Kaon physics

B. Sciascia
Laboratori Nazionali di Frascati - INFN, via E. Fermi 40, 00044 Frascati (Rome) Italy

At present, the main topics addressed by kaon physics are the unitarity test of CKM matrix via precision measurements of the Cabibbo angle as well as precision tests of discrete symmetries: in particular, study of possible CPT violations in a model-independent way through the Bell-Steinberger relation, or through the measurement of charge asymmetries. Other interesting topics are related to the test of predictions from chiral perturbation theory. Also status and prospects of the $K\rightarrow\pi^0\nu\bar{\nu}$ and $K_L\rightarrow\pi^0\nu\bar{\nu}$ decays are discussed.

I. INTRODUCTION

Last year has been full of new results in the kaon physics, which continues to contribute to the development and the understanding of the Standard Model (SM) flavor structure. Given the plenty of experimental results, only a part of these ones will be described in this review. In the first two sections, the NA48/2 results on $K^{\pm}$ decays will be presented: Section II is devoted to the search for direct CP violation in $K^{\pm}\rightarrow\pi^\pm\pi^0\pi^0$ (K$^{\pm}$) [1], while Section III describes the measurement of the $\pi\pi$ scattering length extracted from the $\pi^0\pi^0$ invariant mass distribution of $K^\pm\rightarrow\pi^\pm\pi^0\pi^0$ decays (K$^{\pm}$) [2], a very important and unexpected by product of the CP violation analysis. The status of the determination of $V_{us}$ and the unitarity test of the CKM matrix is presented in Section IV. Section V presents the Bell-Steinberger relation [4], a very important and non-perturbative observable is the asymmetry between $K^+$ and $K^-$ decays into three pions. Direct CP violation manifesting itself as an asymmetry in two CP-conjugate decay amplitude (as in $K^{\pm}_{\pi^3}$ case) is important as a strong qualitative test of the way in which the SM accommodates CP violation. Unfortunately, in general the predicted Dalitz-plot slope asymmetries are in the order of few units per million and the quantitative effort to constrain the fundamental parameters of the theory is difficult due to non-perturbative hadronic effects. An intense theoretical program is under way to improve these predictions, allowing the direct CP violation measurements to be used as strong quantitative constraints on the SM.

The $K^{\pm}_{\pi^3}$ matrix element squared is usually parameterized by a polynomial expansion [14]:

$$|M(u,v)|^2 \sim 1 + gu + hu^2 + kv^2,$$

where $g$, $h$, and $k$ are the so called linear and quadratic Dalitz-plot slope parameters ($|h|,|k| \ll |g|$) and the two Lorentz invariant kinematic variables $u$ and $v$ are defined as

$$u = \frac{s_3 - s_0}{m_\pi^2}, \quad v = \frac{s_2 - s_1}{m_\pi^2},$$
$$s_i = (P_k - P_i)^2, \quad i = 1, 2, 3;$$
$$s_0 = \frac{s_1 + s_2 + s_3}{3}.$$

Here $m_\pi$ is the pion mass, $P_k$ and $P_i$ are the kaon and the pion four-momenta, the indexes $i=1,2$ correspond to the two identical (“even”) pions and the index $i = 3$ to the pion of different charge (the “odd” pion). If CP is conserved, the decay amplitudes must be equal for $K^+$ and for $K^-$ separately. In case of CP violation, the two different amplitudes could be studied through their asymmetry or, equivalently, by means of the asymmetry of the linear parameter $g$:

$$A_g = (g^+ - g^-)/(g^+ + g^-) \approx \Delta g/(2g),$$

where $\Delta g$ is the slope difference and $g$ is the average slope. The slope asymmetry is strongly enhanced.

II. SEARCH FOR DIRECT CP VIOLATION IN K$^+\rightarrow\pi^+\pi^+\pi^-$ DECAYS

CP violation, more than 40 years after its discovery, still plays a central role in present and future investigations of particle physics. For a long time there was no significant progress, until NA48 [1] and KTeV [8] experiments demonstrated the existence of direct CP violation with high significance. This was done by measuring a non-zero $\Re(e'/e)$ parameter in the decays of neutral kaons into two pions. In 2001, the B-factory experiments Babar [9] and Belle [10] measured CP violation in the system of neutral B mesons and in 2004 also the direct CP violation in B decays has been found [11][12].

In kaons, besides the $\Re(e'/e)$ parameter measured in $K_{L,S}$ into two pion decays, a promising complementary observable is the asymmetry between $K^+$ and $K^-$ decays into three pions. Direct CP violation manifesting itself as an asymmetry in two CP-conjugate decay amplitude (as in $K^{\pm}_{\pi^3}$ case) is important as a strong qualitative test of the way in which the SM accommodates CP violation. Unfortunately, in general the predicted Dalitz-plot slope asymmetries are in the order of few units per million and the quantitative effort to constrain the fundamental parameters of the theory is difficult due to non-perturbative hadronic effects. An intense theoretical program is under way to improve these predictions, allowing the direct CP violation measurements to be used as strong quantitative constraints on the SM.

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with respect to the asymmetry of the integrated decay rate [12]. The prediction for \( A_g \) provided at first order by the Chiral Perturbation Theory and in the framework of the SM vary from a few \( 10^{-6} \) to a few \( 10^{-5} \) [13]. Theoretical calculations involving processes beyond the SM allow a wider range for \( A_g \), which can be as high as \( 10^{-4} \) [14].

The NA48/2 experiment in the framework of kaon physics program at the CERN SPS, measured the asymmetries \( A_g \) and \( A_g^0 \) in \( K^+ \rightarrow \pi^+\pi^-\pi^- \) and \( K^+ \rightarrow \pi^+\pi^0\pi^0 \) decays respectively. With \( 1.67 \times 10^9 \) \( K^+ \) decays, the difference in the linear slope parameter of the Dalitz plot for \( K^+ \) and \( K^- \) has been measured [16]:

\[
\Delta g = (-0.7 \pm 0.9_{\text{stat}} \pm 0.6_{\text{trg}} \pm 0.6_{\text{syst}}) \times 10^{-4}.
\]

Using for the Dalitz plot slope the PDG value \( g = -0.2154 \pm 0.0035 \) [14], \( \Delta g \) translates into the direct CP violation charge asymmetry:

\[
A_g = (1.7 \pm 2.1_{\text{stat}} \pm 1.4_{\text{trg}} \pm 1.4_{\text{syst}}) \times 10^{-4}
\]

\[
= (1.7 \pm 2.9) \times 10^{-4}.
\]

Regarding \( K^\pm \rightarrow \pi^\pm\pi^0\pi^0 \) decays, \( 4.7 \times 10^7 \) events have been used to measure the slope difference [16]:

\[
\Delta g^0 = (2.3 \pm 2.8_{\text{stat}} \pm 1.3_{\text{trg}} \pm 1.0_{\text{syst}} \pm 0.03_{\text{ext}}) \times 10^{-4},
\]

the corresponding asymmetry is found to be

\[
A_g^0 = (1.8 \pm 2.2_{\text{stat}} \pm 1.0_{\text{trg}} \pm 0.8_{\text{syst}} \pm 0.2_{\text{ext}}) \times 10^{-4}
\]

\[
= (1.8 \pm 2.6) \times 10^{-4}.
\]

For both measurements the uncertainty due to the trigger is statistical in nature, and the global precision obtained is limited mainly by the available statistics. On the systematics side, a careful control of detector and beam line asymmetries has been done to avoid such effects fake the charge asymmetry. The results have more than one order of magnitude better precision than the previous measurements, and are compatible with the SM predictions. These measurements have been obtained using the 2003 data sample. Analysis of the 2004 data will double the sample and will improve accordingly the uncertainties.

III. CUSP-LIKE EFFECT IN \( K^\pm \rightarrow \pi^\pm\pi^0\pi^0 \)

In searching for direct CP violation in \( K^\pm \rightarrow \pi^\pm\pi^0\pi^0 \), NA48/2 experiment observed for a first time a subtle and interesting phenomenon [3]. In a partial sample of about \( 2.3 \times 10^7 \) \( K^\pm \) decays acquired during 2003 data taking, the \( \pi^0\pi^0 \) invariant mass distribution (\( M_{00}^2 \)) shows an anomaly in the region \( M_{00}^2 = (2m_+)^2 - m_+ \) is the \( \pi^+ \) mass- where a cusp-like behavior is present (see figure 1). This anomaly, observed thanks to the large statistical sample and the excellent \( M_{00}^2 \) resolution, can be described by a re-scattering model [20] dominated by the contribution from the \( K^\pm \rightarrow \pi^\pm\pi^\pi^- \) decay through the charge-exchange reaction \( \pi^\pi^- \rightarrow \pi^\pi^0 \). The amplitude of the cusp is proportional to the \( a_0-a_2 \) difference of the \( \pi\pi \) S-wave scattering lengths. The quantity \( a_0-a_2 \) is one of the few non-perturbative parameters which can be predicted with excellent accuracy from first principles. Recent calculations [21] lead to a theoretical prediction of \( (a_0-a_2)m_{\pi^+} = 0.265 \pm 0.004 \) which has a precision not yet reached by the experiments. The best direct information on \( \pi\pi \) scattering lengths is the one extracted from \( K_{e4} \) decays by the BNL-E865 experiment [22], which is affected by a statistical error of 6%, due to the intrinsic statistical limitation of the \( K_{e4} \) decays with respect to the dominant \( K_{\pi3} \) modes; moreover the BNL and NA48/2 determinations of \( a_0-a_2 \) are affected by different theoretical and systematic errors. Fitting the \( M_{00}^2 \) distribution excluding re-scattering effects gives a \( \chi^2 = 13574 \) for 148 degrees of freedom. A best fit to a re-scattering model gives a good \( \chi^2 \) of 141 for 139 degrees of freedom, and provides a precise determination of the \( a_0-a_2 \) value: \( (a_0-a_2)m_{\pi^+} = 0.268 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}} \), with additional external uncertainty of \( \pm 0.013_{\text{ext}} \) from branching ratio and theoretical uncertainties. If the correlation between \( a_0 \) and \( a_2 \) predicted by chiral symmetry is taken into account, this result becomes \( (a_0-a_2)m_{\pi^+} = 0.264 \pm 0.006_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{ext}} \).

Figure 2 from [23], shows a summary of different theoretical determinations of the S-wave scattering lengths, compared with present experimental measurements from NA48/2, CERN DIRAC and BNL.

FIG. 1: \( M_{00}^2 \) distribution. The insert is an enlargement of a narrow region centered at \( M_{00}^2 = (2m_+)^2 \).
E865: the NA48/2 $a_0 - a_2$ determination has an uncertainty comparable with the two others available measurements.

IV. UNITARITY TEST OF CKM MATRIX AND $V_{us}$ DETERMINATION

In the Standard Model, the quark weak charged current is

$$J_\alpha^+ = (\bar{u} \bar{c} \ell) \gamma_\alpha (1 - \gamma_5) \mathbf{V} \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

where $\mathbf{V}$ is a $3 \times 3$ unitary matrix introduced by Kobayashi and Maskawa [24] in expansion on an original suggestion by Cabibbo [25]. The unitarity condition $\langle \mathbf{V} \mathbf{V}^\dagger \rangle = 1$ is required by the assumption of universality of the weak interactions of leptons and quarks and the absence of flavor-changing neutral currents. The realization that a precise test of CKM unitarity can be obtained from the first-row constraint $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ (with $|V_{ub}|^2$ negligible) has sparked a new interest in good measurements of quantities related to $|V_{us}|$. As we discuss in the following sections, $|V_{us}|$ can be determined using semileptonic kaon decays; the experimental inputs are the Br’s, lifetimes, and form-factor slopes. Both neutral ($K_S$ or $K_L$) and charged kaons may be used and provide independent measurements. Many players have joined the game, as seen from Table IV.

KLOE is unique in that it is the only experiment that can by itself measure the complete set of experimental inputs for the calculation of $|V_{us}|$ using both charged and neutral kaons. This is because the $\phi$ factory is uniquely suited for measurements of the $K_L$ and $K^\pm$ lifetimes. In addition, KLOE is the only experiment that can measure $K_S$ Br’s at the sub-percent level.

1. Semileptonic Kaon Decays

The semileptonic kaon decay rates still provide the best means for the measurement of $|V_{us}|$ because only the vector part of the weak current contributes to the matrix element $\langle \pi | J_\alpha | K \rangle$. In general,

$$\langle \pi | J_\alpha | K \rangle = f_+(t)(P + p)\alpha + f_-(t)(P - p)\alpha,$$

where $P$ and $p$ are the kaon and pion four-momenta, respectively, and $t = (P - p)^2$. The form factors $f_+$ and $f_-$ appear because pions and kaons are not point-like particles, and also reflect both $SU(2)$ and $SU(3)$ breaking. For vector transitions, the Ademollo-Gatto theorem ensures that only to second order in $m_{u,d} - m_{s}$. In particular, $f_+(0)$ differs from unity by only 2–4%. When the squared matrix element is evaluated, a factor of $m^2_{\pi}/m^2_K$ multiplies all terms containing $f_-(t)$, therefore can be neglected for $K_{e3}$ decays. For the description of $K_{\mu3}$ decays, it is customary to use $f_+(t)$ and the scalar form factor $f_0(t) = f_+(t) + [t/(m^2_K - m^2_\pi^0)] f_-(t)$.

The semileptonic decay rates, fully inclusive of radiation, are given by

$$\Gamma_i(K_{e3, \mu3}) = |V_{us}|^2 C_i^2 G^2 M^5 \times \left\{ \frac{1}{768\pi^3} + \delta_{em3} + \delta_{SU(2)} \right\} |f_+(0)|^2,$$

where $C_i$ is the Fermi constant, $M$ is the appropriate kaon mass, and $S_{em}$ is the universal short-distance radiative correction factor. The $\delta$ terms are the long-distance radiative corrections, which depend on the meson and lepton masses, and the $SU(2)$-breaking corrections, which depend on the kaon charge. The form factors are written as $f_+(t) = f_+(0) \hat{f}_+(0)$, with $f_+(0) = 1$. $f_0(t)$ reflects $SU(2)$- and $SU(3)$-breaking corrections and is different for $K^0$ and $K^\pm$. $I_{e3, \mu3}$ is the integral of the Dalitz-plot density over the physical region and includes $|f_+(0)|^2$. $I_{e3, \mu3}$ does not account for photon emission; the effects of radiation are included in the electromagnetic (em) corrections. The numerical factor in the denominator of equation 2 $768 = 3 \times 2^8$, is chosen in such a way that $I = 1$ when the masses of all final-state particles vanish. For $K_{e3}$, $I \approx 0.56$ and for $K_{\mu3}$, $I \approx 0.36$. The vector form factor $f_+$ is dominated by the vector $K\pi$ resonances, the closest being the $K^*(892)$. Note that for $t > 0$, $\hat{f}_+(t) > 1$. The presence of the form factor increases the value of the phase-space integral and the decay rate. The natural form for $\hat{f}_+(t)$ is

$$\hat{f}_+(t) = \frac{M^2_{\pi}}{M^2_V - t}.$$

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TABLE I: Recent world data on $K_{S3}$ decays for calculation of $|V_{us}|$

| Experiment | Measured quantities | References |
|------------|---------------------|------------|
| E865       | $BR(K^+ \to \pi^0 e^+\nu)/BR(K^+ \to \pi^0 X^+)$ (\(\pi^0_D = \pi^0 \to e^+e^-\gamma\)) | [26] |
| KTeV       | $BR(K_{L3}), BR(K_{L3},), \lambda_{\pm}(K_{L3}), \lambda_{\pm,0}(K_{L3})$ | [27, 28] |
| I Stra+    | $\lambda_{\pm}(K_{e3}), \lambda_{\pm,0}(K_{\mu3})$ | [29, 30] |
| NA48       | $BR(K_{L3}), BR(2 \text{ tracks}), BR(K_{S3}), BR(K_{L3}), BR(K_{S3})$, $BR(K_{S3},)$, $BR(K_{S3},)$, $BR(K_{p3},)$, $\lambda_{\pm}(K_{L3})$, $\tau_L$, $\tau^\pm$ | [31, 33, 34] |
| KLOE       | $BR(K_{L3}), BR(K_{L3},), BR(K_{S3},), BR(K_{S3},), BR(K_{p3},)$,  | [36, 38, 39, 40] |

It is also customary to expand the form factor in powers of $t$ as

\[
\tilde{f}_+(t) = 1 + \lambda_1 \frac{t}{m_{\pi^+}^2} + \lambda_2 \left( \frac{t}{m_{\pi^+}^2} \right)^2.
\]

To compare the results obtained from each semileptonic decay mode for both neutral and charged kaons without knowledge of $f_+(0)$, the relation \[2\] is usually used to compute the quantity \(f_+(0)|V_{us}|\). This requires the SU(2) and electromagnetic corrections for all four possible cases.

A. Results on semileptonic neutral kaon decays: KLOE, NA48, KTeV

Measurements of the absolute kaon branching ratios are a unique possibility of the \(\phi\) factory. At a \(\phi\) factory pairs of monochromatic \(K_S\) and \(K_L\) are produced and the \(K_L(K_S)\) can be identified looking at the decay of the companion on the other side (tagging). The \(K_L\) is selected by identification of \(K_S\) decays while a \(K_S\) beam is tagged using the fraction of events in which the \(K_L\) interacts in the calorimeter (\(K_L\) crash). Absolute branching ratios can be determined by counting the fraction of \(K_L\)'s that decay into each channel and correcting for acceptances, reconstruction efficiencies, and background. KLOE has measured the dominant \(K_L\) branching ratios using the \(K_L\) beam tagged by \(K_S \to \pi^+\pi^-\) decays. In the 2001/2002 data sample, \(\sim10^6\) tagged \(K_L\) decays have been used for the measurement, and \(\sim4\times10^6\) to evaluate efficiencies. To measure the BR's for decays to charged particles, a reconstructed \(K_L\) decay vertex is required in the DC fiducial volume. The number of events of each type is obtained by fitting the distribution of missing momentum minus missing energy, in the \(\pi\mu\) mass assignment, with the sum of the MC distributions for each of the decay channels. To select \(K_L \to 3\pi^0\) events, at least three photons are required from the \(K_L\) decay vertex. The reconstruction efficiency and purity of the selected sample are both about 99%. The resulting BR's are:

\[
BR(K_L \to \pi^0\pi^0\pi^0) = 0.4049 \pm 0.0010 \pm 0.0031 \text{,}
\]

\[
BR(K_L \to \pi^0\pi^0\pi^-) = 0.2726 \pm 0.0008 \pm 0.0022 \text{,}
\]

\[
BR(K_L \to \pi^0\pi^0\pi^0) = 0.2018 \pm 0.0004 \pm 0.0026 \text{,}
\]

\[
BR(K_L \to \pi^0\pi^0\pi^-) = 0.1276 \pm 0.0006 \pm 0.0016 \text{,}
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while they disagree with the old PDG values for the BR's. NA48 and KTeV experiments have used a different approach: they have measured ratios of branching ratios since they do not have an absolute normalization. NA48 has normalized \(K_L \to \pi\nu\) events to \(K_L \to 2\) tracks. Using for \(BR(K_L \to 3\pi^0)\), the average of the PDG value and the KTeV measurement, they obtain \(BR(K_L \to \pi\nu) = 0.4007 \pm 0.0006 \pm 0.0014\). KTeV measures 5 ratios of \(K_L\) BR’s \(\Gamma_{\pi3}/\Gamma_{\mu3}, \Gamma_{\pi0}/\Gamma_{\mu3}, \Gamma_{\pi0}/\Gamma_{\nu3}, \Gamma_{\pi0}/\Gamma_{\nu0}\) and \(\Gamma_{\nu0}/\Gamma_{\nu0}\). The 6 decay modes in the above ratios account for 99.93% of the \(K_L\) decays and the ratio can be combined to extract the semileptonic branching ratio \(\Gamma_{\nu0}/\Gamma_{\nu0}\). In Fig. 3 values of the branching ratios from each experiment are reported and compared with values from PDG. The new measurements agrees quite well with each other, while they disagree with the old PDG values for the \(K_L \to \pi\nu\) decays.

A measurement that is unique to KLOE is that of the BR for \(K_S\) semileptonic decays. Using the 2001-2002 data set (410 pb\(^{-1}\)) a sample of 6 500 events has been selected for each charge mode. The distribution of the missing energy minus the missing momentum \(\Delta E_{\pi^-}\) is shown in Fig. 4 for the \(\pi^-e^+\nu\) sample. The signal is centered at zero as expected for a missing neutrino. KLOE finds the value \(BR(K_S \to \pi\nu) = \)
the decays: $K^+_{\mu 2}$, $K^-_{\mu 2}$, $K^+_{\mu 2}$ and $K^-_{\mu 2}$. Two-body decays are rejected applying a cut on the charged decay particle momentum computed in the kaon rest frame $p^*$. The lepton squared mass, $m^2_{lep}$, is obtained from the velocity of the lepton computed from the ToF. The number of $K_{l3}$ and $K_{\mu 3}$ decays is then obtained by fitting the $m^2_{lep}$ distribution to a sum of MC distributions for the signals and various background sources. The BR is evaluated separately for each tag sample. The resulting preliminary BR’s are \[40]:

\[
BR(K_{\mu 3}^\pm) = 0.05047 \pm 0.00046_{stat} \pm 0.00080_{syst},
\]

\[
BR(K_{\mu 3}^0) = 0.03310 \pm 0.00040_{stat} \pm 0.00070_{syst}.
\]

NA48\[41] measures the $K_{\mu 3}^+$ branching ratio normalizing to the $K_{\pi 2}$ decay taken from PDG04. They obtain: $BR(K^{\pm} \rightarrow \pi^0 e^+\nu(\gamma)) = (5.14 \pm 0.02_{stat} \pm 0.06_{syst})\%$. They also measures the $K_{\mu 3}^+/K_{\mu 3}^0$.

ISTRAP\[42] has a similar approach, they obtain: $BR(K^{\pm} \rightarrow \pi^0 e^+\nu(\gamma)) = (5.22 \pm 0.11)\%$. It should be noticed here that PDG06 value for the $BR(K^{\pm} \rightarrow \pi^0 \pi^\pm)$ decreases by $\approx 1\%$ due to the new $K_{e3}$ value from E865 \[20] and $K_{\mu 2}$ from KLOE \[54].

KLOE has also measured the $K^{\pm}$ lifetime. The kaon proper time is determined from the kaon momentum and path length, which is evaluated taking into account the energy losses. The vertex reconstruction efficiency and the resolution functions are measured directly on data by means of the $\pi^0$ vertex reconstruction using only calorimetric information. The preliminary result is: $\tau(K^{\pm}) = 12.367 \pm 0.044_{Stat} \pm 0.065_{Syst}$ ns in agreement with PDG \[14].

C. Form Factor

The momentum dependence of the form factor, which is relevant for the integral over the phase space, is often described in terms of $\lambda$ slopes:

\[
f_+(t) = 1 + \lambda_1 t/m^2_{\pi^+}, \quad \text{linear,}
\]

\[
f_+(t) = 1 + \lambda_\chi t/m^2_{\pi^+} + \frac{1}{2} \lambda_\chi^2 t/m^2_{\pi^+}, \quad \text{quadratic,}
\]

or using the pole model: $f_+(t) = M_{\chi}^2/(M_V^2 - t)$. The slopes and the pole are obtained fitting the t-spectrum with the corresponding expression on form factor.

KLOE experiment has fitted the $K_{e3}$ spectrum using the linear, the quadratic and the pole parameterization. A very slight preference for a small quadratic term is obtained \[37]. The results are: quadratic $\lambda_\chi = (25.5 \pm 1.5 \pm 1.0) \times 10^{-3}$ and $\lambda_\chi^2 = (1.4 \pm 0.7 \pm 0.4) \times 10^{-3}$ with $P(\chi^2) = 92\%$.

pole model $M_\chi = 870 \pm 7$ MeV with $P(\chi^2) = 92.4\%$.

KTeV, ISTRA+ and NA48 experiments have fitted both $K_{e3}$ and $K_{\mu 3}$ spectra. KTeV experiment finds a quadratic slope term different from zero at 4 $\sigma$ level and the slopes are consistent for $K_{e3}$ and $K_{\mu 3}$ decay modes \[28]. NA48 find no evidence of quadratic term in the $K_{e3}$ spectrum, they obtain a $\lambda_0$ consistent with

B. Results on semileptonic charged kaon decays: KLOE, NA48, ISTRA+

In KLOE the measurement of the inclusive $BR(K_{l3})$ uses four independent samples tagged by

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**References:**

1. PDG04
2. PDG06
3. E865
4. KLOE
5. NA48
6. ISTRA+
7. KTeV
8. FRIPL05
KTeV result in the $K_{\mu 3}$ [34]. ISTRA+ experiment finds a quadratic slope term different from zero at 2 $\sigma$ level and a $\lambda_0 = (17.11 \pm 2.31) \times 10^{-3}$ [29, 30]. Fig. 5 compares the KLOE, KTeV, NA48, and ISTRA+ results. The values of the pole masses from the KLOE, KTeV, and NA48 experiments are in agreement, and their average value is $M_V = 875.3 \pm 5.4$ $(\chi^2 = 1.8)$ to which correspond: $\lambda'_+ = (25.42 \pm 0.31) \times 10^{-3}$ and $\lambda''_+ = (1.29 \pm 0.03) \times 10^{-3}$.

### D. $V_{us}$ extraction

As already discussed, from equation (2) $V_{us}$ can be determined using branching ratios, lifetimes and slopes as experimental inputs. $V_{us}$ can be extracted using both charged and neutral modes, allowing for a consistency check between experiment and theory (see figure 4). The value used for $\tau(K_L)$ is the average between KLOE and PDG values. The slopes $\lambda'_+$ and $\lambda''_+$ are those obtained by averaging the pole masses from KTeV, NA48, and KLOE, while the slope $\lambda_0$ is the average from KTeV and ISTRA+.

To extract $V_{us}$ the Leutwyler and Ross estimate of $f_{K^+\pi^-}(0) = 0.961 \pm 0.008$ [47] has been used; this value has been confirmed recently by vastly improved lattice QCD calculations [48].

The grey band in figure 4 is the average value for $|V_{us}f_+(0)| = 0.2164(4)$, from which $V_{us} = 0.2252(18)$ and it confirms unitarity within 1$\sigma$. The yellow band is obtained imposing the unitarity using $V_{ud} = 0.97377(27)$ [47]. The uncertainty on $f_{K^+\pi^-}(0)$ is reflected in the width of the “unitarity” band.

High-precision lattice quantum chromodynamics (QCD) results have recently become available and are rapidly improving [46]. The availability of precise values for the pion- and kaon-decay constants $f_+$ and $f_K$ allows to use of a relation between $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2})$ and $|V_{us}|^2/|V_{td}|^2$, with the advantage that lattice-scale uncertainties and radiative corrections largely cancel out in the ratio [54]:

$$\frac{\Gamma(K_{\mu 2}(\gamma))}{\Gamma(\pi_{\mu 2}(\gamma))} = \frac{|V_{us}|^2}{|V_{td}|^2} \frac{f_K^2}{f_{\pi}^2} \frac{m_K}{m_{\pi}} \left(1 - \frac{m_{\mu}^2}{m_K^2}\right)^2 \times C,$$

where the precision of the numerical factor, $C = (0.9930 \pm 0.0035)$, due to structure-dependent corrections [51] can be improved. Thus, it could very well be that the abundant decays of pions and kaons to $\mu\nu$ ultimately give the most accurate determination of the ratio of $|V_{us}|$ to $|V_{td}|$. This ratio can be combined with direct measurements of $|V_{ud}|$ to obtain $|V_{us}|$ [50, 53]. What is more interesting, however, is to combine all information from $K_{e3}$, $K_{\mu 2}$, $K_{e3}$, $K_{\mu 3}$, and superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$-decays to experimentally test electron-muon and lepton-quark universality, in addition to the unitarity of the quark mixing matrix.

KLOE has measured $BR(K_{\mu 2})$ using 175 pb$^{-1}$ collected in 2002, and found: $BR(K^+ \rightarrow \mu^+\bar{\nu}(\gamma)) = 0.6366 \pm 0.0009_{stat} \pm 0.0015_{syst}$ [51]. From this measurement and using $f_K/f_{\pi}$ from MILC collaboration we derive $V_{us}/V_{td} = 0.2294 \pm 0.0026$. In fig. 6 this result is compared with the one obtained from $K_{e3}$ decay and with the unitarity relation. From the fit, assuming unitarity, we obtain: $V_{us} = 0.2264(9)$.
V. CP AND CPT VIOLATION PARAMETERS USING THE BELL-STEINBERGER RELATION

The three discrete symmetries of quantum mechanics, charge conjugation \((C)\), parity \((P)\) and time reversal \((T)\) are known to be violated in nature, both singly and in pairs. Only CPT appears to be an exact symmetry of nature. Exact CPT invariance holds in quantum field theory which assumes Lorentz invariance, locality and unitarity \([55]\). Testing the validity of CPT invariance therefore probes the most fundamental assumptions of our present understanding of particles and their interactions. These hypotheses are likely to be violated at very high energy scales, where quantum effects of the gravitational interaction cannot be ignored \([57]\). On the other hand, since we still miss a consistent theory of quantum gravity, it is hard to predict at which level violation of CPT invariance might become experimentally observable.

The neutral kaon system offers unique possibilities for the study of CPT invariance. From the requirement of unitarity, Bell and Steinberger have derived a relation, the so called Bell-Steinberger relation \((BSR)\) \([4]\). The most stringent limit on CPT-symmetry violation is provided by the mass difference \(m_{K^0} - m_{\bar{K}^0}\). This difference is related to the quantity \(\delta\) that parametrizes the CPT violation in the mixing:

\[
\delta = \frac{i (m_{K^0} - m_{\bar{K}^0}) + 1/2 (\Gamma_{K^0} - \Gamma_{\bar{K}^0})}{\Delta \Gamma} \times \cos \phi_{SW} e^{i \phi_{SW}}
\]

where \(\tan \phi_{SW} = 2 \Delta \Gamma / \Delta \Gamma\), \(\Delta \Gamma = \Gamma_S - \Gamma_L\), and \(\Delta M = m_L - m_S\). If we assume that the CPT violation is negligible in the decay \((\Gamma_{K^0} - \Gamma_{\bar{K}^0} = 0)\), we have:

\[
\frac{m_{K^0} - m_{\bar{K}^0}}{m_{K^0}} = 3 \times 10^{-14} \delta.
\]

Violation at first order in \(m_K / M_{Planck} \sim 10^{-19}\), coming from quantum gravity effects, can thus be tested by putting an upper limit on \(\Im \delta\) of order \(\sim 10^{-5}\).

\(\Re \delta\) and \(\Im \delta\) are measured in two different ways. \(\Re \delta\) is measured both from the difference \(A_S - A_L\) of the charge asymmetries in \(K_S\) and \(K_L\) semileptonic decays, and from the time-dependent asymmetry

\[
A_{CPT} = \frac{P(K^0 \rightarrow \bar{K}^0(t)) - P(K^0 \rightarrow K^0(t))}{P(K^0 \rightarrow \bar{K}^0(t)) + P(K^0 \rightarrow K^0(t))},
\]

where \(P(K^0 \rightarrow \bar{K}^0(t))\) \((P(K^0 \rightarrow K^0(t)))\) is the probability for a \(K^0\) (\(\bar{K}^0\)) to be observed as a \(K^0\) (\(\bar{K}^0\)) after a time \(t\).

\(\Im \delta\) is obtained from the unitarity relation \((BSR)\):

\[
\left[ \frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{SW} \right] \frac{\Re \delta - \Im \delta}{1 + |\delta|^2} = \frac{1}{\Gamma_S - \Gamma_L} \sum_f a^S(f) a^L(f) = \sum_f \alpha_f
\]

where \(a_{S,L}(f)\) are the \(K_{S,L}\) decay amplitudes to the final state \(f\).

A. Experimental inputs to \(\Re \delta\)

Using the time-dependent asymmetry in semileptonic decays \([53]\) the CPLEAR Collaboration has obtained \(\Re \delta\). They measure the distribution \(A_S(t) = 4 \Re \delta + \mathcal{F} (\Re \delta, y, \Re x_-, \Im x_+)\) \((9)\) that depends, beside the CPT-violating parameter \(\delta\), on the parameters \(y\) and \(x_+\) that describe the CPT violation in the decays and the violation of the \(\Delta S = \Delta Q\) rule, respectively. They obtain:

\[
\begin{align*}
\Re \delta &= (3.0 \pm 3.3 \pm 0.6) \times 10^{-4} \\
\Im \delta &= (-1.5 \pm 2.3 \pm 0.3) \times 10^{-2} \\
\Re x_- &= (0.2 \pm 1.3 \pm 0.3) \times 10^{-2} \\
\Im x_+ &= (1.2 \pm 2.2 \pm 0.3) \times 10^{-2}
\end{align*}
\]

This result is improved by adding as a constraint the measurement \([12, 33]\) \(A_S - A_L = 4 [\Re \delta + \Re x_-] = (-1.8 \pm 10.0) \times 10^{-3}\) to the original CPLEAR fit, obtaining:

\[
\begin{align*}
\Re \delta &= (3.3 \pm 2.8) \times 10^{-4} \\
\Im \delta &= (-1.1 \pm 0.7) \times 10^{-2} \\
\Re x_- &= (-0.03 \pm 0.25) \times 10^{-2} \\
\Im x_+ &= (0.8 \pm 0.7) \times 10^{-2}
\end{align*}
\]
The uncertainty on $3\alpha_+$ is reduced by a factor three. As we will see in the next section, this parameter enters in the unitarity relation and is one of the limiting factors in the determination of $3\delta$ and $Re$.

B. Experimental inputs to $3\delta$ and $Re$

The only decay modes relevant for the evaluation of Eq. (3) are the $\pi\pi(\gamma)$, $\pi\pi\pi$, and semileptonic modes. The product of amplitudes $\alpha_f$ are obtained from the lifetimes of the kaons, the ratios of $K_S$ and $K_L$ amplitudes $\eta$, and the branching ratios, in the following way:

$$\alpha_{\pi\pi} = \eta_{\pi\pi} BR(K_S \to \pi\pi)$$

$$\alpha_{\pi\pi\gamma(DE)} = \sqrt{\frac{T_S}{T_L}} BR(K_L) BR(K_S) e^{i\phi-\gamma}$$

$$\alpha_{\pi\pi\pi} = \frac{T_S}{T_L} \eta_{\pi\pi\pi} BR(K_L \to \pi\pi\pi)$$

where $\pi\pi = \pi^+\pi^-, \pi^0\pi^0$, and $\pi\pi\pi = \pi^+\pi^-\pi^0, \pi^0\pi^0\pi^0$. The $\pi^+\pi^-$ mode includes the inner bremsstrahlung (IB) emission. On the contrary, the contribution of the direct emission DE is estimated separately.

The contribution of semileptonic decays to Eq. (3) is given by:

$$\alpha_{\pi l\nu} = 2\sqrt{\frac{T_S}{T_L}} BR(K_L \to \pi l\nu) \times$$

$$\left[Re - 3Re - i3\delta - i3\alpha_+\right]$$

where $3\alpha_+$ is given by Eq. (11), and $(A_S + A_L)/4 = Re - \Rey = (-4.8 \pm 19.0) \times 10^{-3}$, while $3\delta$ is the unknown of Eq. (3). The determination of $Re$ is of primary importance for the measurement of $A_S$ (3).

Some of the most recent measurements entering in the determination of the $\alpha$ parameters are described in the following sections. The numerical results are shown in Tab. (11).

1. $\pi\pi$

The two body decays give the largest contribution to Eq. (3). The $K_S$ branching ratios are obtained from the recent KLOE measurement of the ratio $\Gamma(K_S \to \pi^+\pi^-)/\Gamma(K_S \to \pi^0\pi^0) = (2.2549 \pm 0.0054)$ with a sample of $400 \times 10^6$ $K_S$ events. The $K_L \to \pi^+\pi^-$ branching ratio has been recently measured by KLOE with $35000$ events, they find $BR(K_L \to \pi^+\pi^- (\gamma_{IB+DE})) = (1.963 \pm 0.021) \times 10^{-3}$, where both the IB and DE components are included. The DE component is then subtracted using the KTeV result described in the following. KTeV measured both the charged and neutral decays with $\sim 10^5$ events each. They find $BR(K_L \to \pi^+\pi^- (\gamma_{IB})) = (1.975 \pm 0.012) \times 10^{-3}$, and $BR(K_L \to \pi^0\pi^0) = (0.865 \pm 0.010) \times 10^{-3}$. The measurement of charged mode includes only the IB component. The two new measurements of $BR(K_L \to \pi^+\pi^-)$ are in agreement but disagree with the PDG value of 2004 (13).

FIG. 8: Spectrum of photon energy in $K_L \to \pi^+\pi^-\gamma$ events, measured by the KTeV Collaboration.

The DE component is measured with two different methods. With the first method, the photon spectrum is measured directly in $K_L \to \pi^+\pi^-\gamma$ decays. KTeV (61) has measured it from a sample of $\sim 10^5 \pi\pi\gamma$ events with $E_\gamma > 20$ MeV (Fig. 8). They find that the fraction of DE component for $E_\gamma > 20$ MeV is $DE/(DE + IB) = 0.689 \pm 0.021$. With the second method, the internal conversion of the photon into a pair $e^+e^-$ is used. The resulting decay $K_L \to \pi^+\pi^-e^+e^-$ shows an asymmetry in the distribution of the angle between the $\pi^+\pi^-$ and $e^+e^-$ planes. This asymmetry is due to the interference between the IB and DE components. KTeV (61) measures this asymmetry with $\sim 5000$ selected events and extracts the information on the DE. The results are in agreement with the previous method, but less precise.

The DE component in $K_S$ decay is suppressed by $CP$ violation. The $K_S$ photon spectrum has been measured in (62).

2. $\pi l\nu$

As has been described in Section [IIX], the semileptonic decays of $K_L$ have been recently measured by KLOE (60), KTeV (27), and NA48 (51) in order to determine the Cabibbo angle. KLOE measured also $BR(K_S \to \pi\pi l\nu)$ and the related charge asymmetry,
The amplitudes for $K \to 3\pi$ decays can be decomposed according to the $CP$ state of the three pion system:

$$a_L = a_L^{CP-}(X,Y)$$
$$a_S = a_S^{CP+}(X,Y) + a_S^{CP-}(X,Y)$$  \hspace{1cm} (13)$$

where $a_s^{CP+(-)}$ is the amplitude for the decay to the final state with $CP = +1(-1)$, and $X \propto E_+ - E_-$ and $Y \propto E_0$. The decay to the $CP = +1$ state is suppressed by the centrifugal barrier. Therefore, $a_L^{CP+}$ is suppressed both by the centrifugal barrier and by $CP$ violation and can be neglected. On the contrary, both $K_S$ amplitudes are retained being one suppressed by $CP$ violation and the other one by the centrifugal barrier. The amplitudes have the following symmetry property:

$$a^{CP\pm}(X,Y) = \mp a^{CP\pm}(-X,Y)$$  \hspace{1cm} (14)$$

Hence, the only contribution to Eq. (8) comes from:

$$\eta_{+0} = \frac{\int dX dY a_L^* a_S^{CP-}}{\int dX dY |a_L|^2}$$  \hspace{1cm} (15)$$

while from (14) follows:

$$\int dX dY a_L^* a_S^{CP+} = 0$$  \hspace{1cm} (16)$$

CPLEAR measures $\eta_{+0}$ from the time-dependent asymmetry \cite{58}:

$$\tilde{N}_{3\pi}(t) - N_{3\pi}(t)$$
$$N_{3\pi}(t) + \tilde{N}_{3\pi}(t)$$  \hspace{1cm} (17)$$

where $N_{3\pi}(t)$ ($\tilde{N}_{3\pi}(t)$) is the number of decays to three pions observed at time $t$ for kaons tagged as $K^0$ ($\bar{K}^0$) at time $t = 0$. They find:

$$\eta_{+0} = (-2 \pm 7) \times 10^{-3} + i(-2 \pm 9) \times 10^{-3}$$  \hspace{1cm} (18)$$
There are two different ways of measuring this quantity. NA48 measures the interference in $K \to 3\pi^0$ decays as a function of the proper time, with $5 \times 10^9$ decays from the near target, normalized to the rate of $10^8 K_L \to 3\pi^0$ from the far target [62]:

$$f_{3\pi^0}(t) \propto 1 + |\eta_{000}|^2 e^{-(\Gamma_S-\Gamma_L)t} + 2D(p) \left[ R_{000} \cos \Delta m t - 3 \eta_{000} \sin \Delta m t \right] e^{-1/2(\Gamma_S-\Gamma_L)t} \tag{22}$$

where $D(p)$ is the dilution factor that describes the momentum dependent production asymmetry between $K^0$ and $\bar{K}^0$ at the target. NA48 measures:

$$\eta_{000} = (-0.002 \pm 0.019) + i(-0.003 \pm 0.021) \tag{23}$$

KLOE uses a pure $K_S$ beam tagged with a $K_L$. This allows KLOE to directly search $K_S \to 3\pi^0$ decays [63]. The selection is done on a sample of $\sim 5 \times 10^7 K_L - K_S$, looking for six photon clusters in the calorimeter and applying a veto for charged tracks in the drift chamber. For each decay both $2\pi^0$ and $3\pi^0$ hypotheses are tested defining two $\chi^2$-like variables. The events are counted in the signal box in the plane defined by these two variables. KLOE finds 2 events with about 3 background events expected, leading to the following 90% limits on the branching ratio and on the module of $\eta_{000}$:

$$BR(K_S \to 3\pi^0) \leq 1.2 \times 10^{-7}$$

$$|\eta_{000}| \leq 0.018 \tag{24}$$

The comparison of NA48 and KLOE results is shown in Fig. [4].

The $K_L$ decays to three pions have been measured by KLOE and KTeV, and a preliminary result for the $3\pi^0$ decay has been also presented by NA48.

C. Results

Following the method outlined in Sections [4A and [4B] and the values in Tab. [4] the KLOE Collaboration together with G. Isidori and G. D’Ambrosio obtained the following results:

$$Re \delta = (161.0 \pm 1.0) \times 10^{-5} \tag{25}$$

$$Im \delta = (1.3 \pm 2.0) \times 10^{-5}$$

and from Eq. [5]:

$$\Gamma_K^0 - \Gamma_{\bar{K}^0} = (5 \pm 4) \times 10^{-18} \text{ GeV} \tag{26}$$

$$m_{K^0} - m_{\bar{K}^0} = (-2.2 \pm 2.0) \times 10^{-18} \text{ GeV}$$

The 90% and 68% C.L. regions are shown in Fig. [10].

Assuming no CPT violation in the decay ($\Gamma_{K^0} - \Gamma_{\bar{K}^0} = 0$), they find the 95% C.L.:

$$-4 \times 10^{-19} < m_{K^0} - m_{\bar{K}^0} < 7 \times 10^{-19} \text{ GeV} \tag{27}$$

to be compared with the previous determination due to the CPLEAR Collaboration:

$$-10 \times 10^{-19} < m_{K^0} - m_{\bar{K}^0} < 17 \times 10^{-19} \text{ GeV} \tag{28}$$

Eq. [24] is at present the most stringent test of CPT symmetry. The uncertainty is dominated by the knowledge of the phases $\phi_{+-}$ and $\phi_{00}$, and of the parameter $3x_+$. Further improvements are expected from the analysis of the complete data sample of KLOE ($\sim 2.5 \text{ fb}^{-1}$), in particular for what concerns the $K_S$ rare decays. However, there are no plans to further improve the determination of $\phi_{+-}$, $\phi_{00}$.

VI. TEST OF LEPTON UNIVERSALITY WITH $K_{L2}$ DECAYS

The measurement of the ratio $R_K = K_{L2}^{\pm}/K_{\mu2}^{\pm}$ between the branching ratios of $K^\pm \to e^\pm \nu$ and $K^\pm \to \mu^\pm \nu$ decays is a sensible tool to test the lepton flavor universality and the V-A structure of the weak interactions; similar arguments hold for pion physics with the ratio $R_\pi = \pi^\pm \to e^\pm \nu/\pi^\pm \to \mu^\pm \nu$. Due to the uncertainties on non perturbative quantities like $f_K$, the relevance of the single decays, like $K_{L2}^{\pm}$, in probing the SM is severely hindered. In the ratio of the electronic and muonic decay modes, the hadronic uncertainties cancel to a very large extent, and the SM prediction of $R_K$ is known with excellent accuracy [26]. This makes possible to exploit the good experimental precision on $R_K$ to constrain new physics effects.

NA48/2 experiment at CERN SPS during 2003 data taking, has already collected more than 4 times the world total $K_{L2}^{\pm}$ statistics [5]. A similar amount of data has been collected in 2004. The 2003 sample has been already analyzed, and 4670 $K_{L2}^{\pm}$ events have been found. In figure [11] the missing mass squared versus E/p distribution for $K_{L2}^{\pm}$ candidates events is shown; background contributions have been identified and subtracted. The $R_K$ measurement is

$$R_K = (2.416 \pm 0.043_{\text{Stat}} \pm 0.024_{\text{Syst}}) \times 10^{-5}. \tag{29}$$

This value significantly improves the previous PDG value $14$ $R_K = (2.44 \pm 0.11) \times 10^{-5}$, and will further improve with NA48/2 current analysis. Another $R_K$ measurement is in progress by the KLOE collaboration: using the complete data set ($2.5 \text{ fb}^{-1}$), which should have a statistical error comparable to the one of the NA48/2 and completely different contributions to the systematic error.

The NA48/2 measurement has to be compared with the SM prediction which reads

$$R_K^{SM} = (2.416 \pm 0.043_{\text{Stat}} \pm 0.024_{\text{Syst}}) \times 10^{-5}. \tag{30}$$

Denoting with $\Delta r_{NP}^e$ the deviation from $\mu - e$ universality in $R_K$ due to new physics, i.e.: $R_K =$
FIG. 10: Allowed region at 68% and 95% C.L. in the $\text{Re}, \text{Im}$ plane, and in the $\Gamma_{K^0} - \Gamma_{\bar{K}^0}$, $m_{K^0} - m_{\bar{K}^0}$ plane.

FIG. 11: Missing mass squared versus $E/p$ distribution of $K^\pm$ candidates events, from NA48/2 experiment.

\[ R_{K}^{SM} \cdot (1 + \Delta r_{NP}^{e-\mu}) \], the NA48/2 result requires at the $2\sigma$ level:

\[ -0.063 \leq \Delta r_{NP}^{e-\mu} \leq 0.017. \quad (31) \]

Two-body kaon decays are helicity suppressed in SM but generally unsuppressed in SM extensions, like the low-energy minimal SUSY extensions of the SM (MSSM) considered in [67]. Here the question addressed is whether the SUSY can cause deviations from $\mu - e$ universality in $K_{12}$ decays at a level which can be probed with the present attained experimental sensitivity (percent level).

VII. CKM UNITARITY AND $K \to \pi \nu \bar{\nu}$ DECAYS

The $K \to \pi \nu \bar{\nu}$ decays in the SM framework are treated in detail in a number of papers and reviews (e.g. see “Rare kaon decays” in [14] and references therein). Here we recall some interesting aspects of these decays, before describing the status of the measurement of the $K_L \to \pi^0 \nu \bar{\nu}$ and $K^\pm \to \pi^\pm \nu \bar{\nu}$ branching ratios.

The unitarity of the CKM matrix assures the absence of Flavor Changing Neutral Current (FCNC) transitions at the tree level. FCNC processes can take place via loop diagrams containing internal quarks and intermediate bosons. The semileptonic rare FCNC transitions $K_L \to \pi^0 \nu \bar{\nu}$ and $K^\pm \to \pi^\pm \nu \bar{\nu}$ are particular and important because of their clean theoretical character. Firstly, the low energy hadronic matrix elements required are just the matrix elements of quark currents between hadron states, which can be extracted with good accuracy from non-rare semileptonic decays, $K_{l3}$. Moreover, the main contribution to the FCNC processes comes from the region of very small distances ($\sim 1/m_t, 1/m_Z$) where accurate estimate of strong interactions effects is possible in the framework of perturbative QCD. Finally these processes are also sensitive to the contributions from new heavy objects, e.g. supersymmetric particles, therefore the comparison of the experimental results with reliable theoretical estimates in the SM framework allows to search for new physics signals in these decays. The $K_L \to \pi^0 \nu \bar{\nu}$ and $K^\pm \to \pi^\pm \nu \bar{\nu}$ completely determine the apex the Unitarity Triangle [68] and the comparison with the Unitarity Triangle measured from B sector could provide decisive tests in the flavor physics. Actually new physics
may differentiate between the kaon and B-mesons sectors. At present the SM predictions of the two

\[ K \to \pi \nu \bar{\nu} \] decay rates are not extremely precise and read \( \text{BR}(K^\pm \to \pi^\pm \nu \bar{\nu})_{\text{SM}} = (8.8 \pm 1.1) \times 10^{-11} \) \[ \quad \text{and} \quad \text{BR}(K_L \to \pi^0 \nu \bar{\nu})_{\text{SM}} = (3.0 \pm 0.6) \times 10^{-11} \].

The two \( K^\pm \to \pi^\pm \nu \bar{\nu} \) candidate events observed by the BNL-787 \[ \text{[21]} \] experiment and the one more candidate event observed by BNL-949 \[ \text{[22]} \] demonstrate that the search for processes with branching ratios below \( 10^{-10} \) with missing energy although very difficult is not impossible. The branching ratio inferred from these candidate events is \( (14.7^{+13.0}_{-8.9}) \times 10^{-11} \), with a central value higher than the SM prediction, but compatible with this latter within the large errors. As sketched in figure \[ \text{[23]} \] a measurement of \( \text{BR}(K^\pm \to \pi^\pm \nu \bar{\nu}) \) at the 10% level is required in order to match the theoretical uncertainty, providing ground for precision tests of the SM flavor structure. An indirect model-independent upper bound on \( K_L \to \pi^0 \nu \bar{\nu} \) branching ratio can be set using the \( \text{BR}(K^\pm \to \pi^\pm \nu \bar{\nu}) \) value \[ \text{[23]} \], the BNL-787/949 measurement gives a \( \text{BR}(K_L \to \pi^0 \nu \bar{\nu}) \) value which is about two order of magnitude greater than the SM expectation:

\[ \text{BR}(K_L \to \pi^0 \nu \bar{\nu}) \simeq 1.4 \times 10^{-9} \quad (90\% \ C.L.) \] \[ \text{(32)} \]

Recently the E391a experiment at the KEK proton synchrotron \[ \text{[24]} \] improved the experimental information on the \( K_L \to \pi^0 \nu \bar{\nu} \) decay setting a new upper limit of \( 2.1 \times 10^{-7} \) at 90% confidence level for the branching ratio, and this using only about 10% of the collected data in the first of the two data taking periods. This limit improves of a factor 2.8 the previous limit \[ \text{[25]} \] by KTeV at Fermilab which pioneered the so called pencil beam technique. The latter is the only method used nowadays to study the \( K_L \to \pi^0 \nu \bar{\nu} \) decay and it consists in using a well collimated neutral kaon beam in order to be able to use the beam direction as a kinematical constrain on the \( K_L \) momentum. Analyzing the whole acquired statistics, the E391a experiment aims to reach the Grossman-Nir limit (see equation \[ \text{[26]} \]). After the stopping of KOPIO experiment by US DoE, the future of \( K_L \to \pi^0 \nu \bar{\nu} \) decay study relies on the step by step approach of J-PARC hadron facility. A Letter of Intent \[ \text{[27]} \] has been presented, which foresees firstly to move the E391a detector at J-PARC, and afterwards the building of a new detector and a dedicated beam line with the aim to collect about 100 \( K_L \to \pi^0 \nu \bar{\nu} \) events, if SM holds.

Because of the BNL-787/949 measurements, the situation is completely different for the \( K \to \pi \nu \bar{\nu} \) charged mode. As already noted, a 100 events measurement could clarify the apparent discrepancy between the BNL measurement and the SM theoretical predictions. Unfortunately the BNL program for the study of \( K^\pm \to \pi^\pm \nu \bar{\nu} \) decay using stopped kaons was terminated prematurely. Five years ago at Fermilab, another initiative \[ \text{[28]} \], called Charged Kaon at the Main injector (CKM) and based on in-flight decay technique to measure \( K^\pm \to \pi^\pm \nu \bar{\nu} \) with a statistics of 100 events, was proposed. The experiment was approved, but never funded and eventually terminated.

The main background in studying \( K^\pm \to \pi^\pm \nu \bar{\nu} \) decays comes from the abundant \( K^\pm \to \pi^\pm \pi^0 \) events. A kinematical constraint can be used (see regions I and II in figure \[ \text{[14]} \]) to separate signal from background. However events in which the \( \pi^0 \) is lost and the the \( K^\pm \) and the \( \pi^\pm \) are not properly reconstructed can fake the signal. Moreover, due to the dependency of the photon detection efficiency from the kaon momentum, the use of a higher kaon beam momentum can help in the \( \pi^0 \) rejection. These considerations drove the proposal P326 (a.k.a. NA48/3) \[ \text{[29]} \] for the CERN SPS which delivers protons of 400 GeV. The P326 strategy relies on a 10^{-8} \( \pi^0 \) rejection (this requires a
single photon rejection of $10^{-5}$ for $E_\gamma > 1$ GeV, on a redundant particle identification, and on the need of tracking and precisely timing about 800 MHz of particles. P326 aims to receive full approval by end of 2006 and expects to collect about 100 events in two years with 10 % of background.

VIII. CONCLUSIONS

There is plenty of new results in kaon physics since the 2005 Physics in Collision edition. In this talk only a selection of these has been reported in. The NA48/2 experiment searching for direct CP violation in charged kaon decays, measured the asymmetries $A_y$ and $A_y^0$ respectively in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays. With the present experimental accuracy, both asymmetries are compatible with zero. The unexpected $\pi \pi$ scattering length measurement in the $K^{\pm} \rightarrow \pi^{\pm} \pi^{\pm} \pi^{\pm} \pi^{\pm}$ channel, opened a new way to test the structure of the $\pi \pi$ interaction at low energies. Using an up-to-date set of experimental measurements of form factors, branching ratios and lifetimes from KLOE, KTeV, BNL E865, and NA48 experiments, a new determination of the Cabibo angle $V_{us}$ has been done proving the unitarity of the CKM matrix at 1σ level. The NA48/2 preliminary measurement of $R_K$ and some theoretical works, renewed the interest in $K^{\pm}$ decays as particular interesting probe of new physics effects. Finally the status and the perspectives for the golden-plated decays $K \rightarrow \pi \nu \bar{\nu}$ have been presented, with the step by step approach followed at KEK for the BR measurement of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay and the proposal to measure the $K^{\pm} \rightarrow \pi^{\pm} \nu \bar{\nu}$ branching ratio at CERN SPS.

Kaons offer a unique playground to the test the Standard Model and to shed light on physics beyond the SM.

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