The Method of Fundamental Solutions using the Vector Magnetic Dipoles for Calculation of the Magnetic Fields in the Diagnostic Problems Based on Full-Scale Modelling Experiment

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Abstract. The article describes the calculation of the magnetic fields in the problems diagnostic of technical systems based on the full-scale modeling experiment. Use of gridless fundamental solution method and its variants in combination with grid methods (finite differences and finite elements) are allowed to considerably reduce the dimensionality task of the field calculation and hence to reduce calculation time. When implementing the method are used fictitious magnetic charges. In addition, much attention is given to the calculation accuracy. Error occurs when wrong choice of the distance between the charges. The authors are proposing to use vector magnetic dipoles to improve the accuracy of magnetic fields calculation. Examples of this approach are given. The article shows the results of research. They are allowed to recommend the use of this approach in the method of fundamental solutions for the full-scale modeling tests of technical systems.

Introduction

The need to save energy and resources leads to creation of systems with different limit modes and high electromagnetic, mechanical, thermal loads on the materials and structures. All this requires reliable identification of research objects, that is determination of the parameters and characteristics of the materials and constructions used in the mathematical modeling with sufficient accuracy. However, a number of parameters and characteristics cannot be measured directly. In such cases, the only way to get information is the solution of inverse problems. They allow define the parameters of the subdomains that are inaccessible to measurement, using the results of measurements in available subdomains of the object [1-4]. For the successful implementation of such an approach is necessary mathematical models and methods of physical fields calculation, focused on the implementation of full-scale modeling test systems that provide the definition of the parameters of systems with high precision and speed.

In recent years, wide application received gridless method of fundamental solutions (MFS) and its variants in combination with grid methods (finite differences and finite elements) for calculating fields [5-8]. The calculation of the magnetic field of the electromagnet, moving along the ferromagnetic plate, considered in article. In this article, the calculation of the dipole magnetic field is used in the auxiliary problem. In research presented in [9], the field in the space around the ferromagnet is...
determined by using a fictitious magnetic charges, the field in the nonlinear medium is calculated by finite element method. This approach is significantly reduces the dimensionality task of calculation of the field and, thereby, reduces calculation time. However, when using dipoles, the field determined with an accuracy that depends on the distance between the charges $+q_i$ and $-q_i$ [10,11].

In this paper, it was proposed to use MFS vector magnetic dipoles to improve the accuracy of magnetic fields calculation in the problems diagnosticof systems based on full-scale modeling experiment.

2. Methodology

For exception of error calculation of the magnetic field by MFS, is used approach needed in the calculation of electrostatic fields [12]. This error caused by the introduction of fictitious magnetic dipoles. If charges $+q$ and $-q$ are positioned as shown in figure 1, then value of the potential at an arbitrary point $P$ is determined by expression

$$\varphi_e(P) = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{r_+} - \frac{q}{r_-} \right) = \frac{q}{4\pi\varepsilon_0} \left( \frac{1}{r_+ + l} - \frac{1}{r_- + l} \right).$$

(1)

Figure 1. To determine the scalar magnetic potential.

By the definition of the gradient, considering that the quantity $l = |l|$ is sufficiently small compared $|\vec{r}|$ we obtain

$$\frac{1}{r_+ + l} - \frac{1}{r_- + l} \approx -l \frac{\nabla \varphi_e}{r_{DP}}. \quad (2)$$

Here, the index $P$ is that in terms of the gradient, coordinates of point $P$ are considered as variables. On the basis of (1) and (2) we obtain

$$\varphi_e(P) = -\frac{q\vec{l}}{4\pi\varepsilon_0} \frac{\nabla \varphi_e}{r_{DP}}. \quad (3)$$

We will reduce the distance $l$ between the charges and to increase $q$ so that $ql = \vec{m}_e$ remain constant.

Then, when $l \to 0$ the expression (3) will give the exact value of the potential in the limit

$$\varphi_e(P) = -\frac{1}{4\pi\varepsilon_0} \vec{m}_e \frac{\nabla \varphi_e}{r_{DP}}. \quad (4)$$

Vector $\vec{m}_e$ as is directed from a negative to a positive charge. The magnitude $\vec{m}_e$ is called an electric dipole moment. Dipole characterized $\vec{m}_e$ is calledavectorelectricdipole.
Considering the analogy of magnetic and electric fields we will determine magnetic dipole moment of the formula

$$\vec{m}_m = q_m \vec{I},$$

where \(q_m\) – fictitious magnetic charges and the dipole is a vector magnetic dipole \(\vec{m}\).

The scalar magnetic potential of such a dipole in a vacuum is defined by analogy with (4)

$$\varphi_m(P) = -\frac{1}{4\pi \mu_0} \vec{m}_m \nabla p \frac{1}{r_{DP}}.$$ (5)

Equation (5) for the scalar magnetic potential of a vector dipole can be in the form

$$\varphi_m(P) = \frac{1}{4\pi \mu_0} \vec{m} \vec{e}_r = \frac{1}{4\pi \mu_0} \frac{m \cos \theta}{r_{DP}^2}.$$ (5)

The potential of a vector dipole is zero in the plane normal to the axis of the dipole and extending through its center.

The magnetic field intensity of a vector dipole determined by the formulas

$$H_r = -\frac{\partial \varphi}{\partial r} = \frac{1}{2\pi \mu_0} \frac{m \cos \theta}{r_{DP}^3}; \quad H_\theta = -\frac{1}{r} \frac{\partial \varphi}{\partial \theta} = \frac{1}{4\pi \mu_0} \frac{m \sin \theta}{r_{DP}^3}; \quad H_a = 0.$$

Figure 2 shows the placement of the normal dipoles to the problem considered in [9]. In the figure positive charges \(+q_i\) are placed above the plane with \(\varphi_m = 0\), the negative charges of the dipoles \(-q_i\) under the plane. Points \(D_i\) are located on the plane with \(\varphi_m = 0\). Figure 3 shows the placement of vector magnetic dipoles \(\vec{m}_j\) for the same problem.

![Figure 2: Placement of points \(D_i\) and dipoles \(q_i\).](image-url)
3. Results and Discussion

3.1. Solution test problem

Consider the use of vector magnetic dipoles to solve the test problem "the continuous ferromagnetic sphere in a homogeneous magnetic field", using the results obtained in [9].

For the calculation of the magnetic field outside of the sphere in figure 4 is used an ordinary dipole and vector magnetic dipole in figure 5.

Figure 3. Placement of vector magnetic dipoles \( \vec{m}_i \).

Figure 4. Location of the charges \(+q\) and \(-q\), used for calculation of the field outside of the sphere.
Analytic solution of a test problem is presented in [13]. The exact value of the z-component of the magnetic field outside of the sphere at the point \( M_0 \) determined by formulas

\[
\vec{H}_z(M_0) = 2 \frac{\mu_s - \mu_0}{\mu_s + 2\mu_0} H_{0z},
\]

where \( H_{0z} \) – z-component of the intensity external homogeneous known magnetic field; \( \mu_s \) – sphere magnetic permeability, where \( \mu_s >> \mu_0 \).

Formulas for determining of the same intensity by MFS (figure 4) has the form

\[
\vec{H}_z(M_0) = q_0 \left( \frac{1}{(R-h)^2} - \frac{1}{(R+h)^2} \right).
\]

Suppose \( h = R/n \). Substituting \( h \) into the equation (7), we obtain

\[
\vec{H}_z(M_0) = q_0 \frac{4n^3}{R^2 (n^2 - 1)^2}.
\]

In [9] the test problem is solved by conventional dipoles (figure 4) using the boundary conditions at the point \( M_0 \). A result we obtain

\[
\vec{H}_z(M_0) = \frac{n^2}{n^2 - 1} 2 \frac{\mu_s - \mu_0}{\mu_s + 2\mu_0} H_{0z}.
\]

Estimate relative error \( \delta \left( \vec{H}_z(M_0) \right) \)

\[
\delta \left( \vec{H}_z(M_0) \right) = \left| \frac{\vec{H}_z(M_0) - H_{0z}}{H_{0z}} \right| 10^2 = \frac{200}{n^2 - 1} \%
\]

Dependence \( \delta(\vec{H}_z(M_0)) \) on \( n = R/h \) is presented in the table 1.

**Figure 5.** Location vector magnetic dipole for the calculation of the field outside of the sphere.
Table 1. Dependence $\delta(\tilde{H}_z^-(M_0))$ on $n = R/h$.

| $n = R/h$ | 2   | 4   | 6   | 8   | 10  |
|-----------|-----|-----|-----|-----|-----|
| $\delta(\tilde{H}_z^-(M_0))$%   | 33  | 6.7 | 2.9 | 1.6 | 1.0 |

With the same precision were defined other quantities problem: $H_0^+(M_0)$; $\varphi_{m}^-(M_0)$; $\varphi_{m}^+(M_0)$.

Consider the solution of a test problem with a vector magnetic dipole (figure 5).

Using the boundary conditions at the point $M_0$, we obtain the ratios of the following form [9]

$$
\begin{cases}
\varphi^+(M_0) = \varphi^-(M_0) + \varphi_{0}^-(M_0); \\
\mu_s H^+_z(M_0) = \mu_0 H^-_z(M_0) + \mu_0 H_{0z},
\end{cases}
$$

where $\varphi^+(M_0) = -H^+_z R$; $\varphi^-_{0}(M_0) = -H_{0z} R$; $\varphi^- (M_0) = -m'_m \nabla_1 \frac{1}{r_{DM0}} = \frac{m'_m}{R^2}$.

$$
H^-_z(M_0) = -\frac{\partial \varphi}{\partial r} = \frac{2m'_m}{R^2}; \quad m'_m = \frac{m_m}{4\pi \mu_0}.
$$

The system (8) are two unknown quantities $H^+_z$ and $m'_m$.

Solving the system (8), we obtain

$$
\tilde{H}_z^+(M_0) = \frac{3\mu_0}{\mu_s + 2\mu_0} H_{0z};
$$

$$
m'_g = \frac{3\mu_0}{\mu_s + 2\mu_0} R^3 H_{0z}.
$$

From the second equation of (8) can be defined $H^-_z(M_0)$ through $H_{0z}(M_0)$:

$$
H^-_z(M_0) = 2 \frac{\mu_g - \mu_0}{\mu_s + 2\mu_0} H_{0z}.
$$

Comparison (9) and (6) shows that the use of vector magnetic dipoles provides a solution of test problem that identical to the analytical solution.

3.2. Theresultsofexperimentalresearch

We choose to research the magnetic system, consisting of a pair of coils, between which is a part of the active element from a ferromagnetic material and its dimensions match the dimensions of the coil windows (figure 6). In the figure: $\tilde{\delta}$ – the current density in the coils; $V^+$ – subdomain occupied by an active element (ferromagnetic with a magnetic permeability $\mu$); $V^-$ – subdomain surrounding the element of space, filled by linear medium with $\mu_0$. The system has the following parameters:

$A_e = A_s = A_c = B_e = 2 \times 10^{-3}$ m; $H_e = 1 \times 10^{-3}$ m; $H_c = 2 \times 10^{-3}$ m; $C_c = 0.5 \times 10^{-3}$ m.
In order to estimate accuracy calculation of the magnetic field by MFS using fictitious magnetic dipoles (figure 2) and the vector magnetic dipoles (figure 3), the problem is solved by finite element method (FEM).

The coils and the active element is placed in the center of the parallelepiped with sides $0.5 \times 0.25 \times 0.25 \text{ m}$, it was filled by 87434 tetrahedra. On the edges of the parallelepiped is given vector magnetic potential $0 \mathbf{H}$. Were determined the values $H_z^0$ when magnetomotive force of each coil $iw_c = 10^3 \text{ A}$ in the points $D_i$. Then there were calculated $\varphi_i^0 (M_i)$ and determined of the values $H_z^+ \varphi^+$ and $\varphi^*$ in the points $D_i$ by numerical integration.

A comparison was made received results and calculations performed by the MFS (table 2, 3). In the calculations is taken into account symmetry of the magnetic system. If the number of collocation points $N = 5$ (figure 2, 3) then due to the symmetry of their position $q_i = q_2 = q_3 = q_4$ and unknown $H_z^+ (M_i)$, $i = 0, 1$; $N = 13$ - by symmetry positions of the points have four unknowns charge $q_0, q_1, q_2, q_3$ and unknown $H_z^+ (M_i)$, $i = 0, 1, 2, 3$.

Table 2. The results of calculations by MFS at the number of collocation points $N = 5$.

| Number of points ($s(i)$) | Calculation using magnetic dipoles | Relative error | Calculation using vector magnetic dipoles | Relative error | FEM calculation |
|---------------------------|-----------------------------------|----------------|------------------------------------------|----------------|----------------|
|                           | $H_z^+$, A/m                       | $\delta(H_z^+)$, % | $H_z^+$, A/m | $\delta(H_z^+)$, % | $H_z^+$, A/m |
| 0                         | 449.2                             | 68.7            | 445.7                                    | 67.4           | 266.2          |
| 1                         | 317.4                             | 49.3            | 315.0                                    | 48.2           | 212.5          |
Table 3. The results of calculations by MFS at the number of collocation points $N = 13$.

| Number of points $(i)$ | Calculation using magnetic dipoles | Relative error | Calculation using vector magnetic dipoles | Relative error | FEM calculation |
|------------------------|------------------------------------|----------------|------------------------------------------|----------------|----------------|
|                        | $H_z^+$, A/m                       | $\delta (H_z^+)$, $\%$ | $H_z^+$, A/m                              | $\delta (H_z^+)$, $\%$ | $H_z^+$, A/m    |
| 0                      | 321.6                              | 20.8            | 320.2                                    | 20.1           | 266.2          |
| 1                      | 247.9                              | 16.6            | 246.9                                    | 16.2           | 212.5          |
| 2                      | 219.3                              | –7.9            | 219.5                                    | 7.8            | 238.0          |
| 3                      | 286.4                              | 16.2            | 285.8                                    | 15.9           | 246.4          |

Comment. The results which are given in the table 2 and 3 are obtained by $\mu = 10^3 \mu_0$.

Results of experimental research show that sufficient accuracy for calculating the parameters of the magnetic field for engineering problems can be obtained with a small number of fictitious magnetic charges and vector magnetic dipoles, which is typical for the MFS. The dimension of the problem is significantly reduced when account is taken the symmetry of the magnetic system.

Application in the MFS of vector magnetic dipoles improves the accuracy calculation of the magnetic fields, and in some cases gives the solution that coincides with the analytical solution.

4. Conclusion

The use of the proposed approach provides a significant reduction in calculation time with the required precision of the magnetic field modeling. This allows to effectively solving the problems of design electrotechnical systems and performing the system diagnostics in the process of full-scale modeling experiment. It also allows performing fault analysis and forecasting of systems in real time.

The obtained results allow recommending the use of MFS with vector dipoles in the calculation of other physical fields (electrical, thermal, etc.).

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