The evolution of helium white dwarfs

III. On the ages of millisecond pulsar systems

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Abstract. We employed recently computed evolutionary white-dwarf models with helium cores, supplemented by heavier models with carbon-oxygen cores, in order to investigate the ages of millisecond pulsar systems based on the cooling properties of the compact companions. Contrary to the behaviour of more massive white dwarfs, the evolutionary speed of low-mass white-dwarf models is substantially slowed down by ongoing hydrogen burning. By comparing the cooling ages of these models with the spin-down ages of the pulsars for those systems for which reasonable information about the compact companions is available, we found good correspondence between both ages. Based on these models any revisions concerning the temporal evolution of millisecond pulsars do not appear to be necessary.

Key words: Stars: evolution – white dwarfs – pulsars: general

1. Introduction

Millisecond pulsars are thought to be components of low-mass binary systems in their final stage of evolution: the neutron star which has been spun up by accretion of matter from a low-mass evolved companion is now being slowed down by emission of magnetic dipole radiation (recycled radio pulsar). The companion, after having transferred most of its envelope mass towards the neutron star, remains as a white dwarf of rather a low mass whose core consists, in the majority of the known cases, of helium. The characteristic age, or so-called spin-down age, of the (recycled) pulsar depends on the physics how the neutron star’s rotational energy is converted into non-thermal emission of electromagnetic energy. On the other hand, the white-dwarf age is ruled by the white dwarf’s thermomechanical structure and the transformation of gravothermal energy content into thermal emission of photons from the surface. Any age determinations of the pulsar and the white-dwarf component should give the same answer, provided our physical understanding of the pulsar’s slow-down processes and the white dwarf’s cooling properties is correct.

So far, no general consensus on this matter has been achieved. Under the assumption that the cooling properties of low-mass white dwarfs are ruled by rather simple laws as is known from evolutionary calculations of more massive white dwarfs with carbon-oxygen cores (cf. Iben & Tutukov 1984; Koester & Schönberner 1986, Blöcker 1995), large age differences between the pulsars and their dwarf companions have been found. In general, the white dwarfs appear to be much younger than the pulsars (cf. Hansen & Phinney 1998a for a recent, detailed account). The best-studied example is the PSR J1012+5307 system, for which Lorimer et al. (1995) determined 7 Gyr for the spin-down age of the pulsar, but only about 0.3 Gyr for the white dwarf’s age. Note that the usual spin-down age determinations are based on the assumption that the initial rotational period after completion of the spin-up by accretion is much smaller than the present one, and that the pulsar emits magnetic dipole radiation (braking index $n = 3$). A summary of the assumptions inherent in the derivation of characteristic or spin-down ages of pulsars is given in Hansen & Phinney (1998). A discrepant result as found for PSR J1012+5307, if true, would have important consequences for the details of the accretion process and the following spin-down phase (cf. Burderi et al. 1996).

A larger sample of millisecond pulsar systems with white-dwarf companions has recently been investigated by Hansen & Phinney (1998), using a grid of low-mass white-dwarf sequences especially computed for this purpose (Hansen & Phinney 1998a). In most cases spin-down and cooling ages appeared to be discrepant to various degrees, and the authors were able to constrain the initial spin periods and spin-up histories for individual systems, especially also for the PSR J1012+5307 system. However, the white-dwarf models which this study is based on, are generated from ad-hoc assumed initial configurations. These configurations appear not to be consistent with respect to the thermo-mechanical structures and un-
processed, hydrogen-rich envelopes with what would be adequate for companions in these pulsar binary systems.

The early investigations concerning the evolution of helium white dwarfs made by Webbink (1975) indicated that the final cooling is slowed down considerably by ongoing hydrogen burning via the pp cycle. Obviously the cooling behaviour of low-mass white dwarfs depends on the size of the still unprocessed hydrogen-rich envelope, i.e. whether this envelope is massive enough as to sustain burning temperatures at its bottom for a long time span. The Webbink (1975) white-dwarf models are, however, just evolved main sequence stars without any consideration of mass loss.

Since white-dwarf envelope masses cannot be guessed from first principles, they must rather be determined by detailed evolutionary calculations. A step in this direction was made by Alberts et al. (1996) and Sarna et al. (1998) who modelled the PSR J1012+5307 system and in particular the evolution of the mass giving companion. It turned out that the donor shrinks below its Roche lobe while still having a rather massive hydrogen-rich envelope which is able to keep hydrogen burning dominant even through the white-dwarf cooling phase. The evolution was slowed down to such an extent that the discrepancy with the spin-down age of the pulsar vanished completely.

Strictly speaking the strength of hydrogen burning, and hence the cooling age of an observed white dwarf, depends on the size of the envelope before entering the cooling path. This envelope mass can be reduced because of thermal instabilities of the burning shell when the CNO rate dies out, namely by

- enhanced hydrogen consumption during the instability (flash) itself, and by
- a possible Roche-lobe overflow driven by the rapid envelope expansion.

The latter case was dominant for the evolution of the Iben & Tutukov (1984) 0.3 M⊙ helium white-dwarf model: Roche-lobe overflow due to the flash-driven envelope expansions reduced the envelope mass below the critical value necessary for hydrogen burning. The white-dwarf models of Webbink (1975) and Sarna et al. (1998) experienced phases of unstable hydrogen burning for $M \lesssim 0.2$ M⊙ (but see Driebe et al. 1999 for a discussion).

Recently Driebe et al. (1998) published a grid of evolutionary tracks for helium white-dwarf models which were generated by enhanced mass loss applied at different positions along the red-giant branch of a 1 M⊙ sequence (see also Iben & Tutukov 1984, Castellani et al. 1994). This method mimicks to some extent the mass transfer in binary systems and allows to get reliable post-red-giant configurations which are very useful for the interpretation of observations. Driebe et al. (1998) covered the whole mass range of interest, and they demonstrated that

- the anti-correlation between core mass and size of envelope (cf. Blöcker et al. 1997) determines later the nuclear activity along the cooling branch, and that
- thermal instabilities of the hydrogen-burning shell appear to be restricted to the mass range of approximately 0.2 to 0.3 M⊙.

The absence of thermal flashes below $M = 0.2$ M⊙ agrees well with the results of Alberts et al. (1996) but disagrees with those of Sarna et al. (1998). Nevertheless, the cooling times of our models are in excellent agreement with both studies. From the given parameters of the white-dwarf component in the PSR J1012+5307 system, Driebe et al. (1998) determined then its age to be of 6 ± 1 Gyr, in good agreement with the pulsar’s spin-down age of 7.0±1.4 Gyr (Lorimer et al. 1997).

The latest effort in a better understanding of the combined pulsar-white dwarf systems is that of Burderi et al. (1998). They took the pulsar spin-down ages at their face value and concluded that the standard assumption for the white-dwarf cooling (i.e. without nuclear burning) complies with the observations, except for masses below approx. 0.2 M⊙. There are, however, some facts that we would like to point out: Burderi et al. (1998) used data ‘renormalized’ to a standard luminosity of $10^{-2}$ L⊙, whereby it remains unclear how ages can be renormalized if the temporal evolution of the systems is not known a priori. Furthermore, they extrapolated existing white-dwarf cooling models into mass regimes where they are not valid anymore.

Because of its importance we felt the necessity to reconsider the whole issue by utilizing more realistic evolutionary models for low-mass white dwarfs. We will show in the next section that with such models a consistent description of those millisecond pulsar binary systems can be achieved for which sufficiently accurate data is available.

2. Pulsar characteristic times and cooling ages of their white-dwarf companions

We started with the sample of millisecond pulsar systems used by Burderi et al. (1998, see their Table 1 and our Fig. 1), but made a few changes: the companion mass for PSR J1012+5307 was updated according to Driebe et al. (1998), and the systems PSR J1640+2224 and PSR J0437-4715 were omitted because of too uncertain pulsar ages. For convenience, the relevant data are collected in Table 1 and all the listed systems are shown in Fig. 1 where the characteristic ages of the pulsars are plotted against the possible mass ranges of their (white-dwarf) companions.

The last column in Table 1 gives the white-dwarf masses according to the binary evolution calculations of Tauris & Savonije (1999). Within these binary calculations a relation between the system’s orbital period $P_{	ext{orb}}$ and the white-dwarf mass can be derived (see e.g. Savonije 1987 and Rappaport et al. 1997). For the systems discussed
here the $P_{\text{orb}} - M_{\text{WD}}$ relation of Tauris & Savonije (1999) predicts masses which are well within the estimated mass limits (see Table 1).

It should be emphasized that the characteristic ages given in Table 1 and plotted in Fig. 1 are based on certain assumptions (see Introduction) which may not be fulfilled in all cases. The corresponding systematic errors are difficult to assess and cannot be accounted for in this study.

Also shown in Fig. 1 are (post-red giant) ages of helium white-dwarf models taken from Driebe et al. (1998), supplemented by ages from evolutionary white-dwarf models with carbon-oxygen cores (Blöcker 1995). The ages are given for four effective temperatures as to simplify the comparisons with the observed systems: the range between 4000 and 20 000 K embraces roughly the estimated effective temperatures of the companions of the systems PSR J0034–0534, PSR J1713+0747, PSR J1012+5307 and PSR B0820+02 (cf. Hansen & Phinney 1998). Only for the PSR J1012+5307 companion exists a rather accurately determined effective temperature (8 600 K, van Kerkwijk et al. 1996, Callanan et al. 1998).

Note that the model ages are counted from the beginning of the post red-giant phase, i.e. they include also the contraction towards the white-dwarf regime. For white dwarfs of low mass this contraction from a giant towards a white-dwarf configuration makes up for a significant fraction of their ages and must be accounted for in younger systems like PSR B0820+02.

The temporal behavior of the models, as shown in Fig. 1, is determined by the following facts:

- The compositional differences between the lighter and heavier white dwarfs causes the obvious age break around $M_{\text{WD}} = 0.5 M_{\odot}$.
- Hydrogen burning in the helium white dwarfs is, for a given temperature, responsible for the strong increase in age with decreasing mass.

- At high temperatures, the lighter models ($\lesssim 0.23 M_{\odot}$) are still in a pre white-dwarf phase very close to the turn-around point with rather low (post red-giant) ages.

Fig. 1 clearly demonstrates that for an effective temperature of 4 000 K our low-mass white-dwarf models exceed ages of 10 Gyr, roughly consistent with the pulsar characteristic ages. From the positions of individual pulsars (with error bars) we can estimate effective temperature and gravity ranges, and also the internal composition, to be expected for the white-dwarf companions. The results are collected in Table 2. Since for some of the white dwarfs temperature estimates based on photometry are available (Hansen & Phinney 19981), consistency checks are possible.

2.1. PSR J1012+5307

The white dwarf in PSR J1012+5307 is so far the best studied companion of all known systems (van Kerkwijk et al. 1996, Callanan et al. 1998). With its surface parameters known, and together with our evolutionary white-dwarf models, a consistent description of the whole system is found (Fig. 2 upper two panels). Since the mass ratio of both components is known, the system’s inclination can be determined as well. The course of the inclination angle with white-dwarf companion mass is shown in the lower panel of Fig. 2 for two limiting pulsar masses. The inclination can be expected to lie between 40 and 50 degrees.

2.2. PSR B0820+02

The mass of the companion is rather ill-defined, and it could have a helium or carbon/oxygen core (cf. Table 2). However, the temperature estimate, $15 250 \pm 250 K$
The temperature of the companion has recently been photometrically determined by van Kerkwijk et al. (2000) to be $T_{\text{eff}} = 4800 \pm 800$ K. Given its accurately known mass of $0.258^{+0.020}_{-0.016}$ M\(_\odot\), this low temperature corresponds to a cooling age of 10 Gyr using our models and 3 Gyr using the models of Hansen & Phinney (1988) with their smaller envelope masses ($\lesssim 3 \times 10^{-4}$ M\(_\odot\)). The characteristic age of the pulsar is, however, 5 Gyr (cf. Table 1), a fact which on one hand might be a hint for a smaller braking index of the pulsar as discussed by van Kerkwijk et al. (2000). On the other hand this result strengthens the dependence of age on the thickness of the hydrogen envelope.

According to our 0.250 M\(_\odot\) model, hydrogen burning ceases at about 10 Gyr, leaving a final unprocessed envelope of $5 \cdot 10^{-4}$ M\(_\odot\). The envelope mass right after the last shell flash is $\approx 2 \cdot 10^{-3}$ M\(_\odot\), a value which is, however, subject to uncertainties like flash strength and metallicity. In additional calculations we artificially reduced this envelope mass by invoking mass loss on the upper cooling branch and followed the evolution of the models in the usual manner. It turned out that an earlier reduction of the envelope mass to $\approx 5 \cdot 10^{-4}$ M\(_\odot\) after $\approx 0.5$ Gyr (at $T_{\text{eff}} \approx 10^4$ K) would be sufficient to give a cooling age of 5 Gyr at the desired effective temperature of 4800 K.

We note that with this reduced envelope mass, hydrogen burning becomes insignificant below $T_{\text{eff}} \approx 10^4$ K.

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Fig. 1. Time-scales vs. white-dwarf masses. Boxes indicate millisecond pulsar systems with reasonably known spin-down ages and companion masses. The sizes of the boxes indicate the errors listed in Table 1. The lines with symbols (+: helium white dwarfs, ×: carbon/oxygen white dwarfs) mark the post red-giant ages of white-dwarf models at four different temperatures along their cooling tracks taken from Driebe et al. (1998) and Blöcker (1995). For more details see text.

(Hansen & Phinney 1988), allows only a C/O white dwarf with at least $\sim 0.5$ M\(_\odot\) and a rather high gravity ($\log g \approx 8$), in agreement with the results of Tauris & Savonije (1999) (cf. Table 1).

2.3. PSR B1855+09

The temperature of the companion has recently been photometrically determined by van Kerkwijk et al. (2000) to be $T_{\text{eff}} = 4800 \pm 800$ K. Given its accurately known mass of $0.258^{+0.020}_{-0.016}$ M\(_\odot\), this low temperature corresponds to a cooling age of 10 Gyr using our models and 3 Gyr using the models of Hansen & Phinney (1988) with their smaller envelope masses ($\lesssim 3 \times 10^{-4}$ M\(_\odot\)). The characteristic age of the pulsar is, however, 5 Gyr (cf. Table 1), a fact which on one hand might be a hint for a smaller braking index of the pulsar as discussed by van Kerkwijk et al. (2000). On the other hand this result strengthens the dependence of age on the thickness of the hydrogen envelope.

According to our 0.250 M\(_\odot\) model, hydrogen burning ceases at about 10 Gyr, leaving a final unprocessed envelope of $5 \cdot 10^{-4}$ M\(_\odot\). The envelope mass right after the last shell flash is $\approx 2 \cdot 10^{-3}$ M\(_\odot\), a value which is, however, subject to uncertainties like flash strength and metallicity. In additional calculations we artificially reduced this envelope mass by invoking mass loss on the upper cooling branch and followed the evolution of the models in the usual manner. It turned out that an earlier reduction of the envelope mass to $\approx 5 \cdot 10^{-4}$ M\(_\odot\) after $\approx 0.5$ Gyr (at $T_{\text{eff}} \approx 10^4$ K) would be sufficient to give a cooling age of 5 Gyr at the desired effective temperature of 4800 K.

We note that with this reduced envelope mass, hydrogen burning becomes insignificant below $T_{\text{eff}} \approx 10^4$ K.

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The mass of the chemically homogeneous envelope is defined as the total mass of the layers above the hydrogen exhausted core.
2.4. PSR J0034-0534

Hansen & Phinney (1998) estimated a very low temperature limit of $T_{\text{eff}} < 3500$ K for the white dwarf, yielding ages of more than 10 Gyr if helium models are used, which is to be compared with the pulsar’s characteristic age of 6.8 ± 2.4 Gyr (Fig. 2). Consistency between the pulsar’s and the white dwarf’s age can only be achieved if we assume the white dwarf to have a carbon/oxygen core and a mass of ≈ 0.5 M⊙. Such a model cools considerably faster and reaches 3500 K well within about 6 Gyr. We note that this mass is noticeably larger than estimates of other studies. The $P_{\text{orb}} - M_{\text{WD}}$ relation of Tauris & Savonije (1999) gives $M \approx 0.21$ M⊙, and the photometric measurements of Lundgren et al. (1996) yield $M \approx 0.23$ M⊙. The cooling models of Hansen & Phinney (1998) predict $M_{\text{WD}} \approx 0.32$ M⊙ as an upper limit. The discrepancy between these results and our mass estimate (C/O-white dwarf) might be related to the same problem as in the case of PSR B1855+09. If the companion is a helium-white dwarf with $M \approx 0.2\ldots0.3$ M⊙ it is prone to hydrogen shell flashes with the corresponding uncertainties.

2.5. PSR 1713+0747

Hansen & Phinney (1998) give a white-dwarf effective temperature of 3400 ± 300 K which is just slightly lower than the one predicted by our helium models (see Table 3). The white-dwarf mass is not very sensitive to the temperature value and consistent with the estimate of $M_{\text{WD}} \approx 0.33$ M⊙ from Tauris & Savonije (1999).

2.6. Other millisecond pulsar systems

In addition to the sample discussed above we investigated some other MSP systems with respect to the possible $(g, T_{\text{eff}})$ combinations for the white-dwarf components (see Table 3). The results are illustrated in Fig. 3 where effective temperature and surface gravity are plotted as a function of the white-dwarf mass for the systems listed in Table 3. The white-dwarf masses can be estimated from the $P_{\text{orb}} - M_{\text{WD}}$ relation of Tauris & Savonije (1999). Taking these masses, it is straightforward to determine effective temperatures and surface gravities of the white-dwarf companions (see Table 3).

We find the systems in the first row of Fig. 3 (J1640+2224, J0751+1807, J1045-4509) to be consistent with He-WD companions if $T_{\text{eff}} \gtrsim 4000$ K. At this temperature our helium models reach cooling ages comparable with the age of the galactic disk (≈ 10 Gyr). For the white dwarf in J1640+2224 a temperature estimate is available: 4500 ± 1100 K (Hansen & Phinney 1998). This value implies, for 10 Gyr, a white-dwarf mass of ≈ 0.35 ± 0.05 M⊙ with a surface gravity of $g \approx 7.6$ (cf. Fig. 3). This mass is in good agreement with the result of Tauris & Savonije (1999) ($M_{\text{WD}}^{\text{T99}} \approx 0.37$ M⊙).

For the other systems the temperature ambiguity (more than one white dwarf mass for a given temperature, see Fig. 3) does not allow to exclude the companions to be C/O-white dwarfs, but the mass values of Tauris & Savonije (1999) indicate that all these systems should contain helium white dwarfs.

3. Conclusions

From the present paper, together with previous efforts which concentrated solely on the PSR J1012+5307 system (Alberts et al. 1996; Sarna et al. 1998; Driebe et al. 1998), it becomes obvious that a consistent description...
of the millisecond binary systems with compact companions can only be achieved by using evolutionary model calculations of white dwarfs which include their complete pre-white-dwarf history. The key to the solution of the apparent age paradoxon between the pulsar and its white-dwarf companion is the fact that low-mass white dwarfs have massive, still unburnt envelopes that sustain hydrogen burning at their bases for a long time. Hydrogen burning slows down the cooling of a low-mass white dwarf to such an extent that cooling ages become comparable to, or may even exceed, observed pulsar spin-down ages.

Employing our evolutionary helium white-dwarf models, supplemented by those with carbon-oxygen cores, we demonstrated that, next to the already studied PSR J1012+5307 system, also in other millisecond pulsar binary systems with reliable information on pulsar age and companion properties, as mass and temperature, reasonable agreement between the components' ages is achieved. The use of white-dwarf models with ad hoc assumed envelope masses may lead to erroneous interpretations since these envelope masses are usually much smaller than those which follow from complete evolutionary calculations.

It appears to us that upon using realistic white-dwarf models in interpreting millisecond binary systems there is no need to modify existing ideas of the spin-down process of pulsars. Clearly a larger sample of well-studied systems like PSR J1012+5307 would be very important in investigating more precisely the cooling theory of white dwarfs and the braking of radio pulsars.

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Table 3. Data for selected millisecond pulsars. $P_{\text{spin}}$: pulsar’s spin period; $\dot{P}$: change of $P_{\text{spin}}$; $\tau_c$: characteristic age; $f(M)$: mass function; $M_{\text{WD}}^{\text{TS99}}$: white-dwarf mass according to the $P_{\text{orb}} - M_{\text{WD}}$ of Tauris & Savonije (1999); $T_{\text{eff}}, g$: predicted effective temperature and surface gravity for $M_{\text{WD}} = M_{\text{WD}}^{\text{TS99}}$ (compare Fig. 3).

| PSR         | $P_{\text{spin}}$ [ms] | $\dot{P}$ [$10^{-20}$ ss$^{-1}$] | $\tau_c$ [Gyr] | $f(M)$ [$10^{-3}$ $M_\odot$] | Ref. | $M_{\text{WD}}^{\text{TS99}}$ [$M_\odot$] | $T_{\text{eff}}$ [K] | $\log g$ | |
|-------------|-------------------------|---------------------------------|----------------|------------------------------|------|-------------------------------------|-------------------|---------|
| B1953+29    | 6.1332                  | 2.95                            | 3.3            | 2.417                        | 1    | 0.35                                 | 8500$^{+4500}_{-1500}$ | 7.4     |
| J0751+1807  | 3.4788                  | 0.8                             | 7.3            | 0.967                        | 2    | 0.18                                 | 7600$^{+500}_{-500}$ | 6.8     |
| J1045-4509  | 7.4742                  | 1.9                             | 6.3            | 1.765                        | 1    | 0.23                                 | 7300$^{+1000}_{-800}$ | 7.1     |
| J1640+2224  | 3.1633                  | 0.29                            | 7.3            | 0.783                        | 1    | 0.37                                 | 4000$^{+3000}_{-2000}$ | 7.6     |
| J1643-1224  | 4.6216                  | 3.3                             | 6.8            | 1.049                        | 1    | 0.244                                | 9500$^{+8000}_{-4000}$ | 7.0     |
| J1804-2718  | 9.3430                  | 4.2                             | 7.7            | 3.347                        | 3    | 0.26                                 | 9000$^{+6000}_{-1000}$ | 7.1     |
| J1911-1114  | 3.6257                  | 1.34                            | 6.7            | 1.239                        | 4    | 0.22                                 | 9000$^{+2000}_{-1000}$ | 7.0     |
| J0437-4715  | 5.7575                  | 5.73                            | $\leq$ 6      | 1.239                        | 3    | 0.244                                | 9000$^{+2500}_{-1000}$ | 7.0     |
| J2129-5721  | 3.7263                  | 2.0                             | 6.8            | 1.049                        | 3    | 0.244                                | 9000$^{+2500}_{-1000}$ | 7.0     |

1: Taylor et al. (1993, 1995 (unpubl. work, available via anonymous ftp from pulsar.princeton.edu (128.112.84.73)))
2: Lundgren et al. (1996b)
3: Lorimer et al. (1996)
4: Sandhu et al. (1997)

Fig. 3. Possible effective temperatures (left y-axis, solid lines) and surface gravities (right y-axis, dashed lines) as a function of white-dwarf mass for the millisecond pulsar systems of Table 3 for three different cooling ages. The middle lines belong to the given characteristic age (see Table 3), except for J1640+2224 where the white-dwarf properties are shown for $\tau = 10$ Gyr. The upper and lower lines refer to ages $\tau_c \pm 2$ Gyr. For J0437-4715, where only an upper limit for $\tau_c$ is known, lines are given for $\tau = 1, 3$ and 6 Gyr. Note that the lower age limit is represented by the upper lines for $T_{\text{eff}}$ and the lower lines for $\log g$. 