FORECAST FOR THE PLANCK PRECISION ON THE TENSOR-TO-SCALAR RATIO
AND OTHER COSMOLOGICAL PARAMETERS

C. Burigana\textsuperscript{1}, C. Destri\textsuperscript{2}, H. J. de Vega\textsuperscript{3,4}, A. Gruppuso\textsuperscript{1}, N. Mandolesi\textsuperscript{1}, P. Natoli\textsuperscript{5}, and N. G. Sanchez\textsuperscript{4}

\textsuperscript{1} INAF/IASF, Istituto di Astrofisica Spaziale e Fisica Cosmica di Bologna, Istituto Nazionale di Astrofisica, via Gobetti 101, I-40129 Bologna, Italy
\textsuperscript{2} Dipartimento di Fisica G. Occhialini, Università Milano-Bicocca and INFN, Sezione di Milano-Bicocca, Piazza della Scienza 3, 20126 Milano, Italy
\textsuperscript{3} LPTHE, Laboratoire Associé au CNRS UMR 7589, Université Pierre et Marie Curie (Paris VI) et Denis Diderot (Paris VII), Tour 24, 5 ème étage, 4 Place Jussieu, 75252 Paris, Cedex 05, France
\textsuperscript{4} Observatoire de Paris, LERMA, Laboratoire Associé au CNRS UMR 8112, 61 Avenue de l’Observatoire, 75014 Paris, France
\textsuperscript{5} Dipartimento di Fisica, Università di Roma Tor Vergata and INFN, Sezione di Tor Vergata, Via della Ricerca Scientifica 1, I-00133 Roma, Italy

Received 2010 March 29; accepted 2010 September 16; published 2010 November 3

ABSTRACT

The Planck satellite, successfully launched on 2009 May 14 to measure with unprecedented accuracy the primary cosmic microwave background (CMB) anisotropies, is operating as expected. The Standard Model of the Universe (“concordance” model) provides the current realistic context to analyze the CMB and other cosmological/astrophysical data, inflation in the early universe being part of it. The Planck performance for the crucial primordial parameter \( r \), the tensor-to-scalar ratio related to primordial B-mode polarization, will depend on the quality of data analysis and interpretation. The Ginzburg–Landau (G–L) approach to inflation allows us to take high benefit of the CMB data. The four-degree double-well inflaton potential gives an excellent fit to the current CMB+LSS data. We evaluate the Planck precision to the recovery of cosmological parameters, taking into account a reasonable toy model for residuals of systematic effects of instrumental and astrophysical origin based on publicly available information. We use and test two relevant models: the \( \Lambda \)CDM model, i.e., the standard \( \Lambda \)CDM model augmented by \( r \), and the \( \Lambda \)CDM/T model, where the scalar spectral index, \( n_s \), and \( r \) are related through the theoretical “banana-shaped” curve \( r = r(n_s) \) coming from the G–L theory with a double-well inflaton potential. In the latter case, the analytical expressions for \( n_s \) and \( r \) are imposed as a hard constraint in a Monte Carlo Markov Chain (MCMC) data analysis. We consider two \( C_r \)-likelihoods (with and without B modes) and take into account the white noise sensitivity of Planck (LFI and HFI) in the 70, 100, and 143 GHz channels as well as the residuals from systematic errors and foregrounds. We also consider a cumulative channel of the three mentioned. We produce the sky (mock data) for the CMB multipoles as well as the residuals from systematic errors and foregrounds. We also consider a cumulative channel of the three models. Foreground residuals affect only the cosmological parameters sensitive to the B modes. As expected, the likelihood \( r \) distribution is more clearly peaked near the fiducial value (\( r = 0.0427 \)) in the \( \Lambda \)CDM/T model than in the \( \Lambda \)CDM model. The best value for \( r \) in the presence of residuals turns out to be about \( r \approx 0.04 \) for both the \( \Lambda \)CDM and the \( \Lambda \)CDM/T models. The \( \Lambda \)CDM/T model is very stable; its distributions do not change by including residuals and the B modes. For \( r \) we find \( 0.028 < r < 0.116 \) at a 95\% confidence level (CL) with the best value \( r = 0.04 \). We also compute the B mode detection probability by the most sensitive HFI-143 channel. At the level of foreground residual equal to 30\% of our toy model, only a 68\% CL (1\( \sigma \)) detection is very likely. For a 95\% CL detection (2\( \sigma \)), the level of foreground residual should be reduced to 10\% or lower of the adopted toy model. The lower bounds (and most probable value) we infer for \( r \) support the searching of CMB B-mode polarization in the current data as well as the planned CMB missions oriented toward B polarization.

Key words: cosmic background radiation – cosmological parameters – inflation – methods: data analysis – space vehicles

Online-only material: color figures

1. INTRODUCTION AND WORK OUTLINE

The Planck satellite\textsuperscript{6} was successfully launched on 2009 May 14 to measure the primary cosmic microwave background (CMB) temperature and polarization anisotropies on the whole sky with unprecedented accuracy. It is now in normal operation with the expected performance (Planck Collaboration 2006; Bersanelli et al. 2010; Mandolesi et al. 2010; Lamarre et al. 2010; Maffei et al. 2010). Planck will improve the measurement of most cosmological parameters by several factors with respect to current experiments, in particular the Wilkinson Microwave Anisotropy Probe (WMAP) satellite.\textsuperscript{7} The expected CMB polarization measurements from Planck will allow us to push both \( E \) and B polarization results well beyond the present knowledge and considerably constrain the tensor (B modes)-to-scalar ratio parameter \( r \), if not obtain a detection of it. In this respect, the way of extracting and physically interpreting cosmological parameters (once the CMB data is cleaned from the different astrophysical foregrounds) will be important. In other words, the actual Planck performance for the crucial primordial parameter \( r \) will depend on the adopted physical modeling and the quality of data analysis and interpretation. It is then important and timely to make forecasts for the Planck determination of \( r \) and other cosmological parameters taking into account the theoretical progress in the field and WMAP results.

The Standard Model of the Universe (or “concordance” model) provides the current realistic context for analyzing...
the CMB and other cosmological/astrophysical data. Inflation (quasi-exponential accelerated expansion) of the early universe is a part of this model and one important goal of CMB experiments is probing the physics of inflation itself. Inflation solves the shortcomings of the decelerated expanding cosmology (horizon problem, flatness, entropy of the universe), and explains the observed CMB anisotropies providing the mechanism for the generation of scalar and tensor perturbations seeding the large-scale structures (LSS) and primordial (still undetected) gravitational waves (B-mode polarization).

The current CMB + LSS data support the standard inflationary predictions of a nearly spatially flat universe with adiabatic and nearly scalar-invariant initial density perturbations. These data are validating the single-field slow-roll inflationary scenario (Komatsu et al. 2009). Single-field slow-roll models provide an appealing, simple, and fairly generic description of inflation (Dodelson 2003; Boyanovsky et al. 2009). The inflationary scenario is implemented using a scalar field, the inflaton with a potential \( V(\phi) \), self-consistently coupled to the space-time metric. In the effective theory based on the Ginzburg–Landau (G–L) approach to inflation (Boyanovsky et al. 2009), the potential is a polynomial in the field starting with a constant term. Linear terms can always be eliminated by a constant shift of the inflaton field. The mass (quadratic) term can have a positive or a negative sign associated with unbroken symmetry (chaotic inflation) or with broken symmetry (new inflation), respectively. The fourth-degree double-well inflaton potential gives an excellent fit to the present CMB + LSS data (Boyanovsky et al. 2009). A cubic term does not improve the fit and can be omitted (Destri et al. 2008a). Adding higher order terms with additional parameters does not significantly improve the fits (Destri et al. 2009). The G–L framework is not just a class of physically well-motivated inflaton potentials, among which are the double- and single-well potentials, but also provides the effective theory for inflation with a powerful increase in physical insight and analysis of the data. The present set of data with the effective theory of inflation favor the double-well potential (Boyanovsky et al. 2009; Destri et al. 2008a). Analyzing the present data without the relation between \( r \) and \( n_s \) does not allow us to discriminate among different classes of models for the inflaton potential in the considered framework. Although the G–L effective theory approach to inflation is quite general, it predicts precise order of magnitude estimates for \( n_s \) and \( r \), and the running of the spectral index \( dn_s/d \ln k \) (Boyanovsky et al. 2009):

\[
n_s - 1 = O \left( \frac{1}{N} \right), \quad r = O \left( \frac{1}{N} \right), \quad \frac{dn_s}{d \ln k} = O \left( \frac{1}{N^2} \right);
\]

here, \( N \sim 60 \) is the number of e-folds from the time the cosmologically relevant modes exit the horizon until inflation ends. The WMAP values for \( n_s \) and the upper bounds for \( r \) and \( dn_s/d \ln k \) agree with these estimates. Since in this framework the estimated running, \( dn_s/d \ln k \sim 3 \times 10^{-4} \), is very small, in this paper we will concentrate on \( n_s \) and \( r \).

In this work, we evaluate the accuracy in the recovery of the cosmological parameters expected from the Planck data. First, we do this forecast without including the systematic effects of instrumental and/or astrophysical origin or their coupling, affecting the Planck measurements, and then by including the systematic effects. In this study, we exploit the Planck sensitivity and resolution at its three favorite cosmological channels, i.e., at the frequencies of 70, 100, and 143 GHz. Table 1 reports the Planck performance at these frequencies, based on the

| Frequency Channel | 143 GHz | 100 GHz | 70 GHz |
|-------------------|---------|---------|--------|
| \( \delta T \) per FWHM\(^2 \) pixel (\( \mu K \)) | 4.2     | 4.8     | 17.2   |
| \( \delta Q \), \( \delta U \) per FWHM\(^2 \) pixel (\( \mu K \)) | 8.1     | 7.7     | 24.3   |

**Note.** The average sensitivity per FWHM\(^2 \) resolution element \((\delta T, \delta Q, \delta U)\) is given in CMB temperature units (i.e., equivalent thermodynamic temperature) for 28 months of integration, almost corresponding to four sky surveys.

Planck Collaboration (2006) and, for the LFI channel at 70 GHz, as updated by Mandolesi et al. (2010), Bersanelli et al. (2010), and Sandri et al. (2010). These sensitivities do not include the degradation in accuracy that could come from various sources of systematic effects, of both instrumental and/or astrophysical origin, or their coupling. In Section 4, we discuss the current published estimates for the residuals of systematic effects and foreground affecting the Planck CMB measurements: stray light, main beam asymmetry, leakage, time constants, glitches, and foreground. In general, in this work we do not use a precise description of the considered systematic effects, but only suitable representations of them, as described in Section 4. This is done in a parametric approach, identifying the corresponding levels at which the control of the systematic effects is necessary so as to not to spoil scientific accuracy of the Planck data. We technically implement this rescaling with a multiplicative constant on the residuals of the systematic effects on the CMB multipoles \( C_\ell \). Obviously, the real analysis of Planck data will have to properly consider all possible systematic effects of optical, thermal, and instrumental (radiometric and bolometric) origin, with an even better accuracy than those achieved in past projects. In parallel, a significantly improved separation of CMB from astrophysical components will be needed, a task that is possible for Planck in principle thanks to its wide frequency coverage.

Instrumental systematics on the CMB tensor-to-scalar ratio have been studied by Hu et al. (2003), Shimon et al. (2008), and Yadav et al. (2010).

We use and test two relevant models: the \(^{\Lambda}\)CDM model, which is the standard \(^{\Lambda}\)CDM model augmented by the tensor-to-scalar ratio \( r \), and the \(^{\Lambda}\)CDM/T model, which is the \(^{\Lambda}\)CDM model in which the double-well inflaton potential (see Equation (1)) is imposed. Namely, \( n_s \) and \( r \) are constrained by the analytic relation \( r = r(n_s) \) on the theoretical banana-shaped curve (the upper border of the banana-shaped region in Figure 1). The novelty in the Monte Carlo Markov Chain (MCMC) analysis of the CMB data with the \(^{\Lambda}\)CDM/T model is in the fact that we impose the analytical expressions for \( n_s \) and \( r \) derived from the inflaton potential as a hard constraint (Destri et al. 2008a). We take both models, \(^{\Lambda}\)CDM and \(^{\Lambda}\)CDM/T, as fiducial models in our MCMC simulations to produce the corresponding skies (mock data). In the \(^{\Lambda}\)CDM model, the independent cosmological parameters are \( \Omega_b h^2 \), \( \Omega_c h^2 \), \( \theta \), \( \tau \), \( A_s \), \( n_s \), and \( r \), while all other independent parameters are assumed to vanish, e.g., \( \Omega_{\Lambda} = 0 \), or have the standard values, e.g., \( w = -1 \). The aforementioned \(^{\Lambda}\)CDM/T model includes the same parameters but with \( n_s \) and \( r \) not being independent, but related by the curve \( r = r(n_s) \) as discussed in Section 2. We produce one sky (mock data) for the anisotropy CMB multipoles \( C_\ell^{TT}, C_\ell^{EE}, C_\ell^{TB}, \) and \( C_\ell^{BB} \) from the \(^{\Lambda}\)CDM model and from the \(^{\Lambda}\)CDM/T model with the parameters listed in Table 2. We describe the detailed procedure in Section 5. We run MCMCs
from this sky and obtain the marginalized likelihood distributions for the cosmological parameters \(\Omega_0, h^2, \Omega_\Lambda, \theta, \omega_0, n_s, i, r\) in the two test models \(\Lambda\)CDM and \(\Lambda\)CDM/T. We study the independent \(\Lambda\)CDM parameters with the mock data produced from \(\Lambda\)CDM (first row of Table 2) and the independent parameters of both \(\Lambda\)CDM and \(\Lambda\)CDM/T with the mock data produced from \(\Lambda\)CDM/T (second row in Table 2). The fiducial values \(r = 0.0427\) and \(n_s = 0.9614\) correspond to the best fit to the CMB–LSS data with the \(\Lambda\)CDM/T model using the double-well inflaton potential expressed by Equation (1).

Namely, these are the best-fit values to \(r\) and \(n_s\) within the G–L effective theory approach. Without using the G–L approach, lower bounds for \(r\) are not obtained and the best-fit value for \(r\) can be much smaller than \(r = 0.04\) (Kinney et al. 2008; Peiris & Easther 2008).

We consider two choices for the \(C_L\) likelihood, one without the \(B\) modes and one with the \(B\) modes, and take into account the white noise sensitivity of Planck (LFI and HFI) in the 70, 100, and 143 GHz channels (Planck Collaboration 2006). We also consider a cumulative channel whose \(\chi^2\) is the sum of the \(\chi^2\)'s of the three channels above. When using different channels in the MCMC analysis, we use different noise realizations while keeping the same noise, i.e., the same realization of the Gaussian process that generated the primordial fluctuations. In our MCMC analysis, we always take standard flat priors for the cosmological parameters. In particular, we assume the flat priors \(0 \leq r < 0.2\) in the \(\Lambda\)CDM model and \(0 \leq r < 8/60\), where \(8/60 \simeq 0.133\) is the theoretical upper limit for \(r\) in the \(\Lambda\)CDM/T model.

We performed the MCMC simulations using the publicly available CosmoMC code8 (Lewis & Bridle 2002) interfaced to the Boltzmann code CAMB9 (see Lewis et al. 2000 and references therein).

Our findings without including the systematic effects are summarized in Figures 3–6 where the marginalized likelihood distributions of the cosmological parameters are plotted for several different setups. In Tables 3–5, we list the corresponding relevant numerical values. Clearly, in the case of the ratio \(r\), due to the specific form of its likelihood distribution, it is more interesting to exhibit upper and lower bounds rather than mean values and standard deviations as in Tables 3 and 4. We report the upper bounds and, when present, the lower bounds in Tables 5 and 6. Our conclusions without including the systematic effects are as follows.

### Table 2

| \(\Omega_0, h^2\) | \(\Omega_\Lambda, h^2\) | \(\theta\) | \(r\) | \(\log(10^{10} A_s)\) | \(n_s\) | \(r\) | \(\Omega_\Lambda\) | \(H_0\) | \(z_{\text{re}}\) |
|---|---|---|---|---|---|---|---|---|---|
| \(\Lambda\)CDM | 0.0223 | 0.1079 | 1.0387 | 0.0864 | 3.0561 | 0.9613 | 0 | 0.7463 | 71,628 | 10.399 |
| \(\Lambda\)CDM/T | 0.0224 | 0.1112 | 1.0410 | 0.0821 | 3.0629 | 0.9615 | 0.0427 | 0.7364 | 71,228 | 10.062 |

Note. In the \(\Lambda\)CDM/T model, we constrain \(r = r(n_s)\) by the double-well inflaton potential given in Equation (1) as depicted on the upper border of the banana-shaped region in Figure 1.

![Figure 1. Universal banana region B in the \((n_s, r)\)-plane setting \(N = 60\). The upper border of the region B corresponds to the fourth-order double-well potential expressed by Equation (1). The lower border is described by the potential \(V(\psi) = \frac{1}{2}m^2(\frac{\psi^2}{\lambda} - \psi^2)\) for \(\psi^2 > m^2/\lambda\) and \(V(\psi) = \infty\) for \(\psi^2 < m^2/\lambda\) (Destri et al. 2009). We display in the vertical full line the \(\Lambda\)CDM value \(n_s = 0.968 \pm 0.015\) using the WMAP+BAO+SN data set. The broken vertical lines delimit the \(\pm 1\sigma\) region.](http://camb.info/) (A color version of this figure is available in the online journal.)

1. The upper bound on \(r\) and the best value of \(n_s\) do not require us to include the \(B\) modes in the likelihood, and can be obtained with the \(\Lambda\)CDM model alone (i.e., \(r < 0.068\) and \(n_s = 0.9549\) at a 95% CL). See Tables 3 and 5 and Figure 3. The inclusion of \(B\) modes, for a non-vanishing fiducial value \(r = 0.0427\), allows peaked marginalized distributions for \(r\) and a lower bound for \(r\). See Table 6 and Figures 3–5. We obtain \(0.013 < r < 0.045\) at a 95% confidence level (CL) in the \(\Lambda\)CDM model, with the best values \(r = 0.0240, n_s = 0.9597\). This shows a substantial progress in the forecasted bounds for \(r\) with respect to the WMAP+LSS data set for which \(r < 0.20\) in the pure \(\Lambda\)CDM model (Komatsu et al. 2009, 2010).

2. Lower bounds on \(r\) and most probable \(r\) values are always obtained (with or without the \(B\) modes) with the \(\Lambda\)CDM/T model. See Tables 3, 4, and 6 and Figure 5. The \(\Lambda\)CDM/T model provides well-peaked distributions for \(r\) on non-zero values \(r \simeq 0.04\). We obtain \(r > 0.039\) at a 68% CL and \(r > 0.030\) at a 95% CL in the \(\Lambda\)CDM/T model.

In Section 7, we include the systematic effects discussed in Sections 3 and 4 in the forecasts. Our conclusions including the systematic effects and foreground residuals are as follows.

1. The likelihood distributions with and without \(B\) modes are almost the same when including the residuals. Only the cosmological parameters sensitive to the \(B\) modes appear to be affected by the residuals, namely, \(\sigma_8, z_{\text{re}}, r\) and \(\tau\). The main numbers are displayed in Tables 5 and 6.

2. The marginalized likelihood \(r\) distribution for fiducial ratio \(r = 0.0427\) is more clearly peaked on a value of \(r\) near the fiducial one in the \(\Lambda\)CDM model than in the \(\Lambda\)CDM/T model.

---

8. http://cosmologist.info/cosmomc/

9. http://camb.info/
Table 3
Best Fits, Mean Values, and Standard Deviations for Cosmological Parameters When $B$ Modes Are Not Included in the $C_l$-likelihood

| Sky Data | Test Model | $\Lambda$CDM: Fiducial $r = 0$ | $\Lambda$CDM$\tau$: Fiducial $r = 0.0427$ |
|----------|------------|--------------------------------|----------------------------------|
| Without $B$ Modes | $\Omega_m h^2$ | 0.0223 ± 0.0001 | 0.0225 ± 0.0001 |
| | $\Omega_b h^2$ | 0.0105 ± 0.0007 | 0.0118 ± 0.0007 |
| | $\Theta$ | 1.0389 ± 0.0022 | 1.0411 ± 0.0022 |
| | $\tau$ | 0.0832 ± 0.0027 | 0.0858 ± 0.0027 |
| | log($10^{10} A_s$) | 3.0479 ± 0.0054 | 3.0703 ± 0.0056 |
| | $n_s$ | 0.9549 ± 0.0021 | 0.9604 ± 0.0022 |
| | $r$ | 0.0041 ± 0.0206 | 0.0594 ± 0.0277 |
| | $\Omega_x h^2$ | 0.7436 ± 0.0038 | 0.7344 ± 0.0040 |
| | $H_0$ | 71.437 ± 0.3610 | 71.111 ± 0.3638 |
| | $z_w$ | 10.128 ± 0.2267 | 10.375 ± 0.2328 |

Note. All values are rounded to order $10^{-4}$ to the nearest value and correspond to the cumulative channel whose $\chi^2$ is the sum of the $\chi^2$ of the three channels HFI-100, HFI-143, and LFI-70.

Table 4
Best Fits, Mean Values, and Standard Deviations for Cosmological Parameters When $B$ Modes Are Included in the $C_l$-likelihood

| Sky Data | Test Model | $\Lambda$CDM: Fiducial $r = 0$ | $\Lambda$CDM$\tau$: Fiducial $r = 0.0427$ |
|----------|------------|--------------------------------|----------------------------------|
| With $B$ Modes | $\Omega_m h^2$ | 0.0223 ± 0.0001 | 0.0225 ± 0.0001 |
| | $\Omega_b h^2$ | 0.1081 ± 0.0007 | 0.1118 ± 0.0007 |
| | $\Theta$ | 1.0389 ± 0.0002 | 1.0412 ± 0.0002 |
| | $\tau$ | 0.0834 ± 0.0027 | 0.0866 ± 0.0027 |
| | log($10^{10} A_s$) | 3.0620 ± 0.0053 | 3.0721 ± 0.0054 |
| | $n_s$ | 0.9606 ± 0.0021 | 0.9597 ± 0.0021 |
| | $r$ | 0.0010 ± 0.0050 | 0.0240 ± 0.0096 |
| | $\Omega_x h^2$ | 0.7456 ± 0.0038 | 0.7342 ± 0.0040 |
| | $H_0$ | 71.624 ± 0.3556 | 71.089 ± 0.3570 |
| | $z_w$ | 10.126 ± 0.2250 | 10.443 ± 0.2285 |

Note. All values are rounded to order $10^{-4}$ to the nearest value and correspond to the cumulative channel whose $\chi^2$ is the sum of the $\chi^2$ of the three channels HFI-100, HFI-143, and LFI-70.

Table 5
Upper Bounds on $r$ With All Figures Rounded Upward to Order $10^{-3}$

| Sky Data | Test Model | $\Lambda$CDM: Fiducial $r = 0$ | $\Lambda$CDM$\tau$: Fiducial $r = 0.0427$ |
|----------|------------|--------------------------------|----------------------------------|
| Without $B$ modes | LFI-70 | $r < 0.2$ | $r < 0.2$ |
| | HFI-100 | $r < 0.068$ | $r < 0.097$ |
| | HFI-143 | $r < 0.070$ | $r < 0.108$ |
| Cumulative | $r < 0.036$ | $r < 0.066$ |
| With $B$ modes | LFI-70 | $r < 0.074$ | $r < 0.151$ |
| | HFI-100 | $r < 0.012$ | $r < 0.037$ |
| | HFI-143 | $r < 0.008$ | $r < 0.041$ |
| Cumulative | $r < 0.008$ | $r < 0.032$ |

Notes. The bound $r < 0.2$ is just the assumed prior, which gets saturated by the $\Lambda$CDM$\tau$ test model in the LFI-70 channel when $B$ modes are absent. The limits in the case of the $\Lambda$CDM$\tau$/T test model with fiducial $r = 0.0427$ in the LFI-70 channel are not really significant in view of the shape of the corresponding likelihood distribution (see Figure 5).

model (compare Figures 7 and 8). In any case, the best value for $r$ in the presence of residuals is about $r \approx 0.04$ (near the fiducial value) for both the $\Lambda$CDM$\tau$ and the $\Lambda$CDM$\tau$/T models. The $\Lambda$CDM$\tau$/T model turns out to be robust; it is very stable (its distributions do not change) with respect to the inclusion of residuals (and they do not change with respect to the inclusion of $B$ modes). The main numbers are included in Tables 5 and 6. With the $\Lambda$CDM$\tau$/T model we have for $r$ at a 95% CL:

$$0.028 < r < 0.116 \text{ with the best values } r = 0.04, n_s = 0.9608.$$
It must be stressed that, in the ΛCDM/T model, future improvements in the precision δ on the measured value of n_s alone will immediately give an improvement dr/dn_s δ on the prediction for r as well as for its lower bound. Better measurements for n_s will thus improve the prediction on r from the T, TE, and E modes even if a secure detection of the B modes is still lacking.

In order to assess the probability for Planck to detect r, we also compute the B mode detection probability by the most sensitive HFI-143 channel; this is done in Section 7.2. We extract 10^3 skies obtaining the corresponding multipoles A_{lm} from the ΛCDM/T model according to the procedure described in Section 5, adopting r = 0.0427 as a fiducial value. We compute all the corresponding likelihood profiles only for the departure from a Gaussian likelihood; Figure 9). We finally compute their skewness, and the kurtosis (which measures the statistics of the shape of the corresponding likelihood distribution as can be seen from Figure 5. In the ΛCDM model, the entries left empty in the table correspond to the cases where there are no lower bounds on r (as can be seen from Figures 3 and 4).

| Sky Data | Test Model | ΛCDM | ΛCDM/T |
|----------|------------|------|--------|
|          |            | 68% CL | 95% CL | 68% CL | 95% CL |
| LFI-70   | Without    |       |        |        |        |
| HFI-100  | r > 0.051  | r > 0.067 | r > 0.035 |
| B modes  | HFI-143    | r > 0.024 | r > 0.024 |
| Cumulative | r > 0.024 | r > 0.024 |
| LFI-70   | With       |       |        |        |        |
| HFI-100  | r > 0.020  | r > 0.046 | r > 0.024 |
| B modes  | HFI-143    | r > 0.026 | r > 0.013 |
| Cumulative | r > 0.022 | r > 0.013 |

Notes. These bounds are assumed significant only when the likelihood at r = 0 is less than \( \exp(-1/2) = 0.6065 \) of its maximum for 68% CL or less than \( \exp(-1) = 0.3678 \) for 95% CL. The limits in the case of the ΛCDM/T test model with fiducial r = 0.0427 in the LFI-70 channel are not really significant in view of the shape of the corresponding likelihood distribution as can be seen from Figure 5. In the ΛCDM model, the entries left empty in the table correspond to the cases where there are no lower bounds on r (as can be seen from Figures 3 and 4).

The forecasted probability of detecting r is based on the statistics of the shape of the r-likelihood. This shape determines whether a detection of r can be claimed with a given confidence level. Real CMB experiments can observe only one sample: the observed sky. Thus, the possibility of inferring r from one single (albeit very large) sample depends on the sample itself, and therefore, whether r will or will not be detected also depends on luck. In addition, the results for many skies presented in Section 7.2 show the consistency of our whole approach in determining r.

Finally, in Section 7.3 we consider the bias effect in the foreground residuals implemented as a linear perturbation affecting the C_l and explore how the cosmological parameter distributions are affected by the bias. We implement two extreme cases: in case (1) the bias fluctuates randomly around zero and in case (2) the bias fluctuates around a non-zero value, staying significantly non-zero. In case (1) the cosmological parameters are practically unaffected, while in case (2) the peaks of the cosmological parameter distributions are shifted within 1σ or 2σ of the WMAP values. In particular, r is no longer detected in case (2).

The best and mean values reported here for r and the other cosmological parameters do not correspond to the true sky data but to the mock skies generated from the MCMC simulations as explained above. Nevertheless, the deviations between the best and fiducial values are relevant indicators for r as well as for the lower and upper bound and the standard deviation. The fact that the fiducial and mean values of r are very close and that Δ_{max} coincides with the mean value of the standard deviation of r indicates that Planck can provide detections of high quality.

More generally, our results support the quest for B-mode polarization in the current CMB data and future B-oriented polarization missions under study by both ESA\(^{10}\) and NASA\(^{11}\) (De Bernardis et al. 2009; Bock et al. 2006).

2. FITTING CURRENT CMB + LSS DATA WITH THE GINZBURG–LANDAU EFFECTIVE THEORY OF INFLATION

As discussed in the introduction, the effective theory of inflation within the G–L approach gives precise order of magnitude estimates for the spectral index n_s, the ratio of tensor-to-scalar fluctuations r, and the running of the spectral index dn_s/d ln k (Boyanovsky et al. 2009).

Within the context of the G–L effective theory of inflation, the work in Boyanovsky et al. (2006), Destri et al. (2008a, 2008b, 2009), and Boyanovsky et al. (2009) showed that

1. The small inflaton self-coupling arises naturally as the ratio of the inflation energy scale and the Planck energy. The inflaton mass is small compared with the inflation energy scale.
2. The amplitude of the CMB anisotropies sets the energy scale of inflation to be \( M \sim 10^{16} \) GeV for all generic slow-roll inflationary potentials.

\(^{10}\) http://www.b-pol.org/index.php
\(^{11}\) http://cmbpol.uchicago.edu/
3. Double-well inflaton potentials give the best fit to CMB+LSS data. Basically, the inflaton potential must have a negative second derivative at horizon exit which favors double-well potentials over single-well potentials.

4. For double-well quartic inflaton potentials, the best value for the tensor-to-scalar ratio fluctuations is \( r \simeq 0.05 \) with the lower bound \( r > 0.023 \) (95\% CL) in the case of the quartic double-well potential. The novelty in the MCMC analysis of the CMB+LSS data that leads to these results is in the fact that we imposed the analytical expressions for \( n_s \) and \( r \) derived from the inflaton potential as a hard constraint (Destri et al. 2008a).

5. Higher order double-well inflaton potentials are investigated in Destri et al. (2009). All \( r = r(n_s) \) curves for double-well even potentials of high order fall inside a universal "banana-shaped" region \( B \) (Figure 1).

The fourth-order binomial potential provides the simplest double-well potential best reproducing the CMB+LSS data within the G–L effective theory approach:

\[
V(\varphi) = \frac{\lambda}{4} \left( \varphi^2 - \frac{m^2}{\lambda} \right)^2, \quad \lambda = \frac{y}{8N} \left( \frac{M}{M_{\text{Pl}}} \right)^4, \quad m = \frac{M^2}{M_{\text{Pl}}},
\]

\[ (1) \]

where \( \varphi \) is the inflaton field, \( \lambda \) stands for the quartic coupling (\( y \) being the corresponding coupling of order one), and \( m \) is the inflaton mass. Note that the quartic coupling \( \lambda \) is proportional to the ratio \( M/M_{\text{Pl}} \) to power 4 and hence very small for all the inflaton self-couplings as stated above.

Adding higher order terms with additional parameters does not significantly improve the fits (Destri et al. 2009).

In Destri et al. (2009), it is found that the \( r = r(n_s) \) curves for double-well inflaton potentials in the G–L spirit fall inside the universal banana region \( B \) depicted in Figure 1.

The lower border of the universal region \( B \) is particularly relevant since it gives a lower bound for \( r \) for each observationally allowed value of \( n_s \). For example, from Figure 1 the best value \( n_s = 0.964 \) implies that \( r > 0.021 \). The upper border of the universal region \( B \) tells us the upper bound \( r < 0.053 \) for \( n_s = 0.964 \). Therefore, we have within the large class of potentials inside the region \( B \)

\[
0.021 < r < 0.053 \quad \text{for} \quad n_s = 0.964.
\]

Moreover, the fourth-order double-well potential represented by Equation (1) is the simplest and G–L stable inflaton potential reproducing very well the present CMB+LSS data.

Without using the G–L approach, lower bounds for \( r \) are not obtained. Kinney et al. (2008) and Peiris & Easther (2008) do not use the G–L approach. As a result, they do not find lower bounds for \( r \), and cannot exclude values for \( r \) much smaller than \( r = 0.0427 \).

It must be noted that our present analysis shows that values \( r < 0.0427 \) (and hence very small \( B \) modes) are outside the possibility of detection by Planck.

Future improvements on the precise value of \( n_s \) alone will immediately give an improvement on the theoretical prediction for \( r \) as well as for its lower bound. An improvement \( \delta \) on the precision of \( n_s \) implies an improvement

\[
\frac{dr}{dn_s} \delta
\]

on the precision of \( r \). According to Destri et al. (2009), at \( n_s = 0.964 \) from \( r = r(n_s) \), we have

\[
\frac{dr}{dn_s} = 4.9 \quad \text{on the upper border of} \quad B
\]

(fourth-degree double well) and

\[
\frac{dr}{dn_s} = 1.35 \quad \text{on the lower border of} \quad B.
\]

Better values for \( n_s \) will thus improve the prediction on \( r \) from the \( T, TE \), and \( E \) modes while a secure detection of \( B \) modes is still lacking.

3. PLANCK SENSITIVITY

In this study, we exploit the Planck sensitivity and resolution at its three cosmological channels, i.e., at the frequencies of 70, 100, and 143 GHz.

Table 1 reports the Planck performance at these frequencies, based on the Planck Collaboration (2006) but consistent with the most recent pre-launch measurements of the HFI channels at 100 GHz and 143 GHz (Lamarre et al. 2010; Maffei et al. 2010), and, for the LFI channel at 70 GHz, as updated in Mandolesi et al. (2010), Bersanelli et al. (2010), and Sandri et al. (2010).

Note that the LFI sensitivity reported here also includes the fluctuations of the 4K reference load, since it is obtained on ground-based calibration performed under realistic conditions. The resolution at the various frequencies comes from accurate optical simulations. Also note that these numbers are likely conservative, i.e., in principle a further refinement of tuning could return into an improvement of in-flight sensitivity. About four surveys have been adopted in this work.

4. RESIDUALS FROM SYSTEMATIC EFFECTS AND FOREGROUND: TOY MODEL

The sensitivities presented above do not include the degradation in accuracy that could come from various sources of systematic effects, of both instrumental and/or astrophysical origin.

In this section, we discuss the current estimates publicly available for the systematic effects affecting Planck measurements.

4.1. Stray Light

Planck achieves very good side-lobe rejection thanks to its telescope design (Sandri et al. 2004, 2010; Maffei et al. 2010; Tauber et al. 2010). In spite of this, the main source of contamination at large angular scales or at low multipoles comes from the so-called stray-light effect, i.e., the signal entering in the lobes at various angular distances from the main beam. It can be distinguished in stray light from the intermediate beam, i.e., at angular distance of a few degrees from the main beam, and from the far beam, i.e., at an angular distance from the main beam some degrees larger. The main sources of stray light are the Galactic emission and the CMB dipole.

The stray light from the intermediate beam introduces a sort of smearing of signal around that observed by the main beam. Detailed studies show that it is important only close to the Galactic plane, a region typically excluded from scientific measurements. However, Galactic emission may create a sort of smearing of signal around that observed by the main beam. Detailed studies show that it is important only close to the Galactic plane, a region typically excluded from scientific measurements. However, Galactic emission may create a sort of smearing of signal around that observed by the main beam.
Note that if the optical behavior is well known it is possible to subtract this effect from the data to high precision by simply evaluating it on the observed sky by means of convolution codes taking into account the effective observational strategy with the only (small) limitaiton introduced by the receiver noise. In practice, this correction is limited by the accuracy in the knowledge of optical behavior. For numerical estimates we will assume an effective uncertainty of $\sim 30\%$ in the beam response in the side lobes, implying that the amplitude of the spurious effect remaining in the data is about one order of magnitude smaller than the original effect.

Therefore, assuming the (conservative) side-lobe levels computed for Planck at frequencies of 70 and 100 GHz, a reasonable estimate for the residual stray light from the Galaxy, rescaled from the computations carried out for the original effect (Burigana et al. 2001b, 2004; Sandri et al. 2010), is

$$C_\ell \frac{\ell (\ell + 1)}{2\pi} \sim 8 \times 10^{-4} \mu K^2 \quad \text{for} \quad \ell \leq 10$$

and

$$C_\ell \frac{\ell (\ell + 1)}{2\pi} \sim 2.5 \times 10^{-4} \mu K^2 \quad \text{for} \quad \ell \geq 11.$$  

Note that at frequencies $\nu \geq 70$ GHz dust emission is the main Galactic diffuse foreground, while at 30 and 44 GHz free–free and synchrotron emission are also relevant. Of course, the stray-light effect is larger at 30 and 44 GHz but we could neglect those frequencies in this study for cosmological parameter estimation. Assuming similar side-lobe levels at 143 GHz, given the typical dust frequency behavior at millimeter wavelengths $T_\nu \propto \nu^{-2}$ for $\ell \geq 10$, and $T_\nu \propto \nu^{-4}$ for $\ell \leq 10$, and

$$C_\ell \frac{\ell (\ell + 1)}{2\pi} \sim 3.2 \times 10^{-3} \mu K^2$$

for $\ell \leq 10$, and

$$C_\ell \frac{\ell (\ell + 1)}{2\pi} \sim 1 \times 10^{-3} \mu K^2$$

for $\ell \geq 11$.

The other relevant source of stray-light contamination is the CMB dipole (Burigana et al. 2006; Gruppuso et al. 2007). Note that, for symmetry reasons, this effect is significant only at even multipoles while it is negligible in practice at odd multipoles. Again, rescaling the results obtained for the original effect, we derive a suitable range for the estimate of the residual contamination

$$C_\ell \frac{\ell (\ell + 1)}{2\pi} \sim 0.016 \div 0.16 \mu K^2$$

for even multipoles (with a typical value of $0.048 \mu K^2$) and

$$C_\ell \frac{\ell (\ell + 1)}{2\pi} \simeq 0$$

for odd multipoles. The larger values apply at lower multipoles (up to about 10) and the lower ones apply at higher multipoles. Although the exact value depends on the particular considered receiver, we assume these estimates are constant with frequency, as in the case of side-lobe levels approximately constant with frequency, the CMB signal frequency being independent (in equivalent thermodynamic temperature).

Note that, at frequencies of 70–143 GHz, for even multipoles dipole stray light is larger than Galactic stray light.

In general, the contamination from stray light can be modeled to first approximation as an additional spurious excess of power. In principle, one could also include perturbations of the above estimates multipole by multipole (or multipole band by multipole band) to avoid a modeling in terms of a simple analytical form for the spurious additional power.

The stray-light effect in polarization mainly depends on the (non-perfect) balance of the stray light in total intensity in the coupled receivers used to extract the $Q$ and $U$ Stokes parameters (Burigana et al. 2004). On the basis of optical simulations, we could assume relative differences of few tens of percent, which should reduce about one order of magnitude of the original effect with respect to that in total intensity. On the other hand, the modeling and verification of optics in polarization are much more complex than in total intensity. We then expect that it will be more difficult to use optical predictions to subtract this effect into the data and assume a residual effect similar to the original one. Therefore, we estimate that the amplitude of the residual effect in polarization will be similar to that in total intensity.

4.2. Main Beam Asymmetry, Leakage, Time Constants, and Glitches

Another potential systematic effect that needs to be kept under control is the effect of the antenna beam profile in the main beam. Beam profiles exhibit a deviation from perfect circular symmetry, in the range from a few percent up to $\sim 30\%$ in the case of the lowest frequency channels (Sandri et al. 2010; Maffei et al. 2010). Main beam distortions are in principle a source of concern as they can bias the estimated power spectra in the high $\ell$ regime, and hence affect the likelihood models and cosmological parameters. This happens for two reasons. Planck’s scanning strategy is not isotropic but has a preferred direction, roughly coincident with ecliptic meridians. First, this fact makes the $\ell$ space equivalent window function of the beam rather non-trivial and difficult to estimate analytically (though approximate analytical solutions do exist, see, e.g., Fosalba et al. 2002). Second, and more important, Planck estimates the Stokes linear polarization parameters by combining measurements taken from different detectors. The beam asymmetry renders the contribution to the intensity $I$ unbalanced even when the same pixel is observed, because of the different orientation and shape of the beams. In turn, this can create $I$ to $Q$, $U$ leakage (Ashdown et al. 2009) and produce biases in the polarization power spectra.

Fortunately, the beam profiles for Planck have been measured very well during ground testing campaigns and will be cross checked in flight (see Burigana et al. 2001a; Naselsky et al. 2007; Huffenberger et al. 2010, and references therein). Furthermore, analytical and semi-analytical machinery exist to estimate the $\ell$ equivalent window function asymmetric beams, once a beam profile is known and a scanning strategy assumed. These methods compute, for each multipole $\ell$, the beam coupling matrix between all power spectra (thus taking leakage into account), starting from an approximate model of the scanning strategy (Ashdown et al. 2009) that can be refined performing signal-only Monte Carlo simulations for the CMB component (M. A. J. Ashdown et al. 2010, in preparation). While a thorough analysis of the accuracy of these procedures has not been performed yet, it is fair to expect that main beam distortions will not be a major source of systematic contamination for Planck (see, e.g., Rocha et al. 2010).
Not even a satellite experiment such as Planck can safely assume to use the entire sky for CMB analysis. Incomplete sky coverage can induce leakage of the $E$ polarization modes into $B$ modes if a sub-optimal power spectrum estimator is employed. While this effect is not connected to the beam, but rather is of geometrical origin, it is worth mentioning here because it may trigger spurious detection of $B$ modes. In a realistic analysis, the leakage effect is corrected from the beginning by using pseudo-$C_{\ell}$ methods (Hivon et al. 2002) which are the standard choice for power spectrum estimation in the high $\ell$ ($\gtrsim 30$). Pseudo-$C_{\ell}$ methods correct for leakage by means of coupling kernels; in particular, the $E$ and $B$ mode pseudo-spectra exhibit correlations that need to be accounted for (see, e.g., Appendix A in Kogut et al. 2003). At low multipoles, pixel-based methods are normally used to directly compute the likelihood function without assuming power spectrum estimation as an intermediate step. Pixel-based methods do not suffer from leakage (Tegmark & de Oliveira-Costa 2001).

The bolometric detectors of HFI exhibit a non-trivial transfer function that distorts the signal both in amplitude and in phase. Qualitatively, the effect on amplitude is akin to a first-order low-pass filter arising from the detector’s intrinsic time constant modified by electrothermal effects (Lamarre et al. 2010). Knowledge of the filter function allows one to deconvolve its effect on timelines at the price of (slightly) increasing and distorting the high-frequency noise level, which becomes non-white (“colored noise”). These measurements will be performed in flight (Lamarre et al. 2010). Any residual error would have an impact similar to beam smearing along the scan direction, so it contributes to beam asymmetry which, as stated above, can be accounted for with high confidence.

Another potential source of concern for bolometric detectors is that they are sensible to cosmic-ray hits that create glitches in the timeline, i.e., they are always seen as positive spikes in the bolometer signal (Lamarre et al. 2010), followed by a tail also due to the bolometer’s time constant. These events can be detected and flagged on the timelines. Residual effects (due to undetected glitches) can be kept under control (Masi et al. 2010) for the cosmological analysis by relying on angular power spectra obtained cross-correlating different detectors (also known as cross-spectra, see, e.g., Polenta et al. 2005).

4.3. Residuals from Foreground

The most important source of contamination in a CMB experiment like Planck will come from the residual of astrophysical foregrounds. In fact, although the wide Planck frequency coverage, possibly complemented by WMAP maps and balloon-borne experiment data, is particularly advantageous for a precise removal of astrophysical signals from the maps and the accurate mapping of CMB anisotropies, nevertheless we expect that a certain level of residual contamination will remain into CMB maps, particularly in polarization. Many methods of component separation, each with its own pros and cons, have been and are continuously elaborated for the analysis of Planck multifrequency maps (see Leach et al. 2008 and references therein). In the present work, we are interested in the residuals from astrophysical foregrounds affecting the recovery of the CMB angular power spectrum. It is typically given as a difference between the input CMB angular power spectrum and the CMB angular power spectrum estimated after the component separation layer. In general, it is not so meaningful to provide a description of different foreground residuals at different frequencies, since, by definition, the component separation layer exactly exploits the multifrequency mapping of the sky. Thus, the estimate adopted in this work has to apply to the whole set of frequency channels.

Different methods show different residuals at various ranges of multipoles. The multipole dependence, or, in other words, the shape of this residual also depends on the method considered. Concerning residuals for the $T$ mode, recent simulations (Leach et al. 2008) show residual shapes only slightly dependent on the multipole, with amplitudes in the range

\[
\frac{C_{\ell}}{2\pi} \sim \frac{10^2}{2\pi} \mu K^2,
\]

the exact value depending on the method and, for each method, on the particular multiple band, with typical variations of about 30%-40%. In this work, we model this spurious power as flat in $C_{\ell} (\ell + 1)$.

Galactic polarized foreground (mainly from diffuse synchrotron and dust emission) affects CMB angular power spectrum recovery more significantly in polarization than in temperature. We expect that their residuals after component separation will partially take memory of the original shape of the foreground power spectrum, in particular at large scales where they show much more power than the CMB. Again, different methods give different residuals, regarding both multipole dependence and amplitude. In this work, we model the foreground residual for the $TE$ mode and $E$ and $B$ modes (assumed to be equal, $B = E$) as the sum of two shapes, the first one (dominant at low multipoles) described by the foreground shape properly rescaled in amplitude, and the second one constructed from the foreground shape properly rescaled in amplitude and changed in slope.

Figure 2 displays our “starting” conservative models for the residuals in the $TE$ mode and in polarization modes. We have the freedom to simply rescale them with multiplicative factors in order to address the typical level of foreground residuals for which the impact on our cosmological aim is not critical (see also Section 4.5).
4.4. Additional Noise Versus Bias

All the systematic effects discussed above, coming from an instrumental effect, sky signal, or from their coupling, can be considered in two different schemes.

In the first, simplest case, they can be treated as sources of spurious additional noise power, i.e., they do not introduce a bias affecting the recovery of the estimation of the CMB angular power spectrum but they increase our uncertainty in its recovery. Therefore, the effect can be modeled adding in quadrature the quoted $C_\ell$ of the power of the residual systematics to those coming from sensitivity, resolution, and cosmic plus sampling variance. This approach is equivalent to assuming that we will be able to properly model and subtract a correct estimation for the systematic effects so that only a statistical uncertainty in their subtraction will affect the data.

In another more critical approach, one can assume to miss the correct estimation of the spurious effects. Their systematic effects will then be much more dramatic, i.e., they will also introduce a bias in the estimation of the CMB angular power spectrum. This case can be modeled as “perturbing” the $C_\ell$ linearly adding the additional spurious power as described above. It is this sum that will be compared with the exact model.

4.5. Parametric Approach to Systematic Effects

In general, in this work we do not use a precise (still not completely available) description of the considered systematic effects, but only suitable representations of them. Therefore, we will use our estimations to understand if the considered classes of systematic effects may significantly affect the cosmological exploitation of Planck data with respect to the determination of cosmological parameters possibly by rescaling the estimation quoted above. This is done with the aim of identifying the corresponding levels at which it is necessary to control the systematic effects in order to avoid spoiling the scientific accuracy of the Planck data. We technically implement this rescaling with a multiplicative constant on the residuals of systematic effects on the $C_\ell$ described in the previous sections.

5. MOCK DATA PRODUCTION AND LIKELIHOODS

In this section, we describe the theoretical basis for our simulations when experimental errors are treated as statistical noise. This includes the instrumental white noise as well as the residuals from systematic errors and foregrounds as described in the previous section. In other words, we assume that the noise contribution to the observed CMB skies due to systematic errors can be precisely assessed, thanks to suitable procedures such as cleaning simulations in the case of foreground residuals.

Let us denote the fiducial theoretical multipoles as $\hat{C}_\ell^{T}$, $\hat{C}_\ell^{E}$, $\hat{C}_\ell^{B}$, $\hat{C}_\ell^{N_T}$, and the (possibly $\ell$-dependent) noise covariances as $N_\ell^{TT}$, $N_\ell^{EE}$, and $N_\ell^{BB}$. For instance, in the case of foreground residuals, we would have

$$N_\ell^{XX} = w_\ell R_\ell^{XX} + n_\ell^{XX}, \quad X, X' = T, E, B,$$

where $w_\ell$ is the window function in multipole space, $n_\ell^{XX}$ is the white instrument noise, and $R_\ell^{XX}$ are appropriate quantities which can be estimated through map cleaning simulations. In any case, here we assume that $N_\ell^{BB} = N_\ell^{EE}$ and that $N_\ell^{TE} = N_\ell^{BT} = 0$.

Thus, the full covariances of the $T-E$ fluctuations read

$$\begin{pmatrix} w_\ell \hat{C}_\ell^{TT} + N_\ell^{TT} & w_\ell \hat{C}_\ell^{TE} \\ w_\ell \hat{C}_\ell^{ET} & w_\ell \hat{C}_\ell^{EE} + N_\ell^{EE} \end{pmatrix} = R_\ell \begin{pmatrix} \hat{C}_\ell \delta_{\ell\ell} & 0 \\ 0 & \hat{C}_\ell \end{pmatrix} R_\ell^t,$$

where $R_\ell$ are suitable rotation matrices and $R_\ell^t$ stands for the transposed matrix of $R_\ell$.

The $B$ fluctuations are decoupled, i.e., $\hat{C}_\ell^{BB} = \hat{C}_\ell^{BT} = 0$, and have full covariance $w_\ell \hat{C}_\ell^{BB} + N_\ell^{BB}$.

With these notations, a possible observed set of fluctuation amplitudes reads

$$\begin{pmatrix} A_\ell^T \\ A_\ell^E \end{pmatrix} = R_\ell \begin{pmatrix} \sqrt{C_\ell^{TT}} g_\ell^{Tm} \\ \sqrt{C_\ell^{EE}} g_\ell^{Em} \end{pmatrix}, \quad A_\ell^B = \sqrt{w_\ell \hat{C}_\ell^{BB} + N_\ell^{BB}} g_\ell^{Bm},$$

where $g_\ell^{Xm}, X \equiv +, -, B$, are independent centered unit Gaussians, that is,

$$\{g_\ell^{Xm}\} = 0, \quad \{g_\ell^{Xm} g_\ell^{X'm}\} = \delta_{\ell\ell'} \delta_{mm'} \delta^{XX'}.$$

The amplitudes $A_\ell^X$ are assumed to be real (which is always possible for integer weights $\ell$).

The corresponding observed multipoles $\bar{C}_\ell$ (sometimes called pseudo-$C_\ell$) are

$$\bar{C}_\ell^{XX'} = \frac{1}{2\ell + 1} \sum_m A_{\ell m}^{X'} A_{\ell m}^X.$$

Now consider the multipoles $C_\ell^{XX'}$ produced by a test cosmological model. In the approximation exploited in this work (all-sky coverage and uniform sensitivity), the likelihood of such multipoles, given the observed $\bar{C}_\ell^{XX'}$, can be written as

$$L = \exp \left( -\frac{1}{2} \chi_\ell^2 T,E - \frac{1}{2} \chi_\ell^2 B \right),$$

where

$$\chi_\ell^2 T,E = \sum_\ell (2\ell + 1) \left[ \text{tr} \left( C_\ell^{-1} \bar{C}_\ell \right) - \log \frac{\text{det} C_\ell}{\text{det} \bar{C}_\ell} - 2 \right]$$

$$\chi_\ell^2 B = \sum_\ell (2\ell + 1) (x_\ell - \log x_\ell - 1)$$

and $C_\ell$, $\bar{C}_\ell$ are the $2 \times 2$ matrices

$$C_\ell = \begin{pmatrix} w_\ell \hat{C}_\ell^{TT} + N_\ell^{TT} & w_\ell \hat{C}_\ell^{TE} \\ w_\ell \hat{C}_\ell^{ET} & w_\ell \hat{C}_\ell^{EE} + N_\ell^{EE} \end{pmatrix},$$

$$\bar{C}_\ell = \begin{pmatrix} \hat{C}_\ell^{TT} & \hat{C}_\ell^{TE} \\ \hat{C}_\ell^{ET} & \hat{C}_\ell^{EE} \end{pmatrix},$$

while

$$x_\ell = \frac{\hat{C}_\ell^{BB}}{w_\ell \hat{C}_\ell^{BB} + N_\ell^{BB}}.$$
matrices \( \hat{R}_\ell \) and eigenvalues \( \hat{C}_\ell \) so that now only the \( T-E \) signal covariances are diagonalized:

\[
\begin{pmatrix}
\hat{C}_{TT}^\ell & \hat{C}_{TE}^\ell \\
\hat{C}_{ET}^\ell & \hat{C}_{EE}^\ell
\end{pmatrix} = \hat{R}_\ell \begin{pmatrix}
\hat{C}_\ell^+ & 0 \\
0 & \hat{C}_\ell^-
\end{pmatrix} \hat{R}_\ell^\dagger.
\]

Then, we double all Gaussian extractions by writing the fluctuations as

\[
\begin{align*}
A_{Tm}^T &= \sqrt{w_\ell} a_{Tm}^T + \sqrt{N_{TT}^\ell} h_{Tm}^y, \\
A_{Em}^T &= \sqrt{w_\ell} a_{Em}^T + \sqrt{N_{EE}^\ell} h_{Em}^y, \\
A_{Bm}^T &= \sqrt{w_\ell} a_{Bm}^T + \sqrt{N_{BB}^\ell} h_{Bm}^y,
\end{align*}
\]

where \( T, E, \) and \( B \) stand for temperature, \( E \) polarization, and \( B \) polarization, respectively, and the amplitudes \( a_{Tm} \) are given by

\[
\begin{pmatrix}
a_{Tm} \\
a_{Em} \\
a_{Bm}
\end{pmatrix} = \hat{R}_\ell \begin{pmatrix}
\hat{C}_\ell^{-1/2} \\
\hat{C}_\ell^{-1/2} \\
\hat{C}_\ell^{-1/2}
\end{pmatrix} \begin{pmatrix}
\tilde{C}_\ell \\
\tilde{C}_\ell \\
\tilde{C}_\ell
\end{pmatrix},
\]

The new independent centered unit Gaussians \( h_{Tm}^y, Y \equiv T, E, B \), are independent of the previous set \( g_{Tm}^X, X \equiv +, -, B \).

The pseudo-C\( \ell \) (that is \( \tilde{C}_\ell \)) can now be written as

\[
\tilde{C}_\ell^{XX} = w_\ell \tilde{C}_\ell^{XX} + 2 \sqrt{w_\ell} N_{XX}^\ell Q_{XX}^{\ell} + N_{XX}^{\ell} P_{XX}^{\ell},
\]

\[
X = T, E, B
\]

\[
\hat{C}_\ell^{TE} = w_\ell \tilde{C}_\ell^{TE} + \sqrt{w_\ell} N_{EE}^\ell Q_{EE}^{\ell} + \sqrt{w_\ell} N_{TT}^\ell Q_{TT}^{\ell} + \sqrt{w_\ell} N_{EE}^\ell P_{TE}^{\ell},
\]

where

\[
\begin{align*}
\tilde{C}_\ell^{XX} &= \frac{1}{2\ell + 1} \sum_m a_{Tm}^X h_{Tm}^Y, \\
Q_{XX}^{\ell} &= \frac{1}{2\ell + 1} \sum_m a_{Em}^X h_{Em}^Y, \\
P_{XX}^{\ell} &= \frac{1}{2\ell + 1} \sum_m a_{Bm}^X h_{Bm}^Y,
\end{align*}
\]

with \( X, Y \equiv T, E, B \).

This second approach allows us to use the same sky and different noises, which is needed when cumulative channels are considered to reduce noise effects. For instance, a cumulative channel formed by the LFI at 70 GHz and the two HFI channels at 100 GHz and 143 GHz is obtained by simply summing the different channels. For instance, \( \chi^2 \) of Equation (15) relative to these three channels.

The above setup is based on the assumption that the noise contribution of systematic errors is precisely assessed. If this was not the case, bias effects would be induced. This can be simulated in the likelihood \( \chi^2 \) of Equation (15) by using different noises in the \( \tilde{C}_\ell^{XX} \) and in the covariance built with the test multipoles \( C_{XX}^{\ell} \). That is, one should make (small) variations from \( N_{XX}^{\ell} \) to some \( N_{XX}^{\ell} \) in Equation (16) while keeping them fixed in Equation (19) (or vice versa) and study their impact on the parameter determination of the test cosmological model.

6. FORECAST PRECISION OF PLANCK MEASUREMENTS FOR COSMOLOGICAL PARAMETERS WITHOUT SYSTEMATICS

In our MCMC simulations we take as a fiducial model the \( \Lambda \)CDM\( r \) model, which is the standard \( \Lambda \)CDM model augmented by the tensor-to-scalar ratio \( r \) as described in the introduction. We consider MCMC simulations with both the \( \Lambda \)CDM\( r \) and the \( \Lambda \)CDM\( T \) model. We denote by \( \Lambda \)CDM\( T \) the \( \Lambda \)CDM\( r \) model in which we impose the double-well inflaton potential given in Equation (1), as described in the introduction and in Section 2.

We consider two sets of best-fit fiducial values for our parameters, as listed in Table 2, where the values of a few other derived parameters are also shown for illustrative purposes. Since \( r = 0 \) in the first set, the model is just the \( \Lambda \)CDM model. In the second set, the values \( r = 0.0427 \) and \( n_s = 0.9614 \) are chosen to lay on the theoretical curve \( r = r(n_s) \) dictated by the double-well inflaton potential and they correspond to the best-fit value \( \chi^2 = 1.26 \) for the coupling (Destri et al. 2008a) within the G–L effective theory approach (see Equation (1)).

We then provide estimates of the errors in the measurements of the cosmological parameters in the following way.

1. We produce one sky (mock data) for \( C_{TT}^\ell, C_{EE}^\ell, \) and \( C_{BB}^\ell \) from the \( \Lambda \)CDM or \( \Lambda \)CDM\( T \) models (see Table 2) according to the procedure described in Section 5.

2. We run MCMCs from this sky and obtain the marginalized likelihood distributions for the cosmological parameters \( \Omega_0 h^2, \Omega_0 \Delta^2, \theta, \Omega_\Lambda, \) and the Age of the Universe, \( \Omega_0 h, \Omega_0 \Delta, n_s, \) and \( r \) in the two models \( \Lambda \)CDM and \( \Lambda \)CDM\( T \). To be precise, we study the independent \( \Lambda \)CDM\( r \) parameters with the mock data produced from \( \Lambda \)CDM (first row of Table 2) and the independent parameters of both \( \Lambda \)CDM and \( \Lambda \)CDM\( T \) with the mock data produced from \( \Lambda \)CDM/T (second row in Table 2).

We consider two choices for the \( C_\ell \)-likelihood, one without the \( B \) modes and one with the \( B \) modes and take into account the white noise sensitivity of \( \text{Planck} \) (LFI and HFI) in the 70, 100, and 143 GHz channels. We also consider a cumulative channel whose \( \chi^2 \) is the sum of the \( \chi^2 \)s of the three channels above. When using different channels in the MCMC analysis, we use different noise realizations while keeping the same sky, that is the same realization of the Gaussian process that generated the primordial fluctuations.

In our MCMC analysis, we always take standard flat priors for the cosmological parameters. In particular, we assume the flat priors \( \Theta \leq 0.2 \) in the \( \Lambda \)CDM\( r \) model and \( \Theta \leq 0.8/60 \) according to the theoretical upper limit for \( r \) in the \( \Lambda \)CDM\( T \) model.

Our findings are summarized in Figures 3–6 where the marginalized likelihood distributions of the cosmological parameters are plotted for several different setups. In Tables 2–4, we list the corresponding relevant numerical values.

Clearly, in the case of the ratio \( r \), due to the specific form of its likelihood distribution it is more interesting to exhibit upper and lower bounds rather than mean value and standard deviation as in Tables 3 and 4. We report the upper bounds and, when present, the lower bounds in Tables 5 and 6.

Note that the fiducial and mean values reported here for \( r \) and the other cosmological parameters do not correspond to the real sky but to mock MCMC generated skies as explained above. However, the deviations between the best and the fiducial values are relevant indicators as well as the lower and upper bounds on \( r \) and the standard deviations.

The fact that the fiducial and mean values of \( r \) are very close and that the standard deviation \( \Delta r_{\text{max}} \) of the distribution of maximum values \( r_{\text{max}} \) coincides with the mean value of the standard deviation of \( r \) indicate that \( \text{Planck} \) can provide measurements of high quality for \( r \).
Figure 3. Marginalized likelihood distributions, without including $B$ modes, of the cosmological parameters for the $\Lambda$CDM$r$ model. We display the distributions for each of the three channels HFI-100, HFI-143, and LFI-70 and for the cumulative of the three channels. The fiducial values are indicated by a vertical thin black line. The fiducial value for the ratio is $r = 0$ in the upper panel and $r = 0.0427$ in the lower panel. Note that the latter fiducial value is smaller than the peaks of the marginalized distribution. This is due just to statistical fluctuations since we are considering only one sky. The upper panel figures imply the upper bound $r < 0.068$ at a 95% CL without $B$ modes.

As expected from the relative difference in sensitivity, typically the distributions obtained with the HFI-100 and HFI-143 channels agree very well while differing markedly from those obtained with the LFI-70 channel. Quite often the higher noise level in LFI-70 also determines shifts in the peak positions with respect to the other two channels. These shifts are within a $1\sigma$ deviation in the LFI-70 distributions and represent therefore normal statistical fluctuations. The LFI-70 distribution on $r$ when the fiducial value is $r = 0.0427$ does not exhibit a peak due to the sensitivity of this channel.

As expected, whenever the probability distribution for a given parameter is close to Gaussian, the cumulative channel produces a distribution that is narrower than the narrowest distribution produced by any individual channel. This applies to all parameters in the $\Lambda$CDM$r$ model, including $r$, which has a distribution close to a left-truncated Gaussian for both fiducial values used.

In the $\Lambda$CDM$r$T model, the relation between $n_s$ and $r$ is nonlinear and there are theoretical upper limits on $n_s$ and $r$ (see Figure 5). These features introduce non-Gaussianities in the...
distributions and eventually also affect any other cosmological parameter having a sensitive correlation with \( n_s \) and/or with \( r \), such as \( \Omega_b \ h^2 \), \( A_s \), or some other derived parameters. Thus, the cumulative channel provides some parameter distributions which are larger than those of the HFI-143 channel, because it is affected by the LFI-70 channel, which is less sensible to constrain \( r \) well within its theoretical prior. This effect can be very well appreciated from Figure 6 in which the likelihood distributions of the coupling constant \( y \) are plotted for the various cases considered.

The very limited relevance of the \( B \) modes for the \( \Lambda CDMrT \) model is in principle expected because the \( n_s \) value fixed by the \( T \) modes essentially determines \( r \) through the theoretical constraint. This property of the \( B \) modes shows up clearly from the figures and the tables.

6.1. Forecasts of Planck Measurements with the \( \Lambda CDMr \) Model

The obtained best fits, mean values, and standard deviations for the cosmological parameters are presented in Tables 3 and 4 for the two cases considered: without \( B \) modes and with \( B \)
Figure 5. Marginalized likelihood distributions of the cosmological parameters for the $\Lambda$CDM$^r_T$ model in which the double-well inflation theoretical model is imposed. The MCMC analysis includes (does not include) $B$ modes in the lower (upper) panel. We display the distributions for each of the three channels and for the cumulative of the three channels. The fiducial values are indicated by a vertical thin black line. The fiducial value of $r$ is well reproduced by the peak of the $\Lambda$CDM$^r_T$ distribution in the case of HFI-100, HFI-143, and the cumulative of the three channels both with and without $B$ modes. Considerable gain is obtained with respect to the $\Lambda$CDM model. Upper and lower panels show quite similar results showing the stability of the $\Lambda$CDM$^r_T$ model with respect to the inclusion of the $B$ modes. Comparison with the lower panel of Figure 4 shows that considerable gain is obtained with respect to the $\Lambda$CDM model.

(A color version of this figure is available in the online journal.)

From Tables 3–6 we see that for the $\Lambda$CDM$r_T$ model the best result, namely a peaked distribution for $r$, is obtained for a non-zero fiducial value for $r (r = 0.0427)$ and with the $B$ modes included in the likelihood. In this case, upper and lower bounds on $r$ are obtained

$$0.013 < r < 0.045$$

at 95% CL

with the best values $r = 0.0594$ and $n_s = 0.9604$ (without $B$ modes); $r = 0.0240$ and $n_s = 0.9597$ (with $B$ modes).
For a fiducial value \( r = 0 \), with or without \( B \) modes, the \( \Lambda \)CDM\( r \) distributions peak at the value \( r = 0 \), as seen from the upper panels of Figures 3 and 4. Upper bounds on \( r \) are obtained in this case. They result in

\[
\begin{align*}
r < 0.068 & \quad \text{without } B \text{ modes}; \\
r < 0.016 & \quad \text{with } B \text{ modes}
\end{align*}
\]

with the best values to 95% CL; \( r = 0.0041 \) and \( n_s = 0.9549 \) (without \( B \) modes); \( r = 0.001 \) and \( n_s = 0.9606 \) (with \( B \) modes).

The results on \( n_s \) practically do not change whether the \( B \) modes are included or not (compare the upper panels of Figures 3 and 4).

The upper bound on \( r \) and the best value of \( n_s \) do not need the inclusion of \( B \) modes and can be obtained for fiducial \( r = 0 \). These values can be obtained and trusted without including the \( \Lambda \)CDM\( r \) model, the pure \( \Lambda \)CDM\( r \) model is enough to obtain them.

We see a substantial progress in the forecasted bounds for \( r \) with respect to the WMAP+LSS data set for which \( r < 0.20 \) in the pure \( \Lambda \)CDM\( r \) model (Komatsu et al. 2009, 2010). For Planck, with the \( B \) modes included and a non-zero \( r \)-fiducial value, we get peaked distributions for \( r \) with a non-zero most probable value, lower bounds for \( r \) (and an improvement of the upper bound). This is obtained by only using the \( \Lambda \)CDM\( r \) model alone without any input from the inflation model. We see now in the following subsection how these forecasts can still be considerably improved by using the \( \Lambda \)CDM\( r \)T model.

6.2. Forecasts of Planck Measurements with the \( \Lambda \)CDM\( r \)T Model

With the \( \Lambda \)CDM\( r \)T model (namely when the double-well inflaton potential is imposed), with or without \( B \) modes included, well-peaked distributions for \( r \) are obtained together with upper and lower bounds and best \( r \) values. We get a considerable gain for \( r \) with respect to the pure \( \Lambda \)CDM\( r \) model, as can be seen from Tables 3 to 6 and Figure 5.

The fiducial value for \( r (r = 0.0427) \) is well reproduced by the peak of the \( \Lambda \)CDM\( r \)T distribution both with and without \( B \) modes. The \( \Lambda \)CDM\( r \)T distribution for \( r \) peaks at the non-vanishing value theoretically associated with the fiducial value of \( n_s \). We get (at a 95% CL):

\[
0.030 < r < 0.113 \quad \text{(without } B \text{ modes)} \quad \text{and} \quad 0.030 < r < 0.114 \quad \text{(with } B \text{ modes)}
\]

with the best values and 95% CL errors:

\[
\begin{align*}
r & = 0.0463 \pm 0.0231 \quad \text{and} \quad n_s = 0.9625 \pm 0.0035 \quad \text{(without } B \text{ modes)} \\
r & = 0.0405 \pm 0.0230 \quad \text{and} \quad n_s = 0.9608 \pm 0.0033 \quad \text{(with } B \text{ modes)}
\end{align*}
\]

The results with the \( \Lambda \)CDM\( r \)T model practically do not change by including the \( B \) modes or not in the likelihood as can be seen from the figures (compare, for instance, the upper panels of Figures 3 and 4) and from the tables. This is so since the \( \Lambda \)CDM\( r \)T model intrinsically carries a non-vanishing ratio prediction, which shows up in agreement with the obtained marginalized distributions even without the inclusion of \( B \) modes.

6.3. Conclusion

The upper bound on \( r \) and the best value of \( n_s \) do not require us to include the \( B \) modes in the likelihood, and can be obtained with the \( \Lambda \)CDM\( r \) model alone (see Figure 3).

For a fiducial value \( r = 0 \), mock Planck data with the \( \Lambda \)CDM\( r \) model alone, with or without \( B \) modes in the likelihood, provide only upper bounds on \( r \) and most probable values for \( n_s \).

The same conclusions are true for a non-vanishing fiducial value \( (r = 0.0427) \) without \( B \) modes and the \( \Lambda \)CDM\( r \) model alone.

The inclusion of \( B \) modes for a non-vanishing fiducial value \( (r = 0.0427) \) allows peaks marginalized distributions for \( r \) with the \( \Lambda \)CDM\( r \) model alone and a lower bound for \( r \) (see Figures 3–5).

Lower bounds on \( r \) and most probable \( r \) values are always obtained (with or without the \( B \) modes) for the \( \Lambda \)CDM\( r \)T model (see Figure 5).

In all cases the \( \Lambda \)CDM\( r \)T model provides well-peaked distributions for \( r \) on non-zero values \( r \approx 0.04 \), with or without the \( B \) modes included.

In summary, we find that the inclusion of the theoretical model greatly helps the recovery of the \( r \) parameter. We also remark that the model is falsifiable in the case of constraints on \( n_s \) and \( r \) which are not compatible with the banana shape of the framework considered.

Figure 6. Marginalized likelihood distributions of the coupling constant \( y \) of the double-well quartic inflaton potential in the \( \Lambda \)CDM\( r \)T model. The MCMC analysis includes (does not include) \( B \) modes in the right (left) panel. We display the distributions for each of the three channels and for the cumulative of the three channels. The fiducial values are indicated by a vertical thin black line. The fiducial value of \( y \) is relatively well reproduced by the peak of the distribution in the case of the HFI-100, HFI-143, and cumulative channels.

(A color version of this figure is available in the online journal.)
as expected, it does not significantly change the conclusions derived taking the discussion and the toy model presented in Section 4 in which a statistical error. We evaluate these statistical errors following the

We computed the cumulative marginalized likelihoods from

Note. All figures are rounded to order $10^{-4}$ to the nearest value.

| Sky Data | Test Model | ACDM: Fiducial $r = 0$ | ACDM/T: Fiducial $r = 0.0427$ |
|----------|------------|------------------------|-------------------------------|
|          |            | Best | Mean | stddev |          | Best | Mean | stddev |          |
| No residuals | 0.0010 | 0.0060 | 0.0050 | 0.0240 | 0.0275 | 0.0096 | 0.0405 | 0.0516 | 0.0230 |
| Best case smooth | 0.0040 | 0.0222 | 0.0160 | 0.0448 | 0.0504 | 0.0238 | 0.0465 | 0.0516 | 0.0235 |
| Middle case rugged | 0.0024 | 0.0230 | 0.0188 | 0.0431 | 0.0472 | 0.0261 | 0.0344 | 0.0513 | 0.0234 |
| Worst case smooth | 0.0083 | 0.0250 | 0.0160 | 0.0436 | 0.0480 | 0.0275 | 0.0387 | 0.0518 | 0.0250 |

Note. All figures are rounded upward to order $10^{-3}$.

| Sky Data | Test Model | ACDM: Fiducial $r = 0$ | ACDM/T: Fiducial $r = 0.0427$ |
|----------|------------|------------------------|-------------------------------|
|          |            | 68% CL | 95% CL |          | 68% CL | 95% CL |          |
| No residuals | $r < 0.008$ | $r < 0.016$ | $r < 0.032$ | $r < 0.045$ | $r < 0.052$ | $r < 0.114$ |
| Best case smooth | $r < 0.028$ | $r < 0.053$ | $r < 0.062$ | $r < 0.091$ | $r < 0.052$ | $r < 0.115$ |
| Middle case rugged | $r < 0.029$ | $r < 0.058$ | $r < 0.059$ | $r < 0.094$ | $r < 0.052$ | $r < 0.115$ |
| Worst case smooth | $r < 0.032$ | $r < 0.062$ | $r < 0.060$ | $r < 0.097$ | $r < 0.052$ | $r < 0.116$ |

Note. These bounds are assumed significant only when the likelihood at $r = 0$ is less than $\exp(-1/2) = 0.6065 \ldots$ of its maximum for 68% CL or less than $\exp(-1) = 0.3678 \ldots$ for 95% CL. Otherwise the entry is left empty in this table. All figures are rounded downward to order $10^{-3}$.

7. FORECAST PRECISION OF PLANCK MEASUREMENTS FOR THE COSMOLOGICAL PARAMETERS WITH TOY MODEL SYSTEMATICS

7.1. Foreground Residuals Without Bias

We computed the computed marginalized likelihoods from the three channels including foreground residuals for the cosmological parameters in the ACDM$\tilde{r}$ models with B modes and fiducial ratios $r = 0$ and $r = 0.0427$.

The foreground residuals are introduced as an additional statistical error. We evaluate these statistical errors following the discussion and the toy model presented in Section 4 in which a worst case scenario for considering the residuals is derived (as well as a best case scenario, and an intermediate or middle case scenario for the residuals).

We plot the likelihoods for the cumulative of the three channels in four cases (see Figures 7 and 8):

1. without residuals;
2. with 30% of the toy model residuals in the $TE$ and $E$ modes displayed in Figure 2 and $16 \mu K^2$ in the $T$ modes;
3. with the toy model residuals in the $TE$ and $E$ modes displayed in Figure 2 and $160 \mu K^2$ in the $T$ modes;
4. with 65% of the toy model residuals in the $TE$ and $E$ modes displayed in Figure 2 and $88 \mu K^2$ in the $T$ modes rugged by Gaussian fluctuations of 30% relative strength.

The likelihood distributions with and without B modes are almost the same when including the residuals. Only the cosmological parameters sensitive to the B modes do appear to be affected by the residuals, namely, $\tau$, $z_{\text{re}}$, and $r$. This is so in the ACDM$\tilde{r}$ model in which for a fiducial value $r = 0$, the upper bound in $r$ does change by including the residuals (see Figure 7).

This change is smaller for a fiducial ratio $r = 0.0427$. In this case, the presence of a lower bound for $r$ (at 68% CL) remains even by including the residuals. The lower bound remains at 95% CL in the best case smooth residuals. The main numbers are displayed in Tables 7–9.

The $r$ distribution for fiducial ratio $r = 0.0427$ is more clearly peaked on a value of $r$ near the fiducial one in the ACDM$\tilde{r}$ model than in the ACDM$\tilde{T}$ model (compare Figures 7 and 8).
In any case, the best value for $r$ in the presence of residuals is about $r \simeq 0.04$ (near the fiducial value) for both the $\Lambda$CDM$_r$ and the $\Lambda$CDM$_T$ models.

The $\Lambda$CDM/T model turns out to be robust; it is very stable with respect to the inclusion of the residuals as its marginalized likelihood distributions do not change (and we have seen in Section 5 that they also do not change whether the $B$ modes are excluded or not). The main numbers are included in Tables 7–9.

We see in the $\Lambda$CDM$_r$ model that we again have for $r$ at a 95% CL:

$$0.028 < r < 0.116$$

with the best value $r = 0.04$.

In summary, foreground residuals only affect $B$ modes and therefore only the cosmological parameters sensitive to $B$ modes are affected.
We use the following method.

1. We extract $10^5$ skies obtaining the corresponding multipoles $A_{lm}$ from the $\Lambda$CDM/T model according to the procedure described in Section 5. We choose $r = 0.0427$ as a fiducial value.
2. We compute all the corresponding likelihood profiles only for $r$. That is, we freeze out all the other parameters to their fiducial values.
3. We compute the interesting properties of each likelihood profile, like the most likely value $r_{\text{max}}$, the mean value $r_{\text{mean}}$, the standard deviation $\Delta r_{\text{max}}$ of the $r_{\text{max}}$ distribution, the skewness, and the kurtosis. This last measures the departure from a Gaussian likelihood.
4. We finally compute the 99% CL, 95% CL, and 68% CL lower bounds for $r$: $r_{99}$, $r_{95}$, and $r_{68}$.

In the five panels of Figure 9, we plot the likelihood profiles for the different skies, $r_{\text{max}}$ and $\delta r_{\text{max}} \equiv r_{\text{max}} - r_{\text{mean}}$. Note that $r_{\text{max}}$ is an unbiased estimator of the true value, since its expectation value $r_{\text{mean}}$ throughout many skies coincides with the fiducial $r$. $\Delta r_{\text{max}}$ is the standard deviation of the $r_{\text{max}}$ distribution. We find that $\Delta r_{\text{max}}$ always agrees extremely well with the mean value of the standard deviation of $r_{\text{max}}$ in each likelihood profile for each different sky. This fact means that asymptotically for a large number of skies the width of the $r$ profile is an unbiased estimator of the actual uncertainty in $r$.

All these results were obtained for a level of foreground residual equal to 30% of the toy model displayed in Figure 2.

We plot in Figure 10 the 99% CL, 95% CL, and 68% CL lower bounds for $r$: $r_{99}$, $r_{95}$, and $r_{68}$, respectively, as functions of the fraction of foreground residual of the worst case. These lower bounds are consistent, since they fail more or less 99%, 95%, and 68% of the sky extractions. This last property is true only if the prior $r > 0$ is not enforced in the likelihood. That is why we get a non-zero likelihood on negative values of $r$. Of course, only positive values of $r$ are meaningful and this allows us to define the probability of detection of $r$, to a 99% CL, 95% CL, and 68% CL, as the fraction of skies which gives a positive 99% CL, 95% CL, and 68% CL lower bounds, respectively.

These probabilities of detection are displayed in Figure 10. At the level of foreground residual equal to 30% of the considered toy model, only a 68% CL (1$\sigma$) detection is very likely. For a 95% CL detection (2$\sigma$), the level of foreground residual should be reduced to 10% or lower of the toy model displayed in Figure 2.

Lensing was not considered in the analysis on $r$-detection probability. Residuals and lensing affect the detection of $B$ modes in complementary ways, with the effect of residuals being stronger than that of lensing. As a consequence, lensing plus residuals can spoil the detection of $r$ even when residuals are assumed at the 30% level of the toy model displayed in Figure 2. In contrast, lensing in the absence of residuals still allows a detection of $r$. For example, several MCMC simulations show that our lower bounds on $r$ are not significantly affected by lensing in the absence of foreground residuals. It should be clear that if the theoretical constraint $r = r(n_s)$ of the $\Lambda$CDM/T model is imposed on the MCMC analysis, $r$ always has well-defined lower bounds regardless of lensing and/or residuals.

The forecasted probability of detecting $r$ is based on the statistics of the shape of the $r$-likelihood. This shape determines whether a detection of $r$ can be claimed with a given confidence.

---

**Figure 8.** Cumulative marginalized likelihoods from the three channels for the cosmological parameters for the $\Lambda$CDM/T model including $B$ modes and fiducial ratio $r = 0.0427$ and the foreground residuals. We plot the cumulative likelihoods in four cases: (a) without residuals, (b) with 0.3 of the worst case residuals in the $TE$ and $E$ modes and $16 \mu K^2$ in the $T$ modes, (c) with the worst case residuals in the $TE$ and $E$ modes and $160 \mu K^2$ in the $T$ modes, and (d) with 65% of the toy model residuals in the $TE$ and $E$ modes displayed in Figure 2 and $88 \mu K^2$ in the $T$ modes rugged by Gaussian fluctuations of 30% relative strength.

(A color version of this figure is available in the online journal.)

7.2. Probability to Detect $r$ from $B$ Modes in the HFI-143 Channel

Figures 9 and 10 describe the probability of the detection of $r$ from $B$ modes in the HFI-143 channel.

In order to assess the probability for Planck to detect $r$, we consider the $B$ mode detection by the most sensitive HFI-143 channel. We use the following method.

1. We extract $10^5$ skies obtaining the corresponding multipoles $A_{lm}$ from the $\Lambda$CDM/T model according to the procedure described in Section 5. We choose $r = 0.0427$ as a fiducial value.
2. We compute all the corresponding likelihood profiles only for $r$. That is, we freeze out all the other parameters to their fiducial values.
3. We compute the interesting properties of each likelihood profile, like the most likely value $r_{\text{max}}$, the mean value $r_{\text{mean}}$, the standard deviation $\Delta r_{\text{max}}$ of the $r_{\text{max}}$ distribution, the skewness, and the kurtosis. This last measures the departure from a Gaussian likelihood.
4. We finally compute the 99% CL, 95% CL, and 68% CL lower bounds for $r$: $r_{99}$, $r_{95}$, and $r_{68}$.
Figure 9. Upper left panel: the likelihood profiles for the different skies. Upper right panel: $r_{\text{max}}$ and $\delta r_{\text{max}} \equiv r_{\text{max}} - r_{\text{mean}}$ and the fiducial $r$, $r_{\text{fid}}$. Lower left panel: the skewness and the kurtosis. Lower middle panel: the 99% CL, 95% CL, and 68% CL lower bounds for $r$: $r_{99}$, $r_{95}$, and $r_{68}$. Lower right panel: the standard deviation (std) of the $r$ distributions for each sky and the standard deviation ($\Delta r_{\text{max}}$) of the $r_{\text{max}}$ distribution. These results correspond to a level of foreground residual equal to 30% of the toy model displayed in Figure 2. The sensitivity of the 143 GHz channel is exploited here.

(A color version of this figure is available in the online journal.)

Figure 10. 99% CL, 95% CL, and 68% CL lower bounds for $r$ as functions of the fraction of foreground residual of the worst case. For 30% foreground residual case, only a 68% CL detection is very likely. For a 95% CL detection, the level of foreground residual should be reduced to 10% or lower of the toy model displayed in Figure 2. The sensitivity of the 143 GHz channel is exploited here.

(A color version of this figure is available in the online journal.)

level, but real CMB experiments can observe only one realization (only one sample): the actual observed sky from which a single likelihood for the $B$-mode multipoles (and hence the value of $r$) is derived. So, the possibility of correctly inferring the value of $r$ from one single (albeit very large) sample strongly depends on the sample itself, and therefore, in view of the detection probability found here, whether $r$ will or will not be detected also depends on luck.
In addition, the results for many skies presented in this section show the consistency of our whole approach to determining $r$. Similar results are valid for the other cosmological parameters.

### 7.3. Bias Effect in the Foreground Residuals

Here we consider two extreme cases of the bias, modeled as being an imprecise determination of the foreground residuals $R_\ell$. We keep the residuals introduced in the noise of the test covariances fixed, while changing the residuals $R_{XX}$ to some $R_{XX}'$ in the noise of the observations, that is in Equation (9). Then, we write

$$R_{XX}' = R_{XX} \left(1 + \beta_X^X\right), \quad X = T, E, B,$$

where we consider the two extreme cases for the numbers $\beta_\ell$.

1. Independent flat random numbers $\beta_\ell$ from $-0.5$ to $0.5$. Since in this case $\beta_\ell$ fluctuates randomly around zero, the effect of the bias mostly cancels out, and the cosmological parameters suffer little change as depicted in Figure 11.

2. Uniform ramps from $-a^X$ to $a^X$ as $\ell$ varies from 2 to 2100, $a^X$ varying randomly up to 20% around 0.5 with $X = T, E, B$. This means choosing

$$\beta_\ell^X = a^X \left(\frac{\ell - 1051}{1049}\right).$$

Note that there is always a non-zero value for $a^X$ despite fluctuations and therefore there is a significant bias effect over the modes. In this case, the bias depresses the estimated multipoles at low $\ell$ and increases them at high $\ell$, thereby increasing the expected values of $n_s$ and depressing those of $r$. We see from Figure 11 that $r$ is no longer detected in this case despite the fact that its fiducial value is always $r = 0.0427$.

We consider a level of 30% of the toy model of foreground residuals displayed in Figure 2 for the bias effect. In this case we change the overall sign of $\beta_\ell^X$ in Equation (24), and we introduce an additional spurious power in the estimation at low $\ell$ and a depression of the power at high $\ell$. This would erroneously increase the probability of detecting $r$.

The peaks in the cosmological parameters (with the exception of $r$) get shifted mainly due to the bias from the $T$ modes. However, they stay within $1\sigma$ or $2\sigma$ of the WMAP values.

In case we do not add the bias in the $T$ modes, only $r$ is affected significantly by the bias. Namely, in case (2) without bias in the $T$ modes, all cosmological parameters except $r$ peak practically at the same value as in the absence of bias. In contrast, the likelihood distribution for $r$ is determined by the bias on the $B$ modes and turns out to be similar to the one in Figure 11 for the bias case (2).

The bias introduced in case (2) goes in the opposite direction to the theoretical double-well models where $n_s$ increases with $r$ (see Figure 1).

We present here only the bias for the $\Lambda$CDM$r$ model. The likelihood distributions for the $\Lambda$CDM/$T$ model including bias are similar to those of the $\Lambda$CDM$r$ model except for $r$ where a lower bound shows up due to the theoretical constraint.

We consider here only two extreme cases of bias: case (1) where the bias is practically harmless and case (2) where it significantly distorts the cosmological parameters, especially $r$ which is no longer detected.

### 8. FINAL CONCLUSION

In this paper, we provide a precise forecast for the Planck results on cosmological parameters, in particular for the tensor-to-scalar ratio $r$. These new forecasts go far beyond the published ones (see, e.g., Planck Collaboration 2006; Colombo et al. 2009) and pave the road for a promising scientific exploitation and...
interpretation of the Planck data (once cleaned from the different astrophysical foregrounds).

We appropriately combined the following, as main ingredients: the current publicly available knowledge of Planck instrument sensitivity and a reasonable toy model estimation of the residuals from systematic errors and foregrounds; the highly predictive theory setup (Boyanovsky et al. 2009; Destri et al. 2008a, 2008b) provided by the G–L approach to inflation to produce and analyze the skies (mock data) which allows a decisive gain in the physical insight and data analysis; and precise data (once cleaned from the different Planck instru-

ments: the current publicly available knowledge of astrophysical foregrounds).

It must also be stressed that, in the considered framework, better measurements for ns will improve the predictions on r from the T, TE, and E modes even if a secure detection of B modes will still be lacking. We also remark that the model is falsifiable in the case where constraints on ns and r are not compatible with the banana shape of the considered framework.

The lower bounds and most probable value inferred from WMAP for r (r ≃ 0.04) in the considered framework support the search for B-mode polarization in Planck data and the future CMB B-oriented polarization missions.

We thank Maria Cristina Falvella for her invaluable help and useful stimulating discussions. We thank the anonymous referee for the constructive comments. We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA). Support for LAMBDA is provided by the NASA Office of Space Science. Some of the results in this paper have been derived using the HEALPix (Górski et al. 2005) package. This work has been conducted in the framework of the Planck LFI Activity, the ASI/INAF Agreement I/072/09/0 for the Planck LFI Activity of Phase E2 and I/016/07/0 for COFIS. The Monte Carlo simulations were performed on the Turing cluster of the Dipartimento di Fisica “G. Occhialini,” Università Milano-Bicocca.

REFERENCES

Ashdown, M. A. J., et al. 2009, A&A, 493, 753
Bersanelli, M., et al. 2010, A&A, 520, A4
Bock, J., Hinshaw, G. F., & Timpie, P. T. 2006, BAAS, 38, 963
Boyanovsky, D., Destri, C., de Vega, H. J., & Sanchez, N. G. 2009, Int. J. Mod. Phys. A, 24, 3669

Boyansovskv, D., de Vega, H. J., & Sanchez, N. G. 2006, Phys. Rev. D, 73, 023008
Burgiana, C., Gruppuso, A., & Finelli, F. 2006, MNRAS, 371, 1570
Burgiana, C., Natoli, P., Vittorio, N., Mandolesi, N., & Bersanelli, M. 2001a, Exp. Astron., 12/2, 87
Burgiana, C., Sandri, M., Villa, F., Maino, D., Paladini, R., Baccigalupi, C., Bersanelli, M., & Mandolesi, N. 2004, A&A, 428, 311
Burgiana, C., et al. 2001b, A&A, 373, 345
Colombo, L. P. L., et al. 2009, MNRAS, 398, 1621
De Bernardis, P., Bucher, M., Burgiana, C., & Piccirillo, L. 2009, Exp. Astron., 23, 5
Destri, C., de Vega, H. J., & Sanchez, N. G. 2008a, Phys. Rev. D, 77, 043509
Destri, C., de Vega, H. J., & Sanchez, N. G. 2008b, Phys. Rev. D, 78, 023013
Destri, C., de Vega, H. J., & Sanchez, N. G. 2009, arXiv:0906.4102
Dodson, S. 2003, Modern Cosmology (San Francisco, CA: Academic Press)
Fosalba, P., Doré, O., & Bouchet, F. R. 2002, Phys. Rev. D, 65, 063003
Górski, K. M., Hivon, E., Banday, A. J., Wandelt, B. D., Hansen, F. K., Reinecke, M., & Bartelmann, M. 2005, ApJ, 622, 759
Gruppuso, A., Burgiana, C., & Finelli, F. 2007, MNRAS, 376, 907
Hivon, E., Górski, K. M., Netterfield, C. B., Brill, P. P., Prunet, S., & Hansen, F. 2002, ApJ, 567, 2
Hu, W., Hedman, M. M., & Zaldarriaga, M. 2003, Phys. Rev. D, 67, 043004
Huffenberger, K. M., Crill, B. P., Lange, A. E., Górski, K. M., & Lawrence, C. R. 2010, A&A, 510, A58
Kinney, W. H., et al. 2008, Phys. Rev. D, 78, 087302
Kogut, A., et al. 2003, ApJS, 148, 161
Komatsu, E., et al. (WMAP Collaboration) 2009, ApJS, 180, 330
Komatsu, E., et al. (WMAP Collaboration) 2010, arXiv:1001.4538
Lamarre, J.-M., et al. 2010, A&A, 510, A9
Leach, S. M., et al. 2008, A&A, 491, 597
Lewis, A., & Bridle, S. 2002, Phys. Rev. D, 66, 103511
Lewis, A., Challinor, A., & Lasenby, A. 2000, ApJ, 538, 473
Maffei, B., et al. 2010, A&A, 520, A12
Mandolesi, N., et al. 2010, A&A, 520, A3
Masi, S., et al. 2010, A&A, 519, A24
Naselsky, P. D., Verkhodanov, O. V., Christensen, P. R., & Chiang, L.-Y. 2007, Astrophys. Bull., 62, 285
Peiris, H. V., & Easther, R. 2008, J. Cosmol. Astropart. Phys., JCAP07(2008)024
Planck Collaboration. 2006, The Scientific Programme of Planck, arXiv:astro-ph/0604069, also available at http://www.rssd.esa.int/SA/PLANCK/docs/Bluebook-ESA-SCI(2005)1_V2.pdf
Polenta, G., Marinucci, D., Balbi, A., De Bernardis, P., Hivon, E., Masi, S., Natoli, P., & Vittorio, N. 2005, J. Cosmol. Astropart. Phys., JCAP11(2005)001
Rocha, G., Pagano, L., Górski, K. M., Huffenberger, K. M., Lawrence, C. R., & Lange, A. E. 2010, A&A, 513, A23
Sandri, M., Villa, F., Nesti, R., Burgiana, C., Bersanelli, M., & Mandolesi, N. 2004, A&A, 428, 299
Sandri, M., et al. 2010, A&A, 520, A7
Shimon, M., Keating, B., Ponthieu, N., & Hivon, E. 2008, Phys. Rev. D, 77, 083003
Tauber, J., et al. 2010, A&A, 520, A2
Tegmark, M., & de Oliveira-Costa, A. 2001, Phys. Rev. D, 64, 063001
Yadav, A. P. S., Su, M., & Zaldarriaga, M. 2010, Phys. Rev. D, 81, 063512