Stable Wireless Network Control Under Service Constraints

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Abstract

We consider the design of wireless queueing network control policies with particular focus on combining stability with additional application-dependent requirements. Thereby, we consequently pursue a cost function based approach that provides the flexibility to incorporate constraints and requirements of particular services or applications. As typical examples of such requirements, we consider the reduction of buffer underflows in case of streaming traffic, and energy efficiency in networks of battery powered nodes. Compared to the classical throughput optimal control problem, such requirements significantly complicate the control problem. We provide easily verifyable theoretical conditions for stability, and, additionally, compare various candidate cost functions applied to wireless networks with streaming media traffic. Moreover, we demonstrate how the framework can be applied to the problem of energy efficient routing, and we demonstrate the application of our framework in cross-layer control problems for wireless multihop networks, using an advanced power control scheme for interference mitigation, based on successive convex approximation. In all scenarios, the performance of our control framework is evaluated using extensive numerical simulations.

I. INTRODUCTION

Since the seminal work of Tassiulas and Ephremides [1], throughput optimal control of stochastic networks by MaxWeight-type policies has gained considerable attention. However, soon it was discovered that MaxWeight can lead to significant delays [2], which may render the practical application of such control strategies impossible. As a result, significant efforts followed in order to reduce the delay of MaxWeight-type policies [3], [4], [5].

Notably, delay is not the only relevant performance measure. For example, in networks with size-limited receive buffers, a policy that only considers delay can be even harmful, since it might be better to absorb traffic peaks at intermediate buffers instead of routing packets as fast as possible towards the end user. Moreover, when the offered performance already meets application targets, it is unnecessary to invest further in incremental improvements of particular performance measures. Consequently, a sophisticated control approach should be able to flexibly cope with specific, possibly time-varying, requirements that are dictated by the actual applications (streaming services, gaming, etc.) [6]. As an example, consider multimedia streaming traffic, which constitutes a large fraction of the traffic observed in wireless networks nowadays. Here, we have to ensure that application buffers do not drain in order to avoid interruptions of the stream. This results in minimum buffer size constraints. On the other hand, it should be avoided that buffers grow too large, since (1) buffers could be limited in size and (2) the user might want to switch the video stream, which implies a waste of already invested network resources.

Recently in [7], we introduced a control framework called $\mu$-MaxWeight which is specifically suited to address such policy design challenges in a joint fashion. In general, our approach not only guarantees stability but it additionally allows to incorporate certain application requirements derived from an underlying cost function. Moreover, it can flexibly react to changing user requirements by applying different cost functions. Based on this framework, we approach the design of wireless networks and corresponding control policies from a service centric –or application centric– point of view. This may also imply a change of viewpoint: from stabilizing transmit buffers to controlling the application buffers at the user side. We assume that the service layer constraints can be expressed as requirements on buffer states, i.e., applications may require maximum and minimum buffer states (which, however, may differ for different network entities). While the first type of constraints is well investigated, the second type bears serious challenges, especially in multihop networks.

A. Contributions

We provide general sufficient conditions on the stability of a $\mu$-MaxWeight based control policy, and we prove two simpler stability results that are based on additional assumptions on the network model. Moreover, we discuss several exemplary fields of application for our control framework.
In the first application, we show that with a sophisticated cost function choice, buffer sizes of particular queues can be steered towards beneficial operating points. The choice of the underlying cost function is crucial for the performance of the resulting control policy. Therefore, to numerically compare different cost functions, we define the notion of queue outage, which both incorporates buffer overflows to capture effects of high packet holding costs or even dropped packets as well as buffer underflows to capture service interruptions.

In the second application, we use our framework to design a control policy for an energy constrained wireless network, where traffic is routed via battery limited intermediate nodes. We assume that when a node has no traffic in each of its queues, it can switch to an energy-saving mode. Therefore, the goal is to maximize the “down time” of as many nodes as possible, while guaranteeing stability of the network.

Eventually, in the third application, we consider cross-layer optimization of wireless networks, and we combine network-layer control with physical layer interference mitigation. Thereby, we demonstrate how our framework can be combined with physical layer power control based on successive convex approximation (SCA).

B. Related Work

After the seminal work in [1], significant research activities followed. Many focussed on delay reduction of backpressure based policies, such as [3]. A general class of throughput optimal policies with improved delay performance has been recently presented in [5], and a general survey on recent policy synthesis techniques can be found, for example, in [8].

Considering queueing networks, previous works consider underflow constraints mainly in broadcast or simpler networks [9]. For example, in [9], the problem is investigated in a network of multiple transmitter-receiver pairs with cooperating transmitters. However, since it requires that buffer levels for each user progress independently of other users, it cannot be easily generalized to the case of arbitrary (multihop) networks (not even to a simple tandem network).

Recently, there has been some interest in queueing network control with an arbitrary underlying cost-metric [10], allowing to incorporate application-dependent constraints in the control policy. However, stability and also cost performance crucially depend on the parameter choice, and this parameter choice has to be found in simulations.

The MaxWeight policy is also frequently applied in wireless cross-layer control problems, in particular, in joint routing and power control problems (see, e.g., [11]). A common assumption on the physical layer is a fully coupled wireless network, where nodes treat interference as noise. In this case, power control results in a weighted sum-rate maximization problem which is known to be NP-hard [12]. In [13], an SCA method was introduced as a means to treat the inherent non-convexity. Compared to traditional power control in wireless networks [14], it eliminates the need to use the common unrealistic high-SINR approximation together with a log-transform of variables (see, e.g., [15]). In [16], a cross-layer control formulation was used based on the MaxWeight technique.

C. Organization

The rest of the paper is organized as follows. The system model is described in Section II and important preliminaries are summarized in Section III. The $\mu$-MaxWeight policy, together with important stability results, is introduced in Section IV. In Section V we treat the important issue of cost-function choice, and we show, how an appropriate choice can be used to steer buffers towards certain target values. In Section VI we apply our framework in different scenarios, such as multimedia traffic, energy constrained nodes, and cross-layer network control. In Section VII we draw some relevant conclusions.

D. Notation

We use boldface letters to denote vectors as well as matrices, and common letters with subscript are the elements, such that $A_i$ is the $i$-th element of vector $A$ and $B_{ij}$ is the element in row $i$ and column $j$ of matrix $B$. Moreover, $A^T$ refers to the transpose of $A$. $E\{X\}$ denotes the expected value of random variable $X$. Let $I$ denote the identity matrix of appropriate dimension. Furthermore, we denote $\mathbf{1}$ the vector of all ones. $\text{diag}(a_1, a_2, \ldots)$ refers to a diagonal matrix built from the elements $a_1, a_2, \ldots$. $\| \cdot \|_i$ denotes the $l_i$ vector norm, and $\| x \|$ is an arbitrary norm. Furthermore, we use $A^c$ to denote the complement of a set $A$. The probability of $A$ is denoted as $\Pr\{A\}$. $\langle \cdot, \cdot \rangle$ denotes the scalar product of two vectors. The indicator $\mathbb{I}\{ \cdot \}$ takes the value 1 if the argument is true, and it takes the value 0 otherwise.
II. System Model

In order to model queueing networks, we use a simple, discrete time, stochastic network model, called Controlled Random Walk (CRW) model. We consider a queueing network with \( m \) queues in total that represent \( m \) physical buffers with unlimited storage capacity. We arrange the queue backlog in the vector \( Q \), such that \( Q = [Q_1, \ldots, Q_m]^T \), which we refer to as the queue state. Let \( M \) be the set of queue indices. Suppose that the evolution of the queueing system is time slotted with \( t \in \mathbb{N}_0 \). Then, our model is defined by the queueing law:

\[
Q(t+1) = [Q(t) + B(t+1) U(t)]^+ + A(t+1),
\]

where \([x]^+ := \max\{0, x\}\). The vector process \( A(t) \in \mathbb{N}_0^m \) (vector of arrival rates in packets per slot) is the (exogenous) influx to the queueing system with mean \( \alpha \in \mathbb{R}_+^m \) and \( B(t) \in \mathbb{Z}_0^{m \times l} \) is a matrix process with average \( B \in \mathbb{Z}_0^{m \times l} \), where \( l \in \mathbb{Z}_+ \) is the number of control decisions to be made (that is, the number of links in the network). The matrix \( B(t) \) contains both information about the network topology (that is, about connectivity or possible routing paths in between queues) and service rates along the specific links. We assume that the control vector \( u = U(t) \) in slot \( t \) is an element of the set \( \{0,1\}^l \). Moreover, we can impose further (linear) constraints on the control using the binary constituency matrix \( C \in \mathbb{Z}_0^{l \times l} \) (with \( l_m > 0 \) being the number of constraints), where each row of \( C \) corresponds to a particular controllable network resource. More precisely, we require that \( C u \leq 1 \), such that \( u(t) \in \{u \in \{0,1\}^l : C u \leq 1\} \). For the sake of notational simplicity, we omit the time index in the following if possible. Throughout the entire paper, \( x \in \mathbb{N}_0^m \) denotes the instantaneous backlog.

In the following, the queueing system \([1]\) is assumed to be a \( \delta_0 \)-irreducible Markov chain (with \( \delta_0 \) being the point measure at \( x = 0 \)).

A. Stability

Throughout this paper we use the following definition of stability.

**Definition 1.** A Markov chain is called \( f \)-stable, if there is an unbounded function \( f : \mathbb{R}_+^m \to \mathbb{R}_+ \), such that for any \( 0 < B < +\infty \) the set \( \mathcal{B} := \{x : f(x) \leq B\} \) is compact, and furthermore it holds

\[
\limsup_{t \to +\infty} \mathbb{E}\{f(Q(t))\} < +\infty. \tag{2}
\]

In Definition \([1]\) the function \( f \) is unbounded in all positive directions so that \( f(x) \to \infty \) if \( \|x\| \to \infty \), regardless of the actual path taken in \( \mathbb{R}_+^m \). Choosing directly \( f(x) = \|x\| \), Definition \([1]\) is equivalent to the definition of strongly stable \([17]\), which implies weak stability. Clearly, for any \( f(x) \) that grows faster than \( \|x\| \), inequality \([2]\) implies that the Markov chain is strongly stable.

Using \( f \)-stability, we can define our notion of throughput optimality. For this we will call a vector of arrival rates \( \alpha \in \mathbb{R}_+^m \) stabilizable, when the corresponding queueing system, driven by some specific scheduling policy, is \( f \)-stable.

**Definition 2.** A scheduling policy is called throughput-optimal if it keeps the Markov chain \( f \)-stable for any vector of arrival rates \( \alpha \) which is stabilizable by some policy.

III. Preliminaries

Let us introduce a cost function

\[
c : \mathbb{N}_0^m \to \mathbb{R}_+, x \mapsto c(x),
\]

assigning any queue state a non-negative number. Typically, the goal is to minimize the average cost over a given finite or infinite time period, or some discounted cost criterion. The optimal solution to the resulting problems (which in discrete time can be modeled as a Markov Decision Problem) can be found by dynamic programming, which is, however, infeasible for large networks. A simple approach to queueing network control is the myopic or greedy policy. Such a policy selects the control decision that minimizes the expected cost only for the next time slot.

\footnote{1 in the numerical evaluations in this paper, we assume Poisson distributed arrival processes}
In [10], a cost function based policy design framework called $h$-MaxWeight is introduced, which is a generalization of the MaxWeight policy. Meyn considers a slightly different definition of the CRW model, which is characterized by

$$Q(t+1) = Q(t) + B(t+1)U(t) + A(t+1).$$

(3)

The control vector $U(t) \in \mathbb{R}^n_0$ is an element of the region $U^*(x) := U(x) \cap \{0, 1\}^l$, with

$$U(x) := \{ u \in \mathbb{R}^l_+ : Cu \leq 1, |Bu + \alpha|_i \geq 0 \text{ for } x_i = 0 \}.$$

A $h$-MaxWeight policy based on [10] chooses the control vector according to

$$\arg\min_{u \in U^*(x)} \langle \nabla h(x), Bu + \alpha \rangle.$$

(4)

Thus, the policy is myopic with respect to the gradient of some perturbation $h$ of the underlying cost function. Meyn develops two main constraints on the function $h$. The first constraint requires the partial derivative of $h$ to vanish when queues become empty, i.e.,

$$\frac{\partial h}{\partial x_i}(x) = 0, \text{ if } x_i = 0.$$

(5)

Moreover, the dynamic programming inequality $\min_{u \in U(x)} \langle \nabla h(x), Bu + \alpha \rangle \leq -c(x)$ has to hold for the function $h$. When $h$ is non-quadratic, the derivative condition (5) is not always fulfilled. Therefore a perturbation technique is used, where $h(x) = h_0(\tilde{x})$, hence it is a perturbation of a function $h_0$. Two perturbations are proposed: an exponential perturbation with $\theta \geq 1$, given by

$$\tilde{x}_i := x_i + \theta \left( e^{\frac{\theta}{x_i}} - 1 \right),$$

and a logarithmic perturbation with $\theta > 0$, defined as

$$\tilde{x}_i := x_i \log \left( 1 + \frac{x_i}{\theta} \right).$$

(6)

While the first approach shows better performance in simulations, the stability of the resulting policy depends on the parameter $\theta$ being sufficiently large (determined by the considered network setting). This is overcome by the second perturbation, which is stabilizing for each feasible $\theta$. However, it comes with the additional constraint

$$\frac{\partial h_0}{\partial x_i}(x) \geq cx_i, \text{ } \forall i \in M,$$

(7)

which is a significant limitation on the space of functions that can be chosen as $h_0$ (apart from the additional condition that $\nabla h_0$ has to be Lipschitz-continuous). Moreover, we would like to emphasize that, beyond these specific two examples, there is no general framework that can be applied to explore the stability properties of other choices of perturbations. This is now provided with the $\mu$-MaxWeight framework.

IV. $\mu$-MAXWEIGHT

In what follows, we consider and investigate scheduling policies of the form

$$u^*(x) = \arg\min_{u \in \mathbb{R}^n_+: Cu \leq 1} \langle \mu(x), Bu + \alpha \rangle,$$

(8)

where $\mu : \mathbb{R}^m_+ \to \mathbb{R}^m_+$ is a vector valued function, and $\mu(x)$ is called the weight vector for an instantaneous queue state $x \in \mathbb{R}^m_+$. Note that $\mu$ is reminiscent of a vector field, and it can thus be interpreted as a scheduling field for which we subsequently present a stability characterization. By construction of the policy, we can, without loss of generality, normalize the weight vector as

$$\bar{\mu}(x) := \frac{\mu(x)}{||\mu(x)||_1},$$

and hence $||\bar{\mu}(x)||_1 = 1$. Furthermore, we assume that the resulting policy is non-idling, i.e., $||\mu(x)||_1 = 0$ if and only if $x = 0$. Below, we provide generalized sufficient conditions for throughput optimality of the systems (1) and (3) under control.
Theorem 1. Consider the queueing system (1), driven by the control policy (8) with some scheduling field \( \mu \). The policy is throughput optimal (with respect to the Definition 2) if the corresponding normalized scheduling field given in (9) fulfills the following conditions:

(A1) Given any \( 0 < \epsilon_1 < 1 \) and \( C_1 > 0 \), there is some \( B_1 > 0 \), so that for any \( \Delta x \in \mathbb{R}_+^m \) with \( \| \Delta x \| < C_1 \), we have 
\[
\left| \frac{\partial h}{\partial x} (\bar{x} + \Delta x) - \frac{\partial h}{\partial x} (\bar{x}) \right| \leq \epsilon_1, \text{ for any } x \in \mathbb{R}_+^m \text{ with } \| x \| > B_1, \text{ and } \forall i \in M.
\]

(A2) Given any \( 0 < \epsilon_2 < 1 \) and \( C_2 > 0 \), there is some \( B_2 > 0 \), so that for any \( x \in \mathbb{R}_+^m \) with \( \| x \| > B_2 \) and \( x_i < C_2 \), we have 
\[
\frac{\partial h}{\partial x} (x) \leq \epsilon_2, \text{ for any } i \in M.
\]

Moreover, for any stabilizable arrival process, the queueing system is f-stable under the given policy according to Definition 4. The exact formulation of f depends on the field \( \bar{\mu}(x) \).

Proof. The proof is provided in full detail in [18].

These conditions can be significantly simplified, assuming that \( B_1 \) is sufficient for stability.

Theorem 2. Consider the queueing system (3) driven by the control policy (4) with some cost function \( h \). Suppose the corresponding scheduling field \( \mu(x) := \nabla h(x) \) is continuously differentiable, and Condition (10) on the network topology \( \{B(\cdot)\} \) holds. Then, the following conditions are sufficient for throughput optimality:

(C1) For any \( \epsilon > 0 \), there is some \( C_1^* > 0 \), so that for all \( \| x \| \geq C_1^* \) holds \( \| \nabla \log(\mu_i(x)) \|_1 \leq \epsilon \) or the weaker condition (i.e. it covers more policies) \( \| \nabla \mu_i(x) \|_1 \leq \epsilon \| \mu(x) \|_1 \) for all \( i \in M \).

(C2) If \( x_i = 0 \), then \( \mu_i(x) = 0 \), \( \forall i \in M \).

Proof. The proof can be found in Appendix A.

Theorem 3. Suppose, everything is as in Theorem 2. Let the scheduling field be defined as \( \mu(x) := \nabla h_0(\bar{x}) \) for some given simple perturbation \( \bar{x} \). Then, for some \( \epsilon > 0 \),

(D1) \( \frac{\partial \bar{h}_0}{\partial x_i} \) is Lipschitz, and \( \frac{\partial \bar{h}_0}{\partial x_i} \to \infty, x_i \to \infty \),

(D2) \( \frac{\partial \bar{h}_0}{\partial x_i} \) is Lipschitz, and \( \frac{\partial \bar{h}_0}{\partial x_i} (\bar{x}) \geq \left( \frac{\partial \bar{h}_0}{\partial x_i}(\bar{x}) \right)^{1+\epsilon}, x_i \to \infty \), is sufficient for stability.

Proof. The proof can be found in Appendix B.

Remark 1. (On necessary conditions) For the weaker part in C1) in Theorem 2 it was shown in [19], in a wireless broadcast setting, that the conditions are also necessary if the boundary of the feasible (rate) region contains differentiable parts, i.e. parts where the hyperplanes defined through the scheduling field are uniquely supported.

It is important to note that the conditions in Theorem 2 and Theorem 3 cover indeed a larger class of throughput optimal policies compared to the perturbation in (6) as follows: Observe that for (6) we have \( \frac{\partial \bar{h}_0}{\partial x_i} = \log \left( 1 + \frac{x_i}{\theta} \right) + \frac{x_i}{\theta+x_i} \), which is indeed 1) Lipschitz and 2) it holds \( \frac{\partial \bar{h}_0}{\partial x_i} \to \infty, x_i \to \infty \). Hence, we have from D2) in Theorem 2 that
\[
\frac{\partial \bar{h}_0}{\partial x_i} (\bar{x}) \geq \log^{1+\epsilon}(x_i)
\]
is required in each component which is much weaker that Meyn’s condition (7) (note: both approaches require Lipschitz-continuity of \( h_0 \)).
A. Reducing Complexity by Pick and Compare

Usually, MaxWeight-type policies and corresponding algorithms have a high complexity, and this also applies to a policy based on the $\mu$-MaxWeight framework. As the number of control vectors grows exponentially in $l$, a large computational burden arises from (8), which is carried out in every time slot. Popular approaches to tackle the complexity issue include randomized pick-and-compare based methods [20], which reduce complexity at the expense of higher delay, and Greedy/Maximal scheduling [21], which has good delay performance but achieves only a fraction of the throughput region (i.e., the set of all arrival rate vectors for which the network is stabilizable).

In this paper, we apply a randomized version of $\mu$-MaxWeight, based on the pick-and-compare approach, which preserves throughput optimality [20][22]. Tailored to $\mu$-MaxWeight, the approach can be summarized as follows: We apply $u(0) \in U^*$ arbitrarily. Afterwards, in each timeslot $t > 0$, we first pick a control $\hat{u} \in U^*$ randomly. Second, we let $u(t) = \hat{u}$, if $\langle \mu(x), B\hat{u} + \alpha \rangle < \langle \mu(x), Bu(t-1) + \alpha \rangle$, and otherwise, we choose $u(t) = u(t-1)$. This algorithm is throughput optimality as long as $\Pr (\hat{u} = u^*) \geq \delta$, for $\delta > 0$ [20] (which is trivially satisfied). The reduced complexity, however, comes at the expense of a higher convergence time. Yet, a tradeoff can be achieved by repeatedly applying the pick and compare steps in every particular timeslot.

Note that the randomized algorithm was used as a basis for decentralized throughput optimal control policies [22][23].

V. Cost-Function Choice

A vital design choice in the $\mu$-MaxWeight framework is the underlying cost-function. Different applications have different requirements and consequently require different cost functions. Subsequently, we will consider both minimum and maximum buffer state constraints.

A straightforward choice is the $l_1$-norm, i.e., a linear cost function of the form
\[
c(Q) = \sum_i c_i Q_i.
\]
(11)

It is used to minimize the total buffer occupancy, which is corresponding to the overall end-to-end delay. However, this cost function is unsuitable to avoid buffer underflows, since it does not penalize buffer states below the target level.

Assume we want to find a cost function that steers the particular buffer levels towards a target buffer state $\tilde{Q}$. A simple cost function choice that penalizes deviations from target buffer state $\tilde{Q}$ in both directions is the shifted quadratic cost function
\[
c(Q) = \sum_i c_i (Q_i - \tilde{Q})^2.
\]
(12)

However, (12) naively treats all buffers in the network equally, although, most likely, only application buffers have minimum state constraints.

Therefore, we combine (11) and (12), such that only application buffers have quadratic cost terms. Thereby, we obtain the composite cost function, given by
\[
c(Q) = \sum_{i \in I_u} c_i (Q_i - \tilde{Q})^2 + \sum_{j \notin I_u} c_j Q_j,
\]
(13)

where $\tilde{Q}$ denotes the desired target level for application buffers, and $I_u$ is the set of application buffer indices. For simplicity, we assume all application buffers have the same target buffer level.

A. Policy Design

Having chosen an appropriate cost function, we now show how to construct a corresponding weight function $\mu$ in (8). In order to guarantee stability, the weight function needs to fulfill the stability conditions of Theorem 1. For this purpose, we employ a perturbation technique (cf. [10]). In [7], we pointed out that a simple way to construct a weight function is
\[
\mu(x) = \nabla_0 h_0(x),
\]
(14)
where a perturbation matrix $P_\theta(x) := \text{diag}\left(1 - \exp\left(-\frac{x_i}{\theta(1 + \sum_{j \neq i} x_j)}\right)\right)$ is used. It is based on the following perturbation of variables:

$$\tilde{x}_i := x_i + \exp\left(-\frac{x_i}{\theta(1 + \sum_{j \neq i} x_j)}\right).$$

It can be easily verified that the conditions from Theorem 2 hold for our cost function choice (13) which results in the policy

$$\mu_i(x) = 2c_i(x_i - \bar{q})\left(1 - \exp\left(-\frac{x_i}{\theta(1 + \sum_{j \neq i} x_j)}\right)\right).$$

The gradient is given by $g(x) := \exp(-x_i/\theta(1 + \sum_{j \neq i} x_j))$

$$[\nabla \mu_i(x)]_k = \begin{cases} 2c_i g(x) + \frac{2c_i(x_i - q)g(x)}{\theta(1 + \sum_{j \neq i} x_j)} & k = i \\ -2c_i(x_i - q)g(x) \frac{1}{\theta(1 + \sum_{j \neq i} x_j)^2} & \text{else} \end{cases}$$

and we can see that it contains functions like $ye^{-y}$ or $y^2e^{-y}$, $y > 0$, where $y(x) := x_i/\theta(1 + \sum_{j \neq i} x_j)$ which at best just cannot decrease for any path $y(x)$ in the $m$-dimensional positive orthant. On the other hand, we have from (16) that $\|\mu(x)\|_1$ grows at least linear in any of the $x_i$ so that we have proved $\|\nabla \mu_i(x)\|_1 \leq \epsilon\|\mu(x)\|_1$ which is exactly the condition of throughput optimality in Theorem 2.

As we will demonstrate now, the choice of the underlying cost function is crucial for the performance of the resulting control policy. However, it should be pointed out that, in network design, it is important to keep the practical limitations of a control framework in mind. For example, although, owing to the cost-funtion based approach, the end-to-end delays can be better controlled than in classical MaxWeight, hard delay guarantees cannot be given. This may limit the application of the control framework for certain delay-critical network applications (e.g., in the context of transport networks with so-called mission-critical communication traffic).

**B. Example: Controlling a network of queues in tandem**

Let us first consider a very simple network, known as tandem queue, comprising a number of $m$ buffers in series. We assume traffic arrives at the first buffer with mean rate $\alpha$, and the application removes traffic from the $m$th buffer at a constant rate $R_a$ (this models, e.g., a streaming service). The output of buffers 1 through $m - 1$ can be regulated by the control policy. While tandem queue networks have been thoroughly investigated (see, e.g., [8]), we have two additional difficulties here. First, we have no explicit control over the rate at which data is extracted from application buffer $Q_m$. Second, in addition to stability, we require also a minimum buffer state at the application buffer.

Consider now the most simple network of $m = 2$. Here, the only remaining control decision is whether to send traffic from $Q_1$ to $Q_2$, or not. Figure 4 shows, how we can steer the buffer state towards a target buffer level, using an appropriate cost function. It depicts the queueing trajectories of the two queues in the tandem network, i.e., the buffer sizes over time, (exponentially averaged over a window of 100 time slots to increase readability) for 20000 time slots using two different cost functions. The dashed lines represent policies based on the simple linear cost function in (11), which, obviously, does not stop the second buffer from growing. By contrast, the solid lines represent the composite cost function (13), which stops $Q_2$ from sending further traffic to $Q_2$ when the latter reaches a certain level. Instead, the excessive traffic is buffered at $Q_1$, since it generates lower costs.

**VI. Applications**

In the following, we discuss several exemplary fields of application for the control framework:

1. the control of a multimedia network carrying streaming traffic, (2) the design of an energy efficient policy for energy constrained networks, and (3) the cross-layer control of wireless networks for interference management.

**A. Application I: Control of multimedia networks**

As a first practical application for our framework, we evaluate the $\mu$-MaxWeight approach in a network designed for entertainment purposes. Therefore, the network is expected to carry mostly streaming traffic (and can, for example, be used to...
model a wireless entertainment system in an aircraft cabin, as in [24]). Figure 2 depicts the considered network schematically. We have several wireless access points (depicted as smaller rectangles in Figure 2), each serving a certain number of user terminals in its area (depicted as circles in Figure 2). The user terminals are assumed to run streaming-based applications. Some terminals, which are located in between several access points, can potentially be served by more than one access point. The access points are connected by a wired backbone network to a central application server (depicted as large rectangle in Figure 2). The server itself is connected to the Internet, thus, traffic for each user arrives in a random fashion. Each component in the system has a number of queues with different requirements.

To account for the anticipated multimedia applications, we define the notion of queue outage as a measure of a control policy’s performance (in addition to the average cost performance metric). We denote the total number of buffer underflow events and buffer overflow events up to time $T$ as $F_{i}^{\text{min}}(T)$ and the $F_{i}^{\text{max}}(T)$, respectively. The total sum of buffer outages is consequently $\bar{F}_{i}^{\text{out}}(T) = \sum_{i \in I_u} (F_{i}^{\text{min}}(T) + F_{i}^{\text{max}}(T))$, and the relative frequency of queue outage events is defined as

$$\bar{P}_{i}^{\text{out}}(T) = \frac{1}{T \cdot |I_u|} \bar{F}_{i}^{\text{out}}(T).$$  \hspace{1cm} (17)$$

Assume, we want to keep the buffer states between a minimum buffer state $Q^{(1)}$ and a maximum buffer state $Q^{(2)}$, which
can be flexibly adopted to the requirements of the desired application. Then, a reasonable choice for the target buffer state is \( \tilde{Q} = \frac{1}{2} (Q^{(1)} + Q^{(2)}) \).

Usually, queueing network models, such as the CRW model, assume static links. However, we require wireless links between access points and terminals. For this, we model the wireless link capacities by applying a result from [25], which determines the mutual information distribution of a multi-antenna OFDM-based wireless system. To obtain rate expressions, we use (similar to [25] Sec. IV.B) the notion of outage capacity \( I_{\text{out},p_o} \), which is given by [26 Eq. (26)]

\[
I_{\text{out},q} = \mathbb{E}[I_{\text{OFDM}}] - \sqrt{\text{Var}[I_{\text{OFDM}}]} Q^{-1}(p_o),
\]

for a given outage probability \( p_o \), and with \( Q(\cdot) \) being the Gaussian Q-function. This outage probability (not to be confused with the queue outage in (17)) is defined as the maximum rate that is guaranteed to be supported for 100(1 − \( p_o \))% of the channel realizations. \( \mathbb{E}[I_{\text{OFDM}}] \) and \( \text{Var}[I_{\text{OFDM}}] \) are determined according to [25].

Subsequently, we evaluate the earlier defined cost functions with respect to the queue outage performance, and with varying traffic intensities, in order to assess the robustness of the different cost functions. In particular, we show results (after \( T = 100000 \) time slots) in a network according to Figure 2 with 3 access points and 10 users per access point. We assume the capacity of the wired links is 100 Mbit/s, the target buffer size is \( \tilde{Q} = 20 \) Mbit/s, and the rate at which the application drains the user buffers is 3 Mbit/s. Moreover, we allow a wireless link outage probability of \( p_o = 0.01 \). Moreover, we apply 100 pick-and-compare iterations per timeslot (cf. Section IV-A).

First, we consider the buffer underflow probability, since our main motivation is to prevent service interruptions due to low buffers. Since the application drains the application queues at a fixed rate, intuitively, one can expect that at traffic lower than this value, the influence of underflows dominates, while at traffic rates larger than this value overflows are more likely to occur. Consider Figure 3(a), which compares the underflow probability of various cost functions, and, as a baseline, that of MaxWeight. When the mean arrival rate equals the application service rate \( R_a \) (or is larger), all policies produce low underflow frequencies, however, we already observe small gains from cost functions based approach. The gain significantly grows when the arrival rates are below \( R_a \). While MaxWeight and the linear CF [11] show almost the same high underflow frequency (since both policies do not penalize low buffer states at the application buffers), already the shifted quadratic CF [12] shows improvements. The best performance is obtained with the composite CF in [13], which assign different costs to application buffers and non-application buffers.

However, not only buffer underflows have to be avoided, but it may be desirable to avoid large queues as well. Therefore, we subsequently investigate the performance with respect to the queue outage probability defined in (17). Figure 3(b) summarizes the results. As long as the arrival rates are below \( R_a \), all cost functions produce a decreasing outage frequency when the arrival rates increase, mainly since queue underflows are less frequent. Beyond the applications service rate, the policies that are not
based on a sophisticated cost function rapidly increase the queue outages with increasing traffic, owing to a higher number of overflows. Only the composite cost function can further decrease the queue outages, since exceeding traffic is stored at buffers that generate lower costs.

B. Application II: Energy Efficient Network Control

As another application, we subsequently demonstrate how the $\mu$-MaxWeight framework can be used for energy efficient (“green”) routing. In particular, wireless ad hoc networks, such as sensor networks, often comprise energy limited, battery operated nodes. In energy constrained networks, routing policies may require significant modifications, such that they extend the lifetime of nodes. Although a considerable amount of work has been conducted on energy efficient routing, only few studies consider the combination of energy efficiency and throughput optimality, e.g., [27].

Consider the exemplary system depicted in Figure 4. We have a source of data, a corresponding destination, and three different routes in between (a similar topology was investigated, e.g., in [28]). Let us assume that the intermediate notes are battery powered devices. Moreover, we assume that a node can switch to an energy saving mode whenever there is no traffic in its queues. In this case, an energy efficient routing algorithm tries to maximize the overall total idle time. However, as in the previous application, stability shall be guaranteed nevertheless. Naturally, when the network load is low, a suitable control policy should route as much traffic as possible using only a single route. This allows the devices that constitute the other routes to be switched off. Only if the network load is high, additional routes should be used to avoid queue instability.

This required behaviour is certainly not achieved when the MaxWeight policy is used, which, by contrast, uses all routes in an attempt to balance buffer states. Therefore, we apply the $\mu$-MaxWeight framework with a suitable cost function in order to obtain the desired behaviour at low traffic load. A straight forward cost function choice in this setup is the linear cost function $\mu$. Let $c_i$ be the cost factor associated with the buffer of node $i$. In order to force the traffic towards the central route, we should choose our cost weights as $c_j >> c_i$, for $j \in \{2, 4, 5, 7\}, i \in \{3, 6\}$.

We simulate the system of Figure 4 with the following parameters. The arrival rate $\alpha$ to the queue associated with node 1 is varied in the interval of $\alpha \in [0.1, 0.5]$. All other queues have no external arrivals. As a benchmark, we use the MaxWeight policy again. The results are depicted in Figure 5. The figure shows the total aggregated idle time, defined as

$$\sum_{t=1}^{T} \sum_{m} \mathbb{1}\{q_m(t) = 0\}$$

over the load $\alpha$. Thereby, we consider only the energy-limited intermediate nodes, that is, $m \in [2, 7]$ in (18). In Figure 5(a) we set the capacities of all links to 0.5, while in Figure 5(b) we set the capacities of the central route higher than those of the upper and lower route.

Clearly, the proposed control framework outperforms MaxWeight in terms of energy efficiency. Even in case of equal routes, where the relative gains are smaller, the performance is still significantly better. Note that as the traffic load increases, the behaviour of $\mu$-MaxWeight gets closer to that of MaxWeight, which is necessary to ensure stability.
C. Application III: Cost-Based Cross-Layer Control

As a last example, we show how the proposed network control framework can be combined with physical layer resource allocation in a common cross-layer control problem. While in network models with fixed link capacities the coupling of link rates due to interference is usually not considered, we here assume a fully coupled wireless network, where nodes treat interference as noise. To cope with the non-convex power control problem, we apply the SCA method from [13]. In [16], cross-layer control based on MaxWeight was already successfully combined with the SCA technique.

Consider a wireless multihop network comprising a set of nodes $N$. Traffic can potentially be generated at all nodes in the network and is categorized by its destination. That is, traffic for node $c$ is called commodity-$c$ traffic. Nodes are connected by links $l \in L$, which are defined as ordered node tuples $(i, j)$, with $i, j \in N$. Furthermore, we denote $O(l)$ the set of all outgoing links of node $j$. The set of all incoming links is accordingly defined as $I(l)$. We define $a(l)$ as the transmitter of link $l$ and $d(l)$ as the receiver of link $l$. Let $g_{lm}$ be the path gain from node $o(l)$ to node $d(m)$. Thus, the gain of link $l$ is denoted $g_{ll}$. Accordingly, let $d_{ij}$ be the distance between node $i$ and node $j$. A frequently used simplification for static gains is that, depending on their distance, the link between two nodes can be modeled as:

$$g_{ij} = \frac{1}{(d_{ij})^\rho},$$

with $\rho \in \mathbb{R}$ being the path-loss exponent. Below, we assume a path-loss exponent $\rho = 4$, which corresponds to a rather lossy environment (e.g., an urban or sub-urban environment).

Each node $j$ assigns a power value $p_l$ to each of its outgoing links $l \in O(j)$. Let $p \in \mathbb{R}^L$ be the vector comprising the current power allocation of all $L$ links in the network. Here, we consider per-node power constraints, where all nodes have a common maximum available power value of $P_{\text{max}}$, thus $\sum_{l \in O(j)} p_l \leq P_{\text{max}}$. However, all algorithms can be easily applied in case of per-link power constraints as well. Let

$$\gamma_l(p) = \frac{g_{ll}p_l}{\sigma_l^2 + \sum_{j \neq l} g_{jl}p_j}$$

be the SINR experienced at node $d(l)$, with $\sigma_l^2 \in \mathbb{R}$ being the noise power. Consequently, the rate of link $l$, depending on the current power allocation $p$, is given by

$$r_l(p) = W \cdot \log_2 \left(1 + \gamma_l(p)\right),$$

where $W \in \mathbb{R}$ is the available system bandwidth.

Our approach is based on the well-known BPPC algorithm (see [11][16] for details on this algorithm), which is known to be throughput optimal [11].

Subsequently, we show how to combine the BPPC approach with $\mu$-MaxWeight. The main difference to the procedure above is the generation of link weights. As before, and in contrast to a MaxWeight-based algorithm that uses only the buffer.
states to determine the weights, we determine the weights using our weight functions $\mu$. Using, for example, a linear cost function, this results in $[\mu(x)]_i = c_i \left( 1 - \exp \left( -\frac{\bar{z}_i}{\bar{\theta} + \sum_{j \neq i} x_{ij}} \right) \right)$, such that

$$w_i^*(t) = \left( [\mu(x)]_{o_i(t)}^{(c)} - [\mu(x)]_{d_i(t)}^{(c)} \right) u_i^*.$$  

(21)

The optimization problem at the physical layer is now

$$\max_p \sum_l w_l^*(t) \cdot \log (1 + \gamma_l(p))$$  

(22)

subject to $p_l \leq P_{\text{max}} \; (\forall l)$, and with weights defined in (21). Using rates defined in (20), we still have a difficult non-convex problem, making a global optimization prohibitively complex in large networks.

Therefore, we apply the SCA algorithm of [13][16], which can be shown to converge to at least a KKT point of (22) [13]. Core of the SCA algorithm is to convexify (22) by iteratively applying an appropriate lower bound, given by

$$\alpha_l \log(\gamma_l(p)) + \beta_l \leq \log (1 + \gamma_l(p)),$$  

(23)

to the link capacity, followed by a logarithmic change of variables, i.e., $\tilde{p}_l := \log(p_l)$. Parameters $\alpha_l \in \mathbb{R}$ and $\beta_l \in \mathbb{R}$ are chosen according to $\alpha_l = \frac{z_0}{\gamma_l} - \frac{\bar{z}_0}{\bar{\theta} + \log(z_0)}$, with some $z_0 \in \mathbb{R}$. Note that the bound (23) is tight for $z_0 = \gamma_l(p)$. The algorithm is initialized according to $\alpha(0) = 1$ and $\beta(0) = 0$, which is equivalent to the high-SINR approximation. Each iteration of the algorithm first includes a maximize-step

$$\hat{p}^*(t) \in \arg \max_p \sum_l w_l^*(t)\tilde{\gamma}(\tilde{p}),$$  

(24)

subject to $\exp(\tilde{p}_l) \leq \tilde{p}_l$, whose solution is then used in a tighten-step to update $\alpha_l$ and $\beta_l$ for each link according to $\alpha_l(t+1) = \frac{\gamma_l(p^*(t))}{1 + \gamma_l(p^*(t))}$ and $\beta_l(t+1) = \log (1 + \gamma_l(p^*(t))) - \alpha_l \log(\gamma_l(p^*(t)))$. When $\hat{r}_l(p^*(t), \alpha(t), \beta(t)) \approx \hat{r}_l(p^*(t), \alpha(t+1), \beta(t+1))$, the algorithm terminates.

To evaluate the performance, we conduct numerical simulations over 10000 time slots and compare to various baselines. A main advantage of the cost function based approach is that every buffer can be weighted with different coefficients $c_i$. Doing so, the use of specific buffers (or entire routes) can be discouraged, for example, in cases where buffer space is more expensive at selected nodes than elsewhere. We perform simulations both with unequal and equal weights for all buffers in a network with $N = 9$ nodes connected by $L = 10$ links. Furthermore, we assume a single source node and 5 possible destination nodes, with the same average arrival rate for all traffic streams. The network has a layered topology that is larger, but structurally similar to the network that is depicted in Figure 2, however with 3 nodes in the second layer and with 5 nodes in the third layer. A bandwidth of 20 MHz is available for the wireless links.

In Figure 3 we compare the average sizes of all queues in the network over the first 200 time slots, both with (left) and without (right) power control. In the latter case, each node simply distributes the available power equally over the currently active outgoing links. In case SCA-based power control is used, all queues converge to a steady mean, while in the no-power-control case some queues grow without bound. Therefore, we can conclude that the stability region is larger due to the convex approximation based power control algorithm. In Figure 7 we compare the time averaged cost incurred by the $\mu$-MaxWeight and SCA based policy with that of the corresponding MaxWeight-based policy (with and without power control based on SCA). In addition, we show the performance of the cross-layer algorithm when relying on the high-SINR assumption in the optimization. Similar to what we saw before, the stability region without power control is much smaller than that based on the SCA algorithm. Although using the high-SINR assumption in the algorithm can stabilize the network at higher arrival rates (unlike no power control), the performance is still considerably worse than that of the SCA-based algorithms. To allow a comparison of the SCA-based cross-layer algorithms using $\mu$-MaxWeight with that using classical MaxWeight (whose curves are very close in the top half of Figure 7), in the lower half of Figure 7 we show the average cost for a relatively high arrival rate over time. After the network reaches a steady state, significant gains can be observed compared to MaxWeight, and these gains are larger when the mean arrival rate is higher. The more traffic traverses the network, the higher will be the interference between active links, and the more beneficial becomes power control.
Fig. 6. Average buffer states over time obtained by $\mu$-MaxWeight using SCA-based power control (left) and no power control (right). The figure shows the trajectories (buffer size over time) of all ($|Q| = 17$) queues in the network.

Fig. 7. Performance comparison of different control policies. The $\mu$-MaxWeight based policies are based on a cost function with equal weights. The top figure depicts the average cost over the mean arrival rate after a simulation time of 10000 time slots, while the bottom figure depicts the average cost over time for a selected average arrival rate of $\alpha = 2.5 \text{ Mbit/s}$.

VII. CONCLUSIONS

We presented and evaluated a framework for the design and control of queueing networks, which combines throughput optimality with the minimization of an underlying cost function, thereby taking into account potential service-dependent constraints. We demonstrated how this framework can be used to flexibly adapt the network control to different applications with corresponding requirements. Moreover, we adapted a well-known cross-layer control algorithm to our control framework, and demonstrated how this non-convex optimization problem can be efficiently approximated using SCA. In all evaluated applications, numerical evaluations indicate high gains of the proposed cost function based approach over classical throughput optimal control policies, such as MaxWeight.
APPENDIX A

PROOF OF THEOREM 2

By Condition 2) of Theorem 2 we can assume that the random walk evolves on $\mathbb{R}^m$. Hence, we can skip Condition 2) of Theorem 1 since this condition (as its counterpart in Theorem 2) ensures positivity of the random walk. We need to show that from

$$\|\nabla \log \mu_i (x)\| \leq \epsilon, \quad \forall i \in \mathcal{M}, \|x\| > C_0 (\epsilon), \quad (25)$$

(where $C_0 (\epsilon)$ is sufficiently large) follows:

$$\frac{\mu_i (x + \Delta x)}{\sum_{j \in \mathcal{M}} \mu_j (x + \Delta x)} - \frac{\mu_i (x)}{\sum_{j \in \mathcal{M}} \mu_j (x)} \leq \epsilon. \quad (26)$$

For orientation, let us assume more restrictive conditions first: take $\mu_i, \forall i \in \mathcal{M}$, Lipschitz continuous, and let $\sum_{j \in \mathcal{M}} \mu_j (x) \to \infty$, if $\|x\| \to \infty$. Note that these conditions already encompasses Meyn’s perturbation $\Theta$ together with, e.g., a linear cost function.

It is easy to prove the theorem with these assumptions: by the mean value theorem, we have $\mu_i (x + \Delta x) = \mu_i (\bar{x}) + \nabla_{\bar{x}} \mu_i (\bar{x}) \Delta \bar{x}$, where $\bar{x}$ is an (arbitrary) point on the line connecting $x$ and $x + \Delta x$, whereas $\bar{x}$ is a point on the line connecting $\bar{x}$ and $x + \Delta x$. Since the field is Lipschitz, we have $\nabla_{\bar{x}} \mu_i (\bar{x}) \Delta \bar{x} \leq C_7$ uniformly. Furthermore, since the policy is non-idling $\sum_{j \in \mathcal{M}} \mu_j (x + \Delta x) \geq C_8$ where the normalization constant $C_8$ can be chosen as large as possible without altering the policy (by the construction of the policy). Moreover, since $\sum_{j \in \mathcal{M}} \mu_j (x) \to \infty, \|x\| \to \infty$, condition $\Theta$ is equivalent to $|\mu_i (x + \Delta x) - \mu_i (x)| \leq \epsilon \sum_{j \in \mathcal{M}} \mu_j (\bar{x})$ and, again, by the mean value theorem: $|\nabla^T \mu_i (\bar{x}) \Delta \bar{x}| \leq \epsilon \sum_{j \in \mathcal{M}} \mu_j (\bar{x})$. Here, we tacitly assumed that we have selected $\bar{x}$ accordingly. Since $\Delta x$ is fixed and by the positivity of $\mu_i$ it is sufficient that $|\nabla \mu_i (x)\| \leq \frac{1}{\Delta x} \mu_i (x)$, which is equivalent to condition $\Theta$ with some $\|x\| > C_0 (\epsilon)$ ($\epsilon'$ slightly smaller).

Let us now prove the general case. By the derivation above, condition $\Theta$ can be written as

$$\frac{1}{\sum_{j \in \mathcal{M}} \mu_j (x)} \nabla^T \mu_i (\bar{x}) \Delta \bar{x} = \epsilon_n,$$

for some $x$ with $\|x\| > C (\epsilon_n)$ where $\epsilon_n$ is a zero sequence and $C (\epsilon_n)$ is strictly increasing for any fixed $\Delta x \in \mathbb{R}^m$. Now, again, by the mean value theorem $|\sum_{j \in \mathcal{M}} \mu_j (x + \Delta x) - \sum_{j \in \mathcal{M}} \mu_j (x)| \leq \epsilon$, where we set $\bar{x}$ as before and let $x + \Delta x = \bar{x}$ and $x + \Delta x = x + \Delta x$. $\bar{x}, \bar{x}$ are points on the lines connecting $x$ with $\bar{x}$ and $\bar{x}$ with $x + \Delta x$, respectively. Note that $\mu_j (\bar{x})$ is zero if and only if $\mu_i (x + \Delta x)$ and $\mu_i (x)$ are both zero since otherwise by condition $\Theta$ the gradient would be zero as well. Since in this case the condition is trivially satisfied so that we exclude it. Hence, it follows that

$$|\mu_i (x + \Delta x) - \mu_i (x)| \leq \epsilon \sum_{j \in \mathcal{M}} \mu_j (\bar{x}) \left(1 + \frac{\nabla_{\bar{x}} \mu_j (\bar{x}) \Delta \bar{x}}{\mu_j (\bar{x})} \right), \quad (A)$$

and

$$|\mu_i (x + \Delta x) - \mu_i (x)| \leq \epsilon \sum_{j \in \mathcal{M}} \mu_j (\bar{x}) \left(1 + \frac{\nabla_{\bar{x}} \mu_j (\bar{x}) \Delta \bar{x}}{\mu_j (\bar{x})} \right), \quad (B)$$

We can prove that, because of condition $\Theta$, (A) and (B) are zero sequences: suppose $\nabla^T \mu_j (\bar{x})$ is non-zero (then we can stop anyway) then by the repeated application of the mean value theorem, the denominator of, say, (A) can be written as $\mu_j (\bar{x}) = \mu_j (\bar{x}) + \nabla^T \mu_j (\bar{x}) \Delta \bar{x}$. This process generates sequences in $\mathbb{R}^m$ with $\bar{x} = x_1, x_2, \ldots$ and $\Delta \bar{x} = \Delta \bar{x}_1 \subset \Delta \bar{x}_2, \ldots$ which are bounded and hence we can pick subsequences converging to some set of limit points $x^{(k)}_n, k = 1, 2, \ldots$. Note that we can restrict the number of limit points to at most two since by definition every limit point is visited arbitrarily often and infinitely close and by construction of the sequence there is no possibility of more than two limit points which neither contain the other in between them. Take these two limit points with corresponding subsequence $x^{(k)}_n, k = 1, 2$: by continuous differentiability we have $\mu_j (x^{(k)}_n) \to \mu_j (x^{(k)}_{\infty})$ and $\nabla \mu_j (x^{(k)}_n) \to \nabla \mu_j (x^{(k)}_{\infty}), k = 1, 2$. It must also hold in the limit that
\[ \mu_j(x^{(1)}) + \nabla^T \mu_j(x^{(2)}) (x^{(1)} - x^{(2)}) = \mu_j(x^{(2)}) \] (and vice versa). Since then
\[ \frac{\nabla^T \mu_j(x^{(2)}) (x^{(1)} - x^{(2)})}{\mu_j(x^{(2)})} \leq \epsilon, \]
and (vice versa) where \( \epsilon > 0 \) is arbitrarily small by condition \( 25 \) we conclude that \( \mu_j(x^{(1)}) = \mu_j(x^{(2)}) \) (but not necessarily \( x^{(1)} = x^{(2)} \)).

Now, we can proceed the process sufficiently often as
\[ \frac{\nabla^T \mu_j(x_{n+1}) \Delta x_{n+1}}{\mu_j(x_{n+1})} \leq \frac{\nabla^T \mu_j(x_1) \Delta x_1}{\mu_j(x_1)} \leq \ldots \]
such that in the final step
\[ \frac{\nabla^T \mu_j(x_{n+1}) \Delta x_{n+1}}{\mu_j(x_{n+1})} = \frac{(\nabla^T \mu_j(x_\infty) + \epsilon') \Delta x_{n+1}}{\mu_j(x_\infty) + \epsilon'} \leq \epsilon, \quad k, l = 1, 2, \]
by condition \( 25 \). Hence, we have
\[ \sum_{j \in M} \mu_j(\bar{x}) \left( 1 + \frac{\nabla^T \mu_j(\bar{x}) \Delta x}{\mu_j(\bar{x})} \right) = \sum_{j \in M} \mu_j(\bar{x}) \left( 1 + \frac{\epsilon'}{1 + \epsilon'} \right) = 1 + \epsilon', \]
\( \epsilon' \), \( \epsilon'' \) zero sequences, and further \( |\mu_i(x + \Delta x) - \mu_i(x)(1 + \epsilon'')| \leq |\mu_i(x + \Delta x) - \mu_i(x)| + \mu_i(x)\epsilon'' \leq \epsilon \sum_{j \in M} \mu_j(\bar{x})(1 + \epsilon') \), which is equivalent to:
\[ |\mu_i(x + \Delta x) - \mu_i(x)| \leq \epsilon \sum_{j \in M} \mu_j(\bar{x})(1 + \epsilon') - \epsilon'' \mu_i(x). \]

Since \( \bar{x} \) is arbitrary and can be suitably choose, condition \( 25 \) with some \( \|x\| > C_0 (\epsilon''') \) is sufficient for the latter to hold.

**APPENDIX B**

**PROOF OF THEOREM 3**

We can write
\[ \mu_i(x) = \frac{\partial h_i}{\partial x_i}(x) = l(x_i) \frac{\partial h_0}{\partial x_i}(\bar{x}) \]
where we defined \( l := \frac{\partial \bar{x}}{\partial x_i} \). Note, that here \( \bar{x}_i \) only depends on \( x_i \). The gradient of the weight \( \mu_i(x) \) is given by:
\[ \frac{\partial \mu_i}{\partial x_j}(x) = \begin{cases} \frac{\partial l}{\partial x_i}(x_i) \frac{\partial h_0}{\partial x_i}(\bar{x}) + l(x_i) \frac{\partial h_0}{\partial x_i}(\bar{x}) & i = j \\ \frac{\partial l}{\partial x_i}(x_i) \frac{\partial h_0}{\partial x_i}(\bar{x}) & i \neq j \end{cases} \]
Define \( x^\Delta := x + \Delta x \) and \( \bar{x}^\Delta := \bar{x}(x^\Delta) \). From the proof of Theorem 2 it is clear that we only have to show that
\[ \frac{\nabla^T \mu_i(x) \Delta x}{||\mu(x^\Delta)||} \leq \epsilon, \]
for some \( \epsilon > 0 \) arbitrarily small. This can be rewritten as:
\[ \frac{\partial l(x_i) \frac{\partial h_0}{\partial x_i}(\bar{x}) \Delta x_i}{\sum_{j \in M} l(x_j) \frac{\partial h_0}{\partial x_j}(\bar{x}) \Delta x_j} + \frac{l(x_i) \sum_{j \in M, j \neq i} \frac{\partial}{\partial x_j}(\bar{x}^\Delta) \Delta x_i}{\sum_{j \in M} l(x_j) \frac{\partial h_0}{\partial x_j}(\bar{x}) \Delta x_j} \leq \epsilon \]
Since \( \frac{\partial h_0}{\partial x_i}, l \) are Lipschitz, thus \( \frac{\partial}{\partial x_j}(\bar{x}^\Delta) \) are uniformly bounded, and \( l(x_i) \frac{\partial h_0}{\partial x_i}(\bar{x}) \geq l^{1+}(x_i) \to \infty \) when \( x_i \to \infty \), the effect of \( \Delta x \) vanishes in the denominator. The condition \( \frac{\partial h_0}{\partial x_i}(\bar{x}) \geq l^{1+}(x_i) \) is required since we have expressions of the form
\[ \frac{l(x_i) l(x_j) \frac{\partial h_0}{\partial x_i}(\bar{x})}{l(x_j) \frac{\partial h_0}{\partial x_j}(\bar{x})} \]
which then become arbitrarily small.
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