Models of maximal atmospheric neutrino mixing and leptogenesis‡

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Abstract

We discuss two extensions of the Standard Model based on the seesaw mechanism and on non-abelian family symmetry groups $O(2)$ and $D_4$, respectively. Both models have a twofold-degenerate neutrino Dirac mass matrix $M_D$, a Majorana mass matrix invariant under a $\mu-\tau$ interchange symmetry and the predictions of maximal atmospheric neutrino mixing and vanishing mixing angle $\theta_{13}$. Leptogenesis can naturally be incorporated if $10^{-3} \lesssim m_1 \lesssim 10^{-2}$ eV where $m_1$ the mass of the lightest neutrino and if the relevant heavy neutrinos are in the range $10^{11}$ to $10^{12}$ GeV. The $D_4$ model is more constrained and leptogenesis requires $m_1$ to be in the vicinity of $4 \times 10^{-3}$ eV.

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1 Introduction

The exploration of neutrino masses and mixing has made tremendous progress in recent years—for a review see, e.g., Ref. [1]. It has turned out that lepton mixing is very different from quark mixing, with two angles of the $3 \times 3$ mixing matrix $U$ being large. These angles are the atmospheric neutrino mixing angle $\theta_{23}$ with $\sin^2 2\theta_{23} > 0.9$ (90\% CL) or $\theta_{23} = 45^\circ \pm 9^\circ$, and the solar mixing angle $\theta_{12}$, which is large but significantly smaller (at the 5$\sigma$ level) than $45^\circ$. On the other hand, the third angle $\theta_{13}$ is small, with $\sin^2 \theta_{13} \lesssim 0.05$ (3$\sigma$ level). Though the data do not show compelling evidence for maximal atmospheric neutrino mixing, they nevertheless warrant the search for models where $\theta_{23} = 45^\circ$ is enforced by symmetries and the features of small $\theta_{13}$ and large but non-maximal solar mixing come out in a natural way. For a general assessment of neutrino physics with respect to experiment and theory see Ref. [2].

In this report we discuss models which are “simple” extensions of the Standard Model (SM) in the sense that the gauge group remains the same, but some multiplets of the SM gauge group are added and the seesaw mechanism [3] is incorporated. We consider only the lepton sector, though there might be interesting relations between mixing in the lepton and quark sectors [4]. Enforcement of exact (or near exact) maximal atmospheric neutrino mixing suggests to make use of a non-abelian family symmetry group $G$ [5, 6]. In this report we discuss two models, one based on $G = O(2)$, which we call $\mathbb{Z}_2$ model [7] for reasons to become clear in Section 3, and another one based on $D_4$ [8], which is discussed in Section 4. For a review on those models see Ref. [9]. In Section 5 we focus on leptogenesis in these models.

Other non-abelian family symmetries which have been considered are $A_4$ [10] and $S_3$ [11]. For a general review on models for neutrino masses and mixing see Ref. [12].

We want to stress the difference between the effect of non-abelian and abelian family symmetries. Considering any entry of a Yukawa coupling matrix, the latter ones allow this entry to be either arbitrary or zero. Moreover, given any distribution of “texture zeros” in fermion mass matrices, there is always an abelian family symmetry and a scalar sector such that this distribution of “texture zeros” is reproduced by a symmetry [13]. Though exact maximal atmospheric neutrino mixing cannot be enforced by abelian family symmetries, interesting models with $\theta_{13} = 0$ can be produced [6].

2 Framework

A suitable framework for constructing models with maximal atmospheric neutrino mixing is given by the SM supplemented by the seesaw mechanism and soft family-lepton number breaking [7, 14]. We denote these lepton numbers by $L_\alpha$ with $\alpha = e, \mu, \tau$. We allow for an arbitrary number $n_H$ of Higgs doublets. Then the Lagrangian is given by

$$\mathcal{L} = \cdots - \sum_j \left( \ell_R j_R \Gamma_j + \nu_R \phi_j^* \Delta_j \right) D_L + \text{H.c.} + \left( \frac{1}{2} \nu_C^T C^{-1} \nu_R + \text{H.c.} \right).$$

(1)

The charged-lepton singlets are denoted by $\ell_R$ and the lepton doublets by $D_L$. We have three right-handed neutrino singlets $\nu_R$ whose Majorana mass matrix $M_R$ must be sym-
metric. The seesaw mechanism assumes that the scale \( m_R \) of the mass eigenvalues of \( M_R \) is much higher than the electroweak scale. With the vacuum expectation values (VEVs) \( \langle \phi_j \rangle_0 = v_j \), the mass matrix of the charged leptons and the so-called Dirac mass matrix in the neutrino sector are, respectively, given by

\[
M_\ell = \sum_j v_j^* \Gamma_j, \quad M_D = \sum_j v_j \Delta_j,
\]

and the mass matrix of the light neutrinos is given by the seesaw formula \[3\]

\[
M_\nu = -M_D^T M^{-1}_R M_D.
\]

The main points in our framework are the following:

- \( L_\alpha \)-conservation \( \Rightarrow \Gamma_j, \Delta_j \) diagonal \( \forall j \);
- Soft \( L_\alpha \)-breaking by terms of dimension three in \( \mathcal{L} \), i.e., by a non-diagonal \( M_R \).

Several remarks are at order:

1. The theory introduced here is renormalizable and all flavour-changing 1-loop amplitudes are finite.
2. Since the Yukawa coupling matrices are diagonal, the mass matrices \( M_\ell, M_D \) are diagonal as well; therefore, \( M_R \) is the only source of neutrino mixing.
3. The soft \( L_\alpha \)-breaking by \( \nu_R \) mass terms occurs at the high scale \( m_R \), nevertheless this theory is in perfect agreement with the data, as shown in [14].

This last point is connected with an interesting non-decoupling property [14] in the scalar sector for \( n_H > 1 \) in the limit \( m_R \to \infty \): flavour-changing vertices \( \ell^\pm \to \ell^\pm + S^0 \), where \( S^0 \) is any neutral physical scalar of the theory, do not vanish in that limit, a property which stems from charged-scalar vertex corrections. As a consequence, the amplitude of, e.g., \( \mu^- \to e^- e^+ e^- \) approaches a constant for large \( m_R \), suppressed by a product of four Yukawa couplings; though small, it might be within the reach of future experiment. On the other hand, amplitudes of \( \mu^- \to e^- \gamma, Z \to e^- \mu^+, \tau^- \to \mu^- \mu^- e^+, \) etc., drop like \( 1/m_R^2 \) and are completely inaccessible experimentally. The same happens for all flavour-changing amplitudes if \( n_H = 1 \). Details can be found in [14].

3 The \( \mathbb{Z}_2 \) model

The \( \mathbb{Z}_2 \) model of [7] is a model of the type described in the previous section, with three Higgs doublets. The model is defined via the following family symmetries:

\[- U(1)_L \alpha (\alpha = e, \mu, \tau); \]

\[- \mathbb{Z}_2^{(tr)}: D_{\mu L} \leftrightarrow D_{\tau L}, \mu_R \leftrightarrow \tau_R, \nu_{\mu R} \leftrightarrow \nu_{\tau R}, \phi_3 \to -\phi_3; \]

\[1\]The amplitude of this process is given by a box diagram, not by a vertex correction.
The groups $U(1)_{L_\alpha}$ are the groups associated with the lepton numbers, broken softly by the Majorana mass terms of $\nu_R$, which also break the total lepton number $L = \sum_\alpha L_\alpha$. The $\mu - \tau$ “interchange symmetry” $Z_2^{(tr)}$ is broken by the VEV of $\phi_3$; this breaking is necessary to allow for $m_\mu \neq m_\tau$. The symmetry $Z_2^{(aux)}$ is an auxiliary symmetry which ensures that $Z_2^{(tr)}$ is intact in the neutrino sector at tree level. The model is dubbed $Z_2$ model because of the $Z_2^{(tr)}$ symmetry. The Yukawa Lagrangian which follows from the above symmetries is given by

$$\mathcal{L}_Y = -y_1 \bar{D}_e L \nu_e R \tilde{\phi}_1 - y_2 \left( \bar{D}_\mu L \nu_\mu R + \bar{D}_\tau L \nu_\tau R \right) \tilde{\phi}_1 - y_3 \bar{D}_e L e_R \tilde{\phi}_1 - y_4 \left( \bar{D}_\mu L \mu_R + \bar{D}_\tau L \tau_R \right) \phi_2 - y_5 \left( \bar{D}_\mu L \mu_R - D_\tau L \tau_R \right) \phi_3 + \text{H.c.}$$  \hfill (4)

For the problem of obtaining “naturally” $m_\mu \ll m_\tau$ with this $\mathcal{L}_Y$ see Ref. [15].

As discussed before, an abelian group cannot enforce exact maximal atmospheric neutrino mixing. Indeed the symmetries listed in the beginning of this section generate a symmetry group which contains a non-abelian part because $U(1)_{L_\mu} \times U(1)_{L_\tau}$ and $Z_2^{(tr)}$ do not commute and together these symmetries generate $O(2)$. The full symmetry group is given by [16]

$$U(1)_{L_\mu} \times U(1)_{(L_\mu + L_\tau)/2} \times O(2)_{(L_\mu - L_\tau)/2} \times Z_2^{(aux)}.$$  \hfill (5)

In this equation the lepton numbers associated with the groups are indicated by subscripts.

Now the mass matrices in the neutrino sector of the $Z_2$ model are readily found to be

$$M_D = \text{diag} \left( a, b, b \right) \quad \text{and} \quad M_R = \begin{pmatrix} m & n & n \\ n & p & q \\ n & q & p \end{pmatrix}.$$  \hfill (6)

Note the $Z_2^{(tr)}$ symmetry in these matrices, which via Eq. (3) is transferred to the mass matrix

$$\mathcal{M}_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} \quad (x, y, z, w \in \mathbb{C})$$  \hfill (7)

of the light neutrinos.

The neutrino mixing matrix is found by the diagonalization procedure

$$U^T \mathcal{M}_\nu U = \hat{m} = \text{diag} \left( m_1, m_2, m_3 \right)$$  \hfill (8)

with positive neutrino masses $m_j$. The key to the determination of $U$ is the observation that $(0, -1, 1)^T$ is an eigenvector of $\mathcal{M}_\nu$ of Eq. (7), whence we also obtain $m_3 = |z - w|$. Then one can show that $U$ has the form

$$U = \text{diag} \left( 1, e^{i\alpha}, e^{i\alpha} \right) \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta / \sqrt{2} & \cos \theta / \sqrt{2} & -1 / \sqrt{2} \\ -\sin \theta / \sqrt{2} & \cos \theta / \sqrt{2} & 1 / \sqrt{2} \end{pmatrix} \text{diag} \left( e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3} \right).$$  \hfill (9)
Thus from the mass matrix (7) we have the following results [7]: \( \theta_{13} = 0^\circ, \theta_{23} = 45^\circ, \theta_{12} \equiv \theta \) is arbitrary, however, without finetuning it will be large. As for the phases, there is no CKM phase since \( U_{e3} = 0 \); the physical Majorana phases are given, e.g., by \( \Delta \equiv 2(\beta_1 - \beta_2) \) and \( 2(\beta_1 - \beta_3) \). The phase \( \alpha \) is non-physical and can be absorbed into the charged lepton fields. There is no prediction for the masses from Eq. (7). However, by convention we always assume \( m_1 < m_2 \) for the masses involved in solar neutrino oscillations.

That the mass matrix (7) does not predict the neutrino masses, follows easily from parameter counting. Taking into account that two unphysical phases can be removed from Eq. (7), while preserving its form, we see that the matrix (7) has six real physical parameters. On the other hand, we have three neutrino masses, three mixing angles and three physical phases, thus Eq. (7) has three predictions, namely those for \( \theta_{13}, \theta_{23} \) and the CKM phase which vanishes by virtue of \( \theta_{13} = 0^\circ \). Consequently, parameter counting does not allow further relations.

### 4 The \( D_4 \) model

The \( D_4 \) model of Ref. [8] contains the same multiplets as the \( \mathbb{Z}_2 \) model plus two heavy real scalars \( \chi_{1,2} \). The family symmetry is the discrete group \( D_4 \) and the pairs \( (D_\mu L, D_\tau L), (\mu_R, \tau_R), (\nu_{\mu R}, \nu_{\tau R}), (\chi_1, \chi_2) \) transform according to its 2-dimensional irreducible representation. The auxiliary symmetry \( \mathbb{Z}_2^{(\text{aux})} \) here is the same as for the \( \mathbb{Z}_2 \) model, however, we have a discrete \( \mathbb{Z}_2^{(\alpha)} \) version of the lepton numbers, such that \( \mathbb{Z}_2^{(\mu,\tau)} \) together with the \( \mu-\tau \) interchange symmetry generate the \( D_4 \). All symmetries are broken spontaneously in this model. For details we refer the reader to Ref. [8].

The Yukawa Lagrangian of the \( D_4 \) model is given by

\[
L_Y' = L_Y + \left[ \frac{1}{2} y_\chi \nu^T_{\tau R} C^{-1} (\nu_{\mu R} \chi_1 + \nu_{\tau R} \chi_2) + \text{H.c.} \right]
\]  

(10)

where \( L_Y \) is is the Lagrangian of Eq. (4) and \( y_\chi \) is a coupling constant. Moreover, the \( D_4 \) symmetry admits \( \nu_R \) mass terms with non-zero masses \( (M_R)_{ee} \) and \( (M_R)_{\mu\tau} = (M_R)_{\tau\mu} \). As shown in [8], the VEVs of \( \chi_1 \) and \( \chi_2 \) of order \( m_R \) are equal up to very small corrections of order \( (100 \text{ GeV}/m_R)^2 \), which we neglect in the following. Taking all the facts of this paragraph together, after spontaneous symmetry breaking we are lead to a mass matrix \( M_R \) of the right-handed neutrino singlets as given by Eq. (6), except that in the \( D_4 \) model we have \( q = 0 \).

Integrating out the heavy degrees of freedom, below the seesaw scale \( m_R \) the \( D_4 \) model is identical with the \( \mathbb{Z}_2 \) model apart from \( q = 0 \). As demonstrated in Ref. [8], from \( q = 0 \) it follows that only the normal spectrum with \( m_1 < m_2 < m_3 \) is allowed. Furthermore, \( q = 0 \) induces two more relations among the physical quantities, namely the two Majorana phases are determined by the light neutrino masses and the solar mixing angle. In particular, we find [8, 17]

\[
\cos \Delta = \frac{(m_1m_2/m_3)^2 - c^4m_1^2 - s^4m_2^2}{2c^2s^2m_1m_2} \quad \text{and} \quad |\langle m \rangle| \equiv |x| = m_1m_2/m_3.
\]

(11)
Figure 1: $\eta_B$ as a function of the lightest neutrino mass $m_1$. We have used as input $	heta_{12} = 33^\circ$, $\Delta m^2 = 7.1 \times 10^{-5}$ eV$^2$, $M_2/M_1 = 10$, $|v_1| = 50$ GeV and $\Delta = 90^\circ$. Curves 1, 2, 3 refer to the $\mathbb{Z}_2$ model with $M_1 = 1, 2.5, 5$ in units of $10^{11}$ GeV, respectively. Curve 4 refers to the $D_4$ model with $M_1 = 5 \times 10^{11}$ GeV; its small range in $m_1$ is the effect of Eq. (11). The horizontal lines indicate the experimental value of $\eta_B$ taken from Ref. [20].

In this equation, $s \equiv \sin \theta_{12}$, $c \equiv \cos \theta_{12}$ and $|\langle m \rangle|$ is the effective Majorana mass appearing in neutrinoless $\beta\beta$-decay (for $x$ see Eq. (7)). With the experimental values of the neutrino mass-squared differences and the solar mixing angle, Eq. (11) constrains the allowed range of the lightest neutrino mass $m_1$: $\cos \Delta \geq -1$ is only fulfilled [8] for $3 \times 10^{-3}$ eV $\lesssim m_1 \lesssim 7 \times 10^{-3}$ eV or $m_1 \gtrsim 1.5 \times 10^{-2}$ eV.

5 Leptogenesis in the $\mathbb{Z}_2$ and $D_4$ models

The seesaw mechanism allows to incorporate baryogenesis via leptogenesis [18], where the CP asymmetry of the decays of the heavy Majorana neutrinos is responsible that in the present universe we have a tiny fraction of baryons over photons, denoted by $\eta_B$; for reviews see Ref. [19].

The suitable basis for the calculation of the CP asymmetry is the physical basis of the heavy Majorana neutrinos. Defining $V^T M_R V = \text{diag} (M_1, M_2, M_3)$, where the $M_j$ are the masses of the heavy Majorana neutrinos and $V$ is a unitary matrix, $V$ has the same structure as $U$ of Eq. (9), since $M_R$ and $M_\nu$ have the same structure. Thus $V$ is a function of $\theta'$, $\chi$, $\gamma_{1,2,3}$ corresponding to $\theta$, $\alpha$, $\beta_{1,2,3}$ in $U$, respectively. The relevant quantity which appears in the CP asymmetry is $\text{Im} \left[ (R_{ij})^2 \right]$ with the matrix $R$ defined
by \( R \equiv V^T M_D M_D^\dagger V^* \) \cite{18, 19}. In the \( \mathbb{Z}_2 \) and \( D_4 \) models this matrix is given by

\[
R = \begin{pmatrix}
|a|^2 c^2 + |b|^2 s^2 & c' s' \left(|b|^2 - |a|^2\right) e^{i(\gamma_1 - \gamma_2)} & 0 \\
|a|^2 s'^2 - |b|^2 c^2 & |b|^2 s'^2 + |b|^2 c^2 & 0 \\
0 & 0 & |b|^2
\end{pmatrix}.
\]  

(12)

We observe that the third heavy Majorana neutrino does not contribute to leptogenesis. This is a consequence of the structure of \( V \) and of \( M_D = \text{diag}(a, b, b) \) being degenerate.

The \( \mathbb{Z}_2 \) model is constrained to such an extent that it allows to calculate \( \theta', 2(\gamma_1 - \gamma_2), |a|, |b| \) as functions of the physical parameters

\[
m_{1,2}, M_{1,2}, \theta_{12} \equiv \theta, \Delta \equiv 2(\beta_1 - \beta_2).
\]  

(13)

With the convention \( M_2 > M_1 \), one can give an explicit analytic expression for the CP asymmetry \( \epsilon_1 \) of the decay of \( N_2 \) into \( N_1 \) in terms of the parameters of Eq. (13). Some numerical results for \( \eta_B \) are displayed in Fig. 1. For the details of the calculation and further numerical results we refer the reader to Ref. \cite{17}.

6 Summary

In this report we have reviewed the \( \mathbb{Z}_2 \) and \( D_4 \) models of Refs. \cite{7, 8}, respectively. Both models are simple extensions of the SM with a non-abelian family symmetry. They predict the neutrino mixing angles \( \theta_{23} = 45^\circ \) and \( \theta_{13} = 0^\circ \), whereas \( \theta_{12} \) remains free but will in general be large. The \( \mathbb{Z}_2 \) model does not make any prediction for the neutrino mass spectrum. However, the requirement of successful leptogenesis forces \( m_1 \) to be rather small, roughly between \( 10^{-3} \) and \( 10^{-2} \) eV; this forbids the inverted spectrum. On the other hand, even without the requirement of successful leptogenesis, the \( D_4 \) model predicts the normal spectrum and leptogenesis restricts the range of \( m_1 \) further due to Eq. (11). This effect is clearly visible in Fig. 1.

The twofold degeneracy of \( M_D \), imposed by the symmetries of the models, plays a crucial role for the CP asymmetry in \( \eta_B \), which can naturally be accommodated in the two models. Reproducing \( \eta_B \) fixes the orders of magnitude of both light and heavy neutrino mass spectrum.

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