On the estimate of the $\sigma_{KN}^{(I=1)}(0)$–term value from the energy level shift of kaonic hydrogen in the ground state

A. N. Ivanov *,†, M. Faber ‡, V. A. Ivanova §, J. Marton ¶, N. I. Troitskaya ∥

September 24, 2018

Abstract

Using the experimental data on the energy level shift of kaonic hydrogen in the ground state (the DEAR Collaboration, Phys. Rev. Lett. 94, 212302 (2005)) and the theoretical value of the energy level shift, calculated within the phenomenological quantum field theoretic approach to the description of strong low–energy $\bar{K}N$ and $\bar{K}NN$ interactions developed at Stefan Meyer Institut für subatomare Physik in Vienna, we estimate the value of the $\sigma_{KN}^{(I=1)}(0)$–term of low–energy $\bar{K}N$ scattering. We get $\sigma_{KN}^{(I=1)}(0) = (433 \pm 85)$ MeV. This testifies the absence of strange quarks in the proton structure.

PACS: 11.30.Rd, 13.75.Jz, 25.80.Nv, 36.10.Gv
1 Introduction

Recently in Refs.\[1\]–\[4\] (see also \[4\]) we have proposed a phenomenological quantum field theoretic model for strong low–energy $KN$ and $KNN$ interactions at threshold for the analysis of the experimental data by the DEAR Collaboration Refs.\[6\]–\[7\] on the energy level displacement of the ground state of kaonic hydrogen

\[- \epsilon_{1s}^{\text{(exp)}} + i \frac{\Gamma_{1s}^{\text{(exp)}}}{2} = (-193 \pm 37) + i(125 \pm 56) \text{ eV}. \tag{1.1}\]

According to the DGBTT formula \[5\], the energy level displacement of the ground state of kaonic hydrogen is related to the S–wave amplitude $f_0^K(p)$ of elastic $K^{-}p$ scattering at threshold as

\[- \epsilon_{1s} + i \frac{\Gamma_{1s}}{2} = 2 \alpha^3 \mu^2 f_0^K(p) = 412.13 f_0^K(p), \tag{1.2}\]

where $2 \alpha^3 \mu^2 = 412.13 \text{ eV/fm}$, $\mu = m_K m_p / (m_K + m_p) = 323.48 \text{ MeV}$ is the reduced mass of the $K^{-}p$ pair, calculated for $m_K = 493.68 \text{ MeV}$ and $m_p = 938.27 \text{ MeV}$, and $\alpha = 1/137.036$ is the fine–structure constant Ref.\[9\]. An accuracy of the DGBTT formula Eq.(1.2) is about 3\% including the vacuum polarization correction Ref.\[10\].

The real part $\Re f_0^K(p)$ of the S–wave amplitude $f_0^K(p)$ defines the energy level shift of kaonic hydrogen in the ground state. It has been calculated in \[1\]–\[3\] up to next–to–leading order in chiral expansion

$$
\Re f_0^K(p) = a_0^K(p) - \frac{1}{4 \pi F_\pi^2} \frac{\mu}{m_K} \left[ \sigma_{KN}^{(i=1)}(0) - \frac{m_K^2}{4 m_N} i \int d^4 x \langle p | T(J_{50}^{4+i5}(x)) \langle p | [4] \right],
$$

where $a_0^K(p) = (-0.54 \pm 0.05) \text{ fm}$ is the S–wave scattering length of elastic $K^{-}p$ scattering calculated to leading order in chiral expansion \[4\]–\[5\] (see also \[11\]), including the contribution of strange baryon resonances such as the $\Lambda(1405)$ resonance \[4\] and the corrections, caused by inelastic channels $K^{-}p \to \Sigma \pi$ and $K^{-}p \to \Lambda \pi$ \[4\]–\[5\].

The two last terms in Eq.(1.3) are next–to–leading order corrections in chiral expansion, where $F_\pi = 92.4 \text{ MeV}$ is the PCAC constant \[9\]. The second one is the $\sigma_{KN}^{(i=1)}(0)$ term of $KN$ scattering. It is defined by \[12\]–\[14\]

$$
\sigma_{KN}^{(i=1)}(0) = \frac{m_u + m_s}{4 m_N} \langle p | \bar{u}(0) u(0) + \bar{s}(0) s(0) | p \rangle,
$$

where $u(0)$ and $s(0)$ are current quark fields, $m_u$ and $m_s$ are their masses. Since the $\sigma_{KN}^{(i=1)}(0)$ term is proportional to $(m_u + m_s)$, so at leading order in chiral expansion the matrix element $\langle p | \bar{u}(0) u(0) + \bar{s}(0) s(0) | p \rangle$ should be calculated in the chiral limit.

The chiral order of the third term is defined by the squared $K^-\pi$ meson mass $m_K^2 = O(m_u + m_s)$. This implies that the matrix element $\langle p | T(J_{50}^{4+i5}(x),J_{50}^{4-i5}(0)) | p \rangle$, where $J_{50}^{4+i5}(x)$ are time–components of axial hadronic currents $J_{50}^{4+i5}(x)$ changing strangeness $|\Delta S| = 1$, as well as $\langle p | \bar{u}(0) u(0) + \bar{s}(0) s(0) | p \rangle$ has to be calculated in the chiral limit.

The real part of the S–wave amplitude, given by Eq.(1.3), determines the energy level shift $\epsilon_{1s}^{(th)}$ of the ground state of kaonic hydrogen. It is equal to \[2\]–\[4\]

$$
\epsilon_{1s}^{(th)} = \epsilon_{1s}^{(0)} + \frac{\alpha^3 \mu^3}{2 \pi m_K F_\pi^2} \left[ \sigma_{KN}^{(i=1)}(0) - \frac{m_K^2}{4 m_N} i \int d^4 x \langle p | T(J_{50}^{4+i5}(x)) \langle p | \right],
$$

1
where $\epsilon_1^{(0)} = (238 \pm 21)$ eV is caused by $\sigma_0^{K^-p} = (-0.54 \pm 0.05)$ fm and the isospin–breaking corrections, which contribute only to the energy level displacement of the ground state of kaonic hydrogen but not to the S–wave amplitude $f_0^{K^-p}(0)$ of elastic $K^-p$ scattering at threshold [3, 4].

The theoretical estimates of $\sigma_{KN}^{(I=1)}(0)$, carried out within ChPT [15] with a dimensional regularization of divergent integrals, converge to the number $\sigma_{KN}^{(I=1)}(0) = (200 \pm 50)$ MeV [13]. The values of the $\sigma_{KN}^{(I=1)}(0)$–term, calculated in ChPT with a cut–off regularization, are by a factor 2 larger than $\sigma_{KN}^{(I=1)}(0) = (200 \pm 50)$ MeV [13].

The aim of this paper is to estimate the value of the $\sigma_{KN}^{(I=1)}(0)$–term from the experimental data by the DEAR Collaboration Eq.(1.1) using the theoretical value of the energy level shift given by Eq.(1.5).

2 Estimate of the $\sigma_{KN}^{(I=1)}(0)$–term

In order to extract the value of the $\sigma_{KN}^{(I=1)}(0)$–term from the experimental data on the energy level shift of kaonic hydrogen in the ground state Eq.(1.1) by using the theoretical value of the energy level shift Eq.(1.5) we have to calculate the contribution of the matrix element $\langle p| J_{50}^{4+i5}(x) J_{50}^{4-i5}(0) | p \rangle$. As has been mentioned above, the contribution of this matrix element should be taken in the chiral limit, i. e. to leading order in ChPT [15].

Following [1,16] we can transcribe the third term in Eq.(1.5) as follows

$$i \int d^4 x \langle p| T(J_{50}^{4+i5}(x) J_{50}^{4-i5}(0)) | p \rangle = \frac{1}{2} \sum_{\sigma_p=\pm 1/2} \sum_x (2\pi)^3 \delta^{(3)}(\vec{p}_X) \frac{|\langle X| J_{50}^{4-i5}(0) | p(\bar{0}, \sigma_p) \rangle|^2}{E_X(\vec{p}_X) - m_N} , (2.1)$$

where $X$ is a hadronic state with baryon number $B = 1$ and strangeness $S = -1$. The lowest states contributing to the sum over the intermediate states $X$ are $X = MB$, where $MB = \Lambda^0\pi^0, \Sigma^0\pi^0, \Sigma^+\pi^-, K^-p$ and $K^0n$. This gives

$$i \int d^4 x \langle p| T(J_{50}^{4+i5}(x) J_{50}^{4-i5}(0)) | p \rangle = \frac{1}{2} \sum_{\sigma_p=\pm 1/2} \sum_{MB} \sum_{\sigma=\pm 1/2} \frac{1}{32\pi^3}$$

$$\times \int \frac{d^3 k}{E_M(k)E_B(k)} \frac{|\langle M(-\vec{k}) B(\vec{k}, \sigma) | J_{50}^{4-i5}(0) | p(\bar{0}, \sigma_p) \rangle|^2}{E_M(k) + E_B(k) - m_N} , (2.2)$$

where $\vec{k}$ is the relative momentum of the $MB$ pairs. The calculation of the matrix elements of $\langle M(-\vec{k}) B(\vec{k}, \sigma) | J_{50}^{4-i5}(0) | p(\bar{0}, \sigma_p) \rangle$ we carry out in the soft–meson limit (to leading order in ChPT) using the PCAC hypothesis and Current Algebra [17], and in the heavy–baryon limit, accepted in ChPT for the analysis of baryon exchanges [15]. This gives

$$i \int d^4 x \langle p| T(J_{50}^{4+i5}(x) J_{50}^{4-i5}(0)) | p \rangle =$$

$$= \frac{1}{2} \sum_{\sigma_p=\pm 1/2} \sum_{\sigma=\pm 1/2} \frac{1}{32\pi^3} \frac{1}{F_M^2} \frac{1}{m_N} \int \frac{d^3 k}{k^2} \frac{1}{k^2} |\langle B(\vec{k}, \sigma) | | Q_5^M(0) | J_{50}^{4-i5}(0) | p(\bar{0}, \sigma_p) \rangle|^2 , (2.3)$$
where we have neglected the mass differences of baryons. This is valid in the chiral limit, since the mass differences of baryons, according to ChPT, are proportional to current quark masses and do not contribute in the chiral limit. $Q_5^M(0)$ is the axial–vector charge operator with quantum numbers of the $M$–meson and $F_M$ is the PCAC constant of the $M$–meson: $\sqrt{2} F_{\pi^0} = F_{\pi^-} = F_{\bar{K}^-} = F_{K^0} = \sqrt{2} F_\pi$.

In the framework of Gell–Mann’s current algebra [17] the equal–time commutators $[Q_5^M(0)^\dagger, J_{50}^{-i\tau}(0)]$ amount to

$$[Q_5^3(0), J_{50}^{i\tau}(0)] = -\frac{1}{2} J_{60}^{i\tau}(0),$$

$$[Q_5^{+1/2}(0), J_{50}^{i\tau}(0)] = J_{60}^{i\tau}(0),$$

$$[Q_5^{0+}(0), J_{50}^{i\tau}(0)] = J_0^7(0) + \sqrt{3} J_0^6(0),$$

$$[Q_5^{6+}(0), J_{50}^{i\tau}(0)] = J_0^{1\tau}(0).$$

(2.4)

The matrix elements of the vector currents are defined by

$$\langle \Lambda^0(\bar{k}, \sigma) | J_0^{i\tau}(0) | p(\bar{0}, \sigma_p) \rangle = -\sqrt{\frac{3}{2}} F_V(k^2) \bar{u}_\Lambda^0(\bar{k}, \sigma) \gamma^0 u_p(\bar{0}, \sigma_p),$$

$$\langle \Sigma^0(\bar{k}, \sigma) | J_0^{i\tau}(0) | p(\bar{0}, \sigma_p) \rangle = -\sqrt{\frac{1}{2}} F_V(k^2) \bar{u}_\Sigma^0(\bar{k}, \sigma) \gamma^0 u_p(\bar{0}, \sigma_p),$$

$$\langle \Sigma^+(\bar{k}, \sigma) | J_0^{i\tau}(0) | p(\bar{0}, \sigma_p) \rangle = -F_V(k^2) \bar{u}_{\Sigma^+}(\bar{k}, \sigma) \gamma^0 u_p(\bar{0}, \sigma_p),$$

$$\langle p(\bar{k}, \sigma) | J_0^6(0) + \sqrt{3} J_0^5(0) | p(\bar{0}, \sigma_p) \rangle = 2 F_V(k^2) \bar{u}_p(\bar{k}, \sigma) \gamma^0 u_p(\bar{0}, \sigma_p),$$

$$\langle n(\bar{k}, \sigma) | J_0^{1\tau}(0) | p(\bar{0}, \sigma_p) \rangle = F_V(k^2) \bar{u}_n(\bar{k}, \sigma) \gamma^0 u_p(\bar{0}, \sigma_p).$$

(2.5)

where $F_V(k^2)$ is a form factor. In the limit of the $SU(3)$ flavour symmetry the vector form factor $F_V(k^2)$ should be the same for all components of the octet of ground baryons. In the heavy–baryon limit we can drop the contribution of the “magnetic” form factor [18].

The result of the calculation of the r.h.s. of Eq. (2.3) is

$$i \int d^4 x \langle p | T(J_{50}^{i\tau}(x) J_{50}^{i\tau}(0)) | p \rangle = \frac{7m_N}{4\pi^2 F_\pi^2} \int_0^\infty dk \, F_V^2(k^2).$$

(2.6)

The contribution of the term (2.1) to the energy level shift is equal to

$$\delta \epsilon_{1s} = -\frac{7\alpha^3 \mu^3 m_K}{32\pi^3 F_\pi^4} \int_0^\infty dk \, F_V^2(k^2).$$

(2.7)

For the numerical estimate we identify $F_V(k^2) = 1/(1 + k^2/M_V^2)$, where $M_V^2 = 0.71 \text{ GeV}^2$ is the squared slope parameter [19]. After the integration over $k$ we get

$$\delta \epsilon_{1s} = -\frac{35\alpha^3 \mu^3 m_K M_V}{1024\pi^2 F_\pi^4} = -260 \text{ eV}.$$  

(2.8)

Assuming that the theoretical expression for the energy level shift of kaonic hydrogen in the ground state Eq. (1.5) fits the experimental value Eq. (1.4) we estimate the $\sigma_{KN}^{(t=1)}(0)$–term. It is equal to

$$\sigma_{KN}^{(t=1)}(0) = (193 + 260 - 238) \frac{2\pi}{\alpha^3 \mu^3} m_K F_\pi^2 = (433 \pm 85) \text{ MeV},$$

(2.9)
where the uncertainty $\pm 85 \text{ eV}$ is defined by the experimental and theoretical uncertainties $\pm 37 \text{ eV}$ and $\pm 21 \text{ eV}$ of the energy level shift, respectively. According to the proposal of the SIDDHARTA Collaboration [7], the experimental uncertainty of the energy level shift should be substantially diminished in the new set of experiments on the energy level displacement of kaonic hydrogen.

3 Discussion

Using the theoretical value of the energy level shift of kaonic hydrogen in the ground state, calculated within the phenomenological quantum field theoretic model of strong low–energy $KN$ interactions [1, 4], and the experimental data by the DEAR Collaboration [6] we have extracted the value of the $\sigma_{KN}^{(I=1)}(0)$–term: $\sigma_{KN}^{(I=1)}(0) = (433 \pm 85) \text{ MeV}$. The obtained result is by a factor 2 larger the estimate $\sigma_{KN}^{(I=1)}(0) = (200 \pm 50) \text{ MeV}$, carried out within ChPT with a dimensional regularization of divergent diagrams [13]. For the cut–off regularization of divergent integrals in CHPT the value of the $\sigma_{KN}^{(I=1)}(0)$–term agrees qualitatively with our estimate (see [13] (Borasoy)).

The value $\sigma_{KN}^{(I=1)}(0) = (433 \pm 85) \text{ MeV}$ agrees well with (i) the value $\sigma_{\pi N}(0) = 61^{+2}_{-1} \text{ MeV}$ of the $\sigma_{\pi N}$–term of $\pi N$ scattering, extracted from the experimental data on the energy level displacement of pionic hydrogen in the ground state [20] and (ii) the vanishing contribution of the strange quarks to the proton structure. The $\sigma_{\pi N}$–term is defined by [21]:

$$\sigma_{\pi N}(0) = \frac{m_u + m_d}{4 m_p} \langle p | \bar{u}(0) u(0) + \bar{d}(0) d(0) | p \rangle. \quad (3.1)$$

Following to the naive quark counting and assuming that the proton has the quark structure $|p\rangle = |uud\rangle$ [22], we can calculate the matrix elements $\langle p | \bar{u}(0) u(0) | p \rangle$ and $\langle p | \bar{d}(0) d(0) | p \rangle$ in terms of the $\sigma_{\pi N}(0)$–term. We get

$$\langle p | \bar{u}(0) u(0) | p \rangle = \frac{8}{3} \frac{m_p}{m_u + m_d} \sigma_{\pi N}(0), \quad \langle p | \bar{d}(0) d(0) | p \rangle = \frac{4}{3} \frac{m_p}{m_u + m_d} \sigma_{\pi N}(0). \quad (3.2)$$

The contribution of the strange quarks to the proton structure is defined by the quantity [14]

$$y = \frac{2 \langle p | \bar{s}(0) s(0) | p \rangle}{\langle p | \bar{u}(0) u(0) + d(0) d(0) | p \rangle}. \quad (3.3)$$

The matrix element $\langle p | \bar{s}(0) s(0) | p \rangle$ is equal to

$$\langle p | \bar{s}(0) s(0) | p \rangle = \frac{4 m_p}{m_u + m_s} \left[ \sigma_{KN}^{(I=1)}(0) - \frac{2}{3} \frac{m_u + m_s}{m_u + m_d} \sigma_{\pi N}(0) \right] = (-2.19 \pm 2.43) \times 10^3 \text{ MeV}. \quad (3.4)$$

where we have used the current quark masses $m_u = 4 \text{ MeV}$, $m_d = 7 \text{ MeV}$ and $m_s = 135 \text{ MeV}$ [23], obtained at the normalization scale $\mu = 1 \text{ GeV}$ commensurable with the scale of spontaneous breaking of chiral symmetry [24].
For the matrix elements \( \langle p | \bar{q}(0) q(0) | p \rangle \), where \( q = u, d \) or \( s \), defined by Eqs. (3.2) and (3.4), we obtain \( y = -0.21 \pm 0.23 \) \cite{14}. This agrees well with the absence of the strange quarks in the proton structure.

Since the contributions of the other states \( X \) in the sum (2.1) should be calculated in the chiral limit and in the heavy–baryon limit, most of these contributions vanish. For example, one can show that the contribution of the states \( X = BMM \) vanishes in the heavy–baryon limit as \( O(1/m_N^2) \). This implies the dominant role of the lowest states \( X = BM \), which we have taken into account.

4 Acknowledgement

We are grateful to Torleif Ericson for helpful discussions and constructive criticism.

References

[1] A. N. Ivanov et al., Eur. Phys. J. A 21, 11 (2004); Eur. Phys. J. A 23, 79 (2005); Phys. Rev. A 72, 022506 (2005).

[2] A. N. Ivanov et al., Phys. Rev. A 71, 052508 (2005).

[3] A. N. Ivanov et al., J. of Phys. G 31, 769 (2005); Theoretical Studies on Hadronic Atoms, nucl-th/0505022.

[4] A. N. Ivanov et al., Eur. Phys. J. A 25, 329 (2005), nucl-th/0505078.

[5] V. A. Ivanova and Ya. A. Berdnikov, (submitted to Yad. Fizika), 2005.

[6] G. Beer et al. (the DEAR Collaboration), Phys. Rev. Lett. 94, 212302 (2005).

[7] C. Guaraldo et al. (the DEAR/SIDDHARTA Collaborations), in Recent Achievements and Perspectives in Nuclear Physics, Naples, Italy, November 3-7, 2004.

[8] S. Deser et al., Phys. Rev. 96, 774 (1954); T. L. Trueman, Nucl. Phys. 26, 57 (1961).

[9] S. Eidelman et al. (The Particle Data Group), Phys. Lett. B 592, 1 (2004).

[10] T. E. O. Ericson, B. Loiseau, and S. Wycech, Phys. Lett. B 594, 76 (2004).

[11] The S–wave scattering length \( a_0^{K^-p} \) is defined by the S–wave scattering lengths \( a_0^I \) of \( \bar{K}N \) scattering with isospin \( I = 0 \) and \( I = 1 \): \( a_0^{K^-p} = (a_0^0 + a_0^1)/2 \). In the chiral limit \( a_0^0 \) and \( a_0^1 \) satisfy the low–energy theorem \( a_0^0 + 3a_0^1 = 0 \) with the numerical values \( a_0^0 = (-1.50 \pm 0.05) \) fm and \( a_0^1 = (0.50 \pm 0.02) \) fm \cite{4}. The contribution of inelastic channels \( \delta a_0^{K^-p} = (-0.04 \pm 0.01) \) fm \cite{3} does not violate the low–energy theorem \( a_0^0 + 3a_0^1 = 0 \) within theoretical uncertainties.

[12] E. Reya, Rev. Mod. Phys. 46, 545 (1974).
[13] V. Bernard, N. Kaiser, and Ulf-G. Meiβner, Z. Phys. C 60, 111 (1993); B. Borasoy, Ulf–G. Meißner, Ann. of Phys. 254, 192 (1997); B. Borasoy, Eur. Phys. J. C 8, 121 (1999).

[14] J. Gasser and M. E. Sainio, hep-ph/0002283.

[15] J. Gasser, Nucl. Phys. Proc. Suppl. 86, 257 (2000). H. Leutwyler, PiN Newslett. 15, 1 (1999); E. Jenkins and A. V. Manohar, Phys. Lett. B 225, 558 (1991); J. Gasser, Nucl. Phys. B 279, 65 (1987); J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985); Ann. of Phys. 158, 142 (1984).

[16] T. E. O. Ericson and A. N. Ivanov, hep–ph/0503277.

[17] S. L. Adler and R. F. Dashen, in Current Algebras, W. A. Benjamin, Inc., New York, 1968.

[18] The matrix element of the vector current $J^{\mu}_{\nu} - i^{b}(0)$ between the ground baryon states can, in principle, contain the contribution with the structure of the magnetic moment

$$F_M(k^2) \bar{u}_B \frac{\sigma^\mu \gamma^\nu}{2m_B} u_p = F_M(k^2) \frac{E_B - m_B}{2m_B} \bar{u}_B \gamma^0 u_p \rightarrow F_M(k^2) \frac{k^2}{4m_B^2} \bar{u}_B \gamma^0 u_p,$$

where $\sigma^\mu = (\gamma^0 \gamma^\rho - \gamma^\rho \gamma^0)/2$ and $k^\nu = (E_B, -k)$ and $F_M(k^2)$ is a “magnetic” form factor. In the heavy–baryon limit $m_B \rightarrow \infty$ the contribution of the “magnetic” form factor vanishes as $O(1/m_B^2)$.

[19] M. M. Nagels et al., Nucl. Phys. B 147, 189 (1979).

[20] H.–Ch. Schröder et al., Eur. Phys. J. C 21 (2001) 473.

[21] G. Höhler, in Pion–Nucleon Scattering, Landolt–Börnstein, Vol. 9b2 Springer, Berlin, 1983), Sect. 2.5.1 and references therein, T. E. O. Ericson, Phys. Lett. B 195, 116 (1987). J. Gasser et al., Phys. Lett. B 213, 85 (1988); A. N. Ivanov et al., Phys. Lett. B 235, 331 (1990); J. Gasser, H. Leutwyler, and M.E. Sainio, Phys. Lett. B 253, 252, 260 (1991); M. Sainio, PiN Newslett. 16, 138 (2002); Int. J. Mod. Phys. A 20, 1872 (2005).

[22] F. E. Close, in AN INTRODUCTION TO QUARKS AND PARTONS, Academic Press, New York, 1979.

[23] J. Gasser and H. Leutwyler, Nucl. Phys. B 94, 269 (1975); Phys. Rep. 87, 77 (1982).

[24] A. N. Ivanov, M. Nagy, and N. I. Troitskaya, Phys. Rev. C59, 451 (1999) and references therein.