Dimension-5 operators and the unification condition in SO(10) and E(6)

Joydeep Chakrabortty\textsuperscript{1} and Amitava Raychaudhuri\textsuperscript{1,2}

\textsuperscript{1)} Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211 019, India
\textsuperscript{2)} Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700 009, India

Abstract

Effective dimension-5 operators which modify the gauge kinetic term in Grand Unified Theories may arise as a consequence of quantum gravity or string compactification. We exhaustively calculate the modification of the gauge unification condition due to such operators for all viable rank-preserving symmetry breakings of SO(10) and E(6) grand unified models.

Key Words: Grand Unified Theories

I Introduction

The Standard Model (SM) is based on the gauge symmetry $G_{SM} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ which has three independent couplings $g_3, g_2,$ and $g_1$ of different strength. If the strong and electroweak interactions merge at high energy within a grand unified theory (GUT) framework then at that stage they will be described by a single gauge coupling $g_{GUT}$. This will necessitate a correlation among the strengths of the three forces measured at lower energies ($\sim M_Z$). GUTs would also interrelate fermion masses because quark-lepton unification is an essential ingredient of the programme [1, 2].

The current low energy measured values of the gauge couplings are no longer consistent with unification in a minimal $SU(5)$ GUTs nor is the key prediction of proton decay so far observed. Further, in $SU(5)$, the SM Higgs doublet ($H_2$) sits in a GUT multiplet with a colour triplet ($H_3$). While $H_2$ should be at the electroweak scale, $H_3$ needs to be very heavy to avoid rapid proton decay – the so-called doublet-triplet splitting. Another shortcoming of the minimal GUT concerns the fermion mass relations. Though $b - \tau$ unification occurs naturally, the light quark-lepton mass ratios, such as $m_s/m_\mu$, do not follow. This has encouraged interest in GUT models based on larger groups such as $SO(10)$ and $E(6)$ which provide the option of a higher unification scale suppressing proton decay along with a richer Higgs structure and several intermediate symmetry-breaking mass scales leading to testable consequences at colliders or via the observation of $n - \bar{n}$ oscillations. Needless to say, unification of all interactions with gravity is the final objective and grand unification is a step in this direction.

In the absence of a full quantum theory of gravity it has been a useful exercise to explore some of its implications on grand unification through higher dimension gauge invariant effective operators, suppressed by powers of the Planck mass, $M_{Pl}$. In string theory, similar effective operators could arise from string compactification, $M_{Pl}$ being then replaced by the compactification scale $M_c$.
In this work we focus on the corrections to the gauge kinetic term:

\[ \mathcal{L}_{\text{kin}} = -\frac{1}{4c} \text{Tr}(F_{\mu\nu} F^{\mu\nu}). \]  

where \( F_{\mu\nu} = \sum_i \lambda_i F_i^{\mu\nu} \) is the gauge field strength tensor with \( \lambda_i \) the matrix representations of the generators of \( G_{\text{GUT}} \) normalized to \( \text{Tr}(\lambda_i \lambda_j) = c \delta_{ij} \). The \( \lambda_i \) are often chosen in the fundamental representation with \( c = 1/2 \). In the following, other representations are sometimes found convenient.

The dimension-5 term from quantum gravity (or string compactification) examined here is \[ \mathcal{L}_{\text{dim-5}} = -\frac{\eta}{M_{\text{Pl}}} \left[ \frac{1}{4c} \text{Tr}(F_{\mu\nu} \Phi_D F^{\mu\nu}) \right] \]

where \( \Phi_D \) stands for the \( D \)-component scalar multiplet and \( \eta \) parametrises the strength of this interaction. A gauge invariant of the form in eq. (2) is allowed if \( \Phi_D \) is in any representation included in the symmetric product of two adjoint representations of the group. For example, in \( SO(10) \), \( (45 \otimes 45)_{\text{sym}} = 1 \oplus 54 \oplus 210 \oplus 770 \) and \( \Phi_D \) may be 54- or 210- or 770-dimensional.

When \( \Phi_D \) develops a vacuum expectation value (vev) \( v_D \), which sets the scale of grand unification \( M_X \) and drives the symmetry breaking \[ G_{\text{GUT}} \rightarrow G_1 \otimes G_2 \otimes \ldots G_n, \]

an effective gauge kinetic term is generated from eq. (2). Depending on the structure of the vev, this additional contribution, in general, will be different for the kinetic terms for the subgroups \( G_1, \ldots, G_n \). After an appropriate scaling of the gauge fields this results in a modification of the gauge coupling unification condition to:

\[ g_1^2(M_X)(1 + \epsilon \delta_1) = g_2^2(M_X)(1 + \epsilon \delta_2) = \ldots = g_n^2(M_X)(1 + \epsilon \delta_n), \]

wherein the \( \delta_i, i = 1, 2, \ldots, n \), arise from eq. (2) and \( \epsilon = \eta v_D/2 M_{\text{Pl}} \sim O(M_X/M_{\text{Pl}}) \). Thus, the presence of the dimension-5 terms in the Lagrangian modify the usual boundary conditions on gauge couplings, namely, that they are expected to unify at \( M_X \). The \( \delta_i \) were calculated for a set of phenomenologically interesting breaking sequences for \( SU(5), SO(10), \) and \( E(6) \) in our earlier work \[4\]. It is not impossible that the modification in eq. (3) will enable the unification programme to succeed with the current low energy values of the coupling constants. This was also examined in the context of the above GUT groups \[1\] to \[5\]. In supersymmetric GUTs, the ratio of these \( \delta_i \) determine the non-universal gaugino mass ratios \[1\] to \[7\] whose detectability at high energy colliders has been investigated in detail.

In earlier works the factors \( \delta_i \) arising from the dim-5 operators were obtained for the \( SU(5), SO(10), \) and \( E(6) \) GUT groups for some selected breaking patterns \[2\] to \[4\]. For \( SO(10) \) and \( E(6) \) those chains which admit a left-right symmetry were picked. In this note we complete the exercise; we work out the unification conditions in the presence of dim-5 operators for \textit{all} phenomenologically viable symmetry descendents of the parent group, which could be either \( SO(10) \) or \( E(6) \), so long as the first step in rank preserving. Thus for \( SO(10) \) the options for the first step of symmetry breaking are \( SU(5) \otimes U(1)_X, SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \) and \( SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \). For \( E(6) \) these could be any one of \( SU(3)_c \otimes SU(3)_L \otimes SU(3)_R, SU(2) \otimes SU(6), SO(10) \otimes U(1)_\eta, SU(5) \otimes U(1)_\xi \otimes U(1)_\eta, SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_\xi \otimes U(1)_\eta \). This work provides a compendium of the results for all options. After noting the known consequences for \( SU(5) \), we present the results for the different alternatives possible with \( SO(10) \) and \( E(6) \) in the succeeding sections. We end with our conclusions. The detailed forms of the vevs are relegated to an Appendix.

\[1\] Since \( \Phi_D \) arises from the symmetric product of two adjoint representations the symmetry breaking is rank preserving.

\[2\] The \( \delta_i \) presented here differ from our earlier results in the magnitudes. The relative values are unchanged.
### II SU(5) GUT

The case of SU(5) has been discussed earlier in the literature \[3, 4\]. For completeness we summarise the results here. \( \Phi_D \) can be in the 24-, 75- or 200-dimensional representation of SU(5) and the symmetry is broken to SU(3)_c \( \otimes \) SU(2)_L \( \otimes \) U(1)_Y.

The procedure to obtain these results \[4\] is to express \( \langle \Phi_D \rangle \) as a diagonal matrix of dimensionality of some SU(5) irreducible representation. From the G_{SM} structure of this representation, the \( \delta_i \) can be read off. We list the \( \langle \Phi_D \rangle \) for the various cases in the Appendix (see sec. A.1).

The \( \delta_i \) arising in the different cases are listed in Table 1.

| SU(5) Representations | \( \delta_1 \) | \( \delta_2 \) | \( \delta_3 \) | \( N \) |
|-----------------------|--------------|--------------|--------------|--------|
| 24                    | 1            | 3            | -2           | 2/\sqrt{15} |
| 75                    | 5            | -3           | -1           | 8/15 \sqrt{3} |
| 200                   | 10           | 2            | 1            | 1/35 \sqrt{21} |

Table 1: Effective contributions (see eq. (3)) to gauge kinetic terms from different Higgs representations in eq. (2) for SU(5) \( \rightarrow \) SU(3)_c \( \otimes \) SU(2)_L \( \otimes \) U(1)_Y. \( N \) is an overall normalisation which has been factored out from the \( \delta_i \).

### III SO(10) GUT

SO(10) \[8\] is now the widely preferred model for grand unification, offering the option of descending to G_{SM} through a left-right symmetric route \[9\] – the intermediate Pati-Salam \( G_{PS} \equiv SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \) – or via SU(5) \( \otimes \) U(1)_Y or in one step to SU(3)_c \( \otimes \) SU(2)_L \( \otimes \) U(1)_Y \( \otimes \) U(1)_X. The effect of dimension-5 interactions in the first case has been reported upon before \[4\] and here we will only recapitulate those results and lay primary emphasis on the other options.

#### III.1 SO(10) \( \rightarrow \) SU(5) \( \otimes \) U(1)

Under SU(5) \( \otimes \) U(1)_X the SO(10) spinorial representation decomposes\[3\] as follows: 16 \( \equiv \) (1,-5) + (5,3) + (10,-1). The SM families belong to this representation. The particle assignments within the 16-plet can be chosen in two distinct ways with different physics consequences: (a) conventional SU(5): U(1)_X commutes with the SM, so the low scale hypercharge (U(1)_Y) is the same as the U(1)_Y in SU(5); e.g., for the (5,3) multiplet \( T_{Y'} \equiv \sqrt{3/5} \text{ diag}(1/3, 1/3, 1/3, -1/2, -1/2) \). The SM generators are entirely within the SU(5) and a singlet under it is uncharged. Therefore, the (1,-5) submultiplet has to be identified with the neutral member in the 16-plet, the \( \nu_i^c \) (\( i = 1,2,3 \)). The other option is (b) flipped SU(5): Here U(1)_Y' and U(1)_X combine to give U(1)_Y: \( T_{Y} = -(2\sqrt{5} T_X + T_{Y'})/5 \) \[10\]. The difference can be illustrated using the (5,3) multiplet. For it the charges are obtained by multiplying the displayed quantum numbers by a factor of \( \frac{1}{2\sqrt{5}} \).
Table 2: Effective contributions (see eq. (3)) to gauge kinetic terms from different Higgs representations in eq. (2) for $SO(10) \to SU(5) \otimes U(1)$. $N$ is an overall normalisation which has been factored out from the $\delta_i$.

| $SO(10)$ Representations | $\delta_5$ | $\delta_1$ | $N$ |
|--------------------------|-----------|------------|-----|
| 210                      | -1        | 4          | $1/4\sqrt{5}$ |
| 770                      | 1         | 16         | $-1/24\sqrt{5}$ |

$U(1)_Y$ assignment is, as before, $T_Y = \sqrt{\frac{3}{5}}\text{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2})$ while the normalised $U(1)_X$ is $\frac{3}{2\sqrt{10}}$ so that $T_Y = \sqrt{\frac{3}{5}}\text{diag}(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{1}{2}, \frac{1}{2})$. Thus, this submultiplet now contains $(u^c_i, L_i)$ rather than the usual $(d^c_i, L_i)$. The $(1,-5)$ state is $SU(3)_c \otimes SU(2)_L$ singlet but carries a non-zero hypercharge, $Y = 1$. The only particle that satisfies this requirement is $l^c_i$.

The complete particle assignments for the first generation in the two options are:

(a) For conventional $SU(5)$

$$ (1, -5) = \nu^c_1, \quad (5, 3) = (d^c_1, l_1), \quad (10, -1) = (q_1, u^c_1, e^c_1), $$

and (b) for flipped $SU(5)$:

$$ (1, -5) = e^c_1, \quad (5, 3) = (u^c_1, l_1), \quad (10, -1) = (q_1, d^c_1, \nu^c_1), $$

where $q$ and $l$ are respectively the left-handed quark and lepton doublets, $u^c$, $d^c$, $e^c$, and $\nu^c$ are the $CP$ conjugated states corresponding to the right-handed up-type quark, down-type quark, lepton, and neutrino, respectively.

In $SO(10)$ GUT, at the unification scale one has $g_5 = g_1$. The presence of any dim-5 effective interactions of the form of eq. (2) will affect this relation generating corrections as shown in eq. (3) which in this case will involve two parameters $\delta_5$ and $\delta_1$.

As noted earlier, $\Phi_D$ can be chosen only in the 54, 210, and 770-dimensional representations. Of these, the 54 does not have an $SU(5) \otimes U(1)$ singlet. So, only the 210- and 770-dimensional cases need examination.

Using $(16 \otimes 16) = 1 \oplus 45 \oplus 210$, $<\Phi_{210}>$ can be expressed as a 16-dimensional traceless diagonal matrix. The form of this $vev$ for this symmetry breaking is given in (A.9). It yields $\delta_5 = -\frac{1}{4\sqrt{5}}$ and $\delta_1 = \frac{1}{\sqrt{3}}$.

In a similar fashion the $vev$ of $\Phi_{770}$ can be written as the $45 \times 45$ diagonal traceless matrix in (A.11). This results in $\delta_5 = -\frac{1}{24\sqrt{5}}$, $\delta_1 = -\frac{2}{3\sqrt{5}}$.

The results for this chain of symmetry breaking are summarised in Table 2. The $\delta_i$ are completely group theoretic in nature and obviously do not depend on whether the particle assignments follow the conventional or flipped $SU(5)$. 
III.2 \( \text{SO}(10) \rightarrow \text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \otimes \text{U}(1)_X \)

The unification condition in the presence of dimension-5 effective interactions of the form of eq. (2) will now involve the parameters \( \delta_i \) \((i = 1, 2, 3)\) as for \( \text{SU}(5) \) and an additional one \( \delta_{1X} \).

In order to break \( \text{SO}(10) \) directly to \( \mathcal{G}_{3211} \equiv \text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \otimes \text{U}(1)_X \) the vev must be a non-singlet not just under \( \text{SO}(10) \) but also under \( \text{SU}(5) \). The decompositions of \( \text{SO}(10) \) representations under \( \text{SU}(5) \otimes \text{U}(1) \) are useful for identifying these vevs. The calculation can be considerably simplified by using the \( \text{SU}(5) \) symmetry breaking patterns at our disposal from sec. II. One simply has to look for 24, 75, and 200 submultiplets within the 54, 210, and 770 \( \text{SO}(10) \) multiplets.

The 54 representation of \( \text{SO}(10) \) has a singlet under \( \mathcal{G}_{3211} \) which is contained in a 24 of \( \text{SU}(5) \). The vev for this case is shown in (A.12) and the contributions to the \( \delta_i \) can be immediately read off from the \( \text{SU}(5) \) result in Table 1. These \( \delta_i \) are listed in Table 3.

Notice, that in this case the effect of dimension-5 terms cannot distinguish between an \( \text{SU}(5) \) theory with \( < \Phi_{24} > \) driving the symmetry breaking and an \( \text{SO}(10) \) one with \( < \Phi_{54} > \). For \( \Phi_{210} \) or \( \Phi_{770} \) the situation is different as they have multiple \( \mathcal{G}_{3211} \) singlet directions.

\( \Phi_{210} \) has three directions which are all singlets under \( \mathcal{G}_{3211} \). Of these one is also an \( \text{SU}(5) \) singlet. In the subspace defined by them, three convenient orthogonal directions can be identified, all singlets under \( \text{U}(1)_X \), and corresponding to 1-, 24-, and 75-directions of the \( \text{SU}(5) \) subgroup. If the vev is along one of these directions it can be simply read off from the results of section II. The vevs corresponding to the 24 and 75 directions are given in (A.13) and (A.14). The \( \delta_i \) derived therefrom are shown in Table 3. In general, we have \( < \Phi_{210} >= \alpha_1 < \Phi_{210,1} > + \alpha_{24} < \Phi_{210,24} > + \alpha_{75} < \Phi_{210,75} > \), where the \( \alpha_i \) are complex numbers and the concomitant \( \delta_i \) will be appropriately weighted combinations of the above results.

\( < \Phi_{770} > \) has four \( \mathcal{G}_{3211} \) invariant directions which can be classified under the \( \text{SU}(5) \) representations 1, 24, 75, and 200. The results for these are also shown in Table 3. Here again, in general, the vev may lie in an arbitrary direction in the space spanned by the four \( \text{SU}(5) \)-identified ones and the resultant \( \delta_i \) can be readily obtained from the above.

| \( \text{SO}(10) \) Representations | \( \delta_1 \) | \( \delta_2 \) | \( \delta_3 \) | \( \delta_{1X} \) | \( N \) |
|-----------------------------------|-------------|-------------|-------------|-------------|-------|
| 54 (24)                           | 1           | 3           | -2          | 0           | \( 1/2\sqrt{15} \) |
| 210 (24)                          | 1           | 3           | -2          | 0           | \( 1/4\sqrt{15} \) |
| 210 (75)                          | 5           | -3          | -1          | 0           | \( 1/12 \) |
| 770 (24)                          | 1           | 3           | -2          | 0           | \( 2\sqrt{15} \) |
| 770 (75)                          | 5           | -3          | -1          | 0           | \( 8/15\sqrt{3} \) |
| 770 (200)                         | 10          | 2           | 1           | 0           | \( -1/8\sqrt{21} \) |

Table 3: Effective contributions (see eq. (3)) to gauge kinetic terms from different Higgs representations in eq. (2) for \( \text{SO}(10) \rightarrow \text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \otimes \text{U}(1)_X \). \( \text{SU}(5) \) subrepresentations are indicated within parentheses. \( N \) is an overall normalisation which has been factored out from the \( \delta_i \).

\( ^4 \)For the \( \text{SU}(5) \) singlet direction the \( \delta_i \) are all equal. A vev in this direction alone will not break \( \text{SO}(10) \) to \( \mathcal{G}_{3211} \).
Table 4: Effective contributions (see eq. (3)) to gauge kinetic terms from different Higgs representations in eq. (2) for \(SO(10) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y\) (flipped \(SU(5))\). \(SU(5)\) subrepresentations are indicated within parentheses. \(N\) is an overall normalisation which has been factored out from the \(\delta_i\).

 Unlike the case of \(<\Phi_{54}\>\) where the singlet direction is unique, the \(<\Phi_{210}\>\) and \(<\Phi_{770}\>\) provide a more general option and therefore the predictions for the \(\delta_i\) are not unique but cover a range. In this sense the model becomes less predictive.

 This route of symmetry breaking of \(SO(10)\) does not admit the flipped \(SU(5)\) option by itself since in that case the \(U(1)_X\) combines with a \(U(1)\) subgroup of \(SU(5)\) to produce \(U(1)_Y\) and thus \(SO(10)\) is broken to \(G_{SM}\), which is of rank 4, not 5. So this symmetry breaking will have to be through some other \(SO(10)\) scalar multiplet. Nonetheless, assuming that such a symmetry breaking is operational, we may ask what would be the impact of the \(vev\)s of \(\Phi_{54}, \Phi_{210}, \text{and} \Phi_{770}\) of this subsection on the unification parameters \(\delta_i, \ i = 1, 2, 3\). Using the \(vev\)s used before and noting that \(U(1)_Y: T_Y = -(2\sqrt{6} T_X + T_{Y'})/5\) one finds the results presented in Table 4.

III.3 \(SO(10) \rightarrow SU(4)_c \otimes SU(2)_L \otimes SU(2)_R\)

 The left-right symmetric route of descent of \(SO(10)\) has been examined by us earlier in detail \[4, 5\]. Here for the sake of completeness we briefly recall the results for the symmetry breaking \(SO(10) \rightarrow G_{PS}\). As noted in eq. (3), we will be interested in the factors \(\delta_{4c}, \delta_{2L}, \text{and} \delta_{2R}\).

 As before, \(\Phi_D\) can be chosen only in the 54-, 210-, and 770-dimensional representations ensuring that \(<\Phi_D>\) leaves \(G_{PS}\) unbroken.

 For \(\Phi_{54}\) the appropriate \(vev\) is given in (A.16) and this results in \(\delta_{4c} = -\frac{1}{\sqrt{15}}\) and \(\delta_{2L} = \delta_{2R} = \frac{3}{2\sqrt{15}}\). Notice that this correction to unification ensures that \(g_{2L}(M_X) = g_{2R}(M_X)\), i.e., D-parity [11] is preserved.

 A 16\times 16 form of \(<\Phi_{210}>\) is given in (A.17) from which one can calculate \(\delta_{4c} = 0\) and \(\delta_{2L} = -\delta_{2R} = \frac{1}{2\sqrt{2}}\). D-parity is broken through \(<\Phi_{210}>\) and thus \(g_{2L}(M_X) \neq g_{2R}(M_X)\) though \(SU(2)_L \otimes SU(2)_R\) remains unbroken at \(M_X\).

\[\text{This also applies to the non-universality options for gaugino masses in supersymmetric theories.}\]
Table 5: Effective contributions (see eq. (5)) to gauge kinetic terms from different Higgs representations in eq. (2) for $SO(10) \rightarrow SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$. $N$ is an overall normalisation which has been factored out from the $\delta_i$.

| $SO(10)$ Representations | $\delta_{4c}$ | $\delta_{2L}$ | $\delta_{2R}$ | $N$ |
|---------------------------|----------------|----------------|----------------|-----|
| 54                        | -2             | 3              | 3              | 1/2\sqrt{15} |
| 210                       | 0              | 1              | -1             | 1/2\sqrt{2}   |
| 770                       | 2              | 5              | 5              | 1/24\sqrt{5}  |

Table 6: Effective contributions (see eq. (3)) to gauge kinetic terms from different Higgs representations in eq. (2) for $E(6) \rightarrow SU(2) \otimes SU(6)$. $N$ is an overall normalisation which has been factored out from the $\delta_i$.

| $E(6)$ Representations | $\delta_2$ | $\delta_6$ | $N$ |
|------------------------|------------|------------|-----|
| 650                    | 5          | -1         | 1/6\sqrt{5} |
| 2430                   | -35        | -9         | 1/12\sqrt{910} |

The final option is $\Phi_{770}$. One can write the vev in terms of a 45-dimensional diagonal traceless matrix and this is given in (A.18). From this one finds $\delta_{4c} = \frac{1}{1/2\sqrt{5}}$ and the D-parity conserving $\delta_{2L} = \delta_{2R} = \frac{5}{24\sqrt{5}}$.

The results for this chain of $SO(10)$ breaking are collected together in Table 5.

IV E(6) GUT

The exceptional group $E(6)$ has been proposed as a viable GUT symmetry [12]. Among its subgroups of same rank are $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$, $SU(3)_c \otimes SU(2)_L \otimes U(1)_L \otimes SU(2)_R \otimes U(1)_R$, $SU(2) \otimes SU(6)$, $SO(10) \otimes U(1)_\eta$, $SU(5) \otimes U(1)_\xi \otimes U(1)_\eta$, and $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_\xi \otimes U(1)_\eta$. All of these intermediate gauge groups accommodate $G_{SM}$ as a subgroup and lead to different low scale phenomenology. We have examined the first option in detail earlier [4, 5]. Here we will concentrate on the remaining breaking chains recalling the first only for the sake of completeness.

For $E(6)$ the adjoint representation is 78-dimensional. Noting that $(78 \otimes 78)_{sym} = 1 \oplus 650 \oplus 2430$ it is clear that $\Phi_D$ can be either 650- or 2430-dimensional. Both of them contain singlets under the above mentioned intermediate gauge groups we are interested in.

IV.1 E(6) $\rightarrow$ SU(2)$\otimes$SU(6)

The inconsistency of the Georgi-Glashow $SU(5)$ model with the proton decay and gauge unification requirements has been a motivation to seek alternative GUT models. $SU(6)$ is one of them. It can naturally guarantee strong-CP invariance and a supersymmetrised version implements doublet-triplet splitting by the missing vev mechanism [13].
The subgroups from the breaking $E(6) \to SU(2) \otimes SU(6)$ have been identified in several physically distinct manners: $SU(2)_R \otimes SU(6)'$, $SU(2)_L \otimes SU(6)'$, and $SU(2)_X \otimes SU(6)$. The results that we discuss are valid irrespective of these alternative interpretations.

The contributions from the 650-dimensional representation for this symmetry breaking chain can be obtained from eq. (A.19). One finds $\delta_2 = \frac{5}{6\sqrt{5}}$ and $\delta_6 = -\frac{1}{6\sqrt{5}}$.

For the 2430-dimensional $E(6)$ representation the vev is given in (A.20). From it we get $\delta_2 = -\frac{35}{12\sqrt{910}}$, $\delta_6 = -\frac{9}{6\sqrt{910}}$. The results for this symmetry breaking chain can be found in Table 6.

### IV.2 $E(6) \to SO(10) \otimes U(1)_\eta$

$E(6)$ contains $SO(10) \otimes U(1)$ as a maximal subgroup. Breaking patterns based on this are well discussed in the literature [12]. Here, we consider the effect of dim-5 operators on the gauge unification condition.

$<\Phi_{650}>$ is given in (A.21). From it one obtains $\delta_{10} = -\frac{1}{6\sqrt{5}}$, $\delta_1 = \frac{5}{6\sqrt{5}}$. Using (A.22) for $<\Phi_{2430}>$ one can similarly get $\delta_{10} = \frac{1}{72\sqrt{26}}$, $\delta_1 = \frac{3}{8\sqrt{26}}$. These results are listed in Table 7.

| $E(6)$ Representations | $\delta_{10}$ | $\delta_1$ | $N$       |
|------------------------|--------------|------------|-----------|
| 650                    | -1           | 5          | 1/6\sqrt{5} |
| 2430                   | 1            | 27         | 1/72\sqrt{26} |

Table 7: Effective contributions (see eq. (3)) to gauge kinetic terms from different Higgs representations in eq. (2) for $E(6) \to SO(10) \otimes U(1)_\eta$. $N$ is an overall normalisation which has been factored out from the $\delta_i$.

### IV.3 $E(6) \to SU(5) \otimes U(1)_\xi \otimes U(1)_\eta$

The results in this case are very similar to that for sec. III.1. There it was noted that the $SO(10)$ 210 and 770 representations contain singlets under $SU(5) \otimes U(1)_X$ and the $\delta_5$ and $\delta_1$ in the two cases were presented in Table 2. These results can be immediately taken over for the current case with the change that the $U(1)_X$ is here termed $U(1)_\xi$ and that $\delta_\eta = 0$ in all cases.

The two relevant representations of $E(6)$ are 650 and 2430. Of these, 650 contains a $(210,0)$ submultiplet under $SO(10) \otimes U(1)_\eta$. So for the 650 the $\delta_i$ will be exactly as for the 210 in Table 2.

The $E(6)$ 2430 representation contains both the $(210,0)$ as well as the $(770,0)$ within it. If the vev is assigned to any one of these directions the resultant $\delta_i$ will be as in the respective case in Table 2. In general, the vev will be a superposition of these two and so the $\delta_i$ will be the appropriately weighted value.

### IV.4 $E(6) \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_\xi \otimes U(1)_\eta$

As for the previous subsection, this alternative can be disposed off straightforwardly using the results of sec. III.2. This time there is one extra step. In sec. III.2 results are presented for the $SO(10)$
Table 8: Effective contributions (see eq. (3)) to gauge kinetic terms from different Higgs representations in eq. (2) for $E(6) \rightarrow SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$. Note that there are two $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ singlet directions in 650. $N$ is an overall normalisation which has been factored out from the $\delta_i$.

| $E(6)$ Representations | $\delta_{3c}$ | $\delta_{3L}$ | $\delta_{3R}$ | $N$ |
|-------------------------|---------------|---------------|---------------|-----|
| 650                     | -2            | 1             | 1             | -1/6$\sqrt{2}$ |
| 650'                    | 0             | 1             | -1            | 1/2$\sqrt{6}$  |
| 2430                    | 1             | 1             | 1             | -1/4$\sqrt{26}$ |

representations 54, 210, and 770. They can be immediately taken over by noting that the $E(6)$ 650 contains (54,0) and (210,0) submultiplets while the 2430 contains (54,0), (210,0), and (770,0).

The main changes compared to sec. III.2 are that the $U(1)_X$ there is called $U(1)_\xi$ here and for all cases $\delta_\xi = \delta_\eta = 0$. For the 650 representation if the $vev$ is chosen along either the (54,0) or the (210,0) directions then the results of Table 3 apply directly. In general, of course, the $\delta_i$ will be a weighted combination of these. Similar conclusions can be drawn about the $< \Phi_{2430} >$ except that here, in general, the $\delta_i$ will be a linear combination of the ones in Table 3.

IV.5 $E(6) \rightarrow SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$

This breaking chain [14] is exhaustively considered in [4, 5] in the light of left-right symmetry [9]. A $Z_2$ symmetry – D-Parity – is assumed to be active between $SU(2)_L$ and $SU(2)_R$. The $vev < \Phi >$ can be classified by its D-Parity behaviour. $< \Phi_{650} >$ has two directions which are singlets under $G_{333}$ which are even and odd under D-Parity.

The form of $< \Phi_{650} >$ is given in (A.26) for the D-Parity even case while (A.27) is for the D-parity odd $vev$. This results in $\delta_{3c} = -1/3\sqrt{2}$ and $\delta_{3L} = \delta_{3R} = 1/6\sqrt{2}$ for the former and $\delta_{3c} = 0$ and $\delta_{3L} = -\delta_{3R} = 1/2\sqrt{6}$ for the latter. $< \Phi_{2430} >$ is listed in (A.28). From it one can readily read off $\delta_{3c} = \delta_{3L} = \delta_{3R} = -1/4\sqrt{26}$. In Table 8 we collect the findings for the different representations of $E(6)$.

It is worth remarking that the three $SU(3)$ subgroups in this chain are on an equal footing. It is possible to relate any two of them through a $Z_2$-type discrete symmetry. For the purpose of illustration and for phenomenological interest we have identified it with D-Parity. Obviously, one could just as well choose the $Z_2$-type symmetry to be between $SU(3)_c$ and $SU(3)_L$ (or $SU(3)_R$). The symmetry breaking $vevs$ of $\Phi_{650}$, either even or odd under this changed parity-like symmetry, are simply linear combinations between the $vevs$ which are odd and even under D-Parity discussed above.

V Conclusions

Higher dimensional non-renormalisable interactions can partially mimic the effects arising from quantum gravity or string compactification. We have considered a class of such operators, see eq. (2), those that modify the gauge kinetic term in GUTs. An operator of this type involves a scalar multiplet which, it turns out, can only break the symmetry in a rank preserving manner. After spontaneous
symmetry breaking such terms affect the unification of coupling constants in a calculable manner. The modifications depend on the subgroup to which the GUT is broken at the first stage and are quantified in terms of group theoretic factors $\delta_i$. In supersymmetric gauge theories the same factors are of interest as they characterise non-universality of gaugino masses at the GUT scale.

For $SO(10)$ and $E(6)$ GUTs we have obtained the $\delta_i$ for all allowed operators and all rank-preserving symmetry breakings. For $SU(5)$ the symmetry breaking is unique. For some symmetry breaking chains of $SO(10)$ (e.g., $SO(10) \rightarrow SU(5) \otimes U(1)$) and $E(6)$, (e.g., $E(6) \rightarrow SO(10) \otimes U(1)$) there is exactly one direction in a scalar multiplet to which the $vev$ can be ascribed and here again the predictions are one-to-one. This is not so for some other possibilities, (e.g., $SO(10) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$) where multiple directions within a scalar multiplet can accomplish the desired symmetry breaking. Here, the predictions for gauge coupling unification are more flexible, as are the implications for gaugino mass non-universality.

Acknowledgements This research has been supported by funds from the XIth Plan ‘Neutrino physics’ and RECAPP projects at HRI.

A Appendix: The vacuum expectation values

In this Appendix we collect the different vacuum expectation values which are used in this work.

A.1 $SU(5)$

For $SU(5)$ $\Phi_D$ can be in the 24, 75, and 200 representations.

The prototype example of the vacuum expectation values found useful in the calculations is in the case of $SU(5)$ with a $\Phi_{24}$ scalar. The $vev$ of this field can be represented as a traceless $5 \times 5$ diagonal matrix $(Tr(\lambda_i \lambda_j) = 1/2 \delta_{ij})$:

$$<\Phi_{24}> = v_{24} \frac{1}{\sqrt{60}} diag(3,3,-2,-2,-2) \equiv v_{24} < 24 >_5 . \quad (A.1)$$

In addition 10- and 15-dimensional forms of the $vev$, identified through the property that the resulting $\delta_i$ are the same as from (A.1), are also found useful. Under $G_{SM}$ the $SU(5)$ 10 = $(1,1)_2 + (3,1)_{-\frac{4}{3}} + (3,2)_{\frac{1}{3}}$. Noting that $Tr(\lambda_i \lambda_j) = 3/2 \delta_{ij}$, one finds:

$$<\Phi_{24}> = v'_{24} \frac{1}{\sqrt{60}} \underbrace{diag(-4,-4,-4,1,\ldots,1)}_{6 \text{ entries}} \equiv v'_{24} < 24 >_{10} . \quad (A.2)$$

Under $G_{SM}$ the 15 of $SU(5)$ is $(6,1)_{-\frac{4}{3}} + (3,2)_{\frac{1}{3}} + (1,3)_2$ and one has $(Tr(\lambda_i \lambda_j) = 7/2 \delta_{ij})$:

$$<\Phi_{24}> = v''_{24} \frac{1}{\sqrt{60}} \underbrace{diag(-4,\ldots,-4,1,\ldots,1,6,6,6)}_{6 \text{ entries} \ 6 \text{ entries}} \equiv v''_{24} < 24 >_{15} . \quad (A.3)$$

(A.1), (A.2), and (A.3) yield the same $\delta_i$ if $v_{24} = v'_{24} = 9v''_{24}$. 

10
Similarly, diagonal matrix. It is readily checked that the normalisation condition is as:
\[ \text{Tr}(\lambda_i \lambda_j) = 5 \delta_{ij}. \]
The 24 of \( SU(5) \) is \((1,1)_0 + (1,3)_0 + (8,1)_0 + (3,2)_{\frac{2}{3}} + (3,2)_{\frac{1}{3}} \). Thus
\[ < \Phi_{24} > = v_{24}'' \sqrt{\frac{5}{252}} \begin{pmatrix} 2,6,6,6,-4,\ldots,-4,1,\ldots,1,1,\ldots,1 \end{pmatrix} \equiv v_{24}'' < 24 >_{24}. \] (A.4)

For the vev of the 75-dimensional representation one uses the \( SU(5) \) relation: \( 10 \otimes \overline{10} = 1 \oplus 24 \oplus 75 \). Taking into consideration that \(< \Phi_{75} >\) must be orthogonal to \(< \Phi_{24} >\), i.e., \((\Lambda.2)\), it can be expressed as:
\[ < \Phi_{75} > = v_{75} \frac{1}{\sqrt{12}} \begin{pmatrix} 3,1,1,1,-1,\ldots,-1 \end{pmatrix} \equiv v_{75} < 75 >_{10}. \] (A.5)
The 24×24 form of \(< \Phi_{75} >\) which yields the same \( \delta_i \) as \((\Lambda.5)\) is:
\[ < \Phi_{75} > = v_{75}' \sqrt{\frac{5}{72}} \begin{pmatrix} 5,-3,-3,-3,-1,\ldots,-1,1,1,1,1,\ldots,1 \end{pmatrix} \equiv v_{75}' < 75 >_{24}. \] (A.6)

Similarly, the relation \( 15 \otimes \overline{15} = 1 \oplus 24 \oplus 200 \) permits the vev for \( \Phi_{200} \) to be written as a 15×15 traceless diagonal matrix. Ensuring orthogonality with \(< \Phi_{24} >\), i.e., \((\Lambda.3)\), one has:
\[ < \Phi_{200} > = v_{200} \frac{1}{\sqrt{12}} \begin{pmatrix} 1,\ldots,1,-2,\ldots,-2,2,2,2 \end{pmatrix} \equiv v_{200} < 200 >_{15}. \] (A.7)
\(< \Phi_{200} >\) can be also cast in a 24×24 form. Keeping \((\Lambda.4), (\Lambda.6), \) and \((\Lambda.7)\) in mind, it is found to be:
\[ < \Phi_{200} > = v_{200}' \sqrt{\frac{5}{168}} \begin{pmatrix} 10,2,2,2,1,\ldots,1,-2,\ldots,-2,-2,\ldots,-2 \end{pmatrix} \equiv v_{200}' < 200 >_{24}. \] (A.8)

**A.2 SO(10)**

For \( SO(10) \) the possible choices for \( \Phi_D \) are the 54-, 210-, and 770-dimensional representations.

The \( SO(10) \) relation \( 10 \otimes 10 = 1 \oplus 45 \oplus 54 \) ensures that \(< \Phi_{54} >\) can be expressed as a 10×10 traceless diagonal matrix. It is readily checked that the normalisation condition is \( \text{Tr}(\lambda_i \lambda_j) = \delta_{ij}. \)

Similarly, \( \overline{16} \otimes 16 = 1 \oplus 45 \oplus 210 \) permits \(< \Phi_{210} >\) to be represented in a 16×16 traceless diagonal form. For the 16×16 matrices \( \text{Tr}(\lambda_i \lambda_j) = 2 \delta_{ij}. \)

Finally, \(< \Phi_{770} >\) can be written as a 45×45 matrix which is traceless and diagonal since \((45 \otimes 45)_{\text{sym}} = 1 \oplus 54 \oplus 210 \oplus 770 \). Note that \(< \Phi_{54} >\) and \(< \Phi_{210} >\) can also be written in a similar form and orthogonality with them has to be ensured when obtaining \(< \Phi_{770} >\). For these matrices \( \text{Tr}(\lambda_i \lambda_j) = 8 \delta_{ij}. \)

The above observations for \( SO(10) \) are valid no matter which chain of symmetry breaking is under consideration.
A.2.1 \( \text{SO(10)} \rightarrow \text{SU(5)} \otimes \text{U(1)} \)

For \( \Phi_{54} \) there is no \( \text{SU}(5) \otimes \text{U(1)}_X \) invariant direction.

Under \( \text{SU}(5) \otimes \text{U(1)}_X \), 16 = (1, -5) + (5,3) + (10,1). Further, the diagonal matrix \( \langle \Phi_{210} \rangle \) must be orthogonal to the one corresponding to \( \text{U(1)}_X \), i.e., \( \frac{1}{2\sqrt{10}} \text{diag}(-5,3,3,3,3,1,1,1,1,1,-1,-1,-1,-1,-1,-1) \).

Satisfying these, we find:

\[
\langle \Phi_{210} \rangle = v_{210} \frac{1}{\sqrt{20}} \text{diag}(5, 1, 1, 1, 1, 1, -1, \ldots, -1, 1) \equiv v_{210} < 210 >_{16} . \tag{A.9}
\]

Under \( \text{SU}(5) \otimes \text{U(1)}_X \) 45 = (1, 0) + (10, 4) + (24, 0). Asking the results from (A.9) be reproduced one arrives at:

\[
\langle \Phi_{210} \rangle = v_{210}' \frac{1}{\sqrt{15}} \text{diag}(-4, -1, \ldots, -1, -1, \ldots, -1, 1, \ldots, 1) \equiv v_{210} < 210 >_{45} . \tag{A.10}
\]

\( \langle \Phi_{770} \rangle \) is chosen to be singlet under \( \text{SU}(5) \otimes \text{U(1)}_X \) and orthogonal to \( \langle \Phi_{210} \rangle \) in (A.10). It is:

\[
\langle \Phi_{770} \rangle = v_{770} \frac{1}{\sqrt{15}} \text{diag}(16, -2, \ldots, -2, -2, \ldots, -2, 1, \ldots, 1) \equiv v_{770} < 770 >_{45} . \tag{A.11}
\]

A.2.2 \( \text{SO(10)} \rightarrow \text{SU(3)}_c \otimes \text{SU(2)}_L \otimes \text{U(1)}_Y \otimes \text{U(1)}_X \)

The case we consider in this subsection is a typical example of several symmetry breaking chains (see the \( E(6) \) cases below) where the \textit{vevs} can be easily written down using the \textit{vevs} for GUT groups which are themselves subgroups of the one under consideration. Here we exploit the findings of sec. A.1 on \( \text{SU}(5) \) symmetry breaking to obtain the required results providing enough details. In subsequent subsections we simply write down the results since the method is the same.

To accomplish the desired symmetry breaking the vev has to be assigned to a component of \( \Phi_D \) which is not only a non-singlet under \( \text{SO(10)} \) but also under its subgroup \( \text{SU}(5) \). In fact, from the discussions in sec. A.1 it must transform as a 24, 75, or 200 of \( \text{SU}(5) \).

The 54-dimensional \( \text{SO(10)} \) representation contains an \( \text{SU}(5) \otimes \text{U(1)}_X \) (24,0) which is appropriate for the symmetry breaking under consideration. Under \( \text{SU}(5) \otimes \text{U(1)}_X \) 10 = (5,2) + (5,-2). Using (A.1) one finds

\[
\langle \Phi_{54,24} \rangle = v_{54}' \frac{1}{\sqrt{60}} \text{diag}(3, 3, -2, -2, -2, 3, 3, -2, -2, -2) \\
= v_{54}' \text{diag}(< 24 >_5, < 24 >_5) \equiv v_{54}' < 54, 24 >_{10} . \tag{A.12}
\]

Under \( \text{SU}(5) \otimes \text{U(1)}_X \) 210 \( \supset \) (24,0) + (75,0). Bearing in mind 16 = (1,-5) + (5,3) + (10,-1) and employing (A.1) and (A.2)

\[
\langle \Phi_{210,24} \rangle = v_{210}' \frac{1}{\sqrt{60}} \text{diag}(0, 3, 3, -2, -2, -2, 6, -4, -4, -4, 1, \ldots, 1) \\
= v_{210}'(0, < 24 >_5, < 24 >_{10}) \equiv v_{210}' < 210, 24 >_{16} . \tag{A.13}
\]
Ensuring orthogonality and using (A.5) one has:
\[
\langle \Phi_{210,75} \rangle = v''_{210} \frac{1}{\sqrt{3}} \begin{bmatrix} diag(0,0,\ldots,0,3,1,1,-1,\ldots,-1) \\ 5 \text{ entries} \\ 6 \text{ entries} \end{bmatrix} = v'_{210} \frac{2}{\sqrt{3}} (0,0,\ldots,0, <75>_{>10}) \equiv v'_{210} <210,75>_{>16}.
\] (A.14)

The 770 representation of $SO(10)$ contains within it $(24,0)$, $(75,0)$, and $(200,0)$ submultiplets under $SU(5) \otimes U(1)_X$. As already discussed, $\langle \Phi_{770} \rangle$ can be expressed as a traceless, diagonal $45 \times 45$ matrix. Further $45 = (1,0) + (10,4) + (10,-4) + (24,0)$. Using (A.8) one gets:
\[
\langle \Phi_{770,200} \rangle = v'''_{770} \frac{\sqrt{8}}{5} (0,0,\ldots,0,0,\ldots,0, <200>_{>24}) \equiv v'''_{770} <770,200>_{>45}.
\] (A.15)

A.2.3 $SO(10) \rightarrow SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$

Under $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$, $10 \equiv (1,2,2) + (6,1,1)$. From the tracelessness condition one can immediately obtain
\[
\langle \Phi_{54} \rangle = v_{54} \frac{1}{\sqrt{60}} \begin{bmatrix} diag(3,3,3,3,-2,\ldots,-2) \\ 6 \text{ entries} \end{bmatrix}.
\] (A.16)

As noted earlier, $\langle \Phi_{210} \rangle$ can be represented as a traceless and diagonal $16 \times 16$ matrix. Since $16 \equiv (4,2,1) + (4,1,2)$ one can readily identify
\[
\langle \Phi_{210} \rangle = v_{210} \frac{1}{\sqrt{8}} \begin{bmatrix} diag(1,\ldots,1,-1,\ldots,-1) \\ 8 \text{ entries} \\ 8 \text{ entries} \end{bmatrix}.
\] (A.17)

Similarly, noting $45 \equiv (15,1,1) + (1,3,1) + (1,1,3) + (6,2,2)$ under $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$, one can write $\langle \Phi_{770} \rangle$ as:
\[
\langle \Phi_{770} \rangle = v_{770} \frac{1}{\sqrt{180}} \begin{bmatrix} diag(-4,\ldots,-4,-10,\ldots,-10,5,\ldots,5) \\ 15 \text{ entries} \\ 3+3 \text{ entries} \\ 24 \text{ entries} \end{bmatrix}.
\] (A.18)

The $\langle \Phi_{54} \rangle$ and $\langle \Phi_{210} \rangle$ can also be written in a similar $45 \times 45$ form and care must be taken to ensure that $\langle \Phi_{770} \rangle$ is orthogonal to them.

A.3 $E(6)$

The options available for $\Phi_D$ for $E(6)$ GUTs are 650- and 2430-dimensional.

In $E(6)$ $\overline{27} \otimes 27 = 1 \oplus 78 \oplus 650$. So, $\Phi_{650}$ can be expressed as a $27 \times 27$ traceless diagonal matrix. For this case $Tr(\lambda_i \lambda_j) = 3 \delta_{ij}$.

Also, $(78 \otimes 78)_{sym} = 1 \oplus 650 \oplus 2430$. Hence both $\langle \Phi_{650} \rangle$ and $\langle \Phi_{2430} \rangle$ can be represented as $78 \times 78$ diagonal traceless matrices. For them $Tr(\lambda_i \lambda_j) = 12 \delta_{ij}$.
A.3.1 E(6) → SU(2)⊗SU(6)

Both 650 and 2430 have directions which are singlets under \(SU(2) \otimes SU(6)\). Under \(SU(2) \otimes SU(6)\) 
\(27 = (2, \bar{6}) + (1, 15)\). Therefore one can readily write \(\langle \Phi_{650} \rangle\) for this channel of symmetry breaking as the \(27 \times 27\) diagonal traceless matrix:

\[
\langle \Phi_{650} \rangle = v_{650} \frac{1}{\sqrt{180}} \text{diag}(5, \ldots, 5, -4, \ldots, -4).
\]

(A.19)

The 2430 vev can be written down using \(78 = (3,1) + (1, 35) + (2, 20)\) and maintaining orthogonality with \(\langle \Phi_{650} \rangle\) one can write

\[
\langle \Phi_{2430} \rangle = v_{2430} \frac{1}{\sqrt{3640}} \text{diag}(70, 70, 70, 18, \ldots, 18, -21, \ldots, -21).
\]

(A.20)

A.3.2 E(6) → SO(10)⊗U(1)_η

The 650 representation has a singlet under \(SO(10) \otimes U(1)\) which as before can be expressed as a \(27 \times 27\) matrix. Under \(SO(10) \otimes U(1)\) 
\(27 = (1, 4) + (10, -2)\). Using this one finds

\[
\langle \Phi_{650} \rangle = v_{650} \frac{1}{\sqrt{12 \times 5}} \text{diag}(40, -5, \ldots, -5, 4, \ldots, 4).
\]

(A.21)

To write down \(\Phi_{2430}\) we note that the decomposition under \(SO(10) \otimes U(1)\) is \(78 = (1, 0) + (45, 0) + (16, -3) + (16, 3)\). Ensuring the requirements of orthogonality to \(\langle \Phi_{650} \rangle\) and tracelessness we have

\[
\langle \Phi_{2430} \rangle = v_{2430} \frac{1}{4 \sqrt{78}} \text{diag}(-108, -4, \ldots, -4, 9, \ldots, 9, 9, \ldots, 9).
\]

(A.22)

A.3.3 E(6) → SU(5)⊗U(1)_ξ⊗U(1)_η

The results for this option of symmetry breaking can be obtained by referring to those in sec. [A.2.1](#) for \(SO(10) \rightarrow SU(5) \otimes U(1)_X\). Here the vev must be assigned to a direction which is a singlet under \(SU(5) \otimes U(1)_\xi \otimes U(1)_\eta\) but not under \(SO(10) \otimes U(1)_\eta\). Such possibilities are the following: 650 of \(E(6)\) contains the submultiplets \((54,0)\) and \((210,0)\) under the latter group and the 2430 of \(E(6)\) includes \((210,0)\) and \((770,0)\). As already noted the \(SO(10)\) 54 does not have a singlet direction of \(SU(5) \otimes U(1)_X\). So, we need to consider only the other possibilities.

It is useful to recall the decomposition \(27 = (1,4) + (10,-2) + (16,1)\) under \(SO(10) \otimes U(1)_\eta\). Then from (A.9) we have

\[
\langle \Phi_{650,210} \rangle = v_{650} \frac{\sqrt{3}}{2} \text{diag}(0, 0, \ldots, 0, < 210 >_{16}).
\]

(A.23)

< \(\Phi_{650}\) can also be expressed as a \(78 \times 78\) traceless diagonal matrix. Here one uses \(78 = (1,0) + (45,0) + (16, -3) + (16,3)\) under \(SO(10) \otimes U(1)_\eta\). Then using (A.9) and (A.10):

\[
\langle \Phi_{2430,210} \rangle = v'_{650} \text{diag}(0, < 210 >_{45}, < 210 >_{16}, < 210 >_{16}).
\]

(A.24)
The remaining vev is \( < \Phi_{2430} > \) which can be written down using (A.11)
\[
< \Phi_{2430,770} >= v_{2430} \sqrt{\frac{3}{2}} \, \text{diag}(0, < 770 >_{45}, 0, \ldots, 0, 0, \ldots, 0). \quad (A.25)
\]

### A.3.4 \( E(6) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_\xi \otimes U(1)_\eta \)

For this symmetry breaking we can utilise the results in sec. A.2.2 for \( SO(10) \rightarrow SU(3) \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \). The relevant submultiplets are the following: 650 of \( E(6) \) contains \((54,0)\) and \((210,0)\) under \( SO(10) \otimes U(1)_Y \) and the 2430 of \( E(6) \) includes \((54,0), (210,0)\) and \((770,0)\). The desired symmetry breaking can occur through the further \( SU(5) \) 24, 75, or 200 content of the \( SO(10) \) multiplets, viz., \( 54 \supset 24; 210 \supset 24 \) and \( 75 \); and \( 770 \supset 24, 75 \) and \( 200 \).

The explicit forms of the vevs are not of much use since ultimately it is the \( SU(5) \) representation which fixes the \( \delta_i \) following the results of sec. II. So, we refrain from displaying the vevs in this case.

### A.3.5 \( E(6) \rightarrow SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \)

As before, \( < \Phi_{650} > \) can be written as a \( 27 \times 27 \) matrix. It turns out that the 650 representation has two directions which are singlet under \( SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \equiv G_{333} \). Of course, a vev in any one of these directions or linear combinations thereof may be chosen to break the symmetry. In particular, two linear combinations may be identified which respect \( \delta_3 L = \pm \delta_3 R \). These are of interest from the physics standpoint as they are respectively even or odd under D-Parity.

In this option of \( E(6) \) symmetry breaking to \( SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \) one has \( 27 = (1, \bar{3}, 3) + (3, 1, \bar{3}) + (\bar{3}, 3, 1) \). The D-even case is:
\[
< \Phi_{650} >= v_{650} \frac{1}{\sqrt{18}} \, \text{diag}(-2, \ldots, -2, 1, \ldots, 1, 1, \ldots, 1). \quad (A.26)
\]

while the D-odd vev is
\[
< \Phi'_{650} >= v'_{650} \frac{1}{\sqrt{6}} \, \text{diag}(0, \ldots, 0, 1, \ldots, 1, -1, \ldots, -1). \quad (A.27)
\]

As in the other cases, \( < \Phi_{2430} > \) can be written as a \( 78 \times 78 \) traceless diagonal matrix. Noting that \( 78 = (8,1,1) + (1,8,1) + (1,1,8) + (3,3,3) + (3,3,3) \) and maintaining orthogonality with \( < \Phi_{650} > \) and \( < \Phi'_{650} > \) one can write
\[
< \Phi_{2430} >= v_{2430} \frac{1}{\sqrt{234}} \, \text{diag}(9, \ldots, 9, 9, \ldots, 9, 9, \ldots, 9, -4, \ldots, -4, -4, \ldots, -4). \quad (A.28)
\]

### References

[1] J. C. Pati and A. Salam, Phys. Rev. D10 (1974) 275; H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 428; G. G. Ross, Grand Unified Theories (Benjamin/Cummings, Reading, USA,
1984); R. N. Mohapatra, *Unification and Supersymmetry. The frontiers of quark-lepton physics* (Springer, Berlin, Germany, 1986).

[2] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. **33** (1974) 451; D. R. T. Jones, Phys. Rev. **D25** (1982) 581.

[3] Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52** (1984) 875; C. T. Hill, Phys. Lett. **B135** (1984) 47; L. J. Hall and U. Sarid, Phys. Rev. Lett. **70** (1993) 2673.

[4] J. Chakrabortty and A. Raychaudhuri, Phys. Lett. **B673** (2009) 57.

[5] J. Chakrabortty and A. Raychaudhuri, Phys. Rev. **D81** (2010) 055004.

[6] S. P. Martin, Phys. Rev. **D79** (2009) 095019.

[7] S. Bhattacharya and J. Chakrabortty, Phys. Rev. **D81** (2010) 015007.

[8] H. Georgi, in *Particles and Fields – 1974*, ed. C. A. Carlson (AIP, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93** (1975) 193; T. Clark, T. Kuo and N. Nakagawa, Phys. Lett. **B115** (1982) 26; C. S. Aulakh and R. N. Mohapatra, Phys. Rev. **D28** (1983) 217.

[9] J. C. Pati and A. Salam, Phys. Rev. **D10** (1974) 275; R. N. Mohapatra and J. C. Pati, Phys. Rev. **D11** (1975) 566; R. N. Mohapatra and J. C. Pati, Phys. Rev. **D11** (1975) 2558; G. Senjanović and R. N. Mohapatra, Phys. Rev. **D12** (1975) 1502.

[10] S.M. Barr, Phys. Lett. **B112** (1982) 219; I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. **B194** (1987) 231.

[11] D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **52** (1984) 1072; Phys. Rev. **D30** (1984) 1052.

[12] F. Gürsey, P. Ramond and P. Sikivie, Phys. Lett. **B60** (1976) 177; Y. Achiman and B. Stech, Phys. Lett. **B77** (1978) 389; J. L. Hewett, T. G. Rizzo and J. A. Robinson, Phys. Rev. **D33** (1986) 1476; R. Howl and S. F. King, JHEP **0801** (2008) 030.

[13] J. E. Kim, Phys. Lett. **B107** (1981) 69; A. Sen, Phys. Rev. **D31** (1985) 900; Z. Chacko and R. N. Mohapatra, Phys. Lett. **B442** (1998) 199.

[14] An early work in this direction is U. Sarkar and A. Raychaudhuri, preprint CUPP/82-5 (unpublished).