We present cosmic solutions corresponding to universes filled with dark and phantom energy, all having a negative cosmological constant. All such solutions contain infinite singularities, successively and equally distributed along time, which can be either big bang/crunches or big rips singularities. Classically these solutions can be regarded as associated with multiverse scenarios, being those corresponding to phantom energy that may describe the current accelerating universe.

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Just like the word atom designated what in principle was thought to be indivisible and finally turned out not to be the case, the word universe, which was originally intended to describe the whole, has recently been reinterpreted to be just a single causally disconnected part from the whole spacetime. Different spacetimes could exist and our universe would be just one more among that ensemble of completely causally disconnected spacetimes. Actually, it was Giordano Bruno who first realized that many other worlds other than ours could exist [1]. This idea has triggered, several centuries later, the development of different theories of the multiverse, this time with fewer risks. Quite possibly, the best known is the many-universes theory, derived from the relative-state-formulation of Everett, when applied to cosmology [2, 3]. This states that all branches of a wavefunction for the universe correspond to equally real different universes existing in parallel within an overall multiverse.

But there are also multiverse models that appear outside the quantum realm, in the framework of general relativity. One example of a multiverse that does not make explicit recourse to a quantum formalism is the chaotic inflationary multiverse [4]. In every flat space that has an event horizon, such as in the inflationary universe, a closed causal region of spacetime is settled which can be influenced by observers. Since the universe is flat, it is infinite, so for observers who are space-like-separated by distances greater than the sum of their respective distances to the event horizons, their respective causal domains are disjointed and therefore every inflationary domain can be interpreted as a single universe in the framework...
of this classical multiverse. Another possible multiverse may appear when we consider the current accelerated expansion of the universe. If we choose as dark energy phantom energy [5], then a singularity is predicted to occur in the finite future [6]. This singularity divides the universe into two classically non-connected regions, before and after the singularity [7]. Here an idea of the multiverse would also appear because that model requires a precise discretization of the parameter in the equation of state, if one wants to consider the region after the big rip as a part of the whole spacetime. Each value of the discretized parameter of the equation of state would then describe a single universe in the context of an infinite multiverse.

Recently, string theory has also resorted to the multiverse idea to interpret the multiplicity of positive-energy vacua which rises up to order $10^{100}$ to $10^{500}$ [8]. The different subuniverses described by this string landscape [9] could be different regions of space, different eras of time in a single big bang, different regions of spacetime or different parts of quantum mechanical Hilbert space (these alternatives are not mutually exclusive) [10].

Furthermore, another multiverse model has been discussed by Smolin [11], who conjectured that new universes are spawned within black holes, and that this kind of baby universe will inherit the physics of the parent universe but with small random variations. The process could continue ad infinitum. Universes that produce many black holes would induce more progeny too, representing the largest volume of space.

On the other hand, in the ekpyrotic model of Steinhard and Turok [12], a brane collides with a confining three-dimensional boundary to a four-dimensional space to create the big bang. The four-dimensional space can be foliated with any number of branes, each of which, in the absence of collisions, constitutes a universe.

Within the framework of the current accelerated expansion of the universe mentioned above, we have considered in this paper a new model in which we have taken into account the existence of a negative cosmological constant. A spacetime with a negative cosmological constant is worth investigating, since it allows a consistent physical interpretation and naturally appears in elementary particle theories. Indeed, in string theory, as in supergravity theories, the vacuum has a negative energy density, which means that it is described by anti-de Sitter (AdS) spacetime. An important advance in understanding quantum issues in strong gravitational fields was conjectured in 1997, namely that string theory in an AdS background is equivalent to a conformal field theory (CFT) [13]. This is a beautiful and concrete example of the holographic principle in quantum gravity [14]. It is with this motivation that we consider a cosmic model of dark energy with a negative constant vacuum energy in this paper.

If we consider a quintessence field to describe dark energy, one can describe it as a perfect fluid with an equation of state $p = w\rho = w\rho_0(a(t)/a_0)^{3(1+w)}$, where $p$ and $\rho$ are the pressure and energy density of the fluid respectively, and $w$ a constant parameter. The Friedmann equation for this flat model, which contains a negative cosmological constant $\Lambda$, can be written as

$$H^2 = -\frac{\lambda}{3} + Ca^{-3\beta},$$

with $\lambda = |\Lambda|/3$, where $\lambda < 8\pi\rho_0/3$ in order for $H_0$ to be real; $C = 8\pi\rho_0/(3a_0^{-3\beta})$ and $\beta = 1 + w$. By integrating equation (1), we can obtain the cosmic scale factor, yielding

$$a(t) = a_0 \left[ \cos \left( \frac{3\beta}{2} \lambda^{1/2}(t - t_0) \right) + \left( \frac{C}{\lambda} a_0^{-3\beta} - 1 \right)^{1/2} \sin \left( \frac{3\beta}{2} \lambda^{1/2}(t - t_0) \right) \right]^{-\frac{2}{3w}},$$

where derivation obviously leads back to equation (1). On the other hand, to keep equation (2) physically meaningful we need $2/(3\beta)$ to be an even integer number which would be positive for dark energy and negative for phantom energy; that is to say a discretization of the equation of state parameter $w = \frac{1-\lambda}{3m}$ is required to ensure $a(t) > 0$ everywhere [7, 15, 16].
For the case in which the dark energy is phantom energy, that is, when $\beta < 0$, this scale factor is converted into

$$a(t) = a_0 \left[ \cos(\alpha(t - t_0)) - b \sin(\alpha(t - t_0)) \right]^{-\frac{2}{3} |\beta|}, \quad (3)$$

where $\alpha = \frac{3|\beta|}{2} \lambda^{1/2}$ and $b = \left( \frac{8\pi}{3} \rho_0 - 1 \right)^{1/2}$. It is easy to see that the scale factor diverges an infinite number of times along the full time interval. Each of these divergences actually describes a big rip singularity (i.e., the place where both the scale factor and the energy density diverge) that takes place at

$$t_{br} = t_0 + \frac{2}{3|\beta| \lambda^{1/2}} \arctg \left[ \left( \frac{8\pi \rho_0}{3\lambda} - 1 \right)^{-1/2} \right] + \frac{2n\pi}{3|\beta| \lambda^{1/2}}, \quad (4)$$

with $n$ again any integer number. We recover the expression for the big rip time obtained in a quintessence model of phantom energy without cosmological constant [6], when we set $n = 0$ in expression (4) and expand it for $\lambda \ll 1$

$$t_{br} = t_0 + \frac{1}{|\beta| \left( 6\pi \rho_0 \right)^{1/2}}. \quad (5)$$

In the light of equation (4) we can in fact see that this model will have infinite big rip singularities. This can be interpreted as follows: classically, a singularity cuts off the spacetime, so the different regions between big rips would be isolated from each other. Thus, each of them would actually correspond to a different universe, independent of the rest, i.e., another spacetime. In fact, since when $a^{-3|\beta|/2}$ goes to zero (which it does many times) then $a(t)$ and $H$ both blow up at the same place, thus $a(t)$ does not go to finite size through the singularity where $H \to \infty$ which means that we cannot have a bounce at the singularities and therefore the solution cannot be analytically continued beyond each of them. On the other hand, unphysical jumps in entropy would be required if one wanted to continue solution (1) along a common time from $-\infty$ to $\infty$. As equation (3) tells us, the independent universes are identical among them and have the same physical characteristics. All of them begin at a big rip singularity, and then progressively contract until a given, constant, minimum value of the scale factor,

$$a_{\min} = a_0 \left( \frac{8\pi \rho_0}{3\lambda} \right)^{1/3|\beta|} > 0, \quad (6)$$

after which the given universe starts expanding, all the way in an accelerated fashion, to again reach the next big rip singularity (see figure 1). The minimum value in equation (6) has been obtained from the extremum value that corresponds to equating to zero equation (1). The lifetime of every one of these universes is given by

$$t_u = \frac{2\pi}{3|\beta| \lambda^{1/2}}. \quad (7)$$

It follows from equation (7) that the smaller $\lambda$ the longer the universe life $t_u$. It can be seen that if $\lambda = 0$, where we recover the quintessence model of phantom energy, these time differences are infinite, as in the model of the usual phantom, there is a unique big rip.

Given that, as we have said before, the infinite singularities have cut off the spacetime generating infinite causally disconnected spacetimes, we can re-scale and redefine the time in each of these spacetimes, in some appropriate form, independently in each of them. This way, the scale factor reaches its minimum value in the zero of the obtained new symmetrical time of symmetry. The aforementioned scale factor can be written in a more compact form as

$$a(\tau) = a_{\min} \left( \cos \tau \right)^{-\frac{2}{3|\beta|}}, \quad (8)$$
Figure 1. Time evolution corresponding to a universe equipped with a negative cosmological constant and (a) quintessential dark energy with \(\beta > 0\) and (b) phantom energy with \(\beta < 0\).

with the new time \(\tau\) covering the interval \((-\pi/2, \pi/2)\) in every universe, reaching the initial and final big rips at the extrema. Each of the universes in the multiverse is as though it were a faster-expanding de Sitter space defined along a finite time interval.

If we assumed that all these universes are classically identical and that our universe is in fact described by this model, we could dare to claim that such universes are governed by the same physical laws as ours, given that all of them would then be exactly physically equivalent. Classically, the existence of life in our universe might be justified as a byproduct of the anthropic principle in its various formulations. If we think that life exists because the initial conditions of our universe allow it to occur, the physical equivalence of the various universes would imply that, classical life existed such as we know it in all of them. But if we considered the emergence of life as a process somehow dependent on quantum effects, as seems to be the case, it would no longer be consistent to extrapolate ideas about such existence based on a classical extension of the physical laws.

We can envisage a model where the expansion is not caused by a phantom fluid, but by dark energy itself, i.e., \(\beta > 0\) in the equation of state. In this case, we would also obtain a multiverse scenario with the same characteristics among the universes, but these would now be closed universes that would decelerate from a big bang until its scale factor reached a finite maximum value (given by equation (6) with \(\beta > 0\)). From that value onwards, the universe would contract in size until finally it died in a big crunch singularity (see figure 1); being therefore unable to explain the current accelerated expansion of our universe.

In view of the results obtained in this work, it is worth mentioning that whereas the insertion of a negative cosmological constant in a phantom energy model has the effect of repeating the big rip singularity an infinite number of times, the analogous consideration of this in a model with dark energy slows down the accelerated expansion caused by this fluid, in such a way that it would cause not just one but infinite big crunches. Hence in both cases we obtain a classical multiverse scenario, in which the universes are identical among them. This scenario could be altered if we included the evolution of astronomical objects in this model [17].

As we said before, the models suggested in the present paper are purely classical, therefore considering quantum effects would probably smooth out the singularities [18], in such a way
that we would no longer have an infinite set of isolated spacetimes, so implying the loss of the multiverse scenario.

The appeal of the multiverse model is that it points towards a less predominant position of what we call our universe in nature. It could well be that, once again, we would have missed the denomination of a physical system and, in a similar way to terms such as atom or elementary particles were once wrongly used to denote what it turned out to be essentially divisible systems, we could well be applying the term ‘universe’ to what is nothing but just a single part or product of it [19].

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