Mass Deformation of the Multiple M2 Branes Theory

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ABSTRACT

Based on recent developments, in this letter we study the one parameter deformation of 2 + 1 dimensional gauge theories with scale invariance and $\mathcal{N} = 8$ supersymmetry, which is expected to be the field theory living on a stack of M2 branes. The deformed gauge theory is defined by a Lagrangian and is based on an infinite set of novel 3-algebras constructed by relaxing the assumption that the invariant metric is positive definite. Under the Higgs mechanism, we can obtain the D-branes world volume theory in the presence of background fluxes.

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1 Introduction

M-branes are mysterious objects and virtually little is known about their underlying dynamics. This is in sharp contrast to D-branes, where a microscopic description in terms of open strings has driven a huge amount of progress in string theory and gauge theory. The three-dimensional superconformal field theory which is supposed to describe multiple coincident M2 branes may lead to profound new insight in our understanding of M-theory. Recently Bagger and Lambert [1, 2, 3] and Gustavsson [4, 5], proposed a new set of 2 + 1 dimensional field theory (henceforth called the BLG theory) which is supposed to describe low energy world volume theory of multiple coincident M2 branes. The BLG theory was constructed in frame of so-called 3-algebra, a generalization of the Lie algebra with triple bracket replacing the commutator and the 4-index structure constant replacing the usual 3-index structure constant of the Lie algebra. There are two requirements of 3-algebra in the BLG theory: one is called the fundamental identity which is the generalization of the Jacobi identity of the Lie algebra; the other is that the metric of 3-algebra is positive definite. Recently the BLG theory was generalized to a novel 3-algebra by relaxing the assumption that the metric on 3-algebra is positive definite [6][7][8]. Henceforth we will call this theory generalized BLG theory. The generalized BLG theory has many features which suggest that it is related to M2 branes. For example, the gauge interaction term is the BF-type which do not admit a tunable coupling constant and this property extends to the full generalized BLG theory. The key point is that this construction starts from the arbitrary Lie algebra, so that we can construct theory of arbitrary number of M2 branes which makes the well-developed large N tool [9] possible in this field.

In this letter we construct the one parameter deformation of the generalized BLG theory and then we use the strategy of [10] to show the relation between the deformed M2 branes theory and the induced D2 branes system. The deformed of BLG theory was considered in [3] [11] [12]. The rest of this letter is organized as follows. In section 2, we give a brief review of the BLG theory and its generalized form. In section 3, we consider the deformation of the generalized BLG theory. In section 4, following Mukhi and Papageorgakis, we consider the reduction of M branes to D branes in the presence background fluxes. In section 5, we will give some discussions. For other recent developments of the BLG theory, see [14]-[25].

Note added: After this letter was finished, the preprint [13] focusing on Janus field theory appeared on arXiv with substantial overlap with our results.

2 Brief Review of Generalized BLG Theory

The BLG theory is based on 3-algebra, which is the generalization of Lie algebra. A 3-algebra is a N dimensional vector space with basis $T^A$ ($A = 1, 2, ..., N$) which is endowed with a trilinear antisymmetric product

$$[T^A, T^B, T^C] = f^{ABC}_D T^D$$ (2.1)
where \( f^{ABC}D \) is the structure constant. From (2.1) it is clear that \( f^{ABC}D = f^{[ABC]}_D \). Then further suppose there is a trace form providing a metric

\[
h^{AB} = \text{Tr}(T^A, T^B) \tag{2.2}\]

In order to serve as the gauge symmetry algebra of M2 branes world volume theory, namely that for the equations of motion to be consistent with gauge symmetry and supersymmetry, the fundamental identity need to be imposed to the 3-algebra:

\[
[T^A, T^B, [T^C, T^D, T^E]] = [[T^A, T^B, T^C], T^D, T^E] + [T^C, [T^A, T^B, T^D], T^E] + [T^D, [T^A, T^B, T^C], T^E] \tag{2.3}\]

which extends the Jacobi identity to the 3-algebra and is equivalent to

\[
f^{EFG}_D f^{ABC}G = f^{EFA}_G f^{BCG}D + f^{EFB}_G f^{CAG}D + f^{EFC}_G f^{ABG}D \tag{2.4}\]

In order to derive equations of motion from the Lagrangian description, a bi-invariant metric \( h^{AB} \) on the 3-algebra is needed which requires

\[
\text{Tr}([T^A, T^B, T^C], T^D) + \text{Tr}(T^A, [T^B, T^C, T^D]) = 0 \tag{2.5}\]

This implies the tensor \( f^{ABCD} \equiv f^{ABC}D^h^{ED} \) is totally antisymmetric.

The BLG theory enjoys the classical conformal invariance and \( \mathcal{N} = 8 \) supersymmetry, which has 16 supersymmetries. The action also has a manifest SO(8) R-symmetry that acts on the scalars \( X^{(I)} \). It has no free parameters and the structure constant of the 3-algebra is quantized [3], which strongly suggests the conformal invariance is exact at the quantum level. The elegant and unique structure of the BLG theory makes it a very compelling candidate of the multiple M2 branes theory. The BLG theory encodes the interactions of three dimensional \( \mathcal{N} = 8 \) multiplet. The fermionic field \( \Psi \) is a Majorana spinor in 10 + 1 dimensions satisfying the chirality condition \( \Gamma \bar{v}^{012} \Psi = -\Psi \) while the SUSY parameter \( \epsilon \) satisfies \( \Gamma \bar{v}^{012} \epsilon = \epsilon \). As a result, \( \Psi \) has 16 real fermionic components equivalent to 8 bosonic degrees of freedom. The bosonic fields include 8 real scalar fields \( X^{(I)}_A \), (where \( I = 1, ... 8 \) specifying the transverse directions of M2 branes) and a gauge field \( A_\mu \) (where \( \mu = 0, \hat{1}, \hat{2} \) describing the longitudinal directions). In 2+1 dimensions, an ordinary gauge field has one propagating degree of freedom. However, in the BLG theory the gauge field \( A_\mu \) has only a Chern-Simons term rather than canonical kinetic terms and hence it has no propagating degree of freedom. Matter fields in the BLG theory take values in 3-algebra, so that we have \( X^{(I)} = X^{(I)}_A T^A, \Psi = \Psi^A T^A \). The BLG Lagrangian is given by [2]

\[
\mathcal{L} = -\frac{1}{2} D_\mu X^{(I)} A^{(J)} D^\mu X^{(J)} + \frac{i}{2} \bar{\Psi}^A \Gamma^A D_\mu \Psi_A + \frac{i}{4} f^{ABCD} \bar{\Psi}^B \Gamma^{IJ} X^{(C)} X^{(D)} \Psi^A \nonumber \\
- \frac{1}{12} (f^{ABCD} X^{(A)} X^{(B(J)} X^{(C(K))} (f^{EFG}_D X^{(E)} X^{(F(J)} X^{(G(K))}) \nonumber \\
+ \frac{1}{2} \epsilon^{\mu\nu\lambda} (f^{ABCD} A_\mu^{(A|B} \partial_{\nu} A_{\lambda}^{CD} + \frac{2}{3} f^{AEF}_G f^{CDGB} A_\mu^{AB} A_\nu^{CD} A_{\lambda}^{EF}) \tag{2.6}\]
The theory is invariant under the $\mathcal{N} = 8$ SUSY transformations:

$$\delta X^{A(I)} = i\bar{\epsilon} \Gamma^I \Psi^A$$  \hspace{1cm} (2.7)

$$\delta \Psi^A = D_\mu X^{A(I)} \Gamma^\mu \Gamma_I \epsilon + \frac{1}{6} X^{B(I)} X^{C(J)} X^{D(K)} f^{A}_{BCD} \Gamma_{IJK} \epsilon$$  \hspace{1cm} (2.8)

$$\delta (\tilde{A}_\mu)^A_B = i\bar{\epsilon} \Gamma^\mu \Gamma^I \epsilon \Psi^C f^{A}_{BCD}$$  \hspace{1cm} (2.9)

and the gauge transformations:

$$\delta X^{A(I)} = \tilde{\Lambda}^A_B X^{B(I)} , \quad \delta \Psi^A = \tilde{\Lambda}^A_B \Psi^B , \quad \delta (\tilde{A}_\mu)^A_B = D_\mu \tilde{\Lambda}^A_B .$$ \hspace{1cm} (2.10)

where $\tilde{\Lambda}^A_B = \Lambda_{MN} f^{MNA}_B$ and $(\tilde{A}_\mu)^A_B = (A_\mu)^M_{MN} f^{MNA}_B$. The gauge group is generated by the $\tilde{\Lambda}^A_B$, while the antisymmetric $\Lambda_{MN}$ are auxiliary parameters. The gauge group is thus a subgroup of $GL(N)$ where $N$ is the dimension of 3-algebra. If we add a metric of signature $(N - k, k)$ on the 3-algebra, then we can say that the gauge group is a subgroup of $SO(N - k, k)$. The closure of the SUSY transformations implies the equations of motion.

In most of physical theories, a positive definite metric is required to preserve unitarity, that is the theory has positive definite kinetic terms preventing the propagation of ghost degrees of freedom. In the BLG theory, the positive definite metric requirement is very strong: it was conjectured in [28] and then proved in [29][30] that there is only one non-trivial 3-algebra $\mathcal{A}_4$ satisfying positive definite metric requirement. 3-algebra $\mathcal{A}_4$ is 4-dimensional and defined by structure constants $f^{ABC}_{\hphantom{ABC}D} = \epsilon^{ABC}_{\hphantom{ABC}D}$, where $\epsilon^{ABC}_{\hphantom{ABC}D}$ is the 4-dimensional Levi Civita symbol. New constructions are possible if we do not require the existence of Lagrangian but only of the equations of motion [31][32], which can be written without the help of metric in the algebra. Note that in the Bagger-Lambert work at the level of equation of motion, the metric is not used. The metric is needed in order to have a Lagrangian and gauge invariant local operators. Recently there is a breakthrough in constructing new 3-algebra [6][7][8]. The novel construction of 3-algebra $\mathcal{A}_G$ is based on an arbitrary compact and semi-simple Lie algebra $G$. These new constructions relax the requirement that the metric on the 3-algebra is positive and definite. The direction of relaxing positive definite metric requirement has been pursued in some earlier papers [33][28]. Following the convention of [6], the metric on the 3-algebra $\mathcal{A}_G$ is

$$h^{AB} = \eta^{AB} , \quad A, B = 0, 1, ..., n + 1 ,$$ \hspace{1cm} (2.11)

where $N = n + 2$ is the dimension of $\mathcal{A}_G$ and $\eta^{AB} = \text{diag}(-1,1,...,1)$ is the Minkowski metric on 3-algebra $\mathcal{A}_G$. Then we split the 3-algebra indices $(A, B, ....)$ into $(a, b, ..., \phi)$, where $a, b = 1, ..., n$ and $\phi \equiv n + 1$. The following form of the totally antisymmetric structure constants satisfies the fundamental identity (2.4):

$$f^{0abc} = f^{\phi abc} = f^{abc} , \quad f^{0\phi ab} = f^{abcd} = 0$$ \hspace{1cm} (2.12)

3-index structure constants $f^{abc}$ are the structure constants of a compact semi-simple Lie algebra $G$ and satisfy the usual Jacobi identity. It is convenient to change the generators to the light-cone form:

$$T^\pm = \pm T^0 + T^\phi$$ \hspace{1cm} (2.13)
In this base, the metric of $A_G$ is given by

$$h^{+-} = h^{--} = 2, \quad h^{++} = h^{--} = 0, \quad h^{ab} = \delta^{ab}, \quad h^{\pm a} = h^{a\pm} = 0 \quad (2.14)$$

and the structure constants are

$$f^{+abc} = -f^{a+bc} = f^{ab+c} = -f^{abc+} = 2f^{abc}$$
$$f^{-abc} = -f^{a-bc} = f^{ab-c} = -f^{abc-} = f^{abc}$$
$$f^{-abc} = f^{+abc} = 0$$

It is easy to see that the generator $T^-$ is central, viz. that the trilinear antisymmetric product vanishes whenever $T^-$ appears. The Lagrangian based on $A_G$ is given by

$$L = \frac{1}{2} h^{AB} D_\mu X_A^{(I)} D^\mu X_B^{(I)} + \frac{i}{2} h^{AB} \bar{\Psi}_A \Gamma^\mu D_\mu \Psi_B$$
$$- \frac{1}{12} h^{MN} f^{ABC} M f^{EFG} N X_A^{(I)} X_B^{(J)} X_C^{(K)} X_E^{(I)} X_F^{(J)} X_G^{(K)}$$
$$- \frac{i}{4} h^{DE} f^{ABC} E X_A^{(I)} X_B^{(J)} \bar{\Psi}_C \Gamma_I \Psi_D + 4\epsilon^{\mu\nu\lambda} \text{Tr} \left( B_\lambda (\partial_\mu A_\nu - [A_\mu, A_\nu]) \right) \quad (2.15)$$

It is important to note that the Lagrangian should be derived directly based on the new algebra $A_G$ rather than from the result of Bagger and Lambert [2]. Bagger-Lambert Lagrangian depends on the assumption that the metric is positive and definite. Henceforth we will call this theory generalized Bagger-Lambert theory which is based on 3-algebra $A_G$.

The generalized Bagger-Lambert theory does not admit any tunable coupling constant which hints that the generalized Bagger-Lambert theory is related to M2 branes.

## 3 Deformation of the Generalized BLG Theory

In ref. [26], it was argued that in the presence of a particular background four form flux, M2 branes preserve four supersymmetries and exhibit an SO(4) R-symmetry. Furthermore, the flux induces a supersymmetric mass term for the world volume scalars and fermions. It was also argued that in this background, the vacuum of $n$ M2 branes is a state in which the scalars describe a fuzzy three-sphere in spacetime. The M2 branes puff up so that their world volume is of form $\mathbb{R}^{1,2} \times \tilde{S}^3$, where $\tilde{S}^3$ is a fuzzy three sphere which becomes a normal $S^3$ as $n \to \infty$. This setup provides an M-theory analog of Myers effect which occurs for D-branes in the presence of background fluxes [27]. Following this argument, the fuzzy sphere solution of the BLG theory was found in [3] based on $A_4$ algebra. The more general deformation was considered in [11] in which two terms are added to the BLG theory. One is the mass term for all the scalars and fermions,

$$L_{mass} = -\frac{1}{2} h^{AB} D_\mu X_A^{(I)} D^\mu X_B^{(I)} + \frac{i}{2} h^{AB} \bar{\Psi}_A \Gamma_{1234} \Psi_B \quad \text{(3.1)}$$
The other is a Myers-like \( \text{SO}(4) \times \text{SO}(4) \) invariant scalar potential induced from background fluxes,

\[
\mathcal{L}_{\text{flux}} = -\frac{1}{6} \mu e^{IJKL} h^{AB}[X^{(I)}, X^{(J)}, X^{(K)}] A X_B^{(L)} - \frac{1}{6} \mu e^{I'J'K'L'} h^{AB}[X^{(I')}, X^{(J')}, X^{(K')}] A X_B^{(L')}
\]

(3.2)

where \( I', J', K', L' = 1, 2, 3, 4 \) and \( I, J, K, L = 5, 6, 7, 8 \) representing the transverse directions. It was proven in [11] that the BLG theory with the above two deformation terms remains fully supersymmetric. Notice that this kind of supersymmetric deformation applies to any 3-algebra with totally antisymmetric structure constants satisfying the fundamental identity. Along this line, we consider the deformation of the generalized BLG theory. The Lagrangian of the deformed theory is:

\[
\mathcal{L} = \mathcal{L} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{flux}}
\]

(3.3)

\[
\mathcal{L} = -\frac{1}{2} h^{AB} D_\mu X_A^{(I)} D^\mu X_B^{(I)} + \frac{i}{2} h^{AB} \bar{\Psi}_A \Gamma_\mu D_\mu \Psi_B
\]

\[
- \frac{1}{12} h^{MN} f^{ABC} M F E G N X_A^{(I)} X_B^{(J)} X_C^{(K)} X_E^{(I')} X_F^{(J')} X_G^{(K')}
\]

\[
- \frac{i}{4} h^{DE} f^{ABC} E X_A^{(I)} X_B^{(J)} \bar{\Psi}_C \Gamma_{IJ} \Psi_D + 4 \epsilon^{\mu\nu\lambda} \text{Tr} \left[ \mathcal{B}_X (\partial_\mu A_\nu - [A_\mu, A_\nu]) \right]
\]

(3.4)

\[
\mathcal{L}_{\text{mass}} = -\frac{1}{2} \mu^2 h^{AB} X_A^{(I)} X_B^{(I)} + \frac{i}{2} \mu h^{AB} \bar{\Psi}_A \Gamma_{1234} \Psi_B
\]

(3.5)

\[
\mathcal{L}_{\text{flux}} = -\frac{1}{6} \mu e^{IJKL} h^{AB}[X^{(I)}, X^{(J)}, X^{(K)}] A X_B^{(L)} - \frac{1}{6} \mu e^{I'J'K'L'} h^{AB}[X^{(I')}, X^{(J')}, X^{(K')}] A X_B^{(L')}
\]

(3.6)

The deformed theory (3.3) breaks the \( \text{SO}(8) \) R-symmetry of the BLG theory down \( \text{SO}(4) \times \text{SO}(4) \). It is invariant under 16 supersymmetries. The supersymmetry transformations of the deformed theory are given by

\[
\delta X^{A(I)} = i \epsilon^{I} \Psi^{A}
\]

\[
\delta \bar{\Psi}^{A} = D_\mu X^{A(I)} \Gamma_\mu \epsilon + \frac{1}{6} X^{B(I)} X^{C(J)} X^{D(K)} f^{A}_{BCD} \Gamma_{IJK} \epsilon - \mu \Gamma_{1234} \Gamma^{J} X^{A(I)} \epsilon
\]

(3.7)

\[
\delta (\bar{A}_\mu)^{A} B = i \epsilon^{I} \Gamma_{J} X^{C(I)} \Psi^{D} f^{A}_{BCD}
\]

By setting \( \mu \rightarrow 0 \), we recover the supersymmetry transformations (2.7) of the BLG theory. The generalized BLG theory is invariant under another 16 non-linearly realized supersymmetries due to the existence of the central generator \( T^- \). For the later convenience, let us clarify the notation. \((+, -, A, B, ...)\) denotes the index of 3-algebra \( \mathcal{A}_g \). \((a, b, ...)\) denotes the index of Lie algebra \( \mathcal{G} \). \((X^{(i)}, X^{(j)}, ...)\) denotes the eight transverse direction and one of them \( X^{(8)} \) denotes the compactified direction. \((X^{(i)}, X^{(j)}, ...)\) denotes the left transverse
direction. Other convention is:

\[ A^{-a}_\mu = A^a_\mu, \quad \frac{1}{2}f^{abc} A_{\mu bc} = B^a_\mu \]

\[ A_\mu = A^a_\mu T^a, \quad B_\mu = B^a_\mu T^a \]

\[ T^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - 2f^{abc} A^b_\mu A^c_\nu \]

\[ X^{\pm (I)} = \pm X^{0(I)} + X^\phi(I) \]

\[ \Psi^\pm = \pm \Psi^0 + \Psi^\phi \]

\[ \bar{\Psi}^\pm = \pm \bar{\Psi}^0 + \bar{\Psi}^\phi \]

(3.8)

\[ D_\mu X^A(I) = \partial_\mu X^A(I) + f^A_{\phantom{A}BCD} A^B_\mu X^C(I) \]

\[ D_\mu \Psi^A = \partial_\mu \Psi^A + f^A_{\phantom{A}BCD} A^B_\mu \Psi^C \]

\[ D_\mu X^a(I) = \partial_\mu X^a(I) + 2f^{abc} A^b_\mu X^c(I) - 4\mathcal{B}^a_\mu X^{-I} \]

\[ D_\mu X^{+I} = \partial_\mu X^{+I} + 4\mathcal{B}^a_\mu X^a(I) \]

\[ D_\mu X^{-I} = \partial_\mu X^{-I} \]

(3.9)

Note that \( D_\mu \Psi^a \), \( D_\mu \Psi^+ \) and \( D_\mu \Psi^- \) have the same forms as \( D_\mu X^a \), \( D_\mu X^+ \) and \( D_\mu X^- \) and \( D_\mu \bar{\Psi} \) has the same form as \( D_\mu \Psi \). Using the above convention, the BLG theory Lagrangian (2.15) can be rewritten in the \( \mathcal{G} \) invariant form:

\[ \mathcal{L} = -\frac{1}{2} D_\mu X^a(I) D^\mu X^a(I) - \frac{1}{2} \left( \partial_\mu X^{+I} + 4\mathcal{B}^a_\mu X^a(I) \right) \partial^\mu X^{-I} \]

\[ + \frac{i}{2} \bar{\Psi}^a \Gamma^\mu D_\mu \Psi_a + \frac{i}{4} \bar{\Psi}^+ \Gamma^\mu \partial_\mu \Psi^+ - \frac{i}{4} \bar{\Psi}^- \Gamma^\mu \left( \partial_\mu \Psi^+ + 4\mathcal{B}^a_\mu \Psi^a \right) \]

\[ + \frac{i}{4} f^{abc} \bar{\Psi}^a \Gamma^{IJ} X^b(I) X^c(J) \Psi^a \]

\[ + \frac{i}{4} f^{abc} \bar{\Psi}^a \Gamma^{IJ} X^{-(I)} X^{c(J)} \Psi^a \]

\[ - \frac{1}{2} f^{abc} X^a(I) X^b(J) X^c(K) \]

\[ - \frac{1}{2} f^{abc} X^a(I) X^c(K) X^b(J) \]

\[ - \frac{1}{2} f^{abc} X^a(I) X^b(J) X^c(K) \]

\[ + 2 \epsilon^{\mu\nu\lambda} \mathcal{B}^a_\mu \mathcal{F}^a_{\nu\lambda} \]

(3.10)

The mass deformation term of Lagrangian takes the form:

\[ \mathcal{L}_{\text{mass}} = -\frac{1}{2} \mu^2 X^a(I) X^a(I) - \frac{1}{2} \mu^2 X^{+I} X^{-I} \]

\[ + \frac{i}{2} \mu \bar{\Psi}^a \Gamma_{1234} \Psi^a + \frac{i}{4} \mu \bar{\Psi}^+ \Gamma_{1234} \Psi^+ + \frac{i}{4} \mu \bar{\Psi}^- \Gamma_{1234} \Psi^- \]

(3.11)

The flux-inducing potential part of Lagrangian can be written:

\[ \mathcal{L}_{\text{flux}} = -\frac{2}{3} \mu e^{IJKL} f^{bcd} X^b(J) X^c(K) X^d(L) X^{-I} + (IJKL \rightarrow I'J'K'L') \]

(3.12)
Notice that in (3.3), the mode $X^{+(I)}$ appears only through linear form, so that it can be integrated out exactly. The integration freezes the mode $X^{-(I)}$ to the value of the free theory with a source-like term due to the presence of the mass term.

\[
(\partial^2 - \mu^2)X^{-(I)} = 0 \tag{3.13}
\]

\[
(\Gamma^\nu \partial_\nu + \mu \Gamma_{1234})\Psi^- = 0 \tag{3.14}
\]

This is a new feature of (3.3) and hints that the negative norm states may be consistently decoupled from the physical Hilbert space.

4 From M2 to D2

In this section, we show how the deformed generalized BLG theory, which is interpreted as a theory of coinciding membranes in particular background, is related to the low energy description of multiple D2 branes. The general solution to this problem is tricky and we need to resort to Janus field theory. The reader who is interested in this approach is referred to recent paper [13] and references therein. In this letter, we are not ambitious to deal with the general solution. Instead we will fix the problem in the specific limit, the weak background flux limit which means that $\mu^2$ is very small. In other words $X^{-(8)}$ varies slowly along with worldvolume coordinates. In this limit, Mukhi-Papageorgakis Higgs mechanism is easy to deal with. Following the strategy of [10], we make one of the scalar fields acquire the expectation value with the restriction of (3.13) and (3.14). Later we will make $X^{-(8)}$ acquire the expectation value so we solve $X^{-(8)}$ only and keep VEV of other fields vanish. The general solution is,

\[
X^{-(8)} = Ae^{p_\mu x^\mu} + Be^{-p_\mu x^\mu} \tag{4.1}
\]

where $A$ and $B$ are integral constants and $p_\mu$ satisfies $p^2 = \mu^2$. For simplicity, we will work in the weak background flux limit. We expect from this limit we can get some hints to the general solution. In this limit we propose that

\[
<X^{-(8)}> = \frac{\sqrt{2}R}{\ell_p^2} \tag{4.2}
\]

where $R$ is the radius of a circle on which we compactify M-theory to get type IIA string theory and $\ell_p$ is the 11 dimensional Planck length scale. We have $R = g_s \ell_s = g_s^{1/2} \ell_p$ from the string theory dualities and $g_Y^2 = 2(2\pi)^{p-2}g_s \ell_p^{p-3}$ from $D_p$ brane world volume theory, where $g_s$, $\ell_s$ are string coupling and string length and $g_Y$ is $D_p$ brane world volume theory coupling which is just $p+1$ dimensional SYM coupling. Combining the above results, we find that $<X^{-(8)}> = g_Y M$. Notice that $g_Y$ must be function of world volume coordinates in order to satisfy the constraint equation (3.13). $\partial_\mu g_Y$ can be neglected compared with $g_Y$ in the above limit. In order to get the super Yang-Mills theory smoothly from its

\[\text{I am grateful to Zhao-Long Wang for pointing this.}\]
strong coupling limit, membranes worldvolume theory, we need to take the limit $g_{YM} \gg 1$. For the novel 3-algebra, the fundamental identity implies that $\mathcal{A}_G$ reduces to the algebra $G \times U(1)$. It is easy to see from structure constants $f_{A B C}^{d e}$. If $A = +$, we get the general Lie algebra $\mathcal{G}$. Otherwise structure constants vanish and $U(1)$ is reduced. Notice that a VEV $<X^{(8)}> \geq 0$ does not preserve all the supersymmetries due to the $\mu$ term in (5.7). This suggests that the reduced D2 branes system does not preserve all the supersymmetries in the reduced background. Now let us show how various terms in (3.10) (3.11) (3.12) reproduce the SYM theory in the reduced background. Under the novel Higgs mechanism, the bosonic kinetic term of (3.10) becomes:

\[ \mathcal{L}_{\text{kineticB}} = -\frac{1}{2} D_\mu X^{a(i)} D^\mu X^{a(i)} - \frac{1}{2} D'_\mu X^{a(8)} D'^\mu X^{a(8)} - 2B_\mu^{a(8)} X^{(8)} - 2D'_\mu X^{a(8)} B^{a(8)} \]

\[ = -\frac{1}{2} \left( \partial_\mu X^{+ (i)} + 4B^{a(8)} X^{a(8)} \right) \partial^\mu X^{- (i)} - \frac{1}{2} \left( \partial_\mu X^{+ (8)} + 4B^{a(8)} X^{a(8)} \right) \partial^\mu X^{- (8)} \]

\[ = -2g_{YM} B_\mu^{a(8)} B^{a(8)} + 2g_{YM} D'_\mu X^{a(8)} B^{a(8)} \]

\[ = -\frac{1}{2} D'_\mu X^{a(i)} D^\mu X^{a(i)} - \frac{1}{2} D'_\mu X^{a(8)} D'^\mu X^{a(8)} \]

\[ = -\frac{1}{2} \partial_\mu X^{+ (i)} \partial^\mu X^{- (i)} - \frac{1}{2} \partial_\mu X^{+ (8)} \partial^\mu X^{- (8)} + \text{higher order} \] (4.3)

The fermionic kinetic term becomes:

\[ \mathcal{L}_{\text{kineticF}} = i \frac{\sqrt{2}}{2} \bar{\Psi}^a \Gamma^\mu D'_{\mu} \Psi_a + i \frac{\sqrt{2}}{4} \bar{\Psi}^+ \Gamma^\mu \partial_\mu \Psi^- + i \frac{\sqrt{2}}{4} \bar{\Psi}^- \Gamma^\mu \partial_\mu \Psi^+ + \text{higher order} \] (4.4)

The Yukawa term of (3.10) becomes:

\[ \mathcal{L}_{\text{Yukawa}} = i g_{YM} f_{abc} \bar{\Psi}^b \Gamma^{a(8)} X^{c(i)} \Psi^a + \text{higher order} \] (4.5)

The sextic potential term of (3.10) becomes:

\[ \mathcal{L}_{\text{potential}} = -\frac{g_{YM}^2}{4} f_{abc} f_{ef} C X^{a(i)} X^{b(j)} X^{e(i)} X^{f(j)} + \text{higher order} \] (4.6)

The Chern-Simons terms of (3.10) is:

\[ \mathcal{L}_{\text{CS}} = 4 \epsilon^{\mu \nu \lambda} \text{Tr} \left( B_\mu (\partial_\lambda A_\nu - [A_\lambda, A_\nu]) \right) \]

\[ = 2 \epsilon^{\mu \nu \lambda} B_\mu \delta^{a}_{\nu \lambda} \] (4.7)

The mass term of deformed Lagrangian (3.11) becomes:

\[ \mathcal{L}_{\text{mass}} = -\frac{1}{2} g_{YM} \mu^2 X^{(8)} + \frac{1}{2} \mu^2 X^{a(i)} X^{a(i)} \]

\[ + \frac{1}{2} \mu \bar{\Psi}^a \Gamma_{1234} \Psi^a + \frac{i}{4} \mu \bar{\Psi}^+ \Gamma_{1234} \Psi^- + \frac{i}{4} \mu \bar{\Psi}^- \Gamma_{1234} \Psi^+ \] (4.8)
The flux-inducing term (3.12) becomes:

\[
\mathcal{L}_{\text{flux}} = -\frac{2}{3} g_{YM} \mu \epsilon^{ijkl} f^{bcd} X_{b}^{(j)} X_{c}^{(k)} X_{d}^{(l)} + \text{higher order} \quad (4.9)
\]

In the above expressions, we have define a new covariant derivative:

\[
D'_{\mu} X^{a(i)} = \partial_{\mu} X^{a(i)} - 2 f^{a}_{\ bc} X^{c(l)} A_{\mu}^{b} = \partial_{\mu} X^{a(i)} + 2 f^{a}_{\ bc} X^{b(l)} A_{\mu}^{c} \quad (4.10)
\]

Notice that the contributions of higher order terms are suppressed in the strong coupling limit. By virtue of the nature of Lagrangian, we find that \( B_{\mu}^{a} \) is an auxiliary field appearing without derivatives. It therefore can be eliminated via its equation of motion. We therefore consider the Lagrangian involving \( B_{\mu}^{a} \):

\[
\mathcal{L} = -2 g_{YM}^{2} B_{\mu}^{a} B^{\mu a} + 2 g_{YM} D'_{\mu} X^{a(8)} B^{\mu a} + 2 \epsilon^{\mu\nu\lambda} B_{\mu}^{a} F_{\nu\lambda}^{a} + \text{higher order} \quad (4.11)
\]

and equation of motion for \( B_{\mu}^{a} \):

\[
B_{\mu}^{a} = \frac{1}{2 g_{YM}^{2}} \epsilon^{\mu\nu\lambda} F_{\nu\lambda}^{a} + \frac{1}{2 g_{YM}} D'_{\mu} X^{a(8)} \quad (4.12)
\]

Inserting this back into the Lagrangian (3.3):

\[
\mathcal{\tilde{L}} = -\frac{1}{g_{YM}^{2}} F_{\mu\nu}^{a} F^{a\mu\nu} - \frac{1}{2} D'_{\mu} X^{a(i)} D'_{\nu} X^{a(i)} - \frac{1}{2} \partial_{\mu} X^{-}(I) \partial_{\nu} X^{+(I)}
\]

\[
+ \frac{i}{2} \bar{\Psi}_{a} \Gamma^{\mu} D'_{\mu} \Psi_{a} + \frac{i}{4} \bar{\Psi}_{a} \Gamma^{\mu} \partial_{\mu} \Psi_{a} + \frac{i}{4} \bar{\Psi}_{a} \Gamma^{\mu} \partial_{\mu} \Psi_{a}
\]

\[
+ \frac{i}{2} g_{YM} f_{abc} \bar{\Psi}_{a} \Gamma^{8i} X^{c(i)} \Psi_{b} - \frac{g_{YM}^{2}}{4} f_{abc} f_{ef} \epsilon^{c(i)} X^{a(j)} X^{b(j)} X^{c(i)} X^{d(j)}
\]

\[
- \frac{1}{2} g_{YM}^{2} \mu^{2} X^{+(8)} - \frac{1}{2} \mu^{2} X^{a(I)} X^{a(I)} + \frac{i}{2} \bar{\Psi}_{a} \Gamma_{1234} \Psi_{a} + \frac{i}{4} \bar{\Psi}_{a} \Gamma_{1234} \Psi_{a}
\]

\[
+ \frac{1}{2} \bar{\Psi}_{a} \Gamma_{1234} \Psi_{a} - \frac{2}{3} g_{YM} \mu \epsilon^{ijkl} f^{bcd} X_{b}^{(j)} X_{c}^{(k)} X_{d}^{(l)} + \text{higher order} \quad (4.13)
\]

A re-definition \( A \to \frac{1}{2} A \) leads to

\[
D'_{\mu} X^{a(i)} \to \partial_{\mu} X^{a(i)} + f^{a}_{\ bc} X^{b(i)} A_{\mu}^{c} \equiv D_{\mu} X^{a(i)} \quad (4.14)
\]

\[
\mathcal{F}_{\mu\nu}^{a} \to \frac{1}{2} (\partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} - f^{abc} A_{\mu}^{b} A_{\nu}^{c}) \equiv \frac{1}{2} F_{\mu\nu}^{a} \quad (4.15)
\]

Notice that the Lagrangian (4.13) can be written in form:

\[
\mathcal{\tilde{L}} = \mathcal{\tilde{L}}_{\text{decoupled}} + \mathcal{\tilde{L}}_{\text{coupled}} \quad (4.16)
\]
For the coupled part, we re-scale the fields as $(X, \Psi) \rightarrow (X/g_Y M, \Psi/g_Y M)$, and then the Lagrangian becomes:

$$\tilde{\mathcal{L}}_{\text{coupled}} = \frac{1}{g_Y^2} \tilde{\mathcal{L}}_0 + \mathcal{O}(\frac{1}{g_Y^3}) \quad (4.17)$$

$$\tilde{\mathcal{L}}_0 = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2} D_\mu X^a X^{a(8)} + \frac{i}{2} \bar{\Psi}^a \Gamma_\mu D_\mu \Psi_a$$

$$+ \frac{i}{2} f_{abc} \bar{\Psi}^b \Gamma^{8i} X^c X^{a(i)} X^{b(j)} X^{c(i)} X^{f(j)}$$

$$- \frac{1}{2} \mu^2 X^{a(i)} X^{a(i)} + \frac{i}{2} \bar{\Psi}^a \Gamma_\mu D_\mu \Psi^a - \frac{2}{3} \mu \epsilon_{kjl} f^{bc} X^b X^c X^d$$

$$- \frac{1}{2} g_Y^2 (X^{0(8)} + X^{\phi(8)}) - \frac{1}{2} \mu^2 X^{a(8)} X^{a(8)} \quad (4.18)$$

This is just the SU(N) SYM theory with the mass term for scalars and fermions and an induced scalar potential term which is just the Myers term for D-branes in the presence of background fluxes. For the decoupled part, we have:

$$\tilde{\mathcal{L}}_{\text{decoupled}} = -\frac{1}{2} \partial_\mu X^{-(I)} \partial^\mu X^{+(I)} + \frac{i}{4} \bar{\Psi}^+ \Gamma_\mu \partial_\mu \Psi^- + \frac{i}{4} \bar{\Psi}^- \Gamma_\mu \partial_\mu \Psi^+$$

$$- \frac{1}{2} g_Y M \mu^2 X^{+(8)} - \frac{1}{2} \mu^2 X^{a(8)} X^{a(8)} + \frac{i}{4} \bar{\Psi}^+ \Gamma_{1234} \Psi^- + \frac{i}{4} \bar{\Psi}^- \Gamma_{1234} \Psi^+$$

$$- \frac{1}{2} \partial_\mu X^{\phi(I)} \partial^\mu X^{\phi(I)} + \frac{1}{2} \partial_\mu X^{0(I)} \partial^\mu X^{0(I)} + \frac{i}{2} \bar{\Psi}^0 \Gamma_\mu \partial_\mu \Psi^0 - \frac{i}{2} \bar{\Psi}^0 \Gamma_\mu \partial_\mu \Psi^0$$

$$+ \frac{i}{2} \bar{\Psi}^0 \Gamma_{1234} \Psi^0 - \frac{i}{2} \bar{\Psi}^0 \Gamma_{1234} \Psi^0 - \frac{1}{2} g_Y M \mu^2 (X^{0(8)} + X^{\phi(8)}) - \frac{1}{2} \mu^2 X^{a(8)} X^{a(8)} \quad (4.19)$$

$$+ \frac{i}{2} \bar{\Psi}^0 \Gamma_{1234} \Psi^0 - \frac{i}{2} \bar{\Psi}^0 \Gamma_{1234} \Psi^0 - \frac{1}{2} g_Y M \mu^2 (X^{0(8)} + X^{\phi(8)}) - \frac{1}{2} \mu^2 X^{a(8)} X^{a(8)} \quad (4.20)$$

where we have used (3.8) to rewrite the above expression. Notice that the modes $X^{\phi(8)}$ and $X^{0(8)}$ are free and can be dualised to two U(1) gauge fields by Abelian duality. Thus (4.19) of the decoupled Lagrangian tell us that there are two copies of U(1) gauge theory. One U(1) theory is normal, and the other is ghost. The rest part of decoupled Lagrangian (4.20) adds some extra terms to the two copies of U(1) theories. The first two terms in (4.20) are the mass terms for two U(1) theories respectively. The second term in (4.20) contains two source-like terms for two U(1) theories respectively. The last term in (4.20) is a non-propagating term and does not affect the dynamics of the theory. Therefore we find that in the limit we take above, the ghost Lagrangian is completely decoupled from the SU(N)×U(1) and it does not affect the unitary.

5 Discussion

In this letter, we start from the multiple M2 branes world volume theory which is based on a novel 3-algebra without the positive definite metric assumption. Then we deform the world volume theory with background fluxes and the induced mass terms. The deformed
theory is supersymmetric and without tunable coupling constant. These properties are expected for the field theory living on multiple M2 branes. Frankly speaking, the approach of this letter is just an approximate approach of the complete solution. The motivation of this letter is that the study of M2 branes model may give us some hints about the Chern-Simons like term of multiple M2 branes in flux background. My colleagues have done some research work on this topic [36]. We expect that the result of our approximate approach will give some hints to their research. In [34][35], the theory of coincident M2 branes on $R \times T^2$ was argued to provide the Matrix theory description of Type IIB string theory, and the correspondence in some limits have been tested [11]. The study of the deformed M2 branes theory may lead new insights in understanding the stringy physics in Type IIB Matrix theory. We have shown that when one component of the scalar fields develops a VEV, the ensuing Higgs mechanism produces a strong coupled SYM on arbitrary number D2 branes in reduced background. It is interesting to find that after Higgsing, the ghost part of original M2 branes theory is consistently decoupled. Though there are hints suggesting that the negative norm states can be consistently decoupled from the physical Hilbert space, the complete analysis remains to be uncovered.

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