Bell Correlations and the Common Future

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Reichenbach's principle states that in a causal structure, correlations of classical information can stem from a common cause in the common past or a direct influence from one of the events in correlation to the other. The difficulty of explaining Bell correlations through a mechanism in that spirit can be read as questioning either the principle or even its basis: causality. In the former case, the principle can be replaced by its quantum version, accepting as a common cause an entangled state, leaving the phenomenon as mysterious as ever on the classical level (on which, after all, it occurs). If, more radically, the causal structure is questioned in principle, closed space-time curves may become possible that, as is argued in the present note, can give rise to non-local correlations if to-be-correlated pieces of classical information meet in the common future — which they need to if the correlation is to be detected in the first place. The result is a view resembling Brassard and Raymond-Robichaud's parallel-lives variant of Hermann's and Everett's relative-state formalism, avoiding "multiple realities."

I. INTRODUCTION

Reichenbach's principle [1] states that, in a fixed causal structure, correlations either stem from a common cause or from one part directly influencing the other. The principle looks natural since it imagines a mechanism that leads to the correlations. Bell's theorems [2, 3] limit the explanatory power of a common cause in the form of classical information while quantum theory predicts correlations beyond that — so-called "Bell non-local correlations" (see also [4]). But then, strangely enough, not all correlations compatible with no-signaling are attainable in nature: An example of an idealization beyond what quantum physics predicts are PR correlations [5], i.e., binary inputs $X, Y$ and outputs $A, B$ satisfying $A \oplus B = XY$.

The absence of a mechanism behind Bell non-local correlations is disturbing, and several patches have been proposed: One can loosen Reichenbach's principle and simply regard the quantum state as a common cause (then, no further mechanism is to be expected) [6, 7]; one can resort to one measurement event influencing another (that would have to be an instantaneous fine-tuned influence in a preferred frame [8–13]); one can assume signals to travel to the past [14, 15] or suppose the existence of multiple realities [16] — but even at that price, no striking story has been told yet.

If the data in question are never brought together, no correlation can be seen [17]. At the occasion of that necessary rendez-vous in the future, the (physically represented) pieces of information locally interact — and this detection procedure of the correlation may be considered its origin: Such violations of causality can become possible, e.g., through closed time-like curves (CTCs). CTCs are world-lines closed in time: A system traveling along it can meet its "younger self." CTCs are consistent with general relativity [18, 19]. We consider the case where all information is classical and all interactions local: The established correlations are sent to the past via a CTC as described by Deutsch [20], ending up in a classical story behind Bell non-local correlations.

This text is organized as follows. Sections II and III review specific modifications of causality, namely Hermann's relative causality and Costa de Beauregard’s retro-causality. Section III B presents a new formulation of this “zigzag” model through relaxing measurement independence. Section IV describes a simple classical mechanism simulating Bell non-local correlations. Section V discusses relations to previous stories (parallel lives, retro-causality, and Viennese “process matrices”).

II. RELATIVE CAUSALITY

The works of Grete Hermann, physicist and philosopher, have been strangely overlooked. Not only did she spot a mistake (later called “silly” by Bell) in von Neumann’s [21] “proof” that quantum theory cannot be extended to yield deterministic predictions, she also provided her own arguments against such an extension — besides contributing to a better understanding of causality. In this context, she described the measurement process in the spirit of the relative-state interpretation [22, 23] of quantum theory — ten years before Hugh Everett III did.

Hermann’s 1948 article [24] looks into the process of measuring an electron’s position as described quantum-physically: If that electron interacts with a photon (described quantum-physically as well), the result is

"a new wave-function which is uniquely determined by the given wave-functions: It does, therefore, not contain the uncertainty that we would have to attribute to that mystic process."1

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1"eine neue Wellenfunktion, die eindeutig durch die beiden gegebenen Wellenfunktionen […] bestimmt ist. […] [S]ie enthält also nicht die Unbestimmtheit, die wir jenem mystischen […] [F]prozess zuschreiben müßten."
At this stage, one would have to measure this new wave-function to determine the position of the electron.

"Without any such new observations, the quantum-mechanical formalism leads to a progressing not visualizable braiding of the fundamental particles."²

According to Hermann, this means that

"the electron, after colliding with the photon, is described by a wave-function with a sharp position only relatively to the new measurement,"³

and in that perspective (the key of the argument)

"it, therefore, constitutes an autonomous physical system characterized by its own wave-function immediately after the collision with the photon."⁴

In other words, after the interaction, the measured system’s wave-function relates to the measurement outcome — which is in what she sees the cause for the electron to be at that position. Such a cause, however, cannot be brought in to give better predictions as it is only accessible to the experimenter after the measurement. In [25], she writes:

"These causes could not have been used for predictions; they determine the system in a relative way, relatively to the observation which was obtained only at the moment of the measurement. They, therefore, could be accessed after this observation only and do, hence, not allow to predict the outcome."⁵

Through these thoughts, Hermann anticipates Everett’s formalism [22, 23] and, at the same time, disentangles causality and predictability. More specifically, Hermann [26] describes an ontology in which a measurement entangles the observed object to the apparatus, and only relative statements are possible. Everett goes beyond Hermann’s view by invoking the wave-function of the whole universe. His formalism has often been called “many worlds:” Whenever a system gets entangled with the apparatus, all possible results are realized in parallel universes — a view brought forward, for example, by DeWitt and Deutsch [27, 28]. This is a left-over of classical concepts: “The coexisting branches […] can only be related to ‘worlds’ described by classical physics. […] [T]he […] meaning of Everett’s ideas is not the coexistence of many [classical] worlds, but on the contrary, the existence of a single quantum one” [29].

A variation of the Hermann/Everett theme are “parallel lives” [16]: Instead of globally, the individual experimenters split locally, into “bubbles” that are later only visible to each other if the quantum predictions result: The model is local and realistic.

If one incorporates time into the description, then a timeless wave-function of the universe as a whole can be imagined [30, 31]. The state of one part of the universe is determined relatively to another, called “clock.” By that, all dynamics (the Schrödinger equation) can be cast in static form: Relatively to the clock, the systems undergo the quantum dynamics.

III. FROM MEASUREMENT-DEPENDENCE TO RETRO-CAUSALITY

One way of relaxing the causal structure is by a retro-causal effect: The “Parisian zigzag” was introduced by Costa de Beauregard seventy years ago [14] and recently revived by Price [15]; we relate it to measurement-dependence.

A. “Parisian zigzag”

In 1947,⁶ Olivier Costa de Beauregard questioned the no-signaling assumption (no instantaneous causality) made by Einstein, Podolsky, and Rosen (“EPR” for short) in 1935 and considered actions to and from the common past: “[A]ll the weight of Einstein’s argument is moved from instantaneous causality to retroactive causality”⁷ [14]. This represents a reply to EPR — circumventing the claim to augment quantum theory by hidden variables — that can even be seen as a reply to Bell’s later reply to EPR.

²"Ohne solche neuen Beobachtungen führt der quantenmechanische Formalismus zu einer immer weitergehenden, aber ganz unanschaulichen Verflechtung der Elementarteilchen."

³"Erst relativ zu der neuen Messung wird der Zustand des Elektrons nach seinem Zusammenstoß mit dem Lichtquant durch eine Wellenfunktion mit scharfer Ortsangabe […] beschrieben."

⁴"bildet es also unmittelbar nach dem Zusammenstoß mit dem Lichtquant durchaus ein für sich bestehendes, durch seine eigene Wellenfunktion charakterisiertes physikalisches System."

⁵"Zu einer Voraussage […] wären jene Gründe […] nicht zu gebrauchen; denn auch sie bestimmen […] das System nur relativ, und zwar relativ zu der Beobachtung, die bei der Messung selber erst gemacht wurde. Sie konnten also dem Physiker erst nach dieser Beobachtung zur Verfügung stehen und ihm somit keine Vorausberechnung von deren Ergebnis gestatten."

⁶In 1947, Olivier Costa de Beauregard shared this idea with Louis de Broglie who disapproved. It was published in 1953 [32].

⁷"[T]out le poids de l’argument d’Einstein est ainsi transporté du paradoxe de la causalité immédiate à la causalité rétroactive."
FIG. 1. Bell’s locality: Time flows from top to bottom. Alice (Bob) inputs X (Y) and obtains A (B). Alice and Bob share an infinite amount of randomness Λ distributed independently of Alice’s and Bob’s inputs.

The “Parisian zigzag” [15, 33, 34] gives a description of Bell non-local correlations via retro-causation, i.e., causation from the future to the past. Assume an experiment in which Alice and Bob each get a photon to be measured. In that model, the photons “do not possess polarizations of their own,” but rather “borrow one later” [34]: When Alice performs the measurement on her photon, it gets a random polarization that is then sent to the photon’s source in the past, from where the “borrowed” polarization travels on to Bob in the future (this is why the speculation is called “zigzag”). In that model, “Einstein’s prohibition to ‘telegraph to the past’ does not hold at the level” of the photons, but at the one of macroscopic (in other words: classical) objects only [34]. A crucial point is that no photon travels directly from one party to the other (a path that is “physically empty”). Instead, it goes “along the Feynman-style zigzag […] made of two timelike vectors (which is physically occupied).” The view is related to models [36] with measurement-dependence through a retro-causal effect, perfectly simulating a singlet. Section III B links retro-causal approaches to measurement-dependence.

B. Retro-causal models

The retro-causality of the “Parisian zigzag” is related to the relaxation of measurement independence in the Bell model: We denote by X (Y) Alice’s (Bob’s) input, the outputs being A and B. The behavior of interest is a conditional distribution $P_{AB|XY}$. Bell-locality allows Alice and Bob to share an infinite amount of randomness Λ (see Figure 1): A distribution $P_{AB|XY}$ is called Bell-local if it can be written as

$$P_{AB|XY} = \sum_{\lambda} P_A(\lambda)P_{A|X,A=\lambda}P_{B|Y,A=\lambda}.$$

It is remarkable that quantum theory is consistent with correlations which are not Bell-local [2]. The definition of Bell-locality decomposes into three conditions.

1. **No-signaling**: Alice’s output is independent of Bob’s input, and vice versa.
2. **Locality**: The correlations between Alice and Bob stem from a shared random variable Λ.
3. **Measurement independence**: The shared randomness is independent of Alice’s and Bob’s inputs.

The third assumption is usually implicit as Λ is understood to root in the common past of Alice and Bob. If we allow the shared randomness to depend on Alice’s and Bob’s inputs, it is possible to reproduce the joint distribution of a Bell non-local quantum

\[8\]In quantum information, Schumacher [35] suggested such a “zigzag” for interpreting superdense coding: “[O]ne of the two bits is sent forward in time through the treated particle, while the other bit is sent backward in time to the EPR source, then forward in time through the untreated particle, until finally it is combined with the bit in the treated particle to reconstitute the two-bit message. Because the bit ‘sent backward in time’ cannot be used to transmit a meaningful message without the help of the other particle, no opportunity for time travel or superluminal communication is created, just as none is created in the classic EPR experiment in which simultaneous measurements are used to establish non-message-bearing correlations over a spacelike interval.”
FIG. 2. A Bell-like model with measurement-dependence and retro-causal influence: The distribution of the shared random variables depends on Alice’s and Bob’s inputs.

state [37]. Relaxing this assumption can mean, e.g., that the shared randomness influences the measurement settings of Alice and Bob. This does not change the causal structure: The common cause is in the past, and there is no retro-causal effect. (This is a flavor of determinism or the fully causal hidden variable approach of Brans [37, 38]). Alternatively, it can mean that the inputs influence the shared randomness. This points to the distributed-sampling problem [39] and retro-causal models as discussed above.

Here, we are interested in a model with relaxed causal structure and focus on the second option: A Bell-local model with measurement dependence through a retro-causal influence (Figure 2). This model can reproduce no-signaling correlations: A first idea is to send Alice’s input retro-actively to the common past and to share it with Bob. Alternatively, Alice’s input can be retro-causally used to bias the uniform distribution of the shared randomness (Λ \sim P_{\text{uniform}}). This is the retro-causal model of Feldman [36], solving a distributed-sampling problem, and it has been shown to allow Alice and Bob for simulating a singlet [39]: The Toner-Bacon protocol with one bit of communication translates to a “zigzag” with a retro-causal bit (from Alice to the common past). Note that in these models, the mutual information between the shared randomness Λ and Alice’s input is nonzero.\(^9\) It means that there is a hidden influence going from Alice to Bob via a fine-tuned “zigzag” [12]. Alternatively, we can build a mechanism without signaling between Alice and Bob. We use the fact [39] that the protocol reproducing a maximally entangled state with the help of one PR-box [44] leads to a “zigzag-style protocol” with two “retro-causal bits”: One bit each travels from Alice and Bob, respectively, to their common past, the PR correlation is established there, and the singlet can be simulated.

IV. FROM RELATIVE AND RETRO-CAUSALITY TO CLOSED TIME-LIKE CURVES

Both models of Section III B have in common that the future affects the past. In “parallel lives,” this manifests itself in the parties meeting-up for the correlation to be established/detected. In the “zigzag” models, the inputs of Alice and Bob are sent to the common past and the correlation is established there. A combination of these pictures sees the respective data meet in the future, and the local computation necessary for the verification of the correlation is at the same time its origin — if the data can travel back in time via a closed time-like curve (CTC).\(^10\)

A. Closed time-like curves with classical information

The idea of Deutsch’s model for CTC dynamics is that two systems undergo a joint evolution after which one of them travels back to the past and re-enters. Whereas Deutsch described his model for quantum states, we use classical information [45]. The

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\(^9\) There are numerous results on calculating and minimizing its amount [38, 40–43].

\(^10\) The latter have already been widely discussed in quantum information [20, 45–47].
FIG. 3. A causality-respecting system jointly evolves with a causality-violating one: The latter system’s state is the fixed-point with maximal entropy.

\[
\begin{align*}
P_{R}^\text{cons} \quad & \quad P_{V}^\text{cons} \\
P_{R}^\text{fin} \quad & \quad P_{V}^\text{fin}
\end{align*}
\]

FIG. 4. A PR box with binary inputs \( (X, Y) \) and outputs \( (A, B) \) satisfying \( A \oplus B = XY \).

causality-respecting system is denoted by \( R \), the causality-violating one by \( V \), and the joint evolution is \( \varepsilon = P_{R}^V \). For an initial state \( P_{R}^\text{init} \) of \( R \) and given \( \varepsilon \), Deutsch’s consistency condition is

\[
P_{V}^\text{cons} = \sum_{r', r, v'} P_{R}^{r'} V = r, V = v, P_{R}^\text{init} P_{V}^\text{cons},
\]

i.e., the states of \( V \) before and after the evolution are identical (see Figure 3). Generally, several consistent states can exist: In that case, Deutsch suggests to choose the one maximizing the entropy, avoiding the information antinomy. The final state \( P_{R}^\text{fin} \) of the causality-respecting system is then

\[
P_{R}^\text{fin} = \sum_{v', r, v'} P_{R}^{v'} V = r, V = v, P_{R}^\text{init} P_{V}^\text{cons},
\]

B. No-signaling correlations from closed time-like curves

We present the setup with Deutsch CTCs based on random variables to reproduce any no-signaling correlation. We show the representative example of the Popescu/Rohrlich (PR) box \([5]\) defined as (see Figure 4):

\[
P_{\text{PR}}^{AB\mid XY}(a, b, x, z) = \frac{\delta_{x, a \oplus b}}{2}.
\]

If Alice and Bob want to simulate the PR box with shared classical randomness, they can reach a success probability of \( 3/4 \). When, instead, Alice and Bob share a quantum state, then (at most \([48]\)) roughly 85% is possible. If, on the other hand, Alice and Bob have access to a classical CTC, they can perfectly simulate a PR box by local interactions. The idea is that Alice and Bob send their inputs \( X, Y \) to the common future to have them interact locally, resulting in outputs according to the PR condition, and let them travel back along the CTC (see Figure 5).

The setup uses an “open” time-like curve \([47, 49]\): The systems traveling to the past do not self-interact. When the local operations are swaps, then a single fixed point of the evolution exists (the PR box); this avoids the information antinomy. Note that the setup does not become signaling even if Alice or Bob choose to apply a different operation locally. First, the joint distribution \( P_{\text{PR}}^{AB\mid XY}(a, b, x, z) \) of the PR box is no-signaling and second, the state of all systems just before the parties apply their local operations is \( P_{X} P_{Y} \rho \), where Alice’s part of \( \rho \) contains no information on \( Y \), and vice versa: Deutsch CTCs are no-signaling preserving. (They are, however, still an “overkill” because they allow for reproducing any no-signaling distribution. A question worth exploring is to find a consistent mechanism, weaker than classical Deutsch CTCs, restricting the resulting correlations — ideally to exactly the quantum correlations.)
FIG. 5. A PR box is simulated locally in the common future of Alice and Bob. The outputs of the PR box travel back in time through a Deutsch closed time-like curve. The dashed lines represent the light cones. Time flows bottom-up.

V. RELATION TO THE OTHER MODELS

We discuss the relation of our speculation to previously considered “stories” behind the emergence of Bell correlations.

A. Hermann/Everett and “parallel lives”

The parallel-lives model [16] (see also [50]) assumes that every party, when performing a measurement, splits into “bubbles” in different realities, labeled by the measurement outcome. When Alice and Bob meet in the common future to compare their results, only those “bubbles” are visible to each other for which the labels reproduce the desired correlation. CTCs are an alternative to such a matching rule.

B. Retro-causality and distributed sampling

In the retro-causal “Parisian zigzag” [14, 15, 33, 34, 51], the input of Alice is sent to the past where it influences the shared random variable of Bob. According to Section III B, this model is fine-tuned [12] and harmonizes with our own speculation in the sense that a CTC can be seen as a mechanism for achieving retro-causality operationally and in a consistent way, i.e., without time-travel antinomies.

C. The “process-matrix” framework

In [52], no causal structure is assumed a priori but only local assumptions are made: The parties receive a system from the environment, interact with it, and output it back to the environment. The latter is non-causal: The inputs to the parties can depend on their outputs. The framework allows for correlations incompatible with definite orders of the parties. Since such correlations can be obtained even for the classical case [53], one might wonder whether also Bell-like correlations can. This is, however, not so. More specifically, the question is whether Bell non-local correlations can be seen as arising from some classical (as opposed to quantum) mechanism which is non-causal. The answer to this question is: “If yes, then only at the expense of signaling.” Thus, the classical variant of the process-matrix framework does not allow for a non-signaling explanation of Bell non-local correlations. The reasoning is straight-forward: Non-signaling correlations are causal (they can be simulated in a causal way); and the contra-positive thereof is that non-causal correlations are signaling. In other words, any classical process matrix that allows for non-causal correlations — this is the surplus of the process-matrix framework — allows for signaling as well.
VI. A LOOK BACK AND A LOOK FORWARD

In Sections II and III, we revisit two routes — proposed more than twenty years before Bell’s argument — to relaxing the causal structure for reconciling Reichenbach’s principle and quantum correlations: Hermann’s relative causality and Costa de Beauregard’s retro-causality. In Section III B, we show a new formulation of the “zigzag” model rooted in measurement-dependence and shedding light on the hidden signaling involved. In Section IV, we realize the retro-causal effect with closed time-like curves, hereby speculating about a classical mechanism establishing Bell non-local correlations.

When the parts of a system in an entangled quantum state are measured, then shared classical information can be insufficient for explaining the observed correlations: John Stewart Bell’s “exploit” in 1964 questioned fundamentally the validity of an attack to quantum theory by Einstein, Podolsky, and Rosen in 1935 — but the puzzle is, after Bell, as unsolved as ever: What could be a classical mechanism leading to correlations of classical information? Reichenbach’s principle states that in a given causal structure, this can root either in a common cause in the common past or in a direct influence from one of the correlated events to the other. Various results question its applicability in the light of Bell correlations — not only but in particular in the multipartite scenario [8–12]. There are at least three escapes from the dilemma: First, Reichenbach’s principle is declared wrong, Bell correlations being an counterexample. It could then, second, be replaced by a modified — quantum — principle accepting as a reason for correlations of classical pieces of information also an entangled state. Third, we can drop the assumption of a fundamental causal structure (Reichenbach’s principle’s basis). With the first two “emergency exits,” the story ends here; we consider the third option: Drop fundamental causality.11 This text is concerned with how loopholes of rigid causality, such as closed timelike curves, can be used to obtain Bell violations. Let us finish with a wilder speculation: What if, in the spirit of Wheeler’s “It from Bit” [54], space-time causality emerges from “laws of large numbers” at the macroscopic level of the thermodynamic limit hand in hand with — and not prior to — the classical? Information so strangely correlated? (Can we come up with a coherent combinatorial canvas comprehending the creation of classicality, correlations, and causality — combined?)

ACKNOWLEDGMENTS

We thank Andrei Khrennikov, Nicolas Gisin, and Nicolas Brunner for their kind invitation to the fascinating event in Växjö. This research is supported by the Swiss National Science Foundation (SNF), the National Centre of Competence in Research Quantum Science and Technology (QSIT), the COST action on Fundamental Problems in Quantum Physics, and by the Hasler Foundation.

11 Are we throwing out the baby with the bath water? Maybe — at least, this has some tradition: “The law of causality […] is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm.” (Bertrand Russell, 1913.)

12 Classicality is an idealized notion existing in the thermodynamic limit and related to macroscopicity and redundancy (i.e., work value [55]). Violations of Bell’s inequalities indicate the emergence, in a space-like separated way, of identical classical bits. This is the strangeness about the missing classical story. The same decision seems to be taken twice!

The measurement is the transition from quantum to classical information, so one may look for the key to the quantum measurement problem — and now also: the emergence of spacetime causality when seen as a feature of the classical realm — within thermodynamics (combinatorics), well aware of the fact that none of the present interpretations of quantum theory harmonizes with Bell non-locality. (Surely, the correlations are sometimes used by camp X to question camp Y’s interpretation and vice versa — but this often pairs with blindness for the weakness of one’s own favorite metaphysics in the face of the Bell curse; or as Nicolas Gisin put it: “Bell, c’est difficile pour tout le monde.”)
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