Numerical Simulation of the Influence of Taylor Vortex on the Apparent Viscosity Measurement of Semi-Solid Metallic Based On ANSYS Fluent

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Abstract. The apparent viscosity of semi-solid metallic slurry with a low solid fraction, which is one of the most essential parameters for representing the rheological behavior, is mainly measured by the concentric cylinder rotational approach. The principle of this method is based on the assumptions that the fluid is in an ideal laminar flow state and obey the Newton's internal friction law. However, as the angular velocity $\omega$ increases, the fluid undergoes a transition from a stable laminar flow state to a Taylor vortex and turbulent flow state. These unstable flow conditions such as Taylor vortex and turbulence have a severe impact on the accuracy of apparent viscosity measurement. However, these unstable flow conditions are difficult to monitored and analyzed in real time through experimental methods. Computer numerical simulation technology provides the possibility and convenience for the visualization of the flow state of the semi-solid metallic slurry in the measurement system. In this work, ANSYS Fluent was used to simulate the apparent viscosity measurement process of semi-solid slurry, and the flow state transition process of the semi-solid slurry in the measurement system was successfully visualized and analyzed. In order to avoid the influence of Taylor vortex, combined with the measurement principle of the concentric cylinder rotational rheometer and Taylor's study on flow stability, the empirical equation of limiting speed to avoid Taylor vortex in the process of Searle rheometer viscosity measurement is given.

Keywords: ANSYS Fluent; numerical simulation; semi-solid metallic; apparent viscosity; Taylor vortex.

1. Introduction
Semi-solid processing (SSP) technology combines the advantages of solidification forming and plastic forming. The quality of parts produced by SSP is comparable to plastic forming and the cost is close to solid forming. Therefore, it can meet the increasing demand for low-cost, complex structures and excellent mechanical properties in automotive and communication industries [1-4]. The rheological behavior of semi-solid metal is one of the most important research contents and theoretical basis of the SSP technology. Apparent viscosity is an important factor for quantitatively semi-solid metal and
characterization of the apparent viscosity has an essential academic significance and practical engineering guidance value [5-10].

The concentric cylinder rotational rheometer is the most commonly used device in semi-solid metal rheological behavior research [11-14]. The schematic diagram of the rheometer is shown in Fig.1. Rotational rheometers are usually divided into Searle type (the inner cylinder rotates while the outer cylinder is stationary) and Couette type (the outer cylinder rotates while the inner cylinder is stationary). The Searle type is the most common type of rheometer due to its simple structure and low cost [15-17].

![Figure 1. Schematic diagram of concentric cylinder rotational rheometer](image)

In the case of a small annular space, according to Newton’s internal friction law, the viscosity of the slurry in this measurement system and its corresponding shear rate can be calculated by Eq. (1) and (2) [17]:

\[ \eta = \frac{M}{4\pi h \omega} \left( \frac{1}{R_i^3} - \frac{1}{R_o^3} \right) = \frac{M(R_o^2 - R_i^2)}{4\pi R_o a h \omega} \]  

(1)

\[ \dot{\gamma} = \frac{2R_a^2}{R_o^2 - R_i^2} \omega \]  

(2)

where \( M \) is the torque and \( \omega \) is the rotation velocity of the cylinder.

The measurement principle mentioned above is derived based on the assumption that the slurry in the measurement system is in an ideal laminar flow state. However, as the angular velocity \( \omega \) increases, the flow state of the slurry undergoes a transition from a stable laminar flow state to the Taylor vortex and turbulent flow. These unstable flow states such as Taylor vortex and turbulence play an important role in viscosity measurement errors [18,19]. According to Taylor’s research on Couette flow, only when the dimensionless parameter Taylor number \( T_a \) was less than a certain critical value \( T_a = 1708 \), the Couette flow was stable. If \( T_a \) exceeds this critical value, the Couette flow will lose its stability and transforms into a new flow state that a series of circular secondary flows called "Taylor vortexes" will appear in the annular space between the inner and outer cylinder. If this unstable flow state continues to develop, the flow state will turn into turbulence eventually. The Taylor vortex is the intermediate state between the steady circumferential flow and turbulence [18,20-22].

The dimensionless parameter Taylor number \( T_a \) can be calculated by Eq. (3):

\[ T_a = \frac{2(\omega R_i^2 - \omega_a R_o^2)\omega \delta^3}{\nu^2 R_i} \]  

(3)
where \( \omega_i \) and \( \omega_a \) are the angular velocities of the inner and outer cylinders respectively, \( \bar{\omega} \) is the average angular velocity \( \bar{\omega} = (\omega_a + \omega_i) / 2 \), \( \delta \) is the annular gap between the inner and outer cylinder \( \delta = R_a - R_i \).

\( T_a \) always less than zero when the outer cylinder rotates while the inner cylinder remains stationary according to Eq. (3). Therefore, the Couette type rheometer does not produce Taylor vortex in any case. As for the Searle type rheometer (the inner cylinder rotates while the outer cylinder remains stationary, \( \omega_a = 0 \)), the flow state of the fluid is not always stable. When the angular velocity \( \omega_i \) is sufficiently small, \( T_a < T_c \), the flow remains stable. As \( \omega_i \) continues increasing, the Taylor vortex appears when \( T_a > T_c \). The neutral boundary curve obtained by Taylor, as shown in Fig. 2 strongly proves these conclusions [22].

**Figure 2.** The neutral boundary curve of Couette flow

In this work, ANSYS Fluent software was used to simulate measuring the apparent viscosity of semi-solid slurry with two types of rheometers and visualize the flow state of the slurry in the measurement system. The influence mechanism of the Taylor vortex on the accuracy of apparent viscosity measurement is investigated, and a solution is given.

**2. Simulation Process**

**Physical model.** The numerical simulation process involved in this work is all carried out in ANSYS Fluent 18.0. The black box model is used to preserve the interaction relationship between the rotating cylinder and the slurry. It’s a simplified model that only retains the internal portion of the slurry and the contact wall of the slurry and the rheometer [23-25]. Meanwhile, the parts that are not in contact with the slurry, such as the outer cylinder wall, the rotating shaft, and the motor are deleted and simplified. The simplified physical model used in the simulation is shown in Fig.3
The inner and outer cylinder wall and the upper ring wall are parts of the slurry. The inner and outer cylinders wall are set to no-sliding wall conditions with the cylinder, and the ring wall is set as a free surface condition. The rotational motion is achieved by setting the angular velocity of the rotating wall surface (the inner cylinder wall or the outer cylinder wall), and thus the slurry rotation required in this simulation is achieved and the interaction between the rotating cylinder and the slurry is reserved.

Structured hexahedral meshes are used in all simulations involved in this work to consider the need for higher accuracy. Use the multi-area sweep method to generate mesh and use surface size to control the mesh quality. The grid independence test has been completed, and the minimum mesh size is set at 0.3mm, and the maximum is 0.8mm. The velocity gradient of the slurry in the rheometer’s annular space changes greatly because of the shear-thinning characteristics of the semi-solid metal. Meanwhile, to study the extreme shearing situation, a large rotating speed is needed here. Therefore, the RNG k-ε model is selected in this simulation. In addition, the fundamental governing equations of fluid mechanics including continuity equations, momentum equations, energy conservation equations and state equations are also considered.

In the simulation process mentioned in this paper, the semi-solid slurry is modelled as a custom shear-thinning non-Newtonian fluid with a density of 2700kg/m³. The apparent viscosity for the slurry conforms to the power-law model and can be expressed by Eq. (4) according to Hu et al. [26].

\[ \eta_a = K\dot{\gamma}^{n-1} = 316\dot{\gamma}^{0.01-1} \]  \hspace{1cm} (4)

**Numerical Analysis.** Firstly, the apparent viscosity measurement simulation process was carried out using a Searle type rheometer, and the torque value \( M \) under a series of shear rates \( \dot{\gamma} (1, 5, 20, 50, 200, 500, 1000, 1300s^{-1}) \) was recorded. The simulated apparent viscosity \( \eta \) was calculated using the Eq. (1), and the relative error \( \epsilon \) is calculated by Eq. (5) in comparison with the theoretical value \( \eta_a \) calculated in Eq. (4):

\[ \epsilon = \frac{\eta - \eta_a}{\eta_a} \times 100\% \]  \hspace{1cm} (5)

In order to visualize the fluid flow in the annular between the inner and outer cylinders of the rheometer, the cloud map of the velocity streamlines in the meridional plane in the rheometer was

![Figure 3. Physical model of the rheometer used in the simulation](image)
obtained using CFD-Post. Then the above simulation procedures were repeated using a Couette type rheometer. The shape parameters of the rheometer model used in this simulation are listed in Table 1.

### Table 1. The shape parameters of the rheometer model

| Outer cylinder radius | Inner cylinder radius | Immersion depth | Bottom height |
|-----------------------|-----------------------|-----------------|--------------|
| \( R_a \) (mm)       | \( R_i \) (mm)        | \( h \) (mm)    | \( H \) (mm)  |
| 50                    | 40                    | 150             | 30           |

3. Results and Discussion

The Taylor number \( T_a \) of Searle and Couette type rheometer can be calculated by Eq. (6) according to Eq. (3) respectively:

\[
T_a = \frac{\omega_i^2 R_i \delta^3}{\nu^2}
\]

The Taylor number \( T_a \) of the Searle type rheometer is calculated as shown in Fig. 4. As the kinematic viscosity \( \nu \) is used in the Eq. (6), but the viscosity of the slurry is unknown during the actual measurement process. To evaluate the relationship between the Taylor number \( T_a \) and the slurry flow state more intuitively, the relationship between the angular velocity \( \omega \) (or the shear rate \( \dot{\gamma} \)) and the Taylor number \( T_a \) must be established. The angular velocity \( \omega \) can be calculated by Eq. (7) according to Eq. (2):

\[
\omega = \frac{\dot{\gamma}(R_a^2-R_i^2)}{2R_a^2}
\]

According to the principle of concentric cylinder rotational rheometer Eq. (1) and the relationship between the kinematic viscosity \( \nu \) and the dynamic viscosity \( \eta \), the kinematic viscosity \( \nu \) can be calculated by Eq. (8).

\[
\nu = \frac{\eta}{\rho} = \frac{M(R_a^2-R_i^2)}{4\pi R_a^4 R_i^2 h \omega \rho}
\]

Combined Eq. (6), Eq. (7) and Eq. (8), \( T_a \) can be calculated as:

\[
T_a = \frac{16\pi^2 R_a^4 R_i^2 h^2 \delta^3 \rho^4}{M^2 (R_a^2-R_i^2)^2} \cdot \omega^4 = \frac{\pi^2 R_i^2 h^2 \delta^3 \rho^2 (R_a^2-R_i^2)^2}{M^2 R_a^4} \cdot \dot{\gamma}^4
\]

Therefore, the Taylor number \( T_a \) of the slurry inside the Searle rheometer is proportional to the fourth power of the inner cylinder angular velocity \( \omega \) (or the shear rate \( \dot{\gamma} \)). That is, the Taylor number \( T_a \) of the slurry will increase rapidly as the angular viscosity (or the shear rate) increases. The critical shear rate of the Searle type rheometer mentioned in this simulation process is 374s\(^{-1}\) when the Taylor number reaches 1708 according to Eq. (9), that is, the Taylor vortex has appeared in the meridional plane. The relative error \( \epsilon \) of the Searle and Couette type rheometers and the Taylor number of the Searle type rheometer are shown in Fig. 4. The straight line is the critical shear rate \( \dot{\gamma}=374\text{s}^{-1} \). That is, the flow state of the slurry is stable in the Searle type rheometer in the left area of the critical shear rate line and the relative errors \( \epsilon \) of these two type rheometers are close. But the flow state is unstable in the right area of the critical line based on Taylor’s research. That is, the Taylor vortex begins to appear in the meridional plane of the Searle type rheometer. The relative error \( \epsilon \) of the Searle type rheometer is much significantly larger than that in the Couette rheometer case. Obviously, compared with the Couette type rheometer without Taylor vortex, the appearance of Taylor vortex caused the relative error of Searle type rheometer to increase significantly.
Figure 4. The relative error $\varepsilon$ of the Searle and Couette type rheometers and the Taylor number of the Searle type rheometer

What should also worth noting that even in the Couette type rheometer case that without Taylor vortex has a large apparent viscosity relative error ($\varepsilon=54\%$ when $\dot{\gamma}=1300\text{s}^{-1}$), which is due to a relatively large inner and outer cylinder annular value and shear rate range were choose during the numerical simulation process in order to make the research results applicable to various extreme situations. For example, a quite large annular value is required in the semi-solid apparent viscosity measurement cases, the petroleum industry needs to measure the apparent viscosity at large shear rates. However, a large annular often bring an unexpected apparent viscosity relative error. This is because, unlike Newtonian fluids, the velocity change of the shear-thinning fluids in the annular is not linear, but non-linear. The shear rate calculated by Eq. 2 is an average level of the shear rate experienced by the fluids in the annular of the measurement system. When the annular is quite small, the velocity change of the fluid in the annular is approximately linear in the shear-thinning fluids case. The average shear rate at this time can represent the shear rate experienced by the fluid in the measurement system. But when the annular is relatively large, the average shear rate cannot represent the shear rate experienced by the shear-thinning fluids in the measurement system. Using the apparent viscosity corresponding to this average shear rate as the theoretical value will result in unexpected relative errors. Therefore, it is necessary to comprehensively consider the fluid properties and measurement accuracy requirements when determining the inner and outer cylinder annular value of the rheometer.

The cloud maps of the velocity streamline in the meridian plane of these two types of rheometers are shown in Fig. 5 and Fig. 6, respectively, when the shear rate is 200, 500, 1000, and 1300s$^{-1}$. As we can see from Fig. 5, there is no Taylor vortex in the annular of the Couette type rheometer, which is consistent with the conclusions obtained from the previous analysis. As for the Searle type rheometer, there is no Taylor vortex in the annular when the shear rate is under the critical shear rate $374\text{s}^{-1}$ (Fig. 6 (a)), which is also consistent with the previous analysis. However, no Taylor vortex has been seen at the shear rate $500\text{s}^{-1}$ (Fig. 6 (b)) as the shear rate has exceeded the critical shear rate of $374\text{s}^{-1}$. That is because Taylor assumes that the fluid is a Newtonian fluid in his study of the flow stability and the shear rate in the annular space between two cylinders is a constant which is independent with the fluid layer position. But the velocity distribution is not linear in the case of non-Newtonian fluids. As for the shear-thinning fluids, the fluid layer near the rotating cylinder has the maximum shear rate and the minimum apparent viscosity. The Taylor vortex thus has the largest chance to appear here. On the contrary, the layer away from the rotating cylinder has the minimum shear rate but the maximum
apparent viscosity. Therefore, the Taylor vortex has a smaller chance to appear. Due to the interaction of these two states, no obvious Taylor vortex is observed when the shear rate is $500 \text{s}^{-1}$. It can be believed that the critical Taylor number $T_c$ for the shear-thinning fluids such as semi-solid metal is larger than that of the Newtonian fluids’ case. As the angular velocity $\omega$ increases, the formation of Taylor vortex can be observed (Fig. 6 (c)) and finally obvious paired Taylor vortex can be seen in the annular of Searle type rheometer when the shear rate is $1300 \text{s}^{-1}$ (Fig. 6 (d)).

![Figure 5. The cloud map of the velocity streamline in the meridian plane (Couette type) (a) 200s$^{-1}$; (b) 500s$^{-1}$; (c) 1000s$^{-1}$; (d) 1300s$^{-1}$](image)

![Figure 6. The cloud map of the velocity streamline in the meridian plane (Searle type) (a) 200s$^{-1}$; (b) 500s$^{-1}$; (c) 1000s$^{-1}$; (d) 1300s$^{-1}$](image)

To analyze the reason for this significant difference of relative errors, the ANSYS post-processing software CFD-Post is used. The pressure distribution of the outer wall on these two type rheometers under the condition of the shear rate is $1300 \text{s}^{-1}$, shown in Fig. 7 and Fig. 8 respectively. By comparing these two figures, the flow state of the Couette type rheometer is stable and the pressure distribution is substantially uniform. However, as for the Searle type rheometer, a high-pressure strip can be seen in the middle of each pair of Taylor vortex. That is, the appearance of the Taylor vortex changes the pressure distribution on the cylinder of the rheometer, forming a series of staggered high-pressure and low-pressure belts. This uneven pressure distribution also means an uneven distribution of shear stress. This uneven distribution of shear stress caused a significant increase in the apparent viscosity relative error of the Searle type rheometer after the Taylor vortex appeared.
In the actual apparent viscosity measurement process, the Searle type rheometer is widely used because of its simple structure and low manufacturing cost. However, the unstable flow states such as Taylor vortex and turbulence will appear in Searle type rheometer as the angular velocity $\omega$ increases. These unstable flow states will seriously affect the apparent viscosity measurement accuracy. Therefore, it is necessary to maintain a stable flow state of the slurry during the apparent viscosity measurement using a Searle type rheometer. Based on Taylor's research, the limit angular velocity $\omega_{limit}$ to prevent the Searle type rheometer from forming Taylor vortex can be calculated by Eq. (10) using the critical Taylor number 1708 according to Eq. (9).

$$\omega_{limit} = 4 \frac{\tau M^2(R_0^2-R_i^2)^2}{16\pi^2R_0^2R_i^2h^2\rho^2\delta^4} \approx 1.8 \frac{M^2(R_0^2+R_i^2)^2}{R_0^2R_i^2h^2\rho^2\delta^4}$$

(10)

where $M$ is the torque when the angular velocity is $\omega$.

The torque value monitored on the inner cylinder of the rheometer is $M$ under a certain angular velocity $\omega$. Suppose the angular velocity $\omega$ satisfies $\omega<\omega_{limit}$, the flow state is stable in the Searle type rheometer, and the Taylor vortex will not appear. According to the previous analysis, the critical Taylor number of the shear-thinning fluids such as semi-solid metal is larger than that of Newtonian fluids
(T_c=1708), so the limit angular velocity \( \omega_{\text{limit}} \) calculated by Eq. (10) is less than the actual limit angular velocity. Therefore, the Taylor vortex will never appear as long as the \( \omega<\omega_{\text{limit}} \) is satisfied in the Searle type rheometer.

4. Summary and Conclusions

A 3D model of the rheometer was established and applied to the simulation of the apparent viscosity measurement process of semi-solid metallic slurry based on ANSYS Fluent. The flow state of the semi-solid slurry in the measurement system was successfully visualized and analyzed. The flow state and measurement error of the slurry in the measurement system of the Searle and Couette type rheometers are focused on, and the following conclusions are obtained:

(1) The appearance of Taylor vortex will seriously affect the apparent viscosity measurement error. The apparent viscosity relative error \( \varepsilon \) of the Searle type rheometer (Taylor vortex is observed in the meridian plane) is more than 3 times that of the Couette rheometer (Taylor vortex is not observed in the annular of the inner and outer cylinder of the rheometer) under certain circumstances.

(2) The appearance of the Taylor vortex changes the pressure distribution on the cylinder of the rheometer, forming a series of staggered high-pressure and low-pressure belts.

(3) The critical Taylor number of the shear-thinning fluids such as semi-solid metal is larger than that of Newtonian fluids (\( T_c=1708 \)). In the actual apparent viscosity measurement process, the limit angular velocity \( \omega_{\text{limit}} \) was proposed to avoid the forming of Taylor vortex.

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References

[1] J. Wannasin, R. Canyook, R. Burapa, L. Sikong, M.C. Flemings, Evaluation of solid fraction in a rheocast aluminum die casting alloy by a rapid quenching method, Scr. Mater. 2008, 59, 1091–1094.

[2] M.F. Qi, Y.L. Kang, Q.Q. Qiu, Industrialized Application of Rheo-HPDC Process for the Production of Large Thin-Walled Aluminum Alloy Parts, Solid State Phenom. 2019, 285, 453–458.

[3] M. Modigell, A. Pola, M. Tocci, Rheological Characterization of Semi-Solid Metals: A Review, Metals. 2018, 8, 245.

[4] G. Li, H. X. Lu, X.G. Hu et al, Current Progress in Rheoforming of Wrought Aluminum Alloys: A Review, Metals. 2020, 10, 238.

[5] X. K. Liang, Study on manufacturing technology and application of semi-solid slurry od Al-Si alloys, Doctor, General Research Institute for Nonferrous Metals, Beijing. 2017-5-7.

[6] M. C. Flemings, R. G. Riek, K. P. Young, Rheocasting, Materials Science and Engineering. 1976, 25, 103-117.

[7] D. B. Spencer, R. Mehrabian, M. C. Flemings, Rheological behavior of Sn-15%Pb in the crystallization range, Metallurgical Transaction. 1972, 3, 1925-1932.

[8] M. C. Flemings, Behavior of metal alloy in the semi-solid state, Metallurgical Transaction A. 1999, 22A, 957-981.

[9] M. A. Taha, N. A. Mahahallawy, A. M. Assar, Control of the continuous rheocasting process, Journal of Materials Science. 1988, 23, 1385-1390.

[10] K. P. Young, R. G. Riek, M. C. Flemings, Structure and properties of Thixocast steels, Metal Science Journal. 1979, 6, 130-137.

[11] M. Tocci, A. Pola, M. Modigell, Rheological investigation of semi-solid AlSi7 alloys by means of oscillation experiments, Solid State Phenomena. 2018, 285, 385-390.
[12] M. Modigell, M. Hufschmidt, Dynamic and static yield stress of metallic suspensions, Solid State Phenomena. 2006, 116, 587-590.
[13] C. Lin, S. S. Wu, S. S. Lu, Effect of high pressure on Fe-rich phases and mechanical properties of Al-14Si alloys with rheo-squeeza casting, Solid State Phenomena. 2018, 285, 57-62.
[14] K. Lu, S. S. Wu, S. S. Lu, C. Lin, Apparent viscosity and rheological behavior of aluminum alloy slurry containing nano-size SiC particles, Solid State Phenomena. 2018, 285, 391-397.
[15] D. Bell, Flow stability, In Non-Newtonian fluids, 2nd ed., Physics Education: U.S.A., 1979, 7, pp. 432-436.
[16] J. P. Hartnett, Y. I. Cho, Non-Newtonian fluids, Handbook of Heat Transfer. 1998, 5, 126-131.
[17] ISO 3219. Plastics-Polymers/resins in the liquid state or as emulsions or dispersions Determination of viscosity using a rotational viscometer with defined shear state.
[18] H. Masude, T. Horie, R. Hubacz, Prediction of onset of Taylor-Couette instability for shear-thinning fluids, Rheol Act. 2017, 56, 73-84.
[19] A. Zell, C. Wagner, Polymer solutions in co-rotating Taylor–Couette flow without vorticity, Physica A. 2012, 391, 464-473.
[20] G. I. Taylor, Stability of a Viscous Liquid Contained Between Two Rotating Cylinders, Phil. Trans. A. 1923, 5, 223.
[21] S. Chandrasekhar, The stability of viscous flow between rotating cylinders, Mathematika. 1954, 1, 9-13.
[22] L. X. Zhuang, X. Y. Yin, Flow stability. In Fluid mechanics, 2nd ed., University of Science and Technology of China Press, China, 2009, 10, pp. 372-373.
[23] P. S. Harvey, M. Graves, Turbulent flow in an agitated Vessel. Pan II: Numerical solution and model predictions, Trans Inst Chem Eng. 1982, 60, 201-210.
[24] K. V. Riet, W. Bruijn, J. M. Smith, Real and pseudo-turbulence in the discharge stream from a Rushton turbine, Chemical Engineering Science. 1976, 31, 407-412.
[25] J. V. Luo, A. D. Gosman, R. I. Issa, J. C. Middleton, M. K. Fitzgerald, Full flow field computation of mixing in baffled stirred tractors, Trans Chime. 1993, 71A, 342-344.
[26] X. G. Hu, Q. Zhu, H.V. Atkinson et al, A time-dependent power law viscosity model and its application in modelling semi-solid die casting of 319s alloy, Acta Mater. 2017, 124, 410-420.