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A simulation of nonlinear thermally radiative hydrodynamic stagnation point flow of nanomaterial in view of a stretchable surface

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Abstract: This article aims to explore the mathematical and computational communication of transverse magnetic field interaction to stagnation point viscoelastic nanofluid flow over convectively heated stretching surface accompanied with a heat source, magnetohydrodynamics, and viscous dissipation. The mathematical framework is established for mass conservation, momentum, energy conservation, and concentration of nanoparticles is implemented. The constitutive nonlinear partial differential flow expressions are reduced by utilizing compatible similarity transformations. The non-dimensionless flow laws of (PDEs) are changed into nonlinear dimensionless governing ordinary differential flow laws and then the bvph2 numerical technique is employed for its solution. The consequences of innumerable governing flow parameters are explicitly deliberated and plotted graphically. The physical such as drag force and heat transfer rate are taken into the account and evaluated accordingly. To confirmed the legitimacy and reliability of the upcoming numerical results were compared with homotopic solution (HAM) and an outstanding promise was perceived.

Keywords: Numerical solution, Homotopic solution, MHD, Joule heating, nanofluid, heat source.

1. Introduction

Many researchers have recently paid serious attention to hydromagnetic boundary layer nanofluid flow in view of convectively heated stretching surface, because of their important and distinctive process in the production of various engineering products. The outcomes of thermally radiative and viscous dissipation on electrically conductive flow of nanomaterial flow by a nonlinear stretching surface with MHD interaction were studied numerically by Mabood et al. [1]. The influence of heat transfer and radiation on a rotating stretching sheet was considered by Wahiduzzaman et al [2]. Further, they perceived that the rise in temperature and concentration distributions, subject to increment in magnetic, radiation parameter and Eckert numbers, but the reverse effect was achieved by increase in the stretching parameter. Choi [3] developed the concept of nanoparticles in 1995 to enhance the thermal behavior of base fluid by incorporating these nanoparticles in base liquid called nanofluid. Moreover, they performed experimentally to show that the embedding of nano sized particles to common liquids such as mixture of water, ethylene glycol, oil and many more base liquids escalating the thermal characteristic of these common fluids intensely. In order to observe the flow characteristics of nanomaterials in various flow situations, a variety of experimental and theoretical studies were conducted based on their wide range of applications. For example, Ibrahim and Tulu [4] present the phenomena of thermal transfer of nanofluid moving on a wedge surface. Zhang et al. [5] considered the impact of multi-dimensional nanomaterial flow and radiation and viscous dissipation effects between two rotating spherical shape. In the occurrence of nanomaterial, Arain et al. [6] inspected the consequences of Carreau
fluid flow in view of suspension of nano size particles along with boundary layer features. In addition, Alwawi et al. [7] have examined the effects of Casson nanofluids in the boundary flux of magnetic fields and nanoparticles. Several scholars and engineers have carried out to investigated the influence of stagnation point nanomaterial flow subject to radiation effects with different geometries. Nagendramma et al. [8] discussed the characteristics of thermophoresis and multi slips effects on viscoelastic Maxwell nanomaterial flow by a stretching permeable surface with heat transfer and viscous dissipation. Ramesh et al. [9] accomplished numerical solution to analyzed Maxwell nano liquids past a stretching surface embedded in porous medium along with thermal radiations. The thermophoresis and Brownian motion effect in the enhancement of heat transfer on MHD two-dimensional viscoelastic stagnation-point nanofluid flow in view of stretching surface with convective boundary conditions is examined by Bai et al. [10]. Haritha et al. [11] have studied the mutual effect of convective boundary conditions and Navier slip on an unsteady Maxwell fluid flow by a stretching sheet subject to magnetic influence, radiation effect, heat reservoir and chemical reaction is analyzed. Mushtaq et al. [12] investigated buoyancy effects of Maxwell fluid in stagnation-point towards a vertical stretching sheet with transverse magnetic influence. The influence of heat mass diffusion of the upper convected Maxwell nanofluid flow over a stretching near the stagnation point with thermal effects is analyzed by Khan et al. [13]. Aziz and Shams [14] inspected thermal radiation processes for heat transference from the flow of electrically conducting Maxwell nanofluid with variable thermal conductivity. The consequences, of MHD stagnation point flow and thermal radiation effects due to upper-convected Maxwell nanofluid by passed a stretching sheet is investigated by Ibrahim [15]. Stability analysis and dual solutions of MHD stagnation point Casson fluid with thermal radiation and viscous dissipation are analyzed by Lund et al. [16]. The results of electrically conducted stagnation point Jeffrey nanofluid flow by a stretching surface in view of Joule heating and radiations effects subject to uniform magnetic field is evaluated by Hayat et al. [17]. Recently several engineers and mathematician investigated [18-23] explored the distinctive features of heat transfer problems and boundary layer flow of non-Newtonian fluids over a stretching surface with different geometry subject to viscous dissipation and Joule heating. Goyal and Bhargava [24] highlighted the impact of velocity slip condition on heat flow of nanofluid over stretching surface with heat reservoir and uniform magnetic field. The impact of nonlinear thermal radiations on MHD nanofluid past a stretching surface is evaluated by Farooq et al. [25]. The impact of magnetohydrodynamics two-dimensional stagnation point flow of Maxwell nanofluid towards a vertically stretching surface embedded in porous medium with radiation and chemical reaction is investigated by Walelign et al. [26].

2. Mathematical Formulation

In this study, it is considered in this model two-dimensional viscoelastic stagnation point nanofluid flow in view of convectively heated stretching surface. Also, a uniform magnetic field $B$ of strength $B_0$ is employed normally to sheet and Joule heating and viscous dissipation is taken into the account. As the surface is stretched because of two equal and opposite forces acting in the stagnation point $(0,0)$ with the velocities $u_w(x) = ax$ and $u_x(x) = bx$ shown in Fig.1. The
magnetic Reynolds number is considered sufficiently small to ignored the effect of induced magnetic field.

Fig. 1. Coordinate system and sketch of the flow problem

Governing partial differential equations by [10].

Mass conservation expression

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(1)

Momentum conservation expression

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_\infty \left( \frac{\partial u_\infty}{\partial x} \right) + v_f \left( \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma_e H_0^2 (u-u_\infty)}{\rho_f} + k_0 \left[ \frac{\partial^3 u}{\partial x \partial y^2} + \left( \frac{\partial u}{\partial x} \right) \frac{\partial^2 u}{\partial y^2} \right] \]
(2)

Energy conservation expression
Concentration expression:

\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_b \frac{\partial^2 C}{\partial y^2} + \left( \frac{D_b}{T} \right) \frac{\partial^2 T}{\partial y^2} + \frac{Q_0 (T - T_f)}{(\rho C_p)_f}
\]

The particular extreme values are

\[
u = u_w(x) = ax, \quad v = 0, \quad v = -k \frac{\partial T}{\partial y} = (T - T_f)h, \quad C_w = C \text{ for } y = 0
\]

\[	ext{and } u = u_\infty(x) = bx, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ for } y \rightarrow \infty
\]

Here, \((u, v)\) are components of velocity in \(x-axis\) and \(y-axis\) respectively. The relation \(u_w(x) = ax\) denote velocity of the stretching sheet, \(v_f\) kinematic-viscosity, \(\sigma_e\) denote fluid electrical-conductivity nature, \(D_r\) thermophoresis-diffusion, \(D_b\) Brownian diffusion, \(M\) magnetic parameter, \(T\) is the temperature, \(u_\infty\) free stream velocity, \(k_o\) represent viscoelastic parameter, \(C\) concentration, \(\alpha\) thermal diffusion, \(Q_0\) heat source parameter, \(\rho_f\) fluid density, \(a, b\) are constants, \(h\) denote convective heat transport coefficient, \(k\) thermally radiative parameter, \(C_w\) volume of the nanomaterial at wall.

By introducing similarity solutions

\[
\eta = \frac{a}{\sqrt{v_f}} y, \quad u = axf'(x), \quad v = -\sqrt{av_f} f (\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \text{ and } \varphi(\eta) = \frac{C - C_w}{C_w - C_\infty}
\]

In view of Equation (6) the governing flow expressions (2)–(4) are changed into system of (ODEs):

\[
f'' + \frac{ff'' - (f')^2}{2} + K \left( 2f' f'' - ff'' - (f')^2 \right) + M (\lambda - \lambda') + \lambda^2 = 0
\]

\[
\theta'' + Pr \left[ f \theta' + Ec (f')^2 + MEc (\lambda - \lambda') + Nb \theta' \varphi' + Nt (\theta')^2 + \gamma \theta \right] = 0
\]

\[
\varphi'' + Le Pr f \varphi' + \frac{Nt}{Nb} \theta'' = 0
\]
The boundary conditions in (4) are changed into dimensionless form:
\[ f(0) = 0, \ f'(0) = 1, \ \theta'(0) = -Bi(1 - \theta(0)), \ \phi(0) = 1 \text{ and } f'(\infty) = \lambda, \ \theta(\infty) = 0, \ \phi(\infty) = 0 \quad (10) \]

Here, \( \theta(\eta) = T - T_{\infty}/T_{f} - T_{w} \) is dimensionless temperature with \( T = T_{\infty} \left[ (1 + (\theta_{w} - 1)\theta) \right] \) and \( \theta_{w} = T_{f}/T_{\infty} \) is the temperature ratio parameter.

The governing variables appearing in Equations (7)–(10) are defined by:
\[
K = \frac{ak_{0}}{\nu_{f}}, \quad M = \frac{\sigma_{e}H_{0}^{2}}{a\rho_{f}}, \quad \lambda = \frac{b}{a}, \quad Pr = \frac{\nu_{f}}{\alpha}, \quad Ec = \frac{u_{w}^{2}}{C_{pf}(T_{f}-T_{\infty})}, \quad Nb = \frac{\tau D_{B}(C_{w}-C_{\infty})}{\nu_{f}}, \\
Nt = \frac{\tau D_{t}(T_{f}-T_{\infty})}{\nu_{f}T_{\infty}}, \quad \gamma = \frac{Q_{0}H_{0}^{2}}{a(\rho C_{p})_{f}}, \quad Le = \frac{\alpha}{D_{B}}, \quad Bi = \sqrt{\frac{\nu_{f}}{a} \left( \frac{h}{k} \right)}
\]

Elucidates viscoelastic parameter, magnetic parameter, ratio parameter, Prandtl number, Eckert number, Brownian motion, thermophoresis parameter, heat/source parameter, Lewis number and Biot number respectively.

Expressions of physical quantities having significance importance are the drag force coefficient, Nusselt and Sherwood numbers defined by:
\[
Cf_{x} = \frac{\tau_{w}}{\rho u_{w}^{2}}, \quad Nu_{x} = \frac{xq_{w}}{k(T_{f}-T_{w})}, \quad Sh_{x} = \frac{xq_{m}}{D_{B}(\phi_{f}-\phi_{w})} \quad (11)
\]

Here, \( \tau_{w} \) denote shear stress at sheet, \( q_{w} \) represent heat flux and \( q_{m} \) signify mass flux of nanoparticle is expressed as:
\[
\tau_{w} = \left[ \mu_{f} \frac{\partial u}{\partial y} + k_{0}\rho_{f} \left( u \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} \right) + v \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) + 2 \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} \right) \right]_{y=0} \quad (12)
\]
\[
q_{w} = -k \frac{\partial T}{\partial y}|_{y=0} \quad (13)
\]
\[
q_{m} = -D_{B} \frac{\partial \phi}{\partial y}|_{y=0} \quad (14)
\]

The simplified form of \( \left( Cf_{x}, Nu_{x}, Sh_{x} \right) \) with the aid of (6) are as follows:
\[
\sqrt{Re}Cf_{x} = f \"(0) + K \left[ 3f \'(0) f \"(0) - f(0) f\"(0) \right] \quad (15)
\]
\[
\frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0)
\]  
\[
\frac{Sh_x}{\sqrt{Re_x}} = -\varphi'(0)
\]

Whereas \( Re_x = ax^2/\nu \) signified local Reynolds number.

3. **Numerical solution and convergence analysis**

In order to addressed the nonlinear system of (ODEs) in (07)-(09) for numerical solution subject to particular extreme values in (10). The flow domain \([0, \infty]\) is replaced with bounded domain \([0, \eta_\infty]\) such that \(\eta_\infty\) take any finite real value should be selected in such a way that the solution verifies the entire domain. Furthermore, the equations (07)-(09) constitute an extremely high nonlinear coupled initial boundary value problem of third and second-order differential expressions. The system has been transferred into seven first order initial value problems.

The new alteration variables \( l, m, p \) and \( q \) then the system of nonlinear differential expressions (07)–(10) are converted to first order linear differential equations as:

\[
f'(\eta) = l
\]

\[
l'(\eta) = m
\]

\[
m' = -\left[ fm - l^2 + K(2lm - fm - m^2) + M(\lambda - l) + \lambda^2 \right]
\]

\[
\theta'(\eta) = p
\]

\[
p' = -Pr\left[ fp + Ecm^2 + MEc(\lambda - l)^2 + Nbpq + Ntp^2 + \gamma \theta \right]
\]

\[
\phi'(\eta) = q
\]

\[
q' = -\left[ LePr fq + \frac{Nt}{Nb}p' \right]
\]

with altered initial and boundary conditions,

\[
f(0) = 0, \ l(0) = 1, \ q(0) = -Bi(1 - \theta(0)), \ \phi(0) = 1, \ l(\infty) = \lambda, \ \theta(\infty) = 0, \ \phi(\infty) = 0
\]
Consequently, we developed the most efficient and validated numerical algorithm of bvp4c tool in line with Mathematica software. The step size $\Delta \eta = 0.001$ is utilized to attain the computational results correct to six decimals $10^{-6}$ place set as a criterion for computational convergence. Moreover, numerical results for velocity, temperature and concentration fields were compare with homotopic method (HAM) to confirmed the effectiveness of the numerical technique. The results are tabularized and shown in Tables 1-3. Plot for the total residual error is evaluated in Fig. 2. We perceived a diminishing tendency in the average squared residual error for higher-order deformations.

| $\eta$ | HAM solution | Numerical solution | Absolute error |
|-------|--------------|--------------------|----------------|
| 0.0   | 1.0000000    | 1.0000000          | 0.0000000      |
| 0.5   | 0.566402     | 0.565980           | 0.000422       |
| 1.0   | 0.345148     | 0.344302           | 0.000845       |
| 1.5   | 0.222820     | 0.221469           | 0.001351       |
| 2.0   | 0.151288     | 0.149336           | 0.001953       |
| 2.5   | 0.107037     | 0.104409           | 0.002628       |
| 3.0   | 0.077603     | 0.074247           | 0.003356       |
| 3.5   | 0.056064     | 0.051932           | 0.004131       |
| 4.0   | 0.038463     | 0.035305           | 0.004959       |
| 4.5   | 0.022495     | 0.016652           | 0.005843       |
| 5.0   | 0.006738     | 6.312960$\times 10^{-8}$ | 0.006738      |

| $\eta$ | HAM solution | Numerical solution | Absolute error |
|-------|--------------|--------------------|----------------|
| 0.0   | 1.133840     | 1.155850           | 0.022006       |
| 0.5   | 1.135380     | 1.145830           | 0.031880       |
| 1.0   | 1.030410     | 1.067950           | 0.037542       |
| 1.5   | 0.858672     | 0.896354           | 0.037681       |
| 2.0   | 0.663212     | 0.696330           | 0.033117       |
| 2.5   | 0.477568     | 0.503349           | 0.025781       |
| 3.0   | 0.320321     | 0.337953           | 0.017632       |
| 3.5   | 0.197477     | 0.207587           | 0.010109       |
| 4.0   | 0.107336     | 0.111340           | 0.004004       |
| 4.5   | 0.044669     | 0.044222           | 0.000448       |
| 5.0   | 0.003369     | 1.802650$\times 10^{-8}$ | 0.003369      |

| $\eta$ | HAM solution | Numerical solution | Absolute error |
|-------|--------------|--------------------|----------------|
| 0.0   | 1.0000000    | 1.0000000          | 0.0000000      |
| 0.5   | 0.717707     | 0.714669           | 0.003037       |
### Results and graphical analysis

In this segment, we exhibit the physical behavior of innumerable influential flow parameters involved in (07)-(10) on stream velocity, thermal field, and concentration profile are debated and plotted through Figs. 3–28.

Figures 3–5, demonstrates the impact of stream velocity parameter $\lambda$ on velocity, thermal field and nanoparticles concentration. It is obvious fluid velocity $f'(\eta)$ enhances subject to increment in $\lambda$. For $\lambda = 1$ we perceive that both fluid and stretching sheet varies with the same velocity and indicates no boundary layer case are disclosed in Fig. 3. As anticipated fluid temperature $\theta(\eta)$ and nanoparticles concentration $\varphi(\eta)$ dwindle subject to larger velocity ratio parameter are

|   | 1.0 | 0.491238 | 0.486369 | 0.004868 |
|---|-----|----------|----------|----------|
|   | 1.5 | 0.327860 | 0.322413 | 0.005447 |
|   | 2.0 | 0.216270 | 0.210943 | 0.005327 |
|   | 2.5 | 0.141594 | 0.136508 | 0.005086 |
|   | 3.0 | 0.091584 | 0.086536 | 0.005048 |
|   | 3.5 | 0.057704 | 0.052421 | 0.005284 |
|   | 4.0 | 0.034408 | 0.028690 | 0.005717 |
|   | 4.5 | 0.018170 | 0.011936 | 0.006233 |
|   | 5.0 | 0.006738 | $-1.114810 \times 10^{-8}$ | 0.006738 |
shown in Figs. 4 and 5. Attributes of viscoelastic parameter \((K)\) on \(f'(\eta)\), \(\theta(\eta)\) and \(\varphi(\eta)\) are interpreted in Figs. 6–8. Here, velocity profile \(f'(\eta)\) and thermal field \(\theta(\eta)\) augments via higher viscoelastic parameter are designed in Figs. 6 and 7. The curves of concentration profile \(\varphi(\eta)\) for various values of \((K)\) are plotted in Fig. 8. Higher estimations of \((K)\) yield \(\varphi(\eta)\) diminished. Physically, elasticity is upsurges by the velocity trouble in material. Fig. 9 discloses variations in velocity field subject to magnetic parameter \((M)\). Here, \(f'(\eta)\) enhances via higher estimation of magnetic factor. Physically, the presence of magnetic parameter generates resistive obstruction force to fluid flow. Such resistive force is called Lorentz force. In consequence fluid movement reduces. Impact of \((M)\) is interpreted in Fig. 10. Clearly thermal field boots through larger magnetic parameter. physically, such scenario is perceived due to Lorentz force. A fractional resistive power which opposes fluid movement which escalates the kinetic energy of fluid particles. Thus, fluid thermal energy is augmented. Hence, stronger the magnetic field, the thicker the thermal boundary. Fig. 11 portrays \((Ec)\) impact of Eckert number on temperature distribution \(\theta(\eta)\). As expected, thermal field increases subject larger \((Ec)\) parameter. physically, Eckert number explains relation in enthalpy and kinetic energy in fluid moment. One can perceive that higher \((Ec)\) parameter, internal system develops an additional heat energy due to which fluid particles have higher kinetic energy releases and consequently, \(\theta(\eta)\) upsurges. Attributes of Eckert number on concentration profile are interpreted in Fig. 12. Clearly, \(\varphi(\eta)\) diminishes when \((Ec)\) is augmented. In reality, the frictional heating is accountable for thermal energy which is left in the fluid. The influence of Lewis number on \(\varphi(\eta)\) is plotted in Fig. 13. Here, we found lower \(\varphi(\eta)\) subject to \((Le)\) increase. Since Lewis number \((Le)\) is defined as ratio of thermal diffusivity to Brownian diffusion, and then it cannot be equivalent to zero. Fig. 14 depicts Brownian motion parameter \((Nb)\) impact on \(\theta(\eta)\). As expected, thermal field upsurges. In nano liquids, the Brownian motion increases by incorporating nanoparticles count and particle size and movement at that point play significant contribution regarding to transference of thermal radiations. An increase in \((Nb)\) perceived an effective yield in nanoparticles movement within the flow. Fig. 15 explains \((Nb)\) impact on \(\varphi(\eta)\). Clearly an increase in \((Nb)\) increases nanoparticles movement, inconsequence, particles freely move with arbitrary velocities in random direction owing to Brownian aspect. Consequently, larger \((Nb)\) estimations boost in \(\varphi(\eta)\). Fig. 16 exposes variations in \(\theta(\eta)\) subject to thermophoresis \((Nt)\) parameter. Here, thermal field enlarges via larger \((Nt)\) parameter. Physically, the thermophoretic force increases subject to increment in \((Nt)\) parameter, this force assists to escape particles by warmer towards colder region and ultimate
thermal field $\theta(\eta)$ augments. In reality, growing thermophoresis force reasons to transfer of nanoparticles from heat to cold parts and subsequently augment the magnitude of $\varphi(\eta)$ is plotted in Fig. 17. The contribution of Prandtl number ($Pr$) on temperature field is evaluated through Fig. 18. Here, we perceived that thermal diffusivity diminishes when Prandtl number upsurges. Hence, $\theta(\eta)$ decays. Fig. 19 exhibits thermal field $\theta(\eta)$ effect for unlike values ($Bi$) parameter. Here, we perceived that thermal field diminishes subject to larger dimensionless parameter ($Bi$). Physically, convection reduces due to the high resistance and conduction upsurge because of low thermal resistance. Attributes of $\gamma$ parameter is interpreted in Fig. 20. Clearly thermal field upsurges subject to positive change in heat source parameter, whereas an opposite result perceived in nanofluid temperature with negative values of heat source parameter are revealed in Fig. 21.

Figures 22 and 23 highlights $M, K$ and $\lambda$ impacts on skin friction. Here, $\sqrt{Re_x}Cf_x$ dwindles when $M$ and $K$ are increased. However, reverse features are found for $M$ and $\lambda$. Attributes of $\lambda$, $\gamma$, Ec and $Bi$ effects on Nusselt number $-\theta'(0)$ are particularized in Figs. 24–26. These figures highlight a decline in $-\theta'(0)$ subject to larger estimations these parameters. Figs. 27 and 28 visualize $-\varphi'(0)$ analysis subjected to $Nt, Nb$ and $Le$. As expected, Sherwood number augment when these parameters are augmented.
Fig. 3 Result of $\lambda$ on velocity $f'(\eta)$

Fig. 4 Result of $\lambda$ on temperature $\theta(\eta)$
**Fig. 5** Result of $\lambda$ on concentration $\varphi(\eta)$

**Fig. 6** Result of $K$ on velocity $f'(\eta)$
**Fig. 7** Result of $K$ on temperature $\theta(\eta)$

**Fig. 8** Result of $K$ on concentration $\varphi(\eta)$
Fig. 9 Result of $M$ on velocity $f'(\eta)$

Fig. 10 Result of $M$ on temperature $\theta(\eta)$
Fig. 11 Result of $Ec$ on temperature $\theta(\eta)$

Fig. 12 Result of $Ec$ on concentration $\phi(\eta)$
Fig. 13 Result of $Le$ on concentration $\varphi(\eta)$

Fig. 14 Result of $Nb$ on temperature $\theta(\eta)$
Fig. 15 Result of $Nb$ on concentration $\phi(\eta)$

Fig. 16 Result of $Nt$ on temperature $\theta(\eta)$
Fig. 17 Result of $Nt$ on temperature $\phi(\eta)$

Fig. 18 Result of $Pr$ on temperature $\theta(\eta)$
**Fig. 19** Result of $Bi$ on temperature $\theta(\eta)$

**Fig. 20** Result of $\gamma$ on temperature $\theta(\eta)$
Fig. 21 Result of $\gamma$ on temperature $\theta(\eta)$

Fig. 22 $K$ and $M$ versus skin friction
Fig. 23 $\lambda$ and $M$ versus skin friction

Fig. 24 $\lambda$ and $\gamma$ versus Nusselt number
**Fig. 25** $Ec$ and $\gamma$ versus Nusselt number

**Fig. 26** $Ec$ and $Bi$ versus Nusselt number
**Fig. 27** $Nb$ and $Nt$ versus Sherwood number

**Fig. 28** $Le$ and $Nb$ versus Sherwood number

5. **Final remarks**
The forthright aim of this communication is to investigate the effect of two-dimensional incompressible magneto stagnation point nanofluid flow by convectively heated stretchable surface with thermal radiation and viscous dissipation effects are investigated and plotted graphically. Following are the key findings of the present outlined as underneath.

- As perceived that larger stream velocity parameter corresponds to upsurge $f'(\eta)$ and lower thermal field $\theta(\eta)$.
- An increment in viscoelastic parameter yields higher $f'(\eta)$ and $\theta(\eta)$.
- Larger magnetic field influence agrees to lower $f'(\eta)$ and higher $\theta(\eta)$.
- Both $\theta(\eta)$ and $-\theta'(0)$ are diminished for larger Biot number.
- Thermal field $\theta(\eta)$ incremented with positive heat source parameter and reverse influence found for negative values.
- Both $\varphi(\eta)$ and $-\varphi'(0)$ are augmented for higher Brownian parameter.

**Data availability statement**

The data used to support the findings of this study are included within the article.

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**Authors Contribution Statement**

H.R and Z.K wrote main manuscript file S.I and W.K give simulation of the problem. All authors contributed equally.
Figures

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