Geometric and Electronic Properties of Monolayer HfX₂ (X = S, Se, or Te): A First-Principles Calculation

Thi My Duyen Huynh¹, Duy Khanh Nguyen², Thi Dieu Hien Nguyen¹, Vo Khuong Dien¹, Hai Duong Pham¹* and Ming-Fa Lin¹,³*

¹Department of Physics, National Cheng Kung University, Tainan, Taiwan, ²Institute of Applied Technology, Thu Dau Mot University, Binh Duong, Vietnam, ³Hierarchical Green Energy Materials, Hi-Research Center, National Cheng Kung University, Tainan, Taiwan

The essential properties of monolayer HfX₂ (X = S, Se, or Te) are fully explored by first-principles calculations. The optimal lattice symmetries, sublattice buckling, electronic energy spectra, and density of states are systematically investigated. Monolayer HfS₂, HfSe₂, and HfTe₂ respectively, belong to middle-gap semiconductor, narrow-gap one and semimetal, with various energy dispersions. Moreover, the van Hove singularities (vHs) mainly arise from the band-edge states, and their special structures in the density of states strongly depend on their two or three-dimensional structures and the critical points in the energy-wave-vector space. The above-mentioned theoretical predictions are attributed to the multi-orbital hybridizations of \([dx^2−y^2, d_{xy}, d_{x^2−z^2}, d_{y^2}−s, px, py, pz}\) in the Hf-X chemical bonds. The diversified physical phenomena clearly indicate a high potential for applications, as observed in MoS₂-related emergent materials ions.

Keywords: first-principles, two-dimensional materials, transition metal dichalcogenides, HfX₂ (X = S, Se, or Te), geometry, multi-orbital hybridization, electronic properties

INTRODUCTION

In recent decades, two-dimensional (2D) materials, including metals, semiconductors, and insulators, emerged as a focus of the scientific community because of a wide range of properties (Novoselov et al., 2016). They can often present essential differences of properties compared to their bulk phase (Kolobov, 2016). Since graphene was isolated from graphite in 2004, it has become an interesting and unique material, which is a gapless semiconductor (Balandin et al., 2008; Castro Neto et al., 2009; Das et al., 2015; Novoselov et al., 2016). Graphene exhibits noticeably essential properties, such as quantum Hall effect (Ma et al., 2009; Zhang et al., 2011), magnetic quantization (Guinea et al., 2006; Huang et al., 2014; Lin et al., 2015) and devoured optical selection (Chung et al., 2011). In order to extend the applications, 2D materials based on graphene have been investigated and offer a significant number of potential candidates (Fiori et al., 2014; Das et al., 2015; Kim et al., 2015; Novoselov et al., 2016; Zhou et al., 2018). Among these materials, 2D transition metal dichalcogenides (2D-TMDs) recently covered a number of applications in optical and electronic devices (Lebègue et al., 2013; Kaul, 2014; McDonnell and Wallace, 2016; Novoselov et al., 2016; Zhou et al., 2018). Although the bulk of TMDs that have been constructed and explored (Kolobov, 2016) show various properties, the electronic properties of layered TMDs are dominated by quantum confinement. It should be noted that the geometry and the electron count play an essential role in their electronic properties (Kolobov, 2016).
HfX₂ (X = S, Se, or Te) is a class in the group IV chalcogenides of the 2D-TMDs family whose general properties were reported in previous studies (Hussain Reshak and Auluck, 2005; Jiang, 2011; Abdulsalam and Joubert, 2016; Toh et al., 2016; Cheng et al., 2018; Salavati, 2018). Experimental investigations (Roubi and Carlone, 1988; Aretouli et al., 2015; Yue et al., 2015; Aminalragia-Giamini et al., 2016; Kanazawa et al., 2016; Yin et al., 2016; Mangelsen et al., 2017; Mleczko et al., 2017; Tsai et al., 2018) on Hf-based have been conducted in bulk and monolayer. HfS₂ is anticipated to have high electron mobility and sheet current density, proposing a candidate for electronic devices (Fiori et al., 2014; Zhang et al., 2014). Moreover, HfS₂ was experimentally investigated in a flake form as promising for use in transistors (Roubi and Carlone, 1988; Traving et al., 2001; Xu et al., 2015; Kanazawa et al., 2016; Mirabelli et al., 2016; Nie et al., 2017). Monolayer and bilayer HfSe₂ exhibit low current density, proposing a candidate for electronic devices (Aretouli et al., 2015; Kang et al., 2015; Abdulsalam and Joubert, 2016; Toh et al., 2016; Cheng et al., 2018; Salavati, 2018). Experimental investigations (Roubi and Carlone, 1988; Traving et al., 2001; Xu et al., 2015; Kanazawa et al., 2016; Mirabelli et al., 2016; Nie et al., 2017). Epitaxial thin films of HfTe₂ are grown in AlN substrates using angle-resolved photoemission spectroscopy provided that HfTe₂/AlN is an epitaxial topological semimetal (Aminalragia-Giamini et al., 2016).

Moreover, theoretical studies via first-principles calculations on HfS₂ show a slow increase in the bandgap with increasing tensile strain and thickness (Kreis et al., 2003; Wu et al., 2017; Salavati, 2018). Similar electronic characteristics are also observed in HfSe₂ (Yun et al., 2012; Setiyawati et al., 2019). In-depth investigations of the electronic band structures of HfS₂ and HfSe₂, reveal that sulfur and selenium atoms have a primary effect on the valence band, while the hafnium atoms play a dominant role in the conduction band (Zhao et al., 2017). The 1T phase structures are found in among the possible phases HfX₂ could exist in, predict that T phase is the most favorable phase (Hussain Reshak and Auluck, 2005; Jiang, 2011; Abdulsalam and Joubert, 2016; Ding et al., 2016; Wu et al., 2017; Zhao et al., 2017; Nakata et al., 2019; Yan et al., 2019). Previous studies show that monolayer 1T-HfTe₂ are metallic (Sun and Wang, 2017), while those of 1T-HfS₂ (Abdulsalam and Joubert, 2016; Wu et al., 2017; Zhao et al., 2017) and 1T-HfSe₂ (Abdulsalam and Joubert, 2016; Ding et al., 2016; Zhao et al., 2017; Setiyawati et al., 2019; Yan et al., 2019) are indirect-gap semiconductors.

From a theoretical point of view, the first-principles calculations for layered materials pose challenges in investigating structural properties because of interactions between layers that are dominated by van der Waals force. Furthermore, the intra-layer chemical bonding (M-X bonding), which is mainly covalent in nature, predicts that the interactions play important roles in the structural features (Kumar and Ahluwalia, 2012). Even though many three-dimensional TMDs have been reported and numerous research papers published on 2D-TMDs, especially MoS₂ (Xiao et al., 2012; Zeng et al., 2012; He et al., 2014; Jiang et al., 2014; Zeng and Cui, 2015; Kang et al., 2016; Kolobov, 2016; Lin et al., 2016; Mirabelli et al., 2016; Hu et al., 2019), the available information on geometric and electronic properties of layered group IV chalcogenides, especially monolayer HfX₂, severely limited. While most numbers of the HfX₂ family that have been constructed to date show promise for applications, but the detailed characteristics might be lacking.

In this work, the geometric and electronic properties of layered HfX₂ (X = S, Se, or Te) are investigated in the T and H phases using first-principles calculations. The optimal lattice symmetries, sublattice buckling, electronic energy spectra, and the density of states are systematically calculated. HfS₂, HfSe₂, and HfTe₂ are respectively middle-gap semiconductor, narrow-gap one and semimetal, with various energy dispersions. The electronic properties strongly depends on the geometry and the multi-orbital hybridizations in the Hf-X bondings. Besides, the van Hove singularities mainly arise from the band-edges state. These physical phenomena clearly indicate a high potential for applications.

**METHOD**

The first-principles calculations based on the density functional theory (DFT) framework (Hohenberg and Kohn, 1964) are implemented in the Vienna Ab initio Simulation Package (VASP) (Kresse and Hafner, 1993) emerged as a dominant method in quantum mechanics simulation. In this work, the essential properties are investigated by DFT using VASP with the generalized gradient approximation (GGA) (Perdew et al., 1993; Perdew and Wang, 2018) of the Perdew-Burke-Ernzerhof (PBE) exchange-correlation functional (Ernzerhof and Scuseria, 1999) and projector-augmented-wave (PAW) potentials (Kresse and Joubert, 1999). The kinetic energy cutoff is set to 400 eV. A vacuum of 15 Å is added along the z-direction to avoid interactions between layers due to the periodic boundary condition. The structures are relaxed until the residual force is less than 0.001 eV/Å, and the self-consistent electronic convergence criterion is set at 10⁻⁶ eV. A Brillouin zone is sampled by 21×21×1 within the Gamma scheme for geometric optimizations and further calculations on electronic structures.

**RESULTS AND DISCUSSION**

Optimized Crystal Structures

Two-dimensional TMDs are characterized by layered structures forming a hexagonal packing in which each layer has a sandwich-like structure of transition metal and chalcogen atoms (Kolobov, 2016). Metal atoms are six-fold coordinated, as shown in Figure 1. Depending on the relative position of the chalcogen atoms, the coordination can be octahedral or trigonal prismatic exhibiting various polymorphs. In their bulk phase, more than 16 polytypes have been defined (Kolobov, 2016; Zhou et al., 2018), which suggests versatile and flexible applications. In the two kinds of symmetry coordination, the two chalcogen planes form a stagger in the octahedral arrangement (Figure 1A), while a slab is directly stacked above each other in the trigonal prismatic arrangement (Figure 1B). As a result, the cleavage properties are caused by the lopsided coordination around the chalcogens, which is perpendicular to the hexagonal or trigonal...
symmetry axis. Furthermore, the dangling bonds are absent, which leads to the surface is very stable and nonreactive.

Monolayer TMDs have only two polymorphs, namely the T and H phases (Kolobov, 2016), depending on their hexagonal or trigonal prismatic coordination. In Figure 2, the crystal optimization for three compounds in the monolayer HfX₂ family is performed with the top (xy plane) and side (yz plane) views. Hafnium (Hf) and two chalcogen atoms (X) form the X-Hf-X layered unit per unit cell, where the Hf atom is sandwiched between two X atoms referred to as a monolayer. In other words, each layer consists of three atomic planes that include one hafnium plane and two chalcogen planes through ionic-covalent interaction (see side view). From the top view, the spatial position of the chalcogen atoms is additionally filled with the light yellow triangles that show the different stacking in the two forms. The structural arrangements are constructed in the T and H phases with the period layer and trigonal symmetry being formulated by the octahedrally-coordinated stacking.

The formation energy is calculated using Eq. 1

$$E_f = E_{tot} - n_{Hf} \mu_{Hf} - n_X \mu_X$$ (1)

where $E_f$ is the formation energy; $E_{tot}$ is the total energy of monolayer HfX₂ (see in Table 1); $n_{Hf}$ and $n_X$ are, respectively, the number of atoms of hafnium and chalcogen atoms per unit cell; $\mu_{Hf}$ and $\mu_X$ are, respectively, the chemical potential of hafnium and chalcogen atoms in their bulk phases. As shown in Table 1 and Figure 3, the formation energy of the T phase is slightly smaller than that of the H phase; this indicates that the T phase structures are stable, while the H phase structures are metastable. This finding is in good agreement with experimental studies (Roubi and Carlone, 1988; Xu et al., 2015; Yue et al., 2015; Aminalragia-Giamini et al., 2016; Kanazawa et al., 2016; Yin et al., 2016; Mleczko et al., 2017; Tsai et al., 2018; Nakata et al., 2019) that reveal most monolayer HfX₂ exist in the T phase. To give further information, we have calculated the formation energy of monolayer MoS₂ in both T and H phases (shown in Table 1). In contrast to our materials, the total and formation energies in the T phase are larger than the H phase shows that the H phase is probably more stable. In addition, all formation energies of both materials are negative indicating the probability to construct them experimentally.

The calculated lattice constant, bond length, and buckling of monolayer T and H phases are also shown in Table 1. As shown in this table, the lattice constants of HfS₂, HfSe₂, and HfTe₂ are, respectively, 3.64, 3.75, 3.96 Å in the T phase, and 3.55, 3.63, 3.92 Åin the H phase. These parameters in the T phase are lightly

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**Figure 1** | The local coordination of the two types (A) octahedral and (B) trigonal prismatic.

**Figure 2** | The top (xy plane) and side (yz plane) views of HfX₂ (X = S, Se, or Te) in the (A) T phase and (B) H phase.
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Hf-X interaction that suppresses the height between hafnium atoms is larger than that of carbon atoms; it is respectively, 2.55, 2.60, 2.87 transfer to the two chalcogen atoms. The bond lengths are, bond forces the four valence electrons of the Hafnium atom to chalcogens. To fully

Compared to graphene (Castro Neto et al., 2009), the distance band, which is discussed in the electronic properties section.

d-orbital in hafnium plays an essential role in the energy on the sp (Balandin et al., 2008)-hybridized orbitals. In addition, the d-orbital in hafnium pure p-orbital takes part in forming these bonds and respectively, there is a gradation of increasing bond lengths that chalcogens S, Se, and Te are respectively 3s (Kolobov, 2016), 3p4, 4s24p4, and 5s (Kolobov, 2016), 5p4. The formal charge of +4 suggests that the lone-pair electrons (Kolobov, 2016) are located

larger than those of the H phase. Compared to the experimental lattice constants in previous studies (Greenaway and Nitsche, 1965; Bayliss and Liang, 1982) which are listed in Table 1, the structure is changed from bulk to monolayer by adding the vacuum, the lattice constant is insignificantly change. The electronic configuration of Hf is 5d6f6s2, and that of the chalcogens S, Se, and Te are respectively 3s (Kolobov, 2016), 3p4, 4s24p4, and 5s (Kolobov, 2016), 5p4. The formal charge of +4 and –2 can be respectively ascribed to the transition metal and chalcogens. To fully fill the valence shell, the relatively ionic Hf-X bond forces the four valence electrons of the Hafnium atom to transfer to the two chalcogen atoms. The bond lengths are, respectively, 2.55, 2.60, 2.87 Å in the T phase and 2.57, 2.58, 2.91 Å in the H phase for HfS2, HfSe2, and HfTe2. With varying the chalcogen atoms going from sulfur to selenium and tellurium, respectively, there is a gradation of increasing bond lengths that anticipates the features of electronegativity and orbital radii. The chalcogen pure p-orbital takes part in forming these bonds and suggests that the lone-pair electrons (Kolobov, 2016) are located on the sp (Balandin et al., 2008)-hybridized orbitals. In addition, the d-orbital in hafnium plays an essential role in the energy band, which is discussed in the electronic properties section. Compared to graphene (Castro Neto et al., 2009), the distance between hafnium atoms is larger than that of carbon atoms; it is Hf-X interaction that suppresses the height fluctuation of hafnium atoms. The chalcogen-metal (Hf-X) bonds are mainly determined by both hafnium and chalcogen atoms.

Furthermore, monolayer HfX2 exists in a bucked form (Δ) (see Figure 2) of approximate 2.89, 2.91, and 3.47 Å, respectively, for monolayer HfS2, HfSe2, and HfTe2 in the T phase. With the H phase, for HfS2, HfSe2, and HfTe2, these values are respectively 3.12, 3.02, and 3.66 Å. The Hf-X interaction suppresses the height fluctuation of the hafnium atom in these monolayers. As a result, the buckling in monolayer HfX2 is sharper than in silicene (Meng et al., 2013), which can be attributed to the sandwiched structure of these materials. Notice that the height fluctuations (buckling) are presented at equilibrium that provides the broad distribution of height displacement. This phenomenon predicts the phenomenological theories of thermal fluctuations in flexible membranes.

Electronic Properties

In the Brillouin zone, the electronic band structures have been calculated along with the high symmetry point Γ-M-K-Γ belonging to the hexagonal lattice. The system band gaps of monolayer HfX2 with the T and H phases are shown in Table 2. Both HfS2 and HfSe2 are semi-conducting with each material exhibiting distinct properties. HfS2 shows a middle-gap of 1.2 and 0.92 eV in the T and H phases, respectively. HfSe2 with these phases remains semi-conducting and belongs to a narrow-gap of 0.86 and 0.4 eV, respectively. In contrast, HfTe2 is semi-metallic. The values of approximately zero reveal it to be a gapless semimetal similar to graphene (Castro Neto et al., 2009). The experimental gaps of the monolayer T phase are also listed in Table 2. However, these values are smaller than the calculated band gaps, in which respectively exists a difference of approximate 0.6 and 0.3 eV for HfS2 and HfSe2. In this work, the band gaps are obtained from PBE functional, which notes that the approaches in calculating might cause the differences that are

**TABLE 1** Total energy (E_{tot}, eV), formation energy (E_r, eV), the lattice constant (a, Å), chemical bonding between hafnium and chalcogen atoms (d_{Hf-X}, Å), buckling (Δ, Å) of monolayer HfX2 in the T and H phases.

| X | T phase | H phase |
|---|---|---|
| | E_{tot} | E_r | a (Å) | d_{Hf-X} (Å) | Δ (Å) | E_{tot} | E_r | a (Å) | d_{Hf-X} (Å) | Δ (Å) |
| | Cal | Exp* | | | | Cal | Exp* | | | |
| S | –22.92 | –5.59 | 3.64 | 3.63^{63} | 2.55 | 2.89 | –22.28 | –4.94 | 3.53 | 2.57 | 3.12 |
| Se | –20.98 | –4.06 | 3.75 | 3.74^{63} | 2.60 | 2.91 | –20.4 | –3.48 | 3.63 | 2.58 | 3.02 |
| Te | –18.66 | –2.45 | 3.96 | 3.95^{63} | 2.87 | 3.47 | –18.21 | –2.01 | 3.92 | 2.91 | 3.66 |
| MoS2 | –21.01 | –1.87 | – | – | – | –21.84 | –2.71 | – | – | – |

*They all are experimentally the lattice constants in the bulk phase.

**TABLE 2** The calculated band gap (E_g, eV) of monolayer HfX2 with the T and H phases and experimental values of the bandgap in the T phase monolayer.

| Chalcogen | T phase | H phase |
|---|---|---|
| | Exp | Exp | Exp | Exp |
| | T phase | H phase | T phase | H phase |
| | 1.2 | 0.92 | 1.8^{66} | – | 0.86 | – | 1.1^{63} | 0.02 | – | 0.4 | – | 0.02 | – |

Note: —, no experimental report.

**FIGURE 3** Formation energy of HfX2 in the T (black line) and H (red line) phases.
significant when comparing them to experimental values. Because most members of group IV (Ti, Zr, and Hf) TMDs are experimentally found in the T phase (Greenaway and Nitsche, 1965; Terashima and Imai, 1987), there is no recorded measurement for making a comparison for the H phase in this work.

The system band energy spectra of monolayer HfX₂ are shown in Figure 4, which presents the atom dominances of hafnium and chalcogens. The left panel (Figures 4A–C) describes the system band of the T phase structures, and the right panel (Figures 4D–F) shows the system band of the H phase. All three materials share a common feature that the valence band maximum (VBM) is located at Γ point (k = 0, 0, 0) and the conduction band minimum (CBM) is at M point (k = ½, 0, 0) of the high symmetry k-points in the T phase. While those of the H phase have a VBM between K and Γ point corresponding to the CBM is located between Γ and M point. The unoccupied conduction bands are asymmetric to the occupied valence bands about the Fermi level. Most energy bands have a strong dispersion relation, with the parabolic dispersion being exhibited in both the conduction and...
the valence bands. However, each material is predicted to have an unusual energy band. There are many distinguishing features between the T and H phases in the electronic band structures that are discussed below.

As previously mentioned, monolayer HfX₂ (X = S, Se, or Te) are constructed with two phases that reveal distinct properties. The electronic bands of the T phase with atom dominance are described in Figures 4A–C (left panel). The red and blue circles respectively represent the hafnium and chalcogen atoms. They all show an indirect bandgap with the direction of the gap shifted from Γ point to M point. The occupied hole band is strongly asymmetric to the unoccupied electron band. The low-lying valence bands exhibit valleys between Γ and M point, as well as K and Γ point. The low-lying valence bands also change into parabolic bands. The parabolic dispersions have band-edge states at M point belonging to saddle points, which results in van Hove singularities (vHs). In Figure 4A, the σ and σ* bands (in the valence and conduction bands) of HfS₂ initiate from Γ point at −2.5, 0 and 2 eV, while the σ and σ* bands of HfSe₂ are at −3, 0 and 1.5 eV (Figure 4B). The energy band of HfTe₂ reveals contrasting features compared to HfS₂ and HfSe₂, as shown in Figure 4C. A semi-metallic property of HfTe₂ has been shown, which is investigated by the pressure-dependence conductivity and Hall coefficient (Aminalragia-Giamini et al., 2016). Apart from the
special valleys located in the low-lying valence band that is similar to HfS$_2$ and HfSe$_2$, there exist flat bands in the low-lying valence bands. There are $\sigma$ and $\sigma'$ bands that initiate from r point at $-3, 0$, and $0.5$ eV. The difference between HfTe$_2$ and both HfS$_2$ and HfSe$_2$ is that all of the VBMs and CBMs are nearly at the Fermi level, which presents a gapless material. With regard to atom dominance in bands, the chalcogen atoms in all structures dominate in valence band ranging from 0 to $-4$ eV, while the contribution of hafnium atoms is smaller than in this band. In contrast, hafnium contributes to the conduction band more than the valence band. In valence band, this atom dominates in bands of a scale of 0 to $-2$ eV. Both hafnium and chalcogens contribute to the whole band with ranges from $-4$ to 4 eV, but chalcogens show a sharper dominance compared to hafnium.

In order to compare with the T phase structures, the band energy spectra of the H phase are presented in Figures 4D–F (right panel). The green and magenta circles represent chalcogen and hafnium atoms. The occupied valence band and the unoccupied conduction band are asymmetric. Both HfS$_2$ and HfSe$_2$ still exhibit semiconducting with narrow-indirect gaps, but the direction is shifted from between K and r point to the point near M point. Similar to the T phase, HfTe$_2$ remains semimetal with the same bandgap value (see Table 2). All the H phase structures exist in approximate flat bands near the Fermi level while the degeneracy lies below it, this leads to additional valleys in the low-lying valence bands. In Figure 4D, two $\sigma$ bands of HfS$_2$ initiate from r point at -1 and -2 eV in the valence band, while the $\sigma$ and $\sigma'$ bands of HfSe$_2$ and HfTe$_2$, respectively, are located at $-1, -2.5, 3.5$ eV and $-0.8, -2.2, 3.2$ eV. Furthermore, all materials in the H phase show the crossing dispersion in the valence band, which is between r and M point. The most important atom dominances are at low-lying valence bands, while those of the T phase is at the edges of the valence band. The chalcogen atoms also dominate in valence bands like in the T phase, but they contribute in a lower energy range from -1 to $-2$ eV. In the conduction band, the dominance of chalcogen atoms is almost absent, while hafnium exists in both the valence and conduction bands. Although all of the chalcogens control the valence bands, they the different contributions in each band. The tellurium atom in HfTe$_2$ has the smallest contribution, followed by Se and S.

In general, the band energy spectra exhibit a lot of asymmetric peaks and few symmetric ones due to the parabolic form of energy bands. A pair of asymmetric peaks exists near the Fermi level that defines the indirect energy gaps. The bandgap opening is associated with the quantum confinement effect. Although all three materials are members of the same chalcogen group, HfS$_2$, HfSe$_2$, and HfTe$_2$ are different in their low-lying peaks and degenerate with varying the stacking configuration between the two phases. Moreover, most van Hove singularities (vHs) come from the parabolic energy dispersions. The peaks near the Fermi level are found to arise from the band-edge states mainly. These peaks could be a signature feature unique to vHs.

**FIGURE 6** | Local density of states (LDOS) of (A) hafnium and (B) chalcogen atoms with multi-orbital hybridization in the T phase.
The density of states (DOS) is shown in Figure 5 for the T and H phases, which directly reflect the main features of the band structures. DOS is ranged from −4 to 4 eV and shows a pair of asymmetric peaks centered at the Fermi level, which characterizes the energy gap. The large energy difference between the valence and conduction peaks obviously demonstrates the asymmetry of the two bands. These structures originate from linear bands, parabolic bands near saddle points, and initial band-edge states of the parabolic bands. In the T phase structures shown in Figure 5A, DOS exhibits a logarithmic divergence at middle energy and shoulder structures at deeper and higher energy. Partially flat energy is exhibited in the valence bands of all structures and in the conduction bands of HfSe2 and HfTe2, while shoulder structures evidently appear in the conduction band of HfS2. Moreover, the low DOS at the Fermi level illustrates the low free carrier density. In contrast, a linear energy dependence near the Fermi level is shown in the H phase structures (Figure 5B). The DOS of the H phase is much larger than that of the T phase at the Fermi level, indicating a higher free carrier density. At deeper and higher energy, DOS reveals peaks, as well as divergent and shoulder structures. Notice that the peak and shoulder structures correspond to the 3p0, 4p0, 5p0, and (3p0,3p0), (4p0,4p0), (5p0,5p0) of HfS2, HfSe2, and HfTe2, respectively, dominating energy bands as indicated in the orbital-projected DOS.

To further verify the various chemical bondings and examine the curvature effect, the orbital-projected DOS could be used as described in Figures 6 and 7. The figures show the local density of states with multi-orbital hybridization in the T and H phases. They show that the main contributions are due to two orbitals of the hafnium and two orbitals of the chalcogens, corresponding to the s, d orbitals of hafnium, and the s, p orbitals of chalcogens. With the T phase structures, the local density of states with multi-hybridization is shown in Figure 6, presenting the dominance of the multi-orbital hafnium and chalcogen atoms. At high energy, hafnium atoms are dominated by 5dz2 and 5dx2−y2 orbitals, as clearly indicated in Figure 6A. But for low energy, DOS depends on the bondings of (3px,3py), (4px,4py) and (5px,5py) of HfS2, HfSe2 and HfTe2, respectively (Figure 6B).

Similar to the T phase, the H phase structures are shown in Figure 7 and indicate that the d-orbitals of hafnium and the p-orbitals of chalcogens, respectively, play essential roles in the conduction and valence bands. The 5dz2, 5dxy, and 5dx2−y2 orbitals dominate at high energy but exist with different features in each material (Figure 7A). HfS2 is dominated by 5dz2 at high energy, while 5dx2−y2 mainly dominates in the conduction band of HfSe2. HfTe2 shows the dominance of all 5dz2, 5dxy, and 5dx2−y2 orbitals. With chalcogen atoms (Figure 7B), 3p0, 4p0, and 5p0, respectively, reveal a prominent dominance at low energy near the Fermi level and lower energy in HfS2, HfSe2, and HfTe2. At middle energy in the valence band, they are respectively based on the (3px,3py), (4px,4py), and (5px,5py) orbitals. In summary, their contributions...
are comparable to each other in that the feature-rich energy bands are mainly determined by the d and p orbitals. There exist strong hybridizations of \(5d_{x^2}, 5d_{z^2-\epsilon^2}\) Hf-orbitals and \((3p_x, 3p_y, 3p_z), (4p_x, 4p_y, 4p_z), (5p_x, 5p_y, 5p_z)\) chalcogen-orbitals. In addition, the p-orbitals of the chalcogens make significant contributions to the \(\sigma\) bonding. These orbitals are evident for the multi-orbital hybridization in these materials.

CONCLUSION

The first-principles calculations are used to investigate the geometric and electronic properties of monolayer HfX\(_2\) (X = S, Se, or Te). Our calculated band structures in the T and H phases can modulate their properties. All structures exist with a sharp buckle that shows height fluctuations at equilibrium. The d-orbitals and p-orbitals of the hafnium and chalcogen atoms, respectively, take part in the chemical bondings of Hf-X. As reported, both HfS\(_2\) and HfSe\(_2\) are semiconductors with an indirect middle-gap and narrow-gap, respectively, while HfTe\(_2\) is a gapless semimetal. The energy gap trend suggests that the atomic species play a dominant role. A lot of asymmetric peaks and few symmetric ones due to the parabolic form are shown in the energy bands and densities of states. The van Hove singularities mainly arise from the band-edge states in the DOS. The logarithmic divergence at middle energy and shoulder fluctuations at equilibrium. The atomic orbitals hybridization is a gapless semimetal. The energy gap trend suggests that the atomic species play a dominant role. A lot of asymmetric peaks and few symmetric ones due to the parabolic form are shown in the energy bands and densities of states. The van Hove singularities mainly arise from the band-edge states in the DOS. The logarithmic divergence at middle energy and shoulder fluctuations at equilibrium. The atomic orbitals hybridization is a gapless semimetal.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author/s.

AUTHOR CONTRIBUTIONS

TH: Conceptualization, Model building, Methodology, Software, Validation, Writing- Reviewing and Editing. DN: Model building, Data curation, Visualization, Original draft preparation. TN: Model building, Data curation, Visualization, Original draft. VD: Model building, Data curation, Visualization, Original draft preparation. NH: Model building, Data curation, Visualization, Original draft preparation. HP: Conceptualization, Model building, Methodology, Software, Validation, Reviewing and Editing, Supervision.

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**Conflict of Interest:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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