Holographic Consistency and the Sign of the Gauss-Bonnet Parameter

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If Einstein-Gauss-Bonnet gravity is obtained as a low energy limit of string theory, then the Gauss-Bonnet parameter $\alpha$ is essentially the inverse string tension and thus necessarily positive. If one treats Einstein-Gauss-Bonnet gravity as a modified theory of gravity in the anti-de Sitter bulk in the bottom-up approach of holography, then there is no obvious restriction on the sign of the parameter a priori, though various studies involving boundary causality have restricted the possible range of $\alpha$. In this short note, we argue that if holographic descriptions are to be consistent, then the Gauss-Bonnet parameter has to be positive. This follows from a geometric consistency condition in the Euclidean picture. From the Lorentzian signature perspective, black holes with a negative $\alpha$ lead to uncontrolled brane nucleation in the bulk and so the supposedly static geometry is untenable. In fact, even the ground state without a black hole is problematic. In other words, the bottom-up approach agrees with the top-down approach on the sign of the parameter. Some possible loopholes of the conclusion are discussed.

I. INTRODUCTION: CONSTRAINTS ON THE GAUSS-BONNET PARAMETER

The Einstein-Gauss-Bonnet (EGB) gravity in D-dimensions in the presence of a negative cosmological constant $\Lambda$ is given by the action [1] (in the units $c = G = 1$)

$$I_{EGB} = \frac{1}{16\pi} \int d^D x \sqrt{g} \left( R - 2\Lambda + \frac{\alpha}{D-4} G \right),$$

where $G := R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$ is the Gauss-Bonnet term, which is a topological invariant in $D = 4$. Therefore we only consider EGB gravity in $D \geq 5$. The AdS curvature length scale $L$ is related to the cosmological constant via $\Lambda = -(D-1)/(D-2)/L^2$. We will refer to $\alpha$ as the Gauss-Bonnet parameter. This theory can be interpreted as a higher curvature correction to general relativity (GR), and in fact can be derived as a low energy limit of heterotic string theory [1, 11–13], in which the Gauss-Bonnet parameter is proportional to the inverse string tension, or related to the coupling constant $\alpha$ in the string worldsheet action [11]. From this “top-down” view, $\alpha$ is clearly positive. Black holes in the EGB theory have been studied in details, see for example, [14–16].

In gauge/gravity correspondence (hereinafter, “holography”), it has become a common practice to employ modified gravity theories in the anti-de Sitter (AdS) bulk to model field theories with certain desired properties. Strictly speaking, the bulk physics is described by string theory. However, under certain well-defined circumstances (the string coupling, and the string length to AdS length scale ratio, are small), one hopes that stringy effects can be neglected. If so, then it suffices to deal with (mostly) semi-classical gravity. In such a “bottom-up” approach, one can often find analytic black hole solutions to work with. The hope is that such effective theories can in fact be embedded into string theory. For examples, attempts have been made to embed massive gravity [17], bimetric gravity [18], Hořava gravity [19] and Lifshitz gravity [20] in string theory, which provide some justifications for their applications in the literature. However, not all effective field theories can be embedded into string theory, as expressed by the “Swampland Conjecture” [21, 22].

On the other hand, from the bottom-up perspective, the Gauss-Bonnet parameter is just a coupling constant. It is not obvious that it has to be positive. This is not to say that there is no constraint at all for the range of $\alpha$. For example, assuming that $\alpha > 0$, various causality considerations (including: scattering amplitude, requirement that graviton Shapiro time delay as opposed to time advance, shock wave characteristic analysis, hyperbolicity, quantum entanglement constraints) [25–28] require $\alpha$ to be small. By small, we mean $\alpha \ll \ell_p^2$, the square of the Planck length. This is not automatically satisfied in string theory: in the weakly coupled case we can...
have $\alpha \gg \ell_p^2$. However, the existence of higher spin massive particles in the full string theory protects causality [25]. From these constraints, $\alpha < 0$ is not ruled out, as shown by the hyperbolicity and boundary causality analysis in [29]. (See also [2, 30–32] for related discussions and constraints.) Boundary causality, as stated in the Gao-Wald theorem [33], essentially forbids signals to take a short-cut through the AdS bulk, so in particular, points which are not causally related on the boundary cannot be causally related through the bulk\(^3\) (see also [36] and a recent lecture note by Witten [37]). The Gauss-Bonnet parameter $\alpha$ has effects on the thermodynamics of black holes and thus on the dual field theory. In particular the Kovtun-Son-Starinets (KSS) viscosity bound [38, 39] is modified [40–42]. Interestingly, such an entropy bound is closely related to the aforementioned causality issue [43, 44]. Furthermore, eikonal instability develops if $\alpha$ is not small enough [45, 46].

Various applications of EGB black holes in holography had been studied, see, e.g., [47–60]. In fact the Gauss-Bonnet term is closely related to the central charges of the conformal field theory [31, 40]. Pathological behavior in the evolution of entanglement entropy and mutual information in the case of $\alpha < 0$ (the mutual information becomes discontinuous during thermalization) did led Sircar et al. [61] to conjecture that EGB theory with $\alpha < 0$ may be inconsistent\(^4\). In view of these, we raise the following question: is EGB gravity with $\alpha < 0$ a consistent effective theory, with some possible embedding into string theory, or does it belong to the Swampland? We would argue in favor of the latter case.

Note that in the asymptotically flat case, working in the effective theory regime, Cheung and Remmen have shown that $\alpha$ is nonnegative for any unitary tree-level ultraviolet completion of the Gauss-Bonnet term, which is free of ghosts or tachyons [63]. Our work, which utilizes a completely distinct method, provides strong evidence for $\alpha \geq 0$ in the AdS case as well.

\section{II. Holographic Consistency and Brane Nucleation}

Holography relates gravitational theories in the AdS bulk to non-gravitational field theory on the boundary (see [64, 65] for a review). Such a surprising relation between two radically different types of theory is highly nontrivial and requires consistency conditions to be valid. A particularly deep condition of this kind in the Euclidean formulation that relates the minimum of a probe brane action to a gravitational bulk action [66–69]:

$$ S^* = \frac{N}{\gamma} S^b, $$

where $S^*$ is the on-shell gravitational action in the bulk, $N$ and $\gamma$ are the number of colors and the scaling exponent for the free energy of the boundary field theory, respectively, and $S^b$ is the probe brane action. Surprisingly, this consistency condition holds if a certain “isoperimetric inequality” [68, 70] (the superscript “E” denotes a Euclidean quantity; likewise superscript “L” denotes a Lorentzian one – not to be confused with the AdS curvature length scale $L$)

$$ \mathcal{E}^E := A(\Sigma) - \frac{D - 1}{L} V(M_\Sigma) \geq 0, $$

holds for any co-dimension 1 hypersurface $\Sigma$ in the bulk homologous to the boundary. Here $A(\Sigma)$ and $V(M_\Sigma)$ denote the area and the volume enclosed by $\Sigma$, respectively. We see that $\mathcal{E}^E$ measures the competition between the area and the volume. Up to a brane tension constant coefficient, $\mathcal{E}^E$ is the action of a probe BPS (Bogomol'nyi–Prasad–Sommerfield) brane that wraps around the black hole at a constant coordinate radius $r$. If the Wick-rotated spacetime is an Einstein manifold, this inequality is related to the topology at infinity and the Yamabe invariant, as discussed in the theorem of Wang [71, 72]. In holographic settings, many manifolds are not Einstein, and so inequality (3) must be checked on a case by case basis. As mentioned in [73], generalizations to Wang’s theorem do exist – for example, the works of Witten-Yau [74] and Cai-Galloway [75] – but they are in general not useful in deciding whether inequality (3) holds. The reason inequality (3) is referred to as an “isoperimetric inequality” can be appreciated as follows [70]: in planar geometry, the area bounded by a closed curve satisfies $A \leq \ell^2/4\pi$, where $\ell$ is the length of the curve, and with equality holds if and only if the closed curve is a circle. This is the usual isoperimetric inequality. We can call this a “consistency condition” for a geometry to be embedded in $\mathbb{R}^2$, i.e., a closed curve cannot bound too large a volume, otherwise it ceases to be planar. Likewise, inequality 3 is a geometric consistency condition for asymptotically hyperbolic manifolds, i.e., the Euclidean signature version of AdS spacetime employed in holography (we refer the readers to [70] and [73] for the details and subtleties).

The Lorentzian version of inequality (3) has a clear physical picture: if $\mathcal{E}^L$ becomes negative at large $r$, the spacetime will nucleate a copious amount of branes via a Schwinger-like process in the bulk (see, for example, [76] and the Appendix of [77]). This would back-react on the metric, which in turn means that the original...
spacetime cannot be static. Any static black hole solution in classical gravity theories that suffer from this “Seiberg-Witten instability” [78, 79] is therefore not a consistent object in the bulk. Note that as with the Schwinger pair-production, this effect is non-perturbative [79]. We remark that it is possible for $\mathcal{S}^E$ to be everywhere non-negative but $\mathcal{S}^L < 0$ at some locations. Holography should be regarded as fully consistent when both inequalities are valid (although $\mathcal{S}^b < 0$ is acceptable if the boundary system only exists for a sufficiently short period so that any Lorentzian pathology would not have the time to influence the black hole geometry). See [73] for detailed discussions.

A classic example is that of toral Reissner-Nordström black holes [85–87]. Although they are valid solutions to Einstein-Maxwell gravity for all values of the charge $Q$, they are not a consistent solution in the bulk when the electrical charge becomes near-extremal (for AdS they are not a consistent solution in the bulk when the boundary system only exists for a sufficiently short period so that any Lorentzian pathology would not have the time to influence the black hole geometry). See [88] for discussions.

We will show that asymptotically locally AdS static black holes in EGB theory is not consistent in the aforementioned sense, even for the neutral case (for which $\mathcal{S}^E$ is equivalent to $\mathcal{S}^L$).

The black hole solution of EGB gravity in $D$-dimensions is [14, 15, 34]

$$ds^2 = -f(r) \, dt^2 + f(r)^{-1} \, dr^2 + r^2 \, d\Omega_{D-2}^2,$$  

where $\Omega_{D-2}^2$ is a metric on a $(D-2)$-dimensional manifold $\Sigma_{k,D-2}$ with constant sectional curvature $k = \{-1, 0, +1\}$, and

$$f(r) = \frac{r^2}{2\alpha(D-3)} \left[ -1 + \sqrt{1 - 4\alpha(D-3)g(r)} \right],$$

with

$$g(r) = \frac{1}{L^2} - \frac{16\pi M}{(D-2)\Omega_{D-2}^2r^{D-1}},$$

where $M$ is proportional to the black hole mass. Here $\Omega_{D-2}$ denotes the unit area of $\Sigma_{k,D-2}$. For example, for a 3-sphere, $\Sigma_{1,3} = 2\pi^2$, whereas for a cubic torus with periodicity $2\pi K$ on each copy of its $S^1$, we have $\Sigma_{0,3} = 8\pi^3 K^3$. In the presence of a Maxwell field, $g(r)$ would contain an additional charge term $2Q^2/[(D-3)(D-2)r^{2(D-2)}]$; its inclusion does not change the qualitative results discussed below.

The brane action $\mathcal{S}_E$ is evaluated as follows:

$$\mathcal{S}_E := r^{D-2} \int d\Omega_{k,D-2} \int d\tau \sqrt{g_{\tau\tau}}$$

$$\quad - \frac{D-1}{L} \int d\tau \int d\Omega_{k,D-2} \int_{r_h}^r dr' r'^{D-2} \sqrt{g_{\tau\tau}} g_{\tau\tau'},$$

where $g_{\tau\tau}$ is the Wick-rotated metric coefficient $g_{\tau\tau}$. For the uncharged case $g_{\tau\tau} = f(r)$. Here $r_h$ denotes the (Euclidean) event horizon. (In fact, the area and volume terms correspond to the Dirac-Born-Infeld (DBI) term and Chern-Simons term, this can potentially generalize the consistency conditions to a more general backgrounds [88]). The normalized time coordinate $t/L$ is periodically identified with a periodicity $2\pi P$, so

$$\mathcal{S}_E = 2\pi P \Omega_{D-2} \left[ r^{D-2} \sqrt{g_{\tau\tau}} - \frac{r^{D-1} - r_h^{D-1}}{L} \right].$$  

The sign of $\mathcal{S}_E$ depends on the competition between the terms in $\mathcal{S}_E$. For a sufficiently large $r$ (so that the constant term involving $r_h$, and the sectional curvature $k$ can be neglected), it is straightforward to show that for
\( \alpha < 0 \), the bracket terms grow like
\[
\mathfrak{g}(r) \sim \frac{r^{D-1}}{\sqrt{2|\alpha|(D-3)}} \left[ \left( \sqrt{1 + \frac{4|\alpha|(D-3)}{L^2}} - 1 \right)^{\frac{1}{2}} \right. \
- \frac{\sqrt{2|\alpha|(D-3)}}{L} \left. \right],
\]
which is negative by elementary algebra (similarly, the corresponding quantity for \( \alpha > 0 \) case is positive).

It is helpful to see visually the behavior of the function \( \mathcal{S}^E(r) \) as we vary \( \alpha \), so let us consider the case \( k = 0 \) in 5-dimensions and let \( L = 1 \). Suppose the topology of the horizon is a cubic torus such that each copy of \( S^1 \) has periodicity \( 2\pi \). To ensure that the metric function Eq.(5) is real for all values of \( M \), and in particular for the ground state without a black hole (i.e. \( M = 0 \)), we must have \( \alpha < L^2/[4(D - 3)] \). In our example, this means \( \alpha < 1/8 \). We show in Fig.(1) the behavior of \( \mathcal{S}^E \); it is negative for most values of \( \alpha < 0 \), and positive for \( \alpha \in (0, 1/8) \). (Actually, eikonal instability would rule out \( \alpha > |1/8| \), at least in the planar limit [46].)

It turns out that the brane action is still positive for some negative values of \( \alpha \) sufficiently close to the horizon. In Fig.(2) we plot the curves along which \( \mathcal{S}^E(r) \) vanishes for various values of the mass. The region above a given curve corresponds to positive brane action, and likewise the region below it corresponds to negative brane action. We note that as \( M \) increases, the curve is shifted towards the right, but so does the event horizon. If we denote the position where the action starts to become negative by \( r_{\text{neg}} \), then \( (r_{\text{neg}} - r_h)/r_h \) is constant (independent of the mass) for fixed \( \alpha \). For \( \alpha = -0.1 \), this ratio is about 0.0617. In fact, these curves exhibit a scaling symmetry under \( M \mapsto M/r_h^4 \) and \( r \mapsto r/r_h \), which yields Fig.(3).

The fact that the Euclidean brane action is negative already violates the consistency condition discussed in Sec.(II) [73]. But even if we only consider the Lorentzian picture, in which branes are nucleated in the region where the action is negative, we still have a problem. Causality dictates that the branes will back-react on the black hole geometry only after some time has passed. This is roughly the time it takes for a brane to free-fall towards the black hole (see, e.g., [91]). The reason is that as branes are copiously produced in the bulk, if they are far away, they do not immediately affect the black hole in the bulk, whereas if they fall towards the black hole, then the event horizon, being teleological in nature, would start to expand outward in anticipation of the falling branes – and thus the spacetime is no longer static. Holography would be consistent if the field theory dual has a typical time scale larger than the instability time scale. This does not help us since the branes are nucleated close to the horizon, i.e., the instability time scale is short.

The situation is actually worse: in the case without a black hole (\( M = 0 \)), the brane action is negative for all \( \alpha < 0 \). Thus the ground state is itself unstable. Much of the above discussion also holds for the \( k = 1 \) case, al-
though the brane action remains positive near the origin $r = 0$ when $M = 0$. However, the ground state has no distinguished center, so $r = 0$ is arbitrary. This implies that branes can be nucleated everywhere. The $k = -1$ case already suffers from brane production instability when the bulk gravity is GR (corresponds to the well known tachyonic instability of the boundary field scalar\footnote{\`{In this way, string theory reproduces the instability that is evident in the field theory.” – Seiberg and Witten\cite{78}}, so we do not analyze it here. In any case, $k$ cannot affect the leading term of the EGB brane action at sufficiently large $r$, so the Euclidean brane action is still negative.

IV. DISCUSSION: GAUSS-BONNET COUPLING CONSTANT IS POSITIVE

Boulware and Deser already commented in 1985\cite{1} that a negative Gauss-Bonnet parameter $\alpha$ is pathological as it leads to a more singular behavior of the spacetime: a new curvature singularity arises at some nonzero value of $r$. However, since this singularity is behind the horizon (see also\cite{92}), one may still argue that this does not immediately rule out the possibility of $\alpha < 0$ EGB black holes (except perhaps for the reason raised in Footnote 4).

In the bottom-up approach of holography, the $\alpha < 0$ case has also been considered in the literature. Often one hopes that in the low energy limit of string theory, stringy phenomena like effects of branes can be neglected. However, this is not always the case. This is reminiscent of the fact that some properties of the full theory can survive down to the infrared and imply constraints on the interactions that can be consistently added to the low energy EFT Lagrangian: see\cite{93} for an application in scalar-EGB theory. Holography consistency conditions impose constraints that render otherwise valid solutions (such as near extremal toral Reissner-Nordström black hole) in the anti-de Sitter bulk inconsistent even as effective low energy solutions. This rules out the case for $\alpha < 0$, which is conjectured to be inconsistent by Sircar et al. in\cite{61} on phenomenological ground in holography.

Since $\alpha > 0$ is a consequence of inverse string tension being positive in the top-down approach, it is also a remarkable consistency check that holography requires precisely this even if in the bottom-up approach one only takes EGB gravity as a modified theory of gravity and knows nothing about the relationship between $\alpha$ and string coupling in certain heterotic string theory embeddings. As discussed in\cite{2}, there are many different possibilities of realizing curvature squared corrections to Einstein-Hilbert action from string theory and M-theory\cite{95,96}, so it is not obvious whether $\alpha < 0$ is consistent with some version of string theory. However, if the holographic consistency condition in the form of “isoperimi-
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