Synchronization effects in a strongly coupled nanomechanical single-electron transistor

M. R. Kolahchi
Institute for Advanced Studies in Basic Sciences, P.O. Box 45195-1159, Zanjan, Iran
E-mail: kolahchi@iasbs.ac.ir

Abstract. The tunneling of electrons in a single-electron transistor (SET) that has a movable gate can be predictable when the energy associated with the motion of the resonating gate is nearly the same as the energy gained by the tunneling electron. This occurs as the tunneling events and the motion of the resonating gate become synchronized. Ideas from stochastic resonance can explain this phenomenon.

1. Introduction
To observe single charge tunneling onto a metallic island, the charging energy of the island must dominate the thermal energy available to the electrons. Even for a metallic island of nanometer size, the screening length is comparatively small enough so that the charging energy can be calculated by associating a capacitance to the island. The number of electrons on the island needs to be well defined too, and this requires the wavefunction of the excess electrons to be localized on the island. This second condition can be met if the tunneling resistance, \( R \), is well beyond the resistance quantum \( h/e^2 \).[1] Single-electron charging effects were first observed by Fulton and Dolan in small tunnel junctions.[2] In their experiment, the island was made of Al with capacitance, \( C \), of a fraction of fF.

In the SET with a movable (resonating) gate, Fig. 1, the motion of the gate can affect the tunneling rate and in turn is influenced by the tunneling events. This is an example of systems where a micron sized mechanical resonator is coupled electrostatically to a mesoscopic conductor. Such systems have been studied extensively.[3-7] In case of the nanomechanical SET, when the coupling is weak, the resonator is effectively damped and its behavior is as if it were in a heat bath.[8] The experimental verification of such a phenomenon in case of a superconducting SET equipped with a nanomechanical resonator is now available.[9]

Here, we devote our attention to a feature of the system that is specific to the strong coupling regime. In this regime, the phase space of the resonator becomes qualitatively different from the weak coupling regime. The tunneling probabilities are also greatly affected, and the motion of the resonator can even block tunneling. The phenomenon of our interest is the synchronization between the motion of the resonator and the tunneling events.
2. Characteristics of the model

In the *Orthodox theory* of single electron tunneling, tunneling is treated perturbatively to derive the transition rates.[10] At low temperatures, and assuming low-impedance circuitry, the tunneling rates are proportional to the difference in the energy of the system, before and after the tunneling event. For instance, the tunneling rate $\Gamma$ in the forward direction, taking the island (or electron box) via the left junction from state $N + 1$ to $N + 2$, can be written as [11]

$$\Gamma_{N+1 \rightarrow N+2} \propto 0.5 - \frac{2E_c}{eV_{ds}} + \zeta + \kappa x/x_0,$$

(1)

where $V_{ds}$ is the drain-source voltage, $x$ is the position of the gate (scaled by $x_0$), and $E_c = e^2/(4C_j + 2C_g)$ ($C_g$ being the capacitance of the gate and $C_j$ that of the junction).

The coupling of the mechanical degree of freedom to the electrical tunneling across SET is given by the ratio of the resonator characteristic energy to that of the tunneling electron [8],

$$\kappa = \frac{m \omega_0^2 x_0^2}{eV_{ds}},$$

(2)

where $m$ is the mass of the resonator and $\omega_0$ its natural frequency, when isolated.

The role of the gate voltage in the dynamics of the SET-resonator is given as [11],

$$\zeta = \frac{2E_c}{eV_{ds}}(N_g - N - 1/2) + \frac{m \omega_0^2 x_0^2}{eV_{ds}}(N + C_j/C_g N_g),$$

(3)

where $N_g = C_g V_g/e$ is the polarization charge on the gate capacitor; hence, not necessarily an integer. The significance of $\zeta$ is manifest specially at large coupling: the change in the polarization charge due to the motion of the resonator, changes the energy of the SET, this change does not lower the Coulomb barrier enough to allow tunneling. The gate voltage as represented by $\zeta$ serves this function of providing enough energy to overcome the barrier.
3. Elements leading to synchronization

Using the tunneling rates, such as Eq. (1), it is possible to study the motion of the resonator and the current in the SET. The weak coupling analytic result for current [9] can be extended to the strong coupling limit [11] resulting in $< I > = e^{-\frac{1}{\tau}(-\zeta - \kappa + 1/2)(\zeta + 1/2/(1 - \kappa))}$. But as Fig. 2 shows the parabolic dependence on $\zeta$ underestimates the current at large coupling $\kappa$, as well as missing the on/off behavior of the current.

The phase space of the resonator which for low $\kappa$ consists of an overlap of the phase spaces pertaining to the two neighboring charge states of the electron box, becomes more and more separated as $\kappa$ is increased.[11] The phase space getting divided at $\kappa \approx 0.6$ or larger (when $E_c/eV_d = 1$), means that for stronger coupling there is little overlap between the individual distribution functions, so the bimodality ensues.[11] This, on the face of it, leads one to conclude that the rate of decrease of current with increasing coupling will at least keep steady, or even bring about a more rapid drop in the tunneling current. In fact, the opposite happens, and it has to do with the emergence of the correlations amongst the tunneling events.

At the point of degeneracy, $\zeta = -\kappa/2$, the resonator moves on average in a symmetric double well potential. (It is also possible to have a multiple well potential indicating the importance of multiple charge states.[11]) However, its motion can be correlated with the motion of the resonator as Fig. 3 shows. This means that the double well potential in effect changes becoming unsymmetric favoring motion to the neighboring well, or for the SET, tunneling to the neighboring charge state. The phenomenon is reminiscent of stochastic resonance.[13] The only other element needed for this scenario to go through is the frequency of oscillations of the resonator which should be close to this average tunneling rate.

To study stochastic resonance for our system, we look at the distribution of waiting times between successive tunneling events. The distribution is not keeping track of forward or backward tunneling, or the junction (left or right) at which the tunneling occurred (in the full counting statistics such details are recorded). Figure 4 shows such a distribution for $\omega_0 = 2.4$ for ten coupling constants. For strong coupling, there is clear evidence that twice in each period of the resonator the chance of tunneling is maximum. In other words, statistically, the peaks (and troughs) in the distribution are spaced by $T = 2\pi/\omega_0$: if the opportunity to escape a potential well is missed, the next such chance comes a period later.
In a more quantitative analysis, the oscillatory function is described by

\[ N(t) \sim [1 - a \times \kappa \sin(2\pi t + \phi)] \times \exp(-t/\tau), \tag{4} \]

where \( t \) denotes the scaled time, and \( \tau \) is the average waiting time. This function is a fairly good fit for all \( \kappa \). (Actually the coefficient \( a \) varies with \( \kappa \), and is \( 1.33 \pm 0.07 \).) Equation (4) is for \( \omega_0 = 2.4 \). Figure 5 gives further support for the stochastic resonance type behavior.

4. Conclusion
In the SET-resonator system, the electron tunneling rate involves the term \( \kappa x \), and so is affected by the position of the oscillator as well as the coupling constant. When the SET is strongly coupled to a nanomechanical oscillator, which also serves as its gate, it becomes possible for the electron tunneling events to become statistically synchronized with the motion of the resonator.

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