Abstract: We describe briefly the recent advances in understanding the distributed nature of computations in the (neural) network structure of the brain. We discuss if such artificial networks will be able to perform mathematics and natural sciences. The problem of consciousness in such machines is addressed. Ancient Indian ideas regarding mind-body relations and J. C. Bose’s experimental observations regarding the highly distributed computations in the plant body is discussed.

I. Mathematics and logic:
What is the nature of mathematical truths? Is mathematical knowledge true a priori? Independent of experience and observation? Does one have to verify mathematical truths in any (mathematical) laboratory? If not, what makes it true?

Philosophers and mathematicians have thought about it for a long time: see e.g., Whitehead and Russell [1,2] for some discussions on these thoughts by western thinkers (a comparative study of eastern and Indian thoughts seems to be missing – at least not known to this author). Although several ideas had been developed over the ages, Whitehead and Russell proposed very forcefully, following Hilbert (1862 - 1943), the idea that mathematical statements are true because of their internal consistency and as they do not convey anything new; they are in fact tautologies. Mathematics is just a condensed form of logic. Mathematical proof of a proposition is just an elaboration of the proposition itself; nothing new is conveyed or introduced by mathematical proof. That is why, there is no need to have a laboratory checking the truth of a mathematical statement. Two plus two is four because the concept of (the set of) four contains the concept of (the sets of) two. That fifty minus fifty makes it zero need not be checked by pushing all the fifty odd audience out of this lecture hall! It is true just like the truth of a statement “A bachelor does not have any wife”; one need not check if each individual bachelor satisfies it or not. To prove the truthfulness of these statements, one just needs to look at the meaning of the words involved!

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If it is true that mathematics is just a condensed form of logic, it is certainly formalizable. In that case, is the human brain unique or even necessary for doing and developing mathematics? Or, can a machine do that? May not be today in a complete form, but in some future day? Mathematics is certainly necessary; but are human mind or the mind of a mathematician absolutely necessary? May be, we do not have the machine or the computer yet to replace the mathematicians; but in the future, can we dispense with them?

Although the debate still continues, it appears that a major part of mathematics like arithmetic is already formalizable and indeed a computer can do it and do it better. Question arises, what about analytical mathematics and geometry? Intuitionists like Kronecker (1823-1891), Poincaré (1854-1912), Borel (1871-1956), Weyl (1885-1955) and others chiefly argued against such formalized view of mathematics on the ground of its inappropriateness in analysis and geometry (see e.g., [2]). For example, how does such formalization work when a function or a number is expressed as an infinite series? When all the terms in the series, which are part of the function or the number, can not be enumerated, as in many such series forwarded by Ramanujam (1887-1920)? Similar are the cases of geometrical analysis. A celebrated demonstration of the problem came from Gödel (1906 - 1978). Seizing upon paradoxical situations like “The statement I am making now is untrue”, Gödel was able to show that if a formalized mathematics, as proposed in Principia Mathematica is wide enough, then (a) the system is necessarily incomplete in the sense that there exists a formula $F$ of the system such that neither $F$ nor its negation is derivable, and (b) if the system is consistent, then no proof of its consistency is possible which can be formalized within it (cf. [2,3]). Later works of people like Feigenbaum on the bifurcation route to chaos in some nonlinear maps and its universality class obtained using renormalization group technique (a physical or inductive principle, not a mathematical one), or like Witten on the string-dynamical description of elementary particles indicated the existence and need of (computational or physical) laboratory of mathematics (cf. [4]).

II. Inductive logic and pattern recognition:

As discussed in the previous section, mathematics is thought primarily to be based on deductive logic. Physics or for that matter other natural sciences are based on inductive logic. Here, based on careful observations and ‘inductions’ from there, one formulates the basic statements or truths. From such inductive truths, one then deduces the special statements or truths appropriate for specific situations in physical or natural sciences. This deductive (logic) part in natural sciences is of course an integral part and naturally involves mathematics. Unlike most of mathematics (if not all) these
inductive truths are obtained from the observations in various ‘laboratories’. Looking at the sun rising on the east every morning, we come to the inductive truth “The sun rises in the east every morning”. This helps us to predict almost certainly, that the sun will come up in the east tomorrow. But unlike the mathematical or deductive truths, this inductive truth and its predictions are provisional; an observation next day might force us to update, refine or change the truth. Compared to this, the prediction that two stars and two others in sky in the night tomorrow will make four stars, is not provisional and one need not have to check if it was valid yesterday!

The search and application of these inductive logic are admittedly the hallmark of human brain; more specifically of the brains of great scientists. Unlike mathematics, which is thought to be mostly formalizable and based on deductive logic, the natural sciences are thought to be essentially based on inductive logic or truth. Unlike the major part of mathematics therefore, which can be performed by a machine or computer, natural science is expected to be essentially developed by and dependent on the human mind or brain!

Recently, the computer scientists have developed algorithms to ‘search or recognise patterns’ in seemingly unrelated situations or sequences. These mechanical processes of pattern recognition are indeed similar in spirit to the search of inductive truths by human mind or brain! For example, by recognising the ‘abnormalities’ in the regular pattern of blood flow through our veins, the physicians diagnose our illnesses. Such an expertise of a medical doctor is of course much too rudimentary compared to that of natural scientist recognising, for example, that the same pattern is involved in the motion of the planets around the sun and the apple falling on the ground from the tree! Nevertheless, in the extended language of the computer science, all these are pattern recognition problems; some are much simpler compared to others. Basically, the problems of the medical doctor and of Newton are the same; pattern recognition. Indeed, the above mentioned simple pattern recognition of blood flow pulses can often be performed these days by ‘expert computer algorithms’ as well. Slightly more complicated recognition problems like that of elementary musical rhythms and of hand-writing etc are now analyzable using computers; several algorithms are already developed to help solving such problems. The recently developed ‘associative memory’ algorithms by Hopfield (in 1984; see the next section and ref. [5]) and of ‘learning’ by reorganising the connections through interactive minimization of errors by the multi-layer perceptrons, originally deviced by Rosenblatt (in 1962; see next section and ref. [5]) are indeed very encouraging. To some natural scientists, these are clear indications (inductive truth?) that all our deductive and inductive logics, and hence entire mathematics and the natural sciences, can in principle be performed by machines: computers
or artificial neural networks. This is a very recent and highly exciting problem posed in the literature of philosophy of science, often known as the ‘Strong Artificial Intelligence (AI)’ hypothesis; see e.g., Crick [6] supporting the hypothesis and the excitement and Penrose [3], Chalmers [7] et al, contradicting parts of it (essentially arguing from the Gödel’s theorem). We will continue with the discussion on this issue at the end of the next section.

III. Neural network modelling of the brain:

Although we differ from the animals in almost all the parts of our bodies, our essential difference is rightly identified in our respective brains. We are known by our brains; a mathematician, a scientist or an artist differ essentially in their respective brains. Although even in the physiques of mine and of mathematicians-scholars like Aryabhatta II (b. 476 A.D.) or Bhaskaracharyya II (b. 1114 A.D.) there must have been a lot of differences, the essential difference, as we are all too aware, have been in our brains! But what is the precise physical difference between my brain, and that of Aryabhatta or for that matter that of a cow? We do not know yet about all the differences, except for their size and the structure.

We know today, at least in principle, the structure and working mechanisms of almost all the parts of our body, except for the brain, and have in fact developed some artificial supplements for them. They are used when parts of our body fail to function normally. We know that the outer-retina part of our eye is like a camera, heart is like a pump, etc and we can be provided with artificial supports like spectacles, pacemakers, etc to supplement their partial failures. In case of the brain, however, we are still helpless, even if it fails minimally. Our interest in the brain structure and function is therefore not just of epistemological, mathematical, computational or physical curiosity or interest, medical support possibility in future is of extreme importance and can hardly be overemphasized.

During evolution, the animal bodies developed their brains to perform the primary task of helping the body act according to the changes in the environment: to analyse the signals received from the environment and to respond accordingly. As the body surface receives the external signals, each portion of it is mapped in the brain. In fact, as the body surface grows with the body volume to the power 2/3, the brain mass varies with 2/3rd power of the animal body mass; bigger brain of the elephant is required for the control of the bigger body. By injecting coloured stains inside a dead brain, about hundred and fifteen years back, the spanish doctor Ramon y Cajal (nobel prize in Medicine in 1906) showed that the brains are made up of many tiny cells, called since then neurons, which are connected to each other through synaptic
junctions separated by semipermeable membranes. We now know almost certainly that human brain contains about $10^{12}$ neurons and we are all born with them; Aryabhatta, Newton, Einstein, Tagore and myself were all born with more or less this number of neuronal cells in the brain. This number does not differ much within the species, but differs considerably from species to species; e.g., the birds have about $10^8$ neurons. The physical structure of a neuron is indicated in Fig. 1. Each neuron is an electrical device capable, in principle, of a very simple (electrical) operation. It collects electrical pulses (of millivolt order) from $10^4$ to $10^6$ other neurons connected to it through its dendrites. These pulses, collected over a synaptic period of a few milliseconds, are then summed-up in the cell body. If the resultant sum exceeds a threshold voltage, a few tens of millivolt order, the neuron fires and a millivolt order electrical pulse propagates (at a speed of few meters per second) through the axon (cable). It then passes over to the other connected neurons through the respective synaptic junctions. As noted by several neurophysiologists, including Hebb (in 1949), these synaptic connections between the neurons develop with training and learning. We are not born with all these electrical wirings (synaptic connections) among the components (neurons), although a significant fraction of them seem indeed to be determined by hereditary factors. Needless to mention here that according to this picture, I differ from Aryabhatta in developing my inter-neuronal connections; not in our brain size or neuron number. These synaptic connections may be both excitatory (where a positive pulse flows across it keeping its phase unchanged) and inhibitory (where a positive pulse passes over to the connected neuron as negative pulse, with changed phase). In fact, such random millivolt order $10^4$ to $10^6$ incoming pulses add up to only about $10^{-2}$ or $10^{-1}$ volts in the cell body of a single neuron. For all excitatory or all inhibitory connections, this sum in a single cell would go to an extremely high value and cause the failure of the cell. More importantly, as we will see later, the absence of ‘frustration’ (see e.g., [5]) in the cases of all excitatory or all inhibitory connections would reduce enormously the brain memory capacity. As may be noted from Fig. 2, these synaptic connections develop with appropriate signals to the brain (received in appropriate time), and it takes maximum time (about 26 to 30 years) for human. Compared to this, the animal brain development (development of their inter-neuron connections) takes very little time and in fact it ceases almost immediately after their birth. It appears therefore that I differ from great artists, scientists or mathematicians, mostly in our respective developments (of synaptic connections) after our births!

As mentioned already, neurons are electrical devices and can be in two functional states: firing state (if the aggregate synaptic voltage in the cell exceeds the threshold) or quiescent state (otherwise). Neurophysiologists McCullogh and Pitts (see e.g., [5])
therefore proposed the idea of functional modelling of a single neuron by a two state
device like an electrical valve or an electronic transistor. The consequent excitement
came from the realization that the present day computer workstations already employ
about $10^8$ transistors (comparable to the neuron number in a pigeon’s head) and the
transistors, being electronic systems, work much faster (typical time scale being $10^{-8}$
seconds) while the ionic flow rate in the neurons are much slower (with typical time scale
of the order of $10^{-1}$ to $10^{-3}$ seconds). Once the inter-neuronal connection architecture
in the brain is understood, its artificial implementation on a silicon device may become
extremely powerful!

As mentioned before, although we know now a little bit about the structure and
function of a single neuron, we are still in the dark about the growth of the neural
network through the inter-neuron synaptic connections. We do not know yet the precise
algorithms followed during the learning processes to develop these connections.

One can use a digital or binary representation of any pattern using pixel decomposi-
tions. Each such pattern can then be made an ‘attractor’ configuration of the network
(of binary neurons) following a network dynamics. Starting from any ‘corrupted’ or
distorted version of that pattern then the dynamics of the network brings back the
‘learned’ pattern as the dynamics get attracted towards that. The dynamical matrix
elements, representing the synaptic interaction between the neurons, depend on the
pattern the network intends to remember or get attracted to. Two independent pat-
terns then demand differently for these matrix elements. Memory of a large number
of such patterns then demand conflicting or frustrating requirements for the synaptic
connections or the matrix elements. This is a generic feature for such networks. In
fact, this frustration leads to a macroscopic number of local attractors of the dynamics
of the network, which helps large memory size etc; without frustration, the network
would have only two attractors (and hence two memory states). In the Hopfield model,
one defines an energy function in the pattern configuration space such that the learned
patterns correspond to local energy minima, whereas the corrupted or distorted pat-
terns correspond to higher energies. Any dissipative energy minimisation dynamics
then brings the system to the local minima or memory state if the starting configura-
tion was within its domain of attraction. In this model, the synaptic connections are
taken, following Hebb’s, symmetric and its magnitude given by the algebraic sum of the
inter-neuron interactions required for each of the pattern to be learned or memorised.
The resultant interactions then become random not only in magnitude but also in sign.
This frustration leads to a maximum memory size of the network (capable of recalling
from distorted patterns) about 14% of the network size, given by the number of neu-
rons in the network. The network gets confused if more patterns are put in it! In the
Rosenblatt perceptron model, these dynamical matrix elements (synaptic connections) evolve dynamically by minimising errors in predicting the ‘unseen’ part of the pattern. After some initial ‘supervision’, such networks perform various pattern recognition jobs satisfactorily (see e.g., [5]).

**Critisms:** As mentioned in the previous section, although there have been intriguing developments and consequent excitement regarding the possibility of artificial intelligence and mind, as good as those of the human, severe criticisms of such Strong AI hypothesis have been forwarded by several scientists. The Strong AI states (cf. Crick [6]) that a “Computer will not only have mental states as its emergent property, the implemented program will by itself constitute the mind”. Penrose [3] argues that such a machine can not have consciousness (cf. emperor’s new cloth) which in his view is complicated by quantum mechanical entanglements. Chalmers [7] argues that even if such a machine performs all these (computations and pattern recognitions), it can not be ‘self-conscious’. The argument is that a computer program is defined purely syntactically, and that the syntax itself is not enough to guarantee the presence of mind. My stomach pain is my personal feeling and I am conscious of that, while the physiological disorder and neurological processes following that are objective facts for a physician identifying the cause of my pain; they are not identical. Searle [8] developed a ‘chinese room’ argument to refute the Strong AI hypothesis. The argument runs as follows: Even if I do not understand chinese, I can behave like a chinese by following a set of preassigned (say translated) rules or programs. Within these set of rules (program) I will appear to behave as understanding it; although I do not! Thus (a) programs are entirely syntactical, (b) minds have semantics and (c) syntax is not the same as, nor by itself sufficient, for semantics. This three step chinese room argument therefore proves “Programs are not minds” (cf. [8]).

**IV. Indian Concept of mind & Bose’s nervous mechanism of plants:**

In Upanishad (1500 B.C. - 1000 B.C.), the mind was argued to be composed of the heart and the brain. In fact, in Praśna, one gets even a description of the physiological structure of the mind [9]. From the heart, 101 ‘nadi’ or ‘dhamani’ gets out, each of which apparently branches out in 100 thinner and tinier branches, and so on. It says, in total about 72000 ‘nadi’ or ‘dhamani’ are spread all throughout our body and the brain. Proper function of our brain depends on all of them [9]. This 3000 year old crude and speculative model might be compared with our present (established) knowledge of about $10^{12}$ neurons in the human brain! Upanishad then argues that the external objects or processes then induce, through the senses and conveyed through the ‘nadi’s,
‘Manas’ (or sense-data) which in turn induces ‘Buddhi’ (becomes unmanifest) giving rise to ‘Purusa-Atman’ (self or ego) which finally melts down to ‘Chitta’ (consciousness). These ideas about the consciousness of our mind were later evolved in Bhagavad Gita (300 B.C.) and in particular in Buddhism (273 B.C. - 200 B.C.) [10].

The idea or the philosophical doctrine that the mind is not essentially confined only to a small part part of the body (for example the brain), and that it disperses all over the body seemed to be a dominating one in ancient Indian thoughts. In fact, even the treatment and control of the mental processes, as advocated and prescribed in Susrut (500 B.C.) and in Yoga and Tantraloka (cf. [10]), involved some thoughtful and thorough exercise of the various parts of our body! It is indeed unfortunate that scholastic follow-ups, scientific investigations following these ideas and their refinements are nonexistent or insignificant. Even documents and books on these developments are scarce (cf. [9, 10]).

It is particularly heartening to ‘discover’ in this context, the experimental work of Jagadish Bose in the last century on the nervous mechanism of plants [11]. It is well known, plants do not have brains, and hence do not have any neuronal cells or their network like us. Yet, the plants do indeed perform computations for adjusting and responding to the changing environments. Plants do these calculations slowly, but surely. Imagine the response, say within a week, of a plant in a suddenly darkened area with sunlight coming only from an angle, or take the case of a creeper plant climbing up a window grill or a pillar with its tentacles or branches! Imagine the amount of computations involved in ‘recognising’ the structure of the neighbouring posts or grilles, in ‘finding’ their minimum cross-sections and in holding them by growing around the necks of the neighbouring structures. Do they also have personal feelings? Are they self-conscious? We do not know.

Through his pioneering experiments, J. C. Bose [11] showed about hundred years ago that the plant cells are excitable and can transmit millivolt order electrical signals at about 10-40 millimeter per second speed. Through these electrical signals, these cells communicate in coordinating their responses to the environment (see Fig. 3). This analysis and ‘recognition’ of the changes in the external environment is therefore performed by the plants, according to Bose, through its extended (nervous) cellular network all across its trunks, branches and leaves. It may be mentioned that this partial electrical signalling between the plant cells, like those in the neurons of the animals, is now a fairly established fact; although, for a long period after Bose’s pioneering work, the plant physiologists did not accept it and considered the inter-cell signalling to be purely chemical diffusion in origin (see e.g., Shephard [11]). This observation of extended computation or processing of the environmental signals all over the living
body of the plant or the animal is in fact very much in conformity with the ancient
Indian idea of mind-body relationship. Again, not much development has taken place
in this direction.

Concluding remarks:

Present analysis of the brain structure and of the neural computation process indicates
how a collective computing property (like consciousness) might emerge out of a network
of (about $10^{12}$) neurons or transistors. Unlike the present day computers, our brain
calculates in a distributed way. It employs parallel processing involving almost all the
neurons in the brain for each computing operation. The observation by Bose [11] on
the plant nervous system suggests that computations can be much more distributed
than we can think today. Plants do not have any brain and yet they compute using
their cells all over the plant body. Can individual brains interact? Electrical contacts
or interactions are not possible, but perhaps socially? World population today is about
$10^{10}$, and every one of us has got a brain to perform simple tasks. Is it possible that
collective computational capacity of many such brains might give rise to higher order
computational abilities and the (social) consciousness we are so familiar with? Isolated
person like Robinson Crusoe’s brain may not generate it. But an interactively evolving
society perhaps develops it necessarily? Partial or indifferent participation may then
lead to different perceptions, ‘value judgements’ and ethics. Compared to the innate
history of the universe, human history therefore becomes accessible to value judgements
and conscious evaluation (cf. [12]). Such a possibility seems to be pretty close to the
ideas floated by the major Indian schools of thought, starting from Upanishad. If this
is true, machines can also have such consciousness; only perhaps collectively!

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**Figure captions:**

Fig. 1. Schematic structure of a neuron.

Fig. 2. Development of inter-neuron synaptic connections in the human visual cortex after the birth: (from left to right) newborn, three month old and two year old infant [From T. H. Bullock, R. Orkand and A. Grinnel, *Introduction to Nervous Systems*, Freeman, San Francisco (1977)].

Fig. 3. Electrical pulsations in Desmodium, measured by inserting the probe slowly (0.1 mm per turn) within the tissues [from Bose [11]].
