Tunable linear and quadratic optomechanical coupling for a tilted membrane within an optical cavity: theory and experiment

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Abstract
We present an experimental study of an optomechanical system formed by a vibrating thin semi-transparent membrane within a high-finesse optical cavity. We show that the coupling between the optical cavity modes and the vibrational modes of the membrane can be tuned by varying the membrane position and orientation. In particular, we demonstrate a large quadratic dispersive optomechanical coupling in correspondence with avoided crossings between optical cavity modes weakly coupled by scattering at the membrane surface. The experimental results are well explained by a first order perturbation treatment of the cavity eigenmodes.

Keywords: optical cavity, optomechanical systems, radiation pressure interaction

(Some figures may appear in colour only in the online journal)

1. Introduction

The study of cavity optomechanics has recently sparked the interest of a broad scientific community due to its different applications, ranging from sensing of masses, forces and displacements at the ultimate quantum limits [1, 2], to the realization of quantum interfaces for quantum information networks [3–6], up to tests of the validity of quantum mechanics at the macroscopic level [7, 8]. A large variety of optomechanical devices have been recently proposed and tested, in which a driven cavity mode interacts with a mechanical resonator due to the fact that the cavity mode frequency $\omega$ depends on the effective position of the mechanical element $z$ [9, 10, 2, 11–13]. In most cases, one has a linear dependence between $\omega$ and $z$, so that the optical field exerts a homogeneous force on the resonator, associated with either the radiation pressure or the gradient dipole force, which can be used to cool the resonator motion [14–20]. In this case the phase shift of the output light is proportional to the mechanical displacement and one can implement highly sensitive readout of forces and displacements [21, 22]. In the so-called membrane-in-the-middle scheme of [12], the position dependence of the cavity mode frequency is caused by a semi-transparent thin membrane placed within the cavity. In such a case, when the membrane is placed at a node or at an antinode of the cavity field, $\omega(z)$ is quadratic in the position of the mechanical element, and one has a dispersive interaction, which allows for new nonlinear optomechanical phenomena [23], as well as for quantum non-demolition detection of the vibrational quanta of the membrane motion [12, 24]. This dispersive regime is analogous to the one already studied in atomic systems coupled to both microwave and optical cavity fields, either in...
the case of single atoms [25–27], or with ultracold ensembles of atoms [28–30].

Here, we present an experimental study of how the optomechanical coupling can be tuned in a membrane-in-the-middle scheme by varying the position and orientation of the membrane with respect to the cavity axis. In particular, we show that by appropriately tilting the membrane, the Hermite–Gauss modes of the cavity can be coupled and avoided crossings between the cavity modes can be created.

When the mode splitting at an avoided crossing is very small, the corresponding value of the second order derivative of the frequency with respect to the position, \( \partial^2 \omega(z)/\partial z^2 \), becomes very large and one achieves a significant quadratic dispersive optomechanical interaction. The first demonstration of the possibility of tuning and increasing the quadratic coupling in a membrane-in-the-middle system was recently carried out in [31]. Here, we consider an experimental setup similar to the one of [31], performing a systematic study of two different parameter sets, one at small tilting angles \((\leq 0.2 \text{ mrad})\), and one at intermediate tilting angles \((\sim 0.8 \text{ mrad})\). In this latter regime we experimentally achieve \( \partial^2 \omega(z)/\partial z^2 = 2\pi \times 4.46 \text{ MHz nm}^{-2} \), which is comparable to the value achieved in [31]. We also provide a theoretical explanation of the experimental results by means of a first order perturbation treatment showing how the tilted membrane determines the new cavity eigenmodes and their frequency shifts. Based on the approach of [32], we analytically evaluate all the relevant matrix elements, including a larger number of modes for a significantly improved description of the frequency shifts. A precise knowledge of the dependence on the membrane position and orientation of these shifts, and in particular of their difference, is of fundamental importance for all applications in which two cavity modes are resonantly coupled to the mechanical resonators, as in [33, 34].

The outline of the paper is as follows. In section 2 we introduce the main aspects of the membrane-in-the-middle scheme, while in section 3 we present our experimental setup. Section 4 is dedicated to the experimental results on the tunability of the optomechanical coupling and the realization of a strong quadratic coupling, while section 5 gives our concluding remarks. In the appendix we provide details of the perturbation theory which explains qualitatively and quantitatively the modification of the cavity modes caused by the membrane.

2. The membrane-in-the-middle scheme

The membrane-in-the-middle system is formed by a thin semi-transparent membrane within a high-finesse Fabry–Perot optical cavity. The empty cavity supports an infinite set of optical modes, conveniently described by the Hermite–Gauss modes [35]. The thin membrane is a dielectric slab of thickness \( d_\text{m} \) and complex index of refraction \( n_\text{m} = n_R + i n_I \). When the membrane is placed within the cavity, the mode functions and their frequencies change in a way that is dependent on the position and orientation of the membrane with respect to the cavity [12, 36, 31, 37].

Consideration of the membrane motion means assuming that its mean center-of-mass position \( z_0 \) along the cavity axis oscillates in time, \( z_0 \rightarrow z_0 + z(x, y, t) \), where \( z(x, y, t) \) represents the membrane’s transverse deformation field, given by a superposition of the vibrational normal modes. One typically assumes the high stress regime of a taut membrane, in which bending effects are negligible and the classical wave equation describes these normal modes well [38, 37]. The membrane vibrational modes are coupled to the optical cavity modes by radiation pressure, and therefore one has, in general, a multimode bosonic system in which many mechanical and optical modes interact in a nonlinear way. However, one can often adopt a simplified description based on a single cavity mode interacting with a single mechanical mode [12, 37, 39]. This is possible when (i) the driving laser mainly populates a single cavity mode (with annihilation operator \( \hat{a} \)), and scattering into other modes is negligible [40]; and (ii) the detection bandwidth is chosen so that it includes only a single, isolated, mechanical resonance with frequency \( \Omega_m \) (described by dimensionless position \( \hat{q} \) and momentum \( \hat{p} \) operators, such that \( [\hat{q}, \hat{p}] = i \)). By explicitly including cavity driving by a laser with frequency \( \omega_\text{L} \) and input power \( P \), one ends up with the following cavity optomechanical Hamiltonian:

\[
H = \frac{\hbar \Omega_m}{2} (\hat{q}^2 + \hat{q}_t^2) + \hbar \omega (\hat{q}) \hat{a}^\dagger \hat{a} + i \hbar E (\hat{q}^\dagger e^{i\omega_\text{L} t} - \hat{a} e^{-i\omega_\text{L} t}),
\]

where \( E = \sqrt{2P \kappa_0 / \hbar \omega_\text{L}} \), with \( \kappa_0 \) the cavity mode bandwidth in the absence of the membrane. In equation (1) we have included the radiation pressure interaction within the cavity mode energy term, by introducing a position-dependent cavity frequency \( \omega(\hat{q}) \), which can be written as

\[
\omega(\hat{q}) = \omega_0 + \text{Re} \{\delta \omega[z_0(\hat{q}), \alpha_x, \alpha_y]\},
\]

where \( \omega_0 \) is the cavity mode frequency in the absence of the membrane, and \( \text{Re} \{\delta \omega[z_0(\hat{q}), \alpha_x, \alpha_y]\} \) is the frequency shift caused by the insertion of the membrane. This shift depends on the membrane’s position along the cavity axis, \( z_0(\hat{q}) \), which in turn depends on the coordinate \( \hat{q} \), because \( z_0(\hat{q}) = z_0 + \hat{x}_0 \Theta \hat{q} \), where \( \hat{x}_0 = \sqrt{\hbar/m \Sigma_m} \) is the spatial width of the mechanical zero-point motion (\( m \) is the effective mass of the mechanical mode), and \( \Theta \) is the dimensionless overlap integral between the transverse mode functions of the selected mechanical and optical modes [37]. The frequency shift also depends on \( \alpha_x \) and \( \alpha_y \), the tilting angles around the \( x \) and \( y \) axes respectively. The \( (x, y, z) \) axes form a left-handed Cartesian frame with the origin at the center of the cavity; \( x \) and \( y \) are otherwise arbitrary, due to the cylindrical symmetry of the optomechanical system around the cavity axis \( z \).

When the thin membrane is perfectly aligned, so that \( \alpha_x = \alpha_y = 0 \), and it is placed very close to the cavity waist, the Hermite–Gauss modes represent the cavity eigenmodes to a very good approximation, because their wavefronts fit well with the membrane. The frequency shift in this case is a simple periodic function of \( z_0 \), which is maximum at the antinodes and minimum at the nodes of the intracavity field, and mode degeneracy is not removed [39, 37]. When the membrane is appreciably shifted from the waist and/or...
is tilted, the light scattering of the Hermite–Gauss modes at the membrane surface is no longer negligible and the cavity eigenmodes are significantly modified [31]. If the distance from the waist and the tilting angles are sufficiently small, the system can be described by means of a degenerate first order perturbation theory of the wave equation within the cavity. In this perturbation limit, the new eigenmodes are linear combinations of a few Hermite–Gauss modes, and the corresponding frequency shifts $\text{Re}\{\delta\omega(z_0, \alpha_x, \alpha_y)\}$ can be correspondingly evaluated [32]. In this paper we study both experimentally and theoretically the behavior of these frequency shifts; we will see, in particular, that the cavity mode frequencies, and therefore the optomechanical coupling as well, can be fine tuned and controlled by varying the position and orientation of the membrane.

The optomechanical coupling between the cavity and membrane vibrational modes is provided by the first order (and eventually higher order) term in the expansion of $\text{Re}\{\delta\omega(z_0, \alpha_x, \alpha_y)\}$ as a function of $\dot{q}$. At most membrane positions $z_0$, the first order expansion in $\dot{q}$ of equation (2) provides an accurate description of the physics: in this case one has a standard radiation pressure optomechanical interaction with a single-photon optomechanical coupling strength given by

$$G_0 = \frac{\partial \omega(\dot{q})}{\partial \dot{q}} = \frac{\partial \omega}{\partial z_0} x_0 \Theta,$$

where $\partial \omega/\partial z_0$ can be directly measured experimentally, while $x_0$ and $\Theta$ depend on the chosen optical and mechanical modes.

In contrast, when the membrane center $z_0$ is placed exactly at a node or antinode of the cavity field, or at an avoided crossing between nearby frequencies, the first order term in the expansion of $\omega(\dot{q})$ vanishes, and the higher order quadratic term must be considered. This term describes a dispersive interaction between the optical and vibrational modes whose coupling rate is given by the second order derivative $\partial^2 \omega(\dot{q})/\partial \dot{q}^2$. This unique property of the membrane-in-the-middle scheme has been discussed in [12, 39, 24], where the authors pointed out that, when the quadratic coupling is sufficiently strong, the system can be exploited for a quantum non-demolition measurement of the vibrational energy. Very recently, an alternative optomechanical device able to show strong quadratic coupling, based on a coupled microdisk resonator composed of two stacked silica microring resonators separated by a thin inner amorphous silicon support pillar, has been proposed [41]. Due to its nanometer size, this device is able to show a very large quadratic optomechanical coupling, $\partial^2 \omega(\dot{z})/\partial \dot{z}^2 \sim 2 \pi \times 6 \text{ THz nm}^{-2}$.

3. The experimental setup

Our membrane-in-the-middle setup is schematically shown in figure 1. The laser source is a Nd:YAG laser (Innolight) with a wavelength of $\lambda = 1064$ nm. The light passes through an optical isolator, and is sent to the optomechanical cavity via two steering mirrors. The cavity of length $L = 9$ cm is formed by two dielectric mirrors with radius of curvature $R_1 = R_2 = 9$ cm tilted at an angle $\alpha = \pi/4$.

In this perturbation limit, the new eigenmodes are linear combinations of a few Hermite–Gauss modes, and the corresponding frequency shifts $\text{Re}\{\delta\omega(z_0, \alpha_x, \alpha_y)\}$ can be correspondingly evaluated [32]. In this paper we study both experimentally and theoretically the behavior of these frequency shifts; we will see, in particular, that the cavity mode frequencies, and therefore the optomechanical coupling as well, can be fine tuned and controlled by varying the position and orientation of the membrane.

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Figure 2. The predicted frequency shift, $\Delta \nu = \delta \omega / 2 \pi$, of the Hermite–Gauss modes close to the selected $\text{TEM}_{00,p}$ (blue curve, $p =$ longitudinal number) versus the membrane position along the cavity axis, for a perfectly aligned membrane around the waist. The green curve refers to the $\text{TEM}_{20,p-1}$, $\text{TEM}_{11,p-1}$, $\text{TEM}_{11,p-1}$ degenerate triplet, and the violet curve to the $\text{TEM}_{10,p-2}$, $\text{TEM}_{13,p-2}$, $\text{TEM}_{22,p-2}$, $\text{TEM}_{13,p-2}$, $\text{TEM}_{04,p-2}$ degenerate quintuplet. See section 3 for the other system parameters.

4. Measurement of the cavity mode frequency shifts

We measure the optical cavity modes excited by the driving laser by monitoring the optical power transmitted through the cavity using the photodiode, as in figure 1. We can also determine the transverse profile of the cavity mode by imaging the transmitted beam with a video camera. The laser is tuned and mode matched to one of the cavity’s $\text{TEM}_{00}$ modes, and its frequency is then scanned over a prefixed range. If the input power is not too low, higher order transverse $\text{TEM}_{mn}$ modes (with not too large $m$ and $n$), which are close in frequency to the driven $\text{TEM}_{00}$ mode, are also populated and can be detected. For the system parameters given in section 3, the relevant cavity modes are shown in figure 2, which refers to a perfectly aligned membrane, $\alpha_x = \alpha_y = 0$, whose position is scanned for one half of the laser wavelength around the cavity waist.

As discussed above, if the membrane is slightly misaligned, $\alpha_x, \alpha_y \neq 0$, the Hermite–Gauss $\text{TEM}_{mn}$ modes are no longer cavity eigenmodes, and the frequency shifts are consequently modified. In particular, (i) degeneracies are removed, and (ii) avoided crossings appear in correspondence with cavity modes which are coupled by the perturbation caused by the tilted membrane. We have experimentally observed both these effects, as shown in figure 3, where the optical power transmitted through the cavity as a function of the laser frequency and the membrane position is represented. Figure 3 refers to a 0.5 mm shift from the cavity waist and to tilting angles of $\alpha_x = -0.21$ mrad and $\alpha_y = 0.15$ mrad. The crossings of the $\text{TEM}_{20,p-1}$ triplet with the $\text{TEM}_{00,p}$ mode and the $\text{TEM}_{40,p-2}$ quintuplet are visible, together with the removed degeneracies. In the zoomed part, avoided crossings are visible, and their behavior can be mapped to the theoretical prediction of the perturbation theory developed in the appendix.

Figure 3. The upper figure shows the optical power, in logarithmic scale, transmitted through the cavity as a function of the laser frequency and the membrane position, in the case of membrane tilting angles of $\alpha_x = -0.21$ mrad, $\alpha_y = 0.15$ mrad, and for the membrane shifted by 0.5 mm from the waist. The crossings of the $\text{TEM}_{20,p-1}$ triplet with the $\text{TEM}_{00,p}$ mode and the $\text{TEM}_{40,p-2}$ quintuplet are visible, together with the removed degeneracies and the presence of avoided crossings. In the zoomed part, the dots refer to the corresponding measured central peak frequency shifts, $\Delta \nu = \delta \omega / 2 \pi$. The four different colors, gray, red, green and blue, correspond to the modes coinciding, on the right hand side of the zoomed figure, with the $\text{TEM}_{00,p}$, $\text{TEM}_{02,p-1}$, $\text{TEM}_{11,p-1}$, and $\text{TEM}_{20,p-1}$ modes respectively. The full lines with the same colors correspond to the theoretical prediction of the perturbation theory developed in the appendix.
purposes, for not too large misalignments, a single tilting angle around an appropriate transverse axis, $\alpha_{\text{eff}} = \sqrt{\alpha_x^2 + \alpha_y^2}$, can be used to describe the system.

Figure 3 shows that, as expected, for most membrane positions along the cavity axis, the frequency shift $\Delta \nu = \delta \omega/2\pi$ of the various optical modes has a linear dependence on $z_0$ and therefore on $q$, corresponding to the usual radiation pressure optomechanical interaction. Figure 3 also shows that the largest coupling is achieved for the TEM$_{00,p}$ mode for $z_0$ values halfway between a node and an antinode (and out of the avoided crossing region), and it is estimated to be $|\delta \omega/\delta z_0| = 2\pi \times 2.8$ MHz nm$^{-1}$, which essentially coincides with the maximum achievable value of $|\delta \omega/\delta z_0|_{\text{max}} = 2\alpha_{\text{opt}} \sqrt{R}/L$ (see [37]). The membrane under consideration has a vibrational mode with an effective mass of $m \simeq 34$ ng and $\Omega_m \simeq 380$ kHz, corresponding to a zero-point motion width of $x_0 \simeq 1.11 \times 10^{-6}$ nm, which, assuming an optimized overlap integral $\phi \simeq 1$, gives a single-photon optomechanical coupling of $G_0/2\pi \simeq 3.3$ Hz.

At membrane positions $z_0$ exactly corresponding to nodes, antinodes and avoided crossings, the linear term in $z$ is zero and one has a purely quadratic dispersive optomechanical interaction,

$$H_{\text{disp}} = \hbar G_2 \hat{a}^\dagger \hat{a} q^2, \quad (4)$$

$$G_2 = \frac{\partial^2 \omega}{\partial q^2} \bigg|_{z_0}. \quad (5)$$

From figure 3 one gets $|\partial^2 \omega/\partial q^2| \simeq 2\pi \times 24$ kHz nm$^{-2}$ at nodes and antinodes of the TEM$_{00,p}$ mode, while one has a significant increase, by almost two orders of magnitude, at the avoided crossing between the TEM$_{00,p}$ mode and the triplet, where we measured $|\partial^2 \omega/\partial q^2| \simeq 2\pi \times 1.0$ MHz nm$^{-2}$. These values of the second order derivatives were obtained numerically from the best fit curve of the corresponding frequency shift. These results show that avoided crossings allow one to achieve significant values of the dispersive optomechanical coupling $G_2$ [31], and suggest that by appropriately adjusting the membrane shift from the cavity waist and its tilting angle, one could ‘engineer’ the spectrum of the optical cavity modes and generate avoided crossings with a very large value of the second order derivative $|\partial^2 \omega/\partial q^2|$. In fact, $|\partial^2 \omega/\partial q^2|$ is inversely proportional to the frequency splitting at the avoided crossing point, and therefore one should look for avoided crossings with a very small splitting, that is, between two cavity modes that are very weakly coupled by light scattering at the membrane surface.

We first experimentally investigated how the frequency splitting and the associated second order derivative at the avoided crossing vary as a function of the tilting angle $\alpha_x$, while keeping all the other parameters fixed. The coupling between the TEM$_{00,p}$ and TEM$_{20,p-1}$ modes decreases for decreasing $|\alpha_x|$, and therefore one expects to find a minimum of the splitting $\Delta \omega_c$ and a maximum of the second order derivative at $\alpha_x = 0$. This behavior is confirmed by the experimental data shown in figure 4, which refer to the avoided crossing splitting $\Delta \omega_c$ between the TEM$_{00,p}$ and TEM$_{20,p-1}$ modes for the same parameters as figure 3, except that we have varied $\alpha_x$ around $\alpha_x = 0$. In particular, we see that at $\alpha_x \simeq 0$ we achieve $|\partial^2 \omega/\partial q^2| \simeq 2\pi \times 2.0$ MHz nm$^{-2}$. We did not attain splittings below 4 MHz, and therefore higher values of the quadratic coupling. The residual nonzero values of the tilting angle $\alpha_x$, and of the position shift from the cavity waist, and the nonperfect cylindrical symmetry, may explain the residual symmetry breaking and nonzero splitting.

In the case of perfect symmetry, the avoided crossing (and the corresponding quadratic coupling) would have disappeared when the splitting approached the mode bandwidth, which in our case is approximately equal to 30 kHz. However, it is worth mentioning that if the splitting is of the order of the mechanical frequency, the cavity will undergo Landau–Zener transitions [43].

We then looked for larger values of the quadratic optomechanical coupling in different parameter regions. We found an interesting configuration by combining the effect of increasing the membrane distance from the cavity waist with the membrane tilting. When the membrane is appreciably shifted from the waist, the curved wavefronts of the cavity modes do not fit anymore with the flat membrane surface, and one has an additional perturbation which, differently from
membrane configuration, which is \(\alpha\) perturbation theory developed in the appendix for the corresponding
The full lines correspond to the theoretical prediction of the
crossing points (see the corresponding transverse mode images).

\[ \Delta \nu = \frac{\delta \omega}{2\pi}, \] of the appreciably populated cavity modes versus the membrane position along the cavity axis. The four
different colors, gray, red, blue and green, correspond to the
populated modes and can be associated with the TEM\(_{00,p}\) mode and the triplet modes, TEM\(_{20,p-1}\), TEM\(_{11,p-1}\), TEM\(_{02,p-1}\), only far from
crossing points (see the corresponding transverse mode images).

The full lines correspond to the theoretical prediction of the
perturbation theory developed in the appendix for the corresponding membrane configuration, which is \(\alpha\) = 0.77 mrad, \(\alpha_z \approx 0\), and the
membrane shifted by 1.2 mm from the waist. The quintuplet modes are not appreciably populated in this case, but they need to be taken
into account in the perturbation theory in order to reproduce the
frequency shifts satisfactorily (some of them are shown in the plot). The avoided crossing of interest, with a very small splitting, is the
one between the gray and red curves around \(z_0 = 0\).

membrane tilting, does not break the cylindrical symmetry
around the cavity axis. The combined effects of the two
perturbations may lead to very small couplings and therefore to
avoided crossings with large second order derivatives. This
situation is shown in figure 5, where we chose tilting angles of \(\alpha_x = 0.77\) mrad and \(\alpha_y \approx 0\), and a membrane shift of
1.2 mm from the waist. The dots refer to the measured values of the
frequency shifts of the appreciably populated modes. The prediction of the perturbation theory developed in the
appendix, given by the full lines in figure 5, reproduces the
frequency shifts well provided that the quintuplet modes are
included, even if they are never significantly populated.

The interesting region in figure 5 is the avoided crossing between the gray and red curves, around \(z_0 = 0\), which is
characterized by a very small splitting. A zoom of this part is shown in figure 6 where, together with the measured and
predicted frequency shifts, we show the theoretical intensity
profiles of the transverse pattern of the modes at various
membrane positions. The splitting at the avoided crossing can be estimated to be of the order of 1 MHz and, more
importantly, one can infer at this point, from the theoretical
fitting curve, the value \(|\delta^2 \omega / \delta z_0^2| = 2\pi \times 4.46\) MHz nm\(^{-2}\), which is significantly larger than the values at nodes and
antinodes, and is comparable to the value measured in [31].
At this avoided crossing point, the splitting is very small because the combined effect of tilting and shift from the waist
causes a small but nonzero coupling between two orthogonal linear combinations of the triplet and TEM\(_{0}\) modes. These
two cavity eigenmodes have the transverse pattern shown in
figure 6, and with a good approximation they can be written as

\[ |\phi\rangle_{\text{red}} = \frac{1}{\sqrt{3}} (|\text{TEM}_{20,p-1}\rangle + |\text{TEM}_{02,p-1}\rangle - |\text{TEM}_{00,p}\rangle) \] and

\[ |\phi\rangle_{\text{gray}} = \frac{2}{\sqrt{6}} (|\text{TEM}_{02,p-1}\rangle - |\text{TEM}_{20,p-1}\rangle + |\text{TEM}_{00,p}\rangle) \] in the vicinity of the position \(z_0 = 0\). The measured frequency shifts
(dots) and the theoretical predictions (full lines) are shown, together
with the image of the transverse pattern of the new cavity
eigenmodes in various membrane positions.

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(dots) and the theoretical predictions (full lines) are shown, together
with the image of the transverse pattern of the new cavity
eigenmodes in various membrane positions.

5. Conclusions

We have studied both theoretically and experimentally a
membrane-in-the-middle setup formed by a high-finesse Fabry–Perot cavity with a thin semi-transparent SiN membrane inside. We have demonstrated that the position and
orientation of the membrane within the cavity allow one to
fine tune the frequencies of the cavity modes and, through
them, the optomechanical coupling of these modes with the
vibrational modes of the membrane. In most membrane
positions the frequency shifts of the cavity modes depend
linearly on the membrane deformation and therefore one has
the traditional radiation pressure coupling between optical
and mechanical modes. However, at the nodes and antinodes
of the cavity field the linear term vanishes and one has
a dispersive interaction, which is quadratic in the position
operator of the mechanical mode. We have shown that such
a quadratic coupling can be enhanced by two orders of
magnitude in correspondence with avoided crossings with
small frequency splittings. We have demonstrated a quadratic
coupling comparable to the value achieved in [31], associated
with an avoided crossing between two new cavity eigenmodes
which are linear combinations of the TEM\(_{00,p}\), TEM\(_{20,p-1}\),
and TEM\(_{02,p-1}\) modes.

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Appendix. First order degenerate perturbation theory

To describe the intracavity electromagnetic field in the
presence of a slightly tilted membrane, we adopt a degenerate
first order perturbation treatment of the homogeneous Helmholtz wave equation. It is based on the approach of [32], and we analytically evaluate without approximations the relevant matrix elements, including a larger number of modes in order to describe the frequency shifts in a wider parameter region in a satisfying way. We consider a symmetric cavity formed by two identical spherical mirrors with radius of curvature \( R \), separated by a distance \( L \), in the coordinate system with the \( z \) axis along the cavity axis and centered at the cavity center. The thin mirror is a dielectric slab of thickness \( t_d \) and complex index of refraction \( n_M = n_R + i n_t \).

The time-independent wave equation of the empty intracavity electromagnetic field is

\[
(V^2 + k^2)\tilde{\phi}_j(r) = 0,
\]

where we have defined \( \tilde{\phi}_j(r) = \sqrt{2}\text{Re}(\phi_j(r)) \) for future convenience. The index \( j \) is a collective index corresponding to the triplet of natural numbers \((l_i, m_j, n_j)\), which determines the \( j \)th Gauss–Hermite mode of the empty cavity [35], and which represents the zeroth order solution.

Defining \( \phi_j^{(s)} = \frac{\tilde{\phi}_j(r)}{\phi_j(r)} \) with

\[
\phi_j^{(s)}(r) = \phi_j(r)e^{-\eta_0 r},
\]

we have

\[
\phi_j^{(s)}(r) = \phi_j(r)e^{-\eta_0 r},
\]

with

\[
\theta_j(r) = k_j z - (m_j + n_j + 1) \arctan \left( \frac{z}{R} \right) + \frac{x^2 + y^2}{w_j(z)^2} - \frac{(l_j - 1)\pi}{2}.
\]

The function \( H_m \) is the \( m \)th Hermite polynomial, \( w_j(z) = \sqrt{2}\sqrt{1 + (z^2/R^2)} \) gives the \( j \)th Gaussian beam’s transverse shape along the \( z \) axis, \( w_j(0) = \sqrt{2}\sqrt{1 + (z^2/R^2)} \) is the \( j \)th Gaussian beam’s radius at the cavity waist, with \( z_R = (L/2)/(1 + g)/(1 - g) \) the Rayleigh range of the optical cavity, and \( k_j = (\pi/L)[l_j + (m_j + n_j + 1)\arccos(g)/\pi] \) is the norm of the \( j \)th wavevector [35]. The set of functions \( \{\tilde{\phi}_j^{(s)}\} \) forms an approximate complete set of orthonormal functions for the space region \( G \) within the Fabry–Perot cavity, which also satisfies the null boundary conditions at the cavity mirrors’ surfaces \( M_{\pm} \), that is,

\[
\int_G d^3r \tilde{\phi}_j(r)\tilde{\phi}_j(r) \approx \delta_{ij} = \delta_{l_i,l_j}\delta_{m_j,m_j}\delta_{n_j,n_j},
\]

\[
\tilde{\phi}_j(r)|_{r \in M_{\pm}} \approx 0.
\]

The insertion of the membrane in a tilted and shifted position modifies the time-independent electromagnetic field wave equation as

\[
(V^2 + k^2[1 + V(r)])\tilde{\psi}(r) = 0.
\]

In equation (A.7), \( V(r) = (n_M^2 - 1)V(r) \) and \( V(r) = \text{rect}(z - z_c(x, y))/t_{as} \), where \( \text{rect}(\zeta) \) is defined as \( \text{rect}(\zeta) = 0 \) if \( |\zeta| > 1/2 \), and \( \text{rect}(\zeta) = 1 \) if \( |\zeta| < 1/2 \). The corrected thickness \( t_{as} \), is defined as \( t_{as} = \frac{Ld}{\cos\left[\left(\alpha_x^2 + \alpha_y^2\right)^{1/2}\right]} \), while \( z_c(x, y) = z_0 + \alpha_x x + \alpha_y y \) gives approximately the \( z \)-axis projection of a point with transverse coordinates \((x, y)\) belonging to the plane of symmetry of the membrane almost orthogonal to the \( z \)-axis. The \( V \) function gives, therefore, the perturbation caused by the presence of the membrane within the cavity.

We write the solution \( \tilde{\psi} \) of the equation (A.7) as a linear combination of the orthonormal basis functions \( \{\phi_j\} \),

\[
\tilde{\psi} = \sum_j c_j\phi_j.
\]

Substituting (A.8) into (A.7), using (A.1), multiplying by \( \phi_j \), integrating over \( G \) and using (A.5), we obtain the equations for each \( i \),

\[
c_i(1 - k_j^2/k^2) + \sum_{j=1}^n c_j V_{ij} = 0,
\]

\[
c_i(1 - k_j^2/k^2) + \sum_{j=1}^n c_j V_{ij} = 0,
\]

\[
i \in \{1, 2, \ldots, n\}.
\]

We have \( k_j^2/k^2 = 1 \) for \( j \in \{1, 2, \ldots, n\} \), and we define \( \eta = k_j^2/k_j^2 = k_j^2/k_j^2 \) for \( j \in \{1, 2, 3, \ldots, n\} \), where typically \( \eta \approx 1 \). Defining \( \lambda^{-1} = k_j^2/k_j^2 \) and \( W_{ij} = 1 + V_{ij} \) for \( j \in \{1, 2, \ldots, n\} \), we can write (A.10) in matrix form as

\[
\begin{pmatrix}
W_{1,1} - \lambda & \cdots & V_{1,n} \\
\vdots & \ddots & \vdots \\
V_{n,1} & \cdots & W_{n,n} - \eta
\end{pmatrix}
\begin{pmatrix}
c_1 \\
\vdots \\
c_n
\end{pmatrix}
= \begin{pmatrix}
0 \\
\vdots \\
0
\end{pmatrix},
\]

which is a sort of eigenvalue problem with respect to \( \lambda \). Solving (A.11) gives \( \lambda \) as a function of the membrane position specified by \((z_0, \alpha_x, \alpha_y)\). In particular, we obtain the frequency shifts with respect to the frequency of the driving laser \( \omega_0 \),

\[
\delta \omega(z_0, \alpha_x, \alpha_y) = c_k[\lambda(z_0, \alpha_x, \alpha_y)]^{1/2} - \omega_0.
\]

As the extinction coefficient \( \kappa = n_t/n_R \) is \( \kappa \ll 10^{-6} \), we can exploit the first order
approximation
\[ \lambda [nM(k)] \approx \lambda [nM(0)] + \{\partial \lambda [nM(k)]/\partial nM\} n'(k) |_{k = 0} = 0 \]
\[ \lambda (n_k) + 2\nu \lambda (\nu_k). \quad (A.13) \]

Using this approximation, we can calculate the \( \lambda s \) as the real roots of the determinant of the matrix of coefficients in (A.11) with real entries (each element of the matrix with \( nM = nR \)).

### A.1. A singlet–triplet case study

We apply now the general method exposed above to the particular case of a singlet mode coupled to a triplet of modes. We assume that the input laser drives a singlet mode TEM\(_{00}\) with longitudinal index \( l \), which has \( n1 = 1 \) and is associated with the frequency \( \omega_1 = (\pi c/L) \times \left[ l + \arccos(g)/\pi \right] \). This mode is coupled to the triplet of modes \( \{l-1, 1, l, (l+1, 0, 2)\} \), which has \( n2 = 3 \) and is associated with the three-fold degenerate frequency \( \omega_2 = (\pi c/L) \times \left[ (l+1) + 3 \arccos(g)/\pi \right] \).

We have \( \omega_1 \approx \omega_2 \approx \omega_0 \) by hypothesis. As discussed in the main text, due to the cylindrical symmetry with respect to the cavity axis, we can consider without loss of generality one of the two angles, \( \alpha_x \), and \( \alpha_y \), to be zero (we choose \( \alpha_x = 0 \)). In particular, by defining \( \zeta_0 = 0/\rho R \), we consider two sub-cases: (a) small arbitrary \( \zeta_0 \) and \( \alpha_x \); (b) small arbitrary \( \zeta_0 \) with \( \alpha_x = 0 \).

#### A.1.1. One-angle-tiled and shifted membrane

If the membrane is generally not aligned (with \( \alpha_x = 0 \)) and is not centered at the waist (small arbitrary \( \zeta_0 \) and \( \alpha_x \)), by applying (A.11) we obtain the eigenvalue equation \( \det[V_\eta(\lambda)] = 0 \) for \( \lambda \), where

\[ V_\eta(\lambda) = \begin{pmatrix}
W_{11} - \lambda & V_{12} & 0 & V_{14} \\
W_{12} & W_{22} - \eta \lambda & 0 & V_{24} \\
0 & 0 & W_{33} - \eta \lambda & 0 \\
V_{14} & V_{24} & 0 & W_{44} - \eta \lambda
\end{pmatrix}. \quad (A.14) \]

The solution of the eigenvalue equation gives \( \lambda_1 = (1 + V_{33})/\eta \) and three other values \( \{\lambda_2, \lambda_3, \lambda_4\} \), which are given by the roots of the cubic equation \( \sum_{i=0}^{3} (\eta \lambda)^{3-i} = 0 \), where

\[ a_0 = \eta^2, \]
\[ a_1 = -\eta(W_{11} + W_{22} + W_{44}), \]
\[ a_2 = -\eta W_{12}(V_{12} + V_{14}) - V_{24} \]
\[ + \eta W_{11} W_{33} + \eta W_{11} W_{22} + \eta W_{12} W_{22}, \]
\[ a_3 = V_{12}^2(W_{44} - V_{24}) + V_{14} V_{12}(W_{22} - V_{24}) \]
\[ + W_{11}(V_{24}^2 - W_{22} W_{44}), \]
\[ W_{ii} = 1 + V_{ii}, \quad i \in \{1, \ldots, 4\}. \quad (A.19) \]

#### A.1.2. Aligned and shifted membrane

If the membrane is perfectly aligned (\( \alpha_x = \alpha_y = 0 \)) and not centered at the cavity waist (small arbitrary \( \zeta_0 \)), the eigenvalue equation simplifies because only one off-diagonal matrix element is nonzero, corresponding to the equal coupling of the TEM\(_{00}\) mode with the TEM\(_{20}\) and TEM\(_{02}\) modes, i.e.,

\[ V_\eta(\lambda) = \begin{pmatrix}
W_{11} - \lambda & V_{12} & 0 & V_{14} \\
V_{12} & W_{22} - \eta \lambda & 0 & 0 \\
0 & 0 & W_{33} - \eta \lambda & 0 \\
V_{14} & V_{24} & 0 & W_{44} - \eta \lambda
\end{pmatrix}. \quad (A.20) \]

The eigenvalues can be explicitly obtained and are given by

\[ \lambda_1 = \lambda_2 = 1 + V_{33}/\eta, \quad (A.21) \]
\[ \lambda_{3,4} = \frac{1}{2} \left( \left( 1 + V_{11} + \frac{1 + V_{22}}{\eta} \right) \pm \sqrt{\left( 1 + V_{11} + \frac{1 + V_{22}}{\eta} \right)^2 - 4(1 + V_{33})^2/\eta^2} \right)^{1/2}. \quad (A.22) \]

Besides the eigenvalue equation simplification, the matrix elements \( V_\eta \) are less involved than in the previous case with nonzero tilting angle, which brings a further simplification.

#### A.2. Approximate evaluation of the matrix elements

It is possible to derive an approximated explicit expression for the matrix elements \( V_{ij} \),

\[ V_{ij}(\zeta_0, \alpha_x, \alpha_y) = (n_M^2 - 1) \int_d d^3r \text{Re}[\sqrt{2} \phi_i(r) \tilde{V}(r) \text{Re}[\sqrt{2} \phi_j(r)] \]
\[ = (n_M^2 - 1) \text{Re} \left[ \int_d d^3r \phi_i(r) \tilde{V}(r) \phi_j(r) \right] \]
\[ + \int_d d^3r \phi_i(r) \tilde{V}(r) \phi_j^*(r) \]
\[ = (n_M^2 - 1) \text{Re} \{ I_{ij}^{(s=1)}(\zeta_0, \alpha_x, \alpha_y) \}
\[ + I_{ij}^{(s)}(\zeta_0, \alpha_x, \alpha_y) \}, (A.23) \]

where we have defined

\[ I_{ij}^{(s)}(\zeta_0, \alpha_x, \alpha_y) = \int_d d^3r \phi_i^{(s=1)}(r) \tilde{V}(r) \phi_j^{(s)}(r). \quad (A.24) \]

The problem is now reduced to evaluation of the integral (A.24). Using the fact that \( \zeta_0 \) is small one obtains (see also [32])

\[ I_{ij}^{(s)}(\zeta_0, \alpha_x, \alpha_y) \approx \left( \pi^2 q_0^{i} q_0^{j} m_0 m_j |n_1 n_1 y_1| \right)^{-1/2} (t_{a_M/L}) \]
\[ \times \text{sinc}(K_s^{(s)} t_{a_M/L}) e^{-K_s^{(s)} (q_0^{i} q_0^{j} + \nu) \nu} \]
\[ \times e^{-i\left(2K_s^{(s)} \zeta_0 + \pi \theta_0^{(s)} \right)} \Gamma_s^{(s)} - i \Delta K_s^{(s)} \]
\[ \times \left[ (r_0^{(s)} + \Gamma_s^{(s)} \zeta_0 + \Gamma_s^{(s)} \alpha_{ij}) \right] \]
\[ + (\Gamma_s^{(s)} \zeta_0 + \Gamma_s^{(s)} \alpha_{ij}) \alpha_{ij} \}
\[ + (\Gamma_s^{(s)} \zeta_0 + \Gamma_s^{(s)} \alpha_{ij}) \alpha_{ij} \}, \quad (A.25) \]
where, by introducing $\delta k_{ij} = (k_i - k_j)/(k_i + k_j)$, we have used the notations $m_{ij}^2 = (m_i + m_j)/2$, $n_{ij}^z = (n_i + n_j)/2$, $K_{ij}^{(s)} = (1 + s)[k_{ij}^2 - (m_{ij}^2 + n_{ij}^z + 1)]^2 / 2$, $\Delta K_{ij}^{(s)} = \left[ 1 + s + (1 - s)\delta k_{ij} \right] / 2$, $\alpha_{sij} = \alpha_s / \sqrt{k_{ij}^2}$, and $\alpha_{sij} = \alpha_s / \sqrt{k_{ij}^2}$. Also defined

$$\Gamma_{ij}^{(s)}(\alpha_s, \alpha_t) = J(m_t, m_j, K_{ij}^{(s)}\alpha_{sij}, p, \delta k_{ij}), \quad (A.26)$$

where

$$J(m, n, c, q, \delta) = \int_{-\infty}^{\infty} dx e^{-x^2} (x - ic)^q H_m(\sqrt{1 + \delta (x - ic)}) H_n(\sqrt{1 - \delta (x + ic)}) \times \sqrt{1 + \delta (x - ic)} \times \sqrt{1 - \delta (x + ic)}, \quad (A.27)$$

This integral can be explicitly evaluated using the properties of the Hermite polynomials, and one obtains

$$J^R(m, n, c, q, \delta) = \delta_{m,n}^2 q^n,$$

$$J^L(m, n, c, q, \delta) = \delta_{m,n}^2 q^n,$$

$$R_k(q, f_s, c) = \sum_{k=0}^{\lfloor q/2 \rfloor} (-1)^k \binom{q}{2k} S_{q-2k}(f_s) c^{2k}, \quad (A.30)$$

$$R_0(q, f_s, c) = \sum_{k=0}^{\lfloor (q-1)/2 \rfloor} (-1)^k \binom{q}{2k+1} S_{q-(2k+1)}(f_s) c^{2k+1}, \quad (A.31)$$

with $S_k(f_s) = \sum_{j=0}^{\lfloor k/2 \rfloor} \left( \frac{k}{2} \right)_j \Gamma(j + 1/2) f_s^{k-2j}$. Using these results, one finally gets the following expression for the relevant integral of equation (A.25):

$$\text{Re}[I_{ij}^{(s)}(\xi_0, \alpha_s, \alpha_t)] = x_1 \cos(x_2) \text{Re}\{\Gamma_{ij}^{(s)}(\xi_0) + \Delta K_{ij}^{(s)}(\alpha_s, \alpha_t)\} \times \left[ \text{Im}\{\Gamma_{ij}^{(s)}(\xi_0) + \Delta K_{ij}^{(s)}(\alpha_s, \alpha_t)\} \right] \times \left[ 1 + \text{Im}\{\Gamma_{ij}^{(s)}(\xi_0) + \Delta K_{ij}^{(s)}(\alpha_s, \alpha_t)\} \right] \times x_1 \sin(x_2) \left[ \text{Im}\{\Gamma_{ij}^{(s)}(\xi_0) + \Delta K_{ij}^{(s)}(\alpha_s, \alpha_t)\} \right] \times x_2 \left[ \text{Im}\{\Gamma_{ij}^{(s)}(\xi_0) + \Delta K_{ij}^{(s)}(\alpha_s, \alpha_t)\} \right], \quad (A.32)$$

where we have defined

$$x_1 = (\pi^{1/2} m_{ij}^2 + m_i m_j m_t m_n) 1/2 = \Gamma_{ij}^{(s)}(\xi_0) + \Delta K_{ij}^{(s)}(\alpha_s, \alpha_t),$$

$$x_2 = 2\Gamma_{ij}^{(s)}(\xi_0)^2 + \xi.$$ 

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