On thermal force from holographic action

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Abstract: Applying the relation between Euclidean on-shell action in the bulk and free energy on the holographic screen to a test charged particle in the charged R-N black hole, we show that not only gravity but also electromagnetic force can be regarded as a sort of thermal force, which accomplishes the unification of gravity and electromagnetism to some extent. In addition, taking into account the fact that both the temperature and thermal force of the dual system are measured at infinity, we argue that the dual holographic screen may be located at infinity.
1. Introduction

Inspired by the emergent phenomenon in the holographic scenario, Erik Verlinde proposed that the origin of gravity can be interpreted as a thermal force caused by changes in the information associated with the positions of material bodies, i.e.,

$$f = T \frac{\Delta S}{\Delta x},$$

(1.1)

where $T$ and $S$ are the temperature and entropy related to the holographic screen, respectively. Since it is related to variation of entropy, this thermal force is usually called entropic force. Such an entropic force proposal has stimulated much effort on its further clarifications and possible applications.

In particular, recently Yue Zhao provides a clear-cut support for such a hand-waving proposal by exploiting the relation between on-shell Euclidean action of gravity theory and partition function for the dual theory.

In this paper, employing such a holographic duality, we shall further show that gravity and electromagnetic force experienced by a charged particle in the charged R-N black holes can be unified as a sort of thermal force. As a bonus, our results also suggests that the dual holographic screen may be located at infinity.

Before proceeding, it is noteworthy that the above thermal formula (1.1) is only valid for the micro-canonical ensemble. When the canonical ensemble is concerned, it follows from thermodynamics that the thermal force should be replaced by

$$f = -\frac{\Delta F}{\Delta x},$$

(1.2)

where $F$ is the corresponding free energy. In addition, Natural Units will be employed here, i.e., $k_B = G = c = \hbar = 1$.

\footnote{Electromagnetic force as a thermal force is also reached by Tower Wang, but the methodology is different.}
2. Thermal force from holographic action

Start from the charged R-N black hole

\[ ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + r^2 d\Omega^2, \]  

(2.1)

where

\[ g_{tt} = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right), \]

\[ g_{rr} = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} \]  

(2.2)

with \(M\) and \(Q\) the mass and charge of black hole, respectively. The corresponding electromagnetic potential reads

\[ A_a = -\frac{Q}{r}(dt)_a. \]  

(2.3)

Now consider a test particle of mass \(m\) and charge \(q\) in such a background with the action given by

\[ S_p = \int d\lambda (-m\sqrt{-g_{\mu\nu}} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} + qA_\sigma \frac{dx^\sigma}{d\lambda}), \]  

(2.4)

which does not depend on the choice of parameter \(\lambda\). In what follows, we will choose \(\lambda = t\). In addition, in order to hold the dual system in equilibrium, we also require the worldline of this charged particle to be the orbit of the time-like Killing vector field \((\frac{\partial}{\partial t})^a\). Thus the on-shell action of the particle can be written as\(^2\)

\[ S_p = \int dt (-m\sqrt{-g_{tt}} + qA_t). \]  

(2.5)

To proceed, we first go to the Euclidean section from the Lorentzian section by setting \(\tau = it\). Then the Euclidean R-N black hole is given by

\[ ds^2 = g_{\tau\tau} d\tau^2 + g_{rr} dr^2 + r^2 d\Omega^2 \]  

(2.6)

with \(g_{\tau\tau} = -g_{tt}\). As is well known, to regularize the conical singularity appearing in this section, \(\tau\) need to have a period of \(\beta\) given by

\[ \beta = 2\pi \sqrt{g_{\tau\tau}} |_{\text{horizon}} = 4\pi \sqrt{g_{\tau\tau} g_{rr}} |_{\text{horizon}}. \]  

(2.7)

\(^2\)Note that in later calculations the zero component of four potential \(A_t\) will be regarded as a scalar field.
which actually corresponds to the inverse of temperature $T$ of the dual system, measured at infinity. Since the back-reaction of the test particle is neglected here, the temperature of the dual system keeps unchanged, which implies the system should be described by the canonical ensemble. Now by the holographic duality, the free energy of the dual system is related to the on-shell Euclidean action of the gravity system as

$$F = T(S_G + S_p), \quad (2.8)$$

where the Euclidean action of the test particle $S_p$ is given by

$$S_p = \int d\tau (-m\sqrt{-g_{tt}} + qA_t) = \frac{-m\sqrt{-g_{tt}} + qA_t}{T}, \quad (2.9)$$

and $S_G$ is the holographic action for the Euclidean background, independent of the position of the test particle$^{[27, 28, 29, 30]}$. Thus Eq.(1.2) follows that the thermal force induced by the change of position of the test particle can be written as

$$f_{\text{th}}^a = -\nabla_a F = \nabla_a (m\sqrt{-g_{tt}} - qA_t) = (\frac{m}{2}\frac{\partial_t g_{tt}}{\sqrt{-g_{tt}}} + q\partial_t A_t)(dr)_a, \quad (2.10)$$

On the other hand, taking into account that the total force and electromagnetic force experienced by the test particle read$^{[31]}$

$$f_{\text{tot}}^a = m\frac{1}{\sqrt{-g_{tt}}}\nabla_a \sqrt{-g_{tt}},$$

$$f_{\text{em}}^a = qF_{ab}\frac{1}{\sqrt{-g_{tt}}} (\frac{\partial}{\partial t})^b = q\frac{1}{\sqrt{-g_{tt}}} \nabla_a A_t, \quad (2.11)$$

we can obtain the external force exerted on the test particle as

$$f_{\text{ex}}^a = f_{\text{tot}}^a - f_{\text{em}}^a = -f^g - f_{\text{em}} = \frac{1}{\sqrt{-g_{tt}}} f_{\text{th}}^a, \quad (2.12)$$

where we have used $f^g = -f_{\text{tot}}^a$ when we view the curved spacetime as an effect of force induced by gravity. Note that the thermal force differs from the force exerted locally by the redshift factor, so it can be identified as the force exerted at infinity$^{[31]}$, i.e.,

$$F_{\text{th}} = F_{\text{ex}}^\infty, \quad (2.13)$$

which along with the temperature of the dual system suggests that the dual holographic screen where the dual system lives may be located at infinity. In addition, if we further associate the gravity and electromagnetism relevant free energy with the first and second term in Euclidean action (2.9), then the above equality implies that not only gravity but also electromagnetic force can be regarded as a sort of thermal force. In this sense, the unification of gravity and electromagnetism is realized on the dual holographic screen, although it does not mean that we does not need quantum gravity in the bulk, as we have had quantum electrodynamics.
3. Discussions

Applying the holographic duality, we have demonstrated that gravity and electromagnetic force can be unified as a sort of thermal force on the dual screen. In addition, both the temperature and thermal force suggest that the dual holographic screen may be located at infinity.

Although we focus ourselves onto the charged R-N black holes for simplicity, our result should be applicable to general K-N black holes. In addition, note that the temperature drops out in our calculation of the thermal force, thus it is interesting to investigate whether the thermal force interpretation may also be generalized to arbitrary asymptotically flat stationary spacetime with stationary electromagnetic field, even more general spacetime. On the other hand, since the weak and strong force share the same structure as the electromagnetic force, it is not difficult to imagine that the weak and strong force can also be interpreted as a sort of thermal force where the concept of force is suitable for use. We expect to report all of these issues in the near future.

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