Compact model* for Quarks and Leptons
via flavored-Axions

Y. H. Ahn

Center for Theoretical Physics of the Universe,
Institute for Basic Science (IBS), Daejeon, 34051, Korea

Abstract

We show how the scales responsible for Peccei-Quinn (PQ), seesaw, and Froggatt and Nielsen (FN) mechanisms can be fixed, by constructing a compact model for resolving rather recent, but fast-growing issues in astro-particle physics, including quark and leptonic mixings and CP violations, high-energy neutrinos, QCD axion, and axion cooling of stars. The model is motivated by the flavored PQ symmetry for unifying the flavor physics and string theory. The QCD axion decay constant congruent to the seesaw scale, through its connection to the astro-particle constraints of both the stellar evolution induced by the flavored-axion bremsstrahlung off electrons $e + Ze \rightarrow Ze + e + A_i$ and the rare flavor-changing decay process induced by the flavored-axion $K^+ \rightarrow \pi^+ + A_i$, is shown to be fixed at $F_A = 3.56^{+0.84}_{-0.84} \times 10^{10}$ GeV (consequently, the QCD axion mass $m_a = 1.54^{+0.48}_{-0.29} \times 10^{-4}$ eV, Compton wavelength of its oscillation $\lambda_a = 8.04^{+1.90}_{-1.90}$ mm, and axion to neutron coupling $g_{Ann} = 2.14^{+0.66}_{-0.41} \times 10^{-12}$, etc.). Subsequently, the scale associated to FN mechanism is dynamically fixed through its connection to the standard model fermion masses and mixings, $\Lambda = 2.04^{+0.48}_{-0.48} \times 10^{11}$ GeV, and such fundamental scale might give a hint where some string moduli are stabilized in type-IIB string vacua. In the near future, the NA62 experiment expected to reach the sensitivity of $\text{Br}(K^+ \rightarrow \pi^+ + A_i) < 1.0 \times 10^{-12}$ will probe the flavored-axions or exclude the model, if the astrophysical constraint of star cooling is really responsible for the flavored-axion.

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* Here ‘compact’ model means a model that provides only requisite parameters it is easy to disprove.
†Electronic address: axionahn@naver.com
I. INTRODUCTION

For all the success of the Standard Model (SM), it is on the verge of being surpassed. Until now, symmetries has played an important role in physics in general and in quantum field theory in particular. The SM as a low-energy effective theory has been very predictive and well tested, due to the symmetries satisfied by the theory - Lorentz invariance plus the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry in addition to the discrete space-time symmetries like P and CP. However, it leaves many open questions for theoretical and cosmological issues that have not been solved yet (e.g., see [1, 2]). These include the following: inclusion of gravity in gauge theory, instability of the Higgs potential, cosmological puzzles of matter-antimatter asymmetry, dark matter, dark energy, and inflation, and flavor puzzles associated with the SM fermion mass hierarchies, their mixing patterns with the CP violating phases, and the strong CP problem. Moreover, there is no answer to the question: why there are three generations in the SM. The SM, therefore, cannot be the final answer. So it is widely believed that the SM should be extended a more fundamental underlying theory. If nature is stringy, string theory, the only framework we have for a consistent theory with both quantum mechanics and gravity, should give insight into all such fundamental issues.

Neutrino mass and mixing is the first new physics beyond SM and adds impetus to solving the open questions in astro-particle physics and cosmology. Seesaw mechanism [3] has been the most promising one responsible for the neutrino mass. Moreover, a solution to the strong CP problem of QCD through Peccei-Quinn (PQ) mechanism [5] \(^1\) may hint a new extension of gauge theory realized in gauge/gravity duality [1, 7]. If the QCD axion as a solution to the strong CP problem exists, it can easily fit into a string theoretic framework and appears cosmologically as a form of cold dark matter \(^2\). Flavor puzzle for the SM charged-fermion mass hierarchies could be solved by implementing Froggatt and Nielsen (FN) mechanism [8]. If those mechanisms are realized in nature at low energies, finding those scales responsible for the seesaw, PQ, and FN mechanisms could be one of big challenges as a theoretical guideline to aforementioned fundamental issues.

\(^1\) See, most recent its related simple toy models (supersymmetric and non-supersymmetric versions) in Ref. [6], see also Ref. [4].

\(^2\) On this issue we will consider flavored-axion as a cold dark matter in the next project. The scale in Eq. (56) we found is available for explaining dark matter, and finding it in experiments can change the fundamental understanding of the universe.
Many of the outstanding mysteries of astrophysics may be hidden from our sight at all wavelengths of the electromagnetic spectrum because of absorption by matter and radiation between us and the source. So, data from a variety of observational windows, especially, through direct observations with neutrinos and axions, may be crucial. Hence, axions and neutrinos in astro-particle physics and cosmology could be powerful sources for a new extension of SM particle physics [1, 2], in that they stand out as their convincing physics and the variety of experimental probes. Fortunately, most recent analyses on the knowledges of neutrino (low-energy neutrino oscillations [9] and high-energy neutrino [10]) and axion (QCD axion [11, 12] and axion-like-particle(ALP) [13, 14]) enter into a new phase of model construction for quarks and leptons. In light of finding the fundamental scales, interestingly enough, there are two astro-particle constraints coming from the star cooling induced by the flavored-axion bremsstrahlung off electrons \( e + Z e \rightarrow Z e + e + A_i \) [13] and the rare flavor-chanting decay process induced by the flavored-axion \( K^+ \rightarrow \pi^+ A_i \) [15], respectlely,

\[
6.7 \times 10^{-29} \lesssim \alpha_{Aee} \lesssim 5.6 \times 10^{-27} \quad \text{at } 3\sigma, \quad \text{Br}(K^+ \rightarrow \pi^+ A_i) < 7.3 \times 10^{-11}, \quad (1)
\]

where \( \alpha_{Aee} \) is the fine-structure of axion to electron.

String theory when compactified to four dimensions can generically contain \( G_F = \text{anomalous gauged } U(1) \) plus \( \text{non-Abelian (finite) symmetries} \). In this regard, in order to construct a model for the aforementioned fundamental issues one needs more types of gauge symmetry beside the SM gauge theory. One of simple approaches to a neat solution for those issues could be accommodated by introducing a type of symmetry based on seesaw [3] and Froggatt-Nielsen (FN) [8] frameworks, since it is widely believed that non-renormalizable operators in the effective theory should come from a more fundamental underlying renormalizable theory by integrating out heavy degrees of freedom. If so, one can anticipate that there may exist some correlations between low energy and high energy physics.

As shown in Ref. [1], the FN mechanism formulated with global \( U(1) \) flavor symmetry could be promoted from the string-inspired gauged \( U(1) \) symmetry. Such flavored-PQ symmetry \( U(1) \) acts as a bridge for the flavor physics and string theory [1, 16]. Even gravity (which is well-described by Einstein’s general theory of relativity) lies outside the purview of the SM, once the gauged \( U(1) \)s are introduced in an extended theory its mixed gravitational-anomaly should be free. Flavor modeling on the non-Abelian finite group has been recently singled out as a good candidate to depict the flavor mixing patterns, \( e.g., \) Ref. [1, 17, 18],
since it is preferred by vacuum configuration and string theory for flavor physics. In the so-called flavored PQ symmetry model where the SM fermion fields as well as SM gauge singlet fields carry PQ charges but electroweak Higgs doublet fields do not [1, 7, 17], the flavored-axions (one linear combination QCD axion and its orthogonal ALP) couple to hadrons, photons and leptons, and its PQ symmetry breaking scale is congruent to the seesaw scale. Hence, flavored-PQ symmetry modeling extended to $G_F$ could be a powerful tool to resolve the open questions for astro-particle physics and cosmology.

Since astro-particle physics observations have increasingly placed tight constraints on parameters for flavored-axions, it is in time for a compact model for quarks and leptons to mount an interesting challenge on fixing the fundamental scales such as the scales of seesaw, PQ, and FN mechanisms. The purpose of the present paper is to construct a flavored-PQ model along the lines of the challenge, which naturally extends to a compact symmetry $G_F$ for new physics beyond SM. Remark that [7] in modeling the $U(1)$ mixed-gravitational anomaly cancellation is of central importance in constraining the fermion contents of a new chiral gauge theory and the flavor structure of $G_F$ is strongly correlated with physical observables. Here the flavored-PQ symmetry together with the non-Abelian finite symmetry is well flavor-structured in a unique way that domain-wall number $N_{DW} = 1$ with the $U(1)_X \times \text{[gravity]}^2$ anomaly-free condition demands additional Majorana fermion and the flavor puzzles of SM are well delineated by new expansion parameters expressed in terms of $U(1)_X$ charges and $U(1)_X - [SU(3)_C]^2$ anomaly coefficients, providing interesting physical implications on neutrino, QCD axion, and flavored-axion$^3$.

The rest of this paper is organized as follows. In Sec. II we construct a compact model based on $SL_2(F_3) \times U(1)_X$ in a supersymmetric framework. Subsequently, we show that the model works well with the SM fermion mass spectra and their peculiar flavor mixing patterns. In Sec. III we show that the QCD decay constant (congruent to the seesaw scale) is well fixed through constraints coming from astro-particle physics, and in turn the FN scale is dynamically determined via its connection to the SM fermion masses and mixings. And we show several properties of the flavored-axions. What we have done is summarized in Sec. V, and we provide our conclusions. In Appendix we consider possible next-to-leading order corrections to the vacuum expectation value (VEV).

$^3$ Recently, studies on flavored-axion are gradually becoming amplified [19].
II. FLAVORED $SL_2(F_3) \times U(1)_X$ SYMMETRY

As mentioned in the Introduction, finding the scales responsible for seesaw\textsuperscript{3}, PQ\textsuperscript{5}, and FN\textsuperscript{8} mechanisms, as a theoretical guideline to the aforementioned fundamental issues, could be one of big challenges. To resolve such interesting challenge, we construct a neat and economical model based on the flavored-PQ symmetry $U(1)_X$ embedded in the non-Abelian finite group, which may provide a hint and/or framework to accommodate all the fundamental issues on astro-particle physics and cosmology. Along this line, the $G_F$ quantum number of the field contents is assigned in a way that (a) the $G_F$ requires a desired vacuum configuration to compactly describe the quark and lepton masses and mixings, (b) the $G_F$ fits in well with the astro-particle constraints induced by the flavored-axions, and (c) the $U(1)_X$ mixed-gravitational anomaly-free condition with the SM flavor structure demands additional Majorana fermions as well as no axionic domain-wall problem.

Similar to Ref.\textsuperscript{7} it is followed by the model setup: Assume we have a SM gauge theory based on the $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group, and that the theory has in addition a $G_F \equiv SL_2(F_3) \times U(1)_X$ for a compact description of new physics beyond SM. Here we assume that the symmetry group of the double tetrahedron $SL_2(F_3)$\textsuperscript{18, 20, 21} is realized in field theories on orbifolds and a subgroup of a gauge symmetry that can be protected from quantum-gravitational effects. Since chiral fermions are certainly a main ingredient of the SM, the gauge- and gravitational-anomalies of the gauged $U(1)_X$ are generically present, making the theory inconsistent. Hence some requirements needed for the extended theory are:

(i) The mixed $G_{SM} \times U(1)_X$ and cubic $U(1)_X \times [U(1)_X]^2$ anomalies should be cancelled by the Green-Schwarz (GS) mechanism\textsuperscript{22} (see Ref.\textsuperscript{1}).

(ii) The non-vanishing anomaly coefficient of the quark sector $\{U(1)_X \times [gravity]^2\}$\textsubscript{quark} constrains the quantity $\sum_{i,j}^{N_f} X_{\psi_j}^i$ in the gravitational instanton backgrounds (with $N_f$ generations well defined in the non-Abelian discrete group), and in turn whose quantity is congruent to the $U(1)_X \times [SU(3)_C]^2$ anomaly coefficient

$$\delta_k^G \delta_{ab} = 2 \sum_{\psi_i} X_{k\psi_i}^\dagger \text{Tr}(t^a t^b), \quad (2)$$

\textsuperscript{4} The details of the $SL_2(F_3)$ group are shown in Appendix\textsuperscript{A}
in the QCD instanton backgrounds, where the $t^a$ are the generators of the representation of $SU(3)$ to which Dirac fermion $\psi_i$ belongs with $X$-charge. Thanks to the two QCD anomalous $U(1)$ we have a relation
\[
|\delta_1^G / \delta_2^G| = |f_{a_1} / f_{a_2}|, \tag{3}
\]
indicating that the ratio of QCD anomaly coefficients is fixed by that of the decay constants $f_{a_i}$ of the flavored-axions $A_i$. Here $f_{a_i}$ set the flavor symmetry breaking scales, and their ratios appear in expansion parameters of the quark and lepton mass spectra (see Eqs. (38), (39), and (40)).

(iii) The mixed-gravitational anomaly $U(1)_X \times [gravity]^2$ must be cancelled to consistently couple gravity to matter charged under $U(1)_X$. Since a heavy Majorana neutrino (necessary to implement the seesaw and PQ mechanisms, simultaneously) with $U(1)_X$ charge $X_1/2$ does not have a vanishing $U(1)_X \times [gravity]^2$ anomaly, its anomaly should be cancelled by another contribution of $U(1)_X \times [gravity]^2$ anomaly. Hence, the $U(1)_X$ charges of SM fermions and new fermions including heavy Majorana neutrinos must be commensurate through the $U(1)_X \times [gravity]^2$ anomaly satisfying a condition
\[
k_1 X_1/2 = k_2 X_2 \tag{4}
\]
where $k_i (i = 1, 2)$ are nonzero integers, which is a conjectured relationship between two anomalous $U(1)$s. The $U(1)_{X_i}$ is broken down to its discrete subgroup $Z_{N_i}$ in the backgrounds of QCD instanton, and the quantities $N_i$ (nonzero integers) associated to the axionic domain-wall are given by
\[
\left| \frac{\delta_1^G}{X_1/2k_2} \right| = N_1, \quad \left| \frac{\delta_2^G}{X_2/k_1} \right| = N_2. \tag{5}
\]

(iv) The $U(1)_X$ invariance forbids renormalizable Yukawa couplings for the light families, but would allow them through effective non-renormalizable couplings suppressed by $(F/\Lambda)^n$ with a flavon field $\mathcal{F}$ and positive integer $n$. Then the SM gauge singlet flavon field $\mathcal{F}$ is activated to dimension-four(three) operators with different orders [1, 8, 17, 23]
\[
c'_1 \mathcal{O}_3 (\mathcal{F})^1 + \mathcal{O}_4 \sum_{n=0} \mathcal{F}^n, \quad \text{with} \quad \frac{1}{\sqrt{10}} \lesssim |c'_1|, \quad |c_n| \lesssim \sqrt{10}, \tag{6}
\]
where $\mathcal{O}_4(3)$ is a dimension-4(3) operator, and all the coefficients $c_n$ and $c'_1$ are complex numbers with absolute value of order unity. Here the flavon field $\mathcal{F}$ is a scalar field.
which acquires a VEV and breaks spontaneously the flavored-PQ symmetry $U(1)_X$. And the scale $\Lambda$, above which there exists unknown physics, is the scale of flavor dynamics, and is associated with heavy states which are integrated out. Such fundamental scale may come from where some string moduli are stabilized.

The flavored-PQ symmetry $U(1)_X$ is composed of two anomalous symmetries $U(1)_{X_1} \times U(1)_{X_2}$ generated by the charges $X_1 \equiv -2p$ and $X_2 \equiv -q$. Here the global $U(1)$ symmetry\(^5\) including $U(1)_R$ is remnants of the broken $U(1)$ gauge symmetries which can connect string theory with flavor physics \([1, 16]\). Hence, the spontaneous breaking of $U(1)_X$ realizes the existence of the Nambu-Goldstone (NG) mode (called axion) and provides an elegant solution to the strong CP problem.

**A. Vacuum configuration**

In this section we will review the fields contents responsible for the vacuum configuration since the scalar potential of the model is the same as in Ref. \([7]\). Apart from the usual two Higgs doublets $H_{u,d}$ responsible for electroweak symmetry breaking, which transform as $(1,0)$ under $SL_2(F_3) \times U(1)_X$ symmetry, the scalar sector is extended via two types of new scalar multiplets that are $G_{SM}$-singlets: flavon fields $\Phi_T, \Phi_S, \Theta, \tilde{\Theta}, \eta, \Psi, \tilde{\Psi}$ responsible for the spontaneous breaking of the flavor symmetry, and driving fields $\Phi_T^0, \Phi_S^0, \eta_0, \Theta_0, \Psi_0$ that are to break the flavor group along required VEV directions and to allow the flavons to get VEVs, which couple only to the flavons. The electroweak Higgs fields $H_{u,d}$ are enforced to be neutral under $U(1)_X$ not to have an axionic domain-wall problem.

Under $SL_2(F_3) \times U(1)_X$ the flavon fields $\{\Phi_T, \Phi_S\}$ transform as $(3,0)$ and $(3,X_1)$, $\eta$ as $(2',0)$, and $\{\Theta, \tilde{\Theta}, \Psi, \tilde{\Psi}\}$ as $(1,X_1), (1,X_1), (1,X_2)$, and $(1,-X_2)$, respectively; the driving fields $\{\Phi_T^0, \Phi_S^0\}$ transform as $(3,0)$ and $(3,-2X_1)$, $\eta_0$ as $(2'',0)$, and $\{\Theta_0, \Psi_0\}$ as $(1,-2X_1)$ and $(1,0)$, respectively. For vacuum stability and a desired vacuum alignment solution, the flavon fields $\{\Phi_T, \eta\}$ are enforced to be neutral under $U(1)_X$.

In addition, the superpotential $W$ in the theory is uniquely determined by the $U(1)_R$ symmetry, containing the usual $R$-parity as a subgroup: \{$\text{matter fields} \rightarrow e^{i\xi/2} \text{matter fields}$\} and \{$\text{driving fields} \rightarrow e^{i\xi} \text{driving fields}$\}, with $W \rightarrow e^{i\xi}W$, whereas flavon and Higgs fields

\(^5\) It is likely that an exact continuous global symmetry is violated by quantum gravitational effects \([24]\).
remain invariant under an $U(1)_R$ symmetry. As a consequence of the $R$ symmetry, the other superpotential term $\kappa_{\alpha}L_{\alpha}H_u$ and the terms violating the lepton and baryon number symmetries are not allowed. In addition, dimension 6 supersymmetric operators like $Q_i Q_j Q_k L_l$ ($i, j, k$ must not all be the same) are not allowed either, and stabilizing proton.

The superpotential dependent on the driving fields having $U(1)_R$ charge 2, which is invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times SL_2(F_3)$, is given at leading order by

$$W_v = \Phi_T^0 (\mu_T \Phi_T + g_T \Phi_T \Phi_T) + \Phi_S^0 (g_1 \Phi_S \Phi_S + g_2 \Theta \Phi_S) + \eta_0 (\mu_\eta \eta + g_\eta \eta \Phi_T)$$
$$+ \Theta (g_3 \Phi_S \Phi_S + g_4 \Theta \Theta + g_5 \Theta \Theta) + g_T \Psi_0 (\Psi \Psi - \mu_\Psi^2) + g_8 \Phi_T^0 \eta \eta,$$

where higher dimensional operators are neglected, and $\mu_{T, \Psi, \eta}$ are dimension parameters and $g_{T, \eta, g_{1,...,8}}$ are dimensionless coupling constants. The fields $\Psi$ and $\bar{\Psi}$ charged by $X_2$, $-X_2$, respectively, are ensured by the $U(1)_X$ symmetry extended to a complex $U(1)$ due to the holomorphy of the superpotential. So, the PQ scale $\mu_\Psi = \sqrt{v_\Psi v_{\bar{\Psi}}/2}$ corresponds to the scale of spontaneous symmetry breaking of the $U(1)_X$ symmetry. Since there is no fundamental distinction between the singlets $\Theta$ and $\tilde{\Theta}$ as indicated in Table I, we are free to define $\tilde{\Theta}$ as the combination that couples to $\Phi_T^0 \Phi_S$ in the superpotential $W_v$ [25]. At the leading order the usual superpotential term $\mu H_u H_d$ is not allowed, while at the leading order the operator driven by $\Psi_0$ and at the next leading order the operators driven by $\Phi_T^0$ and $\eta_0$ are allowed

$$\left(g_{\Psi_0} \Psi_0 + \frac{g_{\eta_0}}{\Lambda} (\Phi_T^0 \Phi_T)_1 + \frac{g_{\eta_0}}{\Lambda} (\eta_0 \eta)_1 \right) H_u H_d,$$

which is to promote the effective $\mu$-term $\mu_{\text{eff}} \equiv g_{\Psi_0} \langle \Psi_0 \rangle + g_{\eta_0} (\Phi_T^0) v_T/(\sqrt{2} \Lambda) + g_{\eta_0} (\eta_0) v_\eta/(\sqrt{2} \Lambda)$ of the order of $m_S$, $m_S v_T/\Lambda$, and $m_S v_\eta/\Lambda$, where $\langle \Psi_0 \rangle$, $\langle \Phi_T^0 \rangle$, and $\langle \eta_0 \rangle$: the VEVs of the scalar components of the driving fields, $m_S$: soft SUSY breaking mass. It is
interesting that at the leading order the electroweak scale does not mix with the potentially large scales, the VEVs of the scalar components of the flavon fields, \( v_S, v_T, v_\Theta, v_\eta \) and \( v_\Psi \). Actually, in the model once the scale of breakdown of \( U(1)_X \) symmetry is fixed by the constraints coming from astrophysics and particle physics, the other scales are automatically fixed by the flavored model structure. And it is clear that at the leading order the scalar supersymmetric \( W(\Phi_T \Phi_S) \) terms are absent due to different \( U(1)_X \) quantum numbers, which is crucial for relevant vacuum configuration in the model to produce compactly the present lepton and quark mixing angles.

The vacuum configuration of the flavon fields, \( \Phi_T, \Phi_S, \eta, \tilde{\Theta}, \Psi, \) and \( \tilde{\Psi} \), is obtained from the minimization conditions of the \( F \)-term scalar potential\(^6\). At the leading order the global minima of the potential are given\(^7\) by

\[
\langle \Phi_T \rangle = \frac{v_T}{\sqrt{2}} (1, 0, 0), \quad \langle \Phi_S \rangle = \frac{v_S}{\sqrt{2}} (1, 1, 1), \quad \langle \eta \rangle = \frac{v_\eta}{\sqrt{2}} (1, 0), \\
\langle \Psi \rangle = \langle \tilde{\Psi} \rangle = \frac{v_\Psi}{\sqrt{2}}, \quad \langle \Theta \rangle = \frac{v_\Theta}{\sqrt{2}}, \quad \langle \tilde{\Theta} \rangle = 0,
\]

where \( v_\Psi = v_\tilde{\Psi} \) and \( \kappa = v_S/v_\Theta \) in SUSY limit.

### B. Quarks, Leptons, and flavored-Axions

Under \( SL_2(F_3) \times U(1)_X \) with \( U(1)_R = +1 \), the SM quark matter fields are sewed by the five (among seven) in-equivalent representations \( 1, 1', 1'', 2' \) and \( 3 \) of \( SL_2(F_3) \), and assigned as in Table II and III. Because of the chiral structure of weak interactions, bare fermion masses are not allowed in the SM. Fermion masses arise through Yukawa interactions\(^7\). Through

\(^6\) The vacuum configuration of the driving fields is not relevant in this work. And we will not consider seriously the corrections to the VEVs due to higher dimensional operators contributing to Eq. (7) since their effects are expected to be only few percents level, see Appendix B.

\(^7\) Since the right-handed neutrinos \( N^c \) (\( S^c \)) having a mass scale much above (below) the weak interaction scale are complete singlets of the SM gauge symmetry, they can possess bare SM invariant mass terms. However, the flavored-PQ symmetry \( U(1)_X \) guarantees the absence of bare mass terms \( M N^c N^c \) and \( M_S S^c S^c \).
TABLE II: Representations of the quark fields under $SL_2(F_3) \times U(1)_X$ with $U(1)_R = +1$.

| Field          | $Q_1, Q_2, Q_3$ | $D^c, b^c$ | $U^c, t^c$ |
|----------------|----------------|------------|------------|
| $SL_2(F_3)$    | $1, 1', 1''$   | $2', 1'$   | $2', 1'$   |
| $U(1)_X$       | $10p - 4q, 8p - 2q, 0$ | $3q - 8p, 3q$ | $-8p, 0$   |

Eq. (6) the Yukawa superpotential for quark sector invariant under $G_{SM} \times G_F \times U(1)_R$ is sewed as

$$W_q^u = \hat{y}_t t^c Q_3 H_u + y_c (\eta U^c) 1^c Q_2 \frac{H_u}{\Lambda} + \hat{y}_c [(\eta U^c) 3 \Phi_T] 1^c Q_2 \frac{H_u}{\Lambda^2}$$

$$+ y_u [(\eta U^c) 3 \Phi_T] 1^c Q_1 \frac{H_u}{\Lambda^3}.$$  \hspace{1cm} (10)

$$W_q^d = y_d b^c Q_3 H_d + y_s (\eta D^c) 1^c Q_2 \frac{H_d}{\Lambda} + \hat{y}_s [(\eta D^c) 3 \Phi_S] 1^c Q_2 \frac{H_d}{\Lambda^2} + y_d [(\eta D^c) 3 \Phi_S] 1^c Q_1 \frac{H_d}{\Lambda^2}$$

$$+ Y_d b^c Q_1 (\Phi_S \Phi_S) 1^{\nu} \frac{H_d}{\Lambda^2} + \hat{y}_d [(\eta D^c) 3 \Phi_T] 1^c Q_1 \frac{H_d}{\Lambda^2}.$$  \hspace{1cm} (11)

In the above superpotential, under $SL_2(F_3) \times U(1)_X$ the left-handed quark $SU(2)_L$ doublets $Q_1, Q_2, Q_3$ transform as $(1, 10p - 4q), (1', 8p - 2q)$ and $(1'', 0)$, respectively, while the right-handed up-type quark $SU(2)_L$ singlets denoted as $U^c = \{u^c, c^c\}$ and $t^c$ transform as $(2', -8p)$ and $(1', 0)$, respectively, and the right-handed down-type quarks $D^c = \{d^c, s^c\}$ and $b^c$ as $(2', 3q - 8p)$ and $(1', 3q)$, respectively. The up-type quark superpotential in Eq. (10) does not contribute to the Cabbibo-Kobayashi-Maskawa (CKM) matrix due to the diagonal form of mass matrix, while the down-type quark superpotential in Eq. (11) does contribute the CKM matrix. According to the assignment of the $U(1)_X$ quantum numbers to the matter fields content as in Table II, the Yukawa couplings of quark fermions are visualized as a function of the SM gauge singlet flavon fields $\Psi(\tilde{\Psi})$ and/or $\Theta(\Phi_S)$, except for the top Yukawa coupling:

$$y_c = \hat{y}_c \left( \frac{\Psi}{\Lambda} \right)^2, \quad \hat{y}_c = \hat{y}_c \left( \frac{\Psi}{\Lambda} \right)^2, \quad y_u = \hat{y}_u \left( \frac{\Psi}{\Lambda} \right)^4 \frac{\Theta}{\Lambda}, \quad \hat{y}_u = \hat{y}_u \left( \frac{\Psi}{\Lambda} \right)^4 \frac{\Theta}{\Lambda},$$

$$y_b = \hat{y}_b \left( \frac{\Psi}{\Lambda} \right)^3, \quad y_s = y_s \left( \frac{\Psi}{\Lambda} \right), \quad y_d = y_d \left( \frac{\Psi}{\Lambda} \right), \quad \hat{y}_d = \hat{y}_d \left( \frac{\Psi}{\Lambda} \right) \frac{\Theta}{\Lambda},$$

$$Y_s = \hat{Y}_{s1} \left( \Theta \Lambda \right)^{2} \Psi \Lambda + \hat{Y}_{s2} \left( \Phi_S \Lambda \right)^{2} \Psi \Lambda, \quad Y_d = \hat{Y}_{d1} \left( \Theta \Lambda \right)^{3} \Psi \Lambda + \hat{Y}_{d2} \left( \Phi_S \Lambda \right)^{2} \Psi \Lambda.$$  \hspace{1cm} (12)

where the hat Yukawa coupling denotes order of unity i.e., $1/\sqrt{10} \lesssim |\hat{y}| \lesssim \sqrt{10}$.

As discussed in Refs. [1, 7, 17], with the condition of $U(1)_X$-[gravity]$^2$ anomaly cancellation new additional Majorana fermions $S_{e, \mu, \tau}$ besides the heavy Majorana neutrinos can
be introduced in the lepton sector. Hence, such new additional Majorana neutrinos can play a role of the active neutrinos as pseudo-Dirac neutrinos. Under $SL_2(F_3) \times U(1)_X$ with $U(1)_R = +1$, the quantum numbers of the lepton fields are summarized as in Table III.

**TABLE III:** Representations of the lepton fields under $SL_2(F_3) \times U(1)_X$ with $U(1)_R = +1$. And here $r \equiv Q_{y_\beta} + p$ is defined.

| Field | $L$ | $e^c, \mu^c, \tau^c$ | $N^c$ | $S^c_{\mu}, S^c_{\mu}, S^c_\tau$ |
|-------|-----|---------------------|------|---------------------|
| $SL_2(F_3)$ | 3 | 1, 1'', 1' | 3 | 1, 1'', 1' |
| $U(1)_X$ | $-r$ | $r - Q_{y_e}, r - Q_{y_\mu}, r - Q_{y_\tau}$ | $p$ | $r - Q_{y_1}, r - Q_{y_2}, r - Q_{y_3}$ |

The lepton Yukawa superpotential, similar to the quark sector, invariant under $G_{SM} \times G_F \times U(1)_R$ reads at leading order

$$W_\ell = y_{\tau} \tau^c (L \Phi_T)_{1''} \frac{H_d}{\Lambda} + y_\mu \mu^c (L \Phi_T)_{1'} \frac{H_d}{\Lambda} + y_e e^c (L \Phi_T)_{1} \frac{H_d}{\Lambda}, \quad (13)$$

$$W_\nu = y_3^{s} S_{\tau}^c (L \Phi_T)_{1''} \frac{H_u}{\Lambda} + y_2^{s} S_{\mu}^c (L \Phi_T)_{1'} \frac{H_u}{\Lambda} + y_1^{s} S_{e}^c (L \Phi_T)_{1} \frac{H_u}{\Lambda} + \frac{1}{2}(\hat{y}_\tau \Theta + \hat{y}_e \Theta) (N^c N^c)_{1} + \frac{\hat{y}_\mu}{2} (N^c N^c)_{3} \Phi_S$$

$$+ \frac{1}{2} \{(y_1^{ss} S_{e}^c S_{e}^c + y_2^{ss} S_{\mu}^c S_{\mu}^c + y_2^{ss} S_{\tau}^c S_{\tau}^c) \Theta\}.$$

(14)

In the above charged-lepton Yukawa superpotential, $W_\ell$, it has three independent Yukawa terms at the leading: apart from the Yukawa couplings, each term involves flavon field $\Phi_T$. Under $SL_2(F_3) \times U(1)_X$, the left-handed charged lepton $SU(2)_L$ doublet denoted as $L$ transforms as $(3, -r)$, while the right-handed charged leptons denoted as $e^c, \mu^c$ and $\tau^c$, the electron flavor transforms as $(1, r - Q_{y_e})$, the muon flavor as $(1'', r - Q_{y_\mu})$, and the tau flavor as $(1', r - Q_{y_\tau})$. Here $Q_{y_\beta}$ denotes the $U(1)_X$ quantum number of Yukawa coupling $y_\beta$ which appears in the superpotentials (13) and (14).

In the neutrino Yukawa superpotential, $W_\nu$, two right-handed Majorana neutrinos $S^c$ and $N^c$ are introduced to make light neutrinos pseudo-Dirac particles and to realize an exact tri-bimaximal mixing (TBM) at leading order, respectively. Such additional Majorana fermion $S^c$ plays a role of making no axionic domain-wall problem, which links low energy neutrino oscillations to astronomical-scale baseline neutrino oscillations. The different assignments of $SL_2(F_3) \times U(1)_X$ quantum number to Majorana neutrinos guarantee

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8 We will study on neutrino in detail including numerical analysis in the next project.
the absence of the Yukawa terms $S^c N^c \times \mathcal{F}$. Consequently, two Dirac neutrino mass terms are generated; one is associated with $S^c$, and the other is $N^c$. The right-handed neutrino $SU(2)_L$ singlet denoted as $N^c$ transforms as the $(3, p)$ and additional Majorana neutrinos denoted as $S^c_\mu$, $S^c_\tau$ transform as $(1, r - Q_{\gamma_2})$, $(1'', r - Q_{\gamma_3})$ and $(1', r - Q_{\gamma_3})$, respectively. Below the cutoff scale $\Lambda$, the mass term of the Majorana neutrinos $N^c$ comprises an exact TBM pattern. Imposing the $U(1)_X$ symmetry explains the absence of the Yukawa terms $L N^c \Phi_S$ and $N^c N^c \Phi_T$ as well as does not allow the interchange between $\Phi_T$ and $\Phi_S$, both of which transform differently under $U(1)_X$, so that the exact TBM is obtained at leading order. With the desired VEV alignment in Eq. (9) it is expected that the leptonic Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix at the leading order is exactly compatible with a TBM

$$\theta_{13} = 0 , \quad \theta_{23} = \frac{\pi}{4} = 45^\circ , \quad \theta_{12} = \sin^{-1} \left( \frac{1}{\sqrt{3}} \right) \simeq 35.3^\circ .$$

In order to explain the present terrestrial neutrino oscillation data, non-trivial next leading order corrections should be taken into account: for example, considering next leading order Yukawa superpotential in the Majorana neutrino sector triggered by the field $\Phi_T$ are written as $(N^c N^c \Theta \Phi_T)_1/\Lambda$ and $(N^c N^c \Phi_S \Phi_T)_1/\Lambda$, and in the Dirac neutrino sector $(L N^c \Phi_T)_1 H_u/\Lambda$. (For neutrino phenomenology we will consider in detail in the next project. See also an interesting paper [27].) After including the higher dimensional operators there remain no residual symmetries.

Remark that, as in the SM quark fields since the $U(1)_X$ quantum numbers are arranged to lepton fields as in Table III with the condition [1] (or Eq. (19)) satisfied, it is expected that the SM gauge singlet flavon fields derive higher-dimensional operators, which are eventually visualized into the Yukawa couplings of leptons as a function of flavon fields. Quarks and leptons are conserved in strong and electromagnetic but not in weak interactions. First, the $U(1)_X$ quantum numbers associated to the charged-leptons are assigned in a way that (i) the charged lepton mass spectra are described and (ii) the ratio of electromagnetic $U(1)_X$-[\( U(1)_{EM} \)]\(^2 \) and color anomaly $U(1)_X-[SU(3)_c]^2$ coefficients lies in the range\( ^{10} 0 < E/N < 4, \)

\( ^9 \) Clearly, the flavor symmetry $SL_2(F_3)$ is broken down to its subgroup: at the leading order, the charged-lepton mass terms and the Dirac neutrino mass terms containing $S^c L$ are invariant under the subgroup $G_T$, while the Majorana mass terms containing $N^c N^c$ are invariant under the subgroup $G_S$.

\( ^{10} \) This range is derived from the bound ADMX experiment $^{12}$ \( (g_{\gamma\gamma}/m_a)^2 \leq 1.44 \times 10^{-19} \text{GeV}^{-2} \text{eV}^{-2} \).
where \( E = \sum_f (\delta^G_1 X_{1f} + \delta^G_2 X_{2f}) (Q^\text{em}_f)^2 \) and \( N = 2\delta^G_1 \delta^G_2 \):

\[
\begin{align*}
\frac{E}{N} &= \frac{23}{6}, \quad \text{for } Q_{y_1} = -3q, \ Q_{y_2} = -6q, \ Q_{y_3} = 11q; \text{ case-I} \quad (16) \\
\frac{E}{N} &= \frac{1}{2}, \quad \text{for } Q_{y_1} = 3q, \ Q_{y_2} = 6q, \ Q_{y_3} = -11q; \text{ case-II} \quad (17) \\
\frac{E}{N} &= \frac{5}{2}, \quad \text{for } Q_{y_1} = 3q, \ Q_{y_2} = 6q, \ Q_{y_3} = -11q; \text{ case-III}. \quad (18)
\end{align*}
\]

Similarly, the \( U(1)_X \) quantum numbers associated to the neutrinos can be assigned by the anomaly-free condition of \( U(1)_X \times \text{[gravity]}^2 \) together with the measured active neutrino observables:

\[
U(1)_X \times \text{[gravity]}^2 \propto 3 \{4p - 3q\}_\text{quark} + \{3p - Q_{y_1^*} - Q_{y_2^*} - Q_{y_3^*} - Q_{y_e} - Q_{y_\mu} - Q_{y_\tau}\}_\text{lepton} = 0. \quad (19)
\]

This vanishing anomaly, however, does not restrict \( Q_{y_\nu} \) (or equivalently \( Q_{y_\nu^*} \)), whose quantum numbers can be constrained by the new neutrino oscillations of astronomical-scale baseline, as shown in Refs. [1, 7, 28]. With the given above \( U(1)_X \) quantum numbers, such \( U(1)_X \times \text{[gravity]}^2 \) anomaly is free for

\[
21 \frac{X_1}{2} = k_2 X_2 \quad \text{with } k_2 = \begin{cases} 11 - \tilde{Q}_{y_1^*} - \tilde{Q}_{y_2^*} - \tilde{Q}_{y_3^*}; \text{ case-I} \\ 1 - \tilde{Q}_{y_1^*} - \tilde{Q}_{y_2^*} - \tilde{Q}_{y_3^*}; \text{ case-II} \\ 7 - \tilde{Q}_{y_1^*} - \tilde{Q}_{y_2^*} - \tilde{Q}_{y_3^*}; \text{ case-III} \end{cases} \quad (20)
\]

where \( \tilde{Q}_{y_i^*} = Q_{y_i^*} / X_2 \). We choose \( k_2 = \pm 21 \) for the \( U(1)_{X_i} \) charges to be smallest making no axionic domain-wall problem, as in Ref. [1, 7]. Hence, for the case-I \( \tilde{Q}_{y_1^*} + \tilde{Q}_{y_2^*} + \tilde{Q}_{y_3^*} = -10 \) \((32)\); for the case-II \(-20 \) \((22)\); for the case-III \(-14 \) \((28)\), respectively, for \( k_2 = 21(-21) \). Then, the color anomaly coefficients are given by \( \delta^G_1 = 2X_1 \) and \( \delta^G_2 = -3X_2 \), and subsequently from Eq. \([5]\) the axionic domain-wall condition as in Ref. [7] is expressed with the reduced \( k_1 = \pm k_2 = 1 \) as

\[
N_1 = 4, \quad N_2 = 3. \quad (21)
\]

Clearly, in the QCD instanton backgrounds since the \( N_1 \) and \( N_2 \) are relative prime there is no \( Z_{\text{DW}} \) discrete symmetry, and therefore no axionic domain-wall problem occurs.

The model incorporates the SM gauge singlet flavon fields \( F_A = \Phi_S, \Theta, \Psi, \tilde{\Psi} \) with the following interactions invariant under the \( U(1)_X \times SL_2(F_3) \) and the resulting chiral symmetry,
In our superpotential, the superfields $\Phi_{11}$ Note that the massless modes are not contained in the fields $\tilde{\Theta}_{12}$ in the SUSY limit, and $\psi$ stands for all Dirac fermions. The kinetic terms $= \frac{\partial^2 K}{\partial F_A \partial F_A} \partial_{\mu} F_A^\dagger \partial^\mu F_A$ with the Kahler potential $K \supset |F_A|^2$+higher order terms for canonically normalized fields are written as

$$\partial_{\mu} \Phi^\dagger S \partial^\mu S + \partial_{\mu} \Theta \partial^\mu \Theta + \partial_{\mu} \Psi \partial^\mu \Psi + \partial_{\mu} \tilde{\Psi} \partial^\mu \tilde{\Psi}.$$  

(23)

The scalar fields $\Phi_S, \Theta$ and $\Psi(\tilde{\Psi})$ have $X$-charges $X_1$ and $X_2(-X_2)$, respectively, that is $\Phi_{Si} \rightarrow e^{i\xi_1 X_1} \Phi_{Si}, \ \Theta \rightarrow e^{i\xi_1 X_1} \Theta; \ \Psi \rightarrow e^{i\xi_2 X_2} \Psi, \ \tilde{\Psi} \rightarrow e^{-i\xi_2 X_2} \tilde{\Psi}$  

(24)

where $\xi_k (k = 1, 2)$ are constants. So, the potential $V_{\text{SUSY}}$ has $U(1)^X$ global symmetry. In order to extract NG modes resulting from spontaneous breaking of $U(1)^X$ symmetry, we set the decomposition of complex scalar fields as follows$^{12}$

$$\Phi_{Si} = \frac{e^{i\frac{\phi_S}{v_S}}}{\sqrt{2}} (v_S + h_S), \ \Theta = \frac{e^{i\frac{\phi}{v}}}{\sqrt{2}} (v + h),$$

$$\Psi = \frac{v_\Psi}{\sqrt{2}} e^{i\frac{\phi_\Psi}{v_\Psi}} \left(1 + \frac{h_\Psi}{v_\Psi}\right), \ \tilde{\Psi} = \frac{v_\Psi}{\sqrt{2}} e^{-i\frac{\phi_\Psi}{v_\Psi}} \left(1 + \frac{h_\Psi}{v_\Psi}\right),$$

(25)

in which we have set $\Phi_{S1} = \Phi_{S2} = \Phi_{S3} \equiv \Phi_{Si}$ and $h_\Psi = h_\tilde{\Psi}$ in the SUSY limit, and $v_\Psi = \sqrt{v_{\Psi}^2 + v_{\tilde{\Psi}}^2}$. And the NG modes $A_1$ and $A_2$ are expressed as

$$A_1 = \frac{v_S \phi_S + v_\Theta \phi_\Theta}{\sqrt{v_S^2 + v_\Theta^2}}, \ \ A_2 = \phi_\Psi$$

(26)

with the angular fields $\phi_S, \phi_\Theta$ and $\phi_\Psi$. With Eqs. (23) and (25), the derivative couplings of $A_k$ arise from the kinetic terms

$$\partial_{\mu} F_k^\dagger \partial^\mu F_k = \frac{1}{2} (\partial_{\mu} A_1)^2 \left(1 + \frac{h_F}{v_F}\right)^2 + \frac{1}{2} (\partial_{\mu} A_2)^2 \left(1 + \frac{h_\Psi}{v_\Psi}\right)^2 + \frac{1}{2} (\partial_{\mu} h_F)^2 + \frac{1}{2} (\partial_{\mu} h_\Psi)^2 + \ldots$$

(27)

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11 In our superpotential, the superfields $\Phi_S, \Theta$ and $\Psi(\tilde{\Psi})$ are the SM gauge singlets and have $-2p$ and $-q(q)$ $X$-charges, respectively. Given soft SUSY-breaking potential, the radial components of the $X$-fields $|\Phi_S|, |\Theta| |\Psi|$ and $|\tilde{\Psi}|$ are stabilized. The $X$-fields contain the axion, saxion (the scalar partner of the axion), and axino (the fermionic superpartner of the axion).

12 Note that the massless modes are not contained in the fields $\tilde{\Theta}, \Phi_T, \Phi_T^S, \Theta_0, \Psi_0$ in SUSY limit.
where \( v_F = v_\Theta (1 + \kappa^2)^{1/2} \) and \( h_F = (\kappa h_S + h_\Theta)/(1 + \kappa^2)^{1/2} \), and the dots stand for the orthogonal components \( h_F^\perp \) and \( A_1^\perp \). Recalling that \( \kappa \equiv v_S/v_\Theta \). Clearly, the derivative interactions of \( A_k (k = 1, 2) \) are suppressed by the VEVs \( v_F \) and \( v_\Psi \). From Eq. (27), performing \( v_F, v_\Psi \to \infty \), the NG modes \( A_{1,2} \), whose interactions are determined by symmetry, are invariant under the symmetry and distinguished from the radial modes, \( h_F \) and \( h_\Psi \).

In Eq. (22) the relevant matter fields interaction terms are given as follows. Once the scalar fields \( \Phi_S, \Theta, \tilde{\Theta}, \Psi \) and \( \tilde{\Psi} \) get VEVs, the flavor symmetry \( U(1)_X \times SL_2(F_3) \) is spontaneously broken\(^{13} \). And at energies below the electroweak scale, all quarks and leptons obtain masses. The relevant Yukawa interaction terms with chiral fermions \( \psi \) charged under the flavored \( U(1)_X \) symmetry is given by

\[
-\mathcal{L}_{YW} = \bar{q}^u_R M_u q^u_L + \bar{q}^d_R M_d q^d_L + \ell_R M_\ell \ell_L + \frac{g}{\sqrt{2}} W^+_{\mu} \bar{q}^u_L \gamma^\mu q^d_L + \frac{g}{\sqrt{2}} W^-_{\mu} \bar{\ell}_L \gamma^\mu \nu_L
\]

\[
+ \frac{1}{2} \left( \begin{array}{ccc}
1 & \bar{S}_R & \bar{N}_R \\
0 & m_{D_S} & m_D \\
m_D & 0 & 0
\end{array} \right)
\left( \begin{array}{c}
0 \\
m_{D_S} e^{i\frac{A_L}{2}} M_S \\
0
\end{array} \right)
\left( \begin{array}{c}
\nu_L \\
S_R \\
N_R
\end{array} \right)
\]

where \( g \) is the \( SU(2) \) coupling constant, \( q^u = (u, c, t) \), \( q^d = (d, s, b) \), \( \ell = (e, \mu, \tau) \), and \( \nu_L = (\nu_e, \nu_\mu, \nu_\tau) \).

1. Quarks and CKM mixings, and flavored-Axions

Now, let us move to discussion on the realization of quark masses and mixings, in which the physical mass hierarchies are directly responsible for the assignment of \( U(1)_X \) quantum numbers. The axion coupling matrices to the up- and down-type quarks, respectively, are diagonalized through bi-unitary transformations: \( V^\psi_R \mathcal{M}_\psi V^\psi_L = \hat{\mathcal{M}}_\psi \) (diagonal form), and the mass eigenstates \( \psi'_R = V^\psi_R \psi_R \) and \( \psi'_L = V^\psi_L \psi_L \). These transformation include, in particular, the chiral transformation necessary to make \( \mathcal{M}_u \) and \( \mathcal{M}_d \) real and positive. This induces a contribution to the QCD vacuum angle. Note here that under the chiral rotation of the quark fields the effective QCD vacuum angle is invariant, see Refs. \[1, 17\]. With the desired VEV directions in Eq. (9), in the above Lagrangian (28) the mass matrices \( \mathcal{M}_u \) and

\(^{13} \) If the symmetry \( U(1)_X \) is broken spontaneously, the massless modes \( A_1 \) of the scalar \( \Phi_S \) (or \( \Theta \)) and \( A_2 \) of the scalar \( \Psi(\tilde{\Psi}) \) appear as phases.
\[ M_u = \begin{pmatrix}
(i y_u \nabla T - \tilde{y}_u \nabla^2 \eta) \nabla \eta e^{i(\frac{\alpha_1}{\sqrt{2}} - \frac{\alpha_2}{\sqrt{2}})} & 0 & 0 \\
0 & (y_c + \frac{1}{2} \tilde{y}_c \nabla T) \nabla \eta e^{-2i \frac{\alpha_2}{\sqrt{2}}} & 0 \\
0 & 0 & \tilde{y}_t
\end{pmatrix} v_u ,
\]

\[ M_d = \begin{pmatrix}
(i y_d \nabla S + \tilde{y}_d \nabla T) \nabla \eta e^{i(\frac{\alpha_1}{\sqrt{2}} - \frac{\alpha_2}{\sqrt{2}})} & 0 & 0 \\
\frac{1-i}{2} y_d \nabla \eta \nabla S e^{i(\frac{\alpha_1}{\sqrt{2}} - \frac{\alpha_2}{\sqrt{2}})} & y_s \nabla \eta e^{\frac{i \alpha_2}{\sqrt{2}}} & 0 \\
3 Y_d \nabla^2 S e^{i(\frac{\alpha_1}{\sqrt{2}} + \frac{\alpha_2}{\sqrt{2}})} & 3 Y_s \nabla^2 S e^{i(\frac{\alpha_1}{\sqrt{2}} - \frac{\alpha_2}{\sqrt{2}})} & y_b e^{-\frac{i \alpha_2}{\sqrt{2}}}
\end{pmatrix} v_d ,
\]

where \( v_d \equiv \langle H_d \rangle = v \cos \beta/\sqrt{2} \) and \( v_u \equiv \langle H_u \rangle = v \sin \beta/\sqrt{2} \) with \( v \approx 246 \) GeV, and

\[ \nabla Q \equiv \frac{v Q}{\sqrt{2} \Lambda} \quad \text{with } Q = \eta, S, T, \Theta, \Psi, \bar{\Psi} .
\]

In the above mass matrices the corresponding Yukawa terms for up- and down-type quarks are given by

\[ y_u = \tilde{y}_u \nabla \eta \nabla^2 \eta , \quad \tilde{y}_u = \tilde{y}_u \nabla \eta \nabla^2 \eta , \quad y_c = \tilde{y}_c \nabla^2 \eta , \quad \tilde{y}_c = \tilde{y}_c \nabla^2 \eta ,
\]

\[ y_d = \tilde{y}_d \nabla \psi , \quad \tilde{y}_d = \tilde{y}_d \nabla \psi , \quad y_s = \tilde{y}_s \nabla \psi , \quad y_b = \tilde{y}_b \nabla^2 \psi .
\]

One of the most interesting features observed by experiments on the quarks is that the mass spectrum of the up-type quarks exhibits a much stronger hierarchical pattern to that of the down-type quarks, which may indicate that the CKM matrix is mainly generated by the mixing matrix of the down-type quark sector. Moreover, due to the diagonal form of the up-type quark mass matrix in Eq. (28) the CKM mixing matrix \( V_{\text{CKM}} \equiv V_L^{\text{ud}} V_L^{\text{d}} \) coming from the charged quark-current term in Eq. (28) is generated from the down-type quark matrix in Eq. (30):

\[ V_{\text{CKM}} = V_L^{\text{d}} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & \lambda \lambda^3 (\rho + i \eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & -\lambda \lambda^2 \\
\lambda \lambda^3 (1 - \rho + i \eta) & -\lambda \lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4) ,
\]

in the Wolfenstein parametrization and at higher precision, where \( \lambda = 0.22509^{+0.00091}_{-0.00071} \), \( A = 0.825^{+0.020}_{-0.037} \), \( \rho = \rho/(1 - \lambda^2/2) = 0.160^{+0.034}_{-0.021} \), and \( \eta = \eta/(1 - \lambda^2/2) = 0.350^{+0.024}_{-0.024} \) with 3σ errors.

The quark mass matrices \( M_u \) in Eq. (29) and \( M_d \) in Eq. (30) generate the up- and down-type quark masses:

\[ \widetilde{M}_u = P_u^* M_u Q_u = \text{diag}(m_u, m_c, m_t) , \quad \widetilde{M}_d = V_R^{\text{d}} M_d V_L^{\text{d}} = \text{diag}(m_d, m_s, m_b) ,
\]
where $P_u$ and $Q_u$ are diagonal phase matrices, and $V_L^d$ and $V_R^d$ can be determined by diagonalizing the matrices for $\mathcal{M}^d_d\mathcal{M}_d$ and $\mathcal{M}_d\mathcal{M}_d^\dagger$, respectively. The physical structure of the up- and down-type quark Lagrangian should match up with the empirical up- and down-type quark masses and their ratios calculated from the measured PDG values \([29]\):

\[
\frac{m_d}{m_b} \simeq 1.12^{+0.13}_{-0.11} \times 10^{-3}, \quad \frac{m_s}{m_b} \simeq 2.30^{+0.21}_{-0.12} \times 10^{-2}, \quad \frac{m_u}{m_t} \simeq 2.41^{+0.03}_{-0.03} \times 10^{-2}, \\
\frac{m_u}{m_d} \simeq 0.38 - 0.58, \quad \frac{m_c}{m_t} \simeq 7.39^{+0.20}_{-0.20} \times 10^{-3}, \quad \frac{m_u}{m_c} \simeq 1.72^{+0.52}_{-0.34} \times 10^{-3}, \tag{35}
\]

\[
m_b = 4.18^{+0.04}_{-0.03} \text{ GeV}, \quad m_c = 1.28 \pm 0.03 \text{ GeV}, \quad m_t = 173.1 \pm 0.6 \text{ GeV}, \tag{36}
\]

where $c$- and $b$-quark masses are the running masses in the $\overline{\text{MS}}$ scheme, and the light $u$, $d$, $s$-quark masses are the current quark masses in the $\overline{\text{MS}}$ scheme at the momentum scale $\mu \approx 2 \text{ GeV}$. So, the following new expansion parameters are defined in a way that the diagonalizing matrix $V_L^d$ satisfies the CKM matrix as well as the empirical quark masses and their ratios in Eqs. (35) and (36):

\[
\nabla_T = \frac{\kappa}{|\hat{y}_d|} \left| \hat{y}_d \right| \quad \text{with} \quad \phi_\hat{d} = -\phi_d - \frac{\pi}{2}, \tag{37}
\]

\[
\nabla_\Theta = \frac{1}{\kappa} \nabla_S = \left| \frac{X_2\delta_1^G}{X_1\delta_1^G} \right| \sqrt{\frac{2}{1 + \kappa^2}} \nabla_\Psi, \tag{38}
\]

\[
\nabla_\eta = \left( \frac{m_c}{m_t} \right)_{\text{PDG}} \left| \frac{\hat{y}_t}{\hat{y}_c + \hat{y}_t} \right| \frac{1}{\nabla_T}, \tag{39}
\]

\[
\nabla_\Psi \simeq \frac{\lambda}{6} \left| \frac{X_1\delta_2^G}{X_2\delta_1^G} \right| \left( B \frac{1 + \kappa^2}{6\kappa^2} \frac{|\hat{y}_b|}{|\hat{Y}_{d_1} + 3\kappa^2\hat{Y}_{d_2}|} \right) \frac{1}{\nabla_T}. \tag{40}
\]

Then, the mixing matrix $V_{L}^{d\dagger} = V_{\text{CKM}}$ is obtained by diagonalizing the Hermitian matrix $\mathcal{M}_d^\dagger\mathcal{M}_d$:

\[
V_{L}^{d\dagger} \mathcal{M}_d^\dagger \mathcal{M}_d V_{L}^{d} = \text{diag}(|m_d|^2, |m_s|^2, |m_b|^2). \tag{41}
\]

The CKM mixing angles in the standard parametrization \([33]\) can be roughly described as

\[
\theta_{12}^q \simeq \frac{1}{\sqrt{2}} \frac{|\hat{y}_d|}{|\hat{y}_s|} \nabla_S, \quad \theta_{23}^q \simeq 3\kappa^2 \frac{\nabla_4}{|\hat{y}_b|} \frac{\nabla_4}{\nabla_2^\Psi}, \quad \theta_{13}^q \simeq 3 \frac{\nabla_{d_1} + 3\kappa^2\nabla_{d_2}}{|\hat{y}_b|} \nabla_\Psi. \tag{42}
\]

And with the quark fields redefinition the CKM CP phase is given as

\[
\delta_{CP}^q \equiv \tan^{-1} \left( \eta/\rho \right) = \phi_2^d - 2\phi_3^d, \tag{43}
\]

where $\phi_2^d \simeq \arg\{(\hat{Y}_{d_1} + 3\kappa^2\hat{Y}_{d_2})\hat{y}_b\}/2 - \phi_1^d/2$ and $2\phi_3^d \simeq \arg\langle \hat{y}_s^*\hat{y}_b \rangle + \phi_1^d - \phi_2^d + \pi/4$, and $\phi_1^d = \arg\langle \hat{Y}_{s_1}^* + 3\kappa^2\hat{Y}_{s_2}^* \rangle \hat{y}_b \rangle/2$. As designed, the CKM matrix is well described with
\( J_{CP}^{\text{quark}} = \text{Im}[V_{us}V_{cb}V_{ub}V_{cs}] \simeq A^2 \lambda^6 \sqrt{\rho^2 + \eta^2} \sin \delta_{CP} \). Subsequently, the up- and down-type quark masses are obtained as

\[
\begin{align*}
    m_t & \simeq |\hat{y}_t| v_u, \\
    m_c & \simeq \left| \hat{y}_c + \frac{1 - i}{2} \hat{\gamma}_e \nabla_T \nabla^2 \nabla_\eta v_u \right|, \\
    m_u & \simeq \nabla^4 \nabla_\eta \nabla_\Theta |i \hat{y}_u \nabla_T - \hat{\gamma}_u \nabla_\eta^2| v_u, \\
    m_b & \simeq |\hat{y}_b| \nabla^3_\psi v_b, \\
    m_d & \simeq 2 |\hat{y}_d \sin \phi_d| \nabla_\psi \nabla_\eta \nabla_S v_d. \\
\end{align*}
\]

And the parameter of \( \tan \beta \equiv v_u/v_d \) is given in terms of the PDG value in Eq. (36) by

\[
\tan \beta \simeq \left( \frac{m_t}{m_b} \right)_{\text{PDG}} |\hat{y}_t| \nabla^3_\psi. \tag{44}
\]

Since all the parameters in the quark sector are correlated with one another, it is very crucial for obtaining the values of the new expansion parameters to reproduce the empirical results of the CKM mixing angles and quark masses. Moreover, since such parameters are also closely correlated with those in the lepton sector, finding the value of parameters is crucial to produce the empirical results of the charged leptons (see below Eq. (48)) and the light active neutrino masses in our model. In the following subsequent subsection we will perform a numerical simulation for quark sector.

2. Numerical analysis for Quark masses and CKM mixing angles

We perform a numerical simulation\(^{14}\) using the linear algebra tools of Ref. [34]. With the inputs

\[
\tan \beta = 4.7, \quad \kappa = 0.33, \tag{46}
\]

and \( |\hat{y}_d| = 1.1 \) (\( \phi_d = 3.070 \) rad), \( |\hat{y}_d| = 1.194 \), \( |\hat{y}_s| = 0.370 \) (\( \phi_s = 4.920 \) rad), \( |\hat{y}_b| = 2.280 \) (\( \phi_b = 0 \)), \( |\hat{y}_u| = 0.400 \) (\( \phi_u = 0 \)), \( |\hat{y}_c| = 1.0 \) (\( \phi_c = 0 \)), \( |\hat{y}_c| = 2.800 \) (\( \phi_c = 3.600 \) rad), \( |\hat{y}_c| = 1.000 \) (\( \phi_c = 0 \)), \( |\hat{y}_u| = 1.017 \) (\( \phi_u = 0 \)), \( |\hat{Y}_{d1}| = 0.900 \) (\( \phi_{Y_{d1}} = 4.800 \) rad), \( |\hat{Y}_{d2}| = 0.800 \) (\( \phi_{Y_{d2}} = 0 \)), \( |\hat{Y}_{s1}| = 2.600 \) (\( \phi_{Y_{s1}} = 6.500 \) rad), \( |\hat{Y}_{s2}| = 1.900 \) (\( \phi_{Y_{s2}} = 0.117 \) rad), leading to

\[
\nabla_\Psi = 0.370, \quad \nabla_S = 0.109, \quad \nabla_T = 0.304, \quad \nabla_\eta = 0.020, \tag{47}
\]

\(^{14}\) Here, in numerical calculation, we only have considered the mass matrices in Eqs. (29) and (30) since it is expected that the corrections to the VEVs due to dimensional operators contributing to Eq. (7) could be small enough below a few percent level, see Appendix [15].
we obtain the mixing angles and Dirac CP phase $\theta_{12}^q = 12.98^\circ$, $\theta_{23}^q = 2.32^\circ$, $\theta_{13}^q = 0.22^\circ$, $\delta_{CP}^q = 65.18^\circ$ compatible with the 3σ Global fit of CKMfitter \[32\]; the quark masses $m_d = 4.49$ MeV, $m_s = 101.62$ MeV, $m_b = 4.18$ GeV, $m_u = 2.57$ MeV, $m_c = 1.28$ GeV, and $m_t = 173.1$ GeV compatible with the values in PDG \[29\].

3. charged-Leptons and flavored-Axions

According to the $U(1)_X$ charge assignment of the charged-leptons in Eqs. (16), (17), and (18), the charged-lepton mass matrix in the Lagrangian (28) is written as

$$ M_\ell = \begin{pmatrix}
  y_e e^{\frac{Q_e A_2}{\sqrt{g}}} & 0 & 0 \\
  0 & y_\mu e^{\frac{Q_\mu A_2}{\sqrt{g}}} & 0 \\
  0 & 0 & y_\tau e^{\frac{Q_\tau A_2}{\sqrt{g}}}
\end{pmatrix} v_d, \\
(48)
$$

where $Q_e = -11$, $Q_\mu = 6$, $Q_\tau = 3$ for the case-I ($E/N = 23/6$); $Q_e = 11$, $Q_\mu = -6$, $Q_\tau = -3$ for the case-II ($E/N = 1/2$); $Q_e = 11$, $Q_\mu = -6$, $Q_\tau = -3$ for the case-III ($E/N = 5/2$). The corresponding Yukawa terms are expressed in terms of Eqs. (31) and (40) used in the quark sector as

$$ y_e = \hat{y}_e \nabla^{11}_\psi, \quad y_\mu = \hat{y}_\mu \nabla^{6}_\psi, \quad y_\tau = \hat{y}_\tau \nabla^{3}_\psi, \\
(49)$$

where $\nabla_\psi = \nabla_{\tilde{\psi}}$ in SUSY limit is used. And the hat Yukawa couplings $\hat{y}_{e,\mu,\tau}$ are fixed\[15\] as $\hat{y}_e = 0.793152$, $\hat{y}_\mu = 1.137250$, $\hat{y}_\tau = 0.968747$ by using the numerical values of Eq. (17) in quark sector via the empirical results $m_e = 0.511$ MeV, $m_\mu = 105.683$ MeV, and $m_\tau = 1776.86$ MeV \[29\].

III. SCALE OF PQ PHASE TRANSITION AND QCD AXION PROPERTIES

The couplings of the flavored-axions and the mass of the QCD axion are inversely proportional to the PQ symmetry breaking scale. In a theoretical view of Refs.\[1, 7, 17\], the scale of PQ symmetry breakdown congruent to that of the seesaw mechanism can push the scale much beyond the electroweak scale, rendering the flavored-axions very weakly interacting particles. Since the weakly coupled flavored-axions (one linear combination QCD

\[15\] The charged lepton sector, in common with the quark sector, has VEV corrections in Eq. (17) and the hat Yukawa couplings are corrected.
axion and its orthogonal ALP) could carry away a large amount of energy from the interior of stars, according to the standard stellar evolution scenario their couplings should be bounded with electrons\textsuperscript{16}, photons, and nucleons. Hence, such weakly coupled flavored-axions have a wealth of interesting phenomenological implications in the context of astroparticle physics\textsuperscript{[1, 7]}, like the formation of a cosmic diffuse background of axions from the Sun\textsuperscript{[35, 36]}; from evolved low-mass stars, such as red-giants and horizontal-branch stars in globular clusters\textsuperscript{[37, 38]}, or white dwarfs\textsuperscript{[39, 40]}; from neutron stars\textsuperscript{[41]}; and from the duration of the neutrino burst of the core-collapse supernova SN1987A\textsuperscript{[42]} as well as the rare flavor changing decay processes induced by the flavored-axions $K^+ \rightarrow \pi^+ A_i$\textsuperscript{[15, 43]} and $\mu \rightarrow e + \gamma + A_i$\textsuperscript{[43, 45]} etc..

Such flavored-axions could be produced in hot astrophysical plasmas, thus transporting energy out of stars and other astrophysical objects, and they could also be produced by the rare flavor changing decay processes. Actually, the coupling strength of these particles with normal matter and radiation is bounded by the constraint that stellar lifetimes and energy-loss rates\textsuperscript{[46]} as well as the branching ratios for the $\mu$ and $K$ flavor changing decays\textsuperscript{[15, 45]} should not be counter to observations. Interestingly enough, the recent observations also show a preference for extra energy losses in stars at different evolutionary stages - red giants, supergiants, helium core burning stars, white dwarfs, and neutron stars (see Ref.\textsuperscript{[13]} for the summary of extra cooling observations and Ref.\textsuperscript{[1]} on the interpretation to a bound of the QCD axion decay constant); the present experimental limit, $\text{Br}(K^+ \rightarrow \pi^+ A_i) < 7.3 \times 10^{-11}$\textsuperscript{[15]}, puts a lower bound on the axion decay constant, and in the near future the NA62 experiment expected to reach the sensitivity $\text{Br}(K^+ \rightarrow \pi^+ A_i) < 1.0 \times 10^{-12}$\textsuperscript{[47]} will probe the flavored-axions or put a severe bound on the QCD axion decay constant $F_A$ (or flavored-axion decay constants $F_{a_i} = f_{a_i}/\delta_i^G$). According to the recent investigation in Ref.\textsuperscript{[1, 7]}, the flavored-axions (QCD axion and its orthogonal ALP) would provide very good hints for a new physics model for quarks and leptons. Fortunately, in a framework of the flavored-PQ symmetry the cooling anomalies hint at an axion coupling to electrons, photons, and neutrons, which should not conflict with the current upper bound on the rare $K^+ \rightarrow \pi^+ A_i$ decay. Remark that once a scale of PQ symmetry breakdown is fixed the other is automatic including the QCD axion decay constant and the mass scale of heavy neutrino

\textsuperscript{16} The second ($\mu$) and third ($\tau$) generation particles are absent in almost all astrophysical objects.
associated to the seesaw mechanism.

In order to fix the QCD axion decay constant $F_A$ (or flavored-axion decay constants $F_{a_i} = f_{a_i}/\delta_i^G$), we will consider two tight constraints coming from astro-particle physics: axion cooling of stars via bremsstrahlung off electrons and flavor-violating processes induced by the flavored-axions.

### A. Flavored-Axion cooling of stars via bremsstrahlung off electrons

In the so-called flavored-axion framework, generically, the SM charged lepton fields are non-trivially $U(1)_X$-charged Dirac fermions, and thereby the flavored-axion coupling to electrons are added to the Lagrangian through a chiral rotation.

In the present model since the flavored-axion $A_2$ couples directly to electrons, the axion can be emitted by Compton scattering, atomic axio-recombination and axio-deexcitation, and axio-bremsstrahlung in electron-ion or electron-electron collision [37]. The flavored-axion $A_2$ coupling to electrons in the model reads

$$g_{Aee} = \frac{X_e m_e}{\sqrt{2} \delta_2^G F_A}$$

where $m_e = 0.511$ MeV, $F_A = f_{a_i}/\sqrt{2} \delta_i^G$, $\delta_2^G = -3X_2$ and $X_e = -11X_2$. Indeed, the longstanding anomaly in the cooling of WDs (white dwarfs) and RGB (red giants branch) stars in globular clusters where bremsstrahlung off electrons is mainly efficient [39] could be explained by axions with the fine-structure constant of axion to electrons $\alpha_{Aee} = (0.29 - 2.30) \times 10^{-27}$ [48] and $\alpha_{Aee} = (0.41 - 3.70) \times 10^{-27}$ [40, 49], indicating the clear systematic tendency of stars to cool faster than predicted. It is recently reexamined in Ref. [13] as Eq. (1) where $\alpha_{Aee} = g_{Aee}^2/4\pi$, which is interpreted in terms of the QCD axion decay constant in the present model as

$$5.02 \times 10^9 \lesssim F_A[\text{GeV}] \lesssim 4.4 \times 10^{10}. \quad (51)$$

### B. Flavor-Changing process $K^+ \rightarrow \pi^+ + A_i$ induced by the flavored-axions

Below the QCD scale ($1 \text{ GeV} \approx 4\pi f_\pi$), the chiral symmetry is broken and $\pi$ and $K$, and $\eta$ are produced as pseudo-Goldstone bosons. Since a direct interaction of the SM gauge singlet flavon fields charged under $U(1)_X$ with the SM quarks charged under $U(1)_X$ can
arise through Yukawa interaction, the flavor-changing process $K^+ \to \pi^+ + A_i$ is induced by the flavored-axions $A_i$. Then, the flavored-axion interactions with the flavor violating coupling to the $s$- and $d$-quark is given by

$$\mathcal{L}^{A_{sd}}_Y \simeq i \left( \frac{|X_1| A_1}{2 f_{a_1}} - \frac{|X_2| A_2}{f_{a_2}} \right) \bar{s} d (m_s - m_d) \lambda \left( 1 - \frac{\lambda^2}{2} \right),$$  \hspace{1cm} (52)

where $V_L^{d\dagger} = V_{\text{CKM}}$, $f_{a_1} = |X_1| v_x$, and $f_{a_2} = |X_2| v_y$ are used. Then the decay width of $K^+ \to \pi^+ + A_i$ is given by

$$\Gamma(K^+ \to \pi^+ + A_i) = \frac{m_{K^\pm}^3}{16 \pi} \left( 1 - \frac{m_{\pi}^2}{m_K^2} \right)^3 |\mathcal{M}_{dsi}|^2,$$  \hspace{1cm} (53)

where $m_{K^\pm} = 493.677 \pm 0.013$ MeV, $m_{\pi \pm} = 139.57018(35)$ MeV [50], and

$$|\mathcal{M}_{ds1}|^2 = \left| \frac{X_1}{2\sqrt{2}\delta^G_{sd}} F_A (1 - \frac{\lambda^2}{2}) \right|^2, \hspace{1cm} |\mathcal{M}_{ds2}|^2 = \left| \frac{X_2}{\sqrt{2}\delta^G_{sd}} F_A (1 - \frac{\lambda^2}{2}) \right|^2,$$  \hspace{1cm} (54)

where $F_A = f_{a_1}/(\delta^G_{sd}\sqrt{2})$ is used. From the present experimental upper bound in Eq. (14), $\text{Br}(K^+ \to \pi^+ A_i) < 7.3 \times 10^{-11}$, with $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu}) = 1.73^{+1.15}_{-1.05} \times 10^{-10}$ [51], we obtain the lower limit on the QCD axion decay constant

$$F_A \gtrsim 2.72 \times 10^{10} \text{GeV}.$$  \hspace{1cm} (55)

Hence, from Eqs. (51) and (55) we can obtain a strongest bound on the QCD axion decay constant

$$F_A = 3.56^{+0.84}_{-0.84} \times 10^{10} \text{GeV}.$$  \hspace{1cm} (56)

Interestingly enough, from Eqs. (47) and (56) the scale $\Lambda = 3F_A/(\sqrt{2}\nabla_\psi)$ responsible for the FN mechanism can be determined

$$\Lambda = 2.04^{+0.48}_{-0.48} \times 10^{11} \text{GeV}.$$  \hspace{1cm} (57)

In the near future the NA62 experiment will be expected to reach the sensitivity of $\text{Br}(K^+ \to \pi^+ A_i) < 1.0 \times 10^{-12}$ [47], which is interpreted as the flavored-axion decay constant and its corresponding QCD axion decay constant

$$f_{a_1} > 9.86 \times 10^{11} \text{GeV} \Leftrightarrow F_A > 2.32 \times 10^{11} \text{GeV}.$$  \hspace{1cm} (58)

Clearly, the NA62 experiment will probe the flavored-axions or exclude the present model.

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17 In the standard parametrization the mixing elements of $V^d_R$ are given by $\theta^R_{23} \simeq A\lambda^2(\nabla_\eta/\kappa^2\nabla_\phi) |\hat{y}_s/\hat{y}_b|$, $\theta^R_{13} \simeq ABL^5 |\sin \phi_d| |\hat{y}_{de}/(\hat{Y}_{s1} + 3\kappa^2\hat{Y}_{s2})| (2\nabla_\eta/3\kappa^2\nabla_\phi)$, and $\theta^R_{12} \simeq 2\sqrt{2} |\sin \phi_d| \lambda^2$. Its effect to the flavor violating coupling to the $s$- and $d$-quark is negligible: $(V^d_R \text{Diag.}(-4A_1^{\hat{v}_f}, -4A_1^{\hat{v}_f}, 0) V^d_R)_{12} = 0$ at leading order.
C. QCD axion interactions with nucleons

Below the chiral symmetry breaking scale, the axion-hadron interactions are meaningful (rather than the axion-quark interactions) for the axion production rate in the core of a star where the temperature is not as high as 1 GeV, which is given by [17]

$$\mathcal{L}^{a-\psi_N} = \frac{\partial_{\mu}a}{2F_A} \bar{\psi}_N \gamma_{\mu} \gamma^5 \psi_N$$

(59)

where $a$ is the QCD axion, its decay constant is given by

$$F_A = f_A/N$$

and

$$f_A = \sqrt{2} \delta_1 N = \sqrt{2} \delta_1 f_{a_1} = \sqrt{2} \delta_2 f_{a_2},$$

and $\psi_N$ is the nucleon doublet $(p, n)^T$ (here $p$ and $n$ correspond to the proton field and neutron field, respectively). Recently, the couplings of the axion to the nucleon are very precisely extracted as [52]

$$X_p = -0.47(3) + 0.88(3) \frac{\tilde{X}_u}{N} - 0.39(2) \frac{\tilde{X}_d}{N} - 0.038(5) \frac{\tilde{X}_s}{N}$$

$$-0.012(5) \frac{\tilde{X}_c}{N} - 0.009(2) \frac{\tilde{X}_b}{N} - 0.0035(4) \frac{\tilde{X}_t}{N},$$

(60)

$$X_n = -0.02(3) + 0.88(3) \frac{\tilde{X}_d}{N} - 0.39(2) \frac{\tilde{X}_u}{N} - 0.038(5) \frac{\tilde{X}_s}{N}$$

$$-0.012(5) \frac{\tilde{X}_c}{N} - 0.009(2) \frac{\tilde{X}_b}{N} - 0.0035(4) \frac{\tilde{X}_t}{N},$$

(61)

where $N = 2\delta_1 \delta_2$ with $\delta_1 = 2X_1$ and $\delta_2 = -3X_2$, and $\tilde{X}_q = \delta_2 X_{1q} + \delta_1 X_{2q}$ with $q = u, d, s$ and $X_{1u} = X_1, X_{1d} = X_1, X_{1s} = 0, X_{1c} = 0, X_{1b} = 0, X_{1t} = 0, X_{2u} = -4X_2, X_{2d} = -X_2, X_{2s} = X_2, X_{2c} = -2X_2, X_{2b} = 3X_2, X_{2t} = 0$. And the QCD axion coupling to the neutron is written as

$$g_{Ann} = \frac{|X_n|m_n}{F_A},$$

(62)

where the neutron mass $m_n = 939.6$ MeV. The state-of-the-art upper limit on this coupling, $g_{Ann} < 8 \times 10^{-10}$ [53], from the neutron star cooling is interpreted as the lower bound of the QCD axion decay constant

$$F_A > 9.53 \times 10^7 \text{GeV}.$$  

(63)

Clearly, the strongest bound on the QCD axion decay constant comes from the flavored-axion cooling of stars via bremsstrahlung off electrons in Eq. (51) as well as the flavor-changing process $K^+ \rightarrow \pi^+ + A_i$ induced by the flavored-axions in Eq. (55).
Using the state-of-the-art calculation in Eq. (61) and the QCD axion decay constant in Eq. (56), we can obtain
\[ g_{Ann} = 2.14^{+0.66}_{-0.41} \times 10^{-12}, \] (64)
which is incompatible with the hint for extra cooling from the neutron star in the supernova remnant “Cassiopeia A” by axion neutron bremsstrahlung, \( g_{Ann} = 3.74^{+0.62}_{-0.74} \times 10^{-10} \).

This huge discrepancy may be explained by considering other means in the cooling of the superfluid core in the neutron star, for example, by neutrino emission in pair formation in a multicomponent superfluid state \( ^3P_2 (m_j = 0, \pm 1, \pm 2) \).

D. QCD axion mass and its interactions with photons

With the well constrained QCD axion decay constant in Eq. (56) congruent to the seesaw scale we can predict the QCD axion mass and its corresponding axion-photon coupling.

As in Refs. [1, 17], the axion mass in terms of the pion mass and pion decay constant is obtained as
\[ m_a^2 F_A^2 = m_{\pi^0}^2 f_\pi^2 F(z, w), \] (65)
where
\[ f_\pi = 92.21(14) \text{ MeV} \] and
\[ F(z, w) = \frac{z}{(1+z)(1+z+w)} \quad \text{with} \quad z = \frac{m_{\pi^0}^{\text{MS}}(2\text{GeV})}{m_a^{\text{MS}}(2\text{GeV})} = 0.48(3) \quad \text{and} \quad \omega = 0.315 z. \] (66)

Note that the Weinberg value lies in \( 0.38 < z < 0.58 \) [29, 56]. After integrating out the heavy \( \pi^0 \) and \( \eta \) at low energies, there is an effective low energy Lagrangian with an axion-photon coupling \( g_{a\gamma\gamma} \):
\[ \mathcal{L}_{a\gamma\gamma} = \frac{1}{4} g_{a\gamma\gamma} a_{\text{phys}} F^{\mu\nu} \tilde{F}_{\mu\nu} = -g_{a\gamma\gamma} a_{\text{phys}} \vec{E} \cdot \vec{B}, \] (67)
where \( \vec{E} \) and \( \vec{B} \) are the electromagnetic field components. And the axion-photon coupling can be expressed in terms of the QCD axion mass, pion mass, pion decay constant, \( z \) and \( w \):
\[ g_{a\gamma\gamma} = \frac{\alpha_{\text{em}}}{2\pi f_\pi m_{\pi^0}} \sqrt{F(z, w)} \frac{1}{N} \left( \frac{E}{3} - \frac{4 + z + w}{3 (1 + z + w)} \right). \] (68)

\[ ^{18} \text{Here} \ F(z, \omega) \text{can be replaced in high accuracy as in Ref. [52] by} \ F(z) = \frac{z}{(1+z)^2} \left\{ 1 + 2 \frac{m_{\pi^0}^2}{f_\pi^2} \left( h_r + \frac{z^2 - 6 z + 1}{(1+z)^2} l_r \right) \right\}, \] where \( h_r = (4.8 \pm 1.4) \times 10^{-3} \) and \( l_r = 7(4) \times 10^{-3} \).
The upper bound on the axion-photon coupling is derived from the recent analysis of the horizontal branch (HB) stars in galactic globular clusters (GCs) [57], which translates into the lower bound of decay constant through Eq. (65), as

\[ |g_{a\gamma\gamma}| < 6.6 \times 10^{-11} \text{ GeV}^{-1} \quad (95\% \text{ CL}) \iff F_A \gtrsim \begin{cases} 3.23 \times 10^7 \text{ GeV} & \text{case-I} \\ 2.64 \times 10^7 \text{ GeV}, & \text{case-II} \\ 8.84 \times 10^6 \text{ GeV}, & \text{case-III} \end{cases} \quad (69) \]

where in the right side \( E/N = 23/6 \) (case-I), 1/2 (case-II), 5/2 (case-III) for \( z = 0.48 \) are used. Subsequently, the bound in Eq. (69) translates into the upper bound of axion mass through Eq. (65) as

\[ m_a \lesssim \begin{cases} 1.42 \times 10^9 \text{ GeV} & \text{case-I} \\ 1.16 \times 10^9 \text{ GeV}, & \text{case-II} \\ 3.89 \times 10^8 \text{ GeV}, & \text{case-III} \end{cases} \quad (70) \]

The bounds of Eqs. (69) and (70) are much lower than that of Eq. (56) coming from the present experimental upper bound \( \text{Br}(K^+ \rightarrow \pi^+ A_i) < 7.3 \times 10^{-11} \) [15] as well as the axion to electron coupling \( 6.7 \times 10^{-29} \lesssim \alpha_{Aee} \lesssim 5.6 \times 10^{-27} \) at 3σ [13].

Hence, from Eqs. (56) and (65) the QCD axion mass and its corresponding axion-photon couplings are model predicted for \( z = 0.48 \):

\[ m_a = 1.54^{+0.48}_{-0.29} \times 10^{-4} \text{ eV} \iff |g_{a\gamma\gamma}| = \begin{cases} 5.99^{+1.85}_{-1.14} \times 10^{-14} \text{ GeV}^{-1}, & \text{case-I} \\ 4.89^{+1.51}_{-0.93} \times 10^{-14} \text{ GeV}^{-1}, & \text{case-II} \\ 1.64^{+0.51}_{-0.31} \times 10^{-14} \text{ GeV}^{-1}, & \text{case-III} \end{cases} \quad (71) \]

Note here that, if 0.38 < \( z < 0.58 \) is considered for the given axion mass range, the ranges of \( |g_{a\gamma\gamma}| \) in Eq. (71) can become wider than those for \( z = 0.48 \). The corresponding Compton wavelength of axion oscillation is

\[ \lambda_a = (2\pi\hbar/m_a)c \quad \text{with} \quad c \simeq 2.997 \times 10^8 \text{ m/s and} \quad \hbar \simeq 1.055 \times 10^{-34} \text{ J s}; \]

\[ \lambda_a = 8.04^{+1.99}_{-1.90} \text{ mm}. \quad (72) \]
The QCD axion coupling to photon $g_{a\gamma\gamma}$ divided by the QCD axion mass $m_a$ is dependent on $E/N$. Fig. 1 shows the $E/N$ dependence of $(g_{a\gamma\gamma}/m_a)^2$ so that the experimental limit is independent of the axion mass $m_a$ [17]: for $0.38 < z < 0.58$, the value of $(g_{a\gamma\gamma}/m_a)^2$ for the case-II and -III are located lower than that of the ADMX (Axion Dark Matter eXperiment) bound [12], while for the case-I is marginally lower than that of the ADMX bound, where $(g_{a\gamma\gamma}/m_a)^2_{\text{ADMX}} \leq 1.44 \times 10^{-19} \text{GeV}^{-2} \text{eV}^{-2}$. The gray-band represents the experimentally excluded bound $(g_{a\gamma\gamma}/m_a)^2_{\text{ADMX}}$, while the cyan-band curve stands for $0.38 < z < 0.58$. For the Weinberg value $z = 0.48^{+0.10}_{-0.10}$, the anomaly values $E/N = 23/6$, 1/2, and 5/2 predict $(g_{a\gamma\gamma}/m_a)^2 = 1.507^{+0.126}_{-0.137} \times 10^{-19} \text{GeV}^{-2} \text{eV}^{-2}$ (case-I), $1.003^{+0.382}_{-0.368} \times 10^{-19} \text{GeV}^{-2} \text{eV}^{-2}$ (case-II), and $1.128^{+0.163}_{-0.252} \times 10^{-20} \text{GeV}^{-2} \text{eV}^{-2}$ (case-III), respectively. Clearly, as shown in Fig. 1, the uncertainties of $(g_{a\gamma\gamma}/m_a)^2$ for the case-II and -III are larger than that of case-I for $0.38 < z < 0.58$.

Fig. 2 shows the plot for the axion-photon coupling $|g_{a\gamma\gamma}|$ as a function of the axion mass.
FIG. 2: Plot of $|g_{a\gamma\gamma}|$ versus $m_a$ for the case-I (slanted red-solid line), case-II (slanted blue dashed line), and case-III (slanted black-dotted line) in terms of $E/N = 23/6, 1/2$ and $5/2$, respectively. Especially, the QCD axion mass $m_a = 1.54^{+0.48}_{-0.29} \times 10^{-4}$ eV is equivalent to the axion photon coupling $|g_{a\gamma\gamma}| = 5.99^{+1.85}_{-1.14} \times 10^{-14}$ GeV$^{-1}$ (horizontal light-red band), $4.89^{+1.51}_{-0.93} \times 10^{-14}$ GeV$^{-1}$ (horizontal light-blue band), and $1.64^{+0.51}_{-0.31} \times 10^{-14}$ GeV$^{-1}$ (horizontal light-black band), which corresponds to the case-I, -II, and -III, respectively.

$m_a$ in terms of anomaly values $E/N = 23/6, 1/2, 5/2$ which correspond to the case-I, -II, and -III, respectively. Especially, in the model, for $F_A = 3.56^{+0.84}_{-0.29} \times 10^{10}$ GeV and $z = 0.48$ we obtain the QCD axion mass $m_a = 1.54^{+0.48}_{-0.29} \times 10^{-4}$ eV and the axion photon coupling $|g_{a\gamma\gamma}| = 5.99^{+1.85}_{-1.14} \times 10^{-14}$ GeV$^{-1}$ (horizontal light-red band), $4.89^{+1.51}_{-0.93} \times 10^{-14}$ GeV$^{-1}$ (horizontal light-blue band), and $1.64^{+0.51}_{-0.31} \times 10^{-14}$ GeV$^{-1}$ (horizontal light-black band), which corresponds to the case-I, -II, and -III, respectively. As the upper bound on Br($K^+ \to \pi^+ + A_i$) gets tighter, the range of the QCD axion mass gets more and more narrow, and consequently the corresponding band width on $|g_{a\gamma\gamma}|$ in Fig. 2 is getting narrower. In Fig. 2 the top edge of the bands comes from the upper bound on Br($K^+ \to \pi^+ + A_i$), while the bottom of the bands is from the astrophysical constraints of star cooling induced by the flavored-axion bremsstrahlung off electrons $e + Ze \to Ze + e + A_i$.

The model will be tested in the very near future through the experiment such as CAPP (Center for Axion and Precision Physics research) as well as the NA62 experiment expected to reach the sensitivity of Br($K^+ \to \pi^+ + A_i$) $< 1.0 \times 10^{-12}$. 

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IV. SUMMARY AND CONCLUSION

Motivated by the flavored PQ symmetry for unifying the flavor physics and string theory [1, 16], we have constructed a compact model based on $SL_2(F_3) \times U(1)_X$ symmetry for resolving rather recent, but fast-growing issues in astro-particle physics, including quark and leptonic mixings and CP violations, high-energy neutrinos, QCD axion, and axion cooling of stars. Since astro-particle physics observations have increasingly placed tight constraints on parameters for flavored-axions, we have showed how the scale responsible for PQ mechanism (congruent to that of seesaw mechanism) could be fixed, and in turn the scale responsible for FN mechanism through flavor physics. Along the lines of finding the fundamental scales, the flavored-PQ symmetry together with the non-Abelian finite symmetry is well flavor-structured in a unique way that domain-wall number $N_{DW} = 1$ with the $U(1)_X \times [\text{gravity}]^2$ anomaly-free condition demands additional Majorana fermion and the flavor puzzles of SM are well delineated by new expansion parameters expressed in terms of $U(1)_X$ charges and $U(1)_X-[SU(3)_C]^2$ anomaly coefficients, providing interesting physical implications on neutrino, QCD axion, and flavored-axion.

In the concrete, the QCD axion decay constant congruent to the seesaw scale, through its connection to the astro-particle constraints of stellar evolution induced by the flavored-axion bremsstrahlung off electrons $e+Ze \rightarrow Ze+e+A_i$ and the rare flavor-changing decay process induced by the flavored-axion $K^+ \rightarrow \pi^+ + A_i$, is shown to be fixed at $F_A = 3.56^{+0.84}_{-0.84} \times 10^{10}$ GeV (consequently, the QCD axion mass $m_a = 1.54^{+0.48}_{-0.29} \times 10^{-4}$ eV, wavelength of its oscillation $\lambda_a = 8.04^{+1.90}_{-1.90}$ mm, axion to neutron coupling $g_{A\text{nn}} = 2.14^{+0.66}_{-0.41} \times 10^{-12}$, and axion to photon coupling $|g_{a\gamma\gamma}| = 5.99^{+1.85}_{-1.14} \times 10^{-14}$ GeV$^{-1}$ for $E/N = 23/6$ (case-I), $4.89^{+1.51}_{-0.93} \times 10^{-14}$ GeV$^{-1}$ for $E/N = 1/2$ (case-II), $1.64^{+0.51}_{-0.31} \times 10^{-14}$ GeV$^{-1}$ for $E/N = 5/2$ (case-III), respectively, in the case $z = 4.8$). Subsequently, the scale associated to FN mechanism is automatically fixed through its connection to the SM fermion masses and mixings, $\Lambda = 2.04^{+0.48}_{-0.48} \times 10^{11}$ GeV, and such fundamental scale might give a hint where some string moduli are stabilized in type-IIB string vacua.

We may conclude that in an extended SM framework by a compact symmetry $G_F = SL_2(F_3) \times U(1)_X$, if the scale responsible for the FN mechanism (whose scale is associated to some string moduli stabilization) is fixed, the scales responsible for seesaw and PQ mechanisms are dynamically determined in way that the SM fermion (including neutrino) masses...
and mixings are well delineated, which in turn provides predictions on several properties of the flavored-axions.

In the very near future, the NA62 experiment expected to reach the sensitivity of $\text{Br}(K^+ \rightarrow \pi^+ + A_i) < 1.0 \times 10^{-12}$ will probe the flavored-axions or exclude the model.
Appendix A: The $SL_2(F_3)$ group

The $SL_2(F_3)$ is the double covering of the tetrahedral group $A_4[18, 20, 21]$. It contains 24 elements and has three kinds of representations: one triplet $3$ and three singlets $1, 1'$ and $1''$, and three doublets $2, 2'$ and $2''$. The representations $1', 1''$ and $2', 2''$ are complex conjugated to each other. Note that $A_4$ is not a subgroup of $SL_2(F_3)$, since the two-dimensional representations cannot be decomposed into representations of $A_4$. The generators $S$ and $T$ satisfy the required conditions $S^2 = R$, $T^3 = 1$, $(ST)^3 = 1$, and $R^2 = 1$, where $R = 1$ in case of the odd-dimensional representation and $R = -1$ for $2, 2'$ and $2''$ such that $R$ commutes with all elements of the group. The matrices $S$ and $T$ representing the generators depend on the representations of the group [21]:

$$
S = \begin{cases} 
1 & T = 1 \\
1' & T = \omega \\
1'' & T = \omega^2 \\
2 & T = \omega A_1 \\
2' & T = \omega^2 A_1 \\
2'' & T = A_2 \\
3 & S = \frac{1}{3} \begin{pmatrix} 
-1 & 2\omega & 2\omega^2 \\
2\omega^2 & -1 & 2\omega \\
2\omega & 2\omega^2 & -1 
\end{pmatrix} \\
& T = \begin{pmatrix} 
1 & 0 & 0 \\
0 & \omega & 0 \\
0 & 0 & \omega^2 
\end{pmatrix}
\end{cases}
$$

where we have used the matrices

$$
A_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 
i & \sqrt{2}e^{i\pi/12} \\
-\sqrt{2}e^{-i\pi/12} & -i 
\end{pmatrix}, \quad A_2 = \begin{pmatrix} 
\omega & 0 \\
0 & 1 
\end{pmatrix}.
$$

The following multiplication rules between the various representations are calculated in Ref. [21], where $\alpha_i$ indicate the elements of the first representation of the product and $\beta_i$ indicate those of the second representation. Moreover $a, b = 0, \pm 1$ and we denote $1^0 \equiv 1$, $1^1 \equiv 1'$, $1^{-1} \equiv 1''$ and similarly for the doublet representations. On the right-hand side the sum $a + b$ is modulo 3.

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The multiplication rules with the 1-dimensional representations are the following:

\[ 1 \otimes \text{Rep} = \text{Rep} \otimes 1 = \text{Rep} \quad \text{with} \quad \text{Rep} \text{ whatever representation} \]

\[ 1^a \otimes 1^b = 1^b \otimes 1^a = 1^{a+b} \equiv \alpha \beta \]

\[ 1^a \otimes 2^b = 2^b \otimes 1^a = 2^{a+b} \equiv (\alpha \beta_1, \alpha \beta_2) \]

\[ 1' \otimes 3 = 3 = (\alpha \beta_3, \alpha \beta_1, \alpha \beta_2), \quad 1'' \otimes 3 = 3 = (\alpha \beta_2, \alpha \beta_3, \alpha \beta_1). \]

The multiplication rules with the 2-dimensional representations are

\[ 2 \otimes 2 = 2' \otimes 2'' = 2'' \otimes 2' = 3 \oplus 1 \]

with \[ 3 = \left( \frac{1 - i}{2} (\alpha_1 \beta_2 + \alpha_2 \beta_1), \ i \alpha_1 \beta_1, \ \alpha_2 \beta_2 \right), \quad 1 = \alpha_1 \beta_2 - \alpha_2 \beta_1; \]

\[ 2 \otimes 2' = 2'' \otimes 2'' = 3 \oplus 1' \]

with \[ 3 = \left( \alpha_2 \beta_2, \ \frac{1 - i}{2} (\alpha_1 \beta_2 + \alpha_2 \beta_1), \ i \alpha_1 \beta_1 \right), \quad 1' = \alpha_1 \beta_2 - \alpha_2 \beta_1; \]

\[ 2 \otimes 2'' = 2' \otimes 2' = 3 \oplus 1'' \]

with \[ 3 = \left( i \alpha_1 \beta_1, \ \alpha_2 \beta_2, \ \frac{1 - i}{2} (\alpha_1 \beta_2 + \alpha_2 \beta_1) \right), \quad 1'' = \alpha_1 \beta_2 - \alpha_2 \beta_1; \]

\[ 2 \otimes 3 = 2 \oplus 2' \oplus 2'' \]

with \[ 2 = \left( 1 + i \alpha_2 \beta_2 + \alpha_1 \beta_1, \ 1 - i \alpha_1 \beta_3 - \alpha_2 \beta_1 \right) \]

\[ 2' = \left( 1 + i \alpha_2 \beta_3 + \alpha_1 \beta_2, \ 1 - i \alpha_1 \beta_1 - \alpha_2 \beta_2 \right) \]

\[ 2'' = \left( 1 + i \alpha_2 \beta_1 + \alpha_1 \beta_3, \ 1 - i \alpha_1 \beta_2 - \alpha_2 \beta_3 \right); \]

\[ 2' \otimes 3 = 2 \oplus 2' \oplus 2'' \]

with \[ 2 = \left( 1 + i \alpha_2 \beta_1 + \alpha_1 \beta_3, \ 1 - i \alpha_1 \beta_2 - \alpha_2 \beta_3 \right) \]

\[ 2' = \left( 1 + i \alpha_2 \beta_2 + \alpha_1 \beta_1, \ 1 - i \alpha_1 \beta_3 - \alpha_2 \beta_1 \right) \]

\[ 2'' = \left( 1 + i \alpha_2 \beta_3 + \alpha_1 \beta_2, \ 1 - i \alpha_1 \beta_1 - \alpha_2 \beta_2 \right); \]

\[ 2'' \otimes 3 = 2 \oplus 2' \oplus 2'' \]

with \[ 2 = \left( 1 + i \alpha_2 \beta_3 + \alpha_1 \beta_2, \ 1 - i \alpha_1 \beta_1 - \alpha_2 \beta_2 \right) \]

\[ 2' = \left( 1 + i \alpha_2 \beta_1 + \alpha_1 \beta_3, \ 1 - i \alpha_1 \beta_2 - \alpha_2 \beta_3 \right) \]

\[ 2'' = \left( 1 + i \alpha_2 \beta_2 + \alpha_1 \beta_1, \ 1 - i \alpha_1 \beta_3 - \alpha_2 \beta_1 \right). \]
The multiplication rule with the 3-dimensional representations is
\[ 3 \otimes 3 = 3_S \oplus 3_A \oplus 1 \oplus 1' \oplus 1'' \]
where
\[ 3_S = \frac{1}{3} \left( 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2, \ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1, \ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \right) \]
\[ 3_A = \frac{1}{2} \left( \alpha_2\beta_3 - \alpha_3\beta_2, \ \alpha_1\beta_2 - \alpha_2\beta_1, \ \alpha_3\beta_1 - \alpha_1\beta_3 \right) \]
\[
\begin{align*}
1 &= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\
1' &= \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1 \\
1'' &= \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 .
\end{align*}
\]

Appendix B: Higher order corrections to vacuum configuration

We consider possible next-to-leading order corrections to the vacuum configuration in Eq. (9). Higher-dimensional operators induced by \( \Phi_T, \Phi_S, \Theta, \Psi, \eta \) invariant under \( SL_2(F_3) \times U(1)_X \) symmetry, suppressed by additional powers of the cutoff scale \( \Lambda \), could be added to the leading order terms in the driving superpotential \( W_v \). They can lead to small deviations from the leading order vacuum configurations. The next leading order superpotential \( \delta W_v \), which is linear in the driving fields and invariant under \( SL_2(F_3) \times U(1)_X \times U(1)_R \), is given by

\[
\delta W_v = \frac{1}{\Lambda} \left\{ a_1(\Phi_T\Phi_T)_{3s}(\Phi_T\Phi_T^T)_{3a} + a_2(\Phi_T\Phi_T)_{1}(\Phi_T\Phi_T^T)_{1} + a_3(\Phi_T\Phi_T)_{1'}(\Phi_T\Phi_T^T)_{1''} \\
+ a_4(\Phi_T\Phi_T)_{1'}(\Phi_T\Phi_T^T)_{1'} + a_5\Psi\tilde{\Psi}(\Phi_T\Phi_T^T)_{1} + a_6(\eta\Phi_T)_{2}(\eta\Phi_T^T)_{2} + a_7(\eta\Phi_T)_{2'}(\eta\Phi_T^T)_{2''} \right\} \\
+ \frac{1}{\Lambda} \left\{ b_1(\Phi_S\Phi_S)_{3a}(\Phi_T\Phi_T^T)_{3a} + b_2(\Phi_S\Phi_S)_{3a}(\Phi_T\Phi_T^T)_{3s} + b_3(\Phi_S\Phi_S)_{1}(\Phi_T\Phi_T^T)_{1} \\
+ b_4(\Phi_S\Phi_S)_{1}(\Phi_T\Phi_T^T)_{1'} + b_5(\Phi_S\Phi_S)_{1'}(\Phi_T\Phi_T^T)_{1'} + b_6(\Phi_S\Phi_T)_{3s} \Theta \\
+ b_7(\Phi_0^S)(\Phi_S\Phi_T)_{3s} \tilde{\Theta} + b_8(\Phi_0^S)(\Phi_S\Phi_T)_{3a} \tilde{\Theta} + b_9(\Phi_0^S)(\Phi_S\Phi_T)_{3s} \tilde{\Theta} \\
+ b_{10}(\Phi_0^S)(\Phi_T\Phi_T^T)_{1} \Theta \Theta + b_{11}(\Phi_0^S)(\Phi_T\Phi_T^T)_{1} \Theta \tilde{\Theta} + b_{12}(\Phi_0^S)(\Phi_T\Phi_T^T)_{1} \tilde{\Theta} \tilde{\Theta} \right\} \\
+ \frac{\Theta_0}{\Lambda} \left\{ c_1(\Phi_S\Phi_S)_{3s} \Phi_T + c_2(\Phi_S\Phi_T)_{1} \tilde{\Theta} \right\} + \frac{\Psi_0}{\Lambda} d_1(\Phi_T\Phi_T)_{3s} \Phi_T \\
+ \frac{1}{\Lambda} \left\{ f_1(\eta\eta)_{3}(\eta\eta_0)_{3} + f_2(\Phi_T\Phi_T)_{3s}(\eta\eta_0)_{3} + f_3(\Phi_T\Phi_T)_{1}(\eta\eta_0)_{1} + f_4(\Psi\tilde{\Psi})(\eta\eta_0)_{1} \right\} .
\]

(B1)
The corrections to the VEVs, Eq. (9), are of relative order $1/\Lambda$ and affect the flavon fields $\Phi_S$, $\Phi_T$, $\Theta$, $\bar{\Theta}$, $\eta$ and $\Psi$, and the vacuum configuration can be modified with relations among the dimensionless parameters $(a_1...a_7, b_1...b_{12}, c_1, c_2, d_1, f_1...f_4)$. By keeping only the first order in the expansion, the minimization equations become

$$\delta v_\Phi + \frac{d_1}{\sqrt{2}} \nabla_\Phi \left( \frac{\mu_\Phi}{\sqrt{2}} + 3\delta v_{T_1} \right) = 0,$$

(B2)

$$\delta v_\eta \frac{A_\eta \sqrt{2} V_T}{\nabla_\eta} + \delta v_{T_1} B_\eta + \delta v_\phi f_4 \sqrt{2} \nabla_\phi = -v_T A_\eta,$$

$$\delta v_\eta \frac{3}{\sqrt{2}} f_1 \nabla_\eta - \delta v_{T_1} \nabla_\eta \eta_{21} - \delta v_{T_3} \nabla_\eta \eta_{22} = -\frac{f_{1\eta}}{2} \nabla_\eta,$$

(B3)

$$\delta v_S T_{11} + (\delta v_S + \delta v_S) \Theta_{22} + \left( g_4 \delta v_\phi + \frac{g_2}{2} \delta v_\phi \right) \sqrt{-6 \frac{g_2}{g_4}} = 0,$$

(B4)

$$\delta v_{T_1} T_{11} + \delta v_{T_1} T_{12} + \delta v_\phi T_{13} = C_\eta v_T,$$

$$\delta v_{T_1} T_{31} + \delta v_{T_2} T_{31} = 0,$$

$$\delta v_{T_2} = 0,$$

(B5)

$$\delta v_{S_1} S_{11} + (\delta v_{S_1} + \delta v_{S_3}) S_{12} + \delta v_{T_1} S_{13} + \delta v_{T_3} S_{14} + \delta v_\phi S_{15} + \delta v_\phi S_{16} = -\frac{\nu_\phi}{\sqrt{2}} S_{13},$$

$$\delta v_{S_1} S_{21} + \delta v_{S_2} S_{22} + \delta v_{S_3} S_{23} + \delta v_{T_1} S_{24} + \delta v_{T_3} S_{25} + \delta v_\phi S_{26} + \delta v_\phi S_{27} = -\frac{\nu_\phi}{\sqrt{2}} S_{24},$$

$$\delta v_{S_1} S_{31} + \delta v_{S_2} S_{32} + \delta v_{S_3} S_{33} + \delta v_{T_1} S_{34} + \delta v_{T_3} S_{35} + \delta v_\phi S_{36} + \delta v_\phi S_{37} = -\frac{\nu_\phi}{\sqrt{2}} S_{34},$$

(B6)

where the parameters $A_\eta, B_\eta, C_\eta, \eta_{ij}, \Theta_{ij}, T_{ij}, S_{ij}$ are dimensionless:

$$A_\eta = \frac{1-i}{6} f_2 \nabla_T + \frac{f_4}{\sqrt{2}} \nabla_T + \frac{f_3}{\sqrt{2}} \nabla_T,$$

$$B_\eta = \frac{5(1-i)}{12\sqrt{2}} g_\eta + \frac{g_4(1-i)}{3} f_4 \nabla_T + \sqrt{2} f_3 \nabla_T,$$

$$C_\eta = -a_2 \frac{\nabla_T}{\sqrt{2}} - \frac{a_7(1-i)}{\sqrt{2}} \frac{\nabla_T}{\sqrt{2}} - \frac{a_6}{\sqrt{2}} \nabla_T,$$

$$\eta_{21} = \frac{5(1-i)}{6\sqrt{2}} g_\eta + \frac{g_4(1-i)}{6} f_4 \nabla_T - \frac{f_3}{\sqrt{2}} \nabla_T - \frac{f_3}{\sqrt{2}} \nabla_T,$$

$$\eta_{22} = \frac{i}{3} \left( \frac{5}{\sqrt{2}} g_\eta - \sqrt{2} f_2 \nabla_T \right),$$

$$\Theta_{11} = \sqrt{2} g_3 + \frac{2\sqrt{2}}{3} c_1 \nabla_T,$$

$$\Theta_{12} = \sqrt{2} g_3 - \frac{2\sqrt{2}}{3} c_1 \nabla_T,$$

$$T_{11} = \frac{\nabla_T}{\sqrt{2}} g - \frac{g_\eta}{\sqrt{2}} \frac{\nabla_T}{\sqrt{2}} + \frac{3}{\sqrt{2}} a_6 \nabla_T + \frac{1-i}{\sqrt{2}} a_7 \nabla_T + \frac{g_4(1-i)}{\sqrt{2}},$$

$$T_{12} = \frac{\nabla_T}{\sqrt{2}} (\sqrt{2} (1-i) a_7 \nabla_T + i \sqrt{2} g_3),$$

$$T_{13} = a_5 \sqrt{2} \nabla_T,$$

$$T_{31} = -\frac{2\sqrt{2}}{3} g - \frac{g_\eta}{\sqrt{2}} \frac{\nabla_T}{\sqrt{2}} - \frac{a_5 \nabla_T}{3} + \frac{3}{\sqrt{2}} a_7 \nabla_T + a_4 \sqrt{2} \nabla_T + \frac{1-i}{\sqrt{2}} a_6 \nabla_T - \frac{1-i}{\sqrt{2}} a_7 \nabla_T + \frac{a_6}{\sqrt{2}} \nabla_T,$$

$$S_{11} = 2 \frac{\nabla_T}{\sqrt{2}} g_1 + \frac{2\sqrt{2}}{3} b_3 \nabla_T + b_3 \sqrt{2} \nabla_T + \frac{2b_3}{3} \nabla_T,$$

$$S_{12} = -\frac{\nabla_T}{3} + \frac{b_3 \sqrt{2}}{3} \nabla_T + b_3 \sqrt{2} \nabla_T,$$

$$S_{13} = \nabla_T \left( \frac{3b_4 \sqrt{2}}{3} + \frac{\sqrt{2} b_5}{3} \nabla_T + \frac{b_{10}}{\sqrt{2}} \right),$$

$$S_{14} = \nabla_T \left( \frac{3b_4 \sqrt{2}}{3} + \frac{b_6}{\sqrt{2}} \nabla_T \right),$$

$$S_{15} = \nabla_T \left( \frac{b_4 \sqrt{2}}{3} + \frac{b_{10} \sqrt{2}}{\sqrt{2}} \right),$$

(B7)
\[ S_{16} = \frac{\sqrt{3}g_1}{\sqrt{\kappa}} \nabla_T + \frac{b_1}{\sqrt{\kappa}} \nabla_T + \frac{g_3}{\sqrt{\kappa}}, \quad S_{21} = -\frac{\sqrt{3}}{3} g_1 - \frac{b_1}{3 \sqrt{\kappa}} \nabla_T + \frac{b_2}{3 \sqrt{\kappa}} \nabla_T + b_5 \sqrt{2} \nabla_T, \]
\[ S_{22} = \frac{2\sqrt{2}}{3} g_1 + \frac{b_1 \sqrt{2}}{3} \nabla_T - \frac{b_2 \sqrt{2}}{3} \nabla_T + b_5 \sqrt{2} \nabla_T, \quad S_{23} = S_{21} + \nabla_T \left( \frac{b_6}{2} - \frac{b_7}{3} \right), \]
\[ S_{24} = \nabla_S \left\{ \frac{3b_3}{\sqrt{2}} + \nabla_T \frac{1}{\sqrt{\kappa}} \left( \frac{b_6}{2} - \frac{b_7}{3} \right) \right\}, \quad S_{25} = \nabla_S \left\{ \frac{3b_3}{\sqrt{2}} - \frac{1}{\sqrt{\kappa}} \left( \frac{b_6}{2} + \frac{b_7}{3} \right) + \frac{b_{10}}{\sqrt{\kappa}} \right\}, \]
\[ S_{26} = \nabla_T \left( \frac{b_6}{2 \sqrt{2}} + \frac{b_7}{3 \sqrt{2}} \right), \quad S_{27} = -\frac{b_6}{2 \sqrt{2}} \nabla_T - \frac{b_7}{3 \sqrt{2}} \nabla_T + \frac{g_2}{\sqrt{2}}, \]
\[ S_{31} = -\frac{\sqrt{3}}{3} g_1 + \frac{b_1}{3 \sqrt{2}} \nabla_T + \frac{b_2}{3 \sqrt{2}} \nabla_T + b_4 \sqrt{2} \nabla_T, \quad S_{32} = S_{31} + \nabla_T \left( \frac{b_6}{2} + \frac{b_7}{3} \right), \]
\[ S_{33} = \frac{2\sqrt{2}}{3} g_1 - \frac{b_1 \sqrt{2}}{3} \nabla_T - \frac{b_2 \sqrt{2}}{3} \nabla_T + b_4 \sqrt{2} \nabla_T, \]
\[ S_{34} = \nabla_S \left\{ \frac{3b_3}{\sqrt{2}} - \frac{1}{\sqrt{\kappa}} \left( \frac{b_6}{2} + \frac{b_7}{3} \right) \right\}, \quad S_{35} = \nabla_S \left( \frac{3b_3}{\sqrt{2}} + \frac{b_7 \sqrt{2}}{3} \right), \]
\[ S_{36} = -\nabla_T \left( \frac{b_6}{2 \sqrt{2}} + \frac{b_7}{3 \sqrt{2}} \right), \quad S_{37} = \frac{b_6}{2 \sqrt{2}} \nabla_T - \frac{b_7}{3 \sqrt{2}} \nabla_T + \frac{g_2}{\sqrt{2}}. \]  

In Eqs. (B7) and (B8) \( v_S/v_\Theta = \kappa \) with \( 1/\kappa = \sqrt{-3g_3/g_4} \) is used. From Eqs. (B2), (B3), and (B5) one can obtain the modified vacuum configurations

\[ \langle \Phi_T \rangle \rightarrow \left( \frac{\nu_T}{\sqrt{2}} + \delta \nu_T, 0, \delta \nu_T \right), \quad \langle \eta \rangle \rightarrow \left( \pm \frac{\nu_\eta}{\sqrt{2}}, \delta \nu_\eta, \delta \nu_\eta \right), \quad \langle \Psi \rangle \rightarrow \frac{\nu_\Psi}{\sqrt{2}} + \delta \nu_\Psi. \]  

And from Eqs. (B4) and (B6), with \( \delta v_{S_2} = -\delta v_{S_3} \) one can obtain

\[ \langle \Phi_S \rangle \rightarrow \left( \frac{\nu_S}{\sqrt{2}} + \delta \nu_{S_1}, \frac{\nu_S}{\sqrt{2}} + \delta \nu_{S_2}, \frac{\nu_S}{\sqrt{2}} - \delta \nu_{S_2} \right), \quad \langle \Theta \rangle \rightarrow \frac{\nu_\Theta}{\sqrt{2}} + \delta \nu_\Theta, \quad \langle \hat{\Theta} \rangle \rightarrow \delta \hat{\Theta}. \]  

Given the values for \( \nabla Q \) with \( Q = \eta, S, T, \Theta, \Psi \) in Eq. (47), one can expect that the shifts \( |\delta \hat{\Theta}|, |\delta v_\Theta|/v_\Theta, |\delta v_{S_1}|/v_S, |\delta \nu_T|/\nu_T, |\delta \nu_\eta|/\nu_\eta, |\delta \nu_\Psi|/\nu_\Psi \) can be kept small enough, below a few percent level.

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