Bichromatic Local Oscillator for Detection of Two-Mode
Squeezed States of Light

Alberto M. Marino, C. R. Stroud, Jr., Vincent Wong, Ryan S. Bennink, and Robert W. Boyd

The Institute of Optics, University of Rochester, Rochester, New York 14627, USA

Abstract

We present a new technique for the detection of two-mode squeezed states of light that allows for a simple characterization of these quantum states. The usual detection scheme, based on heterodyne measurements, requires the use of a local oscillator with a frequency equal to the mean of the frequencies of the two modes of the squeezed field. As a result, unless the two modes are close in frequency, a high-frequency shot-noise-limited detection system is needed. We propose the use of a bichromatic field as the local oscillator in the heterodyne measurements. By the proper selection of the frequencies of the bichromatic field, it is possible to arbitrarily select the frequency around which the squeezing information is located, thus making it possible to use a low-bandwidth detection system and to move away from any excess noise present in the system.

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I. INTRODUCTION

The initial interest in squeezed states was stimulated by the possibility of increasing the sensitivity of interferometers for applications such as gravitational wave detection and verification of relativistic effects. Since then, the field has expanded to other areas such as quantum optics and atomic physics with the development of squeezed states of the electromagnetic field [1, 2] and spin squeezed states [3, 4].

In recent years, multi-mode squeezed states [5, 6] have gained much attention due to the fact that they contain quantum correlations between the different modes that make up the field. A specific case of such states is the two-mode squeezed state (TMSS) whose importance resides in the fact that it is the main source of continuous-variable entanglement and Einstein-Podolsky-Rosen (EPR) type correlations. As a result, this quantum state of the electromagnetic field has found its way into applications such as continuous-variable teleportation [7, 8], quantum key distribution [9, 10, 11], verification of EPR correlations [12, 13], etc.

Although the TMSS has become a fundamental tool for the study of continuous-variable entanglement, its experimental characterization still remains a problem. A number of papers [14, 15, 16] have focused on improving either the temporal or spatial character of the local oscillator (LO) used in heterodyne detection in order to optimize the degree of squeezing measured. However, no attention has been given to the necessary requirements of the detection system needed for such measurements.

The usual detection scheme used for the characterization of a TMSS is based on balanced heterodyne measurements. It requires the use of a LO with a frequency equal to the mean of the frequencies of the two modes of the squeezed field. As a result, the squeezing information is located around the beat note frequency between the LO and either of the field modes that constitute the squeezed state. In general the beat note frequency can be arbitrarily large, thus requiring a high-bandwidth shot-noise-limited detector to perform the measurement. The combined requirements of high bandwidth and low noise place difficult constraints on the detection system, since the electronic noise of the system usually increases as the bandwidth of the detection system increases.

In this paper we present a simple scheme based on a bichromatic local oscillator (BLO) that makes it possible to greatly reduce the bandwidth requirements of the detection system.
Characterization of a TMSS is then much more accessible, for example by using the simple low-frequency design of Gray et al. As will be shown, by the proper selection of frequencies of the bichromatic field it is possible to arbitrarily select the frequency around which the squeezing information is located, thus making it possible to use a low-bandwidth detection system to characterize a TMSS source. Since the measurement frequency can be arbitrarily selected, it is also possible to move away from any excess noise present in the system.

The rest of the paper is organized as follows. In Sec. II we give a general overview of the basic theory of heterodyne detection for the characterization of a TMSS. Then, in Sec. III we introduce the concept of the BLO and show the advantages and limitations of such a detection technique.

II. BALANCED HETERODYNE DETECTION

The most commonly used technique for the detection of a TMSS is balanced heterodyne detection. In general, this technique consists of combining the squeezed field being measured with a strong LO and detecting each of the resulting fields with a photodetector, as shown in Fig. 1. Combining the two detector signals we obtain the difference signal and analyze the noise in this signal with a spectrum analyzer.

\[ \hat{d}_1 = t\hat{E}_S + r\hat{E}_{LO} \]
\[ \hat{d}_2 = r\hat{E}_S + t\hat{E}_{LO}, \]

FIG. 1: Balanced heterodyne detection scheme used for the characterization of squeezed light. Notation: LO = local oscillator; BS = beam splitter.

The fields after the beam splitter are given by
where $\hat{E}_S$ and $\hat{E}_{LO}$ are the positive frequency parts of the squeezed and local oscillator fields and $t$ and $r$ are the transmissivity and reflectivity of the beam splitter. In general, $t$ and $r$ satisfy the relations $|t|^2 + |r|^2 = 1$ and $t^*r = i|rt|$. In order to have a balanced detection scheme, the beam splitter must satisfy the condition $|t| = |r| = 1/\sqrt{2}$, such that the difference signal from the balanced heterodyne detection takes the form

$$\hat{I}_{12} = d_1^\dagger d_1 - d_2^\dagger d_2 = i(\hat{E}_S^\dagger \hat{E}_{LO} - \hat{E}_{LO}^\dagger \hat{E}_S).$$

(2)

As can be seen from this equation, only the interference terms are left for the balanced case.

For a TMSS, the field takes the form [21]

$$\hat{E}_S = \hat{a}_+ e^{-i\omega_Lt} + \hat{a}_- e^{-i\omega_Lt}$$

(3)

and the quadratures are defined according to

$$\hat{X} = \frac{1}{2\sqrt{2}}(\hat{a}_+ + \hat{a}_-^\dagger + \hat{a}_- - \hat{a}_-^\dagger)$$

$$\hat{Y} = \frac{1}{i2\sqrt{2}}(\hat{a}_+ - \hat{a}_+^\dagger + \hat{a}_- - \hat{a}_-^\dagger).$$

(4)

With these definitions and the properties of the TMSS, the variance of the quadratures can be shown to be given by [18]

$$\langle (\Delta \hat{X})^2 \rangle = \frac{1}{4} \left( e^{-2s} \cos^2 \theta + e^{2s} \sin^2 \frac{\theta}{2} \right)$$

$$\langle (\Delta \hat{Y})^2 \rangle = \frac{1}{4} \left( e^{-2s} \sin^2 \frac{\theta}{2} + e^{2s} \cos^2 \frac{\theta}{2} \right),$$

(5)

where $s$ is the degree of squeezing and $\theta$ is the squeezing angle. As a result, the variance along the major and minor axes of the squeezing ellipse is given by

$$\langle (\Delta \hat{X})^2 \rangle_{\text{min}} = \frac{1}{4} e^{-2s}$$

$$\langle (\Delta \hat{Y})^2 \rangle_{\text{max}} = \frac{1}{4} e^{2s}.$$  

(6)

In the standard heterodyne technique, the LO is taken to be of the form

$$\hat{E}_{LO} = \hat{b} e^{-i\omega_Lt}$$

(7)

and is assumed to be in a coherent state, such that $\langle \hat{b} \rangle = |\beta| e^{i\chi}$. In order to obtain a measurement that is time independent, the frequency of the LO has to be selected between
FIG. 2: Frequency components involved in a heterodyne measurement of a two-mode squeezed state. The frequencies of the two modes of the squeezed state are given by $\omega_-$ and $\omega_+$ while the frequency of the LO is represented by $\omega_L$. In order to get a measurement that is independent of time, the frequency of the LO has to be selected between the frequencies of the two modes of the squeezed state.

the frequencies of the two modes of the squeezed state, as shown in Fig. 2, that is $\omega_L = (\omega_+ + \omega_-)/2$.

In the ideal case, the variance of the difference signal, Eq. (2), is proportional to the noise in the quadratures of the measured field, thus giving a direct measure of the noise properties of the squeezed field. That is, in the limit that the LO is much stronger than the squeezed state, the variance of the measured signal has the form \[ \langle (\Delta \hat{I}_{12})^2 \rangle = 2|\beta|^2 \left[ e^{2\omega_+ \cos^2 \left( \chi - \frac{\theta}{2} \right)} + e^{-2\omega_- \sin^2 \left( \chi - \frac{\theta}{2} \right)} \right]. \] (8)

As can be seen from Eqs. (5) and (8), the measured quadrature variance can be selected by changing the phase of the LO, $\chi$. Apart from giving a signal that is proportional to the noise of the squeezed field, the balanced heterodyne detection has the additional advantages of amplifying the measured signal by the strength of the LO, as can be seen in Eq. (8), and of eliminating both the quantum and excess noise contributions of the LO.

Once the heterodyne measurement is performed, the squeezing information is centered around the beat note frequency between the LO and squeezed field. As mentioned above, for the case of a TMSS, the frequency of the LO has to be selected such that $\omega_L = (\omega_+ + \omega_-)/2$. As a result, the squeezing information will be centered around the beat note frequency $\delta = (\omega_+ - \omega_-)/2$. This technique is specially useful when the frequencies of the modes of the squeezed state are close together. However, in general $\delta$ can be arbitrarily large, requiring as a result a large bandwidth for the shot-noise-limited detection system.
III. BICHROMATIC LOCAL OSCILLATOR

As seen in Sect. II, one of the main disadvantages of the standard balanced heterodyne technique is that the squeezing information is located around the beat note frequency $\delta$. Unless the two modes are quite close together, this frequency will be large making it difficult, and in some cases impossible, to characterize the TMSS. In order to alleviate the large bandwidth requirements for the detection system, we propose the use of a bichromatic field as the LO in the balanced heterodyne technique described above. As we will see, by the proper selection of frequencies of the bichromatic field it is possible to perform exactly the same measurement as with the standard technique while making it possible to use a low-bandwidth detection system for characterizing a TMSS.

In this new scheme, the local oscillator is taken to be a bichromatic field of the form

$$\hat{E}_{LO} = \hat{b}_1 e^{-i\omega L_1 t} + \hat{b}_2 e^{-i\omega L_2 t},$$

such that the frequency of each of the local oscillators is taken close to one of the modes of the squeezed field, as shown in Fig. 3. That is, $\Delta_1 = \omega L_1 - \omega_-$ and $\Delta_2 = \omega L_2 - \omega_+ \ll \Delta = \omega_+ - \omega_-$. In most cases, such a BLO can easily be generated from the laser fields used for the generation of the TMSS with the help of either acousto-optic or electro-optic modulators. Both fields of the BLO are taken to be in coherent states, such that $\langle \hat{b}_1 \rangle = \beta_1$ and $\langle \hat{b}_2 \rangle = \beta_2$. Using this form for the LO, the variance of the measured signal is now given by

$$\langle (\Delta \hat{I}_{12})^2 \rangle = -\langle (\Delta \hat{E}_S)^2 \rangle (\beta_1^2 e^{-i2\omega L_1 t} + \beta_2^2 e^{-i2\omega L_2 t} + 2\beta_1 \beta_2 e^{-i(\omega L_1 + \omega L_2)t})$$
\[-\langle \hat{E}_S^\dagger \hat{E}_S \rangle - \langle \hat{E}_S \rangle \langle \hat{E}_S^\dagger \rangle \rangle (|\beta_1|^2 + |\beta_1|^2 + \beta_1 \beta_2 e^{-i(\omega_{L1}-\omega_{L2})t} + \beta_1^* \beta_2 e^{i(\omega_{L1}-\omega_{L2})t}) + h.c. \} + 2 \langle \hat{E}_S^\dagger \hat{E}_S \rangle, \tag{10}\]

where the last term on the right hand side results from the quantization of the BLO and, as will be seen, can be neglected if the BLO is taken to be sufficiently strong.

For the general case of the BLO shown in Fig. 3 it is necessary to include the image band for each of the modes of the squeezed state [19, 20]. Due to the fact that it is not possible to distinguish between the positive and negative frequency beat note signals when looking at the heterodyne signal, it is necessary to take into account frequencies that lie symmetrically on either side of the LO, as shown in Fig. 3. The mode opposite to the squeezed field is known as the image band. As a result, the field that is measured is not the TMSS given by Eq. (3); instead it takes the form

\[\hat{E}_S = \hat{a}_+ e^{-i\omega_+ t} + \hat{a}_{v+} e^{-i\omega_{v+} t} + \hat{a}_- e^{-i\omega_- t} + \hat{a}_{v-} e^{-i\omega_{v-} t}, \tag{11}\]

where modes \(\hat{a}_{v+}\) and \(\hat{a}_{v-}\) are the image bands and are taken to be in vacuum states. The frequency of these modes is such that \(\omega_{L1} - \omega_- = \omega_{v-} - \omega_{L1}\) and \(\omega_{L2} - \omega_+ = \omega_{v+} - \omega_{L2}\).

From the definition of the image band, it follows that there are two specific cases in which Eq. (11) needs to be modified. Referring to Fig. 3, the first is when \(\Delta_1 = \Delta_2 = 0\). In this case there are no image bands, and the measured field again takes the form given by Eq. (3). This is equivalent to the difference between using homodyne or heterodyne detection to measure single mode squeezing [20]. The other case is when \(\Delta_1 = -\Delta_2 = \Delta/4\). In this case, the image band of each LO coincide, so that it is only necessary to take into account a single vacuum field mode in Eq. (11).

Using the properties of the TMSS [19], the different parts of Eq. (10) can be shown to be of the form

\[\langle (\Delta \hat{E}_S^\dagger)^2 \rangle = -2e^{-i\theta} e^{i(\omega_- + \omega_+)t} \sinh s \cosh s \tag{12}\]

\[\langle (\Delta \hat{E}_S)^2 \rangle = -2e^{i\theta} e^{-i(\omega_- + \omega_+)t} \sinh s \cosh s \tag{13}\]

\[\langle \hat{E}_S^\dagger \hat{E}_S \rangle + \langle \hat{E}_S \hat{E}_S^\dagger \rangle - 2 \langle \hat{E}_S \rangle \langle \hat{E}_S^\dagger \rangle = 4 \sinh^2 s + 2 + \langle \hat{a}_{v+} \hat{a}_{v+}^\dagger \rangle + \langle \hat{a}_{v-} \hat{a}_{v-}^\dagger \rangle. \tag{14}\]

Since the image band modes are in the vacuum state, \(\langle \hat{a}_{v+} \hat{a}_{v+}^\dagger \rangle = \langle \hat{a}_{v-} \hat{a}_{v-}^\dagger \rangle = 1\); however, the value of these expressions is not substituted until the final result in order to see their effect and make the necessary changes depending on the cases described above.
The BLO technique becomes useful when $\Delta \gg \Delta_1, \Delta_2$. In this case the bandwidth of the detection system can be designed such that the terms with frequency of the order of $\Delta$ can be neglected. Under this approximation, Eq. (11) and Eq. (12) can be combined to obtain the variance of the difference signal, which can be shown to be given by

$$
\langle (\Delta \hat{I}_{12})^2 \rangle = (|\beta_1|^2 + |\beta_2|^2)(4 \sinh^2 s + 2 + \langle \hat{a}_{v+} \hat{a}_{v+}^\dagger \rangle + \langle \hat{a}_{v-} \hat{a}_{v-}^\dagger \rangle)
+ 4 \beta_1 \beta_2 e^{-i(\Delta_1 + \Delta_2)t} e^{-i\theta} \sinh s \cosh s + 4 \beta_1^* \beta_2^* e^{i(\Delta_1 + \Delta_2)t} e^{i\theta} \sinh s \cosh s. \quad (15)
$$

In order to make the measurement time independent, it is necessary to select the frequency of the fields of the BLO such that $\Delta_1 = -\Delta_2$. By making $\beta_1 = |\beta_1| e^{i\chi_1}$ and $\beta_2 = |\beta_2| e^{i\chi_2}$ and assuming that $|\beta_1| = |\beta_2| \equiv |\beta|$, we can simplify Eq. (15) to the form

$$
\langle (\Delta \hat{I}_{12})^2 \rangle = 4|\beta|^2 \left[ 2 e^{2s} \cos^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) + 2 e^{2s} \sin^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right)
+ \langle \hat{a}_{v+} \hat{a}_{v+}^\dagger \rangle + \langle \hat{a}_{v-} \hat{a}_{v-}^\dagger \rangle + \frac{\langle \hat{a}_{v+}^\dagger \hat{a}_+ \rangle + \langle \hat{a}_{v-}^\dagger \hat{a}_- \rangle}{2|\beta|^2} \right]. \quad (16)
$$

As described above, the last term in Eq. (16) is due to the quantization of the BLO. This additional term is a phase independent noise term that can limit the minimum amount of squeezing that can be measured. However, it can be neglected when $|\beta|^2 \gg (\langle \hat{a}_{v+}^\dagger \hat{a}_+ \rangle + \langle \hat{a}_{v-}^\dagger \hat{a}_- \rangle)/2$, that is, the intensity of the BLO is much greater than the intensity of the TMSS being measured. This is usually the case for balanced heterodyne detection.

As mentioned above, due to the image bands, there are three different cases to consider depending on the values of $\Delta_1$ and $\Delta_2$. Once the appropriate image bands are taken into account, the variance in the difference signal is given by

$$
\langle (\Delta \hat{I}_{12})^2 \rangle =
\begin{cases}
4|\beta|^2 \left[ 2 e^{2s} \cos^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) + 2 e^{2s} \sin^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) \right] & \text{if } \Delta_1 = \Delta_2 = 0, \\
4|\beta|^2 \left[ 2 e^{2s} \cos^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) + 2 e^{2s} \sin^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) + \frac{1}{2} \right] & \text{if } \Delta_1 = -\Delta_2 = \Delta/4, \\
4|\beta|^2 \left[ 2 e^{2s} \cos^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) + 2 e^{2s} \sin^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) + 1 \right] & \text{otherwise.}
\end{cases}
$$

(17)

Except for a scaling factor, the first of these results is exactly the same as the result obtained with the usual balanced heterodyne technique, Eq. (8). However, the squeezing information is now centered around DC, making it possible to use a low-bandwidth detection system.

The other two cases of Eq. (17) contain an extra noise term due to the image bands. This is exactly the situation that results when using heterodyne detection for measuring a single mode squeezed state [20]. This extra noise term limits the amount of squeezing that can be
measured. In the limit of infinite squeezing, $s \rightarrow \infty$, the second cases in Eq. (17) will give a signal that is 6 dB below the classical level, while the third case will only give 3 dB below. However, for the third case it is possible to arbitrarily select the measurement frequency, thus making it possible to move away from the $1/f$ noise and any technical noise present in the detection system.

Up to now we have considered only the case in which both fields of the BLO have exactly the same amplitude. In practice this is not always possible, so it is necessary to consider the situation in which the amplitudes are not properly matched. In order to consider this case, the amplitudes of the two fields of the BLO are taken to be $|\beta_1| = |\beta|$ and $|\beta_2| = |\beta| + \delta \beta$, such that Eq. (15) now takes the form

$$\langle (\Delta \hat{I}_{12})^2 \rangle = 4|\beta|^2 \left\{ \left( 1 + \frac{\delta \beta}{|\beta|} \right) \left[ e^{2s} \cos^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) + e^{-2s} \sin^2 \left( \frac{\chi_1 + \chi_2 - \theta}{2} \right) \right] + \frac{1}{2} \left( \frac{\delta \beta}{|\beta|} \right)^2 \left[ \cosh 2s + \frac{\langle \hat{a}_{v+} \hat{a}_{v+}^\dagger \rangle + \langle \hat{a}_{v-} \hat{a}_{v-}^\dagger \rangle}{2} \right] \right\}.$$  

(18)

As can be seen from Eq. (18), the imbalance in amplitudes leads to an extra source of noise. However, this extra noise term is phase independent and of second order in $\delta \beta/|\beta|$, so that its contribution can easily be made negligible. Thus, to first order, the imbalance in amplitudes has no effect on the measurement other than an overall scaling factor.

The main advantage gained by using a BLO is that, independently of the frequency separation between the modes of the squeezed field, it is possible to characterize a TMSS without the need for a high-frequency shot-noise limited detector. Another property of using a BLO, as can be seen from Eq. (16), is that it is possible to select the measured quadrature variance by changing the phase of either one of the modes of the BLO, which might have some application in the detection of correlations in the different quadratures of the field.

IV. CONCLUSION

We have shown that by using a bichromatic local oscillator in a heterodyne detection scheme it is possible to use a low-frequency detection system for the characterization of a TMSS, independently of the frequency separation between the two modes of the squeezed state. The BLO required for this type of measurement can easily be generated from the
laser fields used for the generation of the TMSS with the help of either acousto-optic or electro-optic modulators. This allows for the use of a simple detection system for the characterization of any TMSS source. In order to get the same measurement result as with the standard balanced heterodyne detection technique, it is necessary to select the frequencies of the BLO to coincide with the frequencies of the two modes of the squeezed state. However, analogous to the case of single mode squeezing, it is possible to arbitrarily select the detection frequency at the expense of extra noise. This freedom to select the desired detection frequency makes it possible to move away from the $1/f$ noise or any technical noise present in the detection system. In principle, it is possible to extend this idea to the general case of multi-mode squeezed states.

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[1] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. 55, 2409 (1985).
[2] R. M. Shelby, M. D. Levenson, S. H. Perlmutter, R. G. DeVoe, and D. F. Walls, Phys. Rev. Lett. 57, 691 (1986).
[3] M. Kitagawa and M. Ueda, Phys. Rev. A 47, 5138 (1993).
[4] A. Kuzmich, L. Mandel, J. Janis, Y. E. Young, R. Ejnisman, and N. P. Bigelow, Phys. Rev. A 60, 2346 (1999).
[5] X. Ma and W. Rhodes, Phys. Rev. A 41, 4625 (1990).
[6] C. F. Lo and R. Sollie, Phys. Rev. A 47, 733 (1993).
[7] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. 80, 869 (1998).
[8] A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science 282, 706 (1998).
[9] S. F. Pereira, Z. Y. Ou, and H. J. Kimble, Phys. Rev. A 62, 042311 (2000).
[10] M. D. Reid, Phys. Rev. A 62, 062308 (2000).
Throughout the paper, the fields are expressed in units of $E_0 = \sqrt{\hbar \omega / 2 \epsilon_0 V}$. The frequency difference between the modes is assumed to be much smaller than the optical frequency so that $E_0$ can be taken as a constant.