Can particles with strength-dependent masses form
dark matter of galaxies?

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Abstract

The basic idea in this Letter is the assumption that masses of the galactic constituents (particles of short-living fluctuations) may be functions of strength of the gravitational field. They may be in galactic space heavier than in the neighborhood of the earth. In the favorable case the total contribution of these constituents can be large enough to form the main part of the galactic dark matter.

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1 Introduction

One of the most important problems in astrophysics concerns the nature of the dark matter (DM) in galaxies [1][2][3][4]. There are many independent lines of evidence that most of the matter in the universe is dark, e.g. (i) rotation curves in spiral galaxies, (ii) velocities of galaxies in clusters, (iii) gravitational lensing, (iv) hot gas in galaxies and clusters, (v) large scale-motion [5][6]. There exit, in principle, two types of hypotheses for understanding the phenomenon of the galactic DM: the particle and field hypotheses. In the particle hypotheses one assumes that DM consists of the massive particles occurring in the galactic space. The prevailing view is that these particles are neutral interacting only gravitationally. Such are, e.g., the weakly interacting massive particles (WIMPs), additional neutrinos, supersymmetric particles, and a host of others [7]. These particles can be detected experimentally. Here, the problem is whether they have sufficient large masses in the appropriate abundance at the right localization of
a galaxy [8]. The field hypotheses base on the assumptions that DM is created by an interaction of the gravitational field with the galactic quantum vacuum or particles within a galaxy. For example, there is the assumption that DM is formed by short-lived particles that are created continuously from the quantum fluctuations [9] which gain for a very short time a real gravitated mass due to the galactic field [10]. Under certain circumstances the total mass of these short-living particles may form a main part of the non-baryonic matter. Another field hypothesis supposes that the gravitationally charged particles and its antiparticles, occurring in the galactic space, form dipoles whose interacting with the gravitational field become polarized. The end effect is that the set of polarized dipoles manifest itself as DM [11] [12] [13]. Besides this approach there are attempts to explain the above list of phenomena without existing DM by modifying the Newtonian law of gravity on large scale [14]. Since there is no observational conformation that the gravitational force falls as $1/r^2$ on large scales the Newton approach to the gravitational force is modified in that way that it describes, at least, some of the above phenomena. Many authors pointed out that a modified gravitational force law with the gravitational acceleration $a'$ given by the formula

$$a' = \frac{GM_v}{r^2} \left(1 + \frac{r}{k}\right),$$

where $k$ represent a constant, could be an alternative to dark matter in galactic halos as an explanation of the constant-velocity rotation curves of spiral galaxies. However, this modification of gravitational force do not satisfied the Tully-Fisher law. Milgrom [15] proposed an alternative idea, that the separation between the classical and modified regime is determined by the value of the 'critical' gravitational acceleration rather than the distance scale. He proposed that

$$a = \frac{GM_v}{r^2}, \quad a \gg a_c, \quad a'^2 = \left(\frac{aGM}{r^2}\right), \quad a \ll a_c,$$

where the value of the critical acceleration $a_c \approx 8.10^{-8} \text{cm}/\text{s}^2$ is determined for large spiral galaxies with $M \sim 10^{44} \text{g}$. This equation implies that for $a \ll a_c$ the velocity of galaxies is constant given by the equation $v^4 = a_0^2 GM_v$. This satisfies the Tully-Fisher law. Although Milgrom’s modification of gravity law is consisted with a large amount of data connected with dark matter it is entirely ad hoc. In what follows, we determine the amount of DM in galactic space provided that masses of galactic constituents are direct proportional to strength of the gravitational field whose source is the baryonic matter.
2 The field of strength-dependent constituents in a galaxy

If $\rho(r)$ is the density of baryonic matter of a spherically symmetrical galaxy then the strength of its field $E_g(r)$ is

$$\nabla E_g(r) = -G\rho(r). \quad (3)$$

Providing that the masses of galactic constituents are directly proportional to $E_g$ we have

$$\nabla E_v = KE_g(r), \quad (4)$$

where $E_v(r)$ is field whose source are the strength-dependent constituents and $K$ is the proportional constant. The solution of Eq.(4) can be written in the form

$$E_v = -\frac{1}{r^2} \int KE_g r^2 dr. \quad (5)$$

For a spherically symmetrical galaxy whose all barionic matter is concentrated in the center, we have

$$E_g(r) = -\frac{GM}{r^2}. \quad (6)$$

Inserting $E_g(r)$ into Eq.(4) we have

$$\nabla E_v = -\frac{KGM}{r^2}, \quad (7)$$

where $E_v(r)$ is the field strength due to the constituents and $K$ is a constant to be specified. The simplest form of this constant $K$, containing the mass of galaxy $M$ gravitational constant $G$ and Milgrom’s critical constant $a_c$, appears to be the expression

$$K = \sqrt{\frac{a_c}{GM}}$$

where $a_c \sim 10^{-8}cms^{-2}$. It is worth to remark that, according to Massa [16], there are several reason to think that the acceleration $a_c = 10^{-8}cms^{-2}$ plays a fundamental role in the cosmology: (i) $a_c$ is the surface gravity of a pion (mass $\sim 10^{-25}gr$, radius $\sim 10^{-13}cm$), (ii) the surface gravity of a protostar (mass $\sim 10^{34}gr$, radius $\sim 10^{17}cm$), (iii) the surface gravity of a typical spiral galaxy (mass $\sim 10^{44}gr$, radius $\sim 10^{22}cm$), (iv) the surface gravity of the observable universe (mass $\sim 10^{56}gr$, radius $\sim 10^{28}cm$). Coincidentally, the
value of said acceleration is roughly equal to $a_c = 10^{-8} \text{cm}\cdot\text{s}^{-2}$, where according to Milgrom, the Newton law is substituted by a modified gravitational law. Eq.(7) has the solution

$$E_v(r) = -\frac{\sqrt{a_c}GM}{r}. \quad (8)$$

Since numerically $a_c \approx G$ and $\sqrt{a_c}G \approx G$ we can approximately write $K \approx 1/\sqrt{M}$

$$\nabla E_v(r) = -\frac{\sqrt{a_c}GM}{r} \approx \frac{G\sqrt{M}}{r}. \quad (9)$$

The total strength of galactic field $E_t(r)$ is the sum of $E_g(r)$ and $E_v(r)$

$$E_t(r) = E_g(r) + E_v(r), \quad (10)$$

where

$$E_t(r) = -\frac{GM}{r^2} \quad E_2 = -\frac{\sqrt{a_c}GM}{r}. \quad (11)$$

$E_t(r)$ represents the modified gravity law. The first term expresses the familiar field due to barionic matter while the second term gives the field whose source are the quantum fluctuations or enlarged masses of particles in the gravitational field. The distance $r_e$ where $E_g$ and $E_v$ are equal is

$$r_e = \sqrt{\frac{GM}{a}} \approx \sqrt{M} \quad (12)$$

so we have

$$E_g \approx -\frac{GM}{r^2} \left(1 + \frac{r}{\sqrt{M}}\right) \quad (13)$$

Near the galactic center the field $E_g$ prevails while for $r > \sqrt{a_c}GM$ the field $E_v(r)$ is dominated. Next we are looking for effective mass $m_e$ of galaxy whose gravitational field in $r$ is identical with $E_t$ if no dark matter would exists, i.e. when

$$\frac{GM}{r^2} \left(1 + \frac{r}{\sqrt{M}}\right) = \frac{Gm}{r^2}. \quad (14)$$

From Eq.(14) it follows

$$m_e \approx M \left(1 + \frac{r}{\sqrt{M}}\right). \quad (15)$$

For example, the effective mass $m_e$ of a typical spiral galaxy with mass $M \approx 10^{44} g$ and the radius $r \approx 10^{22} \text{cm}$ is, according Eq.(15), equal to $2 \times M$. 


3 Two-term gravitational law

As said above Milgrom suggested that Newton’s second law of motion should be reexamined in the case of galactic motions. His basic idea was that at very low accelerations, corresponding to large distances, the second law broke down. To make it work better, he added a new constant into Newton law called the critical acceleration $a_c$. Milgrom’s [17] modified Newtonian dynamics (MOND) has been critiqued mainly in two point [4]. Why this modification works by galactic but not in earth gravitational field which are sometimes much smaller then field at the edge of galaxies and why the transition between both is so abrupt. As shown in previous chapter we modified the gravitational law which represents a continuous transition between the regime near galaxy center and that on the edge of galaxy in the form

$$F_m(r) = -\frac{GM}{r^2} - \sqrt{a_c GM} \frac{r}{r}.$$  \hspace{1cm} (16)

Let us suppose that $F_m(r)$ is valid generally for all gravitating systems and one may it apply to the gravitating system outside the galaxies. For example, to the solar system. The two-term law for the solar system has the form

$$F_\odot = -\frac{G M_\odot}{r_g^2} - \sqrt{a_c G M_\odot} \frac{r_e}{r_g},$$

where $M_\odot \approx 2.10^{30} kg$ is the mass of the sun and $r_g = AU \approx 1.5.10^{11} m$ is the distance between the earth and sun. Now, we can find whether the field $T_2$ has observable effect in distance $r_g$. Inserting these values into Eq.(12) we can determine the distance $r_e$ where $T_1 = T_2$. Doing this we find

$$r_e = \sqrt{\frac{G M_\odot}{r_c}} \approx 10^{15} m.$$  

Inserting $r_e$ into Eq.(16) we get

$$T_1 = \frac{G M_\odot}{r_e^2} = 6.2.10^{-3} msec^{-2}$$

and

$$T_2 = \frac{\sqrt{a_c G M_\odot}}{r_e} = \frac{7.10^{-11} \sqrt{2.10^{30}}}{1.5.10^{11}} = 6.5.10^{-7} msec^{-2}.$$  

The ratio $T_1/T_2 \approx 10^4$, i.e the field $T_1$ in $r_e$ is much stronger then the field $T_2$. Hence, $T_2$ is in distance $r_g = AU$ virtually negligible. Only behind the distance $r_e = 10^{15} m$, i.e. far from the solar system, $T_2$ prevails $T_1$. 


From what has been said so far it follows: (i) the assumption that masses of certain galactic constitutions are strength-dependent combining with Milgrom’s critical acceleration, build in the proportional constant $K$, leads to the two-term gravity law. The first term of this law expresses the familiar force while the second term gives the force whose source are the constituents generated by galactic field itself. The two-term law guarantees that the field given by first term of Eq.(16) is dominant in the neighborhood of the galactic center while at the edge of galaxy prevails the second term. (ii) The two-term law is smooth in contrast with the original Milgrom’s law which is abrupt. (iii) The effective mass of a galaxy is linear function of galactic radius. (iv) The two-term law may be verified in the conditions of solar system. For example, through the influence of second term on the orbits of planets in solar system.

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