Resonant Pair Tunneling in Double Quantum Dots

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Introduction. Measurements of nonequilibrium shot noise in current fluctuations in electronic devices has become a practical tool to probe strongly correlated systems with elementary excitations whose charge, $e^*$, possibly differs from the electron charge $e$, the prominent examples being the observation of the Cooper-pair charge $e^* = 2e$ in normal metal-superconductor junctions, or fractional charges in quantum Hall samples. Remarkably, many-body physics with unusual emergent excitations arises even when the interactions occur in a single point, e.g., in an impurity in a metal, or in a nanoscale quantum dot (QD) connecting few leads. Theoretical studies of various quantum impurity problems encountered the notion of non-Fermi liquid (NFL) behavior, with a fully inelastic scattering of an incoming electron into an out-going scattering state which does not include any single electron component. It is of general interest to study shot noise is QD systems showing such elusive NFL behavior.

Nontrivial effective charges emerge even for quantum impurity problems showing regular Fermi liquid (FL) behavior, for example in the basic single impurity Kondo model, realized by a single magnetic (Kondo) QD coupled to leads, studies of shot noise lead to a prediction of a universal fractional charge $e^* = 5e/3$ in the low temperature regime which was detected experimentally, reflecting a combination of single electron and two-electron backscattering. The crossover which is typically addressed in experiment is very rarely understood theoretically. In this paper we find that a simple and yet unusual “noninteracting-like” picture for transport of particles with effective charge $e^* = 2e$ emerges along an entire crossover from NFL to FL behavior occurring in double QDs in series exhibiting the physics of the 2-impurity Kondo model (2IKM).

The simplest 2IKM consists of two impurity spins ($S_L, S_R$), coupled to two channels of conduction electrons and interacting with each other through an exchange interaction $K$. After the standard “unfolding transformation” reducing the two spin-1/2 channels to four chiral Dirac fermions, $\psi_{\alpha}(x)$, $i = 1, 2 = L, R$, $\alpha = \uparrow, \downarrow$, $x \in \{-\infty, \infty\}$, the Hamiltonian becomes $H = H_0 + H_K$

where $H_0 = \sum_{j,\alpha} \int dx \psi_{j\alpha}^\dagger i \partial_x \psi_{j\alpha}$ and

$$H_K = J_L (\psi_L^\dagger \sigma \psi_L) \cdot \vec{S}_L + J_R (\psi_R^\dagger \sigma \psi_R) \cdot \vec{S}_R + K \vec{S}_L \cdot \vec{S}_R \ (1)$$

where $\sigma(\vec{r})$ is a vector of Pauli matrices acting in spin(channel) space. For this model a NFL quantum critical point (QCP) was found at $K = \nu \sim T_K$ separating a local singlet FL phase at $K > K_c$ from a Kondo screened FL phase at $K < K_c$. However, more realistic models containing inter-channel tunneling,

$$H_{FS} = V_{LR} \psi_L^\dagger \psi_R + H.c. \ (2)$$

[or, $H_{FS} = \text{Re} V_{LR} (\psi^\dagger \tau^0 \psi) - \text{Im} V_{LR} (\psi^\dagger \tau^2 \psi)$ with implicit sum over spin and channel indices] do not show a critical point. The reason for this is that Eq. (2) results in a relevant perturbation with dimension 1/2 at the QCP, leading to an energy scale $T^* \sim T_K |\nu V_{LR}|^2 + (K - K_c)^2 / T_K$ which is finite even at $K = K_c$, below which an effective FL theory takes over. Here $\nu$ is the density of states of the conduction electrons. This crossover from NFL to FL behavior is reflected in the conductance of double QDs exhibiting a crossover from NFL to FL below which an effective FL theory takes over. In this paper we study the full counting statistics (FCS) for charge transfer through a series double QD along the full NFL to FL crossover. In general, charge is transferred in units of $e$ or $2e$. A peculiar situation occurs at $K = K_c$, where $2e$ becomes the basic charge unit along the full crossover. This striking behavior is not captured in a slave-boson mean field calculation.

We also derive a local Fermi liquid Hamiltonian governing the physics below $T^*$. Using an enhanced understanding of this crossover we go beyond previous works in determining all coupling constants in this effective Hamiltonian and obtain a universal theory depending only on an energy scale, $T^*$, similar to Nozières FL theory (FLT) for the single impurity problem, and on the new FL boundary condition associated with the ratio $|K - K_c| / (\nu V_{LR} T_K)$. This approach helps to understand the charge $2e$ carriers.
In the geometry proposed by Zarand et al. \[17\], where transport proceeds between two leads connected via one QD side coupled to a second QD coupled to another lead, exact results on the crossover are not available. Nonetheless, we use our FLT to calculate universal non-equilibrium transport and noise properties at low energies when the NFL critical behavior is destabilized by a non-zero $K - K_c$. Our predictions can be probed experimentally \[25\].

**Full counting statistics.** We will obtain the full charge transfer distribution in a series double QD tuned to the 2IKM regime, along the crossover from NFL to FL behavior, using the formulation we developed in Ref. \[11\].

In terms of abelian bosonization one can write the original free fermion theory with $H_K \to 0$ and $H_{PS} \to 0$ in terms of 8 chiral Majorana fermions $\chi^A_0, \chi^A_1 = \frac{\psi_i + \psi_i \gamma_5}{\sqrt{2}}$, $\chi^A_2 = \frac{\psi_i - \psi_i \gamma_5}{\sqrt{2}}$, associated with the real ($i = 1$) and imaginary ($i = 2$) parts of the charge, spin, flavor and spin-flavor fermions ($A = c, s, f, X$); for a definition of these fermions, see Ref. \[11\]. Then the free Hamiltonian is $H_0[\{\chi\}] = \frac{i}{2} \sum_i \int dx \chi^*_i \frac{d}{dt} \chi^i_j$, where $\{\chi\} = \{\chi^X_0, \chi^X_1, \chi^F_2, \chi^N_1, \chi^F_1, \chi^N_2, \chi^N_2, \chi^F_2\}$. The Fermi operator $\psi_i$ gives rise to charge $e$ (and no spin) tunneling from left to right, and changes $Y = (N_L - N_R)/2$ by 1, $N_i$ being the total fermion number in lead $i = L, R$.

Turning on $H_K$, the QCP is obtained at $K = K_c$ from the free case by a change in boundary condition (BC) occurring only for the first Majorana fermion, $\chi_1(0^-) = -\chi_1(0^+)$. For $\varepsilon \ll K_c$, the leading terms in the Hamiltonian describing deviations $K - K_c$ as well as finite $V_{LR}$ can be written \[11\] in a new basis $\{\chi\}$, where $\chi_1(x) = \chi_1'(x) \text{sgn}(x)$ and $\chi_i = \chi_i', (i = 2, \ldots, 8)$, as $H_{QCP} = H_0[\{\chi\}] + \delta H_{QCP}$ where \[26\]

$$
\delta H_{QCP} = i \sum_{i=1}^2 \lambda_i \chi_i(0)a. \tag{3}
$$

Here $a$ is a local Majorana fermion, $a^2 = 1/2$, and

$$
\lambda_1 = c_1 \frac{K - K_c}{\sqrt{K}}, \quad \lambda_2 = c_2 \sqrt{\frac{\varepsilon}{v_{LR}}} |v_{LR}|, \tag{4}
$$

where $c_1$ and $c_2$ are constant factors of order 1. Those couplings determine two energy scales $\lambda_1^2, \lambda_2^2$, and the total crossover scale is $\lambda^2 = \lambda_1^2 + \lambda_2^2 = T^*$. The operators in $\delta H_{QCP}$ have scaling dimension 1/2, hence they destabilize the QCP; below the crossover scale $T^* = \lambda_1^2 + \lambda_2^2$ the system flows to FL fixed points whose nature depend on the ratio $\lambda_1/\lambda_2$.

By definition the FCS is obtained from the cumulant generating function $\chi(\mu)$ for the probability distribution function $P(Q)$ to transfer $Q$ units of charge during the waiting time $T$ (which is sent to infinity), $\chi(\mu) = \sum_Q e^{\mu Q} P(Q)$. The cumulants ($\delta^n Q$) can be found from $\langle \delta^n Q \rangle = (-i)^n \frac{\partial^n}{\partial \mu^n} \ln \chi(\mu) |_{\mu = 0}$. In fact, due to a formal equivalence of our non-equilibrium formulation and that of Schiller and Hershfield \[27\] for a single QD tuned to the Toulouse limit, we can borrow directly the results of Gogolin and Komnik for the FCS for that model \[28\]; translating between the parameters of the two models in the limit $T^* \ll T_K$, we obtain

$$
\ln \frac{\chi(\mu)}{T} = \int_{-\infty}^{\infty} \frac{d\mu}{4\pi} \ln \left[ 1 + \sum_{n=2}^2 A_n(\epsilon) (\epsilon^{2\mu} - 1) \right]. \tag{5}
$$

Here $A_1(\epsilon) = \frac{2\lambda_1^2}{4\epsilon^2 + 1} [n_F(1 - n_L) + n_R(1 - n_F)], A_2(\epsilon) = \frac{\lambda_2^2}{4\epsilon^2 + 1} n_L(1 - n_R)$. $A_{-n} = A_n|_{L \leftrightarrow R}$, $n_F = (1 + e^{\epsilon/T})^{-1}$, $n_L = n_F(\epsilon + eV)$. The presence of one particle as well as two particle transport processes in our model is apparent from the $\mu$ dependence of the two terms $\propto (e^{\pm 2\mu} - 1)$ and $\propto (e^{\pm 2\mu} - 1)$ in Eq. (5), respectively. At $K = K_c$, giving $A_1 = A_{-1} = 0$, Eq. (5) is equivalent to the formula for the FCS of spinless noninteracting fermions of charge 2e transmitted though a resonant level of width $\sim T^*$, namely the noninteracting formula \[18\] \[28\] is obtained from Eq. (5) by the replacement $2\mu \to \mu, n_{L,R} = n_F(\pm eV/2)$ and adding an overall factor of 2.

The emergence of two particle resonant tunneling at $K = K_c$ follows from Eq. (4). In this case $\delta H_{QCP} = \frac{i}{\lambda_1^2} \lambda_2 \psi^*_i + \psi_j|a$ has the form of a Majorana resonant level $27$. This operator changes $Y$ by $\pm 1$, while a noninteracting resonant level $d$ with $\delta H = \lambda_2(\psi^*_L + \psi^*_R)d + H.c.$ changes $Y = (N_L - N_R)/2$ by $\pm 1$. In both models, transport is given by processes of even order in $\lambda_2$, giving $\Delta Y$ integer in the resonant level model but $\Delta Y$ even-integer in the 2IKM. In the limit of large $V$, $\lambda_2$, in either non-interacting resonant level model or 2IKM, one can do perturbation theory in $\lambda_2$. The $T = 0$ conductance, in this limit, is given by \[11\], \[27\] $G \propto \lambda_2^2/2V^2$, implying a second order process. This follows since the first order tunneling between a fermionic state [either $\psi_i$ or $\psi_i$ ($i = L, R$) in the two models] with energy of order $eV$, and the zero energy level (a or d) does not conserve energy. In the small $V$ limit of the 2IKM we can understand the charge 2e using the FLT developed below.

The two-particle processes can be probed by looking simultaneously at the current $I = e\langle \delta^3 Q \rangle / T$ and noise $S = 2e^2\langle \delta^2 Q \rangle / T$. Eq. (5) gives the $T = 0$ conductance $G = \frac{dI}{dV} = gat$ where $t = \lambda_2^2/\lambda^2 = |v_{LR}|/(\parallel v_{LR}^2 + (c_1/c_2)^2(\Delta T_K)^2\parallel)$ and $g_0 = 2e^2/h$ (or setting $h = 1$ as in the rest of the paper, $g_0 = e^2/\pi$). Using Eq. (5), in Fig. \[1\] we plot shot noise $S(V)$ along the crossover from FL ($eV \gg T^*$) to NFL ($eV \gg T^*$) regimes for various values of $K - K_c$ (determined by $t$). Experiments \[7\] extract effective charges by fitting shot noise measurements with the formula

$$
S_{fit} = 2e^* g_0 \int_0^V t(V')[1 - t(V')] dV', \tag{6}
$$

where $t(V)$ is extracted from nonlinear conductance measurements for non-interacting charge $t(V) = \frac{1}{g_0} \frac{d}{dV}$, $t(0) = 1$, and $e^*$ is the effective
a unitary S-matrix,
\[ \psi_{\alpha}(0^-) = s_{ij} |\alpha\rangle |\psi_{j\alpha}(0^+) \] (repeated indices summed).

We will express \( H_{FL} \) in terms of single particle scattering states \( \Psi_{i\alpha} \) incoming from channel (lead) \( i = 1, 2 = (L, R) \) with spin \( \alpha = \pm 1 \),
\[ \Psi_{j\alpha}(x) = \theta(x) |\psi_{j\alpha}(x) + \theta(-x) s_{j'\alpha}|\psi_{j'\alpha}(x), \] (9)
satisfying \( \Psi_{i\alpha}(0^+) = \Psi_{i\alpha}(0^-) \). In our left moving convention, the region \( x > 0 \) \((x < 0)\) corresponds to the incoming(outgoing) part of the field.

To find the S-matrix we should relate the BC of the \( \eta \)'s, \( \eta_i(0^+) = \eta_i(0^-) \), \((i = 1, 8)\), to the BC of the \( \psi_{i\alpha} \)'s, Eq. (8). The representation of the \( \psi_{i\alpha} \)'s in terms of the \( \eta \)'s (or the \( \chi \)'s or \( \chi' \)s) is fairly complicated, however quadratic forms of those different fermions are linearly related. In particular, consider \( J_M = |\psi\rangle \langle M\psi| : \] where \( M \) acts in the channel space. It is straightforward to find the coefficients \( c_{ij} \) such that \( J_M = \sum_{i,j=1}^{8} c_{ij} \chi_i \chi'_j \). Now consider \( J_M(x) \) at \( x = 0^+ \). Using these linear relations together with the BC for the \( \chi \)'s (which depends on \( \lambda_1/\lambda_2 \)), one can find \( M' \) as function of \( M \) and \( \lambda_1/\lambda_2 \), such that \( J_M(0^-) = J_M(0^+) \). This relation between \( M \) and \( M' \) can be used to find the S-matrix, since Eq. (8) implies \( M' = s^{4}M \). Using this scheme, starting with \( H_{PS} \) with real \( V_{LR} \) we obtain: \( s|\alpha \rangle = \cos(2 \delta) - i a \sin(2 \delta) \pi^2 \), where \( \cos \delta = \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1^2 + \lambda_2^2} \). The last equation gives \( 2 \delta \) modulo \( \pi \). However, under the transformation \( V_{LR} \rightarrow -V_{LR} \) in Eq. (2), we have \( 2 \delta \rightarrow 2 \pi - 2 \delta \) [26]. Thus
\[ 2 \delta = \arg(\lambda_1 + i \lambda_2), \] (10)
\( \delta \), which is denoted as the phase shift, changes from \( 0 \) to \( \pi/2 \) as function of \( K \), and it takes the value of \( \pi/4 \) at \( K = K_c \). This first exact result for the phase shift agrees with the numerical results of Jones [13]. To obtain the S-matrix for complex \( V_{LR} \), one can start with a phase rotation of the \( \psi \)'s, rendering \( V_{LR} \) real [26], and in the end, transform back to the original basis leads to a rotation of the S-matrix \( s \rightarrow \exp(i \tau^3 \arg(V_{LR})/2)s \exp(-i \tau^3 \arg(V_{LR})/2) \).

The FL interaction Eq. (4) can be explicitly written in terms of the the scattering states \( \Psi \). To achieve this formally we (i) switch from quadratic derivative forms of the \( \chi \)'s, \( \partial_{x} = \chi_i \partial_{x} \chi_i + \chi_i \partial_{x} \chi_k \), in Eq. (7), to quartic forms, using \( i \chi_x \partial_x \chi_j + i \chi_j \partial_x \chi_k =: \chi_j \chi_k \rightarrow: \chi_j \chi_k \); \((j \neq j')\); (ii) express \( \chi_i \chi_j \) linearly in terms of quadratic forms of the \( \Psi \)'s (these linear relations have the same coefficients \( c_{ij} \)). The result is \( H_{FL}^{FL} = \int dx \partial_x \psi^\dagger i \partial_x \Psi \) (in-

charge. For \( K \neq K_c \) \((t < 1)\), \( S_{\text{fit}} \) with \( e^* \rightarrow e \) gives a good fit for sufficiently small \( V \); see inset of Fig. [1]). When \((K - K_c) \gg T_K/|V_{LR}| \) \((t \ll 1)\), this fit with \( e^* \rightarrow e \) becomes reasonably good along the full crossover; see curve with \( t = 0.15 \) in Fig. [1]. For \( K \rightarrow K_c \) \((t \rightarrow 1)\) the fit with \( e^* \rightarrow e \) works only for an extremely small range \( eV \ll \sqrt{T^*/T_K} |K - K_c| \rightarrow 0 \), and, remarkably, the full curve fits with \( S_{\text{fit}} \) with \( e^* = 2e \); see curve with \( t = 0.99 \).

**Fermi liquid theory.** While the above results were obtained formally from a calculation in terms of reformulation, it is desirable to understand them from an effective interacting theory written in terms of the original fermions. The reader may skip the technical details and go directly to the FL Hamiltonian, Eq. (11).

The crucial observation is that only the linear combination \((\lambda_1 \chi_1 + \lambda_2 \chi_2)/\lambda \) of the 8 Majorana fermions at the QCP participates in this crossover described by Eq. (3). It can be shown that the effect of \( \delta H_{\text{QCP}} \) is to modify the BC for this linear combination by a simple sign change. In order to write down the FL fixed point Hamiltonian, we define a new basis with modified BC, \( \{\eta\} \), where \( \eta_1 = (\lambda_1 \chi_1 + \lambda_2 \chi_2)/\lambda \) \( \text{sgn}(x) \), \( \eta_2 = (-\lambda_2 \chi_1 + \lambda_1 \chi_2)/\lambda \), and \( \eta_i = \chi_i, \) \((i = 3, \ldots, 8)\). Hence we can write the Hamiltonian for the FL fixed points as \( H_{FL} = H_0(\{\eta\}) + \delta H_{FL} \). We expect the local interaction \( \delta H_{FL} \) to involve uniquely \( \eta_i \), which is the only field participating in the crossover in Eq. (3). The only candidate for the leading FL operator with scaling dimension 2 is
\[ \delta H_{FL} = \lambda_{FL} i\eta \partial_x \eta_i |x = 0, \] (7)
with \( \lambda_{FL} \propto 1/T^* \). Comparison of the scattering phase shift for Eqs. (3) and (7) actually gives \( \lambda_{FL} = 4/T^* \).

Equation (7) for the FL interaction may be written in terms of fields which are simply related to the original fermions. The latter satisfy a FL BC parameterized by

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Shot noise versus voltage for several values of \( K - K_c \) (determined by \( t \)) at \( T = 0 \). Data is fitted in dashed lines using Eq. (9). The inset blows up the FL region, with fits of \( e^* = e \) for \((t = 0.15, 0.5, 0.8)\) and \( e^* = 2e \) for \( t = 0.99 \) \((K \rightarrow K_c, \) red curve).}
\end{figure}
where $J_i = \Psi_i^\dagger \Psi_i, J_i = \Psi_i^\dagger \vec{\delta} \Psi_i (i = L, R)$ and $\epsilon_{\alpha\beta}$ is the antisymmetric tensor. This is the main result of this section. It gives an explicit form of the interactions between the scattering states, related to the original electrons by Eq. (9). This interaction is weak in the FL regime allowing to apply perturbation theory in $1/T^*$. This universal FL Hamiltonian follows by strong restrictions due to a large symmetry emerging close to the QCP \[16\] and leading to the simple form of $H_{QCP}$ in Eq. (3). In practice the symmetry at the QCP is reduced by marginal and irrelevant operators such as the leading irrelevant operator $\partial_{\epsilon\chi} a$ (at the QCP); however they will be associated with a small parameter $1/T_K$, and hence are neglected at low energies for $T^* \ll T_K$. For finite $V_{LR}, K - K_c$ or intra-lead potential scattering $V_{L} \Psi_{L}^\dagger \psi_{L} + V_{R} \psi_{R}^\dagger \Psi_{R}$, additional marginal and irrelevant terms are produced at the QCP, part of which were present before. However close enough to the QCP and starting with a weak coupling problem, namely for $|\nu V_{LR}|, |\nu V_{I}| \ll 1 (i = L, R)$, and $|K - K_c| \ll T_K$, those perturbations can be safely ignored.

The emergence of the basic transport charge $2e$ for $K = K_c$ ($\delta = \pi/4$) in the series geometry follows at low energies as the only term in Eq. (11) which does not conserve the number of $\Psi_L$ fermions minus the number of $\Psi_R$ fermions is $\propto (\Psi_{L}^\dagger \epsilon_{\alpha\beta} \Psi_{L}^\dagger \Psi_{R}^\dagger \epsilon_{\gamma\delta} \Psi_{R}^\dagger) + H.c.$ It converts a spin singlet pair of $\Psi_L$ fermions into a spin singlet pair of $\Psi_R$ fermions (and vice versa). At $K = K_c$ the scattering states $\Psi_L(\Psi_R)$ are waves propagating freely to the right(left), hence the basic transport mechanism is an inelastic $2e$ backscattering.

To calculate transport properties for the Zarand et al. double QD geometry \[17\] which is under current experimental study \[25\], it is necessary to calculate the single particle Green’s function in the 2IKM. Because the electron field cannot be expressed in terms of the Majorana fermions, we have not been able to calculate this throughout the crossover, results being necessarily restricted to the vicinity of the NFL or FL critical points. Here we consider this system at $K$ slightly different than $K_c$, in the FL regimes, $T, eV \ll T^*$, where our FLT can be applied, ignoring particle-hole breaking ($V_{LR} = 0$). The conductance of this system can be expanded as $G_{\text{singlet}} = g_0|\sin^2 \delta_1 + \beta_1 \{ (T/T^*)^2 + \alpha (eV/T^*)^2 \}]$, and $G_{\text{screened}} = g_0|\cos^2 \delta_1 - \gamma_1 \{ (T/T^*)^2 + \alpha' (eV/T^*)^2 \}]$, where $\delta_1$ is a small phase shift associated with marginal potential scattering operators. We assume parity symmetry of the device. Using Eq. (11), after a lengthy but straightforward calculation, we determine universal relations: $\beta_1 = \gamma_1 = O(1), \alpha = \alpha' = 9/10\pi^2$. We also calculate the shot noise in the FL regime, and define effective charges $(e'/e) = S/2\Gamma$ for the local singlet FL regime ($K > K_c$), and $(e'/e)' = S/2(g_0V - \Gamma)$ in the Kondo screened phase ($K < K_c$), defined in the limit $\delta_1 \to 0$. Using Eq. (11) we obtain $(e'/e) = (e'/e)' = 11/9$. Our predictions should be contrasted with the measurements on single QDs with $\alpha = 3/2\pi^2 \[29\]$ and $e'/e = 5/3 \[7\]$. We thank J. Malecki and Y. Oreg for very helpful discussions. This work was supported by NSERC (ES & IA) and CIFAR (IA).

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