Superconductivity and condensation of ordered zero-point oscillations

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Abstract

Currently there is a common belief that the explanation of superconductivity phenomenon lies in understanding the mechanism of electron pairs formation. Paired electrons, however, cannot form a superconducting condensate. These paired electrons perform disorderly zero-point oscillations and have no attractive forces in their ensemble. To create a unified ensemble of particles, the pairs must order their zero-point oscillations so that an attraction between the particles appears. Just the ordering of zero-point oscillations in the electron gas is the cause of the superconducting state arising. The parameters characterizing this order determine the properties of superconductors. The considered model of condensation of ordered zero-point oscillations creates the possibility to obtain estimates for the critical parameters of elementary superconductors, which are also in satisfactory agreement with measured data. The formation of superfluid states in He-4 and He-3 can be explained due to the ordering of zero-point fluctuations of atoms in liquid helium.

1 A mechanism that creates superconductivity

In the century following the discovery of superconductivity, a large number of studies were devoted to its theoretical explanation. Thanks to these studies there has been much progress in understanding superconductivity. However, the physical mechanisms which determine the presence of specific critical parameters for a particular superconductor - its critical temperature and critical magnetic field, is still not able to be fully explained.

There are several theories that describe superconductivity. The microscopic BCS-theory, created in the ’50s, is generally accepted. It is based on the assumption that superconductivity arises from the union of electrons into Cooper pairs, due to their interaction with phonons.

It seems that the phonon mechanism of superconductivity is confirmed by the existence of the isotope effect, i.e. the existence of the superconductor critical temperature dependence from the mass of the isotopes [1, 2]. However, more recent studies have shown that isotopic substitution causes a change in the lattice parameters of metals and thus directly affects properties of their electronic systems.
Dissatisfaction with the BCS theory therefore has lead to a number of other theories, in which the pairing of electrons is explained by alternative mechanisms.

It seems that the association of electron pairs may be due to the magnetic interaction between the electron dipole moments (see below). Generally speaking, the nature of the specific mechanism - that would contain a phonon or nonphonon - which is essentially responsible for the pairing of electrons - is not important. It is important that such a mechanism has worked over the entire all temperature range of all superconductivity existence. A working mechanism of electron pairing is a necessary yet insufficient condition for the existence of the superconducting state.

The reason for this stems from the fact that the paired electrons cannot form the superconducting condensate. The individual pairs differ from each other because they make disordered zero-point oscillations and there are no forces of attraction between them. Only after ordering the zero-point oscillations of the electron pairs and an attraction force appears, the energy of the system can then be lowered and a superconducting condensate can be formed by particles on the minimum energy level.

With this reason the ordering of zero-point oscillations must be considered as the cause of the occurrence of superconductivity.

The calculation of the properties of the condensate of the ordered zero-point oscillations allows us to predict the fundamental properties of superconductors. If the characteristic features of an ordered condensate coincide with the observed properties of superconductors, this builds a case that the phenomenon of superconductivity is due to the zero-point oscillations ordering of the electron gas.

We will therefore consider this process of ordering oscillations in greater detail so as to observe and note characteristics of the condensate, and then to compare them with the characteristics of superconductors.

The roles of zero-point oscillations in the superconducting state formation have been previously considered in the paper [3].

2 The condensate of ordered zero-point oscillations of electron gas

J.Bardeen was first who turned his attention toward a possible link between superconductivity and zero-point oscillations [4].

The special role of zero-point vibrations exists due to the fact that at low temperatures all movements of electrons in metals have been frozen except these oscillations.

Superconducting condensate formation requires two mechanisms: first, the electrons must be united in boson pairs, and then the zero-point fluctuations must be ordered (see Fig.1).
2.1 The electron pairing

The energetically favorable pairing of electrons in the electron gas should occur above the critical temperature.

It seems the pairing of electrons can occur due to the magnetic dipole-dipole interaction.

For the magnetic dipole-dipole interaction to merge two electrons into the singlet pair at the temperature of about 10K, the distance between these particles must be small enough:

\[ r < \left( \frac{\mu_B^2}{kT_c} \right)^{1/3} \approx a_B, \]  

where \( a_B = \frac{\hbar^2}{m_e e^2} \) is the Bohr radius.

I.e. two collectivized electrons must be localized in a volume of one lattice site. It is agreed that the superconductivity can occur only in metals with two collectivized electrons per atom, and cannot exist in the monovalent alkali and noble metals.

It is easy to see that the presence of magnetic moments on ion sites should interfere with the magnetic combination of electrons. This is confirmed by the experimental fact: as there are no strong magnetic substances among superconductors, and an adding iron, for example, to traditional superconducting alloys always leads to a lower critical temperature.

On the other hand, this magnetic coupling should not be destroyed at the critical temperature. The interaction energy between two electrons, with their location near one of the lattice site, can be much greater. This is confirmed by experiments which show that throughout the period of the magnetic flux quantization there is no change at the transition through the critical temperature of superconductor \[ 5, \]\[ 6. \]

The outcomes of these experiments are evidence that the existence of the mechanism of electron pairing is a necessary but not a sufficient condition for the existence of superconductivity.

The above proposed the dipole-dipole mechanism of electron pairing can be seen as an assumption which is consistent with the measurement data and therefore needs a more detailed theoretic consideration and further refinement.

On the other hand, this issue is not very important in the grander scheme, because the nature of the mechanism that causes electron pairing does not paramount importance. Instead, it is important that there is any mechanism in place that converts the electron gas in an ensemble of charged bosons with zero spin in the considered temperature range (as well as in a some range of temperatures above \( T_c \)).

If the temperature is not sufficiently low, the electron pairs still exist but their zero-point oscillations are disordered. Upon reaching the \( T_c \), the interaction between zero-point oscillations should cause their ordering and therefore a superconducting state is created.
The lowering of electron energy at their pairing due to magnetic dipole-dipole interaction

The lowering of electron energy due to the ordering of their zero-point oscillations

The condensate of ordered zero-point oscillations

Figure 1: The schematic representation of the energy levels of conducting electrons in a superconducting metal

2.2 The condensate of zero-point oscillations

2.2.1 The zero-point oscillations amplitude.

The principal condition for the superconducting state formation - the ordering of zero-point oscillations - is realized due to the fact that the paired electrons, which obey Bose-Einstein statistics, interact with each other.

At they interact, their amplitudes, frequencies and phases of zero-point oscillations become ordered.

Let an electron gas have density $n_e$ and its Fermi-energy is $E_F$. Each electron
of this gas can be considered as fixed inside a cell with linear dimension $\lambda_F$:

$$n_e = \frac{1}{\lambda_F^3}. \quad (2)$$

If the interactions of the electron gas are ignored, then its Fermi-energy can be written as [7]:

$$E_F = \frac{(3\pi^2)^{2/3}}{2m_e} n_e^{2/3} \approx \frac{e^2 a_B}{\lambda_F^2}. \quad (3)$$

However, a conduction electron interacts with the ion at its zero-point oscillations. If we consider the ions system as a positive background uniformly spread over the cells, the electron inside one cell has the potential energy:

$$E_p \approx -\frac{e^2}{\lambda_F}. \quad (4)$$

An electron will perform zero-point oscillations inside a cell. If the amplitude of these oscillations is equal $a_0$, their energy is:

$$E_k \approx \frac{1}{2} \frac{\hbar^2}{m_e a_0^2}. \quad (5)$$

In accordance with the virial theorem [11], if a particle executes a finite motion, its potential energy $E_p$ should be associated with its kinetic energy $E_k$ through the simple relation $|E_p| = 2E_k$. In this regard, we find that the amplitude of the zero-point oscillations of an electron in a cell is:

$$a_0 \approx \sqrt{2\lambda_F a_B}. \quad (6)$$

Coming from the quantization condition of zero-point oscillations

$$m_e a_0^2 \Omega_0 = \hbar/2, \quad (7)$$

one can determinate the frequency of zero-point oscillations $\Omega_0$ and their wavelength $L_0 = \frac{2\pi}{\Omega_0}$.

These zero-point oscillations form an oscillating electric dipole moment of the electron with the amplitude value:

$$d_\Omega = ea_0. \quad (8)$$

The interaction of dipole moments at low enough temperatures should lead to a coherent condensate in the zero-point oscillations of the electron gas. As a

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1Of course, the electrons are quantum particles and their fixation cannot be considered too literally. Due to the Coulomb forces of ions, it is more favorable for collectivized electrons to be placed near the ions for the shielding of ions fields. At the same time, collectivized electrons are spread over whole metal. It is wrong to think that a particular electron is fixed inside a cell near to a particular ion. But the spread of the electrons does not play a fundamental importance for our further consideration, since there are two electrons near the node of the lattice in the divalent metal at any given time. They can be considered as located inside the cell as averaged.
result of this interaction, the energy of the interacting particles will decrease in value:

$$\Delta_0 \simeq \frac{d_1^2}{\Lambda_0^3},$$

(9)

where $\Lambda_0 = \frac{1}{n_0}$ is a distance between particles of condensate, and $n_0$ is its density.

### 2.2.2 The interaction of electrically charged quantum oscillators.

Let us consider an electrically charged quantum oscillator at zero-point fluctuations, and estimate the variation of the energy that it gets during an interaction with another similar oscillator.

We will consider two particles with mass $2m_e$ and charge $2e$, spaced at a distance of $L_0$ and oscillating with frequency $\Omega_0$. Their movement will be assumed to be very slow, and we can neglect a retardation of their fields. Let $x$ be the displacement of the considered particle at the oscillations. The Schrodinger equation can be written as:

$$\left[ -\frac{\hbar^2}{2 \cdot 2m_e} \frac{\partial^2}{\partial x^2} + \left( \frac{2m_e \Omega_0^2 x^2}{2} + E_{dd} \right) \right] \Psi(x) = E \Psi(x),$$

(10)

where $E_{dd}$ is the energy of the interaction between two particles which are placed at a distance $L_0$.

This energy is the sum of the interaction energy of a divalent fixed ion ($= \frac{4e^2}{L_0^2}$) and with an oscillating electron pair (at oscillation with the same amplitude in the opposite phase ($= \frac{4e^2}{L_0^2}$)). Under these conditions, and at $L_0 \gg x$ the interaction energy can be written as:

$$E_{dd} \approx -\frac{4e^2 x}{L_0^2},$$

(11)

and the Schrodinger equation:

$$\frac{\partial^2}{\partial x^2} \Psi(x) - \frac{4m_e}{\hbar^2} \left( x^2 \frac{2m_e \Omega_0^2 x^2}{2} - \frac{4e^2 x}{L_0^2} - E \right) \Psi(x) = 0.$$  

(12)

The replacing of the variable

$$\xi = x - \frac{4e^2}{L_0^2 m_e \Omega_0^2}$$

(13)

leads to changing of the Schrodinger equation:

$$\frac{\partial^2}{\partial \xi^2} \Psi(\xi) - \frac{4m_e}{\hbar^2} \left[ \xi^2 m_e \Omega_0^2 - E - \frac{4e^4}{m_e \Omega_0^2 L_0^4} \right] \Psi(\xi) = 0.$$  

(14)
If to note $Q = \frac{4e^4}{\hbar m_e \Omega_0^2} + \mathcal{E}$, as result we obtain:

$$\frac{\partial^2}{\partial \xi^2} \Psi(\xi) - \frac{4m_e}{\hbar^2} \left[ \xi^2 m_e \Omega_0^2 - Q \right] \Psi(\xi) = 0. \quad (15)$$

This equation describes the oscillations of a simple harmonic oscillator with a zero-point energy

$$Q = \frac{\hbar \Omega_0}{2}. \quad (16)$$

Thus, the changing of energy at its interaction with the similar oscillator:

$$\Delta \mathcal{E} = \frac{4e^4}{m_e \Omega_0^2 L_0^2}. \quad (17)$$

Within the oscillations of the electron clouds there is the antiphase, where the distance between them must be equal to the half of the length of the electromagnetic wave:

$$L_0 = \frac{c}{4\pi \Omega_0}. \quad (18)$$

or at taking into account the quantization condition (7):

$$\Omega_0 = \frac{\hbar}{4m_e a_0^2}. \quad (19)$$

at the known amplitude of oscillations (6), we can obtain the condition of the antiphase oscillations:

$$L_0 = a_0^2 m_e c \frac{\pi \hbar}{2 \lambda_F} = \frac{2\lambda_F}{\pi \alpha}. \quad (20)$$

where $\alpha \simeq 1/137$ is the fine structure constant. With using of the Fermi energy determination (3), we can obtain

$$\Delta_0 \equiv \Delta \mathcal{E} \approx 9 \cdot 10^{-7} E_F. \quad (21)$$

Each of the oscillating dipoles has several neighbors. Their oscillating fields affect the ordering of the ensemble in same way. Taking this in to account, the ratio of the critical temperature of zero-point oscillations ordering to the Fermi temperature should be close to

$$\frac{T_c}{T_F} \approx 3 \cdot 10^{-6}. \quad (22)$$

### 2.2.3 The condensate particle density.

From Eq. (18) it follows

$$N_0 \simeq \frac{\lambda_F}{\pi \alpha}. \quad (23)$$
and this forms the ratio of the condensate particle density to the Fermi gas density:
\[
\frac{n_0}{n_e} = \frac{\lambda_F^3}{\Lambda_0^3} \approx (\pi \alpha)^3 \approx 10^{-5}.
\] (24)

When using these equations, we can find a linear dimension of localization for an electron pair:
\[
\Lambda_0 = \frac{L_0}{2} \approx \frac{1}{\pi \alpha (n_e)^{1/3}}.
\] (25)

or, taking into account Eq. (6), we can obtain the relation between the density of particles in the condensate and the value of the energy gap:
\[
\Delta_0 \approx 2 \pi^2 \alpha \frac{\hbar^2}{m_e} n_0^{2/3}
\] (26)

or
\[
n_0 = \frac{1}{\Lambda_0^3} = \left( \frac{m_e}{2 \pi^2 \alpha \hbar^2 \Delta_0} \right)^{3/2}.
\] (27)

It should be noted that the obtained ratios for the zero-point oscillations condensate (of bose-particles) differ from the corresponding expressions for the bose-condensate of particles, which are obtained in many courses (see eg [7]). The expressions for the ordered condensate of zero-point oscillations have an additional coefficient \( \alpha \) on the right side of Eq. (26). The obtained Eqs. (24), (21) and (22) illustrate important characteristics of the zero-point oscillations of the bose-condensate in fermi-gas environment. If there are internal relations between the condensate of zero-point oscillations and superconductivity, these equations should be characteristics of superconductors. The comparison of these equations with measurement data of superconductors is shown in Tables (4.2.11) and (4.2.13).

### 2.2.4 The sound velocity of the zero-point oscillations condensate.

The wavelength of zero-point oscillations \( L_0 \) in this model is a some analogue of the Pippard coherence length in the BCS. As usually accepted \([10]\), the coherence length \( \xi = \frac{\hbar v_F}{4 \pi e} \). The ratio of these lengths, taking into account Eq. (21), is simply the constant:

\[
\frac{L_0}{\xi} \approx 8 \pi^2 \alpha^2 \approx 10^{-3}.
\] (28)

The attractive forces arising between the dipoles located at a distance \( \frac{1}{2} \) from each other and vibrating in opposite phase, create pressure in the system:

\[
P \approx \frac{d \Delta_0}{dV} \approx \frac{d^2 \omega}{L^3}.
\] (29)
In this regard, sound into this condensation should propagate with the velocity:

\[ c_S \simeq \sqrt{\frac{1}{2m_e} \frac{dP}{dm_0}}. \]  

(30)

After the appropriate substitutions, the speed of sound in the condensate can be expressed through the Fermi velocity of electron gas

\[ c_S \simeq \sqrt{2\pi^2 \alpha^3 v_F} \simeq 10^{-2} v_F. \]  

(31)

The condensate particles moving with velocity \( c_S \) have the kinetic energy:

\[ 2m_e c_S^2 \simeq \Delta_0. \]  

(32)

Therefore, by either heating the condensate to the critical temperature when each of its volume obtains the energy \( E \approx n_0 \Delta_0 \), or initiating the current of its particles with a velocity exceeding \( c_S \), can achieve the destruction of the condensate. (Because the condensate of charged particles oscillations is considered, destroying its coherence can be also obtained at the application of a sufficiently strong magnetic field. See below.)

3 The critical parameters of the zero-point oscillations condensate.

The phenomenon of condensation of zero-point oscillations in the electron gas has characteristic features.

There are several ways of destroying the zero-point oscillations condensate in electron gas:

firstly, it can be evaporated by heating. In this case, evaporation of the condensate should possess the properties of an order-disorder transition.

Secondly, due to the fact that the oscillating electrons carry electric charge, the condensate can be destroyed by the application of a sufficiently strong magnetic field.

For this reason, the critical temperature and critical magnetic field of the condensate will be interconnected.

This interconnection should manifest itself through the relationship of the critical temperature and critical field of the superconductors, if superconductivity occurs as result of an ordering of zero-point fluctuations.

3.1 The temperature dependence of the energetic gap in the zero-point oscillations spectrum. The order-disorder transition.

Let us assume that at a given temperature \( T < T_c \) the system of vibrational levels of conducting electrons consists of only two levels:
firstly, basic level which is characterized by an anti-phase oscillation of the electron pairs at the distance $\Lambda_0$, and

secondly, excited, characterized by in-phase oscillation of the pairs.

Let the population of the basic level be $N_0$ particles and the excited level has $N_1$ particles.

Two electron pairs with an in-phase oscillation have a high energy of interaction and therefore cannot form the condensate. The condensate can be formed only by the particles that make up the difference between the populations of levels $N_0 - N_1$. In a dimensionless form, this difference defines the order parameter:

$$\Psi = \frac{N_0}{N_0 + N_1} - \frac{N_1}{N_0 + N_1}. \quad (33)$$

In the theory of superconductivity, by definition, the order parameter is determined by the value of the energy gap

$$\Psi = \Delta_T/\Delta_0. \quad (34)$$

When taking a counting of energy from the level $\varepsilon_0$, we obtain

$$\frac{\Delta_T}{\Delta_0} = \frac{N_0 - N_1}{N_0 + N_1} \approx \frac{e^{2\Delta_T/kT} - 1}{e^{2\Delta_T/kT} + 1} = th(2\Delta_T/kT). \quad (35)$$

Passing to dimensionless variables $\delta \equiv \frac{\Delta_T}{\Delta_0}$, $t \equiv \frac{kT}{kT_c}$ and $\beta \equiv \frac{2\Delta_0}{kT_c}$ we have

$$\delta = \frac{e^{\beta \delta/t} - 1}{e^{\beta \delta/t} + 1} = th(\beta \delta/t). \quad (36)$$

This equation describes the temperature dependence of the energy gap in the spectrum of zero-point oscillations. It coincides in form with other equations describing other physical phenomena, which are also characterized by the existence of the temperature dependence of order parameters [7],[8]. For example, this dependence coincides with temperature dependencies of the concentration of the superfluid component in liquid helium or the spontaneous magnetization of ferromagnetic materials. This equation is common for all order-disorder transitions (the phase transitions of II-type in the Landau classification).

The solution of this equation, obtained by the iteration method, is shown in Fig.(2).

This decision very accurately coincides with the known transcendental equation of the BCS, which was obtained by the integration of the phonon spectrum, and is in a satisfactory agreement with the measurement data.

After numerical integrating we can obtain the averaging value of the gap:

$$\langle \Delta \rangle = \Delta_0 \int_0^1 \delta dt = 0.852 \Delta_0 . \quad (37)$$
3.2 The critical parameters of a zero-point oscillations condensate and superconductivity

To convert the condensate into the normal state, we must raise half of its particles into the excited state (according to Eq. (35), the gap collapses under this condition). To do this, taking into account Eq. (37), the unit volume of condensate should have the energy:

$$E_T \approx \frac{1}{2} n_0 \langle \Delta_0 \rangle \approx \frac{0.85}{2} \left( \frac{m_e}{2 \pi^2 \alpha \hbar^2} \right)^{3/2} \Delta_0^{5/2},$$

(38)

On the other hand, we can obtain the normal state of an electrically charged condensate when applying a magnetic field of critical value $H_c$ with the density of energy:

$$E_H = \frac{H_c^2}{8\pi}.$$  

(39)

As a result, we acquire the condition:

$$\frac{1}{2} n_0 \langle \Delta_0 \rangle = \frac{H_c^2}{8\pi}.$$  

(40)

This created a relation of the critical temperature to the critical magnetic field of the zero-point oscillations condensate of the charged bosons.

The comparison of the critical energy densities $E_T$ and $E_H$ for type I superconductors are shown in Fig. (3). As shown, the obtained agreement between the energies $E_T$ (Eq. (38)) and $E_H$ (Eq. (39)) is quite satisfactory for type I superconductors [9], [10]. A similar comparison for type-II superconductors shows results that differ in approximately by a factor two. The reason for this will be considered below. The correction of this calculation, has not apparently made sense here. The purpose of these calculations was to show that the description of superconductivity as the effect of the condensation of ordered zero-point oscillations is in accordance with the available experimental data. This goal is considered reached in the simple case of I-type superconductors.
3.3 The zero-point oscillations and critical magnetic field of superconductors

The direct influence of the external magnetic field of the critical value applied to the electron system is too weak to disrupt the dipole-dipole interaction of two paired electrons:

\[ \mu_B H_c \ll kT_c. \]  

(41)

In order to violate the superconductivity so as to destroy the ordering of the electron zero-point oscillations. For this the presence of relatively weak magnetic field is required.

By using of Eqs.(40) and (9), we can express the gap through the critical magnetic field and the magnitude of the oscillating dipole moment:

\[ \Delta_0 \approx \frac{1}{2} e a_0 H_c. \]  

(42)

The properties of the zero-point oscillations of the electrons should not be dependent on the characteristics of the mechanism of association and also on the condition of the existence of electron pairs. Therefore, we should expect that this equation would also be valid for type I superconductors, as well as for type-II superconductors (for type-II superconductor \( H_c = H_{c1} \) is the first critical field)

A satisfaction of this condition is illustrated on the Fig.4.
Figure 4: The comparison of the calculated energy of superconducting pairs in the critical magnetic field with the value of the superconducting gap. Here, the following key applies: squares - type-II superconductors, circles - type-I superconductors. On vertical axis - logarithm of the product of the calculated value of the oscillating dipole moment of an electron pair on the critical magnetic field is plotted. On abscissa - the value of the gap is shown.

4 The Sommerfeld constant and critical temperature of superconductor

4.1 The electron states density and specific heat

Let us consider the process of heating the electron gas in metal. When heating, the electrons from levels slightly below the Fermi-energy are raised to higher levels. As a result, the levels closest to the Fermi level, from which at low temperature electrons were forming bosons, become vacant.

At critical temperature $T_c$, all electrons from the levels of energy bands from $E_F - \Delta$ to $E_F$ move to higher levels (and the gap collapses). At this temperature superconductivity is therefore destroyed completely.

This band of energy can be filled by $N_{\Delta}$ particles:

$$N_{\Delta} = 2 \int_{E_F - \Delta}^{E_F} F(\xi) D(\xi) d\xi. \quad (43)$$

Where $F(\xi) = \frac{1}{e^{\frac{\xi - \mu}{\tau}} + 1}$ is the Fermi-Dirac function and $D(\xi)$ is number of states per an unit energy interval, a deuce front of the integral arises from the fact that there are two electron at each energy level.

To find the density of states $D(\xi)$, one needs to find the difference in energy of the system at $T = 0$ and finite temperature:

$$\Delta \xi = \int_0^\infty F(\xi) \xi D(\xi) d\xi - \int_0^{E_F} \xi D(\xi) d\xi. \quad (44)$$

For the calculation of the density of states $D(\xi)$, we must note that two electrons can be placed on each level. Thus, from the expression of the Fermi-energy
we obtain

\[ D(E_F) = \frac{1}{2} \frac{dn_e}{dE_F} = \frac{3n_e}{4E_F} = \frac{3\gamma}{2k^2\pi^2}, \]  

(45)

where

\[ \gamma = \frac{\pi^2k^2n_e}{4E_F} = \frac{1}{2} \left( \frac{\pi}{3} \right)^{3/2} \left( \frac{k}{\hbar} \right)^2 m_e n_e^{1/3} \]  

(46)

is the Sommerfeld constant. Using similar arguments, we can calculate the number of electrons, which populate the levels in the range from \( E_F - \Delta \) to \( E_F \). For an unit volume of material, Eq. (43) can be rewritten as:

\[ n_\Delta = 2kT \cdot D(E_F) \int_{0}^{-\Delta} \frac{dx}{(e^x + 1)}. \]  

(47)

By supposing that for superconductors \( \frac{\Delta_0}{kT_c} = 1.76 \), as a result of numerical integration we obtain

\[ \int_{-\frac{\Delta_0}{kT_c}}^{0} \frac{dx}{(e^x + 1)} = [x - \ln(e^x + 1)]_{-\Delta_0}^{0} \approx 1.22. \]  

(48)

Thus, the density of electrons, which throw up above the Fermi level in a metal at temperature \( T = T_c \) is

\[ n_e(T_c) \approx 2.44 \left( \frac{3\gamma}{k^2\pi^2} \right) kT_c. \]  

(49)

Where the Sommerfeld constant \( \gamma \) is related to the volume unit of the metal.

### 4.2 The type-I superconductors

#### 4.2.1 The critical temperature

The de Broglie wavelengths of Fermi electrons expressed through the Sommerfelds constant

\[ \lambda_F = \frac{2\pi\hbar}{p_F(\gamma)} \approx \frac{\pi}{3} \frac{k^2m_e}{\hbar^2\gamma} \]  

(50)

are shown in Tab. 4.2.1.1.

In accordance with Eq. (43), which was obtained at the zero-point oscillations consideration, the ratio \( \frac{\Delta_0}{kT_c} \approx 2.3 \cdot 10^{-2} \).

In connection with this ratio, the calculated ratio of the zero-point oscillations condensate density to the density of fermions in accordance with Eq. (24) should be near to \( 10^{-5} \).

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2It should be noted that because on each level two electrons can be placed, the expression for the Sommerfeld constant Eq. (46) contains the additional factor 1/2 in comparison with the usual formula in literature.
These ratios for experimental data of type-I superconductors ([9]-[10]) are shown in the right part of Table (4.2.1.1).

It can therefore be seen, that calculated estimations of the condensate parameters are in satisfactory agreement with experimental data of superconductors.

By taking into account Eqs.(46) and (3), we can express the value of the Fermi energy through the measured Sommerfeld constant:

$$E_F(\gamma) = \frac{p_F^2(\gamma)}{2m_e} \simeq \left(\frac{12}{k^2}\right)^2 \left(\frac{k^2}{2m_e}\right)^3 \gamma^2$$  (51)

The performed calculations make it possible to find the direct dependence of the critical temperature superconductor from the experimentally measurable parameter - its electronic specific heat:

$$\Delta_0 \simeq \Theta \gamma^2$$  (52)

where

$$\Theta \simeq 31 \pi^2 \left[ \frac{\alpha h^2}{km_e} \right]^3 \approx 6.55 \cdot 10^{-22} K^4 cm^6 \text{ erg}. \quad (53)$$

The comparison of the calculated critical temperature (Eq.52) and measured data ([9],[10]) is given in Table (4.2.1.2) and in Fig.(7).
In accordance with Eq. (22), the critical temperature of the condensate of zero-point oscillations is related to the Fermi temperature of electron gas is \( T_c/T_F \simeq 3 \cdot 10^{-6} \).

These ratios for I-type superconductors are shown in Table (4.2.1.3). The values of \( T_F \) was obtained from the measured values of the Sommerfeld’s constants \( \gamma \) (Eq. (51)).

The dependence of the critical temperatures \( T_c \) from the Fermi temperatures \( T_F \) for type-I superconductors is shown graphically in Fig. 5. These critical temperatures are taken from measurements ([10], [9]), where the Fermi temperatures are obtained from Eq. (51), the straight line corresponds to Eq. (22), which was obtained at the consideration of the zero-point oscillations condensation.

Based these calculations, it is interesting to compare the density of superconducting carriers \( n_0 \) at \( T = 0 \), which is described by Eq. (27), with the density of normal carriers \( n_e(T_c) \), which are evaporated on levels above \( E_F \) at \( T = T_c \) and are described by Eq. (49).

### Table (4.2.1.2): The comparison of the calculated values of the superconductor critical temperatures with measurement data.

| superconductors | \( T_c \) (meas), K | \( \frac{\gamma}{cm^2 K^2} \) | \( T_c \) (calc).K | \( T_c \) (calc) / \( T_F \) (meas) |
|-----------------|---------------------|-----------------|-----------------|-----------------|
| Cd              | 0.517               | 532             | 0.77            | 1.49            |
| Zn              | 0.85                | 718             | 1.41            | 1.65            |
| Ga              | 1.09                | 508             | 0.70            | 0.65            |
| Tl              | 2.39                | 855             | 1.99            | 0.84            |
| In              | 3.41                | 1062            | 3.08            | 0.90            |
| Sn              | 3.72                | 1070            | 3.12            | 0.84            |
| Hg              | 4.15                | 1280            | 4.48            | 1.07            |
| Pb              | 7.19                | 1699            | 7.88            | 1.09            |

### Table (4.2.1.3): The comparison of the calculated values of the superconductor critical temperatures with measured Fermi temperatures.

| superconductor | \( T_c \),K | \( T_F \),K | \( \frac{T_c}{T_F} \) |
|-----------------|-------------|-------------|-----------------|
| Cd              | 0.51        | 1.81 \cdot 10^5 | 2.86 \cdot 10^{-6} |
| Zn              | 0.85        | 3.30 \cdot 10^5 | 2.58 \cdot 10^{-6} |
| Ga              | 1.09        | 1.65 \cdot 10^5 | 6.65 \cdot 10^{-6} |
| Tl              | 2.39        | 4.67 \cdot 10^5 | 5.09 \cdot 10^{-6} |
| In              | 3.41        | 7.22 \cdot 10^5 | 4.72 \cdot 10^{-6} |
| Sn              | 3.72        | 7.33 \cdot 10^5 | 5.08 \cdot 10^{-6} |
| Hg              | 4.15        | 1.05 \cdot 10^6 | 3.96 \cdot 10^{-6} |
| Pb              | 7.19        | 1.85 \cdot 10^6 | 3.90 \cdot 10^{-6} |
Figure 5: The comparison of critical temperatures $T_c$ of I-type superconductors with their Fermi temperatures $T_F$. The straight line is obtained from Eq.(22).

This comparison is shown in the Table (4.2.1.4) and Fig.6. (Data has been taken from the tables [9], [10]).

| superconductor | $n_0$ | $n_e(T_c)$ | $2n_0/n_e(T_c)$ |
|----------------|-------|------------|-----------------|
| Cd             | $6.11 \cdot 10^{17}$ | $1.48 \cdot 10^{18}$ | 0.83 |
| Zn             | $1.29 \cdot 10^{18}$ | $3.28 \cdot 10^{18}$ | 0.78 |
| Ga             | $1.85 \cdot 10^{18}$ | $2.96 \cdot 10^{18}$ | 1.25 |
| Al             | $2.09 \cdot 10^{18}$ | $8.53 \cdot 10^{18}$ | 0.49 |
| Tl             | $6.03 \cdot 10^{18}$ | $1.09 \cdot 10^{19}$ | 1.10 |
| In             | $1.03 \cdot 10^{19}$ | $1.94 \cdot 10^{19}$ | 1.06 |
| Sn             | $1.18 \cdot 10^{19}$ | $2.14 \cdot 10^{19}$ | 1.10 |
| Hg             | $1.39 \cdot 10^{19}$ | $2.86 \cdot 10^{19}$ | 0.97 |
| Pb             | $3.17 \cdot 10^{19}$ | $6.58 \cdot 10^{19}$ | 0.96 |

Table(4.2.1.4): The comparison of the superconducting carriers density at $T = 0$ with the density of thermally activated electrons at $T = T_c$.

From the data obtained above, we can see the condition of destruction of superconductivity, after heating for superconductors of type I, can really be written as the equation:

$$n_e(T_c) \simeq 2n_0$$  \hspace{1cm} (54)
4.2.2 The relationship $\Delta_0/kT_c$

From Eq. (54) and taking into account Eqs. (27), (49) and (52), which were obtained for condensate, we can say:

$$\frac{\Delta_0}{kT_c} \simeq 1.35.$$  (55)

This estimation of the relationship $\Delta_0/kT_c$ obtained for condensate has a satisfactory agreement with the measured data [9], for type I superconductors as listed in Table 4.2.2.

| superconductor | $T_c$,K | $\Delta_0$,mev | $\Delta_0/kT_c$ |
|----------------|---------|----------------|----------------|
| Cd             | 0.51    | 0.072          | 1.64           |
| Zn             | 0.85    | 0.13           | 1.77           |
| Ga             | 1.09    | 0.169          | 1.80           |
| Tl             | 2.39    | 0.369          | 1.79           |
| In             | 3.41    | 0.541          | 1.84           |
| Sn             | 3.72    | 0.593          | 1.85           |
| Hg             | 4.15    | 0.824          | 2.29           |
| Pb             | 7.19    | 1.38           | 2.22           |

Table 4.2.2: The value of ratio $\Delta_0/kT_c$ obtained experimentally for type-I superconductors.
4.3 The estimation of properties of type-II superconductors

In the case of type-II superconductors the situation is more complicated.

In this case, measurements show that these metals have an electronic specific heat that has an order of value greater than the calculation based on free electron gas model.

The peculiarity of these metals is associated with the specific structure of their ions. They are transition metals with unfilled inner d-shell (see Table 4.3).

It can be assumed that the increase in the electronic specific heat of these metals should be associated with a characteristic interaction of free electrons with the electrons of the unfilled d-shell.

| superconductors | electron shells |
|-----------------|----------------|
| Ti              | 3d² 4s²        |
| V               | 3d³ 4s²        |
| Zr              | 4d² 5s²        |
| Nb              | 4d³ 5s²        |
| Mo              | 4d⁴ 5s²        |
| Tc              | 4d⁵ 5s²        |
| Ru              | 4d⁶ 5s²        |
| La              | 5d¹ 6s²        |
| Hf              | 5d² 6s²        |
| Ta              | 5d³ 6s²        |
| W               | 5d⁴ 6s²        |
| Re              | 5d⁵ 6s²        |
| Os              | 5d⁶ 6s²        |
| Ir              | 5d⁷ 6s²        |

Table 4.3. The external electron shells of elementary type-II superconductors.

Since the heat capacity of the ionic lattice of metals is negligible at low temperatures, only the electronic subsystem is thermally active.

At $T = 0$ the superconducting careers populates the energetic level $\mathcal{E}_F - \Delta_0$. During the destruction of superconductivity through heating, an each heated career increases its thermal vibration. If the effective velocity of vibration is $v_t$, its kinetic energy:

$$\mathcal{E}_k = \frac{mv_t^2}{2} \simeq \Delta_0$$

Equation (56)

Only a fraction of the heat energy transferred to the metal is consumed in order to to increase the kinetic energy of the electron gas in the transition metals.

Another part of the energy will be spent on the magnetic interaction of a moving electron.
At contact with the d-shell electron, a moving free electron induces onto it the magnetic field of the order of value:

$$H \approx \frac{e}{r_c^2} \frac{v}{c}.$$  \hspace{1cm} (57)

The magnetic moment of the d-electron is approximately equal to the Bohr magneton. Therefore the energy of the magnetic interaction between the moving electron of conductivity and the d-electron is approximately equal to:

$$\mathcal{E}_\mu \approx \frac{e^2}{2r_c} \frac{v}{c}.$$  \hspace{1cm} (58)

This energy is not connected with the process of destruction of superconductivity.

Whereas, in metals with a filled d-shell (type-I superconductors), the whole heating energy increases the kinetic energy of the conductivity electrons and only a small part of the heating energy is spent on it in transition metals:

$$\frac{\mathcal{E}_k}{\mathcal{E}_\mu + \mathcal{E}_k} \approx \frac{mv_t}{h} a_B.$$  \hspace{1cm} (59)

So approximately

$$\frac{\mathcal{E}_k}{\mathcal{E}_\mu + \mathcal{E}_k} \approx \frac{a_B}{\Lambda_0}.$$  \hspace{1cm} (60)

Therefore, whereas the dependence of the gap in type-I superconductors from the heat capacity is defined by Eq. (52), it is necessary to take into account the relation Eq. (60) in type-II superconductors for the determination of this gap dependence. As a result of this estimation, we can obtain:

$$\Delta_0 \approx \Theta \gamma^2 \left( \frac{\mathcal{E}_k}{\mathcal{E}_\mu + \mathcal{E}_k} \right) \approx \Theta \gamma^2 \left( \frac{a_B}{\Lambda_0} \right)^{1/2},$$

where $1/2$ is the fitting parameter.

The comparison of the results of these calculations with the measurement data (Fig. (7)) shows that for the majority of type II superconductors the estimation Eq. (61) can be considered quite satisfactory.\[3]\n
\[3\]The lowest critical temperature was measured for Mg. It is approximately equal to 1mK. Mg-atoms in the metallic state are given two electrons into the electron gas of conductivity. It is confirmed by the fact that the pairing of these electrons, which manifests itself in the measured value of the flux quantum $\Phi_0$, is observed above $T_c$. It would seem that in view of this metallic Mg-ion must have electron shell like the Ne-atom. Therefore it is logical to expect that the critical temperature of Mg can be calculated by the formula for I-type superconductors. But actually in order to get the value of $T_c \approx 1mK$, the critical temperature of Mg should be calculated by the formula (61), which is applicable to the description of metals with an unfilled inner shell. This suggests that the ionic core of magnesium metal apparently is not as simple as the completely filled Ne-shell.
Figure 7: The comparison of the calculated values of critical temperatures of superconductors with measurement data. Circles relate to I-type superconductors, squares show II-type superconductors. On the abscissa the measured values of critical temperatures are plotted, on ordinate the calculated estimations are plotted. The calculations of critical temperatures for I-type superconductors were made at using Eq. (52) and the estimations for II-type superconductors was obtained at using Eq. (61).
Alloys and high-temperature superconductors

In order to understand the mechanism of high temperature superconductivity, it is important to establish whether the high-$T_c$ ceramics are the I or II-type superconductors, or whether they are a special class of superconductors.

In order to determine this, we need to look at the above established dependence of critical parameters from the electronic specific heat and also consider that the specific heat of superconductors I and II-types are differing considerably.

There are some difficulties by determining the answer this way - as we do not known truly the density of the electron gas in high-temperature superconductors. However, the densities of atoms in metals do not differ too much and we can use Eq.(52) for the solution of the problem of the I- and II-types superconductors distinguishing.

If parameters of I-type superconductors are inserted into this equation, we obtain quite a satisfactory estimation of the critical temperature (as was done above, see Fig.7). For the type-II superconductors values, this assessment gives an overestimated value due to the fact that type-II superconductors’ specific heat has additional term associated with the magnetization of d-electrons.

This analysis therefore, illustrates a possibility where we can divide all superconductors into two groups, as is evident from the Fig.(8).

It is generally assumed that we will consider alloys $Nb_3Sn$ and $V_3Si$ as the type-II superconductors. This assumption seems quite normal because they are placed in close surroundings of Nb. Some excess of the calculated critical temperature over the experimentally measured value for ceramics $Ta_2Ba_2Ca_2Cu_3O_{10}$ can be attributed to the measured heat capacity that have
been created by electrons of conductivity from non-superconducting elements (layers) of ceramics. It is not a news that it, as well as ceramics $YBa_2Cu_3O_7$, belongs to the type-II superconductors. However, ceramics $(LaSr)_2Cu_4$, Bi-2212 and Tl-2201, according to this figure should be regarded as type-I superconductors, which seems unusual.

5 About the London penetration depth

5.1 The traditional approach to calculation of the London penetration depth

The consideration of the London penetration depth is commonly accepted (see, for example [10]) in several steps:

Step 1. Firstly, the action of an external electric field on free electrons is considered. In accordance with Newton’s law, free electrons gain acceleration in an electric field $E$:

$$a = \frac{eE}{m_e}. \quad (62)$$

The directional movement of the "superconducting" electron gas with the density $n_s$ creates the current with the density:

$$j = en_s v, \quad (63)$$

where $v$ is the carriers velocity.

After differentiating the time and substituting this in Eq.(62), we obtain the first London’s equation:

$$\frac{d}{dt} j = en_s a = n_se^2m_eE. \quad (64)$$

Step 2. After application of operations $rot$ to both sides of this equation and by using the Faraday’s law of electromagnetic induction $rotE = -\frac{1}{c}\frac{dB}{dt}$ we acquire the relationship between the current density and magnetic field:

$$\frac{d}{dt} \left( rot \ j + \frac{n_s e^2}{mc} B \right) = 0. \quad (65)$$

Step 3. By selecting the stationary solution of Eq.(65)

$$rot \ j + \frac{n_s e^2}{mc} B = 0, \quad (66)$$

and after some simple transformations, we can conclude that there is a so-called London penetration depth of the magnetic field in a superconductor:

$$\Lambda_L = \sqrt{\frac{m_e c^2}{4\pi e^2 n_s}}. \quad (67)$$
5.2 The London penetration depth and the density of superconducting carriers

One of the measurable characteristics of superconductors is the London penetration depth, and for many of these superconductors it usually equals to a few hundred Angstroms [14]. In the Table (5.2) the measured values of $\lambda_L$ are given in the second column.

| superconductors | $\lambda_L \times 10^{-6}$ cm measured [14] | $n_s$ according to Eq.(67) | $n_s$ in accordance with Eq.(50) | $n_s/n_e$ |
|-----------------|------------------------------------------|--------------------------|-------------------------------|---------|
| Ti              | 9.2                                      | $3.3 \cdot 10^{21}$      | $1.4 \cdot 10^{23}$          | 0.023   |
| Sn              | 6.4                                      | $6.9 \cdot 10^{21}$      | $3.0 \cdot 10^{23}$          | 0.024   |
| Hg              | 5.1                                      | $1.1 \cdot 10^{22}$      | $3.0 \cdot 10^{23}$          | 0.037   |
| Pb              | 4.2                                      | $1.6 \cdot 10^{22}$      | $4.5 \cdot 10^{22}$          | 0.035   |
| Tl              | 3.9                                      | $1.9 \cdot 10^{22}$      | $1.0 \cdot 10^{24}$          | 0.019   |

Table (5.2)

If we are to use this experimental data to calculate the density of superconducting carriers $n_s$ in accordance with the Eq.(67), the results be about two orders of magnitude larger (see the middle column of Tab. (5.2)).

Only a small fraction of these free electrons can combine into the pairs. This is only applicable to the electrons that energies lie with in the thin strip of the energy spectrum near $E_F$. We can therefore expect that the concentration of superconducting carriers among all free electrons of the metal should be at the level $n_s/n_e \approx 10^{-5}$ (see Eq.(24)). These concentrations, if calculated from Eq.(67), are seen to be about two orders of magnitude higher (see last column of the Table (5.2)).

Apparently, the reason for this discrepancy is because of the use of a nonequivalent transformation. At the first stage in Eq.(62), the straight-line acceleration in a static electric field is considered. If this moves, there will be no current circulation. Therefore, the application of the operation rot in Eq.(65) in this case is not correct. It does not lead to the Eq.(66):

$$\text{rot} \left( \frac{j}{m_e c^2 B} \right) = -1, \quad \text{(68)}$$

but instead, leads to a pair of equations:

$$\text{rot} \left( \frac{j}{m_e c^2 B} \right) = 0$$

$$\frac{n_e c^2}{m_e} B = 0 \quad \text{(69)}$$

and to the uncertainty:

$$\frac{\text{rot} \left( \frac{j}{m_e c^2 B} \right)}{0} = 0.$$
5.3 The adjusted estimation of the London penetration depth

To avoid these incorrect results, let us consider a balance of magnetic energy in a superconductor within magnetic field. This magnetic energy is composed of energy from a penetrating external magnetic field and magnetic energy of moving electrons.

5.3.1 The magnetic energy of a moving electron

By using of formulas of [12], let us estimate the ratio of the magnetic and kinetic energy of the electron (the charge of \(e\) and the mass \(m_e\)) while moving rectilinearly with a velocity \(v \ll c\).

The density of the electromagnetic field momentum is expressed by the equation:

\[
g = \frac{1}{4\pi c} [EH]
\]  

(71)

While moving with a velocity \(v\), the electric charge carrying the electric field with intensity \(E\) creates a magnetic field

\[
H = \frac{1}{c} |Ev|
\]  

(72)

with the density of the electromagnetic field momentum (at \(v \ll c\))

\[
g = \frac{1}{4\pi c^2} [E|vE|] = \frac{1}{4\pi c^2} (vE^2 - E(v \cdot E))
\]  

(73)

As a result, the momentum of the electromagnetic field of a moving electron

\[
G = \int_V g dV = \frac{1}{4\pi c^2} \left( v \int_V E^2 dV - \int_V E \cdot E cos \vartheta dV \right)
\]  

(74)

The integrals are taken over the entire space, which is occupied by particle fields, and \(\vartheta\) is the angle between the particle velocity and the radius vector of the observation point. By calculating the last integral in the condition of the axial symmetry with respect to \(v\), the contributions from the components of the vector \(E\), which is perpendicular to the velocity, cancel each other for all pairs of elements of the space (if they located diametrically opposite on the magnetic force line). Therefore, according to Eq. (74), the component of the field which is collinear to \(v\)

\[
\frac{E cos \vartheta \cdot v}{v}
\]  

(75)

can be taken instead of the vector \(E\). By taking this information into account, going over to the spherical coordinates and integrating over angles, we can obtain

\[
G = \frac{v}{4\pi c^2} \int_r^\infty E^2 \cdot 4\pi r^2 dr
\]  

(76)
If we limit the integration of the field by the Compton electron radius $r_C = \frac{\hbar}{m_e c}$, then $v \ll c$, we can obtain:

$$G = \frac{v}{4\pi c^2} \int_{r_C}^{\infty} E^2 \cdot 4\pi r^2 \, dr = \frac{v \cdot e^2}{c^2 r_C}.$$  

(77)

In this case by taking into account Eq. (72), the magnetic energy of a slowly moving electron pair is equal to:

$$E = \frac{vG}{2} = \frac{v^2 \cdot e^2}{2c^2 r_C} = \frac{\alpha \cdot m_e v^2}{2}.$$  

(78)

5.3.2 The magnetic energy and the London penetration depth

The energy of external magnetic field into volume $dv$:

$$E = \frac{H^2}{8\pi} dv.$$  

(79)

At a density of superconducting carriers $n_s$, their magnetic energy per unit volume in accordance with (78):

$$E_H \simeq \alpha n_s \frac{m_e v^2}{2} = \frac{\alpha m_e j_s^2}{2n_s e},$$  

(80)

where $j_s = 2e n_s v_s$ is the density of a current of superconducting carriers.

Taking into account the Maxwell equation

$$\text{rot} H = \frac{4\pi}{c} j_s,$$  

(81)

the magnetic energy of moving carriers can be written as

$$E_H \simeq \frac{\bar{\Lambda}^2}{8\pi} (\text{rot} H)^2,$$  

(82)

where we introduce the notation

$$\bar{\Lambda} = \sqrt{\frac{\alpha m_e c^2}{4\pi n_s e^2}} = \sqrt{\alpha \Lambda_L}.$$  

(83)

In this case, part of the free energy of the superconductor connected with the application of a magnetic field is equal to:

$$F_H = \frac{1}{8\pi} \int_V \left( H^2 + \bar{\Lambda}^2 (\text{rot} H)^2 \right) dv.$$  

(84)

4Such effects as the pair generation force us to consider the radius of the “quantum electron” as approximately equal to Compton radius [13].
At the minimization of the free energy, after some simple transformations we obtain
\[ H + \tilde{\Lambda} \cdot \text{rot} \cdot \text{rot} \cdot H = 0, \quad (85) \]
thus \( \tilde{\Lambda} \) is the depth of magnetic field penetration into the superconductor.

In view of Eq. (27) from Eq. (83) we can estimate the values of London penetration depth (see table 5.3.2). The consent of the obtained values with the measurement data can be considered quite satisfactory.

| Superconductors | \( \lambda_L, 10^{-6}\text{cm} \) measured | \( \tilde{\Lambda}, 10^{-6}\text{cm} \) calculated \( \text{Eq. (83)} \) | \( \tilde{\Lambda}/\lambda_L \) |
|-----------------|------------------------------------------|------------------------------------------|------------------|
| Ti              | 9.2                                      | 11.0                                    | 1.2              |
| In              | 6.4                                      | 8.4                                     | 1.3              |
| Sn              | 5.1                                      | 7.9                                     | 1.5              |
| Hg              | 4.2                                      | 7.2                                     | 1.7              |
| Pb              | 3.9                                      | 4.8                                     | 1.2              |

The resulting refinement may be important for estimates within the frame of Ginzburg-Landau theory, where the London penetration depth is used as a comparison of calculations and specific parameters of superconductors.

6 Few words to experimenters

6.1 Critical velocity of superconducting carriers and the search for a technological solution

The understanding of the mechanism of the superconducting state should open a way towards finding a solution to the technological problem. This problem was just a dream in the last century - the dream to fabricate a superconductor that was easily produced (in the sense of ductility) and had high critical temperature.

In order to move towards achieving this goal, it important to firstly understand the mechanism that limits of the critical properties of superconductors.

Let us consider a superconductor with a current. The length of their localisation determines the limiting momentum of superconducting carriers:
\[ p_c = \frac{2\pi \hbar}{\Lambda_0}. \quad (86) \]

Therefore, by using the Eq. (83), we can obtain that the critical velocity of superconducting carriers is:
\[ v_c = c_S \quad (87) \]
and this velocity is about a hundred times smaller than the Fermi velocity.

The sound velocity in the crystal lattice of metal \( v_s \), in accordance with the Bohm-Staver relation [15], has approximately the same value:
\[ v_s \simeq \frac{kT_D}{E_F} v_F \simeq 10^{-2} v_F. \quad (88) \]
Figure 9: The schematic representation of the dependence of critical temperature on the speed of sound in superconductors. On the ordinate, the logarithm of the critical temperature of superconductor is shown. On the abscissa, the logarithm of the square of the speed of sound is shown (for Sn and Pb - the transverse velocity of sound is shown, because it is smaller. The speed of sound in a film was used for yttrium-123 ceramics. The dashed line shows the value of the transverse velocity of sound in sapphire, as some estimation of the limit of its value. It can be seen that this estimation leads to the restriction on the critical temperature in the range of 0°C - the dot-dashed line.

This therefore, makes it possible to consider superconductivity being destroyed as a superconducting carrier overcomes the sound barrier. That is, if they moved without friction at a speed that was less than that of sound, after it gained speed and the speed of sound was surpassed, it then acquire a mechanism of friction.

Therefore, it is conceivable that if the speed of sound in the metal lattice is less than \( c_S \), then it would create a restriction on the limiting current in the superconductor.

If this is correct, then superconductors with high critical parameters should have not only a high Fermi energy of their electron gas, but also have a high speed of sound in their lattice.

It is in agreement with the fact that ceramics have higher elastic moduli compared to metals and alloys and also posses much higher critical temperatures (Fig. 6.1).

The dependence of the critical temperature on the square of the speed of sound [16] is illustrated in Fig. 6.1.

This figure, which can be viewed only as a rough estimation due to the lack of necessary experimental data, shows that the elastic modulus of ceramics with
a critical temperature close to room temperature should be close to the elastic modulus of sapphire, which is very difficult to achieve.

In addition, such ceramics would be deprived of yet another important quality - their adaptability. Indeed, in order to obtain a thin wire, we require a plastic superconductor.

A solution of this problem would be to find a material that possesses an acceptably high critical temperature (above 80K) and also experiences a phase transition at an even higher temperature of heat treatment. It would be possible to make a thin wire from a superconductor near the point of phase transition, as the elastic modules are typically not usually very strong at this stage.

6.2 Magnetic electron pairing

This considered formation of mechanism for the superconducting state provides a possibility of obtaining the estimations of the critical parameters of superconductors, which in most cases is in satisfactory agreement with measured data. For some superconductors, this agreement is stronger, and for other, such as Ir, Al, V (see Fig.7), it is expedient to carry out further theoretical and experimental studies due to causes of deviations.

The mechanism of magnetic electron pairing is also of fundamental interest in order to further clarify this.

As was found earlier, in the cylinders made from certain superconducting metals (Al[5] and Mg[6]), the observed magnetic flux quantization has exactly the same period above \( T_c \) and that below \( T_c \). The authors of these studies attributed this to the influence of a special effect. It seems more natural to think that the stability of the period is a result of the pairing of electrons due to magnetic dipole-dipole interaction continuing to exist at temperatures above \( T_c \), despite the disappearance of the material’s superconducting properties. At this temperature the coherence of the zero-point fluctuations is destroyed, and with it so is the superconductivity.

The pairing of electrons due to dipole-dipole interaction should be absent in the monovalent metals. In these metals, the conduction electrons are localized in the lattice at very large distances from each other.

It is therefore interesting to compare the period of quantization in the two cases. In a thin cylinder made of a superconductor, such as Mg, above \( T_c \) the quantization period is equal to \( \frac{2\pi\hbar}{e} \). In the same cylinder of a noble metal (such as gold), the sampling period should be twice as large.

6.3 The effect of isotopic substitution on the condensation of zero-point oscillations

The attention of experimentalists could be attracted to the isotope effect in superconductors, which served as a starting point of the BCS theory. In the '50s it had been experimentally established that there is a dependence of the critical temperature of superconductors due to the mass of the isotope. Because
the effect depends on the ion mass, this is considered to be because of the fact
that it is based on the vibrational (phonon) process.

The isotope effect for a number of I-type superconductors -
Zn, Sn, In, Hg, Pb - can be described by the relationship:

$$\sqrt{M_i T_c} = \text{const},$$  \hspace{1cm} (89)

where $M_i$ - the mass of the isotope, $T_c$ is the critical temperature. The isotope
effect in other superconductors can either be described by other dependencies,
or is absent altogether.

In recent decades, however, the effects associated with the replacement of
isotopes in the metal lattice have been studied in detail. It was shown that the
zero-point oscillations of ions in the lattice of many metals are non-harmonical.
Therefore, the isotopic substitution can directly affect the lattice parameters,
the density of the lattice and the density of the electron gas in the metal, on its
Fermi energy and on other properties of the electron subsystem.

The direct study of the effect of isotopic substitution on the lattice parameters
of superconducting metals has not been carried out.

The results of measurements made on Ge, Si, diamond and light metals,
such as Li (17, 18)(researchers prefer to study crystals, where the isotope
effects are large and it is easier to carry out appropriate measurements) show
that there is square-root dependence of the force constants on the isotope mass,
which was required by Eq. (89). The same dependence of the force constants on
the mass of the isotope has been found in tin [19].

Unfortunately, no direct experiments of the effect of isotopic substitution
on the electronic properties (such as the electronic specific heat and the Fermi
energy), on metals of interest to us have so far been conducted.

Let us consider what should be expected in such measurements. A conven-
ient choice for the superconductor is mercury, as it has many isotopes and their
isotope effect has been carefully measured back in the 50s of the last century as
aforementioned.

The linear dependence of the critical temperature of a superconductor on its
Fermi energy (Eq. (21)) and also the existence of the isotope effect suggests the
dependence of the ion density in the crystal lattice from the mass of the isotope.
Let us consider what should be expected in such measurements.

Even then, it was found that the isotope effect is described by Eq. (89) in
only a few superconductors. In others, it displays different values, and therefore
in a general case it can be described by introducing of the parameter $a$:

$$M_i^4 T_c = \text{Const.}$$  \hspace{1cm} (90)

At taking into account Eq. (21), we can write

$$T_c \sim \mathcal{E}_F \sim n_e^{2/3}.$$  \hspace{1cm} (91)

The parameter $l$ which characterizes the ion lattice obtains an increment $\Delta l$
with an isotope substitution:

$$\frac{\Delta l}{l} = -\frac{a}{2} \cdot \frac{\Delta M_i}{M_i},$$  \hspace{1cm} (92)
Figure 10: The isotope effect in mercury. The solid line is obtained by the sparse-squares technique. In accordance with the phonon mechanism, the coefficient $a$ must be about $1/2$ (the dotted line). As it can be seen, this coefficient is in reality approximately equal to $1/3$.

where $M_i$ and $\Delta M_i$ are the mass of isotope and its increment.

It is generally accepted that in accordance with the terms of the phonon mechanism, the parameter $a \approx \frac{1}{2}$ for mercury. However, the analysis of experimental data [1]-[2] (see Fig. 10) shows that this parameter is actually closer to $1/3$. Accordingly, one can expect that the ratio of the mercury parameters is close to:

$$\frac{(\Delta l)}{(\Delta M_i)} \approx -\frac{1}{6}. \quad (93)$$
7 Superfluidity as a consequence of zero-point oscillations ordering

The main features of the superfluidity of liquid helium became clear few decades ago \[20\], \[21\]. L.D. Landau explains this phenomenon as the manifestation of a quantum behavior of the macroscopic object.

However, the causes and mechanism of the formation of superfluidity are still unclear. There is no explanation as to why the \(\lambda\)-transition in helium-4 occurs at about 2 K, i.e. at almost exactly half of its boiling point:

\[
\frac{T_{\text{boiling}}}{T_\lambda} \approx 1.94, \tag{94}
\]

while for helium-3, this transition is only observed at temperatures of about a thousand times smaller.

The effect of superfluidity can be seen as a consequence of ordering of the zero-point oscillations of He-atoms.

The atoms in liquid helium-4 are electrically neutral, they have no dipole moments and do not form molecules. Yet some electromagnetic mechanism has to be responsible for phase transformations of liquid helium (as well as in other condensed substance where phase transformations are related with changes of energy of the same scale).

In liquid helium, the atom density is \(n_4 \approx 2.2 \cdot 10^{22} \text{ cm}^{-3}\). A separate atom is locked up by neighbors in a volume with linear dimensions roughly equal to \(\Lambda = \frac{1}{n_1^{1/3}} \approx 3.6 \cdot 10^{-8} \text{ cm}\).

According to the reference data \[22\], the effective radius of the atom is \(r_0 \approx 1.2 \cdot 10^{-8} \text{ cm}\). In the volume of localization, an atom undergoes zero-point fluctuations, which conserve helium as a liquid even at \(T = 0\).

The formation of superfluidity suggests the transition of the ensemble of helium atoms into a completely ordered state. The full ordering should include the ordering of zero-point oscillations.

The relationship of the frequency of zero-point oscillations \(\Omega_0\) with their amplitude \(a_0\) can be determined from the condition of quantization:

\[
m_4 a_0^2 \Omega_0 = \frac{\hbar}{2}, \tag{95}\]

where \(m_4 = 6.7 \cdot 10^{-24} \text{ g}\) is the mass of the He-4 atom.

The existence of a superfluid involves the ”stitching” of de Broglie waves of neighboring atoms at the boundary of their cells:

\[
\frac{1}{2} \Lambda = \frac{2\pi \hbar}{p_0}, \tag{96}\]

where \(p_0 = m_4 \Omega_0 a_0\) is the momentum of the particle; the coefficient \(\frac{1}{2}\) arises from the assumption that half of the de Broglie wavelength must be placed on

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\(^5\) The atoms are quantum objects, their behavior is subject to quantum laws. We can only talk about the localization of atoms in a given volume on average.
each cell, to ensure the anti-phase zero-point oscillations of neighboring atoms within the superfluid state.

This condition allows us to estimate the amplitude of the zero-point oscillations of an atom:

\[ a_0 = \frac{\Lambda}{8\pi} \]  

(97)

and the amplitude of the oscillating acceleration experienced by the atom at zero-point oscillations:

\[ g_0 = a_0\Omega_0^2 = 2\pi^3 \frac{\hbar^2}{m^2\Lambda^3}. \]  

(98)

Because of differences in the mass of the nucleus and electron shell, the forces of inertia attached to them will be different. It leads to their mutual oscillations. Force applied to the shell can be neglected. The action of the force of inertia, which is attached to the nucleus, can be represented as an effect of an equivalent oscillating electric field:

\[ E_\Omega = \frac{g_0 m_4}{2e}. \]  

(99)

Thus, an atom under the influence of inertial forces arising from the zero-point oscillations acquires an oscillating dipole moment, whose amplitude is proportional to the polarizability of helium \( \alpha \) [23]:

\[ d_\Omega = \alpha E_\Omega. \]  

(100)

If we take into account the fact that the polarizability of atoms is approximately equal to their volume, we obtain for the liquid helium \( \alpha \approx \frac{4\pi}{3} r_s^3 \approx 7 \cdot 10^{-24} \text{cm}^3 \), and as a result

\[ d_\Omega \approx 10^{-19} \frac{\text{cm}^{5/2} \text{g}^{1/2}}{\text{s}} \]  

(101)

The energy of interaction between two dipoles depend on their relative orientation [24]:

\[ E_{dd} = \frac{(d_1 \cdot d_2)\Lambda^2 - 3(d_1 \Lambda)(d_2 \Lambda)}{\Lambda^3}. \]  

(102)

where \( \Lambda \) is the vector connecting these dipoles.

As the temperature decreases, the longitudinal component of oscillations is first ordered due to the interaction of oscillating dipoles. The directivity of the longitudinal component coincides with the direction of the vector connecting the dipoles. The negative energy of this interaction speaks about an occurrence of attraction between the atoms at this particular ordering:

\[ E_\parallel = -2\frac{d_{13}^2}{\Lambda^3}. \]  

(103)

There is no long-range ordering in liquids. But partial ordering of zero-point vibrations of atoms in a short-range order usually lead to a lowering of their energy in general, which makes this state an energetic utility.
Once these attractive forces become dominant, there will be a partial ordering of the zero-point oscillations of the atoms that occurs at temperature:

\[ T_{∥} = \frac{\mathcal{E}_{∥}}{k} \approx 4K \]  

(104)

which is in agreement with the temperature of helium liquefaction.

The full ordering in the system happens when dipoles with a perpendicular orientation to the vector joining them become ordered. The energy of this interaction can be expressed as:

\[ \mathcal{E}_{⊥} = -\frac{d^2}{\Omega} \frac{\Lambda^3}{\Omega_a} \]  

(105)

(The minus sign is taken away here because in this case the opposite direction of ordered dipoles is energetically favorable). Thus, a total ordering of the zero-point fluctuations system of helium atoms occurs at half the liquefaction temperature:

\[ T_{⊥} = \frac{\mathcal{E}_{⊥}}{k} \approx 2K. \]  

(106)

These findings agree with the measurements of the transition temperature to the superfluid state.

The ratio of these two temperatures

\[ \frac{T_{∥}}{T_{⊥}} = 2 \]  

(107)

explains why the measured ratio of the boiling temperature to the temperature of \( \lambda \)-transition is obtained as close to two (Eq.(94)).

The considered mechanism gives the possibility to estimate yet one characteristic parameter of the superfluid state. Substitution of obtained parameters in formulas of Sec.(2.2.4) allows us to estimate the speed of sound in superfluid helium. The obtained value of this velocity \( v_s \approx 56m/s \) is in good agreement with the L.D. Landau estimation.

A similar explanation can be given to the transition of helium-3 to a superfluid state. In view of the fact that the properties of liquid helium-3 are close to the properties of helium-4, it is possible to think that the ordering of zero-point oscillations in helium-3 should occur during approximately the same conditions. The difference is that for the complete ordering, the electromagnetic interaction has to additionally order the magnetic moments of the nuclei He-3.

We can estimate the temperature at which this ordering happens. Due to the ordered zero-point oscillations, each electron shell creates an oscillating magnetic field on "its" nucleus:

\[ H_{Ω} \approx \frac{dΩ}{\Omega_a} \cdot \frac{\Omega_a \mu_n}{c}. \]  

(108)

Because the value magnetic moments of the nuclei He-3 is approximately equal to the nuclear Bohr magneton \( \mu_{nB} \), the ordering in their system must occur
below the critical temperature

\[ T_c = \frac{\mu_{n_d} H_\Omega}{k} \approx 8 \cdot 10^{-4} K. \]  \hspace{1cm} (109)

This finding is in strong agreement with the measurement data.
8 Conclusion

Until now it has been commonly thought that the existence of the isotope effect in superconductors leaves only one way for explanation of the superconductivity phenomenon - the way based on the phonon mechanism.

Over fifty years of theory development based on the phonon mechanism, has not lead to success. All attempts to explain why some superconductors have certain critical temperatures (and critical magnetic fields) have failed.

This problem was further exacerbated with the discovery of high temperature superconductors. How can we move forward in HTSC understanding, if we cannot understand the mechanism that determines the critical temperature elementary superconductors?

In recent decades, experimenters have shown that isotopic substitution in metals leads to a change in the parameters of their crystal lattice and thereby affect the Fermi energy of the metal. As results, the superconductivity can be based on a nonphonon mechanism.

The theory proposed in this paper suggests that the specificity of the association mechanism of electrons pairing is not essential. It is merely important that such a mechanism was operational over the whole considered range of temperatures. The nature of the mechanism forming the electron pairs does not matter, because although the work of this mechanism is necessary it is still not a sufficient condition for the superconducting condensate’s existence. This is caused by the fact that after the electron pairing, they still remain as non-identical particles and cannot form the condensate, because the individual pairs differ from each other as they commit uncorrelated zero-point oscillations. Only after an ordering of these zero-point oscillations, an energetically favorable lowering of the energy can be reached and a condensate at the level of minimum energy can then be formed. Due to this reason the ordering of zero-point oscillations must be considered as the cause of the occurrence of superconductivity.

Therefore, the density of superconducting carriers and the critical temperature of a superconductor are determined by the Fermi energy of the metal. The critical magnetic field of a superconductor is given by the mechanism of destruction of the coherence of zero-point oscillations.

In conclusion, the consideration of zero-point oscillations allows us to construct the theory of superconductivity, which is characterized by the ability to give estimations for the critical parameters of elementary superconductors. These results are in satisfactory agreement with measured data.
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