Comparative study of laser pulses guiding in capillary waveguides and plasma channels at conditions of non-perfect focusing

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Abstract. For producing of high-quality accelerated electron bunches the structure of laser fields and accelerating wakefields inside the guiding structure should be regular enough to conserve a high value of accelerating longitudinal field throughout the propagation and avoid strong defocusing transverse fields. We had compared the efficiency of capillary waveguides and plasma channels in achieving of this goal, taking in mind different possible nonsymmetric conditions (like non-symmetric shape of laser spot, non-zero angle of incidence of a laser pulse or deviation of a focusing point relatively to the guiding structure axis) of laser pulses focusing in a guiding structure, always available in real experiments. The model for laser pulses propagation in guiding structures for the case of arbitrary dissymmetry of laser focusing is presented.

1. Introduction
Obtaining of high-quality accelerated electron bunches [1, 2] is an essential task for high-energy physics [3, 4].

The characteristic effective length of acceleration of electrons in laser wakefields \( L_{\text{acc}} \approx 0.6L_{\text{ph}} \), \( L_{\text{ph}} = \lambda_0 \gamma^3 \) [5–7] considerably exceeds Releigh length \( Z_R = k_0 r_0^2/2 \), where \( \lambda_0 \) is laser wavelength, \( k_0 = \omega_0/c \), \( \omega_0 \) is laser pulse frequency, \( c \) is light velocity, \( \gamma = \omega_0/\omega_p = \sqrt{n_e/n_{e,0}} \) is relativistic gamma-factor of plasma wave, \( \omega_p = \sqrt{4\pi n_{e,0} e^2/m} \) is plasma frequency (and below \( k_p = \omega_p/c \) is plasma wave vector), \( n_{e,0} \) is concentration of background electrons in plasma, \( r_0 \) is characteristic radius of transverse cross-section of laser pulse envelope in vacuum, \( n_c = m \omega_0^2/(4\pi e^2) \).

Therefore for effective acceleration of electron bunches in laser wakefield accelerator one should use some guiding structure like evacuated capillary waveguide with inner radius comparable to laser pulses spot radius, or plasma channel formed in gas jet or gas-filled capillary with much wider inner radius [8–16].

Use of capillary waveguides with inner radius \( R \approx r_0 \) has its own advantages. For example, one can create plasma inside evacuated capillary by ablating of capillary walls due to action of transverse energy flux from laser fields [17]. Capillaries are more convenient for controlling parameters of plasma inside. On the other hand, modern experiments with gas discharge plasma channels had demonstrated possibility to obtain accelerated electron bunches with energies up...
to several GeV in one acceleration cascade [18, 19], thus revealing possible effectiveness of plasma channels.

In order to describe acceleration of electrons in laser wakefields inside capillary waveguides or plasma channels one has to know the squared module of amplitude of transverse laser field \( |a_\perp|^2 \) and its angular harmonics \( |a_\perp|^2 \), which serve as sources of wakefields [15, 20]. Below we propose theoretical approach for calculation of these quantities in both capillary waveguides and plasma channels under conditions close to real experimental ones, when some nonlinearity in laser pulse shape (say, deviation of laser pulse transverse envelope from cylindrically symmetric one) or in conditions of laser pulse focusing into a guiding structure (like focusing point deflection from the guiding structure axis or non-zero angle between laser pulse and guiding structure axes) is present.

Below we consider laser pulses impinging from the left on the entrance of capillary waveguide or plasma channels at \( z = 0 \). The axis 0\( z \) is along guiding structure axis, the guiding structure is at \( z > 0 \). The laser pulse axis can make some non-zero angle \( \theta \) with axis 0\( z \). When describing propagation in capillary waveguides, we treat plasma density and permittivity of plasma inside capillary as constant, \( \varepsilon = 1 - n_{e,0}/n_c = \text{const}, \ |1 - \varepsilon| \ll 1 \). The plasma density inside plasma channels can be described as

\[
n_e(r) = n_0[1 + (r/r_{ch})^2],
\]

where \( n_0 \) is minimum plasma density at the axis and \( r_{ch} \) is the radius of the channel.

In both cases of capillary waveguides and plasma channels the permittivity of plasma is considered to be independent of time. That means that we disregard nonlinear effects of laser pulse action on plasma density, in comparison with the main effect of laser pulse reflection from plasma channel or capillary walls. This is possible for laser pulses with moderate intensities: at least, the laser pulse power \( P \) should be less than the critical power for relativistic self-focusing, \( P < P_{cr} = 0.017\sigma^2 \) TW, or \( I_0 < I_{cr} = 4.3 \times 10^{19} \text{ W/cm}^2(k_p r_0)^{-2}(\lambda_0/1 \text{ \mu m})^{-2} \), where \( I_0 \) is the peak intensity of laser pulse in vacuum at the entrance of the guiding structure.

2. Basic equations

In order to formulate equations describing laser pulses propagation in different guiding structures: capillary waveguides and plasma channels, we start with consideration of laser fields dynamic in capillary tubes filled by uniform plasmas. The equations describing vector structure of laser fields: capillary waveguides and plasma channels, we start with consideration of laser fields dynamic in capillary tubes filled by uniform plasmas. The equations describing vector structure of laser fields:

\[
\begin{align*}
\{ \mathbf{E} \} &= \i \eta E_{\text{max}} \left[ \mathbf{e}_r \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \mathbf{e}_\varphi \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \mathbf{e}_z \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right] \text{div}_\rho Y_{01}, \\
+ \mathbf{E}_{\text{max}} \left[ \mathbf{e}_r \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \mathbf{e}_\varphi \begin{pmatrix} 0 \\ 1 \end{pmatrix} + i \mathbf{e}_z \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \text{div}_\rho Y_{0-1}, \\
\{ \mathbf{E} \} &= \mathbf{E}_{\text{max}} \sum_\sigma \left[ \mathbf{e}_r \begin{pmatrix} 1 \\ i \sigma \end{pmatrix} + \mathbf{e}_\varphi \begin{pmatrix} 0 \\ 1 \end{pmatrix} + i \mathbf{e}_z \sigma \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \left( \text{div}_\rho - \frac{\sigma l}{\rho^2} \right) Y_{l\sigma},
\end{align*}
\]

where \( E_{\text{max}} = \sqrt{8\pi I_0/c} \), \( I_0 \) is the peak intensity of laser radiation in vacuum, \( \eta \) is parameter of ellipticity of laser polarization (\( \eta = 0 \) for linear polarization, \( \eta = \pm 1 \) for circular polarization); dimensionless units are used hereafter: \( \xi = k_0(z - ct) \), \( \zeta = k_0 z \), \( \rho = k_0 r \) where \( r \) is transverse coordinate, \( \varphi \) is azimuthal angle (cylindrical coordinates with axis \( Oz \) along capillary axis are used); \( \text{div}_\rho = \rho^{-1} + \partial_\rho \) (where \( \partial_\rho \equiv \partial/\partial_\rho \)); \( \mathbf{e}_r, \mathbf{e}_\varphi \) and \( \mathbf{e}_z \) are unit vectors; \( \Delta_\rho = \partial^2_\rho + \rho^{-1}\partial_\rho \).
scalar functions $Y_{l\sigma}$ are solutions of the equations

\[
\left[ \Delta_{\rho} - \frac{k^2}{\rho^2} + \varepsilon - 1 + 2i \frac{\partial}{\partial \xi} + 2 \frac{\partial^2}{\partial \xi^2} \right] Y(\rho, \xi, \zeta) = 0, \quad k = \begin{cases} 1, & l = 0, \\ l - \sigma, & l \neq 0 \end{cases}
\]

with boundary conditions, formulated in such a way to ensure proper transverse energy flux into capillary walls and proper attenuation of modes inside the capillary [17, 21]:

\[
i\sqrt{\varepsilon - 1} - \left. \frac{\text{div}_{\rho} Y_{0\pm1}}{Y_{0\pm1}} \right|_{\rho = R} = 0, \quad i\sqrt{\varepsilon - 1} - \left. \frac{(\text{div}_{\rho} - \sigma l / \rho) Y_{l\neq0,\sigma}}{Y_{l\neq0,\sigma}} \right|_{\rho = R} = 0,
\]

where $\varepsilon = \varepsilon(r)$ is the permittivity of the medium inside the capillary or inside capillary walls.

The above equations (3)–(5) are strictly grounded for the capillary with uniform plasma inside, $\varepsilon(r < R) = $ const, under conditions $R^{-1} \ll 1$ and $r < R$. It is reasonable to assume, that the conditions $R^{-1} \ll 1, |\varepsilon - 1| \ll 1$ and $r < R$ are enough for validity of the equations (3)–(5) also for non-uniform plasma inside the capillary. Taking in mind, that for parabolic plasma channel inside wide enough capillary with radius $R \gg r_0$ capillary walls do not influence on the conditions of laser fields propagation, one can assume that equations (3)–(5) are also suitable for approximate (at zero order on the parameter $[k_0 r_0]^{-1}$) description of dynamics of laser fields inside parabolic plasma channels, if one consider these fields at $r \lesssim r_0$.

The boundary conditions for plasma channels can be formulated as

\[
Y_{l\sigma}(r \to \infty) = 0.
\]

From expressions (3) and solutions of equations (4) with boundary conditions (5) and (6) for capillary waveguides and plasma channels, respectively, one can write down the following expressions for angular harmonics of the squared module of laser fields $|a_\perp|^2$ near the axis $r = 0$:

\[
|a_\perp|^2 = a_0^2 \sum_{p=0}^{\infty} \left[ S_{r,p} S_{r,p}^* + S_{c,p} S_{c,p}^* \right], \quad S_{r,p} = \sum_{\sigma = \pm 1} \sum_{n} \tilde{c}_{\sigma,n}(\xi, z) D_n^p(\xi) \quad S_{c,p} = \sum_{\sigma = \pm 1} \sum_{n} \sigma \tilde{c}_{\sigma,n}(\xi, z) D_n^p(\xi),
\]

where

\[
\tilde{c}_{\sigma,n}(\xi, z) = C_{\sigma,n} F_\parallel(\xi + \Phi_{\sigma,n}(z)) \exp(-i\Phi_{\sigma,n}(z))
\]

are modes coefficients dependent on $\xi$ and $z$; $a_0 = eE_{max}/(m\omega_0 c)$; $F_\parallel(\xi)$ is longitudinal envelop of laser pulse before the entrance into a guiding structure; $D_n^p$ are radial functions and $\Phi_{\sigma,n}$ are phases.

For capillary waveguides with the permittivity of the walls $\varepsilon(r \geq R) = \varepsilon_w$, filled by uniform plasma with permittivity $\varepsilon(r < R) = \varepsilon_c$, one have the following expressions:

\[
D_n^l = J_l(u_{l,n}r / R),
\]

\[
\Phi_{l\sigma n} = (z/2)(k_{l\sigma n}^2 + 1 - \varepsilon_c),
\]

\[
k_{l\sigma n} = \frac{u_{l,n}}{R} \left( 1 - i \mu_B / R \right), \quad k_{l\sigma n} = \frac{u_{l,n}}{R} \left( 1 - i \mu_E / R \right), \quad k_{l\sigma n} = \frac{u_{l,n}}{R} \left( 1 - i \mu_+ / R \right),
\]

where $J_l$ are Bessel functions of the 1-st kind of the $l$-th order and $u_{l,n}$ are their $n$-th roots; $k_{l\sigma n}$ are modes transverse wave vectors,

\[
\mu_B = \frac{1}{\sqrt{\varepsilon_w - 1}}, \quad \mu_E = \varepsilon_w \mu_B, \quad \mu_+ = \mu_B + \mu_E.
\]
are factors dependent on capillary wall properties.

For plasma channels, one has the following expressions for radial functions \( D_n^l \) and phase factors \( \Phi_{l\sigma n} \):

\[
D_n^l = L_n^l(\kappa) e^{-\kappa^2/2} \kappa^l/2, \quad \kappa \equiv k_p r^2 / r_{ch},
\]

where \( L_n^l \) are general Laguerre polynomials:

\[
\Phi_{l\sigma n} = \frac{z n_0}{2 n_e} \left[ 1 + \frac{2n + l + 1}{k_p r_{ch}} \right].
\]

The expressions for modes coefficients \( C_{l\sigma n} \) can be written in the following forms.

- For capillaries:

\[
C_{0\sigma n} = -N_{n-1\sigma} \sum_{k=\pm 1} \frac{1+k_0}{2\sqrt{1+\eta^2}} \left[ k - \frac{1 - \eta + \frac{1}{2}}{2} \right] Z_{k,n},
\]

\[
C_{l\sigma n} = N^{l\sigma}_{n-1\sigma} \frac{1+\eta}{2\sqrt{1+\eta^2}} Z_{l\sigma n}, \quad N_{k,n} = \int_0^1 y^2 J_k(u_{k,n} y)^2 dy,
\]

\[
Z_{k,n} = \int_0^1 y F_k(y) J_k(u_{k,n} y) dy, \quad F_k(y) = (2\pi)^{-1} \int_0^{2\pi} e^{-ik_\varphi} F_\varphi (y R, \varphi) d\varphi,
\]

where \( F_\varphi (r, \phi) \) is laser pulse transverse envelop at \( z = 0 \).

- For plasma channels:

\[
C_{l\sigma n} = N^{l\sigma}_{n-1\sigma} \frac{1+\eta}{2\sqrt{1+\eta^2}} Z_{l\sigma n}, \quad N_{k,n} = \int_0^\infty e^{-y} y^{k|l|} \left[ L_n^{|l|}(y) \right]^2 dy,
\]

\[
Z_{k,n} = 2^{k|l|/2+1} \int_0^\infty F_k \left( \sqrt{2y/(k_p r_{ch})} \right) e^{-y} y^{k|l|/2} L_n^{|l|}(2y) dy.
\]

3. Simulation results

In figure 1, the dependencies of maximum (on \( \xi = z - ct \)) amplitude \( |a_\perp| = |e| E_\perp |/(m\omega_0 c) \) (where \( E_\perp \) is transverse component of electric field) and maximum on \( \xi \) amplitude of wakefield \( \Phi \) (calculated similarly as in papers [15, 20]) on laser pulse propagation distance are shown for the case of laser pulse with full width at half of maximum intensity \( \tau_{FHM} = 56 \) fs, characteristic radius \( r_0 = 50 \mu m \) (\( k_p r_0 = 4.96 \)) and \( \lambda_0 = 0.8 \mu m \), dimensionless amplitude \( a_0 = e|E_\perp|/(m\omega_0 c) = 0.5 \) and maximum power \( P_0/P_{cr} = 0.14 \), propagating in silicon capillary waveguide with inner radius \( R = 82 \mu m \), filled by uniform plasma with \( \gamma = k_p / k_0 = 80 \), or propagating in plasma channel (1) with \( \gamma_0 = \omega_p / \omega_0 = 80 \) (where \( \omega_p = \sqrt{4\pi n_0 e^2 / m} \)) and \( r_{ch} = 1; 1.2; 0.8 \times R_{fit} \), where \( R_{fit} = Z_R / \gamma_0 \) is the matched (or fitted) radius of plasma channel, i.e. the radius, at which in the considered linear regime (without backward influence of laser pulse on the plasma channel) the laser pulse does not change its form while propagating through the channel (see curves by open cycle markers at figure 1); the length \( z \) is normalized by the dephasing length \( L_{ph} = \lambda_0 \gamma_0^{-3} = 41 \) cm.

It is seen from figure 1 that in all cases (perfect focusing and nonzero angle of incidence \( \theta \) and even for non-matched plasma channels, i.e. for channels with \( r_{ch} \neq R_{fit} \)) both \( |a_\perp| \) and \( \Phi \) have higher maximum values and undergo more smooth oscillations with \( z \), in comparison with the case of laser propagation inside capillary waveguide. This is stipulated by the more smooth transverse boundary conditions in the case of plasma channels.

In figure 2, 2D distributions (over longitudinal \( \xi \) and transverse \( x = r \cos(\varphi) \)) coordinates) of laser field amplitude \( |a_\perp|^2 \) and longitudinal and transverse forces \( F_x \) and \( F_r \), which are respective gradients of the wakefield potential [15, 20] are shown for the case of nonzero \( \theta = 1.2 \) mrad and \( \theta = 2.4 \) mrad for capillary waveguides and plasma channels, respectively.
Figure 1. Maximum on ξ amplitude of laser field |a| and wakefield potential Φ as functions of capillary or plasma channel depth z, for different cases of laser pulse focusing (depicted at the legend). Parameters of calculations are described in the text.

One can see, that laser pulse amplitude and longitudinal and transverse forces in the case of plasma channel undergo similar transverse oscillations, as in capillary waveguides, but for angles θ that are about twice of those for capillary waveguides. Besides that, the transverse smearing of laser pulse spot is lower for plasma channels. For the same angles θ transverse oscillations of laser and wakefields are much higher in the case of capillary waveguides, in comparison with plasma channels. Similar conclusions follow from calculations with zero θ, but non-zero displacements δr of laser radiation focusing point relatively to the axis 0z: the transverse oscillations arising due to such displacement are about factor of 2 higher in the case of capillary waveguides, in comparison with plasma channels.

4. Conclusions
We had proposed a simple model for laser pulses propagation in capillary waveguides and plasma channels, which make it possible to study the propagation of moderately intense laser pulses in capillary waveguides and parabolic plasma channels with account of arbitrary dissymmetry of laser pulse focusing into the guiding structure and dissymmetry in the laser pulse shape. This model can be used to study wakefields excitation behind laser pulses, propagating in different guiding structures, under conditions close to experimental ones, and acceleration of electron bunches in those wakefields. This will be the subject of our further activity.

The results of simulations have shown, that despite the fact that plasma channels are more difficult to utilize in experiment and to control their parameters (in comparison with capillary waveguides), plasma channels permit one to guide laser pulses with the conservation of higher maximum amplitude and with more smooth oscillations of laser pulse and wakefield envelopes at the axis of a guiding structure. Moreover, the transverse oscillations of the laser pulse and wakefields are also less pronounced in the case of parabolic plasma channels.
From this one can suppose, that with the use of matched plasma channels one can obtain accelerated electron bunches with better quality: with at least higher energy of accelerated electrons and higher number of trapped and accelerated particles at the same tolerance to the accuracy of laser pulse focusing.

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