Magneto-thermal reconnection processes, related mode momentum and formation of high energy particle populations

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Abstract

In the context of a two-fluid theory of magnetic reconnection, when the longitudinal electron thermal conductivity is relatively large, the perturbed electron temperature tends to become singular in the presence of a reconnected field component and an electron temperature gradient. A finite transverse thermal diffusivity removes this singularity while a finite ‘inductivity’ can remove the singularity of the relevant plasma displacement. Then (i) a new ‘magneto-thermal’ reconnection producing mode, is found with characteristic widths of the reconnection layer remaining significant even when the macroscopic distances involved are very large; (ii) the mode phase velocities can be both in the direction of the electron diamagnetic velocity as well in the opposite (ion) direction. A numerical solution of the complete set of equations has been carried out with a simplified analytical reformulation of the problem. A sequence of processes is analyzed to point out that high-energy particle populations can be produced as a result of reconnection events. These involve mode-particle resonances transferring energy of the reconnecting mode to a superthermal ion population and the excitation of lower hybrid waves that can lead to a significant superthermal electron population. The same modes excited in axisymmetric (e.g. toroidal) confinement configurations can extract angular momentum from the main body of the plasma column and thereby sustain a local ‘spontaneous rotation’ of it.

Keywords: magneto-thermal reconnection, electron temperature gradient, high energy particle populations

(Some figures may appear in colour only in the online journal)
with a simplified analytical reformulation of the problem. The mode growth rate is related to the effects of a finite viscous diffusion coefficient or to those of a small electrical resistivity.

The features that can lead to a possible explanation of the fact that high energy particle populations are produced during reconnection events involve mode-particle resonances [6] producing the transfer of energy to super-thermal particle populations and the spatial near-singularity of the electron temperature that can enhance the thermal energy of particles in one region while depleting that of particles in a contiguous region [5] may be an additional factor to be taken into account within this context.

The same low collisionality modes that produce magnetic reconnection can extract momentum from the plasma sheet and when excited in axisymmetric toroidal confinement configurations can sustain a ‘spontaneous rotation’ [7] of the plasma column by extracting angular momentum from it. This process is to be considered in addition to that of ejection of angular momentum [7] from the edge of the plasma column resulting for instance from the local excitation of electrostatic modes.

2. Initial equilibrium configuration

For the sake of simplicity, we simulate current carrying well-confined plasmas with a variety of geometries by a plane configuration with the confining field \( \mathbf{B} = B_x \mathbf{e}_x + B_y(x) \mathbf{e}_y \), where \( B_y^0 \ll B_x^0 \). The profiles of \( B_y(x) \) related to those of the current density \( J_x(x) \) that have been examined correspond to those produced in experiments. The density and temperature gradients are in the \( x \) direction and no equilibrium electric field \( \mathbf{E} \) is present in the considered frame of reference. Then the equilibrium state is described simply by

\[
\frac{d}{dx}(\rho_e + p_i) + \frac{1}{c} J_x(x) B_y(x) = 0. \tag{1}
\]

In order to study magnetic reconnection process in the considered sheared magnetic field configuration, all perturbations from the considered equilibrium state are taken to be of the form \( \tilde{A} = A(x) \exp(-i \omega t + ik_x x + ik_z z) \), where \( k_x \) and \( k_z (\ll k_y) \) are such that \( k_y \equiv (k \cdot \mathbf{B})/B = 0 \) at \( x = x_0 \) and \( k_y \equiv (k \cdot B_y)/B(x - x_0) \) near \( x = x_0 \). Here \( B_y^0 \equiv dB_y/dx \). The surface \( x = x_0 \) is chosen to correspond to one on which the electron temperature gradient \( dT_e/dx \) is significant.

3. ‘Outer’ asymptotic region and singularities

The layer in which magnetic reconnection takes place is centered around \( x = x_0 \) and has a width \( \delta_0 \) that is much smaller than all the scale distances associated with the gradients of the plasma pressure and the current density. Outside the layer (the ‘outer’ asymptotic region), inertia is negligible and the perturbed plasma is described by

\[
0 = -\nabla \left( \bar{p}_e + \bar{p}_i \right) + \frac{1}{c} \left( \mathbf{J} \times \mathbf{B} \right), \tag{2}
\]

Since \( \mathbf{J} = (e/4\pi) \nabla \times (\mathbf{B} + \mathbf{b}) \), equation (2) becomes

\[
0 = -\nabla \left( \bar{p}_e + \bar{p}_i + \frac{1}{4\pi} \mathbf{B} \cdot \mathbf{B} \right) + \frac{1}{4\pi} \mathbf{B} \cdot \nabla \mathbf{B} + \mathbf{b} \cdot \nabla \mathbf{B}. \tag{3}
\]

Then, considering perturbations that only bend the field lines, i.e. \( B_z \equiv 0 \), and using \( \nabla \times \mathbf{B} = 0 \), the \( \mathbf{e} \cdot \nabla \times \) component of equation (3) gives the (radial) profile of the transverse perturbed magnetic field \( \tilde{B}_z \) as

\[
(k \mathbf{B}) \left( \frac{d^2 \tilde{B}_z}{dx^2} - k_x^2 \tilde{B}_z \right) - k_y B_y^0 \tilde{B}_z = 0. \tag{4}
\]

Here, \( B_y^0 \equiv dB_y/dx \). The solution \( \tilde{B}_z(x) \) is assumed to be continuous at the regular singular point \( x = x_0 \) but its derivative, in general, can be discontinuous. The discontinuity parameter \( \Delta \), concerning \( (dB_y/dx) \tilde{B}_z \) at \( x = x_0 \) is defined by

\[
\Delta \equiv \frac{1}{B_{y,0}} \left( \frac{d \tilde{B}_z}{dx} \right)_{x=x_0} - \frac{d \tilde{B}_z}{dx} \left. \right|_{x=x_0}, \tag{5}
\]

where \( B_{y,0} \equiv B_y(x = x_0) \neq 0 \). Clearly, the parameter \( \Delta \) is a function of the equilibrium current density gradient, \( dJ_x/dx \), and \( k_x \). We choose to include in our analyses equilibrium current density profiles and values of \( k_x \) for which \( \Delta \) can have a relatively wide range of values. We further add that a positive \( \Delta \) is the driving factor of the drift-tearing types of mode identified in [1].

Next, the ‘frozen-in’ condition that characterizes the outer region is

\[
\tilde{E}_\| + \frac{1}{c} (\tilde{u}_e \times \mathbf{B}) = 0, \tag{6}
\]

which implies that \( \tilde{E}_\| \equiv (\tilde{E} \cdot \mathbf{B})/B = 0 \) and that

\[
\tilde{u}_e = \frac{c}{B^2} (\tilde{E} \times \mathbf{B}) \equiv \tilde{u}_e. \tag{7}
\]

Writing \( \tilde{E} = \tilde{E}_\perp + \tilde{E}_\| \) where \( \tilde{E}_\perp = (\mathbf{B} \times \tilde{u}_e)/c \) and \( \tilde{E}_\| = (\mathbf{B} \cdot \mathbf{B}) \tilde{E}_\| \) the \( x \)-component of

\[
\frac{\partial}{\partial t} \tilde{B} = -c (\nabla \times \tilde{E}), \tag{8}
\]

gives

\[
\tilde{E}_\| \cong \omega/c k_x (\tilde{B}_z - i(k \cdot \mathbf{B}) \tilde{z}_x), \tag{9}
\]

where we have considered \( |(k_x, k_y, k_z)| (B_y/B_x) | \ll 1 \) and have introduced the radial plasma displacement \( \tilde{z}_x \equiv \tilde{u}_e l(-i \omega) \). Then, by the ‘frozen-in’ condition (\( \tilde{E}_\| = 0 \)), \( \tilde{z}_x \) is related to \( \tilde{B}_z \) in the outer region as

\[
\tilde{B}_z \cong i(k \cdot \mathbf{B}) \tilde{z}_x. \tag{10}
\]

Additionally, we find it important to include the ‘infinite longitudinal electron thermal conductivity’ condition in the outer region. This condition, referring to the electron thermal energy balance equation, implies
\[ \mathbf{B} \cdot \nabla \mathbf{T}_e + \mathbf{B} \cdot \nabla T_e = 0, \]

and it gives

\[ \mathbf{B}_i = -i(n_c) \mathbf{T}_{Te}^e, \]

where \( \mathbf{T}_{Te}^e \) is the electron temperature perturbation and \( T_e^e \equiv dT_e/dx \). Comparison with equation (10) shows that \( \mathbf{T}_{Te}^e = -T_e \dot{\xi}_e \) in the outer region.

It is evident from equations (10) and (12) that, for \( B_\parallel \neq 0 \) and \( T_e^e \neq 0 \), both \( \dot{\xi}_e \) and \( \mathbf{B}_i \) tend to become singular as \( x \to x_0 \) as \( \mathbf{k} \cdot \mathbf{B} \to 0 \). These singularities are removed by adopting a two-fluid description [1, 3] in dealing with an ‘inner’ region corresponding to \( |x - x_0| \sim \delta_x \). A new ‘magneto-thermal’ reconnection-producing mode, driven by the electron temperature gradient and involving a considerable range of scale distances, is thus found [5].

4. Basic equations for the ‘inner’ region

We derive the basic equations for the inner region that are capable of describing the reconnection-producing modes of interest by following the procedure developed in [1, 3]. We outline the derivation here.

The quasi-neutrality condition given by \( \nabla \cdot \mathbf{j}_e + \nabla \cdot \mathbf{j}_i \equiv 0 \). The additional term \( \mathbf{B} \cdot \nabla J/B \) involving the gradient of the equilibrium current density, which is the driving factor of the relevant instability and is important in the outer region (see equation (4)), can be neglected in the inner region as it does not contribute to the dispersion relation as indicated by the analysis that follows. In the guiding-center description

\[ \mathbf{j}_L \equiv -i en \left( 1 - \omega \frac{\delta \mathbf{B}}{\omega} \right) \left( \mathbf{B} \times \mathbf{U}_E \right), \]

when polarization drift, finite gyroradius correction and momentum diffusion due to viscosity are added to the ion guiding-center drift velocity \( \mathbf{U}_i \equiv c(\mathbf{k} \times \mathbf{B})/B \). Here \( \Omega_i \equiv eB/(mc) \) is the ion cyclotron frequency, \( \omega_d \equiv k_x c/\omega(\omega B) \) is the ion diamagnetic frequency, and \( \mathbf{D}_i \) is the transverse (to the magnetic field) momentum diffusion coefficient. Electrons are taken to move with the drift velocity \( \mathbf{u}_E \). We also find from the Ampère’s law that

\[ \mathbf{j}_L \equiv \frac{i e}{4\pi k_x} \frac{\partial \mathbf{B}}{\partial x}. \]

when we consider \( B \gtrsim B_\parallel (k_y/k_x)(B_\perp/B) \ll 1 \), and use \( \frac{\partial^2 \mathbf{B}}{\partial x^2} \gtrsim k_y^2 \) in the inner region. Then, the quasi-neutrality condition gives

\[ -(\omega - \omega_d) \left( \omega - i \frac{\partial D_i}{\partial x} \right) \frac{\partial^2 \dot{\xi}_e}{\partial x^2} \equiv i(n_c) \frac{\partial \mathbf{B}_i}{4\pi m_i \partial x}. \]

The other equation to be coupled with equation (15) is derived from the longitudinal electron momentum conservation equation

\[ 0 \cong -\nabla \mathbf{p}_e - n_c \mathbf{T}_e \nabla \mathbf{T}_e \equiv \left( \frac{dp_e}{dx} + n_c \frac{dT_e}{dx} \right) \frac{\mathbf{B}_i}{B}, \]

\[ -en(\mathbf{E} - \mathbf{j}_L + \mathbf{j}_i) \equiv \frac{d\mathbf{B}_i}{dx} + \mathbf{B}_i \equiv 0, \]

where \( \mathbf{E}_e = T_e \mathbf{e}_x + n T_e \mathbf{e}_y /\xi_e \) is a numerical coefficient characterizing the thermal force, \( \eta \equiv \eta_\parallel + i\omega L \mathbf{j}_L \) represents the induced electric field, \( L \) being the relevant plasma inductivity. Clearly, the inductivity is introduced to break the frozen-in condition, while leading to results consistent with experimental observations [8] concerning modes driven by the current density gradient. In fact, a finite inductivity could represent the coupling of the plasma current channels produced inside the reconnection layer with the effects of other current channels outside it.

Next we determine \( n_e \), from the electron equation of continuity

\[ -\omega \dot{\mathbf{n}}_e + \mathbf{U}_E \frac{\partial n_e}{\partial x} + ink \mathbf{E} \equiv 0, \]

where the term [d(\mathbf{j}_e)/dx][\mathbf{B}_i/B] has been omitted since, as indicated by the analysis that follows, it does not contribute to the dispersion relation due to mode parity (in \( x - x_0 \) consideration. Assuming \( \dot{\mathbf{n}}_e \equiv -\mathbf{E}_e(\eta) \), introducing \( \dot{\xi}_e \equiv \mathbf{U}_E \mathbf{d} /(-\omega) \), we obtain

\[ \dot{\mathbf{E}} \equiv \mathbf{E}_e \mathbf{d} \frac{\partial n_e}{\partial x} + \frac{i c \mathbf{k}_y \mathbf{P}_L}{4\pi e \eta_\parallel} \frac{d \mathbf{B}_i}{dx}. \]

We point out that, in reality, a particle transport term represented by \( (\delta \eta \mathbf{D}_e) \left( \frac{\partial^2 \mathbf{E}}{\partial x^2} \right) \), for \( \frac{\partial^2 \mathbf{E}}{\partial x^2} \gg k_y^2 \), should also be included in the expression of \( n_e \), where the particle diffusion coefficient \( \mathbf{D}_e \) can have a classical component (due to electron-ion collisions) and an anomalous component (due to collective processes). The effect of this term on the reconnection process is considered to be less important than that of the plasma inductivity and has been neglected for simplicity. Then, using equations (9), (14) and (18) in equation (16) we find

\[ \omega = \omega_d - \omega_\tau T_e \mathbf{B}_i \equiv i(n_c) \mathbf{B}_i \left( \omega - \omega_\tau \right) \mathbf{E}_e \mathbf{d} \frac{\partial n_e}{\partial x} + \mathbf{E}_e \mathbf{d} \frac{\partial n_e}{\partial x}. \]

Here \( \omega \equiv \omega_\tau \) \( \cong (c^2/4\pi) \mathbf{L} \). Clearly, the two characteristic frequencies appearing in equation (19) are

\[ \omega_\tau \equiv -k_y \mathbf{c T}_e \frac{\partial n_e}{\partial x} \] and \( \omega_\tau \equiv \omega_\tau \left( 1 + \alpha_\tau k_y \mathbf{c T}_e \right) \mathbf{d} \frac{\partial n_e}{\partial x}. \]

Lastly, for the electron temperature fluctuation \( \mathbf{T}_e \), we consider the following electron thermal energy balance equation that is relevant in the large parallel thermal conductivity limit \( (D_\parallel T_e \gg \omega) \).

\[ \omega = \omega_d = \omega_\tau \mathbf{B}_i \equiv i(n_c) \mathbf{B}_i \left( \omega - \omega_\tau \right) \mathbf{E}_e \mathbf{d} \frac{\partial n_e}{\partial x} + \mathbf{E}_e \mathbf{d} \frac{\partial n_e}{\partial x}. \]
\[ i\omega T^2 \xi_k \xi_k + D_{\text{le}} \frac{\partial^2 \xi}{\partial x^2} - D_{\text{ll}} k^2 \xi_k \xi_k \cong -i\kappa_{\text{f}} P_{\text{f}} \xi_k \frac{B}{B}, \]  
where \( D_{\text{le}} \) and \( D_{\text{ll}} \) are the transverse and longitudinal diffusion coefficients for the electron thermal energy. By using equations (15), (21) and (19) can be written in the alternative form

\[ k_{\text{f}}(\omega - \omega_{\text{e}}) \left( \frac{\partial B}{\partial x} - i k_{\text{f}} \xi_k \right) + \frac{\omega L_{\text{f}}^T \xi_k \xi_k}{D_{\text{ll}}} \cong \frac{1}{\gamma_{\text{le}}} \left( S_k - \frac{c^2}{\omega_{\text{pe}}^2} \right) - i \kappa_{\text{f}} P_{\text{f}} \frac{B}{B}, \]  

where \( \gamma_{\text{le}} \equiv B^2/(4\pi n_{\text{e}}). \) We shall use equations (15), (21) and (22) for the description of plasma in the inner region where reconnection occurs. We recall here that \( k_{\text{f}} \cong (k, B)|/B(x - x_0) \) in the inner region. The considered width of the ‘intrinsic’ reconnection layer is represented by \( \delta \equiv \delta_1^2, \) and the resistive diffusion coefficient \( D_{\text{le}} \) is considered to be small relative to \( \omega_\perp S_k. \) We also point out that the electron inertia term is omitted in equation (16) under the assumption \( \mathcal{L} \gg 4\pi c \omega_{\text{pe}} \) that also means \( \delta_1^2 > (c^2/\omega_{\text{pe}}^2) \equiv \delta_2^2. \) The ratio \( D_{\text{le}}/D_{\text{ll}} \) takes into account that \( D_{\text{le}} \) resulting from the excitation of relatively small scale electrostatic modes can exceed its collisional value by large factors, as is well known from the experiments on magnetically confined plasmas.

The distance that characterizes the ‘innermost region’, as obtained from equation (21), is

\[ \delta_\text{in} = \left( \frac{D_{\text{le}}}{D_{\text{ll}}} \right)^{1/4} \left( \frac{L_{\text{e}}}{k} \right)^{1/2}, \]  

where \( 1/L_{\text{e}} \equiv B'/B, \) and we note that it remains significant when large macroscopic scale distances are considered. We assume that \( \rho_i < \delta_\text{in} < \delta_m < \delta_t, \) where \( \rho_i \) is the average ion gyroradius.

### 5. Range of phase velocities

The analysis of the modes that can be found involves matching the solution for \( B_\perp \) of the inner region equations (15), (21) and (22), with that of the outer region equation by the asymptotic matching condition given by equation (5). The analysis indicates that the nature and the phase velocity of the relevant modes change significantly when the parameter \( D_0 \), defined by

\[ D_0 \equiv \frac{\delta_2^2}{\delta_m^2} \Delta, \]  

is varied. In particular, when the value of \( D_0 \) is positive and finite, the phase velocity of the modes that are found is in the direction of the ion diamagnetic velocity as they become of the type reported in [3]. When, instead, \( D_0 \) is small such as that for which \( \delta_m < \delta_t, \) as is relevant for applications to plasma configurations for which the involved macroscopic distances are very large (e.g. after space and astrophysics), the direction of the mode phase velocity can be in that of the electron diamagnetic velocity.

For our analysis, we choose a set of appropriate dimensionless quantities and rewrite the equations for the inner region (15), (21) and (22) as

\[ \frac{\omega(\omega - \omega_{\text{e}})}{\omega_\perp^2} \left( 1 - i\kappa_{\text{f}} \right) = \frac{\partial^2 \xi}{\partial x^2} \cong \frac{\partial^2 \tilde{B}}{\partial x^2}, \]  

and

\[ \frac{\omega_\perp \alpha_F \partial^2 \tilde{B}}{\partial x^2} + (\omega - \omega_{\text{e}}) (\tau \tilde{B} - \tau^2 \tilde{Z}_s) - i \varepsilon_{\text{f}} n_{\text{e}} \varepsilon_{\text{f}} \tilde{Z}_s, \]

respectively. Here

\[ \tilde{x} \equiv \frac{x - x_0}{\delta_t}, \quad \tilde{B} \equiv \frac{B}{B_0}, \quad \tilde{Z} \equiv \frac{\xi}{\delta_t}, \quad \tilde{T} \equiv -i \frac{T}{\tau_{\text{ce}}}, \]  

\[ \frac{\alpha_F}{D_{\text{le}} \delta_t^2} \left( \frac{\Delta_0}{k} \right)^2 \cong \left( \frac{\delta_m}{\delta_t} \right)^4, \quad \eta_{\text{le}} \equiv \frac{\omega L_{\text{f}}^T}{\omega_{\text{pe}}}, \]  

\[ \frac{\omega_\perp \alpha_F}{\delta_t^2} \cong \frac{(B'_c)^2}{4\pi n_{\text{e}}}, \quad \tilde{\beta}_p \equiv \frac{\omega_{\text{e}}}{\omega_\perp k^2 \rho_i^2}, \quad \rho_i^2 \equiv \frac{T}{m \Omega_{\text{ce}}}, \]  

\[ \varepsilon_{\text{f}} \equiv \frac{\kappa_{\text{f}}}{\delta_t^2}, \quad \varepsilon_{\text{f}} \equiv \frac{D_0}{\omega_\perp^2 \omega_{\text{pe}}}, \quad \varepsilon_{\text{e}} \equiv \frac{\omega_\perp \delta_m}{\delta_t^2} D_{\text{le}} \delta_t^2. \]  

We note that \( \tilde{\beta}_p \) can be considered finite while \( \delta_1^2 \gg \rho_i^2 \) for the validity of the adopted two-fluid equations. Moreover, with the adopted choice of the variables we may consider that \( \tilde{B} - \tilde{T} \) when \( \tilde{x} \sim 1, \) while \( \varepsilon_{\text{f}} \ll 1, \varepsilon_{\text{f}} \ll 1, \) and \( \varepsilon_{\text{f}} \ll 1. \)

In order to show the possibility that the phase velocity of the relevant modes can change sign, we adopt the constant-\( \tilde{B}_\perp \) approximation, that is, \( B_\perp \equiv \tilde{B}_\perp \equiv \tilde{B}_0/(B'_c k^2 \delta_1^2), \) which is valid for \( (\omega - \omega_{\text{e}})/\omega_{\text{e}} \delta_1^2 < 1, \) and assume for simplicity that \( \varepsilon_{\text{f}} = \varepsilon_{\text{f}} = \varepsilon_{\text{f}} = 0. \) It is also convenient to use the dimensionless variables

\[ \tilde{x} \equiv \frac{x - x_0}{\delta_t}, \quad \tilde{B} \equiv \frac{B}{B_0}, \quad \tilde{T} \equiv \frac{T}{\tau_{\text{ce}}}, \quad \tilde{Z} \equiv \frac{\xi}{\delta_t}, \quad \tilde{W} \equiv \frac{\xi}{\delta_t} \frac{Z}{\delta_t}. \]  

Then, equations (25)–(27) can be written as

\[ \frac{-\Delta_0 \omega - \omega_{\text{e}} \delta_m^2}{\omega} \frac{\partial^2 \tilde{W}}{\partial \tilde{x}^2} \cong \tilde{x} \frac{\partial^2 \tilde{B}}{\partial \tilde{x}^2}, \]
\[
\frac{d^2 U}{d\tau^2} - \pi^2 U \simeq -\pi, \quad (34)
\]
\[
\lambda_0^2 \frac{d^2 W}{d\tau^2} - \pi^2 W \simeq -\pi - \frac{\omega_0}{\omega_{\text{ue}}} \frac{d^2 U}{d\tau^2}, \quad (35)
\]
where
\[
\lambda_0^2 \equiv -\frac{\omega_0^2(\omega - \omega_{\text{ue}})}{\omega_0^2(\omega - \omega_{\text{ue}})} k_0^2 \delta_{\text{in}}^2, \quad (36)
\]
and
\[
\tilde{S} \equiv 1 - \frac{\omega_0^2}{\omega^2} \left( \frac{\delta_{\text{in}}}{\beta_p} \right)^2 \frac{\pi^2}{\delta_{\text{in}}}. \quad (37)
\]
Then, by virtue of equations (33)–(35), the matching condition leading to the dispersion relation can be expressed as
\[
\Delta \frac{\delta_{\text{in}}^2}{\delta_{\text{in}}} = \int_{-\infty}^{\infty} \frac{d\omega}{\omega_{\text{ue}}} \left[ (\omega - \omega_{\text{ue}})(1 - \pi W) - \omega_{\text{I}}(1 - \pi U) \right]. \quad (38)
\]
With the definitions
\[
I_0^0 \equiv \int_{-\infty}^{\infty} \frac{d\tau}{\tilde{S}} (1 - \pi W), \quad \text{and} \quad I_0^1 \equiv \int_{-\infty}^{\infty} \frac{d\tau}{\tilde{S}} (1 - \pi U), \quad (39)
\]
Equation (38) can be rewritten as
\[
\omega \{D_0^0 - T(\omega)\} = -\{\omega_{\text{I}} + \omega_{\text{ue}} T(\omega)\}, \quad (40)
\]
where
\[
D_0^0 \equiv \frac{D_0}{T_0} \quad \text{and} \quad T(\omega) \equiv \frac{T_0^1}{T_0}. \quad (41)
\]
The possibility that \(\omega\) can change sign, when the relative magnitudes of \(D_0^0\) and \(T\) are varied, is evident from equation (40), provided that \(T > 0\). For \(\omega_{\text{I}} = 0\) and \(\tilde{S}_{\text{L}} = 1\), we know from our previous work [3] that \(I_0^0 = \sqrt{2} \Gamma(3/4)/2 \gtrsim Q > 0\) and \(I_0^1 = \lambda_0^2 Q > 0\) so that \(T > 0\), and that ‘drift-tearing’ types of mode but with frequency \(\omega \approx \omega_{\text{ue}} < 0\) are realized. Clearly, these modes correspond to \(D_0^0 > T\), and have phase velocities in the direction of the ion diamagnetic velocity. We then argue that \(T(\omega)\) remains positive for \(\omega_{\text{I}} > 0\) and for \(\tilde{S}_{\text{L}}\) given by equation (37).

Hence, according to equation (40), by lowering the value of \(D_0^0\) (i.e. by lowering the value of \(\Delta\)) so that \(D_0^0 < T\), modes with frequency \(\omega > \omega_{\text{ue}} > 0\) can be obtained. By examining the order of magnitude of the terms in equations (25)–(27) we conclude that \(\omega \approx \omega_{\text{ue}}\) is a significant limit to investigate. These modes will have phase velocities in the direction of the electron diamagnetic velocity.

In order to facilitate the investigation of \(\omega \approx \omega_{\text{ue}}\) modes, we substitute \(\omega = \omega_{\text{ue}} + \delta \omega\), where \(|\delta \omega| < |\omega_{\text{ue}}|\) in equations (25)–(27) and keeping the leading order terms arrive at the following set of equations to analyze for the inner region
\[
\varepsilon \left(1 - i\varepsilon \frac{d^2}{d\tau^2}\right) \frac{d^2 \tilde{Z}}{d\tau^2} \simeq \frac{d^2 \tilde{B}_0}{d\tau^2}, \quad (42)
\]
and
\[
\frac{\eta \varepsilon \alpha \tau}{\varepsilon_0} \frac{d^2 T}{d\tau^2} - \lambda_0 (\pi^2 \tilde{Z} - \pi \tilde{B}_0) - 1 - i\varepsilon = \frac{d^2 \tilde{Z}_0}{d\tau^2}, \quad (43)
\]
and
\[
\frac{\eta_0 \varepsilon_0 \alpha \tau}{\varepsilon_0} \frac{d^2 T}{d\tau^2} - \lambda_0 (\pi^2 \tilde{Z} - \pi \tilde{B}_0) - 1 - i\varepsilon = \frac{d^2 \tilde{Z}_0}{d\tau^2}, \quad (44)
\]
Here
\[
\varepsilon_s \equiv \frac{\beta_0^2}{\beta_{ps}}, \quad \beta_{ps} \equiv \frac{\omega_0(\omega_{\text{ue}} - \omega_{\text{ei}})}{\omega_0 k_0^2 \beta_p^2}, \quad \lambda_0 \equiv \frac{\kappa \omega}{\varepsilon_0 \omega_{\text{ue}}}, \quad \tilde{S} \equiv 1 + i\varepsilon - \frac{\pi^2}{\beta_{ps}}, \quad (45)
\]
and other quantities have been defined earlier. We note that \(\beta_{ps}\) can be considered finite while \(\delta_0^2 \gg \beta_p^2\) for the validity of the adopted two-fluid equations, and, as mentioned earlier, with the adopted choice of the variables we may consider that \(\tilde{B}_0 \equiv \tilde{T}_0\) when \(\tau \approx 1\), while \(\varepsilon_s \ll 1, \varepsilon_0 \ll 1, \text{and} \varepsilon_0 \ll 1\).

6. Some analytical considerations

The solutions that we consider, in the limit where \(\varepsilon_s\) can be neglected, are characterized by \(\tilde{B}_0\), being an even function of \(\pi\) while \(\tilde{Z}\), \(\tilde{B}_0\), as odd functions of \(\pi\) such that \(\tilde{Z}(\pi \to \infty) \to \tilde{B}_0 / \pi\) and \(\tilde{T}(\pi \to \infty) \to \tilde{B}_0 / \pi\). Here \(\tilde{B}_0 \equiv \tilde{T}_0(\pi = 0)\). In particular, the boundary conditions, under which equations (42)–(44) are solved and \(\lambda_0\) is determined, are \(\tilde{B}_0 = 1, \quad \tilde{B}_0 / \pi \to 0, \quad \tilde{Z} = 0\), and \(\tilde{T} = 0\) at \(\pi = 0\), together with \(d\tilde{B}_0 / d\tau \to \tilde{B}_0 (\Delta \delta_0) \pi / \pi^2, \quad \tilde{Z} \to 0, \quad \text{and} \quad \tilde{T} \to 0\) for \(\pi \to \infty\). We note that the resistive term \(\varepsilon_0\) has an important role in removing the singularity at \(\pi^2 = \beta_{ps}\), that would affect strongly the solution of equation (44). Thus we are led to take
\[
\tilde{B}_0 = \tilde{B}_0 (1 + \varepsilon_0 \phi(\pi) + \Delta \delta_0 |\pi|), \quad (46)
\]
considering \((\omega_0 \alpha \tau / \varepsilon_0)\) as a finite quantity. It is clear that there is another ‘imbedded’ region around \(\pi^2 = \beta_{ps}\), and that the width of this region also has to exceed the ion Larmor radius.

When analyzing the ‘innermost region’, in the limit where \(\varepsilon_s\) can be neglected, it is convenient to use the dimensionless variables
\[
\tilde{\pi} \equiv \frac{\pi - \pi_0}{\delta_{\text{in}}}, \quad \tilde{B} (\tilde{\pi}, \pi) \equiv \frac{\tilde{B}_0}{\tilde{B}_0}, \quad \tilde{U} (\tilde{\pi}, \pi) \equiv \frac{\tilde{T}_0}{\tilde{B}_0}, \quad (47)
\]
where \(\delta_{\text{in}}\) is given by equation (23), and \(\varepsilon_{\text{in}} = \delta_{\text{in}} / \beta_{\text{in}} < 1\). Then, equation (43) becomes
\[
\frac{\partial^2 \tilde{U}}{\partial \tilde{\pi}^2} \simeq -\tilde{\pi}^2 \tilde{U} \simeq -\tilde{U} (\tilde{\pi}, \pi). \quad (48)
\]
If the constant-\(\tilde{B}_0\) approximation \((\tilde{B}_0 \approx \tilde{B}_0)\) is adopted, \(\tilde{B} \approx 1\) in equation (48) and the analytic solution of the inhomogeneous equation for \(\tilde{U}\) is given by
and corresponding to a dissipative instability, examples in equation (44) and derive modes with for that are obtained for , and for values of the other parameters as: \(\alpha_T = 1.0\), \(\eta_{\text{ad}} = 1.0\), \(\beta_{\text{ps}} = 0.16\), \(\epsilon_{\mu} = \epsilon_{\eta} = 0.01\), \(\epsilon_{e} = 0.2\), and \(\epsilon_{x} = 0\).

\[
\mathcal{U} = \frac{1}{2} \int_0^1 dt (1 - t^2)^{-1/4} \exp(-t^2/2) \quad (49)
\]

with the asymptotic behaviors: \(\mathcal{U} \approx \alpha \mathcal{T}\), for \(|\mathcal{T}| \ll 1\), and \(\mathcal{U} \approx 1/\mathcal{T}\), for \(|\mathcal{T}| \gg 1\), where \(\alpha = (\Gamma(3/4))^2/\sqrt{2\pi}\). On the basis of this, we may introduce an approximate function representing \(\partial^2 \mathcal{T}_e/\partial x^2\) in equation (44) and derive \(\alpha = Z_M/\mathcal{B}_0\) as the solution of a relatively simple inhomogeneous equation. In particular, an expression for \(\mathcal{B}\) that is consistent with equation (44) and simulates that derived numerically is

\[
\mathcal{B} \approx 1 + (\epsilon_{\eta} B_\text{in}) \phi_{\eta}(\mathcal{X}) + \epsilon_{e} \phi_{e}(\mathcal{X}) + \Delta \delta t/|\mathcal{T}| \quad (50)
\]

then the proper value of \(\Lambda_s\) is obtained by satisfying the asymptotic matching conditions among the considered regions as \(\Lambda_s = \Lambda_s(\eta_{\text{ad}}, \alpha_T, \epsilon_{\eta}, \epsilon_{e}, \epsilon_{\mu}, \Delta_0)\), where \(\Delta_0 \equiv \Delta \delta t/\epsilon_x \ll 1\).

7. Results of the numerical analysis

Numerical solutions of equations (42)–(44), under the boundary conditions stated earlier, have been obtained for \(\text{Im}(\Lambda_s) > 0\), corresponding to a dissipative instability, examples being represented by figures 1 and 2. In particular, the mode growth rate is seen to increase with \(\epsilon_{\eta}\) and with \(1/\alpha_T\), that is, with the peak value of \(\mathcal{T}_e\). A (positive) growth rate is also obtained when \(\epsilon_{\mu}\) is neglected and the effects of a finite viscosity represented by \(\epsilon_{\mu}\) are taken into account.

We note that under conditions where \(\omega \approx \omega_{\text{dr}}\), where the analysis reported in [3] is valid, the mode growth rate is associated with the effects of a finite viscosity, that is \(\epsilon_{\mu} = 0\), an example being represented by figure 3.

A different case to be considered is the following. In the case where the current density \(J_z\) is peaked at \(x = 0\) and \(dJ_z/dx = 0\) at \(x = x_0\), and the appropriate solution of equation (4) in the outer region is obtained, the solution for the inner region equations will involve an even and an odd component of \(\mathcal{B}_0(\mathcal{X})\) unlike the case considered here for simplicity.

8. Formation of high energy electron and ion populations

We propose that a sequence of mode-particle resonances [6] starting from the excitation of oscillatory modes associated with magnetic reconnection is responsible for the generation of experimentally observed high energy electron populations following magnetic reconnection events. In particular, the mode-particle resonances involving reconnecting modes concern frequencies of the order of \(\omega_{\text{Rec}} = \epsilon_{\omega_{\text{Rec}}} \omega_{\text{dr}}\), where

\[
|\omega_\text{d}| = k_y v_{\text{th}}(\rho_f/2 \rho) \quad \text{and} \quad 1/\rho_f \equiv -(dp_f/dx)/p_f.
\]
A theoretical scenario that can be analyzed within the limits of quasi linear theory is the formation of a ring, in the velocity space $v_{th}, v_{L}$, of a highly superthermal ion population, by a pitch angle scattering process driven by excited reconnecting modes. Indicating their frequency by $\omega_{\text{Rec}}$, it is realistic to consider $|\omega_{\text{Rec}}| \ll \Omega_{ci}$ and the mode-particle resonance

$$\Omega_{ci} - k_{\|}^{(2)} \omega_{LH}^{\text{res}} \simeq 0$$

(51)

producing a pitch angle scattering from the parallel direction to a perpendicular direction to the magnetic field. Here $k_{\|}^{(2)} = k_{\|}(x^{(1)} - x_{0}/L_{c})$. Another mode particle resonance involving values of $v_{L}$ close to those of $v_{th}$ and involving the supply of longitudinal energy to make the process represented by equation (51) significant is

$$\omega_{\text{Rec}} - k_{\|}^{(2)} \nu_{\text{th}}^{\text{res}} = 0.$$  

(52)

We may argue that this can be responsible for providing the longitudinal energy relevant to equation (51). If we consider the maximum value of $k_{\|}^{(2)}$ as being of the order of $k_{\|} L_{c}$ we have $v_{th}^{\text{res}}/\nu_{\text{th}} \sim L_{c} / k_{\|} \rho_{L} \gg 1$.

Then lower hybrid (LH) modes excited by the superthermal ion distribution can resonate with the ions and the superthermal electrons simultaneously. The relevant mode-particle resonance conditions involve

$$\omega_{\text{LH}} = k_{\|}^{\text{LH}} \nu_{\text{LH}},$$

(53)

where $k_{\|}^{\text{LH}} > k_{\|}$ and $\nu_{\text{LH}} > v_{\text{th}}$. The transverse ion kinetic energy is thus transferred to the longitudinal electron kinetic energy creating a high energy electron population.

A second envisioned scenario excludes the presence of a pitch angle scattering represented by the mode particle resonance (51) and consider the formation of a superthermal spike in $v_{L}$ out of the tail of the thermal ion distribution due to the mode-particle resonance (52). Then we may argue that a LH mode is excited through the resonance condition

$$\omega_{\text{LH}} = k_{\|}^{\text{LH}} \nu_{\text{LH}},$$

(54)

where $\nu_{\text{LH}}$ is located in the region where the slope of the superthermal spike is positive. Following this, the excited lower hybrid modes may undergo non-linear Landau damping [9] interacting with the superthermal tail of the electron distribution and transfer energy to the resonating superthermal electrons. The relevant non-linear Landau damping process can be represented by

$$\omega_{\nu} + \omega_{\nu} = (k_{\|} + k_{\|}') \cdot v = 0.$$  

(55)

Here $\omega_{\nu} + \omega_{\nu} \simeq 2\omega_{\text{LH}}$ and we may take $k_{\|} + k_{\|}' \simeq 0$. Therefore the resonance condition (55) reduces to

$$2\omega_{\text{LH}} \simeq (k_{\|} + k_{\|}') \nu_{\text{LH}},$$

(56)

with $\nu_{\text{LH}} > v_{\text{th}}$, leading to the formation of a high energy electron population.

9. Momentum of reconnecting modes

The modes that we have found for sheared field configurations and that can produce magnetic reconnection in low collisionality regimes are of the oscillatory type and have characteristic magnitudes and directions of their phase velocities. These modes acquire momentum extracting from the main body of the plasma column. As a consequence, they can induce a ‘spontaneous rotation’ within the main body of the plasma column [10] and the experimental observations are consistent with this theoretical indication.

The procedure to be followed in order to evaluate the momentum of plasma modes that can be described as waves is well known [11]. On the other hand, referring to the simplified plane geometry considered earlier, reconnecting modes are standing in the $x$-direction and propagating in the $y$ and $z$ directions. Nevertheless, we adopt here the procedure given in [11].

We consider that the reconnecting modes evolve adiabatically from zero amplitudes at $t = -\infty$ to some finite amplitudes at $t = 0$. From $t = -\infty$ to $t = 0$, the modes have an infinitesimal growth rate $\gamma$ and are driven by an externally produced charge separation with charge density $\rho_{c}$. The current density $\mathbf{j}^{\text{ext}}$ is related to $\rho_{c}$ by

$$\frac{\partial}{\partial t} \rho_{c} + \nabla \cdot \mathbf{j}^{\text{ext}} = 0.$$  

(57)

Accordingly, the momentum $\mathbf{P}$ of the considered electromagnetic modes is given by

$$\mathbf{P} = -\int \mathbf{E} \, dV = -\int_{-\infty}^{0} dt \left[ \rho_{c}^{\text{ext}} \mathbf{E} + \frac{1}{c} (\mathbf{j}^{\text{ext}} \times \mathbf{B}) \right].$$

(58)

The equations relating $\rho_{c}^{\text{ext}}$ and $\mathbf{j}^{\text{ext}}$ to the total electric field ($\mathbf{E}$) and magnetic field ($\mathbf{B}$) are

$$\rho_{c}^{\text{ext}} = \frac{1}{4\pi} \nabla \cdot \mathbf{E} - \rho_{\text{i}},$$

(59)

$$\mathbf{j}^{\text{ext}} = \frac{c}{4\pi} \left( \nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \right) - \mathbf{j}.$$  

(60)

Here $\rho_{\text{i}}$ is the induced charge density, and it is related to the induced current density $\mathbf{j}$ through the charge conservation equation

$$\frac{\partial \rho_{\text{i}}}{\partial t} + \nabla \cdot \mathbf{j} = 0.$$  

(61)

while $\mathbf{E}$ and $\mathbf{B}$ are related through the induction equation

$$\nabla \times \mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}.$$  

(62)

For electrostatic modes, considered in [11], $\mathbf{B} = 0$, $\mathbf{E} = -\nabla \varphi$, where $\varphi$ is the electrostatic potential, and the expression for momentum $\mathbf{P}$ reduces to

$$\mathbf{P}_{\text{es}} = \int dV \int_{-\infty}^{0} dt (\rho_{c}^{\text{ext}} \nabla \varphi),$$

(63)
with $\rho_{\text{ext}}^z$ given by equation (59).

In the case of reconnecting modes the relevant perturbations are taken to be of the form

$$\tilde{f} = \frac{1}{2} \{ \tilde{f}(x) \exp[-i(\omega + i\gamma)t + ik_x y + ik_z z] + \text{c.c.} \}$$

then we are led to refer to the $k$-component of the momentum surface density $\tilde{P}_k$, given by

$$\frac{1}{2} \int_{-\infty}^{\infty} dx \int_{-\infty}^{0} d\omega \left[ \left( \rho_{\text{ext}}^z # + \frac{1}{c} J_k^\text{ext} \times B_k^* + \text{c.c.} \right) \right] \exp(2i\gamma)$$

$$\frac{1}{2} \gamma \int_{-\infty}^{\infty} dx \left( \rho_{\text{ext}}^z # + \frac{1}{c} J_k^\text{ext} \times B_k^* \right)$$

in particular, we are interested in the $z$-component of $\tilde{P}_k$, that becomes

$$\tilde{P}_k \approx -\frac{1}{2 \gamma} \Re \int_{-\infty}^{\infty} dx \left( ik_z \rho_{\text{ext}}^z + \frac{1}{c} J_k^\text{ext} \times B_k^* \right)$$

as $\tilde{P}_k \approx -ik_z \rho_{\text{ext}}^z + i(\omega/c) A_k^z$ and, for the considered modes, $\tilde{P}_k \approx -ik_z \rho_{\text{ext}}^z + i(\omega/c) A_k^z$, implying that $|\omega A_k^z/c| \ll k_z \rho_{\text{ext}}^z$. A detailed evaluation of $\tilde{P}_k$ will require the analysis of specific significant cases.

We note that these considerations can be extended to the reconnecting modes that can be excited in axisymmetric toroidal configurations described in [12]. In that case, instead of $\tilde{P}_k$, we have to deal with the toroidal angular momentum, and we consider the analyzed modes, propagating along $z$, as a simulation of those propagating along the toroidal direction. Clearly a complete extension of this theory to axisymmetric toroidal confinement configurations requires a further development parallel to that for the theory of resistive one-fluid modes described in [12].

We also note that, as pointed out in [13, 14], the spontaneous generation of magnetic fields in laser-produced plasmas is associated with sharp discontinuities in plasma properties such as sharp temperature gradients and the excitation of plasma surface modes localized around the surface on scale lengths of the order of a collisionless skin depth. The analogy to the theory presented here is that the key role of electron temperature gradient and of the inductive skin depth, with the generation of new magnetic field topologies, are features that the two classes of process have in common.

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