The Number of Neutrinos and the Left-Right Symmetric Model

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Abstract
The meaning of $N_{\nu}$ in the Left-Right Symmetric Model (LRSM) is discussed. In its original definition $N_{\nu}$ is the number of neutrinos or generations of leptons, so, in the Standard Model its value is three. However the determination of $N_{\nu}$ in experiments and in astrophysical observations is subject to different theoretical interpretations. We present arguments that gives $N_{\nu}$ as a parameter of the LRSM. Using an experimental value for the rate $\Gamma_{inv}/\Gamma_{l\bar{l}}$ we calculate the bound $2.90 \leq N_{\nu_{LR}} \leq 3.04$ (90 \% C. L.), where $N_{\nu_{LR}}$ is a function of the left-right mixing angle $\phi$. This range is less restrictive than the one of the standard model.

\textit{to be published in Z. Phys. C}
1 Introduction

Many models have been proposed in the literature that give the standard model (SM) [1] in the low energy limit. One of these models is the left-right symmetric model (LRSM) [2], with a $SU_L(2) \times SU_R(2) \times U(1)$ gauge group. In this class of models the gauge symmetry is spontaneously broken giving different masses to the left and right-handed gauge bosons. In the low-energy limit the LRSM becomes the SM and small deviations could be observed making high precision experiments. In order to know how many light neutrinos have the SM we need to know the value of the partial decay width $\Gamma(Z \rightarrow \nu \bar{\nu})$. However, this process is such that no final states are observed. Hence, the only way to get information about this partial decay is from the invisible width $\Gamma_{\text{inv}} = \Gamma_Z - (\Gamma_{\text{had}} + \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau})$. The number $N_\nu$ is defined as a combination of the experimental and the theoretical magnitudes of $\Gamma_{\text{inv}}$, $\Gamma_{\ell\bar{\ell}}$ and $\Gamma_{\nu\bar{\nu}}$ and it is then identified with the number of light neutrinos.

Recent experimental analysis on $N_\nu$, in the framework of the SM, gives as a result $N_\nu = 2.983 \pm 0.034$. In this paper we make the calculation of $N_\nu$ in the LRSM. We found that in this case the quantity defined as $N_\nu$ is not a constant but depends on the mixing angle $\phi$ and therefore is not necessarily an integer number. We give an estimation of the value $N_\nu$ in this model using experimental data and also from previous constraints on the $\phi$ angle.

In Sec. 2 we describe the model, whereas in Sec. 3 we discuss the meaning
of $N_\nu$ in the LRSM and give the results of the calculation and also our conclusions.

2 The Left-Right Symmetric Lagrangian

In the LRSM the Lagrangian that describes the interaction between the leptons and the neutral gauge bosons is

$$\mathcal{L}_N = g J_3^L W_3^L + g J_3^R W_3^R + \frac{1}{2} g' J_Y B$$  \hspace{1cm} (1)

where $W_3^R$ is the neutral gauge boson of the $SU_R(2)$ sector of the model that couples with the right-handed current $J_3^R$ of leptons. We are interested in the LRSM with a bidoublet and two doublets in the Higgs sector; this Higgs content necessarily implies Dirac Neutrinos. After the spontaneous breakdown of the symmetry, the gauge bosons become mass eigenstates and therefore linear combinations of $W_3^L, W_3^R$ and $B$. The relation between the mass eigenstates and the interaction eigenstates is given through a mixing matrix. This mixing matrix depends on two angles: $\theta_W$, the SM electroweak parameter, and $\phi$, that mixes the left and right handed gauge bosons, after imposing the condition that the electromagnetic current has to couple to the photon. The mixing can be realized in two steps [4]:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix} = \begin{pmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_W & -s_W t_W & -t_W r_W \\ 0 & r_W / c_W & -t_W \\ s_W & s_W & r_W \end{pmatrix} \begin{pmatrix} W_3^L \\ W_3^R \\ B \end{pmatrix}$$ \hspace{1cm} (2)
with \( s_\phi = \sin \phi \), \( c_\phi = \cos \phi \), \( s_W = \sin \theta_W \), \( c_W = \cos \theta_W \) and \( r_W = \sqrt{\cos 2\theta_W} \).

In terms of the mass eigenstates the general Lagrangian responsible for the neutral current interactions is written as

\[
\mathcal{L}_N = e J_{em} A - \frac{e}{c_\theta} (a_1 J^Z_L + b_1 J^Z_R) Z_1 + \frac{e}{c_\theta} (a_2 J^Z_L + b_2 J^Z_R) Z_2 \tag{3}
\]

where

\[
egin{align*}
a_1 &= \frac{s_W s_\phi - c_\phi}{r_W} - \frac{s_\phi}{s_W} \\
b_1 &= \frac{c^2_W s_\phi}{s_W r_W} \\
a_2 &= \frac{s_W c_\phi + s_\phi}{r_W} \\
b_2 &= \frac{c^2_W s_\phi}{s_W r_W} \tag{4}
\end{align*}
\]

In the limit \( \phi \to 0 \) and \( M_{Z_2} \to \infty \) we get the SM Lagrangian. In Eq. (3) we have all the information we need to calculate the process \( Z_1 \to f \bar{f} \).

### 3 \( N_\nu \) as a function of \( \phi \)

If we are working at the \( Z \) peak, the amplitude for the decay \( Z_1 \to l \bar{l} \) comes from the second term of (3) and is given by [4]

\[
M = \frac{g}{c_W} \left[ \bar{u}_\gamma \gamma^\mu (g_{V_{LR}} - g_{A_{LR}} \gamma_5) v \right] \epsilon_\mu^\lambda \tag{5}
\]

where \( \epsilon_\mu^\lambda \) is the \( Z_1 \) boson polarization, \( u \) (\( v \)) is the lepton (antilepton) spinor and

\[
g_{V_{LR}} = \left[ c_\phi - \frac{s^2_W}{r_W} s_\phi \right] g_V - \frac{c^2_W}{r_W} s_\phi g_{V_R}, \tag{6}
\]
Here $\bar{g}_V$ ($\bar{g}_A$) is the vector (axial-vector) coupling constant for charged leptons with radiative corrections that comes from the left-handed sector of the model:

$$\bar{g}_V = \sqrt{\rho_f} \left( \frac{1}{2} + 2\kappa_f \sin^2 \theta_W \right)$$
$$\bar{g}_A = \sqrt{\rho_f} \left( -\frac{1}{2} \right)$$

while $g_{V_R}$ ($g_{A_R}$) is the same coupling constant as before but without radiative corrections and has its origin in the right-handed sector.

For the decay of $Z_1$ in neutrinos we also have corrections for the coupling constants that comes only from the LRSM:

$$g_{\nu V LR} = \left( c_\phi - \frac{s_\phi}{r_W} \right) g'$$
$$g_{\nu A LR} = \left( c_\phi + s_\phi r_W \right) g'$$

where $g' = \frac{1}{2}$. 

In the framework of the LRSM model, the width for the process $Z \rightarrow l\bar{l}$ is written as

$$\Gamma(Z \rightarrow l\bar{l}) = \frac{G_F M_Z^2}{6\pi \sqrt{2}} \left[ g_{V LR}^2 + g_{A LR}^2 \right]$$
In Eq. (11) we have neglected the mass of the leptons. In the case of the decay \( Z \to \nu \bar{\nu} \) the expression (11) becomes

\[
\Gamma(Z \to \nu \bar{\nu}) = \frac{G_F M_Z^3}{6\pi \sqrt{2}} \left[ \left( c_\phi - \frac{s_\phi}{r_W} \right)^2 + \left( c_\phi + r_W s_\phi \right)^2 \right]
\]

(12)

In the limit when \( \phi \) goes to zero we obtain the SM tree level partial decay width:

\[
\Gamma_0(Z \to \nu \bar{\nu}) = \frac{G_F M_Z^3}{12\pi \sqrt{2}}
\]

(13)

The partial widths (11) and (12) are applicable to all charged leptons and all neutrinos respectively. In order to use the expressions (11) and (12) for comparing with the experimental result for the number of light neutrinos \( N_\nu \) we recall the experimental definition for \( N_\nu \) in a SM analysis [7],

\[
N_\nu = R_{\text{exp}} \left( \frac{\Gamma_{\ell \bar{\ell}}}{\Gamma_{\nu \bar{\nu}}} \right)_{\text{SM}}.
\]

(14)

Here, the quantity in parenthesis is the standard model prediction and the \( R_{\text{exp}} \) factor is the experimental value of the ratio between the widths \( \Gamma_{\text{inv}} \) and \( \Gamma_{\ell \bar{\ell}} \) [5],

\[
R_{\text{exp}} = \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell \bar{\ell}}} = 5.942 \pm 0.067.
\]

(15)

The definition (14) for \( N_\nu \) replaces the expression \( N_\nu = \Gamma_{\text{inv}}/\Gamma_{\nu \bar{\nu}} \), since (14) reduces the influence of the top quark mass.

If we want to get information about what is the meaning of \( N_\nu \) in the LRSM model we should define the corresponding expression,

\[
N_{\nu LR} = R_{\text{exp}} \left( \frac{\Gamma_{\ell \bar{\ell}}}{\Gamma_{\nu \bar{\nu}}} \right)_{LR}
\]

(16)
This new expression will be a function of $\phi$, so in this case the quantity defined as the number of light neutrinos is not a constant and not necessarily an integer. Also $N_{\nu_{LR}}$ in formula (16) is independent from the $Z_2$ mass and therefore depends on only one parameter of the LRSM. Experimental values for $\Gamma_{\text{inv}}$ and for $\Gamma_{\bar{\nu}}$ are reported in literature which in our case, can give a bound for the angle $\phi$. However, we can look to these experimental numbers in another way. The partial widths $\Gamma_{\text{inv}} = 499.9 \pm 2.5$ and $\Gamma_{\bar{\nu}} = 83.93 \pm 0.14$ were reported recently [8], but we use here the value (15) for the $R_{\text{exp}}$ rate of Ref. [5]. All these measurements are independent of any model and can be fitted with the LRSM parameter $N_{\nu_{LR}}$ in terms of $\phi$. We can plot the expression (16) to see the general behavior of the $N_{\nu_{LR}}(\phi)$ function. The Fig. (1) is this plot. We can observe that for some values of $\phi$, around 0.6 rad, $N_{\nu_{LR}}$ can be as high as 5.9, and for values of $\phi$ around $-0.9$ rad, $N_{\nu_{LR}}$ is as low as 0. This indicates a strong dependence on $\phi$ for leptonic decays of $Z$. Therefore, according to the above discussion, if we consider $N_{\nu_{LR}}$ as the number of neutrinos, the restriction on the number of generations can be "softened" if we consider a LRSM model.

However $\phi$ is severely bounded, so it is $N_{\nu_{LR}}$. In Fig. (2) we show the allowed region for $N_{\nu_{LR}}$ with 90% C. L. The allowed region is the inclined band that is a result of both factors in Eq. (16): $(\Gamma_{\bar{\nu}}/\Gamma_{\nu\bar{\nu}})_{LR}$ gives the inclination and $R_{\text{exp}}$ gives the broading. The analysis was done using the experimental
value (15) for $R_{\text{exp}}$ reported by ALEPH Collaboration [5] with a 90% C. L. The region for $\phi$ is given from a previous constraint on this angle [4]: $-0.009 \leq \phi \leq 0.004$, obtained from the LEP experimental value of $g_A$ [7]. In the same figure we show the SM ($\phi = 0$) result at 90% C. L. with the solid horizontal lines. As we can see, the allowed region in the LR model (dotted line) for $N_{\nu_{LR}}$ is wider that the one for the SM, and is given by:

$$2.90 \leq N_{\nu_{LR}} \leq 3.04.$$  \hspace{3cm} (17)

Although the allowed region is wider, the fact that the mixing angle has been extremely restricted forces $N_{\nu_{LR}}$ to be near 3, excluding the possibility to get a number such as 2.6 which has been reported as an upper bound from big bang nucleosynthesis [9]. In order to get this value we need $\phi = -0.063$, which is in contradiction with the present restrictions on $\phi$.

Finally, and just for completeness, we can reverse the arguments, that is, we fix the number of neutrinos in the LRSM to be three then the theoretical expression for $R$ will be given by

$$R = \frac{3 \Gamma_{\nu\bar{\nu}}}{\Gamma_{\bar{l}l}}.$$  \hspace{3cm} (18)

If we make a plot of this quantity in terms of $\phi$ we have the curve shown in Fig.(3). The horizontal lines give the experimental region at 90 % C. L. As we can see from the figure the constraint for the $\phi$ angle is: $-0.006 \leq \phi \leq 0.011$. This restriction is in close agreement with the constraint previously obtained
in Ref. [4].

As a conclusion we can say that in the left-right symmetric model we can obtain from experimental results a value for $N_\nu$ different from 3 (not necessarily an integer number). In particular for the LRSM with two doublets, and hence with Dirac neutrinos, $N_{\nu_{LR}}$ is in the neighborhood of three, However, if new precision experiments find small deviations from 3, this model could explain very well these deviations with a small value of $\phi$.

Acknowledgments

This work was supported in part by CONACYT (México).
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Figure Captions

Fig. 1 \( N_{\nu LR} \) as a function of \( \phi \) (rad).

Fig. 2 Allowed region for \( N_{\nu LR} \) from the experimental value \( R_{\text{exp}} \) and from a previous constraint for \( \phi \). The dashed line shows the SM allowed region for \( N_{\nu} \) at 90\% C. L. while the dotted line shows the same result for the LRSM.

Fig. 3 The curve shows the shape for \( R \) in the LRSM. The dashed line shows the experimental region at 90\% C. L.
