Detecting core-periphery structures by surprise

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Abstract – Detecting the presence of mesoscale structures in complex networks is of primary importance. This is especially true for financial networks, whose structural organization deeply affects their resilience to events like default cascades, shocks propagation, etc. Several methods have been proposed, so far, to detect communities, i.e., groups of nodes whose internal connectivity is significantly large. Communities, however, do not represent the only kind of mesoscale structures characterizing real-world networks: other examples are provided by bow-tie structures, core-periphery structures and bipartite structures. Here we propose a novel method to detect statistically significant bimodular structures, i.e., either bipartite or core-periphery ones. It is based on a modification of the surprise, recently proposed for detecting communities. Our variant allows for bimodular nodes partitions to be revealed, by letting links to be placed either 1) within the core part and between the core and the periphery parts or 2) between the layers of a bipartite network. From a technical point of view, this is achieved by employing a multinomial hypergeometric distribution instead of the traditional, binomial hypergeometric one; as in the latter case, this allows a p-value to be assigned to any given (bi)partition of the nodes. To illustrate the performance of our method, we report the results of its application to several real-world networks, including social, economic and financial ones.

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Introduction. – Detecting the presence of mesoscale structures in complex networks is of primary importance \([1,2]\). This is especially true for financial networks, whose structural organization deeply affects their resilience to shocks propagation, node failures, etc. \([3–6]\). Several methods have been proposed, so far, to detect communities, i.e., groups of nodes whose “internal” connectivity is significantly large. Communities, however, do not represent the only kind of mesoscale structures characterizing real-world networks: other examples are provided by bow-tie structures, core-periphery structures and bipartite structures. In what follows, we will focus on the last two types of topological structures.

The intuitive notion of core-periphery network, as a configuration consisting of a densely connected bunch of nodes (i.e., the core) and low-degree nodes preferentially connected to the core (i.e., the periphery ones) has been firstly formalized by Borgatti and Everett: in \([7]\) a score function indicating the extent to which a given graph partition deviates from an ideal core-periphery configuration (where the core is fully connected and the peripheral nodes are only linked to the core ones) was defined. Several later works adopted the same approach \([4,5,8]\), accompanying the error score with a significance level, computed on a properly generated ensemble of networks (see \([9]\) for a review on the topic). Detection of bipartiteness has been approached similarly, by quantifying the deviation of an observed graph partition from the ideal bipartite configuration (where edges exist only between layers and not within them) \([10,11]\).

Conversely, in recent years the detection of mesoscale structures has been faced by adopting a bottom-up approach, i.e., by defining a benchmark model against which to compare the actual network structure: in \([12]\) the authors aim at identifying the most likely generative model that may have produced a given partition; in \([13,14]\) the
authors compare the likelihood values of a stochastic block model tuned to reproduce either a core-periphery or a bipartite structure; similarly, in [15] the authors adopt a random graph model to find multiple core-periphery pairs in networks and in [16] the same authors employ the configuration model as a benchmark, showing that a single core-periphery structure can never be significant under it, seemingly confirming recent findings by the authors of the present paper [4,17].

We contribute to this stream of research by proposing a novel method to detect statistically significant bimodular structures (i.e., either bipartite or core-periphery ones). To this aim, we build upon the results of the papers [18–20] and on the very last comment that can be found in [21], by adopting a surprise-like score function. Our choice is dictated by the versatility of this kind of quantity that allows us to consider undirected as well as directed (binary) networks, a desirable feature that many of the aforementioned algorithms do not have.

Methods. – Let us start by recalling the definition of traditional surprise [18–20]:

$$S \equiv \sum_{i \geq 1} \frac{V_{int}^i}{V^i} \frac{V^{i+1} \cdot (V \cdot L)^i}{L^i} ;$$

(1)

the sum runs up to the value $i = \min \{L, V_{int}\}$, where $V$ is the volume of the network, coinciding with the total number of nodes pairs (i.e., $V = \frac{N(N-1)}{2}$ in the undirected case and $V = N(N-1)$ in the directed case), $V_{int}$ is the total number of intracluster pairs (i.e., the number of nodes pairs within the individuated communities), $L$ is the total number of links and $l^*$ is the observed number of intracluster links (i.e., within the individuated communities).

The hypergeometric distribution describes the probability of observing $i$ successes in $L$ draws (without replacement) from a finite population of size $V$ that contains exactly $V_{int}$ objects with the desired feature (in our case, being an intracluster pair), each draw being either a success or a failure: surprise is the p-value of such a distribution, testing the statistical significance of the observed partition against the null hypothesis that the intracluster link density $p_{int} = \frac{1}{V_{int}}$ is compatible with the density $p = \frac{l^*}{L}$ characterizing the (directed) random graph model.

A bimodular surprise. In order to overcome the limitations of traditional surprise whenever employed to detect bimodular structures (see the Supplementary Material Supplementarymaterial.pdf (SM) for a detailed discussion), we introduce a variant of it that is specifically designed to detect this kind of mesoscale structures.

Whenever community detection is carried out by maximizing the surprise, links are understood as belonging to two different categories, i.e., the internal ones (the ones within clusters) and the external ones (the ones between clusters). On the other hand, whenever one is interested in detecting bimodular structures (be they bipartite or core-periphery), three different “species” of links are needed (e.g., core, core-periphery and periphery links). This is the reason why we need to consider the multinomial version of the surprise, whose definition reads

$$S_{||} \equiv \sum_{i \geq l^*} \sum_{j \geq l^*_{cp}} \frac{V_i}{V} \frac{V^{-(V_i+V_j)}}{L^{-1-(i+j)}} \frac{V_{int}^i}{V^i} \frac{V^{i+1} \cdot (V \cdot L)^i}{L^i} ;$$

(2)

and that we will refer to as the bimodular surprise. The presence of three different binomial coefficients allows three different kinds of links to be accounted for. From a technical point of view, $S_{||}$ is a p-value computed on a multivariate hypergeometric distribution describing the probability of $i + j$ successes in $L$ draws (without replacement), from a finite population of size $V$ that contains exactly $V_c$ objects with a first specific feature and $V_{cp}$ objects with a second specific feature, wherein each draw is either a success or a failure. Although $i$ and $j$ are respectively bounded by the values $V_c$ and $V_{cp}$, analogously to the univariate case, $i + j \in [l^*_c + l^*_{cp}, \min \{L, V_c + V_{cp}\}]$.

The index $c$ in eq. (2) labels the core part and the index $cp$ labels the core-periphery part; whenever considering bipartite networks, the core-periphery portion will be assumed to indicate the inter-layer portion.

Bipartite networks. Let us now calculate $S_{||}$ for a purely bipartite, undirected network whose first and second layer consist of $N_1$ and $N_2$ nodes, respectively. In this case, the values of the parameters read $V_c = \frac{N_1(N_2-1)}{2}$ (here, the label $c$ indicates the internal volume of one of the two layers), $V_{cp} = N_1N_2$, $l^*_c = 0$ and $l^*_p = L$. The latter condition implies that only the addendum corresponding to $i = 0$, $j = l^*_p$ = $L$ survives; thus, our bimodular surprise reads

$$S_{||} = \left(\frac{V_{cp}}{V} \right) \left(\frac{V}{l^*_p} \right) = \frac{\min \{N_1, N_2\}}{\frac{V_{cp}}{V} \left(\frac{V}{l^*_p} \right) \left(\frac{N_1 + N_2}{N_1 + N_2 - 1}\right)} \frac{V_{int}^i}{V^i} \frac{V^{i+1} \cdot (V \cdot L)^i}{L^i} ;$$

(3)

which can be significant, as it should be: in fact, a number of inter-layer links exists, above which the observed bipartite structure is significantly denser than its random counterpart (see also fig. 1). Notice that eq. (3) can be directly employed to test the significance of any bipartite configuration with no intra-layer links, against the null hypothesis that such a configuration is compatible with the random graph model: eq. (3) shows that, in the considered case, the computation of the searched p-value boils down to calculate the ratio between the number of bipartite networks with $l^*_p$ links and the number of generic configurations with the same number of connections.

Star-like networks. Let us now implement our bimodular surprise $S_{||}$ for undirected, star-like configurations, with a fully connected core plus a periphery of nodes, each of which is connected to just one core node. For the moment, let us suppose that the number of core nodes coincides with the number of periphery nodes. The core portion is identified with a clique of $N_1$ nodes: our parameters, thus, read $V_c = \frac{N_1(N_1-1)}{2}$ and $V_{cp} = N_1^2$. Since,
however, \( t^*_{cp} = \frac{N_1(N_1-1)}{2} \), the (only) sum indexed by \( j \) reduces to the single addendum

\[
S_\parallel = \frac{\left(\binom{N_2}{N_1} \right)}{\left(\frac{N_1(N_1+1)}{2}\right)}
\]

which is \( \approx 10^{-2} \) for \( N_1 = 3 \) and decreases (the corresponding partition, thus, becomes more and more significant) as \( N_1 \) increases. Notice that the traditional surprise would identify a community structure where each peripheral node is counted as a community on its own with a comparable significance (see also the SM): \( S_\parallel \), however, is able to recover the ground-truth structure of the observed network.

**k-star networks.** Let us now generalize the star-like network model, by considering a graph with \( k \) peripheral nodes linked to each core node. Our parameters now read

\[
V = \frac{(N_1+kN_k)(N_1+kN_k-1)}{2}, \quad V_c = \frac{N_1(N_1-1)}{2}, \quad V_{cp} = kN_k^2, \quad t^*_{cp} = \frac{N_1(N_1-1)}{2}, \quad \text{and} \quad t^*_{cp} = kN_1.
\]

Again, thus, the (only) sum indexed by \( j \) reduces to just one addendum, i.e.,

\[
S_\parallel = \frac{\left(\binom{kN_k}{N_1} \right)}{\left(\frac{N_1(N_1+kN_k-1)}{2} \right)}
\]

whose behavior is shown in Fig. 1: briefly speaking, both when the number \( N_1 \) of core nodes rises, while keeping the number of leaves fixed, and when the number \( k \) of leaves rises, while keeping the number of core nodes fixed, the bimodular surprise becomes increasingly significant, always recovering the ground-truth partition.

**Asymptotic results.** The presence of binomial coefficients in the definition of \( S_\parallel \) may cause its explicit computation to be demanding from a purely numerical point of view. This subsection is devoted to derive some asymptotic results, in order to speed up the computation of \( S_\parallel \). Similar calculations for what concerns the traditional surprise have been carried out in [21].

Let us start by considering eq. (3). By Stirling-expanding the binomial coefficients appearing in it, one obtains the expression

\[
S_\parallel = \frac{(\frac{V_{cp}}{t^*_{cp}})}{\left(\frac{t^*_{cp}}{p_{cp}}\right)} \approx \frac{p_{cp}^{t^*_{cp}}(1-p_{cp})^{V_c-t^*_{cp}}}{p_{cp}^{t^*_{cp}}(1-p_{cp})^{V_{cp}-t^*_{cp}}}
\]

having defined \( p \equiv \frac{t^*_{cp}}{V_c} \) and \( p_{cp} \equiv \frac{t^*_{cp}}{V_{cp}} \) (see the SM for the details of the calculations). In the sparse case, i.e., when \( p \ll 1 \) and \( p_{cp} \ll 1 \), eq. (6) reduces to \( S_\parallel \approx \left(\frac{p}{p_{cp}}\right)^{t^*_{cp}} \). This expression makes it explicit that a given (bi)partition is statistically significant if its link density, \( p_{cp} \), is large enough to let it be distinguishable from a typical configuration of the random graph model, characterized by link density \( p \).

Let us now move to the core-periphery case and consider partitions satisfying the condition \( t^*_{cp} + t^*_{cp} = L < V_c + V_{cp} \); in this case, one can derive the result

\[
S_\parallel = \frac{(\frac{V_{cp}}{t^*_{cp}})}{\left(\frac{t^*_{cp}}{p_{cp}}\right)} \approx \frac{p^{t^*_{cp}}(1-p)^{V_c-t^*_{cp}}}{p_{cp}^{t^*_{cp}}(1-p_{cp})^{V_{cp}-t^*_{cp}}}
\]

having defined \( p \equiv \frac{t^*_{cp}}{V_c} \), \( p_{cp} \equiv \frac{t^*_{cp}}{V_{cp}} \) and \( p_{cp} \equiv \frac{t^*_{cp}}{V_{cp}} \) (7)

Even if interpreting eq. (7) is less straightforward, it is, however, clear that the significance of the observed partition is a consequence of the interplay between the link density of the core and core-periphery regions (the link density of the periphery has been supposed to be zero; see also the SM for the details of the calculations).

**Results.** Let us now move to analyze some real-world systems: we will employ our novel definition of surprise to understand if the considered networks have a significant bimodular structure (i.e., either bipartite or core-periphery). To this aim, we will search for the (optimal) partition that minimizes \( S_\parallel \) by employing a modified version of the PACO algorithm [20] whose pseudocode is explicitly shown in the
SM and a Python version of which is freely available at the URL https://github.com/jeroenvldj/bimodular_surprise. In what follows we will consider directed as well as undirected networks.

Social networks. Let us start our analysis by considering a number of undirected social networks (see fig. 2). As a first example, let us consider the Zachary Karate Club. Although the latter is commonly employed as a benchmark for community detection, it is also characterized by a clear bimodular structure whose core nodes are represented by the masters, their close disciples and a fifth node “bridging” the two masters. Upon looking at the subgraphs constituted by the masters’ ego-networks, almost ideal (i.e., à la Borgatti) core-periphery networks are observable. A similar comment can be done when considering the network of relationships among the characters of “Les Misérables”: the main characters (e.g., Valjean, Javert, Cosette, Marius) belong to the core, while the large number of secondary characters linked to them constitute the periphery of such a network (see, for example, the nodes linked to Valjean); intuitively, again, core nodes are very inter-connected while the link density of the periphery is very low. As for the Zachary Karate Club network, there seem to be (core) nodes bridging two dense core subsets.

Let us now consider the connected component of the NetSci co-authorship network\(^1\). A core-periphery structure is, again, recovered (although the core is not very dense) where core nodes represent senior scientists (e.g., Stanley, Barabási, Watts, Kertész) and periphery nodes represent younger colleagues, students, etc. It is interesting to observe that the senior scientists share relatively few direct connections, while being connected to a plethora of younger collaborators; even more so, the structure of the co-authorship network seems to reflect the structure of the underlying collaboration network, with each research group seemingly being quite separated from the others.

Economic networks. Let us now consider an economic network, i.e., the directed representation of the World Trade Web (WTW) in the years 1950–2000: as usual, nodes are world countries and links are trade relationships (i.e., exports, imports) between them. Upon running our bimodular surprise optimization we find a clear core-periphery structure with the core including the richest countries and several developing nations and the periphery including some of the poorest nations (e.g., several African nations throughout our dataset —see also fig. 4 where only the years 1960, 1980 and 2000 are shown).

We also observe an interesting dynamics, causing the core size to rise (it represents the ≃ 30% of nodes in 1992 and the ≃ 60% of nodes in 2002) and progressively include countries previously belonging to the periphery. Such a dynamics—that can be interpreted as a signal of ongoing

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\(^1\)The NetScience collaboration network is available at: http://vlado.fmf.uni-lj.si/pub/networks/data/collab/netscience.htm (accessed 2018-05-01).

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Fig. 3: Left panel: core-periphery structure of US political blogs [22]: a core of the most influential blogs (be they republican or democratic), surrounded by a periphery of loosely connected, less important blogs is clearly visible. Notice that blogs are grouped independently of their political orientation. Right panel: core-periphery structure of US airports. As for the NetSci co-authorship network, each core airport seems to be surrounded by a quite large number of periphery airports, sharing relatively few connections between themselves.

Fig. 4: Core-periphery structure of the World Trade Web (black: core nodes; gray: periphery nodes). Loosely speaking, while the richest and several developing countries are found to belong to the core, the poorest nations belong to the periphery (e.g., several African nations, throughout our dataset). Notice that the core size increases with time: apparently, thus, the system becomes increasingly integrated, confirming a result found in [17], where it was shown that the size of the WTW strongly connected component increases with time as well.

integration— confirms the results found in [17], where it was shown that the size of the WTW strongly connected component (SCC) increases with time as well. Although the SCC and the core portion of the World Trade Web do not perfectly overlap, many similarities between the two structures are indeed observable.

Financial networks. Let us now consider a directed financial network, i.e., e-MID, the electronic Italian Interbank Market. Here, we compare two different datasets: the first one collects the 2005–2010 interbank transactions during the so-called maintenance periods [24]; the second one collects interbank transactions on a daily basis from 1999 to 2012 [13,14]. The main difference between the two datasets lies in their level of aggregation: notice, in fact, that the first one basically collects data on a monthly basis.

Let us start by analyzing the first dataset. As fig. 5 shows, its structure undergoes an interesting evolution: after an initial period of two years, where a large periphery of loosely connected nodes (≈ 70%) exists, a transient period of one year (i.e., 2007) during which the percentage of nodes belonging to the core rises, is visible. Afterwards, an equilibrium situation seems to be re-established with the percentage of core and periphery nodes basically coinciding. Even if the total number of banks registered in the dataset steadily decreases after 2007, this does not seem to affect the type of banks belonging to the core and to the periphery, i.e., Italian and foreign banks, respectively.

Let us now move to the analysis of the second dataset. As fig. 6 shows, the analysis of the link density of the portions in which $S_{\parallel}$ partitions the network reveals that, overall, a core-periphery structure seems to characterize the daily data better than a bipartite structure. This picture, however, seems to be less correct from 2008 on: as the last portion of the first panel of fig. 6 shows, a bipartite structure occurs more often than a core-periphery structure during this period. Two snapshots of the network are also explicitly shown, illustrating the values of link density characterizing the different network portions.

Other kinds of networks. As a last example, let us consider the US airports network in its directed representation (see fig. 3). Examples of core airports are the ones of New York, Indianapolis, Salt Lake City, Seattle, etc. The periphery airports are preferentially attached to the core ones. This system shares interesting similarities with the NetSci co-authorship network: each core airport, in
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Fig. 5: Core-periphery structure of e-MID maintenance periods (gray: core nodes; white: periphery nodes). After an initial period of two years characterized by an approximately constant value of the core and periphery size, a structural change took place in 2007 and the percentage of nodes belonging to the core steadily rose until 2008. Afterwards, an equilibrium seemed to be re-established. This may be due to a decrease in the total number of nodes which, however, does not affect the type of banks belonging to the core (Italian banks) and to the periphery (foreign banks). Networks are directed but we have omitted the link directionality for the sake of readability.

fact, seems to be surrounded by a quite large number of periphery airports, sharing few internal connections.

Discussion. – It is hard to underestimate the importance of the presence of bimodal mesoscale structures in real-world networks: while the authors in [25] show that the most robust topology against random failures is the core-periphery one, understanding the relationship between a given node systemicness and its coreness is of paramount importance in finance [6]. In the same field, a core-periphery structure is believed to reflect the “essential” function of banks: the core ones tie the periphery ones into a single market through their intermediation activity [3]. On the other hand, a bipartite structure would reflect the absence of intermediation, i.e., a market displaying preferential trading [13].

In this paper we have proposed a novel measure for bimodal mesoscale structures detection. To this aim, we have adopted a surprise-like score function, by considering the multivariate version of the quantity proposed in [20]. Employing this kind of quantities means implementing a bottom-up approach, i.e., letting the modular structure to be extrapolated from the data and not imposed a priori as in previous approaches [7,14].

Most importantly, such a comparison is based on a properly defined null model, allowing the significance of a given partition to be quantifiable via a p-value. As for the traditional surprise, the reference model is the (directed) random graph model that constrains the total number of observed connections, while randomizing everything else. The choice of employing such a benchmark is dictated by a number of recent results, pointing out that several mesoscale structures of interest (e.g., the core-periphery one, the bow-tie one, etc.) are actually compatible with—and hence undetectable under—a null model constraining the entire degree sequence(s) [4,16].

While solving the problem of consistently comparing an observed structure with a “random” model of it, our approach also solves a second drawback affecting the methods in [3,7] and pointed out in [16]: ideal structures as the ones searched by algorithms à la Borgatti are very reliant on the nodes degree, with the core often composed of just the nodes with the largest number of neighbors. This is not necessarily true when a benchmark is adopted for comparison [12]: as previously discussed, the significance of a given partition detected by surprise results from the

Fig. 6: Mesoscale structure of e-MID daily data. Although for the vast majority of snapshots a core-periphery structure seems to better represent the e-MID network, the number of times a bipartite structure is observed increases after 2008. The middle and bottom panels explicitly show two different snapshots of e-MID: the first one is characterized by the chain of inequalities \( c_p < c_{cp} < c_c \); the second one, instead, shows a configuration for which the values \( c_p \approx c_c < c_{cp} \) are observed, indicating the presence of a bipartite structure (when referring to bipartite structures, the label \( c_p \) is assumed to indicate the inter-layer portion). Networks are directed but we have omitted the link directionality for the sake of readability.
interplay between the link density values of the different network areas.

This also sheds light on the relationship between apparently conflicting structures co-existing within the same network configuration: generally speaking, traditional and bimodal surprise optimization should be considered complementary — rather than mutually exclusive — steps of a more general analysis. As the example of the US political blogs confirms, it is indeed possible that a community structure co-exists with a core-periphery structure; a second, less trivial, example is provided by the World Trade Web, whose community structure has been studied in [26] but whose significance has, then, been questioned [27].

As a last comment, we would like to stress that the two approaches to mesoscale structures detection that have been proposed so far — comparing an observed structure with a benchmark [15,16] and searching for the model best fitting a given partition [12–14,23] — can be supposed to be complementary, since a non-significant structure under a given benchmark is surely more compatible with it. Employing a benchmark, however, provides an advantage, i.e. making the statistical significance of a given structure explicit — something that remains “implicit” when employing the fitting procedure. In other words, searching for the best fit may push one to enrich a model with an increasing amount of information whose relevance cannot be easily clarified. Such a problem seems to affect all likelihood-based algorithms unless a more refined criterion to judge the goodness of a fit is employed: solutions like the one of adopting criteria like the Akaike information criterion et similia have been proposed [28].

The present work calls for a generalization to weighted mesoscale structures detection, a field where relatively little has been done so far [29,30].

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