Improving efficiency of flexible manufacturing system based on optimal readiness of technologies

A M Pishchukhin and G F Akhmedyanova
Orenburg State University, 13, Victory Ave., Orenburg, 460018, Russia
E-mail: pishchukhin55@mail.ru, ahmedyanova@bk.ru

Abstract. The efficiency of using flexible manufacturing systems (FMS) depends on their workload. However, in order for the FMS not to work for the warehouse, it is necessary to coordinate its production with the requirements of the market. Both the market and the production process are stochastic, so the probabilistic matching of these two processes based on the Kolmogorov equation is considered in the paper. The level of approval is estimated by the difference in the probability density of the market price for a product exceeding the level of its production cost using the technology implemented in FMS. The latter probability density assesses the readiness of technologies for implementation, including the availability of raw materials, materials, energy, serviceable equipment and devices – all of this is spent on managing resources. The optimal allocation of these control resources in the preparation of technologies can significantly improve the efficiency of FMS. Of practical significance is the identified logarithmic relationship between the probability density of the demand for this product on the market and the probability density of the readiness of the technology implemented in this FMS for the production of this product in the required amount and allocated financial means to improve this readiness.

1. Introduction
A flexible production system (FMS) is generally intended for the implementation of a number of manufacturing techniques for a product of a selected range. Typically, FMS operates according to a predetermined schedule [1, 2]. However, in order to increase efficiency, FMS should produce exactly the products that are in demand in the market at a given point in time and in the required volumes. For the formation of prices and sales, the company monitoring is very careful [3-5].

Obviously, both the pricing process in the market and the volume of production at the enterprise due to the influence of a large number of random factors are stochastic in nature. On the other hand, the technologies implemented in the FMS should be maintained at a given level of readiness, which is also stochastically evaluated.

The readiness of technology for use, in addition to the readiness of the control program, provides for the availability of raw materials, energy, serviceable equipment and accessories - all of this is spent on managing resources. Of course, the even distribution of these resources across all the technologies used is not the best solution - the funds should be invested cumulatively in accordance with the requirements of the market. In this connection, we investigate the problem of optimal control of probabilistic matching of two processes – market and production.
2. Theory

Probabilistic description of market and production processes is found in the literature quite often [6-10]. In this work, the managerial task is posed: to increase the efficiency of the FMS, primarily due to an increase in the load factor [11, 12] by the manufacture of products in demand on the market, based on the probability matching of the two above processes.

Let the probability of exceeding the market price for the i-th type of production cost of its production with the help of this FMS have Markov nature and be described by the Kolmogorov equation [13], presented below in a canonical form:

$$\frac{\partial \omega_{i}}{\partial t} = b \frac{\partial^{2} \omega_{i}}{\partial x^{2}}$$

where $\omega_{i}$ is the density of the probability described above, depending on the market demand for product $x$ and time $t$; $b$ is the diffusion coefficient.

The readiness of the i-th technology that creates these products should be controlled by control actions using an equation similar to (1). Let $u_{i}(x,t)$ be the number of shares of control resources directed to the organization of readiness, and since all the addends of equation (1) have the dimension of inverse seconds, we introduce the coefficient of the rate flow of funds $c_{i}$. Denoting the probability density of availability of the i-th technology to satisfy the arising demand $\omega_{2i}$, we have:

$$\frac{\partial \omega_{2i}}{\partial t} = b \frac{\partial^{2} \omega_{2i}}{\partial x^{2}} + c_{i} u_{i}(x,t).$$

The task of managing the readiness of technology is to ensure that the possibility of producing products as closely as possible correspond to a favorable period of over-demand in the market over production costs using this technology. This approach dominates the choice of various management strategies [14].

We subtract from (1) equation (2) and get:

$$\frac{\partial(\omega_{i} - \omega_{2i})}{\partial t} = b \frac{\partial^{2}(\omega_{i} - \omega_{2i})}{\partial x^{2}} - c_{i} u_{i}.$$ (3)

We introduce the designation of probable unavailability of the i-th technology:

$$s_{i} = \omega_{i} - \omega_{2i}$$

Then (3) is transformed as follows:

$$\frac{\partial s_{i}}{\partial t} = b \frac{\partial^{2} s_{i}}{\partial x^{2}} + c_{i} u_{i}.$$ (5)

Let it be possible to implement n technologies in FMS and spend on $u_{i}$ a share of resources for each technology to coordinate its readiness with the market situation. The overall probability of technology availability in the first approximation can be defined as the product of the partial probabilities of the availability of each technology, considering them as independent events. The unavailability of the i-th technology decreases after each dosed management impact on the value of $q_{i}$. Considering the managerial impact as independent, the overall change in technology availability will be obtained in the form of a work or an exponential function $q_{i}$. Then the total probability describing the readiness of the FMS as a whole will be expressed as:

$$P = \prod_{i=1}^{n} (1 - q_{i} u_{i}).$$ (6)

To simplify expression (6), let us suppose that $P_{i} = 1 - q_{i} \rightarrow 1$. Making the multiplication and getting rid of the values of the second and higher orders of smallness, we get the total unavailability of the FMS:

$$Q(\vec{u}) = \sum_{i=1}^{n} q_{i} u_{i},$$ (7)

where $\vec{u} = \{ u_{1}, u_{2}, ..., u_{n} \}$ is the vector of shares of control actions.

The total costs of organizational measures related to improving the availability of FMS are expressed as a linear relationship:

$$C = C(\vec{u}) = \sum_{i=1}^{n} c_{i} u_{i},$$ (8)

where $c_{i}$ is the cost of a single organizational measure that increases the availability of the i-th technology.

The control task is set as follows: to find the optimal distribution of the shares of control actions $u_{i}$ to ensure minimum costs at a given level of unavailability of the FMS.
To solve the optimal control problem by the Euler – Lagrange method, we compose the Lagrangian:

\[ F(\mathbf{u}) = \sum_{i=1}^{n} c_i u_i + \sum_{i=1}^{n} \psi_i \left( \frac{\partial s_i}{\partial t} - b \frac{\partial^2 s_i}{\partial x^2} - c_i u_i \right) + \sum_{i=1}^{n} \epsilon_i (Q_i^\epsilon - q_i^{u_i}). \]  

(9)

where \( Q_i^\epsilon \) is the specified value of the unavailability of the FMS to meet market demand. Here, the first term is the integrand function, the second one requires the satisfaction of the equations of the control object, and the last is the fulfillment of the stated constraints.

To ensure the extremum \( F(\mathbf{u}) \), we compile the Euler equations for all variables:

\[
\begin{cases}
\frac{\partial F(\mathbf{u})}{\partial u_i} = c_i - \psi_i c_i - \epsilon_i q_i^{u_i} \ln q_i = 0 \\
\frac{\partial s_i}{\partial t} - b \frac{\partial^2 s_i}{\partial x^2} - c_i u_i = 0 \\
-\frac{a}{\epsilon_i} \frac{\partial \psi_i}{\partial c_i} = 0 \\
Q_i^\epsilon - q_i^{u_i} = 0, \quad i = 1, \ldots, n
\end{cases}
\]  

(10)

From the third equation we get that \( \psi_i = const, \quad i = 1, \ldots, n..\)

From the first equation of the system we find \( u_i \):

\[
u_i = \frac{\ln c_i - \psi c_i}{\ln q_i} = \frac{a_i}{\ln q_i},
\]  

(11)

where \( a_i = \frac{c_i - \psi c_i}{\ln q_i} \).

Let us find the Lagrange multiplier \( \epsilon_i \), substituting \( u_i \) from (11) into the fourth equation of system (10):

\[ Q_i^\epsilon = \sum_{i=1}^{n} q_i^u = \sum_{i=1}^{n} a_i = \frac{1}{\epsilon_i} \sum_{i=1}^{n} a_i, \]  

(12)

from where:

\[ \epsilon_i = \frac{\sum_{i=1}^{n} a_i}{Q_i^\epsilon}. \]  

(13)

In the final form, the expression for determining the optimal proportions of control actions to increase the availability of the \( i \)-th technology is:

\[
u_i = \frac{a_i}{\ln q_i} = \frac{a_i - \ln q_i}{\ln q_i} = \frac{\ln a_i - \ln \left( \frac{a_i Q_i^\epsilon}{q_i} \right)}{\ln q_i} = \frac{1}{\ln q_i} \ln \left( \frac{a_i Q_i^\epsilon}{\sum_{i=1}^{n} a_i} \right). \]  

(14)

Now we substitute the obtained expression into the second equation of the system (10). This equation can be solved using the Green function [15]. The resulting equation is inhomogeneous. Let us consider the solution when the initial conditions:

\[ s_i(x, 0) = s_{i0}; \quad s_i(0, t) = 0. \]  

(15)

Introducing in this case \( l \) as the boundary of the enterprise’s ability to meet the demand for its products, and \( t_f \) as the final control time, we obtain a solution that is written in the form:

\[ s_l = \frac{1}{2\sqrt{\pi}} \int_{0}^{t_f} \frac{1}{\sqrt{b_1(t-\tau)}} \left( e^{-\frac{(x-\xi)^2}{4b_1(t-\tau)}} - e^{-\frac{(x+\xi)^2}{4b_1(t-\tau)}} \right) c_i u(\xi, \tau) d\xi d\tau. \]  

(16)

Substituting here the expression (14) for the share of control, we get:

\[ s_l = \frac{c_i}{\ln q_i} \ln \left( \frac{a_i Q_i^\epsilon}{\sum_{i=1}^{n} a_i} \right) \frac{1}{2\sqrt{\pi}} \int_{0}^{t_f} \Phi \left( \frac{x}{2\sqrt{b_1(t-\tau)}} \right) d\tau, \]  

(17)

where:

\[ \Phi(z) = \int_{0}^{z} e^{-a^2} da \]

- error integral

From these equations, by specifying the finite control time \( t_f \), we can determine the optimal value of the probability of decreasing the unavailability of the FMS \( Q_i^\epsilon \), thereby associating it with the nature of the market process or vice versa.
3. Data and method
To test the efficiency of the method for determining the optimal resources for improving the availability of technologies, let us set the unavailability level reached at the end of control $\tilde{Q}_i = 0.01$.

The funds are allocated to individual events to improve preparedness for the five technologies: 1 - 20000; 2-80000; 3-40000; 4-30000; 5-70000.

We calculate the proportion of control resources allocated to each technology:

$$m_1 = \frac{1}{\ln q_i} \ln \frac{a_1 * Q_{ай}^2}{\sum_{i=1}^{m} a_i} = 2.331$$

- $m_2 = 1.641$
- $m_3 = 2.213$
- $m_4 = 2.742$
- $m_5 = 1.338$.

Funds allocated in accordance with the calculated shares (1-46620 2-131280 3-88520 4-82260 5-93660) are plotted on the histogram of figure 1.

4. Results and discussion
The total amount of allocated funds is 442340. After dividing them into five technologies, you can get an average amount of 88468. Now you can compare the total unavailability of technologies with the optimal solution and equal distribution of funds. These values are $Q(\bar{u}_{opt}) = 0.00268$ and $Q(\bar{u}_{equal}) = 0.00903$ in the first and second cases, respectively.

The developed technique can be used in a simplified version. If we take the target setpoints of technology unavailability proportional to the initial differences of probability densities (designation (4)) - $Q_i^2 = d_i \varepsilon_{i0}$, then from the last equation of system (10) it follows that:

$$u_i = \frac{1}{\ln d_i} (\ln d_i + \ln \varepsilon_{i0}).$$

This means that the proportions of control actions should be proportional to the logarithm of the initial differences of probability densities. These differences can be easily determined by an expert method and divide the funds in accordance with the identified logarithmic dependence.
5. Conclusion
Thus, the optimal allocation of control resources in the preparation of technologies implemented by the FMS in accordance with the requirements of the market can significantly improve its efficiency. The identified logarithmic relationship between the probability density of the demand for this product on the market and the probability density of the readiness of the technology implemented in this FMS for the production of this product in the required amount and allocated to increase this availability is of practical significance.

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