PID Sliding Mode Control for Electro-hydraulic Servo System

Wang Xingxu, He Guang a and Gao Rui
Science and Technology on Electromechanical Dynamic Control Laboratory, Beijing Institute of Technology, Beijing, 100081
a Corresponding author: heguang@bit.edu.cn

Abstract. In general, an advanced control strategy is necessary for electro-hydraulic servo system to obtain fast and accurate system response. A novel control scheme called PID sliding mode control was proposed by combining conventional sliding mode control and PID control strategy. The linear dynamic mode of electro-hydraulic servo system was established precisely. The PID sliding mode control law was deduced in detail. The chattering problem caused by original sliding mode control was weakened by a continuous PID algorithm. The simulation results of electro-hydraulic servo system demonstrated the effectiveness of the proposed method.

1. Introduction
Electro-hydraulic servo system belongs to nonlinear system essentially and it involves dead zone, saturation, friction and other nonlinear factors, plus the uncertainties and disturbances [1-2], it is difficult to obtain satisfactory control effect by conventional PID control methods, while nonlinear control methods such as robust control, adaptive control, sliding mode control (SMC), inversion control and intelligent control have proved more suitable for electro-hydraulic servo system [3-4].

In this paper, we developed a novel SMC method, which combined with PID control strategy (named PIDS) to overcome the chattering problem of original SMC. In more detail, the original SMC discontinuous control input caused by the high-frequency switching gain is replaced with a continuous control input determined by a proportional-integral-derivative (PID) algorithm, which is consisted of the sliding surface and its derivative. Thus, we can give full play to the advantages of sliding mode control for the electro-hydraulic servo system.

2. System Dynamic
2.1 Dynamic of hydraulic cylinder
There basic equations of hydraulic cylinder as follow:

\[ q_L = K_q x_v - K_p p_L \]  \hspace{0.5cm} (1)

\[ q_L = A_p \frac{dx_d}{dt} + C_p P_L + \frac{V_L}{4\beta_p} \frac{dp_L}{dt} \]  \hspace{0.5cm} (2)

\[ p_L A_p = m_i \ddot{x}_d + \beta_p \dot{x}_d + Kx_d + F_L \]  \hspace{0.5cm} (3)
Where, \( q_L \) is load flow; \( K_q \) is flow gain; \( x_v \) is displacement of spool (m); \( p_L \) is load pressure (Pa); \( A_p \) is effective area of hydraulic cylinder piston (m\(^2\)); \( x_d \) is displacement of piston rod (m); \( C_t \) is total leakage coefficient; \( V_t \) is total volume of hydraulic cylinder (m\(^3\)); \( \beta_e \) is equivalent volume modulus of oil ( \( \text{Pa} \)); \( m_t \) is total quality (kg); \( B_p \) is the viscous damping coefficient of piston; \( K \) is load spring stiffness; \( F_L \) is outside load bearing on the piston (N).

After Laplace transformations for equation (1), (2), (3), we can get transfer function as follow:

\[
x_d(s) = \frac{\frac{K_q}{A_p} x_v(s) - \left(\frac{K_q}{A_p} + \frac{V_t}{4 \beta_p A_p^2}\right) F_L(s)}{\frac{m_t V_t}{4 \beta_p A_p^4} s^3 + \left(\frac{1}{4 \beta_p A_p^2} + \frac{m_t K}{4 \beta_p A_p^2}\right) s^2 + \left(1 + \frac{K_v}{4 \beta_p A_p^2}\right) s}
\]  

(4)

Where, \( K_{ce} = k_c + C_p \), total pressure-flow coefficient. Consider the load as inertia load \( K = 0 \); meanwhile \( K_{ce} B_p / A_p^2 << 1 \), the formula (4) can be simplified as

\[
x_d(s) = \frac{\frac{K_q}{A_p} x_v(s) - \left(\frac{K_q}{A_p} + \frac{V_t}{4 \beta_p A_p^2}\right) F_L(s)}{\left(\frac{s^2}{\omega_n^2} + \frac{2 \xi \omega_n}{\omega_n} s + 1\right) s}
\]  

(5)

Where, \( \omega_n = \sqrt{4 \beta_p A_p^2 / V_t m_t} \): hydraulic cylinder inherent frequency; \( \zeta = \frac{K_{ce}}{A_p} \sqrt{\beta_p m_t / V_t} + \frac{B_p}{4 A_p} \sqrt{V_t / \beta_p m_t} \): damping ratio.

The transfer function of hydraulic cylinder is showed as:

\[
G_q(s) = \frac{x_d(s)}{x_v(s)} = \frac{\frac{K_q}{A_p}}{s^2 + \frac{2 \xi \omega_n}{\omega_n} s + 1}
\]  

(6)

2.2 Electronic control system transfer function

The transfer function of electro-hydraulic servo valve is considered as the second order oscillation when inherent frequency less than 50Hz, it is described as follow:

\[
G_v(s) = \frac{x_v(s)}{I(s)} = \frac{K_v}{s^2 + \frac{2 \xi \omega_v}{\omega_v} s + 1}
\]  

(7)

Where, \( K_v \) is flow gain; \( \omega_v \) is inherent frequency; \( \zeta_v \) is damping ratio.

2.3 System transfer function

The mathematic model of whole electro-hydraulic system can be presented as a 5 order transfer function, as shown in Figure 1.

Figure 1. Model block diagram of system
It can be expressed in the form of differential equations as follow:
\[
y^{(5)} = -a_1\ddot{y} - a_2\dot{y} - a_3\ddot{y} - a_4\dot{y} + bu + f(t) \tag{8}
\]
Where, \(a_1, a_2, a_3, a_4, b\) are constants, \(f(t)\) is load disturbances, it can be expressed as follow,
\[
f(t) = \frac{K_{\omega c}}{A_p^2} F_L + \frac{V_i}{4\beta A_p^2} \dot{F}_L \tag{9}
\]

The system parameters are showed in table 1.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \(\xi_v\) | 0.7   | \(V_i\) | 1.6\times10^{-3} \text{m}^3 |
| \(\omega_1\) | 610 \text{rad/s} | \(\beta_1\) | 0.9\times10^{-9} \text{pa} |
| \(K_r\) | 2.1\times10^{-3} \text{m}^3/(\text{s} \cdot \text{A}) | \(K_a\) | 1.32 |
| \(\omega_2\) | 23.7 \text{rad/s} | \(a_1\) | 1267.4 |
| \(\xi_p\) | 0.2   | \(a_2\) | 430489.9 |
| \(K_q\) | 1.9   | \(a_3\) | 18323058 |
| \(A_p\) | 2\times10^{-4} \text{m}^2 | \(a_4\) | 209082990 |
| \(K_{\omega i}\) | 12.6\times10^{-10} \text{m}^3/(\text{W} \cdot \text{s}) | \(b\) | 5505991458 |

3. Design of Sliding Mode Controller

3.1 Equivalent Control Law

According formula (8), the system dynamic equation can be modeled as following for convenience,
\[
x^{(5)} = f(X,t) + g(X,t)u(t) + f(t) \tag{10}
\]
Where, \(X=[x,\dot{x},...,\dot{\ddot{x}}]^T\), \(f(X,t) = -a_1\ddot{x} - a_2\dot{x} - a_3\ddot{x} - a_4\dot{x} + bu\), \(g(X,t) = b\).

It should be noted that a large class of processes can be represented with this type of model structure [5]. The control objective is to force the plant state vector \(X\) to follow a specified desired trajectory vector \(X_r\).

Where, \(X_r=[x_r,\dot{x}_r,...,\dot{\ddot{x}}_r]^T\).

Under the condition that the tracking error vector is defined as \(x_e=X_r-X\), the problem is thus to design a control law \(u(t)\) to ensure that \(x_e=0\), as \(t \to \infty\).

A sliding surface (or a switch function), which determine the closed-up response in the sliding phase, can be defined as follow:
\[
s(X_e,t) = 0 \tag{11}
\]
Where,
\( s(X_e, t) = [\Lambda^T \ 1]X_e = 0 \)  

(12)

And \( \Lambda = [c_1 \ c_2 \ \ldots \ c_l]^T \) is a properly chosen coefficient vector such that \( X_e \) approaches the state plane origin exponentially when \( s(X_e, t) \) equals zero. \( s^{(t)} + c_1s + c_2 \int s + c_3 \) is Hurwitz. With any arbitrary initial states, the sliding mode exists if the following reachability condition is satisfied:

\[ \dot{s} \leq 0 \]  

(13)

For \( \dot{s} = 0 \),

\[ \dot{s}(X_e, t) = [\Lambda \ 1]^T \dot{X}_e = 0 \]  

(14)

Also

\[ x_r^{(s)} = f(X, t) - bu(t) + [0 \ \Lambda]^T X_e = 0 \]  

(15)

Then the equivalent control law can be obtained:

\[ u_{eq} = \frac{1}{b} (x_r^{(s)} - f(X, t) + [0 \ \Lambda]^T X_e) \]  

(16)

3.2 PID Sliding Mode Control Law

Usually, SMC method adopted the discontinuous control term to compensate the uncertainty [6]. It can be described as follow:

\[ u_{Sign} = \frac{K}{b} \text{sgn}(s) + u_{eq} \]  

(17)

Consider the following Lyapunov function:

\[ V = \frac{1}{2} s^2 \]  

(18)

Derivation along \( V \) results in:

\[ \dot{V} = s \cdot \dot{s} \]  

(19)

Where,

\[ s = [\Lambda^T \ 1] \cdot X_e \]  

(20)

\[ \dot{s} = [\Lambda^T \ 1] \cdot \dot{X}_e \]

\[ = [\Lambda^T \ 0] \cdot \dot{X}_e + x_r^{(s)} - x_r^{(s)} \]

\[ = [\Lambda^T \ 0] \cdot \dot{X}_e + x_r^{(s)} + 0 \cdot A^T - b] \cdot [X \ u_{eq}] \]

\[ = [\Lambda^T \ 0] \cdot \dot{X}_e + x_r^{(s)} + 0 \cdot A^T - b] \cdot [X \ u_{eq}] - K \cdot \text{sgn}(s) \]  

(21)

Where, \( A^T = [a_4 \ a_3 \ a_2 \ a_1]^T \)

Substituting (16) into (21), we have

\[ \dot{s} = -K \cdot \text{sgn}(s) \]  

(22)

Substituting (22) into (19), we have
\[
\dot{V} = -K \cdot s \cdot \text{sgn}(s) = \begin{cases} 
-K \cdot s, & s \geq 0 \\
K \cdot s, & s < 0
\end{cases}
\] (23)

Then from (23) \( \dot{V} \leq 0 \).

Therefore, the stability of the electro-hydraulic servo system is guaranteed. But, the frequently switching gain is undesirable for controller. The chattering can be reduced somewhat by introducing a bound region containing the switching surface to smooth the control behavior. In this case, the function \( \text{sgn}(s) \) is replaced by a saturation function,

\[
sat(s) = \begin{cases} 
1 & s > \Delta \\
ks & \|s\| \leq \Delta, k = 1/\Delta \\
-1 & s < -\Delta
\end{cases}
\] (24)

Where, \( \Delta \) is the boundary layer thickness.

Then, \( u_{eq} \) in equation (16) becomes

\[
u_{Sat} = \frac{K}{b} sat(s) + u_{eq}
\] (25)

In this condition,

\[
s = -K \cdot sat(s)
\] (26)

\[
\dot{V} = -K \cdot s \cdot sat(s) = \begin{cases} 
-K \cdot s, & s > \Delta \\
-K \cdot \frac{s}{\Delta}, & \|s\| \leq \Delta \\
K \cdot s, & s < -\Delta
\end{cases}
\] (27)

Then from (27) \( \dot{V} \leq 0 \)

The continuous control approximating the switching signal is obtained by a saturation function. It does reduce the degree of chattering in the control input, but it can not eliminate the chattering. At the same time, the excellent feature of insensitivity to uncertainties and disturbances is lost because of this change. So it is necessary to adopt a novel control method.

\[
u_{pid} = k_p \cdot e(t) + k_i \cdot \int e(t) dt + k_d \cdot \frac{de(t)}{dt}
\] (28)

To overcome the chattering associated with two methods mentioned, we propose a controller combined SMC with PID method. The switching function is replaced with a continuous input determined by a PID algorithm. It takes the sliding mode function as the input, it is showed as follow:

\[
u_{PID-SMC} = k_p \cdot \dot{s} + k_i \cdot \int \dot{s} dt + k_d \cdot \frac{ds}{dt} + u_{eq}
\] (29)

In this condition,

\[
\dot{s} = -b \cdot (k_p \cdot \dot{s} + k_i \cdot \int \dot{s} dt + k_d \cdot \frac{ds}{dt})
\] (30)

\[
\dot{V} = -b \cdot s \cdot (k_p \cdot \dot{s} + k_i \cdot \int \dot{s} dt + k_d \cdot \frac{ds}{dt})
\] (31)
Where, $k_p > 0$, $k_i > 0$, $k_d = 0$

When $s \geq 0$, we can get
\[
k_p s + k_i \int s dt \geq 0 \quad (32)
\]

From (30) and (31), we can get $\dot{v} \leq 0$

When $s \leq 0$, we can get
\[
k_p s + k_i \int s dt \leq 0 \quad (33)
\]

From (30) and (31), we can get $\dot{v} \leq 0$. Therefore, the stability of the electro-hydraulic servo system is guaranteed. The PID proportional term drives the states to the neighborhood of the sliding surface. The PID integral action forces the states onto the sliding surface irrespective of the bounds of the uncertainties and disturbances, while the PID derivative action provides a stabilizing effect to counter the possible excessive control produced by the integral action. In a word, the PID term drive the process states onto the sliding surface without loss of robustness.

### Table 2. System Parameters.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $c_1$     | 240^4 | $b$       | 5505991458 |
| $c_2$     | 4×240^3 | $K$       | 1×10^8 |
| $c_3$     | 6×240^2 | $\Delta$  | 1×10^6 |
| $c_4$     | 960   | $k_{p1}$  | 1.8 |
| $a_1$     | 1267.4 | $k_{i1}$  | 1.5 |
| $a_2$     | 430489.9 | $k_{d1}$  | 0.1 |
| $a_3$     | 18323058 | $k_{p2}$  | 5×10^-9 |
| $a_4$     | 209082990 | $k_{i2}$  | 1×10^-9 |

### 4. Simulation Results

In order to validate the performance of the proposed PID sliding mode control method for the electrohydraulic servo system, we develop the simulation analysis in Matlab software. The plant and the control law of the simulation are described in section 2 and 3 respectively. Taking the sampling time as 10ms normally, and using RK4 for accurately numerical solution. The aim of the control strategy is to make the actuator move according the desired displacement and velocity. The desired displacement is described as follow:
\[
s = 30.44(t + 2e^{-0.7t} - 2e^{-3.5t}) - 59.04 \quad (34)
\]

The desired velocity is the derivative of the displacement. Then we do the simulation experiment with random disturbance from $1\times10^8$ to $2\times10^8$, using a uniformly distributed random function. The results of PID-SMC method are compared with those of Sign, Sat and traditional PID methods. The
parameters of the system are shown in Table 1 and the proposed controller’s parameters are given in Table 2.

It can be observed from Figure 2 to Figure 5 that in a comprehensive view, the PID-SMC has the optimal control effect that it can achieve an excellent position tracking and speed tracking. In spite of the fact that the PID-SMC has a fluctuation at the beginning of the speed tracking, it behaves superbly in any other places later. In contrast, we can easily find that the traditional PID control method has a large position and speed tracking error at first, while approximately 2 seconds later its control effect becomes better than before and is able to make the system have a good response. In addition, it is obvious that neither the Sign nor the Sat is capable of tracking the ideal curve tightly; besides, the tracking error is so large that it is unacceptable in practice. From Figure 6 to Figure 9, it can be concluded that the Sign, Sat and PID-SMC control the system with chattering under the random disturbance. Nevertheless, the buffeting amplitude of PID-SMC is within an acceptable range. Therefore, the PID-SMC control strategy can generate a good system tracking response with less chattering under the action of random disturbance signal. Specific control indicators are shown in Table 3.

Figure 2. Position tracking

Figure 3. Speed tracking

Figure 4. Position tracking error
Figure 5. Speed tracking error

Figure 6. $u_{\text{Sign}}$ Control Input

Figure 7. $u_{\text{Sat}}$ Control Input

Figure 8. $u_{\text{pid}}$ Control Input

Figure 9. $u_{\text{pid,SMC}}$ Control Input
Table 3. Control indicators.

| Control Input | Maximum displacement error (m) | Maximum speed error (m/s) | Buffeting amplitude |
|---------------|---------------------------------|---------------------------|---------------------|
| Sign          | 0.063800                        | 0.013770                  | 0.00141             |
| Sat           | 0.064040                        | 0.014790                  | 0.00122             |
| PID           | 0.014720                        | 0.196400                  | 0                   |
| PID-SMC       | 0.001572                        | 0.024870                  | 0.00119             |

5. Conclusion
In this paper, a PID sliding mode control method is proposed for the electro-hydraulic servo system. The simulation results show that the proposed method has excellent stability, robustness and accuracy. Nevertheless, it doesn’t eliminate chattering completely under random disturbances. This defect needs further study to enhance the control performance of the system.

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