The Vacuum Frame

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One of the most fundamental questions in cosmology is if dark energy is related just to a constant or it is something more complex. In this work, we call the attention to the fact that, under very general conditions, dark energy can be identified with a cosmological constant. Indeed, this fact defines what we call Vacuum Frame. In general, this frame does not coincide with the Jordan or Einstein frame, defined by the invariant character of particle masses or the Newton constant, respectively. We illustrate this question by the introduction of a particular scalar-tensor model where the different hierarchies among these energy scales are dynamically generated.

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I. INTRODUCTION

The present expansion of the Universe is accelerated due to a non-standard component called Dark Energy (DE). One of the most basic questions in cosmology is to characterize its fundamental nature. The most simple model of DE is just a cosmological constant $\Lambda$. Indeed, it determines (part of) the standard model of cosmology, named $\Lambda$-CDM. However, the uncertainties in this respect are important and many observational efforts are and will be dedicated to elucidate this question.

In this work, we would like to point out that the constant nature of DE can be imposed in the majority of cases by taking advantage of a conformal transformation that determines the measurement pattern for energy scales. We have the freedom of choosing such a pattern in any point of our spacetime. If we choose it precisely proportional to the DE, such DE will be constant. This is what defines what we name Vacuum Frame (VF). We can summarize the two basic necessary conditions in order to can properly define the VF, at least, in a given spacetime domain:

1.- DE cannot be conformal invariant. This condition is obviously needed in order to modify the DE profile with a conformal transformation.

2.- DE cannot be zero in any point of such domain. This fact is also necessary to can define an energy pattern within the entire domain.

Working within the VF provides an interesting different approach to the DE problem. We will illustrate it with a particular scalar-tensor theory (STT). This type of models refers to any gravitational theory, whose interaction is mediated not only by the standard spin-2 graviton, but also by a spin-0 scalar. Under this definition is possible to classify many different extensions of General Relativity (GR) \cite{1,2}. The action of the model can be written in different forms that are equivalent as a classical field theory. For example, it is usually written in the so called Jordan Frame (JF), in which the matter content is coupled to the metric in the minimal way, or in the Einstein Frame (EF), where the action for the spin-2 graviton is standard. The metric tensors in these frames are related by a simple conformal transformation \cite{1,2}:

$$ g_{\mu\nu} = A^2(\varphi) g_{\mu\nu}^* , $$

where $g_{\mu\nu}$ denotes the metric in the JF and $g_{\mu\nu}^*$, in the EF (we will use $^*$ for EF quantities). An interesting property of these theories is that physical scales are not absolute or constant. On the contrary, they evolve due to the presence of the new spin-0 field: $\varphi$. In the EF, the Planck scale is constant ($M_{Pl}^*$) but the scales associated with the particle content (that we will denote by $m^*$, in general) evolves proportional to the conformal factor: $m^* = A(\varphi) m$. In the JF, the scales associated with the particle content are fixed (m) but not the Planck scale: $M_{Pl} = M_{Pl}^*/A(\varphi)$.

II. VACUUM FRAME: AN EXAMPLE

In this work we are interested in written the model within the VF, where the scale associated with the vacuum density is constant. We will denote the quantities associated with this frame with $\tilde{}$. Therefore, the action in the VF for the STT can be written as:

$$ S = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{\dot{\tilde{\varphi}}^2}{2} - \frac{\tilde{\omega}(\tilde{\varphi})}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\varphi} \partial_\nu \tilde{\varphi} - \tilde{\Lambda}^4 \right] + \tilde{S}_{SM}[\tilde{g}_{\mu\nu}; \tilde{\varphi}; \tilde{\psi}], $$

where $\tilde{\psi}$ differs the indices of the different fields of the standard model (SM). We will be able to generate the Planck scale dynamically by the vacuum expectation value (asymptotic value, strictly speaking) of the scalar $\tilde{\varphi}_{\text{asy}} = M_{Pl}^*$. Therefore, the only explicit scale in the theory is the associated with the vacuum energy or cosmological constant: $\tilde{\Lambda} \simeq 2.3 \times 10^{-3} \text{ eV}$. With respect to the matter sector, for simplicity, we will assume massless neutrinos and a conformal kinetic term for the Higgs doublet ($\tilde{\Phi}$),

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This type of couplings have been already proposed to describe the dilaton dynamics in non-perturbative string effective actions \[ 3, 4, 5 \]. It has been studied in several works since it is the minimal version of the coupling which provides an attractor toward GR \[ 3, 4, 6 \]. In our case, the coupling given by Eq. \[ 5 \] is even more interesting, since it is able to drive the value of the scalar field to the Planck scale without introducing any fundamental hierarchy in the action, just by tuning \( b = \ln[ M^*_{\text{Pl}} / \Lambda] \simeq 69 \). In addition, if \( \beta \) is large enough, this scale can be generated dynamically. We take \( \beta = 80 \) for Figs. 1 \[ 1, 2 \] and 3. In particular, Fig. 1 shows that the present value of the Planck scale reaches its asymptotic limit: \( M^*_{\text{Pl}} \equiv (8 \pi G_N)^{-1/2} \simeq 2.4 \times 10^{18} \text{ GeV} \).

It is simple to understand the dynamical generation of the Planck scale from Eq. \[ 4 \]. At the initial time in the VF, any scale is expected to be of order \( \Lambda \). For the sake of concreteness, we assume \( \bar{\varphi}_\text{ini} = \Lambda \), which implies \( M^*_{\text{Pl}}(\bar{\varphi}_\text{ini}) = \Lambda \). However, the effective coupling of the scalar mode is intense, given by \( \bar{\alpha}(\bar{\varphi}_\text{ini} / \Lambda) = \sqrt{\beta} \). It implies that \( \bar{\varphi} \) is attracted efficiently toward its asymptotic value: \( \bar{\varphi}_\text{asy} = \Lambda e^b = M^*_{\text{Pl}} \). In this limit, \( M^*_{\text{Pl}}(\bar{\varphi}_\text{asy}) = M^*_{\text{Pl}} \) and \( \bar{\alpha}(\bar{\varphi}_\text{asy} / \Lambda) \rightarrow 0 \), which means that the scalar mode is decoupled and the model can be close enough to GR to satisfy precision constraints from Post-Newtonian parameters (PPNs) or Big Bang Nucleosynthesis (BBN). For simplicity, we will not take into account an stage of inflation, but it should be taken into account that this stage would increase the GR attraction. Therefore, the bounds found in this analysis can be qualified as conservative.

### III. JORDAN FRAME: THE STANDARD HIGGS SECTOR

The linear coupling between the scalar mode and the Higgs doublet looks beyond the standard STT approach, but it is not. The question is that the above action is not written in either the JF or the EF. The electroweak scale is determined by the vacuum expectation value of the Higgs doublet, that depends on the scale: \( \sqrt{\Lambda} \bar{\varphi} \).

We can define the JF as the frame in which the scale in front of \( \Phi^4 \Phi \) is constant:

\[
S_{\text{SM}}[g_{\mu \nu}; \psi; \varphi] = S_{\text{Cond}}[g_{\mu \nu}; \psi] + \int d^4 x \sqrt{-g} \frac{(\alpha \mu)^{2}}{2} \Phi^4 \Phi. \tag{6}
\]

This frame is determined by the conformal transformation defined by the conformal factor, \( B^2(\tilde{\varphi}) = \tilde{\varphi}/M^*_{\text{Pl}} \):

\[
g_{\mu \nu} = (\bar{\varphi}/M^*_{\text{Pl}})^2 g_{\mu \nu}, \quad \varphi / \Lambda \equiv \varphi / \mu, \tag{7}
\]

which leads to the following action:

\[
S = \int d^4 x \sqrt{-g} \left[ \mu \varphi \frac{R}{2} - \mu \frac{\omega(\varphi)}{2 \varphi} g_{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi - \mu \frac{\beta}{2 \varphi^2} \right] + S_{\text{SM}}[g_{\mu \nu}; \psi]. \tag{8}
\]

Although both assumptions can be trivially generalized. In this example, the only non-conformal term in the matter action is the quadratic Higgs term. In order to generate the electroweak scale at the same time than the Planck scale, we write this non-conformal term as:

\[
S_{\text{SM}}[g_{\mu \nu}; \tilde{\varphi}; \tilde{\varphi}] = S_{\text{Cond}}[g_{\mu \nu}; \tilde{\varphi}] + \int d^4 x \sqrt{-g} \frac{\Lambda}{2} \tilde{\varphi} \Phi^4 \Phi, \tag{3}
\]

where \( \varsigma \) is a dimensionless number, whose value needs to be fixed to \( \varsigma \simeq 0.037 \).

Although it is not evident from Eqs. 2 and 3, we will show that the new scalar mode is coupled to the matter sector with a coupling \( \alpha(\tilde{\varphi} / \Lambda) \), which is fixed by \( \tilde{\omega}(\tilde{\varphi}) \):

\[
\tilde{\omega}(\tilde{\varphi}) = \frac{1}{2} \left( \alpha^2(\tilde{\varphi} / \Lambda) - 12 \right). \tag{4}
\]

Therefore, to determine the STT we need to specify the function: \( \alpha^2(x) \). For example, in the classical Brans-Dicke theory: \( \alpha^2(x) \equiv \alpha_0^2 \) is constant. Unfortunately, in this case, an effectively decoupled scalar field is necessary to be consistent with present constraints: \( \alpha_0 \lesssim 10^{-2} \). In this work we will focus on a different \( \alpha^2(x) \) in order to show the potential of STT for dynamical generation of hierarchies. In particular we will focus on the function:

\[
\alpha^2(x) = \beta (b - \ln x). \tag{5}
\]
We have introduced the scale: \( \mu \equiv \sqrt{\Lambda M_{\text{Pl}}^*} \approx 2.4 \text{ TeV} \), and normalize the scalar mode in this frame such as \( \varphi_{\text{ini}} = \mu \). In this way:

\[
\omega(\varphi) = \frac{1}{2} \left( \alpha^{-2}(\varphi/\mu) - 3 \right),
\]

and again, the action depends only on one scale: \( \mu \), that is the scale associated with the particle sector, constant in this frame. In this form, it is easy to recognize a standard STT in the JF, where the vacuum energy can be interpreted as a particular potential for \( \varphi \).

### IV. EINSTEIN FRAME: EVOLUTION

For computation purposes, it is more convenient to write the action in the EF. By Assuming a flat FRW metric: \( ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2) \), and a perfect fluid for the matter content (characterized by its energy density, \( \rho \) and its pressure \( P \)) we can write the Friedmann equations in any frame. In particular, for the EF we can write: \( \rho_* = A^4 \rho \) and \( P_* = A^4 P \), where \( A \) is the conformal factor that defines the transformation from the JF. In our case:

\[
A^2 = M_{\text{Pl}}^* (\mu \varphi) = \exp(\beta \varphi^2),
\]

and the action in this frame reads:

\[
S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^*}{2} \left[ R_* - 2g_{\mu\nu} \partial_{\mu} \varphi_* \partial_{\nu} \varphi_* - 4V_*(\varphi_*) \right] + S_{\text{SM}}[A^2(\varphi_*)g^*_{\mu\nu}; \psi(\varphi_*, \psi_*)].
\]

where we are using a dimensionless scalar field \( \varphi_* \) defined by \( \varphi = (M_{\text{Pl}}^* / \mu) \exp(-\beta \varphi^2) \). Its potential reads:

\[
2V_*(\varphi_*) \equiv \frac{\hat{V}_*(\varphi_*)}{M_{\text{Pl}}^2} = M_{\text{Pl}}^2 4^\beta (\varphi^2 - \varphi^2_{\text{ini}}).
\]

where \( \varphi^2_{\text{ini}} \equiv b/\beta \sim 1 \). In this frame, the non-conformal part of the particle action depends also on \( \varphi_* \):

\[
S_{\text{SM}}[A^2(\varphi_*)g^*_{\mu\nu}; \psi(\varphi_*, \psi_*)] = S_{\text{Conf}}[g^*_{\mu\nu}; \psi_*] + \int d^4x \sqrt{-g_*} \left( \frac{\varsigma}{2} M_{\text{Pl}}^* \right)^2 \Phi_* \Phi_* \exp(\beta \varphi^2 - \varphi^2_{\text{ini}}).
\]

In this frame, it is the exponential dependence of the conformal factor on \( \varphi^2_{\text{ini}} \), what allows to write \( M_{\text{Pl}}^* \) as the unique scale.

The agreement with BBN constraints forces to an effective attraction during the radiation dominated Universe. In this stage, it is easier to find the approximated evolution in the EF. In fact, since pure radiation does not interact with the scalar degree of freedom at first order, it is a good approximation to assume that the equations of motion for radiation are standard. On the other hand, the EF is defined by the fact that the gravitational equations for the metric have their standard form. For these reasons, inside the EF, the evolution deviates minimally from the standard case.

As it has been done in previous works for the same model (with \( V_*(\varphi_*) = 0 \)), it is convenient to use the e-folds of expansion \( p \) in the EF for studying the cosmological evolution (not to be confused with the pressure \( P \)). Denoting by \( R_{*,\text{ini}} \) the value of the scale factor at the initial time (defined by \( T_* = M_{\text{Pl}}^* \)), the number of
e-folds is \( p \equiv \ln(R_e/R_{e,\text{ini}}) \), which implies:

\[
\frac{d}{dp} = \frac{1}{H_e} \frac{d}{dt} = A \frac{d}{H_e} \frac{d}{dt},
\]

where prime shall denote differentiation wrt \( p \). \( R_e \) grows monotonically with time, since \( H_e \) is governed by the standard Friedmann equation and it is always positive. On the contrary, the Hubble rate \( H \) in the JF is related to that of the EF by

\[
H = \frac{H_e}{A} (1 + \alpha_s \phi'_s),
\]

with \( \alpha_s = d \ln[A(\phi_s)]/d \phi_s = \beta \phi_s \). \( H \) can become negative leading to a contracting universe in the JF. In a similar way, the evolution of the Jordan Temperature of the relativistic thermal bath is given by \( \mathcal{I} \):

\[
(\ln T)' = -\frac{1 + \alpha_s \phi'_s}{\mathcal{I}} \approx -(1 + \alpha_s \phi'_s),
\]

since \( \mathcal{I} - 1 \lesssim 10^{-3} \). On the other hand, the equation for the field is given by:

\[
\phi'_s = -\frac{3 - \phi'^2}{2} \left[ S_1(\phi_s, \rho_s, P_s) \phi'_s + S_2(\phi_s, \rho_s, P_s) \right],
\]

where the functions \( S_1 \) and \( S_2 \) depend on the density and pressure of the thermal bath and on \( \phi_s \) trough the potential \( V(\phi_s) \) and the conformal factor \( A(\phi_s) \). In particular:

\[
S_1(\phi_s, \rho_s, P_s) = \frac{\rho_s - P_s + 2V(\phi_s)}{\rho_s + V(\phi_s)},
\]

\[
S_2(\phi_s, \rho_s, P_s) = \frac{\alpha_s (\rho_s - 3P_s) + dV(\phi_s)/d\phi_s}{\rho_s + V(\phi_s)}.
\]

We have computed the evolution by assuming an ideal relativistic thermal bath: \( \rho = g_f T^4 \pi^2/30 \) and \( P = g_f T^4 \pi^2/90 \) except for the source term: \( \rho - 3P \). This term is the dominant source for the evolution of the scalar mode in the radiation dominated regime as it has been discussed in \( \mathcal{J} \). In our case, we have included a potential that is only important in a very first stage, when the scales unify; and in the very late Universe, when the other energy content has been diluted and the potential behaves as a cosmological constant. In particular, for the standard model content: \( g_f \approx 427/4 \) and \( \rho - 3P \approx 140 \rho \rho_\Lambda^2(\mathcal{T})/(61 \pi^2) \), where we are just keeping the leading modification to the non-interacting bath coming from QCD (read \( \mathcal{J} \) for a detailed discussion).

In the early evolution, the weak symmetry is not broken since \( T \) is bigger than the critical temperature:

\[
T_c^2 = \frac{8(\zeta \mu)^2}{4\lambda + 3g^2 + 3g'^2} = \frac{(348 \text{ GeV})^2}{1 + 3g^2/4\lambda + 3g'^2/4\lambda},
\]

V. DISCUSSION AND SIGNATURES

The present constraints and observational signatures of this model are resumed in Fig. 4. On the one hand, the modification of the standard expansion given by Eq. (15) originates potential deviations in standard nucleosynthesis. The uncertainties on these observations are typically of order 10% and it roughly implies: \( \beta|\phi_{s,\text{nuc}}^*| \lesssim 0.2 \) and \( |\beta \phi_{s,\text{nuc}}^*| \lesssim 0.1 \), although both constraints are degenerated (read Ref. \( \mathcal{J} \) for a detailed analysis in the same model used in this work but with negligible vacuum energy).

The other important signature of these models is their present modifications to GR \( \mathcal{J} \). The bounds are most conveniently imposed on the so called post-Newtonian parameters (PPN), which in the context of STTs, can in turn be related to the parameter of the model: \( \beta \), and the present value of the scalar: \( \beta^0 \). Limits from the Very Long Baseline Interferometer \( \mathcal{J} \) and radio links with the Cassini spacecraft \( \mathcal{J} \) enforce \( (\beta \phi_s^*)^2 \lesssim 10^{-5} \).
The perihelion shift of Mercury \cite{12} and the Lunar Laser Ranging experiment \cite{13} instead constrain the combination $(\beta + 16)(\beta \varphi^2)^2 < 10^{-3}$. Therefore, $\beta$ can be large enough in order to have an effective attraction, and reduce the present value of $\varphi_*$. Indeed, it is easy to estimate the decrease of $\alpha$ during matter domination \cite{4} since the linear approximation leads to a simple damped oscillatory solution. Just for $\beta \sim 3/8$, $\varphi_*$ decreases by a factor of about two orders of magnitude during the matter dominated era \cite{7}.

Other possible signatures in gravitational lensing and the cosmic microwave background (CMB) have been studied \cite{14, 15}, although they are less promising. In the latter, the main modification is also related to a difference in the expansion rate, which affects the angular scale of the anisotropies. On the other hand, the decoupling time might be significantly more perturbed and displace the peaks towards higher multipoles for faster expansions. In any case, due to the well known degeneracy of the CMB spectrum with respect to cosmological parameters, present measurements of the peak locations \cite{16} do not translate straightforwardly into a bound on the conformal factor $\Lambda$, but on a general reanalysis of data. In any case, once the BBN bound on $\Lambda$ has been imposed, the resulting values for $\Lambda$ at photon decoupling are so close to unity as to give shifts in the peak multipoles smaller than the experimental error. Thus, the CMB spectrum does not provide significant bounds to the model under discussion.

In conclusion, we have introduced the concept of Vacuum Frame as an alternative framework to analyze the DE problem. We have illustrated such a concept by studying the phenomenology of a simple STT, which is able to generate the large hierarchies among the Planck, electroweak and cosmological constant scale in a dynamical way.

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