Interacting holographic dark energy in Brans-Dicke theory

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We study cosmological application of interacting holographic energy density in the framework of Brans-Dicke cosmology. We obtain the equation of state and the deceleration parameter of the holographic dark energy in a non-flat universe. As system’s IR cutoff we choose the radius of the event horizon measured on the sphere of the horizon, defined as $L = ar(t)$. We find that the combination of Brans-Dicke field and holographic dark energy can accommodate $w_D = -1$ crossing for the equation of state of noninteracting holographic dark energy. When an interaction between dark energy and dark matter is taken into account, the transition of $w_D$ to phantom regime can be more easily accounted for than when resort to the Einstein field equations is made.

I. INTRODUCTION

Recent data from type Ia supernova, cosmic microwave background (CMB) radiation, and other cosmological observations suggest that our universe is currently experiencing a phase of accelerated expansion and nearly three quarters of the universe consists of dark energy with negative pressure \[1\]. Nevertheless, the nature of such a dark energy is still the source of much debate. Despite the theoretical difficulties in understanding dark energy, independent observational evidence for its existence is impressively robust. Explanations have been sought within a wide range of physical phenomena, including a cosmological constant, exotic fields, a new form of the gravitational equation, new geometric structures of spacetime, etc, see \[2\] for a recent review. One of the dramatic candidate for dark energy, that arose a lot of enthusiasm recently, is the so-called “Holographic Dark Energy” (HDE) proposal. This model is based on the holographic principle which states that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume \[3\] and it should be constrained by an infrared cutoff \[4\]. On these basis, Li \[5\] suggested the following constraint on its energy density $\rho_D \leq 3c^2m_p^2/L^2$, the equality sign holding only when the holographic bound is saturated. In this expression $c^2$ is a dimensionless constant, $L$ denotes the IR cutoff radius and $m_p^2 = (8\pi G)^{-1}$ stands for the reduced Plank mass. Based on cosmological state of holographic principle, proposed by Fischler and Susskind \[6\], the

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HDE models have been proposed and studied widely in the literature \cite{7, 8, 9, 10, 11, 12}. The HDE model has also been tested and constrained by various astronomical observations \cite{13, 14} as well as by the anthropic principle \cite{15}. It is fair to claim that simplicity and reasonability of HDE model provides more reliable frame to investigate the problem of dark energy rather than other models proposed in the literature. For example, the coincidence problem can be easily solved in some models of HDE based on the fundamental assumption that matter and HDE do not conserve separately \cite{16}.

On the other side, scalar-tensor theories of gravity have been widely applied in cosmology \cite{17}. Scalar-tensor theories are not new and have a long history. The pioneering study on scalar-tensor theories was done by Brans and Dicke several decades ago who sought to incorporate Mach’s principle into gravity \cite{18}. In recent years this theory got a new impetus as it arises naturally as the low energy limit of many theories of quantum gravity such as superstring theory or Kaluza-Klein theory. Because the holographic energy density belongs to a dynamical cosmological constant, we need a dynamical frame to accommodate it instead of general relativity. Therefore it is worthwhile to investigate the HDE model in the framework of the Brans-Dicke theory. The studies on the HDE model in the framework of Brans-Dicke cosmology have been carried out in \cite{19, 20, 21}. The purpose of the present paper is to construct a cosmological model of late acceleration based on the Brans-Dicke theory of gravity and on the assumption that the pressureless dark matter and HDE do not conserve separately but interact with each other. Given the unknown nature of both dark matter and dark energy there is nothing in principle against their mutual interaction and it seems very special that these two major components in the universe are entirely independent. Indeed, this possibility is receiving growing attention in the literature \cite{22, 23, 24, 25, 26} and appears to be compatible with SNIa and CMB data \cite{27}. On the other hand, although it is believed that our universe is spatially flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large \cite{7}. Besides, some experimental data has implied that our universe is not a perfectly flat universe and recent papers have favored the universe with spatial curvature \cite{28}.

In the light of all mentioned above, it becomes obvious that the investigation on the interacting HED in the framework of non-flat Brans-Dicke cosmology is well motivated. We will show that the equation of state of dark energy can accommodate $w_D = -1$ crossing. As systems’s IR cutoff we shall choose the radius of the event horizon measured on the sphere of the horizon, defined as $L = ar(t)$. Our work differs from that of Ref. \cite{19} in that we take $L = ar(t)$ as the IR cutoff not the Hubble radius $L = H^{-1}$. It also differs from that of Ref. \cite{20}, in that we assume the pressureless
dark matter and HDE do not conserve separately but interact with each other, while the author of [20] assumes that the dark components do not interact with each other.

This paper is outlined as follows: In section II, we consider noninteracting HDE model in the framework of Brans-Dicke cosmology in a non-flat universe. In section III, we extend our study to the case where there is an interaction term between dark energy and dark matter. We summarize our results in section IV.

II. HDE IN BRANSE-DICKE COSMOLOGY

The action of Brans-Dicke theory is given by

$$ S = \int d^4x \sqrt{|g|} \left( -\varphi R + \frac{\omega}{\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + L_M \right). $$

The above action can be transformed into the standard canonical form by re-defining the scalar field $$\varphi$$ and introducing a new field $$\phi$$, in such a way that

$$ \varphi = \frac{\phi^2}{8\omega}. $$

Therefore, in the canonical form, the action of Brans-Dicke theory can be written

$$ S = \int d^4x \sqrt{|g|} \left( -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right), $$

where $$R$$ is the scalar curvature and $$\phi$$ is the Brans-Dicke scalar field. The non-minimal coupling term $$\phi^2 R$$ replaces with the Einstein-Hilbert term $$R/G$$ in such a way that

$$ G_{\text{eff}}^{-1} = \frac{2\pi \phi^2}{\omega}, $$

where $$G_{\text{eff}}$$ is the effective gravitational constant as long as the dynamical scalar field $$\phi$$ varies slowly.

The signs of the non-minimal coupling term and the kinetic energy term are properly adopted to the metric signature. The HDE model will be accommodated in the non-flat Friedmann-Robertson-Walker (FRW) universe which is described by the line element

$$ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), $$

where $$a(t)$$ is the scale factor, and $$k$$ is the curvature parameter with $$k = -1, 0, 1$$ corresponding to open, flat, and closed universes, respectively. A closed universe with a small positive curvature ($$\Omega_k \simeq 0.01$$) is compatible with observations [28]. Varying action (3) with respect to metric (4) for a universe filled with dust and HDE yields the following field equations

$$ \frac{3}{4\omega} \phi^2 \left( H^2 + \frac{k}{a^2} \right) - \frac{1}{2} \dot{\phi}^2 + \frac{3}{2\omega} H \dot{\phi} \phi = \rho_M + \rho_D, $$
\[-\frac{1}{4\omega} \dot{\phi}^2 \left( 2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) - \frac{1}{\omega} H \dot{\phi} \dot{\phi} - \frac{1}{2\omega} \ddot{\phi} \dot{\phi} - \frac{1}{2} \left( 1 + \frac{1}{\omega} \right) \dot{\phi}^2 = p_D, \quad (6)\]

\[\ddot{\phi} + 3H \dot{\phi} - \frac{3}{2\omega} \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) \phi = 0, \quad (7)\]

where \( H = \dot{a}/a \) is the Hubble parameter, \( p_D \) and \( \rho_D \) are, respectively, the energy density and pressure of dark energy. We further assume the energy density of pressureless matter can be separated as \( \rho = \rho_{BM} + \rho_{DM} \), where \( \rho_{BM} \) and \( \rho_{DM} \) are the energy density of baryonic and dark matter, respectively. We also assume the holographic energy density has the following form

\[\rho_D = \frac{3c^2 \phi^2}{4\omega L^2}, \quad (8)\]

where \( \phi^2 = \omega/(2\pi G_{\text{eff}}) \). In the limit of Einstein gravity where \( G_{\text{eff}} \to G \), the above expression reduces to the holographic energy density in standard cosmology

\[\rho_D = \frac{3c^2}{8\pi G L^2} = \frac{3c^2 m_p^2}{L^2}, \quad (9)\]

The radius \( L \) is defined as

\[L = ar(t), \quad (10)\]

where the function \( r(t) \) can be obtained from the following relation

\[\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^\infty \frac{dt}{a} = \frac{R_h}{a}. \quad (11)\]

It is important to note that in the non-flat universe the characteristic length which plays the role of the IR-cutoff is the radius \( L \) of the event horizon measured on the sphere of the horizon and not the radial size \( R_h \) of the horizon. Solving the above equation for the general case of the non-flat FRW universe, we have

\[r(t) = \frac{1}{\sqrt{k}} \sin y, \quad (12)\]

where \( y = \sqrt{k} R_h/a \). Now we define the critical energy density, \( \rho_{\text{cr}} \), and the energy density of the curvature, \( \rho_k \), as

\[\rho_{\text{cr}} = \frac{3\phi^2 H^2}{4\omega}, \quad \rho_k = \frac{3k \phi^2}{4\omega a^2}. \quad (13)\]

We also introduce, as usual, the fractional energy densities such as

\[\Omega_M = \frac{\rho_M}{\rho_{\text{cr}}} = \frac{4\omega \rho_M}{3\phi^2 H^2}, \quad (14)\]

\[\Omega_k = \frac{\rho_k}{\rho_{\text{cr}}} = \frac{k}{H^2 a^2}, \quad (15)\]

\[\Omega_D = \frac{\rho_D}{\rho_{\text{cr}}} = \frac{c^2}{H^2 L^2}. \quad (16)\]
For latter convenience we rewrite Eq. (16) in the form

$$HL = \frac{c}{\sqrt{\Omega_D}}.$$  \hspace{1cm} (17)

Taking derivative with respect to the cosmic time $t$ from Eq. (10) and using Eqs. (12) and (17) we obtain

$$\dot{L} = HL + a\dot{r}(t) = \frac{c}{\sqrt{\Omega_D}} - \cos y. \hspace{1cm} (18)$$

Consider the FRW universe filled with dark energy and pressureless matter which evolves according to their conservation laws

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = 0, \hspace{1cm} (19)$$

$$\dot{\rho}_M + 3H\rho_M = 0, \hspace{1cm} (20)$$

where $w_D$ is the equation of state parameter of dark energy. We shall assume that Brans-Dicke field can be described as a power law of the scale factor, $\phi \propto a^\alpha$. A case of particular interest is that when $\alpha$ is small whereas $\omega$ is high so that the product $\alpha \omega$ results of order unity \[19\].

This is interesting because local astronomical experiments set a very high lower bound on $\omega$ \[30\]; in particular, the Cassini experiment implies that $\omega > 10^4$ \[31, 32\]. Taking the derivative with respect to time of relation $\phi \propto a^\alpha$ we get

$$\dot{\phi} = \alpha H \phi, \hspace{1cm} (21)$$

$$\ddot{\phi} = \alpha^2 H^2 \phi + \alpha \phi \dot{H}. \hspace{1cm} (22)$$

Taking the derivative of Eq. (8) with respect to time and using Eqs. (18) and (21) we reach

$$\dot{\rho}_D = 2H\rho_D \left( \alpha - 1 + \frac{\sqrt{\Omega_D}}{c} \cos y \right). \hspace{1cm} (23)$$

Inserting this equation in conservation law (19), we obtain the equation of state parameter

$$w_D = \frac{1}{3} - \frac{2\alpha}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y. \hspace{1cm} (24)$$

It is important to note that in the limiting case $\alpha = 0$ ($\omega \to \infty$), the Brans-Dicke scalar field becomes trivial and Eq. (24) reduces to its respective expression in non-flat standard cosmology \[7\]

$$w_D = \frac{1}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y. \hspace{1cm} (25)$$
We will see that the combination of the Brans-Dicke field and HDE brings rich physics. For \( \alpha \geq 0 \), \( w_D \) is bounded from below by

\[
w_D = -\frac{1}{3} - \frac{2\alpha}{3} - \frac{2\sqrt{\Omega_D}}{3c}.
\]

If we take \( \Omega_D = 0.73 \) for the present time and choosing \( c = 1 \), the lower bound becomes \( w_D = -\frac{2\alpha}{3} - 0.9 \). Thus for \( \alpha = 0.15 \) we have \( w_D = -1 \). The cases with \( \alpha > 0.15 \) and \( \alpha < 0.15 \) should be considered separately. In the first case where \( \alpha > 0.15 \) we have \( w_D < -1 \). This is an interesting result and shows that, theoretically, the combination of Brans-Dicke scalar field and HDE can accommodate \( w_D = -1 \) crossing for the equation of state of dark energy. Therefore one can generate phantom-like equation of state from a noninteracting HDE model in the Brans-Dicke cosmology framework. This is in contrast to the general relativity where the equation of state of a noninteracting HDE cannot cross the phantom divide. In the second case where \( 0 \leq \alpha < 0.15 \) we have \( -1 < w_D \leq -0.9 \). Since \( \alpha \approx 1/\omega \) and for \( \omega \geq 10^4 \) the Brans-Dicke theory is consistent with solar system observations, thus practically \( \alpha \approx 10^{-4} \) is compatible with recent cosmological observations which implies \( w_D \approx -0.903 \) for the present time in this model. In both cases discussed above \( w_D < -1/3 \) and the universe undergoing a phase of accelerated expansion. It is worthwhile to note that since \( \alpha \approx 1/\omega \) and \( \omega > 10^4 \), therefore for all practical purposes Brans-Dicke theory reduces to Einstein gravity as one can see from the above discussion.

For completeness, we give the deceleration parameter

\[
q = -\frac{\dot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2},
\]

which combined with the Hubble parameter and the dimensionless density parameters form a set of useful parameters for the description of the astrophysical observations. Dividing Eq. (6) by \( H^2 \), and using Eqs. (3), (17), (21) and (22), we find

\[
q = \frac{1}{2\alpha + 2} \left[ (2\alpha + 1)^2 + 2B+1 + \Omega_k + 3\Omega_D w_D \right].
\]

Substituting \( w_D \) from Eq. (24), we get

\[
q = \frac{1}{2\alpha + 2} \left[ (2\alpha + 1)^2 + 2\alpha(\alpha\omega - 1) + \Omega_k - (2\alpha + 1)\Omega_D - \frac{2}{c}\Omega_D^{3/2}\cos y \right].
\]

If we take \( \Omega_D = 0.73 \) and \( \Omega_k \approx 0.01 \) for the present time and choosing \( c = 1 \), \( \alpha\omega \approx 1 \), \( \omega = 10^4 \) and \( \cos y \approx 1 \), we obtain \( q = -0.48 \) for the present value of the deceleration parameter which is in good agreement with recent observational results. When \( \alpha \to 0 \), Eq. (29) restores the deceleration parameter for HDE model in Einstein gravity [25]

\[
q = \frac{1}{2} (1 + \Omega_k) - \frac{\Omega_D}{2} - \frac{\Omega_D^{3/2}}{c}\cos y.
\]
III. INTERACTING HDE IN BRANSE-DICKE COSMOLOGY

In this section we extend our study to the case where both dark components— the pressureless dark matter and the HDE— do not conserve separately but interact with each other. Although at this point the interaction may look purely phenomenological but different Lagrangians have been proposed in support of it [35]. Besides, in the absence of a symmetry that forbids the interaction there is nothing, in principle, against it. Further, the interacting dark energy has been investigated at one quantum loop with the result that the coupling leaves the dark energy potential stable if the former is of exponential type but it renders it unstable otherwise [36]. Therefore, microphysics seems to allow enough room for the coupling. With the interaction between the two dark constituents of the universe, we explore the evolution of the universe. The total energy density satisfies a conservation law

\[
\dot{\rho} + 3H(\rho + p) = 0.
\]  

(31)

where \( \rho = \rho_M + \rho_D \) and \( p = p_D \). However, since we consider the interaction between dark energy and dark matter, \( \rho_{DM} \) and \( \rho_D \) do not conserve separately. They must rather enter the energy balances [16]

\[
\dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \quad (32)
\]

\[
\dot{\rho}_{DM} + 3H\rho_{DM} = Q, \quad (33)
\]

\[
\dot{\rho}_{BM} + 3H\rho_{BM} = 0, \quad (34)
\]

where we have assumed the baryonic matter does not interact with dark energy. Here \( Q \) denotes the interaction term and we take it as \( Q = 3b^2H(\rho_{DM} + \rho_D) \) with \( b^2 \) is a coupling constant. This expression for the interaction term was first introduced in the study of the suitable coupling between a quintessence scalar field and a pressureless cold dark matter field [22, 23]. The choice of the interaction between both components was meant to get a scaling solution to the coincidence problem such that the universe approaches a stationary stage in which the ratio of dark energy and dark matter becomes a constant. In the context of HDE models, this form of interaction was derived from the choice of Hubble scale as the IR cutoff [16].

Combining Eqs. (13) and (21) with the first Friedmann equation (5), we can rewrite this equation as

\[
\rho_{cr} + \rho_k = \rho_{BM} + \rho_{DM} + \rho_D + \rho_\phi, \quad (35)
\]
where we have defined
\[
\rho_\phi \equiv \frac{1}{2} \alpha H^2 \phi^2 \left( \alpha - \frac{3}{\omega} \right). \tag{36}
\]
Dividing Eq. (35) by \( \rho_{cr} \), this equation can be written as
\[
\Omega_{BM} + \Omega_{DM} + \Omega_D + \Omega_\phi = 1 + \Omega_k, \tag{37}
\]
where
\[
\Omega_\phi = \frac{\rho_\phi}{\rho_{cr}} = -2\alpha \left( 1 - \frac{\alpha \omega}{3} \right). \tag{38}
\]
Thus, we can rewrite the interaction term \( Q \) as
\[
Q = 3b^2 H (\rho_{DM} + \rho_D) = 3b^2 H \rho_D (1 + r), \tag{39}
\]
where \( r = \rho_{DM}/\rho_D \) is the ratio of the energy densities of two dark components,
\[
r = \frac{\Omega_{DM}}{\Omega_D} = -1 + \frac{1}{\Omega_D} \left[ 1 + \Omega_k - \Omega_{BM} + 2\alpha \left( 1 - \frac{\alpha \omega}{3} \right) \right]. \tag{40}
\]
Using the continuity equation (34), it is easy to show that
\[
\Omega_{BM} = \Omega_{BM0} a^{-3} = \Omega_{BM0} (1 + z)^3, \tag{41}
\]
where \( \Omega_{BM0} \approx 0.04 \) is the present value of the fractional energy density of the baryonic matter and \( z = a^{-1} - 1 \) is the red shift parameter. Inserting Eqs. (23), (39) and (40) in Eq. (32) we obtain
the equation of state parameter
\[
w_D = -\frac{1}{3} - \frac{2\alpha}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y - b^2 \Omega_D^1 \left[ 1 + \Omega_k - \Omega_{BM} + 2\alpha \left( 1 - \frac{\alpha \omega}{3} \right) \right]. \tag{42}
\]
If we define, following [12], the effective equation of state as
\[
w_{eff} = w_D + \frac{\Gamma}{3H}, \tag{43}
\]
where \( \Gamma = 3b^2 (1 + r)H \). Then, the continuity equation (32) for the dark energy can be written in the standard form
\[
\dot{\rho}_D + 3H \rho_D (1 + w_{eff}^D) = 0. \tag{44}
\]
Substituting Eq. (42) in Eq. (43), we find
\[
w_{eff}^D = -\frac{1}{3} - \frac{2\alpha}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y, \tag{45}
\]
From Eq. (45) we see that with the combination of Brans-Dicke field and HDE, the effective equation of state, \( w_{\text{eff}}^D \), can cross the phantom divide. For instance, taking \( \Omega_D = 0.73 \) for the present time and \( c = 1 \), the lower bound of Eq. (45) is \( w_{\text{eff}}^D = -\frac{2\alpha}{3} - 0.9 \). Thus for \( \alpha > 0.15 \) we have \( w_{\text{eff}}^D < -1 \). Therefore, the Brans-Dicke field plays a crucial role in determining the behaviour of the effective equation of state of interacting HDE. It is important to note that in standard HDE (\( \alpha = 0 \)) it is impossible to have \( w_{\text{eff}}^D \) crossing \(-1\) [12]. Returning to the general case (42), we see that when the interacting HDE is combined with the Brans-Dicke scalar field the transition from normal state where \( w_D > -1 \) to the phantom regime where \( w_D < -1 \) for the equation of state of interacting dark energy can be more easily achieved for than when resort to the Einstein field equations is made. In the absence of the Brans-Dicke field (\( \alpha = 0 \)), Eq. (42) restores its respective expression in non-flat standard cosmology [25]

\[
w_D = -\frac{1}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y - b^2 \Omega_D^{-1} (1 + \Omega_k - \Omega_{BM}) .
\] (46)

Next, we examine the deceleration parameter, \( q = -\ddot{a}/(aH^2) \). Substituting \( w_D \) from Eq. (42) in Eq. (28), one can easily show

\[
q = \frac{1}{2\alpha + 2} \left[ (2\alpha + 1)^2 + 2\alpha(\alpha\omega - 1) + \Omega_k - (2\alpha + 1)\Omega_D - \frac{2}{c} \Omega_D^{3/2} \cos y - 3b^2 \left( 1 + \Omega_k - \Omega_{BM} + 2\alpha \left( 1 - \frac{\alpha\omega}{3} \right) \right) \right].
\] (47)

If we take \( \Omega_D = 0.73 \) and \( \Omega_k \approx 0.01 \) for the present time and \( c = 1, \alpha \approx 1/\omega, \omega = 10^4, \cos y \approx 1, \Omega_{BM} \approx 0.04 \) and \( b = 0.1 \), we obtain \( q = -0.5 \) which is again compatible with recent observational data [34]. When \( \alpha = 0 \), Eq. (47) reduces to the deceleration parameter of the interacting HDE in Einstein gravity [25]

\[
q = \frac{1}{2} (1 + \Omega_k) - \frac{\Omega_D}{2} - \frac{\Omega_D^{3/2}}{c} \cos y - \frac{3b^2}{2} \left( 1 + \Omega_k - \Omega_{BM} \right).
\] (48)

We can also obtain the evolution behavior of the dark energy. Taking the derivative of Eq. (16) and using Eq. (18) and relation \( \dot{\Omega}_D = H\Omega'_D \), we find

\[
\Omega'_D = 2\Omega_D \left( -\frac{\dot{H}}{H^2} - 1 + \frac{\sqrt{\Omega_D}}{c} \cos y \right),
\] (49)

where the dot is the derivative with respect to time and the prime denotes the derivative with respect to \( x = \ln a \). Using relation \( q = -1 - \frac{\dot{H}}{H^2} \), we have

\[
\Omega'_D = 2\Omega_D \left( q + \frac{\sqrt{\Omega_D}}{c} \cos y \right),
\] (50)
where \( q \) is given by Eq. (47). This equation describes the evolution behavior of the interacting HDE in Brans-Dicke cosmology framework. In the limit of standard cosmology (\( \alpha = 0 \)), Eq. (50) reduces to its respective expression in HDE model \[25]\]

\[
\Omega_D' = \Omega_D \left[ (1 - \Omega_D) \left( 1 + \frac{2\sqrt{\Omega_D}}{c} \cos y \right) - 3b^2(1 + \Omega_k - \Omega_{BM}) + \Omega_k \right].
\]

(51)

For flat universe, \( \Omega_k = 0 \), and Eq. (51) recovers exactly the result of \[24].\]

### IV. SUMMARY AND DISCUSSION

In summary, we studied the interacting holographic model of dark energy in the framework of Brans-Dicke cosmology where the HDE density \( \rho_D = 3c^2/(8\pi GL^2) \) is replaced with \( \rho_D = 3c^2\phi^2/(4\omega L^2) \). Here \( \phi^2 = \omega/(2\pi G_{\text{eff}}) \), where \( G_{\text{eff}} \) is the time variable Newtonian constant. In the limit of Einstein gravity, \( G_{\text{eff}} \to G \). With this replacement in Brans-Dicke theory, we found that the accelerated expansion will be more easily achieved for than when the standard HDE is considered. We obtained the equation of state and the deceleration parameter of the HDE in a non-flat universe enclosed by the event horizon measured on the sphere of the horizon with radius \( L = ar(t) \). Interestingly enough, we found that the combination of Brans-Dicke and HDE can accommodate \( w_D = -1 \) crossing for the equation of state of noninteracting dark energy. For instance, taking \( \Omega_D = 0.73 \) for the present time and \( c = 1 \), the lower bound of \( w_D \) becomes \( w_D = -\frac{2\alpha}{3} - 0.9 \). Thus for \( \alpha > 0.15 \) we have \( w_D < -1 \). This is in contrast to Einstein gravity where the equation of state of noninteracting HDE cannot cross the phantom divide \( w_D = -1 \) \[5\].

When the interaction between dark energy and dark matter is taken into account, the transition from normal state where \( w_D > -1 \) to the phantom regime where \( w_D < -1 \) for the equation of state of HDE can be more easily accounted for than when resort to the Einstein field equations is made.

In Brans-Dicke theory of interacting HDE, the properties of HDE is determined by the parameters \( c, b \) and \( \alpha \) together. These parameters would be obtained by confronting with cosmic observational data. In this work we just restricted our numerical fitting to limited observational data. Giving the wide range of cosmological data available, in the future we expect to further constrain our model parameter space and test the viability of our model. The issue is now under investigation and will be addressed elsewhere.
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[1] A. G. Riess, et al., Astron. J. 116, 1009 (1998);
S. Perlmutter, et al., Astrophys. J. 517, 565 (1999);
S. Perlmutter, et al., Astrophys. J. 598, 102 (2003);
P. de Bernardis, et al., Nature 404, 955 (2000).
[2] T. Padmanabhan, Phys. Rep. 380 (2003) 235;
P. J. E. Peebles, B. Ratra, Rev. Mod. Phys. 75 (2003) 559;
E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D 15 (2006) 1753.
[3] G. t Hooft, {	exttt{gr-qc/9310026}}
L. Susskind, J. Math. Phys. 36 (1995) 6377.
[4] A. Cohen, D. Kaplan, A. Nelson, Phys. Rev. Lett. 82 (1999) 4971.
[5] M. Li, Phys. Lett. B 603 (2004) 1.
[6] W. Fischler, L. Susskind, {	exttt{hep-th/9806039}}
[7] Q. G. Huang, M. Li, JCAP 0408 (2004) 013.
[8] S. D. H. Hsu, Phys. Lett. B 594 (2004) 13.
[9] E. Elizalde, S. Nojiri, S.D. Odintsov, P. Wang, Phys. Rev. D 71 (2005) 103504;
B. Guberina, R. Horvat, H. Stefancic, JCAP 0505 (2005) 001;
B. Guberina, R. Horvat, H. Nikolic, Phys. Lett. B 636 (2006) 80;
H. Li, Z. K. Guo, Y. Z. Zhang, Int. J. Mod. Phys. D 15 (2006) 869;
Q. G. Huang, Y. Gong, JCAP 0408 (2004) 006;
J. P. B. Almeida, J. G. Pereira, Phys. Lett. B 636 (2006) 75;
Y. Gong, Phys. Rev. D 70 (2004) 064029;
B. Wang, E. Abdalla, R. K. Su, Phys. Lett. B 611 (2005) 21.
[10] M. R. Setare, S. Shafei, JCAP 09 (2006) 011;
M. R. Setare, E. C. Vagenas, Phys. Lett. B 666 (2008) 111;
H. M. Sadjadi, arXiv:0902.2462
M. R. Setare, E. N. Saridakis, Phys. Lett. B 671 (2009) 331;
M. Jamil, E. N. Saridakis, M. R. Setare, arXiv: 0906.2847.
[11] M. R. Setare, Eur. Phys. J. C 50 (2007) 991;
M. R. Setare, JCAP 0701 (2007) 023;
M. R. Setare, Phys. Lett. B 654 (2007) 1;
M. R. Setare, Phys. Lett. B 642 (2006) 421.

[12] M. R. Setare, Phys. Lett. B 642 (2006) 1.

[13] X. Zhang, F. Q. Wu, Phys. Rev. D 72 (2005) 043524;
    X. Zhang, F. Q. Wu, Phys. Rev. D 76 (2007) 023502;
    Q. G. Huang, Y. G. Gong, JCAP 0408 (2004) 006;
    K. Enqvist, S. Hannestad, M. S. Sloth, JCAP 0502 (2005) 004;
    J. Y. Shen, B. Wang, E. Abdalla, R. K. Su, Phys. Lett. B 609 (2005) 200.

[14] B. Feng, X. Wang, X. Zhang, Phys. Lett. B 607 (2005) 35;
    H. C. Kao, W. L. Lee, F. L. Lin, Phys. Rev. D 71 (2005) 123518;
    J. Y. Shen, B. Wang, E. Abdalla, R. K. Su, Phys. Lett. B 609 (2005) 200.

[15] Q. G. Huang, M. Li, JCAP 0503 (2005) 001.

[16] D. Pavon, W. Zimdahl, Phys. Lett. B 628 (2005) 206.

[17] V. Faraoni, Cosmology in Scalar-Tensor Gravity, Kluwer, Boston, (2004);
    E. Elizalde, S. Nojiri, S. D. Odintsov, P. Wang, Phys. Rev. D 71 (2005) 103504;
    S. Nojiri, S. D. Odintsov, Gen. Relativ. Gravit. 38 (2006) 1285;
    R. Gannouji, et al., JCAP 0609 (2006) 016.

[18] C. Brans and R. H. Dicke, Phys. Rev. 124 (1961) 925.

[19] N. Banerjee, D. Pavon, Phys. Lett. B 647 (2007) 447.

[20] M. R. Setare, Phys. Lett. B 644 (2007) 99.

[21] H. Kim, H. W. Lee, Y. S. Myung Phys. Lett. B 628 (2005) 11;
    Y. Gong, Phys. Rev. D 70 (2004) 064029;
    B. Nayak, L. P. Singh, arXiv:0803.2930;
    L. Xu, J. Lu and W. Li, arXiv:0804.2925;
    L. Xu, J. Lu and W. Li, arXiv:0905.4174.

[22] L. Amendola, Phys. Rev. D 60 (1999) 043501;
    L. Amendola, Phys. Rev. D 62 (2000) 043511;
    L. Amendola and C. Quercellini, Phys. Rev. D 68 (2003) 023514;
    L. Amendola and D. Tocchini-Valentini, Phys. Rev. D 64 (2001) 043509;
    L. Amendola and D. T. Valentini, Phys. Rev. D 66 (2002) 043528.

[23] W. Zimdahl and D. Pavon, Phys. Lett. B 521 (2001) 133;
    W. Zimdahl and D. Pavon, Gen. Rel. Grav. 35 (2003) 413;
    L. P. Chimento, A. S. Jakubi, D. Pavon and W. Zimdahl, Phys. Rev. D 67 (2003) 083513.

[24] B. Wang, Y. Gong and E. Abdalla, Phys. Lett. B 624 (2005) 141.

[25] B. Wang, C. Y. Lin and E. Abdalla, Phys. Lett. B 637 (2005) 357.

[26] B. Wang, C. Y. Lin, D. Pavon and E. Abdalla, Phys. Lett. B 662 (2008) 1;
    W. Zimdahl and D. Pavon, Class. Quantum Grav. 24 (2007) 5461;
    D. Pavon and A. A. Sen, arXiv:0811.1446.
[27] G. Olivares, F. Atrio, D. Pavon, Phys. Rev. D 71 (2005) 063523.

[28] D. N. Spergel, Astrophys. J. Suppl. 148 (2003) 175;
  C. L. Bennett, et al., Astrophys. J. Suppl. 148 (2003) 1;
  M. Tegmark, et al., Phys. Rev. D 69 (2004) 103501;
  U. Seljak, A. Slosar, P. McDonald, JCAP 0610 (2006) 014;
  D. N. Spergel, et al., Astrophys. J. Suppl. 170 (2007) 377.

[29] M. Arik, M.C. Calik, Mod. Phys. Lett. A 21 (2006) 1241;
  M. Arik, M. C. Calik and M. B. Sheftel, gr-qc/0604082.

[30] C. M. Will, Theory and Experiment in Gravitational Physics,
  Cambridge University Press, Cambridge, (1993).

[31] B. Bertotti, L. Iess and P. Tortora, Nature, 425 (2003) 374.

[32] V. Acquaviva, L. Verde, JCAP 12 (2007) 001.

[33] Since $\omega \geq 10^4$ we find that $\alpha \approx 1/\omega = 10^{-4}$, thus Eq. (26) reduces practically to
  $w_D = -\frac{1}{3}(1 + 2\alpha) - \frac{2\sqrt{\Omega_D c}}{3c} \approx -\frac{1}{3} - \frac{2\sqrt{\Omega_D c}}{3c}$ which is exactly the Li’s result. Thus, the Li’s argument [5] in favor of $c = 1$ holds
  here.

[34] R.A. Daly et al., Astrophysics J. 677 (2008) 1.

[35] S. Tsujikawa, M. Sami, Phys. Lett. B 603 (2004) 113.

[36] M. Doran, J. Jackel, Phys. Rev. D 66 (2002) 043519.