A short review of problems with parton distribution functions in nucleons, non-polarized and polarized, is given. The main part is devoted to the transversity distribution its possible measurement and its first experimental probe via spin asymmetry in semi-inclusive DIS. It is argued that the proton transversity distribution could be successfully measured in future DIS experiments with longitudinally polarized target.

1 Parton characteristics of hadrons

It is well known that three most important (twist-2) parton distributions functions (PDF) in a nucleon are the non-polarized distribution function $f_1(x)$, the longitudinal spin distribution $g_1(x)$ and the transverse spin distribution $h_1(x)$ [1]. The first two have been more or less successfully measured experimentally in classical deep inelastic scattering (DIS) experiments but the measurement of the last one is especially difficult since it belongs to the class of the so-called chiral-odd structure functions and can not be seen there.

The non-polarized PDF’s was measured for decades and are rather well known in wide range of $x$ and $Q^2$. Its behavior in $Q^2$ is well described by the QCD evolution equation (DGLAP) and serves as one of the main source of $\alpha_s(Q^2)$ determination. The most recent parametrization of these functions can be found in [2].

One of the main problem here is the very small $x$ behavior. Summing the leading $\log x$ with the help of the BFKL equation predicts a rather quick rise of $xf_1(x) \propto x^{-0.55}$ which seems to find some experimental support. Recent calculations of NLL corrections, however, cardinaly change this situation [3]. Another problem is the flavor asymmetry of the sea quarks connected with the break down of the Gottfried sum rule [4].

The longitudinal spin PDF’s draw common attention during last decade in connection with the famous ”Spin Crisis”, i.e. astonishingly small portion of the proton spin carried by quarks (see [5] and references therein). The most popular explanation of this phenomenon is large contribution of the gluon spin $\Delta G(x)$. The direct check of this hypothesis is one of the main problem of future dedicated experiments like COMPASS at CERN. Even now, however, there are some indication to considerable value of $\Delta G(x)$ coming from the $Q^2$ evolution of the polarized PDF’s [6] with the result $\int_0^1 dx \Delta G(x) = 0.58 \pm 0.12$ at $Q^2 = 1 GeV^2$ and from the first direct experimental probe of $\Delta G(x)$ by HERMES collaboration [7] with the result $\Delta G(x)/G(x) = 0.41 \pm 0.18$ in the region $0.07 < x < 0.28$ (see Fig. 1). The latter

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is in reasonable agreement with large \( N_c \) limit prediction \( \Delta G(x)/G(x) \approx 1/N_c \) for not very small \( x \).

Another problem here is the sea quark spin asymmetry. It is usually assumed in fitting the experimental data that \( \Delta \bar{u} = \Delta \bar{d} = \Delta \bar{s} \). This \textit{ad hoc} assumption however contradicts with large \( N_c \) limit prediction \( \Delta \bar{u} \approx -\Delta \bar{d} \). This was previously discovered for the instanton model \[9\] and supported by calculations in the chiral quark soliton model \[10,11\]. Also this agrees with the phenomenological approach based on "Pauly blocking principle" \[12\]. An indication to a nonzero value for \( \Delta \bar{u} - \Delta \bar{d} \) was also observed in \[13\].

Turn now to DIS with transversely polarized target. A new result on the structure function \( g_2 \) was recently reported by E155x collaboration \[14\]. Its twist-2 part is well defined by the structure function \( g_1 \) via Wandzura-Wilczek relation \[15\]

\[
g_{2}^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_{x}^{1} \frac{dy}{y} g_1(y, Q^2) \tag{1}
\]

and the deflection off this is just pure dynamical twist-3 contribution. This deflection have to be small however due to two exact sum rules automatically valid for the WW-part: the Burkhardt-Cottingham \[16\]

\[
\int_{0}^{1} dx g_1(y, Q^2) = 0 \quad \text{(exper. } -0.01 \pm 0.02) \tag{2}
\]
Old and new PDF’s and PFF’s

and Efremov-Leader-Teryaev \[17\]

\[
\int_0^1 dx x (g_1^{val}(y, Q^2) + 2g_2^{val}(x, Q^2)) = 0 \quad \text{(exper. } -0.004 \pm 0.008) \quad (3) 
\]

Concerning the transversity distribution it was completely unknown experimentally till recent time. The only information comes from the Soffer inequality \[18\]

\[ |h_1(x)| \leq \frac{1}{2}[f_1(x) + g_1(x)] \]

which follows from density matrix positivity. To access these chiral-odd structure functions one needs either to scatter two polarized protons and to measure the transversal spin correlation \(A_{NN}\) in Drell-Yan process (what is the problem for future RHIC) or to know the transverse polarization of the quark scattered from transversely polarized target. There are several ways to do this:

1. To measure a polarization of a self-analyzing hadron to which the quark fragments in a semi inclusive DIS (SIDIS), e.g. \(\Lambda\)-hyperon \[19\]. The drawback of this method however is a rather low rate of quark fragmentation into \(\Lambda\)-particle (\(\approx 2\%\)) and especially that it is mostly sensitive to \(s\)-quark polarization.

2. To measure a transverse handedness in multi-particle parton fragmentation \[20\], i.e. the correlation of the quark spin 4-vector \(s_\mu\) and particle momenta \(k_\nu^\mu, \epsilon_{\mu\nu\sigma\rho}s^\nu k_1^\sigma k_2^\rho (k = k_1 + k_2 + k_3 + \cdots \text{is a jet 4-momentum}).\)

3. To use a new spin dependent T-odd parton fragmentation function (PFF) \[21\] responsible for the left-right asymmetry in one particle fragmentation of transversely polarized quark with respect to quark momentum–spin plane. (The so-called "Collins asymmetry" \[22\].)

The last two methods are comparatively new and only in the last years some experimental indications to the transversal handedness and to the T-odd PFF have appeared \[23, 24\]. The latter result was used \[27\] to extract the information on the proton transversity distribution \[7\] from recently observed azimuthal asymmetry in SIDIS by HERMES \[21\] collaboration and by SMC \[30\]. This will be the subject of the rest of my talk.

2 T-odd PFF’s

Analogous of PDF’s \(f_1, g_1\) and \(h_1\) are the PFF’s \(D_1, G_1\) and \(H_1\), which describe the fragmentation of a non-polarized quark into a non-polarized hadron and a longitudinally or transversely polarized quark into a longitudinally or transversely polarized hadron, respectively.

These PFF’s are integrated over the transverse momentum \(P_{h\perp}\) of a hadron with respect to a quark. With \(P_{h\perp}\) taken into account, new PFF’s arise. Using

\[A\]

1) A similar work was done also in the paper \[28\] where some adjustable parametrization for the T-odd PFF and some estimations for \(h_1(x)\) were used. Our approach is free of any adjustable parameters.
the Lorentz- and P-invariance one can write in the leading twist approximation 8 independent spin structures \[21, 22\]. Most spectacularly it is seen in the helicity basis where one can build 8 twist-2 combinations, linear in spin matrices of the quark and hadron \(\sigma, S\) with momenta \(k, P_h\).

Especially interesting is a new chiral-odd structure that describes a left–right asymmetry in the fragmentation of a transversely polarized quark:

\[
H_1^\perp \sigma (k \times P_{h\perp})/k\langle P_{h\perp}\rangle,
\]

where \(H_1^\perp\) is a function of the longitudinal momentum fraction \(z\) and quark transverse momentum \(k_2^T\). The \(\langle P_{h\perp}\rangle\) is the averaged transverse momentum of the final hadron \[3\]. Since the \(H_1^\perp\) term is chiral-odd, it makes possible to measure the proton transversity distribution \(h_1\) in semi-inclusive DIS from a transversely polarized target by measuring the left-right asymmetry of forward produced pions (see \[23, 31\] and references therein). It serves as analyzing power of the Collins effect.

The problem is that, first, this function was completely unknown till recent time both theoretically and experimentally. Second, the function \(H_1^\perp\) is the so-called T-odd fragmentation function: under the naive time reversal \(P, k, S\) and \(\sigma\) change sign, which demands a purely imaginary (or zero) \(H_1^\perp\) in the contradiction with naive hermiticity. This, however, does not mean the break of T-invariance but rather the presence of an interference of different channels in forming the final state with different phase shifts, like in the case of single spin asymmetry phenomena \[32\]. A simple model for this function could be found in \[24\] \[3\]. (In this aspect they are very different with the T-odd PDF’s which can not exist since they are purely real. Interaction among initial hadrons which could brings an imaginary part breaks the factorization and the whole parton picture.) Thus, the situation here is far from being clear.

Meanwhile, the data collected by DELPHI (and other LEP experiments) give a possibility to measure \(H_1^\perp\). The point is that despite the fact that the transverse polarization of quarks in \(Z^0\) decay is very small \(O(m_q/M_Z)\), there is a non-trivial correlation between transverse spins of a quark and an antiquark in the Standard Model: \(C_{TT}^{u,c} = (v_2^2 q - a_2^2 q)/(v_2^2 q + a_2^2 q)\), which are at \(Z^0\) peak: \(C_{TT}^{u,c} \approx -0.74\) and \(C_{TT}^{d,s,b} \approx -0.35\). With the production cross section ratio \(\sigma_u/\sigma_d = 0.78\) this gives for the average over flavors value \(\langle C_{TT}\rangle \approx -0.5\).

The transverse spin correlation results in a peculiar azimuthal angle dependence of produced hadrons, if the T-odd fragmentation function \(H_1^\perp\) does exist \[24, 34\]. A simpler method has been proposed by an Amsterdam group \[22\]. They predict a specific azimuthal asymmetry of a hadron in a jet around the axis in direction of a second hadron in the opposite jet \[7\].

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where $\theta_2$ is the polar angle of the electron and the second hadron momenta $P_2$, and $\phi_1$ is the azimuthal angle counted off the $(P_2, e^{-})$-plane. This asymmetry was recently measured \cite{26} using the DELPHI data collection. For the leading charged particles (mostly pions) in each jet of two-jet events, summed over $z$ and averaged over quark flavors (assuming $H_1^q = \sum H_1^q / H_1^p$ is flavor independent), the most reliable preliminary value of the analyzing power is found to be

\[
\left| \frac{\langle H_1^+ \rangle}{\langle D_1 \rangle} \right| = (6.3 \pm 2.0)\% , \tag{5}
\]

with presumably large systematic errors. \footnote{Close value was also obtained from pion asymmetry in inclusive $pp$-scattering.}

### 3 Azimuthal asymmetries

The azimuthal asymmetries measured by HERMES for SIDIS with $\pi^+$ and $\pi^-$ production on longitudinally polarized target are

\[
A_{UL}^W = 2 \int \frac{d\phi dy W(dN^+/S^+dyd\phi - dN^-/S^-dyd\phi)}{\int d\phi dy(dN^+/dyd\phi + dN^-/dyd\phi)} , \tag{6}
\]

where $W = \sin \phi$ or $\sin 2\phi$ and $S_1^\pm$ is the nucleon polarization. It consists of two sorts terms \cite{23}: a twist-2 asymmetry $\sin 2\phi$

\[
A_{UL}^{\sin 2\phi} \propto - \sum_a e^2_a h_1^{(1)a}(x) \langle H_1^{\alpha/\pi}(z) \rangle 
\sum_a e^2_a f_1^a(x) \langle D_1^{\alpha/\pi}(z) \rangle , \tag{7}
\]

and a twist-3 asymmetry $\sin \phi$.

\[
A_{UL}^{\sin \phi} \propto \frac{8M}{Q} \sum_a e^2_a \left( x h_2^a(x) \langle H_1^{\alpha/\pi}(z)/z \rangle - h_1^{(1)a}(x) \langle \tilde{H}_1^{\alpha/\pi}(z)/z \rangle \right) 
\sum_a e^2_a f_1^a(x) \langle D_1^{\alpha/\pi}(z) \rangle . \tag{8}
\]

Here $\phi$ is the azimuthal angle around the $z$-axis opposite to direction of virtual $\gamma$ momentum in the Lab frame, counted from the electron scattering plane. The first asymmetry is proportional to the $p_T$-dependent transverse quark spin distribution in longitudinally polarized proton, $h_1^{(1)}(x, p_T)$, while the second one contains two parts: one term is proportional to the twist-3 distribution function $h_L(x)$ and the
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second one proportional to the twist-3 interaction dependent correction to the fragmentation function $\tilde{H}$. We will systematically disregard this interaction dependent correction (just as in the Wandzura-Wilczek relation)\(^6\).

In the same approximation, the integrated functions over the quark transverse momentum, $h_{1L}(x, p_T)$ and $h_L(x)$, are expressed through the transversity $h_1$

$$h_{1L}^{(1)}(x) \equiv \int d^2 p_T \left( \frac{p_T^2}{2M^2} \right) h_{1L}^+(x, p_T) = -x^2 \int_x^1 \frac{d\xi h_1(\xi)}{\xi^2} = -(x/2)h_L(x),$$

(9)

Assume now that only the favored fragmentation functions $D_a^u/\pi$ and $H_a^{\perp u}/\pi$ will contribute this ratios, i.e. $D_1^u/\pi^+ (z) = D_1^d/\pi^- (z) = D_1^\perp (z)$ and similarly for $H_1^+(z)$. This would allow us to extract from the observed HERMES asymmetries an information on $h_1^+(x) + (1/4)h_1^\perp(x)$ and to compare with some model prediction. Instead, we use the prediction of the chiral soliton model \(^8\) for $h_1^+(x)$ and the GRV parametrization \(^9\) for unpolarized PDF’s $f_1^a (x)$ to calculate the asymmetries $A_{UL}^{\sin \phi}$ and $A_{UL}^{\sin 2\phi}$ for $\pi^+$ and $\pi^-$. The comparison of the asymmetries thus obtained with the HERMES experimental data is presented on Fig. 2.

![Fig. 2. Single spin azimuthal asymmetry for $\pi^+$ (left) and $\pi^-$ (right): $A_{UL}^{\sin \phi}$ (squares) and $A_{UL}^{\sin 2\phi}$ (circles) as a functions of x. The solid ($A_{UL}^{\sin \phi}$) and the dashed lines ($A_{UL}^{\sin 2\phi}$) correspond to the chiral quark-soliton model calculation at $Q^2 = 4 GeV^2$. The shaded areas represent the uncertainty in the value of the analyzing power (3).](image)

The agreement is good enough though the experimental errors are yet rather large. The sign of the asymmetry is uncertain since only the modulus of the analyzing power (3) is known. Fig. 2 gives evidence for positive sign. Notice that

\(^6\) The calculations in the instanton model of QCD vacuum supports disregard of $\tilde{H}$ \(^[36]\). As for $\tilde{H}$, it disappears after integration over $z$ due to relation \(^[37]\) $\tilde{H} \propto z \frac{d}{dz}(z H_1^+(z))$.\(^6\) Czech. J. Phys. 51 (2001)
in spite of the factor $M/Q$ in the Exp. it is several times larger than for moderate $Q^2$. That is why this asymmetry prevails for the HERMES data where $\langle Q^2 \rangle \approx 2.5 \text{GeV}^2$. One can thus state that the effective chiral quark soliton model gives a rather realistic picture of the proton transversity $h_1^a(x)$.

The interesting observable related to $h_1(x)$ is the proton tensor charge. The calculation in this model yields $h_1 \equiv \sum_a \int_0^1 dx \left( h_1^a(x) - \bar{h}_1^a(x) \right) = 0.6$. Compare this with most recent experimental value of the proton axial charge $a_0 = 0.28 \pm 0.05$ and the value obtained in the same model $a_0 = 0.35$. These very different values for the axial and tensor charges of the nucleon are in contradiction with the nonrelativistic quark model prediction.

Concerning the asymmetry observed by SMC on transversely polarized target one can state that it agrees with result of HERMES (see [27] for more details).

In conclusion, using the preliminary estimation for Collins effect analyzing power from DELPHI data and the effective chiral quark soliton model for the proton transversity distribution we obtain a rather good description of the azimuthal asymmetries in semi-inclusive hadron production measured by HERMES and SMC, though the experimental errors are yet large. This, however, is only the first experiments! We would like to stress that our description has no free adjustable parameters. Probably the most useful lesson we have learned is that to measure transversity in SIDIS in the region of moderate $Q^2$ it is not necessary to use a transversely polarized target. Due to approximate Wandzura-Wilczek type relations one can explore the longitudinally polarized target also. This is very important for future experiments, like COMPASS at CERN since the proton transversity could be measured simultaneously with the spin gluon distribution $\Delta G(x)$.

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