Routing and wavelength assignment with protection: A quadratic unconstrained binary optimization approach enabled by Digital Annealer technology

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ABSTRACT
Routing and wavelength assignment with protection is an important problem in telecommunications. Given an optical network and incoming connection requests, a commonly studied variant of the problem aims to grant a maximum number of requests by assigning lightpaths with minimum network resource usage, while ensuring the provided services remain functional in the case of a single-link failure through dedicated protection paths. We consider a version where alternative lightpaths for requests are assumed to be given as a precomputed set and show that it is NP-hard. We formulate the problem as an Integer Programming (IP) model and also use it as a foundation to develop a Quadratic Unconstrained Binary Optimization (QUBO) model. We present necessary and sufficient conditions on objective function parameters to prioritize request granting objective over wavelength-link usage for both models, and a sufficient condition ensuring the exactness of the QUBO model. Moreover, we implement a problem-specific branch-and-cut algorithm for the IP model and employ a new quantum-inspired technology, Digital Annealer (DA), for the QUBO model. We conduct computational experiments on a large suite of nontrivial instances to assess the efficiency and efficacy of all of these approaches as well as two problem-specific heuristics. Although the objective penalty coefficient values that guarantee the exactness of the QUBO model were outside the acceptable range for DA, it always yielded feasible solutions even with smaller values in practice. The results show that the emerging technology DA outperforms the considered techniques coupled with GUROBI in finding mostly significantly better or as good solutions in two minutes compared to two-hour run time, whereas problem-specific heuristics fail to be competitive.

1. Introduction
An optical network is a medium for information transmission via signals encoded in light pulses. It connects devices that generate or store data through optical fibers carrying light channels. With the Wavelength Division Multiplexing (WDM) technology allowing multiple signals to be simultaneously transmitted on the same fiber, optical networks have become particularly potent in conveying high volumes of information reliably at high speeds. As such, they are being increasingly deployed to meet the rapidly growing demand in many high-bandwidth applications such as real-time multimedia streaming, cloud computing and mobile network services (Majumdar, 2018; Chadha, 2019).

Wavelength-routed networks form a broad class of WDM networks, and can be considered as a set of nodes joined by fiber links. The communication between a pair of nodes is established through lightpaths; this is referred to as connecting the two nodes. A lightpath is an optical communication channel between two nodes in the network, and is comprised of a path (route) and a wavelength. A typical problem arising in wavelength-routed networks is the Routing and Wavelength Assignment (RWA) problem. Given a set of connection requests between pairs of nodes, the RWA problem decides which requests to grant, i.e., provision a lightpath, such that no two allocated lightpaths with the same wavelength traverse a common fiber link, to prevent interference.

There are many variants of the RWA problem, which mainly differ in their objective and the nature of the connection demand process, as well as extensions incorporating additional concerns such as failures in the network, service quality and resource usage profile (Bandyopadhyay, 2007). In general, they are all difficult to solve, due to the inherent computational complexity of the RWA problem (Erlebach and Jansen, 2001).

In this article, we study an RWA problem whose features are motivated by practical concerns (e.g., fast recovery from failures) and some common goals in the telecommunications industry (e.g., granted request maximization and resource usage minimization). In what follows, we first provide all the relevant background information and motivate our problem setup (Section 1.1); then formally define our problem, introduce our solution approach and summarize our contributions (Section 1.2).
1.1. Background information

The information provided in this section is mostly based on the books by Mukherjee (2006), Bandyopadhyay (2007) and Chatterjee et al. (2016).

1.1.1. Network failures and recovery schemes

While it is desirable to maximize the number of granted requests in RWA, it is also often necessary to provide some degree of protection for them against potential failures in the network. In a WDM network, any of the components may fail, and one failure may disrupt multiple connections. Link failure is the most frequently encountered type of fault (Bandyopadhyay, 2007), which may, for instance, arise from fiber cuts due to human errors during construction operations, or natural calamity such as earthquakes. Also, the probability of having multiple link failures simultaneously, or having additional failure(s) before one has been repaired, is considered to be negligible. Thus, the literature is mostly concerned with the case of a single-link failure, which we follow as well.

As each fiber link can carry terabits of data per second, even a brief disruption of a connection can result in a large amount of data loss. As such, fault management mechanisms play an important role in WDM network survivability (Zhang and Mukherjee, 2004). Link failures can be handled at the optical or a higher layer, but the time to detect and repair a failed link ranges from tens of seconds to a few days. Since even the shortest repair time is still too long relative to the rate of data transfer, some fault recovery strategies, which re-route the broken connections using the available network resources, are usually adopted to keep the services functional while the repair process is in progress. There are two main categories of such strategies, protection and restoration. In the protection scheme, backup (protection) lightpaths are computed and reserved in advance along with the primary (working) lightpaths. This ensures fast recovery of all the affected connections by replacing the primary lightpaths with the backups (in milliseconds upon failure), at the expense of increased network resource usage. In the restoration scheme, on the other hand, backup capacity is not provisioned prior to the occurrence of failure, instead new lightpaths are discovered dynamically upon the interruption of connections. Despite being less demanding in resource usage, restoration schemes fail to guarantee resource availability, and lead to higher recovery times.

Protection and restoration schemes for link failures can be categorized with respect to being path- or link-based, i.e., whether they re-route the whole path or only the failed link(s). Path protection schemes can be dedicated or shared. When it is dedicated, each backup lightpath is reserved for only one primary lightpath, and two backup lightpaths with the same wavelength cannot have a common link on their routes. Two commonly used subcategories of this scheme are denoted by 1:1 and 1 + 1. The former transmits data through only primary lightpaths before failure, whereas the latter allows the use of both primary and backup lightpaths simultaneously.

1.1.2. Static and dynamic RWA

The underlying demand process in RWA problems is considered as static or dynamic. The static case assumes that connection requests with their associated source and destination nodes are known in advance. In the dynamic case, on the other hand, requests arrive one by one and are provisioned a lightpath in real time. In our study, we aim to efficiently solve a static RWA problem.

We note that a dynamic RWA problem can be approximated by discretizing the time horizon into intervals and solving a static RWA problem for each using the batch of requests that arrived during the corresponding interval, as typically done for dynamic network reconfiguration problems (Zhang et al., 2007; Wu et al., 2012; Grover, 2013). Such an approach would usually require solving the static problems quickly, e.g., for some bandwidth on demand services, their service level agreement between the provider and the customer guarantees connections to be established (with a certain level of protection) in a matter of minutes (Fawaz et al., 2004; Losego et al., 2005). In that regard, the methods solving the static RWA problem efficiently may well benefit dynamic settings. In fact, one method we propose can generate good-quality solutions in up to 2 minutes.

1.1.3. Precomputed paths

One way to improve solution times for RWA problems is to employ a two-phase framework; generating a set of paths between all or some potential source and destination pairs in the first phase, and picking one from the precomputed alternatives for each request and doing the wavelength assignment in the second phase, for instance as adopted in Li and Simha (2000) and Noronha and Ribeiro (2006). In cases where an RWA problem needs to be solved repeatedly over time, this strategy can be made more efficient by performing the first phase only once at the beginning, i.e., using the same set of precomputed paths throughout the horizon. Granting requests from their precomputed set of alternatives may also provide more control to decision makers, in the sense that they can disperse the demand over the network in a balanced way if desired and can have a better idea on the state of the network for subsequent decision-making stages well in advance.

1.2. Our work

1.2.1. Problem definition

In the light of discussions in the background section, in this study, we consider the RWA problem with static demand and a 1:1 dedicated path protection scheme against single-link failures, for a given network with a set of precomputed alternative primary and backup paths and a number of available wavelengths. We call this the Routing and Wavelength Assignment with Protection (RWA-P) problem, where the aim is to primarily maximize the number of granted requests, since this brings the actual gains for the service providers (Shen et al., 2005), while minimizing the
wavelength-link usage as a secondary goal to save network resources for future demands.

1.2.2. Outline of our work
We show that RWA-P is NP-hard, propose mathematical models and evaluate their solution performance using promising technologies. More specifically, we propose an Integer Programming (IP) model for RWA-P as well as a strengthened version of it, and we use it as a basis to develop a novel Quadratic Unconstrained Binary Optimization (QUBO) model. We present conditions for model parameters to achieve the intended objective prioritization of granted requests over resource usage and to ensure that the QUBO model is exact. We use a state-of-the-art exact solver, GUROBI, to solve the IP models, both by directly feeding them to the solver and by applying a problem-specific branch-and-cut method. In order to derive solutions for the QUBO model, we employ a new technology called Digital Annealer (DA) and also test GUROBI. We conduct computational experiments on a large suite of instances based on commonly used networks, compare the efficiency and efficacy of our methods and two problem-specific heuristics from the literature, and analyze the effect of the penalty coefficient parameter of the QUBO model on DA’s performance.

1.2.3. Choice of solution technologies
IP has been a widely adopted modeling framework for discrete optimization problems, including RWA problems. This can be attributed to the development of many efficient solution methodologies and enhancements such as the branch-and-cut algorithm, presolve techniques and heuristics. Despite their advanced status, state-of-the-art IP solvers may take prohibitive times to yield optimal solutions, especially for large-scale problems, because run times are typically exponential in the size of the input model. However, for many practical problems, it is sufficient to obtain a good-quality solution, as also noted in Giovanni (2017), but preferably in a reasonable/short amount of time. Although IP solvers can be tuned to generate feasible solutions faster, they may fail to address both of these practical considerations simultaneously.

A plausible alternative to tackle such problems is to formulate them as QUBO models and generate solutions via novel computational architectures and new technologies, such as adiabatic quantum computing (e.g., Papalitsas et al., 2019), neuromorphic computing (e.g., Corder et al., 2018), and optical parametric oscillators (e.g., Inagaki et al. 2016), which have recently attracted significant attention due to their capability in tackling combinatorial optimization problems. A promising example to these new technologies is DA (Aramon et al., 2019), which is a computer architecture that rivals quantum computers in utility (Boyd, 2018). DA is designed to solve QUBO models, and uses an algorithm based on simulated annealing. In many applications, such as the minimum vertex cover problem (Javad-Kalbsasi et al., 2019), maximum clique problem (Naghsh et al., 2019) and outlier rejection (Rahman et al., 2019), it has been shown to significantly improve upon the state of the art and yield high-quality solutions in radically short amount of time.

1.2.4. Contributions
The contributions of this paper are as follows:

- We prove that RWA-P is NP-hard.
- We develop IP and QUBO models, and propose conditions on parameter values to ensure their validity in the sense that the intended objective prioritization is achieved. Moreover, we propose sufficient conditions establishing the exactness of the QUBO model.
- Our QUBO approach is the first of its kind in the vast RWA literature and observed to be highly efficient and efficacious in solving a large suite of nontrivial test instances.
- We conduct a detailed computational analysis and compare the performance of the emerging DA technology to established IP methods. Considering that DA is rarely employed in the operations research literature and only few studies compare it with the state of the art, e.g., (Naghsh et al., 2019; Ohzeki et al., 2019; Matsubara et al., 2020; Şeker et al., 2022), our study serves as a step to bridge the gap between the use of the established and new promising solution technologies.
- We show through sensitivity analysis that smaller penalty coefficient values lead to better solutions for DA, which has been mentioned in few studies previously, e.g., (Cohen, Mandal, Ushijima-Mwesigwa, and Roy 2020) and Cohen, Senderovich and Beck (2020), and recently tested more thoroughly in (Şeker et al., 2022) for different problems.

1.2.5. Article outline
The rest of this article is organized as follows. We review the related literature in Section 2. In Section 3, we present an IP formulation for RWA-P, provide a prioritization condition and discuss problem complexity. In Section 4, we introduce a QUBO model for RWA-P, derive a condition that renders it exact, and overview the operating principles of DA. In Section 5, we present our computational study, and finally in Section 6, we conclude with a brief summary.

2. Related literature
There are many variants of the RWA problem, and a vast number of studies on each type. Here, we restrict our review to the RWA problem with dedicated path protection scheme and static demand, and summarize the most relevant studies below.

Ramamurthy et al. (2003) examine different protection approaches for single-link failures, and develop IP formulations for path- and link-based schemes, assuming that a precomputed set of alternate routes are given. Their model for dedicated path protection aims to minimize the total number of wavelengths used over all links in the network, while
enforcing that all the demand is satisfied and the wavelength capacity on the links is not exceeded. Using test instances generated for a representative network topology, they compare the performance of the proposed IP models. Their problem setup differs from ours in that they do not allow unsatisfied demand assuming sufficient capacity in the network, as such use a different objective than we do.

Azodolmolky et al. (2010) study the RWA problem with dedicated path protection, where they assume a precomputed set of pairs of primary and backup paths per request, i.e., each primary path has an associated unique backup path. They present two IP models, which form the basis of their heuristic algorithm designed for the impairments aware RWA problem. The first model considers the requests that require only a primary lightpath, and aims to minimize the total number of requests that are not accepted. The second model extends the first one by: (i) adding another demand category that necessitates backup lightpaths as well; and (ii) minimizing a combined sum of the former objective and the maximum number of times a wavelength is used on a link, in a way that acceptance of the requests requiring protection are prioritized over the ones that do not, and the wavelength usage is of least priority. The problem setting used for their IP models is similar to ours, especially when it is assumed that no unprotected demand exists. However, it does not allow the primary and backup path options of a request to be arbitrarily combined; they are instead considered in predefined pairs only. The authors test the performances of their proposed heuristic algorithm, two versions of the heuristic from the work of Ezzahdi et al. (2006) that they enhance with quality of transmission considerations, and solving of the presented IP formulations via an exact solver. In our study, we also consider Ezzahdi et al.’s (2006) heuristic and adapt an additional one for RWA-P by combining Azodolmolky et al.’s (2010) proposed heuristic strategy with another one used in various previous works, e.g., (Mokhtar and Azizoglu, 1998; Koganti and Sidhu, 2014), and we compare these two heuristics to our presented methods in Section 5.

We note that, in addition to the aforementioned problem setup differences, our study stands apart from those works in terms of modelling. While the previous IP formulations are link-based, our models are path-based. Also, we present an exact QUBO model, the first for an RWA problem although a constrained binary quadratic model has been used in Ebrahimizadeh et al. (2013).

There are some other relevant works that consider the RWA problem with protection for single-link (or node) failures, either with precomputed paths (Lee and Park, 2006), or by solving both the routing and the wavelength assignment problems simultaneously (Wang et al., 2001; Li et al., 2002) or sequentially (Kokkinos et al., 2010).

Lastly, we note that for many variants of the RWA problem, the complexity has been established to be NP-hard, see for instance (Chlamtac et al., 1992; Chiu and Modiano, 2000; Li and Simha, 2000; Erlebach and Jansen, 2001). Despite some structural similarities between those variants and our problem, the complexity of RWA-P has remained open.

3. IP formulation and problem complexity

We first present an IP formulation for the RWA-P problem in Section 3.1, followed by Section 3.2 where we propose values for the weight parameters used in combining two objectives, namely granted request maximization and link usage minimization, into a single one in order to provably achieve the desired prioritization of the former over the latter. In Section 3.3, we introduce a strengthened version of a group of constraints in the IP formulation, and lastly in Section 3.4, we show the complexity of RWA-P through a reduction from a well-known NP-complete problem.

3.1. IP formulation

As formally defined in Section 1.2, the RWA-P problem aims to grant a maximum number of requests by properly assigning a working and a protection lightpath to each from a precomputed collection, while minimizing the wavelength-link usage as a secondary goal, which we hereafter refer to as link usage for simplicity.

We model an optical network as a directed graph $G = (V, E)$, with $V$ and $E$ respectively denoting the set of nodes and the set of directed edges that join ordered pairs of nodes, where it is possible to have multiple arcs with the same start and end nodes. In the telecommunications context, we refer the directed edges of the input graph as links. We denote the set of requests by $R$, and the set of wavelengths by $\Lambda$. For each request $r \in R$, we represent the set of alternative working and protection lightpaths with $W^r$ and $P^r$, respectively, which are obtained by combining the available precomputed set of paths and wavelengths. The length of a working (protection) lightpath $w(p)$ for request $r \in R$, i.e., the number of the links it contains, is denoted by $B_w^r(B_p^r)$. For convenience, we use $E[\ell]$ to represent the set of links that a given lightpath $\ell$ contains, and $\Lambda[\ell]$ for the wavelength associated with $\ell$.

In order to help compactly represent the constraints of RWA-P, we define four conflict sets, $C_1, \ldots, C_4$. The first conflict set $C_1$ serves to enforce the pair of working and protection lightpaths for a given request to be link-disjoint. It is comprised of $(r,w,p)$ triplets such that the working lightpath $w$ and the protection lightpath $p$ for request $r$ have at least one link in common. Namely,

$$C_1 := \{ (r,w,p) : r \in R, w \in W^r, p \in P^r, E[w] \cap E[p] \neq \emptyset \}.$$

The remaining three conflict sets are used to prevent the concurrent use of lightpaths having the same wavelength and sharing a link. Considering such lightpaths in pairs, there can be one working and one protection, two working, or two protection lightpaths fulfilling these criteria, which we address through sets $C_2$, $C_3$, and $C_4$, respectively. Let $C_2$ be the set of $(r_1,r_2,w,p)$ quadruplets such that the working and protection lightpaths $w$ and $p$ for distinct requests $r_1$
and \( r_2 \) have the same wavelength and at least one link in common:
\[
C_2 := \{(r_1, r_2, w, p) : r_1, r_2 \in R, r_1 \neq r_2, w \in W^w, p \in P^p, \Lambda[w] = \Lambda[p], E[w] \cap E[p] \neq \emptyset\}.
\]

The sets \( C_3 \) and \( C_4 \) contain a similar collection of quadruplets as \( C_2 \) does, but with only working and only protection lightpaths, respectively:
\[
C_3 := \{(r_1, r_2, w_1, w_2) : r_1, r_2 \in R, w_1 \in W^w, w_2 \in W^w, \Lambda[w_1] = \Lambda[w_2], E[w_1] \cap E[w_2] \neq \emptyset\},
\]
\[
C_4 := \{(r_1, r_2, p_1, p_2) : r_1, r_2 \in R, p_1 \in P^p, p_2 \in P^p, \Lambda[p_1] = \Lambda[p_2], E[p_1] \cap E[p_2] \neq \emptyset\}.
\]

**Example 1.** In Figure 1, an example RWA-P network is illustrated with two requests together with their working and protection lightpath alternatives. The source and destination nodes of the two requests are those with \( r_1 \) and \( r_2 \) labels for \( r \in \{1, 2\} \), respectively. The lightpaths are shown with red and green, where each color symbolizes a distinct wavelength, and the lines being solid or dashed indicate whether the lightpath is in the working or protection set, respectively. The request and working/protection indices of the lightpaths are shown beside them in the same color as the lines representing them. For request 1, there are three working and one protection lightpaths, and for request 2, there is one working and two protection lightpaths. (In the instances we use in our computational experiments, working and protection lightpaths of each request are formed by combining each precomputed path with every one of the available wavelengths; see Section 5.1.)

Let us give some example tuples for the conflict sets using the network in Figure 1. The link \((c, t')\) is common in the lightpaths labeled with \( r = 1, w = 2 \) and \( r = 1, p = 1 \), which yields \((r, w, p) = (1, 2, 1) \in C_1\). Furthermore, the link \((s', b)\) being contained in two lightpaths having the same wavelength (red) makes \((r_1, r_2, w_1, w_2) = (1, 2, 3, 1) \in C_3\), and \((s', c)\) being shared by two protection lightpaths with the same wavelength (green) leads to \((r_1, r_2, p_1, p_2) = (1, 2, 1, 2) \in C_4\). Since there is no pair of distinct requests whose working and protection lightpaths have the same wavelength, \( C_2 = \emptyset \) here.

In this example, it is possible to accept both requests by selecting the working and protection lightpaths \( w = 1 \) and \( p = 1 \) for request \( r = 1 \), and also for \( r = 2 \). This solution is indeed the best for link usage as well, because having granted all of the given requests with lightpaths of length two, it is not possible to use any fewer links as each lightpath is of length at least two in this example.

Using the notation introduced above and two sets of binary decision variables defined as
\[
x'_{w} = \begin{cases} 1, & \text{if working lightpath } w \in W^w \text{ is assigned to request } r \in R \\ 0, & \text{otherwise} \end{cases}
\]
\[
y'_{p} = \begin{cases} 1, & \text{if protection lightpath } p \in P^p \text{ is assigned to request } r \in R \\ 0, & \text{otherwise} \end{cases}
\]
we now present a novel IP formulation as follows:

\[
\min \ f(x, y) := \alpha \sum_{r \in R} \left( \sum_{w \in W^w} B^w_{r} x'_{w} + \sum_{p \in P^p} B^p_{r} y'_{p} \right)
\]
\[
- \beta \sum_{r \in R} \sum_{w \in W^w} x'_{w}
\]
\[
\text{s.t. } \sum_{w \in W^w} x'_{w} - \sum_{p \in P^p} y'_{p} = 0 \quad r \in R \tag{1b}
\]
\[
\sum_{w \in W^w} x'_{w} \leq 1 \quad r \in R \tag{1c}
\]
\[
x'_{w} + y'_{p} \leq 1 \quad (r, w, p) \in C_1 \tag{1d}
\]
\[
x'_{w} + y'_{p} \leq 1 \quad (r_1, r_2, w, p) \in C_2 \tag{1e}
\]
\[
x'_{w_1} + x'_{w_2} \leq 1 \quad (r_1, r_2, w_1, w_2) \in C_3 \tag{1f}
\]
\[
y'_{p_1} + y'_{p_2} \leq 1 \quad (r_1, r_2, p_1, p_2) \in C_4 \tag{1g}
\]
\[
x'_{w}, y'_{p} \in \{0, 1\} \quad r \in R, \ w \in W^w, \ p \in P^p \tag{1h}
\]
where \( \alpha \) and \( \beta \) are predetermined positive constants.

Constraint set (1b) enforces that the same number of working and protection lightpaths are selected to grant a request, and (1c) ensures that at most one working lightpath is assigned to each request. Constraint set (1d) guarantees that the selected working and protection lightpaths for each request are link-disjoint, while (1e)–(1g) make sure that the lightpaths having the same wavelength and sharing a link are not chosen simultaneously. Finally, constraint set (1h) states the domains of the decision variables.

The objective function (1a) combines the two goals of RWA-P, minimizing the number of links used and maximizing the number of requests granted, as a weighted sum. The former goal is associated with the economical use of network resources in order to leave more resources for future demands. The latter, on the other hand, is a commonly used goal relating to revenue maximization for the service provider when network resources are not sufficient to accommodate all connection requests, as in our case. Our aim is to primarily maximize the number of granted requests, but we also incorporate link usage as a secondary goal. Therefore, the latter goal must be prioritized over the former, which we detail next.

![Figure 1](image-url)
3.2. Objective prioritization

We now formally define what prioritization of request granting over link usage means, and propose $\alpha$ and $\beta$ values that serve the purpose in (1a). We first introduce some notation to be used in the rest of this section. Let $f_x(x, y)$ and $f_\beta(x)$ be two functions respectively corresponding to the number of links used and the number of requests granted at a solution, i.e.,

\[
    f_x(x, y) := \sum_{r \in R} \left( \sum_{w \in W_r} B_w^r x_w + \sum_{p \in P} B_p^r y_p \right),
\]

\[
    f_\beta(x) := \sum_{r \in R} \sum_{w \in W_r} x_w^r,
\]

so that the objective function (1a) can be equivalently written as

\[
    f(x, y) = \alpha f_x(x, y) - \beta f_\beta(x).
\]

Furthermore, to ease the presentation, for any given feasible solution $(\hat{x}, \hat{y})$ (where $\hat{
}$ represents any operator such as hat, tilde, and bar), we define the associated IP objective value and its components as

\[
    \hat{f} := f(\hat{x}, \hat{y}), \quad \hat{f}_x := f_x(\hat{x}, \hat{y}), \quad \hat{f}_\beta := f_\beta(\hat{x}),
\]

and the worst-case and best-case link usage of a feasible solution granting the same number of requests as

\[
    f_x^\max := \max \left\{ f_x(x, y) \mid (x, y) \text{ satisfies } (1b) - (1h), f_\beta(x) = \hat{f}_\beta \right\},
\]

\[
    f_x^\min := \min \left\{ f_x(x, y) \mid (x, y) \text{ satisfies } (1b) - (1h), f_\beta(x) = \hat{f}_\beta \right\}.
\]

Definition 1 (Prioritization Condition). Request granting is prioritized over link usage, if for any pair of feasible solutions $(\hat{x}, \hat{y})$ and $(\tilde{x}, \tilde{y})$ with $\tilde{f}_\beta > \hat{f}_\beta$, we have $\hat{f} < \tilde{f}$, i.e., the marginal contribution of granting a request to the objective function (1a) is always negative. That is, $\hat{f}_x^\max - \hat{f}_\beta < \tilde{f}_x^\max - \tilde{f}_\beta$ for all feasible $(\hat{x}, \hat{y})$ and $(\tilde{x}, \tilde{y})$ with $\tilde{f}_\beta > \hat{f}_\beta$.

Considering the largest and smallest realizations of the left- and right-hand sides in terms of link usage, respectively, the prioritization condition can be equivalently written as

\[
    \hat{f}_x^\max - \hat{f}_\beta < \hat{f}_x^\min - \hat{f}_\beta \quad \text{for all feasible } (\hat{x}, \hat{y}) \text{ and } (\tilde{x}, \tilde{y}) \text{ with } \tilde{f}_\beta > \hat{f}_\beta.
\]

This condition can also be expressed with the help of an optimization model:

\[
    \frac{\delta}{\alpha} > \Omega^\min := \max \left\{ \frac{f_x^\max - f_x^\min}{f_\beta - \hat{f}_\beta} : (\hat{x}, \hat{y}) \text{ and } (\tilde{x}, \tilde{y}) \text{ are} \right\},
\]

\[
    \text{feasible with } \hat{f}_\beta > \tilde{f}_\beta.
\]

Indeed, it is possible define another optimization model by only considering the solutions differing by one in their number of granted requests,

\[
    \Omega^\min := \max \left\{ \frac{f_x^\max - f_x^\min}{f_\beta - \hat{f}_\beta} : (\hat{x}, \hat{y}) \text{ and } (\tilde{x}, \tilde{y}) \text{ are feasible with } \hat{f}_\beta > \tilde{f}_\beta + 1 \right\}.
\]

which would achieve what (3) does, as provided in the proposition below.

Proposition 1. The optimization models in (3) and (4) yield the same optimal values; that is, $\Omega^\min = \Omega^\min$. This implies that the prioritization condition given in (3) can also be achieved by setting $\alpha, \beta > 0$ such that $\frac{\delta}{\alpha} > \Omega^\min$.

Proof. See Appendix A.1. \qed

For practical purposes, we assume $\alpha = 1$ and that $\beta$ can only take integer values. In this case, letting $\beta^{\text{Tight}}$ denote the smallest integer $\beta$ value satisfying the prioritization condition in (4), we have $\beta^{\text{Tight}} = 1 + \Omega^\min$. However, obtaining $\beta^{\text{Tight}}$ or at least a reasonable upper bound for it necessitates solving of a non-trivial optimization problem, which could be computationally even more expensive than the original RWA-P problem. Next, we derive sufficient condition for prioritization in terms of given instance parameters.

Proposition 2 (IP objective weight selection). Selecting $\alpha, \beta > 0$ such that

\[
    \frac{\delta}{\alpha} > |R| (M - 2) + 2
\]

prioritizes request granting over link usage in (1a) for any feasible solution to the IP, i.e., solutions accepting more requests yield lower objective values, where $M = \max_{r \in R} \{ \max_{w \in W_r} \{ B_w^r \} + \max_{p \in P} \{ B_p^r \} \}$.

Proof. See Appendix A.2. \qed

Proposition 2 provides a lower bound on $\frac{\delta}{\alpha}$ that is sufficient to make request granting the primary goal. Note that computing this lower bound does not involve solution of an optimization problem, and hence, makes it easy to decide on safe objective parameter combinations for a given instance. Letting $\beta^{\text{Base}}$ denote the least possible $\beta$ value from the condition in (5) (assuming $\alpha = 1$), we have $\beta^{\text{Base}} = |R| (M - 2) + 3$. Note that by definition $\beta^{\text{Base}} > \beta^{\text{Tight}}$.

We now show that there exist examples where the bound in (5) is indeed tight.

Proposition 3 (Tight example for the weight selection). There exist RWA-P instances for which the lower bound provided in Proposition 2 is necessary to prioritize request granting over link usage.

Proof. See Appendix A.3. \qed

3.3. Strengthened conflict constraints

We can strengthen the set of conflict constraints in (1d)–(1g) by identifying larger groups of variables that are
mutually in conflict, i.e., groups in which at most one variable can take value one. To this end, we first construct the set \( P^r_w \) of all protection lightpaths for request \( r \) that have at least one common link with working lightpath \( w \), as an extension of the lightpath pairs defined in \( C_1 \). That is, for every \( w \in W^r \) and request \( r \in R \), we define
\[
\bar{P}^r_w := \{ p \in P^r : E[w] \cap E[p] \neq \emptyset \}.
\]

Next, we define the sets of working and protection lightpaths that contain link \( e \) and wavelength \( \lambda \), for every wavelength \( \lambda \in \Lambda \), link \( e \in E \), and request \( r \in R \), which will extend the lightpath pairs defined in conflict sets \( C_2, C_3 \) and \( C_4 \). Namely,
\[
\bar{W}^r_{e, \lambda} := \{ w \in W^r : \Lambda[w] = \lambda, e \in E[w] \},
\bar{P}^r_{e, \lambda} := \{ p \in P^r : \Lambda[p] = \lambda, e \in E[p] \}.
\]

Using these sets, we can write a strengthened form of our conflict constraints as
\[
x^r_w + \sum_{p \in \bar{P}^r_w} y^r_p \leq 1 \quad r \in R, w \in W \tag{6a}
\]
\[
\sum_{r \in R} \sum_{w \in \bar{W}^r_{e, \lambda}} x^r_w + \sum_{r \in R} \sum_{p \in \bar{P}^r_{e, \lambda}} y^r_p \leq 1 \quad e \in E, \lambda \in \Lambda. \tag{6b}
\]

In the rest of this article, we refer to the IP model in (1) as \( \text{IPBase} \), and to that with the conflict constraints in (1d)–(1g) being replaced with the ones in (6a)–(6b) as \( \text{IPStrong} \).

### 3.4. Complexity

See Appendix B for a detailed complexity discussion concluding with the NP-hardness result of RWA-P through a reduction from the maximum stable set problem.

### 4. QUBO formulation and solution method

We now present our proposed modeling and solution approach for RWA-P. In Section 4.1, we develop a QUBO model by dualizing the constraints of \( \text{IPBase} \), i.e., by adding them as a penalty term to the objective, and we give a condition for the penalty parameters to ensure the exactness of the model. In Section 4.2, we give an overview of the Digital Annealer technology and its operating principles.

#### 4.1. Transformation to QUBO

As the first step of obtaining an exact QUBO formulation, we dualize the constraints of our IP formulation given in (1), \( \text{IPBase} \), in such a way that any infeasible solution to it, i.e., any constraint violation, yields a strictly positive penalty term in the objective function of our QUBO model. This is easy to achieve for equality constraints; any linear equality constraint can be transformed into a penalty term by simply taking the square of the difference of its left and right-hand sides, so that any constraint violation translates into a positive penalty value, and hence can be avoided through the minimization of the objective. Therefore, for our only set of equality constraints (1b), the corresponding penalty term includes for each \( r \in R \) the following squared violation expression:
\[
\left( \sum_{w \in W^r} x^r_w - \sum_{p \in P^r} y^r_p \right)^2,
\]

which amounts to a positive value when more working lightpaths than protection lightpaths are selected for the request \( r \), or vice versa.

In case of inequality constraints, however, more custom-tailored approaches are needed, because violations occur in one direction only. In order to transform the inequality constraints in (1c)–(1g) into penalty terms, we first reformulate them as quadratic equality constraints in (7):
\[
\left( \sum_{w \in W^r} x^r_w \right) \left( \sum_{p \in P^r} y^r_p - 1 \right) = 0 \quad r \in R \tag{7a}
\]
\[
x^r_w y^r_p = 0 \quad (r, w, p) \in C_1 \tag{7b}
\]
\[
x^r_{w1} x^r_{w2} = 0 \quad (r_1, r_2, w_1, w_2) \in C_2 \tag{7c}
\]
\[
x^r_{w1} x^r_{p2} = 0 \quad (r_1, r_2, w_1, p_2) \in C_3 \tag{7d}
\]
\[
y^r_{p1} y^r_{p2} = 0 \quad (r_1, r_2, p_1, p_2) \in C_4 \tag{7e}
\]

Constraint set (7a) is the quadratic equivalent of (1c) ensuring that at most one lightpath is selected per request. As the decision variables are binary, the left-hand side of (1c), i.e., the expression denoting the total number of working lightpaths assigned to request \( r \), can take value either zero or one, in which case the left-hand side of (7a) becomes zero. So, (7a) holds only when the corresponding original constraint (1c) is satisfied, otherwise, i.e., when \( \sum_{w \in W^r} x^r_w \geq 2 \), the left-hand side of (7a) takes a strictly positive value. Therefore, the left-hand side of (7a) can be used as a penalty term for violations of constraints (1c). Similarly, constraints (1d)–(1g), which ensure that two conflicting lightpaths cannot be selected simultaneously, are violated only when both variables on the left take value one; all other configurations of the two binary variables are feasible. The quadratic constraints (7b)–(7e) take advantage of the fact that all feasible configurations involve at least one variable having value zero, and force the product of the two to be zero. So, when the associated constraints are violated, the left-hand sides of (7b)–(7e) take strictly positive values, namely value one, thus serve as penalty terms to be added to the objective function of our QUBO model. Note that the magnitude of violation that an infeasible binary solution creates in any one of the constraints (1b)–(1g) is at least one, which has a useful role in rendering our QUBO formulation exact, as we will see when we specify possible values of the penalty coefficient in the following.
We present our QUBO formulation for RWA-P in (8):

\[
\begin{align*}
\min_{\mathbf{r} \in R} & \quad \sum_{w \in W'} \left( \sum_{\mathbf{w} \in W'} B_{w}^r x_{w}^r + \sum_{p \in P'} B_{p}^r y_{p}^r \right) - \beta \left( \sum_{\mathbf{r} \in R} \sum_{w \in W'} x_{w}^r \right) \\
& + \rho \sum_{\mathbf{r} \in R} \left( \sum_{w \in W'} y_{w}^r - \sum_{p \in P'} y_{p}^r \right)^2 \\
& + \rho \sum_{\mathbf{r} \in R} \left( \sum_{w \in W'} x_{w}^r - 1 \right) \left( \sum_{w \in W'} x_{w}^r \right) + \rho \sum_{(r, w, p) \in C_1} x_{w}^r y_{p}^r \\
& + \rho \sum_{(r_1, r_2, w, p) \in C_2} x_{w}^{r_1} y_{p}^{r_2} + \rho \sum_{(r_1, r_2, w_1, w_2) \in C_3} x_{w_1}^{r_1} x_{w_2}^{r_2} \\
& + \rho \sum_{(r_1, r_2, p_1, p_2) \in C_4} y_{p_1}^{r_1} y_{p_2}^{r_2},
\end{align*}
\]

(8a)

\[
s.t. \quad x_{w}^r, y_{p}^r \in \{0, 1\} \quad r \in R, w \in W', p \in P',
\]

(8b)

where \( \rho > 0 \) is the penalty coefficient for the dualized constraints. We note that different penalty coefficients can be used for different terms; however, we choose them to be all the same, \( \rho \), to simplify our derivation of a valid lower bound for it.

We note that IP\textsuperscript{Strong} can be transformed into a QUBO model too, in which case the strengthened conflict constraints in (6a)–(6b) would simply be dualized in the same manner constraints in (1c) are dualized (see quadratic equalities in (7a)), yielding sums of bilinear penalty terms in the objective similar to those corresponding to the conflict constraints in (1d)–(1g). Thus, the two QUBO models can be made equivalent by customizing the penalty coefficient values for the related terms.

Next, we investigate conditions ensuring that an optimal solution to the QUBO model is also optimal for the RWA-P problem.

**Definition 2** (Exactness).

Let \( \mathcal{P} \) be a problem with optimal objective value \( f^* \). A model \( \mathcal{M} \) for problem \( \mathcal{P} \) is exact if any optimal solution to \( \mathcal{M} \) is feasible for \( \mathcal{P} \) and has objective value \( f^* \).

By construction, IP\textsuperscript{Base} (and also IP\textsuperscript{Strong}) is an exact model for RWA-P. For the QUBO model to be exact, however, the penalty coefficient \( \rho \) should be chosen "sufficiently large". Since high valued parameters may lead to numerical issues, smaller "safe" values are desirable. In that regard, we provide a lower bound for \( \rho \) which is sufficient to guarantee that the QUBO model is exact.

**Proposition 4** (QUBO penalty selection). When

\[
\rho > \beta(|R| + 1) - \alpha \left( 1 + \sum_{\mathbf{r} \in R} \left( B_{w_{\text{min}}}^r + B_{p_{\text{min}}}^r \right) \right),
\]

(9)

(8) is an exact QUBO model for the RWA-P problem, where \( B_{w_{\text{min}}}^r = \min_{w \in W'} \{ B_{w}^r \} \) and \( B_{p_{\text{min}}}^r = \min_{p \in P'} \{ B_{p}^r \} \).

**Proof.** See Appendix A.4. \qed

It is important to note that the condition in (9) not only guarantees the exactness of the QUBO model, but also ensures that any infeasible solution for the problem is inferior to the feasible ones. We chose to impose this stronger requirement in deriving the lower bound on the penalty coefficient in order to establish a clear dominance relationship between the classes of feasible and infeasible solutions, which we believe leads to a conceptually better QUBO model. We also note that the resulting lower bound is not tight, as far as the original definition of exactness is concerned. If \( \rho_{\text{base}} \) is a penalty coefficient abiding (9), then similar to the objective weight parameter discussion in the IP case, we can actually design an optimization model to obtain the smallest possible penalty coefficient value, \( \rho_{\text{Tight}} \). However, the resulting model would be much more complex (e.g., a 0-1 quadratic fractional programming model).

Given a QUBO model, we can optimally solve it using the state-of-the-art solvers such as GUROBI (Gurobi Optimization LLC, 2020) and CPLEX (IBM ILOG, 2019). However, if the model is originally constrained and linear, as it is in our case, a more favorable approach would be to use these solvers to solve the IP formulations, in which they are particularly successful. The performance of IP solvers tend to deteriorate as the number of variables and constraints increases, with an exponential rise in solution times typically. For problems suitable to be formulated as a QUBO model, a promising alternative is the DA technology, which demonstrates a robust level of performance across instances of different sizes, as long as the number of variables does not exceed the allowed variable capacity. Next, we provide some information on this technology.

**4.2. The Digital Annealer**

The Digital Annealer (DA) is a quantum-inspired computer architecture designed to derive solutions for combinatorial optimization problems formulated as a QUBO model. It consolidates the merits of both quantum and general-purpose computers, and takes advantage of the massive parallelization that its hardware allows (Aramon et al., 2019; Sao et al., 2019). The first generation of DA is capable of solving problems with up to 1024 variables, however, this number has increased to 8192 in the second generation (Fujitsu Limited, 2020a).

The algorithm of DA is based on simulated annealing. Simulated Annealing (SA) is a probabilistic method for finding solutions to combinatorial optimization problems that aim to minimize some cost function, by making an analogy to the physical process of annealing whereby a heated material is slowly cooled until it reaches a state of minimum energy (Kirkpatrick et al., 1983; Bertsimas et al., 1993). The idea underpinning SA is to propose a random perturbation to the current solution at each iteration, evaluate the consequent change in the objective function, and decide whether or not to move to the proposed solution. If the proposed solution results in a lower objective value, it is always accepted; otherwise, i.e., if it is a “uphill” move, it is accepted with a probability that is a function of the change in the objective value and the current temperature. While higher temperatures more likely permit uphill moves to let
the algorithm explore a larger region of the objective function and to help escape from local optima, the search intensifies around a narrower area with lower temperatures. Under certain conditions, SA asymptotically converges to a global optimum, yet, it may necessitate infinitely many iterations. So, in practice, it is very well possible to converge to a local optimum in SA (Kirkpatrick et al., 1983; Rutenbar, 1989; Glover and Kochenberger, 2006; Gendreau and Potvin, 2010).

To apply SA-based algorithms, one needs to define a solution representation as well as a move operation to propose a new candidate solution at each iteration (Kirkpatrick et al., 1983). In DA, a solution (to a QUBO problem) is represented with a vector of binary variable values, and the move operation is defined as the flip of a variable value, i.e., changing the value of a variable from one to zero or vice versa.

While being grounded in SA, DA’s algorithm differs from it in some key aspects. First, it uses a parallel trial scheme, where it evaluates all possible moves in parallel at each iteration, as opposed to the classic way of considering one random move only. When more than one flip is eligible for acceptance, one of them is chosen uniformly at random. Second, it utilizes a dynamic offset mechanism to escape from local optima, such that if no flip is accepted in the current iteration, the acceptance probabilities in the subsequent iteration are artificially increased. Specifically, when no candidate variable to flip can be found, a positive offset value is added to the objective function, equivalent to multiplying the acceptance probabilities with a coefficient that is a function of the current temperature and the magnitude of the offset. Otherwise, the offset value is set to zero. Third, DA has the parallel tempering option, also referred to as the replica exchange method, where multiple independent search processes (replicas) are initiated in parallel with a different temperature each, and states (solutions) are probabilistically exchanged between them. This way, each replica performs a random walk in the temperature space, helping to avoid being stuck at a local minimum (Hukushima and Nemoto, 1996; Aramon et al., 2019; Matsubara et al., 2020). In our computational experiments, we utilize DA in parallel tempering mode.

5. Computational study

In this section, we present the results of our computational study. We generated a large suite of RWA-P instances using known networks from the literature and conducted a detailed analysis. Our experimental setting can be summarized as follows:

- **Solvers.** We used the second generation of DA (Matsubara et al., 2020) and GUROBI 9.0 (Gurobi Optimization LLC, 2020). For DA experiments, we used the Digital Annealer environment prepared exclusively for research at the University of Toronto. For GUROBI experiments, we used a MacOS computer with 3 GHz Intel Core i5 CPU and 16 GB memory.

- **Methods.** While we (i) provided the QUBO formulation to DA, we (ii) employed GUROBI in three different ways; (i) to directly solve the IP formulations, both IP\(^{\text{Base}}\) and IP\(^{\text{Strong}}\), (ii) to solve IP\(^{\text{Strong}}\) via Branch-and-Cut (B&C) using the lazy callback feature, and (iii) to directly solve the QUBO formulation. We note that we sometimes refer to (i) as "GUROBI as IP solver", and to (ii) as "GUROBI as QUBO solver". In addition, we solve RWA-P via two different heuristic methods from the literature. One of them, which we refer to as RS-Heur, is a random-search-based heuristic from Ezzahdi et al., (2006), which was also tested in Azodolmolky et al., (2010). The other one, which we call Hybrid-Heur, is a mixture of two heuristic methods from the literature where we combine the longest-shortest-path-first request selection strategy from Azodolmolky et al., (2010) with the most-utilized-first wavelength selection strategy from various previous works, e.g., (Mokhtar and Azzoğlu, 1998; Koganti and Sidhu, 2014). See Appendix C for further details of these two heuristic methods.

- **Time limit.** We used three different time limits; 120 seconds, which is approximately the highest run time DA takes for our particular problem, 600 seconds to compare the longer run time performance of GUROBI to that of DA, and 7200 seconds (2 hours) to see what GUROBI can achieve in cases where hours of run times are tolerable.

- **Experiments.** We carried out three main groups of analyses; (1) performance comparison of the five alternative methods, (2) effect of using solutions from DA as an initial solution for GUROBI, as well as a run time analysis for GUROBI to reach DA’s performance level, and (3) sensitivity analysis of DA’s performance to values of the penalty coefficient \(\rho\).

- **Implementation details.** First, we implemented a B&C algorithm using the callback features of GUROBI to pass the conflict constraints as needed. We tried various cut selection and management strategies for user and lazy callbacks, the best of which yielded no better performance than using the lazy callbacks, i.e., adding a conflict constraint each time a feasible solution violating it is encountered, which, therefore, is the one we utilize in our experiments. Second, although it is not possible to explicitly impose a time limit for DA, after some preliminary testing, we set the number of iterations to an appropriate value yielding the desired execution time of 120 seconds, which indeed was the highest time we observed for our particular problem, RWA-P. Once the number of iterations are fixed, DA’s execution times show almost no variability across different instances (which we observed in other problem classes too; see Şek et al. (2022)). Third, we were unable to use the penalty coefficient values suggested in Proposition 4 because they were outside the acceptable range for DA. Thus, we used smaller penalty coefficient values that do not necessarily guarantee the exactness of the model but always yielded feasible solutions in practice.
The remainder of this section is organized as follows. In Section 5.1, we provide some details about the networks used and the way we generated our problem instances. We present our main set of experimental results in Section 5.2, follow it by a run time and initial solution analysis in Section 5.3, and then provide the results of our penalty coefficient analysis in Section 5.4. Finally in Section 5.5, we summarize our key observations as a result of our computational study.

### 5.1. Problem instances

In order to generate our test instances, we made use of four different network topologies of varying sizes and densities from the literature, namely EON (Tornatore et al., 2007), Brazil (Jaumard et al., 2006), USA and China (Hwang et al., 2009). In Table 1, we provide descriptive information about the networks and the test instances we generated from them.

For all the networks, we assume that each edge \( \{u, v\} \) is represented with a pair of links \((u, v)\) and \((v, u)\). We use three different wavelength capacities on links, \(|\Lambda| \in \{5, 10, 15\}\), and three different numbers for requests, \(|R| \in \{60, 80, 100\}\). We generate five random instances for each parameter combination, which makes a total of 180 test instances. For each instance, we selected a distinct source and destination pair for every request among all possible ordered node pairs in the network. For each request, we randomly selected four working and four protection paths between the source and destination nodes, except when \(|\Lambda| = 15\) and \(|R| \in \{80, 100\}\) where we decreased the number of working/protection paths to three and two, so that the sizes of those instances become eligible for DA, which can handle at most 8192 variables. We formed the working and protection lightpaths of every request by combining each generated path with every one of the available wavelengths.

As indicators of instance sizes, we provide the number of variables (“# vars”) as well as the ratio of the number of constraints to the number of variables (“# cons/# vars”) both for IP\(^{\text{Strong}}\) and IP\(^{\text{Base}}\) formulations. It is noteworthy that the strengthened set of our conflict constraints leads to a remarkable decrease in the total number of constraints; the ratio of the number of constraints to the number of variables is 170 to 350 times higher in IP\(^{\text{Base}}\) than in IP\(^{\text{Strong}}\).

### 5.2. Performance comparison of the methods

In this section, we report the results of our main set of experiments and compare the performances of all the methods under consideration. Table 2 reports the number of granted requests for all methods averaged over all wavelength and request numbers, and the average number of links used per lightpath of a granted request in parentheses in a second row for each network, under two groups of columns corresponding to the experiments with 120- and 600-second time limits. We have an additional column reserved for Hybrid-Heur, which does not belong to the other two column groups for the two time limits because the average execution time of this method was less than 0.1 second. For direct solving of IP\(^{\text{Strong}}\) with GUROBI, we present additional sets of results obtained by setting the solver’s emphasis on search for feasible solutions (“IP\(^{\text{Strong}}\) (f)”) for both time limits, whereas all other GUROBI-related results are obtained under default settings.

We observe from Table 2 that the number of granted requests obtained from DA in 120 seconds outperforms those obtained from 120- and even 600-second experiments with GUROBI for all networks. Among the GUROBI-based alternatives, on the other hand, solving of IP\(^{\text{Strong}}\) directly with GUROBI yields the best results, mostly slightly better when the emphasis is on finding feasible solutions. Even though the results from RS-Heur and Hybrid-Heur are better than those obtained from solving IP\(^{\text{Strong}}\) through B&C (with callback) and the QUBO formulation directly with GUROBI, both are still far from being comparable to the other alternatives. Also, IP\(^{\text{Strong}}\) yields considerably better results than IP\(^{\text{Base}}\), as expected, and increasing the time limit from 120 to 600 seconds leads to a relative improvement of only 0.4 to 1.9 in the average number of granted requests for IP\(^{\text{Strong}}\). As for the link usage, since it is our secondary objective, two solutions with different numbers of granted requests cannot be compared in that regard. Nevertheless, in terms of the average number of links used per lightpath of a granted request, DA in 120 seconds and GUROBI in 600 seconds perform almost always the same, even though DA achieves a higher number of granted requests.

Surprisingly, the B&C method mostly performs worse than directly solving the QUBO formulation with GUROBI. In order to see whether it could provide good-quality solutions when longer run times are allowed, we also tested it with a 600-second time limit on a selected set of 30 representative instances based on USA and Brazil networks, with \(|\Lambda|, |R| \in \{(5, 60), (10, 80), (15, 100)\}\), where \(|\Lambda|\) denotes the number of wavelengths and \(|R|\) the number of requests. For each pair of parameter values, we have 10 instances in this selected set, five per network. For half of these instances based on USA network, the average number of granted requests rises from 3.3 to 7.5, and for the remaining ones based on Brazil network it rises from 4.8 to 10.2.
Consequently, we conclude that the B&C method fails to be a favorable method for further consideration, as do solving IPBase and QUBO formulations directly with GUROBI. We also tested the heuristic RS-Heur under a 600-second time limit, and observed that the increase in the average number of granted requests is less than one, confirming that this method too is not a favorable option for further consideration. Similar efforts for Hybrid-Heur did not improve its performance either.

Next, we analyze the performances of solving IPStrong using GUROBI and solving the QUBO formulation with DA in more detail, which appear to be the most promising two alternatives according to the average results given in Table 2. Figure 2 compares the average number of granted requests for every \(|\mathcal{A}|,|R|\) pair, using 120- and 600-second time limits for DA and GUROBI, respectively. For GUROBI, results obtained with and without feasibility emphasis on the solver are provided. The four plots demonstrate that DA outperforms GUROBI for each parameter combination and every one of the four networks, and the difference is particularly evident when the number of wavelengths is 10, i.e., when \(|\mathcal{A}| = 10\). In fact, DA not only outperforms in terms of the average values, but also provides predominantly superior or otherwise as good results in almost all individual instances. These results show that for our test set comprising instances that are hard to optimally solve, DA yields solutions within only two minutes which are better than or as good as the ones GUROBI attains in 10 minutes.

### 5.3. Run time and initial solution analysis

We now present the results of our analyses regarding the run times and the effect of providing initial solutions to GUROBI. We first analyze the best solution progress over time for DA, GUROBI as IP solver, and the two heuristics RS-Heur and Hybrid-Heur. Since GUROBI gives slightly better solutions when the feasibility emphasis is active, we continue with that setting. We experimented with different time limits from 5 to 120 seconds using the aforementioned 30 representative instances, which cover both the extremes and the intermediate portion of the parameter space.

Figure 3 presents the results in six different groups, each of which corresponds to a different combination of network and instance generation parameters in the representative instance set. Each plot in the figure shows the highest number of granted requests that DA, RS-Heur, and GUROBI as IP solver achieve within the considered time limits, as well as those from GUROBI under the 600-second time limit and Hybrid-Heur as constant reference lines. We see that there is no apparent outperformance of DA and GUROBI over one another through the considered time limits when the six groups of results are taken into account altogether. Also, network type or the number of requests and wavelengths do not seem to have an effect on comparative performances over time.

We confine the rest of our analysis to the most promising two alternatives; namely, solving IPStrong using GUROBI with an emphasis on finding feasible solutions and solving the QUBO formulation with DA. Having observed that GUROBI cannot much improve the solution quality despite increasing the time limit to 10 minutes, we now investigate whether feeding a DA solution to GUROBI as an initial (warm start) solution would improve its performance, and also how long it would take GUROBI to reach DA’s solution quality. We again use the set of 30 representative instances for these analyses. For the initial solution analysis, we feed the DA solutions to GUROBI as an initial solution in solving IPStrong with a time limit of 600 seconds and emphasis on finding feasible solutions. In order to look into the possible effect of incorporating link usage in the objective, we additionally carry out the same set of experiments by maximizing the number of granted requests only, that is, by setting \(\alpha = 0\) and \(\beta = 1\). For the run time analysis, we set the objective values of solutions from 120-second DA runs as a stopping condition for GUROBI, along with a time limit of 7200 seconds (2 hours). Note that the objective value of a solution with a certain number of granted requests cannot be attained or surpassed without granting at least as many requests due to the way we set the value of the \(\beta\) coefficient to prioritize request granting over link usage.

Table 3 contains the results of the above-mentioned experiments. The results of the initial solution experiments are provided under a five-column block (“IPStrong (f) w/initial sol (600 sec)”), and is divided into two groups; one for the results of the experiments with the original objective function that incorporates both the request granting and link usage components (“Weighted obj”), and the other for the case when only the request granting component of the objective is considered (“Request obj”). Each group contains the number of granted requests and used links...
averaged over the five instances associated with the parameter pair \((|\Lambda|, |R|)\) and network in each row. For the experiments with the original weighted objective, average optimality gap is also reported, defined as \(\% \text{ gap} = \frac{\text{UB} - \text{LB}}{\text{UB}} \times 100\) where UB and LB respectively denote the upper and lower bounds for the objective value. The last block of columns contains the results of the run time analysis experiments (“IP\text{Strong} (f) until DA obj val (7200 sec)”). This block additionally includes the average run times in the last column (“Time”), as well as the number of granted requests GUROBI attains when the experiments are
allowed to run for a full two hours, i.e., when the objective level of DA is not imposed as a stopping condition, provided in parentheses.

We see from Table 3 that GUROBI is mostly unable to improve the initial solution from DA (or to prove the optimality of the solution) within the 600-second time limit, both in terms of granted requests and link usage. The situation remains the same when the objective is reduced into the maximization of request granting only, indicating that the link usage component in the objective does not really affect the performance, and thus a hierarchical solution approach for RWA-P to handle the two separate objectives would not perform any better than solving it with a weighted objective. As for the run time analysis, in 66.7% of the selected instances (20 out of 30), GUROBI cannot reach the 120-second objective value of DA after 2 hours. The time for the remaining 10 instances to reach DA’s objective value is 1233 seconds on the average, an order of magnitude higher than DA’s run time approximately. Also, as a result of the full 2-hour experiments, the average number of granted requests (shown in parentheses) either does not change or increases by 0.6 only. In terms of average optimality gaps, the values from the initial solution experiments are 6.2% better than those of the run time experiments. This is mainly due to the solutions from DA, which yield 4.3% better upper bounds than the ones from the run time experiments, while lower bounds are only 0.1% worse. These results indicate that DA is capable of delivering good-quality solutions in only 2 minutes, which typically cannot be attained by a state-of-the-art solver after 2 hours of run time.

5.4. Penalty coefficient analysis

As mentioned before, penalty coefficient values rendering our QUBO formulations exact go beyond the acceptable ranges for DA. Nevertheless, we observed in our preliminary experiments that by setting the penalty coefficient values with respect to the β values (which we compute using Proposition 2), DA can always deliver solutions that are feasible for the IP models. Specifically, we set \( q = b + 100 \), yielding IP-feasible solutions throughout all of our experiments. While testing different penalty coefficient values, however, we noticed that it can have a considerable impact on DA’s performance, which we investigate in the following.

Figure 4 contains the results from 120-second DA experiments with different penalty coefficient values, using the same set of 30 representative instances mentioned above. The x-axes of the two plots show the penalty coefficient values as a function of the β parameter, whereas the y-axes display the average number of granted requests. The lower ends of the penalty coefficient values are those used in our main set of experiments, whereas the higher end is set close to the values before passing beyond the acceptable ranges for DA.

We observe that decreasing the penalty coefficient values improves DA’s performance, and the difference is more marked for the instances based on Brazil network. In comparison to the 2-hour performance of GUROBI, not only does DA outperform with the best possible penalty coefficient values for it (i.e., when \( \rho = b + 100 \)), it yields better or at least as good results when \( b + 100 \leq \rho \leq b + 1000 \). Thus, even without much tuning of the penalty coefficient parameter, DA in 2 minutes delivers better results than a state-of-the-art solver does after hours of run times.

Table 3. Initial solution and run time analysis results for the selected set of representative instances.

| Network | # granted | # links | # granted | # links | % gap | # granted | # links | % gap | Time |
|---------|-----------|---------|-----------|---------|-------|-----------|---------|-------|------|
| (5, 60) USA | 13.6 | 227.8 | 13.6 | 227.6 | 30.6 | 13.6 | 227.8 | 13.4 | (13.4) |
| Brazil | 24.0 | 379.0 | 24.0 | 379.0 | 50.4 | 24.0 | 379.0 | 22.8 | (22.8) |
| (10, 80) USA | 26.6 | 485.0 | 26.6 | 485.0 | 35.1 | 26.6 | 485.0 | 25.2 | 443.2 |
| Brazil | 47.4 | 777.6 | 47.4 | 777.0 | 47.8 | 47.6 | 781.6 | 42.8 | 706.2 |
| (15, 100) USA | 29.6 | 589.8 | 30.2 | 600.8 | 12.0 | 30.2 | 606.2 | 57.2 | (57.8) |
| Brazil | 58.6 | 985.8 | 58.6 | 983.4 | 17.6 | 58.6 | 985.8 | 20.6 | 5556.8 |

Figure 4. Results of the penalty coefficient trials with DA using the representative instances.
5.5. Key observations

We lastly summarize some main findings arising from our computational study:

- For our nontrivial set of test instances, the quality of solutions that DA yields within 2 minutes is better than or as good as those a state-of-the-art solver provides after 2 hours of run time.
- DA’s performance is robust across different instances and networks.
- Feeding solutions obtained from DA as initial solutions to GUROBI does not improve its performance; it cannot find any better solution or prove the optimality of the initial solution if it is so, even after a considerable amount of run time.
- Solving RWA-P by only considering the prioritized objective of request granting and ignoring the secondary one (link usage) does not perform any better than solving it with a weighted objective.
- Penalty coefficient values affect DA’s solution quality, but even without particularly tuned values DA can outperform the two-hour results from a state-of-the-art solver.

6. Conclusion

In this study, we consider the routing and wavelength assignment problem with protection, RWA-P. Through complexity analysis and computational experiments, we show that this problem is difficult to solve both in theory and practice. We propose a viable approach, formulating RWA-P as a QUBO model and employing the DA, as a new promising solution technology and test its performance in comparison to exact and heuristic methods. Even though it is not possible to directly impose a time limit for DA, we adjusted the input number of iterations to achieve the desired run times, which typically do not depend on the size of the problem instance. We find that the proposed approach outperforms established methods in handling our nontrivial set of test instances by delivering solutions within 2 minutes that are superior to or as good as the ones other methods yield in 2 hours, as well as two heuristic methods from the literature. Also considering that future generations of DA are planned to achieve megabit-class performance (Fujitsu Limited, 2020b) and offer higher coefficient precision (Nakayama et al., 2021) which can remedy the current limitation on parameter value ranges, we believe that the proposed approach has potential to be utilized widely in practice. As such, future research directions involve considering large-scale cases of RWA-P, adaptation and application of this emerging approach to other RWA problems.

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Data Availability Statement

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