Superpropagators for explicitly broken \(3D\)-supersymmetric theories

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Abstract

A systematic algorithm to derive superpropagators in the case of either explicitly or spontaneously broken supersymmetric three-dimensional theories is presented. We discuss how the explicit breaking terms that are introduced at tree-level induce 1-loop radiative corrections to the effective action. We also point out that the renormalisation effects and the breaking-inducing-breaking mechanism become more immediate whenever we adopt the shifted superpropagators discussed in this letter.

Supergraph techniques have shown their efficacy since the early works on superfield perturbation theory introduced by Salam and Strathdee [1]. They also appeared to be an essential tool for the proof of the finiteness of the \(N=2\) and \(N=4\) super-Yang-Mills theories to all orders in perturbation theory [2, 3]. Power-counting, the analysis of the ultraviolet behaviour of globally supersymmetric and supergravity theories, and loop computations by means of super-Feynman rules, are much more compact and have been employed in a number of works to detect at which order in the perturbative series the S-matrix may indicate the appearance of divergences [4].

Supersymmetry however is not an exact symmetry of the low-energy world. The breaking, either spontaneous or explicit, must be thoroughly studied not only for phenomenological purposes, but also to check till which extent deviations from exact supersymmetry may still be compatible with the taming of the divergences imposed by such a symmetry. Along this line of thought, Girardello and Grisaru [5] have sorted out a detailed classification of all soft and hard breakings of supersymmetry in four dimensions; their work triggered a whole line of investigation on the issue of explicit breaking of global supersymmetry [6]. Explicitly broken two-dimensional supersymmetric gauge models have been widely studied in [7]. In three dimensions, by following similar strategy of [6], Gates and Nishino have been classified the soft breakings terms of \(N=2\) supersymmetry [8]. The issue of partial spontaneous supersymmetry breaking \(N=2\rightarrow N=1\) in \(D=3\) has been studied in [9].

Recently, supersymmetry in three dimensions has been reconsidered in connection with Yang-Mills-Chern-Simons gauge theories, which display remarkable features as long as their ultraviolet properties are concerned, namely their finiteness at all orders in perturbation theory [10]. Also, with the raising...
of interest on supermembranes, three-dimensional supersymmetry becomes a major field of investigation \[1\].

Our letter sets out to reassess superfield Feynman rules whenever supersymmetry is broken (spontaneously or explicitly) in three dimensions. Indeed, spontaneous breakdown may always be rephrased as explicit breakings, with explicit \( \theta \)-dependence, after superfields are shifted by their vacuum expectation values.

So, for the sake of setting a systematic procedure to derive superpropagators in the case of broken supersymmetry, we concentrate on the explicit breakings since they naturally account for the case of spontaneous breaking. With the results we shall present in the sequel, the reassessment of supergraph supersymmetry, we concentrate on the explicit breakings since they naturally account for the case of spontaneous breakdown. With the results we shall present in the sequel, the reassessment of supergraph calculations for 3D broken supersymmetric models becomes more systematic and approximations introduced by simply treating the breakings as insertions are by-passed, since we are able to sum up the latter to all orders and so modify the superpropagators with all powers in the breaking parameters, rather than viewing the breakings as new vertices that correct the exact superpropagators.

We consider an explicitly broken supersymmetric theory of a complex scalar superfield, \( \Phi \), minimally coupled to a real spinor gauge superfield, \( \Gamma_\alpha \).

In three dimensions, the most general complex scalar superfield may be \( \theta \)-expanded in component fields as follows
\[
\Phi(x, \theta) = A(x) + \theta^\alpha \psi_\alpha(x) - \theta^2 F(x) ,
\]
with \( A \) and \( \psi_\alpha \) being respectively complex scalar and two-component spinor fields, while \( F \) is a complex auxiliary scalar field. On the other hand, a three-dimensional supersymmetric gauge field theory may be described by a real spinor supermultiplet,
\[
\Gamma_\alpha(x, \theta) = \chi_\alpha(x) - \theta^\gamma [C_{\gamma\alpha} B(x) - i V_{\gamma\alpha}(x)] - \theta^2 [2 \lambda_\alpha(x) - i \partial_\gamma \chi_\gamma(x)] ,
\]
where \( \chi_\alpha \) and \( \lambda_\alpha \) are (real) Majorana spinors, \( B \) is a real scalar, whereas \( V_\beta^\alpha = (\gamma^\alpha)_\beta V_\alpha \) is the gauge potential and \( V_\alpha \) being the gauge field. Also, we define the gauge-invariant field-strength superfield:
\[
W_\alpha = \frac{1}{2} D^\beta D_\alpha \Gamma_\beta ,
\]
constrained by \( D^\alpha W_\alpha = 0 \).

The minimal coupling between matter and gauge superfields is accomplished by means of the covariant supersymmetric gauge derivative:
\[
\nabla_\alpha \Phi = D_\alpha \Phi - ig \Gamma_\alpha \Phi \quad \text{and} \quad \nabla_\alpha \overline{\Phi} = D_\alpha \overline{\Phi} + ig \Gamma_\alpha \overline{\Phi} ,
\]
where \( g \) is the coupling constant.

Then, we start from an action describing the broken theory of a complex matter superfield minimally coupled to the gauge superfield in three dimensions:
\[
S = S_m + S_\xi ,
\]
\[
S_m = \int dv \left\{ -\frac{1}{2} (1 + 2 \theta^2 m_\psi)(\nabla_\alpha \overline{\Phi})(\nabla_\alpha \Phi) + (m + \theta^2 m_\lambda^2) \overline{\Phi} \Phi \right\} ,
\]
and
\[
S_\xi = \frac{1}{2} \int dv \left\{ (1 - 2 \theta^2 m_\lambda) W^\alpha W_\alpha + \mu \Gamma^\alpha W_\alpha \right\} ,
\]
where \( m \) and \( \mu \) are the mass parameters, whereas \( m_\psi, m_\lambda^2 \) and \( m_\lambda \) are the coefficients for the broken terms in the matter and gauge sectors (an explicit breaking associated to the Chern-Simons term, namely \( \theta^2 \Gamma^\alpha W_\alpha \), has not been considered for such a term also explicitly breaks gauge invariance). Besides the matter broken terms, \( S_m \) contains kinetic and massive terms for the matter superfield along with the minimal coupling to the gauge superfield, while \( S_\xi \) contains kinetic and the topological gauge-invariant mass terms for the gauge superfield.

Furthermore, the action \( [2] \) is invariant under the following gauge transformations:
\[
\delta \Phi = i K \Phi \quad \text{and} \quad \delta \Gamma_\alpha = \frac{1}{g} D_\alpha K ,
\]
\[4\]The notations and conventions adopted throughout this work are those of ref.\[1\], and the superspace measure adopted is \( dv = d^3 x d^2 \theta \). The representation for the \( \gamma \)-matrices is taken as \( \gamma^a = (\sigma_y, i \sigma_z, i \sigma_x) \), where \( \gamma^a \equiv (\gamma^a)_\beta^\alpha \) and \( (\gamma^a, \gamma^b) = -2 \eta^{ab} \), with the metric being given by \( \eta^{ab} = \text{diag}(−+, +) \).
$K = K(x, \theta)$ is a real scalar superfield. In order to obtain the superpropagators, one has to fix this gauge invariance; we add the following gauge-fixing term to the action of eq. (5):

$$S_{gf} = -\frac{1}{4\alpha} \int dv \left(D^a \Gamma_\alpha D^b \Gamma_\beta \right) .$$

Now we turn to the attainment of the superpropagators for the matter and gauge sectors by taking the inverse of the wave operators.

The bilinear piece that stems from the action for the matter superfields is the following:

$$S^0_m = \int dv \left\{ -\frac{1}{2} (D^a \Phi)(D_a \Phi) + m \Phi \Phi - \theta^2 m_\psi (D^a \Phi)(D_a \Phi) + \theta^2 m_A \Phi \Phi \right\}$$

$$= \int dv \Phi \mathcal{K} \Phi ,$$

where the operator $\mathcal{K}$ reads as below:

$$\mathcal{K} = D^2 + m + m_\psi (2\theta^2 D^2 + \theta^2 D_\alpha) + m_A^2 \theta^2 .$$

In order to invert the above wave operator and consequently obtain the superpropagator, we shall use the projection operator formalism. The operators associated to the scalar superfield are classified as follows:

$$P_1 = D^2 , \quad P_2 = \theta^2 , \quad P_3 = \theta^a D_\alpha , \quad P_4 = \theta^2 D^2 \quad \text{and} \quad P_5 = i \partial_{\alpha \beta} \theta^a D^\beta ,$$

and their operator algebra is displayed in Table 1. Moreover, we present some useful relations:

$$\{D_\alpha, \theta_\beta\} = C_{\alpha \beta} , \quad \{D_\alpha, \theta^\beta\} = \delta^\beta_\alpha , \quad \{D^\alpha, \theta_\beta\} = -\delta^\beta_\alpha , \quad \{D^a, \theta^\beta\} = C^{a \beta} ,$$

$$[D^2, \theta_\alpha] = D_\alpha , \quad [D_\alpha, \theta^\beta] = \theta_\beta \quad \text{and} \quad [D^2, \theta^2] = -1 + \theta^a D_\alpha .$$

Thus, rewriting $\mathcal{K}$ in terms of the operators $P_i$ ($i = 0, 1, \ldots, 5$), we have

$$\mathcal{K} = mP_0 + P_1 + m_\psi^2 P_2 + m_\psi P_3 + 2m_\psi P_4 ,$$

where $P_0 \equiv 1$.

Using the algebra of Table 1, we readily obtain the superpropagator:

$$\langle 0 | T[\Phi(x_1, \theta_1)\Phi(x_2, \theta_2)] | 0 \rangle = iK^{-1}_\theta^1 \delta^3(x_1 - x_2)\delta(\theta_1 - \theta_2) ,$$

where we are using

$$\delta^3(x_1 - x_2) = \int \frac{d^3k}{(2\pi)^3} e^{-ik(x_1 - x_2)} .$$

In momentum space, (13) is given by:

$$\langle \Phi(k, \theta_1)\Phi(k, \theta_2) \rangle = \frac{-i}{k^2 + m^2 + m_A^2} \left\{ mP_0 - P_1 + \right.$$

$$\left. - \frac{1}{k^2 + (m + m_\psi)^2} \left[ k^2 (m_\psi^2 + mm_\psi) + m_A^2 (m + m_\psi)^2 - m_A^2 k^2 \right] P_2 + \right.\$$

$$\left. + k^2 m_\psi + m_A^2 (m + m_\psi) - m (m_\psi^2 - mm_\psi) \right\} (P_3 + 2P_4) +$$

$$\left. + (m_A^2 - m_\psi^2 - mm_\psi) P_5 \right\} \delta(\theta_1 - \theta_2) .$$

5 Products of the type $XY = Z$, shall always be assumed to be contracted as $X^{\alpha \gamma}Y^\beta = Z^{\alpha \beta}$. 

Table 1: Multiplicative table fulfilled by $P_1$, $P_2$, $P_3$, $P_4$ and $P_5$. The products are supposed to be in the ordering “row times column”.

|   | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_3$ |
|---|---|---|---|---|---|
| $P_1$ | $\Box$ | $-1 + P_3 + P_4$ | $2P_1 + P_5$ | $-P_1 + \Box P_2 - P_5$ | $\Box (-2 + P_3)$ |
| $P_2$ | $P_3$ | 0 | 0 | 0 | 0 |
| $P_3$ | $-P_3$ | $2P_2$ | $P_3 - 2P_4$ | $2P_4$ | $2\Box P_3 + P_5$ |
| $P_4$ | $\Box P_2$ | $-P_2$ | $2P_1$ | $-P_4$ | $-2\Box P_2$ |
| $P_5$ | $-\Box P_3$ | 0 | $-2\Box P_2 + P_5$ | 0 | $\Box (P_3 + 2P_4)$ |
From the poles in \( k^2 \), it becomes clear that the physical scalar and the fermion field have their masses shifted with respect to the degenerated value corresponding to exact supersymmetry.

The propagators for the component fields can be read off by making use of the following relations:

\[
\begin{align*}
\delta(\theta_1 - \theta_2) &= - (\theta_1 - \theta_2)^2 , \\
\theta_1^2 \delta(\theta_1 - \theta_2) &= - \theta_1^2 \theta_2^2 , \\
\theta_1^2 D_1 \alpha \delta(\theta_1 - \theta_2) &= - 2 \theta_1^2 + \theta_1 \theta_2 , \\
\theta_1^2 D_2 \overline{\alpha} \delta(\theta_1 - \theta_2) &= \theta_1^2 , \\
k_{\alpha \beta} \theta_1^2 D_3 \overline{\beta} \delta(\theta_1 - \theta_2) &= k_{\alpha \beta} \theta_1^2 \theta_2^2 + 2k^2 \theta_1^2 \theta_2^2 .
\end{align*}
\]

The non-vanishing component-field propagators may be extracted out of \( \langle \Phi(k, \theta_1) \Phi(k, \theta_2) \rangle \):

\[
\begin{align*}
\langle \overline{A}(k) A(k) \rangle &= \frac{-i}{k^2 + m^2 + m_A^2} , \\
\langle \overline{F}(k) F(k) \rangle &= i \frac{k^2 + m_A^2}{k^2 + m^2 + m_A^2} , \\
\langle \overline{A}(k) F(k) \rangle &= \frac{i m}{k^2 + m^2 + m_A^2} , \\
\langle \overline{\psi}^\lambda (k) \psi^\beta (k) \rangle &= - i \frac{k_{\alpha \beta} - (m + m_\psi) C_{\alpha \beta}}{k^2 + (m + m_\psi)^2} ,
\end{align*}
\]

which agree with the propagators calculated from the component-field action stemming from eq. (13).

As for the gauge sector, similarly to the matter sector, we may find the superpropagator for the gauge superfield. The bilinear piece of (13) for the gauge superfield plus the gauge-fixing term, (14), reads

\[
S^0_g = \frac{1}{2} \int dv \left\{ \frac{1}{4} (D^\alpha D^\beta \Gamma_\beta) (D^\gamma D_\alpha \Gamma_\gamma) + \frac{\mu}{2} \Gamma_\alpha (D^\beta D_\alpha \Gamma_\beta) - \frac{1}{2 \alpha} (D^\alpha \Gamma_\alpha) (D^2 D^\beta \Gamma_\beta) + \theta^\alpha \overline{\theta}^\beta \frac{m_\lambda}{2} (D^\beta D^\alpha \Gamma_\beta) (D^\gamma D_\alpha \Gamma_\gamma) \right\}

= \int dv \Gamma_\alpha \mathcal{K}^{\alpha \beta} \Gamma_\beta ,
\]

where the operator \( \mathcal{K}^{\alpha \beta} \) can be written as

\[
\mathcal{K}^{\alpha \beta} = - \frac{1}{4} \left[ \frac{1}{2} D^\gamma D^\alpha D^\beta D_\gamma + \frac{1}{2} D^\alpha D^2 D^\beta + \mu D^\beta D^\alpha - m_\lambda D^\alpha D^\beta + \theta^\alpha \theta^\beta D_\gamma \right] .
\]

Here, we must introduce other twelve superspace operators coming from the gauge sector, which can be expressed in terms of the \( P_1 \)'s as follows:

\[
R^\alpha_1 = i P_1 \theta^{\alpha \beta} \quad \text{and} \quad S_{i}^{\alpha \beta} = P_1 C^{\alpha \beta} ,
\]

where \( i = 0, 1, \ldots, 5 \). Their algebra is presented in Table 2.

Table 2: Multiplicative table fulfilled by \( R^\alpha_1 \) and \( S_{i}^{\alpha \beta} \). The ordering is “row times column”.

| \( S_0 \) | \( S_1 \) | \( R_0 \) | \( R_1 \) |
|---|---|---|---|
| \( S_0 \) | \( S_0 \) | \( S_1 \) | \( R_0 \) | \( R_1 \) |
| \( S_1 \) | \( P_1 P_1 C^{\alpha \beta} \) | \( R_i \) | \( i \square P_1 P_1 \delta^{\alpha \beta} \) |
| \( R_0 \) | \( R_1 \) | \( i \square P_1 P_1 \delta^{\alpha \beta} \) | \( \square S_0 \) | \( \square S_1 \) |
| \( R_1 \) | \( R_1 \) | \( i \square P_1 P_1 \delta^{\alpha \beta} \) | \( \square S_1 \) | \( \square P_1 P_1 C^{\alpha \beta} \) |

By using the property \( P_1 P_1 = \sum a_k P_k \) (see Table 1) and (22), where \( i, j, k = 1, 2, \ldots, 5 \), we may check that the algebra presented in Table 2 is in fact closed. Thus, the wave operator \( \mathcal{K}^{\alpha \beta} \) can be rewritten as

\[
\mathcal{K}^{\alpha \beta} = - \frac{1}{4} \left[ \frac{\alpha + 1}{\alpha} \square S^\alpha_0 - (\mu + m_\lambda) S^\alpha_1 - 2m_\lambda \square S^\alpha_1 + m_\lambda S^\alpha_1 + (\mu + m_\lambda) R^\alpha_0 + \alpha - \frac{1}{\alpha} R^\alpha_1 - m_\lambda R^\alpha_3 - 2m_\lambda R^\alpha_4 \right] .
\]
The superpropagator

$$\langle 0 \mid T[\Gamma^\alpha(x_1, \theta_1)\Gamma^\beta(x_2, \theta_2)] \mid 0 \rangle = i \delta^\alpha_\beta \delta^3(x_1 - x_2) \delta(\theta_1 - \theta_2) ,$$

(24)

exhibits the following structure in terms of the superspace operators:

$$\langle 0 \mid T[\Gamma^\alpha(x_1, \theta_1)\Gamma^\beta(x_2, \theta_2)] \mid 0 \rangle = \sum_{i=0}^{5} (s_i S_0^{\alpha\beta} + r_i R_1^{\alpha\beta}) \delta^3(x_1 - x_2) \delta(\theta_1 - \theta_2) ,$$

(25)

where the coefficients $s_i$ and $r_i$ are to be (uniquely) determined by a system of twelve equations. The latter result can be readily attained if we compute a 1-loop diagram with the matter superpropagators. Conversely, it is noteworthy to remark that the gaugino mass breaking at tree-level induce 1-loop radiative corrections to the effective action. We adopt the superfield Feynman rules as presented and discussed in ref. 4 and make use of the superpropagators derived in our paper.

If there are matter superfields in sufficient number such that $\Phi^\beta$-interaction vertices do not break gauge invariance, tadpole supergraphs with a $\Phi$-superfield on the external leg may induce a loop correction to the $F$-term, as a result of the term with the operator $P_4$ present in the $\Phi\Phi$-propagator. $F$-terms are not radiatively induced as a result of the gauge interaction, since the gauge couplings do not allow a $\Phi$-tadpole with a loop where the gauge superfield flow inside alone. It is the matter self-interaction and the explicit breakings governed by $m_\psi$ and $m_A$ the responsible for the 1-loop generation of an $F$-term.

Also, it is interesting to notice that, if we consider the 2-point function with $\Gamma_\alpha$ and $\Gamma_\beta$ on the external legs and the matter superpropagators running inside the loop, the matter breaking terms induce a 1-loop correction to the supersymmetric Chern-Simons mass term; as for the gauge superfield kinetic term, no correction arises that comes from the breakings. The technical reason to understand these results is a simple counting of covariant derivatives inside the loop (some are brought by the propagators, others appear as vertex factors).

The gauge-invariant term that splits the scalar mass inside the matter supermultiplet $(\theta^2 \Phi \Phi)$ induces a 1-loop correction to the breaking term that splits the gaugino mass in the gauge supermultiplet $(\theta^2 W^\alpha W_\alpha)$: such a term appears as the result of the interference between breaking terms present in the matter superpropagators. Conversely, it is noteworthy to remark that the gaugino mass breaking term yields a 1-loop correction that explicitly breaks supersymmetry and splits the scalar mass inside the matter multiplet. The latter result can be readily attained if we compute a 1-loop diagram with the matter superpropagators.
matter superfields on the external legs and a gauge superpropagator appearing as an internal line of the corresponding 1-loop graph.

Concluding these comments, one of the advantages of working with this somewhat complicated superpropagators is that, once the breaking parameters are taken into account to all orders, we get a safe and systematic algorithm for deriving radiative corrections to the breakings as induced from one another. The investigation of the effective action, renormalisation effects and breaking-inducing-breaking mechanism become more automatic if we adopt to work with these full superpropagators. In situations where a spontaneous breaking of supersymmetry takes place, and shifts have to be performed around the true ground state, explicit breakings as the ones collected above show up (spontaneous supersymmetry breaking appears in superspace as $\theta$-terms) and our computations may become useful to compute radiative corrections to the effective action and to physical quantities derived from the effective potential.

In the case supersymmetry is broken for a gauge model, we have to generalize the $R_\xi$-gauge with now $\theta$-dependent present, since the superfields $\Phi$ and $\Gamma_\alpha$ mix up with a $\theta^2$-factor. This problem is now under investigation, and we shall soon report our results elsewhere.

Acknowledgements: Two of us (L.P.C.) and (O.M.D.C.) thank The Erwin Schrödinger International Institute for Mathematical Physics (ESI-Vienna) for the kind hospitality, they also would like to express their gratitude to Manfred Schweda and Wolfgang Kummer. (L.P.C.) thanks also the Institut für Theoretische Physik of the TU-Wien. (O.M.D.C.) acknowledges the Departamento de Teoria de Campos e Partículas (DCP) of the Centro Brasileiro de Pesquisas Físicas (CBPF) for the warm hospitality during all his visits. He dedicates this work to his wife, Zilda Cristina, to his kids, Vittoria and Enzo, and his mother, Victoria.

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