η transitions between charmonia with meson loop contributions

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We study the η transitions between ψ(4040/4160) and J/ψ by introducing charmed meson loops in an effective Lagrangian approach to enhance the decay amplitudes. The branching fractions \( B[\psi(4040) \to J/\psi\eta] \) and \( B[\psi(4160) \to J/\psi\eta] \) estimated in this paper can remarkably explain the experimental measurements of Belle and BESIII within a reasonable parameter range. The η′ transition between ψ(4160) and J/ψ is also investigated, and the branching fraction is under the upper limit of CLEO, which can be tested by future experiments.

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I. INTRODUCTION

Among many heavy quarkonia, the charmonium sector especially has much abundant spectroscopy and decay modes observed [1]. In their mass range, there are also many discrepancies between theoretical predictions and experimental measurements because of the non-perturbative property of Quantum Chromodynamics (QCD). The study of spectroscopy and decay behavior of the charmonia undoubtedly enrich our knowledge of how QCD works in hadron physics.

When considering the decay process \( \psi_1 \to \psi_2 P \), the direct coupling among the initial and final charmonia, \( \psi_1 \) and \( \psi_2 \), and a light chiral meson, \( P \), is highly suppressed due to the OZI rule. Because Belle and BESIII have recently observed branching fractions for some processes of this kind, of the order of 10^{-3}, hence we need to consider other mechanism which enhances this type of decay amplitude. The charmonia, whose masses are above the threshold of a charmed meson pair, dominantly decay into a pair of charmed mesons, which can couple with a light meson and a charmonium by exchanging a proper charmed meson in the final states. The contribution from such a structure, i.e., meson loop contribution, is dominant in this decay and becomes important in understanding decay behaviors of higher charmonia.

Taking \( \psi(3770) \) as an example, which is the first charmonium above the threshold of open charmed mesons, the BES Collaboration announced that the branching fraction of its non-\( D\bar{D} \) decay is \( B[\psi(3770) \to non\-D\bar{D}] = (14.7 \pm 3.2)\% \) [2–5]. Such a large non-\( D\bar{D} \) branching fraction is several times larger than expected in theory [6]. To resolve this discrepancy, the authors in Refs. [7, 8] have taken account of the charmed meson loop, where the initial charmonium \( \psi(3770) \) decays into a charmed meson pair \( D\bar{D} \) and this pair couples to a vector and/or pseudoscalar meson by exchanging a \( D \) meson. After including the meson loop contributions, the large non-\( D\bar{D} \) branching fraction of \( \psi(3770) \) can be nicely explained.

The decay mode \( \psi(3770) \to J/\psi P \) and the lineshape around \( \psi(3770) \) have been studied with meson loop contributions in Ref. [9].

Even though the measurements of the higher charmonia are not yet enough to discuss η transitions, there appear some experiments of this kind. In the International Conference on High Energy Physics, the Belle Collaboration reported their measurements for η transitions between \( \psi(4160/4040) \) and \( J/\psi \) [10, 11]. They announced that \( B[\psi(4040) \to \eta J/\psi] \cdot \Gamma_{\psi J/\psi}(\psi(4160)) = 4.8 \pm 0.9 \pm 1.4 \text{ eV} \) or \( 11.2 \pm 1.3 \pm 1.9 \text{ eV} \) with different fitting parameters to the data. The corresponding results for \( \psi(4160) \) are \( B[\psi(4160) \to \eta J/\psi] \cdot \Gamma_{\psi J/\psi}(\psi(4160)) = 4.0 \pm 0.8 \pm 1.4 \text{ eV} \) or \( 13.8 \pm 1.3 \pm 2.0 \text{ eV} \). Taking \( \Gamma_{\psi J/\psi}(\psi(4040)) = (0.86 \pm 0.07) \text{ keV} \) and \( \Gamma_{\psi J/\psi}(\psi(4160)) = (0.83 \pm 0.07) \text{ keV} \), one obtains the branching ratios as \( B[\psi(4040) \to J/\psi\eta] = (0.56 \pm 0.10 \pm 0.17)\% \) or \( (1.30 \pm 0.15 \pm 0.24)\% \) and \( B[\psi(4160) \to J/\psi\eta] = (0.48 \pm 0.10 \pm 0.17)\% \) or \( (1.66 \pm 0.16 \pm 0.28)\% \). Before this measurement, only the upper limits of branching ratios for \( \psi(4040) \to \eta J/\psi \) and \( \psi(4160) \to \eta J/\psi \) were reported by the CLEO Collaboration, which are \( < 7 \times 10^{-3} \) and \( < 8 \times 10^{-3} \) [12], respectively. Recently, the BESIII Collaboration also analyzed the production of \( e^+ e^- \to \eta J/\psi \) at a center-of-mass energy of \( \sqrt{s} = 4.099 \text{ GeV} \). Because the Born cross section is reported to be \( (32.1 \pm 2.8 \pm 1.3) \text{ pb} \) [13], the corresponding fractional transition rate is \( B[\psi(4040) \to \eta J/\psi] = (5.2 \pm 0.5 \pm 0.2 \pm 0.5) \times 10^{-3} \), which is consistent with the first solution of the Belle Collaboration and measurement by CLEO. With regard to experimental data, in this paper we will use the old data by CLEO [12] and others as well as the most recent data given by Belle [11] and BESIII [13]. This is because PDG has not yet included the most recent data by Belle and BESIII.

Similar to the case of \( \psi(3770) \), \( \psi(4040) \) and \( \psi(4160) \) are above the threshold of charmed meson pairs, and dominantly decay into these. The experimental measurements stimulate us to study the η transition between \( \psi(4040/4160) \) and \( J/\psi \) with the meson loop mechanism, which is essential to understand the hidden charm decay behavior like higher charmonia.

This paper is organized as follows. After introduction, a brief review of meson loop mechanism is presented and the corresponding amplitudes are calculated using an effective Lagrangian in Section II. Our numerical results of the branching ratios are given in Section III. Section IV is devoted to

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The meson loop effect plays a crucial role in understanding the \( \eta \) transition between the higher charmonia and \( J/\psi \). Taking \( \psi(4040) \) as an example, involvement of the meson loop in the decay is depicted in Fig. 1. The charmonium \( \psi(4040) \) is decomposed into \( D^{(*)}D^{(*)} \) and by exchanging \( D \) or \( D^{*} \) meson, the charmed meson pair converts itself into \( J/\psi \eta \).

FIG. 1: The typical meson loop diagrams contributing to \( \psi(4040) \rightarrow J/\psi \eta \). The initial \( \psi(4040) \) couples with a charmed meson pair \( D^{(*)}D^{(*)} \), and by exchanging the \( D \) meson (left) or \( D^{*} \) meson (right), the charmed meson pair converts itself into \( J/\psi \eta \) in the final state.

The effective Lagrangian approach is adopted to evaluate the meson loop contributions to higher charmonia decay into \( J/\psi \eta \) as shown in Fig. 1. Utilizing the heavy quark limit and chiral symmetry, the effective Lagrangians, which involves interactions among \( J/\psi \), pseudoscalar meson, and charmed mesons, read as [14, 15]:

\[
\mathcal{L}_{J/\psi DD^{(*)}D^{(*)}} = ig_{J/\psi DD} \bar{\psi} \left( \bar{D} \gamma^\mu D - D \gamma^\mu \bar{D} \right) - g_{J/\psi DD} \bar{\psi} D^{(*)} \left( \partial^\mu \eta \right) \partial_\mu \eta + \left( \partial^\mu \eta \right) \partial_\mu \eta
\]

\[
\mathcal{L}_{DD^{(*)}P} = -ig^{D^{(*)}D} \bar{D} \gamma^\mu \partial_\mu \eta \partial_\mu \partial_\mu \eta + \frac{1}{2} g^{D^{(*)}D} \eta \partial_\mu \partial_\mu \partial_\mu \eta \partial_\mu \partial_\mu \eta,
\]

with \( D^{(*)} = \left( D^{(*)0}, D^{(*)+}, D^{(*)-} \right) \). Considering \( \eta \) and \( \eta' \) mixing, one has \( \mathcal{P} \) in the form,

\[
\mathcal{P} = \begin{pmatrix}
\frac{\alpha + \beta \eta}{\sqrt{2}} & \pi^+ & K^+
\frac{\pi^-}{\sqrt{2}} & \alpha + \beta \eta' & K^0
\frac{\gamma + \delta \eta'}{\sqrt{2}} & K^- & \gamma \eta + \delta \eta'
\end{pmatrix},
\]

and we adopt \( \theta = -19.1^\circ \) in the present work [16, 17].

Since \( J/\psi \) cannot decay into charmed mesons due to the phase space restriction, we need to consider the symmetric limit of heavy quark effective theory to determine relations among these coupling constants. In this limit, the coupling constants between \( J/\psi \) and charmed mesons satisfy [18, 19]

\[
8_{J/\psi DD} = 8_{J/\psi D^{(*)}D^{(*)}} m_D m_{DD} = 8_{J/\psi DD} \sqrt{m_D m_{DD}} = m_{1/2}/f_{1/2} \text{ with } m_{1/2} \text{ and } f_{1/2} \text{ being the mass and decay constant of } J/\psi.
\]

The adopted values of these coupling constants are \( g_{J/\psi DD} = 7.44, g_{J/\psi D^{(*)}D^{(*)}} = 8.00 \) and \( g_{J/\psi D^{(*)}D^{(*)}} = 3.84 \text{ GeV}^{-1} \), which are determined by the vector meson dominance [18, 19]. On the other hand, the couplings between pseudoscalar meson and charmed mesons can be related to the gauge coupling constant \( g \) by [20]

\[
g_{PD^D} = g_{PD^{(*)}D^{(*)}} = 8g_{PD} m_D = 2g/f_{2} \text{ with } g = 0.59 \text{ and } f_{2} = 132 \text{ MeV referring to [21], which were obtained using the full width of } D^{(*)} \text{ [22].}
\]

The coupling constants between higher charmonia, such as \( \psi(4040) \) and \( \psi(4160) \), and charmed mesons are evaluated by the partial decay width, assuming that these two charmonia dominantly decay into \( D \bar{D}, D \bar{D}^* + h.c. \), and \( D \bar{D}^* \) due to the phase space restriction. Here, the Lorentz structure of interaction between \( \psi(4040)/\psi(4160) \) and charmed mesons is the same as that between \( J/\psi \) and charmed mesons since \( \psi(4040)/\psi(4160) \) and \( J/\psi \) are vector charmonia. However, the relative sign of these coupling constants of \( \psi(4040)/\psi(4160) \) with charmed mesons cannot be constrained and hence, in this work we need to consider the effects due to different signs of these coupling constants. Furthermore the BaBar Collaboration has measured the ratios between these decay modes for \( \psi(4040) \) and \( \psi(4160) \) [23], which are \( \mathcal{B}(\psi(4040) \rightarrow D \bar{D})/\mathcal{B}(\psi(4040) \rightarrow D \bar{D}) = 0.24 \pm 0.05 \pm 0.12, \mathcal{B}(\psi(4040) \rightarrow D \bar{D}^* + h.c.)/\mathcal{B}(\psi(4040) \rightarrow D \bar{D}) = 0.18 \pm 0.14 \pm 0.03, \mathcal{B}(\psi(4160) \rightarrow D \bar{D}^* + h.c.)/\mathcal{B}(\psi(4160) \rightarrow D \bar{D}) = 0.02 \pm 0.03 \pm 0.02, \mathcal{B}(\psi(4160) \rightarrow D \bar{D}^* + h.c.)/\mathcal{B}(\psi(4160) \rightarrow D \bar{D}) = 0.34 \pm 0.14 \pm 0.05 \). Given the total decay width \( \Gamma_{\psi(4040)} = 80 \pm 10 \text{ MeV and } \Gamma_{\psi(4160)} = 103 \pm 8 \text{ MeV, one obtains} |g_{\psi(4040)D}| = 2.13 \pm 0.36, |g_{\psi(4160)D}| = 1.70 \pm 0.15 \text{ GeV}^{-1}, |g_{\psi(4040)D^*}| = 3.34 \pm 1.00, |g_{\psi(4160)D^*}| = 0.57 \pm 0.47, |g_{\psi(4160)D^*}| = 0.77 \pm 0.11 \text{ GeV}^{-1}, \) and \( |g_{\psi(4160)D^*}| = 2.23 \pm 0.15 \).

With the Lagrangians listed above, we can obtain the hadronic decay amplitudes for \( \psi(4040)(p_0) \rightarrow \left[ D^{(*)} (p_1) D^{(*)} (p_2) \right] (g) \rightarrow J/\psi(p_3) \eta(p_4) \),

\[
\mathcal{A}^{D}_{DD^*} = (i)^3 \int \frac{d^4 q}{(2\pi)^4} \left[ -g_{\psi DD^*} \epsilon_{\alpha}^{D^*} (ip_{13} - ip_{23}) \right] \left[ -g_{\psi DD^*} \epsilon_{\alpha}^{D^*} (ip_{14} - ip_{24}) \right]
\]

\[
\times \left[ e_{\mu \nu \rho \sigma} \epsilon_{\alpha}^{D^*} (ip_{14} - ip_{24}) \right] \left[ -g_{\psi DD^*} \epsilon_{\alpha}^{D^*} (ip_{13} - ip_{23}) \right]
\]

\[
\times \frac{1}{p_1^2 - m^2_{D^*}} \frac{1}{p_2^2 - m^2_{D^*}} \frac{1}{q^2 - m^2_{D^*}} \mathcal{F}(q^2, m^2_{D^*}),
\]

\[
\mathcal{A}^{D^*}_{DD^*} = (i)^3 \int \frac{d^4 q}{(2\pi)^4} \left[ -g_{\psi D^* D^*} \epsilon_{\mu \nu \rho \sigma} (-ip_{14} - ip_{24}) \right] \left[ -g_{\psi D^* D^*} \epsilon_{\mu \nu \rho \sigma} (-ip_{13} - ip_{23}) \right]
\]

\[
\times \frac{1}{p_1^2 - m^2_{D^*}} \frac{1}{p_2^2 - m^2_{D^*}} \frac{1}{q^2 - m^2_{D^*}} \mathcal{F}(q^2, m^2_{D^*}),
\]
\[ \mathcal{A}_{DD}^{D} = (i)^{3} \int \frac{d^{4}q}{(2\pi)^{4}} [-g_{\psi'D'DE_{\text{perp}}}(-ip_1^\nu)\epsilon_{\nu}^\mu(pp_2^3) ]
\times \left[ -g_{J/\psi'D'DE_{\text{perp}}} \left( ip_3^3 \right) \epsilon_{\nu}^\mu i(p_2^3) \right] \frac{1}{2} g_{\psi'D'DE_{\text{perp}}}
\times (ip_1^3(-iq^3 + ip_2^3)) p_1^C m_D^2 p_2^C m_D^2
\times -g^{\mu\nu} + \frac{p_1^C p_3^C}{m_D^2} \frac{1}{q^2 - m_D^2} F^2(q^2, m_D^2),
\]
\[ \mathcal{A}_{D'D\bar{D}}^{D} = (i)^{3} \int \frac{d^{4}q}{(2\pi)^{4}} [-g_{\psi'D'DE_{\text{perp}}}(-ip_1^\nu)\epsilon_{\nu}^\mu(pp_2^3) ]
\times \left[ -g_{J/\psi'D'DE_{\text{perp}}} \left( ip_3^3 \right) \epsilon_{\nu}^\mu i(p_2^3) \right] \frac{1}{2} g_{\psi'D'DE_{\text{perp}}}
\times (ip_1^3(-iq^3 + ip_2^3)) p_1^C m_D^2 p_2^C m_D^2
\times -g^{\mu\nu} + \frac{p_1^C p_3^C}{m_D^2} \frac{1}{q^2 - m_D^2} F^2(q^2, m_D^2),
\]
\[ \mathcal{A}_{D'D}^{D'} = (i)^{3} \int \frac{d^{4}q}{(2\pi)^{4}} [-g_{\psi'D'DE_{\text{perp}}}(-ip_1^\nu)\epsilon_{\nu}^\mu(pp_2^3) ]
\times \left[ -g_{J/\psi'D'DE_{\text{perp}}} \left( ip_3^3 \right) \epsilon_{\nu}^\mu i(p_2^3) \right] \frac{1}{2} g_{\psi'D'DE_{\text{perp}}}
\times (ip_1^3(-iq^3 + ip_2^3)) p_1^C m_D^2 p_2^C m_D^2
\times -g^{\mu\nu} + \frac{p_1^C p_3^C}{m_D^2} \frac{1}{q^2 - m_D^2} F^2(q^2, m_D^2),
\]

In the above expressions, \( g_{\psi'D'DE_{\text{perp}}} \) is defined in Eqs. (6)-(8) which are evaluated by estimating the corresponding loop integrals. In Fig. 2, we show the absolute values of the coupling constants corresponding to different intermediates. The grey bands are the uncertainties of \( g_{\psi'D'DE_{\text{perp}}} \), which are resulted from the errors of the coupling constants of \( \psi(4400)/\psi(4160) \) interacting with charmless meson pair. We notice that there exits large uncertainty for \( g_{\psi'D'DE_{\text{perp}}} \) compared with \( g_{\psi(4400)/\psi(4600)} \) and \( g_{\psi(4400)/\psi(4600)} \). For \( \psi(4160) \), the coupling \( g_{\psi(4160)/\psi(4600)} \) has a large error as shown in Fig. 2. In the \( \psi(4040) \rightarrow J/\psi \eta \) process, the coupling constant corresponding to \( D^*D + h.c. \) channel is about 3 times larger than that for \( D\bar{D} \) channel, while the one for \( D^*D \) is nearly 1 order larger than that for \( D\bar{D} \) channel. The coupling constants for \( \psi(4160)/J/\psi \eta \) behave very similar to those for \( \psi(4040) \rightarrow J/\psi \eta \). The dominant contribution comes from the process of the \( D^*D \) channel while the one corresponding to \( D\bar{D} \) is very small and can be neglected.

Having the couplings evaluated above, one can estimate the branching ratios of \( \psi(4040)/\psi(4160) \rightarrow J/\psi \eta \) directly result in those among \( g_{\psi'D'DE_{\text{perp}}} \) listed
in Eqs. (6)-(8). As shown in Fig. 2, the contributions via the intermediate $D\bar{D}$ channel to $\psi(4040)/\psi(4160) \to J/\psi\eta$ processes can be ignored compared with those of other intermediate channels, which enables us to further simplify our calculation. Here, we take the same sign for $g_{\psi'}^{\psi D\bar{D}}$ and $g_{\psi'\psi'}^{\psi D\bar{D}}$. Next, we must consider two typical cases, i.e., the relative sign between $g_{\psi'\psi'}^{\psi D\bar{D}}$ and $g_{\psi'\psi'}^{\psi D\bar{D}}$ is plus or minus. Finally, we present the results of the branching ratios for $\psi(4040)/\psi(4160) \to J/\psi\eta$ in Fig. 3. Because of the uncertainties of $g_{\psi'\psi'}^{\psi D\bar{D}}$, the results in Fig. 3 are drawn using their central values with errors.

To compare our results with the experimental measurements, we have presented the experimental value of $\psi(4040) \to J/\psi\eta$ given by BESIII [13] and Belle [10, 11] in Fig. 3. Here, the branching ratios $\mathcal{B}(\psi(4040) \to J/\psi\eta)$ corresponding to solutions I and II from Belle are $(0.56 \pm 0.10 \pm 0.17)\%$ and $(1.30 \pm 0.15 \pm 0.24)\%$ [10, 11], respectively, while the BESIII measurement gives $\mathcal{B}(\psi(4040) \to J/\psi\eta) = (5.2 \pm 0.5 \pm 0.2 \pm 0.5) \times 10^{-3}$ [13]. Since the second solution of the Belle Collaboration is not consistent with the one of the BESIII and CLEO Collaboration, we consider only the first solution from the Belle Collaboration and results of the BESIII Collaboration, which are the cyan and yellow bands in Fig. 3. Even though there exists uncertainty in our theoretical results for $\psi(4040) \to J/\psi\eta$ due to the large error and undetermined sign of $g_{\psi'\psi'}^{\psi D\bar{D}}$, we can find that our theoretical curves overlap with the experimental measurements in a reasonable parameter region for $\alpha$. Thus, we can conclude that the meson loop effects can provide sizable contributions to $\psi(4040) \to J/\psi\eta$ and explain the experimental data for this process.

FIG. 2: (color online) The $\alpha$ dependence of the absolute values of coupling constants $g_{\psi'/\psi}\eta$ derived from the meson loop contributions to $\psi(4040) \to J/\psi\eta$ (upper panel) and $\psi(4160) \to J/\psi\eta$ (lower panel). Here, we consider different intermediate state contributions to $g_{\psi'/\psi}\eta$, i.e., the green dotted, red dashed and black solid curves correspond to the contributions from intermediate $D^*\bar{D}^*$, $D^*\bar{D} + h.c.$ and $D\bar{D}$, respectively. The grey bands denote the uncertainties caused by the error of the coupling between $\psi(4040)/\psi(4160)$ and charmed meson pairs.

FIG. 3: (color online) The comparison of the branching ratios obtained for $\psi(4040) \to J/\psi\eta$ (upper panel) and $\psi(4160) \to J/\psi\eta$ (lower panel) with the experimental data from Belle (the cyan bands) [10, 11] and BES (the yellow band) [13]. Here, the black solid curves with errors are the results when taking the relative sign of $g_{\psi'\psi}\eta$ given by BESIII [13] and Belle [10, 11] and BES (the yellow band) [13]. The grey bands denote the uncertainties caused by the error of the coupling between $\psi(4040)/\psi(4160)$ and charmed meson pairs.
In this work, we also study $\psi(4160) \to J/\psi \eta$ decay. The results shown in Fig. 3 indicate that we can well explain the Belle’s data, i.e., our results overlap with the experimental measurement in the range of $0.50 < \alpha < 0.80$ and $0.53 < \alpha < 1.20$ under two typical cases of relative signs, where the values $\alpha$ obtained are reasonable. In addition, we also notice that these $\alpha$ ranges for $\psi(4040) \to J/\psi \eta$ and $\psi(4160) \to J/\psi \eta$ overlap with each other, which can be due to the similarity existing in these two processes. This observation also gives an extra test to our theoretical calculation and the hadron loop effects on $\psi(4040)/\psi(4160) \to J/\psi \eta$.

Other than the $\eta$ transition between $\psi(4040)/\psi(4160)$ and $J/\psi$, we also calculate the meson loop contributions to $\psi(4160) \to J/\psi \eta'$. This prediction can be an important test to the meson loop mechanism proposed in this work. The formalism for $\psi(4160) \to J/\psi \eta'$ is similar to that for $\psi(4160) \to J/\psi \eta$, where we only need to make some replacements of the corresponding parameters and masses. Similar to the way applied to Fig. 2, we present the obtained coupling constants $\mathcal{B}_{\psi(4160)\to D^+ D^-}(\psi(4160)\to J/\psi \eta')$ in the upper panel of Fig. 4, where the $\psi(4160) \to J/\psi \eta'$ process via intermediate $D^+ D^-$ state is dominant, while the $D\bar{D}$ channel can be negligible. With these obtained coupling constants, we predict the branching ratio for $\psi(4160) \to J/\psi \eta'$, where we also consider two typical cases since we consider the effects of different signs of $\mathcal{B}_{\psi(4160)\to D^+ D^-}$, which is similar to the treatment when calculating $\psi(4040) \to J/\psi \eta$. In Fig. 4, we do not directly give the branching ratio for $\psi(4160) \to J/\psi \eta'$, instead, we show the ratio of the branching ratios for $\psi(4160) \to J/\psi \eta'$ and $\psi(4160) \to J/\psi \eta$, where this ratio is denoted as $R_{\psi(4160)\to D^+ D^-}(\psi(4160))$. In Fig. 4, we show the variation of $R_{\psi(4160)\to D^+ D^-}(\psi(4160))$ in $\alpha$. Even in a large region $0.5 < \alpha < 1.5$, the obtained ratio $R_{\psi(4160)\to D^+ D^-}(\psi(4160))$ is $(6.96 \pm 7.05)\%$ and $(6.83 \pm 6.89)\%$ for two typical cases as mentioned above. We find that this ratio is not strongly dependent on $\alpha$. Therefore, in this approach, we can well control the uncertainty of our prediction. Using the experimental data of $\psi(4160) \to J/\psi \eta$ (Belle) measurement with 90% confidence level [12], this ratio is $0.48 \pm 0.10 \pm 0.17\%$ [10, 11] and this obtained ratio, we can predict $\mathcal{B}(\psi(4160) \to J/\psi \eta') = (2.0 \sim 4.8) \times 10^{-5}$, which is consistent with the upper limit $(< 5 \times 10^{-5})$ for $\mathcal{B}(\psi(4160) \to J/\psi \eta')$ given by the CLEO measurement with 90% confidence level [12].

### IV. SUMMARY

The charmonium above the threshold of a pair of charmed mesons dominantly decays into charmed mesons, which can couple with charmonium and light meson by exchanging a proper charmed meson, such a mechanism, i.e., the meson loop effect, is essential to understand the decay behavior of higher charmonia above the thresholds in obtaining the enhanced decay amplitudes.

Stimulated by the experimental measurements by the Belle and BESIII Collaborations [10, 11, 13], we introduce the meson loop mechanism to study the $\eta$ transitions between $\psi(4040)/\psi(4160)$ and $J/\psi$ in an effective Lagrangian approach. The theoretical estimates have shown overlaps with the experimental measurements in a reasonable parameter range. More over, we have predicted the branching ratio of $\psi(4160) \to J/\psi \eta'$ of the order of $10^{-4}$, which can be tested by future experiments. We should emphasize that if we include finite-width effects [26] of $\psi(4040)$ and $\psi(4160)$ in this paper, we might be able largely to improve our results, and in addition, we may calculate a nonzero decay ratio of $\psi(4040) \to J/\psi \eta'$ even though this is kinematically forbidden. However, inclusion of the finite-width effects introduces another ambiguity that is out of our control, and hence we have not included this type of effects in this paper.

Before closing this section, we would like to discuss the possible extension of our work. We notice possible radiative decays of $\psi(4040)/\psi(4160)$ into $\gamma \eta$ or $\gamma \eta'$, which are similar to the $\psi(4040)/\psi(4160) \to J/\psi \eta'$ processes discussed in this work, where $\psi(4040)/\psi(4160) \to \gamma \eta^{(')}$ can also occur through triangle charmed-meson loops. So far experiments have not yet observed $\psi(4040)/\psi(4160) \to \gamma \eta^{(')}$. Thus, theoretical estimate of the branching ratios for $\psi(4040)/\psi(4160) \to \gamma \eta^{(')}$
may provide important information of further experimental search for these interesting radiative decay channels. What is more important is that $\psi(4040)/\psi(4160) \rightarrow \gamma \eta(\prime)$ can be an important test to the hadronic loop effects on the higher charmonium decays.

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[1] J. Beringer et al. [Particle Data Group Collaboration], Phys. Rev. D 86 (2012) 010001.
[2] M. Ablikim et al. [BES Collaboration], Phys. Rev. Lett. 97, 121801 (2006) [hep-ex/0605107].
[3] M. Ablikim et al. [BES Collaboration], Phys. Lett. B 641, 145 (2006) [hep-ex/0605105].
[4] M. Ablikim et al. [BES Collaboration], Phys. Lett. B 659, 74 (2008).
[5] M. Ablikim et al. [BES Collaboration], Phys. Rev. D 76, 122002 (2007).
[6] Z.-G. He, Y. Fan and K.-T. Chao, Phys. Rev. Lett. 101, 112001 (2008) [arXiv:0802.1849 [hep-ph]].
[7] X. Liu, B. Zhang and X.-Q. Li, Phys. Lett. B 675, 441 (2009) [arXiv:0902.0480 [hep-ph]].
[8] Y.-J. Zhang, G. Li and Q. Zhao, Phys. Rev. Lett. 102, 172001 (2009) [arXiv:0902.1300 [hep-ph]].
[9] Q. Wang, X.-H. Liu and Q. Zhao, Phys. Rev. D 84 (2011) 014007 [arXiv:1103.1095 [hep-ph]].
[10] Bruce Yabsley et al. [Belle Collaboration], in 36th Internation conference on High Energy Physics, Melbourne.
[11] X. L. Wang et al. [Belle Collaboration], arXiv:1210.7550 [hep-ex].
[12] T. E. Coan et al. [CLEO Collaboration], Phys. Rev. Lett. 96 (2006) 162003 [hep-ex/0602034].
[13] M. Ablikim, et al. [The BESIII Collaboration], arXiv:1208.1857 [hep-ex].
[14] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. 281, 145 (1997) [arXiv:hep-ph/9605342].
[15] Y. S. Oh, T. Song and S. H. Lee, Phys. Rev. C 63, 034901 (2001) [arXiv:nucl-th/0010064].
[16] D. Coffman et al. [MARK-III Collaboration], Phys. Rev. D 38 (1988) 2695 [Erratum-ibid. D 40 (1989) 3788].
[17] J. Jousset et al. [DM2 Collaboration], Phys. Rev. D 41 (1990) 1389.
[18] N. N. Achasov and A. A. Kozhevnikov, Phys. Rev. D 49, 275 (1994).
[19] A. Deandrea, G. Nardulli and A. D. Polosa, Phys. Rev. D 68, 034002 (2003) [hep-ph/0302273].
[20] H.-Y. Cheng, C.-K. Chua and A. Soni, Phys. Rev. D 71 (2005) 014030 [hep-ph/0409317].
[21] C. Isola, M. Ladisa, G. Nardulli and P. Santorelli, Phys. Rev. D 68, 114001 (2003) [hep-ph/0307367].
[22] S. Ahmed et al. [CLEO Collaboration], Phys. Rev. Lett. 87, 251801 (2001) [hep-ex/0108013].
[23] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 79, 092001 (2009) [arXiv:0903.1597 [hep-ex]].
[24] C. Itzykson and J. B. Zuber, “Quantum Field Theory,” New York, Usa: Mcgraw-hill (1980) 705 p.
[25] M. E. Peskin and D. V. Schroeder, “An Introduction to quantum field theory,” Reading, Usa: Addison-Wesley (1995) 842 p.
[26] F. Giacosa and G. Pagliara, Phys. Rev. C 76, 065204 (2007) [arXiv:0707.3594 [hep-ph]].