Direct Design of T-Beams for Combined Load of Bending and Torsion

A. S. Alnuaimi*, K.S. Al-Jabri and A.W. Hago

Department of Civil and Architectural Engineering, Sultan Qaboos University, P. O. Box 33, Al Khodh 123, Muscat, Oman

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Abstract: Tests were conducted on five reinforced concrete T-beams subjected to combined load of bending and torsion. Elastic stress field in conjunction with Nielsen's 2D yield criterion for reinforced concrete subjected to in-plane forces were used in the Direct Design Method for the design of reinforcement. The beam dimensions were: flange width = 600mm, flange thickness = 150mm, web width = 200mm, total depth = 600mm and beam length = 5.2m. Required reinforcement calculated using the Direct Design Method was compared with the ACI and BSI codes. It was found that the Direct Design Method requires longitudinal reinforcement similar to the ACI code but less than the BSI code. In the transverse direction, the Direct Design Method requires much less reinforcement than both codes. The main variable studied was the ratio of the maximum twisting moment to the bending moment which was varied between 0.6 and 1.5. Good agreement was found between the design and experimental failure loads. Most of the longitudinal and transverse steel yielded or reached near yield stress when the design load was approached. All beams failed near the design loads and undergone ductile behaviour until failure. The results indicate that the Direct Design Method can be successfully used to design reinforced concrete T-beams for the combined effect of bending and torsion loads.

Keywords: T-beams, Direct design method, Bending, Torsion, Reinforced concrete

Notations

| Symbol | Description |
|--------|-------------|
| Ac     | gross area of the whole cross-section |
| Ao     | area of the cross-section enclosed by the shear flow centreline |
| Ax, Ay | steel areas per unit width in x and y directions respectively |
| E      | Young's modulus of elasticity |
| fcu    | concrete cube compressive strength |
| fsx    | steel stress in the longitudinal direction |
| fsy    | steel stress in the transverse direction |

*Corresponding author’s e-mail: alnuaimi@squ.edu.om
1. Introduction

The present code design equations for concrete structures (especially for shear and torsion) are largely based on experimental results which lead to continuous changes in the code. Calculation of reinforcement for each load is done separately and then summed algebraically which leads to conservative design.

For combined loading, several interaction theories based on ultimate strength criterion have been developed (ie. theories on Truss-Analogy and Skew-Bending), but the major weakness of this approach lies in the fact that it does not present a general design procedure capable of being applied to design for a given general stress state.

Many researchers investigated the behaviour of reinforced and pre-stressed concrete T-beams under different loading conditions (ie. bending, torsion and shear) and the contribution of the sections' elements (ie. flange and web) on resisting applied forces has been also reported. The experimental behaviour of twenty six reinforced concrete T-beams without web reinforcement under combined bending, shear and torsion was reported by Farmer and Ferguson, (1967). Torque-to-shear ratios of 1.5, 3 and 6 were investigated. Results indicated that span length and cross-sectional warping was not found to influence the behaviour greatly. It also suggested that torsion lowered the shear strength of the beam much less than ordinary plastic theory would suggest. Nawy and Potyondy (1971) studied the flexural cracking behaviour of pre-tensioned I- and T-beams. They tested twenty two I- and T-beams with a span of 9-ft (2.74m) and pre-stressed with 1/4 in (6.35mm) diameter 7-stranded wires. From the results, it was concluded that the confining reinforcement had no influence on the magnitude of load at first crack and T-sections cracked at a lower load than I-sections. Kirk and Loveland (1972) observed, after testing eighteen unsymmetrical reinforced T-beams under bending and torsion, that the location of longitudinal steel at the top or the bottom of the section can have significant influence on the torsional resistance. The effect of reinforced flanges on the torsional capacity of reinforced concrete T-beams was investigated by Zararis and Penelis, (1986). Eighty two T-beams with various flange widths and percentages of reinforcement were tested in combined torsion and bending. Results showed that there is significant contribution from the reinforced flanges to the torsional capacity of the T-beams especially when torsion prevailed. Experimental studies by Kotsovos et al. (1987) also showed that the shear resistance of T-beams with shear-span-to-depth ratios greater than 2.5 is provided by the flange and not as widely accepted by the web. Gurfinkel and Fonseca (1996) investigated the ACI code requirement for additional ductility in the T-beam sections and reported that "There is little justification for code-imposed additional demand on minimum ductility of T-beams sections". Recently experimental investigation was carried out by Rahal and Collins, (2003) in order to evaluate the ACI and AASHTO-LRFD design provisions for combined torsion and shear. It was found that the ACI provisions give very conservative results if the recommended value of 45-degree is used for the inclination of the compression diagonals.
It is clear from the above literature and others that large area of disagreement exists among the researchers on the design of T-beams when shear and torsion are involved. One reason for this disagreement is the fact that the design equations used in the code are not based on sound theoretical principles like the theories of mechanics (ie. Theory of Plasticity).

The Direct Design Method, which is based on the Lower Bound Theorem of the Theory of Plasticity, presents a solution to some of the above pitfalls. This procedure satisfies the fundamental requirements of classical Theory of Plasticity (equilibrium condition, yield criterion and mechanism condition) and it reduces the ductility demand assumed for metals. In contrary to the existing codes of practice, this method precludes the use of empirical equations and therefore, it is less vulnerable to changes and may form a basis for a unified design method.

Previous work by Bhatt and Ebireri (1989) had shown that the 'direct design method' could be applied to the design of hollow and solid beams with square cross-section subjected to combined bending and twisting moments. Alnuami and Bhatt (2004a, b) used the direct design method for the design of eight reinforced concrete square hollow beams subjected to combined bending, torsion and shear. The main variables studied were the ratio of the twisting moment to the bending moment,$ \frac{T_d}{M_d}$, which varied between 0.6 and 1.5. Figure 1 shows the geometry and dimensions of the tested beam.

| Beam No. | $T_d$ kN.m | $M_d$ kN.m | $\frac{T_d}{M_d}$ Ratio |
|----------|------------|------------|-------------------------|
| TB1      | 36         | 60         | 0.60                    |
| TB2      | 75         | 100        | 0.75                    |
| TB3      | 50         | 60         | 0.83                    |
| TB4      | 72         | 72         | 1.00                    |
| TB5      | 108        | 72         | 1.50                    |

| Sec. A-A | Sec. B-B |
|----------|----------|
| A        | A        |

Figure 1. Beam geometry and dimensions

4. Comparison Between the ACI and BSI Codes and the Direct Design Method

Table 2 shows the required reinforcement for the T-beams used in this research based on the ACI and BSI codes and the direct design method, DDM. It can be seen that the direct design approach leads to longitudinal steel requirement close to the ACI code while the BSI code is more conservative. In the transverse reinforcement, the direct design procedure required less reinforcement than both codes in most cases, especially the BSI code. The direct design method was used for the design of this research beams.

5. Design of T-beams

The following steps were used for the design of T-beams, and a numerical example is given in appendix A. Detailed derivation of equations can be found elsewhere Almuami and Bhatt, (2004a, b).
The required reinforcement at mid-span was provided over a test span length of 1200mm in the middle of beam. Figure 3 shows typical reinforcement arrangements in a tested T-beam and Fig. 4 shows details of provided reinforcement in the test span of each beam. The solid dots in the figure indicate the locations of bars, which were strain-gauged.

1. Calculate the ultimate (factored) design bending moment $M_d$ and torsion $T_d$ at the section based on the loading condition, support locations and geometry of the beam.

2. Divide the cross-section into different levels (regions) as shown in Fig. 2.

3. Find the neutral axis, the moment of inertia and the area enclosed by the shear flow centreline.

4. Calculate normal stress $\sigma_x$ due to applied load at the centre of each region (level) using elastic stress distribution, $\sigma_x = M/1$.

5. Calculate the shear stresses due to torsion, $\tau_{xy} = T/2d$. 

6. Assume $\sigma_y = 0$ (No out of plane bending is considered).

7. Calculate the ratios $\sigma_x/\tau_{xy}$ and $\sigma_y/\tau_{xy}$ for each region.

8. Select design equations as explained in the numerical example (Appendix A).

9. Calculate the required reinforcement.

| Beam No. | ACI $A_s$ (mm$^2$) | ACI $A_{sv}$ (mm$^2$/m) | BSI $A_s$ (mm$^2$) | BSI $A_{sv}$ (mm$^2$/m) | DDM $A_s$ (mm$^2$) | DDM $A_{sv}$ (mm$^2$/m) |
|----------|---------------------|------------------------|-------------------|------------------------|------------------|------------------------|
| TB1      | 1136                | 914                    | 1325              | 978                    | 1091             | 606                    |
| TB2      | 1903                | 1848                   | 2479              | 1978                   | 1908             | 1226                   |
| TB3      | 1484                | 1269                   | 1746              | 1358                   | 1516             | 842                    |
| TB4      | 1841                | 1741                   | 2457              | 1956                   | 1932             | 1155                   |
| TB5      | 2633                | 2611                   | 3542              | 2934                   | 2897             | 1732                   |

Figure 2. Regions (Levels) in the cross-section

The required reinforcement at mid-span was provided over a test span length of 1200mm in the middle of beam. Figure 3 shows typical reinforcement arrangements in a tested T-beam and Fig. 4 shows details of provided reinforcement in the test span of each beam. The solid dots in the figure indicate the locations of bars, which were strain-gauged.

To ensure failure at mid-span, additional reinforcement was provided in the regions outside the test span. Further, to reduce the stresses due to distortion of the cross section and produce good gripping mechanism in the torsional arms, the end section was made 600x600 mm solid square for a length of 400mm from each end.

6. Casting of Beams

After the steel cage was erected including all longitudinal and transverse reinforcement, the beam was cast in a
specially designed wooden and Glass Reinforced Plastic (GRP) formwork. One day after casting, the sides of the mould were disassembled leaving the bottom part below the beam. Six concrete cubes (100mm) and six cylinders (150 x 300mm) were cast with each beam from the concrete used in the beam. A vibrating table was used for compaction of the cubes and cylinders while the beam was vibrated using an external vibrator attached to the shuttering. The beam and the cubes and cylinders were cured using wet Hessian cloth for about one week and then in room temperature for another three weeks. All cubes and cylinders were tested on the day the beam was tested to determine the cube and cylinder compressive strengths and cylinder tensile strength. Only 10mm and 12mm High Yield steel bars were used as longitudinal reinforcement and 10mm bars were used for stirrups. From the tensile test results, the average yield stress of steel was 545N/mm² for longitudinal reinforcement and 513N/mm² for stirrups. Ordinary Portland cement (OPC), fine aggregate, coarse aggregate (10mm) and fresh water were used in the concrete mix. Table 3 shows the average of some measured material properties.

### Table 3. Measured average material properties

| Beam No. | $f_{cu}$ (N/mm²) | $f_{y}$ (N/mm²) | $f_{uy}$ (N/mm²) |
|----------|----------------|----------------|----------------|
| TB1      | 36             | 500            | 500            |
| TB2      | 30             | 596            | 515            |
| TB3      | 45             | 500            | 500            |
| TB4      | 29             | 565            | 525            |
| TB5      | 30             | 565            | 525            |

7. Experimental Set-up

Figure 5 shows the testing rig with a typical beam installed. The installation process required inserting the beam square ends into the torsional arms (Fig. 6) which are attached to shafts and bearings resting on pedestals (Fig. 7). The torsional arm ends are hooked to the strong floor by inextensible cables with hydraulic actuators and load cells. On the top face of the beam, a 1200mm long spreader was installed. The spreader was centred at the mid-span of the beam with its centreline coinciding with beam centreline. The bending moment in the middle 1200mm test span is acquired by applying a point load at the middle of the spreader. The torsional moment which is constant over the entire length of the beam, was applied through torsional arms clamped at the ends. The load from the spreader was transferred to the beam through two perpendicular rollers in one side while a roller and a pin were installed at the other side. This support and loading arrangements produced a simply supported condition allowing free rotation about the longitudinal axis of the beam and horizontal displacements with a rigid body motion being prevented by the pin on the top face. The test span region is mainly subjected to bending and twisting moments while outside the test span shear is added to the twisting and bending moments.

![Figure 5. Test rig with a typical beam installed](image5)

![Figure 6. Torsional arm while testing](image6)

![Figure 7. Close lookup of end support and torsional arm with actuator and load cell](image7)

![Figure 8. Legend for strain gauge locations in longitudinal and transverse steel (see Table 4)](image8)

Strains in the reinforcement were monitored using electrical resistance strain gauges.
and arrangement of the strain gauges in the longitudinal and transverse reinforcement and Table 4 gives the location of strain gauges for each beam. At each specified location on each bar, two strain gauges were installed opposite to each other. Figure 9 shows typical locations of the longitudinal bars and Table 5 shows vertical distances between the longitudinal bars.

Linear variable differential transducers (LVDT) were used to measure the beam deflection (Fig. 10). The concrete surface strains were measured by DEMEC (digital) gauges. All strain gauges and LVDTs were hooked to a data logger. The crack widths were measured using a crack width measuring microscope.

For each experiment, the design load was divided into load increments, 10% of the design load for each of the first three increments and 5% for each of the following increments until failure. The first step was to zero all load cells and record instrument readings with minimum possible loads on the model. Flexural loads were normally applied before torsional load at each load level. To allow for stable deformation to take place after each load increment, an interval of about two minutes was used before recording the instrument readings. After the steel strains and displacement readings were recorded, cracks were marked and DEMEC readings were taken before the next load was applied. The beam was considered to have collapsed when it could resist no more loads. This usually happened after a major crack spiralled around the beam near the mid-span dividing the beam into two parts connected by the longitudinal reinforcement.

### 8. Experimental Observations

This section summarizes the observed behaviour in the test span region of the tested T-beams at significant stages in the behaviour. The angle of crack was measured from the horizontal axis. Vertical displacement was measured at the mid-span at the bottom of the web. Strain ratios in the bottom layer of the longitudinal steel and in the transverse steel of mid-span are presented. The quoted load factor is a percentage of the experimentally measured load at each increment to the design load, \( \text{L.F.} = (M_i/M_d + T_i/T_d)/2 \) including the self-weight. The failure load is a percentage of the experimentally measured failure load to the design load, \( \text{L.e}/L_d = (M_e/M_d + T_e/T_d)/2 \).

### Table 4. Location of strain gauges (see Figure 8)

| Space | a | b | c | d | e | f | g | h | Vertical dist. for the stirrup’s strain gauges is 300mm from the bottom of the web. |
|-------|---|---|---|---|---|---|---|---|----------------------------------|
| Beam  | mm | mm | mm | mm | mm | mm | mm | mm |                                  |
| TB1   | 260| 260| 130| 130| 260| 260| 130| 130|                                  |
| TB2   | 128| 128| 64 | 64 | 128| 128| 64 | 64 |                                  |
| TB3   | 186| 186| 93 | 93 | 186| 186| 93 | 93 |                                  |
| TB4   | 136| 136| 68 | 68 | 136| 136| 68 | 68 |                                  |
| TB5   | 90 | 90 | 45 | 45 | 90 | 90 | 45 | 45 |                                  |

### Table 5. Vertical clear distances between longitudinal bars (see Fig. 9)

| Location | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|-----------|----|----|----|----|----|----|----|
| Beam      | mm | mm | mm | mm | mm | mm | mm |
| TB1       | 20 | 100| 100| 120| -  | 20 | 20 |
| TB2       | 20 | 80 | 80 | 100| 100| 20 | 20 |
| TB3       | 20 | 100| 100| 120| -  | 20 | 20 |
| TB4       | 20 | 100| 100| 120| -  | 20 | 20 |
| TB5       | 20 | 100| 100| 120| -  | 20 | 20 |

### Figure 9. Typical vertical clear spaces between longitudinal bars (see Table 5)

### Figure 10. Locations of LVDT

a: Bottom side transducers  B: Front side transducers  C: Rear side transducers
The average strain ratio in the longitudinal $\varepsilon/\varepsilon_y$ or transverse $\varepsilon/\varepsilon_{yy}$ reinforcements is the average of the measured strain at each load increment to the yield strain. At each strain-gauge location on the bar, the average of the readings from the two opposite strain gauges is considered.

TB1: $T_d=36kNm$, $M_d=60kNm$, $T_d/M_d=0.6$  \hspace{1cm} (1)

The first few cracks were noticed at 45% of load at the bottom of the web near mid-span. The cracks were about 80° inclined. More cracks developed by the increase of the load until about 90% of the design load with angle of inclination being reduced to an average of 50° and average width of crack of about 0.4 mm. After this stage, few major inclined cracks started out side the test span. Near failure, prominent cracks spiralled round the beam meeting near the mid-span. The failure happened at 110% of the design load. The average crack width just before failure was about 0.6 mm. Figure 11 shows the displacement at mid-span of the beam and Figs. 12-15 show strain ratios in the longitudinal and transverse reinforcement. The average strain ratio reached near failure in the longitudinal steel was 0.94 and in the stirrups, the ratio was 0.92. Figure 16 shows crack development in the test-span of the beam.

TB2: $T_d=75kNm$, $M_d=100kNm$, $T_d/M_d=0.75$  \hspace{1cm} (2)

At 40% of load, several inclined hair cracks (about 60°) were noticed in the test span region close to the bottom of the web. With the increase of the load, many fine cracks developed until about 90% of load when two major cracks widened up (45° inclination) and spiralled around the beam causing failure at 105% of the design load. The average crack width near failure load was about 0.5 mm.
Figure 17 shows the displacement at mid-span and Figs. 18-19 show the strain ratios in the longitudinal and transverse bars. The average strain ratio in the longitudinal bars was 0.9 and in the stirrups was 0.61. Figure 20 shows crack development in this beam.

\[
TB3: T_d=50\text{kNm}, M_d=60\text{kNm}, T_d/M_d=0.83 \quad (3)
\]

The development of cracks in this beam was similar to that of beam TB2 with number of cracks being larger in TB3 than TB2. Failure happened at 112% of the design load. Figure 21 shows displacement at mid-span and Figs. 22-25 show steel strain ratios in the longitudinal and transverse steel. Figure 26 shows crack development in this beam. The average width of cracks near failure was 0.45 mm and the average strain ratio near failure was 0.91 in the longitudinal steel and 0.85 in the stirrups.
TB4: $T_d=72\,\text{kNm}$, $M_d=72\,\text{kNm}$, $T_d/M_d=1.0$  

At 25% of load, few inclined cracks (about 45°) started in the lower part of the web. The number of cracks increased largely with the increase of the load but the width was smaller than in the previous beams. The beam failed at 100% of the design load with few cracks widened slightly larger than the others did. Figure 27 shows displacement at mid-span and Figs. 28-31 show steel strain ratios in the longitudinal and transverse steel. Figure 32 shows crack development in this beam. The average width of cracks near failure was 0.4 mm and the average strain ratio near failure was 1.05 in the longitudinal steel and 0.94 in the stirrups.

TB5: $T_d=108\,\text{kNm}$, $M_d=72\,\text{kNm}$, $T_d/M_d=1.5$  

This beam behaved in a similar manner like TB4 but...
with larger number and finer cracks. There was no major cracks but the beam failed by losing strength due to the large number of cracks at a load of 98% of the design load. Figure 33 shows displacement at mid-span and Figs. 34-35 show steel strain ratios in the longitudinal and transverse steel. Figure 36 shows crack development in this beam. The average width of cracks near failure was 0.3 mm and the average strain ratio near failure was 1.04 in the longitudinal steel and 0.92 in the stirrups.

9. Discussion of Results

In all beams, the longitudinal and transverse steel either yielded or reached nearly the yield strain. This is to be expected since the beams were designed according to the lower bound theory of plasticity. Beams loaded dominantly in bending (i.e., low $T_d/M_d$ ratio) showed more ductile behaviour than those with large $T_d/M_d$ ratios (Table 7). In general, all beams showed ductile behaviour as expected for under-reinforced sections. Furthermore, there was no crushing of concrete before yielding of steel. All beams failed near their design loads for all variations of $T_d/M_d$ ratios (Column 3 of Table 6).

**Table 6. Experimental/design failure load ratios**

| Load at first crack | $T_d/M_d$ Ratio | $L_e/L_d$ Ratio |
|---------------------|-----------------|-----------------|
| Beam No. TB1 | 0.60 | 1.1 |
| TB2 | 0.75 | 1.05 |
| TB3 | 0.83 | 1.12 |
| TB4 | 1.00 | 1.00 |
| TB5 | 1.50 | 0.98 |
| Average | | 1.05 |

**Table 7. Load-crack and displacement measurement**

| Beam No. | Load at first crack | Average max. Crack width near failure | Max. Disp. Near failure |
|----------|---------------------|---------------------------------------|------------------------|
| TB1      | 0.45 | 0.6 | 29.6 |
| TB2      | 0.4 | 0.5 | 25.3 |
| TB3      | 0.4 | 0.45 | 27.6 |
| TB4      | 0.25 | 0.4 | 13.3 |
| TB5      | 0.25 | 0.3 | 25.5 |
| Average  | 0.35 | 0.39 | 24.26 |

Beams loaded dominantly in bending ($T_d/M_d < 1$) had vertical cracks started to develop first at the bottom of the web, then progressed as inclined cracks in succeeding load increments until reaching the flange at about 80% of the design load.

Beams loaded dominantly in torsion ($T_d/M_d > 1$) had inclined cracks started at mid-depth in the web before they extend into the bottom of the web. Generally, cracks formed earlier in beams loaded dominantly in torsion ($T_d/M_d > 1$) than other beams. In these beams, cracking started at a load as early as 25% of design load, while in other beams, they started at about 40% of design.
In beams where bending was dominant, the cracks in the test span were less in number and wider than in the beams where torsion was dominant. This is due to the fact that torsion was constant throughout the entire length while maximum bending moment existed over a small distance at the middle of the beam.

10. Conclusions

From the results shown in this paper, it can be concluded that the direct design method produces a safe design for reinforced concrete T-beams under combined loading of bending and torsion. Most of the beams failed near design loads and undergone ductile behaviour until failure. The reinforcement yielded or reached near yield when the design load was approached. The direct design method required less longitudinal reinforcement than the BSI code and close to the reinforcement required by the ACI code. It required less transverse reinforcement than both BSI and ACI codes.

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Appendix

Example: Design of T-beams using Direct Design Method

TB3: \( T_d = 50 \text{kN.m} \quad M_d = 60 \text{kN.m} \quad T_d/M_d = 0.83 \)

See Figs: 1 and 2.

- Gross-sectional area (Fig. A1):
  \[ A = 600 \times 150 + 200 \times 450 = 180000 \text{mm}^2 \]
- Depth of neutral axis from top of flange:
  \[ 600 \times 150 \times 150/2 + 200 \times 450 \times 375 = (600 \times 150 + 200 \times 450) \bar{X} \]
  \[ \bar{X} = 225 \text{mm} \]

For torsion assume 50mm effective wall thickness (Fig. A2):

- Area of section within the shear flow centerline,
  \[ A_o = 100 \times 550 + 150 \times 425 = 11.875 \times 10^4 \text{mm}^2 \]

- Moment of inertia, \( I = \frac{1}{3} [600 \times 225^2 + 200(600-225)^2 - 75^3(600-200)] = 5.7375 \times 10^4 \text{mm}^4 \)
- Normal stress due to bending at the centroid of each level: \( \sigma_i = \frac{M_{y_i}}{I} \), \( i \) = level reference and \( y \) = distance between level centroid and neutral axis (Column 3 of Table A1).
- Assume \( \delta_{yi} = 0 \), No out of plane bending (Column 4 of Table A1).

- Calculate shear stress due to torsion, wall thickness 50mm (Column 5 of Table A1):
  \[ \tau_{xy} = \frac{T}{2A_o} = \frac{50 \times 10^6}{2 \times 50 \times 11.875 \times 10^4} = 4.21 \text{N/mm}^2 \]
- Calculate \( \delta_{xi}/\delta_{xy} \) and \( \delta_{yi}/\delta_{xy} \) (columns 6 and 7 of Table A1).
- Select the design case from the boundary graph for Nielsen’s design equations shown in Fig. A4 (Column 8 of Table A1).
- Calculate forces in the longitudinal direction per unit length: \( N_x^L = N_x + |N_{xy}| \) (Column 9 of Table A1).
- Calculate forces in the transverse direction per unit length: \( N_y^T = N_y + |N_{xy}| \) (Column 10 of Table A1).
- Calculate steel areas for each region in the x and y directions, assume \( f_y = f_{xy} = 500 \text{N/mm}^2 \) (columns 11 and 12).
- Select the number of bars and bar size based on the available sizes and required areas.
- Detail calculation for level A is shown below Table A1.

Fig. A1: Geometry of cross-section

Fig. A2: Shear flow centerline

Fig. A3: Levels (regions) in the cross section and normal stress.
Table A1: Calculation of reinforcement for Example A

| Level | Width (mm) | $\delta_1$ (N/mm$^2$) | $\delta_2$ (N/mm$^2$) | $\delta_3$ (N/mm$^2$) | $\delta_4$ (N/mm$^2$) | $\delta_5$ (N/mm$^2$) | $\delta_6$ (N/mm$^2$) | Case | $N_x$ (N/mm) | $N_y$ (N/mm) | $A_x$ (mm$^2$) | $A_y$ (mm$^2$) |
|-------|------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|------|---------------|---------------|----------------|----------------|
| A     | 600        | -2.09               | 0.421               | -0.496              | 0                   | 3                   | 106                 |     | 210          | 127.2         | 420            |                |
| B     | 600        | -1.57               | 0.421               | -0.373              | 0                   | 3                   | 132                 |     | 210          | 158.4         | 420            |                |
| C     | 600        | -1.05               | 0.421               | -0.249              | 0                   | 3                   | 158                 |     | 210          | 189.6         | 420            |                |
| D     | 200        | -0.52               | 0.421               | -0.124              | 0                   | 3                   | 184.5               |     | 210          | 73.8          | 420            |                |
| E     | 200        | 0                   | 0.421               | 0                   | 0                   | 3                   | 210                 |     | 210          | 84            | 420            |                |
| F     | 200        | 0.52                | 0.421               | 0.124               | 0                   | 3                   | 236.5               |     | 210          | 94.6          | 420            |                |
| G     | 200        | 1.05                | 0.421               | 0.249               | 0                   | 3                   | 263                 |     | 210          | 105.2         | 420            |                |
| H     | 200        | 1.57                | 0.421               | 0.373               | 0                   | 3                   | 289                 |     | 210          | 115.6         | 420            |                |
| I     | 200        | 2.09                | 0.421               | 0.496               | 0                   | 3                   | 315                 |     | 210          | 126           | 420            |                |
| J     | 200        | 2.61                | 0.421               | 0.62                | 0                   | 3                   | 341                 |     | 210          | 136.4         | 420            |                |
| K     | 200        | 3.14                | 0.421               | 0.746               | 0                   | 3                   | 367.5               |     | 210          | 147           | 420            |                |
| L     | 200        | 3.66                | 0.421               | 0.869               | 0                   | 3                   | 393.5               |     | 210          | 157.4         | 420            |                |

Detail Calculation for level A:

Col. 3: $\sigma_A = \frac{-My_A}{I} = \frac{-60 \times 10^6 \times 200}{5.7375 \times 10^9} = -2.09 \text{ N/mm}^2$ (C)

Col. 8: From columns 6 & 7: we have: $\delta_1 / \delta_2 = -0.496$, and $\delta_3 / \delta_4 = 0$ from Fig A4 the location of this point lies in the region of case 3.

Col. 9: The force in x direction calculated as given in Fig. A4: $N_x = N_x + |N_y| = 50(-2.09 + 4.21) = 106 \text{ N/mm}$.

Col. 10: The force in y direction calculated as given in Fig. A4: $N_y = N_y + |N_y| = 50(0 + 4.21) = 210 \text{ N/mm}$.

Col. 11: The steel area in x direction is given by: $A_x = \frac{N_x}{f_y} = \frac{106}{500} = 0.212 \text{ mm}^2/mm \Rightarrow 0.212 \times 600 = 127.2 \text{ mm}^2$.

Col. 12: The steel area in y direction is given by: $A_y = \frac{N_y}{f_{yy}} = \frac{210}{500} = 0.42 \text{ mm}^2/mm \Rightarrow 0.42 \times 1000 = 420 \text{ mm}^2$.
CASE 1
Y-steel only required

\[
N_x^s = 0
\]

\[
N_y^s = N_y - \frac{N_{xy}^2}{N_x}
\]

\[
N_2 = N_x + \frac{N_{xy}^2}{N_x}
\]

CASE 2
X-steel only required

\[
N_x^s = N_x + |N_{xy}|
\]

\[
N_y^s = N_y + |N_{xy}|
\]

\[
N_2 = -2|N_{xy}|
\]

CASE 3
X- and Y- steel required

\[
N_x^s = N_x + |N_{xy}|
\]

\[
N_y^s = N_y + |N_{xy}|
\]

\[
N_2 = -2|N_{xy}|
\]

CASE 4
No steel required

\[
N_x^s = 0
\]

\[
N_y^s = 0
\]

\[
N_1 = \frac{N_x + N_y}{2}
\]

\[
N_2 = \pm \sqrt{\left(\frac{N_x - N_y}{2}\right)^2 + N_{xy}^2}
\]

Fig. A4: Boundary graph for Nielson's 2D design equations