Return Period of Low Tide Level in the Yangtze Estuary based on Nonstationarity Analysis

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Abstract. Due to the influence of climate change and human activities, stationarity of hydrologic time series is being challenged. The Yangtze Estuary is a region with highly developed hydraulic structures and shipping. Stationarity analysis of water level in the Yangtze Estuary is of great significance. In this study, the return period of low tide level in the Yangtze Estuary is estimated with nonstationarity considered. Conventional frequency analysis and the time-varying moment method are used to analyze the annual minimum tide level (AMTL) records of Wusongkou Station and Baozhen Station on the basis of temporal change analysis. Abrupt changes are detected at 1996 and 1990 for Wusongkou Station and Baozhen Station separately. The GEV distribution with linear time-varying parameter fits best for Wusongkou Station and Baozhen Station. The AMTL series of both stations reveal a slight increasing trend. The 100-year low tide level of Wusongkou Station is approximately 0.261 meters, which is about 0.041 meters for Baozhen Station. The corresponding return period is 150 years and 119 years respectively, which indicates the existing channel standard more secure and the recalculation of design water level necessary in the Yangtze Estuary.

1. Introduction
In recent years, extreme hydrometeorological events occur more frequently across the world [1,2], which has received great attention by hydrometeorologists. The causes of the increase are complicated, among which climate change and human activities are the major influencing factors. According to former studies [3,4], the stationarity of hydrological time series is being challenged. Nonstationarity means the hydrologic time series obeys a time-varying distribution, i.e., the original time series could have trends or abrupt changes. Temporal change analysis of the observed hydrological time series should be conducted first. At present, it’s convenient and effective to use statistical tests for temporal change analysis [5]. The design value of certain frequency will change in nonstationary conditions and needs to be recalculated. Hydrological frequency analysis based on nonstationarity will help to improve water security management capabilities.

The Yangtze Estuary is located near Shanghai, one of the most developed and urbanized cities in China. The shipping of the Yangtze Estuary has greatly promoted the economy and international trade of Shanghai. Meanwhile, hydraulic engineering constructions are highly developed in this area. High speed urbanization, frequent human activities along with the global climate change all make the basic assumption of stationarity for hydrological frequency analysis unreliable in this area. Thus, the
revaluation of the magnitude and frequency of water level in Yangtze Estuary is extremely urgent.

There are many dynamic factors in the Yangtze Estuary, among which tide is the main driving force. Low tide in the Yangtze Estuary has negative impacts on navigation. Hydrological frequency analysis of low tide level in this area will help to adjust design water level values and improve water resource management abilities. At present, conventional frequency analysis is the most widely used method to evaluate the tide level of certain return period. The most frequently selected probability distributions (PDs) are Gumbel, Pearson-III, Weibull, Normal, Lognormal and GEV distribution [6,7,8,9].

As for extreme tide level series which may not meet the requirement of stationarity, the nonstationary frequency analysis methods for extreme precipitation and streamflow series should be introduced. The time-varying moment method is getting more and more attention nowadays, which allows the parameters of the PDs varying over time or other meteorological variables [10,11]. It's worth noting that the return period is no longer the reciprocal of exceedance probability under nonstationary conditions. By now, the most frequently used methods for nonstationary return period estimation are the expected waiting time (EWT) and expected number of exceedance (ENE) methods [12].

At present, nonstationarity is seldom considered when estimating the return period of tide levels in the Yangtze Estuary. In this study, temporal change characteristics of tide levels in the Yangtze Estuary are analyzed at first. The conventional frequency analysis is conducted to choose an optimal distribution model. Then, the time-varying moment method is used to evaluate the frequency of different tide levels. Finally, return periods under stationary and nonstationary conditions are estimated and compared to analyze the influences of nonstationarity to design tide levels.

2. Study area and data sets
The Yangtze Estuary is the area where the Yangtze River runs into the East China sea. According to statistics, the Yangtze Estuary is the medium-intensity tidal estuary, where the effect of tide on water level cannot be ignored. In this study, Wusongkou Station and Baozhen Station were selected as the representative stations, whose locations are shown in figure 1. For Wusongkou Station, AMTL records from 1978 to 2015 were used, while data during 1965-2016 were used for Baozhen Station.

![Figure 1. The location of Wusongkou Station and Baozhen Station.](image)
3. Methodology

3.1 Conventional Frequency analysis
In this study, the Gumbel, Pearson-III, Weibull, Normal, Lognormal and GEV distribution are used to fit the AMTL records in the Yangtze Estuary. For parameter estimation, the maximum likelihood method (MLM) and the linear moment method (LMM) are used.

3.2 Time-varying Moment Method
The time-varying moment method uses the conventional PDs, while the location, scale or shape parameter varies with time or other meteorological variables. Most often, a trend component is added to the first moment or secondary moment, namely, mean or variance of the distribution. Assuming that the sample size of the time series \( X(t) \) is \( n \), and \( X(t) \) follows a distribution with time varying parameters \( \theta(t) \), the time-varying moment model can be expressed as

\[
X(t) \sim D(\theta(t))
\]

where \( D \) is the distribution \( X(t) \) follows. Akaike Information Criterion (AIC) and Schwarz Bayes Criterion (SBC) are used to select the optimal nonstationary model. AIC and SBC are calculated as

\[
\text{AIC} = -2\ln L(\theta_\mu, \theta_\sigma, \theta_\xi) + 2df
\]

\[
\text{SBC} = -2\ln L(\theta_\mu, \theta_\sigma, \theta_\xi) + \log(n)df
\]

where \( L(\theta_\mu, \theta_\sigma, \theta_\xi) \) is the maximum likelihood function of the models with time-varying parameters \( \theta_\mu, \theta_\sigma \) and \( \theta_\xi \), \( df \) is the number of independently adjusted parameters of the model and \( n \) is length of the time series.

3.3 Return Period Estimation Under Nonstationarity
In this study, the EWT method is adopted for nonstationary return period estimation, which defines the return period as the expected waiting time until the critical value occurs. The EWT method focuses on the time-varying exceedance probability \( P_t \), which is defined as follows

\[
P_t = P(X_t > x) = F_t(x)
\]

where \( X_t \) is the time-varying tide level, \( x \) is the tide level of certain frequency, and \( F_t(x) \) is the time-varying CDF. Based on \( P_t \), the return period \( T \) under nonstationarity is

\[
T = 1 + \sum_{x=1}^{\infty} \prod_{t=1}^{x} (1 - P_t)
\]

It is worth noting that equation (5) is suitable to the annual maximum time series, while for the annual minimum time series, \( P_t = P(X_t < x) = 1 - F_t(x) \). The trend under nonstationarity extends indefinitely when using EWT method. On this condition, \( P_t \) tends to 0 or 1 prematurely, which does not conform to the change of hydrological processes. Therefore, this study adopts the EWT method which introduces trend duration \( M \) [13]. After the duration, the probability distribution of the newly reached stable state of the river basin is \( P_M \), and the calculation formula of EWT return period is

\[
T = \sum_{t=1}^{M} t \cdot P_t \cdot \prod_{i=1}^{t-1} (1 - P_i) + \prod_{i=1}^{M} (1 - P_i) \cdot (M + \frac{1}{P_M})
\]
4. Results and discussion

4.1 Temporal change analysis
In this study, Pettitt test is used for abrupt change detection. The results show that the abrupt change of the AMTL series of Wusongkou Station and Baozhen Station happened in 1996 and 1990 respectively. The abrupt variation could be caused by the change of station setting, datum or measurement method. It is necessary to check and correct the measured low tide records. Besides, abrupt change of series with small sample size could be a local feature of hydrological processes, which requires analysis of series with more records. Considering that recent hydrological situation has a great influence on the future tide level [14], the series before the point of variation was corrected to the level after the change point. The AMTL series after the revision is shown in figure 2.

\[ Y_t = S_t + Kt \quad (t = 1, 2, \ldots, n) \]  

where \( Y_t \) is the generated time series with a linear trend, \( S_t \) is white noise, \( K \) is the slope representing the trend component, \( t \) is the time order and \( n \) is the length of the time series. In this study, \( n \) was set as 10, 20, 30, 40, 50, 70, 90, 100 and 200, and \( K \) was set as 0.1, 0.01 and 0.001. The rejection rate [15] of the Mann-Kendall test is calculated as

\[ ERI = \frac{N_{\text{rej}}}{N} \]  

where, ERI is the rejection rate, \( N_{\text{rej}} \) is the number of times when null hypothesis is rejected and \( N \) is the number of simulations. The results show that the rejection rate decreases with the decrease of \( K \) and \( n \). When \( n \) is 40 and \( K \) is 0.001, the rejection rate is only 0.158. Under the circumstances, the results of Mann-Kendall test are unreliable and insufficient to judge whether a time series has a trend or not.

4.2 Conventional frequency analysis
The revised AMTL series meet the requirements of stationarity. Based on the parameter estimates, the cumulative distribution functions (CDFs) are used to compare the goodness of fitting and the CDFs of Wusongkou Station are shown in figure 3. For Wusongkou Station, the difference between the CDFs using MLM and LMM is small and the fitting effect is good for median records. For extremes, the Gumbel and Lognormal distribution have better fitting effect in the high end, while GEV, Pearson-III and Normal distribution fit the AMTL records better in the low end. While for Baozhen Station (the CDFs is omitted to save space), the difference between the CDFs using MLM and LMM cannot be ignored for Gumbel and Lognormal distribution, whose fitting effect are not satisfactory. The

![Figure 2. The original and revised AMTL series of Wusongkou Station and Baozhen Station.](image-url)
remaining four distributions perform well for median and large records. For extremes in the low end, Pearson-III, Weibull and GEV distribution have better fitting effect.

![CDFs of the AMTL series at Wusongkou Station.](image)

K-S test is used to test whether the time series is subject to a theoretical distribution. The statistical value $D_n$ of each distribution is shown in table 1. For Wusongkou Station, all the distributions pass the K-S test and the $D_n$ value of MLM is always smaller than that of LMM. The AMTL series of Baozhen Station obeys all the distributions used except Lognormal distribution. Besides, Gumbel, Pearson-III, Weibull and GEV distribution have smaller $D_n$ value when using MLM. In general, the MLM performs better and is more robust as a parameter estimation method in this study. It can be drawn that in the fitting of low tide level in the Yangtze Estuary, the MLM is more suitable, which is consistent with the current parameter estimation method used in extreme water level frequency analysis [9].

| Distribution | Wusongkou Station | Baozhen Station |
|--------------|-------------------|-----------------|
|              | $D_n$ (MLM)       | $D_n$ (LMM)     | $D_n$ (MLM)   | $D_n$ (LMM)   |
| Gumbel       | 0.131             | 0.135           | 0.1346        | 0.1424        |
| Pearson-III  | 0.094             | 0.102           | 0.0668        | 0.0712        |
| Weibull      | 0.103             | 0.109           | 0.0656        | 0.0666        |
| Normal       | 0.113             | 0.114           | 0.0805        | 0.0791        |
| Lognormal    | 0.102             | 0.105           | 0.1886        | 0.1154        |
| GEV          | 0.101             | 0.105           | 0.0680        | 0.0734        |

In this study, goodness of fit of different PDs are evaluated by five indexes, i.e., the correlation coefficient $r$, root mean square error $RMSE$, sum of absolute deviations $ABS$, sum of square deviations $OLS$, and sum of relative square deviations $WLS$. The results of goodness of fit are shown in table 2. In terms of the overall fitting effect, the GEV, Weibull and Lognormal distribution perform better in the case of Wusongkou Station. As for Baozhen station, the GEV, Weibull and Pearson-III distribution have smaller index values.

| Distribution | Wusongkou Station | Baozhen Station |
|--------------|-------------------|-----------------|
|              | $r$ | RMSE | ABS | OLS | WLS | $r$ | RMSE | ABS | OLS | WLS |
Based on the analysis above, the three parameter GEV distribution has good fitting effects in the low end of the AMTL series. Besides, the 95% confidence interval of its parameter is small, which means the parameters are sensitive to the AMTL series and the fitting error is small. With these aspects considered, the AMTL series of Wusongkou station and Baozhen Station obey the GEV distribution.

4.3 Nonstationary frequency analysis based on the time-varying moment method

The GEV distribution is selected when applying the time-varying moment method. Since the shape parameter of GEV distribution is quite sensitive and the estimation of which can hardly make the model converge, the shape parameter is assumed to be constant. In this study, three cases are considered, namely, the position parameter varies linearly with time, the scale parameter varies linearly with time and both position and scale parameters vary with time linearly.

The parameters estimated using MLM and the values denoting the goodness of fit are listed in Table 3. The GEV model with a linear time-varying position parameter has the best fitting effect in the case of Wusongkou Station. As for Baozhen Station, the position parameter of the optimal GEV model has a slight rising trend, while the scale parameter is with a slight falling trend. The decrease of scale parameter will lead to a reduction in variance of the time series, which means the distribution of low tide level will be more concentrated for Baozhen Station in the future. Increasing position parameter along with decreasing scale parameter will further increase the low tide level of certain frequency at Baozhen Station. The increase of low tide level in the Yangtze Estuary could be caused by the slow rise of sea level and the reduction of cross section caused by the construction of water conservancy projects.

| Parameter variation | Wusongkou Station | Baozhen Station |
|---------------------|-------------------|-----------------|
|                     | Estimated parameters | AIC values | SBC values | Estimated parameters | AIC values | SBC values |
| \( \mu, \sigma, \xi \) are constants | \( \mu = 0.383, \sigma = 0.069, \xi = -0.200 \) | -163.016 | -158.103 | \( \mu = 0.232, \sigma = 0.110, \xi = -0.352 \) | -76.753 | -77.605 |
| \( \mu = \mu_0 + \mu_1 t \) | \( \mu = 0.193 + 3 \times 10^{-4} t \) | -166.395 | -159.845 | \( \mu = 0.562 - 1.677 \times 10^{-4} t \) | -77.232 | -77.801 |
| \( \sigma, \xi \) are constants | \( \sigma = 0.068, \xi = -0.219 \) | -160.223 | -153.673 | \( \sigma = 0.110, \xi = -0.352 \) | -5.369 | -6.366 |
| \( \sigma = \sigma_0 + \sigma_1 t \) | \( \sigma = 0.109 - 9 \times 10^{-5} t \) | -160.223 | -153.673 | \( \sigma = 5.369 - 2.676 \times 10^{-3} t \) | -81.256 | -81.541 |
| \( \mu, \xi \) are constants | \( \mu = 0.384, \xi = -0.219 \) | -160.223 | -153.673 | \( \mu = 0.232, \xi = -0.352 \) | -81.256 | -81.541 |
| \( \mu = \mu_0 + \mu_1 t \) | \( \mu = 0.374 + 4.653 \times 10^{-6} t \) | -159.248 | -151.060 | \( \mu = 0.183 + 2.498 \times 10^{-5} t \) | -81.256 | -81.541 |
| \( \sigma = \sigma_0 + \sigma_1 t \) | \( \sigma = 0.089 - 1.044 \times 10^{-5} t \) | -159.248 | -151.060 | \( \sigma = 0.235 - 6.366 \times 10^{-5} t \) | -81.256 | -81.541 |
| \( \xi \) is a constant | \( \xi = -0.219 \) | -160.223 | -153.673 | \( \xi = -0.352 \) | -81.256 | -81.541 |
4.4 Return period estimation

Based on the parameter values of the time-varying GEV models, the variation with time of low tide level corresponding to different return periods is shown in figure 4. For Wusongkou Station, the lines showing the relationship between low tide level and study year have a positive slope and are parallel to each other, since only the position parameter is with a linear trend. While for Baozhen Station, the upward trend of low tide level is more obvious with the increase of return period, in accordance with the estimated decreasing scale parameter. The increasing trend is smaller at Baozhen Station, which could be caused by the difference of geographical location. Baozhen Station is closer to the river mouth and is less affected by water conservancy projects.

When the return period is 50 years, the low tide level is about 0.275 meters in 1978 and is about 0.286 meters in 2015 for Wusongkou Station. When it comes to Baozhen Station, the low tide level is about -0.011 meters in 1965 and is about -0.004 meters in 2016. The design water level of the channel will be surplus as a result of the increasing trend of low tide level, which will benefit shipping in the Yangtze Estuary.

![Figure 4](image1.png)

**Figure 4.** The variation with time of low tide level corresponding to different return periods.

With the construction of hydraulic engineering getting more and more complete in the Yangtze Estuary, the hydrological condition will remain basically stable in the near future. It is assumed that the low tide level series had a slowly rising trend since 1981 and reached a new stable state by 2010. The return period under nonstationary conditions is calculated using equation (6) and compared with that under stationary conditions, as shown in figure 5. The return period of the low tide level is larger with nonstationarity considered. The 100-year low tide level of Wusongkou Station is approximately 0.261 meters, and that of Baozhen Station is about -0.041 meters. The corresponding return period grows to 150 years and 119 years respectively. The growth of the return period means the current channel standard more secure and the recalculation of design low water level necessary in the Yangtze Estuary.

It can also be found out that as the low tide level decreases, the increase of the return period becomes larger, and the difference between the return periods under stationary and nonstationary conditions becomes larger, indicating that a slight rising trend will have a great impact on return period when the low tide level is small.

![Figure 5](image2.png)

**Figure 5.** Return period corresponding to the low tide level under stationary and nonstationary conditions.
5. Summary and conclusions
In this study, the temporal change characteristics of AMTL series is firstly analyzed through statistical test. Then, conventional frequency analysis method and the time-varying moment method are used to select the best model for the AMTL series. Finally, the return period under nonstationary conditions is calculated. The results can be summarized as follows:

1. The AMTL series of Wusongkou Station and Baozhen Station has abrupt changes at 1996 and 1990 respectively. After the revision to the level after the change point, the AMTL series show no significant trend according to the Mann-Kendall test.

2. When applying the conventional frequency analysis method, GEV distribution performs best for the AMTL series at Wusongkou and Baozhen Station and the MLM is more suitable for parameter estimation. When using the time-varying moment method, the AMTL series of Wusongkou Station obeys the GEV distribution with a time-varying position parameter, while the GEV distribution of time-varying position and scale parameters shows the best fitting effect for Baozhen Station. Both low tide level series have an increasing trend.

3. With the increasing trend of AMTL series considered, the return period of the low tide level becomes larger under nonstationary conditions. When the return period is 100 years, the low tide level at Wusongkou and Baozhen Station is 0.261 meters and -0.041 meters respectively under stationarity. When nonstationarity is considered, the corresponding return periods grows to 150 years and 119 years, which means the current channel standard is more secure than the design standard at both station.

Further study could focus on the nonstationarity of other hydrological series in the Yangtze Estuary, which will help to analyze the impact of the existing water conservancy projects on hydrological processes and provide suggestions for the planning and management of water resources.

Acknowledgements
The authors are thankful to the National Key Research and Development Plan during the 13th Five-Year plan period (Grant No. 2018YFD1100401) which supported the study.

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