Generalized fuzzy number and graph theory matrix approach in fuzzy decision making of smartphone selection

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Abstract. A number of studies have proposed a method of ranking a smartphone due to the fact that newly smartphones in the market are offering more features which could give more benefit to the user and thus become increasingly important as a tool in helping them to make everyday decisions. But most of the studies have exhibited shortcoming associated to level of confidence of decision makers’ in giving a judgement, which significantly influences final ranking result. This study propose Graph Theory and Matrix Approach (GTMA) on ranking smartphone alternatives that ensures consideration of uncertainty and confidence level upon their judgement towards smartphone selection. A Generalized Triangular Fuzzy Number (GTFN) is used in the calculation of interrelation matrix which leads to give more meaningful value of importance weights of the attributes. Selection index is then calculated to rank the smartphone through fuzzy determinant. A numerical example is provided to illustrate the proposed approach whereby five well-known smartphones brand are considered which are apple, Samsung, Huawei, OPPO and Asus. The result shows that Huawei is the most preferred brand followed by OPPO and Apple in selecting the best smartphone.

1. Introduction
A mobile phone that applies computing technology for a wide range of capabilities, including internet access, databases, video and still photography can be defined as a smart phone [1]; [2]. It is a tool for users to keep connected to the world through messaging service, email, video calls, social networking apps and a tool for accessing and browsing the web. With penetration rates that exceeded hundred percent in most industrial countries, over 93 percent of the world’s population owned cellular phones at the end of 2014 [3]. Mobile phones have become an essential part of human’s lives due to having a significant impact on society, including on individuals’ daily movement and mobility patterns.

However, consumers have their own preferences when they want to buy a smartphone. Some of the consumer will consider brand, product feature and the quality of the product as their preferences to buy a smartphone. Continuous growing of technological advances resulted in continuous application of new features on this device which contributes to more difficulties for people in selecting a new smartphone since the new models are launched with many new features [4]. Thus, customers are more likely to consider several criteria’s before choosing a smartphone, making it a multi-criteria decision making problem.

Evaluation on smartphone selection over multiple criteria’s using various multi-criteria decision making method has been investigated previously. For example, [5] analyzed the consumer’s purchase
evaluation factors of smartphone using Analytic Hierarchy Process (AHP). [4] utilized Analytic Network Process (ANP) and Generalised Choquet Integral (GCI) in selecting the best smartphone while [6] utilized TOPSIS with Entropy. Subsequently, the method is integrated with fuzzy theory so as to cater uncertainty in human opinion in order to determine a better result of ranking alternative [7]; [8]; [9]; [10]. In the context of choosing the best smartphone, [11] used intuitionistic Fuzzy TOPSIS since it able to eliminate uncertainty and provides better representation of the decision makers’ preferences while [12] study the measure of consumers’ evaluation of purchasing smartphone using Fuzzy Elimination and Choice Expressing Reality (ELECTRE1) method. Here, the MCDM methods is considered as an advanced method since it cater the element of uncertainty while making a decision over multiple criteria, but there still an assumption of interdependencies been made to the criteria. In this situation, the criteria is assumed independent to other criteria even though it is positively related.

To deal with this problem, another MCDM approach named Graph Theory and Matrix Approach (GTMA) is proposed since this method can capture the information of interrelationship between criteria through digraph representation [13], [14] had ranked the smartphone alternatives using Graph Theory and Matrix Approach (GTMA) and found out that Apple is the most preferred brand among Malaysian. However, the GTMA does not include the measure of vagueness. Besides, confidence level of decision makers’ while making a judgment over a set of alternative is not yet investigated. Here, Fuzzy Digraph and Matrix Approach can be used to solve the problem.

Fuzzy Digraph and Matrix Approach is one of Multi-Criteria Decision Making (MCDM) method which integrates fuzzy theory into graph theory to solve decision problem involving many interrelated criteria. According to [15], fuzzy graph model is designed when there is vagueness in the description of object or in its relationship or in both. In decision-making environment, decision makers or experts commonly expressed their opinions on qualitative characteristic in linguistic terms rather than in crisp value and the evaluation of parameters has to be measured but it tends to be difficult because of the imprecision of the characteristic. Therefore, in this paper, Fuzzy Digraph and Matrix Approach by using Generalized Triangular Fuzzy Number (GTFN) which include the information of uncertainty given by decision maker through the confidence level is proposed to assist the customers in making more meaningful selection of smartphone.

2. Triangular Fuzzy Number
Fuzzy number is useful in processing information since it deals with uncertainty issues. As presented by [16], a triangular fuzzy number can be defined by a triplet \( A = (a_1, a_2, a_3) \) and the membership can be determined as the following equation:

\[
\begin{align*}
  f(x) &= \begin{cases} 
    \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\
    \frac{x - a_3}{a_2 - a_3}, & a_2 \leq x \leq a_3 \\
    0, & \text{otherwise}
  \end{cases}
\end{align*}
\]

where \( a_1, a_2 \) and \( a_3 \) are real numbers. According to [16], the extended algebraic operations on triangular fuzzy numbers can be expressed as follows:

- Changing sign: \(- (a, b, c) = (- c, - b, - a)\)
- Inverse sign: \((a, b, c)^{-1} = (1/c, 1/b, 1/a)\)
- Addition: \((a, b, c) \oplus (d, e, f) = (a + d, b + e, c + f)\)
- Subtraction: \((a, b, c) \ominus (d, e, f) = (a - f, b - e, c - d)\)
- Multiplication: \((a, b, c) \otimes (d, e, f) \equiv (ad, be, cf) \) or \( k \otimes d, e, f = ka, kb, kc\)
- Division: \((a, b, c) \oslash (d, e, f) \equiv (a/d, b/e, c/f)\) where \( d, e, f > 0 \) and \( a, b, c \geq 0 \)
3. Generalized Triangular Fuzzy Number (GTFN)

[17] in their research stated that a generalized triangular fuzzy number can be defined by \( \tilde{A} = (a_1, a_2, a_3; k) \) and the membership can be determined as in the following equation:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0, & x < a_1 \\
\frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\
\frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\
x, & x > a_3
\end{cases}
\]

Let \( k \) be the confidence level where \( 0 \leq k \leq 1 \). The value of \( k \) represents the confidence level of decision maker showing how confidence they are during evaluation process. Let \( \tilde{A} = (a_1, a_2, a_3; k_\tilde{A}) \) and \( \tilde{B} = (b_1, b_2, b_3; k_\tilde{B}) \) be any two generalized triangular fuzzy numbers, then basic operations on GTFN can be expressed as follows:

\[
\tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3; k_\tilde{A}) + (b_1, b_2, b_3; k_\tilde{B}) = (a_1 + b_1, a_2 + b_2, a_3 + b_3; \min\{k_\tilde{A}, k_\tilde{B}\})
\]

\[
\tilde{A} - \tilde{B} = (a_1, a_2, a_3; k_\tilde{A}) - (b_1, b_2, b_3; k_\tilde{B}) = ((a_1 - b_1), (a_2 - b_2), (a_3 - b_3); \min\{k_\tilde{A}, k_\tilde{B}\})
\]

\[
\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3; k_\tilde{A}) \otimes (b_1, b_2, b_3; k_\tilde{B}) = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3; \min\{k_\tilde{A}, k_\tilde{B}\})
\]

\[
\tilde{A} / \tilde{B} = (a_1, a_2, a_3; k_\tilde{A}) / (b_1, b_2, b_3; k_\tilde{B}) = (a_1 / b_1, a_2 / b_2, a_3 / b_3; \min\{k_\tilde{A}, k_\tilde{B}\})
\]

4. Confidence Level

The level of confidence is used in evaluating the attributes or alternatives by the decision maker. Each of the decision maker give their level of confidence based on scale between 0.1 until 1.0 as shown in Table 1. According to [18], \( k \) indicates the level of confidence and degree of uncertainty of information obtained. The higher the value of \( k \), the higher the level of confidence and the lower the degree of uncertainty.

| Level of Confidence, \( k \) |
|-------------------------------|
| Low Confidence |                  | High Confidence |
| 0.1             | 0.2              | 0.3             | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |

5. Proposed Method

The proposed method for the selection of smartphone consists of several steps starting from data collection of possible smartphone attributes to the calculation of Selection Index through fuzzy determinant. The details for each step is as follows:

**Step 1:** Smartphone attributes collection

All possible attributes of smartphone selection which had been analyzed from previous studies are identified and significant attributes are selected based on the most frequent attributes appeared in the literature.
Step 2: Collection of Data

i. Questionnaire

In order to collect the information for the evaluation and selection of the smartphone, a questionnaire consist of three parts, Part A, Part B and Part C is modified from [19] which require the information on the importance of weights of attributes and the evaluation of attributes on different application. Part A is the demographic profile of decision maker, Part B consist of the information on the importance of attributes’ weights, whereas Part C consist of the information on the suitability of selected attributes on each alternative smartphone. Both Part B and Part C include the scale of confidence level in order to help in improving the result of this study.

The smartphone experts whose evaluate the questionnaire are the smartphone vendors who have a lot of experiences and knowledge of smartphone and have dealt with smartphone for more than 5 years. They have to assigned weight to the attributes using linguistic values from the term set “Preference” as in table 2 and set “Suitability” as in table 3.

ii. Weight of Attributes, $W_{nm}$

Weight of attributes, $W_{nm}$ is one of the important elements to compute relative importance of attributes. $W_{nm}$ is assigned by $m_{th}$ decision maker for $n$ number of attributes. All the smartphone experts are being asked to assign the weight of each attributes in Part B of questionnaire by using linguistic values from set ‘Preference’ as in table 2.

Table 2. Membership Function of the Linguistic Values Sets ‘Preference’ [16]

| Importance Weight | Fuzzy Linguistic Variables |
|-------------------|----------------------------|
| Very Low (VL)     | (0,0,0.3)                  |
| Low (L)           | (0.0,3,0.5)                |
| Medium (M)        | (0.2,0.5,0.8)              |
| High (H)          | (0.5,0.7,1)                |
| Very High (VH)    | (0.7,1,1)                  |

iii. Evaluation of Attributes, $ES_{nrnm}$

The evaluation on the suitability of the smartphone alternatives on each attribute, $ES_{nrnm}$ is made by decision maker in Part C of questionnaire by using linguistic values from set ‘Suitability’ as in table 3.

Table 3. Membership Function of the Linguistic Values Sets ‘Suitability’ [16]

| Importance Weight | Fuzzy Linguistic Variables |
|-------------------|----------------------------|
| Very Poor (VP)    | (0,0,0.2)                  |
| Poor (P)          | (0.0,2,0.4)                |
| Fair (F)          | (0.3,0.5,0.7)              |
| Good (G)          | (0.6,0.8,1)                |
| Very Good (VG)    | (0.8,1,1)                  |
Step 3: Processing and Normalizing Data
i. Computation of Average Weights, \( AW_n \)
By considering that \( W_{nm} \) is the weight of attribute \( n \) with inclusion of level of confidence, \( k_m \) determined by \( m \) decision maker, the Average Weight of each attributes that is in the form of \( AW_n = (o_n, p_n, q_n; k_m) \) can be computed by using following formula:

\[
AW_n = \frac{\sum_{d=1}^{d} W_{nm}}{d} \quad (3.1)
\]

\[
NewAW_n = (o_n \times \sqrt{\min(k_n)}, p_n \times \sqrt{\min(k_n)}, q_n \times \sqrt{\min(k_n)})
\]

ii. Diagonal Elements, \( MS_{nr} = (y_1, y_2, y_3; \min(k_m)) \)
Assume that \( ES_{nrm} \) is the value of attribute \( n \), given by decision maker \( m \) for smartphone alternative \( r \). The mean value of each alternative and for each attribute is computed by using following formula:

\[
MS_{nr} = \frac{\sum_{d=1}^{d} ES_{nrm}}{d} \quad (3.2)
\]

iii. Off-diagonal Elements, \( a_{ij} \) or \( a_{ji} \)
Off-diagonal elements represented by the relative importance of attribute \( i \) relatively to the cumulative importance of attributes \( i \) and \( j \), \( a_{ij} \). In order to compute off-diagonal element which is \( a_{ij} \) and \( a_{ji} \), two elements required which are Normalized Average Weight of attribute \( i \), \( NAW_i = (e_i, f_i, h_i) \) and Normalized Average Weight of attribute \( j \), \( NAW_j = (e_j, f_j, h_j) \). \( NAW_n \) can be calculated by using the following equation where \( S_n^* \) is the defuzzified value of \( S_n \):

\[
NAW_n = \frac{S_n}{\sum_{n=1}^{N} S_n^*} \quad (3.3)
\]

\[
S_n = \frac{NewAW_n}{\sum_{n=1}^{N} q_n} = (S_{n1}, S_{n2}, S_{n3}) \quad (3.4)
\]

Let \( S_n^* = (S_{n1} + S_{n2} + S_{n3})/3 \) \quad (3.5)

Let \( NAW_n = (e_n, f_n, h_n) \)

\[
NAW_n^* = \frac{(e_n + f_n + h_n)}{3} \quad (3.6)
\]

By using the procedures above, the following rule must be satisfied.

\[
\sum_{n=1}^{\infty} NAW_n^* = 1 \quad (3.7)
\]

Then, the following equation is the formula to compute \( a_{ij} \) and \( a_{ji} \), where \( UI^* \) and \( UF^* \) are the defuzzified value of \( UI \) and \( UF \) which can be determined by using following formula:

\[
a_{ij} = \frac{UI}{UI^* + UF^*}, \quad a_{ji} = \frac{UF}{UI^* + UF^*} \quad (3.8)
\]

\[
UI = \frac{NAW_i}{h_i + h_j} = (UI_1, UI_2, UI_3) \quad (3.9)
\]
\[ U_J = \frac{NAW_j}{h_i + h_j} = (UJ_1, UJ_2, UJ_3) \]

\[ UI^* = (UJ_1, + UI_2 + UI_3)/3 \]
\[ UJ^* = (UJ_1, + UJ_2 + UJ_3)/3 \] \hspace{1cm} (3.10)

In this study, by using above procedure, another rule must be satisfied which is the sum of defuzzified values of \( a_{ij} \) and \( a_{ji} \) are equal to 1:

\[ a_{ij}^* + a_{ji}^* = 1 \] \hspace{1cm} (3.11)

**Step 4: Development of Interrelation Matrix and Fuzzy Weighted Directed Graph**

\[ \text{i. Development of Interrelation Matrix} \]

The matrix is a square matrix with size of \( n \times n \) and can be represented as Matrix \([A]\). Matrix \([A]\) is developed by substituting the diagonal elements with the New Mean Values of Attributes, \( NMS_{nr} \) and the off-diagonal elements, \( a_{ij} \) and \( a_{ji} \). For each alternative, the diagonal elements of Matrix \([A]\) is different whereas all the off-diagonal elements remain same. Therefore, the matrix \([A]\) is written as:

\[
[A] = \begin{bmatrix}
NMS_1 & a_{12} & \ldots & a_{1n} \\
a_{21} & NMS_2 & \ldots & a_{2n} \\
\vdots & \ddots & \ddots & \vdots \\
a_{n1} & a_{n2} & \ldots & NMS_{nn}
\end{bmatrix} \] \hspace{1cm} (3.12)

\[ \text{ii. Development of Fuzzy Weighted Directed Graph} \]

A fuzzy weighted directed graph is developed based on the value of off-diagonal elements in the matrix \([IM]\) obtained in Step 4(i). The values are transformed in the form of defuzzified values. The function of weighted directed graph is to show the visualization of the relative importance between attributes. The weighted directed graph consists of nodes \( C = \{ Ci \} \) for \((i = 1, 2, 3, \ldots, n)\) and a set of directed edges \( E = \{ a_{ij} \} \) for \((i, j = 1, 2, 3, \ldots, n)\). The number of nodes \( C \) represent the number of attributes whereas, the directed edges \( a_{ij} \) represents the relative importance between attribute \( i \) to attribute \( j \). If the edge is directed from node \( i \) to node \( j \), then attribute \( i \) is important than attributes \( j \), and if the edge is directed from node \( j \) to node \( i \), attribute \( j \) is important than attributes \( i \). Therefore, the visualization fuzzy weighted directed graph is shown in figure 1 as below:

Let \( Ci \) be a set of nodes \( C \) represent number of attributes where \((i = 1, 2, 3)\) and \( E = \{a_{12}, a_{21}, a_{13}, a_{31}, a_{23}, a_{32}\} \) be a set of edge \( E \) that represent the relative importance between attributes which is in the form of triangular fuzzy number.
Figure 1: Fuzzy Digraph Representation of Relative Importance between Attributes

**Step 5:** Defuzzification of Determinant and Computation of Selection Index, \( S_I_r \)

To compute Selection Index, \( S_I_r \), the following fuzzy determinant must be presented in the following form of triangular fuzzy number:

\[
Det_r = \begin{bmatrix}
det_r(1) & det_r(2) & det_r(3)
\end{bmatrix}
\]  
(3.13)

In order to compute the fuzzy determinant, cofactor expansion technique is applied with operator of triangular fuzzy number. The illustration of the technique is as follows:

Let \( [A] = \begin{bmatrix}
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3 \\
c_1 & c_2 & c_3
\end{bmatrix} \)

Thus, \( |A| = \begin{bmatrix}
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3 \\
c_1 & c_2 & c_3
\end{bmatrix} \)

\[
= a_1 |b_2, b_3; c_2, c_3| - a_2 |b_1, b_3; c_1, c_3| + a_3 |b_1, b_2; c_1, c_2|
\]

where \( a_i, b_i \) and \( c_i \) which \( i = 1, 2, 3 \) are all triangular fuzzy number.

**Step 6:** Rank the Smartphone based on Value of Selection Index

The defuzzified positive determinants for each alternative, \( S_I_r \) are developed in order to evaluate and select the appropriate smartphone. The larger the value of \( S_I_r \), the most appropriate the alternative smartphone to be choose where it can be performed by using following formula:

\[
S_I_r = \frac{\text{det}_r(1) \times 0.5 + \text{det}_r(2) \times 1 + \text{det}_r(3) \times 0.5}{0.5 + 1 + 0.5}
\]  
(3.14)

6. **Illustrative Example**

One case study of smartphone selection in Malaysia context is studied. In this study, four of the attributes are selected namely Price (C1), Brand Name (C2), Features that also known as Convenience or Usefulness (C3) and Social Influence (C4). The questionnaire is given and evaluated by six decision makers who are smartphone experts defined as \( DM_1, DM_2, DM_3, DM_4, DM_5 \) and \( DM_6 \) which includes their level of confidence, \( k_1, k_2, k_3, k_4, k_5 \) and \( k_6 \) respectively. The criteria of the experts are the smartphone vendors who have a lot of experiences and knowledge of smartphone and have dealt with smartphone for more than 5 years. This study considers five major brands of smartphones that are well
known brand in Malaysia which are Apple, Samsung, ASUS, OPPO and Huawei which are the top five smartphones in Malaysia in year 2017 [20]. Each of these alternative smartphones are denotes as $A_1$, $A_2$, $A_3$, $A_4$ and $A_5$ respectively. Both values of Off-diagonal elements and Diagonal Elements obtained from the questionnaire are tabulated in table 4 and table 5 respectively.

**Table 4. Values of Off-diagonal Elements $a_{ij}$ and $a_{ji}$**

| i | j | $a_{ij}$           | $a_{ji}$           |
|---|---|-------------------|-------------------|
| 1 | 2 | (0.3771, 0.5606, 0.5912) | (0.3378, 0.5012, 0.6320) |
| 1 | 3 | (0.4584, 0.6815, 0.7186) | (0.2094, 0.3875, 0.5445) |
| 1 | 4 | (0.4134, 0.6145, 0.6480) | (0.2896, 0.4551, 0.5793) |
| 2 | 3 | (0.4205, 0.6239, 0.7867) | (0.2145, 0.3968, 0.5576) |
| 2 | 4 | (0.3714, 0.5511, 0.6948) | (0.3714, 0.5511, 0.6948) |
| 3 | 4 | (0.2408, 0.4454, 0.6260) | (0.3692, 0.5802, 0.7385) |

**Table 5. Values of Diagonal Elements, $NMS_{nr}$**

| Alternative | $Cn$                      | Mean Values, $NMS_{nr}$           |
|-------------|---------------------------|-----------------------------------|
| $A_1$       | $C1$                      | (0.4147, 0.5963, 0.7155)          |
|             | $C2$                      | (0.6857, 0.8646, 0.8944)          |
|             | $C3$                      | (0.4243, 0.5657, 0.6364)          |
|             | $C4$                      | (0.5159, 0.6833, 0.7948)          |
| $A_2$       | $C1$                      | (0.2357, 0.3771, 0.4950)          |
|             | $C2$                      | (0.4462, 0.6136, 0.7530)          |
|             | $C3$                      | (0.4741, 0.6414, 0.7530)          |
|             | $C4$                      | (0.4462, 0.6136, 0.7530)          |
| $A_3$       | $C1$                      | (0.2510, 0.3904, 0.5578)          |
|             | $C2$                      | (0.1255, 0.2649, 0.4323)          |
|             | $C3$                      | (0.1673, 0.3068, 0.4741)          |
|             | $C4$                      | (0.0837, 0.2231, 0.3904)          |
| $A_4$       | $C1$                      | (0.6112, 0.7901, 0.8497)          |
|             | $C2$                      | (0.6410, 0.8199, 0.8497)          |
|             | $C3$                      | (0.5814, 0.7603, 0.8497)          |
|             | $C4$                      | (0.5516, 0.7304, 0.8497)          |
| $A_5$       | $C1$                      | (0.5857, 0.7530, 0.8367)          |
The formation of interrelation matrix for each alternative smartphone, \([A_r]\) is developed by substituting the diagonal elements with the New Mean Values of Attributes, \(NMS_{nr}\) and the off-diagonal elements, \(a_{ij}\) and \(a_{ji}\). For each alternative, the diagonal elements of the interrelation matrix are different whereas all the off-diagonal elements remain same.

\[
[A_r] = \begin{bmatrix}
NMS_{1r} & a_{12} & a_{13} & a_{14} \\
a_{21} & NMS_{2r} & a_{23} & a_{24} \\
a_{31} & a_{32} & NMS_{3r} & a_{34} \\
a_{41} & a_{42} & a_{43} & NMS_{4r}
\end{bmatrix}
\]

Then, the corresponding fuzzy weighted directed graph is developed based on edge weight tabulated in Table 6. The defuzzified values of off-diagonal elements of the matrix are also computed and tabulated in Table 6 to show the value of relative importance between attributes.

**Table 6.** Values of Relative Importance between Attributes

| \(i, j\) | \(a_{ij}\) or \(a_{ji}\) | Triangular Fuzzy Number | Defuzzified Value |
|----------|--------------------------|------------------------|------------------|
| \(i = 1\) | \(a_{12}\) | (0.3771, 0.5606, 0.5912) | \(a_{12}^* = 0.5096\) |
| \(j = 2\) | \(a_{21}\) | (0.3378, 0.5012, 0.6320) | \(a_{21}^* = 0.4904\) |
| \(i = 1\) | \(a_{13}\) | (0.4584, 0.6815, 0.7186) | \(a_{13}^* = 0.6195\) |
| \(j = 3\) | \(a_{31}\) | (0.2094, 0.3875, 0.5445) | \(a_{31}^* = 0.3805\) |
| \(i = 1\) | \(a_{14}\) | (0.4134, 0.6145, 0.6480) | \(a_{14}^* = 0.5586\) |
| \(j = 4\) | \(a_{41}\) | (0.2896, 0.4551, 0.5793) | \(a_{41}^* = 0.4414\) |
| \(i = 2\) | \(a_{23}\) | (0.4205, 0.6239, 0.7867) | \(a_{23}^* = 0.6104\) |
| \(j = 3\) | \(a_{32}\) | (0.2145, 0.3968, 0.5576) | \(a_{32}^* = 0.3896\) |
| \(i = 2\) | \(a_{24}\) | (0.3714, 0.5511, 0.6948) | \(a_{24}^* = 0.5391\) |
| \(j = 4\) | \(a_{42}\) | (0.3453, 0.4565, 0.5810) | \(a_{42}^* = 0.4609\) |
The fuzzy weighted directed graph is illustrated as in Figure 2 below:

![Fuzzy Weighted Directed Graph](image)

**Figure 2. Fuzzy Weighted Directed Graph of Relative Importance Between Attributes**

From Table 6 and Figure 2, defuzzified value of C1 is higher compared to attribute C2, C3 and C4. Next, defuzzified value of attribute C2 is also higher as compared to attribute C3 and C4. Whilst, defuzzified value of attribute C4 is higher than C3. Thus, it shows that Price is the most important attribute as compared to Brand, Feature and Social Influence respectively. In addition, Brand (C2) is second most important attribute as compared to Feature (C3) and Social Influence (C4). Moreover, Social influence (C4) is more important than Feature (C3).

Fuzzy determinant is then presented in Table 7. It is calculated using cofactor expansion method using relevant operations on triangular fuzzy numbers.

**Table 7. Fuzzy Determinant for each Alternative**

| Alternative | Fuzzy Determinant                |
|-------------|----------------------------------|
| A₁          | $Det_1 = (-1.1652, 0.0053, 1.2567)$ |
| A₂          | $Det_2 = (-0.0112, -0.0031, 1.0447)$ |
| A₃          | $Det_3 = (-0.7412, -0.0180, 0.7220)$ |
| A₄          | $Det_4 = (-1.3536, 0.0405, 1.4639)$ |
| A₅          | $Det_5 = (-1.3739, 0.0496, 1.5452)$ |
Next, the values of Selection Index, $SI_r$, is calculated and the rank of alternative smartphone is obtained by looking at the Selection Index value which is tabulated in Table 8. The larger the $SI_r$, the most appropriate the alternative smartphone is to choose.

Table 8. Ranking of Alternative Smartphone

| Alternative Smartphone | Selection Index, $SI_p$ | Ranking |
|------------------------|-------------------------|---------|
| Apple ($A_1$)          | $SI_1 = 0.0258$         | 3       |
| Samsung ($A_2$)        | $SI_2 = 0.0068$         | 5       |
| ASUS ($A_3$)           | $SI_3 = 0.0138$         | 4       |
| OPPO ($A_4$)           | $SI_4 = 0.0478$         | 2       |
| Huawei ($A_5$)         | $SI_5 = 0.0676$         | 1       |

Table 8 shows the ranking of alternative smartphone by using Fuzzy Digraph and Matrix Approach. From the ranking it can be identified that $A_5$ is the first rank followed by $A_4$, $A_1$, $A_3$ and the lastly is $A_2$. The highest Selection Index is 0.0676 representing Huawei brand and be the most preferred brand. This result also represents OPPO brand as second preferred brand followed by Apple brand and ASUS brand with selection index of 0.0478, 0.0258 and 0.0138 respectively. Samsung brand is the last choice among the five alternatives with selection index of 0.0068.

Huawei has grown from a company that offered cheap smartphone in order to appeal to customers to a global leader in the smartphone industry by providing a brand that can compete with Apple and Samsung [21]. The author also stated that converting the world into a digital globe by providing safety, flexibility and open ICT infrastructure platform to industry and organization is a factor of the success of Huawei. Thus, this shows that Huawei is the most preferred one compared to other brands in this study.

From the data collected, the ranking of alternative smartphone by using Fuzzy Digraph and Matrix Approach without the inclusion of confidence level’s values also be analysed. The result is tabulated in the Table 9 as follows.

Table 9. Ranking of Alternative Smartphone Without Values of Confidence Level

| Alternative Smartphone | Selection Index, $SI_p$ | Ranking |
|------------------------|-------------------------|---------|
| Apple ($A_1$)          | $SI_1 = 0.0548$         | 2       |
| Samsung ($A_2$)        | $SI_2 = 0.0375$         | 3       |
| ASUS ($A_3$)           | $SI_3 = 0.0051$         | 5       |
| OPPO ($A_4$)           | $SI_4 = 0.0306$         | 4       |
| Huawei ($A_5$)         | $SI_5 = 0.1271$         | 1       |

From the result in Table 9, it is identified that $A_5$ is the ranked as number one followed by $A_1$, $A_2$, $A_4$ and $A_3$. The highest Selection Index is 0.1271 representing Huawei brand and be the most preferred brand. This result also represents Apple brand as second preferred brand followed by Samsung brand and OPPO brand with selection index of 0.0548, 0.0375 and 0.0306 respectively. ASUS brand is the last choice among the five alternatives with selection index of 0.0051.
Table 10. Comparison of the smartphone’s ranking with and without confidence level.

| Alternative Smartphone | Ranking with Confidence Level | Ranking without Confidence Level |
|-------------------------|-------------------------------|---------------------------------|
| Apple ($A_1$)           | 3                             | 2                               |
| Samsung ($A_2$)         | 5                             | 3                               |
| ASUS ($A_3$)            | 4                             | 5                               |
| OPPO ($A_4$)            | 2                             | 4                               |
| Huawei ($A_5$)          | 1                             | 1                               |

Table 10 above gives a comparison result of smartphone ranking with and without confidence level. It shows that the most preferred smartphone brand is Huawei brand although there is slight difference in the ranking order for other brand. According to [2], observation of the market on smartphone have revealed that a remarkable growth of OPPO and Huawei brands which overtook the achievement of ASUS and Apple.

7. Conclusion
In this study, the combination of fuzzy set theory using generalized fuzzy number (GFN) in processing data with digraph and matrix technique is successfully applied to rank a selection of smartphone. Thus, it could possibly be used to solve any other decision-making problems involving many criteria. Fuzzy Digraph and Matrix Approach with GFN is not only could accommodate issue of uncertainty in human judgement but also take into account the element of confidence level of the decision maker which could later leads to the realistic ranking order of alternative. Other than that, this study also gives information to mobile manufacturing sector in improving their smartphone product to satisfy current needs of smartphones’ users based on the selected attributes preferred by them.

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References
[1] Lu, M.-T., Hu, S.-K., & Tzeng, G.-H. (2014). Exploring smart phone improvements based on a hybrid MCDM model. Expert Systems with Applications, 41(9), 4401-4413.
[2] Smart, N. (2012). A survey of smartphone and tablet computer use by colorectal surgeons in the UK and Continental Europe. Colorectal Disease, 14(9), 535-538.
[3] Birenboim, A., & Shoval, N. (2016). Mobility research in the age of the smartphone. Annals of the American Association of Geographers, 106(2), 283-291.
[4] Yildiz, A., & Ergul, E. U. (2015). A two-phased multi-criteria decision-making approach for selecting the best smartphone. South African Journal of Industrial Engineering, 26(3), 194-215.
[5] Hu, Y.-C., & Liao, Y.-L. (2013). Utilizing analytic hierarchy process to analyze consumers’ purchase evaluation factors of smartphones. International Journal of Economics and Management Engineering, 7(6), 1556-1561.
[6] Akyene, T. (2012). Cell phone evaluation base on entropy and TOPSIS. Interdisciplinary Journal of Research in Business, 1(12), 9-15.
[7] Mohamad D., Sulaiman N. H. and Abu Bakar A. S. (2011). Solving industrial decision making problems using fuzzy numbers. 2011 IEEE Symposium on Business, Engineering and Industrial Applications (ISBEIA), Langkawi, 271-274.
[8] Kamis N. H., Mohamad D., Sulaiman N. H., Abdullah K. and Ibrahim I (2012). *An integrated fuzzy approach to solving multi-criteria decision making problems*. 2012 IEEE Symposium on Humanities, Science and Engineering Research, Kuala Lumpur, 2012, pp. 1591-1596.

[9] Mohd Hanif, H, Mohamad, D., Sulaiman N. H. (2013). *Solving decision making problems using fuzzy numbers with area dominance approach*. AIP Conference Proceedings 1522(1), 229.

[10] Ramli, N., Mohamad, D. and Sulaiman N. H (2010). Evaluation of Teaching Performance with Outliers Data using Fuzzy Approach. *Procedia-Social Behavioral Sciences*, 8, 190-197.

[11] Büyüköztan, G., & Güleryüz, S. (2016). Multi criteria group decision making approach for smart phone selection using intuitionistic fuzzy TOPSIS. *International Journal of Computational Intelligence Systems*, 9(4), 709-725.

[12] Belbag, S., Gungordu, A., Yumusak, T., & Yilmaz, K. G. (2016). The evaluation of smartphone brand choice: An application with the fuzzy Electre I method. *International Journal of Business and Management Invention*, 5(3), 55-63.

[13] Rao, R. V., & Padmanabhan, K. (2016). Selection, identification and comparison of industrial robots using digraph and matrix methods. *Robotics and Computer-Integrated Manufacturing*, 22(4), 373-383.

[14] Sumarni A. B., Noor Ainy H., Kahartini A. R., Mohd Agos S. N., Herniza M. T., Elsie J. (2019). Application of Graph Theory and Matrix Approach as Decision Analysis Tool for Smartphone Selection. *ASM Science Journal*, 12(6), 53-59.

[15] Sunitha, M., & Mathew, S. (2013). Fuzzy Graph Theory: A Survey. *Annals of Pure and Applied Mathematics*, 4(1), 92-110.

[16] Koulouriotis, D. E., & Ketipi, M. K. (2011). A fuzzy digraph method for robot evaluation and selection. *Expert Systems with Applications*, 38(9), 11901-11910.

[17] Goala, S., & Dutta, P. (2018). Detection of area under potential threat via an advanced aggregation operator on generalized triangular fuzzy number. *Journal of Taibah University for Sciences*, 12(5), 536-544.

[18] Chen, Y., Fung, R. Y., & Tang, J. (2006). Rating Technical Attributes in Fuzzy Qfd by Integrating Fuzzy Weighted Average Method and Fuzzy Expected Value Operator. *European Journal of Operational Research*, 174(3), 1553-1566.

[19] Azman, N. I. (2018). *Fuzzy Inferior Ratio Method with Similarity-Based Compromise Solution*. (Master’s Thesis), Universiti Teknologi MARA, Shah Alam, Selangor.

[20] Al-Koliby, I. S., & Rahman, M. A. (2018). Influence Dimensions of Brand Equity on Purchase Intention Toward Smartphone in Malaysia. *VFAST Transactions on Education Social Sciences*, 15(1), 7-19.

[21] Alkhawajah, W. (2019). Huawei: An Information and Communications Technology Company. *Journal of Information Technology Economic Development*, 10(1), 1-10.