Water quality reliability based on information entropy technology in water distribution system

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Abstract. Reliability of Water Distribution System (WDS) had been widely studied from mechanical and hydraulic aspects for a long time. Compared with them, water quality reliability (WQR) of WDS was not widely-presented by now. In this paper, WQR model of WDS based on entropy technology was proposed and applied to two typical water distribution systems. The results obtained showed that the water quality reliability had relationship with water sources, which was also important for hydraulic reliability of WDS. As a result, the presented model was suitable for assessing the water quality reliability of WDS.

1. Introduction

Municipal Water Distribution System (MWDS) played an irreplaceable role in serving people with safe and clear drinking water. So far, many researches concerning reliability of WDS had been carried out from two aspects: one is mechanical reliability on account of the structure, the other is hydraulic reliability reflecting hydraulic conditions [1]. However, with the improvement of people’s living standards, water quality in WDS turned out to be an important aspect of the municipal water supply works. Ostfeld combined the fraction of delivered quality (FDQ) into measurement of reliability in WDS by simulating stochastically based on EPANET [2]. Zhao defined water quality reliability as the fraction of sum of the days that the residual chlorine fulfills required local standard during the simulation period [3]. Ataoui developed the overall reliability of WDS concerning water quality utility based on cumulative utility model, fuzzy logic technology, and fuzzy set method. The water quality in WDS deteriorated with physical, chemical and biological processes by travelling through complex pipe systems, especially at the extreme ends [4]. Consequently, study of water quality reliability is becoming much more essential in WDS than usual. In 1990, the entropy concept was first introduced by Awumah into reliability assessment of WDS [5]. Ostfeld presented an assumption that water distribution systems carrying maximum entropy flows were generally reliable, which had been verified afterwards [6][7][8]. Tiku T. Tanyimboh & YohanSetiadi analyzed the relationship between the statistical entropy and hydraulic reliability of WDS [9]. Creaco compared entropy technology with resilience indices based on energetic concept by multi-objective optimization methodology to determine which method can represent reliability of WDS better [10]. Entropy-based reliability evaluation had been found to be less computationally intensive than other models. In a nutshell, entropy concept had been applied to hydraulic reliability of WDS successfully for many years, but seldom had it ever been used to assess the WQR of WDS.

In this paper, the author tried to take relative water quality entropy model based on EPANET as a surrogate method for assessing water quality reliability, which was applied to two WDS examples.
First, the relative water quality entropy model was proposed in section 2. Second, the applications of the model to two case studies with different complexity were presented in section 3. Third, discussions and conclusions were drawn in section 4.

2. Methodology

Water quality reliability in water distribution system was defined as the ability of the whole system meeting the water quality demand, which was described by relative water quality entropy in this paper. Since chlorine protected water quality against contamination by limiting bacteria regrowth in water distribution systems, it became most widely used disinfectant worldwide. However, chlorine will be consumed in reactions with other substances remaining in the water after treatment, especially dissolved organic matter (DOM) [11]. As a result, residual chlorine level can represent water quality in water distribution system to some extent, which was selected as representative indicator in relative water quality entropy model. In 《Standards for drinking water quality》 (Chinese) (GB5749-2006), residual chlorine level was set to be not less than 0.3 mg/L at the entrance of the WDS, not less than 0.05 mg/L at the extremities of the water distribution system. During the simulation executed by EPANET, the mixing of flow from pipe segments at junctions was assumed to be complete and instantaneous.

2.1. Water quality entropy (WQE)

Water quality entropy (WQE) measured water quality of water distribution system based on information entropy technology, expressed by Eq. (1) as follows:

\[ S = S_0 + S_{NJ} \]  

Where \( S \) is the overall WQE, \( S_0 \) is the WQE of water sources, \( S_{NJ} \) is the WQE of all the \( NJ \) nodes, and \( S_0 \) is defined by Eq. (2) as follows:

\[ S_0 = -\sum_{i=1}^{NR} \frac{Q_i C_i}{\overline{QC}} \ln \left( \frac{Q_i C_i}{\overline{QC}} \right) \]  

Where \( Q_i \) is the flow of the \( r \)-th water source, \( C_i \) is the chlorine concentration of the \( r \)-th water source, \( \overline{QC} \) is the total chlorine content of all the water sources (\( \overline{QC} = \sum_{i=1}^{NR} Q_i C_i \)), \( NR \) is the number of water sources, and \( S_0 \) is equal to 0 when there was only one water source, which means that WQE concerning water source was 0.

The WQE of node \( j \) is denoted by \( S_j \) as Eq. (3):

\[ S_j = -\sum_{i=1}^{NI} q_i^j C_i^j \ln \left( \frac{q_i^j C_i^j}{Q_j C_j} \right) \]  

Where \( q_i^j \) is the segment flow from node \( i \) to node \( j \), \( C_i^j \) is the residual chlorine concentration at the end of segment from node \( i \) to node \( j \), \( Q_j \) is the flow entering into node \( j \), \( C_j \) is the residual chlorine concentration at node \( j \) (\( C_j = \sum_{i=1}^{NI} q_i^j C_i^j \)), \( NI \) is the number of node set entering into node \( j \), and \( S_j \) is combined into entropy analysis considering the mass balance as well as residual chlorine decay.

For the whole WDS, the mass relationship of chlorine is expressed by Eq. (4) as follows:
\[
C_u = \frac{\sum_{j=1}^{N_j} Q_j C_j}{\sum_{j=1}^{N_j} Q_j} = \frac{\sum_{j=1}^{N_j} Q_j C_j}{Q_0}
\] (4)

Where \(Q_0\) is the total flows of all the segments in the system, \(C_0\) is the average chlorine concentration of all the nodes in the system.

Since first-order chlorine decay model is most widely used in network, the residual chlorine relationship between \(C_j\) and \(C_i\) can be expressed as Eq. (5):

\[
C_j = C_i \exp(-k_0 t_{ij}) = C_i \exp(-k_0 L_j / v_j) = C_i \exp(-k_0 L_j \pi d_j^2 / 4 q_j')
\] (5)

Where \(C_i\) is the chlorine concentration at node \(i\), \(k_0\) is the residual chlorine decay coefficient, \(t_{ij}\) is the water travel time from node \(i\) to node \(j\), and \(L_j\), \(v_j\), \(q_j'\), \(d_j\) were length, velocity, flow and diameter of segment from node \(i\) to node \(j\), respectively.

When \(\eta_j'\) is expressed by Eq. (6), then Eq. (5) is transferred into Eq. (7).

\[
\eta_j' = \exp(-k_0 L_j \pi d_j^2 / 4 q_j')
\] (6)

\[
C_j = C_i \eta_j'
\] (7)

Combining Eq. (7) with Eq. (3), Eq. (8) is deduced as follows:

\[
S_j = -\sum_{i=1}^{N_j} q_j' C_j \ln \frac{q_j' C_j}{Q_j C_j} = -\sum_{i=1}^{N_j} q_j' C_j \ln \frac{q_j' C_j \eta_j'}{Q_j C_j} \eta_j'
\] (8)

If we define \(P_i\) with Eq. (9), then Eq. (8) can be described as Eq. (10).

\[
P_i = \frac{q_j' C_j}{Q_j C_j}
\] (9)

\[
S_j = -\sum_{i=1}^{N_j} p_i \eta_j' \ln p_i \eta_j'
\] (10)

The WQE of node \(j\) \(S_j\) implies the local relative reliability of node \(j\). As regards to the overall network, the relative reliability of node \(j\) is described by \(\overline{S_j}\) in Eq. (11), which represents the individual contribution of node \(j\) to the whole network.

\[
\overline{S_j} = -\sum_{i=1}^{N_j} q_j' C_j \ln \frac{q_j' C_j}{Q_j C_0} - \sum_{i=1}^{N_j} q_j' C_j \ln \frac{q_j' C_j}{Q_j C_j} - \sum_{i=1}^{N_j} q_j' C_j \ln \frac{q_j' C_j}{Q_j C_0} - \sum_{i=1}^{N_j} q_j' C_j \ln \frac{q_j' C_j}{Q_j C_j} = \frac{Q_j C_j}{Q_j C_0} (-\sum_{i=1}^{N_j} q_j' C_j \ln \frac{q_j' C_j}{Q_j C_0} - \sum_{i=1}^{N_j} q_j' C_j \ln \frac{q_j' C_j}{Q_j C_j} + \sum_{i=1}^{N_j} \ln \frac{Q_j C_j}{Q_j C_0} \ln \frac{Q_j C_j}{Q_j C_0})
\] (11)
Where $\bar{S}_j$ is the relative reliability of node $j$ of the overall WDS, the WQEs of all $NJ$ nodes $S_{nj}$ can be described by Eq. (12) as follows:

$$S_{nj} = \sum_{j=1}^{NJ}[\bar{S}_j] = \sum_{j=1}^{NJ}[\frac{Q_C}{Q_0C_0}S_j] = \sum_{j=1}^{NJ}(\frac{Q_C}{Q_0C_0})\ln(\frac{Q_C}{Q_0C_0})$$

Combining Eq. (2), (12) into Eq. (1), the WQE of WDS is expressed by Eq. (13) as follows:

$$S = -\sum_{r=1}^{NR}p_r\ln p_r + \sum_{j=1}^{NJ}p_jS_j - \sum_{j=1}^{NJ}p_j\ln p_j$$

When $p_r$ is expressed by Eq. (14), and $p_j$ is defined by Eq. (15), then Eq. (16) is deduced.

$$p_r = \frac{Q_C}{Q_0C_0}$$

$$p_j = \frac{Q_C}{Q_0C_0}$$

Combining Eq. (10) with Eq. (16), the WQE of the network could be expressed by Eq. (17) as follows:

$$S = -\sum_{r=1}^{NR}p_r\ln p_r + \sum_{j=1}^{NJ}p_j\ln[p_j\eta_j\ln(p_j\eta_j)] - \sum_{j=1}^{NJ}p_j\ln p_j$$

2.2. Maximum water quality entropy (MWQE)

For a certain WDS, it is beneficial that all the segments carry the same quantity of chlorine from water quality point of view. With regard to a specific network topology, the above-mentioned entropy is an indirect WQR measurement model. When all the segments bring the same amount of chlorine, the WQE $S$ reaches the maximum value $S_{max}$. Such a character explains why WQE can be presented as a surrogate for WQR of the WDS. In the condition of uniform distribution of chlorine among segments, the corresponding results could be obtained as follows:

$$p_r = \frac{1}{NR}$$

$$p_j = \frac{1}{NJ}$$

then $S_{max}$ is expressed by Eq. (21) as follows:

$$S_{max} = -\sum_{r=1}^{NR}(\frac{1}{NR})\ln(\frac{1}{NR}) - \sum_{j=1}^{NJ}(\frac{1}{NJ})\ln(\frac{1}{NJ}) + \sum_{j=1}^{NJ}(\frac{1}{NJ})[\sum_{j=1}^{NJ}p_j\ln[p_j\eta_j\ln(p_j\eta_j)]]$$

$$= \ln NR + \ln NJ + \sum_{j=1}^{NJ}p_j\ln[p_j\eta_j\ln(p_j\eta_j)]$$
2.3. Relative water quality entropy (RWQE)

The approaching degree between WQE $S$ and MWQE $S_{\text{max}}$ represents the WQR of WDS, which is defined as the RWQE $R$ by Eq. (22).

$$ R = \frac{S}{S_{\text{max}}} \quad (22) $$

In Eq. (22), $R$ refers to surrogate of WQR in WDSs.

3. Application and discussion

3.1. Case study 1

The first case is a simple network made up of 11 nodes, 3 loops and a single water source shown in Figure 1. The hydraulic and water quality simulations of the network had been executed by EPANET software, in which Hazen-Williams roughness coefficient was set to be 100, and the chlorine decay coefficient was set to be -1.55/day [12]. The 24-h demand pattern characteristic for case 1 was shown in Table 1. The hydraulic time step was an hour, while the water quality time step was 10 minutes.

![Figure 1. Case1 (Rossman, 1994).](image)

| Time of day | 24-2 | 2-4 | 4-6 | 6-8 | 8-10 | 10-12 | 12-14 | 14-16 | 16-18 | 18-20 | 20-22 | 22-24 |
|-------------|------|-----|-----|-----|------|------|------|------|------|------|------|------|
| Multiplier of based water demand | 1.0  | 1.2 | 1.4 | 1.6 | 1.4  | 1.2  | 1.0  | 0.8  | 0.6  | 0.4  | 0.6  | 0.8  |

| Time of day | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   |
|-------------|------|------|------|------|------|------|------|------|------|------|------|------|
| Water quality entropy $S$ | 1.29 | 1.39 | 1.39 | 1.65 | 1.63 | 1.69 | 1.68 | 1.68 | 1.67 | 1.67 | 1.67 | 1.66 |

| Time of day | 13   | 14   | 15   | 16   | 17   | 18   | 19   | 20   | 21   | 22   | 23   | 24   |
|-------------|------|------|------|------|------|------|------|------|------|------|------|------|
| Water quality entropy $S$ | 1.65 | 1.63 | 1.62 | 1.58 | 1.56 | 1.53 | 1.51 | 1.48 | 1.46 | 1.45 | 1.45 | 1.45 |

| Time of day | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   |
|-------------|------|------|------|------|------|------|------|------|------|------|------|------|
| Relative water quality entropy $R$ | 0.64 | 0.67 | 0.67 | 0.64 | 0.63 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.64 |

| Time of day | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   |
|-------------|------|------|------|------|------|------|------|------|------|------|------|------|
| Relative water quality entropy $R$ | 0.61 | 0.64 | 0.63 | 0.63 | 0.62 | 0.61 | 0.60 | 0.59 | 0.58 | 0.57 | 0.57 | 0.56 |
The relative water quality entropy $R$ and water quality entropy $S$ were calculated by Eq. (17) and Eq. (22), which was shown in Table 2.

In Figure 2, although water demand changed with time, the RWQE $R$ varied between 0.55 and 0.7. When water demand decreased from 1.6 at the 8th hour to 0.4 at the 20th hour, $R$ also decreased from 0.65 to 0.59. However, when water demand increased from 1 at the 1th hour to 1.6 at the 8th hour, $R$ almost remained stable round 0.65. In addition, when water demand increased from 0.4 at the 20th hour to 0.8 at the 24th hour, $R$ decreased slowly from 0.59 to 0.56. As we all know, flow velocity in water distribution turned faster in case of high water demand, which leads to quicker chlorine decay. As a result, the RQWE representing WQR should decrease.

Figure 2. 24-h Relative water quality entropy of Case 1.

3.2 Case study 2

In this case, the concept of RWQE was applied to a “real-life” network shown in Figure 3 - Example 3 of EPANET software [12]. The system consists of two sources, three elevated tanks, 117 pipes, 97 demand nodes, and two pumps (the complete data used was exactly as that of Example 3, in Rossman [12], and thus is not repeated here). The coefficient of chlorine decay was set to be -1.55/day.

The demand pattern characteristic in 24 hours for Case 2 was shown in Table 3.

The RWQE $R$ as well as WQR $S$ were calculated by Eq. (2), Eq. (17), Eq. (21) and Eq. (22), and were shown in Table 4. From Table 4 we could conclude that RWQE $R$ is greater in case 2 than that in case 1, which means WDS in case 2 is more reliable than WDS in case 1 because that WDS in case 2 is supplied by multiple water sources.

The results were also shown in Figure 4, from which we can find that the curve of RWQE $R$ was also not relevant to the characteristic of 24-h demand pattern to a great extent, and RWQE $R$ had a sharp decrease at the 16th hour. The reason was that the water flow from water source “Lake” turns to be zero at that time. The WDS with double water sources became single-source WDS during the time of the 16th hour to the end of simulation period which leads to the sharp decrease of relative water quality entropy.

Figure 3. Case 2 (Rossman, 1994).
Table 3. 24-h demand pattern characteristic for Case 2 (Rossman, 1994).

| Time of day | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Multiplier of based water demand | 1.34 | 1.94 | 1.46 | 1.44 | 0.76 | 0.92 | 0.85 | 1.07 | 0.96 | 1.10 | 1.08 | 1.19 |

| Time of day | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  | 23  | 24  |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Multiplier of based water demand | 1.16 | 1.08 | 0.96 | 0.83 | 0.79 | 0.74 | 0.64 | 0.64 | 0.85 | 0.96 | 1.2  | 1.67 |

Table 4. 24-h Relative water quality entropy of Water Distribution System (WDS).

| Time of day | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Water quality entropy $S$ | 4.10 | 4.38 | 4.34 | 4.39 | 4.52 | 4.51 | 4.56 | 4.41 | 4.43 | 4.43 | 4.42 | 4.41 |
| Relative water quality entropy $R$ | 0.76 | 0.80 | 0.80 | 0.81 | 0.83 | 0.83 | 0.84 | 0.81 | 0.82 | 0.82 | 0.81 | 0.81 |

| Time of day | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  | 23  | 24  |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Water quality entropy $S$ | 4.40 | 4.36 | 3.52 | 3.42 | 3.35 | 3.29 | 3.24 | 3.20 | 3.16 | 3.13 | 3.10 | 3.17 |
| Relative water quality entropy $R$ | 0.81 | 0.80 | 0.65 | 0.63 | 0.62 | 0.61 | 0.60 | 0.59 | 0.58 | 0.58 | 0.57 | 0.58 |

Figure 4. 24-h Relative water quality entropy (RWQE) $R$ of Case 2.

4. Discussion and conclusions
In this paper, the concept of relative water quality entropy (RWQE) corresponding to reliability of WDS from water quality point of view was presented. It was derived from the entropy function of Shannon (1948) and considered both water sources and junctions of WDS. Two WDSs had been taken as examples to simulate the RWQE $R$. The results of the analysis indicate that the RWQE model was not only reasonable and feasible, but also could be well applied into practice.

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