ABSTRACT
Session-based recommendation (SBR) aims to predict the user’s next action based on the ongoing sessions. Recently, there has been an increasing interest in modeling the user preference evolution to capture the fine-grained user interests. While latent user preferences behind the sessions drift continuously over time, most existing approaches still model the temporal session data in discrete state spaces, which are incapable of capturing the fine-grained preference evolution and result in sub-optimal solutions. To this end, we propose Graph Nested GRU ordinary differential equation (ODE), namely GNG-ODE, a novel continuum model that extends the idea of neural ODEs to continuous-time temporal session graphs. The proposed model preserves the continuous nature of dynamic user preferences, encoding both temporal and structural patterns of item transitions into continuous-time dynamic embeddings. As the existing ODE solvers do not consider graph structure change and thus cannot be directly applied to the dynamic graph, we propose a time alignment technique, called t-Alignment, to align the updating time steps of the temporal session graphs within a batch. Empirical results on three benchmark datasets show that GNG-ODE significantly outperforms other baselines.

CCS CONCEPTS
• Information systems → Recommender systems.

KEYWORDS
Session-based Recommendation; Graph Neural ODE; Graph Neural Networks

1 INTRODUCTION
Recommender systems can help provide users with personalized information according to their preferences reflected in the historical interactions [19], which are widely applied in e-commerce websites, web searches, and so forth [34, 70]. However, in some scenarios where only the user’s recent interactions within a narrow time range are available, the general recommenders are not applicable since the collaborative signal is scarce, leading to the obscure of user preferences [21]. Thus, session-based recommendation (SBR) is proposed to detect the user intent from the limited interactions in the current session and make recommendations, where the session is defined as the user’s actions within a period of time [21, 29].

Most existing SBR methods focus on modeling sequential pattern among items of a session by using Recurrent Neural Networks (RNNs) [21, 29, 53] or Graph Neural Networks (GNNs) [15, 42, 55, 57]. However, these works view a session as a short sequence and assume that the primary intention of the user in a session usually remains the same, and try to capture the user’s preference directly from the entire session. Consequently, they often ignore the fact that a user’s fine-grained preference can drift over time, even in a relatively short-term session. Although the temporal pattern is crucial in capturing the fine-grained user preference, research on utilizing temporal information in SBR is still in the early stage.

Fortunately, multiple lines of recent studies in SBR have aimed to embrace this challenge by incorporating additional temporal information. The first line of works [38, 71] models the evolution of user preference in a discrete-time setting. They model a session as a sequence of user interactions [19], which are widely applied in e-commerce websites, web searches, and so forth [34, 70]. However, these works view a session as a short sequence and assume that the primary intention of the user in a session usually remains the same, and try to capture the user’s preference directly from the entire session. Consequently, they often ignore the fact that a user’s fine-grained preference can drift over time, even in a relatively short-term session. Although the temporal pattern is crucial in capturing the fine-grained user preference, research on utilizing temporal information in SBR is still in the early stage.

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Another line of works integrates the time dimension by considering timestamps information as a contextual dimension [46, 72]. However, these methods generate discrete user preferences that ignore the time elapse effect. Consider a user who makes a purchase today and her preference representation is updated. The representation will stay the same regardless of when she returns (i.e., a day later, a month later). As a result, the same recommendations will be offered when she returns next time. However, user preferences may change over time [7, 27]. The time elapse effect on user preferences should be considered and thus the preference representation needs to be updated to the query time. In this paper, we argue that the user’s preference is a continuous concept evolving as time progresses. As shown in Figure 1, item interactions can be interpreted as the observations of the latent continuous user preference at a specific timestamp. By modeling the preference dynamics in the continuous-time setting, we no longer need the equal-length time slice segmentation of the whole timeline and manage to consider the time elapse effect to predict the future embedding trajectories of items as time progresses.

In particular, modeling the user preference in a fully continuous manner is challenging. As most neural networks are discrete, where the iterative update of hidden states between two layers is a discretization of a continuous transformation [4, 18, 35], they cannot model user preference in the continuous-time setting. To handle this challenge, in this paper, we propose to utilize neural ODE to complete this task. Owing to its intrinsic continuous nature, neural ODE enables tracking of the evolution of the underlying system. It is expected to offer improved performance compared with using discrete methods to model a continuous dynamical system [22, 23]. However, directly applying neural ODE models in SBR is still inapplicable. As user-item interactions occur irregularly along time, the neural ODE model should be theoretically continuous to guarantee stability. Moreover, due to the dynamic nature of a temporal session graph, the updating time steps of the temporal session graph within a batch may not be consistent. Thus, the existing ODE solvers are inapplicable in the batch update process since the solvers could accept only a one-time step parameter for each calculation.

To address these issues, we propose a novel ODE-based model for modeling the dynamics of user preference along time in a fully continuous manner. Different from previous snapshots-based methods, given an ongoing session, we transform the session into a fully continuous temporal session graph without using snapshots, which builds the potential structural and temporal relations between items. Afterward, we employ Graph Gated Neural Network (GGNN) [32, 57] to encode the item embeddings and transition patterns simultaneously to infer the latent initial states for all items. We further derive a Graph Nested GRU [28, 39, 45, 48] inspired continuous-time Ordinary Differential Equation network (GNG-ODE) that propagates the latent states of the items between different time steps as time progresses. Different from most existing temporal SBR models that learn the dynamics by employing recurrent model structures with discrete depth, our model coincides the time domain with the depth of a neural network and takes advantage of ODEs to steer the latent user/item features between two timestamps smoothly. As the existing ODE solvers are inapplicable in SBR where the graph is dynamic, we further propose a time alignment algorithm, called t-Alignment, to adapt the existing ODE solvers onto our dynamic graph setting by aligning the updating time steps of the dynamic session graphs within a batch. We conduct extensive experiments on three real-world public benchmarks. Comprehensive experimental results verify that GNG-ODE significantly outperforms the competitive baselines.

Our primary contributions can be summarized as follows:

- We propose a novel GNG-ODE to effectively consider the intrinsic complex nature of user-item interactions by modeling a session as a continuous-time temporal session graph. In this carefully designed graph structure, the temporal information of item transitions is preserved in a fully continuous manner. To the best of our knowledge, this is the first work to model the continuous evolution of user preference using neural ODE in SBR.
- We show that GNG-ODE is theoretically well-posed, i.e., its solution always exists and is unique (see Section 4.2). Besides, it enjoys several good numerical properties. We also propose the t-Alignment algorithm to make existing ODE solvers applicable to dynamic environments in SBR.
- Extensive experiments on three public datasets demonstrate the effectiveness of our GNG-ODE model. Compared with all competitive baselines, the improvements brought by modeling the continuous evolution of user preferences are at most 6.05%, according to the ranking metric on average.

2 RELATED WORKS

Session-based Recommendation. Following the development of deep learning, many neural network based approaches have been proposed for SBR. Hidasi et al. [21] first propose to leverage the recurrent neural networks (RNNs) to model users’ preferences. Afterward, attention-based mechanisms are incorporated into the system and significantly boost performance. NARM [30] utilizes attention on RNN models to enhance the captured features while STAMP [33] captures long and short-term preferences relying on a simple attentive model. Convolution Neural Networks (CNNs) are also leveraged. Tang et al. [50] try to embed item session as a matrix and perform convolution on the matrix to get the representation.

To better model the transitions within the sessions, most recent developments focus on leveraging Graph Neural Networks (GNNs) to extract the relationship between sessions. Wu et al. [58] first propose to capture the complex transitions with graph structure. Afterward, Pan et al. [37] try to avoid overfitting through highway

Figure 1: Illustration of the complex structural and temporal patterns in the session. Arrows denote that nearby and distant transition dependencies co-exist. The color gradient represents the continuity change of user preferences as time progresses. Item clicks can be interpreted as the observations of latent continuous user preferences.
Temporal Information in SBR. Temporal information plays a vital role in user preference modeling. Although there are a few works in other recommendation areas [1, 6, 12, 27, 31, 31, 51] utilizing the temporal information to facilitate recommendation, the temporal-related method has not been fully explored in SBR. Some prior efforts reduce the temporal information into the relative order/position information. For example, Yu et al. [65] use RNN to capture the sequential signal, which reveals the user’s future dynamic preference in the next-basket recommendation. Pan et al. [38] further model the evolution of item transitions by constructing a sequence of dynamic graph snapshots which contains the graphs transformed from the session at different timestamps. Zhou et al. [46, 72] integrate the time dimension by considering timestamps information as a contextual dimension. The observations of user clicks are put into bins of fixed duration, and the latent dynamics are discretized in the same way. To characterize the dynamics from both the user side and the item side, Zeyuan et al. [6] propose to build a global user-item graph for each time slice and exploit time-sliced graph neural networks to learn user/item embeddings.

As outlined above, we find previous works on SBR have some limitations. First, temporal information is rarely or crudely exploited in these works. Second, existing methods model structural and temporal patterns separately without considering their interactions, which restricts the capacity of the models. Third, some methods rely on the segmentation of the whole timeline into a specified number of equal-length time slices, resulting in the temporal information loss problem [6, 71]. Finally, these methods generate discrete user preference representations that ignore the time elapse effect on user preferences. The representation will stay the same regardless of when the user returns to the platform, i.e., a day later, a week later, or even one month later, thus limiting the performance.

Neural Ordinary Differential Equation. Neural ODE is a continuous approach to model the discrete sequence governed by a time-dependent function \( T \rightarrow \mathbb{R} \) of a continuous time variable \( t \).

\[
\frac{dh(t)}{dt} = f_\theta(h(t), t) \tag{1}
\]

where \( \theta \) is the parameter of the differential function. Eq. (1) drives the system state forward in infinite steps over time. The differential function \( f : \mathbb{R}^d \times \mathbb{R} \rightarrow \mathbb{R}^d \) induces a differential field that covers the input space. Given initial state \( h(t_0) \), we can derive the state of time \( T \) by a black-box differential equation solver, which evaluates the hidden unit dynamics \( f \) wherever necessary to determine the solution with the desired accuracy:

\[
h(T) = h(t_0) + \int_{t_0}^{T} f_\theta(h(t), t) dt \tag{2}
\]

There is a rich body of literature on Neural ODE recently. Ricky TQ et al. [4] first propose the Neural ODE framework and develop an adjoint method to solve the ode function, which is memory efficient. To improve the expression ability of Neural ODE, Junteng et al. [24] provide a data-driven approach to learn continuous and discrete dynamic behaviors, Emilien et al. [11] add extra dimensions.

3 PROBLEM DEFINITION

Assume the item set is \( V = \{ v_1, v_2, ..., v_{|V|} \} \), where \( v_i \) indicates item \( i \) and \( |V| \) is the number of all items. Given an ongoing session denoted as \( S = \{ v_1, v_2, ..., v_n \} \), the aim of a session-based recommendation is to predict the items that the user will interact with at the next timestamp, that is, \( v_{n+1} \). Specifically, the session-based recommender system takes the session \( S \) as input and outputs the prediction scores on all candidate items, then the items ranked at the top \( K \) positions will be recommended to the user.

4 APPROACH

In this section, we describe our proposed Graph Nested GRU Ordinary Differential Equation for Session-based Recommendation (GNG-ODE) in detail, which is constituted of three main components, that is, (i) the temporal session graph construction, (ii) the dynamic item representation learning and (iii) the user preference generation and prediction. The framework of the proposed GNG-ODE is schematically shown in Figure 3. Given an ongoing session, we first construct a temporal session graph which contains the graphs transformed from the sessions at different timestamps. Then, we learn the dynamic item representations through GNG-ODE. Finally, we generate the hybrid user preference, which is utilized to make predictions on candidate items.

4.1 Temporal Session Graph Construction

Given a session \( S = \{ v_1, v_2, ..., v_n \} \), we first generate the updating time point of the session and its corresponding target items at different timestamps as \( (S_1, v_2), (S_2, v_3), ..., (S_{n-1}, v_n) \), where \( S_i = \{ v_1, v_2, ..., v_i \} \). This is similar to the data augmentation method widely applied in SBR [36, 57]. However, different from the existing methods which shuffle the augmented samples and utilize them.
4.2 Dynamic Item Representation Learning

User click history can be seen as irregularly-sampled data that represents the observations of latent user interests. Typically, observations of user clicks are put into bins of fixed duration [46, 72], and the latent dynamics are discretized in the same way. This leads to difficulties with missing data (e.g., when there is no clicks at some time points) and ill-defined latent variables [4]. To handle these challenges, our representation learning part consists of three components: (i) A GNN-inspired encoder that transforms the transition structure of observed items into initial hidden states. (ii) A hidden trajectory prediction model characterized by the GNG-ODE function and t-Alignment technique to learn the latent dynamics of the transition evolution. (iii) An attention-based decoder that generates the distribution of the next item that the user may click.

4.2.1 Initial Latent State Encoder. Items in a session can be regarded as observations of the latent user preference. To capture the dynamics of latent user preference, we first transform the raw embeddings of items in a session and their static transitions into the initial latent representations by GNN. GGNN is widely used in session-based recommendation tasks [5, 57]. Given a static session graph \( G = (V, E) \), the GNN first aggregates neighborhood information to form a neighborhood representation for a node, then applies GRU to combine the original node representation and the neighborhood representations:

\[
\vec{h}_u^{(l)} = \sum_{v \in N_u} w_{vu} h_u^{(l)}
\]

where \( h_u^{(l)} \) is the hidden representation of item \( u \) in layer \( l \), while \( \vec{h}_u^{(l)} \) denotes the neighborhood representation of item \( u \) in layer \( l \). \( w_{vu} \) is the edge weight of edge \( vu \). Details of constructing the static session graph can be found in [34]. By applying GNN we can infer a initial states jointly considering both item attributes and transition patterns, thus benefits the preference modeling capacity. The output of the last layer is then normalized by \( l_2 \)-norm to make the value between \([-1, 1]\) to ensure the stability of ODE solvers. We denote it as \( h_u \) for simplicity.

4.2.2 Graph Nested GRU ODE. After computing the latent initial states for items, we now define the Graph Nested continuous-time ODE (GNG-ODE) function that drives the system to move forward. Graph Nested GRU (GNG) is widely applied in dynamic graph learning settings [28, 39, 45, 48], by

\[
\begin{align*}
    r_u^t &= \sigma \left(W_G^r x_u^t + U_G^r h_{u}^{t-1} + b_r \right) \\
    z_u^t &= \sigma \left(W_G^z x_u^t + U_G^z h_{u}^{t-1} + b_z \right) \\
    g_u^t &= \tanh \left(W_G^g x_u^t + U_G^g (r_u^t \odot h_{u}^{t-1}) + b_g \right) \\
    h_u^t &= z_u^t \odot h_{u}^{t-1} + (1 - z_u^t) \odot g_u^t
\end{align*}
\]

where \( r_u^t \) and \( z_u^t \) are the reset gate and select gate at time step \( t \) respectively. Let \( x_u^t \) and \( h_u^t \) denote the input embeddings and hidden state of item \( u \) of time step \( t \), respectively. \( b_r, b_z, b_g \in \mathbb{R}^d \) are the parameters and \( W_G, U_G \) denote one layer of graph convolutional networks [25] to aggregate neighborhood information of item \( u \). GNG models the structural and temporal dependency and performs well on discrete-time dynamic graphs. Here we show how to derive a continuous-time GNG-ODE. Specifically, we firstly show that the form of GNG can be written as a difference equation. Given the standard update for the hidden state \( h_u^t \) of the GNG in Eq. (4):

\[
\begin{align*}
    h_u^t &= z_u^t \odot h_{u}^{t-1} + (1 - z_u^t) \odot g_u^t
\end{align*}
\]

...
We can obtain a difference equation by subtracting $h^\Delta t_u$ from this state update equation and factoring out $(1 - z^\Delta t_u)$:

$$
\Delta h^\Delta t_u = h^\Delta t_u - h^{-\Delta t}_u
= (1 - z^\Delta t_u) \circ (g^\Delta t_u - h^{-\Delta t}_u)
$$

(6)

This difference equation naturally leads to the following ODE for $h(t)$ when $\Delta t \to 0$:

$$
\frac{dh_u(t)}{dt} = (1 - z_u(t)) \circ (g_u(t) - h_u(t))
$$

(7)

with $r_u(t), z_u(t), g_u(t)$ the following forms:

$$
r_u(t) = \sigma \left( W^x_u x_u(t) + U^r_u h_u(t) + b_r \right)
$$

$$
z_u(t) = \sigma \left( W^x_u x_u(t) + U^z_u h_u(t) + b_z \right)
$$

(8)

$$
g_u(t) = \tanh \left( W^x_u x_u(t) + U^g_u (r_u(t) \circ h_u(t)) + b_h \right)
$$

The form of derived GNG-ODE is in line with GRU-ODE family. Based on the above corollaries, our GNG-ODE enjoys the following properties:

Continuity. It means that our training procedure is further tractable. Specifically, The Cauchy–Kowalevski theorem [14] states that, given $f = \frac{dh_u(t)}{dt}$, there exists a unique solution of $h$ if $f$ is analytic (or locally Lipschitz continuous), i.e., the ODE problem is well-posed if $f$ is analytic. In our case, as the GNG-ODE is Lipschitz continuous with constant $K = 2$, there will be only a unique optimal ODE for $h_u(t)$. Owing to the uniqueness of the solution, we could find a good solution for GNG-ODE function. Our method becomes fully continuous that can derive item representations at any given timestamps and any time granularity. In this way, we further avoid generating discrete user preference representations and manage to model the time elapse effect to predict the future embedding trajectories of items as time progresses.

Robustness. The continuous nature of our model allows it to track the evolution of the underlying system from irregular observations, and no longer need the equal-length slice segmentation of the whole timeline, which empowers our method to perceive more fine-grained temporal information compared with previous methods.

We can then apply various ODE solvers to integrate the ODE function in Eq. (7). ODE solvers discretize time variable $t$ and convert an integral into many steps of additions. Widely used solvers are fix-step solvers like explicit Euler and fourth-order Runge–Kutta (RK4) method or adaptive step solvers like Dopri5. Then the item representation at time $T$ can be inferred by:

$$
h_u(T) = h_u(t_0) + \int_{t_0}^{T} \frac{dh_u(t)}{dt} dt
$$

(12)

where $h_u(t_0)$ is the initial hidden state of node $u$ derived by the encoder in Section 3.2.1.

Figure 4: An illustration of t-Alignment.
once to get the hidden state of the end of the session. Besides, we
do not need to store all snapshots of a temporal session graph or
interrupt the integral process of ODE solvers to update the graph.

4.3 User Preference Generation and Prediction
After obtaining the item representations right after the last up-
date time of a temporal session graph $G_{t_n}$, which are denoted as $h_{i}(t_n^*)$, we generate the hybrid preference representation to represent the current user interests. Specifically, we combine the recent interest and the long-term preference in the ongoing session to obtain the user’s preference. We use the vector of the last item as the recent interest, that is, $\hat{z}_r = h_{i_n}(t_n^*)$, where $\hat{z}_r \in \mathbb{R}^d$.

For long-term interest, we consider all items in the session and
utilize an attention mechanism to determine the weights for com-
bining the historical item vectors, as follows:

$$\hat{z}_t = \sum_{i=1}^{l_{tn}} y_i h_{i}(t_n^*)$$
$$y_i = \text{Softmax}(a_i)$$

where $\hat{z}_t \in \mathbb{R}^d$ is the generated long-term preference at the $t^{th}$ timestamp, $a_i$ and $y_i$ are the importance scores of item $i$ before and after normalization, respectively, and $W_1 \in \mathbb{R}^{d \times d}$, $W_2, W_3 \in \mathbb{R}^{dc \times d}$ are learnable parameters. $\sigma$ is the sigmoid function.

Then, we generate the dynamic hybrid user preference by tak-
ing into account the long-term and recent interests, which can be
denoted as:

$$\hat{z}_h = W_4 [\hat{z}_t; \hat{z}_r],$$

where $\hat{z}_h \in \mathbb{R}^d$ is the final generated user preference at the times-
tamp $t_n$, and $W_4 \in \mathbb{R}^{dc \times d}$ is the learnable parameters.

After that, we can make predictions by computing a probabil-
ity distribution of the candidate items to be clicked at the next
timestamp through the multiplication operation between the user
preference and the embeddings of each item in $V$:

$$\hat{y}_i = ||\hat{z}_h||^2 ||x_i||,$$

where $|| \cdot ||$ denotes the $L_2$ normalization operation.

Finally, we compute the normalized score for each candidate
item, as follows:

$$\hat{y}_i = \text{Softmax}(\hat{y}_i)$$

where $\hat{y} = \{\hat{y}_1, \hat{y}_2, ..., \hat{y}_|V|\}$ are the normalized prediction scores vector for all candidate items.

4.4 Training
After obtaining the preferential scores, we adopt cross-entropy as
the optimization objective to learn the parameters following [5, 17, 34, 60]. The loss function is:

$$L(\hat{y}) = - \sum_{i=1}^{|V|} y_i \log(\hat{y}_i) + (1 - y_i) \log(1 - \hat{y}_i) + \lambda ||\Theta||^2_2$$

where $y_i \in y$ reflects the appearance of an item in the one-hot encoding vector of the ground truth, i.e., $y_i = 1$ if the $i$-th item is the target item of the given session; otherwise, $y_i = 0$. $\Theta$ is the model parameter. $\lambda$ is a scalar to control the influence of $L_2$ regularization. We use scaled softmax at the normalization stage Eq. (17) to prevent over smoothing of relevance scores.

4.5 Computational Complexity Analysis
Here we analyze the complexity of the initial latent state encoder
and the dynamic representation learning module. Given the item
set in the session as $V$, the transitions (edges) in the session as $E$. Then the time complexity of initial latent state encoder GGNN
is $O((|E| + |V|)|I|)$, where $I$ is the number of layers of GGNN. The time complexity of the dynamic representation learning module
is $O((|E| + |V|)T/\Delta t)$ where $T$ is the time duration of the whole
session and $\Delta t$ is the average integration step size of ODE solvers.

Then the overall time complexity of the two modules is
$O((I + T/\Delta t)(|E| + |V|))$, which is a linear combination of $|E|$ and $|V|$. As $I$ and $T/\Delta t$ are relatively small, we find that the total time complexity
increases but is still acceptable.

5 EXPERIMENT
In this section, we have conducted extensive experiments, and
analyzed the performance of the proposed GNG-ODE method by
addressing the following key research questions as follows:

- **RQ1:** Can our proposed GNG-ODE outperform the state-of-
  the art baselines for session-based recommendation?
- **RQ2:** How does GNG-ODE perform with different encoders?
- **RQ3:** How does GNG-ODE perform comparing with other
  Neural ODE model? Is t-Alignment useful to help GNG-ODE
  jointly capture structural and temporal pattern?
- **RQ4:** How well does GNG-ODE perform with different ODE
  solvers from the effectiveness perspective?
- **RQ5:** How is the scalability of GNG-ODE?
- **RQ6:** How do different hyper-parameters affect GNG-ODE?

| Table 1: Statistics of datasets. |
|----------------------------------|
| Gowalla  | Tmall  | Nowplaying |
| #clicks | 1,122,788 | 818,479 | 1,367,963 |
| #train sessions | 675,561 | 351,268 | 825,304 |
| #test sessions | 155,332 | 25,898 | 89,824 |
| #items | 29,510 | 40,727 | 60,416 |
| Average length | 4.32 | 6.69 | 7.42 |
| Average Interval | 11.07h | 1.49s | 4.36h |

5.1 Datasets and Preprocessing
We evaluate GNG-ODE and the baselines on the following three
publicly available benchmark datasets, which are commonly used
in the literature of session-based recommendation [5, 17, 30, 37, 42, 43, 57, 61, 67]:

- **Gowalla**¹ is a dataset that contains users’ check-in infor-
mation for point-of-interest recommendation. Following [5, 16, 50], we keep the 30,000 most popular locations and set
  the splitting interval to 1 day, and consider the last 20% of
  sessions for testing.
- **Tmall**² is a user-purchase data (only purchase records are
  utilized) obtained from Tmall platform. We also use the last
  20% of sessions as the test sets.

1https://snap.stanford.edu/data/loc-gowalla.html
2http://ocelma.net/MusicRecommendationDataset/lastfm-1K.html
We consider the following baselines to evaluate the performance of GNG-ODE over the best baseline using a paired t-test (*p < 0.01).

| Model     | Gowalla HR@10 | Gowalla HR@20 | Gowalla MRR@10 | Gowalla MRR@20 | Tmall HR@10 | Tmall HR@20 | Tmall MRR@10 | Tmall MRR@20 | Nowplaying HR@10 | Nowplaying HR@20 | Nowplaying MRR@10 | Nowplaying MRR@20 |
|-----------|---------------|---------------|----------------|----------------|-------------|-------------|----------------|----------------|-----------------|-----------------|------------------|------------------|
| NARM      | 40.71         | 42.18         | 44.92          | 42.19          | 21.97       | 24.79       | 29.81         | 24.26          | 19.11           | 22.19           | 21.11            | 22.19            |
| SR-GNN    | 40.71         | 42.18         | 44.92          | 42.19          | 21.97       | 24.79       | 29.81         | 24.26          | 19.11           | 22.19           | 21.11            | 22.19            |
| SGNN-NH   | 40.71         | 42.18         | 44.92          | 42.19          | 21.97       | 24.79       | 29.81         | 24.26          | 19.11           | 22.19           | 21.11            | 22.19            |
| LESSR     | 42.87         | 45.38         | 45.43          | 45.46          | 30.06       | 33.42       | 35.28         | 32.33          | 19.11           | 22.19           | 21.11            | 22.19            |
| GCE-GNN   | 45.38         | 45.43         | 45.46          | 45.46          | 30.06       | 33.42       | 35.28         | 32.33          | 19.11           | 22.19           | 21.11            | 22.19            |
| DAT-MDI   | 45.46         | 53.77         | 53.98          | 53.98          | 30.06       | 33.42       | 35.28         | 32.33          | 19.11           | 22.19           | 21.11            | 22.19            |

**Table 2**: Results(%) of main experiments. The numbers of HR@10, HR@20, MRR@10 and MRR@20 are reported. * denotes a significant improvement of GNG-ODE over the best baseline using a paired t-test (*p < 0.01).

- **Nowplaying** is a comprehensive implicit feedback dataset consisting of user-song interactions crawled from Twitter. For each music, we randomly select 20% of users who have played the music as the test sets, and the remaining users for training.

- Following [5, 30, 34, 42, 43, 57, 69], we filter the sessions containing merely an item and the items appearing less than five times for each dataset. We further make data augmentation that has been widely applied in [5, 30, 34, 57] after filtering short sessions and infrequent items. The statistics of the datasets are shown in Table 1.

### 5.2 Baseline Models

We consider the following baselines to evaluate the performance of the proposed model.

- **NARM** [30] is a RNN-based method for session-based recommendation. It utilizes RNNs with attention to model user action sequences.

- **SR-GNN** [57] is a GNN-based method for session-based recommendation. It applies GNNs to extract item features and obtains session representation through traditional attention network [32].

- **NISER** [17] employs normalized item and session embedding based on graph neural network to alleviate popularity bias problem in session-based recommendation.

- **SGNN-NH** [37] solves the long-range information propagation problem by adding a star node to take the non-adjacent items into consideration with gated graph neural networks.

- **LESSR** [5] transforms the sessions into directed multi-graphs and propagates information along shortcut connections to solve the lossy session encoding problem.

- **GCE-GNN** [56] transforms the sessions into global graph and local graphs to enable cross session learning.

- **DAT-MDI** [2] combines the GNN and GRU to learn the cross session enhanced session representation.

- **TiasASRec** [31] introduces to use relationship matrix to model the temporal relations for items in the sequence.

- **TGSRec** [12] uses a user collaboratively continuous-time transformer for sequential recommendation.

- **TMI-GNN** [47] uses temporal information to guide the multi-interest network to capture more accurate user interests.

### 5.3 Implementation

We apply grid search to find the optimal hyper-parameters for each model. We use the last 20% of the training set as the validation set. The ranges of other hyper-parameters are [46, 128, 256, 512] for hidden dimensionality $d$ and $e=3$ chosen for learning rate $\eta$. The weight decay rate $\lambda$ is set to $10^{-4}$. We use the Adam optimizer to train the models. The batch size is set to 512. We run all models five times with different random seeds and report the average. We use the same evaluation metrics HR@K (Hit Rate) and MRR@K (Mean Reciprocal Rank) following previous studies [5, 17, 30, 37, 42, 43, 57, 61]. The implementation of our model can be found at https://github.com/SpaceLearner/GNG-ODE.

### 5.4 Overall Comparison (RQ1)

To demonstrate the overall performance of the proposed model, we compare it with the state-of-the-art recommendation methods. They include the static models NARM, SR-GNN, NISER+, SGNN-NH, LESSR, GCE-GNN and DAT-MDI, and temporal models like TiasASRec, TGSRec and TMI-GNN. The experimental results of all compared methods are shown in Table 2, from which we have the following observations.

Compared with RNN, GNN-based models have a stronger ability to model the temporal relations for items in the sequence. The weight decay rate $\lambda$ is set to $10^{-4}$. We use the Adam optimizer to train the models. The batch size is set to 512. We run all models five times with different random seeds and report the average. We use the same evaluation metrics HR@K (Hit Rate) and MRR@K (Mean Reciprocal Rank) following previous studies [5, 17, 30, 37, 42, 43, 57, 61]. The implementation of our model can be found at https://github.com/SpaceLearner/GNG-ODE.

1. [https://dbis.ubik.at/node/263#nowplaying](https://dbis.ubik.at/node/263#nowplaying)
2. [https://github.com/lijingsdu/sessionRec_NARM](https://github.com/lijingsdu/sessionRec_NARM)
3. [https://github.com/CRIPAC-DIG/SR-GNN](https://github.com/CRIPAC-DIG/SR-GNN)
4. [https://github.com/johnny12150/NISER](https://github.com/johnny12150/NISER)
5. [https://github.com/twenen/lessr](https://github.com/twenen/lessr)
6. [https://github.com/CCIIPLab/GCE-GNN](https://github.com/CCIIPLab/GCE-GNN)
Table 3: Results of Different Initial State Encoders.

| Dataset | Gowalla MRR@20 | Tmall MRR@20 | Nowplaying MRR@20 |
|---------|---------------|-------------|-------------------|
| HR@20  | HR@20  | HR@20  |
| Identity | 53.84  | 26.80  | 35.03  | 15.11  | 22.75  | 9.23  |
| MLP     | 53.86  | 26.35  | 34.75  | 14.58  | 21.45  | 8.39  |
| GGNN    | 54.58  | 26.91  | 37.66  | 17.25  | 22.83  | 9.45  |

verifies the importance of capturing the complex structural pattern across sessions. We also observe that temporal information helps capture user preference, as temporal baselines all achieve comparable performance. Among temporal baselines, TMI-GNN performs the best, indicating that decomposing temporal information into different interests captures more fine-grained user preference.

Next, we zoom in on the performance of our proposed GNG-ODE. First, we can observe that GNG-ODE can achieve state-of-the-art performance on all cases for three datasets. In particular, GNG-ODE outperforms the existing temporal baselines (i.e., TISASRec, TGSRec and TMI-GNN). We attribute the improvements of GNG-ODE against the baselines to two factors: One is that GNG-ODE can take the continuous evolution of the session graph structures into consideration, and the other one is that GNG-ODE solves the continuous concept modeling problem using the continuous ODE function. In addition, the improvements of GNG-ODE over the best baselines (i.e., DAT-MDI, GCN-GNN and TMI-GNN) in terms of HR@20 and MRR@20 are 2.14% and 6.05% on Tmall, respectively, and the corresponding improvements are 1.69% and 3.82% on the Nowplaying dataset. We can observe that on all datasets, GNG-ODE brings more performance gain over original models when K in evaluation when N=K alone is smaller. A small value of K means the target items stay in the top positions of the recommendation list. Due to the position bias [3] in recommendation that users tend to pay more attention to the items in a higher position of the recommendation list, our framework can help original models to produce more accurate and user-friendly recommendations.

5.5 Impact of Encoders for Initial State Inference (RQ2)
In this experiment, we compare GNG-ODE with different initial state encoders to investigate the contribution of our encoder design. The following variants are tested on all datasets, where the results are reported in Table 3:

(1) Identity: GNG-ODE with raw one-hot embeddings as the initial hidden state.
(2) MLP: GNG-ODE with the output of a two-layer MLP as the initial hidden state.
(3) GGNN: GNG-ODE with the output of a GGNN as the initial hidden state.

From Table 3, we can observe that GGNN achieves best performance on all the three datasets. GGNN encoder emphasizes more on capturing structural information, replacing this with raw embedding or MLP will significantly decrease the recommendation performance on the Tmall dataset. For Gowalla and Nowplaying, compared with the results on Tmall, the structural information contributes less on both HR@20 and MRR@20 metrics. Our analysis is that the difference may be caused by how the influence of the structural and temporal factors in the e-commerce and check-in as well as interest-based scenarios varies. Specifically, in the e-commerce dataset, i.e., Tmall, the structural information is relatively more important, since the transition relation between items is much more complicated than the simple sequential signal [42, 57].

5.6 Impact of ODE Functions (RQ3)
To verify the effectiveness of GNG-ODE and t-Alignment, we replace the GNG-ODE with several widely used ODE functions and compare their recommendation performance. The variants of GNG-ODE are listed as follows. For none t-Alignment version, we use the static session graph as input.

(1) GNG-ODE: the model proposed in this paper.
(2) GCN-ODE: use a two layer graph convolutional network [26] as the ODE function.
(3) GRU-ODE: use a one layer gated recurrent unit [8] as the ODE function.
(4) MLP-ODE: use a two layer linear network with GELU activation [20] as the ODE function.

Table 4: Impact of ODE Function Module.

| Dataset | Gowalla MRR@20 | Tmall MRR@20 | Nowplaying MRR@20 |
|---------|---------------|-------------|-------------------|
| HR@20  | HR@20  | HR@20  |
| GNG-ODE | 54.58  | 26.91  | 37.36  | 17.23  | 22.83  | 9.45  |
| GCN-ODE | 53.44  | 26.63  | 37.45  | 17.83  | 22.65  | 9.25  |
| GRU-ODE | 54.41  | 26.76  | 37.33  | 17.30  | 22.59  | 9.23  |
| MLP-ODE | 54.34  | 26.81  | 37.17  | 17.63  | 22.34  | 9.18  |
| GNG-ODE w/o t-Align | 54.16  | 25.76  | 37.22  | 17.86  | 22.70  | 9.17  |
| GCN-ODE w/o t-Align | 53.85  | 25.57  | 37.18  | 17.37  | 22.62  | 9.14  |
| GRU-ODE w/o t-Align | 54.19  | 26.25  | 36.88  | 17.28  | 22.36  | 9.06  |
| MLP-ODE w/o t-Align | 53.99  | 26.12  | 36.89  | 17.36  | 22.10  | 9.04  |

Table 4 shows the performance of different GNG-ODE variants. We observe that without t-Alignment the performance will degrade on all datasets. This confirms that building continuous session graphs could enable our model to capture the evolution of the session graphs over multi-time steps. This further demonstrates the indispensability of t-Alignment in expanding the applicability of those ODE solvers. Moreover, we find that the method jointly considering structural and temporal patterns (GNG-ODE) outperforms its counterparts that only consider structural patterns (GCN-ODE) or that only consider temporal patterns (GRU-ODE), demonstrating the superiority of GNG-ODE at capturing both information.

5.7 Analysis of ODE Solvers (RQ4)
In this section, we investigate the effect of different ODE solvers. ODE solvers play a central role in the performance of GNG-ODE. Widely used numerical ODE solvers are fixed-step solvers like explicit Euler (Euler), fourth-order Runge–Kutta (RK4) and adaptive step size solvers like Dopri5.

5.7.1 Influence of ODE Solvers. The numbers of the best results for each solver are listed on the head of each subfigure in Figure 5. Adaptive step solver Dopri5 outperforms fixed-step solvers on the three datasets as adaptive step solvers adjust integration steps more
5.8 Running Time Comparison (RQ5)

The computation time per epoch for GNG-ODE is summarized in Figure 6. We also give the running time of DAT-MDI, one recent baseline that does not consider temporal information and TMI-GNN, the best baseline that considers temporal information. For fixed-step solvers we choose $T/\Delta t = 7$. We find that the efficiency of GNG-ODE with Euler solver is on par with DAT-MDI and TMI-GNN. Although RK4 and Dopri5 take more time to compute, they achieve better performance and the time costs are still acceptable.

5.9 Hyper-parameter Study (RQ6)

To answer RQ6, we conduct experiments to study the sensitivity of GNG-ODE on the embedding dimension and the GGNN encoder layers. Specifically, we tune the embedding dimension in $\{64, 128, 256, 512\}$ and search the GGNN encoder layers in $\{1, 2, 3, 4, 5\}$. The ODE solver is set to RK4. The performance of GNG-ODE with different hyper-parameters is presented in Figure 7.

5.9.1 Embedding Size. From Figure 7, we can observe that when the number of encoder layers is small, increasing the embedding dimension can generally improve the recommendation performance, especially from dimensions 64 to 128. This is because a large embedding dimension has a relatively better representation ability of item characteristics. However, there is a merely limited promotion of the performance when the dimension increases from 256 to 512. As increasing the embedding dimension will consume more computation resources, the dimension 256 is a proper choice considering both the effectiveness and efficiency of the recommender.

5.9.2 Number of GGNN Encoder Layers. As shown in Figure 7, increasing the number of layers does not always result in better performance. For example, on Tmall and Nowplaying dataset, the optimal layer number is less than 3. The performance decreases quickly when the layer number exceeds this optimal value because of the over-smoothing problem [63].

6 CONCLUSION

In this paper, we design a new SBR model, GNG-ODE, to model the continuity of user preference along the time in a fully continuous manner with Neural ODE. GNG-ODE works upon our defined continuous-time temporal session graph. We employ the GGNN to encode the structural patterns to infer the initial latent states of the items between different time steps in time. We also propose a time alignment algorithm, called t-Alignment, to adapt the existing ODE solvers onto our dynamic graph setting. Extensive experiments on three real-world datasets demonstrate the effectiveness of GNG-ODE. Moreover, the ablation study and analysis verify the efficacy of those components in GNG-ODE. In conclusion, GNG-ODE is a novel model to solve the SBR problem with temporal information.

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