Mitigating large adversarial perturbations on X-MAS (X minus Moving Averaged Samples)

Woohyung Chun 1 Sung-Min Hong 1 Junho Huh 1 Inyup Kang 1

Abstract

We propose the scheme that mitigates the adversarial perturbation $\epsilon$ on the adversarial example $X_{adv} (= X \pm \epsilon, X$ is a benign sample) by subtracting the estimated perturbation $\hat{\epsilon}$ from $X + \epsilon$ and adding $\hat{\epsilon}$ to $X - \epsilon$. The estimated perturbation $\hat{\epsilon}$ comes from the difference between $X_{adv}$ and its moving-averaged outcome $W_{avg} * X_{adv}$ where $W_{avg}$ is $N \times N$ moving average kernel that all the coefficients are one. Usually, the adjacent samples of an image are close to each other such that we can let $X \approx W_{avg} * X$ (naming this relation after X-MAS [X minus Moving Averaged Samples]). By doing that, we can make the estimated perturbation $\hat{\epsilon}$ falls within the range of $\epsilon$. The scheme is also extended to do the multi-level mitigation by configuring the mitigated adversarial example $X_{adv} \pm \hat{\epsilon}$ as a new adversarial example to be mitigated. The multi-level mitigation gets $X_{adv}$ closer to $X$ with a smaller (i.e. mitigated) perturbation than original unmitigated perturbation by setting the moving averaged adversarial sample $W_{avg} * X_{adv}$ (which has the smaller perturbation than $X_{adv}$ if $X \approx W_{avg} * X$) as the boundary condition that the multi-level mitigation cannot cross over (i.e. decreasing $\epsilon$ cannot go below and increasing $\epsilon$ cannot go beyond). With the multi-level mitigation, we can get high prediction accuracies even in the adversarial example having a large perturbation (i.e. $\epsilon > 16$). The proposed scheme is evaluated with adversarial examples crafted by the FGSM (Fast Gradient Sign Method) based attacks on ResNet-50 trained with ImageNet dataset.

1. Introduction

Adversarial perturbations (Kurakin et al., 2016a; Carlini & Wagner, 2016; Guo et al., 2018) for a CNN (Convolutional Neural Network) are crafted to make a CNN classifier mispredict its input image. Usually, perturbations are imperceptible to human eyes especially when their size (as known as $\epsilon$) are small (i.e. $\epsilon \leq 16$ in the FGSM [Fast Gradient Sign Method] based attack (Kurakin et al., 2016a)). However, even in the case that an adversarial perturbation on an input image is large ($> 16$), human beings can classify the image with a correct label.

For the defense of a CNN against the adversarially perturbed images, there have been the approaches (Buckman et al., 2018; Guo et al., 2017; Papernot et al., 2015; Kurakin et al., 2016b) to make neural networks robust against the perturbations. Fundamentally, the approaches need to have a full data set of neural networks for a re-training. Also, there are some other approaches which do not require any modification of neural networks (Kurakin et al., 2016a; Dzigaite et al., 2016; Prakash et al., 2018). The approaches utilize the property that adversarial perturbations tend to exist on a high-frequency region (Guo et al., 2018; Sharma et al., 2019). Thus, the low-pass image filter (i.e. DCT [Discrete Cosine Transform] or wavelet denoising) deletes the perturbations on a high-frequency region by cutting off high-frequency components. However, they are not so effective for the adversarial example having a large perturbation (Dzigaite et al., 2016).

The image transformations such as DCT and wavelet were developed to make the image with less information (i.e. compressed one with less bits) seemingly have a good quality comparable to original one for human beings. They utilize that human beings are not sensitive to small changes of an image. If the techniques originally developed for the less-sensitive human beings (not for the defense against the adversarial perturbations) can be used to invalidate a relatively small perturbation (i.e. $\epsilon \leq 16$), it would be able to nullify a large perturbation if we can make the large perturbation become a smaller one that the image transformation can delete.

For the mitigation of an adversarial perturbation $\epsilon$ of the

---

1Samsung Electronics System LSI Division, Hwaseong-si, Gyeonggi-do, South Korea. Correspondence to: Woohyung Chun <wh.chun@samsung.com>.
adversarial example $X_{adv} (= X \pm \epsilon)$, we find the estimated perturbation $\hat{\epsilon}$ and add it to $X_{adv}$ in the direction of making $|\epsilon|$ small (i.e. $X_{adv} - \hat{\epsilon}$ for $+\epsilon$, $X_{adv} + \hat{\epsilon}$ for $-\epsilon$). The estimated perturbation comes from the difference between $X_{adv}$ and its moving-averaged outcome $W_{avg} \ast X_{adv}$ where $W_{avg}$ is $N \times N$ moving average kernel that all the coefficients are one. Since $X \approx W_{avg} \ast X$ among the adjacent samples of an image, we can make the difference between $X_{adv}$ and $W_{avg} \ast X_{adv}$ less than $\epsilon$. For a large adversarial perturbation (e.g. $\epsilon > 16$), the mitigation scheme is extended to do the multi-level mitigation by configuring the mitigated adversarial example as a new adversarial example to be mitigated further. Also, in order to guarantee that more mitigation steps get the adversarial perturbation smaller, the less perturbed moving averaged sample (i.e. $W_{avg} \ast X_{adv}$ is closer to $X$ than $X_{adv}$, if $X \approx W_{avg} \ast X$) is set as the boundary condition that the mitigation by subtraction ($X_{adv} - \hat{\epsilon}$) does not go below and the mitigation by addition ($X_{adv} + \hat{\epsilon}$) does not go beyond.

This paper is organized as follows: Section 2 sets up the function $M(\cdot)$ which mitigates a large perturbation that JPEG encoding cannot nullify. $M(\cdot)$ is dedicated to making the large perturbation small such that JPEG encoding is separated as the soothing function from $M(\cdot)$. In Section 3, the way of estimating perturbation is introduced on X-MAS (i.e. when $X \approx W_{avg} \ast X$) and it is extended to the multilevel mitigation for a large perturbation. In the end of the section, the proposed mitigation schemes are built in an algorithm with some relevant parameters. Section 4 evaluates the algorithm with some representing adversarial examples having large perturbations. Finally, Section 5 concludes with the summary of our contribution.

## 2. Problem Setup

Previous researches (Kurakin et al., 2016a; Dziugaite et al., 2016; Guo et al., 2018; Sharma et al., 2019) identified that JPEG encoding is able to nullify the impact of the adversarial perturbations on the prediction accuracy of a CNN. When the adversarial example $X_{adv}$ is JPEG-encoded and then fed into a CNN which recognizes the benign version of $X_{adv}$, $X$ as $Y_{true}$,

$$Pr[Y_{true}|X] \approx Pr[Y_{true}|JPEG(X_{adv})]$$

$$X_{adv} = X \pm \epsilon, \epsilon \geq 0$$

(1)

where $Pr[Y_{true}|X]$ is the accuracy that a CNN predicts $X$ as the true label $Y_{true}$ and $\epsilon$ is the adversarial perturbation applied to all the samples of an input image for a CNN. Usually, the relation ”≈” of Equation (1) works well for the small perturbations but it is not valid for the large perturbations as shown in Figure 1.

In Figure 1, the perturbations $\epsilon$ are crafted by the basic iterative FGSM attack (Kurakin et al., 2016a). Figure 1a satisfies the relation ”≈” of Equation (1) but Figure 1b does not. $JPEG(\cdot)$ compresses the image with the quality of 20 (out of 100). In order to make Figure 1b have the prediction accuracy comparable to Figure 1a, we need to mitigate $X_{adv}$ of Figure 1b to the level of $X_{adv}$ in Figure 1a. Let the mitigation function $M(\cdot)$ that mitigates the adversarial perturbation $\epsilon$ of Equation (1) be

$$M(X_{adv}) = X_{adv} \pm \hat{\epsilon}$$

$$= X \pm \epsilon \mp \hat{\epsilon}, \quad 0 \leq \hat{\epsilon} \leq \epsilon$$

(2)

where $\hat{\epsilon}$ is the estimated perturbation which mitigates the perturbation by subtracting $\hat{\epsilon}$ for $\epsilon > 0$ and adding $\hat{\epsilon}$ for $\epsilon < 0$. To make the difference between $M(X_{adv})$ and $X_{adv}$ be imperceptible to human eyes, $\hat{\epsilon}$ must be within the range of $\epsilon$. That is, $\hat{\epsilon}$ should work on the direction of decreasing the range of $\epsilon$ from $[-\epsilon, \epsilon]$ to $[-\epsilon + \hat{\epsilon}, \epsilon + \hat{\epsilon}]$ where $0 \leq \hat{\epsilon} \leq \epsilon$. In Equation (2), $\hat{\epsilon}$ is subtracted for $X_{adv} = X + \epsilon$ and it is added to $X_{adv} = X - \epsilon$. By applying the mitigation function $M(\cdot)$, Equation (1) can be expressed with the terms as below

$$Pr[Y_{true}|X] \approx Pr[Y_{true}|SF(M(X_{adv}))]$$

$$X_{adv} = X \pm \epsilon, \quad M(X_{adv}) = X_{adv} \pm \hat{\epsilon}, \quad 0 \leq \hat{\epsilon} \leq \epsilon$$

(3)
where $SF(\cdot)$ is the soothing function corresponding to JPEG encoding in Equation (1). Soothing function reduces the impact of the perturbation on the prediction accuracy. As a soothing function, JPEG encoding removes the high-frequency perturbations through DCT (Discrete Cosine Transform) and quantization (Guo et al., 2018; Sharma et al., 2019). Also, the simple moving average filter (which runs the convolutional computations with the weight kernel where all the coefficients are one) can be used as the soothing function by smoothing the perturbations through the spatial average. In Section 3, we propose the method of estimating $\hat{\epsilon}$ in Equation (2) and the way of mitigating the adversarial perturbation using $\epsilon$.

3. Proposed method

3.1. Estimated perturbation $\hat{\epsilon}$

In order to find $\hat{\epsilon}$, we use the moving average filters that make $X_{adv}$ converge some value ranging from $[X - \epsilon, X + \epsilon]$. That is,

$$W_{avg} * X_{adv} = W_{avg} * X \pm W_{avg} * \epsilon$$

(4)

where $\epsilon \geq 0$ and $W_{avg}$ is $N \times N$ ($N \geq 2$) moving average window whose samples are convolved with the kernel that all the coefficients are one. The moving average operation "$W_{avg}$" decreases the difference between adjacent samples. Thus, the moving-average operation mitigates the perturbation $\epsilon$ as below.

$$W_{avg} * X_{adv} - W_{avg} * X = \pm W_{avg} * \epsilon,$$

$$W_{avg} * |\epsilon| < |\epsilon| \quad \text{if} \quad (\frac{1}{|W_{avg}|} \sum_{n=0}^{\frac{|W_{avg}|-1}{2}} \epsilon_n) \neq \epsilon,$$

$$W_{avg} * \epsilon = \epsilon \quad \text{if} \quad (\frac{1}{|W_{avg}|} \sum_{n=0}^{\frac{|W_{avg}|-1}{2}} \epsilon_n) = \epsilon$$

(5)

where $|W_{avg}|$ is the number of coefficients for $N \times N$ $W_{avg}$. $N^2$ and $\epsilon_n$ is the perturbation assigned to each sample that $W_{avg}$ covers. In Equation(5), the cases satisfying $W_{avg} * |\epsilon| < |\epsilon|$ are more probable than the cases that meet $W_{avg} * \epsilon = \epsilon$. For example, in order to make $W_{avg} * \epsilon = \epsilon$ in FGSM (Fast Gradient Sign Method) attack (Goodfellow et al., 2014), all the samples in the coverage of $W_{avg}$ have $\epsilon$. The probability that each sample has the same $\epsilon$ in FGSM is $\frac{1}{3}$ (i.e. $+\epsilon$ among $-\epsilon$, 0 and $+\epsilon$). So, the probability that the result of $N \times N$ moving-average computation becomes $\epsilon$ is $\frac{1}{3^2}$. Its value is about $5 \times 10^{-5}$ when $3 \times 3$ kernel is used for $W_{avg}$. In the same manner, we can find the probability that the result of $N \times N$ moving-average computation becomes $-\epsilon$ and it is also $\frac{1}{3^2}$. Thus, when $\epsilon$ is generated by the rule of FGSM and the moving average kernel $W_{avg}$ is $3 \times 3$, the probability that the inequality $W_{avg} * |\epsilon| < |\epsilon|$ happens is $1 - \frac{2}{3^2}$ ($\approx 0.9999$). Since it is highly probable that $W_{avg} * |\epsilon| < |\epsilon|$, we can use $|\epsilon| - W_{avg} * |\epsilon|$ as the estimated perturbation $\hat{\epsilon}$ in order to make $\epsilon$ smaller. When $X \approx W_{avg} * X$, $\hat{\epsilon}$ can be found by

$$\forall X_{adv} > W_{avg} * X_{adv},$$

$$\hat{\epsilon} = X_{adv} - (W_{avg} * X_{adv})$$

$$\approx \epsilon - W_{avg} * \epsilon \quad (\therefore X \approx W_{avg} * X),$$

$$\forall X_{adv} < W_{avg} * X_{adv},$$

$$\hat{\epsilon} = (W_{avg} * X_{adv}) - X_{adv}$$

$$\approx \epsilon - W_{avg} * \epsilon \quad (\therefore X \approx W_{avg} * X)$$

(6)

where $\epsilon > 0$, $X_{adv} = X + \epsilon$ if the moving average output of $X_{adv}$ is smaller than $X_{adv}$ (i.e. $X_{adv} > W_{avg} * X_{adv}$) and $X_{adv} = X - \epsilon$ if the moving average output of $X_{adv}$ is larger than $X_{adv}$ (i.e. $X_{adv} < W_{avg} * X_{adv}$). $X \approx W_{avg} * X$ can be satisfied by controlling $|W_{avg}|$ (usually, adjacent samples are close to each other).

In Equation (6), $0 < \hat{\epsilon} < \epsilon$ such that $\epsilon$ can mitigate $\hat{\epsilon}$ by either being subtracted from $\epsilon$ or being added to $\epsilon$. That is, when $X_{adv} > W_{avg} * X_{adv}$, $\hat{\epsilon}$ is subtracted from $X_{adv}$ and $\hat{\epsilon}$ is added to $X_{adv}$ if $X_{adv} < W_{avg} * X_{adv}$. Figure 2 illustrates how $\hat{\epsilon}$ mitigates $\epsilon$.

In Figure 2, the solid black line shows the upper and lower limits where the adversarial perturbation $\epsilon$ works on $X$. Thus, the red line which denotes the adversarial example $X_{adv}$ having $\pm \epsilon$ as the adversarial perturbation, does not get out of the black line. Also, the solid green line indicates the upper and lower boundaries that $\hat{\epsilon}$ works on $X_{adv}$. If $(X + \epsilon - \hat{\epsilon}) < X_{adv} < (X + \epsilon)$, $X_{adv}$ has the positive $\epsilon$ such
that \( \hat{\epsilon} = (\epsilon - W_{avg} \ast \epsilon) \) should be subtracted from \( X_{adv} \) to reduce \( +\epsilon \). On the other hand, if \( (X - \epsilon) < X_{adv} < (X - \epsilon + \hat{\epsilon}) \), \( X_{adv} \) has the negative \( \epsilon \). Thus, \( \hat{\epsilon} \) should be added into \( X_{adv} \) to increase \( -\epsilon \). Both \( X_{adv} + \hat{\epsilon} \) and \( X_{adv} - \hat{\epsilon} \) are closer to \( X \) than \( X + \epsilon \) and \( X - \epsilon \) are. In the same manner with \( X_{adv} \), the solid blue line that represents the adversarial example \( X^1_{adv} \), having \( \pm (\epsilon - \hat{\epsilon}) \) as the adversarial perturbation, does not get out of the green line. If \( \hat{\epsilon} = 0 \) (i.e. \( X_{adv} = W_{avg} \ast X_{adv} \)), all the samples within the coverage of \( W_{avg} \) have the same \( \epsilon \). As discussed earlier, the probability that all the samples have the same \( \epsilon \) is very low even in the small \( 3 \times 3 \) \( W_{avg} \). It is much smaller than the probability that \( \epsilon \) of \( X_{adv} \) is 0 (e.g. \( \frac{1}{2} \) in the FGSM based attack). Thus, when \( \hat{\epsilon} = 0 \), it is more probable that the perturbation of \( X_{adv} \) is zero rather than that all the samples have the same \( \epsilon \). Since we do not have to find \( \hat{\epsilon} \) for the sample having no perturbation, we do not change \( X_{adv} \) in case that \( \hat{\epsilon} = 0 \).

In Equation (6), \( \hat{\epsilon} \) can be larger than \( \epsilon \) if \( X \neq W_{avg} \ast X \). It is the case that samples within the moving average kernel are very different to each other. Then, \( \hat{\epsilon} \) may work as another perturbation. Also, \( \hat{\epsilon} \) can be too small to improve the prediction accuracy \( Pr[Y_{true}|X_{adv} \pm \hat{\epsilon}] \). For both large and small \( \epsilon \), more mitigation steps would be required. Figure 3 shows the impact of the single-level mitigation on the prediction accuracy according to the sizes of \( \epsilon \) in \( X_{adv} \).

\[
\begin{align*}
Pr[Y_{true}|X] &= 0.9763 \\
Pr[Y_{true}|W_{avg}(X_{adv} + /- \epsilon)] &= 0.6874 \\
Pr[Y_{true}|W_{avg}(X_{adv} + /- \epsilon)] &= 0.0023 \\
\end{align*}
\]

(a) Single-level mitigation can well mitigate the small perturbations crafted by the basic iterative FGSM attack with \( \epsilon = 2 \).

\[
\begin{align*}
Pr[Y_{true}|X] &= 0.9763 \\
Pr[Y_{true}|W_{avg}(X_{adv} + /- \epsilon)] &= 0.0023 \\
\end{align*}
\]

(b) Single-level mitigation may not work well on large perturbations crafted by the basic iterative FGSM attack with \( \epsilon = 32 \).

\[
\begin{align*}
\epsilon_1 &= (X_{adv} - \epsilon_0) - W_{avg} \ast (X_{adv} - \epsilon_0) \\
&= (X + \epsilon - \epsilon_0 - W_{avg} \ast (X + \epsilon - \epsilon_0) \\
&\approx \epsilon - \epsilon_0 - W_{avg} \ast (\epsilon - \epsilon_0) \quad (\because X \approx W_{avg} \ast X),
\end{align*}
\]

\[
\begin{align*}
\epsilon_1 &= W_{avg} \ast (X_{adv} + \epsilon_0) - (X_{adv} + \epsilon_0) \\
&= W_{avg} \ast (X - \epsilon + \epsilon_0) - (X + \epsilon_0) \\
&\approx \epsilon - \epsilon_0 - W_{avg} \ast (\epsilon - \epsilon_0) \quad (\because X \approx W_{avg} \ast X)
\end{align*}
\]

\[
\begin{align*}
\epsilon_1 &= W_{avg} \ast (\epsilon - W_{avg} \ast \epsilon) \\
&= W_{avg} \ast \epsilon_0.
\end{align*}
\]

Since it is more probable that \( W_{avg} \ast |\epsilon| < |\epsilon|, |\epsilon_1| < |\epsilon_0| \). Thus, \( |\epsilon_1| < |\epsilon_0| < |\epsilon| \) since \( |\epsilon_0| < |\epsilon| \). Figure 4 illustrates the multi-level mitigation that estimates \( \epsilon_1 \).

In Figure 4, the grey line is the boundary condition for \( X_{adv} \) (corresponding to the solid black line of Figure 2), the solid
black line denotes the range of $X_{adv}^1$ and the green line represents the upper and lower boundaries that $X_{adv}^2$ can reach. As the level of mitigation goes deep, the accumulated sum of the estimated perturbations grows even though the estimated perturbation per each level decreases. Therefore, we should control $\epsilon$ to satisfy the following.

$$\sum_{n=0}^{p} |\hat{\epsilon}_n| < |\epsilon| , \quad |\hat{\epsilon}_p| < \ldots < |\hat{\epsilon}_1| < |\hat{\epsilon}_0| < |\epsilon|$$  \hspace{1cm} (7)

where $p$ is the level of mitigation. However, in Equation (7), $\epsilon$ cannot be directly handled as a single term because it is hidden in $X_{adv}$. In order to control $\hat{\epsilon}$, Equation (7) should be rephrased as below.

$$X + \epsilon - \hat{\epsilon}_0 - \hat{\epsilon}_1 \ldots - \hat{\epsilon}_{p-1} - \hat{\epsilon}_p > X - \epsilon,$$

$$X - \epsilon + \hat{\epsilon}_0 + \hat{\epsilon}_1 \ldots + \hat{\epsilon}_{p-1} + \hat{\epsilon}_p < X + \epsilon.$$ \hspace{1cm} (8)

In case that adjacent samples are very different to each other, $X \neq W_{avg} * X$ such that the estimated perturbation at the $p$-th step of the multi-level mitigation, $\hat{\epsilon}_p$ can be too large to satisfy Equation (8). Since usually $X > \epsilon$, the large difference that breaks the relation of Equation (8) may come from the difference between the benign parts (i.e. $X$ or $X_{adv}$). To relax the impact of the large differences on the estimated perturbations, we normalize the estimated perturbation at the $p$-th step of the multi-level mitigation as below.

$$\hat{\epsilon}_p = E[|X_{adv}^p - (W_{avg} * X_{adv}^p)|]$$ \hspace{1cm} (9)

where $E[|\cdot|]$ indicates both $E[X_{adv}^p - (W_{avg} * X_{adv}^p)]$ (to be subtracted from $X_{adv}^p$) and $E[(W_{avg} * X_{adv}^p) - X_{adv}^p]$ (to be added into $X_{adv}^p$). This normalization also prevents $\hat{\epsilon}$ from working as a serious perturbation for $X_{adv}$ having $\epsilon = 0$ (i.e. the mitigation scheme should work for the benign example).

In Equation (8), the boundary condition for both decreasing perturbation $X_{adv}^{p-1} - \hat{\epsilon}$ and increasing perturbation $X_{adv}^{p-1} + \hat{\epsilon}$ better be replaced with the value closer to $X$. As discussed earlier with Equation (5), the probability for $W_{avg} * |\epsilon| < |\epsilon|$ is much larger than $W_{avg} * |\epsilon| = |\epsilon|$ such that it is more probable that $W_{avg} * X_{adv}$ is closer to $X$ than $X \pm |\epsilon|$ when $W_{avg} * X \approx X$. That is,

$$\forall X_{adv} = X + \epsilon, \quad W_{avg} * X_{adv} = W_{avg} * (X + \epsilon) = X + W_{avg} * \epsilon (\therefore W_{avg} * X \approx X) < X + \epsilon (\therefore W_{avg} * |\epsilon| < |\epsilon|),$$

$$\forall X_{adv} = X - \epsilon, \quad W_{avg} * X_{adv} = W_{avg} * (X - \epsilon) = X - W_{avg} * \epsilon (\therefore W_{avg} * X \approx X) > X - \epsilon (\therefore W_{avg} * |\epsilon| < |\epsilon|).$$

Thus, both $X - \epsilon$ and $X + \epsilon$ of Equation (8) can be replaced with $W_{avg} * X_{adv}$ as following.

$$X_{adv}^{p-1} - \hat{\epsilon}_p > W_{avg} * X_{adv}, \quad X_{adv}^{p-1} + \hat{\epsilon}_p < W_{avg} * X_{adv}.$$ \hspace{1cm} (10)

Equation (10) guarantees that the proposed multi-level mitigation gets $X_{adv}$ closer to $X$ with the smaller (i.e. mitigated) perturbation having the same polarity with the original (i.e. unmitigated) perturbation if $X \approx W_{avg} * X$. That is, the term of decreasing $\epsilon$, $X_{adv}^{p-1} - \hat{\epsilon}_p$ does not go below $W_{avg} * X_{adv}$ and the term of increasing $\epsilon$, $X_{adv}^{p-1} + \hat{\epsilon}_p$ does not go below $W_{avg} * X_{adv}$. To this end, $W_{avg} * X_{adv}$ can be used as the decision boundary that determines the maximum mitigation step where the prediction accuracy does not change any longer. Figure 3 shows that the 70-step multi-level mitigation satisfying Equation (10) is applied to the adversarial example having $\epsilon = 32$ of Figure 3.
Mitigating large adversarial perturbations on X-MAS (X minus Moving Averaged Samples)

boundary conditions of Equation (10) guarantee a certain level of prediction accuracy if the number of mitigation steps are large enough like the 70-step multi-level mitigation of Figure 5. Thanks to Equation (10), we can build the algorithmic state machine having the point where the multi-level mitigation stops at.

3.3. Algorithm

Algorithm 1 summarizes the proposed method of mitigating adversarial perturbations.

Among the inputs of Algorithm 1, \(X_{adv}\) and \(W_{avg}\) are used to estimate \(\dot{\epsilon}_j\) and other parameters are related to the condition to stop the multi-level mitigation. As demonstrated in the previous sections, the proposed mitigation scheme has no dependency on the soothing filter and neural network architecture. However, the soothing filter and a CNN inference are required to check if the prediction accuracy gets saturated or not. Also, there should be an array having the size of \(k\) (which stores the previous prediction results) as the input parameter of the algorithm in order to see if the prediction accuracy of the current mitigation step is saturated or not.

4. Evaluation of proposed method

For the evaluation of the proposed mitigation scheme described in Algorithm 1, \(\epsilon\) is crafted by FGSM (Fast Gradient Sign Method) attacks (Kurakin et al., 2016a) on ResNet-50 (He et al., 2015) trained with ImageNet (Deng et al., 2009). Thus, the input parameter "CNN" of Algorithm 1 is ResNet-50. Also, both JPEG encoding and \(3 \times 3\) moving average filter for the variable "SF". We repeat the mitigation until the prediction accuracy gets into a saturated point such that we do not specify the parameter \(k\) for this evaluation. The core part (i.e. estimating perturbations and checking the boundary condition for multi-level mitigation) of Algorithm 1 are written in "convert" script of ImageMagick (Unknown, 2013). The script to find \(\dot{\epsilon}\) with \(3 \times 3\) \(W_{avg}\) is found in Appendix A.

First of all, the proposed mitigation scheme should work for the case where \(\epsilon = 0\) since it does not know if its input image has any adversarial perturbation or not. Algorithm 1 keeps mitigating the perturbation unless it crosses over the boundary condition of \(W_{avg} \times X_{adv}\). When \(X_{adv}\) has no perturbation, the boundary condition becomes \(W_{avg} \times X\). Also, the estimated perturbation \(\dot{\epsilon}\) comes from the difference between \(X\) and \(W_{avg} \times X\). Since the proposed scheme is valid for the case \(X \approx W_{avg} \times X\), \(\dot{\epsilon}\) for \(\epsilon\) does not make the

Algorithm 1 Mitigating adversarial perturbations

```plaintext
1: //\(X_{adv}\) is the adversarial example.
2: //\(\dot{\epsilon}\) is the estimated perturbation.
3: //\(W_{avg}\) is \(N \times N\) moving average kernel.
4: //\(X_{adv}\) is the adversarial example at the j-th step mitigation
5: //\(\dot{\epsilon}_j\) is the estimated perturbation at the j-step mitigation
6: //\(W_{avg} \times X_{adv}\) is the local expectation that \(W_{avg}\) covers.
7: //\(k\) is the size of an array having CNN inference results from \(X_{adv}^{j-(k-1)}\) to \(X_{adv}^j\)
8: //\(SF\) is the soothing filter like JPEG encoding
9: //\(CNN\) is a convolutional neural network
10: //\(X\) is the soothing filter like JPEG encoding
11: Input: \(X_{adv}, W_{avg}, k, SF, CNN\)
12: //initial step j is set to zero
13: j = 0;
14: //initialize the variable that counts the number of consecutive k-steps
15: //whose prediction accuracies are same
16: equal_count = 0;
17: repeat
18: if \(X_{adv}^j \geq (W_{avg} \times X_{adv}^j)\) then
19: //Find \(\dot{\epsilon}_j\) to be subtracted from \(X_{adv}^{j-1}\)
20: \(\dot{\epsilon}_j = X_{adv}^j - (W_{avg} \times X_{adv}^j)\);
21: //Mitigate the perturbation with \(\dot{\epsilon}_j\) iff \(\dot{\epsilon}_j \leq \dot{\epsilon}_{j-1}\)
22: if \((j > 0) \&\& (\dot{\epsilon}_j \leq \dot{\epsilon}_{j-1})\) then
23: //Subtract \(\dot{\epsilon}_j\) from \(X_{adv}^{j-1}\)
24: //If and only if this subtraction meets Equation (10)
25: if \((X_{adv}^{j-1} - \dot{\epsilon}_j) > W_{avg} \times X_{adv}\) then
26: \(X_{adv}^{j-1} = X_{adv}^{j-1} - \dot{\epsilon}_j\);
27: else
28: \(X_{adv}^{j-1} = X_{adv}^{j-1}\);
29: end if
30: end if
31: else
32: //Find \(\dot{\epsilon}_j\) to be added to \(X_{adv}^{j-1}\)
33: \(\dot{\epsilon}_j = (W_{avg} \times X_{adv}^j) - X_{adv}^j\);
34: //Mitigate the perturbation with \(\dot{\epsilon}_j\) iff \(\dot{\epsilon}_j \leq \dot{\epsilon}_{j-1}\)
35: if \((j > 0) \&\& (\dot{\epsilon}_j \leq \dot{\epsilon}_{j-1})\) then
36: //Add \(\dot{\epsilon}_j\) to \(X_{adv}^{j-1}\)
37: //If and only if this addition satisfies Equation (10)
38: if \((X_{adv}^{j-1} + \dot{\epsilon}_j) < W_{avg} \times X_{adv}\) then
39: \(X_{adv}^{j-1} = X_{adv}^{j-1} + \dot{\epsilon}_j\);
40: else
41: \(X_{adv}^{j-1} = X_{adv}^{j-1}\);
42: end if
43: end if
44: end if
45: //Stop the mitigation if the results of consecutive k-steps are same
46: //Pop-and-Push to store new result
47: for \(i = 0\) to \(k - 1\) do
48: result[k-i] = result[k-(i+1)];
49: end for
50: result[0] = CNN(SF(X_{adv}^j));
51: //Check if the results of consecutive k-steps are same or not
52: for \(n = 0\) to \(k - 2\) do
53: if result[n] != result[n+1] then
54: break;
55: end if
56: end for
57: equal_count = n;
58: until equal_count is \(k - 2\)
```

\(^1\)Other scripts and adversarial examples used for this evaluation are available at https://github.com/stonylinux/mitigating_large_adversarial_perturbations_on_X-MAS
Mitigating large adversarial perturbations on X-MAS (X minus Moving Averaged Samples)

prediction accuracy $Pr[Y_{true}|Algorithm1(X)]$ far different from $Pr[Y_{true}|W + X]$ (i.e. $Pr[Y_{true}|Algorithm1(X)] \approx Pr[Y_{true}|W + X]$) as shown in Figure 6.

(a) $Pr[Y_{true}|Algorithm1(X)] \approx Pr[Y_{true}|X]$ when $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} + X]$.

(b) $Pr[Y_{true}|Algorithm1(X)] \approx Pr[Y_{true}|X]$ but $Pr[Y_{true}|Algorithm1(X)] \approx Pr[Y_{true}|W_{avg} + X]$ when $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} + X]$.

Figure 6. Algorithm 1 makes $Pr[Y_{true}|Algorithm1(X)] \approx Pr[Y_{true}|W_{avg} + X]$.

In Figure 6a, $Pr[Y_{true}|Algorithm1(X)]$ is slightly better than $Pr[Y_{true}|W_{avg} + X]$. This means that $X$ is purely mitigated (i.e. perturbed) by the difference of benign parts among adjacent samples but the mitigation does not work as a serious perturbation since the difference of (benign parts among adjacent samples) cannot make $X$ cross over the boundary condition $W_{avg} + X$ according to Equation (10). When $X \neq W_{avg} + X$ in Figure 6b, Algorithm 1 keeps mitigating the perturbation (i.e. adding/subtracting the difference between $X$ and $W_{avg} + X$ to/from $X$) until $X_{adv}^p$ ($X_{adv}$ in p-step mitigation, actually $X^p$ for the case $\epsilon = 0$) has the minimum distance from $W_{avg} + X$. As the result, $Pr[Y_{true}|Algorithm1(X)] \approx Pr[Y_{true}|W_{avg} + X]$ even though $Pr[Y_{true}|Algorithm1(X)] \neq Pr[Y_{true}|X]$.

Most moving average outcomes for the input images of a CNN are predicted as the same label with their original images (i.e. $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} + X]$) because the moving average convolution of the input image works as the low-pass filter like human eyes. Figure 7 shows that Algorithm 1 well estimates $\hat{\epsilon}_j$ when $X \approx W_{avg} + X$.

In Figure 7, the rank of prediction accuracy should be carefully handled. When JPEG encoding is used as the soothing filter without the mitigation steps (i.e. for the case of $Pr[Y_{true}|JPEG(X_{adv})]$), it achieves top-2 accuracy for Figure 7a as well as it gets top-3 accuracy for Figure 7b.

However, both cases work well as the adversarial examples having dominant prediction accuracies. Especially for $Pr[Y_{true}|JPEG(X_{adv})]$ of Figure 7a, no one can say it is okay to reach the top-2 accuracy with the probability of 0.0285 because the wrong recognition can be fatal to a human life (i.e. fire truck cannot replace the ambulance). Thus, when we talk about the prediction accuracy, the number in probability can be much more important than the rank of the accuracy. Figure 8 shows the cases that the number in probability plays an important role in the proposed mitigation.

Figure 7. The proposed mitigation contributes to the enhancement of prediction accuracy much more than JPEG encoding does.

(b) Algorithm 1 improves the prediction accuracy from 0.0699 to 0.8349 when the basic iterative FGSM attack with $\epsilon = 32$ is applied.

Figure 8 shows the cases that $Pr[Y_{true}|X] \neq Pr[Y_{true}|W_{avg} + X]$. Figure 8a does not have $Y_{true}$ (i.e. ImageNet dataset does not have the label corresponding to sunflower) and Figure 8b has far different numbers for $Pr[Y_{true}|X]$ and $Pr[Y_{true}|W_{avg} + X]$. In Figure 8a, $Pr[Y_{daisy}|X]$ becomes the ground truth because the CNN inference having no label for sunflower recognizes $X$ as daisy (i.e. $Y_{daisy}$). Algorithm 1 increases the prediction accuracy for $Y_{daisy}$ from 0.1187 to 0.4805 because $Pr[Y_{daisy}|X] \approx Pr[Y_{daisy}|W_{avg} + X]$. However, in Figure 8b, even though both ranks of the prediction accuracies $Pr[Y_{true}|X]$ and $Pr[Y_{true}|W_{avg} + X]$ are dominant (i.e. top-1 rank), the proposed mitigation does not improve the prediction accuracy very much because $Pr[Y_{true}|X] \neq Pr[Y_{true}|W_{avg} + X]$ (i.e. the difference between $X$ and $W_{avg} + X$ works as a perturbation). Also, larger $|W_{avg}|$ is better for Algorithm 1 if the corresponding $W_{avg}$ satisfies $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} + X]$ with a high probability. It is because $\hat{\epsilon}$ (i.e. the difference between $X_{adv}$ and $W_{avg} + X_{adv}$) can be better normalized with
Mitigating large adversarial perturbations on X-MAS (X minus Moving Averaged Samples)

Figure 8. The proposed mitigation requires that $Pr[Y|X] \approx Pr[Y|W_{avg} * X]$ rather than that the ranks of prediction accuracies are close.

(a) For the CNN recognizing sunflower as daisy, $e$ can be found if $Pr[Y_{daisy}|X] \approx Pr[Y_{daisy}|W_{avg} * X]$ ($X_{adv}$ is generated by the basic iterative FGSM attack with $\epsilon = 32$).

(b) The proposed mitigation might not increase the prediction accuracy very much if $Pr[Y_{true}|X] \neq Pr[Y_{true}|W_{avg} * X]$ ($X_{adv}$ is generated by the basic iterative FGSM attack with $\epsilon = 32$).

Figure 9. Algorithm 1 works better for the adversarial example which has the smaller number of out-of-bound samples.

For the FGSM attack without clipping function (“fast” attack in (Kurakin et al., 2016a)), the large perturbation can seriously distort an image by generating out-of-bound samples. That is, it can make the samples in the range $(0, \epsilon)$ become zero as well as it can increase the samples within $(255 - \epsilon, 255)$ to have the maximum value, 255. Then, Algorithm 1 would not well mitigate the perturbations because the size of estimated perturbation $|e|$ can be reduced by the amount of $|\epsilon - X|$ for the sample $X$ in the range $(0, \epsilon)$ or by the amount of $|X - (255 - \epsilon)|$ for $X$ in the range $(255 - \epsilon, 255)$. Figure 9 shows how the performance of Algorithm 1 varies according to the numbers of out-of-bound samples.

In Figure 9, fast FGSM attack is used to generate two distinct numbers of out-of-bound samples, which show the impact of the out-of-bound samples on the performance of Algorithm 1. The statistical numbers of (Red, Green, Blue) tuples come from the ImageMagick convert script with the option “-format %c histogram:info: “. In fast FGSM attacks, $\epsilon$ is the knob to control the number of out-of-bound samples. The number of out-of-bound samples increases when $\epsilon$ goes up. Also, the number of out-of-bound samples decreases as $\epsilon$ goes down. In Figure 9, the adversarial example generated by the fast FGSM attack with $\epsilon = 16$ has the smaller number of out-of-bound samples than the adversarial example that the fast FGSM attack with $\epsilon = 64$ is applied to (i.e. 682 vs 3573 for (0,0,0) tuples and 216 vs 3147 for (255,255,255) tuples). Larger number of out-of-bound samples makes more perceptible perturbations. Algorithm 1 shows the better performance (i.e. prediction accuracy represented as $Pr[Y_{true}|Algorithm1(X_{adv})]$) when the number of out-of-bound samples is small (i.e. when the perturbation is imperceptible). The number of out-of-bound samples can be very different according to the attack methods. For example, the adversarial example crafted by the iterative least-likely FGSM attack with $\epsilon = 64$ has the smaller number of out-of-bound samples than the adversarial example generated by the fast FGSM attack with $\epsilon = 16$. It is because the iterative least-likely FGSM attack has the clipping function that reduces the number of out-of-bound samples.

5. Conclusion

In this paper, we propose the scheme that mitigates the perturbations on an adversarial example through the estimation of the perturbations on X-MAS (X minus Moving Average Samples i.e. when $X \approx W_{avg} * X$). For large perturbations (i.e. $\epsilon > 16$), the scheme is further developed to run the multi-step mitigation that has $W_{avg} * X_{adv}$ as the boundary condition to prevent the p-step mitigated adversarial example $X_{adv}^p$ from getting worse by crossing over the boundary. The multi-level mitigation gets $X_{adv}$ closer to $X$ for the most adversarial examples whose benign part have the relation that $X \approx W_{avg} * X$. We evaluate the proposed algorithm with some representing examples that have the different relations between $X$ and $W_{avg} * X$. In the evalu-
tion, our proposed scheme well mitigates the imperceptibly crafted large adversarial perturbations (through iterative FGSM attacks with a clipping function) such that it gets the adversarial examples with the mitigated perturbations have high prediction accuracies when $X \approx W_{\text{avg}} \ast X$.

References

Buckman, J., Roy, A., Raffel, C., and Goodfellow, I. Thermometer encoding: One hot way to resist adversarial examples. 2018. URL https://openreview.net/pdf?id=S18Su--CW.

Carlini, N. and Wagner, D. A. Towards evaluating the robustness of neural networks. CoRR, abs/1608.04644, 2016. URL http://arxiv.org/abs/1608.04644.

Deng, J., Dong, W., Socher, R., Li, L.-J., Li, K., and Fei-Fei, L. ImageNet: A Large-Scale Hierarchical Image Database. In CVPR09, 2009.

Dziugaite, G. K., Ghahramani, Z., and Roy, D. M. A study of the effect of JPEG compression on adversarial images. CoRR, abs/1608.00853, 2016. URL http://arxiv.org/abs/1608.00853.

Goodfellow, I. J., Shlens, J., and Szegedy, C. Explaining and harnessing adversarial examples. arXiv preprint arXiv:1412.6572, 2014.

Guo, C., Rana, M., Cissé, M., and van der Maaten, L. Countering adversarial images using input transformations. CoRR, abs/1711.00117, 2017. URL http://arxiv.org/abs/1711.00117.

Guo, C., Frank, J. S., and Weinberger, K. Q. Low frequency adversarial perturbation. CoRR, abs/1809.08758, 2018. URL http://arxiv.org/abs/1809.08758.

He, K., Zhang, X., Ren, S., and Sun, J. Deep residual learning for image recognition. CoRR, abs/1512.03385, 2015. URL http://arxiv.org/abs/1512.03385.

Kurakin, A., Goodfellow, I. J., and Bengio, S. Adversarial examples in the physical world. CoRR, abs/1607.02533, 2016a. URL http://arxiv.org/abs/1607.02533.

Kurakin, A., Goodfellow, I. J., and Bengio, S. Adversarial machine learning at scale. CoRR, abs/1611.01236, 2016b. URL http://arxiv.org/abs/1611.01236.

Papernot, N., McDaniel, P. D., Wu, X., Jha, S., and Swami, A. Distillation as a defense to adversarial perturbations against deep neural networks. CoRR, abs/1511.04508, 2015. URL http://arxiv.org/abs/1511.04508.

Prakash, A., Moran, N., Garber, S., DiLillo, A., and Storer, J. A. Deflecting adversarial attacks with pixel deflection. CoRR, abs/1801.08926, 2018. URL http://arxiv.org/abs/1801.08926.

Sharma, Y., Ding, G. W., and Brubaker, M. A. On the effectiveness of low frequency perturbations. CoRR, abs/1903.00073, 2019. URL http://arxiv.org/abs/1903.00073.

Unknown. Imagemagick website, 2013. URL http://www.imagemagick.org/. Accessed 2013-05-07.
A. The script for 100-step multi-level mitigation

```bash
#!/bin/bash
#
# Version for 3x3 estimation
# # To mitigate the adversarial example "adversarial_example.png"
# ./mitigating_adversarial_with_3x3_estimation.sh adversarial_example.png
# # Caution !!! All "convert" commands better be typed in one line
# # because --fx content within "" does not support bash typed linking with ""
# # The reason we split convert command here is just for the better display
#
# Initial mitigation starts
#
# Find W
convert $1.png -fx '((p[-1,-1]+p[-1,0]+p[-1,1]+p[0,-1]+p[0,0]+p[0,1]+p[1,-1]+p[1,0]+p[1,1])+(p[0,0])/9' > $1_ma0.png

# Find X
convert $1.png -fx 'u[0].p[0,0]-u[1].p[0,0]' > $1_meps0.png

# Check the upper and lower bound of the mitigated samples
# Following is just for displayed
convert $1.png $1_meps0.png $1_peps0.png --fx '((u[1].p[0,0]>0) && (u[2].p[0,0]==0) && ((u[0].p[0,0]-u[1].mean)>(u[0].maxima)) ||(u[0].p[0,0]-u[1].mean)<(u[0].minima))?'u[0].p[0,0]-u[1].p[0,0]' > $1_mitigated_step0.png

# Initial mitigation ends

# Iterative mitigation starts
for i in {0..99}
do
# DO NOT mitigate X, adv[0..p-1]
# if $((!hat(\epsilon).p) \geq \hat{\epsilon}.p[0..p-1])
# or
# if $((\epsilon).p \geq \hat{\epsilon}.p[0..p-1])
# or
# if X, adv[0..p-1] - \hat{\epsilon}.p < W,avg.X, adv (for X, adv[0..p-1] > W, avg + X, adv)
# or
# if X, adv[0..p-1] + \hat{\epsilon}.p > W, avg + X, adv (for X, adv[0..p-1] < W, avg + X, adv)
convert $1_mitigated_step$i.png $1_meps$i.png $1_peps$i.png $1_mitigated_step$i.png
convert $1_mitigated_step$i.png -fx 'u[0].p[0,0]-u[1].p[0,0]' > $1_mitigated_step$i.png

# Check the upper and lower bound of the mitigated samples
# if
convert $1_mitigated_step$i.png $1_meps$i.png $1_peps$i.png $1_mitigated_step$i.png
convert $1_mitigated_step$i.png $1_meps$i.png $1_peps$i.png $1_mitigated_step$i.png

# JPEG encoding with the quality 20 (out of 100)
convert $1_mitigated_and_soothed_by_JPEG.png
```

Mitigating large adversarial perturbations on X-MAS (X minus Moving Averaged Samples)
Mitigating large adversarial perturbations on X-MAS (X minus Moving Averaged Samples)

B. Evaluation of Algorithm 1 related to a moving-average kernel

Since larger $|W_{avg}|$ can control the difference of estimated perturbations between consecutive steps (i.e. $\Delta \epsilon_{j, j-1} = \epsilon_j - \epsilon_{j-1}$) in a finer-granule manner, Algorithm 1 with a large $|W_{avg}|$ can get $X_{adv}$ closer to $X$ so that it achieves a higher prediction accuracy than Algorithm 1 with the smaller $|W_{avg}|$. Figure 10 shows that Algorithm 1 with a large $|W_{avg}|$ gets the high prediction accuracy for the adversarial example having a large perturbation (i.e. $\epsilon = 64$) when $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} \ast X]$ with a high probability.

(a) $|W_{avg}|$ contributes more to the prediction accuracy when $|W_{avg}|$ is large and $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} \ast X]$ ($X_{adv}$ is generated by the basic iterative FGSM attack with $\epsilon = 64$).

(b) Soothing filter (i.e. JPEG) contributes more to the prediction accuracy when $|W_{avg}|$ is small and $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} \ast X]$ ($X_{adv}$ is generated by the basic iterative FGSM attack with $\epsilon = 64$).

Figure 10. Large $|W_{avg}|$ can make the proposed mitigation get a high prediction accuracy without soothing filter if $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} \ast X]$ with a high probability.

In Figure 10, both Figure 10a and 10b satisfy $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} \ast X]$ with a high probability. However, Algorithm 1 mitigates Figure 10a better than Figure 10b. In Figure 10a, Algorithm 1 can get some high prediction accuracy even without JPEG encoding ($Pr[Y_{true}|Algorithm 1(X_{adv})] = 0.6118$) but it cannot get any prediction accuracy ($Pr[Y_{true}|Algorithm 1(X_{adv})] = 0.0000$) in Figure 10b. In case that $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} \ast X]$ and both prediction accuracies are not so high, we can improve the prediction accuracy of the adversarial example mitigated by Algorithm 1 by changing the coefficients of $W_{avg}$. Figure 11 shows that changing the coefficients of $W_{avg}$ can improve the prediction accuracy of the adversarial example mitigated by Algorithm 1 when $Pr[Y_{true}|X] \neq Pr[Y_{true}|W_{avg} \ast X]$.

(a) Algorithm 1 cannot improve the prediction accuracy of the adversarial example $X_{adv}$ (generated by the basic iterative FGSM attack with $\epsilon = 64$) very much when $Pr[Y_{true}|X] \neq Pr[Y_{true}|W_{avg} \ast X]$.

(b) Changing the coefficients of $W_{avg}$ in Figure 11a by putting more weights on the coefficients closer to the centroid of $W_{avg}$.

(c) Algorithm 1 works well on the adversarial example $X_{adv}$ (generated by the basic iterative FGSM attack with $\epsilon = 64$) after the coefficients of $W_{avg}$ is changed to satisfy $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} \ast X]$.

In Figure 11a, $Pr[Y_{true}|X] \neq Pr[Y_{true}|W_{avg} \ast X]$ when the larger (i.e. $7 \times 7$) $W_{avg}$ is used to find $\hat{c}$. In order to make $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} \ast X]$ for $7 \times 7 W_{avg}$, $7 \times 7 W_{avg}$ needs to be changed to have the coefficients which are similar to $3 \times 3 W_{avg}$. To get the same result with $3 \times 3 W_{avg}$ by using $7 \times 7 W_{avg}$, $3 \times 3$ coefficients of the centroid in $7 \times 7 W_{avg}$ are filled with ones and other coefficients are set as zeros. Thus, in order to make the prediction accuracy of $W_{avg} \ast X$ with $|W_{avg}| = 7 \times 7$ ($\approx 0.1515$) grow toward that of $W_{avg} \ast X$ with $|W_{avg}| = 3 \times 3$ ($\approx 0.8329$), in Figure 11b, $3 \times 3$ coefficients of the centroid in $7 \times 7 W_{avg}$ are filled with ones and other coefficients except the

\[
\begin{align*}
Pr[Y_{true}|X] &= 0.6997 \\
Pr[Y_{true}|W_{avg} \ast X] &= 0.8630
\end{align*}
\]

\[
\begin{align*}
0.8630 &- \approx "n02701002 ambulance" \\
0.1338 &- \approx "n03345487 fire engine, fire truck"
\end{align*}
\]

\[
\begin{align*}
Pr[Y_{true}|Algorithm 1(X_{adv})] &= 0.0000 \\
&= 0.0016
\end{align*}
\]

\[
\begin{align*}
&\text{Coefficient is }1\text{ in }7x7 \text{ weight} \\
&\text{Coefficient is }0.5^2\text{ in }7x7 \text{ weight} \\
&\text{Coefficient is }0.5^4\text{ in }7x7 \text{ weight}
\end{align*}
\]

\[
\begin{align*}
Pr[Y_{true}|Algorithm 1(X_{adv})] &= 0.5377 \\
&\approx "n03345487 fire engine, fire truck" \\
&\approx "n02701002 ambulance"
\end{align*}
\]
$3 \times 3$ coefficients are set as small values (i.e. $< 1$) as the distance from the centroid increases. As the result, in Figure 11c, $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} * X]$ and Algorithm 1 mitigates $X_{adv}$ very well to have the good prediction accuracy (i.e. $Pr[Y_{true}|JPEG(Algorithm1(X_{adv}))] = 0.5377$). However, we do not know the size of the $W_{avg}$ (where all coefficients are ones) to make $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} * X]$ as well as which coefficients are the best suited for the mitigation of $X_{adv}$ when $Pr[Y_{true}|X] \neq Pr[Y_{true}|W_{avg} * X]$ because $X_{adv}$ is just given for the mitigation process (corresponding to Algorithm 1 in this paper). Therefore, the ways to make $Pr[Y_{true}|X] \approx Pr[Y_{true}|W_{avg} * X]$ for any given $X_{adv}$ need to be further studied.