Research on mechanical resonance of Precise cable drive systems

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Abstract: Precise cable drive is a light, concise and efficient spatial friction flexible transmission mode, which has the characteristics of flexible layout, lightweight and miniaturization. It is a good choice for Acquiring tracking and pointing applications (ATPs). But the cable drive mechanism (CDM) is easy to cause large-scale speed oscillation in the process of performing tasks as cable compliance. This paper focused on the mechanical resonance in a practical full-closed loop positioning control system. A dynamic model with the electro-mechanical and rigid-flexible coupling characteristics is proposed, which is capable to analyze the global influence factors on speed oscillation. The parametric sensitivities have been conducted to investigate the effects of design parameters on mechanical resonance.

1. Introduction
Precise cable drive[1-3] is a light, concise and efficient spatial friction flexible transmission mode, which has the characteristics of flexible layout, lightweight and miniaturization. It is a good choice for Acquiring tracking and pointing applications (ATPs) [4-5]. However, the cable drive mechanism (CDM) is easy to cause large-scale speed oscillation in the process of performing tasks as cable compliance. It is the most significant obstacle in achieving high-speed high-tracking-accuracy motion of the gimbals.

For CDM system, it is usually abstracted as the combination of axially moving string and rotating mass, in which the vibration includes the coupling of transverse vibration of string and rotating vibration of mass. Axially moving materials tend to vibrate in the process of motion transmission. Linear vibrations of axially moving materials have been studied extensively[6-8]. Although linear theory provides natural frequency, vibration mode and critical speed, it has some limitations. The nonlinear vibration of axially moving materials has attracted the attention of researchers in recent years. Vetyukov [9], [10] presented a nonlinear model of an extensible string at contour motion to analyze the variation of the motion and tension in the transient processes such as startup and accelerated motion. Dufva[11] developed a nonlinear dynamic formulations based on the absolute nodal coordinate formulation. It has the characteristics of constant mass matrix, no Coriolis force and centrifugal force, and is suitable for modeling under large deformation. Other researchers have also extended the model( [12]–[16]). But these studies mainly focused on the mechanism design factors.

This paper is contributed to analyze the mechanical resonance taking control parameters into consideration. A dynamic model with the electro-mechanical and rigid-flexible coupling characteristics is proposed, which is capable to analyze the global influence factors on speed oscillation. The parametric sensitivities have been carried out to investigate the effects of design parameters on mechanical resonance.
2. Modeling of the CDM

A typical CDM is composed of input pulley, output drum, steel cable and preloading mechanism. The steel cable is usually wound on the input pulley in the shape of “8” for several turns and then fixed on the output drum, as shown in Fig.1. The function of the preloading mechanism is to pre-apply an appropriate tension on the steel cable to improve the transmission capacity of the CDM unit.

Figure 1. Basic component of the CDM     Figure 2. Dynamic model of the CDM system

Figure 2 shows a lumped dynamic model of an axis of the ATPS with electro-mechanical and rigid-flexible coupling characteristics. It consists of a motor inertia, $J_m$, and a link inertia, $J_l$, connected by cables with stiffness, $K_s$, and damping, $C_s$. The reduction ratio of the CDM is $i_c$. The input to the system is the motor torque command, $u_a$, and its output is the motor speed, $\omega_m$, and the speed of the output shaft, $\omega_l$.

The Lagrange's equations, describing the motion of the system of n particle in conservation force field, are used to develop the dynamic model from energy view.

\[
\frac{d}{dt} \left( \frac{\partial(T - V)}{\partial \dot{q}_i} \right) - \frac{\partial(T - V)}{\partial q_i} + \frac{\partial Q}{\partial \dot{q}_i} = M_i \quad (i = 1,2)
\]

\[
T = \frac{1}{2} J_m \omega_m^2 + \frac{1}{2} J_l \omega_l^2
\]

\[
V = \frac{1}{2} K_s \left( \frac{\theta_m}{i_c} - \theta_l \right)^2
\]

\[
Q = \frac{1}{2} B_m \omega_m^2 + \frac{1}{2} C_s \left( \frac{\omega_m}{i_c} - \omega_l \right)^2 + \frac{1}{2} B_l \omega_l^2
\]

where, $T$, $V$, $Q$, $M$, represent the total kinetic energy, potential energy, dissipation energy and generalized torques, respectively. The generalized torques of the system includes the motor output torque, $T_m$, and the external disturbance in the form of friction, $T_f$. They can be obtained as

\[
T_m = u_a k_a k_m
\]

\[
T_f = F_c \text{sign} (\omega_l) + F_v \omega_l
\]

where, $k_a$ denotes the amplifying coefficient from command signal to motor current, $k_m$ is the motor output torque coefficient, $F_c$ is a type of Coulomb friction, and $F_v$ denotes viscous friction. Substituting Eq.(2) into Eq.(1), one obtains,

\[
j \ddot{\theta} + B \dot{\theta} + K \theta = T_m - T_d
\]

\[
\theta = \begin{bmatrix} \theta_m \\ \theta_l \end{bmatrix}; J = \begin{bmatrix} J_m & 0 \\ 0 & J_l \end{bmatrix}; B = \begin{bmatrix} B_m + \frac{C_s}{i_c} & -\frac{C_s}{i_c} \\ -\frac{C_s}{i_c} & B_l + C_s \end{bmatrix};
\]

\[
K = \begin{bmatrix} K_s & -\frac{K_s}{i_c} \\ -\frac{K_s}{i_c} & K_s \end{bmatrix}; T_m = \begin{bmatrix} U_{cmd} k_a k_m \\ 0 \end{bmatrix}; T_d = \begin{bmatrix} 0 \\ T_f \end{bmatrix}
\]
We write the equations in state-space form, where the state variation \( \Theta = [\omega_m, \omega_l, \theta_m, \theta_l]^T \), the state space model can be described as

\[
\dot{\Theta} = A\Theta + B(U - D)
\]

\[
Y = C\Theta
\]

where

\[
A = \begin{bmatrix}
\frac{\theta_m}{f_m} - \frac{c_s}{J_m\ell^2} & \frac{c_s}{J_m\ell^2} & -\frac{K_s}{J_m\ell^2} & \frac{K_s}{J_m\ell^2} \\
\frac{c_s}{J_m\ell^2} & \frac{\theta_l + c_s}{J_l\ell^2} & -\frac{K_s}{J_l\ell^2} & \frac{K_s}{J_l\ell^2} \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}; \quad
B = \begin{bmatrix}
\frac{1}{f_m} \\
0 \\
\frac{1}{J_l} \\
0
\end{bmatrix}
\]

\[
U = \begin{bmatrix} T_m \n 0 \n\end{bmatrix}; \quad D = \begin{bmatrix} 0 \\
T_f \n\end{bmatrix}; \quad C = I_{4 \times 4}
\]

In the above formula, \( A, B, U, D, Y, C \) denote the state matrix, input matrix, control instruction matrix, disturbance signal matrix, measurement signal output vector and the output matrix, respectively.

The block diagram of the dynamics in s-domain can be shown in Fig.3.

Figure 3. Block diagram of the dynamics in s-domain

If the damping terms \((B_m, B_l)\) are ignored, the dynamic model in complex field can be deduced as

\[
\begin{align*}
\omega_m(s) &= G_{m1}(s)T_m(s) - G_{m2}(s)T_f(s) \\
\omega_l(s) &= G_{m3}(s)T_m(s) - G_{m4}(s)T_f(s)
\end{align*}
\]

\[
G_{m1}(s) = \frac{1}{f_m s^2 + 2k\xi_\omega_0s + k}\omega_0^2
\]

\[
G_{m2}(s) = \frac{1}{f_m s^2 + 2\xi_\omega_0s + \omega_0^2}
\]

\[
G_{m3}(s) = \frac{1}{J_s s^2 + 2\xi_\omega_0s + \omega_0^2}
\]

\[
G_{m4}(s) = \frac{1}{J_s s^2 + 2\xi_\omega_0s + \omega_0^2}
\]

where, we use the resonant frequency \(\omega_m\), the resonant damping coefficient \(\xi_\omega\), the anti-resonant frequency \(\omega_\omega\), the anti-resonant damping coefficient \(\xi_\omega\), and the scale factor \(k\) to simplify the expressions.

The parameters could be expressed as,

\[
\begin{align*}
\omega_n &= \sqrt{\frac{K_s}{J_l}} \\
\xi_n &= \frac{c_s}{2\sqrt{K_sJ_l}} \\
\omega_d &= \sqrt{\frac{K_sJ_s}{f_mJ_l}} \\
\xi_d &= \frac{c_s}{2\sqrt{K_sJ_sJ_l}} \\
k &= \frac{f_m}{J_l}
\end{align*}
\]

Let \(Q_s = 1 + 2\xi_\omega(s/\omega_n) + (s/\omega_n)^2\), \(Q_s = 1 + 2\xi_\omega(s/\omega_\omega) + (s/\omega_\omega)^2\), the expressions of transfer function model could be further simplified as,
where, $J_m+Jl/i_2^2$ denotes the lumped inertias of the system. A conventional closed-loop control system, which employs PI control of motor speed and P control of load position, as shown in Fig.4.

Figure 4. Position control system of the CDM system

In this figure, $K_p$, $K_i$, and $K$ represent the proportional feedback coefficient of position loop, the proportional and integral feedback coefficient, respectively. The closed-loop transfer function from the reference input to the load position is given by

$$
\theta_\text{ref} = \frac{K_p k d k_m (k_m + K)}{j i_2 Q_d s^3 + k_d k_m (k_m + K)} \left( Q_n - \frac{1}{Q_n} \right)^2
$$

(14)

It is also sorted into the form of cascade of basic links,

$$
\theta_\text{ref} = \frac{(s+pi_1)(s+pi_2)}{(s+pi_3)(s+pi_4)}
$$

(15)

where, $K$ is the proportional element. $p_{i1}, p_{i2}$ are two zeros and $p_{i3}$ is the pole. $\omega_1$, $\omega_2$ are the two natural angular frequencies under the second order oscillation link. $\zeta_1$, $\zeta_2$ are the damping coefficient at the corresponding frequency. The center frequency of mechanical resonance under closed-loop condition is $\min(\omega_1, \omega_2)$.

3. Analysis of speed oscillations in Precise cable drive system

By analyzing Eq.(8), the response characteristic of the motor speed $\omega_m$ can be regarded as composed of two parts: the output generated by the system $G_{m1}(s)$ under the action of $T_m$ and the output generated by the system $G_{m2}(s)$ under the action of $T_f$. Similarly, the response characteristic of the load speed $\omega_l$ can be regarded as composed of the output generated by the system $G_{m1}(s)$ under the action of $T_m$ and the output generated by the system $G_{m2}(s)$ under the action of $T_f$.

In the system $G_{m1}(s)$, the transfer function $G_{r1}(s)$ of the mechanical transmission part is

$$
G_{r1}(s) = \frac{s^2+2k\xi_1\omega_1 s+k\omega_1^2}{s^2+2\xi_1\omega_1 s+\omega_1^2}
$$

(16)

In the system $G_{m2}(s)$, the transfer function $G_{r2}(s)$ of the mechanical transmission part is

$$
G_{r2}(s) = \frac{2k\xi_1\omega_1 s+k\omega_1^2}{s^2+2\xi_1\omega_1 s+\omega_1^2}
$$

(17)

We defined the parameters for the simulation as shown in Tab.1.

| TABLE I. REQUIRED PARAMETERS OF THE EXPERIMENTAL SYSTEM |
|--------------------------------------------------------|
| Parameters                        | Values     |
|-----------------------------------|------------|
| The inertias of the input shaft, $J_m$(Kg m$^2$)    | 0.0003     |
| The damping coefficient of the input shaft, $d$     | 0.02       |
| The inertias of the output shaft, $J_l$(Kg m$^2$)   | 0.008      |
| The damping coefficient of the output shaft, $1/d$  | 0.6        |
Transmission stiffness, $K_s/(N \cdot m \cdot rad^{-1})$  1500
Transmission damping, $C_s/(N \cdot ms \cdot rad^{-1})$  0.8
Transmission ratio, $i$  5.4
Proportional coefficient of velocity loop, $K_v/(N \cdot ms/\text{rad})$  0.05
Integral coefficient of velocity loop, $K_i/(s^{-1})$  12
Proportional coefficient of position loop, $K_p/(s^{-1})$  470

The frequency response characteristic curves of $G_{r1}(s)$ and $G_{r2}(s)$ are shown in Fig.5

![Frequency response characteristic curves](image)

Figure 5. The frequency response characteristic curves of $G_{r1}(s)$ and $G_{r2}(s)$

The amplitude frequency response of $G_{r1}(s)$ includes resonance point and anti-resonance point. At the resonance point, the amplitude gain of $G_{r1}(s)$ is the maximum; At the anti-resonance point, the amplitude gain of $G_{r1}(s)$ is a minimum. The mathematical expression of the amplitude frequency response of $G_{r1}(s)$ is

$$|G_{r1}(j\omega)| = \sqrt{\frac{(k\omega_n^2-\omega^2)^2+4k^2\xi_r^2\omega_n^2\omega^2}{(\omega_n^2-\omega^2)^2+4\xi_r^2\omega_n^2\omega^2}}$$

(18)

Therefore, the problem of finding the resonant point and anti-resonant point of the system model can be transformed into solving the extreme value problem of the system $|G_{r1}(j\omega)|$. Namely,

$$\frac{\partial |G_{r1}(j\omega)|}{\partial \omega} = 0$$

(19)

The resonant point and anti-resonant point of the system can be obtained as,

$$\omega_{ar} = \sqrt{\frac{(1+k)-\sqrt{(1-k)^2+4k^2\xi_r^2\omega_n^2\omega^2}}{2k-4k\xi_r^2-4k^2\omega_n^2}}$$

$$\omega_{an} = \sqrt{\frac{(1+k)+\sqrt{(1-k)^2+4k^2\xi_r^2\omega_n^2\omega^2}}{2-4k\xi_r^2-4k^2\omega_n^2}}$$

(20)

Generally, the viscous damping coefficient $\xi_r$ of mechanical transmission device is much smaller than its transmission stiffness. So the contains $\xi_r$ could be negligible, then

$$\omega_{ar} \approx \omega_{an}$$

$$\omega_r = \omega_n$$

(21)

Therefore, the amplitude gain at the resonant frequency of the system is

$$|G_{r1}(j\omega_n)| = \sqrt{\frac{(k\omega_n^2-\omega_n^2)^2+4k^2\xi_r^2\omega_n^2\omega_n^2}{(\omega_n^2-\omega_n^2)^2+4\xi_r^2\omega_n^2\omega_n^2}} = \sqrt{\frac{(k-1)^2+4\xi_r^2k^2}{4\xi_r^2}}$$

(22)

According to the Eq.(22), the resonant peak of the system is determined by the characteristic parameters $\xi_r$ and $k$. 

5
As can be seen from Fig.5, it shows that the amplitude gain of the transfer function $G_{r2}(s)$ of the load friction torque to the motor speed response is less than 0 dB. Therefore, the load friction torque $T_f$ will not cause the motor speed to oscillate.

4. Parameter sensitivities

Based on the Eq.(15), parametric sensitivities are conducted to analyze how the parameters affect the resonance characteristics under closed-loop condition, and the parameters include $K_p$, $K_v$, $J_l$, $K_s$, as shown in Tab. 1.

![Parameter Sensitivities](image)

Figure 6 Effect of the parameters on the resonance characteristics

When the proportional coefficient of position loop $K_p$ is taken $[350, 400, 450, 500, 550, 600, 650, 700]$ ($s^{-1}$) respectively, the simulation results of the closed-loop frequency characteristics are shown in Fig. 6(a). With the increase of the proportional coefficient of the position loop, the resonant frequency point increases slightly and the resonant peak increases significantly, indicating that the oscillation amplitude of the response speed increases accordingly. At the same time, with the increase of the proportional link of the position loop, the response bandwidth increases, indicating that the time-domain response speed of the system is accelerated. However, the phase angle margin is also significantly reduced, resulting in system instability.

When the proportional coefficient of velocity loop $K_v$ is taken $[0.042, 0.044, 0.046, 0.048, 0.05, 0.052, 0.054, 0.056]$ ($N\cdot ms/\text{rad}$) respectively, the simulation results of the closed-loop frequency characteristics are shown in Fig. 6(b). It can be seen that with the increase of $K_v$, the resonant peak increases significantly, the phase angle margin decreases accordingly, and the bandwidth is almost unchanged.

Figure 6(c) exhibits that the resonant peak amplified with the increasing of the inertias of the output shaft, $J_l$. The inertias of the output shaft is taken $[0.042, 0.048, 0.054, 0.06, 0.066, 0.072, 0.078, 0.084]$ (Kg\cdot m$^2$), respectively.

When the transmission stiffness $K_s$ is taken $[1200, 1400, 1600, 1800, 2000, 2200, 2400, 2600]$ (N\cdot m/rad$^2$) respectively, the simulation results of the closed-loop frequency characteristics are shown in Fig. 6(d). By increasing the stiffness of the CDM, the resonant frequency and resonant peak value of the
system become smaller. With the increase of transmission stiffness, the influence of transmission stiffness on frequency characteristics tends to be gentle.

In general, due to the elastic connection of flexible cable drive, there is an inevitable resonance point in the closed-loop characteristics. The mechanical resonance will be more serious with the increasing of the position loop proportional gain $K_p$, the speed loop proportional gain $K_v$, the inertias of the output shaft $J_o$, or the decreasing of the transmission stiffness $K_T$. This resonance will be reflected in the same frequency oscillation of transmission torque, speed and displacement in the closed-loop system.

5. Conclusions
This paper focused on modeling and analysis of mechanical resonance in CDM for ATPs. A theoretical model with the electro-mechanical and rigid-flexible coupling characteristics is established taking into account cable compliance, control parameters and input signals. Then, parameter sensitivities have been conducted to analyze the relationship between the resonance properties and dynamic parameters. Results showed that the mechanical resonance will be more serious with the increasing of the position loop proportional gain, speed loop proportional gain, the inertias of the output shaft, or the decreasing of the transmission stiffness. The research of this paper is helpful to the design of mechanical resonance suppression controller of the CDM.

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