Fatigue Modeling for Carbon Fiber/Epoxy Laminated Composites Considering Voids’ Effect

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Abstract. In this article, experimental tests under static tensile loadings and tension-tension cyclic loadings were conducted for T300/924 unidirectional laminated composites at different porosity levels. On the basis of the experimental tests, a physical-based residual stiffness model for porous CFRP composites was put forward. The present model describes the deterioration of composites under cyclic loading in perspective of the initiation and propagation of cracks in the matrix, and is capable of capturing the effect of voids on fatigue behaviors of the composites. Lastly, the stiffness degradations of laminates with different void contents under various stress levels were predicted, and the predicted stiffness reduction as well as fatigue life of the material agreed well with the experimental data.

Keywords. Carbon fiber reinforced polymer composite, residual stiffness model, voids’ effect.

1. Introduction

Composites have drawn much attention throughout the world due to their excellent mechanical properties and corrosion resistance. Fiber reinforced polymer (FRP) composites, in particular, have become a promising material for replacement of metal in industrial manufacture. Since failure mechanisms as well as microstructures are more complex for FRP composites in comparison with those of metals, numerous researchers have dedicated to investigate fatigue behaviors of FRP composites.

Early studies tried to characterize fatigue of polymer composites based on S-N curve with incorporation of various types of failure criteria. Hashin [1] proposed a fatigue model for unidirectional (UD) composites, he reported that the ultimate strengths (tension, compression, etc.) of the material were closely associated with mean stress, stress amplitude, and applied number of cycles. Afterwards, more fatigue models were proposed [2-5]. The impact of predominant failure mechanisms on fatigue life of composites has been reported in the study of Barnard [6], in which a discontinuity in S-N curves was observed when the predominant failure modes changed due to changes in stress level.

Some phenomenological models were also proposed in which the fatigue damage of composites was described by degradation of stiffness/strength [7, 8]. In these studies, fatigue fracture was expected to happen as the residual stiffness reduced to a certain value. Critical residual stiffness was defined by investigators based on different assumptions. Recent years, progressive damage models that correlated deterioration of composites to specific damage mechanisms were proposed by investigators [9-11]. Several different damage variables were used in these models to quantitatively
describe the damage modes and damage extent of the material.

However, as a result of the complex failure mechanisms and neglect of interactions between different failure modes in previous studies, further study needs to be done; in addition, some inevitable defects such as porosity have been reported to have a significant impact on the fatigue resistance [12]. so far, a fatigue model which is capable of evaluating the influence of voids on fatigue behaviors of FRP composites has not been seen.

In this article, Experimental tests under both quasi-static loadings and tension-tension cyclic loadings were conducted for T300/924 composites at various porosity levels. On the basis of the experimental results, a new fatigue model for FRP composites considering the impacts of void defects was put forward. This model correlates the damage evolution of composite laminates to the rate of transverse crack initiation and propagation, and is capable of capturing the impact of the presence of voids in matrix. The stiffness degradations under cyclic tension-tension loadings for UD composites with various content of voids were predicted, and the predicted results showed good agreement with the test data.

2. Experimental

2.1. Materials
T300/924 carbon fiber/epoxy UD laminated composites with different number of plies and void contents were manufactured by compression molding at different pressures. The numbers of ply were 12 and 16, respectively. The void contents were measured based on digital microscopy images of the samples’ cross-sections. Information of the materials can be found in table 1.

| Plaque No. | Number of layer | Compressive pressure/MPa | Porosity level |
|------------|----------------|--------------------------|----------------|
| Plaque-A   | 16             | 0.50                     | 0.80%          |
| Plaque-B   | 12             | 0.50                     | 0.70%          |
| Plaque-C   | 12             | 0.30                     | 2.20%          |
| Plaque-D   | 12             | 0.10                     | 4.00%          |

2.2. Testing Configuration
Rectangular samples were fabricated in this study. To protect the gripping section, tabs made of glass fiber composite were glued by resin adhesive to the gripping areas of samples according to ASTM D3039.

The experiments were carried out in MTS 647 testing machine. In fatigue tests, frequency of the applied fatigue loading was 7Hz, and The load ratio R=0.1. Stiffness degradation of sample during fatigue tests was measured by calculating the secant slope of hysteresis loop at different stage of fatigue tests. The density of cracks in the matrix was also studied by recording the crack initiation and propagation on lateral surface of samples at certain intervals.

3. Fatigue Modeling
Fatigue damage index (noted as D) can usually be defined based on stiffness degradation of material to describe the damage accumulation under cyclic loading. Based on experimental results of the present study, it can be found that the stiffness reduction in loading direction of 0-degree UD laminated composites can be attributed to evolution of the density of transverse cracks (total length of cracks in unit volume). Since transverse crack density evolution is closely related to crack initiation rate (which is the dominant factor in stage I of multi-stage fatigue process) and crack propagation rate (which reflects the fatigue process of stage II/III), the index D should be capable of capturing the transverse initiation/propagation rate.

Based on analyses of the number of transverse cracks in unit volume in different stages of fatigue
tests, transverse crack initiation is found to be associated with stress level $\mu$ and void content $v_v$, and can be well fitted by an exponential form of function $D_{cr} = \beta N^{\psi}$, where parameter $\beta$ is a material constant, and $\psi(\mu, v_v)$ is function of $\mu$ and $v_v$. For the material tested in this article, $\beta = 0.015$, the values of $\psi(\mu, v_v)$ for different $\mu$ and $v_v$ are plotted in figure 1.

The transverse crack initiation rate can be obtained by differentiating the function $D_{cr} = \beta N^{\psi}$ with respect to $N$, as shown in equation (1):

$$\frac{\partial D_{cr}}{\partial N} = \beta \cdot \psi(\mu, v_v) N^{\psi(\mu, v_v)-1}$$

(1)

![Figure 1](image.png)

**Figure 1.** Value of $\psi(\mu, v_v)$ for composites with various void contents at different stress levels.

For cyclic loadings parallel to fiber orientation, bridging cracks have been proved to be the dominant failure mode that resulting stiffness reduction in loading direction. B. N. Cox [13] proposed an analytical model for the growth of bridging cracks based on the MCE model. He reported that the initiated matrix cracks could experience steady state propagation, during which process the crack propagation rate is independent of crack length. The steady-state crack tip stress intensity factor $\Delta K_{ss}$ is shown in equation (2):

$$\Delta K_{ss} = \left( \frac{2 \alpha^2 E}{(\alpha + 1) \sqrt{\beta_{MCE}}} \Delta \sigma^{(\alpha+1)/\alpha} \right)^{1/2}$$

(2)

where $\alpha$ stands for the exponent of the fiber traction law. For fiber sliding in the surrounding matrix, $\alpha = 0.5$ can be use. $E$ is the modulus of the material. $\beta_{MCE}$ stands for the bridging stiffness; the expression of $\beta_{MCE}$ is shown in equation (3) [13]:

$$\beta_{MCE} = 2v_f \left( \frac{\pi E_f v_f + E_m (1 - v_f)}{(1 - v_f)E_m r_f} \right)^{1/2}$$

(3)

where $v_f$ is the fiber volume fraction. $E_f$ and $E_m$ represent the stiffness of fiber and matrix, respectively. $r_f$ is the average fiber radius. $\tau$ is the interfacial frictional stress. A typical interfacial
frictional stress of a carbon fiber-matrix interface (T300/epoxy) τ=89MPa was used according to the experiments conducted by W. D. Bascom [14].

Lingzi Zhuang [15] and Mauro Ricotta [16] both found that the void defects can influence the fracture toughness of composites, and subsequently has an impact on the transverse crack propagation rate. An effective steady-state crack tip stress intensity factor $\Delta K_{ss}^{eq}$ was defined in this article, 

$$\Delta K_{ss}^{eq} = \Delta K_{ss}^{k(v_r)}$$

where $k(v_r)$ is a function of $v_r$. The average crack propagation rate can then be calculated based on Paris Law:

$$\frac{da}{dN} = C(\Delta K_{ss}^{eq})^m = C\left(\frac{2\alpha^2}{\alpha + 1}\sqrt{\beta_{MCE}}\Delta \sigma^{(\alpha+1)/\alpha}\right)^{f(v_r)}$$

(4)

where $f(v_r) = k(v_r)m/2$.

In load control tests, as fatigue damage accumulates, the actual stress would increase as a result of the decrease in load-bearing area, which would lead to acceleration of damage evolution. To take into consideration of the relationship between effective stress and the damage accumulation, A. Movaghghar [17] proposed a fatigue model for composite materials with introduction of maximum elastic strain energy $W_{e,\text{max}}$. The difference between maximum and minimum elastic strain energy of a single loop is expressed as equation (5):

$$\Delta W_e = \frac{\Delta \sigma^2}{2E(1-D)}$$

(5)

In this article, to capture the effect of voids and stress level, an equivalent elastic strain energy was defined as $\Delta W_{e,\text{eq}} = \Delta W_e^{\delta(\mu,v_r)}$, where $\delta(\mu,v_r)$ is a function of $\mu$ and $v_r$.

It is assumed in this paper that the fatigue damage factor $D = D(N,E,\Delta W_e,R,\mu,v_r,...)$. As aforementioned, the damage evolution is associated with crack initiation rate, crack propagation rate as well as elastic strain energy, therefore, the fatigue damage evolution rate can be assumed to be expressed as equation (6):

$$\frac{\partial D}{\partial N} = A\psi(\mu,v_r)N^{\psi(\mu,v_r)-1}\left(\frac{2\alpha^2}{\alpha + 1}\sqrt{\beta_{MCE}}\Delta \sigma^{\alpha+1}\alpha^{(\alpha+1)/\alpha}\right)^{f(v_r)}\left(\frac{\Delta \sigma^2}{2E(1-D)}\right)^{\delta(\mu,v_r)}$$

(6)

where A is material constant.

Note that for a given fatigue test with constant stress level, the variables $\mu$ and $v_r$ are both constants, the integration of Equation (6) can therefore be simplified as equation (7):

$$\int (1-D)^{M_2} \partial D = \int M_0 N^{M_1} \partial N$$

(7)

Where $M_0$, $M_1$, and $M_2$ are constants (see equation (8)).

$$\begin{cases}
M_0 = A(M_1 + 1)\left(\frac{2\alpha^2}{\alpha + 1}\sqrt{\beta_{MCE}}\Delta \sigma^{\alpha+1}\alpha^{(\alpha+1)/\alpha}\right)^{f(v_r)}\left(\frac{\Delta \sigma^2}{2E}\right)^{M_2} \\
M_1 = \psi(\mu,v_r)-1 \\
M_2 = \delta(\mu,v_r)
\end{cases}$$

(8)

By integrating Equation (7), the expression of fatigue damage index $D$ can be obtained as follows:
\[ D = 1 - \left(1 - \frac{A}{\chi(\mu, v_v)} \left(\frac{2\alpha^2}{\alpha+1} \frac{E}{\Delta \sigma} \right)^{\alpha+1} \right) ^{f(v_v)} \left(\frac{\Delta \sigma}{2E} \right) \left(\frac{1}{\chi(\mu, v_v)} \right) ^{N^{\mu(v_v)}} \]  

(9)

where \( \chi(\mu, v_v) = 1/(\delta(\mu, v_v) + 1) \).

Equation (9) describes the fatigue damage evolution with respect to \( N \). Note that when \( N = N_f \), there is \( D = 1 \), the fatigue life can then be expressed as:

\[ N_f = \frac{A}{\chi(\mu, v_v)} \left(\frac{2\alpha^2}{\alpha+1} \frac{E}{\Delta \sigma} \right)^{\alpha+1} \left(\frac{\Delta \sigma}{2E} \right) \left(\frac{1}{\chi(\mu, v_v)} \right) ^{-1} \]

(10)

By combining equations (9) and (10), the number of cycle \( N \) can be normalized with respect to \( N_f \), as shown in equation (11). It can be found that the normalized residual stiffness reduction of the composite is controlled by both \( \chi(\mu, v_v) \) and \( \psi(\mu, v_v) \).

\[ D = 1 - \left(1 - \frac{N}{N_f} \right) \]

(11)

Meanwhile, \( D \) can also be defined by stiffness degradation, thus equation (11) can then be written as:

\[ \frac{E_0 - E(N)}{E_0 - E(N_f)} = 1 - \left(1 - \frac{N}{N_f} \right) \]

(12)

The stiffness reduction of the composite can be expressed by the following equation:

\[ \frac{E(N)}{E_0} = 1 - \left(1 - \frac{E(N_f)}{E_0} \right) \left(1 - \left(1 - \frac{N}{N_f} \right) \right) \]

(13)

the value of \( \chi(\mu, v_v) \) and \( \psi(\mu, v_v) \) for certain \( \mu \) and \( v_v \) can be obtained based on data fitting of the stiffness reduction data. Quadratic polynomial is proved to be efficient in the curve fitting of the function \( \chi(\mu, v_v) \) and \( \psi(\mu, v_v) \) (see equation (14)). The values of all parameters can be found in table 2.

\[ \psi(\mu, v_v) = a_0 + b_1v_v + c_2\mu + d_2 v_v \mu + e_2 \mu^2 \]

(14)

By incorporating the expression of \( N_f \), the residual stiffness can be expressed as equation (15):

\[ \frac{E(N)}{E_0} = 1 - \left(1 - \frac{E(N_f)}{E_0} \right) \left(1 - \left(1 - \frac{N}{N_f} \right) \right) \left(\frac{A}{\chi(\mu, v_v)} \left(\frac{2\alpha^2}{\alpha+1} \frac{E}{\Delta \sigma} \right)^{\alpha+1} \right) ^{f(v_v)} \left(\frac{\Delta \sigma}{2E} \right) \left(\frac{1}{\chi(\mu, v_v)} \right) ^{-1} \]

(15)

With the identification of \( \chi(\mu, v_v) \) and \( \psi(\mu, v_v) \), according to equation (10), the expression of \( f(v_v) \) can also be determined through curve fitting based on the fatigue test results for a carefully
chosen constant $A$. By analyzing tests data, it proves that a linear function could generate a good result for low void contents ($v_v \leq 5\%$).

**Table 2.** Parameters of shape functions of fatigue evolution.

| Parameter | $a_i$  | $b_i$  | $c_i$  | $d_i$  | $e_i$  |
|-----------|--------|--------|--------|--------|--------|
| $i=1$     | 1.789  | 3.125  | -6.083 | -0.71  | 5.276  |
| $i=2$     | 1.82   | 0.00489 | -6.302 | 4.844  | 5.802  |

4. **Simulation Results**

The stiffness degradations under cyclic loading for UD laminates with different void contents were predicted based on the present model. 80% of the test data were utilized to acquire the material parameters in the model, and 20% of the test data were used as comparison. Figure 2 shows comparisons between the predicted results and test data. In figure 2, a good correlation can be seen between experimental stiffness reductions and simulation results. The voids’ effects, as well as the impact of stress level on the fatigue damage accumulation of composites are well captured by the present model.

![Figure 2](image1)

**Figure 2.** Comparisons between predicted stiffness reductions and experimental results for samples at porosity levels of (a) 0.8%, and (b) 2.2%.

Figure 3 shows the predicted fatigue life of UD composites with various void contents under different stress levels and the test results. It can be found that the predictions show a good agreement with the test data.
Figure 3. The comparison between predicted fatigue life for UD laminates with various void contents and experimental data.

Figure 4 shows the prediction results of normalized stiffness degradations for T300/924 UD laminates with various void contents (from 1% to 4%) under longitudinal cyclic loading at 900MPa. According to figure 4, porosity level has a remarkable impact on stiffness degradation rate of the material. In detail, laminates with 4% and 3% voids have a much higher stiffness degradation rate than those with lower void contents. It proves that voids in the matrix could stimulate new cracks to initiate, thus accelerate the cracks initiation process, and as a result, much quicker stiffness reduction can be observed in early stage of fatigue; in addition, the presence of voids can also accelerate crack propagation by influencing the fracture toughness in front of the crack tip, which has been revealed by the studies of Linqi Zhuang [15] and Mauro Ricotta [16].

Figure 4. Prediction results of normalized stiffness reduction for T300/924 composites with various void contents under longitudinal cyclic loading.
Evolutions of the damage index $D$ for T300/924 laminated composites under different stress levels were simulated by the present model, with number of cycles being normalized by the fatigue life so as to allow an intuitive comparison between different cases. Figure 5(a) depicts the fatigue damage evolution rates with respect to $N/N_f$ for UD composites under cyclic loading in the fiber orientation, and figure 5(b) presents the evolution of fatigue damage index $D$; stress levels of 85%, 80%, 75%, 70%, and 65% of the UTS were selected in this analysis. It can be found that fatigue damage developed rapidly both at the beginning and at the very end of the fatigue process, whereas a relatively slow damage development can be observed in the middle section. It can be found in figure 5(b) that for high stress level cases, no palpable turning point can be seen between stage I and stage II, fatigue damage developed continuously at a noticeable rate. In contrast, a relatively evident turning point in early stage can be observed for low stress levels, including $\mu = 70\%$ and $\mu = 65\%$.

![Figure 5. Simulation results for fatigue damage evolution of UD laminates ($v_v=0.8\%$) under longitudinal cyclic loading: (a) fatigue damage evolution rate, and (b) fatigue damage index versus normalized number of cycles.](image)

5. Conclusions
In the study of this paper, both quasi-static tensile tests as well as tension-tension fatigue tests were first conducted for FRP UD laminated composites. Based on the experiments, a new fatigue model for FRP composites with void defects was thereby proposed from the perspective of crack initiation rate and crack propagation rate. The model is capable of capturing voids’ effects and the impact of stress level on fatigue behaviors of the material. Based on the present model, fatigue life and stiffness degradation of laminates with different void contents under various stress levels were predicted, and good agreements can be seen between prediction results and the experimental data.

Lastly, the results also reveal that for the fatigue at high stress level, no palpable changing in the evolution rate of fatigue damage can be seen in early stage of fatigue process, fatigue damage developed continuously at a noticeable rate. In contrast, a relatively evident turning point in early stage can be observed for low stress levels.

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