RUNAWAY HEATING BY $r$-MODES OF NEUTRON STARS IN LOW-MASS X-RAY BINARIES

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ABSTRACT

Recently Andersson et al. and Bildsten have independently suggested that an $r$-mode instability might be responsible for stalling the neutron star spin-up in strongly accreting low-mass X-ray binaries (LMXBs). We show that if this does occur, there are two possibilities for the resulting neutron star evolution. If the $r$-mode damping is a decreasing function of temperature, then the star undergoes a cyclic evolution: (1) accretional spin-up triggers the instability near the observed maximum spin rate; (2) the $r$-modes become highly excited through gravitational radiation reaction, and in a fraction of a year ($0.13 \text{ yr}$ in a particular model that we have considered) they viscously heat the star up to $T \sim 2.5 \times 10^9 \text{ K}$; (3) $r$-mode gravitational radiation reaction then spins the star down in $t_{\text{spindown}} \approx 0.08(f_{\text{final}}/130 \text{ Hz})^{-6} \text{ yr}$ to a limiting rotational frequency $f_{\text{final}}$, whose exact value depends on the not fully understood mechanisms of $r$-mode damping; (4) the $r$-mode instability shuts off; and (5) the neutron star slowly cools and is spun up by accretion for $\sim 5 \times 10^6 \text{ yr}$, until it once again reaches the instability point, closing the cycle. The shortness of the epoch of $r$-mode activity makes it unlikely that $r$-modes are currently excited in the neutron star of any galactic LMXBs, and unlikely that advanced LIGO interferometers will see gravitational waves from extragalactic LMXBs. Nevertheless, this cyclic evolution could be responsible for keeping the rotational frequencies within the observed LMXB frequency range. If, on the other hand, the $r$-mode damping is temperature independent, then a steady state with constant angular velocity and $T_{\text{core}} \approx 4 \times 10^8 \text{ K}$ is reached, in which $r$-mode viscous heating is balanced by neutrino cooling and accretional spin-up torque is balanced by gravitational radiation reaction spin-down torque. In this case (as Bildsten and Andersson et. al. have shown) the neutron stars in LMXBs could be potential sources of periodic gravitational waves, detectable by enhanced LIGO interferometers.

Subject headings: binaries: close — stars: evolution — stars: neutron — stars: oscillations — stars: rotation — X-rays: stars

1. INTRODUCTION

Most of the rapidly accreting neutron stars in low mass X-ray binaries (LMXBs) are observed to rotate in a strikingly narrow range of frequencies—from 260 to 330 Hz (see, e.g., van der Klis 1999). A natural explanation for this could be some mechanism that prevents further neutron star spin-up once the rotational frequency is sufficiently high. Recently several such mechanisms were proposed.

White & Zhang (1997) suggested that magnetic braking could be responsible for halting the spin-up; this idea will not be discussed here. Bildsten (1998) pointed out that because gravitational radiation reaction is a sharply increasing function of rotational frequency, it might halt the spin-up. In his original manuscript, Bildsten (1998) identified one mechanism for triggering the necessary gravitational waves: lateral density variations caused by temperature dependence of electron capture reactions. While his manuscript was being refereed, Bildsten learned of the discovery that an $r$-mode instability, driven by gravitational radiation, can be very strong in spinning neutron stars (Andersson 1999; Friedman & Morsink 1999; Lindblom, Owen, & Morsink 1998; Andersson, Kokkotas, & Schutz 1998a; Owen et al. 1999), and the $r$-mode experts learned of Bildsten’s (1998) gravitational-wave idea for saturating LMXB spin-up. Both groups independently saw the connection: Bildsten (1998) and Andersson, Kokkotas, & Stergioulas (1998b) proposed that the $r$-mode instability could provide enough gravitational radiation reaction to halt the LMXB spin-up. In this paper we examine the consequences of this proposal.

Our conclusions depend crucially on whether the dissipation of the $r$-modes decreases with temperature (e.g., as is the case when shear viscosity dominates the $r$-mode damping), or instead is temperature-independent (e.g., as is the case when the mutual friction of proton and neutron superfluids dominates the damping). In the former case (see § 2), we find that the neutron star will undergo a spin-up heating/spin-down cooling cycle; in the latter case (§ 3), it will probably settle down to a stable equilibrium state with an internal neutron star temperature of about $4 \times 10^8 \text{ K}$.

2. VISCOS $r$-MODE DAMPING

Let us consider first the case when dissipation is a decreasing function of temperature. We show that if some $r$-modes become unstable in a neutron star spun up by accretion, then they heat up the neutron star through shear viscosity. As the neutron star heats up, the $r$-modes become more unstable. A thermogravitational runaway takes place in which the $r$-mode amplitude grows; as a result of this growth, the temperature of the star rises, the dissipation becomes weaker, and the instability becomes stronger. Within a fraction of a year the $r$-modes’ gravitational radiation reaction spins the star down to a rotation frequency that is close to the minimum of the critical stability curve (probably around 100–150 Hz, but the exact value depends on poorly understood dissipation mechanisms—see below), with a final temperature of about $2 \times 10^8 \text{ K}$. The instability then shuts off and the star begins a several million year epoch of neutrino cooling and accretional spin-up, leading back to the original instability point.
Figure 1 shows a typical evolutionary trajectory, A → B → C → D → B, of the neutron star in the log \(T_8 - \tilde{\Omega}\) plane, where \(T_8\) is the temperature of the star's core measured in units of \(10^8\) K, and \(\tilde{\Omega} = \Omega/(\pi G \tilde{\rho})^{1/2}\). Here \(\Omega\) is the angular velocity of the neutron star and \(\tilde{\rho}\) is its mean density. The portion A → B of the curve represents the accretional spin-up of the neutron star to the critical angular frequency \(\Omega_{cr}(T)\); B → C represents the heating stage in which the \(r\)-modes become unstable, grow, and heat up the neutron star; C → D shows the spin-down stage in which the \(r\)-mode amplitude saturates because of poorly understood nonlinear effects, and the angular velocity decreases because of the emission of gravitational radiation; and D → B represents cooling back to the equilibrium temperature with simultaneous spin-up by accretion. All four stages are discussed in more detail below.

The initial (steady state) temperature \(T_0\) of the neutron star core in steadily accreting LMXBs is somewhat uncertain; according to Brown & Bildsten (1998), who analyzed heat transport during steady thermonuclear burning of the accreted material and nuclear reactions in the deep ocean, \(T_0 = (1-4) \times 10^8\) K. In Figure 1 we assume \(T_0 = 10^8\) K.

The curve K–L–M is the so-called \(r\)-mode stability curve (Lindblom et al. 1998). If the neutron star is represented by a point above the curve, then some \(r\)-modes in the star are unstable and grow. Otherwise, all \(r\)-modes decay. The portion K–L of the stability curve is determined by the shear viscosity, or by mutual friction if part of the star is superfluid. Its exact location is uncertain precisely because the dissipation of the \(r\)-modes at the relevant temperatures is poorly understood. If shear viscosity dominates the dissipation, then the equation of the K–L portion of the stability
curve is given by
\[
\Omega_\nu = 0.1 \left( \frac{\eta}{\eta_0} \right)^{\frac{1}{6}} T^{-\frac{1}{3}},
\]
where \( \eta \) is the shear viscosity of the neutron star material, and \( \eta_0 \) is the shear viscosity due to electron-electron scattering in the neutron star (we have used eqs. [2.10], [2.14], [2.15], and Table 1 of Owen et al. 1999 in order to work out eq. [1]). If only shear viscosity due to electron-electron scattering were operating, with the shear viscosity given by
\[
\eta = 347 \rho^{\frac{1}{4}} T^{-2},
\]
where all quantities are in cgs units (see Cutler & Lindblom 1987, and references therein), then the critical rotational frequency at \( T = 10^8 \) K would be 130 Hz, which is much less than observed values (van der Klis 1999). However, the friction is probably larger than this (and therefore \( \Omega_\nu \) is also larger) because of interaction of the core fluid with the crust and maybe mutual friction in a superfluid state. The emphasis of this paper is not to figure out whether the \( r \)-mode instability is relevant for LMXBs, but to investigate the consequences if it is relevant. For purpose of illustration, we assume that \( \eta = 244 \times \eta_0 \); this makes the critical rotational frequency 330 Hz at \( T = 10^8 \) K, which is consistent with observations (van der Klis 1999). This choice of viscosity is a cheat since we do not yet know the \( T \) and \( \rho \) dependence of \( \eta \). However, unless the damping is due to mutual friction, \( \eta \) is likely to decrease with increasing temperature, which is a sufficient condition for thermogravitational runaway. Our choice of viscosity possesses this feature; therefore, we believe it has a good chance of representing the real physics.

The portion L–M of the stability curve is determined by bulk-viscosity dissipation. Of the two current estimates (Lindblom et al. 1998; Andersson et al. 1998a) of the bulk-viscosity contribution to damping of the \( r \)-modes, we have chosen the one that gives the higher values of \( \Omega_\nu \), thus maximizing its effect.\(^{1}\) For the evolution curve shown in Figure 1, the fact that no part is in the region where the bulk viscosity dominates suggests that the details of the bulk viscosity will not be of particular importance.

In this work for concreteness we specialize to a polytropic model of a neutron star with \( p \propto \rho^2 \) and consider the \( r \)-mode with \( l = m = 2 \), which is expected to have the strongest instability in such polytropes (Friedman & Morsink 1999; Lindblom et al. 1998). We assume that the time evolution of the normalized angular velocity \( \Omega = \Omega/(\pi G \rho)^{1/2} \) of the star and the dimensionless amplitude \( \alpha \) of the \( r \)-mode are given by phenomenological equations (3.14), (3.15), (3.16), and (3.17) of Owen et al. (1999):
\[
\frac{d\Omega}{dt} = \frac{2\alpha^2 Q}{1 + \alpha^2} \frac{\Omega}{\tau_v} + \frac{4}{3} \sqrt{\frac{1}{T} \frac{M}{T} \times p},
\]
\[
\frac{d\alpha}{dt} = -\frac{1}{\tau_{\text{grav}}} \left[ \frac{1}{\tau_v} \frac{1}{1 + \alpha^2 \Omega} \right] \alpha,
\]
when \( \alpha^2 < k \) (the saturation value of \( \alpha^2 \), which we assume to be \( k = 1 \)), and by
\[
\alpha^2 = k,
\]
\(^{1}\) Lindblom et al. (1998) probably overestimate the effect of bulk viscosity by using an Eulerian density perturbation instead of a Lagrangian one (this point is discussed in Andersson et al. 1998a). For an updated calculation, which uses Lagrangian density perturbations, see Lindblom et al. (1999).

\[
\frac{d\Omega}{dt} = \frac{2\Omega}{\tau_{\text{grav}}} + \frac{kQ}{1 - kQ},
\]
when \( \alpha \) is saturated due to not yet understood nonlinear effects. Here \( \alpha \) is the dimensionless amplitude of the \( r \)-mode defined by equation (1) of Lindblom et al. (1998), and \( \tau_v \) and \( \tau_{\text{grav}} \) are the viscous and gravitational timescales for the \( r \)-mode dissipation and are given by equations (2.14) and (2.15) of Owen et al. (1999):
\[
\tau_{\text{grav}} = -3.26 \Omega^{-6} s,
\]
\[
\frac{1}{\tau_v} = \frac{1}{\tau_{8}} \left( \frac{10^8 K}{T} \right)^2 + \frac{1}{\tau_{8}} \left( \frac{T}{10^8 K} \right)^6 \Omega^2.
\]
In the above equation the viscous damping rate is a sum of contributions from the shear and the bulk viscosities; the former is determined by \( \tau_v \), which we took to be \( 1.03 \times 10^4 \) s in order to fit the observed data; the latter is determined by \( \tau_{\text{grav}} \), which is taken to be \( 6.99 \times 10^{14} \) s, in agreement with Owen et al. (1999).

Note that \( \tau_{\text{grav}} \) is negative since gravitational radiation always amplifies the \( r \)-mode. The second term in equation (2) represents the neutron star spin-up by accretion; \( M \) and \( \dot{M} \) are the mass of the neutron star and its accretion rate, respectively, and \( p \) is a factor of order unity that depends on the accretion radius and the angular velocity of the neutron star; its exact value is not essential for the physics discussed here, and we set \( p = 1 \) from here onward. The numerical parameters \( Q \) and \( J \) are given by 0.094 and 0.261, respectively, for a polytrope star of adiabatic index \( \gamma = 2 \) (Lindblom et al. 1998). For the evolution shown in Figure 1, we took \( M = 1.4 M_\odot \) and \( \dot{M} = 10^{-8} M_\odot \) yr\(^{-1} \), and we assumed a random initial perturbation of magnitude \( \alpha = 10^{-8} \) when the neutron star reaches the stability curve K–L.

Now consider the thermal evolution of the star. The \( r \)-mode deposits heat into the star at the rate
\[
W_{\text{diss}} = \frac{2E_{r-\text{mode}}}{\tau_v} = \frac{\alpha^2 \Omega^2 M R^2 J}{\tau_v},
\]
where \( E_{r-\text{mode}} \) is the energy in the \( r \)-mode (cf. eq. [3.11] of Owen et al. 1999). Here \( R \) is the radius of the neutron star taken to be 12.53 km, and \( J = 1.635 \times 10^{-2} \) for the polytropic model considered here. At the relevant temperatures the neutron star is expected to cool predominantly by the modified URCA process. (This is not entirely true, since close to \( 10^8 \) K neutrino bremsstrahlung cooling from the crust and radiative cooling by photons might become significant. However, their cooling rates are not significantly larger than that of the modified URCA process at \( 10^8 \) K, and they become negligible at higher temperatures. For simplicity, we assume in this work that modified URCA is the only cooling process; the inclusion of other processes would not change the general evolutionary picture.) The modified URCA cooling rate, reduced by heating from nuclear reactions in the deep crust, is given by (Shapiro & Teukolsky 1983)
\[
L_{\text{cool}} = 7 \times 10^{31} (T_8^8 - T_8^8) \text{ ergs s}^{-1}.
\]
Here the subscript 8 indicates that the temperature is measured in units of \( 10^8 \) K, and \( T \) is the equilibrium temperature of the neutron star before the \( r \)-mode heating starts, taken to be \( 10^8 \) K for our calculation. The thermal
evolution equation is then given by
\[ \frac{dT}{dt} = \frac{W_{\text{diss}} - L_{\text{cool}}}{C_v}, \]  
(9)

where \( C_v \) is the heat capacity of the neutron star, taken to be \( 1.4 \times 10^{38} \) ergs K\(^{-1}\) \( \times T_8 \) from Shapiro & Teukolsky (1983, eq. [11.8.2]). However, the heat capacity of a neutron star with a superfluid core is less.

Equations (2), (3), (5), (4), and (9) determine the time evolution of the angular velocity \( \Omega \) and temperature \( T \). Figure 1 shows the predicted evolution for the representative parameter values introduced above.

The evolution consists of four stages. The first stage \( A \rightarrow B \) is the spin-up of the neutron star, during which its angular velocity \( \Omega \) is increasing toward the critical angular velocity, and the \( r \)-mode instability is suppressed by viscosity; since we assume that the star begins at its equilibrium temperature \( T_k = T_k = 1 \), its temperature changes little during the spin-up. For an assumed accretion rate of \( 10^{-6} \) \( M_\odot \) yr\(^{-1}\), this stage takes \( \sim 5 \times 10^6 \) yr.

When the angular velocity reaches its critical value, the \( r \)-mode starts to grow, and the second stage \( B \rightarrow C \) begins. The neutron star gets heated up by the \( r \)-mode through viscosity, the \( r \)-mode becomes more unstable, and thermon- gravitational runaway follows. It takes 0.13 yr for the \( r \)-mode's amplitude to evolve from \( \alpha = \alpha_w \) to \( \alpha = 1 \), where \( \alpha_w = 1.2 \times 10^{-5} \) is the value of the \( r \)-mode amplitude at which the accretional torque is exactly compensated by the gravitational radiation reaction (see Wagoner 1984). For our intuition, it is useful to define two characteristic \( \tau \)-dependent timescales for stage \( B \rightarrow C \): the thermal timescale (cf. eq. [7]),
\[ t_{\text{th}} = \frac{dt}{d \log T} = \frac{C_v T}{W_{\text{diss}}} \sim 3.7 \times 10^{-5} T_k \frac{\tau_v}{\alpha^2}, \]  
(10)

and the timescale for the decrease of angular velocity (cf. eq. [2]),
\[ t_\Omega = \frac{dt}{d (\log \Omega)} \sim \frac{1}{2Q} \frac{\tau_v}{\alpha^2} \sim 5 \times \tau_v \frac{\alpha^2}{\alpha^2}. \]  
(11)

Clearly the neutron star heats up much faster than it spins down because of gravitational radiation. Therefore, during this stage the angular velocity of the star decreases by only a small amount, \( \Delta \Omega = 0.0003 \). Physically the reasons for such little change in \( \Omega \) are that the \( r \)-mode amplitude grows so quickly, and that in this phase the angular momentum loss is not manifested in a reduction of the angular velocity, but instead in the growth of the \( r \)-mode itself (the \( r \)-mode, which is driven by gravitational radiation reaction, carries a negative angular momentum).

Eventually the \( r \)-mode amplitude saturates due to nonlinear effects. This initiates the third stage of the evolution, in which all of the angular momentum loss is manifested by reduction of angular velocity (since the \( r \)-mode cannot grow any more), and the star spins down \( C \rightarrow D \) to the critical angular velocity. At point \( C_1 \), the temperature of the neutron star is such that the neutrino cooling exactly compensates the dissipative heating from the \( r \)-modes. After that the temperature does not change much until the spin-down stage is terminated. The physical reason for this is that even though the thermal timescale at \( C_1 \rightarrow D \) is comparable or smaller than the spin-down timescale, the rate of dissipative heating does not change much. If the heat capacity \( C_v \) of the neutron star were zero, we would have \( W_{\text{diss}} = L_{\text{cool}} \) at all points of \( C_1 \rightarrow D \). This would imply \( T \propto \Omega^{-1/2} \), so even then the temperature would not change significantly over this last part of the spin-down.

An analytical expression for the duration of this rapid spin-down stage can be derived from equations (5) and (6):
\[ t_{\text{spindown}} = 0.08(1/k)\bar{\Omega}_f /0.1 - 6 \text{ yr}, \]  
(12)

where \( \Omega_f \) is the angular velocity at the end of the spin-down. In our simulations \( t_{\text{spindown}} \) is about 0.14 yr.

After the neutron star reaches the stability curve, the \( r \)-mode is damped by viscosity stronger than it is driven by gravitational-radiation reaction; therefore, its amplitude decreases and the neutron star cools back to its original equilibrium temperature while being spun up by accretion. This part of the evolution is represented by \( D \rightarrow B \) on Figure 1; its timescale is the same as that for the original accretional spin-up, i.e., \( \sim 5 \times 10^6 \) yr. After this the cycle is closed and can repeat itself as long as the accretion continues.

We believe that the sharp kink at point \( C \) is not a real physical effect, but a result of our poor understanding of the nonlinear saturation of the \( r \)-mode; however, this artificial feature of our simulations does not seem to affect the existence of the thermon- gravitational runaway and the subsequent rapid spin-down to a lower angular velocity. Despite a large number of uncertainties in the details of the evolution, we believe that this scenario is robust so long as the \( r \)-mode instability does occur in LMXBs, and the damping of the \( r \)-modes decreases with temperature.

If the above described evolutionary scenario is generic, it is then clear that none of the currently observed LMXBs can possess an actively operating \( r \)-mode instability—otherwise we would observe a rapid spin-down on a timescale less than 1 yr. However, it is conceivable that many of the neutron stars in these LMXBs have undergone the \( r \)-mode instability at some stage of their evolution and are currently below the stability curve evolving along leg \( D \rightarrow B \) of Figure 1.

From equations (12) and (2) we can estimate the fraction \( r \) of neutron stars in extragalactic LMXBs that are in the phase of active emission of gravitational waves:
\[ r = \frac{t_{\text{spindown}}}{t_{\text{accretion}}} \sim 1.6(1/k) \times 10^{-8} \left( \frac{\bar{\Omega}_f}{0.1} \right)^{-6}. \]  
(13)

The quantity \( \bar{\Omega}_f \) is bounded from below by the rotational frequencies of young pulsars (this statement is true only if the \( r \)-mode damping is the same for young and old pulsars at the same temperatures). The rotational frequency of the recently discovered N157B (Marshall et al. 1998) is 62.5 Hz. Using the braking index theory, one can project the initial rotational frequency of this pulsar to be no smaller than 100 Hz, which implies \( \bar{\Omega}_f > 0.8 \). Therefore, only \( r < \left( 6/k \right) \times 10^{-8} \) of neutron stars in extragalactic LMXBs are in the phase of rapid gravitational wave emission, which implies that to catch one star in this phase, gravitational-wave detectors must reach out through a volume large enough to encompass \( \sim (0.1 - 0.01)/r \sim 10^6 \) galaxies, like our own (this assumes that there are 10–100 strongly accreting neutron stars in LMXBs in our galaxy). An analysis similar to that of Owen et al. (1999) shows that even “advanced LIGO” detectors are unlikely to be able to see these sources at such great distances.
3. TEMPERATURE-INDEPENDENT \( r \)-MODE DAMPING

There is a possible alternative evolutionary scenario that is similar to the one proposed by Andersson et al. (1998b), Bildsten (1998), and L. Lindblom (1998, private communication). It may be that the \( r \)-mode damping is dominated not by normal dissipative processes, but by mutual friction in the neutron-proton superfluid. Detailed calculations of the effect of such friction on the \( r \)-mode damping are in progress (Lindblom & Mendell 1999); however, for our analysis the essential feature of this dissipative process is already known—it is temperature independent. Therefore, if this process dominates one would not expect a thermogravitational runaway; instead the neutron star will reach a state of threefold equilibrium. The neutron star will “sit” on the stability curve \([1/\tau_{\text{grav}}]+(1/\tau_\alpha) = 0\], the amplitude of the \( r \)-mode will adjust so that the accretional torque is compensated by the gravitational radiation reaction torque \((\alpha = \alpha_w \simeq 1.2 \times 10^{-5} \text{ for our model})\), and the temperature of the neutron star will adjust so that the cooling compensates the frictional heating from the \( r \)-mode: \( W_{\text{diss}} = L_{\text{cool}} \). From equations (2), (6), (7), and (8), one can work out the equilibrium temperature:

\[
T_{eq} = 4.2 \times 10^8 K \left( \frac{f}{330 \text{ Hz}} \right)^{1/8} \left( \frac{M}{10^{-5} M_\odot \text{ yr}} \right)^{1/8} \left( \frac{1.4 M_\odot}{M} \right)^{1/8},
\]

(14)

where \( f \) is the rotational frequency of the star.

It is interesting to examine how (and whether) the star reaches this equilibrium point. For temperature-independent damping, equations (2) and (3) form a closed system with two independent variables, \( f \) and \( L \). To investigate the behavior of the star after it reaches the stability curve at \( f = f_{eq} \), we set \( f = f_{eq} + \Omega_1 \) and expand equations (2) and (3) to first order in \( \Omega_1 \). After trivial algebraic manipulations, we can then reduce the system of two first-order differential equations to a single second-order differential equation:

\[
d^2x/dt^2 + \gamma(x) dx/dt = -\partial V(x)/\partial x.
\]

(15)

Here \( x = \ln \alpha \), and \( \gamma(x) \) and \( V(x) \) are given by

\[
\gamma(x) = \frac{2Q \exp (2x)}{\tau_v},
\]

(16)

and

\[
V(x) = \frac{6}{\tau_{\text{grav}}} \left[ \frac{Q \exp (2x)}{\tau_v} - \frac{x}{\tau_{\text{acc}}} \right].
\]

(17)

In the above equations \( \tau_{\text{grav}} \) is given by equation (6), and \( \tau_{\text{acc}} = (1/p)(3/4)^{1/2} \Omega_{eq} T M / M \) is the timescale for the neutron star to be spun up by accretion to the angular frequency \( \Omega_{eq} \).

Clearly equation (15) can be thought of as an equation of motion for a particle of unit mass in the potential well given by \( V(x) \) and with the damping \( \gamma(x) \). The bottom of the potential well corresponds to the equilibrium state described above, and the damping insures that the “particle” gets there (i.e., that the neutron star settles into the equilibrium state). However, the damping is small. To see this, consider damped oscillatory motion close to the bottom of the well. The complex angular frequency of this motion is given by

\[
\omega = \sqrt{12/(\tau_{\text{acc}} \tau_{\text{grav}})} - i/(2\tau_{\text{acc}}).
\]

(18)

The period of these small oscillations is

\[
P \sim 230 \left( \frac{M}{1.4 M_\odot} \right)^{1/2} \left( \frac{10^{-8} M_\odot \text{ yr}^{-1}}{M} \right)^{1/2} \times \left( \frac{f}{330 \text{ Hz}} \right)^{-5/3} \text{ yr}^{-1},
\]

(19)

but the timescale on which they are damped (i.e., the timescale on which the equilibrium is reached) is \( \tau_{eq} \sim 2\tau_{\text{acc}} \sim 10^7 \text{ yr} \).

Since the damping is so small, fluctuating disturbances may keep this nonlinear oscillator off its equilibrium position. For example, in our evolutionary scenario we have assumed that there is a mechanism that gives \( \alpha \) some nonzero initial value. Presumably, the same mechanism could keep the oscillator in an excited state. Then the amplitude of the \( r \)-mode, and hence the temperature of the star’s core, would vary on the timescales of hundreds of years. Detailed investigation of these issues is a subject for further work. However, it is clear that the time-averaged temperature should be close to the equilibrium value given by equation (14).

If the \( r \)-mode damping does not depend on temperature, we can expect \( r \)-modes to be excited in many of the rapidly rotating neutron stars in LMXBs. These presumably superfluid steady gravitational-wave emitters could be detected by enhanced LIGO gravitational wave detectors, as discussed in Bildsten (1998) and Andersson et al. (1998b). Recently, Brady & Creighton (1998) have considered the computational cost of such detection. Their conclusion was that with the enhanced LIGO sensitivity and available computational capabilities one could detect gravitational-wave emitters in LMXBs that are as bright in X-ray flux as SCO-X1.

If the rotational frequency of the emitting neutron star is localized to within a few 10 s of Hz using astronomical observations (e.g., by quasi-periodic oscillation), one could narrow band the interferometer response around the frequency of \( r \)-mode oscillations (see, e.g., Meers 1988). This could allow LIGO to detect gravitational-wave emitters in LMXBs that are 10–100 times dimmer in X-ray flux than SCO-X1.

Positive detection of gravitational waves at the \( r \)-mode oscillation frequency would make a strong case for the superfluid nature of the \( r \)-mode damping.

4. CONCLUSIONS

In this paper we have investigated the recent proposal that the accretional spin-up of the neutron star in an LMXB is stopped by \( r \)-mode gravitational radiation reaction. There are two possible evolutionary scenarios. In the first scenario, the neutron star goes through cycles such as that shown in Figure 1. The necessary condition for this scenario to be relevant is that \( r \)-mode damping should decrease with increasing temperature. In this case, it is very unlikely that any of the currently observed neutron stars in LMXBs in our galaxy are in the \( r \)-mode excited phase of the cycle. The detection of gravitational radiation from extragalactic
LMXBs in the \( r \)-mode excited phase is also not likely, even with advanced LIGO interferometers.

In the second scenario, \( r \)-mode damping is temperature-independent, and a steady state equilibrium is probably reached where both angular velocity and temperature stay constant or are oscillating with periods of several hundreds of years. Equation (14) makes a robust prediction for the temperature of these objects to be \( \approx 4 \times 10^8 \) K; this temperature is on the high end of what is typically expected, and it might be possible to test this prediction by observations. In this case the neutron stars are emitters of periodic gravitational waves, which could be detected by interferometers like enhanced LIGO.

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