PROBABILISTIC CHARACTERISTICS OF WIRELESS NETWORKS WITH INFRASTRUCTURE TOPOLOGY

Abstract. Based on the analysis of the operation of networks IEEE 802.11 DCF, a function is proposed for determining the probability of frame transmission to a central node depending on the number of stations operating in saturation mode. The probabilities of collisions are calculated. Using a polynomial approximations an expression is obtained for the network throughput, which explicitly depends on the number of the simultaneously operating stations.

Keywords: wireless network IEEE 802.11, DCF mode, collisions, throughput, probabilistic characteristics.

Introduction. Wireless networks with infrastructure topology of IEEE 802.11 standard are intensity developed. The data exchange speeds of several gigabits per second, which have already really been reached at the present time, make it possible to use them instead of wired networks, especially on twisted pair. This opens up the possibility of using wireless networks in the shops of industrial enterprises, at production sites. But the high level of electromagnetic interference, which is characteristic for these industrial premises, leads to a distortion of a significant number of frames moving from peripheral stations to the central node. Herewith these stations do not receive confirmations from the central node on sent frames. The collision compensation method CSMA/CA used in this case does not distinguish between the frame damage due to external interference from operating technology equipment and the collision that occurs when frames are mutually damages by two stations operating in the same wireless segment. In both cases the contention windows mechanism is activated, which delays the frames sent by the stations, and thereby significantly reduces the speed of information exchange in a wireless network.

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Fundamental analysis of IEEE 802.11 network using Distributed Coordination Function (DCF) was carried out in article [1] and later updated in [2]. This analysis is based on the use of bidimensional Markov chain model with ideal channel conditions and saturated load. Each frame needs to wait for a random backoff time before transmitting. The backoff is performed in discrete time units called slots and the stations are synchronized on the slot boundaries. As a result of this analysis, it was obtained that the probability $\tau$ that a station transmits in a randomly chosen slot time is

$$\tau = \frac{2(1-2p)}{(1-2p)(W_0+1)+pW_0[1-(2p)^{m-1}]}$$  \hspace{1cm} (1)

where $W_0=CW_{\text{min}}$ is minimum contention window, $CW_{\text{max}} = 2^m W_0$.

The probability $p$ that a transmitted frame encounters a collision is defined as the probability that, in a time slot, at least one of the $n-1$ remaining stations transmit

$$p = 1 - (1-\tau)^{n-1}$$  \hspace{1cm} (2)

As can be seen from expressions (1) and (2) the dependence of the collision probability $p$ on the number of simultaneously operating stations $n$ at a given initial window width $W_0$ can be obtained only by rather cumbersome numerical calculations. At the same time it is useful for engineering calculations to have analytical expressions for explicitly calculating the probability of collisions and the network throughput based on this, depending on the number of stations operating in saturation mode. The proposed article is devoted to solving this problem.

**Problem statement.** The purpose of this article is to determine the dependence of throughput of an infrastructure of wireless network IEEE 802.11 operating in DCF mode on the number of workstations in an explicit analytical form suitable for engineering calculations.

**Main part.** If $p$ is the collision probability, then an arbitrary frame is successfully transmitted with probability $(1-p)$, and the average backoff window is $(W_0-1)/2$. If the first transmission fails, the frame is successfully transmitted on the second attempt with probability $p(1-p)$. The average backoff window $W_{\text{avg}}$ in this case is $(2W_0-1)/2$. On the third attempt to transfer $W_{\text{avg}} = (4W_0-1)/2$ and so on. It is logical to assume [3] that the probability of
successful frame transfer $\tau$ back is proportional to the average window width, i.e. $\tau = 1/W_{\text{avg}}$.

For the probability of frame transmission to the central node of the segment, we propose the following expression

$$
\tau = \frac{2}{2\left(\frac{n}{N}\right)^{q} \cdot W_{0} - 1},
$$

where $n$ is the number of stations operating in saturation mode, $N$ is the maximum number of stations, $W_{0}$ is the initial window width, $q$ is the variable parameter.

The collision probability, defined by expression (2), for fixed values $N$ and $W_{0}$, depends only on the number of operating stations.

When operating a wireless network in a production environment with a high level of electromagnetic interference due to operating equipment, parameter BER (Bit Error Rate) [4] can reach $\text{BER} = 10^{-3}...10^{-4}$. Under these conditions, it is advisable to limit $N$ to 20 ($N = 20$).

The graphs of the dependences of $\tau(n)$ and $p(n)$ for different values of $q$ are presented in Figs. 1 and 2, respectively.

The dependencies shown in the Fig.1 and Fig.2 are well approximated by parabolas:

for $q=2$ $\tau(n)=10^{-4} (-0,5n^{2} - 7n + 659), \quad R^{2} = 0,9984$;

for $q=3$ $\tau(n)=10^{-4} (-n^{2} + 6n + 636), \quad R^{2} = 0,9961$; \hfill (4)

for $q=4$ $\tau(n)=10^{-4} (-n^{2} + 15n + 616), \quad R^{2} = 0,9951$;

for $q=2$ $p(n)=10^{-5} (-2,2n^{2} + 67,2n - 60,2), \quad R^{2} = 0,9996$;

for $q=3$ $p(n)=10^{-5} (-2,3n^{2} + 71,9n - 70,4), \quad R^{2} = 1,0$; \hfill (5)

for $q=4$ $p(n)=10^{-5} (-2,2n^{2} + 73,1n - 73,9), \quad R^{2} = 0,9998$. 
It should be noted that the approximation of the functions $p(n)$ is carried out in the range $n = 1...16$. Further, with an increase in $n$ to the limiting value $n = N$, a decrease in the functions $p(n)$ is observed for $q = 3$ and $q = 4$. Such decrease for $q = 2$ is observed after $n = 18$. 
In accordance with [1] the network throughput $S$ is defined as ratio of the average frame payload of information transmitted in a time interval to the average length of the interval in form

$$S = \frac{P_{tr} \cdot P_s \cdot E[Fr]}{(1 - P_{tr}) \eta \sigma + P_{tr} P_s T_s + P_{tr} (1 - P_s) T_c}, \quad (6)$$

where $E[Fr]$ is the average frame payload; $P_{tr}$ is the probability that there is at least one transmission in a considered time interval, i.e. $P_{tr} = 1 - (1 - \tau)n$; $P_s$ is the probability that a transmission occurring in the channel is successful, i.e. $P_s = 1 - p$ and

$$P_s = \frac{n \tau (1 - \tau)^{n-1}}{1 - (1 - \tau)^n}; \quad (7)$$

$\eta$ is the number of the empty slots; $\sigma$ is the slot duration; $T_s$ and $T_c$ are the average times the channel is busy because of successful transmission and collision correspondingly.

Due to DCF

$$T_s = T_{phy} + \frac{E[Fr]}{R} + SIFS + \delta + ACK + DIFS + \delta \quad (8)$$

$$T_c = T_{phy} + \frac{E[Fr]}{R} + DIFS + \delta, \quad (9)$$

where $T_{phy}$ is the duration of physical header transmission, $R$ is the information transmission rate, $SIFS$ – Short Interframe Space, $DIFS$ – DCF Interframe Space, $ACK$ – Acknowledgment, and $\delta$ is the propagation delay.

The calculation results show that expression (6) can be represented as

$$S[\text{Mbps}] = \frac{10^{-7} n \cdot L_4(n) \cdot E[Fr]}{B_2(n) \cdot \eta \sigma + 10^{-7} n L_4(n)(T_s - T_c) + C_2(n) T_c} \quad (10)$$

where

$$L_4(n) = \sum_{k=0}^{4} a_{4-k} n^k, \quad B_2(n) = \sum_{k=0}^{2} b_{2-k} n^k, \quad C_2(n) = \sum_{k=0}^{2} c_{2-k} n^k.$$

The values of the coefficients of polynomials $L_4(n)$, $B_2(n)$, and $C_2(n)$ are given in Tabl.1.
As follows from the graphs presented in Fig.1 and Fig.2, expressions (4) and (5), as well as coefficients of the approximating polynomials in Tabl.1, the value \( q = 3 \) is optimal.

For this value \( q \), the dependence of network throughput \( S \) on the number of stations operating in the saturation mode was calculated. The dependency graph is shown in Fig.3. The calculation was carried out using the following parameter values:

\[
T_{\text{phy}} = 48\mu s, \quad E[Fr] = 8 \cdot 10^3 \text{ bit}, \quad \eta = 1, \quad R = 100 \text{ Mbps}, \quad \sigma = 9 \mu s, \quad SIFS = 10 \mu s, \quad DIFS = 50 \mu s, \quad \text{ACK} = 1,12 \mu s, \quad \delta = 0,33 \mu s.
\]

Figure 3 - Dependence of the throughput \( S \) on the number \( n \) of working stations

The nature of the presented dependence and its main parameters correspond to the throughput validation results obtained using the simulator ns-3 and presented in [7]

**Conclusions.** Based on the analysis of the operation of networks IEEE 802.11 DCF, a function is proposed for determining the probability of frame

| \( q \) | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( b_0 \) | \( b_1 \) | \( b_2 \) | \( c_0 \) | \( c_1 \) | \( c_2 \) |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 2   | -0,95 | 18,55 | 1172,4 | -49337 | 692741 | 1,9  | -63,7 | 1051 | -2,2 | 67,2 | -60,2 |
| 3   | -2,3  | 85,8  | -40,2  | -39366 | 681156 | 2,5  | -72   | 1071 | -2,3 | 71,9 | -70,4 |
| 4   | -2,4  | 112,6 | -753,7 | -50939 | 667190 | 2,4  | -76,6 | 1083 | -2,2 | 73,1 | -73,9 |
transmission to the central node of a wireless network IEEE 802.11 DCF, depending on the number of peripheral stations in the network operating in saturation mode. Collision probabilities calculated.

Using approximated polynomials of the second and fourth degrees, an expression is obtained for the network throughput in an explicit form depending on the number of simultaneously operating stations. The nature of this dependence and its main parameters correspond to the results presented in literature and obtained using the simulator ns-3.

REFERENCES
1. Bianchi G. Performance analysis of the IEEE 802.11 distributed coordination function / G. Bianchi // IEEE Journal on Selected Areas in Communications. – 2000. – Vol. 18, No. 3 – P. 535–547.
2. Tinnirello I. Refinements on IEEE 802.11 distributed coordination function modeling approaches / I. Tinnirello, G. Bianchi, Y. Xiao // IEEE Transactions on Vehicular Technology. – 2010. – Vol. 59, No. 3, P. 1055-1067.
3. Tay Y.C. A capacity analysis for the IEEE 802.11 MAC protocol / Y.C. Tay, K.C. Chua // Wireless Networks. – 2001. – Vol. 7, P. 159–171.
4. Tanenbaum A.S. Computer Networks / A.S. Tanenbaum, D.J. Wetherall. – 5th ed. – Pearson, 2010. – 960 p.
5. Khandetskyi V.S. Traffic routing in distributed wireless computer networks / V.S. Khandetskyi, D.P., Sivtsov // System technologies: Reg. interuniversity compendium of scient. works. – Dnipro, 2018. – No 1(114). – P. 27-34.
6. Khandetskyi V.S. Performance analysis of IEEE 802.11 networks / V.S. Khandetskyi, D.P., Sivtsov // System technologies: Reg. interuniversity compendium of scient. works. – Dnipro, 2019. – No 1(120). – P. 156-163.
7. Validation of Wi-Fi network simulation on ns-3: technical report WA 98185 / R. Patidar, S. Roy, T.S. Henderson, M. Mehrnoush. – University of Washington, Aug., 2017. – 7-13 p.
Вероятностные характеристики работы беспроводных сетей с инфраструктурной топологией

Базируясь на анализе функционирования сетей IEEE 802.11 DCF, предложена функция для определения вероятности передачи фрейма центральному узлу в зависимости от количества станций, работающих в режиме насыщения. Рассчитаны вероятности коллизий. С использованием полиномиальной аппроксимации получено выражение для полосы пропускания сети, которая в явном виде зависит от количества одновременно работающих станций.

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