THE HE I 584 Å FOREST AS A DIAGNOSTIC OF HELIUM REIONIZATION

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\section*{ABSTRACT}

We discuss the potential of using the He I 584 Å forest to detect and study He II reionization. Significant 584 Å absorption is expected from intergalactic He II regions, whereas there should be no detectable absorption from low density gas in He III regions. Unlike He II Lyα absorption (the subject of much recent study), the difficulty with using this transition to study He II reionization is not saturation but rather that the absorption is weak. The Gunn-Peterson optical depth for this transition is \( \tau \approx 0.1 \times \Delta_{\text{HeII}} \Delta^2 [(1 + z)/5]^{9/2} \), where \( \Delta_{\text{HeII}} \) is the fraction of helium in He II and \( \Delta \) is the density in units of the cosmic mean. In addition, He I 584 Å absorption is contaminated by lower redshift H I Lyα absorption with a comparable flux decrement. We estimate the requirements for a definitive detection of redshifted He I absorption from low density gas (\( \Delta \approx 1 \)), which would indicate that He II reionization was occurring. We find that this objective can be accomplished (using coeval H I Lyα absorption to mask dense regions and in cross correlation) with a spectral resolution of 10\,4 and a signal-to-noise ratio per resolution element of \( \sim 10 \). Such specifications may be achievable on a few known \( z \sim 3.5 \) quasar sightlines with the Cosmic Origins Spectrograph on the Hubble Space Telescope. We also discuss how He I absorption can be used to measure the hardness of the ionizing background above 13.6 eV.

\textit{Subject headings:} cosmology: theory – cosmology: intergalactic medium – quasars: absorption lines

\section{1. INTRODUCTION}

Starlight produced by the first galaxies is the leading candidate for ionizing the hydrogen as well as singly ionizing the helium at \( z \sim 6 \). It takes a harder source of radiation to doubly ionize the helium, so the reionization of this species is likely deferred until \( z \sim 3 \) when quasars produce a sufficient hard UV background (Madau et al. 1999; Furlanetto & Oh 2008; McQuinn et al. 2009). However, the helium could have been doubly ionized at nearly the same cosmic time that hydrogen was reionized if more exotic sources ionized the hydrogen, such as the first generation of metal-free stars (Bromm et al. 2001; Venkatesan et al. 2003; Tumlinson et al. 2004) or miniquasars (Madau et al. 1999; Volonteri & Gnedin 2009). In a third potential scenario, early sources doubly ionized the helium and then shut off. Afterward, the He II recombined such that quasars could again reionize it at \( z \sim 3 \) (Wyithe & Loeb 2003; Venkatesan et al. 2003).

If He II reionization were ending at \( z \sim 3 \), it should be an easier task to definitively detect this process compared to detecting \( z \gtrsim 6 \) reionization processes. Furthermore, if He II reionization were ending at \( z \sim 3 \), it should have significantly affected the temperature of the intergalactic gas and the ultraviolet radiation background. These motivations, along with recent additions to the Hubble Space Telescope (HST), have inspired a significant effort of late to understand the signatures and the detection prospects of He II reionization (Furlanetto & Oh 2008; McQuinn et al. 2009; Lidz et al. 2009; Bolton et al. 2009; McQuinn 2009; Dixon & Furlanetto 2009; Furlanetto & Dixon 2009; Syphers et al. 2009a,b; McQuinn & Switzer 2009).

Three separate observations of the \( z \sim 3 \) IGM suggest that He II reionization was ending around this redshift: First, several studies have measured the temperature of the intergalactic gas from the widths of the narrowest lines in the H I Lyα forest, and the majority of these studies have found evidence for an increase in the IGM temperature of \( \sim 10^4 \) K between \( z \approx 4 \) and \( z \sim 3 \), before a decline to lower redshift (Schaye et al. 2000; Ricotti et al. 2000; Lidz et al. 2009). These trends have been attributed to the heating from He II reionization. Second, observations of He II Lyα absorption from gas at \( 2.8 < z < 3.3 \) show tens of comoving Mpc (cMpc) regions with no detected transmission (Reimers et al. 1997; Heap et al. 2000), which may indicate that He II reionization was not complete. Thirdly, Songaila (1998) and Agafonova et al. (2007) detected evolution in the column density ratios of certain highly ionized metals at \( z \approx 3 \), which they argued was due to a hardening in the ionizing background above 50 eV and, thus, the end of He II reionization.

However, the interpretations of all of these indications for He II reionization are controversial. Temperature measurements of the IGM are difficult, and not all measurements detected the aforementioned trends. It is often argued that He II Lyα absorption saturates at He II fractions that are too small (\( \sim 10^{-3} \) at the mean density) to study He II reionization (although, see McQuinn 2009). Lastly, inferences from metal lines require significant modeling, and studies have reached different conclusions regarding the degree of their evolution at \( z \sim 3 \) (Boksenberg et al. 2003).

This paper discusses intergalactic absorption by the
$1s^2 \, 1S - 1snp \, 1P_o$ transitions of He I as an unsaturated observable of He II reionization. We primarily focus on the strongest and longest wavelength of these absorption lines, the He I $1s^2 \, 1S - 1s2p \, 1P_o$, 584 Å line. For a given optical depth in the H I Ly$\alpha$ forest, the amount of absorption in the He I forest can be directly estimated if both the fraction of helium that is He II and the ratio of the H I and He I photoionization rates are known. In addition, because the He I ionization edge is relatively close to that of hydrogen, the photoionization rate that He I experiences is similar to this for hydrogen. Thus, the most important determinant of the amount of He I 584 Å absorption is the He II fraction.

Other studies have discussed the He I forest, but did not focus on its usefulness as a probe of helium reionization. Tripp et al. (1990) was the first to discuss intergalactic absorption by the He I 584 Å, and this study attempted unsuccessfully to detect this absorption in the spectrum of a $z = 1.7$ quasar. Reimers & Vogel (1993) targeted He I absorption from $z \approx 2$ H I Lyman-limit systems, and reported the first (and at present only) detection of intergalactic He I 584 Å absorption. Finally,Miralda-Escude & Ostriker (1992) showed that the He I forest could be a useful probe of the hardness of the ultraviolet background, and they focused on using this absorption to rule out a particular nonstandard model for the dark matter. Santos & Loeb (2003) followed up on this idea, arguing that the $z \approx 5$ He I forest could be a useful diagnostic of the hardness of the ultraviolet background after hydrogen reionization.

As with the He II Ly$\alpha$ transition at 304 Å, the He I 584 Å transition falls blueward of the hydrogen Lyman limit and, therefore, is subject to continuum absorption by hydrogen, in this case from H I systems with $z > z' = 3.2 (1 + z_{HeI})/5 - 1$. This continuum absorption may even be worse in terms of obscuring the He I 584 Å forest compared to the He II Ly$\alpha$ forest because this spectral region is more affected by higher redshift H I continuum absorbers. It is unlikely that there are more than a handful of quasar sightlines at $z > 4$ with sufficient near ultraviolet (NUV) flux for detection with the present generation of instruments (and almost certainly not at $z > 6$, during He I reionization; Santos & Loeb 2003). Therefore, our focus is on applications of He I absorption at $z \lesssim 4$.

We have performed a cursory search for candidate He I sightlines among the relatively few published HeII Lya forest spectra that extend redward to 584 ($1 + z_{QSO}$) Å. Of note, QSO OQ 172 ($z = 3.54$) has $F_\lambda \approx 2 \times 10^{-16}$ erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$ at 584 ($1 + z$) Å (Lyons et al. 1993), and QSO 0055-269 ($z = 3.67$) has $F_\lambda \approx 1 \times 10^{-16}$ (Gabor Worseck, priv. com.). In fact, we were surprised to find that most He II sightlines in our search had significant flux in the relevant band.

Many of the existing He II sightlines had been selected by their far ultraviolet flux. NUV selection would be more optimal for identifying candidate He I forest sightlines. For example, HS 1140 +3508 ($z = 3.15$) is obscured in the far ultraviolet by a Lyman-limit system, but has a NUV flux of $F_\lambda \approx 2 \times 10^{-16}$ (Gabor Worseck, priv. com.). Another difficulty with the He I forest is that line absorption from foreground systems can contaminate the He I forest, the most important of which is H I Ly$\alpha$ absorption. H I Ly$\alpha$ absorption from a system with a redshift of $z_{HI,1216} = 2.4 (1 + z_{HeI,584})/5 - 1$ falls directly on top of the He I absorption from a redshift of $z_{HeI,584}$. Fortunately, the Ly$\alpha$ forest is quite thin at relevant $z_{HI,1216}$ (with a flux decrement of several percent), and we find that both this contaminant’s mean transmission and variance tend to be comparable to that of the He I forest. Also, the He I forest correlates strongly with the coeval H I Ly$\alpha$ forest, which allows it to be extracted in cross correlation despite this contamination.

This study is timely because the HST resurfacing mission installed the Cosmic Origins Spectrograph (COS) in May 2009. COS is capable of measurements of He I 584 Å absorption at redshifts relevant to He II reionization (2.8 $\lesssim z < 4.5$), and ground-based spectrographs can cover higher redshifts ($z \gtrsim 4.3$, corresponding to $z \gtrsim 3100$ Å). COS is able to achieve higher signal-to-noise ratios than previous instruments in the ultraviolet. A 60 hr exposure with COS would achieve a signal-to-noise ratio of 10 at $R \approx 30,000$ for a flux of $F_\lambda = 2 \times 10^{-16}$ erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$ (the flux of QSO OQ 172 and HS 1140 +3508) at 2600 Å (He I absorption from $z = 3.5$). A 40 hr exposure with COS would achieve a signal-to-noise ratio of 10 for this flux at $R \approx 20,000$ at 2300 Å ($z = 2.9$).

This paper is organized as follows. Section 2 discusses the physics of the He I forest. Section 3 contrasts simulated spectra for this forest under different assumptions regarding the ionization state of the helium. Section 4 quantifies the spectral quality an observation must achieve to verify whether He II reionization was occurring from the He I forest. Appendix A outlines how to measure the hardness of the ionizing background with He I absorption, and Appendix B derives formulae for the significance with which He I absorption can be detected in cross correlation with the coeval Ly$\alpha$ forest.

This paper assumes a flat ΛCDM cosmology with $H = 0.7$, $\Omega_b = 0.046$, $\Omega_m = 0.28$, $\sigma_8 = 0.82$, $n_s = 1$, and $Y_p = 0.24$, consistent with recent measurements (Komatsu et al. 2009). However, the simulation used to calculate the Ly$\alpha$ forest spectra at $z < 1.5$, the D5 simulation in Springel & Hernquist (2003), assumes a slightly different cosmology, with the most notable differences being $\Omega_b = 0.04$ and $\sigma_8 = 0.9$. The photoionization and recombination rates used in this study are from Hui & Gnedin (1997).

2. THE He I FOREST

In photoionization equilibrium, the He I fraction is determined by the relation

$$x_{HeI} \approx \frac{\alpha_{HeI} n_e}{\bar{g}_{HeI}} x_{HeII}. \quad (1)$$

Photoionization equilibrium is a good approximation because the timescale to reach equilibrium ($\tau_{HeI} \approx \Gamma_{HeI}^{-1} \approx 3 \times 10^4$ yr at $z \sim 3$; e.g., Faucher-Giguère et al. 2008).

3 http://etc.stsci.edu/webetc/. As of mid-January 2010, the dark current for the COS NUV gratings is 5 times higher than specifications (and than what is assumed by this ETC), and, thus, a significantly longer observation is required unless this issue is resolved.
was much shorter than all other relevant timescales. Here, \( \alpha_x, n_e, x_e, \) and \( \Gamma_X \equiv \int_{E_{\text{ion}}}^\infty dE/E \sigma_X(E) J(E) \) are respectively the Case A recombination coefficient, \( n_e \) the number density, the ionization fraction, and the photoionization rate for ion \( X \) (or subscript \( e \) for electrons), and \( E_{\text{ion}} \) and \( \sigma_X \) are the ionization potential and the photoionization cross section. Lastly, \( J(E) \) is the incident specific intensity integrated over solid angle. We will sometimes imprecisely write \( x_{\text{He}I} = 1 \) for a gas parcel even though a small fraction (\( \sim 10^{-5} \Delta \phi \)) of the helium is in \( x_{\text{He}I} \).

If the density and \( \Gamma_{\text{He}I} \) are known (since \( n_e \) is effectively known in the IGM after hydrogen reionization up to density), a measurement of \( x_{\text{He}I} \) can be used to determine \( n_e \) (equation 1). Fortunately, co-evolution Lyman-series absorption provides an estimate for the density of a He I absorber. In addition, the He I photoionization rate (like this for the H I) is expected to have been essentially spatially independent owing to the long mean free path of He I-ionizing photons \( (\Gamma_{\text{He}I}) \) and the large number of sources in a volume of \( \sim V_{\text{He}I} \). It is estimated that \( n_{\text{He}I} > n_{\text{He}I} \sim 300 \text{ cm}^3 \text{ at } z \sim 3 \) (Madau et al. 1999; Faucher-Giguère et al. 2008a; Prochaska et al. 2009). Therefore, \( \Gamma_{\text{He}I} \) was effectively just a single number in all of the IGM.

This paper focuses primarily on the longest wavelength allowed ground-state transition for He I, the 1s\(^2\)S\(^1\) - 1s2p\(^1\)P\(^o\), 584 Å transition. Its oscillator strength is 3.8 times larger than the next strongest transition of He I, the 1s\(^2\)S\(^1\) - 1s3p\(^1\)P\(^o\), 537 Å transition. He I 584 Å absorption is observable in a suitable quasar absorption \( (\Gamma_{\text{He}I}) \) and the large column of sources in a volume of \( \sim V_{\text{He}I} \). It is estimated that \( n_{\text{He}I} > n_{\text{He}I} \sim 300 \text{ cm}^3 \text{ at } z \sim 3 \) (Madau et al. 1999; Faucher-Giguère et al. 2008a; Prochaska et al. 2009). Therefore, \( \Gamma_{\text{He}I} \) was effectively just a single number in all of the IGM.

To redshift the optical depth from 584 Å in the Hubble flow to redshift across the 584 Å transition, a photon experienced the optical depth from gas in the Hubble flow of

\[
\tau_{\text{He}I,584}^{\text{GP}} \approx 0.07 x_{\text{He}I} \Delta_b^2 T_4^{-0.7} \left( \frac{10^{-12}s^{-1}}{\Gamma_{\text{HI}}} \right) \left( \frac{1+z}{5} \right)^{9/2},
\]

\[
\approx 0.025 x_{\text{He}I} \frac{\Gamma_{\text{HI}}}{\Gamma_{\text{He}I}} \tau_{\text{HI},1216}^{\text{GP}},
\]

what is termed the He I Gunn-Peterson optical depth; Gunn & Peterson (1965; Tripp et al. 1990) where

\[
\tau_{\text{HI},1216}^{\text{GP}} \approx 2.7 \Delta_b^2 T_4^{-0.7} \left( \frac{10^{-12}s^{-1}}{\Gamma_{\text{HI}}} \right) \left( \frac{1+z}{5} \right)^{9/2}
\]

is the H I Ly\( \alpha \) Gunn-Peterson optical depth, \( \Delta_b \) is the gas density in units of the cosmic mean, \( T_4 \) is the temperature in units of 10\(^4\) K, and the -0.7 exponent owes to the temperature dependence of the recombinations coefficient (cf. equation 1).

Overdensities of a few and greater at \( z \sim 3 \) had decoupled from the Hubble flow and were collapsing or had collapsed. In these regions, the Gunn-Peterson optical depth no longer describes the absorption. Instead, such regions appear as distinct absorption lines with widths of 10s of km s\(^{-1}\), and their optical depth in He I 584 Å is

\[
\tau_{\text{He}I,584}^{\text{GP}} \approx 0.11 x_{\text{He}I} \frac{\Gamma_{\text{HI}}}{\Gamma_{\text{He}I}} \left( \frac{30 \text{ km s}^{-1}}{\Delta v_{\text{He}I}} \right) \left( \frac{N_{\text{HI}}}{10^{14} \text{ cm}^{-2}} \right),
\]

\[
\approx 0.025 x_{\text{He}I} \frac{\Gamma_{\text{HI}}}{\Gamma_{\text{He}I}} \left( \frac{\Delta v_{\text{He}I}}{\Delta v_{\text{HI}}} \right) \tau_{\text{HI},1216}^{\text{GP}},
\]

where \( \tau_{\text{He}I,584}^{\text{GP}} \) is the H I Ly\( \alpha \) optical depth, and we have approximated the line profile as a tophat with velocity width \( \Delta v_{\text{He}I} \) (and \( 1 \leq \Delta v_{\text{He}I}/\Delta v_{\text{HI}} \leq 2 \), with 2 being the limit of pure thermal broadening). Systems with \( N_{\text{HI}} > 10^{14} \text{ cm}^{-2} \) are common in the Ly\( \alpha \) forest. Each sightline intersects \( \sim 100 \) such systems between \( z = 3 \) and \( z = 4 \) (e.g., Press & Rybicki 1993).

2.1. He I Photoionization Rate

The He I 584 Å optical depth depends on the value of \( \Gamma_{\text{He}I} \) in addition to the field of interest, \( x_{\text{He}I} \). The calculations in this paper assume \( \Gamma_{\text{He}I} = \Gamma_{\text{HI}} \), and \( \Gamma_{\text{He}I} \) is chosen to match measurements of the H I Ly\( \alpha \) mean transmission (requiring \( \Gamma_{\text{HI}} \sim 10^{-12} \text{ s}^{-1} \); e.g., Faucher-Giguère et al. 2008a). This choice is motivated by ultraviolet background models, which find \( \Gamma_{\text{He}I} \approx \Gamma_{\text{HI}} \) at \( z = 2 - 4.5 \) (Haardt & Madau 1996; Faucher-Giguère et al. 2009). The calculation of \( \Gamma_{\text{HI}} \) and \( \Gamma_{\text{He}I} \) in ultraviolet background models requires two inputs: (1) a model for the source emissivity at frequencies near the ionization potential for H I and He I, and (2) the measured H I column density distribution, which determines the attenuation by H I continuum absorption.

The Faucher-Giguère et al. (2009) model predicts \( \Gamma_{\text{He}I} = 0.8 - 1.0 \Gamma_{\text{HI}} \) at \( 2 < z < 5 \). This model assumes that both stars and quasars contributed to the meta-galactic ultraviolet output, with an increasingly important stellar contribution with redshift at \( z \geq 2.5 \). This model uses the result of the stellar population synthesis code PEGASE for the spectral index of starlight between the H I and H II ionization edges. This code yields a similar value for stars (\( \alpha \approx -1.6 \), defined as the power-law slope of the specific intensity; Kewley et al. 2001) to what is expected for quasars (\( \alpha \approx -1.6 \), Telfer et al. 2002). These similar \( \alpha \) result in the near constancy of \( \Gamma_{\text{He}I}(z) \) in the Faucher-Giguère et al. (2009) model. Given these input \( \alpha \), H I continuum absorption then resulted in the spectral index of the average radiation background being hardened above 13.6 eV by roughly 3 (\( \beta - 1 \)) over the input \( \alpha \) for \( \beta < 2 \), where \( \beta \) is the power-law index of the H I column-distribution. The background model of Faucher-Giguère et al. (2009) assumes \( \beta \approx 1.5 \), resulting in a hardening of \( \Delta \alpha \approx 1.5 \).

However, there is modeling uncertainty in the input \( \alpha \) for stars that stems from uncertainty in the spectrum of Wolf-Rayet stars (Kewley et al. 2001), in the average stellar metallicity, and in the stellar initial mass function. If the spectral index of stars were significantly softer, the ratio \( \Gamma_{\text{He}I}/\Gamma_{\text{HI}} \) would be smaller than in the fiducial model in Faucher-Giguère et al. (2009). In the extreme
example that the stellar contribution dominated the ionizing background and had a spectral index of $-3$ instead of $-1$, $\Gamma_{\text{He}}$ would have been reduced relative to $\Gamma_{\text{HI}}$ by a factor of $\approx 3$. This boosts $\tau_{\text{He}1.584}$ by a factor of $3$. In addition, there is uncertainty in $\beta$ at the $\pm 0.3$-level (Prochaska et al. 2009) that affects $\Gamma_{\text{He}}/\Gamma_{\text{HI}}$ at the factor of two-level.

Previous calculations of the ionizing background included only $\text{HI}$ continuum absorption to calculate (given a source emissivity model) the ionizing background between $E_{\text{ion}}^\text{HI}$ and $0.75 E_{\text{ion}}^\text{HeI}$. These models ignored He $\text{I}$ continuum absorption. This approximation may have affected their estimate for $\Gamma_{\text{He}}$ because the He I continuum absorption above $E_{\text{ion}}^\text{HeI}$ is comparable to that of H I for optically thin systems. Namely, for gas of primordial composition in photoionization equilibrium

$$\tau_{\text{HeI}}^\text{cont} (E_{\text{ion}}^\text{HeI}) \approx 0.5 \frac{\Gamma_{\text{HI}}}{\Gamma_{\text{HeI}}} \tau_{\text{HeI}} (E_{\text{ion}}^\text{HeI}), \quad (5)$$

where $\tau_{\text{HeI}}^\text{cont} (E)$ is the continuum absorption of ion X at energy $E$.

However, most of the absorption of background photons with energy $\approx E_{\text{ion}}^\text{HeI}$ at $z \sim 3$ derived from systems with $N_{\text{HI}} \approx 10^{18}$ cm$^{-2}$ (for which $\tau_{\text{HI}}^\text{cont} (E_{\text{ion}}^\text{HeI}) \approx 1$). These systems were much more self-shielded to H I Lyman-limit photons than to those at the the He I ionization edge. Thus, the effective value of $\Gamma_{\text{HI}}/\Gamma_{\text{HeI}}$ in equation (5) was much lower in these dense absorbers, resulting in $\Gamma_{\text{HI}}$ continuum absorption dominating over that of He I at $\approx E_{\text{ion}}^\text{HeI}$. We estimate in calculations not included that the break at $E_{\text{ion}}^\text{HeI}$ owing to He I continuum absorption in the spatially-averaged UV background at $z \sim 3$ amounted to only a couple percent. This justifies our use of a background model that does not include He I continuum absorption.

### 3. Calculations

Figures 1 and 2 show calculations of the He I forest at $z = 4$ and $z = 3$, respectively. These calculations use a randomly selected skewer of length $\approx 60$ cMpc ($\Delta z \approx 0.1$) through the 25 cMpc/$h, 2 \times 512^3$ particle SPH simulation described in Lidz et al. (2009). The D5 simulation in Springel & Hernquist (2003) is also used for calculating the low-z Lyman forest. The mean transmission in the H I Lyα forest has been rescaled in these calculations by adjusting $\Gamma_{\text{HI}}$ to match the observed values. In particular, we use the $\Gamma_{\text{HI}}$-values of Faucher-Giguère et al. (2008) for $z > 2$ and those in Kirkman et al. (2007) for $z < 2$. The He I 584 A forest is calculated in post-processing with the conservative assumption that $\Gamma_{\text{He}} = \Gamma_{\text{HI}}$ (Section 2.1).

The three large panels in Figures 1 and 2 make different assumptions regarding the ionization state of the helium. The leftmost assumes that the He II fraction is given by $x_{\text{HeII}} = 1$ everywhere, the middle that the helium is mostly He III and that the fraction in He II versus He III is determined by a weak He II-ionizing background with $\Gamma_{\text{HeII}} = 5 \times 10^{-16}$ s$^{-1}$, and the rightmost assumes an even weaker background with $\Gamma_{\text{HeII}} = 10^{-16}$ s$^{-1}$. The value $\Gamma_{\text{HeII}} = 5 \times 10^{-16}$ s$^{-1}$ is approximately the upper bound that has been set from observations of the most opaque regions in the He II Lyα forest at $z \approx 3$ (McQuinn 2009).

The densest absorbers may become self-shielded to He II-ionizing photons and can experience even smaller $\Gamma_{\text{HeII}}$. Self-shielding is included in these calculations with a simplified 1-D algorithm for the radiative transfer. This algorithm assumes that regions with $\Delta s < 1$ experience the quoted $\Gamma_{\text{HeII}}$. For the denser regions, this algorithm locates segments along a given 1-D skewer that are bounded by $\Delta s = 1$. Each segment is composed of tens to hundreds of grid cells, each with width $\approx 1$ km s$^{-1}$.

For each overdense segment, our scheme attenuates the ionizing background (half of which is assumed to enter from each side) based on the amount and distribution of He II. The scheme starts with the assumption that $x_{\text{HeII,i}} = 1$ and $J_i (E) = J_i (E)\text{e}^\text{oid} \exp [-\tau_{\text{HeII}}^\text{cont} (E) N_{\text{HeII,i}}]$ for all cells across the segment, where $i$ labels the cell number, and $N_{\text{HeII,i}}$ and $J_i (E)$ are respectively the He II column density and incident intensity to cell $i$ from $i$. Similarly, there is a contribution to $J_i (E)$ from a $i$.

The scheme iterates to converge to $x_{\text{HeII,i}}$ and $x_{\text{HeII,i+1}}$, assuming photoionization equilibrium. For our calculations, $J_i (E)\text{e}^\text{oid}$ is set to have a spectral index of $-1.5$, as expected for quasars.

He II Lyα forest sightlines show that $\Gamma_{\text{HeII}}$ fluctuates wildly on scales of $\gtrsim 10$ cMpc (Zheng et al. 2004; Fechner & Reimers 2007). We do not attempt to model these fluctuations here, but will comment on how they could affect our conclusions. These fluctuations will spatially modulate the number of dense self-shielding regions compared to the uniform-$\Gamma_{\text{HeII}}$ case. However, because the He I forest is sensitive to $x_{\text{HeII}} \sim 1$, $\Gamma_{\text{HeII}}$ fluctuations are less of a concern for studying He II reionization with the He I forest compared to with the He II Lyα forest (which is sensitive to $x_{\text{HeII}} \ll 1$).

The top subpanels in Figures 1 and 2 show the value of $x_{\text{HeII}}$ that results from this algorithm, where Hubble’s law is used to relate position to velocity. The blue curve is $4 \times x_{\text{HeII}}$ and the black curve is $\Delta s$. The value of $x_{\text{HeII}}$ can be quite large for the weak backgrounds that are assumed. For $\Delta s = 1, T = 10^4$ K, and $\Gamma_{\text{HeII}} = 5 \times 10^{-16}$ s$^{-1}$ $(1 \times 10^{-16}$ s$^{-1})$, $x_{\text{HeII}}$ equals 0.06 (0.25) at $z = 3$. This number becomes 0.11 (0.39) at $z = 4$.

The bottom subpanels in each larger panel in Figures 1 and 2 zoom in on the residual transmission in the He II Lyα forest (absorption at wavelengths of 304 [1 + z] Å). Note that in all of the cases there is minimal transmission in this forest because the He II Lyα transition saturates for $x_{\text{HeII}} \sim 10^{-3} \Delta s^{-1}$. There is only a detectable amount of transmission in the case with $\Gamma_{\text{HeII}} = 5 \times 10^{-16}$ s$^{-1}$ and $z = 3$ (middle panel, Fig. 2). Because He II reionization was patchy, regions of transmission in the He II Lyα forest can occur even if reionization was not complete. In contrast, He I absorption has the potential to reveal whether those opaque neighboring regions were in fact He II regions because it is sensitive to $x_{\text{HeII}} \sim 1$.

The middle subpanels in Figures 1 and 2 show the transmission in the He II Lyα forest (blue curves) and He I
Fig. 1.— Plot of different quantities from a simulated skewer through the IGM at \( z = 4 \) with length \( \approx 60 \) cMpc (\( \Delta z = 0.08 \)). The three large panels make different assumptions about the ionization state of the He ii. The leftmost panel assumes \( \bar{x}_{\text{He} \, \text{II}} = 1 \), the middle that the He ii is kept ionized by the background \( \Gamma_{\text{He} \, \text{II}} = 5 \times 10^{-16} \) s\(^{-1}\), and the right the same but \( \Gamma_{\text{He} \, \text{II}} = 10^{-16} \) s\(^{-1}\). For the latter two cases, self-shielding by dense regions is included with an approximate radiative transfer method that is described in the text. In each larger panel, the top subpanel shows \( \Delta_b \) (black curve) and \( 4 \times x_{\text{He} \, \text{II}} \) (blue curve), the second subpanel down shows the transmission in the He i 584 Å (black curve) and H i 1216 Å (blue curve) forests, and the third subpanel down zooms in on the He i 584 Å transmission (black curve). The third panel also shows the observed transmission (blue curve), which, in addition to He i 584 Å absorption, includes the contamination from foreground H i Ly\( \alpha \) absorption. The highlighted regions represent denser gas in which \( \tau_{\text{HI}, 1216} > 3 \) in the coeval Ly\( \alpha \) forest. Lastly, the bottom subpanel zooms in on the transmission in the He ii Ly\( \alpha \) forest.

Fig. 2.— Same as Figure 1 but at \( z = 3 \).

584 Å forest (black curves). These panels demonstrate that the transmission in the H i and He i forests is highly correlated. The He i absorbers that correspond to the deepest H i Ly\( \alpha \) forest lines are the most visible, and the weaker He i lines (systems that are not dense enough to self-shield) disappear in the cases in which the helium is mostly doubly ionized.

The third subpanel down in each larger panel in Figures 1 and 2 zooms in on the the He i 584 Å transmission (solid black curves). These subpanels also include mock realizations of the observed spectrum at these wavelengths (i.e., He i 584 Å plus foreground H i Ly\( \alpha \) absorption; solid blue). Each panel uses a different skewer through the IGM to calculate this foreground absorption. The highlighted regions in these panels represent the locations with \( \tau_{\text{HI}, 1216} > 3 \) for absorption coeval to that of the He i. Outside of the highlighted regions, the amount of He i absorption is substantially different between the three cases in each figure. Trace amounts of absorption remain in essentially just the \( \bar{x}_{\text{He} \, \text{II}} = 1 \) case. A detection of He i absorption in these regions would indicate that He ii reionization was occurring. The next section quantifies the prospects for detecting this absorption.

The top panel in Figure 3 plots the effective optical
depth (defined as $\tau_{\text{eff}} = -\log \mathcal{T}$, where $\mathcal{T}$ is the average transmission). This quantity is plotted for both the He I 584 Å forest and the foreground Lyα forest. The He I 584 Å value of $\tau_{\text{eff}}$ is shown for the model with $x_{\text{HeII}} = 1$ (thick dashed black curve) and the model with $\Gamma_{\text{HeII}} = 10^{-16} \text{s}^{-1}$ (thick solid red curve). The value of $\tau_{\text{eff}}$ is comparable in these two models and comparable to that of the foreground Lyα absorption (thick green dashed curve). Yet, $\tau_{\text{eff}}$ is 3 times larger at $z = 3$ in the $x_{\text{HeII}} = 1$ model compared to the $\Gamma_{\text{HeII}} = 10^{-16} \text{s}^{-1}$ model, which could potentially allow these models to be distinguished. Also, the value of $\tau_{\text{eff}}$ in the case with $\Gamma_{\text{HeII}} = 5 \times 10^{-16} \text{s}^{-1}$ (not shown in Fig. 3) is a factor of 10 smaller at $z = 3$ than the $x_{\text{HeII}} = 1$ case. If $\Gamma_{\text{HeII}}$ fluctuates spatially such with $\Gamma_{\text{HeII}} > 5 \times 10^{-16} \text{s}^{-1}$ (approximately the minimum value derived from He II Lyα analyses at $z \approx 3$; [McQuinn 2009], the signal will be even smaller than in this case.

The thin curves that represent He I 584 Å absorption in the top panel of Figure 3 are the same as the thick, except regions are masked with $\tau_{\text{HeI,1216}} > 3$ in the coeval Lyα forest (and the thin curves for the model with $\Gamma_{\text{HeII}} = 10^{-16} \text{s}^{-1}$ are multiplied by 10). This figure also demonstrates that the transmission is much different between the $x_{\text{HeII}} = 1$ and $\Gamma_{\text{HeII}} = 10^{-16} \text{s}^{-1}$ models if denser regions that have $\tau_{\text{HeI,1216}} > 3$ are masked.

3.1. Recombining Regions and Double Reionization Scenarios

We have shown that the amount of He I absorption is substantially different between the case where the He II was ionized by a weak background and the case $x_{\text{HeII}} = 1$. There is also the exciting possibility that the He II was recombining after being ionized by an early generation of sources [Venkatesan et al. 2003] or after a nearby quasar had turned off. In this case, the He II fraction of a gas parcel is $x_{\text{HeII}} = 1 - \exp(-\Delta t/t_{\text{rec}})$, where $\Delta t$ is the time since the He II-ionizing background turned off, $t_{\text{rec}} = (\sigma_{\text{HeII}}^2 n_e)^{-1}$, and $\sigma_{\text{HeII}}^2$ is the Case B recombination coefficient. At $z = 3$ and $T = 20,000 \text{ K}$, $t_{\text{rec}}$ is roughly equal to 0.8 of the Hubble time at the mean density.

Thus, the helium in underdense regions at $z \approx 3$ would have remained doubly ionized for $\Delta t \gtrsim H(z)^{-1}$ after the background turned off. The density dependence of $x_{\text{HeII}}$ is different in this case than in the other cases we have considered, such that there is the possibility that it also can be distinguished using He I absorption. However, it takes an extremely weak background with $\Gamma_{\text{HeII}} > \sigma_{\text{HeII}}^2 n_e \approx 1.2 \times 10^{-17} \Delta_b \text{s}^{-1}$ to counteract recombination in a region. [McQuinn et al. 2009] found in simulations that such a background developed soon after He II reionization by quasars was underway. Therefore, we consider the recombining scenario to be less likely than the others at $3 \lesssim z \lesssim 4$, but nevertheless a tantalizing possibility.

4. VERIFYING THAT He II REIONIZATION WAS OCCURING

The $z \approx 3$ He II reionization process is expected to have been very patchy. If quasars are the source of the ionization, models predict large-scale patches of primarily He II or He III gas that spanned many tens of Mpc

![Figure 3](image-url)

**Fig. 3.** Effective optical depth ($\tau_{\text{eff}}$) and standard deviation in the transmission ($\sigma$) as a function of the redshift of the 584 Å absorption ($\tau_{\text{584}}$). These calculations assume $\Gamma_{\text{HeII}} = \Gamma_{\text{HI}}$, and $\Gamma_{\text{HI}}$ is chosen to match observations of the mean H I Lyα absorption. The curves represent He I 584 Å absorption for the cases (1) an IGM with $x_{\text{HeII}} = 1$ (thick long-dashed curve with pluses), (2) the same but masking regions that correspond to $\tau_{\text{HeI,1216}} > 3$ in the coeval Lyα forest (thin long-dashed with x’s), (3) the He I 584 Å line using our model with $\Gamma_{\text{HeII}} = 10^{-16} \text{s}^{-1}$ in the voids (thick solid with filled squares), and (4) the same but again masking regions that have $\tau_{\text{HeI,1216}} > 3$ and multiplying by 10 in both panels (thin solid with open squares). Also shown are $\tau_{\text{eff}}$ and $\sigma$ for the foreground Lyα absorption that falls on top of the 584 Å signal from $z_{\text{qso}}$ (thick short-dashed curve), and the same but masking regions with $\tau > 0.2$ to suppress this Lyα foreground (thin short-dashed curve).

[Furlanetto & Oh 2008, McQuinn et al. 2009]. Even in He III regions, the densest gas parcels would have self-shielded such that the helium remains largely singly ionized. Therefore, a detection of He I absorption will not necessarily indicate that He II reionization was occurring, but a detection of He I absorption from low density gas would. The lower the density in which this absorption is detected, the stronger the constraint on $\Gamma_{\text{HeII}}$ and $\Gamma_{\text{HI}}$ in models where the gas was ionized by a weak background. Furthermore, the H I Lyman forest provides a measure of the density of an absorber, which allows one to target low density gas parcels. We showed in Section 3 that the amount of He I absorption in regions with $\tau_{\text{HI,1216}} < 3$ (which roughly correspond to regions with $\Delta_b < 1$) is drastically different between such models.

If absorption from low density gas were detected, it would need to be proven that it is not due to a low-z interloper in order to claim a detection of a large-scale He II region. This section discusses two methods to establish that He II reionization was occurring at $z \approx 3$: (1) a direct method that involves identifying individual weak He I lines, and (2) a statistical method that uses the cross correlation with the Lyα forest to detect the He I absorption. Unlike the former method, the latter method can be performed even when the signal-to-noise
ratio (SNR) on individual He I lines is much less than unity.

4.1. Direct Identification

With a high enough SNR per resolution element, it may be possible to directly detect individual He I absorbers as was done in Reimers & Vogel (1993) (there, for relatively dense systems). Once a candidate absorber is identified, it then needs to be ruled out that the absorption is not from a foreground interloper. To detect He II gas from regions near the cosmic mean density requires targeting He I absorbers with $\tau_{\text{HeI},584} \approx 0.1$ ($\tau_{\text{HeII},1216} \approx 3$). Here is an example of how direct identification might be performed: For wavelengths that correspond to He I absorption at $z \approx 3$, we count $\approx 10$ H I Ly$\alpha$ systems in $10^4$ km s$^{-1}$ of spectrum with a maximum flux decrement between 5 and 10 percent. The probability that one of these systems falls within 50 km s$^{-1}$ of a single He I absorber with the same decrement is 10% and of 2 He I absorbers is 1%. Thus, by targeting He I absorption from coeval H I systems, one can rule out chance contamination from foreground Ly$\alpha$ once a few systems are detected.

An alternative method for direct identification would be to find a second He I line (such as the 537 Å line) from the same absorption system. However, because the absorption in other He I lines will be at least several times weaker, it will be difficult to detect a second line from systems near the cosmic mean density (requiring SNR $\sim 100$ per spectral pixel).

4.2. Cross Correlation

It is most likely that the SNR per resolution element will not be high enough to directly identify He I lines in regions with $\Delta z \sim 1$ with present instruments. Since $\tau_{\text{HeI},584} \times x_{\text{HeI}}/\tau_{\text{HI},1216}$, with $0 < x_{\text{HeI}} < 1$ and potentially more structure in $\tau_{\text{HI},1216}$ than in $x_{\text{HeI}}$, a measurement of $\tau_{\text{HI},1216}$ may be used to construct a model for the He I signal that can be used to extract it in cross correlation. In what follows, we denote the He I transmission field as $\tilde{T}_{\text{HeI},i}$, where $i$ labels the spectral pixel. Furthermore, we assume $\Gamma_{\text{HeI}}/\Gamma_{\text{HI}} = 1$ and $x_{\text{HeI}} = 1$ to estimate the He I transmission from $\tau_{\text{HI},1216}$. Thus, our estimate for the He I transmission in a pixel is $\tilde{T}_{\text{HeI}} = \exp(-0.025 \tau_{\text{HI},1216,1})$ (cf. equations 2 and 4). However, in the limit $\tau_{\text{HeI},584} \ll 1$, the SNR of the cross correlation is not affected by what we assume for the proportionality between $\tau_{\text{HeI},584}$ and $\tau_{\text{HI},1216}$. In fact, we find that it is only marginally sensitive to this proportionality in realistic cases because the He I absorption is weak and because the SNR in cross correlation depends more on the phase of modes than on their amplitude. We discuss in Sec. 4.2.2 how an imperfect estimate for $\tilde{T}_{\text{HeI}}$ affects our estimates.

An estimator for detecting this signal in cross correlation is

$$\left(\frac{S}{N}\right)^2 = \sum_{vk} \left(\frac{\tilde{T}_{\text{HeI}}(k) \tilde{T}_{\text{HI}}(k)}{|\tilde{T}_{\text{HeI}}(k)|^2} \tilde{T}_{\text{HeI}}(k) \tilde{T}_{\text{HI}}(k) + P_{\text{inst}} \right)$$

where $D$ represents the data, tildes denote a quantity in Fourier space (a convenient basis because the covariance matrix of the noise is diagonal), $P_{\text{HI}}$ is the power spectrum of the foreground H I absorption, and $P_{\text{inst}}$ is the power spectrum of the instrumental noise (which we assume is white). Conveniently, the variance on this estimator owing to noise is its average, making it an estimate for the square of the SNR. This estimator is unbiased in the sense that components that do not correlate with $\tilde{T}_{\text{HeI}}$ do not contribute. This estimator assumes that the signal is known since it requires $\tilde{T}_{\text{HeI}}$ as input. In practice, this assumption can be avoided because for interesting cases we find that $\tilde{T}_{\text{HeI}} \propto \tilde{T}_{\text{HeII}}$ (and the normalization factor can be determined in a measurement to fractional precision $\approx 0.5$ SNR$^{-1}$).

The ensemble average of this estimator over the noise and foreground absorption yields

$$\left\langle \frac{S}{N} \right\rangle^2 = \sum_{vk} \frac{r(k)^2 |\tilde{T}_{\text{HeI}}(k)|^2}{P_{\text{HI}}(k) + P_{\text{inst}}}.$$  \hspace{1cm} (7)

$$\sim N_{\text{res}} \frac{\sigma_{\text{HeI}}^2}{\sigma_{\text{HI}}^2 + \sigma_{\text{inst}}^2}.$$  \hspace{1cm} (8)

The cross correlation coefficient $r(k)$ is bounded such that $-1 < r(k) < 1$ and is defined as

$$r(k) \equiv \frac{\tilde{T}_{\text{HeI}}(k) \tilde{T}_{\text{HeI}}(k)}{(|\tilde{T}_{\text{HeI}}(k)|^2 |\tilde{T}_{\text{HeI}}(k)|^2)^{1/2}}.$$  \hspace{1cm} (9)

The approximate equality given by equation (3) is a rough estimate for the SNR of detection if $r \approx 1$ and if the information is coming from modes near the reso-
tion in $\mathcal{R} \gtrsim 20,000$ spectra. The H I foreground absorption is also highly non-Gaussian, which could affect the probability distribution of the estimated SNR from equation (9), especially for short skewers that do not sample a representative set of foreground absorption lines. We have calculated the probability distribution of the estimated SNR from mock observations constructed from our simulations. In practice we find that the probability distribution is relatively Gaussian even for $25/h$ cMpc skewers.

4.2.1. Masking to measure He I absorption in low density regions

To isolate the He I absorption from low density gas, we mask pixels in the simulated He I forest with coeval Lyα absorption of $\tau_{\text{He I},1216} > 3$. This mask covers 10% of the pixels at $z = 3$ and 25% at $z = 4$. The calculation of the SNR in the presence of a mask is more complicated and is no longer diagonal in Fourier space. In addition, the mask artificially correlates noise wavevectors with those of the signal. To avoid such issues, our calculations only mask the signal (by setting $\tau_{\text{He I},1216} = 0$) and not the noise to calculate the SNR via equation (9). This estimate for the SNR likely underestimates the value that an optimal analysis could achieve because it involves adding additional noise to the signal. (See Appendix B for additional discussion with regard to the effect of a mask.)

The full calculation in the presence of masks is computationally tractable (with just the additional complication of inverting $N_{\text{pixel}} \times N_{\text{pixel}}$ matrices, where $N_{\text{pixel}}$ is the number of spectral pixels). In addition, a simplified analysis that applies the cross-correlation statistic (eqn. (9) to just the intervals between masked regions, and sums the SNR^2 from each of these intervals, would likely provide a reasonable estimate for the SNR of detection.

For the mask with $\tau_{\text{He I},1216} > 3$, the standard deviation of the He I absorption is reduced by approximately a factor of 10 for the model with $\Gamma_{\text{He II}} = 1$ relative to the unmasked model, and by a factor of more than 100 for the model with $\Gamma_{\text{He II}} = 10^{-16}$ s$^{-1}$ (Fig. 3). However, even when masking $\tau_{\text{He I},1216} > 3$, it is still possible to detect the signal in the model with $\Gamma_{\text{He II}} = 1$ with $N_{\text{res}} \sim 10^3$ for $\sigma_{\text{inst}} = 0.1$ (the dashed green curve in the bottom panel of Fig. 4). This value $\sigma_{\text{inst}} = 0.1$ may be achievable with COS on several existing sightlines, as discussed in the introduction.

Thus far we have discussed masking regions that correspond to $\tau_{\text{He I},1216} > 3$. Such a mask requires a SNR $\approx 50$ on the H I Lyα forest continuum at $\mathcal{R} \geq 2 \times 10^4$ to be able to detect flux in pixels with $\tau_{\text{He I},1216} = 3$ at $2\sigma$. Such a SNR may not be achievable on all He I forest quasars. If instead only SNR $\approx 20$ can be achieved, then masking $\tau_{\text{He I},1216} > 2$ is more reasonable. The signal can still be detected at $3\sigma$ with such a mask with $\sigma_{\text{inst}} \approx 0.06$, $\mathcal{R} = 20,000$, and $N_{\text{res}} = 1000$, as illustrated by the red solid curve in the bottom panel of Figure 4. In addition, if Lyβ information is used, masking only regions with $\tau_{\text{He I},1216} > 10$ is even possible. Such a mask results in much more signal for the $x_{\text{He I}} = 1$ model (orange dot-dashed curves), with a $3\sigma$ detection possible even at $\sigma_{\text{inst}} \approx 0.3$. Even though this mask includes denser regions than the mask with $\tau_{\text{He I},1216} > 3$, the expected SNR

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7 The discussion in this section could equivalently be phrased in terms of optimal signal extraction with a matched filter, where the matched filter is $M_i \equiv \langle \mathcal{F}^{-1} \mathcal{T} \rangle_{\text{inst}}$, and $\mathcal{C}_{\text{inst}}$ is the covariance matrix of the noise (composed of both the instrumental noise and low-$z$ Lyα absorption). For $\mathcal{T} = \mathcal{T}_{\text{inst}}$ or $\mathcal{T}_{\text{H I}}$, $M_i$ is the filter that, when its dot product is taken with the signal, maximizes in the filtered data the ratio of power in the signal to the average power in the noise. Furthermore, equation (7) is the SNR that such a filter can achieve.
indicates that noise in the reconstruction of \( \tau_{\text{HeI,1216}} \) does not significantly degrade the measurement.

The He I absorption signal can also differ from the estimate for this signal from \( \tau_{\text{HeI,1216}} \) because of the patchy structure of He II reionization. The signal in the cross correlation is coming from \( k \sim 1 \frac{c}{\text{Mpc}} \), whereas the structure in the He II fraction is thought to be modulated at scales of \( R_{\text{bubble}} \sim 30 \frac{c}{\text{Mpc}} \), the size of the He III bubbles around quasars (Furlanetto & Dixon 2009; McQuinn et al. 2009). The wavevectors affected by the structure of He II reionization \( k \sim R_{\text{bubble}}^{-1} \) will be more than an order of magnitude smaller than those that contribute to the SNR. If HeII reionization is perfectly patchy such that the helium is either singly ionized or doubly ionized and a skewer of length \( R_{\text{bubble}} \) is measured, then \( r \approx \bar{x}_{\text{HeII}}^{1/2} / k \gg R_{\text{bubble}}^{-1} \), where \( \bar{x}_{\text{HeII}} \) is the volume-averaged He II fraction. Therefore, the total SNR of the cross correlation is reduced on average by the factor \( \bar{x}_{\text{HeII}} \).

Also note that in the contrasting toy case that He II reionization was perfectly homogeneous, the SNR is also reduced by \( \bar{x}_{\text{HeII}} \).

A detection of He I absorption from low density gas would translate to a \( 1 \sigma \) error on \( \delta \bar{x}_{\text{HeII}} \) of \( \delta \bar{x}_{\text{HeII}} \approx \bar{x}_{\text{HeII}} \text{SNR} / \text{SNR} \) for a given estimated SNR of detection in cross correlation with \( \text{exp}(-0.025 \tau_{\text{HeI,1216}}) \). Thus, a \( 4 \sigma \) detection from low density gas would yield a 25% constraint (ignoring uncertainty in \( \Gamma_{\text{HeI}} \)). This constraint assumes that the low density gas during He II reionization was composed of large-scale regions in which the helium was either He II or He III, which is expected theoretically and is what is seen in simulations (McQuinn et al. 2009).

5. CONCLUSIONS

This paper discussed the usefulness of the He I 584 Å forest to study He II reionization at \( 3 < z < 4.5 \). The optical depth of the He I 584 Å line is proportional to the optical depth in H I Ly\( \alpha \) by the factor \( 0.025 \bar{x}_{\text{HeII}} \Gamma_{\text{HI}} / \Gamma_{\text{HeI}} \) (ignoring differences in the amount of thermal broadening between H I and He I). The factor \( \Gamma_{\text{HeI}} / \Gamma_{\text{HI}} \) should have been spatially variable, but the ratio \( \Gamma_{\text{HeI}} / \Gamma_{\text{HI}} \) should have been essentially spatially independent and roughly equal to unity at relevant redshifts. Therefore, this absorption can be used to study He II reionization through its dependence on \( \tau_{\text{HeII}} \).

Our best method at present to probe He II reionization, He II Ly\( \alpha \) absorption, saturates at neutral fractions of a part in a thousand at the cosmic mean density. In contrast, He I 584 Å absorption is unsaturated in all except the densest regions, even for \( x_{\text{HeII}} = 1 \). This absorption provides a complementary window into the ionization

\[ \text{SNR} = \sqrt{\text{dimensionless absorption power}}. \]

The data can be coaxed further to improve the significance of detection. For example, much of the contamination from foreground H I absorption can be eliminated by removing all pixels that have \( \tau > 0.2 \) (see the thin green dashed curves in Fig. 4). The motivation for this additional mask is that the foreground absorption is characterized by rare absorbers that typically have larger optical depths than the He I absorbers. This operation improves the SNR further (blue dotted curve in the bottom panel of Fig. 4).

Although, such an operation would make the full analysis more complicated because it correlates the foregrounds with the mask. In addition, masking lines based on their associated Lyman-series and other metal lines would further improve the SNR.
state of intergalactic helium at $z \sim 3$ that can definitively test whether an opaque region in the He $\Pi$ Ly$\alpha$ forest was due to a large-scale He $\Pi$ region. Even for realistic amounts of instrumental noise and foreground H $\alpha$ Ly$\alpha$ absorption, we showed that the coeval H $\alpha$ Ly$\alpha$ forest absorption can be used to construct a matched filter that can detect the He $\Pi$ absorption at high significance from a single quasar sightline.

A detection of He $\Pi$ absorption from $\Delta z \sim 1$ gas at $z \sim 3$ would definitively indicate that He $\Pi$ reionization was occurring. If a significant fraction of the intergalactic helium was in He $\Pi$, we found that He $\Pi$ 584 Å forest absorption from low density gas could be identified in a quasar spectrum with SNR $\sim 10$ and $\mathcal{R} \sim 10^4$ in an interval of $\Delta z = 0.2$. These specifications may be achievable with the COS instrument on the HST for a few known $z > 3$ targets.

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APPENDIX

A. MEASURING THE HARDNESS OF THE IONIZING BACKGROUND WITH He $\Pi$ ABSORPTION

We argued in Section 2.1 that the spectrum between 13.6 eV and He $\Pi$ Ly$\alpha$ at 40.8 eV (and, thus, the ratio $\Gamma_{\text{HI}}/\Gamma_{\text{HeI}}$) depends only on the H $\Pi$ column density distribution and the spectrum of the sources. Since the H $\Pi$ column density distribution is constrained by observations, a measurement of $\Gamma_{\text{HI}}/\Gamma_{\text{HeI}}$ will constrain the spectrum of the sources in this band (Miralda-Escude & Ostriker 1992; Santos & Loeb 2003). This ratio can be measured by comparing the continuum absorption at 912 (1 + $z$) Å – the H $\Pi$ limit – and 504 (1 + $z$) Å – the He $\Pi$ limit (Miralda-Escude & Ostriker 1992). These two optical depths are related via

$$\tau_{\text{HeI}}^{\text{cont}}(E_{\text{ion}}) \approx x_{\text{HeI}} \left( \frac{n_{\text{He}}}{n_{\text{HI}}} \right) \alpha_{\text{HeI}} \Gamma_{\text{HeI}}^{\text{ion}} \frac{E_{\text{ion}}}{E_{\text{HeI}}} \frac{\sigma_{\text{HeI}}(E_{\text{ion}})}{\sigma_{\text{HI}}(E_{\text{ion}})} \tau_{\text{HI}}^{\text{cont}}(E_{\text{ion}}),$$

$$\approx 0.10 x_{\text{HeI}} \frac{\Gamma_{\text{HeI}}^{\text{ion}}}{\Gamma_{\text{HI}}^{\text{ion}}} \tau_{\text{HeI}}^{\text{cont}}(E_{\text{ion}})/\tau_{\text{HI}}^{\text{cont}}(E_{\text{ion}}).$$

(A1)

where primes denote the average value for the quantity inside the absorber. This ratio of continuum optical depths can be measured most easily in systems with $N_{\text{HI}} \sim 10^{17}$ cm$^{-2}$, corresponding to systems with $\tau_{\text{HI}}^{\text{cont}}(E_{\text{ion}}) \sim 1$. These
column densities will be extremely self-shielded to He II-ionizing photons such that \( x_{\text{HeII}} = 1 \) is a good assumption. Thus, the ratio of the continuum optical depths can be used to measure \( \Gamma_{\text{HI}}^\prime / \Gamma_{\text{HeI}}^\prime \). Because this method relies on detecting smooth spectral features, this science can be pursued even in much lower resolution spectra than are required to study He II reionization, and a < 10% constraint on the continuum is required to detect the weak breaks from He I continuum absorption in systems with \( \tau_{\text{cont}}^* (E_{\text{HeI}}) \sim 1 \) and, thereby, constrain \( \Gamma_{\text{HI}}^\prime / \Gamma_{\text{HeI}}^\prime \).

The primary complication with this continuum-absorption method to constrain \( \Gamma_{\text{HI}}^\prime / \Gamma_{\text{HeI}}^\prime \) is that He I self-shields at H I column densities that are a decade larger than those for which the H I self-shields. Thus, in systems with \( \tau_{\text{cont}}^* (E_{\text{HeI}}) \sim 1 \), \( \Gamma_{\text{HI}}^\prime < \Gamma_{\text{HeI}} \) even though \( \Gamma_{\text{HeI}}^\prime \approx \Gamma_{\text{HeI}} \). However, if the absorber is a slab of material for which both the line of sight and the incident radiation is perpendicular to the plane of the slab, then \( \Gamma_{\text{HI}}^\prime = \Gamma_{\text{HI}} [1 - \exp(-\tau_{\text{cont}}^*/\tau_{\text{HI}}^*)] / \tau_{\text{HI}}^* \). The difference in this case between \( \Gamma_{\text{HI}}^\prime \) and \( \Gamma_{\text{HI}} \) is relatively small for \( \tau_{\text{cont}}^* \lesssim 1 \). (In fact, under most circumstances, the correction will be smaller than in this toy example because there will be other orientations with smaller optical depths to locations within the absorber.) Therefore, a measurement of \( \Gamma_{\text{HeI}} / \Gamma_{\text{HI}}^\prime \) (which yields \( \Gamma_{\text{HI}}^\prime / \Gamma_{\text{HeI}}^\prime \)) can be mapped to a constraint on \( \Gamma_{\text{HI}}^\prime / \Gamma_{\text{HeI}}^\prime \) without significant error.

A second method to constrain \( \Gamma_{\text{HI}}^\prime / \Gamma_{\text{HeI}}^\prime \) is to measure the equivalent width of a line in the He I forest and to compare this with the equivalent width of this system’s H I Lyman-series absorption lines [Santos & Loeb 2003]. This method can be employed on systems that have much smaller H I column densities than the above continuum absorption method. What this method requires is absorption systems that have \( x_{\text{HeII}} \approx 1 \) in order to directly measure \( \Gamma_{\text{HI}}^\prime / \Gamma_{\text{HeI}}^\prime \) (equation [4]). For \( \Gamma_{\text{HeII}} = 10^{-15} \) s\(^{-1}\), systems with \( N_{\text{HI}} \gtrsim 10^{15} \) cm\(^{-2}\) at relevant z self-shield to He II-ionizing photons and can have \( x_{\text{HeII}} \approx 1 \) [McQuinn et al. 2009]. At lower \( \Gamma_{\text{HeII}} \), even smaller H I column densities self shield. (Note that, for a fixed \( \Gamma_{\text{HeII}} \), the He II is likely to transition from ionized to neutral over a relatively narrow range in \( N_{\text{HI}} \). The same does not happen to the hydrogen once it starts to self-shield, and it is like to remain highly ionized for \( N_{\text{HI}} \lesssim 10^{18} \) cm\(^{-2}\) at \( z \sim 3 \).

The major difficulty with the second method is that it will be challenging to measure the H I column density for systems that have a significant optical depth in He I 584 Å. The use of higher H I Lyman series resonances, or continuum absorption for the largest column densities, will be required for such measurements. For example, systems with \( N_{\text{HI}} \sim 10^{15} \) cm\(^{-2}\) and \( x_{\text{HeII}} = 1 \) have \( \gamma_{\text{HeII},584} \approx 1 \) and H I Ly\( ^\gamma \) absorption of \( \gamma_{\text{HI},973} \approx 2 \), assuming a linewidth of 30 km s\(^{-1}\). Even if \( x_{\text{HeII}} = 1 \) is not assumed, such a measurement would place an upper bound on \( \Gamma_{\text{HI}} / \Gamma_{\text{HeI}}^\prime \).

### B. Signal-to-Noise of Cross Correlation

Denote the measurement and estimated templates with subscript \( m \) and \( e \), respectively. Both the template and measurement have some underlying signal and noise, which we represent here for the measurement as \( T_m (k) = T_{m,t} (k) + n_m (k) \), where \( T_{m,t} \) is the underlying He I signal and \( n_m (k) \) is the measurement noise. A similar decomposition applies for the estimated signal: \( T_e (k) = T_{e,t} (k) + n_e (k) \), where \( T_{e,t} \) is the component of the estimate that correlates with the signal and \( n_e (k) \) is the component that does not. The covariance of the cross-power between the He I measurement and the template estimate, \( H_{m,e} \), is then

\[
\langle \delta H_{m,e} (k_1) \delta H_{m,e}^* (k_2) \rangle = \langle H_{m,e} (k_1) H_{m,e}^* (k_2) \rangle - \langle H_{m,e} (k_1) \rangle \langle H_{m,e}^* (k_2) \rangle,
\]

\[
= \langle T_{m,t} (k_1) T_{m,t}^* (k_2) \rangle - \langle T_{m,t} (k_1) \rangle \langle T_{m,t}^* (k_2) \rangle + \langle T_{m,t} (k_1) \rangle \langle n_m (k_1) \rangle \langle T_{m,t}^* (k_2) \rangle + \langle T_{m,t} (k_1) \rangle \langle n_m (k_1) \rangle \langle n_m (k_2) \rangle + \langle n_m (k_1) \rangle \langle n_m (k_2) \rangle \langle T_{m,t}^* (k_2) \rangle + \langle n_m (k_1) \rangle \langle n_m (k_2) \rangle \langle n_m (k_2) \rangle,
\]

\[
= \delta_{k_1,k_2} [\langle T_{m,t} (k_1) \rangle^2 N_m (k_1) + 2 \langle T_{m,t} (k_1) \rangle \langle T_{e,t} (k_1) \rangle N_m (k_1) N_e (k_1) + \langle T_{e,t} (k_1) \rangle^2 N_m (k_1) + N_m (k_1) N_e (k_1)]
\]

where \( N_X (k) = \langle n_X (k) n_X^* (k) \rangle \) and in general, \( T_{X,t} (k_1) T_{X,t}^* (k_2) \) is non-diagonal, but is multiplied here by \( \langle n_X^* (k_1) n_X (k_2) \rangle \), which is \( \delta_{k_1,k_2} N_X (k_1) \). The significance with which we can measure the amplitude of the cross-power is

\[
SNR^2 = \sum_{k_1,k_2} T_{e,t} (k_1) T_{m,t}^* (k_1) \langle \delta H_{m,e} (k_1) \delta H_{m,e} (k_2) \rangle^{-1} T_{e,t}^* (k_2) T_{m,t} (k_2),
\]

\[
= \sum_k \frac{|T_{e,t} (k) T_{m,t}^* (k)|^2}{|T_{m,t} (k)|^2 N_e (k) + 2 \langle T_{e,t} (k) \rangle \langle T_{e,t} (k) \rangle N_m (k) + N_m (k) N_e (k)}
\]

\[
\approx \sum_k \frac{|T_{e,t} (k) T_{m,t}^* (k)|^2}{|T_{e,t} (k)|^2 N_m (k)}
\]

where the last line assumed that template estimate noise \( N_e (k) \) is negligible. Ignoring the additional He I ionization structure from fluctuations in \( x_{\text{HeII}} \) (which we argued in Section [12] mostly affects the amplitude of small-scale modes in \( T_{m,t} \)), one has \( N_m (k) = P_{\text{inst}} (k) + P_{\text{HI}} (k) \). With this replacement, Eq. [B2] coincides with Eq. [7].

In the text, we estimated the SNR in the case where only the signal is masked but not the noise. The derivation of Eq. [B2] holds in this case, except that \( T_{e,t} (k) T_{m,t}^* (k) \) is replaced with this cross correlation between the (noiseless)
estimated and measured signal, including the masked regions. Furthermore, one could imagine adding mock noise in masked regions to real observations to replicate this case.

In a proper analysis of observations, the noise will be masked along with the signal. Since the mask covers a relatively small fraction of space at relevant redshifts, the estimate for the SNR given by Eq. B2 should approximately hold. This can be noted by first considering the SNR one can achieve if the segments between masks were considered separately. In this case, the SNR$^2$ is the sum of Eq. B2 computed from each segment. The difference in this case is (1) that the SNR is decreased by $\approx (1 - f_{\text{mask}})^{1/2}$ where $f_{\text{mask}}$ is the mask covering fraction because there are $f_{\text{mask}}$ fewer $k$ values, and (2) $T_{e,i}(k)T_{m,i}(k)$ in Eq. B2 is replaced with its value in the unmasked regions (which is slightly different than the replacement discussed in the previous paragraph, which included the power from the mask). This argument ignores the correlations that occur between the noise from the foreground H I absorption in different segments. However, these correlations should be small since the low-z forest is characterized by discrete, largely uncorrelated lines.