Correspondence between Holographic and Gauss-Bonnet dark energy models

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Abstract

In the present work we investigate the cosmological implications of holographic dark energy density in the Gauss-Bonnet framework. By formulating independently the two cosmological scenarios, and by enforcing their simultaneous validity, we show that there is a correspondence between the holographic dark energy scenario in flat universe and the phantom dark energy model in the framework of Gauss-Bonnet theory with a potential. This correspondence leads consistently to an accelerating universe. However, in general one has not full freedom of constructing independently the two cosmological scenarios. Specific constraints must be imposed on the coupling with gravity and on the potential.

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1 Introduction

Nowadays it is strongly believed that the universe experiences an accelerated expansion. Recent observations from type Ia supernovae [1] in association with Large Scale Structure [2] and Cosmic Microwave Background anisotropies [3] have provided main evidence for this cosmic acceleration. In order to explain this interesting behavior, many theories have been proposed. Although it is widely accepted that the cause which drives the acceleration is the so-called dark energy, its nature and cosmological origin still remain enigmatic at present. One recent proposal is the dynamical dark energy scenario (see [4] and references therein), since the cosmological constant puzzles may be better interpreted by assuming that the vacuum energy is cancelled to exactly zero by some unknown mechanism and introducing a dark energy component with a dynamically variable equation of state. The dynamical dark energy paradigm is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving under a suitable potential.

In addition, many string theorists have devoted to understand and shed light on the cosmological constant or dark energy within the string framework. The famous Kachru-Kallosh-Linde-Trivedi (KKLT) model [5] is a typical example, which tries to construct metastable de Sitter vacua in the light of type IIB string theory. Furthermore, string landscape idea [6] has been proposed for shedding light on the cosmological constant problem based upon the anthropic principle and multiverse speculation. Although we are lacking a quantum gravity theory today, we still can make some attempts to probe the nature of dark energy according to some principles of quantum gravity. Currently, an interesting attempt in this direction is the so-called “holographic dark energy” proposal [7, 8, 9]. Such a paradigm has been constructed in the light of the holographic principle of quantum gravity theory, and thus it presents some interesting features of an underlying theory of dark energy [10]. Furthermore, it may simultaneously provide a solution to the coincidence problem, i.e why matter and dark energy densities are comparable today although they obey completely different equations of motion [9]. The holographic dark energy model has been extended to include the spatial curvature contribution [11] and it has also been generalized in the braneworld framework [12]. Lastly, it has been tested and constrained by various astronomical observations [13, 14, 15, 16, 17, 18].

Since holographic energy density corresponds to a dynamical cosmological constant, we need a dynamical framework, instead of general relativity, to consistently accommodate it. Therefore, it is interesting to investigate it under the Brans-Dicke theory [19, 20, 21, 22]. As it is known, Einstein’s theory of gravity may not describe it correctly at very high energy. The simplest alternative to general relativity is Brans-Dicke scalar-tensor theory [23], and amongst the most popular modified-gravity attempts, which may successfully describe the cosmic acceleration, is the \( f(R) \)-gravity. Very simple versions of such a theory, like \( 1/R \) [24] and \( 1/R + R^2 \) [25], may lead to the effective quintessence/phantom late-time universe. Another proposal, closely related to the low-energy string effective action, is the scalar-Gauss-Bonnet gravity [26], which can be considered as a form of gravitational dark energy.

In the present paper we are interested in investigating the conditions under which we can obtain a correspondence between holographic and Gauss-Bonnet models of dark energy, i.e to examine holographic dark energy in a spatially flat Gauss-Bonnet universe.
2 Gauss-Bonnet Dark Energy

In this section we formulate a Gauss-Bonnet model for dark energy [26, 27, 28]. As usual, as a candidate for dark energy we consider a scalar field $\phi$, which is moreover coupled to gravity through the higher-derivative (string-originated) Gauss-Bonnet term. The corresponding action is given by

$$S = \int d^4x \sqrt{g} \left[ \frac{1}{2\kappa^2} R - \frac{\gamma}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + f(\phi)G \right],$$  \hspace{1cm} (1)$$

where $\kappa^2 = 8\pi G$ and $\gamma = \pm 1$. For a canonical scalar field $\gamma = 1$, but we extend the model to $\gamma = -1$ which corresponds to phantom behavior. In (1) $G$ stands for the Gauss-Bonnet combination which is explicitly given as:

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma},$$  \hspace{1cm} (2)$$

where $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ are respectively the Riemann and Ricci tensors and $R$ is the curvature scalar of the spacetime with metric $g_{\mu\nu}$. Finally, the coupling with gravity constitutes of a function $f(\phi)$. In the following we will concentrate on the spatially flat Robertson-Walker universe with

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2d\Omega^2),$$  \hspace{1cm} (3)$$

thus we impose $k = 0$ in (1).

The equations of motion can be easily derived from (1), and the result is [27]:

$$\frac{\gamma}{2} \ddot{\phi}^2 - V(\phi) + 16f'(\phi)\dot{\phi}H \frac{\ddot{a}}{a} + 8 \left[ f'(\phi)\ddot{\phi} + f''(\phi)\dot{\phi}^2 \right] H^2 = p_\Lambda$$  \hspace{1cm} (4)$$

for the scale factor, and

$$\gamma \left[ \frac{\ddot{\phi} + 3H\dot{\phi} + \frac{V'(\phi)}{\gamma}}{\gamma} \right] = 24f'(\phi)H^2 \frac{\ddot{a}}{a}$$  \hspace{1cm} (5)$$

for the scalar field. Furthermore, we obtain a constraint equation, namely:

$$\frac{\gamma}{2} \ddot{\phi}^2 + V(\phi) - 24f'(\phi)\dot{\phi}H^3 = \rho_\Lambda.$$

In the expressions above, $p_\Lambda$ and $\rho_\Lambda$ are the pressure and energy density due to the scalar field and the Gauss-Bonnet interaction [28], which are identified as the corresponding quantities of dark energy.

3 Holographic Dark Energy

Let us describe briefly the holographic dark energy model [7, 8, 9]. In this dark-energy-paradigm one determines an appropriate quantity to serve as an infrared cut-off for the theory, and imposes the constraint that the total vacuum energy in the corresponding maximum volume must not be greater than the mass of a black hole of the same size. By saturating the inequality one identifies the acquired vacuum energy as holographic dark energy. Although the choice of the IR cut-off has raised a discussion in the literature
[9, 19, 29], it has been shown, and it is generally accepted, that in the case of a flat Universe the most suitable ansatz is the use of the event horizon $R_h$ [8]:

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2},$$

which leads to results compatible with observations. The holographic energy density $\rho_\Lambda$ is given by

$$\rho_\Lambda = \frac{3c^2}{R_h^2},$$

in units where $M_p^2 = 8\pi$, and $c$ is a constant which value is determined by observational fit. Furthermore, we can define the dimensionless dark energy as:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} = \frac{c^2}{R_h^2H^2}.$$  

In the case of a dark-energy dominated universe, dark energy evolves according to the conservation law

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0,$$

or equivalently:

$$\dot{\Omega}_\Lambda = H\Omega_\Lambda(1 - \Omega_\Lambda) \left( 1 + \frac{2\sqrt{\Omega_\Lambda}}{c} \right),$$

where the equation of state is

$$p_\Lambda = -\frac{1}{3} \left( 1 + \frac{2\sqrt{\Omega_\Lambda}}{c} \right) \rho_\Lambda,$$

which leads straightforwardly to an index of the equation of state of the form:

$$w_\Lambda = -\frac{1}{3} \left( 1 + \frac{2\sqrt{\Omega_\Lambda}}{c} \right).$$

As we can clearly see, $w_\Lambda$ depends on the parameter $c$. In recent fit studies, different groups have ascribed different values to $c$. A direct fit of the present available SNe Ia data indicates that the best fit result is the best-fit value $c = 0.21$ within 1-$\sigma$ error range [14]. In addition, observational data from the X-ray gas mass fraction of galaxy clusters lead to $c = 0.61$ within 1-$\sigma$ [15]. Similarly, combining data from type Ia supernovae, cosmic microwave background radiation and large scale structure give the best-fit value $c = 0.91$ within 1-$\sigma$ [16], while combining data from type Ia supernovae, X-ray gas and baryon acoustic oscillation lead to $c = 0.73$ as a best-fit value within 1-$\sigma$ [17]. Finally, the study of the constraints on the dark energy arising from the holographic connection to the small $l$ CMB suppression, reveals that $c = 2.1$ within 1-$\sigma$ error [18]. In conclusion, $0.21 \leq c \leq 2.1$, and holographic dark energy provides the mechanism for the $w = -1$ crossing and the transition to the accelerating expansion of the Universe.
4 Correspondence between Holographic and Gauss-Bonnet Dark Energy models

The main goal of this work is to investigate the conditions under which there is a correspondence between the Gauss-Bonnet dark energy model and the holographic dark energy scenario, in the flat Universe case. In particular, to determine an appropriate Gauss-Bonnet potential which makes the two pictures to coincide with each other.

Let us first consider the simple Gauss-Bonnet solutions acquired in [27, 28]. In this case \( f(\phi) \) is given as [26]

\[
f(\phi) = f_0 e^{\frac{2\phi}{\phi_0}}. \tag{14}
\]

In addition, we assume that the scale factor behaves as \( a = a_0 t^{h_0} \), and similarly to [27] we will examine both \( h_0 \)-sign cases. However, when \( h_0 \) is negative the scale factor does not correspond to expanding universe but to shrinking one. If one changes the direction of time as \( t \to -t \), the expanding universe whose scale factor is given by \( a = a_0 (-t)^{h_0} \) emerges. Since \( h_0 \) is not an integer in general, there is one remaining difficulty concerning the sign of \( t \). To avoid the apparent inconsistency, we may further shift the origin of the time as \( t \to -t \to t_s - t \). Then the time \( t \) can be positive as long as \( t < t_s \), and we can consistently write \( a = a_0 (t_s - t)^{h_0} \). Thus, we can finally write [27]

\[
H = \frac{h_0}{t}, \quad \phi = \phi_0 \ln \frac{t}{t_1} \tag{15}
\]

when \( h_0 > 0 \) or

\[
H = -\frac{h_0}{t_s - t}, \quad \phi = \phi_0 \ln \frac{t_s - t}{t_1} \tag{16}
\]

when \( h_0 < 0 \), with an undetermined constant \( t_1 \).

Let us first investigate the positive-\( h_0 \) case. If we establish a correspondence between the holographic dark energy and Gauss-Bonnet approach, then using dark energy density equation (6) and relation (9), together with expressions (15), we can easily derive the scalar potential term as

\[
V = e^{\frac{2\phi}{\phi_0}} \left( \frac{3\Omega_\Lambda h_0^2}{t_1^2} + \frac{48 f_0 h_0^3}{t_1^2} - \frac{\gamma \phi_0^2}{2} \right). \tag{17}
\]

Note that expressions (15) allow for an elimination of time \( t \) in terms of the scalar field \( \phi \). Furthermore, by substituting \( \phi \), and \( H \) from (15), \( f(\phi) \) from (14) and \( V(\phi) \) from (17) into (5) we obtain:

\[
-3\gamma h_0 \phi_0 + \frac{6\Omega_\Lambda h_0^2}{\phi_0} + \frac{96 f_0 h_0^3}{\phi_0 t_1^2} - 3h_0^2 \frac{d\Omega_\Lambda}{d\phi} + \frac{48 f_0 h_0^3 (h_0 - 1)}{\phi_0 t_1^2} = 0 \tag{18}
\]

where

\[
\frac{d\Omega_\Lambda}{d\phi} = \frac{d\Omega_\Lambda}{dt} \frac{t}{\phi_0} = \frac{d\Omega_\Lambda}{dt} \frac{t_1}{\phi_0} e^{\frac{\phi}{\phi_0}}, \tag{19}
\]

with \( \dot{\Omega}_\Lambda \) given by (11).

Now, under the ansatz \( a = a_0 t^{h_0} \) it is easy to see from (7) that in order for \( R_h \) to be finite, \( h_0 \) has to be greater than 1. In such a case we straightforwardly find:

\[
R_h = \frac{t}{h_0 - 1}, \tag{20}
\]
\[ \Omega_\Lambda = \frac{c^2(h_0 - 1)^2}{h_0^2}, \]  
\[ \text{and} \]
\[ w_\Lambda = \frac{2}{3h_0} - 1. \]

Lastly, in order to obtain a complete consistency in equations (18)-(22), we have to impose the constraint:
\[ h_0 = \frac{c}{c - 1}, \]  
for \( c \neq 1 \), which is moreover consistent with \( h_0 > 1 \) (when \( c = 1 \) we need \( h_0 = 1/2 \) which has been excluded due to \( R_h \) convergence, i.e in this case there is no accepted solution).

Note however that when \( c \neq 1 \) we must have \( c > 1 \) in order to “remain” in the positive-\( h_0 \) case. As we observe, the case under examination leads to a correspondence between Gauss-Bonnet and Holographic dark energy models, where \( \Omega_\Lambda = 1 \), and \(-\frac{1}{3} \geq w_\Lambda \geq -1 \) (as it is implied by (22) under \( h_0 > 1 \)). Thus, it is not too practical since it corresponds to a Universe with complete dominance of dark energy, and which is not accelerating.

Let us proceed to the investigation of the negative-\( h_0 \) case. Repeating the same steps, but imposing relations (16) we find that
\[ V = e^{-\frac{3t}{h_0}} \left( 3\Omega_\Lambda h_0^2 - \frac{48f_0h_0^3}{t_1^2} - \frac{\gamma\phi_0^2}{2} \right), \]
\[ \text{and} \]
\[ -2\gamma\phi_0 + 3\gamma h_0\phi_0 + \frac{6\Omega_\Lambda h_0^2}{\phi_0} + \frac{96f_0h_0^3}{\phi_0t_1^2} - 3h_0^2 \frac{d\Omega_\Lambda}{d\phi} + \frac{48f_0h_0^3(h_0 - 1)}{\phi_0t_1^2} = 0, \]
where
\[ \frac{d\Omega_\Lambda}{d\phi} = -\frac{d\Omega_\Lambda}{dt} \frac{(t_s - t)}{\phi_0} = -\frac{d\Omega_\Lambda}{dt} \frac{t_1}{\phi_0} e^{\phi_0}. \]

Now, under the ansatz \( a = a_0(t_s - t)^{h_0} \) we can see from (7) that \( R_h \) is always finite if \( h_0 \) is negative, which is just the case under investigation. Then we have:
\[ R_h = \frac{t_s - t}{1 - h_0}, \]
\[ \Omega_\Lambda = \frac{c^2(h_0 - 1)^2}{h_0^2}, \]  
and therefore
\[ w_\Lambda = \frac{2}{3h_0} - 1. \]

Finally, in order to obtain a complete consistency in equations (25)-(29), we have to impose the constraint:
\[ h_0 = \frac{c}{c - 1}, \]  
while there is no accepted solution if \( c = 1 \). Furthermore, since we are in the negative-\( h_0 \) case we must have \( c < 1 \). Thus, we conclude that we acquire a correspondence between Gauss-Bonnet and Holographic dark energy models. In addition, it is interesting that in this case we get \( w_\Lambda \leq -1 \) which corresponds to an accelerating universe.
As we can see so far, under the ansatz $a = a_0 t^{h_0}$ (for $h_0 > 0$) or $a = a_0 (t_s - t)^{h_0}$ (for $h_0 < 0$), which is usually assumed in the Gauss-Bonnet models of dark energy [27, 28], we do obtain a correspondence with holographic dark energy scenario. Although this correspondence might look rather trivial, we do acquire an accelerating universe and moreover an interesting classification in terms of the parameter $c$ of holographic dark energy, since $c > 1$ corresponds to $h_0 > 1$ and $c < 1$ to $h_0 < 0$.

Let us now make some comments about the freedom of construction of Gauss-Bonnet or holographic dark energy models separately. In the basic work [28], as well as in [30], the authors examine several examples of scalar-Gauss-Bonnet gravity, with multiple fields (phantom or canonical). Starting from various ansätze for the form of $f(\phi)$, they study the cosmological evolution. However, simple algebraic calculations reveal that these models, although correct from the Gauss-Bonnet dark energy point of view, are not consistent with the holographic dark energy framework. Specifically, we find that the evolution for $\Omega_\Lambda$ is not consistent with (11). Similarly, starting with conventional holographic dark energy models it is not a priori ensured that these models fit within the Gauss-Bonnet framework. Therefore, one must be careful in constructing either of the two scenarios. In particular, he must ensure the simultaneous validity of relations (5), (6) and (11). Thus, these relations correspond to specific constraints for the function $f(\phi)$, which describes the coupling with gravity, and for the potential $V(\phi)$.

An enlightening contribution to the aforementioned discussion is the following. The peculiar feature of holographic dark energy, as long as one accepts its (still controversial in the literature) framework, is that it correlates dark energy with the event horizon of the universe, i.e. with the scale factor (or equivalently with the Hubble parameter). Thus, apart from the Friedmann equations, and the evolution equations for the scalar fields, one has to fulfill this additional correlation, which is quantitatively expressed by relation (9) (or (11)). In other words, this relation can be considered as an extra constraint, imposed externally to the system of Einstein equations. On the other hand, in Gauss-Bonnet framework, dark energy acquires a contribution from the scalar field and its potential, and from the Gauss-Bonnet coupling, namely relation (6). In [28], the authors reconstruct the scalar-Gauss-Bonnet gravity for general cosmological evolutions, i.e they reconstruct the scalar field potential $V(\phi)$ and the coupling $f(\phi)$ for arbitrary $H(t)$ and $\phi(t)$. Although this procedure is efficient for any conventional cosmological evolution, it is not adequate for the peculiar and non-conventional holographic dark energy evolution, due to the external, additional and non-trivial constraint. In particular, in order to satisfy relation (6) in full generality, and for an arbitrary $\rho_\Lambda$ (which will be later identified with relation (11)), one has to specify $H(t), f(\phi), \phi(t)$ and $V(\phi)$ independently, which cannot be performed using the procedure of [28]. The peculiar nature of holographic dark energy framework can be embedded in Gauss-Bonnet gravity only by the simultaneous satisfaction of relations (5), (6) and (11).

5 Conclusions

In the present paper, we considered separately the holographic and Gauss-Bonnet dark energy models. Then we investigated the conditions under which the two scenarios can be simultaneously valid. In particular, by considering holographic dark energy density as a dynamical cosmological constant, we obtained its equation of state in the Gauss-Bonnet
framework. Thus, we have suggested a correspondence between the holographic dark energy scenario in flat universe and the phantom dark energy model in the framework of Gauss-Bonnet theory with a potential. This correspondence can lead to an accelerating universe, under a special ansatz for the function $f(\phi)$, which describes the coupling with gravity, and for the scale factor evolution. However, in general one has not full freedom of constructing independently the two cosmological scenarios. If he requires their consistent unification, then specific constraints must be imposed in $f(\phi)$, as well as in the potential $V(\phi)$.

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