Correlation dynamics of a two-qubit system in a Bell-diagonal state under non-identical local noises

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The property of quantum correlation has been studied in recent years, especially for the quantum and classical correlations affected by environment. The dynamics of quantum and classical correlations in two-qubit system under identical local noise channels have been investigated recently. Here we will consider the dynamics of quantum and classical correlations when the local noise channels of two sides are not identical. We investigate the dynamics of quantum and classical correlations with three types of local noise channels in both Markovian and non-Markovian conditions, and show the decay rules of quantum and classical correlations with different types and parameter times of local noise channels.

Keywords Quantum correlations · dynamics · two-qubit system · Bell-diagonal state · non-identical local noises

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1 Introduction

Quantum entanglement is a kind of quantum correlation and it plays an important role in quantum information and communication [1]. However, entanglement is not the only type of correlation useful for quantum technology, and there are other nonclassical correlations apart from the entanglement, which are also responsible for the quantum advantage over their classical counterparts, such as deterministic quantum computation with one pure qubit [2–4], quantum phase transition [5] and quantum Grover research algorithm without entanglement [6, 7]. Therefore, it is desirable to investigate, characterize, and quantify quantum correlations.

It is widely accepted that mutual information is the total correlation contained in a bipartite system, which is the sum of classical correlation and quantum correlation [8–11]. Therefore, how to distinguish between the quantum and the classical aspects of the total correlation is an outstanding question. In 2001, based on the distinction between the quantum information theory and classical information theory, Olliver and Zurek [12] first defined a quantifier, called the quantum discord, as a measure for all quantum correlations in bipartite systems. Other algorithms to evaluate quantum correlations have also been investigated [13–18]. For pure states, quantum discord exactly equals to the entanglement, which implies that there is no quantum correlation for separable pure states [8, 9]. However, for general two-qubit mixed states, the situation is more complicated. Up to now, quantum discord is also an immature field. There are few analytical expressions, including special cases for two-qubit system [19–21]. For multiqubit and high-dimension systems, it is still an open issue as how to define quantum discord.

It is well known that a quantum system inevitably interacts with its environment, which is responsible for the loss of quantum properties initially presenting in the system. That is, it is important to investigate the behaviors of these quantum properties (quantum entanglement and quantum correlations) under the action of decoherence. For the entanglement dynamics of quantum systems, many works have been done under the influence of both Markovian and non-Markovian environments [22–26]. It has been shown that if the environment is a non-Markovian one, entanglement can suddenly disappear at a finite time, which was named as "entanglement sudden death" (ESD). However, for a Markovian one, ESD may occur for a suitable choice of the noise channel and initial states, but ESB can not be observed [23, 26]. In recent years, quantum correlation dynamics has received increasing attention, and similar works about the entanglement dynamics have been investigated about the quantum correlation [21, 27, 32]. The previous works showed that quantum discord and entanglement behave differently under the influence of the environment. In particular, the phenomenon of ESD does not occur for quantum discord, which disappears only asymptotically.

The dynamics of the discord in the presence of the environments acting on each subsystems has, until now, been studied only for the same type. In this paper, we consider the correlation dynamics of two-qubit system under the effect of two non-identical independent local Markovian and non-Markovian environments. When the Markovian
environments acting on two subsystems have the same type, there are three regimes for different decay rules of quantum
and classical correlations with different relations of coefficients in initial two-qubit state. For the case with Markovian
environments acting on each subsystem belonging to different types, three are still three coefficients regimes for decay
rules of quantum and classical correlations. However, if the environment is non-Markovian, there are three coefficients
regimes of decay rules for two subsystems with the same type of local noise channels and only one coefficients regime
of decay rules for two subsystems with different types of local noise channels. We focus on the coefficients regimes
with sudden change points on decay rates of quantum and classical correlations, which are decided by the types and
time parameters of local noise channels.

This article is organized as follows. In Sec.II, we introduce the property of Bell-diagonal state and noise channels.
The dynamics of the quantum correlation under local Markovian environment will be discussed in Sec.III. For non-
Markovian case, it is discussed in Sec.IV. Finally, we conclude our work in Sec.V.

2 Correlations and channels

2.1 Correlations

The total information (mutual information) of a two-qubit bipartite system is defined as

$$I(\rho) = S(\rho_A) + S(\rho_B) - S(\rho),$$

where $\rho$ is the density matrix of the whole system $AB$, and $\rho_A = \text{tr}_B \rho$ and $\rho_B = \text{tr}_A \rho$ are the reduced density matrices
of the subsystems $A$ and $B$, respectively.

$$S(\rho) = -\text{tr} \rho \log_2 \rho$$

is the von Neumann entropy for the matrix $\rho$. Quantum discord is defined as

$$Q(\rho) = I(\rho) - C(\rho),$$

where

$$C(\rho) = \text{Max}_{\{B_k\}}(S(\rho_A) - S(\rho|\{B_k\}))$$

is the classical correlation of the state when acting the measurements $\{B_k\}$ on the subsystem $B$.

$$S(\rho|\{B_k\}) = \sum_k p_k S(\rho_k)$$

is the conditional entropy of the subsystem $A$. Here

$$\rho_k = \frac{(I \otimes B_k) \rho (I \otimes B_k)}{\text{tr}(I \otimes B_k) \rho (I \otimes B_k)}$$

is the state of the system under the measurements $\{B_k\}$. Here $I$ is the identity matrix.

In this work, we investigate the dynamics of two-qubit systems in a Bell-diagonal state which has three parameters
[33]. It includes subsets of separable states, classical states, and entangled states with maximally mixed marginal
($\rho_{A(B)} = I_{A(B)}/2$), and it can be described as:

$$\rho = \frac{1}{4}(I + \sum_{i=1}^{3} c_i \sigma_i \otimes \sigma_i),$$

where $\sigma_i$ ($i = 1, 2, 3$) are the three Pauli operators and $c_i$ represent the three parameters for describing Bell-diagonal states.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. $$

Here $i^2 = -1$. The state $\rho$ can also be written as

$$\rho = \lambda_{\psi^+}\langle \psi^+ | \psi^+ \rangle + \lambda_{\psi^-}\langle \psi^- | \psi^- \rangle + \lambda_{\phi^+}\langle \phi^+ | \phi^+ \rangle + \lambda_{\phi^-}\langle \phi^- | \phi^- \rangle,$$
where $|\psi^\pm\rangle$ and $|\phi^\pm\rangle$ are the four Bell states for two-qubit systems, that is,

$$
|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle),
$$
$$
|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).
$$

The classical correlation and quantum correlation of these states can be derived from Eqs. (3) and (4) \[21\],

$$
C(\rho) = \sum_{j=1}^{2} \frac{1 + (-1)^j \chi}{2} \log_2 [1 + (-1)^j \chi],
$$
$$
Q(\rho) = 2 + \sum_{k=1}^{4} \lambda_k \log_2 \lambda_k - C(\rho),
$$

where $\chi = \text{Max}\{|c_1|, |c_2|, |c_3|\}$. Classical correlation and quantum correlation of this two-qubit system depend on the coefficients $c_i$ ($i = 1, 2, 3$).

### 2.2 Markovian noise channels

We consider three kinds of Markovian noise channels on each qubit in the dynamics, that is, a bit-flip channel, a phase-flip channel, and a bit-phase-flip channel. A bit-flip channel can be described with the two Kraus operators as follows:

$$
E_1 = \sqrt{1 - \frac{p}{2}} I, \quad E_2 = \sqrt{\frac{p}{2}} \sigma_1.
$$

The Kraus operators for a phase-flip channel can be written as

$$
E_1 = \sqrt{1 - \frac{p}{2}} I, \quad E_2 = \sqrt{\frac{p}{2}} \sigma_3.
$$

A bit-phase-flip channel can be described with the following Kraus operators,

$$
E_1 = \sqrt{1 - \frac{p}{2}} I, \quad E_2 = \sqrt{\frac{p}{2}} \sigma_2.
$$

Here $p$ is the probability that the noise acts on the qubit, and $0 \leq p \leq 1$.

The evolved state of a two-qubit system under local environments can be described with a completely positive trace-preserving map, which can be written in the operator-sum representation \[1\]:

$$
\rho(t) = \sum_{i,j} E_i^{(A)} E_j^{(B)} \rho(0) E_j^{(B)\dagger} E_i^{(A)\dagger},
$$

where $E_i^{(k)}$ ($k = A, B$) is the Kraus operator which is used to describe the channel $A$ or $B$, and $\sum_i E_i^{(k)\dagger} E_i^{(k)} = 1$.

### 2.3 Non-Markovian noise channels

Depolarizing channel is a Markovian noise channel based on bit-flip error, phase-flip error, and bit-phase-flip error with complete positive map (CPM)

$$
\Phi(\rho) = (1 - \rho) + \frac{p}{3} (\sigma_1 \rho \sigma_1 + \sigma_2 \rho \sigma_2 + \sigma_3 \rho \sigma_3),
$$

where $0 \leq p \leq 1$. If this noise channel is generalized to non-Markovian condition, the memory effects of system-environment interaction should be introduced. The non-Markovian depolarizing channel can be derived from generic memory master equation

$$
\dot{\rho} = K \mathcal{L} \rho,
$$
where $\mathcal{L}$ is the Lindblad superoperator describing the dissipative dynamics of system-environment interaction, $K$ is an integral operator with $K \phi = \int_0^t k(t-t')\phi(t')dt'$, and kernel function $k(t-t')$ determines the type of memory of system-environment interaction.

The derivation of memory master equation can be illustrated by the model with a spin in a magnetic field [34]. The time-dependent phenomenological Hamiltonian of this model is

$$H(t) = \hbar \Gamma_i(t) \sigma_i,$$

(19)

Here $\Gamma_i(t) = a_i n_i(t)$ is independent random variable, $a_i$ is a coin-flip random variable taking the values $\pm |a_i|$, and $n_i$ is a random variable having a Poisson distribution with the mean value $t/2\tau_i$. With the von Neumann equation

$$\dot{\rho}(t) = -i/\hbar [H, \rho],$$

the memory kernel master equation is obtained [34],

$$\dot{\rho}(t) = -\int_0^t \sum_k \exp(-\gamma t') a_k^2 [\sigma_k, [\sigma_k, \rho(t)]]dt',$$

(20)

where $< \Gamma_j(t)\Gamma_k(t') > = a_k^2 \exp(-\gamma (t-t')/\tau_k)\delta_{jk}$ is correlation function of the random telegraph signal, $k = 1, 2, 3$ represent the general case of noise in three directions.

The linear map $\Phi_t(\rho) : \rho \rightarrow \rho_t$ is required to be a completely positive, trace-preserving map, which has a Kraus decomposition

$$\Phi_t(\rho) = \sum_k A_k^t \rho A_k.$$  

In Ref.[34], this particular condition had been studied in detail, and the CPM is assured when two of the $a_i$ are zero. The physical situation of noise is reduced to one direction, and the Kraus operators of the direction $a_i = a$ are

$$A_i = \sqrt{1 - \Lambda(\nu)/2} \sigma_i, \quad A_j = 0, \quad A_k = 0, \quad A_4 = \sqrt{1 + \Lambda(\nu)/2} I.$$  

(21)

Here $\Lambda(t) = \exp(-\nu)|\cos(\mu t) + \sin(\mu t)/\mu|$ is damped harmonic oscillator, $\mu = \sqrt{(4\nu^2 - 1}$ is frequency, and $\nu = t/(2\tau)$ is dimensionless time. The non-Markovian depolarizing channel becomes bit-flip channel, phase-flip channel, and bit-phase-flip channel with $p = 1 - \Lambda(t)$ in Eqs. (13)-(15).

3 Correlation dynamics of a two-qubit system in a Bell-diagonal state under Markovian noise

3.1 Bi-side Markovian noise channels with the same type

In this section, we discuss the dynamics of correlations of a two-qubit system which is originally in a Bell-diagonal state and interacts with two independent local Markovian channels with the same type but different decoherence rates. There are three types of noise for local Markovian channels: bit-flip noise, phase-flip noise, and bit-phase-flip noise.

(1) Two independent local phase-flip channels

When two local noise channels are both phase-flip ones, the Kraus operators for the two-qubit system are

$$E_0^{(A)} = \sqrt{1 - p/2} I_A \otimes I_B,$$

$$E_1^{(A)} = \frac{p}{2} \sigma_3^A \otimes I_B,$$

$$E_0^{(B)} = I_A \otimes \sqrt{1 - p'/2} I_B,$$

$$E_1^{(B)} = I_A \otimes \frac{p'}{2} \sigma_3^B,$$

(22)

where $p = 1 - \exp(-\gamma t)$ and $p' = 1 - \exp(-\gamma' t)$, and $\gamma$ and $\gamma'$ are the phase damping rates for the channels $A$ and $B$ (i.e., the qubits $A$ and $B$), respectively. Any Bell-diagonal state $\rho$ shown in Eq. (7) evolves to another Bell-diagonal state under this kind of noise channels,

$$\rho(t) = \frac{1}{4} (I + \sum_{i=1}^3 c_i(t) \sigma_i \otimes \sigma_i).$$

(23)

Here the three coefficients are

$$c_1(t) = (1 - p)(1 - p')c_1,$$

$$c_2(t) = (1 - p)(1 - p')c_2,$$

$$c_3(t) = c_3.$$  

(24)
The dynamics of classical correlation and quantum correlation under this noise channel are shown in Eq.(11) and Eq.(12), respectively. Here $\chi = \text{Max}\{|c_1(t)|, |c_2(t)|, |c_3(t)|\}.

(2) Two independent local bit-flip channels

When the two local noise channels are both bit-flip channels, the Kraus operators of channels for the system are given by

$$E^{(A)}_0 = \sqrt{1 - \frac{p}{2}} I_A \otimes I_B, \quad E^{(A)}_1 = \sqrt{\frac{p}{2}} \sigma_A^1 \otimes I_B,$$

$$E^{(B)}_0 = I_A \otimes \sqrt{1 - \frac{p}{2}} I_B, \quad E^{(B)}_1 = I_A \otimes \sqrt{\frac{p}{2}} \sigma_B^1. \quad (25)$$

If the initial state is Bell-diagonal state (Eq.(7)), under this noise channel, the coefficients may decay to be

$$c_1(t) = c_1, \quad c_2(t) = (1 - p)(1 - p')c_2, \quad c_3(t) = (1 - p)(1 - p')c_3. \quad (26)$$

(3) Two independent local bit-phase-flip channels

When the two local noise channels are both bit-phase-flip channels, the Kraus operators are

$$E^{(A)}_0 = \sqrt{1 - \frac{p}{2}} I_A \otimes I_B, \quad E^{(A)}_1 = \sqrt{\frac{p}{2}} \sigma_A^2 \otimes I_B,$$

$$E^{(B)}_0 = I_A \otimes \sqrt{1 - \frac{p}{2}} I_B, \quad E^{(B)}_1 = I_A \otimes \sqrt{\frac{p}{2}} \sigma_B^2. \quad (27)$$

The initial state shown in Eq.(7) under these noise channels can be evolved to another Bell-diagonal state (Eq.(23)) with the coefficients

$$c_1(t) = (1 - p)(1 - p')c_1, \quad c_2(t) = c_2, \quad c_3(t) = (1 - p)(1 - p')c_3. \quad (28)$$

The three noise channels discussed above evolve one Bell-diagonal state to another Bell-diagonal state by decaying coefficients $\{c_i\}$. From Eq.(11) and (12), one can see that the classical correlation is dependent on the maximal value of coefficients $\{|c_i|\}$, and the quantum correlation is the difference of mutual information and classical correlation. The decay rate of classical correlation depends on the decay rate of maximal coefficients $\{|c_i(t)|\}$. For example, if the noise of local channels are both phase-flip noise, the dynamics of classical and quantum correlations have three conditions as in Ref.[24].

(a) When $|c_3| = 0$, the quantum correlation and classical correlation decay monotonically.

(b) When $|c_3| \geq \{|c_1|, |c_2|\}$, the classical correlation will not change in this process, and quantum correlation has the same decay rate as mutual information.

(c) When $|c_2| \geq \{|c_1|, |c_3|\}$, and $|c_3| \neq 0$, the classical correlation decays with the diminishing of $c_2(t)$ before the point $c_3 = (1 - p)(1 - p')c_2(3) = (1 - p)(1 - p')c_1$, and then classical correlation suddenly changes to a constant. While the quantum correlation in this condition has the decay rate which is suddenly changed at the same point $c_3 = (1 - p)(1 - p')c_2(3) = (1 - p)(1 - p')c_1$. Figure 1 shows the dynamics of mutual information, quantum correlation and classical correlation for the symmetry phase-flip channels. The relation of decay rates of the two local channels is $p' = xp$. The decay rates of quantum and classical correlations become larger and the sudden change point is moved forward with $x$ changing from 0 to 1. For some special cases $c_1 = k, c_2 = -c_3k$ and $|k| > |c_3|$, the sudden transition between classical and quantum decoherence takes place, as shown in Fig 2.

The three symmetry noise channels with phase-flip, bit-flip and bit-phase-flip are equivalent to local unitary operations. Therefore, the dynamics of classical and quantum correlations under the other two channels with the time parameters $p$ and $p'$ are similar to those in the case with symmetry phase-flip channels. For bit-flip channels, the relation of coefficients for three regimes are $|c_1| = 0, |c_1| \geq \{|c_2|, |c_3|\}$ and $|c_2| \geq \{|c_1|, |c_3|\}$ ($|c_3| \geq \{|c_1|, |c_2|\}$). For bit-phase-flip channels, the relation of coefficients for three regimes are $|c_2| = 0, |c_2| \geq \{|c_1|, |c_3|\}$ and $|c_1| \geq \{|c_2|, |c_3|\}$ ($|c_3| \geq \{|c_1|, |c_2|\}$).
FIG. 1: (Color online) The dynamics of mutual information (a), classical correlation (b), and quantum correlation (c) of a two-qubit system in the state \( \rho \) (Eq.(23)) with initial coefficients \( c_1 = 0.1, c_2 = 0.5, c_3 = 0.3 \) under the bi-side phase-flip channels. Here \( x = p'/p \) represents the relation of two parameter times of two phase-flip channels.

FIG. 2: (Color online) The dynamics of mutual information (a), classical correlation (b), and quantum correlation (c) of a two-qubit system in the state \( \rho(t) \) (Eq.(23)) with initial coefficients \( c_1 = 1, c_2 = -0.5, c_3 = 0.5 \) under the bi-side phase flip channels. Here \( x = p'/p \) represents the relation of two parameter times of two phase-flip channels.

3.2 Two local channels are different types

In this section, we discuss the dynamics of correlations of a two-qubit system in a Bell-diagonal state under two different local Markovian noise channels. That is, the noise channels interacting with the two qubits are different types and have different decoherence rates. The local Markovian noise channels discussed here are still phase-flip channel, bit-flip channel, and bit-phase-flip channel, respectively.

(1) A bit-flip channel and a phase-flip channel

When the two local Markovian noise channels are a bit-flip channel and a phase-flip channel, the Kraus operators for two-qubit system can be written as

\[
E_0^{(A)} = \sqrt{1 - \frac{p}{2}} I_A \otimes I_B, \quad E_1^{(A)} = \sqrt{\frac{p}{2}} \sigma_A^1 \otimes I_B, \\
E_0^{(B)} = I_A \otimes \sqrt{1 - \frac{p'}{2}} I_B, \quad E_1^{(B)} = I_A \otimes \sqrt{\frac{p'}{2}} \sigma_B^3. \tag{29}
\]
The coefficients of the final two-qubit Bell-diagonal state are
\[
\begin{align*}
c_1(t) &= (1 - p')c_1, \\
c_2(t) &= (1 - p)(1 - p')c_2, \\
c_3(t) &= (1 - p)c_3.
\end{align*}
\] (30)

(2) A bit-flip channel and a bit-phase-flip channel

When the two local channels are a bit-flip one and a bit-phase-flip one, the Kraus operators for these channels are
\[
\begin{align*}
E_0^{(A)} &= \sqrt{1 - \frac{p}{2}} I_A \otimes I_B, & E_1^{(A)} &= \sqrt{\frac{p}{2}} \sigma_A^1 \otimes I_B, \\
E_0^{(B)} &= I_A \otimes \sqrt{1 - \frac{p}{2}} I_B, & E_1^{(B)} &= I_A \otimes \sqrt{\frac{p'}{2}} \sigma_B^3.
\end{align*}
\] (31)
The coefficients of the final two-qubit Bell-diagonal state decay to be
\[
\begin{align*}
c_1(t) &= (1 - p')c_1, \\
c_2(t) &= (1 - p)c_2, \\
c_3(t) &= (1 - p)(1 - p')c_3.
\end{align*}
\] (32)

(3) A phase-flip channel and a bit-phase-flip channel

When the two local channels are a phase-flip channel and a bit-phase-flip channel, the Kraus operators of these channels are
\[
\begin{align*}
E_0^{(A)} &= \sqrt{1 - \frac{p}{2}} I_A \otimes I_B, & E_1^{(A)} &= \sqrt{\frac{p}{2}} \sigma_A^3 \otimes I_B, \\
E_0^{(B)} &= I_A \otimes \sqrt{1 - \frac{p}{2}} I_B, & E_1^{(B)} &= I_A \otimes \sqrt{\frac{p'}{2}} \sigma_B^2.
\end{align*}
\] (33)
The coefficients of the final two-qubit Bell-diagonal state are
\[
\begin{align*}
c_1(t) &= (1 - p)(1 - p')c_1, \\
c_2(t) &= (1 - p)c_2, \\
c_3(t) &= (1 - p')c_3.
\end{align*}
\] (34)
The two-side noise channels discussed above have different types and decoherence rates for the two qubits in the system. These three kinds of noise channels evolve one Bell-diagonal state to another Bell-diagonal state and change the coefficients \(\{c_i\}\) yet. Different from the cases under the noises with the same types, the correlations are not symmetry for the parameter times \(p\) and \(p'\). Here the relation of parameter times of two local channels is \(p' = xp\) and \(0 \leq x \leq 1\).

If the noise channels of the two sides are a bit-flip channel and a phase-flip channel, there are still three conditions with the different relation of coefficients \(\{c_i\}\).

(a) When \(|c_1| \geq \{|c_2|, |c_3|\}\), the relation of the coefficients \(\{c_i(t)\}\) are \(|c_1(t)| \geq \{|c_2(t)|, |c_3(t)|\}\), and both the quantum correlation and classical correlation decay monotonically.

(b) When \(|c_3| \geq \{|c_1|, |c_2|\}\) \((|c_2| \geq |c_1| \geq |c_3|)\), the decay rates of the quantum correlation and classical correlation are both suddenly changed at the point \(c_1(1 - p') = (1 - p)c_3\) \((c_2(1 - p) = c_1)\). In the case \(|c_3| \geq \{|c_1|, |c_2|\}\), the sudden change point is moved backward and the decay rates of quantum correlation and classical correlation become larger with \(x\) changing from 0 to 1. In the case \(|c_2| \geq |c_1| \geq |c_3|\), the sudden change point is not affected by \(x\), while the decay rates of quantum correlation and classical correlation still become larger with \(x\) changing from 0 to 1.

(c) When \(|c_2| \geq |c_3| \geq |c_1|\), the sudden change points are determined by \(x\). If \(x \leq (c_2 - c_3)/(c_2 - c_1)\), the decay rates of quantum correlation and classical correlation are suddenly changed at the point \(c_2(1 - p) = c_1\). This sudden change point is not affected by \(x\), while the decay rates of quantum correlation and classical correlation become larger with \(x\) changing from 0 to \((c_2 - c_3)/(c_2 - c_1)\). If \(x \geq (c_2 - c_3)/(c_2 - c_1)\), the decay rates of quantum correlation and classical correlation have two sudden change points at \(c_2(1 - p') = c_3 \) and \(c_3(1 - p) = c_1(1 - p')\). The sudden...
Fig. 3: (Color online) The dynamics of mutual information (a), classical correlation (b), and quantum correlation (c) of a two-qubit system in the state $\rho(t)$ with the parameters $c_1 = 0.1$, $c_2 = 0.5$, $c_3 = 0.3$ under the bi-side Markovian noise composed of a bit-flip channel and a phase-flip channel. Here $x = p'/p$ represents the relation of two parameter times of two local noise channels.

change point $c_3(1 - p') = c_3$ is moved forward and the decay rates of quantum correlation and classical correlation become larger with $x$ changing from $(c_2 - c_3)/(c_2 - c_1)$ to 1. The sudden change point $c_3(1 - p) = c_1(1 - p')$ is moved backward and the decay rates of quantum correlation and classical correlation become larger with $x$ changing from $(c_2 - c_3)/(c_2 - c_1)$ to 1.

The three noise channels with phase-flip, bit-flip, and bit-phase-flip are equivalent to local unitary operations. Therefore, the dynamics of classical and quantum correlations under the other two channels with parameter times $p$ and $p'$ are similar to the noise channels with a bit-flip channel and a phase-flip channel. For the condition of a bit-flip and a bit-phase-flip channels, the relation of coefficients for three regimes are $|c_1| \geq |c_2|, |c_3|$, $|c_2| \geq |c_1|, |c_3|$, $|c_3| \geq |c_1|, |c_2|$, and $|c_3| \geq |c_2| \geq |c_1|$. For the condition of a phase-flip and a bit-phase-flip channels, the relation of coefficients for three regimes are $|c_3| \geq |c_1|, |c_2|$, $|c_2| \geq |c_1|, |c_3|$, $|c_1| \geq |c_3| \geq |c_2|$, and $|c_1| \geq |c_2| \geq |c_3|$.

4 Correlation dynamics of a two-qubit system in a Bell-diagonal state under non-Markovian noise

As introduced in Sec. 2.3, the Markovian channel with phase-flip noise, bit-flip noise, and bit-phase-flip noise can be generalized to non-Markovian channel with parameter time oscillating as $p = 1 - \Lambda(t)$. Due to the Kraus composition form of non-Markovian channels, the finial state after operating on Bell-diagonal state satisfies Eq.(23). Here we still investigate the noise channels of two sides in two conditions, bi-side non-Markovian noise channels with the same type or different types.

(1) Bi-side non-Markovian noise channels with the same type

Here the three local non-Markovian channel are bit-flip, phase-flip, and bit-phase-flip channels oscillating with time parameter $\nu$. The non-Markovian noise channels of two sides are the same type with different time parameter $\nu' = x\nu$. As discussed in Sec. 3.1, as the three types of noise channels are equivalent to each other, we focus on the condition of bi-side non-Markovian phase-flip channels.

Using the Kraus operators in Eq.(21), the finial two-qubit Bell-diagonal state in Eq.(23) can be calculated with coefficients evolving as

$$
c_1(t) = \Lambda(\nu)\Lambda(\nu')c_1,
$$

$$
c_2(t) = \Lambda(\nu)\Lambda(\nu')c_2,
$$

$$
c_3(t) = c_3.
$$

The dynamics of correlations can still be distinguished to three regimes, as the same as Markovian channels corresponding to the relation of coefficients $\{c_i\}$.

(a) $|c_3| = 0$. The mutual information, quantum correlation and classical correlation display damped oscillations and decay asymptotically to zero.
FIG. 4: (Color online) Dynamics of mutual information (a), classical correlation (b), and quantum correlation (c) of two-qubit system under noise channel of which two sides are both non-Markovian phase-flip channels. The initial coefficients of two-qubit system are $c_1 = 0.1$, $c_2 = 0.5$, $c_3 = 0.3$, and the parameters of non-Markovian phase-flip channel are $\tau = 5s$, $a = 1s^{-1}$. Here $x = \nu'/\nu$ represents the relation of two parameter times of two non-Markovian phase-flip channels.

FIG. 5: (Color online) Dynamics of mutual information (a), classical correlation (b), and quantum correlation (c) of two-qubit system under noise channel of which two sides are both non-Markovian phase-flip channels. The initial coefficients of two-qubit system are $c_1 = 1$, $c_2 = -0.5$, $c_3 = 0.5$, and the parameters of non-Markovian phase-flip channel are $\tau = 5s$, $a = 1s^{-1}$. Here $x = \nu'/\nu$ represents the relation of two parameter times of two non-Markovian phase-flip channels.

(b) $|c_3| \geq \{|c_1|, |c_2|\}$. The classical correlation will not change in this regime, and quantum correlation has the same damped oscillations with mutual information.

(c) $|c_2| \geq \{|c_1|, |c_3|\} (|c_1| \geq \{|c_2|, |c_3|\})$ and $|c_3| \neq 0$. The classical correlation decays with damped oscillation of $c_2(t) (c_1(t))$ and changes to be a constant at sudden change points $c_3 = \Lambda(\nu)\Lambda(\nu')c_2$ ($c_3 = \Lambda(\nu)\Lambda(\nu')c_1$). The quantum correlation has decay rate changed suddenly at these sudden change points. As shown in Fig.5 the classical and quantum correlations decrease to the sudden change point, and then classical correlation becomes constant while quantum correlation has a discontinuous change with the changing of damping oscillation rule. There is a special case with $c_1 = k$, $c_2 = -c_3 k$ and $|k| > |c_1|$ as shown in Fig.5, the quantum correlation may become constant at a sudden change point which is called frozen discord in Ref. [35]. Corresponding to the sudden transition between classical and quantum decoherence of Markovian channels, the transition between classical and quantum damping oscillations happens.

(2) Bi-side non-Markovian noise channels with different types
Now we discuss the dynamics of correlations of two local non-Markovian noise channels with different types operating on a two-qubit Bell-diagonal state. The three types of non-Markovian channels are bit-flip, phase-flip, and bit-phase-flip channels oscillating with parameter time $\nu$. We focus on two-side noise channels composed of a bit-flip and a phase-flip channel with different parameter time $\nu' = x\nu$.

The final Bell-diagonal state after this noise channel is

$$
c_1(t) = \Lambda(\nu')c_1,
$$

$$
c_2(t) = \Lambda(\nu)\Lambda(\nu')c_2,
$$

$$
c_3(t) = \Lambda(\nu)c_3. \tag{36}
$$

The dynamics of correlations is determined by the coefficients $\{c_i\}$ as we discussed above. If $x = 0$, the parameter time of a phase-flip channel is $\Lambda(0) = 1$, and the noise channel becomes a one-side noise channel. For this condition, the dynamics of correlations in three regimes are the same as bi-side non-Markovian noise channels with the same type. If $x \neq 0$, the oscillation periods of $\Lambda(\nu)$ and $\Lambda(\nu')$ are decided by the parameter times of the two local channels $\nu$ and $\nu'$. The different oscillation periods of $\Lambda(\nu)$ and $\Lambda(\nu')$ implies the uncontinuous changes of both classical and quantum correlations may happen in any relation regime of coefficients $\{c_i\}$.

5 Discussion and conclusion

In this article, we have studied the dynamics of correlations of a two-qubit system in a Bell-diagonal state under independent local noise channels of which the two sides are not identical. With the anti-commute relation of Pauli operators, the Bell-diagonal state has the coefficients $\{c_i\}$ decreased when the bi-side local noise channels are phase-flip channel, bit-flip channel, and bit-phase-flip channel, respectively. The classical correlation is determined by the maximal value of $\{|c_i(t)|\}$.

Under Markovian noise channels, the dynamics of correlations decrease monotonically. If the two sides of noise channels are the same type, there are three regimes for the decay rates, dependent on the coefficients $\{c_i\}$. In the first regime, both the classical and quantum correlations decay monotonically asymptotically to zero. In the second regime, the classical correlation keeps to be a constant, while the quantum correlation decreases with the same decay rate as mutual information. In the third regime, classical correlation decreases monotonically till the sudden change point changing to a constant, while the quantum correlation has the decay rate changed suddenly at the sudden change point. If the two sides of noise channels are the different types, there are also three regimes for the decay rates with relation of coefficients $\{c_i\}$. In the first regime, both the classical and quantum correlations decay monotonically. In the second regime, both the classical and quantum correlations have decay rates sudden changed at the sudden change point. In the third regime, there appears the condition of two sudden change points. The sudden change point is affected by the parameter $x$, which describes the relation of two time parameters $p$ and $p'$, and the regime of $x$ for two sudden change points is calculated.

When the two sides are non-Markovian channels, correlations may display damped oscillations. If the two sides of noise channels are the same type, there are still three regimes for the decay rates, dependent on the coefficients $\{c_i\}$. In the first regime, both the quantum correlation and classical correlation display damped oscillations and decay asymptotically to zero. In the second regime, the classical correlation keeps a constant while the quantum correlation has the same damped oscillations with mutual information. In the third regime, classical correlation may be a constant in some time intervals between sudden change points, while quantum correlation has uncontinuous changes with the changing of damping oscillation rule at the sudden change points. If the two sides of noise channels are the different types, both classical and quantum correlations have uncontinuous changes with the changing of damping oscillation rule at the sudden change points when the parameter times of the two local channels are different. The sudden change points are affected by parameter $x$ which describes the relation of time parameters $\nu$ and $\nu'$.

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