Dipole-exchange spin waves in Fibonacci magnetic multilayers

J. Milton Pereira Jr. and R. N. Costa Filho
Departamento de Física, Universidade Federal do Ceará, Caixa Postal 6020
Campus do Pici, 60451 – 970 Fortaleza, Ceará, Brazil

A microscopic model is employed to calculate the spectrum of spin waves in quasiperiodic magnetic multilayers in the dipole-exchange regime. Results are presented for structures in which thin ferromagnetic films are separated by non-magnetic spacer following a Fibonacci sequence and extend previous magnetostatic calculations. The results show the splitting of the frequency bands and the mode mixing caused by the dipolar interaction between the films as a function of spacer thickness, as well as the fractal aspect of the spectrum induced by the non-periodic aspect of the structure.

I. INTRODUCTION

The development of ultrathin film deposition techniques has allowed the production of layered materials with properties that are not found in bulk media. Layered magnetic structures, in particular, have attracted a great deal of attention due to their possible use in novel microelectronic devices [1]. The transport properties, as well as the collective excitations, of magnetic structures in which magnetic films are interspersed with non-magnetic layers were shown to be quite distinct from those of single films. These new properties are a consequence of the fact that the interactions between microscopic elements of the system can be influenced by the structure of the multilayer. Thus, by adjusting the composition and the thickness of the films, one gains the ability to modify the spectrum of excitations of the structure. One way of achieving that control is by changing the thickness of the non-magnetic layers, since that can alter the strength of the dipolar field between the magnetic films. Several studies have investigated the effect of this long range interlayer interaction on the magnetostatic modes of magnetic structures [2, 3, 4]. Recently, a microscopic theory was developed for the regime in which the dynamic aspect of the exchange interaction becomes relevant [5]. In this dipole-exchange regime, the dipolar interaction acts to couple the volume spin wave (SW) modes of the magnetic layers and thus causes significant modifications of the SW frequency branches, in comparison with the results for the magnetostatic case.

So far, the microscopic dipole-exchange SW theory has been applied to layered media in which the thickness of the spacers is assumed to be constant throughout the structure. On the other hand, several studies have focused on the properties of layered structures in which the width of the elements is varied along its thickness. The interest on these structures was motivated by reports on the existence of a quasicrystalline phase in metallic alloys [6], which have been shown to display long range orientational order, but are not invariant under lattice translations. One type of artificial quasiperiodic structured considered is the Fibonacci multilayer, i.e. a multilayer with elements that have widths that vary according to a binary substitutional sequence [8, 9]. The elementary excitations of these multilayers have been extensively studied, both theoretically as well as experimentally (for a review, see [10]).

To date, work on spin excitations in quasiperiodic magnetic media has been restricted to the magnetostatic regime, in which the spins of the magnetic layers precess uniformly [11, 12]. These calculations, therefore, do not address the influence of the multilayer structure on the standing volume SW modes of each layer, which have amplitudes that vary along the thickness of the films. The influence of the exchange interaction is also expected to become particularly relevant for structures with ultrathin layers (i.e., films with thickness of a few monolayers). Moreover, the dipole-exchange wave vector regime in thin films has recently become experimentally accessible, due to the development of new experimental techniques that allow the observation of the SW dispersion across the whole Brillouin zone [13].

In this paper we use the microscopic model presented in Ref. [6] to the study of magnetic excitations in Fibonacci multilayers in the dipole-exchange regime. The description takes into account the position dependence of the microscopic spins in each layer. This model has been applied previously to obtain the spin wave spectrum of ultrathin films of ferromagnets [14] and antiferromagnets [15]. The paper is structured as follows: in section 2 the structure of the multilayer is described and a microscopic Hamiltonian for the system is introduced. In section 3, numerical results are presented and discussed for multilayers of GdCl₃ and EuO. In section 4 the main results are summarized and a brief conclusion is presented.

II. MODEL

Let us consider a system containing \( N_f \) ferromagnetic films separated by non-magnetic films (spacers), as shown in Fig. 1. In the following discussion, the ferromagnets and spacers are referred to as films, whereas the spin layers in each ferromagnet are referred to as atomic layers. The films have a simple cubic crystal structure, with lattice constant \( a \) and have ideal interfaces corresponding to (001) crystal planes. Each of the magnetic films is a single domain and contains \( N_m \) atomic layers (in the
for the system is localized spins is included explicitly. The Hamiltonian formalism in which the dipolar coupling between $J_{ij}$ and $J_{jk}$ is the exchange between nearest-neighbor sites $i$ and $j$ in the same film $H_0$ is the Zeeman field, taken as parallel to the $z$-direction. The last term in the Hamiltonian represents the contribution of the dipolar interaction. The $D_{lm}^{\alpha\beta}$ are the long-range dipolar coupling coefficient between any sites $l$ and $m$ in the same or different films. The $\alpha$ and $\beta$ indices denote components $x$, $y$ or $z$; $g$ is the Landé factor and $\mu_B$ is Bohr’s magneton. The expression for the dipolar factors is

$$D_{lm}^{\alpha\beta} = \frac{|\mathbf{r}_{lm}|^2 \delta_{\alpha\beta} - 3 \mathbf{r}_{lm}^{\alpha} \mathbf{r}_{lm}^{\beta}}{|\mathbf{r}_{lm}|^5},$$

which, for low temperatures (i.e. $T << T_c$, where $T_c$ is the Curie temperature of the ferromagnets) can be written as $S_i^z \approx \sqrt{2S} S_i^z$, $S_i^z \approx \sqrt{2S} a_i^\dagger a_i$, where $a_i^\dagger$ and $a_i$ are boson creation and annihilation operators, respectively.

The system is

$$\mathcal{H} = -\sum_{i,j} J_{ij} S_i \cdot S_j - g\mu_B \sum_i H_0 S_i^z + (g\mu_B)^2 \sum_{\alpha,\beta} \sum_{l,m} D_{lm}^{\alpha\beta} S_l^{\alpha} S_m^{\beta},$$

where $\mathbf{r}_{lm} = \mathbf{r}_l - \mathbf{r}_m$ connects magnetic sites in the lattice. In order to calculate the SW frequencies, one can write down the equations of motion for the operators in the films. These are then transformed to a representation involving a two-dimensional in-plane wave vector $\mathbf{k} = (k_x, k_y)$ parallel to the film surfaces. The resulting system of equations can be solved numerically for the frequency $\omega$ of the modes. The Fourier amplitudes of the dipolar terms can be expressed in terms of rapidly converging summations, as shown in Ref \cite{14}. For multilayers, the expressions for the amplitudes are modified by the presence of the spacers. By increasing the thickness of the spacers, the dipolar coupling between the ferromagnetic films can eventually become negligible, and the resulting SW spectrum must then reproduce the results for the single film. The spin Hamiltonian can be rewritten in terms of boson creation and annihilation operators by means of the Holstein-Primakoff transformation \cite{16} which, for low temperatures (i.e. $T << T_c$, where $T_c$ is the Curie temperature of the ferromagnets) can be written as $S_i^z \approx \sqrt{2S} S_i^z$, $S_i^z \approx \sqrt{2S} a_i^\dagger a_i$, where $a_i^\dagger$ and $a_i$ are boson creation and annihilation operators, respectively.

Next, the equations of motion for the creation and annihilation operators can be obtained, and be transformed to a representation involving a two-dimensional (2D) in-plane wave vector $\mathbf{k}_n = (k_x, k_y)$, where $n$ is an index assigned to each magnetic atomic layer of the system. The Fourier transforms of the dipole sums in the Hamiltonian are similar to the terms calculated for a single ferromagnetic film \cite{14}, with additional terms including extra distance factors, which correspond to the dipolar interaction between spins in different films. In the present case, the different values of thickness of the spacers correspond to a quasiperiodic modulation of the distance factors. This contrasts with the previous theories of Fibonacci multilayers, which were based on transfer matrix formalisms. The present method allows us to introduce the quasiperiodic aspect in the structure in a more straightforward way, by means of a simple modification of the dipole sums expressions of a single film. Specifically, the expressions

![FIG. 1: Schematic depiction of a Fibonacci magnetic multilayer. The black dots represent ferromagnetic spins, and the white dots are non-magnetic sites. In this figure, the magnetic films have thickness $D = 2a$, the B spacers have widths $d = 2a$ and the A spacers have thickness $d + \delta$, with $\delta = 2a$.](image)
for the dipole sums that describe the interactions between spins in different atomic layers contain exponential terms of the type

\[ e^{-2|y|\gamma_{ab}}, \]

where \(|y|\) is the distance between the layers, \(\gamma_{ab}\) is a function of the in-plane wavevectors and \(a\) and \(b\) are dummy indices \(\text{[14]}\). Thus, for a pair of spins located in atomic layers of different films, an extra distance, corresponding to the thickness of the spacers located between said films, must be added to \(|y|\), which would otherwise correspond to the distance between the spins if they were located in the same film. Thus, when writing the expressions for the dipole sums, one must keep track of the distances between each pair of spins. If one labels the films from 1 to \(F_u + 1\), the extra separation between two spins in two given films with labels \(t\) and \(u\) is

\[ |\Delta(t, u)| = |t - u|(D + d) + L(t, u) \delta. \]

where \(L(t, u)\) is a function that gives the number of \(A\) blocks found between the films \(t\) and \(u\), and has the following properties: \(L(t, u) = L(u, t)\), \(L(t, t) = 0\), \(L(t, u) - L(t, u') = L(u, u')\) and (for \(u > t\)), \(L(t, u + 1) = L(t, u) + L(t + 1, u + 1) - L(t + 1, u)\).

Having calculated the expressions for the dipole sums, one can write down the equations of motion for the SW modes in matrix form. The SW dispersion can then be calculated numerically as the eigenvalues of a \((2N_l N_m)\times (2N_l N_m)\) matrix. the generation number.

III. NUMERICAL RESULTS

Solutions for SW dispersion relations were calculated numerically for Fibonacci multilayers with parameters corresponding to the ferromagnet EuO \((T_c \approx 69\, \text{K})\) and GdCl\(_3\) \((T_c \approx 2.2\, \text{K})\). The parameters for the dipolar coupling strength (given in terms of the bulk saturation magnetization) and the exchange field were, for the GdCl\(_3\), \(4\pi M = 0.82\, \text{T}\) and \(H_{ex} = 0.54\, \text{T}\), respectively, and \(4\pi M = 2.4\, \text{T}\) and \(H_{ex} = 38\, \text{T}\) for the EuO, where in both cases \(g\mu_B H_{ex} = 6SJ\).

Figure 2 shows the lowest SW frequency bands as a function of reduced wavevector, for a EuO structure with 35 ferromagnetic thin films \((n = 8)\), each comprising 4 atomic layers \((D = 4a)\), with the \(B\) type spacers having thickness \(d = 5a\), and the \(A\) spacers having \(d = 20a\) \((\delta = 15a)\), with zero external field. The frequencies were calculated in the Voigt geometry (i.e. when both the magnetization and the field are on the plane and \(k_y = 0\)) and given in units of GHz (using \(\gamma = 28\, \text{GHz/T}\)). In the absence of the dipolar coupling, the dispersion would display four branches (each consisting of 35 degenerate modes), with the characteristic sinusoidal dependence on \(k\). These modes arise due to the quantization of the volume modes in each magnetic film. Since we assumed that the ferromagnets have a simple cubic crystal structure,

\[ H_{ex} = 38\, \text{T} \]

In this case each ferromagnet has 4 atomic layers, whereas the spacers have thicknesses \(d = 20a\) (A) and \(d = 5a\) (B), where \(a\) is the lattice parameter of the ferromagnets.

![FIG. 2: Spin wave dispersion relation for a EuO structure with 35 films (34 spacers), for an external field \(H_0 = 0.0\, \text{T}\).](image1)

![FIG. 3: Spin wave dispersion relation for a GdCl\(_3\) structure with 22 films (21 spacers), for an external field \(H_0 = 0.36\, \text{T}\).](image2)
and also that the strength of the exchange interaction was the same for all film layers, the exchange-dominated regime does not lead to the existence of surface SW modes. As in the periodic case, by introducing the dipolar interaction, the volume SW modes become coupled, which lifts their degeneracy and causes the appearance of frequency bands at small wavevectors. Furthermore, the dipole-dipole coupling causes the appearance of surface SW bands, which result from a widening of the Damon-Eshbach modes of a single film. However, in contrast with the periodic multilayer results, the non-periodic aspect of the present structure causes the appearance of small gaps and the splitting of the branches into sets of subbands. This behavior can be observed in Fig. 2, where the inset shows the structure of minigaps and subbands. According to previous continuum calculations for thick films, the number and distribution of these subbands tend to form a Cantor set, with a hierarchy of bands and gaps. This Cantor set-like structure can also be observed in Fig. 2. A further consequence of the quasiperiodic structure is the appearance of isolated modes, which are found between subbands. One of such isolated modes can be identified in the inset in Fig. 2. These isolated branches are usually found in quasiperiodic structures and have been shown previously to correspond to excitations that are neither extended nor localized, but have instead a critical behavior, with amplitudes that display a power-law aspect. 

The SW dispersion for a GdCl$_3$ structure is shown in Fig. 3. In this case, the multilayer contains 35 magnetic films and 34 spacers. The magnetic films have thickness $D = 5a$, whereas for the spacers the values $d = 20a$ (A spacers) and $d = 5a$ (B spacers) were used, for an external field $H_0 = 0.36$ T. The figure shows the surface SW band, along with two volume bands. Not shown in the figure are the two higher frequency bands, which display the same pattern of gaps and subbands as the lower ones. As in the previous figure, the results show the formation of frequency bands at small wavevectors, with the appearance of small gaps inside the bands. The present results, however, also show a strong mixing of the surface SW band (which corresponds to a broadening of the Damon-Eshbach modes) and the lowest lying bulk band. A consequence of this mixing is the presence of a frequency gap at small wave vectors, induced by a mode repulsion effect. This feature has also been reported for the periodic multilayer case. 

Figure 4 shows a plot of the lowest SW frequencies as a function of the thickness parameter $\delta$ for a EuO multilayer containing 35 magnetic films, for $k_x = 0.03\pi/a$, $D = 5a$ and zero external field. For $\delta = 0$, the results correspond to the spectrum of a periodic structure with $d = 5a$. As the thickness of the A spacers increases, the results show the appearance of gaps and the formation of subbands. A similar behavior is found for a multilayer with 35 films of GdCl$_3$, as shown in Fig. 5, for $k_x = 0.05\pi/a$. In this case the B spacers have thickness $d = 5a$, and the ferromagnets have $D = 8a$. For the periodic case ($\delta = 0$), the results correspond to 8 frequency bands, which arise from the 8 SW modes of the films. As the thickness of the A spacers increases, the figure shows the splitting of the four lowest bands. The same effect is observed in the upper bands. For larger values of $\delta$, the greater distance between magnetic films separated by the A spacers means that the influence of the dipolar interaction across them becomes negligible, and the results match the SW spectrum of bilayers (separated by the B spacers) and single films (i.e. films located between A spacers).

IV. CONCLUSIONS

We presented the first results for the dipole-exchange spin wave spectrum of quasiperiodic magnetic multilayers in which thin ferromagnetic films are separated by non-magnetic spacers. The thickness of the spacers was assumed to follow a Fibonacci binary string created by a substitutional rule. Results were obtained by means of a microscopic spin Hamiltonian that includes the short-range exchange interaction and the long-range dipole-dipole coupling between localized spins. The dipolar interaction, by coupling the magnetic films, acts to lift the degeneracy of the surface and standing volume modes of the films, thus creating frequency bands, which are dis-
FIG. 5: Spin wave frequency branches as a function of the spacers thickness ratio, for a GdCl₃ structure with 35 films (34 spacers), for an external field \( H_0 = 0.36 \) T and \( k_x = 0.05\pi/a \). In this case each ferromagnet has 8 atomic layers and the \( B \) spacers have thicknesses \( d = 5a \), where \( a \) is the lattice parameter of the ferromagnets.

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