DETECTION OF EARTH-LIKE PLANETS USING APODIZED TELESCOPES

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ABSTRACT

The mission of NASA’s Terrestrial Planet Finder (TPF) is to find Earth-like planets orbiting other stars and characterize the atmospheres of these planets using spectroscopy. Because of the enormous brightness ratio between the star and the reflected light from the planet, techniques must be found to reduce the brightness of the star. The current favorite approach to doing this is with interferometry: interfering the light from two or more separated telescopes with a π phase shift, nulling out the starlight. While this technique can, in principle, achieve the required dynamic range, building a space interferometer that has the necessary characteristics poses immense technical difficulties. In this paper, we suggest a much simpler approach to achieving the required dynamic range. By simply adjusting the transmissive shape of a telescope aperture, the intensity in large regions around the stellar image can be reduced nearly to zero. This approach could lead to construction of a TPF using conventional technologies, requiring space optics on a much smaller scale than the current TPF approach.

Subject headings: instrumentation; interferometers — planetary systems — stars: imaging — techniques: interferometric

1. INTRODUCTION

The mission of the Terrestrial Planet Finder (TPF) is to find Earth-like planets orbiting other stars by imaging and to determine their atmospheric constituents by spectral analysis of their reflected light (Beichman, Woolf, & Lindemth smith 1999). Imaging provides separation of the light reflected from the planet from that of the star. Because the star is so bright compared with the planet, the dynamic range required is of order 10^9 at visible wavelengths. The “book” approach proposed for TPF is to operate in the infrared (7–14 μm) where the dynamic range required is “only” 10^6. However, working in the IR increases the resolution requirement of the telescope or interferometer needed to separate the image of the planet from the image of the star. A 20 m interferometer baseline or telescope diameter may be sufficient in the visible band, but, in the IR, a 200 m telescope or 200 m baseline interferometer is required. Such a large telescope is out of the question, and a 200 m interferometer cannot be built on a single structure, so free flyers are needed, making for very difficult construction and operation. The proposed interferometer would be used in a mode where the starlight is “nulled” out in a narrow fringe. This fringe is then swept around by rotating the interferometer, allowing the much fainter planet to be detected.

An optical telescope produces an image of an unresolved star that forms the familiar Airy pattern, which consists of a bright central peak surrounded by concentric rings of ever decreasing brightness. These rings are due to diffraction from the sharp circular aperture of the telescope. Apodization is a technique that can be used to decrease the brightness of the diffraction rings. It can be accomplished by either altering the shape or by modifying the transmission of the aperture, or both.

In this paper we consider a simpler approach to TPF that achieves the dynamic range required to separate the light reflected from an Earth-like planet from that of its star. We suggest that apodizing a telescope with a filter having an optimized shape and transmission would sufficiently reduce the diffraction lobes of the telescope to allow detection of Earth-like planets orbiting Sun-like stars. Techniques for reducing optical diffraction lobes have been in use since the 19th century. Analogous algorithms are commonly used for sideband reduction in radio astronomy and electronic signal processing applications (Harris 1987), and an excellent review of optical techniques is given in Jacquinot & Roizen-Dossier (1964). Oliver (1975) suggested apodizing the still-to-be-built Large Space Telescope to allow that telescope to detect Jupiter-sized exoplanets. In 1980, Black (1980) carried out laboratory experiments, showing that apodizing could be used to separate two point sources that had a 10^3 ratio and a 1" separation, comparable to the magnitude ratio and separation of Sirius and its white dwarf companion. Watson et al. (1991) suggest that apodization could be combined with a coronagraph in a space-based telescope to directly detect exoplanets.

2. APPROACH

We propose to use an approach that does not require nulling interferometry to achieve the dynamic range required for planet detection. This approach provides sufficient dynamic range at separations of a few times the diffraction limit of a telescope at visible or near-IR wavelengths, so telescope size (or interferometer baseline) is greatly reduced. Our approach is elegantly simple. We place a square, transmission-weighted apodizing aperture in the telescope pupil or at a relay image of the telescope pupil. The advantage of a square aperture was suggested by Zanoni & Hill (1965), who were interested in measuring the gravitational deflection of starlight by the Sun without having to wait for a solar eclipse. The square aperture results in most of its extended diffraction occurring along the x- and y-axes perpendicular to the edges of the square and strong suppression around the diagonal directions. We then apodize the aperture with a transmission function that has peak transmission in the center of the aperture and drops off toward the edges of the aperture. This results in producing extended regions around the diagonals of the point-spread function (PSF) that drop precipitously in intensity. In those regions, the intensity is less than 10^-9 of the PSF peak, even for realistic levels of random wave front error in the pupil (1/72 wave of...

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random roughness in the surface finish) and bandpasses (3000 Å). However, it is obvious that any telescope that can directly image exoplanets will have to be of extremely high quality. Ripple and imperfections in the mirror, spiders, or mirror supports will all diffract light at angles comparable to the expected angular separation of a planet (Angel, Cheng, & Woolf 1986; Breckinridge, Kuper, & Shack 1982). Trauger et al. (2000) suggest that high-order, very small throw adaptive optics may be needed to correct imperfections in the mirror.

The format of the transmission function is critical to achieving the full dynamic range. That transmission function is formed by taking the product of two functions, one aligned with the vertical edges of the square aperture and the other aligned with the horizontal edges. Using these crossed one-dimensional functions provides 3–4 orders of magnitude better suppression of the diffraction sidelobes than using a circular symmetric apodization, even when it is combined with a square aperture.

The intensity of the PSF of a diffraction-limited telescope is just the square modulus of the Fourier transform of the pupil function of the telescope. A circular aperture having a constant transmission inside the pupil and zero outside has a PSF in the familiar Airy pattern of a bright central peak surrounded by alternate dark and bright rings.

For an unapodized rectangular aperture of width \( w \) and height \( h \),

\[
I(x, y) = I(0, 0) \left( \frac{\sin \alpha}{\alpha} \right)^2 \left( \frac{\sin \beta}{\beta} \right)^2,
\]

where \( \alpha = \pi x w / \lambda R \), \( \beta = \pi y h / \lambda R \), and \( R \) = distance from pupil to image.

Along the diagonal (\( |x| = |y| \)) of a square aperture (\( h = w \)),

\[
I(x, y) = I(0, 0) \left( \frac{\sin \alpha}{\alpha} \right)^2 \left( \frac{\sin \alpha}{\alpha} \right)^2,
\]

where \( \alpha = \pi x w / \lambda R \), \( \beta = \pi x w / \lambda R \), and \( R \) = distance from pupil to image.

Fig. 2.—Image of two point sources through an apodized and unapodized square aperture. Upper left: 10⁶ point source ratio. Upper right: 10⁹ ratio, 1/100 wave rms wave front error. Lower left: 10⁹ ratio and 1/72 wave rms wave front error. Lower right: 10⁹ ratio and 1/50 wave rms wave front error.

Fig. 3.—Image of two point sources through an apodized and unapodized circular aperture. Upper left: 10⁰ : 1 point source ratio, no apodization. Upper right: 10⁰ : 1 ratio with sonine apodization (\( \varphi = 5 \)). Lower left: 10⁰ ratio, no apodization. Lower right: 10⁰ ratio with sonine apodization.
Fig. 4.—Contour plots of two point sources with 10° ratio and 1/72 wave rms wave front error imaged with a square aperture and different apodization. Upper left: Sonine apodization, \( n = 4 \). Upper right: Sonine apodization, \( n = 5 \). Lower left: \( \cos^2 \) apodization. Lower right: \( \cos^4 \) apodization.

\[
I(x, x) = I(0, 0) \left( \frac{\sin \alpha}{\alpha} \right)^4.
\]

The intensity in the diffraction pattern drops rapidly along the diagonal directions. If we also apodize the square with crossed transmission functions perpendicular to the edges of the square, the intensity in the diagonal drops even faster with increasing distance from the central peak. The transmission of the aperture must have the general form \( T(x, y) = F(x)F(y) \), providing rapid attenuation along the square aperture diagonal.

We have numerically tested the combination of a square aperture and several different transmission functions, and the results appear to be quite insensitive to the functional form of the apodization. The (slightly) best transmission functions tested so far are the so-called sonine functions suggested by Oliver (1975). Crossed sonine functions have a transmission function of the form

\[
T(x, y) = \begin{cases} 
(1 - x^2)^{-\nu}(1 - y^2)^{-\nu} & \text{if } -1 \leq x \leq 1, -1 \leq y \leq 1, \\
0 & \text{otherwise},
\end{cases}
\]

where \( \nu \) is a small integer such as 3, 4, or 5.

One trade-off with apodization is that the overall aperture transmission is reduced below 20%, so the optimum apodizer is one that combines the highest transmission with the best reduction of diffraction sidelobes. We tested the case of combining a square aperture and a circularly symmetric transmission sonine function, but this results in 10^3 less dynamic range.

Fig. 5.—Diagonal cuts through monochromatic PSFs: (1) unapodized circular aperture; (2) square aperture, no apodization; (3) circular aperture, sonine apodization \( (\nu = 4) \); (4) square aperture, sonine apodization.

Fig. 6.—Diagonal cuts through PSF images for polychromatic light: (1) circular aperture, no apodization; (2) circular aperture, sonine apodization; (3) square aperture, sonine apodization; (4) square aperture, sonine apodization, plus planet down 10° from the star. The vertical dashed line shows the point in the cut that is separated by three diffraction-limited elements from the central peak.
We also tried various values for $\nu$ with the sonine function and two other transmission functions: $\cos^2(x) \cos^2(y)$ and $\cos^2(x) \cos^4(y)$ (evaluated from $-\pi/4$ to $\pi/4$). Comparisons of using the different-shaped functions are shown in our simulation results in the next section.

3. SIMULATIONS

The dynamic range achieved by using a square aperture combined with crossed transmission functions is demonstrated by results shown in Figures 1–6.

Figure 1 shows the square aperture embedded in a larger field (upper left), the image of two point sources (100 : 1 ratio) through the square aperture with no apodization (upper right), the (sonine, $\nu = 4$) apodized aperture (lower left), and the image of the same two point sources through the apodized aperture (lower right). The images of the point sources have been expanded around the region where the sources are located. The point-source images all are displayed using the fourth root of the intensity, scaled up with a gamma of 0.25, using an interactive data language color display table. Obviously, the apodization has dramatically reduced the diffraction sidelobes. For the cases shown here, the “planet” is separated from the star by 6 times the diffraction-limited element set by the aperture size.

Figure 2 shows four images through the sonine-apodized aperture ($\nu = 5$), again expanding the region around the point sources. The upper left is for a $10^7$ brightness ratio of star to planet; upper right is a $10^5$ ratio of star to planet, no wave front aberration; lower left has random error of 1/72 wave rms in the aperture with a $10^5$ ratio; and lower right is 1/50 wave rms random error, again with a $10^5$ ratio of star to planet. Very high optical quality is obviously required to detect Earth-like planets.

For comparison, in Figure 3, we show the equivalent imaging of a pair of point sources through a circular aperture, with and without sonine ($\nu = 5$) apodization. The upper two images are for a 100 : 1 ratio of “star” to “planet.” The lower images are for a 10 ratio of star to planet. Obviously, apodization of a circular aperture does not come close to providing the dynamic range required for Earth-like planet detection.

Figure 4 consists of four contour plots of the region around the point sources, for different apodizing functions. All plots are for the case where the point source ratio is $10^6$ and the aberration is about 1/72 wave. The contours are on a logarithmic scale with the lowest contour at $10^{-10}$ and the peak of the star at $10^6$. The two upper plots use crossed sonine functions for apodization with $\nu = 4$ (upper left) and $\nu = 5$ (upper right). The lower plots use cross $\cos^2$ apodization (lower left) and $\cos^4$ (lower right).

There are small differences in the results. The $\nu = 5$ and $\cos^4$ images are slightly cleaner, but the transmission of the apertures is less, as noted on the plots. These results show that the dynamic range is only a weak function of the specific apodizing shape. They also show that there is a very large region outside the central diffraction spikes that has sufficient dynamic range to detect planets, even terrestrial-sized planets.

Figure 5 shows diagonal cuts through the monochromatic PSF for four cases: (1) circular aperture, no apodization; (2) square aperture, no apodization; (3) circular aperture with sonine apodization; and (4) square aperture with crossed sonine apodization.

The results in Figures 1–5 were all calculated for monochromatic light. In Figure 6, we show cuts through the diagonals of the PSFs for the case where we have integrated over a 3000 Å bandpass. This has the effect of reducing the deep nulls in the diffraction patterns. The vertical dashed line shows the position of a point in the cuts that is separated from the central peak by three diffraction-limited elements. Curve 1 shows a cut through the PSF for an unapodized circular aperture. Curve 2 shows the effect of sonine apodization on the PSF for the circular aperture. Curve 3 shows the apodized square aperture (sonine apodization) without the planet (down $10^7$ from the star), and curve 4 has a planet down $10^9$ from the star. Again, these are calculated with about 1/72 wave rms aberration in the pupil. Broadening the bandpass seems to enhance, rather than degrade, the performance of the apodizers.

4. DISCUSSION

These results clearly demonstrate the power of the square aperture when combined with crossed transmission function apodization. One can apodize in a relay plane of the pupil, allowing use of any shape telescope aperture initially. The apodizing mask can then be rotated to sweep out a full field, obviating the need to rotate the telescope or telescope array. The technique could be combined with the hypertelescope (Labeyrie 1996; Guyon & Roddier 1999) approach to building a large telescope and apodizing the densified pupil. Because of the simplicity of the technique, one could easily imagine that it could be used on a precursor mission to TPF. It might be possible to detect Earth-like planets around the nearest stars with only a modest aperture, operating at visible or near infrared wavelengths. At a minimum, giant planets could be imaged and their spectra measured.

For all of the examples that we have shown, we have located the planet six diffraction-limited elements from the star, along the diagonal, where it is well separated. If we move the planet in closer to the star, the minimum separation where the planet signal is just above the wings of the star diffraction is about three diffraction-limited elements (this may be deduced from the plots in Fig. 6). One can extrapolate this scale to the minimum sizes of telescopes needed to separate out planets from stars for TPF. For example, if we wish to image in the wavelength region around 7600 Å where the Oxygen “A” line is located, then the diffraction limit of a 2 m aperture telescope with a square aperture (given by $\lambda/D$, where $D$ is the telescope diameter) is 0.076. Since we can just detect an Earth-like planet spaced 3 times the diffraction limit from the star, this would allow detection of an Earth in a 1 AU orbit at a little over 4 pc. Obviously, planets in larger radius orbits or larger planets could be detected around more distant stars. Using larger apodized telescopes would increase the distance and therefore the number of Sun-like stars that one could survey for planets in 1 AU or larger orbits.

The nearby (3.2 pc) star ε Eridani has recently (Hatzes et al. 2000) been reported to have a Jupiter-like planet orbiting it. An Earth-like planet in a 1 AU orbit around ε Eridani imaged by a 2 m apodized telescope would be separated by 073. 2200 photons would be detected from the planet in 1 hr for a 3000 Å bandwidth, 90% QE detector, and 15% throughput for the apodized telescope. In the same integration time, the Jupiter-like planet, orbiting at 3.3 AU, would give 275,000 detected photons (assuming it has a similar albedo to Jupiter).

There are many additional areas to be explored. Can we combine square apodization with nulling, or with coronagraphy to allow detection of planets in closer to the stellar peak, thereby reducing the required aperture size for detecting planets around more distant stars? What is the optimum apodization function,
making the narrowest diffraction spikes or maximizing the transmission? Can we combine apodization with dilute array (interferometric) telescope concepts, in place of a large monolithic telescope? What is the best layout for a telescope array? Can we build a single structure to rigidly support the array? What are the detailed specifications on optical quality, at all spatial scales? Finally, if apodization allows operation at shorter wavelengths, can spectra at these wavelengths definitively detect the signatures of life?

We plan to carry out lab tests to demonstrate the effectiveness of apodization techniques. Relatively straightforward experiments using superpolished spherical mirrors and pairs of point sources with a controlled flux ratio should allow feasibility testing of this approach.

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