Effects of heterogeneous self-protection awareness on resource-epidemic coevolution dynamics

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Abstract

Recent studies have demonstrated that the allocation of individual resources has a significant influence on the dynamics of epidemic spreading. In the real scenario, individuals have a different level of awareness for self-protection when facing the outbreak of an epidemic. To investigate the effects of the heterogeneous self-awareness distribution on the epidemic dynamics, we propose a resource-epidemic coevolution model in this paper. We first study the effects of the heterogeneous distributions of node degree and self-awareness on the epidemic dynamics on artificial networks. Through extensive simulations, we find that the heterogeneity of self-awareness distribution suppresses the outbreak of an epidemic, and the heterogeneity of degree distribution enhances the epidemic spreading. Next, we study how the correlation between node degree and self-awareness affects the epidemic dynamics. The results reveal that when the correlation is positive, the heterogeneity of self-awareness restrains the epidemic spreading. While, when there is a significant negative correlation, strong heterogeneous or strong homogeneous distribution of the self-awareness is not conducive for disease suppression.

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We find an optimal heterogeneity of self-awareness, at which the disease can be suppressed to the most extent. Further research shows that the epidemic threshold increases monotonously when the correlation changes from most negative to most positive, and a critical value of the correlation coefficient is found. When the coefficient is below the critical value, an optimal heterogeneity of self-awareness exists; otherwise, the epidemic threshold decreases monotonously with the decline of the self-awareness heterogeneity. At last, we verify the results on four typical real-world networks and find that the results on the real-world networks are consistent with those on the artificial network.

**Keywords:** Coevolution dynamics, Epidemic spreading, Resource allocation, Self-protection awareness, Complex networks

1. Introduction

Resources, such as funds, medical and protective equipment, play a vital role in constraining or mitigating the outbreak of an epidemic. However, the pandemics always announced themselves with a sudden explosion of cases, inducing a severe shortage of public resources [1, 2]. Examples including the SARS coronavirus in 2003 [3] and Ebola virus in 2014 [4] etc. At the end of 2019, a new type of coronavirus called COVID-19 broke out in Wuhan, China, and it quickly spreads around the world. By the end of April 2020, more than three million cases worldwide have been officially reported [5]. The increasing demands for protection and treatment have led to a severe shortage of public resources [6].

Facing the shortage of public resources, the topic of optimal resource allocation in suppressing disease spreading has aroused extensive attention from varies communities [7, 8, 9, 10, 11, 12]. For example, Andrey et al. [8] solved the problem of optimal deployment of limited resources by studying the interplay between network topology and spreading dynamics. Based on a scalable dynamic message-passing approach [9], they got the optimal distribution of available resources and demonstrated the universality of the method on a variety of real-world examples. Preciado et al. [7] researched the problem of how to optimally allocate the vaccination resources on complex networks. Through a convex framework, they found the cost-optimal distribution of the resources. Nicholas et al. [13] developed a framework to find an optimal strategy of resource allocation to eliminate one of the epidemics when two competitive epidemics spreading on a bilayer network. Besides, Nowzari et al. [14] studied the problem of containing an initial epidemic outbreak under budget constraints based on the analysis of a generalized
epidemic model over arbitrary directed graphs with heterogeneous nodes. Chen et al. [15] studied the problem of optimally allocate the limited resources to minimize the prevalence. They solved the problem under the premise of the positive correlation between node degree and resource own by each node.

The previous works only considered the optimization of public resources and addressed the problem from a mathematical perspective. However, during the outbreak of epidemics, public resources such as medical staff, protective equipment are in a severe shortage. Usually, public resources are allocated preferentially to meet the needs of severe and critical patients [16]. The treatment and protection of mild or susceptible individuals mainly depend on the accumulation of individual resources and the support of resources among individuals. The topic that the influence of individual resource on the epidemic spreading has attracted wide attention in physic community [17, 18, 19]. For instance, Böttcher [17] considered that the healthy individuals could contribute resources during an outbreak of an epidemic. By studying the coevolution of the resource and disease, they found an “explosive” increase of infected nodes induced by resource constraints. Chen et al. [18] studied the interplay between resource allocation and disease spreading on top of multiplex networks, and found a hybrid phase transition. In this multiplex network framework, they further investigated the impact of preferential resource allocation on the dynamics of epidemic spreading [19].

In real scenario, the diffusion of information/awareness can change human behaviors, such as wearing masks or staying at home to reduce the frequency of face-to-face contact [20, 21]. The interplay between information/awareness diffusion and epidemic spreading is another topic that has inspired a wide range of research by scholars [22, 23, 24, 25]. For example, Granell et al. [26] studied the dynamical interplay between the epidemic spreading and information diffusion on top of multiplex networks. Funk et al. [27] studied the coevolution of information and disease on well-mixed populations and lattices, and found that in a well-mixed population, the information about the disease can suppress the disease. Kabir et al. [28] proposed a two-layer susceptible-infected-recovered/unaware-aware (SIR-UA) epidemic model to investigate the impact of awareness on epidemic spreading on top of different heterogeneous networks. Moreover, Kabir et al. [29] further studied the effects of awareness on the epidemic spreading based on a metapopulation model combined with a SIS-UA (susceptible-infected-susceptible-unaware-aware) epidemic model.

Information or awareness can also change the individuals’ willingness of resource donation [30]. In reality, the susceptible individuals would weigh whether to help others or protect themselves when they are aware of the disease. Since
individuals are in various circumstances and have different personalities, there is a heterogeneous distribution of awareness for self-protection. For example, a cautious person is more likely to reserve resources for self-protection than a generous person during an outbreak, or a person who has received help from others will have a stronger willingness to donate resources than others. There remains a question that how the heterogeneous distribution of awareness for self-protection influence the epidemic dynamics. To answer this question, a resource-epidemic coevolution model is proposed, which is based on the following assumption: Namely, each susceptible individual has both a probability of resource donation and a certain level of self-awareness. A larger self-awareness of an individual indicates a stronger sense of self-protection and a lower willingness of resource donation. Besides, we also consider that the susceptible individuals can perceive the disease from immediate neighbors. With the increase of infected neighbors, the susceptible individuals will reduce the probability of resource donation, since the donation behavior will lead to fewer resources for self-protection and a higher probability of been infected. Then the interplay between resource allocation and epidemic spreading is studied in our paper.

We first study the effects of degree and self-awareness heterogeneity on epidemic dynamics on the artificial networks. Through extensive Monte Carlo simulations, we find that the heterogeneity of degree distribution enhances the epidemic spreading, which is consistent with the classical results on scale-free networks [31, 32]. Besides, we find that the self-awareness heterogeneity suppresses the outbreak of an epidemic. Next, we study the influence of the correlation between node degree and self-awareness on the epidemic dynamics on artificial networks. We find that when there is a positive correlation between node degree and self-awareness, the heterogeneity of self-awareness restrains the outbreak of the epidemic. While, when there is a strong negative correlation, the epidemic threshold first increases and then decreases with the decline of the self-awareness heterogeneity. Furthermore, we find an optimal self-awareness distribution, at which the disease can be suppressed to the most extent. By exploring the relationship between the epidemic threshold and correlation coefficient, we reveal that the more positive correlation between node degree and self-awareness, the better the disease can be suppressed. Besides, a critical value of the correlation coefficient is found. When the coefficient is below the critical value, an optimal heterogeneity of self-awareness exists. In contrast, the epidemic threshold decreases monotonously with the decline of the self-awareness heterogeneity. At last, we verify the results on four typical real-world networks, and find that the results on the real-world networks are consistent with those on the artificial network.
2. Model descriptions

In this section, a resource-epidemic coevolution model that is named as the resource-based epidemiological susceptible-infected-susceptible model (r-SIS) is proposed.

2.1. Epidemic spreading model

For the epidemic spreading model, each node has two possible states: the infected (I) and the susceptible (S) state [33]. At any time step \( t \), each I-state node \( i \) can transmit the disease to its susceptible neighbors, at the same time it recoveries to S-state with the rate \( \mu_i(t) \), which depends on the resource quantity \( \omega_i(t) \) received from its healthy neighbors. We consider that the resource, such as funds and medical, can promote the recovery of the I-state individuals [34, 35]. Thus, the recovery rate \( \mu_i(t) \) is assumed to be proportional to resource quantity \( \omega_i(t) \) in this paper and is defined as:

\[
\mu_i(t) = 1 - (1 - \mu_0)^\varepsilon \omega_i(t),
\]

where \( \mu_0 \) is the spontaneous recovery rate that is independent of resource. Since the value of \( \mu_0 \) does not qualitatively affect the results [36], without loss of generality, it is set at a small value \( \mu_0 = 0.1 \) in this paper. Besides, resource wastes widely exist in the real scene of the medical system during the treatment process [37]. To mimic the phenomenon of resource wasting, a parameter \( \varepsilon \) is introduced to represent the resource utilization rate, which has been proved to not qualitatively affect the dynamical properties [18, 36]. Thus, without loss of generality, it is set to be \( \varepsilon = 0.6 \) in this paper. In the spreading process of the epidemic, every S-state node has a probability of being infected by its I-state neighbors. We consider that the healthy (S-state) nodes are the source of resources, they can both generate new resources and donate them to the I-state neighbors. If an S-state node chooses to donate its resources, it will have fewer resources for self-protection, which leads to a greater risk of being infected. Otherwise, if it refuses to donate resources for self-protection, the infection rate will reduce by a factor \( c \). As there is a heterogeneous distribution of self-awareness in populations, the infection rate varies from node to node. To reflect the relationship between the donation behavior and infection probability, we denote the basic infection rate as \( \beta \), and define the actual infection of node \( i \) as:

\[
\beta_i(t) = \begin{cases} 
\beta, & \text{if donating resource;} \\
c \beta, & \text{otherwise.}
\end{cases}
\]
According to the individual-based mean-field theory \[38, 39\], the dynamical process of the r-SIS model can be expressed as:

\[
\frac{d\rho_i(t)}{dt} = -\mu_i(t)\rho_i(t) + \beta_i(t)[1 - \rho_i(t)] \sum_{j=1}^{N} a_{ij}\rho_j(t),
\]

(3)

where \(a_{ij}\) is the element of adjacency matrix \(A\). If there is an edge between nodes \(i\) and \(j\), \(a_{ij} = 1\), otherwise, \(a_{ij} = 0\). Besides, to calculate the spreading size of the epidemic, a parameter \(\rho_i(t)\) is introduced to represent the probability that node \(i\) is in I-state. Thus we can calculate the fraction of infected nodes in a network of size \(N\) at time \(t\) by averaging overall \(N\) nodes:

\[
\rho(t) = \frac{1}{N} \sum_{i=1}^{N} \rho_i(t).
\]

(4)

At last, the prevalence of the epidemic in the stationary state is defined as \(\rho \equiv \rho(\infty)\).

### 2.2. Resource allocation model

As the statements above, the healthy individuals can generate and donate resources to support the recovery of their I-state neighbors during the outbreak of an epidemic. For the sake of simplicity, we assume that each susceptible node will generate one unit resource at each time step. Besides, the S-state nodes can perceive the severity of an outbreak from the state of infection in the neighborhood. The parameter \(m_i\) is defined to represent the amount of the I-state neighbors of node \(i\). Generally, the larger the value of \(m_i\), the lower probability of resource donation of node \(i\) \[40, 41\]. Besides, the parameter \(\alpha_i\) is defined to represent the awareness of self-protection for each node \(i\). A larger value of \(\alpha_i\) indicates more sensitive of node \(i\) to the disease, and a lower intention to donate resource. We consider that \(\alpha_i\) obeys the heterogeneous distribution \(g(\alpha) \sim \alpha^{-\gamma_\alpha}\), where \(\gamma_\alpha\) is the self-awareness exponent. Thus, the resource donation probability of a healthy node \(i\) is related to its intrinsic self-awareness and a total of \(m_i\) infected neighbors, which is defined as:

\[
q_i(t) = q_0(1 - \alpha_i)^{m_i(t)},
\]

(5)

where \(q_0\) is a basic donation probability. We consider that the resource of an S-state node will be distributed equally among neighbors.
Based on the above resource allocation scheme, the amount of resource that node \( i \) donates to node \( j \) at a time can be expressed as:

\[
\omega_{i\rightarrow j}(t) = q_i(t) \frac{1}{m_i(t)}. \tag{6}
\]

With the resource allocation scheme defined in Eq. (6), we can further define the resource quantity \( \omega_i(t) \), which is expressed as:

\[
\omega_i(t) = \sum_{j \in N_i} [1 - \rho_j(t)] \omega_{j\rightarrow i}(t)
= \sum_{j \in N_i} [1 - \rho_j(t)] \frac{q_j(t)}{m_j(t)}. \tag{7}
\]

where \( N_i \) represents the neighbor set of node \( i \), and the expression \( [1 - \rho_j(t)] \) represents the probability that a neighbor \( j \) is in S-state. Combining Eq. (6), we can get the expression of infection rate of any node \( i \) at time \( t \) as:

\[
\beta_i(t) = q_i(t) \beta + [1 - q_i(t)]c\beta. \tag{8}
\]

3. Simulation results

Although various theoretical methods such as the heterogeneous mean-field (HMF), quench mean-field (QMF) and dynamical message passing (DMP) approaches have been proposed to analyze both the single dynamical process [42] and the multiple coupled dynamical processes [25, 43], the nonlinearity of the model described in section 2 and the strong dynamic correlations make it infeasible to obtain precise theoretical solutions for epidemic size and threshold by utilizing the existing theoretical methods. Therefore, extensive Monte Carlo simulations are carried out to study the coevolution of resource allocation and epidemic spreading in this section. First, we study the impact of heterogeneous distributions of self-awareness and node degree on the epidemic dynamics, and then investigate the effects of correlation between node degree and self-awareness on the spreading dynamics on artificial networks through Monte Carlo simulations. At last, we will verify the results by conducting simulations on several typical real-world networks.

A specific simulation is carried out as follows [44]: during each time time interval \([t, t + \Delta t]\), each S-state node \( i \) changes to I-state in rate \( \beta_i \), which can be
defined as \[45\]

\[
\beta_i(t) = \lim_{\Delta t \to 0} \frac{P(S_i^{t+\Delta t} = I \text{ infected by } j|S_i^t = S, S_j^t = I)}{\Delta t},
\] (9)

where \(S_i^t\) is denoted as the state of node \(i\) at time \(t\), and \((S_i^{t+\Delta t} = I \text{ infected by } j)\) represents that node \(i\) is infected by an I-state neighbor \(j\) [46]. At the same time, the I-state node will recover to S-state with rate of \(\mu_i(t)\), which defines as

\[
\mu_i(t) = \lim_{\Delta t \to 0} \frac{P(S_i^{t+\Delta t} = S|S_i^t = I)}{\Delta t}.
\] (10)

The infection rate \(\beta_j(t)\) and recovery rate \(\mu_i(t)\) are dependent on the resource donation probability \(q_j(t)\) and resource quantity \(\omega_i(t)\) respectively. The process of resource allocation takes place with the propagation of disease. In synchronous updating, the \(\Delta t\) is finite, and the infection and recovery probability of node \(i\) is \(\tilde{\beta}_i = \beta_i \Delta t\), and \(\tilde{\mu}_i = \mu_i \Delta t\). According to Eqs. (9) and (10), the transition probabilities can be expressed as:

\[
\beta_i \Delta t = P(S_i^{t+\Delta t} = I \text{ infected by } j|S_i^t = S, S_j^t = I),
\] (11)

\[
\mu_i \Delta t = P(S_i^{t+\Delta t} = S|S_i^t = I).
\] (12)

At the end of each time step, the state of all nodes in the network update synchronously. To ensure that the dynamical processes enter a stationery, in which the prevalence fluctuates within a small range, each simulation will run a sufficiently long time. Besides, in order to avoid the influence of other factors on the results, without loss of generality, we set the coefficient \(c\) at a constant value \(c = 0.05\), such that if any healthy individual \(j\) chooses to reserve its resource, the probability that it is infected in one contact with an infected neighbor reduces to \(\beta_j = 0.05 \beta\).

3.1. Effects of heterogeneous self-awareness and degree distributions

In order to investigate the effects of heterogeneous distributions of self-awareness and node degree on the epidemic dynamics, we first generate networks with degree distribution \(P(k) \sim k^{-\gamma_D}\) using the uncorrelated configuration model (UCM) [47,48], where \(\gamma_D\) is the degree exponent. The maximum and minimum degree of the network are set to be \(k_{\text{max}} \sim \sqrt{N}\) and \(k_{\text{min}} = 3\) respectively, which assures no degree correlation of the network when \(N\) is sufficient large [49,50], and the mean
degree is set to be $\langle k \rangle = 8$. Then we generate a self-awareness sequence $\{\alpha_i\}_{i=1}^N$ according to the distribution $g(\alpha) \sim \alpha^{-\gamma_\alpha}$. The maximum and minimum values are set to be $\alpha_{\text{max}} \sim N^{1/2}$, $\alpha_{\text{min}} = 5$ respectively. To ensure $\alpha \in [0, 1]$, we rescale each value as $\alpha/\alpha_{\text{max}}$. At last, each node is assigned an value of self-awareness randomly.

In addition, we employ the susceptibility measure $[51]$ $\chi$ to numerically determine the epidemic threshold:

$$\chi = N \frac{\langle \rho^2 \rangle - \langle \rho \rangle^2}{\langle \rho \rangle},$$

where $\langle \cdots \rangle$ is the ensemble averaging. To obtain a reliable value of $\chi$, we perform at least $2 \times 10^3$ independent realizations on a specific network with fixed self-awareness distribution for each basic infection rate $\beta$. At the threshold $\beta_c$, the value of $\chi$ exhibits a maximum value. And then, by performing the simulations on 100 different networks, we can obtain the average value of $\beta_c$.

Fig. 1(a) displays the prevalence $\rho$ in the stationary state as a function of basic infection rate $\beta$ at different degree and awareness exponents. We can observe that the dynamic process converges to two possible stationary states: the completely healthy state when $\beta < \beta_c$, and nearly all infected state $\beta > \beta_c$, which indicates a first-order transition at $\beta_c$. As shown in Fig. 1(a), the epidemic threshold $\beta_c$ decreases with the heterogeneity of degree distribution, which is consistent with the epidemic outbreak on networks without self-awareness $[52]$. The phenomenon is induced by the hub nodes that exist on strong heterogeneous networks. When a given heterogeneity of degree distribution, the value of $\beta_c$ increases with the heterogeneity of self-awareness distribution, which is in contrast to the effects of degree heterogeneity. For instance, when $\gamma_D = 2.1$, the $\beta_c$ for $\gamma_\alpha = 2.1$ is larger than that for $\gamma_\alpha = 4.0$. Fig. 1(b) exhibits the value of relative threshold $\beta_c/\beta_c^0$ as a function of $\gamma_\alpha$ for three degree exponents $\gamma_D = 2.1$, $\gamma_D = 2.5$ and $\gamma_D = 4.0$ respectively. We can observe that the epidemic threshold $\beta_c$ decreases gradually with the increases of $\gamma_\alpha$, which reveals that the self-awareness heterogeneity restrains the epidemic spreading.

Next, we explain qualitatively the results above by exploring the time evolutions of the critical parameters when the basic infection rate is set to be $\beta = 0.04$. Fig 2(a) plots the time evolution of the average donation probability $\langle q \rangle$ (top pane) and average number of infected neighbors $\langle m \rangle$ for varies values of $\gamma_D$ and $\gamma_\alpha$ (bottom panel). It shows that when $\gamma_D = 2.1$, the donation probability $\langle q \rangle$ first decreases and then increases with time $t$ for $\gamma_\alpha = 2.1$ (see blue upper-circles). While, for $\gamma_\alpha = 4.0$ (see red circles), the value of $\langle q \rangle$ first decreases, and then
Figure 1: The influence of degree and awareness distributions on the spreading dynamics without correlations. (a) The prevalence $\rho$ in the stationary state versus the basic infection rate $\beta$ for $\gamma_D = 2.1$, $\gamma_\alpha = 2.1$ (red up-triangles), $\gamma_D = 2.1$, $\gamma_\alpha = 4.0$ (blue circles), $\gamma_D = 4.0$, $\gamma_\alpha = 2.1$ (orange squares), and $\gamma_D = 4.0$, $\gamma_\alpha = 4.0$ (yellow rhombus) respectively. (b) The relative epidemic threshold $\beta_c/\beta_c^0$ as a function of awareness exponent $\gamma_\alpha$ for degree exponent $\gamma_D = 2.1$ (blue circles), $\gamma_D = 2.5$ (orange squares), and $\gamma_D = 4.0$ (yellow right-triangles), where $\beta_c^0 \approx 0.041$ is the threshold at $\gamma_D = 4.0$, $\gamma_\alpha = 2.1$. Data are obtained by averaging over 500 independent simulations.
Figure 2: Plots of the critical parameters versus time $t$. (a) Top pane: time evolution of the average resource donation probability $\langle q \rangle$ for $\gamma_D = 2.1, \gamma_\alpha = 2.1$ (blue up-triangles), $\gamma_D = 2.1, \gamma_\alpha = 4.0$ (orange circles), $\gamma_D = 4.0, \gamma_\alpha = 2.1$ (yellow squares), and $\gamma_D = 4.0, \gamma_\alpha = 4.0$ (green snowflakes) respectively; Bottom pane: the corresponding time evolution of the average number of infected neighbors $\langle m \rangle$. (b) The average recovery rate $\langle \mu \rangle$ versus $t$. (c) The time evolution of average effective infection rate $\langle \lambda \rangle$. (d) The time evolution of the fraction of infected nodes $\rho(t)$. Data are obtained by averaging over 500 independent Monte Carlo simulations.
increases slightly in the middle time, at last, it declines with time $t$. This phenomenon can be qualitatively explained as follows: When there is a strong heterogeneity of the self-awareness distribution, for instance, $\gamma_\alpha = 2.1$, the network has a larger number of nodes with very low self-awareness, and more nodes with high self-awareness. Since the more nodes with large self-awareness means a lower willingness to donation resources, and a lower donation probability in the early stage, thus the value of $\langle q \rangle$ drops more abruptly than that for $\gamma_\alpha = 4.0$, which induces a larger number of infected neighbors $\langle m \rangle$, as less resource is donated from healthy nodes to their neighbors. This phenomenon can be verified by studying the bottom pane of Fig. 2(a). With less resource, the I-state nodes will recovery with a relatively lower recovery rate [see blue upper-triangles and red circles in Fig. 2(b)]. With the spread of the disease, more nodes with a very small value of $\alpha$ participate in the behavior of donating resources. The smaller value of $\alpha$ means a larger willingness to donate resource, which leads to a greater growth of $\langle q \rangle$ for $\gamma_\alpha = 2.1$ that $\gamma_\alpha = 4.0$ [see the blue upper-triangles and red circles in the top pane of Fig 2(a)]. Consequently, with the increase of donation probability of the entire network, the value of $\langle m \rangle$ decreases gradually, which leading to a slower decrease of $\langle \mu \rangle$ for $\gamma_\alpha = 2.1$ than $\gamma_\alpha = 4.0$ [see the blue upper-triangles and red circles in Fig 2(b)]. The relative larger recovery rate leads to a lower effective infection rate, which is defined as $\langle \lambda \rangle = \langle \beta \rangle / \langle \mu \rangle$, as shown in Fig. 2(c). Thus the prevalence $\rho(t)$ increases slower for $\gamma_\alpha = 2.1$ than $\gamma_\alpha = 4.0$, as shown in Fig 2(d). From the statement above, we can explain the reason why the heterogeneity of self-awareness distribution can suppress the outbreak of an epidemic.

When the degree heterogeneity of the network decreases, for instance, $\gamma_D = 4.0$, the donation probability for both $\langle q \rangle$ for $\gamma_\alpha = 2.1$ that $\gamma_\alpha = 4.0$ increases with time $t$ [see the yellow squares and green snowflakes in the top pane of Fig 2(a)], which leads to an increase of the recovery rate of the whole network, as shown in Fig. 2(b). Consequently, the effective infection rate of the network $\langle \lambda \rangle$ decreases gradually to a very small value with time $t$. Thus we see that the prevalence decreases gradually to zero [see Fig 2(d)]. From the statement above, we can also explain the phenomenon that the threshold $\beta_c$ decreases with an increase of $\lambda_D$.

### 3.2. Effects of degree-awareness correlations

In this section, we focus on how the correlation between the node degree and self-awareness affects the spreading dynamics. A network with a given correlation coefficient is built as follows:

- A network with degree distribution $P(k) \sim k^{-\gamma_D}$ is built by the steps in Section 3.1.
• A self-awareness sequence is generated from the distribution \( P(\alpha) \sim \alpha^{-\gamma_\alpha} \);

• Sorting the nodes of the network by the degree in ascending order, and then sorting the self-awareness sequence in ascending or descending order, respectively.

• Rearranging the order of each self-awareness value with a given probability \( \pi \), and then assigning each self-awareness value to the corresponding node.

According to the steps above, we can get a network with a given awareness-degree correlation. The correlation coefficient is:

\[
\sigma = 1 - \pi.
\] (14)

If the self-awareness sequence is sorted in ascending order, we can get a positive awareness-degree correlation with coefficient \( \sigma \); otherwise, we can get a negative awareness-degree correlation with coefficient \( \sigma \).

First of all, we study the case when there is a positive degree-awareness correlation, i.e., \( \sigma = 0.8 \). Figs. 3(a), (b) and (c) exhibit the prevalence \( \rho \) in the stationary as a function of \( \beta \) when degree exponent is \( \gamma_D = 2.1, \gamma_D = 2.5 \) and \( \gamma_D = 4.0 \), respectively. It shows that for a network with a fixed structure, the epidemic threshold \( \beta_c \) increases with self-awareness heterogeneity. To verify the results obtained in Figs. 3(a) to (c), we explore the relationship between \( \beta_c \) and the awareness exponent \( \gamma_\alpha \) in Fig. 3(d). Obviously, for each fixed \( \gamma_D \), the value of \( \beta_c \) decreases gradually with the \( \gamma_\alpha \), which suggests that when node degree and self-awareness correlated positively, the heterogeneity of self-awareness inhibits the epidemic spreading.

Next, we explore the case when there is a negative degree-awareness correlation. Figs. 4(a) to (c) display the value of \( \rho \) versus \( \beta \) for \( \gamma_D = 2.1, \gamma_D = 2.5 \) and \( \gamma_D = 4.0 \) when \( \sigma = -0.8 \) respectively. It shows that for a fixed network, for instance \( \gamma_D = 2.1 \), the epidemic threshold \( \beta_c \) first increases when \( \gamma_\alpha \) increase from \( \gamma_\alpha = 2.1 \) to \( \gamma_\alpha = 2.5 \), and then it decreases when \( \gamma_\alpha \) increases from \( \gamma_\alpha = 2.5 \) to \( \gamma_\alpha = 4.0 \). We next study systematically the effects of negative correlation between node degree and self-awareness on the spreading dynamics by exploring the relationship between threshold \( \beta_c \) and awareness exponent \( \gamma_\alpha \) in Fig. 4(d) for \( \gamma_D = 2.1 \) (green upper-triangles), \( \gamma_D = 2.5 \) (red circles), and \( \gamma_D = 4.0 \) (yellow squares) respectively. It shows that the threshold \( \beta_c \) first increases and then decreases with \( \gamma_\alpha \), and an optimal value \( \gamma_\alpha^{\text{opt}} \) (\( \gamma_\alpha^{\text{opt}} \) is around 2.6) exists, at which the value of \( \beta_c \) reaches maximum. This result can be qualitatively explained as follow: When the self-awareness heterogeneity is very strong, e.g., \( \gamma_\alpha = 2.1 \), there is
Figure 3: Impacts of self-awareness heterogeneity on spreading dynamics with positive correlation between node degree and self-awareness. (a) The prevalence \( \rho \) in the stationary state as a function of basic infection rate \( \beta \) for \( \gamma_\alpha = 2.1 \) (red upper-triangles), \( \gamma_\alpha = 2.5 \) (blue circles), \( \gamma_\alpha = 2.9 \) (orange squares), and \( \gamma_\alpha = 3.5 \) (yellow rhombuses) respectively when degree exponent is fixed at \( \gamma_D = 2.1 \). (b) The value of \( \rho \) versus \( \beta \) for the corresponding \( \gamma_\alpha \) when \( \gamma_D = 2.5 \). (c) The value of \( \rho \) as a function of \( \beta \) for the corresponding \( \gamma_\alpha \) when \( \gamma_D = 4.0 \). (d) The epidemic threshold \( \beta_c \) as a function of \( \gamma_\alpha \) for \( \gamma_D = 2.1 \) (orange upper-triangles), \( \gamma_D = 2.5 \) (green squares), and \( \gamma_D = 4.0 \) (yellow circles) respectively. The correlation coefficient is \( \sigma = 0.8 \). Data are obtained by averaging over 500 independent Monte Carlo simulations.
Figure 4: The influence of self-awareness heterogeneity on spreading dynamics with negative correlation between node degree and self-awareness. (a) The prevalence $\rho$ in the stationary state as a function of basic infection rate $\beta$ for $\gamma_\alpha = 2.1$ (blue upper-triangles), $\gamma_\alpha = 2.5$ (orange circles), $\gamma_\alpha = 2.9$ (yellow squares), and $\gamma_\alpha = 3.5$ (purple rhombuses) respectively when degree exponent is fixed at $\gamma_D = 2.1$. (b) The value of $\rho$ versus $\beta$ for the corresponding $\gamma_\alpha$ when $\gamma_D = 2.5$. (c) The value of $\rho$ as a function of $\beta$ for the corresponding $\gamma_\alpha$ when $\gamma_D = 4.0$. (d) The epidemic threshold $\beta_c$ as a function of $\gamma_\alpha$ for $\gamma_D = 2.1$ (green upper-triangles), $\gamma_D = 2.5$ (orange cirlces), and $\gamma_D = 4.0$ (yellow squares) respectively. The correlation coefficient is $\sigma = -0.8$. Data are obtained by averaging over 500 independent Monte Carlo simulations.
a large number of nodes with very small values of self-awareness $\alpha$ (strong willingness of resource donation), and many nodes with very large values $\alpha$ (weak willingness of resource donation). When the coefficient $\sigma = -0.8$, there is a strong negative correlation between node degree and self-awareness. Under this circumstance, the large degree nodes will have a strong willingness to donate resource, while the small degree nodes (covering most nodes of the network) have a weak willingness to donate resource, which leads to a high infection rate of these hub nodes. Besides, the epidemic spreading dynamics exhibits hierarchical features [32]. That is to say, the hubs with large degrees are more likely to be infected firstly, and then the disease propagates from hubs to the intermediate nodes, and finally to nodes with small degrees. Therefore, the large numbers of small degrees will be infected rapidly by the hub nodes in this situation, and the epidemic will outbreak easily.

When the heterogeneity of self-awareness distribution is weak, i.e., $\gamma_\alpha = 4.0$, there is a small fraction of nodes with very large or small value of $\alpha$, many nodes have an intermediate $\alpha$ around the mean value $\langle \alpha \rangle$. The awareness level of the small degree nodes reduces compared to the case of $\gamma_\alpha = 2.1$, which leads to a raise of both the donation probability $\langle q \rangle$ and effective infection rate $\langle \lambda \rangle$ of these nodes. As the strong negative correlation between node degree and self-awareness, the hub nodes still have a very small value of $\alpha$ (large value of donation probability $\langle q \rangle$). During the outbreak of an epidemic, these hubs nodes are more likely to be infected in the early stage, and then transmit the disease to those small degree nodes rapidly as they have a highly effective infection rate $\langle \lambda \rangle$. Thus the epidemic will break out more easily than the case of $\gamma_\alpha = 2.1$ in this situation.

According to the above statement, we have qualitatively explained the optimal phenomenon by explaining why diseases are more likely to break out when there is a strong heterogeneity ($\gamma_\alpha = 2.1$) and weak heterogeneity ($\gamma_\alpha = 4.0$).

Next, we study the effects of correlation between node degree and self-awareness by exploring the relationship between epidemic threshold $\beta_c$ systematically and correlation coefficient. Fig. 5 displays the $\beta_c$ as a function of $\sigma$ for $\gamma_\alpha = 2.1$, $\gamma_\alpha = 2.5$, $\gamma_\alpha = 3.1$ respectively. We find that for each fixed heterogeneity of self-awareness, the epidemic threshold $\beta_c$ increases monotonously with correlation coefficient $\sigma$. For instance, for $\gamma_\alpha = 2.1$, the threshold increases from $\beta_c \approx 0.027$ to $\beta_c \approx 0.044$, which suggests that the more positive of the correlation between node degree and self-awareness, the better the disease can be suppressed. This phenomenon can be qualitatively explained as follows: When there is a larger positive correlation, the hub nodes will have a larger value of $\alpha$, which indicates a smaller probability of resource donation $\langle q \rangle$, and consequently a lower effective
Figure 5: The epidemic threshold $\beta_c$ as a function of correlation coefficient $\sigma$ for $\gamma_\alpha = 2.1$ (blue circles), $\gamma_\alpha = 2.5$ (red squares), and $\gamma_\alpha = 3.1$ (yellow upper-triangles). The degree exponent is fixed at $\gamma_D = 2.5$. Phase I and phase II are separated by critical value $\sigma_c \approx -0.4$. Data are obtained by averaging over 500 independent Monte Carlo simulations.

infection rate $\langle \lambda \rangle$ of the hubs. This phenomenon reduces the infection probability of the hub nodes in the early stage. Meanwhile, the small degree nodes have a larger probability of resource donation, which increases the recovery rate of the I-state neighbors, including the hub nodes. Thus the outbreak of the epidemic is effectively delayed. In addition, it also shows that the parameter plane $(\sigma - \beta_c)$ is separated into two phases: phase I and phase II, by a critical value of $\sigma \approx 0.4$. In phase I, the threshold $\beta_c$ first increases and then decreases with the increase of $\gamma_\alpha$, namely the optimal value $\gamma_\alpha^{opt}$ exists in this region, as shown the curves in Fig. 4 (d) for $\sigma = -0.8$. In phase II, the threshold $\beta_c$ decreases monotonously with $\gamma_\alpha$, as shown the curves in Fig. 1(b) for $\sigma = 0$, and the curves Fig. 3(d) for $\sigma = 0.8$ respectively.

3.3. Verification on real-world networks

In this section, we verify the results obtained on artificial networks by conducting the simulations on the real-world networks. The following four typical real-world networks are chosen in our paper: (i). The OpenFlights network [53]. This network describes the flights between airports in the world. The nodes represent a portion of the world’s airports, and edges represent flights from one airport
to another. There are $N = 2939$ nodes and $V_e = 30501$ edges in the network with maximum degree $k_{\text{max}} = 473$ [54]. (ii) The Euroroad network [55] is an international road network. Most road in the network is located in Europe. The nodes of the network represent cities, and the edge between two nodes denotes that an E-road connects them. There are $N = 1174$ nodes and $V_e = 1417$ edges with $k_{\text{max}} = 10$. (iii) The face-to-face contact network [56]. Nodes in this network represent the individuals who attended the exhibition INFECTIONOUS: STAY AWAY in 2009 at the Science Gallery in Dublin, edges describe the face-to-face contacts that were active for at least 20 seconds among individuals during the exhibition [57]. There are a total numbers of $N = 410$ nodes and $V_e = 17298$ edges (contacts) in the network. The maximum degree is $k_{\text{max}} = 294$. (iv) The Facebook network [58]. This dataset consists of ‘circles’ (or ‘friends lists’) from Facebook. Facebook data was collected from survey participants using this Facebook app. The dataset includes node features (profiles), circles, and ego networks. There are $N = 4039$ nodes and $V_e = 88234$ edges in the network.

Figs. 6 (a) to (d) display the prevalence $\rho$ in stationary state as a function of $\beta$ when there is no degree-awareness correlation on OpenFlights network, Euroroad network, face-to-face contact network and the Facebook network respectively for different heterogeneities of self-awareness. We find that on the OpenFlights network, the threshold $\beta_c$ decreases with the increase of $\gamma_{\alpha}$, which is consistent with the result on artificial networks [see Fig. 1 (a)], and the prevalence $\rho$ increases with $\gamma_{\alpha}$ for a fixed basic infection rate $\beta$. However, there is a difference when disease propagates on the other three real-world networks, i.e., the awareness heterogeneity does not alter the threshold of $\beta_c$. Besides, it shows that the prevalence increases with the decreases of awareness heterogeneity, which is consistent with the result on OpenFlights network. The results above reveal that the awareness heterogeneity can suppress the outbreak of epidemic on real-world networks when there is no correlation between node degree and self-awareness.

At last, we study the effects of degree-awareness correlation on the epidemic spreading on the four real-world networks. The awareness exponent is fixed at $\gamma_{\alpha} = 2.5$. It shows that when disease propagates on the OpenFlights and face-to-face contact networks, the threshold $\beta_c$ does not be altered, but the prevalence $\rho$ decreases with correlation coefficient $\sigma$. This result indicates that a more positive correlation between node degree and self-awareness can better suppress the disease spreading, which is consistent with the result of artificial networks. When disease propagates on the Euroroad network, the correlation does not alter both threshold $\beta_c$ and prevalence $\rho$. At last, when disease propagates on the Facebook
Figure 6: The effects of awareness distribution on the epidemic spreading on real world networks. (a) The prevalence $\rho$ as a function of basic infection rate $\beta$ on the OpenFlights network for awareness exponent $\gamma_\alpha = 2.1$ (blue upper-triangle), $\gamma_\alpha = 2.5$ (red circles), and $\gamma_\alpha = 4.0$ (yellow squares) respectively. (b) The prevalence $\rho$ versus $\beta$ for the corresponding values of awareness exponent on the Euroroad network. (c) The value of $\rho$ as a function of $\beta$ for the four typical values of $\gamma_\alpha$ on the face-to-face contact network. (d) The results on Facebook network. The correlation coefficient between self-awareness and node degree is $\sigma = 0$. 
Figure 7: The influence of correlation between self-awareness and node degree on the epidemic spreading on real world networks. (a) The prevalence $\rho$ as a function of basic infection rate $\beta$ on the OpenFlights network for $\sigma = -0.8$ (blue upper-triangle), $\sigma = -0.4$ (red circles), $\sigma = 0.4$ (yellow squares), and $\sigma = 0.8$ (green rhombuses) respectively. (b) The prevalence $\rho$ versus $\beta$ for the corresponding values of $\sigma$ on the Euroroad network. (c) The value of $\rho$ as a function of $\beta$ for the four typical values of $\sigma$ on the face-to-face contact network. (d) The results on Facebook network. The awareness exponent is set to be $\gamma_\alpha = 2.5$. 
network, the threshold $\beta_c$ increases with $\sigma$, which is consistent with the result on artificial networks [see Fig. 5].

Based on the above research, we can see that the results on the real-world networks are consistent with those on the artificial network. However, the complex structural features of the real networks, such as clustering, community structure, and small-world characteristics, have an important impact on the results, and needs to be further studied in our future research.

4. Discussion

In summary, we have studied systematically the impact of heterogeneous awareness of self-protection on the dynamics of epidemic spreading. A coevolution dynamical model of resources and epidemic on complex networks has been proposed. The two processes of resource allocation and disease spreading are coevolving in such a way that the generation and allocation of resource depend on the S-state nodes, and the recovery rate of I-state nodes rely on the resources from their S-state neighbors. Both the effective infection rate and the recovery rate of the nodes will be alerted due to the resource factor. First of all, we have studied the effects of the heterogeneity of both degree and self-awareness distributions on the epidemic dynamics on artificial networks. Through extensive Monte Carlo simulations, we have found that the degree heterogeneity enhances the epidemic spreading, which is consistent with the classical results on scale-free networks. Besides, the threshold $\beta_c$ increases with the growth of self-awareness heterogeneity for a fixed network structure, which indicates that the self-awareness heterogeneity suppresses the outbreak of an epidemic.

Then we have studied the impact of correlation between node degree and self-awareness on the epidemic dynamics on artificial networks. We have found that when there is a positive correlation between node degree and self-awareness, the threshold $\beta_c$ increases monotonously with the heterogeneity of self-awareness. When there is a strong negative correlation, e.g., $\sigma = -0.8$, the epidemic threshold $\beta_c$ first increases and then decreases with $\gamma_\alpha$. An optimal value $\gamma_\alpha^{\text{opt}}$ have been found, at which the disease can be suppressed to the most extent. Besides, we have studied systematically the effects of degree-awareness correlation on the epidemic dynamics by exploring the relationship between $\beta_c$ and correlation coefficient $\sigma$. We have found that the epidemic threshold $\beta_c$ increases monotonously with $\sigma$ in $[-1.0, 1.0]$, which reveals that the stronger the positive correlation, the more likely the disease can be suppressed. Besides, we have also found a critical value $\sigma_c \approx 0.4$, when $\sigma < \sigma_c$, an optimal value $\gamma_\alpha^{\text{opt}}$ exists, the threshold $\beta_c$ in-
creases with $\gamma_\alpha$ before $\gamma_\alpha^{\text{opt}}$ and decreases after $\gamma_\alpha^{\text{opt}}$. When $\sigma > \sigma_c$, $\beta_c$ decreases monotonously with the increase of $\gamma_\alpha$.

At last, to verify our results, we have conducted Monte Carlo simulations on four typical real-world networks. It reveals that the results on the real-world networks are consistent with those on the artificial network. However, the sophisticated structural features of the real networks, such as clustering, community structure, and small-world characteristics, have an essential impact on the results. For instance, the self-awareness heterogeneity does not alter the threshold $\beta_c$ on the *Euroroad* network, face-to-face contact network, and *Facebook* network. Besides, the correlation does not change both the threshold $\beta_c$ and prevalence $\rho$, when disease propagates on the *Euroroad* network.

Our findings make a substantial contribution to the understanding of how the heterogeneous distribution of individual awareness for self-protection influences the dynamics of epidemic spreading. The results in this paper are of practical significance for controlling the outbreak of infectious diseases, especially in the context of the outbreak of *COVID-19*. It will also guide us to make the most reasonable choice between resource contribution and self-protection when perceiving the threat of disease, and also have a direct application in the development of strategies to suppress the outbreaks of epidemics.

The present work mainly focus on the spreading dynamics of infectious diseases that can be described by the *susceptible-infected-susceptible* model, such as seasonal influenza. However, the findings obtained in this paper could still shed light on the control of the diseases with similar characteristics, such as the SIRS and SIR-like epidemics that have been widely studied in recent years [59, 60, 61, 62]. There are still some limitations of our work. For example, as the SIS model can not describe the spread of an irreversible epidemic, there would be difference in the dynamical characteristics such as the transition type and phase diagram between the SIS model and the irreversible epidemic models. Thus, a coupled dynamic model of resource allocation and epidemic spreading based on the SIR,SIRV and SEIR models will be studied in our future works. In addition, the theoretical analysis will also be researched.

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