_2}(z_1, z_2) = \frac{z_1^4}{4z_1z_2} \int \frac{d^2q_1}{4(2\pi)^2} \int \frac{d^4l}{(2\pi)^4} \delta \left( z - \frac{p_{\perp}}{l} \right) \text{Tr} \left[ \frac{\gamma^-}{2p_{\perp}} \right] \int d^4xe^{i\mathbf{p}\cdot\mathbf{x}} \sum_{S=2} \langle 0| \psi_q(x) | p_1, p_2, S - 2 \rangle \langle p_1, p_2, S - 2 | \bar{\psi}_q(0) | 0 \rangle. \quad (13)$$

In the above expression, $q_{\perp} = p_{1\perp} - p_{2\perp}$. This expression is identical to that presented in Ref. [11] with the obvious switch in the direction of the final momentum from the (+) direction to the (−) direction. Also, Eq. (12) may be directly generalized from the expression for single inclusive hadron production presented in Ref. [4]. The resulting picture in terms of Feynman diagrams is thus that of Fig. [2]. Hence, it may be stated that the factorization of the dihadron fragmentation from the LO diagram in DIS may be easily generalized from the factorization of the dihadron fragmentation function in $e^+e^-$ annihilation.

The method of factorization of the dihadron fragmentation function followed here may be easily repeated in the case at next-to-leading order to which we now generalize. The case at higher twist will be discussed in the next subsection. At next-leading-order in the strong coupling constant $\alpha_s$, the dominant contribution at the leading log level (in $n \cdot A = 0$ gauge) comes from final state gluon radiation as shown in Fig. [3]. Specifically, these large logarithms result as the transverse momentum of the radiated gluon vanishes ($i.e.$, $l_{\perp} \to 0$). In this limit, the two detected hadrons may materialize in the fragmentation of the struck quark or from the radiated gluon. It is also possible that one hadron originates in the fragmentation of the quark while the other originates in the fragmentation of the gluon. In all these cases, the fragmentation functions may be factorised as a convolution with the hard cross section and a set of splitting functions, which encode the probability of a quark radiating a gluon which carries away a certain fraction of its forward momentum.

No doubt, these contributions result in the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution of the dihadron fragmentation functions. These have been discussed at length in Refs. [11, 12] for the case of jets in $e^+e^-$ collisions. Using the methodology in these references and the preceding discussion at leading order, the leading log contributions to the hadronic tensor, as the radiated gluon becomes collinear to the struck quark, of the semi-inclusive DIS process may be expressed as

$$\frac{dW^{(1)}_{\mu\nu}}{dz_1dz_2} = \sum_q \int dx_q f_q(x) H^{(0)\mu\nu}_q \int \frac{d^2q_1}{4(2\pi)^2} \int \frac{d^4l}{(2\pi)^4} \delta \left( z - \frac{p_{\perp}}{l} \right) \text{Tr} \left[ \frac{\gamma^-}{2p_{\perp}} \right] \int d^4xe^{i\mathbf{p}\cdot\mathbf{x}} \sum_{S=2} \langle 0| \psi_q(x) | p_1, p_2, S - 2 \rangle \langle p_1, p_2, S - 2 | \bar{\psi}_q(0) | 0 \rangle.$$

In the above equation, the $H^{(0)}_{\mu\nu}$ represents the leading order hard cross section in Eq. (12). The quantities $P_{q_{\perp}g}(z)$ represent the probabilities that the struck quark radiated a gluon and retained a fraction $z$ of its energy. They have the well known expressions [24, 25, 26].
\begin{equation}
P_{q\rightarrow qg}(z) = C_F \left( \frac{1 + z^2}{1 - z} \right)_+.
\end{equation}

The subscript ‘+’ indicates that the negative contribution from a virtual correction has been added within the splitting function. The splitting function in the second line of Eq. (14), \( \hat{P}_{q\rightarrow qg} \), is identical to the above equation except that it lacks the negative virtual correction. The other splitting function \( P_{q\rightarrow qg}(1 - z) \) and also does not admit a virtual correction.

The gluon dihadron fragmentation function \( D_{g\rightarrow h_1 h_2}(z_1, z_2) \) is obtained as

\begin{equation}
D_g(z_1, z_2) = \frac{z_1^3}{2z_1^2z_2^2} \int \frac{dq^2}{8(2\pi)^2} \int \frac{dl^4}{(2\pi)^4} \delta \left( z_h - \frac{p_h}{l^-} \right) \int dq^4 x e^{i q \cdot x} \sum_{S-2} \langle 0| A_0^a(x)| p_1, p_2, S-2 \rangle \langle p_1, p_2, S-2| A_0^a(0)| 0 \rangle \frac{\delta^{ab} q^{\mu\nu}(l)}{16},
\end{equation}

where \( d^{\mu\nu}(l) \) is the gluon’s polarization tensor in the light-cone gauge and sum over the color indices of the gluon field is implied. The meaning of the various momenta and momentum fractions in Eq. (16) is identical to the case of the quark fragmentation function. The single fragmentation functions \( D_q(z), D_\bar{q}(z) \) have the usual expressions \[13, 27, 28].

The NLO modification at leading twist presented above suggests the formulation of an effective next-leading-order and leading twist hard part. This includes the imaginary part of the photon nucleus forward scattering amplitude at order \( \alpha_s \). In particular, the hard part represents the process of a quark in the nucleus being struck by a hard photon into an off-shell intermediate state which then radiates a gluon,

\begin{equation}
H^{(1)d}_{\mu\nu}(x, p, q, z) = H^{(0)}_{\mu\nu}(x, p, q) \int_0^{\mu^2} \frac{dl^T}{l^T} \frac{\alpha_s}{2\pi} C_F \frac{1 + z^2}{1 - z}.
\end{equation}

This hard part is represented by the diagram in Fig. 3 One can in principle deal solely with this hard part. The requisite double and single fragmentation functions may be convoluted with this hard part as a separate final step. This allows one to focus solely on the derivation of the hard part at the leading twist. Note that this hard part is identical to that derived for the case of the modification of the single fragmentation functions in Ref. \[1]. There remains the question as to whether such a separation may be achieved at higher twist. This is the subject of the next subsection.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{The hard part of the next-to-leading order contribution to \( W^{\mu\nu} \).}
\end{figure}

B. Higher Twist

In the calculation of cross sections which involve a hard scale, it often suffices to simply calculate the leading twist part. Higher twist contributions are suppressed by powers of the hard scale. It was demonstrated in Ref. \[22], that a class of higher twist operators are enhanced by the size of the target, \( A^{1/3} \), where \( A \) is the atomic mass. Specifically these contributions arise from the multiple scattering encountered by the struck quark off the soft gluons.
as it bore through the nucleus. Such contributions are also important in the medium modification of the single hadron fragmentation functions [4]. In this section we isolate the hard part of the diagrams involved in the modification of the dihadron fragmentation functions.

In the previous subsection, we demonstrated the factorization of the fragmentation and parton distribution functions from the hard scattering cross section at leading and next-to-leading order and at leading twist. It was demonstrated that the hard part is the same as in the case of the single fragmentation functions. The focus will now be on a similar proof at next-to-leading twist. At leading order the diagrams to be considered are illustrated those in Fig. 4.

![FIG. 4: The leading order and next to leading twist contribution to $W^{\mu\nu}$.](image)

The reader will immediately note that the procedure involved in the factorization of the dihadron fragmentation function from the hard part is entirely similar to that at leading twist, and also to the case of the single fragmentation functions. The resulting hard part has no radiated gluon and therefore no medium modification of the fragmentation functions. The sole effect of the double scattering indicated by the diagrams in Fig. 4 is the appearance of a net transverse momentum $k_\perp$ in the outgoing jet brought in by the average transverse momentum of the gluons. In the limit of vanishing $k_\perp$, the contribution of the above diagrams is an overall Eikonal phase that will be absorbed into a gauge invariant definition of the quark distribution function.

Medium modifications to the semi-inclusive cross section at higher twist in the limit of very small transverse momentum of the soft gluons in a nucleus emanate specifically at next to leading order. The contributions are illustrated by diagrams in Fig. 5. Both diagrams represent the case where the struck quark undergoes scattering off a soft gluon in the nucleus and also radiates a fraction of its forward momentum into a collinear gluon. Both the quark and gluon then exit the medium and fragment. The diagram on the left corresponds to the case where both detected hadrons emanate from the fragmentation of the quark, while the diagram on the right corresponds to the case where one of the detected hadrons originates in the fragmentation of the quark and the other in the fragmentation of the gluon. In Fig. 5 we show diagrams that arise at next-to-leading order and at next-to-leading twist in the evaluation of the medium modification of the dihadron fragmentation function. The dominant contributions, however, come from gluon-gluon secondary scattering [4].

![FIG. 5: A contribution at next-to-leading order and next-to-leading twist to $W^{\mu\nu}$.](image)
As an example, we focus on the diagram on the right: the case of two independent fragmentations. The factorization of the dihadron fragmentation function from a quark line, as is the case for the diagram on the left in Fig. 5, is a simpler generalization from the case of the single inclusive cross section. Thus, the case of two single fragmentations represents the new addition to the type of diagrams involved in the calculation of the medium modification of the dihadron fragmentation functions. It will be demonstrated that the next-to-leading twist hard part, which arises from the evaluation of this contribution, is no different than the hard part, at next-to-leading twist, in the case of the single fragmentation functions. The difference lies in the convolutions involved: besides the usual convolution with the quark and gluon dihadron fragmentation functions, one also needs to convolute with a product of two single fragmentation functions as in Eq. (14).

As an illustration, the contribution to the hadronic tensor from the right-hand diagram in Fig. 5 may written in general as

\[
W^{\mu\nu} = e^2 g_{\alpha}^{d} e^{\nu b} \int d^4 ye^{ig\cdot Y} d^4 y_1 \sum_{S_1 S_2 S_2'} \sum_{\gamma_1 h_2} \langle p|\bar{\psi}(y)|Y \rangle \sum_{\gamma}(Y|A_{\alpha}^{c}(y_1)|S_1) \gamma^\nu(0|A_{\beta}^{d}(0)|S_2 p_2) \\
\times \langle S_2^b p_2|A_{\alpha}^{b}(0)|0 \rangle \int \frac{dk_2}{(2\pi)^4} \frac{k_2 + l}{k_2^2 + i\epsilon} e^{-iy \cdot (k_2 + l)} e^{iy \cdot (k_2 - l)} \gamma^\mu(0|\bar{\psi}(0)|S_2^a p_1) \\
\times \langle S_2^a p_1|\psi(0)|0 \rangle \gamma^\sigma \int \frac{dk_1}{(2\pi)^4} e^{iy_2 \cdot (l - k)} \frac{\rho}{k_2^2 + i\epsilon} \gamma^\rho \frac{k + l}{(k + l)^2 + i\epsilon} \gamma^\nu \langle S_1 |A_{\beta}^{d}(y_2)|X\rangle \sum_{\gamma} \langle X | \psi(0) | p \rangle. \tag{18}
\]

In the above equation, the gluon momentum \( l = P_{p_2} + p_2 \) and quark momentum \( q = P_{S_2} + p_1 \) are implicit. The sums \( \sum_{\gamma}, \sum_{\gamma} \) represent intermediate states assumed by the target after the initial quark is struck by the hard photon. The final state sums are \( \sum_{S_1} \) in the direction of the target, \( \sum_{S_2} \) in the direction of the photon. The latter is split into two parts \( S_2^a \) emanating from the fragmentation of the quark and \( S_2^b \) emanating from the fragmentation of the gluon. Momentum \( k_2 \) and \( k_2' \) represent the momentum of the quark between the radiation of the gluon and scattering off the soft gluon field.

We introduce the following factor of unity:

\[
\int d^4 l \int \frac{d^4 x_1}{(2\pi)^4} e^{i x_1 \cdot (l - p_1 - p_{22})} \int d^4 l \int \frac{d^4 x_2}{(2\pi)^4} e^{i x_2 \cdot (l - p_2 - p_{22})} \int dz_1 \delta \left( z_1 - \frac{p_1}{l} \right) \int dz_2 \delta \left( z_2 - \frac{p_2}{l} \right),
\]

followed by a reorganization of the spinor matrix structure as in Eq. (6) which leads to the trace over spinor indices. The introduction of the various integrals and \( \delta \)-functions allow for a simple procedure of indentifying and extracting the matrix elements in the fragmentation functions. Using the \( \delta \)-functions over the momentum fractions, one may re-express the partonic momenta \( q, l \) in terms of the rescaled hadronic momenta \( p_1, p_2 \). As a result the quark fragmentation function may be isolated as,

\[
D_{q}^{h_1} (z_1^2) = \frac{z_1^3}{2} \int \frac{d^4 l}{(2\pi)^4} \delta \left( z_1^2 - \frac{p_1}{l} \right) \int d^4 x e^{ix_1 \cdot l} \text{Tr} \left[ \frac{z_1^2}{2p_1} (0|\bar{\psi}(x)|S_2^a p_1) \langle S_2^b p_1 | \psi(0) | 0 \rangle \right], \tag{19}
\]

and the gluon fragmentation function as,

\[
D_{g}^{h_2} (z_2^2) = \frac{z_2^2}{2} \int \frac{d^4 l}{(2\pi)^4} \delta \left( z_2^2 - \frac{p_2}{l} \right) \int d^4 x e^{ix_2 \cdot l} \langle 0|A_{\beta}^{d}(x_2)|S_2^b p_2 \rangle \langle S_2^b p_2 | A_{\alpha}^{b}(0) \rangle d^{a\beta} \delta^{b0} \frac{d^{a\beta} \delta^{b0}}{8}. \tag{20}
\]

Utilizing the above relations, one obtains the differential hadronic tensor as

\[
\begin{align*}
W^{\mu\nu} &= dz'_{1} dz'_{2} e^2 g^{d} \text{Tr}[e^{d} e^{\nu b}] \int d^4 y d^4 y_1 d^4 y_2 \sum_{\gamma_1 h_2} \text{Tr} \left[ \sum_{S_1} \langle S_1 |A_{\alpha}^{c}(y_1) \psi(0)|P\rangle \langle P|\bar{\psi}(y)|A_{\beta}^{d}(y_1)|S_1 \rangle \right] \gamma^\nu \\
&\times \int \frac{dk_2}{(2\pi)^8} \frac{k_2 + l}{(k_2^2 + i\epsilon)} \gamma^\beta \frac{k_2 - l}{k_2^2 + i\epsilon} e^{iy_2 \cdot (l - k)} e^{iy_2 \cdot (l - k')} \frac{D_{q}(z'_{1}) D_{g}(z'_{2})}{(z'_{1})^3 (z'_{2})^2}.
\end{align*}
\tag{21}
\]
The factorization of the fragmentation functions from the hard part is completed by reexpressing the hadronic momentum integrals in terms of their rescaled partonic counterparts i.e., \( d^3p_1 \simeq d^3q(z_1')^3 \) and \( d^3p_2 \simeq d^3l(z_2')^3 \). We also introduce the variable \( z \), the fraction of momentum that remains in the quark after it has radiated a gluon. In terms of \( z \) the hadronic momenta are rescaled to \( z_1 = z_1'z \) and \( z_2 = z_2'(1 - z) \). Stipulating that the matrix elements of the initial states do not depend strongly on \( y_\perp, y_\perp + y_\perp, \) the integrals over the transverse coordinates may be completed yielding \( \delta \)-functions over the transverse momenta. The matrix elements involving the initial states are dominated by the \((-)\) component of their coordinates. Integrating over these transverse momenta under the influence of the above mentioned \( \delta \)-functions, the double differential hadronic tensor may be expressed as

\[
\frac{d^2W^{\mu\nu}}{dz_1dz_2} = \int \frac{dz}{z(1-z)} D_q \left( \frac{z_1}{z} \right) D_g \left( \frac{z_2}{1-z} \right) H^{(1)}(p, q, z) \int dy^- dy^+ dy^z d^2y_\perp \\
\times \frac{1}{2} e^{i(y_\perp \cdot k_\perp / P)} \langle \psi(y^-) \gamma^+ A^{\alpha\alpha'}(y^+, y_\perp) A^{\alpha\alpha'}(y^+, 0) \rho(0) | P \rangle.
\]

(22)

The reader will note that the double differential cross section has been expressed as a convolution of an initial state quark-gluon correlation in the incoming nucleon(nucleus), a hard scattering piece \( (H^{(1)}(p, q, z)) \) and final state quark and gluon fragmentation functions \( (D_q(x), D_g(x)) \). Isolation of the initial state piece involves the well known decomposition of the gluon vector potentials \( 2^2 \),

\[
A^a_\perp \simeq \omega_{\alpha',\sigma} A^{\alpha\sigma'} + p_\alpha \frac{A^a \cdot n}{p \cdot n},
\]

where,

\[
\omega_{\alpha,\beta} = g_{\alpha\beta} - p_\alpha n_\beta + p_\beta n_\alpha.
\]

Within the given kinematics, the contribution from the second term in the Eq. (23) far out weighs that from the first term. This is followed by the usual approximation of isolating the leading twist piece of the quark gluon correlation function. The hard part in Eq. (22) is obtained as

\[
H^{(1)}(p, q, z) = g^4 \frac{\text{Tr}[\rho^a \rho^b \rho^c \rho^d]}{N_c^2 - 1} \text{Tr} \left[ \frac{\gamma}{2} \right] \sqrt{1 - \frac{2}{q}} \int \frac{dx_1 dx_2 dx_4 dk^2}{(2\pi)^3} \frac{d^4l}{(2\pi)^4} \delta^+(l^2) \delta^+(l^2) \frac{\hat{q} + x_4 \hat{p}}{2q \cdot p(x_B - x_4 + i\epsilon)} \\
\times \frac{\gamma^\beta}{2q \cdot p(x_B - x_4 + i\epsilon)} \left( \hat{q} + x_4 \hat{p} - l \right) \frac{\gamma^\alpha \gamma^\mu}{2q \cdot p(x_B - x_1 - i\epsilon)} \delta_{\alpha \beta} \left( 1 - \frac{l^-}{q^-} \right).
\]

(24)

In the above equation, \( x_1, x_2, x_4 \) represent the forward momentum fractions of the incoming quark and gluon and outgoing quark in Fig. 7. The reader will also note that the hard part presented above in the evaluation of the double inclusive cross section is identical to the hard part in the evaluation of the single inclusive cross section [4].

The remaining analysis of the cross section involves expanding the hard part as a Taylor expansion around the point \( k^\perp = 0 \). The leading term in such an expansion yields the first term in the Eikonal expansion of the intermediate quark propagator in Fig. 3. The second term which involves a linear derivative of the transverse momentum is zero identically for unpolarized targets. The third term survives and represents the first contribution to the nuclear enhanced higher twist contributions to semi-inclusive deep inelastic scattering. The evaluation of the higher twist contribution, factorized from the fragmentation functions, is in every way identical to that for the single inclusive cross section [4].

Extracting the leading order hard part \( H^{(0)} \) from \( H^{(1)} \) and reorganizing the double inclusive cross section in the form of Eq. (11), i.e.,

\[
\frac{dW^{\mu\nu}}{dz_1dz_2} = \int dx_B f(x_B) H^{(0)\mu\nu} \tilde{D}_q^{h_1h_2}(z_1, z_2),
\]

one obtains the medium modified dihadron fragmentation function \( \tilde{D}_q^{h_1h_2}(z_1, z_2) \).
The modification of the fragmentation function in this sense depends on the scattering of the struck quark off the gluons in the medium and is thus dependent on the initial state. Following the methods employed in the computation of the medium modification of the single fragmentation functions, one obtains the medium modification of the dihadron fragmentation functions as the sum of higher twist contributions extracted from the sum of multiple diagrams at next-to-leading order. The presence of multiple fragmentation options allows for a greater number of contributions to the equations governing the medium modification of the dihadron fragmentation functions. These may however be grouped together in a similar fashion to the vacuum evolution contributions to the dihadron fragmentation functions.

\[
\bar{D}_{q}^{b_1,h_2}(z_1,z_2) = D_{q}^{b_1,h_2}(z_1,z_2) + \int_{0}^{Q^2} \frac{d^2 l_\perp}{2\pi} \frac{\alpha_s}{z_2} \left[ \int_{z_1}^{1} \frac{dz}{z} \left( \Delta P_{q-gq}(z,x_B,x_L,l_\perp^2)D_{q}^{b_1,h_2} \left( \frac{z_1}{z},\frac{z_2}{z} \right) \right) + \int_{z_1}^{1-z_2} \frac{dz}{z(1-z)} \Delta \bar{P}_{q-gq}(z,x_B,x_L,l_\perp^2) \right] \times D_{g}^{b_1} \frac{z_1}{z} \left( \frac{z_2}{1-z} \right) \text{Re} \left[ \frac{h_1 \rightarrow h_2}{} \right] \text{\quad (26)}
\]

In the above \( z_h = z_1 + z_2 \), and the switch \((h_1 \rightarrow h_2)\) is only meant for the last term. \( \Delta P_{q-gq}, \Delta P_{q-gq} \) represent the medium modified splitting functions where a momentum fraction \( z \) is left in the quark and the gluon respectively. Their expressions are identical to the modified splitting functions derived in Ref.\[4\]. In the above, \( x_L = l_\perp^2/(2p^+ q^- z(1-z)) \), where the radiated gluon or quark carries away a transverse momentum \( l_\perp \).

As in the case for the evolution in the vacuum (Eq.\[14\]), the splitting function \( \bar{P}_{q-gq} = \text{Re}[P_{q-gq}] \) has no virtual counterpart and is given as

\[
\Delta \bar{P}_{q-gq} = \frac{1 + z^2}{1 - z} \frac{C_A 2\pi \alpha T_{qg}(x,x_L)}{\langle k_T^2 \rangle \left( \frac{1}{\langle k_T^2 \rangle} + \langle k_T^2 \rangle \right) N_c f_{q}^A(x,\mu_f^2)}. \text{\quad (27)}
\]

In the above equation, \( C_A = 3, N_c = 3 \). The scale \( \mu_f^2 \) represents the scale at which the quark distribution functions are factorized from the hard cross section at leading order and leading twist. The mean transverse momentum of the soft gluons is represented by the factor \( \langle k_T^2 \rangle \). The term \( T_{qg}^A \) represents the quark gluon correlation in the nuclear medium and includes contributions from multiple higher twist diagrams. The formal derivation of \( T_{qg}^A \) identical to the case for the single fragmentation functions is outlined in Ref.\[4\]. The final result which includes squares of amplitudes of soft-hard scatterings, hard double scattering and their interferences results in the simplified form,

\[
T_{qg}^A(x_B,x_L) = \int \frac{dy}{2\pi} dy_1 dy_2 e^{i(x_B x_L)p^+ y} (1 - e^{-i\pi LP^+ y_i})(1 - e^{-i\pi LP^+(y^- - y_{1i})}) \times \frac{1}{2} (p|\bar{\psi}(0)\gamma^+ F_\sigma^+(y_2) F_\sigma^+(y_{1i}) \psi(y^-) |p)(-y^-) \theta(y_2 - y_{1i}). \text{\quad (28)}
\]

Up to this point the state \( |p \rangle \) was generically referred to as that of a nucleus with momentum \( Ap \). Indeed, the isolation of the higher twist piece in DIS so far has been independent of the nuclear content. If \( |p \rangle \) represented a nucleon with momentum \( p \) then the above equation would represent the medium modification factor for DIS off a nucleon. The enhancement of such higher twist objects in extended nuclear media will be the subject of the subsequent sections. This will entail the decomposition of the nuclear medium into nucleons and will directly involve the nuclear density distribution. The evaluation of \( T_{qg}^A \) will expose the eventual nuclear size enhancement and as a result justify the incorporation of the higher twist contributions in the evaluation of inclusive hadron production in the DIS off a large nucleus.

**III. EVALUATION OF THE NUCLEAR MODIFICATION FACTOR**

In the preceding section, the higher-twist contribution to the inclusive hadron production in deep-inelastic scattering off a nucleus was expressed as the convolution of the quark distribution function in a nucleus, a hard LO photon-quark scattering cross section and a nuclear modified fragmentation function. The equations governing the modification of the dihadron fragmentation function in medium (Eq.\[26\]) depend on the nuclear modification factor (Eq.\[28\]). This quantity essentially represents a quark-gluon correlation function within the nucleus. The evaluation of this correlation function is the subject of this section.
The essential kinematics for the process of DIS off a large nucleus has been outlined in Sec. II. A virtual photon with momentum \( \mathbf{q} \) strikes a large nucleus with atomic mass \( A \) with a momentum \( A \mathbf{p} \). In the evaluation of the multiple scattering of the struck quark in a large nucleus we will invoke the convolution model, which decomposes the deep-inelastic scattering off a nucleus in terms of the scattering off its constituent nucleons. In this paper, we will follow the version of the convolution model outlined in Ref. [24]. The evaluation of the quark-gluon correlation function which arises as the struck quark traverses such a nucleus commences with the decomposition of the nuclear ket \( |p; A\rangle \) in terms of nucleons,

\[
|p; A\rangle = \int \prod_{i=1}^{A} d^3p_i \delta(p_i) \Phi(\{p_i\}) (2\pi)^3 2p_i^+ \delta^3(\sum_i \mathbf{p}_i - A \mathbf{p}).
\]  (29)

In the above equation, \( \Phi(\{p_i\}) \) represents the nuclear wavefunction; the ket \( |\{p_i\}\rangle = |p_1, p_2, \ldots, p_A\rangle \) represents a particular ket where the nucleons \( 1, \ldots, A \) assume momenta \( p_1, \ldots, p_A \). The overall three momentum conservation is enforced via the \( \delta \)-function. In the notation employed in this article, a single nucleon state is normalized as

\[
\langle p_i | p_j \rangle = 2p_i^+ (2\pi)^3 \delta^3(\mathbf{p}_i - \mathbf{p}_j),
\]  (30)
as a result the \( n \)-nucleon state is normalized as

\[
\langle \{p_i\} | \{p_j\} \rangle = \prod_{i=1}^{n} 2p_i^+ (2\pi)^3 \delta^3(\mathbf{p}_i - \mathbf{p}_j).
\]  (31)

Defining the nuclear momentum \( P_A = A \mathbf{p} \), the normalization of the nuclear state may be stipulated as

\[
\langle p; A|p'; A\rangle = 2p^+ (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}').
\]  (32)

Substituting the expression for \( |p; A\rangle \) from Eq. (29) into Eq. (32) leads to the normalization condition on the nuclear wavefunction (where we use the simplified notation \( d^A = \prod_{i=1}^{A} d^3p_i \theta(p_i^+) / ([2\pi]^3 2p_i^+) \)):

\[
\int d^A \prod_{i=1}^{A} \Phi^*(\{p_i\}) \Phi(\{p_i\}) (2\pi)^3 2p^+ \delta^3(\sum_i \mathbf{p}_i - P_A) = 1.
\]  (33)

Halting one of the momentum integrations on the l.h.s. of the above equation leads to the momentum space one-nucleon density, which represents the probability of finding a nucleon with momentum \( k \),

\[
\rho(k) = \int d^A \prod_{i=1}^{A} \Phi(k, \{p_i\})^2 (2\pi)^3 2p^+ \delta(Ap^+ - k^+ - \sum_{i=2}^{A} p_i) \delta^2(\mathbf{k}_\perp - \sum_{i=2}^{A} \mathbf{p}_i),
\]  (34)

where, we have decomposed the 3-\( \delta \) function into its longitudinal and transverse components. Halting two integrations leads to the two-nucleon momentum correlator \( \rho(k_1, k_2, \Delta) \), which contains information regarding the sharing of momentum between nucleons in a nucleus,

\[
\rho(k_1, k_2, \Delta) = \int d^A \prod_{i=3}^{A} \Phi^*(k_1 + \Delta/2, k_2 - \Delta/2, \{p_i\}) \Phi(k_1 - \Delta/2, k_2 + \Delta/2, \{p_i\})
\]

\[
\times (2\pi)^3 2p^+ \delta(Ap^+ - k_1^+ - k_2^+ - \sum_{i=3}^{A} p_i) \delta^2(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp} - \sum_{i=3}^{A} \mathbf{p}_i).
\]  (35)

Both these and the higher correlations obey the obvious sum rules, e.g.,

\[
\int d\mathbf{k} \rho(k) = 1, \quad \int d\mathbf{k}_1 d\mathbf{k}_2 \rho(k_1, k_2, 0) = 1,
\]  (36)
where we have used the short hand \( \hat{d}k = d^3k\theta(k^+)/(2\pi)^32k^+ \).

The matrix element included in the definition of the modification factor in Eq. (28) involves interferences between the different processes which contribute to different phase factors in Eq. (28). The interference between the various processes is thus dependent on these factors and ultimately on the locations \( y, y_1, y_2 \) where the various scatterings have taken place. Hence, the nuclear wavefunctions discussed above need to be transformed to position space. In the notation adopted in this paper, the position wavepackets demonstrate the following outer product and normalization,

\[
\int \prod_i d^3x_i |\{x_i\}\rangle \langle \{x_i\}| = 1, \quad \langle x_i|p_i \rangle = e^{i\vec{p}_i\cdot\vec{x}_i}\sqrt{2p_i^+}. \tag{37}
\]

Within this basis, the nuclear ket may be expressed as

\[
|p; A\rangle = \int \prod_i^{A-1} d^3 x_i \int \prod_i^{A-1} dp_i \frac{\delta(p_A^+)}{\sqrt{2p_A^+}} \Phi(\{p_i\}, p_A)e^{i\sum_i p_i\cdot x_i} 2p_i^+|\{x_i\}\rangle.
\tag{38}
\]

By mere inspection, the nuclear wave function in position space may be expressed as

\[
\Phi(\{x_i\}) = \int \prod_i^{A-1} dp_i \frac{\theta(p_A^+)}{\sqrt{2p_A^+}} \Phi(\{p_i\}, p_A) e^{i\sum_i p_i\cdot x_i} 2p_i^+,
\tag{39}
\]

where, once again, it is understood that \( p_A = P_A - \sum_i^{A-1} p_i \). As a result the wavefunction in position space has the right translational properties, \( i.e. \),

\[
\Phi(\{x_i + \Delta x\}) = e^{iP_A\cdot \Delta x} \Phi(\{x_i\}).
\]

The inverse transform is obtained as

\[
\Phi(\{p_i\}) = \frac{1}{p^+} \int \prod_i^{A-1} d^3 x_i \sqrt{2p_i^+} e^{i\sum_i p_i\cdot x_i} \Phi(\{x_i\}) \sqrt{2p_A^+ - \sum_i^{A-1} p_i^+}.
\tag{40}
\]

Armed with the decompositions above for the nuclear wavefunction, one can now evaluate the nuclear modification factor in Eq. (28). The primary object to be evaluated in Eq. (28) is the matrix element

\[
\mathcal{M} = \langle p; A|\bar{\psi}(0)\gamma^+ F^+_\sigma(y^+) F^{+\sigma}(y^-) \psi(y^-)|p; A\rangle.
\]

The quark and gluon operators carry color. However, the expectation of this operator is evaluated in a nucleus which is color neutral. In the convolution model outlined above the nucleus is decomposed into its constituent color neutral nucleons. The various position variables \( 0, x', y', y_2 \) represent the locations in the nucleus where the quark was struck by the hard photon and where the soft gluon was encountered by the struck quark. As the quark and gluon carry color in different representations of SU(3), both quark operators must be confined within the same nucleon, as must both gluon operators. It is possible that all four operators are confined within the same nucleon. This is definitely the case for the higher twist contribution to DIS off a nucleon. In a nucleus, the two quarks operators and the two gluon operators may be confined to two different nucleons. In the limit in which the struck quark carries a large forward momentum \( x_1 p^+ \) and the soft gluon carries a very small transverse momentum \( k_\perp \), the struck quark follows an almost linear trajectory as it burrows through the nucleus. As a result the struck gluon may be located in any of the nucleons that lie directly behind the nucleon which is struck by the photon. This leads to an enhancement of the order of one nuclear dimension \( i.e., \sim A^{1/3} \). In this and the next section, this factor of \( A^{1/3} \) will be explicitly extracted. This nuclear size enhancement, \textit{aposteriori}, justifies the following approximation for the matrix element,
\[ M = \left( \frac{A}{2} \right) \int \prod_{i=1}^{A-1} d\hat{p}_i \int d^3k_1d^3k_2d^3\theta (k_1^+ + \Delta^+/2)\theta (k_1^- - \Delta^-/2)\theta (k_2^+ + \Delta^+/2)\theta (k_2^- - \Delta^-/2) \frac{2p_A^+}{(2\pi)^9 2^2 \sqrt{(k_1^+ - \Delta^+/2) (k_2^- - \Delta^-/2)}} \times \Phi^*(k_1 - \Delta/2, k_2 + \Delta/2, \{p_i\}_{A-1}^A) \Phi (k_1 + \Delta/2, k_2 - \Delta/2, \{p_i\}_{A-1}^A) (2p_A^+)^2 \times \langle k_1 - \Delta/2 | \bar{\psi}(0) \gamma^+ \psi(y^-) | k_1 + \Delta/2 \rangle \langle k_2 + \Delta/2 | F_{\sigma}^+(y^-) F_{\sigma}^+(y_1^-) | k_2 - \Delta/2 \rangle. \] (41)

In the above equation, we have simply reexpressed the nuclear ket in terms of nucleon kets in momentum space as required by Eq. (40). The set of states \( \{p_i\}_{A-1}^A \) represents 3 to \( A - 1 \) nucleons, the momenta of the first two nucleons are \( k_1 \pm \Delta/2 \) and \( k_2 \pm \Delta/2 \). Due to color confinement the two quark operators in \( M \) act on the same nucleon, as do the two gluon operators. The factor of \( (\frac{A}{2}) \) originates in the denominator of the two nucleons removed left as in the equation above. The momenta of the two nucleons on the r.h.s. of the Eq. (41) which form the bras and kets of the two partonic operators are such that the sum of the momenta of the two nucleons that form the bras are exactly equal to the sum of the momenta of the two nucleons that form the kets and each is equal to \( k_1 + k_2 \). This mismatch in momentum is the reason for the appearance of the two off-forward parton distributions \( 30 \) which will result from the fourier transform of the following shifted matrix elements of the partonic operators,

\[ \langle k_1 - \Delta/2 | \bar{\psi}(0) \gamma^+ \psi(y^-) | k_1 + \Delta/2 \rangle \langle k_2 + \Delta/2 | F_{\sigma}^+(y^-) F_{\sigma}^+(y_1^-) | k_2 - \Delta/2 \rangle. \]

Utilizing the expressions afforded by the preceding discussion, the matrix element in Eq. (41) may be transformed to position space. The \( A - 2 \) \( \theta \)-functions which contain all but the mismatched momenta i.e. \( p_A^1, \ldots, p_A^+ \) are Fourier transformed,

\[ \theta(p_i^+) = \int \frac{dz_i e^{iz_i p_i^+}}{2\pi i z_i - i\epsilon}. \] (42)

The \( A - 2 \) momentum integrations can now be carried out setting the position of the \( i^{th} \) nucleon in \( \Phi^* \) to \( x'_i = x_i + z_i \). Utilizing the fact that the nuclear wavefunctions are rapidly dropping functions as any one of the coordinates tends to infinity allows the evaluation of the \( A - 2 \) contour integrations over the variables \( z_i \), yielding the expression

\[ M = \left( \frac{A}{2} \right) \int d^3x_1d^3x_2d^3x'_1d^3x'_2 \Phi*(x'_1, x'_2, \{x_i\}) \Phi(x_1, x_2, \{x_i\}) \times \exp \left[ -i(\vec{k}_1 + \vec{\Delta}/2) \cdot x_1 - i(\vec{k}_2 - \vec{\Delta}/2) \cdot x_2 + i(\vec{k}_1 - \vec{\Delta}/2) \cdot x'_1 + i(\vec{k}_2 - \vec{\Delta}/2) \cdot x'_2 \right] \times \langle k_1 - \Delta/2 | \bar{\psi}(0) \gamma^+ \psi(y^-) | k_1 + \Delta/2 \rangle \langle k_2 + \Delta/2 | F_{\sigma}^+(y^-) F_{\sigma}^+(y_1^-) | k_2 - \Delta/2 \rangle. \] (43)

\[ T = \int d\frac{y^-}{2\pi} d\frac{y^-}{2\pi} e^{ix_A p^+ y^- + ix_B p^+ (y_1^- - y_2^-)} e^{ix_C p^+ y_2^-} \frac{1}{2} M(y^-; y_1^-; y_2^-) \theta(-y_2^-) \theta(y^- - y_1^-), \] (44)

where, \( x_A, x_B, x_C \) represent different combinations of momentum fractions, and \( M(y^-; y_1^-; y_2^-) \) represents the same matrix elements as those of Eqs. (41) and (43) with the dependence on the partonic position variables shown explicitly. As the reader will note, the full form of \( T \) is essentially a convolution between two factors one that contains an integral over nucleon position variables and another that contains an integral over partonic position variables. These two terms are convoluted through the nucleon momenta \( k_1, k_2, \Delta \) i.e.

\[ T = \int d^3k_1d^3k_2d^3\Delta \left[ N(\vec{k}_1, \vec{k}_2, \vec{\Delta}) P(\vec{k}_1, \vec{k}_2, \vec{\Delta}) \right], \] (45)
where,

\[ N(\vec{k}_1, \vec{k}_2, \vec{\Delta}) = \int d^3x_1 d^3x_2 d^3x'_1 d^3x'_2 N(\vec{k}_1, \vec{k}_2, \vec{\Delta}, \vec{x}_1, \vec{x}_2, \vec{x}'_1, \vec{x}'_2) \]  

and

\[ P(\vec{k}_1, \vec{k}_2, \vec{\Delta}) = \int dy_1^+ dy_2^- d\vec{y}_2 P(\vec{k}_1, \vec{k}_2, \vec{\Delta}, y^+, y^-_1, y^-_2). \]  

New partonic position variables with unit Jacobian \( u^- = y^-_1 - y^-_2 \) and \( w^- = (y^-_1 + y^-_2)/2 \) are introduced. In terms of these the partonic matrix elements of the gluon operators may be simplified as

\[
\langle k_2 + \Delta/2 | F^+_{\sigma}(y^-_2) F^{\sigma+}(y_1^-) | k_2 - \Delta/2 \rangle = \langle k_2 + \Delta/2 | F^+_{\sigma} \left( \frac{-u^-}{2} \right) F^{\sigma+} \left( \frac{u^-}{2} \right) | k_2 - \Delta/2 \rangle e^{i\Delta \cdot w^-}. \]  

Similarly, the \( \theta \)-functions over the partonic locations may be reexpressed as

\[
\theta(-y^-_2) \theta(y^- - y^-_1) = \theta(u^-/2 - w^-) \theta(y^- - w^- - u^-/2) = \int \frac{dkdl}{(2\pi i)^2} \frac{e^{ik(u^- - u^-/2)}}{k - i\epsilon} \frac{e^{il(y^- - w^- - u^-/2)}}{l - i\epsilon}. \]  

Integrating out \( w^- \) one obtains the constraint \( 2\pi \delta(\Delta^+ - (k + l - x_C p^+)) \). This \( \delta \)-function is used to perform the integral over the longitudinal off-set in the momenta i.e., \( \Delta^+ \). It has already been stated that the matrix elements of the partonic operators in Eqs. (41,43) are, under the action of Fourier transforms, related to the off-forward parton distributions (OFPD) \([31]\). However, as was pointed out in Ref. \([31]\), MIT bag models of the OFPD’s suggest that the variation with \( \Delta \) is rather slow and thus we will assume that the OFPD’s remain more or less constant with \( \Delta \). It will also be assumed that the matrix elements are almost independent of the transverse momentum of the nucleons i.e., \( k_{1\perp}, k_{2\perp} \). This allows the integration over \( k_{1\perp}, k_{2\perp} \) and \( \Delta \), which leads to three sets of \( \delta \)-functions over the transverse components of the nucleons involved: \( \delta^2(\vec{x}_{1\perp} - \vec{x}'_{1\perp}), \delta^2(\vec{x}_{2\perp} - \vec{x}'_{2\perp}) \) and \( \delta^2(\vec{x}'_{1\perp} - \vec{x}_{2\perp}) \). Thus all the transverse positions of the nucleons are set to be the same. As a result, the simple linear trajectory of the propagation of the struck quark as presented in Secs. II & III is reinstated. The representative matrix element, with the incorporation of the above simplifications, assumes the form

\[
T = \int \frac{dkdl}{(2\pi i)^2} \int \frac{dk_1^+ dk_2^+}{(2\pi)^2} \frac{\theta(k_1^+ + \Delta^+/2) \theta(k_1^+ - \Delta^+/2) \theta(k_2^+ + \Delta^+/2) \theta(k_2^+ - \Delta^+/2)}{4\sqrt{(k_1^+ - \Delta^+/2)(k_2^+ - \Delta^+/2)}} \times \left( A \right) \int \frac{dx^-_1 dx_2^- dx_1^- dx_2^- dx_{1\perp}^- dx_{2\perp}^-}{(2\pi)^{A-1}} \prod_{i=3}^{A} \frac{d^3x_i e^{ik_1 i} e^{ik_2 i} e^{i\Delta \cdot (x_{2\perp} - x_{1\perp})}}{	ext{\Phi}^*\left(\{x_1^- + \delta_1^-/2, x_1\perp\}, \{x_2^- + \delta_2^-/2, x_2\perp\}, \ldots\right) \Phi\left(\{x_1^- - \delta_1^-/2, x_1\perp\}, \{x_2^- - \delta_2^-/2, x_2\perp\}, \ldots\right)} \times \frac{1}{2} \int \frac{dy^- u^- e^{i\gamma^\perp (x_{AP^+ + t^+}) e^{i\alpha^- (x_{AP^- - l^-})}}}{(k^+ - i\epsilon)(l^+ - i\epsilon)} \times \langle k_1 + \Delta/2 | \psi(0, x_{1\perp})^\perp \psi(y^-, x_{1\perp}) | k_1 + \Delta/2 \rangle \langle k_2 + \Delta/2 | F^+_{\sigma}(-u^-/2, x_{1\perp}) F^{\sigma+}(u^-/2, x_{1\perp}) | k_2 - \Delta/2 \rangle. \]

In the above equation, \( \Delta^+ = k^+ + l^+ - x_C p^+ \), we continue to use \( \Delta^+ \) to save writing. The only unphysical variables remaining are the two momenta \( k^+, l^+ \). Again, following Ref. \([31]\), we note that the the OFPD’s also demonstrate minimal variation with \( \Delta^+ \) and as a result that dependence may be ignored. Restricting attention solely on the part that depends on \( k^+, l^+ \) we obtain the integrals

\[
\int \frac{dkdl}{(2\pi i)^2} \frac{e^{ik^+ (x_{2\perp}^- - x_{1\perp}^-)} e^{il^+ (x_{2\perp}^- - x_{1\perp}^-)} e^{i\gamma^\perp (y^- - u^-)}}{(k^+ - i\epsilon)(l^+ - i\epsilon)}. \]

This yields the obvious condition \( \theta(x_{2\perp}^- - x_{1\perp}^-) \theta(x_{2\perp}^- - x_{1\perp}^- + y^- - u^-) \). Within the picture adopted by the convolution model, \( x_1, x_2 \) are the locations of the centers of the two nucleons that contain the struck quark and the soft gluon which scatters off the struck quark, where as \( u, y \) are locations within a given nucleon. As a result in the case where the struck quark and the soft gluon originate within two separate nucleons (as is the case in this calculation), the
second $\theta$-function is identical to the first and the overall integration over $k^+ l^+$ simply yields the physical condition that the nucleon containing the struck quark be situated spatially ahead of the nucleon containing the soft gluon. The contour integration over the complex space of $k^+ l^+$ sets their values to zero, as a result $\Delta^+ = x_C p^+$. By inspection of Eq. (23), one notes that the only values that may be assumed by $x_C$ are 0 or $x_L$. As a result, we make the further approximation that $x_C p^+ \ll k_1^+, k_2^+$ and neglect $x_C$ in the denominators of the top line of Eq. (50). Corrections to this approximation are either vanishing or suppressed by powers of $x_1^2$. Following Ref. [29], and given that the nuclear wavefunction is peaked for values of $k_1^+ \approx p^+$ we make the approximation of replacing the off-forward parton distributions with the regular distributions,

\[
\frac{p^+}{k_1^+} \int \frac{dk}{2\pi} e^{ikx} \langle k_1 - \Delta/2 | \hat{\psi}(0, x_\perp) \gamma^+ \psi(y^-, x_\perp) | k_1 + \Delta/2 \rangle = 2Q \left( \frac{xp^+}{k_1^+}, \frac{\Delta^+}{p^+}, \frac{M^2 - k_1^2}{M^2} \right) \approx 2f_q(x) \tag{51}
\]

\[
\frac{du}{2\pi} e^{iu x} \langle k_2 + \Delta/2 | F_\pi^+(-u/2, x_\perp) F^{\sigma+}(u/2, x_\perp) | k_2 - \Delta/2 \rangle = 2G \left( \frac{xp^+}{k_2^+}, \frac{\Delta^+}{p^+}, \frac{M^2 - k_2^2}{M^2} \right) \approx 2f_q(x) \tag{52}
\]

It should be pointed out that the above approximations for the parton distribution functions have only been demonstrated to hold in convolution with nuclear density distributions which are peaked around the mean values of the momenta of the nucleons (see Ref. [29] for further details).

Incorporation of the above simplifications leads to the following expression for the representative matrix element,

\[
T \simeq \int_{0}^{P_{\text{max}}} \frac{dk_1^+dk_2^+}{(2\pi)^2} A^2 \int dx_1^- dx_2^- d\delta_1 d\delta_2 d^2x_\perp \prod_{i=3}^{A-1} q^3 x_i e^{ik_1 x_i} e^{ik_2 x_i} e^{i\varepsilon p^+ x_i \cdot x_i} \rho \left( x_1, x_\perp \right) \rho \left( x_2, x_\perp \right)
\]

\[
\times \Phi^* \left( \{ x_1^- + x_1^-/2, x_\perp \}, \{ x_2^- + x_2^-/2, x_\perp \}, \ldots \right) \Phi \left( \{ x_1^- - x_1^-/2, x_\perp \}, \{ x_2^- - x_2^-/2, x_\perp \}, \ldots \right)
\]

\[
\times 2\pi f_q(x_A) f_q(x_B). \tag{53}
\]

In the above equation, $P_{\text{max}}$ is the maximum allowed momentum in the nucleus, in principle this should be weakly dependent on the total number of nucleons in the nucleus, we will ignore this dependence in this effort. The integrals

\[
\int_{0}^{P_{\text{max}}} \frac{dk_n}{2\pi} e^{ik_n x_n},
\]

are sharply peaked around the point $\delta_n = 0$ with a height proportional to $P_{\text{max}}$. We thus replace these integrals with a saddle point approximation around the point $\delta_n = 0$. All factors of the width of the integrals will be included within an overall unknown normalization constant $B$. This approximation simplifies the product of the nuclear wave-function and its complex conjugate $\Phi^* \left( \{ x_1^- + x_1^-/2, x_\perp \}, \{ x_2^- + x_2^-/2, x_\perp \}, \ldots \right) \Phi \left( \{ x_1^- - x_1^-/2, x_\perp \}, \{ x_2^- - x_2^-/2, x_\perp \}, \ldots \right) \rightarrow \Phi^* \left( \{ x_1^-, x_\perp \}, \{ x_2^- , x_\perp \}, \ldots \right) \Phi \left( \{ x_1^- , x_\perp \}, \{ x_2^- , x_\perp \}, \ldots \right) = \rho \left( x_1, x_\perp \right) \rho \left( x_2, x_\perp \right)$. This is the two nucleon correlation function in a nucleus. We make the last approximation of replacing this with a product of two single nucleon densities and an unknown constant $D$. i.e.,

\[
\rho(x_1, x_\perp) = D \rho(x_1) \rho(x_\perp). \tag{54}
\]

this leads to the final simplified expression for the representative matrix elements which has already appeared previously in Refs. [21–24],

\[
T = \int dx_1^- dx_2^- d^2x_\perp C \rho(x_1, x_\perp) \rho(x_2, x_\perp) e^{ix p^+ (x^- - x^-)} \theta(x^- - x^-) A^2 f_q(x_A) f_q(x_B), \tag{55}
\]

where all normalization constants including factors of $2\pi$ and $p^+$ have been absorbed into the dimensionful constant $C$.

Such a constant may not be determined from first principles in our approach, but should, in principle, depend on the kinematics of a given experiment. As our derivation has demonstrated, it also may have a mild variation on the nucleus chosen. However, we will assume that the overall constant be independent of the nucleus chosen. Ostensibly, the overall constant also has no dependence on the fragmentation of the escaping jet and thus may not depend on the momentum fractions of the detected hadrons $z_1, z_2$ nor on the number of hadrons detected. Once a data point for a choice of momentum fraction (fractions) of a detected hadron (hadrons) is described by tuning $C$, it is set for
all other momentum fractions and nuclei that may be used in the given experimental kinematics. The constant has a strong dependence on $p^\perp$, however, as in most experimental setups the energy per nucleon in an accelerated nucleus is held fixed for different nuclei, there is no variation across nuclear targets for different total momenta $P_A^+$. In the present section, we have introduced a number of approximations in the evaluation of the medium modification factor outlined in Eq. (25). The approximations have resulted in a very much simplified expression for the modification factor. In the subsequent section we will evaluate the modification of the dihadron fragmentation functions using the expression from Eq. (55). Such a computation will require a certain model of the nuclear density distribution $\rho(\vec{x})$. Calculations for two separate distributions (gaussian, hard-sphere) will be presented.

IV. NUMERICAL RESULTS

The modification of single and double hadron fragmentation functions in nuclei has been studied experimentally by the HERMES experiment at DESY [19, 21, 22]. The modification is presented as a ratio of the fragmentation of a hard parton produced in the DIS off a large nucleus, versus a deuterium nucleus. Within the kinematics of the experiment, the DIS off the deuterium nucleus leads to minimal modification of the fragmentation process. Fragmentation of the parton produced in the DIS off a large nucleus, versus a deuterium nucleus. Within the kinematics of the experiment, the HERMES experiment at DESY [19, 21, 22]. The modification is presented as a ratio of the fragmentation of a hard particle produced in DIS off a deuterium nucleus:

$$R_1(z) = \frac{N_A(z)}{N_D(z)}.$$  

(56)

Theoretically this ratio is equated with the ratio of the medium modified fragmentation function versus that in the vacuum, i.e.,

$$R_1(z) = \frac{\tilde{D}(z,\nu,Q^2,A)}{\tilde{D}(z,\nu,Q^2)}.$$  

(57)

Mesurements for a Nitrogen (N) and Krypton (Kr) nucleus are presented as the square and circular points in Figs. 6-8.

The medium modification of the associated hadron fragmentation function is obtained by measuring the number of events with at least two hadrons with momentum fractions $z_1, z_2$ ($N_A^{(2)}(z_1, z_2)$), and the number of events with at least one hadron with momentum fraction $z$ ($N_A(z)$). As in the case of the single fragmentation function a double ratio is presented using similar measurements off deuterium,

$$R_2(z_2) = \frac{\sum_{z_1>0.5} N_A^{(2)}(z_1, z_2)}{\sum_{z_1>0.5} N_A^{(2)}(z_1, z_2)}.$$  

(58)

This double ratio for different $z_2$ is plotted in Figs. 7 and 9 as the square points for nitrogen and the circular points for Krypton. In the limit of low multiplicity per event, and the assumption that there is minimal modification in Deuterium the above ratio is theoretically estimated as

$$R_2(z_2) = \frac{\int_0^{z_2} dz_1 D_A(z_1, z_2, \nu, Q^2, A)}{\int_{0.5}^{0.5} dz_1 D_A(z_1, z_2, Q^2)}.$$  

(59)

In the remainder of this section we focus on the evaluation of the medium modification factor and the medium modification of single and double fragmentation functions. In the preceding section, a set of approximations to the medium modification factor of Eq. (28) were carried out. There exist four terms in Eq. (28), each of which may be expressed in general as in Eq. (44). The generic term, under the approximations instituted in the previous section...
may be expressed as Eq. (50). Substituting Eq. (50) back into the complete expression for the modification factor Eq. (28) leads to the expression,

\[
T^A_{gg}(x, x_L) = \int dx_1 dx_2 d^2 x_\perp C \rho(x_1, x_\perp) \rho(x_2, dx_\perp) \theta(x_2^2 - x_1^2) A^2 \\
\times \left[ f_q(x + x_L) x_T f_g(x_T) \left( 1 - e^{-ix_L p^+ (x_2^2 - x_1^2)} \right) + f_q(x) x_L f_g(x_L) \left( 1 - e^{ix_L p^+ (x_2^2 - x_1^2)} \right) \right],
\]

which contains all the four terms of Eq. (28). To proceed further, an expression for the nuclear density distribution has to be substituted in the above equation. Two different choices of such a distribution will be studied in this article: a Gaussian distribution, which essentially concentrates most of the nucleons towards the center of the nucleus and a hard sphere distribution where the nucleons are distributed evenly over the entire nuclear volume. No doubt, most nuclei lie somewhere in between these two extremes. We refrain from using a Woods-Saxon distribution [32], as it becomes analytically intractable. The hard-sphere distribution is a good approximation of the Woods-Saxon distribution for large nuclei. In the following we present results for each of the different cases in turn.

The computation of the modification factor will be followed by the calculation of the modification of the fragmentation functions in Eq. (28) for the dihadron fragmentation function and for the single fragmentation function following Ref. [4]. As is clear from Eq. (28), the evaluation of the medium modified fragmentation function also requires the input of a vacuum dihadron fragmentation function and vacuum single fragmentation functions. The vacuum dihadron fragmentation functions may not be predicted entirely with QCD but require a measurement at a given energy scale. In the absence of experimental data we follow Ref. [12] and parametrize two-particle correlations in the tuned Monte-Carlo event generator JETSET [33]. The vacuum single fragmentation functions are tabulated and parametrized by various collaborations. We choose the parametrization of Ref. [34]. This parametrization is also consistent with single particle distribution from our chosen event generator.

A. Gaussian distribution

Imagine a large nucleus with \( A \) nucleons at rest, at a given time \( t_0 \). The mass of each of the nucleons is \( M \), and the nuclear radius is \( R_A \). The spatial distribution of nucleons is assumed to be that of a Gaussian, i.e.,

\[
\rho(y, x_\perp) = \rho_0 \exp \left[ - \frac{y^2 + x_\perp^2}{2R_A^2} \right] = \rho_0 \exp \left[ - \frac{x_\perp^2}{2R_A^2} \right] \exp \left[ - \frac{y^2}{2R_A^2} \right],
\]

where \( \rho_0 \) is the normalization factor, \( y \) is the coordinate in the 3-direction and \( x_\perp \) is the two dimensional vector orthogonal to it. The lightcone vector is \( y^- = t_0 - y \) at \( t_0 = 0 \). The density is normalised to unity, i.e.,

\[
\int dy^- d^2 x_\perp \rho_0 e^{-\frac{y^2}{2R_A^2} e^{-\frac{x_\perp^2}{2R_A^2}}} = 1 \implies \rho_0 = \frac{1}{(2\pi)^{3/2} R_A^3}
\]

We then boost the nucleus in the \( y^- \)-direction to a frame where its momentum \( p_A = A[\sqrt{M^2 + p^2}, 0, 0, p] \) is very large. For a large enough boost, we may approximate the light cone coordinates in the boosted frame \( y_b = y^-/2\gamma \), where \( \gamma = p^+/M \). The integral of the density in the coordinates of the boosted frame becomes, (dropping the subscript \( b \)),

\[
\int dy^- d^2 x_\perp 2\gamma \rho(y^-, x_\perp) = \int dy^- d^2 x_\perp 2\gamma \rho_0 e^{-\frac{x_\perp^2}{2R_A^2} e^{-\frac{y^2}{2R_A^2}}} = 1
\]

The overall factor of \( 2\gamma \) may be absorbed into a redefinition of \( \rho_0 \). This expression for the density is then substituted into Eq. (60). There exist two kinds of terms in Eq. (60); those with and without the phase factor \( \exp[\pm ix_L p^+ (x_2^2 - x_1^2)] \). Summing the terms without the phase factor, one obtains,

\[
T^{A-1}_{gg}(x, x_L) = \int dx_1 dx_2 d^2 x_\perp 2\gamma \frac{2\gamma}{(2\pi)^3 R_A^6} \theta(x_2^2 - x_1^2) A^2 \\
\times \rho_0 e^{-\frac{x_\perp^2}{R_A^2} e^{-\frac{y^2}{2R_A^2}}} f_q(x + x_L) x_T f_g(x_T)
\]
It is now trivial to carry out the Gaussian integrals over the position variables $x_1, x_1', x_2'$ which yields factors of $2\pi, \gamma$ and $R_A^4$. Each power of $R_A$ yields a factor of $A^{1/3}$. Counting the power of $A$ and absorbing all other factors of $p^+$ and $2\pi$ into the overall normalization constant, we obtain the form as in Ref. [4]

$$T_{qq}^{A(1)}(x, x_L) = \hat{C}MR_A f_q^A(x + x_L)\gamma(xT) \times \rho_0 e^{-x^2/2A^2} f_q(x + x_L) x_T f_g(x_T) [1 - e^{-y^2/2y_A^2}],$$

(65)

where we have approximated the quark structure function in a nucleus to be simply $A$ times that in a nucleon $f_q^A(x + x_L) = Af_q(x + x_L)$.

The part of the above exponentials that depend on the positions of the two nucleons may now be integrated out as $x_1, x_1', x_2'$.

The terms with the phase factor undergo a similar integration, these may be summed to obtain

$$T_{qq}^{A(1)}(x, x_L) = -\int dx_1^- dx_2^- d^2x_\perp 2C \frac{2\gamma}{(2\pi)^3 R_A^3} \theta(x_2^- - x_1^-) A^2 \times \rho_0 e^{-x^2/2A^2} f_q(x + x_L) x_T f_g(x_T) [1 - e^{-y^2/2y_A^2}],$$

(66)

The part of the above exponentials that depend on the positions of the two nucleons may now be integrated out as before. Adding the the contributions (both with and without the phase factor) results in the final expression for the medium modification function, in the Gaussian density approximation,

$$T_{qq}^A(x, x_L) = \frac{\hat{C}}{x_A} \left[ f_q^A(x) x_T^2 f_g^N(x_T) + f_q^A(x) (x_L + x_T) f_g^N(x_L + x_T) \right] (1 - e^{-y^2/2y_A^2}),$$

(67)

where $x_A = 1/(MR_A)$. The above expression for the modification factor is then used in Eq. (27) to obtain the modified splitting functions and the medium modified fragmentation functions in the Gaussian density approximation. There still remains the overall factor $\hat{C} \sim C[xf_g^N(x)]$ which represents the correlation between the nucleons and the gluon distribution at small $x$. We will consider this factor a fit parameter. It is determined for one measurement of the fragmentation function (single or double) at one value of momentum fraction of the hadrons in the DIS off one nucleus. We will use that value to predict medium modification of both single and dihadron fragmentation functions.

The modification of the single and double fragmentation functions in the DIS off nuclei ($D(z), D(z_1, z_2)$) has been measured by the HERMES experiment at DESY. The modification for the single inclusive distribution is plotted in Fig. [6] for Nitrogen (squared points) and Krypton (circular points) targets. These are then divided by the identical measurement on the deuteron nucleus, assuming minimal modification of the fragmentation function in deuterium. One notes a continuous suppression with increasing momentum fraction $z$ and increasing nuclear size. The suppression factor is expressed in our calculation as the ratio of the medium modified to the vacuum fragmentation functions. The value of $\hat{C}$ is set by fitting the point at the lowest $z$ for Nitrogen targets. Three different choices for $\hat{C} = 0.006, 0.007, 0.008$ are presented. Arguably the best fit is with $\hat{C} = 0.006$. The variation with increasing $z$ is then a prediction from Eq. (20) and is shown as the solid line for the N nucleus. The calculation of the suppression factor in $Kr$ requires nothing more than a change of the number $A$ from 14 to 81. One notes good agreement with the data. However the agreement seems to deteriorate for smaller $z$.

The modification of the dihadron fragmentation function as a function of momentum fraction for $N$ is presented in Fig. [6]. It should be stressed once again that there are no free parameters in the calculation of the dihadron fragmentation function. The sole parameter $\hat{C}$ was set by comparison to the medium modified single fragmentation function (Fig. [6]). It turns out that within the choice of parameters the comparison with the $N$ nucleus is very good. The modified dihadron fragmentation function for $K\pi$ is shown in Fig. [6]. One notes the agreement deteriorates for increasing $z$. The best fit to the data is obtained, once again with $\hat{C} = 0.006$.

It would seem that the Gaussian approximation for the nuclear density distribution provides a good fit to the data for both single and double pion distributions in the case of $N$. The comparison is not so good in the case of $Kr$. However, it should be noted, that the Gaussian density approximation, which is close to the actual density distribution in $N$ is far from the actual nuclear density distribution in a larger nucleus such as $Kr$. In the next subsection, we will use a hard sphere distribution which is a better approximation for large nuclei.

### B. Hard-sphere distribution

In this subsection nuclear density distribution will be assumed to be a hard-sphere,

$$\rho(y, x_\perp) = \rho_0 \theta(R_A - \sqrt{y^2 + x_\perp^2}).$$

(68)
The normalisation,

\[ \int dyd^2x_\perp \rho(y, x_\perp) = \rho_0 \frac{4\pi}{3} R_A^3 = 1, \]  

implies \( \rho_0 = \frac{3}{(4\pi R_A^3)} \). As before, we now boost the nucleus to a frame where its forward momentum fraction is \( \gamma p^+ \). The nuclear density in terms of the variables \( y^-, x_\perp \) is given as

\[ \rho(y^-, x_\perp) = 2\gamma \rho_0 \theta(R_A - \sqrt{y^-2\gamma^2 + x^2_\perp}). \]
We undertake the approximation that \((x_L + x_T)f_0^N(x_L + x_T) \simeq x_T f_0^N(x_T)\) for the gluon densities in Eq. (60). This leads to the simplified expression,

\[
T_{qg}^A(x, x_L) = \int dx_1 dx_2 \, d^2 x_L \, 2C \frac{18\gamma}{(4\pi)^2} R_A^0 \theta(x_L^2 - x_1^2) A^2 \rho_0 \theta(R_A - \sqrt{x_1^2 4\gamma^2 + x_2^2}) \times \rho_0 \theta(R_A - \sqrt{x_2^2 4\gamma^2 + x_1^2}) f_q(x + x_L) x_T f_g(x_T) [1 - \cos\{x_L p^+(x_2^2 - x_1^2)\}].
\]

Integrating over the transverse coordinate, one obtains

\[
T_{qg}^A(x, x_L) = \int_{-R_A/2\gamma}^{R_A/2\gamma} dx_1 \int_{x_1}^{R_A/2\gamma} dx_2 \left[ R_A^0 - (x_2^2 2\gamma)^2 \right] \times \frac{C}{A}\rho_0^2 f_q^A(x + x_L) x_T f_g(x_T) [1 - \cos\{x_L p^+(x_2^2 - x_1^2)\}].
\]

In the above equation, one of the factors of \(A\) has been absorbed into the definition of the quark distribution function in a nucleus \(f_q^A(x) = A f_q(x)\). As in the case for the Gaussian density distribution, one may also absorb the gluon density \(x_T f_g(x_T)\) into the over all normalization constant \(\tilde{C} \sim C x_T f_g(x_T)\). Integrating over \(x_2\) and \(x_1\) and absorbing all constants except for factors of mass number \(A\), one obtains the final expression for the medium modification factor in the case of a hard-sphere distribution as

\[
T_{qg}^A(x, x_L) = \tilde{C} \frac{f_q^A(x)}{x_A} \left[ \frac{1}{3} + 4 \frac{x_A^3}{x_L^3} \sin \left( \frac{x_L}{x_A} \right) - 8 \frac{x_A^4}{x_L^4} \left\{1 - \cos \left( \frac{x_L}{x_A} \right)\right\} \right].
\]

Substituting the above result into the modified splitting functions in Eq. (21), one can calculate the medium modified fragmentation functions for a hard-sphere nuclear density distribution. There is the overall constant \(\tilde{C}\) as in the case of the Gaussian density distribution. This is set by fitting to the data. Results for the medium modification of the single hadron fragmentation function in the hard sphere approximation are presented in Fig. 8 for three different values of the parameter \(\tilde{C} = 0.022, 0.024, 0.030\) as compared with the identical data set as in Fig. 6. The data seem to prefer the value for \(\tilde{C} = 0.022\). This fits both the lowest momentum fraction point in \(N\) and provides a somewhat better description of the large \(z\) region in \(Kr\). In this sense a hard-sphere approximation seems to work better for the case of a large nucleus such as \(Kr\).

![FIG. 8: The medium modification of the single fragmentation functions compared to data from the HERMES collaboration. A hard-sphere distribution of nucleons in a nucleus is used. The dotted lines indicate a \(\tilde{C} = 0.022\), the dot-dashed lines a \(\tilde{C} = 0.026\) and the solid line a \(\tilde{C} = 0.030\).](image-url)
The modification of the dihadron fragmentation function as a function of the momentum fraction in the hard-sphere density approximation for \( N \) is presented in Fig. (9). Once again, there are absolutely no free parameters in the calculation of the dihadron fragmentation function. The sole parameter \( \tilde{C} \) was set by comparison to the medium modified single fragmentation function (Fig. 8). It turns out that within the choice of parameters the comparison with the \( N \) nucleus is not as good as the case for the Gaussian density distribution. This is to be expected as the Gaussian distribution is indeed closer to the actual density distribution in \( N \).

The modified dihadron fragmentation function for \( Kr \) is presented in Fig. (9). One notes that while the agreement with the data still deteriorates for increasing \( z \), there exits an overall quantitative improvement in the fit over that obtained from the Gaussian density distribution. The best fit to the data is obtained, once again with \( \tilde{C} = 0.03 \). It should be pointed out, in passing, that besides the slight improvement in the results for the case of the \( Kr \) nucleus there remains no qualitative difference in the modification of the single and double fragmentation functions between the cases of a Gaussian distribution of nucleons and a hard sphere distribution.

V. DISCUSSIONS AND CONCLUSIONS

The focus of this paper has been on the medium modification of dihadron fragmentation functions in the semi-inclusive DIS off a large nucleus. We have first generalized the formalism for the modification of the single fragmentation function in a dense medium to the modification of the dihadron fragmentation function. The modification arises from the inclusion of next-to-leading twist contributions to the double differential inclusive cross section for observing two hadrons within a jet produced via leptoproduction from a nucleus. Higher twist contributions are suppressed by powers of the hard scale \( Q^2 \) and are thus ignored in the DIS off a nucleon target. A class of these higher twist
contributions are however enhanced by nuclear size $A^{1/3}$ and lead to the medium modification of both single and dihadron fragmentation functions.

We have demonstrated (in Sec. II) that the medium modification of the dihadron fragmentation functions may be simply computed as a sum of convolutions involving medium modified splitting functions and vacuum dihadron fragmentation functions and a qualitatively new contribution involving a new modified splitting function $P_{qg}$ and a product of quark and gluon single hadron fragmentation functions [Eq. (29)]. Two of the modified splitting functions were shown to be identical to that in the case of the single fragmentation functions (i.e., $\Delta P_{qg}$ and $\Delta P_{gq}$). The new splitting function $\Delta P_{qg}$ was shown to be equivalent to $\Delta P_{gq}$ but without any virtual correction. Thus all modified splitting functions depend on the same medium modification factor $T_{qg}^A$ [Eq. (28)], identical to the case of the single fragmentation functions.

These higher-twist contributions to the semi-inclusive DIS cross section involves multiple scattering and gluon bremsstrahlung encountered by the struck quark and includes the well known Landau-Pomeranchuck-Migdal (LPM) interference effect. This interference effect leads to the appearance of the off-forward parton distribution functions (OFPD) [29] in the nuclear modification factor $T_{qg}^A$. Based on phenomenological models of OFPD’s we approximated these by the regular forward parton distributions. The final result depends on the density distribution of the nucleons in the nucleus [Eq. (60)]. Results for two different distributions were presented: Gaussian (Figs. 6,7) and hard-sphere (Figs. 8,9). The two distributions demonstrated qualitatively similar effects on both single and double inclusive spectra.

The evaluation of the modification factor involved an overall normalization constant which represents the correlation between the nucleon containing the struck quark and the nucleon containing the soft gluon which scatter off the struck quark. This was set by fitting to the overall experimental data on single inclusive distribution for DIS off $N$. The computation of the modification of the single fragmentation function on $K^0$ and the dihadron fragmentation function on both $N$ and $K^0$ thus involves no free parameters. The comparison of the theoretical prediction for the medium modified fragmentation functions (both single and double) with the experimental data for DIS off a $N$ nucleus is very good. The comparison for the case of the $K^0$ nucleus is somewhat satisfactory. A possible cause for this may lie in the inclusion of only the twist four contributions. These are contributions proportional to $A^{1/3}/Q^2$ that are suppressed by a power of $Q^2$ yet enhanced by a factor of $A^{1/3}$. Physically this means that the struck quark may undergo at most two more scatterings off soft gluons prior to exiting the nucleus and fragmenting. Further scatterings necessarily involve higher twist parton correlation functions and require the inclusion of further powers of $A^{1/3}/Q^2$. While, such contributions may be unimportant in the case of $N$, they may become necessary for the case of $K^0$ and heavier nuclei. A systematic inclusion of all orders of $A^{1/3}/Q^2$ in the modification of the fragmentation process will involve a far more complicated calculation and is beyond the scope of this paper.

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