The cosmic $^6\text{Li}$ and $^7\text{Li}$ problems and BBN with long-lived charged massive particles

Karsten Jedamzik

Laboratoire de Physique Mathématique et Théorique, C.N.R.S.,
Université de Montpellier II, 34095 Montpellier Cedex 5, France

Charged massive particles (CHAMPs), when present during the Big Bang nucleosynthesis (BBN) era, may significantly alter the synthesis of light elements when compared to a standard BBN scenario. This is due to the formation of bound states with nuclei. This paper presents a detailed numerical and analytical analysis of such CHAMP BBN. All reactions important for predicting light-element yields are calculated within the Born approximation. Three priorly neglected effects are treated in detail: (a) photodestruction of bound states due to electromagnetic cascades induced by the CHAMP decay, (b) late-time efficient destruction/production of $^3\text{H}$, $^7\text{Li}$, and $^7\text{Li}$ due to reactions on charge $Z = 1$ nuclei bound to CHAMPs, and (c) CHAMP exchange between nuclei. Each of these effects may induce orders-of-magnitude changes in the final abundance yields. The study focuses on the impact of CHAMPs on a possible simultaneous solution of the $^6\text{Li}$ and $^7\text{Li}$ problems. It is shown that a priorly suggested simultaneous solution of the $^6\text{Li}$ and $^7\text{Li}$ problems for a relic decaying at $\tau_x \approx 1000$ sec is only very weakly dependent on the relic being neutral or charged, unless its hadronic branching ratio is $B_h \ll 10^{-4}$ very small. By use of a Monte-Carlo analysis it is shown that within CHAMP BBN the existence of further parameter space for a simultaneous solution of the $^6\text{Li}$ and $^7\text{Li}$ problem for long decay times $\tau_x \gtrsim 10^8$ sec seems possible but fairly unlikely.

I. INTRODUCTION

Big Bang nucleosynthesis (BBN) is one of the standard pillars of modern cosmology. In its simplest version, reduced to a model with only one parameter, i.e. the contribution of baryons to the critical density, $\Omega_b h^2 \approx 0.0224$ [1], standard BBN predicted and observationally inferred primordial light element abundances are very close. This holds particularly true for $^2\text{H}$, and with somewhat less confidence also for $^4\text{He}$. However, when the $A > 4$ elements are considered agreement is less convincing. The observationally inferred $^7\text{Li}/\text{H}$ ratio is about a factor three smaller than that predicted in SBBN [2]. Moreover, $^6\text{Li}$ which is known to only be synthesized at the level $^6\text{Li}/\text{H} \sim 10^{-15} - 10^{-14}$ during SBBN has been recently observed in about a dozen metal-poor halo stars with abundance $^6\text{Li}/\text{H} \sim 3 - 5 \times 10^{-12}$ [3]. It is tantalizing that these observations indicate a plateau-structure, similar to that observed in $^7\text{Li}$, i.e. $^6\text{Li}$ abundance independent of metallicity of the star, for stars at the lowest metallicities. A $^6\text{Li}$ plateau, should point to a pregalactic or primordial origin of this isotope, since the $^6\text{Li}$ had already been in place before stars produced metallicity (and cosmic rays). However, it is cautioned that fairly uncertain stellar pre-main-sequence (PMS) destruction of $^6\text{Li}$ could contrive to give an apparent plateau [3].

$^7\text{Li}$ (as well as $^6\text{Li}$) are observed in the atmospheres of metal-poor halo stars. When transported to the hotter interior of the star, by either convection or turbulence, both isotopes may be destroyed. It is thus possible that atmospheric $^7\text{Li}$ has been depleted by some factor though standard stellar models do not forsee this. A number of groups have recently re-studied this possibility [3, 7, 8]. Postulating stellar turbulence with a parametrised magnitude, but of unknown origin, Korn et al. [8] claim that a star-to-star homogeneous factor 1.95 depletion is possible and even favorable when observations of the metal-poor globular cluster NGC6397 are considered. If true, the remaining factor $\sim 1.5$ could be either due to systematic errors in the effective stellar temperature calibration or due to an overestimate of the SBBN predicted $^7\text{Li}$ abundance due to systematic errors in nuclear reaction data. Concerning the second possibility, a recent remeasurement of the key $^7\text{Li}$ producing reaction $(^3\text{He}(\alpha, \gamma)^7\text{Be})$ seems to rather indicate a slight underestimate of the synthesized $^7\text{Li}$ [9].

$^6\text{Li}$ is known to be produced by spallation ($p + \text{CNO} \rightarrow \text{LiBeB}$) and fusion ($\alpha + \alpha \rightarrow \text{Li}$) reactions by standard cosmic ray primaries scattering off nucleons and nuclei in the intergalactic medium [10]. Though this process may explain the observed $^6\text{Li}$ at solar metallicity, it is clear, however, that it falls short by a large factor ($\sim 50$) to explain the $^6\text{Li}$ observed at low metallicity. Similar holds true for putative cosmic ray populations due to shocks developed during structure formation [11]. In order to produce $^6\text{Li}/\text{H} \sim 5 \times 10^{-12}$ an early cosmic ray population of $\sim 100$ eV/nucleon is required [12]. Most candidate sources fall short of this. The few viable remaining sources are due to accretion on the central Galactic black hole, albeit with an efficiency a factor $10^4$ larger than that presently observed, or due to a significant fraction $\sim 0.1$ of all baryons forming supermassive stars (and cosmic rays) [12]. It may also be that our galaxy was host to a radio-loud quasar some time ago [13]. The energetic problem becomes even exaggerated when likely $^6\text{Li}$ destruction during the stellar PMS [3] and putative $^6\text{Li}$ destruction during the stellar main-sequence phases are considered, possibly solving the $^7\text{Li}$ discrepancy. Finally, it has also been suggested that the $^6\text{Li}$ may result in situ from production by solar flares within the first billion of years of the star’s life [14]. Though this seems possible, it is hard to evaluate if a sufficient fraction of the freshly synthesized $^6\text{Li}$ falls back into the stellar atmosphere rather than being expelled by the solar wind.
It is entirely possible that the $^7\text{Li}$ and $^6\text{Li}$ anomalies are signs of physics beyond the standard model possibly connected to the quest for the cosmic dark matter. Even very small non-thermal perturbations in the early Universe may lead to a significant and observable $^6\text{Li}$ abundance, without overly perturbing other light elements. It had thus been suggested that an anomalous high $^6\text{Li}$ abundance is due to non-thermal nuclear reactions (i.e. $^3\text{H} (\alpha, n)^4\text{Li}$ …) induced by the late-time $t \geq 10^3$ s electromagnetic [13, 16] or hadronic [17] decay of a relic particle, as for example the gravitino. $^6\text{Li}$ in abundance as observed in old stars may also be synthesized due to residual dark matter annihilations during the BBN epoch [18]. In particular, a standard thermal freeze-out due to residual dark matter annihilations during the BBN dance as observed in old stars may also be synthesized as a relic particle, as for example the gravitino. $^6\text{Li}$ in abundance as observed in old stars may also be synthesized due to residual dark matter annihilations during the BBN epoch [18]. In particular, a standard thermal freeze-out process of weak scale particle dark matter (such as supersymmetric neutralinos) is concomitant with the production of $^6\text{Li}$ in the right amount, given the dark matter mass falls in the range $20 \lesssim m_\chi \lesssim 90$ GeV, and annihilation is to a significant fraction hadronic and s-wave. Concerning a solution to the $^7\text{Li}$ problem, early attempts utilising the electromagnetic decay of a relic and the induced $^7\text{Be}$ photodisintegration [19] ($^7\text{Li}$ is mostly synthesized as $^7\text{Be}$, which later on electron-captures) have not proven viable due to unacceptable perturbations in the $^2\text{H}/\text{H}$ and $^4\text{He}/^3\text{H}$ ratios [20]. However, it has been shown that the hadronic decay of a relic during BBN, and the induced excessive neutron abundance may prematurely convert $^7\text{Be}$ to $^7\text{Li}$ which is then destroyed by proton capture. When $\Omega_\chi B_\chi \sim 1 - 5 \times 10^{-4}$, where $B_\chi$ is the hadronic branching ratio, a factor 2 - 4 destruction of $^7\text{Li}$ results [21]. For relic decay times $\approx 1000$ s, it is moreover possible to synthesize all the observed $^6\text{Li}$ by non-thermal nuclear fusion. This has been the first, and so far only, known simultaneous solution to the $^6\text{Li}$ and $^7\text{Li}$ problems. It is noted that such a decay also leads to a possibly problematic 30% - 50% increase in the synthesized $^3\text{H}/\text{H}$ ratio.

Within the context of minimal supersymmetric extensions of the standard model of particle physics, a simultaneous solution is nicely realised, either by heavy gravitino decay, or in the case that gravitinos are the lightest supersymmetric particles (LSPs) by the supersymmetric partner of the tau-lepton (the stau) decaying into gravitinos [21]. In the second scenario, an added benefit is that for the right parameters to solve the $^6\text{Li}$ and $^7\text{Li}$ problems, TeV staus left over from a thermal freeze-out at higher temperature, and decaying at $\tau_\tau \approx 1000$ s into $50 - 100$ GeV gravitinos produce naturally about the right amount of gravitinos to explain the dark matter and of a warmness interesting to the formation of large scale structure formation [22]. Unfortunately, staus of mass 1 TeV are too heavy to be discovered at the LHC.

Recently, it has been realised that the existence of electrically charged massive particles (CHAMPs) during the BBN epoch may lead to modifications of the synthesis of light elements [23, 24, 25], beyond those simply due to their decay. Since for gravitino LSPs, the next-to-LSP (NLO) is long-lived and in about half of the supersymmetric parameter space it is the electrically charged stau, such effects are important to consider. Other metastable charged relic particles possibly existing during BBN have been also proposed [26]. Modifications to BBN occur due to the formation of electrically bound states between the negatively charged CHAMPs and the positively charged nuclei. The realization that (meta)-stable weak-scale mass charged particles enter into bound states during and after BBN had already been made in the late eighties [27, 28, 29], when the possibility of charged dark matter was analyzed. Nevertheless, the influence of bound states on BBN had not been much discussed.

In this paper results of the up-to-now most detailed calculations of BBN nucleosynthesis in the presence of decaying negatively charged particles are presented. The analysis attempts to reveal all key processes important for a reliable prediction of light element yields, thereby revealing, heretofore neglected effects, which make orders of magnitude changes in the predicted BBN yields for much of the parameter space. These changes are found mostly for late decaying $\tau_\tau \sim 10^6$ s CHAMPs. The aim of the paper is to analyze the potential of bound-state nucleosynthesis to solve the cosmic $^7\text{Li}$ and $^6\text{Li}$ problems.

The outline of the paper is as follows. In Section 2 a discussion/analysis of all priorly suggested solutions to the $^7\text{Li}$ problem within bound-state nucleosynthesis is presented, whereas in Section 3 details of the present calculations are given. In Section 4 it is shown that BBN continues to very low temperatures $T \ll 1$ keV in the presence of bound states. Section 5 shows that bound states are efficiently photodisintegrated already at high temperature due to the decay of the relic. Section 6 stresses the importance of CHAMP transfer reactions at late times. Finally In Section 7 possible further solutions to the $^6\text{Li}$ and $^7\text{Li}$ problems for late-decaying CHAMPs $\tau_\tau \gtrsim 10^6$ sec are discussed. Section 8 draws the conclusions. An appendix gives some detail on the determination of reaction rates in the Born approximation.

II. BOUND-STATE BBN AND PRIOR SUGGESTED SOLUTIONS TO THE $^7\text{Li}$ PROBLEM

Modifications to BBN occur due to the formation of electrically bound states between the negatively charged CHAMPs and the positively charged nuclei. Since bound state binding energies may be appreciable (cf. Table 1), a significant fraction of $^7\text{Be}$ may be captured by CHAMPs at temperatures as high as $T \lesssim 30$ keV, whereas the same occurs at $T \lesssim 10$ keV for $^4\text{He}$. This may be seen in Fig. 11 which shows the fractions $f_{\text{tot}} = n_{(N,X)}/n_{(N)}^X$ of $^7\text{Be},^7\text{Li},^6\text{Li}$, and $^4\text{He}$ locked up within bound states. On first sight, the most important effect of bound states during BBN is a reduction of the Coulomb barrier [23, 24]. Nevertheless, since SBBN is essentially finished at $T \approx 10$ keV, Coulomb barrier modifications of reactions rates involving $^4\text{He}$ should be hardly important (even though,
ad hoc, speculated otherwise in Ref. [24]). However, as shown by Pospelov [23] there is a non-trivial catalytic effect on reactions involving photons in the final state. SBBN reaction rates involving dipole radiation (EI: e.g. $^3\text{He}(^4\text{He}, \gamma)^7\text{Be}$) scale as $\lambda^{-3}$, whereas reaction rates forbidden at the dipole approximation but allowed at quadrupole (E2; e.g. $^2\text{H}(^4\text{He}, \gamma)^6\text{Li}$) scale as $\lambda^{-5}$, where $\lambda$ is the wavelength of the emitted photon. This, in both cases is around $\sim 130$ fm. In the presence of a $^4\text{He}$-CHAMP bound state the reaction may proceed photonless (e.g. $^2\text{H}(^4\text{He}-X^-, X^-)^6\text{Li}$) and $\lambda$ is approximately replaced by the Bohr radius $a_{\text{Bohr}}$ of the $^4\text{He}$-CHAMP bound system. Since $a_{\text{Bohr}} \approx 4.8$ fm (cf. Table 1) very large enhancement factors of $7 \times 10^7$ and $3 \times 10^9$ [23, 30] for the S-factors of the $^2\text{H} + ^4\text{He}$, and $^3\text{He} + ^4\text{He}$ reactions, respectively, have been estimated.

A recent more detailed three-body nuclear reaction calculation of the $^2\text{H} + ^4\text{He}$ reaction, has reduced this estimate by a factor $\sim 10$ [21]. Such large enhancement factors are important as they lead to excessive $^7\text{Li}$ (and $^7\text{Li}$) production for any weak scale charged particles which are sufficiently long-lived $\tau_d \gtrsim 4 \times 10^6$ s, unless $\Omega_X \lesssim 3 \times 10^{-6}$. They have thus been utilised to place a stringent upper limit on the reheating temperature in the early Universe $T \lesssim 10^6$ GeV in the case when the supersymmetric gravitino exists and when it is the LSP [32]. Nevertheless, it seems somewhat premature to the put such upper limits, as the BBN with charged long-lived particles for decay times $\tau_d \gtrsim 10^6$ s had priorly not been investigated (cf. Section 8).

The putative existence of bound states during BBN has also led to a flood of claims of possible solutions to the $^7\text{Li}$ and/or $^6\text{Li}$ anomalies. In Ref. [24] it was realized that significant fractions of the $^7\text{Be}$ and $^7\text{Li}$ isotopes are within bound states during BBN. This has lead the authors to arbitrarily enhance certain reactions rates involving mass-7 elemental destruction processes by large factors, leading to the claim that the existence of bound states may solve the $^7\text{Li}$ overproduction problem. However, these claims are, up to now, unfounded [32] (see also below). In Ref. [25] it was noted that during the decay of $X^-$, when residing in a bound state with $^4\text{He}$, the $^4\text{He}$ nucleus could break up. The resultant energetic $^3\text{He}$ and $^3\text{He}$ could then fuse on $^4\text{He}$ to produce $^6\text{Li}$, in a similar to what had been proposed in [15, 17]. Though the suggestion is correct, the authors calculate the break-up probability to be very small (cf. also Ref. [35]), such that the $^6\text{Li}$ synthesis by catalytic $^2\text{H}(^4\text{He}-X^-, X^-)^6\text{Li}$ is by far dominant. The analysis of Ref. [30] (and Ref. [34]) essentially confirms the simultaneous solutions to the $^6\text{Li}$ and $^7\text{Li}$ problems as given in Ref. [21, 22], even when bound state effects are included. In Ref. [30] the case of almost degenerate NLSP staus $\tilde{\tau}$ and LSP neutralinos $\tilde{\chi}$ has been considered. Here mass splittings smaller than $\delta m = m_{\tilde{\tau}} - m_{\tilde{\chi}} \lesssim 1$ GeV have been assumed. In this region of $\tilde{\tau}$-\tilde{\chi} parameter space, motivated by the well-known $\tilde{\tau}$-\tilde{\chi} coannihilation region for neutralino dark matter, the stau is relatively long-lived due to final phase space supression of the decay. It is claimed, that the $^7\text{Li}$ overproduction problem may be solved by internal conversion of staus in bound states with $^7\text{Be}$, to neutralinos, e.g. $(\tilde{\tau} \rightarrow ^7\text{Be}) \rightarrow \tilde{\chi} + \nu_\tau + ^7\text{Li}$ and the subsequent decay of $^7\text{Li}$ by protons. If this is true, the $^7\text{Li}$ problem may be solved with $\delta m \gtrsim 100$ MeV even for the smallest abundances of staus. A more detailed analysis of the $^7\text{Be}$-bound state fraction via the Boltzmann equation shows, however, that only a very small fraction of $^7\text{Be}$ are within bound states, thus making modifications of the $^7\text{Li}$ abundance at low stau-density negligible. At larger stau-densities some effect may result.

Ref. [33] make the interesting suggestion that the $^7\text{Li}$ overproduction problem may be solved by partial decay of $^7\text{Li}$ via $(^7\text{Be}-X^-)(p, \gamma)(^8\text{B}-X^-)$, (and subsequently $^8\text{Be} \rightarrow ^8\text{Be} + e^+ + \nu_e$) nucleus. This reaction would mostly occur via a $p-^7\text{Be}$ resonance in the $^8\text{B}$ nucleus which, in the absence of bound states lies at 769.5 keV relative to the $p-^7\text{Be}$ continuum. This line would be produced via a $^8\text{Be}$ bound resonance in the $^8\text{B}$ nucleus which, in the absence of bound states lies at 769.5 keV relative to the $p-^7\text{Be}$ continuum. The catalysis in the reaction would occur by a shifting of the resonance to $\sim 167$ keV relative to the $p-^7\text{Be}$-continuum since the $^8\text{Be}-X^-$ bound state binding energy ($E_{^8\text{Be}-X^-} \approx 2.0$ MeV) is larger than that of $^7\text{Be}$ ($E_{^7\text{Be}-X^-} \approx 1.39$ MeV), making the resonance available at only slightly supra-thermal energies. Moreover, apart from the decrease in the resonance energy they also deduce a factor $\sim 10^3$ larger reaction rate coefficient. Adopting their calculated rates for $^7\text{Be}-X^-$ bound state formation and the $(^7\text{Be}-X^-)(p, \gamma)(^8\text{B}-X^-)$ reaction, I partially confirm this effect by full numerical analysis. For example, for $\tau_\chi = 1.5 \times 10^3$ s and the number ratio

FIG. 1: Bound state fractions $f_i^b \equiv n_{(N_i,X^-)}/n_{N_i}^{\text{tot}}$ of nuclei $N_i$ bound to CHAMP $X^-$ as a function of temperature $T$, for a model with $M_X = 100$ GeV and $\Omega_X h^2 = 0.1$ (corresponding to a CHAMP-to-baryon ratio $Y_{X^-} = 4.26 \times 10^{-10}/2$). Shown are $f_2^b$ for $^7\text{Be}$ solid (red), $^7\text{Li}$ long-dashed (green), $^7\text{Li}$ short-dashed (blue), and $^4\text{He}$ dotted (purple), respectively. Nuclear destruction of bound states results in a behaviour of $f_i^b$ different than that expected from simple estimates by the Saha equation. This is particularly seen in $f_2^b$ for $^7\text{Li}$ due to the $^4\text{He}$($^7\text{Li} - X^-, X^-)^4\text{He} + ^4\text{He}$ reaction.
of $X^{-} s$ to baryons $Y_{X} \approx 0.2$, I find a reduction of the $^7$Li abundance by 33%, and a $^6$Li/H ratio of $2 \times 10^{-11}$. However, the effect is not as strong as initially imagined, since by the reciprocity theorem the inverse rate is also enhanced. The inverse rate $1/\tau_{\text{inv}}$ is thus around $10^{5}$ times larger at $T \approx 32.5$ keV than the beta decay rate $1/\tau_{\beta}$ of $^8$B (half-time of 770 ms), converting $^8$B-$X^{-}$ rapidly back to $^7$Be-$X^{-} + p$, before $^8$B can beta-decay. The effect is therefore essentially absent at early times (i.e. small $\tau_{X}$). Nevertheless, the inverse rate quickly drops below the beta decay rate (i.e. $\tau_{\beta}/\tau_{\text{inv}} \approx 0.1$ at $T \approx 24.1$ keV). For the same parameters as above, I still find a 14% reduction of the final $^7$Li. This drops to 7% for $\tau_{X} = 10^{3}$ and $7 \times 10^{2}$ s, respectively.

It is interesting to know if the solution of the lithium problems proposed in Ref. [21] is changed when the decaying relic is charged, such as the stau. In Fig. 2 the parameter space solving either the $^7$Li problem, or both the $^6$Li and $^7$Li problems, is shown. The upper panel shows results for a charged relic and the lower panel for a neutral relic. Here observational limits as discussed in Ref. [27] have been applied and the $^6$Li, $^7$Li problems are assumed to be reconciled with observational data for $^6$Li/$^7$Li $\sim 0.03$ and $^7$Li/$^6$H $\sim 2.5 \times 10^{-10}$. The assumed parameters of the model are a hadronic branching ratio $B_{h} = 10^{-4}$ and relic mass $M_{X} = 1$ TeV. It is seen that even at $B_{h}$ as small as $10^{-4}$ the $^7$Li-solving region is essentially unmodified, whereas some changes are observed in the $^6$Li and $^7$Li solving regions. These latter are mostly due to excessive $^6$Li production when the relic is charged, disallowing some of the larger life times $\tau_{X} \sim 2 \times 10^{4}$ s. Bound state effects are nevertheless important when the hadronic branching ratio is very small. This may be seen in the lowest panel of Fig. 2 where $B_{h} = 0$ has been assumed. When only bound state effects are operative, the $^3$H/$^1$H ratio is essentially unmodified. This is in contrast to the solution of the lithium problems with a hadronic decay, as seen by the dotted (blue) lines in the upper two panels, beyond which $^2$H/$^1$H is larger than $4 \times 10^{-5}$. It is intriguing that both processes, hadronic decay and bound state effects, have the same preferred $\tau_{X}$ for a simultaneous solution of the lithium problems.

### III. DETAILED BOUND-STATE BBN CALCULATIONS

The calculations presented here attempt to take proper account of the influence of singly bound states on the nucleosynthesis for elements with nucleon number $A \leq 7$. Heavier elements as well as the formation of molecules, such as $(X^{-} - ^4\text{He} - X^{-})$, are not considered. All effects of electromagnetic and hadronic cascade nucleosyntheses are included and treated as presented in Ref. [37]. The fractions of individual nuclei $i$ in bound states $f_{i}^{b} = n_{i}(N,X^{-})/n_{N}^{\text{tot}}$ are computed by full numerical integration of the Boltzmann equation. This is required since estimates by the Saha equation are only very approximate, due to the relatively early freeze-out of the CHAMP-nuclei recombination process [24]. Except of the recombination rate of $X^{-}$ on $^3$He, which is taken from Ref. [35], all other recombination rates are computed by a numerical integration of the Schrödinger equation. This may make difference up to a factor two in $f_{i}^{b}$ since the recombination rates as given in Ref. [24] only apply asymptotically at low temperature $T$. Bound state wave functions and bound-state energies are also computed by an integration of the Schrödinger equation, assuming realistic charge radii for the nuclei as measured by experiment. The reader is referred to Table 1, for some of the bound state properties. Finally, it is important, to take into account the nuclear destruction of bound states. Nuclear rates are very fast at early times, and for reaction which are sufficiently exothermic, the electric bound between the final nucleus (nuclei) ought to be destroyed [39]. This often changes $f_{i}^{b}$ by orders of magnitude.

A proper evaluation of BBN yields with bound states is only possible when somewhat realistic nuclear reaction rates for nuclei within bound states are present. With the exception of the reaction $^2$H$(^4\text{He} - X^{-}, X^{-})^6$Li a more detailed evaluation of such reactions had been absent of the literature so far. Improving over simple scaling relations [24, 30] seems important also, since nuclear reactions including bound states contain three quantities of similar magnitude, $a_{B}$ the Bohr radius, $a_{\text{nucl}}$ the nuclear radius, and $k_{f}$ the momentum of the outgoing nucleus. All three quantities are in the several Fermi range, thus leading potentially to important cancellation effects. More importantly, estimates via simple scaling relations adopt the Born approximation, which is known to fail at low energies and strong perturbations [40]. This is essentially the case for all reactions of importance to bound-state BBN. The failure of the Born approximation had been seen, for example, by the reduction of the $^2$H$(^4\text{He} - X^{-}, X^{-})^6$Li rate by a factor $\sim 10$, when a more detailed evaluation [31] is compared to a simple scaling result.

I have identified all key reactions in bound-state BBN.
These are shown in Table 2. It is completely beyond the scope of the present paper to evaluate all these reaction rates more properly, i.e., beyond the Born approximation, a task which is formidable in particular when the important CHAMP-exchange reactions (cf. Section 6) are also considered. For the $^2\text{H}(^4\text{He}-X-, X^-)^6\text{Li}$ process the rate as given by Ref. [31] was adopted. For other reactions, as a starting point, I have thus nevertheless, evaluated rates in the Born approximation. These rates will serve as benchmarks later on. For details concerning these calculations the reader is referred to Appendix A. Results for the in such a way obtained $S$-factors are shown in Fig. 3 and Fig. 4 respectively.

### IV. LATE-TIME BIND-STATE BIG BANG NUCLEOSYNTHESIS

The reader may have noted that Table 2 also includes reactions with bound states on elements $^1\text{H}, ^2\text{H}, ^3\text{H}$ with only one charge number $Z = 1$. In fact, such reactions are extremely important at low temperatures $T \lesssim 3, 2, 1 \text{ keV}$.
when one after the other, non-negligible fraction of $^3\text{H}$, $^2\text{H}$, and $^1\text{H}$ enter into bound states. This may be seen in Fig. [5]. It is noted here, that a possible impact of such reactions has been pointed out before [24], albeit in a very approximative way. It was not clear, a priori, if the Coulomb barrier between, for example, $^1\text{H}$ and $^6\text{Li}$ is sufficiently supressed in order to make reactions such as $^6\text{Li}(^1\text{H}→^1\text{H})^3\text{He} + ^3\text{He}$ efficient enough to substantially reduce any priorly synthesized $^6\text{Li}$. This is because, on first sight, Coulomb shielding of the proton could only be partial, due to the fairly extended Bohr radius $a_B ≈ 29$ fm of the $^1\text{H}→^1\text{H}$ system. In Fig. [6] $l = 0$ spherical wave without any Coulomb repulsion, i.e. $V_c = 0$, is compared to the spherical Coulomb wave functions between the $^6\text{Li}$ and the $^1\text{H}→^1\text{H}$ bound state with $l = 0$ and $l = 1$ initial angular momentum, respectively. It is seen that essentially no Coulomb supression exists. Rather, the incoming wave function of the $^6\text{Li}$ nuclei is even strongly enhanced at the center. This is not surprising, as by assumption, the $X^−$ resides at the center, and due to the significant spread in the wave function of the proton ($a_B ≈ 29$ fm) the effective proton charge density at the center is low. The Coulomb potential for the $^6\text{Li}$ nucleus is $\phi_{6\text{Li}} = -3e^2\exp(-2r/a_B)(1/r + 1/a_B)$, thus very attractive at the center and approaching zero at large distances. Nuclear reactions between such bound states and bare nuclei, are therefore not Coulomb supressed. It is rather conceivable, that Coulomb focussing occurs at low energies, even enhancing the reaction rates over the $V_C = 0$ case. This may be observed in the $S$-factor for the $(^1\text{H}→^1\text{H}) + ^6\text{Li}→^4\text{He} + ^3\text{He} + X^−$ reaction as shown in Fig. [4]. It is noted here, that due to an anomalously low $7\text{Li}(^1\text{H}→^1\text{H})^8\text{Be}$ rate found in the Born approximation the rate for $7\text{Li}(^1\text{H}→^1\text{H})^8\text{Be}$ has been computed and utilised in the calculations.

Thus, $Z = 1$ bound states at $T ≈ 1$ keV behave almost as neutrons (with the exception that they are stable). Already very small fractions of these bound states induce therefore a second round of late-time nucleosynthesis, capable of destroying all the synthesized $^6\text{Li}$,$^8\text{Be}$, and some of the $^7\text{Li}$. This may be seen in Fig. [7] where the $^{6}\text{Li}/^7\text{Li} + ^7\text{Li}/^8\text{Be}$, and $^3\text{H}/^2\text{H}$ ratios are shown for a CHAMP with $\Omega_\chi h^2 = 0.01$, $\Omega_b h^2 = 100$ GeV, and decay time $\tau_\chi = 10^{10}$ s, where $h$ is the dimensionless present-day Hubble parameter, and $\Omega_\chi$ the fractional contribution of CHAMPs to the present critical density, would they not have decayed. Note, that this is easily converted to the CHAMP-to-baryon ratio $Y_\chi = (\Omega_\chi h^2/\Omega_b h^2)(m_p/m_\chi)$ which is $Y_\chi ≈ 4.26 \times 10^{-3}$ for the adopted parameters. The calculations presented in Fig. [7] (as well as Figs. [8] and [9]) are performed under the assumption that the $X^−$ decay is not associated with any electromagnetic- or hadronic- energy release and in the absence of $X$-exchange reactions (cf. Section 6). This is done to isolate the effects of the bound states. At early times, towards the end of conventional BBN, when a significant fraction of $^4\text{He}$ enters bound states, the reactions $^2\text{H}(4\text{He}→X^−, X^−)^6\text{Li}$, $^3\text{H}(4\text{He}→X^−, X^−)^7\text{Li}$, and $^3\text{He}(4\text{He}→X^−, X^−)^7\text{Be}$, synthesize significant, and observationally completely unacceptable abundances of the $A > 4$ isotopes. However, when bound states of the $Z = 1$ elements form at $T ≈ 1$ keV, essentially all the synthesized $^6\text{Li}$ and $^7\text{Li}$ may be rapidly destroyed by the reactions $^6\text{Li}(^1\text{H}→^1\text{H})^8\text{Be}$, $^3\text{He}$, and $^7\text{Li}(^1\text{H}→^1\text{H})^8\text{Be}$. The situation appears different for $^7\text{Li}$, due to the small estimate for

![FIG. 4: Nuclear reaction $S(E)$-factors as function of energy computed in the present analysis. The most important $S$-factors for nuclear reactions involving bound states with $Z = 1$ nuclei are shown: (from top to bottom at the highest energies) $^6\text{Li}(^1\text{H}→^1\text{H})^3\text{He} + ^3\text{He}$ double-dotted (black), $^7\text{Be}(^1\text{H}→X^−, X^−)^4\text{Be}$ long-dashed (green), $^4\text{He}(^1\text{H}→X^−, X^−)^6\text{Li}$ dotted (purple), $^4\text{He}(^1\text{H}→X^−, X^−)^6\text{Li}$ dash-dotted (light-blue), $^7\text{Li}(^1\text{H}→X^−, \gamma)^6\text{Be} + X^−$ short-dashed (blue), and $^3\text{He}(^1\text{H}→X^−, X^−)^7\text{Be}$ solid (red).](image)

![FIG. 5: Bound state fractions $f^b_i$ for $^3\text{H}$ (solid - red), $^2\text{H}$ (dashed - green), and $^1\text{H}$ (blue - dotted) as a function of temperature $T$. Adopted model parameters are as in Fig. [4] with a $X$ decay time $\tau_\chi = 10^{10}$ s. For illustrative purposes photodisintegration of bound states due to $X$-decay (cf. Section 5) and $X$-exchange reactions (cf. Section 6) have not been taken into account.](image)
the $^7\text{Li}$(H−X−,X−)$^8\text{Be}$ and $^7\text{Li}$(H−X−,$\gamma$)$^8\text{Be}$-X− cross sections, implying that almost all initially synthesized $^7\text{Li}$ is left intact. The abundance of $^7\text{Li}$/H is found at an observationally friendly $2.7 \times 10^{-10}$. It is noted that $^2\text{H}$ is also destroyed, though to a much smaller degree, mostly by the reactions

$$^3\text{H}(2\text{H}−X−,n)^3\text{He}+X−,$$  
$$^3\text{He}(2\text{H}−X−,p)^4\text{He}+X−,$$  
$$^4\text{H}(3\text{H}−X−,n)^4\text{He}+X−,$$  
$$^5\text{He}(3\text{H}−X−,p)^4\text{He}+X−,$$  
$$^7\text{Li}(1\text{H}−X−,X−)^7\text{Be}.$$  

The reader is referred to Table 3 concerning assumptions about the rate of these, and some other reactions involving only $A \leq 4$ elements. Furthermore, when regarding Fig. 7 in more detail, one also notes late-time production of $^6\text{Li}$ and $^7\text{Be}$ at some level due to the $^4\text{He}(^2\text{H}−X−,X−)^6\text{Li}$ as well as the $^6\text{Li}(1\text{H}−X−,X−)^7\text{Be}$ reactions.

It is thus premature to conclude, that extreme $^6\text{Li}$ overproduction, rules out the existence of CHAMPS with long life times. The model shown above, at CHAMP densities many (five !) orders above those already claimed to be ruled out by $^6\text{Li}$ overproduction is observationally viable in all abundances. Constraints on the existence of CHAMPS in the early Universe could therefore, in principle, be much milder for long X− life times than initially predicted. Nevertheless, they is further important physics entering the calculations discussed in the next two sections.

| No. | $(AX) + B \rightarrow C + X$ | enhancement |
|-----|----------------------------|-------------|
| 10  | $^2\text{H}$(H−X−,X−)$^3\text{He}$ | $1.25 \times 10^2$ |
| 11  | $^1\text{H}$(H−X−,X−)$^3\text{He}$ | 10.7 |
| 12  | $^3\text{H}$(H−X−,X−)$^4\text{He}$ | 1 |
| 13  | $^1\text{H}$(H−X−,X−)$^3\text{He}$ | 1 |

V. PHOTODISINTEGRATION OF BOUND STATES BY THE DECAY OF THE CHAMPS

There is another effect, heretofore overlooked, which may significantly reduce the in catalytic BBN at $T \approx 10\text{ keV}$ synthesized $^6\text{Li}$ (and $^7\text{Li}$) abundance. CHAMP decays are typically accompanied by the injection of electromagnetically interacting particles, with total energy comprising often a large fraction of the X rest mass. It is well-known, that such particles ($e^−$, $e^+$, and $\gamma$’s) induce a rapid cascade on the cosmic blackbody photons, due to $\gamma\gamma_{\text{BB}}$ pair creation and inverse Compton scattering $e^\pm + \gamma$ processes, until the energy of any remaining $\gamma$’s is too low to further pair-produce, i.e. for $E_{\gamma} \lesssim E_{\text{th}} \approx m_{\gamma}^2/2T \approx 1.2\text{ MeV}(T/10\text{keV})^{-1}$. It is
seen, that this energy is above the binding energy of $^4\text{He}-X^-$ (and $^4\text{H}-X^-$) even at temperatures as high as $T \approx 30$ keV, making possible the $^4\text{He}-X^-$ and ($^4\text{H}-X^-$) bound state photodisintegration before any significant $^6\text{Li}$ synthesis (destruction) has occurred. In Fig. 3 the resultant photon spectrum due to the injection of energetic electromagnetically interacting particles at cosmic epochs with temperature $T = 10, 1$, and 0.1 keV is shown. The shown spectrum $\varepsilon_\gamma \, d\varepsilon_\gamma /d\varepsilon_\gamma$ is generated by a Monte-Carlo simulation taking account, not only of $e^\pm$ pair production and inverse Compton scattering, but also $\gamma\gamma$ scattering (important at high $E_\gamma \lesssim E_{\text{th}}$, Bethe-Heitler pair production $\gamma + p, ^4\text{He} \rightarrow p, ^4\text{He} + e^- + e^+$, Compton scattering of the produced $e^\pm$, as well as the important Thomson (Klein-Nishina) scattering of $\gamma$'s on thermal electrons. It is based on the calculations presented in Ref. 39, with the Thomson scattering process extended to energies as low as $E_\gamma \approx 25$ keV, to account for $^1\text{H}-X^-$ destruction.

Following secondary and tertiary, etc. generations of scattered photons to obtain the correct photon spectrum for the bound state destruction process is mandatory. For example, the injection of 1 TeV of electromagnetically interacting energy at $T = 1$ keV is associated with injection of $N_\gamma \approx 3.3 \times 10^6$ primary photons with energy $E_\gamma \gtrsim 25$ keV, resulting from the initial cascade on the blackbody. When further interactions of these $\gamma$'s are considered the number rises to $N_\gamma \approx 1.1 \times 10^8$. In other words, an injected photon takes about 30 interactions before dropping below the threshold for $^1\text{H}-X^-$ photodisintegration. This exemplifies the importance of subsequent $\gamma$ interactions. In Fig. 3 one may note a "pile-up" of photons at low $E_\gamma$. This is due to the typical fractional loss of $\gamma$'s in the Thomson regime $E_\gamma \lesssim m_e$ being small, such that it takes several Thomson scatterings for a photon to have dropped below $E_\gamma \lesssim E_{\text{th}}^b$ 25 keV. A similar pile-up does not exist at $E_\gamma \lesssim E_{\text{th}}^{1\text{He}} \approx 350$ keV since during scatterings of $\gamma$'s with energy $E_\gamma \sim m_e$ on electrons the $\gamma$'s may loose a significant fraction of their energy. We thus expect the effects of photodisintegration of bound states have a larger impact on the $^1\text{H}-X^-$ bound state fraction than on that of $^4\text{He}-X^-$. This effect is not only due to the above, but also due to the photodisintegration cross section of $^1\text{H}-X^-$, $\sigma_{^1\text{H}-X^-}$ being larger than the one for $^4\text{He}-X^-$. Note that all calculations below, include numerically evaluated cross sections for the photodisintegration of all $A \leq 7$ nuclear bound states.

In Fig. 4 the bound state fractions in two scenarios: (a) of $^4\text{He}$ for a model with $\Omega_X h^2 = 0.1$ and $\tau_X = 3 \times 10^8$ (and electromagnetic decay), and (b) of $^1\text{H}$ for $\Omega_X h^2 = 5 \times 10^{-3}$ and $\tau_X = 3 \times 10^8$, are shown in the same graph. Here the solid lines show $f_{^1\text{H}}^b$ ($f_{^4\text{He}}$) when non-thermal bound state photodestruction is included, whereas the dotted lines show results when it is neglected. It is seen that realistic bound state fractions are significantly lower. In scenario (a) a $^6\text{Li}/^1\text{H}$ ratio $\sim 10^5$ to $\sim 10^8$ is lower, compared to when photodestruction is neglected, whereas in scenario (b) the $^6\text{Li}/^1\text{H}$ ratio is

$$\sim 100$$ times higher. Here case (b) is affected by a reduced efficiency of $^6\text{Li}(^1\text{H}-X^-, X^-)^4\text{He} + ^3\text{He}$, whereas in case (a) the reaction $^2\text{H}(^4\text{He}-X^-, X^-)^6\text{Li}$ is rendered less dominant. For sufficiently high $\Omega_X$, and when thermal photodisintegration is unimportant, the resultant bound state fraction may be estimated by a steady state between the recombination rate, i.e. $\langle \sigma v \rangle_{\text{rec}} n_{^4\text{He}} n_{X^-}$ and the photodisintegration rate, i.e. $\langle \sigma v \rangle_{\text{ph}} n_{^6\text{Li}} n_{X^-}$. Here $n_{^4\text{He}}, n_{^6\text{Li}}, n_{X^-}$, and $n_{X^-}$ are free $^4\text{He}$, bound $^4\text{He}$, $X^-$, and nonthermal photon number densities, respectively. The nonthermal photon number density $n_{X^-}$ may be obtained from $n_{X^-} \approx d\varepsilon_X / d\varepsilon_{\text{th}} n_{^4\text{He}}$ where $d\varepsilon_X / d\varepsilon_{\text{th}} \approx n_{X^-} / \tau_X$ before substantial decay, $\tau_{\text{th}}$ is the life time of photons against Thomson scattering (i.e. the typical survival time), and $N_{^4\text{He}}$ is the typical number of photons per particle decay with energy above the photodisintegration threshold $E_{\text{th}}$ (including secondary generations). This, for example at $T = 1$ keV, is approximately $4 \times 10^6$ and $1 \times 10^8$ for $^4\text{He}$ and $^1\text{H}$ bound state photodisintegration, respectively, per 1 TeV of electromagnetically interacting energy injected into the plasma. It is thus found

$$f_{^4\text{He}}^b \approx \frac{n_{^6\text{Li}} n_{X^-}}{n_{^4\text{He}}} \approx \frac{\langle \sigma v \rangle_{\text{rec}} \tau_{X^-}}{\langle \sigma v \rangle_{\text{ph}} \tau_{\text{th}} n_{^4\text{He}}^2}$$

It may be noted that this expression, which is valid only for large $X^-$ is independent of the CHAMP-tobaryon ratio, but dependent on the CHAMP life time.

VI. CHAMP EXCHANGE REACTIONS

It has been shown in Section 4 that the existence of only small fractions $f_P^b \sim 10^{-5}$ of protons in bound states,
forming below $T < 1$ keV, may efficiently destroy again any priorly synthesized $^6\text{Li}$ and $^7\text{Be}$. In Section 5 it has been seen that the efficiency of this destruction may be significantly reduced when non-thermal photodestruction of bound states is taken into account. In this section, a further important process reducing late-time $^6\text{Li}$ and $^9\text{Be}$ destruction is discussed. CHAMPs in bound states may exothermically transfer to heavier nuclei of equal or higher charge. In particular, $(^1\text{H}^-X^-) + ^3\text{He}$ is of interest (Eq. A4) as that shown in Fig. 5. Bound states could be removed by the $(^1\text{H}^-X^-) + ^4\text{He}^-X^- + ^4\text{He}^-X^- + ^3\text{He}$ charge exchange process. Charge exchange reactions turn out to be very important. In Table 4 the most important of these processes are presented. Rates for these processes were calculated in a very similar way, i.e.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
No. & $(AX) + B \rightarrow C + X$ & rate [cm$^3$s$^{-1}$] \\
\hline
14 & $(^1\text{H}^-X^-) + ^3\text{He} \rightarrow (^2\text{H}^-X^-) + ^4\text{He}^-X^- + ^3\text{He}$ & 8.8 \times 10^{-12} \\
15 & $(^1\text{H}^-X^-) + ^3\text{He} \rightarrow (^2\text{H}^-X^-) + ^4\text{He}^-X^- + ^3\text{He}$ & 1.0 \times 10^{-12} \\
16 & $(^2\text{H}^-X^-) + ^3\text{He} \rightarrow (^3\text{H}^-X^-) + ^2\text{H}$ & 1.4 \times 10^{-12} \\
17 & $(^3\text{H}^-X^-) + ^4\text{He}^-X^- + ^3\text{He}$ & 3.6 \times 10^{-12} \\
18 & $(^3\text{H}^-X^-) + ^4\text{He}^-X^- + ^3\text{He}$ & 2.9 \times 10^{-12} \\
19 & $(^3\text{H}^-X^-) + ^4\text{He}^-X^- + ^3\text{He}$ & 8.0 \times 10^{-12} \\
\hline
\end{tabular}
\caption{Rates for CHAMP-exchange reactions computed in the Born approximation.}
\end{table}

in the Born approximation, to those of nuclear reactions involving bound states, as presented in Appendix A. Here the dipole (quadrupole) operators Eq. (A4) (Eq. A5) replaced by the electromagnetic potential between the bound state and the heavier nucleus. The same arguments as presented in Section 3 apply concerning the failure of the Born approximation. In particular, rates given in Table 4 should be only considered as benchmarks, with the true rates possibly deviating significantly.

Fig. 10 shows bound state fractions for the same model as that shown in Fig. 9 but now with CHAMP exchange reactions included (photodisintegration of bound states is neglected). From the comparison of these two figures it is evident that whereas bound state fractions of $^1\text{H}$ in the absence of exchange reactions reach levels close to $f^b_p \approx 10^{-3}$, they are two orders of magnitude below when exchange reactions are present. This is mostly due to the $(^1\text{H}^-X^-) + ^4\text{He}^-X^- + ^1\text{H}$ reaction. A for the final BBN yield almost equally important change is the elevated $^2\text{H}$ (and $^3\text{H}$) bound state fraction when
the reactions in Table 4 are included. Though most $^1\text{H}$ exchange their CHAMPs with $^4\text{He}$, due to the large $^4\text{He}$ abundance, a large fraction $\sim 1$ of $^2\text{H}$ enter bound states by capture of CHAMPs from protons as well. The $^2\text{H}$ bound state fraction in Fig. 10 (as well as Fig. 11) is only small $f_B^2 \ll 1$, simply because once a $^2\text{H}$ (and $^3\text{H}$) bound state has formed, its life time against destruction by reactions shown in Table 3, is very short. In other words, essentially each $^2\text{H}$ which enters a bound state will be subsequently destroyed, leading to the production of $^4\text{He}$ and $^3\text{He}$. This will have important consequences for bounds on CHAMPs at larger CHAMP density, since either the lower bound on $^2\text{H}$ or the upper bound on $^3\text{He}/^2\text{H}$ may be violated. Fig. 11 shows the abundance evolution corresponding to Fig. 10 and is the equivalent to Fig. 7 but now with exchange reactions switched on. Several trends are visible: With charge exchange reactions the $^2\text{H}/^4\text{He}$ ratio has fallen below the observational lower limit, i.e. $7.4 \times 10^{-6}$ compared to $2 \times 10^{-5}$ in Fig. 7, the final $^7\text{Li}/^4\text{He}$ ratio is larger, i.e. $9.5 \times 10^{-10}$ compared to $2.7 \times 10^{-10}$, and the $^6\text{Li}/^4\text{He}$ ratio is much larger, i.e. $3.3 \times 10^{-10}$ compared to $\sim 4 \times 10^{-14}$. Here $^7\text{Li}$ is larger due to reduced $^7\text{Be}(^2\text{H}^-X^-)^7\text{Be}$ and enhanced $^4\text{He}(^2\text{H}^-X^-)^4\text{Li}$ efficiencies, and $^6\text{Li}$ is larger due to reduced $^6\text{Li}(^2\text{H}^-X^-)^6\text{Li} + ^4\text{He}$ and enhanced $^4\text{He}(^2\text{H}^-X^-)^4\text{Li}$. When doing bound-state BBN computations with reactions on $Z = 1$ bound states as well as CHAMP exchange reactions included often very counter-intuitive results are obtained. As only one example, when the $^2\text{H}(^1\text{H}^-X^-)$ rate is increased the $^6\text{Li}$ (and $^7\text{Be}$) abundance may be reduced drastically. This is not what is expected since a lower $^2\text{H}(^1\text{H}^-X^-)$ and higher $^2\text{H}(^1\text{H}^-X^-)$ fraction ought to lead to a higher $^6\text{Li}$ abundance via enhanced $^4\text{He}(^2\text{H}^-X^-)^6\text{Li}$ and reduced $^6\text{Li}(^1\text{H}^-X^-)^6\text{He}$ abundance. Nevertheless, this is not what happens, due to a higher $^2\text{H}(^1\text{H}^-X^-)$ fraction more $^2\text{H}$ is destroyed initially, rendering the $^4\text{He}(^2\text{H}^-X^-)^4\text{Li}$ less effective at late times. Since the final abundance yield is given by the balance of the still fast processes of $^4\text{He}(^2\text{H}^-X^-)^4\text{He}$ production and $^6\text{Li}(^1\text{H}^-X^-)^6\text{Li} + ^4\text{He}$ destruction at late times less $^6\text{Li}$ results. Due to a lower $^6\text{Li}(^1\text{H}^-X^-)^7\text{Be}$. Efficiency less $^7\text{Be}$ results. Late-time bound-state BBN is very non-linear requiring full numerical integration up to late times to obtain reliable predictions.

\section{VII. Solutions to the $^6\text{Li}$ and $^7\text{Li}$ problems due to bound-state BBN for long-lived $\tau_x \gtrsim 10^6\text{sec}$ CHAMPs \footnote{In Section 2 priorly proposed solutions to the $^7\text{Li}$ overabundance and $^6\text{Li}$ underabundance resulting within BBN in the presence of (relatively) short-lived CHAMPs have been discussed. Notwithstanding possible astrophysical explanations of these deviations between theory and observation, it has been shown that both problems may be solved at once in the presence of a decaying particle with decay time $\tau_x \approx 1000\text{s}$. This is possible in either case, a charged relic or a neutral relic. In subsequent sections it has been seen that late-time nucleosynthesis in the presence of charged weak-scale mass particles may lead to orders-of-magnitude modifications of the $^6\text{Li}$, $^7\text{Li}$, and ($^2\text{H}$) abundances. It would be interesting to know}}

\begin{table}
| Model | $Y_x$ | Reactions modified |
|-------|-------|--------------------|
| A     | $4.3 \times 10^{-4}$ | #4: 0.1 #7: 2. #14: 0.1 #17: 0.3 |
| B     | $4.3 \times 10^{-4}$ | #4: 0.3 #9: 0.3 #17: 0.1 #18: 30. |
| C     | $4.3 \times 10^{-6}$ | #5: 3. #7: 30. #14: 0.03 #17: 0.03 |
\end{table}

\begin{table}
| Model | $^2\text{H}/^4\text{He}$ | $^7\text{Li}/^4\text{He}$ | $^6\text{Li}/^4\text{He}$ |
|-------|--------------------------|--------------------------|--------------------------|
| A     | $2.6 \times 10^{-5}$     | $2.8 \times 10^{-10}$    | $0.3 \times 10^{-12}$    |
| B     | $2.4 \times 10^{-5}$     | $2.3 \times 10^{-10}$    | $3.7 \times 10^{-11}$    |
| C     | $2.7 \times 10^{-5}$     | $1.5 \times 10^{-10}$    | $3.2 \times 10^{-11}$    |
\end{table}
if CHAMPs with long life times \(\tau_x \gtrsim 10^6\)s may reconcile the \(^6\)Li and \(^7\)Li discrepancies.

In Fig. 12 abundance yields for \(\tau_x = 10^{12}\)s and varying \(Y_x\) are shown. Here reaction rates in the Born approximation were adopted and electromagnetic- and hadronic-energy injection due to the \(X\) decay was neglected, corresponding to, for example, an invisible decay or a decay to a neutral daughter particle almost degenerate in mass with the CHAMP. The model also approximates well the case of no decay, i.e. a stable CHAMP. At low CHAMP-to-baryon ratio \(Y_x\) only \(^6\)Li is modified. Here most of the \(^6\)Li is synthesized not at early times due to \(^2\)H(\(^4\)He--\(^X\)\(^--\)\(^X\))\(^6\)Li but rather at late times due to \(^4\)He(\(^2\)H--\(^X\)\(^--\)\(^X\))\(^6\)Li. A small CHAMP density may therefore easily account for \(^6\)Li in Pop II stars. When \(Y_x\) increases to \(10^{-8}\) too much \(^6\)Li is synthesized. For larger \(Y_x \gtrsim 10^{-3}\) \(^6\)Li destruction due to a high (\(^1\)H--\(^X\)\(^--\)) fraction reduces \(^6\)Li again to observationally friendly levels. However, such models are then ruled out by \(^7\)Li overproduction and \(^2\)H underproduction, due to high (\(^2\)H--\(^X\)\(^--\)) and (\(^3\)H--\(^X\)\(^--\)) fractions, with \(^7\)Li produced by \(^3\)H(\(^4\)He--\(^X\)\(^--\))\(^7\)Li and \(^2\)H destroyed by reactions given in Table 3. When the decay is electromagnetic or hadronic, with a large fraction \(f_{EM}\) \(\sim 1\) of rest mass of \(X\) converted to electromagnetically interacting particles such high \(Y_x\) should in any case be ruled out due to elevated \(^3\)He/\(^2\)H-ratios (cf. Ref. [27]).

Nevertheless, significant uncertainties exist due to the uncertainties in the bound-state nuclear reactions and charge exchange reactions. In Tables V and VI three (somewhat randomly chosen) models which do solve the \(^6\)Li and \(^7\)Li problems are shown. Here a number of reaction rates were scaled up (or down) from the Born approximation in order to arrive at an observationally satisfying result. It is seen that even at low \(Y_x\) such models may exist, depending on the exact magnitude of rates for a variety of reactions. It is also seen that, when going to lower \(Y_x\), rates have to deviate more drastically from the Born approximation in order to solve the \(^6\)Li and \(^7\)Li problems. The corresponding abundance yields for Model C, where at low \(Y_x\) observationally satisfying results are obtained, are shown in Fig. 13. The figure clearly indicates that parameter space for a reduction of \(^7\)Li production and some \(^6\)Li exists.

In the absence of reliable estimates for reaction rates it is difficult to assess quantitatively if significant parameter space for simultaneous solutions for the \(^6\)Li and \(^7\)Li discrepancies for late decaying \(\tau_x \gtrsim 10^6\)sec CHAMPs exist. In particular all nuclear reactions shown in Table III and Table IV as well as the charge exchange reactions

| Reac. | \(f_{cut}^{EM}\) | Reac. | \(f_{cut}^{EM}\) |
|-------|-----------------|-------|-----------------|
| 1     | 3               | 11    | 30              |
| 2     | 30              | 12    | 30              |
| 3     | 30              | 13    | 10              |
| 4     | 30              | 14    | 100             |
| 5     | 30              | 15    | 100             |
| 6     | 30              | 16    | 100             |
| 7     | 30              | 17    | 100             |
| 8     | 30              | 18    | 100             |
| 9     | 30              | 19    | 100             |
| 10    | 30              | 30    | 100             |
shown in Table IV i.e. a total number of nineteen reactions. Though all rates have been determined numerically in the Born approximation in this paper, as the Born approximation is likely to fail badly, results become uncertain. In order to still arrive at a reliable result one is thus forced to perform a Monte-Carlo analysis, varying all ill-determined reaction rates within conservative ranges. This has been done in the present paper. In particular, the Born approximation values of the rates shown in Figs. 3 and 4 as well as given in Tables II and IV have been taken as benchmarks. For each reaction a random generator determined a factor $f_i$ with which the benchmark rate was multiplied. These factors where generated with a probability distribution flat in logarithmic space, and between values $1/f_{cut}^i \leq f_i \leq f_{cut}^i$. For the reaction-rate dependent conservatively chosen $f_{cut}^i$ the reader is referred to Table VII. For each point in parameter space, i.e. for $Y_e$ and $\tau_\sigma$, this procedure was repeated a 1000 times in order to arrive with one thousand different randomly chosen sets for the 19 ill-determined reaction rates. For each realization of reaction rates an independent BBN calculation was then performed and compared to the observational constraints.

The results of this Monte-Carlo analysis are shown in Fig. 14. Here dark (dark-blue) area indicates the probability that between 1% - 5% (i.e. 10-50) of all independent 1000 BBN calculations with randomly varied rates respect the abundance limits on other light elements (as given in Ref. [27]) while fulfilling $^7Li/^4H < 2.5 \times 10^{-10}$ and $0.66 > ^6Li/^7Li > 0.03$. Similarly, light (light-blue) areas indicate the same, but with now only the $^7Li$ discrepancy solved (i.e. $^6Li/^7Li < 0.03$) is acceptable. It is noted that in none of the parameter space a probability > 5% for $^6Li + ^7Li$ (or only $^7Li$) solving areas is found, indicating that the reaction rate combinations which may yield such solutions are rather rare. The liklihood for such scenarios is even further diminished when electromagnetic- and/or hadronic- energy injection due to the decay of the particle is considered. In fact, when $f_{EM} \sim 1$ all of the parameter space shown in Fig. 14 capable of solving the $^6Li+^7Li$ problems simultaneously (though at a $< 5\%$ liklihood), would be completely eliminated. Only when $f_{EM}$ is rather small, some area remains. This is shown by the (red) line for $f_{EM} = 3 \times 10^{-2}$ corresponding, for example, to the decay of a stau $\tau$ to a tau and gravitino, with the gravitino only $10\%$ lighter than the tau. The area above the line is ruled out by overproduction of the $^3He/^2H$ ratio due to $^4He$ photodisintegration. On the other hand, not shown in Fig. 14 are areas where only the $^6Li$ abundance as observed in Pop II stars may be produced. These exist plentiful, and at high probability, in particular at lower $Y_e < 10^{-5}$. It thus seems unlikely that CHAMP's with $\tau_{\sigma} \gtrsim 10^8$sec may resolve the $^7Li$ problem, though they could possibly constitute the source for the observed $^6Li$ at low metallicly.

VIII. CONCLUSIONS

In summary, I have presented results of a very detailed study of BBN in the presence of negatively charged massive particles (CHAMPS). Such particles have been shown to form bound states with nuclei towards the end of a conventional BBN epoch $[23, 24]$. They may alter BBN yields due to the catalysis of nuclear reactions $[24]$. The present analysis attempts to take into account of all relevant effects for making precisely predictions of catalytic light-element nucleosynthesis for nuclei with $A \lesssim 7$, but excluding the formation of molecules. It includes numerical evaluations in the Born approximation of all key nuclear cross sections, where one of the nuclei is in a bound state. Bound-state recombination and photodisintegration cross sections are also determined numerically. Furthermore, three very important and priorly not treated effects for the CHAMP BBN at late times $\tau \gtrsim 10^5$sec are included: (a) rapid nuclear reactions including charge $Z = 1$ nuclei in bound states, (b) the photodisintegration of bound states due to $\gamma$- and $\nu$- rays generated during the decay of the CHAMPS, and (c) CHAMP-exchange reactions from a bound state within a lighter nucleus to a bound state within a heavier nucleus. Light element abundances and bound state fractions are computed without approximations. The effects of hadronic and electromagnetic cascades due to CHAMP disintegration on light element abundances are properly taken into account.

The present detailed study reveals that bound-state BBN proceeds very differently than initially forecasted $[23, 24]$. At low temperatures $T \lesssim 1$ kV, a large number $\sim 20$ of Coulomb-barrier unsupressed nuclear reactions and charge exchange reactions become operative and are capable, in most of the parameter space, to change $^6Li$, $^7Li$, and $^2H$ abundances by orders of magnitude. Unfortunately, reaction rates for these processes are not well approximated by the Born approximation, such that for CHAMP life times $\tau_{\sigma} \gtrsim 10^5$sec one has to resort to a Monte-Carlo analysis.

The purpose of this study is to investigate the potential of CHAMP BBN to resolve the current $^6Li$ and $^7Li$ discrepancies between standard BBN and observations. It is shown, that a priorily proposed simultaneous solution of the $^6Li$ and $^7Li$ problems with a relic particle decaying at $\tau_{\sigma} \approx 1000$sec $[21]$, is not very dependent on the decaying relic being charged $[25]$ or not, unless its hadronic branching ratio is well below $B_h \lesssim 10^{-4}$. A solution with $B_h \ll 10^{-4}$ has, however, the advantage to not change much the $^3He/^2H$ ratio from its respective standard BBN value. Since $^6Li$ and $^7Li$ may be rapidly destroyed at late times one generically expects further simultaneous solutions of the $^6Li$ and $^7Li$ problems for $\tau_{\sigma} \gtrsim 10^8$sec. Nevertheless, even given the current reaction rate uncertainties, a Monte-Carlo shows that only a very small fraction $\lesssim 5\%$ of reaction rate combinations may lead to such solutions. Since such possible solutions occur at relatively high CHAMP-to-baryon ratio $3 \times 10^{-5} \lesssim Y_e \lesssim 10^{-2}$ they
are further constrained by the effects of electromagnetic energy injection and possible $^3\text{He}^3/2\text{H}$ overproduction, requiring the decay to be invisible, or mother and daughter particle to be somewhat degenerate in mass $\sim 10\%$. On the other hand, CHAMPs may well be the source of the observed $^6\text{Li}$ at low metallicity.

I acknowledge helpful discussions with and M. Asplund, S. Bailly, O. Kartavtsev, K. Kohri, A. Korn, G. Moultaeva, M. Pospelov, J. Rafelski, G. Starkman, V. Tatischeff, and T. Yanagida.

**APPENDIX A: THERMONUCLEAR REACTIONS IN THE PRESENCE OF BOUND STATES IN THE BORN APPROXIMATION**

Consider the three-body system of nuclei $A$, $B$, and CHAMP $X^-$. Since for weak scale mass CHAMPs and light nuclei $M_X \gg M_A, M_B$, it is an excellent approximation to assume $X^-$ to be at rest at the origin, effectively acting as an external potential which absorbs momentum but not energy. The Hamiltonian of the system is then given by

$$ H = \frac{1}{2} M_A \mathbf{r}_A^2 + \frac{1}{2} M_B \mathbf{r}_B^2 + V_C(\mathbf{r}_A - \mathbf{r}_B) + V_{\text{NUC}}(\mathbf{r}_A, \mathbf{r}_B) - \frac{Z_A e^2}{r_A} - \frac{Z_B e^2}{r_B}, \quad (A1) $$

where $\mathbf{r}_A$, $\mathbf{r}_B$ represent the position vectors of nuclei $A$ and $B$, $r_A$, $r_B$ their magnitudes, and $Z_A e$, $Z_B e$ their respective charges. In Eq. (A1) the first two terms represent kinetic energies, the second and third term, Coulomb and nuclear potentials between $A$ and $B$, and the last terms, the Coulomb potentials between the (assumed singly charged) CHAMP $X^-$ and the nuclei. This Hamiltonian will be split into a dominant contribution $H_0$ and a perturbative contribution $H_1 \ll H_0$. In a rearrangement reaction of the type $(A - X^-) + B \rightarrow C + X^-$, where $C$ is a nuclear bound state between $A$ and $B$, the unperturbed and perturbed Hamiltonians for initial and final states are different, i.e. $H_0 \neq H_0'$, $H_1 \neq H_1'$. In particular, whereas in the initial state the perturbation is best chosen as the nuclear attraction between $A$ and $B$, i.e. $H_1 = V_{\text{NUC}}$ and $H_0 = H - H_1$, in the final state it will be the differential Coulomb force of $X^-$ on the nucleus bound state $C = (A - B)$. When initial and final states are chosen as eigenstates to $H_0$ and $H_1'$, respectively, standard methods show that, in the Born approximation the transition amplitude may be computed by either $\langle i | H_1' f \rangle$ or $\langle f | H_0' i \rangle$. The initial and final states are chosen as

$$ |i \rangle = |\Phi_{(A-X^-)}(r_A)\rangle |\Phi_{\text{Coul}}(r_B)\rangle \quad (A2) $$

$$ |f \rangle = |\Phi_{(A-B)}(\rho)\rangle |\Phi_{\text{Coul}}(s)\rangle \quad (A3) $$

where $s$ and $\rho$ are the $A - B$ center of mass and relative coordinates, respectively. Coulomb wave functions $|\Phi_{\text{Coul}}\rangle$ and the $A - X^-$ bound state wave function $\Phi_{(A-X^-)}$ were determined numerically with realistic charge distributions. The nuclear wave function $\Phi_{(A-B)}$ was parametrised by $\Phi = 2\sqrt{\gamma^5/3} \rho \exp(-\gamma\rho)$ with $\gamma$ adjusted such that in the absence of $X^-$ the correct experimentally determined cross section results. The perturbation $H_1'$ was chosen as the first non-vanishing element in the expansion of the last two terms of Eq. (A1) in terms of relative coordinate $\rho$. For dipole transitions this results into

$$ H_1' = -(Z_A R_A + Z_B R_B) e^2 s_i |P_i/ s^3 \rangle \quad (A4) $$

whereas for quadrupole transitions

$$ H_1' = -(Z_A R_A^2 + Z_B R_B^2) e^2 (\frac{3}{2} s_i s_j \rho_i \rho_j - \frac{1}{2} \rho^2 / s^3) \quad (A5) $$

where $R_A = M_B/(M_A + M_B)$ and $R_B = -M_A/(M_A + M_B)$. Rates were evaluated by numerical integration of the matrix elements $\langle i | H_1' f \rangle$ employing Fermi’s Golden rule

$$ \sigma_v = \frac{2\pi}{\hbar} V \int dN_f \delta(E_i - E_f) |\langle i | H_1' f \rangle|^2 \quad (A6) $$

where $V$ is a normalization volume, $v$ relative velocity, $\delta$ the Delta-function, and

$$ dN_f = \frac{V}{(2\pi\hbar)^3} \rho^2 d\rho d\Omega \quad (A7) $$

a measure of the final phase space for nucleus $C$. For the evaluation of the matrix elements, six-dimensional integrals over the coordinates of two nuclei could be analytically reduced to three-dimensional integrals which were numerically evaluated. Similar to Ref. [31] I have not considered internal spin of the nuclei, except for the obvious total angular momentum degeneracy factors. Finally cross sections $\sigma(E)$ were converted to S-factors $S(E)$. They are related by

$$ \sigma(E) = (S(E)/E) \exp(-G(E)), \quad (A8) $$

where $E$ is center-of mass (CM) energy and $\exp(G)$ with

$$ G(E) = \frac{2\pi(Z_A - 1) Z_B \alpha c}{\rho_{\text{CM}}} \quad (A9) $$

is the Coulomb repulsion factor. In the above $\rho_{\text{CM}}$ is the relative velocity ($\rho_{\text{CM}} \approx \rho_B$ for bound states), and $\alpha$, $c$ fine structure constant and speed of light, respectively. For assumptions concerning the angular momentum of the final $A-B$ nucleus, the number of multipoles included in the calculation, and the assumed S-factor in the absence of bound states the reader is referred to Table 2. The determined S-factors were subsequently integrated over a thermal distribution to derive thermal nuclear rates in the presence of bound states.
[1] D. N. Spergel et al., astro-ph/0603449
[2] F. Spite and M. Spite, Astronomy & Astrophysics 115, 357 (1982); P. Bonifacio and P. Molaro, MNRAS 285, 847 (1997); S. G. Ryan, T. C. Beers, K. A. Olive, B. D. Fields and J. E. Norris, Astrophys. J. Lett. 530, L57 (2000); P. Bonifacio et al., Astronomy & Astrophysics 390, 91 (2002); J. Melendez and I. Ramirez, Astrophys. J. 615, L33 (2004); C. Charbonnel and F. Primas, Astronomy & Astrophysics 442, 961 (2005).
[3] M. Asplund, D. L. Lambert, P. E. Nissen, F. Primas and V. V. Smith, Astrophys. J. 644, 229 (2006).
[4] for former 4Li detections cf. to: V. V. Smith, D. L. Lambert, and P. E. Nissen, Astrophys. J. 408, 262 (1993); 506, 405 (1998); L. M. Hobbs and J. A. Thorburn, Astrophy. J. 491, 772 (1997); R. Cayrel, M. Spite, F. Spite, E. Vangioni-Flam, M. Cassé, and J. Audouze, Astron. & Astrophys. 343, 923 (1999); F. E. Nissen, M. Asplund, V. Hill, and S. D’Odorico, Astr. & Astrophys. 357, L49 (2000).
[5] calculations by Richard et al. [6] presented in Ref. [3].
[6] 0. Richard, G. Michaud, and J. Richer, Astrophys. J., 619, 538 (2005).
[7] M. Salaris and A. Weiss, Astron. Astrophys. 376, 955 (2001); M. H. Pinsoneault, G. Steigman, T. P. Walker, K. Narayanan and V. K. Narayanan, Astron. J. 574, 398 (2002); S. Talon and C. Charbonnel, Astronomy & Astrophysics 418, 1051 (2004); A. M. Boesgaard, A. Stephens and C. P. Deliyannis, Astrophys. J. 633, 398 (2005); L. Piau, arXiv:astro-ph/0511402.
[8] A. J. Korn et al., Nature 442, 657 (2006).
[9] F. Confortola et al. [LUNA Collaboration], Phys. Rev. C 75, 065803 (2007)
[10] cf., for example, to E. Vangioni-Flam, M. Casse and J. Audouze, Phys. Rept. 333, 365 (2000); R. Ramaty, S. T. Scully, R. E. Lingenfelter and B. Kozlovsky, Astronomy & Astrophys. J. 534, 747 (2000).
[11] S. Inoue and T. K. Suzuki, Nucl. Phys. A 718, 69 (2003).
[12] N. Prantzos, arXiv:astro-ph/0510122
[13] B. B. Nath, P. Madau and J. Silk, Mon. Not. Roy. Astron. Soc. Lett. 366, L35 (2006)
[14] V. Tatischeff and J. P. Thibaud, arXiv:astro-ph/0610756
[15] K. Jedamzik, Phys. Rev. Lett. 84, 3248 (2000).
[16] For a recent re-analysis, cf. to M. Kusakabe, T. Kajino and G. J. Mathews, Phys. Rev. D 74, 023526 (2006).
[17] S. Dimopoulos, R. Esmailzadeh, L. J. Hall and G. D. Starkman, Astrophys. J. 330, 545 (1988).
[18] K. Jedamzik, Phys. Rev. D 70, 083510 (2004).
[19] J. L. Feng, A. Rajaraman and F. Takayama, Phys. Rev. D68, 063504 (2003).
[20] J. R. Ellis, K. A. Olive and E. Vangioni, Phys. Lett. B 619, 30 (2005).
[21] K. Jedamzik, Phys. Rev. D 70, 063524 (2004).
[22] K. Jedamzik, K. Y. Choi, L. Roszkowski and R. Ruiz de Austri, JCAP 0607, 007 (2006)
[23] M. Pospelov, arXiv:hep-ph/0603215
[24] K. Kohri and F. Takayama, arXiv:hep-ph/0605243
[25] M. Kaplinghat and A. Rajaraman, Phys. Rev. D 74, 103004 (2006).
[26] D. Fargion, M. Khlopov and C. A. Stephan, Class. Quant. Grav. 23, 7305 (2006)
[27] A. De Rujula, S. L. Glashow and U. Sarid, Nucl. Phys. B 333, 173 (1990).
[28] S. Dimopoulos, D. Eichler, R. Esmailzadeh and G. D. Starkman, Phys. Rev. D 41, 2388 (1990).
[29] J. Rafelski, M. Sawicki, M. Gajda and D. Harley, Phys. Rev. A 44, 4345 (1991).
[30] R. H. Cyburt, J. R. Ellis, B. D. Fields, K. A. Olive and V. C. Spanos, JCAP 0611, 014 (2006).
[31] K. Hamaguchi, T. Hatsuda, M. Kamimura, Y. Kino and T. T. Yanagida, arXiv:hep-ph/0702274
[32] J. Pradler and F. D. Steffen, Phys. Lett. B 648 224 (2007).
[33] Reaction rates, for example, for neutrons on 7Be, are given by the product of their reaction cross section σ and the relative velocity between the reacting nuclei, v. It is argued that due to the fast motion of the 7Be nucleus around the CHAMP, the relative velocity v could be enhanced by orders of magnitude. This is clearly not so, as v is, simply, part of the flux factor at large distances, defining the flux of incoming neutrons in the cosmic rest frame, which is essentially the rest frame of the heavy 7Be-X− bound system. Due to the small thermal velocities of the heavy bound state, the relative velocity, and thus the reaction rate, is even reduced, but only about a factor of order unity. See also Ref. [35] for a more detailed discussion.
[34] D. Cumberbatch, K. Ichikawa, M. Kawasaki, K. Kohri, J. Silk and G. D. Starkman, arXiv:0708.0095 [astro-ph].
[35] C. Bird, K. Koopmans and M. Pospelov, arXiv:hep-ph/0703096.
[36] T. Jittoh, K. Kohri, M. Koike, J. Sato, T. Shimomura and M. Yamanaka, arXiv:0704.2914 [hep-ph].
[37] K. Jedamzik, Phys. Rev. D 74, 103509 (2006).
[38] The Bohr radius is only given for illustrative purposes, all calculations assume bound state wave functions determined by the Schroedinger equation, and deviating significantly from the 1s Bohr wave function. The approximative Bohr radius was determined by requiring the same asymptotic slope at larger distances.
[39] For example, the rapid 4Li(p,α)7He rate for T ≳ 10 keV, continuously destroys 4Li-X− bound states, such that the 6Li bound state fraction is below that expected from naive estimates.
[40] A necessary requirement for the validity of the Born approximation is H∥/Ei ≪ 1, where H∥ is the perturbative Hamiltonian and Ei is the initial energy. I acknowledge private communication with T. Yanagida on this point.
[41] Throughout the paper, it is assumed that only half of the stated Ω abundances is within negatively charged species X−, with the other half in the antiparticle of X, i.e. X+, such that electrical neutrality is kept.
[42] Naively thought, some of the 7Li (synthesized as 7Be) could survive the rapid 7Be(H−−X−−X−−)3B destruction reaction at T ≈ 1 keV if 7Be would be converted to 7Li at τ ≲ 10s. In fact, the half life for 7Be against electron capture is τBe ≈ 4.6 × 1010s. Nevertheless, the binding energy of an electron to the 7Be nucleus is Ee ≈ 0.21 keV, such that the 7Be nucleus stays bare until much lower temperatures T ≲ 1 keV. Since the electron plasma density is well below the effective density of electrons in a recombined 7Be nucleus, the 7Be nuclei has to either avoid...
recombination or $\beta^+$-decay to convert to $^7$Li. Due to the rapidness of the $^7$Be destruction process, it is therefore expected that $^7$Be $\rightarrow$ $^7$Li at low temperatures plays hardly a role.

[43] M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B 649, 436 (2007)

[44] The effects of annihilation of $X^-$ with $X^+$ in $X^- - X^+$ bound states forming to some degree are subdominant compared to those of the decay itself.