Cosmic accelerated expansion and the entropy-corrected holographic dark energy

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Abstract

By considering the logarithmic correction to the energy density, we study the behavior of Hubble parameter in the holographic dark energy model. We assume that the universe is dominated by interacting dark energy and matter and the accelerated expansion of the universe, which may be occurred in the early universe or late time, is studied.
I. INTRODUCTION

To explain the cosmic accelerated expansion of the universe [1], and motivated by the holographic principle [2], a model of dark energy has been proposed [3–9] which has been tested and constrained by various astronomical observations [10]. This proposal is generically known as the “Holographic Dark Energy” (HDE). Its definition is originally extracted from the entropy-area relation which depends on the theory of gravity. In the thermodynamics of black hole horizons, there is a maximum entropy in a box of length $L$, commonly termed, the Bekenstein-Hawking entropy bound, $S \simeq m_p^2 L^2$, which scales as the area of the box $A \sim L^2$ rather than the volume $V \sim L^3$. Here $m_p^2 = (8\pi G)^{-1}$ is the reduced Planck mass. In this context, Cohen et al. [3] proposed that in quantum field theory a short distance cutoff $\Lambda$ is related to a long distance cutoff $L$ due to the limit set by formation of a black hole, which results in an upper bound on the zero-point energy density. In line with this suggestion, Hsu and Li [4, 5] argued that this energy density could be viewed as the holographic dark energy density satisfying $\rho_d = 3n^2 m_p^2 / L^2$, where $L$ is the size of a region which provides an IR cut-off, and the numerical constant $3n^2$ is introduced for convenience.

It is essential to notice that in the literature, various scenarios of HDE have been studied via considering different system’s IR cutoff. In the absence of interaction between dark matter and dark energy in flat universe, Li [5] discussed three choices for the length scale $L$ which is supposed to provide an IR cutoff. The first choice is the Hubble radius, $L = H^{-1}$ [4], which leads to a wrong equation of state for dark energy ($\omega = 0$), namely that for dust. The second option is the particle horizon radius. In this case it is impossible to obtain an accelerated expansion. Only the third choice, the identification of $L$ with the radius of the future event horizon gives the desired result, namely a sufficiently negative equation of state to obtain an accelerated universe. However, as soon as an interaction between dark energy and dark matter is taken into account, the first choice, $L = H^{-1}$, in flat universe, can simultaneously drive accelerated expansion and solve the coincidence problem [11]. It was also demonstrated that in the presence of an interaction, in a non-flat universe, the natural choice for IR cutoff could be the apparent horizon radius [12].

As we earlier mentioned that the black hole entropy $S$ plays a central role in the derivation of HDE. Indeed, the definition and derivation of holographic energy density ($\rho_d = 3n^2 m_p^2 / L^2$) depends on the entropy-area relationship $S \sim A \sim L^2$ of black holes in Einstein’s gravity, where $A \sim L^2$ represents the area of the horizon. However, this definition can be modified from the inclusion of the effects of thermal fluctuations around equilibrium [15], quantum fluctuations [16], or by considering the loop quantum gravity (LQG) [17], all leading almost to the same result.
The corrections provided to the entropy-area relationship leads to the curvature correction in the Einstein-Hilbert action and vice versa \[13\]. The corrected entropy takes the form \[14\]

\[ S = \frac{A}{4G} + \gamma \ln \frac{A}{4G} + \delta, \]

where \(\gamma\) and \(\delta\) are dimensionless constants of order unity. The exact values of these constants are not yet determined and still debatable. Taking the corrected entropy-area relation \[11\] into account, the energy density of the HDE will be modified as well. On this basis, Wei \[18\] proposed the energy density of the so-called “entropy-corrected holographic dark energy” (ECHDE) in the form

\[ \rho_d = 3m_p^2 m_p^2 L^{-2} + \gamma L^{-4} \ln(m_p^2 L^2) + \delta L^{-4}. \]

In the special case \(\gamma = \delta = 0\), the above equation yields the well-known holographic energy density.

To solve some essential problems in standard cosmology, it is believed that there was also another stage of accelerated expansion in the early universe known as inflation \[19\]. In the same way that some models, proposed to explain the inflation (such as a scalar field in slow roll model \[20\]), have been used to explain the present acceleration of the universe, one can examine different models of dark energy, to explain the evolution of the universe in the inflation era.

In this paper we study all possible behaviors of the Hubble parameter in a universe dominated by (ECHDE) and a barotopic matter and study some consequences of this model in accelerated expansion in late time and also in the inflation era.

The plan of the paper is as follows: In section 2 we construct a cosmological model and derive some useful expressions for our further uses. In section 3, we obtain different possible behaviors of Hubble parameter in the interacting ECHDE model. In section 4 we discuss the possible implications of this model in the accelerated expansion of the universe in the early stage and in the present epoch separately (we do not intend to study the whole history of the universe in a same framework). We also check the conditions under which the universe will undergo multiple acceleration-deceleration phases. Finally we conclude this paper in the last section. We use units \(\hbar = G = c = k_B = 1\).
II. THE MODEL

We consider a spatially flat Friedman Robertson walker (FRW) universe dominated by dark energy and matter (this can be dark matter, radiation and so on). The Friedman equations read

\[ H^2 = \frac{8\pi}{3}(\rho_d + \rho_m) \]
\[ \dot{H} = -4\pi(P_d + \rho_d + \rho_m), \]

where \( \rho_d \) and \( \rho_m \) are densities of dark energy and matter respectively. \( P_d \) is the pressure of dark energy and the Hubble parameter, \( H \), is assumed to be differentiable. As we have noticed in the introduction, the corrections (2), can be obtained from the loop quantum gravity, as well as by the inclusion of the effects of thermal fluctuations around equilibrium, quantum fluctuations, or by considering charge or mass fluctuations. So it is safe to study this problem in the framework of FRW cosmology, using ordinary Friedman equations as was mentioned in [18].

The matter and dark energy are allowed to exchange energy [21] via the source term \( Q \):

\[ \dot{\rho}_d + 3H(w_d + 1)\rho_d = -Q \]
\[ \dot{\rho}_m + 3H(1 + w_m)\rho_m = Q. \]

Because of this interaction term, we have not the conservation of partial stress-energy tensors of matter and dark energy : \( T^{\mu\nu}_{(\text{matter})};\mu} = -T^{\mu\nu}_{(\text{dark})};\mu} \neq 0. \) It is assumed that the matter component and dark energy have the same velocity which is the velocity of the whole fluid, \( V \). We can write

\[ T^{\mu\nu}_{(\text{matter});\mu} V_\nu = -T^{\mu\nu}_{(\text{dark});\mu} V_\nu = 0. \]

In the scalar field model of inflation, after the slow roll regime, the scalar field, whose energy density is \( \rho_d \), decays to radiation via a rapid coherent oscillation. The source term for this decay which allows the radiation creation from inflaton is taken as [22]

\[ Q = \alpha H \rho_d, \]

where \( \alpha \) is a constant. Also, in dark energy models different interactions between matter and dark energy are assumed. As the nature of dark energy has not yet been known, these interactions are taken from other models such as string theory and scalar tensor theory and so on, or as

\[ Q = \beta H(\rho_d + \rho_m) \]
\[ Q = \eta H \rho_m, \]
where $\beta$ and $\eta$ are real constants, are proposed phenomenologically to alleviate the coincidence problem and also to prevent the universe to undergo the big rip \[23\]. To study the evolution of the universe we are obliged to make use of a specific interaction, which we choose a general form as \[24\],

$$Q = 3H(\tilde{\lambda}_m \rho_m + \lambda_d \rho_d) \tag{8}$$

where $\tilde{\lambda}_m$ and $\lambda_d$ are real constants. \[8\] reduces to \[5\] and \[7\] for $\tilde{\lambda}_m = 0$, $\lambda_d = \tilde{\lambda}_m$ and $\lambda_d = 0$ respectively. Note that \[8\] is the same as the scalar

$$Q = \frac{1}{3} V_\mu V_\nu V_\gamma \left( \tilde{\lambda}_m T_{(\text{matter})}^{\mu\nu} + \lambda_d T_{(\text{dark})}^{\mu\nu} \right), \tag{9}$$

written in the comoving frame. The equation of state (EoS) parameter of dark energy, $w_d$, is defined by $P_d = w_d \rho_d$ and the (EoS) parameter of matter, $0 \leq w_m = \frac{P_m}{\rho_m}$, is assumed to be a constant, e.g. for cold dark matter we have $w_m = 0$ and for radiation $w_m = \frac{1}{3}$.

Different models have been proposed for dark energy, hereinafter we adopt the (ECHDE) model \[18\] for which infrared cutoff is taken as $L = \frac{1}{H}$. In this model, the dark energy density may be expressed as

$$\rho_d = \frac{3}{8\pi} \left( \frac{c^2}{L^2} + \frac{\alpha}{L^4} \ln(L^2) + \frac{\beta}{L^2} \right), \tag{10}$$

e, $\alpha$ and $\beta$ are dimensionless real constants and their values are still in debates in the literature \[18\].

When $\frac{\alpha}{L^4} \ln(L^2) + \frac{\beta}{L^2} \ll \frac{c^2}{L^2}$, the model reduces to the ordinary holographic dark energy model. The correction terms are relevant in the early universe, and also in the late time provided that $\frac{H^2}{m_p^2}$ becomes larger with respect to the present time. Note that in the units used in this paper the reduced Planck mass is given by $m_P = (8\pi)^{(-1/2)}$, so $c^2$ is the same as $n^2$ in \[18\]. The ratio of dark energy density to critical density is then

$$\Omega_d = c^2 - \alpha H^2 \ln(H^2) + \beta H^2. \tag{11}$$

By construction we must have

$$0 \leq c^2 - \alpha H^2 \ln(H^2) + \beta H^2 \leq 1. \tag{12}$$

Time derivative of $\Omega_d$ is obtained as

$$\dot{\Omega}_d = -2\dot{H} H^{-1}(\alpha H^2 - \Omega_d + c^2). \tag{13}$$
In terms of the (EoS) of the universe,

\[ w = \frac{P_d + P_m}{\rho_d + \rho_m} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}, \]  

(14)

(13) can be written as

\[ \dot{\Omega}_d = 3H(w + 1)(\alpha H^2 - \Omega_d + c^2). \]  

(15)

From (13) one can derive

\[ \dot{r} = 3Hr \left( w_d - w_m + (\lambda_d + r\tilde{\lambda}_m) \left( \frac{r + 1}{r} \right) \right), \]  

(16)

where \( r = \frac{\rho_m}{\rho_d} \). Using \( r = \frac{1 - \Omega_d}{\Omega_d} \), and

\[ w = w_d\Omega_d + w_m\Omega_m, \]  

(17)

we obtain

\[ w = -\frac{1}{3H} \frac{\dot{\Omega}_d}{1 - \Omega_d} - \frac{\lambda_d\Omega_d}{1 - \Omega_d} - \tilde{\lambda}_m + w_m. \]  

(18)

For \( \alpha = \beta = 0 \), we have \( \Omega_d = c^2 \) and \( w = w_m \). If \( w_m = 0 \) (e.g., when the matter is cold dark matter) and in the absence of interaction we get \( w = w_d = 0 \), implying that \( \rho_d \) is the same as dark matter. This was the motivation of taking another infrared cutoff for the model in [5]. By taking the interaction into account, the \( \alpha = \beta = 0 \) model can describe an accelerating universe with a non-dynamical \( \Omega_d \) corresponding to a scaling solution.

From (15) and (18) we obtain

\[ w = \frac{(\tilde{\lambda}_m - \lambda_d - w_m + 1)\Omega_d - \alpha H^2 - c^2 - \tilde{\lambda}_m + w_m}{\alpha H^2 - 2\Omega_d + c^2 + 1}, \]  

(19)

which results in

\[ \dot{H} = \frac{3H^2}{2} \frac{(\lambda_d - \lambda_m + 1)\Omega_d + \lambda_m - 1}{\alpha H^2 - 2\Omega_d + c^2 + 1}, \]  

(20)

where \( \lambda_m = \tilde{\lambda}_m - w_m \). (20) can be written as an autonomous first order differential equation

\[ \dot{H} = G(H) := \frac{3}{2} H^2 \frac{f(H)}{g(H)}, \]  

(21)

where

\[ f(H) = (\lambda_m - 1) + (\lambda_d - \lambda_m + 1) \left( c^2 + \beta H^2 - \alpha H^2 \ln(H^2) \right) \]

\[ g(H) = -c^2 + 1 + (\alpha - 2\beta)H^2 + 2\alpha H^2 \ln(H^2). \]  

(22)
Note that equation (21) requires that $\dot{H}$ and all of the higher order time derivatives of $H$ be zero at the time when $\dot{H} = 0$, i.e. $\dot{H} = 0$ can occur only asymptotically. As a result $H$ can not cross $H = 0$ (note that at $H = 0$, $\dot{H} = 0$). So we may assume $H(t) > 0, \forall t$ in the domain of validity of our model. We also assume that $H$ is differentiable, therefore $g(H) \neq 0$ and the sign of $g(H)$ does not change.

III. CLASSIFICATION OF THE HUBBLE PARAMETER BEHAVIORS

Obtaining an analytical general solution to (21) is not possible. In this part, instead of solving (21), by using some mathematical methods based on the properties of Lambert functions, we discuss and classify different possible behaviors of the Hubble parameter dictated by this model, in terms of its parameters ($\lambda_m, \lambda_d, c^2, \alpha, \beta$). As we do not fix the parameters, various behaviors for the model are derived. Note that the results obtained in this section are general and do not necessarily match with our present or early universe. We will discuss this issue in the next section where as we will see only some cases in the classification have necessary (although not sufficient) conditions to describe the early and late time acceleration of our universe.

For $\dot{H} + H^2 > (>)0$ the universe is accelerating (decelerating). For $\dot{H} > 0$ the universe is in super- acceleration (phantom) phase and for $\dot{H} < 0$ the accelerated universe is in the quintessence phase. Following the discussion in the last part of the previous section, the sign of $\dot{H}$ is unchangeable, hence the system is still in quintessence or phantom phase and quintessence-phantom crossing does not occur.

By defining $u = H^2$, we have

$$\dot{H} + H^2 = \frac{F(u)}{g(u)} u,$$

where $F(u) := A + Bu + Cu \ln(u)$, and

$$A = 2(1 - c^2) + 3p$$
$$B = 2(\alpha - 2\beta) + 3q$$
$$C = 4\alpha + 3s.$$  \hspace{1cm} (24)

We have defined

$$p = \lambda_m - 1 + (\lambda_d - \lambda_m + 1)c^2$$
$$q = (\lambda_d - \lambda_m + 1)\beta$$
$$s = -(\lambda_d - \lambda_m + 1)\alpha.$$  \hspace{1cm} (25)
So \( f \) in \(^{(22)}\) can be written as \( f(u) = p + qu + su \ln(u) \).

To obtain the number of critical (fixed) points of the equation \(^{(21)}\), and to get some insights about the behavior of the system, we must find the number of zeroes and the behavior of \( f(H) \) in terms of the parameters of the model. In the same manner, we must also study the behavior of \( F(u) \). \( f \) and \( g \) in \(^{(22)}\) and \( F \) in \(^{(23)}\) have the functional form \( K(u) := a + bu + cu \ln(u) \). The general behavior of \( K \) in terms of its parameters is discussed in detail in the appendix.

\( G(H) \) (or \( \dot{H} \)) in \(^{(21)}\) has a zero at \( H = 0 \) and at most two other zeroes at \( H_1 = \sqrt{u_1} \) and \( H_2 = \sqrt{u_2} \) determined in terms of Lambert W function: \( W(x) \) \(^{[24]}\), as follows (the real branches of \( W(x) \) are denoted by \( W_0 \) and \( W_{-1} \)):

\[
\begin{align*}
    u_1 &= \exp \left( W_{-1} \left( -\frac{p}{s} \exp \left( \frac{q}{s} \right) \right) - \frac{q}{s} \right) \\
    u_2 &= \exp \left( W_0 \left( -\frac{p}{s} \exp \left( \frac{q}{s} \right) \right) - \frac{q}{s} \right).
\end{align*}
\]

(26)

These are fixed points of the equation \(^{(21)}\), hence they can not be crossed. We consider three domains: \( D_1 = (0, H_1) \), \( D_2 = (H_1, H_2) \) and \( D_3 = (H_2, \infty) \). If \( \exists t \), such that \( H(t) \in D_i \), then \( H(t) \) will restricted to \( D_i \). The sign of \( \dot{H} \) depends on the sign of \( G \) in \( D_i \), if \( G(H) < (>)0 \) for \( H \in D_i \), then \( \dot{H} < (>)0 \). The stability of the model at critical points depends also on the sign of \( g(H) \) in the region where \( H \) is restricted. We assume that \( H \) is differentiable (note that by construction \( g(H) \)) is well defined: \( \forall H \in \mathbb{R}, g(H) \in \mathbb{R} \). Hence \( g(H) \) has no roots in the region where \( H \) is restricted, and therefore its sign does not change. For simplicity we only study the cases with \( g(H) > 0 \). The cases with \( g(H) < 0 \) can be treated in the same manner. Indeed, \( \frac{df}{dH}(H_i) < 0 \) implies that \( \dot{H} > 0 \). This requires that \( \frac{df}{dH}(H_i) \) and \( g(H_i) \) have different signs \( (H_i \) is the critical point). For a \( g(H_i) \) with opposite sign, \( \dot{H} < 0 \). Note that \( g(H) > 0 \) is the only physical choice in regions including \( H = 0: \lim_{H \to 0} g(H) = 1 - \Omega_d(H = 0) > 0 \).

Using the results obtained in the appendix and the above arguments, we can classify behaviors of the Hubble parameter as follows:

For the very special case \( s = 0, \alpha \neq 0 \) implies \( \lambda_d - \lambda_m + 1 = 0 \). In this situation the only fixed point of the autonomous differential equation \(^{(21)}\) is \( H = 0 \). If \( \lambda_m > 1 \), then \( \dot{H} > 0 \) which gives rise to a super accelerated expansion. For \( \lambda_m < 1 \), \( H = 0 \) becomes stable fixed point and for \( \lambda_m = 1 \) we obtain a de Sitter space time \( \dot{H} = 0 \).

According to the appendix we have the following possibilities (in situations where \( H_1 = H_2 \) we denote the root of \( f \) by \( H_1 \)):

\( (1,1) \left( s > 0, p > 0, q > -s \left( \ln \left( \frac{p}{s} \right) + 1 \right) \right) \): We have \( \dot{H}(t) > 0 \), so \( H = 0 \) is an unstable critical point and \( \lim_{t \to \infty} H = \infty \).
(I,2) \( s > 0, p > 0, q = -s (\ln(\frac{a}{s}) + 1) \): If \( H(t) \in (0, H_1) \), then \( \dot{H}(t) > 0 \) and \( \lim_{t \to \infty} H(t) = H_1 \). If \( H(t) > H_1 \), \( \dot{H}(t) > 0 \) and \( \lim_{t \to \infty} H = \infty \).

So (I,1) and (I,2) indicate that the expansion of the universe is super-accelerated.

(II,1) \( s < 0, p < 0, q < -s (\ln(\frac{a}{s}) + 1) \): We have \( \dot{H}(t) < 0 \) so \( \lim_{t \to \infty} H(t) = 0 \).

(II,2) \( s < 0, p < 0, q = -s (\ln(\frac{a}{s}) + 1) \): If \( H(t) \in (0, H_1) \), then \( \dot{H}(t) < 0 \) and \( \lim_{t \to \infty} H(t) = 0 \). If \( H(t) > H_1 \), then \( \dot{H}(t) < 0 \) and \( \lim_{t \to \infty} H(t) = H_1 \).

(II,3) \( s < 0, p < 0, q > -s (\ln(\frac{a}{s}) + 1) \): If \( H(t) \in (0, H_1) \), then \( \dot{H}(t) < 0 \). For \( H(t) \in (H_1, H_2) \), we have \( \dot{H}(t) > 0 \) and finally if \( H(t) \in (H_2, \infty) \), then \( \dot{H}(t) < 0 \). In this situation \( H_2 \) is a stable critical point.

(II,4) \( s < 0, p \geq 0 \): If \( H(t) \in (0, H_1) \) then \( \dot{H}(t) > 0 \). But if \( H(t) \in (H_1, \infty) \) then \( \dot{H}(t) < 0 \). Here \( H_1 \) is a stable critical point.

In the above, situations corresponding to \( \dot{H} > 0 \) are related to the phantom phase. We remind that in an open set \((0, H_1), (H_1, H_2) \) or \((H_2, \infty) \), the sign of \( \dot{H} \) does not change and, e.g. if the universe is in phantom phase \((\dot{H} > 0)\) in some time of inflation, it will remain in this phase as long as the model is valid. The same is true for \( \dot{H} < 0 \). Note that the cases corresponding to \( \dot{H} < 0 \) do not necessitate an accelerated expansion. In this case to see whether there is an acceleration phase we must study \( \dot{H} + H^2 \).

So let us examine \( \dot{H} + H^2 \). This expression, besides at \( H = 0 \), has at most two positive zeroes at \( H_3 \) and \( H_4 \):

\[
\begin{align*}
H_3^2 &= \exp\left( W_{-1} \left( -\frac{A}{C} \exp \left( \frac{B}{C} \right) \right) - \frac{B}{C} \right) \\
H_4^2 &= \exp\left( W_0 \left( -\frac{A}{C} \exp \left( \frac{B}{C} \right) \right) - \frac{B}{C} \right).
\end{align*}
\]

As these are not fixed points, they can be crossed allowing consecutive acceleration deceleration phases, but note that these transitions may only occur when \( \dot{H} < 0 \).

Again we can classify generally the model as follows (to avoid any confusion, we use III and IV instead of I and II used above, but note that \((III, \text{i}), \) and \((IV, \text{i})\), again, correspond to \((I, \text{i})\)
and (I, i) in the appendix respectively, and in situations where \( H_3 = H_4 \) we denote the root by \( H_3 \):

(I,1) and (II,2) \((C > 0, A > 0, B \geq -C \left( \ln \left( \frac{A}{C} \right) + 1 \right))\): We have \( \dot{H} + H^2(t) > 0 \).

(II,3) \((C > 0, A > 0, B < -C \left( \ln \left( \frac{A}{C} \right) + 1 \right))\): If \( H(t) \in (0, H_3) \), then \( \dot{H} + H^2(t) > 0 \). For \( H(t) \in (H_3, H_4) \), we have \( \dot{H} + H^2(t) < 0 \). For \( H(t) \in (H_4, \infty) \), \( \dot{H} + H^2(t) > 0 \) holds.

(III,4) \((C > 0, A \leq 0)\): If \( H(t) \in (0, H_3) \), then \( \dot{H} + H^2(t) < 0 \) and for \( H(t) > H_3 \): \( \dot{H} + H^2(t) > 0 \).

In summary by studying all various possible behaviors of the Hubble parameter as the solution of autonomous differential equation (21), in (I-II, 1-4) the conditions leading to \( \dot{H} > 0 \) and \( \dot{H} < 0 \) and all possible late time behaviors of \( H \) are specified. In this model the transition from phantom to quintessence and vice versa are forbidden. This is due to the fact that the points where \( \dot{H} = 0 \), are critical points of the theory. Although the conditions to have a phantom phase \( \dot{H} > 0 \) can be read from (I-II,1-4) but they don’t elucidate the conditions required to have the quintessence phase. So by studying all possible behaviors of \( \dot{H} + H^2 \), conditions needed for accelerated and decelerated expansion were specified.

### IV. PHYSICAL DISCUSSION AND RESULTS

In this part we try to study physical implications of our model in the late time and separately in the inflation era of our universe.

#### A. Late time acceleration

From I and II, we find out that all the possible late time solutions in this model are de Sitter space-time and the asymptotic values of the Hubble parameter may be the critical points: \( H = 0 \), \( H = H_1 \) or \( H = H_2 \) specified by [21]. We remind that the system is restricted to the regions \( D_i = (H_i, H_j) \) bounded by critical points and the sign of \( \dot{H} \) is unchangeable in \( D_i \). So if \( \dot{H} < (>) 0 \) in \( D_i \), then \( \lim_{t \to \infty} H = H_i(H_j) \). The model is stable at \( H_i \) where \( f(H_i) = 0 \) provided that
\( \frac{1}{g(H)} \frac{df}{dH}(H) < 0 \), and a necessary condition for the system to tend to \( H = 0 \) is \( \frac{f(0)}{g(0)} < 0 \). For \( \lim_{t \to \infty} H = 0 \), we have \( \frac{dG}{dH}(0) = 0 \) and \( \frac{d^2G}{dH^2} = 3 \lambda m - 1 + (\lambda d - \lambda m + 1) \omega^2 < 0 \). In the absence of interaction this inequality is satisfied but the interaction may prevent the model to go asymptotically to \( H = 0 \), and instead, forces it to tend to \( H_1 \).

In the present era we have \( \frac{H^2}{m_p^2} \ll 1 \), therefore \( \frac{\alpha}{L^4} \ln(L^2) + \frac{\beta}{L^4} \ll \frac{c^2}{L^2} \) and the correction terms have not important role. So at first sight, it seems that the corrections are not relevant in the late time acceleration, but this is not true. To emphasize this via a simple example, let us take the ordinary holographic dark energy model. In the absence (or smallness) of the correction terms, the only critical point of (21), is \( H = 0 \). If we ignore the interaction, we have \( \dot{H} = -\frac{3}{2}H^2 < 0 \) and \( \lim_{t \to \infty} H(t) = 0 \) and the universe tends to a static space time at late time, besides, \( \dot{H} + H^2 < 0 \) and the acceleration does not occur. During this evolution \( \Omega_d \) has the constant value \( \Omega_d = c^2 \).

Here the correction terms will be also irrelevant in the future evolution of the universe. Now let us take the interaction into account and let \( (\lambda m - 1) + \frac{\lambda m c^2}{1 - c^2} < 0 \) (this corresponds to \( p \geq 0 \) cases in (I) and (II). Then \( \dot{H} > 0 \) and the correction terms will become relevant at late time.

As in our epoch \( \frac{H^2}{m_p^2} \ll 1 \), the present time belongs to the region \( (0, H_1) \) and if \( \dot{H} < 0 \), then there is no need to correction terms to study the future evolution of the universe and we have \( \lim_{t \to \infty} H(t) = 0 \), so the evolution of the universe can be explained in the same manner as (11). If, instead, \( \dot{H} > 0 \) which is related to (I,1), (I,2), (I,3) or (II,4) the correction terms become important and the universe tends to a de Sitter space time characterized by : \( \lim_{t \to \infty} H(t) = H_1 \). (I,1) implies \( \lim_{t \to \infty} H = \infty \) and is excluded by (12). So among various cases in our classification in the previous section we are left only with \( s > 0, p > 0, q \leq -s (\ln(\frac{L}{s} + 1)) \) and the region \( (0, H_1) \).

The value of \( \Omega_d \) at late time is

\[
\lim_{t \to \infty} \Omega_d = \frac{1 - \lambda m}{\lambda d - \lambda m + 1}.
\] (28)

So the interaction, besides preventing the model to tend to \( H = 0 \), via the energy exchange, alleviates the coincidence problem. In this case it is worth noting that if \( \alpha = \beta = 0 \), then \( \lim_{t \to \infty} H(t) = \infty \). So the correction terms prevent the Hubble parameter to become very large asymptotically.

**B. Inflation**

Among various possible acceleration phases, it seems that the phantom phases, reported through the situations I and II, are not consistent with a non eternal inflationary phase. Indeed as long
as the universe is dominated by (ECHDE) this inflationary phase continues, i.e. the system is restricted to the domain specified by critical points. So if the phantom phase occurs, can not be ceased. Although crossing the critical points, \( H_1 \) and \( H_2 \), is not possible but the system can cross \( H_3 \) and \( H_4 \) and therefore transition from acceleration to deceleration and its inverse are possible. To see this, as an example, let us take the case (III,3) where for \( H(t) \in (H_3, H_4) \), we have \( \dot{H} + H^2 < 0 \) therefore \( \dot{H} < 0 \) and \( H \) is decreasing. But \( H_3 \) is not a fixed point and the Hubble parameter crosses \( H_3 \) and enters in accelerating domain.

To describe the temporary inflationary phase, we must consider \( \dot{H} < 0 \) situations in I and II and then investigate the cases reported in III and IV. If we expect that the inflation be ended, we must select the situations allowing the transition from acceleration to declaration phase. This is related to the presence of the source term \( Q \), allowing the energy exchange between dark energy and matter. This is only allowed in (III,3) with \( H_{inf.f.} > H_4 \), (III,4) with \( H_{inf.f.} > H_3 \), and (IV,3) with \( H_{inf.f.} \in (H_3, H_4) \), provided that the \( H \) belongs to cases in I and II where \( \dot{H} < 0 \). \( H_{inf.f.} \) denotes the value of the Hubble parameter during inflation. At the end of inflation, \( H \) is determined by \( H_3 \) (cases : (IV,3)and (III,4)) or \( H_4 \) (case (III,3)) determined by (27). The corresponding values of \( \Omega_d \) can then be read from (11). In the above cases it is straightforward to see that (IV) corresponds to an inflation which is not past eternal.

Until now, for the sake of generality we did not fix the values of parameters \( \{ \alpha, \beta, c^2, \lambda_m, \lambda_d \} \). To be more specific, and as an illustration of our results, let us take the case (II,1) and (III,4) characterized by

\[
\begin{align*}
\lambda_d - \lambda_m + 1 & \left( \beta - \alpha \left( 1 + \ln \left( \frac{\lambda_m - 1 + (\lambda_d - \lambda_m + 1)c^2}{\alpha(\lambda_m + \lambda_d + 1)} \right) \right) \right) < 0 \\
(\lambda_m - \lambda_d - 1)\alpha & < 0 \\
(\lambda_m - 1)(1 - c^2) + \lambda_dc^2 & < 0 \\
\alpha & > 0 \\
(3\lambda_d - 3\lambda_m + 1)c^2 + 3\lambda_m - 1 & \leq 0,
\end{align*}
\]

and

\[
(1 - 3\lambda_d + 3\lambda_m)\alpha > 0
\]

\[
(3\lambda_d - 3\lambda_m + 1)c^2 + 3\lambda_m - 1 \leq 0,
\]

respectively. This situation describes a deceleration followed by an acceleration phase as can be seen from the diagram of \( \dot{H} + H^2 \) depicted in fig.(1) for the optional choice \( \{ \alpha = 4.18, \beta = -10.71, c^2 = 0.7, \lambda_m = 0, \lambda_d = 0.1, \} \) which satisfies (29) and (30) (note that the same behavior is expected for all models whose parameters satisfy (29) and (30)). The inflation ends at \( H^2 = 0.02(\simeq 0.6m_P^2) \) and (12) restricts \( H^2 \) to \( H^2 < 0.18(0.95m_P^2) \).
It is also interesting to see if the model permits to have consecutive acceleration and deceleration phases. We remind that $\dot{H} + H^2$ has at most two zeroes besides $H = 0$. So in general, besides acceleration-deceleration or deceleration-acceleration transitions, it is also possible to have successive deceleration-acceleration phases in this model. Acceleration-deceleration-acceleration corresponds to (III,3) and deceleration-acceleration-deceleration corresponds to (IV,3), provided that the zeroes of $\dot{H} + H^2$ belongs to domain $D_i$ (specified by critical points) where the Hubble parameter is restricted. Now to illustrate this result let us take the case (II,1) and (III,3) characterized by (29) and:

$$
\alpha(3\lambda_m - 3\lambda_d + 1) > 0
$$

$$
3\lambda_m - 1 + (3(\lambda_d - \lambda_m) + 1)c^2 > 0
$$

$$
3(\lambda_m - \lambda_d + 1)(\alpha - \beta) + 2\beta < \alpha(3\lambda_d - 3\lambda_m - 1) \times 
\ln \frac{(3\lambda_m - 1)(1 - c^2) + 3\lambda_d c^2}{\alpha(3\lambda_m - 3\lambda_d + 1)}. 
$$

(31)

$\dot{H} + H^2$ is depicted in terms of $u = H^2$, for the choice $\{\alpha = 0.5, \beta = -0.5, \lambda_d = \lambda_m = 0.103, c^2 = 0.7\}$, which satisfy (29) and (31), in fig. (2) (again note that the same qualitative behavior is expected for all models whose parameters satisfy (29) and (31)). The model has an acceleration phase for $H^2 > 0.0218$ (or in $h = c = 1$ units $H^2 > 0.548m_P^2$) a deceleration phase for $0.0150 (= 0.377m_P^2) < H^2 < 0.0218 (= 0.548m_P^2)$ and again an acceleration phase for $0 < H^2 < 0.0150 (= 0.377m_P^2)$. Note that (12) restricts the model to $H^2 < 1.19 = (29.9m_P^2)$. Models with double inflation (i.e. two stages of inflation) were studied and reported before in the literature [26].

The scope of our discussion was only restricted to the study of possible accelerations of the universe in early and late times. However a realistic general model of inflation must also be capable to describe physical problems such as the reheating and the cosmological perturbations.
FIG. 2: $\dot{H} + H^2$ as a function of $u = H^2$, for $\alpha = 0.5$, $\beta = -0.5$, $\lambda_d = \lambda_m = 0.103$, $c^2 = 0.7$

and so on.

In the inflationary era, the universe is dominated by $\rho_d$ for which the pressure is negative and by giving rise to $\dot{H} + H^2 > 0$ drives the inflation. In different models, there may be different scenarios for the reheating. As an example, in the scalar field model and in the slow roll approximation, after the inflation the scalar field decays to other particles during a rapid coherent oscillation in the bottom of the potential slope [22]. In our model and in the absence of scalar fields, there can be another possibilities for reheating:

By taking $w_m = 1/3$ and $\lambda_m = 0$, the model acts similar to warm inflation models where the inflaton decays to radiation (ultra-relativistic particles) during inflation [27]. This is due to the source term, $3H\lambda_d\rho_d$, which allows energy exchange between these components. In the inflationary era, the universe is dominated by $\rho_d$, and this source term is significant in (4). In contrast, after the inflation, the contribution of $\rho_m$ becomes more significant and the universe becomes radiation dominated for $\dot{H} + 2H^2 \approx 0$ which occurs in deceleration era. In this era the radiation creation from $\rho_d$ is not significant. Note that the radiation dominated era occurs for $\dot{H} + 2H^2 \approx 0$. The corresponding value of the Hubble parameter can be obtained by solving the equation $A_r + B_r u + C_r u \ln(u) = 0$, where $A_r = 4(1-c^2)+3p$; $B_r = 4(\alpha-2\beta)+3q$ and $C_r = 8\alpha+3s$.

One can also assume another possibility for reheating by taking $w_m \neq \frac{1}{3}$. This assumption may be valid provided that $\rho_m$ decays to ultra-relativistic particles at the end of inflation, giving rise to the preheating or the reheating. However the study of this era requires that we consider the contribution of baryonic matter in (4) and add the corresponding interaction terms to our equations.

At the end it is worth to note that the above conditions posed on the parameters of the model, $(\alpha, \beta, \lambda_d, \lambda_m)$, in (III,3),(III,4), and (IV,3) although are necessary conditions for transient inflation
but are not sufficient. For example for a given model with specified parameters the e-folds number
\[ N := \ln \left( \frac{a(t_{\text{end}})}{a(t_i)} \right) \]
must be calculated. It is given by
\[ N = \frac{1}{2} \int_{u_i}^{u_{\text{end}}} \frac{du}{G(u)}, \tag{32} \]
where as before \( u := H^2 \), \( i \) and \( \text{end} \) denote the beginning and the end of the inflation and \( G(u) \) is
given by (21). In the case (III,4); (III,3); and (IV,3) we have \( u_{\text{end}} = H_3^2 \); \( u_{\text{end}} = H_4^2 \); and \( u_{\text{end}} = H_3^2 \)
respectively. In the case (IV,3), \( H_3^2 < u_i \leq H_4^2 \). Note that in other two cases \( u_i \in (u_{\text{end}}, V) \) where \( V \) is the maximum value of \( u \) satisfying (12). For a viable model we must have \( 60 \leq N \). This put
more constraints on the parameter of the model. However for a general model, analytically solving (32) or obtaining a lower bound for it, are very complicated tasks.

V. CONCLUSION

In this paper we considered a spatially flat FRW universe dominated by (ECHDE) and a
barotopic matter interacting via a more general source term with respect to other papers in this
subject. We took the apparent horizon as the infrared cutoff (this is the more natural choice
adopted in the literature). Considering the effects of thermal fluctuations around equilibrium,
quantum fluctuations, or charge and mass fluctuations, modifies the entropy attributed to the
horizon. We applied these corrections to the apparent horizon entropy and achieved to obtain an
autonomous differential equation for the Hubble parameter. Then we obtained the critical points
of the model and classified the behavior of the system in terms of the parameters of the interaction
and (ECDHE). For this purpose we used algebraic features of the autonomous differential equation
and \( \text{LambertW} \) functions.

Although in the present time the corrections are marginal but they may play an important
role in the early and late times. We deduced that the correction terms may force the universe to
tend to a de Sitter space-time at late time. We obtained the possible ultimate value of the Hubble
parameter and also derived the corresponding dark energy density. We showed that the coincidence
problem is alleviated in this model.

In addition we studied some necessary (although not sufficient) conditions for the model to
describe the acceleration phase in inflation era. The inflation was assumed to be transient and the
possible values of the Hubble parameter at the end of inflation were derived.

Appendix
In this part we study the behavior and the properties of the roots of \( K(u) := a + bu + cu \ln(u) \).

\[
\frac{dK(u)}{du} \text{ has and only has a root at } \tilde{u} = \exp\left(-\frac{b+c}{c}\right), \text{ so following the Rolle’s theorem } K(u) \text{ has at most two positive roots which we denote } u_1 \text{ and } u_2 > u_1.
\]

We have also \( \frac{d^2K(u)}{du^2}(\tilde{u}) = \frac{c}{\tilde{u}} \). Generally our model can be classified into three classes: \( c = 0 \), I: \( c > 0 \) and II: \( c < 0 \).

In the case I:

1) for \( K(0) > 0 \) and \( K(\tilde{u}) > 0 \), \( K \) has no roots and \( K(u) \) is always positive.

2) For \( K(0) > 0 \) and \( K(\tilde{u}) = 0 \), \( K \) has only one root and \( K(u) \) is positive.

3) For \( K(0) > 0 \), and \( K(\tilde{u}) < 0 \), \( K \) has two roots: \( 0 < u_1 < u_2 \), and \( K(u > u_2) > 0 \), \( K(u_1 < u < u_2) < 0 \) and \( K(0 < u < u_1) > 0 \). In this case \( \frac{dK(u_1)}{du} < 0 \) and \( \frac{dK(u_2)}{du} > 0 \).

4) For \( K(0) \leq 0 \) and \( K(\tilde{u}) < 0 \), \( K \) has only one non-zero root \( u_1 \). We have also \( K(u > u_1) > 0 \), \( K(0 < u < u_1) < 0 \) and \( \frac{dK(u_1)}{du} > 0 \).

The cases (I,1), (I,2), (I,3), and (I,4) correspond to

\[
c > 0, \ a > 0, \ b > -c\left(\frac{\ln\frac{a}{c} + 1}{c}\right);
\]
\[
c > 0, \ a > 0, \ b = -c\left(\frac{\ln\frac{a}{c} + 1}{c}\right);
\]
\[
c > 0, \ a > 0, \ b < -c\left(\ln\frac{a}{c} + 1\right), \text{ and};
\]
\[
c > 0, \ a \leq 0,
\]
respectively.

In the case II:

1) for \( K(0) < 0 \) and \( K(\tilde{u}) < 0 \), \( K \) has no roots and \( K(u) \) is always negative.

2) For \( K(0) < 0 \) and \( K(\tilde{u}) = 0 \), \( K \) has only one root and \( K(u) \) is always negative.

3) For \( K(0) < 0 \), and \( K(\tilde{u}) > 0 \), \( K \) has two roots: \( 0 < u_1 < u_2 \), and \( K(u > u_2) < 0 \), \( K(u_1 < u < u_2) > 0 \) and \( K(0 < u < u_1) < 0 \). In this case \( \frac{dK(u_1)}{du} > 0 \) and \( \frac{dK(u_2)}{du} < 0 \).

4) For \( K(0) \geq 0 \) and \( K(\tilde{u}) > 0 \), \( K \) may have only one nonzero root \( u_1 \), and \( K(u > u_1) < 0 \), \( K(0 < u < u_1) > 0 \) and \( \frac{dK(u_1)}{du} < 0 \).

The cases (II,1), (II,2), (II,3), and (II,4) correspond to

\[
c < 0, \ a < 0, \ b < -c\left(\ln\frac{a}{c} + 1\right);
\]
\[
c < 0, \ a < 0, \ b = -c\left(\ln\frac{a}{c} + 1\right);
\]
\[
c < 0, \ a < 0, \ b > -c\left(\ln\frac{a}{c} + 1\right), \text{ and};
\]
\[
c < 0, \ a \geq 0,
\]
respectively.
The numbers of the roots of $K$ can also be explained in terms of Lambert W function: $W(x)$, in a more straightforward way. The real branches of $W(x)$ are denoted by $W_0$ and $W_{-1}$. For real $x$, if $-\frac{1}{e} < x < 0$ there are two possible real values for $W(x)$: $-1 < W_0(x)$, and $W_{-1}(x) < -1$. We have also $W_0(-\frac{1}{e}) = W_{-1}(-\frac{1}{e}) = -1$.

For $\frac{a}{c}e^{\frac{b}{c}} \leq 0$ (the cases 4 in I and II), the solution of the equation $K = 0$ in terms of Lambert W function is

$$u = \exp \left( W_0 \left( -\frac{a}{c} \exp \left( \frac{b}{c} \right) \right) - \frac{b}{c} \right)$$

For $\frac{1}{e} < \frac{a}{c}e^{\frac{b}{c}}$, $K$ has no real roots (the cases 1 in I and II), and for $0 < \frac{a}{c}e^{\frac{b}{c}} < \frac{1}{e}$, $K$ has two roots (the cases 3 in I and II):

$$u_1 = \exp \left( W_{-1} \left( -\frac{a}{c} \exp \left( \frac{b}{c} \right) \right) - \frac{b}{c} \right)$$

$$u_2 = \exp \left( W_0 \left( -\frac{a}{c} \exp \left( \frac{b}{c} \right) \right) - \frac{b}{c} \right) .$$

For $\frac{1}{e} = \frac{a}{c}e^{\frac{b}{c}}$, $u_1 = u_2$ corresponding to the cases 2 in I and II.

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