Radiative corrections to $e^+e^- \rightarrow WW \rightarrow 4f$ with \textsc{RacoonWW}$^\dagger$

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Abstract
\textsc{RacoonWW} is the first Monte Carlo generator for $e^+e^- \rightarrow WW \rightarrow 4f(+\gamma)$ that includes the electroweak $O(\alpha)$ radiative corrections in the double-pole approximation completely. Some numerical results for LEP2 energies are discussed, and the predictions for the total W-pair cross section are confronted with LEP2 data.

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Radiative corrections to $e^+e^- \rightarrow WW \rightarrow 4f$ with RACOONWW

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RACOONWW is the first Monte Carlo generator for $e^+e^- \rightarrow WW \rightarrow 4f(\gamma)$ that includes the electroweak $\mathcal{O}(\alpha)$ radiative corrections in the double-pole approximation completely. Some numerical results for LEP2 energies are discussed, and the predictions for the total W-pair cross section are confronted with LEP2 data.

1. W-PAIR PRODUCTION AT LEP2

The investigation of W-pair production at LEP2 plays an important role in the verification of the Electroweak Standard Model (SM). Apart from the direct observation of the triple-gauge-boson couplings in $e^+e^- \rightarrow W^+W^-$, the increasing accuracy in the W-pair-production cross-section and W-mass measurements has put this process into the row of SM precision tests.

To account for the high experimental accuracy on the theoretical side is a great challenge: the W bosons have to be treated as resonances in the full four-fermion processes $e^+e^- \rightarrow 4f$, and radiative corrections need to be included. While several lowest-order predictions are based on the full set of Feynman diagrams, only very few calculations include radiative corrections beyond the level of universal effects (see Refs. [2, 3] and references therein). Fortunately, to match the experimental precision for W-pair production at LEP2 a full one-loop calculation for the four-fermion processes is not needed, and it is sufficient to take into account only those radiative corrections that are enhanced by two resonant W bosons. The neglected corrections are of the order $(\alpha/\pi)(\Gamma_W/M_W)$, i.e. below 0.5% even if possible enhancement factors are taken into account. The theoretically clean way to carry out this approximation is the expansion about the two resonance poles, which is called double-pole approximation (DPA) [4]. A full description of this strategy and of different variants used in the literature [5, 6, 7] (some of them involving further approximations) is beyond the scope of this article. We can only briefly sketch the approach pursued in RACOONWW [8, 9].

2. RADIATIVE CORRECTIONS WITH RACOONWW

In DPA, $\mathcal{O}(\alpha)$ corrections to $e^+e^- \rightarrow WW \rightarrow 4f$ can be classified into two types: factorizable and non-factorizable corrections. We first focus on virtual corrections.

Factorizable corrections are those that correspond either to W-pair production or to W decay. Virtual factorizable corrections are represented by the schematic diagram of Fig. 1, in which the shaded blobs contain all one-loop corrections to the on-shell production and on-shell decay processes, and the open blobs include the corrections to the W propagators. For the corresponding matrix element $\mathcal{M}$ the application of the DPA
amounts to the replacement
\[
M = \frac{R(k_{W+}^2, k_{W-}^2)}{(k_{W+}^2 - M_W^2)(k_{W-}^2 - M_W^2)} \frac{R(M_W^2, M_W^2)}{(k_{W+}^2 - M^2)(k_{W-}^2 - M^2)},
\]

where the originally gauge-dependent numerator \(R(k_{W+}^2, k_{W-}^2)\) is replaced by the gauge-independent residue \(R(M_W^2, M_W^2)\), and \(M^2 = M_W^2 - iM_W\Gamma_W\) is the location of the poles in the complex \(k_{W^\pm}\) planes. The one-loop corrections to this residue can be deduced from the known results for the pair production [10] and the decay [11] of on-shell \(W\) bosons. However, the spin correlations between the two \(W\) decays should be taken into account.

Non-factorizable corrections [12, 13] comprise all those doubly-resonant corrections that are not yet contained in the factorizable ones, and include, in particular, all diagrams involving particle exchange between the subprocesses. Such diagrams only lead to doubly-resonant contributions if the exchanged particle is a photon with energy \(E_\gamma \approx \Gamma_W\); all other non-factorizable diagrams are negligible in DPA. A typical diagram for a virtual non-factorizable correction is shown in Fig. 2, where the full blob represents tree-level subgraphs. We note that diagrams involving photon exchange between the \(W\) bosons contribute to both factorizable and non-factorizable corrections; otherwise the splitting into those parts would not be gauge-invariant.

In RacoonWW the virtual corrections are treated in DPA, including the full set of factorizable and non-factorizable \(O(\alpha)\) corrections. The real corrections are calculated from full matrix elements for \(e^+e^- \rightarrow 4f\gamma\), as described in Ref. [14], i.e. the DPA is not used in this part. In this way, we avoid potential problems in the definition of the DPA for the emission of photons with energies \(E_\gamma \sim \Gamma_W\). On the other hand, this asymmetry in the calculation of virtual and real corrections requires particular care concerning the structure of IR and mass singularities. The singularities have the form of a universal radiator function convoluted with the respective lowest-order matrix element squared \(|M_0|^2\) of the non-radiative process. Since the virtual corrections are calculated in DPA, but the full matrix element is used for the real photons, a simple summation of virtual and real corrections would lead to a mismatch in the singularity structure and eventually to totally wrong results. Therefore, we extract those singular parts from the real photon contribution that exactly match the singular parts of the virtual photon contribution, then replace in these terms the full \(|M_0|^2\) by the one calculated in DPA, and finally add this modified part to the virtual corrections. This modification is allowed within DPA and leads to a proper matching of all IR and mass singularities. This treatment has been carried out in two different ways, once following phase-space slicing, once using the subtraction formalism of Ref. [15].

Beyond \(O(\alpha)\), RacoonWW includes soft-photon exponentiation and leading higher-order ISR effects in the structure-function approach. Using \(G_\mu\) as input parameter instead of \(\alpha(0)\) in the lowest-order cross section, also the leading higher-order effects from \(\Delta\alpha\) and \(\Delta\rho\) are taken into account. On the other hand, in the relative correction factor we use \(\alpha(0)\) in order to describe the couplings of the real photons correctly.

3. NUMERICAL RESULTS

3.1. Predictions from RacoonWW
A survey of numerical results obtained with RacoonWW has already been presented in Ref. [5] for LEP2 and linear-collider energies. Here we only review the \(W\) invariant-mass distribution given there, extend the results for the total cross section, and add some studies of the intrinsic ambiguities in the DPA.

Figure 3 shows the invariant-mass distribution of the \(d\bar{u}\) quark pair for the semi-leptonic channel \(e^+e^- \rightarrow \nu_\mu\mu^+d\bar{u}\) at \(\sqrt{s} = 200\) GeV at tree-level and with electroweak \(O(\alpha)\) corrections for two different recombination cuts, \(M_{rec} = 5\) GeV and 25 GeV. The recombination of photons with final-state charged fermions is performed as described in Ref. [5]: we first determine the lowest invariant mass \(M_{zf}\) built by the emitted photon and a charged final-state fermion. If \(M_{zf}\) is smaller than \(M_{rec}\), the photon momentum is added to the
Figure 3. Invariant-mass distribution of the $d\bar{u}$ pair for $e^+e^- \rightarrow \nu_\mu\mu^+d\bar{u}$ and $\sqrt{s} = 200$ GeV (taken from Ref. [8a])

Figure 4. Relative corrections to the invariant-mass distribution of the $d\bar{u}$ pair for $e^+e^- \rightarrow \nu_\mu\mu^+d\bar{u}$ and $\sqrt{s} = 200$ GeV (taken from Ref. [8a])

Figure 5. Total WW production cross section at LEP2, as given by the LEPEWWG [16].

As expected, there is a tendency to shift the maxima to larger invariant masses if more and more photons are recombined. In Fig. 4 we display the relative corrections $\delta = \frac{d\sigma}{d\sigma_0} - 1$ for the two values of $M_{rec}$, which illustrates the strong dependence of the corrected invariant-mass distributions on the treatment of the real photons. We obtain consistent results for the phase-space “slicing” and the “subtraction” methods. The size of the shown effects demonstrates that a careful treatment of real photons is mandatory in the W-mass reconstruction at LEP2 accuracy.

Figure 5 shows a comparison of RacoonWW results and of other predictions with recent LEP2 data, as given by the LEP Electroweak Working Group [1, 16]. The data are in good agreement with the predictions of RacoonWW and YFSWW3 [5]. The predictions of these two generators differ between 0.5–0.7%. More details on the conceptual differences of the two generators,

†Meanwhile the dominant source of this difference has been found, and the new results of YFSWW3 agree within 0.3% with the results of RacoonWW. Details on the new YFSWW3 predictions can be found in Ref. [3].
as well as a detailed comparison of numerical results, can be found in Ref. [ 3]. Figure 5 also includes the prediction provided by GENTLE [ 17], which differs from the RACOONWW and YFSW3 results by 2–2.5%. This difference is due to the neglect of non-leading, non-universal $O(\alpha)$ corrections in GENTLE. Consequently, the comparison between SM predictions with the precise measurements of the $W$-pair production cross section at LEP2 reveals evidence of non-leading electroweak radiative corrections beyond the level of universal effects.

### 3.2. Intrinsic ambiguities of the DPA

In order to investigate the accuracy of the DPA quantitatively, we have performed a number of tests. The implementation of the DPA has been modified within the formal level of $\alpha\Gamma_W/M_W$ and the obtained results have been compared. Because in RACOONWW only the virtual corrections are treated in DPA, while real photon emission is based on the full $e^+e^- \rightarrow 4f_\gamma$ matrix element with the exact five-particle phase space, only the $2 \rightarrow 4$ part is affected by the following modifications. We consider three types of uncertainties (see Ref. [ 8] for more details):

- **Different on-shell projections:**
  For the DPA one has to specify a projection of the physical momenta to a set of momenta for on-shell $W$-pair production and decay\(^5\), in order to define a DPA. This can be done in an obvious way by fixing the direction of one of the $W$ bosons and of one of the final-state fermions originating from either $W$ boson in the CM frame of the incoming $e^+e^-$ pair. The default in RACOONWW is to fix the directions of the momenta of the fermions (not of the anti-fermions) resulting from the $W^+$ and $W^-$ decays ("def"). A different projection is obtained by fixing the direction of the anti-fermion from the $W^-$ decay ("proj") instead of the fermion direction.

- **Treatment of soft photons:**
  As explained above, the matching of IR and mass singularities between virtual and real corrections requires a redistribution of the singular parts. This redistribution fixes only the universal, singular terms, while the redistribution of non-singular terms are mere convention. Owing to the asymmetric treatment of the corrections (virtual in DPA, real from full matrix elements), different redistributions of non-singular contributions change the result by terms of the order $(\alpha/\pi)(\Gamma_W/M_W)$, i.e. these redistributions are equivalent within the accuracy of the DPA. In RACOONWW two different schemes for this redistribution are implemented. As default, the endpoint contributions of the subtraction functions, as defined in Ref. [ 13], are calculated in DPA and added to the virtual photon contribution. In the other scheme, the universal IR-sensitive part is extracted from the virtual photon contribution in the YFS [ 13] and added to the real photon contribution. The resulting soft+virtual part of the photonic correction is, thus, treated off shell ("eik"). The difference between the two described treatments is that certain terms of the form $(\alpha/\pi) \times \pi^2 \times O(1)$ are either multiplied with the DPA ("def") or with the full off-shell Born cross sections ("eik").

- **On-shell vs. off-shell Coulomb singularity:**
  The Coulomb singularity is (up to higher orders) fully contained in the virtual $O(\alpha)$ correction in DPA. Performing the DPA to the full virtual correction leads to the on-shell Coulomb singularity, which is a simple factor of $\alpha\pi/(2\beta)$, where $\beta$ is the velocity of an on-shell $W$ boson. However, since the Coulomb singularity is an important correction in the LEP2 energy range and is also known beyond DPA [ 19], RACOONWW includes this extra off-shell Coulomb correction factor as default. This replacement of the Coulomb singularity is performed by adding and subtracting the corresponding contributions in the virtual non-factorizable corrections, as described in Ref. [ 22]. Switching the extra off-shell Coulomb terms off ("Coul"), yields an effect of the order of the uncertainty of the DPA.

In the following table and figures the total cross section and two distributions for $e^+e^- \rightarrow u\bar{d}\gamma\bar{\nu}_\mu(\gamma)$ have been compared for the different versions of the DPA defined above. The results have been obtained using a photon-recombination procedure that is similar to the one described in the previous section. The precise definition can be found in Ref. [ 8].

The results for the total cross section are shown in Table 1. We find relative differences at the level of $1\%$. As expected, the prediction that is based on the on-shell Coulomb correction is somewhat higher than the exact off-shell treatment, since off-shell effects screen the positive

\(^{5}\text{This option only illustrates the effect of different on-shell projections in the four-particle phase space. If real photon corrections were treated in DPA the impact of different projections could be larger.}\)
Table 1

| √s/GeV | δ_{proj}/% | δ_{eik}/% | δ_{Coul}/% |
|--------|------------|------------|-----------|
| 172    | -0.03      | -0.09      | 0.79      |
| 200    | -0.03      | -0.01      | 0.13      |
| 500    | -0.01      | 0.08       | 0.01      |

Intrinsic DPA ambiguities δ = σ/σ_{def} − 1 of the RACOONWW predictions for the total cross section of e^+e^- → udμ^-\bar{ν}_μ(γ) (based on the results of Ref. [9]).

Coulomb singularity. Note that for the low CM energy of 172 GeV the difference between on-shell and off-shell Coulomb singularity, which is quite large (0.79%), cannot be viewed as a measure of the theoretical uncertainty, since the on-shell Coulomb singularity is not adequate near threshold.

In Figs. 6 and 7 we show the differences of the "proj", "eik", and "Coul" modifications to the default version of the DPA for the distributions in the W-production angle θ_{W+} and in the ud invariant-mass M_{ud}, both at √s = 200 GeV. For the θ_{W+} distribution the relative differences are of the order of 0.1–0.2% for all angles, which is of the expected order for the intrinsic DPA uncertainty. For the M_{ud} distribution the DPA uncertainties are at the level of 0.1–0.3% within a window of 2Γ_{W} around the W resonance. The uncertainties grow with the distance from the resonance point, as expected, since the DPA runs out of its range of validity away from the resonance region.

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