Changes to preservice elementary teachers’ beliefs about mathematics and the teaching and learning of mathematics: How and why?

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Changing the beliefs of preservice teachers is a prominent topic in mathematics education. In this paper, we present the results of a study that focuses on the changes in preservice elementary-school teachers’ beliefs during a method course and use the theory of conceptual change to explore the mechanisms that underlie these changes. The results indicate not only that broad changes are possible, but also that they happen as a result of juxtaposing their lived experiences as K-12 students with their current experiences as students immersed in a course designed around problem-solving. In this regard, the results also indicate that their beliefs about mathematics change differently from their beliefs about the teaching and learning of mathematics. The conclusions pose some important questions for future research and further discussion.

Keywords: teacher beliefs; beliefs about mathematics; beliefs about teaching and learning mathematics; conceptual change

INTRODUCTION

“Prospective elementary teachers do not come to teacher education feeling unprepared for teaching” (Feiman-Nemser, McDiarmid, Melnick, & Parker, 1987). During their time as students of mathematics, they first formulated and then concretized, deep-seated beliefs about mathematics and what it means to teach and learn mathematics (Ball, 1988; Lortie, 1975). These beliefs form the foundation upon which they will eventually build their own practice as teachers of mathematics (Skott, 2001) and will serve as the primary regulators for their professional behavior in the classrooms (Ernest, 1989).

“Beliefs form the bedrock of teachers’ intentions, perceptions, and interpretations of a given classroom situation and the range of actions the teacher considers in responding to it” (Chapman, 2002, p. 180)

Unfortunately, these beliefs often run counter to contemporary research on what constitutes good practice. As such, it is one of the roles of the teacher education programs to reshape these beliefs and correct misconceptions that could impede effective teaching in mathematics (Green, 1971). In this article, we look closely not only at what it means to reshape the beliefs of preservice teachers but also at the underlying mechanisms of such change.

Mathematics teachers’ beliefs

Within the field of mathematics education, researchers first focused their attention on beliefs as a way to explain the discordance between teachers’ knowledge of mathematics, their teaching capacity, and their demonstrated abilities in the classroom (McLeod, 1992; Philipp, 2007; Thompson, 1992). Although beliefs have often been referred to as “messy constructs” (Furinghetti & Pehkonen, 2002; Pajares, 1992), there is a consensus that beliefs are considered as personal philosophies about mathematics as well as the teaching and learning of mathematics.

Some of this messiness can be reduced, however, if we focus on the composition of these beliefs. For example, Green (1971) organizes beliefs “along a central-peripheral dimension that reflects psychological strength or degree of nearness to self” (Chapman, 2002, p. 179). Green (1971) distinguishes between beliefs that are primary and derived. “Primary beliefs are so basic to a person’s way of operating that she cannot give a reason for holding those beliefs: they are essentially self-evident to that person” (Mewborn, 1999). Derived beliefs, on the other hand, are identifiable related to other beliefs. Green (1971) also partitions beliefs according to the psychological conviction with which an individual adheres to them. Core beliefs are strongly held and are central to a person’s personality, while less strongly held beliefs are referred to as peripheral. Finally, Green distinguishes between evidential and non-evidential beliefs. Evidential beliefs are formed, and held, either on the basis of evidence or logic. Non-evidential beliefs are grounded neither in evidence nor logic but reside at a deeper, tacit level.

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In an altogether different organization, Dionne (1984) suggests that beliefs are composed of three basic components called the traditional perspective, the formalist perspective, and the constructivist perspective. Similarly, Ernest (1991) describes three philosophies of mathematics as instrumentalist, Platonist, and problem-solving, whereas Törner and Grigutsch (1994) name the three components as toolbox aspect, system aspect, and process aspect. In the toolbox aspect, mathematics is seen as a set of rules, formulae, skills, and procedures, whereas mathematical activity is deemed to involve calculating, using rules, procedures, and formulae. In the system aspect, mathematics is characterized by logic, rigorous proofs, exact definitions, and a precise mathematical language, and doing mathematics consists of accurate proofs as well as of the use of a precise and rigorous language. Finally, in the process aspect, mathematics is considered as a constructive process where relations between different notions and sentences play an important role. Here, the mathematical activity involves creative steps, such as generating rules and formulae, thereby inventing or reinventing the mathematics (Liljedahl, Rösken, & Rolka, 2006).

Changes in mathematics teachers’ beliefs

Changes in a mathematics teacher’s beliefs can be seen as transition of beliefs between these three categories – toolbox aspect, system aspect, and process aspect. However, research tells us that teachers’ beliefs are often robust and therefore difficult to change (Coomen, 2001; Kaastra, Hassuula, Laine, & Pehkonen, 2005, 2006; Stuart & Thurlow, 2000). Schommer-Aikins (2004) points out that beliefs are “like possessions. They are like old clothes; once acquired and worn for awhile, they become comfortable. It does not make any difference if the clothes are out of style or ragged. Letting go is painful and new clothes require adjustment” (p. 22).

Having said that, an abundance of research purports to produce changes in preservice teachers of mathematics. Prominent in this research is an approach through which preservice teachers’ beliefs are challenged (Feiman-Nemser et al., 1987). Another prominent method for evoking change in preservice teachers is by involving them as learners of mathematics (and mathematics pedagogy), usually submersed in a constructivist environment (Ball, 1988; Feiman-Nemser & Featherstone, 1992). A third method for producing changes in belief structures has emerged out of the work of one of the authors in which it has been shown that preservice teachers’ experiences with mathematical discovery have a profound, and immediate, transformative effect on the beliefs regarding the nature of mathematics, as well as their beliefs regarding the teaching and learning of mathematics (Liljedahl, 2005; Smith, Williams, & Smith, 2005).

All three of these approaches are combined in the design and teaching of the aforementioned mathematics methods course in which the data were collected.

METHODS

The participants

The participants in this study are preservice elementary school enrolled in an Elementary Mathematics Teaching Methods course for which the first author was the instructor. This particular offering of the course enrolled 39 students (37 females and 2 males), the vast majority of whom were extremely fearful of having to learn mathematics and even more so of having to teach mathematics. This fear resided, most often, within their negative beliefs and attitudes about their ability to learn and do mathematics. However, as apprehensive and fearful of mathematics as these students were, they were extremely open to, and appreciative of, any ideas that may have helped them to become better mathematics teachers.

The program

The aforementioned Elementary Mathematics Teaching Methods course is embedded within a 4 + 1 teacher education program. That is, the students come to the program with a 4-year undergraduate degree already in hand, and the program itself is only 1 year (three terms). Preservice teachers enter the program either in September or January, and in their first term they do a 4-week practicum placement. For those who enter in September, this is followed by a term in which they do a 13-week practicum and then by the term in which they take the teaching methods courses. Those who enter in January take the methods courses in the second term and the 13-week practicum in their third term. This means that the aforementioned Elementary Mathematics Teaching Methods course enrolled students who were either in their second or third term and who had between 4 and 17 weeks of teaching practicum experience.

The course

During the course, the participants were immersed into a problem-solving environment in which problems were used as a way to introduce concepts in mathematics, pedagogy, and theories of learning. There were problems that were assigned to be worked on in class, as homework, and as a project. Problems assigned to be worked on in class were done in visibly random groups of three (Liljedahl, 2014), whereas problems assigned as homework and projects were worked on in self-selected groups. However, these groups were not rigid, and as the weeks passed, the class became a very fluid and cohesive entity that tended to work on problems as a collective whole. Communication and interaction between participants were frequent and whole-class discussions with the instructor were open and frank.

The data

Throughout the course, the participants kept a reflective journal in which they responded to assigned prompts. These prompts varied from invitations to think about assessment to instructions to comment on curriculum. One set of prompts, in particular, were used in the first and last week of the course to assess each participant’s beliefs about mathematics, and teaching and learning mathematics and to gauge how these beliefs had changed over the course.

- What is mathematics?
- What does it mean to learn mathematics?
- What does it mean to teach mathematics?
Journal writing in mathematics education has a long and diverse history of use. While journaling helps students learn mathematical concepts (Chapman, 1996; Ciochine & Polivka, 1997; Dougherty, 1996), it has also been shown to be an effective tool for facilitating reflection among students (Mewborn, 1999) as well as an effective communicative tool between students and teachers (Burns & Silbey, 2001). More relevant to this study, journaling has become an accepted method for qualitative researchers to gain insights into their participants’ thinking (Mewborn, 1999; Miller, 1992). In particular, reflective journals have been shown to be a very good method for soliciting responses pertaining to beliefs, even when such responses are not explicitly asked for (Koirala, 2002; Liljedahl, 2005).

The analysis

The three authors (PL, BR, and KR) independently coded the data according to Törner and Grigutsch’s (1994) three aspects of beliefs—toolbox aspect, system aspect, and process aspect. The independent results were then compared, discrepancies were discussed, and pertinent entries were recoded. This process (Huberman & Miles, 1994) led to a more elaborate understanding of the framework, as well as a more consistent coding of the data. In the following, we use excerpts from the participants’ journals to exemplify our shared understanding of each aspect of beliefs with respect to mathematics as well as the teaching and learning of mathematics.

Beliefs about mathematics: Toolbox aspect

“My first impression is that math is numbers, quantities, units. In math there is always one right answer. [...] Math is about [...] memorizing formulas that yield the right answer.” (Stephanie)

“When first pondering the question “What is mathematics?” I initially thought that mathematics is about numbers and rules. It is something that you just do and will do well as long as you follow the rules or principles that were created by some magical man thousands of years ago.” (David)

Beliefs about mathematics: System aspect

“Mathematics is the science of pattern and structure. It uses number sense and mathematical concepts to develop a flexible understanding of the world around us.” (Nora)

“Mathematics is a universal language. It is the study of numbers, proportions, relationships, patterns and sequences. Becoming literate in this language is important in order to understand space and time; to develop logical thinking and reasoning; [...]” (Rachel)

Beliefs about mathematics: Process aspect

“For me, math has truly transformed from being a skill or procedure that can be used merely for efficiency to being imbedded within a process of meaning-making that goes on inside the individual, a construction of understanding that we make up.” (Becky)

Beliefs about the teaching and learning of mathematics: Toolbox aspect

“For me math is a puzzle to figure out. All of the questions or problems we were given in school had a solution that I just needed to apply a formula or rule to and the answer would be clear.” (Chealsy)

Beliefs about the teaching and learning of mathematics: System aspect

“To learn mathematics is to learn how numbers are used to represent concepts and matter, as well as show relationships and solve problems.” (Diana)

“Learning math means understanding patterns, quantities, shapes, [...] To teach mathematics is to teach fundamental number concepts [...]” (Lorena)

Beliefs about the teaching and learning of mathematics: Process aspect

“The other thing that stands out is the difference between formally teaching students, and actually facilitating learning. By being a facilitator of the learning process, we are able to choose situations, activities and problems for the students to work on either individually or in groups, and through this approach, students are able to [...] try different ideas, and develop strategies.” (Robyn)

“I think to teach mathematics you need to let the thinking be put in your students’ hands. You need to give them ownership of ideas and let them feel safe and free within the classroom.” (Michelle)

These excerpts represent only a small portion of all the student journals. A wide range of statements supporting each category were found in the data. It should be noted, however, that not all excerpts were so easy to categorize. We are following Dionne’s (1984) suggestion that mathematical beliefs constitute a mixture of the aforementioned aspects, and as such, clear classification cannot always be made. As a result, many journal entries were coded for more than one aspect. For example, in the following journal entry, the system aspect is intertwined with the toolbox aspect in beliefs about mathematics.

“I think [mathematics] has to do with the complex relationships between numbers and the symbols we use to make sense of the world among us. More and more I see maths as a system put in place to help us better [...] make sense of the world around us. Maths allows us to group things, to calculate, to categorize. It’s a great way to bring order from chaos.” (Heng-Zi)

In addition, the data were checked for comments that were indicative of rhetoric and were attentive to comments that were hollow echoes of conventional beliefs about mathematics and the teaching and learning of mathematics such as those made by Leslie and Reine.
“Math is the study of numbers and patterns and the relationship between them.” (Leslie)

“Mathematics is the study of numbers.” (Reine)

Overall, there were three participants whose data were deemed to be rhetoric, and as such excluded from the aggregation.

RESULTS

The coded data from the journal entries at the beginning of the course were aggregated and compared to the aggregated data from the journal entries at the end of the course. This allowed us to make the comparisons necessary to produce a holistic picture of the evolving beliefs of the class as a whole. The results of this aggregation are displayed in Table 1 and Figs 1–4.

The most obvious change visible in these data is the degree to which a process aspect of mathematics and the teaching and learning of mathematics have been introduced into the collective beliefs of the class and how this has been accompanied by a general migration away from the toolbox and system belief aspects. This shift is overwhelmingly significant within the beliefs about the teaching and learning of mathematics. However, to see what gave way to this change a different sorting of the data is necessary.

For each participant, the data were examined to see how their beliefs at the beginning of the course compared to their beliefs at the end of the course. For example, in her first entry, Becky’s beliefs about mathematics correspond to the toolbox aspect.

“So, all I can think of when asked this question [what is mathematics?] is numbers, the study of numbers and perhaps how math exists in this world all around us, how it encompasses us, from everything from our 10 fingers and toes to the products we bought for groceries to the angle of the sun or the curvature of the earth.”

In Becky’s last entry, however, her beliefs about mathematics correspond to the process aspect.

“I was focused on seeing math as a tool; that its benefit was in its uses. I put the stress on the applications […] and equated meaning to its usefulness as a tool. Throughout the course, this has been the biggest evolution in my thinking, as I have moved from this definition of math towards one that focuses more on math as the thought processes or reasoning that goes on inside of us to make sense and give meaning to number relationships and patterns. […] For me, math has truly transformed from being a skill or procedure that can be used merely for efficiency to being imbedded within a process of meaning-making that goes on inside the individual, a construction of understanding that we make up.”

Table 1. Aggregation of coded data

|                        | Beliefs about mathematics (Before) | Beliefs about mathematics (After) | Beliefs about teaching and learning mathematics (Before) | Beliefs about teaching and learning mathematics (After) |
|------------------------|-----------------------------------|----------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| Toolbox                | 23                                | 11                               | 33                                                     | 4                                                      |
| System                 | 22                                | 19                               | 18                                                     | 8                                                      |
| Process                | 0                                 | 12                               | 6                                                      | 30                                                     |

Fig. 1. Beliefs about mathematics at the beginning of the course

Fig. 2. Beliefs about mathematics at the end of the course
The changes in Becky’s beliefs are coded as \([x, y]\), where \([x, y]\) is an ordered pair denoting \([\text{initial belief, final belief}]\). For most of the participants \((n = 36)\), more than one belief was expressed in their responses to at least one of the journal prompts. This resulted in more than one ordered pair being assigned for a participant.

The results of coding the data in this fashion are presented in Tables 2 and 3 and Figs 5 and 6. Table 2 and Fig. 5 present the changes in beliefs about mathematics across all the participants, whereas Table 3 and Fig. 6 present the changes in beliefs about the teaching and learning of mathematics across all the participants. Each of the cells in these tables is to be read as the number of ordered pairs of that type. For example, in Table 2, the cell at the bottom left has a number 8 in it, which corresponds to the column in the top left corner of Fig. 5. This means that there were eight ordered pairs of the type \([\text{toolbox, process}]\), meaning that there were eight participants who expressed toolbox beliefs about mathematics at the beginning of the course and then process beliefs about mathematics at the end of the course.

|                        | Toolbox before | System before | Process before |
|------------------------|----------------|---------------|----------------|
| Becomes toolbox        | 7              | 7             | 0              |
| Becomes system         | 8              | 14            | 0              |
| Becomes process        | 8              | 7             | 0              |

|                        | Toolbox before | System before | Process before |
|------------------------|----------------|---------------|----------------|
| Becomes toolbox        | 3              | 1             | 2              |
| Becomes system         | 14             | 5             | 1              |
| Becomes process        | 29             | 16            | 6              |
From these figures, a number of more nuanced results emerge – the results that are also supported within the fine-grain analysis of individual participants’ journal entries. One of these has already been alluded to above and pertains to the predominant shift toward a process belief about the teaching and learning of mathematics. In all, there were 56 instances where initial beliefs were either replaced by, or complemented with, a process belief. This can be seen in the third row of Table 3 as well as the back row of Fig. 6. Not seen in these tables and figures, however, is the result that this adoption of a process belief was embodied within 24 of the 30 (80%) participants who did not initially profess a process belief.

A second result that emerges from this data is the robustness of the systems belief about mathematics. Fourteen out of 28 participants who had a demonstrated systems belief of mathematics at the beginning of the course retained that belief at the end of the course. This is by far the greatest retention of mathematics at the beginning of the course retained that belief by the fact that 7 of the 14 participants who retained a systems language. This resilience is further accentuated as logic, rigorous proofs, exact definitions, and a precise mathematical language. This resilience is further accentuated by the fact that 7 of the 14 participants who retained a systems belief of mathematics simultaneously shifted from a systems belief about the teaching and learning of mathematics to a process belief. This bifurcation of beliefs about mathematics from beliefs about the teaching and learning of mathematics speaks to the nature of beliefs to cluster (Green, 1971) and that these participants’ beliefs about mathematics reside within a different belief structure from their beliefs about the teaching and learning of mathematics.

DISCUSSION

There are two main conclusions emerging from these two ways of coding the data – the first of which pertains to the effectiveness that the problem-solving environment had on the recasting of these preservice teachers’ beliefs about what it means to teach and learn mathematics. Through their own experiences with mathematics in a non-traditional setting, most of the participants come to see, and furthermore to believe, in the value of teaching and learning mathematics as a process. This is an important shift in which it most closely aligns their beliefs with contemporary theories of learning as well as contemporary ideas about what constitutes effective practice (National Council of Teachers of Mathematics, 2000). There is an important subtlety here, though, that is not to be overlooked. Beliefs are not in practice. Although there is strong evidence to suggest that beliefs govern practice (Chapman, 2002), we are not to be fooled into thinking that beliefs automatically translate into practice. Research has repeatedly shown that the adversity faced in the early years of teaching can have profoundly detrimental effects on novice teachers’ best intentions of practice (Berliner, 1987). Further research is needed to determine the robustness of these beliefs in the face of such adversity and to closer examine the transition from intentions of practice to actual practice.

The second conclusion pertains to the robustness of the systems belief about mathematics. For many of the participants in this study, it appears these beliefs were impervious to their recent experiences with mathematics within the context of the course that the study was situated. Robust beliefs are to be expected. What was unique within this study, however, was that these beliefs remained unchanged, whereas other (closely related) beliefs did not. We conjecture that this robustness is, in fact, due to the resilience of the particular belief structures that beliefs about mathematics reside within (Liu, 2011). Again, further research is needed to closer examine this conjecture.

All of this is to say that the beliefs of these preservice teachers changed. What this research has failed to show, however, is how and why these changes are occurring. In the following, we examine the idea of changing beliefs in preservice elementary-school teachers vis-à-vis a theory of conceptual change (Posner, Strike, Hewson, & Gertzog, 1982; Vosniadou & Verschaffel, 2004).

Conceptual change

The theory of conceptual change emerges out of Kuhn’s (1970) interpretation of changes in scientific understanding through history. Kuhn proposes that progress in scientific understanding is not evolutionary, but rather a “series of peaceful interludes punctuated by intellectually violent revolutions,” and in those revolutions “one conceptual world view is replaced by another” (p. 10). That is, progress in scientific understanding is marked more by theory replacement than theory evolution. Kuhn’s ideas form the basis of the theory of conceptual change, which has been used to hypothesize about the teaching and learning of science (c.f. Posner et al., 1982) and mathematics (c.f. Tirosh & Tsamir, 2006; Vosniadou & Verschaffel, 2004).

The theory of conceptual change starts with an assumption that in some cases students form misconceptions about phenomena based on lived experience, that these misconceptions stand in stark contrast to the accepted scientific theories that explain these phenomena, and that these misconceptions are robust. For example, many children believe that heavier objects fall faster. This is clearly not true and a rational explanation as to why this belief is erroneous is unlikely to correct a child’s misconceptions. First, it would require far too much specialized knowledge to access any of the explanations that could be given. Second, such an explanation would be attempting to replace the understanding developed through lived experiences with an understanding developed in rational thought. In the theory of conceptual change, however, there is a mechanism by which such theory replacement can be achieved – the mechanism of cognitive conflict.

Cognitive conflict works on the principle that before a new theory can be adopted, the current theory needs to be rejected. Cognitive conflict is meant to create the impetus to reject the current theory. Therefore, in the aforementioned example, a simple experiment to show that objects of different mass actually fall at the same speed may be enough to prompt a child to reject their current understanding.

The theory of conceptual change is not a theory that applies to learning in general. It is highly situated, applicable only in those instances where misconceptions are formed through lived experiences and in the absence of formal instruction.
“the kind of learning required when the new information to be learned comes in conflict with the learners’ prior knowledge usually acquired on the basis of everyday experiences.” (Vosniadou & Verschaffel, 2004, p. 445)

In such instances, the theory of conceptual change explains the phenomenon of theory rejection followed by theory replacement. The theory of conceptual change, although focusing primarily on cognitive aspects of conceptual change, is equally applicable to meta-conceptual, motivational, affective, and sociocultural factors as well (Liljedahl, 2011; Vosniadou, 2006). In fact, Pehkonen (2006) suggests that the theory of conceptual change could explain the difficulties regarding teachers' change in beliefs. He points to the complex situation of a teacher who possesses new pedagogical knowledge but simultaneously does not change his or her beliefs about teaching. In this paper, we provide a detailed analysis of change in beliefs as conceptual change.

Changes in beliefs as conceptual change

The theory of conceptual change, as presented in the context of science and mathematics education, is equally applicable to some instances of change in preservice teachers' beliefs about mathematics and the teaching and learning of mathematics. In particular, the theory of conceptual change can be used to more closely examine instances of belief rejection and belief replacement. In so doing, we open up the possibility of more closely examining the mechanisms associated with the changes in beliefs presented above.

The theory of conceptual change, as the explanatory framework described above, has three primary criteria for relevance – (a) it is applicable only in those instances where misconceptions are formed through lived experiences and in the absence of formal instruction, (b) there is a phenomenon of theory rejection, and (c) there is a phenomenon of theory replacement. We propose that each of these criteria is equally relevant to instances of replacement of preservice teachers' beliefs about mathematics, as well as beliefs about the teaching and learning of mathematics.

In the context of preservice teachers, the relevant lived experience occurs in their time as students. As learners of mathematics they have both experienced the learning of mathematics and the teaching of mathematics, and these experiences have impacted their beliefs about the teaching and learning of mathematics (Chapman, 2002; Feiman-Nemser et al., 1987; Lortie, 1975; Skott, 2001). The question is – Can these experiences be viewed as having happened outside of the context of formal instruction? Although their experiences as learners of mathematics are situated within the formal instructional setting of a classroom, the object of focus of that instruction is on mathematics content. That is, while content is explicitly dealt with in such a setting, theories of learning, methodologies of teaching, and philosophical ideas about the nature of mathematics are not.

Analysis

As with the initial coding of the data, the three authors independently reanalyzed the data for evidence of the three stages of conceptual change – formation of beliefs through lived experience, belief rejection, and belief replacement – as well as evidence of cognitive conflict. The independent results were then compared, discrepancies were discussed, and pertinent entries were recoded. As before, this process gave us a more nuanced understanding of the framework, which allowed us to more clearly see data through lens of conceptual change.

RESULTS

Through this analysis, we found 25 cases of conceptual change within the journals of the participants. In the following, we present five of these cases organized according to the four themes of the analysis formation of beliefs through lived experience, belief rejection, belief replacement, and cognitive conflict.

Formation of beliefs through lived experience

Some participants are very explicit about the fact that their initial conceptions of what mathematics is and what it means to teach and learn mathematics come from their lived experiences as mathematics students. For example, David articulates that his lived experience as a student of mathematics is now informing his “teacherly” understanding of mathematics as numbers and rules.

“When first pondering the question, “What is mathematics?” I initially thought that mathematics is about numbers and rules. It is something that you just do and will do as long as you follow the rules or principles that were created by some magical man thousands of years ago. That is a struggling student’s point of view. To be honest, I don’t like math. […] I found it so boring and so robotic. Lessons were even set up in a robotic way. The teachers would show us the principles and then we would do the exercises.” (David)

In so doing, David subtly hints that his beliefs about the teaching of mathematics – as robotic – have also been informed by these same experiences. Similarly, Lorena indicates in her final journal entry that her initial beliefs about the teaching of mathematics were also informed by her lived experiences as a student.

“Throughout the journey in this course, my thinking towards mathematics has changed. […] my approach towards math was very teacher-centered because that was all I knew.” (Lorena)

Other participants, on the other hand, are much more subtle about where their beliefs come from.

“Mathematics is a subject area that is most commonly seen as the subject of logic, calculations and one answer. […] It is a language that can be understood by all because the symbols are universal. […] It is an intricate system of numbers, correlations, patterns and values that is used to better understand the world around us.” (Catherine)
Although she does not explicitly state this, Catherine’s articulation of mathematics as a system can only have been acquired during her lived experience as a student. Similarly, Kalpna’s comments that mathematics is about numbers is likely informed by her earlier experiences as a student.

“What I can say about my understanding of mathematics, at this point, is that it is about numbers and how they relate to one another.” (Kalpna)

In her last entry, Kalpna reflects on the origin of beliefs.

“I started with a definition that was short and vague, like my understanding in math.” (Kalpna)

For Kalpna, her lived experience of mathematics had left her with not only a deficit understanding of mathematics, but also an impoverished belief as to what it was or could be.

Belief rejection

Many of the participants discussed the transformational nature of their experience in the course. These discussions were not limited to the new experiences and new insights they gained. They also commented extensively on how their experiences with the course helped them to cast aspersions on their previous experiences as learners of mathematics. For many, these comments were accompanied by terminologies such as definitions that was short and vague, like ‘mathematics is about numbers is.

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Many of the participants discussed the transformational nature of their experience in the course. These discussions were not limited to the new experiences and new insights they gained. They also commented extensively on how their experiences with the course helped them to cast aspersions on their previous experiences as learners of mathematics. For many, these comments were accompanied by terminologies such as definitions that was short and vague, like ‘mathematics is about numbers is.

Belief replacement

To complete the changes to his beliefs, David needs to encounter mathematics and the teaching and learning of mathematics in ways that are sufficiently different from his lived experiences.

“However, after experiencing a couple of challenging problems and exciting classes, I have to say that my definition of mathematics can be summed up very simply. To me, mathematics is not about answers, it’s about process. Mathematics is about exploring, investigating, representing, and explaining problems and solutions.” (David)

The changes to David’s beliefs about mathematics are accompanied by changes to his beliefs about the teaching and learning of mathematics – changes that were similarly held back by his lack of alternative experiences.

“Learning math is about inquiry and the development of strategies. It is about using your intuition, experimenting with strategies and discussing the outcome. It is about risk taking and experimenting. To teach mathematics is to welcome all ideas that are generated and facilitate discussion. It is about letting the students make sense of the math in their own way, not ‘my way’. The teacher’s role is about guiding the process, but handing the problem over to the students.” (David)

Similarly, Kalpna has found a new depth to her understanding of, and beliefs about, mathematics.

“My understanding of math now, is so much deeper and it is not something I want to avoid, but rather deal with head on.” (Kalpna)

In many ways, the data about belief replacement are redundant with the data that were reported earlier in Figs 1–6 and Tables 1 and 2. The new beliefs that participants espouse are evidence of belief replacement. It is a deeper understanding of lived experiences and belief rejection that helps us nuance our understanding of not only the changes that happen, but also what they are changing from and how that change is mediated by a priori belief rejection. As mentioned earlier, however, the process of belief rejection is, itself, mediated by cognitive conflict.

Cognitive conflict

Jordan articulated nicely one of the experiences that provided the cognitive conflict for her to begin to reject her initial beliefs.

“I love the insight you gave regarding ‘You always find something the last place you look.’ This related to teaching was a bit ‘earth shaking’ for me.” (Jordan)
Although seemingly minor, this experience was earth shattering for her. Similarly, Kalpna talks about her experience in the course as being life changing.

“I have developed a totally new way of looking at math. I have had a life changing experience with this class.”  
(Kalpna)

However, in Kalpna’s case, the experience is not so much a well-defined moment, as it is with Jordan, but is more a culmination of the experiences throughout the entirety of the course coupled with the reflective affordances offered through the journal writing.

“Writing this reflective journal on math has been a helpful, meaningful, and valuable experience. I didn’t realize until I began writing how much I had to say about math.”  
(Kalpna)

DISCUSSION

There are two major conclusions that can be drawn from the reanalysis of the data through the theory of conceptual change. The first is that conceptual change is an effective theory for uncovering the blunt awareness that belief change has occurred and allows us to see that any changes in beliefs are preceded by belief rejection – and that any cases of belief rejection are, themselves, preceded by the phenomenon of cognitive conflict.

The more interesting conclusion, however, is that belief rejection, in and of itself, is not sufficient to precipitate a belief replacement. Although a necessary precursor, preservice students must also be afforded alternate experiences to their K-12 learning from which to construct their new beliefs. That is, dissatisfaction with one’s beliefs about mathematics or the teaching and learning of mathematics is not enough to move away. There must also be somewhere to move toward.

CONCLUSIONS

To say that preservice elementary teachers in a mathematics methods courses changed their beliefs is neither helpful nor illuminating. The goal of this paper was to get past that beliefs changed and get to a deeper understanding of exactly what has changed and how these changes were mediated.

In the first part of the paper, we used Törner and Grigutsch’s (1994) three aspects of beliefs – toolbox aspect, system aspect, and process aspect – to more closely document what change looks like. From this we learned that, through their participation in the aforementioned methods course, the preservice teachers shifted their beliefs toward a process understanding of both mathematics and the teaching and learning of mathematics. This shift was more prominent for their beliefs about the teaching and learning of mathematics.

To better understand how these changes occurred, we appropriated the theory of conceptual change from science education and argued that it should apply equally well to instances where a priori beliefs about mathematics and the teaching and learning of mathematics have been developed through preservice teachers’ lived experiences as learners of mathematics. Through a reanalysis of the data, we saw 25 cases of belief change being preceded by belief rejection. We also saw the importance that alternative experiences played in creating cognitive conflict, belief rejection, and belief replacement. This, we believe, explains why the changes toward process beliefs were more prominent for the teaching and learning of mathematics than they were for mathematics. The course, being a methods course, was specifically designed to provide alternative experiences as learners of mathematics as well as explicate alternative teaching methods. The fact that beliefs about mathematics also changed within this context is, we believe, a by-product of the problem-solving context in which experiences of teaching and learning were situated.

Whether or not the theory of conceptual change can be equally well applied to populations of in-service teachers remains to be seen. While preservice teachers’ beliefs emerge from their unexamined experiences as learners of mathematics as well as their informal observations of teaching, in-service teachers’ conceptions may have been more formally shaped by their teacher education experiences as well as their immersion in a field rife with discourse around the teaching and learning of mathematics. The degree to which this still constitutes lived experiences within the context of conceptual change remains to be seen.

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