Weak radiative hyperon decays, Hara’s theorem and the diquark

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Abstract

Weak radiative hyperon decays are discussed in the diquark-level approach. It is pointed out that in the general diquark formalism one may reproduce the experimentally suggested pattern of asymmetries, while maintaining Hara’s theorem in the SU(3) limit. At present, however, no detailed quark-based model of parity-violating diquark-photon coupling exists that would have the necessary properties.

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Weak radiative hyperon decays (WRHD’s) have proven to be a challenge to our theoretical understanding. Despite many years of theoretical studies, a satisfactory description of these processes is still lacking. For a review see ref.[1] where current theoretical and experimental situation in the field is presented.

The puzzle posed by weak radiative hyperon decays contains a couple of ingredients, most of which relate to the issue of Hara’s theorem [2]. This theorem, originally formulated at the hadron level, states that the parity-violating amplitude of decay $\Sigma^+ \to p\gamma$ should vanish in the limit of SU(3) flavour symmetry. Since SU(3) symmetry is expected to be weakly broken, the parity-violating amplitude in question and, consequently, the $\Sigma^+ \to p\gamma$ decay asymmetry should be small. Experiment [3] shows, however, that the asymmetry is large:

$$\alpha(\Sigma^+ \to p\gamma) = -0.72 \pm 0.086 \pm 0.045$$  (1)

Satisfactory explanation of such a large value of this asymmetry constitutes a theoretical problem, which is even more difficult when one demands a successful simultaneous description of the experimental values of the asymmetries of three related WRHD’s, namely $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma^0\gamma$.

Theoretical calculations may be divided into those ultimately carried out at hadron level (eg. [4]) and those performed totally at quark level (eg. [5, 6]). Hadron-level calculations such as those of Gavela et al. [4] do satisfy Hara’s theorem in the SU(3) limit and may yield fairly large negative $\Sigma^+ \to p\gamma$ asymmetry but have problems with a simultaneous description of all four asymmetries [1] which are predicted to be negative. The data, on the other hand, seem to indicate that the $\Xi^0 \to \Lambda\gamma$ asymmetry is positive. Strict quark-level calculations and the hadron-level VMD approach of ref.[7] describe the whole body of data significantly better [1]. However, they achieve this at the cost of apparently violating Hara’s theorem in the SU(3) limit. Unless some other mechanism or effect intervenes, it seems therefore that the data indicate that Hara’s theorem is broken.
Very small size of the experimental $\Xi^+ \rightarrow \Sigma^- \gamma$ branching ratio relative to those of the $\Sigma^+ \rightarrow p\gamma$, $\Lambda \rightarrow n\gamma$, $\Xi^0 \rightarrow \Lambda\gamma$, and $\Xi^0 \rightarrow \Sigma^0\gamma$ processes means that $s \rightarrow d\gamma$ single-quark transitions are negligible in $\Sigma^+$, $\Lambda$, and $\Xi^0$ decays. Thus, as already indicated by the analysis of Gilman and Wise, the dominant contribution must come from processes involving at least two quarks. In both hadron- and quark-level approaches it is the $W$-exchange between two quarks (one of which emits a photon) that provides the dynamics underlying such two-quark processes. Both types of existing approaches use a more or less complicated prescription for the calculation of the relevant amplitudes. However, the basic transition is essentially a diquark $\rightarrow$ diquark $+ \gamma$ process, and thus a simple description of amplitudes (essentially a counterpart of model-independent single-quark analysis of Gilman and Wise) should be possible. It is the purpose of this note to fill in this gap and to analyse the diquark transitions in a manner somewhat similar to GW analysis, while attempting to maintain Hara’s theorem in the SU(3) limit.

To begin with, let us observe that in the $\Sigma^+$, $\Lambda$, and $\Xi^0$ radiative decays under consideration the basic diquark $\rightarrow$ diquark $+ \gamma$ transition is everywhere the same $(us) \rightarrow (ud) + \gamma$ process. The $(us)$ or $(ud)$ diquark may be in spin 0 or spin 1 state. Let us therefore denote possible diquark states by $A(q_1q_2)$ and $S(q_1,q_2)$ ($q_1q_2 = ud$ or $us$):

\[
\begin{align*}
|A(q_1q_2)\rangle &= \frac{1}{\sqrt{2}}(q_1q_2 - q_2q_1) \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \\
|S^{+1}(q_1q_2)\rangle &= \frac{1}{\sqrt{2}}(q_1q_2 + q_2q_1) \uparrow\uparrow \\
|S^{0}(q_1q_2)\rangle &= \frac{1}{\sqrt{2}}(q_1q_2 + q_2q_1) \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\
|S^{-1}(q_1q_2)\rangle &= \frac{1}{\sqrt{2}}(q_1q_2 + q_2q_1) \downarrow\downarrow
\end{align*}
\]

We now rewrite the SU(6) wave functions of relevant initial and final baryons (containing $us$ and $ud$ diquarks respectively) in terms of a diquark and a spec-
tator quark:

\[ \langle p \uparrow \rangle = \frac{1}{\sqrt{2}} \langle A(ud) \rangle \langle u \uparrow \rangle + \frac{1}{3\sqrt{2}} \langle S^0(ud) \rangle \langle u \uparrow \rangle - \frac{1}{3} \langle S^+ (ud) \rangle \langle u \downarrow \rangle + \ldots \]

\[ |\Sigma^+ \downarrow \rangle = -\frac{1}{\sqrt{2}} \langle A(us) \rangle \langle u \downarrow \rangle + \frac{1}{3\sqrt{2}} \langle S^0(us) \rangle \langle u \downarrow \rangle - \frac{1}{3} \langle S^- (us) \rangle \langle u \uparrow \rangle + \ldots \]

\[ \langle n \uparrow \rangle = -\frac{1}{\sqrt{2}} \langle A(ud) \rangle \langle d \uparrow \rangle + \frac{1}{3\sqrt{2}} \langle S^0(ud) \rangle \langle d \uparrow \rangle - \frac{1}{3} \langle S^+ (ud) \rangle \langle d \downarrow \rangle + \ldots \]

\[ |\Lambda \downarrow \rangle = \frac{1}{2\sqrt{3}} \langle A(us) \rangle \langle d \downarrow \rangle + \frac{1}{2\sqrt{3}} \langle S^0(us) \rangle \langle d \downarrow \rangle - \frac{1}{\sqrt{6}} \langle S^- (us) \rangle \langle d \uparrow \rangle + \ldots \]

\[ \langle \Lambda \uparrow \rangle = -\frac{1}{\sqrt{3}} \langle A(ud) \rangle \langle s \uparrow \rangle + \ldots \]

\[ |\Xi^0 \downarrow \rangle = \frac{1}{\sqrt{2}} \langle A(us) \rangle \langle s \downarrow \rangle + \frac{1}{3\sqrt{2}} \langle S^0(us) \rangle \langle s \downarrow \rangle - \frac{1}{3} \langle S^- (us) \rangle \langle s \uparrow \rangle + \ldots \]

\[ \langle \Sigma^0 \uparrow \rangle = -\frac{1}{9\sqrt{2}} \langle S^+ (ud) \rangle \langle s \downarrow \rangle - \frac{1}{3} \langle S^0 (ud) \rangle \langle s \uparrow \rangle + \ldots \] (3)

In Eq.(3) we have explicitly written down only those diquarks within which the weak \( us \rightarrow ud \) transition may take place. Denoting weak + electromagnetic diquark \( \rightarrow \) diquark + \( \gamma \) transition amplitudes by

\[ t_{+1} = \langle S^+ (ud) \rangle \gamma \langle T | A(us) \rangle \]

\[ t_{-1} = \langle A(ud) \rangle \gamma \langle T | S^- (us) \rangle \]

\[ v = \langle S^0 (ud) \rangle \gamma \langle T | S^- (us) \rangle + \langle S^+ (ud) \rangle \gamma \langle T | S^0 (us) \rangle \] (4)

(the momenta of final baryon and photon define the axis of spin quantisation) and using Eqs.(3), we may express the amplitudes of WRHD’s in terms of \( t_{+1}, t_{-1}, \) and \( v \):

\[ \langle p \uparrow \gamma | T | \Sigma^+ \downarrow \rangle = \frac{1}{3\sqrt{2}} t_{+1} - \frac{1}{3\sqrt{2}} t_{-1} - \frac{1}{9\sqrt{2}} v \]

\[ \langle n \uparrow \gamma | T | \Lambda \downarrow \rangle = -\frac{1}{6\sqrt{3}} t_{+1} + \frac{1}{2\sqrt{3}} t_{-1} - \frac{1}{6\sqrt{3}} v \]

\[ \langle \Lambda \uparrow \gamma | T | \Xi^0 \downarrow \rangle = \frac{1}{3\sqrt{3}} t_{-1} \]

\[ \langle \Sigma^0 \uparrow \gamma | T | \Xi^0 \downarrow \rangle = \frac{1}{3} t_{+1} + \frac{1}{9} v \] (5)

The above formulas are valid both for parity-violating and for parity-conserving amplitudes (with different parameters), for any two-quark \( us \rightarrow ud + \gamma \) processes. With six parameters it should not be difficult to fit the four experimental
asymmetries and four branching ratios if the two-quark transitions are indeed dominant. We have not attempted such a fit since 1) the experimental numbers still carry quite significant errors, and 2) we are more interested in the theoretical problem which manifests itself in parity-violating amplitudes only. From now on we will accept that the parity-conserving amplitudes are well described by the standard pole model prescription (eg. [4]). Thus, only three parameters are at our disposal. Please note also that the $\Xi^0 \rightarrow \Lambda \gamma$ asymmetry is especially interesting as it provides a direct measure of a single diquark amplitude $t_{-1}$.

For the sake of a subsequent discussion let us restrict the meaning of Eq.(5) to the parity-violating sector and let us re-express the amplitudes in terms of three parameters $P$, $Q$ and $v$, where $P$ and $Q$ are given by:

$$P = \frac{1}{2}(t_{+1} + t_{-1}) - \frac{1}{6}v$$

$$Q = \frac{1}{2}(t_{+1} - t_{-1}) - \frac{1}{6}v$$

(6)

The amplitudes in question expressed in this way are given in the second column of Table 1. Phase convention used in our formulas is such that the signs of parity-violating amplitudes in Table 1 are automatically equal to the sign of asymmetries once the common sign of parity-conserving amplitudes is fixed as positive. From Table 1 it can be seen that pole and quark models correspond to a different choice of diquark parameters $P$, $Q$, and $v$. For the pole model [4] we have $P = C$ and $Q = Cx$ with $C = 1/(1 - x^2)$ (see ref.[1]), where $x$ is the SU(3) breaking parameter estimated in [1] to be $x = \delta s/\Delta \omega \approx 1/3$ ($\delta s = m_s - m_d \approx 190 MeV$, $\Delta \omega = m(1/2^-) - m(1/2^+) \approx 570 MeV$). In the pole model, Hara’s theorem is satisfied in the SU(3) limit $x \rightarrow 0$ and, consequently, in this limit only the $P$ parameter is nonzero. For the quark model/VDM approach [3, 4] $P = Cx$ and $Q = C$. If formulas of these approaches really describe parity-violating amplitudes in full, one obtains violation of Hara’s theorem as $Q$ does not vanish in the SU(3) limit. Since explicit calculations of diquark-photon couplings in pole or quark models yield either $v = 0$ or
$v \approx 0$, we will neglect $v$ in the following. For an explicit calculation of $v$ in the constituent quark model of a diquark see ref. [10], where a very small $v$ proportional to $(\delta s/m_d)^2$ is obtained.

To proceed let us consider now the most general gauge-invariant $diquark \rightarrow diquark + \gamma$ parity-violating strangeness-changing interaction:

$$\mathcal{L} \propto (J_\mu + J_\mu^\dagger) A^\mu \quad (7)$$

with $J_\mu$ being the strangeness-changing $0^+ \rightarrow 1^+$ diquark current

$$J^\mu = g(q^2) [(q \cdot k)\epsilon^*_S - p^\mu \epsilon^*_S \cdot q] \quad (8)$$

where $\epsilon^*_S$ describes polarization of the final $1^+$ diquark, $g$ is a real function of $q^2$, and $p$, $k$, and $q$ are momenta of the initial diquark, final diquark and photon respectively. $J^{\mu\dagger}$ is obtained from Eq. (8) by changing $q \rightarrow -q$, $\epsilon^*_S \rightarrow \epsilon_S$, while leaving $p$ and $k$ unchanged (together with their interpretation of initial and final diquark momenta). For real transverse photons there will be no contribution from the $p^\mu$ term since, upon integration over diquark momentum in the initial baryon, the terms of opposite $p^k$ will cancel.

Using the diquark-photon interaction of Eqs.(7–8) we find that the $diquark \rightarrow diquark + \gamma$ transition amplitudes are proportional to:

$$t_{+1} \propto g(0) [m^2(A(u, s)) - m^2(S(u, d))]$$

$$t_{-1} \propto g(0) [m^2(A(u, d)) - m^2(S(u, s))] \quad (9)$$

where $m(A(u, q))$, $m(S(u, q))$ are masses of spin 0 and spin 1 diquarks.

Let us now assume that diquarks composed of $us$ are heavier than those made of $ud$:

$$m^2(A(u, s)) = m^2(A(u, d)) + \Delta s$$

$$m^2(S(u, s)) = m^2(S(u, d)) + \Delta s \quad (10)$$
with $\Delta s$ vanishing in the SU(3) limit. We then find

$$P \propto g(0) \left[ m^2(A(u,d)) - m^2(S(u,d)) \right]$$

$$Q \propto g(0) \Delta s$$

Ways of breaking SU(3) symmetry that are different in detail from the simple version of Eq.(10) may be also considered. As long as in the SU(3) limit one has $m(A(u,s)) \rightarrow m(A(u,d))$ and $m(S(u,s)) \rightarrow m(S(u,d))$, the qualitative results of our discussion will not change. One has to remember, however, that if function $g$ also depends on diquark masses, this might cancel the $m$-dependence of Eq.(9). For example, in the VMD-based approach to parity-violating amplitudes in $B \rightarrow K^{*}\gamma$ decay (analogous to our diquark decay), Golowich and Pakvasa use the gauge-invariant coupling $\epsilon_\mu(\gamma) = g' \epsilon_\mu(\gamma)(\epsilon^{\ast \mu}(K^{*}) - \frac{1}{q^{\nu}} \epsilon^{\ast}(K^{*}) \cdot q p^\nu)$.

Upon inspection of Table 1 we see that in the gauge-invariant diquark-level approach Hara’s theorem is recovered in the SU(3) limit ($Q \rightarrow 0$). In addition we see that one may obtain $P \approx 0$ if masses of $A$ and $S$ diquarks are similar, i.e. in the spin symmetry limit. Thus, it is possible to obtain the signature $(-,+,+,–)$ of the $\Sigma^+ \rightarrow p\gamma$, $\Lambda \rightarrow n\gamma$, $\Xi^0 \rightarrow \Lambda\gamma$, and $\Xi^0 \rightarrow \Sigma^0\gamma$ asymmetries (characteristic of the quark model/VDM approach and also suggested by experiment), and yet maintain Hara’s theorem in the SU(3) limit. In other words, large asymmetries with signature $(-,+,+,–)$ are compatible with Hara’s theorem, provided the SU(3)-breaking term is much larger than the SU(3)-symmetric term ($Q \gg P$).

The diquark approach of this paper seems to suggest that although $Q \rightarrow 0$ in the SU(3) limit, it may happen that $Q \gg P$ for realistic SU(3) breaking. However, the problem remains how to achieve this in a microscopic model of the diquark. Explicit calculations in the pole model give just the opposite: $Q/P \approx 1/3$. One can obtain $Q \gg P$ in the pole model provided that $x = \delta s/\Delta \omega \gg 1$. This corresponds to the Li-Liu proposal which was
opposed by Gaillard [13].

On the other hand, explicit calculations at quark level (such an approach to diquark lies at the basis of ref. [10]) yield in the SU(3) limit

$$t_{+1} = -t_{-1} \neq 0$$

leading therefore to an apparent violation of Hara’s theorem. Technically, the origin of the above result is obviously the same as in the quark model calculations of Kamal and Riazuddin [5]. The diquark approach of ref. [10] is conceptually identical to the quark-level approach of [5, 6].

Recently it was argued [14] that the $\bar{\psi}_p \gamma_5 \gamma_\mu \psi_{\Sigma^+} A^\mu$ term, to the appearance of which the quark-model Hara’s-theorem-violating results for $\Sigma^+ \to p\gamma$ were assigned, may be renormalized away. It would be interesting to study if and how such a renormalization procedure affects quark model predictions for the asymmetries of four related WRHD’s ($\Sigma^+ \to p\gamma$, $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma^0\gamma$), and how it compares with the general diquark-level approach which provides an after-renormalization description.

In summary, the diquark-level gauge-invariant approach to parity-violating amplitudes of WRHD’s is in principle capable of explaining the experimentally suggested pattern of asymmetries without violating Hara’s theorem in the SU(3) limit. However, in order to achieve this one needs large SU(3)-breaking and small SU(3)-symmetric terms in parity-violating diquark-photon couplings. At present no existing quark-based model of a diquark exhibits this property.

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Table 1

Parity-violating amplitudes in the general diquark approach, and the pole and quark models. Expressions in the last two columns are taken from ref. [1] with $x$ being the SU(3) breaking parameter and $C = 1/(1 - x^2)$.

| process | diquark | pole model | VDM/quark model |
|---------|---------|------------|-----------------|
| $\Sigma^+ \rightarrow p\gamma$ | $-\frac{2}{3\sqrt{2}}Q$ | $-\frac{2}{3\sqrt{2}}Cx$ | $-\frac{2}{3\sqrt{2}}C$ |
| $\Lambda \rightarrow n\gamma$ | $-\frac{1}{3\sqrt{3}}P + \frac{2}{3\sqrt{3}}Q + \frac{2}{9\sqrt{3}}v$ | $-\frac{1}{3\sqrt{3}}C + \frac{2}{3\sqrt{3}}Cx$ | $-\frac{1}{3\sqrt{3}}Cx + \frac{2}{3\sqrt{3}}C$ |
| $\Xi^0 \rightarrow \Lambda\gamma$ | $-\frac{1}{3\sqrt{3}}P + \frac{1}{3\sqrt{3}}Q$ | $-\frac{1}{3\sqrt{3}}C + \frac{1}{3\sqrt{3}}Cx$ | $-\frac{1}{3\sqrt{3}}Cx + \frac{1}{3\sqrt{3}}C$ |
| $\Xi^0 \rightarrow \Sigma^0\gamma$ | $-\frac{1}{3}P - \frac{1}{3}Q - \frac{2}{9}v$ | $-\frac{1}{3}C - \frac{1}{3}Cx$ | $-\frac{1}{3}Cx - \frac{1}{3}C$ |