Classification of Tidal Disruption Events Based on Stellar Orbital Properties

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Abstract

We study the rates of tidal disruption of stars on bound to unbound orbits by intermediate-mass to supermassive black holes using high-accuracy direct N-body experiments. Stars from the star cluster approaching the black hole can have three types of orbit: eccentric, parabolic, and hyperbolic. Since the mass fallback rate shows different variabilities depending on the orbital type, we can classify tidal disruption events (TDEs) into three main categories: eccentric, parabolic, and hyperbolic. The respective TDEs are characterized by two critical values of the orbital eccentricity: the lower critical eccentricity is the one below which stars on eccentric orbits cause finite, intense accretion, and the upper critical eccentricity is the one above which stars on hyperbolic orbits cause no accretion. Moreover, we find that parabolic TDEs can be divided into three subclasses: precisely parabolic, marginally eccentric, and marginally hyperbolic. We analytically derive that the mass fallback rate of marginally eccentric TDEs can be flatter and slightly higher than the standard fallback rate proportional to \( \dot{M} \propto t^{-5/3} \), whereas it can be flatter and lower for marginally hyperbolic TDEs. We confirm using N-body experiments that only a few eccentric, precisely parabolic, and hyperbolic TDEs can occur in a spherical stellar system with a single intermediate-mass to supermassive black hole. A substantial fraction of the stars approaching the black hole would cause marginally eccentric or marginally hyperbolic TDEs.

Key words: accretion, accretion disks – black hole physics – galaxies: nuclei – galaxies: star clusters: general – methods: numerical – stars: kinematics and dynamics

1. Introduction

Tidal disruption events (TDEs) are thought to be key phenomena in the search for dormant supermassive black holes (SMBHs) at the centers of the inactive galaxies or for intermediate-mass black holes (IMBHs) at the centers of star clusters. Most TDEs take place when a star at a large separation (~1 pc) is perturbed onto a parabolic orbit approaching close enough to the SMBH to be ripped apart by its tidal force. The subsequent accretion of stellar debris falling back to the SMBH causes a characteristic flare with a luminosity large enough to exceed the Eddington luminosity for a timescale of weeks to months (Rees 1988; Evans & Kochanek 1989; Phinney 1989). Such flares have been discovered at optical (Gezari et al. 2012; Arcavi et al. 2014; Rolofien et al. 2014; Hung et al. 2017), ultraviolet (Gezari et al. 2006; Chornock et al. 2014; Vinkò et al. 2015), and soft X-ray (Komossa & Bade 1999; Saxton et al. 2012; Maksym et al. 2013; Auchettl et al. 2017) wavelengths with inferred event rates of \( 10^{-2} \)–\( 10^{-5} \) per year per galaxy (Donley et al. 2002; Wang & Merritt 2004; van Velzen & Farrar 2014; Stone & Metzger 2016). The other, high-energy jetted TDEs have been detected through non-thermal emissions in radio (Zauderer et al. 2011; Alexander et al. 2016; van Velzen et al. 2016) or hard X-ray (Burrows et al. 2011; Brown et al. 2015) wavelengths with a much lower event rate (Farrar & Piran 2014).

TDEs can largely contribute to the growth of relatively low-mass SMBHs (\( \lesssim 10^6 M_\odot \)) or IMBHs because of the lack of large amounts of gas in their environments, although the rate of tidal disruption is relatively low. The growth rate depends on the stellar density profile (Bahcall & Wolf 1976) and timescale of mass supply in the star cluster based on the classical loss-cone theory (Frank & Rees 1976). Baumgardt et al. (2004) examined the cluster density profile and the effect of the TDEs on black hole growth by performing self-consistent N-body simulations of star clusters composed of equal-mass stars and a central IMBH. Subsequently, Brockamp et al. (2011) calculated the tidal disruption rate of stars by SMBHs by performing higher-resolution N-body simulations. They concluded that relaxation-driven stellar feeding cannot let the black hole grow to more than \( 10^7 M_\odot \). Although the standard two-body scattering mechanism for generating TDEs (Magorrian & Tremaine 1999; Wang & Merritt 2004) predicts effectively parabolic trajectories, recent high-accuracy direct N-body simulations show that a significant amount of stars entering the tidal disruption radius has orbital eccentricities less and more than 1.0 (Zhong et al. 2014).

It still remains under debate how the standard, theoretical mass fallback rate proportional to \( t^{-5/3} \) (Rees 1988; Evans & Kochanek 1989; Phinney 1989) translates into the observed light curves. While most of the soft X-ray TDEs appear to follow the \( t^{-5/3} \) power-law decay curve proportional to the fallback rate (see Komossa 2015 for a review), optical to ultraviolet TDEs exhibit a different decay curve (Gezari et al. 2012; Arcavi et al. 2014; Chornock et al. 2014; Holoien et al. 2014).

Lodato et al. (2009) numerically showed that the fallback rate depends on the internal structure of the tidally disrupted stars, leading to early-time deviations from the standard fallback rate.
The centrally condensed core that survives because of the partial disruption of the star can make the resultant light curves steeper (Guillochon & Ramirez-Ruiz 2013). The accretion of clumps formed by the self-gravity of the debris stream causes significant variations of the light curve around the $r^{-5/3}$ average at late times (Coughlin & Nixon 2015). The outflows or winds caused during the super-Eddington accretion phase makes the optical to ultraviolet light curves deviate from the standard $r^{-5/3}$ curve (Strubbe & Quataert 2009; Lodato & Rossi 2011). There have been some arguments that the energy dissipated by stream–stream collisions during the debris circulation powers the observed optical to ultraviolet TDEs (Piran et al. 2015; Jiang et al. 2016; Bonnerot et al. 2017).

Recent hydrodynamic simulations have shown that observable properties of these “eccentric” TDEs significantly deviate from those of standard TDEs; in particular, the rate of mass return is substantially increased by being cut off at a finite time, rather than continuing indefinitely as a power-law decay (Hayasaki et al. 2013, 2016). This suggests that the variability of TDE light curves also depends on the orbital type of approaching stars, especially orbital eccentricity and penetration factor (which is the ratio of the tidal disruption radius to the pericenter distance of the star) of stars.

In this paper, we classify the TDEs by the type of orbits of stars approaching SMBHs or IMBHs, and examine each occurrence rate in the dense star cluster system modeled by N-body experiments. In Section 2, we give the condition for classifying the TDEs by the type of the stellar orbit, and analytically derive the mass fallback rate of each TDE based on the condition, which can have a different time dependence from the standard fallback rate proportional to $r^{-5/3}$. In Section 3, we describe our numerical approach and simulation results, where we mainly focus on the eccentricity distribution of N-body particles over their penetration factor. We discuss the relevance of our simulation results by using the scaling method to extrapolate them in Section 4. Finally, Section 5 is devoted to the conclusion of our scenario.

### 2. Type of Tidal Disruption Events

As a star approaches and enters into the tidal disruption radius of the SMBH or IMBH, it is disrupted by the tidal force of the black hole which dominates the stellar self-gravity and pressure forces at the tidal disruption radius:

$$r_{\text{t}} = \left( \frac{M_{\text{bh}}}{m_*} \right)^{1/3} r_\star \approx 24 \left( \frac{M_{\text{bh}}}{10^6 M_\odot} \right)^{-2/3} \times \left( \frac{m_*}{M_\odot} \right)^{-1/3} \left( \frac{r_\star}{R_\odot} \right) r_\odot. \quad (1)$$

Here, we denote the black hole mass with $M_{\text{bh}}$, the stellar mass with $m_*$, the radius with $r_\star$, and the Schwarzschild radius with $r_\odot = 2GM_{\text{bh}}/c^2$, where $G$ and $c$ are Newton’s gravitational constant and the speed of light, respectively. The tidal force then produces a spread in the specific energy of the stellar debris:

$$\Delta \epsilon \approx \frac{GM_{\text{bh}}r_\star}{r_{\text{t}}^2} \quad (2)$$

(Evans & Kochanek 1989).

#### 2.1. Critical Value of the Orbital Eccentricity and Semimajor Axis

The specific energy of the tidally disrupted star ranges over

$$-\Delta \epsilon + \epsilon_{\text{orb}} \leq \epsilon \leq \Delta \epsilon + \epsilon_{\text{orb}}. \quad (3)$$

Here, $\epsilon_{\text{orb}}$ is the specific orbital energy of the star approaching the black hole:

$$\epsilon_{\text{orb}} = \begin{cases} -\frac{GM_{\text{bh}}}{2r_{\text{t}}} \beta (1 - e) & \text{eccentric or circular orbit;} \\ 0 & (0 \leq e < 1) \\ \frac{GM_{\text{bh}}}{2r_{\text{t}}} \beta (e - 1) & \text{hyperbolic orbit: } (e > 1). \end{cases} \quad (4)$$

where $e$ and $\beta$ are the orbital eccentricity of the approaching star and the penetration factor, respectively. The penetration factor is defined by $r_{\text{t}}/r_p$, where $r_p$ is the pericenter distance: $r_p = a(1 - e)$ for eccentric orbits and $r_p = a(e - 1)$ for hyperbolic orbits. In the standard TDE scenario where a star is disrupted from a parabolic orbit, the debris mass will be centered on zero and distributed over $-\Delta \epsilon \leq \epsilon \leq \Delta \epsilon$ because $\epsilon_{\text{orb}} = 0$ (Evans & Kochanek 1989; Rees 1988).

Since the stellar debris with negative specific energy is bound to the black hole, it returns to pericenter and will eventually accrete onto the black hole. For eccentric orbits, if $-\Delta \epsilon + \epsilon_{\text{orb}} \leq 0$ in Equation (3), all of the stellar debris should be bound to the black hole even after tidal disruption, and eventually falls back into the black hole. The condition $\epsilon_{\text{orb}} = -\Delta \epsilon$ therefore gives a critical value of the orbital eccentricity of the star,

$$\epsilon_{\text{crit},1} = 1 - \frac{2q^{-1/3}}{\beta}, \quad (5)$$

below which all of the stellar debris should remain gravitationally bound to the black hole, where the ratio of the black hole to the stellar mass is defined by $q = M_{\text{bh}}/m_*$.  

In contrast, if $-\Delta \epsilon + \epsilon_{\text{orb}} \leq 0$ in Equation (3) for hyperbolic orbits, part of the stellar debris should be bound to the black hole and eventually fall back into the black hole. The condition $\epsilon_{\text{orb}} = \Delta \epsilon$ also gives a critical value of the orbital eccentricity of the star,

$$\epsilon_{\text{crit},2} = 1 + \frac{2q^{-1/3}}{\beta}, \quad (6)$$

below which part of the stellar debris should remain gravitationally bound to the black hole.

These critical eccentricities give us the conditions for the tidal disruption flare to happen in terms of the orbital eccentricity of the star:

$$\begin{cases} 0 \leq e < \epsilon_{\text{crit},1} & \text{eccentric TDEs} \\ \epsilon_{\text{crit},1} \leq e \leq \epsilon_{\text{crit},2} & \text{parabolic TDEs} \\ \epsilon_{\text{crit},2} < e & \text{hyperbolic TDEs}. \end{cases} \quad (7)$$
Alternatively, we can define critical values for classifying TDEs using the semimajor axis as follows:

\[
\begin{cases}
0 < a < a_c \quad \text{eccentric TDEs} \\
a_c \leq a \quad \text{parabolic TDEs} \\
0 < a < a_c \quad \text{hyperbolic TDEs},
\end{cases}
\]  

(8)

where \(\epsilon_{\text{orb}} < 0\) for eccentric TDEs and \(\epsilon_{\text{orb}} > 0\) for hyperbolic TDEs, and \(a_c\) is defined by

\[
a_c \equiv \frac{q^{1/3}}{2} \approx 50 \left( \frac{q}{10^6} \right)^{1/3} \eta.
\]  

(9)

Panel (a) of Figure 1 shows the dependence of critical eccentricities on the penetration factor \(\beta\) with the fixed value of \(M_{\text{bh}} = 10^6 M_\odot\), whereas panel (b) shows the dependence of the critical eccentricities on the mass ratio \(q\) with the fixed value of \(\beta = 1\). In both panels, the red and blue shaded areas show the regions of eccentric and hyperbolic TDEs, respectively. The white shaded region between the blue and red solid lines show the region of parabolic TDEs.

2.2. Modification of Mass Fallback Rates

Following Evans & Kochanek (1989), the mass fallback rate is given by

\[
\frac{dM}{dt} = \frac{dM}{d\epsilon} \frac{d\epsilon}{dt},
\]  

(10)

where \(dM/d\epsilon\) is the differential mass distribution of the stellar debris with specific energy \(\epsilon\). Because the thermal energy of the stellar debris is negligible compared with the binding energy, \(\epsilon \approx \epsilon_d\), where \(\epsilon_d\) is defined as the specific binding energy of the stellar debris,

\[
\epsilon_d \equiv \frac{GM_{\text{bh}}}{2a_d},
\]  

(11)

and by applying Kepler’s third law to it, we obtain

\[
\frac{d\epsilon_d}{dt} = 1 \left( \frac{d\epsilon_d}{dt} \right) \left( \frac{GM_{\text{bh}}}{2a_d} \right)^{3/2} r^{5/3}.
\]  

(12)

Here, we newly assume that

\[
\frac{dM}{d\epsilon} \equiv \frac{\eta(\alpha, a) m_\bullet}{2\Delta \epsilon} \left( \frac{-\epsilon_d}{\Delta \epsilon} \right) (\epsilon_d < 0),
\]  

(13)

where \(\alpha\) is the power-law index and \(\eta(\alpha, a)\) is the normalization coefficient obtained by the finite integral

\[
\int_{-\epsilon_d + \epsilon_{\text{orb}}}^{\epsilon_d} (dM/d\epsilon) d\epsilon = m_\bullet/2 \Delta \epsilon,
\]  

as

\[
\eta(\alpha, a) \equiv (\alpha + 1) \left[ (1 + \frac{a_\alpha}{a})^{\alpha+1} - \left( \frac{a_\alpha}{a} \right)^{\alpha+1} \right]^{-1}.
\]  

(14)

It is required that \(\alpha + 1\) should be greater than zero because \(0 \leq \eta(\alpha, a) < \infty\). If \(\alpha = 0\) is adopted, Equation (13) is reduced, independently of the semimajor axis of the approaching star, to the top-hat distribution around zero specific energy, \(dM/d\epsilon = m_\bullet/(2\Delta \epsilon)\), proposed by Rees (1988). The non-zero value of \(\alpha\) represents the effect of the density profile of the star on \(dM/d\epsilon\). In the limit of \(a \rightarrow \infty\), Equation (13) can be used to estimate the \(dM/d\epsilon\) of the centrally condensed stars on parabolic orbits (Lodato et al. 2009) or of the partially disrupted stars on parabolic orbits (Guillochon & Ramirez-Ruiz 2013). In the case of eccentric TDEs, \(dM/d\epsilon\) has a different distribution from top-hat (Hayasaki et al. 2013). This implies that the \(dM/d\epsilon\) of non-parabolic TDEs can deviate from the standard, top-hat distribution. Since \(dM/d\epsilon\) is a decreasing function of \(-\epsilon_d\), \(\alpha\) should be less than or equal to zero. The possible range of \(\alpha\) is therefore given by \(-1 < \alpha \leq 0\).

The specific binding energy of the most tightly bound debris is given by

\[
\epsilon_{\text{mb}} = -\Delta \epsilon \pm \frac{GM_{\text{bh}}}{2a}.
\]

where the negative and positive signs of the second term of the right-hand side originate from the stars originally approaching on eccentric and hyperbolic orbits, respectively. It is easily confirmed that \(\epsilon_{\text{mb}}\) is reduced to be that of a precisely parabolic orbit (\(e = 1\)) in the limit of \(a \rightarrow \infty\). The orbital period of the most tightly bound debris is proportional to \(\epsilon_{\text{mb}}^{-3/2}\) from
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### Table 1
Summary of the Classification of TDEs

| Type of TDEs                  | \(e\)                  | \(a\)          | \(\alpha\) | \(\frac{dM}{dt}\)          |
|------------------------------|------------------------|----------------|------------|----------------------------|
| Eccentric                    | \(0 \leq e < e_{crit,1}\) | \(0 < a < a_c\) | ...        | Intense accretion*          |
| Marginally eccentric         | \(e_{crit,1} \leq e < 1\) | \(a_{c} \leq a < a_c\) | \(-1 < \alpha \leq 0\) | \((\eta_2(a, \alpha)/3)(m_*/t_{mb})/t_{mb}\)^{−2\alpha/3−5/3} |
| Precisely parabolic          | \(e = 1\)              | \(a \to \infty\) | \(-1 < \alpha \leq 0\) | \((1/3)(\alpha + 1)(m_*/t_{mb})/t_{mb}\)^{−2\alpha/3−5/3} |
| Marginally hyperbolic        | \(1 < e \leq e_{crit,2}\) | \(a_{c} \leq a < a_c\) | \(-1 < \alpha \leq 0\) | \((\eta_2(a, \alpha)/3)(m_*/t_{mb})/t_{mb}\)^{−2\alpha/3−5/3} |
| Hyperbolic                   | \(e_{crit,2} < e\)     | \(0 < a < a_c\) | ...        | No accretion               |

**Notes.** The first column represents the type of TDE. The second column shows the condition for classifying TDEs using the orbital eccentricity, \(e\), with two critical eccentricities \(e_{crit,1}\) and \(e_{crit,2}\) (see Equations (5)-(7)). The third column shows the condition using the semimajor axis, \(a\), with the critical semimajor axis \(a_c\) (see Equations (8) and (9)). The fourth column denotes the possible range of \(\alpha\) for each TDE. The final column represents the mass fallback rate for each TDE, where \(\eta_2(a, \alpha)\) is the proportionality coefficient given by Equation (17), and \(t_{mb}\) is the orbital period of the most tightly bound orbit given by Equation (15).

* See Hayasaki et al. (2013).

\(\beta\) it corresponds to Equation (3) of Evans & Kochanek (1989) if \(\alpha = 0\) is adopted.

Kepler’s third law,

\[
t_{mb} = \sqrt{\frac{4\pi^2}{GM_\ast}} a_c^{3/2} \left(1 + \frac{a_c}{a}\right)^{-3/2} \ (a \geq a_c).
\] (15)

Substituting Equations (12) and (13) into Equation (10) with Equations (11) and (15), we obtain the modified fallback rate,

\[
\frac{dM}{dt} = \frac{\eta_2(\alpha, a)}{3} \left(\frac{m_*}{t_{mb}}\right) t_{mb}^{−2\alpha/3−5/3},
\] (16)

where \(\eta_2(\alpha, a)\) is the proportionality coefficient defined by

\[
\eta_2(\alpha, a) \equiv (\alpha + 1) \left(1 + \frac{a_c}{a}\right)^{-\alpha} \times \left[\left(1 + \frac{a_c}{a}\right)^{\alpha + 1} - \left(\frac{a_c}{a}\right)^{\alpha + 1}\right]^{-1}.
\] (17)

with the upper and lower signs corresponding to the hyperbolic and eccentric orbit cases, respectively. Note that \(\eta_2(\alpha, a)\) should be greater than or equal to zero in order for \(\frac{dM}{dt} \geq 0\).

The relation between \(\eta_2\) and \(\eta_1\) is given by \(\eta_2(\alpha, a) = \eta_1(\alpha, a)(1 \mp a_c/a)^{\alpha+1}\) from Equations (14) and (17). The possible range of \(\alpha\) for a given value of \(a\) is therefore \(-1 < \alpha \leq 0\) in Equation (16). For both the eccentric and hyperbolic orbit cases, the possible range of \(a\) is \(a_c \leq a < \infty\). In the limit of parabolic orbit \((a \to \infty)\), \(\eta_2(\alpha, a)\) is reduced to \(\alpha + 1\). For the hyperbolic orbit case, \(\eta_2(\alpha, a)\) is always smaller than unity, and \(\eta_2(\alpha, a_c) = 0\) at the equality \(a = a_c\). This equality means that the star approaches the black hole on a hyperbolic orbit such that no debris falls back after tidal disruption. These arguments imply that parabolic TDEs can be divided into three subclasses: marginally eccentric \((e_{crit,1} \leq e < 1)\); standard, precisely parabolic \((e = 1)\); and marginally hyperbolic \((1 < e \leq e_{crit,2})\). We summarize the classification of TDEs in Table 1.

For marginally eccentric TDEs, the mass fallback rate is maximum at \(a = a_c\) and \(\alpha = 0\). While the mass fallback rate is proportional to \(t^{-5/3}\) for \(\alpha = 0\), it more loosely decays with time for \(-1 < \alpha < 0\). For precisely parabolic TDEs \((a \to \infty)\), Equation (16) reduces to \(\frac{dM}{dt} = (\alpha + 1)/(3m_*/t_{mb}) (t/t_{mb})^{-2\alpha/3−5/3}\). If \(\alpha = 0\) is adopted, it corresponds to Equation (3) of Evans & Kochanek (1989): \(\frac{dM}{dt} = 1/3 (m_*/t_{mb}) (t/t_{mb})^{-5/3}\). For marginally hyperbolic TDEs, the mass fallback rate takes a maximum at \(a \to \infty\) and \(\alpha = 0\), and is close to zero as \(a \to a_c\). It more loosely decays with time for \(-1 < \alpha < 0\), as is the case with the marginally eccentric TDEs. For hyperbolic TDEs \((e_{crit,2} < e)\), the mass fallback rate should be zero. In other words, none of the debris mass is bounded to the black hole. The hyperbolic TDEs thus cannot contribute to the event rate of the tidal disruption, even if they might occur. The current formula for \(\frac{dM}{dt}\) significantly underestimates the mass fallback rate of eccentric TDEs \((0 \leq e < e_{crit,1})\), because \(\frac{dM}{dt}\) would have a Gaussian-like distribution rather than a simple power-law one if the specific binding energy of the most loosely bound orbit is negative enough beyond \(-\Delta\varepsilon\) (Hayasaki et al. 2013). Our conjecture given by Equation (13) should therefore be inapplicable to eccentric TDEs.

### 3. N-body Experiments

In this section, we present a scaling study using N-body experiments of whether there are five (three plus two) types of TDEs from the viewpoint of stellar orbits, and estimate the fractional number of each event rate. All of the simulations are performed using the massively parallel \(\phi\)-GRAPE code (Harfst et al. 2007), with high performance up to 1.5 T flop/s per GPU on our HPC clusters in Beijing (NAOC/CAS) and Heidelberg (ARI/ZAH; Berczik et al. 2011; Spurzem et al. 2012; Berczik et al. 2013a, 2013b). The code is a direct N-body simulation package, with a high-order Hermite integration scheme and individual block time steps. A direct N-body code evaluates in principle all pairwise forces between the gravitating particles, and its computational complexity scales asymptotically with \(N^2\); however, it is not to be confused with a simple brute force shared memory code, due to the block time steps. The present code is well-tested and has already been used to obtain important results in our earlier large-scale (up to few million body) simulation (Khan et al. 2012; Zhong et al. 2014; Khan et al. 2016; Li et al. 2017).

#### 3.1. Method

We use the same simulation method as Zhong et al. (2014) here. In our simulations, \(G = 1\), \(M_\ast = 1\), \(r_c = 1\), and \(E_c = −1/4\) (Hénon units) are adopted for our purposes (Hénon 1971; Heggie & Mathieu 1986), where \(G\), \(M_\ast\), \(r_c\), and \(E_c\) are the gravitational constant, the total mass of the cluster, the virial radius, and the energy of the star cluster, respectively.

We choose different values of \(N\) and introduce the normalized accretion radius \(\xi_{acc} \equiv r_{acc}/r_c\) to evaluate the physical scaling behavior of our system and extrapolate to the real...
system. Here, \( N = M_c / m \) is the particle number, which defines the ratio between the particle mass \( m \) and the total mass of the system \( M_c \). Note that \( m \) does not have to be identical to the stellar mass. The currently adopted \( N \) is from 128 to 512 K (see Table 2), where we define \( 1 \) K = 1024 in this paper due to technical reasons, and a Plummer model is adopted for the initial stellar distribution (Aarseth et al. 1974).

The normalized accretion radius \( \xi_{\text{acc}} \) is another dimensionless number which defines the radius at which simulation particles are going to be disrupted by the tidal forces of a central black hole, relative to the virial radius of our system, which is used as the standard unit. Extrapolation to the real system means that \( N \) is approaching real particle (star) numbers (say 10^8 in galactic nuclei) and \( \xi \equiv r_{\text{acc}} / r_e \) is close to 1 at the same time. We also have a third dimensionless parameter in our models, which is \( \mu = M_{\text{bh}} / M_c \). From the standard relations between galactic bulges and central massive black holes (Magorrian et al. 1998; McConnell & Ma 2013), it should be up to \( \sim \)0.006. However, we choose higher values because we only simulate part of the central star cluster mass, \( \mu = (0.01, 0.05) \).

In our simulations, there are two types of sink particles. The first is the black hole that remains stationary at the center of mass with no accretion. In this case, the stars entering a finite accretion radius, which corresponds to the tidal disruption radius, does not contribute to the growth of the black hole and does not add linear momentum to the black hole, and are removed from the stellar system right away. It looks unphysical but is enough to test which orbit the stars are tidally disrupted on. The second type is the black hole particle simply gaining the masses of the removed stars without incurring their linear momentum. Once the star arrives at the tidal disruption radius, it will be removed from the stellar system. We already implemented similar approaches in the \( \phi \)-GRAPE/GPU code and tested them well against all the energy and momentum conservations in our earlier works (Just et al. 2012; Kennedy et al. 2016). The initial density profile of the Plummer model has a central flat core, which adjusts to the gravity of the central black hole during a few dynamical orbits, as is the case in Zhong et al. (2014). In any case, all of the stars have equal mass and form no binary stars through the simulations. We adopt three fixed accretion radii in \( N \)-body units; \( \xi_{\text{acc}} = 10^{-3}, 10^{-4}, \) and \( 10^{-5} \). We also run the model with \( \xi_{\text{acc}} = 5 \times 10^{-5} \), which are used to extrapolate our simulation models to a realistic system (see Section 4). The accretion radius we used here is larger than the tidal disruption radius, typically boosted by a factor of \( 10^2 - 10^3 \) for the SMBH cases, because of our scaling requirements. We discuss this in Section 4.

Table 2 shows the simulation parameters and results. The first column shows each simulated model. The second, third, fourth, and fifth columns are the total number of \( N \)-body particles \( N \) in units of \( K = 1024 \), the accretion radius \( \xi_{\text{acc}} \), the number of accreted particles \( N_{\text{acc}} \) and the simulation run time \( t_{\text{end}} \). The sixth and seventh columns describe the normalized initial and final masses of the black hole, respectively. The last five columns represent the fractional number of accreted particles on the respective orbits in percent (\%), where \( f_{e, f_{\text{acc}}}, f_p, f_{\text{min}}, \) and \( f_{\text{th}} \) are the fractional number of the eccentric, parabolic, marginally hyperbolic, and hyperbolic TDEs, respectively (see Table 1 for the definition of each TDE).

| Model | \( N/K \) | \( \xi_{\text{acc}} \) | \( N_{\text{acc}} \) | \( t_{\text{end}} \) | \( m_{\text{ini}} \) | \( m_{\text{end}} \) | \( f_e \) | \( f_{\text{acc}} \) | \( f_p \) | \( f_{\text{min}} \) | \( f_{\text{th}} \) |
|-------|---------|----------------|----------------|----------------|----------------|----------------|------|--------------|------|-------------|------|
| 1     | 128     | \( 10^{-3} \) | 449            | 1500           | 0.01           | 0.01           | 0    | 95.6         | 0.2  | 4.2         | 0    |
| 2     | 128     | \( 10^{-4} \) | 1972           | 700            | 0.01           | 0.01           | 0    | 28.9         | 0.0  | 71.1        | 0    |
| 3     | 256     | \( 10^{-5} \) | 712            | 1500           | 0.01           | 0.01           | 0    | 75.3         | 0.0  | 25.7        | 0    |
| 4     | 256     | \( 10^{-4} \) | 1035           | 400            | 0.01           | 0.01           | 0    | 19.1         | 0.0  | 80.9        | 0    |
| 5     | 512     | \( 10^{-5} \) | 693            | 1000           | 0.01           | 0.01           | 0    | 54.3         | 0.0  | 45.7        | 0    |
| 6     | 128     | \( 10^{-5} \) | 1141           | 900            | 0.05           | 0.05           | 0    | 91.1         | 0.0  | 8.9         | 0    |
| 7     | 128     | \( 10^{-4} \) | 2433           | 500            | 0.05           | 0.05           | 0    | 29.82        | 0.08 | 70.1        | 0    |
| 8     | 256     | \( 10^{-5} \) | 1318           | 700            | 0.05           | 0.05           | 0    | 67.5         | 0.0  | 32.5        | 0    |
| 9     | 256     | \( 10^{-4} \) | 3763           | 500            | 0.05           | 0.05           | 0    | 22.3         | 0.0  | 77.7        | 0    |
| 10    | 512     | \( 10^{-5} \) | 1854           | 600            | 0.05           | 0.05           | 0    | 45.95        | 0.05 | 54.0        | 0    |
| 11    | 128     | \( 10^{-5} \) | 1171           | 2200           | 0.01           | 0.01           | 0    | 54           | 0.0  | 46.0        | 0    |
| 12    | 128     | \( 10^{-4} \) | 8288           | 2450           | 0.01           | 0.037          | 0.2   | 87.8         | 0.0  | 12.0        | 0    |
| 13    | 128     | \( 10^{-5} \) | 16620          | 2200          | 0.01           | 0.14           | 0.6   | 22.8         | 0.0  | 76.4        | 0.2  |
| 14    | 256     | \( 10^{-5} \) | 1627           | 2522           | 0.01           | 0.018          | 0    | 39.3         | 0.0  | 60.7        | 0    |
| 15    | 256     | \( 10^{-4} \) | 6651           | 1400           | 0.01           | 0.035          | 0    | 13.5         | 0.0  | 86.5        | 0    |

Note. The first column shows each model. The second, third, fourth, and fifth columns are the total number of \( N \)-body particles \( N \) in units of \( K = 1024 \), the accretion radius \( \xi_{\text{acc}} \), the number of accreted particles \( N_{\text{acc}} \) and the simulation run time \( t_{\text{end}} \), respectively. The sixth and seventh columns describe the normalized initial and final masses of the black hole, respectively. The last five columns represent the fractional number of accreted particles on the respective orbits in percent (\%), where \( f_e, f_{\text{acc}}, f_p, f_{\text{min}}, \) and \( f_{\text{th}} \) are the fractional number of the eccentric, parabolic, marginally hyperbolic, and hyperbolic TDEs, respectively (see Table 1 for the definition of each TDE).
3.2. Results

In this section, we describe the results of the $N$-body simulations. The rate of accreted stars is given by $\langle M_{\text{acc}} \rangle = (M_c/\ln t_{\text{end}})(N_{\text{acc}}/N)$, which is estimated to be $10^{-5}$ to $10^{-6}$ in simulation units for all of the models. All of our simulations do not reach the steady state for the rate. This is because the state of the loss cone, which controls the rate of accreted stars, changes with time. In the early phase, the loss cone is full and a density cusp forms around the central black hole, leading to an enhancement of the accretion rate. On the other hand, the empty loss cone leads to a reduction of the accretion rate in the late phase when a few half-mass relaxation times elapse (see also Figure 1 of Zhong et al. 2014).

Figures 2–4 show the dependence of the orbital eccentricity of the $N$-body particles, which accrete inside the accretion radius, on the penetration factor, $\beta$. Hereafter, we call it the $e-\beta$ distribution of the accreted stars. In these figures, the small black circles represent the $e-\beta$ distribution of the accreted $N$-body particles, whereas the black dashed line denotes $e = 1$. The red and blue solid lines show the critical eccentricities that are analytically expected from Equations (5) and (6) with the fixed value of the mass ratio of the black hole to $N$-body particles, while the small red and blue circles show the two critical eccentricities of each $N$-body particle, which are numerically determined by substituting both the $\beta$ of each $N$-body particle and the mass ratio of the black hole to the $N$-body particles into Equations (5) and (6).

Figure 2 shows the $e-\beta$ distribution in Models 1–5. We confirm that the numerically calculated critical eccentricities are in good agreement with the analytically expected ones. In addition, almost all of the accreted particles are distributed closely around $e = 1$ between the two critical eccentricities. This means that eccentric and hyperbolic TDEs occur extremely rarely. For $\xi_{\text{acc}} = 10^{-4}$ cases (Models 2 and 4), the $N$-body particles are clearly distributed in the range of $e_{\text{crit},1} < e < e_{\text{crit},2}$. We note that a significant fraction of the accreted particles will undergo marginally eccentric and marginally hyperbolic TDEs. We also note from Figure 3 that the $e-\beta$ distributions of Models 6–10 qualitatively correspond to those of Models 1–5.

The $e-\beta$ distribution of the accreted stars in the growing black hole case is different from that of the fixed black hole mass case mainly in the two following points. Figure 4 represents the $e-\beta$ distribution of the accreted stars in Models 11–15. The first point is that the numerically calculated critical eccentricities deviate from the analytically expected ones. In Models 11–15, the black hole mass increases with time because of the accreted particles during the simulations. As seen in panel (b) of Figure 1, both critical eccentricities are closer to unity ($e = 1$) with the growth of the black hole particle. The second point is that the number of more strongly bound $N$-body particles is larger than that in the fixed black hole case by comparing panel (d) of Figure 2 with panel (d) of Figure 4. This is because the deeper gravitational potential of the black hole captures more particles at the same distance from the black hole than in the non-growth case. Moreover, some $N$-body particles clearly have an orbital eccentricity beyond the two critical eccentricities, as seen in panel (e). This is because the larger cross-section makes it possible for the particles with a larger angular momentum than the $\xi_{\text{acc}} = 10^{-4}$ and $\xi_{\text{acc}} = 10^{-5}$ cases to accrete onto the black hole.

Finally, let us see how the fraction of accreted particles is assigned to eccentric, marginally eccentric, precisely parabolic, marginally hyperbolic, and hyperbolic TDEs. Because $f_c$, $f_p$, and $f_h$ are very tiny as seen in the last five columns of Table 2, the eccentric, precisely parabolic, and hyperbolic TDEs are extremely rare events. Almost all of the accreted particles originate from $N$-body particles on marginally eccentric ($e_{\text{crit},1} \leq e < 1$) or marginally hyperbolic orbits ($1 < e \leq e_{\text{crit},2}$). We also find from the last five columns of Table 2 that the ratio of $f_{\text{me}}$ to $f_{\text{mh}}$ changes drastically. This can be interpreted as follows: while all of the stars inside the influence radius of the central black hole, $r_h = GM_{\text{bh}}/\sigma^2$, where $\sigma$ is the cluster’s velocity dispersion, are bound to the black hole, the stars outside the influence radius are not bound to the black hole. According to the loss-cone theory (Frank & Rees 1976; see also Merritt 2013 for a review), stars are supplied to the black hole mainly from the critical radius, $r_{\text{crit}}$, where the opening angle of the loss-cone angle $\theta_{\text{lc}} \approx \sqrt{1/r_{\text{crit}}}$ for $r \lesssim r_h$ is equal to the diffusion angle $\theta_d \propto \ln(N/N)$. Because $r_{\text{crit}}$ is proportional to $(N/\ln N)r_h$, it depends on each model. If $r_{\text{crit}}$ is smaller than $r_h$, most of the accreted stars would be bound, causing marginally eccentric TDEs. Otherwise, they would be unbound, causing marginally hyperbolic TDEs. We will have a more detailed discussion about this speculation in a forthcoming paper (S. Zhong et al. 2018, in preparation).

4. Discussion

In a realistic intermediate-mass to supermassive system, the tidal disruption radius should be smaller than $\xi_{\text{acc}} = 10^{-5}$, which is the smallest normalized accretion radius among our simulations, if the accretion radius is equal to the tidal disruption radius. Therefore, we extrapolate from the simulation data the orbital eccentricities at the realistic value of $\xi_{\text{acc}}$ and the higher particle resolution using the linear least-squares fitting method, $y = cx + d$, where the fitted values are plugged into $c$ and $d$, the mean value $e_{\text{mean}}$ or standard deviation $e_{\text{std}}$ of the orbital eccentricity are plugged into $\gamma$, and the normalized accretion radius and the number of $N$-body particles are plugged into $x$. We define $e_{\text{mean}}$ and $e_{\text{std}}$ as follows: first, we divide the respective orbital eccentricities into some subsamples using a certain range of $\beta$, and then compute $e_{\text{mean}}$ and $e_{\text{std}}$ in each subsample. As one can see in the lower-$\beta$ region around $\beta \sim 1$, the different models show significant variations, while in the higher-$\beta$ region they take almost the same values. Therefore, the data points within the range of $1 < \beta < 1.2$ are used to calculate $e_{\text{mean}}$ and $e_{\text{std}}$.

Figure 5 shows the dependence of the mean value and the standard deviation of the simulated orbital eccentricity on the number of $N$-body particles for the fixed value of $\xi_{\text{acc}} = 10^{-5}$ and $\mu = 0.01$ (Models 1, 3, and 5). The left and right panels are for $e_{\text{mean}}$ and $e_{\text{std}}$, respectively. The eccentricity increases slightly with higher mass resolution, whereas the standard deviation is smaller as the number of $N$-body particles increases. Figure 6 shows the dependence of the mean value and the standard deviation of the simulated orbital eccentricity on the normalized accretion radius for the fixed value of $N = 256$ K and $\mu = 0.01$ (Models 3 and 4). Note that we used the $\xi_{\text{acc}} = 5 \times 10^{-5}$ case to make the argument more reliable. Both the mean value and standard deviation of the orbital eccentricity decrease with the normalized accretion radius. Overall, it is noted from the figures that the orbital eccentricity deviates little from the mean value with the accretion radius
and the number of $N$-body particles. This tendency can be adopted for the realistic extrapolated region, because the variation of the standard deviation is less than 0.1% for the given variables.

Next, in order to discuss how realistic the extrapolated values are, we introduce

$$\rho_{bh} = \frac{M_{bh}}{r_t^3}$$ \hfill (18)

as the black hole density estimated at the tidal disruption radius. With the three dimensionless parameters we previously defined, $\mu = M_{bh}/M_c$, $\xi_{acc} = r_{acc}/r_c$, and $\zeta = r_t/r_{acc}$, the black hole density can be rewritten as

$$\rho_{bh} = \mu \left( \frac{\zeta}{\xi_{acc}} \right)^3 \rho_c,$$ \hfill (19)

where $\rho_c \equiv M_c/r_c^3$ is defined as the mean stellar density of the cluster. Substituting Equation (1) into Equation (18), the black hole density is equivalent to the mean star density, $\rho_{bh} \equiv m_b/r_t^3$.

Therefore, the normalized mean star density can be given in

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**Figure 2.** Dependence of critical and orbital eccentricities on the penetration factor $\beta$ in the case of $\mu = 0.01$ (Models 1–5). The small red and blue circles represent the critical eccentricities of each accreted $N$-body particle for eccentric and hyperbolic TDEs, respectively. The small black circles show the orbital eccentricities of the accreted $N$-body particles. The black dashed line denotes $e = 1$, while the red and blue solid lines denote the corresponding analytically expected critical eccentricities.
The upper equation shows the normalized mean star density obtained from our simulation parameters, where we adopt \( \zeta = 1 \), which means that the accretion radius corresponds to the tidal disruption radius. In the lower equation, the normalized star density we estimate straightforwardly is constant for the accretion radius.

Figure 7 shows the dependence of the normalized mean star density on \( \xi_{\text{acc}} \). The solid and dashed black lines are \( \rho^*/\rho_c \), which is given by the upper part of Equation (20), with \( \mu = 0.01 \) and \( \mu = 0.05 \), respectively. Assuming that \( \rho_c = 10^8 \, M_\odot \, \text{pc}^{-3} \), the red and blue lines are \( \rho^*/\rho_c \), which is given by the lower part of Equation (20), with \( (m_*, r_*) = (1 \, M_\odot, 1 \, R_\odot) \) and \( (m_*, r_*) = (10 \, M_\odot, 10 \, R_\odot) \), respectively. The shaded area is the region where the cluster density would be realistic. Our simulation models range from \( \xi_{\text{acc}} = 10^{-5} \) to \( 10^{-3} \), whereas the extrapolated range is less than \( \xi_{\text{acc}} = 10^{-5} \). From the figure, we note that the range of \( \xi_{\text{acc}} \leq 10^{-5} \) should
be realistic, if the averaged density of the realistic star cluster composed mainly of early-type stars is equal to $10^{8} \, M_{\odot} \, pc^{-3}$. This is independent of whether the cluster has an SMBH or IMBH.

Let us discuss whether our extrapolation method is applicable to all of the models we have created. To resolve the transition from full to empty loss cone, as predicted by the loss-cone theory (Frank & Rees 1976; Merritt 2013), in direct $N$-body simulations, $\xi_{\text{acc}}$ has to be consistent with the limited resolution due to the finite particle number in the model. A too large $\xi_{\text{acc}}$ means all loss cones are too large and never completely empty (always $\theta_{lc} > \theta_{D}$), and a too small one means that we are always in the pinhole regime where $\theta_{D} > \theta_{lc}$. For a given particle number $N$, only a certain range of $\xi_{\text{acc}}$ allows the correct full to empty loss cone transition at $\theta_{lc} = \theta_{D}$ to be resolved. Zhong et al. (2014) confirmed that our simulations are consistent with the loss-cone theory, if the normalized accretion radius is less than $\xi_{\text{acc}} = 10^{-4}$. Therefore, we applied our extrapolation method only to the simulation models with a normalized accretion radius less than $\xi_{\text{acc}} = 10^{-4}$. Model 15 should be excluded from the extrapolation argument noted above, although it produces a tiny but interesting possibility of causing both eccentric and hyperbolic TDEs, as shown in Table 2.

As seen in panel (b) of Figure 1, the critical eccentricities are also closer to unity as the ratio of the central black hole to the stellar mass is larger. This tendency can be seen in Figure 4: the simulated critical eccentricities is close to unity as the black

Figure 4. Same format as Figure 2, but for the case of the growing black hole initially from $\mu = 0.01$ (Models 11–15).
hole mass increases with time, although it is limited to the very narrow range of the mass ratio. In a forthcoming paper, we will examine the broader range of the mass ratio. It is interesting to see which marginally eccentric or hyperbolic TDEs occur more preferentially. As discussed in Section 3.2, the source of marginally eccentric TDEs is stars.
falling to the black hole mainly from the critical radius inside the influence radius, whereas the source of marginally hyperbolic TDEs is stars falling to the black hole mainly from the critical radius outside the influence radius. Therefore, the ratio of $f_{\text{ne}}$ to $f_{\text{mp}}$ should be determined by the location of the critical radius relative to the influence radius. This suggests that $f_{\text{ne}}/f_{\text{mp}}$ is close to unity if the stars have a recessed distribution symmetrically around the radius, where the influence radius is accordingly equal to the critical radius. Models 5 and 10 correspond to this case. Whether this argument is robust will be confirmed by performing higher particle resolution N-body experiments with a smaller accretion radius.

The deviation between some observed optical–UV TDEs light curves and the $r^{-5/3}$ decline rate is currently a topic of debate (e.g., Gezari et al. 2012). Also, the soft X-ray TDE candidate represents a slightly different power-law decay from $r^{-5/3}$ (Maksym et al. 2013, although it appears to correspond to the $r^{-5/3}$ curve overall. Assuming that the observed luminosity is simply proportional to $r^{-n}$, we find

$$n = \frac{2\alpha + 5}{3}$$  (21)

from our conjecture of the mass fallback rate given by Equation (16). We note from Table 1 that the possible range of $n$ is $1 < n \leq 5/3$ for marginally eccentric and marginally hyperbolic TDEs. Gezari et al. (2012) discussed that the value of $n$ fitted to the decay of PS1-10jh was estimated to be $n = 5/9, 35/36$, and $12/15$ for the respective flaring phases. Because these indices are less than unity, our conjecture is not appropriate for the PS1-10jh case. The other optical–UV TDE candidate, J0225–0432, showed that a best fit for the value of $n$ to the UV data gives $n \approx 1.1$ (Gezari et al. 2008). In this case, we cannot reject the possibility that J0225–0432 is a candidate for marginally eccentric or marginally hyperbolic TDEs. This is also consistent with the light curve of J0225–0432 being shallower than the $r^{-5/3}$ profile because of the internal structure of the star, as argued by Lodato et al. (2009). If the star is partially disrupted, the range of $n$ can be from 2.2 to 4 because of the centrally condensed mass distribution, leading to a steeper mass fallback rate (Guillochon & Ramirez-Ruiz 2013). This case is also beyond the scope of our conjecture. It suggests that the differential mass distribution should not follow a simple power law of the specific energy. We need to rebuild the conjecture by taking account of the detailed internal structure of the star or the stellar debris.

Although these arguments seem to be independent of the semimajor axis and orbital eccentricity of the star approaching the black hole, the difference between precisely parabolic and marginally eccentric/hyperbolic TDEs is shown in the magnitude of the mass fallback rate for a given value of $\alpha$. In addition, the value of $\alpha$ can depend on the semimajor axis and the orbital eccentricity as Hayasaki et al. (2013) implied. Little is known whether and how it can depend on them, and there is no direct estimation of $\alpha$. Therefore, it is desired to examine the dependence of $\alpha$ on the given semimajor axis and orbital eccentricity in detail using hydrodynamic simulations.

5. Conclusions

We have investigated the distribution of the orbital eccentricity of stars approaching intermediate to supermassive black holes using N-body experiments. Since our N-body models do not reach a realistic resolution in particle number $N$ for galactic nuclei and consequently also cannot resolve the realistic value of the tidal disruption radius, we have used the method of scaling to extrapolate our results to the situation in a real galactic nucleus or nuclear stellar cluster. We have also found the condition for categorizing TDEs into three types: eccentric, parabolic, and hyperbolic, using the orbital eccentricity, $e$, and the semimajor axis of the originally approaching star, $a$. Based on this condition, we have analytically derived the mass fallback rates of the respective TDEs. Our main conclusions are summarized as follows:

1. Parabolic TDEs are moreover divided into three subclasses: TDEs from stars on precisely parabolic orbits ($e = 1$), marginally eccentric TDEs ($e_{\text{crit},1} \leq e < 1$), and marginally hyperbolic TDEs ($1 < e \leq e_{\text{crit},2}$). While the mass fallback rate of marginally eccentric TDEs can be flatter and slightly higher than the standard fallback rate proportional to $r^{-5/3}$, it can be flatter and lower for marginally hyperbolic TDEs. The details are summarized in Table 1.

2. We find that there are two critical values of the orbital eccentricity: $e_{\text{crit},1} = 1 - 2q^{-1/3}/\beta$, below which eccentric TDEs occur, and $e_{\text{crit},2} = 1 + 2q^{-1/3}/\beta$, above which hyperbolic TDEs occur, where $q$ is the ratio of the black hole to the stellar mass and $\beta$ is the penetration factor. As the mass ratio becomes more extreme and the pericenter distance closer to the Schwarzschild radius, these critical eccentricities get closer to 1. We confirm from our simulations that these critical eccentricities vary as the black hole grows.

3. Alternatively, there is a critical value of the semimajor axis: $a_c = 50(q/10^6)^{1/3}r_t$, where $r_t$ is the tidal disruption radius. If $a \leq a_c$, then eccentric and hyperbolic TDEs would occur. However, we confirm by N-body experiments that eccentric, precisely parabolic, and hyperbolic TDEs occur extremely rarely in a spherical stellar system with a single intermediate-mass to supermassive black hole. Instead, a substantial fraction of the stars causes marginally eccentric or marginally hyperbolic TDEs.

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