Mid-infrared quantum optics in silicon

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The mid-infrared presents a brave new frontier for optics, with applications from remote sensing to communications. Applied quantum optics stands to revolutionise many aspects of information technology, provided performance can be maintained when scaled up. Silicon quantum photonics satisfies the scaling requirements of miniaturisation and manufacturability, but suffers from high linear and nonlinear loss. Here, we show that silicon quantum photonics can be translated to the mid-infrared, resulting in a technology platform which simultaneously maximises manufacturability and miniaturisation, while minimising loss. We demonstrate the necessary ingredients for a new quantum photonic platform: photon-pair generation, single-photon detection, and quantum interference, all at wavelengths beyond 2 μm. Across various regimes, we observe a maximum coincidence rate of 275±5 Hz, a coincidence-to-accidental ratio of 25.7±3.3, and a two-photon quantum interference visibility of 0.985±0.015 (net). Mid-infrared silicon quantum photonics will bring new quantum applications within reach.

The mid-infrared (MIR) is the energy band of vibrations. The molecular ‘fingerprint’ region, 2–20 μm, is characterised by sharp molecular transitions. Lab-on-chip sensors use this region to spectrally target molecular species in liquid and gaseous analytes. Lidar systems exploit atmospheric transparency and reduced scintillation in the MIR, as well as the high-power handling and reduced phase error of MIR optical phased arrays for improved reliability. Much work has been done to bring integrated optics to the MIR. In the short-wave part of the band (the short-wave infrared), up to about 4 μm, silicon-on-insulator photonics dominates. Here, silicon benefits from reduced two-photon absorption, facilitating nonlinear optical applications: optical parametric oscillators and amplifiers, supercontinuum sources, and frequency combs have all been developed.

Quantum photonic technology promises to revolutionise how we measure, communicate, and ultimately process information, by harnessing quantum superposition and entanglement. Quantum sensors can overcome the classical shot-noise limit. Quantum communications systems offer physically guaranteed security, and may one day provide the fabric for networked quantum computation. Quantum computers promise to exponentially accelerate specific tasks, and optics remains a contender. All these applications are exquisitely performance sensitive, requiring a huge scaling up for integration or real-world deployment. As classical optics ventures into the MIR, quantum optics is close behind. Bulk-crystal photon-pair sources have been designed; experiments with one, and two MIR photons have been shown. These experiments have used upconversion and avalanche photodiode detectors, which have the benefit of room-temperature operation, but suffer from low sensitivity and intrinsic thermal noise.

Silicon photonics, benefiting from a global manufacturing capability, and operating mainly around the 1.5-μm telecommunications band, has exploded in scale and functionality. Quantum silicon photonics has grown rapidly in tandem. In silicon, quantum-correlated photon pairs are

![Figure 1: Dispersion of key optical phenomena, relative to 1.55 μm values.](image)
scattered from a bright pump laser via the refractive nonlinearity, by the spontaneous four-wave mixing (SFWM) process. Increasingly large interferometers have used photon pairs to power increasingly complex proof-of-concept quantum protocols\textsuperscript{28–30}. To go beyond a handful of photons, though, ultra-low optical loss is needed.

Propagation and device losses have steadily fallen\textsuperscript{31–35}, but two-photon absorption (TPA) is intrinsic. TPA allows a crystal electron to be promoted to the conduction band by absorbing two photons. It is stimulated absorption, growing with optical intensity. TPA clearly limits pump power, but it also limits the heralding efficiency of single photon sources\textsuperscript{36}, and so is a fundamental limit to the large-scale viability of silicon quantum photonics\textsuperscript{1}. Beyond silicon’s two-photon band edge, around 2.1 μm, two photons carry insufficient energy to excite a crystal electron, and TPA becomes impossible. A resonant peak occurs in the refractive nonlinearity around the two-photon band edge, making photon-pair sources more efficient. 2.1 μm is still well within the transparency window of conventional silica cladding, and environmental black-body noise is not too large. Linear losses are reduced at long wavelengths by the $\lambda^{-4}$ dependence of Rayleigh scattering from etched waveguide side-walls\textsuperscript{37,38}, and subwavelength features are more readily manufactured\textsuperscript{35,39}. We plot the TPA coefficient $\beta_{\text{TPA}}$, nonlinear refractive index $n_2$, and Rayleigh scattering cross-section in Fig. 1. One common approach to avoiding TPA is to swap out the guiding material (e.g. silicon nitride\textsuperscript{40,41}). In this work, we investigate the potential of the 2.1-μm band as a platform for low-loss quantum optics in crystalline silicon.

Any new quantum photonic platform needs: a source of quantum light, a way to detect that light, and provable quantum interference. We report on all three ingredients here. We design and characterise a silicon waveguide able to generate entangled photon pairs, centred on 2.071 μm, and use classical nonlinear optics to verify its design. We deploy a new detector system, optimised for the 2-μm band, and verify its performance. We then drive SFWM in the designed waveguide and observe quantum-correlated photon pairs using the single-photon detectors. Finally, we embed two such photon-pair sources in a reconfigurable on-chip inter-
Figure 3: Measurement of correlated photons and characterisation of superconducting detectors: a, Dark field optical micrograph of the waveguide (WG) source with vertical grating couplers (VGC); scale bar 50 µm. b, Experimental configuration for correlated photon measurement. Polarisation controller (PC), input optical tap (9:1), photodiode (PD), beam splitter (1:1), output optical tap (99:1), superconducting nanowire single photon detector (SNSPD), time interval analyser (TIA). c, System detection efficiency (SDE) and dark count rate (DCR) with change in bias current measured at λ = 2.07 µm wavelength on detector A. Error bars are due to systematic error in the estimation of the number of launched photons. d, SDE and DCR for detector B. e, Spectral response of detector efficiencies at a fixed bias current of 8.4 and 7.9 µA for detectors A and B, respectively. A moving average window of five points has been applied to data and the error bars are the standard deviation of the points in the sampled moving average window. f, Sample coincidence histogram integrated for 540 seconds. The peak at zero delay corresponds to photon pairs generated in the same SFWM event. The star on the plot in g denotes the datapoint that produced this histogram. g, Measured coincidence to accidental ratio, net and raw coincidence rate with varying launched pump power. Error bars are one standard deviation of the random error in the measurement.

Waveguide design for MIR SFWM

Arrival-time coincidence measurements are the main method of verifying quantum correlations of photon pairs. SFWM, where two pump photons are scattered to higher and lower frequencies via a virtual energy level, conserves...
energy and momentum. For efficient SFWM, the phase-matching condition between the four fields must be satisfied. The total wave-vector mismatch is given by

$$\Delta k = \Delta k_{\text{lin}} - 2\gamma P,$$  \hspace{1cm} (1)

where $P$ is the peak pump power in the waveguide, and $\gamma = n_2 P_{\text{eff}}/A_{\text{eff}}$ is the waveguide nonlinear parameter, here $n_2$ and $A_{\text{eff}}$ are the nonlinear refractive index and effective modal area, respectively. The linear wave-vector mismatch is

$$\Delta k_{\text{lin}} = 2k_p - k_s - k_i \approx -\beta_2 (\omega_p - \omega_s)^2,$$  \hspace{1cm} (2)

where the subscripts $p$, $s$, and $i$ denote the pump, signal, and idler frequencies, respectively, and $\beta_2 = (1/2)\partial^2 k/\partial \omega^2$ is the waveguide group-velocity dispersion (GVD). For efficient four-wave mixing, $\beta_2 \leq 0$, i.e., the GVD must be anomalous or zero.

We designed the waveguide, shown in Fig. 2a, with a 510 x 340 nm$^2$ cross-section and a 15° side-wall angle, as specified by the foundry. We target the TE$_{0}$ fundamental mode (Fig. 2b) which has a small anomalous $\beta_2$ and is highly confined at 2.071 µm. We plot the calculated variations of $\beta_2$ and $A_{\text{eff}}$ versus waveguide width in Fig. 2c.

To confirm the phase matching of our source, and estimate its bandwidth, we measure classical stimulated four-wave transmission at longer wavelengths (Fig. 2e). We observe phase-matched four-wave mixing over at least 60 nm, with the emission limited by grating coupler cut-off. The sub-wavelength fundamental mode (Fig. 2b) which has a small anomalous $\beta_2$ and is highly confined at 2.071 µm. We plot the calculated variations of $\beta_2$ and $A_{\text{eff}}$ versus waveguide width in Fig. 2c.

To confirm the phase matching of our source, and estimate its bandwidth, we measure classical stimulated four-wave mixing. Fig. 2f shows a spectral map of the pump and stimulated emission, as the seed laser wavelength is varied. We observe phase-matched four-wave mixing over at least 60 nm, with the emission limited by grating coupler transmission at longer wavelengths (Fig. 2e).

To compare the nonlinearity in our waveguided device with bulk silicon, we use the Gerchberg-Saxton optical phase-retrieval algorithm. Our results are summarised in Fig. 2d. We find an effective waveguide nonlinearity of $\gamma = 176 \pm 31$ W$^{-1}$m$^{-1}$ ($n_2 = 13.3 \pm 2.3 \times 10^{-18}$ m$^2$W$^{-1}$ in bulk) and waveguide nonlinear absorption coefficient $\alpha_{\text{TTPA}} = 24.8 \pm 3.2$ W$^{-1}$m$^{-1}$ ($\beta_{\text{TTPA}} = 0.557 \pm 0.07$ cm$^{-1}$·GW$^{-1}$ in bulk) in agreement with measurements in bulk silicon. Here, $\partial \alpha/\partial P = \alpha_{\text{TTPA}}$ and $\partial \alpha/\partial P = \beta_{\text{TTPA}}$, where $\alpha$ is the wave loss coefficient, and the intensity $I = P/A_{\text{eff}}$ for a waveguided power $P$.

**Observation of correlated photon pairs**

The first ingredient of our quantum photonic platform is a source of quantum correlated photon pairs, from SFWM. To drive SFWM in our designed waveguide, we start with a picosecond-pulsed pump laser, centred at 2.0715 µm, which we filter to a width of 1.0 nm using a double-pass grating monochromator. Controlling polarisation, we inject this pump into the fundamental TE mode of the waveguide via vertical grating couplers (VGC). The waveguide, shown in Fig. 3a, is wrapped into a 7.2-mm square spiral with 10-µm radius Euler bends. Signal and idler photons are emitted in the same spatial mode at the output of the chip and are probabilistically separated by a 1:1 fibre beam splitter. Both channels are filtered with back-to-back grating monochromators to achieve the > 100 dB pump rejection required to attenuate the pump down to the single photon level.

The signal and idler photon collection filters are separated ±1.46 THz ~ ±20.8 nm from the pump.

To detect single-photons from SFWM, sensitive detectors are also essential. Superconducting nanowire single photon detectors (SNSPD) have timing jitter, dark count rates (DCR), and system detection efficiency (SDE) unrivalled by other technologies. This type of detector has been shown to be sensitive to single photons up to 5 µm. We incorporate superconducting nanowires into a dielectric stack optimised for absorption into the nanowave cavity at 2.1 µm. The SNSPDs were fabricated from a 4-nm thick niobium nitride film, deposited using magnetron sputtering, and patterned using electron-beam lithography into a meander of 100-nm-wide superconducting wire. The two detectors had different latching and non-latching designs. At an operating temperature of 780 mK, we find that the SDE of the two detectors plateaus with bias currents around 8 µA, with peak SDE values of 47 ± 19% and 41 ± 17%, for detectors A and B, respectively. In the following experiments, we operate with detector bias currents of 8.1 µA and 7.9 µA. We measure a timing jitter of 215 ps (full width at half maximum). The dark count rate and efficiency as a function of bias at $\lambda = 2.071$ µm is shown in Figs. 3c and 3d. We use an attenuated tuneable laser to measure the SDE as a function of wavelength, and plot the results in Fig. 3e.

The correlated photon-pairs are detected with the electrical output pulses time correlated using standard coincidence counting logic. We observe a characteristic peak at zero relative delay from photon pairs produced in the same SFWM event. An example is shown in Fig. 3f. For each optical power launched, the time correlation histograms are fitted with a Gaussian function, and integrated over 3 standard deviations about the peak centre, ~ 389 ps. Our analysis gives a maximum raw coincidence rate of 275 ± 5 Hz and peak coincidence to accidental ratio (CAR) of 25.7 ± 3.3 at a coincidence rate of 1.1 Hz (Fig. 3g). We define $\text{CAR} = (X_{\text{raw}} - X_{\text{acc}})/X_{\text{acc}}$ where $X_{\text{raw}}$ is the integrated counts in the histograms central peak and $X_{\text{acc}}$ is taken as the average number of integrated counts in the side peaks. On a longer 1.75-cm waveguide, a peak raw coincidence rate was measured at 1.2 kHz—shown in Supplementary Fig. S1. Due to the simple separation of the signal and idler with a beam splitter, the measured coincidence rates are 1/4 of the actual pair-production rate. These measurements translate to true rates of 1.1 kHz and 4.8 kHz, respectively.

**On-chip quantum interference**

High-visibility quantum interference is an essential component for any quantum optics platform. The Hong-Ou-Mandel (HOM) effect causes two completely indistinguishable single photons, when injected into the two ports of a balanced beam-splitter, to strictly bunch at the outputs producing the path-entangled output state $(|20\rangle - |02\rangle)/\sqrt{2}$. Here, we achieve on-chip quantum interference using a time-reversed HOM experiment, with experimental setup and waveguide circuit shown in Figs. 4a and 4b.

After filtering the pump as before, we couple it onto the chip through a vertical grating coupler. We set the on-
chip peak pump power to 0.65 W, corresponding to a CAR of 17.1. The pump field is evenly split between the two photon-pair sources, with a balanced $1 \times 2$ multimode interference coupler (MMI). Both 7.39-mm sources are coherently pumped and the relative phase $\phi$ between the two arms is varied with a thermo-optic phase modulator. The biphoton state then passes through a second 1:1 (directional) coupler. Coherent pumping of both sources produces SFWM photon pairs in a superposition, and the quantum state at the directional coupler output is

$$|\psi\rangle = \cos \phi |\psi_{\text{bunch}}\rangle + \sin \phi |\psi_{\text{split}}\rangle,$$

where the two-photon Fock states

$$|\psi_{\text{bunch}}\rangle = \frac{|1,0\rangle_a |0,1\rangle_b + |0,0\rangle_a |1,1\rangle_b}{\sqrt{2}},$$

$$|\psi_{\text{split}}\rangle = \frac{|1,0\rangle_a |0,1\rangle_b - |0,1\rangle_a |1,0\rangle_b}{\sqrt{2}},$$

denote the non-degenerate frequency signal and idler ($s, i$), and the spatial modes ($A, B$). SFWM photons are then frequency demultiplexed on-chip with asymmetric Mach-Zehnder interferometers. After coupling back into fibre, photons pairs pass through back-to-back monochromator filters to reject the pump laser. Finally, we detect coincidences of the $|\psi_{\text{split}}\rangle$ state.

Characteristic half-period interference fringes are observed\textsuperscript{49,50} in the coincidences as we vary the on-chip phase $\phi$, demonstrating quantum interference with a net visibility of $V = 0.985 \pm 0.015$ (0.88 $\pm$ 0.01 raw). We calculate the visibility as $V = (N_{\text{min}} - N_{\text{max}})/(N_{\text{min}} + N_{\text{max}})$, where $N_{\text{max}}$ and $N_{\text{min}}$ are the maximum and minimum values of the coincidence count rate of the sinusoidal fit, equivalent to a HOM dip visibility of 0.984 $\pm$ 0.015 (0.76 $\pm$ 0.01 raw)\textsuperscript{49}.

This compares favourably to a recently reported HOM interference dip with photon-pairs from bulk down-conversion crystal\textsuperscript{51}. Projecting onto $|1,0\rangle_A |0,1\rangle_B$, we observe coincidence rates of up to 5.5 $\pm$ 0.2 Hz at the interference peak. If we could simultaneously measure all four chip outputs, the observed rates would double.

**Discussion and conclusions**

Despite operating in the 2- $\mu$m band, we see from Fig. 2d that two-photon absorption remains, albeit reduced from its strength in the telecommunications band. This is to be expected at room temperature and at 2.07 $\mu$m, as silicon’s indirect band gap gives a TPA cut-off around 2.21 $\mu$m (Fig. 1). Our estimates of both nonlinear absorption and refraction are in broad agreement with literature values for bulk silicon\textsuperscript{4,5}. At low temperatures, growth in the band gap causes a blue shift in the TPA cut-off\textsuperscript{52}, to 2.15 $\mu$m. Future room-temperature experiments will benefit from 2.2-$\mu$m laser development (e.g. semiconductor disk lasers\textsuperscript{53}), while experi-
ments integrating SNSPDs on-chip will also benefit from this low-temperature shift of the TPA cut off.

Black-body radiation from our room-temperature apparatus was an additional source of photon noise in our measurements. The ‘dark’ count rate, collected with the lights and laser turned off, plateaus rather than growing exponentially with bias, in Figs. 3c,d. Despite the darkness of these measurements, MIR black-body photons from the environment nonetheless continue to couple into and illuminate the detectors—a challenge of quantum optics in the MIR. Black-body noise would be suppressed by a cold filter—another motivation for integrating detectors with optics on a chip.

In demonstrating a bright source of photon pairs, efficient single-photon detectors, and high-visibility quantum interference, we have provided all the necessary ingredients for a dense, manufacturable, and high-performance platform. The mid-infrared presents a disruptive new approach to scalable optical quantum information processing in silicon.

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ansatz giving an input pulse duration of $\tau = 4.82$ ps. The laser is then connected to the 17.5 – mm spiral with an 9:1 optical tap to monitor the input power. The optical power launched into the chip is controlled with a knife-edge, allowing the intensity-dependent transmission and spectrum of the output pulse to be measured. For each power launched, a frequency spectrum is recorded on an OSA (Yokogawa AQ6375) at the chip output.

To retrieve nonlinear phase from the output spectrum, sequential Fourier and inverse Fourier transforms are performed to transform between the time and frequency domains (Gerchberg-Saxton algorithm)\textsuperscript{33}. By retaining the phase information, but replacing the amplitude with our measured secant-pulse envelope (time domain) or power spectrum (frequency domain), the algorithm is found to converge on a steady state. Using this algorithm the nonlinear phase corresponding to the measured spectral broadening is retrieved for each pulse. The lowest power scan is taken as the reference and is subtracted from all higher-power phase profiles. Finally, the phase profiles are fitted with a secant-shaped phase model and combining this with the free carrier dispersion and absorption effects found from fitting the transmission data, the nonlinear phase and hence the waveguide nonlinearity are found. To determine the value of $n_2$ in Fig. 1, we model solved effective mode area calculated using the method from\textsuperscript{34}. In addition to determining the refractive nonlinearity, the transmitted power is also used to determine the TPA parameter by using a fit to the inverse transmission against the input power, as described in\textsuperscript{35}.

Stimulated four-wave mixing We pump the waveguide source with a filtered picosecond pulsed laser centred at $\lambda = 2.0715$ $\mu$m and a tunable continuous wave (CW) laser (Sacher Lion) as the stimulating field. The two lasers are combined on a 1:1 fibre beam splitter before the chip with power monitoring optical taps to normalise the stimulated FWM at the chip output. Tuning the frequency of the CW laser, the stimulated FWM from a 7.2-mm spiral is measured on an OSA.

Detector characterisation Electrical output traces of the RF pulses from the SNSPDs are recorded for several bias settings. The peak voltage of the pulse changes linearly as a function of detector bias and a linear model is fit to this data—10.84 and 7.46 mV/μA. This is combined with a constant offset of 20 and 15 mV, determining the discrimination voltage as a function of bias for detectors A and B, respectively.

Bright light of a tunable CW laser is measured on a calibrated Thorlabs InGaAs photodiode (S148C) with a knife edge variable optical attenuator to control the incident photon flux. Individual transmission of four neutral density filters are then recorded before inserting them in the beam path and reducing the net photon flux down to the single photon regime. Finally, the photodiode input fibre is connected to the superconducting detector, and singles counts from a PicoHarp timer tagger are integrated for 0.5 s for each optical power launched. The estimated input and output photon flux is fit with a linear slope model and is used to determine the system detection efficiency.

Correlated photon pair measurement We pump the waveguide source shown in Fig. 3a with a Ho-doped fibre laser (AdValue Photonics) centred on 2.0715 μm, producing 2.9 ps pulses, at a 40 MHz rate, with 486 W peak power coupled into fibre from free space. The laser is filtered with a monochromator in a double pass configuration (Fig. 3b), producing 5.78 ps pulses, 24.4 W peak power. Filtered pump is then coupled into the waveguide with a V-groove array and vertical grating couplers. After propagating through the waveguide spiral, the signal and idler photons are passively demultiplexed on a fibre beam splitter. Demultiplexing the signal and idler incurs a 6 dB penalty on coincidences since only 1/4 of events are incident on the correct off-chip filters. Back-to-back monochromators suppress the pump down to the single photon level. A double pass configuration is not used as the extinction is less than two monochromators in series. Finally, the photon pairs are detected by SNSPDs. A PicoQuant PicoHarp with a 4-ps resolution correlates detector output pulses. The main coincidence peak at $\tau = 0$ s in Fig. 3f corresponds to photon pairs generated in the same SFWM event. The side peaks are either from accidental events due to leaked pump and multi-pair emission clocked at 25 ns intervals corresponding to the repetition rate of the pump. For each power launched, a histogram was recorded for 540 and 180 seconds in the low and high power regimes, respectively. Each histogram was subdivided into 20 second integration intervals to obtain statistics of the random error in the measurement.
**Thermo-optic phase modulator calibration** We used a 16-bit DC voltage source (Qontrol Systems) to drive the on-chip thermo-optic phase modulators. Injecting CW laser light into the input vertical grating coupler, the optical output of the chip was monitored with a photodiode whilst the phase-voltage was varied. The interferometers voltage-squared versus transmission was fit with a sinusoidal envelope. In the time-reversed HOM experiment, 30 evenly spaced squared-voltages were set on the source phase shifter.