Black Holes in Brane Worlds

P. Suranyi and L.C.R. Wijewardhana
Department of Physics, University of Cincinnati, Cincinnati, Ohio, 45221

March 27, 2022

Abstract

In a Randall-Sundrum theory (RS1) 3+1 dimensional black holes and higher dimensional black holes are not the natural continuations of each other. 3+1 dimensional black holes decay into a large number of 4+1 dimensional black holes at a critical mass, \( M_{\text{crit}} \sim 10^{32} \text{TeV} \). Those black holes themselves may become unstable above another, albeit much smaller critical mass, \( M_0 \sim 10^3 \text{TeV} \).

Models of the universe with extra dimensions larger than the Planck length have been under intense investigation during the last few years [1, 2, 3, 4]. The general feature of these models is that standard model particles are compelled to live on 3-branes, to satisfy momentum conservation in 3+1 dimensions and to conform to other phenomenological bounds, while gravity pervades all dimensions. Many of these models predict the observation of black holes at future accelerators [1, 5, 6, 7]. The models either use flat but compact extra dimensions(ADD scenario) [1] or a number of branes embedded in AdS space, with warped extra dimension(s) [2, 3].

Black holes in theories with extra dimensions have been studied widely. The classic paper of Myers and Perry [8] found solutions in \( D \)-dimensional flat space. Black hole solutions were also found in AdS space [4, 11]. No non-trivial black hole solutions have been found in closed form in brane theories of the Randall Sundrum type. Yet, it is important to learn as much as possible about black holes in such models. The black string solution [11] that extends in a uniform manner from the brane into the extra dimension has the Gregory-Laflamme instability in the ADD scenario [11, 12]. It is easy to invoke an entropy argument [13], to show that an instability will occur at a critical mass. An alternative interpretation is given in [14].

To understand the arguments by Gregory and Laflamme, compare the entropies of standard 3+1 and 4+1 dimensional Schwarzschild black holes of the same mass. Then one obtains a critical mass

\[
M_{\text{crit}} \sim \frac{M_4^4}{M_5^3},
\]
where $M_D$ is the Planck mass in $D$ dimensions. At this mass the radius of the horizon of the 4 dimensional black hole is approximately the same as that of the 5 dimensional one. When a 4 dimensional black hole in the process of its Hawking radiation \cite{15, 16} passes this critical mass, the entropy of a 5 dimensional black hole with the same mass becomes larger. Then, according to Gregory and Laflamme, under the influence of quantum fluctuations, the black string breaks up into a large number of small 5 dimensional black holes which then unite into a single 5 dimensional black hole, having a larger entropy than the collection of smaller black holes.

In what follows we would like to investigate the question of stability of black holes in models with warped extra dimensions. We will consider both of the models of Randall and Sundrum (RS1 and RS2) \cite{2, 3}. We will use an expansion technique to find such black hole solutions and then discuss their properties.

First we will investigate black holes in RS1 and find a markedly different behavior from ADD. One important goal of RS1 is to solve the hierarchy problem by making the fundamental gravity scale 1 TeV. This is achieved by requiring an exponential relation between the scales on the TeV brane, the home of standard model particles, and on the Planck brane.

We start form the Randall-Sundrum classical action $S$ for the system consisting of two 3-branes with brane tensions $\sigma_{Planck} = -\sigma_{sm} = \frac{12M_3^3}{\lambda}$. The branes are fixed at the points $y = 0$ and $y = y_{\text{max}} = \pi r_c$.

\[
S = \frac{M_5^2}{16\pi} \int d^4x \int dy \sqrt{-g} \left( \frac{12}{\lambda^2} + R \right) + \int d^4x \sqrt{-\eta} \sigma_{sm} + \int d^4x \sqrt{-\eta} \sigma_{Planck}
\]

where $R$ is the 5-d Ricci scalar, $M_5$ is the fundamental gravity scale, $\lambda$ is the curvature length of the AdS space and the brane tensions $\sigma_{TeV}$ and $\sigma_{Planck}$ are selected such that the metric satisfies the correct induced Einstein equations on the branes.

In RS1 the range of the variable $y$ of \cite{4} is limited to $0 \leq y \leq y_{\text{max}}$. As we are interested in objects living on the TeV brane, located at $y = y_{\text{max}}$ rather than on the Planck brane located at $y = 0$ it is convenient to introduce the conformal variable $w$ \cite{4} defined by $|w| = z_c - \lambda \exp\{y/\lambda\}$ where $z_c = \lambda \exp\{y_{\text{max}}/\lambda\}$. Note that unlike in Ref.\cite{4} we use the convention $G = M_d^{-d+2}$. The range of variable $w$ is $0 < |w| < z_c - \lambda$. The location of the TeV brane, where standard model particles live, is $w = 0$. The parameter $z_c = O(TeV^{-1})$.

The metric in this coordinate system takes the form

\[
ds^2 = \left( \frac{z_c}{z_c - |w|} \right)^2 \left( \eta_{\mu\nu} dx^\mu dx^\nu - dw^2 \right)
\]

where the rescaled ‘brane variables’ are defined as $dx^\mu = \lambda dx^\mu / z_c$. The new 5-d gravity action is given by
\[ S = \frac{\tilde{M}_5^3}{16\pi} \int d^4x' \int dw \sqrt{-G} \left( \frac{12}{z_c^2} + R_G \right) + S_{\text{brane}}, \]  

(3)

where \( \tilde{M}_5 = M_5 \lambda \) and \( R_G = \lambda^2 R / z_c^2 \). The crucial difference between the actions (1) and (3) is that the gravitational constant, \( M_5^3 \), is rescaled to \( \tilde{M}_5^3 = (\lambda M_5 / z_c)^3 \), where \( \tilde{M}_5 = O(1 \text{TeV}) \) can be chosen and the new effective AdS curvature length is \( z_c \). As we will deal with this rescaled 5 dimensional Planck mass only, we will use the notation \( M_5 \) for \( \tilde{M}_5 \).

Let us now consider a black hole in the RS1 scenario bound to the TeV brane with its singularity on the brane. As long as the radius \( R \) of the horizon of such a black hole is substantially smaller than \( z_c \), the effect of the AdS term of (3) is negligible. Thus, such an object could very well be described by a flat 5 dimensional black hole solution. Then the constraint \( R << z_c \) can be translated to the relation

\[ M << M_0 = M_5^3 (z_c M_5)^2 / 8. \]

Using phenomenological constraints, the dimensionless constant \( z_c M_5 \) was estimated \[7\] to be \( z_c M_5 \geq 20 \). Setting the Planck mass at \( 1 \) TeV this bound for the black hole mass is \( M << 500 \) TeV. As quantum gravity sets in at around \( 1 \) TeV black holes satisfying this bound do exist and can presumably be produced at future accelerators \[7\].

Consider now a primordial RS1 black hole of mass 1-10 TeV, produced when the temperature of the universe was \( T \sim 1 \) TeV \[17\]. The Hawking temperature of the black hole,

\[ T_{BH} = \sqrt{3M_5^3 / 32\pi M_{BH}}, \]

for \( D = 5 \), will satisfy \( T_H < T \) and consequently it will accrete plasma rather than decaying with plasma emission. At a timescale \( t \sim 1 \) TeV \(^{-1} \) its will acquire mass and reach the \( M = M_0 \) limit. Then it will not be able to expand into the \( w \) direction anymore, so beyond this point its horizon area will be of the size \( A \sim 4\pi R^2 z_c \), rather than \( A = 2\pi^2 R^3 \). Since the relationship between mass and radius is unchanged, \( R^2 \simeq 8M / 3\pi M_5^3 \), the area of the horizon and the entropy of the black hole will be proportional to \( M \). Then, unlike for AdS black holes, where \( S \sim M^{3/4} \), for RS1 black holes in the above mass range, \( S \sim M \). Then the Hawking temperature remains constant \( T_H \sim 0.2 \) TeV during further accretion. These black holes could, in principle, grow until the temperature of the outside world catches up with them.

The heuristic scenario outline above can be made more precise by solving the Einstein equation in a perturbative fashion, in an asymptotic expansion in powers of \( r^{-2} \). The procedure is similar to the one previously applied to investigate corrections to 4 dimensional black hole solutions in brane worlds \[18\]. The details of this calculation will be presented in a future publication. \[19\] The results of our study can be
summarized as follows: An ansatz for the metric tensor is taken as
\[ ds^2 = \left( \frac{z_c}{z_c - |w|} \right)^2 (g_{tt} dt^2 - g_{ww} dw^2 - g_{rr} dr^2 - 2g_{rw} dr dw - r^2 d\Omega^2), \]
where
\[ g_{ii} = 1 + \sum_{n=1} f_i^n(w)/r^{2n}, \]
where \( i = t, w, r \) and
\[ g_{rw} = \sum_{n=1} f_{rw}^n/r^{1+2n}. \]
The power of \( r \) in the leading terms was chosen such that the solution would go over smoothly into the Myers-Perry black hole solution when the mass is sufficiently small. The coefficient of \( r^2 d\Omega^2 \) was chosen to be unity. This choice establishes the scale of the radial coordinate \( r \). This choice is also consistent with the Myers-Perry solution at \( w \ll z_c \). Finally, in agreement with the Myers-Perry solution we set \( f_1^w = f_0^{rw} = 0 \). Then we solve the Einstein equation order by order of the asymptotic series [20]. We also impose the junction conditions for empty brane [21]. Then we obtain a unique solution in second order: \( f_1^t = -f_1^t = 8M/3\pi M_5^3 \). To make \( f_2^t \) unique one has to fix the scale for the \( w \) coordinate, as well. This can be done by requiring that the leading contributions, \( f_2^w \) and \( f_1^{rw} \), are exactly equal to the ones predicted by the Myers-Perry solution. Then one obtains
\[ f_2^t(w) = \frac{8M}{3\pi M_5^3} \left( -w^2 + \frac{2w^3}{3z_c} \right). \]
Now the area of the horizon is given by
\[ A = 4\pi \int_0^{z_c - \lambda} dw r^2(w) \theta(r(w)) \sqrt{1 + (dr/dw)^2}, \]
where \( r(w) \) is the radius of the horizon, which is the solution of the equation \( g_{tt} = 0 \). The result of this calculation is, as expected, that \( A \sim M^{3/2} \) for \( M \ll M_0 \), while \( A \sim M + c + O(M^{-1}) \) for \( M > M_0 \), where \( c < 0 \) is a calculable constant. We call the region \( M < M_0 \) the low mass region and \( M_0 < M < M_{\text{crit}} \) the intermediate region. \( M_{\text{crit}} \) will be determined below.

Let us consider now the consequences of the dependence of the entropy of the black hole on mass. Using (4) we obtain the following form for the entropy in the intermediate region (\( M \gg M_0 \))
\[ S \simeq \frac{8}{3} M z_c - \frac{3\pi}{20} (M_5 z_c)^3 + O(M^{-1}). \]
As the Hawking temperature is constant primordial black holes might accrete matter until the universe cools to \( T = T_H \). We will see later that this will not really happen.
Still, assuming such an accretion is possible it is interesting to see at what mass value would the entropy of the 5 dimensional black hole, \( S_4 = \frac{4\pi M_4^2}{M_5^2} \), equal to the entropy \( S = \frac{4\pi M^2}{M_4^2} \) of a 4 dimensional black hole with the same mass. Equating these two expressions one obtains

\[
M_{\text{crit}} \simeq \frac{2}{3\pi^2} M_4^2 z_c \sim 10^{32}\text{TeV}.
\]

It turns out that the Schwarzschild radius of such a four dimensional black hole is \( R = O(\text{TeV}^{-1}) \) and its Hawking temperature is \( T_H = O(0.1\text{TeV}) \). This is a region where quantum effects are supposed to enter in a world with a Planck mass of \( M_5 \simeq 1\text{TeV} \). Quantum instability is supposed to transform the 4 dimensional black hole into a 5 dimensional black hole as a Gregory-Laflamme type instability sets in. Though an \( N \)-black hole configuration has almost the same entropy due to the linear dependence of the entropy on mass, the negative constant term makes the decay into a single 5 dimensional black hole slightly more favorable. We shall see that this does not happen.

The first warning sign about the validity of the above simple minded scenario comes from considering the radii of these black holes. While the radius of the horizon of the four dimensional black hole is \( R \sim 1\text{TeV}^{-1} \), the decay product, the 5 dimensional black hole has a macroscopic, \( R \sim M_4/M_5^2 \simeq 10^{16}\text{TeV}^{-1} \simeq 1\text{mm} \) radius of horizon. It is physically difficult to imagine such a transformation. It would be aesthetically much more pleasing if the \( R = 1\text{TeV}^{-1} \) four dimensional black hole would decay into black holes of similar radius. To see that this is what really happens we should realize that black holes of the intermediate region, \( M_0 < M < M_{\text{crit}} \) are not stable themselves. We can envision this in the following manner. As soon as the size of black holes reaches the size of space in the 5th dimension when their masses reach \( M_0 \), they are forced to expand along the brane only. As soon as they are large enough along the brane to break up into two more or less spherically symmetric 5 dimensional black holes, that barely fit the fifth dimension, they will do so, thereby increasing their entropy. In fact, if we consider the relationship of entropies of \( n \) 5 dimensional spherical black holes and one flattened 5 dimensional black hole then we obtain, in a rough approximation

\[
n S_5(M/n) = n\sqrt{2\pi} \left( \frac{4M}{3nM_5} \right)^{3/2} = 8M z_c/3
\]

Then we find that for every \( n \) we find a mass \( M_n \) for which this equality is satisfied

\[
M_n = nM_5^3 z_c^2/3/2\pi \sim nM_0.
\]

For \( M > M_n \) a black hole of the intermediate region is unstable against decaying into \( n \) five dimensional black holes.

Then the fate of 4 dimensional black holes is also clear. As soon as they reach the critical mass of \( M_{\text{crit}} \) where their radius is \( R \sim 1\text{TeV}^{-1} \) and Hawking temperature
$T_H \simeq 1$ TeV they become unstable due to quantum fluctuations and decay into a large number ($N$) of 5 dimensional black holes, 

$$N \simeq \frac{M_{\text{crit}}}{M_0} \simeq \frac{16}{9\pi^2} \frac{M_4^2}{M_5^3 z_c} \sim 10^{20}.$$ 

The radius of the horizon of these black holes is $R \sim z_c$, very similar to that of the decaying 4 dimensional black hole. Similarly, all the black holes have a Hawking temperature of a fraction of 1 TeV.

The moral of these considerations is that in RS1 there are two distinct types of black holes that have nothing to do with each other: 4 dimensional ones that become unstable at a minimal mass, $M \sim 10^{32}$ TeV, at which the size of their horizon is reduced to the quantum scale (the inverse of the mass of KK modes) and 5 dimensional black holes that have maximal masses of $M \sim 10^3$ TeV. In between these two extremal masses there is no stable black hole centered at the TeV brane.

Note that the instability we have studied differs substantially from that of Gregory and Laflamme [12, 13], which transforms 4 dimensional black holes into a five dimensional black hole of the same mass (not considering intermediate states). The reason for the difference is that in flat space, the heavier 5 dimensional black holes are, the more stable they become. Contrast this behavior with what we have studied in RS1, where 5 dimensional black holes of the size $R \sim z_c$ are the most stable.

If after inflation the universe reheats to $T \simeq 1$ TeV then one expects that a large number of 5 dimensional brane black holes are produced by the collision of standard model particles. The Hawking temperature of these black holes is lower than the ambient temperature and they are expected to rapidly accrete matter until they reach $M \sim 2M_0 \sim 10^3$ TeV, when they decay into two black holes of mass $M_0$. This process then repeats itself until most of the particles are transformed into black holes. The black holes will start to decay only after the outside temperature reaches their Hawking temperature, in about $10^{-13}$ s.

Finally let us briefly consider RS2 (single brane world) black holes. Here the physical brane is at $y = 0$ and the second brane is taken to infinity. In conformal coordinates $z$ it is obvious that for large $|z|$, far away from the brane, the conformal factor $\lambda^2/(\lambda + |z|)^2 \simeq \lambda^2/z^2$ and the metric approaches that of the AdS space without a brane. Therefore, far away from the brane, black hole solutions of the AdS type should exist. Conversely, when $|z| \ll \lambda$ then the space looks flat. Thus, close to the brane Myers and Perry type black holes should exist. We are interested in black holes centered on the brane.

It was shown by Randall and Sundrum [3] that normal Newtonian gravity prevails on the brane at distances $R \gg \lambda$. This constraint puts an upper limit on $\lambda = M_4^2/M_5^2 \leq 1 nm \simeq 10^{16}$ TeV$^{-1} \sim 10^{32} M_4^{-1}$. This restricts the five dimensional Planck scale to be between $10^5$ TeV and the 4-d Planck scale of $10^{16}$ TeV. Black hole masses are bounded below by this five dimensional Planck scale. For $R \gg \lambda$, black holes, which are effectively 3+1 dimensional, with a small extension to the bulk,
should exist, with the standard connection between the mass and the radius of the horizon: $R = 2M/M^4_4$. Such pancake shaped black hole solutions have been found using an expansion technique. The entropy of such a black hole is

$$S = \frac{4\pi M^2}{M^4_4}. \quad (6)$$

where $M_4 = (G_4)^{-1/2}$.

The evaporation of such four dimensional RS2 black holes is similar to ADD. The size of the evaporating black hole approaches the AdS curvature length scale, $\lambda$, at a mass $M_{\text{crit}} = \lambda M_4^2 \simeq 10^{32} M_4$. It can be shown that at that point it becomes entropically favorable for it to decay into an almost flat 5 dimensional black hole centered on the brane. This phenomenon could also be studied using an expansion method, similar to the one applied to RS1.

It would be interesting to discuss the stability and the possible growth of TeV-range brane black holes from the point of view of the AdS-CFT correspondence. The growth of these black holes by accretion in the early universe and the possibility of creating a baryon asymmetry by such accreting black holes will be discussed elsewhere.

The authors are indebted to P. Argyres, M.Bowick, F.P. Esposito, A. Shapere and J. Terning for valuable discussions. The support of the U.S. Department of Energy under Grant No. DE-FG02-84ER40153 is gratefully acknowledged.

References

[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B 429, 263 (1998); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D 59, 086004, 1999.

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83:3370-3373, 1999

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. 83:4690-4693, 1999

[4] See also the review: V.A.Rubakov, hep-ph/0104152, Phys. Usp. 44:871-893, 2001.

[5] S.Dimopolous and G.Landsberg, Phys. Rev. Lett. 161602(2001), S.B.Giddings and S.Thomas, Phys. Rev. D 5:056010, 2002; D.M. Eardley and S.B.Giddings, gr-qc/0201034.

[6] P. Argyres, S. Dimopoulos, and J. March-Russell, Phys. Lett. B 441, 96 (1998), T. Banks and W. Fischler, hep-th/9906038, R. Emparan, G. Horowitz and R. Myers, P.R.L. 85, 499 (2000).

[7] L. A. Anchordoqui, H. Goldberg and A. D. Shapere, e-Print Archive: hep-ph/0204228

[8] R. C. Myers and M. J. Perry, Annals Phys. 172, 304 (1986).
[9] S.W. Hawking and D.N. Page, Commun.Math.Phys.87:577,1983

[10] D. Birmingham, Class.Quant.Grav.16:1197-1205,1999

[11] A. Chamblin, S.W. Hawking, H.S. Reall, Phys.Rev.D61:065007,2000

[12] R. Gregory and R. Laflamme, Phys.Rev.Lett.70:2837-2840,1993;

[13] R. Gregory and R. Laflamme, Waterloo 1993, General relativity and relativistic astrophysics* 190-19

[14] G. T. Horowitz and K. Maeda, Phys.Rev.Lett.87:131301,2001

[15] S. W. Hawking, M.N.R.A.S. 152, 75 (1970).

[16] Also see, B.J.Carr and S.Hawking, M. N. A. R. S. 168, 399; I.Novikov and K.Thorne in Black Holes ed. C.Dewitt and B.Dewitt, p.343, Gordon Breach:New York(1973).

[17] Unlike in ADD in RS1 the universe could be heated to a temperature of a TeV. See Ref.[4].

[18] R. Casadio and L. Mazzacurati, e-Print Archive: gr-qc/0205129

[19] P. Suranyi and L.C.R. Wijewardhana, in preparation.

[20] For convenience we rearrange the series, such that it is an expansion in powers of \( r^{-2} \) = \( (w^2 + r^2)^{-1} \), rather than \( r^{-2} \).

[21] W.Israel, Nuovo Cimento B44,1(1966);B 48,463(1966). See also R.Cassadio and L.Mazzacurati, gr-qc/0205129;T.Shiromizu, K.Maeda and M.Sakai, Phys.Rev.D 62,024012(2000).