Traceable calibration of a fibre-coupled superconducting nano-wire single photon detector using characterized synchrotron radiation

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Abstract
Radiometric calibrations of fibre-coupled single photon detectors are experiencing growing demand, especially at the telecommunication wavelengths. In this paper, the radiometric calibration of a fibre-coupled superconducting nano-wire single photon detector at the telecom wavelength 1.55 µm by means of well-characterized synchrotron radiation is described. This substitution method is based on the unique properties of synchrotron radiation and the Metrology Light Source, the dedicated electron storage ring of the Physikalisch-Technische Bundesanstalt, and is suitable for fibre-coupled single photon detectors. The Metrology Light Source is used as a light source with a high dynamic range of the radiant power to bridge the radiometric gap occurring in the transition from radiant power measurements and the counting of photons with single photon detectors. Very low uncertainties below 2% have been achieved in the measurement of the detection efficiency of a fibre-coupled superconducting nano-wire single photon detector.

Keywords: metrology, quantum communication, synchrotron radiation, single photon detector, radiometry

((Some figures may appear in colour only in the online journal)

1. Introduction
Maintaining an uninterrupted traceability chain for radiometrically calibrated single photon detectors to the primary standards, i.e. cryogenic radiometers, sees growing demand. Traceability is a prerequisite for applications, such as remote sensing and quantum communications. However, achieving this aim is difficult due to the very different radiant power levels of power measuring classical radiation detectors and single photon detection systems. Usually the so-called substitution method or the so-called correlated-photon method are used in the visible for high accuracy radiometric detector calibrations in the free space regime. These two independent methods have achieved very low uncertainties of 0.17% [1] for the substitution method and 0.18% for the correlated-photon method [2]. Furthermore, the resulting detection efficiencies obtained with these two methods agreed within 0.14% [2].

Recently, a calibration method based on synchrotron radiation (SR) has been shown to achieve even lower uncertainties of 0.16% [3]. For fibre-coupled single photon detectors, currently only the substitution method is available to obtain the lowest possible uncertainties. The so-called quantum cloning technique for radiometry [4] is promising but suffers at the moment from rather high uncertainties of the order of 4%. However, to our knowledge, no uncertainty budget for a calibration of fibre-coupled single photon detectors has been published so far. Furthermore, the metrological meaning of terms, such as detection efficiency, has not been consolidated yet.
In this work, the term detection efficiency is related to the efficiency of a detector to measure the radiant power coming out of the fibre. At the telecom wavelengths, the achievable calibration uncertainties are limited by the single photon detector characteristics, which are inferior to those of single photon detectors, designed for operation at visible wavelengths. Thus, the achievable uncertainties for radiometric calibrations in the NIR are significantly higher than those shown in the visible.

In this paper, we report how the SR based calibration method described in [5] and [3] has been adopted to be suitable for a fibre-coupled superconducting nano-wire single photon detector (SNSPD) [6]. First, the measurement setup is described followed by the characterization of the SR and the detector properties, a prerequisite to achieve low uncertainties. Finally, the experimental results, i.e., the measured detection efficiency of a SNSPD, are shown and an uncertainty budget is compiled.

2. Measurements

The measurement principle for fibre-coupled detectors is similar to the principle for the free space detectors described in [3]. The storage ring current ranges used for this calibration method are substantially different as compared with the free space calibration described in [3]. The Metrology Light Source (MLS) [7], the dedicated electron storage ring of the Physikalisch-Technische Bundesanstalt (PTB), is used as a radiation source with a high dynamic range. The emitted radiation of an electron storage ring can be calculated from basic storage ring parameters, such as the magnetic field at the source point and the vertical source size, using the Schwinger equation [8]. According to the Schwinger equation, the photon flux emitted by an electron in a storage ring’s bending magnet is directly proportional to the number of stored electrons. At the MLS, the ring current can be varied from 1 stored electron (approximately 1 pA) to $10^{11}$ stored electrons without changing the spectral distribution of the emitted spectrum [8, 9]. Thus, the MLS is used in this work as a radiation source with a dynamic range of the emitted radiant power of up to 11 orders of magnitude. Using this calibration method for fibre-coupled detectors, the vertical source size of the SR becomes a critical parameter and limits the maximum ring current to values of approximately 1 mA. At higher ring currents the coupling between the electrons in the circulating electron bunches is current dependent and, thus, changes the source size. Hence, the coupling efficiency of the SR into the fibre is ring current dependent for ring currents above 1 mA. Furthermore, the minimum ring current is limited to values not smaller than several nA to achieve sufficient count rates of the devices under test because of coupling losses into the optical fibre. To increase the available radiant power of the MLS per stored electron, the undulator U180 [9], tuned to the telecom wavelengths, was used in this calibration campaign. A schematic of the setup is shown in figure 1. The detection efficiency of the SNSPD is polarization dependent. To overcome this issue a fibre-coupled polarizer was used for this calibration. The polarization was adjusted to reach the maximum count rate of the SNSPD.

To perform the calibration of the SNSPD, the first harmonic of the undulator radiation was further monochromatized by a filter with a central wavelength of 1551.97 nm and fed into a single mode optical fibre (SMF-28) for a wavelength of 1550 nm. This fibre was then connected to the reference detector, an InGaAs photodiode, and the SNSPD sequentially (see the schematic of the setup in figure 1). The SNSPD was current biased to 90% of the critical bias current. The absolute photon rate per stored electron that was coupled into the fibre was determined at a ring current $I_{\text{high}}$ of about 1 mA using the reference InGaAs detector that was calibrated traceable to a cryogenic radiometer of PTB. At this level, the ring current could be measured with a relative standard uncertainty of $5 \times 10^{-3}$ by cooled photodiodes with aluminum filters. The ring current was then reduced to approximately 5 nA. At this ring current $I_{\text{low}}$, corresponding to about 5000 stored electrons, the count rate of the SNSPD (approximately 630,000 counts s$^{-1}$), normalized to the ring current, was measured. At this level, the ring current was determined from counting the electrons as described in [9]. The uncertainty contribution of the measurement of $I_{\text{low}}$ was 0.5% and is included in the uncertainty of the measured count rate of the SNSPD $N_{\text{CR}}$. A detailed description of the ring current measurement of the MLS can be found in [9]. The uncorrected detection efficiency of the SNSPD $DE_{\text{SNSPD}}$ is, with the count rate of the SNSPD $CR_{\text{SNSPD}}$, $N$ the number of stored electrons, and the photon rate in the low ring current
regime $PR_{l_{\text{low}}}$, determined from

$$DE_{\text{SNSPD}}^{*} = \frac{CR_{\text{SNSPD}} \cdot N_{\text{high}}^{-1}}{PR_{l_{\text{high}}} \cdot N_{\text{high}}^{-1}}.$$  

(1)

The photon rate of the MLS, which has been calculated from the measured photocurrent of the reference detector, per stored ring electron and the measured count rate of the SNSPD is plotted in figure 2. The measurement with the reference detector in the high ring current regime has been repeated several times in order to determine uncertainty contributions such as the reproducibility of the fibre coupling. $DE_{\text{SNSPD}}^{*}$ is a first approximation of the detection efficiency. The corrections and uncertainties to get the precise detection efficiency of the SNSPD are given below.

2.1. Characterization of the SR

The response of single photon detectors to photons is different from that of classical detectors. Classical detectors give an output proportional to the radiant power averaged over the observation time, while single photon detectors give an output signal for each event caused by the absorption of one or more photons. In most cases, the output signal of single photon detectors, related to a particularly detected event, is independent of the number of photons absorbed during this event. Thus, a classical detector measures the radiant power while single photon detectors count events containing one or more photons. The influence of the photon statistics of thermal, coherent and single photon sources on the calibration result has been studied in [10].

The measured count rate of the single photon detectors to be calibrated here has to be corrected for detected events that contained two or more photons. However, the probability that an SR pulse emits more than three photons is negligible in the low ring current regime. The correction factor $c_{\text{at}}$ can be approximated by

$$c_{\text{at}} = 1 + (p2 + p3)p1^{-1},$$

with probabilities $p1$, $p2$, $p3$ that the pulses contain one, two or three emitted photons, respectively. Hence, the photon statistics has to be known to obtain $c_{\text{at}}$. The photon statistics of the emitted SR has been determined to obey a thermal distribution, Poisson distribution or a mix of both distributions depending on the experimental conditions. The corresponding photon number distributions are derived, for instance, in [17]. With the photon number distributions of coherent and thermal radiation the measured count rate can be corrected if the percentage of these distributions of the MLS for the measurement condition is known. As shown in figure 3, the photon statistics correction $[2p(2)+3p(3)]p(1)^{-1}$ can heavily influence the calibration result. Furthermore, if the photon statistic is unknown, it would become the major uncertainty contribution associated with the calibration method itself resulting in relative standard uncertainties higher than 0.5% for rates above 2500 000 photons s$^{-1}$.

To obtain the photon number distribution of the SR for the given experimental conditions, the $g^{(2)}$-function [18], i.e. the intensity correlation function of the MLS, was calculated and was measured with a Hanbury-Brown and Twiss (HBT) interferometer [19] with two SNSPDs as detectors. To correct for the photon statistics of the MLS, a thermal statistic is assumed [11]. However, the thermal behaviour of the emitted radiation is expected to be weakened because the pulse width of the SR is longer than the longitudinal coherence length of the beam. In addition, the cross section of the beam is larger than the transverse coherence length. Taking these effects into account, the estimated value $g^{(2)}_{e}(0)$ for $g^{(2)}(0)$ is given by

$$g^{(2)}_{e}(0) = 1 + F(\pi^{1/2}d_{x}/l_{x})F(\pi^{1/2}d_{y}/l_{y})\tau_{c}/\tau_{p}$$

(2)

with $l_{x}$ and $l_{y}$ the transverse coherence widths in vertical and horizontal directions, $d_{x,y}$ the dimensions of the entrance optics of the fibre-coupling setup for the SNSPDs, $\sigma_{x,y}$ the standard deviations of the Gaussian source size, $\lambda$ the wavelength, $D$ the distance between the source and the detector, $\tau_{c}$ the coherence time of the beam, $\tau_{p}$ the pulse width of the SR with $\tau_{c} \ll \tau_{p}$ and $F(b) = (2/b)\int_{0}^{b} e^{-u^{2}} du - (1 - e^{-b^{2}})/b^{2}.$

(3)

$$l_{x,y} = \lambda D/2\pi^{1/2}\sigma_{x,y}.$$  

(4)

For 1.55 $\mu$m, where the photon statistics was measured and the SNSPD was calibrated $g^{(2)}_{e}(0) = 1.006.$
The precise detection efficiency \( DE \) can be calculated from:

\[
DE_{\text{SNSPD}} = \gamma_{\text{bias}} \cdot \gamma_{\text{deadtime}} \cdot \gamma_{\text{counting}} \cdot \gamma_{\text{st}} \cdot \gamma_{\text{bw}} \cdot DE_{\text{SNSPD}}
\] (5)

with the correction factors for the applied bias current of the SNSPD, the dead time of the SNSPD, the dead time of the counting electronics, the photon statistic of the undulator radiation and the radiation bandwidth (see below).

### Correction for photon statistics

As described above, the Poissonian statistics were used to perform the correction for the influence of the statistical fraction of pulses from a single electron bunch that contained up to three photons. The correction was calculated for the emission characteristics of the MLS, i.e. a pulse rate of 500 MHz, and a photon rate of about 630 000 s\(^{-1}\) at \( I_{\text{low}} \), which gives a mean photon number per pulse of the order of \( \mu = 0.001 \). This minor correction has a value of \( c_\mu = 1.0013 \). The relative uncertainty of the correction factor has been estimated to be 50% of the correction, i.e. \( u(c_\mu) = 0.06\% \).

### Correction for dead time

The recovery time \( dt_{\text{SNSPD}} \) of the SNSPD was determined to be \((10 \pm 2)\) ns by sampling the output pulse of the SNSPD with an oscilloscope and measuring the duration while the output pulse declined to 0 V. The recovery time introduces a correction of \( c_{\text{dtcorr}} = 1.0010 \) obtained from \( c_{\text{dt}} = (1 - dt \cdot CR)^{-1} \). Furthermore, the counting electronics has an additional dead time \( dt_{\text{counting}} \) of \((20 \pm 1)\) ns which introduces a correction of \( c_{\text{dtcounting}} = 1.0019 \). The uncertainties of these corrections are \( u(c_{\text{dtcorr}}) = 0.02\% \) for the recovery time correction of the SNSPD and \( u(c_{\text{dtcounting}}) = 0.01\% \) for the dead time correction of the counting electronics.

### Correction for bandwidth of the SR

To correct for the spectral dependence of the calculated emitted photon flux of the undulator U180 of the MLS \( \Phi_{\text{MLS}}(\lambda) \), of the transmittance of the used interference filter \( F(\lambda) \) and the different spectral shapes of the spectral responsivity of the reference detector \( s_{\text{InGaAs}} \) and the detection efficiency \( DE_{\text{SNSPD}}(\lambda) \) of the SNSPD two correction factors were determined:

\[
c_{\text{SNSPD}} = \frac{\int_\lambda \Phi_{\text{MLS}}(\lambda) \cdot F(\lambda) \cdot DE_{\text{SNSPD}}(\lambda) \cdot d(\lambda)}{DE_{\text{SNSPD}}(1551.97 \text{ nm}) \cdot \int_\lambda \Phi_{\text{MLS}}(\lambda) \cdot F(\lambda) \cdot d(\lambda)}
\] (6)

\[
c_{\text{InGaAs}} = \frac{\int_\lambda \Phi_{\text{MLS}}(\lambda) \cdot F(\lambda) \cdot s_{\text{InGaAs}}(\lambda) \cdot E_{\text{phot}}(\lambda) \cdot d(\lambda)}{s_{\text{InGaAs}}(1551.97 \text{ nm}) \cdot \int_\lambda \Phi_{\text{MLS}}(\lambda) \cdot F(\lambda) \cdot E_{\text{phot}}(\lambda) \cdot d(\lambda)}
\] (7)

Figure 3. Photon statistic correction \([2p(2) + 3p(3)]p(1)^{-1}\) plotted over the photon rate of the MLS for the assumptions, thermal radiation (blue line) and coherent radiation (red line).
Figure 4. Schematic of the setup to measure the photon statistics of the MLS (see 4(a)) to take into account the trivial coincidences due to the electron bunch structure (see text) and the resulting normalized coincidences (4(b)). The spectrally filtered radiation of the MLS has been fed into a fibre-coupled HBT setup [19] with the detectors D1 and D2. The schematic shows the fibre-coupled beam splitter with a splitting ratio of 50/50 (50/50 BS), the two SNSPDs (D1 and D2), and the coincidence counters. The measured value of $g^{(2)}(0)$ is $1.0049 \pm 0.0008$ and is close to the estimated value $g^{(2)}(0) = 1.006$.

For the calibration, only the relative spectral functions of the wavelength-dependent quantities are needed in (6) and (7). The nominal detection efficiency of the SNSPD was taken from the data sheet, $\tilde{D}E_{\text{SNSPD}}(\lambda)$ was used for this correction and the same wavelength-dependent photon flux of the MLS was used in (6) and (7). The correction is then calculated as the ratio of the bandwidth correction of the SNSPD and of the reference detector and results in $c_{\text{bwSNSPD}} = 0.9990$. The uncertainty of $c_{\text{bwSNSPD}}$ is estimated to $u(c_{\text{bwSNSPD}}) = 0.01\%$.

Correction for the applied bias current. SNSPDs are typically operated at a bias current of 90% of the critical bias current value. The minimum current increment that can be adjusted by the controller of the SNSPD system is 0.1 µA with typical critical currents in the range from 20 µA to 30 µA. Hence, the accuracy of the determined critical bias current as well as the applied bias current are limited by the controller. The uncertainty contribution of the measurement of the critical bias current $I_{\text{critical}}$ is determined to be $u(I_{\text{critical}}) = 0.82\%$ using the dependence of the detection efficiency on the bias current (see also figure 5). In addition, the measured detection efficiency was corrected to obtain the value for an applied bias current of 90% of the critical value. Figure 5 shows the measured detection efficiency and the dark count rate of the SNSPD versus the applied bias current normalized to the critical current. The actual applied bias current $I_{\text{bias}}$ during the calibration was set to $I_{\text{bias}} = 0.901 \cdot I_{\text{critical}}$. The correction for the applied bias current was determined from the dependence of the detection efficiency on the applied bias current, shown in figure 5 to $c_{\text{bias}} = 0.9952$ with $u(c_{\text{bias}}) = 0.16\%$.

Polarization dependence of the detection efficiency. To determine the uncertainty contribution associated with the polarization dependence of the detection efficiency of the SNSPD, the reproducibility of the polarization adjustment was investigated with a laser source at a wavelength of approximately 1550 nm.

A fibre-coupled polarizer, that was inserted between the laser and the SNSPD, was used to maximize the count rate of the SNSPD, i.e. to set the polarization of the radiation incident on the SNSPD parallel to the meander wire. The count rate was integrated over a period of 30 s. The bias current was set to 21.6 µA, i.e. to 90% of the critical bias current. The relative standard uncertainty associated with the polarization dependence of the SNSPD was then obtained from the relative standard deviation of the measured counts to $u_{\text{pol}} = 1.35\%$.

2.3. Uncertainty budget

Table 1 shows the uncertainties arising from the measurements and from the introduced corrections. The values given are relative standard uncertainties. The uncertainty for the determination of the count rate of the SNSPD contains the uncertainty in the measurement of the count rate, the
Figure 5. Measured detection efficiency (blue squares) and dark count rate (red circles) of the SNSPD plotted over the bias current normalized to the critical bias current. The uncertainty bars of the measured detection efficiency are about the same size as the blue markers.

Table 1. Relative standard uncertainties and correction factors contributing to the traceable calibration of a SNSPD by means of SR.

| Source of uncertainty                      | Correction factor | SNSPD   |
|--------------------------------------------|-------------------|---------|
| Count rate of SNSPD                        | \( u_{CR_{SNSPD}} \) | 0.075%  |
| Ring current (\( I_{low} \))               | \( u_{I_{low}} \)  | 0.5%    |
| Ratio photocurrent to ring current (\( I_{high} \)) | \( u_{I_{refnorm}} \) | 0.53%   |
| Photocurrent reference detector            | \( u_{ampmeter} \) | 0.05%   |
| Spectral responsivity InGaAs detector      | \( u_{InGaAs} \)  | 0.15%   |
| Fibre-coupling reference detector          | \( u_{fc} \)      | 0.38%   |
| Fibre-coupling SNSPD                      | \( u_{fc} \)      | 0.38%   |
| Source size variation                      | \( u_{ss} \)      | 0.5%    |
| Polarization                               | \( u_{pol} \)     | 1.35%   |
| Fibre connection                           | \( u_{fibre} \)   | 0.36%   |
| Critical bias current                      | \( u_{bc} \)      | 0.82%   |
| Applied bias current \( C_{bias} \)        | \( u_{bias} \)    | 0.9952  |
| Counter dead time \( C_{dead} \)           | \( u_{C_{dead}} \) | 0.01%   |
| SNSPD dead time \( C_{deadSNSPD} \)        | \( u_{C_{deadSNSPD}} \) | 0.02% |
| Photon statistics \( c_{st} \)             | \( u_{st} \)      | 0.06%   |
| Bandwidth \( c_{bw} \)                     | \( u_{bw} \)      | 0.01%   |
| Combined relative standard uncertainty of the detection efficiency | \( u_{SNSPD} \) | 1.9%    |

The uncertainty of the determination of the dark count rate and the uncertainty contribution of the counting electronics as well as the repeatability of the measurement and gives \( u_{CR_{count}} = 0.075\% \). The uncertainty of the ring current measurement in the low ring current regime was determined to be 0.5%. The uncertainty of the calibration of the reference detector is \( u_{S_{InGaAs}} = 0.15\% \). Another source of uncertainty is the determination of the ratio of the photocurrent of the calibrated reference detector and the ring current at \( I_{high} \). The values of the uncertainties connected to these measurements are the systematical uncertainty of the used amperemeter \( u_{ampmeter} = 0.05\% \) and the uncertainty of the measured ratio of the photocurrent of the reference detector and of the ring current \( u_{I_{refnorm}} = 0.53\% \).

To check whether the finite vertical source size of the SR has an influence on the calibration result the change in the coupling efficiency into the optical fibre was determined from measurements of the ratio of the photocurrent to the ring current with the reference detector at different ring currents. For ring currents below 1 mA no dependence of the coupling efficiency on the ring current is expected. Figure 6 shows the measured change in the coupling efficiency plotted over the ring current. Within the uncertainty of the ring current measurement of 0.5% there was no systematical dependence of the coupling efficiency on the ring current, for ring currents below 1 mA. Therefore, this uncertainty contributes with the uncertainty of the ring current measurement to the uncertainty budget, i.e. \( u_{ss} = 0.5\% \) as an upper limit estimate. The uncertainty arising from the reproducibility of the coupling efficiency into the fibre with respect to the SR is \( u_{fibre} = 0.38\% \). The reproducibility of the polarization adjustment in order to maximize the count rate of the SNSPD and, thus, the uncertainty contribution is \( u_{pol} = 1.35\% \).

The combined relative standard uncertainty of this calibration of a SNSPD is \( u_{SNSPD} = 1.9\% \). The main contribution to these overall uncertainties (see Table 1) is the uncertainty associated with the polarization dependence of the detection efficiency of the SNSPD.
Figure 6. Relative change in the normalized photocurrent of the fibre-coupled reference detector at 1.55 µm plotted over the ring current. Within the uncertainties of $I_{\text{ref norm}}$, no systematical dependence of the ratio of photocurrent and ring current and, thus, of the coupling efficiency on the ring current is detected for ring currents below 1 mA.

3. Conclusions

An SNSPD has been calibrated traceable to a primary detector standard, the cryogenic radiometer, by means of the high dynamic range of the radiant power of the undulator radiation. The measured detection efficiency is $DE_{\text{SNSPD}}(1551.97 \text{ nm}) = 0.1501 \pm 0.0028$. The Metrology Light Source was used to bridge the gap in the radiant power needed to operate the classical reference detectors with low uncertainties and the photon flux that allowed low uncertainty measurements with the SNSPD.

Though third-generation electron storage rings such as BESSY II and the MLS are only available to a few national metrology institutes the unique properties of these sources make this method valuable. In particular, the combination of the wavelength range at which calibrations can be performed and the low uncertainties achievable are unrivalled by any other calibration method. In particular, stakeholders in sciences such as astronomy or elementary particle physics could benefit from this highly reliable method where radiation conditions can be adapted to the needs of the experiment, such as, e.g., the simulation of a space-like high energetic radiation background.

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