We consider a charged leptophilic extension of the Standard Model of particle physics as a minimal dark sector. It accommodates a WIMP paradigm at the TeV-scale that is sufficient to solve all small-scale problems of $\Lambda$CDM and explain the excess of highly energetic cosmic ray Standard Model electrons and positrons presented recently by the DAMPE collaboration. The predictive power of this model allows to test it in the near future.

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I. INTRODUCTION

In this work we are interested in the small-scale structures generated by a gauge invariant charged leptophilic ultraviolet-complete extension of the Standard Model (SM), which possibly explains the excess of highly energetic cosmic ray SM-electrons and positrons (CRE) presented recently by the DAMPE collaboration [1]. The CRE spectrum includes a tentative peak excess around energies of 1.4 TeV.

This excess suggests that Dark Matter candidates should be able to annihilate into SM charged leptons, especially into SM-muons and SM-electrons [2]. We study the thermal evolution of the proposed dark sector (DS) and we develop a charged leptophilic theory away from non-gauge invariant milli-charged couplings. We also study the small-scale structure of the proposed DM theory and we show that a specific mass hierarchy setup is able to solve the missing satellite problem together with the cuspy simulated profiles and the too big to fail issue; for a thorough discussion about such theories see [3] and [4]. We point out that since the DAMPE results [1] became public, a whole zoo of publications has appeared, e.g. [5], [6], [7], motivating our work; however, none of them was able to address the enduring small-scale problems of $\Lambda$CDM studying the corresponding small-scale structure. This research follows closely our previous neutrinophilic approach [8] and the general study of the dark QED paradigm [9] together with the work on small-scale structure from neutron dark decays [10].

This paper is organized as follows. In Sec. II we explain the problematic of the construction of the desired partition function and we introduce the proposed interactions in a gauge invariant leptophilic extension of the SM. Then, in Sec. III we compute various relevant observables in this framework and present in Sec. IV a list of strong constraints on the thermal evolution and the parameter set of the theory. Then we test the minimal leptophilic theory in Sec. V to find out whether the small-scale issues are cured successfully. Finally, in Sec. VI we conclude.

II. THE CHARGED LEPTOPHILIA

A. Motivation of the framework

In order to reproduce the peak excess in the CRE spectrum, we introduce a DM candidate at the TeV-scale, which admits non-negligible leptophilic interactions. In other words, these interactions take place between SM charged leptons and the DM particles and cannot be mediated via the SM photon, due to strong constraints on milli-charged theories. Therefore, the stable DM particles can not carry hypercharge and/or other SM-charges and vice versa: no SM-fields are charged under any dark symmetries. In addition, we assume that the DM candidate is a fermion following the stability of matter principle and the need of astrophysically interesting properties [11].

The only dimension-4 operator that realizes all the above considerations is that of a Yukawa type, where the DM fermionic field $e$ couples to a SM-charge lepton $l$. However, gauge invariance demands the presence of an auxiliary dark field $\Delta^+$ carrying exactly the opposite charges of $l$. For simplicity we assume that $\Delta^+$ admits only a hypercharge of +1 and therefore $l \in (1, 1, -1)$ is taken to be right-handed. This is the same type of interactions arising from a supersymmetrical treatment of the SM: a lepton-photon-slepton interaction; nevertheless, we do not consider such extensions in our work, since we are only interested in the consequences of this interaction on the small-scale structure.

B. The partition function

More precisely, the dark sector (DS) is based on a new global symmetry, which for the sake of minimalism of the theory is taken to be an Abelian $U(1)_r$. As explained previously, the minimal leptophilic theory is assumed to
consist of a stable dark electron field, which is charged under the global $U(1)_r$ with charge $r$ and is a SM-singlet with mass parameter $M$. In other words, the $e$-field can not be Majorana and has to be Dirac in accordance to the model independent study in Ref. [12]. We call it dark electron $e$, renaming the known right-handed charged lepton triplet $l_R$ to SM-electron $e_R$, SM-muon $\mu_R$ and SM-tau $\tau_R$. The dark electron is accompanied by an auxiliary scalar doublet $\Delta \equiv (\Delta^+, \sqrt{2}^{-1} \Delta^0)^T$, where the first field $\Delta^+$ is complex and admits a global charge $r$, a $Y$-hypercharge $+1$, and a mass $M'$, while the second component $\Delta^0$ is real and a total singlet with mass $M_0$. It is useful to define $\mathbb{P}_\pm$, the up/down projections in the doublet $\Delta$-space and the vector $\chi = (1, 1)^T$. This SM extension includes modes electrically charged and therefore departs from the so-called “nightmare scenario” of DM. Furthermore, in this work we areagnostic about the origin of the masses of the initial theory.

The UV-complete partition function of the charged leptophilic theory discussed above is given by

$$Z_r = \int D[\Psi_{SM}, \Psi] \exp\{iI[\Psi_{SM}, \Psi]\}$$

with $\Psi \in \{e, \Delta\}$, $\Psi_{SM}$ the SM-fields and $I$ the action functional using the following Lagrangian density

$$L_{DS} = \bar{e}K_e e + \Delta \bar{\chi}K_{\Delta} \chi + L_l + L_1.$$  \quad (2)

Here, we used the useful kinetic kernel abbreviations $K_e \equiv i\not\partial - M$, $K_{\Delta} \equiv \not\partial\not\partial - M_I M$ and the covariant derivative $\not\partial \equiv \not\partial - i\not\gamma Y B$. Furthermore, we consider for simplicity mass eigenstates for the scalar field $M = \text{diag}(M', M_0)$; all mass parameters are positive and $B$ is the SM-gauge boson associated with the hypercharge ($Y$).

The leptophilic virtue of the proposed theory is incarnated in the minimal gauge invariant Yukawa potential discussed previously between SM right-handed charged leptons, the dark electron and the auxiliary charged scalar field, namely

$$L_l = -g_i \bar{e} \left( \chi \not\partial + \Delta \right) l_R^i + \text{h.c.},$$ \quad (3)

with $g_i \in \mathbb{R}$ and $i$ running over all the components of the SM-charged lepton triplet.

Since the $\Delta^0$-field is a total singlet, further dimension-4 operators of purely dark interactions should be present in $L_l$; such terms, to the best of our knowledge, were not included and therefore not studied in the leptophilic-related literature so far. Here, we choose to present the most relevant ones for the small-scale structure formation of the DM cosmology,

$$L_1 \supset \sum a_i \left( \Delta \not\partial + \Delta \right) \left( \chi \not\partial + \Delta \right)$$

$$\frac{-\sqrt{2}}{\mu} \left( \not\partial + \not\partial \Delta \right) \left( \chi \not\partial + \Delta \right)$$

$$- \sqrt{2} g_0 \left( \not\partial + \not\partial \Delta \right) \left( \chi \not\partial + \Delta \right)$$

$$\frac{\sqrt{2}}{g_0} \left( \not\partial + \not\partial \Delta \right) e,$$ \quad (4)

with $a_i g_0 / \mu_+/0 / M' \in \mathbb{R}$. At this point, it is proper to give some hints towards the necessity of each of the above interactions regarding the phenomenological signature of the proposed leptophilic framework. The first term is responsible for the relic density depletion of the auxiliary charged scalars, the second one gives rise to the scenario where the neutral scalar modes are unstable, the third one enables IR-dominant, purely uncharged interactions, and the last one allows attractive self-interactions between the DM modes: it is this very interaction that enables a self-interacting DM (SIDM) scenario leading to a phenomenologically viable theory with clear predictive power.

### III. THE SPECTRUM HIERARCHY AND SCATTERINGS

We continue by presenting the spectrum hierarchy of the theory, which leads to extremely interesting phenomenological results due to a spectrum degeneracy of the participating particles. Furthermore, we compute important scatterings and decays of fields appearing in the DS partition function \footnote{\textsuperscript{[11]}}.

#### A. The spectrum

The dark electron $e$ and the charged scalar $\Delta^+$ admit masses at the TeV-scale. Moreover, as we mentioned before, we entertain the possibility of the presence of a very strong mass degeneracy, namely

$$M' - M = d > 0 \quad \text{with} \quad \delta \equiv \frac{d}{M} \ll 1,$$ \quad (5)

but still $d > m_e$, where $m_e$ is the mass of the SM-electron; furthermore we take $d < m_\mu, m_\tau$. In this work, we consider the benchmark mass $M = 1.4$ TeV to explain the peak-event of the DAMPE CRE measurements. Therefore, $M'$ is fixed to 1.4 TeV up to $O(m_e)$ corrections.

On the other hand, the uncharged scalar $\Delta^0$ should admit a mass well below the TeV-scale as explained in \footnote{\textsuperscript{[11]}}. Later on it will become clear that in order to obtain a viable small-scale structure scenario $M_0 \sim O(1)$ keV.

#### B. Scatterings in the leptophilic theory

We assume that $g_i \gg g_0$ in order to realize the electron/positron excess in the CRE spectrum. This means that the dark electron modes annihilate rapidly at $T < M$ into SM-charged leptons $l_i$ through the dominant $s$-wave channel to dark radiation,

$$\langle v_{\text{rel}} \sigma_{\text{ann}} \rangle e_{-i} = \frac{\pi \alpha_i^2}{8 M \pi} \sqrt{1 - \delta_i^2},$$ \quad (6)

where $\langle \ldots \rangle$ denotes the thermal average using relative velocities, $\alpha_i \equiv g_i^2 / 4\pi$ and $\delta_i \equiv m_i / M$. Here, it becomes
clear why a Majorana dark electron would disable the s-wave channel and lead to p-wave suppressed annihilations only, which is phenomenologically not acceptable [12].

The annihilations of the charged scalars into dark electrons is of no importance, due to the the heavy phase-space suppression, and their annihilations into SM-electrons is p-wave suppressed. However, the $\Delta^+\gamma$ particle number reduces mainly due to the annihilations into uncharged scalars as long as $M' > M_0$ with a thermal averaged cross-section of

$$
\langle v_{\text{rel}} \sigma_{\text{ann}} \rangle_{\Delta^+\rightarrow\Delta^0} = \frac{\pi \alpha_1^2}{4M'^2} \sqrt{1 - \frac{M^2_0}{M'^2}},
$$

assuming $a_1 \gg (\mu/M')^2$ and $a_1 \equiv a_1/4\pi$. Finally, their remaining negligible abundance is eliminated due to their rapid decays into SM-electrons and dark electrons.

We turn now our attention to the elastic scattering between the dark electrons/positrons and the SM-electrons. At temperatures $d, m_e \gg T$ and at lowest order of perturbation theory, the momentum transfer elastic cross-section, which is defined by $\sigma_T \equiv \int d\Omega (1 - \cos \theta) d\sigma_{\text{el}}/d\Omega$, is approximated to

$$
\sigma_T^\mp \approx \frac{\pi \alpha_0^2 \mp \delta}{4M^2},
$$

In the above expression the possible resonant behavior is hidden in the effective coupling constant $\alpha_{\text{res}}$, which is defined as

$$
\alpha_{\text{res}} \equiv \alpha_e \delta' P_{\mp}(\delta, \delta')^{-1}
$$

through the resonance polynomial in $\delta$ and $\delta'$ given by

$$
P_{\mp}(\delta, \delta') = 1 - \left(1 + \frac{\mu^2}{M^2}\right) + \frac{1}{2} (-\delta^2 + \delta'^2),
$$

with $\mp$ denoting the scattering of dark electron/positron modes on SM-electrons and $v$ the SM-electron velocity in the rest frame of the incoming dark electron/positron. Here, we omitted the lepton labels in the $\delta, \delta'$ taking $i = e$, because we consider only resonances near the SM-electron physical mass. One notices that as $\delta$ approaches $\delta'$ the dark positron cross-section $\sigma_T^\pm$ grows rapidly, which could lead to a kinetic freeze-in and therefore to a possible late decoupling regime. However, for temperatures of order $O(1) \text{keV}$ the SM-electron-positron velocity is $v \sim 0.1$, which bounds the cross-section by the SM-electron mass. The dark electron cross-section $\sigma_T^\mp$ could be of similar strength but it is always bounded by the SM-electron mass $m_e$. We note that the above realization is only valid as long as

$$
|P_{\mp}(\delta, \delta')| \gtrsim \frac{4\pi \alpha}{9} \left(\frac{T}{M}\right)^2,
$$

which means that the thermal mass of the SM-electron in the SM-photon-plasma [13] is much smaller than the effective mass of the corresponding $s/u$-channel mediator. Here, $\alpha$ is the SM fine-structure constant. During the above discussion we ignored the optical term $\Gamma/M'$ in the polynomial $P_{\mp}(\delta, \delta')$, due to the total decay width $\Gamma$ of $\Delta^\pm$, since it is negligible compared to the plasma temperature. In this work, we will also investigate, whether this resonant scattering suffices to provide solutions to at least some of the small-scale problems of the $\Lambda$CDM.

Even after the chemical freeze-out of the dark matter modes the scattering between them and the uncharged scalars with energy $E$ becomes also of great importance, since it allows a natural late decoupling scenario. More precisely, one obtains for the IR-dominated part of the momentum transfer elastic cross-section

$$
\sigma_T \approx \pi \alpha_0^2 M^2 \frac{1}{4M^2} \log \frac{[4E^2 - 3M_0^2]}{[E^2 - \frac{M_0^2}{4}]} \frac{E^2 - M_0^2}{8(E^2 - M_0^2)^2},
$$

where we defined $\alpha_0 \equiv g_0^2/4\pi$. One notices that as $T$ approaches the rest mass of the uncharged scalar, the elastic scattering becomes extremely dominant.

The minimum leptophilic theory possesses all properties of a prototype self-interacting dark matter theory (SIDM), which is realized through the uncharged scalar mediator. The SIDM cross-sections per dark matter mass, $\sigma_T$, are purely attractive and for given average velocities are strongly velocity-dependent in the regime $2\alpha_0 M_0/(M v_{\text{rel}}^2) \lesssim 10^3$, where $\alpha_0 \equiv g_0^2/4\pi$. Such interactions appear to be a necessary ingredient in order to resolve small-scale problems that are present during structure formation in non-SIDM. The numerical solutions in various regimes of the momentum transfer cross-sections can be found in Refs. [13], [14].

The charged scalar decays at tree-level exclusively into dark electrons and positrons and SM-charged leptons (here only SM-electrons and positrons) due to the mass gap $d > m_e$. This is described by the decay width

$$
\Gamma_{\Delta^+\rightarrow e_R e_R} \approx \alpha_e \delta' \sqrt{\delta^2 + \delta'^2} M.
$$

This corresponds for example to a life time $\sim 10^{-12}$ s for $M = 1.4$ TeV, when taking conservatively $d = m_e + 0.1\%$ with $\alpha_e = 0.1$.

Concluding, we note that the uncharged scalar is in general unstable and decays, otherwise it would possibly over-close the universe. Moreover, these scalars decay dominantly into photons assuming $M_0 < 2m_e$. The decay width is loop-suppressed and it is given by

$$
\Gamma_{\Delta^0\rightarrow \gamma\gamma} \approx O(1) \alpha^2 \left(\frac{M_0}{M}\right)^4 \left(\frac{\mu_+}{M_0}\right) \mu_+.
$$

For $T > M_0$, the inverse rate $\Gamma_{\Delta^0\rightarrow \gamma\gamma}$ is a bosonic source, which is rescaled by a factor $M_0/2T$ relative to the decay width.
IV. PARAMETERS AND CONSTRAINTS

A dark matter theory is always subject to various constraints in a purely phenomenological approach. Therefore, in this section we discuss such constraints on the parameter space.

A. Particle physics bounds and detection

The physical modes that interact directly with photons and SM-leptons should admit a mass of at least 100 GeV, considering the lowest bound on neutralino and slepton masses \([15]\). This constraint does not restrict the proposed theory, since the dark electrons and the charged scalars are assumed to be responsible for the 1.4 TeV CRE excess measured by the DAMPE experiment.

We assume that all couplings lie in the perturbative domain. For the dimensionful parameters \(\mu_+/0\) this translates to \(\mu_+ \lesssim M'\), which arises from the one-loop vertex correction of the mixed-scalar cubic interaction, and \(\mu_0 \lesssim M_0\). For example, the mass-squared correction of the uncharged scalar after the one-loop calculation due to this mixed interaction is given by

\[
\Sigma_{\Delta \phi}^\pm(q^2)/q^2 \approx -\frac{1}{192\pi^2} \left(\frac{\mu_+/M'}{M'}\right)^2 \tag{15}
\]

for \(q^2 \ll M'\). Therefore, assuming that the new physics regarding the origin of the parameter \(\mu_+\) is present not well above the mass scale of the dark electrons and the charged scalars, we demand conservatively \(\mu_+/M', \mu_0/M_0 \ll 1\) as in \([3]\).

The couplings appearing in the leptonophilic Yukawa sector are subject to further constraints. Firstly a WIMP condition should be present: as we mentioned before, the dark electrons and positrons are thermally produced WIMPs; therefore, the condition on the coupling constants of the theory, \(\alpha_i\), reads

\[
\alpha_i > \left(\frac{M}{M_{Pl}}\right)^{1/2}. \tag{16}
\]

It is important to note that the proposed new interactions between SM-charged leptons should eventually contribute to the anomalous magnetic moment of the SM-lepton. We calculated the corresponding factor due to the one-loop vertex correction at zero momentum transfer as

\[
\lim_{q^2 \to 0} \delta F_2^i = -\frac{\alpha_i}{48\pi} \delta'^2. \tag{17}
\]

The above contribution should be less than the measured values for the leptonic Landé-factors, \(\frac{g-2}{2}\), found in \([16]\). Furthermore, one notices that any perturbative couplings are admissible for all the SM-leptons. If, on the other hand, parity were conserved and the Yukawa portal couples also to left-handed SM-leptons in a similar manner, then the correction would be \(\frac{\alpha_\mu}{2\pi} \delta'_\mu\) and the coupling constants are severely suppressed. In other words, the gauge invariant chiral portal protects the interaction strength and allows the existence of thermally produced WIMPs at the TeV-scale.

Moreover, the presence of the charged scalar induces an effective magnetic moment for the dark electron of order \(\mu_e \approx 10^{-5} \mu_p\), leading to a spin-dependent cross-section

\[
\sigma_{\text{proton}} \approx 3\alpha \mu_e^2 \sim \alpha^2 \sigma_{\text{ann.}e\rightarrow\gamma} \sim \mathcal{O}(10^{-40})\text{cm}^2, \tag{18}
\]

where for simplicity we assumed that \(\alpha_i = \alpha_e \delta_e\). A similar problematic applies to the spin-independent cross-sections. Such cross-sections are in line with the LUX and XENON indirect detection measurements of WIMP-proton elastic scatterings \([17, 18, 19, 20]\).

Ending the discussion about the present bounds on parameters which arise from particle physics, we note that for \(M, M' \approx 1.4\) TeV no constraints on the coupling constant \(\alpha_e\) can be inferred from LEP measurements \([21]\); however, the TeV-scale should be directly testable in the new ILC experiment \([21]\).

B. Bounds on light degrees of freedom

In this work, the benchmark point given by \(M = 1.4\) TeV and \(M_0 \sim \mathcal{O}(1)\) keV is of great interest; these light modes of the uncharged scalars decouple from the SM-plasma when the heavy particles become non-relativistic at \(T \sim M\). However, if these modes freeze-in due to IR-dominant processes before the SM-neutrinos decouple, this could modify the interactions responsible for the big bang nucleosynthesis (BBN). Examining the parameter set of the leptonophilic theory, we find that for perturbative couplings this interference is not expected to be realized. More precisely, we study the deviation of the effective SM-neutrino degrees of freedom, parametrizing the relativistic energy budget of the universe following \([8]\).

For example, for \(M_0 \sim \mathcal{O}(1)\) keV the BBN processes are not affected and only around \(x \sim 1\) with \(x := M_0/T\) the SM-photon-plasma cools down and during the BBN period we obtain \(\Delta N_{\text{eff}} |_{\text{BBN}} \approx 0.03\). Nevertheless, after the decay of the uncharged scalars into photons, the photon-plasma experiences a reheating and the final deviation of the effective SM-neutrino degrees of freedom can be estimated as \(N_{\text{eff}} \approx 2.97\) using equilibrium physics as first order approximation. Concluding, we note that both results are not only perfectly compatible with BBN \([22]\) and CMB \([23]\) 1\sigma measurements but they may lead also to a solution for the recent tension regarding the decrease of \(\Delta N_{\text{eff}}\) from BBN to CMB-based measurements, \(\Delta N_{\text{eff}} |_{\text{CMB}} - \Delta N_{\text{eff}} |_{\text{BBN}} < 0\). Finally, if \(M_0\) is less than the recombination temperature, then \(\Delta N_{\text{eff}} |_{\text{CMB}} = 0.03\), which is still CMB 1\sigma-compatible.
V. THERMAL EVOLUTION TOWARDS A SOLUTION TO THE DM PROBLEM

Until this point, we defined the complete framework of the proposed theory and we set the most recent bounds on the corresponding parameter space. Now we are ready to compute the DM relic density, which consists of electrons and positrons and to study their kinetic decoupling behavior. All the temperatures are given in the SM-photon-plasma frame.

A. The present DM density

In this section the following assumptions has to be made for simplicity: the DM relic abundance consists mostly of the dark electrons and dark positrons. We thus neglect the relic density of the charged scalar $\Delta^+$, i.e. no mixed dark matter is present. This assumption is naturally accommodated in our theory for the given spectrum hierarchy, since the $\Delta^+$ scalars decay before the dark electrons and positrons acquire their relic abundance.

In order to compute the relic density, we followed the same procedure as in [8], i.e. we solved the Boltzmann equation numerically requiring that the dark modes depart significantly from their equilibrium distribution at the time of the chemical freeze-out. The DM relic density is found to be

$$\Omega_{\gamma}h^2 \approx \frac{1}{2} \times 0.12 \left( \alpha_i \right)^2 \left( \frac{M}{1.4 \text{ TeV}} \right)^2,$$  

with the leptophilic coupling constant $\alpha_i \equiv \sqrt{\sum_{\delta \delta} } \alpha_i^2$. The 1/2 factor in front is due to the assumption of a symmetric $e^-, e^+$ ensemble. One finds that the total annihilation cross-section is of order $\sim 10^{-26} \text{cm}^2 \text{s}^{-1}$, which can explain nicely not only the CRE peak of the DAMPE spectrum but also the sizable hidden excess in the DAMPE measurements between 0.6 TeV and 1.1 TeV if the proposed flavor structure is adopted. The constraints and the relic abundance fix the leptophilic coupling constant $\alpha_i$ completely but not the generation-oriented ones, i.e. $\alpha_i$.

B. Multiple kinetic decoupling regimes

There are mainly two different mechanisms in the elastic scattering history of the dark electrons and dark positrons in this minimal leptophilic framework: firstly, the dark positrons are able to scatter resonantly with the SM-electrons, and, secondly, all DM particles could interact efficiently with the uncharged scalars through an IR-dominant $t$-channel process.

We start by studying the first case, assuming that the IR-dominant scalar channel is absent: the elastic scattering of the dark positrons with the SM-electrons. As long as the SM-electrons become non-relativistic the dark positrons depart from their kinetic equilibrium with the SM-electrons and reenter this regime only near the resonance of the corresponding elastic cross-section $\sigma^+_T$. More precisely, the situation appearing in the thermal evolution of the proposed theory is somewhat more intricate: not all dark thermal relics are in LTE at later times with the SM-electrons, only the dark positrons admit this virtue and only for a limited time window: furthermore, the SM-electrons are already thermal relics themselves after their annihilation to SM-photons took place around $T \sim m_e$. Therefore, the SM-electron density is characterized by the present photon to baryon ratio $\eta$ assuming an electrically neutral universe after the BBN epoch as in [10, 24], and, technically, one can estimate the momentum transfer scattering rate [24], using the following expression

$$\Gamma_{\gamma \ell}(T) \approx \frac{5}{\pi} \frac{\delta}{1 + \delta^2} \frac{\sigma^+_T(\delta, \delta)}{m_e} T^{7/2}.$$  

Equating the above rate with the Hubble one, we obtain the approximate kinetic decoupling temperature $T_{kd}$. As a benchmark point we choose thermal relics with $M = 1.4 \text{ TeV}$ and $d = m_e + 0.1\%$, or, equivalently, $(\min \| P_{\ell}(\delta, \delta) \|)_{T=420 \text{eV}}^{1/2} M \approx 720 \text{ keV} \gg T_{\text{BBN}}$. This parameter set yields a temperature for the kinetic decoupling $T_{kd} \approx 420 \text{ eV}$. For this choice of parameters the dark positrons and SM-electrons are in kinetic equilibrium for around two hours. Furthermore, we notice that $T_{kd}$ is extremely sensible to small changes of the mass gap $d$ between $M'$ and $M$, but it changes slowly as $M$ runs in the TeV-regime. This indicates a possibly unstable late kinetic decoupling: for temperatures slightly higher than $T_{kd}$ the dark positrons may not be coupled to the SM-electrons at all, due to the tiny value of $\eta$. Indeed, one finds out that these two hours are not sufficient for the DM modes to enter a kinetic equilibrium with the SM-electrons ruining the hopes for a viable late kinetic decoupling regime of leptophilic DM theory even through a resonant scattering channel; quantitatively only one out of ten DM particles experiences a collision during that time interval.

However, this leptophilic theory provides an alternative way towards $\mathcal{O}(1) \text{keV}$ values of kinetic decoupling temperatures: the elastic scattering of the DM particles with the uncharged scalars, which turns out to be stable enough. The dark electrons and dark positrons are in LTE with the $\Delta^0$-particles at high temperatures; when the temperature falls below the rest mass of the uncharged scalar field, it annihilates and decays into photons. The main consequence is the rapid reduction of the $\Delta^0$-density, which makes the remaining elastic scatterings between the DM particles and the uncharged scalars exponentially suppressed. The rate for the elastic scattering with $\Delta^0$ can be found numerically using the rate
of the averaged momentum cross-section as in \[23\] and is
\[
\Gamma_{0\text{el}}(T) \approx \frac{1}{3\pi^2 M} \int_{M_0}^{\infty} dE f_{\Delta^0}(E) \frac{\partial}{\partial E} \left(\frac{E^2 - M_0^2}{2\mu_0} \sigma_T\right).
\]
(21)
Here, \(f_{\Delta^0}(E)\) is the distribution function per d.o.f. of the uncharged scalar field at temperature \(T\). In order to approximate the time when the these elastic scatterings cease to be efficient, we equate as before the elastic scattering rate with the Hubble one. Therefore, the kinetic decoupling temperature \(T_{\text{kd}}^0\) is to be found. It is interesting to note that \(T_{\text{kd}}^0\) is not very sensible to small changes of \(M_0 \sim \mathcal{O}(1)\) keV. For the benchmark point \(M_0 = 7.1\) keV, \(\alpha_0 = 1.3 \times 10^{-4}\), and \(\mu_0 = \frac{1}{2} M_0\) we obtain \(T_{\text{kd}}^0 \approx 420\) eV.

Finally, we present the above discussed rates and results for the given parameter set in Figure 1. We remark that smaller values of \(M_0\) give rise to lower decoupling temperatures, thus depleting the small-scale structure of dark matter and leading to unacceptable results.

C. The small-scale structure

A late kinetic decoupling has positive effects for structure formation at small scales: it yields easily the correct protohalo masses and helps alleviating the cusp vs. core and too big to fail problems as well, as argued in \[25\] and \[26\]. Moreover, for optimal results the kinetic decoupling should happen after BBN following the discussions in \[8\], \[9\], \[10\] but long before the recombination period, in order to satisfy the strong Lyman-\(\alpha\) constraints \[21\], \[22\]. In other words, the usual solution to the missing satellite problem of \(\Lambda\)CDM-cosmology can be found after suppressing the linear power spectrum at scales as large as that of dwarf galaxies \[26\], \[29\], \[30\], i.e. for damping masses corresponding to the desired temperatures of the late kinetic decoupling.

Furthermore, the combination of the SIDM effects, which lower the inner densities of the galaxies energizing the particle modes, together with the suppression of the linear power spectrum of the leptophilic theory leads to a possible cure of the two remaining small-scale issues of \(\Lambda\)CDM-cosmology in the non-linear regime: the cusp vs. core and the too big to fail problems. Until now, there was no leptophilic theory able to explain the CRE excess in the DAMPE spectrum and at the same time yield the correct small-scale structure.

VI. RESULTS, DISCUSSION AND CONCLUSION

The constrained parameter set of the proposed theory includes naturally values of SIDM elastic cross-sections of order \(\sim 0.1 - 10\) cm\(^2\)g\(^{-1}\) \[31\] as long as \(M_0 \sim \text{keV}\) and \(\alpha_0 \sim \mathcal{O}(10^{-4})\). We choose a benchmark point for thermally produced WIMPs with \(M = 1.4\) TeV, \(M_0 = 22\mu_0 = 7.1\) keV, and a singlet coupling \(\alpha_0 = 1.3 \times 10^{-4}\); therefore, a late kinetic decoupling scenario exists independently from possible resonances, namely \(T_{\text{kd}} \approx 0.42\) keV, and at the same time the thermally averaged velocity dependent SIDM cross-sections are similar to those of the tuned ETHOS-4 model \[26\]. This model is known in the DM community to be compatible with all present constraints and alleviates the missing satellite, the too big to fail and also the cusp vs. core problems \[32\].

In Ref. \[33\] a mysterious photon-ray measurement at 3.55 keV is presented, which indicates the possible existence of lighter particles (well below the TeV-scale) in the DM spectrum. This line is naturally accommodated in our model, since the scale for the decaying \(\Delta^0\) seems to be the keV-scale in order to alleviate the small-scale abundance problem. In other words, the minimal leptophilic theory does not only explain the CRE excess at the TeV-scale in the DAMPE spectrum \[1\], but it also favors at least one much lighter particle to cure all the small-scale problems simultaneously, delivering damping masses and SIDM cross-sections of the desired order and predicting the cold DM present abundance.

In this paper we showed for the first time that charged leptophilic dark matter candidates could solve all enduring small-scale problems of the \(\Lambda\)CDM-cosmology and at the same time stay in line with all recent astro-physical experiments. The proposed theory includes a gauge invariant Yukawa portal, which couples the dark electron field to the SM right-handed electron one. Two auxiliary fields are needed: a heavy charged scalar and a much lighter uncharged scalar. However, even if a strong mass degeneracy between the dark electron and the charged scalar is present, the delivered small-scale structure is
 unacceptable if no uncharged scalars are included in the theory. Therefore, the existence of the uncharged scalars is necessary to realize the SIDM nature of the theory together with a stable conventional late elastic scattering regime. This theory enables a parameter space of the tuned ETHOS-4 model \[20, \] and the properties of late decoupling and magnitudes of SIDM cross-sections are able to potentially solve the enduring small-scale problems of ΛCDM: the satellite abundances, the cuspy profiles and the massive subhalos. In addition, this minimal charged leptophilic UV-complete extension of the Standard Model (SM) possibly explains the excess of highly energetic cosmic ray SM-electrons and positrons (CRE) presented recently by the DAMPE collaboration and its predictive power allows further testing in the near future.

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