A Case of Subdominant/Suppressed “High Energy” Contribution to the Baryon Asymmetry of the Universe in Flavoured Leptogenesis

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The CP-violation necessary for the generation of the baryon asymmetry of the Universe \(Y_B\) in the “flavoured” leptogenesis scenario can arise from the “low energy” PMNS neutrino mixing matrix \(U\) and/or from the “high energy” part of neutrino Yukawa couplings, which can mediate CP-violating phenomena only at some high energy scale. The possible interplay between these two types of CP-violation is analysed. The type I see-saw model with three heavy right-handed Majorana neutrinos having hierarchical spectrum is considered. We show that in the case of inverted hierarchical light neutrino mass spectrum, there exist regions in the corresponding leptogenesis parameter space where the relevant “high energy” phases have large CP-violating values, but the purely “high energy” contribution in \(Y_B\) plays a subdominant role in the production of baryon asymmetry compatible with the observations. In some of these regions the purely “high energy” contribution in \(Y_B\) is so strongly suppressed that one can have successful leptogenesis only if the requisite CP-violation is provided by the Majorana phase(s) in the neutrino mixing matrix.

It is well established at present [1, 2] (see also [3, 4]) that lepton flavour effects can play a very important role in the leptogenesis mechanism [5, 6] of generation of the baryon asymmetry of the Universe, \(Y_B\). In the regime in which the lepton flavour effects in leptogenesis are significant (“flavoured” leptogenesis), the CP-violation necessary for the generation of matter-antimatter asymmetry compatible with the observations can be provided exclusively by [7] the Dirac and/or Majorana [8] CP-violating phases in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix \(U_{\text{PMNS}} \equiv U\) [9]. In the case of the three hierarchical heavy right-handed (RH) Majorana neutrinos \(N_j, j = 1, 2, 3\), and CP violation due to the Majorana phases in \(U\), this typically requires that the mass of the lightest RH Majorana neutrino \(N_1\), satisfies \(M_1 \gtrapprox 4 \times 10^{10}\) GeV [7]. In ref. [10] it was shown that if the requisite CP-violation in “flavoured” leptogenesis is due to the CP violating phases in the PMNS matrix, the baryon asymmetry \(Y_B\) in certain physically interesting cases exhibits strong dependence on the lightest neutrino mass, \(m(\nu_j)\), \(j = 1, 2, 3\). For specific values of \(m(\nu_j)\), in particular, the asymmetry \(Y_B\) can be strongly enhanced (by a factor of \(\sim 100\) or more) with respect to that predicted in the case of \(m(\nu_j) = 0\). This enhancement can make the predicted \(Y_B\) compatible with the observations even when this is not the case for \(m(\nu_j) \approx 0\). Some aspects of the matter-antimatter asymmetry generation in the “flavoured” leptogenesis scenario in the case when the relevant CP-violation is due to the Majorana or Dirac CP-violating phases in \(U_{\text{PMNS}}\), were investigated in refs. [11, 12].

In this work we analyse the more general possibility in which the requisite CP-violation in “flavoured” leptogenesis is provided both by the “low energy” Majorana and/or Dirac CP-violating phases in the neutrino mixing matrix and the “high energy” phases which can be present in the matrix of neutrino Yukawa coupling, \(\lambda\), and can mediate CP-violating processes only at some “high energy” scale. The scheme in which we work is the non-supersymmetric type I see-saw model [15] with three heavy right-handed (RH) Majorana neutrinos, \(N_j\), having masses \(M_j\) with hierarchical spectrum, \(M_1 \ll M_2 \ll M_3\). The see-saw mechanism of neutrino mass generation provides a natural explanation of the smallness of neutrino masses. Moreover, through the leptogenesis theory it allows to relate the generation and the smallness of neutrino masses with the generation of the baryon asymmetry of the Universe, \(Y_B\). In the thermal leptogenesis scenario, the CP-violating asymmetry relevant for leptogenesis in the case of hierarchical heavy Majorana neutrino masses, is generated in out-of-equilibrium decays of the lightest RH neutrino, \(N_1\). The latter is produced by thermal scattering after inflation.

In the basis in which the Majorana mass matrix of the RH neutrinos and the matrix of the charged lepton Yukawa couplings, \(\lambda^{lep}\), are diagonal, the only source of CP-violation in the lepton sector is the matrix of neutrino Yukawa couplings \(\lambda\). The orthogonal parametrization of \(\lambda\) [16], involving a complex orthogonal matrix \(R\), allows to relate in a simple way \(\lambda\) with the neutrino mixing matrix \(U\): \(\lambda = (1/v)\sqrt{M}R\sqrt{m}U^\dagger\), where \(M\) and \(m\) are diagonal matrices formed by the masses \(M_j > 0\) and \(m_k \geq 0\) of \(N_j\) and of the light Majorana neutrinos \(\nu\) respectively, \(j, k = 1, 2, 3\), and \(v = 174\) GeV is the vacuum expectation value of the Higgs doublet field. This parametrization permits to investigate

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the combined effect of the CP-violation due to the “low energy” neutrino mixing matrix $U$ and the CP-violation due to the “high energy” matrix $R$ in the generation of the baryon asymmetry in “flavoured” leptogenesis. The PMNS matrix $U$ is present in the weak charged lepton current and can be a source of CP-violation in, e.g. neutrino oscillations at “low” energies $^1 E \sim M_Z$ (see, e.g. [18–20]). The matrix $R$, as is well-known, does not affect the “low” energy neutrino mixing phenomenology. The two matrices $U$ and $R$ are, in general, independent. It should be noted, however, that in certain specific cases (of, e.g. symmetries and/or texture zeros) of the matrix $\lambda$ of neutrino Yukawa couplings, there can exist a relation between (some of) the CP-violating phases in $U$ and (some of) the CP-violating parameters in $R$ (see, e.g. [21, 22]). For hierarchical heavy Majorana neutrinos, the baryon asymmetry $Y_B$ depends on the CP-violating (complex) elements $R_{ij}$, $j = 1, 2, 3$, of the $R$–matrix.

In the present letter we study certain aspects of the possible interplay between the “low energy” CP-violation due to the Dirac and/or Majorana CP-violating phases in the PMNS matrix $U$, and the “high energy” CP-violation originating from the matrix $R$, in “flavoured” leptogenesis. We concentrate on the case of light Majorana neutrinos with inverted hierarchical (IH) spectrum (see, e.g. [23]), $m_3 \ll m_{1,2} \equiv \sqrt{\Delta m^2_{31}} \approx 0.05 \text{ eV}$. The case of normal hierarchical spectrum has been analysed in detail in [24], were results for the IH spectrum were also presented. Here we investigate in greater detail the case of IH spectrum, thus extending further the analysis performed in [24]. We show that if the light Majorana neutrinos possess IH mass spectrum, there exist significant regions of the corresponding leptogenesis parameter space where the relevant “high energy” $R$–phases have large CP-violating values, but the purely “high energy” contribution in $Y_B$ plays a subdominant role in the production of baryon asymmetry compatible with the observations. The requisite dominant term in $Y_B$ can arise due to the “low energy” CP-violation in the neutrino mixing matrix $U$. In some of these regions the “high energy” contribution in $Y_B$ is so strongly suppressed that one can have successful leptogenesis only if the requisite CP-violation is provided by the Majorana CP-violating phase(s) in $U$.

Negative results regarding the possible effects of “low energy” CP violation in “flavoured” leptogenesis with hierarchical heavy (RH) and light Majorana neutrinos, when the “high energy” CP-violation is also present, were reported in [25] and more recently in [26]. We would like to note that the study performed in [25] was by no means exhaustive. More specifically, the results reported in [25] were obtained either for i) $\min(m_{ij}) = 0$ and specific texture zero in the $R$-matrix, or ii) for a value of the mass of the lightest RH neutrino $^2 M_1 = 10^{10} \text{ GeV}$. In the case ii), the lightest neutrino mass $\min(m_{ij})$ was allowed to vary within the interval $0 \leq \min(m_{ij}) \leq 10^{-3} \text{ eV}$. However, in both these cases the contribution in $Y_B$ due to the “low energy” CP violation is strongly suppressed. The contribution under discussion can be relevant for the production of $Y_B$ compatible with the observations provided $^7 M_1 \gtrsim 4 \times 10^{10} \text{ GeV}$; in the case i) it can be relevant if one considers values of $M_1 \gtrsim 4 \times 10^{10} \text{ GeV}$ and of $\min(m_{ij}) \gtrsim 5 \times 10^{-4} \text{ eV}$ [10]. In the present article we explore the region of the parameter space corresponding to $M_1 \gtrsim 5 \times 10^{10} \text{ GeV}$. In this region the effects of the “low energy” CP violation in flavoured leptogenesis can be significant. In what concerns the analysis performed in [26], it differs substantially from the analysis performed here. In [26] the leptogenesis is considered in the framework of the SUSY extension of the Standard Model, more specifically, in the minimal Supergravity (MSUGRA) scenario with real boundary conditions, in which the dynamics responsible for supersymmetry breaking are flavour blind and all the lepton flavour and CP violation is controlled by the neutrino Yukawa couplings. The leptogenesis parameter space is constrained, in particular, by requiring that the $\mu \to e + \gamma$ decay rate branching ratio, predicted in this scenario, satisfies $BR(\mu \to e + \gamma) \gtrsim 10^{-12}$. We work in the simpler non-SUSY version of leptogenesis. The difference between our results and those found in [26] may reflect the difference in the priors on the scanned leptogenesis parameters, for instance, the range in which the lightest RH neutrino mass $M_1$ is varied.

We use the standard parametrization of the PMNS neutrino mixing matrix:

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13} \end{pmatrix} \text{diag}(1, e^{i \alpha_{21}}, e^{i \alpha_{31}})$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CP-violating phase and $\alpha_{21}$ and $\alpha_{31}$ are the two Majorana CPV phases [8, 27], $\alpha_{21,31} = [0, 4\pi]$. All our numerical results are obtained for the best fit values of the solar and atmospheric neutrino oscillation parameters [28–30]: $\Delta m^2_{\odot} = \Delta m^2_{21} = 7.65 \times 10^{-5} \text{ eV}^2$, $\sin^2 \theta_{12} = 0.30$,$^3$

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$^1$ As is well-known, only the Dirac phase in $U$ can be a source of CP-violation in neutrino oscillations; the probabilities of oscillations of flavour neutrinos do not depend on the Majorana phases in $U$ [8, 17].

$^2$ Private communication by S. Davidson. We thank S. Davidson for clarifications regarding the analysis performed in [25].
\(|\Delta m^2_{31(32)}| = 2.4 \times 10^{-3}\) eV² and \(\sin^2 \theta_{23} = 1\). We also use the upper limit on the CHOOZ mixing angle \(\theta_{13} [28, 30, 31]: \sin^2 \theta_{13} < 0.035 (0.056), \ 95\% (99.73\%)\) C.L.

We work in the “two flavour” regime \([1, 2]\), in which the \(\tau\) flavour interactions are in thermal equilibrium and the Boltzmann evolution of the CP-asymmetry in the \(\tau\) lepton charge, \(\epsilon_\tau\), is distinguishable from the evolution of the \((e + \mu)\)-flavour asymmetry \(\epsilon_2 \equiv \epsilon_e + \epsilon_\mu\). This regime is realised at temperatures \(10^9\) GeV \(\lesssim T \sim M_1 \lesssim 10^{12}\) GeV. In this study we neglect the effects of the lightest neutrino mass \(m_3\) and we set for simplicity \(|R_{13}| = 0\). The latter condition is compatible with the hypothesis of \(N_3\) decoupling in the case of IH spectrum \(^3[21]\). The results of our analysis are valid actually if the following much weaker conditions are fulfilled: if \(|R_{13}|^2 \sin^2 \varphi_{13}^2 \ll \min(|R_{11}|^2 |\sin 2 \varphi_{11}|, |R_{12}|^2 |\sin 2 \varphi_{12}|), and if \(m_3\) is sufficiently small, so that the terms \(\propto m_3 |R_{13}|^2\) and \(\propto m_3^2 |R_{13}|^2\) in the asymmetries \(\epsilon_2\) and \(\epsilon_\tau\) are negligible. The first condition will be satisfied even if \(R_{13}^2\) is not zero, but is just real, i.e. if \(\text{Im}(R_{13}^2) = 0\). The second condition is naturally satisfied in the case of IH spectrum.

From the orthogonality condition for the \(R\)-matrix elements of interest in the general case of \(R_{13} \neq 0\), \(\text{Im}(R_{13}^2) = 0\), \(R_{11}^2 + R_{12}^2 + R_{13}^2 = 1\), \(R_{1j} \equiv |R_{1j}| e^{i \phi_{1j}}, j = 1, 2\), we can express the “high energy” CP-violating phases, \(\varphi_{11}\) and \(\varphi_{12}\), in terms of the absolute values \(|R_{11}|, |R_{12}|\) and of \(R_{13}\) which is real:

\[
\begin{align*}
\cos 2 \varphi_{11} &= \frac{(1 - R_{13}^2)^2 + |R_{11}|^4 - |R_{12}|^4}{2 |R_{11}|^2 (1 - R_{13}^2)}, \\
\cos 2 \varphi_{12} &= \frac{(1 - R_{13}^2)^2 - |R_{11}|^4 + |R_{12}|^4}{2 |R_{12}|^2 (1 - R_{13}^2)},
\end{align*}
\]

(2)

(3)

with \(\text{sgn}(\sin 2 \varphi_{11}) = -\text{sgn}(\sin 2 \varphi_{12})\). In the cases we discuss below (and illustrate in Figs. 1 and 2) we have chosen \(\sin 2 \varphi_{11} < 0\). The CP-violating asymmetry \(\epsilon_\tau\) in the case considered is given by:

\[
\epsilon_\tau \cong -\frac{3 M_1}{16 \pi v^2} \sqrt{|\Delta m^2_{31}|} \left[ |R_{11}|^2 \sin(2 \varphi_{11}) \left[ (|U_{1\tau}|^2 - |U_{\tau\tau}|^2) - \frac{|\Delta m^2_{31}|}{|\Delta m^2_{21}|} |U_{\tau 1}|^2 \right] + |R_{11}| |R_{12}| \left[ \frac{1}{2} \frac{|\Delta m^2_{31}|}{|\Delta m^2_{21}|} \cos(\varphi_{11} + \varphi_{12}) \text{Im}(U_{\tau 1}^* U_{\tau 2}) + 2 \left( 1 - \frac{1}{2} \frac{|\Delta m^2_{31}|}{|\Delta m^2_{21}|} \right) \sin(\varphi_{11} + \varphi_{12}) \text{Re}(U_{\tau 1}^* U_{\tau 2}) \right] \right]
\]

(4)

For \(\varphi_{11} = k \pi/2, \varphi_{12} = k' \pi/k, k, k' = 0, 1, 2, \ldots, R_{11}\) and \(R_{12}\) are either real or purely imaginary and the expression for \(\epsilon_\tau\) reduces to the one derived in [7]. Under these conditions we can have successful leptogenesis for \(R_{13} = 0\) in the case considered only if \(R_{11} R_{12}\) is purely imaginary, i.e. if \(|\sin(\varphi_{11} + \varphi_{12})| = 1\), the requisite CP-violation being provided exclusively by the Majorana or Dirac phases in the PMNS matrix [7]. We remind the reader that i) the \(R\)-matrix will satisfy the CP-invariance constraint if its elements \(R_{ij}\) are real or purely imaginary 4, and ii) in order to have CP-violation, e.g. only due to the Majorana phase \(\alpha_{21}\) in \(U\), both \(\text{Im}(U_{\tau 1}^* U_{\tau 2})\) and \(\text{Re}(U_{\tau 1}^* U_{\tau 2})\) should be different from zero [34, 35], while the Dirac phase \(\delta\) should have a CP-conserving value, \(\delta = k \pi, k = 0, 1, 2, \ldots\) (i.e., the rephasing invariant \(J_{\text{CP}}\) associated with \(\delta [19]\) should satisfy \(J_{\text{CP}} = 0\)). Let us note also that purely imaginary \(R_{11} R_{12}\), i.e. \(|\sin(\varphi_{11} + \varphi_{12})| = 1\), and \(\text{Re}(U_{\tau 1}^* U_{\tau 2}) = 0\), \(J_{\text{CP}} = 0\) corresponds to the case of CP-invariance and \(\epsilon_\tau = 0\). However, purely imaginary \(R_{11} R_{12}\) and \(J_{\text{CP}} = 0\), \(\text{Im}(U_{\tau 1}^* U_{\tau 2}) = 0\), \(\text{Re}(U_{\tau 1}^* U_{\tau 2}) \neq 0\) (i.e. \(\delta = k \pi, \alpha_{21} = 2\pi q, k, q = 0, 1, 2, \ldots\)), corresponds to CP-violation due to the neutrino Yukawa couplings, i.e. due to the combined effect of the matrix \(R\) and of the PMNS matrix \(U\) [7], and \(\epsilon_\tau \neq 0\). It is interesting that in this case both the \(R\)-matrix and the PMNS matrix \(U\) satisfy the CP-invariance constraints (having real and/or purely imaginary elements), while the neutrino Yukawa couplings do not satisfy these constraints. As a consequence, under the indicated conditions i) there will be no CP-violation effects caused by PMNS matrix \(U\) in the low energy neutrino mixing phenomena (neutrino oscillations, neutrinoless double beta decay, etc.), and ii) there will be no CP-violation effects in the “high energy” phenomena which depend only on the matrix \(R\) (i.e. do not depend on the PMNS matrix \(U\)).

Indeed, consider the general case of hierarchical heavy RH Majorana neutrinos with arbitrary light neutrino mass spectrum, non-negligible lightest neutrino mass and matrix \(R\) with non-zero elements. The CP-violating asymmetry in the lepton flavour \(l\), generated in the decays of the lightest RH Majorana neutrino \(N_1\) reads (see, e.g. [7]):

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\(^{3}\) A complete decoupling of \(N_3\) in the case of IH spectrum occurs when the elements of the \(R\)-matrix satisfy \(R_{13} = R_{23} = R_{31} = R_{32} = 0\) [21]. In the context of “flavoured” leptogenesis the case of \(N_3\) decoupling and \(|R_{13}| = 0\) for the IH spectrum was discussed earlier in [2, 7]. However, our analysis practically does not overlap with the analyses performed in [2, 7]. Let us note finally that the \(N_3\) decoupling in the case of NH spectrum was considered, e.g. in [21, 32, 33].

\(^{4}\) For the precise form of the CP-invariance constraint on the elements \(R_{ij}\) of the \(R\)-matrix see [7].
Let us assume next that CP-violating asymmetries in the individual lepton flavours \[1, 36, 37\]:

\[
\epsilon_l = -\frac{3M_1}{16\pi v^2} \Im \left( \frac{\sum_j m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}}{\sum_l m_{1l}} \right), \quad l = e, \mu, \tau. 
\]

(5)

Let us assume next that \(M_1 > 10^{12}\) GeV. In this case the baryon asymmetry \(Y_B\) is determined by the sum of the CP-violating asymmetries in the individual lepton flavours \[1, 36, 37\]:

\[
\epsilon_1 = \sum_l \epsilon_l = -\frac{3M_1}{16\pi v^2} \frac{\Im \left( \sum_j m_j^2 R_{1j}^2 \right)}{\sum_k m_k |R_{1k}|^2}, \quad l = e, \mu, \tau. 
\]

(6)

As it follows from the preceding equation and is well-known, the asymmetry \(\epsilon_l\) does not depend on the PMNS matrix - it depends only on the elements of the \(R\)-matrix. In this sense one can say that the generation of the baryon asymmetry in the regime under discussion is purely a “high energy” phenomenon. It should be obvious, however, from eq. (6) that if the \(R\)-matrix is CP-conserving, i.e. if its elements \(R_{1j}\) are real \[1\] (E. Nardi et al.) or purely imaginary \[7\], we would have \(^5\) \(\epsilon_1 = 0\). In what concerns the “low energy” neutrino mixing phenomena caused by the PMNS matrix \(U\), the CP-symmetry will obviously be unbroken for \(\delta = k\pi, \alpha_{21} = 2\pi q\) and \(\alpha_{31} = \pi q'\), \(k, q, q' = 0, 1, 2, \ldots\)

One can easily show that for the IH light neutrino mass spectrum of interest in the present analysis, the following relation holds \(^6\) : \(\epsilon_2 = -\epsilon_\tau (1 + O(\Delta m^2_5/\Delta m^2_3))\). Thus, the baryon asymmetry \(Y_B\) can be written as a function of \(\epsilon_\tau\) only, like in the case of the matrix \(R\) satisfying the CP-invariance constraints \[7\]:

\[
Y_B = -\frac{12}{37} \frac{\epsilon_\tau}{g_*} \left( \eta \left( \frac{390}{589} \bar{m}_\tau \right) - \eta \left( \frac{417}{589} \bar{m}_2 \right) \right) 
\]

\[
\equiv Y_B^0 (A_{\text{HE}} + A_{\text{MIX}}) 
\]

(7)

where \(A_{\text{HE(MIX)}} \equiv C_{\text{HE(MIX)}}(\eta(0.66\bar{m}_\tau) - \eta(0.71\bar{m}_2)), \eta(0.66\bar{m}_\tau)\) and \(\eta(0.71\bar{m}_2)\) being the efficiency factors for the...

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\(^5\) As can be shown, this result does not depend, in particular, on the assumption about the spectrum of the heavy Majorana neutrinos. More concretely, the CP violating asymmetry \(\epsilon_j\), generated in the decays of the heavy Majorana neutrino \(N_j\) due to the interference of the tree level and one loop contributions to the relevant amplitudes, would be zero if \(M_j > 10^{12}\) GeV, \(j = 1, 2, 3\), and \(R_{jk}\) is either real or purely imaginary, \(k = 1, 2, 3\) and satisfies the CP-invariance constraint \[7\].

\(^6\) Note that this relation is valid not only for \(R_{13} = 0\), but also for nonzero real \(R_{13}^2, R_{13} \neq 0, \Im(R_{13}^2) = 0\).
Figure 2: The dependence of the “high energy” term $|Y_B^0 A_{HE}|$ (blue line), the “mixed” term $|Y_B^0 A_{MIX}|$ (green line) and of the total baryon asymmetry $|Y_B|$ (red line) on $|R_{12}|$ in the case of IH spectrum, CP-violation due to the Majorana phase $\alpha_{21}$ in $U$ and $R$-phases, for $\alpha_{21} = \pi/2$, $|R_{12}| \approx 1$, $M_1 = 10^{11}$ GeV and i) $s_{13} = 0$ (upper panel), ii) $s_{13} = 0.2$, $\delta = 0$ (middle panel), iii) $s_{13} = 0.2$, $\delta = \pi$ (lower panel). The light-blue curve represents the dependence of $Y_B$ on $|R_{12}|$ for the given PMNS parameters and CP-conserving matrix $R$, with $[7] R_{11} R_{12} \equiv ik|R_{11} R_{12}|$, $k = -1$ and $|R_{11}|^2 = |R_{12}|^2 = 1.$
asymmetries $\epsilon_\tau$ and $\epsilon_2$ (see [1, 2]), and

$$Y_B^0 \cong 3 \times 10^{-10} \left( \frac{M_3}{10^9 \text{ GeV}} \right) \left( \frac{\sqrt{\Delta m^2_{\text{atm}}}}{5 \times 10^{-2} \text{ eV}} \right),$$

(8)

$$C_{\text{HE}} = G_{11} \sin 2\varphi_{11} \left[ |U_{\tau 1}|^2 - |U_{\tau 2}|^2 \right],$$

(9)

$$C_{\text{MIX}} \cong 2G_{12} \sin(\varphi_{11} + \varphi_{12}) \Re(U^*_{\tau 1}U_{\tau 2}),$$

(10)

where $G_{11} \equiv |R_{11}|^2/(|R_{11}|^2 + |R_{12}|^2)$, $G_{12} \equiv |R_{12}|/(|R_{11}|^2 + |R_{12}|^2)$ and we have neglected the contributions proportional to the factor $0.5\Delta m^2_{\text{atm}}/|\Delta m^2_{\text{atm}}| \cong 0.016$ in the CP-asymmetry $\epsilon_\tau$. In Eq. (7), $Y_B^0 A_{\text{HE}}$ is the “high energy” term which vanishes in the case of a CP-conserving matrix $R$, while $Y_B^0 A_{\text{MIX}}$ is a “mixed” term which, in contrast to $Y_B^0 A_{\text{HE}}$, does not vanish when $R$ conserves CP: it includes the “low energy” CP-violation, e.g. due to the Majorana phase $\alpha_{21}$ in the neutrino mixing matrix. We recall that the phase $\alpha_{21}$ enters also into the expression for the neutrinoless double beta decay effective Majorana mass in the case of IH light neutrino mass spectrum [38].

In order to have CP-violation due to the Majorana phase $\alpha_{21}$, both $\operatorname{Im}(U^*_{\tau 1}U_{\tau 2})$ and $\Re(U^*_{\tau 1}U_{\tau 2})$ should be different from zero [34, 35].

Using the formalism described above, we have studied the interplay between the CP-violation arising from the “high energy” phases of the orthogonal matrix $R$ and the “low energy” CP-violating Dirac and/or Majorana phases in the neutrino mixing matrix, as well as the relative contributions of the “high energy” and the “mixed” terms $Y_B^0 A_{\text{HE}}$ and $Y_B^0 A_{\text{MIX}}$ in $Y_B$. We have found that there exist large regions of the corresponding leptogenesis parameter space where the “high energy” contribution to $Y_B$ is subdominant, or even strongly suppressed. These results are illustrated in Figs. 1 and 2. Below we discuss two specific examples of such a suppression for $R_{13} = 0$, which can take place even when the “high energy” $R$-phases possess large CP-violating values. In both cases the asymmetry $\epsilon_\tau$ is produced in the regime of mild wash-out ($\bar{m}_\tau \cong (1 - 3) \times 10^{-3} \text{ eV}$), while the asymmetry $\epsilon_2$ is generated with strong wash-out effects (see, e.g. [1, 2]). We note also that in both cases we have $\epsilon_2 = -\epsilon_\tau + O(\Delta m^2_{\text{atm}}/|\Delta m^2_{\text{atm}}|)$. Under these conditions the two-flavour regime in leptogenesis is realised typically for $^7 M_1 \lesssim 5 \times 10^{11} \text{ GeV}$ [12, 13] (see also [14]). The are small subregions of the parameter space explored by us where our results are valid for $M_1 \lesssim 7 \times 10^{11} \text{ GeV}$; in another subregion they are valid for $M_1 \lesssim 5 \times 10^{11} \text{ GeV}$. If, for instance, $|R_{11}| = 1$, the two-flavour regime of leptogenesis is realised for $M_1 \lesssim 5 \times 10^{11} \text{ GeV}$ provided $|R_{12}| \lesssim 0.7$. For $|R_{11}| \leq 0.5$, the same conclusion is valid for $M_1 \lesssim 5 \times 10^{11} \text{ GeV}$ in the whole interval of variability of $|R_{12}|$: for $|R_{11}| = 1.1$ and $|R_{12}| \leq 1$ this is realised for $M_1 \lesssim 3 \times 10^{11} \text{ GeV}$. In the latter case $|R_{12}|$ can vary in the interval $0.45 \lesssim |R_{12}| \lesssim 1.45$.

Consider the term $Y_B^0 A_{\text{HE}}$. One can convince oneself that for sufficiently large $\theta_{13}$, $Y_B^0 A_{\text{HE}}$ depends in a crucial way on the Dirac phase $\delta$ through the following combination of the elements of the neutrino mixing matrix:

$$|U_{\tau 1}|^2 - |U_{\tau 2}|^2 \cong (s_{12}^2 - c_{12}^2)s_{23}^2 - 4s_{12}c_{12}s_{23}c_{21}s_{13}\cos\delta \cong -0.20 - 0.92 s_{13}\cos\delta,$$

(11)

where we have used $s_{23}^2 = 0.30$ and $s_{23}^2 = 0.5$. Indeed, for, e.g. $s_{13} = 0.2$ and the Dirac phase assuming the CP-conserving value $\delta = \pi$, we get $(|U_{\tau 1}|^2 - |U_{\tau 2}|^2) \cong -0.016$. At the same time we have $|Y_B^0 A_{\text{MIX}}| \propto |U^*_{\tau 1}U_{\tau 2}| \cong 0.27$. As a consequence, if the Majorana phase $\alpha_{21}$ has a sufficiently large CP-violating value, the contribution of $|Y_B^0 A_{\text{HE}}| \propto |Y_B^0 A_{\text{HE}}|$ to $|Y_B|$ can be by an order of magnitude bigger than the contribution of the “high energy” term $|Y_B^0 A_{\text{HE}}|$. Actually, for $s_{12}^2 = 0.30$ and $s_{23}^2 = 0.5$, the “high energy” term in $Y_B$ will be strongly suppressed by the factor $(|U_{\tau 1}|^2 - |U_{\tau 2}|^2)$ if $(-\sin\theta_{13}\cos\delta) \gtrsim 0.15$, independently of the values of the “high energy” phases $\varphi_{11}$ and $\varphi_{12}$. Even if the latter assume large CP-violating values, the purely “high energy” contribution to $Y_B$ would play a subdominant role in the generation of the baryon asymmetry compatible with the observations if the above inequality holds. For $(\sin\theta_{13}\cos\delta) > 0.17$ and $M_1 \lesssim 5 \times 10^{11} \text{ GeV}$, the observed value of the baryon asymmetry cannot be generated by the “high energy” term $Y_B^0 A_{\text{HE}}$ alone. One can have successful leptogenesis in this case only if there is an additional dominant contribution in $Y_B$ due to the CP-violating Majorana phase $\alpha_{23}$ in the neutrino mixing matrix. Let us emphasise that this result is valid in the whole range of variability of the parameter $|R_{12}|$, $(1 - |R_{11}|^2) \lesssim |R_{12}|^2 \lesssim (1 + |R_{11}|^2)$, and for $|R_{11}|$ having values in the interval $0.3 \lesssim |R_{11}| \lesssim 1.2$. For values of $|R_{11}|$ outside the indicated interval we cannot have successful leptogenesis in the two-flavour regime for $M_1 \lesssim 5 \times 10^{11} \text{ GeV}$. For the $3\sigma$ allowed values of $s_{23}^2 = 0.38$ and $s_{23}^2 = 0.36$, the same conclusion is valid if $0.06 \lesssim (-\sin\theta_{13}\cos\delta) \lesssim 0.12$. The values of $\sin\theta_{13}$ and $\sin\theta_{13}\cos\delta$, for

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7 We thank A. Riotto for useful discussions of this point.
which we can have the discussed strong suppression\(^8\) of \(Y_B^0 A_{\text{HE}}\), can be probed by the Double CHOOZ and Daya Bay reactor neutrino experiments\(^{41, 42}\) and by the planned accelerator experiments on CP violation in neutrino oscillations\(^{20}\).

The results discussed above are illustrated in Fig. 1, where we show the dependence of \(|Y_B^0 A_{\text{HE}}|, |Y_B^0 A_{\text{MIX}}|\) and \(|Y_B|\) on \(|R_{12}|\) for a fixed value of \(|R_{11}| = 0.7 (R_{13} = 0)\) and \(\alpha_{23} = \pi/2\), \(s_{13} = 0.2, (-s_{13} \cos \delta) = 0.15\) and \(M_1 = 10^{11}\) GeV. Note that varying \(|R_{12}|\) in its allowed range is equivalent to varying the “high energy” CP-violating phases, see eqs. (2) and (3). Shown is also the behavior of the “high energy” term for two additional values of \((-s_{13} \cos \delta)\). As is clearly seen in Fig. 1, for \((-s_{13} \cos \delta) > 0.15\), \(A_{\text{HE}}\) is strongly suppressed and is much smaller than \(A_{\text{MIX}}\) in almost all the range of variability of \(|R_{12}|\). The same conclusion holds if we allow \(|R_{11}|\) to vary in the range \(0.3 \leq |R_{11}| \leq 1.2\).

Another case in which the contribution of the “high energy” term in \(Y_B\) is subdominant is illustrated in Fig. 2. We show the two different contributions to the baryon asymmetry as function of \(|R_{12}|\) which is varied in the interval \(0.05 \leq |R_{12}| \leq 0.65\), in the case of \(|R_{11}| \equiv 1, R_{13} = 0\) and \(|R_{12}| \equiv 1\), \(s_{13} = 0\) (upper panel), ii) \(s_{13} = 0.2, \delta = 0\) (middle panel), iii) \(s_{13} = 0.2, \delta = \pi\) (lower panel). The behavior of the total baryon asymmetry generated when the CP-violation is due exclusively to the Majorana phase \(\alpha_{23}\)\(^7\) is also given. We observe that in most of the chosen range of \(|R_{12}|\), the contribution of the “mixed” term \(Y_B^0 A_{\text{MIX}}\) in \(|Y_B|\) is greater than that of the “high energy” term \(Y_B^0 A_{\text{HE}}\) and plays a dominant role in the generation of baryon asymmetry compatible with that observed. Indeed, it follows from eqs. (2) and (3) that in the case under discussion we have: \(s_{23} \beta_{11} \equiv -|R_{12}|^2\) and \(s_{23} \beta_{12} \equiv (1 - |R_{12}|)^4/8\). This implies \(A_{\text{HE}} \equiv |G_{11} \sin 2 \beta_{11}| \sim |R_{11} R_{12}|^2\), while \(A_{\text{MIX}} \equiv 2 |G_{12} \sin (\phi_{11} + \phi_{12})| \sim \sqrt{2} |R_{11} R_{12}|\), where we have used \(|\sin (\phi_{11} + \phi_{12})| \equiv 1/\sqrt{2}\). The latter approximation is rather accurate for \(|R_{11}| = 1\) and \(|R_{12}| \leq 0.5\). Thus, for \(|R_{12}| = 0.4\), for instance, we have \(\phi_{11} \sim -0.08, \phi_{12} \sim \pi/4\), and correspondingly for \(s_{13} = 0\) and \(\alpha_{23} = \pi/2\) we get \(|A_{\text{MIX}}|/|A_{\text{HE}}| \equiv 2.6\) (Fig. 2, upper panel). Note also that, e.g. in the case of \(s_{13} = 0\), the generated \(|Y_B|\) is largest when the “high energy” R-phases assume CP-conserving values. The same feature is clearly observed also for \(s_{13} = 0.2\) and \(\delta = 0\) at \(|R_{12}| \leq 0.55\). Moreover, for \(0.25 \leq |R_{12}| \leq 0.50\), the baryon asymmetry generated in the case of CP-conserving R-phases is significantly larger in absolute value than the asymmetry produced when the relevant R-phases possess CP-violating values (Fig. 2, middle panel). We note also that for \(s_{13} = 0.2\) and \(\delta = \pi\) (Fig. 2, lower panel), the “high energy” term \(Y_B^0 A_{\text{HE}}\) is strongly suppressed by the factor \((|U_{\tau 1}|^2 - |U_{\tau 2}|^2)^2\) (see eq. (11) and the related discussion) and is hardly visible in the corresponding figure. If, however, \(|R_{12}| \geq 0.8\) and \(M_1 \geq 7 \times 10^{10}\) GeV, the “high energy” term in \(|Y_B|\) is the dominant one and can provide the requisite baryon asymmetry compatible with the observations.

In summary, we have considered a case of subdominant/suppressed “high energy” contribution to the baryon asymmetry of the Universe in thermal flavoured leptogenesis scenario with hierarchical heavy Majorana neutrinos. It can arise if the light neutrino mass spectrum is of inverted hierarchical type and if the \(R_{13}\) element of the complex orthogonal \(\mathbf{R}\)-matrix in the orthogonal parametrisation of neutrino Yukawa couplings satisfies \(\text{Im}(R_{13}^2) = 0\). Under these conditions the “high energy” contribution to the baryon asymmetry, arising in the two-flavour regime in leptogenesis, will be strongly suppressed by the factor \((|U_{\tau 1}|^2 - |U_{\tau 2}|^2)^2\) provided \((-\sin \theta_{13} \cos \delta) \gtrsim 0.15\). The interval of values of \((-\sin \theta_{13} \cos \delta)\) for which the suppression takes place depends on the precise values of \(\sin^2 \theta_{12}\) and \(\sin^2 \theta_{23}\), which are still determined experimentally with non-negligible uncertainty. In the simplified case of \(R_{13} = 0\), this result is valid in the whole range of variability of the parameter \(|R_{12}|\), determined by \(24 \mid 1 - |R_{11}|^2 \mid \leq |R_{12}|^2 \leq (1 + |R_{11}|^2)\), and for \(|R_{11}|\) has a value in the interval \(0.3 \leq |R_{11}| \leq 1.2\). For the indicated ranges of values of \(|R_{11}|\) and \(|R_{12}|\), the “high energy” CP-violating phases are not necessarily small. However, reproducing the observed value of the baryon asymmetry is problematic (or can even be impossible) without a contribution due to the CP-violating phases in the PMNS matrix. The “high energy” contribution can be subdominant also in the case of \(\sin \theta_{13} = 0\). This possibility can be realised for values of the Majorana phase in the PMNS matrix \(0 < \alpha_{23} \lesssim 2\pi/3\) and roughly in half of the parameter space spanned by the relevant elements of the \(\mathbf{R}\) matrix. In both cases the observed value of the baryon asymmetry can be reproduced for values of the lightest RH Majorana neutrino mass lying in the interval \(5 \times 10^{10}\) GeV \(\lesssim M_1 \lesssim 7 \times 10^{11}\) GeV. Similar results hold in the more general case of \(R_{13} \neq 0\), \(\text{Im}(R_{13}^2) = 0\), for \(0 \leq |R_{13}| \lesssim 0.9, 1.05 \lesssim |R_{13}| \lesssim 1.5\), and \(0.3 \lesssim |R_{11}| \lesssim 1.2\).

The results obtained in this study show that in the “flavoured” leptogenesis scenario, the contribution to \(Y_B\) due to the “low energy” CP-violating Majorana and Dirac phases in the neutrino mixing matrix, in certain physically interesting cases, like IH light neutrino mass spectrum, relatively large values of \((-\sin \theta_{13} \cos \delta)\), etc., can be indispensable for the generation of the observed baryon asymmetry of the Universe even in the presence of “high energy” CP-violation.

\(^8\) It is interesting to note that in the recent analysis of the global neutrino oscillation data\(^{39}\), a nonzero value of \(\sin^2 \theta_{13}\) was reported at 1.6σ. The best value and the 1σ allowed interval of values of \(\sin \theta_{13}\) found in\(^{39}\), \(\sin \theta_{13} = 0.126\) and \(\sin \theta_{13} = (0.077 - 0.161)\), are in the range of interest for our discussion. In addition, \(\cos \delta = -1\) is reported to be preferred over \(\cos \delta = +1\) by the atmospheric neutrino data\(^{39, 40}\).
generated by additional physical phases in the matrix of neutrino Yukawa couplings, e.g. by CP-violating phases in the complex orthogonal matrix $R$ appearing in the “orthogonal parametrisation” of neutrino Yukawa couplings.

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ADDENDUM

We have shown in the article that for a certain range of values of $\sin(\theta_{13}) \cos(\delta)$ in the case of IH light neutrino mass spectrum and hierarchical heavy Majorana neutrino mass spectrum, the “high energy” contribution to the baryon asymmetry of the Universe will be strongly suppressed if the complex $R_{13}$ element of the $R$-matrix satisfies the inequality: $|R_{13}|^2|\sin(2\tilde{\phi}_{13})| \ll \min(|R_{11}|^2|\sin(2\tilde{\phi}_{11})|, |R_{12}|^2|\sin(2\tilde{\phi}_{12})|)$. In this Addendum we extend the analysis performed in our article to the case of arbitrary complex $R_{13}$. Hence, we abandon the condition on $|R_{13}|^2|\sin(2\tilde{\phi}_{13})|$ given above and determine the ranges of values of $|R_{13}|$ for which the indicated strong suppression of the “high energy” contribution to the baryon asymmetry still takes place even when the phase of $R_{13}$, $\tilde{\phi}_{13}$, is allowed to assume large CP-violating values. In this case the contribution to the corresponding CP asymmetry due to the low energy Majorana CP violating phases is essential for producing baryon asymmetry compatible with the observations.

It follows from the numerical analysis performed in this Addendum that for, e.g., $M_1 = 10^{11}$ GeV, the scenario described above is realised and we have subdominant (strongly suppressed) “high energy” contribution to the observed value of the baryon asymmetry for arbitrary $\tilde{\phi}_{13}$ if i) for $|R_{11}| < 0.5$, $|R_{13}|$ satisfies $|R_{13}| \lesssim |R_{11}|$; ii) for $0.5 \lesssim |R_{11}| < 1$ we have $|R_{13}| < 0.5$, and if iii) for $|R_{11}| > 1$ we have $|R_{13}| < |R_{11}|/2$. In each of the cases i) - iii) we can have successful leptogenesis, as our results show, due to the contribution to the baryon asymmetry associated with the Majorana CP violating phase(s) in the neutrino mixing matrix.

The results obtained in the present Addendum are illustrated in Fig. 3, where we show the total baryon asymmetry $Y_B$ (red dots/line), the purely “high energy” contribution (blue dots/line) and the “mixed” term as function of $|R_{12}|$ for fixed values of $|R_{11}|$ and $|R_{13}|$. The lightest neutrino mass $m_3$ is set to zero and the lightest RH Majorana neutrino mass corresponds to $M_1 = 10^{11}$ GeV. The figure is obtained for $\sin(\theta_{13}) \cos(\delta) = -0.2$. In the upper left panel, we have set $|R_{11}| = 0.7$ and have taken a real $R_{13}$, with $R_{13} = 0.3$. The phase of $R_{11}$, $\tilde{\phi}_{11}$, was varied in the interval $[3\pi/2, 2\pi]$, while the $R_{12} = |R_{12}|e^{i\tilde{\phi}_{12}}$ is determined by the orthogonal condition: $R_{11}^2 + R_{12}^2 + R_{13}^2 = 1$. In all the other panels of Fig. 1 we consider a complex $R_{13} = |R_{13}|e^{i\tilde{\phi}_{13}}$ and the “high energy” phase $\tilde{\phi}_{13}$ was assumed to take random values in the interval $[0, 2\pi]$. We plot the behavior of each term contributing to the baryon asymmetry as a function of $|R_{12}|$ for fixed $|R_{11}|$ and $|R_{13}|$, following the procedure described above.
Figure 3: Total baryon asymmetry $Y_B$ (red dots), “high energy” term (blue dots) and “mixed” term (green dots) vs $|R_{12}|$ for $M_1 = 10^{11}$ GeV, $m_3 = 0$, $\alpha_{21} = \pi/2$, $s_{13} = 0.2$, $\delta = \pi$, complex $R_{11}$, $R_{12}$, and $i$) $|R_{11}| = 0.7$ and real $R_{13} = 0.3$ (upper left panel); ii) $|R_{11}| = 0.7$ and $|R_{13}| = 0.3$ (upper right panel); iii) $|R_{11}| = 0.4$ and $|R_{13}| = 0.4$ (middle left panel); iv) $|R_{11}| = 0.4$ and $|R_{13}| = 0.2$ (middle right panel); v) $|R_{11}| = 1.3$ and $|R_{13}| = 0.6$ (lower left panel); vi) $|R_{11}| = 1.5$ and $|R_{13}| = 0.7$ (lower right panel). The figures in the upper right panel, the middle and lower panels are obtained for complex $R_{13}$. The horizontal lines indicate the allowed range of the baryon asymmetry: $Y_B = (8.77 \pm 0.24) \times 10^{-11}$. 