GRAIN ALIGNMENT INDUCED BY RADIATIVE TORQUES: EFFECTS OF INTERNAL RELAXATION OF ENERGY AND COMPLEX RADIATION FIELD

THEM HOANG AND A. LAZARIAN

Department of Astronomy, University of Wisconsin, Madison, WI 53706, USA; hoang@astro.wisc.edu, lazarian@astro.wisc.edu

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ABSTRACT

Earlier studies of grain alignment dealt mostly with interstellar grains that have strong internal relaxation of energy which aligns the grain axis of maximum moment of inertia (the axis of major inertia) with respect to the grain’s angular momentum. In this paper, we study the alignment by radiative torques for large irregular grains, e.g., grains in accretion disks, for which internal relaxation is subdominant. We use both numerical calculations and the analytical model of a helical grain introduced by us earlier. We demonstrate that grains in such a regime exhibit more complex dynamics. In particular, if initially the grain axis of major inertia makes a small angle with angular momentum, then radiative torques can align the grain axis of major inertia with angular momentum, and both the axis of major inertia and angular momentum are aligned with the magnetic field when attractors with high angular momentum (high-\(J\) attractors) are available. For alignment without high-\(J\) attractors, beside the earlier studied attractors with low angular momentum (low-\(J\) attractors), there appear new low-\(J\) attractors. In addition, we also study the alignment of grains in the presence of strong internal relaxation, but induced not by a radiation beam as in earlier studies but instead induced by a complex radiation field that can be decomposed into dipole and quadrupole components. We found that in this situation the parameter space \(q_{\text{max}}^\text{max}\), for which high-\(J\) attractors exist in trajectory maps, is more extended, resulting in the higher degree of polarization expected. Our results are useful for modeling polarization arising from aligned dust grains in molecular clouds.

Key words: dust, extinction – ISM: magnetic fields – polarization

Online-only material: color figures

1. INTRODUCTION

Magnetic fields play a crucial role in many astrophysical processes (e.g., star formation, accretion disks, cosmic-ray transport). An important and easily available way to map magnetic fields is to observe the polarization of emission or absorption by aligned grains (Goodman et al. 1995; Hildebrand et al. 2000; Hildebrand 2002; Crutcher et al. 2004). The reliable interpretation of polarimetry in terms of magnetic fields requires, however, a solid understanding of the alignment of grains. This is the major motivation of work in the field of grain alignment. An additional motivation to understand when grains get aligned comes from the necessity of separating the polarization by dust grains from the polarized cosmic microwave background (CMB) signal (see Lazarian 2008).

Grain alignment is a complex area of study related to many physical effects of different timescales (see the review by Lazarian 2007). It is also the research area associated with the names of Lyman Spitzer and Ed Purcell, who decisively contributed to our understanding of a number of fundamental physical processes of grain alignment.

A number of mechanisms have been known to induce grain alignment (see Lazarian 2007). However, recently grain helicity has been discussed as the necessary element of successful alignment. Note that the idea of grains having twist and therefore interacting differently with left and right polarized photons can be traced back to the pioneering work by Dolginov & Mytrophanov (1976). A more recent work in the area showed that a similar effect is present when irregular grains interact with a gaseous flow (Lazarian & Hoang 2007b).

In this paper, we concentrate on studying grain alignment by radiative torques (hereafter RATs). However, the intrinsic similarity of the radiative and mechanical torques acting on helical grains (see the discussion in Lazarian & Hoang 2007b) makes our present results applicable to the alignment by mechanical torques. Note that for the alignment of helical grains the magnetic field acts only through inducing the Larmor precession. Therefore the alignment may potentially occur with both longer and shorter grain axes perpendicular to the magnetic field. Observations of interstellar dust indicate that alignment with the long axes perpendicular to the magnetic field is dominant in the diffuse gas. Therefore we shall call this alignment right alignment, and the opposite type of alignment we shall call wrong alignment. Different alignment processes produce either right or wrong alignment and the goal of grain alignment theory is to predict correctly both the direction and the degree of the expected alignment.

As a dust grain scatters or absorbs photons, it experiences RATs. These torques can be stochastic or regular. Stochastic RATs arise from, for instance, a spheroidal grain randomly emitting or absorbing photons. The latter process, for instance, was invoked by Harwit (1970) in his model of grain alignment based on grains being preferentially spun up in the direction perpendicular to the photon beam. However, Purcell & Spitzer (1971) showed that the randomization arising from the same grain emitting thermal photons makes the achievable degree of grain alignment negligible. In fact, they showed that stochastic RATs arising from grain thermal emission is an important process of grain randomization. More recently, grain thermal emission was analyzed as a source of the excitation of grain rotation as well as its damping in relation to the rotation of tiny spinning grains that are likely to be responsible for the so-called anomalous foreground emission (Draine & Lazarian 1998).

\footnote{Such flows arise naturally in the reference of a grain as the grain interacts with ubiquitous MHD turbulence (Lazarian & Yan 2002; Yan & Lazarian 2003; Yan et al. 2004).}
RATs were first discussed by Dolginov (1972) in terms of chiral, e.g., hypothetical, quartz grains. The idea is that starlight passing through such grains would spin them up. Later, Dolginov & Mytrophanov (1976) considered an irregular grain model of two spheroids twisted to each other, made of more accepted materials, e.g., silicate grains, and claimed that these grains will be both spun up and aligned by regular RATs. This work was, unfortunately, mostly ignored for 20 years.

Grain alignment by RATs drew much attention when Bruce Draine modified the Discrete Dipole Approximation Code (Draine & Flatau 1994, hereafter DDSCAT) and enabled later researchers to calculate RATs for irregular grains.2

Draine & Weingartner (1996, hereafter DW96) treated RATs as a kind of pinwheel torque, and found that RATs can accelerate grains to suprathermal rotations. Later, Draine & Weingartner (1997, hereafter DW97) introduced many essential elements of the modern treatment of RATs, for instance phase trajectory maps. They confirmed the Dolginov & Mytrophanov (1976) hypothesis that RATs can produce alignment by themselves. However, the important moments of grain dynamics, so-called crossovers, were not treated correctly in this paper. Crossovers are special events in the dynamics of grains rotating subject to regular torques. During crossovers, grain angular velocity approaches its minimal value and models of crossovers caused by pinwheel torques in Spitzer & McGlynn (1979) and in Lazarian & Draine (1997) envisaged grain flipping with the direction of the angular momentum staying the same. DW97, however, assumed that RAT crossovers are different and during RAT-induced crossovers angular momentum changes its direction to the opposite. This treatment of crossovers resulted in grains having cyclic trajectories, which were, in fact, an artifact of the adopted treatment, as was shown later (Lazarian & Hoang 2007a, henceforth LH07a). In fact, LH07a showed that instead of cyclic trajectories one gets situations when RATs tend to slow grains down.

A more general analysis for the dynamics of RAT alignment was done in Weingartner & Draine (2003, henceforth WD03), where thermal fluctuations within grains were accounted for. These fluctuations were shown by Lazarian (1994, see also Lazarian & Roberge 1997) to induce grain thermal wobbling, in which the amplitude gets larger as the grain angular momentum approaches its thermal value. For their choice of parameters, WD03 found that a low-J attractor appears on the trajectory phase maps.3 The study, however, was limited in terms of parameter space explored. In fact, it was a numerical study of alignment of a single grain subjected to an electromagnetic wave of a single frequency at a single incident direction. Therefore, implications of the study for the problem of RAT alignment were difficult to evaluate.

We feel that the numerical work above done with DDSCAT should be treated as an experimental attack on the problem of RAT alignment. The limitations of such an approach are self-evident. For instance, to obtain predictions one should analyze many hundreds of different grain shapes, many wavelengths, and many incident directions. The study in DW97 included three shapes and the resulting torques looked very different, leaving one to wonder what causes the alignment.4 At the same time, unfortunately, the analytical study in Dolginov & Mytrophanov (1976) was in error.

The first analytical model (henceforth AMO) of RATs consistent with the numerical calculations was proposed in LH07a. This study provided simple analytical expressions for the RATs (see also Section 2). In particular, the functional forms of the torques acting on grains of different shapes are similar, with the difference between the grains of different shapes amounting to a single parameter, termed the $q^{\text{max}}$ ratio, which is the ratio of the magnitudes of the first two components of torques in the grain symmetry system of reference where the radiation direction is the symmetry axis. AMO allowed theoretical predictions and an analytical treatment of the RAT alignment. As a result, studies of a parameter space proved to be an insurmountable problem. In LH07a, similar to DW97, the thermal fluctuations were intentionally ignored and in such a setup the low-J attractors, rather than cyclic trajectories, were reported. In fact, it became clear that RATs, over a large part of the parameter space, spin down rather than spin up grains. Using AMO, LH07a obtained values of $q^{\text{max}}$ for which grains have only low-J attractors and when they have both low-J and high-J attractors simultaneously.

AMO was extensively used in all our papers that followed. For instance, in Hoang & Lazarian (2008, henceforth HL08), we studied the RAT alignment in the presence of thermal fluctuations using both AMO and DDSCAT for an extended sample of grain shapes, radiation directions, and wavelengths. We found that irregular grains do not stop completely, but rotate at a rate comparable with the rate of thermal rotation at the dust temperature, which agrees with an earlier example in WD03. Importantly, HL08 considered the RAT alignment in the presence of gaseous bombardment and reported a new effect, namely the transfer between the low-J and high-J attractors, in situations when the both low-J and high-J attractors were present simultaneously.5 Thus, counterintuitively, the collisions were found to increase the alignment.

The most important practical implication of the LH07a and HL08 studies was that, over a large range of the parameter space of the $q^{\text{max}}$ ratio and angles between the magnetic field and radiation direction, the grains rotate thermally and therefore, as a result the aforementioned thermal fluctuations of grains exhibit reduced alignment of grain axes with respect to J. This opened possibilities of estimating the expected alignment and comparing it to the alignment inferred from observations. Importantly enough, Lazarian & Hoang (2008, henceforth LH08) reported that in the presence of superparamagnetic inclusions (Jones & Spitzer 1967; Mathis 1986; Bradley 1994; Martin 1995; Goodman & Whittet 1995) the high-J attractors always exist. This finding results in a very nontrivial effect, namely that, in the presence of both superparamagnetic inclusions, grains that otherwise would rotate subthermally get into the state of fast suprathermal rotation. According to HL08, this also means that all grains eventually get into the state of fast rotation and perfect alignment. From the point of view of observations, this opens prospects of testing the existence of superparamagnetic inclusions by measuring the degree of polarization.

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2 The only work on RATs before that was Lazarian (1995) which corrected some of the points in the Dolginov & Mytrophanov (1976) study, but underestimated the importance of RATs.

3 Conventionally, an attractor with angular momentum $J$ larger than the thermal value is called a high-J attractor, and an attractor with $J$ of the order of the thermal value is called a low-J attractor.

4 In relation to this, one of us (A.L.) recalls that Lyman Spitzer, after studying the work on RATs, suggested that one should seek for some simple trigonometric fits to torques that would produce the alignment.

5 Note that the parameter spaces for the existence of low-J and high-J attractors are similar and known through the LH07a study.
This paper continues our studies in LH07a and HL08. The major issues we address below are (1) the effect of internal relaxation, and (2) the effect of a complex radiation field on the grain alignment.

The first issue is important in terms of the alignment of large grains, e.g., grains within protoplanetary accretion disks. The alignment of such grains was assumed in a recent modeling of polarization from T Tauri accretion disks in Cho & Lazarian (2005), Pelkonen et al. (2007), and Bethell et al. (2007). All these studies assumed rather naive models of alignment. In particular, the actual modeling would require taking into account the actual complex radiation field that is being experienced by the grain, rather than assuming that the radiation is coming from a point source.

In what follows, in Section 2 we briefly describe the AMO, grain torque-free motion, and calculations of RATs. In Section 3, we compare the Barnett and nuclear relaxation times with the radiative alignment time and identify grain size when the internal relaxation can be ignored. In Section 4, we study the properties of RATs and the resulting RAT alignment for large grains in the absence of internal relaxation. Grain alignment by RATs arising from dipole and quadrupole components of radiation fields is studied in Section 5. We discuss our findings in Section 6, and our summary is presented in Section 7.

2. RADIATIVE TORQUES

2.1. Definition of Radiative Torques

Considering only the anisotropic component of the radiation field, the radiative torque resulting from the interaction of a radiation beam with a grain is then defined by

$$\Gamma_{\text{rad}} = \frac{u_{\text{rad}}a^2 \lambda}{2} \gamma Q_r(\Theta, \beta, \Phi),$$

(1)

where $a$ is the grain size, $\gamma$ is the degree of anisotropy, $\lambda$ and $u_{\text{rad}}$ are the mean wavelength and total energy density of the radiation field, respectively; $Q_r$ is the RAT efficiency vector depending on angles $\Theta$, $\beta$, and $\Phi$, and its overline denotes averaging over the spectrum of the radiation field (DW96); $\Theta$ is the angle between the axis of maximum moment of inertia (hereafter, the axis of major inertia) $a_1$ and the radiation direction $k$, $\beta$ is the rotation angle about $a_1$, and $\Phi$ is the precession angle of $a_1$ about $k$ (see Figure 1, left panel). In the lab coordinate system $\hat{e}_1\hat{e}_2\hat{e}_3$, $Q_r$ is decomposed as

$$Q_r(\Theta, \beta, \Phi) = Q_{e_1}(\Theta, \beta, \Phi)\hat{e}_1 + Q_{e_2}(\Theta, \beta, \Phi)\hat{e}_2 + Q_{e_3}(\Theta, \beta, \Phi)\hat{e}_3.$$  

(2)

2.2. Analytical Model of RATs

LH07a proposed an analytical model of RATs arising from the interaction of a helical grain which consisted of a mirror attached to an ellipsoid body with a radiation beam. Although this model was derived in the geometric optics approximation, LH07a found that whether the RAT alignment has high-$J$ or low-$J$ attractors depends on the ratio of torque components $q_{\text{max}}^\text{max} = Q_{e_1}/Q_{e_2}^\text{max}$ (see Appendix A). Therefore this parameter was used to characterize the RAT alignment in LH07a. For

Figure 1. Left panel: orientation of a grain, described by three principal axes $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$, in the lab coordinate system $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$, is defined by three angles $\Theta$, $\beta$, and $\Phi$. The direction of the incident radiation beam $k$ is along $\hat{e}_1$. Right panel: the orientation of angular momentum $\mathbf{J}$ in the lab coordinate system; $\psi$ is the angle between the magnetic field $\mathbf{B}$ and $k$, $\xi$ is the angle between $\mathbf{J}$ and $\mathbf{B}$, and $\phi$ is the Larmor precession angle of $\mathbf{J}$ about $\mathbf{B}$.

(A color version of this figure is available in the online journal.)
irregular grains, LH07a found that \( q^{\text{max}} \) depends on the wavelength of radiation, grain shape, size, and composition, ranging from \( 10^{-3} \) to \( 10^{2} \). Thus, we treat \( q^{\text{max}} \) as a variable parameter in this paper.

To conform with numerical calculations using DDSCAT, we combine the functional forms of RATs from AMO with the scaling for the magnitude of RATs as a function of wavelength and grain size. The final RATs used in our series of papers are given in Appendix A. We note that, for the AMO, the functional forms of RATs do not depend on wavelength so that we can simplify the notations by using \( Q_{\Gamma} \) instead of \( \tilde{Q}_{\Gamma} \).

### 2.3. Torque-Free Motion

The motion of a triaxial grain in the absence of external torques, namely torque-free motion, is well known (see Landau & Lifshitz 1976; also WD03). Let us consider an irregular grain with angular momentum \( J \) and rotational energy \( E \). It is convenient to introduce a dimensionless parameter\(^6\)

\[
p = \frac{2I_{1}E}{J^{2}},
\]

where \( I_{1} \) is the inertia moment about the axis of major inertia \( a_{1} \), and other inertia moments are smaller than \( I_{1} \), i.e., \( I_{1} > I_{2} > I_{3} \). Here, \( p \) spans the range 1 to \( I_{1}/I_{3} \) where the lower and upper limits correspond to the grain rotation about the axis of major inertia \( a_{1} \) and the axis of minimum moment of inertia \( a_{3} \), respectively.

For torque-free motion, the angular velocity components \( \omega_{1}, \omega_{2}, \) and \( \omega_{3} \) about the three principal axes can be obtained analytically as functions of \( p \) and time \( t \) (see WD03; HL08 for their expressions).

In Figure 3, we plot \( \omega_{1}, \omega_{2}, \) and \( \omega_{3} \) as functions of time \( t/P_{r} \), with \( P_{r} \) being the rotation period, for different \( p \) for an irregular grain with the ratio of inertia moments \( I_{1} : I_{2} : I_{3} = 2 : 1.5 : 1 \). The plot corresponds to a positive flipping state, i.e., \( \omega_{1} \) and \( \omega_{3} \) both positive for \( p < I_{1}/I_{2} \). Similarly, when \( \omega_{1} \) and \( \omega_{3} \) are both negative for \( p < I_{1}/I_{2} \), we call it a negative flipping state (see HL08 for details).

We can see that when \( p \to 1 \) or \( I_{1}/I_{3} = 2 \) the rotation is stable about the \( a_{1} \) and \( a_{3} \) axes, respectively. However, the rotation is not stable about the intermediate axis \( a_{2} \) when \( p = 1.32 \) and 1.34, which are very close to \( I_{1}/I_{2} = 1.333 \). In fact, the middle panel shows that as \( p \to I_{1}/I_{2} \) the grain gets into a negative state with \( I_{2}/I_{2}/J = -1 \) for almost one half period, and then reverses its rotational state, staying there for another half period. Of course, at \( p = I_{1}/I_{2} \) we have \( I_{2}/I_{2}/J = 1 \) and \( \omega_{1} = \omega_{3} = 0 \).

Because the timescale of torque-free motion is much shorter than the internal relaxation timescale and other dynamical timescales (e.g., Larmor precession, gas damping times), we can average RATs over such a motion to obtain \( \bar{Q}_{\Gamma}(\Theta, \beta, \Phi) \) where the tilde denotes averaging over torque-free motion. The resulting RATs are functions of \( \xi, \phi, \psi, p, \) and \( J \).

### 2.4. Radiative Torques in Alignment Coordinate System

To study the alignment of the angular momentum \( J \) with the magnetic field \( B \), we represent RATs in the spherical coordinate system \( J, \xi, \phi, \) the so-called alignment coordinate system (see the right panel in Figure 1).

In this alignment coordinate system, the RAT \( \Gamma_{\text{rad}} \) can be written as

\[
\Gamma_{\text{rad}} = M[J(\xi, \phi, p, J)\hat{e} + G(\xi, \phi, p, J)\hat{\phi} + H(\xi, \phi, p, J)\hat{\psi}],
\]

\(^6\) This parameter was denoted by \( q \) in WD03; however, we use \( p \) to avoid the possible confusion with \( q^{\text{max}} \).
where the dependence on the angle between the radiation direction \( \mathbf{k} \) and \( \mathbf{B} \), \( \psi \), is omitted; \( M \) is given by

\[
M = \frac{\gamma u_{\text{rad}} a^2 \xi}{2},
\]

(5)

\( F \), which is the torque component parallel to \( \xi \), acts to change the orientation of \( \mathbf{J} \) with respect to \( \mathbf{B} \). \( H \), the component parallel to \( \mathbf{J} \), acts to spin grains up, and \( G \) induces the precession of \( \mathbf{J} \) about the magnetic field or radiation. \( H \) and \( F \) are given by

\[
H(\xi, \phi, p, J) = Q_\xi(\xi, \phi, p, J)(\cos \psi \cos \xi - \sin \psi \sin \xi \cos \phi) \\
+ \frac{Q_\omega(\xi, \phi, p, J)}{\sin \xi \sin \phi}, \\
F(\xi, \phi, p, J) = Q_\omega(\xi, \phi, p, J)(-\sin \psi \cos \xi \cos \phi - \sin \xi \cos \psi) \\
+ \frac{Q_\xi(\xi, \phi, p, J)}{\sin \xi \sin \phi},
\]

(6)

where \( \xi, \psi, \phi \) are angles describing the direction of \( \mathbf{J} \) in the laboratory coordinate system (see Figure 1, right panel). Here, \( Q_{\xi,\psi,\phi}(\xi, \phi, p, J) \) with \( i = 1, 2, 3 \) are the RAT components along the \( \mathbf{e}_i \) axes, respectively, obtained by averaging \( Q_{\xi,\psi,\phi} \) over torque-free motion.

We define also the torque components along the grain axes \( \mathbf{a}_1, \mathbf{a}_2, \) and \( \mathbf{a}_3 \) as

\[
Q_{\xi,\psi,\phi}(\xi, \phi, p, J) = Q_{\xi,\psi,\phi}(I, \mathbf{a}_i) \quad \text{for} \quad i = 1, 2, 3,
\]

(8)

and the work done by RATs per second

\[
Q_{\xi,\psi,\phi}(\xi, \phi, p, J) = Q_{\xi,\psi,\phi}(I, \mathbf{a}_1) \quad \text{for} \quad i = 1, 2, 3.
\]

(9)

Here, \( Q_{\xi,\psi,\phi}(I, \mathbf{a}_i) \) for \( i = 1, 2, 3 \) correspond to the components of angular velocity along three grain axes \( \mathbf{a}_i \). Their expressions for torque-free motion are given in Appendix A of HL08. In addition, from Figure 1, we have

\[
\mathbf{a}_1 = \mathbf{e}_1 \cos \Theta + \mathbf{e}_2 \sin \Theta \cos \Phi + \mathbf{e}_3 \sin \Theta \sin \Phi,
\]

(10)

\[
\mathbf{a}_2 = -\mathbf{e}_1 \sin \Theta \sin \beta + \mathbf{e}_2 \cos \Theta \cos \Phi \cos \beta - \sin \Phi \sin \beta,
\]

(11)

\[
\mathbf{a}_3 = \mathbf{e}_1 \sin \Theta \sin \beta - \mathbf{e}_2 \cos \Theta \cos \Phi \sin \beta + \mathbf{e}_3 \sin \Phi \cos \beta.
\]

(12)

### 2.5. Calculations of RATs

Taking analytical expressions \( Q_{\xi,\psi,\phi}(\Theta, \beta, \Phi = 0) \) with \( i = 1, 2, 3 \) and 3 from AMO (see Appendix A), we first calculate \( Q_{\xi,\psi,\phi}(\Theta, \beta, \Phi = 0) \) by using Equations (A13)–(A15) for \( \Theta \in [0, \pi], \beta \in [0, 2\pi] \), and \( \Phi \in [0, 2\pi] \). Next, we calculate RATs for \( \xi \) from 0.1 to 20\( \mathbf{J} \), \( \xi \) from 0 to \( \pi \), \( \phi \) from 0 to 2\( \pi \), and \( p \) from 1 to \( I/\mathbf{J} \) and for a given light direction \( \psi \). For a given set of parameters \( \mathbf{J} \), \( \xi, \phi, \psi \), we use coordinate transformations to get angles \( \Theta, \beta, \Phi \) as functions of Euler angles (see Appendix A in HL08). Finally, the torque components \( Q_{\xi,\psi,\phi}(\xi, \psi, \phi) \) are obtained using interpolation for \( Q_{\xi,\psi,\phi}(\Theta, \beta, \Phi) \). Then we average \( Q_{\xi,\psi,\phi}(\xi, \psi, \phi) \) over Euler angles, i.e., averaging over torque-free motion, to get \( Q_{\xi,\psi,\phi}(\xi, \psi, \phi) \). Substituting \( Q_{\xi,\psi,\phi}(\xi, \psi, \phi) \) into Equations (6), (7), and (9), we obtain \( F(\xi, \psi, \phi), H(\xi, \psi, \phi), \) and \( Q_{\xi,\psi,\phi}(\xi, \psi, \phi) \). We then average them over angle \( \phi \) to get the Larmor precession averaged values. Since these torques are functions of \( J, \xi, \phi, p \), we denote them by \( F(\phi)(J, \xi, \phi, p), H(\phi)(J, \xi, \phi, p), \) and \( Q_{\xi,\psi,\phi}(\xi, \psi, \phi) \), where the dependence on \( \psi \) is omitted. We will use these results to solve equations of motion for the alignment in Section 4.

### 3. TIMESALES: GAS DAMPING, INTERNAL RELAXATION, AND RADIATIVE ALIGNMENT

#### 3.1. Gas Damping Time

We represent the magnitude of torques in terms of the thermal angular momentum and the gas damping time. The latter, for the sake of simplicity, can be obtained for an oblate spheroid with moments of inertia \( I_1 = I_2 \), \( I_⊥ = I_3 \). The expressions for triaxial ellipsoids are more complex, but do not change the results substantially. Then, the thermal angular momentum is given by

\[
J_θ = \sqrt{I_⊥k_B T_{\text{gas}}},
\]

\[
= 5.89 \times 10^{-20} a_{-5}^{5/2} b_{1/2}^{1/2} \rho_{10}^{1/2} T_{\text{gas}}^{-1/2} \text{ g cm}^{-2} \text{ rad}^{-1},
\]

(13)

with \( \rho \) the density of material within the grain and \( \rho/3 \) \( \text{g cm}^{-3} \), \( a \) the grain size, \( a_{-5} = a/10^{-5} \), \( s = s/0.5 \), and \( T_{\text{gas}} \) the gas temperature and \( T_{\text{gas}} = T_{\text{gas}}/100 \text{ K} \). And the damping time due to gas collision (see Roberge et al. 1993, hereafter RDF93) is

\[
t_{\text{gas}} = \frac{3}{4\sqrt{\pi}} \frac{I_1}{n_{\text{H}} m_{\text{H}} a d \Gamma_1},
\]

\[
= 2.3 \times 10^{12} a_{-5}^{5/2} b_{1/2}^{1/2} \rho_{10}^{-1/2} \text{ cm}^{-3} \text{ s},
\]

(14)

where \( n_{\text{H}} \) is the thermal velocity of a gas particle with density \( n_{\text{H}} \), \( m_{\text{H}} \) is the hydrogen mass, \( \Gamma_1 \) is the geometrical parameter which is unity for a sphere (see RDF93), and we adopted standard parameters in the interstellar medium (ISM; see Table 1 in HL08).

#### 3.2. Internal Relaxation Timescales

A rotating paramagnetic grain experiences internal dissipation of energy due to Barnett and nuclear relaxation (see Purcell 1979; Lazarian & Draine 1999, hereafter LD99, respectively). In addition, superparamagnetic inclusions can increase significantly the rate of internal relaxation (Lazarian & Hoang 2008).

Disregarding other relaxation processes within the grain, e.g., inelastic relaxation (Purcell 1979; Lazarian & Efroimsky 1999) as well as the relaxation due to superparamagnetic inclusions, the internal relaxation rate arising from Barnett and nuclear relaxation for a brick of sides \( 2a \times 2a \times 2b \) (see Figure 18, right panel) is then

\[
t_{\text{int}}^{-1} \approx \frac{1}{t_{\text{Bar}}^{-1} + t_{\text{nucl}}^{-1}}
\]

\[
= 1.310^{-7} \rho^{-2} \sin^2(0.5 + 0.125s^2)^2 a_{-5}^{-5/2} b_{1/2}^{-1/2} T_{\text{gas}}^{-1/2} d_{-5}^{-5/2} f(J)^2 (h - 1) f(J).
\]

(15)

Here the Barnett and nuclear relaxation times \( t_{\text{Bar}} \) and \( t_{\text{nucl}} \) are taken from Hoang & Lazarian (2009, hereafter HL09), \( s = b/a, \)

\( h = I_⊥/I_1, \) and \( \omega = (2kT_{\text{gas}}/I_1)^{1/2} \) is the thermal angular velocity.

\footnote{For the oblate spherical with the major and minor axes \( a \) and \( b \), \( I_1 = 2Ma^2/3 = (8\pi/15)Mb^2, \) and \( I_⊥ = (4\pi/15)Mb^2(a^2 + b^2), \) but sometimes we define the equivalent sphere with the same volume as the grain and use \( I_⊥ = (8\pi/15)M \alpha a_1^3 \) with \( \alpha_i \) for \( i = 1, 2, \) and 3 being dimensionless factors (see WD03).}
of the grain at the gas temperature $T_{\text{gas}}$, $\tilde{T}_d = T_d/15$ K with $T_d$ being the dust temperature, and

$$f(J) = 1.610^5 \left[ 1 + \frac{(\omega_1 T_d)}{2} \right]^{-2} + 0.47 \left[ 1 + \frac{(\omega_1 T_{\text{rad}})}{2} \right]^{-2}.$$  \hspace{1cm} (16)

For grains larger than 1 $\mu$m, $f(J) \approx 1.6 \times 10^5$ for $J = J_{\text{th}}$. In the above equations $\omega_1 = J \cos \theta / I_1$ and $\tau_n$ is given by

$$\tau_n = 1 / \left( \tau_{\text{ne}}^{-1} + \tau_{\text{nn}}^{-1} \right),$$  \hspace{1cm} (17)

where $\tau_{\text{ne}} = 3 \times 10^{-4} (2.7/g_p)^2 (10^{22} \text{ cm}^{-3}/n_e) \text{ s}$ and $\tau_{\text{nn}} = \hbar / (3.8g_p n_e \mu_p) \approx 0.58 \tau_{\text{ne}} (n_e/n_n) \text{ s}$, with electron and nucleus density $n_e$ and $n_n$, are the relaxation times of interaction of nucleus–electron and nucleus–nucleus spins, respectively. Also, $\tau_{\text{rad}} = 2.9 \times 10^{-11} \text{ s}$ is the spin–spin relaxation time of electronic spins, and $\cos \theta = 1/2$ is chosen (see LD99; HL09).

### 3.3. Radiative Alignment Timescale

The characteristic timescale for RATs to accelerate the grain from $J = 0$ to $J = I_1 \omega$ is estimated by

$$t_{\text{rad}} = \frac{I_1 \omega}{\gamma_{\text{rad}}} = \frac{I_1 \omega}{M(H)\phi},$$  \hspace{1cm} (18)

where $\langle H \rangle\phi$ is the spin-up component of RATs that is averaged over the torque-free motion and the precession angle $\phi$ using Equation (6).

Using $M$ given by Equation (5) and $I_1 = 16\rho a^3 s/3$ for the brick of sides $2a \times 2a \times 2b$, we obtain

$$t_{\text{rad}} = \frac{2^{3/2}(16\rho T_{\text{gas}}/\lambda)^{1/2} a^{3/2}}{(\gamma_{\text{rad}} H)\phi}.$$  \hspace{1cm} (19)

For standard parameters of the ISM, this yields

$$t_{\text{rad}} \approx 2.8 \times 10^{10} \frac{\hat{r} \hat{g}_a \hat{d}_{a-5}}{\hat{d}_{a-5}} 1.2 \mu \text{m} \mu_{\text{ISRF}} 10^{-3} \frac{\hat{g}_{a-5}}{\hat{u}_{\text{rad}}} \frac{\gamma_{\text{rad}}}{\gamma(H)\phi} \text{ s},$$  \hspace{1cm} (20)

where $\hat{s} = s/0.5$ with $s = b/a$ being the ratio of the long to short axes; $\mu_{\text{ISRF}}$ is the energy density of the interstellar radiation field (ISRF; see Mathis et al. 1983).

### 3.4. Radiative Alignment Time Versus Internal Relaxation Time

To compare the effect of RATs and internal relaxation, let us estimate the ratio of their timescales. Following Equations (15) and (20), we obtain for a brick grain

$$\frac{t_{\text{rad}}}{t_{\text{int}}} = \frac{3.7 \times 10^3 \hat{g}_{a-5} (0.5 + 0.125s^2)^2 a^{-6.5}}{\hat{l} \hat{u}_{\text{rad}} 10^{-3} \gamma(H)\phi} \left( \frac{\omega}{\omega_T} \right)^3 f(J).$$  \hspace{1cm} (21)

Here, $\hat{G}_a = T_d/15$ K, $\hat{u}_{\text{rad}} = \hat{u}_{\text{rad}}$, and $\hat{l} = \hat{l}_{a-1}/2$. $\mu_{\text{ISRF}} = 8.64 \times 10^{-13} \text{ erg cm}^{-3}$ is the energy density of ISRF (see Mathis et al. 1983) and $\gamma = 0.1$.

Using the scaling of RAT magnitude obtained in LH07a, $(H)\phi \approx |Q_{\text{TR}}| \approx 0.4 (\hat{l}/a)^{-3}$ for $\hat{l} > 1.8a$, we get

$$\frac{t_{\text{rad}}}{t_{\text{int}}} = \frac{0.04 \times 10^5 r^{1-5} (0.5 + 0.125s^2)^2 a^{-6.5}}{\hat{g}_{a-5} \hat{l} \hat{u}_{\text{rad}} 10^{-3} \gamma_{\text{rad}}} \frac{\gamma_{\text{rad}}}{\gamma(H)\phi} \left( \frac{\omega}{\omega_T} \right)^3 f(J).$$  \hspace{1cm} (22)

Increasing both the grain size and the radiation intensity lead to a decrease of $t_{\text{rad}}$.

For a grain size $a = 1.2 \mu$m, $f(J) = J_{\text{th}} \sim 10^5$, so it can be seen from Equation (23) that $t_{\text{rad}}/t_{\text{int}} \sim 0.67$ for the ISRF (see also Figure 4).

Figure 4 shows $t_{\text{gas}}, t_{\text{int}},$ and $t_{\text{rad}}$ as functions of $a$ for the ISM obtained from Equations (14), (15), and (20), respectively. It can be seen that for grains smaller than $\sim 1 \mu$m, $t_{\text{rad}}/t_{\text{int}} \ll 1$ and decreases steeply with decreasing $a$. For grains larger than $\sim 1.2 \mu$m, we see that $t_{\text{rad}}/t_{\text{int}} > 1$ and $t_{\text{rad}}/t_{\text{int}}$ increases rapidly with $a$. When the mean energy density $u_{\text{rad}}$ increases by $10^5$ times, the size corresponding to $t_{\text{int}} = t_{\text{rad}}$ decreases from 1.5 to 0.6 $\mu$m.

Note that earlier works on grain alignment dealt with the alignment of interstellar grains with size in the range from 0.005 to 0.25 $\mu$m. For this range of grain size, internal relaxation is very strong, so that the average of RATs over thermal fluctuations arising from internal relaxation was accounted for (WD03; HL08). In some circumstances, e.g., accretion discs and molecular clouds, where larger grains corresponding to weak internal relaxation are expected, we need to study the internal and external alignment at the same time.

Another characteristic timescale involved in grain dynamics is the Larmor precession time of the grain magnetic moment about an ambient magnetic field. Due to the Barnett effect, a rotating paramagnetic grain develops a magnetic moment $\mu_{\text{Bar}}$ which is proportional to the angular velocity (see Dolginov & Mytrophanov 1976). The value of $\mu_{\text{Bar}}$ is given by

$$\mu_{\text{Bar}} = \chi(0) V h \frac{g}{\mu_B} \omega,$$  \hspace{1cm} (24)

where $\chi(0) = 4.2 \times 10^{-2} f_p \hat{g}_{a-5}$ (with $f_p$ being the fraction of paramagnetic material) is the magnetic susceptibility at zero frequency, $V$ is the volume, $g$ is the gyromagnetic ratio (which is $\sim 2$ for electrons), and $\mu_B = e/2m_e$ is the Bohr magneton.
The Larmor precession time in an external magnetic field $B$ is then

$$t_L = \frac{2\pi I_1 \omega}{\mu_{\text{Bar}} B} \approx 4.2 \times 10^5 a_s^2 \frac{\hat{\omega}}{B \chi(0)} \text{ s},$$

(25)

where $\hat{B} = B/(5\mu G)$ and $\chi(0) = \chi(0)/10^{-3}$, and we used $I_1 = 8\pi\rho a_s^2/15$ for the inertia moment.

It is easy to see that this timescale is shorter than both internal relaxation and radiative alignment timescales for grains larger than about $1 \mu m$ (see Figure 4), so that we can average RATs over the Larmor precession while dealing with the overall alignment.

4. RAT ALIGNMENT IN THE ABSENCE OF INTERNAL RELAXATION

Now let us study grain dynamics for grains larger than $1 \mu m$ in the diffuse ISM.$^8$ For this range of grain size, $t_{\text{rad}} > t_g$, the influence of internal relaxation on grain alignment can be ignored. As a result, we follow both the evolution of angular momentum and grain axes subject to RATs.

4.1. Equations of Motion

The orientation of $J$ with respect to the magnetic field $B$ is described by the following equation:

$$\frac{dJ}{dt} = \Gamma_{\text{rad}} - \frac{J}{t_{\text{gas}}},$$

(26)

where $\Gamma_{\text{rad}}$ is the vector of radiative torque (see Equation (A1)) and $t_{\text{gas}}$ is the gas damping time.

With the use of Equation (4), we can rewrite Equation (26) in the spherical coordinate system $J, \xi, \phi$ assuming that both precession timescales of $a_1$ around $J$ and $J$ around $B$ are smaller than the internal relaxation time:

$$\frac{dJ}{dt} = M((H(J, \xi, p))_\phi - \frac{J}{t_{\text{gas}}},$$

(27)

$$\frac{d\xi}{dt} = \frac{M}{J} (F(J, \xi, p))_\phi,$$

(28)

where $(H(J, \xi, p))_\phi$ and $(F(J, \xi, p))_\phi$ are spin-up and aligning components, respectively, obtained from averaging Equations (6) and (7) over torque-free motion and the precession angle $\phi$, and $M$ is given by Equation (5). Also, we assume that the gas damping is isotropic, and we ignore the alignment by paramagnetic dissipation.

Unlike the axisymmetric grains where the angle $\theta$ between $a_1$ and $J$ is constant during torque-free motion, irregular grains exhibit torque-free wobbling, and the conserved quantities are $E$, the total energy, and $J$, the value of the angular momentum. Therefore for an irregular grain, similar to WD03, we can use $p$ to describe the dynamical evolution of grain axes. Using Equation (3) and taking the time derivative of $p$, we obtain

$$\frac{J^2 dp}{dt} = M[I_1 (Q_w(J, \xi, p))_\phi - pJ (H(J, \xi, p))_\phi]$$

$$- \frac{J(p-1)}{t_{\text{rad}}} \left( \frac{1 - p I_3/I_1}{1 - I_3/I_1} \right),$$

(29)

where $(Q_w)$ is the average of the torque component along the angular velocity given by Equation (9) and $t_{\text{rad}}$ is given by Equation (15) (see Appendix B). Let us define

$$\langle K(J, \xi, p) \rangle_\phi = \frac{Q_w(J, \xi, p)}{pJ};$$

(30)

then Equation (29) becomes

$$\frac{J dp}{dt} = M \left[ (K(J, \xi, p))_\phi - \langle H(J, \xi, p) \rangle_\phi \right]$$

$$- \frac{(p-1)}{t_{\text{rad}}} \left( \frac{1 - p I_3/I_1}{1 - I_3/I_1} \right).$$

(31)

Before studying grain dynamics induced by RATs, we first want to consider how the torque components $(F(J, \xi, p))_\phi$, $(H(J, \xi, p))_\phi$, and $(K(J, \xi, p))_\phi$ change with $p$ and $\xi$. We adopt an AMO that has the inertial property of triaxial ellipsoids with the ratio of inertia moments $I_1 : I_2 : I_3 = 2:1:1$ and $I_1 : I_2 : I_3 = 1.745 : 1.610 : 0.8761$. The latter is identical with the ratio of inertia moments of the shape 1 (see Figure 5) and naturally the parameter $q_{\text{max}}$ can be changed for the AMO (LH07a). A $1.5 \mu m$ grain size and ISRF are adopted in this section.

4.2. Variation of RATs with $p$: Stationary Points for $p$

In Figure 6, we plot $(H(J, \xi, p))_\phi$ and $(K(J, \xi, p))_\phi$ as functions of $p$ for $J = 10 I_{\text{ort}}, \xi = 22^\circ, 140^\circ,$ and $\psi = 0^\circ$ (left panel) and $70^\circ$ (right panel) for the AMO with parameter $q_{\text{max}} = 1.2$.

We see that $(H(J, \xi, p))_\phi$ and $(K(J, \xi, p))_\phi$ decrease as $p$ increases to $I_1/I_2$, and then increase as $p$ increases to $I_1/I_3$. This is because grain axes wobble more rigorously about $J$ as $p \rightarrow I_1/I_2$, which results in the decrease of torques due to averaging over wobbling with large amplitude. As $p$ increases

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$^8$ Although such large grains are rare in the diffuse medium, our study below can be applied to circumstances where $\mu m$ grains are present, e.g., accretion disks, comets.
ψ = internal alignment of without and with high-
remind the reader that these cases correspond to the alignment
$p J$ attractors, there are three stationary points 1, 2, and 3. Let
the left panel, corresponding to the alignment without high-
again, ω rotates stably about $a_3$, and averaged torques increase
because the wobbling amplitude decreases.

In addition, $⟨H(J, ξ, p)⟩_φ$ and $⟨K(J, ξ, p)⟩_φ$ are identical at $p = 1, I_1/I_2 = 1.33$, and $I_1/I_3 = 2$. This arises from the
fact that at these points, ω is parallel to $J$, so the projection
of RATs onto $J$, $⟨H(J, ξ, p)⟩_φ$, is the same with that onto $ω$,
$⟨K(J, ξ, p)⟩_φ$.

The sharp decrease of $⟨K(J, ξ, p)⟩_φ$ when $p → I_1/I_2$ is
associated with the dynamics of a triaxial ellipsoid. Nevertheless,
for $p$ very close to $I_1/I_2$, $ω$ oscillates in the lab system and
gives rise to a sharp change of $⟨K(J, ξ, p)⟩_φ$ (see dashed lines in Figure 6).

Equation (31) shows that stationary points occur when $dp/dt = 0$. In the absence of internal relaxation for large grains,
it requires $⟨K(J, ξ, p)⟩_φ - ⟨H(J, ξ, p)⟩_φ = 0$.

Figure 7 shows $⟨K(J, ξ, p)⟩_φ - ⟨H(J, ξ, p)⟩_φ$ for the same
AMO with $q^{max} = 1.2, ψ = 0^0, 70^0$ (left) and 2.5, $ψ = 0^0$
(right). There we omit the dependence of torques on $J$ and $ξ$. We
remind the reader that these cases correspond to the alignment
without and with high-$J$ attractors, respectively, when the perfect
internal alignment of $a_1$ with $J$ was assumed (see LH07a). In the left panel, corresponding to the alignment without high-$J$
attractors, there are three stationary points 1, 2, and 3. Let $p_1$, $p_2$, and $p_3$ be the values of $p$ at these points. It is easy to see
that $p_1 = 1$, $p_2 = I_1/I_2$, and $p_3 = I_1/I_3$. In the alignment with high-$J$ attractors (see right panel), there exists another stationary
point 4, and its value of $p$ is denoted by $p_4$. Our calculations
show that $p_4$ depends on $q^{max}$ and $ψ$. In Figure 12, we plot
$q^{max}$ as a function of $p_4$ for two radiation directions $ψ = 0^0$
and $70^0$.

In addition, Figure 7 shows that when $p → I_1/I_2$, the figure
exhibits very sharp changes. In fact, we see the fast changes
of $⟨K(J, ξ, p)⟩_φ - ⟨H(J, ξ, p)⟩_φ$ as $p$ increases from slightly
smaller than $I_1/I_2$ to slightly larger than $I_1/I_2$. As $p = I_1/I_2$, it becomes zero.

Similar to Figure 7, Figure 8 shows $⟨K(J, ξ, p)⟩_φ - ⟨H(J, ξ, p)⟩_φ$ for the shape 1 (see Figure 5) and AMO with the same ratio of inertia moments $I_1 : I_2 : I_3 = 1.745 : 1.610 : 0.8761$ and $q^{max} = 2.5$, and for $ψ = 0^0$. It can be seen that the AMO reproduces well the dependence of RATs on $p$ for the
shape 1. For instance, we observe a good correspondence
between the stationary points obtained, but the stationary point
4 in the AMO is very close to the point 1 in this case (point 4 is
seen in the right panel of Figure 7).

4.3. Variation of RATs with $ξ$: Stationary Points for $ξ$

Let us consider the forms of RATs as a function of $ξ$ at particular values $p = 1$ (perfect coupling of $a_1$ with $J$) and
$I_1/I_3 (a_1 \perp J)$ for the same AMO.

In Figure 9, we present the torque components $⟨F(p)⟩$ and
$⟨H(p)⟩$ for the alignment without high-$J$ attractors, i.e., $q^{max} = 1.2, ψ = 0^0$ for $p = 1, I_1/I_2$, and $I_1/I_3$. We see that stationary

Figure 6. Torque components averaged over torque-free motion, $(H(p))$ and $(K(p))$, for different angles $ξ$ and for two radiation directions with the magnetic field, $ψ = 0^0$ and $70^0$. These torques $(H(p))$ and $(K(p))$ coincide for $p = 1, I_1/I_2$, and $I_1/I_3$, corresponding to three stationary points 1, 2, and 3.

Figure 7. Difference between $(K(p))$ and $(H(p))$ for different angles $ξ$ for $ψ = 0^0$ and $70^0$ and $q^{max} = 1.2$ (left panel) and $ψ = 0^0, q^{max} = 2.5$ (right panel). Zero points denoted by filled circles corresponding to $(K) - (H) = 0$ are stationary points of $p$. In the left panel, three stationary points are 1, 2, and 3, corresponding to $p = 1, I_1/I_2$, and $I_1/I_3$. In the right panel, in addition to stationary points 1, 2, and 3, there exists another stationary point 4.

Figure 8. Torque components averaged over torque-free motion, $(H(p))$ and $(K(p))$, for different angles $ξ$ and for two radiation directions with the magnetic field, $ψ = 0^0$ and $70^0$. These torques $(H(p))$ and $(K(p))$ coincide for $p = 1, I_1/I_2$, and $I_1/I_3$, corresponding to three stationary points 1, 2, and 3.
Figure 8. Similar to Figure 7 but for shape 1 and AMO with the same ratio of inertia moments $I_1 : I_2 : I_3 = 1.745 : 1.610 : 0.8761$ and $q_{\text{max}} = 2.5$, and for $\psi = 0^\circ$.

For $p = 1$, we see four stationary points (1,2,7,8), where (8) is a high-$J$ attractor. Other stationary points (1,3,6,8) and (1,4,5,8) are for $p = I_1/I_2$ and $I_1/I_3$, respectively.

4.4. Helicity versus Axis of Rotation

For $p = I_1/I_3$, the rotation of the grain is about the axis of minor inertia $a_3$. In this case, RATs depend on the angle between the grain rotation axis $a_3$ and the radiation direction $k$. This angle is also the angle between $J$ and $B$, $\xi$, when $\psi = 0^\circ$. Therefore, the dashed line in Figure 9 represents RATs as a function of the angle between the grain rotation axis and $k$.

For $p = 1$, the grain rotates about $a_1$ described by the angle $\Theta$. Thus, the solid line describes RATs as a function of $\Theta = \xi$. We can see a large difference of $\langle F(p) \rangle$ when the rotation axis changes from the axis of major inertia to the axis of minor inertia. The right panel is similar to the left one, but it shows $\langle H(p) \rangle$. We see that the helicity of the grain for $p = 1$ is similar to that for $p = I_1/I_3$. In other words (see LH07a), the helicity of the grain is identical for the rotation around the maximal and minimal axes.

4.5. Trajectory Map

To visualize grain alignment, we present trajectory maps using $J$, $\xi$, and $p$ which are the solutions of equations of motion (27)–(31). We adopt the initial condition $J_0 = 4I_1\omega T$, $\xi_0$ spanning from 0 to $\pi$, and $p_0 = 1$. Here we consider only one initial value $p_0 < I_1/I_2$ (i.e., grain initially in positive flipping state).

Figure 9. Torque components $\langle F \rangle$ and $\langle H \rangle$ for $p = 1$, $I_1/I_2$, and $I_1/I_3$: (1,6), (1,2,5,6), and (1,3,4,6) are stationary points for $p = 1$, $I_1/I_2$, and $I_1/I_3$, respectively.

Figure 10. Similar to Figure 9 but for the case $q_{\text{max}} = 2.5$, i.e., alignment with high-$J$ attractor for $\psi = 0^\circ$; (1,2,7,8), (1,3,6,8), and (1,8) are stationary points for $p = 1$, $I_1/I_2$, and $I_1/I_3$, respectively.
In Figure 11, we present the trajectory maps for the RAT alignment of the triaxial AMO induced by the ISRF for $\psi = 0$ and for $q^{\text{max}} = 1.2$ and 4, corresponding to alignment without and with high-$J$ attractors (left and right panels). In the former case, the alignment occurs at two low-$J$ attractors A and B, where A corresponds to the perfect alignment of $\mathbf{J}$ with $\mathbf{B}$, and $\mathbf{B}$ corresponds to $\cos \xi = 0.64$. We see that point B is the new low-$J$ attractor appearing in the absence of internal relaxation.\footnote{LH07a found that for the AMO with $q^{\text{max}} = 1.2$, all grains are driven to one low-$J$ attractor $\mathbf{A}$ for $\psi = 0^\circ$.}

In the case $q^{\text{max}} = 4$, the alignment has a high-$J$ attractor $\mathbf{A}$ at $\cos \xi = 1$, and low-$J$ attractors $\mathbf{C}$ and $\mathbf{D}$ with $\cos \xi = -0.65$ and 0.65, respectively.

In addition, we studied the radiative alignment for the case $q^{\text{max}} = 0.78$ and $\psi = 70^\circ$, which gives the trajectory map with high-$J$ attractor. We observed that the alignment is similar to the right panel in Figure 11.

4.6. Internal Alignment of Grain Axes with $\mathbf{J}$

Let us discuss the internal alignment corresponding to the trajectory maps shown in Figure 11. The initial value of $p$ was assumed to be smaller than $I_{1}/I_{2}$.

For alignment without high-$J$ attractor (the left panel), point $\mathbf{A}$ corresponds to the perfect internal alignment with $p = 1$, while point $\mathbf{B}$ has internal alignment with $\mathbf{J}$ along the $\mathbf{a}_{2}$ axis, which corresponds to $p = I_{1}/I_{2}$.

For the case of alignment with high-$J$ attractor, i.e., $q^{\text{max}} = 4$ and $\psi = 0^\circ$ (right panel of Figure 11), the internal alignment depends on the initial value of $p_0$. If $p_0 < p_4$, then the right panel of Figure 7 shows that $\langle K(p) \rangle - \langle H(p) \rangle < 0$ for $\cos \xi_0 \sim 1$. As a result, after some time, $p$ decreases to $p = 1$, i.e., perfect internal alignment, and the aligning torque $\langle F(p) \rangle$ approaches $\langle F(p = 1) \rangle$. Therefore the alignment returns to the case of perfect coupling of $\mathbf{a}_1$ with $\mathbf{J}$ as studied in DW97 and LH07a, i.e., the alignment of $\mathbf{J}$ with $\mathbf{B}$ is determined by $\langle F(p = 1) \rangle$ and $\langle H(p = 1) \rangle$. LH07a found that for the AMO with $q^{\text{max}} > 2$ and $\psi = 0^\circ$, the alignment has high-$J$ attractors. We can see this in the right panel of Figure 11 for $q^{\text{max}} = 4$. Combined with internal alignment, the high-$J$ attractor corresponds to $\mathbf{J} \parallel \mathbf{B}$ and long grain axes perpendicular to $\mathbf{B}$. For $\cos \xi_0 \sim 1$, we have $\langle K(p) \rangle - \langle H(p) \rangle > 0$, so $p$ increases from $p_0$ to $p_4$ if $p_0 < p_4$. Thus, the low-$J$ attractor corresponds to the alignment with $p = p_4$. As a result, $\mathbf{A}$ and $\mathbf{B}$ correspond to internal alignment with $\mathbf{a}_1 \parallel \mathbf{J}$, and $\mathbf{C}$ corresponds to internal alignment with $\mathbf{a}_1 \perp \mathbf{J}$.

If $p_0 > p_2$, then there are two attractors with $p = p_4$ and $p_2$. Since the stationary point $p_4$ is important in determining the type of alignment, in the left panel of Figure 12 we plot $q^{\text{max}}$ against $p_4$, for two light beam directions $\psi = 0^\circ$ and $70^\circ$. Shaded
areas describe the alignment with high-$J$ attractor corresponding to a perfect alignment of $\mathbf{J}$ with $\mathbf{B}$ and perfect internal alignment, i.e., $\mathbf{a}_1 \parallel \mathbf{J}$.

The right panel of Figure 12 presents the alignment depending on $q_{\text{max}}$ and $\psi$. Moreover, when internal alignment happens with $p = 1$, then we can predict the alignment for different $\psi$ if we know $q_{\text{max}}$. The result is presented in the right panel of Figure 12. The solid curves, characterizing the boundary for the alignment with high-$J$ attractor, are obtained assuming that $P_0 < P_1$. We see that the alignment here is similar to that found in LH07a. We note that at $p = 1$ the average over the wobbling for irregular grains is identical with the averaging over the rotation angle $\beta$ used in LH07a.

We note that internal alignment with $p = I_1/I_2$ corresponds to the rotation of a grain along the axis of intermediate inertia moment $\mathbf{a}_2$. For torque-free motion, this rotation is unstable. Therefore, gas bombardment can destabilize this internal alignment, and grains may return to alignment with short or long axes perpendicular to $\mathbf{J}$.

### 4.7. RAT Alignment in the Presence of Weak Internal Relaxation

Let us consider a regime of weak internal relaxation, i.e., when $t_{\text{rad}} < t_{\text{int}} < t_{\text{gas}}$. Figure 4 shows that grains with size in the range 1.5 to 6 $\mu$m correspond to this situation. We note that these large grains are present in nondiffuse situations, such as protoplanetary disks and comet tails. For this regime, it is convenient to introduce a new radiative timescale, $t'_{\text{rad}}$, which is defined by (using Equation (31))

$$ t'_{\text{rad}} = \frac{I_1 \omega}{M|\langle K(p) \rangle - \langle H(p) \rangle|_{\text{max}}}, $$

where max denotes the maximal value of $|\langle K(p) \rangle - \langle H(p) \rangle|$ as a function of $p$ for given angles $\xi$ and $\psi$.

It can be seen that $t'_{\text{rad}}$ is about one order of magnitude larger than $t_{\text{rad}}$ given by Equation (20). Therefore it is possible to have $t'_{\text{rad}} > t_{\text{int}} > t_{\text{rad}}$. For this case, one interesting effect occurs with the wrong alignment previously discussed in that grains are driven to right alignment on a characteristic time $t_{\text{rad}}$. However, for practical purposes, a change from $t_{\text{rad}}$ to $t'_{\text{rad}}$ is marginal due to a steep dependence.

### 5. RAT ALIGNMENT BY DIPOLE AND QUADRUPOLE COMPONENTS OF THE RADIATION FIELD

All earlier studies of the RAT alignment (DW96, DW97, WD03; LH07, LH08, HL08, HL09) have been done assuming that the radiation field can be described by a unidirectional beam directed along $\mathbf{k}$ (see Figure 1) with a degree of anisotropy $\gamma$. This assumption is consistent with grains near a point radiation source. In many circumstances, e.g., in molecular clouds and even in some diffuse clouds, an approximation of the radiation field using dipole and quadrupole components is more appropriate.

Below, we study the RAT alignment induced by the dipole and quadrupole fields. For the sake of simplicity, we assume that the axis of major inertia $\mathbf{a}_1$ is constrained to be parallel to the angular momentum $\mathbf{J}$ (i.e., thermal fluctuations and thermal flipping are completely ignored; see DW97; LH07a). In fact, our study in HL08 proved that this is a sufficiently good approximation unless we want to calculate the exact degree of alignment.

![Figure 13](image-url) Lab coordinate system $\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3$ for the dipole field: $\mathbf{B}$ is the magnetic field direction, $\mathbf{d}$ is the dipole axis, $\chi$ is the angle between the dipole axis $\mathbf{d}$ and the magnetic field $\mathbf{B}$, and $\mathbf{k}$ is the radiation direction. $\phi_k$, $\psi_k$ describe the direction of the radiation $\mathbf{k}$ in the lab coordinate system.

### 5.1. Coordinate Systems

We define a lab coordinate system $\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3$ (or $\mathbf{e}_i$-system), with $\mathbf{e}_1$ parallel to $\mathbf{B}$, $\mathbf{e}_2 \perp \mathbf{e}_1$ and lying in a plane formed by the magnetic field $\mathbf{B}$ and the dipole axis $\mathbf{d}$ for dipole field or symmetric axis for the quadrupole field, and $\mathbf{e}_3$ perpendicular to the plane $\mathbf{e}_1\mathbf{e}_2$ (see Figure 13). Let $\chi$ be the angle between $\mathbf{d}$ and $\mathbf{B}$, and define a coordinate system $\mathbf{e}_0^\theta \mathbf{e}_1^\theta \mathbf{e}_2^\theta \mathbf{e}_3^\theta$ with $\mathbf{e}_0^\theta \parallel \mathbf{d}$, the so-called $\mathbf{d}$-system. Now, the radiation direction $\mathbf{k}$ in the $\mathbf{d}$-system is determined by two angles $\phi_k$ and $\psi_k$ (see Figure 13). Since RATs depend on the angle between grain axes $\mathbf{a}_i$ and the radiation direction $\mathbf{k}$, let us define a coordinate system $\mathbf{e}_i^\theta$ with $\mathbf{e}_0^\theta \parallel \mathbf{k}$, $\mathbf{e}_3^\theta$ lying in the plane $\mathbf{k}$ and $\mathbf{d}$, and $\mathbf{e}_2^\theta$ perpendicular to this plane. The orientation of a grain in the $\mathbf{e}_i^\theta$ system is described by three angles $\Theta$, $\beta$, and $\Phi$ (see the left panel of Figure 1 with $\mathbf{e}_i$ replaced by $\mathbf{e}_i^\theta$). The transformations between these coordinate systems are described in Appendix D.

### 5.2. Dipole Component

For a given direction of the grain in the lab coordinate system, the net RAT resulting from a dipole radiation field is given by

$$ \Gamma_{\text{rad}} = \sum_{k=1}^{N} Q(\psi_k) \frac{\gamma^2 \lambda^2 u_d(\psi_k) \Delta \Omega_k}{2\pi}, \quad (33) $$

where $\psi_k$ is the angle between the radiation direction $\mathbf{k}$ and the dipole axis $\mathbf{d}$, $\gamma$ is the degree of anisotropy, $u_d(\psi_k)$ is the energy density of the dipole field, $\Delta \Omega_k = \pi/N$ is the element of solid angle in the direction $\psi_k$, and summation (33) is performed in the range $\psi_k$ from $-\pi/2$ to $\pi/2$ (i.e., only outward radiation is accounted for). The spatial distribution of dipole radiation intensity is given by the usual expression

$$ u_d(\psi_k) = u_{\text{ISRF}} \sin^2 \psi_k. \quad (34) $$

We assume in Equation (34) that the energy density of the dipole field has the same amplitude as that of the ISRF and we deal only with its spatial distribution.

To study the RAT alignment, we need to know components of torques in the lab coordinate system, i.e.,

\[ \text{Appendix D.} \]

10 The symmetric axis of the quadrupole field is defined by two dipole axes.
an example, we use the AMO for a grain size terms of phase trajectory maps (see LH07a for more details). As to find \( J,\xi \) details).

\[ Q_{\epsilon}(\xi, \psi, \phi), Q_{\epsilon}(\xi, \psi, \phi), \text{ and } Q_{\epsilon}(\xi, \psi, \phi). \] It is straightforward but tedious to obtain \( Q_{\epsilon}(\xi, \psi, \phi) \) from \( Q_{\epsilon}(\Theta, \beta, \Phi) \) given by Equations (A4)–(A5) through a series of coordinate transformations. First, we perform the transformation from the \( k \)-system to the \( d \)-system, then from the \( d \)-system to the \( e_\gamma \)-system. Finally, we average the resulting torques in the spherical system \( J, \xi, \phi \) over the azimuthal angles of \( k \) about the dipole axis \( \phi_k \) and the Larmor precession angle \( \phi \) (see Appendix E for more details).

Using the obtained RATs, we solve the equations of motion to find \( J, \xi \) as functions of time, and present their evolution in terms of phase trajectory maps (see LH07a for more details). As an example, we use the AMO for a grain size \( a = 0.2 \mu m \) and \( q^\text{max} = 3.5 \). For the dipole radiation field with \( \chi = 0^\circ \), the torque components and the corresponding trajectory map are shown in the left and right panels of Figure 14, respectively. The left panel shows the existence of stationary points at \( \cos \xi \sim \pm 1 \). It is possible to check that the stationary point \( \cos \xi = 1 \) is an attractor because \( \langle F \rangle_{\phi}/\langle H \rangle_{\phi} \big|_{\cos \xi = 1} < 0 \) and \( \langle H \rangle_{\phi} > 0 \). This high-\( J \) attractor is denoted by a circle (A) in the trajectory map (see the right panel). In addition, there exists also a low-\( J \) attractor B, as usual. Both stationary points correspond to a perfect alignment of angular momentum with the magnetic field.

5.3. Quadrupole Component

Let us consider now the RAT alignment by the quadrupole component of the radiation field. The spatial distribution of the energy density for the quadrupole field is given by

\[ u_{\text{quad}} = u_{\text{ISRF}} \sin^2 \psi_k \cos^2 \psi_k, \tag{35} \]

where \( \psi_k \) is the angle between the radiation direction and the symmetric axis of the quadrupole. Following the same procedure as with the dipole field, we obtain results for the AMO with \( q^\text{max} = 0.78 \) (i.e., similar to the value of the shape 2 at \( \lambda = 1.2 \mu m \), see LH07a) and the grain size \( a_{\text{eff}} = 0.2 \mu m \) in Figure 15 for two directions of the quadrupole with the magnetic field: \( \chi = 0^\circ \) (left panel) and \( 45^\circ \) (right panel). The alignment occurs without high-\( J \) attractors in the former case, but with a high-\( J \) attractor in the latter one.

5.4. Simultaneous Action of Dipole and Quadrupole Components

Figure 16 shows the existence of high-\( J \) and low-\( J \) attractors as functions of \( q^\text{max} \) and \( \chi \) for the dipole (left panel) and quadrupole (right panel) fields. For the dipole field, it can be seen that the high-\( J \) attractor appears when \( q^\text{max} > 3.2 \) for \( \chi = 0^\circ \), then increases with increasing \( \chi \), and \( q^\text{max} > 100 \) for \( \chi = 30^\circ \). For \( \chi > 30^\circ \), the high-\( J \) attractor exists when \( q^\text{max} < 1.8 \). On the other hand, for the quadrupole field, the high-\( J \) attractor exists when \( q^\text{max} < 2.5 \) for a wide range \( \chi > 20^\circ \).

LH07a have calculated RATs for a number of irregular grain shapes, grain sizes, and wavelengths, and found that for the majority of shapes we have \( q^\text{max} < 10 \). Thereby, from Figure 16 we see that the possibility of alignment with a high-\( J \) attractor is enhanced for both the dipole and quadrupole fields compared to a single beam.

In the left panel of Figure 17, we present the ratio of energy density of dipole to quadrupole components, \( u_d/u_q \), as a function of \( \chi \) for the AMO with \( q^\text{max} = 1.2 \). For this AMO, the high-\( J \) attractor appears frequently (see Figure 16, left panel). It reveals that the energy density of the dipole component is required to be dominant over that of the quadrupole in order to have a high-\( J \) attractor for \( \chi < 20^\circ \). In the range \( \chi > 30^\circ \), the high-\( J \) attractor does not depend on the ratio of the energy densities because both components produce high-\( J \) attractors.

Apart from the existence of high-\( J \) attractors, the value of the angular momentum \( J \) at these points is essential for calculations of the degree of alignment. It is easy to see that the angular momentum achievable (i.e., maximal value) depends on the angle \( \chi \).

Now, let us adopt an irregular grain (shape 1) with RATs obtained using DDSCAT, and calculate the value of the maximal angular momentum \( J_{\text{max}}(\chi) = l_1 \omega_{\text{max}}(\chi) \) produced by dipole and quadrupole components of the ISRF as a function of the angle \( \chi \). The anisotropy degree of radiation \( \gamma = 0.1 \) is assumed for all components. We compare the results with that induced by a beam of radiation.

In the right panel of Figure 17, we present the resulting value of \( \omega_{\text{max}}(\chi) \). There it can be seen that \( \omega_{\text{max}}(\chi) \) decreases rapidly with increasing \( \chi \) for the dipole component, but it exhibits a slow change for the quadrupole component. In addition, \( \omega_{\text{max}} \) induced by the dipole component is \(~10\) times greater than that by the quadrupole one. Therefore when we focus only on the
effect of spin-up by RATs, the quadrupole component can be neglected. The dipole component results in lower values of $\omega_{\text{max}}$ compared to that produced by the radiation beam (see Figure 17, right panel) as a result of the average over the entire space for the dipole field.

### 5.5. Implications for Grain Alignment

The degree of RAT alignment depends strongly on the possibility of existence of high-$J$ attractors (see HL08). In LH07a, we identified the criteria for the existence of high-$J$
attractors as a function of the angle between the beam direction and the magnetic field. There it is shown that high-J attractors occur when $q^{\max} > 2$ for $\psi < 45^\circ$, and $q^{\max} < 1$ for $\psi > 45^\circ$. In other words, there is a gap of $q^{\max}$ from 1 to 2 in which there are no high-J attractors, irrespective of grain shape and size. Fortunately, the dipole and quadrupole radiation can produce high-J attractors for $q^{\max} < 2$ (see Figure 17) for $\chi > 30^\circ$. This indicates that the dipole and quadrupole radiation can produce the range of high-J attractors and enable us to have high-J attractors for $q^{\max} = 1$ to 2. This range is easily satisfied for irregular grains (see LH07a) in a wide range of $\lambda / a_{e\perp}$. Therefore an enhancement of the degree of RAT alignment is expected for the dipole and quadrupole field compared to the case of a radiation beam.

6. DISCUSSION

In this section we provide a brief account of our accomplishments in this paper and present our outlook on further work in the field of grain alignment.

6.1. Alignment in Environments Different from ISM: Large Grains

Traditionally, grain alignment was the topic of ISM research. The gap between the studies of polarized radiation from aligned dust in environments other than the ISM and the theory of grain alignment grew so wide that researchers outside the ISM domain sometimes write papers about aligned dust and do not refer to any theoretical work on grain alignment. However, there is ample evidence of grain alignment in non-ISM environments (see Tamura et al. 1999; Rosenbush et al. 2007; Hough et al. 2007).

In many of these environments, e.g., for dust in comets and circumstellar dust, the radiation field is stronger than in the ISM, and thus RATs are stronger. One difference that one faces there is that grains may be substantially larger. As we demonstrated in the paper, for larger grains the internal relaxation gets slower, which makes one wonder whether the RAT alignment happens very differently from the ISM case. Our study shows that the direction of alignment of the angular momentum $\mathbf{J}$ with respect to the magnetic field $\mathbf{B}$ is similar to that in the presence of strong internal relaxation. This alignment can be perfect with high-J and/or low-J attractors, depending on the factor $q^{\max}$. However, the alignment of grain axes with respect to $\mathbf{J}$ depends on the initial angle between them—more precisely, on the initial value of the dimensionless parameter $p$. If this angle is smaller than a particular value, $p_\perp$, defined in the main text, which is a function of $q^{\max}$, the grain axis of major inertia $\mathbf{a}_1$ gets aligned along $\mathbf{J}$, and both $\mathbf{a}_1$ and $\mathbf{J}$ become aligned with $\mathbf{B}$. HL09 showed that pinwheel torques (e.g., $H_2$ formation, isotropic radiative torque, etc.) increase rapidly with grain size. As a result, large grains can be spun up to suprathermal rotation with $J > J_{th}$. The direct effect of suprathermal rotation is the alignment of $\mathbf{a}_1$ and $\mathbf{J}$.

Table 1 compares our results for grains without and with internal relaxation using the AMO. We can see that for high-J attractors the alignment is the same in both cases. We note that the results in the latter case were shown to be consistent with the alignment of irregular grains obtained by DDSCAT. However, the general case of alignment of large irregular grains in the absence of internal relaxation requires further studies to confirm the predictions obtained with the AMO. Further studies should consider also the effect of pinwheel torques on the alignment in the absence of internal relaxation.

6.2. Utility of AMO

The analytical model (AMO) was introduced in LH07a to describe the case of alignment in the presence of perfect alignment of $\mathbf{J}$ with the axis of maximal moment of grain inertia. It provided an excellent representation of the alignment of irregular grains within this approximation. Later, in HL08 and HL09, we applied AMO to grains in the presence of thermal fluctuations which at small values of $J$ partially randomize the alignment of $\mathbf{J}$ with the aforementioned axis. Nevertheless, AMO happened to do a nice job in these cases, adequately describing the behavior of irregular grains.

The case of no internal alignment is the extreme case of testing AMO. For instance, AMO has only one helicity axis, while multiple helicity axes are possible for an irregular grain. It is interesting that even for this case our limited testing is indicative of the AMO’s utility.

6.3. Toward Modeling of Polarized Radiation

Similar to the polarimetric work of non-ISM observers, the modeling of polarization arising from aligned grains has been mostly developing without much connection to the theory of grain alignment. Exceptions from this rule include Cho & Lazarian (2005, 2007), Pelkonen et al. (2007), Bethell et al. (2007), and Falceta-Golcaves et al. (2008). However, this modeling was done on the basis of somewhat ad hoc alignment prescriptions which should be improved as the theory gets predictive.

A step toward a more reliable polarization modeling is done in this paper where we considered grain alignment induced by the dipole and quadrupole components of the radiation field. The earlier studies assumed that the radiation was coming from a particular direction, which is, for instance, not the case for most of the grains in starless cores or accretion disks. In the latter cases, it is appropriate to decompose the radiation field in multipoles and consider the effects of the individual components. Our study shows that the parameter space for having high-J attractors differs for the alignment by the different multipole components.

Moreover, we identified the range of torque ratio, $q^{\max}$, for which the presence of dipole and quadrupole components of the

\begin{table}[h]
\centering
\caption{Comparison of Radiative Alignment for AMO}
\begin{tabular}{|c|c|c|c|}
\hline
\text{Without Internal Relaxation (This Work)} & \text{With Internal Relaxation (LH07a)} & \\
\hline
\text{High-J Attractors} & \text{Low-J Attractors} & \\
\hline
J $\parallel$ B & J aligned parallel or at some angle with B & J $\parallel$ B & J aligned parallel or at some angle with B \\
\hline
Long axes $\perp$ B & Long axes $\parallel$ or at some angle with J & Long axes $\perp$ B & Long axes $\parallel$ B \\
\hline
\end{tabular}
\end{table}

\footnote{The alternative to modeling the RAT alignment by the multipoles of the radiation field is to study the alignment numerically at every point of the data cube using the radiation field at the particular point. Naturally, this would entail much more intensive computations.}
radiation field results in alignment with high-$J$ attractors. The required range $q^{\text{max}}$ is fulfilled for irregular grains studied in HL08.

6.4. Credit to Dolginov & Mytrophanov (1976)

In our paper, we showed that the analytical results on RATs in Dolginov & Mytrophanov (1976) were incorrect (see Figure 18). Therefore, naturally, our present results on the RAT alignment obtained for grains in the absence of internal relaxation, which was also the assumption in the Dolginov & Mytrophanov (1976) study, differ from those in that study. This should not, however, undermine the pivotal significance of the Dolginov & Mytrophanov paper. This paper is important as it discovered RATs and discussed the possibility of irregular grains being aligned by RATs, even if it failed to describe it quantitatively. In addition, the notion of grain helicity, which is the central concept of AMO, can be traced back to Dolginov & Mytrophanov’s (1976) work. Moreover, as we mentioned in the introduction, the problem was so hard that the papers that followed Dolginov & Mytrophanov (1976), in all its complexity, were not able to capture correctly the physics of the RAT alignment either.

6.5. Other Processes

Our paper, for the sake of simplicity, does not consider the effect of the pinwheel torques. Such torques, e.g., torques arising due to $\text{H}_2$ formation over catalytic sites over grain surface (Purcell 1979), were discussed in the framework of the RAT alignment in HL09 for grains with strong internal relaxation. A study of the effect of these torques on large grains, for which the effects of internal relaxation are reduced, will be done elsewhere. In addition, in our paper we considered the RAT alignment with respect to the magnetic field. As we discussed in LH07a, for a sufficiently slow rate of Larmor rotation the alignment can happen also with respect to the radiation field. If, however, grains have superparamagnetic inclusions the rate of rotation increases substantially. This, potentially, provides another way of testing whether grains have or do not have superparamagnetic inclusions.

7. SUMMARY

In this paper, we continued our work on the RAT alignment using both AMO and DDSCAT calculations of torques. Our principal results can be summarized as follows.

1. We identified the range of grain size for which the internal relaxation within a normal paramagnetic grain is not important.
2. We demonstrated that, in the absence of internal relaxation, RATs can align the grain’s angular momentum with respect to the magnetic field, which is similar to the alignment in the presence of strong internal relaxation. For the internal alignment of grain axes with the angular momentum, it can be perfect, i.e., $a_1 \parallel J$ when the initial angle between them is small.
3. We studied the RAT alignment induced by dipole and quadrupole components of the radiation field assuming perfect internal alignment of the grain axis of major inertia with the angular momentum due to strong internal relaxation. Using the AMO, we found that the parameter space for the existence of high-$J$ attractors is extended compared to the earlier studied case of a single direction radiation. This parameter space is given by the range of $q^{\text{max}}$ and the angle $\chi$ between the symmetric axis of dipole and quadrupole radiation fields and the magnetic field. Therefore, higher degrees of radiative alignment are expected.
4. Our study for the joint action of dipole and quadrupole components showed that for the angle $\chi$ between the dipole and quadrupole axis and the radiation direction smaller than $\sim 20^\circ$, the dipole component has to be dominant over the quadrupole one in order to align grains with high-$J$ attractors.

Incidentally, the same paper discussed for the first time the Barnett effect in the application to interstellar grains. This induced the all-important notion of fast Larmor precession of grains and helped E. Purcell to discover the effect of Barnett relaxation.
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APPENDIX A
RATS FOR THE ANALYTICAL MODEL: AMO
RAT for the toy model in Figure 2 is given by
\[ \Gamma_{\text{rad}} = \frac{\gamma u_{\text{rad}} \lambda^2}{2} Q_{\Gamma}, \]  
(A1)
where \( Q_{\Gamma} = (Q_{\xi_1} e_1 + Q_{\xi_2} e_2 + Q_{\xi_3} e_3) \) is the vector of RAT efficiency, with \( Q_{\xi_1}, Q_{\xi_2} \) and \( Q_{\xi_3} \) the components of \( Q_{\Gamma} \) in the laboratory system. Here, \( l_2 \) is the size of the squared mirror, \( \lambda \) is the wavelength, and \( u_{\text{rad}} \) is the energy density (in units erg cm\(^{-3}\)) of the radiation field.

Using the self-similar scaling of the magnitude of RAT's obtained for an irregular grain of size \( a \) illuminated by radiation field of wavelength \( \lambda \),
\[ |Q_{\Gamma}| \sim 0.4 \left( \frac{\lambda}{a} \right)^{-3} \quad \text{for} \quad \lambda > 1.8a, \]  
and the functional forms of RATs from the AMO, we can write RAT components as follows:
\[ Q_{\xi_1}(\Theta, \beta, \Phi = 0) = \frac{|Q_{\Gamma}| q_{\text{max}}}{\sqrt{(q_{\text{max}})^2 + 1}} q_{\xi_1}(\Theta, \beta, \Phi = 0), \]  
(A4)
\[ Q_{\xi_2}(\Theta, \beta, \Phi = 0) = \frac{|Q_{\Gamma}| q_{\text{max}}}{\sqrt{(q_{\text{max}})^2 + 1}} q_{\xi_2}(\Theta, \beta, \Phi = 0), \]  
(A5)
\[ Q_{\xi_3}(\Theta, \beta, \Phi = 0) = \frac{|Q_{\Gamma}| q_{\text{max}}}{\sqrt{(q_{\text{max}})^2 + 1}} q_{\xi_3}(\Theta, \beta, \Phi = 0), \]  
(A6)
where
\[ q_{\xi_1}(\Theta, \beta, \Phi = 0) = -\frac{4l_1}{\lambda} C \left( n_1 n_2 \cos \Theta + \frac{n_1^2}{2} \cos \beta \sin 2\Theta - \frac{n_2^2}{2} \cos \beta \sin 2\Theta - n_1 n_2 \sin 2\Theta \cos^2 \beta \right), \]  
(A7)
\[ q_{\xi_2}(\Theta, \beta, \Phi = 0) = \frac{4l_1}{\lambda} C \left( n_1^2 \cos \beta \cos^2 \Theta - \frac{n_1 n_2^2}{2} \cos \beta \sin 2\Theta - \frac{n_1 n_2}{2} \sin 2\Theta + n_2^2 \cos \beta \sin^2 \Theta \right), \]  
(A8)
\[ q_{\xi_3}(\Theta, \beta, \Phi = 0) = \frac{4l_1}{\lambda} C n_1 \sin \beta [n_1 \cos \Theta - n_2 \cos \beta \sin \Theta] + \left( \frac{b}{l_2} \right)^2 \frac{2 \alpha a}{\lambda} (s^2 - 1) K(\Theta) \sin 2\Theta. \]  
(A9)

In Equations (A7)-(A9), \( C \) is a function defined as
\[ C = |n_1 \cos \Theta - n_2 \sin \Theta \cos \beta|, \]  
(A11)
where \( \Theta \) is the angle between the axis of major inertia \( a_i \) and the radiation field \( k \), \( \beta \) is the angle describing the rotation of the grain about \( a_i \) (see Figure 1, right panel); \( n_1 = -\sin \alpha, n_2 = \cos \alpha \) are components of the normal vector of the mirror tilted by an angle \( \alpha \) in the grain coordinate system, \( a, b \) are minor and major semi-axes of the spheroid, \( s = a/b < 1 \), and \( e \) is the eccentricity of the spheroid, \( l_1 \) is the distance from the mirror to the spheroid, and \( l_2 \) is the size of the squared mirror; \( K(\Theta) \) is the fitting function (see also LH07a). The second term of Equation (A9) represents the torque due to the spheroid. Assuming \( l_1 \sim \lambda \) and \( b, a \sim l_2 \ll l_1 \), then this term is subdominant compared to the first term (see Figure 2). Thus, we ignore it in our calculations.

Our calculations for the alignment in the presence of thermal fluctuations showed that the AMO can reproduce the alignment property with low \( J \) as found with RAT's obtained by DDSCAT when \( q_{\xi_1} \) is modified to (see HL08)
\[ q_{\xi_1}(\Theta, \beta, \Phi = 0) = -\frac{4l_1}{\lambda} C \left( n_1 n_2 \frac{3 \cos^2 \Theta - 1}{2} + \frac{n_1^2}{2} \cos \beta \sin 2\Theta - \frac{n_2^2}{2} \cos \beta \sin 2\Theta - n_1 n_2 \cos 2\beta \right). \]  
(A12)

This modification can arise from the imperfect scattering and/or the absorption effect by the mirror (LH07a). Also, the results in LH07a remain unchanged because the averaging over \( \beta \) for the last term goes to zero.

We adopt the AMO with \( \alpha = 45^\circ \) in this paper, unless mentioned otherwise. We also assume the amplitude of \( Q_{\xi_1} \) is comparable to that of \( Q_{\xi_2} \) and \( Q_{\xi_3} \).

RATs at a precession angle \( \Phi \) (see Figure 1, right panel) can be derived from RATs at \( \Phi = 0 \) using the coordinate system transformation as follows:
\[ Q_{\xi_1}(\Theta, \beta, \Phi) = Q_{\xi_1}(\Theta, \beta, \Phi = 0), \]  
(A13)
\[ Q_{\xi_2}(\Theta, \beta, \Phi) = Q_{\xi_2}(\Theta, \beta, \Phi = 0) \cos \Phi + Q_{\xi_1}(\Theta, \beta, \Phi = 0) \sin \Phi, \]  
(A14)
\[ Q_{\xi_3}(\Theta, \beta, \Phi) = Q_{\xi_3}(\Theta, \beta, \Phi = 0) \sin \Phi - Q_{\xi_1}(\Theta, \beta, \Phi = 0) \cos \Phi. \]  
(A15)

A.1. AMO versus the Model of Dolginov & Mytrophanov (1976)

Figure 18 shows the comparison of the RAT components for the AMO and RAT's obtained for the twisted spheroid from Dolginov & Mytrophanov (1976). It can be seen that the torques are radically different. According to LH07a, the AMO corresponds to calculations of torques for irregular grains, and thus we can conclude that the model in Dolginov & Mytrophanov (1976) does not represent adequately RATs.

APPENDIX B

DYNAMICAL EQUATIONS FOR IRREGULAR GRAINS

For irregular grains, we define a parameter which is constant during the torque-free motion:
\[ p = \frac{2l_1 E}{J^2}, \]  
(B1)
where $E$ is the total energy, $I_1$ is the inertia moment along the principal axis $a_1$, and $J$ is the value of angular momentum. Since both $J$ and $E$ are conserved during torque-free motion, $p$ is accordingly conserved. The evolution of $p$ in time due to external torques is then

$$\frac{dp}{dt} = \frac{1}{J^2} \left( 2I_1 \frac{dE}{dt} J - 2I_1 E \frac{dJ}{dt} \right). \quad \text{(B2)}$$

The energy of the grains varies as

$$\frac{dE}{dt} = (J \frac{d\omega}{dt} + \Gamma \omega). \quad \text{(B3)}$$

The first term describes the energy dissipation due to the Barnett and nuclear relaxation. The second term represents the effect of external torques on the grain rotational energy. This equation differs from that in WD03 by a factor of 2. We consider here only RATs, so $\Gamma = \Gamma_{\text{rad}} = M Q_\text{R}$. Substituting $dE/dt$ into Equation (B2), we get

$$\frac{dp}{dt} = \frac{1}{J^2} \left[ I_1 \left( J \frac{d\omega}{dt} + \Gamma \omega \right) - pJ \Gamma \right]. \quad \text{(B4)}$$

Averaging Equation (B4) over the torque-free motion, we obtain

$$\frac{J^2 dp}{dt} = MI_1 \langle Q_\omega \rangle - pJ \frac{dJ}{dt}$$
$$- \left( p - 1 \right) \frac{1 - p I_3 / I_1}{1 / I_3 / I_1}. \quad \text{(B5)}$$

where we used $\Gamma = dJ/dt$, and

$$Q_\omega = Q_{\Gamma, \omega} = Q_{a_1} \omega_1 + Q_{a_2} \omega_2 + Q_{a_3} \omega_3. \quad \text{(B6)}$$

Here, $\omega_i$ for $i = 1, 2, 3$ correspond to the components of angular velocity along three grain axes $a_1$, $a_2$, and $a_3$.

**APPENDIX C**

RAT ALIGNMENT INDUCED BY A SINGLE COMPONENT FOR THE SPHEROIDAL AMO

We study first the alignment with high-$J$ attractors. For this purpose, we consider the alignment by the first component $Q_{e_1}$, which is shown to produce the alignment with high-$J$ attractors (see LH07a). With this simplification, we can obtain the analytical results for the motion. According to Equation (31) in HL08, the averaged value of $Q_{e_1}$ over torque-free motion, and fast Larmor precession is given by

$$\langle Q_{e_1} \rangle = Q_{e_1}^{\text{max}} \left( 3 \cos^2 \xi \cos^2 \theta + \frac{3 \sin^2 \xi \sin^2 \theta}{2} - 1 \right), \quad \text{(C1)}$$

$$\langle Q_{a_1} \rangle = \langle Q_{e_1} \rangle \cos \xi, \quad \text{(C2)}$$

The aligning and spin-up torque components are then

$$\langle F \rangle_\phi = -\langle Q_{e_1} \rangle \sin \xi, \quad \text{(C3)}$$
$$\langle H \rangle_\phi = \langle Q_{e_1} \rangle \cos \xi. \quad \text{(C4)}$$

It can be seen that the torque components are functions of two alignment angles $\theta$ and $\xi$.

Let us investigate the property of these stationary points. Consider first the stationary points $\xi = \pi$ and $\theta = 90^\circ$. Equation (C4) shows that $\langle H \rangle_\phi = \langle Q_{e_1} \rangle > 0$ because $\langle Q_{e_1} \rangle = -Q_{e_1}^{\text{max}}$. In addition, the first derivative of $\langle F \rangle_\phi < 0$ (see Equation (C3)). Therefore, $\xi = \pi$ is a high-$J$ attractor with $\theta = 90^\circ$. Moreover, the stationary point $\xi = 0, \theta = 0$ is still a high-$J$ attractor because $\langle F \rangle_\phi / \langle H \rangle_\phi < 0$. This indicates that in the absence of internal dissipation, the alignment occurs in two types: the longest axis parallel and perpendicular to the magnetic field. The former is consistent with the Davis–Greenstein (1951) prediction, i.e., “right” alignment, while the latter provides “wrong” alignment (see LH07a).

The trajectory map in Figure 19 for the alignment by one component $Q_{e_1}$ exhibits two high-$J$ attractors $A$ and $D$, as expected. In addition, there are two repellors $B$ and $C$. The final state of grains depends on their initial angles $\xi_0$. For instance, grains with initial angles $| \cos \xi_0 | > 0.6$ are aligned on $A$ and $D$, but grains with $| \cos \xi_0 | < 0.6$ are constrained within two repellors $B$ and $C$, and finally damped by gas friction.

The right panel represents the evolution of $\theta$ and $\xi$ as functions of time for a few grains with $\cos \xi_0 < -0.8$. It can be seen

![Figure 19](image-url)
that the angular momentum of these grains is perfectly aligned with respect to the magnetic field, but the shortest axis \( a_1 \) is nearly perpendicular to \( J \), corresponding to “wrong” internal alignment.

**APPENDIX D**

**TRANSFORMATION OF THE COORDINATE SYSTEM**

To find the torques in the lab system \( (e_i) \) when the radiation direction \( k \) varies, we need to implement the coordinate transformation from the system \( kk_xk_y \) to \( e_i \) (see Figure 13). We assume that the magnetic field is directed along \( e_1 \); then \( k_z = k \) and \( k_y \) lies in the plane \( B, k \). Thus, \( k_x \) is perpendicular to the plane \( k, k_y \). Therefore the coordinate system \( k, k_x, k_y \) acts as the system \( e_3 \) in Figure 1.

Denoting \( Q_k \) as the torque components in the \( k \)-system, we need to know the torque components in the lab system. The transformation from the \( k \)-system to the \( e_i \)-system is carried out by three rotations with three Euler angles.

Following Figure 13, we have

\[
e_i^x = e_1 \cos \chi + e_2 \sin \chi,
\]

\[
e_i^y = -e_1 \sin \chi + e_2 \cos \chi,
\]

\[
e_i^z = e_3.
\]

The radiation direction \( k \) is determined by two angles \( \psi_k \) and \( \phi_k \) in the lab system; we therefore have the coordinate transformation from the \( k \)-system to the \( d \)-system:

\[
e_1^0 = e_1^x \cos \psi_k + e_2^x \sin \psi_k \cos \phi_k + e_3^x \sin \psi_k \sin \phi_k,
\]

\[
e_2^0 = -e_1^y \sin \psi_k + e_2^y \cos \psi_k \cos \phi_k + e_3^y \cos \psi_k \sin \phi_k,
\]

\[
e_3^0 = e_3^z \sin \phi_k + e_3^z \phi_k.
\]

Assuming that \( a_1 \) is parallel to \( J \), which is described by \( \xi, \phi \) in the lab system, then we have

\[
a_1 = e_1 \cos \xi + e_2 \sin \xi \cos \phi + e_3 \sin \xi \sin \phi,
\]

\[
a_2 = -e_1 \sin \xi + e_2 \cos \xi \cos \phi + e_3 \cos \xi \sin \phi.
\]

We can find the angles \( \Theta, \Phi, \) and \( \beta \) in the \( e_1^0 \) system:

\[
\cos \Theta = e_1^0 \cdot a_1,
\]

\[
\tan \frac{\Phi}{2} = \frac{\sin \Theta - a_1 \cdot e_2^0}{a_1 \cdot e_3^0}.
\]

\[
\beta = 2\tan^{-1} \left( \frac{\sin \Theta + a_2 \cdot e_2^0}{\sin \Theta (a_2 \cdot e_3^0 \cos - a_2 \cdot e_2^0 \sin)} \right).
\]

By substituting Equations (D4)–(D6) and (D7) and (D8) into (D9) and (D11), we obtain the angles \( \Theta, \Phi, \) and \( \beta \), and the torques \( Q_k^0 \) are interpolated. Finally, we find the corresponding torques, \( \bar{Q}_i(\xi, \phi) \), by coordinate transformation:

\[
\bar{Q}_i = C_{ij} B_{jk} Q_0^{k}.
\]

Here, the matrices \( C, B^T \) are transposal matrices of \( C \) and \( B \), inferred from Equations (D9)–(D11) and (A4)–(B1) as follows:

\[
e_i^0 = e_1^0,
\]

\[
e_j^0 = B_{ij} e_j^0.
\]

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