ANTI-ALIASING ADD-ON FOR DEEP PRIOR SEISMIC DATA INTERPOLATION

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ABSTRACT

Data interpolation is a fundamental step in any seismic processing workflow. Among machine learning techniques recently proposed to solve data interpolation as an inverse problem, Deep Prior paradigm aims at employing a convolutional neural network to capture priors on the data in order to regularize the inversion. However, this technique lacks of reconstruction precision when interpolating highly decimated data due to the presence of aliasing. In this work, we propose to improve Deep Prior inversion by adding a directional Laplacian as regularization term to the problem. This regularizer drives the optimization towards solutions that honor the slopes estimated from the interpolated data low frequencies. We provide some numerical examples to showcase the methodology devised in this manuscript, showing that our results are less prone to aliasing also in presence of noisy and corrupted data.

Index Terms— spatial aliasing, seismic data, interpolation, deep prior, convolutional neural network

1. INTRODUCTION

Economic or environmental constraints, cable feathering in marine acquisitions and the presence of dead or damaged traces result in most seismic datasets being poorly and often irregularly sampled in space. As a result, the vast majority of seismic processing and imaging workflows [1, 2, 3] typically include a preliminary interpolation step. The importance of the issue is confirmed by the vast number of methods proposed and the vitality of the relevant scientific literature.

Besides traditional processing methods such as transform representations [4, 5, 6], low-rank approximation [7], multichannel singular spectrum analysis (MSSA) [8] and its interpolated version I-MSSA [9], many seismic interpolation methods based on Convolutional Neural Networks (CNNs) have been proposed [10, 11, 12, 13]. However, the vast majority of CNN-based methods work according to a training paradigm. In other words, the used CNN learns how to reconstruct missing traces during a training step, in which several pairs of corresponding corrupted and uncorrupted data are fed to the CNN itself. On one hand, this approach needs a huge amount of training data. On the other hand, CNNs performances strongly depend on training examples seen during training, which may prevent the CNN to generalize and work on different kinds of data.

A different approach has been proposed interpreting the CNN architecture as a Deep Prior in the framework of inverse problems to address tasks such as interpolation, denoising or super-resolution [14]. Through this approach, the CNN learns the inner structure of a 2D image from the corrupted data itself, without pre-training: this prevents any over-fitting issue as well as the need for training data. This strategy has been recently explored also for the task of 2D seismic data interpolation using common U-Net as Deep Prior. In particular, Deep Priors have shown effective reconstruction performance when tackling both irregular and regular sampling [15]. However, Deep Prior techniques are not optimally performant when facing highly aliased data.

Inspired by our previous studies [16, 17, 18], we propose to regularize the Deep Prior objective function with a directional Laplacian. Specifically, data orientations are estimated from the low-end spectrum of the CNN output. Then, we build the directional Laplacian that drives the optimization towards solutions that minimizes the energy of events not coherent with estimated slopes.

The rest of the paper is structured as follows. Section 2 provides the theoretical formulation of the interpolation problem and the details of the proposed method. Section 3 contains the results achieved through our numerical simulations of three different scenarios. Finally, Section 4 concludes the paper. The complete code to run our analysis and the datasets used in this manuscript are accessible to the reader in a repository available at https://bit.ly/3a2efl1t.

2. METHODOLOGY

Seismic data interpolation is an ill-posed inverse problem. In order to obtain a reasonable solution it is necessary to constrain the inversion by adding some kind of a priori information. A standard way consists in formulating the inverse problem as finding the solution $x^*$ that minimizes an objec-
The interpolated solution is obtained as
\begin{equation}
J(x) = E(Sx - y_{\text{obs}}) + R(x),
\end{equation}
where $S$ is a linear sampling operator, $y_{\text{obs}}$ are the acquired seismic data, $E(\cdot)$ is the data fidelity term, and $R(\cdot)$ is a regularizer function formalizing the distance of the solution from the space of solutions honoring the desired prior. The choice of $R(\cdot)$ is critical and usually derives from insights of human experts.

In Deep Prior paradigms, feasible solutions are modeled as the application of a CNN to a random noise realization $z$. The CNN acts as a parametric non linear function $f_\theta(\cdot)$ and the problem is recast as finding the set of parameters $\theta^*$ which minimizes the objective function
\begin{equation}
J(\theta) = E(Sf_\theta(z) - y_{\text{obs}}).
\end{equation}

In this work, we propose a modification to the objective cost function (2), adding a regularization term given by the Laplacian operator oriented according to the events slopes.

Since data is aliased due to sub-sampling, we need to estimate the slopes in a robust way. To this end, we first estimate a low-pass version of the data by applying Deep Prior interpolation considering the cost function
\begin{equation}
J(\theta) = E(Sf_\theta(z) - Fr_{\text{obs}}),
\end{equation}
where $F$ is a 1D low-pass filter that acts trace-by-trace, thus removing alias effect that could badly impact on standard Deep Prior inversion.

By minimizing Eq. (3), we estimate a low-pass version of the interpolated data. Then, we employ the structure tensor algorithm [19] to estimate the slope angles $\phi$ and to build the directional Laplacian operator $\nabla_\phi$ [20]. The latter is employed to regularize the inversion of the broadband seismic data, allowing us to recast the interpolation problem as the minimization of the cost function
\begin{equation}
J(\theta) = \|Sf_\theta(z) - y_{\text{obs}}\|_2^2 + \varepsilon \|\nabla_\phi f_\theta(z)\|_2^2,
\end{equation}
where $\varepsilon$ is the vector that collects the anisotropy of the estimated gradient square tensor for each data sample [19]. In the context of seismic data, $\varepsilon$ can be interpreted as a confidence measure of the estimated slopes. Thus, it is added to the cost function to locally weight the directional Laplacian $\nabla_\phi$. Notice that, as the optimization goes on, the interpolated data can be used to produce a better estimate of the slopes, updating the Laplacian directions at runtime.

After the minimization of Eq. (4) has been performed, the interpolated solution is obtained as
\begin{equation}
x^* = f_{\theta^*}(z).
\end{equation}

3. NUMERICAL RESULTS

In this section, we show numerical results obtained with our methodology on three different use cases. The CNN architecture is a standard U-Net design with skip connections. The input noise $z$ is sampled from a Gaussian distribution $\mathcal{N}(0,0.1)$. Optimization is performed through Adam algorithm with learning rate 0.001. Low-pass filtering is performed through a second order Butterworth design. As a reconstruction metrics, we compute the total SNR of the interpolated data with respect to the reference data. We underline the fact that the latter is not included in the optimization process: the network is not fed with the reference full data at any step.

3.1. First use-case: synthetic

First, we employ a synthetic examples made of 4 linear events with different dips. In particular, three events are mildly steep, while the fourth event has been designed to create aliasing effects. This example is simple yet particularly challenging due to the slope of the events. The data is made of 170 time samples with sampling frequency 1 kHz and 100 traces equispaced by 5 m. Data are depicted in Fig. 1b. We build a regular binary mask for simulating the decimation by keeping 1 trace over 3, as depicted in Figure 1a. The cutoff frequency $f_c$ is 50 Hz; the network is optimized for the first 3000 it-
Fig. 3: Results on post-stack field data: input decimated data (a), reference full data (b), results of the proposed method (c) and standard Deep Prior optimization (d), along with the reconstruction errors (e, f).

Fig. 4: FK spectra of post-stack field data: input decimated data (a), reference full data (b), results of the proposed method (c) and standard Deep Prior optimization (d).

Fig. 5: Slopes $\phi$ in degree (a) and related confidence $w$ (b) estimated on the interpolated F3 data.

Fig. 6: Objective function components of post-stack field data interpolation.

shows a more homogeneous reconstruction. This effect is particularly visible also in the spectra depicted in Figure 2d, where we can notice a residual aliasing pattern; on the other hand, in Figure 2c it has been removed.

3.2. Second use-case: post-stack

Encouraged by this result, we processed a post-stack 2D section of the F3 Netherlands field data [21]. This section consists of 448 time samples, with sampling period 16 ms, and 128 traces with 25 m spacing, as depicted in Figure 3b. This example is quite complex, with different slopes and noise residuals coming from previous data processing. The regular binary mask keeps 1 trace over 5, as depicted in Figure 3a. The cutoff frequency $f_c$ is 10 Hz; the network is optimized for the first 1000 iterations to fit the low-passed data, as in Eq. (3). Then, the slopes are estimated and the optimization is performed on the sub-sampled data for 5000 iterations more with regularization weight $\varepsilon = 0.05$. The behavior of the cost function terms at each iteration can be observed in Figure 6. The big discontinuity in the data fidelity term (in blue) arises because the optimization switches from Eq. (3) to Eq. (4). The orange curve depicts the Laplacian regularizer. For the
Fig. 7: Results on pre-stack field data: input decimated data (a), reference full data (b), results of the proposed method (c) and standard Deep Prior optimization (d), along with the reconstruction errors (e, f).

Fig. 8: FK spectra of pre-stack field data: input decimated data (a), reference full data (b), results of the proposed method (c) and standard Deep Prior optimization (d).

first 1000 iterations it has not been taken into account; then, it is included in the optimization. The discontinuities visible at iteration 2000, 3000 etc. are due to the fact that the slopes \( \phi \) and confidence \( \omega \) are refined every 1000 iterations from the low-pass interpolated data.

Figures 3c and 3d show the results of our approach and a standard Deep Prior optimization, respectively. Both techniques produce a good estimate of the flat zones. On the other hand, where the interpolated data in Figure 3d is aliased, the directional Laplacian regularizer has improved the continuity of the steeper reflections. The total SNR of the proposed approach is 6.62 dB, against 5.57 dB obtained without regularization. Moreover, the Deep Prior paradigm produces estimates of the data with less noise content; this is clearly visible in the spectrum depicted in Figure 4c.

Figure 5 depicts the slopes \( \phi \) and its confidence \( \omega \) estimated from the low-pass filtered interpolated data. By smoothing the gradient with \( \sigma = 5 \), we are able to discriminate local dips with a good resolution.

3.3. Third use-case: pre-stack

Finally, we show the limitations of the devised methodology on pre-stack seismic data taken from the Viking Graeben Line 12. This dataset consists of 1408 time samples, with sampling period 4 ms, and 120 traces with 25 m spacing. Moreover, the dataset is affected by noise and aliasing, visible in Figure 7b and its spectrum in Figure 8b. In order to equalize the amplitude of the late events, we apply a linear gain on each trace. The decimation is 50\%, as depicted in Figure 7a. The cutoff frequency \( f_c \) is 20 Hz; the network is optimized for the first 2000 iterations to fit the low-passed data, as in Eq. (3). Then, the slopes are estimated and the optimization is performed on the sub-sampled data for 6000 iterations more with regularization weight \( \varepsilon = 0.05 \).

This example shows clearly the limitation of the standard Deep Prior inversion. Almost every event is poorly reconstructed, as depicted in Figure 7d. The achieved SNR is 4.47 dB. Adopting the proposed technique, data in Figure 7c are better interpolated; the total SNR is 8.34 dB. The residual depicted in Figure 7e is concentrated almost exclusively on the first events. However, a little aliasing effect is still present, as clearly visible in the spectra of Figure 8.

4. CONCLUSIONS

In this manuscript, we propose a Deep Prior method for interpolating seismic data strongly affected by aliasing due to high regular sub-sampling. First, the interpolation problem is cast in the Deep Prior paradigm. This means we make use of a CNN as an optimizer to minimize a cost function, rather than a tool that needs to be trained beforehand. Then, we add a regularization term that relies on directional Laplacian. The latter is built upon data slopes estimated from a low-pass version of the interpolated data produced in a first step.

Numerical simulations have been run on synthetic, post-stack and pre-stack data. Results have proven the proposed methodology to be effective and worth of future efforts, which will be devoted to improving the robustness of the inversion and extending this paradigm to 3D data.
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