Covariant Classification Scheme of Hadrons

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Starting from the multi-local Klein-Gordon equations with Lorentz-scalar squared-mass operator we give a covariant quark representation of the general composite mesons and baryons with definite Lorentz transformation property. The mass spectra satisfy the approximate symmetry under the $\tilde{U}(4)$ transformation group, including the chiral transformation as a subgroup, concerning the spinor freedom of light constituent quarks, and this symmetry predicts the existence of new type of chiral mesons and baryons out of the conventional framework in non-relativistic quark model: For example, for light $q\bar{q}$ systems, the scalar $\sigma$- and axial-vector $a_1$-nonets, and for heavy-light $Q\bar{q}$ and $qQ$ systems the scalar and axial-vector mesons are predicted to exist as relativistic $S$-wave states besides the ordinary $P$-wave state mesons. The existence of two “exotic” $1^{-+}$ meson nonets is predicted as the relativistic $P$-wave states in $q\bar{q}$ systems. For light quark baryons the extra $5\bar{6}$ with positive parity and the extra $7\bar{0}$ with negative parity of the static $SU(6)$ are predicted to exist as the ground state chiral particles.

§1. Introduction

There exist the two contrasting, non-relativistic and relativistic, viewpoints of level-classification. The former is based on the non-relativistic quark model (NRQM) with the approximate $LS$-symmetry and gives a theoretical base to the PDG level-classification. The latter is embodied typically in the NJL model with the approximate chiral symmetry. It is widely accepted that $\pi$ meson nonet has the property as a Nambu-Goldstone boson in the case of spontaneous breaking of chiral symmetry.

| Model       | Non-Relativistic | Relativistic      |
|-------------|------------------|-------------------|
| Approx. Symm.| Non Relat. Q. M. | $LS$-Symm.       |
| Evidence    | Bases for PDG    | $\pi$-nonet as NG boson |

Table 1. Two Contrasting Viewpoints of Level Classification

Owing to the recent progress, both theoretical and experimental, the existence of light $\sigma$-meson as chiral partner of $\pi(140)$ seems to be established especially through the analysis of various $\pi\pi$-production processes. This gives further a strong support to the relativistic viewpoint.

Thus, the hadron spectroscopy is now confronting with a serious problem, existence of the seemingly contradictory two viewpoints, Non-relativistic and Extremely Relativistic ones. The purpose of this talk is to present an attempt for a new level-classification scheme unifying these two viewpoints. The following is an overview of
our attempt, taking an example of the light-quark hadron system:

|                  | NRQM                  | COQM (old)                     | COQM (extended)               |
|------------------|-----------------------|--------------------------------|--------------------------------|
| **Symmetry**     | $SU(6)_SF$            | “boosted” $SU(6)_SF$           | Lorentz scalar                 |
| **(Wave Function)** |                       |                                |                                |
| $(qq)$-meson      | $f(x_1,x_2) \chi_q \bar{\chi}_{\bar{q}}$ Pauli spinor | $f(x_1,x_2) u_q \bar{v}_{\bar{q}}$ Dirac spinor | $u_+(-P)$, $v_-(-P)$          |
|                  |                       |                                | boosted Pauli spin             |
| $(qqq)$-baryon    | $f(x_1,x_2,x_3) \chi_{q_1} \chi_{q_2} \chi_{q_3}$ | $f(x_1,x_2,x_3) u_{q_1} u_{q_2} u_{q_3}$ | $u_+(-P)$, $v_-(-P)$          |
| $(\bar{q}q\bar{q})$-anti-baryon | $f(x_1,x_2,x_3) \bar{\chi}_{\bar{q}_1} \bar{\chi}_{\bar{q}_2} \bar{\chi}_{\bar{q}_3}$ | $f(x_1,x_2,x_3) \bar{v}_{\bar{q}_1} \bar{v}_{\bar{q}_2} \bar{v}_{\bar{q}_3}$ | $v_+(-P)$                       |
| **Space(-time)** | $O(3) \otimes SU(2)_F \otimes SU(3)_F$ | $O(3,1) \otimes \tilde{U}(4)_{D.S.} \otimes SU(3)_F$ |                                |
| **(Spin Wave Function)** | multi-Pauli spinor | multi-boosted Pauli spinor | multi-Dirac spinor |

Table II. Overview of Attempt: example of light quark hadrons

The left column concerns the NRQM, while the right column does the covariant oscillator quark model (COQM) as a basic kinematical framework of our attempt.

In NRQM the confining force is assumed to be spin-independent and the mass spectra have the $SU(6)_SF$ spin-flavor symmetry. In the extended (old) version of COQM the confining force is assumed to be Lorentz-scalar (“boosted-spin” independent) and the mass spectra have the $\tilde{U}(12)_SF$ symmetry (boosted $SU(6)_SF$ symmetry). The 3-dimensional space-coordinates of constituent quarks and/or antiquarks, as variables in the meson and baryon wave functions (WF), in NRQM are extended to the 4-dimensional Lorentz-vectors in COQM. Similarly the multi-Pauli-spinors, as spin WF in NRQM are extended to the covariant multi-Dirac spinors in COQM; where in the old version only the positive-energy spinors $u_+ (P)$ ($\bar{v}_+ (P)$) for quarks (anti-quarks) are considered, and in the extended version the negative-energy spinors $u_- (\bar{P})$ ($\bar{v}_- (\bar{P})$) (the $P$ is the space-momentum of hadrons as a whole-entity) are also taken into account. Thus, the WF of hadrons in the new level-classification scheme become the tensors in the $O(3,1)_{Lorentz} \otimes \tilde{U}(4)_{D.S.} \otimes SU(3)_F$-space (being extended from the ones in the $O(3) \otimes SU(2)_{P.S.} \otimes SU(3)_F$ space of NRQM). The numbers of freedom of spin-flavor WF in NRQM are $6 \times 6^* = 36$ for mesons and $(6 \times 6 \times 6)_{\text{Symm.}} = \underline{56}$ for baryons: These numbers in COQM become $12 \times 12^* = \underline{144}$ for mesons and $(12 \times 12 \times 12)_{\text{Symm.}} = \underline{364} = \underline{182}$ (for baryons) + $\underline{182}$ (for antibaryons).

Inclusion of heavy quarks is straightforward: The WF of general $q$ and/or $Q$ hadrons become tensors in $O(3,1) \otimes [\tilde{U}(4)_{D.S.} \otimes SU(3)_F]_q \otimes [SU(2)_{P.S.} \otimes \tilde{U}(1)_{F}]_q$.

* The $\tilde{U}(12)_SF$ symmetry was first proposed in 1965 as a generalization of the static $SU(6)_SF$ symmetry. However, at that time only the boosted Pauli-spinors are taken as physical components of fundamental representation of $\tilde{U}(4)_{D.S.}$. Now in the extended scheme all general Dirac spinors prove to be physical.
§2. Covariant framework for describing composite hadrons

As WF of mesons and baryons we set up the following field-theoretical expressions, respectively, as

\[
\Phi_A^B(x_1, x_2) = \langle 0|\psi_A(x_1)\bar{\psi}^B(x_2)|M\rangle + \langle M|\psi_A(x_1)\bar{\psi}^B(x_2)|0\rangle, \quad (2.1)
\]

\[
\Phi_{A_1A_2A_3}(x_1, x_2, x_3) = \langle 0|\psi_{A_1}(x_1)\psi_{A_2}(x_2)\psi_{A_3}(x_3)|B\rangle
+ \langle B|\psi_{A_1}(x_1)\psi_{A_2}(x_2)\psi_{A_3}(x_3)|0\rangle, \quad (2.2)
\]

where \(\psi_A\) is the quark field \((A = (\alpha, a); \alpha = 1 \sim 4\) (\(a\) denoting Dirac spinor (flavor) indices) and \(\bar{\psi}^B\) denotes its Pauli-conjugate. We start from the Yukawa-type Klein Gordon equation as a basic wave equation [3]:

\[
[\partial^2/\partial X_\mu^2 - M^2(r_\mu, \partial/\partial r_\mu)]\Phi(X, r, \cdots) = 0, \quad (2.3)
\]

where \(X_\mu(r_\mu)\) are the center of mass (relative) coordinates of hadron systems. The WF are separated into the positive (negative)-frequency parts concerning the CM plane-wave motion and expanded in terms of eigen-states of the squared-mass operator as

\[
\Phi(X, r, \cdots) = \sum_{P_N, N} \left[ e^{iP_N \cdot X} \psi_N^{(+)}(P_N, r, \cdots) + e^{-iP_N \cdot X} \psi_N^{(-)}(P_N, r, \cdots) \right], \quad (2.4)
\]

\[
M^2(r_\mu, \partial/\partial r_\mu, \cdots)\psi_N^{(\pm)} = M_N^2\psi_N^{(\pm)}, \quad (2.5)
\]

\[
M^2 = M_{\text{conf}}^2 + \delta M_{\text{pert. QCD}}^2. \quad (2.6)
\]

The \(M^2\) consists of two parts: The confining-force part \(M_{\text{conf}}^2\) is assumed to be Lorentz-scalar and \(A, (B)\)-independent, leading to the mass spectra with the \(SU(12)\) symmetry and also with the chiral symmetry. As its concrete model we apply the covariant oscillator in COQM, leading to the straight-rising Regge trajectories. The effects due to perturbative QCD \(\delta M^2\) are neglected in this talk.

The internal WF is, concerning the spinor freedom, expanded in terms of complete set of relevant multi-spinors, Bargmann-Wigner (BW) spinors.

- **meson**:
  \[
  \psi_{N, A}^{(\pm)} B(P_N, r) = \sum_W W_{\alpha}^{(\pm)}(P_N) M_{N, a}^{(\pm) b}(r, P_N) \quad (2.7)
  \]

- **baryon**:
  \[
  \psi_{N, A_1 A_2 A_3}^{(\pm)}(P_N, r_1, r_2) = \sum_W W_{\alpha_1 \alpha_2 \alpha_3}^{(\pm)}(P_N) B_{N, a_1 a_2 a_3}(r_1, r_2, P_N). \quad (2.8)
  \]

The BW spinors are defined as multi-Dirac spinor solutions of the relevant local Klein-Gordon equation:

\[
(\partial^2/\partial X_\mu^2 - M^2)W_{\alpha^{\cdots} \beta^{\cdots}}(X) = 0 \quad (2.9)
\]

\[
W_{\alpha^{\cdots} \beta^{\cdots}}(X) \equiv \sum_{\mathbf{P}, P_0 = E} (e^{iPX} W_{\alpha^{\cdots}}^{(+)}(\mathbf{P}) + e^{-iPX} W_{\alpha^{\cdots}}^{(-)}(\mathbf{P})). \quad (2.10)
\]

For mesons and baryons BW spinors are bi-Dirac and tri-Dirac spinors, respectively. We further go into more details of BW spinors. First we define the Dirac spinors for...
constituent quarks and anti-quarks with hadron 4-momentum $P_\mu$ as “mono-index” BW spinors:

$$
(\partial^2 / \partial X_\mu^2 - M^2) \psi_\alpha(X) = 0,
$$

$$
\psi_{q,\alpha}(X) \equiv \sum_{P_\mu} e^{i P \cdot X} u_{q,\alpha}(P_\mu)
= \sum_{P, P_0 = \pm E_P} (u_+(P) e^{i P \cdot X} + u_-(P) e^{-i P \cdot X}),
$$

$$
\psi_{q,\alpha}(X) \equiv \sum_{P_\mu} e^{-i P \cdot X} v_{q,\alpha}(P_\mu)
= \sum_{P, P_0 = \pm E_P} (v_+(P) e^{-i P \cdot X} + v_-(P) e^{i P \cdot X}),
$$

where the hadron 4 momentum $P_\mu$ satisfies the equations

$$
P_\mu^2 + M^2 = 0, \quad P_0 = \pm E_P, \quad E_P \equiv \sqrt{P^2 + M^2}.
$$

Here it is to be noted that all 4-independent solution $u_q(P)$ ($v_q(P)$) with spin $\sigma_3(\sigma_3' = -\sigma_3') = \pm 1$ and $P_0 = \pm E_P$ for quarks(anti-quarks) inside of hadrons.

The BW equations, the BW-spinors as their solutions and their irreducible com-

| Meson | $W^{(+)-}(P)$ | $M^{(+)}(P)$ | BW-Equation | $(P_0 \equiv E_P > 0)$ |
|--------|----------------|-------------|--------------|------------------------|
| $U_\alpha^\beta(P)$ | $u_\alpha(P) \bar{v}^\beta(P)$; | $P_\alpha$, $V_\mu$; | $(i P \cdot \gamma^{(1)} + M)U = 0$, | $U(-i P \cdot \gamma^{(2)} + M) = 0$ |
| $C_\alpha^\beta(P)$ | $u_\alpha(P) \bar{v}^\beta(-P)$; | $S$, $A_\mu$; | $(i P \cdot \gamma^{(1)} + M)C = 0$, | $C(i P \cdot \gamma^{(2)} + M) = 0$ |
| $D_\alpha^\beta(P)$ | $u_\alpha(-P) \bar{v}^\beta(P)$; | $S$, $A_\mu$; | $(-i P \cdot \gamma^{(1)} + M)D = 0$, | $D(-i P \cdot \gamma^{(2)} + M) = 0$ |
| $V_\alpha^\beta(P)$ | $u_\alpha(-P) \bar{v}^\beta(-P)$; | $P_\alpha$, $V_\mu$; | $(-i P \cdot \gamma^{(1)} + M)V = 0$, | $V(i P \cdot \gamma^{(2)} + M) = 0$ |

Table III. Bargmann-Wigner (BW) Equations and Spinors for mesons. Only the positive frequency parts are given. The negative frequency parts $W^{(-)}$ are obtained from the operation $W^{(-)} = W^{(+)} \{ u \leftrightarrow v \}$. For example, $C^{(-)} = C^{(+)} \{ u \leftrightarrow v \} = v_\alpha(P) \bar{u}^\beta(-P)$.

| Baryon | $W^{(+)}_{a_1a_2a_3}(P)$ | $B^{(+)}(P)$ | BW-Equation $(P_0 \equiv E_P > 0)$ |
|--------|----------------|-------------|------------------------|
| $E_{a_1a_2a_3}(P)$ | $u_{a_1}(P)u_{a_2}(P)u_{a_3}(P)$; | $\psi(\frac{1}{2}), \psi_\mu(\frac{1}{2})$. | $(i P \cdot \gamma^{(1,2,3)} + M)E = 0$, |
| $G_{a_1a_2a_3}(P)$ | $u_{a_1}(P)u_{a_2}(P)u_{a_3}(-P)$; | $\psi(\frac{1}{2}), \psi_\mu(\frac{1}{2})$. | $(i P \cdot \gamma^{(1,2)} + M)G = 0$, | $(-i P \cdot \gamma^{(3)} + M)G = 0$, |
| $F_{a_1a_2a_3}(P)$ | $u_{a_1}(P)u_{a_2}(-P)u_{a_3}(-P)$; | $\psi(\frac{1}{2}), \psi_\mu(\frac{1}{2})$. | $(i P \cdot \gamma^{(1)} + M)F = 0$, | $(-i P \cdot \gamma^{(2,3)} + M)F = 0$, |

Table IV. Bargmann-Wigner (BW) Equations and Spinors for baryons. Only the positive frequency parts are given. The negative frequency parts $W^{(-)}$ are obtained from the operation $W^{(-)} = W^{(+)} \{ u \rightarrow v \}$. For example, $E_{a_1a_2a_3}^{(-)} = E_{a_1a_2a_3}^{(+)} \{ u \rightarrow v \} = v_{a_1}(P)v_{a_2}(P)v_{a_3}(P)$.
§3. Transformation rule for hadrons and Chiral symmetry

By using the covariant quark representation of composite hadrons given above we can derive automatically their rule for any (relativistic) symmetry transformation from that of constituent quarks. The rules for chiral transformation of mesons and baryons are, respectively,

\[
\begin{align*}
\text{meson: } & \psi_A^B(P, r) \rightarrow [e^{i\alpha\lambda^a/2} \gamma^5 \psi(P, r)]^B_A, \\
\text{baryon: } & \psi_{A_1A_2A_3}(P, r_1, r_2) \rightarrow [\Pi_{i=1}^3 e^{i\alpha\lambda_{a(i)}^a/2} \gamma_5 \psi(P, r_1, r_2)]_{A_1A_2A_3}.
\end{align*}
\]

(3.1) (3.2)

The physical meaning of chiral transformation are clearly seen from the operations:

\[
\begin{align*}
u(P) \rightarrow u'(P) &= \gamma_5 u(P) = u(-P); \\ u_\pm(P) &\leftrightarrow u_\mp(-P), \\
v(P) \rightarrow v'(P) &= \gamma_5 v(P) = v(-P); \\ v_\pm(P) &\leftrightarrow v_\mp(-P).
\end{align*}
\]

(3.3) (3.4)

That is, the chiral transformation transforms the members of relevant BW-spinors with each other. Accordingly, if \( M^2 \) operator is independent of Dirac indices, the hadron mass spectra have effectively the \( \hat{U}(4) \) symmetry and also the chiral symmetry.

For convenience of later discussions we note further on physical meaning of BW equations and introduce the notion of “exciton-quark”. That is, the BW spinors with the total hadron momentum \( P_\mu \) and mass \( M \) are equivalent to the product of free Dirac spinors of the exciton quark with momentum \( p^{(i)}_\mu \equiv \kappa^{(i)} P_\mu \) and mass \( m^{(i)} \equiv \kappa^{(i)} M \) (\( \sum_i \kappa^{(i)} = 1 \)), as is seen from the equations (in an example of the \( U \)-type (E-type) BW spinors of meson(baryon) systems).

\[
\begin{align*}
\text{meson: } & (iP \cdot \gamma^{(1)} + M)U(P) = 0 \xrightarrow{\times \kappa^{(1)}} (ip^{(1)} \cdot \gamma^{(1)} + m^{(1)})U(P) = 0 \\
& U(P)(-ip \cdot \gamma^{(2)} + M) = 0 \xrightarrow{\times \kappa^{(2)}} U(P)(-ip^{(2)} \cdot \gamma^{(2)} + m^{(2)}) = 0 \\
& p^{(1)} + p^{(2)} = P_\mu; \quad m^{(1)} + m^{(2)} = M, \\
\text{baryon: } & (iP \cdot \gamma^{(i)} + M)E(P) = 0 \xrightarrow{\times \kappa^{(i)}} (ip^{(i)} \cdot \gamma^{(i)} + m^{(i)})E(P) = 0 \\
& p^{(1)} + p^{(2)} + p^{(3)} = P_\mu; \quad m^{(1)} + m^{(2)} + m^{(3)} = M.
\end{align*}
\]

(3.5) (3.6) (3.7) (3.8)

The above consideration is valid through all ground-state and/or excited state hadrons: Accordingly the mass \( M_N \) of the \( N \)-th excited hadron with the 4-momentum \( P_N \) is generally given as a sum of the \( N \)-th excited mass \( m_N \) of the exciton quark with the 4-momentum \( P_N^{(i)} \equiv \kappa^{(i)} P_N \).

\[
M_N = m_N^{(1)} + m_N^{(2)} + \cdots, \quad p_N^{(i)} = \kappa^{(i)} P_N \quad (\sum_i \kappa^{(i)} = 1).
\]

(3.9)

§4. Level structure of mesons

4.1. Phenomenological criterion for chiral symmetry

Considering the physical meaning of BW equations (see, Eqs. (3.5) and (3.7)) mentioned in the last section, we may set up the phenomenological criterion for chiral
symmetry being effective as

\[ m_{q,N}^2 \ll A_{\text{conf}}^2 (\approx A_{\chi SB}^2) \approx 1 \text{GeV}^2. \]  

(4.1)

We can estimate the values of exciton light-quark mass \( m_{q,N} \) by applying the following mass formulas for the light-light \( n\bar{n} \)-meson \((n = u \text{ or } d)\) and the light-heavy \( nQ \)- and \( Q\bar{n} \)-meson systems \((Q = c \text{ or } b)\).

\[
M_N^2 = M_0^2 + N \Omega, \quad M_N = m_{q,N} + m_{q(Q),N} \quad (m_{q,0} = m_q, \ m_{Q,0} = m_Q) \tag{4.2}
\]

\[
M_N^2 = \left( \sqrt{m_q^2 + p^2} + \sqrt{m_{q(Q)}^2 + p^2} \right)^2 + V_N \equiv \left( \sqrt{m_q^2 + A_N^2} + \sqrt{m_{q(Q)}^2 + A_N^2} \right)^2. \tag{4.3}
\]

The equation (4.2) is the conventional formula in COQM, where the \( \Omega^{-1} \) is the inverse Regge-slope and the zero-th exciton quark mass \( m_{q,0}(m_{Q,0}) \) is identified with the corresponding constituent-quark mass \( m_q(m_Q) \). The equation (4.3) comes from the standard bound-state picture of hadrons, where \( V \) is the scalar confining potential and \( A_N \) corresponds to the average value of relative momentum \(|p|\) of constituent quarks in the \( N \)-th excited meson rest-frame. The result of values of light-exciton quark masses, thus estimated using the values of \( \Omega \) and constituent quark masses obtained in the preceding analyses, is collected in Table V.

| \( \Omega \) /GeV | \( n\bar{n} \) | \( n\bar{c} \) | \( nb \) | Chiral symm. |
|------------------|-------------|-------------|------|-------------|
| \( m_{n,0} \)    | N = 0       | 0.38        | 0.38 | \( \bigcirc \) |
| (GeV)            | N = 1       | 0.64        | 0.70 | \( \bigtriangleup \) |
|                  | N = 2       | 0.83        | 0.95 | 1.07 \( \times \) |

Table V. Light exciton-quark mass \( m_{n,N} \) for mesons

By inspecting the values of \( m_{n,N} \) in Table V in relation to the criterion Eq. (4.1) we are able to infer that the chiral symmetry concerning the light quarks is valid (still effective) for the ground (first excited) state of \( n\bar{n} \) and \( nQ \) meson systems, while the symmetry will prove invalid from the \( N \)-th \((N \geq 2)\) excited hadrons.

4.2. Level structure of ground state mesons

In Table VI we have summarized the properties of ground state mesons in the light and/or heavy quark systems. It is remarkable that there appear new multiplets of the scalar and axial-vector mesons in the \( qQ \) and \( Qq \) systems and that in the \( q\bar{q} \) systems the two sets (Normal and Extra) of pseudo-scalar and of vector meson nonets exist. It is also to be noted that the \( \pi \) nonet \((\rho \text{ nonet})\) is assigned to the \( P_s^{(N)} \) \((V_{\mu}^{(N)}\) \) state, whose spin WF is much changed from that in NRQM. We call the new type of particles in the extended COQM (which have never appeared in NRQM) as “chiralons”. 

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*[Additional notes or questions can be added here]*
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### 4.3. Level structure of mesons in general

The mass of the ground and excited state mesons is given by

\[ M_N^2 = M_0^2 + N\Omega = m_N^{(1)} + m_N^{(2)}. \]  

(4.4)

Their quantum numbers are given in Table VII. Here it is to be noted that some chiralons have the “exotic” quantum numbers from the conventional NRQM viewpoint.

| (q̅q) | mass | Approx. Symm. | Spin WF | SU(3) | Meson Type |
|-------|------|--------------|---------|-------|------------|
| N=all | \(P_+(N,E) \otimes \{L, N\}\) | \(P = (-1)^{L+1}\) | \(C = (-1)^L\) | \(P_+, V_+\) |
| \(N = 0\) (and 1) | \(S_{(N,E)} \otimes \{L, N\}\) | \(P = (-1)^L\) | \(C = \pm(-1)^L\) | \(S_{(N,E)}, A_{(N,E)}\) |

| (q̅Q or Q̅q) | mass | Approx. Symm. | Spin WF | SU(3) | Meson Type |
|-------|------|--------------|---------|-------|------------|
| N=all | \(P_+(N,E) \otimes \{L, N\}\) | \(P = (-1)^{L+1}\) | \(C = (-1)^L\) | \(P_+, V_+\) |
| \(N = 0\) (and 1) | \(S_{(N,E)} \otimes \{L, N\}\) | \(P = (-1)^L\) | \(C = \pm(-1)^L\) | \(S_{(N,E)}, A_{(N,E)}\) |

Table VII. Level structure of Mesons in general

The schematic picture of meson spectroscopy is shown in Fig. 1.

### §5. Level Structure of Baryons

The baryon WF Eq. (2.2) should be full-symmetric (except for the color freedom) under exchange of constituent quarks: The full-symmetric total WF in the extended scheme is obtained, in the following three ways, as a product of the sub-space WF with respective symmetric properties:

\[ |\rho F\sigma\rangle_S = |\rho\rangle_S |F\sigma\rangle_S \quad (a); \quad |\rho\rangle_\alpha |F\sigma\rangle_\alpha + |\rho\rangle_\beta |F\sigma\rangle_\beta \quad (b); \quad |F\rangle_A |\rho\sigma\rangle_A \quad (c); \]  

(5.1)

where \(|\rho\rangle_S\) is the full-symmetric \(\rho\)-spin space WF and so on. \(\rho \otimes \sigma = \gamma\) is the conventional two, \(\rho\) and \(\sigma\) spin, 2 by 2 matrix representation of the 4 by 4 Dirac matrix, and \(| \rangle_{\alpha(\beta),A}\) mean the \(\alpha(\beta)\)-type partial symmetric and full anti-symmetric subspace WF, respectively. The intrinsic parity operation is given by \(\hat{P} = \Pi_{i=1}^3 \gamma_4^{(i)}\), that is, the
parity of \((E^{(+)}, G^{(+)}, F^{(+)})\) BW spinors are \((+, -, +)\) and those of \((E^{(-)}, G^{(-)}, F^{(-)})\) BW spinors are \((-+, +, -)\). The symmetry properties of ground state light-quark baryon WF and their level structures thus determined are summarized in Table VIII.

| \(W^{(+)}\) | spin-flavor wave function | \(B^{\mathbb{D}}\) | static | \(SU(6)\) |
|---|---|---|---|---|
| \(E^{(+)}\) : | \(|\rho\rangle S |F\sigma\rangle S = |\rho\rangle S |F\sigma\rangle S \) | \(\Delta^\mathbb{D}_{1/2}\) | 10 \times 2 = 20 | 56 |
| \(|\rho\rangle S (|F\alpha\rangle S |\sigma\rangle _\alpha + |F\beta\rangle S |\sigma\rangle _\beta\) \) | \(N^\mathbb{D}_{1/2}\) | 8 \times 2 = 16 |
| \(G^{(+)}\) : | \(|\rho\rangle S |F\sigma\rangle _\alpha + |\rho\rangle S \) | \(|F\rangle S |\sigma\rangle _\alpha\) | \(\Delta^\mathbb{D}_{1/2}\) | 10 \times 2 = 20 | 70 |
| \(|\rho\rangle S \) | \(|F\rangle S \) | \(|\sigma\rangle _\alpha\) \) | \(|\sigma\rangle _\beta\) \) | \(N^\mathbb{D}_{1/2}\) | 8 \times 2 = 16 |
| \(|\alpha\rangle \) | \(|\beta\rangle \) | \(|\alpha\rangle \) | \(|\beta\rangle \) | \(|\alpha\rangle \) | \(|\beta\rangle \) | \(A_{1/2}^\mathbb{D}\) |
| \(|\alpha\rangle \) | \(|\beta\rangle \) | \(|\alpha\rangle \) | \(|\beta\rangle \) | \(|\alpha\rangle \) | \(|\beta\rangle \) | \(N^\mathbb{D}_{1/2}\) | 8 \times 2 = 16 |

Here it is remarkable that there appear chiralons in the ground states. That is, the extra positive parity \(56^\mathbb{D}\)-multiplet of the static \(SU(6)\) and the extra negative parity \(70^\mathbb{D}\)-multiplet of the \(SU(6)\) in the low mass region. It is also to be noted that the chiralons in the first excited states are expected to exist. The above consideration on the light-quark baryons are extended directly to the general light and/or heavy quark baryon systems: The chiralons are expected to exist also in the \(qqQ\) and \(qQQ\)-baryons, while no chiralons in the \(QQQQ\) system.
§6. Experimental Candidates for Chiral Particles

In our level-classification scheme a series of new type of multiplets of the particles, chiralons, are predicted to exist in the ground and the first excited states of \( q\bar{q} \) and \( q\bar{Q} \) or \( Q\bar{q} \) meson systems and of \( qqq \), \( qqQ \) and \( qQQ \)-baryon systems. Presently we can give only a few experimental candidates or indications for them:

\( (q\bar{q}-\text{mesons}) \) One of the most important candidates is the scalar \( \sigma \) nonet to be assigned as \( S^{(N)}(1S_0) \) : \( [\sigma(600), \kappa(900), a_0(980), f_0(980)] \). The existence of \( \sigma(600) \) seems to be established through the analyses of, especially, \( \pi\pi \)-production processes. A firm experimental evidence \( \square \) for \( \kappa(800–900) \) through the decay process\( \square \( D^+ \to K^-\pi^+\pi^+ \) was reported at this conference.

In our scheme respective two sets of \( P_s \)- and of \( V_\mu \)-nonets, to be assigned as \( P_s^{(N,E)}(1S_0) \) and \( V_\mu^{(N,E)}(3S_1) \), are to exist: Out of the five vector mesons (stressed\( \square \) as problems with vector mesons, \( [\rho'(1450), \rho'(1700), \omega'(1420), \omega'(1600), \phi(1690)] \), the lower mass \( \rho'(1450) \) and \( \omega'(1420) \), and the \( \phi(1690) \) are naturally able to be assigned as the members of \( V_\mu^{(E)} \)-nonets;

Out of the three established \( \eta, [\eta(1295), \eta(1420), \eta(1460)] \) at least one extra, plausibly \( \eta(1295) \) with the lowest mass, may belong to \( P_s^{(E)}(1S_0) \) nonet.

Recently the existence of two “exotic” particles \( \pi_1(1400) \) and \( \pi_1(1600) \) with \( J^{PC} = 1^{-+} \) and \( I = 1 \), observed\( \square \) in the \( \pi\eta, \rho\pi \) and other channels, is attracting strong interests among us. These exotic particles with a mass around 1.5GeV may be naturally assigned as the first excited states \( S^{(E)}(1P_1) \) and \( A_\mu^{(E)}(3P_1) \) of the chiralons.

\( (q\bar{Q} \text{ or } Q\bar{q}-\text{mesons}) \) At this conference some experimental indication for existence of the two chiralons in \( D \)- and \( B \)-meson systems obtained through the \( \Upsilon(4S) \) or \( Z^0 \) decay process, were reported, respectively,

\[ D^+_I = A_\mu^{(3S_1)}, \quad J^P = 1^+ \quad \text{in} \quad D^+_I \to D^* + \pi \]

\[ B^+_I = S^{(1S_0)}, \quad J^P = 0^+ \quad \text{in} \quad B^+_I \to B + \pi, \]

by Yamada K\( \square \) and Ishida M\( \square \).

\( (qqq \text{-baryons}) \) The two facts have been a longstanding problem that the Roper resonance \( N(1440)_{1/2^+} \) is too light to be assigned as radial excitation of \( N(939) \) and that \( A(1405)_{1/2^-} \) is too light as the \( L = 1 \) excited state of \( A(1116) \). In our new scheme they are reasonably assigned to the members of ground state chiralons with \[ SU(6), \quad SU(3), \quad J^P, \]

\( N(1440)_{1/2^+} = F(56, 8, 1/2^+) \), \( A(1405)_{1/2^-} = G(70, 1, 1/2^-) \).

The particle \( \Delta(1600)_{1/2^+} \) which is lighter than \( \Delta(1620)_{1/2^-} \) may also belong to the extra \( 56 \) of the ground state chiralons. This situation is shown in Table IX.

| SU(6) | SU(3), \( J^P \) | SU(3), \( J^P \) |
|---|---|---|
| 56 | 8 \( \frac{1}{2}^+ \) | \( N(939), A(1116), \Sigma(1192), \Xi(1318) \) |
| 56' | 8 \( \frac{1}{2}^+ \) | \( N(1440), \Sigma(1660) \) |
| 70 | 1 \( \frac{1}{2}^- \) | \( A(1405), \Delta(1620) \) |

Table IX. Assignment of \( qqq \)-baryons: The baryons in the boxes are candidates of chiralons.
§7. Concluding Remarks

I have presented an attempt for Level-classification scheme unifying the seemingly contradictory two viewpoints: Non-relativistic one with $LS$-symmetry and Relativistic one with Chiral symmetry.

As results, I have predicted the existence of New Chiral Particles in the lower mass regions “Chiralons”, which had never been appeared in NRQM.

We have several good candidates for chiralons, for example, 

\[
\sigma\text{-nonet } \{ \sigma(600), \kappa(800), a_0(980), f_0(980) \} \text{ as “Relativistic” } S\text{-wave states of } (q\bar{q}).
\]

\[
\pi_1(1400), \pi_1(1600), (1^{-+}) ; \text{ as “Relativistic” } P\text{-wave states of } (q\bar{q}).
\]

Roper resonance $N(1440)_{1/2^+}$ and SU(3) singlet $\Lambda(1405)_{1/2^-}$; as “Relativistic” $S$-wave states of $(qqq)$.

Further search, both experimental and theoretical, for chiralons is necessary and important.

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