T-duality and dimensional reduction of S-brane solutions

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Abstract

We studied dimensional reduction and T-duality in singular, spacelike brane solutions of 10 or 11 dimensional supergravity, including spacelike counterparts of wave and monopole solutions. Dimensional reduction is well-defined if, and only if, the solutions possess static dimensions. However, T-duality is ill-defined for some of these solutions where dilaton expectation values depend on time. This led us to conclude that singular solutions of supergravity should be regarded as low-energy solutions of superstring theory only if dilaton expectation values are independent of time.

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1 Introduction

Time-dependent brane solutions of supergravity have attracted a great deal of attention as models of cosmology[1]-[11]. In most of these models, our spacetime is thought to consist of many branes. In other words, our 4-dimensional spacetime is to be described as a low-energy approximation of a 10 or 11-dimensional M/superstring solution. Unfortunately, however, since we do not understand string field theory

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† Singular does not mean not regular but not general.
or M-theory well, we cannot conclude these models have their origin in M/superstring theories.

If a supergravity solution is an approximation of an M/superstring solution, dimensional reduction from 11- to 10-dimensional spacetime and T-duality operation between IIA and IIB superstring theories should be well-defined. For time-independent brane solutions, dimensional reduction and T-duality operation rules are given[12]. Nevertheless, the situation is rather different when we consider time-dependent solutions. Though dimensional reduction of spacelike brane solutions is discussed in [13][14], it is not well-defined at all times. Furthermore, their T-duality is hardly discussed in any papers. It is the time dependence of the dilaton expectation values, which concern string coupling and the radii of compactified dimensions, that makes this problem so complicated.

In our previous papers[7][8], we constructed singular, spacelike brane solutions of supergravity. Furthermore, most of them possess static, flat dimensions. This is of merit, as it enables us to define the dimensional reduction and T-duality of time-dependent solutions. In this paper, we need to construct singular spacelike counterparts of wave and monopole solutions, called w- and m-solutions, respectively, before discussing dimensional reduction and T-duality. We investigate these solutions and find that reduction of dimensions whose metric tensors depend on time is ill-defined and the time-dependence of dilaton expectation values spoils T-duality. These results lead us to conclude that singular solutions of supergravity should be regarded as low-energy solutions of M/superstring theory only if the dilaton expectation values are independent of time.

The organization of this paper is as follows: our starting point is Einstein gravity coupled to a dilaton field and \( n \)-form fields in 11 dimensions (M-theory) and 10 dimensions (IIA and IIB superstring theories). In section 2 we write a singular, spacelike brane solution by following [8] and construct w- and m-solutions. In section 3, dimensional reduction and T-duality are discussed. All the M, IIA and IIB solutions that possess static flat dimensions and static hyperbolic space are selected and listed in Appendix A and their families depending on dimensional reduction (oxidation) or T-duality are shown in Appendix B.

## 2 Singular spacelike solutions

Firstly, we present singular S-brane solutions according to our previous papers[7][8]. We consider Einstein gravity coupled to a dilaton field \( \phi \) and \( m \) kinds of \( n \)-form field \( F_n \), whose action \( I \) is
\[ I = \frac{1}{16\pi G} \int d^D x \sqrt{-g} \left[ R - \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \sum_{A=1}^{m} \frac{1}{2 \cdot n_A} e^{\alpha_A} F_{n_A} \right], \]  

(1)

where \( \alpha_A \) is the dilaton coupling constant given by

\[
\alpha_A = \begin{cases} 
0 & \text{(M - theory)} \\
-1 & \text{(NS - NS sector)} \\
\frac{5 - n_A}{2} & \text{(R - R sector)}
\end{cases}
\]

and \( D = 11 \) for M-theory and \( D = 10 \) for superstring theories.

We assume the following metric form:

\[ ds^2 = \sum_{i=1}^{p+1} e^{2u_i} dx^i dx^i + \sum_{a,b=p+2}^{D} e^{2v_{ab}} dy^a dy^b, \]

(2)

where

\[ \eta_{ab} = \{ \text{diag.}(+ \cdots +, -) \}. \]

(3)

We use \( x^i, \ i = 1, \ldots, p + 1 \) as the coordinates of the space where the branes exist. General orthogonally intersecting solutions have also been given [15], where the metric functions \( u \) and \( v \) and fields \( \phi \) and \( F \) depend only on \( y^D \). A D-brane solution which depends on all the extra space coordinates has been suggested [16].

We assume the metrics and the fields do not depend only on the timelike coordinate \( y^D \) but also on the other perpendicular coordinates \( y^a, \ a = p + 2, \ldots, D - 1 \), i.e.

\[
\begin{align*}
  u &= u(y) \equiv u(y^{p+2}, \ldots, y^D), \\
  v &= v(y) \equiv v(y^{p+2}, \ldots, y^D), \\
  \phi &= \phi(y) \equiv \phi(y^{p+2}, \ldots, y^D), \\
  F &= F(y) \equiv F(y^{p+2}, \ldots, y^D).
\end{align*}
\]

The field strength for an electrically charged \( Sp \)-brane is given by

\[ (F_n)_{i_1 \cdots i_{n-1} a}(y) = \epsilon_{i_1 \cdots i_{n-1}} \partial_a E(y), \]

(4)

where

\[ n = q + 2. \]

The magnetically charged case is given by

\[ (F_n)^{a_1 \cdots a_n} = \frac{1}{\sqrt{-g}} e^{-\alpha \phi} \epsilon^{a_1 \cdots a_n b} \partial_b E(y), \]

(5)
where
\[ n = D - q - 2. \]

We write a set of solutions of the field equations and the Bianchi identity as
\[ E_A(y) = iH_A(y), \]
\[ u_i(y) = \sum_{A=1}^{m} \frac{\delta_{A,i}}{2(D-2)} \ln H_A(y), \]
\[ v(y) = \sum_{A=1}^{m} \frac{-(q_A + 1)}{2(D-2)} \ln H_A(y), \]
\[ \phi(y) = \sum_{A=1}^{m} \frac{-\varepsilon_A}{2} \ln H_A(y), \]
where
\[ \delta_{A,i} = \begin{cases} D - q_A - 3 & (i \in q_A) \\ -(q_A + 1) & (i \notin q_A) \end{cases}, \]
\[ \varepsilon_A = \begin{cases} +1 & (F_{n A} \text{ is an electric field strength}) \\ -1 & (F_{n A} \text{ is a magnetic field strength}) \end{cases}, \]
\[ H_A(y) \equiv \frac{Q_A}{h(y)}, \]
with \( Q_A \) a constant (in the following, we set \( Q_A = 1 \) and \( H_A = H \)) and
\[ \partial^2 h(y) \equiv \eta^{ab} \partial_a \partial_b h(y) = 0. \]

Next, we construct \( w \)- and \( m \)-solutions, which are spacelike counterparts of wave and monopole solutions, respectively. If a spacelike \( F1 \) solution has two isometric coordinates \((x \text{ and } z)\), and one of them \((z)\) is static, we can apply the time-independent T-duality rule \cite{12} to it to obtain a \( w \)-solution:
\[ ds^2 = H^{-1}[dz + i(H - 1)dx]^2 + H dx^2 + \sum_{i=3}^{p+1} dx^i dx^i + \sum_{a,b=p+2}^{D} \eta^{ab} dy^a dy^b. \]

Similarly, if a spacelike NS5 solution has a static perpendicular direction \((z)\), yielded is an \( m \)-solution:
\[ ds^2 = \sum_{i=1}^{p} dx^i dx^i + H (dz + 2i \tilde{B}_a dy^a)^2 + \sum_{a,b=p+2}^{D} H^{-1} \eta^{ab} dy^a dy^b, \]
where \( \tilde{B}_a \) is \( i \) times an imaginary antisymmetric field defined by
\[ \partial_a \tilde{B}_b - \partial_b \tilde{B}_a = \eta_{ac} \eta_{bd} \epsilon^{cde} H^{-2} \partial_e (H). \]
In both solutions, dilaton coupling $\alpha_A = 0$.

Nevertheless, they include complex metric tensors and their geometric contents are vague. Thus, we transform the coordinate $z \rightarrow iz$ and obtain

$$ds^2 = -H^{-1}dz + (H - 1)dx^2 + Hdx^2 + \sum_{i=3}^{p+1} dx^i dx^i + \sum_{a,b=p+2}^D \eta^{ab} dy^a dy^b,$$

(16)

$$ds^2 = \sum_{i=1}^p dx^i dx^i - H(dz + 2\tilde{B}_a dy^a)^2 + \sum_{a,b=p+2}^D H^{-1} \eta^{ab} dy^a dy^b.$$  

(17)

Looked at this standpoint, the timelike dimension has periodicity so the others are thought to be in a heatbath.

The intersection rule, which has been suggested for general solutions in other papers [15][17][18], should be satisfied:

$$-\varepsilon_A \varepsilon_B \alpha_A \alpha_B - 2(\bar{q} + 1) + 2(q_A + 1)(q_B + 1) = 0,$$

(18)

where $\bar{q} + 1$ is the number of dimensions which $q_A$-brane and $q_B$-brane are crossing on. A $w$-solution can be put in any two isometric directions, while an $m$-solution needs a direction with no other branes.

To discuss dimensional reduction and T-duality, we construct a singular solution with static dimensions. To this end, we consider a metric which depends only on the scale parameter $r$ of the entire, or a part of, spacetime perpendicular to the brane:

$$r \equiv \sqrt{-\eta_{ab} y^a y^b}, \quad -\eta_{ab} y^a y^b > 0,$$

(19)

Note that $r$ is a timelike coordinate. To satisfy (12),

$$h = r^{-(D-p-3)}.$$

(20)

Then, the metric of this spacetime is

$$ds^2 = \sum_{i=1}^{p+1} e^{2u_i} dx^i dx^i - e^{2v} \left( dr^2 - r^2 d\Sigma_{D-p-2}^2 \right),$$

(21)

after some coordinate transformation if $w$- and/or $m$ solutions are included. $d\Sigma_{D-p-2}$ is the line element of a $(D - p - 2)$-dimensional hyperbolic space $H^{D-p-2}$ with unit scale factor.

We define cosmic time (our time) $t$ as

$$dt = e^v dr = r^{(D-p-3)} \sum_{2A+1} \frac{-(q_a+1)}{2D-3} dr,$$

(22)
and impose a condition:

\[(D - p - 3) \sum_{A=1}^{m} \frac{(q_A + 1)}{2(D - 2)} = 1.\]  

(23)

Easily proved from (13) and (14), a \(w\)-solution contributes 0 toward \(q_A + 1\), while an \(m\)-solution contributes \(D - 2\) toward \(q_A + 1\). In this case, since

\[t = \ln r,\]  

(24)

\[e^{2u} = r^{-2}.\]  

(25)

the metric (21) becomes

\[ds^2 = -dt^2 + \sum_{i=1}^{p+1} e^{2u_i} dx^i dx^i + d\Sigma_{D-p-2}^2.\]  

(26)

Note that the scale factor of the extra space is independent of \(t\) in this metric. On the other hand, because

\[e^{2u_i} = e^{\frac{D-p-3}{D-2}} \sum_A \delta_{A,i} t,\]  

(27)

the \(i\)-th dimension exponentially expands if \(\sum_A \delta_{A,i} > 0\) or is static if \(\sum_A \delta_{A,i} = 0\) or shrinks if \(\sum_A \delta_{A,i} < 0\). \(\delta_{A,i}\) is given in (10) for brane solutions and for \(w\)- and \(m\)-solutions,

\[
\delta_{A,i} = \begin{cases} 
-(D - 2) & (i = z) \\
D - 2 & (i \neq z, i \in q_A) \\
0 & (i \neq z, i \notin q_A)
\end{cases}
\]

\[
\delta_{A,i} = \begin{cases} 
D - 2 & (i = z) \\
0 & (i \neq z)
\end{cases}
\]

respectively.

The solutions satisfying the condition (23) and possessing static flat directions are given in Appendix A.

3 Dimensional reduction and T-duality

We expect that the time-independent rule of dimensional reduction can be applied to static flat dimensions of spacelike solutions. In fact, we can easily confirm this. For all the static flat dimensions (\(\bigcirc\)) in Appendix A, dimensional reduction is well-defined and spacetime of the other dimensions behaves in the same way, whether it is done or
not. For example, six dimensions can be reduced in (M-3-1) and the metric of the other five dimensions stays

$$ds^2 = -dt^2 + e^{4t} dx^2 + d\Sigma^2_3,$$

while the spacetime dimensions are reduced from 11 to 5. Oxidation is naturally defined as the inverse manipulation of dimensional reduction. On the other hand, dimensional reduction is ill-defined for dimensions which depend on time. If we reduce a time-dependent dimension (one of $\oplus$, $\ominus$ and $\odot$), values of $\delta A_i$ of the other dimensions change and (23) is not satisfied.

Using dimensional reduction, we can define T-duality between IIA and IIB solutions. If IIA and IIB solutions are reduced to the same 9-dimensional solution, the IIA solution is the T-dual of the IIB solution and vice versa. Obeying these rules, we construct some families. The individual solutions that compose each family are associated with each other depending on dimensional reduction (oxidation) or T-duality. These are given in Appendix B.

Though all the M and IIA solutions listed in Appendix A are included in one of the above families, some of IIB solutions, i.e. (B-3-1), (B-3-4), (B-4-13), (B-4-14), (B-5-1), (B-5-2), (B-5-3) and (B-5-4), are not. Since these solutions possess static flat dimensions, dimensional reduction is well-defined. T-duality, however, is ill-defined for them. For example, one of the T-dual solution of (B-3-1) should be composed of F1, F1 and D2. Such a solution, however, does not appear in Appendix A. In other words, we cannot obtain the IIA F1-F1-D2 solution by oxidation from a 9-dimensional solution to which (B-3-1) reduces, which is a T-dual operation exactly.

It is dilaton expectation values that cause this classification. In our model, the T-duality of a solution is well-defined if the dilaton expectation value is independent of time. If not, T-duality does not exist for this solution. We can be convinced of this reason if we examine T-duality rules within the Einstein frame. One formula for converting a IIB solution to IIA by compactifying $z$ direction is

$$g^A_{\mu\nu} = (g^B_{zz})^{D-2} \exp\left\{ \frac{8}{(D-2)^2} \phi^B \right\} \left[ g^B_{\mu\nu} - g^B_{z\mu} g^B_{z\nu} - 4B^B_{z\mu} B^B_{z\nu} \exp\left\{ -\frac{8}{D-2} \phi^B \right\} \frac{g^B_{zz}}{g^B_{zz}} \right],$$

where $g^A$ and $g^B$ are IIA and IIB metric tensors, respectively. If a IIB metric tensor is static and a dilaton depends on time, a IIA metric tensor must depend on time. That is, T-duality is ill-defined for this solution.
Therefore, we conclude that the singular solutions listed in Appendix A with time-dependent dilaton expectation values \([B-3-1], (B-3-4), (B-4-13), (B-4-14), (B-5-1), (B-5-2), (B-5-3)\) and \((B-5-4)\) have no superstring theoretical origins. A constant dilaton expectation value in 10-dimensional spacetime corresponds to a coupling constant in our 4-dimensional spacetime. Such a condition is agreeable and sometimes imposed by hand on phenomenological models. Nevertheless, taking account of (9) and assuming that string field theory would determine what sets of branes are allowed, we suppose that a solution whose brane content satisfies
\[
\sum_{A=1}^{m} \varepsilon_A \alpha_A = 0, 
\]
has superstring theoretical origins even if dilaton expectation value depend on time whether the solution is singular or general.

Our starting point is the Einstein action, with some branes as the low-energy effective action of supergravity. Nevertheless, we do not know string field theory well or which sets of branes are possible within its limits. Solutions where T-duality is ill-defined are not thought to be solutions of string field theory.

### 4 Appendix A: Singular solutions possessing static flat directions

We use following symbols:

| symbol | dimension                        |
|--------|----------------------------------|
| ⊕      | exponentially expands            |
| ⊖      | exponentially shrinks            |
| ⊙      | static                           |
| •      | constitutes cosmic time and static extra space |
| ◦      | exponentially shrinks            |
## 4.1 M-theory

| M2   | M5   | Other dim. |
|------|------|------------|
| ⊕    | ⊕    | ⊙ ⊙ ⊙ ⊙ ⊙ |
| (M-2-1) |

| M2   | M2   | M2   | Other dim. |
|------|------|------|------------|
| ⊕    | ⊕    | ⊕    | ⊙ ⊙ ⊙ ⊙ ⊙ |
| (M-3-1) |

| M2   | M2   | M2   | Other dim. |
|------|------|------|------------|
| ⊕    | ⊕    | ⊕    | ⊙ ⊙ ⊙ ⊙ ⊙ |
| (M-3-2) |

| M5   | M5   | Other dim. |
|------|------|------------|
| ⊕    | ⊕    | ⊙ ⊙ ⊙ ⊙ ⊙ |
| ⊙ ⊙ ⊙ ⊙ ⊙ |
| (M-3-3) |

| w    | M2   | M5   | Other dim. |
|------|------|------|------------|
| ⊕    | ⊕    | ⊕    | ⊙ ⊙ ⊙ ⊙ ⊙ |
| ⊙ ⊙ ⊙ ⊙ ⊙ |
| (M-3-4) |

| M2   | M5   | m    | Other dim. |
|------|------|------|------------|
| ⊕    | ⊕    | ⊕    | ⊙ ⊙ ⊙ ⊙ ⊙ |
| ⊙ ⊙ ⊙ ⊙ ⊙ |
| ⊙ ⊙ ⊙ ⊙ ⊙ |
| (M-3-5) |

| M2   | M2   | M5   | M5   | Other dim. |
|------|------|------|------|------------|
| ⊕    | ⊕    | ⊕    | ⊕    | ⊙ ⊙ ⊙ ⊙ ⊙ |
| ⊙ ⊙ ⊙ ⊙ ⊙ |
| (M-4-1) |

9
|      | M2 | M2 | M5 | M5 | Other dim. |
|------|----|----|----|----|------------|
| (M-4-2) | ⊕ | ⊕ | ⊕ | ⊕ | · · · |
| w    | ⊕ | ⊕ | ⊕ | ⊕ | z          |
| w    | ⊕ | ⊕ | ⊕ | ⊕ | z          |
| w    | ⊕ | ⊕ | ⊕ | ⊕ | z          |
| m    | ⊕ | ⊕ | ⊕ | ⊕ | z          |
| (M-5-1) | ⊕ | ⊕ | ⊕ | ⊕ | · · · |
| (M-5-2) | ⊕ | ⊕ | ⊕ | ⊕ | · · · |
| (M-6-1) | ⊕ | ⊕ | ⊕ | ⊕ | · · · |
4.2 IIA superstring theory

|   | F1 ⊕ | D2 ⊕ | D4 ⊕ | NS5 ⊕ | Other dim. |
|---|------|------|------|-------|------------|
| A-2-1 | ⊕    | ⊕    | ⊕    | ⊕     | ⊕          |

|   | D2 ⊕ | D4 ⊕ | Other dim. |
|---|------|------|------------|
| A-2-2 | ⊕    | ⊕    | ⊕          |

|   | D0 ⊕ | F1 ⊕ | D1 ⊕ | D4 ⊕ | Other dim. |
|---|------|------|------|------|------------|
| A-3-1 | ⊕    | ⊕    | ⊕    | ⊕    | ⊕          |

|   | F1 ⊕ | D2 ⊕ | D2 ⊕ | Other dim. |
|---|------|------|------|------------|
| A-3-2 | ⊕    | ⊕    | ⊕    | ⊕          |

|   | F1 ⊕ | D2 ⊕ | D2 ⊕ | Other dim. |
|---|------|------|------|------------|
| A-3-3 | ⊕    | ⊕    | ⊕    | ⊕          |

|   | D4 ⊕ | D4 ⊕ | NS5 ⊕ | Other dim. |
|---|------|------|-------|------------|
| A-3-4 | ⊕    | ⊕    | ⊕     | ⊕          |

|   | D2 ⊕ | NS5 ⊕ | D6 ⊕ | Other dim. |
|---|------|-------|------|------------|
| A-3-5 | ⊕    | ⊕     | ⊕    | ⊕          |

|   | w ⊕ | F1 ⊕ | NS5 ⊕ | Other dim. |
|---|-----|------|-------|------------|
| A-3-6 | ⊕    | ⊕    | ⊕     | ⊕          |
| D2       | D4       | Other dim. |
|----------|----------|------------|
|          |          |            |

(A-3-7)

| D2       | D4       | Other dim. |
|----------|----------|------------|
|          |          |            |

(A-3-8)

| F1       | NS5      | Other dim. |
|----------|----------|------------|
|          |          |            |

(A-3-9)

| D0       | D4       | Other dim. |
|----------|----------|------------|
|          |          |            |

(A-4-1)

| F1       | D2       | D4       | Other dim. |
|----------|----------|----------|------------|
|          |          |          |            |

(A-4-2)

| F1       | D2       | D4       | NS5        | Other dim. |
|----------|----------|----------|------------|------------|
|          |          |          |            |            |

(A-4-3)

| D2       | D2       | D2       | D6         | Other dim. |
|----------|----------|----------|------------|------------|
|          |          |          |            |            |

(A-4-4)
### 4.3 IIB superstring theory

|   | F1 | NS5 | Other dim. |
|---|----|-----|------------|
| F1 | ⊕  | ⊕   |            |
| NS5| ⊕  | ⊕   |            |
| Other dim. | ⊕ | ⊕   |            |

|   | D1 | D5 | Other dim. |
|---|----|----|------------|
| D1 | ⊕  | ⊕   |            |
| D5 | ⊕  | ⊕   |            |
| Other dim. | ⊕ | ⊕   |            |
| (B-3-8) |
| --- |
| D3  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| D3  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| m   | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| Other dim. | | | | | | | | | | | | |

| (B-3-9) |
| --- |
| D1  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| D5  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| m   | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| Other dim. | | | | | | | | | | | | |

| (B-3-10) |
| --- |
| w   | ⊕ | ⊕ | c | z | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| F1  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| NS5 | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| Other dim. | | | | | | | | | | | | |

| (B-3-11) |
| --- |
| F1  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| NS5 | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| m   | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| Other dim. | | | | | | | | | | | | |

| (B-4-1) |
| --- |
| F1  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| D1  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| NS5 | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| D5  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול | חול |
| Other dim. | | | | | | | | | | | | |

| (B-4-2) |
| --- |
| F1  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול | חול |חול |
| D3  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול |חול | חול |
| D3  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול |חול | חול |
| NS5 | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול |חול | חול |
| Other dim. | | | | | | | | | | | | |

| (B-4-3) |
| --- |
| F1  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול |חול | חול |
| D3  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול |חול | חול |
| D3  | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול |חול | חול |
| NS5 | ⊕ | ⊕ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | חול |חול | חול |
| Other dim. | | | | | | | | | | | | |

16
|      | D1 | D3 | D3 | D3 | D5 | Other dim. |
|------|----|----|----|----|----|------------|
| (B-4-1) | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |            |

|      | D1 | D3 | D3 | D3 | D5 | Other dim. |
|------|----|----|----|----|----|------------|
| (B-4-5) | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |            |

|      | D3 | D3 | D3 | D3 | Other dim. |
|------|----|----|----|----|------------|
| (B-4-6) | ⊕ | ⊕ | ⊕ | ⊕ |            |

|      | D3 | D3 | D3 | D3 | Other dim. |
|------|----|----|----|----|------------|
| (B-4-7) | ⊕ | ⊕ | ⊕ | ⊕ |            |

|      | w | D3 | D3 | m | Other dim. |
|------|---|----|----|---|------------|
| (B-4-8) | ⊕ | ⊕ | ⊕ | ⊕ |            |

|      | w | D1 | D5 | m | Other dim. |
|------|---|----|----|---|------------|
| (B-4-9) | ⊕ | ⊕ | ⊕ | ⊕ |            |

|      | w | D3 | D5 | NS5 | Other dim. |
|------|---|----|----|-----|------------|
| 17   | |    |    |     |
|   | (B-4-10) |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | z |
| D1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| m  | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |  |
| Other dim. |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

|   | (B-4-11) |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| m  | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| Other dim. |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

|   | (B-4-12) |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D5 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| Other dim. |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

|   | (B-4-13) |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| NS5 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| Other dim. |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

|   | (B-4-14) |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| F1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| Other dim. |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

|   | (B-5-1) |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| Other dim. |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

|   | (B-5-2) |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D1 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| D3 | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ | ⊕ |
| Other dim. |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
5 Appendix B: Families

| M  | M-2-1 |   |   |
|----|-------|---|---|
| IIA| A-2-1 | A-2-2 |
| IIB| B-2-1 | B-2-2 | B-2-3 |

2 branes

| M  | M-3-1 | M-3-4 |   |
|----|-------|-------|---|
| IIA| A-3-1 | A-3-2 | A-3-6 | A-3-7 |
| IIB| B-3-2 | B-3-6 | B-3-7 | B-3-10 |

3 branes, 1 expanding dimension
| M | M-3-3 | M-3-5 |
|---|---|---|
| IIA | A-3-4 | A-3-5 |
| IIB | B-3-5 | B-3-8 |

3 branes, 2 expanding dimensions

| M | M-3-2 |
|---|---|
| IIA | A-3-3 |
| IIB | B-3-3 |

3 branes, 3 expanding dimensions

| M | M-4-1 | M-4-3 | M-4-4 |
|---|---|---|---|
| IIA | A-4-1 | A-4-2 | A-4-6 |
| IIB | B-4-1 | B-4-2 | B-4-4 |

4 branes, 1 expanding dimension

| M | M-4-2 | M-4-5 |
|---|---|---|
| IIA | A-4-3 | A-4-5 |
| IIB | B-4-3 | B-4-5 |

4 branes, 3 expanding dimensions

| M | M-5-1 |
|---|---|
| IIA | A-5-1 |
| IIB | B-5-1 |

5 branes

| M | M-6-1 |
|---|---|
| IIA | A-6-1 |
| IIB | B-6-1 |

6 branes

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