Beat vibration mechanism of a sluice pier under high-speed flood discharge excitation

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Abstract
The sluice pier prototype vibration test conducted in China Shuhe Hydropower Station shows that the vibration response of the measuring points exhibits a special form of vibration, which demonstrates intermittency and impact under high-speed flood discharge excitation. The waveform envelope increases or decreases sharply with time and shows the formation of beat vibration. Research on the mechanism of this type of vibration is rare. To analyze the characteristics and causes of the obvious beat phenomenon in the testing of the sluice pier prototype, we conduct research in the following manner. First, mathematical models of synthetic signals are built to analyze the influences of initial phase difference, frequency ratio, and amplitude ratio and determine the beat vibration formation condition. Second, the stochastic subspace identification method is used to identify the operational modal parameters (including the vibration order, frequency, damping ratio, and mode of vibration) on the basis of the measured prototype dynamic response. Last, the internal causes of the beat vibration phenomenon are analyzed based on the combination of the formation conditions of apparent beat vibration. Results show that the vibration of Shuhe sluice pier is dominated by the first two frequency components. The vibration frequency ratio is between 0.928 and 0.962 and satisfies the necessary conditions of beat vibration. The amplitude ratio is between 0.66 and 1.63 and thus makes the waveforms of beat vibration highly evident.

Keywords
High-speed flood discharge, sluice pier, beat vibration, prototype test, modal identification

Introduction
A sluice guarantees the safe operation of hydro-junctions during flood discharge periods. Random vibration often occurs in sluice piers during flood discharge periods because a sluice is usually designed as a light thin-wall structure. However, a strong beat vibration phenomenon that possesses the features of intermittency and impact was observed in the Shuhe sluice pier prototype vibration test during flood discharge periods, and the waveform envelopes of the vibration response increased or decreased dramatically over time. The vibration amplitude of sluice piers caused by beat vibration increases significantly in certain periods and thus affects the normal operation of the sluice. This type of vibration occurred in the spillway guide wall of the Wujiangdu Hydropower Station (located in the main stream of Wujiang River in Guizhou Province, China) in 1984. Especially for the Shuhe Hydropower Station (located in the main stream of Hanjiang River in Shaanxi Province, China), it has affected the normal operation of this project. Beat vibration is a new form of vibration in hydraulic field, so it is necessary to analyze the causes of it. However, the mechanism of this beat vibration has
not been explained because of the lack of observation data. Thus, exploring the characteristics and formation mechanism of this type of vibration is important for the safe operation and management of the projects and the design of new hydraulic projects.

Several scholars have investigated the physical definition and formation mechanism of beat vibration. The synthesis conditions of beat vibration can be obtained with the rotating vector method.\(^1\) Simulation analysis and experiments have also been conducted on small frequency differences in beat vibration to extend the traditional theory of two-beat vibration to multi-source beat vibration and thereby obtain the rangeability and fundamental frequencies of such vibration.\(^2\) Studies on beat vibration in the Ertan arch dam (located in the Yalong River in Sichuan Province, China) under flood discharge excitation have proven that the phenomenon appearing in a part of the measuring points on the arch ring is caused by the first two mode shapes of the arch dam. The phenomenon of random vibration is generated because of the low frequency of water flow load and the first two working frequencies of the dam being in the scope of the flow load frequency, but the calculation of the first two vibration amplitudes has not been fully explored.\(^3\)–\(^5\) The problems of beat vibration and resonance in hydraulic machinery have been discussed, and the essential features of beat vibration and resonance and the differences in disturbing and inherent frequencies have been analyzed. As a result, a new perspective toward beat vibration and resonance production has been proposed.\(^6\) A previous study conducted a theoretical analysis and calculation of the stability of a hydro-automatic flap gate to determine the causes of self-excited unstable vibration under the hydraulic conditions of sluice gates.\(^7\) In engineering vibration, vibration forms, such as beat, are not new. Studies have proven that when the high-frequency components of the wave force are close to or integral multiples of the natural frequencies of the structure, the structure exhibits high-frequency resonance under the action of extreme waves. This form of vibration is detected regularly in the time history of the measured wind vibration response.\(^8\) The relationship between the vibration characteristics of cables in cable-stayed spatial structures and tower columns, cables, and space structures has also been investigated. When the natural frequencies between tower columns and spatial structures are approximately the same but slightly different from those of cables, the vibrations of the cables present the characteristics of “beat.”\(^9\) The phenomenon of beat vibration in the coupling system can be explained well by introducing the mass matrix into the structure–fluid damping coupling system.\(^10\) In this study, Shuhe Hydropower Station was used to analyze the phenomenon of beat vibration occurring in the measurements of the sluice pier. The formation conditions of beat vibration were determined by constructing a mathematical model, and the characteristics of such vibration were analyzed based on the results of modal parameters identified and measured on the sluice pier. The reasons for such characteristics were also provided.

**Phenomenon of beat vibration on the sluice pier of Shuhe Hydropower Station**

**Basic situation of prototype tests**

Shuhe Hydropower Station is located on Hanjiang River in Shanxi Province, China. It is one of the large-scale step hydropower stations on Hanjiang River. The sluice and its powerhouses are arranged separately, with the powerhouse dam section on the left side and the spillway dam section on the right side. Five sluice gates are arranged from left to right and numbered #1–#5. The prototype hydropower station is shown in Figure 1.

![Figure 1. Shuhe Hydropower Station project.](image)
The width and height of the sluice orifice are 13 and 23.8 m, respectively, and the length of the lock chamber is 54 m. The bottom is designed with a broad crest weir. The altitude of the weir top is 193.5 m, and the thickness and the height of the pier are 4.5 m and 36.5 m.

The right and left sluice piers of the #2 sluice orifice were selected as the test objects in the prototype vibration tests. Sensors are arranged by using QR decomposition and modal assurance criterion (MAC).\textsuperscript{11} As shown in Figure 2, 10 measuring points were arranged on the top of the sluice piers. Six measuring points were on the left side and numbered H1–H6, and the stake numbers were 0+00, 0+05, 0+16, 0+27, 0+39, and 0+49. Four measuring points were on the right side and numbered H7–H10, and the stake numbers were 0+00, 0+16, 0+27, and 0+39. The direction of the dynamic displacement of the test was transverse to the flow direction (lateral). The vibration sensor was a DP-type dynamic displacement sensor (Figure 3). The DP-type is a seismic transducer consisting of a seismic pick-up of moving coil type which has a much higher natural frequency and a low frequency extension (correcting) circuit. The output characteristics of DP-type sensor are similar to those transducer, but the natural frequency is 1/20–1/100 of the pick-up used, and the range of low frequency measurement is effectively extended. Sensor sensitivity can be adjusted according to the requirements. The sensor contains an integrating circuit, and its output will be proportional to the vibration amplitude. The physical principle for this sensor is explained in these works.\textsuperscript{12–14} The frequency response range of the sensor was 0.5–200 Hz, and the sensitivity was 5 mV/μm. The Data Acquisition and Signal Processing (DASP) system (Figure 4) was used at a sampling frequency of 50 Hz. In the three tested working conditions, the duration of the collected time series is 81.9 s, and the time interval of the adjacent sample points is 0.02 s. These conditions are listed in Table 1.

![Figure 2. Sensor arrangement on the sluice pier.](image)

![Figure 3. DP-type dynamic displacement sensor.](image)
Beat vibration waveforms of typical measuring points

By analyzing the time history line of the dynamic displacement responses of the measuring points on the sluice pier under conditions 1–3, we detected a phenomenon similar to beat vibration at several measuring points, including point H4 on the left pier and point H9 on the right pier on condition 1, point H3 on the left pier and point H9 on the right pier on condition 2, and point H3 on the left pier and point H8 on the right pier on condition 3, as shown in Figures 5 to 10. The maximum amplitudes under three conditions are listed in Table 2.

These figures show that the time history of dynamic displacement responses on the measuring points of the left and right sluice pier exhibited considerable synchronization under the same condition. Obvious beat vibration also occurred in the time history curves of response on the selected measuring points under all conditions. Mathematical models of the vibration signals were constructed, the influencing factors were analyzed, and the formation conditions were determined to examine the characteristics and causes of the phenomenon.

Formation conditions of beat vibration waveforms

When two columns of simple harmonic vibration in the same direction with negligible difference in frequency are superimposed, the amplitude of the superimposed vibration waveform changes slowly and cyclically over time.
Figure 6. Dynamic displacement time history and power spectral density curve of measuring point H9 on condition 1.

Figure 7. Dynamic displacement time history and power spectral density curve of measuring point H3 on condition 2.

Figure 8. Dynamic displacement time history and power spectral density curve of measuring point H9 on condition 2.

Figure 9. Dynamic displacement time history and power spectral density curve of measuring point H3 on condition 3.
This phenomenon is called “beat.” The two columns of harmonic vibration signals are defined as \(x_1(t)\) and \(x_2(t)\) in this study to analyze the formation conditions of the beat vibration waveforms

\[
x_1(t) = A_1\cos(\omega_1 t + \phi_{10})
\]

\[
x_2(t) = A_1\cos(\omega_2 t + \phi_{20})
\]

where \(A_1\) and \(A_2\) are the amplitudes, \(\omega_1\) and \(\omega_2\) are the angular frequencies, and \(\phi_{10}\) and \(\phi_{20}\) are the initial phases. By using the rotation vector method to superimpose the signals \(x_1(t)\) and \(x_2(t)\), the amplitude \(A\) and angular frequency \(\omega\) of the synthetic signal are obtained as

\[
A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\omega_2 - \omega_1)t + (\phi_{20} - \phi_{10})]}
\]

\[
\omega = \omega_1 + \frac{1 + \beta\cos(\omega_2 - \omega_1)t + (\phi_{20} - \phi_{10})]}{1 + \beta^2 + 2\beta\cos(\omega_2 - \omega_1)t + (\phi_{20} - \phi_{10})}(\omega_2 - \omega_1)
\]

where \(\beta = A_1/A_2\) is the amplitude ratio.

According to equations (3) and (4), the amplitude \(A\) of the synthetic signal changes periodically with the cycle \(T_p = 2\pi/(\omega_2 - \omega_1)\), where \(T_p\) is the cycle of the “beat.” The synthetic amplitude is related to the amplitudes \(A_1\) and \(A_2\), angular frequency difference \(\omega_2 - \omega_1\), and initial phase difference \(\phi_{20} - \phi_{10}\) of the signal before synthesis. The angular frequency \(\omega\) of the synthesized signal is related to the amplitude ratio \(\beta\) of the signal before synthesis. Then, the influence of the three factors on the waveforms of beat vibration is considered, including the initial phase difference \(\phi_{20} - \phi_{10}\), the frequency ratio \(\xi (\xi = \omega_2 - \omega_1)\), and the amplitude ratio \(\beta\).

**Influence of initial phase difference \(\phi_{20} - \phi_{10}\) on the synthetic waveforms of beat vibration**

This study analyzed the influence of the initial phase difference \(\phi_{20} - \phi_{10}\) on synthetic vibration, with amplitude ratio \(\beta\) being equal to 1 (\(A_1 = A_2 = 12\)) and angular frequencies \(\omega_1\) and \(\omega_2\) being equal to \(4\pi\) and \(4.4\pi\), respectively. The superimposed waveforms when the initial phase difference is equal to 0 and \(\pi\) are shown in Figure 11. The change in the initial phase difference exerted no effect on the amplitude and frequency of the synthesized
Influence of frequency ratio $\zeta$ on the synthetic waveforms of beat vibration

Amplitude ratio $\beta$ should be equal to 1 ($A_1 = A_2 = A_0$). The analysis of the upper section showed that the initial phase difference had no effect on the appearance of beat vibration after synthesis, such that the initial phase difference was 0, i.e. $\phi_{10} = \phi_{20} = 0$. Thus, the equation of the synthesized signals is obtained as follows

$$x(t) = 2A_0\cos[(\omega_2 - \omega_1)/2t] \cdot \cos[(\omega_2 + \omega_1)/2t] \quad (5)$$

Using equation (5), the time-varying cycle $T_1$ of the synthesized signal is $4\pi/(\omega_1 + \omega_2)$. The mathematical model simulation tests showed that the obvious beat vibration occurred in the synthesized waveforms only when a cycle of “beat” $T_p = 2\pi/(\omega_2 - \omega_1)$ contained several synthetic signal waveform cycles $T_1$. Assuming that the minimum value of this number is $n$, the condition expression for an apparent beat vibration of the synthetic signal can be expressed as follows

$$T_p \geq nT_1 \quad (6)$$

According to $\zeta = \omega_2/\omega_1$, the following expression can be obtained

$$\frac{|1 - \zeta|}{1 + \zeta} \leq 2n \quad (7)$$

Then, frequency ratio $\zeta$ should meet the range

$$\frac{2n - 1}{2n + 1} \leq \zeta \leq \frac{2n + 1}{2n - 1} \quad (8)$$

The number $n$ must take different values when observing the waveforms of the synthesized signals to determine the value range of frequency ratio $\zeta$. The superimposed signal waveforms when $n$ equals 2, 3, 4, and 5 were plotted. Figure 12 shows that when $n$ is equal to 2 and 3, the number of $T_1$ cycles in the synthetic signal waveforms is small, and the boundaries of the waveforms are sparse, resulting in an invisible beat vibration. When $n$ is equal to 4 and 5, the waveforms of beat are relatively complete, such that obvious beat vibration is observed. Thus, signal frequency ratio $\zeta$ is the decisive factor in the occurrence of beat vibration after synthesis. According to equation (8), the frequency ratio $\zeta$ of obvious beat vibration is between 0.78 and 1.29 and cannot be equal to 1.

Influence of amplitude ratio $\beta$ on the synthetic waveforms of beat vibration

Amplitude ratio $\beta$ has an effect on the amplitude and frequency of the synthesized signal. Mathematical model simulation tests have shown that the change in the amplitude ratio does not have an effect on the appearance of vibration waveforms and only affects the obvious degree of vibration waveforms by changing the relative
difference between the upper and lower envelopes of the synthetic vibration signals. When the amplitude ratio is slightly different from 1, the relative distance between the upper and lower envelopes is correspondingly reduced, such that obvious beat vibration cannot be observed. Two synthetic signal waveforms with the same frequency ratio but with different amplitude ratios of 0.22 and 1 are shown in Figure 13. When the amplitude ratio is 1, the upper and lower envelopes of the synthetic vibration signal have a large difference, leading to obvious beat vibration. By contrast, when the amplitude ratio is 0.22, the difference is relatively modest; hence, no obvious beat vibration is observed. The amplitude ratio only affects the significance level of beat vibration and is not the dominant factor during the occurrence of vibration. Half the amplitude on the envelope of the waveform is the critical condition that determines whether the phenomenon of beat vibration is obvious or not.1 In other words, beat vibration obviously appears when the amplitude difference between the upper and lower envelopes is greater than half of the upper amplitude; otherwise, the opposite condition is observed. Therefore, amplitude ratio β between 0.3 and 3 for obvious beat vibration can be obtained.

In summary, beat vibration appears in the synthesized signals only when the frequency ratio ξ of the two signals is between 0.78 and 1.29 and is not equal to 1 and when the amplitude ratio β of the two signals is between 0.3 and 3. In the subsequent sections, in allusion to the beat vibration on the sluice of Shuhe Hydropower Station, the modal parameters of the sluice are identified with the stochastic subspace method. The working modal parameters are extracted. Afterward, the causes of this phenomenon that occurred in the prototype test are analyzed.

**Prototype working modal parameter identification of the sluice**

*Stochastic subspace modal identification method based on the order selection of singular entropy*

The stochastic subspace method proposed by Van Overschee15 is an identification method based on linear discrete space equations and is currently one of the most accurate methods to identify structural modal parameters based
on environmental incentives. On this basis, singular entropy was used in this study to determine the vibration order and identify the multi-order modal parameters of the structure accurately. The basic principle is as follows.

The structure being tested is assumed to have $m$ measurement points, and the data length of each measurement point is $j$. These response data are combined to form a $2mi \times j$ Hankel matrix. The row space of the Hankel matrix is divided into two parts, namely, “past” and “future.”

$$\mathbf{Y}_{0|2i-1} = \frac{1}{\sqrt{j}} \begin{bmatrix} y_0 & y_1 & y_2 & \cdots & y_{j-1} \\ y_1 & y_2 & y_3 & \cdots & y_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{i-1} & y_i & y_{i+1} & \cdots & y_{i+j-2} \\ y_i & y_{i+1} & y_{i+2} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & y_{i+3} & \cdots & y_{i+j} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{2i-1} & y_{2i} & y_{2i+1} & \cdots & y_{2i+j-2} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{0|i-1} \\ \mathbf{Y}_{i|2i-1} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_p \\ \mathbf{Y}_f \end{bmatrix}$$ (9)

where $y_i$ is the response of all measuring points at time $i$; subscript $p$ means “past” and subscript $f$ means “future”; and $\mathbf{Y}_{0|i-1}$ means a block of the Hankel matrix consisting of all of the measuring points, with the subscript of the first row of the Hankel matrix starting at the moment 0 and ending at the moment $i-1$.

The QR decomposition method is used to reduce the data of the Hankel matrix, according to projection theory. After the row space of $\mathbf{Y}_f$ is projected to the row space formed by $\mathbf{Y}_p$, the orthogonal projection matrix is expressed as follows

$$\mathbf{O}_i = \mathbf{Y}_f / \mathbf{Y}_p = \mathbf{Y}_p (\mathbf{Y}_p^T \mathbf{Y}_p)^+ \mathbf{Y}_p \in R^{mi \times j}$$ (10)

where $(\mathbf{Y}_p^T \mathbf{Y}_p)^+$ is the Moore–Penrose pseudoinverse of the matrix.

According to stochastic subspace identification (SSI) theory, projection matrix $\mathbf{O}_i$ can be decomposed into the product of observation matrix $\Gamma_i$ and Kalman-filtered state vector $\hat{\mathbf{X}}_i$, and singular value decomposition (SVD) is used to derive projection matrix $\mathbf{O}_i$ as follows

$$\mathbf{O}_i = \mathbf{U} \mathbf{S} \mathbf{V}^T = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix} = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1^T$$ (11)

$$\begin{cases} \Gamma_i = \mathbf{U}_1 \mathbf{S}_1^i \\ \hat{\mathbf{X}}_i = \Gamma_i^+ \mathbf{O}_i \end{cases}$$ (12)

where $\mathbf{U}_1 \in R^{mi \times n}$, $\mathbf{S}_1 \in R^{n \times n}$, $\mathbf{S}_2 = 0$, and $\mathbf{V}_i^T \in R^{n \times j}$.

The state space equations are expressed as follows

$$\begin{bmatrix} \hat{\mathbf{X}}_{i+1} \\ \mathbf{Y}_{i|j} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{X}}_i \\ \mathbf{Y}_{i|j} \end{bmatrix} + \begin{bmatrix} \mathbf{W}_i \\ \mathbf{V}_i \end{bmatrix}$$ (13)

where $\hat{\mathbf{X}}_{i+1}$ is the Kalman-filtered state vector at the next moment, $\mathbf{Y}_{i|j} \in R^{m \times j}$ is the Hankel matrix with only one block, and $\mathbf{W}_i$ and $\mathbf{V}_i$ are residuals.

Given that the Kalman-filtered state vector and output are known and the residual matrix is unrelated to the estimated sequence $\hat{\mathbf{X}}_i$, system matrix $\mathbf{A}$ and output matrix $\mathbf{C}$ are obtained using the state-space equation
(equation (13)), which is solved by means of the least-squares solution, as follows
\[
\begin{bmatrix}
A \\
C
\end{bmatrix} = \begin{bmatrix}
X_{i+1} \\
Y_{i+1}
\end{bmatrix} \bar{X}^+_i
\]  

(14)

The eigenvalue decomposition of system matrix \( A \) is expressed as follows
\[
A = J \Lambda J^{-1}
\]  

(15)

where \( \Lambda = \text{diag}(\lambda_i) \in C^{m \times n}, i = 1, 2, \ldots, n \); \( \lambda_i \) is the eigenvalue of discrete time; and \( J \) is the feature vector matrix of the system, \( J \in R^{n \times n} \).

The relation of eigenvalues between discrete and continuous time systems is expressed as follows
\[
\lambda_{ci} = \frac{\ln \lambda_i}{\Delta t}
\]  

(16)

The relationship among modal eigenvalues \( \lambda_{ci}, \lambda_{ci}^*, \) natural frequency \( k \), and damping ratio \( a \) of the system vibration is expressed as follows
\[
\lambda_{ci} \lambda_{ci}^* = -a_i k_i \pm jk_i \sqrt{1 - a_i^2}
\]  

(17)

The mode shape can be expressed as
\[
H = CJ
\]  

(18)

The excited structural vibration order, which is one of the most important parameters, was obtained using the previously presented method to identify the structural working modal parameters. When the input excitation is unknown, the structural working modal parameters are also unknown. In the SSI method, the structural vibration order is determined by SVD to projection matrix \( O_i \) in equation (11). When the signal is noiseless or has a high signal-to-noise ratio, matrix \( S_1 \) obtained by SVD to projection matrix \( O_i \) can be expressed as follows
\[
S_1 = \text{diag}(\hat{\lambda}_1, \hat{\lambda}_2, \cdots \hat{\lambda}_i, \cdots \hat{\lambda}_{k'}, 0, \cdots 0), \quad (k' < n \text{ and } \hat{\lambda}_i \neq 0, i = 1, 2, \cdots, k')
\]  

(19)

By contrast, when the signal has a low signal-to-noise ratio, matrix \( S_1 \) can be expressed as follows
\[
S_1 = \text{diag}(\hat{\lambda}_1, \hat{\lambda}_2, \cdots \hat{\lambda}_i, \cdots \hat{\lambda}_n), \quad (\hat{\lambda}_i \neq 0, i = 1, 2, \cdots, n)
\]  

(20)

where \( \hat{\lambda}_i (i = 1, 2, \cdots, n) \) is the main diagonal element of the matrix \( S_1 \), i.e. the singular value of projection matrix \( O_i \) in equations (19) and (20). The number of non-zero singular values \( k' \) is regarded as the structural system order in the case of no noise or high signal-to-noise ratio. However, actual engineering signals are disturbed by noise, and all of the main diagonal elements of the matrix \( S_1 \) may be non-zero, which makes it difficult to determine the order.

The amount of information of the structural vibration signal can be objectively reflected by the matrix \( S_1 \), i.e. the more the non-zero main diagonal elements in the matrix are, the more complex the signal components are. Based on this characteristic, singular entropy theory is used to determine the order of the modal system. The definition of singular entropy\(^1^8\) is
\[
E_k = \sum_{i=1}^{k'} \Delta E_i, \quad (k' \leq n)
\]  

(21)
where $k'$ is the order of singular entropy and $\Delta E_i$ is the increment of singular entropy at the order $\hat{i}$, which can be calculated using the following formula

$$\Delta E_i = -\left(\frac{\hat{\lambda}_i}{\sum_{k' = 1}^{n} \hat{\lambda}_{k'}}\right) \cdot \ln \left(\frac{\hat{\lambda}_i}{\sum_{k' = 1}^{n} \hat{\lambda}_{k'}}\right)$$  \hspace{1cm} (22)$$

Assuming that $\sigma_i = \ln(\hat{\lambda}_i/\sum_{k' = 1}^{n} \hat{\lambda}_{k'}), (\hat{i} \leq n)$, the sequence composed of $\sigma_i (\hat{i} = 1, 2, \ldots, n)$ is the singular spectrum obtained by SVD of projection matrix $O$. When the singular entropy increment decreases to the asymptotic value, the characteristic information of the signal is saturated, and the corresponding order of the singular spectrum can be considered an approximation of the system order. In this case, the singular spectrum order is considered the order of the system. For the same vibration response signal, the severity of noise interference exerts no effect on the required order of the singular spectrum for a complete, signal-effective characteristic information extraction, i.e. the order of the system is certain.

### Modal identification results

On the basis of this method, the measured vibration response time history of measuring points H1–H10 on conditions 1–3 of the sluice pier was taken as the input. Then, working modal order determination and modal parameter identification were conducted. The calculated results, including the singular entropy increments changed with orders for the left and right sluice pier of the second hole on the three conditions, are shown in Figures 14(a) to 19(a). From the resultant pictures, the characteristic information of the signals has been saturated when the singular entropy increments are reduced to asymptotic value, and the order of the singular spectrum can be determined to be 7. After eigenvalue decomposition of the system matrix $A$ and removing modeless terms and conjugate terms in the eigenvalues, the conclusion that the order of the system is 3 is obtained. By using frequency stabilization diagrams, the influence of false modes is eliminated. The frequency stabilization diagrams are shown in Figures 14(b) to 19(b). The previously presented three frequencies, mode shapes, and damping ratios for the sluice pier are identified, as shown in Tables 3 to 5.

### Analysis of the causes of beat vibration on the sluice

#### Influence of frequency ratio on the beat vibration of the sluice pier

The modal identification results of the sluice pier on all working conditions showed that the sluice has three main vibration modes, in which the first-order frequency is in the range of 2.31–2.41 Hz, the second-order frequency is in the range of 2.45–2.55 Hz, and the third-order frequency is in the range of 3.76–4.41 Hz. The power spectral density curves of the vibration response on the typical measuring points under each working condition showed that most of the vibration energy on the sluice pier is concentrated in the frequency range of 2.3–2.4 Hz, which means that the sluice pier vibration is mainly dominated by the first two vibration modes. The first two vibration frequency ratios of the sluice pier are shown in Table 6, and the value range is between 0.928 and 0.962. The
frequency coupling conditions (0.78 ≤ ξ ≤ 1.29 and ξ ≠ 1), which meet the conditions of beat vibration about the first two vibration modes, are the decisive factors in the formation. Therefore, the phenomenon of beat vibration occurs in the response.

Influence of amplitude ratio on the beat vibration of the sluice pier

A statistical analysis of the amplitude ratio of the first two main vibration modes on the six selected measuring points was conducted to further explore the influence of amplitude ratio on beat vibration. Given that the amplitude components of each vibration frequency are difficult to separate and the flow-induced vibration
Figure 18. Singular entropy increment changed with orders and frequency stabilization diagram for the left sluice pier on condition 3.

Figure 19. Singular entropy increment changed with orders and frequency stabilization diagram for the right sluice pier on condition 3.

Table 3. Modal parameter identification results of sluice pier on condition 1.

| Modal order | Left pier       | Right pier      |                           |                           |
|-------------|-----------------|-----------------|---------------------------|---------------------------|
|             | Frequency (Hz)  | Mode shape      | Damping ratio (%)         | Frequency (Hz)            |
|             |                 |                 |                           | Mode shape                |
|             |                 |                 |                           | Damping ratio (%)         |
| 1           | 2.309           |                 | 4.103                     | 2.312                     |
| 2           | 2.487           |                 | 2.461                     | 2.477                     |
| 3           | 3.886           |                 | 2.618                     | 3.828                     |

Table 4. Modal parameter identification results of sluice pier on condition 2.

| Modal order | Left pier       | Right pier      |                           |                           |
|-------------|-----------------|-----------------|---------------------------|---------------------------|
|             | Frequency (Hz)  | Mode shape      | Damping ratio (%)         | Frequency (Hz)            |
|             |                 |                 |                           | Mode shape                |
|             |                 |                 |                           | Damping ratio (%)         |
| 1           | 2.303           |                 | 4.606                     | 2.323                     |
| 2           | 2.451           |                 | 3.188                     | 2.461                     |
| 3           | 3.771           |                 | 1.863                     | 3.763                     |
The response of the sluice pier can be regarded as an ergodic stationary random process, the amplitude of the sluice pier can be estimated by the triple dynamic displacement mean square error. Therefore, the ratio of the dynamic displacement mean square deviation of the vibration response components on each measuring point was used as the amplitude ratio in this study.

According to the principle of random vibration, the dynamic displacement mean square deviation of a structural node $K$ can be expressed as

$$
\sigma_K = \sqrt{\int_0^\infty S_{VK}(f) df}
$$

(23)

Table 5. Modal parameter identification results of sluice pier on condition 3.

| Modal order | Left pier |   | Right pier |   |
|-------------|-----------|---|------------|---|
|             | Frequency (Hz) | Mode shape | Damping ratio (%) | Frequency (Hz) | Mode shape | Damping ratio (%) |
| 1           | 2.409     |   | 2.332      |   | 2.389   | 2.871      |
| 2           | 2.504     |   | 2.631      |   | 2.552   | 1.602      |
| 3           | 4.409     |   | 2.975      |   | 4.352   | 2.121      |

Table 6. Vibration frequency ratio of the sluice pier.

| Condition | Condition 1 |   | Condition 2 |   | Condition 3 |   |
|-----------|-------------|---|-------------|---|-------------|---|
|           | Left pier   |   | Right pier  |   | Left pier   |   |
| Frequency ratio | 0.928 |   | 0.940       |   | 0.962       | 0.936    |

Figure 20. Dynamic displacement mean square deviation computation diagram.

Table 7. Dynamic displacement mean square deviation of different points.

| Modal component | Condition 1 |   | Condition 2 |   | Condition 3 |   |
|----------------|-------------|---|-------------|---|-------------|---|
|                | H4 (µm)   | H9 (µm) | H3 (µm)   | H9 (µm) | H3 (µm)   | H8 (µm) |
| First-order component | 31.35 | 31.22 | 24.11      | 32.55 | 25.47      | 16.36   |
| Second-order component | 47.49 | 48.49 | 29.82      | 40.71 | 15.62      | 11.82   |
| Mean square deviation ratio | 0.66 | 0.64 | 0.81       | 0.80 | 1.63       | 1.38    |
where \( S_{VK}(f) \) is the power spectral density function of node \( K \) and \( f \) is the frequency. As a consequence, the dynamic displacement mean square deviation is obtained from the area enclosed by the peak of each frequency and the abscissa in the power spectrum density curve, as shown in Figure 20.

On the basis of the previously presented method, the dynamic displacement mean square deviations of the vibration response components on each measuring point are shown in Table 7. The ratio is in the range of 0.64–1.63, which meets the condition of obvious beat vibration \((0.33 < \beta < 3)\). Therefore, another factor that makes beat vibration highly obvious is the amplitude ratio. Overall, the phenomenon of beat vibration in the prototype vibration tests of the sluice pier was mainly due to the vibration of the sluice pier dominated by the two previous components. The first two mode shapes can satisfy the formation conditions of beat vibration, including the conditions of frequency and amplitude ratios. Consequently, obvious beat vibration occurs.

The external cause of beat vibration on the sluice pier

The formation of the obvious beat vibration waveform is affected by internal and external factors. The frequency ratio and amplitude ratio of the first two main mode shapes of the pier are in the coupling condition of the beat vibration is the internal cause. The vibration responses of the sluice pier of Shuhe Hydropower Station are taken as input, and the water load of the structure is obtained by back analysis based on genetic algorithm. We can conclude that the random beat vibration occurs, when the first three natural frequencies of the pier are within the energy range of the water load \((0–5)\). This is the external cause of beat vibration on the sluice pier. The combination of internal and external factors ultimately lead to the formation of beat vibration.

Conclusion

In this study, the sluice pier of Shuhe Hydropower Station was used as an example to analyze the phenomenon of beat vibration in prototype vibration tests. Mathematical simulation experiments were conducted to determine the formation conditions of beat vibration. On the basis of the conditions and results of operational modal parameter identification in the sluice pier prototype tests, the frequency and amplitude ratios of the vibration response components were discussed, and the causes of beat vibration in the prototype tests were analyzed. The following conclusions were obtained.

1. The mathematical simulation experiments showed that when the beat vibration signal was synthesized by two columns of signals, the initial phase difference of the signals had no effect on the formation of the vibration waveform, and it only changed the position of the synthesized envelopes. The frequency ratio was the decisive factor in the appearance of beat vibration. Beat vibration appeared only when frequency ratio \( \xi \) was between 0.78 and 1.29 and not equal to 1 \((0.78 < \xi < 1.29 \text{ and } \xi \neq 1)\). The amplitude ratio affected the obvious degree of beat vibration by changing the relative difference between the upper and lower envelopes of the synthetic signal. When the amplitude ratio was between 0.33 and 3 \((0.33 < \beta < 3)\), the waveform of beat vibration was obvious.

2. The working modal parameters for the right and left sluice piers of hole #2 were identified using the stochastic subspace method based on the order determination of singular entropy. The results showed that the sluice had three main vibration modes, in which the first vibration frequency was in the range of 2.31–2.41 Hz, the second vibration frequency was in the range of 2.45–2.55 Hz, the third vibration frequency was in the range of 3.76–4.41 Hz. The vibration of the sluice pier was mainly dominated by the first two vibration modes.

3. The causes of beat vibration on the sluice were analyzed based on the modal identification results and power spectral density curve of the measuring points. The results showed that the internal cause of beat vibration is that the vibration frequency ratio was between 0.928 and 0.962, which satisfies the necessary conditions of beat vibration, and the amplitude ratio was between 0.66 and 1.63, which makes beat vibration highly obvious. The external cause of it is that the first three natural frequencies of the pier are within the energy range of the water load which causes random beat vibration.

Data Availability

Data from this manuscript may be made available upon request to the authors.
Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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