GAUGINO CONDENSATION, S-DUALITY
AND
SUPERSYMMETRY BREAKING
IN SUPERGRAVITY MODELS

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ABSTRACT

The status of the gaugino condensation as the source of supersymmetry breaking is reexamined. It is argued that one cannot have stable minima with broken supersymmetry in models where the dilaton is coupled only linearly to the gaugino condensate. We show that the problems of the gaugino condensate mechanism can be solved by considering nonstandard gauge kinetic functions, created by nonperturbative effects. As an example we use the principle of S-duality to modify the coupling of the gaugino condensate to effective supergravity (superstring) Lagrangians. We show that such an approach can solve the problem of the runaway dilaton and lead to satisfactory supersymmetry breaking in models with a single gaugino condensate. We exhibit a general property of theories containing a symmetry acting on the dilaton and also shed some light on the question whether it is generically the auxiliary field of the modulus \( T \), which dominates supersymmetry breaking.
1 Introduction

Understanding the ways supersymmetry can possibly be broken in supersymmetric theories unifying fundamental forces is likely to be the most challenging problem in supersymmetric model building over the coming years. While the agreement that low energy supersymmetry has to be broken on a TeV scale to allow for realistic phenomenology is unanimous, the hunt for a technically satisfactory and esthetically compelling model of supersymmetry breaking continues. It has long been concluded that spontaneous breaking of supersymmetry in the observable sector of supersymmetric models cannot work properly, because of unbroken supersymmetric mass relations between known particles and their superpartners (vanishing supertrace) which practically make superpartners visible in some way in all the models analyzed so far [1]. This makes it necessary to construct hidden sectors in these models, where supersymmetry can be broken spontaneously and the effect of this breaking can then be transmitted by sufficiently weak couplings to the observable sector – where it is seen as explicit (soft) breaking. The problem with this scenario consists in the necessity of providing an explicit (and rather large if the hidden sector is to decouple at low energies) mass scale which can be used to generate a TeV scale in the observable sector. The only known way of generating such an intermediate scale in field theoretical models is to employ nonabelian Yang-Mills theories, the condensation scale of which can easily account for the required order of magnitude of the new scale [4], [3]. In this case the source of really weak couplings is gravity. Thus the nonabelian gauge sector of a superstring model is a natural hidden sector based on well known physics. This path has been explored by many authors over the years [3]–[1], but unfortunately no fully satisfactory solution for realistic SUSY breakdown in supergravity, and in particular in superstring inspired effective supergravities has been found. The main flaw of the models considered in the literature is the difficulty of fixing the the vacuum expectation value of the dilaton, the field which sets the value of the gauge coupling at the unification scale – so crucial for the dynamics of the gauge sector – at a physically acceptable value. Solutions proposed so far in the context of gaugino condensation involve several gaugino condensates and a rather unnatural adjustment of the hidden matter sector [4]. Of course, it may be that because of some unknown and deep reason string theory will generate these special matter sectors with suitably designed superpotentials and couplings. However, at the level of field theory, and gaugino condensation in itself is inherently a field theoretical phenomenon, one would prefer that if condensates break supersymmetry they should do it in a generic way, irrespectively of accidental complications of the matter sector. Further to that, if there is an obvious obstacle for the mechanism to work, as we believe the form of perturbative couplings of the dilaton in stringy Lagrangians to be, its possible absence should be understandable in terms of some fundamental symmetry.

In this work we want to analyze as far as possible the ways the pure gauge hidden sector, coupled eventually to the dilaton and other moduli, can break supersymmetry.
and identify crucial problems. As the mechanism of the formation of condensates is nonperturbative, it is not obvious what technical tools should be used for such an analysis. As we explain in more detail in the text, we decided to use the generalized effective Lagrangian approach, which offers a hope for controlling a variety of dynamics pertaining to the condensation in the presence of a possible back-reaction of gravity, moduli fields and other fields typically present in realistic models. Indeed, it turns out that the role the gaugino condensate can play is rather model dependent. Moreover, the presence and nature of the couplings of dilaton and other moduli to condensates and to gravity appears to be crucial for the supersymmetry breaking with simultaneous stabilization of all the moduli to work. In fact, on the basis of our analysis we tend to argue that the linear coupling of the dilaton to the gaugino condensate — motivated by string tree level amplitudes [6] — and therefore the effective superpotential for the dilaton in its customary form, is highly unsatisfactory and that the dilaton-induced problems we discuss provide an evidence for the need of a more sophisticated structure of the strongly coupled sectors of supersymmetric gauge theories. We suggest that it is possible to solve the problem of the runaway dilaton/SUSY breaking in a fundamental way, for example by postulating a new symmetry of the effective Lagrangian, the so-called S-duality. It turns out that the modification of dilaton couplings induced by even the simplest versions of S-duality is sufficient to stabilize the dilaton at a reasonable value and allows for supersymmetry breaking. Evidence for S-duality in string models has been presented in [7], and it might be that S-duality gives the proper way of promoting the inherently field-theoretical gaugino condensation mechanism into the string theoretical framework.

S-duality invariant effective purely dilatonic superpotentials have been conjectured in [8] (however with no reference to gaugino condensation) and recently reexamined by the authors of [9]. The general form of the superpotential proposed in [8], which was constructed specifically to fix the vev of the dilaton, does not give a free theory in the weak coupling limit. The authors of Ref. [9] note that one can easily modify any effective superpotential in such a way, that it vanishes asymptotically as Re$S \to \infty$ in any direction. Their one-condensate model shows a realistic minimum for the dilaton but, unfortunately, SUSY is unbroken at this minimum.

Our paper is structured as follows: in chapter 2 we discuss the effective Lagrangians for gaugino condensate in globally supersymmetric sigma models containing condensates and the dilaton. In chapter 3 we couple these models to supergravity. In chapter 4 we introduce S-dual effective Lagrangians, in chapter 5 we discuss general aspects of supersymmetry breaking in the class of models of interest, and finally summarize our results in chapter 6.
2 Effective Lagrangian in global SUSY-Yang-Mills theory

Following the pioneering work of [10] we discuss in some detail the construction of the effective Lagrangian which is supposed to describe gaugino condensation in the globally supersymmetric Yang-Mills theory. The underlying principle of this construction is t’Hooft’s anomaly matching condition, which demands that the effective low energy Lagrangian, valid below some scale \( \Lambda \) (which is to be identified with the condensation scale in our case), should reproduce the anomalies of the underlying constituent theory. In the case of the Yang-Mills model it is well known [11] that R-symmetry and supersymmetry currents as well as the energy momentum tensor lie in the same (general) supersymmetry multiplet. Moreover, this implies that the corresponding chiral, \( \gamma \)-trace and trace anomalies lie in another, chiral, multiplet which we will denote by \( U \).

For the sake of convenience we will sometimes use the chiral multiplet \( Y \), defined by

\[
U = Y^3,
\]

which has canonical mass dimension. In terms of constituent fields the lowest component of \( U \) is proportional to the gaugino bilinear \( \lambda \lambda \), so it makes sense to take \( U \) as the (pseudo-)goldstone multiplet entering the low energy Lagrangian. When we define components of \( U \) as

\[
U = A + \sqrt{2} \theta \psi + \theta \theta F
\]

then the R-symmetry acts as

\[
A \rightarrow e^{3i\alpha} A, \quad \psi \rightarrow e^{3i\alpha/2} \psi, \quad F \rightarrow F
\]

which can be also written as

\[
U(x, \theta) \rightarrow e^{3i\alpha} U(x, e^{-3i\alpha/2} \theta)
\]

while the scale symmetry acts as

\[
A(x) \rightarrow e^{3\gamma} A(x e^\gamma), \quad \psi(x) \rightarrow e^{7\gamma/2} \psi(x e^\gamma), \quad F(x) \rightarrow e^{4\gamma} F(x e^\gamma)
\]

which one can write more concisely

\[
U \rightarrow e^{3\gamma} U(x e^\gamma, \theta e^{\gamma/2})
\]

Assuming that the anomalies associated with the above classical invariances are reproduced by the superpotential \( W \), one obtains the (holomorphic) equation for the superpotential

\[
U \frac{\partial W}{\partial U} - W = bU,
\]

which has a general solution of the form

\[
W = aU + bU \log\left(\frac{U}{\mu^3}\right),
\]
where $a$ is in general undetermined - as the variation of $F$ (highest component of $U$) under both symmetries vanishes - and corresponds to the rescaling of the condensation scale $\mu$, and $b$ is some function of the gauge coupling which is fixed by demanding the specific form of the resulting anomaly coefficient. To have the complete model one has to make a choice for the Kähler potential $K$. Usually one demands that the variation of $K_D$ (D-component of $K$) is non-anomalous which fixes it, up to a field-independent coefficient, to be $K = 9(U\bar{U})^{1/3} = 9Y\bar{Y}$. This choice leads to unbroken supersymmetry and nonzero condensate with the expectation value $U = \mu^3e^{-(a+b)/b}$. This agrees with the conclusion of reference [12] where the analysis based on the index theorem implies that supersymmetry is unbroken in this case, and with instanton calculations [13]. However, these methods do not give any information about the form of the kinetic term for the condensate, and in our present approach the conclusion that supersymmetry stays unbroken is almost independent of the form of $K$, no matter whether it breaks classical invariances or not. Indeed, the effective potential, for any $K$, has the form

$$V = (K_{U\bar{U}})^{-1}|U|^2|a + b \log(U/\mu^3)|^2,$$

(7)

$$(K_{U\bar{U}} = \partial^2 K/\partial U \partial \bar{U})$$

and the expression controlling supersymmetry breaking is

$$F_U = (K_{U\bar{U}})^{-1}U(a + b \log(U/\mu^3)),$$

(8)

where $F_U$ denotes the auxiliary field of the condensate superfield $U$. One can see that $V = 0$ implies $F = 0$, regardless of the particular form of $K$ (unless the metric $K_{U\bar{U}}^{-1}$ becomes singular). It should be noted, that the demand that SUSY is unbroken does not fix the form of $K$ at all, does not even imply that $K$ is invariant under classical invariances. We will see however, that the choice of $K$ is not an innocent assumption if one considers supergravity models. It should also be noted, that there is no canonical form of $K$ implied by any reliable calculation.

It is instructive to examine in more detail the variations of $K_D$ under chiral and scale transformations. The infinitesimal change under chiral symmetry (2) is

$$\delta K_D = 3i\alpha(\partial K/\partial U U - \partial K/\partial \bar{U} \bar{U})_D$$

(9)

and under scale symmetry (4)

$$\delta(\int d^4x K_D) = \gamma \int d^4x \left(3\partial K/\partial U U + 3\partial K/\partial \bar{U} \bar{U} - 2K\right)_D$$

(10)

Let us consider two simple examples. If one takes $K = K(U\bar{U})$ then the variation ([2]) vanishes identically, but ([14]) is nonzero unless $K = \text{const.} (U\bar{U})^{1/3}$. If one considers $K = K(U + \bar{U})$ this gives a non-invariant $K_D$ under chiral transformation, but the special choice $K = (U + \bar{U})^{2/3}$ is invariant under the scale transformation ([4]).
Finally, let us note that for the choice $K = c U \bar{U}$, which will be discussed in the next section, the non-vanishing variation (10) is

$$\delta_{\gamma} \left( \int d^4 x (U \bar{U}) \right) = 4 \gamma (|F^U|^2 + \text{fermionic and derivative terms})$$

which vanishes if supersymmetry is unbroken (which is the case for this choice of $K$ in global supersymmetry).

The natural generalization of the above Lagrangian consists in allowing for a dynamical, field-dependent gauge coupling. As the global Yang-Mills Lagrangian contains a term $(1/g^2 W^\alpha W_\alpha)_F + \text{h.c.}$ then it is natural to promote the inverse gauge coupling constant to a chiral superfield, $f$. This extension is well motivated by the superstring effective Lagrangian which gives $f = S$ at the string tree-level [6], where $S$ denotes the dilaton, and also predicts a characteristic no-scale type Kähler function for the dilaton, $K = - \log(S + \bar{S})$. Thus the modified Lagrangian, which can be considered as the flat global limit of some supergravity model dilaton plus gaugino condensate sector is

$$L = K(S, \bar{S})_D + K(U, \bar{U})_D + ((f(S)U + bU \log(U/\mu^3))_F + \text{h.c.})$$

We assume, as it is the case in superstring models, that the dilaton $S$ is dimensionless, which in the context of supergravity implies that the associated dimensionful field is $\dot{S} = S M$ where $M = M_{pl}$. Formally $M$ can be an arbitrary scale, but it is natural to assume that there are no fundamental scales in the model other than $M_{pl}$ and the condensation scale. The introduction of the dilaton, which is a gauge singlet, requires that we extend both chiral and scale symmetries to act on the dilaton in such a way that under R-symmetry $f(x, \theta) \rightarrow f(x, e^{-3i\alpha/2}\theta)$ and under scale transformation $f(x, \theta) \rightarrow f(xe^{\gamma/2}, e^{\gamma/2}).$ In addition it creates an anomalous symmetry which is the global shift of the imaginary part of $f$, $f \rightarrow f + i\Lambda$. This symmetry is an exact symmetry of the corresponding sigma model if $K = K(f + \bar{f})$. Because of the presence of this new shift symmetry and because of the holomorphicity of the superpotential there appear two new exact symmetries of the superpotential. The first, which is generally assumed to be the exact symmetry of the full Lagrangian through the suitable choice of $K(S, \bar{S})$, is the combination of the R-symmetry (2) and the imaginary shift

$$f(x, \theta) \rightarrow f(x, e^{-3i\alpha/2}) - 3ib\alpha$$

The second symmetry, which is violated by usual (for instance, the no-scale form) Kähler functions, is (13) accompanied by

$$f(x, \theta) \rightarrow f(xe^{\gamma/2}, e^{\gamma/2}) - 3b\gamma$$

Even though the second symmetry is not a symmetry of the kinetic terms, its existence implies an additional degeneracy of the manifold of supersymmetric solutions of the effective model [14].
Let us discuss implications of the presence of the dilaton for supersymmetry breaking issue. The general potential we consider has the form

\[ V = g^{U\bar{U}} |f + b \log(\frac{U}{\mu^3})^2 + g^{SS} |U|^2 |\frac{\partial f}{\partial S}|^2 \]

where \( g^3 = (\partial^2 K/\partial z^i \partial \bar{z}^j)^{-1} \) is the inverse Kähler metric. If we assume the most symmetric choices for the Kähler potential, \( K = -\log(S + \bar{S}) + 9(U\bar{U})^{1/3} \) and \( f = S \), then the minimal value of (15) corresponds to \( U = 0 \) with any value of \( S \) which also gives unbroken supersymmetry. Thus the introduction of the dilaton not only prevents supersymmetry breaking, but also ‘undoes’ condensation - one is forced to conclude that the condensate does not form in this case. The general form of (15) suggests however some possibilities. First, one can imagine that \( f \) has such a form, that the second term in (15) cancels without driving \( U \) to zero, fixing at least partially the vev of the dilaton. Then the terms under the sign of the absolute value in the first term adjust themselves to cancel this term as well. In this case supersymmetry is generically unbroken in global models, but at least one can save the condensate creating a new manifold of (supersymmetric) vacua, which is disconnected from \( U = 0 \). The second, interesting, possibility is, at the level of this global model, that one takes a non-symmetric Kähler function for \( U \), for instance \( K = U\bar{U}/\mu^4 \). Then the value of \( U = 0 \) does not automatically cancel the first term in the potential. The only supersymmetric point in the hyperplane \( U - S \) is the asymptotic minimum of the potential at \( U \to 0 \) and \( S \to \infty \). At any finite value of \( S \) and \( U \) supersymmetry is broken and potential is larger than zero. This example suggests a reasonable way to solve the problem of generating SUSY breaking at finite values of both dilaton and condensate in purely dilaton-condensate models. The observation is that there may exist corrections to the global model which prevent the dilaton from running to infinity, but stop it at the Planck scale (in our normalization with \( M = M_{pl} \) around \( S = 1 \)) instead – corrections coming from gravity. In that case one expects the potential (15) to be the leading term in the full potential and supersymmetry to be broken through the auxiliary field of the dilaton (the \( F^U \)-term adjusts itself to zero by means of small corrections when \( S \) changes) with the characteristic value \( F^S \approx \mu^3/M_{pl} \). Unfortunately, we do not know how to construct a model which works this way, at least without introducing any mass scale different from Planck and condensation scales into the dilatonic superpotential \( f \). However, the idea that gravitational corrections can modify the effective Lagrangian in an interesting way is carefully discussed in the next chapters where we couple the dilaton-condensate system to supergravity multiplet.
3 Gaugino condensate coupled to supergravity

There are three different ways of introducing gaugino condensates into supergravity. In the component Lagrangian method, pioneered by [3], one takes the standard Lagrangian of supersymmetric Yang-Mills theory coupled to supergravity and after identifying Lorentz-invariant gaugino bilinears one replaces them by a constant of the order of the condensation scale $\mu^3$. Such a procedure has the drawback of discarding the back-reaction of other fields in the model onto the condensate, hence one cannot determine this way whether the condensate really forms. The formation of the condensate and its magnitude is simply assumed in this approach implicitly relying on the observation made in the global version of the model. However, as we assume that gravitational corrections can play an important part, the internal consistency of such an approach is not entirely clear. Furthermore, to arrange for broken SUSY one needs a nontrivial gauge kinetic function, and then one has to check whether condensates of other fields present in the model do not cancel the contribution of the assumed gaugino condensate to the vacuum expectation values of the auxiliary fields. A refinement of this method which takes into account a possible dependence of condensate on some other fields (like the dilaton) leads to the effective superpotential method: here one makes educated guesses about how the condensate depends on the remaining chiral fields in the model. Using these one can then construct the gaugino induced “nonperturbative” corrections to the original superpotential of the model. For instance, the belief that the condensate dissolves in the weak coupling limit leads to a superpotential which decays exponentially with the increasing value of the dilaton $S$ (large $S$ corresponds to weak coupling) in string inspired supergravities. One then searches for minima of the effective theory and determines whether supersymmetry is broken at these minima. The third method is the effective Lagrangian (Veneziano-Yankielowicz) approach. Generalizing the global-SUSY Veneziano-Yankielowicz type Lagrangians discussed in chapter 2, one should notice that scale invariance may be broken in some sector of the model, in fact gravity itself introduces an explicit mass scale $M_{pl}$, so that non-invariant corrections to the Kähler potential of the gaugino condensate become more natural, and that in the superpotential both terms can get independent chiral-superfield coefficients. Also, one can add an arbitrary constant to the superpotential (introducing the M-term) which formally does not affect the global limit. The standard practical consistency condition is that in the naive limit $M_{pl} \rightarrow \infty$, all fields except the condensate frozen, one should recover the V-Y Lagrangian. However, in view of the above arguments, this condition appears to be too strong, as the models whose naive global limit we take are usually not the pure Yang-Mills models. So, we only demand that in this naive global limit the supersymmetry breaking is not induced by the nonabelian Yang-Mills sector, which is the common conclusion of all different methods used to analyze global Yang-Mills theories: effective Lagrangian, index theorem and instanton calculus. The method we follow here is the effective Lagrangian approach, as it is best suited to take into account.
the dynamics leading to condensation and, perhaps, to supersymmetry breaking.

In supergravity models the effective scalar potential which one has to minimize is

\[ V = g_{ij} F^i F^j - 3 e^G \]  

(16)

where \( G = K + \log(|W|^2) \), \( g_{ij} = \partial^2 K / \partial z^i \partial \bar{z}^j \) and \( F^i \) whose vacuum expectation value signals supersymmetry breaking is defined to be \( F^i = g^{i\bar{j}} \partial G / \partial \bar{z}^j e^{G/2} \). The simplest model one can consider contains a condensate with canonical kinetic term (in the following part we will use the field \( Y \) instead of \( U \), so that the symmetric kinetic term looks canonical [15])

\[ K = Y \bar{Y} \]  

(17)

and a simple Veneziano-Yankielowicz type superpotential

\[ W = Y^3 (3 \ln \frac{Y}{\mu} - 1), \]  

(18)

where \( \mu \) is the scale at which we expect the condensate to form (whenever we perform numerical calculations we choose it to be \( \mu = 10^{-5} M_{Pl} \)). After minimization of the potential (14) one discovers that there are two minima, one at \( Y = 0 \) (vanishing condensate) and one at \( Y \neq 0 \). One can easily see, looking at the values of the \( F^Y \) term, that at both minima supersymmetry is unbroken. This agrees at first glance with the result of the explicit one-instanton calculation of [16], where it is argued that instanton effects do not introduce SUSY breaking even in the local case. However, one should note that this instanton calculation cannot take fully into account the dynamics encoded in the choice of the Kähler function \( K \). In fact, the discrepancy with the one-instanton-induced result reported in [16] arises even without changing the Kähler function, as shown in the example discussed below.

The easiest way to extend the above toy model consists in the incorporation of a constant term in the superpotential, which can be thought of as parameterizing unknown and condensate unrelated effects:

\[ W = Y^3 (3 \ln \frac{Y}{\mu} - 1) + c \]  

(19)

For values of the constant \( c < 1.752 \mu M_{Pl}^2 \) supersymmetry remains unbroken but if the constant exceeds this value, supersymmetry breaking occurs. The breaking scale can be adjusted to any value by choosing an appropriate \( c \). Fig. 1 shows the dependence of the expectation value of the auxiliary field \( F_Y \) at the minimum with respect to \( c \). The apparent non-analyticity in this plot is not a numerical artifact as can be seen by looking at the shape of the potentials at the respective values (Fig. 2). At the critical value for \( c \) the minimum with non-vanishing condensate ceases to exist and reappears
again for larger values of the constant. This shows that some sort of ‘phase transition’ occurs.

Fig. 1 - Scale of supersymmetry breakdown with respect to the constant $c$

Fig. 2 - Shape of potential for different values of $c$
The next possibility of extending the basic model is to take non-minimal kinetic terms for the gaugino condensate. As proposed by [2] one could take

$$W = Y^3 (3 \ln \frac{Y}{\mu} - 1)$$  \hspace{1cm} (20)$$

together with

$$K = 3 \ln \frac{3}{1 - \frac{1}{c} Y^3 \bar{Y}^3 / \mu^4 - \bar{Y} Y},$$  \hspace{1cm} (21)$$

where the expansion with respect to $Y \bar{Y}$ gives minimal kinetic terms at the first order. The general structure of the scalar potential of this class of models parameterized by $c$ can be described as follows: there exists a minimum with $V = 0$ at $Y = 0$, corresponding to a vacuum with vanishing condensate. Supersymmetry is unbroken, of course. For any generic value of $c$ there is also a minimum next to a pole at nonzero $Y$ (Fig. 3). The cosmological constant of this vacuum may be positive or negative depending on $c$ and for $c = 9$ becomes 0 [3]. Whether supersymmetry is broken at this minimum depends on the value of $c$, as in the aforementioned model. This model exhibits a phase transition as well, which is much more pronounced. For $c > 9 + \epsilon$, where $\epsilon \simeq 10^{-6}$, supersymmetry is unbroken, for $c$ smaller than this value, SUSY is broken (meaning that also the zero cosmological constant version exhibits SUSY breaking).

Fig. 4 shows a plot of the SUSY breaking scale with respect to $x$, with $c = 9 + x$. When one considers the global limit of this class of models, SUSY is always unbroken, with the exception of the value $c = 9$. For this special value of $c$ SUSY is broken in the global limit, and the metric $g_{Y \bar{Y}}$ vanishes at this point, which means that small fluctuations of condensate around the minimum do not propagate in the flat limit. As the gaugino condensate is the only field considered in these models, it is of course a trivial statement, that it is the F-term of the condensate which is responsible for SUSY breaking. One should note, that with $Y^3$ being the expectation value of a composite field, also the goldstino is composite here.
Fig. 3 - Shape of $V$ for $c = 15$

Fig. 4 - Scale of supersymmetry breakdown with respect to the constant $c$
Whereas models with only the condensate field are obviously not very realistic\(^1\), these toy models can be studied to see which features are in principle available for SUSY breaking scenarios. Particularly one would like to see a realistic model containing a dilaton and moduli\(^2\), where supersymmetry is broken (preferably by the auxiliary field of the condensate) and the cosmological constant is 0. To determine what can be done to achieve these goals, we proceed to incorporate an additional field into our models, namely the dilaton, which has to be present in any string inspired supergravity model.

The way to incorporate the dilaton is to introduce a nontrivial gauge kinetic function as the coefficient of \(Y^3\) (formerly \(U\)) in the superpotential and to add a suitable term to the Kähler function, exactly as in global effective sigma models discussed in chapter 2, and then to put them into the formula \((1.6)\). In what follows in this chapter we commit ourselves to the no-scale form \(K = -\log(S + \bar{S})\) and \(f = S\) as dictated by perturbative string calculations\(^3\).

Incorporating the dilaton in this way into the models defined by \((17-20)\) makes it clear that their local versions exhibit serious deficiencies. In the case of global SUSY flat directions of vacua with unbroken supersymmetry exist in the models. Locally supersymmetric versions are even worse: the dilaton will either run to 0 or to \(\infty\), resulting in a strongly coupled or free theory, neither of which is a viable solution. This is a general feature of the locally supersymmetric potentials associated with models we discuss here when \(f = S\), which is a consequence of the symmetry \((14)\) of the superpotential. It holds also for the non-minimal kinetic term \((21)\). Hence, the attractive possibility suggested at the end of chapter 2 does not seem to be realized in the actual supergravity Lagrangian.

Incorporating additional moduli and matter fields does not give attractive solutions as well: more than one gaugino condensate is needed in any case and the matter fields have to live in specially chosen representations and have to acquire mass via their Yukawa couplings \(^4\). Of course, there is an important assumption which we make throughout this investigation – we demand a hierarchy between scales \(\mu\), the condensation scale and the Planck scale \(M_{\text{pl}}\). However, the existence of this hierarchy seems to be absolutely necessary if one wants to use gaugino condensates for realistic phenomenology.

Considering the above results one is tempted to conclude that the general condensate-dilaton structure of the hidden sectors discussed so far is insufficient to create an acceptable minimum fixing in a stable way values of all the fields involved (moduli and

\(^1\)As pointed out in \[2\] it is difficult to transmit SUSY-breaking to the matter sector in models whose hidden sector consists exclusively of gaugino condensates, also, string inspired models generically contain dilaton and moduli.

\(^2\)By moduli we mean gauge singlet chiral fields which enter couplings of the effective Lagrangian, but otherwise have no sources of potential – like moduli in string inspired models.

\(^3\)However, the general unpleasant features described below do not depend on the specific form of the Kähler potential for the dilaton, for instance we could have taken equally well \(K = \bar{S}S\).
condensate) and breaking supersymmetry. The last resort which is left is to modify
the superpotential for the dilaton, i.e. to take \( f \neq S \). When one tries, it quickly
becomes obvious that not every modification would work. In this situation the best
one can do is to try to control the modifications through some new symmetry. This is
the approach we discuss in the next chapter.

4 S-dual effective Lagrangians

Here we discuss a viable solution to the problem of the run-away dilaton (proposed in
[17]): taking a non-standard coupling of the condensate to the dilaton in the super-
potential. Guessing naively, and having in mind that a reasonable expectation value
of the dilaton lies somewhere in the neighborhood of unity, one would postulate a
superpotential of the form \( W \sim S + 1/S \). Amazingly enough, this simple superpo-
tential falls close to the superpotentials realizing the principle of S-duality in the low
energy effective Lagrangians. Having amassed in the preceding chapters the evidence
for the need of modifications in the S-dependence of the effective superpotential, we
find that S-duality, discussed recently both in the context of strings and in the context
of N=1, N=2 supersymmetric Yang-Mills theories, is the best motivated candidate to
control the required corrections, and the one which seems naturally suited to fulfill our
expectations.

As pointed out in [17] there are two different nontrivial ways of realizing the S-
duality in the Lagrangians of the type we discuss here. In its simplest realization,
S-duality is an \( SL(2,\mathbb{Z}) \) symmetry generated by \( S \rightarrow 1/S, \ S \rightarrow S + i \). We shall
discuss two physical realizations of S-duality which differ in the way the gaugino sector
transforms under the action of the first generator:

Type-I S-duality: here we assume that the gaugino sector closes under the S-duality
transformation. This states the invariance of \( fY^3 \) under S-duality. We have
shown in [17] that one can then redefine fields and assume that \( Y \) and \( f \) are both
independently invariant under the duality transformation. Thus it is described
by \( S \rightarrow 1/S \) and \( f \rightarrow f \) (or equivalently \( g^2 \rightarrow g^2 \)).

Type-II S-duality: if the gaugino sector (the ‘electric condensate’) does not close
one has to take an additional sector (the ‘magnetic condensate’). Only then
one can have true strong-weak coupling duality. Type-II S-duality is therefore
defined by the condition \( f \rightarrow 1/f \) (or equivalently \( g^2 \rightarrow 1/g^2 \)). Under the same
transformation these condensates would be interchanged.

Type-I S-duality

Because we can assume that the gaugino condensate does not transform under this
S-duality [17], we are forced to consider a \( SL(2,\mathbb{Z}) \)-invariant gauge kinetic function \( f \).
Demanding that for large $S$ the $f$ should behave asymptotically like $S$ (giving the old theory in the weak-coupling limit), we take

$$f = \frac{1}{2\pi} \ln(j(S) - 744), \quad (22)$$

where $j(S)$ denotes the usual generator of modular-invariant functions. Since the superpotential must achieve a modular weight of $-1$ to cancel the contributions of the Kähler potential, we are forced to include an $\eta^2(S)$ prefactor into the superpotential. We choose

$$K = -\ln(S + \bar{S}) + YY \quad (23)$$

and

$$W = \frac{Y^3}{\eta^2(S)} \left( \frac{1}{2\pi} \ln(j(S) - 744) - 3b \ln \frac{Y}{\mu} + c_0 \right) + c, \quad (24)$$

where we have again included a constant $c$ parameterizing unknown effects which do not depend on $S$ and $Y$. Not surprisingly, this superpotential breaks explicitly the “accidental” scale invariance of the superpotential (4), (14). Note that the constant we include into the superpotential breaks S-duality, therefore one can study whether the properties of the potential which are created by S-duality are stable under perturbations.

For $c = 0$ the scalar potential of this model exhibits a well defined minimum, regardless whether one considers the SUGRA case or goes to the global SUSY limit. Unfortunately, however, supersymmetry is unbroken in both cases. Changing $c$ to a non-zero value does not help, either. Up to some critical value of $c$, the minimum continues to exist and supersymmetry stays unbroken, but for larger values of the constant the minimum becomes unstable and vanishes.

We also studied the S-dual extension of the model given by (20, 21), therefore taking non-minimal kinetic terms for the gaugino condensate into account. Nevertheless we could not break supersymmetry regardless of the value of the constant. It is also easy to see, why there is a fundamental difference to the one-field model: the scalar potential (14) can be written as

$$V = e^K \left( g^{SS} G_S G_S + g^{YY} G_Y G_Y - 3WW \right) \quad (25)$$

where $G_x = K_x W - W_x$. It is the term containing the metric $g^{YY} = K_{YY}^{-1}$, which is responsible for the singularity. But whereas $G_Y$ has been a function of only $Y$ before, it is now a function of $S$ and $Y$. This additional freedom allows one to find values for $S$, where $G_Y = 0$ at the singularity in $K_{YY}^{-1}$. Therefore paths exists, along which the vevs can slide around the pole and then fall into the negative side of the pole. By a specific choice of the constant $c_0$ (changing in effect the condensation scale $\mu$) it is
possible to avoid these zeroes of $G_Y$ at the singularity, thus confining the vevs to the left side of the pole as in the $Y$-only model. But further analysis shows that in these cases supersymmetry is unbroken.

If adding the dilaton degree of freedom to a model destroys its supersymmetry breaking properties, one would expect that adding further fields, i.e. the generic modulus $T$ (the ‘breathing mode’ common to all string compactifications) does not change the picture any further. But this is not the case.

Adding a modulus $T$ in the usual $T$-duality invariant way \cite{15} gives rise to a combined model with $S$- and $T$-duality, given by

$$K = -\ln(S+\bar{S}) - 3\ln(T + T - Y\bar{Y}),$$  \hspace{1cm} (26)$$

and

$$W = \frac{Y^3}{\eta^2(S)j(S)^{q/3}}\left(\frac{1}{2\pi} \ln(j(S) - 744) - 3b \ln \frac{Y^2}{\eta} (T)\mu + c_0\right) + c. \hspace{1cm} (27)$$

This is actually a family of models because of the $j(S)^{q/3}$ factor in the denominator of the prefactor. We include this factor because it does not change the $S$-modular weight of $W$. In principle there could be any function of $j(S)$, although one has to watch out for singularities. For $q > 0$ one gets a scalar potential which vanishes for $S \to \infty$ at any fixed value of $T$ and $Y$ (for a discussion of this property, see \cite{17}).

Under $T$-duality we assume the condensate field $Y$ to transform as a modular function of weight $-1$, so that the superpotential has correct modular weight -3 under $T$-duality (if $c = 0$). The constant $c$ breaks both S- and T-dualities. The second constant $c_0$ can in principle be adjusted to change the scale $\mu$ (it can be reabsorbed into it \cite{17}). We adjust the value so that for $S = 1$ and $T = 1$ the minimum of the scalar potential is at $Y = \mu$, thus setting the condensation scale to $\mu^3$ in the case of $c = 0$ and global SUSY (where the actual minimum is at $S = 1, T = 1$).

This model exhibits SUSY breakdown with realistic expectation values of the fields. For $c = 0$ (unbroken S- and T-duality) one finds a minimum at $S = 1, T \simeq 1.23, Y \simeq \mu$. The SUSY breaking scale is determined by the expectation value of the auxiliary field of the modulus $<F_Y> \simeq \mu^3$. This is much larger than the other auxiliary fields: $<F_Y> \simeq \mu <F_T>$ and $<F_S> = 0$.

The cosmological constant is negative and of the order $-<F_T>^2 \simeq -\mu^5$.

For small values of the constant ($c = O(10^{-17})$) this solution is still stable, although several things happen with increasing $c$.

- $F_S$ becomes nonzero and becomes larger when $c$ grows and reaches up to $\simeq 10\%$ of $F_T$,

- $V_0$ increases with $c$ and eventually crosses value 0 and becomes positive. Thus there is the possibility of cancelling the cosmological constant.
For a larger value of $c$ the scalar potential becomes unstable and the dilaton runs away to infinity. Fig. 5 shows roughly the qualitative behaviour of the expectation values of the auxiliary fields and the value of the cosmological constant with respect to $c$. For larger values of $c$ the theory becomes unstable and the dilaton will run away, but in some intermediate region we can make no statement on the existence of a minimum (searching for a minimum is numerically very demanding, because the terms in the scalar potential differ by up to 20 orders of magnitude).

Nevertheless one has for example a satisfactory model with $q = 1$, $c \simeq 7 \times 10^{-18}$, $\mu = 10^{-5}$, where the vacuum is at $S = 1.002$, $T = 1.234$, $Y = 1.127\mu$, which has a vanishing cosmological constant and supersymmetry is broken: $<F_T> = 6 \times 10^{-18}$, $<F_S> = 2 \times 10^{-20}$ and $<F_Y> = 3 \times 10^{-23}$. Therefore the principle of S-duality allows one to break supersymmetry without going to multiple condensates or complicated matter representations, and with a simple constant, parameterizing unknown and duality violating contributions, one can also adjust the cosmological constant to 0.

One should take note of the fact, that contrary to our model with the gaugino condensate only, the constant $c$ in this model is very small in comparison to typical values of the rest of the superpotential. This makes this scenario much more appealing, because we cannot and should not expect that the gaugino superpotential is the only contribution (in principle these contributions could come from the Yukawa sector of
the superpotential, which we consider to be non-existent throughout our analyses).

An interesting observation in itself is the statement, that if one considers a symmetry acting on $S$ (in our case S-duality), this generically forces oneself to postulate the full effective theory to all loops. If one postulates a symmetry (acting on $S$) at the string tree-level, then at one loop this symmetry will typically either be broken due to anomaly cancelling contributions, or the anomaly will stay uncancelled.

Assume an effective supergravity theory at string tree-level. Under S-duality $S \rightarrow (aS - ib)/(icS + d)$ the Kähler potential transforms as

$$K \rightarrow K + \Phi(S) + \Phi(\bar{S}),$$

(28)

with $\Phi(S) = \ln(icS + d)$. To cancel this modular anomaly the gauge kinetic function $f$ has to transform like

$$f/\eta^2(S) \rightarrow f/\eta^2(S) - a\Phi(S),$$

(29)

whereas it was designed to transform like

$$f/\eta^2(S) \rightarrow f/\eta^2(S)(icS + d).$$

(30)

To achieve this, $S$ has to transform in a very complicated way. This will break S-duality and even after changing the Kähler function like in $[18]$, the new Lagrangian can hardly be invariant under the $SL(2,\mathbb{Z})$ transformation on $S$ any longer while still having the same Kähler transformation $\Phi(S)$ under the symmetry.

This seems to be a general principle: if the theory is at tree-level invariant under a symmetry acting on $S$ which produces a Kähler transformation (depending on the dilaton) $\Phi(S)$, then adding the anomaly cancelling terms generically makes it impossible to change the Kähler function and superpotential in such a way, that

1. the theory is still invariant under the S-transformation, and that

2. the Kähler transformation $\Phi(S)_{1-loop}$ associated with the old symmetry and the modified Kähler function is still equal to $\Phi(S)_{tree}$. This has to be the case, because otherwise the theory would not be anomaly free, because the counter-terms designed to cancel the anomaly coming from $\Phi(S)_{tree}$ will not cancel the one coming from $\Phi(S)_{1-loop}$.

Thus it seems not to be possible to promote a specific symmetry on $S$ to higher loops. On the contrary, if one proposes a symmetry like S-duality on the level of the fundamental string theory, this symmetry cannot be present in an effective action unless it is an action to all orders (if it is associated with a Kähler transformation). Therefore, since we want to impose S-duality as a physical principle, we have to demand the $SL(2,\mathbb{Z})_S$ invariance at the level of the full theory to all loops including non-perturbative effects.
Of course this shows, that our model can only be a toy model to show that imposing symmetry constraints can solve the problems of gaugino condensation. In a more realistic example we would expect to have mixing between the dilaton and the modulus, since we know that these are present already at the 1-loop level due to target-space modular invariance [19].

**Type-II S-duality**

The type-II implementation of S-duality takes into account the fact that the gaugino sector of the theory might not close under the S-duality transformation. To write down a model which is invariant one has to include an additional sector, the ‘magnetic condensate’, which is supposed to represent the dual phase of the theory [20].

The simplest toy model which illustrates the idea of type-II S-duality is given by

\[
K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - YY - H\bar{H}),
\]

\[
W = \frac{1}{\eta^2(S)}(Y^3 S + H^3 / S + 3bY^3 \ln \frac{Y\eta^2(T)}{\mu} + 3bH^3 \ln \frac{H\eta^2(T)}{\mu} + Y^3 H^3 / \mu^3).
\]

This model does not exhibit a full $SL(2, \mathbb{Z})$-symmetry, but only the strong-weak-coupling duality

\[
f = S \rightarrow 1/S, \quad Y \leftrightarrow H.
\]

In principle one could with some effort promote this symmetry to a full $SL(2, \mathbb{Z})$, but for illustration of our statements we choose this simple model, especially because both real and imaginary parts of $S$ already become fixed even with this smaller symmetry.

Again, the unwanted scale symmetry \[11, 12\] is not realized in the superpotential. The scalar potential possesses a (although rather hard to find) minimum close to $S = 1, T = 0.560, H = Y = \mu' = 3.64 \times 10^{-2} \mu [17]$. It turns out that at the minimum supersymmetry is broken, with the magnitude of SUSY breaking again determined by $<F_T> \simeq \mu'^3 / M$, where $\mu'$ is the dynamically determined value of the condensate at the minimum (in the previous type-I examples we adjusted (using $c$) $\mu'$ to be of the phenomenologically reasonable value $10^{-5}$). The cosmological constant is again negative and of the order $\simeq -\mu'^6$.

**5 General aspects of supersymmetry breaking**

All our models exhibit one universal feature, which seems to hold in all gaugino condensation models with dilaton and modulus, regardless of whether one works in the
effective Lagrangian or the effective superpotential approach: it is always $F_T$ which achieves the dominant vacuum expectation value of all the auxiliary fields. Of course one would like to know, whether this feature is generic or not. It should be made clear, that most of the models are constructed in a way which makes $<F_S>$ small or zero. In our models of the previous chapter we have assumed S-duality, therefore the scalar potential will always have an extremum at $S = 1$ and in our models this happens to be a minimum. But one can easily calculate that (if $c = 0$) $F_S(S = 1) = 0$. We showed that breaking the S-duality (by taking a constant into the superpotential) increased the value of $F_S$. So we believe that the smallness of $F_S$ is caused by our specific construction. The same is true in models where multiple gaugino condensates are used to fix the vev of the dilaton. These fix the value of the modulus $T$ by looking at only the minima which correspond to minima of the tree-level approximation. These are guaranteed to have $<F_S> = 0$ due to the fact that the superpotential factorizes into $S$ and $T$ dependent parts \[5\]. One-loop corrections are then able to give $F_S$ contributions, but these can be expected to be small. But it is not clear, whether these corrections do not introduce new minima which have $<F_S>$ of the same order as $<F_T>$.

This shows the possibility that $<F_S>$ could in principle become the dominant supersymmetry breaking contribution, and that statements which claim that $F_S$ is shown to stay small generically should be taken carefully.

Finally, let us examine the mass hierarchy of different terms in the potential of a typical condensate-moduli model \[21\]. The superpotential of these models naturally appears in such a form, that the magnitude of the superpotential at the SUSY breaking minimum (if there is any) is of the order $\mu^3$ (condensation scale cubed) which means that the gravitino mass term which usually sets the magnitude of the soft breaking terms in the observable sector is of the order of $\kappa^2 \mu^3$, i.e. lies in the TeV range when $\mu$ is of the order of a typical condensation scale for unification groups. When one writes down the potential and groups together terms of different order in $\mu$ one gets (after careful restitution of powers of the Planck scale)

$$V_1 = g^{UV} W_U \bar{W}_W \sim \mu^4 \quad (34)$$
$$V_2 = \kappa^2 g^{UV} K_U W \bar{W}_U + h.c. \sim \kappa^2 \mu^6 \quad (35)$$
$$V_3 = \kappa^2 (S + \bar{S})^2 e^{\kappa^2 K \frac{1}{S + \bar{S}}} |W + W_S|^2 \sim \kappa^2 \mu^6 \quad (36)$$
$$V_4 = -3 \kappa^2 |W|^2 \sim \kappa^2 \mu^6 \quad (37)$$
$$V_5 = \kappa^4 g^{UV} |K_U W|^2 \sim \kappa^4 \mu^8 \quad (38)$$

Among these terms the first one is dominating the potential, so one could try to find the zeroth-order minimum just minimizing this dominant term. The condition which is used widely to determine this approximate minimum is $W_U = 0$ which in turn allows, if the superpotential is simple enough, to express the condensate through the dilaton in
terms of superfields. Then one introduces this relation into the original superpotential obtaining an effective superpotential for \( S \) only, which is then processed in a usual way in quest for the minimum in \( S \). In simple models this approach, known as effective superpotential method, gives a pretty correct description of the behaviour of \( S \), but one has to be rather careful in drawing conclusions this way in more sophisticated models. First, the ‘new’ effective potential misses the terms (35) and (38), the first of which is formally of the order of the terms which are left. Second, the actual condition which enforces the unbroken global supersymmetry is \( g^U \bar{W}_U = 0 \) which in a case of a general Kähler potential for the condensate may have several solutions, some of them corresponding to vanishing of \( g^U \). Also, in general discussions of the cosmology of potentials induced by gaugino condensation one should study the full model. The condition of the type \( \partial W/\partial U = 0 \) singles out a curved ‘valley’ in the full potential, and in general excitations orthogonal to the valley are possible in the early universe as is the existence of other, disconnected valleys. Finally, as was demonstrated by our examples with constant terms in the superpotential, the existence of terms which are normally irrelevant for global supersymmetry can change dramatically the vacuum properties of local models, effectively changing the above discussed hierarchy of terms in the potential, thus invalidating the integrating-out procedure based on the condition \( \partial W/\partial U = 0 \). Indeed, even naive counting of powers of mass scales shows that if there is a constant of the order of \( \mu/\kappa^2 \) in the superpotential, the omitted terms (35) and (38) are in fact of the order of \( \mu^4 \), i.e. as important as the would-be leading term (although it should not be forgotten, that the constant which we can use to adjust the cosmological constant to 0 is much smaller: \( c \simeq 0.01 \mu^3 \)). Finally, we have shown for type-II S-duality in [17], that in cases where condensates couple to different functions of the dilaton, like \( S \) vs \( 1/S \), solving the simple integrating-out conditions becomes ambiguous if possible at all.

6 Discussion and conclusions

In the present paper we have analyzed the effective Lagrangian pertinent to supergravity hidden sectors composed of gauge fields, dilaton and other moduli.

We tried to construct models which are phenomenologically as realistic as possible, demanding that they possess potentials with stable minima corresponding to reasonable expectation values of all fields and exhibit supersymmetry broken so as to produce an acceptable gravitino mass in the TeV range.

We have been searching for such minima both analytically and using high precision numerical methods. In pure gauge models, without dilaton, we have found that the breaking of supersymmetry requires introduction of non-symmetric (non-minimal) Kähler functions or/and adding a sufficiently large constant to the superpotential (the M-term) in the local case. When one adds the dilaton to these models, the situation
becomes much more difficult. With only the simplest linear coupling of the dilaton to the gaugino condensate we were unable to find a model with a well defined nontrivial minimum, even including an arbitrary constant into superpotential and taking a non-symmetric Kähler function for the condensate. Also, we have confirmed that the global models which without the dilaton featured nontrivial (although supersymmetric) minima become “ill” when one couples to them the dilaton in a linear way - they generally acquire a flat or runaway direction associated with dilaton. Motivated by this evidence we suggest the need to modify the coupling of the dilaton to the gauge sector. We have proposed and discussed in detail modifications of the dilaton-gaugino effective Lagrangian consistent with the principle of S-duality. We have identified two different, physically nontrivial, ways of incorporating S-duality into the low-energy effective Lagrangian, which differ in the way the coupling constant is transformed. Both implementations lead easily to physically satisfying solutions, i.e. stable minima with broken supersymmetry, while employing only a single gaugino condensate and without the need for specially constructed matter sectors. We feel that this solution to the long-standing problem of runaway dilaton vacua is more attractive than the traditional methods relying on multiple condensates. In general, although we cannot prove this in a mathematically rigorous way, it seems reasonably safe to conclude that in the full nonperturbative effective Lagrangian coming from superstring the coupling of the dilaton is likely to be significantly different from the simple linear one implied by perturbative calculations if a hidden gauge sector is the source of supersymmetry breaking and mass hierarchy.

Acknowledgments

This work was supported by the Deutsche Forschungsgemeinschaft and EC grants SC1–CT92–0789 and SC1–CT91–0729. Z.L. has been supported by the A. von Humboldt Fellowship. A. N. has been supported by a PhD scholarship from the Technical University of Munich. We would like to thank Andre Lukas, Peter Mayr and Dimitris Matalliotakis for useful discussions.

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