1. Introduction

Increase in electric power demands results in denser interconnections power grids are subjected to unrestricted electromechanical swings. With random dynamic loads being added the transmission system is incapable to evacuate power which results in a fragile grid, causing theses wings. Without any corrective actions, these unrestricted electromechanical perturbations may lead to complete or limited power interruptions\(^1\)\(^-\)\(^3\).

The development in the renewable energy sector has been high over the past decade. A sizeable amount of renewable power is harvested through wind. The advantages with wind power are, it is renewable in nature, installation time required for Wind energy generators is minimal and cost competitive. However, wind power has some adverse attributes with respect to the electric network operations\(^4\). For instance, wind power developed is not so regular, because wind does not blow at a firm speed. Besides, the superior wind locations are usually located faraway populated areas\(^5\). Hence, it would require considerable improvement in organizational investments to deliver the energy developed from wind, to the end consumer’s located far away.

A strange oscillatory behavior in system voltages was visualized when connecting a variety of wind generators into the transmission infrastructure of a utility company. One solution proposed was to overcome the dynamic instability was to install an under voltage relay at the new wind generator bus. This relay will trip the new generator units when its terminal voltage drops down to the settling value\(^5\)\(^-\)\(^7\).

The conception of the present work is to assess and validate, suitably the influence of singly fed wind generators on the small perturbation stability of power systems\(^8\)\(^,\)\(^9\). The system stability is strengthened by placing shunt FACTS stabilizers at critical sites for damping the power swings caused due to wind generators.

2. Materials and Methods

In stability investigation of a multi-generator system, modeling of all the machines in detailed manner is
exceedingly tedious because of the large number of synchronous generators to be simulated\textsuperscript{11}. Therefore simplifying assumptions and approximations are usually made in modeling the system\textsuperscript{12}. Often only a few machines are modeled in detail, usually those machines located near the disturbances, while other machines are represented by simpler models\textsuperscript{11}. 

In the two-axis machine model, the transient effects are considered, while the sub-transient effects are neglected\textsuperscript{13}. The damper bar effects are not considered. Further assumptions are made in this model that in the stator emf equations, emfs corresponding to flux linkages (\(\lambda_d\) and \(\lambda_q\)) are insignificant compared to the speed emf terms and that \(\omega_0 = \omega = 1.0\) per unit\textsuperscript{11}. With the simplistic assumptions given above the perturbed state equations in per unit form are listed in Equation (1)\textsuperscript{11}.

\[
\Delta E' = \frac{1}{\sigma}(-E_d - (X_c - X_q)I_q)
\]

\[
\Delta E'' = \frac{1}{\sigma}(E_{do} - E_{d}' + (X_d - X_a)I_q)
\]

\[
\Delta \omega = \frac{1}{\tau_f}(T_m - D\omega - T_c)
\]

\[
\Delta \delta = \omega
\]

To formulate the perturbation model of the multi-generator power system it is required to get rid of the algebraic variables like currents, from synchronous machine state equations\textsuperscript{11}. Hence, this section presents the formulation of the network equations (DPD\textsubscript{12}DPD\textsubscript{12}) in terms of the differential variables of the generators\textsuperscript{11}. The network equation derived in prevalent reference frame can be noted down as given in Equation (2)\textsuperscript{11}.

\[
\hat{I} = \hat{Y} \hat{V}
\]

\(\hat{V}\) and \(\hat{I}\) are the node voltage and current phasors and \(\hat{Y}\) is the bus admittance matrix of the network. To express the network variables (voltages, currents) from the prevalent system reference frame to the individual generator d-q coordinates, \(d\) kept\textsuperscript{11} as the phase angle calculated with the q axis of the individual generator and the prevalent network reference frame (Q\textsubscript{prev})\textsuperscript{11}.

\[
V_n + jV_d = (V_n \cos \delta - V_o \sin \delta) + j(V_n \sin \delta + V_o \cos \delta)(3) \hat{V} = \hat{V} e^{j\delta}
\]

Similarly for the node currents we can write, \(\hat{I} = \hat{T} \hat{I}\)

Substituting (3) and (5) in equation (2) we get

\[
\hat{I} = (T^{-1} \hat{Y} T) \hat{V} = \hat{M} \hat{V}
\]

Equation (6) is expressed in matrix form as given below:

\[
\begin{bmatrix}
\hat{I}_1 \\
\hat{I}_n
\end{bmatrix} =
\begin{bmatrix}
\hat{Y}_{11} & \ldots & \hat{Y}_{1n} e^{-j\delta_{1n}} \\
\vdots & \ddots & \vdots \\
\hat{Y}_{n1} e^{-j\delta_{n1}} & \ldots & \hat{Y}_{nn}
\end{bmatrix}
\begin{bmatrix}
\hat{E}_1 \\
\vdots \\
\hat{E}_n
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta I_1 \\
\Delta I_n
\end{bmatrix} =
\begin{bmatrix}
\Delta \hat{E}_1 \\
\vdots \\
\Delta \hat{E}_n
\end{bmatrix}
\begin{bmatrix}
\hat{E}_{k1} e^{-j\delta_{k1}} k_0 n_{k1} \\
\vdots \\
\hat{E}_{nk} e^{-j\delta_{nk}} n_{nk}
\end{bmatrix}
\]

\[
\Delta I_n = \Delta I_{1n} + j\Delta I_{2n}
\]

\[
\Delta I_{1n} = G_e \Delta E_{1n} - B_e \Delta E_{1n} + \sum_{k=1}^{n}(G_{nk} \sin \delta_{nk} + B_{nk} \cos \delta_{nk}) \Delta \delta_{nk} E_{1n} + \sum_{k=1}^{n}(G_{nk} \sin \delta_{nk} + B_{nk} \cos \delta_{nk}) \Delta \delta_{nk} E_{1n}
\]

\[
\Delta I_{2n} = B_e \Delta E_{2n} + G_e \Delta E_{2n} + \sum_{k=1}^{n}(G_{nk} \sin \delta_{nk} + B_{nk} \cos \delta_{nk}) \Delta \delta_{nk} E_{2n} + \sum_{k=1}^{n}(G_{nk} \sin \delta_{nk} + B_{nk} \cos \delta_{nk}) \Delta \delta_{nk} E_{2n}
\]

Substituting (9) and (10) in the differential equations of the synchronous generator\textsuperscript{11} (1), yields the perturbation model\textsuperscript{11} with five differential variables which are

\[
[\Delta E_{1}'', \Delta E_{1}', \Delta \omega, \Delta \delta, \Delta E_{FD}]\textsuperscript{11}
\]

### 2.1 Induction Generator Modeling for Small Perturbation Analysis

In an induction machine, both stator and rotor quantities have to be transformed to a reference frame that is revolving at synchronous speed and which also serves
as the prevalent reference for network voltage and current phasors. For the stator the transformation will yield (Synchronous) speed emf term to consider for the effect of proportional speed change between stator and synchronously revolving observation reference. The rotor is revolving at a speed $\omega_r$ which does not correspond to grid frequency which is connected to the stator. Hence, the transformation to synchronously rotating network reference frame will yield slip speed emf terms\(^{13}\). The d-q reference frame is revolving at synchronous speed with the d-axis 90 degree further forward in space to the q-axis\(^{13}\).

\[
V_{qs} + R_s I_{qs} + P \Psi_{qs} = s \Psi_{ds}\tag{11}
\]

\[
V_{ds} + R_s I_{ds} + P \Psi_{ds} = s \Psi_{qs}
\]

\[
V_{qr} + R_s I_{qr} + P \Psi_{qr} = (\omega_s - \omega_r) \Psi_{dr}
\]

\[
V_{dr} + R_s I_{dr} + P \Psi_{dr} = (\omega_s - \omega_r) \Psi_{qr}
\]

Equation (11) comprises of the full set of mathematical expressions that illustrates the emf equations of the singly excited wind generator. A fixed-speed wind turbine generator is coupled to a wind turbine through a speed changing mechanism, where generator stator winding is connected to the utility grid. The generator slip differs with the generated power, so the speed is not, in fact, constant. However, since the speed variations are very small (just 1-2%), it is prevalently referred to as a ‘fixed-speed’ singly fed or singly excited wind generator.

From the voltage equations given in Equation (11), neglecting stator transients \((P \Psi_{qs} = P \Psi_{ds} = 0)\) and substituting for flux linkages in terms of currents we get the linearized differential equations for a singly fed induction generator given below\(^{13}\).

\[
\Delta E'_q = -\frac{1}{T_o}\left(\Delta E'_q - (X_o - X'_q) \Delta I_q - s \omega_s \Delta E'_d - E'_q \omega_s \Delta s\right)
\]

\[
\Delta E'_d = -\frac{1}{T_o}\left(\Delta E'_d + (X_o - X'_d) \Delta I_q + s \omega_s \Delta E'_q + E'_q \omega_s \Delta s\right)
\]

\[
\Delta s' = -\frac{1}{\tau_o} (\Delta T_m - \Delta T_r)
\]

\[
T'_o = \frac{L_{rr}}{R_r} \quad \text{is the (stator) open circuit time constant}\(^{13}\). \text{Slip}\]

\[
s = \frac{\omega_s - \omega_r}{\omega_s}
\]

The incremental current changes in equation (12) are the same as the linearized current equation of the synchronous machine\(^{11}\) (9-10) with the exception that the diagonal element of the Transformation matrix (T) which converts from the network reference to the individual d-q coordinates is set 1 as the equations for induction generator are written only on the prevalent network reference (Q\(_{ref}\))\(^{13}\).

### 3. Results and Discussion

The problem solving steps for small perturbation analysis is executed for the multi-generator is explained down.

**Step 1:** Get the details of transmission links, node information and generator data for the given system\(^{11}\).

**Step 2:** Form the node admittance matrix from the provided line information\(^{11}\).

**Step 3:** Exclude all the nodes apart from the internal generator nodes\(^{11}\).

**Step 4:** Obtain the Y\(_{min}\) matrix from the network where generator nodes alone are intact\(^{11}\).

\[
Y_{min} = Y_{nn} - (Y_{nr} \ast (Y_{rr})^{-1} \ast Y_{rm}) \quad (13)
\]

**Step 5:** Formulate the differential equations for pE’\(_q\), pE’\(_d\), pd, pw as \(x = [A] \chi\) after eliminating the algebraic equations\(^{11}\).

**Step 6:** Damping ratio is calculated for the oscillatory modes\(^{11}\).

\[
\zeta = -\text{Re (Eigen value)}/ (|\text{Eigen value}|)
\]

### 3.1 Small Perturbation Analysis Results

The multi-generator system considered for small perturbation analysis three generators, nine node system\(^{11}\) which is illustrated in Figure 1.

**Figure 1.** 3 Generator 9 Node systems.

For small perturbation analysis, G1 is modeled with two differential variables corresponding to mechanical system [d,w] (classical model), G2 and G3 are modeled...
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with four state variables \([E'_q, E'_d, \omega, w]\) (Two axis model). Table 1 displays the oscillatory modes of the three generator system.

Table 1. Oscillatory modes the of 3 Generator 9 Node system

| Eigen Values       | Damping Ratio (ζ) | Associated States |
|--------------------|-------------------|-------------------|
| -0.00255 ±         | 0.07345           | δ13, ω3           |
| 0.034735i          |                   |                   |
| -0.0006095 ±       | 0.02651           | δ12, ω2           |
| 0.02298i           |                   |                   |

From Table 1, it is clear that the damping ratios of the oscillatory modes are feeble, which turns out to be the root cause for unrestricted oscillations. The examinations of oscillatory modes are carried out with singly fed wind generator. From the test system chosen above the third generator is replaced with a singly fed wind generator. The state variable used for the singly fed induction generators are \(pE'_q, pE'_d, pw\). The oscillatory mode with singly excited wind generator is noted down in Table 2.

Table 2. Oscillatory modes of the system with singly excited wind generator

| S.No | Eigen value       | Damping ratio(ζ) | Frequency (Hz) |
|------|------------------|------------------|----------------|
| 1    | -0.0005655507 ±  | 0.02833          | 1.1974         |
|      | 0.0199561668i    |                  |                |

To boost the small perturbation response of the power system shunt FACTS stabilizers are used closer to the load buses.

Table 3. Oscillatory modes of the system with singly fed wind generator including FACTS Stabilizers

| Electromechanical Mode | Damping ratio(ζ) |
|------------------------|------------------|
| With SVC               | -0.0005578072 ±  | 0.07894         |
|                        | 0.0070439275i    |                 |
| With STATCOM           | -0.0039071814 ±  | 0.16961         |
|                        | 0.0227030144i    |                 |

From the tabulation of oscillatory modes given in Table 3, it can noted that the shunt FACTS devices in the network at bus 5 the damping ratio enhances to 0.07894 from the base case value of 0.02833. With STATCOM the damping ratio improves to 0.16961. The damping ratio of the electromechanical mode is higher with STATCOM in the network compared to the SVC. This is due to the truth that STATCOM is a voltage source converter based FACTS controller which has better short-term response than a passive thyristor controlled SVC.

4. Conclusions

From the outcomes, it can be concluded that among shunt connected FACTS controllers the STATCOM provides better settling of the oscillations compared to that of the SVC. This is due to the fact that a STATCOM is in essence voltage sourced converter based shunt controller which has superior short term response and SVC is a passive shunt inserted thyristor switched capacitor/reactor. This work can be extended for including different types of wind generator models in a power system and the stabilization could be improved by using a combination of shunt and series FACTS stabilizers.

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