Multi-objective Optimization of Interferometric Array $u-v$ Coverage

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Abstract

A new principle is introduced to optimize the configuration of an interferometric array, based on the trade-off between the uniform and Gaussian $u-v$ distributions. The multi-objective optimization method, nondominated sorting genetic algorithm II (NSGA-II), is applied to achieve the optimal trade-off. The resulting array having a single configuration can meet the observation requirements of both compact and extended sources. This method has been successfully applied to design a 16-element array as the initial stage of the Daocheng Solar Radio Telescope to illustrate its feasibility. NSGA-II is improved by introducing artificial intervention into the genetic operator to solve the equality constraints. The improved NSGA-II is applied to obtain the Pareto optimal set, and a 16-element array configuration is retrieved with the best $u-v$ trade-off between the snapshot mode and Earth rotation synthesis mode.

Key words: Interferometers – Algorithms – Solar radio telescopes

1. Introduction

Array optimization is a critical task for the design of interferometric arrays. The configuration of array elements determines the coverage of $u-v$ sampling points, where the spatial frequency harmonics in the Fourier domain can be sampled for image reconstruction. Therefore, the $u-v$ coverage of an array is always a primary concern in the design of interferometers and a prominent impacting factor for image retrieval (Thompson et al. 2001).

Two typical $u-v$ distribution cases were introduced by Holdaway & Helfer (1999), namely, the Gaussian and the uniform distributions. In the absence of prior information on the radio sources, larger holes in the $u-v$ coverage will contribute to larger errors in the reconstructed image (Keto 1997). The uniform $u-v$ distribution ensures that the sampling points are spread evenly (Cornwell 1988; Jin & Rahmat-Samii 2008). Therefore, this type of error will be reduced by the uniform $u-v$ distribution. Furthermore, the uniform $u-v$ coverage usually yields higher resolution in all directions and a better signal-to-noise ratio (Keto 1997; Su et al. 2004). A uniform $u-v$ distribution is appropriate when the array is used to observe bright and complicated sources. By contrast, the Gaussian $u-v$ distribution shows a centrally condensed sampling coverage with many short baselines. First, many short baselines will improve the reconstruction of the large-scale structures of extended sources. Second, the radially decreasing distribution will reduce the inner sidelobe of the synthesized beam (Su et al. 2004). Finally, a centrally condensed sampling distribution can also be used to optimize radiometric sensitivity (Boone 2002). Faint sources with extended structures can be detected more accurately by the array, where the $u-v$ coverage is the Gaussian distribution (Holdaway & Helfer 1999). In general, the optimization of the $u-v$ distribution is an essential and complicated task and should be driven by the scientific purpose and the observation requirements of the radio source.

Many studies have been conducted to address the optimization principles and methods of an interferometric array to achieve uniform or Gaussian $u-v$ distributions. Cornwell (1988) applied the simulated annealing technique to obtain the configuration associated with uniform $u-v$ coverage, based on the principle of maximization of the distance between samples. Keto (1997) discussed the self-organizing neural networks, which were effective in generating optimal interferometer shapes. To retrieve a Gaussian $u-v$ distribution, Boone (2001) introduced an optimization method which was based on the computation of pressure forces related to the discrepancies between the model and the actual distribution of Fourier samples. Su et al. (2004) proposed a “sieving” algorithm based on the elimination of candidate elements to fit $u-v$ distributions to a predefined shape. The predefined distribution could be a uniform or a Gaussian $u-v$ distribution.

To achieve uniform $u-v$ coverage, particle swarm optimization was utilized to maximize the total number of sampled grids (Jin & Rahmat-Samii 2008). The optimization principles mentioned above were focused on a single objective and were applied to either the uniform or Gaussian distributions. Therefore, published studies have mainly adopted single-objective
optimization algorithms. The optimal array obtained by single-objective optimization can only meet the observation requirements of compact or extended sources, not both.

In practice, the scientific purposes of an interferometric array are diverse. To meet different observation requirements of multiple radio sources, the prevailing approach is to design reconfigurable interferometers with additional antenna spots (Napier et al. 1983; Brown et al. 2004). However, the reconfigurable array is technically challenging on combining data from multiple configurations and requires high costs associated with antenna transporters, additional antenna stations, and workers’ salaries to move the antennas (Holdaway & Helfer 1999). Thus, it is not always feasible.

To observe both compact and extended sources with the interferometric array using a single configuration, a new principle is introduced to optimize the interferometric array, based on the trade-off between the uniform and Gaussian $u$–$v$ distributions. In this case, previous single-objective optimization algorithms are no longer applicable. In this study, a type of multi-objective optimization called nondominated sorting genetic algorithm II (NSGA-II), will be used owing to its excellent performance in achieving the optimal trade-off between multiple competing objectives. The Pareto optimal set is derived based on a trade-off using NSGA-II and consists of many different array configurations. Thus, we need to choose the optimal array configuration from the Pareto optimal set to deliver favorable images for compact and extended sources. Compared with single-objective optimization, the optimal array obtained by multi-objective optimization can meet the observation requirements of both compact and extended sources.

The study is structured as follows: in Section 2, two multi-objective optimization methods are introduced, including the conventional weighted aggregation (CWA) and NSGA-II. The concept of the Pareto optimal set is also introduced. In Section 3, a new optimization principle based on a trade-off between the uniform and the Gaussian $u$–$v$ distributions is derived based on a trade-off using NSGA-II and consists of multiple competing objectives. The Pareto optimal set is then chosen from the Pareto optimal set to deliver favorable images for compact and extended sources. Compared with single-objective optimization, the optimal array obtained by multi-objective optimization can meet the observation requirements of both compact and extended sources.

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2. Multi-objective Optimization Method and Pareto Optimal Set

Multi-objective optimization plays an essential role in addressing the optimization problems involving multiple, conflicting objectives, and has been extensively investigated in recent years (Gunantara 2018). Evolutionary algorithms (EAs) have been widely used for multi-objective optimization owing to their outstanding searching capability (Zhou et al. 2011). There are three main methods associated with the evolutionary multi-objective optimization, namely, weighted aggregation approaches, Pareto-based approaches, and population-based non-Pareto approaches (Jin et al. 2001). Herein, we focus on the first two methods.

We consider a multi-objective optimization problem with $n$ decision variables $(x_i, i = 1,2,...,n)$ and $k$ objectives $(f_k(x), i = 1,2,...,k)$. As a weighted aggregation approaches, CWA aggregates multiple and conflicting objectives in a single function using a set of prescribed weights.

$$ F = \sum_{i=1}^{k} w_if_i(x) $$

where $w_i$ is the weight for the $i$th objective function. It follows that

$$ \sum_{i=1}^{k} w_i = 1. $$

Thus, the multiple-objective optimization problem is reduced to a single-objective optimization problem that can be solved by single-objective optimization algorithms (Coello 1999). Given that the relative importance of each objective relies on prior knowledge, it is challenging to specify the weights of all the objectives.

An improved version of the NSGA, known as the NSGA-II, was proposed by Deb et al. (2002). NSGA-II is a multi-objective genetic algorithm based on Pareto dominance. The genetic algorithm (Holland 1975) is an EA. As compared with the standard genetic algorithm, NSGA-II classifies the candidate solutions in different non-domination levels. Further, the classifying procedure guides the search direction of populations toward the Pareto optimal front. As compared with the NSGA, NSGA-II presents several improvements as following (Deb et al. 2002).

1. A fast, nondominated sorting method was proposed which reduced the complexity of calculation.
2. An elitist-preservation strategy was applied to protect an already found Pareto optimal solution from being deleted during evolution.
3. A diversity-preserving strategy related to the distance metric was used to ensure the diversity of solutions.

To illustrate the functionality of NSGA-II, a test problem described in Zitzler et al. (2000) was solved by NSGA-II to retrieve the Pareto optimal set and compare the outcome with the optimization outcome obtained by the CWA. Two fitness functions are defined as shown below,

$$ f_i = x_i $$

(3)
The size of the population was 500 in the optimization. The entire search process of 2000 iterations and the Pareto optimal set of NSGA-II are depicted in Figure 1(a). The plot shows all the positions visited by the populations during the search process. The populations converge to a front, and those obtained solutions marked as red dots in Figure 1(a) represent the Pareto optimal set.

When conducting the optimization with CWA, two fitness functions are transformed to a single objective function using proper weights

\[
F = w_1f_1 + w_2f_2, \quad w_1 + w_2 = 1.
\]

In the test, three arbitrary weighting combinations are implemented according to: (1) \(w_1 = 0.8, w_2 = 0.2\), (2) \(w_1 = 0.5, w_2 = 0.5\), and (3) \(w_1 = 0.0\) and \(w_2 = 1.0\). Figure 1(b) shows the results of the CWA optimization for a population of 500 and 2000 iterations, as indicated by the three black pentagrams which correspond to the three weighting combinations.

In Figure 1(b), we also show the Pareto optimal sets of the NSGA-II optimization process at the 100th, 500th, and 2000th iteration cycles. As it can be observed from Figure 1(b), three solutions by CMA are located close to the Pareto optimal front. With a predefined weighting combination, CWA obtains at most one solution of the Pareto optimal set. In the absence of the relative importance of objectives, the expected solution requires extensive adjustments of the weighting coefficients. Again, selecting the appreciated coefficients for CWA is an important and complicated problem. NSGA-II is then used to optimize the \(u-v\) coverage of the interferometric array in this study.

For testing function cases, a possible evaluation method of the quality of the actual Pareto optimal set involves the comparison with the theoretical Pareto optimal front. For example, the resulting Pareto optimal set derived by 2000 iterations is indistinguishable from the theoretical Pareto optimal front, as shown in Figure 1(b). However, this approach is not feasible in most practical engineering multi-objective optimal problems because the theoretical Pareto optimal front is not available. Thus, the derived nondominated solutions (an approximation of the theoretical Pareto optimal front) mentioned in this study should be regarded as the quasi-Pareto optimal solution (Zitzler et al. 2003; Jin & Rahmat-Samii 2007).

The quality of the quasi-Pareto optimal solutions can be evaluated from the following aspects. For a good approximation set, few new solutions will be explored after the number of iterations is increased. Meanwhile, the quality of approximation sets scarcely changes as the number of iterations increases. A comprehensive metric called hypervolume (Zitzler et al. 2003; Shang et al. 2021) is used to evaluate the quality variation of approximation sets with different iterations.
Hypervolume indicators can evaluate the convergence, diversity, and spread of the solution sets simultaneously.

For the two-dimensional (two-objective) minimization problem, the hypervolume indicators of the Pareto optimal sets measure the area in the objective space enclosed by the non-dominated solutions and a reference point. The reference point of the minimization problem is given by a vector, the components of which were the maximum values of all the objectives (Knowles & Corne 2003).

3. Optimizing the $u-v$ Coverage of the Circular Array

As mentioned in the Introduction, when the array is expected to observe various radio sources, the $u-v$ coverage of the array is a trade-off between the uniform and the Gaussian distributions. For a compact source, the $u-v$ sampling is desired to be uniformly spread, and the distance between adjacent points should be long enough to achieve a good angular resolution. This is designated as the optimization principle to achieve a uniform $u-v$ distribution. By contrast, the optimization principle for a Gaussian $u-v$ coverage is to aggregate more $u-v$ points in the central region of the $u-v$ plane. In practice, the total number of $u-v$ points is always limited owing to finite antennas. Obviously, when more $u-v$ points concentrate in the central region, fewer $u-v$ points are located in the outer region. Therefore, the uniformity of the Gaussian $u-v$ distribution is poor. If the array is expected to perform well on both compact and extended sources, the mutual restriction between the optimization principles of the uniform and Gaussian distributions must be considered. The NSGA-II is capable of solving the optimization problem involving multiple contradictory issues, as demonstrated in Section 2. The algorithm can be used to achieve a compromise between the conflicting $u-v$ distributions. The trade-off is based on two fitness functions derived from two optimization principles. Details of the derivation are presented below.

3.1. Uniform $u-v$ Coverage

An important question that needs to be addressed is related to the identification of ways to measure the uniformity degree of the $u-v$ distribution. Cornwell introduced an approach based on the computation of the sum of the distance between samples (Cornwell 1988). A larger sum tends to suggest more uniform $u-v$ coverage of an array configuration. However, if the number of elements is increased, the required calculations become prohibitive. Thus, it is only applicable to situations associated with few elements. In 2008, another measurement was proposed for the Y-shaped array based on the statistics of sampled grids (Jin & Rahmat-Samii 2008). Compared with the method proposed by Cornwell (1988), the computational complexity of the second method decreased. In this section, the second method will be applied to measure the uniformity degree of the circular array.

The shape of the entire $u-v$ coverage of a circular array is a disk. First, the bounding square of the disk is divided into $N_{grid} \times N_{grid}$ grids. According to the definition of $N_{grid}$ by Jin & Rahmat-Samii (2008), $N_{grid}$ can be written.

$$N_{grid} = \text{int}\left(\frac{4}{\pi}N(N-1)\right)$$

where $N$ is the number of elements of the circular array, and the $\text{int}[\cdot]$ denotes rounding to the nearest integer. The ratio between the areas of the disk’s bounding square and the disk is $4/\pi$. The snapshot $u-v$ coverage of the circular array is plotted in Figure 2. The white broken white lines show how the plane is divided into cells, and the red cells denote the sampled grids that include at least one $u-v$ point. Given that the most uniform $u-v$ coverage should have only one $u-v$ point in each cell, the $u-v$ distribution becomes more uniform as the number of red cells increases. To quantitatively assess the degree of uniformity of the $u-v$ distribution, the first fitness function $f_1$ is defined as the number of sampled grids, (Jin & Rahmat-Samii 2008),

$$f_1 = -N_{\text{sampled}}.$$  

There is a negative sign in Equation (8) because NSGA-II is a minimizer by default (Jin & Rahmat-Samii 2008). Increasing the value of $N_{grid}$ can improve the degree of uniformity of the $u-v$ distribution.

3.2. Gaussian $u-v$ Distribution

The Gaussian $u-v$ distribution requires denser sampling points in the central region. To derive this type of distribution, the first step is to define the weight for each $u-v$ point. The weighting process is conducted by tapering every point with the Gaussian function as indicated below (Su et al. 2004),

$$r = \sqrt{u^2 + v^2}$$

$$w_i = (1 - \epsilon) e^{-r^2/D^2} + \epsilon$$

where $\epsilon$ represents the spacing of floating point numbers and is used to prevent the weights of long baselines from being equal to zero. In this case, $u$ and $v$ are the coordinates of the sampling points in the Fourier domain, $r$ presents the distance between the corresponding $u-v$ point and the origin of the Fourier domain. Subsequently, the second fitness function $f_2$ is defined as the sum of all weights,

$$f_2 = -\sum_{i=1}^{m} w_i.$$  

A larger value of the sum of $w_i$ means that the $u-v$ coverage is denser in the central region. $D$ is a free parameter and is used for the adjustment of the concentration of the $u-v$ coverage. Different Gaussian functions when $D/r_{max}$ varies from 0.25 to 0.72 are shown in Figure 3. A larger ratio of $D$ and $r_{max}$
increases the weight of the \( u-v \) point located in the edge region, and the \( u-v \) distribution becomes less concentrated.

The trade-off between the two fitness functions (defined by Equations (8) and (11)) with NSGA-II yields the Pareto optimal set. Each solution in the Pareto optimal set represents an array configuration. The last step is to select the optimal array configuration that is favorable for both compact and extended sources. The detailed implementation of the optimization is verified with a 16-element circular array in the next section.

4. Application for the 16-element Initial Array of the DSRT

The DSRT is one of the “giant” equipment of the Meridian Project. Its construction began in the early part of 2020. DSRT is located in Daocheng in the Sichuan Province in China. It consists of 313 parabolic antennas (diameters: 6 m), which are equally spaced around the circumference of a circle with a radius of 500 m. Its scientific objective is to monitor solar radio bursts and deliver synthetic imaging with a high-dynamic range. The high-dynamic range observations demand high consistency between the antennas. A central transmitter located on the top of a tower is adopted for accurate calibration of the circular array. Owing to the requirements on hardware consistency and calibration, the implementation is challenging. For early recognition of the technical risk of the 313-element array, an initial array of 16 elements was constructed first to demonstrate new techniques employed by DSRT. In this section, the NSGA-II was applied to optimize the configuration of the 16-element array.

Requirements for the initial array configuration include the following: (1) antenna coordinates should be selected from 313 predefined spots because the initial 16-element array will be included in the final array. (2) Three out of the 16 antennas should be placed close to each other to demonstrate mutual coupling between adjacent antennas. Furthermore, to inspect antenna aperture shadowing when the Sun appears at a low-elevation angle in the winter, the group which contains the three antennas should be located at the eastern edge of the circle. (3) The 16-element array should realize the positioning of the solar burst in the snapshot mode. (4) The 16-element array should have the ability to synthesize good images of the quiet Sun with the help of the Earth’s rotation.

4.1. \( u-v \) Distribution Required for 16-element Array

The brightness distribution of solar bursts on short timescales can usually be retrieved by snapshot imaging. Because 16 elements are sparsely located along a circle with a radius of 500 m, the \( u-v \) coverage of the snapshot is poor. Although the position of the burst is still retrievable, the image quality is low owing to the severe undersampling at the spatial frequency domain. In this situation, the primary observation requirement for the radio burst is the positioning of the burst source. For the purpose of positioning, high-angular resolution is necessary irrespective of the direction. Technically, this requires that the long baselines are adequate in different directions. Furthermore, the sampling points at the \( u-v \) plane need to be evenly spread so that better images can be retrieved in the absence of a priori knowledge about the source. Therefore, the
requirement on the snapshot is that the \( u-v \) coverage is less directional and more uniform (Keto 1997).

However, the brightness of the quiet Sun is approximately constant. It can be assumed that during the daytime, the quiet Sun is invariable from the point-of-view of radio observations. The technique of Earth rotation synthesis is used to improve the \( u-v \) coverage by accumulating snapshots acquired every 12 h or more (Holdaway & Helfer 1999). In particular, the visibility function of the quiet Sun can be approximated as the Hankel transform of a disk with uniform brightness. Thus, a centrally condensed sampling distribution with more sampling points at low-spatial frequencies offers better visibility of the quiet Sun. Moreover, Gaussian-tapered \( u-v \) coverage reduces the inner sidelobes of the synthesized beam. Consequently, the interferometric array with a Gaussian \( u-v \) distribution is required in the Earth rotation synthesis mode.

Given that observing solar bursts with the snapshot and quiet Sun with Earth rotation proposes conflicting optimization objectives of the \( u-v \) distribution, a trade-off between the uniform \( u-v \) and Gaussian \( u-v \) distributions has to be considered. Additionally, in Earth rotation synthesis mode, Gaussian \( u-v \) coverage can be decomposed into multiple time-varying snapshots, and each snapshot has a centrally condensed \( u-v \) coverage. Thus, the trade-off can be completed using snapshot \( u-v \) distributions.

### 4.2. Equality Constraint

The optimization problem of the 16-element array configuration can be described as an optimization problem related to the locations of the 16 antennas. First, the installation positions of the 16 antennas need to be selected from the predefined spots of 313 antennas. Thus, the optimization problem in this study was simplified to select 16 out of 313 discrete spots. Second, three spots have been determined to be located at the eastern edge of the circular array resulting in three of the 16 discrete points are fixed. The installation positions of the remaining 13 non-fixed antennas are selected from 310 discrete points. Mathematically, the installation positions of 13 non-fixed antennas in each possible array configuration are represented by a vector with 310 elements \( x_i(i = 1, 2, \ldots, 310) \). Each element in the vector is represented by either “1” or “0,” and its value indicates that the corresponding element is turned on or off, respectively. Hence, the summation value of the vector is constant and equal to 13,

\[
    x_1 + x_2 + \ldots + x_{310} = 13. \tag{12}
\]

In the optimization process, the initial vector must set 13 elements to the “on” state. According to Equation (12), the summation of the vector should be equal to 13. Through a series of genetic operations (Katoch et al. 2021), such as crossover, mutation, and selection, the summation may no longer follow the equality constraint.

A relatively simple method for solving this equality constraint introduced by the 16-element array is the penalty function (Yeniay 2005). However, this approach may lead to lower optimization efficiency. Therefore, we execute an improvement over the standard NSGA-II algorithm to solve the equality constraint, in this section. An enhanced genetic operator for NSGA-II is implemented by adding artificial intervention. The relevant flowchart is plotted in Figure 4. Compared with the standard genetic algorithm, an additional evaluation is conducted to determine whether the summation of the candidate solution equals 13 after standard selection, crossover, and mutation operations. If the total number of “1” equals 13, the corresponding candidate solutions will be defined as a new offspring. If not, the difference is calculated and assigned to \( n_{\text{gap}} \). If the summation is larger than 13, \( n_{\text{gap}} \) elements of “1” are selected and converted to “0.” If the summation is smaller than 13, \( n_{\text{gap}} \) elements of “0” are selected and converted to “1.” The above operations can ensure that the new offsprings generated by the genetic operations still follow the equality constraint. The resulting new offsprings represent the installation positions of 13 non-fixed antennas. The 13 selected spots and the installation positions of the three fixed antennas constitute the 16-element array configurations, and the 16-element arrays are applied to the subsequent optimization process. With the above artificial intervention, the improved NSGA-II yields a higher efficiency than the penalty function.
4.3. Trade-off Between Uniform u–v Distribution and Gaussian u–v Distribution

In this section, the improved NSGA-II is applied to optimize the array configuration of the 16-element array to confirm its efficiency. The optimization criterion of the 16-element array design problem applies a trade-off between the uniform and the Gaussian u–v distributions. Two conflicting objectives for this multiple-objective optimization problem are represented by two fitness functions, as indicated by Equations (8) and (11).

Two important parameters, namely, \( N \) and \( D/r_{\text{max}} \), need to be determined. \( N \) denotes the number of array elements and is set to 16. \( D/r_{\text{max}} \) controls the shape of the Gaussian function and further affects the concentration of the u–v distribution. The determination of \( D/r_{\text{max}} \) is based on experimental tests. In the experimental tests, the source is a simulated astronomical region containing a solar disk. By sampling the Fourier transform of the source intensity distribution, the visibility measurements located on the u–v coverage of the 313-element array for DSRT (mentioned in Section 4) are obtained. Subsequently, the simulated visibility measurements covering the circular u–v domain with different cutoff frequencies are used to calculate the source image. The reconstructed images are achieved by DFT (Thompson & Bracewell 1974). Based on comparisons, an acceptable reconstruction can be achieved when the cutoff frequency is greater than 0.4\( r_{\text{max}} \). Therefore, the half value of the full width half maximum (FWHM) of the Gaussian function used to control the u–v distribution is set to be \( >0.4r_{\text{max}} \). Based on the above analysis, \( D/r_{\text{max}} \) should be greater than 0.48. However, given that the u–v coverage of the 16-element array is sparse, a smaller \( D/r_{\text{max}} \) will result in shaping the u–v distribution into a shape similar to “8.” For this u–v coverage, there will be larger gaps in some places in the Fourier plane. Thus, the corresponding array configurations are invalid. Suppose that a Gaussian function with a small \( D/r_{\text{max}} \) was employed to solve the multiple-objective optimization problem of the 16-element array. In that case, there will be many invalid array configurations in the resulting Pareto optimal set, and efficiency will be reduced. Finally, \( D/r_{\text{max}} \) was determined to be 0.6 based on several simulations.

In Figure 5(a), the Pareto optimal set by NSGA-II with a population size of 500 and 2000 iterations is shown. To assess the quality variation of the approximation sets with different iterations, the Pareto optimal set of 6000 iterations was calculated and is also shown in Figure 5(a). It is observed that as the number of iterations increases from 2000 to 6000, very few new solutions are explored. Moreover, the quality of Pareto optimal sets with different iterations is measured by hypervolume indicators (mentioned in Section 2), as shown in Figure 5(b). The reference point in Figure 5(b) was calculated by the CWA method using the two weighting combinations: \( w_1 = 1.0, w_2 = 0.0 \), and \( w_1 = 0.0 \) and \( w_2 = 1.0 \), respectively. The hypervolume indicator for the Pareto optimal set with 6000 iterations is only approximately 1.6% better than that associated with 2000 iteration. Given that the performance associated with 6000 iterations is similar to that for 2000 iterations, we no longer increase the number of iterations and set the Pareto optimal set of 6000 iterations as the final optimization result.

Figure 5(c) shows the entire search process comprising 6000 iterations, and the populations are well converged to the Pareto optimal set. In this case, \(-f_1\) denotes the number of sampled grids, and \(-f_2\) is the sum of weights of all u–v points. Suppose that the solution has a smaller \( f_1 \) value than others in the Pareto optimal set. In that case, this demonstrates that the corresponding array configuration has a more uniform u–v distribution. By contrast, a smaller \( f_2 \) indicates that more u–v points will gather in the central region.

4.4. Optimal Array Selected from the Pareto Optimal Set

Once the Pareto optimal set is obtained, the next step is to select the optimal array configuration from the Pareto optimal set. Because the optimal array is a trade-off between the uniform u–v and the Gaussian u–v distributions, the array is favorable in both snapshot and Earth rotation synthesis modes. The determination of the optimal array is based on the analysis of u–v coverage and synthesized beams. There were 500 solutions in the initial Pareto optimal set. Once the duplicate solutions were eliminated, 89 different array configurations remained.

By comparing the snapshot u–v coverage of the 89 arrays, we find that some arrays are associated with severe spatial undersampling. This undersampling usually occurs in longer baselines, such as the situation plotted in Figure 6(a). The reason for missing longer baselines is attributed to the fact that we prefer to accept the Gaussian u–v distribution rather than the uniform distribution during a trade-off that results in an increased number of u–v points in the center. Thus, the arrays associated with severe undersampling usually have smaller \( f_2 \) values, that is, poor uniformity. To study the undersampling’s impact on positioning capability, we compared the snapshot observations of the arrays with different \( N_{\text{sampled}} \) values.

In simulations, the original source was a simulated astronomical region which contained three point sources with the same intensity, and the observation frequency was 450 MHz. The simulation results of 16-element arrays with \( N_{\text{sampled}} = 161 \) or \( N_{\text{sampled}} = 165 \) are shown in Figure 6 and include snapshot u–v coverage and reconstruction results. The white circle in each reconstruction image denotes the solar disk, and the reconstruction results are achieved by gridding algorithm (Jackson et al. 1991). The imaging algorithm adopted a uniform weighting of the u–v points to obtain the smallest possible angular resolution and reduce the sidelobe level. Figures 6(a) and (c) show the performance comparison between the snapshot u–v coverages of the arrays with
sampled = 161 and \( N_{\text{sampled}} = 165 \). The comparison demonstrates that the \( N_{\text{sampled}} = 161 \) case has a more serious miss of longer baselines in the horizontal direction, which leads to alias pixels with high intensity, as marked in Figure 6(b). In contrast, interference source influence on the positioning is suppressed when \( N_{\text{sampled}} = 165 \). Therefore, for a low \( N_{\text{sampled}} \) value, a nonuniform \( u-v \) coverage that will lead to a more severe miss of baselines may result in larger imaging errors and limit the capacity of the array to locate the targets.

By traversing each array configuration, we exclude the arrays associated with existing serious, incomplete \( u-v \) coverage problems. Correspondingly, ten array configurations are left, and they are numbered from 1 to 10.

After analyzing the \( u-v \) coverage of the 16-element arrays, the synthesized beams in snapshot and Earth rotation synthesis modes are studied. In the study, the synthesized beam in the Earth rotation synthesis mode at 450 MHz is composed of 540 snapshot observations obtained at 1 minute intervals. Arrays with smaller \( f_2 \) values were removed, because they are often associated with severe undersampling, as mentioned in Section 4.4. To compensate for this, a Gaussian tapering (FWHM: 0.78\( r_{\text{max}} \)) giving more weight to short baselines was

Figure 5. Results on the optimization problem of the 16-element array using NSGA-II, and hypervolume metric for the resulting solutions (a) Pareto optimal sets of NSGA-II optimization with 2000 and 6000 iterations. (b) Hypervolume metric of the quasi-Pareto optimal solutions with different numbers of iterations. Partially enlarged view of the area is given to show fine details. (c) Entire search process of 6000 iterations and the Pareto optimal set (red dots) of NSGA-II.
applied to the $u$–$v$ sampling when calculating the synthesized beam in Earth rotation synthesis mode. Figures 7(a) and (b) show the comparison of synthesized beams of ten different 16-element array configurations in snapshot mode, and Figure 7(c) shows the comparison results of the Earth rotation synthesis mode. As shown in Figure 7(a), the synthesized beams of arrays numbered 2, 3, and 4 have a worse circular symmetry in the snapshot mode, whereas the other arrays have small differences. Each point in Figure 7(b) represents a 16-element array configuration, where the ordinate represents the mean of the resolutions in all directions, and the abscissa represents the standard deviation. The mean values vary from 97°78 to 111°32, and the standard deviations range between 0°39 and 12°35. The 16-element array 10 provides a higher resolution, while lower resolutions are provided by the arrays 1, 2, 3, and 4. Figure 7(c) shows the amplitude of the first sidelobe versus distance between the first sidelobe and the phase center of the main lobe. The lowest first sidelobe intensity is yielded by array 6, $-12.59$ dB, and is far from the main lobe. Higher first sidelobes are yielded by arrays 3, 4, 5, 9, and 10. Furthermore, arrays 7 and 8 provide synthesized beams that are associated with a short distance between the first sidelobes and the main lobe’s phase center.

As mentioned earlier, a multi-attribute decision making method called TOPSIS (Yoon & Hwang 1995) was used to evaluate the performance of ten 16-elements arrays and choose the final optimal array configuration. Four important statistical attributes for the array-configuration choice problem are: (a) the mean of the resolutions in all directions of the synthesized beams in the snapshot mode, (b) the standard deviation of the resolutions in all directions (representing circular symmetry) of the synthesized beams in the snapshot mode, (c) the amplitude of the first sidelobe of the synthesized beams in Earth-rotation synthesis mode, and (d) the distance between the first sidelobe and the phase center of the main lobe of the synthesized beams in Earth-rotation synthesis mode. The importance of each attribute is characterized by a set of normalized weights (Behzadian et al. 2012), and the choice of weights will affect the final decision. One of the goals of this study was to obtain synthesized beams with low sidelobes, required for high-dynamic-range imaging. Therefore, higher weights were given to the attributes (c) and (d). The scores obtained by TOPSIS for ten 16-element array configurations when the weights of the four attributes were set to 0.2, 0.2, 0.3, and 0.3 are shown in Figure 8. Array 6 was accordingly selected as the final optimal array.

In the case of the optimal array configuration, array 6 is applied to the first stage of the DSRT array. Figure 9 illustrates the characteristics of array 6, including the array configuration, snapshot $u$–$v$ distribution, $u$–$v$ distribution in Earth rotation synthesis mode, synthesized beam, and synthesized beam slices along different directions.

5. Conclusions

In this study, a new optimization principle and the multi-objective optimization algorithm, NSGA-II, used to optimize interferometric array $u$–$v$ coverage were presented.

In practice, an array with a single configuration is used to observe multiple radio sources. In this study, the optimization principle was implemented by trading-off the uniform and the Gaussian $u$–$v$ distributions to obtain favorable observations for both compact and extended sources. NSGA-II, as an evolutionary multi-objective optimization algorithm, is effective at solving the problem that defines the best trade-off between two conflicting $u$–$v$ distributions. The optimization result of NSGA-II, called the Pareto optimal set, contains multiple array configurations.

The example of designing the 16-element array for the initial stage of the DSRT is presented to illustrate the efficiency of the method. The improved NSGA-II is used to optimize the 16-element array configuration, and a resulting Pareto optimal set is obtained. First, compared with the previous single-objective optimization for obtaining uniform $u$–$v$ coverage or a Gaussian $u$–$v$ distribution, the optimized array obtained by NSGA-II is favorable for both compact and extended sources. This is because the optimized array is a
trade-off between the uniform and the Gaussian $u-v$ distributions. Second, this approach is associated with a lower budget and technical difficulty than a reconfigurable array owing to its single configuration. Moreover, the final optimal array selected from the Pareto optimal set yielded a better performance than the other arrays. Specifically, the optimal array had the lowest first sidelobe intensity of $-12.59$ dB and a slight sacrifice of resolution subject to the observation conditions in this study.

Finally, the method presented in this study was not only applicable to circular arrays but also to other types of configurations. The new optimization principle can be further applied to study performance assessment of interferometric arrays whose $u-v$ coverages gradually change from uniform to
Gaussian distributions, which may be a great help in selecting proper configurations for the reconfigurable array.

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**Figure 8.** Ranking results for the performance of ten 16-elements arrays using TOPSIS.

**Figure 9.** Features of the array numbered 6. (a) Array configuration (red pluses) and snapshot $u$–$v$ distribution (blue dots). (c) $u$–$v$ distribution in Earth rotation synthesis mode. (d) Synthesized beam. (d) Slices of the synthesized beam in (c).