I. INTRODUCTION

The interferometry correlations are a sensitive measure of the space-time evolution of the system formed in heavy-ion collisions. The interferometry, or Handbury-Brown Twiss (HBT) radii, extracted from the same-sign $\pi-\pi$ correlation function give an estimate of the size of the region where the pion pair is emitted [1, 2]. From the studies of Au-Au collisions at BNL Relativistic Heavy Ion Collider energies it has been found that hydrodynamic simulations describe fairly well the experimental data on HBT correlations [3, 4]. Similar agreement is seen for Pb+Pb collisions at the CERN Large Hadron Collider.

From the analysis of the three dimensional $\pi-\pi$ correlation function three radii parameters can be extracted [5, 6]: $R_{out}$, $R_{side}$, and $R_{long}$. A key assumption required to reach agreement with the data is the use of a hard equation of state for the matter created in the collision. A softening of the equation of state leads to an increase of the pion pair is emitted [11, 2]. From the studies of Au-Au collisions at BNL Relativistic Heavy Ion Collider it has been found that hydrodynamic simulations describe fairly well the experimental data on HBT correlations [3, 4]. Similar agreement is seen for Pb+Pb collisions at the CERN Large Hadron Collider.

II. MODEL AND CALCULATIONS

I perform calculations in a 3 + 1-dimensional viscous hydrodynamic model [10]. The shear viscosity to entropy density ratio is $\eta/s = 0.08$. The bulk viscosity to entropy ratio is temperature dependent

$$\zeta/s = \frac{\zeta_H}{s} \frac{1}{1 + \exp((T - T_H)/\Delta T_1)} + c \frac{\zeta_{peak}}{s} \frac{1}{T}$$

where

$$\frac{\zeta_{peak}}{s} (T) = \begin{cases} l_1 \exp \left( \frac{T}{T_H} - 1 \right) + l_2 \exp \left( \frac{T}{T_H} - 1 \right) & \text{for } T < T_a \\ A_0 + A_1 \frac{T}{T_H} + A_2 \left( \frac{T}{T_H} \right)^2 & \text{for } T_a < T < T_b \\ l_3 \exp \left( \frac{1 - T}{T_H} \right) + l_4 \exp \left( \frac{1 - T}{T_H} \right) & \text{for } T > T_b \\ \end{cases}$$

and $T_H = 180\text{MeV}$, $T_a = 179.1\text{MeV}$, $T_b = 189\text{MeV}$, $\Delta T_H = 4\text{MeV}$, $A_0 = -13.45$, $A_1 = 27.55$, $A_2 = -13.77$, $l_1 = 0.9$, $l_2 = 0.26$, $l_3 = 0.9$, $l_4 = 0.256$, $s_1 = 0.0025$, $s_2 = 0.022$, $s_3 = 0.025$, $s_4 = 0.13$. The parametrization of the bulk viscosity peak is similar as in [12]. In the following I study two different cases $c = 0$, $1$ (Fig. 1). The first case corresponds to no bulk viscosity peak, with bulk viscosity appearing only at low temperatures. Such
a small bulk viscosity appears naturally in a mixture of massive particles with nonequilibrium distributions. The parametrization with $c = 1$ includes a bulk viscosity peak around the critical temperature as expected from lattice QCD results [11]. The height of the bulk viscosity peak is also estimated from fits to particle spectra and azimuthal harmonic flow coefficients [15]. In the following I denote the two scenarios by the value of bulk viscosity at $T_H$, that is $\zeta/s = 0.02, 0.35$. An increase in the bulk viscosity implies larger dissipative correction in the hydrodynamic evolution. The corresponding entropy production in the evolution is compensated by a reduction of the initial entropy for calculation with larger bulk viscosity to obtain the same final particle multiplicity at central rapidity.

The results presented further correspond to the freeze-out temperature of 150 MeV. The Cooper-Frye formula for the emission of hadrons from the freeze-out hypersurface includes nonequilibrium corrections from both shear tensor and bulk viscosity. Bulk viscosity corrections are included using the relaxation time approximation [17]. I have checked that varying the freeze-out temperature down to 140 MeV does not change the conclusions.

From pion pairs a Bose-Einstein symmetrized correlation function is constructed [18] and binned in bins of average transverse momentum of the pion pair. The three dimensional correlation function in relative pair momentum $q$ is fitted with a Gaussian formula [5, 6] \[ C(q) = 1 + \lambda \exp \left( -R_{\text{out}}^2 q_{\text{out}}^2 - R_{\text{side}}^2 q_{\text{side}}^2 - R_{\text{long}}^2 q_{\text{long}}^2 \right) . \] The three components of $q$ are defined along the beam axis ($q_{\text{long}}$), along the pair transverse momentum ($q_{\text{out}}$), and transverse to those ($q_{\text{side}}$). The interferometry analysis is summarized by the three HBT radii $R_i$ as functions of the average pair momentum $k_F$.

\section{Smooth Initial Conditions}

To present a simple example I calculate the evolution of the fireball with smooth initial density density. The entropy density is obtained from the optical Glauber model. The density in the transverse plane is proportional to a combination of densities of participant nucleons $\rho_v(x, y)$ and binary collisions $\rho_{\text{bin}}(x, y)$

\[ s(x, y) \propto (1 - \alpha)\rho_v(x, y) + 2\alpha\rho_{\text{bin}}(x, y) , \]

with $\alpha = 0.15$ [19]. The normalization of the initial density is adjusted in order to reproduce the charged particle multiplicity. The calculation is performed for zero impact parameter for illustration.

The bulk viscosity peak reduces the transverse flow at the edge of the fireball. It causes a retardation of the expansion at the edge. The resulting freeze-out hypersurface is slightly modified (Fig. 2). The effect on the freeze-out hypersurface can be described as a change from a scenario with earlier freeze-out in the outer layers (“burning log scenario”) to a freeze-out at constant proper time. The total life-time of the system is similar in the two scenarios.

Both calculations, with and without a bulk viscosity peak, reproduce the HBT radii fairly well (Fig. 3). The differences in the HBT radii between the two scenarios are small, less than 5%. The increase of bulk viscosity around the critical temperature does not modify strongly the dynamics. Note that the $R_{\text{out}}/R_{\text{side}}$ ratio decreases when the bulk viscosity peak is introduced (Fig. 4). This decrease may be understood as due to the observed change in the shape of the freeze-out surface [20]. The ratio $R_{\text{out}}/R_{\text{side}}$ is larger when the outside layers of the fireball freeze-out earlier.
FIG. 3. (color online) The HBT radii for central Pb-Pb collisions at 2760 GeV. ALICE Collaboration data (squares) compared to hydrodynamic calculations with smooth initial conditions, with (dotted lines) and without (solid lines) a peak in the temperature dependence of bulk viscosity.

FIG. 4. (color online) The ratio of HBT radii $R_{\text{out}}/R_{\text{side}}$ for central Pb-Pb collisions at 2760 GeV. ALICE Collaboration data (squares) are compared to hydrodynamic calculations using smooth initial conditions with (dotted line) and without (solid line) a peak in the temperature dependence of bulk viscosity.

IV. EVENT-BY-EVENT HYDRODYNAMIC SIMULATIONS

A realistic modeling of the collective expansion in heavy-ion collisions requires hydrodynamic simulations to be performed for an ensemble of fluctuating initial conditions. I use a Glauber Monte Carlo model to generate the initial entropy density [21]. The density in the transverse plane is written as a sum of Gaussians of width 0.5 fm centered at the positions of the participant nucleons. A contribution of binary collisions is added with $\alpha = 0.15$. In Fig. 5 are shown the freeze-out hypersurfaces corresponding to the same initial Monte Carlo Glauber event (one particular event with 377 participant nucleons) but evolved with hydrodynamics with or without a peak in the temperature dependence of bulk viscosity. The effect of bulk viscosity is qualitatively similar as for smooth initial conditions. Quantitatively the difference between the two scenarios is larger at the very edge of the fireball, due to larger local gradients of the density in event-by-event simulations. The increase in bulk viscosity hinders the expansion in the outer layers of the fireball, where the initial gradients are the largest. It leads to a freeze-out hypersurface, where inner layers freeze-out earlier.

For each event a freeze-out hypersurface is defined and the pion pair correlation function is constructed. After averaging over events, HBT radii are extracted from the Gaussian formula (Eq. 3). For realistic event-by-event simulations one gets similar conclusions as for the ex-

FIG. 5. (color online) Same as Fig. 2 but starting the evolution with one particular initial condition from a Glauber Monte Carlo model.

FIG. 6. (color online) Same as Fig. 3 but for a calculation using an ensemble of fluctuating initial conditions from a Glauber Monte Carlo model.
The relative increase in the life-time of the fireball is small in central Pb-Pb collisions, and limited to the outer layers of the fireball.

The effective softening of the equation of state due to the bulk viscosity peak does not lead to an increase of the $R_{\text{out}}/R_{\text{side}}$ ratio. This observation is contrary to expectation from earlier studies of the sensitivity of the HBT radii to the equation of state \cite{15, 22, 24}. Surprisingly, the ratio $R_{\text{out}}/R_{\text{side}}$ is reduced when the bulk viscosity peak is introduced. Although the expansion is slightly slower with bulk viscosity, the shape of the hypersurface is modified in such a way as to reduce the problematic $R_{\text{out}}/R_{\text{side}}$ ratio by a few percent, making it closer to the data.

It would be interesting to include the interferometry radii in the data constraining the parameters of the matter created in relativistic heavy-ion collisions using exhaustive Bayesian statistical analyses. Existing analysis have studied a matter either without a bulk viscosity peak or they do not take the HBT radii in the data used to constraint the parameters \cite{15, 22, 24}. The present result suggests that the presence of a peak in the temperature dependence of bulk viscosity would be consistent with an enlarged analysis including HBT radii in the data set. Further insight could be gained by studying the effects of bulk viscosity on the HBT radii in small, rapidly expanding systems, as in p+Pb collisions.

\section*{V. CONCLUSIONS}

I have tested the effect of a peak in the temperature dependence of bulk viscosity on the size and the life-time of the source in Pb-Pb collisions at 2760 GeV. The increase of bulk viscosity around the critical temperature slows down the expansion of the fireball in the outer layers, where initially the gradients are the largest and the temperature of matter is close to the critical temperature. The change in the value of the radii is small \cite{15, 22, 24}. The final effect on the HBT radii is to reduce the $R_{\text{out}}/R_{\text{side}}$ ratio by up to 3\% for the calculation with a bulk viscosity peak \cite{7}. This improves the description of the data.

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