Abstract

A new mechanism for spontaneous breaking of flavour symmetry is demonstrated. An exactly flavour symmetric model with degenerate bare nonets and with sufficiently strong tri-linear meson couplings is shown to lead to self-consistency equations which are unstable. Instead there exists a stable solution, which break flavour symmetry spontaneously in the mass spectrum. For a $C$-degenerate meson spectrum the stable mass spectrum obeys the Okubo-Zweig-Iizuka (OZI) rule and the approximate equal spacing rule.

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1. Introduction and the NJL model. Any solution of the hadron mass spectrum, consistent with QCD, must satisfy the constraints of: (i) unitarity, (ii) analyticity, and in the massless quark limit also (iii) chiral symmetry and (iv) flavour symmetry.

Conventionally one breaks (iii-iv) by adding effective non-degenerate quark masses to the Lagrangian, wherby the pseudoscalars obtain (small) masses and the degeneracy of all flavour multiplets is split. The gluon anomaly contributes to the flavourless sector in breaking the symmetry to approximate $SU_{3_f}$, giving especially the $\eta$ mesons part of their masses. Most of the chiral quark masses are assumed to come from a short distance regime, where weak interactions are relevant. Phenomenologically fermion masses can be generated by suitable $ad$ hoc couplings to a Higgs field. Here I shall discuss an alternative way of breaking the flavour symmetry.
From deep inelastic scattering, including the spin problem of the proton, we know that mesons and baryons are not simple $q\bar{q}$ and $qqq$ quark model bound states, but have, in addition to the valence quarks, large components in the form of a multiquark $q\bar{q}$ sea and gluons. Thus a constituent quark of the naive quark model must, in fact, be composed of many quarks, antiquarks and gluons. This requires a multichannel formalism, which couples $q\bar{q}$ and multiquark states.

Recently it was shown that one can obtain a good understanding of the light scalar meson nonet \[3\] when one uses the constraints ($i$-$iv$) above provided one includes in a coupled channel framework all light two pseudoscalar thresholds with flavour symmetry broken mainly by the pseudoscalar masses. Similar models for other multiplets also improve our understanding of the hadron spectrum, even for heavy quarks. One good example is the anomalously large $\Upsilon(5)-\Upsilon(4)$ mass splitting \[4\], where single channel potential models clearly fail, because of the opening of the $B\bar{B}, B\bar{B}^*$ etc. thresholds.

In such models one needs a smaller bare $m_s - m_d$ quark mass term than usual, and this becomes smaller the more hadronic thresholds are included. With sufficient number of higher thresholds (obeying the empirically approximate equal spacing rule) the $SU3_f$ breaking usually attributed to $m_s - m_d$ could be generated by hadronic loops. This opens up the possibility that the bare $m_s - m_d$ could be put equal zero, and that flavour symmetry breaking could be generated spontaneously, through the self-consistency equations for hadron propagators, which include hadronic thresholds.

The well known Nambu–Jona-Lasinio (NJL) model \[1\] is a standard reference model for for chiral symmetry breaking and for dynamical symmetry breaking in general. Although I shall not use this model here the instability mechanism in the original NJL model has similarities with the mechanism I shall discuss, and therefore I recall here some well known results from the NJL model. For reviews see Ref. \[2\]. With a small chiral mass ($\mu$) the well known gap equation for the quark constituent mass ($M$) is

$$\frac{M}{\Lambda} = \frac{\mu}{\Lambda} + \frac{M}{\Lambda} \frac{\alpha_{NJL} \pi}{4} F(0, \frac{M^2}{\Lambda^2}, \frac{M^2}{\Lambda^2}, 1) .$$

Here $\Lambda$ is the cutoff and $\alpha_{NJL}$ measures the strength of the four fermion interaction. The function $F$ takes into account the dependence on the fermion mass in the tadpole-like loop. It will be defined below in a dispersive framework.

Eq. \(1\) has for $\alpha_{NJL} > 1, \mu = 0$, apart from the trivial, unstable solution $M = 0$, a massive and stable solution. This is most easily seen by plotting the left side of the equation,
\( M_{\text{out}}/\Lambda \), against \( M_{\text{in}}/\Lambda \) of the right side as in Fig. 1. For \( \alpha_{NJL} > 1 \) the slope of the curve at the origin is > 1. Then starting at any value for \( M_{\text{in}}/\Lambda \) and iterating the equation one ends up at the massive solution as shown in Fig. 1. On the other hand if \( \alpha_{NJL} < 1 \) one obtains the dashed curve and the trivial massless solution is the stable one.

At the same time as the fermions obtain mass, poles are generated in the pseudoscalar and scalar \( q\bar{q} \) channels. The pion is massless as required by chiral symmetry while the \( \sigma \) has a mass of \( 2M \). In generalizations one expects a massless \( 0^{-+} \) multiplet and massive but degenerate multiplets of other \( J^{PC} \) when the \( SU(N_f) \) remains unbroken. In the following I shall not rely further on the NJL model.

2. A flavour symmetric scalar model. We now construct a meson model, where all flavour related bare \( q_i\bar{q}_j \) states are degenerate, and which couple to each others through flavour symmetric couplings. These mesons can then be described in the ideally mixed reference frame, i.e. the flavourless states are simply unmixed (like \( s\bar{s}, c\bar{c} \)). Consider now meson loops as shown in Fig.2 and assume that disconnected quark-line loop diagrams which violate the OZI rule can be neglected. To be exact this requires that we have at least two degenerate nonets with opposite charge conjugation, and equal F-like and D-like flavour couplings as described in more detail in Ref. [3]. Otherwise singlet states are shifted differently from nonsinglet states and quark-line disconnected loops (like \( \phi \rightarrow \omega \) through strange intermediate states) do not vanish. Here this \( C \)-degeneracy is assumed for simplicity, but it is relaxed in Ref. [4] for a model with three flavours, where the OZI rule is broken, but the isospin subgroup remains unbroken.

Taking into account these loops, or the vacuum polarization diagrams, the meson spectrum should still be consistent with being degenerate, if one disregards possibility of instability. This is satisfied, since also the thresholds \( m_{ik} + m_{kj} \) are degenerate in flavour, and when summed over \( k \), each meson \( ij \) (using the shorthand \( q_i\bar{q}_j \equiv ij \)) gets an equal contribution from the vacuum polarization diagrams. Of course to have this result, not only the couplings but also any cutoff, \( \Lambda \), or subtraction constant involved must be independent of flavour. Then the expression for the self energy, which include quantum loops can be the same for all \( ij \), and also the renormalized masses (which now include ”unitarity effects”) can remain degenerate. Consequently there is one very symmetric situation, where one knows the solution, and our equations must allow for this trivial self-consistent solution. But is this solution stable? I shall show that for sufficiently large coupling it is not!

3. A new mechanism for spontaneous symmetry breaking. The simplest, model is ob-
tained for the case of only two flavours and scalar mesons coupling to two-body thresholds of scalar mesons. Denote the two flavours by 1 and 2. Then there are four mesons, \( i j = 11, 12, 21 \) and \( 22 \), and for each meson there are meson-meson thresholds with different quark content, \( m_{ik} + m_{kj}, \ k = 1, 2 \). The inverse propagators, \( P^{-1} \), get contributions from the meson loops due to these thresholds. One finds a sum of two contributions (cf. Fig. 2)

\[
P^{-1}_{ij}(s) = m_0^2 - s + \frac{g^2}{4\pi} \sum_{k=1}^{N_I=2} F(s, m_{ik}^2, m_{kj}^2, \Lambda),
\]

where \( g \) is the coupling for each threshold, and \( m_0 \) is the common bare mass. The function \( F \) must have the unitarity cut, which is proportional to two-body phase space. A simple model for \( F \), which I use for the demonstration in this paper, has an imaginary part given by

\[
\text{Im}[F] = \lambda^2(s, m_{11}^2, m_{22}^2)\theta(\lambda)\theta(\Lambda^2 - \lambda/(4s)).
\]

It vanishes below threshold (the first \( \theta \) function) and for momenta above the cutoff (the second \( \theta \) function). The real part is determined by the dispersion relation:

\[
\text{Re}[F(s, m_{11}^2, m_{22}^2)] = \frac{1}{\pi} \mathcal{P} \int_{0<\lambda<4s} \frac{\lambda^2(s', m_{11}^2, m_{22}^2)ds'}{s' - s}. \tag{3}
\]

The function \( F \) can be evaluated analytically and is simply related (up to subtractions and factors of \( s \) in the imaginary part) to the Chew-Mandelstam, and \( H(s) \) functions, which also appear in the literature. With this \( F \) the coupling constant \( g \) is dimensionless.

The physical meson masses (poles) are given by the zeroes of the inverse propagators of Eq.(2). Now eliminate the universal bare mass \( m_0 \) by fixing the mass of one of the mesons, say \( m_{11} \). One then obtains the following self-consistency equations (cf. Fig. 2):

\[
0 = P^{-1}_{ij}(s = m_{ij}^2) = m_{11}^2 - s + \frac{g^2}{4\pi} \sum_{k=1}^{N_I=2} \left[ F(s, m_{ik}^2, m_{kj}^2, \Lambda) - F(m_{11}^2, m_{ik}^2, m_{kj}^2, \Lambda) \right]. \tag{4}
\]

By construction \( m_{11} \) is always the solution of \( P^{-1}_{11}(m_{11}^2) = 0 \). The two other masses, \( m_{12} \) and \( m_{22} \), are then determined by the remaining two equations. From the above discussion the degenerate solution with equal \( m_{ij} = m_{11} \) is one solution. But this need not be the only solution, and furthermore the symmetric solution need not be stable. Denoting a small variation from the symmetric solution in the threshold masses by \( \delta_{ij} = m_{ij}^2 - m_0^2 \), which results in a shift \( \delta_{ij}^{\text{out}} \) in the pole positions at the right hand side of Eq. (3), self-consistency
of course requires $\delta_{ij}^{\text{out}} = \delta_{ij}$ and stability $\delta_{ij}^{\text{out}} < \delta_{ij}$. I.e., if the latter inequality is satisfied then starting from some $\delta_{ij} \neq 0$ and iterating one converges towards the symmetric solution $\delta_{ij}^{\text{out}} = \delta_{ij} = 0$. On the other hand, if $\delta_{ij}^{\text{out}} > \delta_{ij}$ the symmetric solution is unstable. Denoting by $F_s = \partial F/\partial s|_{s=m_1^2=m_2^2}$ and by $F_{m^2} = \partial F(s, m_1^2, m_2^2)/\partial m_1^2|_{s=m_1^2=m_2^2}$ the self-consistency equations can be written

$$
\delta_{ij}^{\text{out}} \left[ N_f \frac{g^2}{4\pi} F_s - 1 \right] + \frac{g^2}{4\pi} F_{m^2} \sum_k [\delta_{ik} + \delta_{jk}] = 0 .
$$

(5)

This self-consistency requires that small deviations from the symmetric solution must satisfy the equal spacing rule

$$
\delta_{ij}^{\text{out}} - \delta_{ii}^{\text{out}} = \frac{1}{2} (\delta_{jj}^{\text{out}} - \delta_{ii}^{\text{out}}) ,
$$

(6)

while the symmetric solution is unstable if

$$
r = \frac{\delta_{ii}^{\text{out}}}{\delta_{ii}} = \frac{N_f F_{m^2}}{-N_f F_s + 4\pi/g^2} > 1 .
$$

(7)

This quantity is plotted in Fig. 3 for large values of the coupling constant. Here $r$ is the slope at origin of the function plotted in Fig. 4. Equivalently one can write this condition as a bound on $g^2/(4\pi)$:

$$
N_f \frac{g^2}{4\pi} > [F_s + F_{m^2}]^{-1} .
$$

(8)

The left hand side is always positive for any reasonable function $F$, with correct threshold behaviour. Thus the instability occurs for sufficiently large coupling $g$. In [6] it was found that typical coupling constants such as $g_{\rho\pi\pi}$ and $g_{\sigma\pi\pi}$ very well satisfy this instability condition, indicating that the instability we discuss actually occurs in Nature. Since this instability depends only on the threshold behaviour of relativistic phase space, it is a fundamental property related only to the underlying Lorenz invariance and flavour symmetry of the model. When $g$ satisfies the condition one must look for another stable solution. This can only be done numerically since the function $F$ is nonlinear already in the simplest possible models.

To find the stable solution one must solve the nonlinear equations. This can be done by iterating the Eqs. (4), starting with some value off the symmetric solution (See Fig. 4), in a way analogous to Fig. 1. Using the input masses $m_{ij,\text{in}}$ for the threshold masses in the loop one calculates for which $s = m_{ij,\text{out}}^2$ one has zeroes in the inverse propagators Eq. (4). The latter are then in the next iteration are used as input masses for the thresholds. Of course, a direct way is to solve without iterations the nonlinear equations on a computer. These
solutions are shown in Fig. 5a for the function $F$ defined as above and in the limit when $g$ is very large.

4. *The symmetry is broken also in the wave functions.* How can this unsymmetric solution arise from a completely flavour symmetric theory? All four mesons are apart from the ”bare $q\bar{q}$ seed” composed of clouds of multi-quark pairs in the form of meson-meson pairs. But these clouds can be different! One can write for the wave functions decompositions:

$$|ij > \propto \sqrt{\frac{g^2}{4\pi}}[\sqrt{z_{ij}^1} |i1, 1j> + \sqrt{z_{ij}^2} |i2, 2j>] + |ij>.$$  \hspace{1cm} (9)

The coefficients $z_{ij}^{ij}$ are generally different and can be computed from the relative slopes with respect to $s$ of the function $F$ evaluated at the stable solution $z_{ij}^k \propto -\frac{\partial}{\partial s} F(s = m_{ij}^2, m_{ik}^2, m_{kj}^2, \Lambda)$. The normalized probabilities $Z_{ij} = z_{ij}^1/(z_{ij}^1 + z_{ij}^2)$ for the stable solution of the model discussed are shown in Fig. 5b. Only for the unstable, symmetric, solution are the wave functions the same for all $ij$. The equations (9) are also self-consistency equations, i.e., the quantities $ij$ on the right hand side stand for a collective multiquark state obtained when this quantity is iterated into these quantities on the right hand sides. Thus the true physical Fock states have components with an arbitrary number of virtual quarks or mesons in their wave functions.

5. *Concluding remarks.* The output stable spectrum is ideally mixed and obeys approximately the equal spacing rule $m_{22} - m_{11} \approx 2(m_{21} - m_{11})$, as one should expect from Eq. (6). Thus just as in the real world one can define the constituent quark mass as approximately $M_i \approx m_{ii}/2$, whereby $m_{ij} \approx M_i + M_j$.

A natural very important question which arises in this connection is: Where are the Goldstone degrees of freedom and the Goldstone bosons expected whenever a symmetry is spontaneously broken? In Ref. [6] I argue within a scalar QCD model that actually the scalar or longitudinal confined gluons are the would-be Goldstone bosons, not scalar mesons carrying flavour as is usually expected.

In the model presented above there were only S-wave thresholds. But the instability increases ($r$ grows) if the threshold involve angular momentum factors $k^{2L}$, since this increases the sensitivity to the thresholds and in particular $F_{m^2}$ in Eq. (4) grows. Above only scalar mesons were considered. Adding other multiplets of different $J^P C$ does not change the picture qualitatively, since most multiplets obey approximately the equal spacing rule. After summing over all thresholds one has a similar behaviour in $P^{-1}$. Effectively this is approximately equivalent to increasing $g$ and $\Lambda$ in the present model.
The instability occurs for any number of flavours $N_f$. Of course, how the symmetry group is broken down to a lower symmetry depends on details of the function $F$ as one moves off the symmetry point, and how the mixing between the flavourless states evolves. In numerical experiments with $N_f = 3$ I find that generally an SU2 subgroup remain unbroken (See Ref. [3,4].

The assumption of $C$-degeneracy of multiplets which was made above is certainly not exactly true in reality. In the real world this is certainly broken, although on the average there can be an almost equal contribution from F- and D-coupled flavour thresholds. Therefore the OZI rule is approximately valid for most multiplets. I relax the $C$-degeneracy in Ref. [3] using a model for the ground state mesons, including the pion. Then the OZI rule must be violated by loop diagrams, but the isospin subgroup remains unbroken.

Since the present mechanism requires smaller quark masses it may be important in the resolving of the strong CP problem, for which one resolution would require a vanishing quark mass. Also, there need be no contradiction with standard relations between pseudoscalar masses and usual quark masses [7], since the usual quark masses are ”measured” through these relations. Including the loop effects discussed here, the scale of the quark masses decreases while $B$ in $m^2(0^{-+}) = B(m_{q1} + m_{q2})$ increases. In lattice gauge theory [8] one also calculates light quark masses, but there one either uses a quenched approximation, or make crude approximations for quark loops. In the present work the loops play a crucial role, without which the mechanism of spontaneous breaking would not work. Therefore there is no contradiction with present lattice calculations.

Finally it should be pointed out that this work does revive old ideas from the 60’ies and 70’ies, where one hoped to bootstrap the hadron spectrum through similar self-consistency equations [9,10], but to my knowledge no demonstration like the one I have presented here was ever presented.

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FIG. 1. Graphical solution of the gap equation. For $\alpha_{NJL} > 1$ the slope of the solid curve at the origin is $> 1$. Then the stable quark mass $M$ is massive, otherwise one finds the trivial massless solution (dashed curve). Here $\alpha_{NJL} = 1.30$ from which $M/\Lambda = 0.48$, which fits $f_\pi = 93$ MeV. Then $\Lambda = 0.653$ MeV if $M = M_N/3$.

FIG. 2. The self-consistency equation diagrammatically. Iterating the equation gives a sum of multiloop diagrams.
FIG. 3. The ratio $r$ at the symmetric point ($x = y = z = \frac{m_0^2}{\Lambda}$) is shown when $g$ is very large. The fact that $r > 1$ implies instability of flavour symmetry for all $\Lambda$ and spontaneous flavour symmetry breaking occurs.
FIG. 4. The spontaneous breaking of flavour symmetry for two flavours, when $m_{11}/\Lambda = 0.5$ and $g$ very large. Compare this figure with Fig. 1. The slope of the curve at the origin is given by $r$, which is always $> 1$ for all $\Lambda$ (Fig. 3). The symmetric point of equal quark masses is unstable (like the massless point in Fig. 1), while the stable solution has unequal masses.
FIG. 5. a) The spontaneous splitting of meson masses $m_{ij}/\Lambda$ as a function of the lightest meson mass in units of $\Lambda$. Note that the stable solution approximately satisfies the equal spacing rule, $m_{22} - m_{12} \approx m_{12} - m_{11}$, in accord with physical mass splittings. The unstable solution with degenerate masses, and same subtraction constant. is shown by the dashed curve. b) The normalized probabilities $Z_{1}^{ij}$ of Eq. (6).