Singular-optical coloring of regularly scattered white light

Oleg V. Angelsky*, Peter V. Polyanskii*, and Steen G. Hanson#

*Department of Correlation Optics, Chernivtsi National University, Chernivtsi 58012, Ukraine
# Optics and Plasma Research Department, Risø National Laboratory, P.O. Box 49, DK-4000 Roskilde, Denmark
oleg@optical.chernivtsi.ua

Abstract: When the surface roughness is comparable with the wavelength of the probing radiation, the scattered field contains both the regular (forward-scattered) component of coherent nature and the diffusely scattered part. Coloring of the regular component of white light scattered by a colorless dielectric slab with a rough surface is considered as a peculiar effect of singular optics with zero (infinitely extended) interference fringes. To explain the observed alternation of colors with respect to the increasing depth of the surface roughness, we apply a model of transition layers associated with the surface roughness. By applying the chromascopic technique, it is shown that the modifications of the normalized spectrum of the forward-scattered white light can be interpreted as the effect of a quarter-wavelength (anti-reflecting) layer for some spectral component of a polychromatic probing beam.

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1. Introduction

Phase singularities in polychromatic (white-light) fields recently have become the subject of extensive studies [1-13] giving rise to numerous new concepts and experimental techniques. Most of the mentioned investigations are related to phase singularities in the complex amplitude of the field associated with some spectral (monochromatic) component of polychromatic radiation. Lately, correlation singular optics [14], i.e. a singular optics approach of non-uniformly polarized, partially spatially coherent and polychromatic fields have revealed phase singularities intrinsic to arbitrary complex parameter of an optical field. It is important that such singularities can occur, when the common phase singularities of the field’s complex amplitude are absent both in the complex beam as a whole and in some of its components [15-20]. This important extension of the subject of singular optics is in good agreement with Wolf’s methodology in optics for observable quantities [21].

In this paper, we consider the phenomenon of coloring of the regularly scattered component of a white-light probing beam passing a dielectric slab with a rough surface whose inhomogeneities are comparable with some wavelength portion of the probing radiation (i.e. partly, coloring of the image of a white-light source) from the point of view of the paradigm of singular optics. To be more precise, we show that the diffraction-induced or scattering-induced modifications of the normalized spectrum of the regular beam are governed by the phase singularity, viz. an amplitude zero with changing of the sign of the reflectivity of the transition layer, which can be related to the surface roughness. The applied model of transition layers with a “diluted” index of refraction naturally explains the alternation of the colors with respect to the roughness depth observed in practice. The proposed singular-optical model of the effect of coloring of the regularly scattered component of a white-light beam is proved by the use of Berry’s chromascopic technique. As an illustration, we demonstrate the fascinating effect of “blue Moon” and “red Moon” carried out by the use of samples of one-sided ground glass.

2. Basic observations and interpretation

Observing a white-light source through a colorless slightly rough surface obtained e.g. by one-sided grinding of a glass plate with corundum with a mean size of grains ~7 μm to 10 μm, one can notice surprisingly intense coloring of the source, which varies from turquoise to magenta. This effect cannot be explained as a result of selective absorption. So, coloring is rather pronounced, when a scatterer is positioned in free space, but disappears if the same
scatterer coincides with the source or with the image plane, or, generally, with the plane of any field-of-view diaphragm of the imaging system. The observed modifications of the normalized spectrum of the forward-scattered component of a polychromatic beam certainly originate from interference, thus being closely connected with the phenomena of singular optics [22].

The presence of the regular component in the scattered radiation implies that the heights of surface inhomogeneities, \( h \), are comparable with the wavelength of the probing beam, here the wavelengths of all spectral components of the polychromatic beam. Thus, the assumption of mutual incoherence of partial waves scattered by various surface inhomogeneities is violated, and the phase relations between such waves must be taken into account both for the forward-scattered and for the specularly reflected radiation even in the case of a white-light probing beam from an extended source, when both spatial and temporal coherence is extremely low.

Note that under fully developed light scattering, when the regular component is absent, one can observe another singular optical effect of coloring. Namely, if a rough surface with large surface inhomogeneities is illuminated by a white-light beam and the illuminated area is small enough to provide high spatial coherence, the scattered field manifests polychromatic (colored) speckles, and the phase singularities in the spectral components of such pattern can be diagnosed using edge diffraction [10] or interferometrically [12], by applying a chromascope [5-7] or an inverted chromascope [11].

It is instructive to consider the solution to the problem of diffraction of a polychromatic radiation at bleached (phase-only) relief holographic gratings. It is known [23] that the diffraction efficiency, \( \eta \), for the \( l \)-th diffraction order of a phase hologram with a harmonic relief (including the zero order, \( l = 0 \)) characterized by the amplitude transmittance,

\[
T_j = \exp \left[ iq \cos \left( \frac{2\pi}{p} x \right) \right] = \sum_{l=-\infty}^{\infty} i^l J_l(q) \exp \left( il \frac{2\pi}{p} x \right),
\]

equals the squared \( l \)-th order Bessel function of the first kind: \( \eta = \left| J_l(q) \right|^2 \). Here \( q = \frac{2\pi}{\lambda} \left( n - n_0 \right) H \) is the phase modulation, \( n \) and \( n_0 = 1 \) are the indices of refraction of the emulsion and air, respectively, and \( p \) being the grating’s period along the \( x \)-axis. It is not surprising that the governing parameter for the spectral changes for the forward-diffracted component of a polychromatic radiation contains the ratio \( H / \lambda \) for the phase-only holographic gratings. Consequently, any limited area of a rough surface can be represented as a set of harmonic gratings with an amplitude transmittance

\[
T_r = \exp \left[ i \sum_{m} q_m \cos \left( \frac{2\pi}{p_m} x \right) \right] = \prod_{n} \left[ \sum_{m=-\infty}^{\infty} i^l J_l(q_m) \exp \left( il \frac{2\pi}{p_m} x \right) \right],
\]

where \( q_m = \frac{2\pi}{\lambda} \left( n - n_0 \right) H \), so that one obtains for the forward-diffracted component in the first approximation:

\[
\eta_0 = \prod_{m} J_0^2 \left( q_m \right).
\]

Obviously, the vanishing of any factor in Eq. (3) for any \( \lambda \) leads to vanishing of the product as a whole for the specified wavelength. It means that for an arbitrary height distribution function, a phase singularity associated with the crossing of zero magnitude of the zero-order
Bessel function of the first kind certainly takes place for some wavelength of the polychromatic probing radiation and for the corresponding term of the expansion (2). When \( \eta_0 \) approaches zero (viz., undergoes a phase singularity and changes its sign crossing the corresponding amplitude zero) for any \( \lambda_i \), its magnitude also becomes small over a finite spectral domain adjacent to this wavelength. In this way, the entire normalized spectrum of the forward-scattered radiation is considerably modified, and the complementary color, \( \lambda_c \) (with respect to \( \lambda_i \), including the adjacent spectral domain will prevail in the observed image. Thus, one can observe more or less intense (though always mixed - not pure) colors of the forward-scattered radiation.

3. Surface roughness as a transition layer

For interpretation of the observed alternation of colors induced by scattering of a white-light beam with respect to increasing roughness, we apply the model of a rough surface as a transition layer with the “diluted” index of refraction [24], which extends the well-known analogy from light-scattering from particles and layers [25, 26].

The real height distribution function of the inhomogeneities characterizing the given rough surface is generally unknown. However, irrespective of the specific functional form of such distribution function, one can consider a surface roughness as an irregular transition layer with a “diluted” index of refraction, whose magnitude is the geometrical mean of the indices of refraction of glass and air [27, 28]. Of course, the effective thickness of this layer, \( H \), depends on the real height distribution function. As a result, the analysis of the spectral modifications of the forward-scattered component of a polychromatic radiation passing a rough surface is reduced to the problem of matching of impedances of three media [28], viz., in the context of the optical problem, matching of the refraction indices of a glass \( n_1 \), a transition layer \( n_2 = \sqrt{n_1 n_3} \), and the environment \( n_3 \). If the optical thickness of the transition layer, \( n_2 H \), equals \( \lambda/4 \), \( \lambda \) being a wavelength in the media with a refraction index \( n_2 \), for some spectral component of the probing beam, this layer acts similar to an anti-reflection coating for this component, while under the assumed relation between the indices of refraction of the three media, the waves reflected from two boundaries of the transition layer are in opposite phases and interfere destructively. This certainly happens for some wavelength due to the condition \( \lambda < H \) for all spectral components. As a result, this spectral component and its spectral vicinity will prevail in the forward-scattered light.

Proceeding from this model for determining the color of the forward-scattered component of a white-light probing beam, one must first compute the relative intensity of the back-scattered (specularly reflected) light, as a function of the wavelength [28]:

\[
\frac{I_r}{I_i} = 4 \left[ \frac{1 - \sqrt{n_1}}{1 + \sqrt{n_1}} \right]^2 \sin^2 \left[ \frac{\pi \lambda}{2 \left( \frac{\lambda_i}{\lambda_c} - 1 \right)} \right],
\]

where \( I_r \) and \( I_i \) are the intensities of the reflected and the incident beams, respectively, \( \lambda_i \) is the specified wavelength of the incident beam within the spectral range of the probing radiation, and \( \lambda_c \) is the wavelength, the amplitude of which vanishes for the reflected radiation. Then the relative intensity of the forward-scattered component at the same wavelength \( \lambda_i \) is determined by the difference:

\[
\frac{I_f}{I_i} = 1 - \frac{I_r}{I_i}.
\]
In writing Eq. (5), we neglected light scattering in all other directions. However, such approximation is reliably justified for small heights of the roughness; cf. the estimations of the effective thickness of the transition layer below.

To illustrate the sequence of colors following from the model of transition layers, we apply the novel technique of chromoscopic processing of colored optical fields introduced by Berry [5, 6] and first experimentally implemented by Leach and Padgett [7] for observing the chromatic effects near an isolated white-light vortex.

Following [5, 6], to reveal the colors, the RGB values of the tested field are scaled to isoluminance by the transformation:

\[
\begin{pmatrix} R \\ G \\ B \end{pmatrix} \Rightarrow \begin{pmatrix} R \\ G \\ B \end{pmatrix} / \max (R, G, B).
\]

This procedure preserves the ratios between the three RGB values while making the strongest one equal to unity. The main difference between our approach and the one realized in [5-7] is the following. The authors of papers [5-7] apply the chromoscopic technique to the non-uniform “isolated” amplitude zero of the spectral complex amplitude, which varies linearly with the position of the point of observation and with wavenumber over the visible range [11]. In contrast, we implement the chromoscopic processing of a uniform color image of a white-light source formed by the forward-scattered component of radiation. We apply the chromoscopic processing following Eq. (6) both to the specularly reflected and to the forward-scattered components of the white-light probing beam.

Figure 1 illustrates the colors of the forward-scattered component (fragments a, b, c) and the colors of the back-scattered (specularly reflected) component (fragments d, e, f) for the cases of vanishing of blue \( \lambda_b = 435.8 \text{ nm} \) (Fig. 1, fragments a and d), or green \( \lambda_g = 546.1 \text{ nm} \) (Fig. 1, fragments b and e), or red \( \lambda_r = 700 \text{ nm} \) (Fig. 1, fragments c and f). The pairs of fragments a and d, b and e, and c and f correspond to the effective depths of the transition layer of 88.36 nm, 110.73 nm, and 141.93 nm, respectively, which are close to 0.1 of the mean diameter of the corundum assumed to be used for obtaining the color effects (\( \sim 10 \mu \text{m} \)). The results of simulation are in agreement with the alternation of the colors observed experimentally. That is, blue shift takes place for smaller depths of the transition layer, and reddening of the forward-scattered light is observed for larger depths of this layer. It is obvious that the inverse sequence of colors is observed in the specularly reflected light.
Fig. 1. Chromascopic simulation of the spectral changes of the forward-scattered component of a white light induced by a colorless glass rough surface (n_1=1.52, n_2=1.233, and n_3=1) following the model of the transition layer.

Let us formulate some precautions concerning the evaluation of the results represented in Fig. 1.

1. Simulation is performed for a discrete set of spectral components, while in practice one operates with a continuous spectrum. That is why the represented data are only of instructive nature: real colors are strongly dependent on the actual spectral density function of the source, so that one observes different colors induced by a given sample of a rough surface illuminated by sources with different color temperatures. Nevertheless, the general tendency (blue shift to the reddening of the forward-scattered component) is truly predicted by the model of transition layers.

2. Comparing the upper and the lower rows of Fig. 1, an inexperienced observer can conclude that the intensity of colors [29] in the specularly reflected radiation is much higher than in the forward-scattered component. However, the apparent higher intensity of colors of the specularly reflected component is the result of the normalization procedure, cf. Eq. (6). Therefore, one must take into account that the colored specularly reflected component is much lower in intensity than the forward-scattered one. This follows from the fact that for a fixed ratio...
\( H/\lambda \), the forward-scattered radiation is governed by the multiplier \( \pi(n - n_0) \) (see Section 2), while the specularly reflected component of the normally incident beam is governed by the multiplier \( 4\pi \), so that the effective depth of the transition layer in reflection exceeds its effective depth in transmission (for glass) by almost one order of magnitude. As a consequence, the relative intensity of the specularly reflected radiation is much lower (by two orders of magnitude, approximately), than the intensity of the forward-scattered component. It is evident that a surface, which can be regarded as slightly rough for transmitted radiation, cannot be slightly rough for the reflected one. This is the reason, why the possibility for observation of the colored beam specularly reflected from a rough surface was questioned earlier [30].

4. Experiments

Proceeding from the conceptual background represented in Section 3, one expects that the effect of coloring of the image of a white-light source formed by a beam scattered at a slightly rough surface can be observed under controlled conditions using the samples of rough surfaces prepared by a one-sided mechanical grinding of a dielectric slab.

We have performed a set of observations, the typical example of which is represented in Fig. 2. This figure illustrates the scattering-induced spectral modifications in the forward-scattered component of polychromatic light, viz. in the image of the Moon observed without (fragment a) and with (fragments b and c) the samples of a ground glass positioned in front of the camera’s aperture. These photos have been obtained in Chernivtsi, in March 21, 2006, from 5.00 to 5.10 a.m. The samples were prepared by grinding the glass with corundum with a mean size of grains of 10 \( \mu \)m. Depending on the strength of roughness, one observes a blue or a red Moon. Note, any software color correction for the fragments of Fig. 2 was not performed; only the brightness of the fragments (b) and (c) was increased. When the strength of roughness changes rapidly over the specified area of a surface, one can observe a blue-to-red shift of the perceived color of the Moon, without intense green in between these colors. The explanation of this interesting circumstance is the same as given by Berry [6] for the case of colored phase singularities in polychromatic speckle fields as well as for the sequence of colors in the natural rainbow. Namely, prevailing green requires that both the red and violet domains of the spectrum are suppressed simultaneously. But even in this case, an observer perceives a sensation of “unsaturated white”, rather than of intense green [6], although green really dominates in the resulting (modified) spectrum. The main contributor in estimation of colors pertains to subjective sensation.

![Fig. 2. Photos of a natural Moon (a), a blue Moon (b), and a red Moon (c).](image-url)
The photos in Fig. 2 show that the enigmatic [31] effect of “a blue Moon” (as well as “a blue Sun”) can be observed by any interested observer. To do this, one must prepare properly ground samples of glass through which to observe the Moon. Needless to say, the observed coloring is here brought about by the use of colorless glass.

We also observed the scattering-induced spectral changes in a white-light beam specularly reflected from a slightly rough surface for very large angles of incidence and reflection. Note, to perform such an observation one must prepare such a fine roughness (“hoarfrost” [30]), which does not provide the coloring considered above for the forward-scattered component in the transmitted radiation. A detailed theory and experimental results will be published elsewhere. Here, we only specify the sequence of colors of the specular component, which is observed as the angle of incidence of a white-light probing beam decreases gradually from the maximal one, i.e. 90 deg: white-yellow-orange-red-crimson-blue-yellow-orange-red, and so on. In such a manner, changing synchronically the angles of incidence and observation, we were in a position to observe up to four sequences of the reflected “rainbow”.

Another observation verifying the introduced model of the scattering-induced spectral modifications is seen when using an immersion liquid with a refraction index \( n_i \) such that \( n_i < n_r < n_l \). If the immersion rapidly dries out, one observes the alternation of colors in the forward-scattered component of a white-light beam passing the sample as predicted in Section 3, here observed in real time.

5. Conclusions

We have studied the effect of interference coloring of the regular component of a polychromatic (white-light) beam scattered off a rough surface with height inhomogeneities comparable with (but slightly less than) any wavelength of the spectral components of the probing beam. The main result of our study consists in substantiation of the model of the transition layer with a “diluted” index of refraction that provides prevailing transmission of the radiation with some specific wavelength, for which the back-scattering (specular reflection) vanishes. This model leads to a sequence of colors of the image of a white-light source, which is observed in practice. The effect of interference coloring induced by light scattering at slightly rough surfaces will facilitate measurement of the effective thickness of the transition layer. This can be found from the spectral modifications of the regularly scattered component and compared with results derived from contact profilometry and/or atomic force microscopy. These results will be published elsewhere.

In our opinion, the represented results are of interest not only for opticians, but also for the readers involved in astrophysics, meteorology, quantum mechanics, nanotechnologies, and biomedical diagnostics, due to the universal character of scattering. Observing the changeable coloring can be applied for practical control of growing of thin films to modeling of the spectral modification of cosmological radiation.