Abelian Gauge Theory in de Sitter Space

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Abstract

Quantization of spinor and vector free fields in 4-dimensional de Sitter space-time, in the ambient space notation, has been studied in the previous works. Various two- points functions for the above fields are presented in this paper. The interaction between the spinor field and the vector field is then studied by the abelian gauge theory. The $U(1)$ gauge invariant spinor field equation is obtained in a coordinate independent way notation and their corresponding conserved currents are computed. The solution of the field equation is obtained by use of the perturbation method in terms of the Green’s function. The null curvature limit is discussed in the final stage.

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1 Introduction

Quantum field theory in de Sitter space-time has evolved as an exceedingly important subject, studied by many authors in the course of the past decade. Historically the de Sitter space-time, with maximum symmetry in the curved space-time manifold, was introduced as a solution of the positive cosmological Einstein’s equations. It has the same degree of symmetry as the flat Minkowski space solution [1]. The interest in the de Sitter space increased tremendously when it turned out that it could play a central role in the inflationary cosmological paradigm [2]. Very recently, a non-zero cosmological constant has been proposed to explain the luminosity observations of the farthest supernovae [3]. If this hypothesis is validated in the future, our ideas on the large-scale universe needs to be changed and the de Sitter metric will play a further important role.

All these developments make it more compelling than ever to find a formulation of de Sitter quantum field theory with the same level of completeness and rigor as for its Minkowskian counterpart. In Minkowski space, a unique Poincaré invariant vacuum can be fixed by imposing the positive energy condition. In curved space-time, however, a global time-like Killing vector field does not exist and therefore the positive energy condition cannot be imposed. Thus symmetry alone is not sufficient for determination of a suitable vacuum state. In de Sitter space, however, symmetries identify the vacuum only in relation to a two parameter ambiguity $|\alpha, \beta\rangle$, corresponding to a family of distinct de Sitter invariant vacuum states (see [4] and references there in). Only the one parameter family $|\alpha, 0\rangle$, is invariant under the CPT transformation [5, 6, 7]. By imposing the condition that in the null curvature limit, the Wightman two-point function become exactly the same as Minkowskian Wightman two-point function, the other parameter ($\alpha$) can be fixed as well. This vacuum, $|0, 0\rangle$, is called Euclidean vacuum or Bunch-Davies vacuum. It should be noted that this condition is different with the Hadamard condition, which requires that the leading short distance singularity in the Hadamard function $G^{(1)}$ should take its flat space value. The leading singularity of the Hadamard function is $\cosh 2\alpha$ times its flat space value.

Bros et al. [8, 9] presented a QFT of scalar field in de Sitter space that closely mimics QFT in Minkowski space. They have introduced a new version of the Fourier-Bros transformation on the hyperboloid [10], which allows us to completely characterize the Hilbert space of “one-particle” states and the corresponding irreducible unitary representations of the de Sitter group. In this construction the correlation functions are boundary values of analytical functions. It should be noted that the analyticity condition is only preserved in the Euclidean vacuum. In a series of papers we generalized the Bros construction to the quantization of the various spin free fields in de Sitter space [11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

In the case of the interaction fields, the tree-level scattering amplitudes of the scalar field, with one graviton exchange, has been calculated in de Sitter space [21]. Recently, the electromagnetic classical fields produced by the geodesic and uniformly accelerated discrete charges in de Sitter space-time have been constructed by Bicak and Krtous [22, 23]. In this paper, we show that the $U(1)$ gauge theory in 4-dimensional de Sitter space describe the interaction between the spinor field (“electron”) and the massless vector fields (“photon”). The ambient space notation i.e. a coordinate independent way, has been used throughout this study.

In section 2, we briefly recall the ambient space notation and the analysis of the quantum free spinor field [11, 12, 13], and the quantum free vector field [11, 14, 16] in de Sitter space.
Various two-point functions for free spinor and vector fields in de Sitter space are then presented. In section 3, the $U(1)$ gauge invariant spinor field equation is obtained in the ambient space notation. Their corresponding conserved current is calculated in a coordinate independent way. The solution of the field equation is obtained by the use of the perturbation method. It has been shown that in the ambient space notation, the formalism of the quantum field in de Sitter space is very similar to the quantum field formalism in Minkowski space. The null curvature limit is discussed in the final stage. A brief discussion in section 4 concludes this paper.

2 Quantum free fields

2.1 Spinor fields
de Sitter space-time is visualized as the hyperboloid with equation:

$$X_H = \{ x^\alpha \in \mathbb{R}^5 : x.x = \eta_{\alpha\beta} x^\alpha x^\beta = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 - (x^4)^2 = -H^{-2} \}$$

$$\eta^{\alpha\beta} = \text{diag}(1, -1, -1, -1, -1); \ \alpha, \beta = 0, 1, \ldots, 4.$$ (1)

The kinematical group of the de Sitter space-time is $G_H = SO_{0}(1, 4)$. In this space, we need five $\gamma$ matrices instead of the usual four in Minkowski space-time. They are defined by the Clifford algebra:

$$\{ \gamma^\alpha, \gamma^\beta \} = \gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\eta^{\alpha\beta}, \ \gamma^{\alpha\dagger} = \gamma^0 \gamma^\alpha \gamma^0.$$ (2)

An explicit quaternion representation, which is suitable for symmetry consideration, is provided by [11, 12]

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^4 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & i\sigma^1 \\ i\sigma^1 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & -i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & i\sigma^3 \\ i\sigma^3 & 0 \end{pmatrix}$$ (2)

in terms of the $2 \times 2$ unit $I$ and Pauli matrices $\sigma^i$. Starting from the Casimir operator and using the infinitesimal generators, the Casimir eigenvalue equation and some algebraic relation dS-Dirac field equation is obtained [13, 24]

$$(-i \not x \gamma \bar{\partial} + 2i + \nu)\psi(x) = 0, \ \nu \in \mathbb{R}, \ \not x = x.\gamma,$$ (3)

where $\bar{\partial}_\alpha = \theta_{\alpha\beta} \partial^\beta = \partial_\alpha + H^2 x_\alpha (x.\partial)$. dS-Dirac plane waves solutions are [11, 12, 13]

$$\psi_1^{\xi, \nu}(x) = (H x.\xi)^{-2+i\nu} \mathcal{V}(x, \xi),$$

$$\psi_2^{\xi, \nu}(x) = (H x.\xi)^{-2-i\nu} \mathcal{U}(\xi),$$

where $\mathcal{V}$ and $\mathcal{U}$ are the polarization spinors and

$$\xi \in \mathbb{C}^+ = \{ \xi : \eta_{\alpha\beta} \xi^\alpha \xi^\beta = (\xi^0)^2 - \xi.\bar{\xi} - (\xi^4)^2 = 0, \ \xi^0 > 0 \}.$$ (3)

Due to the singularity and sign phase ambiguity, the solution are defined in the complex de Sitter space [8],

$$z \in X_H^{(c)} \equiv \{ z = x + iy \in \mathbb{C}^5 ; \ \eta_{\alpha\beta} z^\alpha z^\beta = (z^0)^2 - \bar{z}.z - (z^4)^2 = -H^{-2} \},$$
\[ \psi_1^\xi (z) = (Hz, \xi)^{-2+iv} \mathcal{V}(z, \xi), \]
\[ \psi_2^\xi (z) = (Hz, \xi)^{-2-iv} \mathcal{U}(\xi). \]

The spinor field operator is defined by the boundary value of complex solutions

\[ \psi(x) = \int_T \sum_{a=1,2} \{ a_a(\xi, \nu) \mathcal{U}^a(\xi)[(x,\xi)^2-2-i\nu + e^{-i\pi(-2+iv)}(x,\xi)^2-2+iv] + d_a^\dagger(\xi, \nu) H / \mathcal{V}^a(\xi)[(x,\xi)^2+2+i\nu + e^{-i\pi(2+iv)}(x,\xi)^2+2+iv] \} d\mu_T(\xi), \tag{4} \]

where \( T \) denotes an orbital basis of \( \mathbb{C}^+ \) and \cite{25}.

\[ d\mu_T(\xi) \] is an invariant measure on \( \mathbb{C}^+ \) \cite{9}. The vacuum state, which is fixed by imposing the condition that in the null curvature limit the Wightman two-point function become exactly the same as Minkowskian Wightman two-point function, is defined as follows

\[ a_a(\xi, \nu)|\Omega >= 0 = d_a(\xi, \nu)|\Omega >. \]

This vacuum, \( |\Omega > \), is equivalent to the Euclidean vacuum \( |0,0> \). “One particle” and “one anti-particle” states are

\[ d_a^\dagger(\xi, \nu)|\Omega >= |\xi, a, \nu >, \quad a_a^\dagger(\xi, \nu)|\Omega >= |\xi, a, \nu >. \tag{5} \]

Various two-point functions for scalar field can be calculated in terms of the Wightman two point function, \( \mathcal{W}^{\nu}(x, x') \) \cite{9},

- commutator function
  \[ \mathcal{C}^{\nu}(x, x') = \mathcal{W}^{\nu}(x, x') - \mathcal{W}^{\nu}(x', x), \]

- retarded propagator
  \[ G_r^{\nu}(x, x') = i\theta(x^{(0)} - x'^{(0)}) \mathcal{C}^{\nu}(x, x'), \]

- advanced propagator
  \[ G_a^{\nu}(x, x') = G_r^{\nu}(x, x') - i\mathcal{C}^{\nu}(x, x'), \]

- chronological propagator
  \[ \mathcal{T}^{\nu}(x, x') = -iG_r^{\nu}(x, x') + \mathcal{W}^{\nu}(x, x') = -iG_a^{\nu}(x, x') + \mathcal{W}^{\nu}(x, x'), \]

- anti-chronological propagator
  \[ \bar{\mathcal{T}}^{\nu}(x, x') = iG_r^{\nu}(x, x') + \mathcal{W}^{\nu}(x, x') = iG_a^{\nu}(x, x') + \mathcal{W}^{\nu}(x, x'). \tag{6} \]
where all above functions have been presented in the coordinate independent way notation. For the spinor field, the matrix Wightman two point function is \([11, 12, 13]\)

\[
W_\nu(x, x') = \langle \Omega, \psi(x) \bar{\psi}(x') \Omega \rangle = D(x', \partial_{x'}) W_0(x, x') = \frac{i\nu(1 + \nu^2)}{64\pi \sinh(\pi\nu)} \times \\
[ (2 - i\nu)P_{-2-i\nu}(Z(x, x')) \not{x} - (2 + i\nu)P_{-2+i\nu}(Z(x, x')) \not{x}' ] \gamma^4,
\]

where \(\bar{\psi} = \psi^\dagger \gamma^0 \gamma^4\). \(W_0(x, x')\) is the two-point function of the scalar field and \(D(x', \partial_{x'})\) is the matrix differential operator \([11]\)

\[
D(x', \partial_{x'}) = \frac{1}{\nu + i}(-i \not{x'} \partial_{x'} + i + \nu).
\]

\(P_{-2-i\nu}(Z(x, x'))\) is a generalized Legendre function \([9]\) and

\[
Z(x, x') = -H^2 x.x' = 1 + \frac{H^2}{2} (x - x')^2 \equiv \cosh H\sigma(x, x').
\]

Similar to the case of Minkowski space \([26]\), various two-point functions for spinor field are obtained from the corresponding functions of the scalar field by:

\[
S^Z_\nu(x, x') = D(x', \partial_{x'}) W^Z_0(x, x'),
\]

where \(W^Z_0\) are the various two-point functions for scalar field eq. (6). \(Z\) stands for retarded, advanced, chronological and anti-chronological propagators. For example, the anti-commutator function is given by \([13]\)

\[
\mathcal{A}'(x, x') = \langle \Omega, \{ \psi(x) \bar{\psi}(x') \} \Omega \rangle = \frac{i\nu(1 + \nu^2)}{32\pi R^2} \epsilon(x^0 - x'^0) \theta(Z - 1) \times \\
[ (2 - i\nu)P_{-2-i\nu}(Z(x, x')) \not{x} + (2 + i\nu)P_{-2+i\nu}(Z(x, x')) \not{x}' ] \gamma^4,
\]

where \(\theta\) is the Heaviside step function and

\[
\epsilon(x^0, x'^0) = \begin{cases} 
1 & x^0 > x'^0 \\
0 & x^0 = x'^0 \\
-1 & x^0 < x'^0.
\end{cases}
\]

### 2.2 Vector fields

Starting from the Casimir operator and using the infinitesimal generators, the Casimir eigenvalue equation and some algebraic relation the field equation for a massless vector field in the ambient space notation is obtained \([11, 14, 16]\)

\[
(H^{-2}(\bar{\partial})^2 + 2) K(x) - 2x\bar{\partial}.K(x) - H^{-2}\partial x.\partial K = 0.
\]

\(5\)
This five-component vector field quantity has to be viewed as a homogeneous function of the $\mathbb{R}^5$-variables $x^\alpha$ with some arbitrarily chosen degree $\sigma$ [24]

$$x^\alpha \frac{\partial}{\partial x^\alpha} K_\beta(x) = x \cdot \partial K_\beta(x) = \sigma K_\beta(x).$$

(10)

The direction of $K_\alpha(x)$ lies in the de Sitter space if we require the condition of transversality $x \cdot K(x) = 0$. The above field equation is gauge invariant, i.e.

$$K \rightarrow K' = K + H^{-2} \partial \phi_g$$

(11)

$$(H^{-2}(\partial)^2 + 2)K'(x) - 2x\partial.K'(x) - H^{-2}\partial \phi_g.K'(x) = 0,$$

(12)

where $\phi_g$ is an arbitrary scalar field. Similar to the flat space massless vector field, the gauge fixing is accomplished by adding to (9) a gauge fixing term. We obtain [11, 16]

$$(H^{-2}(\partial)^2 + 2)K(x) - 2x\partial.K(x) - cH^{-2}\partial \phi_g.K = 0,$$

(13)

where $c$ is the gauge fixing parameter. The simple choice for $c$ is $\frac{2}{3}$. The plane wave solutions in this gauge, are [11, 16]

$$K_1^{\xi,\lambda}(z) = \mathcal{E}_1^{(\lambda)}(z, \xi)(Hz, \xi)^{-1},$$

$$K_2^{\xi,\lambda}(z) = \mathcal{E}_2^{(\lambda)}(z, \xi)(Hz, \xi)^{-2},$$

(14)

where $\mathcal{E}_{1,2}^{(\lambda)}(z, \xi)$ are the polarization vectors. In this gauge, we can write the ”massless” vector field in terms of the polarization vectors and a ”massless” conformally coupled scalar field.

In this gauge, the Wightman vector two-point function is [11, 16]

$$W_{\alpha\alpha'}(x, x') = \langle \Omega, K_\alpha(x)K_{\alpha'}(x')\Omega \rangle = D_{\alpha\alpha'}(x, x', \partial_x)W_0(x, x'),$$

where $W_0(x, x')$ is the Wightman two-point function of the conformally coupled scalar field [27, 11]

$$W_0(x, x') = -\frac{1}{8\pi^2} \left[ \frac{H^2}{1 - \mathcal{Z}(x, x')} - i\pi H^2 \epsilon(x^0 - x'^0)\delta(1 - \mathcal{Z}(x, x')) \right],$$

(15)

and $D_{\alpha\alpha'}(x, x', \partial_x)$ are the bi-tensor differential operator

$$D_{\alpha\alpha'}(x, x', \partial_x) = \theta_\alpha \cdot \theta_{\alpha'} + H^{-2} \partial_\alpha \left[ \bar{\partial} \theta_{\alpha'} + H^2 x \theta_{\alpha'} \right].$$

(16)

Similar to the spinor field, the various two point functions for massless vector field are obtained from the corresponding functions of the scalar field according to the formula

$$G_{\alpha\alpha'}^Z(x, x') = D_{\alpha\alpha'}(x, x', \partial_x)G_0^Z(x, x'),$$

(17)

where $G_0^Z$ stands for the two point functions of scalar fields eq. (6). For example, the commutator function is

$$G_{\alpha\alpha'}(x, x') = \langle \Omega, [K_\alpha(x)K_{\alpha'}(x')][\Omega \rangle = D_{\alpha\alpha'}(x, x', \partial_x)G_0(x, x')$$

$$= \left( \theta_\alpha \cdot \theta_{\alpha'} + H^{-2} \partial_\alpha \left[ \bar{\partial} \theta_{\alpha'} + H^2 x \theta_{\alpha'} \right] \right) - \frac{H^2}{8\pi} \epsilon(x^0 - x'^0)\delta(1 - \mathcal{Z}(x, x')),$$

(18)

Where $G_0(x, x')$ is the commutator function of the conformally coupled scalar field [27, 11].
3 Gauge invariant field equation

dS-Dirac field equation is invariant under the $U(1)$ global symmetry,

$$\psi(x) \longrightarrow \psi'(x) = e^{-i\Lambda}\psi(x)$$

(19)

$$(-i \not\! x \gamma. \bar{\partial} + 2i + \nu)\psi'(x) = 0,$$

where $\Lambda$ is a constant. This equation is not invariant under the $U(1)$ local gauge symmetry

$$\psi(x) \longrightarrow \psi'(x) = e^{-i\Lambda(x)}\psi(x),$$

(20)

$$(-i \not\! x \gamma. \bar{\partial} + 2i + \nu)\psi'(x) \neq 0.$$  

(21)

The notation of local gauge symmetry with its space-time-dependent transformation can be used to generate the gauge interaction. The abelian $U(1)$ local symmetry is defined by the interaction between the electron field and the electromagnetic field i.e. “electron-photon” interaction.

For obtaining a dS-Dirac local gauge invariant equation, it is necessary to replace the covariant derivative $\bar{\partial}_\beta$ with the gauge-covariant derivative $D_\beta$ which is defined by

$$D_\beta = \bar{\partial}_\beta + iqA_\beta,$$

where $A_\beta$ is a new vector field or the gauge field, and $q$ is a free parameter which can be identifies with the electric charge in the null curvature limit. If the gauge field $A_\beta(x)$ has the transformation property

$$A_\beta \longrightarrow A'_\beta = A_\beta + \frac{1}{q} \bar{\partial}_\beta \Lambda(x),$$

(22)

the gauge-covariant derivative of the spinor field has the following simple transformation

$$D_\beta \psi(x) \longrightarrow [D_\beta \psi(x)]' = e^{-i\Lambda(x)}D_\beta \psi(x),$$

(23)

Comparing equation (22) with the equation (11), the simplest choice of $A_\beta$ (gauge-invariant vector field) is the massless vector field $K(x)$, which was presented in the previous section. Thus the dS-Dirac local gauge invariant equation is

$$(-i \not\! x \gamma. D + 2i + \nu)\psi(x) = 0,$$

$$(-i \not\! x \gamma. \bar{\partial} + q \not\! x \gamma. K + 2i + \nu)\psi(x) = 0.$$  

(24)

This equation is simultaneously invariant under the two transformations (20) and (22)

$$(-i \not\! x \gamma. D' + 2i + \nu)\psi'(x) = 0,$$

where

$$D'_\beta = \bar{\partial}_\beta + iqK'_\beta.$$

The Noether’s theorem states that any lagrangian invariant under a continuous one-parameter transformation, is associated with a local conserved current. By using the dS-Dirac equation for $\psi$ and $\bar{\psi}$ [13],

$$\bar{\psi}\gamma^\lambda(i \bar{\partial} \not\! x - 2i + \nu) = 0,$$
the free field lagrangian is defined by
\[ L_0 = H \bar{\psi} \gamma^\alpha (-i \not{\partial} + 2i + \nu) \psi(x). \]
Thus the invariance of the free field lagrangian under the transformation (19) leads to the following conserved current
\[ J_\alpha(x) = \frac{\delta L_0}{\delta \partial^\alpha \psi} \frac{\delta \bar{\psi}}{\delta \Lambda} = -H \bar{\psi}(x) \gamma^\alpha \not{\partial} \psi(x). \] (25)
It is easily verified that this current satisfies the following conditions
\[ \bar{\partial} \cdot J(x) = 0, \quad x \cdot J(x) = 0. \]
The field equation (24) gives the interaction between the spinor field (electron) and the massless vector field (photon). We can write the interaction Lagrangian between electron and photon in the following way
\[ L_{\text{int}} = qH \bar{\psi}(x) \gamma^\alpha \not{\partial} \psi(x). \]
By choosing electromagnetic field \( K(x) \), we can obtain the solution of the field equation (24) by perturbation method. It can be written in the following form
\[ (-iH \not{\partial} + 2iH + \nu H) \psi(x) = -qH \not{\partial} \psi(x). \] (26)
In practice, this equation explains the interaction of an electron field \( \psi(x) \) and an electromagnetic field \( K(x) \) in de Sitter space. Its solution is discussed by the use of the Green function method. The matrix Green function, is solution of the following equation
\[ (-iH \not{\partial} + 2iH + \nu H) S^\nu_{\nu'}(x, x') = \delta_H(x - x'), \]
where \( S^\nu_{\nu'}(x, x') \) is retarded or advance Green function, obtained in the previous section (eq. (8)). \( \delta_H(x - x') \) is the Dirac delta function in the ambient space notation on the de Sitter hyperboloid
\[ \int d\sigma(x') \delta_H(x - x') f(x') = f(x), \] (27)
where \( d\sigma(x') \) is the dS-invariant volume [9] and \( f(x) \) is a homogeneous function (with some arbitrarily chosen degree) of the \( \mathbb{R}^5 \)-variables \( x^\alpha \). Therefore the solution of the field equation can be written in the following form
\[ \psi(x) = \psi_0(x) + (-qH) \int d\sigma(x') S^\nu(x, x') \not{\partial} \psi(x'), \]
where \( \psi_0(x) \) is a free field solution (3). In the perturbation theory, solution is
\[ \psi(x) = \psi_0(x) + (-qH) \int d\sigma(x') S^\nu(x, x') \not{\partial} \psi_0(x') \]
\[ + (-qH)^2 \int d\sigma(x') d\sigma(x'') S^\nu(x, x') \not{\partial} \psi_0(x') S^\nu(x', x'') \not{\partial} \psi_0(x'') + O(q^3). \] (28)
In this notation, the de Sitter QFT formalism is very similar to the Minkowskian counterpart.

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Finally, we consider the Minkowskian limit \((H = 0)\). The dS point \(x = x_H(X)\) has been expressed in terms of the Minkowskian variable \(X = (X_0 = ct, \vec{X})\) measured in units of the dS radius \(H^{-1}\):

\[
x_H(X) = \left( x^0 = H^{-1} \sinh HX^0, \bar{x} = H^{-1} \frac{\vec{X}}{||\vec{X}||} \cosh HX^0 \sin H ||\vec{X}||, \right.
\]
\[
x^4 = H^{-1} \cosh HX^0 \cos H ||\vec{X}||. \quad (29)
\]

Note that \((X^0, \vec{X})\) are global coordinates. In the null curvature limit, we have \(x^\alpha = (X^0, \vec{X}, H^{-1}) = (X^\mu, x^4 = H^{-1})\). In this limit the field equation (3) reads \([11, 13]\)

\[
(i\gamma^\mu \gamma^4 \partial_{\mu} - q\gamma^\mu \gamma^4 K^\mu(x) - \nu H)\psi(x) = 0, \quad (30)
\]

where in the null curvature limit \(\nu H \rightarrow m\) and \(q \rightarrow e\). They are mass and electric charge of the electron, respectively, in the Minkowski space. \(\gamma^\mu \gamma^4 = \gamma^\mu\) are the usual Dirac matrices in the Minkowski space. This is exactly the Dirac equation in the presence of an electromagnetic field in Minkowski space

\[
(i\gamma'_\mu \partial_{\mu} - e\gamma'_\mu A^\mu(X) - m)\psi(X) = 0. \quad (31)
\]

4 Conclusions

The formalism of the quantum field in de Sitter universe, in ambient space notation, is very similar to the quantum field formalism in Minkowski space. The Fourier-Bros transformation on the hyperboloid allows us to write the fields in terms of two separate parts, a polarized and “plane wave” part. In this notation the concept of the “particle states”, contrary to the concept of energy, is defined globally. The Fock space is constructed in terms of the five-vector \(\xi\), which in the Minkowskian limit is the four energy-momentum vector \((p^\mu)\). The interaction between the quantum charge particle and a classical electromagnetic field can be described by the perturbation method in terms of Green function, very similar to the Minkowski space.

Similarly nonabelian gauge fields can be constructed as well. The importance of this formalism may be shown further by the consideration of the linear quantum gravity and supergravity in de Sitter space, which lays a firm ground for further study of universe.

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