Interpreting experimental bounds on $D^0 - \bar{D}^0$ mixing in the presence of CP violation

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We analyse the most recent experimental data regarding $D^0 - \bar{D}^0$ mixing, allowing for CP violation. We focus on the dispersive part of the mixing amplitude, $M_{12}^D$, which is sensitive to new physics contributions. We obtain a constraint on the mixing amplitude: $|M_{12}^D| \leq 6.2 \times 10^{-11}$ MeV at 95% C.L. This constraint is weaker by a factor of about three than the one which is obtained when no CP violation is assumed.

I. INTRODUCTION

The ongoing searches for $D^0 - \bar{D}^0$ mixing [1, 2, 3, 4, 5, 6, 7, 8] have not yet detected a signal of such mixing. Thus, the experimental data place an upper bound on the mixing amplitude. The value of this upper bound, however, depends on the assumptions one makes when analysing the experimental results. Specifically, the question of CP violation in $D^0 - \bar{D}^0$ mixing has an important impact on the final answer. Most often, $D^0 - \bar{D}^0$ mixing experiments are analysed assuming no CP violation. While this assumption is valid for the standard model, it does not hold for many new physics models.1 (See, for example, the supersymmetric models in [10, 11].) Obviously, if the constraint on $D^0 - \bar{D}^0$ mixing is to be used to test such new physics models, the experimental data should be interpreted in an appropriate framework [12]. We therefore present here the analysis of experimental results

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1 In fact, since it has been recently suggested that the $D^0 - \bar{D}^0$ mixing amplitude may be large even in the standard model [9], CP violation may be the most valuable clue for new physics in this system.
allowing for CP violation in mixing.\textsuperscript{2}

The organization of this work is as follows: In section \textbf{II} we present our formalism. We review the most recent experimental data in section \textbf{III}, and perform the analysis in section \textbf{IV}. We conclude in section \textbf{V}.

\section{II. NOTATION AND FORMALISM}

We follow mostly the formalism of ref. \cite{14}. The mass eigenstates are given by

\[ |D_{1,2}\rangle = p |D^0\rangle \pm q |\bar{D}^0\rangle , \]

(1)

The mass and the width differences are parameterized as follows:

\[ x \equiv \frac{m_2 - m_1}{\Gamma} , \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma} , \]

(2)

with the average mass and width defined as

\[ m \equiv \frac{m_1 + m_2}{2} , \quad \Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2} . \]

(3)

We define the $D^0$ and $\bar{D}^0$ decay amplitudes by

\[ A_f \equiv \langle f | \mathcal{H}_d | D^0 \rangle , \quad \bar{A}_f \equiv \langle f | \mathcal{H}_d | \bar{D}^0 \rangle , \]

(4)

and the complex observable $\lambda_f$ as

\[ \lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} . \]

(5)

In almost all models, CP violation in decay of the relevant modes can be safely neglected \cite{13,14} (for an exception, see \cite{15}), leading to

\[ A_f = \bar{A}_f . \]

(6)

Now we can parametrize the effects of indirect CP violation in the relevant decay processes: The doubly-Cabibbo-suppressed (DCS) $D^0 \rightarrow K^+\pi^-$, the singly-Cabibbo-suppressed (SCS) $D^0 \rightarrow K^K^-$, the Cabibbo-favoured (CF) $D^0 \rightarrow K^-\pi^+$, and the

\textsuperscript{2} We do not consider, however, CP violation in $D^0$ decays, which is absent in most new physics extensions \cite{13}.
three conjugate processes. We denote

$$|q/p|^2 = 1 + 2A_m,$$  \hspace{1cm} (7) 

$$\lambda_{K^+\pi^-}^{-1} = \sqrt{R_D} (1 - A_m) e^{-i(\delta + \phi)} ,$$  \hspace{1cm} (8) 

$$\lambda_{K^-\pi^+} = \sqrt{R_D} (1 + A_m) e^{-i(\delta - \phi)} ,$$  \hspace{1cm} (9) 

$$\lambda_{K^+K^-} = -(1 + A_m) e^{i\phi} ,$$  \hspace{1cm} (10) 

where the $\phi$ and $\delta$ are the weak phase and the strong phase, respectively, and

$$R_D = \left| \frac{A_{K^+\pi^-}}{A_{K^-\pi^+}} \right|^2 = \left| \frac{\bar{A}_{K^-\pi^+}}{A_{K^-\pi^+}} \right|^2.$$  \hspace{1cm} (11) 

Next we define

$$x' \equiv x \cos \delta + y \sin \delta,$$  \hspace{1cm} (12) 

$$y' \equiv y \cos \delta - x \sin \delta.$$ 

The rates of the DCS, SCS and CF decays are expanded for short times $t \lesssim 1/\Gamma$ as

$$\Gamma[D^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} \left| A_{K^-\pi^+} \right|^2 
\times \left[ R_D + \sqrt{R_D} (1 + A_m)(y' \cos \phi - x' \sin \phi) \Gamma t + \frac{1 + 2A_m}{4}(y^2 + x^2)(\Gamma t)^2 \right] ,$$  \hspace{1cm} (13) 

$$\Gamma[D^0(t) \rightarrow K^-\pi^+] = e^{-\Gamma t} \left| A_{K^-\pi^+} \right|^2 
\times \left[ R_D + \sqrt{R_D} (1 - A_m)(y' \cos \phi + x' \sin \phi) \Gamma t + \frac{1 - 2A_m}{4}(y^2 + x^2)(\Gamma t)^2 \right] ,$$  \hspace{1cm} (14) 

$$\Gamma[D^0(t) \rightarrow K^+K^-] = e^{-\Gamma t} \left| A_{K^+K^-} \right|^2 \times \left[ 1 - (1 + A_m)(y' \cos \phi - x' \sin \phi) \Gamma t \right] ,$$  \hspace{1cm} (15) 

$$\Gamma[D^0(t) \rightarrow K^-\pi^+] = \Gamma[D^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} \left| A_{K^-\pi^+} \right|^2.$$

Several experiments measure the parameter $y_{CP}$, defined by

$$y_{CP} = \frac{\tau(D^0 \rightarrow K^-\pi^+)}{\tau(D^0 \rightarrow K^+K^-)} - 1 ,$$  \hspace{1cm} (17)

[3] Note that $A_m$ in our definition is twice smaller than the $A_m$ used by CLEO [2].
with $\tau$ being the measured lifetime fitted to a pure exponential decay rate for the specific modes \cite{[1]} \cite{[4]}. If CP is a good symmetry in the relevant processes, this definition of $y_{CP}$ corresponds to

$$y_{CP} \equiv \frac{\Gamma(\text{CP even}) - \Gamma(\text{CP odd})}{\Gamma(\text{CP even}) + \Gamma(\text{CP odd})},$$

since then the $K^+K^-$ state is an even CP state and the $K^-\pi^+$ state is an equal mixture of CP even and CP odd states. By fitting the decay rates in (13) and (14) to exponents, and expanding for small $A_m$ we get \cite{[4]}:

$$y_{CP} = y \cos \phi - A_m x \sin \phi .$$

We are interested in the dispersive part of the mixing amplitude, $M_{12}^D$: Short distance contribution from new physics can affect $M_{12}^D$ in a significant way. In terms of measurable quantities, $|M_{12}^D|$ is given by \cite{[13]}

$$|M_{12}^D|^2 = \frac{4(\Delta m)^2 + A_m^2(\Delta \Gamma)^2}{16(1 - A_m^2)},$$

or, using eq. (2),

$$|M_{12}^D|^2 = \frac{\Gamma^2 x^2 + A_m^2 y^2}{4(1 - A_m^2)} .$$

III. EXPERIMENTAL DATA ON $D^0 - \overline{D^0}$ MIXING

The neutral $D$ system is studied by various experiments. First, the CLEO experiment \cite{[2]} measures the rates (13), (14):

$$R_D = (0.48 \pm 0.13)\% ,$$

$$y' \cos \phi = (-2.5^{+1.4}_{-1.6})\% ,$$

$$x' = (0.0 \pm 1.5)\% ,$$

$$2A_m = 0.23^{+0.63}_{-0.80} ,$$

$$\sin \phi = 0.00 \pm 0.60 .$$

The FOCUS experiment \cite{[3]} provides a measurement of the ratio between the branching ratio of the DCS and CF decays. This measurement is consistent with CLEO data at the
level of $\sim 0.8\sigma$. However, as no direct measurement of the parameters is done, no stronger bounds on the parameters result.

The value of $y_{CP}$ is measured by the various experiments. Table I presents the various results. The world weighted average of $y_{CP}$ is hence:

$$y_{CP} = (1.0 \pm 0.7)\%.$$ (23)

### IV. INTERPRETATION OF THE EXPERIMENTAL DATA

Our aim is to constrain the $D^0 - \bar{D}^0$ mixing amplitude $M_{12}^D$. First we combine (12) and (19) to get

$$y_{CP} + A_m \sin \phi (x' \cos \delta + y' \sin \delta) = y' \cos \phi \cos \delta - x' \cos \phi \sin \delta.$$ (24)

The measured values of (22) and (23) can be used to constrain $\cos \delta$. Assuming first $A_m = 0$ and also $|\sin \phi| \approx 0$ we find

$$(1.0 \pm 0.7)\% = (-2.5^{+1.4}_{-1.6})\% \cos \delta - (0.0 \pm 1.5)\% \sin \delta,$$ (25)

which implies a certain distribution for $\cos \delta$. Due to the sign difference between $y_{CP}$ and $y'$ and due to the relative smallness of $x'$ it is expected that this distribution of $\cos \delta$ will be biased to negative values. By a full analysis, considering the measured values of $A_m$ and $\sin \phi$ we can characterize the bias by stating the total confidence level value:

$$\cos \delta \lesssim 0.7 \quad (95\% \text{ C.L.}),$$ (26)

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4 A similar procedure was followed in ref. [14].
TABLE II: Comparison between mass and width difference parameters at 95% C.L. with different assumption on mixing parameters.

| Assuming $\cos \delta = 1, \cos \phi = 1$ | No assumption |
|------------------------------------------|---------------|
| $|x| \lesssim 2.9\%$                       | $|x| \lesssim 6.3\%$ |
| $-5.8\% \lesssim y \lesssim 1.0\%$      | $|y| \lesssim 4.6\%$ |

(and $\cos \delta \lesssim 0.0$ at 68% C.L.).

Since we have now a distribution for $x'$, $y'$ and $\cos \delta$, we may invert (12) to solve for $x$ and $y$:

$$x = x' \cos \delta - y' \sin \delta,$$

$$y = y' \cos \delta + x' \sin \delta .$$

We note that the signs of $x$ and $y$ in (27) are not measured by current experimental results. Since the measured value for $x'$ is distributed around zero the sign for $y$ is determined by the sign of $y'$ which, in turn, depends on the sign of $\cos \phi$. This sign is not provided by any measurement (all we know is that $|\cos \phi| \approx 1$). Similarly, the sign of $x$ is determined by the sign of both $y'$ and $\sin \delta$, which are not measured.

The resulting distributions for $x$ and $y$ are therefore in the form of two superimposed distributions for the two possible sign choices (denoted by the $\pm$ sign). We obtain:

$$x \approx (\pm 2.8 \pm 2.5)\% ,$$

$$y \approx (\pm 0.9 \pm 3.6)\% .$$

We note that these values are different from those quoted in [17] where it is assumed that $\delta = \phi = 0$. When we consider the distribution of $\cos \delta$, the bound on $x$ (and hence the bound on $\Delta m_D$) is weakened by a factor of about 2.2. The bound on $y$, however, (and hence the bound on $\Delta \Gamma$) is strengthened. For comparison, table I shows the 95% C.L. ranges for $x$ and $y$ in the two cases: One which assumes $\cos \delta = 1$ and $\cos \phi = 1$, and one which takes the values mentioned.

We evaluate now the $D^0 - \bar{D}^0$ mixing amplitude. Taking the average decay width [17]

$$\Gamma_D = (1.595 \pm 0.011) \times 10^{-9} \text{ MeV} ,$$

(29)
and using (21), we obtain a distribution for $M_{12}$ which is maximal near zero:

$$|M_{12}^D| \leq 6.2 \times 10^{-11} \text{ MeV} \quad (95\% \text{ C.L.}) ,$$

(30)

(and $|M_{12}^D| \leq 3.3 \times 10^{-11} \text{ MeV at 68% C.L.}$).

It is interesting to compare this value to the ones obtained by using some simplifying assumptions. First, assuming no CP violation in mixing, we set $A_m = 0$ but allow for $\delta, \phi \neq 0$. We get

$$|M_{12}^D| \leq 5.4 \times 10^{-11} \text{ MeV} \quad (95\% \text{ C.L.}) .$$

(31)

Second, we set $A_m = \phi = 0$ and allow $\delta \neq 0$. We get

$$|M_{12}^D| \leq 4.0 \times 10^{-11} \text{ MeV} \quad (95\% \text{ C.L.}) .$$

(32)

Third, we set $\delta = 0$, but allow $A_m, \phi \neq 0$. We get

$$|M_{12}^D| \leq 3.9 \times 10^{-11} \text{ MeV} \quad (95\% \text{ C.L.}) .$$

(33)

Last, we set $A_m = \phi = \delta = 0$ and get\(^5\)

$$|M_{12}^D| \leq 2.3 \times 10^{-11} \text{ MeV} \quad (95\% \text{ C.L.}) .$$

(34)

This is the value which appears in [17]. Thus, allowing CP violation, the resulting constraint is about 2.7 weaker (i.e. larger) than the one which is obtained with the maximal set of assumptions.

V. CONCLUSIONS.

We interpret the most recent data from the experimental searches for $D^0 - \bar{D}^0$ mixing. Allowing CP violation in mixing, we obtain the upper bound

$$|M_{12}^D| \leq 6.2 \times 10^{-11} \text{ MeV} \quad (95\% \text{ C.L.}) ,$$

(35)

which is 2.7 times weaker than the naive calculation.

\(^5\)Actually, it is enough to assume $A_m = \delta = 0$ since, in this case, the value of $\phi$ affects only $y$, which does not contribute to $M_{12}^D$. 

The actual upper bound for $D^0 - \bar{D}^0$ mixing amplitude depends, therefore, on the model in question. Assuming that CP is conserved in $D^0 - \bar{D}^0$ mixing, as is the case in the standard model, the bound is the one in (32). (If, in addition, one is willing to assume that $SU(3)$-flavour symmetry holds in $D$ decays, the bound is given by (34).) For a more general model, with new sources of CP violation, eq. (35) gives the present bound. Taking into account this weaker bound leads to modifications \cite{18} compared to analyses that consider only the CP conserving bound \cite{13, 20, 21}.

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