Research on the Establishment of Robot Workpiece Coordinate System and Its Approximation Method

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Abstract. This paper studies the coordinate system of the KUKA robot workpiece. According to the mathematical model of the robot system workpiece coordinate system, with the help of the CATIA software, the workpiece coordinate system is established. Through the solutions of the direct and inverse kinematics problems of the robot, the method of approaching the workpiece coordinate system is obtained. By setting the angles of A1, A2 and A3, the robot end actuator can reach the corresponding position of the workpiece coordinate system, and by setting the angles of A4, A5 and A6, the robot end actuator and the workpiece coordinate system can be adjusted to form the attitude relationship. Finally, the feasibility of the above method is verified by applying the program of KUKA robot to cut automobile ceiling panel. The results show that the robot workpiece coordinate system can be established offline by using 3D CAD software, by adjusting the angle of six axes of the robot, the robot end actuator can reach the designated position of the workpiece coordinate system and get the required attitude relationship.

Keywords: KUKA robot; workpiece coordinate system; robot pose; automotive interior cutting.

1. Introduction

In order to make KUKA robot complete a task, the establishment of world coordinate system, robot coordinate system, flange coordinate system, tool coordinate system, and the workpiece coordinate system are required [1], all of which belong to the Cartesian coordinate system. The robot user program is to control the robot tool coordinate system, so that it moves on the workpiece coordinate system in accordance with the scheduled trajectory and completes various tasks. As shown in Fig.1, the robot coordinate system is the base coordinate system, which is established on the robot base; the world coordinate system is usually set to coincide with the robot coordinate system; the flange coordinate system is established on the sixth-axis end face; the tool coordinate system is the position where the robot end actuator in the flange coordinate system, the tool coordinate system can be calibrated after the end actuator is selected; the workpiece coordinate system is established on the workpiece[2], which is determined by the users. The KUKA robot carries high-pressure water to cut automobile interior parts which is a complicated work[3], the plane where the cutting task lies is often located at any position in space, in order to help programming and ensure the cutting quality, size and shape, the XOY plane of workpiece coordinate system should coincide with the cutting task plane, the origin is best set at the
starting point of cutting, and the robot end actuator must be perpendicular to the cutting task plane of
the workpiece [4]. Therefore, the establishment of workpiece coordinate system and robot end actuator
approaches the workpiece coordinate system during teaching programming; and they are the key
problems to be solved when programming user programs [5].

In the system motion planning field of robot, for example, multi-axis coordinated control [6],
trajectory tracking control, tool coordinate system calibration, etc., many scientific research results have
been published. There is little systematic research for the establishment and approximation method of
workpiece coordinate system in the user program [7], which is also the working background of this
paper. Namely the definition based on workpiece coordinate system of robot system, the workpiece
coordinate system is established offline with CATIA software; the method of approaching workpiece
coordinate system is obtained by analyzing and solving the forward and inverse problems of robot
kinematics.

Fig.1 KUKA robot coordinate system

2. Establishment of Workpiece Coordinate System Based on 3D Drawing Software
The KUKA robot system defines workpiece coordinate system as BASE_DATA[], and the X, Y, Z, A,
B, and C value of BASE_DATA[] are obtained by some way, then the workpiece coordinate system
can be established and defined. At present, the product design is made mostly based on 3D software
(such as CATIA software, etc.), moreover, the mathematical model of product is formed. The
mathematical model of receiving model can be generated in accordance with mathematical model of
product. According to the actual positional relationship between the receiving mold and the robot world
coordinate system, a virtual robot world coordinate system can be established. This virtual robot world
coordinate system is translated and rotated, and finally its XOY plane is placed on the plane where
product mathematical model need to be processed, and the origin is placed at the cutting starting point.
By recording the data of translation and rotation, the X, Y, Z, A, B, and C value of workpiece coordinate
system BASE_DATA[] can be established with the 3D software, then the workpiece coordinate system
can be established.

3. Approximation of Workpiece Coordinate System
After the workpiece coordinate system is established, it is necessary to adjust the rotation angle of each
axis of robot in the teaching programming process, so that the robot end actuator reaches a certain
position of the established workpiece coordinate system, and forms a certain attitude relationship with
the workpiece coordinate system, namely pose, in this way, the task can be completed, this process can
be regarded as the approach of the robot end actuator to the workpiece coordinate system. The KUKA
robot cuts automotive interior; the water gun barrel (end actuator) is required to be adjusted on the XOY
plane perpendicular to the workpiece coordinate system, the center line of the barrel coincides with the
cutting start point, so as to ensure the cutting size and quality.

3.1. Description of pose matrix
The position and attitude of rigid body can be determined by the relative relationship between any point
on it and the coordinate system passing through this point relative to the reference coordinate
system. Assume that there is an object A, and a point $a_d$ on it is selected as a benchmark. The
coordinate system \( S_d(o_d-x_d y_d z_d) \) is set on the \( o_d \). Another reference coordinate system \( S_o(o_o-x_o y_o z_o) \) is selected, as shown in Fig.2. Therefore, the space position and attitude of object A can be determined by the relationship of the vector \( P_{oA} (= o_o o_d) \) and the vector basis \( i_d, j_d, k_d \) of \( S_d \) relative to \( S_o \).

![Fig.2 pose determination of rigid body A](image)

Namely:

\[
P_{oA}^0 = (O_o O_d)^T = \begin{bmatrix} x_{oA} \\ y_{oA} \\ z_{oA} \end{bmatrix} = \begin{bmatrix} P_{o0} \\ P_{00} \end{bmatrix}
\]

(1)

And

\[
R_d^0 = \begin{bmatrix} i_{d0} & j_{d0} & k_{d0} \\ i_{o0} & j_{o0} & k_{o0} \\ i_{e0} & j_{e0} & k_{e0} \end{bmatrix} = \begin{bmatrix} \cos i_{i0} & \cos j_{i0} & \cos k_{i0} \\ \cos i_{00} & \cos j_{00} & \cos k_{00} \\ \cos i_{k0} & \cos j_{k0} & \cos k_{k0} \end{bmatrix}
\]

(2)

In the formula, \( P_{oA}^0 \) - the representation of radial direction of \( O_d \) point in the coordinate system \( S_o \), it is \( 3 \times 1 \) array (position array); \( R_d^0 \) - the representation of coordinate system \( S_d \) in the coordinate system \( S_o \), it is \( 3 \times 3 \) square array (attitude array). In the formula, \( i_{d0} \Lambda \Lambda j_{d0} \Lambda \Lambda k_{d0} \Lambda \Lambda \) represent the projection of the base vector \( i_d, j_d, k_d \) in the \( S_o \) along \( i_0(x_o), j_0(y_o), k_0(z_o) \), respectively; \( \cos i_{i0} \Lambda \Lambda \cos j_{i0} \Lambda \Lambda \cos k_{i0} \Lambda \Lambda \) represent the angle cosine of the base vector \( i_d \) and \( i_0, j_d \) and \( j_0, k_d \) and \( k_0 \), respectively.

For convenience, the \( 4 \times 4 \) square matrix can be used to represent the position and attitude at the same time, it is recorded as \( T_d^0 \), it is called as the pose matrix, namely:

\[
T_d^0 = \begin{bmatrix} R_d^0 & P_{oA}^0 \\ 000 & 1 \end{bmatrix}
\]

(3)

3.2. Pose matrix \( T_i^{-1} \) between two rods

The robot is a multi-rod system, and the pose matrix between two rods is the basis for obtaining the end pose matrix of robot. The pose matrix between two rods depends on the structural parameters, motion forms and motion parameters between two rods, and the geometric models established with these parameters in different orders.

The two adjacent rods connected by the rotary pair are taken (as shown in Fig.3), one is the \( i-1 \) rod \( (L_{i-1}) \), the other is the \( i \) rod \( (L_i) \). The former is close to the base, and the latter is close to the tool. The rotary pair that connects the two rods is called the joint, the number is: the joint of \( L_{i-1} \) and \( L_i \) is \( i \) joint; the joint of \( L_i \) and \( L_{i+1} \) is \( i+1 \) joint. The fixed coordinate \( S_i \) of connecting rod \( L_i \) can be formulated in two different ways, one is that the axis \( z_i \) of \( S_i \) is placed on the rotation axis of \( i \)
joint, at this time, the origin \( o_i \) of \( S_i \) is on the axis of \( i \) joint, namely the coordinate system \( S_i \) is placed on the joint of the rod \( L_i \) close to the base, so it is called the forward fixed coordinate system.

![Fig.3 two rods connected by rotary pair](image)

The concept of pose matrix and the definition of multi-rod pose matrix are extended to each rotation axis of robot; namely the pose relationship of each axis joint can be calculated in accordance with the structural parameters of the robot. Since the structure of each axis joint of robot is concatenation, the
relative coordinate system among them is also concatenation, so the pose matrix of the robot end relative to the robot base can be obtained from the following formula:

\[ T_n^0 = T_1^0 T_2^1 \cdots T_{n-1}^{n-1} \]  

(5)

3.4. End pose matrix of KUKA robot

KUKA robot is a robot with six degree of freedom joint, its structural parameters and joint variables are shown in Table.1. It can be obtained that the end pose matrix is

\[
T_6^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5 = \begin{bmatrix}
   n_x & o_x & a_x & p_x \\
   n_y & o_y & a_y & p_y \\
   0 & 0 & 0 & 1
\end{bmatrix}
\]

(6)

In the formula:

\[
\begin{align*}
n_x &= c_1 \left( c_{23} \left( c_4 c_5 c_6 - s_4 s_6 \right) - s_{23} s_5 s_6 \right) + s_1 \left( s_4 c_5 c_6 + c_4 s_6 \right) \\
n_y &= s_1 \left( c_{23} \left( c_4 c_5 c_6 - s_4 s_6 \right) - s_{23} s_5 s_6 \right) - c_1 \left( s_4 c_5 c_6 + c_4 s_6 \right) \\
n_z &= -s_{23} \left( c_4 c_5 c_6 - s_4 s_6 \right) - c_{23} s_5 c_6 \\
o_x &= c_1 \left( c_{23} \left( -c_4 c_5 s_6 - s_4 c_6 \right) - s_{23} s_5 s_6 \right) + s_1 \left( c_4 c_5 - s_4 c_6 \right) \\
o_y &= s_1 \left( c_{23} \left( -c_4 c_5 s_6 - s_4 c_6 \right) - s_{23} s_5 s_6 \right) + c_1 \left( c_4 c_5 - s_4 c_6 \right) \\
o_z &= -s_{23} \left( c_4 c_5 s_6 - s_4 c_6 \right) + c_{23} s_5 s_6 \\
a_x &= -c_1 \left( c_{23} c_5 s_6 + s_{23} c_5 \right) + s_1 s_4 s_5 \\
a_y &= -s_1 \left( c_{23} c_4 s_5 + s_{23} c_5 \right) + c_1 s_4 s_5 \\
a_z &= -s_{23} c_4 s_5 - c_{23} c_5 \\
p_x &= c_1 \left( a_x c_2 + a_x c_3 - d_4 s_{23} \right) - d_3 s_1 \\
p_y &= s_1 \left( a_x c_2 + a_x c_3 - d_4 s_{23} \right) + d_3 c_1 \\
p_z &= -a_3 s_{23} - a_2 s_2 - d_4 c_{23}
\end{align*}
\]

The positive solution, namely the structural parameters and joint variables of each rod are known, and the space position and pose of the end rod actuator is solved, they are called the positive problem of robot kinematics. The inverse solution is when knowing the requirements of a certain operation is met, the space position and attitude of the end actuator, as well as the structural parameters of each rod, the joint variables is solved, they are called as the inverse problem of robot kinematics.

| \(i\) | \(a_{i-1}\) | \(a_i\) | \(d_i\) | \(\theta_i\) |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | \(\theta_1\) |
| 2 | -90° | 0 | 0 | \(\theta_2\) |
| 3 | 0 | \(a_3\) | \(d_4\) | \(\theta_3\) |
| 4 | -90° | 0 | 0 | \(\theta_4\) |
| 5 | 90° | 0 | 0 | \(\theta_5\) |
| 6 | -90° | 0 | 0 | \(\theta_6\) |

3.5. Solve inverse solution of KUKA robot

If the pose matrix \(T_6^0\) of KUKA robot and the structural parameters of each rod are known, the inverse solution of pose matrix can be solved to obtain the joint variables, and it is the basis for the robot end actuator to approach the workpiece coordinate system.
\[
\begin{align*}
\theta_1 &= \tan^{-1} \frac{p_y}{p_y} - \tan^{-1} \left( \frac{d_3}{\pm \sqrt{d_3^2 - d_2^2}} \right) \\
\theta_2 &= \tan^{-1} \left( \frac{p_z(-a_1 - a_2 c_3) + a_3 d_3}{p_z(a_3^2 - d_4) - A(-a_1 - a_2 c_3)} \right) - \left( \tan^{-1} \frac{a_1}{d_4} - \tan^{-1} \frac{k}{\pm \sqrt{a_1^2 + d_4^2 - k^2}} \right) \\
\theta_3 &= \tan^{-1} \frac{a_1}{d_4} - \tan^{-1} \frac{k}{\pm \sqrt{a_1^2 + d_4^2 - k^2}} \\
\theta_4 &= \tan^{-1} \frac{-a_x c_1 + a_y c_2}{-a_x c_1 c_2 - a_y c_3 + a_z s_3} \\
\theta_5 &= \tan^{-1} \frac{-s_5}{c_5} \\
\theta_6 &= \tan^{-1} \frac{-s_6}{c_6}
\end{align*}
\]

It can be seen that only formula (7), (8) and (9) have \( \rho, p_y, p_z \), so they determine the space position of the origin \( O_6 \) of end frame. Formula (10) (11) and (12) have \( n_x, n_y, A \), so they determine the attitude of the end rod frame. Therefore, when the axes of the back three joints of the six-axis robot intersect at a point, the front and rear three joints have different functions. The front three joints and the members can be regarded as the position mechanism, and the rear three joints and the member can be regarded as the attitude mechanism.

Therefore, the robot end actuator can reach the corresponding position of workpiece coordinate system by setting the angles of \( A_1, A_2 \) and \( A_3 \), and the pose relationship between the end actuator and the workpiece coordinate system can be adjusted by setting the angles of \( A_4, A_5 \) and \( A_6 \).

4. Establishment of Workpiece Coordinate System and Approximation Verification

This paper takes KUKA robot cutting the ceiling interior panel visor hole program of Zhonghua car as an example, the workpiece coordinate system is established and each axis angle of the robot are adjusted and make the end actuator approach the established coordinate system.

The CATIA software is used to call the mathematical model of ceiling interior panel products, as shown in Fig.4-A; according to the actual size of the water cutting receiving mold, the product mathematical model generate the receiving mold mathematical model, as shown in Fig.4-b; according to the actual position relationship between the water cutting receiving mold and the robot base, the virtual robot world coordinate system is generated, as shown in Fig.4-c; the virtual robot world coordinate system is translated and rotated, so that the XOY surface of the robot world coordinate system coincides with the visor hole, and the origin and cutting starting point are located in the center of the hole, as shown in Fig.4-d.

The translation and rotation data of the coordinate system along the XYZ three coordinate system in the process of 4-c to 4-d is read, and the established workpiece coordinate system BASE\_DATA[*] is obtained. The specific data are as follows, \( a=-158.8^\circ, b=-117.4^\circ, x=1054.4, y=455.8, z=-1065.8 \).

Namely: \[ \text{BASE\_DATA[*].A=-158.8; BASE\_DATA[*].B=117.4; BASE\_DATA[*].C=0; BASE\_DATA[*].X=1054.4; BASE\_DATA[*].Y=455.8; BASE\_DATA[*].Z=-1065.8. \]
Fig. 4 use CATIA software to establish the workpiece coordinate system.

In the water cutting workshop, the ceiling interior receiving mold is installed on the workbench, the angles of robot A1, A2 and A3 are adjusted to make the robot end actuator (water gun) reach the position of the visor hole generally, and then the angles of A4, A5 and A6 are adjusted to make the end actuator vertical to the plane where the visor hole is located. Finally, $base = base\_ Data\[*\]$ is called to make the end actuator approach the workpiece coordinate system accurately.

As shown in Fig. 5, the each axis angle of the robot is adjusted when KUKA robot carries the high-pressure water cutting, $A1=35.9\degree, A2=-48.7\degree, A3=-42.9\degree, A4=212.4\degree, A5= 64.0\degree, A6=0.2\degree$, and the end actuator reaches the required pose, that is, it approaches the established workpiece coordinate system $Base\_ DATA[*]$.

Fig. 5 approximation of workpiece coordinate system

5. Conclusion
As can be seen from the work of this paper: (1) on the basis of definition of the workpiece coordinate system of robot system, the relevant functions of CATIA software are used, the tool mathematical model can be generated based on the product mathematical model; according to the actual installation relationship between the tool and the robot, the world coordinate system of virtual robot can be established, this coordinate system are translated and rotated, and the workpiece coordinate system $BASE\_ DATA[*]$ can be established; (2) through the analysis and solution of the robot kinematics
forward and inverse problems, the method of approaching the workpiece coordinate system is obtained, namely the robot end actuator reaches the corresponding position of the workpiece coordinate system by setting angle of A1 and A2, A3, the end actuator and workpiece coordinate system are adjusted to form attitude relationship by setting the angle of A4, A5, A6, and finally, $BASE=BASE\_DATA[*]$ is called to achieve accurate approximation of the workpiece coordinate system.

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