Algorithm that transfer between different energy metering methods for simplification of energy trading and unified billing

Netzah Calamaro, Moshe Donko, Doron Shmilovitz *

Faculty of Electrical and Electronics Engineering, Tel-Aviv University, Tel-Aviv, Israel

ARTICLE INFO

Keywords:
Smart metering
Algebraic metering method
Vector metering method
Smart grid

ABSTRACT

The paper settles a discrepancy between two smart-metering methods. The issue bears on a billion installed smart meters and $36B electric-energy trading market. The contemporary problem of the energy-metering registration gap between the methods, relates to energy-trading unified-billing, and it will aggravate in the future. A mathematical definition for active and reactive, import/export energy-metering formulas is given for vector/arithmetic common metering models employing energy formulas rather than power, energy being the language of smart metering design/usage/billing. Economic reasoning, difficulty of implementation, and required regulator’s flexibility at establishing different import/export tariffs-per different tariff program-types are explained. Innovations in the presented work include: (1) mathematical formulation of metering-methods-definition that is validated over: one EU organization, twelve major meter manufacturers, twenty model-types, is presented and tested. (2) formulation of drivers for 100% accurate conversion from one method to another is developed. (3) fifteen real-life experiments covering the entire problem spectrum are conducted with new results, discovering new energy/tariff “conservation-rules”, relevant to manufacturers/utility companies/regulators/customers. (4) a correct segmentation of customers to energy/tariff registration-gap is generated. (5) an algebra that is suitable to energy metering/conversion/tariff computation-and-design is presented. (6) research reduces the cost of a contemporary solution by 98%. (6) Advantages/disadvantages of each method are named. (7) scenarios where one of the energy methods may be incorrect are considered-and-rejected. Eleven theorems formulated and proved, and fifteen field test cases covering the entire electricity market. (8) regulators may maintain arithmetic meters, enjoying their added value, and manage precise arithmetic/vector metering using these meters—especially using load profile and potentially satisfying with billing registers.

1. Introduction

Smart energy metering methods (MM) are roughly split into two groups: (i) The arithmetic metering and (ii) The vector metering [1]. Herein energy registration gap shall be terminated brief energy gap. The meaning is: metrology speaking energy is same, registration is different. In the vector metering method, the energy “injected from the load into the grid” is subtracted from “the energy injected from the grid into the load”. This subtracted energy results in a NET energy. If it is positive, it is registered as import, and if negative, it is registered as export. “Arithmetic metering” or algebraic metering is separately registering the “sum of phases energies, where energy is flowing in from the grid to the load” as Active import. It then registers” the sum of phase energies where energy flow is from the load to the grid” as Active export. It shall be explained that when the electric energy flow is simultaneous bidirectional, meaning import in some phases and export in other phases, there is a significant energy gap between the two metering methods, and that is not because one of the measurements is wrong, but because they measure different aspects of the same phenomena. That phenomenon is being encountered all around the world, and the response is usually to adhere to a single metering method, usually reversioning to vector, which is very costly. Clarification: no difference at metrological energy measurement. Energy is registered differently at the various metering methods. Therefore, while writing “energy metering methods energy gap” intention is “energy metering methods energy registration gap”. This is also recognized by European countries as explained by the European forum for energy Business Information eXchange (Ebx) [2]. A survey of power definitions at IEEE standards is available at [3].

* Corresponding author.
E-mail addresses: shmilo@tauex.tau.ac.il (D. Shmilovitz).

https://doi.org/10.1016/j.heliyon.2022.e11542
Received 5 January 2022; Received in revised form 22 May 2022; Accepted 1 November 2022

2405-8440/© 2022 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
While the gap is common in non-residential customers, it is still less common in residential customers. Examples of devices generating such an energy gap are, for example, renewable energy farms, welding transformers [4], and industrial induction ovens [5]. Renewables are gaining momentum which means an apparent gap. In addition, elevators while slowing down, the motor turns into a generator [6], and electric trains have the same effect while slowing down [7]. There are quite a few works regarding this issue. Regeneration causes bi-directional energy flow and therefore potential metering method energy gap. That potential shall be investigated. This energy metering gap has raised a three-fold problem. (1) For example, in Denmark after installing arithmetic metering method meters, due to public complaints, they are in the process of reverting to vector metering methods [2]. This is due to some customers who pay a larger bill using the arithmetic method and shall be explained in the theoretical section. (2) This migration is complex and expensive, the current-day solution is to replace meters in the field, or firmware update. (3) The migration back to the vector metering method is taking back all the advantages of this method, such as fast detection of fraud, of reverse phase connect, and ease of bi-directional energy management of distributed energy resources. The problem of two groups of metering methods in energy trading is that causing an energy gap will aggravate with time due to the increase of renewable licenses and increase of inverters’ deployment in electric devices, and other devices that inject also active energy to the grid. The cost saving proposition of this work is ~98% of handling issue with the present-day solution, and it is the first paper to confront this subject.

Identification of gap: (1) If a software solution exists that may be located at the data center, this shall prevent replacing firmware which is costly because it is multiplied by meters count. The current day solution is meters’ firmware update, or meters replacement of arithmetic to vector, either 100% of customers or only prosumers, and this work shall demonstrate that it is inaccurate. The research shall go beyond solving that. The research shall attempt to investigate exactly what is percentile of meters and what are customer segments that generate the gap phenomena, and in which conditions does it occur. Present knowledge states that the problem exists for prosumers. This activity, if successful, may save a lot of money and effort. Research shall investigate whether it’s possible to maintain arithmetic metering benefits for distributed energy resources market and enable the regulators to implement arithmetic or vector metering method for billing while using the same meters. That is the most important contribution because up to this paper the common knowledge is that regulator has to select either arithmetic or vector metering method and cannot enjoy both. (2) Proof of universality is required in all aspects because there are approximately 1 billion smart meters world-wide: (2.i) This research shall cover a list of fifteen electric devices and field scenarios, which physically are suspected that they may inject energy from the load to the grid. (2.ii) In order to perform that, the research must be universal. It must cover a very wide list of meter model types, herein twenty are presented, cover a wide list of manufacturers, herein twelve; (2.iii) It shall cover eleven complex theorems covering all the electric devices scenarios; (2.iv) It shall cover fifteen field test cases through remote meter reading. (3) An algebra is proposed herein that enables to discuss mathematically in the areas of meter design, and design of metering schematics for new scenarios at micro-grids/renewables/plants/and industry, including the tariff program discussion. This article must not be observed as an “old metering in-attractive school”, but of a significant solver of a contemporary problem. Additional objectives of the article are: (3.i) To propose a 100% mathematical model for arithmetic and vector metering methods; (3.ii) To propose a 100% valid driver for load profile level. Research into a possible solution at the billing registers level, which present day knowledge considers as impossible; (3.iii) To propose an algebra that potentially shall be shown to enable discussing both energy metering and tariff programs mathematically. (4) Split the energy market into segments and find out which are affected by the energy gap between two metering methods. (5) Demonstrate how the non-negligible “arithmetic metering method” smart grid benefits may be maintained and still manage billing using the vector metering method over the arithmetic metering method. (5) Enable a techno-economic report generation per each country. Exactly how many meters are involved and how much of national energy percentile is involved, and what are the relevant segments and what is their exact volume.

If obtained, what would these goals accomplish? (1) This would enable trading between two companies and two nations with “different metering methods”. Electric energy trade is $35.7B. In 2019, the top “exporters” of electricity were Germany ($4.56B), France ($3.5B), Canada ($1.9B), Sweden ($1.6B), and China ($1.59B) [8]. In 2019 the top “importers” of electricity were Italy ($2.95B), Germany ($2.13B), United States ($2.03B), Switzerland ($1.49B), and United Kingdom ($1.33B) [8, 9]. That does not include the China electric energy trade in favor of other resources such as water [10]. A survey over ten European countries reveals a ratio of 50:50 between metering methods, as described in [2], in smart metering projects. It must be mentioned that all electro-mechanical meters are the vector method. Basic electronic meters, meaning meters without tariff, remote reading and load profile, are split between the vector method and the absolute method. The smart metering split arithmetic and vector, is relevant to energy management twofold: (1) Arithmetic energy seems to enable better a “distributed electricity” market type (DER). This shall be further detailed in the background section in the theoretical section. (2) Suppose there are two different customers of the same tariff program type with two metering methods. This is common in countries with arithmetic meters [2], and this research group experienced it from close involvement. That would no longer be a problem since an accurate conversion from one method to another would become possible.

There are world-wide nearly one billion smart meters [11, 12], with both metering methods, and $36B trading, and very few papers or international reports on bridging the gap between two metering methods [13]. Some of this is maybe due to discreteness by utility companies. Peer-to-peer energy trading at DER market is becoming significant now and is described in the work [14] by Soto et al. Work [15] by Weikel describes advantages of arithmetic and vector metering registration methods. Effect on power factor definition is discussed in the work [16] by Chi-Jui Wu et al. A survey of ten states in Europe shows roughly 50:50 between these methods, as reported by European forum for energy Business Information eXchange (EbIX) [2]. When energy flow is simultaneously bi-directional, as shall be proved, there shall be differences between the methods in energy metering. It is not intended that metering is a subjective issue, or that one of the methods is inaccurate. It is a standard opinion [17, 18] that the two metering methods measure differently the same phenomena, and that currently both are considered as correct. There are further gaps. This variation of energy metering poses several serious unanswered issues that are addressed by this paper:

(i) Up until 2022 there was no standard definition of metering methods. This year only recently standard IEC 62052-41 was pre-release published with a standard definition of metering methods.

a) prior to that standard, that there is no standard energy definition of the arithmetic (or algebraic) and vector energy metering method, nor can the definitions accepted by industry/DSOs/regulators be deduced from the electric parameters defining standards: IEEE 1459 [17], and its EU matching DIN 40110-1.3 [18]. There is no straight-forward relation between the vector/algebraic apparent power definition and the vector/algebraic active/reactive energy metering method. That is important to state since there is confusion here. Maybe some hidden connection exists, but it is not evident.

b) What is existent is three-fold:

1) An official EU organization the European forum for energy Business Information eXchange (EbIX) who published “a tabular formulation” of the energy metering methods in an official document [2]. Up to the proposed research herein, that document is the most accurate formal definition.
2) Industry by personal knowledge, but not so published. It shall be shown using experiments, request-for-information over twelve "meter manufacturers", twenty "meter model types" direct and CT, three DSOS, that the definition is identical in all implementations in all energy quadrants. That implies that the non-official knowledge is identical for all.

3) The DLMS/COSEM blue book [33] “mentions” the arithmetic and vector metering methods but “does not” define them. It does “mention and define” a third method, absolute metering. The DLMS/COSEM standard defines the absolute metering-method based on the active import/export definitions. That definition relies on the selected main method being the arithmetic/vector metering method. Again, there are missing definitions from the standard.

c) What is required:
1) It is required to define by this research, mathematically and not “tabular, over a specific single example”, the metering methods formula.
2) It is required to validate their exact adaptation a-priori, over a considerable industry spectrum of existing meters.
3) It is required to verify that they do not contradict the formulas standards: IEEE 1459 [17], and its EU matching DIN 40110-1,3 [18]. Clarification: the physical formulas are defined at DIN and IEEE. The registered OBIS codes are defined by DLMS/COSEM.

(ii) Conversion drivers, feasibility study, and definition, validation:
Over these mathematical proposed definitions of metering methods, there are no contemporary drivers that convert from one metering method to another. If it is possible to implement such drivers that nobody thought about previously, or at least did not publish, then it is required by this paper:
1) To define the drivers.
2) To prove theoretically that they are correct in accordance with the previous section’s definitions.
3) To experimentally validate the drivers over as much load-profile data as possible. This is performed herein over twenty meter-model types and twelve manufacturers and fifteen use cases.

4) Up to proposed research it is considered impossible to obtain a driver at billing registers level. It shall be investigated whether there can be some driver to obtain that conversion goal.
(iii) To whom shall such a driver assist? If that research idea of “conversion drivers between metering methods” is sustainable, then it shall result in a simplification and improvement of the following four niches:
1) Accurate trading between nations. While trading with other countries, Italy and France are selling electrical energy to their neighbors [8, 11]. Suppose that one nation is using arithmetic metering, and another is using vector metering. Can an algorithm be developed that is capable of conversion from the arithmetic metering method (MM) into the vector MM, and vice versa, while being located at the data center?
2) Enable equivalent billing between prosumers because that is where the problem exists: bi-directional energy flow results in the gap. That is not manipulation of billing, it is equating two metering methods.
3) Provide flexibility to the regulator to select the desired metering method for that country.
4) Provide flexibility to enjoy all the significant benefits of arithmetic MM, as shall be shown, and maintain an equivalent billing to vector MM. Provide the regulator the flexibility to define various tariff programs, and separate tariffs per energy flowing from the grid to the load, and from the load to the grid. Can that algorithm be 100% accurate? This paper is going to present an algorithm that is capable to withstand that objective.

(iv) The research must figure out how wide is the phenomena of energy gap between two meters installed at the same premises with two different MM. That is a research objective. Does it exist in residential and industrial premises, as renewables, in railways, elevators, factories with a furnace arc, or with capacitors banks? The research must draw out a segmentation of the problem according to various parameters. Now all that is known is that it exists with prosumers and that is a partial answer.

(v) It is the objective of the research to forecast whether in ten years from now, with the evolvement of inverters in electric devices, and induction oven-spots, and additional renewables shall it become more common?

(vi) Testing and results:
1) This research defines a set of fifteen test cases, that according to a literature survey, there is identified missing information which shall be conducted within the research. The test suit is designed to cover an entire spectrum of the energy gap problem. Energy gap between MM and billing speaking: how does a train behave, how does an elevator behave, how does a capacitor bank at the entry to a factory behave, how does a furnace arc behave? How does a residential apartment behave, how does an open-delta connection behave, how do renewable prosumers behave? All this is unknown; it is believed to be known but the research shall demonstrate that it is partially known.

(vii) The test suit contains fifteen interesting phenomena occurring in real-life and challenging the energy MM gap issue.

(viii) Coverage of legacy non-smart meters metering methods. When delving into the metering at various instances, we would like, as much as possible, to convert from any method to any method. Here we add legacy meters, electro-mechanical and some third and fourth metering methods, named absolute arithmetic and absolute vector. A question arises: could all be converted to all? It is necessary to finally make some order regarding this issue.

(ix) Correct method per specific measurement scenarios. Is there such a thing as the correct metering method, vector or arithmetic, for a specific load schematic? In this paper, it will be demonstrated that there are instances where it is correct to use the vector metering method and maybe priory incorrect to use the arithmetic metering method directly onto the billing formula. This caused some nations to revert to vector metering. But if a converter from arithmetic into vector exists, then there is no problem that an arithmetic method is used. And if the opinion of wrong is incorrect then also no problem.

(x) Bridging the standard power language with the industry energy language.
1) The formation of a new “metering algebra” facilitating discussion of various metering aspects, including tariff programs design.
2) Another issue is that researchers and engineers are accustomed to arithmetic and vector power definitions from standards IEEE 1459 and DIN 40110-1,3, and the electric industry is using the arithmetic and vector energy metering method. However, vector energy and arithmetic energy do not look at all as time integral operation over power of their power counterpart. They appear to be different definitions.
3) A comparative discussion shall occur as regards to the relationship between power and energy definitions. Are they equivalent or not?

(xii) A standards gap goes further: this section is complementary to clause (i) and it does not overlap clause (ix).
1) Addressing the issue that standards recognize vector and arithmetic metering. The meter metrology standards IEC/EN 62052-11, 62053-21 to [24, 26] and for accuracy class 0.2s meters IEC 62053-22-24 [27], reactive meters IEC 62053-23 [28] and IEC 62053-24 reactive meter accuracy class 0.5 [29] and EN 50470-1,3 [30, 31] do not include a segment on arithmetic and vector MM. The former standards published a new revision in 2020.

2) The functionality definition standards DLMS/COSEM – IEC 62056, and the blue book, and the DLMS companion standards for PLC: Prime + T5, and DIS S-FSK/G3 mention the two methods, but not an explicit mathematical formula or table defining these methods.
3) An objective: present to the engineering community the necessity of an industry standard. This paper shall present findings, possibly valid for all of the 1B smart meters, based on testing and request for information (RFI). International EBIIX org. published a use-case table, [2], without mathematical formulas. Such an economical and measurement issue deserves a definition and validation procedures. This paper
attempts to address that gap two-fold: (a) Mathematically define every-thing that is already implemented, innovate in real-life use cases, publish the results to the energy industry and the regulation decision maker’s knowledge. (b) Emphasize the significance of such a standard.

(xii) The advantages and disadvantages of each method shall be mentioned for the first time collaboratively.

(xiii) “Tariff program”, “mathematical formulation”, inclusion in the comparative study between MM. Another question raised is: is the issue of correct metering only determined by metering or also by billing? Meaning by the type of tariff program set by the regulator on the consumer/producer. In this paper, the intention is to show that it is also a matter of regulation and not only of physics. Standard IEEE 1459 [1, 17] specifically emphasizes literally that present day knowledge is that both power metering methods, arithmetic and vector, are valid. It is a matter of a point of view of the same phenomena. The tariff program mathematical formula affects the energy gap relevancy, and a proposed algebra should be capable to research problems also there. All this shall motivate us to conclude that there is a necessity to define an algebraic system with theorems that enables: algorithms for migration from one metering method to another, and for deducing when there shall be no difference between metering methods taking the billing formula into consideration. Then it is a matter of installing these drivers and DSO companies are no longer worried which metering method to use and how to trade between nations uniformly.

(xiv) A software validation tool: Such a tool must be presented to enable validation of the correct metering method over the electric load profile, which is the data sent to the billing software and sent by the distributed system operator (DSO) to suppliers and customers. Contemporary validation is mainly of pulses from meter optical port.

(xx) Some surprising results shall be proved, and field tested. Is it impossible according to present knowledge, as reflected by state-of-the-art [25], to convert from arithmetic to vector at the billing register level. A surprising out-of-the-box answer.

(xvi) a market analysis of Israel in terms of which energy segments precisely generate the energy gap, which means simultaneously inject active energy to grid, while consuming energy from grid shall be performed as systematic examples. An OECD country located at its advanced preliminary smart metering deployment stage which has performed all technology researches, deployed a smart metering system, has deployed 250,000 smart meters, and is at the urge of starting full deployment. No previous publication was found that specifies the entire definitions, theorems and measurements, and conclusions, and covering such a wide set of field tests. That is the gap that this paper is addressing.

2. Background

2.1. Metering method definition

Two highlights: internationally, arithmetic is considered a synonym with algebraic. The following knowledge was verified through test case 10, at the results sections.

For the standards power definitions of arithmetic and vector methods definitions, the reader is referred to IEEE 1459 [17], DIN 40110-1.3 [18] and a book by the initial convener of IEEE 1459 committee, Chapter 4 of that book facilitates the access to these standards. There are also three papers [17, 19, 20] reviewing the issue and coherent to the standards. Ref. [21] is IEEE and Wiley reference, and includes a chapter on power and energy metering. Observing the IEEE 1459 standard, it is apparent that there is no separate vector and arithmetic active power MM definition. There are only separate vectors and arithmetic MM apparent power definitions. The active power that is of interest to this work is periodic-averaged and not instantaneous. Following is not a review on power definitions, only essentials. For a three-phase system, the apparent power vector and arithmetic MM definitions are specified as eq. (1):

\[ S_Y = S_A = S_A = S_A = S_A \]

\[ \begin{align*}
\bar{S}_Y &= P + jQ, \\
S_Y &= \sqrt{P^2 + Q^2}, \\
S_A &= S_a + S_b + S_c, \\
S_A &= \sqrt{P_a^2 + Q_a^2}
\end{align*} \] (1)

Where:

\[ S_Y \rightarrow \text{total three-phase vector apparent power}, \ S_A \rightarrow \text{arithmetic total three-phase apparent power scalar} \]

\[ P \rightarrow \text{total three-phase active power} \]

\[ Q \rightarrow \text{total three-phase reactive power} \]

The total active and total reactive powers standard definition is formulated at eq. (2):

\[ P = \sum_{n=a,b,c} \bar{v}_n \cdot \bar{i}_n = \sum_{n=a,b,c} v_n \cdot i_n \cos(\phi_n) = \sum_{n=a,b,c} P_n \]

\[ Q = \sum_{n=a,b,c} v_n \cdot i_n \sin(\phi_n) = \sum_{n=a,b,c} Q_n \] (2)

Where:

\[ \bar{v}_n \rightarrow \text{voltage vector of phase } n, \ v_n \rightarrow \text{voltage scalar of phase } \]

\[ \bar{i}_n \rightarrow \text{current vector of phase } n, \ i_n \rightarrow \text{current scalar of phase } n \]

\[ P_n \rightarrow \text{phase } n \text{ active power} \]

\[ Q_n \rightarrow \text{phase } n \text{ reactive power} \]

\[ \phi_n \rightarrow \text{relative phase angle between phase } n \text{ current and its voltage} \]

The fact that no standard definition separating between vector and arithmetic active power, enforces us to put some order in our knowledge: the energy “vector and arithmetic” MM do not derive from the metering definition standards, at least not straightforwardly. In addition, these definitions do not contradict the standards. Maybe the definitions may be derived by some thought extrapolation of active energy from apparent energy, maybe not. Referring now to the functionality standard DLMS/COSEM to the blue book [33] 13th edition (2015), there is no definition of active vector and active arithmetic energy, only of “active import” and “active export” OBIS code, and they are not the same. The companion “communication-protocol specific” (PLC variants) standards follow the DLMS/COSEM standards. We then arrive to the conclusion that arithmetic and vector metering are industry definitions deriving from technology capability, immersing with the arrival of the smart meter: the ability to digitally separate the “energy flow direction” registers. However: (1) In Results Section 5, test case 10, it shall be shown that all inspected smart meters follow these definitions. Therefore, the EBIX organization has defined the arithmetic and vector energy MM [2] and is so far the only source. That definition (i) is tabular (ii) and is a specific example, and this is one of the issues this paper is aiming to generalize. This paper defines arithmetic and vector MM at energy level, initially a verbal definition. For mathematical definition see Sections 3.3–3.6.

(a) Vector meter definition: (a.1.) Sum of positive energy flow phases is not stored in a separate register from negative energy flow phases, only the NET. (a.2.) Vector metering method is: (1) When “the energy injected from the load into the grid” is subtracted from “the energy injected from the grid into the load”. This subtracted energy is net energy. (2) If it is positive, then it is registered as “Active import”. (3) If the difference is negative, then it is registered as “Active export”. Fig. 1 illustrates the vector apparent power and vector MM definitions graphically. (a.3.) Observe in Fig. 1, that in the left schematic, all the phases are vector summed in a single register. The phases are not separated. The first clause’s sentence states that. For vector power definitions, the reader is referred to paper [19]. Left: the blue “active power” phase components exactly follow the energy vector active power definition: may be regarded that vector active energy is following the standard.

Observing Fig. 2 may be again noticed that energy definition according to [2] and power definition[1, 17, 18] are not straightforwardly matching, nor they are matching at any way. (b) Arithmetic metering method definition: (b.1) A sum of positive energy flow phases is stored in a separate register, and from negative energy flow phases, in a separate register. (b.2) By this method, the energy “injected from the grid to load/customer” is registered at a register named, “Active import”, separate from “the energy injected from grid/customer to grid”. That
2. Arithmetic metering

Table 1. World survey of metering methods deployment according to nation.

| No | State     | Metering method development                                                                 |
|----|-----------|---------------------------------------------------------------------------------------------|
| 1  | Preliminary historical state | Metering method development for energy Business Information eXchange. This organization’s official objective in Europe is defined: “to advance, develop and standardize the use of electronic information exchange in the energy industry”. At EU-28, the presented data is based on report [2], that includes information collected from EU countries distributed system operators (DSO). Other countries and other data is derived from other reports. Table 1 shows a survey of metering methods according to nations. In Europe, over ten countries the distribution is about 33:33:33 arithmetic/vector/mixed. Internationally, it is estimated mostly vector. World-wide most of the smart meters are vector. |
Table 2. Advantages of the vector metering method over the arithmetic metering method.

| No | issue                                                                 | advantage                                                                                     | arithmetic counterpart                                                                 |
|----|-----------------------------------------------------------------------|----------------------------------------------------------------------------------------------|-----------------------------------------------|
| 1  | Modern meters enable measurement of primary coil energy through secondary coil via mathematic compensation of transformer effect | The energy measured at transformer primary and secondary coils is equivalent because primary (import − export) and secondary is the same for the vector method. | The energy measured at transformer primary and secondary coils is not equivalent unless conversion of the arithmetic to the vector method at the secondary coil: at primary it is (import − export). At secondary it separates “energy from grid to customer” and “energy from customer to grid” |
| 2  | Energy conservation law                                              | In vector metering it naturally exists                                                       | In arithmetic metering it requires to perform subtraction (import − export). |
| 3  | Equity to legacy electro-mechanical meters, installed as reference to utility smart meter, by customer | exist                                                                                         | Does not exist. Electro-mechanical meters are by physics vector. |
| 4  | Open-delta connection import energy                                   | No metering issue exists                                                                     | might accidentally be measured, as export at one biphasic, such as S-T load at three phases R-S-T. |

Table 3. Advantages of the arithmetic metering method absolute and over the vector metering method (DER stands for distributed energy resources, and that is why it means bi-directional energy flow).

| No | issue                                                                 | advantage                                                                                     | disadvantage                                                                                   |
|----|-----------------------------------------------------------------------|----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|
| 1  | Easily identify technical/non-technical losses                        | Easily identify meter phase disconnect, reverse phase, “export energy for residential” is zero | Open delta meter connection might pause a problem – to be researched herein                   |
| 2  | Serve bi-directional flow DER market                                   | A single meter at a node easily identifies the import, export                               |                                                                                                |
| 3  | Enables payment on active export based on tariff program − residential – not pay on export, prosumer – pay | Enables                                                                                       | Does not enable unless use a second generation-only meter                                      |
| 4  | Enables asymmetric tariff for import and export                       | Enables                                                                                       |                                                                                                |
| 5  | Enables transfer to any other metering method in software level system | Enables through conversion formula to vector, absolute.                                       | Does not enable to arithmetic except with two meters on-spot                                    |

2.3. Presentation of the advantages and disadvantages of the vector metering method

It must be notified as a preliminary place holder, that a survey performed using the meter data management system (MDM) over 150,000 residential customers and 100,000 industrial, that no residential meter showed export energy with the exception of an elevator meter. While slowing down the motor acts as generators and injects active energy to grid. Table 2 lists the advantages of vector metering, with comparison to arithmetic metering.

2.4. Presentation of the advantages and disadvantages of the arithmetic metering method

Table 3 lists the advantages of arithmetic metering, sometimes with comparison to vector metering. It is evident that arithmetic metering has many important advantages for enablement of a modern energy market. Especially enabling to the regulator liberty through a tariff program to avoid payment for customers not registered as prosumers. Especially that the active energy that is injected into the grid is with harmonics and obligates the utility company (DSO, TSO) to filter out that energy in order to withstand power quality standard EN 50160 [24]. A harmonic energy does not efficiently feed electric devices. Railway energy is considered regenerative. The regulator may approve this customer as a prosumer.

2.5. A background survey of devices that may inject active energy from customer to grid

Arithmetic meters enable a separate energy flow measurement and not only “a subtracted (energy + −energy−) energy” metering, (energy) is energy flowing “from the grid to the load”, (energy−) is energy flowing “from the load to the grid”. Intentionally the terminology [import, export] registers are not used. These registers play a different role per each MM. The expressions {(energy+) , (energy−)} has same role for both MM. Arithmetic MM ability poses significant advantages to the distributed energy resources (DER) multi-player energy market, as presented in Table 3. Notable for potential public opinion effect on the regulator, is Denmark, that due to prosumer public opinion and some say to reasoning, has reverted to vector metering. But there are countries reverting to arithmetic metering, such as Denmark (mixture), Germany, Poland and Slovenia, due to its advantages. Fig. 3 shows electric devices regenerating “active” energy flow from the device to the grid, as they are becoming more and more common, as indicated in Section 2.3. These devices are referenced worldwide and have been recorded by our group. The energy flowing to the grid if it is harmonic, then it does not assist in device’s operation, and pollutes the grid and enforces the utility company to make an effort to withstand power quality standard EN-50160 [24].

2.6. Problem statement and solution directing

Fig. 4 demonstrates a smart metering data chain at a very high-level description, as defined for example by the European union [32]. The blue dot marked modules are candidates where a proposed “Energy metering method conversion” (MMMC) module, may be installed. It may be installed anywhere where billing registers data and load-profile exists. Chain 1 is a twenty-four-hour latency data chain. Chain 2 is near-real-time data chain implemented either with standard communication port, such as P1-dsmr or with web portal.

3. A mathematical definition of metering methods

The current chapter focuses on generic mathematical definitions of arithmetic and vector MM. When it is said “according to EBIX document” [2], it is meant also that the equations were tested in Test case 10, over twenty “meter model types” from twelve known manufacturers, and knowledge of implementation verified over twelve “meter manufacturers” and three DSO companies in different countries.

3.1. A collaborative energy table definition of all metering methods

Table 4 is taken in accordance with sample measurements on actual meters and according to the EBIX document [2]. It shows “load profile periods” or use cases and measurement results of each system. Reasoning: Table 4 is initially presented since it is universally accepted, and it matches the verbal definitions provided by our research in Section 2.1.
Fig. 3. Example of electric devices generating active export energy: (a) train during deceleration, (b) elevator during deceleration, (c) industrial induction heater, (d) Welding transformer, (e) renewable wind-turbine inverter, (f) PV inverter.

Fig. 4. Smart metering modern data chain in a liberalized market. Potential points to insert equalization of billing for different metering methods: (a) top right: meter to suppliers shared data-hub; (b) top left: suppliers to end customers, (c) bottom: chain 1, chain 2 data-chains as defined by EU-28 [32].

Table 4. Use cases and energy values per each metering method. Partially taken from [2] and complemented with additional data.
is negative then active export register is incremented. The export equation is the complementary eq. (4):
\[
E_{\text{active.export.Total} (2.80 \ \text{register})} : \begin{cases} 
\text{if } (E_{A,1} + E_{A,2} + E_{A,3}) < 0 \\
\text{else } E_{\text{active.export.Total} = \text{unchanged}}
\end{cases}
\]

Where:
\[
E_{\text{active.export.Total} (2.80 \ \text{register})} \text{ – is the register of the NET energy of the difference between energy flowing from the grid to the customer and energy flowing from the customer to the grid, providing that in current load profile period, it is negative. Fig. 5 shows the sub-algorithm flow diagram that matches eq. (3). Fig. 5-b is a more intuitive representation of vector metering method.}
\]

The active to reactive mapping of registers, as defined by DLMS/COSEM standard [33], is defined in Table 7. Mapping is generated by the presented paper. Registers are defined by the standard, regardless of the metering method.

### 3.4. Arithmetic metering method metering equations for billing registers

The following equations constitute a definition of the arithmetic metering method, not contradictory with standards [17, 18], and according to the international document [2] for active/reactive import energy. Following Fig. 2, the sum of active positive phases is incrementing the active import, while the sum of negative phases is incrementing the active export. The equations are:
\[
E_{\text{active.import.Total} (1.80 \ \text{register})} = \sum_{\text{all phases}(i)} E_{A,i} \\
E_{\text{active.export.Total} (2.80 \ \text{register})} = \sum_{\text{all phases}(i)} E_{A,i} \quad (5)
\]

Where:
\[
E_{\text{active.import.Total} (1.80 \ \text{register})}, E_{\text{active.export.Total} (2.80 \ \text{register})} \text{ were defined and are the same registers as in section 3.2 Table 4 and in section 2.1.}
\]

The flow diagram that is equivalent to eq. (5) is described in Fig. 6. It may be more simplified because the flow is the same for the group of all positive energy phases and all negative energy phases. In Fig. 6 (\(\cup E_{A,i}\)) – implies the sum over all phases of active energy with sign meaning. Fig. 6-b is a more intuitive graphical representation of arithmetic MM. The green arrows reflect the total sum of all phases with energy flow in the same direction. The blue arrows reflect the phase current flows. The active-import is sum of all phases with energy+ energy flow from grid to load. The active export is sum of all phases with energy– energy flow from load to grid.

Further research not presented at current paper, shows that even if Fig. 6 schematic and eq. (5) is implemented at some meters instantaneously, The load profile shall behave also such as Fig. 6 schematic and eq. (5), is observed for 98% of scenarios at load profile level.

### Table 5. Active Energy Registers (taken from [34]).

| Registration | OBIS code |
|--------------|-----------|
| Active import Rate 1 | 1-0:3.8.1*255 |
| Active import Rate 2 | 1-0:3.8.2*255 |
| Active import Rate 3 | 1-0:3.8.3*255 |
| Active import Rate 4 | 1-0:3.8.4*255 |
| Active export Rate 1 | 1-0:2.8.1*255 |
| Active export Rate 2 | 1-0:2.8.2*255 |
| Active export Rate 3 | 1-0:2.8.3*255 |
| Active export Rate 4 | 1-0:2.8.4*255 |
| Active import total over all rates | 1-0:3.8.0*255 |
| Active export total over all rates | 1-0:2.8.0*255 |

### Table 6. Reactive Energy Registers (taken from [34]).

| Registration | OBIS code |
|--------------|-----------|
| Reactive import Rate 1 | 1-0:3.8.1*255 |
| Reactive import Rate 2 | 1-0:3.8.2*255 |
| Reactive import Rate 3 | 1-0:3.8.3*255 |
| Reactive import Rate 4 | 1-0:3.8.4*255 |
| Reactive export Rate 1 | 1-0:4.8.1*255 |
| Reactive export Rate 2 | 1-0:4.8.2*255 |
| Reactive export Rate 3 | 1-0:4.8.3*255 |
| Reactive export Rate 4 | 1-0:4.8.4*255 |
| Reactive import total over all rates | 1-0:3.8.0*255 |
| Reactive export total over all rates | 1-0:4.8.0*255 |

### Table 7. Active to Reactive parameters mapping in the vector metering method definition of reactive energy.

| Active Energy parameter | Reactive Energy parameter |
|-------------------------|--------------------------|
| \(E_{\text{active.import.Total}(1.8.0 \ \text{register})}\) | \(E_{\text{active.export.Total}(5.8.0 \ \text{register})}\) |
| \(E_{\text{active.export.Total}(2.8.0 \ \text{register})}\) | \(E_{\text{active.export.Total}(4.8.0 \ \text{register})}\) |
| \(E_{A,1}\) | \(E_{A,1}\) |
| \(E_{A,2}\) | \(E_{A,2}\) |
| \(E_{A,3}\) | \(E_{A,3}\) |

### 3.2. Smart meter active and reactive registers regardless of metering method

It is important to note that the DLMS/COSEM standard, blue book, defines registers but does not define the metering method. In both metering methods, the explicit registers are described in Table 5. Table 4 lists the active energy registers OBIS codes as defined by DLMS/COSEM [34] and its blue book, where rate stands for tariff. Registers index at DLMS/COSEM (OBIS code) is regardless of the metering method.

Table 6 lists the reactive registers as starting point.

### 3.3. Vector metering method metering equations for billing registers

The following equations constitute definition of the vector metering method not contradictory with standards [17, 18], and according to Table 3, the international document [3, and Section 2.1, verbal definitions for active/reactive import energy, and standard IEC 62052-41. A reader referring to standards shall not observe something that looks like the following, and that is paper’s objective, to complement the data. The equations are valid for billing registers and for load profile buffer periodic values. For reactive energy, it is necessary simply to replace active with reactive and \(E_{A}\) with \(E_{R}\). Table 3 is an example dependent on specific values. The definition is generic. Rule is simple and follows Fig. 1: if signed sum of all active energy phase registers is positive, then active import is incrementally otherwise active export.

\[
E_{\text{active.import.Total} (1.8.0 \ \text{register})} : \text{if } (E_{A,1} + E_{A,2} + E_{A,3}) > 0 \\
\text{then } E_{\text{active.import.Total} = |E_{A,1} + E_{A,2} + E_{A,3}|}.
\]

\[
E_{\text{active.import.Total} (1.8.0 \ \text{register})} = \sum_{\text{all phases}(i)} E_{A,i} \\
E_{\text{active.export.Total} (2.80 \ \text{register})} = \sum_{\text{all phases}(i)} E_{A,i} \quad (5)
\]

Where:
\[
E_{\text{active.import.Total} (1.8.0 \ \text{register})} - \text{is the register of the NET energy of the difference between energy flowing from grid to customer and energy flowing from the customer to the grid, providing that in current load profile period, it is positive.}
\]

\[
E_{A,i} (i = 1, 2, 3) - \text{active energy of phase } i \text{ in aggregative format.}
\]

\[
E_{A,i} \text{ is always incremental reflecting energy flow through phase } i \text{ up to the present moment.}
\]

Billing registers are incremental energy, load-profile periods are sometimes differential energy, if signed sum of phases active energy
Fig. 5. (a) Flow diagram definition of active and reactive (import, export) billing registers. (b) flow examples demonstrating the principle of vector metering as NET energy measurement.

Fig. 6. (a) The arithmetic metering method flow diagram. (b) flow examples demonstrating the principle of arithmetic metering as separate energy measurement of energy from grid, and energy to grid.

3.5. Vector metering method metering equations for load profile channels

If the load profile channels are aggregative then the formulas are the same as defined in Section 3.3, except for replacement of OBIS codes from billing registers to matching load profile channels.

If the load profile channels are differential, it is required to replace unchanged with 0.

It is then also required to replace $E_{AI}$ – from aggregative to differential. There is no standard obligation as to the load profile being aggregative or differential. The equations and figures are not repeated herein.

Same as active channels and must be applied to reactive channels if they are implemented.

3.6. Arithmetic metering method metering equations for load profile channels

The same modifications as formulated in Section 3.5 are valid here.

4. Drivers for transferring between the metering methods

Up to now chapter 3 focused on mathematical definitions of the metering methods. The current chapter focuses on the “drivers” definitions. In results section, all the mathematical definitions are tested through fifteen field tests, and over twenty “meter types”.

4.1. Drivers for transferring from arithmetic to vector metering methods – billing registers and load profile channels

The following formula is for conversion from arithmetic metering to vector metering. It is not contradictory to IEEE 1459 [17] and DIN 40110-1,3 [18] standards and in accordance to the EBIX document [2] and in accordance with Sections 3.3, 3.4 definitions in this paper. The standard defines them as active, reactive and apparent power as at eq. (1) and eq. (2), and herein and at EBIX document, they are specified energetically and register-ally.

$$\text{if } (E_{A,\text{import}} - E_{A,\text{export}}) > 0 \text{ then: } E^{\text{import}}_{V} = (E_{A,\text{import}} - E_{A,\text{export}}). E^{\text{export}}_{V} = 0$$

$$\text{else if } (E_{A,\text{import}} - E_{A,\text{export}}) < 0 \text{ then: } E^{\text{export}}_{V} = (E_{A,\text{export}} - E_{A,\text{import}}). E^{\text{import}}_{V} = 0$$

(6)

Where: $E_{A,\text{import/export}}$ – implies active arithmetic method import/export total energy

$E^{\text{import/export}}_{V}$ – implies active vector method import/export total energy

Fig. 7 shows flow diagram of sub-module of driver. The high level design.

That formula does not appear in any of the reference documents, it is derived from the EBIX document [2] and from Sections 3.3, 3.4 herein. The registers are defined in the DLMS/COSEM blue book [34], the formulas are not defined.
4.2. Conversion from vector to arithmetic using a common two meters connection scheme

There is no ability to always transfer from vector MM to arithmetic MM, since arithmetic is two parameters information equivalent to one vector parameter. At vector MM, either the import or the export is different than zero, the export or import accordingly is zero. However, for PV or wind-turbine, there is a common meter installation scheme that exists for example in several countries, and it has a conversion formula. Fig. 8 shows an illustration of the installation scheme. Bi-directional vector meter (navy blue arrows) is showing energy ingoing from grid to premise, and outgoing from premise to grid. The outgoing energy flow is what remains from solar manufacturing after the domestic self-consumption from self-generation plus electric grid. The single directional (dark blue arrows) meter at parallel to inverter measures exactly the energy generated by PV. That component is the self-generation. Conversion from vector to arithmetic is as defined at eq. (7):

\[
\begin{align*}
(1) & \quad E_{\text{generation}} = E_{A,\text{export}} \\
(2) & \quad E_{V,\text{import}} = \begin{cases} 
E_{A,\text{import}} - E_{A,\text{export}} & \text{if } E_{A,\text{import}} - E_{A,\text{export}} > 0 \\
0 & \text{otherwise} 
\end{cases} \\
(3) & \quad E_{V,\text{export}} = \begin{cases} 
E_{A,\text{export}} - E_{A,\text{import}} & \text{if } E_{A,\text{export}} - E_{A,\text{import}} > 0 \\
0 & \text{otherwise} 
\end{cases}
\end{align*}
\]

The unknowns are colored blue. But arithmetic active export is the generation meter. Substituting (5.1.) in (5.2.)-(5.3.) enables extracting arithmetic active import yields per each load profile period eq. (8) is formulated herein, using the registers defined at eq. (4).

\[
\begin{align*}
(2) & \quad E_{A,\text{import}} = \begin{cases} 
E_{V,\text{import}} + E_{A,\text{export}} & \text{if } E_{A,\text{import}} - E_{A,\text{export}} > 0 \\
E_{V,\text{import}} + 0 & \text{otherwise} 
\end{cases} \\
(3) & \quad E_{A,\text{export}} = E_{\text{generation}}
\end{align*}
\]

Fig. 7. (a) Arithmetic to vector 100% accurate conversion formula both at billing registers level and at load profile periods level. (b) a graphical representation reflecting the NET flow nature of vector metering and the counter separated directional flows of arithmetic metering.

Fig. 8. An example of a PV or wind-turbine bi-meter installation scheme.

Fig. 9. (a) The arithmetic metering method to the absolute metering method conversion (b) for non ‘smart electronic meter’ if a residential customer reverses one of phases polarity energy would still be measured as consumption.

4.3. Conversion from the arithmetic metering method to the absolute energy metering method

The conversion formula is straightforward from the arithmetic method to absolute method metering definitions. Absolute energy is not a major section of the current paper. It is important for nations that did not fully deploy smart metering and wish to protect their assets from fraud. Most of the world is not fully covered with smart metering. The formula enables regulator to decide on unification to absolute metering by partially using smart meters as arithmetic meters. Eq. (9):

\[
E_{\text{Absolute}} = |E_{A,\text{import}}| + |E_{A,\text{export}}|
\]

Fig. 9 shows flow diagram of conversion algorithm.

4.4. Mathematical identities and their effect on billing formulas – development of a metering method algebra

This section has two objectives:

(i) To prove some important results.

(ii) To show that the algebraic language developed herein enables the discussion of various scenarios in metering and in billing, including Tariff.
Theorem 1. Tariff and net energy balance indiffer-ence. The difference (import − export) is the same for vector and arithmetic MM, both at load-profile level and at billing registers level.

Proof. For the arithmetic meter, the accumulative energy in billing registers for import is

\[ E_{\text{Active import, Total}}(\text{Arithmetic}) = \sum_{n=1}^{N} (\text{Energy}^+)_n, \]

\[ (\text{Energy}^+)_n = \sum_{i, \text{all positive}} E_{A,i}(n) \]

\[ E_{\text{Active export, Total}}(\text{Arithmetic}) = \sum_{n=1}^{N} (\text{Energy}^-)_n, \]

\[ (\text{Energy}^-)_n = \sum_{i, \text{all negative}} E_{A,i}(n) \] (10)

Where:

Energy+ = energy flow over all phases from the grid to the customer
Energy− = energy flow over all phases from the customer to the grid
n = index of load profile period

\[ E_{A,i}(n) = \text{Active energy of phase at period index } n \]

For the vector meter, the summation is different but over the same components. Summation of import is only when the difference is positive, and the same for export when the difference is negative.

\[ E_{\text{Active import, Total}}(\text{vector}) = \sum_{n=1|\text{Energy}^+, n = (\text{Energy}^-), n > 0}^{N} (\text{Energy}^+)_n - (\text{Energy}^-)_n \]

\[ E_{\text{Active export, Total}}(\text{vector}) = \sum_{n=1|\text{Energy}^+, n = (\text{Energy}^-), n < 0}^{N} (\text{Energy}^-)_n - (\text{Energy}^+)_n \]

\[ (\text{Energy}^+)_n = \sum_{i, \text{Energy A,i}, n (0 \neq 0)} E_{A,i}(n), \]

\[ (\text{Energy}^-)_n = \sum_{i, \text{all negative}} E_{A,i}(n) \] (11)

Differentiating eq. (11) and rearranging the components, a differenti-ated form of eq. (10) is obtained. □

The result is important and four-fold: (i) It is correct at load profile periods level. (ii) It is correct for energy registers level. (iii) Tariff programs with payment according to formula \( A \text{(import − export)} + B \times \text{generation} \) named NET tariff program, is indifferent to the vector or arithmetic metering method, although it has a separate tariff for generation and a separate tariff for consumption. (iv) The result means that conversion according to the tariff being different than the NET shall not! work at billing register level, but only at load profile periods level. (v) Aggregated energy registers shall maintain the relationship. According to (iv), billing according to registers monthly, the conversion from arithmetic to vector MM, shall not work and that is a very important result. It bounds the customer’s community where the gap shall be evident.

Theorem 2. “An example of a common tariff program where there is no dif-ference between the metering methods”. The billing formula is also affecting, the effect of the differences between the metering methods may not be relevant to the tariff program. For the following formula according to the above Theorem 1, there is no difference between arithmetic and vector, despite the tariff for the generation being different than the tariff for (import − export). The reasoning of the formula is that payment is only for the energy difference between the flow inside and outside the premises during the nights when there is no generation. When there is generation, there is payment for generation according to a second tariff as described at eq. (12):

\[ \text{payment} = A(E_{A,\text{import}} - E_{A,\text{export}}) + B(E_{\text{generation}}) \] (12)

Proof. Proof is in Theorem 1. □

Theorem 3. “Bi-directional but not simultaneously”. If a load is bi-directional energy “consumption and generation”, and during any moment, all the phases are in the same direction (symmetrical) then; a) there is no energy gap between the metering methods (MM); and b) there shall be as a result no billing gap regardless of tariff.

Proof. According to conversion definition by eq. (13):

\[ E_{V, \text{Active import, total}}(t_n) = \begin{cases} E_{A,\text{Active import, total}}(t_n) - E_{A,\text{Active export, total}}(t_n) & \text{import − export} > 0 \\ 0 & \text{import − export} < 0 \end{cases} \]

\[ E_{V, \text{Active export, total}}(t_n) = \begin{cases} E_{A,\text{Active export, total}}(t_n) - E_{A,\text{Active import, total}}(t_n) & \text{export − import} > 0 \\ 0 & \text{export − import} < 0 \end{cases} \] (13)

Where:

\( t_n \) = nth moment period. If periods are of quarter of an hour, this means complete quarter hours: 12:00, 12:15 etc.

But at any moment either \( E_{\text{Active import, total}}(t_n) = 0 \) or \( E_{\text{Active export, total}}(t_n) = 0 \).

Therefore eq. (13) converges to the degenerate form is defined by eq. (14):

\[ E_{V, \text{Active import, total}}(t_n) = \begin{cases} E_{A,\text{Active import, total}}(t_n) & \text{import − export} > 0 \\ 0 & \text{import − export} < 0 \end{cases} \]

\[ E_{V, \text{Active export, total}}(t_n) = \begin{cases} E_{A,\text{Active export, total}}(t_n) & \text{export − import} > 0 \\ 0 & \text{export − import} < 0 \end{cases} \] (14)

And that is the exact identity from both sides. Since there is no energy gap, then for any tariff function, the billing is identical. The theorem modifies current day knowledge that prosumers are always generating MM gap. Next theorem shows when does the gap occur. □

Theorem 4. 4.1) The only way that energy flow is bi-directional, but not simultaneous, is if all phases simultaneously are flowing in the same direction at any moment. 4.2) The conversion formula is not violated at any case. 4.3) For some electric load connection schemes the energy direction is reversed at some phases, generating bi-directional energy flow. Open-delta load connection at high-voltage with start connected meter.

Proof. Let us initially identify electrical situations in which this may occur by observing Fig. 10. As for the proof. Observing eq. (13) and (14) this is the case stated in Theorem 4. That’s the end of proof of section 4.1. □ The conversion from arithmetic into vector is due to metering mode design and is independent of connection scheme. That is compilation of section 4.2. □ Observing three-phase connection schemes, there are three schemes: Fig. 10-a star, Fig. 10-b delta, Fig. 10-c: open delta. Star and Delta due to closed circuit (star, delta) the energy flow is not reverse even by non-symmetrical loads, even complex \( R + jX \). A vector thinking exercise may demonstrate that. Open Delta the current enters as energy followed by phase A and goes out as energy at phase C. That complements proof of 4.3. □ As simple as it sounds many debates were performed over Theorem 4 by this group. Now for field test demonstration – at all the instances found with bi-directional flow without regenerative component, such as train or elevator, all were open delta high voltage connections: (i) Furnace arc oven, (ii) a large electricity three-phase boiler, (iii) at entry to plant a three phase compensating capacitor with disconnected phase, (iv) An electric train not
during regenerative cycle at slow-down – it is 3 phase delta but at any instance only a single bi-phase is connected and it is 161 kV. The only three phase connection that this research group located is three phase open delta. □

4.5. Theorem 5: the saw-tooth effect – unavoidable results

The following theorem discusses an observed phenomenon while comparing arithmetic to vector MM at same measurement point.

**Theorem 5.** At conversion from an arithmetic metering method, of meter type A, to a vector metering method, and comparison to the vector metering method meter type B, it is expected that the percentile error shall have a saw-tooth shape ripple. Observe Fig. 11 for comprehension of proof.

**Proof.** It could be mathematically formulated but it is explained herein. Fig. 11 illustrates the meter internal structure and shall be verified experimentally in the "Results".

1) The “implicit cell accumulating energy” until registering at “next load profile period” has a specific “kWh size” quanta. That is a small bucket of water shown in Fig. 11.

2) When that bucket fills it is poured into the load-profile period (LP) bucket and the billing register “energy up to now” bucket, one period at a time.

3) If for example during “fifteen minutes” period, three and a half (3.5) implicit buckets are filled in Fig. 11, then three buckets shall be poured into the load profile period. The explicit bucket, and half an “implicit bucket” shall remain filled and continue being filled in the next period, but not poured into the large bucket until full.

4) Now meter A has a bucket of “size a” Watt-hour, and meter B has a “size b” that equals for example three times: “size A” Wh. Assuming that the meters are time synchronized, then meter A shall be filled three quanta of minimal registered energy, while meter B is filled more slowly.

5) When the energy flow is low, it shall be observed that one meter is filled while the other is not. But periodically, after 3 periods the less accurate meter shall equate in energy to the more accurate meter. □

That shall be demonstrated in the Results, Section 0.

**Conclusion:** Therefore, the “metering method conversion module” has three sub-modules: (i) The decision making, which formula to select; (ii) the conversion formula; (iii) the saw-tooth synchronization module. This is shown in Fig. 12.

**Theorem 6.** In single phase meters there is no energy MM gap between arithmetic and vector metering.

**Proof.** For a single phase, there shall be only one registering in any load profile period, either import or export. This can be proved easily with mathematical formalism, which is not required. The conversion formula from arithmetic to vector shall converge to eq. (14). That concludes the proof. □

**Theorem 7.** At Open-delta load connection at high-voltage with start connected meter, where there is an apparent exception of bi-directional energy flow while other connections same flow direction for passive loads, there is no exception at all: a unidirectional energy flow.
Table 8. Use cases and energy values per each metering method. Partially taken from [2] + complemented with additional data.

| Period index | Phase R (Wh) | Phase S (Wh) | Phase T (Wh) | Arithmetic import/export | Billing register | Billing register | Billing register | Billing register |
|--------------|--------------|--------------|--------------|---------------------------|-----------------|-----------------|-----------------|-----------------|
|              |              |              |              | Arithmetic | Import/export | Arithmetic | Import/export | Arithmetic | Import/export | Arithmetic |
| 1            | 2            | -1           | -1           | 2                       | 0               | 0              | 2               | 2               |
| 2            | 4            | 4            | -8           | 8                       | 0               | 0              | 10              | 0               |
| 3            | -3           | 3            | 4            | 3                       | 0               | 4              | 17              | 13              |
| 4            | 2            | -2           | 4            | 2                       | 6               | 0              | 4               | 19              |
| 5            | 7            | 4            | -2           | 11                      | 2               | 9              | 0               | 21              |
| Total        |              |              |              | 30                      | 21              | 13             | 4               | 30              | 21             |

Fig. 12. Entire driver system high level block diagram.

Fig. 13. Open-delta load connection to grid.

Proof. The issue of open-delta connection of load is illustrated in Fig. 13. Thick arrows indicate the energy flow. It may be observed that although current lines (thin arrows) are all in the same direction, then the energy flow at the load, and measured by the meter, flows from the grid to the load at bi-phase B-A, and flows from the load to the grid at bi-phase A-C. Measuring it as arithmetic might yield an export energy without knowledge whether it is a current source at A-C really injecting export, or the in-going energy reversing and reverting to the grid. The above line of argument is incorrect because it is incomplete. The meter is connected to the load via a measurement current transformer. The phase voltage polarity is marked by the plus sign, the current at the load AC and BA is always moving from the plus sign to the plus sign. There shall be no error using the arithmetic MM. One may notice that ingoing current at phase B is outgoing at phase C.

Theorem 8. The billing level conversion inability. a) The conversion of arithmetic to vector does not work at the billing registers level for the proposed formula (6). b) It does not work for any other formula.

Proof. Relevancy. This is one of the most important theorems, because it may accidentally be considered that if at load-profile periods level there is equality, then also the summation shall be equal. This is a very rational reasoning, but it does not work. While at the load profile level the conversion works fine, at the period’s level, the arithmetic MM when converted equals the vector MM. The aggregation does not work efficiently. 8.1. proof by example. By observing Table 3, which is especially constructed with bi—directional simultaneous energy flow, if it is written with billing registers or only observing the total sum, then the translation law is stated. Table 7 is partially overlapping Table 3. 8.2. formal and intuitive comprehension is showing bellow at Table 8. It is sufficient to show a single case where the conversion formula does not hold. If for “arithmetic” MM the import is always larger than the export, then the vector active export billing register shall be zero. Then situation is reversed now. The total billing active import is still larger, the vector active export billing register shall be zero. There is no way of tracking the history where the active export was different than zero. Mathematically: referring to the conversion driver procedure, it follows in eq. (6), at any minute in time, the “Active import” and “Active export” billing registers, and operating it to temporary vector “Active import, export” registers. As defined by eq. (15):

$$E_{V,\text{import}} = \sum_{n=1}^{N} E_{V,\text{import period}}(t_n)$$

$$E_{V,\text{export}} = \sum_{n=1}^{N} E_{V,\text{export period}}(t_n)$$

At the periodic level, there are separate instances both of import and export, having non-zero values at the periodic level, and as a result have non-zero values at both import and export at the most general case, which is sure to occur for bi-directional simultaneous energy flow, such as renewables. Eq. (17) describes the simultaneity:

$$E_{V,\text{import}} > 0 \quad \text{and} \quad E_{V,\text{export}} > 0$$

That concludes the proof for Section (8-a).

8.3. Proving the second part. To prove that there is no deterministic formula is much more difficult than proving that some formula is not suitable. In order to prove that the formulation of the conversion formula from arithmetic into vector is written with a mathematical function instead of an if-then:

$$E_{V,\text{import}} = \frac{\Delta E + \text{sgn}(\Delta E) \cdot \Delta E}{2} = f(\Delta E)$$

$$E_{V,\text{export}} = \frac{-\Delta E + \text{sgn}(-\Delta E) \cdot (-\Delta E)}{2} = f(-\Delta E)$$

$$\Delta E = E_{A,\text{import}} - E_{A,\text{export}}$$

Where:

- \(\text{sgn}(\Delta E)\) – sign function of \(\Delta E\).
- \(\Delta E\) is the energy increment from one time point \(t_i\) to another time point \(t_{i+1}\).
- \(A\) – indicates “Arithmetic” and “V” indicates “Vector.”
That writing takes an advantage of a known formula that when the sign value is 1, then it is $\Delta E/2$ and when the sign is $-1$ then the value is zero. That manipulation transfers from an if-then formulation into an entirely mathematical formulation. Observing eq. (18), the sign() function is like a dice, determining for each period if a non-zero value be registered, the import or the export. When that 1, 0 is not known by “an observer”, especially when every possible scenario in the world is considered, that procedure is said to be random, and that function “f” is a random variable and the variable $\Delta E$ is a random variable. Now, for a specific distribution, the aggregation or summation converges into a known pattern.

$$E_{\text{import-total}} = \sum_{n=1}^{N} f(\Delta E) \cdot \text{p}(\Delta E)$$

$$E_{\text{export-total}} = \int_{0}^{\infty} x \cdot \text{p}(x)\text{d}x$$

(19)

The latter expression is computable when the distribution is known. But all the distributions in the world are permitted because this is a theorem. Therefore, there is no limit to the aggregation value. If the history of the random variable is unknown, assuming a known distribution, then the exact billing register value is unknown. That means that in general there is no formula that may convert from arithmetic into vector. □

**Theorem 9** (Forecasting.). Solving the conversion equation with an estimator. It is possible to know, based on an arithmetic billing register, and arithmetic/vector forecasted functions, the vector billing register within a predetermined uncertainty boundary of the forecasting function. Aggregation (billing) registers level – the accuracy is converging to zero with time.

**Proof.** After proving that deterministically there is no conversion formula, then statistically there may be a formula. There is an arrangement that may enable knowledge of the vector billing registers, provided the arithmetic billing registers are known. That knowledge is within an uncertainty limit and not 100% accurate. This requires a little thinking out of the box, which is not characteristic of the “billing” industry. But the electric market changes. Let us describe the forecasting function according to the following eq. (20):

$$f_{\text{arithmetic-or-vector}}(t) = f(E_{\text{import}}\text{forecast}(t))$$

– some forecasting function of $E$

Let: $\Delta \beta$ be the percentile uncertainty of that function. Then the following machine shall provide the vector billing import, export registers provided the arithmetic import/export registers are known. Equation (6) or eq. (19) are operated on the arithmetic and vector forecasting functions. There are load forecasting algorithms capable of forecasting fifteen days, with periodic time granularity, such as fifteen minutes. This may all be formulated formally, but it is omitted here. We implemented 3 types of electricity load forecasting: ANN – Artificial neural networks (shallow deep learning), Decision tree, and LSTM – long short-term memory (deep learning). Fig. 14 shows LSTM forecast. Observing Fig. 14, the medium-term real-time accuracy of that algorithm is ~15% for residential and ~5% for industrial customers. But turning the problem into aggregating. Aggregating first, the inaccuracy shall decline by a factor $\sqrt{\text{time}/2}$ and that diverges. In a single day, there are ninety-six periods, meaning inaccuracy is 10 times smaller than a single period, and monthly accuracy shall increase by a factor of $\sqrt{3000} = 55$. Customer shall on occasion pays 5 cents more, and then pay 5 cents less. On the average customer shall pay exactly the sum as if periodic conversion is as close to 100% accurate as 99%. A formal proof is that each load profile period is considered a random variable $X(t_n)$. The cluster of load profile periods, are constructed of a collection of independent random variables of the same type, and two rules abide to such as defined at eq. (21):

$$\sigma^2(\sum_{n} x_n) = \sum_{n=1}^{N} \sigma_n^2 = N \sigma_0^2$$

$$\mu_{\text{total}} = N \mu_0$$

$$x = \sum_{n=1}^{N} \frac{x_n}{N}, \sigma(x) = \frac{\sigma_0}{\sqrt{N}}, \mu_0 = \mu_0$$

(21)

Where: $\sigma$ – variance of sum of identical random variables $x_n$. Since law of large numbers this is expected to be a normal distribution of sum of Gaussians $x_n$ – random variable representing a specific load period occurring at $t = t_n$.

$N$ – load profile periods count

$\mu_{\text{total}}$ – mean of sum of identical random variables $x_n$, 

$\sigma_0$ – variance of each random variable $x_n$ each representing a specific load profile.

$\mu_0$ – mean of each random variable $x_n$ each representing a specific load profile. □

A short discussion on Theorem 9 results. Although the conversion formula at the billing level is extremely accurate, knowing the billing industry, this proposed formula is not suitable to the present state, although it is 97–100% accurate. This is to some extent a revolutionary theorem because to everyone’s knowledge, it is not possible to compute a conversion at billing level including the EBIX organization, which represents the EU-28 to some extent. Here it is shown that a statistical estimator that is sufficiently accurate, within a meter type labeled accuracy, to be no less efficient than a deterministically correct conversion. Fig. 14 shows accuracy. The conversion equation is solved with an estimator, the load forecasting. But it takes a single day for the solution to be more accurate than meter registered accuracy. Billing usually is monthly or bi-monthly. The work and load-forecasting application is by the authors.

**Theorem 10.** The billing level conversion ability at constrained scenarios using eq. (6). When energy flow is bi-directional but not simultaneous, then it is possible to use the conversion formula at energy from arithmetic MM to vector MM for 100% precision. The statistical formula is not meant here, but eq. (6). Corollary: speaking of daily billing, if most of the duration is bi-directional energy flow but not simultaneous. Then approximately the percentile energy gap from total consumption shall be small. In terms of billing computation, billing is meant “daily or monthly” or another duration of aggregation of the entire energy consumption during that period.

**Proof.** When the flow is bi-directional but not simultaneous then there is no difference at any load profile period between arithmetic MM and vector MM. Since there is no different at the load profile level, there is no difference at the billing level: the billing registers are aggregations of import/export channels accordingly. The second part of the theorem. For example, many PV manufacturers without storage. When PV is active it is dominant over the consumption. When there is no generation then there is only consumption. That leaves a few daily hours when there is overlap. In addition, any non-symmetrical three phase delta connection may inject energy from load to grid. □

Collaboratively: 97% quarter hourly precision, thirty days forecast. The aggregation of customers is equivalent to an aggregation of 250 periods and then load-forecasting. Aggregation first, estimation second increases accuracy. Fig. 15 shows PV farms research results of two characteristic load profile graphs of self-consumption vs. generation, taken from two research works. Research by Tzanova et al. [35] from 2021, and work by Zucker et al. [36] from 2016.

Fig. 14 implements three forecasting algorithms by this group following.

Graphs are taken from the work because they exemplify the real-life condition. The graphs show that for small PV prosumers, the generated energy is of an order of magnitude of the self-consumption, and for large
PV, the eq. (20) of forecaster. The first two algorithms are: Fig. 14-b, ANN – artificial neural network, defined as shallow-deep learning. This is not CNN – no convolution. The second algorithm is decision tree. These two are used as cheap 5 min/customer algorithms. They excel for industrial customers but insufficiently accurate for residential. The third algorithm is LSTM multi-variable input single variable output, is an exclusive algorithm, that runs one hour/customer at GPU machine which is 8,000 times faster than core i7 (Terra Flops). It excels in accurate forecasting. The high accuracy of meter’s cluster load forecasting by ANN, decision tree – is validating eq. (21) section 4.5. The other option of separately forecasting each customer then aggregate was implemented, yields inaccurate results. All this is described also by Fig. 14(a, b).

**Theorem 11.** The three-phase load star and delta connection vs. open delta connection and energy gap between MM. 11.1. When a three-phase load is star or delta there is no simultaneous energy flow to the grid. When such a load is an open delta load, then there is an energy flow to the grid simultaneous to energy flow from the grid.

**Proof.** This is a small expansion of Theorem 4.3 proof A note; this theorem significance. Present day knowledge up to this paper is that prosumers are generating MM energy gap [2]. No other knowledge. This theorem and result at section 5.2.9 changes the status of when do MM energy gap occur which equals when do simultaneous bi-directional energy flow occur at separate phases: 1) for passive 3 phase loads only open delta load connection with meter star connection, is generating MM energy gap, while star or delta is not generating MM an energy gap due to bi-directional energy flow at separate phases. 2) The other option is a generating plant such as PV and renewable. 3) last option is harmonic generating loads that generate harmonic active energy, such as converters. That is considered one of the most important results of current paper, due to exact comprehension of phenomenon and fencing. □
5. Results

5.1. Test setup

1. Test setup in lab: a “meters pallet” where meters are installed, and in the lab a three-phase calibrator of omicron 256+ connected to a smart meter were used. For direct connected meters, an additional insulating transformer was added to the calibrator. Fig. 16 shows the test setups. Case 2 is the common test case.

2. Test setup in field: a test is being performed in a smart metering system over meters installed in the field.

1) If it is desired to study the energy gap in renewables such as PV, then what is required is to allocate farms with one of the meter’s arithmetic and the other vector.

2) If PV is with a single meter, then a secondary meter of another metering method must be installed.

3) Reading is performed from the meter data management system (MDM). No need to install a setup in the lab or go out to the field. Upon reading meters’ load profile, it is run through the drivers and the “metering method validation tool”. That is the test.

4) Various test cases, such as to study the effect of different methods for various use cases are conducted from the MDM. That means that field tests are fast and various scenarios are at hand.

We used mostly data obtained through smart metering system of field deployed meters. Fig. 16 describes two lab setups. Lab setup (17-a) describes an Omikron 256+ three phase calibrator used vs. indirect CT or CT connected meter. Indirect meters have separate channels for each phase current and for each phase voltage. Directly connected meters are connected to calibrator through a 3-phase insulation transformer 0–20 Amperes due to phase current and voltage entering the meter through same channels. The Calibrator enables a lot of things: simulation of field conditions is possible through injecting energy in different directions for different phases in any desired energy quanta. Lab setup (17-a) are meter boards capable of feeding 54 meters each. The advantage is operating meters nearby at large quantity. The disadvantage is much less controllability over supply power.

5.2. An experimental study of the conversion from the arithmetic to the vector metering method

5.2.1. Test cases 1 + 2: two current transformers (CT) connected meters one arithmetic and one vector – at a PV farm – load profile scenario and billing register scenario

Objective: PV is the ultimate bi-directional simultaneous energy flow, and therefore, the best exercise load profile level arithmetic to vector conversion. The theory shall be tested in two scenarios: 1) In load profile level, which means conversion of load profile periods from arithmetic into vector first, then aggregation from load profile into billing registers. 2) Billing registers level, which means, aggregation of periods first, and then conversion. Setup: the two meters are connected to a single measurement point. The arithmetic meters load profile and billing data are converted according to eq. (6) using a software written driver. The results are shown in Fig. 17. This is the percentile energy balance when operating the research proposed “energy metering methods conversion” (EMMC) module. Both meters are class 0.5%, which means each ±0.5% accurate. That means a 0.70% collaborative error.

Analysis of results. The average error at load-profile level is 0.03 much below two-meters system accuracy. The reason that the load profile accuracy is better than the billing accuracy is because the load profile signal is closer to the continuous time random variable $$\varepsilon(t)$$, although it is actually a time-series closer than the billing random variable. The billing is an aggregation of these random variables. Each day is with an error of 96 periods which is $$\sqrt{96} \cdot \text{periodic accuracy}$$. The measured average error in billing is 0.45%, roughly multiplied by 10, the load profile error. The longer the billing duration, the higher the inaccuracy. That means that after one month, the error is 1.6% after two months 3.2% etc. The billing error is aggregating. Conversion arithmetic to vector is performed using eq. (7) section 4.2. Conversion from vector to arithmetic using eq. (8) is performed when using two meters at renewables as defined at section 4.2. Figs. 18–22 demonstrate eq. (14)–(17) of load profile, of section 4.2.

This is a very important conclusion. It means that load-profile based billing is preserving the meters accuracy. It means that active import is also preserving accuracy within meters’ accuracy. Fig. 18 shows the distribution. The Y axis is frequency of occurrence; the x axis is the accuracy. Fig. 18 shows load profile comparison, as compared to Fig. 16.
Fig. 17. Active consumption energy gap arithmetic vs. vector metering method; (a) quarter hourly load profile; (b) daily sampled energy billing registers.

Fig. 18. Distribution of gap percentile for active import load profile of test case 1. x axis is error measure; y axis is frequency of occurrence.

Fig. 19. Comparison of arithmetic load profile after conversion (blue) to vector load profile (red) at same measurement point; (a) is shorter duration (hours); (b) is longer duration (months). Experiment shows that vector and arithmetic meters: same measurement – load profile coincides.
that showed a percentile gap. It is obvious from Fig. 19 that the energy metering method conversion algorithm works accurately, certainly within the accuracy class boundaries. But the theory is tested to its strongest conclusions by the following graphs. Fig. 20 is showing at four energy quadrants (active, reactive) X (import, export) how the conversion works at the load profile level. It is in accordance with Theorem 9, the conversion formula is working for all quadrants. The graphs of converted arithmetic and vector are so close that it appears as if a single graph exists. It is remarkable that although the reactive channels are not so much used, they are implemented exactly the same as the metering method formula. At the billing registers level, the result is that for active import conversion works. In the reactive import/export, the precision of the conversion is pretty good. The Active export conversion at the “billing level” is not working accurately, as shown in Fig. 21. That is in accordance with Theorem 8, that states conversion at the “billing level” using the formula does not always work, but also states according to Theorem 10 that it may work pretty well if the consumption/generation regime is bi-directional, but not simultaneous “most of the time”. For the active import, actually herein the active export generation is dominant and for it the consumption is minor. For “Active export”, herein import is always dominated by the consumption and is always zero by the conversion and is not zero by vector metering during the nights. Fig. 21: mostly active export is inaccurate a billing level. 

That is the meter connection style. Fig. 19 is similar results as Fig. 21 and therefore mutual validation exists.

5.2.2. Test case 2: investigation of load profile accuracy of conversion module at the rest of the load profile channels/energy quadrants – a PV farm with one arithmetic and one vector meter

Test case 1 tested only the “active import” load profile and billing register MM mismatch. But there are four quadrants. Fig. 22 shows the accuracy of conversion module at the four energy channels: (Active, Reactive)X (import, Export). Notice that each graph is a different scale. Results are as follows. Active import: 0.03%, Active export: 1.5%, Reactive import: 0.03% with bursts of average 2%, Reactive export: 0.03% with mild bursts of average 0.5%. The conclusion is that implementations of reactive energy are the same as active in smart meters, based on the proposed conversion driver. Reactive mismatches between arithmetic and vector meters are larger because inherent reactive energy inaccuracy is usually larger.

Fig. 23 shows four quadrants at billing registers level. Active import is accurate but active export is not, because at any “billing instant” if the total $\text{import}_{\text{arithmetic}} - \text{export}_{\text{arithmetic}}$ is for example positive, then the active export shall be zero and comparing it with non-zero “vector billing”, active export yields a non-zero error value. Therefore, the formula (6) cannot be used and only a solution with a forecasting model, according to Theorem 10, may be used.

5.2.3. Test case 3: the saw-tooth effect, two meters, one with different inherent load-profile period energy quanta, a PV farm with one arithmetic and one vector meter

To investigate the need for the “saw-tooth synchronization” sub-module, the following load-profile was downloaded from two meters. Fig. 24 shows the saw-tooth in time axis and Fig. 25 shows the time differences in the load profile buffer numerical file. Analysis: the saw-tooth effect is clearly evident from Figs. 25, 26: exactly every four periods of the arithmetic meter, which is inherently more sensitive, the registration is identical for both meters. This is exactly in accordance with Theorem 5: saw-tooth theorem.

5.2.4. Test case 4: an example of three phase power transformer at power plants that show “irrational” results in arithmetic metering

Fig. 26 shows load profile registration of active export energy at a power plant at three generation tariff payment rates for different time segments: high, medium, low.

What could have been accidentally considered as meter failure, is not. The power transformer has one phase reversed by reversing coil
polarity. During inactive hours, the energy arriving to the transformer from some inner plant grid reverses by that phase and is recorded. Accumulating Theorem 4 and present test case there are two methods to obtain bi-directional simultaneous flow: open delta and a transformer with reverse single phase.

5.2.5. Test case 5: inspection over a large residential segment with arithmetic meters and at contrast an industrial stove

The experiment was of course run from the meter data management system (MDM). An entire population segment containing 35,000 residential meters, all of them being the arithmetic metering method. The meters were inspected for recording of “Active export” energy because
Fig. 23. Four quadrants consumption percentile gap arithmetic vs. vector metering method at load profile level. Active import is accurate, but the active export is not accurate. (a) Reactive export, (b) reactive import, (c) active import, (d) active export.

Fig. 24. Demonstration of percentile gap with the saw-tooth synchronizing component of the algorithm deactivated.

Table 9. Experiment of export energy detection at residential meters.

| Phenomenon                                    | Number of meters |
|-----------------------------------------------|------------------|
| Meters without export energy                 | 34496            |
| Meters with export energy due to wrong phase connect | 3                |
| Elevator meters                              | 1                |
| **Total**                                    | **35000**        |

none of these customers is defined as a prosumer. Is there an “arithmetic export energy” at residential apartments? Fig. 27 shows the energy flow in an induction stove top that was suspected as a possible injector to the grid. Table 9 summarizes the results.

Preliminary expectation from the test – comprehending the “gap” scenario:

1) 3 phase induction stoves are not expected to energy flow energy + to the grid, because at low voltage connection is not open-delta. Single phase devices are not expected because there is no metering gap at single phase. Therefore, in general no expectancy to MM gap.

2) Elevators at three-phase may cause energy + and there-wise export active arithmetic energy due to regenerative nature during de-acceleration.

At contrast to residential test, Fig. 28 demonstrates industrial high-voltage induction ovens for melting metal and glass. At these device types, the energy metering method gap was observed. Industrial ovens are connected at our country as open delta load connection, star connected meter. Energy from grid to load is directed also from load to grid.

20
The results from this experiment are conclusive and they are important conclusions:

1. Although there is information on penetration of electric cooking “induction stove-top”, and this new electric device may be 3 phase, there was not even a single apartment which injects energy into the grid. They are not three phase open delta because they are not high voltage. At high voltage industrial premises open-delta connection, certainly a metering gap was consistently observed, due to simultaneous bi-directional energy flow at two phases. Contemporarily: residential to not generate energy flow to grid for non-prosumer customers.

2. The billing is still monthly, and arithmetic and vector MM billing is the same because residential did not generate energy injected to the grid, and not because metering methods are the same, and they do not yield the same energy reading for simultaneous bi-directional energy flow.

3. Elevators do inject export energy to the grid, and in that case, a vector method bill would be cheaper. The selection of the metering method for a customer is a regulatory issue. Results are in accordance with Theorem 4 as regards to option when this is not 3 phase open delta load and star connected meter.

4. It is evident here how fast do arithmetic metering catches the wrong meter phase connect. This demonstrates the arithmetic metering method capability for non-technical and technical loss detection.

5. Active export energy may occur in three-phase loads. But only open delta load star connected meter, or harmonic generating load, or renewable. Then the energy gap between two metering methods shall occur.

6. When residential have three phase induction-stove tops, which is expected according to industrial experience there shall be no arithmetic export energy and no gap, because load is open-delta connection. For star or delta connections not expected a bi-directional energy flow at separate phases, therefore not expected metering gap. The induction stove tops at Fig. 27(a, b) and induction oven at Fig. 28 (a, b), exhibit the energy MM gap phenomena and the active energy from load to grid, due to self-generated harmonics and active energy, as measured by arithmetic and vector meter.

It should be notified that by decision, the entire national smart meters in our homeland are now vector. This was performed using remote firmware update. Metering business is traditional: if there is a 1% gap, there shall be no gap at all. This implies a significant cost of remote and proxy firmware update for all meters. However nationwide all smart meters are configurable arithmetic or vector MM – that is left to regulator’s decision.

5.2.6. Test case 6: a sample of thirteen renewable PV farms – with two different tariff programs at load profile level – inclusion of tariff program into metering method algebra

A renewable small/medium farm is the best location to search for gaps, because there is “simultaneous” bi-directional energy flows: to and from the grid. That is when the measurement gap emerges. The setup requires two meters installed at the same point, one arithmetic and the other vector. There are three different tariff programs: the NET Tariff program defined by equations set (23). That formula was developed long before arithmetic metering was implemented. Seven meters were according to the NET Tariff program, and six meters according to the non-NET. The non-NET tariff program is defined in eq. (22):

\[ \text{Bill} = A \cdot E_{\text{Active, import}} - B \cdot E_{\text{Active, export}} \]

Where:
- \( A \) – active, import tariff.
- \( B \) – active, export tariff

The NET tariff program may have two tariff programs, one for dual meter configuration displayed in Fig. 8. The other to a single contract is for a bi-directional meter as defined by eq. (23).

\[ \text{Bill(dual – meter)} = A \cdot \text{(import – export)} + B \cdot \text{generation} \]

\[ \text{Bill(single – meter)} = A \cdot \text{import – export} \]

Where:
- \( \text{dual – meter} \) – a system of two meters at same premise not connected at same location. Usually for renewable.
- \( \text{import, export} \) – import energy, export energy generation - energy generated by the PV or other renewable

The results of the experiment are summarized as follows at Table 10. They are in accordance with section 4.4 Theorem 10. These experiments validate the non-net tariff formula by eq. (22).

Conclusions from this experiment result:

1. The NET result is with system inaccuracy. There is a significant metering energy gap. Due to tariff programs of eq. (23), the bill is indifferent to the arithmetic or vector MM. No MM energy gap exists.

2. The MM energy gap phenomenon exposed at Tariff formula (22) is resolved only by conversion from arithmetic to vector following eq. (6): meaning at load-profile level, or following eq. (6) and Fig. 7 a, b.

3. It is observed that the proposed algebra covers the tariff program as well as the metering, enables bi-directional tariff.

5.2.7. Test case 7: comparison of metering methods at billing registers level – as compared to the load-profile channels level

The following experiment was performed on actual meters at the SAP ERP billing system on PV meters, one arithmetic and one vector at the same measurement point. The results reveal that at the billing register level, the conversion does not work so efficiently as the load profile. The solution is limited only to load profile aggregated metering which is the smart metering solution. Fig. 29 shows the energy gap percentile error distribution at the billing level monthly bill, and Fig. 30 shows the energy gap percentile error distribution at the load profile level, both for the four quadrants. It is evident that the billing registers are not accurate at all quadrants, especially active export. The accuracy for some
billing registers is according to Theorem 10 and inexact according to
Theorem 8. For PV billing, both active export and import are required.
It is evident that for the load profile level, which for smart meters ex-
ist also at the billing level for many countries, accuracy enables precise
unified billing regardless of the metering method.

5.2.8. Test case 8 - electric railway station: comparison of methods at load
profile energy measurement and at two different tariff programs

The electric railway experiment was conducted in order to decide
whether it generates an energy gap or not. Following the theorems of
the theoretical section chapter, translating the problem it means bi-
directional.

A model type SATECINC EM 720 of a local utility company was
used for this experiment. A similar model type SatecINC PM 180 is
used at several countries in Europe. Fig. 31 shows the accelerating and
decelerating train and energy flow direction. The experiment was con-
ducted at four railway stations. A meter with two metering methods
registers sets, and two methods of load profile channels – at same me-
ter were used. The meter default method for payment is vector at local
utility company. The billing is configured to be according to the vector
method. Such meters are located at European railways in Spain, Turkey,
Netherlands and possibly Russia, but of a slightly different type: PM 180 –
with identical registers/load profile structure. Important conclusions
from the conducted experiment as shown at Fig. 31:

1. The railway is an example of regenerating energy while operating
the break.
2. There is “simultaneous bi-directional” energy flow. There is “bi-
directional flow” and at some moments it is simultaneous. That
is an important result – prior to conducting the experiment this
was unknown to the utility company. The result is in accordance
with Theorems 3, 4 which where chronologically proved after the
experiment.
3. At the railway station at each moment only a single phase load is
fed.

5.2.9. Test case 9: an elevator at a residential or commercial building

After test case 6 on “residential premises” was conducted, the fo-
cus turned onto stairways meters, a focused experiment of elevators.
Why did some elevators generate MM energy gap while others did not?
The test result is summarized in Table 11 herein. Fig. 32 is a heuris-
tic demonstration of an elevator as a motor at acceleration and steady
state, and a generator at deceleration.

Conclusions are evident and important from test setup by Fig. 32
and from Table 11:

[1] Elevators are regenerating in deceleration periods. The fact that it
is not open-delta load connection plus meter star connection – rules
out the possibility that connection scheme is relevant. A check of
elevator scheme is in accordance with the deduction – not an open
delta, not high-voltage for residencials.

[2] The first row shows a zero energy-gap. Observing load profile that
is similar behavior to the railway system: bi-directional flow is ei-
ther from grid or to grid but never simultaneous for the phases
split.

5.3. Test case 10: testing and requesting information from meter
manufacturers on a large array of twelve manufacturers and twenty model
types from various manufacturers

There is great doubtfulness as regards to how accurate can this for-
mula be. In order to respond to that challenge, the following experiment
was conducted. They are aimed to show universal:

1. Smart meters of various manufacturers existing at a local util-
ity company: (i) Landis & Gyr, (ii) Elgama-Elektronika, (iii) Dr.
Neuhaus, (iv) Kaifa, (v) Ziv, (vi) SatecINC – was conducted for arith-
metic and vector metering methods.
3. Regarding the meters existing at the local utility company (i)–(vi): for CT connected meters, there were already existing pairs: primary and secondary and already field reports of gaps at the prosumers. As a clarification, a secondary meter is installed for redundancy, beyond 100 Amperes, two meters are installed, so if one fails the other remains. For direct meters, there were no pairs, and a secondary meter was installed on the “opposite” metering method.

The following meters manufacturers and model types and quantities were tested, as described in Table 12. Table 13 demonstrates single phase arithmetic meters verified to be identical to vector single phase meters. Kaifa meters have sold at least 28 M meters to EU.

Comments: (i) Kaifa meters have generated more than 28 M meters to EU and meters are MID notified body blocks b and d. (ii) Prime protocol is PLC OFDM communication type.

Table 14 details: CT, CT-VT connected three-phase meters used for industrial premises of consumption load, larger than 100 Amperes.

It must be mentioned that currently all the above meters are dual arithmetic and vector, with configuration of metering method selection, with a default vector metering method used for billing. That is obtained by firmware version. Table 14 lists three-phase direct and single-phase absolute vector meters that were validated. All are basic non-smart meters: billing only, no load-profile, no remote communication, no tariff, class 2 accuracy. There are about twelve-year-old meters to be replaced by smart metering deployment. The conclusion from Table 15 is that all tested meter model type manufacturers, and all tested twelve manufacturers, and in addition eight additional manufacturers, proposed specification.

In addition, Group 4: All electro-mechanical meters were validated as vector in nature.

Behave in accordance with metering methods formula (9) defined by the current paper and Fig. 9. The paper reflects to the author’s opinion the international state. The models are universal, the algebra is universal, and the testing performed in local utility reflects, to the author’s opinion, the universal state. The results are in accordance with Theorems 1-11.
Fig. 30. Energy gap histogram at four quadrants of load-profile. (a) Reactive export, (b) reactive import, (c) active import, (d) active export.

Fig. 31. (a) Train A accelerating and consumes active energy flow from the grid – marked red arrow. Train B is breaking and during deceleration the train motor turns into a generator that injects active energy flow to the grid, marked green arrow. (b) EM 720 CT-VT connected meters used in railways (c) energy load profile of arithmetic active + (import) red, active- (export) blue load profile. Taken from national railway meters.
Fig. 32. (a) Elevator (1) accelerating and consuming energy (left) (2) while decelerating is generating energy (right) (b) Kaifa direct three-phase meter used in the experiment.

Table 12. Three-phase direct smart meters model types with accuracy class 1 that were tested for being arithmetic in accordance with the paper’s defined specifications.

| Model type Communication method | Quantity scanned | Manufacturer | Manufacturer headquarters country | Suitability of metering method to arithmetic formula |
|---------------------------------|------------------|--------------|----------------------------------|-----------------------------------------------|
| Cellular                        | ~4000            | Elgama       | Lithuania                        | V                                             |
| PLC IDIS S-FSK                  | ~10000           | Landis & Gyr | Switzerland                      | V                                             |
| PLC IDIS S-FSK other type       | 175              | Landis & Gyr | Switzerland                      | V                                             |
| PLC IDIS S-FSK class 0.5 s      | 7                | Landis & Gyr | Switzerland                      | V                                             |
| PLC IDIS G3                     | ~150             | Landis & Gyr | Switzerland                      | V                                             |
| PLC Prime 1.3.6                 | ~300000          | Kaifa meters | China                            | V                                             |
| PLC Prime 1.3.6.                | ~50,000          | Ziv automation | Spain                           | V                                             |

Table 13. Single-phase accuracy class 1 direct smart meter model types with accuracy class 1 that were tested for being arithmetic in accordance with the paper’s defined specifications.

| Model type Communication method | Quantity scanned | Manufacturer | Manufacturer headquarters country | verified no gap from vector method |
|---------------------------------|------------------|--------------|----------------------------------|---------------------------------|
| PLC Prime 1.3.6                 | ~10000           | Kaifa meters | China                            | V                               |
| Cellular                        | ~1500            | Kaifa meters | China                            | V                               |

Table 14. CT, CT-VT connected three-phase arithmetic meter model types with accuracy class 0.5 s, and above.

| Model type Communication method | Quantity scanned | Manufacturer | Manufacturer headquarters country | verified no gap from vector method |
|---------------------------------|------------------|--------------|----------------------------------|---------------------------------|
| PLC Prime 1.3.6                 | ~10000           | Kaifa meters | China                            | V                               |
| Cellular                        | ~1500            | Kaifa meters | China                            | V                               |
| Cellular CT                     | ~1500            | Dr. Neuhaus/ Reallin | China                        | V                               |
| Cellular CT                     | ~3500            | Elgama       | Lithuania                        | V                               |
| Cellular CT-VT                  | ~100             | Elgama       | Lithuania                        | V                               |
| Cellular CT                     | ~5000            | Elgama       | Lithuania                        | V                               |
| Cellular CT-VT                  | ~1000            | Kaifa meters | China                            | V                               |
| PLC Prime 1.3.6                 | ~1000            | Elgama       | Lithuania                        | V                               |
| PLC Prime 1.3.6.                | ~1000            | ZIV automation | Spain                          | V                               |
| Cellular CT-VT class 0.2 s for high-voltage customers and plants | ~400 | Satec INC | USA | V |
5.3.1. Test case 11: a single-phase meter – testing whether energy gap between metering methods occur

Through MDM, single phase arithmetic meters were read. For residential, no active export was documented. In a lab setup, as described in Fig. 16, two meters of different energy MM were simultaneously installed with an Omikron 256+ calibrator, operating various energy flow directions. The result showed no energy MM gap, in accordance with Theorem 6. There is no other phase with opposite energy flow. Table 11 and Table 12 reflect the experiments. Theoretically for single phase meters there can be no MM energy gap in accordance with Theorem 6.

5.4. Test case 12: a plant with a furnace arc – verification whether active energy is injected to the grid and does the energy gap between metering methods occur

At a single spot at an industrial plant, a difference was registered at an industry area named “Ramat-Hovav”, southern national district. Furthermore, this occasionally burns the district’s “three phase transformer” at a one its three phases, the transformer is located at the distribution sub-station. Fig. 33 shows an electric scheme taken from [37]. That paper insists on a filter at exit from the factory. That was not performed at that specific factory. Fig. 34 shows measurement of energy load profile with arithmetic meter for a steel welding plant located at local nation. An evident simultaneous bi-directional energy flow. We looked for reasoning in the academic literature. Injecting “reactive and active” energy non-prime harmonics to the grid from the furnace arc is described in the work by Redlarski et al. [37].

That work also explains that the third grid frequency harmonic penetrates the grid because it is 120° phase shift, while the third phase of a three phase is also 120°. That work also recognizes that active energy may be injected by the arc to the grid of a third harmonic and that is not healthy for the devices, and further cannot be used for consumption. To explain the results, it is required to use Theorem 4. When the connection a) is asymmetrical b) and in addition contains a resistive component, that is the case where a furnace arc shall inject active energy to the grid. Herein it makes sense that the regulator shall not require to define that customer as a consumer, and an arithmetic meter is implied. Not only does the active export assist, it enforces the utility or factory owner to install a filter so as to maintain grid power quality. Fig. 34 is the results of test setup described at Fig. 33. A formula for harmonic active power and active energy [1], [17] is shown as eq. (24):

\[ P = \sum \frac{v_{a,b} \cos(\phi_a)}{i_n} \quad E_A = \int P(t)dt^{1/2} \]

\[ P(t) \] being the slowly varying envelope and \( i_n, i_a \) derived from 20 msec periodic assessment. From herein same procedure applied at arithmetic and vector metering shall yield energy+, energy−. A meter that’s implemented at time axis shall yield same results – this may be theoretically shown by integrating Fourier series of \( v(t), i(t) \). A paper by this group [38] explains how to separate net current into current+ and current- components using Current physical components theory (CPC). Eq. (24) may be applied to Fig. 34(b-d) graphs for energy to power and to power harmonics.

From current into power, and the power direction is the energy direction. CPC is suitable for waveform sensor:

\[ I_{energy+} = I_{Active} + I_{Scattered} \]

\[ I_{energy-} = \text{Real}\{I_{customer}\} \]

---

Table 15. Three-phase meters with absolute vector metering method validated as compared to the paper’s.

| Model type | Quantity installed | Manufacturer | Manufacturer headquarters country | Suitability of metering method to arithmetic formula |
|------------|--------------------|--------------|----------------------------------|-----------------------------------------------|
| Single phase | ~1000000 | Iskraemeko | Slovenia | V |
| Three phases | ~1500000 | Iskraemeko | Slovenia | V |
| Three phases | ~25000 | Holley | China | V |
| Three phases | ~100000 | Dr. Neuhaus | China/Germany | V |
| Single phase | ~2000 | Elster | UK | V |
| Single phase | ~40000 | Elster | UK | V |
| Single phase | ~50000 | Hexing | China | V |
| Single phase | ~25000 | Dr. Neuhaus | China/Germany | V |
| Single phase | ~10000 | Holley | China | V |

---

Fig. 33. (a) Furnace arc; (b) Active power of a higher harmonic discharged on the damping resistor \( R_T \), and in the transmission system as a function of the resistance of the damping resistor \( R_T \). Furnace electric schematic is taken from [37]; (c) Elgama CT-VT connected meter used in the experiment as arithmetic vs. secure meter manufactured in the year 2009, used as a vector.
5.5. Test case 14: an open delta connection – is arithmetic metering correct or is it wrong there?

There is doubt as regards to correctness of measurement of an open delta connected load with arithmetic meters. A metering array, including one arithmetic meter and one vector meter, is installed in the field. Since open delta is accustomed in our homeland for low voltage larger than 100 Ampere load. The experiment shows perfect match after conversion from arithmetic to vector. It also shows the same metering without the conversion. That is in accordance with Theorem 7.

5.5.1. Test case 15: detection of reverse phase connectivity using arithmetic active export energy

The following rule has been asserted over the residential population: (i) Detect a reverse phase connect event through smart meter; (ii) Logic or: detect active export energy. Section (ii) is in case the event occurrence in the event log file is double checked in case it is missed. The result was seven meters of a certain meter manufacturer, and its name is not published herein. All meters are verified phase connect. That exemplifies how arithmetic metering may be used for technical non-technical loss detection, as stated at Table 4.

5.5.2. Test case 16: demonstration of energy gap at billing energy load profile channels level – as observed by SAP ERP billing system – accumulated by Tariff registers

The following two real-life examples, just rounded to “nice” kilowatt-hour figures, demonstrate energy gap that exists but is transparent to NET tariff program, but is not to a tariff program with different tariff per generation and consumption. Table 16 is an effective numeric example of difference between metering methods and between tariff programs, dependent on metering method. It is observed that NET tariff is indifferent to metering method although a MM gap exists, in accordance with Theorem 2. It is observed that an energy metering gap exists, and billing gap exists at case 2. Table 16 case 1 scenarios 1.1–1.3
are in validating eq. (12) for Tariff, and eq. (13)–(17) for load profile. The billing is aggregation of load profile. If dominant direction of energy flow is dominant in one direction all/most of time, then billing follows similar formulas as load-profile.

Cases 1, 2 scenarios demonstrate Fig. 15(a, b) graphs of load profile, for the case of dominant consumption or generation. Some of test cases could be repeated with Fig. 16(a, b) equipment using meter board, and Omikon 256 + calibrator, provided recorded waveforms on field using waveform recorders such as SATEC EM720, or HIOKI rk6600, with large memory bank. The waveforms transferred to Matlab and injected by Omikon 256+. That form of lab reproduction of field scenarios is known at smart metering projects.

6. A techno-economic model for computation of energy-gapped market segments based on the presented theory – an example of analysis of the Israeli market

The set of fifteen conducted tests have covered the issue from all angles. Engineering issues that appear to the research group as relevant and according to the literature survey are not reported, and no answer is documented, and no questions were asked. What is the world status regarding that issue? What are the boundaries of the problem? This paper attempts to encapsulate the experimental and theoretical findings in some summarizing figures that are of interest to the engineering community of smart metering manufacturers, users, regulators, and utility companies. Table 17 summarizes the segmentation parameters, as collected from data center of local Distributed system operator (DSO). Presented state is an example of the results presented herein, exemplifying the methodology: eight million citizens, generation capacity 16,905 Giga Watt, 3.2 million meters. It is an OECD country in the midst of an energy market reform and national deployment with already ~500,000 meters deployed.

Fig. 36 provides a classification of customer segments according to this research, and this is the first published paper or document over the issue, making a lot of order in a 1 B meters and 36B$ energy trading. Assuming 100% of smart metering deployment. Majority of problem is located at high voltage open-delta, regenerative railway and PV farms. Currently residential segment is outside the game with exception of a few elevators. There are ~200,000 PV licenses plan in coming years ~6.2% of future meters count, but certainly ~45% of future national energy market. With increase of EV this is going to change.

Conclusions observing Fig. 36-a: (1) Most residential exhibit no export energy except elevators 0.2%, 66.2% of Israel customers are without energy gap as to 2022. (2) The highest energy gap segment is PV, and then induction oven/stove tops, and then all the rest. Conclusions from Fig. 36-b: Again most energy involved at energy MM gap at Israel is renewables, then ovens, stove tops. Then all the rest. Conclusions from Fig. 36-c: Meter count of MM energy gap is fenced such that 95% is without the MM energy gap phenomena. Renewables are majority of meters at locations with gap, then elevators, then only faulty capacitor banks with disconnected phase.

7. Conclusions

The research has initially presented the knowledge and technology and economy gaps:

1) MM energy gap is precisely fenced with Theorem 4: (i) either 3 phase open delta, or renewables, (ii) or harmonic generating load such as furnace arc and converters, (iii) or renewable such as PV, (iv) or power transformer with single phase reversed. (v) regenerative devices such as elevators and electric trains. There was a when not rule: not a single phase meter/load and not at three phase all phases star/delta load connection with passive loads. That modifies present day knowledge of prosumers.

2) Whenever there is a national dispute between the arithmetic and vector metering methods, after being detected by customers or utility companies billing departments, a dispute which may be regulatory or due to political pressure by customers. The current policy is meters’ replacement in the field, or firmware updates, which are very costly operations for PLC/RF mesh. The companies would prefer arithmetic metering due to being significantly more suitable to distributed energy resources (DER), and a liberalized market conduct, and technical/non-technical simple loss detection. Customers usually would prefer the vector metering method since it is less costly for some segments due to decrement of export energy from import.

3) It is impossible up to present day knowledge, to use arithmetic meters, enjoying the significant benefits of arithmetic MM for DER energy market, while maintaining vector MM billing – preferred by some regulators (not by all). The research investigated the enablement of using vector or arithmetic MM for billing while using arithmetic meters. This was researched herein and found suitable.
### Table 16. Difference at billing dependent on metering method and on.

| Case 1: NET contract | Scenario 1.1: consumption larger than generation | \( A(\text{import} - \text{export}) + B(\text{Generation}) \) |
|----------------------|-----------------------------------------------|--------------------------------------------------|
| Generation point meter | 500 Arithmetic gen 500 Vector gen 500 | |
| Bi-directional meter energy+ | 300 Arithmetic import: 300 Vector Import = 100 = 300-200 (since energy+\text{energy}+) | |
| Bi-directional meter energy− | 200 Arithmetic export: 200 Vector export = 0 (since energy+\text{energy}−) | |
| Billing over arithmetic method | 500 + (200-300) = 400 Billing over vector 500 + (0-100) method | |

| Case 1: NET contract | Scenario 1.2: consumption = generation | \( A(\text{import} - \text{export}) + B(\text{Generation}) \) |
|----------------------|-----------------------------------------------|--------------------------------------------------|
| Generation point meter | 500 Arithmetic gen 500 Vector gen 500 | |
| Bi-directional meter energy+ | 500 Arithmetic import: 500 Vector Import = 500-500 = 0 (since energy+\text{energy}+) | |
| Bi-directional meter energy− | 500 Arithmetic export: 500 Vector export = 0 (since energy+\text{energy}−) | |
| Billing over arithmetic method | 500 + (500-500) = 500 Billing over vector 500 + (0-0) = 500 method | |

| Case 1: NET contract | Scenario 1.3: Generation larger than consumption | \( A(\text{import} - \text{export}) + B(\text{Generation}) \) |
|----------------------|-----------------------------------------------|--------------------------------------------------|
| Generation point meter | 500 Arithmetic gen 500 Vector gen 500 | |
| Bi-directional meter energy+ | 200 Arithmetic import: 200 Vector Import = 0 (since energy+\text{energy}+) | |
| Bi-directional meter energy− | 300 Arithmetic export: 300 Vector export = 100 (since energy+\text{energy}−) | |
| Billing over arithmetic method | 500 + (300-200) = 600 Billing over vector 500 + (100-0) = 600 method | |

| Case 2: flow direction variable tariff | Scenario 2.1: consumption larger than generation | \( A(\text{import}) + B(\text{export}) \) |
|--------------------------------------|-----------------------------------------------|--------------------------------------------------|
| Generation point meter | 500 Arithmetic gen 500 Vector gen 500 | |
| Bi-directional meter energy+ | 300 Arithmetic import: 300 Vector Import = 100 = 300-200 (since energy+\text{energy}+) | |
| Bi-directional meter energy− | 200 Arithmetic export: 200 Vector export = 0 (since energy+\text{energy}−) | |
| Billing over arithmetic method | Generation: 300 Consumption: 200 Billing over vector method Generation:0 Consumption: 100 | |

| Case 2: flow direction variable tariff | Scenario 2.2: consumption = generation | \( A(\text{import}) + B(\text{export}) \) |
|--------------------------------------|-----------------------------------------------|--------------------------------------------------|
| Generation point meter | 500 Arithmetic gen 500 Vector gen 500 | |
| Bi-directional meter energy+ | 500 Arithmetic import: 500 Vector (since energy+\text{energy}+) | |
| Bi-directional meter energy− | 500 Arithmetic export: 500 Vector export = 0 (since energy+\text{energy}−) | |
| Billing over arithmetic method | Generation: 500 Consumption: 500 Billing over vector method Generation:0 Consumption: 0 | |

| Case 2: flow direction variable tariff | Scenario 2.3: Generation larger than consumption | \( A(\text{import}) + B(\text{export}) \) |
|--------------------------------------|-----------------------------------------------|--------------------------------------------------|
| Generation point meter | 500 Arithmetic gen 500 Vector gen 500 | |
| Bi-directional meter energy+ | 200 Arithmetic import: 200 Vector Import = 0 (since energy+\text{energy}+) | |
| Bi-directional meter energy− | 300 Arithmetic export: 300 Vector export = 100 (since energy+\text{energy}−) | |
| Billing over arithmetic method | Generation: 300 Consumption: 200 Billing over vector method Generation:100 Consumption:0 | |

4) This was shown initially as a problem in trading between companies and between nations with different metering methods. Trading is already enabled as the companies did not wait for the proposed algorithm. However, it is simplified and improved by the proposed solution. World-wide energy trading was estimated at $36 B annually and smart meters count around $1B, as for 2022. The cost of present-day handling was shown to be dramatic. It is of the order of magnitude of a second-generation smart metering deployment,
Table 17. Segmentation of metering method gap, by example of an example of a developed OECD country of 3.2 M households. The renewables are 6% of generation.

| Market segment name                  | Is there metering method gap | Energy Is energy flow bidirectional, simultaneous? | Percentile of customers in Israel | Percentile of annual energy consumption (assuming 67% is consumed by 100,000 customers) |
|--------------------------------------|-----------------------------|--------------------------------------------------|---------------------------------|----------------------------------------------------------------------------------|
| Residential                          | No                          | No                                               | 98.44% (3.15 x 10^6)           | 33%                                                                              |
| Renewables                           | Yes                         | Yes                                              | 1.5625% (50,000)               | 10% (30% by 2030)                                                               |
| Furnace-arc                          | Yes                         | Yes                                              | 0.00625% (Estimated 200)       | 0.2%                                                                            |
| Three phase induction ovens          | Yes                         | Yes when asymmetric load                         | 0.00625% (Estimated 200)       | 0.2%                                                                            |
| Elevators                            | Yes                         | Yes when asymmetric load                         | 1.953% (Estimated 62500)       | 0.644%                                                                          |
| railways                             | No                          | No                                               | 0.00019% (25)                  | 0.0375%                                                                         |
| Three phase arithmetic               | Dependent on situation      | Dependent on situation                           |                                 |                                                                                  |
| Single phase                         | No                          | No                                               |                                 |                                                                                  |
| Open-delta connection (High voltage) | No                          | No                                               | 0.15% (4800)                   | 4.8%                                                                            |
| Capacitors bank when factory is off  | Yes                         | Yes when asymmetric load                         | 1.5625% ~50,000               | Theoretically ~10% of energy market                                             |
| Low voltage non-residential (> 100  | Yes                         | Yes when asymmetric                              | 0.0156% (estimated 500)       | 0.5%                                                                            |
| Amperes consumption                  |                              |                                                  |                                 |                                                                                  |
| Very high voltage (120KV-400kV)      | Yes                         | Yes                                              | 0.0156% (estimated 500)       | 0.5%                                                                            |
| Railway                              | Yes                         | Yes                                              | 0.000125% (Negligible. Only one company, 20 meters, 4 stations) |                                                                                  |

1 Based on data from local DSO.

Fig. 36. (a) Top left: segmentation based on percentile energy segments of Energy gap problematic segments in Israel. It includes the residential segment which is 33% of total energy consumption and is 66.2% of all the problematics + residential; (b) Top right: all the problematic segments only, percentile of energy consumption. High voltage consumers is 50% of entire “problematic”: (c) Bottom: meter count regarding all the problematics and residential which are not. 5% of meters actually reveal a problem.

which takes between 5-15 years. This issue cannot be ignored. The problematic energy segments were accurately mapped and quantified and no other significant relevant segments exist.

5) It was also shown that for renewables and industrial premises, the scenario of “(energy-)” generation simultaneously to “(energy+)” already exists, and for the residential premises, only part of the elevators and that’s it. It is more known that devices inject reactive energy to the grid due to inverters. It is much less known or not known at all those devices inject active energy to the grid, which is useless, due to being harmonic.

6) Standardization gap. It has been discussed that current Metrology standards IEC/EN 62052, 62053-21 to 24, although renewed in year 2020, that the MID standard EN 50470-1,3, do not address the issue. It was shown that DLMS/COSEM standard, IEC 62056 and the 4 colored books, and the PLC companion standards PRIME + T5 and IDIS G3/S-FSK, that all these standards define the energy registers but do not refer to metering methods expressions, except for absolute metering. Therefore, a problem exists of validation of the metering method, which is measurement correctness validation.
It was investigated whether there are known electric schematics which yield an incorrect result with arithmetic metering – this is not an issue if conversion to the correct result is possible.

Now transferring from gaps to results:

(i) A survey of metering method status and gap status around the world was presented. It is not an entire world survey, that information is occasionally discrete, but it is the only survey that is published currently and it provides a good image of the world status.

(ii) Eleven “energy gap between metering methods” theorems were formulated and proved.

(iii) Fifteen interesting test cases were conducted, covering all relevant segments, and some results were new, at least to this group, and they are not reported in published papers, to the best of our knowledge or we could not find them. These tests shed light on the spread of energy gap phenomena and to its boundaries and are effective to regulation and utility companies in decision making. Tests are described at sections 5.1, 5.2.

(iv) A techno-economical report evaluating precisely: (1) All relevant segments to the problem; (2) Quantifying energy percentile share per segment; (3) Quantifying meters count percentile share per segment. That report enables, per nation, to precisely draw the boundaries of the phenomena, from 100% to 5%, as shown in the example. It may be performed periodically to detect changes in market.

(v) The proposed algorithms enable usage of arithmetic metering in favor of DER and technical and non-technical loss detection, and yet maintain the option of vector metering billing in case that it is the regulator’s decision. The regulator has freedom.

(vi) The proposed algorithm as a simulator, or by comparing to a reference arithmetic meter was shown to be an accurate validation to the metering method implementation of a meter model type under test. Further this group reports the detection of 3 conceptual arithmetic implementation defects that are detected only with the driver used as validation tool, and possibly exists world-wide and cause wrong billing. That is beyond paper’s scope.

(vii) It was shown that at load profile level all quadrants that the formula is correct. At the billing level, it was shown to be incorrect, but approximate for premises where energy flow is bi-directional but usually not simultaneous.

(viii) The gap between arithmetic and vector metering methods results was well bounded from the above by the research. The exact rule is: 1) either open-delta load connection with star connected meter, or regenerative electric devices 3) or harmonic generating loads such as AC/DC, DC/AC converters, 4) or other active loads. That is work’s most important rule that modifies current knowledge of prosumers. The licenses count in the world for PV, and EV charging pillars count expected to grow. Whoever shall ignore that and the benefits of arithmetic metering might encounter problems at DER grid.

(ix) For residential customers, there were no gaps except for the elevator. For PV with a NET tariff program, it was both proven and tested that there was no difference between the metering methods. There remains renewables and industrial energy generators with asymmetrical tariff between import and export. That puts many DSO’s around the world on safe ground. They may now install arithmetic meters and only the prosumer segment shall show a gap from previous electro-mechanical and basic electronic meters. These construct a small segment of the customers. However, the electricity market goes on 100% accuracy because it is bound to liability and standards. A regulator insisting on that shall require the proposed algorithm for 100% of customers.

(x) An algebra defining “metering method” formulas was developed as energy mathematical equations, for the first time published or otherwise. The algebra was demonstrated to be suitable as “meter design” and “smart metering measurement and billing issues”. It includes visibility to both energy and billing tariff programs, that the standards do not. The algebra was further developed to “energy conversion formulas”, one from the arithmetic metering method to the vector metering method, and for renewable bi-meter deployment, also from vector to arithmetic. It was shown that the arithmetic method may be converted at the load profile level, 100% accurate in all of the use cases, the entire time, to all the other methods. Had the proposed algorithms and module not been first time published herein, it would have been used by organizations to accurately resolve this significant economic and regulatory issue, and possibly published in some paper. It would have been used to validate the meter functionality.

Finally, the algebra was used to prove some theorems regarding energy measurement, and billing payment, issues that are completely not addressed by standards and engineering and are of heavy economic implication.

(xi) According to algebra and field experiments it was shown by (Active, Reactive)X(import, export) channels of energy load profile periods that the algorithm is accurate. For active import, it is accurate at the billing register also, but for the rest of channels, it is meter type dependent. The algorithm is always correct for all channels if the billing formula is of the NET Tariff program, meaning: payment = A(import – export) + B. The NET Tariff program is a common international tariff program but is not the only tariff program. There are various other tariff programs, therefore entire renewables and the prosumers community should be considered, and this is the recommended best practice.

(xii) Arithmetic metering through property (iv) above facilitates smart grid management. (1) If the regulator, for example, wants a different tariff for energy flow to the grid and from the grid, he may perform so through arithmetic metering. (2) If it is required to pay for (energy−) active energy only for a tariff program defined for prosumers, then arithmetic enables that. (3) If it is required to differentiate grid management for (energy−), (energy+) the same applies. (4) If it is required to measure vector metering for a segment of tariff programs, then the proposed paper answers that issue. (5) If it is required to easily detect technical/non-technical losses which constitute around 15% per nation annually of total generation, then arithmetic metering enables that. Therefore, although it is painful, probably in the far future, meters shall be entirely arithmetic. The current paper proposes a complete solution.

(xiii) The research demonstrates beyond any doubt, to our comprehension, that the conversion formulas work 100% accurately for load profile. It was also proved that at the billing level with asymmetrical tariff weights for import and export or for (import-export) shall not work efficiently. Although it works for load profile, which is the modern-day billing computation method, for ‘residential and non-residential premises’

(xiv) The percentile gap is within the systems error. The new proposed conversion module enables unification of billing that is an identical result to meter replacement of the metering method, either by fleet replacement or by remote firmware update, which are extremely costly in the field. The research has pointed out advantages and disadvantages of both metering methods and that selection has been notified.

Declarations

Author contribution statement

Netzah Calamaro: Conceived and designed the experiments; performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.
Moshe Donko: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.
Doron Shmilovitz: Conceived and designed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.

**Funding statement**

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

**Data availability statement**

Data will be made available on request.

**Declaration of interests statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

**Acknowledgements**

Thanks to Mr. Avraham Calamaro for helpful insights.

**References**

[1] Power Definitions and the Physical Mechanism of Power Flow, IEEE, chapter 4, Prof. Alexander Eagles Emanuel.
[2] Smart meter reading method, in: European Union Document by Regulatory Organization “European Forum for Energy Business Information Exchange” (EBIX), 2017.
[3] P.S. Filipski, Y. Baghrouz, M.D. Cox, Discussion of power definitions contained in the IEEE dictionary, IEEE Trans. Power Deliv. 9 (3) (July 1994) 1237–1244.
[4] K. Sowa, M. Banzytki, S. Piróg, One phase active filter with energy storage for active power surge compensation in feed line, Arch. Electr. Eng. 65 (2) (2016) 221–234.
[5] D. Rodríguez, J. Martínez Gómez, G. Guerrón, A. Ríoñio, Impact of induction stoves penetration over power quality in Ecuadorian households, Rev. ESPACIOS 40 (13) (2019).
[6] D. Vollrath, Regenerative elevator drives: what, how and why, Elevator World 58 (2010) 95–99.
[7] Y. Jiang, J. Liu, W. Tian, M. Shahidehpour, M. Krishnamurthy, Energy harvesting for the electrification of railway stations: getting a charge from the regenerative braking of trains, IEEE Electrif. Mag. 2 (3) (Sept. 2014) 39–48.
[8] https://oeo.world/en/profile/hr92/electricity.
[9] S. Vükkel, H. Döger, V. Meral, Financial analysis of international energy trade: a strategic outlook for EU-15, Energies 12 (2019) 431.
[10] Y. Liu, B. Chen, W. Wei, L. Shao, Z. Li, W. Jiang, G. Chen, Global water use associated with energy supply, demand and international trade of China, Appl. Energy 257 (2020) 113992.
[11] T. Sirojan, S. Lu, B.T. Phung, E. Ambikairajah, Embedded edge computing for real-time smart meter data analytics, in: 2019 International Conference on Smart Energy Systems and Technologies (SEST), 2019, pp. 1–5.
[12] T. Wilson, Smart grid & smart meter architecture, in: Wireless Safety Summit, Oct. 5, 2011.
[13] W. Wichakool, Z. Remscrim, U.A. Orji, S.B. Leeb, Smart metering of variable power loads, IEEE Trans. Smart Grid 6 (1) (Jan. 2015) 189–198.
[14] E.A. Soto, L.B. Bosman, E. Wollega, W.D. Leon-Salas, Peer-to-peer energy trading: a review of the literature, Appl. Energy 283 (2021) 116268.
[15] S.J. Weikel, Measurements of electricity meters beyond kWh, in: 2003 IEEE Power Engineering Society General Meeting (IEEE Cat. No. 03CH37491), vol. 1, 2003, pp. 92–94.
[16] Chi-Jui Wu, Cheng-Ping Huang, Tzu-Hsun Fu, Tzu-Chih Zhao, Hong-Shian Kuo, Power factor definitions and effect on revenue of electric arc furnace load, in: Proceedings. International Conference on Power System Technology, vol. 1, 2002, pp. 93–97.
[17] IEEE 1459-2010: IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Non sinusoidal, Balanced, or Unbalanced Conditions.
[18] DIN 40110-1, 3: QUANTITIES USED IN ALTERNATING CURRENT THEORY, TWO-LINE CIRCUITS.
[19] Y. Beck, N. Calamaro, D. Shmilovitz, A review study of instantaneous electric energy transport theories and their novel implementations, Renew. Sustain. Energy Rev. 57 (May 2016) 1428–1439.
[20] N. Calamaro, Y. Beck, D. Shmilovitz, A review and insights on Pouyning vector theory and periodic averaged electric energy transport theories, Renew. Sustain. Energy Rev. 42 (Feb. 2015) 1279–1289.
[21] S.A. Dyer (Ed.), Survey of Instrumentation and Measurement, Wiley-IEEE, 2001, page 875.
[22] J. Augusti, R. Krikštolaitytė, Lithuanian energy security level assessment based on indicator dependence, in: Safety and Security Engineering IV, 2011, pp. 71–82, Retrieved from https://www.witpress.com/docs/91118.pdf.
[23] DLMS spec for bulk meters in India.
[24] Standard EN 50160 – Voltage Characteristics of Public Distribution Systems.
[25] https://www.dhis.org/, EBIX European forum for energy Business Information e-change.
[26] IEC 62052-11: Electricity metering equipment - General requirements, tests and test conditions - Part 11: Metering equipment, IEEE org.
[27] IEC 62053-22: 2020 Electricity Metering Equipment - Particular Requirements - Part 22: Static Meters for AC Active Energy (Classes 0, 1S, 0, 2S and 0, 5S), IEEE org.
[28] IEC 62053-23: 2020 Electricity Metering Equipment (A.C.) - Particular Requirements - Part 23: Static Meters for Reactive Energy, (Classes 2 and 3).
[29] IEC 62053-24: Corrigendum- 1 - Amendment – 1 - Electricity Metering Equipment (A.); Particular Requirements - Part 24: Static Meters for Reactive Energy at Fundamental Frequency (Classes 0, 5 S, 1 S and 1.
[30] EN 50470-1, Electricity metering equipment (a.c.) Part 1: General requirements, tests and test conditions - Metering equipment (class indexes A, B and C), CENELEC.
[31] EN 50470-3, Electricity metering equipment (a.c.) Part 3: Particular requirements - Static meters for active energy (class indexes A, B and C), CENELEC.
[32] C. Alaton, F. Touquet, Benchmarking smart metering deployment in the EU-28 final report, Directorate-General for Energy (European Commission), Tractebel Impact.
[33] DLMS User Association, COSEM Architecture and Protocols, Seventh Edition.
[34] DLMS User Association, Identification System and Interface Classes, 13th Edition.
[35] A. Gioia, A. Amato, P. Di Leo, S. Fichera, G. Malgaroli, F. Serrino, S. Tzanova, Self-consumption and self-sufficiency in photovoltaic systems: effect of grid limitation and storage installation, Energies 14 (2021) 1591.
[36] S. Quaslin, K. Kavvadas, A. Mercier, I. Pappone, A. Zucker, Quantifying self-consumption linked to solar home battery systems: statistical analysis and economic assessment, Appl. Energy 182 (2016) 58–67.
[37] A.G. Lange, G. Redlarski, Selection of C-type filters for reactive power compensation and filtration of higher harmonics injected into the transmission system by arc furnaces, Energies 13 (2020) 2230.
[38] N. Calamaro, A. Ofir, D. Shmilovitz, Application of enhanced CPC for load identification, preventive maintenance and grid interpretation, Energies 14 (2021) 3275.