Research Article

Stability and Global Sensitivity Analysis for an Agree-Disagree Model: Partial Rank Correlation Coefficient and Latin Hypercube Sampling Methods

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In this paper, we present a new mathematical model that describes agree-disagree opinions during polls. We first present the model and its different compartments. Then, we use the next-generation matrix method to compute thresholds of equilibrium stability. We perform the stability analysis of equilibria to determine under which conditions these equilibrium points are stable or unstable. We show that the existence and stability of these equilibria are controlled by the calculated thresholds. Finally, we also perform several computational and statistical experiments to validate the theoretical results obtained in this work. To study the influence of various parameters on these thresholds and to identify the most influential parameters, a global sensitivity analysis is carried out based on the partial rank correlation coefficient method and the Latin hypercube sampling.

1. Introduction

Participation in political life requires citizens’ attention, time, knowledge, money, and motivation. Citizens will participate if they receive benefits commensurate with the cost of participation. Perhaps, the debate on voting is the subject of the most studied political behavior. Individual economic, social, and psychological costs and voting benefits are well known [1]. Candidates and the media often use polls in election campaigns to determine which candidates are in the lead and who are likely to emerge victoriously. Opinion polls are surveys of intent for a sample of voters, while future results allow participants to negotiate and discuss opinions based on a particular outcome. Opinion polls revealed the expected voting quotas. Public opinion polls are widely used to identify people’s political positions, vote, and other behaviors by asking questions about their opinions, activities, and personal characteristics. Answers to these questions are then counted, statistically analyzed, and interpreted. Polls’ result also provides parties with the opportunity to refine their campaign strategies. In the long run, parties change their positions in public opinion [2].

Polls then play a key role in contemporary political campaigns. Party performance updates receive considerable media attention and often serve as a basis for political commentary in the weeks leading up to the election day. We know that knowing the positions of the electorate can affect voter behavior [3, 4]. However, it is unclear how voting shapes party leaders’ strategies during the election campaign. Public opinion is often criticized for influencing voter perceptions and focusing on popularity rather than politics. By strategically communicating the poll results, parties can shape voters’ vision of competition and encourage popular mobilization [5]. This explains why, at the beginning of the 20th century, more than thirty democratic countries banned the publication of opinion polls near elections [6, 7].

To fully understand the role of polls before the elections, it is necessary to take into account the offer of campaigns. This is particularly important in contexts of increased polarization, where holidays play an increasing role in determining the type of information that reaches voters [8]. At any time, party leaders may choose to speak of candidates from their party or other candidates for election, focusing on various political or gender issues. Voter signals in opinion
polls can influence party leaders’ decisions to balance these different elements of campaign speech [9].

In the electoral process, where there are only two political parties, the chances of creating strong political polarization are increasing. For example, U. S. citizens face a polarizing scenario during a presidential election, where they must vote for a two-party decision: Republicans and Democrats. Previous studies have shown that U. S. elections are attracting the Internet, where both Twitter and blogs are showing high political polarization [10, 11].

Other political scenarios requiring a bilateral decision are second-round electoral processes. While citizens of the first round can vote from a wide range of different political parties, in the last round, they can vote only for the two final candidates. Voters may not be fully identified with either side, but they still have to take a side. Previous articles have shown that this second round increases political polarization in the country [12]. In [13], the authors analyzed the 2017 presidential election in Chile and measured the resulting political polarization. Of a minority of qualified users, they could estimate the opinion of the majority.

In this contribution, we examine how voters’ preferences and expectations compete with each other when dealing with voter support information to candidates. So, we start by developing a mathematical model to describe the evolution of opinions to predict the probability of results, and then we calculate and analyze the equilibrium points of the model by deriving the important stability thresholds for each equilibrium state.

Through the development of more efficient models, reliable statistical and mathematical methods are needed to improve the accuracy of modeling. One of the most recently popular ways is sensitivity analysis (SA) methods. Sensitivity analysis is used for a variety of reasons, such as developing decisions or recommendations, communicating, understanding or quantifying the system, and developing models. In the development of the model, it can be used to verify the validity or accuracy of the model, to simplify, calibrate, weak process, or lost data, and even to identify the important parameter for other studies [14].

The paper is organized as follows: Section 2 introduces our new model, giving some details about interactions between different compartments and parameters of the model. In Section 3, we derive basic reproduction numbers. Section 4 provides the results of the stability analysis of equilibria. In Section 5, sensitivity analysis is performed to identify the most important parameter in the proposed model, and Section 6 concludes the paper.

2. Presentation of the Model

There are many scenarios involving a binary decision. The poll model that we present here describes the opinions of agreement (or approval) and disagreement (or disapproval), regarding a candidate or idea, in polls preceding the elections. Note that the model can also describe situations in which there are more than two candidate parties because we can always reduce the situation to two decisions. For example, if there are four parts A, B, C, and D, we wish to study the political position of party A, and we can then examine the two subsets \{A\} and \{B, C, D\} of the investigation. Votes for A are considered agree, and votes for B, C, or D are considered disagree. More simply, we consider the opinion poll of voters on the performance of the party studied. As an example, we can cite the “approve or disapprove” poll done by Thomson Reuters on how Donald Trump fulfills his role as the president [15].

Without loss of generality, we devise a mathematical model describing the evolution of agree and disagree opinions during polls (surveys), and the types of surveys we consider here are surveys that can be answered in agreement, disagreement, or otherwise. Thus, the population targeted by the poll is regrouped into three groups: agree, disagree, and ignorant individuals.

The model herewith has been formulated using three compartments. Each of them has been described as follows:

1. Ignorant (I): people who do not know about the poll or those who abstain from voting for personal reasons
2. Agree (A): people in agreement with the idea being studied
3. Disagree (D): people in disagreement with the idea being studied

All contacts are modeled by the standard incidence rate. For the modeling processes, a set of assumptions has been used. These are as follows:

1. The targeted population is well mixed, that is, the ignorant individuals are homogeneously spread throughout the population
2. Recruitment and mortality are negligible under the temporal scale consideration; therefore, no individual is recruited and no individual dies during the poll
3. Individuals have the right to communicate with each other and can thus convince one another
4. People who are unsure of their opinion are ignorant
5. People who abstain from voting are ignorant

Everyone has their reasons for agreement or disagreement. An ignorant person can be persuaded by someone who agrees at a rate \( \beta_1 \) or by someone who does not agree with the opinion at a rate \( \beta_2 \). A person agreeing with the opinion may be persuaded by someone who does not agree at a rate \( \alpha_2 \), or a person who disagrees with the opinion may be persuaded by someone who agrees at a rate \( \alpha_1 \). People can abstain from voting or lose interest without any direct contact with individuals from the opposite opinion group, then agree people become ignorant at the rate \( \gamma_1 \), and disagree people become ignorant at the rate \( \gamma_2 \). A flowchart describing different interactions between the compartments of the model is presented in Figure 1.

All these assumptions and considerations are written as the following system of ordinary differential equations:
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\[ I' = -\beta_1 \frac{AI}{N} - \beta_2 \frac{DI}{N} + \gamma_1 A + \gamma_2 D, \]
\[ A' = \beta_1 \frac{AI}{N} + \alpha_1 \frac{AD}{N} - \alpha_2 \frac{AD}{N} - \gamma_1 A, \]
\[ D' = \beta_2 \frac{DI}{N} + \alpha_1 \frac{AD}{N} - \alpha_2 \frac{AD}{N} - \gamma_2 D, \]

where \( I(0) \geq 0, A(0) \geq 0, D(0) \geq 0, \) and \( N = I + A + D. \) Note that \( N' = I' + A' + D' = 0; \) thus, the population size \( N \) is considered as a constant in time. We can easily prove that, for nonnegative initial conditions, the solutions of systems (1)–(3) are nonnegative. To do this, recall that by \([16]\) the system of equation

\[ x' = f(x_1, x_2, \ldots, x_k), \]

with

\[ x(0) = x_0 \geq 0, \]

is a positive system if and only if \( \forall i = 1, 2, \ldots, k. \)

Thus, for models (1)–(3), it is easy to verify that

\[ I = 0 \implies I' \geq 0, \]
\[ A = 0 \implies A' \geq 0, \]
\[ D = 0 \implies D' \geq 0. \]

Therefore, all the solutions of systems (1)–(3) are nonnegative.

It is also clear that the solutions of models (1)–(3) are bounded based on the fact that \( N = I + A + D \) is constant, and then \( S \leq N, A \leq N, \) and \( D \leq N. \) Therefore, we will focus to study models (1)–(3) in the closed positively invariant feasible set given by

\[ \Omega = \left\{ (I, A, D) \in \mathbb{R}_+^3 \mid \frac{I}{A + D} = N \right\}. \]

A summary of parameter description is given in Table 1.

### Table 1: Parameter descriptions.

| Parameter | Description |
|-----------|-------------|
| \( \beta_1 \) | Ignorant to agree transmission rate |
| \( \beta_2 \) | Ignorant to disagree transmission rate |
| \( \alpha_1 \) | Disagree to agree transmission rate |
| \( \alpha_2 \) | Agree to disagree transmission rate |
| \( \gamma_1 \) | Interest loss factor of agree individuals |
| \( \gamma_2 \) | Interest loss factor of disagree individuals |

3. Thresholds: Basic Reproductive Numbers

In epidemiology, the basic reproductive number \( R_0 \) (or epidemic threshold) is defined as the average number of secondary cases of an infection produced by a “typical” infected individual during his/her entire life as infectious when introduced in a population of susceptibles \([17–21]\). The threshold \( R_0 \) is mathematically characterized in terms of infection transmission as a “demographic process,” but offspring production is not seen as giving birth in a demographic sense, but it causes new infections through transmission \([22–24]\). Thus, the infection process can be considered as a “consecutive generation of infected individuals.” The following growing generations indicate a growing population (i.e., an epidemic), and the growth factor for each generation indicates the potential for growth. So, the mathematical characterization of \( R_0 \) is this growth factor \([22]\). Generally, if \( R_0 > 1 \), an epidemic occurs, whereas if \( R_0 < 1 \), there will probably be no epidemic.

Following this definition, we will define our thresholds as follows: \( R_{D0} \) is the average number of new disagreements produced by an individual disagreeing introduced in a population of ignorant people during the period in which he or she was in this opinion. And, \( R_{A0} \) is the average number of new agreements produced by an individual in agreement and that was introduced into a population of ignorant people during the period in which he or she was in that opinion.

For the analysis of epidemic models, the first step is to calculate the disease-free equilibrium (DFE). This equilibrium point is then used to calculate the basic reproductive number using the next-generation matrix method. The objective of this section is only the calculation of the thresholds and not the equilibrium states of the model. But, in this method, we have to determine the equilibrium states when \( A = 0 \) and when \( D = 0 \).

In this contribution, let \( R_{X0} \) be the threshold of growing of the opinion \( X \) (either \( X = A \) “agree” or \( X = D \) “disagree”). Then, \( R_{X0} \) is the threshold associated with the disagree-free equilibrium, while \( R_{A0} \) is the one associated with the agree-free equilibrium.

We calculate the equilibria for the aforementioned model, and based on the next-generation matrix approach, we derive associated thresholds.

The points of equilibrium of systems (1)–(3) indicate the potential for growth. So, the mathematical characterization of \( R_0 \) is this proof.

\[ I' = A' = D' = 0, \]

for the disagree-free equilibrium when there is no negative opinion, i.e., if we put \( D = 0 \). This gives
where $I^*$ and $A^*$ represent the numbers of ignorant and agree individuals, respectively, in the absence of disagree people.

Therefore, for the system governed by (1)–(3), the disagree-free equilibrium is

$$e_1 = \left( \frac{y_1}{\beta_1}, N, 0, \frac{N(\beta_1 - y_1)}{\beta_1} \right).$$

Following the second generation approach [22], we compute the threshold $R_{D0}$ associated to the disagree-free equilibrium, which is

$$R_{D0} = \frac{\alpha_1 \beta_1 - \alpha_2 y_1 + \beta_2 y_2}{\alpha_1 \beta_1 - \alpha_1 y_1 + \beta_1 y_2}$$

for the agree-free equilibrium when there is no positive opinion, i.e., if we put $A = 0$. This gives

$$I^* = \frac{y_2}{\beta_2} N,$$

$$D^* = \frac{N(\beta_2 - y_2)}{\beta_2},$$

where $I^*$ and $D^*$ represent the numbers of ignorant and disagree individuals, respectively, in the absence of agree people.

Therefore, for the system governed by (1)–(3), the agree-free equilibrium is

$$e_2 = \left( \frac{y_2}{\beta_2} N, 0, \frac{N(\beta_2 - y_2)}{\beta_2} \right).$$

By also following the second generation approach [22], we compute the threshold $R_{A0}$ associated to the agree-free equilibrium, which is

$$R_{A0} = \frac{\alpha_1 \beta_2 - \alpha_2 y_2 + \beta_1 y_1}{\alpha_2 \beta_2 - \alpha_1 y_2 + \beta_2 y_1}.$$

Throughout this paper, we consider the following assumption: the model parameters verify

$$\beta_1 - y_1 > \frac{\beta_1 y_2}{a_1},$$

$$\beta_1 - y_1 > \frac{\beta_2 y_1}{a_2},$$

$$\beta_2 - y_2 > \frac{\beta_2 y_1}{a_2},$$

$$\beta_2 - y_2 > \frac{\beta_1 y_2}{a_1}.$$

Therefore, from the above assumption, we get

$$R_{D0} = \frac{\alpha_1 \beta_1 - \alpha_2 y_1 + \beta_2 y_2}{\alpha_1 \beta_1 - \alpha_1 y_1 + \beta_1 y_2} > 0,$$

$$R_{A0} = \frac{\alpha_1 \beta_2 - \alpha_2 y_2 + \beta_1 y_1}{\alpha_2 \beta_2 - \alpha_1 y_2 + \beta_2 y_1} > 0.$$

### 4. Stability Analysis

In order to analyze in terms of the proportions of ignorant, agree, and disagree individuals, let $i = (I/N), a = (A/N)$, and $d = (D/N)$ denote the fraction of the classes $I, A, \text{and} D$ in the population, respectively. After some calculations and replacing $I$ by $i, A \text{by} a, \text{and} D \text{by} d$, equations (1)–(3) can be written as

$$i' = -\beta_1 ai - \beta_2 di + \gamma_1 a + \gamma_2 d,$$

$$a' = \beta_1 ai + a_1 ad - \alpha_2 ad - \gamma_1 a,$$

$$d' = \beta_2 di + \alpha_2 ad - \alpha_1 ad - \gamma_d d.$$  

From the fact $N = I + A + D$, we have $i + a + d = 1$. Then, model systems (18)–(20) will be reduced to the following two differential equations:

$$a' = \beta_1 a (1 - a - d) + a_1 ad - \alpha_2 ad - \gamma_1 a,$$

$$d' = \beta_2 d (1 - a - d) + a_1 ad - \alpha_2 ad - \gamma_d d,$$

which can be reduced to

$$a' = \beta_1 a (1 - a - d) + a_1 ad - \gamma_1 a,$$

$$d' = \beta_2 d (1 - a - d) + a_1 ad - \gamma_d d,$$

where $\alpha = \alpha_1 - \alpha_2$.

#### 4.1. Steady States.

The steady states of system (22) are obtained by solving the system of equations

$$0 = \beta_1 a (1 - a - d) + a_1 ad - \gamma_1 a,$$

$$0 = \beta_2 d (1 - a - d) + a_1 ad - \gamma_d d.$$  

This system has four equilibrium points, and the trivial equilibrium $E_0 = (0, 0, 0)$ is an equilibrium that exists always without any condition. It means there is no survey and there is no need to opinions.

One disagree-free equilibrium $E_1 = (1 - (y_1/\beta_1), 0, 0)$ exists if the condition $\beta_1 > y_1$ holds, and the agree-free equilibrium $E_2 = (0, 1 - (y_2/\beta_2))$ exists if $\beta_2 > y_2$ holds.

The fourth and the positive equilibrium $E^* = (a^*, d^*)$, where

$$a^* = \frac{a_1 \beta_2 - \alpha_2 y_2 + \beta_2 y_1}{a_2 (\alpha_1 - \beta_1 + \beta_2)},$$

$$d^* = \frac{a_1 \beta_1 - \alpha_1 y_1 + \beta_1 y_2}{a_2 (\alpha_1 - \beta_1 + \beta_2)},$$

exists if one of the following conditions holds: $R_{D0} > 1$ and $R_{A0} > 1$ and $a (\alpha_1 - \beta_1 + \beta_2) > 0$ or $R_{D0} < 1$.
and $R_{A0} < 1$ and $\alpha (\alpha - \beta_1 + \beta_2) < 0$. In fact, by a simple calculation, we get

$$a\beta_2 - \alpha y_2 + \beta_1 y_2 - \beta_2 y_1 > 0 \Longleftrightarrow R_{A0} > 1,$$

$$a\beta_1 - \alpha y_1 + \beta_1 y_2 - \beta_2 y_1 < 0 \Longleftrightarrow R_{D0} > 1.$$  \hspace{1cm} (25)

Thus, from (25), we deduce that $\alpha^* > 0$ and $d^* > 0$. Throughout the article and without loss of generality, we just consider the first condition as a sufficient condition for the existence of $E_*$.

### 4.2. Stability of Steady States

The Jacobian matrix of system (22) is

$$J = \begin{pmatrix}
J_{11} & a\alpha - a\beta_1 \\
-a d - \beta_2 d & J_{22}
\end{pmatrix},$$

where

$$J_{11} = a d - a \beta_1 - \gamma_1 - \beta_1 (a + d - 1),$$

$$J_{22} = -\gamma_2 - a a - \beta_2 d - \beta_2 (a + d - 1).$$  \hspace{1cm} (27)

**Proposition 1.** The equilibrium $E_0 = (0, 0)$ is unstable if $\beta_1 > \gamma_1$ or $\beta_2 > \gamma_2$. Otherwise, it is stable.

**Proof.** The Jacobian matrix at this equilibrium is

$$J(E_0) = \begin{pmatrix}
\beta_1 - \gamma_1 & 0 \\
0 & \beta_2 - \gamma_2
\end{pmatrix}.$$  \hspace{1cm} (28)

It is clear that if $\beta_1 > \gamma_1$ or $\beta_2 > \gamma_2$, we get one positive eigenvalue of $J(E_0)$, and then $E_0$ is unstable. Else, we have all eigenvalues of $J(E_0)$ having the negative real part, which completes the proof. \hfill $\square$

**Remark 1.** Note that the conditions in the previous proposition imply the existence of $E_1$ or $E_2$. Therefore, $E_0$ is unstable whenever there exists $E_1$ or $E_2$.

**Proposition 2.** The equilibrium $E_1 = (1 - (\gamma_1/\beta_1), 0)$ is unstable if $R_{D0} > 1$. Otherwise, it is stable.

**Proof.** The Jacobian matrix at this equilibrium is

$$J(E_1) = \begin{pmatrix}
\gamma_1 - \beta_1 & (\alpha - \beta_1)(\beta_1 - \gamma_1) \\
0 & \alpha (\gamma_1/\beta_1 - 1) - \gamma_2 + \beta_2 y_1/eta_1
\end{pmatrix}.$$  \hspace{1cm} (29)

The eigenvalues of $J(E_1)$ are

$$\lambda_1 = \gamma_1 - \beta_1,$$

$$\lambda_2 = \alpha (\gamma_1/\beta_1 - 1) - \gamma_2 + \beta_2 y_1/eta_1.$$  \hspace{1cm} (30)

It is clear from the existence condition of this equilibrium that $\gamma_1 - \beta_1 < 0$. Thus, the stability of the point of equilibrium $E_1$ is based on the eigenvalue $\lambda_1$ of the matrix $J(E_1)$. By a simple calculation, we have

$$a\beta_1 - \alpha y_1 + \beta_1 y_2 - \beta_2 y_1 < 0 \Longleftrightarrow R_{D0} > 1.$$  \hspace{1cm} (31)

This implies that if $R_{D0} > 1$, $E_1$ is unstable; else, $E_1$ is stable. \hfill $\square$

**Proposition 3.** The equilibrium $E_2 = (0, 1 - (\gamma_2/\beta_2))$ is unstable if $R_{A0} > 1$. Otherwise, it is stable.

**Proof.** The Jacobian matrix at this equilibrium is

$$J(E_2) = \begin{pmatrix}
\beta_1 y_2/\beta_2 - \alpha (\gamma_2/\beta_2 - 1) - \gamma_1 & 0 \\
-(\alpha + \beta_2)(\beta_2 - y_2)/\beta_2 & \gamma_2 - \beta_2
\end{pmatrix}.$$  \hspace{1cm} (32)

The eigenvalues of $J(E_2)$ are

$$\lambda_1 = \beta_1 y_2/\beta_2 - \alpha (\gamma_2/\beta_2 - 1) - \gamma_1,$$

$$\lambda_2 = \gamma_2 - \beta_2.$$  \hspace{1cm} (33)

It is clear from the existence condition of this equilibrium that $\gamma_2 - \beta_2 < 0$. Thus, the stability of the point of equilibrium $E_2$ is based on the eigenvalue $\lambda_1$ of the matrix $J(E_2)$. By a simple calculation, we have

$$a\beta_2 - \alpha y_2 + \beta_1 y_2 - \beta_2 y_1 > 0 \Longleftrightarrow R_{A0} > 1.$$  \hspace{1cm} (34)

This implies that if $R_{A0} > 1$, $E_2$ is unstable; else, $E_2$ is stable. \hfill $\square$

**Proposition 4.** The equilibrium $E^* = (\alpha^*, d^*)$, where

$$\alpha^* = \frac{a\beta_2 - \alpha y_2 + \beta_1 y_2 - \beta_2 y_1}{\alpha (\alpha - \beta_1 + \beta_2)},$$

$$d^* = \frac{-a\beta_1 y_2 + \beta_1 y_2 - \beta_2 y_1}{\alpha (\alpha - \beta_1 + \beta_2)},$$

is stable if $\beta_1 < \gamma_1$ and $\gamma_2 < \beta_2$ and $\alpha > 0$ or $\beta_1 > \gamma_1$ and $\gamma_2 > \beta_2$ and $\alpha < 0$.

**Proof.** The Jacobian matrix at this equilibrium is

$$J(E^*) = \begin{pmatrix}
J_{11} & \alpha^* \alpha - \alpha^* \beta_1 \\
-\alpha d^* - \beta_2 d^* & J_{22}
\end{pmatrix},$$

where

$$J_{11} = a\alpha^* - \alpha^* \beta_1 - \gamma_1 - \beta_1 (\alpha^* + d^* - 1),$$

$$J_{22} = -\gamma_2 - a^* \alpha - \beta_2 d^* - \beta_2 (a + d - 1).$$  \hspace{1cm} (37)

Using fact (23), we have
where the characteristic polynomial of is
\[ \lambda^2 + c_1\lambda + c_2, \]
\[ (40) \]

where
\[ c_1 = -\frac{\beta_1\gamma_2 - \beta_2\gamma_1}{\alpha}, \]
\[ c_2 = \frac{-1}{\alpha (\alpha - \beta_1 + \beta_2)} \times (a^2 \beta_1\beta_2 - a^2 \beta_1\gamma_2 - a^2 \beta_2\gamma_1 + a^2 \gamma_1\gamma_2 + \alpha \beta_1\gamma_2 - \alpha \beta_2\gamma_1 + \alpha \beta_1\gamma_2 - \alpha \beta_2\gamma_1 - \alpha \beta_1\gamma_2 + \alpha \beta_2\gamma_1 + \alpha^2 \gamma_1\gamma_2 + \beta_1\gamma_2 - \beta_2\gamma_1 - \beta_1\gamma_2 + \beta_2\gamma_1). \]
\[ (41) \]

By the conditions “\( \beta_1 < \gamma_1 \) and \( \gamma_2 < \beta_2 \) and \( \alpha > 0 \)” or “\( \beta_1 > \gamma_1 \) and \( \gamma_2 > \beta_2 \) and \( \alpha < 0 \),” we have \( -(\beta_1\gamma_2 - \beta_2\gamma_1/\alpha) > 0 \), and by existence conditions and
\[ a\beta_2 - a\gamma_2 + \beta_2\gamma_1 - \beta_2\gamma_1 > 0 \iff R_{D0} > 1, \]
\[ a\beta_1 - a\gamma_1 + \beta_1\gamma_2 - \beta_1\gamma_2 < 0 \iff R_{A0} > 1, \]
\[ (42) \]
we have \( c_1 > 0 \) and \( c_2 > 0 \). Using the Routh–Hurwitz stability criterion, we conclude that the equilibrium point \( E^* \) is locally asymptotically stable.

Remark 2

(1) Note that the conditions \( R_{D0} > 1 \) and \( R_{A0} > 1 \) lead to the instability of \( E_3 \) and \( E_2 \) and help in the existence of \( E^* \).

(2) Sufficient conditions in the above proposition could be reduced to \( \alpha (\beta_1\gamma_2 - \beta_2\gamma_1) < 0 \).

A summary of sufficient existence and stability conditions of the steady states of model (11) is given in Table 2.

4.3. Examples. It can be seen from Table 3 that the equilibrium \( E_0 \) exists and it is stable in the four cases: example 1: \( R_{D0} > 1 \) and \( R_{A0} > 1 \), example 2: \( R_{A0} < 1 \) and \( R_{D0} < 1 \), example 3: \( R_{D0} < 1 \) and \( R_{A0} > 1 \), and example 4: \( R_{D0} > 1 \) and \( R_{A0} < 1 \).

Example 5 shows the existence and the stability of the steady state \( E_1 \) in the case of \( R_{A0} < 1 \) and \( R_{D0} < 1 \) and in the case of \( R_{A0} > 1 \) and \( R_{D0} < 1 \) in example 6. Examples 7 and 8 show the existence and the stability of the equilibrium \( E_2 \) in the cases of “\( R_{A0} < 1 \) and \( R_{D0} < 1 \)” and “\( R_{A0} < 1 \) and \( R_{D0} > 1 \),” respectively.

Examples 9 and 10 show the possibility of the existence and stability of the two steady states \( E_1 \) and \( E_2 \) at the same time, that is, “\( \beta_1 > \gamma_1 \) and \( \beta_2 > \gamma_2 \) and \( R_{A0} < 1 \) and \( R_{D0} < 1 \)”.” These examples give an insight about the stability of each equilibrium for given parameters’ values. After some numerical calculations, we noted that, in this situation, the parameters \( \gamma_1 \) and \( \gamma_2 \) may switch from \( E_1 \) to \( E_2 \) and the inverse, while if \( \gamma_1 > \gamma_2 \), then \( E_1 \) will be more attractive, and if \( \gamma_2 > \gamma_1 \), then \( E_2 \) will be more attractive and that with the same initial conditions. We simulate the model with different initial conditions to illustrate the impact of initial conditions on the stability of \( E_1 \) and \( E_2 \) in this situation. Figure 2 depicts the stability of \( E_1 \) and \( E_2 \) at the same time, where we consider the same set of parameters from example 9 in Table 3. By changing the initial conditions, we can see that \( E_1 \) is stable when \( I(0) = 100, A(0) = 100 \), and \( D(0) = 80 \), while it can be seen that \( E_2 \) is also stable with this set of parameters but after choosing \( I(0) = 100, A(0) = 80 \), and \( D(0) = 100 \).

Example 11 shows the existence and the stability of the equilibrium \( E_+, \) where \( “R_{D0} > 1 \) and \( R_{A0} > 1 \) and \( \alpha (\alpha - \beta_1 + \beta_2) > 0 \)” and \( “\beta_1 < \gamma_1 \) and \( \gamma_2 < \beta_2 \) and \( \alpha > 0 \),” while example 12 shows the existence and the stability of \( E_+, \) where \( “R_{D0} > 1 \) and \( R_{A0} > 1 \) and \( \alpha (\alpha - \beta_1 + \beta_2) > 0 \)” and \( “\beta_1 > \gamma_1 \) and \( \gamma_2 > \beta_2 \) and \( \alpha < 0 \).”

Figure 3 depicts examples of the existence and stability of the equilibrium state \( E_0 \) for different parameters’ values and threshold \( R_{D0} \) and \( R_{A0} \) values simulated with initial conditions and parameters’ values from Table 3. Figure 4 depicts examples of the existence and stability of the equilibrium state \( E_1 \) for different parameters’ values and threshold \( R_{D0} \) and \( R_{A0} \) values simulated with initial conditions and parameters’ values from Table 3. It can be seen from Figure 4(b) that the function \( D \) decreases towards zero, but it will take a long time. In the Figure 4(c), we have considered 1,200 hours to show that the function \( D \) will go to zero but very slowly. Figure 5 depicts examples of the existence and stability of the equilibrium state \( E_1 \) for different

\[ J_{11} = -\beta_1\alpha^*, \]
\[ J_{22} = -\beta_2\alpha^*. \]
\[ (38) \]
Table 2: Summary of sufficient existence and stability conditions of equilibria.

| Equilibrium   | Existence conditions | Stability conditions |
|---------------|----------------------|----------------------|
| $E_0$         | $\beta_1 > \gamma_1$ | $R_{D0} < 1$         |
| $E_1 = (1 - (\gamma_1/\beta_1), 0)$ | $\beta_1 > \gamma_1$ | $R_{D0} < 1$         |
| $E_2 = (0, 1 - (\gamma_2/\beta_2))$ | $\beta_2 > \gamma_2$ | $R_{A0} < 1$         |
| $E_4 = (a^*, d^*)$ | $R_{D0} > 1$ and $R_{A0} > 1$ | $\beta_1 < \gamma_1$ and $\beta_2 < \gamma_2$ |

Table 3: Steady states and parameters’ values used in example simulations, where $I(0) = A(0) = D(0) = 100$.

|   | $\beta_1$ | $\beta_2$ | $\alpha_1$ | $\alpha_2$ | $\gamma_1$ | $\gamma_2$ | $R_{D0}$ | $R_{A0}$ |
|---|-----------|-----------|-------------|-------------|------------|------------|----------|----------|
| 1 | 0.0010    | 0.1010    | 0.1010      | 0.3010      | 0.5010     | 0.3010     | 1.9901   | 2.0730   |
| 2 | 0.0010    | 0.2010    | 0.2010      | 0.3010      | 0.5010     | 0.5010     | 0.5000   | 0.8543   |
| 3 | 0.0010    | 0.0010    | 0.1010      | 0.2010      | 0.1010     | 0.1010     | 0.5000   | 2.0001   |
| 4 | 0.0010    | 0.1010    | 0.1010      | 0.2010      | 0.1010     | 0.1010     | 2.0001   | 0.5000   |
| 5 | (2.9703, 297.0297, 0) | 0.1010 | 0.0010 | 0.2010 | 0.1010 | 0.0010 | 0.0010 | 0.6634 | 0.4925 |
| 6 | (2.9703, 297.0297, 0) | 0.1010 | 0.0010 | 0.2010 | 0.1010 | 0.0010 | 0.0010 | 0.5025 | 101 |
| 7 | (2.9703, 297.0297) | 0.0010 | 0.1010 | 0.0010 | 0.2010 | 0.1010 | 0.0010 | 0.4925 | 0.6634 |
| 8 | (2.9703, 297.0297) | 0.0010 | 0.1010 | 0.0010 | 0.2010 | 0.1010 | 0.0010 | 0.4925 | 0.6634 |
| 9 | (120.3593, 179.6407, 0) | 0.5010 | 0.1010 | 0.1010 | 0.2010 | 0.2010 | 0.1010 | 0.6688 | 0.5196 |
| 10 | (120.3593, 179.6407) | 0.1010 | 0.5010 | 0.0010 | 0.1010 | 0.0010 | 0.2010 | 0.5196 | 0.6688 |
| 11 | (133.3333, 10.5556, 156.1111) | 0.1010 | 0.7010 | 0.3010 | 0.0010 | 0.2010 | 0.3010 | 467.7774 | 1.0672 |
| 12 | (214.2857, 80.4643, 5.2500) | 0.3010 | 0.4010 | 0.0010 | 0.8010 | 0.2010 | 0.5010 | 1.0649 | 300.8004 |

Figure 2: Illustration of the bistability of $E_1$ and $E_2$ at the same time using the set of parameters of example 9 from Table 3. Stability of $E_1$ (a) when $I(0) = 100$, $A(0) = 100$, and $D(0) = 80$. Stability of $E_2$ (b) when $I(0) = 100$, $A(0) = 80$, and $D(0) = 100$.

5. Analysis of Thresholds

5.1. No Abstention: No Loss of Interest. In most of the situations, people abstain from voting for personal reasons. Here, we discuss the situation when there is no abstention and there is no loss of interest of voting, i.e., $\gamma_1 = \gamma_2 = 0$. Therefore, $R_{D0}$ and $R_{A0}$ become

$$R_{D0} = \frac{\alpha_2}{\alpha_1},$$

$$R_{A0} = \frac{\alpha_1}{\alpha_2},$$

which means that thresholds $R_{D0}$ and $R_{A0}$ depend only on $\alpha_1$ and $\alpha_2$, and the equilibria become

$$E_1 = (1, 0),$$

$$E_2 = (0, 1),$$

and from the existence conditions in Table 2, we can deduce that there is no $E_*$. $E_1$ exists without any condition, and it is stable if $\alpha_1 < \alpha_2$, and $E_2$ exists without any condition, and it is stable if $\alpha_1 < \alpha_2$.

This result explains the effect of the polarization on the outcome of polls. When there is no abstention from voting and people keep their interest in voting, then the most influential parameters on the outcome of the polls are the polarization factors $\alpha_1$ and $\alpha_2$. For example, during a poll determining the political position of one candidate Y by voting with Approve or Disapprove or otherwise, if candidate Y can convince people by his political vision and/or by other ways, then he can change the course of events in his favor. Mathematically, he makes $\alpha_2 < \alpha_1$. But, if somehow he contributes to making $\alpha_1 < \alpha_2$, then things could spin out of control on the election day.
Figure 3: Examples of the steady state $E_0$ simulated with initial conditions $I(0) = 100$, $A(0) = 100$, and $D(0) = 100$ and parameters’ values from Table 3. (a) Example 1. (b) Example 2. (c) Example 3. (d) Example 4.

Figure 4: Continued.
5.2. One Chance. In some situations, voters are not allowed to modify their choice; then, they could take one side until the end of the survey. In this section, we discuss the situation when there is no persuasion and then there is no polarization, i.e., $\alpha_1 = \alpha_2 = 0$. Therefore, $R_{D0}$ and $R_{A0}$ become

$$R_{D0} = \frac{\beta_2 y_1}{\beta_1 y_2}$$

$$R_{A0} = \frac{\beta_1 y_2}{\beta_2 y_1}$$

(45)
which means that thresholds $R_{D0}$ and $R_{A0}$ depend only on $\beta_1$, $\beta_2$, $\gamma_1$, and $\gamma_2$, and there is no change in the equilibria $E_1$ and $E_2$. From the conditions of the existence in Table 2, we can deduce that there is no $E_1$. A sufficient condition to make $E_1$ stable is $\beta_2 < \beta_1$ and $\gamma_1 < \gamma_2$, and the sufficient condition to make $E_2$ stable is $\beta_2 > \beta_1$ and $\gamma_1 > \gamma_2$. This result explains the efficiency of election campaigns. For instance, if there are two candidate parts $X$ and $Y$ and the poll is carried out to study the political position of $Y$, then votes for $Y$ are considered as agree and votes for $X$ are considered as disagree. If candidate $Y$ presents a successful election campaign, it will attract more people alongside him and increase the number of people agreeing (i.e., $\beta_2 < \beta_1$), which may lead disagree people to lose interest or abstain from voting (i.e., $\gamma_1 < \gamma_2$).

5.3. Statistical Analysis. Here, we use probability distribution functions of the six parameters given in Table 4 sampled by using the Latin hypercube sampling, see Figure 7. We compute the probabilities of equilibria existence and stability conditions. It can be seen from Table 5 that the probability of $E_1$ to exist is about 0.5960, while its probability of stability is about 0.4040. The probability of the existence of $E_2$ is about 0.6250, while the probability of its stability is about 0.5370. The probability of $E_1$ to exist and to be stable is 0.43. The equilibrium $E_1$ has a probability of existence about 0.0240 and a probability of stability about 0.2190, while $E_2$ exists, and it is stable with a probability of 0.53.

6. Sensitivity Analysis

Global sensitivity analysis (GSA) approach helps to identify the effectiveness of model parameters or inputs and thus provides essential information about the model performance. Out of the many methods of carrying out sensitivity analysis is the partial rank correlation coefficient (PRCC) method that has been used in this paper. The PRCC is a method based on sampling. One of the most efficient methods used for sampling is the Latin hypercube sampling (LHS), which is a type of Monte Carlo sampling [25] as it densely stratifies the input parameters. As the name suggests, the PRCC measures the strength between the inputs and outputs of the model using correlation through the sampling done by the LHS method [26–28].

Parameters $\beta_1$ and $\beta_2$ follow normal distribution with mean and standard deviation 0.5 and 0.01, respectively, while parameters $\alpha_1$, $\alpha_2$, $\gamma_1$, and $\gamma_2$ follow triangular distribution with minimum, maximum, and mode as 0.02, 0.8, and 0.51, respectively. A summary of probability distribution functions is given in Table 4. In Latin hypercube sampling approach, probability density function (given in Table 4) for each parameter is stratified into 100 equiprobable (1/100) serial intervals. Then, a single value is chosen randomly from each interval. This produces 100 sets of values of $R_{D0}$ and $R_{A0}$ from 100 sets of different parameter values mixed randomly, calculated by using equations (4) and (5), respectively.

Sensitivity analysis has been done concerning the basic reproduction numbers $R_{D0}$ and $R_{A0}$ and the main objective of this section is to identify which parameters are important in contributing variability to the outcome of basic reproduction numbers based on their estimation uncertainty. To sort the model parameters according to the size of their effect on $R_{D0}$ and $R_{A0}$, a partial rank correlation coefficient is calculated between the values of each of the six parameters and the values of $R_{D0}$ and $R_{A0}$ in order to identify and measure the statistical influence of any one of the six input parameters on thresholds $R_{D0}$ and $R_{A0}$. The larger the partial rank correlation coefficient, the larger the influence is on the input parameter affecting the magnitude of $R_{D0}$ and $R_{A0}$.

![Figure 6: Examples of the steady state $E_*$ simulated with initial conditions $I(0) = 100$, $A(0) = 100$, and $D(0) = 100$ and parameters’ values from Table 3. (a) Example 11. (b) Example 12.](image_url)
As shown in Table 6, the transmission rate, $\beta_1$, and parameters $c_1$ and $c_2$ are highly correlated with the threshold $R_{D_0}$ with corresponding values $-0.1374$ and $0.1613$ and $-0.2101$, respectively. Moderate correlation exists between transmission rates $\alpha_1$ and $\alpha_2$ and $R_{D_0}$ with corresponding values as $-0.0633$ and $0.0669$, respectively. Weak correlations have been observed between the transmission rate $\beta_2$ and $R_{D_0}$ with the corresponding value $0.0025$.

As shown in the sensitivity index column of Table 6, the parameter $c_2$ accounts for the maximum variability 0.8499 in the outcome of basic reproduction number $R_{D_0}$. The parameter $\beta_1$ is then the next to account for the variability 0.7492 in the outcome of $R_{D_0}$. Then, the transmission rate $\beta_1$ accounts for the variability 0.5596 in the outcome of $R_{D_0}$ followed by the transmission rate $\alpha_2$ that accounts for the variability 0.5269 in the outcome of $R_{D_0}$. Transmission parameters $\alpha_1$ and $\beta_2$ account for the least variability 0.0192 and 0.0192 in the outcome of basic reproduction number $R_{D_0}$, respectively. Hence, parameters $\gamma_1$ and $\gamma_2$ and the transmission rate $\beta_1$ are the most influential parameters in determining $R_{D_0}$.

It can be seen from Table 7 that the parameter $\gamma_1$ is highly correlated with the threshold $R_{E_0}$ with the corresponding value $-0.3343$ and $6.7425e-04$, respectively. Moderate correlation exists between transmission rates $\alpha_1$ and $\alpha_2$ and $R_{E_0}$ with corresponding values as $0.0857$ and $0.0382$, respectively. Weak correlations have been observed between the transmission rate $\beta_2$ and $R_{E_0}$ with the corresponding value $0.2523$ and $0.2076$, respectively.

**Table 5:** Summary of probabilities of existence and stability conditions of equilibria.

| Equilibrium | Probability of existence | Probability of stability | Probability of existence and stability |
|-------------|--------------------------|--------------------------|--------------------------------------|
| $E_1$       | 0.5960                   | 0.5300                   | 0.4040                               |
| $E_2$       | 0.6250                   | 0.5370                   | 0.4300                               |
| $E_*$       | 0.0240                   | 0.2190                   | 0.001                                |

**Table 6:** PRCCs for $R_{D_0}$ and six input parameters and the corresponding sensitivity index.

| Parameters | Sampling | PRCCs | p value | Sensitivity index |
|------------|----------|-------|---------|-------------------|
| $\beta_1$  | LHS      | -0.1374 | 0.1728 | 0.5596            |
| $\beta_2$  | LHS      | 0.0025  | 0.9800 | 0.0192            |
| $\alpha_1$ | LHS      | -0.0633 | 0.5318 | 0.1065            |
| $\alpha_2$ | LHS      | 0.0669  | 0.5085 | 0.5269            |
| $\gamma_1$ | LHS      | 0.1613  | 0.1090 | 0.7492            |
| $\gamma_2$ | LHS      | -0.2101 | 0.0359 | 0.8499            |

**Table 7:** PRCCs for $R_{E_0}$ and six input parameters and the corresponding sensitivity index.

| Parameters | Sampling | PRCCs | p value | Sensitivity index |
|------------|----------|-------|---------|-------------------|
| $\beta_1$  | LHS      | 0.2137 | 0.0328  | 0.5753            |
| $\beta_2$  | LHS      | -0.1928| 0.0546  | 0.2421            |
| $\alpha_1$ | LHS      | 0.0857 | 0.3967  | 0.0159            |
| $\alpha_2$ | LHS      | -0.2076| 0.0382  | 0.3872            |
| $\gamma_1$ | LHS      | -0.3343| 6.7425e-04| 0.8004            |
| $\gamma_2$ | LHS      | 0.2523 | 0.0113  | 0.7122            |

**Figure 7:** Probability distribution functions of the six sampled input parameters’ values. These results have been obtained from Latin hypercube sampling using a sample size of 1000. (a) $\beta_1$. (b) $\beta_2$. (c) $\alpha_1$. (d) $\alpha_2$. (e) $\gamma_1$. (f) $\gamma_2$.
Figure 8: Scatter plots for the basic reproduction number $R_{D0}$ and six sampled input parameters’ values. These results have been obtained from Latin hypercube sampling using a sample size of 100. (a) $\beta_1$. (b) $\beta_2$. (c) $\alpha_1$. (d) $\alpha_2$. (e) $c_1$. (f) $c_2$.

Figure 9: Continued.
of these points of equilibrium is controlled by the threshold $\lambda$ and the transmission rate $\beta$. The parameter $\gamma$ accounts for the variability $0.5753$ in the outcome of this threshold followed by the transmission rate $\alpha$ that accounts for the variability $0.3872$ in the outcome of $R_{A0}$. The parameter $\gamma_1$ and $\gamma_2$ are also the most influential parameters in the determination of thresholds $R_{A0}$ and $R_{D0}$. If the threshold, $R_{D0}$, is less than one, the disagree opinion dies out and the disagree-free equilibrium is stable. If $R_{D0}$ is greater than one, the disagree opinion persists and the disagree-free equilibrium is unstable. If the threshold, $R_{A0}$, is less than one, the agree opinion dies out and the agree-free equilibrium is stable. If $R_{A0}$ is greater than one, the agree opinion persists and the disagree-free equilibrium is unstable. We simulated some examples with different parameter values to show the existence and the stability of such equilibria.

Weak correlations have been observed between the transmission parameters $\beta_1$, $\beta_2$, and $\alpha$ and the parameter $\gamma_2$ and $R_{A0}$ with corresponding values as $0.2137$, $-0.1928$, $-0.2076$, and $0.2523$, respectively. Weak correlations have been observed between the transmission rate $\alpha$ and $R_{A0}$ with the corresponding value $0.0857$.

One can see in the sensitivity index column of Table 7 that the parameter $\gamma_1$ accounts for the maximum variability $0.8004$ in the outcome of basic reproduction number $R_{A0}$. The parameter $\gamma_2$ is the next to account for the variability $0.7122$ in the outcome of $R_{A0}$. The transmission rate $\beta_1$ accounts for the variability $0.5753$ in the outcome of this threshold followed by the transmission rate $\alpha$ that accounts for the variability $0.3872$ in the outcome of $R_{A0}$. Transmission parameters $\beta_2$ and $\alpha$ account for the least variability $0.2421$ and $0.0159$ in the outcome of basic reproduction number $R_{A0}$, respectively. Hence, parameters $\gamma_1$ and $\gamma_2$ and the transmission rate $\beta_1$ are also the most influential parameters in determining $R_{A0}$.

Scatter plots comparing the basic reproduction numbers $R_{D0}$ and $R_{A0}$ against each of the six parameters: $\beta_1$, $\beta_2$, $\alpha_1$, $\alpha_2$, $\gamma_1$, and $\gamma_2$ are shown in Figures 8 and 9, respectively, based on Latin hypercube sampling with a sample size of 100. These scatter plots clearly show the linear relationships between outcome of $R_{D0}$ and $R_{A0}$ and input parameters.

7. Conclusion

In this paper, an IAD model-type compartmental model has been considered to explore agree-disagree opinions during polls. The equations governing the system have been solved to compute equilibrium states, and the next-generation matrix method is used to derive basic reproduction numbers $R_{A0}$ and $R_{D0}$.

The model exhibits four feasible points of equilibrium, namely, the trivial equilibrium, agree-free equilibrium, disagree-free equilibrium, and the positive equilibrium. Sufficient equilibrium conditions of existence are given, and stability analysis is performed to show under which conditions equilibrium states are stable or unstable. The stability of these points of equilibrium is controlled by the threshold number $R_{A0}$ and $R_{D0}$. The parameter $\gamma_1$ is the next to account for the variability $0.5753$ in the outcome of $R_{A0}$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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