Spatial MAC in MIMO Communications and its Application to Underlay Cognitive Radio

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Abstract

We propose a learning technique for MIMO secondary users (SU) to spatially coexist with Primary Users (PU). By learning the null space of the interference channel to the PU, the SU can utilize idle degrees of freedom that otherwise would be unused by the PU. This learning process does not require any handshake or explicit information exchange between the PU and the SU. The only requirement is that the PU broadcasts a periodic beacon that is a function of its noise plus interference power, through a low rate control channel. The learning process is based on energy measurements, independent of the transmission schemes of both the PU and SU, i.e. independent of their modulation, coding etc.. The proposed learning technique also provides a novel spatial division multiple access mechanism for equal-priority MIMO users sharing a common channel that highly increases the spectrum utilization compared to time based or frequency multiple access.

I. INTRODUCTION

The emergence of Multiple Input Multiple Output (MIMO) communications opens new directions and possibilities for spatially sharing wireless channels [1-3]. Consider a scenario of two independent MIMO communication systems that share the same flat fading MIMO channel

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Blind spatial division multiple access for MIMO users with equal priority. The matrix $H_{i,j}, i \neq j \in \{1, 2\}$ are unknown to both users. The objective of the two users is to learn the null space of $H_{i,j}, i \neq j \in \{1, 2\}$.

as depicted in Figure [1] Assuming that each user has more antennas at the transmitter then the maximum number of antennas that each one has at the receiver, they can share the channel without interfering to each other by using orthogonal spatial dimension. This spatial sharing is even more appealing in MIMO Cognitive Radio (CR) networks [4–6] since it enables a CR MIMO Secondary User (SU) to transmit a significant amount of power simultaneously as the PU without interfering with him by utilizes spatial dimensions that are not used by the PU. This spatial separation requires, in both CR and MAC, that the interference channel be known. In the MAC (see Fig. [1]), it means that $H_{21}$ and $H_{12}$ be known to user 1 and 2 respectively, while in the CR case it is sufficient that the SU, say user 2, knows $H_{21}$. This information can be achieved by conventional channel estimation techniques that require a high level of cooperation, including handshake, transition of a known synchronous training sequence and the use of matched filters for each receiver antenna. In the MAC scenario, this process needs to be applied twice were at the first time one of the systems transmits a training sequence while the second estimates the channel and transmits the estimation back to the other system, then it is repeated where the two systems exchange roles. During these processes, each system must stop its data flow unless it is capable of full duplex, i.e. transmitting and receiving simultaneously at the same time and on the same frequency band. Although complicated, this channel estimation can be carried out in MAC since both users are equal priority. In CR on the other hand, this is far more complicated
since nobody expect PUs to stop their reception and perform channel estimation for unlicensed secondary users. Thus, acquiring and/or operating without knowing the interference matrix to the PU is a major issue of active research \cite{7,11} in CR. Note that every solution that is good for CR problem can be utilized to the MAC problem. Henceforth, we consider the problem of interference channel learning in the context of CR.

We consider the underlay CR paradigm \cite{12}, that is, the SUs are constrained not to exceed a maximum interference level at the PU. The optimal power allocation for the case of a single SU who knows the matrix $H_{21}$ in addition to its own Channel State Information (CSI) was derived by Zhang and Liang \cite{4}. In the case of multiple SUs, Scutari at al. \cite{6} formulated a competitive game between the secondary users. Assuming that the interference matrix to the PU is known by each SU, they derived conditions for the existence and uniqueness of a Nash Equilibrium point to the game. Zhang et al. \cite{8} were the first to take into consideration the fact that the interference matrix $H_{12}$ may not be perfectly known (but is partially known) to the SU. They proposed Robust Beamforming to assure compliance with the interference constraint of the PU while maximizing the SU’s throughput. Another work for the case of an unknown interference channel with known probability distribution is due to Zhang and So \cite{9} who optimized the SU throughput under a constraint on the probability that the interference to the PU be above a given threshold.

A very appealing solution concept for CR in general and MIMO CR in particular, is that the SU would be able to mitigate the interference to the PU blindly without a handshake and without using conventional channel estimation techniques. Yi \cite{11} Proposed such a solution in the case where there is a channel reciprocity between the PU’s transmitter and receiver in which the SU listens to the PU signal and estimates $H_{12}$’s null space from its second order statistics. This work was enhanced by Chen et al. in \cite{10}. Both works require channel reciprocity and therefore are restricted to a PU that uses Time Division Duplexing (TDD). Once the SU obtains the null space of $H_{12}$ it can transmit within this null space without interfering with the PU.

Beside the channel reciprocity case, obtaining the value of $H_{1j}$ by the SUs (i.e. the interference channel to the PU) requires the PU to participate in the SU’s estimation task. This task requires that the SU transmits a training sequence, from which the PU estimates $H_{1j}$ and feeds it back to the SU. Such cooperation increases system complexity overhead, since it requires a handshake between both systems and in addition, the PU needs to be synchronized to the SU’s training
The addressed cognitive radio scheme. The matrix $H_{12}$ is unknown to the secondary transmitter and $v_1(k)$ is a stationary noise (which may include stationary interferences). The interference from the SU, $H_{12}x_2(k)$, is treated by the PU as noise, i.e. no interfere cancellation is performed. The SU obtains a closed form expression for the null space of the interference channel to the PU $H_{12}$ by measuring the variation of $q(k)$ resulting of finite set of transmitted signals $\{x_2(k)\}_{t=1}^T$.

sequence. This required cooperation is one of the major obstacles to deployment of MIMO CR systems.

The objective of this paper is to design a simple procedure based on minimal cooperation by the PU such that a MIMO SU will be able to meet its interference constraint without explicitly estimating the matrix $H_{1j}$ and without burdening the PU with any handshake, estimation and/or synchronization associated with SUs. Consider the problem depicted in Fig. 2. In this scheme the PU, although active, is not necessarily aware of the SU. Its role in the SU’s learning process is limited to broadcasting a single one-dimensional beacon through a low rate control channel. This beacon is a function of the PU’s noise plus interference. The advantage of this technique over conventional channel estimation techniques is that it does not require a handshake and synchronization between the secondary and the PUs and can be implemented using only energy measurements. This is also a very appealing property for interference mitigation between two MIMO users (i.e. “multiple access”) since it makes the information exchange mechanism between the two users that is needed for them to share the same channel very simple.

The remainder of this paper is organized as follows: Section II formulates the problem. Section III presents the Energy Based Cannel Learning (EBCL) algorithm for interference mitigation in the primary -secondary user CR scenario. Section IV discusses the implementation of the EBSL algorithm in spatial channel sharing between two independent MIMO users of equal priority. Section V presets numerical results.
II. PROBLEM FORMULATION

Consider a flat fading MIMO interference channel with a single PU and a single SU without interference cancellation, i.e. each system treats the other system’s signal as noise. User’s $i$’s $i \in \{1, 2\}$ received signal is given by

$$y_i(k) = H_{ii}x_i(k) + H_{ij}x_j(k) + v_i(k), \quad k \in \mathbb{N}$$

(1)

where $j \in \{1, 2\}, \ j \neq i$, $H_{ij} \in \mathbb{C}^{r_i \times t_q}$ and $v_i(k)$ is a zero mean stationary noise. In this paper all vectors are column vectors. Let $A$ be an $l \times v$ complex matrix, then, its null space is defined as $N(A) = \{y \in \mathbb{C}^v : Ay = 0\}$ where $0 = [0, ..., 0]^T$ and its column space $C(A) = \text{span}(a_1, \ldots, a_v) \subseteq \mathbb{C}^l$. We assume that user 1 is the PU. The secondary user (user 2) is allowed to transmit as long as the interference does not exceed a maximum level at the PU, i.e.

$$\|H_{12}x_2(k)\|^2 \leq \eta,$$

(2)

where $\eta = 0$ represents the case where the SUs are allowed to transmit only in the null space of the matrix $H_{12}$.

Since the secondary user is MIMO it can share the channel without interfering with the PU if it uses spatially orthogonal degrees of freedom. In particular, the SU will not interfere with the PU if its transmitted signal $x_2$ satisfies $x_2 \in N(H_{12})$. The main obstacle in using this technique is that it requires knowledge of $N(H_{12})$. The matrix $H_{11}$ is known only to the PU, and the matrix $H_{12}$ is unknown to both the PU and the SU; hence its estimation requires cooperation between the two users. The state of the art in MIMO channel estimation techniques requires that the SU transmits a training signal that is known to the PU. The PU then estimates the channel using a matched filter. Other techniques that are not based on a known deterministic signal waveform are the blind MIMO channel estimation techniques [13–15, e.g.] in which the receiver uses the received signal statistics, i.e covariance matrices and higher order comulant tensors, to estimate the channel. These approaches require an extensive set of measurements and processing at the receiver side (the PU’s receiver in this case). After the PU obtains an estimate of $H_{12}$ he transmits it to the SU. This kind of “service” provided by the PU to the SU is highly undesirable due to the overhead and cooperation required on the part of the PU. Thus, reducing the role of the PU in this channel learning phase will make CR technology more attractive for practical applications.
Our objective is to derive a simple procedure for the SU to learn the null space of the matrix $H_{12}$ such that the PU would not need matched filters or to make extra measurements other than those required for its usual operation. We would also like to reduce the amount of processing at the PU and above all we would like the SU to obtain the null space of $H_{12}$ without having a handshake with the PU and even without the PU being aware of the SU. We denote $G \triangleq H_{12}^*H_{12} \in \mathbb{C}^{t_2 \times t_2}$ (3) and divide time into $N$-length intervals referred to as transmission cycles. In each transmission cycle, the SU transmits a constant signal (this is required only during the learning process), i.e.

$$x_2((n-1)N) = x_2((n-1)N + 1) = \cdots = x_2(Nn - 1) \triangleq \tilde{x}_2(n)$$ (4)

while the PU measures its total noise plus interference. It then broadcasts to all of the users in its vicinity the one dimensional signal $q(n)$ that satisfies the following assumption.

Assumption 1: There exist some $K \in \mathbb{N}$ such that the value of $\|H_{12}\tilde{x}_2(n)\|^2$ can be extracted up to an arbitrary scalar factor $\alpha > 0$ from $\{q(l)\}_{l=0}^n$, for every $1 \leq n \leq K$.

Note that from the SU point of view, knowing $H_{12}$ at transmitter is equivalent to knowing $G$, which is defined in (3). The problem of learning the $G$ from $\{q(l)\}_{l=0}^n$, referred to as the energy based channel learning problem, is depicted in Figure 3. Note that as long as $\alpha$ is constant for every $1 \leq n \leq K$, the function $q(n)$ can be measured via energy detectors since $\alpha$ is arbitrary.

A natural choice for a beacon that satisfies Assumption 1 is the following:

$$q(n) = \frac{1}{N} \sum_{k=(n-1)N+1}^{Nn} \mathbb{E} \left\{ \|y_1(k) - H_{11}\tilde{x}_1(k)\|^2 \right\}$$ (5)

where $\tilde{x}_1(k)$ is the decoded signal. This beacon is transmitted at time instances $k = nN$, $n \in \mathbb{N}$. If we neglects the decoding errors, (i.e. $\tilde{x}_1(k) = x_1(k)$) we obtain

$$q(n) = \frac{1}{N} \sum_{l=(n-1)N}^{Nn-1} \mathbb{E} \left\{ \|H_{12}\tilde{x}_2(n) + v(k)\|^2 \right\} = \|H_{12}\tilde{x}_2(n)\|^2 + \text{Tr}(\mathbb{E}\{v_1(k)v_1^*(k)\})$$ (6)

$$= \tilde{x}_2^*(n)G\tilde{x}_2(n) + c$$

We will now show that this beacon satisfies Assumption 1 i.e. that the secondary user can extract $\alpha\|H_{12}\tilde{x}_2(n)\|^2$ from $\{q(l)\}_{l=0}^n$. This is done as follows: At the beginning of the learning process ($n = 0$) the SU transmits $\tilde{x}_2(0) = 0$, that is, it does not transmit. Let $\alpha > 0$ be the
magnitude of the control channel from the PU to the SU. Then, at time \( k = 0 \) the SU measures \( \alpha q(0) \) where \( q(0) = \text{Tr}(E\{v_1(k)v_1^*(k)\}) \). For \( n > 0 \), the SU transmits the signal \( x_2(n) \) and at time \( k = nN \) it measures the \( \alpha q(n) \) broadcast by the PU. The SU then obtains \( \alpha \|H_{12}x_2(n)\|^2 \) by subtracting \( \alpha q(0) \) from \( \alpha q(n) \). Note that \( \alpha \) may be unknown to the SU and that the only requirement is that it be constant during the learning process.

In practice, the beacon will be based on the sample average

\[
q(n) = \frac{1}{N} \sum_{k=(n-1)N}^{Nn-1} \|y_1(k) - H_{11}\hat{x}_1(k)\|^2
\]

which depends on the averaged value of \( \|z(k)\| \) at the \( n \)th cycle where

\[
z(k) = H_{12}x_2(k) + v_1(k)
\]

It is important to stress that the function \( q(n) \) is calculated entirely from \( y_1(k) \). Therefore it is calculated by the PU processing unit after decoding its signal \( \hat{x}_1(k) \) without any additional measurements.

In the next section we will show how the SU can learn the null space of the matrix \( H_{12} \) from the measurements \( \{q(n)\}_{n=1}^{t_2} \), where \( t_2 \) is the SU’s number of transmit antennas.

### III. THE ENERGY BASED CANNEL LEARNING ALGORITHM

In order to obtain \( H_{12} \)’s null space it is sufficient to calculate \( G \)’s null space (where \( G \) is defined in (3)). The following proposition expresses the matrix \( G \) as a function of \( \{x_2(n)G\tilde{x}_2(n)\}_{n=1}^{t_2} \), where each \( \tilde{x}_2(n) \) is a different transmitted signal.
Proposition 1: Let \( S(A, x) \overset{\Delta}{=} x^* A x \) and \( r_{l,m}(\theta, \phi) \) be a \( t_2 \)-dimensional column vector whose entries are all equal to zero except of the \( l \)th and \( m \)th entry, which are equal to \( \cos(\theta) \) and \( e^{-i\phi} \sin(\theta) \), respectively, i.e.

\[
\begin{align*}
 r_{l,m}(\theta, \phi) &= [0, \ldots, 0, \cos(\theta), \\
                        &\quad 0, \ldots, 0, e^{-i\phi} \sin(\theta), 0, \ldots, 0]^T
\end{align*}
\]  

(9)

The entries \( \{g_{l,m}\}^{t_2}_{l,m=1} \) of the matrix \( G = H_{12}^* H_{12} \) are given by

\[
\begin{align*}
 g_{l,l} &= S(G, r_{l,m}(0, 0)) \\
 \Re(g_{l,m}) &= c_{l,m}(\pi/4, 0) \\
 \Im(g_{l,m}) &= -c_{l,m}(\pi/4, \pi/2)
\end{align*}
\]

(10) (11) (12)

where

\[
 c_{l,m}(\theta, \phi) = (g_{l,l}\cos^2(\theta) + g_{m,m}\sin^2(\theta)) \\
 - S(G, r_{l,m}(\theta, \phi))
\]

(13)

Proof: Note that

\[
 S(G, r_{l,m}(\theta, \phi)) = \cos^2(\theta) |g_{l,l}| \sin^2(\theta) |g_{m,m}| \\
 - |g_{l,m}| \sin(2\theta) \cos(\phi + \angle g_{l,m})
\]

(14)

from which (10) follows. By substituting (14) into (13) we obtain

\[
 c_{l,m}(\theta, \phi) = \sin(2\theta) |g_{l,m}| \cos(\phi + \angle g_{l,m})
\]

(15)

from which (11) and (12) follow.

The EBCL algorithm provides a closed form expression for the matrix \( G \). For every \( \tilde{x} \), the value of \( \|H\tilde{x}\|^2 \) can be obtained by transmitting \( \tilde{x}(n) \), receiving \( q(n) \) and subtracting \( q(0) \) from it, i.e.

\[
\|H\tilde{x}(n)\|^2 = q(n) - q(0)
\]

(16)

From Proposition 1 it follows that the matrix \( G \) can be obtained precisely by \( t_2^2 \) transmission cycles. The CF-BNSL algorithm is described in Table I. After obtaining the matrix \( G \), its null space can be calculated offline at the secondary transmitter’s processing unit. Once the SU knows the null space of the interference channel to the PU’s transmitter it can transmit freely as long as its transmitted signal is restricted to its null space, i.e. \( x_2 \in N(H_{12}) \).
function \textbf{G=EBCL}

Set \( b = S(\textbf{G}, 0) \);

for \( l = 2, ..., t \)

Set \( g_l = S(\textbf{G}, r_l(0, 0)) - b \);

end for

for \( l = 1, ..., t - 1 \)

for \( m = l + 1, ..., t \)

Set \( \alpha_{l,m} = S(\textbf{G}, r_{l,m}(\pi/4, 0)) - b \);

Set \( \beta_{l,m} = S(\textbf{G}, r_{l,m}(\pi/4, \pi/2)) - b \);

Set \( e_1 = \text{Calc}_c(g_l, g_m, \alpha_{l}, \pi/4) \);

Set \( e_2 = \text{Calc}_c(g_l, g_m, \beta_{l}, \pi/4) \);

end for

end for

end \textbf{EBCL}

function: \( e = \text{Calc}_c(g_1, g_2, \alpha, \theta) \)

\[
e = g_1 \cos^2(\theta) + g_2 \sin^2(\theta) - \alpha;
\]

end \textbf{Calc}_c

The advantage of the proposed scheme (see Fig. 2) is that the PU, although active, does not have to be aware of the SU. Its role in the SU’s learning process is limited to broadcasting periodically the beacon \( q(n) \) through a low rate control channel to all of the secondary users in its vicinity. Thus, in order to implement the EBCL algorithm, the secondary user needs only to detect and measure \( q(n) \)’s energy in every transmission cycle without having a handshake with the PU. Recall that the only condition required for the EBCL is that Assumption 1 holds. This assumption holds even if there are multiple secondary users in the system as long as their interference to the PU is stationary. Thus a new secondary user can join the network while multiple SUs coexist with the PU in a steady state, i.e. they are not varying the spatial orientation or their transmitted signal.
IV. EBCL ALGORITHM FOR SPATIAL DIVISION MULTIPLE ACCESS

The fact that the CF-BNSL algorithm is based entirely on energy measurement and not on matched filters makes it very appealing for implementation as a blind spatial division multiple access technique for MIMO users with equal priority (see Figure 3), that is, (2) is no longer required. This simplifies the coordination between the two users as follows: At the first stage, there is a handshake between the two systems in which it is decided which system begins with learning and which provides feedback. Assume that system 2 begins with learning while system No. 1 feeds back its measurements. Then system No. 2 transmits a signal $\tilde{x}_2(n)$ while system No. 1 measures and feeds back the beacon in (7). This way, system 2 learns the matrix $G_1$ by applying the CF-BNSL algorithm. This process is then repeated where both systems exchange roles such that system 1 learns $G_2$. Thus, if system 1 and 2 restrict their transmission to $\mathcal{N}(H_{21})$ and $\mathcal{N}(H_{12})$ respectively, they do not interfere with each other and create in effect a Spatial Channel Sharing (SCS).

An important question that arises is whether the spatial channel sharing is worth the effort of null space learning. Recall that in the primary-secondary user CR scenario the SU must be invisible to the PU. This fact makes the learning of $\mathcal{N}(H_{12})$ worthwhile because, as long as the channel remains unchanged, the SU is operating freely without colliding. This is not the case for MIMO users of equal priority. They can choose not to mitigate interference at all or to share the channel using a much simpler multiple access scheme such as Frequency Division Duplexing (FDD), which is static and does not require null space learning. In the sequel it is shown that the SCS provides a much better spectrum utilization (in terms of degrees of freedom) than FDD if both systems have a sufficient number of antennas at the transmitter.

In the sequel it is assumed that $1 \leq i \neq j \leq 2$, $t_i > r_j$ and that the EBCL algorithm is performed by both users. Let

$$G_i = H_{ji}^* H_{ji}$$

and let

$$W_i \Lambda_i W_i^* = G_i$$

be its eigenvalue decomposition. Then user $i$’s pre-coding matrix $T_i$ is given by

$$T_i = [w_{q_1}, ..., w_{q_{t_i-r_j}}]$$
where \( w_q^k \) is \( W_i \)'s \( q \)th column and \( q_1, q_2, ..., q_t-r_j \) are the indexes that chose the eigenvectors that correspond to \( G_i \)'s Null space, i.e.

\[
w_{q_1}^* G_i w_{q_1} = \cdots = w_{t_i-r_j}^* G_i w_{t_i-r_j} = 0 \tag{20}
\]

The following proposition shows that for Zero-Mean Spatially White (ZMSW) channels that satisfy \( t_i \geq r_j \), the EBCL results in a free interference \( r_i \times (t_i - r_j) \)-ZMSW channel for each user.

**Proposition 2:** Assume that \( H_{iq}, q, i \in \{1, 2\} \) are \( r_i \times t_q \) (ZMSW) channels that are independent of each other and satisfy \( t_i \geq r_j \). Let \( \tilde{H}_{ii} \) be user \( i \)'s equivalent channel when both users apply the CF-BNDL algorithm i.e. \( \tilde{H}_{ii} = H_{ii} T_i \) where \( T_i \) is users \( i \)'s pre-coding matrix defined in (19). Then, \( \tilde{H}_{ii} \) is an \( r_i \times (t_i - r_j) \) ZMSW channel.

**Proof:** See Appendix A.

Proposition 2 implies that if \( t_i \geq r_i + r_j \) for \( i \neq j \in \{1, 2\} \), the difference between SCS using the EBCL algorithm compared to the case where there is no interference is equivalent to not using \( r_j \) antennas. Furthermore, both users would not lose degrees of freedom compared to the case where there is no interference since \( \text{rank}(H_{ii}) = r_i \) a.s., and \( \text{rank}(\tilde{H}_{ii}) = \min\{r_i, t_i - r_j\} \) a.s. which are equal if \( t_i - r_j \geq r_i \). The following theorem extends the last statement for a wider range of channel types.

**Theorem 3:** Assume that \( H_{iq}, i, q \in \{1, 2\} \) are independent (i.e. independent of each other) random matrices defined on the same probability space \( (\Omega, \mathcal{F}, P) \) such that \( \text{vec}(H_{ii}) \) is a continues random vector \(^2\) for \( i = 1, 2 \). Let \( d_i = \text{rank}(H_{ii}) \) be user \( i \)'s number of degrees of freedoms if he is operating alone, and let \( d_i^n \) be user \( i \)'s number of degrees of freedom when both users apply the EBCL algorithm, i.e. \( d_i^n = \text{rank}(\tilde{H}_{ii}) \), where \( \tilde{H}_{ii} = H_{ii} T_i \) and \( T_i \) is users \( i \)'s pre-coding matrix defined in (19). Then, \( d_i = d_i^n \) a.s. if \( t_i \geq r_i + r_j \).

**Proof:** See Appendix B.

\(^1\)It means the the entries of the matrix \( H \) are i.i.d. zero-mean unit-variance circular Gaussian random variables [see e.g. \(^6\) Section 10.1].

\(^2\)A \( t \)-dimensional complex random vector \( x \) is said to be continuous if it can be written as \( x = x_{Re} + i x_{Im} \) where \( \hat{x} = [x_{Re}^T, x_{Im}^T] \) such that \( \hat{x} \) is a continuous \( 2t \)-dimensional random vector, i.e. \( \hat{x} \) has a probability density function with respect to the Lebesegue measure.
Fig. 4. Graphical illustration of the space $A_2$. Assuming that all matrices are full rank and that the secondary user has more antennas at the receiver than at the transmitter, i.e. $r_1 > t_1$, then $C^\perp(H_{11}) \neq 0$. Then, the subspace $A_2$ that maps signals to $C^\perp(H_{11})$ can be used by the SU without interfering with the PU. A necessary and sufficient condition is that $t_2 > t_1$.

V. Obtaining Additional Degrees of Freedom

Constraining the SU to transmit only in $N(H_{12})$ may be inefficient in some cases. Consider a scenario where the PU has more antennas at the receiver than at the transmitter i.e. $t_1 < r_1 = \text{rank}(H_{11})$ and full CSIR of its own channel $H_{11}$. Then, the PU’s signal of interest at the receiver, that is $H_{11}x_1$, can lie only in the $r_1$-dimensional subspace $C(H_{11}) \subset \mathbb{C}^{r_1}$. This redundancy can be utilized by the SU to obtain additional degrees of freedom by transmitting $x_2 \in N(H_{12}) + A_2$ where $A_2 = \{x_2 \in \mathbb{C}^{t_2} : H_{12}x \in C^\perp(H_{11})\}$ (see Fig. 4 for illustration). If all matrices are full rank, a necessary and sufficient condition for $N(H_{12}) + A_2 \neq 0$ is that $r_2 > r_1$. Note that the subspace $N(H_{12}) + A_2$ is equal to $N(P_{H_{11}}H_{12})$ where $P_B = B^* (BB^*)^+ B$ is the projection matrix into the column space of $B$ (which is equal to the range of $B$) and $(\cdot)^+$ represents the pseudo inverse operation. These extra degrees of freedom can be obtained by the EBCL algorithm with no additional cost. The only modification required is for the PU to project $z(t)$, defined in (8), into $C(H_{11})$ while the rest of the algorithm remains the same, i.e. to replace $z(t) = y_1(t) - \hat{x}_1(t)$ with $\tilde{z}(t) = P_{H_{11}} z(t)$. This idea can also be implemented in the case of two users with equal priority that is described in Section IV.

The sum of two vector subspaces is the vector space created by the sum of all the vectors in these two subspaces, i.e. let $B$ be a vector space and let $B_1, B_2$ be two vector subspaces of $B$, then $B_1 + B_2 = \{x \in B : x = y + z, y \in B_1, z \in B_2\}$. 

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VI. NUMERICAL EXAMPLES

To determine the value of null space learning in this setting we turn to simulations. Figure 5 compares the rate gain of SCS over that of FDD in a two-user symmetric MIMO interference channel without interference cancellation. By symmetric we mean that $t_1 = t_2$ , $r_1 = r_2$ and that $H_{ii}, i = 1, 2$ are ZMSW channels as well as $H_{ij}, i \neq j, \in \{1, 2\}$. Figure 5(a) shows that for $t = 4$ and $r = 2$ , the SCS outperforms the FDD, i.e. the SCS’s rate gain is higher than that of the FDD. Furthermore, in the high SNR regime the SCS rate converges to the channel capacity without interference, i.e. the rate of a single user occupying the entire channel, as long as $t \geq 2r$, as shown in Figure 5(b). From this we conclude that in the FDD scheme, each user exploits only half of its degrees of freedom, whereas in the SCS scheme both users exploit all of their degrees of freedom (as long as $t_i \geq r_i + r_j$) and the only performance loss is due to the restriction of the transmit signal to $\mathcal{N}(H_{ji})$.

It is important to stress that knowing $G$ can be utilized for a more sophisticated channel sharing than the SCS. For example, suppose that in addition to transmitting in $\mathcal{N}(P_{H_{jj}}H_{ji})$, system $i$ wishes to use also part of its orthogonal compliment $\mathcal{N}^\perp(P_{H_{jj}}H_{ji})$. This of course creates interference to system $j$. However by choosing eigenvectors that correspond to $G$’s lowest eigenvalues, system $i$ can balance between its performance gain and the interference to system $j$. To show that explicitly, let $V \Sigma V^*$ be the eigenvalue decomposition of $G$, where $\Sigma$ is a real nonnegative diagonal matrix that contains $G$ eigenvalues in decreasing order, i.e. $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_d > 0$ where $d < t_i$. Then the eigenvector that corresponds to $\sigma_d$ (i.e. $V d$ ’s column) produces minimum interference to system $j$. This way, system $i$ can balance between choosing eigenvectors that provide it with the best performance gain and minimizing the interference to system $j$.

VII. CONCLUSIONS

We proposed a blind technique for MIMO SUs to spatially coexist with PUs based on minimal cooperation from the PU. This cooperation does not require additional sensing by the PU and is carried out by calculating the power of the PU’s total noise plus interference. This value is broadcast via a low rate control channel to all of the SUs in its vicinity (beacon). By doing so, the PU enables the SU to utilize unused degrees of freedom.

The advantages of the proposed technique are:
Fig. 5. Comparison between blind spatial division and FDD/TDD in a symmetric MIMO interference interaction. The matrices $H_{i,q}$, $i, q \in \{1, 2\}$ are i.i.d. complex Gaussian with zero mean and unit variance. Both user's number of received antennas is 2. The interference expected power is 10.5dB lower than the expected signal power for both users. The vertical axis represents the ratio between the achievable rate to the rate obtained via uniform power allocation over the entire band/time. In Subfigure (a) the horizontal axis represents expected received SNR while the number of transmit antennas for each user is 2. In Subfigure (b) the horizontal axis represents the number of antennas at the transmitter while the expected power at each receiver is 140 dB.

1) The SU operates autonomously and independently of the PU (as long as the PU transmits the defined beacon).

2) The PU produces the beacon from information that already exists in all communication systems, i.e. from the PU’s decoded signal and its received signal.

3) The entire learning process is based on energy measurements, independent of the transmission schemes of both the PU and SU, i.e. independent of their modulation, coding etc.

   This flexibility is very important in CR networks which are inherently ad-hoc.

4) The entire learning process takes $t_2^2$ transmission cycles where $t_2$ is the number of the
SU’s transmit antennas.

5) The proposed technique is easily applicable to CR networks with one PU and multiple SUs as long as only one SU performs the learning procedure at a time while the other SUs don’t change their spatial power allocation. In practice, this is not a problem since the learning process takes only $t^2_2$ transmission cycles.

For the same reasons the proposed scheme can be easily implemented for spatial channel sharing of two independent MIMO secondary users of equal priority. We demonstrated that if both users share the channel using the CF-BNSL algorithm:

1) They don’t loss degrees of freedom while gaining an interference free MIMO channel.
2) In case of for zero-mean spatially-white Gaussian channels and $t_i > r_j$, then the SCS results in a free interference $r_i \times (t_i - r_j)$-zero-mean spatially-white Gaussian channel for each user.

**APPENDIX A**

**PROOF OF PROPOSITION 2**

Without loss of generality we set $i = 1, j = 2$ and denote $\tilde{H}_1 = H_{11} W_1$. Since $H_{21}$ is ZMSW channel, the random matrix $G_2$ (defined in (17)), by definition, is a central Wishart Matrix. Thus, $W_1$ (defined in (18)) is a unitary matrix that is uniformly distributed over the manifold of unitary matrices in $\mathbb{C}^{t_1 \times t_1}$ [see e.g., 17, Lemma 2.6]. Since the channel $H_{11}$ is ZMSW it is bi-unitary invariant [17], that is $UH_{11}V$’s distribution is unchanged for any unitary matrices $U, V$. Thus, for every $W_1$, the conditional distribution of $\tilde{H}_1$’s is equal to $H_{11}$, i.e. $P(\tilde{H}_1 / W_1) = P(H_{11})$. Therefore, given $W_1$, $H_{11}$ entries are i.i.d. zero-mean unit-variance complex Gaussian random variables (i.e. ZMSW channel) and because this distribution is not a function of $W_1$, the marginal distribution of $\tilde{H}_1$ is the same, i.e. $P(\tilde{H}_1) = P(\tilde{H}_1 / W_1)$. It follows that $\tilde{H}_1$ is a $r_1 \times t_1$ ZMSW channel and therefore $\tilde{H}_1$ (which is composed of some $t_1 - r_2$ columns of $\tilde{H}_1$) is an $r_1 \times (t_1 - r_2)$ ZMSW channel.

**APPENDIX B**

**PROOF OF PROPOSITION 3**

In this proof we shall use some special notation. Matrices will be denoted by italic upper case letters (i.e. the channels $H_{iq}, i, q = 1, 2$ are now denoted by $H_{iq}, i, q = 1, 2$) while
random matrices will be denoted by boldface upper case letters. We will make notational distinction between scalars and vectors and denote both with lower case italic letters. Random vectors/variables will be denoted by boldface lowercase letters. Without loss of generality, we set $i = 1$ and denote $H_{11}^T = H$ and $H_{12}^T = \tilde{H}$. Let $h_q, \tilde{h}_q$ be $H$’s and $\tilde{H}$’s $q$th columns respectively and $H_{-q}$ be the $t_1 \times (r_1 - 1)$ matrix that results from deleting $H$’s $q$th column.

The Theorem is first proven for real matrices. In this case $h_q, \tilde{h}_q : \Omega \rightarrow \mathbb{R}^{t_1}$ are Borel measurable functions. If $r_1 \leq t_1 - r_2$, user 1 losses at least one degree of freedom iff there exists a sequence of scalars $\{a_q\}_{q=1}^{r_1}$ not all zero such that $\sum_{q=1}^{r_1} a_q h_q \in \mathcal{N}^\perp(H_{21}) = \text{span}(\tilde{h}_1, ..., \tilde{h}_{r_2})$. The later is equivalent to the following statement: There exists $1 \leq q \leq r_1$ such that $h_q \in C(B_{-q})$ where $B_{-q} \triangleq [H_{-q}, \tilde{H}]$. Using the sub-additivity of measures

$$P(d_1^N < d_1) \leq P \left( \bigcup_{q=1}^{r_1} h_q \in C(B_{-q}) \right) \leq \sum_{q=1}^{r_1} P(h_q \in C(B_{-q}))$$

(21)

Note that

$$P(h_q \in C(B_{-q})) = \int_{\Omega} P(h_q \in C(B_{-q})|\tilde{H})dP(\omega)$$

(22)

It remains to show that $P(h_q \in C(B_{-q})|\tilde{H}) = 0$, a.s. By hypothesis, $H$ is independent of $\tilde{H}$, thus $P(h_q \in C(B_{-q})|\tilde{H}) = P(h_q \in C(B_{-q}))$, a.s. Now recall that $P_H$ is absolutely continuous with respect to the Lebesgue measure, that is, $P_H \ll m^{t_1r_1}$ where $m^k$ is the $k$-dimensional Lebesgue measure. Let $Q(Z) = \{ [x, Y] : x \in C([Y, Z]), Y \in \mathbb{R}^{t_1 \times (r_1 - 1)}, Z \in \mathbb{R}^{t_1 \times r_2} \}$ and let $Q_Y(Z) = \{ x : [x, Y] \in Q(Z) \}$ be $Q(Z)$’s $Y$-section. Then for every $Z \in \mathbb{R}^{t_1 \times r_2}$

$$m^{t_1 \times r_1}(Q(Z)) = \int_{\mathbb{R}^{t_1 \times (r_1 - 1)}} m^{t_1}(Q_Y(Z))dm^{t_1 \times (r_1 - 1)}(Y)$$

(23)

[see e.g. 19, Theorem 2.36] and since for every $Z, Y$, $Q_Y(Z)$ is a vector subspace of $\mathbb{R}^{t_1}$ whose dimension is at most $r_1 + r_2 - 1$ it satisfies $m^{t_1}(Q_Y(Z)) = 0$ (recall that $r_2 + r_1 \leq t_1$). This establishes the desired result for real channel matrices.

To extend this result to complex matrices, note that $h_q = h_{q, \text{Re}} + ih_{q, \text{Im}}$, and $\tilde{h}_q = \tilde{h}_{q, \text{Re}} + i\tilde{h}_{q, \text{Im}}$ where $h_{q, \text{Re}}, h_{q, \text{Im}}, \tilde{h}_{q, \text{Re}}, \tilde{h}_{q, \text{Im}} : \Omega \rightarrow \mathbb{R}^{t_1}$ are Borel measurable functions. Furthermore, the vector space $\mathbb{C}^{t_1}$ is isomorphic to $\mathbb{R}^{2t_1}$, that is, there exists a bijective mapping (one to one and on

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4The existence of a conditional probability measure $P(\cdot|h_q)(\omega)$ for each $\omega \in \Omega$ is due to the fact that all random vectors are assumed to be $\mathbb{R}^{t_1}$-Borel measurable. Such probability measure is termed regular conditional probability [see e.g. 18].

5Let $\mu, \nu$ be two measure defined on the same measurable space $(X, \mathcal{M})$, then $\mu \ll \nu$ if $\nu(A) = 0 \Rightarrow \mu(A) = 0$. 
to) from one to the other which in this case is given by \( \psi(x) = [\text{Re}(x^T), \text{Im}(x^T)]^T \) where \( x \in \mathbb{C}^{t_1} \).

Let \( \tilde{\psi}(x) = [-\text{Im}(x^T), \text{Re}(x^T)]^T \) then \( \mathcal{C}(B_q) \) is mapped into \( \mathcal{V}_q = \text{span}(\tilde{\psi}(h_1), \tilde{\psi}(h_1), ..., \tilde{\psi}(h_{r_2}), \tilde{\psi}(h_{r_2}), \tilde{\psi}(h_3), \tilde{\psi}(h_3), ..., \tilde{\psi}(h_{q-1}), \tilde{\psi}(h_{q-1}), \tilde{\psi}(h_{q+1}), \tilde{\psi}(h_{q+1}), ..., \tilde{\psi}(h_{r_1}), \tilde{\psi}(h_{r_1})) \). Thus, \( h_q \in \mathcal{C}(B_q) \) iff \( \psi(h_q) \in \mathcal{V} \) or \( \tilde{\psi}(h_{q1}) \in \mathcal{V} \). Because \( \tilde{\psi}(h_{q1}) \) and \( \psi(h_{q1}) \) are orthogonal, \( h_q \in \mathcal{C}(B_q) \) is equivalent to \( \psi(h_q) \in \mathcal{V}^\perp \) or \( \tilde{\psi}(h_q) \in \mathcal{V} \). Henceforth the proof is identical to the real case since \( m_{2t_1}(\mathcal{V}) = m_{2t_1}(\mathcal{V}^\perp) = 0 \) and because \( \mathbf{H} \) is a continuous random matrix.

REFERENCES

[1] S. Jafar and M. Fakhereddin, “Degrees of freedom for the MIMO interference channel,” IEEE Transactions on Information Theory, vol. 53, pp. 2637 –2642, july 2007.

[2] Q. Spencer, A. Swindlehurst, and M. Haardt, “Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels,” IEEE Transactions on Signal Processing, vol. 52, pp. 461 – 471, feb. 2004.

[3] L. Ruan and V. Lau, “Dynamic interference mitigation for generalized partially connected quasi-static MIMO interference channel,” IEEE Transactions on Signal Processing, vol. 59, pp. 3788 –3798, aug. 2011.

[4] R. Zhang and Y.-C. Liang, “Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks,” IEEE Journal of Selected Topics in Signal Processing, vol. 2, pp. 88 –102, Feb 2008.

[5] G. Scutari, D. Palomar, and S. Barbarossa, “Cognitive MIMO radio,” IEEE Signal Processing Magazine, vol. 25, pp. 46 –59, December 2008.

[6] G. Scutari and D. Palomar, “MIMO cognitive radio: A game theoretical approach,” IEEE Transactions on Signal Processing, vol. 58, pp. 761 –780, Feb. 2010.

[7] S. Huang, X. Liu, and Z. Ding, “Decentralized cognitive radio control based on inference from primary link control information,” IEEE Journal on Selected Areas in Communications, vol. 29, pp. 394–406, February 2011.

[8] L. Zhang, Y.-C. Liang, Y. Xin, and H. V. Poor, “Robust cognitive beamforming with partial channel state information,” IEEE Transactions on Wireless Communication, vol. 8, pp. 4143–4153, August 2009.

\(^6\) or in other words \( h_q \in \mathcal{C}(\mathbf{H}_{2t_1}) \) iff \( \mathcal{H}_{\mathcal{V}}(\psi(h_q)) = \psi(h_q) \) or \( \mathcal{H}_{\mathcal{V}^\perp}(\tilde{\psi}(h_q)) = \tilde{\psi}(h_q) \) where \( \mathcal{H}_{\mathcal{V}} \) is the projection operator into \( \mathcal{V} \).
[9] Y. J. Zhang and A. M.-C. So, “Optimal spectrum sharing in MIMO cognitive radio networks via semidefinite programming,” IEEE Journal on Selected Areas in Communications, vol. 29, pp. 362–373, February 2011.

[10] Z. Chen, C.-X. Wang, X. Hong, J. S. Thompson, S. A. Vorobyov, F. Zhao, H. Xiao, and X. Ge, “Interference mitigation for cognitive radio MIMO systems based on practical precoding,” Arxiv preprint arXiv:1104.4155, vol. abs/1104.4155, 2011.

[11] H. Yi, “Nullspace-based secondary joint transceiver scheme for cognitive radio MIMO networks using second-order statistics,” in IEEE International Conference on Communications (ICC), 2010, pp. 1 –5, May 2010.

[12] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, “Breaking spectrum gridlock with cognitive radios: An information theoretic perspective,” Proceedings of the IEEE, vol. 97, pp. 894 –914, May 2009.

[13] S. Zhou, B. Muquet, and G. Giannakis, “Subspace-based (semi-) blind channel estimation for block precoded space-time OFDM,” IEEE Transactions on Signal Processing, vol. 50, pp. 1215 –1228, May 2002.

[14] Z. Ding and L. Qiu, “Blind MIMO channel identification from second order statistics using rank deficient channel convolution matrix,” IEEE Transactions on Signal Processing, vol. 51, pp. 535 – 544, Feb 2003.

[15] C. Shin, R. Heath, and E. Powers, “Blind channel estimation for MIMO-OFDM systems,” IEEE Transactions on Vehicular Technology, vol. 56, pp. 670 –685, March 2007.

[16] A. Goldsmith, Wireless Communications. Cambridge University Press, 2005.

[17] A. Tulino and S. Verdú, Random Matrix Theory and Wireless Communications, vol. 1. Now Publishers Inc, 2004.

[18] K. Athreya and S. Lahiri, Measure Theory and Probability Theory. Springer-Verlag New York Inc, 2006.

[19] G. Folland, Real Analysis: Modern Techniques and their Applications. New York, NY: John Wiley & sons, 1984.