QCD Corrections to Radiative $B$ Decays in the MSSM with Minimal Flavor Violation

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Abstract

We compute the complete supersymmetric QCD corrections to the Wilson coefficients of the magnetic and chromomagnetic operators, relevant in the calculation of $b \rightarrow s\gamma$ decays, in the MSSM with Minimal Flavor Violation. We investigate the numerical impact of the new results for different choices of the MSSM parameters and of the scale where the quark and squark mass matrices are assumed to be aligned. We find that the corrections can be important when the superpartners are relatively light, and that they depend sizeably on the scale of alignment. Finally, we discuss how our calculation can be employed when the scale of alignment is far from the weak scale.
1 Introduction

More than a decade after their first direct observation, radiative $B$ decays have become a key element in the program of precision tests of the Standard Model (SM) and its extensions. The inclusive decay $B \to X_s \gamma$ is particularly well suited to this precision program thanks to its low sensitivity to non-perturbative effects. The present experimental world average [1] for the branching ratio of $B \to X_s \gamma$ has a total error of less than 10% and agrees well with the SM prediction, that is subject to a similar uncertainty [2]. In view of the final accuracy expected at the $B$ factories, about 5%, the SM calculation needs to be improved. It presently includes next-to-leading order (NLO) perturbative QCD corrections as well as the leading non-perturbative and electroweak effects (see [3] and [4] for a concise discussion and a complete list of references). The calculation of next-to-next-to-leading order (NNLO) QCD effects is currently under way [5] and is expected to bring the theoretical accuracy to the required level.

The theoretical accuracy of the predictions in the context of new physics models may have important consequences on model building. This is particularly true for radiative $B$ decays, where higher order corrections can be enhanced by large factors: in this case the current status of theoretical calculations is not always satisfactory. While the NLO corrections have been extensively studied in the context of Two Higgs Doublet Models [2,6,7], in the Minimal Supersymmetric Standard Model (MSSM) the complete leading order (LO) result is known [8,9] but the NLO analysis is still incomplete to date. The main reason is that new sources of flavor violation generally arise in the MSSM, making a general analysis quite complicated even at the leading order [9]. Experimental constraints on generic $b \to s$ flavor violation have been recently studied in [10]: radiative decays play a central role in these analyses, and the constraints are strong only for some of the flavor violating parameters.

A simplifying assumption frequently employed in supersymmetric analyses is that of *Minimal Flavor Violation* (MFV), according to which the only source of flavor (and possibly of CP) violation in the MSSM is the CKM matrix [11,12]. It can be implemented by assuming that the squark and quark mass matrices can be simultaneously diagonalized and, as a consequence, it implies the absence of tree-level flavor-changing gluino (FCG) interactions. The MFV hypothesis certainly represents a useful and predictive approximation scheme and seems to be favored by the present absence of deviations from the SM. However, because the weak interactions affect the squark and quark mass matrices in a different way [13], their simultaneous diagonalization is not preserved by higher order corrections and can be consistently imposed only at a certain scale $\mu_{MFV}$, complicating the study of higher order contributions in this framework. The NLO study of radiative decays in the MFV scenario has been pioneered in [14] (see also [15]), where the gluonic corrections to chargino contributions have been computed, while those involving a gluino
were computed in the heavy gluino limit, in which case FCG effects can be consistently neglected.

An alternative possibility is to include only the potentially large contributions beyond the leading order: they originate from terms enhanced by tan $\beta$ factors, when the ratio between the two Higgs vacuum expectation values is large, or by logarithms of $M_{\text{SUSY}}/M_W$, when the supersymmetric particles are considerably heavier than the $W$ boson. Compact formulae that include both kinds of higher-order effects within MFV are given in ref. [16]. Indeed, tan $\beta$-enhanced terms at the next-to-leading order do not only appear from the Hall-Rattazzi-Sarid effect (the modified relation between the bottom mass and Yukawa coupling) [18], but also from an analogous effect in the top-quark Yukawa coupling [11,16, 17] and in effective flavor-changing $\bar{s}_L b_R$ neutral heavy Higgs vertices [11]. In the effective theory approach first employed in [19] the dominant terms enhanced by tan $\beta$ can be taken into account at all orders. A generalization beyond MFV has been proposed in [20].

In the limit of heavy superpartners, in particular, the Higgs sector of the MSSM is modified by non-decoupling effects and can differ substantially from the type-II Two Higgs Doublet Model. The charged Higgs contribution therefore receives two-loop contributions enhanced by tan $\beta$ that have been computed in [11,16,17] in the limit of heavy gluino. Interestingly, the explicit calculation of the relevant two-loop diagrams presented in [21] has demonstrated the validity of this approximation even when the charged Higgs is not much lighter than the gluino. However, there is a priori no reason why the results derived in the heavy gluino limit should be a good approximation of the true result for generic values of the relevant mass parameters or in the case of other two-loop contributions.

In this letter we present the results of the full NLO calculation of the supersymmetric QCD corrections to the Wilson coefficients of the two operators that are relevant in the MFV scenario, extending and completing the work of ref. [14]. In particular, we compute all two-loop diagrams that contain a gluino, under the assumption that the gluino couplings to quarks and squarks are flavor conserving at the scale $\mu_{\text{MFV}}$. Our results allow for a consistent and complete NLO analysis of radiative $B$ decays in the MFV framework.

The paper is organized in the following way: in section 2 we describe the calculation, the renormalization procedure, and the checks; in section 3 the numerical impact of our results is discussed; section 4 explains how our results can be employed in the context of realistic models of SUSY breaking and contains our conclusions.

## 2 Gluino contribution to the Wilson coefficients

As we focus here on short-distance contributions with MFV, we can restrict our discussion to the form of the Wilson coefficients of the $\Delta B = 1$ magnetic and chromo-magnetic...
operators\footnote{There are one-loop gluino contributions to the Wilson coefficients of the four-quark operators, but we will not consider them here.}\: Q_7 = (e/16\pi^2)m_b\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu} \text{ and } Q_8 = (g_s/16\pi^2)m_b\bar{s}_L\sigma^{\mu\nu}T^a b_R G^{a}_{\mu\nu} \text{ evaluated at the matching scale } \mu_w \text{ in the effective Hamiltonian:}

\[ \mathcal{H} = \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu_w) Q_i(\mu_w) \quad (1) \]

where \( G_F \) is the Fermi constant and \( V_{ts}, V_{tb} \) are elements of the CKM matrix. We can organize the Wilson coefficients of the operators \( Q_{7,8} \) in the following way:

\[ C_{7,8}(\mu_w) = C_{7,8}^{(0)\text{SM}}(\mu_w) + C_{7,8}^{(0)H^\pm}(\mu_w) + C_{7,8}^{(0)\text{SUSY}}(\mu_w) \]

\[ + \frac{\alpha_s(\mu_w)}{4\pi} \left[ C_{7,8}^{(1)\text{SM}}(\mu_w) + C_{7,8}^{(1)H^\pm}(\mu_w) + C_{7,8}^{(1)\text{SUSY}}(\mu_w) \right], \quad (2) \]

where the various LO contributions are classified according to whether the corresponding one-loop diagrams contain only SM fields (\( C_{7,8}^{(0)\text{SM}} \)), a physical charged Higgs boson and an up-type quark (\( C_{7,8}^{(0)H^\pm} \)), or a chargino and an up-type squark (\( C_{7,8}^{(0)\text{SUSY}} \)). The expressions for \( C_{7,8}^{(0)\text{SM}} \) and \( C_{7,8}^{(0)H^\pm} \) can be found, e.g., in ref. [6], while those for \( C_{7,8}^{(0)\text{SUSY}} \) can be found, e.g., in eq. (4) of ref. [14]. Neutralino and gluino exchange diagrams will be neglected under our MFV assumption. The relation between the LO and NLO Wilson coefficients at \( \mu_w \) and the branching ratio for \( B \to X_s \gamma \) is well known (see for example refs. [2, 6]).

The NLO coefficients \( C_{7,8}^{(1)\text{SM}} \) and \( C_{7,8}^{(1)H^\pm} \) contain the gluonic two-loop corrections to the SM and charged Higgs loops, respectively, and can be found for instance in ref. [6]. At NLO the supersymmetric contribution \( C_{7,8}^{(1)\text{SUSY}} \) can be further decomposed,

\[ C_{7,8}^{(1)\text{SUSY}} = C_{7,8}^{(1)\tilde{g}} + C_{7,8}^{(1)\chi^\pm}, \quad (3) \]

where \( C_{7,8}^{(1)\tilde{g}} \) contains two-loop diagrams with a gluino together with a Higgs or W boson, while \( C_{7,8}^{(1)\chi^\pm} \) corresponds to two-loop diagrams with a chargino together with a gluon or a gluino or a quartic squark coupling. It should be recalled that, unlike \( C_{7,8}^{(1)\text{SM}} \) and \( C_{7,8}^{(1)H^\pm} \), the two-loop gluonic corrections to the chargino loops are not UV finite: as shown in [14], in order to obtain a finite result one has to combine them with the chargino-gluino diagrams. The chargino-gluon two-loop contributions have been fully computed in refs. [14, 15]. On the other hand, two-loop contributions involving gluinos (in both \( C_{7,8}^{(1)\tilde{g}} \) and \( C_{7,8}^{(1)\chi^\pm} \)) have been considered in ref. [14] only in the heavy gluino limit\footnote{In ref. [14] one of the top squarks was also decoupled, but it is straightforward to generalize the formulae for the light stop to the heavy stop.}. We are now going to relax this approximation and to compute \( C_{7,8}^{(1)\text{SUSY}} \) for arbitrary gluino mass in the MFV framework, assuming vanishing flavor-changing gluino couplings.

The two-loop diagrams containing a gluino or a quartic squark coupling that contribute to \( C_{7,8}^{(1)\tilde{g}} \) and \( C_{7,8}^{(1)\chi^\pm} \) are shown in figs. 1 and 2 respectively. Together with the diagrams

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Figure 1: Feynman diagrams containing a gluino and a $W$ or a Higgs boson ($\phi = H^\pm, G^\pm$). A photon or gluon is assumed to attach in all possible ways to the particles in the loops.

Figure 2: Same as fig. 1 for diagrams containing a chargino and a gluino or a quartic squark coupling. The index $i$ labels the three generations of up-type quarks and squarks.
with gluons, they complete the QCD contribution to the Wilson coefficients of $Q_{7,8}$ in the MSSM under the MFV assumption. The effective theory is trivial, and the Wilson coefficients are directly given by the result of the Feynman diagrams. We follow the same methods employed in [6], in particular we perform our calculation in the background-field gauge [22], regularize the ultraviolet divergences using naive dimensional regularization (NDR), and neglect terms suppressed by powers of $m_b/M_W$ or $m_b/M_{SUSY}$ (after factoring out a bottom mass in the definition of the operators $Q_7$ and $Q_8$). The result for each diagram depends on a number of mass and coupling parameters; it can be simplified assuming the up-type squarks of the first two generations to be degenerate in mass, and neglecting the masses of all quarks of the first two generations. This set of assumptions allows us to exploit the unitarity of the CKM matrix and to factor out the combination $V_{ts}V_{tb}$ in the effective Hamiltonian of eq. (1).

The complete calculation of the two-loop gluino contribution presents a novel feature with respect to heavy gluino analysis of [14], namely the need for flavor-changing counterterms. Indeed, there are two-loop gluino diagrams that contain the effective FCG interactions $\tilde{b}\tilde{g}s$ or $b\tilde{s}\tilde{g}$ (see, e.g., diagrams (a) and (b) in fig. 1 and 2 respectively). These one-loop electroweak vertices are divergent and need to be renormalized. The corresponding contributions were irrelevant in [14] because they are suppressed by inverse powers of the gluino mass. We therefore distinguish between flavor-conserving counterterms, already considered in [14], and flavor-changing counterterms of electroweak origin. 

Flavor-conserving counterterms are of $O(\alpha_s)$ and originate from the masses of the bottom and top quarks, from the masses and left-right mixing of the up-type squarks that enter the one-loop diagrams with charginos, and from the flavor diagonal part of the external leg corrections. The finite parts of these counterterms depend on our choice of renormalization scheme for the masses and mixing angles that enter the one-loop results. In order to facilitate the inclusion and resummation of some large higher order effects, one can also distinguish between the top and bottom masses that originate from the loops or from the use of equations of motion, and those arising from Yukawa couplings or their supersymmetric equivalent. 

In the MFV framework, the remaining flavor-changing counterterms are of electroweak origin and arise from the renormalization of the flavor mixing of quarks and squarks and from the flavor changing part of the external leg corrections. To discuss them, we start from the gluino-quark-squark interaction Lagrangian in the super-CKM basis, where the matrices of Yukawa couplings are diagonal and the squarks are rotated parallel to their fermionic superpartners:

$$\mathcal{L} \supset -g_s T^a \sqrt{2} \left( \bar{g}^a b_L \tilde{b}_L - \tilde{g}^a b_R \tilde{b}_R + \bar{g}^a s_L \tilde{s}_L^* - \tilde{g}^a s_R \tilde{s}_R^* \right) + h.c. \quad (4)$$

where $g_s$ is the strong coupling constant and $T^a$ are SU(3) generators. We can restrict to the mixing between second and third generations, and since we are neglecting $m_s$, we need
not consider the terms involving $s_R$ or $\tilde{s}_R$. Upon renormalization of the mixing matrices, the *bare* quark and squark fields are rotated as follows:

$$
\begin{pmatrix}
  \tilde{d}_1 \\
  \tilde{d}_2 \\
  \tilde{d}_3
\end{pmatrix} = (U^r + \delta U)
\begin{pmatrix}
  \bar{b}_L \\
  \bar{b}_R \\
  \tilde{s}_L
\end{pmatrix} , \quad
\begin{pmatrix}
  d_1_L \\
  d_2_L
\end{pmatrix} = (u^{Lr} + \delta u^L)
\begin{pmatrix}
  b_L \\
  s_L
\end{pmatrix} .
$$

(5)

The MFV assumption translates into the requirement that the renormalized mixing matrices be flavor diagonal:

$$
U^r = \begin{pmatrix}
  B & 0 \\
  0 & 1
\end{pmatrix} = \begin{pmatrix}
  \cos \theta_b & \sin \theta_b & 0 \\
  -\sin \theta_b & \cos \theta_b & 0 \\
  0 & 0 & 1
\end{pmatrix} , \quad
u^{Lr} = \begin{pmatrix}
  1 & 0 \\
  0 & 1
\end{pmatrix} ,
$$

(6)

where $B$ is a $2 \times 2$ mixing matrix in the sbottom sector and $\theta_b$ is the sbottom mixing angle. Under this requirement, the mass eigenstates for the down-type squarks relevant to our calculation can be identified with the usual sbottoms $\tilde{b}_1$ and $\tilde{b}_2$ and the left super-strange $\tilde{s}_L$. However, even if we assume that the renormalized mixing matrices for quarks and squarks are flavor-diagonal, this is not the case for the corresponding counterterms $\delta u^L$ and $\delta U$. They generate the FCG interactions:

$$
\mathcal{L} \supset -g_s T^a \sqrt{2} \left[ (\delta U^L_{3i} + B^L_{1i} \delta u_{L21}^L) \bar{s}_L g^a \tilde{b}_i + (\delta U_{31} - \delta u_{L2}^L) \bar{g}^a b_L \tilde{s}_L - \delta U_{32} \bar{g}^a b_R \tilde{s}_L^* \right] + \text{h.c.}
$$

(7)

Additional flavor changing renormalization effects are due to the (on-shell) wave function renormalization (WFR) of external quarks (see diagrams $(f)$ and $(g)$ in figs. 1 and 2). The divergent parts of the mixing counterterms $\delta u^L$ and $\delta U$ are determined in a gauge-invariant way by the requirement that they cancel the divergence of the antihermitian part of the corresponding WFR matrix [23]. Using $m_s \to 0$ and neglecting terms suppressed by $m_b/M_W$, we obtain:

$$
\delta u^L_{21} = -\frac{1}{2} \left[ \Sigma^L_{ab}(0) + 2 \Sigma^S_{ab}(0) \right] ,
$$

(8)

where we have decomposed the generic quark self-energy as

$$
\Sigma_{ij}(p) \equiv \Sigma^L_{ij}(p^2) \not\! p \not\! P_L + \Sigma^R_{ij}(p^2) \not\! p \not\! P_R + \Sigma^S_{ij}(p^2)(m_iP_L + m_jP_R) ,
$$

(9)

$P_L$ and $P_R$ being chiral projectors. The counterterm for the squark mixing matrix is instead

$$
\delta U_{ik} = \frac{1}{2} \Sigma_{j \neq i} \frac{\Pi_{ij}(m_j^2) + \Pi^*_ji(m_i^2)}{m_i^2 - m_j^2} U_{jk} ,
$$

(10)

which, for the terms that appear in eq. (7), specializes to:

$$
\delta U^L_{3i} = -\frac{1}{2} \frac{\Pi^*_{2i}(m_2^2) + \Pi_{1i}(m_1^2)}{m_1^2 - m_2^2} B_{ij} ,
$$

$$
\delta U_{3j} = -\frac{1}{2} \Sigma_{i} \frac{\Pi^*_{2i}(m_2^2) + \Pi_{1i}(m_1^2)}{m_1^2 - m_2^2} B_{ij} .
$$

(11)
We therefore see that the counterterms of the gluino flavor changing couplings are determined by quark and squark flavor-changing two-point functions only. We have checked that the counterterms in eqs. (8) and (11) renormalize correctly the \( \bar{d}d' g^a \) vertex (see [24]) and agree with the known one-loop RGE equations of the MSSM [25].

The finite part of the counterterms in eqs. (8) and (11) is related to the way we interpret the MFV requirement in eq. (6). In particular, if we perform a minimal subtraction we are imposing the MFV condition on the \( \overline{\text{MS}} \)-renormalized parameters of the Lagrangian evaluated at the scale \( \mu_{\text{MFV}} \). An alternative option consists in absorbing also the finite part of the antihermitian WFR: this results in a conventional and gauge-dependent on-shell renormalization scheme [23]. In the following, we will assume the first option and therefore our result will depend on the mass scale \( \mu_{\text{MFV}} \), that we identify with the scale where the quark and squark mass matrices are assumed to be aligned.

Once the flavor-changing vertices of eq. (7) are inserted into one-loop diagrams with a gluino and a down-type squark, the resulting counterterm contributions cancel the UV poles arising from i) the diagrams in figs. 1d, 1e and 2e, ii) the diagrams in figs. 2a and 2b with the photon or gluon attached to the down-type squark or to the gluino and iii) the flavor-changing WFR diagrams in figs. 1f, 1g, 2f and 2g. The remaining UV poles of the diagrams in figs. 1 and 2 are canceled by the flavor-conserving counterterms, but for a residual pole in the diagrams with gluino and chargino of fig. 2. This is the pole that was found in [14] in the limit of heavy gluino; it is compensated by a corresponding pole in the diagrams with gluon and chargino. In the gluonic corrections to the chargino diagrams reported in [14,15] the residual UV divergence has been subtracted either by the heavy gluino effective chargino-quark-squark vertex or in a minimal way. The finite parts related to this subtraction must be taken into account before combining with the gluino contributions. A shift in the \( \chi \bar{b}t \) coupling is also necessary to restore supersymmetric Ward identities that are not respected by NDR (see [14]).

The analytic expressions of \( C_{7,8}^{(1)\text{SUSY}} \) we derived are too long to be reported. However, in view of our choice for the flavor changing counterterms, we can split our result into two pieces

\[
C_{7,8}^{(1)\text{SUSY}}(\mu_W) = C_{7,8}^{(1a)\text{SUSY}}(\ldots, \mu_W) + C_{7,8}^{(1b)\text{SUSY}}(\ldots, \mu_{\text{MFV}}),
\]

where the dots represent the relevant combination of couplings, masses and mixing angles and the \((1a)\) piece can be identified with the contribution that, in the heavy gluino limit, reduces to the result of ref. [14]. The interesting point is that \( C_{7,8}^{(1b)\text{SUSY}} \) contains logarithms of the ratio \( M_{\text{SUSY}}/\mu_{\text{MFV}} \), i.e. of a supersymmetric mass over a mass scale related to the mechanism of supersymmetry breaking. For example, in supergravity models one identifies \( \mu_{\text{MFV}} \) with the Planck mass and therefore the Wilson coefficients contain very large logarithms that need to be resummed. If we were to employ an on-shell definition

\[\text{For the quark mixing matrix a simplification occurs when the external quark masses can be neglected, as in our case, and the gauge dependence drops out.}\]
for the flavor changing counterterms, $C_{7,8}^{\text{(1b)susy}}$ would be independent of $\mu_{\text{MFV}}$ and our result would have no large logarithm. In practice, the use of on-shell mixing counterterms is equivalent to assuming that MFV is valid at the scale of the supersymmetric masses entering the loops.

We performed several checks of our calculation. Ref. [21] presented a calculation of the tan $\beta$-enhanced part of the contribution to the Wilson coefficients coming from the diagrams in fig. 1b that involve a charged Higgs boson. We have verified that, if we restrict our calculation to the same subset of diagrams and adopt the same input parameters as in ref. [21], we can reproduce exactly fig. 8 of that paper. Also, a calculation of the QCD contributions to the Wilson coefficients from the diagrams in fig. 2d, involving a chargino and a quartic squark coupling, has been presented in ref. [15]. We have checked that, if we assume MFV in the up squark sector and perform an $\overline{\text{MS}}$ renormalization, we find complete agreement with the analytical formulae of [15]. On the other hand, the contribution of the diagrams in fig. 2d is removed by the corresponding counterterm contribution if the squark masses and mixing are defined on-shell. As already mentioned, the results for $C_{7,8}$ depend on the renormalization scheme for a number of parameters. In the case all parameters are renormalized in the on-shell scheme, the QCD corrections to the Wilson coefficients still depend on the matching scale $\mu_w$ at which the effective operators $Q_{7,8}$ are renormalized (see eq. (1)). This dependence can be expressed in terms of the LO anomalous dimension matrix [14] and we reproduce it correctly.

3 Numerical Results

We start the discussion of our numerical results by defining the set of input parameters relevant to the calculation of the Wilson coefficients. For the SM parameters we take $M_Z = 91.2$ GeV, $\sin^2 \theta_W = 0.23$ and $\alpha_s(M_W) = 0.12$ and for the top mass we use the SM value in the $\overline{\text{MS}}$ scheme i.e. $m_t(M_W) = 176.5$ GeV (corresponding to a physical top mass of 175 GeV). The soft SUSY-breaking terms that enter the squark mass matrices in the MFV scenario and are relevant to our calculation are: the masses for the SU(2) doublets, $m_{Q_i}$, where $i$ is a generation index; the masses for the third-generation singlets, $m_T$ and $m_B$; the trilinear interaction terms for the third-generation squarks, $A_t$ and $A_b$. We take all of them as running parameters, computed in a minimal subtraction scheme at the renormalization scale $\mu_{\text{SUSY}} = 500$ GeV. We recall that, in the super-CKM basis, the $3 \times 3$ mass matrices for the up-type and down-type left squarks are related by $(M_D^2)_{LL} = V (M_U^2)_{LL} V^\dagger$, where $V$ is the CKM matrix, therefore the two matrices can be both flavor-diagonal at $\mu = \mu_{\text{MFV}}$ only if they are flavor-degenerate. This means that in the MFV scenario we must introduce a common mass parameter for the three generations of SU(2) squark doublets, i.e. $m_{Q_i} \equiv m_Q$ at $\mu = \mu_{\text{MFV}}$. The other MSSM parameters relevant to our calculation, for which we need not specify a renormalization prescription,
are: the charged Higgs boson mass $m_{H^\pm}$; the gluino mass $m_{\tilde{g}}$; the SU(2) gaugino mass parameter $M_2$; the higgsino mass parameter $\mu$, with the same sign convention as in ref. [16]; the ratio of Higgs vacuum expectation values $\tan\beta$.

Some potentially large higher-order corrections can be absorbed in the one-loop results. Following ref. [16], we absorb in the one-loop coefficients the $\tan\beta$-enhanced corrections to the bottom Yukawa coupling [18]. As explained in [16], large logarithms of the ratio $M_{\text{SUSY}}/\mu_W$, induced by gluonic corrections to the one-loop chargino-stop diagrams, could also be resummed to all orders by expressing the higgsino couplings in terms of $m_t(\mu_{\text{SUSY}})$.

In what follows, however, we will use $m_t(M_W)$ for the couplings of charginos (or Higgs bosons) to top quarks and squarks, as well as for the mass of the virtual top quarks in the loops. For consistency with our choice of the SUSY parameters, we will use in the stop mass matrix the $\overline{\text{DR}}$-renormalized top quark mass, computed at the scale $\mu_{\text{SUSY}}$ with the field content of the MSSM.

Leaving a systematic study of the constraints imposed on the MSSM parameters by the $B \to X_s \gamma$ branching ratio to a future publication, we restrict our analysis to two different choices of MSSM parameters:

(I) $m_Q = 230$ GeV, $m_T = 210$ GeV, $m_B = 260$ GeV, $A_t = -70$ GeV, $A_b = 0$, $m_{H^\pm} = 350$ GeV, $m_{\tilde{g}} = M_2 = 200$ GeV, $\mu = 250$ GeV, $\tan\beta = 30$;

(II) $m_Q = 480$ GeV, $m_T = 390$ GeV, $m_B = 510$ GeV, $A_t = -560$ GeV, $A_b = -960$, $m_{H^\pm} = 430$ GeV, $m_{\tilde{g}} = 600$ GeV, $M_2 = 190$ GeV, $\mu = 390$ GeV, $\tan\beta = 10$.

The first set is analogous to “spectrum II” in ref. [21] and is characterized by moderately large $\tan\beta$ and fairly light superpartners. The second set of parameters corresponds broadly to the so-called Snowmass Point SPS1a’ [26], obtained through RG evolution from a set of universal high-energy boundary conditions imposed by the mechanism of gravity-mediated supersymmetry breaking. It is characterized by a smaller value of $\tan\beta$ and somewhat heavier superpartners (well within the reach of future collider experiments). In both cases we impose the MFV relation $m_{Q_i} \equiv m_Q$.

We can now discuss our numerical results for the Wilson coefficients $C_7(M_W)$ and $C_8(M_W)$. To start with, we assume MFV at the level of the running parameters of the MSSM Lagrangian, at the renormalization scale $\mu_{\text{MFV}} = 500$ GeV. In figs. 3 and 4, the left end of each curve corresponds to the choice of MSSM input parameters defined above in the sets I and II, respectively. To study the decoupling behaviour of the corrections, we rescale all the supersymmetric mass parameters – but for the charged Higgs boson mass – by an increasing common factor, and show $C_7(M_W)$ and $C_8(M_W)$ as a function of the resulting value of the gluino mass (i.e. in figs. 3 and 4 all the squark and chargino masses increase together with $m_{\tilde{g}}$). In each plot, the dashed line corresponds to the pure one-loop result (i.e. without resummation of the $\tan\beta$-enhanced corrections to the bottom Yukawa coupling), supplemented with the two-loop gluonic corrections to the
Figure 3: Wilson coefficients $C_7(M_W)$, left, and $C_8(M_W)$, right, as a function of the gluino mass for a choice of MSSM input parameters modeled on set I (see text).

Figure 4: Same as fig. 3 for the MSSM input parameters of set II.
diagrams with SM particles or charged Higgs boson; the dot-dashed line contains in addition the tan $\beta$-enhanced gluino contributions as computed in the effective theory approach in refs. [16, 17]; finally, the solid line corresponds to our complete two-loop diagrammatic calculation. Comparing the solid and dot-dashed curves in figs. 3 and 4, it can be seen that for low values of the superparticle masses the tan $\beta$-enhanced gluino contributions of refs. [16, 17] do not provide a good approximation of our full two-loop result, especially in the case of $C_7(M_W)$. As the superpartners get heavier, however, the effective theory approach becomes more reliable, and the corresponding results get closer to those of the complete calculation. Indeed, for large values of the superparticle masses the difference between the two-loop results (solid and dot-dashed lines) and the one-loop results (dashed lines) is mainly due to the non-decoupling charged Higgs contributions discussed in refs. [16, 17].

As mentioned above, in figs. 3 and 4 we assume that MFV is valid at the level of the running parameters of the Lagrangian, at a renormalization scale of the order of the superparticle masses. The plots in fig. 5, obtained with the MSSM parameters of set II, allow us to appreciate the implications of this assumption. In each plot, the solid line represents our two-loop results for the Wilson coefficients as a function of $\mu_{MFV}$, when the latter is varied between 100 GeV and $10^{16}$ GeV. For comparison, we also plot the one-loop results (dashed lines), defined as in the previous figures, and the scale-independent two-loop results that we obtain by employing an on-shell definition of the flavor changing counterterms (dot-dashed lines). It can be seen that, for values of $\mu_{MFV}$ of the order of the superparticle masses, the results obtained with the minimal definition of the flavor changing counterterms are very similar to those obtained with the on-shell definition. However, when $\mu_{MFV}$ is increased up to the GUT scale, the logarithm of the ratio $M_{SUSY}/\mu_{MFV}$ becomes very large, and the corresponding contribution modifies sensibly the two-loop part of the correction. Of course, in this case a fixed-order calculation does not provide a good approximation to the correct result, and the large logarithmic corrections have to be resummed.

4 Discussion and summary

We have presented a complete calculation of the $\mathcal{O}(\alpha_s)$ supersymmetric corrections to the Wilson coefficients relevant for radiative $B$ decays, assuming MFV (i.e. the vanishing of flavor-changing gluino couplings) at a scale $\mu_{MFV}$. The magnitude of $\mu_{MFV}$ depends on the specific model of supersymmetry breaking, but can be much larger than that of all other mass scales entering the calculation, giving rise to large logarithms that must be resummed. It is important to realize that the logs of $\mu_{MFV}$ are directly related to the running of the flavor-changing gluino-quark-squark couplings that we have required to vanish at that scale. In other words, even if we impose MFV at $\mu_{MFV}$, the MSSM
Wilson coefficients $C_7(M_W)$, left, and $C_8(M_W)$, right, as a function of the scale $\mu_{MFV}$ at which the MFV condition is imposed (see text).

The lagrangian at a scale $\mu \neq \mu_{MFV}$ will contain the interactions

$$-g_s T^a \sqrt{2} \left[ g_{sL}^i(\mu) \tilde{s}_i^a d_i + g_{bL}^i(\mu) \tilde{g}^a b_L d_i^a + g_{bR}^i(\mu) \tilde{g}^a b_R d_i^a \right] + \text{h.c.}, \quad (13)$$

where $\tilde{d}_i$ are the down-type squark mass eigenstates (no longer identified with flavor eigenstates). The couplings $g_{sL}^i$ and $g_{bR,L}^i$ induce $b \to s$ transitions mediated by one-loop gluino diagrams and their evolution follows from the standard RGE of the MSSM (see [13, 25]). In particular, the resummation of the large logs of $\mu_{MFV}$ is accomplished by solving the one-loop RGE for the quark and squark mass matrices, which are then diagonalized at the scale $\mu$. Indeed, the coefficient of $\log \mu_{MFV}$ in eq. (12) can be easily reproduced by expanding the RGE solution for the above couplings in powers of $\alpha_W$.

Even in the case of very large $\mu_{MFV}$, a natural and consistent approximation scheme can be adopted if the $b \to s$ flavor violation generated radiatively at the low scale $\mu_{SUSY}$, though not vanishing, is small (as is generally the case for $\tan \beta$ not too large [27]) or the gluino mass is large. The one-loop gluino diagrams can then be computed using the interactions in eq. (13) at the scale $\mu_{SUSY}$, and it is safe to neglect all QCD corrections to this contribution. The same applies to one-loop diagrams with flavor-changing neutralino-quark-squark couplings (whose contribution gets also suppressed in the $Q_7 - Q_8$ mixing [28]). In addition to these two contributions, we are now able to include all other supersymmetric contributions at $\mathcal{O}(\alpha_s)$. The QCD correction $C_{7,8}^{(1)SUSY}(\mu_{W})$, in particular, should be computed using $\mu_{MFV} = \mu_{SUSY}$, because the radiative effects that generate FCG interactions are already taken into account and resummed by the one-loop gluino diagrams. This strategy allows for a precise calculation of radiative decays in the scenarios.
characterized by MFV at a high scale. A detailed numerical implementation for the main SUSY breaking scenarios will be presented elsewhere.

In summary, we have completed the calculation of the QCD corrections to radiative $B$ decays in supersymmetric models characterized by Minimal Flavor Violation at a scale $\mu_{MFV}$. In the case $\mu_{MFV}$ is much larger than the electroweak scale, we have explained how to resum the ensuing large logs. We have seen that the numerical results based on the new calculation differ significantly from existing partial calculations for relatively light superpartners, though they agree well with [16] in the case of a heavy SUSY spectrum. We believe the new results, soon to be made available as a public computer code, will prove essential for an accurate calculation of radiative $B$ decays in most supersymmetric scenarios.

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