Annotating Derivations: A New Evaluation Strategy and Dataset for Algebra Word Problems

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Abstract

We propose a new evaluation for automatic solvers for algebra word problems, which can identify mistakes that existing evaluations overlook. Our proposal is to evaluate such solvers using derivations, which reflect how an equation system was constructed from the word problem. To accomplish this, we develop an algorithm for checking the equivalence between two derivations, and show how derivation annotations can be semi-automatically added to existing datasets. To make our experiments more comprehensive, we include the derivation annotation for DRAW-1K, a new dataset containing 1000 general algebra word problems. In our experiments, we found that the annotated derivations enable a more accurate evaluation of automatic solvers than previously used metrics. We release derivation annotations for over 2300 algebra word problems for future evaluations.

1 Introduction

Automatically solving math reasoning problems is a long-pursued goal of AI (Newell et al., 1959; Bobrow, 1964). Recent work (Kushman et al., 2014; Shi et al., 2015; Koncel-Kedziorski et al., 2015) has focused on developing solvers for algebra word problems, such as the one shown in Figure 1. Developing a solver for word problems can open several new avenues, especially for online education and intelligent tutoring systems (Kang et al., 2016). In addition, as solving word problems requires the ability to understand and analyze natural language, it serves as a good test-bed for evaluating progress towards goals of artificial intelligence (Clark and Etzioni, 2016).

An automatic solver finds the solution of a given word problem by constructing a derivation, consisting of an un-grounded equation system \( \{ \{ A_m = B_n, C_m + D_n = E \} \) in Figure 1 and alignments of numbers in the text to its coefficients (blue edges). The derivation identifies a grounded equation system \( \{ 5m = 15n, 5m + 5n = 100 \} \), whose solution can then be generated to answer the problem. A derivation precisely describes how the grounded equation system was constructed from the word problem by the automatic solver. On the other hand, the grounded equation systems and the solutions are less informative, as they do not explain which span of text aligns to the coefficients in the equations.

While the derivation is clearly the most informative structure, surprisingly, no prior work evaluates automatic solvers using derivations directly. To the best of our knowledge, none of the current datasets contain human-annotated derivations, possibly due to the belief that the current evaluation metrics are sufficient and the benefit of evaluating on derivations is minor. Currently, the most popular evaluation strategy is to use solution accuracy (Kushman et al., 2014; Hosseini et al., 2014; Shi et al., 2015; Koncel-Kedziorski et al., 2015).
which computes whether the solution was correct or not, as this is an easy-to-implement metric. Another evaluation strategy was proposed in (Kushman et al., 2014), which finds an approximate derivation from the gold equation system and uses it to compare against a predicted derivation. We follow (Kushman et al., 2014) and call this evaluation strategy the equation accuracy.

In this work, we argue that evaluating solvers against human labeled derivation is important. Existing evaluation metrics, like solution accuracy are often quite generous — for example, an incorrect equation system, such as,

\[ \{m + 5 = n + 15, \ m + n = 15 + 5\} \tag{1} \]

can generate the correct solution of the word problem in Figure 1. While equation accuracy appears to be a stricter metric than solution accuracy, our experiments show that the approximation can mislead evaluation, by assigning higher scores to an inferior solver. Indeed, a correct equation system, \((5m = 15n, 5m + 5n = 100)\), can be generated by using a wrong template, \(Am = Bn, km + kn = C\), and aligning numbers in the text to coefficients incorrectly. We show that without knowing the correct derivation at evaluation time, a solver can be awarded for the wrong reasons.

The lack of annotated derivations for word problems and no clear definition for comparing derivations present technical difficulties in using derivation for evaluation. In this paper, we address these difficulties and for the first time propose to evaluate the solvers using derivation accuracy. To summarize, the contributions of this paper are:

- We point out that evaluating using derivations is more precise compared to existing metrics. Moreover, contrary to popular belief, there is a meaningful gap between the derivation accuracy and existing metrics, as it can discover crucial errors not captured previously.
- We formally define when two derivations are equivalent, and develop an algorithm that can determine the same. The algorithm is simple to implement, and can accurately detect the equivalence even if two derivations have very different syntactic forms.
- We annotated over 2300 word algebra problems with detailed derivation annotations, providing high quality labeled semantic parses for evaluating word problems.

## 2 Evaluating Derivations

We describe our notation and revisit the notion of derivation introduced in (Kushman et al., 2014). We then formalize the notion of derivation equivalence and provide an algorithm to determine it.

### Structure of Derivation

The word problem in Table 1 shows our notation, where our proposed annotations are shown in **bold**. Equivalent textual numbers, described in **equivTNum**, are distinguished with subscripts.

| Word Problem | \(x\) | We are mixing a solution of 32\% sodium and another solution of 12\% sodium. How many liters of 32\% and 12\% solution will produce 50 liters of a 20\% sodium solution? |
| Textual Numbers | \(Q(x)\) | \(\{32, 12, 32, 12, 50, 20\} \) |
| Equation System | \(y\) | \(32m + 12n = 20 \times 50, m + n = 50\) |
| Solution | | \(m = 20, n = 30\) |
| Template | \(T\) | \(Am + Bn = C \times D, m + n = C\) |
| Coefficients | \(C(T)\) | \(A, B, C, D\) |
| Alignments | \(A\) | \{32, 12 \rightarrow A, 12, \rightarrow B, 50 \rightarrow C, 20 \rightarrow D\} |
| equivTNum | \(D\) | \{132, 322, 121, 121\} |
| Derivation | | \((T, A)\) |

Table 1: The symbols we used in the paper. Our proposed annotations are shown in **bold**. Equivalent textual numbers, described in equivTNum, are distinguished with subscripts.

Note that an approximation of the derivation is necessary, as there is no annotated derivation. From the brief description in their paper and the code released by Kushman et al. (2014), we found that their implementation assumes that the first derivation that matches the equations and generates the correct solution is the correct reference derivation against which predicted derivations are then evaluated.

\[^{2}\text{available at https://aka.ms/datadraw}^\]
where a tuple \((q, c)\) indicates that the number \(q\) is not relevant to the final equation system.

Note that there may be multiple semantically equivalent textual numbers. e.g., in Figure 1 either of the 32 can be aligned to coefficient slot \(\lambda\) in the template. These equivalent textual numbers are marked in the EquivTNum field in the annotation. If two textual numbers \(q, q' \in\) EquivTNum, then we can align a coefficient slot to either \(q\) or \(q'\), and generate a equivalent alignment.

An alignment \(A\) and a template \(T\) together identify a derivation \(z = (T, A)\) of an equation system. Note that there may be multiple valid derivations, using one of the equivalent alignments. We assume there exists a routine Solve\(y\) that find the solution of an equation system. We use a Gaussian elimination solver for our Solve routine. We use hand-written rules and the quantity normalizer in Stanford CoreNLP (Manning et al., 2014) to identify textual numbers.

**Derivation Equivalence** We define two derivations \((T_1, A_1)\) and \((T_2, A_2)\) to be equivalent if the corresponding templates \(T_1, T_2\) and alignments \(A_1, A_2\) are equivalent.

Intuitively, two templates \(T_1, T_2\) are equivalent if they can generate the same space of equation systems — i.e., for every assignment of values to slots of \(T_1\), there exists an assignment of values to slots of \(T_2\) such that they generate the same equation systems. For instance, template \(2\) and \(3\) below are equivalent

\[
m = A + B n \quad m = C - n \quad (2)
\]

\[
m + n = A \quad m - C n = B. \quad (3)
\]

because after renaming \((A, B, C)\) to \((B, C, A)\) respectively in template \(2\), and algebraic manipulations, it is identical to template \(3\). We can see that any assignment of values to corresponding slots will result in the same equation system.

Similarly, two alignments \(A_1\) and \(A_2\) are equivalent if corresponding slots from each template align to the same textual number. For the above example, the alignment \(\{1 \rightarrow A, 3 \rightarrow B, 4 \rightarrow C\}\) in template \(2\), and alignment \(\{1 \rightarrow B, 3 \rightarrow C, 4 \rightarrow A\}\) in template \(3\) are equivalent. Note that the alignment \(\{1 \rightarrow A, 3 \rightarrow B, 4 \rightarrow C\}\) for \(2\) is not equivalent to \(\{1 \rightarrow A, 3 \rightarrow B, 4 \rightarrow C\}\) in \(3\), because it does not respect variable renaming. Our definition also allows two alignments to be equivalent, if they use textual numbers in equivalent positions for corresponding slots (as described by EquivTNum field).

In the following, we carefully explain how template and alignment equivalence are determined algorithmically. Algorithm 1 shows the complete algorithm for comparing two derivations.

**Template Equivalence** We propose an approximate procedure templequiv(2) that detects equivalence between two templates. The procedure relies on the fact that under appropriate renaming of coefficients, two equivalent templates will generate equations which have the same solutions, for all possible coefficient assignments.

For two templates \(T_1\) and \(T_2\) with the same number of coefficients \(|C(T_1)| = |C(T_2)|\), we represent a choice of renaming coefficients by \(\gamma\), a

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**Algorithm 1 Evaluating Derivation**

**Input:** Predicted \((T_p, A_p)\) and gold \((T_g, A_g)\) derivation

**Output:** 1 if predicted derivation is correct, 0 otherwise

1: if \(|C(T_p)| \neq |C(T_g)|\) then → different # of coeff. slots
2: return 0
3: end if
4: \(\Gamma \leftarrow \text{TEMPL(EQUIV(T_p, T_g))}\)
5: if \(\Gamma = \emptyset\) then → not equivalent templates
6: return 0
7: end if
8: if ALIGN(EQUIV(\(\Gamma, A_p, A_g\))) then → Check alignments
9: return 1
10: end if
11: return 0
12: end if
13: procedure TEMPL(EQUIV(T1, T2))
14: \(\Gamma \leftarrow \emptyset\)
15: for each 1-to-1 mapping \(\gamma : C(T_1) \rightarrow C(T_2)\) do
16: match \(\Gamma \leftarrow \Gamma \cup \{\gamma\}\)
17: for \(t = 1 \cdots R\) do\(\Gamma \leftarrow \Gamma \cup \{\gamma\}\) R : Rounds
18: Generate random vector \(v\)
19: \(A_1 \leftarrow \{(v_i \rightarrow c_i)\}, A_2 \leftarrow \{(v_i \rightarrow \gamma(c_i))\}\)
20: if \(\text{Solve}(T_1, A_1) \neq \text{Solve}(T_2, A_2)\) then
21: \(\Gamma \leftarrow \emptyset\); break
22: break
23: if \(\gamma\) is equivalent then
24: \(\Gamma \leftarrow \emptyset\)
25: if true then \(\Gamma \leftarrow \Gamma \cup \{\gamma\}\)
26: return \(\Gamma\)
27: end if
28: end if
29: procedure ALIGNED(EQUIV(\(\Gamma, A_1, A_2\)))
30: for mapping \(\gamma \in \Gamma\) do
31: \(\Gamma \leftarrow \Gamma \cup \{\gamma\}\) if following holds true,
32: \(\{(q, c) \in A_1 \iff \{(q, \gamma(c))\) \{\gamma', c\} \in A_2\}
33: \(\text{where} \{(q', q) \in \text{EquivTNum}\}
34: \text{then return 1}\)
35: \(\text{return 0}\)
36: end if
37: end for
38: end procedure
Alignment Equivalence The TEMPLEQUIV procedure returns every mapping $\gamma$ in $\Gamma$ under which the templates were equivalent (line 16). Recall that $\gamma$ identifies corresponding slots, $c$ and $\gamma(c)$, in $T_1$ and $T_2$ respectively. We describe alignment equivalence using these mappings.

Two alignments $A_1$ and $A_2$ are equivalent if corresponding slots (according to $\gamma$) align to the same textual number. More formally, if we find a mapping $\gamma$ such that for each tuple $(q, c)$ in $A_1$ there is $(q, \gamma(c))$ in $A_2$, then the alignments are equivalent (line 33). We allow for equivalent textual numbers (as identified by EquivTNum field) to match when comparing tuples in alignments.

The proof of correctness of Algorithm 1 is sketched in the appendix. Using Algorithm 1 we can define derivation accuracy, to be 1 if the predicted derivation $(T_p, A_p)$ and the reference derivation $(T_g, A_g)$ are equivalent, and 0 otherwise.

Properties of Derivation Accuracy By comparing derivations, we can ensure that the following errors are detected by the evaluation.

Firstly, correct solutions found using incorrect equations will be penalized, as the template used will not be equivalent to reference template. Secondly, correct equation system obtained by an incorrect template will also be penalized for the same reason. Lastly, if the solver uses the correct template to get the correct equation system, but aligns the wrong number to a slot, the alignment will not be equivalent to the reference alignment, and the solver will be penalized too.

We will see some illustrative examples of above errors in [3]. Note that the currently popular evaluation metric of solution accuracy will not detect any of these error types.

3 Annotating Derivations

As none of the existing benchmarks contain derivation annotations, we decided to augment existing datasets with these annotations. We also annotated DRAW-1K, a new dataset of 1000 general algebra word problems to make our study more comprehensive. Below, we describe how we reduced annotation effort by semi-automatic generated some annotations.

Annotating gold derivations from scratch for all problems is time consuming. However, not all word problems require manual annotation – sometimes all numbers appearing in the equation system can be uniquely aligned to a textual number without ambiguity. For such problems, the annotations are generated automatically. We identify word problems which have at least one alignment ambiguity – multiple textual numbers with the same value, which appears in the equation system. A example of such a problem is shown in Figure 1, where there are three textual numbers with value 5, which appears in the equation system. Statistics for the number of word problems with such ambiguity is shown in Table 2.

We only ask annotators to resolve such alignment ambiguities, instead of annotating the entire derivation. If more than one alignments are genuinely correct (as in word problem of Table 1), we ask the annotators to mark both (using the EquivTNum field). This ensures our derivation annotations are exhaustive – all correct derivations are marked. With the correct alignment annotations, templates for all problems can be easily induced.

Annotation Effort To estimate the effort required to annotate derivations, we timed our annotators when annotating 50 word problems (all involved alignment ambiguities). As a control, we also asked annotators to annotate the entire derivation from scratch (i.e., only provided with the word problem and equations), instead of only fixing alignment ambiguities. When annotating from scratch, annotators took an average of 4 minute per word problem, while when fixing alignment ambiguities this time dropped to average of 1 minute.
Table 2: Statistics of the datasets. At least 20% of problems in each dataset had alignment ambiguities that required human annotations. The number of templates before and after annotation is also shown (reduction > 20%).

Table: Reconciling Equivalent Templates

The number of templates has been used as a measure of dataset diversity (Shi et al., 2015; Huang et al., 2016), however prior work did not reconcile the equivalent templates in the dataset. Indeed, if two templates are equivalent, we can replace one with the other and still generate the correct equations. Therefore, after getting human judgements on alignments, we reconcile all the templates using TEMPL_EQUIV as the final step of annotation.

TEMPLEQV is quite effective (despite being approximate), reducing the number of templates by at least 20% for all datasets (Table 2). We did not find any false positives generated by the TEMPL_EQUIV in our manual examination. The reduction in Table 2 clearly indicates that equivalent templates are quite common in all datasets, and number of templates (and hence, dataset diversity) can be significantly overestimated without proper reconciliation.

4 Experimental Setup

We describe the three datasets used in our experiments. Statistics comparing the datasets is shown in Table 2. In total, our experiments involve over 2300 word problems.

DOLPHIN-L

DOLPHIN-L is the linear-T2 subset of the DOLPHIN dataset (Shi et al., 2015), which focuses on number word problems – algebra word problems which describe mathematical relationships directly in the text. All word problems in the linear-T2 subset of the DOLPHIN dataset can be solved using linear equations.

DRA-1K

Diverse Algebra Word (DRA-1K), consists of 1000 word problems crawled from algebra.com. Details on the dataset creation can be found in the appendix. As ALG-514 was also crawled from algebra.com, we ensured that there is little overlap between the datasets.

We randomly split DRA-1K into train, development and test splits with 600, 200, 200 problems respectively. We use 5-fold cross validation splits provided by the authors for DOLPHIN-L and ALG-514.

4.1 Evaluation

We compare derivation accuracy against the following evaluation metrics.

Solution Accuracy

We compute solution accuracy by checking if each number in the reference solution appears in the generated solution (disregarding order), following previous work (Kushman et al., 2014; Shi et al., 2015).

Equation Accuracy

An approximation of derivation accuracy that is similar to the one used in Kushman et al. (2014). We approximate the reference derivation \( \tilde{z} \) by randomly chosen from the (several possible) derivations which lead to the gold \( y \) from \( x \). Derivation accuracy is computed against this (possibly incorrect) reference derivation. Note that in equation accuracy, the approximation is used instead of annotated derivation. We include the metric of equation accuracy in our evaluations to show that human annotated derivation is necessary, as approximation made by equation accuracy might be problematic.

4.2 Our Solver

We train a solver using a simple modeling approach inspired by Kushman et al. (2014) and Zhou et al. (2015). The solver operates as follows. Given a word problem, the solver ranks all templates seen during training, \( \Gamma_{train} \), and selects the set of the top-\( k \) (we use \( k = 10 \)) templates \( \Pi \subset \Gamma_{train} \). Next, all possible derivations \( D(\Pi) \) that use a template from \( \Pi \) are generated.

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6These were adjudicated on by the first author.
Table 3: TE and TD compared using different evaluation metrics. Note that while TD is clearly superior to TE due to extra supervision using the annotations, only derivation accuracy is able to correctly reflect the differences.

| Setting | Soln. Acc. | Eqn. Acc. | Deriv. Acc. |
|---------|------------|-----------|-------------|
| **ALG-514** |            |           |             |
| TE      | 76.2       | 72.7      | 75.5        |
| TD      | 78.4       | 73.9      | 77.8        |
| TD - TE | 2.2        | 1.2       | **2.3**     |
| **DRAW-1k** |            |           |             |
| TE      | 52.0       | 48.0      | 48.0        |
| TD      | 55.0       | 48.0      | 53.0        |
| TD - TE | 3.0        | 0         | **5.0**     |
| **DOLPHIN** |            |           |             |
| TE      | 55.1       | 50.1      | 44.2        |
| TD      | 57.5       | 38.8      | 54.9        |
| TD - TE | 2.4        | -13.3     | **10.7**    |

Table 4: When combining two datasets, it is essential to reconcile templates across datasets. Here TE* denotes training on equations after reconciling the templates, while TE simply combines datasets naively. As TE* represents a more appropriate setting, we compare TE* and TD in this experiment.

| Setting | Soln. Acc. | Eqn. Acc. | Deriv. Acc. |
|---------|------------|-----------|-------------|
| **DRAW-1k + Alg-514** |            |           |             |
| TE      | 32.5       | 31.5      | 29.5        |
| TE*     | 60.5       | 56.0      | 54.0        |
| TD      | 62.0       | 53.0      | 59.5        |
| TD - TE* | 1.5       | -3.0      | **5.5**     |
| **DRAW-1k + Dolphin** |            |           |             |
| TE      | 41.0       | 37.5      | 37.5        |
| TE*     | 58.5       | 55.5      | 51.5        |
| TD      | 60.0       | 53.0      | 58.0        |
| TD - TE* | 1.5       | -2.5      | **6.5**     |

5 Experiments

Are solution and equation accuracy equally capable as derivation accuracy at distinguishing between good and bad models? To answer this question, we train the solver under two settings such that one of the settings has clear advantage over the other, and see if the evaluation metrics reflect this advantage. The two settings are,

**TE (Train on Equation)** Only the \((x, y)\) pairs are provided as supervision. Similar to (Kushman et al., 2014; Zhou et al., 2015), the solver finds a derivation which agrees with the equation system and the solution, and trains on it. Note that the derivation found by the solver may be incorrect.

**TD (Train on Derivation)** \((x, z)\) pairs obtained by the derivation annotation are used as supervision. This setting trains the solver on human-labeled derivations. Clearly, the TD setting is a more informative supervision strategy than the TE setting. TD provides the correct template and correct alignment (i.e. labeled derivation) as supervision and is expected to perform better than TE, which only provides the question-equation pair.

We first present the main results comparing different evaluation metrics on solvers trained using the two settings.

5.1 Main Results

We compare the evaluation metrics in Table 3. We want to determine to what degree each evaluation metric reflects the superiority of TD over TE.

We note that solution accuracy always exceeds derivation accuracy, as a solver can sometimes get the right solutions even with the wrong derivation. Also, solution accuracy is not as sensitive as derivation accuracy to improvements in the solver. For instance, solution accuracy only changes by 2.4 on Dolphin-L when comparing TE and TD, whereas derivation accuracy changes...
by 10.7 points. We found that the large gap on Dolphin-L was due to several alignment errors in the predicted derivations, which were detected by derivation accuracy. Recall that over 35% of the problems in Dolphin-L have alignment ambiguities (Table 2). In the TD setting, many of these errors made by our solver were corrected as the gold alignment was part of supervision.

Equation accuracy too has several limitations. For DRAW-1K, it cannot determine which solver is better and assigns them the same score. Furthermore, it often (incorrectly) considers TD to be a worse setting than TE, as evident from decrease in the scores (for instance, on DOLPHIN-L). Recall that equation accuracy attempts to approximate derivation accuracy by choosing a random derivation agreeing with the equations, which might be incorrect.

Study with Combining Datasets With several ongoing annotation efforts, it is a natural question to ask is whether we can leverage multiple datasets in training to generalize better. In Table 4, we combine DRAW-1K’s train split with other datasets, and test on DRAW-1K’s test split. DRAW-1K’s test split was chosen as it is the largest test split with general algebra problems (recall Dolphin-L contains only number word problems).

We found that in this setting, it was important to reconcile the templates across datasets. Indeed, when we simply combine the two datasets in the TE setting, we notice a sharp drop in performance (compared to Table 3). However, if we reconciled all templates and then used the new equations for training (called TE* setting in Table 4), we were able to see improvements from training on more data. We suspect difference in annotation style led to several equivalent templates in the combined dataset, which got resolved in TE*. Therefore, in Table 4, we compare TE* and TD settings.

In Table 4, a trend similar to Table 3 can be observed – solution accuracy assigns a small improvement to TD over TE*. Derivation accuracy clearly reflects the fact that TD is superior to TE*, with a larger improvement compared to solution accuracy (e.g., 5.5 vs 1.5). Equation accuracy, as before, considers TD to be worse than TE*.

Note that this experiment also shows that differences in annotation styles across different algebra problem datasets can lead to poor performance.

### Table 5: Comparison of our solver and other state-of-the-art systems, when trained under TE setting. All numbers are solution accuracy. See footnote for details on the comparison to SWLLR.

| Dataset      | Ours | KAZB | Best Result |
|--------------|------|------|-------------|
| ALG-514      | 76.2 | 68.7 | 79.7 (ZDC)  |
| DOLPHIN-L    | 55.1 | 37.5 | 46.4 (SWLLR) |
| DRAW-1K      | 52.0 | 43.2 | –           |

Table 5: Comparison of our solver and other state-of-the-art systems, when trained under TE setting. All numbers are solution accuracy. See footnote for details on the comparison to SWLLR.

when combining these datasets naively. Our findings suggest that derivation annotation and template reconciliation are crucial for such multi-data supervision scenarios.

### 5.2 Comparing Solvers

To ensure that the results in the previous section were not an artifact of any limitations of our solver, we show here that our solver is competitive to other state-of-the-art solvers, and therefore it is reasonable to assume that similar results can be obtained with other automatic solvers.

In Table 5, we compare our solver to KAZB, the system of Kushman et al. (2014), when trained under the existing supervision paradigm, TE (i.e., training on equations) and evaluated using solution accuracy. We also report the best scores on each dataset, using ZDC and SWLLR to denote the systems of Zhou et al. (2015) and Shi et al. (2015) respectively. Note that our system and KAZB are the only systems that can process all three datasets without significant modification, with our solver being clearly superior to KAZB.

### 5.3 Case Study

We discuss some interesting examples from the datasets, to show the limitations of existing metrics, which derivation accuracy overcomes.

Correct Solution, Incorrect Equation In the following example from the DOLPHIN-L dataset, by choosing the correct template and the wrong alignments, the solver arrived at the correct solutions, and gets rewarded by solution accuracy.

The sum of 2(q1) numbers is 25(q2). 12(q3) less than 4(q4) times one(q5) of the numbers is 16(q6) more than twice(q7) the other number.

Find the numbers.

\[ 2(q1) + 25(q2) = 12(q3) - 4(q4) \cdot 1(q5) + 16(q6) - 2(q7) \]

\[ \text{Solve for } q1, q2, q3, q4, q5, q6, q7. \]

Footnote:

- SWLLR also had a solver which achieves 68.0, using over 9000 semi-automatically generated rules tailored to number word problems. We compare to their similarity based solver instead, which does not use any such rules, given that the rule-based system cannot be applied to general word problems.
Note that there are seven textual numbers \(q_1, \ldots, q_7\) in the word problem. We can arrive at the correct equations \(\{m + n = 25, 4m - 2n = 16 + 12\}\), by the correct derivation,
\[
m + n = q_2 \quad q_4m - q_7n = q_6 + q_3.
\]
However, the solver found the following derivation, which produces the incorrect equations \(\{m + n = 25, 2m - n = 2 + 12\}\),
\[
m + n = q_2 \quad q_1m - q_5n = q_7 + q_3.
\]
Both the equations have the same solutions \((m = 13, n = 12)\), but the second derivation is clearly using incorrect reasoning.

**Correct Equation, Incorrect Alignment** In such cases, the solver gets the right equation system, but derived it using wrong alignment. Solution accuracy still rewards the solver. Consider the problem from the DOLPHIN-L dataset,

The larger of two \((q_1)\) numbers is 2\((q_2)\) more than 4\((q_3)\) times the smaller. Their sum is 67\((q_4)\).
Find the numbers.

The correct derivation for this problem is,
\[
m - q_3n = q_2 \quad m + n = q_4.
\]
However, our system generated the following derivation, which although results in the exact same equation system (and thus same solutions), is clearly incorrect due incorrect choice of "two",
\[
m - q_3n = q_1 \quad m + n = q_4.
\]
Note that derivation accuracy will penalize the solver, as the alignment is not equivalent to the reference alignment \((q_1 \text{ and } q_2 \text{ are not semantically equivalent textual numbers})\).

**Bad Approx. in Equation Accuracy** The following word problem is from the ALG-514 dataset:

Mrs. Martin bought 3\((q_1)\) cups of coffee and 2\((q_2)\) bagels and spent 12.75\((q_3)\) dollars. Mr. Martin bought 2\((q_4)\) cups of coffee and 5\((q_5)\) \(q_7\) bagels and spent 14.00\((q_6)\) dollars. Find the cost of one\((q_7)\) cup of coffee and that of one\((q_8)\) bagel.

The correct derivation is,
\[
q_1m + q_2n = q_3 \quad q_4m + q_5n = q_6.
\]
However, we found that equation accuracy used the following incorrect derivation for evaluation,
\[
q_1m + q_2n = q_3 \quad q_2m + q_5n = q_6.
\]
Note while this derivation does generate the correct equation system and solutions, the derivation utilizes the wrong numbers and misunderstood the word problem. This example demonstrates the needs to evaluate the quality of the word problem solvers using the annotated derivations.

### 6 Related Work

We discuss several aspects of previous work in the literature, and how it relates to our study.

**Existing Solvers** Current solvers for this task can be divided into two broad categories based on their inference approach --- *template-first* and *bottom-up*. Template-first approaches like Kushman et al. (2014), Zhou et al., 2015 infer the derivation \(z = (T, A)\) sequentially. They first predict the template \(T\) and then predict alignments \(A\) from textual numbers to coefficients. In contrast, bottom-up approaches (Hosseini et al., 2014; Shi et al., 2015; Koncel-Kedziorski et al., 2015) jointly infer the derivation \(z = (T, A)\). Inference proceeds by identifying parts of the template (eg. \(3n + Bm\)) and aligning numbers to it \((\{2 \rightarrow A, 3 \rightarrow B\})\). At any intermediate state during inference, we have a partial derivation, describing a fragment of the final equation system \((2m + 3n)\). While our experiments used a solver employing the template-first approach, it is evident that performing inference in all such solvers requires constructing a derivation \(z = (T, A)\). Therefore, annotated derivations will be useful for evaluating all such solvers, and may also aid in debugging errors.

Other reconciliation procedures are also discussed (though briefly) in earlier work. Kushman et al. (2014) reconciled templates by using a symbolic solver and removing pairs with the same canonicalized form. Zhou et al. (2015) also reconciled templates, but do not describe how it was performed. We showed that reconciliation is important for correct evaluation, for reporting dataset complexity, and also when combining multiple datasets.
Labeling Semantic Parses  Similar to our work, efforts have been made to annotate semantic parses for other tasks, although primarily for providing supervision. Prior to the works of Liang et al. (2009) and Clarke et al. (2010), semantic parsers were trained using annotated logical forms (Zelle and Mooney, 1996; Zettlemoyer and Collins, 2005; Wong and Mooney, 2007, inter alia), which were expensive to annotate. Recently, Yih et al. (2016) showed that labeled semantic parses for the knowledge based question answering task can be obtained at a cost comparable to obtaining answers. They showed significant improvements in performance of a question-answering system using the labeled parses instead of answers for training. More recently, by treating word problems as a semantic parsing task, Upadhyay et al. (2016) found that joint learning using both explicit (derivation as labeled semantic parses) and implicit supervision signals (solution as responses) can significantly outperform models trained using only one type of supervision signal.

Other Semantic Parsing Tasks  We demonstrated that response-based evaluation, which is quite popular for most semantic parsing problems (Zelle and Mooney, 1996; Berant et al., 2013; Liang et al., 2011, inter alia) can overlook reasoning errors for algebra problems. A reason for this is that in algebra word problems there can be several semantic parses (i.e., derivations, both correct and incorrect) that can lead to the correct solution using the input (i.e., textual number in word problem). This is not the case for semantic parsing problems like knowledge based question answering, as correct semantic parse can often be identified given the question and the answer. For instance, paths in the knowledge base (KB), that connect the answer and the entities in the question can be interpreted as legitimate semantic parses. The KB therefore acts as a constraint which helps prune out possible semantic parses, given only the problem and the answer. However, such KB-based constraints are unavailable for algebra word problems.

7 Conclusion and Discussion

We proposed an algorithm for evaluating derivations for word problems. We also showed how derivation annotations can be easily obtained by only involving annotators for ambiguous cases. We augmented several existing benchmarks with derivation annotations to facilitate future comparisons. Our experiments with multiple datasets also provided insights into the right approach to combine datasets – a natural step in future work. Our main finding indicates that derivation accuracy leads to a more accurate assessment of algebra word problem solvers, finding errors which other metrics overlook. While we should strive to build such solvers using as little supervision as possible for training, having high quality annotated data is essential for correct evaluation.

The value of such annotations for evaluation becomes more immediate for online education scenarios, where such word solvers are likely to be used. Indeed, in these cases, merely arriving at the correct solution, by using incorrect reasoning may prove detrimental for teaching purposes. We believe derivation based evaluation closely mirrors how humans are evaluated in schools (by forcing solvers to show “their work”).

Our datasets with the derivation annotations have applications beyond accurate evaluation. For instance, certain solvers, like the one in (Roy and Roth, 2015), train a relevance classifier to identify which textual numbers are relevant to solving the word problem. As we only annotate relevant numbers in our annotations, our datasets can provide high quality supervision for such classifiers. The datasets can also be used in evaluation test-beds, like the one proposed in (Koncel-Kedziorski et al., 2016).

We hope our datasets will open new possibilities for the community to simulate new ideas and applications for automatic problem solvers.

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A Creating DRAW-1K

We crawl over 100k problems from [algebra.com](http://algebra.com). The 100k word problems include some problems which require solving non-linear equations (e.g., finding roots of quadratic equations). We filter out these problems using keyword matching. We also filter problems whose explanation do not contain a variable named “x”. This leaves us with 12k word problems.

Extracting Equations A word problem on [algebra.com](http://algebra.com) is accompanied by a detailed explanation provided by instructors. In our crawler, we use simple pattern matching rules to extract all the equations in the explanation. The problems often have sentences which are irrelevant to solving the word problem (e.g., “Please help me, I am stuck.”). During cleaning, the annotator removes such sentences from the final word problem and performs some minor editing if necessary.

1000 problems were randomly chosen from these pool of 12k problems, which were then shown to annotators as described earlier to get the derivation annotations.

B Proof of Correctness (Sketch)

For simplicity, we will assume that EquivTNum is empty. The proof can easily be extended to handle the more general situation.

Lemma 1. The procedure TEMPLEQIV returns $\Gamma \neq \emptyset$ iff templates $T_1$, $T_2$ are equivalent (w.h.p.).

Proof First we prove that with high probability we are correct in claiming that a $\gamma$ found by the algorithm leads to equivalence. Let probability of getting the same solution even when the template are not equivalent be $\epsilon(T_1, T_2, \gamma) < 1$. The probability that solution is same for R rounds for $T_1, T_2$ which are not equivalent is $\leq \epsilon^R$, which can be made arbitrarily small by choosing large R. Therefore, with a large enough R, obtaining $\Gamma \neq \emptyset$ from TEMPLEQIV implies there is a $\gamma$ under which templates generate equations with the same solution, and by definition, are equivalent.

Conversely, if templates are equivalent, it implies $\exists \gamma^*$ such that under that mapping for any assignment, the generated equations have the same solution. As we iterate over all possible 1-1 mappings $\gamma$ between the two templates, we will find $\gamma^*$ eventually.

Proposition Algorithm returning 1 implies derivations $(T_p, A_p)$ and $(T_g, A_g)$ are equivalent.

Proof Algorithm returns 1 only if TEMPLEQIV found a $\Gamma \neq \emptyset$, and $\exists \gamma \in \Gamma$, following holds

$$(q, c) \in A_g \iff (q, \gamma(c)) \in A_p$$

i.e., the corresponding slots aligned to the same textual number. TEMPLEQIV found a $\Gamma \neq \emptyset$ implies templates are equivalent (w.h.p). Therefore, $\exists \gamma \in \Gamma$ such that the corresponding slots aligned to the same textual number implies the alignments are equivalent under mapping $\gamma$. Together they imply that the derivation was equivalent (w.h.p.).