Are direct search experiments sensitive to all spin-independent WIMP candidates?

F. Giuliani

Centro de Física Nuclear, Universidade de Lisboa, 1649-003 Lisboa, Portugal
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The common analysis of direct searches for spin-independent Weakly Interacting Massive Particles (WIMPs) assumes that a spin-independent WIMP couples with the same strength with both nucleons, i.e. that the spin-independent interaction is also fully isospin-independent. Though in a fully isospin-dependent interaction scenario the spin-independent WIMP-nucleus cross section is strongly quenched, the leading experiments are still sensitive enough to set limits 1-2 orders of magnitude less stringent than those traditionally presented. In the isospin-dependent scenario the difference between the limits of CDMS-II and ZEPLIN-I is significantly reduced. Here, a model-independent framework is discussed and applied to obtain the current general model-independent limits.

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Direct searches for dark matter in the form of spin-independent Weakly Interacting Massive Particles (WIMPs) are customarily analyzed assuming that the WIMP couples with the same strength to both protons and neutrons. This assumption, satisfied by various theoretical candidates [1, 2, 3], corresponds to the customary approximation $Z \approx N$, where $Z$ is the neutron number $N$ (is the neutron number) which leads to the prejudice of inherent insensitivity of the detectors to isospin-dependent spin-independent WIMPs. This approximation, valid for light nuclei, breaks down for heavier ones, like $^{127}$I ($\frac{N}{Z} = 1.396$) and $^{131}$Xe ($\frac{N}{Z} = 1.426$), which are used by various current and future experiments. In this Letter, the effects of dropping this approximation are discussed.

Direct searches are based on the detection of WIMP nonrelativistic elastic scattering on nucleons, whose general effective spin-independent lagrangian is [2]:

$$\mathcal{L} = 4\sqrt{2}G_F [\psi^\dagger \psi (g_p p^\dagger p + g_n n^\dagger n)],$$  \hspace{1cm} (1)

where $G_F$ is the Fermi constant, $\psi$, $p$ and $n$ are the WIMP, proton and neutron two-component nonrelativistic spinors, and $g_{p,n}$ are the spin-independent WIMP-proton and WIMP-neutron coupling strengths, respectively.

The general zero momentum transfer elastic scattering WIMP-nucleus cross section $\sigma_A$ for a nucleus of mass number $A$ resulting from Eq. (1) is:

$$\sigma_A = \frac{4}{\pi} G_F^2 \mu_A^2 (g_p Z + g_n N)^2$$  \hspace{1cm} (2)

where $\mu_A$ is the WIMP-nucleus reduced mass. For light nuclei $Z \approx N \approx \frac{A}{2}$, which substituted in Eq. (2) yields

$$\sigma_A = \frac{4}{\pi} G_F^2 \mu_A^2 A^2 \frac{(g_p + g_n)^2}{4},$$  \hspace{1cm} (3)

i.e. light nuclei experiments are only sensitive to $\frac{2g_p + g_n}{2}$, which corresponds to the restriction of isospin-independence: $g_p = g_n = \frac{(g_p + g_n)}{2}$. Even the experiments analyzing their data in a mixed-model framework, e.g. DAMA/NaI or HDMS [1, 2], retain this restriction, which allows a four- instead of five-dimensional parameter space.

If the interaction is fully isospin-dependent, i.e. $g_p = -g_n$, the two terms in Eq. (2) tend to suppress the spin-independent $\sigma_A$, so that $^{127}$I would effectively have only 21 nucleons and $^{131}$Xe 23, causing the $^{127}$I sensitivity to drop by a factor 36 and that of $^{131}$Xe by a factor 32. The spin-independent sensitivity of light nuclei, like F or Si would be negligible.

The impact of this model-dependence is illustrated in Fig. 1 showing on the righthand side the case of a fully isospin-dependent interaction. In this case the limits shift to levels over an order of magnitude higher than the commonly reported limits shown on the left side. Moreover, the relative positions of the various exclusion plots change, as is understood in the light of Table I which reports the maximum value of N-Z for the compositions of various leading experiments. Since Ge has N-Z significantly lower than I or Xe, CDMS and EDELWEISS are shifted more upwards than DAMA/NaI and ZEPLIN-I. The fact that DAMA/NaI is shifted more upwards than ZEPLIN-I is a clear consequence of Na’s insensitivity to the (fully) isospin-dependent interaction scenario, more than of the lower N-Z of I with respect to Xe. The ability of CRESST-II to exclude DAMA/NaI improves, but in spite of the high N-Z of W does not reach ZEPLIN-I nor

| material   | experiments     | max. N-Z | Refs. |
|------------|-----------------|----------|-------|
| NaI        | NAIAD, DAMA/NaI | 21       | [4, 7]|
| Xe         | ZEPLIN-I        | 28       | [8]   |
| Ge         | EDELWEISS, CDMS-II | 12   | [9, 10]|
| CaWO₄      | CRESST-II       | 38       | [11]  |

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TABLE I: maximum N-Z for the detector compositions of the dark matter search experiments included in this Letter.

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CDMS.

The limits in the left-hand Fig. 1 are obtained by observing that for A=1 (proton or neutron) Eq. 3 becomes

$$\sigma = \frac{4}{\pi} G_F^2 \mu_{p,n}^2 \frac{(g_p + g_n)^2}{4},$$

where $\sigma$ is the WIMP-nucleon cross section.

Substituting Eq. 3 in Eq. 4, yields $\sigma = (\mu_{p,n}^2)^2 \frac{2}{\sqrt{A}}$, $f_A$ being the fraction of isotope A and $\sigma_{lim}^A$ the upper limit to $\sigma_A$ obtained by attributing the entire rate to isotope A only. Introducing the new symbol $\sigma_{lim}^{SI(A)} = (\mu_{p,n}^2)^2 \sigma_{lim}^A$ to indicate the overestimated limit obtained by attributing all the observed rate to the isotope A, and recalling that $\sum_A f_A = 1$ results in

$$\sigma = \frac{1}{\sigma_{lim}^{SI(A)}} \leq 1.$$  

In order to use the full Eq. 4, the following auxiliary cross sections are introduced, in analogy with the spin-dependent sector:

$$\begin{cases} \sigma_p^{SI(A)} = \left( \frac{\mu_{p,n}^2}{\mu_{p,n}^A} \right)^2 \frac{2}{\sqrt{A}} \\ \sigma_n^{SI(A)} = \left( \frac{\mu_{p,n}^2}{\mu_{p,n}^A} \right)^2 \frac{2}{\sqrt{A}} \end{cases}$$

With these quantities, Eq. 4 becomes $\sigma_A = \left( \sqrt{\frac{\sigma_p}{\sigma_p^{SI(A)}}} \pm \sqrt{\frac{\sigma_n}{\sigma_n^{SI(A)}}} \right)^2 \sigma_A^{lim} \leq f_A \sigma_A^{lim}$, which implies:

$$\begin{cases} \sum_A \left( \sqrt{\frac{\sigma_p}{\sigma_p^{SI(A)}}} \pm \sqrt{\frac{\sigma_n}{\sigma_n^{SI(A)}}} \right)^2 \leq 1 \\ \sum_A \left( \frac{g_p}{\sqrt{\sigma_p^{SI(A)}}} \pm \frac{g_n}{\sqrt{\sigma_n^{SI(A)}}} \right)^2 \leq \frac{\pi}{4 G_F^2 \mu_{p,n}^2} \end{cases}$$

where the sign of the sum inside the first parenthesis is given by that of $\frac{g_n}{g_p}$, and the second is obtained from the first through $\sigma_{p,n} = \frac{1}{\pi} G_F^2 \mu_{p,n}^2$, neglecting the small difference between the proton and neutron mass. Extending the terminology introduced in Ref. 14, Eq. 4 defines cross section and coupling strength representations for the limits on spin-independent WIMPs. Since $\sigma_{p,n}^{SI(A)}$ depend upon $M_W$, the exclusion plots become three-dimensional, which suggests, for illustrative purposes, to report bidimensional cuts at constant $M_W$.

A cross section representation survey at $M_W = 50$ GeV/c$^2$, where CDMS-II reaches the most stringent limits, is shown in Fig. 2. In the left side $\frac{g_p}{g_n} > 0$, and the boundary of the region allowed by each experiment is a smoothly decreasing convex curve. The region allowed by each experiment lies within this curve, except for DAMA/NaI, whose allowed region lies outside its curve since only the lower limit curve is shown. The straight line is the traditional condition of isospin-independence, whose intersection point with the new exclusion contour is the point of the traditional exclusion plot at the chosen $M_W$. As evident, if $\frac{g_p}{g_n} > 0$ the intersection (shaded) of all allowed areas but DAMA/NaI coincides with the CDMS-II area alone. For $\frac{g_p}{g_n} < 0$ (righthand Fig. 2), instead, the region allowed by each experiment (insert in the righthand Fig. 2) has a generally finite protuberance corresponding to $\frac{g_n}{g_p} \approx -\frac{N}{Z}$, which is the least detectable interaction for its composition: $g_p \approx -1.3 g_n$ for Ge-based experiments, while for I- and Xe-based is $g_p \approx -1.4 g_n$ and for W $g_p \approx -1.5 g_n$. The protuberance of CDMS-II is cut by both ZEPLIN-I and CRESST-II, being slightly tangent to the DAMA/NaI contour, so that the couplings of the least constrained candidate with $M_W = 50$ GeV/c$^2$ are determined by the upper right point of the shaded area in the righthand Fig. 2. Again, the region allowed by DAMA/NaI is the outside of its contour, which en-
limits as follows:

\[ g_p/g_n > 0 \] (left) and \[ g_p/g_n < 0 \] (right) at \( M_W = 50 \text{ GeV/c}^2 \). The overall allowed regions are shaded. The insert in the righthand figure shows the shape of the area allowed by CDMS-II alone.

FIG. 2: cross section representation survey for \( g_p/g_n > 0 \) (left) and \( g_p/g_n < 0 \) (right) at \( M_W = 50 \text{ GeV/c}^2 \). The overall allowed regions are shaded. The insert in the righthand figure shows the shape of the area allowed by CDMS-II alone.

tirely contains the shaded area allowed by all other experiments, making the intersection of the DAMA/NaI region with the shaded area empty. The emptiness of the intersection of the DAMA/NaI region with that allowed by all other experiments for both \( g_p/g_n > 0 \) and \( g_p/g_n < 0 \) translates the incompatibility of the DAMA/NaI spin-independent candidate with the observations of the remaining experiments.

The coupling strength representation corresponding to Fig. 2 is shown in Fig. 3. The parameters for Eq. (7) are obtained from the published traditional spin-independent limits as follows: \( \sigma_N \) is related to \( \sigma_{lim}^N \) by

\[ \frac{1}{\sigma_N} = \sum_A \frac{1}{\sigma_{lim}^{S(A)}} = \sum_A \left( \frac{\mu_A}{\mu_{p,n}} \right)^2 A^2 \sigma_{lim}^N = \sum_A \left( \frac{\mu_A}{\mu_{p,n}} \right)^2 f_A A^2 \frac{1}{\sigma_{lim}^{f_A}} \]

where \( \sigma_{lim}^{f_A} = \frac{R}{f_A^p} \) is the "average" zero momentum transfer nuclear cross section, with \( R \) the upper limit on WIMP-induced recoil rate, \( J_\psi \) the incident WIMP current and \( \rho \) the total number density of the sensitive material. Since \( \sum_A (\mu_{p,n}/\mu_{p,n})^2 f_A A^2 \) is easily computable, \( \sigma_{lim}^N \) can be calculated from the published \( \sigma_N \), and used as starting point for the model independent re-analysis. Once \( \sigma_{lim}^N \) is known, \( \sigma_{lim}^A \) is simply \( \sigma_{lim}^N / f_A \), and Eq. (7) can be applied. In the coupling strength representation, the \( g_p - g_n \) regions at constant \( M_W \) allowed by each experiment are the interior of ellipses, which degenerate to two parallel lines for single nuclei experiments. The most striking difference with the spin-dependent case is that, unlike the proton-to-neutron group spin ratio, \( Z/N \) is always positive and less than 1, so that all ellipses/bands have similar orientations. The overall model-independent limits are still found by intersecting all experimental

conics, but the improvement obtained by intersection is somewhat smaller. EDELWEISS contains entirely the CDMS ellipse, because of the identical detector composition, and for this reason is superseded by the latter. ZEPLIN-I and CRESST-II, instead, have a slightly different orientation and cut part of the CDMS ellipse, resulting in the shaded overall allowed region.

The overall combined limits from the intersection at \( M_W = 50 \text{ GeV/c}^2 \) are \( |g_p| \leq 0.11 \) and \( |g_n| \leq 0.082 \), or
σ_p \leq 6.9 \times 10^{-4} \text{ pb} \text{ and } \sigma_n \leq 4.0 \times 10^{-4} \text{ pb}, about three orders of magnitude less stringent than the usually assumed limits. The coupling strengths corresponding to the traditional isospin-independent limits can be found in Fig. 8 by intersecting the \( g_p = g_n \) line with each experimental ellipse. The strong reduction in constraining power is due to the large elongation of the ellipses, combined with the small difference in their orientation. This translates graphically the fact that each experiment has minimum sensitivity to WIMPs for which \( \frac{g_p}{g_n} \approx -\frac{N}{2} \), while \( \frac{N}{2} < 2 \) for practically all stable or long-lived isotopes.

Fig. 4 shows the locus of the least restrictive intersection points of CDMS-II and either ZEPLIN-I or CRESST-II in the coupling strength representation as a function of \( M_W \). For each WIMP mass, the intersection of two experiments is 2 pairs of points symmetric with respect of the origin. Of these, the pair farther from the origin has been selected, and within this pair the point with \( g_p > 0 \). These intersections have generally \( \frac{g_p}{g_n} < 0 \), because they correspond to small detector sensitivities. As a consequence of the larger angle between the CRESST-II and the CDMS ellipses, ZEPLIN-I, in spite of the lower traditional limits, is generally superseded by CRESST-II when determining the limits by intersection with CDMS.

Concluding, the leading experiments, being based on high mass number isotopes, do not fulfill the traditional prejudice of the direct spin-independent WIMP searches being only sensitive to isospin-independent candidates. Though the experiments do not constrain equally well all candidates with the same \( M_W \), even candidates whose coupling is primarily isospin-dependent (i.e. have \( g_p \approx -g_n \)) are constrained with 1-2 orders of magnitude weaker limits. The exclusion limits on the candidate least constrained by the combination of the leading experiments are 2-3 orders of magnitude less restrictive than usually presented. The DAMA/NaI region obtained assuming the standard halo model of Ref. [12] is excluded for all purely spin-independent WIMP candidates satisfying Eq. (1).

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[franck@cc.fc.ul.pt]

[1] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. 267, 195 (1996).
[2] D. R. Tovey, R. J. Gaitskell, P. Gondolo, et al., Phys. Lett. B 488, 17 (2000).
[3] C. Savage, P. Gondolo, and K. Freese, Phys. Rev. D 70, 123513 (2004).
[4] R. Bernabei, M. Amato, P. Belli, et al., Phys. Lett. B 509, 197 (2001).
[5] A. Kurylov and M. Kamionkowski, Phys. Rev. D 69, 063503 (2004).
[6] V. A. Bednyakov and H. V. Klapdor-Kleingrothaus, hep-ph/0504031.
[7] B. Ahmed, G. J. Akerib, Z. CERN, et al., Phys. Rev. D 54, 691 (2003).
[8] V. A. Kudryavtsev and the Boulby Dark Matter Collaboration, in Proc. of the 5th International Workshop on the Identification of Dark Matter (Edinburgh, 2004).
[9] A. Benoit, L. Berg, A. Broniatowski, et al., Phys. Lett. B 545, 197 (2002).
[10] D. S. Akerib, J. Alvaro-Dean, M. ArmelFunkhouser, et al., Phys. Rev. Lett. 93, 211301 (2004).
[11] G. Angloher, C. Bucci, P. Christ, et al., Astrop. Phys. 23, 325 (2005).
[12] J. D. Lewin and P. F. Smith, Astrop. Phys. 6, 87 (1996).
[13] F. Giuliani, Phys. Rev. Lett. 93, 161301 (2004).
[14] F. Giuliani and T. Girard, hep-ph/0502232.