Transportation problem with interval-valued intuitionistic fuzzy sets: impact of a new ranking

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Abstract
To address uncertainty and hesitation of a real-life problem, interval-valued intuitionistic fuzzy sets (IVIFSs) have received increasing interest among researchers and industrialists. In this paper, we present an advanced illustration of IVIFSs using physical distancing during COVID-19 to understand the deep concept of IVIFSs. Due to special feature of an IVIFSs, it finds a better decision of a real-life problem having uncertainty and hesitation. Here some important arithmetic operations between two IVIFSs are also stated. Ranking of IVIFSs is a valuable tool and it is not easy to rank due to its ill-defined membership and non-membership degrees, and same difficulties arise in a wide variety of real-life problems. To tackle these difficulties, we introduce a new ranking function of IVIFSs, and it follows well to the law of trichotomy. And for its superiority, we compare it with some existing ranking functions by taking a suitable example. Furthermore, its applicability are tested on the basis of an IVIFSs. Further, it is very interesting to note that some unpredicted factors such as road condition, diesel prices, traffic condition and weather condition affect to the cost of transportation, and therefore, decision makers encounter uncertainty and hesitation to estimate cost of transportation. To resolve such issues, we consider transportation problem with IVIFSs parameters, and for its solution, a simple computational method is developed and illustrated.

Keywords Law of trichotomy · Intuitionistic fuzzy sets · Interval-valued intuitionistic fuzzy sets · Transportation problem · Uncertainty

1 Introduction
At present, the role of fuzzy optimization techniques in engineering and management applications has attracted massive attention because of their high accuracy, efficiency and adaptability that provides high-quality realistic results. Fuzzy optimization techniques have been highly explored in health, engineering and industrial sectors. Initially, the concept of mathematical logic was initiated by a greatest philosopher Aristotle. And his law of excluded middle became main tools for proving mathematical assertions. Later Cantor invented the set theory and this theory is presented by characteristic function that uses 0 and 1 only. Many conventional methods of the real-life problems based on fixed data are available in the literature, but due to increasing complexity, the problem based on fixed data cannot present to the situation properly.

The idea of fuzzy sets (FS) was invented by Zadeh [1] which is an important tool to present the uncertainty and has been used by researchers [2,3], etc. in engineering and management sectors. Further, it is observed that the FS does not deal to the situation of uncertainty and hesitation both of a real-life problem.

Atanassov [4] has extended a fuzzy sets to intuitionistic fuzzy sets (IFS) by incorporating an additional degree, called non-membership degree. IFS is a very realistic and recent tool to deal the problem having uncertainty and hesitation both. Recently, several researchers [5–8] have used it in many sectors of engineering and management. Further, it is observed that a single membership degree and non-membership degree does not state properly to the situation of uncertainty and hesitation in the real-life problems due to ill-defined membership and non-membership degrees, and hence, we admit a kind of further uncertainty.

To enhance the capability of handling uncertainty and hesitation of an IFS, Atanassov and Gargov [9] invented an interval-valued intuitionistic fuzzy sets which is a generalization of IFS in which membership and non-membership...
degrees are intervals rather than fixed real number. Current research work has been focusing on operations of IVIFSs and some other interesting properties of IVIFSs [10–12]. Due to increasing complexity of many real-life optimization problems, it is often a challenge for the decision maker to provide the values of parameters in a precise way. Therefore, several research works have been carried out in this direction ranking of FS and IFS. Among several generalizations of FS, the notions of IVIFSs are an interesting and very useful tool in modeling and making decision of real-life problems under uncertainty and hesitation. Many ranking methods of FS and IFS are available and widely used. Vogel’s approximation method (VAM) is a very common method that is used to calculate the IBFS of a TP. 

### Table 1 Comparison of ranking methods

| S.no. | Methods                        | Ranking | Ranking criteria                                      |
|-------|--------------------------------|---------|------------------------------------------------------|
| 1.    | Nayagam and Sivaraman [13]     | $A_1 < A_2$ | Based on new score and accuracy of IVIFSs           |
| 2.    | Lee ranking method [14]        | $A_1 < A_2$ | Based on score and accuracy of IVIFSs               |
| 3.    | Bharati and Singh [46]        | $A_1 < A_2$ | Based on value and ambiguity index of IVIFNs        |
| 4.    | Proposed ranking              | $A_1 < A_2$ | Based on extended Yager’s function of IVIFSs        |

In reality, however, we often encounter the situation that the degrees itself is frequently ill-defined as in the statement that the membership and non-membership degrees are “high,” “low,” “near 0.6,” “middle,” “not high,” “very low,” etc. To explain this fact, Atanassov and Gargov IVIFS in “high,” “low,” “near 0.6,” “middle,” “not high,” “very low,” etc. To explain this fact, Atanassov and Gargov IVIFS in which membership and non-membership degrees are subsets of [0, 1] rather than a point in [0, 1]. Methods based on fuzzy and intuitionistic fuzzy sets can be improved by assigning these parameters as IVIFSs.

In this paper, our efforts are to develop an iterative method of an interval-valued intuitionistic fuzzy transportation problem (IVIFTPP). In IVIFTPP, the cost of the transportation problem is represented by triangular interval-valued intuitionistic fuzzy numbers which includes a triangle membership function and a triangle non-membership function, and it would be capable to tackle uncertainty and hesitation. For the optimal solutions of IVIFTPP, a new technique of ranking is adapted and it will be very simple computational viewpoints. The proposed iterative method of IVIFTPP would be attracted massive attention because of their high accuracy, efficiency and adaptability that searches high-quality realistic solutions than the existing methods of FS and IFS (Figs. 1 and 2).
\[ v_A(x) = \begin{cases} v, & x = b \\ 1, & x \geq c, x \leq a \\ 1 - \frac{(1-v)(x-a)}{b-a}, & a < x < b \\ v - \frac{(1-v)(x-b)}{c-b}, & b < x < c \end{cases} \] (2)

This study presents two main contributions: The first contribution of this study is to deal with the formulation of a new ranking function of interval-valued intuitionistic fuzzy numbers based on Yager’s approach. Furthermore, it is felt that today is highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. The second contribution of this study is to deal with the formulation of a kind of transportation problems, known as interval-valued intuitionistic transportation problem that provide a powerful framework to meet this challenge.

### 2.1 Interval-valued intuitionistic fuzzy sets

The uncertainty and hesitation occur in every real-life problem, and therefore, it is very necessary to explain it. Now, suppose \( X \) represents set of 100 peoples of a village, and if we ask about the number of people who follow physical distancing during COVID-19 pandemic, the natural answer that we get are \([30, 40], [35, 40], \) etc., and the number of people who do not follow physical distancing are \([5, 10], [6, 8], \) etc. In the same manner, let \( X = \{x_1, x_2, \ldots, x_N\} \) be the set of \( N \) people in a village. And let the number of people who follow physical distancing during COVID-19 pandemic be \([m_1(x), m_2(x)]\) and number of people who do not follow be \([n_1(x), n_2(x)]\). Then \([m_1(x), m_2(x)] + [n_1(x), n_2(x)] \leq N\)

\[ \Rightarrow \frac{[m_1(x), m_2(x)] + [n_1(x), n_2(x)]}{N} \leq 1, \text{ since } N > 0; \text{ hence, the following inequalities make sense} \]

\[ \Rightarrow \frac{[m_1(x), m_2(x)]}{N} + \frac{[n_1(x), n_2(x)]}{N} \leq 1 \]

\[ \Rightarrow \frac{m_1(x)}{N}, \frac{m_2(x)}{N} + \frac{n_1(x)}{N}, \frac{n_2(x)}{N} \leq 1 \]

Therefore, \([x \in X : \frac{[m_1(x), m_2(x)]}{N}, \frac{[n_1(x), n_2(x)]}{N}]\) is an interval-valued intuitionistic fuzzy set. Now, we shall represent the formal definition of interval-valued intuitionistic fuzzy sets.

### Definition 3 (Atanassov and Gargov [9]).

Let \( X \) be an universal set. An interval-valued intuitionistic fuzzy A in \( X \) is expressed as \( A = \{(x, [\mu_A^{-}(x), \mu_A^{+}(x)], [v_A^{-}(x), v_A^{+}(x)]) : x \in X\}, \) where \( \mu_A^{-}(x) : X \to [0, 1], \mu_A^{+}(x) : X \to [0, 1] \) define lower and upper degrees of memberships, and \( v_A^{-} : X \to [0, 1], v_A^{+} : X \to [0, 1] \) define lower and upper degrees of non-memberships of the element \( x \in X \). And for every \( x \in X, 0 \leq \mu_A^{+}(x) + v_A^{+}(x) \leq 1. \)
of memberships, and making due its special features. Li [44] proposed representation theorem of IVIFS and defined operations between IVIFS. In this paper, we present arithmetic operations for triangular interval-valued intuitionistic fuzzy numbers. For this, let \( A = \{ (a_1, b_1, c_1), [\mu_A^-, \mu_A^+], [v_A^-, v_A^+] \} \) and \( B = \{ (a_2, b_2, c_2), [\mu_B^-, \mu_B^+], [v_B^-, v_B^+] \} \) be two triangular interval-valued intuitionistic fuzzy numbers, then

\[
A \oplus B = \{ (a_1 + a_2, b_1 + b_2, c_1 + c_2), \\
[\min(\mu_A^-, \mu_B^-), \min(\mu_A^+, \mu_B^+)], \\
[\max(v_A^-, v_B^-), \max(v_A^+, v_B^+)] \}
\]

(7)

\[
A \odot B = \{ (a_1 - c_2, b_1 - b_2, c_1 - c_2), \\
[\min(\mu_A^-, \mu_B^-), \min(\mu_A^+, \mu_B^+)], \\
[\max(v_A^-, v_B^-), \max(v_A^+, v_B^+)] \}
\]

(8)

\[
A^{-1} = \left\{ \left( \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right), [\mu_A^-, \mu_A^+], [v_A^+, v_A^-] \right\} \text{ if } a > 0
\]

(12)

### 2.2 Arithmetic operations

After IFS, IVIFS became a very popular tool in decision making due its special features. Li [44] proposed representation theorem of IVIFS and defined operations between IVIFS. In this paper, we present arithmetic operations for triangular interval-valued intuitionistic fuzzy numbers. For this, let \( A = \{ (a_1, b_1, c_1), [\mu_A^-, \mu_A^+], [v_A^-, v_A^+] \} \) and \( B = \{ (a_2, b_2, c_2), [\mu_B^-, \mu_B^+], [v_B^-, v_B^+] \} \) be two triangular interval-valued intuitionistic fuzzy numbers, then

\[
A \oplus B = \{ (a_1 + a_2, b_1 + b_2, c_1 + c_2), \\
[\min(\mu_A^-, \mu_B^-), \min(\mu_A^+, \mu_B^+)], \\
[\max(v_A^-, v_B^-), \max(v_A^+, v_B^+)] \}
\]

(7)

\[
A \odot B = \{ (a_1 - c_2, b_1 - b_2, c_1 - c_2), \\
[\min(\mu_A^-, \mu_B^-), \min(\mu_A^+, \mu_B^+)], \\
[\max(v_A^-, v_B^-), \max(v_A^+, v_B^+)] \}
\]

(8)

\[
A^{-1} = \left\{ \left( \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right), [\mu_A^-, \mu_A^+], [v_A^+, v_A^-] \right\} \text{ if } a > 0
\]

(12)

(i) **Triangular intuitionistic fuzzy number:**

For a triangular intuitionistic fuzzy number, \( \mu_A^+ = \mu_A^- = \mu_A \) and \( v_A^- = v_A^+ = v_A \). Relations from (7) to (12) become: For this, let \( A = \{ (a_1, b_1, c_1), [\mu_A^-, \mu_A^+], [v_A^-, v_A^+] \} \) and \( B = \{ (a_2, b_2, c_2), [\mu_B^-, \mu_B^+], [v_B^-, v_B^+] \} \) be two triangular intuitionistic fuzzy numbers, then

\[
A \oplus B = \{ (a_1 + a_2, b_1 + b_2, c_1 + c_2), \\
[\min(\mu_A^-, \mu_B^-), \min(\mu_A^+, \mu_B^+)], \\
[\max(v_A^-, v_B^-), \max(v_A^+, v_B^+)] \}
\]

(7)

\[
A \odot B = \{ (a_1 - c_2, b_1 - b_2, c_1 - c_2), \\
[\min(\mu_A^-, \mu_B^-), \min(\mu_A^+, \mu_B^+)], \\
[\max(v_A^-, v_B^-), \max(v_A^+, v_B^+)] \}
\]

(8)

\[
A^{-1} = \left\{ \left( \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right), [\mu_A^-, \mu_A^+], [v_A^+, v_A^-] \right\} \text{ if } a > 0
\]

(12)
\[ A \oplus B = \{(a_1 + a_2, b_1 + b_2, c_1 + c_2); \min(\mu_A, \mu_B), \max(v_A, v_B)\} \]  
\[ A \ominus B = \{(a_1 - c_2, b_1 - b_2, c_1 - a_2); \min(\mu_A, \mu_B), \max(v_A, v_B)\} \]  
\[ A \odot B = \begin{cases} 
\{(a_1 a_2, b_1 b_2, c_1 c_2); \min(\mu_A, \mu_B), \max(v_A, v_B)\} & \text{if } a_1, a_2 \in \mathbb{R}^+ \\
\{(a_1 c_2, b_1 c_2, a_1 a_2); \min(\mu_A, \mu_B), \max(v_A, v_B)\} & \text{if } a_1 < 0 \text{ and } a_2 > 0 \\
\{(c_1 a_2, b_1 a_2, c_1 c_2); \min(\mu_A, \mu_B), \max(v_A, v_B)\} & \text{if } c_1 < 0 \text{ and } c_2 > 0
\end{cases} \]  
\[ A \odot B = \begin{cases} 
\{(\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}); \min(\mu_A, \mu_B), \max(v_A, v_B)\} & \text{if } c_1, c_2 \in \mathbb{R}^+ \\
\{(\frac{c_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}); \min(\mu_A, \mu_B), \max(v_A, v_B)\} & \text{if } c_1 < 0 \text{ and } c_2 > 0 \\
\{(\frac{c_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}); \min(\mu_A, \mu_B), \max(v_A, v_B)\} & \text{if } c_1 < 0 \text{ and } c_2 < 0
\end{cases} \]  
\[ k \hat{A} = \begin{cases} 
\{(ka, kb, kc); \mu_A, v_A\} & \text{if } k > 0 \\
\{(kc, kb, ka); \mu_A, v_A\} & \text{if } k < 0
\end{cases} \]  
\[ A^{-1} = \left(\frac{1}{c}, \frac{1}{b}, \frac{1}{a}\right); \mu_A, v_A\] if \( a > 0 \)

(ii). Triangular fuzzy number:

For fuzzy number, \( \mu_A = \mu_A^+ = 1 \) and \( v_A^+ = v_A^- = 0 \). 

Relations from (7) to (12) become:

For this, let \( A = \{(a_1, b_1, c_1); \{1, 0\}\} \) and \( B = \{(a_2, b_2, c_2); \{\mu_B, v_B\}\} \) be two triangular intuitionistic fuzzy numbers, then

\[ A \oplus B = \{(a_1 + a_2, b_1 + b_2, c_1 + c_2)\} \]  
\[ A \ominus B = \{(a_1 - c_2, b_1 - b_2, c_1 - a_2)\} \]  
\[ A \odot B = \begin{cases} 
\{(a_1 a_2, b_1 b_2, c_1 c_2) & \text{if } a_1, a_2 \in \mathbb{R}^+ \\
\{(a_1 c_2, b_1 c_2, a_1 a_2) & \text{if } a_1 < 0 \text{ and } a_2 > 0 \\
\{(c_1 a_2, b_1 a_2, c_1 c_2) & \text{if } c_1 < 0 \text{ and } c_2 > 0
\end{cases} \]  
\[ A \odot B = \begin{cases} 
\left(\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}\right) & \text{if } c_1, c_2 \in \mathbb{R}^+ \\
\left(\frac{c_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}\right) & \text{if } c_1 < 0 \text{ and } c_2 > 0 \\
\left(\frac{c_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}\right) & \text{if } c_1 < 0 \text{ and } c_2 < 0
\end{cases} \]  
\[ k \hat{A} = \begin{cases} 
\{(ka, kb, kc); \mu_A, v_A\} & \text{if } k > 0 \\
\{(kc, kb, ka); \mu_A, v_A\} & \text{if } k < 0
\end{cases} \]  
\[ A^{-1} = \left(\frac{1}{c}, \frac{1}{b}, \frac{1}{a}\right); \mu_A, v_A\] if \( a > 0 \)

\[ \Rightarrow (x - a) \geq \frac{a}{\mu} (b - a) \]  
\[ \Rightarrow x \geq a + \frac{a}{\mu} (b - a) \]

Now, \( \mu_A^-(x) \geq a \)

\[ \Rightarrow \mu^{-\left(\frac{c - x}{c - b}\right)} \geq \frac{a}{\mu} \]  
\[ \Rightarrow \frac{c - x}{c - b} \geq \frac{a}{\mu} \]  
\[ \Rightarrow (c - x) \geq \frac{a}{\mu} (c - b) \]  
\[ \Rightarrow x \leq c - \frac{a}{\mu} (c - b) \]  
\[ \Rightarrow x \in \left[a + \frac{a}{\mu} (b - a), c - \frac{a}{\mu} (c - b)\right] \]

Hence, we get

\[ \left[a + \frac{a}{\mu} (b - a), c - \frac{a}{\mu} (c - b)\right] \]

Similarly, \( \mu_A^+(x) \geq a \)

\[ \Rightarrow \mu^{\left(\frac{a - x}{b - a}\right)} \geq \frac{a}{\mu} \]  
\[ \Rightarrow \frac{a - x}{b - a} \geq \frac{a}{\mu} \]  
\[ \Rightarrow (x - a) \geq \frac{a}{\mu} (b - a) \]  
\[ \Rightarrow x \geq a + \frac{a}{\mu} (b - a) \]

Now, \( \mu_A^+(x) \geq a \)

\[ \Rightarrow \mu^{\left(\frac{c - x}{c - b}\right)} \geq \frac{a}{\mu} \]  
\[ \Rightarrow \frac{c - x}{c - b} \geq \frac{a}{\mu} \]  
\[ \Rightarrow (c - x) \geq \frac{a}{\mu} (c - b) \]  
\[ \Rightarrow x \leq c - \frac{a}{\mu} (c - b) \]  
\[ \Rightarrow x \in \left[a + \frac{a}{\mu} (b - a), c - \frac{a}{\mu} (c - b)\right] \]
In this section, we extend Yager’s function [45] to help in the ranking of interval-valued intuitionistic fuzzy numbers. This function is the integral of the mean of the level sets associated with lower and upper memberships, and similarly with lower non-membership and upper non-memberships. We also verified some properties of the introduced functions. The merit of this function is that it does not require convexity, nor does it require normality of the interval-valued intuitionistic fuzzy sets ranked. Let $A^{\mu}_{\nu}$ and $A^{\mu\nu}$ be level sets corresponding to lower and upper membership functions, respectively. Similarly, let $A^{\nu\mu}_{\beta}$ and $A^{\nu\mu\beta}$ be level sets corresponding to lower and upper non-membership functions, respectively.

Let $m(A^{\nu\mu}_{\beta})$ and $m(A^{\nu\mu\beta}_{\beta})$ be means of level sets of lower and upper memberships, respectively.

$$f_{i}^{\mu}(A) = \int_{0}^{\max_{\mu}} m(A^{\mu}_{\nu})d\alpha$$
$$= \frac{1}{2} \int_{0}^{\mu^+} a + \frac{\alpha}{\mu^-}(b-a) + c - \frac{\alpha}{\mu^-}(c-b)d\alpha$$
$$= \frac{1}{4}(a+2b+c)\mu^-$$

Similarly for upper membership:

$$f_{i}^{\mu}(A) = \frac{1}{4}(a+2b+c)\mu^+$$

$F^\mu(A^{\mu}_{\nu}) = \sigma f_{i}^{\mu}(A) + (1-\sigma) f_{i}^{\nu}(A), \sigma \in [0,1]$ Since average represents a good choice, we take $\sigma = 0.5$

$$F^\mu(A^{\mu}_{\nu}) = \frac{1}{8}(a+2b+c)(\mu^- + \mu^+)$$ (26)

In the same manner, we can proceed for non-memberships

$$f_{i}^{\nu}(A) = \int_{0}^{\nu^+} m(A^{\nu\mu}_{\beta})d\beta,$$
$$= \int_{0}^{\nu^+} a + \frac{1-\beta}{1-\nu^-}(b-a) + b$$
$$+ \left(1 - \frac{\beta - \nu^-}{1-\nu^-}\right)(c-b)d\beta$$
$$= \frac{1}{4}(a+2b+c)(1-\nu^-)$$

$$f_{i}^{\nu}(A) = \frac{1}{4}(a+2b+c)(1-\nu^-)$$

$$\times f_{u}^{\nu}(A) = \int_{\nu^+}^{1} m(A^{\nu\mu\beta})d\beta,$$
$$= \int_{\nu^+}^{1} a + \frac{1-\beta}{1-\nu^+}(b-a) + b$$
$$+ \left(1 - \frac{\beta - \nu^+}{1-\nu^+}\right)(c-b)d\beta$$
$$= \frac{1}{4}(a+2b+c)(1-\nu^+)$$

$$f_{u}^{\nu}(A) = \frac{1}{4}(a+2b+c)(1-\nu^+)$$
\[ F^v(A^\mu_a) = \sigma f^v_l(A) + (1-\sigma) f^v_u(A), \] \( \sigma \in [0, 1] \]

Since average represents a good choice, we take \( \sigma = 0.5 \)

\[ F^v(A^\mu_a) = \frac{1}{8}(a+2b+c)(2-v^- - v^+) \quad (27) \]

### 3.1 Properties of \( F^\mu \) and \( F^v \)

**Property 1** Let \( A = (a) \) be a crisp number then \( F^\mu(a) = a \)

**Proof** For a crisp number, \( \max \mu = \mu^- = \mu^+ = 1 \), \( \max v = v^- = v^+ = 0 \),

\[ A^\mu_a = a \]

\[ F^\mu(A) = \int_0^{\max \mu} m(A^\mu_a)d\alpha = \int_0^{1} a \, d\alpha = a \]

So, \( F^\mu(a) = a \) and \( F^v(a) = 0 \). \( \square \)

**Property 2** If \( A \) is an ordinary subset of \( \mathbb{R} \), then \( F^\mu(A) = m(A) \) (\( F^v \) does not make sense).

**Proof** Let \( A = [a, b] \) be a subset of \( \mathbb{R} \), then \( m(A) = \frac{a+b}{2} \).

\[ F^\mu(A) = \int_0^{\max \mu} m(A^\mu_a)d\alpha = \int_0^{1} a + b \frac{1}{2} \, d\alpha \]

\[ = \frac{a+b}{2} = \text{mean of } A. \]

\( \square \)

**Property 3** If \( A = (p: \mu(p) = q) \). Then \( F^\mu(A) = pq \).

**Proof** Clearly \( A^\mu_a = p \) and \( \max \mu = q \). Then by definition

\[ F^\mu(A) = \int_0^{\max \mu} m(A^\mu_a)d\alpha = \int_0^{q} p \, d\alpha = pq. \]

\( \square \)

**Property 4** Let \( A \) be interval-valued intuitionistic fuzzy number and \( a \) be a crisp number such that \( a \geq 1 \). Then

\[ F^\mu \left( \frac{A}{a} \right) = \frac{F^\mu(A)}{a}. \]

**Proof** Let \( B = \frac{A}{a} \) and \( B = ([\mu^-(x),\mu^+(x)], [\nu^-(x),\nu^+(x)]) \).

If \( \in A \), then \( \frac{\mu}{a} \in A^\mu_a \) and \( \frac{\nu}{a} \in B^\nu \).

Therefore, \( F^\mu \left( \frac{A}{a} \right) = \int_0^{1} \frac{1}{a} A^\mu_a d\alpha = \frac{1}{a} \int_0^{1} A^\mu_a d\alpha = \frac{F^\mu(A)}{a} \) (Fig. 4).

According to the law of trichotomy, every \( x, y \in \mathbb{R} \) either \( x < y \) or \( x > y \) or \( x = y \). It is pointed out that the law of trichotomy holds in classical logic, and it does not hold in fuzzy logic. In this paper, we introduce a new function \( R : \mathbb{R}^n \rightarrow \mathbb{R} \) that assigns each interval-valued intuitionistic fuzzy number to a real number. The ranking function which is based on (26) and (27) is defined by

\[ R(A) = \eta F^\mu(A^\mu_a) + (1-\eta) F^v(A^\nu_b), \]

\( \eta \in [0, 1], \alpha + \beta < 1 \). \quad (28)

Figure 5 shows the ranking function of a collection of interval-valued intuitionistic fuzzy sets of real numbers. \( \mathbb{N}(\mathbb{R}) \) denotes the collections of all interval-valued intuitionistic fuzzy numbers on \( \mathbb{R} \). Therefore, ranking of interval-valued intuitionistic fuzzy sets is redefined as in the following manner

\[ R(A) = \eta F^\mu(A^\mu_a) + (1-\eta) F^v(A^\nu_b), \eta \in [0, 1] \]

\[ \eta = 0.5 \] represents best compromise choice, and thus, we take the same. \( \square \)

**Lemma** Let \( A \) and \( B \) be two interval-valued intuitionistic fuzzy numbers, and \( R(A) = \frac{(a+2b+c)(\mu+\mu^-+2-\nu^- - \nu^+)}{16} \).

Then exactly one of the following is true:

(i) If \( R(A_1) < R(A_2) \), then \( A_1 < A_2 \).

(ii) If \( R(A_1) > R(A_2) \), then \( A_1 > A_2 \).

(iii) If \( R(A_1) = R(A_2) \), then \( A_1 = A_2 \).

**Some remarks on proposed ranking function:**

**Remark 1** If \( A \) is an intuitionistic fuzzy number, then

\[ R(A) = \frac{(a+2b+c)(\mu+\mu^-+2-\nu^- - \nu^+)}{16} \]

\[ R(A) = \frac{(a+2b+c)(2\mu+2-2\nu)}{16} \]

\[ R(A) = \frac{(a+2b+c)(\mu+1-\nu)}{8} . \]
Remark 2 If $A$ is a triangular fuzzy number, then $R(A) = \frac{(a+2b+c)}{4}$.

To see this, let $A$ be a triangular fuzzy number. We can express $A$ in interval-valued intuitionistic fuzzy sense as $A = \{[a, b, c], [\mu, \mu], [1-\mu, 1-\mu]\}$. For fuzzy set, $\mu + \nu = 1$ or $\nu = 1-\mu$.

Then

\[
R(A) = \frac{(a+2b+c)(\mu + \mu + 2 - (1-\mu) - (1-\mu))}{16}
\]

\[
R(A) = \frac{(a+2b+c)(\mu + \mu + 2 - (1+\mu) - (1+\mu))}{16}
\]

\[
R(A) = \frac{(a+2b+c)(4\mu)}{16}
\]

\[
R(A) = \frac{(a+2b+c)\mu}{4}
\]

which is a very famous ranking of fuzzy numbers that have been utilized by several researchers.

Remark 3 If $A = a$ is a fixed real number, then $R(A) = a$.

To see this, let $A$ be a fixed real number. We can express any fixed real number in interval-valued intuitionistic fuzzy sense as $A = \{(a, a, a), [1, 1], [0, 0]\}$. Then

\[
R(A) = \frac{(a+2a+a)(1+1+2-0-0)}{16}
\]

\[
R(A) = \frac{(16a)}{16}
\]

\[
R(A) = a.
\]

4 Comparison

In this section, the proposed ranking function that is defined by $R(A) = \frac{(a+2b+c)(\mu^+ + \mu^- + 2-\nu^+ - \nu^-)}{16}$ with some existing ranking function of interval-valued intuitionistic fuzzy sets. Nayagam and Sivaraman [13] presented an ranking function to rank IVIF sets, and that is defined as

Let $A = \{[a, b], [c, d]\}$ be an interval-valued intuitionistic fuzzy sets, then the ranking of $A$ is defined as $LG(A) = \frac{\text{membership degree} + \text{hesitancy degree}}{2}$, $\delta \in [0, 1]$. After simplification, we get

\[
LG(A) = \frac{(a+b)(1-\delta) + \delta(2-(c+d))}{2}, \delta \in [0, 1].
\]  

(29)

Lee [14] introduced the concept of novel score and deviation of interval-valued intuitionistic fuzzy sets. Further, he proposed a ranking methodology to rank a collection of interval-valued intuitionistic fuzzy sets based on novel score and deviation and the method are:

Let $A = \{[a, b], [c, d]\}$ be an interval-valued intuitionistic fuzzy sets then novel score $S(A)$ and deviation $D(A)$ are

\[
S(A) = \frac{2+a+b-c-d}{3-a-b-c-d}
\]  

(30)

\[
D(A) = \frac{b+d-a-c}{b+d-a-c}
\]  

(31)

For the comparison, we take two interval-valued intuitionistic fuzzy subsets of real numbers $A_1 = \{(1, 2, 3), [0.1, 0.2], [0.3, 0.5]\}$ and $A_2 = \{(1, 4, 7), [0.1, 0.2], [0.2, 0.3]\}$.

Proposed ranking function:

\[
R(A_1) = \frac{(a+2b+c)(2+\mu^- + \mu^+ - (\nu^- + \nu^+))}{16}
\]

\[
R(A_1) = \frac{(1+2(2)+3)(2+0.1+0.2 - (0.3+0.5))}{16}
\]

\[
R(A_1) = 0.75
\]

and

\[
R(A_2) = \frac{(a+2b+c)(2+\mu^- + \mu^+ - (\nu^- + \nu^+))}{16}
\]

\[
R(A_2) = \frac{(1+2(4)+7)(2+0.1+0.2 - (0.3+0.3))}{16}
\]

\[
R(A_2) = 1.8
\]

Clearly, $R(A_1) < R(A_2)$.

Therefore, $A_1 < A_2$.

Nayagam and Sivaraman ordering (29):

\[
LG(A_1) = \frac{(0.1+0.2)(1-\delta) + \delta(2-(0.3+0.5))}{2}
\]

\[
LG(A_1) = \frac{0.3(1-\delta) + \delta(1.2)}{2}
\]

\[
LG(A_2) = \frac{(0.1+0.2)(1-\delta) + \delta(2-(0.2+0.3))}{2}
\]

\[
LG(A_2) = \frac{0.3(1-\delta) + \delta(1.5)}{2}
\]

It is very clear that $LG(A_1) < LG(A_2)$ for every $\delta \in [0, 1]$.

Therefore, $A_1 < A_2$.

Using Lee ranking (30), we get

\[
S(A_1) = \frac{2+0.1+0.2-0.3-0.5}{3-0.1-0.2-0.3-0.5} = \frac{3.1}{1.2} = 0.78
\]

\[
S(A_2) = \frac{2+0.1+0.2-0.2-0.3}{3-0.1-0.2-0.2-0.3} = \frac{1.5}{1.2} = 0.81
\]

and

\[
D(A_1) = 0.2 + 0.5 - 0.1 - 0.3 = 0.3
\]

\[
D(A_2) = 0.2 + 0.3 - 0.1 - 0.2 = 0.2
\]
Since $S(A_1) < S(A_2)$, $A_1 < A_2$.

Finally we conclude that the proposed ranking function agrees with Nayagam and Sivaraman ranking function and Lee ranking, and the main difference between proposed function and existing function is: In the proposed ranking based on Yager’s function, where as in Nayagam and Sivaraman and Lee function based on score and accuracy of interval-valued intuitionistic fuzzy sets, Bharati and Singh ranking is based on value and hesitation indices of interval-valued intuitionistic fuzzy sets.

5 Interval-valued intuitionistic fuzzy transportation problem

An IVIFTPP is a very special case of interval-valued intuitionistic fuzzy linear programming problem (IVIFLPP), and IVIFLPP is solved by using simplex method. Simplex method provides a very weak initial basic solution to IVIFTPP, and it takes large time of computation. Therefore, for the basic feasible solutions of IVIFTPP, we may use one of the three methods: interval-valued intuitionistic fuzzy northwest corner method (IVIFNWC), interval-valued intuitionistic fuzzy least cost method (IVIFLCM) and interval-valued intuitionistic fuzzy Vogel’s approximation method (IVIFVAM). And its interval-valued intuitionistic fuzzy optimal solution is obtained by using interval-valued intuitionistic fuzzy u–v method. In this paper, we consider an IVIFTPP with $m$ supplies and $n$ demands.

Let $c = \{(c_{1}^{ij}, c_{2}^{ij}, c_{3}^{ij}), [\mu_{c_{1}^{ij}}, \mu_{c_{2}^{ij}}, \mu_{c_{3}^{ij}}], [\nu_{c_{1}^{ij}}, \nu_{c_{2}^{ij}}, \nu_{c_{3}^{ij}}]\}$ be a interval-valued intuitionistic fuzzy numbers representing to the cost of transportation to send one unit of thing from $i^{th}$ place to $j^{th}$ place. Let $a_{i} = \{(a_{i}^{1}, a_{i}^{2}, a_{i}^{3}), [\mu_{a_{i}^{1}}, \mu_{a_{i}^{2}}, \mu_{a_{i}^{3}}], [\nu_{a_{i}^{1}}, \nu_{a_{i}^{2}}, \nu_{a_{i}^{3}}]\}$, and $b_{j} = \{(b_{j}^{1}, b_{j}^{2}, b_{j}^{3}), [\mu_{b_{j}^{1}}, \mu_{b_{j}^{2}}, \mu_{b_{j}^{3}}], [\nu_{b_{j}^{1}}, \nu_{b_{j}^{2}}, \nu_{b_{j}^{3}}]\}$ represent supplies and demands, respectively. Then IVIFTP is presented as:

Minimize $\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{1}^{ij}, c_{2}^{ij}, c_{3}^{ij}, c_{1}^{ij}, c_{2}^{ij}), [\mu_{c_{1}^{ij}}, \mu_{c_{2}^{ij}}, \mu_{c_{3}^{ij}}], [\nu_{c_{1}^{ij}}, \nu_{c_{2}^{ij}}, \nu_{c_{3}^{ij}}] x_{ij}$

S.t. $\sum_{j=1}^{n} x_{ij} \geq (a_{i}^{1}, a_{i}^{2}, a_{i}^{3}), [\mu_{a_{i}^{1}}, \mu_{a_{i}^{2}}, \mu_{a_{i}^{3}}], [\nu_{a_{i}^{1}}, \nu_{a_{i}^{2}}, \nu_{a_{i}^{3}}]$, $i = 1, 2, \ldots, m$

$m \sum_{i=1}^{m} x_{ij} \geq (b_{j}^{1}, b_{j}^{2}, b_{j}^{3}), [\mu_{b_{j}^{1}}, \mu_{b_{j}^{2}}, \mu_{b_{j}^{3}}], [\nu_{b_{j}^{1}}, \nu_{b_{j}^{2}}, \nu_{b_{j}^{3}}]$

$x_{ij} \geq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$.  

(32)

We conclude that the proposed ranking function agrees with Nayagam and Sivaraman ranking function and Lee ranking, and the main difference between proposed function and existing function is: In the proposed ranking based on Yager’s function, where as in Nayagam and Sivaraman and Lee function based on score and accuracy of interval-valued intuitionistic fuzzy sets, Bharati and Singh ranking is based on value and hesitation indices of interval-valued intuitionistic fuzzy sets.

5.1 Interval-valued intuitionistic fuzzy transportation problem of type 1

A transportation problem where costs are interval-valued intuitionistic fuzzy numbers, demands and supplied are real numbers is called IVIFTP of type 1. This type of transportation problem occurs because the cost of the transportation depends on various uncontrollable factors such as weather condition, road condition and traffic. Mathematically, a IVIFTP of type 1 is represented in the following way:

Minimize $\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{1}^{ij}, c_{2}^{ij}, c_{3}^{ij}), [\mu_{c_{1}^{ij}}, \mu_{c_{2}^{ij}}, \mu_{c_{3}^{ij}}], [\nu_{c_{1}^{ij}}, \nu_{c_{2}^{ij}}, \nu_{c_{3}^{ij}}] x_{ij}$

S.t. $\sum_{j=1}^{n} x_{ij} = a_{i}, i = 1, 2, \ldots, m$, 

$\sum_{i=1}^{m} x_{ij} = b_{j}, j = 1, 2, \ldots, n$, 

$x_{ij} \geq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$.  

(33)

5.2 Interval-valued intuitionistic fuzzy transportation problem of type 2

A transportation problem is that in which the demands and supplies are represented by interval-valued intuitionistic fuzzy numbers and cost of transportation is treated as fixed.

Table 2 Uncertain transportation

| Sources $S_i$ | $D_1$ | $D_2$ | $D_3$ | $a'$ |
|---------------|-------|-------|-------|------|
| $S_1$         | $\tilde{c}_{11}$ | $\tilde{c}_{12}$ | $\tilde{c}_{13}$ | $a_1$ |
| $S_2$         | $\tilde{c}_{21}$ | $\tilde{c}_{22}$ | $\tilde{c}_{23}$ | $a_2$ |
| $S_3$         | $\tilde{c}_{31}$ | $\tilde{c}_{32}$ | $\tilde{c}_{33}$ | $a_3$ |

Hitchcock [15] invented the basic transportation problem with fixed parameters, and it was modeled by standard linear programming without uncertainty and hesitation. After Zadeh’s fuzzy sets, several transportation models have appeared in the literature in which uncertainty was a main problem to deal. In this paper, we tackle uncertainty and hesitant of transportation problem that are coming from all directions. All uncertainty and hesitation are dealt well using interval-valued intuitionistic fuzzy sets. Therefore, we focus on interval-valued intuitionistic fuzzy transportation problem (Table 2), and in sort, we call it IVIFTP problem. Here, we can classify interval-valued intuitionistic fuzzy transportation problem into four types that are discussed below:
real numbers and that type of transportation problem is called IVIFTP of type 2.

Let \( a_i = \{ (a_{ij}^1, a_{ij}^2, a_{ij}^3), [\nu_{ij}^1, \nu_{ij}^2], [\mu_{ij}^1, \mu_{ij}^2], [v_{ij}^1, v_{ij}^2] \}, b_j = \{ (b_{ij}^1, b_{ij}^2, b_{ij}^3), [\mu_{bj}^1, \mu_{bj}^2], [\nu_{bj}^1, \nu_{bj}^2] \} \) and \( c_{ij} \) be the cost that spend to transport a product \( x_{ij} \) from the \( i^{th} \) origin to the \( j^{th} \) destination. The reason that appeal IVIFTP of type 2 is due to various uncontrollable factors such as storage capacity and public demand.

Minimize \( \bar{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \)
S.t. \( \sum_{j=1}^{n} x_{ij} = ((a_{ij}^1, a_{ij}^2, a_{ij}^3), [\nu_{ij}^1, \nu_{ij}^2], [\mu_{ij}^1, \mu_{ij}^2], [v_{ij}^1, v_{ij}^2]), i = 1, 2, \ldots, m, \)
\( \sum_{i=1}^{m} x_{ij} = ((b_{ij}^1, b_{ij}^2, b_{ij}^3), [\mu_{bj}^1, \mu_{bj}^2], [\nu_{bj}^1, \nu_{bj}^2], [v_{bj}^1, v_{bj}^2]), j = 1, 2, \ldots, n, \)
\( x_{ij} \geq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \) \( (34) \)

### 5.3 Interval-valued intuitionistic fuzzy transportation problem of type 3

A transportation problem is that in which all costs of transportation, demands and supplies are interval-valued intuitionistic fuzzy numbers. Let \( \{ (a_{ij}^1, a_{ij}^2, a_{ij}^3), [\nu_{ij}^1, \nu_{ij}^2], [\mu_{ij}^1, \mu_{ij}^2], [v_{ij}^1, v_{ij}^2] \}, i = 1, 2, \ldots, m \) be the quantity available at \( i^{th} \) origin, \( \{ (b_{ij}^1, b_{ij}^2, b_{ij}^3), [\mu_{bj}^1, \mu_{bj}^2], [\nu_{bj}^1, \nu_{bj}^2], [v_{bj}^1, v_{bj}^2] \}, j = 1, 2, \ldots, n \) be the quantity needed at \( j^{th} \) destination and \( \{ (c_{ij}^1, c_{ij}^2, c_{ij}^3), [\mu_{cij}^1, \mu_{cij}^2], [\nu_{cij}^1, \nu_{cij}^2], [v_{cij}^1, v_{cij}^2] \} \) be the transportation cost require to send \( x_{ij} \) from \( i^{th} \) origin to \( j^{th} \) destination. The cost of the transportation as in type 1 and 2.

Minimize \( \bar{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^1, c_{ij}^2, c_{ij}^3), [\mu_{cij}^1, \mu_{cij}^2], [\nu_{cij}^1, \nu_{cij}^2], [v_{cij}^1, v_{cij}^2] \) \( x_{ij} \)
S.t. \( \sum_{j=1}^{n} x_{ij} \approx ((a_{ij}^1, a_{ij}^2, a_{ij}^3), [\nu_{ij}^1, \nu_{ij}^2], [\mu_{ij}^1, \mu_{ij}^2], [v_{ij}^1, v_{ij}^2]), i = 1, 2, \ldots, m, \)
\( \sum_{i=1}^{m} x_{ij} \approx ((b_{ij}^1, b_{ij}^2, b_{ij}^3), [\mu_{bj}^1, \mu_{bj}^2], [\nu_{bj}^1, \nu_{bj}^2], [v_{bj}^1, v_{bj}^2]), j = 1, 2, \ldots, n, \)
\( \bar{x}_{ij} \geq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \) \( (35) \)

### 5.4 Interval-valued intuitionistic fuzzy transportation problem of type 4

A transportation problem is that in which all the cost of transportation, demands and supplies are interval-valued intuitionistic fuzzy numbers, and decision variables are interval-valued intuitionistic fuzzy numbers as well. This type of transportation is called a fully interval-valued intuitionistic fuzzy transportation problem a fully IVITPP. Recently, Kumar and Hussain [47] have studied fully intuitionistic fuzzy transportation problems. For the mathematical formulation, let \( \{ (a_{ij}^1, a_{ij}^2, a_{ij}^3), [\mu_{aj}^1, \mu_{aj}^2], [\nu_{aj}^1, \nu_{aj}^2] \}, i = 1, 2, \ldots, m \) be the quantity available at the \( i^{th} \) origin, \( \{ (b_{ij}^1, b_{ij}^2, b_{ij}^3), [\mu_{bj}^1, \mu_{bj}^2], [\nu_{bj}^1, \nu_{bj}^2] \} \) be the quantity needed at the \( j^{th} \) destination and \( \{ (c_{ij}^1, c_{ij}^2, c_{ij}^3), [\mu_{cij}^1, \mu_{cij}^2], [\nu_{cij}^1, \nu_{cij}^2] \} \) be the cost to transport \( \bar{x}_{ij} \) from the \( i^{th} \) origin to the \( j^{th} \) destination.

Minimize \( \bar{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^1, c_{ij}^2, c_{ij}^3), [\mu_{cij}^1, \mu_{cij}^2], [\nu_{cij}^1, \nu_{cij}^2], [v_{cij}^1, v_{cij}^2] \) \( \bar{x}_{ij} \)
S.t. \( \sum_{j=1}^{n} \bar{x}_{ij} \approx ((a_{ij}^1, a_{ij}^2, a_{ij}^3), [\mu_{aj}^1, \mu_{aj}^2], [\nu_{aj}^1, \nu_{aj}^2], [v_{aj}^1, v_{aj}^2]), i = 1, 2, \ldots, m, \)
\( \sum_{i=1}^{m} \bar{x}_{ij} \approx ((b_{ij}^1, b_{ij}^2, b_{ij}^3), [\mu_{bj}^1, \mu_{bj}^2], [\nu_{bj}^1, \nu_{bj}^2], [v_{bj}^1, v_{bj}^2]), j = 1, 2, \ldots, n, \)
\( \bar{x}_{ij} \geq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \) \( (36) \)

### 5.5 Balanced interval-valued intuitionistic fuzzy transportation problem

An IVITPP in which sum of all demands is equal to the sum of all supplies and all these are done after taking ranking is called balanced IVITPP, mathematically if

\[ \sum_{i=1}^{m} R(\tilde{a}_i) = \sum_{j=1}^{n} R(\tilde{b}_j). \] \( (37) \)

Otherwise, it is called unbalanced interval-valued intuitionistic transportation problem.

### 5.6 Interval-valued intuitionistic fuzzy optimal solution

A basis feasible solution that minimizes to the cost of transportation or that maximizes the profit of transportation
problem is called an interval-valued intuitionistic fuzzy optimal solution (IVIFOS).

6 Computational method

The steps of the computational method are given below:

Step 1: In this step, the cost of transportation is expressed as triangular interval-valued intuitionistic fuzzy numbers.

Step 2: Write IVIFTPP in tabular form as in below, where in the table all the parameters are represented by triangular interval-valued triangular intuitionistic fuzzy numbers.

\[
\tilde{c}_{ij} = \{ (c_{ij}^1, c_{ij}^2, c_{ij}^3), [ \mu_{c_{ij}^1}, \mu_{c_{ij}^2}, \mu_{c_{ij}^3}], [\nu_{c_{ij}^1}, \nu_{c_{ij}^2}, \nu_{c_{ij}^3}] \},
\tilde{a}_i = \{ (a_i^1, a_i^2, a_i^3), [ \mu_{a_i^1}, \mu_{a_i^2}, \mu_{a_i^3}], [\nu_{a_i^1}, \nu_{a_i^2}, \nu_{a_i^3}] \},
\tilde{b}_j = \{ (b_j^1, b_j^2, b_j^3), [ \mu_{b_j^1}, \mu_{b_j^2}, \mu_{b_j^3}], [\nu_{b_j^1}, \nu_{b_j^2}, \nu_{b_j^3}] \},
\]

Step 3: Using proposed ordering, we transform to the interval-valued intuitionistic fuzzy transportation problem (8) into its crisp form (Table 3), we get

Step 4: Now check whether it is balanced or not.

If \( \sum_{i=1}^{m} R \tilde{a}_i = \sum_{j=1}^{n} R \tilde{b}_j \), then TP is balanced.

If \( \sum_{i=1}^{m} R \tilde{a}_i \neq \sum_{j=1}^{n} R \tilde{b}_j \), then TP is unbalanced.

Step 5: If the given TP is balanced, then go to step 6 otherwise make it balanced by adding dummy rows or columns as required.

Step 6: In this step, we search the initial basic feasible solutions of the crisp transportation problem by using one of the following methods and methods are given below:

6.1 Interval-valued intuitionistic fuzzy least cost method

Step 1: In this step, the cost of transportation is expressed as interval-valued intuitionistic fuzzy numbers, particularly triangular interval-valued intuitionistic fuzzy numbers.

Step 2: Search a smallest \( \tilde{c}_{ij} \) in the IVIFS cost matrix of the transportation problem. Suppose it be \( \tilde{c}_{ij} \). Allocate \( \tilde{x}_{ij} = \min(\tilde{a}_i, \tilde{b}_j) \) in the cell \((i, j)\).

Step 3: If \( \tilde{x}_{ij} = \tilde{a}_i \) cross off the \( i^{th} \) row of transportation table and decrease \( \tilde{b}_j \) by \( \tilde{a}_i \). Go to next step.

If \( \tilde{x}_{ij} = \tilde{b}_j \) cross off the \( j^{th} \) column of the transportation table and decrease \( \tilde{a}_i \) by \( \tilde{b}_j \). Go to next step.

Step 4: Repeat steps 1 and 2 for the resulting reduced transportation table until all the requirements are satisfied.

6.2 Interval-valued intuitionistic fuzzy northwest corner method

Step 1: In this step, the cost of transportation is expressed as interval-valued intuitionistic fuzzy numbers, particularly triangular interval-valued intuitionistic fuzzy numbers.

Step 2: Select a northwest (upper left-hand) corner cell of the IVIF transportation table and allocate as much as possible so that either the capacity of the first row is exhausted or the destination requirement of the first column is satisfied, i.e., \( \tilde{x}_{11} = \min(\tilde{a}_1, \tilde{b}_1) \).

Step 3: If \( \tilde{b}_1 > \tilde{a}_1 \), then we move down vertically to the second row and make second allocation of magnitude \( \tilde{x}_{21} = \min(\tilde{a}_2, \tilde{b}_1) - \tilde{x}_{11} \) in cell (2, 1).

If \( \tilde{b}_1 < \tilde{a}_1 \), we move right horizontally to the second column and make the second allocation of magnitude \( \tilde{x}_{12} = \min(\tilde{a}_1, \tilde{b}_2) - \tilde{x}_{11} \) in cell (2, 1).

If \( \tilde{b}_1 = \tilde{a}_1 \), there is a tie for the second allocation. One can make the second allocation of magnitude.

Step 4: Repeat step 1 and step 2 moving down towards the lower/right corner of the transportation table until all the requirement satisfied.
6.3 Interval-valued intuitionistic fuzzy Vogel’s approximation method

[16] Vogel’s approximation method (VAM) is the most common method used to search initial basic feasible solution of TP. The demerit of this method is that VAM usually assigns items to the dummy cells before other cell in the table. Further, it was modified by several researchers such as: [17,18].

**Step 1:** In this step, the cost of transportation is expressed as interval-valued intuitionistic fuzzy numbers, particularly triangular interval-valued intuitionistic fuzzy numbers.

**Step 2:** For each row and column of the IVIF transportation table, identify the smallest and next smallest IVIFS cost with the help of the proposed ranking. And then calculate the penalty \( p_i, p_j, i = 1, 2, \ldots m; j = 1, 2, \ldots n \) between them for each row and column.

**Step 3:** Identify the largest penalty \( p_i, p_j, i = 1, 2, \ldots m; j = 1, 2, \ldots n \) among all the rows and columns. If a tie occurs, then choose any arbitrary cell. Let the greatest penalty occur corresponding to \( kth, 1 \leq k \leq m \) row and let \( z^{(k)} \) be the smallest cost in the \( kth \) row. Allocate the minimum of \( \widetilde{a}^i \) and \( \widetilde{b}^j \) or \( \min(\widetilde{a}^i, \widetilde{b}^j) \) to the corresponding to \( kth \) row and \( jth \) column, and cross either the \( kth \) row and \( jth \) column in the usual manner.

**Step 4:** Update the column and row penalties for the reduced transportation table and go to step 2. Repeat the procedure until all the requirements are satisfied.

**Step 5:** In this step, dual variables \( \widetilde{u}^i \) and \( \widetilde{v}^j \) corresponding to the \( i^{th} \) row and \( j^{th} \) column are defined, respectively, such that \( \widetilde{u}^i + \widetilde{v}^j = \widetilde{c}^i \) for each basic cell \( (i, j) \).

**Step 6:** Define \( \widetilde{Z}^i \) for all non-basic variables. Calculate \( \widetilde{Z}^i = \widetilde{u}^i + \widetilde{v}^j \).

**Step 7:** Assign quantity \( \tau \) in the cell \( (i, j) \) for which \( \widetilde{Z}^i - \widetilde{c}^i \) is most positive and make a loop as follows:

**Step 8:** Start from \( \tau \) – cell and move alternatively horizontally and vertically to the nearest basic cell with the restriction that end point of the loop must not lie in any non-basic cell except \( \tau \) – cell. In this way, return to \( \tau \) cell to complete loop.

**Step 9:** Move along loop of \( \tau \) – cell. Add and subtract \( \tau \) successively to/from the allocations in the cell lying at the turning points of the loop. Take the value of \( \tau \) to be minimum of \( x_{ij} \) from which \( \tau \) subtracted.

**Step 10:** Inserting the value of \( \tau \) in the above step, the next basic feasible solution is obtained which improves the interval-valued intuitionistic cost. While inserting value of \( \tau \), a cell assumes 0 value. This cell becomes non-basic. This gives us the improved basic feasible solutions.

**Step 11:** The optimal value of the objective function is calculated by \( \tilde{Z} = \widetilde{c}^j \cdot \widetilde{X}^0 \).

7 Illustration

In this section, numerical example of [34] is taken to verify the proposed computational method of the interval-valued intuitionistic fuzzy transportation problem. Here, cost of transportation is represented by triangular interval-valued intuitionistic fuzzy numbers. It is very interesting to see that cost obtained from proposed approach is minimum that existing.

**Step 1:** The cost of transportation varies due to various uncertain situation like weather condition, traffic condition, petroleum price, etc. The value of cost cannot deal the situation properly; to address this situation, we express parameter by TIVIFNs (Table 4).

| Cell | Cost | Value |
|------|------|-------|
| A    | 10   | 0.10  |
| B    | 15   | 0.15  |
| C    | 20   | 0.20  |

**Step 2:** Identify smallest element and next smallest element in each row and each column (Tables 5, 6).

| Cell | Cost | Value |
|------|------|-------|
| A    | 10   | 0.10  |
| B    | 15   | 0.15  |
| C    | 20   | 0.20  |

Similarly for column

| Cell | Cost | Value |
|------|------|-------|
| A    | 10   | 0.10  |
| B    | 15   | 0.15  |
| C    | 20   | 0.20  |

**Step 3:** Row penalties:

First row \( rp_1 \):

\[
\{(4, 6, 16), [0.3, 0.4], [0.03, 0.07]\}
\]

\[
\{4, 5, 7\}, [0.3, 0.4], [0.01, 0.02]\}
\]

Second row \( rp_2 \):

\[
\{(5, 10, 15), [0.2, 0.5], [0.01, 0.04]\}
\]

\[
\{(-2, 5, 11), [0.2, 0.4], [0.01, 0.04]\}
\]
### Table 4  Interval-valued intuitionistic fuzzy transportation problem

|   | \(D_1\)                                                                 | \(D_2\)                                                                 | \(D_3\)                                                                 | \(a_i\) |
|---|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|---------|
| \(S_1\) | \([1, 4, 9], [0.1, 0.5], [0.01, 0.03]\)                                    | \([3, 13, 14], [0.2, 0.4], [0.02, 0.04]\)                                    | \([4, 6, 16], [0.3, 0.4], [0.03, 0.07]\)                                    | 7       |
| \(S_2\) | \([4, 5, 7], [0.3, 0.4], [0.01, 0.02]\)                                    | \([5, 10, 15], [0.2, 0.5], [0.01, 0.04]\)                                    | \([7, 16, 24], [0.3, 0.5], [0.02, 0.03]\)                                    | 15      |
| \(S_3\) | \([1, 3, 6], [0.4, 0.5], [0.01, 0.02]\)                                    | \([5, 13, 21], [0.3, 0.4], [0.03, 0.04]\)                                    | \([8, 18, 27], [0.4, 0.5], [0.05, 0.05]\)                                    | 10      |
| \(b_j\) | 8                                                                           | 6                                                                           | 18                                                                         |         |

The selected elements are represented by bold.

### Table 5  Interval-valued intuitionistic fuzzy transportation problem cont.

|   | \(D_1\)                                                                 | \(D_2\)                                                                 | \(D_3\)                                                                 | \(a_i\) |
|---|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|---------|
| \(S_1\) | \([1, 4, 9], [0.1, 0.5], [0.01, 0.03]\)                                    | \([3, 13, 14], [0.2, 0.4], [0.02, 0.04]\)                                    | \([4, 6, 16], [0.3, 0.4], [0.03, 0.07]\)                                    | 7       |
| \(S_2\) | \([4, 5, 7], [0.3, 0.4], [0.01, 0.02]\)                                    | \([5, 10, 15], [0.2, 0.5], [0.01, 0.04]\)                                    | \([7, 16, 24], [0.3, 0.5], [0.02, 0.03]\)                                    | 15      |
| \(S_3\) | \([1, 3, 6], [0.4, 0.5], [0.01, 0.02]\)                                    | \([5, 13, 21], [0.3, 0.4], [0.03, 0.04]\)                                    | \([8, 18, 27], [0.4, 0.5], [0.05, 0.05]\)                                    | 10      |
| \(b_j\) | 8                                                                           | 6                                                                           | 18                                                                         |         |

The selected elements are represented by bold.

### Table 6  Interval-valued intuitionistic fuzzy transportation problem cont.

|   | \(D_1\)                                                                 | \(D_2\)                                                                 | \(D_3\)                                                                 | \(a_i\) |
|---|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|---------|
| \(S_1\) | \([1, 4, 9], [0.1, 0.5], [0.01, 0.03]\)                                    | \([3, 13, 14], [0.2, 0.4], [0.02, 0.04]\)                                    | \([4, 6, 16], [0.3, 0.4], [0.03, 0.07]\)                                    | 7       |
| \(S_2\) | \([4, 5, 7], [0.3, 0.4], [0.01, 0.02]\)                                    | \([5, 10, 15], [0.2, 0.5], [0.01, 0.04]\)                                    | \([7, 16, 24], [0.3, 0.5], [0.02, 0.03]\)                                    | 15      |
| \(S_3\) | \([1, 3, 6], [0.4, 0.5], [0.01, 0.02]\)                                    | \([5, 13, 21], [0.3, 0.4], [0.03, 0.04]\)                                    | \([8, 18, 27], [0.4, 0.5], [0.05, 0.05]\)                                    | 10      |
| \(b_j\) | 8                                                                           | 6                                                                           | 18                                                                         |         |

The selected elements are represented by bold.

### Table 7  Basic feasible solutions

|   | \(D_1\)                                                                 | \(D_2\)                                                                 | \(D_3\)                                                                 | \(a_i\) |
|---|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|---------|
| \(S_1\) | \([1, 4, 9], [0.1, 0.5], [0.01, 0.03]\)                                    | \([3, 13, 14], [0.2, 0.4], [0.02, 0.04]\)                                    | \([4, 6, 16], [0.3, 0.4], [0.03, 0.07]\)                                    | 7       |
| \(S_2\) | \([4, 5, 7], [0.3, 0.4], [0.01, 0.02]\)                                    | \([5, 10, 15], [0.2, 0.5], [0.01, 0.04]\)                                    | \([7, 16, 24], [0.3, 0.5], [0.02, 0.03]\)                                    | 15      |
| \(S_3\) | \([1, 3, 6], [0.4, 0.5], [0.01, 0.02]\)                                    | \([5, 13, 21], [0.3, 0.4], [0.03, 0.04]\)                                    | \([8, 18, 27], [0.4, 0.5], [0.05, 0.05]\)                                    | 10      |
| \(b_j\) | 8                                                                           | 6                                                                           | 18                                                                         |         |

The selected elements are represented by bold.

---

**Step 4:** Choose row or column having larger penalty. Third has maximum penalty, and third row is selected for allocation. Continuing in the same ways, we get basic feasible solutions (Table 7).
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\( x_{13} = 7, x_{22} = 6, x_{23} = 9, x_{31} = 8, x_{33} = 2. \) Now we shall use interval-valued intuitionistic fuzzy \( \hat{u}_i - \hat{v}_j \) method to get interval-valued intuitionistic fuzzy optimal solutions.

Here, we calculate interval-valued intuitionistic fuzzy net evaluations (i.e., \( z_{ij} - c_{ij} \)) for all non-basic cell.

**Step 5:**

Put \( \tilde{u}_1 = 0 \) (Table 8), we get

\[
\tilde{u}_2 = \{(-9, 10, 20); [0.3, 0.4][0.03, 0.07]\}
\]

\[
\tilde{u}_3 = \{(-8, 12, 23); [0.3, 0.4][0.05, 0.07]\}
\]

\[
\tilde{v}_1 = \{(-22, 9, 14); [0.3, 0.4][0.05, 0.07]\}
\]

\[
\tilde{v}_2 = \{(-15, 0, 24); [0.2, 0.4][0.03, 0.07]\}
\]

\[
6\tilde{v}_3 = \{(4, 6, 16); [0.3, 0.4][0.03, 0.07]\}
\]

Net evaluations corresponding to all non-basic cells

\[
z_{11} - c_{11} = \tilde{u}_1 + \tilde{v}_1 - \tilde{c}_{11}
\]

\[
= 0 + \{(-22, -9, 14); [0.3, 0.4][0.05, 0.07]\}
\]

\[
-\{(1, 4, 9); [0.1, 0.5], [0.01, 0.03]\}
\]

\[
= \{(-31, -13, 13); [0.1, 0.5],[0.01, 0.03]\} < 0
\]

\[
\therefore \tilde{z}_{11} - \tilde{c}_{11} < 0
\]

\[
z_{12} - c_{12} = \tilde{u}_1 + \tilde{v}_2 - \tilde{c}_{12}
\]

\[
= 0 + \{(-15, 0, 24); [0.2, 0.4][0.03, 0.07]\}
\]

\[
-\{(3, 13, 14); [0.2, 0.4],[0.02, 0.04]\}
\]

\[
= \{(-29, -13, 21); [0.2, 0.4],[0.03, 0.07]\} < 0
\]

\[
\therefore \tilde{z}_{12} - \tilde{c}_{12} < 0
\]

\[
z_{13} - c_{13} = \tilde{u}_2 + \tilde{v}_1 - \tilde{c}_{13}
\]

\[
= \{(-9, 10, 20); [0.3, 0.4][0.03, 0.07]\}
\]

\[
+\{(-22, -9, 14); [0.3, 0.4][0.05, 0.07]\}
\]

\[
-\{(4, 5, 7); [0.3, 0.4],[0.01, 0.02]\}
\]

\[
= \{(-31, -1, 13); [0.3, 0.4],[0.03, 0.07]\}
\]

\[
-\{(4, 5, 7); [0.3, 0.4],[0.01, 0.02]\}
\]

\[
= \{(-38, -6, 4); [0.3, 0.4],[0.03, 0.07]\} < 0
\]

\[
\therefore \tilde{z}_{13} - \tilde{c}_{13} < 0
\]

\[
z_{21} - c_{21} = \tilde{u}_2 + \tilde{v}_1 - \tilde{c}_{21}
\]

\[
= \{(-8, 12, 23); [0.3, 0.4][0.05, 0.07]\}
\]

\[
+\{(-15, 0, 24); [0.2, 0.4][0.03, 0.07]\}
\]

\[
-\{(5, 13, 21); [0.3, 0.4],[0.03, 0.04]\}
\]

\[
= \{(-23, 12, 47); [0.3, 0.4],[0.03, 0.07]\}
\]

\[
-\{(5, 13, 21); [0.3, 0.4],[0.03, 0.04]\}
\]

\[
= \{(-44, -1, 42); [0.3, 0.4],[0.03, 0.07]\} < 0
\]

\[
\therefore \tilde{z}_{21} - \tilde{c}_{21} < 0
\]

**Step 6:** Finally, we get for all non-basic cells: \( \tilde{z}_{ij} - \tilde{c}_{ij} < 0 \)

Stop the process here. Therefore, we skip steps 7-9 and we shall go last step (i.e, step 11).
Step 11: Therefore, \( x_{13} = 7, x_{22} = 6, x_{33} = 9, x_{31} = 8, x_{33} = 2 \) are optimal solutions and minimum cost is:

\[
7 * \{(4, 6, 16); [0.3, 0.4], [0.03, 0.07]\}
+ 6 * \{(5, 10, 15); [0.2, 0.5], [0.01, 0.04]\}
+ 9 * \{(7, 16, 24); [0.3, 0.5], [0.02, 0.03]\}
+ 8 * \{(1, 3, 6); [0.4, 0.5], [0.01, 0.02]\}
+ 2 * \{(8, 18, 27); [0.4, 0.5], [0.05, 0.05]\}
= \{(145, 306, 520); [0.2, 0.4], [0.03, 0.07]\}

Interval-valued intuitionistic fuzzy cost is: \( z^0 = \{(145, 306, 520); [0.2, 0.4], [0.03, 0.07]\} \).

### 8 Conclusions

One of the very interesting problems of decision science is the ranking of interval-valued intuitionistic fuzzy sets. Also, it is very hard to develop a ranking function to rank interval-valued intuitionistic fuzzy sets. In the present paper, we use the law of trichotomy to order interval intuitionistic fuzzy sets, and for this, we introduce a transformation that is called ranking function. The proposed ranking function depends on both value of variable and interval-valued intuitionistic fuzzy degrees, and this is a beauty of proposed ranking function. Also, it distinguishes proposed ranking to existing ranking function. In the present paper, a computational methodology which is based on ranking function is developed and applied to an interval-valued intuitionistic fuzzy transportation problem to get a compromise solution. Further, it is very interesting to note that the proposed computation method predicts a minimum transportation cost as compared to the existing approach (see Tables 9 and 10), and interval-valued intuitionistic fuzzy degree of transportation cost is given in Fig. 6. To check the performance and superiority of the proposed ranking function, an illustrative example is presented (see Table 1). Also we compare our ranking function with existing ranking function and it shows that presented ordering function follows to the existing ranking function.

Due to the appearance of interval-valued intuitionistic fuzzy sets in real-life problems such as decision making, multiattribute decision making with incomplete weight, fuzzy forecasting, risk analysis, etc., clustering and artificial intelligence, the proposed method will be high performance and efficient.

### Acknowledgements

The authors would like to thank to the editor and anonymous referees for various suggestions which have led to an improvement in both the quality and clarity of the paper.
Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Human and animal rights This article does not contain any studies with human or animal subjects performed by any of the authors.

Informed consent Informed consent was not required as no human or animals were involved.

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