SIMILARITY, ENTROPY AND SUBSETHOOD MEASURES BASED ON CARDINALITY OF SOFT HYBRID SETS

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ABSTRACT. The real world is inherently uncertain, imprecise and vague. Soft set theory was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertainties, not clearly defined objects. A soft set consists of two parts which are parameter set and approximate value set. So while talking about any property on a soft set, it is notable to consider that each parts should be evaluated separately. In this paper, by taking into account this case, we firstly define the concept of cardinality of soft hybrid sets which are soft set, fuzzy soft set, fuzzy parameterized soft set and fuzzy parameterized fuzzy soft set. Then we discuss the entropy, similarity and subsethood measures based on cardinality in a soft hybrid set, and investigate the relationships among these concepts as well as related examples. Finally, we present an application which is a representation method based on cardinality of a soft hybrid space.

1. INTRODUCTION

The real world is full of uncertainty, imprecision and vagueness in fields such as medical science, social science, engineering, economics etc. Classical set theory, which is based on the crisp and exact case may not be fully suitable for handling problems of uncertainty in such fields. So many authors have become interested in modeling uncertainty recently and have proposed various theories. Theory of fuzzy sets [14], theory of intuitionistic fuzzy sets [15], theory of vague sets [29] and theory of rough sets [33] are some of the well-known theories. In these theories, the concepts such as cardinality, entropy, distance measure and similarity measure is widely used for the analysis and representation of various types of data information such as numerical information, interval-valued information, linguistic information, and so on.

The concept of cardinality expressing elementary characteristics of a set is commonly used to characterize the concepts such as entropy, similarity, subsethood and comparison between two fuzzy sets. The cardinality of a crisp set is the number of elements in the set. In fuzzy set theory, since an element can partially belong to a fuzzy set, a natural generalization of the classical notion of cardinality is to weigh each element by its membership degree. So cardinality of a fuzzy set is the sum of the membership values of its all elements [2]. In [5], Sostak studied on fuzzy cardinals and cardinality of fuzzy sets.

Key words and phrases. Soft set, Cardinality, Entropy, Similarity measure, Subsethood measure.
Entropy and similarity measure of fuzzy sets are two basic concepts in fuzzy set theory. Entropy, which describes the degree of fuzziness in fuzzy set and other extended higher order fuzzy sets was first mentioned by Zadeh [14]. Then it have been widely investigated by many researchers from different points of view. De Luca and Termini [2] introduced some axioms to describe the fuzziness degree of fuzzy set. Kaufmann [3] proposed a method to measure the fuzziness degree of fuzzy set based on a distance measure between its membership function and the membership function of its nearest crisp set. In [26], Yager introduced the fuzziness degree of fuzzy set on the relationship between the fuzzy set and its complement. On the other hand, a similarity measure is an important tool for determining the degree of similarity between two objects and is applied in many fields including pattern recognition, decision making, machine learning, data mining, market prediction and image processing. Kosko [6] presented a fuzzy entropy based on the concept of cardinality of the fuzzy set. To show relationship between these two concepts, Liu [28] investigated entropy, distance measure and similarity measure of fuzzy sets and their relations. Fan and Xie [11] introduced the similarity measure and fuzzy entropy induced by distance measure. Similarity measures based on union and intersection operations, the maximum difference, and the difference and sum of membership grades is proposed by Pappis and Karacapilidis [8]. Wang [30] presented two similarity measures between fuzzy sets and between elements.

Subsethood is a concept used to measure the degree to which a set contains another set. In classical theory, a set $A$ is called a subset of $B$ and is denoted by $A \subset B$ if every element of $A$ is an element of $B$, whenever $U$ is a universal set and $A, B$ are two sets in $U$. Therefore, subsethood measure should be two valued for crisp sets. That is, either $A$ is precisely subset of $B$ or vice versa. But since an element $x$ in universal set $U$ can belong to a fuzzy set $A$ to varying degrees, it is notable to consider situations describing property being "more and less" subset of a fuzzy set to another and to measure the degree of this subsethood. Fuzzy subsethood allows a given fuzzy set to contain another to some degree between 0 and 1.

Soft set theory [9] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. He showed several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. In recent years the development in the fields of soft set theory is rapidly increasing and its application has been taking place in a wide pace, see [1, 4, 9, 10, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 31, 32]. However, in the literature, there is not a lot of work on the concepts of cardinality, entropy, similarity and subsethood measure of a soft set and its some hybrid structures which are fuzzy soft sets, fuzzy parameterized soft set, fuzzy parameterized fuzzy soft set, intuitionistic fuzzy soft set etc. Majumdar and Samanta [25] introduced the notion of softness of a soft set and used entropy as a measure for this softness. In [24], Majumdar and Samanta defined several types of distances between two soft sets and proposed similarity measures of two soft sets. Also they showed an application of this similarity measure of soft sets. Kharal [4] presented that some definition
and lemma of Majumdar and Samanta contain errors. Unfortunately, several basic properties presented in [31] are not true in general; these have been pointed out and improved by Yang [31]. Moreover, Yang [31] defined a new similarity measure of soft sets, which measure similarity of both the parameter set and approximate value set. If considering that a soft set consists of two parts, it is worth to consider that each parts should be evaluated separately. To define the concepts of cardinality, entropy, similarity and subsethood measure of a soft hybrid set, we will adopt this idea in this paper.

The presentation of the rest of this paper is organized as follows. In section 2, we briefly recall the notions of fuzzy set and soft hybrid sets and their some properties. Also the concepts of cardinality, entropy, similarity and subsethood measure of a fuzzy set are given in this section. In section 3, we give definition of cardinality for soft hybrid sets and investigate basic properties of the cardinality function. In section 4, entropy of soft hybrid sets is presented and a theorem showing relative between entropy and cardinality is provided. We investigate the similarity and subsethood measures on soft hybrid sets in section 5,6 respectively. Moreover, we provide that entropy can be expressed by the similarity measure. In section 7, an application of cardinality is presented as a method for representation of a soft hybrid spaces. Some concluding comments are given in the final section.

2. Preliminaries

In this section we present some basic concepts and terminology that will be used throughout the paper.

In this paper, $U$ is a finite universe set, $P(U)$ its power set and $E$ is always the finite universe set of parameters with respect to $U$ unless otherwise specified.

2.1. Fuzzy sets.

Definition 2.1. Zadeh [14] defined a fuzzy set $A$ in the universe of discourse $U$ as follows:

$$A = \{x, \mu_A(x) : x \in U\}$$

which is characterized by the membership function $\mu_A(x) : U \to [0,1]$, where $\mu_A(x)$ indicates the membership degree of the element $x$ to the set $A$. The complement of a fuzzy set $A$ is defined by $A^c = \{x, 1 - \mu_A(x) : x \in U\}$.

A family of all fuzzy sets in $U$ will be denoted by $\mathcal{F}(U)$.

Definition 2.2. Let $A$ and $B$ be two fuzzy set over $U$. Then

(1) [2] The sigma count of $A$, denoted by $|A|$ is given by

$$|A| = \sum \text{count}(A) = \sum_{x \in U} \mu_A(x).$$
Thus, a soft set $F$ over $U$ can be represented by the set of ordered pairs
$$F = \{(e, f_A(e)) : e \in E, \; f(e) \in P(U)\}$$

Note that the set of all soft sets over $U$ will be denoted by $S(U)$.

**Example 2.1.** Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be a universal set and $E = \{e_1, e_2, e_3, e_4\}$ be a set of parameters. Consider $A = \{e_1, e_2, e_4\}$ and $B = \{e_1, e_2, e_3, e_4\}$. Then
$$F_A = \{(e_1, \{x_3, x_4\}), (e_2, \{x_1\}), (e_4, \{x_2, x_4\})\}.$$ 
$$F_B = \{(e_1, \{x_3, x_4, x_5\}), (e_2, \{x_1, x_3\}), (e_3, \{x_1, x_2, x_4\}), (e_4, U)\}.$$ 

**Definition 2.4.** Let $F_A$ and $G_B$ be soft sets over a common universe set $U$ and $A, B \subseteq E$. Then

1. $F_A$ is called the absolute soft set, denoted by $\tilde{F}$ if $f_A(e) = U$ for all $e \in E$.
2. $F_\emptyset$ is called the null soft set, denoted by $\Phi$ if $f_\emptyset(e) = \emptyset$ for all $e \in E$.
3. $F_A$ is a soft subset of $G_B$, denoted by $F_A \subseteq G_B$, if $f_A(e) \subseteq g_B(e)$ for all $e \in E$.
4. $F_A$ equals $G_B$, denoted by $F_A = G_B$, if $F_A \subseteq G_B$ and $G_B \subseteq F_A$.
5. The complement of $F_A$, denoted by $\tilde{F_A}$, is defined by $f_A(e) = U - f_A(e)$ for all $e \in E$.
6. The intersection of $F_A$ and $G_B$ is a soft set $K_D$ defined by $k_D(e) = f_A(e) \cap g_B(e)$, where $D = A \cap B$. We write $(K_D) = (F_A) \bar{\cap}(G_B)$.
Definition 2.5. \[20\] A fuzzy parameterized soft set \(F_A\) over \(U\) is a set defined by a function \(f_A\) representing a mapping

\[f_A : E \rightarrow P(U) \text{ such that } f_A(e) = \emptyset \text{ if } \mu_A(e) = 0\]

where \(A\) is a fuzzy set over \(E\) with the membership function \(\mu_A : E \rightarrow [0, 1]\).

Thus, a fuzzy parameterized soft set \(F_A\) over \(U\) can be represented by the set of ordered pairs

\[F_A = \{(\mu_A(e)/e, f_A(e)) : e \in E, f_A(e) \in P(U) \text{ and } \mu_A(e) \in [0, 1]\}\]

Note that the set of all fuzzy parameterized soft sets over \(U\) will be denoted by \(FPS(U)\).

Example 2.2. Let \(U = \{x_1, x_2, x_3, x_4, x_5\}\) be a universal set and \(E = \{e_1, e_2, e_3, e_4\}\) be a set of parameters. Consider \(A = \{0.2/e_2, 0.6/e_3, 1/e_4\}\) and \(B = \{0.3/e_1, 0.2/e_2, 0.6/e_3\}\). Then

\[(F_A)_{fps} = \{(0.2/e_2, \{x_2, x_4\}), (0.6/e_3, \{x_1, x_3, x_4\}) (1/e_4, U)\},\]

\[(G_B)_{fps} = \{(0.3/e_1 \{x_1, x_2\}), (0.2/e_2, \{x_4\}), (0.6/e_3, \{x_1, x_4\})\}.

Now, we give some basic definitions on the fuzzy parameterized soft sets as follows;

Definition 2.6. Let \(F_A\) and \(G_B\) be a fuzzy parameterized soft sets over \(U\) and \(A, B \subseteq E\). Then

1. \(F_A\) is called the absolute fuzzy parameterized soft set, denoted by \(\tilde{F}\) if \(f_A(e) = U\) and \(\mu_A(e) = 1\) for all \(e \in E\).
   \(F_\emptyset\) is called the null fuzzy parameterized soft set, denoted by \(\Phi\) if \(f_\emptyset(e) = \emptyset\) and \(\mu_\emptyset(e) = 0\) for all \(e \in E\).
2. \(F_A\) is a fuzzy parameterized soft subset of \(G_B\), denoted by \(F_A \subseteq G_B\) if \(\mu_A(e) \leq \mu_B(e)\) and \(f_A(e) \subseteq g_B(e)\) for all \(e \in E\).
3. \(F_A\) equals \(G_B\), denoted by \(F_A = G_B\), if \(F_A \subseteq G_B\) and \(G_B \subseteq F_A\).
4. The complement of \(F_A\), denoted by \(F_A^c\) is defined by \(f_A^c(e) = U - f_A(e)\) and \(\mu_A^c(e) = 1 - \mu_A(e)\) for all \(e \in E\).
5. The intersection of \(F_A\) and \(G_B\) is a fuzzy parameterized soft set \(K_D\) defined by \(\mu_D(e) = \min\{\mu_A(e), \mu_B(e)\}\) and \(k_D(e) = f_A(e) \cap g_B(e)\) for all \(e \in E\), where \(D = A \cap B\). We write \(K_D = F_A \cap G_B\).
(6) The union of $F_A$ and $G_B$ is a fuzzy parameterized soft set $H_C$ defined by
\[
\mu_C(e) = \max \{\mu_A(e), \mu_B(e)\} \text{ and } h_C(e) = f_A(e) \cup g_B(e) \text{ for all } e \in E, \text{ where } C = A \cup B.
\]
We write $H_C = F_A \cup G_B$.

(7) $F_A \cap G_B$ is a fuzzy parameterized soft set defined by $F_A \cap G_B = (K, A \times B)$, where $\mu_{A \times B}(e, t) = \min \{\mu_A(e), \mu_B(t)\}$ and $k_{A \times B}(e, t) = f_A(e) \cap g_B(t)$ for any $e \in A$ and $t \in B$.

(8) $F_A \setminus G_B$ is a fuzzy parameterized soft set defined by $F_A \setminus G_B = (H, A \times B)$, where $\mu_{A \times B}(e, t) = \max \{\mu_A(e), \mu_B(t)\}$ and $h_{A \times B}(e, t) = f_A(e) \setminus g_B(t)$ for any $e \in A$ and $t \in B$.

**Definition 2.7.** A fuzzy soft set $F_A$ over $U$ is a set defined by a function $f_A$ representing a mapping
\[
f_A : E \rightarrow \mathcal{F}(U)
\]
where $f_A(e)$ is a fuzzy set over $U$ for all $e \in E$.

Thus, a fuzzy soft set $F_A$ over $U$ can be represented by the set of ordered pairs
\[
F_A = \{(e, f_A(e)) : e \in E, \ f_A(e) \in \mathcal{F}(U)\}
\]

Note that the set of all fuzzy soft sets over $U$ will be denoted by $\mathcal{FS}(U)$.

**Example 2.3.** Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be a universal set and $E = \{e_1, e_2, e_3, e_4\}$ be a set of parameters. Consider $A = \{e_2, e_4\}$ and $B = \{e_1, e_2, e_4\}$. Then
\[
(F_A)_{fs} = \{(e_2, \{0.1/x_1, 0.8/x_3, 0.3/x_4\}), (e_4, \{0.3/x_1, 0.4/x_2\})\}
\]
\[
(G_B)_{fs} = \{(e_1, \{0.3/x_1, 0.2/x_2, 0.7/x_4\}), (e_2, \{0.4/x_1, 0.5/x_4\}), (e_4, \{0.3/x_2, 0.2/x_3, 0.8/x_4\})\}
\]

**Definition 2.8.** Let $F_A$ and $G_B$ be two fuzzy soft sets over $U$ and $A, B \subseteq E$. Then

(1) $F_A$ is called the absolute fuzzy soft set, denoted by $\tilde{U}$ if $f_\emptyset(e) = U$ for all $e \in E$, i.e., $f_\emptyset(e)(x) = 1$ for all $e \in E$ and all $x \in U$.

$F_\emptyset$ is called the null fuzzy soft set, denoted by $\Phi$ if $f_A(e) = \emptyset$ for all $e \in E$, i.e., $f_\emptyset(e)(x) = 0$ for all $e \in E$ and all $x \in U$.

(2) $F_A$ is a fuzzy soft subset of $G_B$, denoted by $F_A \subseteq G_B$, if $f_A(e) \subseteq g_B(e)$ for all $e \in E$, i.e., $f_A(e)(x) \leq g_B(e)(x)$ for all $e \in E$ and all $x \in U$.

(3) $F_A$ equals $G_B$, denoted by $F_A = G_B$, if $F_A \subseteq G_B$ and $G_B \subseteq F_A$.

(4) The complement of $F_A$, denoted by $F_A^c$, is defined by $f_A^c(e) = U - f_A(e)$ for all $e \in E$, i.e., $f_A^c(e)(x) = 1 - f_A(e)(x)$ for all $e \in E$ and all $x \in U$.

(5) The intersection of $F_A$ and $G_B$ is a fuzzy soft set $K_D$ defined by $k_D(e)(x) = \min \{f_A(e)(x), g_B(e)(x)\}$ for all $e \in E$, i.e., $k_D(e)(x) = \min \{f_A(e)(x), g_B(e)(x)\}$ for all $e \in E$ and all $x \in U$, where $D = A \cap B$. We write $K_D = F_A \cap G_B$.

(6) The union of $F_A$ and $G_B$ is a fuzzy soft set $H_C$ defined by $h_C(e)(x) = \max \{f_A(e)(x), g_B(e)(x)\}$ for all $e \in E$, i.e., $h_C(e)(x) = \max \{f_A(e)(x), g_B(e)(x)\}$ for all $e \in E$ and all $x \in U$, where $C = A \cup B$. We write $H_C = F_A \cup G_B$. 
(7) $F_A \land G_B$ is a fuzzy soft set defined by $F_A \land G_B = (K, A \times B)$, where $k_{A \times B}(e, t) = f_A(e) \cap g_B(t)$ for any $e \in A$ and $t \in B$, i.e., $k_{A \times B}(e)(x) = \min \{f_A(e)(x), g_B(t)(x)\}$ for all $x \in U$.

(8) $F_A \lor G_B$ is a fuzzy soft set defined by $F_A \lor G_B = (H, A \times B)$, where $h_{A \times B}(e, t) = f_A(e) \lor g_B(t)$ for any $e \in A$ and $t \in B$, i.e., $h_{A \times B}(e)(x) = \max \{f_A(e)(x), g_B(t)(x)\}$ for all $x \in U$.

**Definition 2.9.** A fuzzy parameterized fuzzy soft set $F_A$ over $U$ is a set defined by a function $f_A$ representing a mapping

$$f_A : E \rightarrow \mathcal{F}(U) \text{ such that } f_A(e) = \emptyset \text{ if } \mu_A(e) = 0.$$ 

where $A$ is a fuzzy set over $E$ with the membership function $\mu_A : E \rightarrow [0, 1]$ and $f_A(e)$ is a fuzzy set over $U$ for all $e \in E$.

Thus, a fuzzy parameterized fuzzy soft set $F_A$ over $U$ can be represented by the set of ordered pairs

$$F_A = \{(\mu_A(e)/e, f_A(e)) : e \in E, f_A(e) \in \mathcal{F}(U) \text{ and } \mu_A(e) \in [0, 1]\}$$

Note that the set of all fuzzy parameterized fuzzy soft set over $U$ will be denoted by $\mathcal{FPFS}(U)$.

**Example 2.4.** Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be a universal set and $E = \{e_1, e_2, e_3, e_4\}$ be a set of parameters. Consider $A = \{0.4/e_1, 0.2/e_2\}$ and $B = \{0.4/e_1, 0.2/e_2, 0.6/e_3\}$. Then

$$(F_A)_{fpfs} = \{(0.4/e_1, \{0.3/x_1, 0.1/x_2\}), (0.2/e_2, \{0.1/x_2, 0.4/x_3, 0.6/x_4\})\}.$$ 

$$(G_B)_{fpfs} = \{(0.4/e_1, \{0.2/x_2, 0.5/x_3\}), (0.2/e_2, \{0.6/x_3\}), (0.6/e_3, \{0.2/x_2\})\}.$$ 

**Definition 2.10.** Let $F_A$ and $G_B$ be fuzzy parameterized fuzzy soft set over $U$ and $A, B \subseteq E$. Then

(1) $F_A$ is called the absolute fuzzy parameterized fuzzy soft set, denoted by $\hat{U}$ if $f_A(e) = U$ and $\mu_A(e) = 1$ for all $e \in E$ i.e., $f_A(e)(x) = 1$ for all $e \in E$ and all $x \in U$.

$F_{\emptyset}$ is called the null fuzzy parameterized fuzzy soft set, denoted by $\Phi$ if $f_{\emptyset}(e) = \emptyset$ and $\mu_{\emptyset}(e) = 0$ for all $e \in E$, i.e., $f_{\emptyset}(e)(x) = 0$ for all $e \in E$ and all $x \in U$.

(2) $F_A$ is a fuzzy parameterized fuzzy soft subset of $G_B$, denoted by $F_A \subseteq G_B$, if $\mu_A(e) \leq \mu_B(e)$ and $f_A(e) \subseteq g_B(e)$ for all $e \in E$, i.e., $f_A(e)(x) \leq g_B(e)(x)$ for all $e \in E$ and all $x \in U$.

(3) $F_A$ equals $G_B$, denoted by $F_A = G_B$, if $F_A \subseteq G_B$ and $G_B \subseteq F_A$.

(4) The complement of $F_A$, denoted by $F_A'$, is defined by $f_A'(e) = U - f_A(e)$ and $\mu_A'(e) = 1 - \mu_A(e)$ for all $e \in E$, i.e., $f_A'(e)(x) = 1 - f_A(e)(x)$ for all $e \in E$ and all $x \in U$.

(5) The intersection of $F_A$ and $G_B$ is a fuzzy parameterized fuzzy soft set $K_D$ defined by $\mu_D(e) = \min \{\mu_A(e), \mu_B(e)\}$ and $k_D(e) = f_A(e) \lor g_B(e)$ for all $e \in E$, i.e., $k_D(e)(x) = \max \{f_A(e)(x), g_B(e)(x)\}$ for all $e \in E$ and $x \in U$, where $D = A \cap B$. We write $K_D = F_A \land G_B$. 

(6) The union of $F_A$ and $G_B$ is a fuzzy parameterized fuzzy soft set $H_C$ defined by $\mu_C(e) = \max \{\mu_A(e)eB(e)\}$ and $h_C(e) = f_A(e) \cup g_B(e)$ for all $e \in E$, i.e., $h_C(e)(x) = \max \{f_A(e)(x), g_B(e)(x)\}$ for all $e \in E$ and $x \in U$, where $C = A \cup B$. We write $H_C = F_A \cup G_B$.

(7) $F_A \wedge G_B$ is a fuzzy parameterized fuzzy soft set defined by $F_A \wedge G_B = (K, A \times B)$, where $\mu_{A \times B}(e, t) = \min \{\mu_A(e), \mu_B(t)\}$ and $h_{A \times B}(e, t) = f_A(e) \cap g_B(t)$ for any $e \in A$ and $t \in B$, i.e., $h_{A \times B}(e)(x) = \min \{f_A(e)(x), g_B(e)(x)\}$ for all $x \in U$ (where $\cap$ is the intersection operation of sets).

(8) $F_A \vee G_B$ is a fuzzy parameterized fuzzy soft set defined by $F_A \vee G_B = (H, A \times B)$, where $\mu_{A \times B}(e, t) = \max \{\mu_A(e), \mu_B(t)\}$ and $h_{A \times B}(e, t) = f_A(e) \cup g_B(t)$ for any $e \in A$ and $t \in B$, i.e., $h_{A \times B}(e)(x) = \max \{f_A(e)(x), g_B(e)(x)\}$ for all $x \in U$ (where $\cup$ is the intersection operation of sets).

Here, we mean the concepts of soft set, fuzzy soft set, fuzzy parameterized soft set and fuzzy parameterized fuzzy soft set by soft hybrid sets. By $X(U)$, We consider any from $S(U)$, $FS(U)$, $FPS(U)$ and $FFFS(U)$. According to above-given definitions in same universal set, we have the ranking soft set $\rightarrow$ fuzzy soft set $\rightarrow$ fuzzy parameterized soft set $\rightarrow$ fuzzy parameterized fuzzy soft set. So we will satisfy the proofs over most general set, fuzzy parameterized fuzzy soft set.

3. Cardinality of soft hybrid sets

The cardinality of a set in the crisp sense plays an important role in Mathematics and is defined as the number of elements in the set. In fuzzy set theory, the concept of cardinality of a fuzzy set is an extension of the count of elements of a crisp set. A simple way of extending the concept of cardinality was suggested by Deluca and Termini [2]. In the section, we generalize the concept of cardinality to soft hybrid sets.

**Definition 3.1.** Let $(a_1, b_1), (a_2, b_2) \in \mathbb{R} \times \mathbb{R}$. Then we define

1. $(a_1, b_1) \leq (a_2, b_2)$ iff $a_1 \leq a_2$, $b_1 \leq b_2$ and $(a_1, b_1) = (a_2, b_2)$ iff $a_1 = a_2, b_1 = b_2$.
2. $(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$.

**Definition 3.2.** Let $F_A$ be a soft hybrid set over $U$, i.e., $F_A \in X(U)$. Let $\text{count}(\pi, \sigma)$ be a mapping given by $\text{count}(\pi, \sigma) : X(U) \rightarrow (\mathbb{R}^+ \cup \{0\}) \times (\mathbb{R}^+ \cup \{0\})$, where $\pi : P(E) \rightarrow \mathbb{R}^+ \cup \{0\}$ (or $\pi : F(E) \rightarrow \mathbb{R}^+ \cup \{0\}$) and $\sigma : P(U) \rightarrow \mathbb{R}^+ \cup \{0\}$ (or $\sigma : F(U) \rightarrow \mathbb{R}^+ \cup \{0\}$) are two mappings. Then $|F_A|$ is called the cardinality of $F_A$ and is defined by $|F_A| = \text{count}(\pi, \sigma)(F_A) = (|A|, |F|) = (|A|, |F|)$ where $|A|$ is the cardinality of $A$ and $|F|$ is the sum of cardinalities of $f_A(e), \forall e \in A$. 
Definition 3.3. Let $F_A$ be a soft hybrid set over $U$, i.e., $F_A \in X(U)$.

1. The function $|F_A|$ defined by $|F_A| = \text{count}(\pi, \sigma)(F_A) = (\pi(A), \sigma(F)) = (|A|, |F|) = (|\{e \in A | f_A(e)\}|, \sum_{e \in A} |f_A(e)|)$ where $f_A(e) \in P(U)$ for all $e \in E$, is called the cardinality of soft set $F_A$.

2. The function $|F_A|$ defined by $|F_A| = \text{count}(\pi, \sigma)(F_A) = (\pi(A), \sigma(F)) = (|A|, |F|) = (\sum_{e \in A} |\mu_A(e)|, \sum_{e \in A} |f_A(e)|)$ where $f_A(e) \in F(U)$ for all $e \in E$, is called the cardinality of fuzzy parameterized soft set $F_A$.

3. The function $|F_A|$ defined by $|F_A| = \text{count}(\pi, \sigma)(F_A) = (\pi(A), \sigma(F)) = (|A|, |F|) = (|A|, \sum_{e \in A} \sum_{x \in U} |f_A(e)(x)|)$ where $f_A(e) \in F(U)$ for all $e \in E$, is called the cardinality of soft hybrid sets $F_A$.

4. The function $|F_A|$ defined by $|F_A| = \text{count}(\pi, \sigma)(F_A) = (\pi(A), \sigma(F)) = (|A|, |F|) = (\sum_{e \in A} |\mu_A(e)|, \sum_{e \in A} \sum_{x \in U} |f_A(e)(x)|)$ where $f_A(e) \in F(U)$ and $\mu_A(e) \in F(E)$ for all $e \in E$, is called the cardinality of fuzzy parameterized soft set $F_A$.

Example 3.1. Consider the soft hybrid sets $(F_A)_s, (F_A)_{fps}, (F_A)_{fs}$ and $(F_A)_{fdfs}$. Then

$|(F_A)_s| = (3, 5) \quad |(F_A)_{fs}| = (2, 19) \quad |(F_A)_{fps}| = (1, 10) \quad |(F_A)_{fdfs}| = (0, 6, 15)$

Theorem 3.1. Let $F_A$ and $G_B$ be two soft hybrid sets over $U$, i.e., $F_A, G_B \in X(U)$.

1. If $F_A \subseteq G_B$ then $|F_A| \leq |G_B|$ and if $F_A \subseteq G_B$ then $|F_A| \leq |G_B|$.

2. If $F_A \subseteq G_B$ and $G_B = \tilde{U}$, then $|F_A| \leq |\tilde{U}|$.

3. If $F_A = \emptyset$ then $|F_A| = (0, 0) = 0$.

4. If $|F_A| = (a, b)$ then $(a, b) \leq (m, mn)$, where $|E| = m$ and $|U| = n$.

Remark 3.2. Let $F_A$ and $G_B$ be two soft hybrid sets over $U$, i.e., $F_A, G_B \in X(U)$. Then $F_A$ and $G_B$ need not to be equal even if they have same cardinality.

Definition 3.4. Let $F_A$ and $G_B$ be two soft hybrid sets over $U$, i.e., $F_A, G_B \in X(U)$. We define the soft hybrid sets $F_{AXB}$ and $G_{AXB}$ as follows:

$F_{AXB} = \{((e, t), f_A(e)) : (e, t) \in A \times B\}$,

$G_{AXB} = \{((e, t), g_B(t)) : (e, t) \in A \times B\}$.

In fact, this is a simple operation which is the parameter reduction.

Theorem 3.3. Let $F_A$ and $G_B$ be two soft hybrid sets over $U$, i.e., $F_A, G_B \in X(U)$.

Theorem 3.4. (1) $|F_A \cap G_B| + |F_A \cap \bar{G_B}| = |F_A| + |\bar{G_B}|$ \quad (2) $|F_A \cap G_B| + |F_A \cap \bar{G_B}| = |F_A| + |\bar{G_B}|$

$|F_A \cap G_B| + |F_A \cap \bar{G_B}| = |F_{AXB}| + |G_{AXB}|$ \quad (4) $|F_A \cap G_B| + |F_A \cap \bar{G_B}| = |F_{AXB}| + |G_{AXB}|$

Proof. Let $F_A, G_B \in X(U)$. For all $e \in E$ and all $x \in U$. \hfill \Box
(1) \(|F_A \cup G_B| + |F_A \cap G_B| = \text{count}(\pi, \sigma)(F_A \cup G_B) + \text{count}(\pi, \sigma)(F_A \cap G_B)\)

= \(|A \cup B|, |F \cup G|) + (|A \cap B|, |F \cap G|) = (|A \cup B| + |A \cap B|, |F \cup G| + |F \cap G|)

= (\(|\max \{\mu_A(e), \mu_B(e)\}\| + |\min \{\mu_A(e), \mu_B(e)\}\|, |\max \{f_A(e)(x), g_B(e)(x)\}\| + |\min \{f_A(e)(x), g_B(e)(x)\}\|)

\(|\mu_A(e)|, |f_A(e)(x)|\) + |\mu_B(e)|, |g_B(e)(x)|) = \text{count}(\pi, \sigma)(F_A) + \text{count}(\pi, \sigma)(G_B) = |F_A| + |G_B|.

(2) Similarly, we can easily get that \(|F_A^c \cup G_B^c| + |F_A^c \cap G_B^c| = |F_A^c| + |G_B^c|\).

(3) For all \((e, t) \in A \times B\) and all \(x \in U\),

\(|F_A \cup G_B| + |F_A \cap G_B| = \text{count}(\pi, \sigma)(F_A \cup G_B) + \text{count}(\pi, \sigma)(F_A \cap G_B)\)

= \(|A \times B|, |F \cup G|) + (|A \times B|, |F \cap G|) = (\(|A \times B| + |A \times B|, |F \cup G| + |F \cap G|\))

= (\(|A \times B| + |A \times B|, |\max \{f_A(e)(x), g_B(t)(x)\}| + |\min \{f_A(e)(x), g_B(t)(x)\}|)

\(|\mu_A(e)|, |f_A(e)(x)|\) + |\mu_B(e)|, |g_B(t)(x)|) = \text{count}(\pi, \sigma)(F_A \times B) + \text{count}(\pi, \sigma)(G_A \times B) = |F_A \times B| + |G_A \times B|.

(4) Similarly, we can easily get that \(|F_A^c \cup G_B^c| + |F_A^c \cap G_B^c| = |F_A^c| \times B| + |G_A^c \times B|\).

4. Entropy of soft hybrid sets

Now, we give a new definition to measure the softness of a soft hybrid set.

**Definition 4.1.** Let \(F_A\) be a soft hybrid set over \(U\), i.e., \(F_A \in X(U)\). Let \(\text{ent}(\varepsilon, \kappa)\) be a mapping given by \(\text{ent}(\varepsilon, \kappa) : X(U) \rightarrow [0, 1] \times [0, 1]\) where \(\varepsilon : P(E) \rightarrow [0, 1]\) (or \(\varepsilon : \mathcal{F}(E) \rightarrow [0, 1]\)) and \(\kappa : P(U) \rightarrow [0, 1]\) (or \(\kappa : \mathcal{F}(U) \rightarrow [0, 1]\)) are two mappings. It is called an entropy for \(F_A\), if it satisfies the following axiomatic requirements:

1. \(\text{ent}(\varepsilon, \kappa)(F_A) = (0, 0) = 0\) if and only if \(F_A\) is a soft set.
2. \(\text{ent}(\varepsilon, \kappa)(F_A) = (1, 1) = 1\) if and only if \(A = A^c\) and \(F = F^c\).
3. \(\text{ent}(\varepsilon, \kappa)(F_A) = \text{ent}(\varepsilon, \kappa)(F_A)^c\).
4. \(\text{ent}(\varepsilon, \kappa)(F_A) \leq \text{ent}(\varepsilon, \kappa)(G_B)\) if \(\mu_A(e) \leq \mu_B(e) \leq 0.5\) and \(f_A(e)(x) \leq g_B(e)(x) \leq 0.5\) or \(0.5 \leq \mu_B(e) \leq \mu_A(e)\) and \(0.5 \leq g_B(e)(x) \leq f_A(e)(x)\) for all \(e \in E\) and all \(x \in U\).

**Definition 4.2.** Let \(F_A\) be a soft hybrid set over \(U\), i.e., \(F_A \in X(U)\). Then \(\text{ent}(\varepsilon, \kappa)\) is defined as follows:

\[
\text{ent}(\varepsilon, \kappa)(F_A) = (\varepsilon(A), \kappa(F)) = \left(\frac{\pi(A \cap A^c)}{\pi(A \cup A^c)} \frac{\sigma(F \cap F^c)}{\sigma(F \cup F^c)}\right)
\]
where mapping \( \text{count}(\pi, \sigma) \) is a cardinal function.

**Theorem 4.1.** The above-defined measure \( \text{ent}(\varepsilon, \kappa)(F_A) \) is an entropy of soft hybrid set \( F_A \), i.e., is satisfies all the properties in Definition 4.1.

\[
(1) \text{ent}(\varepsilon, \kappa)(F_A) = (0, 0) \iff \left( \frac{\pi(A \cap A^c)}{\pi(A \cup A^c)}, \frac{\sigma(F \cap F^c)}{\sigma(F \cup F^c)} \right) = 0 \iff \pi(A \cap A^c) = 0 \text{ and } \sigma(F \cap F^c) = 0 \iff |A \cap A^c| = 0 \text{ and } |F \cap F^c| = 0 \iff |\min \{ \mu_A(e), 1 - \mu_A(e) \}| = 0 \text{ and } |\min \{ f_A(e)(x), 1 - f_A(e)(x) \}| = 0 \text{ for all } e \in E \text{ and all } x \in U \iff \mu_A(e) = 0 \text{ or } \mu_A(e) = 1 \text{ and } f_A(e)(x) = 0 \text{ or } f_A(e)(x) = 1 \text{ for all } e \in E \text{ and all } x \in U \iff F_A \text{ is a soft set.}
\]

\[
(2) \text{ent}(\varepsilon, \kappa)(F_A) = (1, 1) \iff \left( \frac{\pi(A \cap A^c)}{\pi(A \cup A^c)}, \frac{\sigma(F \cap F^c)}{\sigma(F \cup F^c)} \right) = 1 \iff \pi(A \cap A^c) = 1 \text{ and } \sigma(F \cap F^c) = 1 \iff |A \cap A^c| = |A \cup A^c| \text{ and } |F \cap F^c| = |F \cup F^c| \iff |\min \{ \mu_A(e), 1 - \mu_A(e) \}| = |\max \{ \mu_A(e), 1 - \mu_A(e) \}| \text{ and } |\min \{ f_A(e)(x), 1 - f_A(e)(x) \}| = |\max \{ f_A(e)(x), 1 - f_A(e)(x) \}| \text{ for all } e \in E \text{ and all } x \in U \iff A = A^c \text{ and } F = F^c.
\]

(3) For all \( e \in E \) and all \( x \in U, \)

\[
\text{ent}(\varepsilon, \kappa)(F_A) = (\varepsilon(A), \kappa(F)) = \left( \frac{\pi(A \cap A^c)}{\pi(A \cup A^c)}, \frac{\sigma(F \cap F^c)}{\sigma(F \cup F^c)} \right)
= \left( \frac{\min \{ \mu_A(e), 1 - \mu_A(e) \}}{\max \{ \mu_A(e), 1 - \mu_A(e) \}}, \frac{\min \{ f_A(e)(x), 1 - f_A(e)(x) \}}{\max \{ f_A(e)(x), 1 - f_A(e)(x) \}} \right)
= \left( \frac{\max \{ 1 - \mu_A(e), 1 - \mu_A(e) \}, \min \{ 1 - f_A(e)(x), 1 - f_A(e)(x) \}}{\max \{ \mu_A(e), 1 - \mu_A(e) \}, \max \{ f_A(e)(x), 1 - f_A(e)(x) \}} \right)
= \left( \frac{\pi((A \cap A^c) \cap (A \cap A^c))}{\pi((A \cup A^c) \cup (A \cup A^c))}, \frac{\sigma((F \cap F^c) \cap (F \cap F^c))}{\sigma((F \cup F^c) \cup (F \cup F^c))} \right)
= \text{ent}(\varepsilon, \kappa)(F_A).
\]

(4) Consider \( \mu_B(e) \leq \mu_B(e) \leq 0.5 \) and \( f_A(e)(x) \leq g_B(e)(x) \leq 0.5 \) or \( 0.5 \leq \mu_B(e) \leq \mu_A(e) \) and \( 0.5 \leq g_B(e)(x) \leq f_A(e)(x) \) for all \( e \in E \) and all \( x \in U. \) Then

\[
\text{ent}(\varepsilon, \kappa)(F_A) = (\varepsilon(A), \kappa(F)) = \left( \frac{\pi(A \cap A^c)}{\pi(A \cup A^c)}, \frac{\sigma(F \cap F^c)}{\sigma(F \cup F^c)} \right)
= \left( \frac{\min \{ \mu_B(e), 1 - \mu_B(e) \}}{\max \{ \mu_B(e), 1 - \mu_B(e) \}}, \frac{\min \{ g_B(e)(x), 1 - g_B(e)(x) \}}{\max \{ g_B(e)(x), 1 - g_B(e)(x) \}} \right)
= \left( \frac{\pi((B \cap B^c) \cap (B \cap B^c))}{\pi((B \cup B^c) \cup (B \cup B^c))}, \frac{\sigma((G \cap G^c) \cap (G \cap G^c))}{\sigma((G \cup G^c) \cup (G \cup G^c))} \right)
= \text{ent}(\varepsilon, \kappa)(G_B).
\]

**Remark 4.2.** Let \( F_A \) be a soft hybrid set over \( U \), i.e., \( F_A \in S(U). \) Then it is clear that \( \text{ent}(\varepsilon, \kappa)(F_A) = (0, 0) = 0. \)

**Definition 4.3.** Let \( F_A \) be a fuzzy parameterized soft set over \( U \), i.e., \( F_A \in X(U). \)

(1) The function \( \text{ent}(\varepsilon, \kappa) \) defined by

\[
\text{ent}(\varepsilon, \kappa)(F_A) = (\varepsilon(A), \kappa(F)) = \left( \frac{\pi(A \cap A^c)}{\pi(A \cup A^c)}, \frac{\sigma(F \cap F^c)}{\sigma(F \cup F^c)} \right) = \left( \frac{|A \cap A^c|}{|A \cup A^c|}, \frac{|F \cap F^c|}{|F \cup F^c|} \right)
\]

(2) The function \( \text{ent}(\varepsilon, \kappa) \) defined by
Example 4.1. Consider the soft hybrid sets $(G_B)_s$, $(G_B)_{fps}$, $(G_B)_f$s and $(G_B)_{fps}$. Then

\[ \text{ent}(\varepsilon, \kappa)(G_B)_s = 0 \), \text{ent}(\varepsilon, \kappa)(G_B)_{fps} = (0.42, 0), \text{ent}(\varepsilon, \kappa)(G_B)_f = (0, 0.42), \]

\[ \text{ent}(\varepsilon, \kappa)(G_B)_{fps} = (0.50, 0.48). \]

Theorem 4.3. Let $F_A$ and $G_B$ be two soft hybrid sets over $U$, i.e., $F_A, G_B \in X(U)$. Then

1. \[ \text{ent}(\varepsilon, \kappa)(F_A) + \text{ent}(\varepsilon, \kappa)(G_B) = \text{ent}(\varepsilon, \kappa)(F_A \cap G_B) + \text{ent}(\varepsilon, \kappa)(F_A \cup G_B). \]

2. \[ \text{ent}(\varepsilon, \kappa)(F_A \times B) + \text{ent}(\varepsilon, \kappa)(G_A \times B) = \text{ent}(\varepsilon, \kappa)(F_A \cap G_B) + \text{ent}(\varepsilon, \kappa)(F_A \cup G_B). \]

Proof. Let $F_A$ and $G_B$ be two soft hybrid sets over $U$, i.e., $F_A, G_B \in X(U)$. For all $e \in E$ and all $x \in U$.

(1) \[ \text{ent}(\varepsilon, \kappa)(F_A \cap G_B) + \text{ent}(\varepsilon, \kappa)(F_A \cup G_B) = \]

\[ \left( \frac{[\pi(A \cap A^c)]}{[\pi(A)]} \right) \left( \frac{[\pi(F \cap F^c)]}{[\pi(F)]} \right) + \left( \frac{[\pi(B \cap B^c)]}{[\pi(B)]} \right) \left( \frac{[\pi(G \cap G^c)]}{[\pi(G)]} \right) \]

\[ = \left( \frac{\min[\min(\mu_A(e), \mu_B(e)), \max[1-\mu_A(e), 1-\mu_B(e)]]}{\max[\min(\mu_A(e), \mu_B(e)), \max[1-\mu_A(e), 1-\mu_B(e)]]} \right) \left( \frac{\min[\min(\mu_A(e), \mu_B(e)), \max[1-\mu_A(e), 1-\mu_B(e)]]}{\max[\min(\mu_A(e), \mu_B(e)), \max[1-\mu_A(e), 1-\mu_B(e)]]} \right) \]

\[ + \left( \frac{\min[\min(\mu_A(e), \mu_B(e)), \max[1-\mu_A(e), 1-\mu_B(e)]]}{\max[\min(\mu_A(e), \mu_B(e)), \max[1-\mu_A(e), 1-\mu_B(e)]]} \right) \left( \frac{\min[\min(\mu_A(e), \mu_B(e)), \max[1-\mu_A(e), 1-\mu_B(e)]]}{\max[\min(\mu_A(e), \mu_B(e)), \max[1-\mu_A(e), 1-\mu_B(e)]]} \right) \]

\[ = \left( \frac{\min[\min(\mu_A(e), 1-\mu_A(e))]}{\max[\min(\mu_A(e), 1-\mu_A(e))]} \right) \left( \frac{\min[\min(\mu_A(e), 1-\mu_A(e))]}{\max[\min(\mu_A(e), 1-\mu_A(e))]} \right) \]

\[ + \left( \frac{\min[\min(\mu_B(e), \max[1-\mu_B(e), 1-\mu_B(e)])]}{\max[\min(\mu_B(e), \max[1-\mu_B(e), 1-\mu_B(e)])]} \right) \left( \frac{\min[\min(\mu_B(e), \max[1-\mu_B(e), 1-\mu_B(e)])]}{\max[\min(\mu_B(e), \max[1-\mu_B(e), 1-\mu_B(e)])]} \right) \]

\[ = \left( \frac{\min[\mu_A(e), 1-\mu_A(e)]}{\max[\min(\mu_A(e), 1-\mu_A(e))]} \right) \left( \frac{\min[\mu_A(e), 1-\mu_A(e)]}{\max[\min(\mu_A(e), 1-\mu_A(e))]} \right) \]

\[ + \left( \frac{\min[\mu_B(e), 1-\mu_B(e)]}{\max[\min(\mu_B(e), 1-\mu_B(e))]} \right) \left( \frac{\min[\mu_B(e), 1-\mu_B(e)]}{\max[\min(\mu_B(e), 1-\mu_B(e))]} \right) \]

\[ + \left( \frac{[\pi(A \cap A^c)]}{[\pi(A)]} \right) \left( \frac{[\pi(F \cap F^c)]}{[\pi(F)]} \right) + \left( \frac{[\pi(B \cap B^c)]}{[\pi(B)]} \right) \left( \frac{[\pi(G \cap G^c)]}{[\pi(G)]} \right) = \text{ent}(\varepsilon, \kappa)(F_A) + \text{ent}(\varepsilon, \kappa)(G_B). \]

(2) Similarly, we can easily get that \( \text{ent}(\varepsilon, \kappa)(F_A \times B) + \text{ent}(\varepsilon, \kappa)(G_A \times B) = \text{ent}(\varepsilon, \kappa)(F_A \cap G_B) + \text{ent}(\varepsilon, \kappa)(F_A \cup G_B). \)
5. Similarity measures in soft hybrid sets

Definition 5.1. Let $F_A$ and $G_B$ be two soft hybrid sets over $U$, i.e., $F_A, G_B \in X(U)$. Let $\text{sim}(\alpha, \beta)$ be a mapping given by $\text{sim}(\alpha, \beta) : X(U) \times X(U) \rightarrow [0,1] \times [0,1]$, where $\alpha : P(E) \times P(E) \rightarrow [0,1]$ (or $\mathcal{F}(E) \times \mathcal{F}(E) \rightarrow [0,1]$) and $\beta : P(U) \times P(U) \rightarrow [0,1]$ (or $\mathcal{F}(U) \times \mathcal{F}(U) \rightarrow [0,1]$) are two mappings. Then $\text{sim}(\alpha, \beta)$ is called the similarity measure of $F_A$ and $G_B$ is defined by

$$\text{sim}(\alpha, \beta) (F_A, G_B) = (\alpha(A, B), \beta(F, G))$$

where $\alpha(A, B)$ is the similarity measure of $A$ and $B$ while $\beta(F, G)$ is the similarity measure of $f_A(e)$ and $f_B(e) \in E$ for all $e \in E$, if it satisfies the following axiomatic requirements:

1. $0 \leq \text{sim}(\alpha, \beta) (F_A, G_B) \leq 1$.
2. $\text{sim}(\alpha, \beta) (F_A, G_B) = 1$ if and only if $F_A = G_B$, i.e., $A = B$ and $F(e) = G(e)$ for all $e \in E$.
3. $\text{sim}(\alpha, \beta) (F_A, G_B) = \text{sim}(\alpha, \beta) (G_B, F_A)$.
4. If $F_A \subseteq G_B \subseteq H_C$, then $\text{sim}(\alpha, \beta) (F_A, H_C) \leq \text{sim}(\alpha, \beta) (F_A, G_B)$ and $\text{sim}(\alpha, \beta) (F_A, H_C) \leq \text{sim}(\alpha, \beta) (G_B, H_C)$.

Definition 5.2. Let $F_A$ and $G_B$ be two soft hybrid sets over $U$, i.e., $F_A, G_B \in X(U)$. Then

$$\text{sim}(\alpha, \beta) (F_A, G_B) = (\alpha(A, B), \beta(F, G)) = \left( \frac{\pi(A \cap B)}{\pi(A \cup B)}, \frac{\sigma(F \cap G)}{\sigma(F \cup G)} \right)$$

where mapping count($\pi, \sigma$) is a cardinal function.

Theorem 5.1. The above-defined measure $\text{sim}(\alpha, \beta) (F_A, G_B)$ for soft hybrid sets is a similarity measure for soft hybrid sets over $U$, i.e., it satisfies all the properties in Definition 5.1.

(1) It is clear.

(2) $\text{sim}(\alpha, \beta) (F_A, G_B) = 1 \iff \text{sim}(\alpha, \beta) (F_A, G_B) = (\alpha(A, B), \beta(F, G)) = \left( \frac{\pi(A \cap B)}{\pi(A \cup B)}, \frac{\sigma(F \cap G)}{\sigma(F \cup G)} \right) = 1 \iff \pi(A \cap B) = 1 \text{ and } \pi(A \cup B) = 1 \iff \pi(A \cap B) = \pi(A \cup B)$ and $\sigma(F \cap G) = \sigma(F \cup G) \iff |A \cap B| = |A \cup B|$ and $|F \cap G| = |F \cup G| \iff |\min \{\mu_A(e), \mu_B(e)\}| = |\max \{\mu_A(e), \mu_B(e)\}|$ and $|\min \{f_A(e)(x), g_B(e)(x)\}| = |\max \{f_A(e)(x), g_B(e)(x)\}|$ for all $e \in E$ and all $x \in U$ $\iff \mu_A(e) = \mu_B(e)$ and $f_A(e)(x) = g_B(e)(x)$ for all $e \in E$ and all $x \in U$.

(3) $\text{sim}(\alpha, \beta) (F_A, G_B) = (\alpha(A, B), \beta(F, G)) = \left( \frac{\pi(A \cap B)}{\pi(A \cup B)}, \frac{\sigma(F \cap G)}{\sigma(F \cup G)} \right) = \text{sim}(\alpha, \beta) (G_B, F_A)$.

Let $F_A \subseteq G_B \subseteq H_C$. Then we have $\mu_A(e) \leq \mu_B(e) \leq \mu_C(e)$ and $f_A(e)(x) \leq g_B(e)(x) \leq h_C(e)(x)$ for all $e \in E$ and all $x \in U$.

(4) $\text{sim}(\alpha, \beta) (F_A, H_C) = (\alpha(A, C), \beta(F, H)) = \left( \frac{\pi(A \cap C)}{\pi(A \cup C)}, \frac{\sigma(F \cap H)}{\sigma(F \cup H)} \right) = \left( \frac{|A \cap C|}{|A \cup C|}, \frac{|F \cap H|}{|F \cup H|} \right)$.
Let \( \alpha \) and \( \beta \) be two soft hybrid sets over \( U \), i.e., \( F_A, G_B \in X(U) \).

(1) The function \( \text{sim}(\alpha, \beta) \) defined by \( \text{sim}(\alpha, \beta)(F_A, G_B) = (\alpha(A,B), \beta(F,G)) \)

\[
\text{sim}(\alpha, \beta)(F_A, G_B) = \frac{|A \cap B|}{|A \cup B|} \times \frac{|f_A(e) \cap g_B(e)|}{\max\{f_A(e), g_B(e)\}}
\]

where \( f_A(e), g_B(e) \in P(U) \) for each \( e \in E \), is a similarity measure of soft sets \( F_A \) and \( G_B \).

(2) The function \( \text{sim}(\alpha, \beta) \) defined by

\[
\text{sim}(\alpha, \beta)(F_A, G_B) = \frac{\sum e \in A \cap B \min\{\mu_A(e), \mu_B(e)\}}{\sum e \in A \cup B \max\{\mu_A(e), \mu_B(e)\}} \times \frac{|f_A(e) \cap g_B(e)|}{\max\{f_A(e), g_B(e)\}}
\]

where \( \mu_A(e), \mu_B(e) \in \mathcal{F}(E) \) and \( f_A(e), g_B(e) \in P(U) \) for all \( e \in E \), is a similarity measure of fuzzy parameterized soft sets \( F_A \) and \( G_B \).

(3) The function \( \text{sim}(\alpha, \beta) \) defined by

\[
\text{sim}(\alpha, \beta)(F_A, G_B) = \frac{\sum e \in A \cap B \min\{\mu_A(e), \mu_B(e)\}}{\sum e \in A \cup B \max\{\mu_A(e), \mu_B(e)\}} \times \frac{|f_A(e) \cap g_B(e)|}{\max\{f_A(e), g_B(e)\}}
\]

where \( f_A(e), g_B(e) \in \mathcal{F}(U) \) for all \( e \in E \), is a similarity measure of fuzzy soft sets \( F_A \) and \( G_B \).

(4) The function \( \text{sim}(\alpha, \beta) \) defined by

\[
\text{sim}(\alpha, \beta)(F_A, G_B) = \frac{\sum e \in A \cap B \min\{\mu_A(e), \mu_B(e)\}}{\sum e \in A \cup B \max\{\mu_A(e), \mu_B(e)\}} \times \frac{|f_A(e) \cap g_B(e)|}{\max\{f_A(e), g_B(e)\}}
\]

where \( \mu_A(e), \mu_B(e) \in \mathcal{F}(E) \) and \( f_A(e), g_B(e) \in \mathcal{F}(U) \) for all \( e \in E \), is a similarity measure of fuzzy parameterized fuzzy soft sets \( F_A \) and \( G_B \).

Example 5.1. Consider the soft hybrid sets \( (F_A)_s, (F_A)_{fps}, (F_A)_{fps}, (G_B)_s, (G_B)_{fps}, (G_B)_{fps}, (G_B)_{fps}, (G_B)_{fps} \). Then

\[
\text{sim}(\alpha, \beta)((F_A)_s, (G_B)_s) = (0.75, 0.38) \quad \text{sim}(\alpha, \beta)((F_A)_{fps}, (G_B)_{fps}) = (0.38, 0.25)
\]

\[
\text{sim}(\alpha, \beta)((F_A)_{fps}, (G_B)_{fps}) = (0.60, 0.15) \quad \text{sim}(\alpha, \beta)((F_A)_{fps}, (G_B)_{fps}) = (0.50, 0.20).
\]

Theorem 5.2. Let \( F_A \) be a soft hybrid set over \( U \), i.e., \( F_A \in X(U) \). Then \( \text{ent}(\varepsilon, \kappa)(F_A) = \text{sim}(\alpha, \beta)(F_A \cap F_A^c, F_A \cap F_A^c) \).

Proof. \( \text{sim}(\alpha, \beta)(F_A \cap F_A^c, F_A \cap F_A^c) = \frac{\pi((A \cap A^c) \cap (A \cup A^c))}{\pi((A \cup A^c) \cap (A \cup A^c))} \times \frac{\sigma((F \cap F^c) \cap (F \cup F^c))}{\sigma((F \cup F^c) \cap (F \cup F^c))} \)

\[
= \frac{\min\{\mu_A(e), 1-\mu_A(e)\}}{\max\{\mu_A(e), 1-\mu_A(e)\}} \times \frac{|f_A(e) \cap g_B(e)|}{\max\{f_A(e), 1-f_A(e)\}} = \text{ent}(\varepsilon, \kappa)(F_A).
\]

\( \square \)
6. Subsethood of soft hybrid sets

**Definition 6.1.** Let \( F_A \) and \( G_B \) be two soft hybrid sets over \( U \), i.e., \( F_A, G_B \in X(U) \). Let \( \text{sub}(\theta, \delta) \) be a mapping \( \text{sub}(\theta, \delta) : X(U) \times X(U) \rightarrow [0, 1] \times [0, 1] \) where \( \theta : P(E) \times P(E) \rightarrow [0, 1] \) (or \( \theta : \mathcal{F}(E) \times \mathcal{F}(E) \rightarrow [0, 1] \)) and \( \delta : P(U) \times P(U) \rightarrow [0, 1] \) (or \( \delta : \mathcal{F}(U) \times \mathcal{F}(U) \rightarrow [0, 1] \)) are two mappings. Then \( \text{sub}(\theta, \delta) (F_A, G_B) \) is called the subsethood measure of \( F_A \) and \( G_B \) is defined by

\[
\text{sub}(\theta, \delta) (F_A, G_B) = (\theta(A, B), \delta(F, G))
\]

where \( \theta(A, B) \) is the subsethood measure of \( A \) and \( B \) while \( \delta(F, G) \) is the subsethood measure of \( f_A(e) \) and \( f_B(e) \) for all \( e \in E \), if it satisfies the following axiomatic requirements:

1. \( \text{sub}(\theta, \delta) (F_A, G_B) = 1 \) if and only if \( F_A \subseteq G_B \).
2. Let \( F_A' \subseteq F_A \). Then \( \text{sub}(\theta, \delta) (F_A, F_A') = 0 \) if and only if \( F_A = \emptyset \).
3. If \( F_A \subseteq G_B \subseteq H_C \), then \( \text{sub}(\theta, \delta) (H_C, F_A) \leq \text{sub}(\theta, \delta) (G_B, F_A) \);
   and if \( F_A \subseteq G_B \), \( \text{sub}(\theta, \delta) (K_D, F_A) \leq \text{sub}(\theta, \delta) (K_D, G_B) \).

**Definition 6.2.** Let \( F_A \) and \( G_B \) be two soft hybrid sets over \( U \), i.e., \( F_A, G_B \in X(U) \). Then

\[
\text{sub}(\theta, \delta) (F_A, G_B) = (\theta(A, B), \delta(F, G)) = \left( \frac{\pi(A \cap B)}{\pi(A)}, \frac{\sigma(F \cap G)}{\sigma(F)} \right)
\]

where mapping \( \pi, \sigma \) is a cardinal function.

**Theorem 6.1.** The above-defined measure \( \text{sub}(\theta, \delta) (F_A, G_B) \) is a subsethood measure for soft hybrid sets over \( U \), i.e., it satisfies all the properties in Definition 6.1.

1. \( \text{sub}(\theta, \delta) (F_A, G_B) = 1 \) if and only if \( F_A \subseteq G_B \) and \( \frac{\pi(A \cap B)}{\pi(A)} = 1 \) and \( \frac{\sigma(F \cap G)}{\sigma(F)} = 1 \) if and only if \( A \cap B = |A| \) and \( |F \cap G| = |F| \) if \( \min \{ (\mu_A(e), \mu_B(e)) \} = |\mu_A(e)| \) and \( |\min(f_A(e)(x), g_B(e)(x))| = |f_A(e)(x)| \) for all \( e \in E \) and all \( x \in U \) if \( \mu_A(e) \leq \mu_B(e) \) and \( \mu_A(e) \leq g_B(e)(x) \) for all \( e \in E \) and all \( x \in U \) then \( F_A \subseteq G_B \).

2. Since \( F_A \subseteq F_A', 1 - \mu_A(e) \leq \mu_A(e) \) and \( 1 - f_A(e)(x) \leq f_A(e)(x) \) for all \( e \in E \) and all \( x \in U \).

3. \( \text{sub}(\theta, \delta) (H_C, F_A) = \left( \frac{\pi(C \cap A)}{\pi(C)}, \frac{\sigma(H \cap F)}{\sigma(H)} \right) = \left( \frac{|C \cap A|}{|C|}, \frac{|H \cap F|}{|H|} \right) \)

Let \( F_A \subseteq G_B \subseteq H_C \). Then we have \( \mu_A(e) \leq \mu_B(e) \leq \mu_C(e) \) and \( f_A(e)(x) \leq g_B(e)(x) \) for all \( e \in E \) and all \( x \in U \). Then

\( \text{sub}(\theta, \delta) (H_C, F_A) = \left( \frac{\pi(C \cap A)}{\pi(C)}, \frac{\sigma(H \cap F)}{\sigma(H)} \right) = \left( \frac{|C \cap A|}{|C|}, \frac{|H \cap F|}{|H|} \right) \)
for all \( e \) sets \( \mu \) and \( \pi \) in \( X \) .

Example 6.1. \( (\mu(e)) = \min\left(\frac{\min(\mu(c), \mu_A(e))}{|\mu(e)|}, \frac{\min(\mu(c), f_A(e)(x))}{|\mu(e)|}\right) \leq \left(\frac{\min(\mu(c), \mu_B(e))}{|\mu(e)|}, \frac{\min(\mu(c), f_A(e)(x))}{|\mu(e)|}\right) \).

So \( \text{sub}(\theta, \delta)(H_C, F_A) \leq \text{sub}(\theta, \delta)(G_B, F_A) \).

Similarly, we can see that \( \text{sub}(\theta, \delta)(K_D, F_A) \leq \text{sub}(\theta, \delta)(K_D, G_B) \) if \( F_A \leq G_B \).

Definition 6.3. Let \( F_A \) and \( G_B \) be two soft hybrid sets over \( U \), i.e., \( F_A, G_B \in X(U) \).

1. The function \( \text{sub}(\theta, \delta) \) defined by

\[
\text{sub}(\theta, \delta)(F_A, G_B) = \left(\frac{|A \cap B|}{|A|}, \frac{|f_A(e) \cap g_B(e)|}{|f_A(e)|}\right),
\]

where \( f_A(e), g_B(e) \in P(U) \) for all \( e \in E \), is a subsethood measure of soft sets \( F_A \) and \( G_B \).

2. The function \( \text{sub}(\theta, \delta) \) defined by

\[
\text{sub}(\theta, \delta)(F_A, G_B) = \left(\frac{\sum_{x \in A} \min(\mu_A(e), \mu_B(e))}{|A|}, \frac{|f_A(e) \cap g_B(e)|}{|f_A(e)|}\right),
\]

where \( f_A(e), g_B(e) \in P(U) \) and \( \mu_A(e), \mu_B(e) \in \mathcal{F}(E) \) for all \( e \in E \), is a subsethood measure of fuzzy parameterized soft sets \( F_A \) and \( G_B \).

3. The function \( \text{sub}(\theta, \delta) \) defined by

\[
\text{sub}(\theta, \delta)(F_A, G_B) = \left(\frac{|A \cap B|}{|A|}, \frac{\sum_{x \in E} \min(f_A(e)(x), g_B(e)(x))}{|E|}\right),
\]

where \( f_A(e), g_B(e) \in \mathcal{F}(U) \) for all \( e \in E \), is a subsethood measure of fuzzy soft sets \( F_A \) and \( G_B \).

4. The function \( \text{sub}(\theta, \delta) \) defined by

\[
\text{sub}(\theta, \delta)(F_A, G_B) = \left(\frac{\sum_{x \in E} \min(f_A(e)(x), g_B(e)(x))}{|E|}, \frac{\sum_{x \in A \cap B} \min(\mu_A(e), \mu_B(e))}{|A \cap B|}\right),
\]

where \( f_A(e), g_B(e) \in \mathcal{F}(U) \) and \( \mu_A(e), \mu_B(e) \in \mathcal{F}(E) \) for all \( e \in E \), is a similarity measure of fuzzy parameterized fuzzy soft sets \( F_A \) and \( G_B \).

Example 6.1. Consider the soft hybrid sets \((F_A)_s, (F_A)_{fps}, (F_A)_{fps}, (G_B)_s, (G_B)_{fps}, (G_B)_{fps}\).

Then \( \text{sub}(\theta, \delta)((F_A)_s, (G_B)_s) = 1 \) and \( \text{sub}(\theta, \delta)((G_B)_s, (F_A)_s) = 0 \) in Molodtsov’s soft subset sense. Here we say that \((F_A)_s\) is precisely a soft subset of \((G_B)_s\). However, it may be situations being “more and less” a subset of a set in another set. For example, since \( \text{sub}(\theta, \delta)((F_A)_{fps}, (G_B)_{fps}) = \).
(0.44, 0.33) and \( \text{sub}(\theta, \delta) \left((G_B)_{f_{ps}}, (F_A)_{f_{ps}}\right) = (0.72, 0.66) \), we can say that \((G_B)_{f_{ps}}\) is much more a soft subset of \((F_A)_{f_{ps}}\).

**Theorem 6.2.** Let \( F_A \) and \( G_B \) be two soft hybrid sets over \( U \), i.e., \( F_A, G_B \in X(U) \). Then

\[
\text{sim}(\alpha, \beta)(F_A, G_B) = \text{sub}(\theta, \delta)(F_A \tilde{\cap} G_B, F_A \tilde{\cup} G_B)
\]

\[
\text{sub}(\theta, \delta)(F_A \tilde{\cap} G_B, F_A \tilde{\cup} G_B) = (\theta(A \cap B, A \cup B), \delta(F \cap G, F \cup G))
\]

\[
= \left( \frac{\pi((A \cap B) \cap (A \cup B))}{\pi(A \cap B) \cap (A \cup B)}, \frac{\sigma((F \cap G) \cap (F \cup G))}{\sigma((F \cap G) \cap (F \cup G))} \right) = \left( \frac{\|A \cap B\|}{\|A\|}, \frac{\|F \cap G\|}{\|F\|} \right) = \text{sim}(\alpha, \beta)(F_A, G_B).
\]

### 7. A REPRESENTATION METHOD BASED ON CARDINALITY OF SOFT HYBRID SPACES

**Definition 7.1.** Let \( F_A \) be a soft hybrid sets over \( U \), i.e., \( F_A, G_B \in X(U) \). Then depth of \( A \) denoted by \( \text{depth}(F_A) \) is given by

\[
\text{depth}(F_A) = (m, mn) - (a_1, a_2)
\]

where \( a_1 = \sum_{i=1}^{m} \mu_A(x_i) \) and \( a_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} f_A(e_i)(x_j) \) for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

It is clear that \( \text{depth}(\hat{U}) = 0 \) and \( \text{depth}(\Phi) = (m, mn) \).

**Definition 7.2.** Let \( F_A \) and \( G_B \) be two soft hybrid sets over \( U \), i.e., \( F_A, G_B \in X(U) \). Then we say \( G_B \) is a better representative of \( \hat{U} \) than \( F_A \) denoted by \( G_B \supset F_A \), if and only if

\[
\|\text{depth}(G_B)\| < \|\text{depth}(F_A)\|
\]

where \( \|(a, b)\| \) is given by \( \frac{|a| + |b|}{2} \).

**Example 7.1.** Let \( F_A, G_B, H_C, K_D \in FPFS(U) \). Suppose that \( U = \{x_1, x_2, x_3, x_4, x_5\} \) be a universal set and \( E = \{e_1, e_2, e_3, e_4\} \) be a set of parameters, \( A = \{0.1/e_2, 0.4/e_3, 0.2/e_4\}, B = \{0.2/e_2, 0.5/e_3, 0.4/e_4\}, C = \{0.3/e_2, 0.3/e_3, 0.1/e_4\} \) and \( D = \{0.4/e_2, 0.1/e_3, 0.2/e_4\} \). We consider the sets given as follows:

\[
F_A = \begin{cases}
(0.1/e_2, 0.5/x_1, 0.1/x_3, 0.7/x_4) \\
(0.4/e_2, 0.2/x_4, 0.4/x_1, 0.3/x_4) \\
(0.2/e_4, 0.5/x_2, 0.1/x_3, 0.7/x_4)
\end{cases}
, \quad G_B = \begin{cases}
(0.2/e_2, 0.3/x_1, 0.2/x_3, 0.5/x_4) \\
(0.5/e_3, 0.2/x_3, 0.4/x_4, 0.3/x_5) \\
(0.4/e_4, 0.5/x_2, 0.1/x_3, 0.7/x_4)
\end{cases}
\]

\[
H_C = \begin{cases}
(0.3/e_2, 0.4/x_1, 0.3/x_3, 0.1/x_4) \\
(0.3/e_3, 0.1/x_3, 0.1/x_4, 0.3/x_5) \\
(0.1/e_4, 0.6/x_2, 0.5/x_3, 0.4/x_4)
\end{cases}
, \quad K_D = \begin{cases}
(0.4/e_2, 0.3/x_1, 0.4/x_3, 0.4/x_4) \\
(0.1/e_3, 0.4/x_3, 0.2/x_4, 0.5/x_5) \\
(0.2/e_4, 0.1/x_2, 0.2/x_3, 0.6/x_4)
\end{cases}
\]

Then \( \|\text{depth}(F_A)\| = \|\langle 4, 20 \rangle - \left\{ \left( \sum_{i=1}^{m} \mu_A(x_i), \sum_{i=1}^{m} \sum_{j=1}^{n} f_A(e_i)(x_j) \right) = (0.7, 3.5) \right\} \| = \|\langle 4, 20 \rangle - (0.7, 3.5)\| = \frac{3.3 + 16.5}{2} = 9.90. \)
Similarity, $\parallel \text{depth} (G_B) \parallel = 9.85$, $\parallel \text{depth} (H_C) \parallel = 10.25$ and $\parallel \text{depth} (K_D) \parallel = 10.10$. So we have the ranking $G_B \supset F_A \supset K_D \supset H_C$. Thus $G_B$ is the best representative of $U$.

8. Conclusion

In this paper, we firstly defined the concept of cardinality of soft hybrid sets. Then we discussed the entropy, similarity and subsethood measures based on cardinality. The relationships among these concepts was investigated as well as related examples. An application of cardinality is presented as a method for representation of a soft hybrid spaces. We hope that the findings in this paper will help the researchers to enhance and promote the further study on this concepts to carry out general framework for the applications in practical life.

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