Flavor analysis of nucleon electromagnetic form factors

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We have performed an evaluation of the individual up- and down-flavor contributions to the nucleon electromagnetic form factors within a relativistic constituent-quark model. It is found that the theoretical parameter-free predictions agree surprisingly well with recent phenomenological data from a flavor decomposition of the proton and neutron electromagnetic form factors for momentum transfers up to \(Q^2 \sim 3.5 \text{ GeV}^2\). It means that in this regime three-quark valence degrees of freedom dominate and other contributions, such as explicit mesonic effects, can at most play a minor role.

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By the advent of more and more precise experimental data on nucleon form factors, especially from Jefferson Lab, it has recently become possible to reliably identify the various \(u\)- and \(d\)-flavor contributions to the elastic proton and neutron electromagnetic form factors \(G_p^E, G_p^M, G_n^E,\) and \(G_n^M\). The corresponding results cover the range of momentum transfers up to \(Q^2 \sim 3.5 \text{ GeV}^2\). Some essential observations thus gained have been described, e.g., in ref. [1] as:

- The \(Q^2\) dependences of the ratios of the Pauli to Dirac form factors \(F_2^q/F_1^q\) for the quark-flavor contributions \(q=u,d\) to the nucleon form factors are practically constant for momentum transfers beyond \(Q^2 \sim 1.5 \text{ GeV}^2\), in contrast to the observed behaviors of the same ratios for both the proton and the neutron.

- The \(d\)-flavor contributions \(F_1^d\) and \(F_2^d\) of the Dirac and Pauli form factors show a scaling behavior with \(Q^4\) starting at momentum transfers of \(Q^2 \sim 1.5 \text{ GeV}^2\). However, this seems not to be confirmed by a very recent analysis in ref. [2].

- On the other hand, the \(u\)-flavor contributions \(F_1^u\) and \(F_2^u\) of the Dirac and Pauli form factors do not scale with \(Q^4\) or drop off in the considered range of momentum transfers.

In refs. [1, 2] one has also made comparisons to selective theoretical results, with no one being able to explain all the particular characteristics of the available phenomenological data. Also some speculations are offered for the interpretation of the new phenomenological insights, like a possible role/interplay of three- and five-quark components in the nucleon states, quark-diquark clustering, or the onset of perturbative quantum chromodynamics.

We have analyzed the various flavor ingredients in the nucleon electromagnetic form factors within the relativistic constituent-quark model (RCQM) whose hyperfine interaction is derived from Goldstone-boson exchange [4]. This type of dynamics stems from the spontaneous breaking of chiral symmetry (\(\text{SB}_\chi\text{S}\)) in low-energy quantum chromodynamics (QCD) and produces a flavor-dependent spin-spin interaction that allows to describe the baryon spectra in agreement with phenomenology, notably also with the right level orderings in the nucleon and other baryon excitation spectra [5].

The GBE RCQM has also been applied to produce covariant results for the nucleon electromagnetic form factors [6, 7], including the electric radii and magnetic moments [8]. The direct predictions of the GBE RCQM have in all
instances been found in remarkably good agreement with existing experimental data without the need of introducing any further parameterizations, such as constituent-quark form factors or the like. These studies have been performed in the framework of Poincaré-invariant quantum mechanics using its point form \([9]\). The approach allows to calculate covariant observables, since the Lorentz transformations (including rotations) can be constructed from interaction-independent generators. This is even true for the electromagnetic current assumed according to a spectator model \([10]\). It has been shown that the relativistic results obtained in this manner do not only fulfill all requirements of the Poincaré algebra but also other necessary constraints, notably current conservation \([11]\). The GBE RCQM has been similarly successful in predicting the axial and induced pseudoscalar form factors of the nucleons \([7, 12]\). Most recently one has obtained also the axial charges of the other baryon ground states and their resonances and found them in reasonable agreement with existing lattice QCD results \([13, 14]\). Similarly, the GBE RCQM has been able to provide a microscopic description of the \(\pi NN\) as well as \(\pi N\Delta\) interaction vertices in accordance with their behavior expected from phenomenology \([15]\).

In view of the recently published phenomenological data, we have put the GBE RCQM to the test of producing the flavor contributions to the nucleon electromagnetic form factors. This is particularly interesting, as there has not yet been any consistent theoretical explanation of the whole set of experimental data. The calculations have been performed in complete accordance with the work in refs. \([6–8]\). There one can also find detailed descriptions of the formalism and the calculations of the nucleon elastic electromagnetic form factors, which cannot be repeated here.

Under charge symmetry the Sachs form factors of the nucleons are constituted as

\[
G_E^p = \frac{2}{3} G_{uE}^p - \frac{1}{3} G_{dE}^p, \quad G_E^n = \frac{2}{3} G_{uE}^n - \frac{1}{3} G_{dE}^n, \\
G_M^p = \frac{2}{3} G_{uM}^p - \frac{1}{3} G_{dM}^p, \quad G_M^n = \frac{2}{3} G_{uM}^n - \frac{1}{3} G_{dM}^n, \tag{1}
\]

where \(G_{u,d}^E,M\) and \(G_{u,d}^{E,M}\) are related to the \(u\)- and \(d\)-flavor contributions in the proton and neutron form factors by:

\[
G_{u,E,M}^u(Q^2) = G_{u,p,M}^u(Q^2) = 2G_{d,n,M}^d(Q^2), \tag{3}
\]

\[
G_{d,E,M}^d(Q^2) = G_{d,p,M}^d(Q^2) = \frac{1}{2} G_{u,n,M}^u(Q^2). \tag{4}
\]

In Fig. 1 we show the results for the proton and neutron electric and magnetic form factors as they are composed of the \(u\)- and \(d\)-flavor contributions defined in Eqs. (1) to (4). Starting out from zero momentum transfer, where the electric radii and magnetic moments are reasonably well described \([8]\), it is nicely seen, how the right \(Q^2\) dependence develops. Slight differences to the phenomenological data are only visible for the anyway very small \(u\)-flavor contribution \(\frac{2}{3} G_{dM}^d\) to the magnetic form factor of the neutron.

The direct predictions of the GBE RCQM for the flavor-separated Dirac and Pauli form factors defined by

\[
F_i^u = 2F_i^p + F_i^n, \quad F_i^d = F_i^p + 2F_i^n, \quad i = 1, 2 \tag{5}
\]

are shown in Fig. 2 in comparison to the phenomenological analyses of refs. \([1, 3]\). Again a reasonable agreement is observed.

Often ratios of electromagnetic form factors are considered, since they are directly accessible by experiment and provide a more discriminative insight into the \(Q^2\) dependences with smaller uncertainties. In Fig. 3 we first show the predictions of the GBE RCQM for the \(F_2/F_1\) ratios for the proton and the neutron. The analogous ratios of the
FIG. 1: $u$- and $d$-flavor contributions to the proton (upper panels) and neutron (lower panels) electric and magnetic form factors as predicted by the GBE RCQM in comparison to phenomenological data from refs. [1] (filled circles) and [3] (vertical bars) as well as global experimental data as indicated. The uncertainties in the data of ref. [1] are practically not visible, as they do not exceed the sizes of the circles; the uncertainties in the data of ref. [3] correspond to the heights of the vertical bars. Notice that in case of the neutron the $u$- and $d$-flavor contributions are here represented by $G_E^n$ and $G_M^n$, respectively; cf. Eqs. (3) and (4).

Flavor-separated form factors are depicted in Fig. 4 by means of the functions

$$S^q(Q^2) = Q^2 \frac{F^q_2(Q^2)}{F^q_1(Q^2)}, \quad q = u, d.$$  \hfill (6)

According to the phenomenological data both functions $S^q(Q^2)$ reflect an almost linear rise with increasing $Q^2$ (i.e. constancy of the $Q^2$ dependences of the ratios $F^q_2/F^q_1$). This behavior is met by the GBE RCQM only for the $u$-flavor. The $d$-flavor ratio starts to depart from the phenomenological data at $Q^2 \sim 1.5$ GeV$^2$.

Regarding the distinct $Q^2$ dependences of the flavor-separated Dirac and Pauli form factors $F^q_1$ and $F^q_2$ we have also examined the ratios $F^q_i/F^{q'}_i$ (see Fig. 5). For both $i = 1, 2$ they reasonably follow the phenomenological data. In the regime of low momentum transfers this is especially true for $F^d_1/F^n_1$, where the data sets of refs. [1] also coincide. This is obviously not the case for $F^d_2/F^n_2$. Our theoretical predictions just fall amidst these two phenomenological data sets. We note that the recent analysis of ref. [2] has yielded data of quite a similar behavior, and our theory perfectly agrees with them (see Fig. 9 of ref. [2]). Anyway, here is a case where the phenomenological analyses [1–3]
In any case, it seems to be well established from experiments that the $d$-contributions $F_i^d$ fall off faster than the $u$-contributions $F_i^u$ towards higher momentum transfers, a behavior that has sometimes been interpreted as an indication for diquark clustering in the nucleons. However, our GBE RCQM, which is a genuine three-quark
microscopic model and has no reference to diquark configurations whatsoever (see its spatial probability distributions depicted in ref. [16]), also produces these fall-offs in overall agreement with the phenomenological data. The above reasoning regarding diquark clustering thus appears questionable.

In view of the reasonable flavor contributions to the electromagnetic form factors found for the GBE RCQM results, let us shortly summarize the main ingredients in our theory. First of all, we have very precise nucleon wave functions generated by an interacting mass operator that contains a linear confinement, with a strength corresponding to the string tension of QCD, and a hyperfine interaction that incorporates $SB\chi S$, leading to a specific flavor dependence, contrary to a one-gluon-exchange hyperfine interaction (see the corresponding discussion in ref. [5]). The nucleon wave functions contain non-zero orbital angular momenta as well as mixed-symmetric spatial wave-function parts; the latter are most important for reproducing the neutron electromagnetic form factors, even though the non-symmetric spatial components are relatively small. Furthermore, due to the use of point-form relativistic quantum mechanics, the matrix elements of the electromagnetic current operator are strictly frame-independent [10]. We may calculate the electromagnetic form factors in the Breit frame, the laboratory frame, or in any other frame, as we can perform the Lorentz transformations exactly in the point form. Our specific construction of the electromagnetic current operator – according to the point-form spectator model – also guarantees current conservation [11].

The GBE RCQM for the nucleons relies on three-quark configurations only. The underlying interaction Lagrangian is just built by coupling valence-quark fields with Goldstone bosons. No explicit mesonic effects or more-quark components are introduced. From our previous studies we have learned that relativistic (boost) effects are most important in the reproduction of the nucleon electromagnetic form factors [6, 7]. This is even true for the quantities extracted at or near zero momentum transfers, i.e. the magnetic moments and electric radii [8]. For the success of the GBE RCQM we identify as the essential symmetry ingredients the respect of $SB\chi S$ of low-energy QCD and Lorentz invariance.

![Graph](image-url)

**FIG. 4:** The functions $S^q(Q^2)$ defined in Eq. (6), representing the ratios of the $u$- and $d$-flavor contributions to the Pauli and Dirac form factors in Fig. 2 as predicted by the GBE RCQM in comparison to phenomenological data from refs. [1, 3]. The symbols are the same as in Fig. 1 where the vertical bars corresponding to ref. [3] are slightly displaced to the right in order to discriminate between the uncertainties in both data sets, in case the $Q^2$ values on the abscissa coincide.
FIG. 5: Ratios of u- and d-flavor contributions to the Dirac and Pauli form factors as predicted by the GBE RCQM in comparison to phenomenological data from refs. 1 and 3. Same symbols and remarks as in Fig. 4.

Acknowledgments

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