SOME IMPLICATIONS OF A SUPERSYMMETRIC MODEL WITH R-PARITY BREAKING BILINEAR INTERACTIONS

Sourov Roy\textsuperscript{1} and Biswarup Mukhopadhyaya\textsuperscript{2}

Mehta Research Institute, 10 Kasturba Gandhi Marg, Allahabad - 211 002, INDIA

ABSTRACT

We investigate a supersymmetric scenario where R-parity is explicitly broken through a term bilinear in the lepton and Higgs superfields in the superpotential. We show that keeping such a term alone can lead to trilinear interactions, similar to those that are parametrized by $\lambda$- and $\lambda'$ in the literature, involving the physical fields. The upper limits of such interactions are predictable from the constraints on the parameter space imposed by the lepton masses and the neutrino mass limits. It is observed that thus the resulting trilinear interactions are restricted to values that are smaller than the existing bounds on most of the $\lambda$- and $\lambda'$-parameters. Some phenomenological consequences of such a scenario are discussed.

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\textsuperscript{1}E-mail : sourov@mri.ernet.in

\textsuperscript{2}E-mail : biswarup@mri.ernet.in
1 INTRODUCTION

It is being increasingly realised by those engaged in the search for supersymmetry (SUSY) that the principle of R-parity conservation, assumed to be sacrosanct in the prevalent search strategies, is not in practice inviolable. The R-parity of a particle is defined as

\[ R = (-1)^{L + 3B + 2S} \]

and can be violated if either baryon (B) or lepton (L) number is not conserved in nature, a fact perfectly compatible with the non-observation of proton decay. This is because, whereas the violation of B or L, taken singly, is inadmissible in the standard model (SM) where all the elementary baryons and leptons are fermions, the SUSY version of the SM allows it by virtue of the scalar quarks and leptons that are part of the particle spectrum.

Under R-parity violation the phenomenology changes considerably, the most important consequence being that the lightest supersymmetric particle (LSP) can decay now. However, the way in which R-parity can be violated is not unique; different types of R-violating interaction terms can be written down, leading to different observable predictions. In addition, R-parity can be violated spontaneously, in stead of explicitly, whence another class of interesting effects are expected. If the phenomenology of R-parity breaking has to be understood, and the consequent modifications in the current search strategies have to be effectively implemented, then it is quite important to explore the full implication of each possible R-breaking scheme. In this paper we probe some aspects of one such scheme, namely, where lepton number violation has its origin in terms bilinear in the lepton and Higgs superfields in the superpotential.

The R-conserving part of the minimal supersymmetric standard model (MSSM) is of the following form in terms of superfields:

\[
W_{MSSM} = \epsilon_{ab} [\mu H_1^a H_1^b + h_{ij}^L L_i^a H_1^b E_j^c + h_{ij}^d Q_i^a H_1^b D_j^c + h_{ij}^u Q_i^a H_2^b U_j^c]
\] (1)
where \((a, b)\) are SU(2) indices, \((i, j)\) are generation indices and the superscript \(c\) denotes right-handed chiral superfields. Here \(Q = \binom{u}{d}\), \(L = \binom{\nu}{l}\) and \(H_1, H_2\) are the Higgs superfields that gives masses to the down- and up-type quark superfields. If now R-breaking interactions are incorporated, the superpotential takes the form \([5]\)

\[
W = W_{MSSM} + W_L + W_B
\]

with

\[
W_L = \epsilon_{ab}[\lambda_{ijk} L^a_i L^b_j E^c_k + \lambda'_{ijk} L^a_i Q^b_j D^c_k + \epsilon_i L^a_i H^b_2]
\]

and

\[
W_B = \lambda''_{ijk} U^c_i D^b_j D^c_k
\]

Obviously, both \(W_L\) and \(W_B\) cannot be present if the proton has to be stable. In the rest of this paper we shall concentrate on the case where only lepton number is violated.

\(W_B\) as well as the first two terms in \(W_L\) have received a lot of attention in recent times, and constraints have been derived on them from existing experimental data \([6]\). However, the term \(\epsilon_i L^a_i H^b_2\) is also a viable agent for R-parity breaking. It is particularly interesting for the fact that it can trigger a mixing between charginos and charged leptons as well as between neutralinos and neutrinos, resulting in observable effects that are not to be seen with the \(\lambda\)-and \(\lambda'\)-terms alone. One of these distinctive effects is that, the lightest neutralino can decay invisibly into three neutrinos, which is not possible if only the first two terms in \(W_L\) are present. Some other implications, especially those in the scalar sector of the theory, have been investigated recently in the literature \([7]\). The significance of such bilinear R-violating interactions is further emphasized by the following observations:
(1) Although it may seem possible to rotate away the $LH_2$-terms by redefining the lepton and Higgs superfields, their effect is bound to show up via the scalar potential [7].

(2) Even if one may rotate away these terms at one energy scale, they reappear at another as the couplings evolve radiatively [8].

(3) The $\lambda$- and $\lambda'$-terms themselves give rise to the bilinear terms at the one-loop level [9].

(4) It has been argued that if one wants to subsume R-parity violation in a Grand Unified Theory (GUT), then the trilinear interactions in $W_L$ naturally come out to be rather small in magnitude ($O(10^{-3})$ or so) [10]. However, the superrenormalizable bilinear terms are not subjected to such requirements a priori.

We perform an analysis here keeping $eLH_2$ as the only R-parity violating term in the theory [11]. Moreover, for reasons that we shall discuss below, we are incorporating such a term only for the third generation lepton superfield $L_3$. We shall see that after one incorporates the effect of mixing, such a term can give rise to trilinear interactions among the physical states, which are very similar in nature to those induced by the $\lambda$’s and the $\lambda'$’s. All these interactions are derived in section 2, together with the gauge boson couplings of the lepton-chargino and neutrino-neutralino physical states. It is interesting to note that the parameters giving rise to these interactions are constrained by the $\tau$- and $\nu_\tau$ masses. Thus it is possible to predict the maximum possible values for the couplings for any given set of parameters of the MSSM. In section 3 we discuss these constraints and some of their phenomenological consequences. Our conclusions are summarised in section 4. The detailed forms of some formulas of section 2 are presented in the appendix.
2 The Formalism

As has been stated before, we consider a superpotential of the form

\[ W = W_{MSSM} + \epsilon L_3 H_2 \]  

(5)

where the SU(2) indices have been suppressed. The simplification achieved by letting only the third generation mix with the Higgs superfield can be justified if one notes that the value of \( \epsilon_i \) for a particular generation is constrained severely by the upper limit on the neutrino mass in that generation. Since the \( \tau \)-neutrino mass has the least restrictive laboratory bound of 24 MeV [12], only \( \epsilon_3 \) (to be called \( \epsilon \) hereafter) can be large enough to be phenomenologically significant.

An immediate consequence of a non-zero \( \epsilon \) is the mixing between the charged leptons and the charginos as well as between neutrinos and neutralinos. The other quantity that can trigger such mixing is a non-zero vacuum expectation value (vev) of \( \tilde{\nu}_\tau \). This vev leads to off-diagonal \( \nu_r - \tilde{Z} \) and \( \tau - \tilde{W} \) terms in the current eigenstate basis [13].

In such a situation, the \((3 \times 3)\) chargino mass matrix is

\[
M_{\tilde{\chi}^\pm} = \begin{pmatrix}
M & -gv_2 & 0 \\
-gv_1 & \mu & fv_3 \\
-gv_3 & \epsilon & -fv_1
\end{pmatrix}
\]

(6)

where \( v_1 = \langle H_1 \rangle \), \( v_2 = \langle H_2 \rangle \), \( v_3 = \langle \tilde{\nu}_\tau \rangle \) and \( f = h_{33} = \frac{m_\tau}{v_3} \), \( M \) being the SU(2) gaugino mass parameter. Here we have assigned \((-i\tilde{W}^-, \tilde{H}^-_1, \tau_L^-)\) along the rows and \((-i\tilde{W}^+, \tilde{H}^+_2, \tau_R^+)\) along the columns. Similarly, the extended neutralino mass matrix in
the basis \((-i\tilde{A}, -i\tilde{Z}, \tilde{H}_1^0, \tilde{H}_2^0, \nu_r)\) is given by

\[
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_{\tilde{A}} & \frac{1}{2}(M_{\tilde{Z}} - M_{\tilde{A}})\tan2\theta_W & 0 & 0 & 0 \\
\frac{1}{2}(M_{\tilde{Z}} - M_{\tilde{A}})\tan2\theta_W & M_{\tilde{Z}} & -\frac{g_{V1}}{\sqrt{2}\cos\theta_W} & \frac{g_{V2}}{\sqrt{2}\cos\theta_W} & -\frac{g_{V2}}{\sqrt{2}\cos\theta_W} \\
0 & -\frac{g_{V1}}{\sqrt{2}\cos\theta_W} & 0 & -\mu & 0 \\
0 & \frac{g_{V2}}{\sqrt{2}\cos\theta_W} & -\mu & 0 & -\epsilon \\
0 & -\frac{g_{V3}}{\sqrt{2}\cos\theta_W} & 0 & -\epsilon & 0
\end{pmatrix}
\]  

(7)

with

\[M_{\tilde{A}} = M' \cos^2\theta_W + M\sin^2\theta_W\]  

(8)

\[M_{\tilde{Z}} = M' \sin^2\theta_W + M\cos^2\theta_W\]  

(9)

\(M'\) and \(M\) being respectively the \(U(1)\) and \(SU(2)\) gaugino mass parameters.

The diagonalisation of \(M_{\tilde{\chi}^\pm}\) and \(M_{\tilde{\chi}^0}\) is straightforward; one thus obtains two \((3 \times 3)\) matrices \(U\) and \(V\) for the right-handed and left-handed chargino respectively and a \((5 \times 5)\) mixing matrix \(N\) for the neutralinos. These correspond to their MSSM forms in the proper limit.

Let us now consider the scalar sector in this scenario. The scalar potential, including the third generation sleptons, is given by

\[
V = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_L^2 \bar{\tau}_L^\dagger \bar{\tau}_L + m_R^2 \bar{\tau}_R^\dagger \bar{\tau}_R + m_{\tilde{\nu}_r}^2 \tilde{\nu}_r^\dagger \tilde{\nu}_r
\]

\[+ f^2 H_1^\dagger H_1 (\tilde{L}^\dagger \tilde{L} + \bar{\tau}_R^\dagger \bar{\tau}_R) + f^2 \tilde{L}^\dagger \tilde{L} \bar{\tau}_R^\dagger \bar{\tau}_R + \mu f [H_1^\dagger \tilde{L} \bar{\tau}_R + \tilde{L}^\dagger H_2 \bar{\tau}_R]
\]

\[-\epsilon f [H_1^\dagger \bar{\tau}_R H_2 + H_1 \bar{\tau}_R^\dagger H_1^\dagger] + \mu [\tilde{L} H_1^\dagger \tilde{L}^\dagger H_1] - f^2 H_1^\dagger \tilde{L} (H_1^\dagger \tilde{L})^\dagger
\]

\[+ B_1 \mu (\phi_0^0 \phi_0^0 - \phi_0^- \phi_0^+ + \phi_2^0 \phi_0^0 - \phi_2^+ \phi_0^-) + Af (\bar{\tau}_L \phi_0^0 - \tilde{\nu}_r \phi_0^-)\tau_1^\dagger R
\]

\[+ B_2 \epsilon (\tilde{\nu}_r \phi_0^- - \bar{\tau}_L^\dagger \phi_2^+ + \phi_2^0 \tilde{\nu}_r - \phi_2^+ \bar{\tau}_L^\dagger) + Af (\tilde{\nu}_r \phi_0^+ - \tilde{\nu}_r \phi_0^-) \tau_1^\dagger R
\]

\[+ \frac{1}{8} (g^2 + g'^2) [\bar{H}_1^\dagger H_1 - H_2^\dagger H_2]^2 + \frac{1}{2} g^2 |H_1^\dagger H_1|^2 - \frac{1}{2} g'^2 |\tilde{L}^\dagger \tilde{L} + H_1^\dagger H_1 - H_2^\dagger H_2|^2
\]
\[
\begin{align*}
&+ \frac{1}{4} g^2 (\bar{\nu}_\tau^T \bar{\tau}_L \bar{\nu}_\tau) + \frac{1}{2} g^2 (\bar{\nu}_\tau^T \bar{\tau}_L \phi_1^0 + \bar{\tau}_L \bar{\nu}_\tau \phi_1^0) \\
&+ \frac{1}{4} g^2 (\bar{\nu}_\tau^T \bar{\nu}_\tau \phi_1^0 + \bar{\tau}_L \bar{\nu}_\tau \phi_1^0 - \bar{\nu}_\tau^T \bar{\nu}_\tau \phi_1^0 - \bar{\tau}_L \bar{\nu}_\tau \phi_1^0) \\
&+ \frac{1}{4} g^2 (\bar{\nu}_\tau^T \bar{\nu}_\tau \phi_2^0 + \bar{\tau}_L \bar{\nu}_\tau \phi_2^0 - \bar{\nu}_\tau^T \bar{\nu}_\tau \phi_2^0 - \bar{\tau}_L \bar{\nu}_\tau \phi_2^0) \\
&+ \frac{1}{2} g^2 (\bar{\nu}_\tau^T \bar{\tau}_L \phi_2^0 + \bar{\tau}_L \bar{\nu}_\tau \phi_2^0) + \frac{1}{4} g^2 \bar{\tau}_L \bar{\tau}_L (H_1^\dagger H_1 - H_2^\dagger H_2) + \frac{1}{4} g^2 (\bar{\nu}_\tau^T \bar{\tau}_L \bar{\nu}_\tau) \\
&+ \frac{1}{8} (g^2 + g^2) \{(\bar{\nu}_\tau^T \bar{\nu}_\tau)^2 + (\bar{\tau}_L \bar{\tau}_L)^2\} \quad (10)
\end{align*}
\]

where \( \bar{L} = (\bar{\nu}_\tau^T)_{L} \), \( H_1 = (\phi_1^0) \), \( H_2 = (\phi_2^0) \). \( A \) is the SUSY breaking trilinear soft term and \( B_1, B_2 \) are the bilinear soft terms.

The mass-squared matrices for the neutral scalars, neutral pseudo-scalars and charged scalars are given respectively by

\[
M_s^2 = \begin{pmatrix}
m_1^2 + 2\lambda c + 4\lambda v_1^2 & -4\lambda v_1 v_2 + B_1 \mu & 4\lambda v_1 v_3 + \mu \epsilon \\
-4\lambda v_1 v_2 + B_1 \mu & m_2^2 - 2\lambda c + 4\lambda v_2^2 & -4\lambda v_3 v_2 + B_2 \epsilon \\
4\lambda v_1 v_3 + \mu \epsilon & -4\lambda v_3 v_2 + B_2 \epsilon & m_{\tilde{\nu}_\tau}^2 + 2\lambda c + 4\lambda v_3^2
\end{pmatrix} \quad (11)
\]

\[
M_p^2 = \begin{pmatrix}
m_1^2 + 2\lambda c & -B_1 \mu & \mu \epsilon \\
-B_1 \mu & m_2^2 - 2\lambda c & -B_2 \epsilon \\
\mu \epsilon & -B_2 \epsilon & m_{\tilde{\nu}_\tau}^2 + 2\lambda c
\end{pmatrix} \quad (12)
\]

and

\[
M_c^2 = \begin{pmatrix}
r - \frac{1}{2} g^2 c & -B_1 \mu + \frac{1}{2} g^2 v_1 v_2 & -B_2 \epsilon + \frac{1}{2} g^2 v_2 v_3 & -\epsilon f v_1 \\
-B_1 \mu + \frac{1}{2} g^2 v_1 v_2 & s + \frac{1}{4} g^2 c & \mu \epsilon + \frac{1}{2} g^2 v_1 v_3 & -\epsilon f v_2 + Af v_3 \\
-B_2 \epsilon + \frac{1}{2} g^2 v_2 v_3 & \mu \epsilon + \frac{1}{2} g^2 v_1 v_3 & p + \frac{1}{4} g^2 t + \frac{1}{4} g^2 c & \mu f v_2 - Af v_1 \\
-\epsilon f v_1 & -\epsilon f v_2 + Af v_3 & \mu f v_2 - Af v_1 & q - \frac{1}{2} g^2 c + f^2 v_3^2
\end{pmatrix} \quad (13)
\]

with

\[
r = m_2^2 + \frac{1}{4} g^2 (v_1^2 + v_2^2 + v_3^2) \\
s = m_1^2 + \frac{1}{4} g^2 (v_1^2 + v_2^2 - v_3^2)
\]

6
\[ p = m_L^2 + f^2 v_1^2 \]
\[ q = m_R^2 + f^2 v_1^2 \]
\[ t = (-v_1^2 + v_2^2 + v_3^2) \]
\[ c = (v_1^2 - v_2^2 + v_3^2) \]
\[ \lambda = (g^2 + g'^2)/8 \]

Here the real and imaginary part of \( \tilde{\nu}_\tau \) enter into \( M_s^2 \) and \( M_p^2 \) respectively. The corresponding diagonalising matrices that control the mixing in those sectors are described here as \( S, P \) and \( C \).

The scalar sector is subject to the following constraints \[\text{[14]}\]:

(1) The extremization of the neutral part of the potential leads to

\[ (m_1^2 + 2\lambda c)v_1 + B_1 \mu v_2 + \mu \epsilon v_3 = 0 \] \hspace{1cm} (14)

\[ (m_2^2 - 2\lambda c)v_2 + B_1 \mu v_1 + B_2 \epsilon v_3 = 0 \] \hspace{1cm} (15)

\[ (m_{\tilde{\nu}_\tau}^2 + 2\lambda c)v_1 + B_1 \mu v_2 + \mu \epsilon v_3 = 0 \] \hspace{1cm} (16)

Furthermore, the second derivatives with respect to the neutral fields at the extremum must be all positive.

(2) The potential must be bounded from below \[\text{[15]}\]. The resulting condition is

\[ m_1^2(v_2^2 - v_3^2) + m_2^2 v_2^2 + m_{\tilde{\nu}_\tau}^2 v_3^2 + 2\mu \epsilon (v_2^2 - v_3^2)^{\frac{3}{2}} v_3 \\
+ 2B_1 \mu (v_2^2 - v_3^2)^{\frac{3}{2}} v_2 + 2B_2 \epsilon v_2 v_3 \geq 0 \] \hspace{1cm} (17)

Note that setting \( v_3 = 0 \) above give us the corresponding condition in MSSM.
(3) Gauge symmetry breaking requires that the minimum of the potential has to be negative [10]. This implies

\[ X_{\text{min}} \leq 0 \tag{18} \]

where \( X_{\text{min}} \) is the lowest eigenvalue of the matrix

\[
\begin{pmatrix}
  m_1^2 & B_1 \mu & \mu \epsilon \\
  B_1 \mu & m_2^2 & B_2 \epsilon \\
  \mu \epsilon & B_2 \epsilon & m_{\tilde{\nu}}^2 \\
\end{pmatrix}
\]

(19)

(4) All the eigenvalues of the \( M_s^2, M_p^2 \) and \( M_c^2 \) have to be non-negative. This leads to the necessary (but not sufficient) conditions that \( B_1 \) and \( \mu \), as also \( B_2 \) and \( \epsilon \), are of opposite signs.

Once the five mass matrices mentioned above are diagonalised, we are in a position to write down all the interactions in terms of the physical fields in the spin-\( \frac{1}{2} \) and spin-0 sectors. We emphasize that it is the couplings of these physical fields that are going to be ultimately related to experimental observables. Hence any phenomenological constraint that is relevant should basically apply to them.

Now, the physical scalar states that are dominantly charged sleptons or sneutrinos have Higgs components in them. Similarly, there are bound to be some gaugino (or Higgsino) admixtures in the states which are mostly \( \tau \) or \( \nu_\tau \). Consequently, the Higgs and gaugino interactions of leptons (in the current eigenstate basis) give rise to trilinear interaction terms involving dominantly leptonic (or sleptonic) fields. Similar interactions of the quarks can also give rise to L-violating interactions. Thus we notice that starting from the bilinear interaction \( LH_2 \), trilinear couplings of physical states very similar to those conventionally parametrized by \( \lambda \) and \( \lambda' \) automatically emerge.

In the interactions presented below, we have designated by \( e^i, \nu^i (\bar{e}^i, \bar{\nu}^i) \) the fermion (scalar) mass eigenstates which are dominantly leptons (sleptons) of the \( i \) th generation.
The scalar (pseudoscalar) dominated by the real (imaginary) part of $\tilde{\nu}_L^i$ is described as $\tilde{\nu}_L^i (\tilde{\nu}_L^j)$. Thus we end up with trilinear terms in the Lagrangian, given by

$$\mathcal{L}_{tr} = \mathcal{L}_1 + \mathcal{L}_2$$

with

$$\mathcal{L}_1 = \rho_{3i} \bar{e}_R^i \nu_L^i + \rho'_{333} \bar{e}_R^i \nu_L^i + \omega_{3i} \nu_L^i e_R^i$$

$$+ \omega'_{3i} \bar{\nu}_L^i e_R^i + \eta_{3i} \bar{e}_R^i \nu_L^i + \xi_{3i} \bar{\nu}_L^i \nu_L^i + \zeta_{3i} \bar{e}_R^i \nu_L^i + \zeta'_{3i} \bar{\nu}_L^i \nu_L^i$$

$$+ \zeta\nu_L^3 + \zeta'\nu_L^3 + \delta_{333} \bar{\nu}_L^i e_R^i e_L^i + \text{H.c.},$$

$$\mathcal{L}_2 = \Omega_{3i} \bar{e}_R^i \nu_L^i \bar{d}_L^i + \Omega'_{3i} \bar{e}_R^i d_L^i \bar{u}_L^i + \Lambda_{3i} \bar{\nu}_L^i \nu_L^i \bar{d}_L^i$$

$$+ \Lambda'_{3i} \bar{d}_L^i \nu_L^i \bar{d}_L^i + \Lambda''_{3i} \bar{u}_L^i \nu_L^i \bar{d}_L^i + \Lambda'''_{3i} \bar{\nu}_L^i \nu_L^i \bar{d}_L^i$$

$$+ \Psi_{ij3} \bar{u}_R^i \nu_L^j \bar{e}_L^j + \Psi'_{ij3} \bar{d}_R^i \nu_L^j \bar{e}_L^j + \Psi''_{3ij} \bar{e}_R^i \nu_L^j \bar{d}_R^j$$

$$+ \Psi'''_{3ij} \bar{\nu}_L^i \nu_L^j \bar{d}_R^j + \Delta_{i3j} \bar{d}_R^i \bar{e}_L^j \bar{u}_L^j + \Delta'_{i3j} \bar{\nu}_L^i \nu_L^j \bar{d}_R^j$$

$$+ \Delta''_{ij3} \bar{d}_R^i \nu_L^j \bar{d}_R^j + \Sigma_{3ij} \bar{d}_R^i \nu_L^j \bar{\nu}_L^j + \Sigma'_{3ij} \bar{u}_R^i \nu_L^j \bar{\nu}_L^j$$

$$+ \Sigma''_{3ij} \bar{d}_R^i \nu_L^j \bar{\nu}_L^j + \chi_{ij3} \bar{u}_R^i \nu_L^j \bar{d}_R^j + \chi'_{ij3} \bar{u}_R^i \nu_L^j \bar{\nu}_L^j$$

$$+ \chi''_{ij3} \bar{u}_R^i \nu_L^j \bar{\nu}_L^j + \text{H.c.},$$

where the notation used for the quark and squark fields is obvious. The detailed expressions for the different couplings in terms of the elements of the mixing matrices will be found in the Appendix. Wherever the index 3 has been kept fixed in the couplings, it is because only the third generation of leptons mixes with Higgs in this picture.

It is instructive to compare the above Lagrangian with that obtained from $\lambda$- and $\lambda'$-type trilinear terms in the superpotential. In the notation of reference [5], such interactions are

$$\mathcal{L}_\lambda = \lambda_{ijk} [\bar{\nu}_L^i e_R^k \nu_L^j + \bar{e}_R^j e_R^k \nu_L^i + e_R^k \nu_L^i e_L^j - (i \leftrightarrow j)] + \text{H.c.},$$

(23)
\[ \mathcal{L}_N = \lambda'_{ijk} \left[ \bar{\nu}_L^i d_R^k d_L^j + \bar{\nu}_L^i d_R^k \nu_L^i + \bar{\nu}_L^i \bar{\nu}_L^i d_R^k d_L^j - \bar{e}_L^i d_R^k u_L^j - \bar{e}_L^i e_L^i - \bar{e}_R^{k*} e_L^i u_L^j \right] + \text{H.c.} \quad (24) \]

First of all, all the terms in equation (23) can be generated in \( \mathcal{L}_1 \) if one allows for mixing of the three leptonic generations and also Yukawa coupling of the first two generations. The fact that we have neglected both of the above features is responsible for the absence of coefficients in equation (21) with all three indices different. On the other hand, \( \mathcal{L}_1 \) can contain coefficients with generation indices \{iii\} (in our case, \{333\} only because of reasons stated above). Such terms are forbidden in equation (23) by gauge invariance of the superpotential, unless there is mixing among the lepton generations.

Next, we note that the terms in \( \mathcal{L}_1 \) proportional to \( \rho, \eta, \xi, \zeta \) and \( \zeta' \) do not arise in (23). The \( \rho \)-term owes its structure to the gaugino and Higgsino couplings in the MSSM part of the Lagrangian. The four remaining terms also could not be allowed in (23) because, again, \( SU(2) \) invariance of the superpotential would forbid terms with either three left chiral fields or one left and two right chiral fields. A particularly interesting consequence of this is the presence of trilinear interaction involving a sneutrino and two neutrino physical fields. This lends considerable additional phenomenology to the scenario under study here. For example, a sneutrino can decay into two neutrinos here \[7\]. Also, as we shall see below, it entails the possibility of invisible decays of the lightest neutralino.

On comparing equations (22) and (24) we find that all the \( \lambda' \)-type terms are generated in \( \mathcal{L}_2 \) as well. In addition, there are several more terms which are prevented in (24) in order to prevent weak isospin and hypercharge violation in the superpotential.

The other novel consequences of the \( LH_2 \) term are the flavor-changing couplings of the \( W \) and the \( Z \). Although it has been sometimes claimed in the literature \[17\] to be a signature of spontaneous \( R \)-parity violation, in practice it follows just from the bilinear terms of the type discussed by us. Diagonalisation of the mass matrices \( M_{\tilde{\chi}^\pm} \) and \( M_{\tilde{\chi}^0} \) immediately imply
that now there can be a tree-level interaction involving a $\tau(\nu_\tau)$ dominated physical state, a neutralino (chargino)-dominated state and a $W$. Similarly, the fact that the neutralinos (charginos) and the $\nu_\tau(\tau)$ differ in $T_3$ and $Y$ implies that their $Z$-couplings can now be non-diagonal. The interactions are given by

$$\mathcal{L}_{W-\tilde{\chi}+\tilde{\chi}^0} = gW_\mu \tilde{\chi}^0 \gamma^\mu [O^L_{ij} P_L + O^R_{ij} P_R] \tilde{\chi}^+_j + \text{H.c.,}$$

(25)

$$\mathcal{L}_{\tilde{\chi}^0 Z} = \frac{g}{\cos\theta_W} [\tilde{\chi}_i \gamma^\mu (O'_{ij}^L P_L + O'_{ij}^R P_R) \tilde{\chi}^0_j] Z_\mu + \text{H.c.,}$$

(26)

$$\mathcal{L}_{\tilde{\chi}^0 \tilde{\chi}^0 Z} = \frac{g}{\cos\theta_W} [\frac{1}{2} \tilde{\chi}_i \gamma^\mu (O''_{ij}^L P_L + O''_{ij}^R P_R) \tilde{\chi}^0_j] Z_\mu + \text{H.c.}$$

(27)

where the detailed forms of the matrices $O$, $O'$ and $O''$ are relegated to the Appendix.

We end this section by re-iterating that the bilinear interaction $LH_2$ is sufficient to generate all the $\lambda$- and $\lambda'$-type terms involving physical fields. They also give rise to other trilinear interaction terms which are otherwise disallowed. Furthermore, the postulate that only the third generation is involved in $L$-violating mixing (partially justified by the observed mass hierarchy) suggests that flavor changing trilinear terms should be smaller in magnitude.

3 Numerical Results

In order to find out the allowed region in the parameter space, one has to take a number of constraints into account. First, we note that $\epsilon$ and $v_3$ are the only parameters outside MSSM that enter into the chargino and neutralino mass matrices. The strongest constraint on them follows from the fact that the $\tau$-mass has been experimentally measured [12]. Therefore, for any combination of the MSSM parameters $(m_\tilde{g}, \mu, \tan\beta)$, the lowest eigenvalue of $m_{\tilde{\chi}^\pm}$ should agree with $m_\tau$ for any combination of $\epsilon$ and $v_3$. Also, $\nu_\tau$ has a laboratory upper limit of 24
MeV on its mass. These two restrictions, taken together, constrain the $\epsilon - v_3$ space in a severe manner.

Figures (1 - 4) show the allowed areas of the $\epsilon - v_3$ parameter space for several combinations of the MSSM parameters. Here, in addition to the constraint $m_{\nu_\tau} < 24$ MeV the lowest eigenvalue of the $m_{\tilde{\chi}^\pm}$ has been allowed at most a $3\sigma$-deviation from the measured central value of $m_\tau$. However, there is an extra parameter here to play with, namely, the third diagonal entry of $m_{\tilde{\chi}^\pm}$. In the figures presented here, we have fixed this term at the central experimental value of $m_\tau$, viz. 1.777 GeV. The allowed area slightly increases on varying this mass parameter, but it is not permissible to drift too much from the value used here. It is easy to check that our allowed region is consistent at 95% confidence level with the restrictions imposed by the global fit of LEP data and low energy experiments on the mixing of the $\tau$ and the $\nu_\tau$ with exotic fermions [18]. In any case, we find that there is no substantial allowed region with $|\epsilon|$ and $v_3$ larger than about 20 and 5 GeV respectively. Sometimes there are extremely narrow allowed bands with one of them of a considerably higher value. Such “fine-tuned” areas are not used in our subsequent calculations.

The next set of constraints arise from the scalar sector where all of the four conditions mentioned in the previous section have to be fulfilled. The scalar potential introduces several new parameters: $m_1, m_2, m_0$ (the slepton/sneutrino mass assuming a degeneracy), $A, B_1$ and $B_2$. Of these, the minimisation conditions imply that only three are independent. We have chosen $A, B_1$ and $B_2$ to be the three independent parameters. Furthermore, we set $A$ equal to zero to simplify our analysis. Now, taking $\epsilon$ and $v_3$ from the allowed regions described above, one gets restricted in the choice of $B_1$ and $B_2$. A definite requirement in this respect is that $B_1(B_2)$ should have a sign opposite to that of $\mu(\epsilon)$.

Having thus been guided to the allowed region in the entire parameter space, we can now compute all the $R$-parity violating couplings in terms of them. We have neglected all
CP-violating phases. The values of these for some sample values of the SUSY parameters are shown in Tables 1 and 2. The numbers indicate the maximum values that the respective couplings can have. Most of the couplings are seen to be on the order of $10^{-3}$ or less, excepting a few on the order of $10^{-2}$ or even $10^{-1}$. It is noticeable that a higher value of $\epsilon$ often raises the couplings. The cases where this does not happen can be ascribed to enhanced cancellations among the different terms that comprise a particular coupling. In particular, an enhancement in some of the terms occurs when the slepton mass $m_0$ is close to one of the Higgs masses, which causes a large slepton(sneutrino)-Higgs mixing. Wherever such mixing terms dominate in any interaction strength, the corresponding strength is large.

In general, the $R$-violating interactions that we obtain here after satisfying all requisite constraints are considerably smaller than the bounds on the analogous $\lambda$- and $\lambda'$-type terms derived in the literature from existing experimental data. The latter includes limits from a wide variety of phenomena, from low-energy weak processes to results from the Large Electron Positron (LEP) collider. This suggests that if indeed bilinear interactions are the real sources of the nonconservation of $R$-parity, then our experimental precision requires considerable improvement before such interactions can be probed.

Finally, let us turn to some processes that can be looked upon as the typical consequences of bilinear $R$-violating terms. Of course, the lightest neutralino $\tilde{\chi}^0$ (the LSP in MSSM) is bound to be unstable. When its mass is less than that of the standard gauge bosons, it can only have three-body decays. The final states for such decays are the same whether $R$-parity is violated originally through bilinear or trilinear interactions.

However, if $m_{\tilde{\chi}^0}$ is larger than $m_Z$, $m_W$, then the bilinear terms in our scenario open up two-body decay channels which are not otherwise possible. These are the channels $\tilde{\chi}^0 \rightarrow \tau W$ and $\tilde{\chi}^0 \rightarrow \nu Z$, controlled by $O_{L(R)}$, $O'_{L(R)}$ and $O''_{L(R)}$ of equations 25-27. In Table 3 we list their values for the same set of input parameters as in Tables 1 and 2.
Figures 5 and 6 contain some plots for the dominant branching ratios, assuming that $\tilde{\chi}^0 \rightarrow \tau W$ and $\tilde{\chi}^0 \rightarrow \nu_\tau Z$ are the only available channels. The former mode is found to dominate in figure 5, while the latter takes over in figure 6. This is because there are essentially two main components in each of the $\chi^0 \tau W$ and $\tilde{\chi}^0 \nu_\tau Z$ interactions. One of these comes from the neutrino-tau(neutrino)-W(Z) gauge couplings, and the other, from the Higgsino-Higgsino-W(Z) coupling. In the area of the parameter space shown in figure 5, it is found that while the two components add up in the former process, they interfere destructively in the latter, causing a large cancellation. Exactly the opposite thing happens in figure 6, where, in particular, $|\mu|$, $|B_1|$ and $|B_2|$ are large. This indicates that pair-produced neutralinos are expected to give rise to signals of the form $\tau \tau WW$ and $ZZ + p_T$. $R$-parity violation through bilinear interaction terms is quite characteristically reflected through such signals.

4 Summary and Conclusions

We have studied the effects of an $R$-parity violating bilinear term $L_3 H_2$ in the superpotential. We find that this term, together with a sneutrino vev, leads to trilinear couplings involving dominantly leptonic and sleptonic (as also two quarks/squarks and one lepton/slepton) physical fields. We emphasize that it is these terms involving physical fields which are of phenomenological significance. The interactions thus generated include the $\lambda$- and $\lambda'$-type ones which follow from trilinear $R$-parity violating terms in the superpotential. In addition, we obtain several terms that are not permitted in the other case. The most noteworthy among them is the one involving a sneutrino and two neutrinos. Also, there arise off-diagonal interactions of charginos and neutralinos with $\tau$ and $\nu_\tau$ coupled to a $W$ or a $Z$. Such interactions are the characteristic features of bilinear $R$-violation. The parameter space of such a scenario
can be best limited by restricting the lowest eigenvalues of the chargino and neutralino mass matrices. Further constraints follow from requirements of electroweak symmetry breaking in the scalar sector. The trilinear couplings thus generated mostly turn out to be small compared to their current phenomenological limits. Thus if bilinear terms are the sole sources of $R$-parity violation, then the restrictions imposed by the lepton and neutrino masses are still more stringent than any other experimental bound. And finally, we have discussed some phenomenological consequences of such a scenario. In particular, we show that if the lightest neutralino is heavier than the weak gauge bosons, then its dominant decay occurs in the channels $\tilde{\chi}^0 \rightarrow \tau W$ and $\tilde{\chi}^0 \rightarrow \nu_\tau Z$ giving rise to rather characteristic signals.

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Appendix A

Here we present the full forms of the various couplings in equations (21) and (22). In obtaining the trilinear interactions, the mixing among leptons and quarks in gaugino couplings have been neglected. Using the notation already established in the text,

\[ \rho_{131} = \rho_{232} = -gU^*_{31} \]  \hspace{1cm} (A.1)

\[ \rho_{333} = -gU^*_{31} + fU^*_{32}N^*_{55}C_{43} \]  \hspace{1cm} (A.2)

\[ \rho'_{333} = fV^*_{33}N^*_{55}C_{23} - fV^*_{33}N^*_{53}C_{33} \]  \hspace{1cm} (A.3)

\[ \omega_{131} = \omega_{232} = -gV^*_{31} \]  \hspace{1cm} (A.4)

\[ \omega_{333} = -gV^*_{31} + \frac{1}{\sqrt{2}} fV^*_{33}U^*_{32}S_{33} - \frac{1}{\sqrt{2}} fV^*_{33}U^*_{33}S_{13} \]  \hspace{1cm} (A.5)

\[ \omega'_{131} = \omega'_{232} = gV^*_{31} \]  \hspace{1cm} (A.6)

\[ \omega'_{333} = gV^*_{31} + \frac{1}{\sqrt{2}} fV^*_{33}U^*_{32}P_{33} - \frac{1}{\sqrt{2}} fV^*_{33}U^*_{33}P_{13} \]  \hspace{1cm} (A.7)

\[ \eta_{131} = \sqrt{2} eN^*_{51} - \frac{\sqrt{2} g}{\cos\theta_W} (-\frac{1}{2} + \sin^2\theta_W)N^*_{52} \]  \hspace{1cm} (A.8)

\[ \eta_{232} = \sqrt{2} eN^*_{51} - \frac{\sqrt{2} g}{\cos\theta_W} (-\frac{1}{2} + \sin^2\theta_W)N^*_{52} \]  \hspace{1cm} (A.9)

\[ \eta_{333} = \sqrt{2} eN^*_{51} - \frac{\sqrt{2} g}{\cos\theta_W} (-\frac{1}{2} + \sin^2\theta_W)N^*_{52} - f N^*_{53}U^*_{33}C_{43} \]  \hspace{1cm} (A.10)

\[ \xi_{131} = -\frac{\sqrt{2} g}{2\cos\theta_W} N^*_{52} \]  \hspace{1cm} (A.11)
\[
\xi_{232} = -\frac{\sqrt{2} g}{2 \cos \theta_W} N^*_{52} \tag{A.12}
\]

\[
\xi_{333} = -\frac{\sqrt{2} g}{2 \cos \theta_W} N^*_{52} \tag{A.13}
\]

\[
\zeta_{113} = -\sqrt{2} e N^*_{51} + \frac{\sqrt{2} g \sin^2 \theta_W}{\cos \theta_W} N^*_{52} \tag{A.14}
\]

\[
\zeta_{223} = -\sqrt{2} e N^*_{51} + \frac{\sqrt{2} g \sin^2 \theta_W}{\cos \theta_W} N^*_{52} \tag{A.15}
\]

\[
\zeta_{333} = -\sqrt{2} e N^*_{51} + \frac{\sqrt{2} g \sin^2 \theta_W}{\cos \theta_W} N^*_{52} + fV^*_{33} N^*_{55} C_{24} - fV^*_{33} N^*_{53} C_{34} \tag{A.16}
\]

\[
\zeta'_{333} = fU^*_{32} N^*_{55} C_{44} \tag{A.17}
\]

\[
\delta_{333} = -fU^*_{33} N^*_{53} C_{44} \tag{A.18}
\]

\[
\Omega_{3ii} = -gU^*_{31} \tag{A.19}
\]

\[
\Omega'_{3ii} = -gV^*_{31} \tag{A.20}
\]

\[
\Lambda_{3ii} = -\sqrt{2}\left\{\frac{g}{\cos \theta_W} \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W\right) N_{52} + \frac{2}{3} g \sin \theta_W N_{51}\right\} \tag{A.21}
\]

\[
\Lambda'_{3ii} = \sqrt{2}\left\{\frac{g}{\cos \theta_W} \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W\right) N_{52} - \frac{1}{3} g \sin \theta_W N_{51}\right\} \tag{A.22}
\]

\[
\Lambda''_{3ii} = -\sqrt{2}\left\{\frac{2}{3} \frac{g}{\cos \theta_W} \sin^2 \theta_W N^*_{52} - \frac{2}{3} g \sin \theta_W N^*_{51}\right\} \tag{A.23}
\]

\[
\Lambda'''_{3ii} = \sqrt{2}\left\{-\frac{1}{3} \frac{g}{\cos \theta_W} \sin^2 \theta_W N^*_{52} + \frac{1}{3} g \sin \theta_W N^*_{51}\right\} \tag{A.24}
\]
\[ \Psi_{ij3} = f_1 C_{23} \] (A.25)

\[ \Psi'_{ij3} = f_1 C_{24} \] (A.26)

\[ \Psi''_{3ij} = f_1 U^*_{32} \] (A.27)

\[ \Psi'''_{3ij} = -f_1 N^*_{53} \] (A.28)

\[ \Delta_{i3j} = f_1 U^*_{32} \] (A.29)

\[ \Delta'_{i3j} = -f_1 N^*_{53} \] (A.30)

\[ \Delta''_{ij3} = -\frac{1}{\sqrt{2}} f_1 S_{13} \] (A.31)

\[ \Delta'''_{ij3} = -\frac{1}{\sqrt{2}} f_1 P_{13} \] (A.32)

\[ \Sigma_{3ij} = f_2 V^*_{32} \] (A.33)

\[ \Sigma'_{3ij} = -f_2 N^*_{54} \] (A.34)

\[ \Sigma''_{ij3} = f_2 C_{13} \] (A.35)

\[ \Sigma'''_{ij3} = f_2 C_{14} \] (A.36)

\[ \chi_{ij3} = -\frac{1}{\sqrt{2}} f_2 S_{23} \] (A.37)

\[ \chi'_{ij3} = -\frac{1}{\sqrt{2}} f_2 P_{23} \] (A.38)
\[ \chi''_{i3j} = f_2 V^*_{23} \]  
(A.39)

\[ \chi'''_{i3j} = -f_2 N^*_{54} \]  
(A.40)

\( f_1 \) and \( f_2 \) are defined above as \( f_1 = h_{33}^d = \frac{m_b}{v_1} \), \( f_2 = h_{33}^u = \frac{m_t}{v_2} \). We have neglected the Yukawa interactions of the remaining quarks.

Next we give detailed forms of the matrices \( O, O' \) and \( O'' \) which appear in equations (25)-(27).

\[ O^L_{ij} = \frac{1}{\sqrt{2}} N_{i4} V^*_{j2} - \cos \theta_W N_{i2} V^*_{j1} - \sin \theta_W N_{i1} V^*_{j1} \]  
(A.41)

\[ O^R_{ij} = -\frac{1}{\sqrt{2}} N^{*}_{i3} U_{j2} - \cos \theta_W N^{*}_{i2} U_{j1} - \sin \theta_W N^{*}_{i1} U_{j1} + \frac{1}{\sqrt{2}} N^{*}_{i5} U_{j3} \]  
(A.42)

\[ O'_{ij}^L = V_{i1} V^*_{j1} + \frac{1}{2} V_{i2} V^*_{j2} - \delta_{ij} \sin^2 \theta_W + V_{i3} V^*_{j3} \sin^2 \theta_W \]  
(A.43)

\[ O'_{ij}^R = U^{*}_{i1} U_{j1} + \frac{1}{2} U^{*}_{i2} U_{j2} - \delta_{ij} \sin^2 \theta_W + U^{*}_{i3} U_{j3} (-\frac{1}{2} + \sin^2 \theta_W) \]  
(A.44)

\[ O''^L_{ij} = \frac{1}{2} N_{i3} N^*_{j3} - \frac{1}{2} N_{i4} N^*_{j4} + \frac{1}{2} N_{i5} N^*_{j5} \]  
(A.45)

\[ O''^R_{ij} = -\frac{1}{2} N^{*}_{i3} N_{j3} + \frac{1}{2} N^{*}_{i4} N_{j4} = -O''_{ij}^L \]  
(A.46)
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Table 1:

Sample values of the couplings in equation 21, for two values of $\epsilon$, with $v_3 = 3.4$, $\mu = 200$, $m_\tilde{g} = 750$, $B_1 = -180$, $B_2 = -160$, $\tan\beta = 2$. All the mass parameters are expressed in GeV.
Table 2:

Sample values of the couplings in equation 22, with the same input parameters as in Table 1.

|        | $\epsilon = 16$ | $\epsilon = 2$ |        | $\epsilon = 16$ | $\epsilon = 2$ |
|--------|-----------------|-----------------|--------|-----------------|-----------------|
| $\Omega_{3ii}$ | 0.00633         | -0.00589        | $\Delta_{i3j}'$ | 0.00464         | 0.00033         |
| $\Omega'_{3ii}$ | -0.00018        | 0.00017         | $\Delta''_{ij3}$ | 0.00309         | -0.00151        |
| $\Lambda_{i3i}$ | 0.00275         | -0.00256        | $\Delta'''_{ij3}$ | 0.00335         | -0.00054        |
| $\Lambda'_{i3i}$ | 0.00584         | -0.00543        | $\Sigma_{3ij}$ | 0.00048         | -0.00045        |
| $\Lambda''_{i3i}$ | 0.00235         | -0.00219        | $\Sigma'_{3ij}$ | -0.00238        | 0.00223         |
| $\Lambda'''_{i3i}$ | 0.00117         | -0.00109        | $\Sigma''_{ij3}$ | 0.01521         | 0.01113         |
| $\psi_{ij3}$ | 0.00308         | -0.00059        | $\Sigma'''_{ij3}$ | -0.01649        | -0.01182        |
| $\psi'_{ij3}$ | -0.00344        | 0.00057         | $\chi_{ij3}$ | -0.01785        | -0.04348        |
| $\psi''_{3ij}$ | -0.00467        | -0.00029        | $\chi'_{ij3}$ | 0.01674         | 0.01170         |
| $\psi'''_{3ij}$ | 0.00464         | 0.00033         | $\chi''_{i3j}$ | 0.00048         | -0.00045        |
| $\Delta_{i3j}$ | -0.00467        | -0.00029        | $\chi'''_{i3j}$ | -0.00238        | 0.00223         |
### Table 3

|       | \( \epsilon = 16 \) | \( \epsilon = 2 \) |
|-------|----------------------|----------------------|
| \( O^L_{51} \) | 0.00412 | -0.00385 |
| \( O^R_{51} \) | 0.06329 | -0.01545 |
| \( O^L_{52} \) | -0.00640 | 0.00595 |
| \( O^R_{52} \) | 0.10156 | 0.00201 |
| \( O^L_{43} \) | 0.00016 | -0.00015 |
| \( O^R_{43} \) | -0.05506 | 0.00550 |
| \( O^L_{32} \) | -0.00023 | 0.00022 |
| \( O^R_{32} \) | -0.05813 | -0.00390 |
| \( O^L_{31} \) | 0.00006 | -0.00006 |
| \( O^R_{31} \) | -0.03097 | 0.01152 |
| \( O''^L_{45} \) | -0.00501 | 0.00467 |
| \( O''^R_{45} \) | -0.01581 | -0.00146 |

Table 3:

Sample values of the couplings in equations 25-27, with the same input parameters as in Table 1.
Figure Captions

Figure 1:

The allowed region (dark) in the $\epsilon$-$v_3$ parameter space, with $B_1 = -180$, $B_2 = -160$, $\mu = 200$, $\tan\beta = 2$, $m_{\tilde{g}} = 750$. All mass parameters are expressed in GeV.

Figure 2:

Same as in Figure 1, with $B_1 = -160$, $B_2 = -170$, $\mu = 200$, $\tan\beta = 10$, $m_{\tilde{g}} = 750$.

Figure 3:

Same as in Figure 1, with $B_1 = 150$, $B_2 = 170$, $\mu = -200$, $\tan\beta = 2$, $m_{\tilde{g}} = 300$.

Figure 4:

Same as in Figure 1, with $B_1 = 150$, $B_2 = 200$, $\mu = -200$, $\tan\beta = 2$, $m_{\tilde{g}} = 750$.

Figure 5:

$B(\tilde{\chi}^0 \rightarrow \tau W)$ plotted against the lightest neutralino mass (in GeV). The bold (thin) line corresponds to $\tan\beta = 2$, $B_1 = -180$, $B_2 = -160$, $\epsilon = 5.0$, $v_3 = 2.0$ $\mu = 200$ ($\tan\beta = 10$, $B_1 = -160$, $B_2 = -170$, $\epsilon = 1.0$, $v_3 = 0.4$, $\mu = 200$). All mass parameters are expressed in GeV.

Figure 6:

$B(\tilde{\chi}^0 \rightarrow \nu_{\tau} Z)$ plotted against the lightest neutralino mass (in GeV). The bold (thin) line corresponds to $\tan\beta = 2$, $B_1 = -350$, $B_2 = -330$, $\epsilon = 10.0$, $v_3 = 3.0$ $\mu = 500$ ($\tan\beta = 10$, $B_1 = -280$, $B_2 = -290$, $\epsilon = 4.0$, $v_3 = 1.0$, $\mu = 500$).
Figure 1

$v_3$
Figure 3

$\mathbf{\nu}_3$ vs $\mathcal{E}$
Figure 5

$B(\tilde{\chi}^0 \rightarrow \tau W)$
Figure 6

$B(\chi^0 \rightarrow \nu Z)$