Mathematical modeling of power loads in the construction of a parallelogram lift

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Abstract. The question of determining the power loads in the elements of the supporting structure of a parallelogram hoist is considered. The design diagram of the lift supporting structure and the mathematical model describing the dependence of the power geometric characteristics of the structure are presented. To simplify the design scheme, not a volumetric, but a flat supporting structure was considered. Equilibrium equations were compiled for each element of the design scheme. The dependence of the angle of inclination of the element of the hydraulic cylinder rod on the angle of lifting of the supporting structure is obtained and the nature of the dependence is shown. The system of equations was solved in matrix form for the given values of the angle of the hydraulic cylinder rod position.

1. Introduction
Modern lifting equipment, due to its specificity, is characterized by a variety of design solutions [1]. This is due to: the used principles of motion realization [2;3]; working conditions; mass-dimensional characteristics of the goods being lifted; characteristics of drives; features of the workplace. Of all the elements of the lift, its supporting structure is the most important part, and the efficiency of its functioning depends on its strength and compactness. In view of the well-known advantages in various industries and fields of activity, lifts of the parallelogram type are actively used [2-4]. Pneumatic drives due to the significant compressibility of the working medium [5,6]. When mechanizing the executive movement of hoists, including the parallelogram type, a hydraulic drive is mainly used [3;4;7-9].

The supporting frame of a parallelogram lift has a complex geometric shape, which complicates its power and strength calculation [7-11]. The use of mathematical calculation methods in combination with CAD / CAE [2;7-11] allows to provide a rational combination of geometric and strength characteristics. The specificity of the parallelogram-type lifting equipment is that the principle of organizing the movement is common to everyone. Therefore, if we use CAD / CAE, then every time we need to create a new bar or solid model [2;12-16]. If a mathematical model is created that links geometry and force factors, then the preliminary and subsequent calculations are greatly simplified [7;10;11;17].

2. Main part
The aim of the proposed work is to obtain a mathematical model for calculating parallelogram lifting mechanisms. To simplify the mathematical model, we represent the two-dimensional design scheme of the parallelogram mechanism in the form of a flat system (figure 1). The flat mechanism of the hydraulic
The parallelogram lift is a system consisting of three components (figure 1): “A” - left tilt frame, “C” - right tilt frame, “B” - hydraulic cylinder rod.

To create a mathematical model, the structure was broken down into elements. For each element, the equilibrium equations were compiled, taking into account the replacement of the component “B” (hydraulic cylinder rod) with the corresponding forces, a system of equations for the two components “A” and “C” of the supporting structure was obtained [7;12;16]:

\[
\begin{align*}
P \cdot \cos \beta + X &= 0 \quad (1.1) \\
R + Y + P \cdot \sin \beta - F &= 0 \quad (1.2) \\
-X \cdot L \cdot \sin \alpha - Y \cdot L \cdot \cos \alpha + 2 \cdot F \cdot L \cdot \cos \alpha - \\
- P x \cdot (b + L) \cdot \sin \alpha - P y \cdot (b + L) \cdot \cos \alpha &= 0 \quad (1.3) \\
Q - X - P \cdot \cos \beta &= 0 \quad (1.4) \\
T - P \cdot \sin \beta - Y - F &= 0 \quad (1.5) \\
X \cdot L \cdot \sin \alpha - Y \cdot L \cdot \cos \alpha - 2 \cdot F \cdot L \cdot \cos \alpha &= 0 \quad (1.6)
\end{align*}
\]

where: \( F \) is the force at the ends of the vertical rods; \( X \) is the effort in the central node along the Ox axis, (N); \( V \) is the effort in the central node along the Oy axis, (N); \( R \) – reaction in the movable support along Ox, (N); \( T \) is the reaction in a fixed support according to Oy, (N); \( Q \) – reaction in a fixed support along Ox, (N); \( L \) – rod length, \( L=1470 \) mm, (figure 2); \( b \) is the length of the small section of the rod, \( b=500 \) mm (figure 2); \( \alpha \) – tilt angle of the mechanism (figure 2); \( \beta \) – angle of inclination of the hydraulic cylinder rod (figure 2).

Equations (1.1), (1.2), (1.3) describe the loading of the element “C”, and equations (1.4), (1.5), (1.6) describe the element “A”. The payload \( F \) is known, \( F = 3125 \) N, the linear dimensions of the support triangle (figure 2): bar length, \( L = 1470 \) mm; length of a small section of the rod, \( b = 500 \) mm. Obviously, the system has solutions only if the angle of inclination of the hydraulic cylinder rod \( \beta \) is known, therefore its dependence on any already known value was determined. The equation of the

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**Figure 1.** Design diagram of a flat element of the supporting structure of a parallelogram type hoist.
dependence of the angle of inclination of the hydraulic cylinder rod $\beta$ on the angle of lifting of the mechanism $\alpha$ was determined. For this, the geometric characteristics of the supporting triangle with sides ($L$, $b$, $c$) of the supporting structure were considered (figure 2).

![Support triangle of the hoist mechanism.](image)

Figure 2. Support triangle of the hoist mechanism.

Taking into account that the length of the stock is variable ($c = \text{var}$), after simple transformations, we get:

$$
\cos \beta = \frac{(b^2 + L^2 - 2L \cdot b \cos (2 \cdot \alpha)) - b^2 + L^2}{2 \cdot (\sqrt{b^2 + L^2 - 2L \cdot b \cos (2 \cdot \alpha)}) \cdot L}
$$

Add an additional angle $\alpha$ to equation (2) and express the numerical value of the angle $\beta$ in degrees:

$$
\beta = \left[ \arccos \left( \frac{(\sqrt{(b^2 + L^2 - 2L \cdot b \cos (2 \cdot \alpha))^2 - b^2 + L^2}) + \alpha}{2 \cdot (\sqrt{b^2 + L^2 - 2L \cdot b \cos (2 \cdot \alpha)} \cdot L)} \right) \right] (3)
$$

The dependence of the lifting angle of the support mechanism $\beta$ on the angle of inclination of the element “B” $\alpha$ has a linear character and is shown in figure 3.

![Dependence of the angle of the element “B” inclination on the angle of ascent of the supporting structure.](image)

Figure 3. Dependence of the angle of the element “B” inclination on the angle of ascent of the supporting structure.
Let us determine the angle of inclination of the element “B” for seven positions of the supporting structure and solve the system of equations (1). The calculation was carried out for the following values of the angle $\beta$: 11.2°; 17.2°; 23.2°; 29.2°; 35.2°; 41.2°; 45°. Taking into account the fact that the force at the ends of the vertical rods, F and the linear dimensions of the support triangle L and b, are known, we find the solution of the system of linear equations (1) by the matrix method. For this, we substitute the known numerical values into the system of equations (1):

$$\begin{align*}
0.4407P + X &= 0 \\
(0.897P+(R+Y)) \cdot 3125 &= 0 \\
-1250.35P + \left( -613.89P + \left( \frac{707 \cdot 1470(-X)}{1000} - 1039.44Y \right) + 6495562.5 \right) &= 0 \\
-0.4407P + (Q - X) &= 0 \\
(-Y + (-0.8976P + T)) - 3125 &= 0 \\
(0.7071 \cdot 1470X - 1039.44Y) - 6439481.25 &= 0
\end{align*}$$

Let us bring the system of equations (4) to the canonical form and represent it in matrix form:

$$\begin{bmatrix}
0.4407x_1 & 0x_2 & 0x_3 & 0x_4 & 1x_5 & 0x_6 \\
0.8976x_1 & 0x_2 & 1x_3 & 0x_4 & 0x_5 & 1x_6 \\
-1864.23x_1 & 0x_2 & 0x_3 & 0x_4 & -1039.29x_5 & -1039.44x_6 \\
-0.4407x_1 & 1x_2 & 1x_3 & 0x_4 & 1x_5 & 0x_6 \\
-0.8976x_1 & 0x_2 & 0x_3 & 0x_4 & 0x_5 & -1x_6 \\
0x_1 & 0x_2 & 0x_3 & 0x_4 & 1039.44x_5 & -1039.44x_6
\end{bmatrix} =
\begin{bmatrix}
0 \\
3125 \\
-6495562.5 \\
0 \\
6496481.25
\end{bmatrix}$$

The results of solving matrix (5) by Cramer's method are summarized in table 1.

**Table 1.** Calculation of the main force factors in the structure at certain angles of ascent.

| Угол подъема $\beta$ | 11.2° | 17.2° | 23.2° | 29.2° | 35.2° | 41.2° | 45° |
|---------------------|-------|-------|-------|-------|-------|-------|-----|
| 0.4407x_1 | 0x_2 | 0x_3 | 0x_4 | 1x_5 | 0x_6 | 3125 | 6496481.25 |
| 0x_1 | 0x_2 | 0x_3 | 0x_4 | 1039.44x_5 | -1039.44x_6 | |
| P | 33000 | 23133 | 18753 | 16409 | 15015 | 14126 | 13724 |
| X | 30617 | 19584 | -14144 | -10847 | -8594 | -6924 | -6063 |
| Y | -12312 | -12312 | -12312 | -12312 | -12312 | -12312 | -12312 |
| R | 3125 | 3125 | 3125 | 3125 | 3125 | 3125 | 3125 |
| T | 3125 | 3125 | 3125 | 3125 | 3125 | 3125 | 3125 |

The results obtained represent the strength characteristics in the bars of the structure. To define force constraints in structural elements using CAD / CAE, it is necessary to create a bar model and specify the assortment type [12;15;16].

The characteristic coincidence of the bending stress plots determined theoretically and using the Structure 3D CAD / CAE APM WinMachine module [18] is shown in figure 4.
3. Conclusions and discussion

The results obtained make it possible to calculate the power parameters of the supporting structure of a parallelogram lift with elements of different geometric parameters, as well as proceed to the calculation of their strength characteristics, search for material and type of profile. Having specified the type of profile and choosing the appropriate assortment, it is possible to determine the stresses acting in dangerous sections of the structure and rationally select the parameters of the section of the element.

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