Theoretical substantiation and calculations of water flow to ranney water intakes and drainages under protection from submergence of the urban territories and buildings by ground water

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Abstract. The methodic of evaluation of the innerdrain hydraulic of the water flow along the length of the drains of the ranney drainage (water uptake) on summary inflow to them is developed. The methodic is based on the mathematical models which allow more fully to take into account the complex real flow character and the hydraulic conditions in the zone of the influence ranney water intakes and drainages. The different cases of solution of joint problem of flow in aquifer and in the drains-rays are considered and presented as the system of the analytic dependences. Obtained results showed that the inflow to drain-ray is distributed uneven along its length and the main inflow is formed in the first half of the drain-ray. Therefore, in some cases the influence of innerdrain hydraulic should be taken into account when calculating the parameters of drainage and evaluation of their operation.

Introduction

When studying the work of underground water intakes and drainages an important problem in the calculation of the inflow of groundwater to them concludes into account the movement of water in these structures. However it should be noted that in most calculations of filtration to horizontal water intakes and drainages the inflow along the length of the drains as well as other hydraulic parameters were assumed to be unchanged and in some cases at small inflows and others the influence of water flow inside the drains will be negligible and can be neglected. These assumptions were used at development of the calculation methods based on the theory flow resistances [1,2]. The efficiency of numerical methods using was presented under solution of the problem of protection from submergence of ground waters urban territory in Kharkov [3]. Comparative analysis of modeling data and regime observations of drawdowns on the protection areas showed a good agreement between ones. But as shown by the results of experiments in more complex filtration schemes of inflow to underground horizontal structures reliable results of calculations of filtration inflow to the drainage can be obtained only by taking into account the hydraulics of water flow inside the drainage. Therefore the methods of filtration calculation of ranney water intakes (drainages) based on the method of filtration resistances and on a position of
uniform distribution of the intensity of inflow \( q(y) \) along drains and in some other cases require additional improving and scientific justification.

**Materials and methods**

As known the main mathematical model of the ajoint flow in aquifer and inside of the horizontal drainages consists of the well-known system of Saint-Venan equations which describes in hydraulic formulation the variable motion of the channel flow [4,5]:

\[
\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA \left( \frac{\partial Z}{\partial x} + S_f + S_e \right) + L + W_f B = 0, \tag{1}
\]

\[
\frac{\partial Q}{\partial x} + \frac{\partial (A q)}{\partial t} = q, \tag{2}
\]

where \( S_f \) – friction, which is determined from the known Manning equation, \( S_e \) – local flow gradient, \( Q(x,t) \) – flow through the cross section, \( A(h) \), \( A_0 \) – respectively the active and inactive cross-sectional areas, \( K_C \) – the coefficient of water conductivity of the channel, \( K_r \) – expansion coefficient (minus sign) and compression one (plus sign), \( B \) – the width of the channel along the water surface, \( W_f \) – correction to the wind, \( q \) – outflow from the channel, \( z(x,t) \) – depth of flow, \( x \) – the distance along the channel, \( R \approx \frac{A}{B} \) – hydraulic radius, \( L \) – lateral possible inflow, \( n \) – roughness.

The value of \( q \) in equation (2) is calculated from the solution of the planned filtration problem which is described by the known equation of transient filtration[6,7]:

\[
\mu \frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \left( T \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left( T \frac{\partial H}{\partial y} \right) + \varepsilon(x,y,t), \tag{4}
\]

where \( H(x,y,t) \) – the level of groundwater, \( \mu(x,t) \) – storage coefficient, \( T \) – coefficient of water transmissivity, \( \varepsilon \) – flow rate per unit surface.

An important problem in the joint interaction of the given equations is in the changing of the model of their conjugation. In [6] the following conjugation criteria are proposed:

a) in the case of hydraulic communication we have:

\[
\frac{\partial H}{\partial u} = \lambda \left( H - (z + z_0) \right), \tag{5}
\]

where \( u \) – normal to the boundary of conjugation, \( z_0 \) – marking of the bottom (river, canal), \( \lambda(x,t) \) – a parameter that takes into account the hydraulic friction which in general will be variable along the length of the drain. Some recommendations relative given parameter are proposed in [4,7].

b) in the absence of hydraulic connection:

\[
Q_n \rightarrow 0 \rightarrow B k_n \frac{z - z_{sp}}{z + z_k}, \quad z > z_{sp}, \tag{6}
\]

\[
Q_n \rightarrow 0 \rightarrow 0, \quad z < z_{sp}, \tag{7}
\]

where \( B(z) \) – the width of the river, \( k_n \) – vertical filtration coefficient, \( z_k \) – the thickness of the impervious layer, \( z_{sp} \)– critical depth below which water infiltration stops.

As a rule the system of equations (1), (2), and (4) is realized numerically under the following initial and boundary conditions:

\[
z(x,0) = z^0(x), \quad Q(x,0) = Q^0(x).
\]
\[ Q(0, t) = Q_0(t), \quad Q(L, 0) = Q_0(0) \]

taking into account the conjugation models.

The joined interaction of the filtration flow to underground water intakes (drainages) and the movement of fluid inside them were investigated when solving various problems of reclamation, protection of areas from flooding, water supply, etc [5]. However mainly these studies were conducted empirically but the impotance of solving this problem lies in the fact that the existing features of the formation of filtration of the uneven inflow to the drains-rays of the finite length can significantly affect on the parameters of the flow inside these drains which must be taken into account in the calculations [7, 8].

With the existing theoretical justification for the calculated dependences on the basis of the known method of filtration resistances it is assumed that the inflow intensity is uniformly distributed along the length of drains \( q(y) \) and at the same time in some cases the equality of the heads along the length of the drainage is not always true [7, 8].

In the first approximation if we accept \( q(y) = \text{const} \) which as shown by field studies in some cases is possible (with a small number of drains – rays, high-quality filter, etc.) the hydraulic losses due the friction and discharge of the drainage area will be next [7, 8]:

\[ \Delta h_y = \frac{Q_0^2}{2gw^2} \left[ 2 \left( \frac{y}{l} \right)^2 + \frac{\lambda y}{3d_o} \left( \frac{y}{l} \right)^2 \right]. \]  
(8)

\[ Q(y) = \frac{Q_0}{l} y, \]  
(9)

and along the full length of the drain \( y = l \) we have

\[ \Delta h_y = \frac{Q_0^2}{2gw^2} \left( 2 + \frac{\lambda l}{3d_o} \right), \]  
(10)

\[ Q(l) = Q_0 \]  
(11)

Analysis of the numerical results of field studies shows that in many practical cases the distribution of the inflow along the length of the drain can be taken by linear law that is in this approximation we have:

\[ q(y) = \frac{2Q_0}{l^2} (l - y). \]  
(12)

where \( Q_0 \) – flow of the entire drain length \( l \).

The water flow (flow rate) in section \( y \) is obtained after integrating the equation (12):

\[ Q(y) = 2Q_0 \left[ \frac{y}{l} - \frac{Q_0}{l} \left( \frac{y}{l} \right)^2 \right]. \]  
(13)

Checking the boundary conditions of equation (13) shows that at the beginning of the drain at \( y = 0 \) we have \( Q = 0 \), and at the end of the drain at \( y = l \) we have \( Q = Q_0 \) that is the equation is correctly integrated. As expected the analysis of equation (13) shows that the most intensive inflow will be from 0 to 0.5 in the first half of the drain which receives 75% \( Q_0 \) about and only 25% \( Q_0 \) in the second half of the drain.

Finally omitting the intermediate laying-outs the head pressure losses on the drain area with a length \( y \) at the linear distribution of the inflow along the drain length are:
\[
\Delta h_y = \frac{Q_y^2}{2gw} \left[ 8 \left( \frac{y}{l} \right)^4 - \left( \frac{y}{l} \right)^3 + \left( \frac{y}{l} \right)^2 \right] + \\
+ \frac{2l}{d_o} \left[ \frac{1}{5} \left( \frac{y}{l} \right)^5 - \left( \frac{y}{l} \right)^4 + \frac{4}{3} \left( \frac{y}{l} \right)^3 \right].
\]

and along the full length \( y = l \) we have
\[
\Delta h_l = \frac{Q_l^2}{2gw} \left( 2 + \frac{8 \lambda l}{15 d_o} \right),
\]

where \( d_o = 2r_0 \), \( w \) – the cross-sectional area of the drain.

To determine the flow rate \( Q_0 \) of each drain-ray the dependencies are used in which the head (level) in the drains is assumed to be constant along their length.

For more accurate calculations of this problem we consider the results based on the solution of the above mathematical models which describe and take into account the joint interaction of water flow in the drain and filtration inflow to it. After linearization and simple transformations of the equations of hydraulics and filtration taking into account of the linear dependence of the evaporation intensity to the depths are reduced to a stationary form:
\[
\pi C d^{5/2} \frac{d^2 H}{ds^2} + 32kh_c \sqrt{\frac{dH}{ds}} \left. \frac{dH}{dx} \right|_{x=0} = 0,
\]

where \( M \) – the ordinate of the surface of the earth, \( h_c \) – the average thickness of the filtration flow, \( h_k \) – critical level below which no evaporation from the groundwater surface occurs, \( \varepsilon_0 \) – intensity of evaporation on the surface of the earth, \( \left. \frac{dH}{ds} \right|_{s_c} \) – the average value of the head gradient along the length of the drain.

The solution of linearized equation (17) with linearized boundary conditions allows to determine the parameters of the filtration flow in the area of the drain and has the form:
\[
h = h_k + \frac{(H - h_k) \sqrt{\alpha(x - L/2)}}{ch(\sqrt{\alpha L/2}) + 2 \sqrt{\alpha} \Phi sh(\sqrt{\alpha L/2})}, \quad \alpha = \frac{\varepsilon_0}{kh_c(M - h_k)}.
\]

From expression (18) we find
\[
\left. \frac{dH}{dx} \right|_{x=0} = \frac{\sqrt{\alpha (H - h_k)} sh(\sqrt{\alpha L/2})}{ch(\sqrt{\alpha L/2} + 2 \Phi \sqrt{\alpha} sh(\sqrt{\alpha L/2}))}.
\]

Substituting expression (19) into (16) we obtain a linear equation with respect to the function \( H(y) \) whose solution under given boundary conditions is obtained in the form:
\[
H(y) = h_k + \frac{(H - h_k) ch(\sqrt{\alpha L/2})}{ch(\sqrt{\beta l_1})} \beta = \frac{32 \sqrt{\alpha} Kh_c}{\pi C d^{5/2} \left[ ch(\sqrt{\alpha L/2}) + 2 \Phi \sqrt{\alpha} sh(\sqrt{\alpha L/2}) \right]}.
\]
Thus by formulas (20) and (18) one can find the values of the piezometric head along the length of the pipe-drain $H(y)$ and the value of the level $h(x)$ in different sections orthogonal to its axis.

The comparative analysis with using the examples of calculations showed that the results of the analytical calculations according to the formulas closely coincide with the results of numerical modeling of the proposed general equations at $t \rightarrow \infty$.

Thus when solving an external (filtration) problem it is necessary to take on the drain ($x = 0$) the value of the head which changes along the coordinate $y$. In this case the groundwater head $h$ value and the drain flow rate $q$ will also change and depend on the coordinate $y$. That to solve the external problem consider the profile (one-dimensional) model of filtration to the imperfect drain in homogeneous soil, namely:

$$T \frac{\partial H}{\partial x} = \mu \frac{\partial H}{\partial t}, \quad T = km_c. \quad (21)$$

Equation (21) is solved under initial and boundary conditions which on an imperfect drainage are taken according to the expression for the head $H_0^0$ which changes inside the drain along its length $y$:

$$x = 0, \quad H = H_0^0 = H_0 - y^2; \quad (22)$$

and the depth (level) of water is assumed to be constant $H_k$ that is at $x = L$ we have a boundary condition $H = H_k$.

The solution of equation (21) under the given boundary conditions is made for the stationary model of filtration in aquifer $\left(\frac{\partial H}{\partial t} = 0\right)$.

The following equation is obtained to determine the level $H$ at section $L$:

$$H = H_0^0(y) + \frac{C_m}{C_i + 1} \left(H_k - H_0^0(y)\right) \quad (23)$$

To determine the flow (magnitude of inflow to the drain in each section) we obtain the dependence:

$$q(y) = \frac{T\left(H_k - H_0^0(y)\right)}{L + 2\Phi_0} \quad (24)$$

where $T = km_c; \quad m_c = m_{oc} + \frac{h_k - m_{oc}}{2}, \quad h_k$ – the depth of water to the ground water level, $m_{oc}$ – the average depth of the drain position with respect to the impermeable layer, $r_{dr}$ filtration resistance.

Thus the total drain-ray discharge of length $l$ will be

$$Q = q_c l. \quad (25)$$

where $q_c$ – the value of the specific inflow to the drain-ray determined by the formula (24) by averaging several values $q(y)$ along the length of the drain $l$.

Recall that in the conditions of lateral inflow from different sources of the flow the flow rate of each i-horizontal ray (drain) of the ranney water intake or drainage is determined by the following universal dependence [1,7,8]:

$$Q_{oi} = q_i l_i = \frac{2\pi T S_{oi} l_i}{F_r + \Phi_r} \quad (26)$$

Then the total inflow of the water intake (drainage) consisting of $N(N \geq 1)$ rays will be equal to

$$Q = \sum_{n=1}^{N} Q_{ni} \quad (27)$$
Here $F_i$ and $\Phi_i$ are known external and internal resistances of the $i$-th drain of length $l$ which do not depend on the regime of water flow in the drains-ray.

The calculation of the drawdown $S_{gi}$ in the $i$-th ray (drain) in the case of taking into account the interdrain hydraulics (parameters of water flow in the drain) in formulas (26) and (27) should be taken as follows:

$$S_{gi}(y) = H_k - H^0_{oi}(y)$$

$$H^0_{oi}(y) = H_{0oi} - \gamma_i \cdot q^2_{oi}.$$ 

Where $H_{0oi}$ and $q_{oi}$ - the mean value of the level in the drain and the specific flow rate determined by the disregard of the interdrain hydraulics, i.e at $\gamma_i q^2_{oi} = 0$.

As in the previous case that to take into account the flow inside the drain we consider a joint solution of the equation in the outer region (filtration problem) and the continuity flow of water in the inner region (hydraulic problem). Then for steady filtration we have the following system of equations, namely: an equation describing the non-stationary fluid flow in a horizontal drain-ray (21) and an equation for discharge:

$$q = \frac{k(H_k - H^0(y))}{\Phi_\rho}.$$ 

(29)

In solving this problem we assume that the filtration resistance $\Phi_\rho$ does not change along the length of the drain-ray, the slope of the drain is taken equal to 0, however if necessary it is easy to take into account in the problem; the drain works by the full cross-section, the inflow to it is distributed evently along the drain, the resistance coefficient $\lambda = const$ and is equal to its average value along the length of the drain.

Thus the radial velocity of water inflow into the drain will be determined as:

$$V_r = \frac{q}{2\pi\rho} = \frac{k(H_k - H^0(y))}{2\pi\rho\Phi_\rho}.$$ 

(30)

The given system of equations is solved under following boundary conditions $y = 0, V = 0, y = l, H^0 = H_{0i}$.

As a result of this solution the following dependencies are proposed to determine the distribution of average velocity and head along the length of the drain – ray $l$:

$$V_\rho = A \left[ \exp(a + b) \frac{y}{2} - \exp(b - a) \frac{y}{2} \right].$$ 

(31)

$$H^0_{oi}(y) = H_k - \left( \frac{aA}{2k} \right) \left[ (a + b) \exp \left( a + b - \frac{y}{2} \right) + (a - b) \exp \left( b - a - \frac{y}{2} \right) \right];$$

(32)

$$A = \frac{2k(H_k - H_{0i})}{\Phi_\rho \omega \left[ (a + b) \exp \left( a + b - \frac{l}{2} \right) + (a - b) \exp \left( b - a - \frac{l}{2} \right) \right]}.$$ 

(33)

$$b = \frac{\sqrt{V_\rho}}{\Phi_\rho \omega}, \quad \xi = \frac{\lambda}{2\rho}, \quad a = \sqrt{b^2 + \xi}.$$ 

If the drain is working in the submerged regime then the average velocity can be determined by the expression:
\[
\bar{V}_{r_0} = \sqrt{\frac{4r_0g(H_k - H_{ao})}{2\pi(h(1 - \frac{1}{ch(ab/2)})}}.
\]  

Since equation (33) is transcendental the velocity \(\bar{V}_{r_0}\) must be determined by selection that is to find such value at which equality (33) is satisfied.

The results of the calculation of the piezometric head \(H\) along the length of the drainage pipe according to formulas (20) and (18) were compared with the corresponding data obtained numerically. Good agreement of the calculative results was obtained [7, 8].

**Conclusions**

The analysis of the existing models and methods of calculation of underground water intakes and drainages shows a significant possible influence on the formation of filtration inflow the hydraulics of the flow inside these structures. It is shown that the special influence of interdrain hydraulics on the formation of filtration inflow can be observed in perforated drain-rays and therefore the influence of interdrain hydraulics should be taken into account when calculating the parameters of drainage and evaluation of their operation [9].

Based on the analysis of existing and the proposed models which take into account the joint influence of water movement in the outer filtration area and in the inner region inside the drain the methodic of calculating the parameters of drains-rays in particular their discharges was developed taking into account the possible loss of pressure due to additional hydraulic resistance arising when water is moving in perforated drains-rays. The results of investigations presented in given work are the part of the general methodic of calculations of the water inflow to ranney water intakes and drainages which includes the calculations on the base of method of flow resistances, numerical modeling of the plane and space problems of water inflow and evaluation of the influence of innerdrain hydraulics on the inflow character to them. Developed methodic allows successfully to solve the complex filtration problems in the areas of influence of ranney water intakes and drainages and may be widely used in projects and building of these structures.

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