Confinement encoded in Landau gauge
gluon and ghost propagators

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Abstract

An introduction is given into current lattice investigations of the
non-perturbative gluon and ghost propagators, in the light of the
Gribov-Zwanziger and Kugo-Ojima scenarios of confinement, in the
context of results obtained from the non-perturbative Dyson-Schwinger
approach in the continuum and in connection with the vortex mecha-
nism of confinement.

1 Introduction

In lattice gauge field theory, confinement of quarks is numerically
proved although the dynamical origin is still under debate [1]. An
exponential area law holds for Wilson loops in the fundamental rep-
resentation. What about gluon confinement? Wilson loops with
(static) adjoint charges do not decay with an area law, but gluons are
confined, too. This talk is an introduction to alternative confinement
ideas [2 3 4] and presents a report on combined efforts by contin-
uum and lattice theorists to understand how they might be realized
in Nature.

The infrared behavior of gluon, ghost and quark propagators is
the focus of a field-theoretic approach [5] to confinement. Green’s
functions carry all information about the structure of a theory. These
propagators, in distinction to hadron propagators, are gauge-variant.
This is the origin of difficulties related to the Gribov ambiguity which
shows up at different places. The first, pioneering study \[6\] of the gluon propagator in Landau gauge (on lattices as small as \(4^3 \times 8\)) dates back to 1987. Gluon and ghost propagators became topics of stronger interest in the middle of the 90-s. A first review about this activity was given in \[7\].

To establish a relation between confinement and the gluon and ghost propagators one mostly concentrates on the infrared momentum range. Is this range, where the asymptotic behavior sets in, \(O(100)\) or \(O(10)\) MeV or smaller? In order to probe small momenta, one needs to control the infinite-volume limit. This makes the problem difficult on the lattice, even with present day lattice sizes and computers. The following are the signatures of confinement from this point of view:

- The gluon propagator should vanish in the limit \(q \to 0\) \[8, 9\]. The gluon dressing function \(Z(q^2)\), defined through
  \[
  D_{\mu\nu}(q) = \frac{Z(q^2)}{q^2} \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right),
  \]
  should behave in the infrared as \(Z(q^2) \propto (q^2)^{\kappa_D}\) with \(\kappa_D > 1\).

- The ghost propagator ought to be more singular at \(q \to 0\) than a free scalar one \[3, 10\]. This is the horizon condition. The ghost dressing function, defined through
  \[
  G_{\mu\nu}(q) = \frac{J(q^2)}{q^2},
  \]
  should behave in the infrared as \(J(q^2) \propto (q^2)^{\kappa_G}\) with \(\kappa_G < 0\).

- Positivity of the spectral function is expected \[3\] to be violated for the gluon propagator, meaning that the weight function \(\rho(m^2)\) in the \(\text{K"allen-Lehmann representation}\)
  \[
  D(q^2) = \int_0^\infty dm^2 \, \rho(m^2) \frac{m^2}{q^2 + m^2}, \quad D(t, \vec{q} = 0) = \int_0^\infty dm^2 \, \rho(m^2) e^{-mt}
  \]
  would no longer be \(\rho(m^2) \geq 0\) for all \(m^2\).

- The Kugo-Ojima (KO) confinement criterion \[11\] is formulated in terms of the ghost propagator \(G_{xy}^{ab} = \langle \bar{c}_x^a c_y^b \rangle\). One defines \(u^{ab}(q^2)\) by
  \[
  \int d^4 x e^{i(q-x)\cdot y} \langle (D^{ac}_\mu c_c)_x (f^{bcd} A^{d \bar{c}}_{\nu})_y \rangle = \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) u^{ab}(q^2)
  \]
and requires that \( u^{ab}(q^2) \to -\delta^{ab} \) in the limit \( q^2 \to 0 \). This guarantees the absence of colored asymptotic states. The criterion was derived from the so-called quartet mechanism within the BRST quantization of Yang-Mills theory.

The practical request from the side of hadron physics has stimulated non-perturbative studies in the continuum of the gluon and ghost propagator that have started ten years ago [5, 6]. The authors were seeking for solutions of the hierarchy of Dyson-Schwinger equations (DSE) (coupled for both propagators) adopting some truncations. In this approach infinite volume presents no problem. More recently one has learned how to solve the DSE in a compactified space, on the 4-torus [11, 12]. The lessons from DSE, for infinite and compactified space, provide a framework to discuss the status of the lattice calculations. It helps to orient oneself on the “long march” to the infinite-volume limit.

The particular value of lattice calculations at first consists in their ability to control the assumptions and truncations made in the DSE approach. At second and even more interesting as I find, they are possible to assess the importance of special confining field excitations (monopoles and vortices, dyons and calorons) and/or external conditions on the functional form of the propagators. At third, from the beginning of the lattice studies it was clear that the Gribov ambiguity would present a hard problem.

Thus, it is left to the lattice studies to elucidate the open theoretical problems how to deal with it. If the lattice discretization is the definition of QCD in the non-perturbative regime, different prescriptions how to take into account the Gribov problem could lead to different versions of QCD requiring verification.

The vanishing (divergence) of the gluon (ghost) propagators can be traced back to the restriction inside the so-called Gribov region \( \Omega \) of the gauge field representants \( A_\mu \) (transverse gauge copies) that are contributing to the path integral. This is the region where the Faddeev-Popov operator \( \mathcal{M} \) is positive. The problem are more than one of such copies. In the infinite-volume limit the tendency emerges that the most important configurations concentrate at the boundary, the Gribov horizon, such that small non-trivial eigenvalues of \( \mathcal{M} \) accumulate close to zero with a finite density. This is the Gribov-Zwanziger confinement scenario [3]. The infrared exponents \( \kappa_D = 2\kappa \approx 1.2 \) and \( \kappa_G = -\kappa \approx -0.595 \) (constrained to \( 1/2 < \kappa < 1 \) [13]) have been obtained both by the DSE approach [14, 13] and by stochas-
tic quantization [15, 16]. A consequence of the interrelation between both infrared exponents is an infrared fix-point of the strong coupling, \( \alpha_s(0) = 8.915/N_c \).

The positivity violation was noticed very soon [6, 17, 18] in lattice simulations, when the “local mass” \( m_{\text{eff}}(t) = -d \log D(t, \vec{q} = 0)/dt > 0 \) was found to increase with increasing \( t \). For a physical particle in the asymptotic Hilbert space, the effective mass \( m_{\text{eff}}(t) \) approaches the actual mass from above.

## 2 The lattice framework

Lattice gauge theory is formulated in a way that circumvents the choice of a gauge. Apart from our task (to calculate Green’s functions) there are many other contexts in which fixing the gauge is necessary or useful. Gauge-fixing usually becomes a very time-consuming part of such calculations and deserves particular attention. The procedure of such a calculation is as follows: An ensemble of gauge configurations \( \{U\} \) is generated with one’s favorite action using the Monte Carlo (MC) method, either without (“quenched”) or with the back-reaction of (“dynamical”) quarks through the fermion determinant taken into account. In the quenched approximation one has just a gluonic inverse “bare coupling” \( \beta \), and the lattice spacing \( a \) is a function of it, \( a(\beta) \), that can be defined by putting the string tension \( \sigma = a^{-2}\sigma_L(\beta) \) equal to some physical value. Up to a global scale, the renormalization of the gluon propagator (matching the propagators measured at different \( \beta \)) is an independent way to define the running lattice scale \( a(\beta) \).

The vector potential needs to be extracted from the basic transporters (“links”) as \( A_{x+\hat{\mu}/2,\mu} = \left(U_{x,\mu} - U_{x,\mu}^\dagger\right)_{\text{traceless}}/(2i\text{ag}) \). In order to implement the gauge in question, every gauge configuration \( \{U\} \) has to be gauge-transformed \( U_{x,\mu} \to U_{2,\mu}^g = g_x U_{x,\mu} g_x^\dagger + \hat{\mu} \) by a suitable \( \{g\} \).

For example, for the Landau gauge an extremization

\[
F_U[g] = \frac{1}{N_c} \sum_{x,\mu} \text{Re} \text{ Tr } g_x U_{x,\mu} g_x^\dagger x+\mu \to \text{Max} \tag{5}
\]

with respect to \( \{g\} \) solves the problem. A local maximum is found when

\[
(\partial_\mu A_{\mu}^g)_x = \sum_\mu \left(A_{x+\hat{\mu}/2,\mu}^g - A_{x-\hat{\mu}/2,\mu}^g\right) = 0 \tag{6}
\]
(the transversality condition) is satisfied with high precision. This defines the recommended stopping criterion for the various iterative gauge-fixing methods. Having found a local maximum \( \{ g \} \), for any infinitesimal \( \tilde{g} \), one has \( F_U[g] < F_U[1] \). For the absolute maximum \( \{ g \} \), this should hold for all \( \tilde{g} \). Thus, the gauge-fixing problem has been put into the form of a disordered spin system. The search for the (classical) ground state of a spin glass is known to be a non-polynomially hard problem.

If extracting physics would depend on the ability to find the absolute maximum one had to stop here. In this case the measure is said to be restricted to the so-called fundamental modular region \( \Lambda \). It is possible, however, to go a bit further and to investigate the convergence of gauge-variant observables with an increasing number \( n_{\text{copy}} \) of Gribov copies. A sequence of replica ensembles labelled by \( n_{\text{copy}} \) is recursively created (with \( n_{\text{copy}} = 0 \) denoting to the original MC ensemble). Each time one steps from \( n_{\text{copy}} \rightarrow n_{\text{copy}} + 1 \), for each MC configuration a new gauge-fixing attempt is made starting from a random gauge transformation. If a better representant of the original MC configuration is found, it replaces the “previously best” copy, such that the \( n_{\text{copy}} \)-th ensemble is an ensemble of “currently best” copies after \( n_{\text{copy}} \) attempts.

On the other side, Zwanziger [19] gave arguments that in the infinite-volume limit an average over all gauge-fixed copies in the Gribov region would be the physically correct prescription. This would make the search for ever better copies obsolete, and it would be just a question of statistics how many gauge copies of one MC configuration are evaluated.

In any case, for the present lattice sizes it is important to assess the gauge copy dependence of the propagators. Following the “best copy vs. first copy” strategy, one sees that the dependence is stronger at small momenta and becomes indeed weaker with increasing volume.

In order to do the maximization, methods like overrelaxation (OR), Fourier accelerated gauge-fixing (FA) and simulated annealing (SA) are practically in use. The latter [20] is a quasi-equilibrium MC process with a probability distribution \( \propto \exp(F_U[g]/T) \). Annealing means that the temperature is guided from \( T_{\text{max}} \) down to \( T_{\text{min}} \). The idea is that OR following the SA (until the transversality is satisfied) finds the finally gauge-fixed copy with only few iterations within one basin of attraction. Therefore, improvement of the gauge-fixing is not mainly aiming to accelerate the relaxation but to increase the yield of “good”
gauge-fixed copies, as close as possible to the best copy. Given this objective, SA strategies become superior on large lattices [21] also in terms of computing time.

The gluon propagator is defined immediately in momentum space by correlating Fourier transforms \( \tilde{A}_g^a \) of the \( A_g^a \) field,

\[
D^{ab}_{\mu\nu}(q) = \langle \tilde{A}_g^a(\mu) \tilde{A}_g^b(-\nu) \rangle ,
\]
where the finite lattice Fourier transform is calculated for integers \( k_\mu \in (-L_\mu/2, L_\mu/2] \). The momentum vector \( q_\mu(k_\mu) = (2/a) \sin (\pi k_\mu / L_\mu) \) is associated to them. If the gluon propagator is to be calculated for many momenta, use of fast Fourier transformation is necessary.

The ghost field is not a \( c \)-number field in the memory, such that the ghost propagator, similar to a quark propagator, must be obtained by inversion of the Faddeev-Popov operator

\[
\mathcal{M}_{xy}(U) = \sum_\mu \left( A_{x,\mu}^{ab}(U) \delta_{xy} - B_{x,\mu}^{ab}(U) \delta_{x,x+\bar{\mu},y} - C_{x,\mu}^{ab}(U) \delta_{x,x,y} - \hat{\mu}_{\mu}^{ab} \right) ,
\]

with \( \mathcal{M}_{xy} \rightarrow -\delta^{ab} \Delta_{xy} \) for \( U_{x,\mu} \rightarrow 1 \). The matrices \( A, B \) and \( C \) are defined in terms of (gauge fixed) links as

\[
A_{x,\mu}^{ab} = \text{Re} \text{ Tr} \left[ T^a, T^b \left( U_{x,\mu} + U_{x-\bar{\mu},\mu} \right) \right] ,
\]
\[
B_{x,\mu}^{ab} = 2 \text{ Re} \text{ Tr} \left[ T^b T^a U_{x,\mu} \right] ,
\]
\[
C_{x,\mu}^{ab} = 2 \text{ Re} \text{ Tr} \left[ T^a T^b U_{x-\bar{\mu},\mu} \right] .
\]

In momentum space the propagator is obtained by inverting \( \mathcal{M} \) on a plane wave source (for \( k \neq (0,0,0,0) \))

\[
\sum_{b,y} \mathcal{M}_{xy}^{ab} \phi^{b(c)}_y = \psi^{a(c)}_x = \delta^{ac} e^{2\pi i k \cdot x} ,
\]

giving

\[
G^{ab}(q) = \frac{1}{V(N_c^2 - 1)} \sum_c \sum_x \psi^{a(c)*}_x \phi^{b(c)}_x .
\]

For the inversion the conjugate gradient algorithm is used. For preconditioning one uses the simple (not the covariant !) Laplacian.

If one defines the strong coupling \( \alpha_s \) through the ghost-gluon vertex, then, knowing the (renormalized) dressing functions \( Z_R \) and \( J_R \) and assuming for the vertex renormalization constant \( Z_1(q^2) \approx 1 \), one obtains [22] in the MOM-scheme the running coupling as follows

\[
\alpha_R(p^2) = \alpha_R(\mu^2) \left[ Z_R(p^2, \mu^2) \right] J_R(p^2, \mu^2) \left[ J_R(p^2, \mu^2) \right]^2 .
\]
3 Some lattice results

The lattice calculations should give an answer to the following questions:

- Do the propagators show the infrared behavior proposed by DSE?
- What is the infrared limit of the MOM-scheme coupling $\alpha_s(q^2)$?
- What is the impact of Gribov copies on the propagators?
- How fast is the infinite-volume limit reached?
- Which propagators are modified by “unquenching”?
- How are the other confinement criteria fulfilled?
- How do Faddeev-Popov eigenvalues and eigenmodes behave?
- Is the ghost propagator in the infrared dominated by the lowest eigenmodes of $M$?

Finally, one might ask:

- Are there modified gauge-fixing conditions, equivalent to the common ones in the infinite-volume limit, that are advantageous for convergence to the infinite volume limit and/or less vulnerable to discretization effects?

I will present some answers in the following. Our studies have included quenched $SU(3)$ QCD on lattices from $12^4$ to $72^4$ generated with Wilson gauge action at $\beta = 5.7$, $5.8$, $6.0$ and $6.2$. The full QCD configurations kindly provided by the QCDSF collaboration are $16^3 \times 32$ and $24^3 \times 48$ lattices created with Wilson gauge action at $\beta = 5.29$ and $5.25$ and $N_f = 2$ clover-improved Wilson fermions of varying mass ($\kappa = 0.135 \ldots 0.13575$). The last question of an improved gauge-fixing was recently investigated in quenched $SU(2)$ gauge theory [23] where the consequences of enlarging the set of admissible gauge transformations by global $Z(N)$ flips (proposed in [24]) were further examined.

In Fig. 1a we show the gluon dressing function for quenched QCD [25, 26]. Characteristic is the intermediate bump of the dressing function. The exact form of the dressing function is not described by the DSE, which pretend to describe only the infrared and ultraviolet behavior. In particular the bump is underestimated. Fig. 2a shows
that this enhancement becomes partly (30\%) depressed by the back-
reaction of dynamical quarks [27]. The same is observed for dynamical
configurations of the MILC collaboration in [28]. In view of the diffi-
culties to determine the infrared exponent $\kappa_D$ (see below) it is prema-
ture to speak about the dynamical-quark effect on $\kappa_D$. Since the main
effect is not in the infrared behavior, the change could be considered
irrelevant for the confinement problem. Indeed, breakdown of gluon
confinement is not realistic in the real world with dynamical quarks.
In contrast to that, it is known that dynamical quarks indeed change
the confinement property of static quarks (“string breaking” [29]). In
the quenched $SU(2)$ theory the so-called “infrared bump” (sitting,
however in fact at 1 GeV!) is entirely the result of the presence of
P-vortices as confining agents seen in Maximal Center Gauge (MCG)
and projection. The enhancement By the same operation confinement
[30], topological charges and chiral symmetry breaking [31] are
destroyed. A natural conjecture is that dynamical quarks to some
extent suppress P-vortices. This hypothesis deserves closer investi-
gation. That the opposite effect of unquenching is observed for the
density of monopoles [32] can be explained that there is an “inert”
component of monopoles [30] not related to P-vortices.

Fig. 1b presents the ghost dressing function [25, 26] for the quenched
theory. The behavior in the infrared is opposite to the gluon dress-
ing function and not incompatible with being divergent. In Fig. 2b
one sees that unquenching [27] has no dramatic effect on the ghost
propagator, except for the smallest momenta accessible, where also
a splitting according to the quark masses (see the legend) becomes
visible.

The infrared increase of the ghost structure function in the quenched
theory is obvious, but the fitting of an infrared exponent does not give
the expected $\kappa$. For $SU(2)$ gauge theory it is known [34] that the re-
moval of P-vortices leads to a global change of the ghost dressing
function $J(q^2)$ which becomes almost constant. One can say that the
global (not only infrared) behavior of the ghost dressing function is
the closest relative to the confinement of quarks. Since vortex re-
moval also removes all non-perturbative attributes [35] (percolating
monopole trajectories, string tension, chiral condensate and the topo-
logical charge [31], it is very likely that the original divergence of the
ghost propagator like $1/(q^2)^{1+\kappa}$ is mainly a result of the topologi-
cal structure leading to an enhanced density of low-lying eigenvalues
of $\mathcal{M}$ as demonstrated for MC [36] and model configurations [37].
We have found, however, that the direct correspondence between the
ghost propagator at lowest momenta and the lowest-lying Faddeev-
Popov eigenmodes is rather weak [38]. The effect of dynamical quarks
is not as strong as vortex removal, but it might be caused indirectly
via the gradual suppression (or pairing) of topological objects by light
sea quarks, too. The quark mass dependence of the effect might be
related to the stronger string breaking induced by lighter sea quarks.

Our data suggest that both in the quenched and the dynamical case
there are apparently no finite-volume effects on the ghost propagator.
This will be made more precise later.

Figs. 1 and 2 for the gluon propagator come from a study where
the Gribov ambiguity was ignored. Only one gauge-fixed copy (“first
copy”) was evaluated. I should remark that this procedure is equiva-
 lent to the prescription of averaging over all gauge-fixed copies within
the Gribov region (justified in [19]) for a given MC configurat-
inon. In order to demonstrate that the propagators are all vulnera-
tble to the Gribov ambiguity, but to a different extent, in Fig. 3 we pr esent
(for smaller lattices) the effect of the Gribov ambiguity on the gluon
and ghost propagator (for the quenched case) [25, 26]. In the subpan-
els (a) and (b) the ratio of the dressing functions calculated in two
different ensembles is shown. The “fc” ensemble is the ensemble of
(arbitrary) first gauge-fixed copies for each MC configuration, “bc”
is the ensemble of the best copies after \( n_c = 20 \) to 30 gauge-fixing
attempts. In the case of the gluon propagator in Fig. 3a we see a
relatively broad band of “Gribov noise” that does not show a d is-
tinct momentum or volume tendency. On the other side, for the ghost
dressing function in Fig. 3b a relatively sharp effect of overestima-
tion for the first copy is seen that becomes stronger towards smaller
momenta. The effect is slightly suppressed with increasing physical
volume (see the data points for the lowest \( \beta = 5.8 \)), an observation
that can be an early hint towards the weakening of the Gribov copy
effect at very large volumes. For smaller lattices, however, the ghost
propagator will be overestimated at the smallest momenta if there is
no systematical search for better Gribov copies, i.e. if one averages
over Gribov copies.

There are not enough data yet in the region of small enough mo-
menta to get stable fits of the infrared exponents. If one attempts
this, \( \kappa \) is found too small. In order to anticipate whether the gluon
propagator finally may turn to zero in the limit \( q^2 \to 0 \), one looks at
the propagator instead of the dressing function. Fig. 4 shows data
for the gluon and the ghost propagator on a $64^4$ lattice at $\beta = 5.7$ obtained at the MVS-15000BM of the Joint Supercomputer Center (JSCC) Moscow. The gluon propagator in Fig. 4a shows at least a kind of plateau. The leftmost data point represents the gluon propagator at zero momentum, $D(0)$. The decreasing tendency of $D(0)$ with the lattice volume (not shown here) suggests that the propagator function $D(p)$ also cannot be taken as the infinite-volume limit. More recent data (on a lattice $80^4$) presented at Lattice 2007 [39] indicate that the plateau extends to $|q|$ below 100 MeV. The ghost propagator in Fig. 4b (shown in a log-log-plot) suggests already something close to a power law, but the corresponding $\kappa$ comes also too small compared with the preferred $\kappa = 0.595$.

That means that the now accessible momentum range is probably still pre-asymptotic. DSE results anticipating the approach to the infinite-volume limit indicate how far lattice calculations are from seeing the asymptotic behavior. The DSE have been formulated and solved on a finite torus [11, 12], and an interesting pattern of finite-volume deviations for the calculated propagators has been found and compared with our lattice data (see Figs. 5a and 5b taken from [12]).

For the gluon propagator the approach to the infinite-volume curve is from above, with an enormous overshooting towards the lowest momentum for any given lattice volume (see Fig. 5a). For the ghost propagator the approach is from below and less dramatic. This is shown in Fig. 5b. The insufficient slope $\kappa$ in the log-log-plot of the ghost propagator in Fig. 4b is well explained by this type of finite volume effect.

Fig. 6a shows the DSE result for the running coupling with the volume dependence induced by the volume dependence of the gluon and ghost propagators. This makes clear that it is illusory to see the running coupling approaching the infrared fix-point before lattices reach a linear size $L = O(15 \text{ fm})$. We have checked [26, 41] on the lattice the assumed $q^2$ independence of the ghost-ghost-gluon vertex renormalization constant, a tacit assumption in deriving Eq. [12]. Fig. 7b shows the result of our calculation of the gluon and ghost dressing functions, giving the running coupling [26]. The volume is just large enough to reveal the turn-over to an apparently decreasing behavior of coupling with $q^2 \to 0$. But this has nothing to do with the true asymptotic behavior. In the light of this observation, the optimism of having seen already the approach to the fix-point [42] seems to be premature.
The violation of positivity and the very slow approach to the KO confinement criterion have been presented at Lattice 2006 [40]. Recently, the Adelaide group [28] has discussed violation of positivity together with scaling and the effect of dynamical quarks in much more detail for lattice ensembles provided by the MILC collaboration.

4 Summary

Various effects on the gluon and ghost propagators have already been studied for quenched QCD, and the effect of dynamical quarks and Gribov copies has been added by our investigations. The infrared exponents characteristic for the way how “ghosts manage to confine gluons” are still elusive. The infrared asymptotic region in momenta (volumes) is not yet reached. There are three extrapolations needed before lattice QCD can be applied to the real world: (a) to take the continuum limit, (b) to control the chiral limit and to extrapolate to the physical pion mass and (c) to take the infinite-volume limit. The latter is probed by the infrared behavior of gluon and ghost propagators, and it turns out that the approach is extremely slow. Discretization effects also show up in the data, but can be easily tamed by suitable momentum cuts. The effects of the vortex mechanism of quark confinement and of dynamical quarks on the form of the propagators are very interesting and worth to be microscopically understood.

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Figure 1: The gluon dressing function (a) and the ghost dressing function (b) for quenched QCD. Data from various lattice sizes and \( \beta \)-values are seen matching on one curve. The little \( q_i^2 \) marks a momentum range \( q^2 < q_i^2 \) where a power fit for \( \kappa \) has been attempted. Both propagators give \( \kappa \approx 0.2 \).
Figure 2: The effect of dynamical quarks (a) on the gluon propagator that becomes depressed in the intermediate momentum range around $O(1 \, \text{GeV})$; (b) on the ghost propagator that becomes depressed only in the infrared region.
Figure 3: The ratio between first and best gauge copies used for calculating the dressing function (a) for the gluon propagator ("Gribov noise"), (b) for the ghost propagator where one sees a systematic Gribov copy effect becoming weaker with increasing physical volume.
Figure 4: (a) The gluon propagator, (b) the ghost propagator, measured on the 64$^4$ lattice at $\beta = 5.7$. Notice that the leftmost data point of the gluon propagator actually refers to zero momentum, $D(0)$. So far there is no indication that $D(p) \to 0$ for $p \to 0$. 

18
Figure 5: The propagators from DSE for a few volumes and for infinite volume shown together with our lattice data of [25, 40], (a) for the gluon propagator, where the lattice data, being tangents to the infinite-volume curve, are strongly deviating upward; (b) for the ghost propagator, where the lattice data, being tangents to the infinite-volume curve, are deviating downward with a pre-asymptotic slope. (Figures taken from [12]).
Figure 6: The MOM-scheme running coupling constant $\alpha_s(q^2)$: (a) from the DSE approach for different box sizes compared with the infinite volume limit (taken from [12]), (b) from our quenched lattice calculations. The little $q^2_c$ indicates the end of a fit range $q^2 > q^2_c$ where the 1- and 2-loop running coupling has been fitted. Notice the weak volume dependence on the right of the peak and the beginning splitting on the left of the peak between the largest volumes.