Analysis of factors affecting steering performance of wheeled skid-steered vehicles

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Abstract: Wheeled skid-steered vehicles have the advantages of compact structure, small steering radius and wide application. However, poor steering performance is an important limitation restricting the development of wheeled skid-steered vehicles. A kinematic model for analysing the relationship between wheels' rotation speed and the motion state of the vehicle's mass centre based on Newton's second law was established, then the correctness of the kinematic model was verified by comparing the calculation results with simulation results. The key design parameters affecting the steering performance were analysed by a control-variable method based on the established kinematic model. The results indicate that the established kinematic model can evaluate the steering performance of skid-steered vehicles correctly and can provide a reference for optimising the design of the vehicles.

1 Introduction

Skid-steering means that the vehicles change the riding direction by differential rotation speeds of the left and right wheels without steering mechanism. Compared with traditional vehicles, wheeled skid-steered vehicles have the advantages of simple structure, low cost, space-saving, high energy utilisation, etc. So it's widely used in lunar rover, reconnaissance aircraft, etc. [1]. It's important to establish the kinematic model of wheeled skid-steered vehicles for optimising the design of the vehicles on the premise of excellent steering performance.

Wheel radius, friction coefficient between wheels and ground, lateral and longitudinal wheelbases will directly affect the force of skid-steered vehicles. So, selecting suitable parameters can improve steering performance by changing the force of the vehicles. Bekker [2] provides a method for analysing the relationship between wheels and ground by experiment. Maclaurink [3] provides a calculation method for predicting the steering performance and power flows of a notional skid steered tracked vehicle. In order to improve motion control and pose estimation, Martinez [4] proposes a kinematic approach for the tracked mobile robots. In [5], kinematic and dynamic model of wheeled skid-steered robots was first presented, and an adaptive control algorithm based on Kalman filter was designed. In [6], Yu et al. developed a dynamic model of wheeled skid-steered vehicle for analysing planar (two-dimensional (2D)) motion and linear 3D motion. Low and Wang [7] provided a way to control the trajectory and the posture of the wheeled skid-steered vehicles based on GPS.

Currently, most researches about skid steering focus on wheel mechanics, dynamic modelling and trajectory control. However, there are few studies about the key parameters such as wheel radius, friction coefficient, lateral and longitudinal wheelbase affecting the steering performance of wheeled skid-steered vehicles. So a kinematic model for analysing the relationship between wheels’ rotation and the movement of skid-steered vehicles was established, then the correctness of the kinematic model was verified by simulation. The influences of wheel radius, friction coefficient, lateral and longitudinal wheelbases on the vehicle's steering performance were analysed by the control-variable method.

2 Dynamic model and model validation

The study project is a four-wheel vehicle without steering mechanism. The simplified skid-steering model of the vehicle is shown in Fig. 1, where o is the vehicle's centroid, W is the lateral wheelbase, L is the longitudinal wheelbase, and r is the wheel radius. Fixed coordinates (XOY) and vehicle coordinates (xoy) of each wheel relative to the centre of the vehicle are established, as shown in Fig. 1. The axis perpendicular to the plane and passing through point o is defined as the z-axis. $\Omega$ is the vehicle's deflection angle formed by an axis parallelled to the X-axis rotating at a certain angle around o, counterclockwise rotation is positive and clockwise rotation is negative. $[F_x,F_y]$ represent the resultant forces along with the z-axis and y-axis, $[F_x,F_z]$ are the resultant forces along with the X-axis and Y-axis, M represents the resultant moment about the z-axis. The number of wheels is i (i = 1, 2, 3, 4), the sliding velocity of wheel i relative to the ground is defined as $\Delta V_i$. Denote $[\Delta V_{i,y}, \Delta V_{i,z}]$ as the longitudinal and lateral sliding speeds of wheel i, and $f_i$ is the friction of wheel i.

When the left and right wheels rotate at a fixed speed difference, the vehicle body will deflect and the driving direction of the vehicle will change. At the moment, the instantaneous motion of the vehicle can be regarded as translational motion in the xoy coordinates and the rotational motion around the point o. The longitudinal and lateral sliding velocities ($\Delta V_{i,y}$ and $\Delta V_{i,z}$) of each wheel make up the skidding velocity ($\Delta V_o$) of each wheel relative to ground. If the rotation speed of the left and right wheels keeps the fixed difference, longitudinal velocity, related to wheel radius, will influence the direction of friction, thus reflecting the motion state of the vehicle. Longitudinal and lateral wheelbases can influence the resultant moment. In summary, wheel radius, friction coefficient longitudinal and lateral wheelbases have an impact on the vehicle's motion state.
2.1 Kinematic model

The process of low speed wheeled skid-steered vehicle was analysed. Based on the summary above, some assumptions are made to simplify the process of steering:

1. The mass centre of the vehicle coincides with the centroid.
2. The contact relationship between the wheels and the ground is simplified into the Coulomb friction model, and the friction coefficient is isotropic.
3. The vehicles are running on a horizontal hard road and the four wheels keep in contact with the ground during steering.
4. In the process of low-speed steering, centrifugal force is negligible.

The relationship between the motion of the vehicle's mass centre and the force in the $XOY$ coordinates is as follows:

$$
\begin{align*}
\ddot{X} &= \frac{F_X}{m} \\
\ddot{Y} &= \frac{F_Y}{m} \\
\dot{\phi} &= \frac{M}{J}
\end{align*}
$$

In (1), $m$ is the mass of the vehicle and $J$ represents the moment of inertia of the vehicle about the $z$-axis. $\phi$ represents the angular velocity of the vehicle. $X\dot{}$, $Y\dot{}$ is the velocity of the vehicle's mass centre along with the $X$-axis and $Y$-axis, $x\dot{}$, $y\dot{}$ is the velocity of the vehicle's mass centre along with the $x$-axis and $y$-axis. The conversion relation between $x\dot{}$, $y\dot{}$ and $X\dot{}$, $Y\dot{}$ is

$$
\begin{pmatrix}
x \\
y
\end{pmatrix}
= \begin{pmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{pmatrix}
\begin{pmatrix}
X \\
Y
\end{pmatrix}
$$

The friction force between wheels and ground can affect the motion state of vehicles directly. The relation between wheels’ rotation speed and the motion state of the mass centre during steering is shown in Fig. 2. The $v_i$ represents the speed of the centre of wheel $i$ fixed on the vehicle body, $v_{ix}$ represents the projection of $v_i$ along with the $x$-axis; $v_{iy}$ represents the projection along the $y$-axis. The $\omega_i$ represents the rotation speed of wheel $i$, which make the vehicle's mass centre produce a positive movement trend towards the $x$-axis is defined as positive, otherwise defined as negative.

The projection of the sliding velocity of wheel $i$ relative to the ground along with the $x$-axis and $y$-axis can be expressed as

$$
\begin{align*}
\Delta v_{ix} &= \dot{x} - \dot{\phi} W - \omega_i r \\
\Delta v_{iy} &= \dot{y} + \dot{\phi} L \\
\Delta v_{2x} &= \dot{x} - \dot{\phi} W - \omega_i r \\
\Delta v_{2y} &= \dot{y} - \dot{\phi} L \\
\Delta v_{3x} &= \dot{x} + \dot{\phi} W - \omega_i r \\
\Delta v_{3y} &= \dot{y} + \dot{\phi} L \\
\Delta v_{4x} &= \dot{x} + \dot{\phi} W - \omega_i r \\
\Delta v_{4y} &= \dot{y} - \dot{\phi} L
\end{align*}
$$

The direction of friction on the wheels is opposite to the direction of relative sliding speed relative to the ground at the contact point of the wheels. Fig. 3 shows the relationship between friction and wheels' sliding speed relative to the ground.

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Table 1 Main parameters

| Vehicle structure parameters and friction coefficient | Vehicle's initial motion state | Speed of wheels during steering |
|-----------------------------------------------------|------------------------------|-------------------------------|
| mass, m/kg | 26,877 | x-coordinate of the mass centre, m | 0 | speed of wheel 1, rad/s | 0 |
| moment of inertia, J/kg*m^2 | 40,309 | velocity along with the X-axis, m/s | 3 | speed of wheel 2, rad/s | 0 |
| longitudinal wheelbase, L/m | 1.8 | y-coordinate of the mass centre, m | 0 | speed of wheel 3, rad/s | 5 |
| lateral wheelbase, W/m | 2.4 | velocity along with the Y-axis, m/s | 0 | speed of wheel 4, rad/s | 5 |
| wheel radius, r/m | 0.6 | deflection angle, ∅/rad | 0 | — | — |
| friction coefficient, μ | 0.2 | angular velocity, rad/s | 0.0000001 | — | — |

Fig. 4 Comparison between simulation results and calculated results
(a) Trajectory of the mass centre of the vehicle, (b) Rule of deflection angle changing with time

The sine and cosine functions of the skidding angle of each wheel can be obtained by substituting (2) and (3) into (4). Then, according to (5) and the function of the last step, the projections of the friction force along with the x-axis and y-axis of each wheel were calculated. Furthermore, the projections, along with the x-axis and y-axis of the resultant force on the vehicle in the coordinates xyO were calculated by substituting the previous result into (6). Finally, formula (7) was used to calculate the projections of the resultant force along the axes of the coordinates XOY and the resultant moment about the vehicle's mass centre. The expression of the resultant force and resultant moment were substituted into (1) to obtain the kinetic model of the vehicle, which is expressed as

\[
\begin{align*}
\sin \theta_i &= \frac{-\Delta v_{ix}}{\sqrt{\Delta v_{ix}^2 + \Delta v_{iy}^2}} (i = 1, 2, 3, 4) \\
\cos \theta_i &= \frac{-\Delta v_{iy}}{\sqrt{\Delta v_{ix}^2 + \Delta v_{iy}^2}} (i = 1, 2, 3, 4)
\end{align*}
\]

(4)

The projection of the friction along with the x and y axes of each wheel is expressed as

\[
\begin{align*}
\dot{f}_{ix} &= f_i \sin \theta_i \\
\dot{f}_{iy} &= f_i \cos \theta_i
\end{align*}
\]

(5)

In the coordinates xyO, the projection of the resultant force along with the x-axis and y-axis and the moment about the mass centre of the vehicle is expressed as follows:

\[
\begin{align*}
F_x &= \sum_{i=1}^{4} f_{ix} \\
F_y &= \sum_{i=1}^{4} f_{iy} \\
M &= (f_{ix} - f_{ix}) \frac{w}{2} + (f_{ix} - f_{ix} - f_{ix} - f_{ix}) \frac{L}{2}
\end{align*}
\]

(6)

The forces at the mass centre of the vehicle in the xyO coordinates and the same in the XOY coordinates are shown in Fig. 2. The conversion relation between [F_x, F_y] and [F_z, F_y] is shown below:

\[
\begin{bmatrix}
F_x \\
F_x
\end{bmatrix} = \begin{bmatrix}
\cos \varphi & -\sin \varphi \\
\sin \varphi & \cos \varphi
\end{bmatrix} \begin{bmatrix}
F_z \\
F_y
\end{bmatrix}
\]

(7)

2.2 Model validation

There was a four-wheel skid-steered vehicle; the main parameters of the vehicle and ground are shown in Table 1. The vehicle's motion parameters were set in MSC.ADAMS according to relevant parameters in Table 1. Meanwhile, relevant data in Table 1 were substituted into (8) and solved the obtained equation by Runge–Kutta method. Motion track of mass centre and the rule of deflection angle changing with time obtained by calculation and simulation are shown in Fig. 4, the time when the vehicle starts to turn is defined as 0.

Fig. 4a shows the trajectory of the vehicle obtained by calculation and simulation. Fig. 4b shows the rule of deflection angle obtained by calculation and simulation. As can be seen from Fig. 4, the calculation results have a small difference compared to the simulation results. So, the established kinematic model can correctly describe the relationship between wheels' rotation speed and motion rule of the vehicle's mass centre.
3 Results and discussion

3.1 Influence of lateral and longitudinal wheelbases

Define two variables to evaluate the steering performance of the vehicle

\[
\begin{align*}
\Delta X(t) &= X(t) - X(0) \\
\Delta \varphi(t) &= \varphi(t) - \varphi(0)
\end{align*}
\]  

(9)

In (9), the \(X(0)\) and \(\varphi(0)\) are the X-coordinate and deflection angle of vehicle’s mass centre at the start of steering, the \(X(t)\) and \(\varphi(t)\) represent the X-coordinate and deflection angle of the vehicle \(t\) seconds later from the start of steering. The \(\Delta X(t)\) is the subtraction between \(X(t)\) and \(X(0)\), \(\Delta \varphi(t)\) is the subtraction between \(\varphi(t)\) and \(\varphi(0)\). Smaller \(\Delta X(t)\) means that the vehicle can complete the steering in a smaller space in the same conditions, larger \(\Delta \varphi(t)\) indicates that the vehicle can obtain a larger deflection angle in the same conditions.

Longitudinal and lateral wheelbases are factors affecting the resultant moment of the vehicles. The \(\Delta X(0)\) and \(\Delta \varphi(0)\) were taken as indicators to evaluate the vehicle’s steering performance. The trajectory of the vehicle’s mass centre and the rule of vehicle’s deflection angle were obtained according to the calculation method in Section 2. Then \(\Delta X(0)\) and \(\Delta \varphi(0)\) were calculated through (9). The results obtained through the control-variable method are shown in Fig. 5.

As shown in Fig. 5, the different combination of lateral and longitudinal wheelbases will make the vehicle have different steering performance. Slight change of longitudinal wheelbase can make a great change of \(\Delta X(0)\) and \(\Delta \varphi(0)\). The \(\Delta X(0)\) decreases with the increase of the longitudinal wheelbase, and the \(\Delta \varphi(0)\) increases with the increase of the longitudinal wheelbase. The \(\Delta X(0)\) and the \(\Delta \varphi(0)\) are not so sensitive to the lateral wheelbase. The \(\Delta X(0)\) decreases firstly and then increases with the increase of the lateral wheelbase, and the rule of \(\Delta \varphi(0)\) changing with the increase of lateral wheelbase is opposite to that of \(\Delta X(0)\). A suitable combination of longitudinal and lateral wheelbase can make the vehicle have the best steering performance.

3.2 Influence of wheels radius

The influence of wheels radius on \(\Delta X(0)\) and \(\Delta \varphi(0)\) within a certain range is analysed through control-variable. The results are shown in Fig. 6.

It can be seen from Fig. 6, the \(\Delta X(0)\) and the \(\Delta \varphi(0)\) are positively correlated with wheel radius. The increasing \(\Delta X(0)\) means that the vehicle needs more space to complete steering, and the increasing \(\Delta \varphi(0)\) indicates that the vehicle can obtain a larger deflection angle. So, it is necessary to select suitable wheels radius according to actual needs.

3.3 Influence of friction coefficient

Different friction coefficients produce different friction forces between wheels and ground. Analysing the influence of friction coefficient on steering performance can provide a reference for selecting a suitable friction coefficient. The influence of friction coefficient on steering performance was analysed through the control-variable method. The results are shown in Fig. 7.

As the index of steering performance, \(\Delta X(0)\) is positively correlated with friction coefficient, while \(\Delta \varphi(0)\) is negatively correlated with the friction coefficient. The larger the friction coefficient is, the smaller the effect of changing the friction coefficient on \(\Delta X(0)\). Increasing the friction coefficient can increase \(\Delta X(0)\) greatly when the friction coefficient is <0.3. The changing
friction coefficient has nearly no influence on $\Delta \phi_3$ when the friction coefficient is $>0.4$.

4 Conclusion

The kinematic model describing the relationship between wheels rotation speed and the motion state of the skid-steered vehicle was established. The correctness of the kinematic model is validated by comparing simulation results and calculation results. Some factors affecting the steering performance of skid-steered vehicles were analysed through the control-variable method, and the results are as follows:

(i) In some conditions, the steering performance is sensitive to the change of longitudinal wheelbase and the slight change of longitudinal wheelbase will make the vehicle's steering performance change greatly. The influence of lateral wheelbase on steering performance is relatively small compared with that of longitudinal wheelbase.

(ii) Within a certain range, as the radius of the wheel increases, the vehicle needs a wider ground to complete the steering, and the vehicle can obtain a larger body angle deflection at the same time.

(iii) Within a certain range, the vehicle can obtain better steering performance with the increase of friction coefficient. The steering performance of the vehicle is greatly improved with the increase of friction coefficient when the friction coefficient is small. If the friction coefficient is $>0.4$, the improvement of steering performance is limited by increasing the friction coefficient.

5 References

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Fig. 7 Influence of friction coefficient on steering performance
(a) Influence of friction coefficient on $\Delta X_3$, (b) Influence of friction coefficient on $\Delta \phi_3$