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Full NLO QCD corrections to off-shell $\bar{t}t\bar{b}b$ production

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In this article, we report on the computation of the next-to-leading order (NLO) QCD corrections to $pp \rightarrow \mu^+\bar{\nu}_\mu e^+\nu_e\bar{b}b\bar{b}$ at the LHC, which is an irreducible background to $pp \rightarrow \bar{t}tH(\rightarrow b\bar{b})$. This is the first time that a full NLO computation for a $2 \rightarrow 8$ process with 6 external strongly interacting partons is made public. No approximations are used, and all off-shell and interference effects are taken into account. Cross sections and differential distributions from the full computation are compared to results obtained by using a double-pole approximation for the top quarks. The difference between the full calculation and the one using the double-pole approximation is in general below 5% but can reach 10% in some regions of phase space.

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I. INTRODUCTION

The physics program of the Large Hadron Collider (LHC) is driven by the measurement of fundamental parameters of the Standard Model (SM) of particle physics. These range from masses and widths to couplings of elementary particles. Such parameters are experimentally measured using specific physical processes that are particularly sensitive to them. Given the complexity of the LHC environment, the sought-after signal is often polluted by background processes that mimic the final state of the signal. Even worse, there are also irreducible backgrounds that have exactly the same final state as the signal and differ only in the order of the strong and electroweak couplings. Thus, the extraction of fundamental parameters requires the subtraction of contributions of background processes from the measurements. Therefore, in order to allow for a precise measurement of parameters, theoretical predictions with high precision are required for both the signals and the backgrounds.

A prime example is the extraction of the Higgs coupling to top quarks from the measurement of $pp \rightarrow \bar{t}tH$. Given the large branching ratio of the Higgs boson into a pair of bottom–antibottom quarks, it is one of the favorite channels for the measurement of $pp \rightarrow \bar{t}tH$. Taking into account the top-quark decay products, the complete signal process reads $pp \rightarrow \mu^+\bar{\nu}_\mu e^+\nu_e\bar{b}b\bar{b}$ at order $O(\alpha_s^3\alpha^4)$. The same process receives contributions at order $O(\alpha_s^4\alpha^2)$, where the bottom–antibottom pair results from a strong interaction. In recent years, much attention has been devoted to the computation of the signal [1–19] as well as the background process [20–29]. In particular, it has been found that theoretical predictions for the background can vary substantially depending on the exact matching and/or parton shower used and tend to overestimate the experimental measurement by 30–50% [30–32]. In such predictions, the process is computed with on-shell top quarks, i.e., $pp \rightarrow \bar{t}t\bar{b}b$, which are subsequently decayed by a parton-shower program. However, top quarks also generate bottom quarks while decaying. Therefore, the physically relevant irreducible-background process is $pp \rightarrow \mu^+\bar{\nu}_\mu e^+\nu_e\bar{b}b\bar{b}$ at order $O(\alpha_s^3\alpha^4)$. The reason why studies have so far focused on an on-shell description of the top quark is the complexity of the above process [33]. Indeed, it is a $2 \rightarrow 8$ process with 6 external strongly interacting particles and multiple intermediate resonances. Such a complex process has never been computed at next-to-leading order (NLO) QCD accuracy.$^2$

Experimentally, the cross section for $pp \rightarrow \bar{t}t\bar{b}b$ has been measured by the ATLAS and CMS collaborations [38,39]. The production of a Higgs boson in association with a

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$^1$Squared Yukawa couplings are understood as order $\alpha$.

$^2$In Refs. [34–36], $2 \rightarrow 8$ computations at NLO have been presented with 4 external strongly interacting partons. The calculation of Ref. [37], on the other hand, involves up to 7 external QCD particles but for a $2 \rightarrow 7$ process.
top–antitop pair was observed by both ATLAS and CMS by combining various Higgs decay channels [40,41]. For the specific channel with the Higgs decaying into a bottom–antibottom pair searches have been performed as well [42,43].

In this article we report on the computation of the NLO QCD corrections to $pp \to \mu^- \bar{\nu}_\mu e^+ \nu_e \bar{b}b\bar{b}b$ at order $O(\alpha_s^4)$ at the LHC. No approximations are used, and all off-shell as well as all interference effects are taken into account. This computation has been made possible by the use of the efficient Monte Carlo integrator MOCANLO in combination with RECOLA [44–47] in association with OTTER [48], a new tensor integral library, and COLLIER [49,50]. In addition to the full computation, a calculation using a double-pole approximation (DPA) for the virtual corrections that retains only contributions with top and antitop quarks decaying into a lepton–neutrino pair and a bottom quark, has been performed [51]. Comparison of the DPA results with those of the full calculation serves as a consistency check and provides an indication of contributions beyond the approximation of on-shell top quarks. The results are presented in the form of cross sections and differential distributions. We emphasize that the present computations certainly do not answer all questions regarding the theoretical description of $tib\bar{b}$ on its own. Nonetheless, they constitute an important piece of information that could serve as a basis for future comparative studies.

The article is split into two main parts. Section II describes the computations carried out, while Sec. III focuses on the presentation of the numerical results. A summary of the main findings of the present work is provided in Sec. IV.

II. DETAILS OF THE CALCULATIONS

A. Process definition

The hadronic process under investigation is the production of a top–antitop pair in association with a bottom–antibottom pair at the LHC. Considering the leptonic decays of the top quarks, the process reads

$$pp \to \mu^- \bar{\nu}_\mu e^+ \nu_e \bar{b}b\bar{b}b + X.$$  (2.1)

At leading order (LO), the dominant contribution is of order $O(\alpha_s^4)$. The process (2.1) constitutes the irreducible-background to $pp \to t\bar{t}H(\to b\bar{b})$, which is of order $O(\alpha_s^5)$. The partonic processes contributing to hadronic events have two gluons, a quark–antiquark pair, and two bottom quarks or two antibottom quarks in the initial state,

$$gg \to \mu^- \bar{\nu}_\mu e^+ \nu_e \bar{b}b\bar{b}b, \quad q\bar{q}/qq \to \mu^- \bar{\nu}_\mu e^+ \nu_e \bar{b}b\bar{b}b, \quad b\bar{b}/bb \to \mu^- \bar{\nu}_\mu e^+ \nu_e \bar{b}b\bar{b}b, \quad bb \to \mu^- \bar{\nu}_\mu e^+ \nu_e \bar{b}b\bar{b}b.$$  (2.2)

On the other hand, the virtual corrections are made of one-loop amplitudes interfered with tree-level ones. The one-loop diagrams are built from the tree-level ones by inserting a virtual gluon and closed quark loops in all possible ways. Note that here no mixed QCD–electroweak (EW) corrections are present at this order as it can be the case in other computations for top–antitop production [13,52,53]. For illustration, the one-loop virtual amplitude of the $gg$ channel involves more than 200000 Feynman diagrams, while the corresponding real tree-level one possesses 41364 diagrams. Moreover, the virtual corrections to the $gg$ channel feature up to rank-6 8-point integrals, involve more than 10000 different tensor integrals, and evaluate in 2.6 seconds per phase space point on average on a Intel(R) Core(TM) i7-7700 CPU @ 3.60 GHz.

B. Description of the computations

1. Computation based on complete NLO matrix elements

The full computation comprises all possible real and virtual corrections mentioned above that contribute to the cross section at order $O(\alpha_s^4)$, i.e., all partonic channels and all Feynman diagrams of order $O(\alpha_s^4)$, contributing to the cross section in the order $O(\alpha_s^3)$, are taken into account. In particular, no approximations are employed, and all off-shell as well as all interference effects are included. Some of the contributing diagrams for the partonic channel $gg \to \mu^- \bar{\nu}_\mu e^+ \nu_e \bar{b}b\bar{b}b$ are shown in Fig. 1. These include diagrams with two top resonances [Figs. 1(a)–1(c) and 1(e)], with
three potential top resonances [Fig. 1(d)], with one top resonance [Figs. 1(f) and 1(g)], and with no top resonance [Fig. 1(h)]. The diagram in Fig. 1(d) contains one antitop and two top propagators. While both top propagators cannot be simultaneously resonant, each one becomes resonant in some part of phase space, corresponding to different on-shell processes, i.e., $t\bar{t}$ production with the subsequent decays $t \to \nu_e e^+ b\bar{b}$ and $\bar{t} \to \bar{\nu}_e e^- \bar{b}$ and $t\bar{t}bb$ production with the subsequent decays $t \to \nu_e e^+ b$ and $\bar{t} \to \bar{\nu}_\mu \mu^- \bar{b}$. On the other hand, the diagram in Fig. 1(e) contributes only to $t\bar{t}$ production but not to $t\bar{t}bb$ production, since there are no $t\bar{t}bb$ intermediate states.

The computation is carried out in the 5-flavor scheme that assumes the bottom quarks to be massless throughout. All leptons and quarks (apart from the top quark) are thus taken to be massless. Also, all potentially resonant particles, i.e., top quark, W boson, and Z boson, are treated within the complex-mass scheme [54–56], ensuring gauge invariance of all the amplitudes.

2. Double-pole approximation

Similar to Refs. [13,52], in addition to the full computation we also perform a calculation based on a DPA. Note that the DPA is only applied to matrix elements, while the
physical observables are calculated with off-shell kinematics. Specifically, we examine the \(tt\)-DPA which consists in retaining only contributions that feature two resonant top quarks and projecting the top-quark momenta on shell, apart from those in the denominators of the resonant propagators, which are kept off shell. At LO, the \(tt\)-DPA is based on the doubly top-resonant contributions in the Born matrix element. More precisely, we include only those Feynman diagrams that contain both the decays \(t \rightarrow \nu_\tau e^+ b\) and \(\bar{t} \rightarrow \bar{\nu}_\mu \mu^- \bar{b}\) as subdiagrams, like those in Figs. 1(a)–1(d), but no diagrams with different top decays like the one in Fig. 1(e). Moreover, only one of the bottoms, say bottom 1, is allowed as decay product of the top quark, while the other one, bottom 2, is not; i.e., diagrams with bottoms interchanged are not contained in the DPA. The same applies to the antibottoms with respect to the antiquips. The DPA is constructed as a check of the full calculation and to provide a good approximation thereof. While a comparison of this approximation with the full calculation cannot yield quantitative results on off-shell top-quark effects, it nevertheless gives an indication on their size. In practice, the actual off-shell effects are often even larger, in particular, in specific regions of phase space.

3. Tools

The numerical integration has been carried out with the help of the multichannel Monte Carlo integration program MOCANLO. This code was developed for the integration of high-multiplicity processes involving top–antitop pairs and has proven to be particularly efficient for those and related processes [8,13,52,53]. It relies on a multichannel phase-space integration following Refs. [54,63,64].

All one-loop amplitudes in the 8-body phase space have been obtained from the matrix-element generator RECOLA2 [44–47] in combination with the OTTER library that is based on the on-the-fly reduction [65] and uses the stability improvements for hard kinematics described in Ref. [66]. By default, OTTER uses double-precision scalar integrals provided by COLLIER [49,50] and for exceptional phase-space points makes targeted use of multiprecision scalar integrals provided by ONELOOP [67]. The computation of the virtual amplitudes has been carried out as well exclusively with the COLI branch of the COLLIER library, yielding perfect agreement. The infrared (IR) singularities in the real and virtual corrections are handled via the Catani–Seymour dipole subtraction formalism [62,68]. We note that the partonic process \(bg \rightarrow \mu^- \bar{\nu}_\mu e^+ bb\) involves 40 Catani–Seymour dipoles, while \(gg \rightarrow \mu^- \bar{\nu}_\mu e^+ bb\) involves 30.

4. Validations

This computation builds on several previous computations for processes involving top–antitop pairs with MOCANLO [8,13,36,52,53] which have themselves been thoroughly checked. Within the dipole-subtraction scheme, the variation of the \(\alpha_{\text{dipole}}\) parameter [62] that narrows the phase space to singular regions has been used. For representative channels a comparison of results for \(\alpha_{\text{dipole}} = 1\) and \(\alpha_{\text{dipole}} = 10^{-2}\) has revealed perfect agreement within statistical errors. The results presented below have been obtained using \(\alpha_{\text{dipole}} = 10^{-2}\). Furthermore, independence on the IR-regulator parameter has been verified for representative channels, proving IR finiteness. Finally, the virtual corrections were computed with RECOLA2 both in the conventional ‘’Hooft–Feynman gauge and within the Background-Field method using the two independent integral-reduction libraries OTTER and COLLIER. Moreover, for the gluon-initiated process we verified that when replacing one of the gluon polarization vectors at a time by its normalized four-momentum via \(e^\mu \rightarrow p^\mu/|p^\mu|\) only in the virtual amplitude, the corresponding contribution to the
cross section integrates to a numerical zero at the relative level of $10^{-8}$. Finally, the calculation based on the DPA for the virtual corrections provides a further validation of the full NLO calculation.

### C. Input parameters and event selection

#### 1. Input parameters

The theoretical predictions presented here are for the LHC at 13 TeV center-of-mass energy. The on-shell values for the masses and widths of the gauge bosons [69],

\[
M_W^0 = 80.379 \text{ GeV}, \quad \Gamma_W^0 = 2.085 \text{ GeV}, \\
M_Z^0 = 91.1876 \text{ GeV}, \quad \Gamma_Z^0 = 2.4952 \text{ GeV},
\]

are converted into pole masses according to [70]

\[
M_V = M_V^0/c_V, \quad \Gamma_V = \Gamma_V^0/c_V, \\
c_V = \sqrt{1 + (\Gamma_V^0/M_V^0)^2}, \quad V = W,Z.
\]

The latter are used in the calculation. The top-quark mass and widths are fixed to

\[
m_t = 173 \text{ GeV}, \quad \Gamma_t^{\text{LO}} = 1.443303 \text{ GeV}, \\
\Gamma_t^{\text{NLO}} = 1.3444367445 \text{ GeV}.
\]

The top-quark width at LO has been computed based on the formulas of Ref. [71], while the NLO QCD value has been obtained upon applying the relative QCD corrections of Ref. [72] to the LO width. The LO top width is utilized for the LO computation, while the NLO one is employed in the NLO calculation (including the Born contributions).

Concerning the electromagnetic coupling $\alpha$, the $G_\mu$ scheme is applied, where $\alpha$ is fixed from the Fermi constant,

\[
\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right).
\]

with

\[
G_\mu = 1.16638 \times 10^{-5} \text{ GeV}^{-2}.
\]

The sets of parton distribution functions (PDF) NNPDF31 LO and NNPDF31 NLO with $\alpha_s = 0.118$ [73] have been used at LO and NLO, respectively. The values of $\alpha_s$ for the dynamical scales have been taken from the PDF sets which are interfaced through LHAPDF [74,75]. Accordingly, a variable-flavor-number scheme with at most 5 flavors is used for the running of $\alpha_s$.

The renormalization and factorization scales, $\mu_{\text{ren}}$ and $\mu_{\text{fact}}$, are set equal to

\[
\mu_0 = \frac{1}{2} \left[ \left( p_T^{\text{miss}} + \sum_{i=J} E_{T,i} \right) + 2m_j \right]^{1/2} \left( \sum_{i=J} E_{T,i} \right)^{1/2},
\]

where $p_T^{\text{miss}}$ is the transverse component of the vector sum of the two neutrino momenta and $J$ denotes all bottom and light jets after jet clustering. The transverse energy $E_{T,j}$ of the other particles is defined as $E_{T,j} = \sqrt{p_{T,j}^2 + m_j^2}$, where $m_j^2$ is the invariant-mass squared of the object considered (which can be a jet resulting from parton recombination and is thus not necessarily zero). Note that this choice of scale has the property not to refer explicitly to a top quark, as it has been done so far in the literature. While the first factor in Eq. (2.9) serves as a proxy for the typical momentum transfer in the strong couplings of the top quarks, the second one mimics the one in the couplings of the bottom quarks in the process. The choice can be viewed as a modification of the renormalization scales used in Refs. [23,24,28] avoiding identification of the top quarks.

#### 2. Event selection

The event selection is generic and assumes a resolved topology (as opposed to boosted kinematics). Quarks and gluons are clustered using the anti-$k_T$ algorithm [76] with a jet-resolution parameter $R = 0.4$. Concerning the flavor, the recombination rules read,

(i) $j + j \rightarrow j$,
(ii) $b + j \rightarrow b$,
(iii) $b + b \rightarrow j$,

where the bottom jet $j_b$ contains at least one $b$ or $\bar{b}$ quark, while $j$ is a light jet. The last combination is implemented in order to account for a proper treatment of the singularity originating from gluon splitting into pairs of bottom–antibottom quarks and effectively leads to the elimination of events where the bottom and antibottom quarks are collinear and thus recombined into one jet. For each bottom jet and charged lepton, a cut on its transverse momentum and its rapidity is applied,

\[
|y_{j_b}| < 2.5,
\]

while $\ell = e^+, \mu^-$. Finally, we require at least 4 bottom jets, a positron, and a muon passing all these cuts in the final state.

\[
\begin{align*}
p_T, j_b &> 25 \text{ GeV}, \\
p_T, \ell &> 20 \text{ GeV},
\end{align*}
\]

(2.10)

while for events without real radiation passing the cuts $J$ involves precisely 4 bottom jets, for real-radiation events the fifth (bottom or light) jet is included as well if it is not recombined during the jet clustering.
III. NUMERICAL RESULTS

A. Cross sections

The LO and NLO cross sections obtained from the full computation read

$$\sigma_{\text{LO}} = 5.203(4)^{+60%}_{-35%} \text{ fb} \quad \text{and} \quad \sigma_{\text{NLO}} = 10.31(8)^{+18%}_{-21%} \text{ fb},$$

(3.1)

respectively. The digits in parentheses indicate the numerical Monte Carlo errors on the predictions. The superscript and subscript represent the percentage scale variations. We use the conventional 7-point scale variation, i.e., we calculate the quantities for the following pairs of renormalization and factorization scales,

$$\left(\mu_{\text{ren}} / \mu_0, \mu_{\text{fact}} / \mu_0\right) = \{(0.5,0.5), (0.5,1), (1,0.5), (1,1), (1,2), (2,1), (2,2)\},$$

(3.2)

with the central scale defined in Eq. (2.9) and use the resulting envelope. The first observation is that the corrections are substantial and amount to 97.8% for the central scale, i.e., the $K$-factor is 1.98. The large $K$-factor is related to our scale choice (2.9) which results in somewhat larger scales than usual.\footnote{We note that the LO cross section scales with $\alpha_s$, which results not only in a scale uncertainty of the order of 50% but also in a large variation of $K$-factors. In fact, a $K$-factor near 2 is not unusual for this process [28,77], if the PDFs used for the LO calculation employ the same values for $\alpha_s$ as those for the NLO calculation.}

The dependence of the results on the scale choice is shown in Fig. 2 for the case when both the renormalization and factorization scales are set to a common value. Based on these results, choosing $\mu_0/2$ as central scale might be preferable, as it is closer to the maximum of the NLO curve and gives rise to smaller NLO QCD corrections ($K = 1.47$) in agreement with results for on-shell top quarks in the literature [23,28,29,78]. In any case, the inclusion of NLO QCD corrections significantly reduces the size of the scale uncertainty from $[+60%, -35%]$ to $[+18%, -21%]$.

Table I shows the cross sections of the different partonic channels. As usual at the LHC, the gluon-initiated contributions largely dominate the partonic cross section. For example, at LO the $gg$ channel represents 96.4% of the hadronic cross section, while the $q\bar{q}$ channels with $q = u,d,c,s$ give 4.2% and $b\bar{b}$, $bb$, and $\bar{b}\bar{b}$, only 0.16%, 0.070%, and 0.055%, respectively. The $gg$ and $q\bar{q}$ channels get NLO QCD $K$-factors 2.05 and 1.30, respectively. Such differences have already been observed for several top–antitop production processes (see for instance Refs. [8,21,53]). We note that for the bottom-induced channels, the $K$-factor is even higher and ranges between 2.18–2.76. However, these contributions are greatly suppressed by their (anti)-bottom-quark PDFs and are below 0.3% of the total cross section at both LO and NLO. At NLO, new partonic channels are opening up. The $gg/g\bar{g}$ channels yield rather small negative corrections (of the order of $-1.8\%$ of the total NLO cross section), while the $gb/g\bar{b}$ contribution is positive and reaches $+0.9\%$. Overall the NLO corrections are dominated by the ones of the $gg$ channel to raise a $K$-factor of 1.98.

FIG. 2. Cross section at LO and NLO in fb for the process $pp \to \mu^+ \nu^- e^+ e^- \nu_e b\bar{b}b\bar{b}$ at $\sqrt{s} = 13$ TeV as a function of the scale $\mu$, which refers to both the factorization and renormalization scales. The central scale $\mu_0$ is defined in Eq. (2.9).

| Ch.   | $\sigma_{\text{LO}}$ [fb] | $\sigma_{\text{NLO}}$ [fb] | $K$-factor | $\delta$ [%] |
|-------|--------------------------|-----------------------------|------------|--------------|
| $gg$  | 4.861(4)                 | 9.958(8)                    | 2.05       | 96.4         |
| $q\bar{q}$ | 0.3298(1)          | 0.431(1)                    | 1.30       | 4.0          |
| $bb$  | 0.00742(1)              | 0.017(2)                    | 2.29       | 0.16         |
| $g\bar{g}$ | $\cdots$              | $-0.19(2)$                  | $\cdots$  | $-1.8$       |
| $gb/g\bar{b}$ | $\cdots$            | $0.094(2)$                  | $\cdots$  | 0.91         |
| $b\bar{b}$ | 0.00263(1)             | 0.0072(9)                   | 2.76       | 0.070        |
| $\bar{b}\bar{b}$ | 0.00262(1)           | 0.0057(8)                   | 2.18       | 0.055        |
| $pp$  | 5.203(4)                 | 10.31(8)                    | 1.98       | 100          |
The cross sections in the tt-DPA, retaining only doubly top-resonant contributions as specified in Sec. II B 2, read
\[
\sigma_{\text{LO}}^{\text{DPA}} = 5.029(2)^{+60\%}_{-35\%} \text{ fb} \quad \text{and} \quad \sigma_{\text{NLO}}^{\text{DPA}} = 10.23(8)^{+19\%}_{-21\%} \text{ fb}.
\]
These values should be compared to the ones of the full computation in Eq. (3.1). First, the scale variation is essentially the same as in the full calculation, indicating that the functional dependence of the cross sections on the renormalization and factorization scales is not significantly modified. Looking at the central values, one observes that the tt-DPA is lower than the full computation by 3.3% at LO. At NLO, the difference between the two cross sections (0.8%) is of the order of the integration error which is 0.7%. This is due to the way the NLO DPA is constructed (see Sec. II B). While the full LO and real contributions are used, only the virtual contributions are computed in the pole approximation. Since the virtual corrections amount to about 30%, the expected error of the tt-DPA at NLO is about 30% of the expected error at LO, i.e., 0.3\(\Gamma_t/m_t\) \approx 0.25%.

For \(tt\) production, finite-width effects at the level of one percent have been found by comparing the off-shell calculation with the narrow-top-width limit in Refs. [79–81]. It is instructive to perform a similar analysis for the process (2.1). To this end, we determine the corresponding narrow-top-width limit in LO as in Ref. [80] by a numerical extrapolation,
\[
\hat{\sigma}(\Gamma_t) = \sigma(\Gamma_t) \left( \frac{\Gamma_t}{\Gamma_t^{\text{phys}}} \right)^2
\]
in the range \(0 < \Gamma_t < \Gamma_t^{\text{phys}}\), where \(\Gamma_t^{\text{phys}}\) is the physical top-quark width from Eq. (2.6), for the dominating gg channel. The factor \((\Gamma_t/\Gamma_t^{\text{phys}})^2\) ensures that effective top-decay branching ratios remain constant in the limit. In Fig. 3 we show results of this extrapolation for the full calculation, for the DPA, and for a calculation (res \(tt\)) where we take into account the same subset of diagrams as in the DPA but do not perform the on-shell projection. This analysis yields two interesting results: First, the difference between the DPA and the full calculation is larger than the difference between the full calculation and its narrow-width approximation. This indicates that contributions of nonresonant diagrams and finite-width effects of the resonant diagrams cancel to some extent, and the generic accuracy of on-shell calculations is worse. Second, the narrow-width limits of the three calculations do not agree, but differ at the level of one percent, the size of finite-width effects. The origin of these differences are contributions of resonant top or antitop quarks that decay via \(t \rightarrow \nu_e e^+ b \bar{b}\) or \(t \rightarrow \nu_\mu \mu^- b \bar{b}\). While some of these contributions, e.g., Fig. 1(d), are included in the subset of diagrams selected for the DPA, others, e.g., Fig. 1(e), are not. This demonstrates that the narrow-width limit of the full calculation differs by contributions of the order of \(\Gamma_t/m_t\) from an on-shell calculation of \(ttbb\) production with subsequent top decays. The construction of a narrow-width approximation based on on-shell calculations that takes into account all these top resonances would be nontrivial. While the extrapolation was performed at LO, the same kind of features persists in an NLO calculation.

The situation is different for \(tt\) production, where no extra top resonances are present. We verified with our codes
that for $gg \rightarrow \mu^+\nu_e e^+\nu_e b\bar{b}$ the DPA and the approximation “res $t\bar{t}$” based on resonant $t\bar{t}$ diagrams approach the same value in the narrow-top-width limit. Also for $t\bar{t}b\bar{b}$ production the cross sections for the different approximations in the narrow-top-width limit coincide, if one eliminates the resonances related to $t \rightarrow \nu_e e^+ b\bar{b}$ or $\bar{t} \rightarrow \bar{\nu}_e \mu^- b\bar{b}$ decays with additional cuts. This is demonstrated in the right plot in Fig. 3, where we imposed the additional cut $M_{bb} > 110$ GeV on all pairs of bottom and/or antibottom jets.

**B. Differential distributions**

Turning to differential distributions, several physical observables are shown in Figs. 4–6. While in the upper panels, the absolute predictions at LO and NLO QCD in the full and in the tt-DPA are displayed, the two lower panels show the same contributions with respect to different normalizations. In these panels the error bars represent the numerical Monte Carlo errors. In the middle insets the size of the QCD corrections in the full computation and in the tt-DPA is compared. Finally, the lower insets illustrate the quality of the approximate computation by normalizing the tt-DPA to the full computation at both LO and NLO QCD.

In Fig. 4, we present the distributions in the transverse momentum and the invariant mass of the bottom–antibottom pair that does not originate from the top-quark decay. In a $t\bar{t}H$ analysis with $H \rightarrow b\bar{b}$, this pair of bottom jets corresponds to the background of the decay products of the Higgs boson. More precisely, the bottom jets are identified as originating from a top quark by maximizing the likelihood function $\mathcal{L}$, defined as a product of two Breit–Wigner distributions corresponding to the top-quark and anti-top-quark propagators,

$$
\mathcal{L}_{ij} = \frac{1}{(p_{\mu}^2 - m_t^2)^2 + (m_t \Gamma_t)^2 + (p_{\nu}^2 - m_t^2)^2 + (m_t \Gamma_t)^2};
$$

where the momenta $p_{abc}$ are defined as $p_{abc} = p_a + p_b + p_c$. The combination of bottom jets $\{b_i, b_j\}$ that maximizes this function defines the two bottom jets originating from top quarks. From the 2 or 3 bottom jets left in the event, the two hardest ones, i.e., those with highest transverse momenta, define the bottom–antibottom pair that does not originate from the top-quark decay and whose transverse-momentum and invariant-mass distributions are shown in Fig. 4. The distribution in the transverse momentum of the two bottom jets not coming from a top decay shows rather stable corrections around 100% apart from low transverse momentum, where the QCD corrections reach 110%. The difference between the full calculation and the one in DPA does not show significant variations over the phase space neither at LO nor at NLO QCD but is largely inherited from the fiducial cross section. In particular, the difference between the tt-DPA and

![Graphs showing differential distributions at LO and NLO for $pp \rightarrow \mu^+\nu_e e^+\nu_e b\bar{b}$. Transverse momentum of the two bottom jets not originating from a top quark, and invariant mass of the two bottom jets not originating from a top quark.](image-url)
the full calculation at NLO is within the integration errors, as for all following distributions. The distribution in the invariant mass of the bottom–antibottom pair, on the other hand, exhibits larger variations between the full computation and the tt-DPA one at LO. The difference between the two computations is about 4% in the first bin, decreases to a few per cent around 100 GeV where the bulk of the cross section is located, and increases to almost 10% at 400 GeV. The additional contributions not contained in the tt-DPA increase the cross section. The largest effects appear for small $m_{bb}$, a region that is enhanced by $b\bar{b}$ pairs resulting from virtual gluons (for instance Fig. 1f), and for large invariant masses, where diagrams with bottom quarks coupling directly to the incoming gluons give sizeable contributions (for instance Fig. 1(g)). The QCD corrections tend to grow when going to higher invariant masses. This is in contrast to the results of on-shell computations.

Note, however, that the jet-resolution parameter $R = 0.4$ together with the transverse-momentum cut on the b jets of 25 GeV imply a minimum invariant mass of two b jets of about 10 GeV.
(see Figs. 6 and 17 of Ref. [23]), where the relative corrections to the invariant-mass distribution tend to decrease with increasing invariant mass. This is most likely due to the different scale choices in the on-shell and off-shell calculations, where the scales in the latter tend to be higher. We mention that the two distributions in Fig. 4 are the only ones that can be qualitatively compared with results of the literature where stable top quarks are used [23,24,28]. Since these computations are, however, done with different event selections and scale choices, a direct comparison is rather difficult. The transverse-momentum distribution is given as well in Fig. 10 of Ref. [23] but with a cut of 100 GeV on the invariant mass of the two bottom jets. Nevertheless, the differential $K$-factor is flat for this distribution above 50 GeV both in the off-shell and on-shell calculation.

In Fig. 5, the distribution in the transverse momentum of the second-hardest $b$ jet is shown. The full corrections are large (about 130\%) at low transverse momentum, then become smaller to finally reach roughly 100\% at 300 GeV. Such a behavior has already been observed in top–antitop production in the lepton $+$ jets channel for the transverse momentum of the hardest $b$ jet [53]. The large corrections were attributed to real radiations that take away momentum.

FIG. 6. Differential distributions at LO and NLO for $pp \rightarrow \mu^+\nu_\mu e^+\nu_e\bar{b}\bar{b}bb$: transverse momentum of the muon, rapidity of the muon, cosine of the angle between the muon and the positron, and azimuthal-angle distance between the muon and the positron.
of the final-state particles. The effect of nondoubly resonant top quarks is rather clear in this distribution at LO. The tt-DPA deviates from the full computation by almost 10% in the first bin. The difference is minimal at 100 GeV but always between 2% and 5%. This indicates significant nondoubly resonant contributions that might originate from multiperipheral diagrams where the bottom quarks couple directly to the incoming gluons [Fig. 1(g)]. Moreover, while bottom jets resulting from top quark decays tend to have transverse momenta of the order of the top mass, this is not the case for bottom jets in general. Looking at the distributions in the transverse momentum of the other b jets (not shown here), the difference between the tt-DPA and the full computation is reduced at low transverse momenta of the third and fourth hardest b jet, but enhanced for the hardest one. In contrast to the case of the hardest and second hardest b jets, for the third and fourth hardest b jets these configurations receive also contributions with doubly resonant top quarks that are included in the tt-DPA. At NLO, the differences are within integration errors owing to the fact that the DPA is only applied to the virtual amplitudes. For the distribution in the rapidity of the hardest b jet, the full and the approximate computation have the same qualitative behavior. The full NLO QCD corrections are essentially flat in this distribution. They are a bit above +100% at rapidity 2.5 and slightly below +100% in the central region. The distribution in the invariant mass of the two hardest bottom jets is depicted in the bottom left of Fig. 5. These bottom jets can either originate from a top-quark decay or are produced directly. The corrections tend to be larger at low and large invariant mass (100% at 0 GeV and 105% at 600 GeV) and reach a minimum around 300 GeV of 95%. The quality of the tt-DPA is rather good in this observable in the sense that no significant shape distortion is observed and only a difference in the overall normalization is present. The last plot in Fig. 5 concerns the distribution in $H_T$, defined as

$$H_T = p_T^{\text{miss}} + \sum_{i=e^\pm, \mu^\pm} E_T^i.$$  

This observable is interesting because it gives an estimate of the typical scale of the process. This is the reason why it enters the definition of the renormalization and factorization scale in Eq. (2.9). Note that as opposed to Eq. (2.9), here only the four hardest bottom jets fulfilling the event selection in Eq. (2.10) are taken into account. While the corrections are of the usual size for low values of this quantity, they steadily increase to exceed 300% above 1200 GeV. Such an effect has already been observed for top-pair production [80]. The quality of the tt-DPA is at the level of −3% at LO around the maximum of the distribution near 500 GeV. Above and below, the difference tends to increase: in the first nontrivial bin it amounts to around −6%, while at high values it steadily reaches −6% at 1.8 TeV.

The full set of distributions shown in Fig. 6 deals with leptonic observables. The corrections to the distribution in the transverse momentum of the muon are larger in the first bin, reach a minimum around 60 GeV and exceed 130% towards high transverse momentum. In the same way, the disagreement between the full computation and the tt-DPA at LO tends to increase slowly from 3% to 5% when going to large momenta. Similarly to the distribution in the rapidity of the hardest bottom jet, the one of the muon also does not feature significant shape distortions in the QCD corrections. The full corrections are, to a large extent, inherited from the total cross section. The difference between the tt-DPA and the full calculation is flat over the kinematic range shown for this observable. For the distribution in the cosine of the angle between the muon and positron, the corrections generally tend to increase with $\cos\theta_{\mu^+\mu^-}$, ranging between 90% and 110%. The difference between the tt-DPA and the full computations is also flat. At last, we show the distribution in the azimuthal angle between the two leptons. The corrections are at the level of 120% at zero degree and steadily decrease to 90% in the back-to-back configuration. Again, the shape of the tt-DPA computation is quite similar to the one of the full calculation over the full range.

**IV. CONCLUSION**

In this article we have presented the first full NLO QCD computation for $pp \rightarrow \mu^-\mu^+e^-\nu_ee^+\nu_\mu\bar{b}b\bar{b}$ at order $\mathcal{O}(\alpha_s^2\alpha^3)$ at the LHC. This final state is of particular interest as it is shared with $pp \rightarrow t\bar{t}H(\rightarrow b\bar{b})$ which is key for the extraction of Higgs coupling to top quarks. The present computation is carried out with full tree and one-loop matrix elements and, thus, includes all off-shell and nonresonant contributions. It therefore goes beyond the state of the art of fixed-order computations, which focused so far on the description of the $t\bar{t}b\bar{b}$ process with stable top quarks. In addition to the phenomenological relevance, the calculation constitutes a significant progress in complexity as it is the first full NLO QCD computation for a $2 \rightarrow 8$ process with 6 external strongly interacting particles. Along with the full computation, we also provide predictions in a double-pole approximation which retains only doubly resonant top contributions in the virtual corrections. Since the Feynman diagrams of the considered process include also resonant top and antitop quarks that are not related to on-shell production of $t\bar{t}b\bar{b}$ and subsequent top decays, the narrow-width limit of the full calculation differs from an on-shell calculation with subsequent decays, i.e., $pp \rightarrow t\bar{t}b\bar{b}$ followed by $t \rightarrow \mu^-\nu_\mub$ and $\bar{t} \rightarrow e^+\nu_e\bar{b}$, by terms of the order of the top width. Recent analyses have revealed differences between various theoretical predictions of $t\bar{t}b\bar{b}$ when including parton-shower effects. While the present computations certainly do not lift all discrepancies, they could serve as a basis for future comparative studies.
At the level of the cross sections, the QCD corrections turn out to be about 100% for our choice of renormalization and factorization scale. At the differential level, on top of this overall shift, shape distortions are present and reach 25% for some distributions. For observables that can be compared with on-shell computations (transverse momentum and invariant mass of the two bottom jets not coming from the top quarks), we observe qualitative differences in the shape of the corrections. This should therefore warrant further investigations in the future. At LO, the difference between the full computation and the one in the double-pole approximation stays below 5% in most distributions but reaches up to 10% in some cases. Our results show that a simplified calculation using the double-pole approximation for the virtual corrections is sufficient at this level of accuracy. While it does not provide a quantitative statement on the size of off-shell top-quark effects at NLO, it nevertheless indicates that these are at least at the level of 5–10% across phase space.

The results shown here provide an important piece of information regarding the theoretical description of $t\bar{t}bb$ production at hadron colliders. They should prove useful for present and upcoming analyses of $pp \to t\bar{t}H(\to b\bar{b})$ and its irreducible background at the LHC.

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Note added.—After finishing this work, a similar calculation for the same process was published by a different group [77]. This calculation fully confirms our results and provides an extensive discussion on scale and PDF uncertainties. This analysis furthermore revealed that excluding the additional jet from the definition of the renormalization and factorization scale increases the NLO cross section and brings it into agreement with the NLO results of other scale definitions within the theoretical uncertainties.

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