Hints within the Standard Model on $m_t$ and $m_H$

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Abstract

If one or more otherwise divergent quantities in the standard model are actually finite, they may be indications of underlying dynamics. In particular, one-loop finiteness of the $m_H$ renormalization is achieved if $m_t^2 \simeq m_H^2 = (2M_W^2 + M_Z^2)/3$.

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1 Introduction

The standard model has a quadratic divergence proportional to

\[ 2\lambda + \frac{1}{2}g_1^2 + \frac{3}{2}g_2^2 - 4 \sum_f \left( \frac{n_f}{3} \right) g_f^2, \tag{1} \]

where \( \lambda \) is the quartic scalar self-coupling, \( g_1 \) the U(1) gauge coupling, \( g_2 \) the SU(2) gauge coupling, and \( g_f \) the Yukawa couplings of the fermions \( f \) to the Higgs boson, with \( n_f \) the number of colors, \( i.e. \) 3 for quarks and 1 for leptons. This may very well be just an artifact of the regularization procedure and we can forget all about it after proper renormalization of all the physical quantities. Alternatively, we may take it seriously as a hint that new physics will come in at some energy scale higher than the electroweak scale of \( 10^2 \) GeV and make it finite. In a scenario involving supersymmetry, new particles will appear below 1 TeV or so and cancel the divergence associated with each term of the above expression. In a scenario without new particles up to an energy scale \( \Lambda \gg 1 \) TeV, it may be conjectured that whatever the underlying dynamics, it should be such that the above expression is suppressed, say of order \( (10^2 \text{GeV}/\Lambda)^2 \), which must then come about from the cancellation among the various couplings.

Attaching \( v^2 \) (square of the Higgs-boson vacuum expectation value) to (1) and setting it equal to zero, we obtain the well-known Veltman condition\[1\]

\[ 4m_t^2 \simeq 2M_W^2 + M_Z^2 + m_H^2, \tag{2} \]

where all other fermion masses have been dropped because their contributions are negligible. This condition is consistent with the present experimental data \( M_Z = 91.175 \pm 0.021 \) GeV, \( M_W = 80.14 \pm 0.27 \) GeV, \( m_t > 91 \) GeV, and \( m_H > 60 \) GeV.
2 A Closer Look

Since $\lambda, g_1^2, g_2^2$, and $g_\phi^2$ change as functions of $q^2$, and the expression (1) is not invariant under this change, the Veltman condition (2) should apply only at one unique mass scale. In the standard model, the presence of spontaneous symmetry breaking implies the existence of a tadpole diagram in which a physical Higgs boson ends up in a loop involving all the massive particles. This diagram is quadratically divergent and contributes to all self-masses. Hence the natural choice is $q^2 = m_H^2$.

3 Recent Conjectures

In addition to the condition (2), is there another hint within the standard model of a possible relationship among couplings? Perhaps a particular logarithmic divergence should be suppressed as well. This is the essence of 3 recent conjectures.

Osland and Wu\cite{2} singled out the $He^+e^-$ coupling and required its logarithmic divergence to be zero. This results in the condition

$$m_t^2 \simeq \frac{5}{2} M_Z^2 - M_W^2.$$  \hspace{1cm} (3)

However it is not clear why this particular coupling should be chosen instead of some other, and once it is chosen, we must still define it at some $q^2$ because this logarithmic divergence cannot be zero at all mass scales.

Blumhofer and Stech\cite{3} proposed to set the logarithmic divergence of the Higgs tadpole also to zero. This has the advantage of a well-defined $q^2$, i.e. $m_H^2$, but the procedure is gauge-dependent and therefore suspect. However, they argued that the choice $\xi = 0$ in the $R_\xi$ gauge would correspond to a gauge-invariant physical quantity having to do with vacuum...
condensates. This then implies

\[ 4m_t^4 \simeq 2M_W^4 + M_Z^4 + \frac{1}{2}m_H^4. \] (4)

Decker and Pestieau\[4\] chose the mass of the electron neutrino and required its logarithmic divergence to be zero, assuming of course that there is a right-handed singlet partner to the observed left-handed neutrino and they combine to allow a Dirac mass. This results in the following condition

\[ 4m_t^4 \simeq 2M_W^4 + M_Z^4 + \frac{1}{2}m_H^4 + \frac{1}{2}(m^2_e - m^2_\nu) m_H^2, \] (5)

which is almost identical to (4). Again it is not clear why this particular mass (which may not even exist) should be chosen instead of some other.

4 The Most Natural Choice

If a particular logarithmic divergence is to be chosen zero in addition to the Veltman condition, the most natural choice is clearly that of the Higgs-boson mass itself.\[5\] After all, it is uniquely defined at \( q^2 = m_H^2 \) as already assumed in (2). It is also gauge-independent. The resulting condition is

\[ 2m_t^2 \simeq 2M_W^2 + M_Z^2 - m_H^2, \] (6)

which, when combined with (2), implies

\[ m_t^2 \simeq m_H^2 = \frac{2}{3}M_W^2 + \frac{1}{3}M_Z^2. \] (7)

If dimensional regularization is used to extract the quadratic divergence of the standard model, the residue of the pole at \( d = 2 \) depends also on \( d \) and the Dirac trace. To get the Veltman condition, we have to set both equal to 4. Perhaps we should\[2\] really use the value 2, then instead of (2), we find

\[ 6m_t^2 \simeq 2M_W^2 + M_Z^2 + 3m_H^2. \] (8)
Remarkably, when combined with (6), the condition (7) is again obtained. Hence the proposed conjecture of one-loop finite $m_H$ renormalization is independent of the regularization procedure for the quadratic divergence.

5 Conclusion

Numerically, the condition (7) implies

\begin{align*}
    m_t & \simeq 84 \text{ GeV} + \text{higher-order corrections}, \quad (9) \\
    m_H & \simeq 84 \text{ GeV} + \text{higher-order corrections}, \quad (10)
\end{align*}

whereas present data require $m_t > 91 \text{ GeV}$, and $m_H > 60 \text{ GeV}$. Hence the above conjecture is on the verge of being ruled out. On the other hand, if either (2) or (6) turns out to be approximately satisfied, it may still be an indication of underlying dynamics.

It should be noted that the above conditions are all based on only one-loop contributions and there is no explicit reference to the mass scale $\Lambda$ of new physics. The higher-order contributions, all defined at $q^2 = m_H^2$, are considered as small corrections, but they will depend on $\Lambda$ logarithmically.

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