Impact vibration response attenuation using four-bar linkage landing gear system

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Abstract. This research proposes a novel landing gear system using a four-bar linkage mechanism. The mathematical model of the landing gear system is calculated analytically. A simulation study is performed to investigate the landing gear performance in reducing the impact vibration response of the main system. It is found from the simulation study that nonlinear characteristic of the landing gear stiffness improves its impact vibration response. The simulation results show that rebound displacement and acceleration response of the main system using the proposed technique is smaller than that of using a conventional landing gear system.

1. Introduction
Safety and comfort are two important factors that should be considered in designing the mechanical system elements. In the impact-induced vibration cases, the system safety and comfort are mainly affected by the large transmitted force and acceleration of the main system due to impact load. Several health problems have been reported in connection with the impact-induced vibration [1-3].

One of the impact-induced vibration cases in the mechanical system is the aircraft landing process. The impact load occurs during landing may result in a damage of the airframe and equipment [4,5]. The landing gear system plays an important role in the aircraft during the landing process. In general, the aircraft landing gear system has two main functions. First, it is used to support the aircraft mass during ground motion. The other function is to reduce the impact-induced vibration response of the aircraft body during landing [6]. A good performance landing gear not only reduces the impact-induced vibration, but also keeping the aircraft wheel contacting with the ground for steering stability [7].

The passive landing gear system is the popular technique used for impact-induced vibration control of small and medium-size aircraft. In this conventional technique, the landing gear suspension system is simply modeled using a combination of linear spring and viscous damping elements [8,9]. Some experimental study has been conducted by researchers to investigate the landing gear performance under impact load using drop test experiment [10,11].

A new configuration of the landing gear system using a four-bar linkage mechanism is proposed in this research. To investigate its performance, a simulation study of the landing gear vibration response is conducted. Comparison of the four-bar linkage and the conventional landing gear performance is performed base on the landing gear vibration response. The evaluation is conducted by considering the maximum rebound displacement and acceleration response of the landing gear system.
2. **Landing gear system**

The conventional configuration of the landing gear system consists of a wheel and suspension system. The landing gear suspension system is simply modeled by linear spring stiffness ($k_s$) and viscous damping ($c_s$) as shown in Figure 1. The wheel system can be expressed by a wheel mass ($m_w$), linear spring ($k_w$) and dashpot ($c_w$). During the landing process, a large impulsive load is received by the wheel. The impact excitation force that received by wheels is transmitted to the main mass via suspension system. The landing gear system is designed to reduce the transmitted force to the main mass. Minimization of the transmitted force is indicated by the reduced of the main mass displacement and acceleration response.

![Figure 1. Conventional configuration of landing gear system](image)

Figure 2 shows the proposed model of the landing gear system using four-bar linkage mechanism. In this model, the bar mass is assumed very small in comparison with that of the wheel and main mass. As the consequence, the inertia force of the linkage is neglected in the dynamic analysis.

![Figure 2. Landing gear system using four-bar linkage mechanism](image)

2.1. **Kinematics analysis of landing gear mechanism**

Figure 3 shows the position analysis of the landing gear kinematic model. By using the position vector relation, the landing gear position vector equations can be written as:

$$FDe^{j\theta_1} = FAe^{j\beta} + ADe^{j\alpha}$$  \hspace{1cm} (1)

$$FEe^{j\theta_2} = FAe^{j\beta} + ADe^{j\alpha} + DEe^{j\beta}$$  \hspace{1cm} (2)
FE in Eq.(2) consist of the initial length $FE_0$ and the relative displacement $x_m-x_w$ as given by:

$$FE = FE_0 + x_m - x_w$$  \hspace{1cm} (3)

$x_m$ and $x_w$ in Eq.(3) describe the main mass and wheel displacement, respectively. Because of the main mass motion is kept in the vertical direction, then Eq.(2) can be rewritten by:

$$\left( FE_0 + x_m - x_w \right) e^{j90} = FAe^{j\beta} + ADe^{j\alpha} + De^{j\beta}$$  \hspace{1cm} (4)

![Figure 3. Schematic diagram of landing gear kinematic](image)

$\alpha$ and $\beta$ in Eq.(4) are the angle between bar $AD$ and $DE$ to the $x$-axis as shown in figure 3. Equation (4) can be expressed in its real and imaginary components as follows:

$$\left( FE_0 + x_m - x_w \right)ej = (FA + DE)\cos \beta + AD\cos \alpha + \left( (FA + DE)\sin \beta + AD\sin \alpha \right)j$$  \hspace{1cm} (5)

For the simplification, it is assumed that

$$a = (FA + DE)$$  \hspace{1cm} (6)

$$b = AD$$  \hspace{1cm} (7)

$$c = (FE_0 + x_m - x_w)$$  \hspace{1cm} (8)

By inserting notations in Eq.(6), (7) and (8) into Eq.(5) and collecting the same terms then the position equations can be rewritten as:

$$a\cos \beta + b\cos \alpha = 0 \rightarrow a\cos \beta = -b\cos \alpha$$  \hspace{1cm} (9)

$$a\sin \beta + b\sin \alpha = c \rightarrow a\sin \beta = c - b\sin \alpha$$  \hspace{1cm} (10)

The solution of the simultaneous equation in Eq.(9) and (10) resulting in $\alpha$ and $\beta$ as follows:

$$\alpha = \sin^{-1} \left( \frac{b^2 + c^2 - a^2}{2bc} \right)$$  \hspace{1cm} (11)

$$\beta = \sin^{-1} \left( \frac{a^2 + c^2 - b^2}{2ac} \right)$$  \hspace{1cm} (12)

The spring length $L$ is calculated using the vector equation as follows:

$$BDe^{j\theta} = ADe^{j\alpha} - AB e^{j\beta}$$  \hspace{1cm} (13)

or

$$BD\cos \theta_3 + BD\sin \theta_3 j = AD\cos \alpha + AD\sin \alpha j - (AB\cos \beta + AB\sin \beta j)$$  \hspace{1cm} (14)
In Eq.(14), \( \theta_3 \) is the angle between \( BD \) and the \( x \)-axis. By replacing \( d = AB \) and \( L = BD \) in Eq.(14) then:

\[
\begin{align*}
bcos \alpha - d \cos \beta &= L \cos \theta_3 \\
bsin \alpha - d \sin \beta &= L \sin \theta_3
\end{align*}
\]

(15)

(16)

From Eq.(15) and (16) the spring length(\( L \)) and angle(\( \theta_3 \)) can be calculated as follows:

\[
L = \sqrt{b^2 + d^2 - 2bd \cos(\alpha - \beta)}
\]

(17)

\[
\theta_3 = \sin^{-1} \left( \frac{b \sin \alpha - d \sin \beta}{L} \right)
\]

(18)

The spring deformation is calculated by:

\[
\Delta L = L - L_0
\]

(19)

2.2. Dynamic analysis of landing gear mechanism

The total force acts on the shock absorber consist of spring and damping elements is calculated as follows:

\[
F_{kc} = k_s \Delta L + c_s \frac{d}{dt} \Delta L
\]

(20)

The governing equations of the landing gear system are calculated from the balance of forces and moment as given by:

\[
\begin{align*}
m_w \ddot{x}_w + k_w x_w + m_w g + \frac{A}{(1 - B)} F_{kc} &= 0 \\
M \ddot{x}_m + B m_w \dot{x}_w + B k_w x_w + Mg + B m_w g - AF_{kc} &= 0
\end{align*}
\]

(21)

(22)

\( M, m_w \) and \( g \) in Eq.(22) are the main mass, wheel mass and the constant of gravity, respectively. \( A \) and \( B \) parameters in Eq.(21) and (22) are calculated as follows:

\[
A = \left( \frac{\sin \left( 180^\circ + \theta_3 - \beta \right) \sin \alpha}{\cos \left( \beta - \alpha - 90^\circ \right)} \right) - \sin \theta_3
\]

(23)

\[
B = \frac{\sin \left( \beta - 90^\circ \right) \sin \alpha}{\cos \left( \beta - \alpha - 90^\circ \right)}
\]

(24)

It can be observed from Eq.(21) to (24) that the landing gear spring characteristic is non-linear.

3. Methodology

Numerical simulation of the system’s response is conducted using MATLAB software utilizing the landing gear dynamic model as depicted in Eq.(21) and (22). Simulation is performed using the nominal parameters as shown in table 1. In the simulation study, the landing gear system is initially in the free fall condition of the initial height \( h = 0.05 \) m. The maximum displacement and acceleration response of the main mass are analyzed to investigate the landing gear performance in reducing the impact-induced vibration response. The landing gear performance is evaluated in two cases, with conventional and four-bar linkage mechanism.

| Variable Name       | Value  | Dimension |
|---------------------|--------|-----------|
| Wheel mass (\( m_w \)) | 1 Kg   |
| Main mass (\( M \))  | 5 Kg   |
| Wheel spring stiffness (\( k_w \)) | 5×10^2 N/m |
4. Results and discussion

Figure 4a and 4b show the simulation results of displacement and acceleration response of the main mass using conventional and four-bar linkage landing gear system. As shown in figure 4a, the system initially released from the initial dropping height $h = 0.1$ m. After contacting with the ground, the landing gear spring is compressed. The compression process occurs until the maximum spring deflection is reached. The landing gear spring releases its potential energy after reaches a maximum compression condition. The main mass rebounds when the spring releasing force is larger than the landing gear weight. Furthermore, when the spring force smaller than the gravitational force, the landing gear system becomes a damped vibration system. It is shown from figure 4a and 4b that the simulation results of maximum rebound displacement and acceleration of the main mass using four-bar linkage landing gear system reduces by 50% and 70% from those obtained by the conventional landing gear system.

![Figure 4a Displacement](image1)

![Figure 4b Acceleration](image2)

**Figure 4.** Displacement and acceleration response

The simulation results of the bar angle variation during the landing process is shown in shown in figure 5. The initial angle of $\alpha$, $\beta$, and $\theta_3$ are 25°, 139° and 7.3°, respectively. After contacting with the ground, at $t = 0.1428$ s, the angle of $\alpha$ and $\theta_3$ decreases but the angle of $\beta$ increases. It is shown from figure 5 that the landing gear oscillates after it is contacting with the ground.

| Parameter                  | Value  | Unit |
|----------------------------|--------|------|
| Absorber stiffness ($k_s$) | $5 \times 10^4$ | N/m  |
| Wheel damping ($c_w$)      | 100    | N.s/m|
| Absorber damping ($c_s$)   | 500    | N.s/m|
Figure 5. Variation of link angle

The results of landing gear spring length variation obtained from simulation are shown in figure 6. As shown in figure 6, the initial spring length is 0.305 m. At the instant after contacting the ground, the spring elongates and its length increases until reaching a maximum value at \( L = 0.316 \) m. The spring length reduces as the vibration time increases due to the damping effect.

Figure 6. Variation of spring length

Figure 7 shows comparison results of the maximum rebound displacement and acceleration calculated by the simulation. In the simulation, the initial height of the landing gear is varied from 0 to 0.15 m. The maximum rebound displacement and acceleration are calculated from the maximum displacement and acceleration response of the landing gear after landing. It is shown from figure 7 that the maximum displacement and acceleration response obtained using four-bar linkage landing gear system is lower than that obtained from conventional landing gear system.
Simulation of the damping factor effect to the maximum rebound displacement and acceleration of landing gear is shown in figure 8. The simulation is conducted using the conventional and four-bar mechanism of landing gear systems. In the simulation, the damping factor ($c_s$) is normalized to the nominal damping factor as depicted in table 1. It is shown from figure 8 that for damping ratio ($c/c_{ref} > 0.5$), the maximum acceleration and rebound displacement obtained using four-bar linkage landing gear system is smaller in comparison those obtained using a conventional landing gear system.

5. Conclusions
Simulation study of impact vibration response of the landing gear system using a four-bar linkage mechanism has been conducted. Some conclusions are obtained as follows:

Figure 7. Effect of initial landing height to the maximum response

Figure 8. Effect of damping to the maximum response
a. The proposed four-bar linkage landing gear system can reduce the maximum rebound displacement and acceleration of the main mass during the landing process.

b. For damping factor \( \frac{c_{ref}}{c} > 0.5 \), the four-bar linkage landing gear performance is better than the conventional method in reducing the impact vibration response.

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