Mixed bino–wino–higgsino dark matter in gauge messenger models

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Abstract. Almost degenerate bino and wino masses at the weak scale constitute one of the unique features of gauge messenger models. The lightest neutralino is a mixture of bino, wino and higgsino and can produce the correct amount of dark matter density if it is the lightest supersymmetric particle. Furthermore, as a result of the squeezed spectrum of superpartners which is typical for gauge messenger models, various co-annihilation and resonance regions overlap and very often the correct amount of neutralino relic density is generated by an interplay of several processes. This feature makes the explanation of the observed amount of the dark matter density much less sensitive to fundamental parameters. We calculate the neutralino relic density assuming a thermal history and present both spin independent and spin dependent cross sections for direct detection. We also discuss phenomenological constraints from \(b \rightarrow s\gamma\) and muon \(g-2\) and compare results obtained using gauge messenger models to well known results for the mSUGRA scenario.

Keywords: dark matter, cosmology of theories beyond the SM, physics of the early universe

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1. Introduction

Models with weak scale supersymmetry are some of the most attractive candidates for extensions of the standard model (SM). Among them the minimal supersymmetric standard model (MSSM) is popular due to its minimality and simplicity. Smallness of the weak scale compared to the Planck scale is nicely explained by the smallness of supersymmetry breaking and the three different gauge couplings meet at the grand unification (GUT) scale, $2 \times 10^{16}$ GeV, which is close to the Planck scale. Furthermore, assuming R-parity, the lightest supersymmetric particle (LSP) is stable and provides a reason for the existence of dark matter.

The lightest neutralino, being weakly interacting, neutral and colorless, and appearing as the LSP in a large class of SUSY breaking scenarios including the most popular one, mSUGRA, is an especially good candidate, since it naturally leads, assuming thermal history, to a dark matter density $\Omega_{DM} h^2 \sim 1$ [1, 2]. This observation was certainly a major success of supersymmetry. However, the precisely measured value of the dark matter density, $\Omega_{DM} h^2 \sim 0.1 \pm 0.01$ [3], together with direct search bounds on superpartners, tightly constrains supersymmetric models and obtaining the correct amount of the dark matter density while evading all experimental constraints on superpartners is no longer trivial. For example, a bino-like lightest neutralino which is typical in the mSUGRA scenario usually annihilates too little which results in too much relic density. The bulk region of mSUGRA scenario where neutralino annihilation is further enhanced by $t$-channel exchange of relatively light sleptons and the correct amount of dark matter density can be obtained has been highly squeezed. Indeed, when the neutralino is mostly bino and $M_1/m_{\tilde{e}_R} \leq 0.9$, the correct relic density constrains $m_{\tilde{e}_R} \leq 111$ GeV at 95% CL [4]. The limits on the Higgs boson mass and $b \to s\gamma$ independently disfavor the bulk region. In the region with small $\mu$ term neutralinos can efficiently annihilate via their higgsino components. This region extends along the line of no EWSB and is referred to as the...
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focus point or hyperbolic branch region. Remaining regions are the regions where special relations between independent parameters occur and the neutralino relic density is further reduced by either co-annihilation with other superpartners, e.g. the stau co-annihilation region in mSUGRA when the stau mass is very close to the neutralino mass, or by the CP odd Higgs boson resonance when the mass of the CP odd Higgs boson is close to twice the mass of the lightest neutralino. These regions require a critical choice of parameters in the sense that the predicted value of the dark matter density is highly sensitive to small variations of parameters [4].

The lightest neutralino in gauge messenger models is typically mostly bino with a sizable mixture of wino and higgsino. The wino and higgsino components enhance the annihilation of the lightest neutralino and the correct amount of the dark matter density is obtained without relying on critical regions of the parameter space. The virtue of the lightest neutralino being a mixture of bino and wino was recognized in studies of unconstrained MSSM [5]–[9]. As already discussed, the bino-like neutralino typically leads to too large relic density. On the other hand both wino-like and higgsino-like LSPs annihilate too fast and the correct amount of $\Omega_{\text{DM}}$ is obtained only if they are very heavy, $m_{\chi_1^0} \sim 1$ TeV for higgsino-like and $m_{\chi_1^0} \sim 2.5$ TeV for wino-like neutralino. Obviously the lightest neutralino which is a proper mixture of bino, wino and higgsino can lead to the correct amount of the dark matter density while avoiding all experimental limits and being fairly light. The only problem is that this situation typically does not happen in widely studied SUSY breaking scenarios. For example, in models with universal gaugino masses at the GUT scale, e.g. mSUGRA, the ratio of bino and wino masses at the weak scale is 1:2 and the sizable bino–wino mixing is not possible. However, in gauge messenger models the sizable bino–wino mixing is a built-in feature. The bino and wino masses are generated with the ratio 5:3 at the GUT scale which translates to the ratio 1:1.1 at the EW scale. The bino and wino masses are almost degenerate and thus, besides sizable mixing, also the chargino co-annihilation is always present and plays an important role.

The gauge messenger model is independently well motivated [10]. The same field as breaks $SU(5)$ to $SU(3) \times SU(2) \times U(1)_Y$, the symmetry of the standard model, is also used to break supersymmetry. The heavy X and Y gauge bosons and gauginos play the role of messengers of SUSY breaking\(^3\). All gaugino, squark and slepton masses are given by one parameter and thus the model is very predictive. Besides already mentioned non-universal gaugino masses at the GUT scale, also the squark and slepton masses squared are non-universal and typically negative with squarks being more negative than sleptons. This feature leads to a squeezed spectrum at the EW scale (the heaviest superpartner has a mass only about twice as large as the lightest one). Negative stop masses squared at the GUT scale are partially responsible for large mixing in the stop sector at the EW scale which maximizes the Higgs mass and reduces fine-tuning in electroweak symmetry breaking [13]. Assuming no additional sources of SUSY breaking the gravitino is the LSP and then, depending on $\tan \beta$, the stau or sneutrino is the next-to-lightest SUSY particle (NLSP). However, the masses of the lightest neutralino, sneutrino, stau, and stop are very close to each other and thus considering small additional contributions to scalar masses, e.g. from gravity mediation the size of which is estimated to be of order 20%–30% of the gauge mediation, the neutralino can become the LSP. In that case we can utilize the bino–
wino–higgsino mixed neutralino feature of gauge messenger models to obtain the correct amount of the dark matter density. Although there is no necessity to rely on special resonance or co-annihilation scenarios, due to the squeezed spectrum of gauge messenger models, these special regions overlap and very often the correct amount of the relic density is generated as an interplay of several processes. This feature makes obtaining the correct amount of the dark matter density much less sensitive to fundamental parameters.

In this paper we consider the lightest neutralino of gauge messenger models as a dark matter of the universe candidate. In section 2 we review basic features of gauge messenger models. We discuss neutralino dark matter in the mSUGRA scenario in more detail in section 3 which will be useful when comparing results of gauge messenger models. In this section we also outline the procedure used to obtain results and summarize experimental constraints used in the analysis. Results from neutralino relic density in gauge messenger models are presented in section 4 together with the discussion of constraints from $b \to s\gamma$ and muon $g-2$. We also give predictions for direct dark matter searches. Finally, we conclude in section 5. For convenience, formulae for the composition of the lightest neutralino in gauge messenger models which are used in the discussion of results are derived in the appendix.

### 2. Gauge messenger model

Let us summarize basic features of gauge messenger models introduced in [10]. The simple gauge messenger model is based on an $SU(5)$ supersymmetric GUT with a minimal particle content. It is assumed that an adjoint chiral superfield, $\Sigma$, gets a vacuum expectation value (vev) in both its scalar and auxiliary components: $\langle \Sigma \rangle = (\Sigma + \theta^2 F_\Sigma) \times \text{diag}(2, 2, 2, -3, -3)$. The vev in the scalar component, $\Sigma \simeq M_G$, gives supersymmetric masses to $X$ and $Y$ gauge bosons and gauginos and thus breaks $SU(5)$ down to the standard model gauge symmetry. The vev in the $F$ component, $F_\Sigma$, splits masses of heavy gauge bosons and gauginos and breaks supersymmetry. The SUSY breaking is communicated to MSSM scalars and gauginos through loops involving these heavy gauge bosons and gauginos which play the role of messengers (the messenger scale is the GUT scale). The gauge messenger model is very economical, all gaugino and scalar masses are given by one parameter,\footnote{The gauge messenger model is very economical, all gaugino and scalar masses are given by one parameter, $M_{\text{SUSY}} = \frac{\alpha_G |F_\Sigma|}{4\pi M_G}$, and it is phenomenologically viable [10].}

\begin{align*}
M_{\text{SUSY}} &= \frac{\alpha_G |F_\Sigma|}{4\pi M_G}, \quad (1)
\end{align*}

and it is phenomenologically viable [10].

A unique feature of the gauge messenger model is the non-universality of gaugino masses at the GUT scale. The bino, wino and gluino masses are generated with the ratio 5:3:2 at the GUT scale:

\begin{align*}
M_1 &= 10 M_{\text{SUSY}}, \quad (2) \\
M_2 &= 6 M_{\text{SUSY}}, \quad (3) \\
M_3 &= 4 M_{\text{SUSY}}. \quad (4)
\end{align*}

As a consequence, the weak scale bino, wino and gluino mass ratio is approximately 1:1.1:2.
Similarly, soft scalar masses squared are non-universal and typically negative at the GUT scale. They are driven to positive values at the weak scale making the model phenomenologically viable. Negative stop masses squared are a major advantage with respect to the electroweak symmetry breaking which requires less fine-tuning and at the same time avoids the limit on the Higgs boson mass by generating large mixing in the stop sector [13]. In the gauge messenger model the GUT scale boundary conditions for squark and slepton masses of all three generations are given as

\[ m^2_Q = -11 M^2_{\text{SUSY}}, \]  
\[ m^2_{\tilde{u}_c} = -4 M^2_{\text{SUSY}}, \]  
\[ m^2_{\tilde{d}_c} = -6 M^2_{\text{SUSY}}, \]  
\[ m^2_{L} = -3 M^2_{\text{SUSY}}, \]  
\[ m^2_{\tilde{e}_c} = +6 M^2_{\text{SUSY}}, \]  
\[ m^2_{H_u,H_d} = -3 M^2_{\text{SUSY}}. \]  

For completeness, the soft tri-linear couplings are given by

\[ A_t = -10 M_{\text{SUSY}}, \]  
\[ A_b = -8 M_{\text{SUSY}}, \]  
\[ A_{\tau} = -12 M_{\text{SUSY}}, \]  

and the same results apply to soft tri-linear couplings of the first two generations. The soft SUSY breaking parameters given in equations (2)–(13) correspond to the simple SU(5) gauge messenger model with minimal particle content. For soft SUSY breaking parameters in extended models see [10].

Gauge mediation does not generate the \( \mu \) and \( B\mu \) terms and they have to be introduced as independent parameters. As we discuss later, they can be generated by gravity mediation through the Giudice–Masiero mechanism [14]. The absolute value of \( \mu \) is fixed by requiring EWSB with the correct value of \( M_Z \) and thus only the sign(\( \mu \)) can be chosen arbitrarily. The other parameter, \( B\mu \), can be replaced by \( \tan \beta = v_u/v_d \). Thus the simple gauge messenger model has one discrete and two continuous parameters:

\[ M_{\text{SUSY}}, \tan \beta, \text{sign}(\mu). \]  

Furthermore, constraints on muon anomalous magnetic moment favor the sign of \( \mu \) being the same as the sign of the wino mass which in our notation is positive.

An example of the spectrum of the gauge messenger model is given in figure 1. For comparison we also give a typical spectrum of the mSUGRA scenario in the same figure. The mass ratio of the gluino and the lightest neutralino is about 2 in the simple gauge messenger model while it is about 6 in the mSUGRA. Assuming no additional sources
Figure 1. The spectrum of the simple gauge messenger model for \( \tan \beta = 10 \) and \( M_{\text{SUSY}} = 80 \) GeV (a) and the spectrum of mSUGRA for \( \tan \beta = 10, m_0 = M_{1/2} = 800 \) GeV, \( A = 0 \) (b). The parameters are chosen such that the lightest neutralino masses are the same in the two cases.

of SUSY breaking the gravitino is the LSP (with the mass of order the EW scale) and then, depending on \( \tan \beta \), the stau or sneutrino is the NLSP\(^4\). However, as we can see in figure 1, the masses of the lightest neutralino, sneutrino, stau, and stop are very close to

\(^4\)Neglecting mixing in the stau sector, the sneutrino would be the NLSP due to the \( D \)-term contribution which is negative for the sneutrino and positive for the stau. The mixing in the stau sector is enhanced by \( \tan \beta \) and for \( \tan \beta \gtrsim 15 \) the stau becomes lighter than the sneutrino \([10]\).
each other and thus considering small additional contributions to soft masses, e.g. from gravity mediation or $D$-term contributions from the breaking of $U(1)$ contained in an extended GUT like $SO(10)$ or $E(6)$, the neutralino can become the LSP. In that case we can utilize the bino–wino–higgsino mixed neutralino feature of gauge messenger models to obtain the correct amount of dark matter.

Since the messenger scale is the GUT scale, and the gauge mediation is a one-loop effect, the naively estimated size of gravity mediation induced by non-renormalizable operators (suppressed by $M_{Pl}$) is comparable to the contribution from gauge mediation. The gauge mediation contribution is given by $M_{SUSY}$:

$$C \frac{\alpha}{4\pi} \left| \frac{F}{M_{GUT}} \right|,$$

where $C$ represents group theoretical factors appearing in equations (2)–(13) and the contribution from gravity mediation is

$$\lambda \left| \frac{F}{M_{Pl}} \right|.$$  

(16)

For a typical $C \sim 5$–10 and $\lambda$ of order 1 we find that the gauge messenger contribution is about five times larger than the contribution from gravity mediation.

Considering the contribution from gravity, the $\mu$ and $B\mu$ terms can be generated \cite{14} together with additional contributions to soft masses of the two Higgs doublets which we parameterize by $c_{H_u}$ and $c_{H_d}$ so that the soft masses of the two Higgs doublets at the GUT scale are given as

$$m^2_{H_u} = -3M^2_{SUSY} + c_{H_u}M^2_{SUSY},$$

(17)

$$m^2_{H_d} = -3M^2_{SUSY} + c_{H_d}M^2_{SUSY}.$$  

(18)

In addition we also consider a universal contribution to squark and slepton masses which we parameterize by $c_0$ so that, e.g.,

$$m^2_{\tilde{Q}} = -11M^2_{SUSY} + c_0M^2_{SUSY},$$

(19)

and similarly for other squark and slepton masses in equations (5)–(9). Thus in the most general case the parameter space of gauge messenger models that we consider is given by five continuous parameters and the sign of the $\mu$ term:

$$M_{SUSY}, \tan \beta, c_0, c_{H_u}, c_{H_d}, \text{sign}(\mu).$$

(20)

A small contribution from gravity mediation, $c_0 > 5$, is enough to make the neutralino lighter than the sneutrino or stau in most of the parameter space. The neutralino is then the LSP or NLSP, depending on the gravitino mass. Making the gravitino heavier is not problematic and it can be done assuming other sources of SUSY breaking which do not contribute to soft SUSY breaking terms of the MSSM sector. In the next section we consider the neutralino LSP as a dark matter candidate.
3. Neutralino dark matter in mSUGRA

In the mSUGRA scenario, or in general in any model with universal gaugino masses at the GUT scale, the lightest neutralino is a mixture of bino and higgsino. The bino-like neutralino typically has a very small annihilation cross section and cannot annihilate efficiently. As a consequence, if the neutralino is the LSP it gives too large a relic density and thus most of mSUGRA parameter space is ruled out by WMAP data. Representative slices through mSUGRA parameter space are shown in figure 2 for tan $\beta = 10$ and 50. The white region represents allowed region after various constraints are imposed (these are indicated in the plots and will be discussed later) and the blue dots represent the region in which the neutralino relic density is calculated to be within the WMAP range. In several specific regions of parameter space the neutralino relic density is further reduced and these regions are compatible with WMAP data. In the ‘bulk region’ (small $m_0$ and $M_{1/2}$) neutralino annihilation is enhanced by t-channel exchange of sleptons. This region is however disfavored by the limit on the Higgs boson mass and $b \to s\gamma$. Contrary to the bino-like neutralino, the Higgsino-like neutralino annihilates too efficiently and the relic density turns out to be smaller than the WMAP value. When the bino mass and the $\mu$ term are comparable, sizable mixing is possible and the correct relic density is obtained. This is the region in figure 2 for larger $m_0$ which goes along the line where EWSB is no longer possible (or the chargino mass limit). In the region where the stau mass is close to the neutralino mass the neutralino relic density is reduced by co-annihilation with stau. For $\tan \beta = 10$, only a tiny stau co-annihilation region is available. The bulk region and the focus point region are excluded by the direct search bound on the lightest Higgs and chargino and the muon anomalous magnetic moment. It is indeed the case that the most of the parameter space producing the correct dark matter density is already ruled out. For $\tan \beta = 50$, in addition to the stau co-annihilation region, the funnel region (pseudoscalar Higgs resonance) appears and also a large part of the mixed Higgsino region (focus point) is not ruled out for larger $m_0$.

In order to compare the result with gauge messenger models that we will discuss later, we fix the ratio $m_0/M_{1/2}$ and present a slice through mSUGRA parameter space in the $m_0$ (or $M_{1/2}$)–tan $\beta$ plane in figure 3. The region producing the correct relic density exists only for large tan $\beta$ ($\tan \beta \geq 45$) which is due to the pseudoscalar Higgs resonance.

3.1. Experimental constraints and procedure used

Before we discuss results for gauge messenger models let us summarize the procedure that we use to calculate the neutralino relic density and the experimental constraints we employ. We obtained our results using SOFTSUSY 2.0.8 [15] for the renormalization group evolution of soft SUSY breaking parameters from the GUT scale to the EW scale and for calculation of the SUSY spectrum. The mass of the lightest CP even Higgs boson is calculated with FeynHiggs 2.4.1 [16]. Indirect constraints from $b \to s\gamma$, muon $g - 2$ and $B \to \mu^+\mu^-$, and the neutralino relic density are obtained using micrOMEGAs 2.0 [17] and the direct detection rates are calculated using DarkSUSY 4.1 [18].

The WMAP result for the dark matter density is [3]

\[
\Omega_{DM} h^2 \sim 0.113 \pm 0.009.
\]
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Figure 2. Slices through the parameter space of mSUGRA in the $m_0$–$M_{1/2}$ plane for $\tan \beta = 10$ (top) and $\tan \beta = 50$ (bottom) with $A_0 = 0$ and $\mu > 0$. Blue dots represent the region in which the neutralino relic density is within the WMAP range. Shaded regions are excluded by various constraints.

In our plots we consider $2\sigma$ bounds for the neutralino relic density, $0.09 \leq \Omega_{DM} h^2 \leq 0.13$, to be in agreement with the observed dark matter density. This region is represented by blue bands in plots.

For $B(b \rightarrow s\gamma)$ we consider the allowed range to be $2.3 \times 10^{-4} \leq B(b \rightarrow s\gamma) \leq 4.7 \times 10^{-4}$ which is obtained by summing the experimental and theoretical error linearly and taking the $2\sigma$ range [19, 20].
Figure 3. A slice through the parameter space of mSUGRA in the $\tan \beta$--$m_0 = M_{1/2}$ plane for $A_0 = 0$ and $\mu > 0$. Blue dots represent the region in which the neutralino relic density is within WMAP range. Shaded regions are excluded by various constraints.

The muon anomalous magnetic moment might be the only indirect evidence for the presence of new physics at around the weak scale. The recent experimental value of $a_\mu = (g - 2)\mu/2$ from the Brookhaven ‘Muon $g - 2$ Experiment’ E821 [21] is

$$a_\mu^{\exp} = (11\,659\,208 \pm 5.8) \times 10^{-10}. \quad (22)$$

The standard model prediction contains QED, EW and hadronic parts. As a result of uncertainties in the hadronic contribution, we quote results of two groups for $\Delta a_\mu = a_\mu^{\exp} - a_\mu^{\text{th}}$ where $a_\mu^{\text{th}}$ stands for the theoretical prediction of the standard model. From results of [22] and [23] we have

$$\Delta a_\mu = (31.7 \pm 9.5) \times 10^{-10}, \quad (23)$$

which indicates a $3.3 \sigma$ deviation from the standard model. In order to explain the experimental result within $2 \sigma$, we need a contribution from the new physics $\Delta a_\mu \geq 13 \times 10^{-10}$. On the other hand, from results of [24] and [25], we have

$$\Delta a_\mu = (20.2 \pm 9.0) \times 10^{-10}, \quad (24)$$

which indicates a $2.1 \sigma$ deviation. In this case we need $\Delta a_\mu \geq 2 \times 10^{-10}$ if we allow for $2 \sigma$ variation. Both groups calculated the hadronic contribution using $e^+e^-$ data. The $\tau$ decay data has not been used because of the uncertainties related to isospin breaking effects. By combining these two results [26, 27], we get

$$\Delta a_\mu = (25.2 \pm 9.2) \times 10^{-10}, \quad (25)$$
which indicates a 2.7σ deviation from the standard model. A contribution from the new physics $\Delta a_\mu \geq 7 \times 10^{-10}$ is necessary in this case to agree with data.

In our plots, we draw all three 2σ bounds, $\Delta a_\mu = (2, 7, 13) \times 10^{-10}$. As we neglected $\tau$ decay data, we take the most conservative bound, $\Delta a_\mu = 2 \times 10^{-10}$, to constrain the parameter space. For illustrative purposes we add two dashed lines corresponding to $\Delta a_\mu = 7 \times 10^{-10}$ and $13 \times 10^{-10}$.

### 4. Neutralino dark matter in gauge messenger models

The squeezed spectrum of gauge messenger models makes the discussion of the neutralino relic density very complex. Various regions with correct relic density which are usually well separated in scenarios with a highly hierarchical spectrum are overlapping here and often there is no single process that would be crucial for obtaining the correct amount of the neutralino relic density.

The lightest neutralino in gauge messenger models is typically mostly bino with a sizable mixture of wino and higgsino. In order to understand the dependence of the neutralino relic density on fundamental parameters it is important to know the composition of the lightest neutralino. The formulae for wino and higgsino components of the lightest neutralino mass eigenstate are derived in the appendix and for $\tan \beta \geq 10$ they can be written as

\[
N_{11} \simeq 1,
\]

\[
N_{12} \simeq \frac{M_2^2 \sin 2\theta_W}{2\epsilon(\mu^2 - M_1^2)},
\]

\[
N_{13} \simeq \frac{\mu M_Z \sin \theta_W}{\mu^2 - M_1^2},
\]

\[
N_{14} \simeq -\frac{M_1 M_Z \sin \theta_W}{\mu^2 - M_1^2},
\]

where $\epsilon$ is defined as $M_2 = M_1(1 + \epsilon)$. The bino/wino mass ratio is fixed in the gauge messenger model. As $M_1/g_i^2$ is RG invariant at the one-loop level, this ratio at the EW scale is

\[
\frac{M_1(M_Z)}{M_2(M_Z)} = \frac{5}{3} \tan^2 \theta_W \frac{M_1(M_{GUT})}{M_2(M_{GUT})} \simeq 0.9,
\]

which means $\epsilon \simeq 0.1$. From the above equations we see that the wino and higgsino mixing is sizable unless the ratio $M_2/\mu$ is too small. For $\mu \geq M_1$, the down type Higgs component, $N_{14}$, is larger than the up type Higgs component, $N_{13}$. The bino–wino mixing, $N_{12}$, is suppressed compared to the bino–higgsino mixing by $M_2/\mu \leq 1$. This is why the bino–wino mixing is negligible in most of SUSY breaking scenarios. However, in gauge messenger models the mixing is enhanced by $1/\epsilon$ thanks to near degeneracy of bino and wino. As a result, the lightest neutralino in gauge messenger models is mostly bino with sizable and comparable wino and higgsino components.

Results for the neutralino relic density in gauge messenger models are given in figures 4–7. We start the discussion with figure 4 in which we present the results for a
Figure 4. The neutralino relic density in the $M_{\text{SUSY}}$–$\tan \beta$ plane of the gauge messenger model with $c_{H_u} = c_{H_d} = 0$, $\mu > 0$ and $c_0 = 10$ (top) and $c_0 = 20$ (bottom). Blue dots represent the region in which the neutralino relic density is within the WMAP range. Regions with too much or too little relic density are indicated. For convenience, the top axis indicates the mass of the lightest neutralino. Shaded regions are excluded by various constraints.

simple gauge messenger model with additional contribution to squark and slepton masses, $c_0 = 10$ (top) and $c_0 = 20$ (bottom). An additional contribution, e.g. from gravity, at this level is enough to push the masses of all squarks and sleptons above the neutralino mass in a large region of the parameter space. Increasing $c_0$ shrinks the region of stau (N)LSP and opens up the region with neutralino (N)LSP. Blue dots represent the region
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Figure 5. The neutralino relic density in the $M_{\text{SUSY}}-c_0$ plane of the gauge messenger model with $c_{H_u} = c_{H_d} = 0$, $\mu > 0$ and $\tan \beta = 10$. Blue dots represent the region in which the neutralino relic density is within the WMAP range. Shaded regions are excluded by various constraints.

in which the neutralino relic density is within the WMAP range. The top part of the blue band corresponds to the region of stau co-annihilation. This is easy to understand because the blue band stretches along the line dividing the neutralino and stau (N)LSP regions. The bottom part of the blue band is due to the CP odd Higgs resonance which is not obvious from the plots but it will become clearer in later discussion. In most of the region for $c_0 = 10$ the co-annihilation with the stop is also important and it is the dominant process in the region where the two bands meet. However, this does not mean that stop co-annihilation and thus the special choice of $c_0$ that we made is crucial for obtaining the correct amount of the neutralino relic density. For $c_0 = 20$ the contribution from stop co-annihilation is no longer significant but the shape of the blue band is very similar, only shifted to the left, to the region of smaller neutralino mass, in which the bino–wino–higgsino mixing and the chargino co-annihilation become important.

For even larger values of $c_0$, see figure 5, the effects coming from the exchange of or co-annihilation with squarks and sleptons disappear as squarks and sleptons become heavy and the band of the correct relic density is independent of $c_0$. The residual small $c_0$ dependence comes from the fact that increasing $c_0$ influences the renormalization group evolution of $m_{H_u}^2$ in such a way that the size of the $\mu$ term increases which consequently reduces the mixture of higgsino and wino in the lightest neutralino. As a result, the correct value of the neutralino relic density is obtained with slightly lighter neutralino.

Let us discuss the neutralino annihilation process in detail for one specific point from figure 5 with $M_{\text{SUSY}} = 42$ GeV and $c_0 = 60$. This point is away from the CP odd Higgs resonance and the relic density, $\Omega_{\text{DM}} h^2 = 0.11$, reflects the composition of the
Figure 6. Top: the neutralino relic density in the \( M_{\text{SUSY}}-c_{H_u} \) plane of the gauge messenger model with \( c_0 = 10, \ c_{H_d} = 0 \), \( \mu > 0 \) and \( \tan \beta = 10 \). Blue dots represent the region in which the neutralino relic density is within the WMAP range. Shaded regions are excluded by various constraints. Bottom: the dependence of various superpartner masses on \( c_{H_u} \) for the choice of parameters corresponding to the plot on the top with \( M_{\text{SUSY}} = 60 \text{ GeV} \).

The lightest neutralino is mostly bino with small mixtures of wino and higgsinos:

\[
N_{11} = 0.95, \quad N_{12} = -0.22, \quad N_{13} = 0.18, \quad N_{14} = -0.09.
\]
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The lightest and the next-to-lightest neutralinos and the light chargino are nearly degenerate:

\[ m_{\chi^0_1} = 167 \text{ GeV}, \]
\[ m_{\chi^0_2} = 193 \text{ GeV}, \]
\[ m_{\chi^+} = 191 \text{ GeV}. \]  

Figure 7. Top: the neutralino relic density in the \( M_{\text{SUSY}}-c_{H_d} \) plane of the gauge messenger model with \( c_0 = 10, c_{H_u} = 0, \mu > 0 \) and \( \tan \beta = 10 \). Bottom: the neutralino relic density in the \( M_{\text{SUSY}}-\tan \beta \) plane of the gauge messenger model with \( c_{H_u} = -50, c_{H_d} = -50, \mu > 0 \) and \( \tan \beta = 10 \). Blue dots represent the region in which the neutralino relic density is within the WMAP range. Shaded regions are excluded by various constraints.
The dominant annihilation channel for this point is $\chi_1^0\chi_1^0 \rightarrow W^+W^-$ which represents 31% of the annihilation cross section at the freeze-out temperature. It is mediated by the t-channel exchange of charginos and thus the wino component of $\chi_1^0$ plays an important role since the light chargino is mostly the wino. Also important as a channel is $\chi_1^0\chi_1^0 \rightarrow bb$ which contributes 24% indicating that the CP odd Higgs mediated s-channel diagram makes a contribution even away from the resonance. This is again a consequence of the wino and higgsino mixing (the higgsino–bino-A and higgsino–wino-A interactions are crucial). The amplitude for this process scales as $N_{14}(N_{12} - \tan \theta_W N_{11})$ and with $N_{12} \sim -0.2$ we see that the wino component enhances this process by $\sim 60\%$. Finally, slepton mediated t-channel diagrams contribute less than 10%.

Chargino co-annihilation is always present since in the gauge messenger model the wino (and thus the lightest chargino) is only about 10% heavier than the bino. The chargino co-annihilation for this point contributes about 20% to the annihilation cross section at the freeze-out temperature. It is mediated mainly by the W boson in the s-channel which contributes about 10% and also, to a smaller extent, by the charged Higgs in the s-channel.

In summary, the wino and higgsino mixing and the chargino co-annihilation play an important role in obtaining the correct amount of the neutralino relic density in gauge messenger models in the region with fairly light superpartners (not ruled out by direct searches or the limit on the Higgs boson mass). With this knowledge we can continue with the discussion of more typical (and more complex) scenarios when additional co-annihilations and/or resonances further enhance the neutralino annihilation cross section.

In figure 6 (top) we study the dependence of the neutralino relic density on the additional contribution to the mass squared for $H_u$. To better understand the behavior of the neutralino relic density we also plot the dependence of the SUSY spectrum on $c_{H_u}$ for fixed value of $M_{\text{SUSY}}$ in figure 6 (bottom). For $c_{H_u} \gtrsim 25$ the lightest stop mass is very close to the lightest neutralino mass and the stop co-annihilation is dominant. The correct amount of the relic density is then obtained in an almost horizontal band at $c_{H_u} \simeq 30$. Going to smaller $c_{H_u}$ the difference between the stop and neutralino masses is increasing and the co-annihilation with the stop is no longer important. The CP odd Higgs resonance takes over for $c_{H_u} \simeq 20$ at $M_{\text{SUSY}} = 60$ GeV and somewhat smaller $c_{H_u}$ for larger $M_{\text{SUSY}}$. The second smaller peak is mainly due to co-annihilation with the lightest chargino through the charged Higgs resonance which happens when $m_{H^\pm} \simeq m_{\chi_1^0} + m_{\chi_1^+}$ and to a smaller extent due to co-annihilation with the second lightest neutralino through the CP odd Higgs resonance which happens when $m_A \simeq m_{\chi_1^0} + m_{\chi_2^0}$. Since the lightest chargino and the second lightest neutralino are mostly winos these two resonances happen in the same region, $c_{H_u} \simeq 0$ at $M_{\text{SUSY}} = 60$, and continue to somewhat smaller $c_{H_u}$ for larger $M_{\text{SUSY}}$. Finally, decreasing $c_{H_u}$ further takes the lightest neutralino away from stop co-annihilation and resonance regions and the blue band of the correct relic density is almost vertical in this region. The residual $c_{H_u}$ dependence comes from the fact that $c_{H_u}$ changes the size of the $\mu$ term which then varies the mixture of higgsino and wino in the lightest neutralino. The correct amount of the neutralino relic density in this region is obtained entirely due to the wino and higgsino mixing and the chargino co-annihilation as discussed in the example above.

The dependence of the neutralino relic density on the additional contribution to the mass squared for $H_d$ is given in figure 7 (top). The $c_{H_d}$ controls masses of the heavy CP
even, charged and CP odd Higgs bosons and only negligibly affects everything else. Thus the region of the correct relic density is a vertical band except for the CP odd and charged Higgs resonances. Finally, in figure 7 (bottom) we chose such values of $c_{H_u}$ and $c_{H_d}$ that the CP odd Higgs resonance does not appear. This plot is similar to those in figure 4, but now the stau co-annihilation region turns into a vertical band signaling independence of $\tan\beta$. A similar vertical band appears also in the mSUGRA scenario, see figure 3, but it is in the region ruled out by direct searches for SUSY and the Higgs boson.

4.1. Discussion of $b \to s\gamma$ and muon $g - 2$

From figures 4–7 we see that the limits on $B(b \to s\gamma)$ are typically the most constraining of all direct and indirect limits. The charged Higgs contribution is additive to the standard model contribution and scales as

$$B(b \to s\gamma)^{H\pm} \propto \frac{m_t^2}{m_{H^\pm}},$$

while the chargino–stop loop contributes as

$$B(b \to s\gamma)^{\tilde{t}} \propto \frac{\mu A_t \tan \beta}{m_{\tilde{t}}^2}.$$  \hspace{1cm} (33)

The chargino–stop loop contributes with opposite sign compared to the charged Higgs diagram if $\mu A_t$ is negative. In gauge messenger models the charged Higgs is typically heavier than stop and the chargino–stop loop dominates the new physics contribution. As a result, the predicted branching ratio becomes lower than the standard model result and the lower bound on $B(b \to s\gamma)$ plays an important role.

In the limit when $M_1, M_2, m_{\tilde{\mu}}$ and $m_{\tilde{\nu}_\mu}$ are approximately equal, which is the case in gauge messenger models, and $\mu > M_1, M_2$, the expression for the supersymmetric contribution to the muon anomalous magnetic moment [28] simplifies to

$$\Delta a_{\mu}^{\text{SUSY}} \simeq \frac{g_2^2 m_{\mu}^2}{32\pi^2} \frac{\mu M}{M^2 (\mu^2 - M^2)} \tan \beta,$$

where $M$ represents sneutrino, smuon, chargino or neutralino masses. It can be rewritten as

$$\Delta a_{\mu}^{\text{SUSY}} \simeq 13 \left(\frac{100 \text{ GeV}}{M}\right)^2 \left(\frac{\mu M}{\mu^2 - M^2}\right) \tan \beta \times 10^{-10}.$$  \hspace{1cm} (35)

As a result, we obtain a relation between $M$ and $\tan \beta$. In most of the parameter space $\mu$ is just about twice as large as the lightest neutralino mass and thus we can set $\mu M/(\mu^2 - M^2) \simeq \frac{2}{3}$ in which case we get

$$\Delta a_{\mu} \times 10^{10} \simeq \frac{26}{3} \left(\frac{100 \text{ GeV}}{M}\right)^2 \tan \beta.$$  \hspace{1cm} (36)

Assuming conservative bounds $2 \times 10^{-10} < \Delta a_{\mu} < 50 \times 10^{-10}$ as discussed in section 3.1 we can derive the lower and upper bounds on $M$ as a function of $\tan \beta$:

$$M_{\text{lower}} \sim 40 \sqrt{\tan \beta} \text{ GeV},$$

$$M_{\text{upper}} \sim 80 \sqrt{\tan \beta} \text{ GeV},$$

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For $\tan \beta = 10$ we find $130 \text{ GeV} \lesssim M \lesssim 630 \text{ GeV}$ and similarly for $\tan \beta = 50$ we have $280 \text{ GeV} \lesssim M \lesssim 1400 \text{ GeV}$. In figures 4–7 the value of $M$ approximately corresponds to the neutralino mass represented by the top axis. It is interesting to note that the indirect bound from the upper limit on the muon anomalous magnetic moment is already well above the direct search limits on superpartners.

As a result of the squeezed spectrum of gauge messenger models the limits on $B(b \to s\gamma)$ and the muon $g - 2$ are almost parallel to each other; see figures 4–7. This is a consequence of the SUSY contribution to both processes scaling approximately as $\tan \beta/M^2$. The limits on $B(b \to s\gamma)$ constrain the SUSY spectrum from below while the limits on $g - 2$ constrain the parameter space from above. The allowed parameter space is then only a strip in between these two bounds. This is a characteristic feature of models with squeezed spectrum. If the required value of $\Delta a_\mu$ turns out to be close to the upper range of current estimates most of the parameter space of gauge messenger models that we considered will be ruled out with only tiny regions remaining; see figures 4–7.

Interestingly, it is still possible to obtain the correct amount of the dark matter density in these tiny regions; see figures 6 and 7 (bottom).

### 4.2. Direct detection of neutralino dark matter

In this section we calculate the spin dependent and spin independent neutralino–nucleon cross sections in gauge messenger models. The spin independent neutralino–nucleon cross section is dominated by (light and heavy) Higgs mediated t-channel diagrams which are controled by the higgsino component of the lightest neutralino:

$$
\sigma_{\chi N} = \frac{g^2 g'^2 m_N^4}{4 \pi M_W^2} \left[ - \frac{X_d \tan \beta N_{13}}{m_H^2} + \frac{X_u N_{14}}{m_h^2} \right]^2 ,
$$

where $X_d = f_{T_d} + \frac{2}{27} f_{TG}$ and $X_u = f_{T_u} + \frac{4}{27} f_{TG}$ [2].

Substituting $N_{13}$ and $N_{14}$ from equations (28) and (29) we find

$$
\sigma_{\chi N} = \frac{g^4 m_N^4}{4 \pi} \left( \frac{\mu^2}{(\mu^2 - M_t^2)^2} \right) \left[ \frac{X_d \tan \beta}{m_H^2} + \frac{M_1}{\mu} \right] \left( \frac{X_u + X_d}{m_h^2} \right) .
$$

Squark exchange diagrams are negligible due to the hypercharge as long as the squark masses are comparable to the heavy Higgs mass. Inserting the numbers $X_u = 0.144$ and $X_d = 0.18$ [2], we get the direct detection rate close to the one we obtained using DarkSUSY.

The detection cross sections for points with the correct neutralino relic density from figure 4 (top), the gauge messenger model with $c_{H_u} = c_{H_d} = 0$, $\mu > 0$ and $c_0 = 10$, that satisfy all direct and indirect constraints are given in figure 8. In gauge messenger models with no additional contribution to Higgs soft masses and only a small contribution to other scalar masses, enough to make them heavier than the lightest neutralino, the direct detection cross section scales as $\sigma_{\chi N} \propto \tan^2 \beta / M_{\text{SUSY}}^6$ for $\tan \beta \geq 10$. This behavior is clearly visible in figure 8. The thickness of the line is determined by the allowed region for $\tan \beta$, in this case $5 < \tan \beta < 25$; see figure 4.

The predicted cross sections are not
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Figure 8. Spin dependent and spin independent neutralino–nucleon cross sections for points with the correct neutralino relic density from figure 4 (top), the gauge messenger model with $c_{H_u} = c_{H_d} = 0$, $\mu > 0$ and $c_0 = 10$, that satisfy all direct and indirect constraints. The lines represent the current CDMS limits [29] and expected limits from CDMSII [30] for the spin independent cross section.

(a) Spin dependent neutralino-nucleon cross section for simple gauge messenger with $c_0 = 10$

(b) Spin independent neutralino-nucleon cross section for simple gauge messenger with $c_0 = 10$
within the reach of CDMSII [30]. Assuming additional contributions to Higgs soft masses allows for a wider range of the higgsino and wino mixing and the range of the predicted detection cross sections spreads as is shown figure 9. Part of the parameter space is within the reach of CDMSII and the whole parameter space of gauge messenger models can be explored at Super-CDMS [30].

5. Conclusions

The lightest neutralino in gauge messenger models is mostly the bino with a sizable mixture of the wino and higgsino. The wino and higgsino components enhance the neutralino annihilation cross section. Furthermore, the splitting between the bino and wino masses is at the level of 10% and thus the co-annihilation with the chargino is contributing in the whole region of the parameter space. These two features, the lightest neutralino being a mixture of the bino, wino and higgsino, and the chargino co-annihilation, are sufficient for obtaining the correct neutralino relic density to explain WMAP results with fairly light neutralino (and other superpartners) while satisfying all the constraints from direct searches for superpartners and the limit on the Higgs boson mass.

This is in contrast with scenarios with the usual hierarchical spectrum, e.g. mSUGRA, in which the properties of the lightest neutralino (being bino-like) typically lead to the correct neutralino relic density in the region which is already ruled out by direct SUSY and Higgs searches or disfavored by $b \rightarrow s\gamma$. In mSUGRA-like models obtaining the correct amount of the neutralino relic density relies on special co-annihilation and resonance regions which are critically sensitive to small variations of independent parameters. Due to a large hierarchy in the spectrum these surviving strips are typically well separated by large regions of the parameter space ruled out by WMAP data.

In gauge messenger models, as a result of the squeezed spectrum of superpartners, various co-annihilation and resonance regions overlap and very often the correct amount of the neutralino relic density is generated as an interplay of several processes. For example the stop co-annihilation contributes significantly in a large region of the parameter space. This can be easily understood from the fact that both stop and neutralino masses are mainly controlled by the same parameter and as it happens the neutralino and stop masses are very close to each other. Varying contributions to scalar masses from other sources is only slowly changing this relation. Furthermore, even if we increase stop masses by assuming an independent additional contribution, which effectively shuts down the stop co-annihilation, the band of the correct neutralino relic density only moves to the region with somewhat lighter neutralino which still satisfies the limits from direct SUSY and Higgs searches. This feature makes the explanation of the observed amount of the dark matter density much less sensitive to fundamental parameters.

In gauge messenger models with no additional contribution to Higgs soft masses and only a small contribution to other scalar masses, enough to make them heavier than the lightest neutralino, the direct detection cross section is predicted to be in the range $10^{-46}$–$10^{-44}$ cm$^2$ which is not within the reach of CDMSII but can be explored at Super-CDMS. Some of the results concerning the neutralino relic density in gauge messenger models, namely the presence of various co-annihilation regions, originate from the squeezed SUSY spectrum. Therefore we expect similar results for other models derived in different
Figure 9. Spin dependent and spin independent neutralino–nucleon cross sections for points with the correct neutralino relic density satisfying all direct and indirect constraints obtained in an extended scan over whole parameter space of gauge messenger models discussed in this paper. The lines represent the current CDMS limits and expected limits from CDMSII for the spin independent cross section.
contexts which lead to squeezed spectrum, e.g. deflected anomaly mediation \cite{31}–\cite{33} and mirage mediation \cite{34}–\cite{39}. However, the special features of the gauge messenger model related to the bino–wino–higgsino mixed dark matter and with the associated chargino co-annihilation depend on details of a model and are not automatically guaranteed by the squeezedness.

In conclusion, let us note that both the natural EWSB and the natural explanation of the correct amount of the dark matter density independently disfavor models with a hierarchical spectrum. Models with a squeezed spectrum seem to be favored and thus it is desirable to explore their phenomenological and collider predictions.

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Appendix

In this appendix we set conventions and derive approximate formulae for the composition of the lightest neutralino in the gauge messenger model which are useful for the discussion of neutralino relic density.

The neutralino mass matrix in the basis \((B, W, h_d, h_u)\) is given by

\[
M_N = \begin{pmatrix}
M_1 & 0 & -M_Zc_\beta s_\theta_W & M_Zs_\beta s_\theta_W \\
0 & M_2 & M_Zc_\beta c_\theta_W & -M_Zs_\beta c_\theta_W \\
-M_Zc_\beta s_\theta_W & M_Zc_\beta c_\theta_W & 0 & -\mu \\
M_Zs_\beta s_\theta_W & -M_Zs_\beta c_\theta_W & -\mu & 0
\end{pmatrix},
\]

where \(s_\theta_W \equiv \sin \theta_W, \ c_\theta_W = \cos \theta_W\) with \(\theta_W\) being the Weinberg angle (weak mixing angle) and similarly \(s_\beta = \sin \beta, \ c_\beta = \cos \beta\) where \(\tan \beta = v_u/v_d\).

In the gauge messenger model, bino and wino masses are comparable. Thus it is convenient to express the wino mass in terms of the bino mass and a small parameter describing the difference,

\[
M_2 = M_1(1 + \epsilon).
\]

Numerically \(\epsilon \simeq 0.09\) and it is almost independent of \(\tan \beta\). Thus, for \(M_1 < |\mu|\) the lightest neutralino is mostly bino and the splitting between the bino and the wino is at the level of 10%.
The neutralino mass matrix can be brought to a diagonal form by an orthogonal transformation,

$$M_{\text{diag}} = N M N^T,$$

where $N_{ij}$, $j = 1, 2, 3$ and $4$, represent the mixtures of $B$, $W$, $h_u$, $h_d$ in the lightest neutralino mass eigenstate.

In order to calculate $N_{ij}$, it is convenient to rotate the neutralino mass matrix to a basis in which the lower right $2 \times 2$ block is diagonal,

$$M = \begin{pmatrix}
M_1 & 0 & -\frac{1}{\sqrt{2}} M_Z s_{\beta} (s_{\beta} + c_{\beta}) & \frac{1}{2} M_Z s_{\beta} (s_{\beta} - c_{\beta}) \\
0 & M_2 & \frac{1}{2} M_Z c_{\beta} (s_{\beta} + c_{\beta}) & -\frac{1}{\sqrt{2}} M_Z c_{\beta} (s_{\beta} - c_{\beta}) \\
-\frac{1}{\sqrt{2}} M_Z s_{\beta} (s_{\beta} + c_{\beta}) & \frac{1}{2} M_Z c_{\beta} (s_{\beta} + c_{\beta}) & \mu & 0 \\
\frac{1}{\sqrt{2}} M_Z s_{\beta} (s_{\beta} - c_{\beta}) & -\frac{1}{\sqrt{2}} M_Z c_{\beta} (s_{\beta} - c_{\beta}) & 0 & -\mu
\end{pmatrix},$$

which is obtained by an orthogonal transformation,

$$M = U M N U^T,$$

with

$$U = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}.$$

The matrix $M$ can be diagonalized by an orthogonal transformation,

$$M_{\text{diag}} = V M V^T.$$

The advantage of $M$ is that we can treat off-diagonal elements as perturbations and calculate eigenvectors (elements of $V$) using the matrix perturbation formalism. Then, the mixing matrix $N$ (the diagonalization matrix in the original basis) is simply given as

$$N = V U.$$

In the leading order, neglecting the off-diagonal elements of $M$, the diagonalization matrix $V$ is an identity matrix. When the mass differences between eigenvalues are not extremely small, $(M_2 - M_1) \mu^2 \geq M_1 M_2^2$, or equivalently $\epsilon \mu^2 \geq M_Z^2$, the non-degenerate perturbation formalism can be applied. At the first order of perturbation theory we have

$$V_{nm}^{(1)} = \frac{M_{mn}}{\Delta_{nm}},$$

where $\Delta_{mn} = M_{mn} - M_{nn}$. Similarly, the second-order corrections are given as

$$V_{nl}^{(2)} = \sum_{m \neq n} \frac{M_{lm} M_{mn}}{\Delta_{ml} \Delta_{nm}}.$$
For $M_1 < |\mu|$ we find
\begin{align}
V_{11}^{(1)} &= 0, \\
V_{12}^{(1)} &= 0, \\
V_{13}^{(1)} &= -\frac{M_{31}}{\mu - M_1} = \frac{M_Z \sin \theta_W (\sin \beta + \cos \beta)}{\sqrt{2}(\mu - M_1)}, \\
V_{14}^{(1)} &= +\frac{M_{41}}{\mu + M_1} = \frac{M_Z \sin \theta_W (\sin \beta - \cos \beta)}{\sqrt{2}(\mu + M_1)},
\end{align}
and thus the higgsino component in the lightest neutralino appears at the first order.

Since $V_{12}^{(1)} = 0$ it is necessary to calculate the contribution from the next order. This contribution is small in general but can significantly alter the result when $M_1 \sim M_2$. The second-order correction is
\begin{align}
V_{12}^{(2)} &= -\frac{M_{23} M_{31}}{(M_2 - M_1)(M_1 - \mu)} - \frac{M_{24} M_{41}}{(M_2 - M_1)(M_1 + \mu)} \\
&= -\frac{M_Z^2 \sin 2\theta_W (\sin \beta + \cos \beta)^2}{4\epsilon M_1 (\mu - M_1)} + \frac{M_Z^2 \sin 2\theta_W (\sin \beta - \cos \beta)^2}{4\epsilon M_1 (\mu + M_1)},
\end{align}
and, since $\epsilon \sim 0.1$, it is comparable to the first-order corrections coming from the higgsino mass. Therefore, we have a sizable bino–wino mixing in addition to bino–higgsino mixing.

The diagonalization matrix $V$ is then approximately given as $V \simeq 1 + V^{(1)} + V^{(2)}$. Finally, we can find the components of the mixing matrix in the original interaction basis. Using equations (A.8) and (A.6) we get
\begin{align}
N_{11} &\simeq 1, \\
N_{12} &\simeq V_{12}^{(2)}, \\
N_{13} &= +\frac{1}{\sqrt{2}} V_{13} + \frac{1}{\sqrt{2}} V_{14} \simeq \frac{M_Z \sin \theta_W (\mu \sin \beta + M_1 \cos \beta)}{\mu^2 - M_1^2}, \\
N_{14} &= -\frac{1}{\sqrt{2}} V_{13} + \frac{1}{\sqrt{2}} V_{14} \simeq -\frac{M_Z \sin \theta_W (M_1 \sin \beta + \mu \cos \beta)}{\mu^2 - M_1^2}.
\end{align}

For $\tan \beta \geq 10$ these formulae can be further simplified:
\begin{align}
N_{11} &\simeq 1, \\
N_{12} &\simeq -\frac{M_Z^2 \sin 2\theta_W}{2\epsilon (\mu^2 - M_1^2)}, \\
N_{13} &\simeq +\frac{\mu M_Z \sin \theta_W}{\mu^2 - M_1^2}, \\
N_{14} &\simeq -\frac{M_1 M_Z \sin \theta_W}{\mu^2 - M_1^2}.
\end{align}
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