Theoretical model of HD 163296 presently forming in-situ planets and comparison with the models of AS 209, HL Tau, and TW Hya

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ABSTRACT

We fit an isothermal oscillatory density model to the disk of HD 163296 in which planets have presumably already formed and they are orbiting at least within the four observed dark gaps. This 156 AU large axisymmetric disk shows various physical properties comparable to those of AS 209, HL Tau, and TW Hya that we have modeled previously, but it compares best to AS 209. The disks of HD 163296 and AS 209 are comparable in size and they share similar values of the power-law index \(p \approx 1\) (a radial density profile of the form \(\rho(R) \propto R^{-p}\)), the rotational parameter \(\beta_0\) (to within a factor of 3); a relatively small inner core radius (although this parameter for HD 163296 is exceptionally small, \(R_1 \approx 0.15\) AU, presumably due to unresolved planets in the inner 50 AU); the scale length \(R_0\) and the Jeans gravitational frequency \(\Omega_J\) (to within factors of 1.4); the equation of state \((\rho_0/c_s^2)\) and the central density \(\rho_0\) (to within factors of 2); and the core angular velocity \(\Omega_0\) (to within a factor of 4.5). In the end, we compare all six nebular disks that we have modeled so far.

Keywords. planets and satellites: dynamical evolution and stability—planets and satellites: formation—protoplanetary disks

1. Introduction

In previous work (Christodoulou & Kazanas 2019a,b,c,d), we presented isothermal models of the solar nebula and four ALMA/DSHARP observed protostellar disks capable of forming protoplanets long before their protostars will actually be formed by gas accretion/dispersal processes. This new “bottom-up” planet formation scenario is currently observed in real time by the latest high-resolution (~1-5 AU) observations of many protostellar disks by the ALMA telescope (ALMA Partnership 2015; Andrews et al. 2016; Ruane 2017; Lee et al. 2017; 2018; Macias et al. 2018; Avenhaus et al. 2018; Clarke et al. 2018; Koppler et al. 2018; Guzman et al. 2018; Isella et al. 2018; Zhang et al. 2018; Dullemont et al. 2018; Favre et al. 2018; Harsono et al. 2018; Huang et al. 2018; Pérez et al. 2018; Kudo et al. 2018; Long et al. 2018; Pineda et al. 2018; van der Marel et al. 2019).

The ALMA/DSHARP observations show many circular protostellar disks with annular dark gaps presumably carved out by protoplanets that have already been formed at a time long before accretion/dispersal processes will dissipate these gaseous disks. Up until now, few disks show asymmetries and spiral arms, signs of developing instabilities (Pérez et al. 2018; Huang et al. 2018; van der Marel et al. 2019; Villenave et al. 2019). Motivated by the DSHARP observations of annular gaps, we have produced theoretical models of the disks of AS 209 (seven gaps), HL Tau (seven gaps), TW Hya (5 gaps), and RU Lup (4 gaps) (Christodoulou & Kazanas 2019b,c,d). In this work, we apply the same theoretical model to the observed disk of HD 163296, a young protostellar system observed by ALMA/DSHARP (Huang et al. 2018). HD 163296 turns out to be difficult to model. It shows only four widely-spaced dark gaps over a radial extent of 156 pc, and it is evident that its inner 50 pc are not adequately resolved by the ALMA observations (Huang et al. 2018). The analytic (intrinsic) and numerical (oscillatory) solutions of the isothermal Lane-Emden equation (Lane 1869; Emden 1907) with differential rotation, and the resulting model of the midplane of the gaseous disk have been described in detail in Christodoulou & Kazanas (2019a) for the solar nebula. Here, we apply in § 2 the same model to the four dark gaps of HD 163296. In § 2 we summarize our modeling results and we compare the inferred physical parameters of HD 163296 against those found previously for AS 209, HL Tau, and TW Hya.

2. Physical Models of the HD 163296 Protostellar Disk

2.1. Numerical Setup

The numerical integrations that produce oscillatory density profiles were performed with the MATLAB ode15s integrator (Shampine & Reichelt 1997; Shampine et al. 1999) and the optimization used the Nelder-Mead simplex algorithm as implemented by Lagarias et al. (1998). This method (MATLAB routine fminsearch) does not use any numerical or analytical gradients in its search procedure which makes it extremely stable numerically, albeit somewhat slow. The boundary conditions for the oscillatory density profiles are, as usual, \(\tau(0) = 1\) and \([\tau/dx](0) = 0\), where \(\tau\) and \(x\) are the dimensionless values of the density and the radius, respectively.
Table 1. Radii of dark gaps in AS 209, HL Tau, TW Hya, and HD 163296 (from Table 1 of Huang et al. 2018)

| Gap Name | AS 209 R (AU) | Gap Name | HL Tau R (AU) | Gap Name | TW Hya R (AU) | Gap Name | HD 163296 R (AU) |
|----------|---------------|----------|--------------|----------|--------------|----------|-----------------|
| D9       | 08.69         | D14      | 13.9         | D1       | 1            | D10      | 10              |
| D24      | 23.84         | D34      | 33.9         | D26      | 25.62        | D48      | 48              |
| D35      | 35.04         | D44      | 44           | D32      | 31.5         | D86      | 86.4            |
| D61      | 60.8          | D53      | 53           | D42      | 41.64        | D145     | 145             |
| D90      | 89.9          | D67      | 67.4         | D48      | 48           | D44      | 44              |
| D105     | 105.5         | D77      | 77.4         |          |              |          |                 |
| D137     | 137           | D96      | 96           |          |              |          |                 |

Table 2. Comparison of the protostellar disk model of HD 163296 against AS 209, HL Tau, and TW Hya

| Property Name | Property Symbol (Unit) | AS 209 Best-Fit Model | HL Tau Best-Fit Model | TW Hya Best-Fit Model | HD 163296 Best-Fit Model |
|---------------|------------------------|-----------------------|-----------------------|-----------------------|-------------------------|
| Density power-law index | $k$ | 0.0 | 0.0 | -0.2 | -0.2 |
| Rotational parameter | $\beta_0$ | 0.0165 | 0.00562 | 0.00401 | 0.0530 |
| Inner core radius | $R_1$ (AU) | 6.555 | 52.04 | 28.67 | 0.1492 |
| Outer flat-density radius | $R_2$ (AU) | 68.96 | 90.55 | ... | 38.79 |
| Scale length | $R_0$ (AU) | 0.01835 | 0.009813 | 0.004100 | 0.01312 |
| Equation of state | $\epsilon / \rho_0$ (cm$^2$ g$^{-1}$ s$^{-2}$) | $6.32 \times 10^{16}$ | $1.81 \times 10^{16}$ | $3.15 \times 10^{15}$ | $3.23 \times 10^{16}$ |
| Minimum core density$^a$ | $\rho_0$ (g cm$^{-3}$) | $5.62 \times 10^{-9}$ | $1.97 \times 10^{-8}$ | $1.13 \times 10^{-7}$ | $1.10 \times 10^{-8}$ |
| Isothermal sound speed$^a$ | $c_0$ (m s$^{-1}$) | 188 | 188 | 188 | 188 |
| Jeans gravitational frequency | $\Omega_j$ (rad s$^{-1}$) | $4.9 \times 10^{-8}$ | $9.1 \times 10^{-8}$ | $2.2 \times 10^{-7}$ | $6.8 \times 10^{-8}$ |
| Core angular velocity | $\Omega_c$ (rad s$^{-1}$) | $8.0 \times 10^{-10}$ | $5.1 \times 10^{-10}$ | $8.7 \times 10^{-10}$ | $3.6 \times 10^{-9}$ |
| Core rotation period | $P_0$ (yr) | 249 | 390 | 228 | 55.3 |
| Maximum disk size | $R_{\text{max}}$ (AU) | 144 | 102 | 52 | 156 |

$^a$Calculated for $T = 10$ K and $\mu = 2.34$

Fig. 1. Equilibrium density profile for the midplane of the HD 163296 disk that has already formed at least four annular dark gaps (presumably protoplanets) (Huang et al. 2018). The best-fit parameters are $k = -0.2$, $\beta_0 = 0.0530$, $R_1 = 0.1492$ AU, and $R_2 = 38.79$ AU. The radial scale length of the disk is $R_0 = 0.01312$ AU. The Cauchy solution (solid line) has been fitted to the dark gaps of HD 163296 (Table 1) so that its density maxima (dots) correspond to the observed orbits of the protoplanets (open circles). The density maximum corresponding to the location of the first maximum was scaled to a distance of 10 AU of the D10 gap. The mean relative error of the fit is 6.3%, most of it coming from the two outer gaps (Table 1). The intrinsic analytical solution (dashed line) and the nonrotating analytical solution (dash-dotted line) are also shown for reference. Notice the exceptionally small inner core ($\ln R_1 = -1.90$) of the model.

Fig. 2. Schematic diagram of the ALMA-observed dark gaps (dots) that we have modeled so far. Key: SS: Solar System; J: Jupiter; P: Pluto. The crosses represent the empty density peaks in which no dark gaps have been observed yet. The dark gaps of HD 163296 are depicted by black dots. The disk of this protostellar system is comparable in size to the disk of AS 209, and the calculated physical properties of the best-fit model indicate that the HD 163296 model is more similar to AS 209 than any other model (Table 2).

2.2. Best-Fit model of HD 163296

The radii of the four dark gaps observed in HD 163296 are shown in Table 1. In Fig. 1, we show the best optimized fit to these four gaps. In the models, we could not produce any acceptable result with only three free parameters ($k$, $\beta_0$, and $R_1$). We had to also introduce a flat-density region starting at radius $R_2$, and it turns out that three of four gaps fall within this outer intrinsically flat region. The mean relative error of the fit is 6.3%
and it comes mostly from the two outermost gaps D86 and D145 (Table 1).

The physical properties of the best-fit HD 163296 model are listed in Table 2 along with the best-fit models of AS 209, HL Tau, and TW Hya. Although the best-fit model of HD 163296 shares similar parameters with all of these other protostellar disks, it is apparent that it is most similar to the best-fit model of AS 209. A detailed comparison between the disks of HD 163296 and AS 209 is made in § 3 below.

3. Summary and Discussion

In § 2 we presented the best-fit isothermal differentially-rotating protostellar model of HD 163296 observed by ALMA/DSHARP (Huang et al. 2018). This model shows four dark gaps in a large disk (Table 1), and it is widely believed that protoplanets have already formed and carved out these gaps in the observed axisymmetric disk. The best-fit model is depicted in Figure 1 and a comparison of its physical properties versus AS 209, HL Tau, and TW Hya is shown in Table 2. The physical properties of HD 163296 are much closer to those of AS 209 than any of the other disks listed in Table 2.

In Figure 2 we show a schematic diagram of dark gaps (dots) in the ALMA-observed disks that we have modeled up until now, and we have also included our outer solar system (Outer SS). There is no physics to be deduced from this figure (as opposed to Table 2), this layout is only a relative comparison of the arrangements of dark gaps in the depicted protostellar systems. In conjunction with the physical properties listed in Table 2, the picture that emerges about HD 163296 is the following:

1. In general, one can identify various isolated properties of HD 163296 that are similar to any of the other of these other disks.
2. However HD 163296 is most similar to AS 209 in its physical properties (Table 2) and its layout of dark gaps (Fig. 2), although it exhibits fewer gaps than AS 209. In particular, we note the following physical properties that are comparable between these two disks:
   2a. The power-law index of HD 163296 is $k = -0.2$, close to zero as in AS 209. This implies a radial density profile of approximately $\rho(R) \propto R^{-1}$ for both disks.
   2b. The inner core radius of HD 163296 has the smallest radius found during all of our modeling efforts. The inner radius is merely $R_1 \approx 0.15 \text{ AU}$, as opposed to the relatively small inner radius of $\approx 0.6 \text{ AU}$ for AS 209 and $\approx 0.82 \text{ AU}$ for the solar nebula.
   2c. The rotational parameter of HD 163296, $\beta_0 = 0.0530$, is so small that it guarantees the stability of the young disk against nonaxisymmetric dynamical instabilities (Christodoulou et al. 1995).
   2d. The scale length $R_0$ and the Jeans gravitational frequency $\Omega_J$ of the two disks are very much comparable to within factors of merely 1.4.
   2e. The equation of state ($\epsilon_1/\rho_0$) and the central density $\rho_0$ of the two disks are comparable to within factors of about 2.
   2f. The only fundamental parameter that is slightly larger in HD 163296 is the core's angular velocity $\omega_1$; it is larger by a factor of 4.5 than in AS 209. We note however that the cores of both systems are not adequately resolved by the current ALMA/DSHARP observations (Huang et al. 2018).

From these comparisons, it is evident that the protostellar disk of HD 163296 is quite similar in structure and physical properties to the disk of AS 209 (Table 2) and quite dissimilar to our solar nebula (Christodoulou & Kazanas 2019a).

References

ALMA Partnership, Brogan, C. L., Perez, L. M., Hunter, T. R., et al. 2015, ApJ, 808, L3
Andrews, S. M., & Williams, J. P. 2007, ApJ, 659, 705
Andrews, S. N., Wilner, D. J., Zhu, Z., et al. 2016, ApJ, 820, L40
Avenhaus, H., Quanz, S. P., Garufi, A., et al. 2018, ApJ, 863, 44
Brown, M. E., & Pan, M. 2007, ApJ, 677, 2418
Christodoulou, D. M., & Kazanas, D. 2019a, arXiv: 1901.02593 (Solar Nebula)
Christodoulou, D. M., & Kazanas, D. 2019b, Part 7, arXiv: 1901.10642 (AS 209 and Tau)
Christodoulou, D. M., & Kazanas, D. 2019c, Part 8, arXiv: 1902.01222 (RU Lup)
Christodoulou, D. M., & Kazanas, D. 2019d, Part 9, arXiv: 1902.04547 (TW Hya)
Christodoulou, D. M., Shlosman, I., & Tohline, J. E. 1995, ApJ, 443, 551
Clarks, C. J., Tazzari, M., Juhasz, A., et al. 2018, ApJ, 866, L6
Dullemond, C. P., Birnstiel, T., Huang, J., et al. 2018, DSHARP VI, ApJ, 869, L46
Ehrenreich, R. 1997, Gaskugeln, Leipzig, B. G. Teubner
Favre, C., Federle, D., Mauz, L., et al. 2018, ApJ, arXiv:1812.04062
Greaves, J. S., & Rice, W. K. M. 2010, MNRAS, 407, 1981
Guzmán, V. V., Huang, J., Andrews, S. M., et al. 2018, DSHARP VIII, ApJ, 869, L48
Harsono, D., Bjerkeli, P., van der Wiel, M. H. D., et al. 2018, Nature Astronomy, 2, 646
Huang, J., Andrews, S. N., Dullemond, C. P., et al. 2018, DSHARP II, ApJ, 869, L42
Hung, C.-L., Lai, S.-P., & Yan, C.-H. 2010, ApJ, 710, 207
Isella, A., Huang, J., Andrews, S. N., et al. 2018, DSHARP IX, ApJ, 869, L49
Keppler, M., Benisty, M., Müller, A., et al. 2018, A&A, 617, A44
Kudo, T., Hashimoto, J., Muto, T., et al. 2018, ApJ, 868, L5
Lagarías, J. C., Reeds, J. A., Wright, M. H., & Wright, P. E. 1998, SIAM Journal of Optimization, 9, 112
Lane, L. J. H. 1869-70, Amer. J. Sci. Arts, 4, 57
Lee, C.-F., Li, Z.-Y., Ho, P. T. P., et al. 2017, ApJ, 843, 27
Lee, C.-F., Li, Z.-Y., Hirano, N., et al. 2018, ApJ, 863, 94
Long, F., Pinilla, P., Herczeg, G. J., et al. 2018, ApJ, 869, 17
Macius, E., Espaillat, C. C., Ribas, A, Schwarz, K. R., et al. 2018, ApJ, 865, 37
Pérez, L. M., Arenou, P., Andrews, S. N., et al. 2018, DSHARP X, ApJ, 869, L10
Pineda, J. E., Szulágyi, J., Quanz, S. P., van Dishoeck, E. F., et al. 2018, ApJ, arXiv:1811.10365
Ruane, G., Mawet, D., Kastner, J., et al. 2017, AJ, 154, 1
Shampine, L.F., Reichelt, M. W. 1997, SIAM Journal on Scientific Computing, 18, 1
Shampine, L.F., Reichelt, M. W., & Kierzenka, J. A. 1999, SIAM Review, 41, 538
Tohline, J. E. 2002, ARA&A, 40, 349
Trujillo, C. A., & Brown, M. E. 2003, Earth, Moon and Planets, 92, 99
van der Marel, N., Dong, R., di Francesco, J., et al. 2019, ApJ, arXiv:1901.03680
Villenave, M., Benisty, M., Dent, W. R. F., Ménard, F., et al. 2019, A&A, in press, arXiv:1902.04612
Zhang, S., Zhu, Z., Huang, J., et al. 2018, DSHARP VII, ApJ, 869, L47