Mobility-Aware Interference Model for 5G Heterogeneous Networks

Yunquan Dong, Zhi Chen, Member, IEEE Pingyi Fan, Senior Member, IEEE
and Khaled Ben Letaief, Fellow, IEEE

{dongyq08, chenzhi06}@mails.tsinghua.edu.cn, fpy@mail.tsinghua.edu.cn, eekhaled@ece.ust.hk

Abstract—To meet the goal of increasing the network throughput by 1000x from 4G to 5G, 5G cellular networks need to be more heterogeneous and much denser by deploying more and more low power base station served small cells. As a result, the interference environment is expected to be much more complicated. Particularly, the number of users in each small cell can change dramatically due to users’ mobility, so does the uplink interference. Thus one question arises naturally: in what way user’s mobility affects the uplink interference in 5G heterogeneous networks? Based on the Levy flight moving model, this paper established a mobility-aware interference model for 5G heterogeneous networks. It is shown that both the average and the variance of the uplink interference will decrease with the increase of user velocity, for both closed and open femto cells, which can guarantee a better system performance. In addition, with a larger pathloss exponent, the performance of closed femto cells becomes worse. However, the performance of open femto cells becomes better.

Index Terms—interference modeling, user mobility, heterogeneous networks, Levy flights.

I. INTRODUCTION

As the 4G LTE/LTE-A (long term evolution/advanced) cellular system has been deployed all over the world and is reaching maturity, the 5G cellular system is believed to appear by the year of 2020. As is expected, the network aggregate data rate will be increased by roughly 1000x from 4G to 5G [1, 2]. To achieve this goal, 5G communication systems need more nodes per unit area besides more HZ and more bit/s/HZ per node [1]. That is, more and more low power base station served small cells such as pico/femto/relay cells are being added to the existing network [3]. It is even predicted that in the not too distant future, the number of base stations may actually exceed the number of cell phone subscribers [3]. A network that consists of a mix of macro cells and low power served cells is referred to as a HetNet (heterogeneous network), or the recently term of DenseNets [4].

Research on HetNets dates back to the discussion on femto cells in 2008 [5] and was admitted by 3GPP LTE-A standard in 2011 [6]. HetNets is also believed to be an important part of the next generation cellular networks. By adding more and more low power small cells, the reuse of spectrum across the space is improved while the number of users competing for resources at each base station is reduced. In fact, the spectral efficiency of modern access technologies such as LTE is already very close to Shannon’s limit. Therefore, enhancing the network efficiency by densifying the network in the spatial domain rather than user efficiency in the frequency domain seems to be one key step to 5G communications.

Due to the scarcity of the spectrum, lower power base stations are preferred to be deployed in the same band as macro base stations. Naturally, the interference management in HetNets becomes an unavoidable issue. In the literature, this problem has been discussed from various aspects. In the physical layer, the downlink cochannel interference can be modeled as an interference channel. However, its capacity region is really difficult to evaluate, especially when the number of users and interferers are large. In [7], the authors proposed an interference canceling block modulation scheme for the interference management in HetNets, which guarantees that the covariance matrix of the interference is rank deficient at each receiver. There were also joint detection algorithms and maximum-likelihood based local detections as proposed in [8, 9]. In the MAC layer or above, common interference management schemes includes: 1) Frequency reuse techniques such as fractional frequency reuse [10], soft frequency reuse [11] and optimum combining [12], and so on; 2) load balancing and power control schemes such as Range Extension technique in LTE Rel 10 systems, user association schemes in [14, 15] and the proportional optimal power control in [16, 17].

The authors of [17] investigated the difference as well as the equivalence among some commonly used interference models for interference limited adhoc/sensor networks, such as the additive interference model, the capture threshold model, the protocol model and the interference range model. As pointed in the paper, different interference models can produce significantly different results. In fact, this paper adopts some basic principles of the interference range model. [18] considered the uplink inter cell interference modeling for HetNets. The key point in [18] is to find the distribution of the location of scheduled users, and then the moment generating function of their interference. Most recently, [19] studied the downlink interference in a HetNet using stochastic geometry theory, in which every interfering base station locates outside of a guard zone and follows the Poisson point process. A dominant interferer was also assumed to locate at the edge of the guard zone. Together with the Gamma approximation method, the Laplace transform of total interference was given, which can be used in evaluating users’ success/outage probability and average rate.

Although very important, previous works were focused...
on static adhoc networks or downlink HetNets, where the interferers are fixed. On the contrary, the uplink interference in 5G HetNets are produced by users with mobility. It is still not clear whether users’ mobility will change uplink interference model or not, which is the motivation of this paper.

Particularly, since more and more base stations are deployed in the network, each cell becomes smaller and smaller. As a result, the number of users in the cell or an interfering area is very limited and will not follow the LLN (law of large numbers) like macro cell scenarios. In this case, users’ mobility will have more impact on the number of users in a cell, which determines their uplink interference to other type of users.

This paper focuses on the mobility-aware uplink interference modeling in HetNets. Particularly, users’ mobility is modeled by Levy flights [20]. In this model, each user takes one flight in every time interval $T_s$, which is defined to be the longest straight line trip of a user from one location to another without a directional change or pause. Some recent studies on human mobility show that flight length distributions have a heavy-tail tendency [20, 21]. By normalizing the flight length with a basic step length $\Delta$, the flight length turns to be the number of basic steps in each flight. Therefore, $\Delta$ can be seen as an indicator of user velocity, that is, a user with a larger $\Delta$ moves more quickly in the average sense.

The main contributions of this paper are summarized as follows.

1) We established a mobility-aware uplink interference model, in which uplink interference is connected with user mobility via a bond of the number of users in the interfering area. This interference model is useful in evaluating the system performance such as success probability in the transmission and the average transmission rate.

2) Some interesting observations were obtained.

- First, the number of users in an interested area was found to follow the Binomial distribution, which can be approximated by Poisson distribution when basic step length $\Delta$ is large, i.e., user velocity is large.
- Second, closed femto cells which admits only authenticated users suffers more interference than open femto cells, which admits any user coming into its coverage. It was seen that the performance of users in an open femto cell is also better than that of closed femto cell users, in both success probability and average rate.
- Third, uplink interference will decrease slightly if user velocity is increased, which improves system performance to some extent, for both open and closed femto cells.
- Finally, when the pathloss exponent is increased, the performance of closed femto cells becomes worse, but the performance of open femto cells gets better.

The rest of this paper is organized as follows. The communication system model is presented in Section IV. User’s average probability of coming into or gonging out of an interested small cell is presented in Section V based on which the corresponding queueing model for the number of users in a small cell is formulated in Section VI. After that, the fluctuation of uplink interference as well as its statistical characterizations are presented in Section VII. In Section VIII the interference model will be used in evaluating user’s success probability in the transmission and average transmission rate per user. The obtained result will also be presented via numerical results in Section VII. Finally, we conclude our work in section VIII.

### II. System Model

Consider a typical HetNet model as shown in Fig. 1. The area is served by a macro-eNB (M-eNB), as well as some low power pico-eNBs, femto-eNBs and relays (collectively referred to as H-eNBs). Femto cells can either be open cells which admit every user coming into its coverage, or closed ones which serves some authenticated users only. Define the radius of the macro cell as $L$ and the radius of an interested small cell (some pico/femto/relay cell) as $R$, where $L \gg R$ holds true.

![5G Heterogeneous Network model](image)

$N$ users are assumed to be distributed evenly within the macro cell. Each of them moves in the network following the Levy flight model. Specifically, each user takes one step towards some uniformly distributed random direction in every time interval $T_s$. When normalized by a basic step length $\Delta$, the flight length $Z$ follows the power law distribution

$$f_Z(z) = \frac{\alpha}{z^{1+\alpha}}; \quad z \in [1, +\infty)$$

where $\alpha$ falls in between 0.53 and 1.81 as shown by many human mobility traces [20, 21].

In this mobility model, each flight length is a random variable and takes a constant time $T_s$ regardless of its flight length. Therefore, for a given $T_s$, the general moving velocity is determined by the basic step length $\Delta$. In this sense, this paper will show how user mobility affects uplink interference by investigating the functional relationships between statistics of uplink interference and basic step length $\Delta$. In addition, it is assumed that users reaching the macro cell edge are reflected back into the macro cell, so that there are always $N$ users in the macro cell.

Users served by low power H-eNBs are denoted as H-UEs. Since these cells are much smaller than macro cells, H-UEs’ transmit power ($P_H^T$) is also lower in the uplink. On the other hand, macro users (M-UEs) will communicate with the M-eNB, which locates much further away. Thus M-UEs have
larger transmit power ($P^m_i$) than H-UEs, and will cause great interference to H-UEs in the uplink.

Due to large scale attenuation and small scale fading, the received signal power at base stations can be expressed as

$$P_r = \frac{\gamma}{d^\beta} P_t,$$

where $d$ is the distance between the user and the base station, $\beta \geq 2$ is the pathloss exponent, and $\gamma$ is a random variable determined the specific fading channel model such as Rayleigh, Rician or Nakagami fading.

It is clear that the received power will decrease rapidly with distance. Therefore, it is reasonable to assume that only M-UEs near from or within the small cell interfere H-UEs in the uplink. Let $R_I$ be the interfering radius. Like the interference range model [17], it is assumed that only interference from M-UEs within the interfering circle will be considered, as shown by Fig. [1].

Besides, the number of users in an interested area will be a random variable due to users’ mobility. Let $C_k$ be an arbitrary chosen small cell with radius $R$ and $\zeta_i$ be the number of users in $C_k$ at the time $nT_s$, it is seen that the process $\{\zeta_i, n \geq 0\}$ will be a Markov chain.

In order to establish this Markov chain, the average probability that a user moves into /out of a small cell needs to be investigated first. If not specified, “the (interested) cell” in the following parts refers to small cells such as femto cell, pico cell or relay cells.

III. AVERAGE PROBABILITY OF MOVING INTO/OUT OF A SMALL CELL

According to power law distribution [1], a user moves at least one basic step $\Delta$ during each time interval $T_s$, and may cover a very large distance with some small probabilities. Therefore, every user outside of $C_k$ has the chance to come into the cell with non-zero probability. On the other hand, any user within the cell may step out. By taking the average over all possible user locations, one will get the average coming in probability $P_i(R, \Delta)$ and average going out probability $P_o(R, \Delta)$, where $R$ is the radius of $C_k$ and $\Delta$ is the basic step length.

A. Average Going out Probability

As shown in Fig. 2, assume that the cell center locates at $(r_0, \pi)$ in the $(r, \theta)$ polar coordinate system which is centered at the user. Due to the isotropic property of the circular cell, the position of a user only depends on its distance to the HeNB, since users are evenly distributed and are symmetric in the angular coordinate. Therefore, we will consider users at different locations by changing $r_0$.

For a given $r_0$, suppose the user moves at direction $\theta$ that will intersect with cell edge at point $(r_\theta, \theta)$, which satisfies

$$(r_\theta + r_0 \cos \theta)^2 + (r_\theta \sin \theta)^2 = R^2.$$

This is equivalent to

$$r_\theta^2 + 2r_0 \cos \theta r_\theta + r_0^2 - R^2 = 0. \quad (2)$$

By solving $r_\theta$ from the equation one has

$$r_\theta = \sqrt{R^2 - r_0^2 \sin^2 \theta - r_0 \cos \theta}.$$

Besides, the flight length $Z$ is power law distributed according to [1]. Define $X = Z\Delta$, then its p.d.f. is given by

$$f_X(x) = \frac{\alpha \Delta^\alpha}{x^{\alpha+1}}, \quad x \in [\Delta, +\infty). \quad (3)$$

Next, the probability of going out can be solved case by case.

1) $\Delta < R$: If $0 < r_0 < R - \Delta$, the user can step out of the cell only if the flight length satisfies $X > r_\theta$, as shown by the black line in Fig. 2. We have

$$P_o\{r_0|0 < r_0 < R - \Delta\} = 2 \int_0^\pi \frac{1}{2\pi} \Pr\{X > r_\theta\} d\theta. $$

However, if $R - \Delta < r_0 < R$, the user will move out of the cell definitely in some scenarios, since the user is very close to the edge and every flight is no shorter than $\Delta$.

Define $\theta_1$ as the angle which enables the user to reach the cell edge when the flight length is exactly $\Delta$. Then point $(\Delta, \theta_1)$ will on the curve defined by (3) and $\theta_1$ can be solved as

$$\theta_1 = \arccos \frac{R^2 - r_0^2 - \Delta^2}{2\Delta r_0}, \quad \theta_1 \in (0, \pi).$$

It is clear that, if $-\theta_1 < \theta < \theta_1$, the user will certainly step out of the cell. For other cases, the user will step out if the flight length satisfies $X > r_\theta$. Then one gets the conditional going out probability as

$$P_o\{r_0|R - \Delta < r_0 < R\} = 2 \int_0^{\theta_1} \frac{1}{2\pi} d\theta + 2 \int_{\theta_1}^\pi \frac{1}{2\pi} \Pr\{X > r_\theta\} d\theta. \quad (2)$$

2) $R \leq \Delta < 2R$: In this situation, the user will step out of the cell in most directions, except for angles $2\pi - \theta_1 > \theta > \theta_1$ and $r_0$ is relatively large, i.e., $r_0 > \Delta - R$. This is because if the user is very close to the cell edge in one direction, it become difficult to step out from the opposite direction. On the contrary, the user will step out in every direction if $r_0 < \Delta - R$.

Specifically, we have

$$P_o\{r_0|0 < r_0 < \Delta - R\} = 1,$$

$$P_o\{r_0|\Delta - R < r_0 < R\} = 2 \int_\theta^{\theta_1} \frac{1}{2\pi} d\theta + 2 \int_{\theta_1}^\pi \frac{1}{2\pi} \Pr\{X > r_\theta\} d\theta.$$
3) $2R \leq \Delta$: In this situation, the user will step out of the cell with probability 1, regardless of its moving direction and location, i.e.,

$$P_r\{r_0|2R < \Delta\} = 1.$$ 

On the one hand, whether a user will step out of the cell is partially determined by the flight length. By the probability density function (p.d.f.) of flight length $X$, we have

$$\Pr\{X > r_0\} = \int_{r_0}^{\infty} f_X(x)dx = \frac{\Delta^\alpha}{r_0^\alpha}.$$ 

On the other hand, the location of the user is also random. Since users are evenly distributed in the cell, the probability that $r_0$ is smaller than $x$ is

$$F(r_0)(x) = \Pr\{r_0 \leq x\} = \frac{x^2}{R^2}.$$ 

Then we get the angle independent p.d.f. of $r_0$ as

$$f(r_0)(x) = \frac{2x}{R^2}, \quad x \in [0, R].$$

By taking the average over $r_0$ and combining analysis above, the proposition below summarizes the average going out probability.

**Proposition 1.** The average probability that a user in the interested cell will step out of the cell is presented at the top of next page.

**B. Average Coming in Probability**

In Fig. 3 the origin of the polar system is a user outside of the interested small cell $C_k$. The distance between the user and small cell center is $d_0$. By changing $d_0$ from $R$ to $L$, we are actually considering users at different locations. Besides, a user’s location mainly depends on the distance and is totally symmetric in the direction.

![Fig. 3. Probability of coming in. The user starts from the origin.](image)

Suppose point $(\rho_0, \theta)$ is on the edge of $C_k$, then it satisfies the equation below.

$$(d_0 - \rho_0 \cos\theta)^2 + (\rho_0 \sin\theta)^2 = R^2,$$

which leads to

$$\rho_0^2 - 2d_0 \cos\theta \rho_0 + d_0^2 - R^2 = 0.$$ 

Solving $\rho_0$ from above equation, we have its two roots

$$\rho_1 = d_0 \cos\theta - \sqrt{R^2 - d_0^2 \sin^2\theta},$$

$$\rho_2 = d_0 \cos\theta + \sqrt{R^2 - d_0^2 \sin^2\theta},$$

which are in fact the distance from origin to the intersection point of cell edge and the line with angle $\theta$.

Since the direction of each flight is evenly distributed, the joint p.d.f. of $X$ and $\theta$ is given by

$$f_{X,\theta}(x, \theta) = \frac{\alpha\Delta^\alpha}{2\pi x^{\alpha+1}}, \quad x \in (\Delta, +\infty), \theta \in [0, 2\pi),$$

which is also the distribution of the end point of a flight. Note that this distribution takes the starting point of a flight as the reference point. However, in the network view, the end point is still uniformly distributed, since each move is completely random.

In the coordinate system in Fig. 3 the coming in probability is also the probability that the end point of a flight falls into the small cell $C_k$, and can be expressed by the integration of $f_{X,\theta}(x, \theta)$ over the shaded areas, like the shaded part of the cell in Fig. 3.

As shown by the example in Fig. 3 the user locates at the origin. If the center of cell $C_k$ is $d_0$ away from the user and $R < d_0 < R + \Delta$ (as shown by the black circle), cell $C_k$ will have a common area with the circular area which is centered on origin and with radius $\Delta$. Particularly, this area is not possible to be the end point of any flight, since its distance to the user is less than $\Delta$.

Define $\theta_2$ as the angular coordinate of the intersection point, it is clear that the corresponding radial coordinate is $\Delta$. Therefore, point $(\Delta, \theta_2)$ satisfies equation (4) and $\theta_2$ can be solved,

$$\theta_2 = \arccos\frac{\Delta^2 + d_0^2 - R^2}{2\Delta d_0}.$$ 

Define $\theta_3$ as the angular coordinate of the tangent line of circle $C_k$ which passes the origin. It is seen that

$$\theta_3 = \arcsin\frac{R}{d_0}.$$ 

It is also noted that $\theta_3 \geq \theta_2$ always holds true for any $d_0 \in [R, L]$. Particularly, the distance $d_0$ that enables $\theta_2 = \theta_3$ can be solve from

$$\arcsin\frac{\Delta^2 + d_0^2 - R^2}{2\Delta d_0} = \arcsin\frac{R}{d_0},$$

from which we get $d_0^* = \sqrt{\Delta^2 + R^2}$.

Now we will discuss the coming in probability case by case.

1) $\Delta < 2R$: In this case, the user has a probability to enter $C_k$ if $d_0 > R$.

First, if $R < d_0 < d_0^*$, the user will be in $C_k$ only if the direction of the flight is $\theta \in (-\theta_2, \theta_2)$ and the flight length is $\Delta < X < \rho_2$. The corresponding conditional coming in probability is

$$P_r\{d_0|R < d_0 < \sqrt{\Delta^2 + R^2}\} = 2\int_{\theta_2}^{\theta_3} \frac{1}{2\pi} \Pr\{\Delta < X < \rho_2\}d\theta.$$ 

Second, if $d_0^* \leq d_0 < R + \Delta$, the user will be in $C_k$ if $\theta \in (-\theta_3, -\theta_2)$ or $\theta \in (\theta_2, \theta_3)$, and the flight length is $\rho_1 < X < \rho_2$; or $\theta \in (\theta_2, \theta_3)$ and the flight length is $\Delta < X < \rho_2$. The corresponding probability is,

$$P_r\{d_0|\sqrt{\Delta^2 + R^2} < d_0 < R + \Delta\}$$

$$= 2\int_{\theta_2}^{\theta_3} \frac{1}{2\pi} \Pr\{\Delta < X < \rho_2\}d\theta + 2\int_{\theta_2}^{\theta_3} \frac{1}{2\pi} \Pr\{\rho_1 < X < \rho_2\}d\theta.$$
Third, for \( d_0 \geq R + \Delta \), the two circles mentioned above will not intersect anymore. So the coming in probability turns to be

\[
P_i\{d_0|R + \Delta \leq d_0\} = 2 \int_0^{\theta_1} \frac{1}{2\pi} \Pr\{\rho_1 < X < \rho_2\} d\theta.
\]

2) \( \Delta \geq 2R \): In this case, the user may step across \( C_k \) unless \( d_0 \) is larger than \( \Delta - R \). Therefore, discussions on coming in probability will replace \( R < d_0 < d_0^* \) by \( \Delta - R < d_0 < d_0^* \), and no other changes.

By assuming that the interested cell \( C_k \) lies at the center of the macro cell, the probability that a user fall into a circle with radius \( x \) is \( \Pr\{X < x\} = \frac{1}{\pi x^2} \), which is also the distribution of \( d_0 \). Therefore, the probability of \( d_0 \) is

\[
f_{d_0}(x) = \frac{2}{\pi L^2}, \quad x \in [0, L].
\]

Finally, by taking the average over \( d_0 \), the coming in probability is presented by the Proposition as follows.

**Proposition 2.** The average probability that a user will come into the interested cell is

\[
P_i(R, \Delta) = \frac{2\Delta^\alpha}{\pi L^2} \int_0^{\theta_2} \int_0^{\theta_1} \left( \frac{1}{\Delta - R} \right) d\theta d\rho_1 d\rho_2
\]

\[
+ \frac{2\Delta^\alpha}{\pi L^2} \int_0^{\theta_2} \int_0^{\theta_1} \left( \frac{1}{\Delta + R} \right) d\theta d\rho_1 d\rho_2
\]

\[
+ \frac{2\Delta^\alpha}{\pi L^2} \int_{\Delta - R}^{\theta_2} \int_0^{\theta_1} \left( \frac{1}{\rho_1^2} \right) d\theta d\rho_1 d\rho_2
\]

\[
+ \frac{2\Delta^\alpha}{\pi L^2} \int_{\Delta + R}^{\theta_2} \int_0^{\theta_1} \left( \frac{1}{\rho_2^2} \right) d\theta d\rho_1 d\rho_2
\]

IV. THE NUMBER OF USERS IN \( C_k \)

Denote the number of users in the interested small cell \( C_k \) at time \( nT_s \) as \( \xi_n \). Due to user mobility, the number of user in \( C_k \) will be a random variable. In fact, the uplink interference is mainly determined by the number of such interferers. In this section, the stochastic characteristics of \( \xi_n \) will be investigated.

A. Queueing Model Formulation

Assume that users leaves \( C_k \) at the beginning of each time interval, i.e., \( n^- \), and arrive at \( C_k \) at \( n^- \). As shown in previous section, \( P_i(R, \Delta) \) and \( P_o(R, \Delta) \) are the average coming in and going out probabilities respectively. In this paper, it is assumed that each user outside of \( C_k \) will step into the cell with probability \( P_o(R, \Delta) \), and each user in the cell will leave with probability \( P_i(R, \Delta) \) in the average sense, denoted by \( P_i \) and \( P_o \) for short, respectively. In this way, a tractable mathematics model can be established.

If \( \xi_{n-1} = k \), we will have \( N - k \) users outside of the cell. Define the probability that there will be \( j \) users coming into the cell at time \( n^- \) as

\[
\nu(j, N - k) = \Pr\{j \text{ users will come in during } (n^-, n)|\xi_{n-1} = k\}
\]

\[
= C_j^k P_i^j (1 - P_i)^{N-k-j} \quad j = 0, 1, 2, \cdots, N - k.
\]

Likewise, define the probability that there are \( j \) users leaving the cell at time \( nT_s \) as

\[
\mu(j, k) = \Pr\{j \text{ users will leave during } (n, n^+)|\xi_{n-1} = k\}
\]

\[
= C_j^k P_o^j (1 - P_o)^{k-j} \quad j = 0, 1, 2, \cdots, k.
\]

Therefore, the transition probability of the Markov chain \( \{\xi_n^+ , n \geq 0\} \) is

\[
p_{kj} = \Pr\{\xi_{n}^+ = j|\xi_{n-1} = k\} \quad (a)
\]

\[
\sum_r \Pr\{r \text{ users leave during } (n, n^+)|\xi_{n-1}^+ = k\}
\]

\[
\times \Pr\{j + r - k \text{ users arrive during } (n^-, n)|\xi_{n-1} = k\}
\]

\[
= \sum_{r=\max(0,k-j)}^{\min(k,N-j)} \mu(r, k)\nu(j + r - k, N - k), \quad j = 0, 1, \cdots, k
\]

for \( N - 1; 0 \leq k \leq N \).

In above equation, the upper limit of the summation in (a) can also be expressed by: \( N - j = k - (j - (N - k)) \). That is to say, \( j - (N - k) \) is the gap to the goal of \( j \) users on condition that all of other \( N - k \) users will come in, and can only be filled by users who will not leave the cell. Therefore, the maximum users can leave is at most \( k - (j - (N - k)) = N - j \).

With \( P = \{p_{kj}\}_{(N+1)\times(N+1)} \), all the statistics of \( \{\xi_n^+ , n \geq 0\} \) are hence determined.

B. Stationary Distribution of \( \xi_n^+ \)

Since both \( P_i \) and \( P_o \) are positive and smaller than 1, and the Markov chain has finite states, it is readily derived that the Markov chain \( \{\xi_n^+, n \geq 0\} \) considered here has a stationary distribution, which is equal to its limiting distribution.

Define \( \pi = \{\pi_j, 0 \leq j \leq N\} \) as the stationary distribution of \( \{\xi_n^+, n \geq 0\} \), namely,

\[
\pi_j = \lim_{n \to +\infty} \Pr\{\xi_n^+ = j\}.
\]
Define the probability generating function (PGF) of $\pi_k$ as
\[ \xi^+(z) = \sum_{j=0}^{N} \pi_j z^j. \]

Once $\xi^+(z)$ have been obtained, the corresponding distribution, average and variance can also be obtained. In fact, $\xi^+(z)$ will be given by the following theorem.

**Theorem 1.** The PGF of the stationary distribution of $\xi^+$, i.e., the number of users in $C_k$, is given by
\[ \xi^+(z) = \left( \frac{P_t z}{P_t + P_o} + \frac{P_o}{P_t + P_o} \right)^N \]
where $P_t$ and $P_o$ are abbreviations for $P_t(R, \Delta)$ and $P_o(R, \Delta)$ respectively.

**Remark 1.** Using some polynomial expansions to $\xi^+(z)$ we will have
\[ \xi^+(z) = \sum_{j=0}^{N} C_N^j \left( \frac{P_t z}{P_t + P_o} \right)^j \left( \frac{P_o}{P_t + P_o} \right)^{N-j} \cdot \]

It is clear that $\pi_j = C_N^j \left( \frac{P_t z}{P_t + P_o} \right)^j \left( \frac{P_o}{P_t + P_o} \right)^{N-j}$. Therefore, the number of users in $C_k$, i.e., $\xi^+_n$, is a Binomial distributed random number in the limitation sense.

**Remark 2.** Denote $\eta = \frac{P_t}{P_t + P_o}$ and $\lambda = N\eta$. It is well known that Binomial distribution can be approximated by Poisson distribution when $N$ is very large and $q$ is very small, which will make further analysis easier.

Actually, the number of users in the macro cell $N$ is about tens of thousands while the coming in probability is less than $10^{-3}$. Thus the approximation usually works well.

**Proof:** Firstly, the stationary distribution $\pi$ satisfies the following equations.
\[ \pi \mathbf{P} = \pi, \quad \pi \mathbf{e} = 1, \]
where $\mathbf{e}$ is a row vector of ones.

For the $j$-th element of stationary distribution $\pi_j$, we have
\[ \pi_j = \sum_{k=0}^{N} \pi_k p_{kj}, \quad j = 0, 1, \cdots, N. \]

By multiplying $z^j$ on both sides and take the summation from 1 to $N$, we have
\[ \sum_{j=1}^{N} \pi_j z^j = \sum_{k=0}^{N} \pi_k \sum_{j=1}^{N} \sum_{r=0}^{\min(k, N-j)} \mu(r, k) \nu(j + r - k, N - k) \]
\[ = \sum_{k=0}^{N} \pi_k \sum_{j=1}^{N} \sum_{r=0}^{\min(k, N-j)} \mu(r, k) \nu(j + r - k, N - k) \]
\[ = \sum_{k=0}^{N} \pi_k (P_o + (1 - P_o) z)^k (P_t z + 1 - P_t)^{N-k} \]
\[ = (P_t z + 1 - P_t)^N \xi^+ \left( \frac{P_o + (1 - P_o) z}{P_t z + 1 - P_t} \right), \]
where the order of the summations is changed in (a) and variable substitution $i = j + r - k$ is used in (b).

By solving $\xi^+(z)$ from the above equation, the theorem will be proved.

It is known that the average and variance of a random variable $X$ are related with its PGF $G_X(z)$ by following equations
\[ \mathbb{E}[X] = G'_X(z)|_{z=1} \]
\[ \mathbb{D}[X] = G''_X(z)|_{z=1} + (G'_X(z)|_{z=1})^2 \]
Then the statistics of the number of users in $C_k$ are given by the following proposition.

**Proposition 3.** The average and variance of number of users in the interested small cell $C_k$ are given by, respectively
\[ \mathbb{E}[\xi^+] = \frac{N P_t}{P_t + P_o}, \]
\[ \mathbb{D}[\xi^+] = \frac{N P_t P_o}{(P_t + P_o)^2}. \]

**Remark 3.** Note that both $P_t$ and $P_o$ are functions of basic step length $\Delta$, which is an index of moving velocity. Therefore, both $\mathbb{E}[\xi^+]$ and $\mathbb{D}[\xi^+]$ are also functions of user velocity.

**V. THE FADING OF UPLINK INTERFERENCE**

In the uplink, M-UEs use much higher transmit power than H-UEs since they are usually very far from the macro eNB. Thus H-UEs’ uplink signal will suffer from serious interference from nearby M-UEs. Besides, the interference will be a random variable due to channel fading and user mobility. In this sense, the uplink interference also has a ‘fading’ property.

First, each interferer is randomly located. Users are assumed to be uniformly distributed and moves randomly. Therefore, their distances to the interfered H-eNB are random variables, which introduces uncertainty to the interference.

Second, each interfering link suffers from small scale fading, which varies much more quickly along time.

Last but not the least, the number of interferers will also be a random variable due to user mobility. Particularly, its fluctuation is further accelerated by the miniaturization of cells.

However, it should be noted that the ‘fading’ of the uplink interference caused by users’ mobility is a kind of large scale fading and also a slow fading. Generally speaking, the velocity of a user is 3 km/h for pedestrians and no more than 120 km/h if the user is in a vehicle. Therefore, the flight time is relatively large, which makes the fluctuation of the uplink interference much slower than small scale fading.

In the following part, the fading property of uplink interference will be characterized in terms of distribution and statistic moments, based on which the impact of user mobility on uplink interference can be revealed. As seen in previous sections, going out probability $P_o(R, \Delta)$ and coming in probability $P_t(R, \Delta)$ are all functions of basic step $\Delta$, which is an indicator of user mobility.
A. Uplink Interference of Closed Femto cells

For a closed femto cell \( C_k \) with radius \( R \), only some authenticated users within the cell coverage are allowed to communicate with the H-eNB. Other UEs have to be linked to the macro eNB, even if it is in the femto cell. As shown in Fig[I] each UE within the circle of interfering radius \( R_i \), which is referred to as \( C' \), is an interferer to femto UEs.

Let \( \xi_3 \) be the number of a macro UEs within the interfering circle \( C' \) in the \( n \)-th time interval. Thus it is a Binomial distributed random variable with its PGE \( \xi_3(x) \) given by (6). Theorem[I] in which \( P_i \) and \( P_o \) are abbreviations of \( P_i(R_i, \Delta) \) and \( P_o(R, \Delta) \), respectively.

Denote the distance between macro user \( j \) and the femto e-NB as \( d_{ij} \). Let \( \gamma_j \) and \( F_j(x) \) be the small scale fading power gain and its cumulative distribution function (CDF). \( P_i = \mathbb{E}[\gamma_j] \) is the average power gain and \( P_i^{(2)} = \mathbb{E}[\gamma_j^2] \) is the second order moment. Then the instantaneous interference can be expressed as \( I_{ij} = \frac{P_i^{(2)} N}{d_{ij}^2} \), where \( P_i^{(2)} \) is macro UE’s transmit power. The moments of \( I_{ij} \) can be given by following proposition.

**Proposition 4.** The first and second order moments of the interference of a uniformly distributed user within the interfering circle are

\[
\mu_c = \mathbb{E}[I_{ij}] = \frac{2P_i^m P_i(P_i^{(2)} - 2)}{(\beta - 2)R_i^{(2) - 2} (R_i^2 - 1)},
\]

\[
\mu_c^{(2)} = \mathbb{E}[I_{ij}^2] = \frac{(P_i^{(2)})^2 P_i^2 (P_i^{(2)} - 2)}{(\beta - 1)R_i^{(2) - 2} (R_i^2 - 1) \text{, (8)}}
\]

**Remark 4.** In the case of \( \beta = 2 \), (8) holds in the limitation sense. That is,

\[
\mu_c = \lim_{\beta \rightarrow 2} \frac{2P_i^m P_i(P_i^{(2)} - 2)}{(\beta - 2)R_i^{(2) - 2} (R_i^2 - 1)} = \frac{2P_i^m P_i}{R_i^2 - 1}\ln R_i.
\]

**Proof:** Since the user is uniformly distributed within the interference circle, the probability that the distance is less than \( x \) is \( \Pr(d_j < x) = \frac{x^2}{R_i^2 \text{, (9)}} \), where it is assumed \( d_j \geq 1 \) so that the transmit power won’t be amplified in the case that \( d_j \) is very small.

In addition, the p.d.f. of \( d_j \) is

\[
f_d(x) = \frac{2x}{R_i^2 - 1}, \quad x \in [1, R_i].
\]

Therefore, the average interference power can be derived as follows

\[
\mu_c = \mathbb{E}[\gamma_j d_{ij}] = \int_0^{\infty} \int_0^{R_i} f_j(x)f_d(y) xP_i^{(2)} y^{-\beta} \text{ dxdy} = \int_0^{\infty} xF_j(x) dx \int_0^{R_i} \frac{2P_i^{(2)} 1}{(\beta - 2)R_i^{(2) - 2} y^{\beta - 1} \text{ dxdy}} = \frac{2P_i^m P_i (R_i^{(2)} - 2)}{(\beta - 2)R_i^{(2) - 2} (R_i^2 - 1) \text{, (10)}}
\]

Likewise, we have

\[
\mu_c^{(2)} = \mathbb{E}[\gamma_j d_{ij}^2] = \int_0^{\infty} \int_0^{R_i} f_j(x)f_d(y) xP_i^{(2)} y^{-\beta} \text{ dxdy} = \int_0^{\infty} x^2F_j(x) dx \int_0^{R_i} \frac{2P_i^{(2)} 1}{(\beta - 1)R_i^{(2) - 2} y^{\beta - 1} \text{ dxdy}} = \frac{(P_i^{(2)})^2 P_i^2 (R_i^{(2)} - 2)}{(\beta - 1)R_i^{(2) - 2} (R_i^2 - 1) \text{, (11)}}
\]

Since there are \( \xi_3 \) macro-UEs within the interfering circle in all, the total interference will be

\[
I_c = \sum_{j=1}^{\xi_3} \sum_{j=1}^{\xi_3} \frac{\gamma_j P_i^{(2)}}{d_{ij}^2}.
\]

Define \( G_{I_j}(s) = \mathbb{E}[e^{sI_j}] \) as the Moment generating function (MGF) of each individual interference, then the MGF of the total interference and its average and variance are summarized in following theorem.

**Theorem 2.** The MGF of the uplink interference to closed femto cell UEs is

\[
G_{I_c}(s) = \left( \frac{P_i G_{I_j}(s)}{P_i + P_o} + \frac{P_o}{P_i + P_o} \right)^N \text{, (9)}
\]

its average and variance are given by respectively

\[
\mathbb{E}[I_c] = \frac{NP_i}{P_i + P_o} \mu_c,
\]

\[
\mathbb{D}[I_c] = \frac{NP_i}{P_i + P_o} \mu_c^{(2)} - \frac{NP_i^2}{(P_i + P_o)^2} \mu_c^2,
\]

where \( P_i \) and \( P_o \) are calculated with \( R_i \) and \( \Delta \).

**Remark 5.** As known to all, the p.d.f. of a random variable is completely determined by its MGF. Thus Theorem 2 gives a full characterization of the uplink interference to a closed femto cell. Besides, explicit expressions for \( G_{I_j}(s) \) can be obtained when \( F_j(x) \) is given.

**Remark 6.** Two key parameters for the results in Theorem 2 are \( R_i \) and \( \Delta \). While \( R_i \) specifies the area interfering H-UEs, \( \Delta \) indicates the mobility of users. Therefore, this theorem has presented how user mobility will affect the uplink interference.

**Proof:** By its definition, one has

\[
G_{I_c}(s) = \mathbb{E}[e^{sI_c}] = \mathbb{E}[e^{s \sum_{j=1}^{\xi_3} I_{ij}}] = \sum_{k=0}^{N} \Pr(\xi_3 = k) \left( \mathbb{E}[e^{sI_j}] \right)^k = (\xi^+ \left( G_{I_j}(s) \right))
\]

where \( \xi^+ \) was given by (6), which proves (9).

Using \( G_{I_c}(s) \), the average uplink interference will be

\[
\mathbb{E}[I_c] = G_{I_c}'(s)_{s=0} = N \left( \frac{P_i G_{I_j}(s)}{P_i + P_o} + \frac{P_o}{P_i + P_o} \right)^N \left( \frac{P_i G_{I_j}(s)}{P_i + P_o} \right)^{N-1} \bigg|_{s=0} = \frac{NP_i}{P_i + P_o} \mu_c.
\]
Similarly, its second moment is
\[
E[I_s^2] = G_{I_s}(s)|_{s=0} = \left[ N \left( \frac{P_i G_{I_o}(s)}{P_i + P_o} + \frac{P_o}{P_i + P_o} \right) + \frac{P_i}{P_i + P_o} G_{I_s}(s) \right] |_{s=0}
\]
\[
= N\left( N - 1 \right) P_i^2 \mu_c^{(2)} + N P_i \mu_c - \frac{N P_o}{P_i + P_o} \mu_c^{(2)}
\]
where \( \mu_c \) and \( \mu_c^{(2)} \) are given by Proposition 4.

Therefore, the variance of uplink interference is
\[
\text{Var}[I_c] = E[I_c^2] - E[I_c]^2
\]
\[
= \frac{N P_i \mu_c - N P_o}{P_i + P_o} \mu_c^{(2)}
\]

B. Uplink Interference of Open Femto cells

Open femto cells or pico cells, relay cells will admit every users coming into their coverage. Therefore, only MUEs outside the cell and within the interfering radius cause interference.

Assume there are \( \xi_3 \) users in all within the circular area \( C' \) of radius \( R_1 \), in which \( \xi_1 \) users are within and served by the open femto cell. Thus the number of interferers in the interfering ring is \( \xi_2 = \xi_3 - \xi_1 \).

Let \( d_j \in (R_1, R_2) \) be the distance between an interferer and the H-eNB. Its interference to H-UEs is \( I_{o,j} = \frac{P_j P_m}{d_j^2} \) and the corresponding first and second moments are given by the following proposition.

**Proposition 5.** The first and second order moments of the interference of a uniformly randomly located user within the interfering ring are
\[
\nu_0 = E[I_{o,j}] = \frac{2 P_j P_m \left( R_1^3 - 2R_3^3 - 2R_3^3 \right)}{(\beta - 2)R_1^2 - 2R_3^3 - 2R_3^3 - R_1^3 - R_2^3)}
\]
\[
\nu_0^{(2)} = E[I_{o,j}^2] = \frac{(\beta - 1)R_1^2 - 2R_3^3 - 2R_3^3 - (R_1^2 - R_2^3)}{(\beta - 1)R_1^2 - 2R_3^3 - 2R_3^3 - (R_1^2 - R_2^3)}
\]

The proof of Proposition 5 is similar to that of Proposition 4 and is omitted here.

For open femto cells, only users within the interfering ring will interfere H-UEs. Besides, these users are linked to the macro eNB. Therefore, the total interference will be
\[
I_o = \sum_{j=1}^{\xi_2} I_{o,j} = \sum_{j=1}^{\xi_2} \sum_{j=1}^{\xi_2} \frac{P_m}{d_j^2}
\]

Define \( G_{I_o}(s) = E[e^{sI_o}] \) as the MGF of the instantaneous interference from macro-UE \( j \), then the MGF of the total interference and its average and variance are given by the following theorem.

**Theorem 3.** The MGF of the uplink interference to open femto cell UEs is
\[
G_{I_o}(s) = \left( \frac{P_i \left( P_i G_{I_o}(s) + 1 - q \right)}{P_i + P_o} + \frac{P_o}{P_i + P_o} \right)^N
\]

and the corresponding average and variance are given by
\[
E[I_o] = \frac{NP_i q}{P_i + P_o} \nu_0
\]
\[
\text{Var}[I_o] = \frac{NP_i q^2}{(P_i + P_o)^2} \nu_0^{(2)} - \frac{NP_i q^2}{P_i + P_o} \nu_0
\]

where \( q = 1 - \frac{R_2^2}{R_1^2} \), \( P_i \) and \( P_o \) are calculated with \( R_1 \) and \( \Delta \).

**Proof:** The total number of users within the interfering circle \( C' \) follows the Binomial distribution defined by Theorem 1. For any user who has stepped into the interfering circle \( C' \), its location will be uniformly distributed in the area due to the randomness of its moves. Thus its probability to lie in the interferer ring is
\[
Pr(\xi_2 = k) = Pr(\xi_3 - \xi_1 = k)
\]
\[
= \sum_{i=0}^{N-k} \Pr(\xi_1 = i, \xi_3 = k + i)
\]
\[
= \sum_{i=0}^{N-k} \Pr(\xi_3 = k + i) \Pr(\xi_2 = k | \xi_3 = k + i)
\]
\[
= \sum_{i=0}^{N-k} \Pr(\xi_3 = k + i) C_k^k q^k (1 - q)^{i-k}
\]

Next, the MGF of \( \xi_2 \) is
\[
G_{\xi_2}(s) = \sum_{k=0}^{N} z^k \Pr(\xi_2 = k)
\]
\[
= \sum_{k=0}^{N} \sum_{i=0}^{N-k} \Pr(\xi_3 = k + i) C_k^k q^k (1 - q)^{i-k}
\]
\[
= \sum_{k=0}^{N} \sum_{j=k}^{N} \Pr(\xi_3 = j) (qz + 1 - q)^k = \xi_3 (qz + 1 - q)^k
\]
where variable substitution \( j = k + i \) is used in (a).

Then the MGF of total interference \( I_o \) will be
\[
G_{I_o}(s) = E[e^{sI_o}] = E[e^{s \sum_{j=1}^{\xi_2} I_{o,j}}] = \sum_{k=1}^{N} \Pr(\xi_2 = k) \left( E[e^{sI_{o,j}}] \right)^k
\]
\[
= G_{\xi_2}(G_{I_o}(s))
\]

which proves (12).

The average interference is
\[
E[I_o] = G_{I_o}'(s)|_{s=0} = G_{\xi_2}'(G_{I_o}(s))|_{s=0} = \frac{NP_i q}{P_i + P_o} \nu_0
\]

The second moment of \( I_o \) can be obtained by
\[
E[I_o^2] = G_{I_o}'(s)|_{s=0} = G_{\xi_2}'(G_{I_o}(s))|_{s=0} = \frac{NP_i q^2}{(P_i + P_o)^2} \nu_0^{(2)} + \frac{NP_i q^2}{P_i + P_o} \nu_0
\]
Finally, (13) will be proved by using $\mathbb{D}[I_o] = \mathbb{E}[I_o^2] - \mathbb{E}^2[I_o]$.

VI. SUCCESS PROBABILITY AND AVERAGE RATE

This section characterizes the system performance as a function of the random signal to noise-interference ratio (SINR), which is given by

$$\rho(\kappa) = \frac{P_f^I(\kappa)}{I + P_n},$$

where $I$ is the uplink interference and $I = I_o$ for closed femto cells, $I = I_o$ for open femto cells, $P_n$ is the noise power. $P_f^I(\kappa)$ is the received power at the H-eNB from a H-UE which is $d = \kappa R$ away, i.e.,

$$P_f^I(\kappa) = \frac{\kappa^2 P_f^I}{R^3}, \quad \kappa \in \left(\frac{1}{R}, 1\right),$$

where $P_f^I$ H-UE’s transmit power. It is assumed in this section that $\gamma$ follows the negative exponential distribution, namely the Rayleigh fading model,

$$f_\gamma(x) = \frac{1}{\gamma} e^{-\frac{x}{\gamma}}.$$

SINR is a useful quantification for performance analysis in cellular systems since system performance is usually interference limited, especially for users at the cell edge. In this formulation, it is assumed that no pre-coding or multi-user detection are used at the H-eNB. Thus signals from unexpected users contribute to interference only.

Two metrics of performance are evaluated: the success probability defined as $\Pr\{\rho(\kappa) \geq T\}$ where $T$ is a given threshold and the average achievable rate is given by

$$C(\kappa) = W \ln(1 + \rho(\kappa)),$$

where $W$ is the system bandwidth.

The success probability is a measure of the impact of interference and channel fading on system performance. The achievable rate indicates the capability of the cell in serving H-UEs.

A. Success Probability

The success probability of both closed and open femto cell UEs can be summarized in the following proposition.

Proposition 6. The success probability of a femto user is given by

$$\Pr\{\rho(\kappa) \geq T\} = \exp\left(-\frac{\kappa^2 R^3 P_f^I T}{P_f^I P_\gamma}\right) G_\gamma\left(\frac{\kappa^2 R^3 T}{P_f^I P_\gamma}\right),$$

where $\kappa = \frac{d}{R} \in \left(\frac{1}{R}, 1\right)$ is an indicator of the distance between the user and the H-eNB, and $P_n$ is the noise power. Particularly, $G_\gamma(s) = G_\gamma(s)$ is given by (7), Theorem 2 for closed femto cells and $G_\gamma(s) = G_\gamma(s)$ is given by (12), Theorem 2 for open femto cells.

Proof: Whether one user can access to the H-eNB successfully or not is determined jointly by the instant channel gain and the instant uplink interference. Thus the success probability will be

$$\Pr\{\rho(\kappa) \geq T\} = \int_0^\infty f_I(x) dx \int_0^\infty f_\gamma(y) dy$$

where

$$G_\gamma(s) = G_\gamma(s)$$

for closed femto cells, $G_\gamma(s)$ is the MGF of the interference from macro-UEs to closed/femto cells, given by (9) and (12), respectively.

In either Theorem 2 or Theorem 3, the MGFs of uplink interference are presented in terms of $G_\gamma(s)$ and $G_\gamma(s)$, respectively, namely the MGF of the interference from a certain macro interferer $j$. For Rayleigh fading and commonly used pathloss exponents, one has

$$G_\gamma(s) = \mathbb{E}[e^{sI_S}] = \mathbb{E} \left[e^{s\gamma^\alpha}\right]$$

where

$$= \exp\left(-\frac{\kappa^2 R^3 P_f^I T}{P_f^I P_\gamma}\right) G_\gamma\left(\frac{\kappa^2 R^3 T}{P_f^I P_\gamma}\right).$$

Besides, simple closed expressions can be obtained for some special cases such as $\beta = 2$ and $\beta = 4$.

$$G_{\Delta \gamma}(s) = 1 + \frac{P_m P_s^I}{R_1^2 - R_2^2} \ln \frac{R_1^2 - R_2^2 - P_m^I P_s^I}{R_1^2 - P_m^I P_s^I}, \quad \beta = 2;$$

$$G_{\Delta \gamma}(s) = 1 + \frac{\sqrt{P_m^I P_s^I}}{2(R_2^2 - R_2^2)} \ln \frac{R_2^2 - P_m^I P_s^I}{R_2^2 - P_m^I P_s^I},$$

Likewise, one has

$$G_{\Delta \gamma}(s) = \mathbb{E}[e^{s\Delta I_S}] = \mathbb{E} \left[e^{s\Delta^\alpha}\right]$$

and two special cases are

$$G_{\Delta \gamma}(s) = 1 + \frac{\sqrt{P_m^I P_s^I}}{2(R_2^2 - R_2^2)} \ln \frac{R_2^2 - P_m^I P_s^I}{R_2^2 - P_m^I P_s^I}, \quad \beta = 2;$$

$$G_{\Delta \gamma}(s) = 1 + \frac{\sqrt{P_m^I P_s^I}}{2(R_2^2 - R_2^2)} \ln \frac{R_2^2 - P_m^I P_s^I}{R_2^2 - P_m^I P_s^I}.$$
B. Average rate

The average rate is another important evaluation of UEs’ performance, especially for those edge UEs. The average rate at position $\kappa$ is integrated over the random interference and the fading channel gain, and is summarized in the following proposition.

Proposition 7. The average rate of a femto user at position $\kappa = \frac{d}{R}$ is

$$\mathbb{E}[C(\kappa)] = W \int_0^{\infty} \frac{P_i^f P_v e^{-P_n x}}{R^\beta + P_i^f P_v x} G_I(-x) dx,$$

where $G_I(s) = G_{I_1}(s)$ for closed femto cells and $G_I(s) = G_{I_u}(s)$ for open femto cells.

Proof: The randomness of $C(\kappa)$ comes from the fading of the channel as well as the randomness of uplink interference. Its CDF is given by

$$F_C(x) = \Pr\{C(\kappa) \leq x\} = \int_0^{\infty} f_I(z) dz \int_{z + P_n}^{\infty} f_v(y) dy = 1 - \exp\left(-\frac{-\kappa^\beta R^\beta P_n}{P_i^f P_v}(e^{\frac{x}{R^\beta}} - 1)\right) G_I\left(\frac{-\kappa^\beta R^\beta}{P_i^f P_v}(e^{\frac{x}{R^\beta}} - 1)\right).$$

Then the average rate will be

$$\mathbb{E}[C(\kappa)] = \int_0^{\infty} x dF_C(x) = -\int_0^{\infty} x \exp\left(-\frac{-\kappa^\beta R^\beta P_n}{P_i^f P_v}(e^{\frac{x}{R^\beta}} - 1)\right) G_I\left(\frac{-\kappa^\beta R^\beta}{P_i^f P_v}(e^{\frac{x}{R^\beta}} - 1)\right) dx + \int_0^{\infty} \exp\left(-\frac{-\kappa^\beta R^\beta P_n}{P_i^f P_v}(e^{\frac{x}{R^\beta}} - 1)\right) G_I\left(\frac{-\kappa^\beta R^\beta}{P_i^f P_v}(e^{\frac{x}{R^\beta}} - 1)\right) dx$$

$$= W \int_0^{\infty} \frac{P_i^f P_v e^{-P_n x}}{R^\beta + P_i^f P_v x} G_I(-x) dx.$$

VII. Numerical Results

Consider a heterogeneous network as shown in Fig. 5. There is a macro-eNB (M-eNB) and $N = 625000$ users in the macro cell, whose radius is $L = 2500$ m. Therefore, we have one user for every 10 m$^2$. There are also many femto/pico/relay cells in the network, which are covered by low power H-eNBs. Assume that transmit power of macro-UEs is $P_i^m = 20$ dBm. The transmit power of those H-eNB served H-UEs is about 23 dB lower, since they are much closer to their base stations, i.e., $P_i^f = -3$ dBm. Assume that the radius of the interested femto cell is $R = 80$ m, and the interfering radius is $R_I = 160$ m. Let $W = 5$ MHz be the system bandwidth and $N_0 = 3.98107 \times 10^{-18}$ be the noise power spectrum density. Thus the noise power is $P_n = W N_0$. If no otherwise specified, the Levy walk parameter is $\alpha = 0.6$ and the pathloss exponent is $\beta = 2$. Suppose the flight time is $T_s = 1$ s, then the user velocity will be $3 \sim 120$ km/h if we set $0.833 \leq \Delta \leq 33.3$, when the flight length equals to one basic step length.

First, the average coming in probability and going out probability are presented in Fig. 4. The coming in probability $P_i$ corresponds to the solid line and vertical axis on the left. In fact, $\Delta = 300$ corresponds to an impossible velocity for H-UEs. However, this figure is meant to show a full picture and tendency of the relationship among them. It is seen that as basic step length $\Delta$ increases, $P_i$ is increasing with $\Delta$ first and then decreasing. In fact, when $\Delta$ is relatively small, users who are far from the interested cell do not have much chance to step into the cell. This situation will be alleviated if $\Delta$ is increased, since more users will have chance to step into the cell. However, when $\Delta$ gets larger and larger, a user may simply step over the cell, which make $P_i$ smaller. On the other hand, the going out probability $P_o$ corresponds to the dashed line and the vertical axis on the left, which is monotonically increasing with $\Delta$. For a user within the cell, it will step out of the cell when $\Delta$ is large, and will definitely step out if $\Delta > 2R$. As is seen, $P_o = 1$ for large $\Delta$.

The statistics of the number of users in the interested small cell are presented in Fig. 5. It is seen that the average number of users in the cell $\mathbb{E}[\xi^+]$ is decreasing linearly if $\Delta$ is increased. This is mainly because the going out probability $P_o$ is increasing fast when $\Delta$ is small and $P_i$ is decreasing for large $\Delta$s. Furthermore, the number of users $\xi^+$ is a Binomial distributed random number according to Theorem 1 and Proposition 5 which turns to be a Poisson distributed one if $N$ is large and $P_i$ becomes smaller and smaller. As shown in Fig. 5, $\mathbb{E}[\xi^+] = \mathbb{D}[\xi^+]$ holds true when $\Delta$ is large, which shows that it is well approximated by Poisson distribution.
However, it is wondered that why \( \mathbb{E}[\xi^+] \neq R^2/L^2 \) when it is assumed that users are uniformly distributed. In fact, if the user velocity is zero, above equation will definitely holds. On the other extreme, i.e., if the user velocity tends to infinity, it can be inferred that the probability that a user stays at some specified area will be zero. Therefore, the average number of users in the area will also be zero, unless the problem is considered on a much larger area.

Fig. 6. The average/variance of the interference of closed and open femto cells v.s. \( \Delta \), presented by (a) and (b), respectively.

Fig. 7. The success probability of closed femto cells. In (a), \( \Delta = 30 \). In (b), \( \kappa = 0.9 \).

Fig. 8. The success probability of open femto cells. In (a), \( \Delta = 3 \). In (b), \( \kappa = 0.9 \).

Fig. 9. The average rate of closed femto cells. In (a), \( \Delta = 3 \). In (b), \( \kappa = 0.9 \).

Fig. 10. The average rate of open femto cells. In (a), \( \Delta = 3 \). In (b), \( \kappa = 0.9 \).

The success probability of a closed femto cell user, which is the probability that the received SINR \( \rho(\kappa) \) is larger than a threshold \( T \), is presented in Fig 7 (a) and (b). Particularly, \( \kappa = \frac{d}{\beta} \) is the ratio between the user’s distance to the H-eNB and the cell radius. It is seen from Fig 7 (a) that the success probability will decrease if \( \kappa \) is increased, which is due to the large scale signal attenuation. In Fig 7 (b), the success probability is increasing with basic step length \( \Delta \) (user velocity), especially when user velocity is small. In fact, the uplink interference will be smaller for larger \( \Delta \), as mentioned previously. In Fig 7 (a), compared with the line marked by circle on the top, the line marked by ‘+’ is evaluated with a larger Levy walk parameter \( \alpha \). Actually, larger \( \alpha \) means that users will take more short flights and be less mobile. As a result, the success probability is smaller. By using a larger threshold \( T = 10^{-5} \), the success probability becomes smaller once again, as shown by the line marked by ‘△’. Finally, the line at the bottom corresponds to a lager pathloss exponent \( \beta = 3 \). Due to the serious attenuation and interference, its success probability is the smallest. Similar observations are obtained in Fig 7 (b), which presents the relationship between success probability and user velocity.

Fig 8 presents the success probability of an open femto user. Likewise, success probability is decreasing with \( \alpha \) and \( T \), and is increasing in \( \Delta \), which is similar to that of closed femto
cells. However, if the pathloss exponent $\beta$ is increased from 2 to 3, the success probability also becomes larger, which is contrary to the case of closed femto cells. In both Fig.8 (a) and Fig.8 (b), the curve corresponds to $\beta = 3$ (labeled by ‘$\Delta$’) states better performance than that corresponds to $\beta = 2$ (labeled by ‘$\nabla$’). In fact, although the signal attenuation is more serious for larger $\beta$s, the uplink interference to open femto cells will be much smaller. As a result, the system performance gets improved.

Fig.9 and Fig.10 shows the relationships between the average uplink transmission rate of a H-UE and distance/moving velocity, for closed femto cells and open femto cells, respectively. In both cases, Levy walk parameter $\alpha$ does not affect the average rate much. On the other hand, average transmission rate is slightly increasing with basic step length. In fact, the fluctuation in the uplink interference affects more on its high order statistics than its average. These two figures also show that the performance of open femto cells will be better for large $\beta$s, which is opposite to closed femto cells.

Finally, the scaling of the performance of both closed and open femto cells with user densities, i.e., number of users per square meter is evaluated in Fig.11. It is seen that, both success probability and average transmission rate will decrease when user density is increased, in which open femto cells perform better. However, it is easy to obtain the total throughput of the network, which is still increasing with user density.

VIII. CONCLUSION

In this work, a mobility-aware interference model for 5G heterogeneous networks was proposed, providing a convenient tool to evaluate the system performance such as success probability and average rate. From our work, some interesting insights can be drawn to help the system designs. As is shown, users with higher velocity produce less uplink interference. Besides, open femto cells suffer less interference compared with closed femto cells, leading to better performance. In addition, large pathloss exponents makes open femto cells perform even better but degrades that of closed femto cells. Intuitively, open femto cells can guarantee better frequency reuse and spectrum efficiency. Therefore, it is suggested that more open femto cells or public pico/relay cells be deployed, other than closed femto cells.

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