Optimization of texture shape to minimize the friction coefficient and consideration of update equations for design variables

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Abstract
In this study, texture shape optimization is performed to minimize the friction coefficient based on the adjoint variable method under constant load. The shape update equation based on the gradient descent method is regarded as a differential equation, and in order to improve the accuracy of the solution of the differential equation, a new shape update equation is proposed that adapts the second-order Taylor expansion. Shape optimization is performed for four different textures, and the results are compared with those based on the gradient descent method and the proposed shape update equation. FreeFEM++ is used for the calculation.

Keywords shape optimization, friction coefficient, texture, finite element method, shape update equation

1. Introduction
Friction and wear occur on the sliding surfaces of machine parts, resulting in energy and material losses. Technologies to reduce such friction and wear are therefore attracting attention. Their advantages include reduced energy consumption and longer life of machine parts. Since all machines rely on mechanical parts, reduction of friction and wear is anticipated to deliver considerable economic benefits [1]. There are two main methods of reducing the friction between sliding surfaces: to improve the materials comprising the sliding surfaces [2, 3] and to improve the surface shape. One method of applying the latter technique is to create "texture" by machining grooves or holes in the sliding surfaces [4–6]. Friction can be reduced by applying a texture to a sliding surface that is fluid-lubricated. The degree of change in friction depends on the shape of the texture. In this study, the shape optimization of textures is performed under constant load based on the adjoint variable method [7,8] aiming to reduce the friction coefficient. A new shape updating equation is proposed, and the results are compared with those obtained when the gradient descent method is used. FreeFEM++ is used for the calculation [9,10].

2. Formulation of the shape optimization problem

2.1 Formulation
This study assumes fluid-lubricated sliding surfaces, in which a fluid is present between two surfaces that are in relative motion without contact. A model is shown in Fig. 1. The whole domain is defined as Ω and its boundary as Γ1. The domain where the texture exists is defined as ω and its boundary as Γ2. Domain Ω includes domain ω. The governing and adjoint equations are calculated for the domain Ω, and the boundary conditions are given in Γ1. The Poisson equation for smoothing is calculated for the domain ω. The boundary conditions are given in Γ2. The friction coefficient µ is obtained by dividing the frictional force \( F \) by the load \( W \), as shown in (1) [11].

\[
J = \mu = \frac{F}{W} = \frac{\int_{\Omega} \left( \frac{\eta U}{h} + \frac{h}{2} \frac{dp}{dx} \right) \, d\Omega}{\int_{\Omega} (p) \, d\Omega},
\]  

(1)

where \( \eta \) and \( U \) are the viscosity and the velocity in the \( x \)-direction of the friction surface and are constants. \( h \) and \( p \) are the oil film thickness and pressure and are variables given at each nodes. When the performance function is defined by the friction coefficient, it is not possible to make an evaluation under constant-load conditions because the load changes with each calculation iteration. In actual experiments, the friction coefficient is compared under constant load. Therefore, as shown in (2), the performance function is defined in terms of frictional force, and a calculation is added to keep the load constant by adjusting the basic oil film thickness.

\[
J = F = \int_{\Omega} \left( \frac{\eta U}{h} + \frac{h}{2} \frac{dp}{dx} \right) \, d\Omega.
\]  

(2)

The Reynolds equation is used as the governing equation, as shown in (3).

\[
\frac{\partial}{\partial x} \left( h^3 \frac{dp}{dx} \right) + \frac{\partial}{\partial y} \left( h^3 \frac{dp}{dy} \right) - 6\eta U \frac{\partial h}{\partial x} = 0 \text{ in } \Omega.
\]  

(3)
The boundary conditions are given by (4).

\[ p = 0 \quad \text{on } \Gamma_1. \]  

(4)

Negative pressure is also eliminated, as shown in (5) \[12\].

\[ p \geq 0 \quad \text{in } \Omega. \]  

(5)

From (2) and (3), the Lagrange function is defined as shown in (6).

\[ J^* = J + \int_\Omega \left[ \lambda \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + \frac{3}{h} \left( \frac{\partial p}{\partial x} \frac{\partial h}{\partial x} + \frac{\partial p}{\partial y} \frac{\partial h}{\partial y} \right) - \frac{6\eta U h}{h^3} \frac{\partial h}{\partial x} \right] d\Omega \]

\[ = \int_\Omega \frac{L}{\partial h} d\Omega. \]  

(6)

From the first variation of the Lagrange function, the adjoint equation and the gradient equation with respected differential term in terms of a difference formula, (14) is finally obtained.

\[ h^{n+1} = h^n - \Delta \alpha \left( \frac{\partial L}{\partial h} \right)^n - \frac{\Delta \alpha}{2} \left( \frac{\partial L}{\partial \alpha} \right)^n \left( \frac{\partial L}{\partial h} \right)^{n-1}. \]  

(14)

Since (14) takes into account even higher order differential terms than (12), it is expected to improve the accuracy of solutions in differential equations for Shape Update Equations. In this study, we compare the results of these equations.

2.2 Computational flow

The computational flow of shape optimization in this paper is shown as follows. In the shape update in Step 8), the Shape Update Equations are applied as shown in (12) and (14).

1) Input of computational conditions and setting a computational model.
2) Pressure field analysis of the governing equation.
3) When the pressure is integrated and the load obtained is close to the set value, proceed to the next step. If not, update the basic oil film thickness and return to Step 2).
4) Compute the performance function and the convergence detection formula \[\left( J^{n+1} - J^n \right) / J^0 \] and end the computation if it is smaller than the convergence criterion \( \epsilon \). If not, proceed to the next step.
5) Compute adjoint variables based on the adjoint equations.
6) Compute the gradient of oil film thickness \( \partial L/\partial h \).
7) Smooth the gradient by applying Poisson equation.
8) Update the shape and return to Step 2).

3. Computational conditions

For the four types of textures shown in Figs. 2, 3, 4 and 5, optimization of the texture depth is performed under the conditions shown in Table 1. In the calculation, the target load is given as 10 [N], the velocity of
Table 1. Computational conditions.

| Target Load W | 10[N] |
|----------------|-------|
| Lower surface velocity in x direction U | 1000.0 [mm/s] |
| Viscosity of lubricating oil η | 0.08 [Pa·s] |
| Initial depth of a texture h_{dep} | 0.01 [mm] |
| Surface area ratio of textures | 35 [%] |

friction surface is given as 1000 [mm/s], the viscosity of lubricating oil is given as 0.08 [Pa·s], the initial depth of the texture is given as 0.01 [mm], and the area fraction is given as 35 [%]. For interpolation of the state and the adjoint variables, the first-order triangular element is employed for the variable \( p \), and the second-order triangular element is used for the variables \( \lambda \) and \( h \).

4. Numerical experiments

4.1 Consideration of friction coefficient

The shape optimization is performed and the results are compared using Shape Update Equations (12) and (14). The history of the friction coefficient is shown in Figs. 6, 7, 8 and 9. The horizontal axis shows the number of iterations and the vertical axis shows the friction coefficient. The orange line shows the results when using Shape Update Equation (14) and the blue line shows the results when using Shape Update Equation (12). The gray line shows the difference between these two results. The difference increases and then reaches a ceiling. It can be seen that the proposed Shape Update Equation rapidly approaches the ceiling at the beginning of the calculation. The final friction coefficients are smaller and the number of calculations is reduced in some cases, when the proposed shape updating equation is used for four different textures.

4.2 Consideration of final shape

The final shape of texture using (14) is shown in Figs. 10, 11, 12 and 13. The final shape using (12) is shown in Figs. 14, 15, 16 and 17. The contours in these figures show the texture depth. For each final shape, a depressed area was obtained in the center. It is also confirmed that the texture of the final shape is deeper when (14) is used.

5. Conclusions

In this study, the finite element method and the adjoint variable method were used to optimize the tex-
texture shape by adjusting the basic oil film thickness and adding a calculation to keep the load constant. The Taylor expansion of the Shape Update Equation based on the gradient descent method is proposed as a new Shape Update Equation. As a result, the friction coefficient was reduced by reducing the frictional force under constant load. When the proposed Shape Update Equation was used, it was confirmed that a lower friction coefficient can be obtained and the number of calculations is reduced in some cases. These results were confirmed for four different textures.

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References

[1] A. Hase, Fundamental of tribology (in Japanese), Journal of the Japan Society for Precision Engineering, 81 (2015), 643–647.

[2] N.A. El-Mahallawy, S.H. Zoalfakar and A.A.G.A. Maboud, Microstructure and mechanical properties of Al/SiC surface composite with different volume fractions using friction stir process, IOP Conf. Ser.: Mater. Sci. Eng., 634 (2019), Code 012046.

[3] K. Dejun and Z. Ling, Tribological properties of chemical vapor deposited diamond film on YT14 cemented carbide under water lubrication condition, J. Test. Eval., 48 (2020), 4767–4779.

[4] A. Codrignani, B. Frohnapfel, F. Magagnato, P. Schreiber, J. Schneider and P. Gumbsch, Numerical and experimental investigation of texture shape and position in the macroscopic contact, Tribol. Int, 122 (2018), 46–57.

[5] S. Sharma, G. Janwal and R.K. Awasthi, Numerical study on steady state performance enhancement of partial textured hydrodynamic journal bearing, Ind. Lubr. Tribol., 71 (2019), 1055–1063.

[6] B. Manser, I. Belaidi, A. Hamrani, S. Khelladi and F. Bakir, Texture shape effects on hydrodynamic journal bearing performances using mass-conserving numerical approach, Tribology - Materials, Surfaces and Interfaces 14 (2020), 53–50.

[7] E. Katamine and H. Azegami, Solution to viscous flow field domain optimization problems (Approach by the traction method) (in Japanese), Trans. Jpn. Soc. Mech. Eng. B., 60 (1994), 3859–3866.

[8] E. Katamine and H. Azegami, Domain optimization analysis of viscous flow field (In the case of considering convective term) (in Japanese), Trans. Jpn. Soc. Mech. Eng. B., 61 (1995), 1646–1653.

[9] F. Hecht, New development in FreeFEM++, J. Numer. Math., 20 (2012), 251–265.

[10] K. Ootsuka and T. Takaishi, Finite element analysis using mathematical programming language FreeFEM++ (in Japanese), KYORITSU SHUPPAN, Tokyo, 2014.

[11] Y. Hori, Hidrodynamic lubrication, Springer-Verlag, Tokyo, 2006.

[12] M. Oda, K. Iwamoto, K. Tanaka and T. Fujino, Lubrication characteristics of sliding surfaces with dimples or grooves - In the case of surface texturing applied on parallel surfaces (in Japanese), Journal of the Japan Institute of Marine Engineering, 53 (2018), 727–736.