B → K∗γ Decay within MSSM

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The paper deals with a next-to-leading order analysis of the radiative B → K∗γ decay. Working in a PQCD approach, we compute the correction to the essential form factor, coming from a single gluon exchange with the spectator. We investigate the supersymmetry effects on the branching ratio and direct CP asymmetry and constrain the squark mixing parameter (δ23)LR.

I. INTRODUCTION

After the Cabibbo-favoured b → sγ mode was first reported, in 1993, by CLEO II [1] and updated in 1995 [2], the exclusive radiative decays, B → K∗γ and B → ργ, as well as the inclusive ones, B → Xs(γ)γ, have become main targets for both experimental and theoretical investigations. The exclusive modes, which are easier to be experimentally investigated [3, 4, 5], but less theoretically clear, have been worked out in different approaches. For example, the spin symmetry for heavy quarks combined with wave function models [6, 7] or the heavy quark effective theory when both b and s are heavy [8] have been used. Also, perturbative QCD (PQCD) formalisms, introduced for exclusive nonleptonic heavy-to-light transitions, have been extended to account for the radiative decays. Recently, detailed analyses of B → K∗γ and B → ργ, in the next-to-leading order (NLO), with the inclusion of hard spectator and vertex corrections, have been performed [9-11] and a consistent treatment, based on a new factorization formula, has been proposed [12]. Besides an independent determination of the |V_{td}/V_{ts}| ratio, the b → sγ decays are suitable for studying the viability of SUSY extensions of the SM, in view of flavour changing neutral currents (FCNC) and CP tests, and for imposing constraints on the supersymmetric benchmark scenarios [13, 14].

The aim of the present paper is to analyse the B → K∗γ decay, in the minimal supersymmetric SM (MSSM) context. First, at next-to-leading order, we compute the hard-spectator correction to the essential form factor. In this respect, we employ the PQCD approach developed by Szczepaniak et al. for decays dominated by tree diagrams [15] and later extended to penguin processes [16]. In order to fit the Br estimation with data and since large CP asymmetries, which are expected to be measured more precisely in the near future, at the hadron machines, seem to have a new physics origin, we extend our analysis beyond the SM. In this respect, we make use of the mass insertion method and include, in the Wilson coefficients C7,8, gluino-mediated FCNC contributions. Finally, the Br data and the BaBar allowed range for direct CP asymmetry are used to constrain the squark mixing parameter (δ23)LR.

II. NLO CORRECTION TO FORM FACTOR

The effective Hamiltonian describing the B → K∗γ radiative decay is given by [9, 10]

\[ H = \frac{G_F}{\sqrt{2}} \lambda_p \left[ C_7 O_7 + C_1 O_1^p + C_8 O_8 \right], \]

where \( \lambda_p \equiv V_{pb}V_{pa}^* \), with p summed over u and c, and C1, C7, C8 are the effective Wilson coefficients at \( \mu = m_b \). The hadronic matrix elements of the four-fermion operator and of the electromagnetic and chromagnetic penguin operators

\[ O_1^p = \frac{m_b}{8\pi^2} \tilde{s}_\gamma \mu (1 - \gamma_5) p \tilde{p} \gamma^\mu (1 - \gamma_5) b, \]

\[ O_7 = \frac{em_b}{8\pi^2} \tilde{s} \sigma^{\mu\nu} (1 + \gamma_5) b F_{\mu\nu}, \]

\[ O_8 = \frac{g_8 m_b}{8\pi^2} \tilde{s} \sigma^{\mu\nu} (1 + \gamma_5) T_i b G^{i\mu\nu}, \]

possess the general Lorentz decomposition:
where $q_\mu$ is the momentum of the photon and $\epsilon^* \epsilon$ is the $K^*$ 4-vector polarization. The form factors are not known from first principles and this imprecise knowledge is a major source of mismatch between theory and data as well as between different theoretical estimations [9].

In the heavy quark limit, $m_b \gg \Lambda_{QCD}$, by neglecting the corrections of order $1/m_b$ and $\alpha_s$, one has the following relation among the form factors [9]

$$ \frac{m_B}{m_B + m_{K^*}} V(0) = \frac{m_B + m_{K^*}}{m_B} A_1(0) = T_1(0) = T_2(0) \equiv F_{K^*}(0) \quad (5) $$

This is broken when one includes QCD radiative corrections coming from vertex renormalization and hard gluon exchange with the spectator. At order $\alpha_s$, the form factors encode strong interaction effects by receiving an additive correction from hard spectator interaction [11]. We recommend [9, 10, 12] for detailed analyses of both factorizable and nonfactorizable vertex and hard-spectator contributions, involving the operators $O_7$, $O_8$ and penguin-type diagrams of $O_1$. However, it has been stated that factorization holds, at large recoil and leading order in $1/m_b$, and quantitative tests for proving QCD factorization at the level of power corrections have been provided [17].

For a consistent treatment of radiative decays, at next-to-leading order in QCD, a novel factorization formula have been proposed in [12]. In this approach, the hadronic matrix elements in (1) are written in terms of the essential form factor, which describes the long-distance dynamics and is a nonperturbative object, and of the hard-scattering kernels, $T^I_i$ and $T^{II}_i$, including the perturbative short-distance interactions, as

$$ \langle K^* | O_1 | B \rangle = [F_{K^*}(0) T^I_i + \phi_B \otimes T^{II}_i \otimes \phi_{K^*}] \cdot \eta, \quad (6) $$

where $\eta$ is the photon polarization. When the dominant contribution comes from $O_7$, we use (4) to write down the decay amplitude as

$$ A^{(0)} = \frac{G_F}{\sqrt{2}} \frac{\lambda_p}{2\pi^2} \frac{m_{b}^2(\mu)}{2\pi^2} C_7(\mu) F_{K^*}(0) \left[ \epsilon_{\mu\nu\rho\sigma} \eta^{\rho} \epsilon^{*\nu} P_{K^*}^\rho P_B^\sigma - i (P_{K^*} \cdot \eta) \epsilon^* + i (\epsilon^* \cdot (\eta P_{K^*} \cdot \eta) \right], \quad (7) $$

and consequently the branching ratio reads

$$ B^{I,LO} = \frac{G_F^2 \alpha_s |\lambda_p|^2 m_{b}^2}{128\pi^4} m_{b}^2 (1 - z^2)^3 |C_7(\mu)|^2 |F_{K^*}(0)|^2, \quad (8) $$

where $z = m_{K^*/m_B}$. At next-to-leading order in $\alpha_s$, one has to consider, in (6), the contributions to the hard scattering kernels $T^I_i$ coming from the operators $O_1$ and $O_8$. These have been evaluated in [12] and bring (7) to the expression.
\[ A = \frac{G_F}{\sqrt{2}} \lambda_\rho \frac{e m_b(\mu)}{2\pi^2} \left[ C_7 + \frac{\alpha_s C_F}{4\pi} (C_1 G_1^p + C_8 G_8) \right] F_K(0) \left[ \varepsilon_{\mu\nu\alpha\beta}(\eta^\ast \epsilon^{\nu\rho} P_K^\ast P_B^\rho - i (P_K \cdot q)(\eta^\ast) + i(\epsilon^\ast)(\eta P_K^\ast) \right], \]

where \( C_F = (N^2 - 1)/(2N) \), \( N = 3 \), and

\[ G_1(s) = -\frac{833}{162} - \frac{20i\pi}{27} + \frac{8\pi^2}{9} s^{3/2} + \frac{2}{9} [48 + 30i\pi - 5\pi^2 - 2i\pi^3 - 36\zeta(3) + (36 + 6i\pi - 9\pi^2) \ln s + (3 + 6i\pi) \ln^2 s + \ln^3 s] s^2 \]

\[ + \frac{2}{9} [18 + 2\pi^2 - 2i\pi^3 + (12 - 6\pi^2) \ln s + 6i\pi \ln^2 s + \ln^3 s] s^2 \]

\[ + \frac{1}{27} [-9 + 112i\pi - 14\pi^2 + (182 - 48i\pi) \ln s - 126 \ln^2 s] s^3, \]

\[ G_8 = \frac{11}{3} - \frac{2\pi^2}{9} + \frac{2\pi}{3}, \]

\[ \times \begin{array}{c|c|c}
\bar{k}_B & \phi & \bar{k}_B \\
B & Q & B \\
\end{array} \]

\[ \begin{array}{c|c|c}
\bar{P}_B & k_b & \bar{k}_s \bar{P}_K \\
\bar{B} & Q & \bar{B} \\
\end{array} \]

\[ \begin{array}{c}
\phi_B = \frac{f_B}{12} \varphi_B(x)(P_B + m_B)\gamma_5 \end{array} \]
\[ Br(B^+ \to K^{*+}\gamma) = \begin{cases} 
(3.83 \pm 0.62 \pm 0.22) \times 10^{-5} \ (\text{BaBar} \ [3]) \\
(3.76_{-0.83}^{+0.89} \pm 0.28) \times 10^{-5} \ (\text{CLEO} \ [4]) \\
(3.89 \pm 0.93 \pm 0.41) \times 10^{-5} \ (\text{Belle} \ [5]) 
\end{cases} \]

\[ Br(B^0 \to K^{*0}\gamma) = \begin{cases} 
(4.23 \pm 0.40 \pm 0.22) \times 10^{-5} \ (\text{BaBar} \ [3]) \\
(4.55_{-0.72}^{+0.86} \pm 0.34) \times 10^{-5} \ (\text{CLEO} \ [4]) \\
(4.96 \pm 0.67 \pm 0.45) \times 10^{-5} \ (\text{Belle} [5]) 
\end{cases} \]

whose world average value, over the \( B^\pm \) and \( B^0 \) decay modes, is

\[ Br_{exp}(B^\pm \to K^{*\pm}\gamma) = (4.22 \pm 0.28) \times 10^{-5} \ (17) \]

For the direct CP asymmetry,

\[ a_{CP} = \frac{\Gamma(B \to K^{*}\gamma) - \Gamma(B \to K\gamma)}{\Gamma(B \to K^{*}\gamma) + \Gamma(B \to K\gamma)}, \]

which is predicted by the SM to be \( a_{CP} < 0.005 \), the BaBar and CLEO data are: \( a_{CP} = -0.044 \pm 0.076 \pm 0.012 \) (BaBar [3]) and \( a_{CP} = 0.08 \pm 0.13 \pm 0.03 \) (CLEO [4], for the sum of neutral and charged \( B \to K^{*}\gamma \) decays). Even though the data look consistent with the SM result within 1\( \sigma \), it is too early to draw a definitive conclusion because of the large errors. With BaBar and Belle large data samples, there is hope for precise measurements of large CP asymmetries and this makes room for new physics. Moreover, it has been stated that CP asymmetries larger than few percent would have a dominantly supersymmetric origin [19].

III. BRANCHING RATIO AND DIRECT CP ASYMMETRY WITHIN MSSM

Following this idea, let us analyse the \( B \to K^{*}\gamma \) decay in the MSSM context. Using the mass insertion approximation, [20], we incorporate, in the Wilson coefficients \( C_7 \) and \( C_8 \), the FCNC SUSY contributions

\[ C_7^{\text{SU}}(M_{\text{SU}}) = \frac{\sqrt{2} \alpha_s}{G_F V^{V}\bar{V} \cdot V^{V}} \left( \delta_{23}^{d} \right)_{LR} \frac{m_{\tilde{g}}}{m_\mu} F_0(x); \]

\[ C_8^{\text{SU}}(M_{\text{SU}}) = \frac{\sqrt{2} \alpha_s}{G_F V^{V}\bar{V} \cdot V^{V}} \left( \delta_{23}^{d} \right)_{LR} \frac{m_{\tilde{g}}}{m_\mu} G_0(x), \]

where

\[ F_0(x) = -\frac{4x}{9(1-x)^4} \left[ 1 + 4x - 5x^2 + 4x \ln(x) + 2x^2 \ln(x) \right], \]

\[ G_0(x) = \frac{x}{3(1-x)^4} \left[ 22 - 20x - 2x^2 + 16x \ln(x) - x^2 \ln(x) + 9 \ln(x) \right] \]

In (20), \( x = m_{\tilde{g}}^2/m_\mu^2 \) is expressed in terms of the gluino mass, \( m_{\tilde{g}} \), and an average squark mass, \( m_{\tilde{s}} \). We underline that, in the expressions of \( C_7^{\text{SU}}(M_{\text{SU}}) \) and \( C_8^{\text{SU}}(M_{\text{SU}}) \), we have kept only the left-right squark mixing parameter \( \left( \delta_{23}^{d} \right)_{LR} = (\Delta_{h_{\mu}})/m_\mu^2 \) since, being proportional to the large factor \( m_{\tilde{g}}/m_\mu \), it has a significant numerical impact on the branching ratio value. The quantities \( \Delta_{h_{\mu}} \) are the off-diagonal terms in the sfermion mass matrices, connecting the flavours \( b \) and \( s \) along the sfermion propagators. In these assumptions, the total Wilson coefficients, encoding the new physics, become

\[ C^{\text{total}}_{7,8}[x,\delta] = C_7(m_\mu) + C_7^{\text{SU}}(m_{\tilde{g}}), \]

\[ C^{\text{total}}_{7,8}[x,\delta] = C_8(m_\mu) + C_8^{\text{SU}}(m_{\tilde{g}}), \]

where \( C_{7,8}^{\text{SU}}(m_{\tilde{g}}) \) have been evolved from \( M_{\text{SU}} = m_{\tilde{g}} \) down to the \( \mu = m_\mu \) scale, using the relations [21]

\[ C_8^{\text{SU}}(m_{\tilde{g}}) = \eta C_8^{\text{SU}}(m_{\tilde{g}}), \]

\[ C_7^{\text{SU}}(m_{\tilde{g}}) = \frac{8}{3} \eta - \eta^2 C_8^{\text{SU}}(m_{\tilde{g}}), \]

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with

\[ \eta = \left( \frac{\alpha_s(m_\tilde{g})}{\alpha_s(m_t)} \right)^{2/21} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{2/23} \]  

(23)

Finally, putting everything together, we replace, in (9), the Wilson coefficients \( C_7 \) and \( C_8 \) respectively by \( C_{total}^{7}[x, \delta] \) and \( C_{total}^{8}[x, \delta] \), the form factor \( F_{K^*}(0) \) by \( F_{K^*}(0) + F^{sp}(a) \) and, consequently, the branching ratio is

\[ Br^{total} = Br^{SM+SUSY} = \tau_B \frac{G_F^2 m_B^2}{32\pi^4} m_B^3 (1 - z^2)^3 |F_{K^*}(0) + F^{sp}(a)|^2 \]

\[ \times \left| \lambda_p \left[ C_{total}^{7}[x, \delta] + \frac{\alpha_s C_F}{4\pi} \left( C_1 G_0^p + C_8^{total}[x, \delta] G_8 \right) \right] \right|^2 \]  

(24)

For a given \( x \) and \( \delta \equiv \rho e^{i\varphi} \), the total branching ratio (24) is depending on three free parameters: \( a, \rho, \varphi \). One can notice that, by including the hard gluon contributions as a correction to \( F_{K^*}(0) \), the direct CP asymmetry parameter, (18), is free of the uncertainties coming from the form factors.

In what it concerns the gluino, as its pair production cross section has large cancellations in the \( e^+e^- \) annihilation, there is hope that the laser-backscattering photons will provide a precise gluino mass determination [22]. For a wide range of squark masses, a gluino mass of 540 GeV may be measured, with a precision of at least \( \pm 2 \ldots 5 \), at the multi-TeV linear collider at CERN.

In the next coming discussion, we use the following input parameters: \( m_b(m_b) = 4.2 \text{ GeV}, \alpha = 1/137, |V_{tb}V_{ts}^*| = 0.0396 \pm 0.002, \tau_B^\rho = (1.546 \pm 0.018) \text{ ps}, \)

\( m_q = 500 \text{ GeV and } x \) has the values \( x_1 = 0.3, x_0 = (540/500)^2 \text{ and } x_2 = 3 \) (where \( l(g) \) comes from \( m_\tilde{g} \) less (greater) than \( m_\tilde{g} \)).

In figure 2, we represent the contour plots on which the \( Br^{total} \), (24), is equal to the world average data (17) and the BaBar constraint [3]

\[ 0.17 < a_{CP} < 0.082 \]  

(25)

When \( \{ \rho, \varphi \} \in [0, 0.03] \times [-\pi/2, \pi/2] \), we get for (17) three dashed lines, with increasing thickness, as \( x \) goes from \( x_1 \) to \( x_0 \). Correspondingly, for \( a_{CP} \), we get three pairs of solid curves: the lower ones, for \( a_{CP} = -0.17 \), and the upper ones, for \( a_{CP} = 0.082 \). These solid contours close inside the values of direct CP asymmetry which do not agree with (25). One is able to constrain \( \left( \delta_{23}^{LR} \right)_{LR} \) to the segments of the \( Br \)-plots outside the solid contours, for each \( x \). For example, when \( m_\tilde{g} > m_\tilde{\tilde{g}} \), all negative phases, with suitable \( \rho \)’s, can accommodate both (25) and (17). Also, \( \rho \) is constrained by the experimental data on the branching ratio of \( B \to X_s \gamma \) to \( \rho < 0.016 \) [20, 23].

In figure 3, we represent the \( Br^{total} \) (in units of \( 10^{-5} \)) and \( a_{CP} \) (in units of 0.1), with respectively dashed and solid lines, as functions of \( \varphi \), for \( x = x_0 \). As \( \rho \) takes the following values: \( \rho \in [0.005, 0.01, 0.012] \), we get three pairs of curves, with increasing thickness. The horizontal dashed band corresponds to the data \( 3.76 \times 10^{-5} < Br_{exp} < 4.68 \times 10^{-5} \) (in 90% C.L.), while the horizontal solid band stands for the constraint (25). We notice that one should avoid the region \( \varphi \in [-\pi/16, \pi/16] \) on the \( \rho \approx 0.01 \) plot, since the CP asymmetry parameter drops quickly, from positive to negative values much outside the constraint (24).

Finally, let us perform a numerical analysis, for \( x = x_0 \), and increasing \( \rho \), starting with \( \rho = 0.005 \). As \( \varphi \in [-8\pi/16, -4\pi/16] \cup [3\pi/16, 7\pi/16] \), the \( Br^{total} \) and the direct CP asymmetry are inside the ranges \( 10^5 \times Br^{total} \in [8.3, 3.3] \) and \[3.1, 8.1\], accommodating data and other theoretical models predic-
FIG. 3: The total branching ratio, in units of $10^{-5}$, (the dashed lines) and $10 \times a_{CP}$ (the solid lines), as functions of $\varphi$, for $x = x_0$. The thickness of plots increases as $\rho$ is: $\rho = \{0.005, 0.01, 0.012\}$. The horizontal dashed band corresponds to the world average branching ratio with 90% C.L. and the horizontal solid one is for the constraint (25).

tions, and, respectively, $a_{CP} \in [-0.054, -0.093] \cup [0.096, 0.062]$. When $\rho$ goes to bigger values, the two $\varphi$ ranges, constrained by the allowed branching ratios, get closer and $a_{CP}$ moves toward much bigger values. For example, for $\rho = 0.01$ and $\varphi \in [-6\pi/16, -4\pi/16] \cup [3\pi/16, 5\pi/16]$, one gets $10^5 \times B_r^{total} \in [8.1, 3.6]$ and $[3.2, 7.02]$ and, respectively, $a_{CP} \in [-0.1, -0.16] \cup [0.21, 0.12]$. We notice that only the negative $\varphi$-values lead to $a_{CP}$ inside the BaBar constraint (25). For $\rho$ close to the upper limit, $\rho = 0.015$, and $\varphi \in [-4\pi/16, -2\pi/16] \cup [\pi/16, 3\pi/16]$, the values $10^5 \times B_r^{total} \in [7.8, 3.6]$ and $[3.3, 7.3]$ are compatible with a measurable $a_{CP} \approx \pm 0.12$.

IV. CONCLUSIONS

In the present paper, we have analysed the radiative $B \to K^*\gamma$ decay in the MSSM framework. First, we have used the PQCD approach, developed by Szczepaniak et al. [15] and extended to “penguin” processes [16], to compute the hard-spectator contribution, $F^{sp}(a)$, to the essential form factor $F_{K^*}(0)$. For the peaking parameter in the $B$ wave function $a = 0.072$ and $F_{K^*}(0) = 0.38$, the branching ratio becomes $B_r^{NLO} = 6.97 \times 10^{-5}$, which is above the experimental data. In order to reach an agreement with data and find large CP asymmetries, which hopefully will be measured with a better precision in the near future, we extend our analyses by including, in the Wilson coefficients $C_{7,8}$, the SUSY contributions coming from squark mixing parameter $(\delta_{23}^d)_{LR} = p e^{i\varphi}$. Consequently, the total branching ratio depends, besides $a$, on three (SUSY) parameters: $x, \rho, \varphi$, while $a_{CP}$ is free of the uncertainties coming from the corrected form factor. Using the graphs displayed in figures 2 and 3, one is able to find ranges for the mass insertion parameter $(\delta_{23}^d)_{LR}$. As an example, for $x = (540/500)^2$, the world average branching ratio, $B_{r_{CP}} = 4.22 \times 10^{-5}$, can be accommodated for $\left\{x, \rho, \varphi\right\} = \{0.005, -\frac{\pi}{13}\}$ or $\{0.01, -\frac{2\pi}{13}\}$. The corresponding asymmetries, $a_{CP} = -0.085$ and respectively $a_{CP} = -0.147$, are inside the BaBar constraint (25).

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