Electron acceleration by cosh-Gaussian laser beam in the presence of axial magnetic field

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Abstract
The present study was conducted to determine electron acceleration by cosh-Gaussian laser beam (CGLB) and axial magnetic field. CGLB possesses higher power and has ability to focus earlier than the Gaussian beam. Nonlinear differential equations are derived for CGLB for energy and velocity. The dependence of electron energy gained for different values of laser intensity parameter ‘a₀’ has been graphically plotted. It is examined that both laser and axial magnetic field significantly affects the electron energy.

1. Introduction
Particle accelerators have become a benchmark for any country’s development status. It not only helps us to understand the origin of matter and the universe but these accelerators are showing huge potential applications in medicine, engineering and in biological, chemical and physical sciences. The basic mechanism of accelerating a charged particle involves subjecting the charged particle in electric field. The positive ions are accelerated in the direction of field whereas electrons get accelerated opposite the direction of field. By increasing the value of electric field the magnitude of accelerating force also increases and by this the energy acquired the electrons or ions also increases.

About forty years back Tajima and Dawson proposed the use of plasma as accelerating medium [1]. Since plasma has a nature of supporting ultra-high accelerating gradient and is an excellent resource of electrons, so laser induced relativistic electron acceleration is studied both theoretically [2-3] as well as experimentally [4-5] in the past and present. Plasma based electron acceleration can be generally divided in two groups. The first is to use driven plasma wave generated by lasers to accelerate electrons called Laser Wake Field Acceleration (LWFA). LWFA uses laser ponderomotive force to generate the plasma wave as dynamic accelerating structure. The structures of laser wakefield strongly depends upon the laser pulse. The second one, Direct Laser acceleration (DLA). In this mechanism, laser pulse directly imparts its electromagnetic energy into electrons. It accelerates electrons in the ion channels or in the plasma bubbles through the Betatron resonance mechanism.

Mourou et al. [6] in 1985 successfully demonstrated the amplification of any short pulse to get high peak power pulses using Chirped Pulse Amplification (CPA). Rosenzweig et al. [7] in 1988 reported the experimental test to accelerate the electrons in the wake of an intense diver-beam pulse in the plasma. One of the initial experimental investigation was presented in 1990 by Nishida et al. [8] in which the acceleration of injected electrons by (vₑ x B) method was demonstrated. Nakajima et al. [9] observed the acceleration of electrons by the wake fields induced in plasma by a short intense laser pulse. Electrons were accelerated upto 1000MeV using LWFA. Malka et al. [5] used an ultra-intense laser pulse in vacuum to generate MeV energy electrons. Tsakiris et al. [10] presented the DLA of electrons in radial electric fields. Niu et al. [11] proposed phase modulation for getting axially-symmetric and radially polarized beam. Singh et al. [3] have studied the effects of laser spot size, initial phase and laser pulse duration on electron acceleration by radially polarized...
laser pulse. Askari et al. [12] have shown that the intensity of the laser pulse and its frequency greatly affects the wake field which is responsible for acceleration. Also, applied magnetic field has a crucial role on electron acceleration [13-14]

The self focusing effect of cosh - Gaussian is observed by Patil et al. [15]. They solved differential equation for beam width parameter by paraxial and WKB approximations. Gaur et al. [16] showed that cosh - Gaussian high power Laser beam profile has strong self focusing effects in collision less plasma.

In present work, the propagation- parameters of a CGL beam are varied to study the effect on the energy gained by electrons.

2. Electron-Dynamics

Consider the electric vector of CGLB as

$$\vec{E} = \hat{x} E(x,0) \exp(-i(\omega_0 t - k_0 x))$$  (1)

Where $\omega_0$ is the laser frequency, $k_0 = \omega_0 / c$ is the propagation constant and $c$ gives the velocity of light in vacuum.

$$E(x,0) = E_0 \exp \left(-\frac{x^2}{w_0^2}\right) \cosh(\Omega_0 x)$$  (2)

is the field distribution of a Cosh Gaussian beam at $z = 0$,

$E_0$ is the amplitude at the central position of $x$ i.e. at $x = 0$ , $w_0$ and $\Omega_0$ are the waist width of Gaussian amplitude distribution, and hyperbolic cosine function.

For $z > 0$ the field distribution is

$$E_x = \frac{E_0}{2} \exp \left(-\frac{b^2}{4}\right) \left[ \exp \left(-\left(\frac{x}{w_0} + \frac{b}{2}\right)^2\right) + \exp \left(-\left(\frac{x}{w_0} - \frac{b}{2}\right)^2\right) \right]^2$$  (3)

$b = \Omega_0 w_0$ is the normalized modal parameter.

The Lorentz force equation is given by

$$\vec{F} = \frac{\text{d}\vec{p}}{\text{d}t} = \frac{d(\gamma \vec{m} \vec{v})}{\text{d}t} = e \left[ \vec{E} + \left(\vec{v} \times \vec{B}\right) \right]$$  (4)

The equations describing electron - momentum and energy are

$$\frac{\text{d}\vec{p}}{\text{d}t} = -e \left[ E_x \left(1 - \frac{v_z}{c}\right) + B_y v_y \right]$$  (5)

$$\frac{dp_x}{dt} = e B_y v_z$$  (6)

$$\frac{dp_y}{dt} = -e \frac{v_z}{c} E_x$$  (7)

$$\frac{dv_y}{dt} = -e \frac{v_z}{m_0 c^2} v_x E_x$$  (8)

Where $\gamma = \sqrt{1 + \left(p_x^2 + p_y^2 + p_z^2\right)/m_0^2 c^2}$ is the Lorentz factor

The normalized equations are given by

$$\frac{dy}{dt} = -\frac{1}{c} \left[ \frac{v_z}{2} \exp \left(\frac{b^2}{4}\right) \left[ \exp \left(\frac{x}{w_0} + \frac{b}{2}\right)^2 + \exp \left(\frac{x}{w_0} - \frac{b}{2}\right)^2 \right] \right]$$  (9)

$$\frac{dv_{1y}}{dt} = -\frac{a_0}{c_1} \left[ \frac{1}{2} \exp \left(\frac{b^2}{4}\right) \right] \left[ \exp \left(-\frac{x}{w_0} + \frac{b}{2}\right)^2 + \exp \left(-\frac{x}{w_0} - \frac{b}{2}\right)^2 \right] \left[1 - v_{1z}\right] - \frac{b_0}{c_1} v_{1y}$$  (10)

$$\frac{dv_{1y}}{dt} = \frac{b_0}{c_2} v_{1y}$$  (11)
\[ \frac{dv_{12}}{dt_1} = -a_0 \frac{v_{1x}}{c} \left[ \frac{b^2}{4} \left( \exp\left( \frac{x_1 + b}{2} \right) + \exp\left( \frac{x_1 - b}{2} \right) \right) \right] \]  

(12)

The normalized parameter used are:-

\[ a_0 = eE_0 / m_0 \omega_0 c, \quad b_0 = eB_0 / m_0 \omega_0, \quad t_1 = \omega_0 t, \quad v_{1x} = v_{x} / c, \quad v_{1y} = v_{y} / c, \quad v_{1z} = v_{z} / c, \]

\[ x_1 = \frac{x}{w_0}, \quad c_1 = \left[ \frac{1}{\sqrt{1 - v_{x}^2}} + \frac{v_{x}^2}{\left(1 - v_{x}^2\right)^{3/2}} \right], \quad c_2 = \left[ \frac{1}{\sqrt{1 - v_{y}^2}} + \frac{v_{y}^2}{\left(1 - v_{y}^2\right)^{3/2}} \right], \]

\[ c_3 = \left[ \frac{1}{\sqrt{1 - v_{z}^2}} + \frac{v_{z}^2}{\left(1 - v_{z}^2\right)^{3/2}} \right] \]

Equations (9)-(12) are ordinary coupled differential equations. By solving these equations numerically using a computer simulation code the variation of electron energy with time have been obtained.

\[ \text{Figure 1. Graphical study of gain in energy (}\gamma\text{) by the electrons with normalised time for a \text{cosh-Gaussian laser beam for different values of laser intensity parameter (a_o)}.} \]

3. Results and discussion

Throughout, we set parameters, \(a_0 = 1, 3, 5\) and 10, corresponding to laser intensity \(I \sim 1.38 \times 10^{18} \text{ W/cm}^2, 7.23 \times 10^{18} \text{ W/cm}^2, 3.45 \times 10^{19} \text{ W/cm}^2\) and \(1.38 \times 10^{20} \text{ W/cm}^2\), \(\beta = 0.0005\) corresponding to a magnetic field of 5.34 kG, \(w_0 = 31\) corresponding to a laser spot size of 3.1μm. The gain in electron energy has been examined for particular value of laser spot ‘\(w_0\)’ with intensity parameter \(a_0 = 1, 3, 5,\) and 10. Electron energy gain of 5GeV is observed for \(a_0 =10\) for the optimized parameters.

4. Conclusion

Present investigation signifies that by using a cosh-Gaussian laser beam profile we can achieve greater electron energy gain at lower laser power. These beams possess higher power and have ability to focus earlier than the Gaussian beam. This can be used to obtain very high accelerations in very small space and time.
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