Features of Motion Around Charged D-Stars

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(Dated: June 11, 2018)

The motion of light and a neutral test particle around the charged D-star has been studied. The difference of the deficit angle of light from the case in asymptotically flat spacetime is in a factor $(1 - \epsilon^2)$. The motion of a test particle is affected by the deficit angle and the charge. Through the phase analysis, we prove the existence of the periodic solution to the equation of motion and the effect of the deficit angle and the charge to the critical point and its type. We also give the conditions under which the critical point is a stable center and an unstable saddle point.

PACS numbers: 11.10.Lm, 04.40.-b

1. Introduction

Astronomical observations on the CMB anisotropy[1-3] and the relation between red-shift and luminosity distance of SNeIa [4-6] depicted that our universe is spatially flat, and about 23 percent and 73 percent of energy density is resulted respectively from cold dark matter and dark energy, respectively. The nature of these substances are quite unusual and there is no justification for assuming that it resembles known forms of matter or energy. On the other hand, phase transitions of quantum fields in the early Universe may produce various kinds of topological defects. The idea that point-like defects known as monopoles ought to exist has proved to be remarkably durable. The global monopole, which has divergent mass in flat space is one of the most important defects. The effects of gravity on the global monopole were firstly studied by Barriola and Vilenkin [7]. When gravity is taken into account the linearly divergent mass of a global monopole has an effect analogous to that of a deficit solid angle plus that of a tiny mass at the origin. It is shown that this small gravitational potential is actually repulsive [8-12].

As possible candidates for the cold dark matter, various kinds of cold stars such as Q-stars have been proposed [13-22]. Later, a new class of cold stars named D-stars (effect stars) have been proposed by Li et al. [23,24]. Compared to Q-stars, the D-stars have a feature, that is, in the absence of the gravitational field the theory has monopole solutions, which makes the D-stars behave very differently from the Q-stars. Recently, these new objects have farther been studied. Their possible interesting astrophysical applications [25,26] and charged D-stars have also been studied. In contrast to ordinary D-stars, there exists a U(1) gauge field outside the charged D-stars [27].

In this paper, we will discuss the behavior of test particle and geodesics in the exterior spacetime of the charged D-stars. With the aid of phase-plane analysis, we carefully analyse the motion of light and a test particle around a charged D-stars and show some unique features of the behavior.

2. The charged D-Stars

To be specific, we shall work with a fixed model, where a global $O(3) \times U_{em}(1)$ symmetry is broken down to $O(3) \times U_{em}(1)$. The Lagrangian density is (we work in units such that $\hbar = c = 1$)[27],

$$L = g^{\mu\nu} D_\mu \phi D_\nu \phi^* + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma^a \partial_\nu \sigma^a - \eta^2 \sigma^a \sigma^a \phi \phi^* - \eta^2 (\phi \phi^*)^2 - \frac{\lambda^2}{8} (\sigma^a \sigma^a - \sigma_0^2)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

(1)

where $D_\mu = \partial_\mu + ieA_\mu$; $F_{\mu\nu}$ is the electric-magnetic field vector, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; $\sigma^a$ is a triplet of scalar fields, isovector index $a = 1, 2, 3$; $\phi$ is a complex scalar field with the $U_{em}(1)$ symmetry $\phi \to e^{-i\theta} \phi$. The field configuration describing a global monopole is

$$\sigma^a = \sigma_0 f(\rho) \frac{x^a}{\rho}, \text{with} x^a x^a = \rho^2$$

(2)
so that we will actually have a monopole solution if \( f \to 1 \) at spatial infinity. There is the current conservation \( j^\mu = 0 \), where

\[
j^\mu = ig^{\mu\nu}(\phi^* \frac{\partial \phi}{\partial x^\nu} - \frac{\partial \phi^*}{\partial x^\nu} \phi)
\]

(3)

This leads to the conservation of the electric charge

\[
Q = \int j^0 \sqrt{-\text{det} g} d^3x
\]

(4)

Introduce

\[
\phi(\rho, t) = \frac{1}{\sqrt{2}} \sigma_0 h(\rho) e^{i\sigma_0 \omega t}
\]

(5)

so that \( f \) and \( h \) are both dimensionless and real. As the lowest energy solution of the theory, there should not exist electric current in and around the charged D-stars, so there should not exist the magnetic field. Therefore, the nonzero component of \( A_\mu \) is \( A_0 \), which can be chosen as

\[
A_0(\rho) = \frac{\sigma_0}{e}[\omega - g(\rho)]
\]

(6)

The general static metric with spherical symmetry can be written as

\[
ds^2 = B(\rho)d\tau^2 - A(\rho)d\rho^2 - \rho^2 (d\theta^2 + \sin^2 \theta d\varphi^2)
\]

(7)

with the usual relation between the spherical coordinates \( \rho, \theta, \varphi \) and the "Cartesian" coordinates \( x^a \). Introducing a dimensionless \( r \equiv \sigma_0 \rho \), from the Lagrangian density (1) and the definitions for \( f, h \) and \( g \), the scalar field equations can be obtained as

\[
f'' + \left( \frac{2}{r} + \frac{B'}{2B} - \frac{A'}{2A} \right) f' = A f \left[ \frac{2}{r^2} + \eta^2 h^2 + \frac{1}{2} \lambda (f^2 - 1) \right]
\]

(8)

\[
h'' + \left( \frac{2}{r} - \frac{B'}{2B} - \frac{A'}{2A} \right) h' = A h \left[ \eta^2 f^2 + \eta^2 h^2 - \frac{1}{B} g^2 \right]
\]

the equation for electric-magnetic field is

\[
g'' + \left( \frac{2}{r} - \frac{B'}{2B} - \frac{A'}{2A} \right) g' = A e^2 h^2 g
\]

(9)

where the prime denotes differentiation with respect to \( r \), and \( A(r) \) and \( B(r) \) are the metric fields. By expressing the energy-momentum tensor of the system, one can reduce Einstein equation: \( G_{\mu\nu} = 8\pi G T_{\mu\nu} \).

The charged D-stars consists of three regions: an interior surface and exterior, which can be discussed respectively [27]. For considering the exterior region of the star, one formally solves the Einstein equations of the static spherically symmetric metric as follows [28]

\[
A^{-1} = 1 - \frac{8\pi G}{\rho} \int_{\rho}^{\rho_*} T_{\mu}^{\mu} \rho^2 d\rho
\]

(10)

\[
B = \frac{1}{A(\rho)} \exp \left[ 8\pi G \int_{\rho}^{\rho_*} (T_{\mu}^{\mu} - T_{\rho}^{\rho}) A(\rho) \rho d\rho \right]
\]

where the time coordinate has been rescaled so that \( B = A^{-1} \) as \( \rho \to \infty \). In terms of the dimensionless variable \( r \) and another dimensionless quantity \( \epsilon \), the Einstein equation can be formally integrated and the solutions read as

\[
A^{-1}(r) = 1 - \epsilon^2 - \frac{2G\sigma_0 M_A(r)}{r} + \frac{G\sigma_0^2 Q_A^2(r)}{4\pi r^2}
\]
\[ B(r) = 1 - \epsilon^2 - \frac{2G\sigma_0 M_B(r)}{r} + \frac{G\sigma_0^2 Q_B^2(r)}{4\pi r^2} \] 

(11)

where

\[ M_A(r) = 4\pi\sigma_0 \exp[-\triangle(r)] \int_0^r dr \exp[\triangle(r)] \times \left\{ f^2 - 1 + r^2 \left[ \frac{1}{2B} g^2 h^2 + \frac{1}{8} \lambda^2 (f^2 - 1)^2 + \eta^2 \left( \frac{1}{2} f^2 h^2 + \frac{1}{4} h^4 \right) \right] + \frac{1}{2} r^2 (1 - \epsilon^2 + \frac{G\sigma_0^2 Q_A^2}{4\pi r^2}) (h'^2 + f'^2) \right\} \]

\[ Q_A^2 = 16\pi^2 r \exp[-\triangle_1(r)] \int_0^r r^2 dr \exp[\triangle_1(r)] (1 - \epsilon^2 - \frac{2G\sigma_0 M_A}{r}) \frac{g'}{e^2 B} \] 

(12)

\[ M_B(r) = M_A(r) \exp[\tilde{\triangle}(r)] + \frac{r(1 - \epsilon^2)}{2G\sigma_0} \{ 1 - \exp[\tilde{\triangle}(r)] \} \]

and

\[ Q_B^2 = Q_A^2 \exp[\tilde{\triangle}(r)] \]

in which

\[ \triangle(r) = \frac{\epsilon^2}{2} \int_0^r dz (h'^2 + f'^2) z \]
\[ \triangle_1(r) = \frac{\epsilon^2}{2} \int_0^r dr \frac{g'}{e^2 B} r \]
\[ \tilde{\triangle}(r) = \epsilon^2 \int_0^\infty dz (h'^2 + f'^2 + 2A \frac{1}{2B} g^2 h^2) z \]

If this convergence is fast enough in asymptotic spacetime, \( M_A(r) \) and \( M_B(r) \) will also quickly converge to finite values. Therefore, one can find the asymptotic expansions:

\[ f(r) = 1 - \frac{1}{r^2} - \frac{3 - 2\epsilon^2}{2r^3} + O(r^{-6}) \]

\[ M_A(r) = M_A + \frac{4\pi\sigma_0}{r} + O(r^{-3}) \] 

(14)

\[ M_B(r) = M_A(r) (1 - \epsilon^2) \frac{4\pi\sigma_0 (1 - \epsilon^2)}{r^3} + O(r^{-7}) \]

Since the effective mass \( M_A(r) \) approaches very quickly its asymptotic value \( M_A \), it is a good approximation to take it as

\[ M_A(r) = M_A + \frac{4\pi\sigma_0}{r} \] 

(15)

Therefore, to investigate the motion of light and a test particle around a charged D-star, we can take the metric coefficients as

\[ A(\rho)^{-1} = B(\rho) = 1 - \epsilon^2 - \frac{2G\tilde{M}}{\rho} + \frac{2GQ^2}{\rho^2} \] 

(16)
where, for convenience, we have rescaled \( \tilde{M} = \sigma_0 M_A \) and \( \tilde{Q}^2 = \sigma_0^2 \frac{Q_A^2 - 32\pi}{8\pi} \).

3. The behavior of null geodesic outside the charged D-stars:

The orbit of light outside of the charged D-star can be obtained by solving the geodesic equation

\[
\frac{\, \! d^2 x^\rho}{d\tau^2} + \Gamma^\rho_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0
\]  

(17)

where \( \tau \) is the affine parameter. But by using the fact that \( g_{ab}\frac{dx^a}{d\tau}\frac{dx^b}{d\tau} = 0 \) for null geodesics and the constant of motion

\[
E = \mathcal{B}(r)\frac{dt}{d\tau}
\]  

(18)

and

\[
L = r^2\frac{d\phi}{d\tau}
\]  

(19)

one can, considering the motion confined on the \( \theta = \frac{\pi}{2} \) plane, easily obtain the equation for geodesics as[29].

\[
\frac{1}{2} \dot{r}^2 + \frac{1}{2} \mathcal{B}(r)\frac{L^2}{r^2} = \frac{1}{2} E^2
\]  

(20)

Introducing the effective potential

\[
V_{eff} = \frac{1}{2} \mathcal{B}(r)\frac{L^2}{r^2} = \frac{L^2(1 - \epsilon^2)}{2r^4} - \frac{L^2G\tilde{M}}{r^3} + \frac{GL^2\tilde{Q}^2}{r^4}
\]  

(21)

the geodesics of light becomes the same as that of a neutral test particle with unit mass moving in the effective potential [30]. Taking \( G = 1 \) for convenience, it is easy to find that the effective potential has a maximum

\[
V_{\text{max}} = \frac{4L^2(1 - \epsilon^2)^3}{2} \left[ 3\tilde{M}^2 - 4\tilde{Q}^2(1 - \epsilon^2) + \tilde{M}\sqrt{9\tilde{M}^2 - 16\tilde{Q}^2(1 - \epsilon^2)} \right]^{\frac{1}{2}}
\]  

(22)

at

\[
r = r_c = \frac{3\tilde{M} + \sqrt{9\tilde{M}^2 - 16\tilde{Q}^2(1 - \epsilon^2)}}{2(1 - \epsilon^2)}
\]  

(23)

When \( \frac{1}{2} E^2 = V_{\text{max}} \), we will have

\[
b_{\text{crit}} = \frac{3\tilde{M} + \sqrt{9\tilde{M}^2 - 16\tilde{Q}^2(1 - \epsilon^2)}}{2\sqrt{2}(1 - \epsilon^2)^{\frac{3}{2}}}
\]  

(24)

where \( b_{\text{crit}} \) is the critical value of \( b \), which is the generalization of the apparent impact parameter and is defined as \( b = \frac{L}{E} \). The capture cross section of the charged D-star will be

\[
\sigma_c = \pi b_{\text{crit}}^2 = \frac{\pi \left[ 3\tilde{M} + \sqrt{9\tilde{M}^2 - 16\tilde{Q}^2(1 - \epsilon^2)} \right]^4}{8(1 - \epsilon^2)^3}\left[ 3\tilde{M}^2 - 4\tilde{Q}^2(1 - \epsilon^2) + \tilde{M}\sqrt{9\tilde{M}^2 - 16\tilde{Q}^2(1 - \epsilon^2)} \right]
\]  

(25)
It is not difficult to prove that the path of light has a turning point at the largest radius, \( R_0 \), where \( \frac{d\varphi}{dr}|_{r=R_0} = 0 \). Using Eqs.(18)-(20), one can obtain the equation which can give the relation between \( R_0 \) and \( b \)

\[
b^{-2}R_0^4 - (1 - \epsilon^2)R_0^2 + 2\bar{M}R_0 - 2\bar{Q}^2 = 0
\]

(26)

In order to compute the deflect angle of the light, we rewrite the Eqs.(18)-(20) as

\[
\frac{d\varphi}{dr} = \left[ r^4b^{-2} - (1 - \epsilon^2)r^2 + 2\bar{M}r - 2\bar{Q}^2 \right]^{-1/2}
\]

(27)

The change of light when passing a charged D-star should be \( \Delta\varphi = \varphi_\infty - \varphi_{-\infty} \). Considering the symmetry, we have

\[
\Delta\varphi = 2\int_{R_0}^{\infty} dr \left[ r^4b^{-2} - (1 - \epsilon^2)r^2 + 2\bar{M}r - 2\bar{Q}^2 \right]^{1/2}
\]

(28)

The deflection of light up to the first order of \( \bar{M} \) (\( \bar{M} \) is small in the unit \( G = 1 \)) is given by [29]:

\[
\delta\varphi = \Delta\varphi - \pi \approx \bar{M} \left. \frac{\partial(\Delta\varphi)}{\partial\bar{M}} \right|_{\bar{M}=0} = \frac{4\bar{M}}{(1 - \epsilon^2)^{3/2}R_0}
\]

(29)

4. The motion of timelike particles around a charged D-star

From Eqs.(17)-(19), the equation of motion for a timelike test particle can be expressed as

\[
\left( \frac{L}{r^2} \frac{d\varphi}{dr} \right)^2 = \bar{E}^2 - \bar{\mu}^2 B(r) - \frac{L^2}{r^2} B(r)
\]

(30)

where \( \bar{\mu} = \frac{\mu}{\sigma_0} \) and \( \bar{E} = \frac{E}{\sigma_0} \) are the rescaled mass and energy of the test particle. Introducing \( \chi = \frac{1}{\bar{\mu}} \), substituting it into Eq.(30) and then differentiating the equation with respect to \( \varphi \), one will obtain the following equation:

\[
\frac{d^2\chi}{d\varphi^2} = \frac{1}{p} - (1 - \epsilon^2 + \gamma)\chi + \alpha\chi^2 + \beta\chi^3
\]

(31)

where \( p, \gamma, \alpha \) and \( \beta \) are dimensionless parameters defined as:

\[
\begin{align*}
\frac{1}{p} &= \frac{G\bar{M}\bar{\mu}^2}{L^2} \\
\gamma &= \bar{\mu}^2 \frac{G\bar{Q}^2}{L^2} \\
\alpha &= \frac{3G\bar{M}}{4\pi} \\
\beta &= -\frac{2G\bar{Q}^2}{4\pi}
\end{align*}
\]

(32)

Noting that Eq.(31) is a nonlinear differential equation, it can be integrated formally as

\[
\varphi - \varphi_0 = \int_{\chi_0}^{\chi} \frac{d\chi}{\sqrt{\int_{\chi_0}^{\chi} \left( \frac{1}{p} - (1 - \epsilon^2 + \gamma)\chi + \alpha\chi^2 + \beta\chi^3 \right) d\chi + \chi_0^2}}
\]

(33)

where \( \chi_0^2 \) is the initial value of \( \frac{d\chi}{d\varphi}|_{\varphi=\varphi_0} \). However, it is impossible to obtain the exact expression by integrating the above equation. In the following, we will gain some qualitative property of the system with the aid of phase-plane analysis without solving the equation numerically. To do so, we introduce two parameters as \( x = \chi \) and \( y = \frac{d\chi}{d\varphi} \) and the autonomous system corresponding to Eq.(31) will be
\[
\frac{dx}{d\varphi} = f(x, y) = y \\
\frac{dy}{d\varphi} = g(x, y) = \frac{1}{p} - (1 - \epsilon^2 + \gamma)\chi + \alpha\chi^2 + \beta\chi^3
\]

Now, we prove the existence of periodic solution. According to the well known Bendixson’s criterion[30], the equation of motion will have periodic solution if the divergence of the functional vector of the autonomous system is vanishing, i.e., \(\nabla \cdot (f, g) = 0\). It is obvious that the functional vector \((f, g)\) corresponding to the Eqs.(34) satisfies the criterion and therefore indicates that Eq.(31) has a periodic solution. One can also find that the periodic solution exists when \(\epsilon = \gamma = \beta = 0\) which is the case of an ordinary star in asymptotically flat spacetime. This shows that the presence of constant and deficit angle and the electric charge will not exclude the existence of periodic solution from the equation of motion.

Next, we analyze the critical points on the phase plane. The critical point is \((x_0, 0)\), where \(x_0\) satisfies

\[
\frac{1}{p} - (1 - \epsilon^2 + \gamma)x_0 + \alpha x_0^2 + \beta x_0^3 = 0
\]

To analyze the type of the critical point, we firstly linearize the Eq.(34) and then do the translation \(x = x - x_0\). Thus the linearized equations should be:

\[
\frac{dx}{d\varphi} = y \\
\frac{dy}{d\varphi} = \delta x
\]

where

\[
\delta = -(1 - \epsilon^2 + \gamma) + 2\alpha x_0 + 3\beta x_0^2
\]

Using Eqs.(35) and (36), Eq.(37) could be rewritten as

\[
\delta = -\frac{3}{px_0} + 2(1 - \epsilon^2 + \gamma) - \alpha x_0
\]

The eigenvalues corresponding to the system of equations will be

\[
\lambda_{1,2} = \pm \sqrt{\delta}
\]

The types of the critical point could be classified according to the eigenvalues as following:

I. when \(\delta > 0\), we have \(\lambda_1 < 0 < \lambda_2\), which indicates that the critical point is an unstable saddle point. Considering the Eq.(38), this case will correspond to the condition that

(1). \(\alpha < 0\) and \(\Delta < 0\).

(2). \(\alpha > 0\), \(\Delta > 0\) and \(\frac{(1-\epsilon^2+\gamma)-\sqrt{\Delta}}{\alpha} < x_0 < \frac{(1-\epsilon^2+\gamma)+\sqrt{\Delta}}{\alpha}\).

(3). \(\alpha < 0\), \(\Delta > 0\) and \(x_0 > \frac{(1-\epsilon^2+\gamma)-\sqrt{\Delta}}{\alpha}\), where \(\Delta = (1 - \epsilon^2 + \gamma)^2 - \frac{2\alpha}{p}\).

II. when \(\delta < 0\), we have two pure imaginary eigenvalues \(\lambda_{1,2} = \pm i\sqrt{\beta}\), which indicates that the critical point is stable center. Considering Eq.(38), this case will correspond to the condition that

(1). \(\alpha > 0\) and \(\Delta < 0\).

(2). \(\alpha > 0\), \(\Delta > 0\) and \(x_0 > \frac{(1-\epsilon^2+\gamma)+\sqrt{\Delta}}{\alpha}\), or \(0 < x_0 < \frac{(1-\epsilon^2+\gamma)-\sqrt{\Delta}}{\alpha}\).

(3). \(\alpha < 0\), \(\Delta > 0\) and \(0 < x_0 < \frac{(1-\epsilon^2+\gamma)-\sqrt{\Delta}}{\alpha}\).

III. when \(\delta = 0\), we have \(\lambda_{1,2} = 0\), which, together with the form of the autonomous system Eqs.(36), indicates that the motion is uniformly on the lines \(y = \text{Constants}\) and all the the points on the lines \(y = \text{Constants}\) are balanced positions. From Fig.1 to Fig.2, we show the phase graph for different initial values and different parameters.

5. Discussion
FIG. 1: The phase graph when $\alpha = 0.100$, $\gamma = 0.100$, $\epsilon = 0.010$, $p = 11$ and $\delta = 0.001$, which corresponds to a charged case.

FIG. 2: The phase graph when $\alpha = 0.100$, $\gamma = 0.000$, $\epsilon = 0.000$, $p = 11$ and $\delta = 0.000$, which corresponds to an uncharged case.

We study the motion of light and a neutral test particle around a charged D-star in this paper. We show that the deflect angle of light is different from the case in asymptotically flat spacetime for the existence of a factor of $(1 - \epsilon^2)$, which stands for the feature of a D-star that the spacetime has a deficit angle.

Using the phase-plane analysis, we investigate the qualitative property of the dynamical equation controlling the motion of a test particle around the charged D-star. We prove that the equation of motion has a periodic solution, the position of whose critical point and type on the phase plane can be affected by the deficit angle and the charge. The conditions under which the critical point is stable center and unstable saddle point have also been given.

ACKNOWLEDGMENTS

This work was partially supported by the Foundation from Science and Technology. Committee of Shanghai under
Grant No.02QA14033, and the Foundation from the Education Committee of Shanghai under Grant No.01QN86.

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