Effective Lagrangian at Cubic Order in Electromagnetic Fields and Vacuum Birefringence

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Abstract

The effective Lagrangian of electromagnetic fields at the cubic order in field strength is considered. This generalized Lagrangian is motivated by electrodynamics on non-commutative spaces. We find the canonical and symmetrical energy-momentum tensors and show that the vacuum in the model behaves like an anisotropic medium. The propagation of a linearly polarized laser beam in the external transverse magnetic field is investigated. We obtain the dispersion relation and refraction indexes for two polarizations. From experimental values of the induced ellipticity, reported by PVLAS collaboration, the constraint on parameters in the effective Lagrangian is evaluated.

1 Introduction

The magnitude of the rotation [1] and ellipticity [2] (reported by the PVLAS Collaboration) of a linearly polarized laser beam propagating through a transverse magnetic field can not be explained within quantum electrodynamics (QED) [3], [4]. This stimulates activities in the theoretical proposals on physics beyond the Standard Model. Possible explanations of the phenomena observed, in the framework of particle physics, are in [5], [6], [7], [8], [9], [10], [11], [12]. The effect of vacuum birefringence takes place also in Lorentz violating electrodynamics [13]. Two popular scenarios, explaining data of the PVLAS experiment, include the existence of a new axion-like (spin-0) particle (ALP) [6] and/or minicharged particles (MCPs) [8] (see [14], [15] for last reviews). The best fit is obtained for MCPs of spin-1/2 [14]. In addition, the parameters of ALP are different as compared with the parameters of a QCD axion. The case when the coupling and the mass of an axion-like particle depend on the temperature and matter density was considered in [16]. This can adjust astrophysical bounds to allow for the PVLAS signal.
In this letter, we study the induced ellipticity of the laser beam (vacuum birefringence) using the effective Lagrangian at cubic order in the electromagnetic field strength which is motivated by electrodynamics on non-commutative (NC) spaces.

Models on NC spaces attract a great interest because NC coordinates appear in the superstring theory with the presence of the external background magnetic field [17]. In the NC field theories the Lorentz invariance is broken due to the fact that the constant parameters $\theta_{\mu\nu}$ are coupled to tensors, but a twisted form of the Lorentz invariance is valid [18]. The NC parameter $\theta$ is extremely small and astrophysical bounds on $\theta^{-1/2}$ are of the order of the Planck scale. Experimental bounds on the NC parameters are discussed in [19]. Coefficients for the Lorentz violation in electrodynamics within the Standard Model Extension (SME) are estimated from Cosmic Microwave Background (CMB) radiation in [20]. The Lorentz violation constraints in the photon sector from measurements of the linear polarization in gamma-ray bursts were considered in [21].

We use the Heaviside-Lorentz system of units, and $\hbar = c = 1$.

2 Lagrangian and Field equations

It was shown that there is no polarization rotation and ellipticity observed in the NC version of electrodynamics [22], [23], [21], [25], [26], [27]. The effective Lagrangian of the NC version of electrodynamics, at first order in the NC parameter $\theta$, is cubic in the electromagnetic field strength. Therefore, to explain the data of the PVLAS experiment, we consider the generalized effective Lagrangian of electromagnetic fields at cubic order in the field strength. Possible structures, including second-rank “tensor” parameters in the Lagrangian, are as follows:

$$\theta^{(1)}_{\alpha\beta} F_{\alpha\beta} F_{\mu\nu}^2, \quad \theta^{(2)}_{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} F_{\mu\nu}, \quad \theta^{(3)}_{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} F_{\mu\nu}^2, \quad \theta^{(4)}_{\alpha\beta} \tilde{F}_{\alpha\beta} F_{\mu\nu} F_{\mu\nu},$$

$$\theta^{(5)}_{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} \tilde{F}_{\mu\nu}, \quad \theta^{(6)}_{\alpha\beta} \tilde{F}_{\mu\alpha} F_{\nu\beta} F_{\mu\nu}, \quad \theta^{(7)}_{\alpha\beta} \tilde{F}_{\mu\alpha} \tilde{F}_{\nu\beta} F_{\mu\nu}, \quad \theta^{(8)}_{\alpha\beta} \tilde{F}_{\mu\alpha} \tilde{F}_{\nu\beta} \tilde{F}_{\mu\nu},$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the strength of the electromagnetic field, $\tilde{F}_{\mu\nu} = (1/2) \varepsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$ ($\varepsilon_{1234} = -i$) is a dual tensor. It is easy to verify that these structures are not independent and may be converted into only the first two terms. So, we have two independent antisymmetric “tensor”-parameters,
or four three-“vectors”. As parameters $\theta_{\mu\nu}$ are not transformed as real tensors, the Lorentz invariance is broken. Here we investigate the “minimal” extension of NC electrodynamics, and therefore, the possible independent structure

$$\theta_{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\gamma} F_{\nu\gamma}$$

(including forth-rank “tensor”-parameters $\theta_{\alpha\beta\mu\nu}$) is not considered here.

The generalized effective Lagrangian of electromagnetic fields at cubic order in the field strength is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{8} \theta_{\alpha\beta}^{(1)} F_{\alpha\beta} F_{\mu\nu}^2 - \frac{1}{2} \theta_{\alpha\beta}^{(2)} F_{\mu\alpha} F_{\nu\beta} F_{\mu\nu}.$$  \hspace{1cm} (1)

Two constant “tensors” $\theta_{\alpha\beta}^{(1)}$ and $\theta_{\alpha\beta}^{(2)}$ are independent. At $\theta_{\alpha\beta}^{(1)} = \theta_{\alpha\beta}^{(2)}$, one comes to NC electrodynamics [28] (using Seiberg-Witten map [17]) with the accuracy of $\mathcal{O}(\theta^2)$. The Lagrangian (1) can also be rewritten as

$$\mathcal{L} = \frac{1}{2} \left( \mathbf{E}^2 - \mathbf{B}^2 \right) \left[ 1 + (\alpha \cdot \mathbf{B}) - (\xi \cdot \mathbf{E}) \right] - (\mathbf{E} \cdot \mathbf{B}) \left[ (\beta \cdot \mathbf{E}) + (\gamma \cdot \mathbf{B}) \right],$$  \hspace{1cm} (2)

where the electric field is $E_i = i F_{i4}$ and the magnetic induction field being $B_i = \epsilon_{ijk} F_{jk}$, $\alpha_i = 2 \theta_i^{(2)} - \theta_i^{(1)}$, $\beta_i = \theta_i^{(2)}$, $\theta_i^{(1,2)} = (1/2) \epsilon_{ijk} \theta_{jk}^{(1,2)}$, and $\gamma_i = i \theta_i^{(1)}$, $\xi_i = 2i \theta_i^{(2)} - i \theta_i^{(1)}$. The parameters $\theta_{\mu\nu}^{(1,2)}$ have the dimension of (length)$^2$. It follows from Eq.(2) that terms containing parameters $\alpha_i$, $\beta_i$ violate CP-symmetry. The Lagrange-Euler equations lead to equations of motion as follows:

$$\partial_{\mu} F_{\nu\mu} + \frac{1}{2} \theta_{\alpha\beta}^{(1)} \partial_{\mu} (F_{\mu\nu} F_{\alpha\beta}) + \frac{1}{4} \theta_{\alpha\beta}^{(1)} \partial_{\mu} (F_{\alpha\beta}^2)$$

$$- \theta_{\alpha\beta}^{(2)} \partial_{\mu} (F_{\alpha\beta} F_{\mu\nu}) + \theta_{\mu\beta}^{(2)} \partial_{\mu} (F_{\alpha\beta} F_{\nu\alpha}) - \theta_{\alpha\beta}^{(2)} \partial_{\mu} (F_{\mu\alpha} F_{\nu\beta}) = 0.$$  \hspace{1cm} (3)

Eq.(3) can be represented as

$$\frac{\partial}{\partial t} \mathbf{D} - \nabla \times \mathbf{H} = 0, \quad \nabla \cdot \mathbf{D} = 0,$$  \hspace{1cm} (4)

where the displacement ($\mathbf{D}$) and magnetic ($\mathbf{H}$) fields are given by

$$\mathbf{D} = \frac{\partial \mathcal{L}}{\partial \mathbf{E}} = \left[ 1 + (\alpha \cdot \mathbf{B}) - (\xi \cdot \mathbf{E}) \right] \mathbf{E} - \left[ (\beta \cdot \mathbf{E}) + (\gamma \cdot \mathbf{B}) \right] \mathbf{B}$$

$$- (\mathbf{E} \cdot \mathbf{B}) \beta - \frac{1}{2} \left( \mathbf{E}^2 - \mathbf{B}^2 \right) \xi,$$  \hspace{1cm} (5)
\[ H = - \frac{\partial L}{\partial B} = B [1 + (\alpha \cdot B) - (\xi \cdot E)] + [(\beta \cdot E) + (\gamma \cdot B)] E + (E \cdot B) \gamma - \frac{1}{2} (E^2 - B^2) \alpha. \] (6)

At the case \( \theta^{(1)}(\alpha) = \theta^{(2)}(\alpha = \beta), (\xi = \gamma) \), we arrive at NC electrodynamics. The second pair of Maxwell equations \( \partial_\mu \tilde{F}_{\mu \nu} = 0 \) reads

\[ \frac{\partial}{\partial t} B + \nabla \times E = 0, \quad \nabla \cdot B = 0. \] (7)

Here we study the propagation of a linearly polarized laser beam in the external transverse magnetic field for the case when the effective Lagrangian of electromagnetic fields is given by Eq.(1). This consideration generalizes the results of [22], [23], [24], [25], [26], [27] in the case of two independent antisymmetric “tensor”-parameters \( \theta^{(1)}_{\alpha \beta} \) and \( \theta^{(2)}_{\alpha \beta} \).

### 3 Energy-Momentum Tensor

Now, we find the energy-momentum tensor to clear up the direction of energy propagation. With the help of the standard procedure [29], we obtain the gauge-invariant canonical energy-momentum tensor of electromagnetic fields

\[ T_{\mu \nu} = -F_{\mu \alpha} F_{\nu \alpha} \left( 1 - \frac{1}{2} \theta^{(1)}_{\gamma \beta} F_{\gamma \beta} \right) + \frac{1}{4} \theta^{(1)}_{\mu \alpha} F_{\nu \alpha} F_{\rho \beta}^2 - \theta^{(2)}_{\mu \beta} F_{\nu \alpha} F_{\rho \beta} F_{\gamma \rho} - (F_{\mu \alpha} F_{\nu \gamma} + F_{\nu \alpha} F_{\mu \gamma}) \theta^{(2)}_{\alpha \beta} F_{\gamma \beta} - \delta_{\mu \nu} L. \] (8)

At \( \theta^{(1)}_{\mu \beta} = \theta^{(2)}_{\mu \beta} \), the canonical tensor (8) converts into one for NC electrodynamics, obtained in [30]. Tensor (8) is non-symmetric, but is conserved, \( \partial_\mu T_{\mu \nu} = 0 \). From Eq.(8), we find the energy density \( E \), and the Poynting vector \( P \):

\[ E = T_{44} = \frac{E^2 + B^2}{2} [1 + (\alpha \cdot B)] - (\xi \cdot E) E^2 - (E \cdot B) (\beta \cdot E), \]

\[ T_{m4} = -i P_m, \quad P = E \times H. \] (9)

so that the four-vector of the energy-momentum is \( P_\mu = (P, iE) \), and the continuity equation \( \partial_\mu P_\mu = 0 \) is valid. With the help of Eq.(6), the Poynting vector can also be written as

\[ P = [1 + (\alpha \cdot B) - (\xi \cdot E)] (E \times B) \]
\[
\frac{1}{2} (B^2 - E^2) (E \times \alpha) + (E \cdot B) (E \times \gamma). \tag{10}
\]

It follows from Eq. (10) that the direction of the Poynting vector (and the energy propagation) is different from the direction of the wave vector \(k\) or \((E \times B)\). So, the vacuum in the model considered, behaves like an anisotropic medium (see [26] for a particular case \(\theta^{(1)}_{\alpha\beta} = \theta^{(2)}_{\alpha\beta}\)).

The symmetric energy-momentum tensor can be obtained by varying the action, corresponding to the Lagrangian (1), on the metric tensor \(g_{\mu\nu}\) [29]. After calculations, we arrive at the symmetric energy-momentum tensor:

\[
T^\text{sym}_{\mu\nu} = T_{\mu\nu} + \frac{1}{4} \theta_{\mu\nu}^{(1)} F_{\mu\alpha} F_{\rho\beta}^2 - \theta_{\mu\nu}^{(2)} F_{\gamma\mu} F_{\rho\beta} F_{\gamma\rho}, \tag{11}
\]

where the conserved tensor \(T_{\mu\nu}\) is given by Eq. (8). As the action corresponding to the Lagrangian (1) is not a scalar, the conservation of the symmetrical energy-momentum tensor obtained (11) is questionable. From Eq. (8), (11), one can obtain non-zero traces of the canonical and symmetrical energy-momentum tensors. For classical electrodynamics, \(\theta_{\alpha\beta}^{(1)} = \theta_{\alpha\beta}^{(2)} = 0\), and therefore the trace of the canonical energy-momentum tensor vanishes. It should be noted that the modified energy-momentum tensor leads to changing the curvature of space-time. This may have an influence on the inflation of the universe.

### 4 Vacuum Birefringence

Now we consider the plane electromagnetic wave \((e, b)\) propagating in \(z\)-direction and perpendicular to the external constant and uniform magnetic field \(\mathcal{B} = (\mathcal{B}, 0, 0)\). Then \(E = e, B = b + \mathcal{B}\). The rotation of the magnetic field, in the PVLAS experiment, does not effect the vacuum birefringence within QED calculations [3], [4], and therefore, we consider the stationary and uniform external magnetic field. After linearizing Eq. (5), (6) around the background magnetic induction field \(\mathcal{B}\), one obtains the linearized equations:

\[
d_i = \varepsilon_{ij} e_j + \rho_{ij} b_j, \quad h_i = (\mu^{-1})_{ij} b_j + \sigma_{ij} e_j \tag{12}
\]

where

\[
\varepsilon_{ij} = \left[1 + (\alpha \cdot \mathcal{B})\right] \delta_{ij} - \beta_i \mathcal{B}_j - \beta_j \mathcal{B}_i, \quad \rho_{ij} = \xi_i \mathcal{B}_j - \delta_{ij} (\gamma \cdot \mathcal{B}) - \mathcal{B}_i \gamma_j,
\]
\( \mu_{ij}^{-1} = \left[ 1 + (\alpha \cdot \mathbf{B}) \right] \delta_{ij} + \alpha_i \mathbf{B}_j + \alpha_j \mathbf{B}_i, \quad \sigma_{ij} = -\mathbf{B}_i \xi_j + \delta_{ij} (\gamma \cdot \mathbf{B}) + \gamma_i \mathbf{B}_j. \) (13)

From Eq.(12) and Maxwell equations

\[ k_i d_i = k_i b_i = 0, \] (14)

we find the equation for the electric field \( \mathbf{e} \):

\[
\left[ \mathbf{k}^2 \left( \mu^{-1} \right)_{bi} + k_a \left( \mu^{-1} \right)_{al} k_l \delta_{ib} - \mathbf{k}^2 \left( \mu^{-1} \right)_{pp} \delta_{ib} - k_l \left( \mu^{-1} \right)_{bl} k_i \right. \\
+ \omega^2 \epsilon_{ib} + \omega \epsilon_{ijk} k_j \sigma_{kb} + \omega \rho_{ij} \epsilon_{jmb} k_m \left] e_b = 0, \right.
\] (15)

where \( \epsilon_{ijk} \) is the antisymmetric tensor \( (\epsilon_{123} = 1) \). The homogeneous Eq.(15) possesses non-trivial solutions when the determinant of the matrix equals zero. To simplify the problem, we consider the case \( \theta_{4\beta}^{(1)} = \theta_{4\beta}^{(2)} = 0 \) \( (\xi = \gamma = 0) \). It should be mentioned that NC field theory preserves unitarity only for non-zero space-space non-commutativity, \( \theta_{0a} = 0 \) [31, 32]. In addition, the bounds on the time-space components \( \theta_{0a} \) are much weaker \( (\theta^{-1/2} > \mathcal{O}(10 \text{ GeV})) \) compared to space-space components \( (\theta^{-1/2} > \mathcal{O}(10 \text{ TeV})) \) [19]. Evaluating the determinant for this case \( (\xi = \gamma = 0) \), we obtain the dispersion relation:

\[ A^2 \left[ A + 2n^2 (\alpha \cdot \mathbf{B}) - 2(\beta \cdot \mathbf{B}) \right] = 0, \] (16)

where

\[ A = 1 + (\alpha \cdot \mathbf{B}) - n^2 \left[ 1 + 3(\alpha \cdot \mathbf{B}) \right], \] (17)

and \( n = k/\omega \) is the index of refraction. There are two solutions to Eq.(16):

\[ A = 0, \quad n^2_\perp = 1 - 2(\alpha \cdot \mathbf{B}) \]

\[ A + 2n^2 (\alpha \cdot \mathbf{B}) - 2(\beta \cdot \mathbf{B}) = 0, \]

\[ n^2_\parallel = 1 - 2(\beta \cdot \mathbf{B}). \] (18)

In Eq.(18), we use the expansion in small parameters \( \alpha, \beta \). The \( n^2_\perp, n^2_\parallel \) correspond to the cases when the electric field of the plane wave \( \mathbf{e} \) is perpendicular \( (\mathbf{e} \perp \mathbf{B}) \) and parallel \( (\mathbf{e} \parallel \mathbf{B}) \) to the background magnetic induction field \( \mathbf{B} \). So,
the speed of light is different for two modes. At the case $\alpha = \beta$, we arrive at the result [22], that the speed of light is shifted equally for both polarizations. Only at the case $\alpha \neq \beta$, we have the effect of the induced ellipticity or birefringence. In the general case $\xi \neq 0$, $\gamma \neq 0$, parameters $\xi$, $\gamma$ contribute to indexes of refraction $n_\perp$, $n_\parallel$. If the angle between the polarization vector $e$ and the external magnetic induction field $\overrightarrow{B}$ is $\theta$, then the polarization vector at $z = 0$ is $e|_{z=0} = E_0(\cos \theta, \sin \theta) \exp(-i\omega t)$. The components of the polarization vector at arbitrary $z$ are given by

$$
e_\perp = E_0 \sin \theta \exp i (k_\perp z - \omega t), \quad e_\parallel = E_0 \cos \theta \exp i (k_\parallel z - \omega t),$$

where $k_\perp = n_\perp \omega$, $k_\parallel = n_\parallel \omega$. We obtain from Eq.(19) [33]

$$\alpha = \theta, \quad \delta = (k_\perp - k_\parallel) z = ((\beta - \alpha) \cdot \overrightarrow{B}) \omega z, \quad \sin 2\chi = (\sin 2\alpha) \sin \delta.$$  

One finds from Eq.(20) the induced ellipticity (the ratio of minor to major axis of the ellipse)

$$\Psi \equiv \tan \chi \simeq \frac{1}{2} \delta \sin 2\theta = \frac{((\beta - \alpha) \cdot \overrightarrow{B}) \pi L}{\lambda} \sin 2\theta,$$

where $\omega = 2\pi/\lambda$, $\lambda$ is a wave length. We have used here the smallness of the $\delta$. As a result, after propagating the distance $L$, initially linearly polarized light becomes elliptically polarized. One obtains to first order in the small parameter $\delta$: $\psi \simeq \theta$ (because the angle of the rotation of the ellipse $\psi$ is given by $\tan 2\psi = (\tan 2\alpha) \cos \delta$). There is no rotation of the polarization axis of the ellipse.

With the help of the preliminary results of the PVLAS experiment [2]

$$\Psi = (-3.4 \pm 0.3) \times 10^{-12} \text{ rad pass}, \quad L = 1 \text{ m},$$

$$\lambda = 1064 \text{ nm}, \quad \theta = \frac{\pi}{4}, \quad \overrightarrow{B} = 5.5 \text{ T}, \quad e\overrightarrow{B} = 3.25 \times 10^{-10} (\text{MeV})^2,$$

one can find from Eq.(21) the constraint for the parameter difference:

$$((\beta - \alpha) \cdot \overrightarrow{B}) \simeq 10^{-9} (\text{MeV})^{-2},$$

where the subscript means the projection on the direction of $\overrightarrow{B}$. The induced ellipticity of the PVLAS experiment can be explained within the effective Lagrangian (1). Possibly the improvement of PVLAS data will change the estimation (23) to the lower value.
5 Conclusion

We suggest the effective Lagrangian at the cubic order in the electromagnetic field strength which contains two “tensors” $\theta^{(1,2)}_{\mu\nu}$. This is a generalization of NC electrodynamics. At the limit $\theta^{(1)}_{\mu\nu} = \theta^{(2)}_{\mu\nu}$, one arrives at electrodynamics on NC spaces (with the help of the Seiberg-Witten map). The Lorentz covariance is broken because parameters $\theta^{(1,2)}_{\mu\nu}$ are not transformed as real tensors. Lorentz violating structures at the quadratic order in field strength, leading to birefringence in a vacuum without a magnetic field, were discussed in [13]. The Lorentz violating operators at quadratic order are constrained by astrophysical data [20], [21], and therefore, we do not include these structure in the Lagrangian investigated.

The density of the energy and momentum, and the canonical and symmetric energy-momentum tensors are found. The canonical energy-momentum tensor is conserved, but the symmetric energy-momentum tensor, obtained by varying the action on the metric tensor, is non-conserved. The traces of the canonical and symmetric energy-momentum tensors do not equal zero, i.e., there is a trace anomaly at the tree level. This anomaly is related to the violation of the Lorentz invariance. We show that the propagation of the electromagnetic wave in the constant magnetic background and the energy propagation have different directions, i.e. the vacuum is similar to an anisotropic medium.

It was proven that the model suggested leads to the induced ellipticity which, at the case $\alpha = \beta$, $\gamma = \xi = 0$, disappears in accordance with the previous results [22], [23], [24], [25], [26], [27]. We have calculated the induced ellipticity through the parameters $\alpha$, $\beta$. This relation allows us to obtain the constraint on parameters introduced to explain the ellipticity observed in the PVLAS experiment. For the case $\xi \neq 0$, $\gamma \neq 0$, the induced ellipticity depends on four “vector”-parameters $\alpha$, $\beta$, $\xi$, $\gamma$.

It should be mentioned that the discussed additions to the Lagrangian can not explain all of the PVLAS observations because they do not lead to a rotation of the polarization (dichroism). We leave the discussion of bounds on parameters introduced coming from astrophysics for further investigations.

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Added note. - After this paper was prepared, I studied the manuscript E. Zavattini et al. [PVLAS Collaboration], arXiv:0706.3419 [hep-ex] with new experimental data. New PVLAS results give limits on magnetically induced
rotation and ellipticity in vacuum and correct Eq.(22) and our estimation (23) to the lower value.

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9
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