Connectors meet Choreographies

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Abstract

We present Cho-Reo-graphies (CR), a new language model that unites two powerful programming paradigms for concurrent software based on communicating processes: Choreographic Programming and Exogenous Coordination. In CR, programmers specify the desired communications among processes using a choreography, and define how communications should be concretely animated by connectors given as constraint automata (e.g., synchronous barriers and asynchronous multi-casts). CR is the first choreography calculus where different communication semantics (determined by connectors) can be freely mixed; since connectors are user-defined, CR also supports many communication semantics that were previously unavailable for choreographies. We develop a static analysis that guarantees that a choreography in CR and its user-defined connectors are compatible, define a compiler from choreographies to a process calculus based on connectors, and prove that compatibility guarantees deadlock-freedom of the compiled process implementations.

1 Introduction

Background. Programming concurrent software is hard: given the specification of the desired interactions among processes, implementation is error-prone even for experienced programmers \cite{35, 38}. The challenge of concurrent programming has driven decades of research on new programming models. A particularly fruitful idea was to provide a native language abstraction for interaction, rather than modelling it as a side-effect. Two research lines in particular were built upon this idea, but following very different directions.

The first research line is that on Choreographic Programming \cite{39}. Central to Choreographic Programming is the choreography, a programming artefact that specifies a concurrent system in terms of the interactions among its constituent processes, by using an “Alice and Bob” notation that disallows writing mismatched I/O actions (e.g., a send without a corresponding receive). Through EndPoint Projection (EPP), choreographies can be compiled to faithful and deadlock-free implementations in process languages \cite{17}.

Previous work studied models for choreographies with different interaction semantics (e.g., synchronous, asynchronous, multi-cast). Common to all existing models of Choreographic Programming is the fact that the nature of interactions is “hardcoded”: each model proposes new syntax and semantics, so results have to be proven from scratch every time and cannot be combined. For example, none of the existing models supports mixing synchronous with asynchronous interactions within the same choreography. This is a serious limitation, which raises the challenge of finding a unifying framework. Furthermore, there is still no indication of how other interesting interaction semantics (e.g., barriers) can be introduced to choreographies.

The second research line is that on Exogenous Coordination \cite{2, 3, 34}, where interaction protocols and process code are developed separately. Process programs are then modularly composed
with protocols, given as _connectors_, which dictate interactions by accepting/offering messages from/to processes. Connectors offer an elegant way of programming different semantics for interactions, starting from basic instances (e.g., synchronous and asynchronous point-to-point channels) and then composing them to create more sophisticated protocols.

The limitation of Exogenous Coordination is that we do not have a global system view when composing processes with connectors: programmers cannot define the intended flow of information among processes when a connector is used many times, or if many connectors are used in different parts of a system. Hence, incompatibilities between process code and connectors may cause deadlocks, which cannot happen in Choreographic Programming. The two paradigms thus have complementary strengths.

**Contribution.** We develop the first integration of the best aspects of Choreographic Programming and Exogenous Coordination: a new calculus of choreographies, called _Cho-Reo-graphe\(\text{s}\) (CR), whereby the process interactions specified in a choreography are animated by arbitrary, user-defined, connectors based on the Exogenous Coordination language Reo. CR allows for mixing different connectors in the same choreography, making it for the first time possible to write choreographies where different interactions can have different communication semantics. Furthermore, by tapping into the expressivity of connectors, we can endow choreographies with hitherto unexplored communication semantics, such as alternators or barriers. This makes CR more expressive and a generalisation of existing models of Choreographic Programming. Through EPP, choreographies can be compiled into concurrent implementations in a process language. In these implementations, the same connectors as in the original choreography animate interactions among processes. We show that these processes are deadlock-free, provided that the original choreography is compatible with the connectors.

Mixing different communication semantics (i.e., connectors) in choreographies produces new challenges of formalisation and decidability. Solving these is our main technical contribution. On the formalisation front, we have to balance expressiveness and comprehensibility: the formal semantics of the calculus should be easy enough to explain and understand, without sacrificing the expressive power of connectors. We address this challenge with a new labelled reduction semantics for choreographic interactions, where labels act as the interface with connectors. On the decidability front, we prove that (1) deadlock-freedom is generally undecidable in our calculus, but (2) we can establish deadlock-freedom for a large subset of the language. Our proof of (1) shows that undecidability is a direct consequence of the expressive power that connectors introduce to choreographies. We address (2) by designing a new decidable static analysis (compatibility) that is made possible by the careful design of labelled reductions for interactions.

By leveraging existing work, distributed implementations of Reo connectors can be automatically generated and deployed on distributed systems (e.g., in Scala \[41, 42\] or Java \[31\]); our approach is compatible with this work. As such, a practical tool based on CR can in principle be built on top of existing code generators for distributed implementations of Reo connectors. In general, we believe that our results represent the beginning of an interesting research line on concurrent programming; we discuss future directions in \$6\).

**Structure.** We motivate our work with an example in \$2\) and introduce connectors in \$3\). CR is described in \$4\) together with results on deadlock-freedom. In \$5\) we introduce the target process language for EndPoint Projection and prove correctness of synthesised process implementations. We conclude in \$6\) discussing directions in which this work can be extended. \$6\) concludes, discussing directions in which this work can be extended.

**Related work.** We already covered the main references to previous work and how it falls short of serving our aim. We briefly recap related work.

Choreographies have been studied in different settings with fixed communication semantics, including synchronous \[15, 22, 32\], asynchronous point-to-point \[17, 24, 27, 40\], one-to-many \[18, 20\], and many-to-one \[16, 30\]. In our model, these semantics are simply instances of what we
can do. But since we tap into the generality of Reo connectors, we can also do more (e.g., we illustrate how to use barriers). CR is not the first model where choreographies may deadlock: this is common for settings with realistic communication semantics (e.g., [23, 37]). However, it is the first model with arbitrary communication semantics. Our development also extends the line of work on out-of-order execution for choreographies, initiated in [17], where non-interfering interactions may proceed concurrently. This style allows for more safe behaviours in the semantics of choreographies (by swapping non-interfering communications), which the programmer gets for free (concurrency is inferred). As in many other works, out-of-order execution also simplifies our syntax: we do not need to provide for a parallel operator in choreographies, since most parallel behaviour is already captured by out-of-order execution (cf. [17] for details). As in previous work, CR supports asynchronous behaviour without requiring the programmer to reason about it in choreographies: communications are still specified as atomic interactions, which may be asynchronously reduced in a safe way (cf. [17, 20, 40]).

In Exogenous Coordination, interaction protocols for communicating processes, called connectors, are programmed separately from the internal code of each process. This enables a compositional approach for developing protocols, where complex protocols can be built by assembling simpler ones. Exogenous Coordination has been studied extensively over the last two decades [2, 3, 34]. Examples of models of Exogenous Coordination are the algebras of connectors [11, 12], the algebra of stateless connectors [13], and constraint automata [8]; examples of languages are (interactions in) BIP [9, 10], Ptolemy [14, 43], and Reo [4, 5].

2 Motivating Example & Approach

We present an example to introduce the concept of choreographies and the problem we are interested in studying. This example will be used as running example throughout the whole article.

Example 1 (Book sale). Alice (a) wants to buy a book from seller Carol (c), facilitated by a bank (b) and a shipper (s). First, Alice sends the title of the book to Carol. Carol then replies to Alice with the price of the book. If Alice is happy with the price, she notifies Carol, the bank, and the shipper that the purchase proceeds. Alice subsequently sends the money to the bank (who transfers it to Carol’s account), and Carol sends the book to the shipper (who dispatches it to Alice). The choreography for this scenario looks as follows:

1. a⟨title⟩→ c; c⟨price⟩→ a;
2. if a.happy then ( a→ c[ok]; a→ b[ok]; a→ s[ok]; a⟨money⟩→ b; c⟨book⟩→ s

In previous choreography models, the nature of the interactions is fixed: depending on the model, these interactions are either all synchronous or all asynchronous. This is not flexible enough, because requirements may be different for each interaction.

Example 2 (Book sale). The book sale scenario has the following requirements:

- Because there are no strict timing constraints, it is reasonable for Alice and Carol to communicate asynchronously in line 1.
- Because the same label (ok or ko) is sent from Alice to Carol, the bank, and the shipper, it makes sense to combine these communications in a multi-cast.
- It is better for Alice to send her money to the bank as late as possible (e.g., because she receives interest on her money). Therefore, Alice does not want to send her money until she knows the others have received her ok-label. Thus, the multi-cast of this label should be synchronous (i.e., handshake between Alice, Carol, the bank, and the shipper).
• Alice and Carol may not trust each other: Alice does not want to send money before Carol has sent the book, and vice versa. To resolve this impasse, Alice and Carol should synchronously (barrier-like) send money and book, to ensure each of them holds her end of the bargain; the receives by the bank and the shipper may subsequently proceed asynchronously. □

As we show in the next sections, our model is powerful enough to express all these interactions (and more). The key idea is to tag interactions with the name of the particular connector through which they transpire. For instance, a value communication from p to q through a synchronous channel sync is expressed as p → q thru sync, while the same communication through an asynchronous channel async is expressed as p → q thru async. If all interactions in a choreography transpire through the same (type of) connector, as in all existing choreography approaches, tags become redundant and can be omitted.

3 Reo and Constraint Automata

We view processes in a concurrent system as black boxes with interfaces consisting of ports. For a process to send (receive) a message to (from) another process, it performs an output action (input action) on one of its own output ports (input ports), but without specifying a receiver (sender). Instead, a separate connector, connected to the ports of the processes, decides how messages flow among them.

Reo [4, 5] is a graphical, dataflow-inspired language for connectors. As we focus primarily on semantics in the rest of this paper, we only present Reo’s semantic formalism: constraint automata [8]. The transitions of a constraint automaton model possible synchronous message flows (i.e., interactions) through a connector in a particular state. Constraint automata are parametrised over a language of constraints Φ to specify transition labels. Depending on the required level of expressiveness, different instantiations of Φ may be considered. In this work, we consider Φ_M, which is a language of constraints over two sets P and M, of ports and memory cells (storage space local to a connector). Constraints are finite sets of formulas of the form p1 → p2 (port p1 passes a message to port p2), m1 → m2 (cell m1 passes a message to cell m2), p → m (port p passes a message to cell m), and m → p (cell m passes a message to port p). Furthermore, we require that all terms on the right-hand side of formulas in the same constraint in a transition label be distinct, to ensure that every port/cell is assigned a unique message (e.g., p1 → m ∧ p2 → m is forbidden, but p → m1 ∧ p → m2 is allowed).

Definition 1. A constraint automaton is a tuple (S, P, M, − − →, s0, μ0), where S is a finite set of states, P is a finite set of ports, M is a finite set of memory cells, − − → ⊆ S × Φ_P,M × S is a transition relation, s0 ∈ S is an initial state, and μ0 is an initial memory snapshot mapping the memory cells in M to their initial content.

A transition (s, φ, s’) ∈ − − →, which we write s ̸→ φ s’ for short, means that, from state s, a subset of the ports in P can interact according to φ and the automaton goes to state s’. Constraints in Φ_P,M also control the evolution of memory snapshots as transitions are made. Instead of formalizing this separately (e.g., in terms of runs and languages [29]), we combine it directly in the semantics of our choreographies, in §4.

In this work, we restrict ourselves to a subset of constraint automata that satisfy two additional assumptions on their transitions. First, the occurrences of each port in transition labels are either all as sender (the port occurs only on the left-hand side of constraints) or all as receiver (it occurs only on the right-hand side of constraints); this constraint is imposed on the whole automaton. For instance, a constraint automaton cannot have two transitions where one is labelled by p1 → p2 and the other by p2 → p3. Secondly, the transition relation of each automaton is deterministic on the ports used, i.e., for any given state s, if there are two distinct transitions s ̸→ φ1 s1 and s ̸→ φ2 s2, then the sets of ports used in φ1 and φ2 must be distinct (but they can overlap, or one be a strict subset of the other). The first assumption simplifies defining the semantics of choreographies in

[1] We can formalise Φ as an equational theory as in [8, 29], but this is unnecessary for our development.
§4; the second assumption ensures that this semantics is deterministic, in line with previous work on choreographies.

Port automata are minimalistic, meaning that a port automaton defines only the orders in which ports may be used. On the contrary, a port automaton says nothing about the actual messages being exchanged through ports. Notably, a port automaton does not define which particular message (sent through an output port by a sender) a receiver receives (through an input port). Port automata, thus, underspecify connector behaviour.

Example 3. Fig. 1 shows example constraint automata for useful connectors. Sync models a synchronous channel, which indefinitely lets two processes synchronously send and receive a message through ports $p_1$ and $p_2$. Async1 models an asynchronous channel with a 1-capacity buffer (using a memory cell $m$). Indefinitely, first, this connector lets a process send a message through port $p_1$; subsequently, it lets a process receive the message through port $p_2$. Async2 models an asynchronous channel with a 2-capacity buffer. SyncMulti2 and SyncMulti3 model synchronous multi-cast connectors for two and three receivers. Indefinitely, this connector lets three processes synchronously send and receive a message, from port $p_1$ to ports $p_2$ and $p_3$. Async1Multi2 models an asynchronous multi-cast connector for two receivers. Indefinitely, first, the connector lets a process send a message (lower middle transition); then, it lets two processes receive the message, either one after the other (top or bottom transitions) or simultaneously (upper middle transition). Barrier models a barrier send/receive connector. Indefinitely, this connector lets two processes synchronously send and receive a message through ports $p_1$ and $p_2$, while it synchronously lets processes synchronously send and receive another message through ports $p_3$ and $p_4$.

Choreography programmers can model connectors by explicitly defining constraint automata. Alternatively, programmers can model connectors by composing constraint automata from basic primitive constraint automata, using a synchronous product operator [8]. A significant advantage of this latter approach is that there exists an intuitive and user-friendly graphical syntax for constraint automaton product expressions. In this graphical syntax, called Reo [4, 5], programmers draw constraint automaton product expressions as data-flow graphs between (ports of) processes. As an example, Figure 2 shows Reo connectors for the constraint automata in Figure 1. Reo conveniently hides from choreography programmers the intimidating act of explicitly composing constraint automata, without sacrificing generality: Reo is complete for constraint automata (under the instantiation of $\Phi$ considered in this paper), meaning that every constraint automaton can be expressed as a Reo connector [6, 7]. Moreover, tooling exists to animate flows of messages through Reo connectors (http://reo.project.cwi.nl).

We use (constraint) automata and (Reo) connectors interchangeably: at the semantics level, we use automata, while at the syntax level, we use connectors.
We now present Cho-Reo-graphies (CR): our choreography calculus that combines choreographies with Reo connectors. A choreography describes the behaviour of a set of processes. For simplicity, we write the semantics, but which are not meant to be used by programmers.

Figure 2: Example Reo connectors for the constraint automata in Fig. 1.

\[
C ::=} \eta \text{ thru } C \mid \text{if } p.e \text{ then } C_1 \text{ else } C_2 \mid \text{def } X = C_1 \text{ in } C_2 \mid X \mid 0
\]

\[
\eta ::=} p(e) \rightarrow q.x \mid p \rightarrow q[\ell] \mid X.\ell\{x\} \mid X.\ell[\{x\}]
\]

Figure 3: Cho-Reo-graphies, syntax. The boxed terms are runtime terms, necessary for defining the semantics, but which are not meant to be used by programmers.

4 Cho-Reo-graphies

We now present Cho-Reo-graphies (CR): our choreography calculus that combines choreographies with Reo connectors. A choreography describes the behaviour of a set of processes. For simplicity, we assume that values are untyped; treating value types is straightforward and analogous to [15, 17, 23].

The syntax of CR is displayed in Fig. 3. We range over choreographies using \(C, C', \ldots\); over processes using \(p, q, \ldots\); over interactions using \(\eta, \eta', \ldots\) (and over sets of them using \(\tilde{\eta}, \tilde{\eta}', \tilde{\eta}_1, \ldots\)); over connector names using \(\gamma, \gamma', \ldots\); over procedure names using \(X;\) over (side-effect free) expressions using \(e, e', \ldots\); over values using \(v, v', \ldots\); and over selection labels using \(\ell, \ell', \ldots\). Each process \(p\) owns a finite set of local variables \(\text{var}_p = \{x_1, \ldots, x_n\}\), and expressions are assumed to be inductively defined including \(\text{var}_p\) as base cases.

We comment briefly on label selections. It is standard practice in Choreographic Programming to distinguish value communications, which are used to exchange data, from label selections, which are used to propagate decisions regarding control flow. In the choreography in Ex. 1, Alice uses label selections to propagate decisions regarding control flow. Although this can be encoded using value communications [22], it is useful to distinguish them, as they are usually treated differently in implementations. Also, as discussed in §5, label selections are instrumental in generating process implementations automatically. The syntax of labels is unspecified.

We write \(p(e) \rightarrow \{q_1, x_1, \ldots, q_n, x_n\}\) (value multicast) and \(p \rightarrow \{q_1, \ldots, q_n\}[\ell]\) (label multicast) to abbreviate \(\{p(e) \rightarrow q_1, x_1, \ldots, p(e) \rightarrow q_n, x_n\}\) and \(\{p \rightarrow q_1[\ell], \ldots, p \rightarrow q_n[\ell]\}\), respectively. Also, we write \(p(x) \rightarrow q\) for \(p(x) \rightarrow q.\ell\) (i.e., if sender \(p\) sends the value in local variable \(x\), and if receiver \(q\) stores the received value in a local variable with the same name, we omit the name). If \(\tilde{\eta}\) is a singleton, we omit curly braces.

Example 4 (Book sale). The choreography for our running example (Ex. 1) can be written as follows in CR.

1. \(a(\text{title}) \rightarrow c \text{ thru a2c;}\)
2. \(c(\text{price}) \rightarrow a \text{ thru c2a;}\)
3. \(\text{if } a.\text{happy} \text{ then } (a \rightarrow \{c, b, s\}[\text{ok}] \text{ thru a2cbs;})\)
4. \(\{a(\text{money}) \rightarrow b, c(\text{book}) \rightarrow s\} \text{ thru ac2bs; 0}\)
5. \(\text{else } (a \rightarrow \{c, b, s\}[\text{ko}] \text{ thru a2cbs; 0})\)
Our notation suggests that Alice multi-casts ok (or ko) to Carol, the bank, and the shipper. It is important to understand, though, that this notation is really just syntactic sugar. Ultimately, only the behaviour of connector a2cbs determines how exactly the three communications represented by a \(\rightarrow \{c,b,s\}\) transpire.

The semantics of most terms is standard. In \(\text{if } p \text{ then } C_1 \text{ else } C_2\), process \(p\) evaluates expression \(e\); if this results in \(\text{true}\), the choreography proceeds as \(C_1\), and otherwise, as \(C_2\). In \(\text{def } X = C_2 \text{ in } C_1\), procedure \(X\) is defined as \(C_2\); it can then be invoked as \(X\) from both \(C_1\) and \(C_2\). \(0\) indicates successful termination.

The interesting new part is the semantics of \(\eta\) thru \(\gamma\), which informally specifies that all communications in \(\eta\) occur through connector \(\gamma\). More precisely, in \(p(e) \rightarrow q.x\) thru \(\gamma\), process \(p\) (the sender) evaluates expression \(e\) and offers the resulting value to connector \(\gamma\). The connector eventually accepts and passes it to process \(q\) (the receiver), who stores it in its local variable \(x\).

The behaviour of \(p \rightarrow q[\ell]\) thru \(\gamma\) is similar, except that \(p\) offers a label instead of a normal value. Label selections do not change the state of the receiving process; their role in synthesizing process implementations is discussed in \(\S 5\).

The boxed terminals in Fig. 3 are runtime terms, meant to be used only in defining the formal semantics and not by programmers. They arise because connectors may have a multi-step semantics (they do not necessarily synchronise sends with receives). In particular, \(q.x?e\) is obtained when a communication \(p(e) \rightarrow q.x\) is partially executed, and \(p\) has already sent its value, but \(q\) has not yet received it; \(q[\ell]\) arises similarly—see Ex. 8 below.

For the semantics of \(\eta\) thru \(\gamma\) to be well-defined, \(\eta\) must satisfy two conditions. First, all interactions in \(\eta\) must have distinct receivers: if \(p_1(e_1) \rightarrow q_1.x_1 \in \eta\) and \(p_2(e_2) \rightarrow q_2.x_2 \in \eta\), then \(q_1 \neq q_2\). This ensures that the value received by a receiver is uniquely defined. Second, all sends must be consistent: if a process \(p\) is involved in multiple interactions in the same set, then they are either all communications of the same expression or all selections of the same label.

**Example 5** (Book sale). In the context of our running example, the following interaction sets are allowed (✓) or disallowed (✗) by our conditions for distinct receivers and consistent sends.

- ✗ \{a\{money\} \rightarrow b, c\{book\} \rightarrow s\} – Alice sends money to the bank, while Carol sends a book to the shipper (distinct receivers; consistent sends).
- ✓ \{a\{money\} \rightarrow b, c\{money\} \rightarrow b\} – Both Alice and Carol send money to the bank, but the bank can receive only from one sender at a time (receivers are not distinct).
- ✗ \{c\{price\} \rightarrow a, c\{book\} \rightarrow s\} – Carol sends both the price to Alice and the book to the shipper, but Carol can send only one value at a time (sends are not consistent).
- ✓ \{a \rightarrow b\{ok\}, a \rightarrow c\{ok\}, a \rightarrow s\{ok\}\} – Alice sends label ok to Carol, the bank, and the shipper (distinct receivers; consistent sends).

**Example 6** (Book sale). Returning to our running example, suppose that Alice wants to buy the book titled Foo.

Let \(G\) denote the connector mapping in Ex. 7 let \(C’\) denote lines 2-3 in Ex. 4 let \(\sigma_0\) denote the initial choreography state function such that \(\sigma_0(a.title) = \"foo\"\), let \(\sigma_0 = \sigma_0[c.title \mapsto \"foo\"]\), and let \(A_0 = \{a2c \mapsto \langle 1, \{m \mapsto \bot\}\rangle\} \cup A_0^{\text{rest}}\) denote the initial automaton state function, where:

\[A_0^{\text{rest}} = \{c2a \mapsto \langle 1, \{m \mapsto \bot\}\rangle, a2cbs \mapsto \langle 1, \emptyset\rangle, ac2bs \mapsto \langle 1, \emptyset\rangle\}\]

Initially, thus, all connectors are in their initial state (state 1). Furthermore, connectors a2c and c2a have an empty memory cell (\(m \mapsto \bot\)); connectors a2cbs and ac2bs have no memory cells (memory snapshot \(\emptyset\)). By rule \(\{C\} Com\) (presented after this example), the choreography in Ex. 4...
reduces under \( \mathcal{G} \) as follows:

\[
\begin{align*}
\text{\texttt{a(title) \to c thru a2c; C', \ \sigma_0, \ \{a2c \mapsto \{1, \{m \mapsto \bot\}\}\} \cup A_{\text{rest}}^c}}
\end{align*}
\]

\( \sim_{\mathcal{G}} \)

In state 1, according to \( \mathcal{G} \), connector \( a2c \) only has a transition that allows Alice to send (asynchronously) to Carol. By performing such a send, Alice enables the choreography to make a reduction, in which the first half of the communication completes; the (asynchronous) receive remains. In the same step, \( a2c \) moves to state 2, and the value sent by Alice is stored in \( a2c \)'s internal memory cell (\( \text{\texttt{title evaluates to "foo", based on } \sigma_0} \)).

\[
\begin{align*}
\text{\texttt{c.title="foo" thru a2c; C', \ \sigma_0, \ \{a2c \mapsto \{2, \{m \mapsto \text{"foo"}\}\}\} \cup A_{\text{rest}}^c}}
\end{align*}
\]

\( \sim_{\mathcal{G}} \)

In state 2, according to \( \mathcal{G} \), connector \( a2c \) only has a transition that allows Carol to receive (asynchronously) from Alice. By performing such a receive, Carol enables the choreography to make a reduction in which the whole communication completes. In the same step, \( a2c \) moves to state 1 (the internal memory cell is not cleared).

\[
\begin{align*}
\emptyset \text{ thru a2c; C', \ \sigma_0, \ \{a2c \mapsto \{1, \{m \mapsto \text{"foo"}\}\}\} \cup A_{\text{rest}}^c}
\end{align*}
\]

4.1 Formal semantics

The semantics of CR is a reduction semantics parametrised over a connector mapping: a function \( \mathcal{G} \) from connector names to automata. Intuitively, \( \mathcal{G}(\gamma) \) denotes the automaton that models connector \( \gamma \) used in the choreography; the set \( P \) of ports in each \( \mathcal{G}(\gamma) \) is simply a one-to-one mapping (i.e., a renaming of) the set of processes that use the connector\(^2\).

**Example 7** (Book sale). The connector names in Ex. 6 are \( a2c, c2a, \text{ac2bs}, \) and \( \text{ac2bs} \). Thus, the requirements in Ex. 3 give rise to the following connector mapping:

\[
\begin{align*}
\{ \\
\text{a2c} \mapsto \text{Async1}[a/p_1, c/p_2], \\
\text{c2a} \mapsto \text{Async1}[c/p_1, a/p_2], \\
\text{ac2bs} \mapsto \text{SyncMulti3}[a/p_1, b/p_2, c/p_3, s/p_4], \\
\text{ac2bs} \mapsto \text{Barrier}[a/p_1, b/p_2, c/p_3, s/p_4]
\}
\end{align*}
\]

where \( \text{Async1}[a/p_1, c/p_2] \) denotes automaton \( \text{Async1} \) in Fig. 3 with \( a \) substituted for \( p_1 \), and \( c \) for \( p_2 \) (and likewise in the other mappings). Under this connector mapping, thus, Alice and Carol communicate via asynchronous channels (\( a2c \) and \( c2a \)) to exchange title and price; Alice, Carol, the bank, and the shipper communicate via synchronous multi-cast (\( \text{ac2bs} \)) to exchange ok or ko, and via barrier sends/receives (\( \text{ac2bs} \)) to exchange money and book.

**Remark 1.** The book sale scenario illustrates an important design decision, namely the separation between intention and realisation: a choreography defines what interactions are intended (e.g., communications of the money from Alice to the Bank and the book from Carol to the shipper), while the connectors define how these communications are realised (e.g., synchronously or asynchronously). As a result, every interaction has to be expressed in two places, serving two complementary purposes: as “specifications” in the choreography and as “implementations” in the connectors (automaton transitions). As usual, implementations should respect specifications; we address this in § 4.3.

The reduction relation for CR under a given \( \mathcal{G} \) is denoted as \( \sim_{\mathcal{G}} \); it ranges over triples \( C, \sigma, \mathcal{A} \), where \( C \) is a choreography, \( \sigma \) is a choreography state function (mapping each process to a mapping of its variables to values, i.e., \( \sigma(p.x) \) is the value stored at variable \( x \) in process \( p \)), and \( \mathcal{A} \) is an automaton state function (mapping each connector name \( \gamma \) in the domain of \( \mathcal{G} \) to a pair \( (s, \mu) \) of the state and memory snapshot of the automaton \( \mathcal{G}(\gamma) \)). Before introducing the formal rule for communications, we give an example that discusses the intuition.

**Example 8** (Book sale). Returning to our running example, suppose that Alice wants to buy the book titled Foo.

\(^2\)This means that each process can interact at most through one port in each automaton.
Let $\mathcal{G}$ denote the connector mapping in Ex. 7, let $C'$ denote lines 2-3 in Ex. 3, let $\sigma_0$ denote the initial choreography state function such that $\sigma_0(\text{a.title}) = "\text{foo}"$, let $\sigma'_0 = \sigma_0[\text{c.title} \mapsto "\text{foo}"]$, and let $A_0 = \{\text{a2c} : \{1, \{m \mapsto \bot\}\}\} \cup A_0^{\text{rest}}$ denote the initial automaton state function, where:

$$A_0^{\text{rest}} = \{\text{c2a} : \{1, \{m \mapsto \bot\}\}, \text{a2cbs} : \{1, \emptyset\}, \text{ac2bs} \mapsto \{1, \emptyset\}\}$$

Initially, thus, all connectors are in their initial state (state 1). Furthermore, connectors a2c and c2a have an empty memory cell ($m \mapsto \bot$); connectors a2cbs and ac2bs have no memory cells (memory snapshot $\emptyset$). By rule $[C|\text{Com}]$ (presented after this example), the choreography in Ex. 4 reduces under $\mathcal{G}$ as follows:

$$\text{a(title)} \rightarrow \text{c thrua2c; C'}, \quad \sigma_0, \quad \{\text{a2c} \mapsto \{1, \{m \mapsto \bot\}\}\} \cup A_0^{\text{rest}}$$

$$\text{c.title} = "\text{foo}" \rightarrow \text{thrua2c; C'}, \quad \sigma_0, \quad \{\text{a2c} \mapsto \{2, \{m \mapsto "\text{foo}"\}\}\} \cup A_0^{\text{rest}}$$

These intuitions are captured in the rule for communications $[C|\text{Com}]$, which is the key rule defining $\leadsto^\mathcal{G}$. We discuss this rule in detail.

$$\frac{\mathcal{A}(\gamma) = \langle s, \mu \rangle \quad \tilde{\eta}, \sigma, \mu \xrightarrow{\phi} \tilde{\eta}', \sigma', \mu' \quad s \xrightarrow{\gamma} s'}{\tilde{\eta} \xrightarrow{\text{thru}} \gamma; C, \sigma, \mathcal{A} \leadsto^\mathcal{G} \tilde{\eta}' \xrightarrow{\text{thru}} \gamma; C, \sigma', \mathcal{A}[\gamma \mapsto \langle s', \mu' \rangle]}{[C|\text{Com}]}$$

Rule $[C|\text{Com}]$ allows some of the communications in $\tilde{\eta}$ to reduce as long as the state of the automaton corresponding to connector $\gamma$ allows it. The rule reads: under $\mathcal{G}$, triple $\tilde{\eta} \xrightarrow{\text{thru}} \gamma; C, \sigma, \mathcal{A}$ can reduce if automaton $\mathcal{G}(\gamma)$ can fire a transition out of its current state that is compatible with the interactions specified in $\tilde{\eta}$.

More formally, the first premise of this rule retrieves the current state $s$ and memory snapshot $\mu$ of the connector $\gamma$ controlling the communication. In the second premise, the labelled reduction $\tilde{\eta}, \sigma, \mu \xrightarrow{\phi} \tilde{\eta}', \sigma', \mu'$ (defined below) states that reducing $\tilde{\eta}$ to $\tilde{\eta}'$ transforms the state of processes $\sigma$ into $\sigma'$ and the memory snapshot of the connector $\mu$ into $\mu'$. The label $\phi$ represents the actions executed in this reduction. The third premise checks that these actions are allowed by the automaton, by checking that $\phi$ labels an outgoing transition of $s$.

The rules defining labelled reductions $\tilde{\eta}, \sigma, \mu \xrightarrow{\phi} \tilde{\eta}', \sigma', \mu'$ are given in Fig. 3. They are obtained by considering the different possible cases for terms $\tilde{\eta}$ and matching them to appropriate constraints $\phi$. Let $e \xrightarrow{\sigma} v$ denote that expression $e$ evaluates to value $v$ under $\sigma$ (i.e., after substituting every free variable $x$ in $e$ by $\sigma(p.x)$). In rule $[C|\text{SyncVal}]$, an entire communication $p(e) \rightarrow q, x$ is executed in one step. Accordingly, the label $p \rightarrow q$ denotes that the automaton should support a synchronous communication between $p$ and $q$. The state of the receiver $q$ is updated with the value sent by $p$. Rule $[C|\text{SendVal}]$ applies in the case where the message from $p$ should be stored in a memory cell $m$ of the automaton (label $p \rightarrow m$). This is used for asynchronous communications, where the message is received later on by the receiver. In the reductum, we keep a runtime term signalling that the receiver is still waiting to receive the message $(q, x?v)$. This kind of runtime terms is handled by $[C|\text{RecvVal}]$, whose label specifies that $q$ should receive the message stored in some memory cell $m$. Its premise checks that the value $q$ is expecting to receive ($v$, defined in the choreography term) is the one stored in $m$ in Rules $[C|\text{SyncSel}]$, $[C|\text{SendSel}]$ and $[C|\text{RecvSel}]$.

---

In other words, the automaton delivers the messages as specified in the choreography.
Figure 4: Semantics of individual communications. The side condition \((\dagger)\) in rule \([C]Join\) reads: if memory cell \(m\) occurs in both \(\phi_1\) and \(\phi_2\), then it is not both written to in \(\phi_1\) and read from in \(\phi_2\).
which is necessary for correct interaction with rule procedure definitions, and close the reduction relation interfere. The most interesting rules deal with communications: ⌊

Example 10 (Book sale). We formally derive the reduction of line 4 in Ex. 4. Let \( G \) denote the connector mapping in Ex. 4 let \( \sigma \) denote a choreography state function such that \( \sigma(a, \text{money}) = 10 \) and \( \sigma(c, \text{book}) = \text{foo.pdf} \), let \( \sigma' = \sigma(b, \text{money} \rightarrow 10) \), let \( \sigma'' = \sigma(s, \text{book} \rightarrow \text{foo.pdf}) \), and let \( A = A^{\text{rest}} \cup \{ac2bs \rightarrow \{1, 0\}\} \) denote an automaton state function, for some \( A^{\text{rest}} \). We derive:

\[
A(\text{ac2bs}) = (1, \emptyset)\]

\[
\xrightarrow{\text{[C]SyncVal}} \begin{cases} a \rightarrow b, c \rightarrow s, \sigma, \emptyset \\ \sigma', \emptyset \end{cases} \]

\[
\xrightarrow{\text{[C]Join}} \begin{cases} a \rightarrow b, c \rightarrow s, \sigma, \emptyset \\ \sigma', \emptyset \end{cases} \]

\[
\xrightarrow{\text{[C]Com}} \begin{cases} a \rightarrow b, c \rightarrow s, \sigma, \emptyset \\ \sigma'', \emptyset \end{cases} \]

Rule \([C]\text{[Com]}\) is the only rule in the semantics that can cause a choreography to get stuck; we discuss this in more detail in the next section.

The remaining rules defining \( \rightarrow \) are standard from other choreography calculi (e.g., [22]), and they are given Fig. 6: they define reductions for conditionals, allow for reductions under procedure definitions, and close the reduction relation \( \rightarrow \) under the structural precongruence \( \equiv \). Rule \([C]\text{[Struct]}\) uses a structural precongruence that allows for actions to be swapped if they do not interfere. The most interesting rules deal with communications: \([C]\text{[Eta-Split]}\) and \([C]\text{[EtaEnd]}\), displayed in Fig. 5 (where \( pn(\eta) \) denotes the process names that occur in \( \eta \)). Rule \([C]\text{[Eta-Split]}\) allows interactions through the same connector to be joined in one \( \eta \) or split among several ones, which is necessary for correct interaction with rule \([C]\text{[Eta-Eta]}\). (See [21] for a similar discussion: this rule is needed whenever one \( \eta \) can specify several communications. Ex. 11 below illustrates this interplay in our language.) Rule \([C]\text{[EtaEnd]}\) removes completed interactions from the head of a choreography. The remaining rules defining \( \preceq \) concern recursion unfolding and swapping of conditionals; these rules are straightforward adaptations of those in [22].
\[
\begin{align*}
i &= 1 \text{ if } e \Downarrow_{g}^\sigma \text{ true, } i = 2 \text{ otherwise} \quad \text{[C]Cond} \\
\text{if } p \cdot e \text{ then } C_1 \text{ else } C_2, \sigma, A \leadsto_{g} C_i, \sigma, A \quad \text{[C]Cond} \\
C_1 \preceq C_2, C_2, \sigma, A \leadsto_{g} C_2', \sigma', A', C_2' \preceq C_1' \quad \text{[C]Struct} \\
C_1, \sigma, A \leadsto_{g} C_1', \sigma', A' \quad \text{[C]Ctx} \\
def X = C_2 \text{ in } C_1, \sigma, A \leadsto_{g} C_i, \sigma, A \quad \text{[C]Ctx}
\end{align*}
\]

Figure 6: Cho-Reo-graphy, semantics.

**Example 11.** Consider the following choreography, where for simplicity we abstract from the actual values being communicated.

\[
C \equiv \{p \rightarrow q \mid \gamma; \{p \rightarrow q, t \rightarrow v\} \text{ thru } \gamma'\}
\]

and assume that both \(G(\gamma)\) and \(G(\gamma')\) allow the interactions between the processes they connect to occur independently and in any order. Then it is actually possible that the communication between \(t\) and \(v\) is the first one to take place. In order for our choreography language to allow this behaviour, we need to use both \([C|Eta-Split]\) and \([C|Eta-Eta]\) to exchange actions, as follows.

\[
C \equiv \{p \rightarrow q, r \rightarrow s\} \text{ thru } \gamma; \{p \rightarrow q, t \rightarrow v\} \text{ thru } \gamma' \\
\equiv \{p \rightarrow q, r \rightarrow s\} \text{ thru } \gamma; t \rightarrow v \text{ thru } \gamma'; p \rightarrow q \text{ thru } \gamma' \quad \text{by } [C|Eta-Split] \\
\equiv t \rightarrow v \text{ thru } \gamma'; \{p \rightarrow q, r \rightarrow s\} \text{ thru } \gamma; p \rightarrow q \text{ thru } \gamma' \quad \text{by } [C|Eta-Eta]
\]

### 4.2 Flexibility

An immediate advantage of CR is that different communication semantics can freely be mixed in the same choreography. A second advantage is that CR enables programmers to change the semantics of a choreography modularly, by altering the behaviour of the connectors through which the processes interact with each other, *without* the need to change the choreography itself.

**Example 12 (Book sale).** The original book sale scenario (Ex. 1 and 2) requires Alice and Carol to send money and book to the bank and the shipper synchronously, as they initially do not trust each other. Now, suppose Alice and Carol establish mutual trust after successfully completing a number of book sales, such that their communications with the bank and the shipper no longer need to occur synchronously. Instead of redeveloping the choreography from scratch, we need to redefine only the connector mapping \(G\) in Ex. 7, as follows:

\[
G := G[ac2bs \mapsto \begin{array}{c}
0 \rightarrow 1 \\
1 \rightarrow 0
\end{array}]
\]

Thus, we updated the mapping for \(ac2bs\); for all other connectors, the mapping remains the same as in Ex. 7. The new automaton for \(ac2bs\) allows either a communication between Alice and the bank, asynchronously followed by a communication between Carol and the shipper (via state 2), or the same two communications in the reverse order (via state 2).

Redefining the connector mapping for \(ac2bs\) is the only change we need to make: the choreography itself is exactly the same as in Ex. 4. This means that also the first reductions remain exactly the same as in Ex. 5 and 6. By contrast, the reduction in Ex. 10 is no longer valid, as it

\[\text{The communications between Alice and the bank, and between Carol and the shipper, are synchronous in this automaton. We can easily make those communications asynchronous as well, but we skip this here to save space (the automaton gets larger).}\]
This connector mapping is still compatible with the choreography in Ex. 4. Thus, as intended, our reduction rules let us derive two separate reductions with one communication each (first Alice and the bank, then Carol and the shipper) instead of one reduction with two communications (Ex. 12). Similarly, we can derive two separate reductions whereby Carol and the shipper communicate first, followed by Alice and the bank.

Example 13. The previous example works also “in the opposite direction”, from a trusting Alice and Carol (using connector mapping $\mathcal{G}$ in Ex. 12) to cautious ones (using connector mapping $\mathcal{G}$ in Ex. 4).

Our choreography is also compatible with the case where we have a trusting Alice and a cautious Carol, who only sends the book after receiving payment. A connector mapping that implements this behaviour is the following.

$$\mathcal{G} := \mathcal{G}[ac2bs \mapsto \begin{array}{c} \mathcal{C}\text{SyncVal} \\ \{c(\text{book}) \rightarrow s\} \text{ thru ac2bs } 1, \sigma', A' \end{array}]$$

This connector mapping is still compatible with the choreography in Ex. 4. The symmetric case where Carol is trusting and Alice is cautious is similar.

Example 14. Note that the changes to connector mappings in Ex. 12 and 13 would still be possible if the programmer had written, e.g.,

$$\ldots; \{a(\text{money}) \rightarrow b\} \text{ thru ac2bs; } \{c(\text{book}) \rightarrow s\} \text{ thru ac2bs; } 0$$

instead of the choreography in Ex. 4. Indeed, these two choreographies are equivalent due to the congruence rule $[\mathcal{C}\text{Eta-Split}]$, and thus the sets of connectors that are compatible with each of them are the same.

This might be surprising at first, but it fits with the view of choreographies as global specifications of independent processes. Specifically in this case, no choreography can impose a causal dependency between the two communications $a(\text{money}) \rightarrow b$ and $c(\text{book}) \rightarrow s$ unless it includes an additional communication in the middle involving a process that can observe both. The lack of causal dependencies in this example thus leaves the connector for $ac2bs$ free to decide the order in which the interactions are performed.

4.3 Deadlock-freedom

Rule $[\mathcal{C}\text{Con}]$ is the only rule in the semantics that can cause a choreography to get stuck: in choreography $\eta$ thru $\gamma; C$, there can be incompatibilities between the communications allowed by connector $\gamma$ and the intended communications in $\eta$, causing none of the communications in $\eta$ to be permitted by $\gamma$. In this case, we say that $\gamma$ does not respect the choreography.

5A choreography expresses the intentions of the programmer. Although she may instantiate connectors however she likes, we assume they do not violate her intentions.
can happen because the transitions available at the current state \( s \) either require communications between processes not involved in \( \eta \) or because there is an incompatibility with messages in transit (the premises of rules \([\mathrm{C}]\mathrm{RecvVal}\) and \([\mathrm{C}]\mathrm{RecvSel}\)).

In more detail, the first premise in rule \([\mathrm{C}]\mathrm{Com}\) always holds (assuming \( A \) is defined for all connector names in \( \eta\mathrm{thru}\gamma; C \)). This gives us unique bindings for \( s \) and \( \mu \). The third premise in \([\mathrm{C}]\mathrm{Com}\) is also always true (assuming every state of a connector has at least one outgoing transition; this can trivially be checked). For every outgoing transition of \( s \), this gives us bindings for \( \phi \) and \( s' \). Now, the choreography gets stuck if for each of those bindings, the second premise in \([\mathrm{C}]\mathrm{Com}\) is false. This can happen in two cases: either \( \tilde{\eta}, \sigma, \mu \nvdash \eta', \sigma', \mu' \) can be derived (using the rules in Fig. 4) and \( \phi \neq \varphi \) for every derivation, or \( \tilde{\eta}, \sigma, \mu \nvdash \tilde{\eta}', \sigma', \mu' \) cannot be derived at all. The former happens if every \( \varphi \) contains different processes than \( \phi \) (see Ex. 15), or the same processes but in different send/receive pairs (see Ex. 16); the latter happens if \( \tilde{\eta} \) contains only asynchronous receives for which rules \([\mathrm{C}]\mathrm{RecvVal}\) and \([\mathrm{C}]\mathrm{RecvSel}\) in Fig. 4 are inapplicable (see Ex. 17).

**Example 15** (Book sale). Suppose we mistakenly redefine the connector mapping \( G \) in Ex. 7 as follows (cf. Ex. 14 i.e., the boxed label is wrong):

\[
G := G[\text{ac2bs} \mapsto \begin{array}{cc}
\text{c} & \rightarrow \text{s} \\
\text{a} & \rightarrow \text{b} \\
\text{a} & \rightarrow \text{c}
\end{array}]
\]

Thus, connector ac2bs initially allows a communication either between Alice and the bank, or between Carol and the shipper, as in Ex. 14. But in the former case, ac2bs subsequently allows a second communication between Alice and the bank (instead of between Carol and the shipper).

The first derivation in Ex. 14 is still valid, but the second derivation is not: rule \([\mathrm{C}]\mathrm{SyncVal}\) is still applied to derive \(\{c(\text{book}) \rightarrow s\}, \sigma', \emptyset \nvdash \eta', \sigma', \emptyset\) to fulfill the second premise of rule \([\mathrm{C}]\mathrm{Com}\), but ac2bs has no transition in state 2 labelled with \(c \rightarrow s\). As there are no other derivations to fulfill the second premise of rule \([\mathrm{C}]\mathrm{Com}\), the choreography gets stuck.

**Example 16** (Book sale). Suppose we mistakenly redefine connector mapping \( G \) in Ex. 7 as follows (i.e., the boxed process names are wrong/swapped):

\[
G := G[\text{ac2bs} \mapsto \text{Barrier}[a/p_1, b/p_2, c/p_3, d/p_4]]
\]

Formally, automaton \(G(\text{ac2bs})\) has the following transition: \(1 \xrightarrow{a \wedge b \wedge c} \text{ac2bs}\). Thus, connector ac2bs allows communications between Alice and the shipper (instead of between the bank, and between Carol and the bank (instead of the shipper).

The derivation in Ex. 14 is no longer valid: rules \([\mathrm{C}]\mathrm{SyncVal}\) and \([\mathrm{C}]\mathrm{Join}\) are still applied to derive \(\{a(\text{money}) \rightarrow b, c(\text{book}) \rightarrow s\}, \sigma, \emptyset \nvdash \eta', \sigma', \emptyset \nvdash \eta, \sigma', \emptyset\) to fulfill the second premise of rule \([\mathrm{C}]\mathrm{Com}\), but ac2bs has no transition labelled with \(a \rightarrow b \wedge c \rightarrow s\). As there are no other derivations to fulfill the second premise of rule \([\mathrm{C}]\mathrm{Com}\), the choreography gets stuck.

**Example 17** (Book sale). Suppose we mistakenly redefine the connector mapping \( G \) in Ex. 7 as follows (i.e., the boxed process names are wrong/swapped):

\[
G := G[\text{ac2bs} \mapsto \begin{array}{ccc}
\text{a} & \rightarrow m_1 \\
\text{m}_2 & \rightarrow f \\
\text{m}_1 & \rightarrow s
\end{array}]
\]

Thus, connector ac2bs allows an asynchronous send by Alice, followed by an asynchronous send by Carol, followed by an asynchronous receive by the shipper, followed by an asynchronous receive by the bank. However, the shipper receives the value sent by Alice (instead of Carol), while the bank receives the value sent by Carol (instead of Alice).

Let \( \sigma, \sigma', \sigma'' \), and \( A \) be defined as in Ex. 14. Furthermore, let \( \mu = \{m_1 \mapsto \perp, m_2 \mapsto \perp\} \), let \( \mu' = \{m_1 \mapsto 10, m_2 \mapsto \perp\} \), and let \( \mu'' = \{m_1 \mapsto 10, m_2 \mapsto \text{foo.pdf}\} \). The following reductions
can be derived using rule $[C|Com]$:

\begin{align*}
\{a\langle \text{money} \rangle \rightarrow b, c\langle \text{book} \rangle \rightarrow s\} & \quad \text{thru}\ ac2bs; 0, \ \sigma, \ A[ac2bs \mapsto \langle 1, \mu \rangle] \\
\sim_{\varnothing} & \quad \{b\langle \text{money}\$10 \rangle, c\langle \text{book} \rangle \rightarrow s\} \quad \text{thru}\ ac2bs; 0, \ \sigma, \ A[ac2bs \mapsto \langle 2, \mu' \rangle] \\
\sim_{\varnothing} & \quad \{b\langle \text{money}\$10, s\langle \text{book} \rangle\langle \text{foo.pdf} \rangle \rightarrow s\} \quad \text{thru}\ ac2bs; 0, \ \sigma, \ A[ac2bs \mapsto \langle 3, \mu'' \rangle]
\end{align*}

At this point, the choreography gets stuck: there are no derivations to fulfill the second premise $\{C|Join\}$. To see this, note that only rule $[C|RecvVal]$ may be applicable (together with rule $[C|Join]$), but $\mu''(m_2) = \langle \text{foo.pdf} \rangle$, whereas the choreography states $b\langle \text{money}\$10 \rangle$. In other words, the choreography expects the bank to receive $\$10$, but connector $ac2bs$ allows the bank only to receive $\text{foo.pdf}$, out of memory cell $m_2$.

None of these situations can arise in existing choreography models, where all (a)synchronous channels are guaranteed to respect their choreographies, because the choreography syntax is carefully tuned to the fixed communication semantics of these channels. In CR, we have no fixed communication semantics: the fact that connectors in CR may not respect their choreography is, thus, a consequence of the added expressiveness and flexibility CR provides.

We proceed with a more formal account.

**Definition 2.** Connector mapping $\mathcal{G}$ in automaton state function $A$ respects choreography $C$ if: for every $\sigma, \eta, \gamma, \sigma'$ and $A'$, if $C, A \sim_{\varnothing} \eta \text{ thru} \gamma; C', \sigma', A'$, then there exist $\sigma''$ and $A''$ such that $\eta \text{ thru} \gamma; C', \sigma', A' \sim_{\varnothing} C'', \sigma'', A''$. Connector mapping $\mathcal{G}$ respects choreography $C$ if $\mathcal{G}$ respects $C$ in initial automaton state function $A_0$ (which assigns each automaton to its initial state and memory snapshot, as specified in $\mathcal{G}$).

**Definition 3.** $C, \sigma, A_0$ is deadlock-free for every $\sigma$ iff $\mathcal{G}$ respects $C$.

We can show respectfulness/deadlock-freedom to be undecidable using a classical recursion-theoretic argument.

**Theorem 1** (Undecidability of Deadlock-Freedom). In general, it is undecidable whether a connector mapping $\mathcal{G}$ respects a choreography $C$.

**Proof.** Let $\eta$ be a communication action that does not respect $A$, and assume that connector $\gamma$ has synchronous links $p2q$ and $q2p$, from $p$ to $q$ and conversely (e.g., $\text{Sync}$ in Fig. 1). Synchronous links are always enabled and do not change $A$.

Let $f$ be a total function implemented at $p$ and consider the choreography

\[ C \equiv \text{def } X = q(y) \rightarrow p.x \text{ thru} \gamma; \]

\[ \text{if } (p.f(x) = 0) \text{ then } p(x + 1) \rightarrow q.y \text{ thru} \gamma; X \]

\[ \text{else } \eta \text{ thru} \gamma \]

\[ \text{in } p(0) \rightarrow q.y \text{ thru} \gamma; X \]

In this choreography, $q$ sequentially sends the natural numbers to $p$, which applies $f$ to its input and proceeds if the result is 0. If $q$ sends a value where $f$ is not 0, the choreography attempts to perform $\eta$ and deadlocks. Then $C$ respects $A$ iff $f$ is constantly equal to 0, which by Rice’s theorem is not decidable.

**Remark 2.** Undecidability of deadlock-freedom arises because of the new ways in which a choreography and connectors can affect each other, which did not exist in previous work. Specifically, deadlock occurs if a connector’s current state has no transitions for the interactions in the choreography’s current $\eta$. In previous models, this can never happen, since the choreography syntax matches the hard-wired communication semantics by definition. Violation of respectfulness is, thus, a unique byproduct of allowing custom communication semantics, through connectors. Concretely, the proof of Thm. 1 relies on the existence of a communication action that does not respect $\mathcal{G}$, which does not exist in previous models.
We can approximate respectfulness by a decidable relation, called compatibility, essentially by abstracting away from data. The key point is that a conditional satisfies compatibility only if both its branches satisfy compatibility.

**Definition 4.** Let $C$ be a choreography, $G$ be a connector mapping, and $A$ be an automaton state function. We say that $C$ and $G$ are compatible by automaton state function $A$ if $\vdash^A C$, where the relation $\vdash$ is defined by the rules in Fig. 7. We say $C$ and $G$ are compatible, written $\vdash C$, if $\vdash^A C$ with $A_0$ as in Definition 3.

Relation $\vdash$ uses a context $\Gamma$, defined inductively as $\Gamma ::= (X : A), \Gamma \mid \cdot$, and an abstraction of the labelled reductions for communications from Fig. 4 $\tilde{\eta}, \tilde{\mu} \xrightarrow{\circ} \tilde{\eta}', \tilde{\mu}'$. The latter models a symbolic execution of communications; it is defined as in Fig. 4 with two differences: (i) $\sigma$ is removed from the domain of the reduction and (ii) in rule $\{C|\text{SendVal}\}$, $v$ is a fresh token.

We comment on this relation. Intuitively, $X : A \in \Gamma$ indicates that procedure $X$ can be called only whenever the automata have current states $A$; this is encoded in rules $\{C|\text{Def}\}$ and $\{C|\text{Call}\}$ (in the former rule, a unique automaton state function $A_X$ is stipulated; in the latter rule, it is checked against the current automaton state function $A$). Together with the fact that we allow actions to be swapped in rule $\{C|\text{Com}\}$, but not recursive calls to be unfolded, this means that the recursive structures of the choreography and the automata in the connector mapping must be similar (i.e., the loops in the automata must match the recursions in the choreography). Furthermore, in order for these rules to ensure respectfulness, the transition relation in the automaton also needs to be confluent (cf. Thm. 2).

**Remark 3.** Compatibility can become more robust/modular by disregarding connectors not occurring in procedure bodies in $\{C|\text{Def}\}$. We chose the current formulation for simplicity.

**Example 18** (Book sale). We illustrate how compatibility works in the context of our running example by revisiting choreography $\{a(money) \to b, c(book) \to s\}$ thru ac2bs: 0 with five different connector mappings from previous examples.

✓ Let $G$ and $A$ be defined as in Ex. 10. Using Fig. 7 we derive:

\[\vdash^A \{a(money) \to b, c(book) \to s\} \text{ thru ac2bs: 0} \quad \{C|\text{Def}\}\]

\[\vdash^A \{a(money) \to b, c(book) \to s\} \text{ thru ac2bs: 0} \quad \{C|\text{Com}\}\]

\[\vdash^A \{a(money) \to b, c(book) \to s\} \text{ thru ac2bs: 0} \quad \{C|\text{Done}\}\]

\[\vdash^A \{a(money) \to b, c(book) \to s\} \text{ thru ac2bs: 0} \quad \{C|\text{Nul}\}\]

\[\vdash^A \{a(money) \to b, c(book) \to s\} \text{ thru ac2bs: 0} \quad \{C|\text{Cond}\}\]
Thus, the choreography and the connector mapping are compatible. Corollary 1 below implies that the connector mapping respects the choreography.

✓ Let $G$ and $A$ be defined as in Ex. 17. Furthermore, let $A_2' = A[ac2bs \mapsto (2, \emptyset)]$, let $A_2'' = A[ac2bs \mapsto (2, \emptyset)]$, and let $A'' = A_2'[ac2bs \mapsto (1, \emptyset)] = A_2''[ac2bs \mapsto (1, \emptyset)] = A$. Using Fig. 7 we derive:

\[
\frac{\vdash_{A_2'} \emptyset \text{thru} \ ac2bs; 0}{A(2, \emptyset)} \quad \frac{\vdash_{A_2''} \emptyset \text{thru} \ ac2bs; 0}{A(2, \emptyset)}
\]

The bottom application of rule $[CC]\text{Com}$ requires two subderivations: one to cover the case where connector $ac2bs$ makes a transition to state $2$ (left subderivation), and another to cover the case where $ac2bs$ makes a transition to state $2$ (right subderivation). In both cases, we have compatibility.

Thus, the choreography and the connector mapping are compatible. Corollary 1 below implies that the connector mapping respects the choreography.

✗ Let $G$ and $A$ be defined as in Ex. 17. Furthermore, let $A_2' = A[ac2bs \mapsto (2, \emptyset)]$, and let $A_2'' = A[ac2bs \mapsto (2, \emptyset)]$. Using Fig. 7 we attempt:

\[
\frac{\vdash_{A_2'} \emptyset \text{thru} \ ac2bs; 0}{A(2, \emptyset)} \quad \frac{\vdash_{A_2''} \emptyset \text{thru} \ ac2bs; 0}{A(2, \emptyset)}
\]

This attempted derivation is the same as in the second $✓$-item, except the subderivation inside the box has become invalid: under our current $G$, connector $ac2bs$ in state $2$ has no transition labelled with $c \rightarrow s$.

Thus, the choreography and the connector mapping are incompatible. In fact, in this case, the choreography may deadlock under $G$.

✗ Let $G$ and $A$ be defined as in Ex. 17. Using Fig. 7 we attempt:

\[
\frac{\vdash_{A_2'} \emptyset \text{thru} \ ac2bs; 0}{A(2, \emptyset)} \quad \frac{\vdash_{A_2''} \emptyset \text{thru} \ ac2bs; 0}{A(2, \emptyset)}
\]

This attempted derivation is the same as in the first $✓$-item, except the subderivation inside the box has become invalid: under our current $G$, connector $ac2bs$ in state $1$ has no transition labelled with $a \rightarrow b$.

Thus, the choreography and the connector mapping are incompatible. In fact, in this case, the choreography deadlocks under $G$.

✗ Let $G$ and $A$ be defined as in Ex. 17. Furthermore, let $\diamond$ and $\heartsuit$ denote two fresh token values, let $\mu = \{m_1 \mapsto \bot, m_2 \mapsto \bot\}$, let $\mu' = \{m_1 \mapsto \diamond, m_2 \mapsto \bot\}$, let $\mu'' = \{m_1 \mapsto \diamond, m_2 \mapsto \heartsuit\}$, let
\[ A' = A[ac2bs \mapsto (2, \mu')] \] and let \[ A'' = A[ac2bs \mapsto (2, \mu'')] \]. Using Fig. 7, we attempt:

\[
\begin{align*}
A''(ac2bs) = (3, \mu'') \\
\vdash_{A''} \{ b.money? \diamond, a.book? \lozenge \} \text{ thru ac2bs; 0} & \quad [CC|Com]
\end{align*}
\]

\[
\begin{align*}
A'(ac2bs) = (2, \mu') \\
\vdash_{A'} \{ b.money? \diamond, a.book? \lozenge \} \text{ thru ac2bs; 0} & \quad [CC|Com]
\end{align*}
\]

\[
\begin{align*}
A(ac2bs) = (1, \mu) \\
\vdash_{A} \{ a\text{money?} \rightarrow b, c\text{(book)} \rightarrow s \} \text{ thru ac2bs; 0} & \quad [CC|Com]
\end{align*}
\]

This attempted derivation fails, because the intended subderivation inside box (the receive of \( \diamond \), followed by the receive of \( \lozenge \)) is invalid: under our current \( G \), connector \( ac2bs \) in state 3 has no transition labelled with \( m_1 \rightarrow b \).

Thus, the choreography and the connector mapping are incompatible. In fact, in this case, the choreography deadlocks under \( G \). \( \square \)

However, the restriction that a choreography and the automata in its connector mapping must have similar recursive structures (for them to be judged compatible) implies there exist connector mappings that respect their choreographies, but that cannot be shown to do so by means of the compatibility relation – which is unavoidable in view of our undecidability result. We illustrate this by some examples.

**Example 19.** Let \( C \) be the simple choreography:

\[
def X = p \rightarrow q \text{ thru } \gamma; p \rightarrow q \text{ thru } \gamma; r \rightarrow s \text{ thru } \gamma; X \text{ in } X
\]

and \( G(\gamma) \) a connector that allows communications from \( p \) to \( q \) to occur simultaneously with communications from \( r \) to \( s \) (e.g., Barrier in Fig. 7). Then \( C \) is deadlock-free, since structural precongruence allows the second communication from \( p \) to \( q \) to be “delayed” and the communication from \( r \) to \( s \) to be “pushed forward”:

\[
\begin{align*}
p \rightarrow q \text{ thru } \gamma; p \rightarrow q \text{ thru } \gamma; r \rightarrow s \text{ thru } \gamma & \\
\equiv p \rightarrow q \text{ thru } \gamma; [p \rightarrow q, r \rightarrow s] \text{ thru } \gamma & \quad \text{by } [C|\text{Eta-Split}]
\end{align*}
\]

\[
\begin{align*}
p \rightarrow q \text{ thru } \gamma; r \rightarrow s \text{ thru } \gamma; p \rightarrow q \text{ thru } \gamma & \\
\equiv p \rightarrow q \text{ thru } \gamma; r \rightarrow s \text{ thru } \gamma; p \rightarrow q \text{ thru } \gamma & \quad \text{by } [C|\text{Eta-Split}]
\end{align*}
\]

However, \( \nvdash^G \diamond \) \( C \), since the second communication from \( p \) to \( q \) in the body of \( X \) cannot be consumed without unfolding the definition of \( X \).

In this example, the second recursive structure of \( X \) (two communications from \( p \) to \( q \) and one from \( r \) to \( s \)) differs from the recursive structure of \( G(\gamma) \) (one communication between each pair of processes). \( \square \)

**Example 20.** Consider now the choreography \( C \) defined as

\[
def X = p \rightarrow q \text{ thru } \gamma; r \rightarrow s \text{ thru } \gamma; X \text{ in } X
\]

where \( G(\gamma) \) only allows communications from \( p \) to \( q \). Again \( C \) is deadlock-free, since structural precongruence allows the communications from \( r \) to \( s \) to be indefinitely postponed. However, \( \nvdash^G \diamond \) \( C \).

In general, compatibility ensures that the choreography is not only deadlock-free, but also that there is a correspondence between the recursive structure of the choreography and the recursive structure of the connectors: the connector must allow all interactions in the body of a definition to be executed before calling other procedures. \( \square \)

**Theorem 2** (Compatibility Preservation). Let \( C \) and \( C' \) be choreographies, \( G \) be a connector mapping, \( \sigma \) and \( \sigma' \) be choreography states, and \( A \) and \( A' \) be automaton state functions. If the transition relation in each \( G(\gamma) \) is confluent, \( \vdash^G_A C \) and \( C, \sigma, A \rightarrow G C', \sigma', A' \), then \( \vdash^{G'}_{A'} C' \).
Proof. Straightforward by case analysis on the reduction from \( C, \sigma, A \) to \( C', \sigma', A' \), using the fact that the automata are confluent (to make sure unfolding cannot add unwanted reductions), and therefore compatibility is preserved by structural precongruence.

The hypothesis that the transition relations of automata are confluent is required to make sure that unfolding cannot add unwanted reductions.

Corollary 1 (Soundness of Compatibility). Under the assumptions of Theorem 2 if \( \vdash^G_A C \), then \( G \) in \( A \) respects \( C \).

Proof. If \( C, \sigma, A \sim^G \eta; C', \sigma', A' \), then, by induction on the length of this sequence of reductions, we use Theorem 2 to show that \( \eta; C', \sigma', A' \sim^G C'', \sigma'', A'' \) for some \( A'' \) and \( \sigma'' \).

Furthermore, compatibility is decidable.

Theorem 3 (Decidability of Compatibility). There is an algorithm that, given \( C, G \) and \( A \), returns yes if \( \vdash^G_A C \) and no if \( \not\vdash^G_A C \) in time \( O(P \times \max(p)_A \times k \times (\sum d_A)^2k) \), where \( p_A \) is the maximum number of ports in any automaton, \( d_A \) is the maximum number of transitions from a state in each automaton, \( k \) is the maximum number of communication actions in any procedure definition (or main choreography) and \( P \) is the total number of procedure definitions (including the main choreography).

A simple finiteness argument suffices for establishing decidability of compatibility, since the number of automaton states is finite, the number of applicable rules at each step is finite, and all rules have a finite number of premises, and the size of the choreographies in the premises is always smaller than the size of the choreographies in the conclusions. Therefore, by non-deterministically guessing the types of all procedures, we can decide whether \( \vdash^G_A C \) or not. However, we provide a more intelligent proof that constructs the types for recursive definitions.

Proof of Theorem 3. We assume that every procedure defined in \( C \) is called at least once outside of its body.

The idea behind our algorithm is to construct a derivation for \( \vdash^G_A C \) by applying the rules in Fig. 7 bottom-up. When we meet a term of the form \( \text{def } X = C_2 \text{ in } C_1 \), we focus on \( C_1 \) first, and leave \( A_X \) (see rule \( \text{CC[Def]} \)) unspecified. We instantiate \( A_X \) later, when we meet \( X \) for the first time inside of \( C_1 \). More precisely:

1. Initialize a list \( L = \emptyset \). \( \vdash^G_A C \).
2. While \( L \) is not empty:
   (a) Remove the first pending judgement \( \Gamma \vdash^G_A C \) from \( L \).
   (b) If \( \Gamma = \emptyset \), proceed to the next iteration.
   (c) If \( \Gamma \) is of the form \( \text{if p.e then } C_1 \text{ else } C_2 \), then add \( \Gamma \vdash^G_A C_1 \) and \( \Gamma \vdash^G_A C_2 \) at the beginning of \( L \).
   (d) If \( \Gamma \) is of the form \( \text{def } X = C_2 \text{ in } C_1 \), then add \( \Gamma, (X : A_X) \vdash^G_A C_1 \) and \( \Gamma, (X : A_X) \vdash^G_A C_2 \), in this order, at the beginning of \( L \). Here, \( A_X \) is a unique variable representing an unknown state function.
   (e) If \( \Gamma \) is of the form \( X \), there are two cases. If \( \Gamma \) contains \( (X : A_X) \) with \( A_X \) instantiated, check whether \( A_X = A \); if so, proceed to the next iteration, otherwise return no. If \( \Gamma \) contains \( (X : A_X) \) with \( A_X \) uninstantiated, replace all occurrences of \( A_X \) in \( L \) by \( A \) and proceed to the next iteration.6
   (f) Otherwise, \( \Gamma \) is of the form \( \hat{\gamma} \text{ thru } \gamma; C' \). Consider all possible ways of rewriting \( C \) as \( \hat{\gamma}' \text{ thru } \gamma; C' \) by swapping independent actions, without unfolding recursive definitions. Let \( A(\gamma) = (s, \mu) \). For each such \( \hat{\gamma}' \), check whether \( \hat{\gamma}', \mu \rightarrow^{\gamma} \hat{\gamma}'' \), \( \mu' \) for some \( \phi \), and in the affirmative case compute \( s' \) such that \( s \rightarrow^{\gamma} s' \) and add \( \Gamma \vdash^G_{A[\gamma \rightarrow (s', \mu')]} \hat{\gamma}'' \text{ thru } \gamma; C' \) at the beginning of \( L \). If no such transitions exist, return no.

6Note that \( \Gamma \) must contain \( (X : A_X) \), otherwise the initial choreography is not well-formed.
Termination of this algorithm is straight-forward: the sum of the sizes of all the choreographies in $\mathcal{L}$ strictly decreases at each iteration, and each step terminates in finite time. (The size of a choreography is the number of nodes in its abstract syntax tree, except that $p(e) \rightarrow q.x$ and $p \rightarrow q[e]$ count as 2, while $q.x?v$ and $q[e]$ count as 1.) Soundness is immediate by observing that the judgements stored in $\mathcal{L}$ are exactly those that are necessary to construct a derivation of $\vdash^\mathcal{A} C$, since at each stage there is only one rule that can be applied to build such a derivation, and this rule is determined by the structure of $C$. If the algorithm returns \textsc{yes}, then a valid derivation for $\vdash^\mathcal{A} C$ can be built. If the algorithm returns \textsc{no} because of a mismatch between the state of the automata and a communication action (Step 2.f), then clearly $\not\vdash^\mathcal{A} C$. If the algorithm returns \textsc{no} because of an incompatibility between the state assigned to a procedure name $X$ in $\mathcal{G}$ and the state in the current judgement (Step 2.e), then this failure means that we constructed two judgements involving $X$ with different automaton state functions, which also implies that $\not\vdash^\mathcal{A} C$.

To obtain the complexity bound, perform step 2f as follows: consider all possible transitions ($\sum d_A$) and check which ones are enabled (naively: go through the current choreography and check each transition, which yields $\max(p)\times k$). Each transition consumes at least half an interaction, so this can be repeated $\leq 2k$ times for each procedure definition.

Since automata are deterministic, $d_A < 2^{p_A}$ (each subset of ports determines at most one transition), and $p_A$ is at most the number of processes in the choreography. Thus, this upper bound can be stated independently of the automata considered.

Although this complexity is high, we believe that compatibility checking is feasible in practice. The bound is an over-approximation, since choreographies typically contain many causal dependencies among communications that reduce non-determinism (as in our examples). Previous works on choreographies proposed algorithms with even worse worst-case complexity, but feasible in practice for the same reason.

**Theorem 4** (Progress). Let $C$ be a choreography, $\mathcal{G}$ be a connector mapping, $\sigma$ be a choreography state and $A$ be an automaton state function such that $\vdash^\mathcal{A} C$. Then, either $C \preceq 0$ ($C$ has terminated) or there exist $C'$, $\sigma'$ and $A'$ such that $C, \sigma, A \leadsto^\mathcal{G} C', \sigma', A'$.

**Proof.** If $C \not\preceq 0$, then $C$ is of the form $\bar{\eta}$ thru $\gamma$; $C'$ or if $p.e$ then $C_1$ else $C_2$, eventually inside some recursive definitions. In the latter case, $C$ can always reduce; in the former case, compatibility guarantees that $C$ can reduce.

**Theorem 5** (Deadlock-Freedom by Design). Let $C$ be a choreography, $\sigma$ be a choreography state function, and $A$ be an automaton state function. If $\vdash^\mathcal{A} C$ and $C, \sigma, A \leadsto^\mathcal{G} C', \sigma', A'$, then either $C' \preceq 0$ or there exist $C''$, $\sigma''$ and $A''$ such that $C', \sigma', A' \leadsto^\mathcal{G} C'', \sigma'', A''$.

**Proof (sketch).** From Theorem\[4\] if $C \not\preceq 0$, then by Theorem\[2\] we also have that $\vdash^\mathcal{A} C'$ whenever $C, \sigma, A \leadsto^\mathcal{G} C', \sigma', A'$. The thesis then follows by induction.

5 Connected Processes

CR shows how choreographies can be combined with connectors, but it does not indicate how we can obtain executable implementations. The missing link is determining how a choreography can be compiled to terms representing executable processes that communicate through connectors. We address this aspect by presenting a process calculus based on standard I/O actions and a translation (compilation procedure) from CR to this calculus.

5.1 Syntax and semantics

We define Connected Processes (CP), the process calculus to represent concrete implementations of choreographies. The syntax of CP is given in Fig.\[8\] A network $N$ is a parallel composition of
whose rules are given in Fig. 9. This rule is reminiscent of rule \(\ell\).

| if \(e\) then \(B_1\) else \(B_2\) | def \(X = B_2\) in \(B_1\) | \(X\) | \(0\)

\[ B ::= \text{o(e); B} \mid i?x; B \mid o \oplus \ell; B \mid i & \{\ell_i : B_i\}_{i \in I} \]

\[ N, M ::= p \triangleright p B \mid (N | M) | 0 \]

**Figure 8:** Connected Processes, Syntax.

\[ e \triangleright^n v \]

\[ p \triangleright_p o(e); B_p \mid q \triangleright_q i?x; B_q; \mu \xrightarrow{\ell \rightarrow v + 1} p \triangleright_p B_p \mid q \triangleright_q p[x \rightarrow v] B_q, \mu \]

**[CP|SyncVal]**

\[ e \triangleright^n v \]

\[ p \triangleright_p o(e); B, \mu \xrightarrow{p \rightarrow m + 1} p \triangleright_p B, \mu; m \mapsto v \]

**[CP|SendVal]**

\[ q \triangleright_q i?x; B, \mu \xrightarrow{m \rightarrow v} q \triangleright_q [x \rightarrow v] B, \mu \]

**[CP|RecvVal]**

\[ j \in I \]

\[ p \triangleright_p o \oplus \ell_j; B \mid q \triangleright_q i \& \{\ell_i : B_i\}_{i \in I}, \mu \xrightarrow{p \rightarrow q + 1} p \triangleright_p B \mid q \triangleright_q p B_j, \mu \]

**[CP|SyncSel]**

\[ p \triangleright_p o \oplus \ell_j; B, \mu \xrightarrow{p \rightarrow q + 1} p \triangleright_p B, \mu \]

**[CP|SendSel]**

\[ q \triangleright_q i \& \{\ell_i : B_i\}_{i \in I}, \mu \xrightarrow{m \rightarrow v} q \triangleright_q B_j, \mu \]

**[CP|RecvSel]**

\[ N_1, \mu \xrightarrow{\ell_j} N_1', \mu' \]

\[ N_2, \mu' \xrightarrow{\phi_j} N_2', \mu'' \]

\[(N_1 \mid N_2), \mu \xrightarrow{\phi_1 \cup \phi_2} (N_1' \mid N_2'), \sigma'', \mu'' \]

**[CP|Join]**

**Figure 9:** Semantics of communications (process level). Side condition (\(\dagger\)) in [CP|Join] is the same as in [C|Join].

Processes. A process is written \(p \triangleright p B\), where \(p\) is its identifier, \(\rho\) its state (mapping variable names to values), and \(B\) its behaviour. Behaviours correspond to local views of choreography interactions. Procedure definitions and calls, conditionals, and termination (\(0\)) follow the same ideas as in CR. Communication actions implement the local behaviour of each process in a choreography interaction: sending a value through an output port (\(o(e)\)); receiving a value through an input port (\(i?x\)); selecting a label through an output port (\(o \oplus \ell\)); and offering a choice on some labels through an input port (\(i \& \{\ell_i : B_i\}_{i \in I}\)).

The key difference from choreographies is that communications now refer to actual ports, instead of to connectors (we have no “thru \(\gamma\)” for communications in the process calculus). This reflects the principle that processes should not know how they are connected \([5, 29, 30]\).

The semantics of CP is parameterised on connectors represented as a set of automata \(\mathcal{C}\) that do not share any ports. Differently from the automata used in CR, the ones in \(\mathcal{C}\) use the names of the actual ports to which they are connected (and which are also used by the processes). Reductions in CP have the form \(N, A \triangleright_C N', A'\), where \(A\) maps each automaton in \(\mathcal{C}\) to a pair \(\langle s, \mu \rangle\) of its state \(s\) and memory snapshot \(\mu\). The key reduction rule of CP is the one for communications.

\[ a \in \mathcal{C} \quad A(a) = \langle s, \mu \rangle \quad N, \mu \xrightarrow{\phi} N', \mu' \quad s \xrightarrow{\phi} s' \]

**[CP|Com]**

This rule is reminiscent of rule [C|Com] for choreographies. In particular, it uses a similar auxiliary reduction relation on pairs of networks and memory snapshots (stated in premise \(N, \mu \xrightarrow{\phi} N', \mu'\)), whose rules are given in Fig. 9.

The remaining rules defining the semantics of CP are standard, and given in Fig. 10. Rule
5.2 EndPoint Projection (EPP)

The EPP of a choreography \( C \) from CR into CP follows the usual construction, but with an additional ingredient: we need to add port names associated with communication actions. This is visible in the rules for projecting the individual behaviour of each process (Fig. 11), notably in the rule for projecting conditionals.

**Remark 4.** In Fig. [11] \( o \) and \( i \) denote variables that range over concrete ports. Thus, a process \( p \) has output ports \( o_{\gamma_1}, o_{\gamma_2}, \ldots \), and input ports \( i_{\gamma_1}, i_{\gamma_2}, \ldots \), where \( o \) and \( i \) actually stand for \( o^p \) ("output port at \( p \)") and \( i^p \) ("input port at \( p \)"), while connector \( \gamma_1 \) knows output ports \( o_{\gamma_1}, o_{\gamma_1'}, \ldots \) and similarly for input ports.

The rule for projecting conditionals uses the standard partial merging operator \( \sqcup \), where \( B \sqcup B' \) is isomorphic to \( B' \) and \( B' \) up to branching with different labels (see [22] for details).

We now define the projection of \( C \) given a state \( \sigma \). As usual, this is the parallel composition of the projections of all processes in \( C \).

\[
[C, \sigma] = \prod_{p \in \mathit{pr}(C)} p \triangleright \rho_p [C]_p \quad \text{where } \rho_p(x) = \sigma(p, x) \text{ for each variable } x \text{ at } p
\]

\( C \) is projectable when \([C, \sigma]\) is defined for some \( \sigma \). This is equivalent to saying that \([C, \sigma]\) is defined for all \( \sigma \). We illustrate endpoint projection in Ex. [21] and [22].

**Example 21** (Book sale). Continuing with our running example, the choreography presented in...
Ex. 4 is projectable, and yields the following network of connected processes each state $\sigma$.

\[
\begin{align*}
\text{a} & \rightarrow p_0 \triangleleft \text{i}_{2cbs}\{\text{title}\}; \text{i}_{2cbs}\{\text{price}\}; \text{if} (\text{happy}) \text{then} (\text{o}_{ac2bs} \oplus \text{ok}) \oplus (\text{o}_{ac2bs} \{\text{money}\}; 0) \text{ else } (\text{o}_{ac2bs} \oplus \text{ok}) \oplus (\text{o}_{ac2bs} \{\text{money}\}; 0) \\
\text{b} & \rightarrow p_0 \triangleleft \text{i}_{2cbs}\{\text{ok} : \text{i}_{ac2bs}\{\text{money}\}; 0 \oplus \text{ok} : 0\} \oplus (\text{o}_{ac2bs} \{\text{book}\}; 0) \oplus (\text{o}_{ac2bs} \{\text{money}\}; 0) \\
\text{c} & \rightarrow p_0 \triangleleft \text{i}_{2cbs}\{\text{title}\}; \text{o}_{ac2bs} \{\text{price}\}; \text{i}_{ac2bs} \{\text{ok} : \text{i}_{ac2bs}\{\text{book}\}; 0 \oplus \text{ok} : 0\} \\
\text{s} & \rightarrow p_0 \triangleleft \text{i}_{2cbs}\{\text{ok} : \text{i}_{ac2bs}\{\text{book}\}; 0 \oplus \text{ok} : 0\}
\end{align*}
\]

Example 22. It is also worthwhile to note that the following choreographies are not congruent, and that they have different EPPs:

\[
\begin{align*}
C_1 &= p \rightarrow \{q, r|\ell\} \text{ thru } \gamma; 0 \\
C_2 &= p \rightarrow \{q|\ell\} \text{ thru } \gamma; p \rightarrow r|\ell\text{ thru } \gamma; 0
\end{align*}
\]

Choreography $C_1$ is syntactic sugar for $\{p \rightarrow q|\ell, p \rightarrow r|\ell\}$ thru $\gamma; 0$. The EPP to $p$, thus, consists of one send; connector $\gamma$ must subsequently ensure that label $\ell$ is replicated to, and received by, both $q$ and $r$ (i.e., formally, $\gamma$ must be compatible with $C_1$^{13}).

Choreography $C_2$, in contrast, is not congruent to $\{p \rightarrow q|\ell, p \rightarrow r|\ell\}$ thru $\gamma; 0$. Specifically, we cannot use rule $[C\text{\text{-}Eta-Split}]$ to merge the two interactions in $C_2$, because its disjointness premise does not hold ($p$ occurs in both interactions). Accordingly, the EPP of choreography $C_2$ on $p$ consists of two sends.

To state the operational correspondence between a choreography and its projection, we need to map the process names used as ports in a connector mapping $\mathcal{G}$ to the actual port names used in networks. We define $[\mathcal{G}]$ to be the set of all automata in the codomain of $\mathcal{G}$, where each output port $p$ in automaton $\mathcal{G}(\gamma)$ becomes $p.o_\gamma$ (and likewise for input ports). We define a similar function for automaton state function $\mathcal{A}$.

Theorem 6 (Operational Correspondence). Let $C$ be a projectable choreography. Then, for all $\sigma$, $\gamma$, and $\mathcal{A}$:

Completeness: If $C, \sigma, \mathcal{A} \sim_\mathcal{G} C', \sigma', \mathcal{A}'$, then $[C, \sigma], [\mathcal{A}] \sim_{[\mathcal{G}]} [C', \sigma'], [\mathcal{A}']$;

Soundness: If $[C, \sigma], [\mathcal{A}] \sim_{[\mathcal{G}]} N, \mathcal{A}'$, then $C, \sigma, \mathcal{A} \sim_\mathcal{G} C', \sigma', \mathcal{A}'$ for some $\sigma'$ and $\mathcal{A}'$ with $[C', \sigma'] \prec N$ and $[\mathcal{A}'] = \mathcal{A}'$.

In the soundness result, the pruning relation $\prec$ states that the processes in $N'$ may offer more branches than those present in $[C', \sigma']$, but these are never selected $[13] [17] [28] [25]$.

In particular, if $\vdash \mathcal{G} C$, then $[C, \sigma]$ is guaranteed to be deadlock-free when executed with all automata in $[\mathcal{G}]$ in their initial states.

Example 23 (Book sale). For any connector mapping $\mathcal{G}$, the process network in Ex. 27 operates under $[\mathcal{G}]$ exactly as the choreography in Ex. 4 under $\mathcal{G}$. In particular, if $\mathcal{G}$ respects the original choreography, then this implementation never deadlocks under $[\mathcal{G}]$.

6 Conclusions

Choreographic approaches to concurrent programming have been heavily investigated $[1, 28]$, but they typically adopt some fixed (and restrictive) communication semantics (like point-to-point synchronous). CR is the first model that modularly integrates choreographies with what runs “under the hood” of communications, allowing for user-defined communication semantics given as connectors. Thanks to compatibility (Definition 4), CR inherits the good properties of both Choreographic Programming and Exogenous Coordination. Thus, we have significantly extended the

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\footnote{This behaviour was the motivation for introducing multicasts as abbreviations: the notation in C$_1$ better conveys how communications really happen; EPP follows this intuition.}
applicability of choreographies: not only can we capture new kinds of behaviours in choreographies (like barriers, cf. Ex. 7, and alternators, cf. Ex. 13), but we can even define systems that integrate different parts with different communication semantics and check whether such integration will lead to deadlocks. This is essential in many real-world scenarios, where different components with different communication semantics are usually combined (e.g., some microservices in a distributed system may asynchronously exchange data to be used later in a synchronous multiparty transition, similarly to our example).

This work lays the foundations for applying the combined power of choreographies and connectors to the challenge of concurrent programming, in that CR contains all the necessary foundations to obtain a concrete implementation. The results in §5 specify how to use CR to obtain code in a process model supported by connectors. Thus, a natural next step will be to implement CR by combining implementations of processes generated from choreographies [19, 25] with distributed implementations of Reo connectors [31, 41, 42]. The main pieces exist; the main challenge lies in their effective composition, and CR is the first essential step towards this objective.

CR also provides a very explicit direction for future developments of this new combined research line: allowing for more kinds of connectors would make the model immediately more expressive. By relaxing the requirements we imposed on the automata in CR (see page 4), we can introduce non-deterministic communication semantics to choreographies, to cater to applications that require lossy channels and safe communication races. Likewise, a more fine-grained semantics that splits communications into two independent send and receive actions (similar to [24]) would enrich the class of behaviours that are captured.

We have followed the traditional approach of viewing choreographies as precise specifications of the intended interactions. However, it would be reasonable to allow choreographies to underspecify communications, such that the underlying connectors were allowed to exchange messages also to participants not defined in the choreography. For example, the semantics for the choreography term $p(e) \rightarrow q.x \text{thru} \gamma$ can allow $\gamma$ to send the message from $p$ to $q$ via an intermediate process $r$ that may perform additional actions (like logging the message, or sharing it through another connector). This generalisation can, in particular, provide a novel way for studying how choreographies can be applied to open-ended systems, where the processes projected from multiple choreographies execute in parallel and share connectors.

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