Enhancement of quantum transport efficiency in a noisy spin channel

Naghi Behzadi\(^1\)\(^\odot\), Bahram Ahansaz\(^2\) and Abbas Ektesabi\(^2\)

\(^1\)Research Institute for Fundamental Sciences, University of Tabriz, Iran
\(^2\)Physics Department, Azarbaijan Shahid Madani University, Iran

E-mail: n.behzadi@tabrizu.ac.ir

Received 22 February 2018, revised 29 April 2018
Accepted for publication 24 May 2018
Published 22 June 2018

Abstract

In this paper, based on the notion of the pseudomode framework introduced by Garraway in (1997 Phys. Rev. A 55 2290), we propose a mechanism for enhancing the efficiency of excitation energy transport in a spin channel which is in contact with a Lorentzian reservoir, through the inclusion of other similar auxiliary spin chains into the reservoir. To this aim, a Lindblad-type master equation for the dynamics of the transport process is provided on the basis of the pseudomode approach. It is ascertained that increasing the number of auxiliary chains in the reservoir enhances the transport efficiency or, equivalently, raises the population of the reaction center attached to the end of the channel. Moreover, it is interesting to note that the mechanism has better efficiency for channels with longer lengths.

Keywords: efficiency of quantum transport, spin channel, auxiliary chains, pseudomode approach, Lorentzian reservoir, Lindblad-type master equation

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum transport of charge or energy plays a central role in many scientific disciplines and is the basis of many physical, chemical or biological processes. The efficient transport of excitation energy through natural or artificial network of many-body interacting quantum systems has attracted the attention of many researchers in recent years [1–7]. However, the dynamics of quantum many-body systems are often investigated in one-dimensional structures, such as spin chains, due to the existence of exact analytical solutions in some cases [9] and efficient numerical methods for their simulation in other cases [11]. Quantum spin chains are the best known among the various physical systems that can serve as quantum channels and also can be realized in different physical systems such as arrays of Josephson junctions [13, 14], cold atoms in optical lattices [15], arrays of quantum dots [16, 17], etc.

In the context of quantum transport of excitation energy, such as in light harvesting (photosynthetic) complexes, manipulated dephasing noises have the ability to improve transport efficiency [18, 19] by removing local or global destructive interferences of coherent hopping [20]. However, in general, any real quantum system suffers dissipation due to unavoidable interaction with its surrounding environment [21, 22], which ultimately causes the irreversible decay of excitation energy into the environment. So, protecting the energy transport process from the noisy effects of the environment to improve its efficiency is a fundamental challenge in this field, as is in some other branches of quantum technology, such as quantum information processing [20–26].

Recently, the possibility of protecting quantum coherence and entanglement from environmental effects was demonstrated in a qubit system [27, 8], as well as in a qutrit system [10], through the entering of auxiliary qubits or qutrits into the related reservoir respectively. In fact, it was observed that by increasing the number of similar auxiliary systems, the desired system is well-protected from its environmental noisy effects. Therefore, motivated by these works, we propose a mechanism for enhancing the efficiency of excitation energy transport in a noisy spin channel which is in contact with a common Lorentzian dissipative reservoir. This procedure works through the inclusion of other similar auxiliary spin chains into the reservoir when the system-environment coupling constants have been designed in a particular way. In fact, via this design, we can exploit the advantages of the...
2. Dynamics of the system

2.1. The Hamiltonian

In this section, we consider $N$ identical linear spin chains interacting with a common structured reservoir, each of which consists of $M$ two-level systems or qubits with uniform nearest-neighbor interaction, as depicted in figure 1. The chains are non-interacting, i.e. they have no direct interaction with each other initially. The Hamiltonian of the whole system is considered as ($\hbar = 1$)

$$H = H_S + H_R + H_{SR},$$

where the system Hamiltonian $H_S$, the reservoir Hamiltonian $H_R$ and the system-reservoir interaction Hamiltonian $H_{SR}$ are given by the following equations, respectively.

$$H_S = \omega_0 \sum_{j=1}^{N} \sum_{l=1}^{M} \sigma_j^z \sigma_{j+1}^z + J \sum_{j=1}^{N} \sum_{l=1}^{M-1} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y),$$

$$H_R = \sum_{k} \omega_k a_k^\dagger a_k,$$

$$H_{SR} = \sum_{j=1}^{N} \sum_{l=1}^{M} (g_{kl}^R \sigma_j^+ a_k + g_{kl}^R \sigma_j^- a_k^\dagger).$$

(2) (3) (4)

In these equations, $\omega_0$ is the transition frequency of two-level atoms and $J$ is the strength of uniform coupling between the nearest-neighbor atoms in the spin chains. Also, $a_k$ ($a_k^\dagger$) is the annihilation (creation) operator of the $k$th field mode with frequency $\omega_k$ and $g_{kl}^R$ is the coupling strength between the $k$th field mode and the $l$th atom of the $j$th chain. The lowering operator $\sigma_j^+ = (\sigma_j^+)^\dagger$ is the annihilation operator of the $j$th chain into the excited state of the $j$th atom. The lowering operator $\sigma_j^- = (\sigma_j^-)^\dagger$ is the creation operator of the $j$th chain into the ground state of the $j$th atom. The lower case letters are used to denote the $k=0$ and $k=M$ modes.

Figure 1. A schematic representation of a spin channel in the presence of, e.g., four similar auxiliary chains, each of which has $M = 4$ spins, and all of them are contained in a common structured reservoir (the yellow circle). The first site of the channel is initially populated (the orange site), and the green arrow indicates an irreversible transfer of excitation from the last site of the channel into the sink.

$$|j, l\rangle \equiv |0\rangle^\otimes M \cdots |0\rangle^\otimes M \otimes \left( 0_{j-1} \otimes 0_{l-1} \otimes 0_0 \right) \otimes |0\rangle^\otimes M \cdots |0\rangle^\otimes M$$

(5)

is a typical standard basis state for the $MN$-dimensional single excitation subspace of the chains. In fact, $|j, l\rangle$ indicates that there exists an excitation in the $j$th site of the $l$th atom with $j = 1, 2, \ldots, N$ and $l = 1, 2, \ldots, M$, and it can be considered as a set of basis for the single excitation subspace of $N$ similar chains, each of which has $M$ identical atoms. In the following, we define another set of basis in terms of the standard basis (5), as

$$|\varphi_j^l\rangle = \sqrt{\frac{2}{M+1}} \sum_{i=1}^{M} \sin(iq_l) |j, i\rangle = \sum_{i=1}^{M} u_{jl|i},$$

(6)

which are eigenstates of the $H_S$ with corresponding
As a further illustration, it is clear from the end part of equation (7) that each field mode of the reservoir has identical interactions with all of the chains. It can depend on the configuration of the chains inside the reservoir. Roughly speaking, the reservoir could be considered as electromagnetic radiations inside an imperfect cavity formed by two identical spherical mirrors [33]. Obviously, the cavity modes have a Lorentzian spectral density [34]. Since the cavity has cylindrical symmetry, all of the chains in the cavity which are parallel to the cavity axis, with equal radius distances and also with equal distances from the mirrors, are coupled to the cavity modes in the same way. Therefore, by this argument, each field mode of a common Lorentzian reservoir, which collectively interacts with a spin chain [32, 35], can be coupled to the arbitrary number of the other similar chains in the same way.

Now let us consider one of the chains as the spin channel under study, and the remaining ones as auxiliary chains. By considering the Schrödinger equation for the Hamiltonian (7), and that the reservoir spectral density is Lorentzian, the exact dynamics for the N-chain open system (as well as for the spin channel) can be obtained in a similar way to [36] carried out for a three-level V-type atom. However, in general, the addition of a sink site as a reaction center to the end of the spin channel causes difficulties in obtaining useful dynamics for describing the excitation transport from the initial site of the channel into the reaction center. In other words, it is not only impossible to obtain an exact master equation for the process of excitation transport into the reaction center, but also it is so difficult to exploit an alternative perturbative Lindblad-type master equation. Nevertheless, from the viewpoint of the pseudomode approach [30], there exists an exact Lindblad-type master equation for the dynamics of a multilevel V-type atomic system interacting with a Lorentzian structured reservoir. In fact, the pseudomode approach enables us to include another Lindblad term corresponding to the irreversible dissipation process arisen from the presence of a reaction center attached to the end of the channel. Therefore, an efficient framework can be provided for analysing the excitation transport process, along with the discussion about the improvement of its efficiency on the basis of pseudomode approximation, which will be illustrated in the next subsection.

2.2. The pseudomode approach

For a given reservoir, pseudomodes are auxiliary variables introduced in one-to-one correspondence with the poles of the respective spectral distribution function in the complex frequency plane. This approach allows us to derive an exact master equation without using the perturbation theory, Born or Markov approximations. Such an exact master equation describes the coherent interaction between the system and the pseudomodes in the presence of decay of the pseudomodes due to the interaction with a Markovian reservoir. For example, this approach has been utilized to develop an equivalent master equation for a three-level V-type system [37], multilevel V-type [30], and a three-level atom in a ladder configuration [39]. Also, the exact entanglement dynamics of
a two-qubit system which is in contact with a common structured reservoir has been studied by using the pseudomode mechanism through establishing a connection with a three-level ladder system [38].

In the following, we briefly review the key features of the pseudomode theory which has been utilized to derive an exact Lindblad-type master equation for the dynamics of a multilevel V-type atomic system, as discussed in [30]. To this aim, we consider a multilevel V-type atomic system coupled to a bath of harmonic oscillators. This atom consists of a single ground state $|0\rangle_a$ coupled to a number of upper excited states such as $|\lambda\rangle_a (\lambda = 1, 2, ..., N)$ with the corresponding energy difference $\omega$. Thus, the Hamiltonian of the system is given by

$$H' = \sum_{i=1}^{N} \omega_i |\lambda\rangle_a \langle i| + \sum_{\lambda} \omega_\lambda a_\lambda^{\dagger} a_\lambda + \sum_{i,\lambda} \Omega_i a_\lambda^{\dagger} |0\rangle_a + \Omega_\lambda^{\ast} a_\lambda |1\rangle_a.$$

where $a_\lambda$ and $a_\lambda^{\dagger}$ are the respective annihilation and creation operators for each oscillation mode of the reservoir with frequency $\omega_\lambda$, and $\Omega_\lambda$ is the coupling strength of the $\lambda$th excited state of the multilevel atom and the $\lambda$th mode of the reservoir. To describe a one-photon excitation process, we work in the single excitation subspace where the state vector of the total system is written as

$$|\psi(t)\rangle = c_0(t)|0\rangle_a |0\rangle_e + \sum_i c_i(t)|i\rangle_a |0\rangle_e + \sum_\lambda c_\lambda(t)|0\rangle_a |1\rangle_e.$$

Here, the ket $|0\rangle_e$ shows that all of the reservoir modes are in their respective vacuum states while the ket $|1\rangle_e$ indicates that all of the reservoir modes, except the $\lambda$ one, are in the ground states.

Substitution of equations (8) and (9) into the time dependent Schrödinger equation $id/dt |\psi(t)\rangle = H' |\psi(t)\rangle$ leads to the following infinite set of coupled differential equations:

$$i\frac{dc_0(t)}{dt} = \omega c_0(t) + \sum_{\lambda} \Omega_\lambda c_\lambda(t),$$

$$i\frac{dc_i(t)}{dt} = \omega_i c_i(t) + \sum_{\lambda} \Omega_\lambda c_\lambda(t).$$

It is clear that the coefficient $c_0(t)$ will be invariant in time, which means $c_0(t) = c_0(0)$. Formally, eliminating the coefficient $c_0(t)$ enables us to derive an integro-differential equation for the amplitudes of the excited states of the atom, i.e. $c_i(t)$. After some algebraic manipulations, a closed set of integro-differential equations for $c_i(t)$, $i = 1, 2, 3, ..., N$, is obtained as

$$i\frac{dc_i(t)}{dt} = -i\omega_i c_i(t) - \int_0^t dt' \sum_{j} G_{ij}(t - t') c_j(t'),$$

where the kernel function $G_{ij}(t-t')$ can be expressed in terms of the spectral density $J_y(\omega)$ of the reservoir as follows:

$$G_{ij}(t-t') = \int_{-\infty}^{\infty} d\omega J_y(\omega)e^{-i\omega(t-t')}.$$

with

$$J_y(\omega) = \frac{\Omega_i \Omega_j}{2\pi} D(\omega).$$

Here, $\Omega_i (\Omega_i^2 = \sum_{\lambda} |\Omega_\lambda|^2)$ is the coupling strength between the $i$th excited state of the $(N + 1)$-level atom and the reservoir, and $D(\omega)$ is the reservoir structure function with the normalization $\int_{-\infty}^{\infty} d\omega D(\omega) = 2\pi$, which enables us to consider the various types of the reservoir.

It is very useful to calculate the integral (12) from a contour in the complex $\omega$ plane which is closed in the lower half plane due to the vanishing of the exponential part of the integrand (because $t > t'$). So, taking a contour in the lower half plane gives

$$G_{ij}(t-t') = -i\Omega_i \Omega_j \int_{r_1} d\omega D(\omega) e^{-i\omega(t-t')}.$$

We suppose that the function $D(\omega)$ has poles in the lower half plane at $z_1, z_2, ..., z_l$ and the residues of $D(\omega)$ are denoted by $r_1, r_2, ..., r_l$. Then, by using the theorem of residues, we have

$$G_{ij}(t-t') = -i\Omega_i \Omega_j \sum r_l e^{-i\omega(t-t')}.$$

As a result, the integral in equation (12) can be converted into a sum over $l$ which makes it easier to calculate if we know the positions and residues of the poles of $D(\omega)$. Each of the poles of the reservoir in the lower half complex $\omega$ plane will be associated with one pseudomode. In the following, by inserting equation (15) into equation (11), we find that

$$i\frac{dc_i(t)}{dt} = \omega_i c_i(t) - \sum_{l} \Omega_i r_l e^{-i\omega(t-t')} \int_{0}^{t} dt'' e^{i\omega(t''-t')} c_j(t'').$$

Now, based on equation (16), we can introduce a fictional pseudomode amplitude as

$$s_i(t) = -i\sum_{l} \sqrt{-ir_l} e^{-i\omega t} \int_{0}^{t} dt'' e^{i\omega(t''-t')} c_j(t'').$$

So, equation (11) can be converted into the following form:

$$i\frac{dc_i(t)}{dt} = \omega_i c_i(t) + \sum_{l} g_{il} s_i(t),$$

$$i\frac{ds_i(t)}{dt} = \omega_i s_i(t) + \sum_{l} g_{il} c_i(t),$$

where the last equation follows from the differentiation of equation (17) and the coupling between the pseudomode $l$ and the atomic level $i$ is $g_{il} = \sqrt{-ir_l}$. Furthermore, it should be noted that equation (12) implies that $G_{ij}(0) = \Omega_i \Omega_j$ and thus, by considering equation (15), we always have

$$\sum_{l} (-ir_l) = 1.$$
take the Lindblad-type master equation [30]
\[ \frac{\partial \rho}{\partial t} = -i[H_0, \rho] + \sum_{l} \left( L^\dagger_l \rho L^\dagger_l - \frac{1}{2} L^\dagger_l L^\dagger_l \rho - \frac{1}{2} \rho L^\dagger_l L^\dagger_l \right), \]
\[ (20) \]
where \( \rho \) is the density matrix of the system comprised of the pseudomodes and the atom and \( H_0 \) is the Hermitian Hamiltonian defined as
\[ \begin{align*}
H_0 &= \sum_{i=1}^{N} \omega_i |i\rangle \langle i| + \sum_{l} \text{Re}(z_l) b^\dagger_l b_l \\
&\quad + \sum_{i=1}^{N} \sum_{l} g_i^l (|i\rangle \langle 0| b_l + |0\rangle \langle i| b^\dagger_l). \end{align*} \]
\[ (21) \]
In equation (21) the operator \( b_l \) \( (b^\dagger_l) \) is the annihilation (creation) operator for the excitation of the fictional mode \( l \). Also, the Lindblad operators involving the pseudomodes take the form
\[ L_l = \sqrt{-2 \text{Im}(z_l)} b_l, \]
\[ (22) \]
In the following, we consider the most straightforward example of a single pseudomode given by the Lorentzian spectral distribution with the structure function
\[ D(\omega) = \frac{\Gamma}{(\omega - \omega_c)^2 + (\Gamma/2)^2}, \]
\[ (23) \]
which has a single pole in the lower half complex frequency plane as
\[ z_l = \omega_c - i \frac{\Gamma}{2}, \]
\[ (24) \]
where the constants \( \Gamma \) and \( \omega_c \) are the decay rate and frequency of the pseudomode, respectively. Since we have a single pole at \( z_l \), then the normalization property in equation (19) gives \((-i\Gamma_l) = 1\). As a result, the single pseudomode coupling is a real quantity and it is given by \( g_0^l = \Omega_s \sqrt{-i\Gamma_l} = \Omega_s \). Now, exploiting the pseudomode approach for our system of \( N \) chains contained in a Lorentzian structured reservoir gives the following master equation:
\[ \frac{\partial \tilde{\rho}}{\partial t} = -i[H, \tilde{\rho}] + \frac{\Gamma}{2} (2b^\dagger \tilde{b} b - b^\dagger b \tilde{b}^\dagger + \tilde{b} b^\dagger b) \equiv \mathcal{L}(\tilde{\rho}), \]
\[ (25) \]
where \( \tilde{\rho} \) is the density matrix of the extended system, i.e., the system comprised of the chains and the pseudomode, and \( \mathcal{H} \) is the Hermitian Hamiltonian given in equation (21) as
\[ \begin{align*}
\mathcal{H} &= \sum_{j=1}^{M} \sum_{j=1}^{N} E_j \langle \varphi'_j | \varphi_j \rangle + \omega_c b^\dagger b \\
&\quad + \sum_{j=1}^{N} \Omega_j \langle \varphi_j | (0) b + |0\rangle \langle \varphi'_j | b^\dagger). \end{align*} \]
\[ (26) \]
Here, \( \Omega_j \) corresponds to the coupling strength between the single pseudomode and the \( j \)th excited state of the \((N + 1)\)-level V-type system. Henceforth, we assume that \( \Omega_1 = \Omega_2 = \ldots = \Omega_N = \Omega_0 \), which means that all of the spin chains have identical interactions with the environment.

3. Results

In the previous section, we obtained a Lindblad-type master equation for the dynamics of the \( N \) identical chains interacting with a common Lorentzian reservoir in a similar way of interaction of the \((N + 1)\)-level V-type system. In this section, based on the pseudomode framework, we explicitly investigate the improvement of the efficiency of excitation transport in the spin channel, specified by \( j = 1 \), through the inclusion of \((N - 1)\) auxiliary sites into the reservoir. The first site of the channel has been initially excited, while its \( M \)th site has been attached to the sink site or reaction center (see figure 1). The Lindblad-type master equation for the excitation transport in the spin channel is as follows:
\[ \frac{\partial \tilde{\rho}}{\partial t} = \mathcal{L}(\tilde{\rho}) + \mathcal{L}_{\text{sink}}(\tilde{\rho}), \]
\[ (27) \]
where \( \rho \) is the density matrix of the extended system comprised of the chains, sink site and the pseudomode. The term \( \mathcal{L}(\tilde{\rho}) \) is same as the equation (25), while \( \mathcal{L}_{\text{sink}}(\tilde{\rho}) \) describes the population of the sink site through an irreversible decay process from the chosen \( M \)th site of the channel, which is given by
\[ \mathcal{L}_{\text{sink}}(\tilde{\rho}) = \frac{\Gamma_{\text{sink}}}{2} (2 \sigma_{\text{sink}}^\dagger \sigma_{\text{sink}}^\dagger \tilde{\rho} \sigma_{\text{sink}}^\dagger \sigma_{\text{sink}} - \{ \sigma_{\text{sink}}^\dagger \sigma_{\text{sink}}^\dagger \tilde{\rho} \sigma_{\text{sink}}^\dagger \sigma_{\text{sink}}, \tilde{\rho} \}). \]
\[ (28) \]
In equation (28), \( \Gamma_{\text{sink}} \) is the rate of the dissipative process that reduces the number of excitations in the channel and traps it in the sink, and \( \sigma_{\text{sink}}^\dagger \) and \( \sigma_{\text{sink}} \) are the raising and lowering operators of the sink site, respectively. By integrating equation (27), the population of the sink or transport efficiency is obtained as
\[ P_{\text{sink}}(t) = 2 \Gamma_{\text{sink}} \int_0^t \rho_{M,M}(t') dt'. \]
\[ (29) \]
Now, our main claim is that the efficiency of excitation transport from the initial site of the channel to the reaction center can be improved by entering the similar auxiliary channels into the reservoir. In other words, the transport efficiency in the spin channel can be well-controlled using the auxiliary chains.

Numerical analysis demonstrates the performance of the introduced protocol. Figures 3 and 4 show the population of the sink for the channels with \( M = 3, 5 \) qubits. For the channel with \( M = 3 \) qubits, it is observed that the quality of quantum transport is seriously degraded due to the dissipation of excitation energy throughout the channel in the absence of auxiliary chains \((N = 1)\), as shown in figure 3. Meanwhile, transport efficiency is considerably improved by entering the auxiliary three-qubit chains \((N = 2, 6)\) into the reservoir (figure 3). Similar behaviors take place for the case of the five-qubit channel (see figure 4). It is interesting to note that the protocol has better efficiency for the channels with longer lengths. This can be easily illustrated using the fact that, in the Hamiltonian (7) and (26), the interaction of the \( N \) chains with the reservoir is established only through the \( N \)-dimensional subspace (the excited states of the \((N + 1)\)-level V-type
configuration), so we have a noise-free subspace of dimension \((M - 1)N\), which increases by increasing the length of the chains. Intuitively, the ratio of the dimension of the coupled subspace to the reservoir and the decoupled (noise-free) one is \(\frac{1}{M-1}\). Evidently, for a channel with longer length, we have a better decoupling from environmental noise effects, so transport efficiency is improved.

As is depicted in figures 3 and 4, for single chains (especially for the channel with \(M = 3\)), the population has been distributed, in a steady way, between the reservoir and the sink. To illustrate why this is so, consider the three-qubit channel whose initial excitation is written as \(|100\rangle = \frac{1}{\sqrt{2}}(|1^{e=1}\rangle + |1^{e=2}\rangle + |1^{e=3}\rangle\), where \(|1^{e=1}\rangle = \frac{1}{\sqrt{2}}(|100\rangle + \sqrt{2}|010\rangle + |001\rangle\), \(|1^{e=2}\rangle = \frac{1}{\sqrt{2}}(|100\rangle - \sqrt{2}|010\rangle + |001\rangle\) and \(|1^{e=3}\rangle = \frac{1}{\sqrt{2}}(|100\rangle - |001\rangle\)

Obviously, \(|100\rangle\) has a support on the decoupled subspace spanned by \(|\varphi_1^{e=1}\rangle, |\varphi_1^{e=2}\rangle\), so the interaction of the channel with the reservoir is possible only through the eigenstate \(\varphi_1^{e=1}\), which is the excited state of a two-level system with transition frequency \(E_{r-1} = \omega_0 + 2J\cos(q_{r-1})\) (see figure 2 for a single chain case). Therefore, it is expected that the sink is ultimately populated at least by 0.75. However, the population of the sink never reaches this value, as shown in figure 3. As an illustration, we know from [4–6] that due to destructive interferences, the eigenstates \(\{|\varphi_1^{e=2}\rangle, |\varphi_1^{e=3}\rangle\}\) are decoupled from the common reservoir. On the other hand, the presence of the sink attached irreversibly to the end site of the channel affects the coherence of these states and so removes the destructive interferences from them, so there is a leakage for trapped excitation in that subspace into the reservoir too. Therefore, the amount of initial excitation trapped in the mentioned decoupled subspace cannot be

**Figure 3.** Population of the sink (or transport efficiency) in terms of time (in units of \(\omega_0^{-1}\)) for a spin channel of length \(M = 3\) spins, without using the auxiliary chains, i.e., \(N = 1\) (the solid green curve), using one auxiliary chain, i.e. \(N = 2\) (the dotted-dashed blue curve) and using five auxiliary chains, i.e. \(N = 6\) (the dashed red curve). These panels are plotted with the fixed parameters \(\Omega_0 = 1.02\) (in units of \(\omega_0\)), \(\Omega_0 = 0.15\) (in units of \(\omega_0\)) and \(\Gamma_{sink} = 0.6\) (in units of \(\omega_0\)).
completely transferred to the reaction center. In addition, it is observed that for a fixed $\Gamma_{\text{sink}}$ transport efficiency in the non-Markovian regime, i.e., $\frac{\Gamma}{\Omega_0} < 4$ (see [21, 38]) is better than the one in the Markovian regime, i.e., $\frac{\Gamma}{\Omega_0} > 4$, as shown in figures 3 and 4. Consequently, it is found out that the method of this paper for the enhancement of the efficiency of energy transport in linear noisy channels works well in both Markovian and non-Markovian regimes. Also, in this regard, increasing the $\Gamma_{\text{sink}}$ yields the improvement of transport efficiency. Another point which should be noted here is that, based on the results obtained in [27, 40], the maximum decay of excitation energy into the environment can occur in resonance conditions; then in order to better observe the role of the auxiliary chains on the transport efficiency, the central frequency of the reservoir, i.e. $\omega_c$, is assumed to be in (almost) resonance with $\omega_0$, i.e. the transition frequency of atoms. Finally, as can be illustrated using [31], the strength of nearest-neighbor coupling $J$ has an instructive role in the short-time population of the sink.

Finally, we explain why the increasing number of the auxiliary chains in the reservoir leads to the improvement of the transport efficiency in the considered channel. To this aim, let us remember from [27–10] that the protection of entanglement or coherence in a qubit (two-dimensional) system or in a qutrit (three-dimensional) system is achieved through entering auxiliary qubits or qutrits into the related reservoir, respectively. In fact, entering auxiliary systems into the respective reservoirs leads to more separation of the system-reservoir bound state, as an isolated eigenstate of the whole system, from the remainder spectrum. This approach can be extended for the protection of an open quantum system with $M$-dimensional Hilbert space, which could be considered as a spin chain with $M$ spin in the single excitation subspace. Therefore, entering the other similar auxiliary chains into the reservoir leads to the improvement of the formed bound state,
i.e., a better separation of this from the remainder spectrum of the whole system. Consequently, by this situation, the mentioned spin channel of length $M$ is protected from the dissipative effect of the reservoir and so the excitation energy can be transferred into the sink with an improved transport efficiency.

4. Conclusions

We presented a method for enhancing the transport efficiency of excitation energy in a noisy spin channel. It was indicated that the transport process in such a channel, whose end has been attached irreversibly to a reaction center and is in contact with a dissipative Lorentzian structured reservoir, can be described well by a Lindblad-type master equation obtained using the pseudomode approach. By using this framework, we observed as a result that the transport efficiency is improved well through the addition of some other similar auxiliary spin chains into the reservoir. Moreover, we showed that this procedure has better efficiency for channels with longer lengths.

ORCID iDs

Naghi Behzadi @ https://orcid.org/0000-0001-7980-9942

References

[1] Ferrari D, Celardo G L, Berman G P, Sayre R T and Borgonovi F 2014 J. Phys. Chem. C **118** 20
[2] Ghosh P K, Smirnov A Y and Nori F 2011 J. Chem. Phys. **134** 244103
[3] Ghosh P K, Smirnov A Y and Nori F 2011 Phys. Rev. E **84** 061138
[4] Plenio M B and Huelga S F 2008 New J. Phys. **10** 113019
[5] Caruso F, Chin A W, Datta A, Huelga S F and Plenio M B 2009 J. Chem. Phys. **131** 105106
[6] Chin A W, Datta A, Caruso F, Huelga S F and Plenio M B 2010 New J. Phys. **12** 065002
[7] Scholak T, Wellens T and Buchleitner A 2011 J. Phys. B: At. Mol. Opt. Phys. **44** 184012
[8] Cui B, Yi X X and Oh C H 2012 J. Phys. B: At. Mol. Opt. Phys. **45** 085501
[9] Takahashi M 2005 Thermodynamics of One-Dimensional Solvable Models (Cambridge: Cambridge University Press)
[10] Caruso F 2010 New J. Phys. **16** 055015
[11] White S R 1992 Phys. Rev. Lett. **69** 2863
[12] Marais A, Sinayskiy I, Kay A, Petruccione F and Ekert A 2013 New J. Phys. **15** 013038
[13] Tsomokos D I, Hartmann M J, Huelga S F and Plenio M B 2007 New J. Phys. **9** 79
[14] Romito A, Fazio R and Bruder C 2005 Phys. Rev. B **71** 100501
[15] Duan L M, Demler E and Lukin M D 2003 Phys. Rev. Lett. **91** 090402
[16] Benjamin S C and Bose S 2003 Phys. Rev. Lett. **90** 247901
[17] Nikolopoulos G M, Petrosyan D and Lambropoulos P 2004 Europhys. Lett. **65** 297
[18] Behzadi N and Ahansaz B 2015 H. kassani Phys. Rev. E **92** 042103
[19] Zhang M, Lee T E and Sadeghpour H R 2015 Phys. Rev. A **91** 052101
[20] Rebentrost P, Mohseni M, Kassal I, Lloyd S and Aspuru-Guzik A 2009 New. J. Phys. **11** 033003
[21] Breuer H P and Petruccione F 2002 The Theory of Open Quantum Systems (Oxford: Oxford University Press)
[22] Gardiner C W and Zoller P 2000 Quantum Noise (Berlin: Springer)
[23] Maniscalco S, Francisa F, Zaffino R, Gullo N and Plastina F 2008 Phys. Rev. Lett. **100** 090503
[24] Hou Y, Zhang G, Chen Y and Fan H 2012 Ann. Phys. **327** 292
[25] Kim Y S, Lee J C, Kwon O and Kim Y H 2011 Nat. Phys. **8** 117
[26] An N Ba 2013 Phys. Lett. A **337** 2520
[27] Behzadi N, Ahansaz B and Faizi E 2017 Eur. Phys. J. D **71** 280
[28] Behzadi N, Ahansaz B, Faizi E and Kassani H 2018 Quantum Inf. Process. **17** 65
[29] Ahansaz B and Behzadi N 2017 arXiv:1707.02625
[30] Garraway B M 1997 Phys. Rev. A **55** 2290
[31] Christandl M, Datta N, Ekert A and Landahl A J 2004 Phys. Rev. Lett. **92** 187902
[32] Cormick C, Bermudez A, Huelga S and Plenio M B 2013 New J. Phys. **15** 1073027
[33] Kimble H J 1998 Phys. Scr. **76** 127
[34] Francica F, Maniscalco S, Piloto J, Plastina F and Suominen K A 2009 Phys. Rev. A **79** 032310
[35] Huelga S F, Rivas A and Plenio M B 2012 Phys. Rev. Lett. **108** 160402
[36] Behzadi N, Ahansaz B, Ektesabi A and Faizi E. 2017 Ann. Phys. **378** 407
[37] Garraway B M and Knight P L 1996 Phys. Rev. A **54** 3592
[38] Mazzola L, Maniscalco S, Piloto J, Suominen K A and Garraway B M 2009 Phys. Rev. A **79** 042302
[39] Dalton B J and Garraway B M 2003 Phys. Rev. A **68** 033809
[40] Xiao X, Li Y, Zeng K and Wu C 2009 J. Phys. B: At. Mol. Opt. Phys. **42** 235502