Strong Security and Separated Code Constructions for the Broadcast Channels with Confidential Messages

Ryutaroh Matsumoto∗ Masahito Hayashi†

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Abstract

We show that the capacity region of the broadcast channel with confidential messages does not change when the strong security criterion is adopted instead of the weak security criterion traditionally used. We also show a construction method of coding for the broadcast channel with confidential messages by using an arbitrary given coding for the broadcast channel with degraded message sets.

1 Introduction

The information theoretic security attracts much attention recently [19], because it offers security that does not depend on a conjectured difficulty of some computational problem. A classical problem in the information theoretic security is the broadcast channel with confidential messages (hereafter abbreviated as BCC) first considered by Csiszár and Körner [11], in which there is a single sender called Alice and two receivers called Bob and Eve. The problem in [11] is a generalization of the wiretap channel considered by Wyner [24]. In the formulation in [11], Alice has a common messages destined for both Bob and Eve and a private message destined solely for Bob. The word “confidential” means that Alice wants to prevent Eve from knowing much about the private message. The coding in this situation has two goals, namely error correction and secrecy.

The traditional criterion of judging the secrecy is the so-called weak security criterion, which requires that the mutual information divided by the code length between the secret message and the adversary’s information converges to zero as the code length goes to the infinity. Suppose that the secret message $S_n$ and the adversary’s message $Z^n$ of length $n$ have the identical part of length $n/\log n$ and that the rests are statistically independent, then the weak security criterion judges this situation as secure, while the adversary knows infinitely much information on the secret message. This example suggests that the weak security criterion is inappropriate for some applications.

To exclude such an insecure situation, Maurer [21] introduced the strong security criterion, which requires the mutual information converges to zero without division by the code length. It is important to study the capacities and the capacity regions of various information theoretical problems under the strong security criterion. The key agreement problem [10, 16, 22] and the wiretap channels [3, 10, 14, 16] have been studied under the strong security criterion. However, the capacity region for the BCC has not been clarified as far as the authors’ knowledge, because the strong security results in [3, 10, 14, 16] for the wiretap channels do not seem to easily generalize to the BCCs. Note that [22] cannot be used to prove the strong security of transmission of secret messages, and that [3] is an adaptation of [22] to the wiretap channels. We shall clarify that the capacity region under the strong security criterion is the same as that under the weak one. Our proof argument just attaches inverses of hash functions to an existing random coding argument for the broadcast chan-
nel with degraded message sets (hereafter abbreviated as BCD). Thus the analysis of decoding error probability in our proof automatically becomes as good as the best analysis for BCD. The idea of attaching inverses of hash functions seems first appeared in Csiszár [10] in the context of information theoretic security.

On the other hand, in a communication system with single sender and single receiver, the source coding and the channel coding are the most classical and fundamental problems. The famous source-channel separation theorem [9, Section 7.13] states that we can get an optimal source-channel joint coding by combining an optimal source coding and an optimal channel coding, at least in the sense of asymptotic information rate. Therefore, it is natural to ask if there is a similar separation theorem between secrecy coding and error correction coding in the information theoretic security. In this direction, recently Csiszár and Narayan [12, Lemma B.2] and Renner [23] Lemma 6.4.1] implicitly proved the separation theorem between secrecy coding and error correction coding in the key agreement problem considered by Maurer [20] and Ahlswede-Csiszár [1], which is also a classical and fundamental problem in the information theoretic security. Specifically, Csiszár, Narayan, and Renner showed that the optimal key rate can be attained for the model SW in [1] with one-way public communication provided that we are given optimal probability distributions of the auxiliary random variables in the key capacity formula, by combining Slepian-Wolf encoder and decoder for error correction and a family of two-universal hash functions (fully random functions in [23]) for secrecy. However, the separation in other problems does not seem to be explored, as far as the authors know.

Although our argument for the capacity region of BCC in Section 3 separates the analysis of the decoding error probability and the mutual information, it does not separate the construction of a code for error correction and provision of secrecy. In Section 4 we introduce another form of the privacy amplification theorem so that we can separate the code constructions for error correction and secrecy, then we clarify which rate pairs can be achieved by our separated code construction.

This paper is organized as follows: Section 2 reviews relevant research results used in this paper. Section 3 proves the capacity region under the strong security criterion is the same as under the weak one. Section 4 presents a computational procedure of an upper bound on the mutual information when inverses of hash functions are attached to an arbitrary given code for BCD. Section 5 concludes the paper.

2 Preliminaries

2.1 Broadcast channels with confidential messages

Let Alice, Bob, and Eve be as defined in Section 1. $X$ denotes the channel input alphabet and $Y$ (resp. $Z$) denotes the channel output alphabet to Bob (resp. Eve). We assume that $X$, $Y$, and $Z$ are finite unless otherwise stated. We shall discuss the continuous channel briefly in Remarks 12 and 14. We denote the conditional probability of the channel to Bob (resp. Eve) by $P_{Y|X}$ (resp. $P_{Z|X}$). The set $S_n$ denotes that of the private message and $E_n$ does that of the common message when the block coding of length $n$ is used. We shall define the achievability of a rate triple $(R_1, R_e, R_0)$, where $R_1$ is the rate of the secret message, $R_e$ is the so-called equivocation rate [11], and $R_0$ is the rate of the common message. For the notational convenience, we fix the base of logarithm, including one used in entropy and mutual information, to the base of natural logarithm. Privacy amplification theorems reviewed later are sensitive to choice of the base of logarithm.

Definition 1 The rate triple $(R_1, R_e, R_0)$ is said to be achievable if there exists a sequence of Alice’s stochastic encoder $f_n$ from $S_n \times E_n$ to $X^n$, Bob’s deterministic decoder $\varphi_n : Y^n \rightarrow S_n \times E_n$ and Eve’s deterministic decoder $\psi_n : Z^n \rightarrow E_n$ such that

$$\lim_{n \to \infty} \Pr[ (S_n, E_n) \neq \varphi_n(Y^n) \text{ or } E_n \neq \psi_n(Z^n) ] = 0,$$

$$\liminf_{n \to \infty} \frac{H(S_n|Z^n)}{n} \geq R_e,$$

$$\liminf_{n \to \infty} \frac{\log |S_n|}{n} \geq R_1,$$

$$\liminf_{n \to \infty} \frac{\log |E_n|}{n} \geq R_0,$$

where $S_n$ and $E_n$ represent the secret and the common message, respectively, have the uniform distribution on $S_n$ and $E_n$, respectively, and $Y^n$ and $Z^n$ are the received
signal by Bob and Eve, respectively, with the transmitted signal \( f_n(S_n, E_n) \) and the channel transition probabilities \( P_{YX}, P_{ZX} \). The capacity region of the BCC is the closure of the achievable rate triples.

**Theorem 2** \([17]\) The capacity region for the BCC is given by the set of \( R_0, R_1, R_e \) such that there exists a Markov chain \( U \rightarrow V \rightarrow X \rightarrow YZ \) and

\[
R_1 + R_0 \leq I(V; Y|U) + \min[I(U; Y), I(U; Z)], \\
R_0 \leq \min[I(U; Y), I(U; Z)], \\
R_e \leq I(V; Y|U) - I(V; Z|U), \\
R_e \leq R_1.
\]

As described in \([19]\), \( U \) can be regarded as the common message, \( V \) the combination of the common and the private messages, and \( X \) the transmitted signal.

If we set \( R_e = R_1 \) then we have \( \lim_{n \to \infty} I(S_n; Z^n)/n = 0 \), which is traditionally called perfect security, because Eve knows little about \( S_n \). However, Maurer \([21]\) and Csiszár \([10]\) observed that \( \lim_{n \to \infty} I(S_n; Z^n) = 0 \) is a better criterion for the secrecy of \( S_n \) from Eve, and this stronger requirement is called the strong security criterion, while the traditional one is called the weak security criterion recently.

**Corollary 3** \([17]\) The notation is same as Theorem 2. If we require \( R_e = R_1 \), the capacity region for \((R_0, R_1)\) is given by the set of \( R_0 \) and \( R_1 \) such that there exists a Markov chain \( U \rightarrow V \rightarrow X \rightarrow YZ \) and

\[
R_0 \leq \min[I(U; Y), I(U; Z)], \\
R_1 \leq I(V; Y|U) - I(V; Z|U).
\]

**2.2 Broadcast channels with degraded message sets**

If we set \( R_e = 0 \) in the BCC, the secrecy requirement is removed from BCC, and the coding problem is equivalent to the broadcast channel with degraded message sets (abbreviated as BCD) considered by Körner and Marton \([18]\).

**Corollary 4** The capacity region of the BCD is given by the set of \( R_0 \) and \( R'_1 \) such that there exists a Markov chain \( U \rightarrow V = X \rightarrow YZ \) and

\[
R_0 \leq \min[I(U; Y), I(U; Z)], \\
R'_1 \leq I(V; Y|U) + \min[I(U; Y), I(U; Z)].
\]

Throughout this paper, the information rate of the private message to Bob without secrecy requirement is denoted by \( R'_1 \) instead of \( R_1 \), to emphasize the difference. One of several typical proofs for the direct part of BCD is as follows \([5]\): Given \( P_{UV} \), \( R_0, R'_1 \), we randomly choose \( \exp(n R_0) \) codewords of length \( n \) according to \( P_{U}' \), and for each created codeword \( u^n \), randomly choose \( \exp(n R'_1) \) codewords of length \( n \) according to \( P_{V|U}'(u^n) \). Over the constructed ensemble of codebooks, we calculate the average decoding probability by the joint typical decoding, or the maximum likelihood decoding, etc.

**2.3 Privacy amplification theorem**

We shall use a family of two-universal hash functions \([8]\) and a privacy amplification theorem obtained by Hayashi \([16]\) based on the work by Bennett et al. \([4]\). So we shall review them.

**Definition 5** Let \( \mathcal{F} \) be a set of functions from \( S_1 \) to \( S_2 \), and \( F \) the not necessarily uniform random variable on \( \mathcal{F} \). If for any \( x_1 \neq x_2 \in S_1 \) we have

\[
\Pr[F(x_1) = F(x_2)] \leq \frac{1}{|S_2|},
\]

then \( \mathcal{F} \) is said to be a family of two-universal hash functions.

**Proposition 6** Let \( L \) be a random variable with a finite alphabet \( \mathcal{L} \) and \( Z \) any random variable. Let \( \mathcal{F} \) be a family of two-universal hash functions from \( \mathcal{L} \) to \( \mathcal{M} \), and \( F \) be a random variable on \( \mathcal{F} \) statistically independent of \( L \). Then

\[
I(F(L); Z|F) \leq \frac{1}{\rho} |\mathcal{M}| E[P_{U|Z}(L|Z)^\rho] 
\]

for \( 0 < \rho \leq 1 \). If \( Z \) is not discrete RV, \( I(F(L); Z|F) \) is defined to be \( H(F(L)|F) - E[H(F(L)|F, Z = z)] \). In addition to the above assumptions, when \( L \) is uniformly distributed, we have

\[
\frac{1}{\rho} |\mathcal{M}| E[P_{U|Z}(L|Z)^\rho] = \frac{|\mathcal{M}| E[P_{UZ}(L|Z)^\rho] P_{L}(L)^{-\rho}}{|\mathcal{L}|^\rho}. 
\]

In addition to all of the above assumptions, when \( Z \) is a discrete random variable, we have

\[
\frac{|\mathcal{M}| E[P_{UZ}(L|Z)^\rho] P_{L}(L)^{-\rho}}{|\mathcal{L}|^\rho}.
\]
Remark 7 It was assumed that $Z$ was discrete in [16]. However, when the alphabet of $L$ is finite, there is no difficulty to extend the original result.

As in [16] we introduce the following two functions.

**Definition 8**

\[
\psi(\rho, P_{Z|L}, P_L) = \log \sum_{\ell} \sum_{z} P_L(\ell) P_{Z|L}(z|\ell)^{1+\rho} P_Z(z)^{-\rho} \leq \log \left( \sum_{\ell} P_L(\ell) \right) \sqrt{\lambda} \tag{3}
\]

\[
\phi(\rho, P_{Z|L}, P_L) = \log \sum_{\ell} \sum_{z} \left( \sum_{|z|=1} P_L(\ell) P_{Z|L}(z|\ell)^{1/(1-\rho)} \right)^{1-\rho} \leq \log \left( \sum_{\ell} P_L(\ell) \right) \sqrt{\lambda} \tag{5}
\]

Observe that $\phi$ is essentially Gallager’s function $E_{\lambda}$ [13].

**Proposition 9** [13] [16] \(\exp(\phi(\rho, P_{Z|L}, P_L))\) is concave with respect to $P_L$ with fixed $0 < \rho < 1$ and $P_{Z|L}$. For fixed $0 < \rho < 1, P_L$ and $P_{Z|L}$ we have

\[
\exp(\psi(\rho, P_{Z|L}, P_L)) \leq \exp(\phi(\rho, P_{Z|L}, P_L)). \tag{6}
\]

3 Calculation of the average mutual information with random coding

In this section we shall prove that the capacity region given in Corollary [3] is also the capacity region of the BCC under the strong security criterion. We do not need the proof for the converse part. We shall prove the direct part. Let the RV $B_n$ on $\mathcal{B}_n$ denote the private message to Bob without secrecy requirement, $E_n$ on $\mathcal{E}_n$ the common message to both Bob and Eve, $F_n$ on $\mathcal{F}_n$ a function in a family $\mathcal{F}_n$ of two-universal hash functions from $\mathcal{B}_n$ to $\mathcal{S}_n$, $\Lambda$ an RV indicating selection of codebook in the way reviewed in Section 2.2. $U^n = \Lambda(E_n)$ on $\mathcal{U}^n$ and $V^n = \Lambda(B_n, E_n)$ on $\mathcal{V}^n$ codewords for the BCD taking the random selection $\Lambda$ taking into account, and $Z^n$ Eve’s received signal, where $n$ denotes the code length. We assume that for every $f_n \in \mathcal{F}_n$ is surjective and for all $s \in \mathcal{S}_n$ the set $\{b \in \mathcal{B}_n \mid f_n(b) = s\}$ has the constant number of elements. Such requirement on $f_n$ is satisfied, for example, when $\mathcal{F}_n$ is the set of all surjective linear maps from $\mathcal{B}_n$ to $\mathcal{S}_n$.

The structure of the transmitter and the receiver is as follows: Fix a hash function $f_n \in \mathcal{F}_n$ and Alice and Bob agree on the choice of $f_n$. Given a secret message $s_n$, choose $b_n$ uniformly randomly from $\{b \in \mathcal{B}_n \mid f_n(b) = s_n\}$, treat $b_n$ as the private message to Bob, encode $b_n$ along with the common message $e_n$ by an encoder for the BCD, and get a codeword $v^n$. Apply the artificial noise to $v^n$ according to the conditional probability distribution $P_{E|V}$ and get the transmitted signal $x^n$. Bob decodes the received signal and get $b_n$, then apply $f_n$ to $b_n$ to get $s_n$. This construction requires Alice and Bob to agree on the choice of $f_n$. We shall show that $I(S_n; Z^n|F_n) = E_{\lambda} I(S_n; Z^n|F_n = f_n)$ to be arbitrary small. This ensures that most choice of $f_n$ makes $I(S_n; Z^n|F_n = f_n)$ small. The same argument was also used in [10].

Let $S_n$ denote the RV of the secret message. Define $B'_n$ to be the RV uniformly chosen from the random set $\{b \in \mathcal{B}_n \mid f_n(b) = S_n\}$. We want to apply the privacy amplification theorem to $I(F_n(B'_n); Z^n|F_n)$. To use the theorem (Proposition 6) we must ensure independence of $F_n$ and $B'_n$. The independence is satisfied by the assumptions on $\mathcal{F}_n$ if $S_n$ is uniformly distributed. In that case $B'_n$ is uniformly distributed over $\mathcal{B}_n$. Denote $B'_n$ by $B_n$. The remaining task is to find an upper bound on $I(F_n(B_n); Z^n|F_n, \Lambda)$. Since the decoding error probability of the above scheme is not greater than that of the code for BCD, we do not have to analyze the decoding error probability.

Firstly, we consider $I(F_n(B_n); Z^n|F_n, \Lambda)$ with fixed selection $\lambda$ of $\Lambda$. In the following analysis, we do not make any assumption on the probability distribution of $E_n$ except that $S_n, E_n, F_n$ and $\Lambda$ are statistically independent.

\[
I(F_n(B_n); Z^n|F_n, \Lambda = \lambda) \leq I(F_n(B_n); Z^n, E_n|F_n, \Lambda = \lambda) = I(F_n(B_n); E_n|F_n, \Lambda = \lambda|F_n, E_n, \Lambda = \lambda) = 0 \tag{7}
\]

\[
\leq \sum_{\epsilon} P_{E_n}(\epsilon) I(F_n(B_n); E_n|\Lambda = \lambda) = \sum_{\epsilon} P_{E_n}(\epsilon) \sum_{b} P_{B_n}(b) \frac{\exp(\eta P_{R_1})}{\rho \exp(\eta P_{R_1})} P_{z|b, e, \Lambda = \lambda}(z) \tag{8}
\]

\[
P_{z|b, e, \Lambda = \lambda}(z) = \epsilon(z|b, e) \Theta_{\epsilon, \eta} P_{z|b, e, \Lambda = \lambda}(z) \tag{9}
\]

1The statistical independence of the corresponding random variables in [3][10] was not discussed in detail.
We can see that (*) → I(V; Z|U) as ρ → 0. by applying the 1'Hôpital's rule to (*).

This shows that the amount \( R'_1 - R_1 \) of random garbage required to make \( S_n = F_n(B_n) \) secret from Eve is \( I(V; Z|U) \) per channel use. By choosing \( R_0 = \min\{I(U; Y), I(U; Z)\} - \delta \) and \( R'_1 = I(V; Y|U) - \delta \), we have completed the direct part proof.

**Remark 10** Our proof does not require the common message \( E_n \) to be decoded by Bob. Our technique can provide an upper bound on the mutual information of \( S_n = F_n(B_n) \) even when \( E_n \) is a private message to Eve.
Remark 11 The (negative) exponential decreasing rate of the mutual information in our argument is
\[ R_1 - R'_1 + \frac{1}{\rho} \log \left( \sum_{u \in U} P_U(u) \exp(\phi(\rho, P_{Z|V}, P_{V|U=u})) \right) \]

Minimizing the above expression over \( 0 < \rho \leq 1 \), \( R'_1 \) and \( U \rightarrow V \rightarrow X \rightarrow YZ \) such that \( R_0 \leq \min(I(U; Y), I(U; Z)) \), and \( R'_1 \leq I(V; Y|U) \) gives the smallest negative exponent. From the form of the mathematical expression, increase in \( R'_1 \) decreases the mutual information and increases the decoding error probability of the secret message to Bob. This suggests that the optimal mutual information and the optimal decoding error probability cannot be realized simultaneously.

Remark 12 We can easily carry over our proof to the case of the channel being Gaussian, because

- we can extend Eq. (3) to the Gaussian case just by replacing the probability mass functions \( P_{Z|L} \) and \( P_Z \) by their probability density functions.
- the random codebook \( \Lambda \) obeys the multidimensional Gaussian distribution,
- the concavity of \( \phi \) is retained when its second argument is conditional probability density,
- and the all mathematical manipulations in this section remains valid when \( U, V, Z, \Lambda \) are continuous and their probability mass functions are replaced with probability density functions, while \( B_n, E_n, F_n \) remain to be discrete RVs on finite alphabets.

4 Separated code construction for the broadcast channel with confidential messages

Suppose that we are given single triple of an encoder and decoders for BCD. We want to construct a code for BCC based on the code for BCD, by attaching the inverse of a randomly chosen two-universal hash function to the given BCD code. If we could do this without loss of any optimality, then the practical study of codes for BCC would become unnecessary, because the study of practical BCC codes can be reduced to that of practical BCD codes. We stress that the random choice of encoder and decoder is widely accepted as a practical method, see e.g., [7, 15, 23].

Let \( X \) be the uniform distribution on the given codebook, and \( Z \) Eve’s received signal given \( X \) as channel input. By simply applying Proposition 6 to \( X \) and \( Z \), the size of secret message set \( S \) has to satisfy
\[ \min_{0 < \rho \leq 1} \frac{|S|^\rho \mathbb{E}[P_{X|Z}(X|Z)^\rho]}{\rho} \leq \text{acceptable value}. \quad (11) \]

When the number of codewords is, say \( 2^{1000} \), evaluation of the left hand side is practically impossible.

We shall introduce another form of the privacy amplification theorem alternative to Proposition 6 whose computation as Eq. (11) is intractable with arbitrary given single BCD encoder, so that we can compute a suitable size of \( S \). What follows is an extension of a result on the wiretap channel [17]. The following theorem is an adaptation of the channel resolvability lemma [14, Lemma 2].

**Theorem 13** Assume that the given family of two-universal hash function \( F \) from \( L \) to \( M \) satisfies that
\[ |F^{-1}(m)| = \frac{|L|}{|M|}, \quad \forall m, \]
the statistically independent random variable \( K \) and \( L \) obey the uniform distributions on \( K \) and \( L \), respectively, and a fixed conditional probability \( Q_{Z|K,L} \) is given. We also assume that \( F \) is statistically independent of \( K \) and \( L \). Then,
\[ I(F(L); Z|F) \leq \frac{|M|^\rho \exp(\phi(\rho, Q_{Z|K,L}, P_{K,L}))}{(|K| \times |L|)^\rho}, \]
for \( 0 < \rho < 1 \).

**Proof.**
\[ I(F(L); Z|F) \]
\[ \leq I(F(L); K, Z|F) \]
\[ = I(F(L); Z|K, F) \]
\[ \leq \sum_k P_k(k) \frac{|M|^\rho}{|L|^{\rho}} \exp(\phi(\rho, P_{Z|K=k,L}, P_{L})) \]
message and Eve’s received signal. Let  

per bound on the mutual information between the secret  

deckers for the BCD. We shall derive a computable up-
plification theorem. Let  

Recall that the function  

hash functions.  

have  

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Suppose that we are given a triple of an encoder and decoders for the BCD. We shall derive a computable upper bound on the mutual information between the secret message and Eve’s received signal. Let  

be the secret message to Bob,  

be the common message to both Bob and Eve. We assume that  

and  

are statistically independent to each other. Let  

be an RV on a family  

of two-universal hash functions.  

is statistically independent of  

and we use the same assumptions on the hash functions as Section  

Let  

be the uniform random on the set  

. As in Section  

,  

and  

are statistically independent and we can apply the privacy amplification theorem. Let  

be Eve’s received signal after encoding  

by the given encoder  

for BCD, applying the artificial noise  

, and transmitting the resulted signal over the given channel. By using Theorem  

we have  

Because  

is concave with respect to  

, its maxi-
mization can be computed in practice, for example by  

. On the other hand,  

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We have to investigate under which conditions the above computational procedure for the secret message size can achieve a rate pair  

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, the generalization of results in Section  

to the Gaussian channels is easy provided that the transmitted signal is chosen from a fixed finite subset of  

for every channel use. When the transmitted signal is chosen from varying finite sets for each channel use, we have difficulty in Eq.  

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5 Conclusion

We argued that the weak security criterion, which requires only the mutual information divided by the code length converges to zero, may be inappropriate in some applications, by explicitly providing an insecure example, and made a case for the strong security criterion introduced by Maurer  

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We argued that the weak security criterion, which requires only the mutual information divided by the code length converges to zero, may be inappropriate in some applications, by explicitly providing an insecure example, and made a case for the strong security criterion introduced by Maurer  

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under the strong security criterion before this paper. We have shown that the capacity region under the strong security is the same as that under the weak one.

On the other hand, the separation between secrecy coding and error correction coding is important from both theoretical and practical viewpoints. We presented a random coding argument and a code construction that separate error correction and secrecy. However, our separations for the broadcast channel with confidential messages are still incomplete compared to the source channel separation [9, Section 7.13] or the separation in the classical [12, 23] and quantum [23] key agreement problem, because we cannot separately and independently construct codes for secrecy and error correction and combine them without losing the optimality in the sense of asymptotic information rate, as done in [9, Section 7.13] and [12, 23].

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