A CP-Violating Kinematic Structure

D. V. Ahluwalia

Escuela de Fisica, ISGBG, Ap. Postal C-600
Univ. Aut. de Zacatecas
Zacatecas, ZAC 98068, Mexico

Abstract. A CP violating kinematic structure is presented. The essential physical input is to question the textbook wisdom, “Now when a particle is at rest, one cannot define its spin as either left- or right-handed, so $\phi_R(0) = \phi_L(0)$,” as found, e.g., in Lewis Ryder’s Quantum Field Theory, and in many other books on the representations of the Lorentz group. It is suggested that this equality is true only up to a phase. The demand of C, P, and T covariances, separately, fixes this phase to be $\pm 1$. If these conditions are relaxed, a natural CP-violating kinematic structure emerges. Having established a CP-violating kinematic structure, we then discuss how Planck scale physics necessarily invokes non-commutative space-time and that such changes in the structure of space-time will force upon us additional violations/deformations of the CPT structure of space-time, and a violation of the principle of equivalence via a violation of the Lorentz symmetries. The latter may carry significant consequences for understanding the data on ultra high energy cosmic rays.

INTRODUCTION

The moment one invokes the Poincaré space-time symmetries the notions of mass, spin, and the existence of two types of matter, i.e. particles and antiparticles, immediately arise. Poincaré symmetries also play equally important role in the gravitational realm. One can even argue that the standard wave-particle duality carries its basis in the generators of space-time translations. The eigenstates of these generators, which can be superimposed to make normalizable physical states, carry a spatial periodicity that, in the presence of a non-vanishing Planck constant, can be identified with the de Broglie wave length.

Here, I argue that the quantum mechanical framework induces new elements in the C, P, and T structure of the representations offered by the Lorentz group. Arguments of this nature were long ago appreciated by Gürsey, Michel, Wigner, and by other physicists of their generation, see, e.g., Ref. [1] and pp. 453-457 of Ref. [2], and also refer to the Lee and Wick paper [3]. The new element that I mentioned in the abstract turns out, in a very precise sense [4], to be a missing link that prevented construction of the Wigner classes (see Wigner in Ref. [1]). These arguments provide unsuspected source of CP violation. It emerges, e.g., that the violation of
CP may also be found in a new quantum-induced CP-violating kinematic structure offered by the Lorentz group, rather than in a new interaction. To clarify, one should note that in the standard gauge theory of the electroweak interactions the underlying kinematic structure manifestly violates parity. The gauge group of the weak interactions serves to provide the gauge bosons that mediate the interaction on this P-violating kinematic structure. The credit on gauge bosons, and the Higgs boson, is to help set the range of this P-violating interaction in a renormalizable theory. Our general results are not in conflict with the common wisdom, see, e.g., Ref. [5], because these works confine to the usual kinematic structure for the spin-1/2 fermions and spin-1 vector bosons (note, just because a particle carries spin one does not necessarily make it a vector object) and which seek CP violation in interactions. Purely on the grounds of representations of the Lorentz group in the quantum realm, we shall discover the possibility of a new phase field (called $\Phi(\theta)$ below). In this talk we shall remain far from identifying the (scalar) phase field with the Higgs, but the existence of the new field arises so naturally that there may be an important place for it in physics.

This also sets the stage to study CPT-related consequences of the possible deformations of the space-time symmetries required by the joint realm of the quantum and gravity. On the empirical side, the data on the ultra high energy cosmic rays has taken us to within nine orders of magnitude of the Planck scale. That data has already raised questions that invoke the violation of the Lorentz symmetries. In this context too, I find the reflections presented here to be more than of academic interest.

**THE QUANTUM-INDUCED CPT STRUCTURE OF SPACE-TIME**

The kinematic structure of the existing quantum theory of fields originates from the Poincaré symmetries [6]. This is well explained in recent books of Ryder, Sterman, and Weinberg [7–9]. For the purpose at hand the reader is also referred to my book reviews [10,11], and Ref. [12]. In these references certain conceptual issues have been clarified and corrected.

I now provide a brief, but careful, review of the spin-1/2 representation space. If we wish to construct a parity covariant spin-1/2 representation space then the fields operators associated with the kinematic structure must be constructed in the $(1/2, 0) \oplus (0, 1/2)$ representation space. One must further decide whether one wishes to describe charged particles in the Dirac sense, or neutral particles in the Majorana sense. To treat both of these constructs at an equal footing the underlying spinors must carry similar C, P, and T properties as the field operators themselves. For the usual Majorana field operator the just-stated requirement is badly violated because it is expanded in terms of the Dirac spinors. In the usual textbook constructs, see, e.g., Ref. [13], it is only in terms of the Fock space creation and annihilation operators that the distinction is made between the Dirac and
Majorana field operators. In these constructs, we repeat, the underlying spinors, for the Majorana field operator, are still the Dirac spinors.

To describe the fundamentally charged particles in the Dirac sense the appropriate \((1/2, 0) \oplus (0, 1/2)\) spinors have the form:

\[
\psi(\vec{p}) \equiv \left( \begin{array}{c} \phi_R(\vec{p}) \\ \phi_L(\vec{p}) \end{array} \right).
\] (1)

The \(\phi_R(\vec{p})\) transforms as a \((1/2, 0)\) spinor,\(^1\) and boosts as

\[
\phi_R(\vec{p}) = \exp \left( \frac{\vec{\sigma} \cdot \vec{\varphi}}{2} \right) \phi_R(\vec{0}).
\] (2)

In the above equation, the \(\vec{\sigma}\) are the usual Pauli matrices, and \(\vec{\varphi}\) is the boost parameter. The definition of \(\vec{\varphi}\) is motivated by the fact that \(E^2 - \vec{p}^2 = m^2\), and that \(\cosh^2 \alpha - \sinh^2 \alpha = 1\) (as an identity). Thus,

\[
\cosh \varphi = \frac{E}{m}, \quad \sinh \varphi = \frac{|\vec{p}|}{m}.
\] (3)

The direction associated with \(\vec{\varphi}\) is that of the three momentum associated with the particle:

\[
\hat{\varphi} = \frac{\vec{p}}{|\vec{p}|}.
\] (4)

We shall assume that \(m \neq 0\).

Whereas, the \(\phi_L(\vec{p})\) transforms is a \((0, 1/2)\) spinor, and boosts with the opposite sign in the exponent:

\[
\phi_L(\vec{p}) = \exp \left( -\frac{\vec{\sigma} \cdot \vec{\varphi}}{2} \right) \phi_L(\vec{0}).
\] (5)

The \(\vec{0}\) corresponds to the momentum vector for the particle at rest.

On the other hand, if one wishes to describe fundamentally neutral particles in the Majorana sense then the correct choice of the \((1/2, 0) \oplus (0, 1/2)\) spinors is:

\[
\lambda(\vec{p}) = \left( \begin{array}{c} \zeta \Theta^{\frac{1}{2}} \phi_L^*(\vec{p}) \\ \phi_L(\vec{p}) \end{array} \right),
\] (6)

\(^1\) Note, identical transformation properties under Lorentz group do not necessarily imply that other transformations properties will be identical as well. The latter, e.g., may refer to transformations under C, P, and T. Thus, the Dirac- and Majorana-\((1/2, 0) \oplus (0, 1/2)\) constructs carry different physical characteristics under operations of C, P, and T, while carrying identical transformations under the Lorentz group.
$$\rho(\vec{p}) = \begin{pmatrix} \phi_R(\vec{p}) \\ (\zeta_\rho \Theta_{[1/2]}^{\dagger})^* \phi_R^*(\vec{p}) \end{pmatrix}. \quad (7)$$

In these expressions $\Theta_{[1/2]}$ is the spin-1/2 Wigner time reversal operator:

$$\Theta_{[1/2]} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (8)$$

The phases factors $\zeta_\lambda$ and $\zeta_\rho$, that appear in Eqs. (6) and (7), are determined by the condition of self(S)/anti-self(A) conjugacy under the operation of charge conjugation. In Refs. [14,15] it was shown that this condition determines these phases to be:

$$\zeta_S^\lambda = +i \rho_S^\lambda, \quad (9)$$
$$\zeta_A^\lambda = -i \rho_A^\lambda, \quad (10)$$

Incorporating the anti-self conjugate spinors is essential for the complete, in the sense of mathematically complete set, description of the neutral $(1/2, 0) \oplus (0, 1/2)$ representation space. The classic 1957 McLennan-Case reformulation of the Majorana theory was incomplete in this aspect. Similarly, there is a widespread, but incorrect, belief that somehow Majorana neutral objects carry half as many degrees of freedom as Dirac neutral objects. The reader who wishes to scrutinize these issues is referred to Ref. [14].

The wave equations satisfied by these spinors follow from: (a) The transformation properties of the $(1/2, 0)$ and $(0, 1/2)$ spinors, and very importantly, (b) The relative phase between the $(1/2, 0)$ and $(0, 1/2)$ spinors at rest. Because this may carry important physical consequences we establish this claim. We shall first carefully examine the $(1/2, 0) \oplus (0, 1/2)$ representation space suitable for describing charged particles. Only later shall we return to the neutral particles very briefly.

Due to the isotropy of the $\vec{p} = \vec{0}$, one may argue that $\phi_R(\vec{0}) = \phi_L(\vec{0})$. In fact that is precisely what is done in the standard textbooks [7,16]. Ryder’s classic book on the theory of quantum fields [7], in fact, argues, “Now when a particle is at rest, one cannot define its spin as either left- or right-handed, so $\phi_R(0) = \phi_L(0)$.” This may be argued to some extent if one was to confine to a purely classical framework. In the process, as it turns out, what one misses are anti-particles! However, if one is to invoke a quantum framework for the interpretation of these spinors then this equality can only be claimed up to a phase:

$$\phi_R(\vec{0}) = e^{i\theta} \phi_L(\vec{0}). \quad (11)$$

This is the central physical input which will yield us the result (21). In fact, in a private communication, Ryder has pointed out that $\theta$ may be a $2 \times 2$ matrix. Here,
we shall confine to the simplest suggestion that $\theta$ is a real angle (possibly carrying a space-time dependence).

Equations (2), (5), and (11) contain essentially\(^2\) the entire kinematic structure of the charged spin-1/2 particles. To see this we follow the footsteps of Lewis Ryder [7], but we now carefully incorporate the important ingredient embedded in a non-vanishing $\theta$.

1. On the right-hand side of Eq. (2), substitute for $\phi_R(\vec{0})$ from (11). This gives

\[
\phi_R(\vec{p}) = e^{i\theta} \exp \left( \frac{\vec{\sigma}}{2} \cdot \vec{\varphi} \right) \phi_L(\vec{0}).
\]  

(12)

2. From Eq. (5) obtain,

\[
\phi_L(\vec{0}) = \exp \left( \frac{\vec{\sigma}}{2} \cdot \vec{\varphi} \right) \phi_L(\vec{p}),
\]

(13)

and insert it into the right-hand side of Eq. (12). This yields:

\[
\phi_R(\vec{p}) = e^{i\theta} \exp \left( \vec{\sigma} \cdot \vec{\varphi} \right) \phi_L(\vec{p}).
\]

(14)

3. Similarly, starting from Eqs. (5) and (11) we obtain:

\[
\phi_L(\vec{p}) = e^{-i\theta} \exp \left( -\vec{\sigma} \cdot \vec{\varphi} \right) \phi_R(\vec{p}).
\]

(15)

4. Now, because $(\vec{\sigma} \cdot \vec{p})^2 = 2 \times 2$ Identity matrix, $I_2$

\[
(\vec{\sigma} \cdot \vec{p})^n = \begin{cases} I_2 & \text{for } n \text{ even} \\ \vec{\sigma} \cdot \vec{p} & \text{for } n \text{ odd} \end{cases}
\]

(16)

This leads to the identities:

\[
\exp(\pm \vec{\sigma} \cdot \vec{\varphi}) = \frac{EI_2 \pm \vec{\sigma} \cdot \vec{p}}{m}
\]

(17)

5. Next, substitute these identities in Eqs. (14) and (15), and re-arrange to obtain:

\[
\begin{pmatrix}
-m e^{-i\theta} & EI_2 + \vec{\sigma} \cdot \vec{p} \\
EI_2 - \vec{\sigma} \cdot \vec{p} & -me^{i\theta}
\end{pmatrix}
\begin{pmatrix}
\phi_R(\vec{p}) \\
\phi_L(\vec{p})
\end{pmatrix} = 0.
\]

(18)

\(^2\) We qualify with “essentially” because while constructing the field operators the Fock space considerations must be invoked, in addition.
6. Finally, with \( p_\mu = (p^0, -\vec{p}) \), \( E = P^0 \), read off the Weyl-representation gamma matrices:
\[
\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix},
\]
(19)
and introduce
\[
\Phi(\theta) = \begin{pmatrix} \exp(-i\theta) & 0 \\ 0 & \exp(+i\theta) \end{pmatrix},
\]
(20)
This yields, the central result of our thesis:
\[
\left( \gamma^\mu p_\mu - m\Phi(\theta) \right) \psi(\vec{p}) = 0.
\]
(21)

The obtained equation is Poincaré covariant and indeed carries the solutions with the correct dispersion relations \( E = \pm \sqrt{\vec{p}^2 + m^2} \), because not only
\[
\text{Det} [\gamma^\mu p_\mu - m\Phi(\theta)] = (\vec{p}^2 + m^2 - E^2)^2
\]
but also because \( \text{Det} [\gamma^\mu p_\mu - m\Phi(\theta)] \) is independent of \( \theta \). Thus, as expected, Poincaré covariance cannot constrain \( \Phi(\theta) \). The \( \Phi(\theta) \) is constrained to be \( \pm 1 \) if one places the extra condition that the resulting equation be covariant separately under operations of C, P, and T. The \( \Phi(\theta) \)'s cannot be “rotated away” particularly if one considers a general system of more than one spin-1/2. Further, in absence of the indicated CPT-related covariances, \( \Phi(\theta) \) may carry a space-time dependence. Note, without the \( \Phi(\theta) = -1 \), the “antiparticle” solutions are missed as was brought to the reader’s attention earlier.

We thus conjecture that \( \Phi(\theta) \) may carry space-time dependence (particularly, in curved space-time) — and even if it were to be taken as a constant matrix — \( \Phi(\theta) \) carries information on the CPT structure of the spin-1/2 charged fields. It is too premature to speculate if \( \Phi(\theta) \) may not be related to the observed CP violation in the universe. However, if one insists on a renormalizable theory with no space-time dependence in \( \Phi(\theta) \), then considerations of Weinberg, see Sec. 12.5 of Ref. [9], could render \( \Phi(\theta) \) physically unobservable. The CP violating kinematic structure obtained here is also contained in the postulated “CP violating Dirac equation” of Funakubo et al. [17]. For non local elements in this structure, and additional kinematic details, the reader is referred to Ref. [18].

Before proceeding further, let’s take note that the existence of \( \Phi(\theta) = -1 \), missed in the well-known classical arguments, and in fact not allowed by the realm of classical framework, is a consequence of the quantum mechanical freedom encoded in Eq. (11). For a C–, P–, and T–covariant theory, existence of two values, rather

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3) We now abbreviate \( I_2 \) by \( I \). The zeros below stand for \( 2 \times 2 \) null matrices, and \( \sigma^1 = \sigma_x \), etc.
than one, for $\Phi(\theta)$ is responsible for the existence of two types of matter, i.e. particles and antiparticles. In the standard treatment of spin-1/2 particles, the quantum-mechanical origin of this fact was well-understood by Dirac and other knowledgeable physicists, even though by no means existence of the phase field $\Phi(\theta)$ was so transparent, or was even suspected. Second, the linearity of Eq. (21) in $p_\mu$ is a consequence of the very specific property of the Pauli matrices contained in Eq. (16). Third, this linearity is not guaranteed if one considers neutral particles in the Majorana sense, see Ref. [14].

When one repeats the same exercise for the charged particles in the $(1,0) \oplus (0,1)$ representation space one obtains a new, and to an extent unexpected, result that such spin-1 particles, as opposed to spin-1 vector particles, are of a Wigner class which was presented only relatively recently in Ref. [4]. In this Wigner class, a boson and its antiboson carry opposite relative intrinsic parity. This arises because the C and P operators of the $(1,0) \oplus (0,1)$ representation space anticommute, rather than commute. No search for such particles in the low energy domain has yet been undertaken. Because the natural counterpart of the spin-1/2 $\gamma^\mu$ in the $(1,0) \oplus (0,1)$ representation space is a set of two-indexed $6 \times 6$ matrix objects $\gamma^{\mu \nu}$ they may provide a natural coupling to the space-time metric $g_{\mu \nu}$. For this reason, these particles may have played an important role in the early universe. In particular, their unexpected properties as regards their C, P, and T transformations make them natural candidates for studying any unusual CP properties of the early phase of the universe. Once again, a phase field similar to $\Phi(\theta)$ above can be introduced for the $(1,0) \oplus (0,1)$ representation space.

Here we shall refrain from further discussing the representation spaces associated with the neutral particles. Once again, one finds that neutral $(1/2,0) \oplus (0,1/2)$ representation space carries distinct, and different, structure under the transformations of C, P, and T. The reader is referred to Refs. [14,15,19,20].

**DISCUSSION: POSSIBLE ROLE OF GRAVITY**

The CPT properties of the underlying kinematic structure of the existing quantum field theories are thus explicitly seen to have their entire roots in the space-time symmetries and the quantum mechanical phase fields, such as $\Phi(\theta)$. A natural thesis thus arises: The observed CP violation may carry its origins in some new CP-violating kinematic structure rather than a new interaction.

Having established these results it is now important to note that if gravitational effects of quantum measurements are not neglected then the resulting space-time is necessarily non-commutative, and brings in certain elements of non-locality [21]. This happens because the quantum mechanical collapse of a wave function also carries with it an unavoidable collapse of the associated energy-momentum tensor. This brings in an interplay of the gravitational and quantum realms. Therefore, at the Planck scale one expects a space-time that is non-commutative and which uses appropriate deformations of the Poincaré group. Such interplay of the
quantum and gravitational realms generically introduce gravitationally-modified wave-particle duality,\(^4\) or equivalently in the context of string-theory suggested modifications in the fundamental uncertainty relations [34]. All this endows space-time with new quantum-induced modifications. This would have immediate consequences for the underlying kinematic structure of the theory as regards its CPT properties. This latter result, on the one hand this seems suggested by the intrinsic non-locality and non-commutative geometry imposed by the interplay of the quantum and gravitational realms [21], and on the other by hand (which seems to contain the same physical origin) by more formal studies devoted to investigating non-commutative space-time. Therefore, it is clear that that deformations of the Poincaré symmetries, whether expected on the grounds of the latest data on ultra high-energy cosmic rays, or on the basis of more theoretical grounds, shall have far reaching consequences for the entire structure on which the present theoretical physics is based [35–39]. In this context, it is relevant to note that the instance Poincaré symmetries are deformed one must expect a modification of the principle of equivalence also.

It is important to emphasize that a \(\Phi(\theta) \neq \pm 1\) may lead to low energy consequences, such as CP violation, whereas deformations of the Poincaré symmetries and the associated change in the underlying CPT structure of the kinematics shall be responsible for additional effects that would be dominant as we approach the Planck scale.

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\(^4\) In this context I refer the reader not only to the more physically motivated works, such as [21–30], but also to more formal and important works represented by Refs. [31–33].
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