Chapter 1

Towards noncommutative gravity

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In this short article accessible for non-experts I discuss possible ways of constructing a non-commutative gravity paying special attention to possibilities of realizing the full diffeomorphism symmetry and to relations with 2D gravities.

1.1. Preliminaries

For the first time I met Wolfgang Kummer in 1992. It happened on my way back from Italy to St.Petersburg. At that time, a hundred of US dollars was a fortune in Russia. Therefore, to save money I took a train going through Vienna, and not a plane flying over it. The most natural decision was to stop in Vienna for a couple of days and give a seminar at TU. This is how one of the most fruitful and exciting collaborations in my life started, and this is also a very rare example of a positive effect of severe financial difficulties.

The Vienna School of 2D gravity was an amazingly successful project, see. To keep it running, new interesting directions of research were always needed. About 2005 I told Wolfgang about my recent work on noncommutative (NC) gravity in two dimensions which almost literally repeated some of the steps done previously in the commutative case. We decided to return to this after completing our current work. Unfortunately, deteriorating health did not allow Wolfgang to take up this job. This short article is a kind of a proposal for a “Vienna-style” NC gravity. This is not a (mini)review, with most visible consequence that the literature is incomplete. I am asking all authors whose papers will not be mentioned for understanding. For a systematic overview of NC gravities the reader may
consult the paper by Szabo.\textsuperscript{3}

Generally speaking, the desire to construct an NC gravity is very natural. One of the main arguments in favor of noncommutativity comes from gravity.\textsuperscript{4} Particular ways to realize noncommutativity differ much from model to model.

To stay closer to Vienna, whenever possible, I will discuss noncommutative counterparts of dilaton gravities in two dimensions (see Ref.\textsuperscript{5} for a review). In the commutative case, the classical first-order action reads

\[
S = \int_M \left[ X_a D e^a + X d \omega + \epsilon (U(X)X^a X_a/2 + V(X)) \right],
\]

where \( a = 0, 1 \) is a Lorentz index, \( e^a \) and \( \omega \) are the zweibein and connection one-forms respectively, \( \epsilon \) is a volume two-form, \( X \) is the dilaton, and \( X^a \) is an auxiliary field which generates the torsion constraint. \( D e^a = d e^a + \epsilon^a_{\ b} \omega \wedge e^b \), where \( \epsilon^{ab} \) is the Christoffel symbol. \( U(X) \) and \( V(X) \) are two arbitrary functions called the dilaton potentials. With the choice \( U(X) = 0, V(X) \propto X \) one obtains the Jackiw-Teitelboim model.\textsuperscript{6} Other choices reproduce all gravity models in two dimensions, see Ref.\textsuperscript{7}

1.2. What can we call a noncommutative gravity?

In principle, any theory containing some effects of noncommutativity of the coordinates and looking more or less like a gravity theory may be called a noncommutative gravity. The problem is that the people working on a particular approach are (naturally) more enthusiastic about it than the rest of the community. Therefore, I asked myself, what kind of noncommutative gravity theory could have a chance to satisfy Wolfgang? An answer to this question seems to be a rather strict point of view on NC gravity.

To construct a gravity one first needs a manifold. NC manifolds may be understood through the Gelfand-Naimark duality. To a manifold \( M \) one can associate a commutative associative algebra \( C^\infty(M) \) of smooth functions. Under certain restrictions, each commutative associative algebra is an algebra of smooth functions on some manifold. In this sense, an algebra \( A \), which is a noncommutative associative deformation of \( C^\infty(M) \) defines an NC deformation of \( M \). Most conveniently the deformation is done by replacing the point-wise product \( f_1 \cdot f_2 \) by a noncommutative star product \( f_1 \ast f_2 \), which can be presented as

\[
f_1 \ast f_2 = f_1 \cdot f_2 + \frac{i}{2} \theta^{\mu\nu}(x) \partial_\mu f_1 \cdot \partial_\nu f_2 + O(\theta^2) .
\]
Towards noncommutative gravity

Because of the associativity, $\theta^{\mu\nu}$ is a Poisson bivector, i.e., it has to satisfy the Jacobi identity. Note, that in two dimensions the Jacobi identity is satisfied any antisymmetric tensor $\theta^{\mu\nu}(x)$.

For a constant $\theta$ there exists a simple (Moyal) formula for the star product

$$(f_1 \star_M f_2)(x) = \exp \left( \frac{i}{2} \theta^{\mu\nu} \partial_\mu \partial_\nu \right) f_1(x) f_2(y)|_{y=x}. \quad (1.3)$$

Next, one has to satisfy the relativity principle, i.e., one should realize the group of diffeomorphisms (or a deformation of this group) on an NC manifold. Then one has to construct invariants which in the commutative limit $\theta \to 0$ reproduce the Einstein-Hilbert action coupled to matter fields. This program, upon completion, should give an NC gravity.

None of the existing approaches to the NC gravity fulfills strictly all the requirements formulated above, but we still can learn a lot from each of them.

1.2.1. Minimalistic approaches

These are approaches which are not even trying to construct a full NC gravity but instead focus on some selected features of NC theories. For example, in one of such approaches, reviewed in Ref., the nonlocality, which is a characteristic feature of NC theories, is modelled by delocalization of sources in otherwise commutative theories. Such approaches are very useful in one wishes to understand what kind of physical effects may follow from the noncommutativity, but they are not designed to check theoretical consistency.

1.2.2. Seiberg-Witten map

In 1999 Seiberg and Witten\(^9\) discovered a map between commutative and noncommutative gauge theories. Due to this map, gauge symmetries, including diffeomorphisms, can be realized by standard commutative transformations on commutative fields. The NC fields are expressed through power series in $\theta$ with growing number of commutative fields and their derivatives. This map was applied also to gravity, and even some physical effects were studied, see e.g.\(^10\) With higher orders of $\theta$ technical difficulties in applying the Seiberg-Witten map grow fast, so that no one was able to go beyond the second order. Because of this, this method can hardly be considered as an ultimate solution of the problem of constructing an NC gravity.
gravity, but it gives a very valuable information: the statement that such a theory does exist at least in the form of power series.

1.2.3. Gauging symplectic diffeomorphisms

Looking at the formula (1.3) one immediately sees a source of the problems with the diffeomorphisms: $\theta^{\mu\nu}$ looks as a tensor, but the formula (1.3) is not tensorial. Then, it is natural to assume that the things become easier with the part of the diffeomorphisms group which does not change $\theta$. For a non-degenerate $\theta^{\mu\nu}$ these diffeomorphisms (symplectomorphisms) are generated by vector fields of the form

$$\xi^\mu(x) = \theta^{\mu\nu} \partial_\nu f(x).$$

Such diffeomorphisms preserve also the volume element, and thus we are dealing with unimodular gravity theories. NC theories based on gauging symplectic diffeomorphisms were indeed constricted and gave rise to many interesting results. Though in our rather strict approach to NC gravities this group looks too small, we again receive an important message that a consistent NC theory may be constructed at least with this small part.

1.2.4. Gravity through Yang-Mills type symmetries

The action of a Yang-Mills gauge transformation can easily be extended to a noncommutative case. Let in a commutative theory $\delta_\alpha \phi = \rho(\alpha) \cdot \phi$, where $\phi$ is a field transformed according to a finite dimensional representation $\rho$ of the symmetry algebra. Then in an NC case one can define $\delta_\alpha^* \phi = \rho(\alpha) \star \phi$. A problem appears with commutators. Let $T^A$ be a basis in the Lie algebra taken in the representation $\rho$. Then

$$\delta_\alpha^* \delta_\beta^* - \delta_\beta^* \delta_\alpha^* = \delta_{[\alpha,\beta]}^*,$$

$$[\alpha,\beta]^* = \frac{1}{2} [T^A, T^B] (\alpha_A \star \beta_B + \beta_B \star \alpha_A)$$

$$+ \frac{1}{2} \{T^A, T^B\} (\alpha_A \star \beta_B - \beta_B \star \alpha_A)$$

The expression on the right hand side of the last line is a gauge generator if both commutator $[T^A, T^B]$ and anticommutator $\{T^A, T^B\}$ belong to the Lie algebra. This imposes severe restrictions on possible gauge groups and their representations. For example, $su(n)$ cannot be extended to NC spaces, while $u(n)$ can.
One can demonstrate, that with the choice of the potentials \( U(X) = 0, \)
\( V(X) \propto X \) corresponding to the Jackiw-Teitelboim model\(^6\) is equivalent to
an \( su(1, 1) \) BF theory. Consequently, extending this symmetry to an NC \( u(1, 1) \) one can construct an NC version of the JT gravity.\(^13\) The model appears to be both classical\(^13\) and quantum\(^2\) integrable. Of course, by extending the gauge symmetry one introduces a new gauge field, which, however, decouples in the commutative limit and does not lead to any contradictions. However, there is a different problem with this approach. One cannot deform the linear dilaton potential \( V(X) \) by adding higher powers of the dilaton and preserving the number of NC gauge symmetries.\(^14\) This means that other interesting dilaton gravity models cannot be constructed in this approach.

1.2.5. Twisted symmetries

Practically all symmetries of commutative theories can be realized on a noncommutative space as twisted symmetries. The twisting is based on an observation that the Moyal product (1.3) can be represented as a composition of the point-wise product and a Drinfeld twist. Indeed, the point-wise product \( \mu : A \otimes A \rightarrow A, \mu(f_1 \otimes f_2) = f_1 \cdot f_2 \) and the Moyal product \( \mu_* : A \otimes A \rightarrow A, \mu_*(f_1 \otimes f_2) = f_1 \star_M f_2 \) are related through \( \mu_* = \mu \circ \mathcal{F}^{-1} \), where

\[
\mathcal{F} = \exp \mathcal{P}, \quad \mathcal{P} = -\frac{i}{2} \theta^{\mu \nu} \partial_{\mu} \otimes \partial_{\nu}
\]  

(1.5)
is a twist.

The way how the symmetry generators act on tensor products is defined by the coproduct \( \Delta \). In commutative field theories one uses a primitive coproduct \( \Delta_0(\alpha) = \alpha \otimes 1 + 1 \otimes \alpha \), so that we have the usual Leibniz rule

\[
\alpha(\phi_1 \otimes \phi_2) = \Delta_0(\alpha)(\phi_1 \otimes \phi_2) = (\alpha \phi_1) \otimes \phi_2 + \phi_1 \otimes (\alpha \phi_2).
\]  

(1.6)

We may define another (twisted) coproduct

\[
\Delta_\mathcal{F} = \mathcal{F} \Delta \mathcal{F}^{-1}
\]  

(1.7)

The action of a generator \( \alpha \) on the star-product of fields is defined as follows

\[
\alpha(\phi_1 \star_M \phi_2) = \mu_*(\Delta_\mathcal{F}(\alpha)\phi_1 \otimes \phi_2) = \mu \circ \mathcal{F}^{-1}(\Delta_\mathcal{F}(\alpha)\phi_1 \otimes \phi_2)
\]  

(1.8)
Twisting, in a sense, pushes the symmetry generator through the star product. This makes it possible to define symmetry transformations without
transforming the star product. In algebraic language, we have a Hopf algebra symmetry instead of a Lie algebra one.

The literature on twisted symmetries is very large. We like to mention an early paper by Oeckl.\textsuperscript{15} The symmetries relevant for our discussion are the Poincare symmetry\textsuperscript{16} (this was the first symmetry to be twisted), diffeomorphisms,\textsuperscript{17} and gauge symmetries.\textsuperscript{18} Moreover, the twist interpretation may be given to some star products other than the Moyal one.

Twisting the diffeomorphism transformations allowed to define a model of NC gravity\textsuperscript{17} invariant under the full diffeomorphism algebra, though this invariance is realized in a non-standard way\textsuperscript{a}.

The twisted symmetries are not \textit{bona fide} physical symmetries. One cannot use them, for example, to gauge away any degrees of freedom. The problem of proper interpretation of twisted local symmetries remains. One possible interpretation is as follows.\textsuperscript{20} Let us replace the partial derivatives $\partial$ in (1.3) and (1.5) with covariant derivatives $\nabla$ with a trivial connection. Since $\nabla_\mu$ commute, the new star product will be again associative. (For non-commuting $\nabla$ the associativity is violated\textsuperscript{21}). If the original theory were twisted gauge invariant, the theory with this new star product will be both twisted gauge invariant and gauge invariant in the ordinary sense. To return back, one has to fix the gauge $\nabla = \partial$. Therefore, twisted gauge invariance is a remnant of ordinary gauge invariance after fixing the gauge by imposing a condition on gauge-trivial covariant derivatives appearing inside the star product.

1.2.6. \textit{NC geometry and spectral action}

A unifying approach to describe \textit{any} NC geometry was introduced by Connes\textsuperscript{22} (see also Ref.\textsuperscript{23} for a recent overview). It is based on the notion of a spectral triple $(A, H, D)$ consisting of an associative algebra $A$ represented by bounded operators on a Hilbert space $H$ and a Dirac operator $D$ acting on $H$. These three object satisfy certain relations and restrictions. As soon as a spectral triple is defined, the corresponding classical action follows from the so-called spectral action principle\textsuperscript{24}

\begin{equation}
S = \text{Tr} \Phi(D/\Lambda),
\end{equation}

where $\Phi$ is a positive even function, and $\Lambda$ is a scale parameter. All unitary symmetries of the operator $D$ are inherited by the spectral action. As an expansion in $\Lambda$ the action (1.9) may be calculated by the heat kernel

\textsuperscript{a}There are also critics of twisting local symmetries, see.\textsuperscript{19}
Towards noncommutative gravity

methods. On Moyal spaces such methods are rather well developed. The problem is “only” to find a corresponding spectral triple.

A similar idea, that the NC gravity may be induced is explored in the emergent gravity approach, see Ref. and references therein.

1.3. The star products

As we have seen above, rigidity of $\theta^{\mu \nu}$ under the diffeomorphism transformations creates a lot of problems. It may be a good idea to transform both $\theta^{\mu \nu}$ and the star product under the diffeomorphisms. To this end, we need general star products.

The modern history of deformation quantization started with the papers. The main part of the deformation-quantization program is a construction of a star product for a given Poisson structure $\theta^{\mu \nu}(x)$. For symplectic manifolds (non-degenerate $\theta^{\mu \nu}$) the existence of a star product was demonstrated by De Wilde and Lecomte, and a very elegant construction was given by Fedosov. For generic Poisson structure the existence of a star product was demonstrated by Kontsevich who also gave an explicit formula (which is, however, too complicated to be used for actual calculations of higher orders in the star product). Such orders of the star product were computed by using the Weyl map and a representation of noncommutative coordinates in the form of differential operators.

A very promising non-perturbative formula for the star product was suggested by Cattaneo and Felder. They took a Poisson sigma model with the action

$$ S_{PSM} = \int \left[ A_\mu dX^\mu + \frac{1}{2} \theta^{\mu \nu} (X) A_\mu \wedge A_\nu \right] $$

(1.10)

defined on a two-dimensional manifold. $X$ and $A$ are the fields on this manifold, which are a zero-form taking values in a Poisson manifold and a one-form with values in the cotangent space to this manifold, respectively. The two-dimensional world-sheet is supposed to be a disc (with suitable boundary conditions imposed on $A$). Three distinct points on the boundary of the disc are selected, denoted 0, 1, and $\infty$. The star product is then given by a correlation function

$$ f \star g(x) = \int dA dX f(X(0))g(X(1)) e^{i S_{PSM}} , $$

(1.11)

where the integration is restricted by the condition $X^\mu(\infty) = x^\mu$. The main advantage of this formula is that it does not imply any expansion in $\theta$.  

What is then the relation to two-dimensional dilaton gravities? The point is that the Poisson sigma models were originally introduced\textsuperscript{33,34} as generalizations of the dilaton gravity action (1.1). Indeed, by identifying $X, X^a$ with $X^\mu$, and $\omega, e^a$ with $A_\mu$ and making a suitable choice of $\theta^{\mu\nu}(X)$ one can reduce (1.10) to (1.1). In the context of two-dimensional gravities rather powerful methods of calculation of the path integral were developed.\textsuperscript{35} At least some of these methods work also for generic Poisson sigma models.\textsuperscript{36} The approach\textsuperscript{35} was specially tailored to study quantum gravity phenomena, like virtual black holes, and not the correlation functions of the type (1.11). However, some steps to adjust that methods to the new tasks have already been done. For example, inclusion of boundaries was considered in a paper,\textsuperscript{37} which was the last publication of Wolfgang Kummer.

1.4. Conclusions

As we have seen, there are many rather successful approaches to NC gravity. One can be optimistic, that soon an NC gravity satisfying our (perhaps, too strict) criteria will be formulated. It is likely, that 2D dilaton gravities will play a prominent role in this process.

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