Fluctuations in the detection of the HOM effect

Dmitry N. Makarov

Hong-Ou-Mandel (HOM) effect is known to be one of the main phenomena in quantum optics. It is believed that the effect occurs when two identical single-photon waves enter a 1:1 beam splitter, one in each input port. When the photons are identical, they will extinguish each other. In this work, it is shown that these fundamental provisions of the HOM interference may not always be fulfilled. One of the main elements of the HOM interferometer is the beam splitter, which has its own coefficients of reflection $R = 1/2$ and transmission $T = 1/2$. Here we consider the general mechanism of the interaction of two photons in a beam splitter, which shows that in the HOM theory of the effect it is necessary to know (including when planning the experiment) not only $R = 1/2$ and $T = 1/2$, but also their root-mean-square fluctuations $\Delta R^2$, $\Delta T^2$, which arise due to the dependence of $R = R(\omega_1, \omega_2)$ and $T = T(\omega_1, \omega_2)$ on the frequencies where $\omega_1, \omega_2$ are the frequencies of the first and second photons, respectively. Under certain conditions, specifically when the dependence of the fluctuations $\Delta R^2$ and $\Delta T^2$ can be neglected and $R = T = 1/2$ is chosen, the developed theory coincides with previously known results.

The HOM effect was first experimentally demonstrated by Hong et al in 1987. HOM interference shows up in many instances, both in fundamental studies of quantum mechanics and in practical implementations of quantum technologies. For example, one of the main practical applications of the HOM effect is to check the degree of indistinguishability of two incoming photons. When the HOM dip reaches all the way down to zero coincident counts, the incoming photons are perfectly indistinguishable, whereas if there is no dip, the photons are distinguishable. A HOM interferometer scheme was presented in, one of the main elements of which was a beam splitter (BS). To observe quantum interference, a BS is chosen close to 1:1 (having coefficients of reflection $R$ and transmission $T$ close to 1/2). A theoretical explanation of the HOM effect based on constant coefficients $R$ and $T$ and boson statistics of photons is quite simple. In this interpretation, we are not interested in what happens to the incident photons in the BS. For this, they consider BS lossless (hereinafter simply BS) as ideal, i.e. with constant coefficients $R$ and $T$ and BS is the source of the other two photons obeying bosonic statistics. In this case, the annihilation operators before entering 1 and 2 photons in BS represent $\hat{a}_1$ and $\hat{a}_2$, respectively, and after exiting BS is $\hat{b}_1$ and $\hat{b}_2$. The transformation from one pair of operators to another is generally described by the BS matrix (denoted as $U_{BS}$) in the form (see, e.g.11,12)

$$
\begin{pmatrix}
\hat{b}_1 \\
\hat{b}_2 
\end{pmatrix}
= U_{BS}
\begin{pmatrix}
\hat{a}_1 \\
\hat{a}_2 
\end{pmatrix},
\quad
U_{BS} = \begin{pmatrix}
e^{i\phi_1}\sqrt{T} & e^{i\phi_2}\sqrt{R} \\
e^{-i\phi_2}\sqrt{R} & e^{-i\phi_1}\sqrt{T}
\end{pmatrix}.
$$

(1)

It is easy to see that for $R = T = 1/2$, the photons at the output (described by the operator $\hat{b}_1, \hat{b}_2$) only come out in pairs from 1 or 2 ports. This analysis is fundamental to understanding the HOM effect and is not subject to any additional research. The basic scheme HOM interferometer for arbitrary photons (including quantum entangled photons) is shown in Fig. 1. In reality, the pair of photons arriving at the BS do not have a set frequency, but have a certain frequency distribution. Nonetheless, in the theoretical description (e.g.13-17) of the experimentally observed value $P$ ($P$ is the joint probability of detecting photons after exiting the BS on the output ports), the frequency distribution does not affect BS matrix $U_{BS}$, because $R$ and $T$ are constant values. Currently, the well-known HOM effect theories are based on calculating the value of $P$ within the constant values of $R = T = 1/2$.

In the work presented the coefficients $R$ and $T$ are variables, which significantly affects the theory of the HOM effect. The problem of interaction of two photons in BS is solved analytically, allowing the determination of the photon statistics after exiting the BS. Within the general form $U_{BS}$ is a BS matrix similar to Eq. (1), where $R$ and $T$ are some functions that depend on the frequencies of incident photons, the interaction time of two photons in BS, and on the BS material. This leads to the value of the coincidence counting probability $P$ being calculated to take into account the dependence on the frequencies of $R$ and $T$. It is shown that even in the case of identical incident photons and their average values $R = T = 1/2$ (averaging over the frequencies of incident photons), a
A but also taking into account the time delay i.e. there is a fluctuation in the reflection and transmission coefficients, had was not earlier taken into account a the electron with number i with the number 1, m, with the number i.

Photons in BS

It is well known that in quantum optics two modes of the electromagnetic field (two input ports) are usually considered, since even if 1 port remains unused, it should be considered as an input for vacuum fluctuations9,10. As a result, the Hamiltonian of Eq. (2) will be 9,10. As a result, the Hamiltonian of Eq. (2) will be

\[ \hat{H}_1 + \hat{H}_2 + \frac{1}{2} \sum_i \left( \hat{p}_i + \frac{\hbar}{c} \hat{A}_i \right)^2 + \sum_i U(r_i) \right) \Psi = i \frac{\partial \Psi}{\partial t}, \]  

where \( \hat{H}_i = \omega_i \hat{a}_i^+ \hat{a}_i \) is the Hamilton operator for the first (\( i = 1 \)) and the second (\( i = 2 \)) photon (\( \omega_i \) is the frequency, and \( \hat{a}_i \) is the annihilation operator of the photon with number \( i \); \( U(r_i) \) is the atomic potential acting on the electron with number \( a \) electric charge, \( m_e \) is the electron mass)

\[ \hat{H} = \sum_{i=1}^{2} \left( \frac{\omega_i}{2} q_i^2 + \frac{\hbar^2}{2m_e} \nabla^2 q_i + \beta_i q_i \xi_i \sum_a \hat{p}_a \right) + N \beta_1 \beta_2 q_1 q_2 \xi_1 \xi_2 + \sum_a \hat{p}_a^2 / 2 + \sum_i U(r_i). \]  

where \( \beta_i = \sqrt{\frac{\omega_i}{m_e c^2}} \), and the value \( N = \sum_a (1) \) is the number of electrons participating in the interaction with photons in a polyatomic system. Eq. (3) can be seen to correspond to the equation for coupled harmonic...
oscillators interacting with the electrons of a polyatomic system. A similar system was considered in \(^{24}\), but without taking into account interaction with electrons, we obtain

\[
\hat{H} = \sum_{i=1}^{2} \frac{1}{2} \left( \sqrt{A_i} \left( \hat{p}_i^2 + \hat{y}_i^2 \right) + 2D_i \sqrt{A_i} \sum_a \hat{P}_a \right) + \sum_a \hat{P}_a^2 + \sum U(r_a),
\]

where \( y_1 = \frac{A_1}{4} \left( q_1 / \sqrt{\omega_1} \cos \alpha - q_2 / \sqrt{\omega_2} \sin \alpha \right) \) and \( y_2 = \frac{A_2}{4} \left( q_1 / \sqrt{\omega_1} \sin \alpha + q_2 / \sqrt{\omega_2} \cos \alpha \right) \) are new variables; \( \hat{p}_1 = -i\hat{\partial} / \partial y_1 \); \( D_1 = \beta_1 A_1 \sqrt{\omega_1} \cos \alpha - \beta_2 A_1 \sqrt{\omega_2} \sin \alpha \); \( D_2 = \beta_1 A_2 \sqrt{\omega_1} \sin \alpha + \beta_2 A_2 \sqrt{\omega_2} \cos \alpha \). The study \(^{24}\) showed that \( \tan(2\omega) = (B_0 - B_1) \), where \( C = 2\beta_1 \beta_2 \sqrt{\omega_1 \omega_2} \). This is an obvious fact, since these quantities are responsible for various inelastic transitions of electrons in an atom under the action of photons, which are usually negligible in lossless BS. As a result, the dynamics of two photons in BS will be described by the wave function

\[
|\Phi(t_{BS})\rangle = e^{-iH_{BS}t_{BS}}|\Phi(0)\rangle; \quad \hat{H}_{BS} = \sum_{i=1}^{2} \sqrt{A_i} \left( \hat{p}_i^2 + \hat{y}_i^2 \right),
\]

where \( t_{BS} \) is the photon interaction time in BS and \( |\Phi(0)\rangle \) is the initial state of the photons before entering the BS. In the future, to calculate the required quantities, we will need the electric field operators \( \hat{E}_1^+(t_1) \) and \( \hat{E}_2^+(t_2) \) and the initial wave function \( |\phi(0)\rangle \) of the photons before entering the BS. Then, as anticipated, the coefficients will not alter. The next problem, we consider is the probability \( P_{12} \) of the joint detection of photons on 1 and 2 detectors (correlation between the two detectors). If our coincidence gate window accepts counts for a time \( T_D \), then the rate of coincidences \( \mathcal{P} \) between detectors 1 and 2 is proportional to (see, e.g.\(^{1,13,14}\))

\[
P_{12} \propto \int^{T_D/2}_{-T_D/2} \int^{T_D/2}_{-T_D/2} \langle \hat{b}_1^+(t_1) \hat{b}_2^+(t_2) \hat{b}_1(t_1) \hat{b}_2(t_2) \rangle dt_1 dt_2.
\]

Let us consider the case where the reaction time \( T_D \) (time resolution) of the detectors \( D_1 \) and \( D_2 \) in the experiment is many times slower than other time scales of the problem \( T_D \gg 1 \): in this case \( T_D \rightarrow \infty \). It should be added that the theory presented below is not difficult to generalize to the case of \( T_D \ll 1 \), which is currently implemented experimentally (e.g.\(^{16,26}\)).

Equation (9) is applicable in the case of monochromatic photons. In reality, they cannot be such and it is necessary to take into account the frequency distribution, and in this case the initial wave function of the photons will be in the form \( |\Psi\rangle = \int \phi(\omega_1,\omega_2) \hat{a}_1^+ \hat{a}_2^+ |0\rangle d\omega_1 d\omega_2 \), where \( \phi(\omega_1,\omega_2) \) is the joint spectral amplitude (JSA) of the two-photon wavefunction (\( \int |\phi(\omega_1,\omega_2)|^2 d\omega_1 d\omega_2 = 1 \)). Further calculations of \( P_{12} \) are similar to those that are generally accepted (e.g.\(^{1,13,14}\)), the only difference being that it is necessary to consider the \( T \) and \( R \) functions depending on the action of the frequencies. As a result, we obtain...
\[
P_{1,2} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\xi_1(t_1, t_2, \tau) - \xi_2(t_1, t_2, \tau, \delta \tau)|^2 dt_1 dt_2,
\]
\[
\xi_1(t_1, t_2, \tau) = \frac{1}{2\pi} \int \phi(\omega_1, \omega_2)T(\omega_1, \omega_2)e^{-i\omega_1 t_1}e^{-i\omega_2 t_2}d\omega_1 d\omega_2,
\]
\[
\xi_2(t_1, t_2, \delta \tau) = \frac{1}{2\pi} \int \phi(\omega_1, \omega_2)R(\omega_1, \omega_2)e^{-i\omega_1 (t_1 + \delta \tau)}e^{-i\omega_2 (t_2 - \delta \tau)}d\omega_1 d\omega_2,
\]
where \(P_{1,2}\) is normalized so that with \(t_{BS} = 0\) the probability is \(P_{1,2} = 1\) (without BS, the probability of joint operation of the detectors is 100%), which corresponds to standard normalization in HOM theory. We then obtain
\[
P_{1,2} = \int \left\{ |\phi(\omega_1, \omega_2)|^2 (T^2(\omega_1, \omega_2) + R^2(\omega_1, \omega_2)) - 2\Re\left\{ \phi(\omega_1, \omega_2)\phi^*(\omega_2, \omega_1)T(\omega_1, \omega_2)R(\omega_2, \omega_1)e^{-i(\omega_2 - \omega_1)(\delta \tau + \tau)} \right\} \right\} d\omega_1 d\omega_2.
\]
It should be added that if \(T\) and \(R\) are assumed to be independent of frequencies and \(T = R = 1/2\), then Eq. (11) corresponds to the well-known equation, e.g.\(^{27-29}\). It is also seen that the time delay of \(\delta \tau + \tau\); therefore, we denote it by \(\Delta \tau = \delta \tau + \tau\).

We next consider the case of identical photons at \(\Delta \tau = 0\), in this case \(\phi(\omega_1, \omega_2) = \phi(\omega_2, \omega_1)\) and \(R(\omega_1, \omega_2) = R(\omega_2, \omega_1)\) (because \(V_1 = V_2\)), and the quantity
\[
P_{1,2}(\Delta \tau = 0) = (T - R)^2 = \int \left\{ |\phi(\omega_1, \omega_2)|^2 (T(\omega_1, \omega_2) - R(\omega_1, \omega_2))^2 \right\} d\omega_1 d\omega_2.
\]
If we assume that photons are completely identical and monochromatic to \(\omega\), and \(\sigma_1 = \sigma_2 = \sigma\) is SPDC of type I, where \(\sigma\) is the bandwidth of the pump beam, \(\sigma_0\) and \(\sigma\) are the central frequency and the bandwidth, respectively, for both the signal and the idle beams.\(^{30}\) The second case, on the other hand, if we consider \(\sigma_p \to \infty\) in Eq. (13), then this will be the case of Fock states (e.g.\(^{35,36}\)). Indeed, in this case, in Eq. (13), the \(\phi(\omega_1, \omega_2)\) function will be factorized, which corresponds to Fock states. Substituting Eq. (13) into Eq. (11) we obtain
\[
P_{1,2} = \int e^{-\frac{V_1^2/4}{\Omega_1}} \{ T(\gamma + 2\delta \gamma) - 2B e^{-\left(\frac{\Omega_1}{2}\right)^2} \cos B(\Delta \tau, \Omega_1 \gamma) \} \frac{dy}{\sqrt{\Omega}}
\]
where \(B = A \left[ \frac{1 + \frac{\Omega_2}{\Omega_1}}{\frac{\Omega_2}{\Omega_1} + \frac{\sigma_p^2 + \sigma_1^2}{\sigma_2^2}} \right] \) and \(A = 2\frac{\Omega_1 \sigma_1}{\sigma_2^2} \cdot \Omega_1 = \sqrt{\frac{4\sigma_1^2 \sigma_2^2 + (\sigma_1^2 + \sigma_2^2) \sigma_p^2}{\sigma_1^2 + \sigma_2^2}} \cdot \Delta \omega = \omega_{02} - \omega_{01} \cdot T(\gamma) \) and \(R(\gamma)\) are determined by the Eq. (8), with the only difference being that
\[
\Omega = \Omega_2/\Omega_1 \ll 1,
\]
If we assume that \(\Omega_2/\Omega_1 \ll 1\), then \(T\) and \(R\) become constant values and they can always be selected in the experiment \(T = R = 1/2\). This is true for photon sources where \(\Omega_2/\Omega_1 \ll 1\). Estimates of the value of \(\Omega_1\) are given in the conclusion, where it is shown that \(\Omega_2/\Omega_1\) can be of the order of \(\Omega_2\); therefore, it is necessary to take into account fluctuations of the coefficients \(R\) and \(T\). The equation for \(P_{1,2}\), in our case Eq. (14) for the constants \(T = R = 1/2\) easily integrates and coincides with the well-known equations (9). Next, we consider how the value of \(P_{1,2}\) will look depending on the value of \(\Delta \tau\) (HOM dip) in the case of \(V_1 = V_2\) and \(\sigma_p \to \infty\) for different values of \(\Omega_2/\Omega_1\) and \(\Delta \omega/\Omega_1\), but for \(\Omega_{BS} \to \infty\) such that \(T = R = 1/2\), see Fig. 2; as \(\Omega_2/\Omega_1\) increases, the value of \(P_{1,2}\) tends to unity. This can be seen in the general analysis of the Eqs. (8) and (11). Figure 2 also shows that when \(T\) and \(R\) are taken into account from the frequency, \(P_{1,2}\) can significantly differ from the previously known theory of HOM interference.

If we assume that photons are completely identical and monochromatic to \(\epsilon = 0\) in Eq. (8) (in reality this does not happen), then we get the BS described in\(^{31}\) (in this work \(R = \sin^2(C_2); P_{1,2} = \cos^2(2C_2); \phi = \pi/2\), where \(C = \Omega/(2\nu)\) is the coupling constant between adjacent waveguides, \(z = vt_{BS}\), \(v\) is wave velocity in a
waveguide). In this case, the frequency dependence of the coefficients $R$ and $T$ disappears and it is always possible to experimentally select $R = T = 1/2$, where there are no fluctuations of these coefficients. In other words, this case corresponds to the standard HOM theory. You can see that the theory developed here is general, including the one suitable for BS in the form of coupled waveguides. In addition, changing the parameter $\Omega$, you can go to a different type of coupling in the waveguide, including the no coupling waveguide i.e. for $\Omega \to 0$ we get $T = 1/2, R = 0$ (photons propagate only along their waveguides). It should be added that such a passage to the limit was not previously in HOM theory (e.g.31).

Let us imagine in the figure (see Fig. 3) that in the case of identical photons (but not monochromatic), i.e. for $\sigma_1 = \sigma_2 = \sigma$ (in this case $\Omega_x = \sqrt{2} \sigma$) and $\Delta \omega = 0$; $\Delta \tau = 0$ the results obtained here may differ from31. If in this case we choose $\Omega_x/\Omega \ll 1$, then the dependences $R$ and $P_{1,2}$ are simplified and the results coincide with31, i.e. $R = \sin^2(\Omega_{BS}/2); P_{1,2} = \cos^2(\Omega_{BS})$. From Fig. 3 you can see that with the selected parameters, HOM interference ($P_{1,2} \approx 1$) can be realized only with $\Omega_x/\Omega = 1$ and one value $\Omega_{BS} \approx 2$. If we take into account that $\Omega_{BS} = z/v$, then we see that HOM interference is possible only for a certain value of length $z$. An estimate will be given below of the value of $\Omega$, where we obtain that $z$ should be of the order of micrometers. In the standard HOM theory, there are no restrictions on the length $z$ of the waveguide coupling.

It should be added that experiments do not always have good coincidences of $P_{1,2} = P_{1,2}(\Delta \tau)$ with theoretical predictions of HOM interference with constant coefficients $T = R = 1/2$. In such experimental studies, additional oscillations of the dependence $P_{1,2} = P_{1,2}(\Delta \tau)$ between the minimum of this function and $P_{1,2} = P_{1,2}(\Delta \tau \gg \tau_c)$ ($\tau_c$ is the coherence time). These oscillations can have a different nature (e.g.32,33), but fluctuations of the coefficients $R$ and $T$ are not studied. We should also add about the observation in the experiment of the studied effect of fluctuations. If the fluctuations are significant, then the visibility of $V$ will be small, although the photons can be identical. Therefore, in an experiment without measuring fluctuations they cannot be seen. In other words, if the visibility of the studied source is small, then following the standard HOM theory, we can conclude that the photons are not identical. In fact, this may not be the case, and photons can be identical with large fluctuations. Therefore, the presented theory is needed to avoid such errors in the interpretation of the HOM effect.

Figure 2. Dependence of $P_{1,2}$ on $\Delta \Omega_x$ (HOM dip). Case (a) corresponds to completely identical photons, and cases (b–d) correspond to non-identical photons. The visibility $V = V(\Omega_x/\Omega)$ depends on the parameter $\Omega_x/\Omega$ (the color of the lines corresponds to: red with $\Omega_x/\Omega = 1$, green with $\Omega_x/\Omega = 0.5$, brown with $\Omega_x/\Omega = 0.25$, blue with $\Omega_x/\Omega = 0$). The case $\Omega_x/\Omega = 0$ and visibility $V(0)$ corresponds to the previously known theory of HOM interference with constant coefficients $T = R = 1/2$.

Figure 3. (a) The dependence of $R$ depending on $\Omega t_{BS}$ for $\Omega_x/\Omega = 1; 2; 5; 10$ (top–down in figure). (b) The dependence of $P_{1,2}$ depending on $\Omega t_{BS}$ for $\Omega_x/\Omega = 1; 2; 5; 10$ (bottom–up in figure).
Discussion and conclusion

Thus, the developed theory shows that for a real BS, the coefficients of transmission $T$ and the reflection $R$ depend on the frequency. This dependence can significantly change the well-known theory of HOM interference. Under certain conditions, when the dependence on frequencies can be neglected (for example, in the case (Eq. 13) for $\Omega_2/\Omega \ll 1$), the coefficients $T = R = 1/2$ can be selected, and the developed theory is the same as that applying to the case of an ideal BS. In the special case of mixed, identical, and separable photons, there is a relationship between the visibility $V$ of the HOM dip and the purity $\mathcal{P}$ of the input photons when $T = R = 1/2$.

$$V = \frac{P_{1,2}(\Delta \tau \gg \tau_c) - P_{1,2}(\Delta \tau = 0)}{P_{1,2}(\Delta \tau \gg \tau_c)} = \text{Tr} \rho_1 \rho_2; \quad V(p_1 = p_2) = \mathcal{P}, \quad (16)$$

where $\rho_1, \rho_2$ are the density matrices of some quantum states of 1 and 2 photons, respectively. If we take into account the dependence of $T$ and $R$ on frequency, it is easy to see that the dependence of $V = \text{Tr} \rho_1 \rho_2$ in Eq. (16) will no longer apply (see Eq. (11), where $T$ and $R$ are present). This leads to the important conclusion that visibility $V$ with significant fluctuations of the coefficients $T$ and $R$ cannot be used to judge quantum interference, and for $p_1 = p_2$ purity $\mathcal{P}$ of the input photons. Therefore, when conducting an experiment, it is necessary to not only choose $T = R = 1/2$, but also minimize fluctuations. It should be added that fluctuations in the HOM interference had not been previously measured, because it was believed that the beam divider had strictly specified coefficients $T$ and $R$ during the experiment. In the case of HOM interference, the coefficients $T = R = 1/2$ were selected, which actually correspond to $T = R = 1/2$ in the experiment. It is quite simple to measure fluctuations at $T = R = 1/2$, for this it is necessary to measure $P_{1,2}$ at $\Delta \tau \gg \tau_c$, because $P_{1,2} = 2T^2 = 2R^2$ (this can be seen from Eq. (11) for $R = T = 1$). If the fluctuations are small, then $P_{1,2}$ will go over to the known value $P_{1,2}(\Delta \tau = 0) = 4(\tau^2/T^2 - \tau^2) = 4R^2/T^2 - 1 = 2T^2/T^2 - 1$.

A new physical quantity appears in the theory presented, which characterizes BS and its interaction with photons (Eq. 8) is $\Omega$. This value depends on the characteristics of the incident photons: $\omega_1, \omega_2, V_1, V_2, u_1, u_2$, and on the characteristics of BS itself $N$. The frequency $\Omega$ can be estimated if we consider a pair of photons that is quite close in characteristics, i.e. $V_1 \approx V_2 \approx V$, as well as $\omega_1 \approx \omega_2 \approx \omega_0$, $u_1 \approx u_2 \approx 1$ and assume that the overlap of the photon wave packets in BS is ideal (all electrons of atoms with the number $N$ are in the volume $V$). With such an estimate, it is easy to obtain that $\Omega_{id} = 4\pi n/\omega_0 n$, where $n = N/V$ is the electron concentration in BS ($\Omega_{id}$ is $\Omega$ in the case of identical photons). It should be added that for BS, the frequency $\Omega_{id}$ can be represented by the well-known value for plasma frequency $\omega_p$, then $\Omega_{id} = \omega_p^2/\omega_0 n$, where $\omega_p = 4\pi ne^2/m_e$ (in the CGS system). If we quantify $\Omega_{id}$ for solid materials and the optical frequency ($\sim 10^{15}\text{rad/s}$) range, we get that $\Omega_{id} \sim (10^{14} - 10^{15})\text{rad/s}$. Of course, the optical frequency is selected as an example, but it is not a defining one. The frequency range where fluctuations must be taken into account is much wider. The higher the frequency, the greater the contribution made by the fluctuations of $R$ and $T$. Obviously, a similar estimate is also valid for non-identical photons; the order in such an estimate will be preserved i.e. $\Omega \sim \Omega_{id}$. In reality, these $\Omega$ values have lower values due to non-ideal overlap of the wave packets of photons in BS. It can be seen that these values of $\Omega$ are essential in the theory of HOM interference. For example, from the Eqs. (14) and (15) it can be seen that the dependence of $T$ and $R$ on frequencies is determined by the relation $\Omega / \Omega_{id}$, considering the case of optical frequencies of photons with $\omega \sim 10^{15}\text{rad/s}$, where $\Omega_0$ is usually less by orders of magnitude than $\omega_0$ (e.g. 1), the $\Omega \approx 2\pi n/\tau_c \approx 10^{14}\text{rad/s}$ value was obtained) we get what could be $\Omega_2 / \Omega_{id} \sim 1$ (this is the second case with $\omega_2 \sim \omega_1 \sim \Omega$). It can be seen from $\Omega_{id} \sim 1/\omega_0$ that the developed theory is especially relevant in the case of the ultraviolet and X-ray frequency ranges, since $\Omega_{id}$ becomes smaller. The frequency $\Omega$ for optical photons can be compared with the spectral line width, for example, for the photon source described in Eq. (13). In other words, compare $\Omega$ with $\sigma_1, \sigma_2$ (for simplicity $\sigma$). Usually, for most photon sources, $\sigma \ll \omega_0$, and as we showed above $\Omega \sim \omega_0$. This means that $\Omega \gg \sigma$, which leads to $\Omega_2 / \Omega_{id} \ll 1$, where $\Omega_2 \sim \sigma$. As mentioned above, if $\Omega_2 / \Omega_{id} \ll 1$ then our theory coincides with the standard HOM theory with constant coefficients $R, T$. This perfectly explains why in the case of BS in the form of prisms, you can usually use the standard HOM theory. In the case when $\Omega \sim \sigma$ this is no longer the case. Such a case can be realized when the coupling $\Omega_{id}$ is quite small, for example, on BS in the form of coupled waveguides.

It should be added that the dependence on the frequencies of reflection and transmission coefficients cannot be represented as a function of the optical spectral filters, which are often used in experiments e.g. 21. Indeed, when using frequency filters, the spectral amplitude (ISA) $\phi(\omega_1, \omega_2)$ is replaced by $\phi(\omega_1, \omega_2) = \int P_{\phi}(\omega_1, \omega_2) d\omega_1 d\omega_2$, where $P_{\phi}(\omega_1, \omega_2)$ is the function optical spectral filters. As can be seen from Eq. (11), the coefficients $R$ and $T = 1-R$, with respect to $\phi(\omega_1, \omega_2)$ enter non-symmetrically, which leads to the above statement. For example, in the case of identical photons, using optical spectral filters, the $P_{1,2}(\Delta \tau = 0) = \int P(\omega_1, \omega_2)[(T^2/R^2)\omega_1 d\omega_2$, for $R = T = 1/2$ we get $P_{1,2}(\Delta \tau = 0) = 0$. This has fundamental differences from the case of the dependence on the frequencies of the R and T coefficients, see Eq. (12).

Received: 23 September 2020; Accepted: 6 November 2020
Published online: 18 November 2020

References

1. Hong, C. K., Ou, Z. Y. & Mandel, L. Measurement of subpicosecond time intervals between two photons by interference. Phys. Rev. Lett. 59, 2044–2046 (1987).
2. Nielsen, M. A. & Chuang, I. L. Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2000).
Acknowledgements

The study was supported by the Russian Science Foundation No. 20-72-10151.

Author contributions

D.N.M. conceived a project and it was carried out by himself.

Competing interests

The author declares no competing interests.

Additional information

Correspondence

and requests for materials should be addressed to D.N.M.

Reprints and permissions information

is available at www.nature.com/reprints.

Publisher’s note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.
Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

© The Author(s) 2020