Spin-Hall interface resistance in terms of Landauer type spin dipoles

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We considered the nonequilibrium spin dipoles induced around spin independent elastic scatterers by the intrinsic spin-Hall effect associated with the Rashba spin-orbit coupling. The normal to 2DEG spin polarization has been calculated in the diffusion range around the scatterer. We found that although around each impurity this polarization is finite, the corresponding macroscopic spin density, obtained via averaging of individual spin dipole distributions over impurity positions is zero in the bulk. At the same time, the spin density is finite near the boundary of 2DEG, except for a special case of a hard wall boundary, when it turns to 0. The boundary value of the spin polarization can be associated with the interface spin-Hall resistance determining the additional energy dissipation due to spin accumulation.

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I. INTRODUCTION

Most of the theoretical studies on the spin-Hall effect (SHE) has been devoted to calculation of the spin current (for a review see Ref. \textsuperscript{11}). Such a current is a linear response to the external electric field \( \mathbf{E} \) which induces a spin flux of electrons or holes flowing in the direction perpendicular to \( \mathbf{E} \). This spin flux can be due either to the intrinsic spin-orbit interaction (SOI) inherent to a crystalline solid\textsuperscript{12}, or to spin dependent scattering from impurities\textsuperscript{13}. The spin-Hall current, as a response to the electric field, is characterized by the spin-Hall conductivity. On the other hand, similar to the conventional Hall effect, one can introduce the spin-Hall resistivity from calculating the local chemical potential difference \( \mu_s = \mu \uparrow - \mu \downarrow \) in a response to the DC electric current. For 2DEG in a local equilibrium this potential difference can be related to the \( z \)-component (perpendicular to 2DEG) of the spin polarization, according to \( \Sigma = N_F \mu_s \), where \( N_F \) is the density of states near the Fermi level. Therefore, the spin-Hall resistivity is closely associated with spin accumulation near interfaces. It should be noted that measuring spin polarization is thus far the only realistic way to detect SHE\textsuperscript{14}. For interfaces of various nature such an accumulation has been calculated in a number of works\textsuperscript{6,7,8,9,10,11,12}. A typical example to study spin accumulation is an infinite along \( x \)-direction 2D strip with a width \( w \) along \( y \)-direction. In this geometry the DC current flows in \( x \)-direction, while the spin-Hall current flows in \( y \)-direction with the spin density accumulating near boundaries. An analog of the Hall voltage could be a difference of \( \mu_s \) on both sides of the strip. There is, however, a fundamental distinction from the charge Hall effect. In the latter case, due to the long-range nature of the electric potential created by conserving electric charges, the Hall voltage is proportional to the width of the strip. In contrast, the spin-Hall electrochemical potential at the interface does not depend on \( w \) as \( w \to \infty \) because spin relaxation essentially suppresses the long range contribution to spin polarization buildup near interfaces. Hence, it is sensible to introduce an interface spin-Hall resistance, which is the proportionality coefficient between the interface value of \( \mu_s \) and the electric current density.

Below, we will consider the spin-Hall resistance from the microscopic point of view. This approach is based on the Landauer’s\textsuperscript{15} idea that at a given electric current each impurity is surrounded by a nonequilibrium charge cloud forming a dipole. Combined together these dipoles create a voltage drop across the sample. Therefore, each impurity plays a role of an elementary resistor. In a similar way, nonequilibrium spin dipoles could be induced subsequent to the spin-Hall current. One may expect that the spin cloud will appear around a spin-orbit scatterer in case of extrinsic SHE, as well as around a spin-independent scatterer, in case of the intrinsic effect. The latter possibility for a 2D electron gas with Rashba interaction has been considered in Ref. \textsuperscript{14}. The perpendicular to 2DEG polarization was calculated within the ballistic range around a scatterer. On the other hand, in order to study spin accumulation and the spin-Hall resistance on a macroscopic scale, one needs to calculate the spin density distribution at distances much larger than the mean free path \( l \) of electrons. Below, we will extend the Green function method of Ref. \textsuperscript{14} to the diffusive range. In Section II the spin density distribution around an individual target impurity will be calculated. In Section III we will consider the interface spin accumulation created by spin dipoles randomly but homogeneously distributed in space. A relation between spin-Hall resistance and energy dissipation will be discussed in Sec. IV. A summary and discussion of results will be presented in Section V.

II. SPIN CLOUD INDUCED BY A SINGLE IMPURITY

As known, the electric field applied to a homogeneous 2DEG with Rashba SOI induces a parallel to 2DEG component of the nonequilibrium spin polarization\textsuperscript{15}. The spin-Hall effect produces, however, a zero spin polarization in its \( z \)-component. This understanding about such
a homogeneous gas has implied an averaging over impurity positions. An impure system, on the other hand, can not be uniform on a microscopic scale. The effect of each impurity on the spin polarization could be singled out by considering an impurity (a target impurity) at a fixed position while taking at the same time the average over positions of other impurities. In such a way the Landauer electric dipole has been calculated\cite{16,17}. The electron density around a target impurity represented by the elastic scatterer was found from the asymptotic expansion of the scattered wave functions of electrons. At the same time, wavevectors of incident particles were weighted with the nonequilibrium part of the Boltzmann distribution function. We will employ another method based on the Green function formalism\cite{18,19}. Within this method the spin density response to the electric field $E$ is given by the standard Kubo formula with the scattering potential of the target impurity incorporated into the Green functions $G^{r/a}(r,r',\omega)$ denoted by the superscripts $r$ and $a$, respectively. As such, the $n$-component of the stationary spin polarization is given by

$$S_n(r) = -\frac{\pi}{m^*} \int d^2r'\int \frac{dn}{2\pi} \frac{dF(\omega)}{d\omega} \times \left\langle \sigma^n G^r(r,r',\omega) (vE) G^a(r',r,\omega) \right\rangle, \quad (1)$$

where the overline denotes averaging over impurity positions, the trace runs through the spin variables and $n_F(\omega)$ is the Fermi distribution function. To avoid further confusion we note that the angular moment is obtained by multiplying $S_n(r)$ by $\hbar/2$ and $e$ is the particle charge, which is negative for electrons. At low temperatures only $\omega$ in close vicinity around $E_F$ contributes to the integral in (1). Therefore, below we set $\omega = E_F$ and omit the frequency argument in the Green functions. Further, $v$ is the particle velocity operator containing a spin dependent part associated with SOI. Writing SOI in the form

$$H_{so} = h_k \cdot \sigma, \quad (2)$$

one obtains the velocity operator

$$v^j = \frac{k^j}{m^*} + \frac{\partial h_k \cdot \sigma}{\partial k^j}, \quad (3)$$

where $\sigma = (\sigma^x, \sigma^y, \sigma^z)$ is the Pauli matrix vector. In case of Rashba interaction the spin-orbit field $h_k$ is given by

$$h_x = \alpha k_y, \quad h_y = -\alpha k_x. \quad (4)$$

We assume that the target impurity, located at $r_i$, is represented by a scattering potential $U(r - r_i)$. The Green functions in (1) have to be expanded in terms of this potential. Up to the second order in $U$ one obtains

$$G^{r/a}(r,r') = G^{r/a(0)}(r,r') + \int ds^2 G^{r/a(0)}(r,s) U(s - r_i) G^{r/a(0)}(s,r') +$$

$$\int ds ds' G^{r/a(0)}(r,s) U(s - r_i) G^{r/a(0)}(s,s') U(s' - r_i) G^{r/a(0)}(s',r'). \quad (5)$$

The unperturbed functions $G^{r/a(0)}$ depend, nevertheless, on scattering from background random impurities. The latter create the random potential $V_{\text{sc}}(r)$ which is assumed to be delta correlated, so that the pair correlator $\langle V_{\text{sc}}(r) V_{\text{sc}}(r') \rangle = \Gamma \delta(r - r') / \pi N_F$, where $\Gamma = 1/2\tau$ is expressed via the mean elastic scattering time $\tau$. The delta correlation means that the corresponding impurity potential is the short range one. In fact, the potential of the target impurity could be different from that of the random impurities. It might be a special sort of impurities added to the system. On the other hand, the target and the random impurities would be identical if one would try to employ the spin dipoles for the interpretation of spin accumulation near interfaces.

After substitution of (5) into (1) one must calculate background impurity configurational averages containing products of several Green functions $G^{0}$. Assuming that the semiclassical approximation $E_F \tau \gg 1$ is valid,
In its turn, $\Sigma_z(q)$ is expressed in terms of the diffusion propagator. Indeed, let us represent this vertex using a basis of four $2 \times 2$ matrices $\tau^0 = 1$ and $\tau^i = \sigma^i$ with $i = x, y, z$. Then, $\Sigma_z(q)$ can be written as

$$\Sigma_z(q) = \sum_{b=0}^3 D^{zb}(q)\tau^b, \quad b = 0, x, y, z$$

(10)

where $D^{zb}(q)$ are the matrix elements of the diffusion propagator satisfying the spin diffusion equation, as it was described in Ref. [4] and references therein. The non-diagonal element $D^{z0}(q)$ appears due to the spin-charge mixing and it is zero for SOI of quite general form, including the Rashba interaction. Finally, from (8), using (9) and (10), we express $S_z(q)$ in the form

$$S_z(q) = \sum_{n=x,y,z} D^{zn}(q)I^n(q),$$

(11)

where $I^n(q)$

$$I^n(q) = \frac{e}{2\pi m^*} \sum_{p,k} (p \cdot E)Tr[G_p^{a\sigma}G_{p+k+q,p}^{a\sigma}].$$

(12)

The function $I^n(q)$ has a simple physical meaning. For $n = x, y, z$ it represents a source of spin polarized particles emitting from the target impurity. Their further diffusion and spin relaxation result in the observable polarization. This source term feature is conceptually similar, though different in its context, to the original charge cloud consideration when SOI is not present and Boltzmann equation is used to describe the subsequent background scattering. For $q \ll U^{-1} \ll k_F$ the source can be expanded in powers of $q$. Therefore, the wavevector independent terms represent the delta source located at $r_i$, while the linear in $q$ terms are associated with the gradient of the delta-function. Below we will keep only the constant and linear terms for each n-th component $I^n(q)$ and assume, for simplicity, the short range scattering potential $U(r)$, so that its $k$-th Fourier transform is simply $U\exp(-ik\cdot r)$, where $U$ is a constant. Further, $I^n(q)$ can be written as

$$I^n(q) = I^n_1(q) + I^n_2(q),$$

(13)

where $I_1$ and $I_2$ are of the first and the second order with respect to the scattering potential $U$, respectively. Accordingly, $I_1$ and $I_2$ are represented by Figs. 1(a),1(b) and Figs. 1(c),1(d), respectively. Using (5) to express Green functions $G_{k,k'}^{\sigma\sigma}$ in (12) we obtain

$$I^n_1(q) = \frac{eU}{2\pi m^*}e^{iq\cdot r} \sum_p (p \cdot E) \times Tr[G_p^{a\sigma}G_{p+k+q,p}^{a\sigma}+G_p^{a\sigma}G_{p-q}^{a\sigma}],$$

(14)

and

$$I^n_2(q) = \frac{eU^2}{2\pi m^*}e^{iq\cdot r} \sum_{pk} (p \cdot E)Tr[G_p^{a\sigma}G_{p+k+q}^{a\sigma} \times \{G_{k,q}^{a\sigma}G_{k+q}^{a\sigma} - \gamma\sigma^{ab}G_{p+q}^{a\sigma} + \gamma G_{p-q}^{a\sigma}\}],$$

(15)

The vertices $\Sigma_z(q)$ and $T(p)$ can be easily calculated. As it was discussed in Ref. [14] due to considerable cancellation of diagrams which is known from literature on the spin-Hall effect, $T(p)$ acquires a quite simple form in a special case of Rashba SOI. Namely,

$$T(p) = \frac{e}{m^*}p \cdot E.$$  

(9)
where
\[ \gamma = i \text{Im} \sum_k G_k^* = i \pi N_F \] (16)

In our following consideration we let the x-axis to be parallel with the electric field, and the z-axis to be perpendicular to the 2DEG. The system Hamiltonian is symmetric under a symmetry operation combining a reflection from the plane perpendicular to the y-axis, that means \( p_y \to -p_y \), and a unitary transformation \( \sigma_i \to \sigma_y \sigma_i \sigma_y \). Applying this transformation to (12) one can easily see that \( I^x(q_x, q_y) = -I^x(q_x, -q_y), I^z(q_x, q_y) = -I^z(q_x, -q_y) \) and \( I^y(q_x, q_y) = I^y(q_x, -q_y) \). Making use of another symmetry operation \( p_x \to -p_x, p_y \to -p_y \) and \( \sigma_i \to \sigma_z \sigma_i \sigma_z \), we obtain \( I^x(q_x, q_y) = I^x(-q_x, -q_y), I^z(q_x, q_y) = -I^z(-q_x, -q_y) \) and \( I^y(q_x, q_y) = I^y(-q_x, -q_y) \). From these relations it is easy to see that expansion of \( I^2 \) into power series starts from linear in \( q \) terms, while the leading term in \( I^y \) is \( \text{const} \) and the next one is quadratic in \( q \). On this reason only \( \text{const} \) will be taken into account in \( I^y \). The expansion of \( I^y \) starts from \( q_y, q_y \), and this source component will be neglected.

Calculation of \( I_1 \) and \( I_2 \) given by Eqs. (14), (15) is based on the standard linearization near the Fermi level, thus ignoring band effects giving rise to small corrections \( \sim h_{\text{F}}/E_F, \Gamma/E_F \). Further, the diffusion approximation is valid at \( q \ll 1/l \). At the same time, the characteristic lengthscale is determined by the spin relaxation length \( l_{so} \), which is the distance a particle diffuses during the D’yakonov-Perel’ spin relaxation time \( \tau_{so} = 4(h_{\text{F}}^2 \tau)^{-1} \). The corresponding diffusion length \( l_{so} = \sqrt{D \tau_{so}} \), where \( D = v_F^2 \tau/2 \) is the diffusion constant. Hence, \( l_{so} = v_F / h_{\text{F}} \). Taking \( q \sim 1/l_{so} \) one finds that the diffusion approximation is valid if \( h_{\text{F}} / \Gamma \ll 1 \). Therefore, within this approximation we will retain only the leading powers of \( h_{\text{F}} / \Gamma \ll 1 \). In such a way, direct calculation of \( I_1^y \) with the Green functions and SOI given by Eqs. (7) and (9), respectively, shows that both \( I_1^y \) and \( I_2^y \) are small by a factor \( \Gamma / E_F \). For example, using the relation
\[ (G_k^{x/a})^2 = -\frac{\partial}{\partial E_F} (G_k^{x/a}) \] (17)
which follows from (7), evaluating \( I_1^y \) at \( q = 0 \), one can represent the corresponding sum in (14) as
\[ -\frac{\partial}{\partial E_F} \sum_p p_x T \{ \tau^{x/a} (G_p^{x/a}) \} = -\frac{\partial}{\partial E_F} \left( \frac{2 \pi}{\Gamma} N_F m^* \frac{\partial h_{h_{\text{F}}}^*}{\partial p_x} \right) . \] (18)

In case of Rashba SOI with the constant coupling strength \( \alpha \) and energy independent parameters \( \Gamma, m^* \) and \( N_F \), the sum (18) is equal to 0. Otherwise, it is finite, but small due to the smooth energy dependence of these parameters. Similar analysis, although not so straightforward, can be applied to \( I_2^y \), which is linear in \( q \). The smallness of \( I_2^y \) can be also seen from Ref. 14 where the linear in \( U \) contribution to the spin density was associated with fast Friedel oscillations. It is clear that their Fourier transform will be small in the range of \( q \ll k_F^{-1} \).

At the same time \( I_1^y \) and \( I_2^z \) are not zero. They are given by
\[ I^y = v_d N_F m^* \alpha h_{h_{\text{F}}}^2 \frac{\Gamma'}{\Gamma} \]  
\[ I^z = -i q_y v_d N_F h_{h_{\text{F}}}^2 \frac{\Gamma'}{2 \Gamma^2} \]  
\[ I^x = 0 , \] (19)

where \( \Gamma' = \pi N_F U^2 \) and \( v_d = eE/\tau m^* \) is the electron drift velocity. If the target impurity is represented by one of the random scatterers, we get \( \Gamma' = \Gamma/n_i \), where \( n_i \) is the density of impurities.

In the above calculation we did not take into account the diagrams shown in Figs. 1(e)-1(f) and those similar to them. It can be easily seen that such diagrams contain \( I_1^y \) as a factor. For example, the sum of diagrams at Fig. 1(e)-1(f) contains a multiplier the sum of diagrams shown in Figs. 1(a)-1(b). Therefore, such diagrams are small by the same reason as \( I_1^y \) are, at least, in the most important range of \( f \ll l^{-1} \). Particularly in this range of small \( f \) the diffusion propagator between the two scattering events in Figs. 1(e)-1(f) becomes large.

Now one can combine the source \( I^y \) with the diffusion propagator to find from Eq. (11) the shape of the spin cloud around a single scatterer. Taking into account (19), Eq. (11) is transformed into
\[ S_z(q) = -v_d N_F h_{h_{\text{F}}}^2 \Gamma' \frac{\Gamma''}{2 \Gamma^3} [i q_y D^{zz}(q) - 2 \alpha D^{xy}(q)] . \] (20)

The matrix elements \( D^{ij}(q) \) satisfy the spin diffusion equation:
\[ \sum_i \left( -\delta^{ij} Dq^2 - \Gamma^{ij} + i \sum_m R^{im} q_m \right) D^{ij}(q) = -2 \Gamma \delta_{ij} , \] (21)

where the matrix \( \Gamma^{ij} \) determining the D’yakonov-Perel’ spin relaxation rates is given by
\[ \Gamma^{ij} = 4 \tau \left( \delta^{ij} h_{h_{\text{F}}}^2 - h_{h_{\text{F}}}^2 h_{h_{\text{F}}}^2 \right) , \] (22)

with angular brackets denoting averaging over the Fermi surface. In the case of Rashba SOI, Eq. (14), one gets \( \Gamma^{zz} = 4 \tau h_{h_{\text{F}}}^2 \) and \( \Gamma^{xx} = \Gamma^{yy} = 2 \tau h_{h_{\text{F}}}^2 \). The last term in lhs of (21) is associated with spin precession in SOI field. It has the form
\[ R^{ilm} = 4 \tau \sum_p e^{ipl} \langle h_{h_{\text{F}}}^p m \rangle . \] (23)

For the Rashba SOI the nonzero components are
\[ i \sum_m R^{izm} q_m = -i \sum_m R^{izm} q_m = 4i D m^* \alpha q_i . \] (24)

We ignored in (21) a small term which gives rise to the spin-charge mixing. This mixing is already taken
III. SPIN ACCUMULATION IN A SEMIINFINITE SYSTEM

In this section we will consider a semiinfinite electron gas $y > 0$ bounded at $y = 0$ by a boundary parallel to the electric field. Our goal is to calculate a combined effect of spin clouds from random impurities. It is important to note that summation of spin dipoles from many scatterers does not result in a magnetic potential gradient in the bulk of the sample. This is principally different from the Landauer charge dipoles which are associated with the macroscopic electric field. The origin of such a distinction can be immediately seen from (20). The magnetic potential, as it was defined in Sec.I, is proportional to $S_z$. Taking its gradient one gets $q_y S_z$. After averaging over impurity positions $q \to 0$, so that $q_y S_z \to 0$. It happens due to spin relaxation, which provides at $q = 0$ a finite value of the denominator in (20). At the same time, in case of the charge cloud, the denominator of the particle diffusion propagator is proportional to $q^2$. Hence, the corresponding gradient of the electrochemical potential (electric field) is finite at $q = 0$. Although the bulk magnetic potential is zero, one can not expect that it will also be zero near an interface. In order to calculate the spin polarization near the boundary, Eq.(21) with $q = -i\nabla$ and $2\Gamma \delta(r)\delta_{ij}$ in the r.h.s. has to be solved using appropriate boundary conditions. With the so obtained $D^{ij}(r)$, the resultant spin density induced by impurities placed at points $r_i$ is given by Eq.(11):

$$S_j(r) = \sum_{n=x,y,z} \int d^2r' D^{jn}(r-r')I_{tot}^n(r'), \quad (28)$$

where the source term is obtained by the inverse Fourier transform of (19):

$$I_{tot}^n(r) = v_d N_F m^* \alpha E_{k_F} \frac{1}{\Gamma (y_i)} \sum_i \delta(r-r_i),$$

$$I_{tot}^r(r) = -v_d N_F h_{k_F} \frac{1}{21\nu y_{i}} \sum_i \partial \frac{\partial \delta(r-r_i)}{\partial y},$$

$$I_{tot}^r(r) = 0. \quad (29)$$

where the relation $\Gamma' = \Gamma / y_i$ is used because we assumed that the target impurities are identical to the random ones. The macroscopic polarization is obtained by averaging of (28) and (29) over impurity positions. After averaging over $x_i$ and the semiinfinite region $y_i > 0$ the spin polarization source (29) transforms to $I_{av}^n(y)$:

$$I_{av}^n(y) = v_d N_F m^* \alpha E_{k_F} \frac{1}{\Gamma^2} \sum_i \delta(y-y_i) \frac{1}{21\nu y_i}, \quad (30)$$

It follows from (28) that the corresponding mean value of the spin polarization $S_{av}(y)$ satisfies the diffusion equation (21) with the source $2\Gamma I_{av}^n(y)$ in its rhs. The so obtained diffusion equation, however, is not complete. One
should take into account that the boundary itself can create the interface spin polarization. Most easily it can be done in the framework of the Boltzmann approach. In terms of the Boltzmann function the spin density is defined as \( S_{av}(y) = \sum_k g_k \), and the charge density as \( \sum_k g_k \). The equation for the Boltzmann function can be written in the form (see e.g. Ref. 25)

\[
v_y \nabla_y g_k + 2(g_k \times h_k) + eE_x \frac{\partial g_k}{\partial k_x} = \frac{1}{\tau} (S_E(y) - g_k), \tag{31}
\]

where \( S_E(y) = \delta(E - E_F)S_{av}(y)/N_F \) and \( g_k^{(0)} = -h_k\delta(E - E_F) \) is the equilibrium Boltzmann function. The terms proportional to the charge component of the Boltzmann function have been omitted in (31) due to the system local electroneutrality, at least in the scale of the mean free path, which is the smallest characteristic scale of \( g_k \) spatial variations. The spin polarization source associated with the boundary is given by a direct action of the electric field, without taking into account secondary scattering from impurities. Hence, the term with the electric field, without taking into account secondary scattering from impurities, that is represented by the integral of \( 2\Gamma \)

This condition means that the spin orientation does not change after specular reflection from the interface. The solution of Eq. (31) satisfying (32) can be easily found. As a result, up to \( o(\alpha^2) \) we obtain

\[
S_{av}^y(y) = S_{av}^x(y) = 0
\]

\[
S_{av}^z(y) = 8v_d\alpha^2\tau m^* \sum_{k_y>0} k_y \delta(E_k - E_F)e^{-y/m^*}, \tag{33}
\]

Within the diffusion approximation the second of these equations represents a delta source of the spin polarization with intensity

\[
1 \int_0^\infty dy S_{av}^y(y) = v_d N_F h_F^{2} \frac{1}{\Gamma}. \tag{34}
\]

This source is exactly of the same magnitude, but opposite in sign to the spin polarization emerging from impurities, that is represented by the integral of \( 2G_{av}^z(y) \), with \( G_{av}^z(y) \) given by Eq. (29). Taking into account that both sources are located at the interface, so that they cancel each other, one sees that only \( y \)-component of the source originating from impurity scattering retains in the diffusion equation which acquires the form

\[
\frac{\partial^2 S_{av}^z}{\partial y^2} - 4m^*\alpha \frac{\partial S_{av}^y}{\partial y} - 8m^*\alpha^2 S_{av}^z = 0
\]

\[
\frac{\partial^2 S_{av}^y}{\partial y^2} + 4m^*\alpha S_{av}^x - 4m^*\alpha^2 S_{av}^y = -\frac{2\Gamma}{D}I_{av}^z. \tag{35}
\]

The bulk solution of this equation is \( S_{av}^z = 0 \) and \( S_{av}^y = S_b = 2\tau e N_F \alpha \), that coincides with the polarization obtained from (20)-(27) at \( q \to 0 \).

In order to calculate the spin polarization near the interface we employ the hard wall boundary conditions for (35). Such boundary conditions can be easily obtained from Eq. (31) by performing its summation over \( k \) and integrating from \( y = 0 \) to some point \( y_0 \), placed at the distance much larger than \( l \), but still small compare to \( l_{so} \). A simple analysis of Eq. (31) shows that up to \( o(\alpha^2) \) the sum over \( k \) of the vector product in the l.h.s. of (31) can be neglected, while the r.h.s. and the term containing the electric field turn to zero identically. As a result, we get

\[
\frac{1}{m^*} \sum_k k_y g_{k_x,k_y}|_{y=y_0} = \frac{1}{m^*} \sum_k k_y g_{k_x,k_y}|_{y=0}. \tag{36}
\]

According to (37), the above sum is zero at \( y = 0 \). Hence, it is also zero at \( y = y_0 \). The latter sum coincides with the spin current within its conventional definition, where a contribution associated with the charge density due to the second term of the velocity operator \( \sum_k \) is ignored in an electroneutral system. Using the gradient expansion of (31) this current can easily be expressed through \( S_{av}^z(y) \) over \( y = 0 \), its \( y \)-derivative and the last term in the l.h.s. of (31). In this way one arrives to the boundary conditions from Ref. 23. We generalize these conditions by adding possible effects of the surface spin relaxation (see also Ref. 10). These additional terms are characterized by the two phenomenological parameters \( \rho_x \) and \( \rho_z \). Finally we obtain

\[
-D\frac{\partial S_{av}^z(y)}{\partial y}|_{y=0} + 2Dm^*\alpha S_{av}^y(0) = -\rho_x S_{av}^z(0)
\]

\[
-D\frac{\partial S_{av}^y(y)}{\partial y}|_{y=0} = -2m^*\alpha S_{av}^x(0) = -\rho_y S_{av}^y(0). \tag{37}
\]

One can easily see from (35), (37) and (38) that at \( \rho_x/y = 0 \) the homogeneous bulk solution \( S_{av}^z = 0 \), \( S_{av}^y(0) = S_b \) turns out to be the solution of the diffusion equation everywhere at \( y > 0 \). Therefore, in this particular case the \( z \)-components of spin clouds from many impurities completely cancel each other and there is no spin accumulation near the interface, in agreement with Refs. 23-25. At the same time, when \( \rho_z \neq 0 \) the out of plane component of the spin density is not zero. In the case of weak surface relaxation \( \rho_z \ll D/l_{so} \) one obtains from (37) (38)

\[
S_{av}^z(0) = 0.35\rho_y \tau e E \frac{1}{2\pi h D}, \tag{38}
\]
where we inserted $\hbar$ to restore conventional units. It is interesting to note that in such a regime of small enough $\rho_s$ the surface polarization does not depend on the spin-orbit constant.

**IV. SPIN-HALL RESISTANCE AND ENERGY DISSIPATION**

As it was defined in the Introduction, the interface spin-Hall resistance is given by

$$R_{sH} = \frac{S_{av}^z(0)}{N_F j},$$

where $j$ is the DC current density, $j = \sigma E$, with the Drude conductivity $\sigma = ne^2\tau/m^*$. The so defined spin-Hall resistance is closely related to the additional energy dissipation which takes place due to spin accumulation and relaxation near interfaces of a sample. Indeed, as was shown in Ref. [6], the spin accumulation is associated with a correction to the electric conductivity of DC current flowing in the $x$ direction. For Rashba SOI the correction to the current density has the form

$$\Delta j(y) = -\frac{e}{4m^*} \frac{\alpha^2 k_F^2}{\Gamma^2} \partial S_{av}^z \partial y.$$  

This expression is finite within the distance $\sim l_{so}$ from the interface. After integration over $y$ one obtains a correction to the electric current

$$\Delta I = \frac{e}{4m^*} \frac{\alpha^2 k_F^2}{\Gamma^2} S_{av}^z(0).$$

The corresponding interface energy dissipation (per the unit of the interface length) can be expressed from (39) and (41) as

$$\Delta W = \Delta I E = \frac{m^*}{\epsilon \hbar} \frac{\alpha^2 \tau R_{sH} j^2}{\gamma}.$$  

In its turn, $R_{sH}$ can be determined from Eq. [35]. It can be easily seen that $\Delta W > 0$ if $\rho_y > 0$.

**V. RESULTS AND DISCUSSION**

Summarizing the above results, within the drift diffusion theory we found out that the intrinsic spin-Hall effect induces in 2DEG a nonequilibrium spin density around a spin independent isotropic elastic scatterer. The $z$-component of this density has a shape of a dipole directed perpendicular to the external electric field, while the parallel to 2DEG polarization is isotropic. Due to the D’yakonov-Perel’ spin relaxation, the spin density decays exponentially at a distance larger than the spin-orbit precession length. Noteworthy, that such a cloud exists even in the case of the Rashba spin-orbit interaction when the macroscopic spin current is absent. We also calculated the macroscopic spin density near an interface from taking the sum of clouds due to many scatterers and independently averaging over their positions. Surprisingly, in the case of the hard wall boundary, the so calculated spin polarization exactly coincides with that found from the drift diffusion or Boltzmann equations.

Besides conventional semiconductor quantum wells, the results of this work can be applied to metal adsorbate systems with strong Rashba type spin splitting in the surface states. In this case the spin cloud can be measured by STM with a magnetic tip.

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