Higher twists in deep inelastic scattering

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Abstract

We perform an exploratory study of higher twist contributions to deep inelastic scattering. We estimate the size of two major sources of higher twist, namely absorptive corrections and the vector meson dominance (VMD) contribution. We find that they give a sizeable higher twist component of $F_2$. For example at $x = 0.01$ it is about 8\% (17\%) at $Q^2 = 10 \text{ GeV}^2$ (4 GeV$^2$), reaching up to 27\% at $x = 10^{-4}$ and $Q^2 = 4 \text{ GeV}^2$. At the smaller $x$ value the largest contribution comes from absorptive corrections, while at the larger values of $x$ the VMD term dominates.
At large $Q^2$ the cross section for deep inelastic scattering (DIS) is to a good approximation described by just the twist-2 component of the structure function. That is

$$\sigma_T(\gamma^* p) = \frac{4\pi^2\alpha}{Q^2} F_T(x, Q^2) \quad (1)$$

with $F_T \simeq F_T^{(2)}$, where $\alpha$ is the electromagnetic coupling. Indeed in extracting parton distributions from DIS data it is commonly assumed that (1) is exact and that $F_T^{(2)}$ is given entirely by twist-2. To leading order in QCD we have

$$F_T^{(2)} = \sum \epsilon_q^2 x[q(x, Q^2) + \bar{q}(x, Q^2)] \quad (2)$$

where $q$ and $\bar{q}$ are the quark and antiquark distributions and $\epsilon q$ is the charge of the quark. For sufficiently small values of $Q^2$ the higher twist ($4, 6, \ldots$) components of $F_T$, defined by

$$F_T = F_T^{(2)} + \frac{F_T^{(4)}}{Q^2} + \frac{F_T^{(6)}}{Q^4} + \ldots \quad (3)$$

would be expected to give noticeable contributions to $\sigma_T$. Surprisingly, even though the DIS data have become much more precise, the recent global analyses still show no necessity for higher twist contributions — despite including data at remarkably low values of $Q^2$. Moreover parametric fits of $F_2$ data \[1\] have found very small values of $F_2^{(4)}$ at low $x$. Our objective is to explore the role, and to estimate the size, of the higher twist terms.

From the point of view of the Wilson Operator Product Expansion (OPE) the higher twist terms correspond to operators describing a larger number of partons. Say, for twist-4 we must consider operators with four quarks $\langle N|\bar{q}qq\bar{q}|N \rangle$ or four gluons $\langle N|gggg|N \rangle$, and so on. Strictly speaking for each new twist and new operator we have to specify a new input function which should be determined by a global fit to the DIS data. Unfortunately the data are not yet precise enough to determine more than the leading twist, $F_T^{(2)}$, decomposition in terms of partons.

Rather we will discuss two effects which are expected to give the dominant twist-4 contributions in the HERA domain. The contributions arise from (a) absorption corrections and (b) the vector meson dominance (VMD) contribution. We estimate their size below, but first we show their $1/Q^4$ behaviour.

At low $x$ the behaviour of $\sigma_T(\gamma^* p)$ is controlled by the evolution of gluons in the $t$ channel. The evolution equation may be written

$$xg(x, Q^2) = xg(x, Q_0^2) + \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{Q^2} \frac{dQ^2}{Q^2} \frac{\alpha_s(Q^2)}{2\pi} \frac{x}{z} g\left(\frac{x}{z}, Q^2\right) P_{gg}(z) \quad (4)$$

where the first two terms correspond to the conventional twist-2 DGLAP evolution with gluon-gluon splitting function $P_{gg}(z)$, while the last term with the negative sign takes into account the higher-twist absorptive corrections\[2\]. The value of $A$ is discussed below. The last term

\[1\]Since we are interested in the possible higher twist effects on parton analyses we concentrate on the cross section for the absorption of transversely polarised photons, $\sigma_T$.

\[2\]Such a form was first introduced in Ref. \[3\], and studied further in Ref. \[4\].
corresponds to gluon recombination and is shown schematically in Fig. 1, where the four gluon twist-4 structure is evident. The extra $Q'^2$ in the denominator of the absorptive term reflects the small probability to find an additional gluon in a small domain of transverse size $1/Q'$. Strictly speaking the last term contains both twist-2 and twist-4 components coming from evaluating the integral at the lower limit $Q'^2_0$ and the upper limit $Q'^2$ respectively. The twist-2 component may be regarded as an extra input for the twist-2 DGLAP evolution. On the other hand the contribution from the upper limit results from the evolution of the four gluon state from $Q'^2_0$ to $Q'^2$, and is therefore manifestly twist-4.

To identify the twist-2 and twist-4 parts of the absorptive correction we simply rewrite the last term in (4) in the form

$$- A \int_{Q'^2_0}^{Q'^2} \cdots = - A \int_{Q'^2_0}^{\infty} \cdots + A \int_{Q'^2_0}^{Q'^2} \cdots .$$

(5)

The negative first term combines with the input distribution, $xg(x, Q'^2_0)$, in (4) to give a new initial condition for the twist-2 evolution, while the positive second term is the twist-4 component. Thus although the whole absorptive correction is negative, the twist-4 component is itself positive.

A second well-known higher twist contribution to $\sigma_T(\gamma^*p)$ comes from the Vector Meson Dominance (VMD) term

$$\sigma_T(\text{VMD}) = \pi \alpha \sum_V \frac{M_V^4 \sigma_V(s)}{\gamma_V^2 (Q'^2 + M_V^2)^2}$$

(6)

where the sum is over vector mesons $V$ of mass $M_V$ and where $\gamma_V$ specifies the $\gamma - V$ coupling. $\sigma_V(s)$ is the total $Vp$ cross section at centre-of-mass energy $\sqrt{s}$. The $\sigma_T(\text{VMD})$ component dominates $\sigma_T(\gamma^*p)$ at very low $Q'^2$. The space-time picture of the VMD contribution is compared with that of the conventional (twist-2) DGLAP contribution in Fig. 2. For the DGLAP contribution of diagram (a) the upper quark propagator $1/(Q + k)^2 \simeq 1/Q^2$, so that the distance between points 1 and 2, $\Delta r_{12} \sim 1/Q$, is very small. Moreover in terms of old-fashioned perturbation theory, the invariant mass of the produced $q\bar{q}$ system $M \sim Q$. In contrast in the small region of phase space corresponding to $M^2 \sim M_V^2 \ll Q^2$ the left-hand diagram of 2(a) may be drawn as in the left-hand diagram of 2(b), where the space-time development of the VMD contribution is evident. The photon first creates a light $q\bar{q}$ pair long before the interaction with the target. The presence of the four quark twist-4 structure is evident in the schematic right-hand diagram of 2(b), and is reflected by the $1/Q^4$ behaviour of (6) at large $Q^2$. In this case, which corresponds to large distances and rather small transverse momentum $k_T$ of the quarks, it is better to deal with constituent quarks with masses $m_{u,d} = 350$ MeV and $m_s = 500$ MeV.

**Absorptive or gluon rescattering effects**

In general the four $t$ channel gluons shown in the lower part of Fig. 1 interact with each other. It is convenient to re-organize the perturbative expansion and to consider first the
interaction between the gluons in each pair separately and then to consider the interactions between the two gluon ladders. The sign and the value of each contribution depends on the colour structure. Each pair of gluons may form a singlet (1), a symmetric or antisymmetric octet \((8_s, 8_a)\), decuplets \((10, 10)\) or a 27 colour state. The various colour configurations have different energy dependences

\[ \sigma \sim (s')^{2\alpha(0)-2} \]  

depending on the intercept \(\alpha(0)\), where \(\sqrt{s'}\) is the energy shown in Fig. 1. Information on the intercepts comes from the BFKL equation. Now the BFKL kernel \(K\) contains two parts — the virtual (one-loop) correction, which results in the reggeization of the \(t\) channel gluons, and a term describing real \(s\) channel gluon emission. In general the intercepts may be written in the form

\[ \alpha_i(0) = 1 + \alpha_S [c_i \langle K_{\text{real}} \rangle - \langle K_{\text{virtual}} \rangle] \]  

where the kernels are averaged over the corresponding BFKL eigenfunctions. The virtual correction is negative and does not depend on the total colour charge of the gluon pair, while the colour factor \(c_i\) for real emission is equal to 3, \((\frac{3}{2}, \frac{3}{2})\), 0 and \(-1\) for the singlet, octets \((8_s, 8_a)\), decuplets and 27-plet configurations respectively. Due to the Regge bootstrap property of the BFKL equation, the octet intercepts are \(\alpha_8(0) = 1\) — the virtual correction cancels the real emission part of the kernel. On the other hand from (8) we see that the intercepts of the 10 and 27 configurations are less than 1 and their contributions therefore decrease with energy. Only the singlet-singlet configuration gives an amplitude which grows faster than the twist-2 contribution as \(x\to 0\), since the singlet intercept \(\alpha_1(0) > 1\).

To estimate the size of the twist-4 term in the gluon evolution equation (4) we therefore need to evaluate the factor \(A\) in the last term for the singlet-singlet configuration. First, it contains a colour factor of \(9/16\) corresponding to the coupling of the four gluon state to the two \(t\) channel gluons. The best way to obtain this factor is to consider the cross section for heavy photon dissociation [4, 5] and to use the AGK cutting rules [6], which have been justified in QCD in refs. [7, 8]. To explain the remaining content of \(A\) it is convenient to write

\[ A = \frac{9}{16} \frac{CK_1}{B}. \]  

The dimensional factor \(B\) (which compensates for the extra \(Q'^2\) in the denominator in the last term of (4)) comes from the integral over the momentum \(t = (p - p')^2\) transferred through the “pomeron” loop (which is indicated by the circular arrow on Fig. 1). In accordance with the measurements of heavy photon dissociation at HERA we use

\[ B = \frac{d\sigma(0)}{dt} / \int d\sigma(t) / dt \ dt \simeq 7.2 \text{ GeV}^{-2}. \]  

Until now we have considered only “elastic” \(\gamma^* p \to X p\) proton interactions. However the probability of dissociation of the target proton is not negligible. It is of order 50-70% of the

In other words the octet BFKL amplitude self-consistently reproduces the original gluon trajectory with \(\alpha_8(0) = 1\).
“elastic” interaction [10]. Thus we include a factor $C = 1.6$ in (9). Finally we include in the coefficient $A$ the effect of pomeron- pomeron rescattering (shown in Fig. 3). Such rescattering partially fills the rapidity gap and so this contribution is not usually included in the diffractive component $F_D^2$ of $F_2$. The contribution increases with energy, but fortunately the dependence is rather slow ($\sim 1 + 0.45(ln1/x)^{1/4}$) and so we may include it as a constant $K$ factor. In the HERA domain $K = 1.63$ to 1.76 [11] and so we let $K = 1.7$ in (9). Interestingly our resultant value of $A$ is in close agreement with the value $(81/16R^2 \simeq 0.2$ GeV$^{-2}$ for $R = 1$ fm) used in phenomenological analyses of absorptive corrections [12].

To compute the higher twist $\Delta F_T$ contribution to $F_T$ we start with an ordinary (twist-2) DGLAP fit to DIS and treat the last term of the evolution equation (4, 5) as a small correction

$$x\Delta g(x, Q^2) = A \int_x^1 \frac{dx'}{x'} \int_{Q^2}^{\infty} \frac{dQ'^2}{Q'^4} [\alpha_s(Q'^2)x'g(x', Q'^2)]^2.$$ (11)

Now at small $x$ the quark density is dominantly driven by the $g \to q\bar{q}$ transition. Though a full calculation is possible, for our exploratory study it is sufficient to use the approximation due to Prytz [13], that is we evaluate

$$\Delta F_T(x, Q^2) \simeq \frac{\Delta g(2x, Q^2)}{g(2x, Q^2)}.$$ (12)

Our pure gluonic estimate of $F_2^{(4)}$ omits the two jet dissociation, $\gamma \to q\bar{q}$, contribution corresponding to Fig. 4. This graph contributes to the diffractive structure function $F_D^2$ but not to the gluon distribution. Rather it contributes to the quark distribution, but with a strength which is suppressed relative to the corresponding gluon distribution by the colour factor $(C_F/C_A)^2 \sim 1/5$. Moreover it gives a higher twist contribution to both $F_L$ and $F_T$. The explicit form of this small correction to $F_2^{(4)}$ has been evaluated in refs. [14, 13], and should be included in a detailed calculation of the higher twist contribution.

**Vector Meson Dominance contribution**

In the vector meson dominance contribution (6) to higher twist, usually only the $\rho, \omega$ and $\phi$ mesons are included

$$\sigma_T(\gamma^*p) = \pi\alpha \sum_{V=\rho,\omega,\phi} \frac{M_V^4 \sigma_V(s)}{\gamma_V^2(Q^2 + M_V^2)^2}.$$ (13)

We evaluate the $\rho p, \omega p$ and $\phi p$ cross sections ($\sigma_V$) at centre-of-mass energy $\sqrt{s}$ as described in the footnote to eq. (22). Of course there will be some contribution from the higher mass vector meson resonances but there are expectations that they will be suppressed by smaller values of $1/\gamma_V^2$ and/or possibly smaller $Vp$ cross sections $\sigma_V$.

We can also make an alternative estimate of this non-perturbative higher twist contribution based on hadron-parton duality. Since for small $x$ the $\gamma^* \to q\bar{q}$ fluctuations occur over a much
longer time scale than the interaction of the $q\bar{q}$ pair with the target proton, we may use hadron-parton duality to write the $\gamma^* p$ cross section in terms of a dispersion relation with respect to the invariant $q\bar{q}$ mass $M$ [10],

$$\sigma(\gamma^* p) = \sum_q \int_0^\infty \frac{dM^2}{(Q^2 + M^2)^2} \rho(s, M^2) \sigma_{q\bar{q}+p}(s, M^2),$$  \hspace{1cm} (14)

where $\sigma_{q\bar{q}+p}$ is the cross section for the scattering of the $q\bar{q}$ system on the proton and where the spectral function $\rho$ represents the density of $q\bar{q}$ states. We may use perturbative QCD to evaluate (14). The cross section is given by the probability $|M|^2$ of the $\gamma^* \rightarrow q\bar{q}$ transition multiplied by the imaginary part of the forward amplitude describing the $q\bar{q}$-proton interaction

$$A_{q\bar{q}+p} = i\pi \sigma_{q\bar{q}+p}. \hspace{1cm} (15)$$

For transversely polarized photons the amplitude of the $\gamma^* \rightarrow q\bar{q}$ transition is

$$M_T = \sqrt{\frac{z(1-z)}{Q^2 + k_T^2}} u_\lambda(\gamma, \epsilon_\pm) u_{\lambda'} = \frac{(\epsilon_{\pm}, k_T)[(1 - 2z)\lambda \pm 1] \delta_{\lambda, -\lambda'} + \lambda m_q \delta_{\lambda, \lambda'}}{Q^2 + k_T^2},$$  \hspace{1cm} (16)

where the $q$ and $\bar{q}$ longitudinal momentum fractions and transverse momenta are $z, k_T$ and $(1 - z), -k_T$. We use the notation of Ref. [15], which was based on the earlier work of Ref. [17]. Namely $Q^2$ and the photon polarization vectors are given by

$$\bar{Q}^2 = z(1 - z)Q^2 + m_q^2$$  \hspace{1cm} (17)

$$\epsilon_T = \epsilon_\pm = \frac{1}{\sqrt{2}}(0, 0, 1, \pm i),$$  \hspace{1cm} (18)

and where $\lambda, \lambda' = \pm 1$ according to whether the $q, \bar{q}$ helicities are $\pm \frac{1}{2}$.

In terms of the quark momentum variables we thus obtain

$$\sigma_T(\gamma^* p) = \sum_q \alpha \frac{e_q^2}{2\pi} \int dz \, dk_T^2 \, \frac{z^2 + (1 - z)^2 k_T^2 + m_q^2}{(Q^2 + k_T^2)^2} N_c \sigma_{q\bar{q}+p}(s, k_T^2)$$ \hspace{1cm} (19)

where the number of colours $N_c = 3$. Before evaluating (19) let us relate this expression to the dispersion relation form of (14). We may use

$$M^2 = \frac{k_T^2 + m_q^2}{z(1-z)} \hspace{1cm} (20)$$

to change the integration variable from $dk_T^2$ to $dM^2$. Then (19) has the dispersion-like form

$$\sigma_T(\gamma^* p) = \frac{\alpha}{2\pi} \sum_q e_q^2 \int dz \frac{dM^2}{(Q^2 + M^2)^2} \left\{ M^2 \left[ z^2 + (1 - z)^2 \right] + 2m_q^2 \right\}. \hspace{1cm} (21)$$

In comparison to (14) we see that (21) is a two-dimensional integral. To see the reason for this let us consider massless quarks. Then $z = \frac{1}{2}(1 + \cos \theta)$ where $\theta$ is the angle between the $q$ and
the $\gamma^*$ in the $q\bar{q}$ rest frame. The $dz$ integration is implicit in (14) as the integration over the quark angular distribution in the spectral function $\rho$.

We use the additive quark model (AQM) to evaluate $\sigma_{q\bar{q}+p}$ in (19) which means that each quark is assumed to interact with the target proton individually. For forward scattering, (15), the momentum of the interacting quark is not changed and thus there is no interference — that is the initial and final $q\bar{q}$ states are exactly the same. Now the cross section $\sigma_T(\gamma^*p)$ of (19) receives contributions from all $M^2$ up to $Q^2$. However to estimate the non-perturbative higher twist contribution we must note that the additive quark model result

$$\sigma_{q\bar{q}+p} \simeq \frac{2}{3}\sigma_{pp},$$

is only valid up to some relatively low mass, $M < M_0$. The AQM hypothesis may be justified if the separation between the $q$ and the $\bar{q}$, $\Delta r \simeq 1/k_T$, is large in comparison with the $q\bar{q}$ interaction radius $R$ defined by

$$\sigma_{q\bar{q}}^{\text{inel}} = \pi R^2 \simeq \sigma_{pp}^{\text{inel}}/9 \simeq 3 -5 \text{mb}. \quad (23)$$

As the separation $\Delta r \simeq 1/k_T$ becomes smaller (that is $k_T > 1/R \simeq 0.5$ GeV) the quarks start to shadow each other and the cross section $\sigma_{q\bar{q}}$ decreases. As a result the integration in (19) is effectively restricted to the region $M < M_0$, where $M_0$ may be estimated from (20) using $k_T \simeq 0.5$ GeV, the constituent quark mass $m_q \simeq 0.35$ GeV, and a typical value of $z(1-z) \simeq 0.2$. In this way we obtain the upper limit $M_0^2 \simeq 2$ GeV$^2$. We impose this limit on the results that we present below.

This part of phase space (that is the domain of small $M^2$, small $k_T^2$, but $z \sim 0.5$) is not that responsible for the leading logarithm behaviour generated by DGLAP evolution. Rather the $dk_T^2/k_T^2$ behaviour of integral comes from the alignment kinematic configuration of small $z \sim k_T^2/Q^2$ (or $1-z \sim k_T^2/Q^2$). Then $Q^2$ of (17) is of the order of $k_T^2$. Hence the cross section for $q\bar{q}$ scattering on the proton at large $k_T$ (that is $k_T^2 \gg \Lambda_{\text{QCD}}^2$, but $k_T^2 \ll Q^2$) has the QCD form $\sigma_{q\bar{q}+p} \sim 1/k_T^2$, and so (19) becomes

$$\sigma_T(\gamma^*p) \sim \frac{\alpha}{Q^2} \int \frac{dk_T^2}{k_T^2}. \quad (24)$$

From (24) we see that this region of phase space, $z \sim k_T^2/Q^2$, corresponds to $M^2 \sim Q^2$.

Thus, in summary, the higher twist contribution comes from the large distance domain, $\Delta r \sim 1/k_T$ of small $k_T$ and $M$, but “large” $z \sim 0.5$, where we have to use a non-perturbative (AQM) estimate of $\sigma_{q\bar{q}+p}$. On the other hand DGLAP arises from the perturbative small distance regime of large $k_T$ and $M \sim Q^2$, but small $z$ (or 1-z).

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4To be precise instead of (24) we take the $\sigma_{q\bar{q}+p}$ cross sections to be $\frac{1}{2}[\sigma(\pi^+p) + \sigma(\pi^-p)]$ for the light $q\bar{q}$ pairs (or the $\rho$ and $\omega$ VMD contributions), and to be $\sigma(K^+p) + \sigma(K^-p) - \frac{1}{2}[\sigma(\pi^+p) + \sigma(\pi^-p)]$ for $s\bar{s}$ pairs (or the $\phi$ contribution). We take the hadronic cross sections from Ref. [18].
Results and Discussion

Fig. 5 shows the estimates of the higher twist components as a fraction of $F_T$ as a function of $x$ for two values of $Q^2$. The lower continuous curve corresponds to the effect of gluon rescattering and is calculated from (12) using the MRS(R2) set of partons [19]. The upper two curves correspond to the two alternative ways to calculate the VMD contribution. The dotted curve corresponds to $\Delta F_T$ calculated from the $\rho, \omega$ and $\phi$ contributions to the standard VMD formula (13), while the dashed curve is calculated from eq. (14), using the AQM formula (22), with the $q\bar{q}$ invariant mass $M$ restricted to the domain $M^2 < M^2_0 = 2$ GeV$^2$. The two estimates of the VMD component are in good agreement with each other. We use constituent quark masses ($m_{u,d} = 350$ MeV and $m_s = 500$ MeV) for the VMD estimates. If we were to use current quark masses then $\Delta F_T$ would be enhanced by about 25%.

The combined effect of gluon rescattering and the VMD contribution is also shown (by the heavy continuous curve) on Fig. 5. To calculate the total effect we use (11) and (12) for the rescattering contribution together with the AQM for the VMD contribution. In the region of $x = \text{few} \times 10^{-4}$ we see that the two higher twist components are of comparable magnitude. However the energy dependence of the two contributions is quite different. The ‘perturbative’ rescattering term involves $(xg(x))^2 \propto s^{0.4}$, but actually increases faster than this due to the $dx'/x'$ integration. On the other hand the ‘non-perturbative’ VMD contribution only grows as $s^{0.08}$. Thus as $x$ increases the VMD component becomes the dominant higher twist component. The behaviour is evident in Fig. 5. In fact at $Q^2 = 4$ GeV$^2$, for example, by $x = 0.1$ the higher twist contribution (dominated by the VMD component) reaches about 17% of the whole value of $F_T$. On the other hand at very small $x$ the main twist-4 contribution comes from gluon rescattering and at $Q^2 = 4$ GeV$^2$ and $x = 10^{-4}$, for example, the total twist-4 exceeds $\frac{1}{4}$ of the $F_T$ value.

These are very large higher twist contributions indeed, and there is a puzzle why they are not evident in fitting to the data. One possibility is that in fitting the data the input distribution together with the twist-2 DGLAP evolution is able in a limited kinematical domain, say $Q^2 \sim 3 - 10$ GeV$^2$, to mimic the contribution coming from higher twists. If this is the case, then we should subtract the higher twist (or at least the VMD contribution which is more or less known) from the DIS data before we perform the DGLAP analysis. In other words in the fit to the data we should include the VMD contribution (and maybe also gluon screening) as well as the conventional DGLAP contributions. An alternative possible resolution of the puzzle is that there exists yet another important source of higher twist which enters with a negative sign such that it partially cancels out the higher twist component. For instance renormalon models have been used to estimate power corrections to deep inelastic scattering [20].

We conclude that there are theoretical reasons to expect very significant higher twist contributions to $F_2$, even for $Q^2$ as high as 10 GeV$^2$. Surprisingly there is almost no evidence for them in the data. We have been able to estimate the size of the two major higher twist contributions

\footnote{In the latest global analysis [21] partons have been obtained by fitting to data with $Q^2 > 2$ GeV$^2$, and then}
coming, first, from vector meson dominance and, second, from gluon rescattering in which the four-gluon state is composed of two colour singlets. These two higher twist contributions have very different energy dependences and there is an increasingly significant resultant higher twist effect as $x$ decreases due to the increasing importance of the gluon rescattering term. Of course it is possible that $F_2$ may be subject to other higher twist contributions which partially cancel the effects of the two components that we have estimated and which are expected to dominate. This exploratory study makes clear that, on the one hand, the higher twist contribution needs more detailed theoretical analysis while, on the other hand, it is important to extract as much experimental information as possible by fitting the $Q^2$ dependence of precise data on $F_2(x, Q^2)$ in the region of $Q^2 \lesssim 10$ GeV$^2$.

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the analysis was repeated with the omission of data below $Q^2 = 10$ GeV$^2$. The gluon obtained at $Q^2 = 10$ GeV$^2$ from the latter analysis is some 5\% higher at small $x$ than the gluon from the former analysis which contained low $Q^2$ data. This could be the first indication that some non-twist-2 contribution is hidden in the data.
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Figure Captions.

Fig. 1 An absorptive or gluon rescattering diagram giving rise to a twist-4 contribution.

Fig. 2 An illustration of the space-time structure of a conventional twist-2 DGLAP contribution and a twist-4 vector meson dominance (VMD) contribution. In the latter case the dotted lines makes the 4 quark structure evident.

Fig. 3 An example of an interaction between the colour singlet two-gluon ladders which gives rise to the factor $K$ in (9).

Fig. 4 A contribution to twist-4, $F_T^{(4)}$, associated with the quark, rather than the gluon, distribution.

Fig. 5 The higher twist, $\Delta F_T$, contributions compared to $F_T$ at $Q^2 = 10$ and 4 GeV$^2$. Two estimates of the VMD fraction are shown; they are essentially coincident at $Q^2 = 4$ GeV$^2$. The fraction obtained from the sum of the gluon rescattering and the VMD(AQM) higher twist contributions is shown as the heavy continuous curve.
\[ s' = \frac{Q'^2}{x'} \]

Fig. 1

(a) DGLAP

(b) VMD

Fig. 2
\[ \frac{\Delta F_T}{F_T} \]

- **Total**
- **VMD(AQM)**
- **VMD(\rho, \omega, \phi)**

\[ Q^2 = 10 \text{GeV}^2 \]

\[ Q^2 = 4 \text{GeV}^2 \]