Cold formability prediction by the modified maximum force criterion with a non-associated Hill48 model accounting for anisotropic hardening

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Abstract. Experimental and numerical investigations on the characterisation and prediction of cold formability of a ferritic steel sheet are performed in this study. Tensile tests and Nakajima tests were performed for the plasticity characterisation and the forming limit diagram determination. In the numerical prediction, the modified maximum force criterion is selected as the localisation criterion. For the plasticity model, a non-associated formulation of the Hill48 model is employed. With the non-associated flow rule, the model can result in a similar predictive capability of stress and r-value directionality to the advanced non-quadratic associated models. To accurately characterise the anisotropy evolution during hardening, the anisotropic hardening is also calibrated and implemented into the model for the prediction of the formability.

1. Introduction

Cold formability, normally characterised as the forming limit curve (FLC), is one of the key characteristics for the sheet metal forming industry. Among many influencing parameters, the plastic anisotropy in terms of both stress and deformation has significant impact on the cold formability behaviour [1, 2]. Various plasticity models have been developed for the accurate description of anisotropy and most of models are based on the associated flow rule [3-6]. Recently, the non-associated quadratic anisotropic models have received increasing attention, although the experimental justification of its application remains an open issue [7]. The use of these models is mainly driven by its efficiency and simplicity. Compared to the associated quadratic models, it improves the anisotropic characteristic predictive capability to a similar level to the advanced non-quadratic associated anisotropic models. During the plastic deformation process of anisotropic materials, the hardening as well as the deformation behaviour is actually evolving. These phenomena are normally referred to as anisotropic (distortional) hardening. However, only very few models consider this factor [8, 9], and most anisotropic plasticity models still use the initial/constant yield stress or r-value to determine the parameters of the yield criterion. In this study, the evolving feature of the yield surface is implemented into a non-associated Hill48 model and it is further used to give the prediction of the cold formability of a ferritic steel in conjunction with the modified maximum force criterion (MMFC) [10].
2. Modelling strategy

The Hill48 plasticity model [3] was the first and the most often employed anisotropic models for sheet metals. The yield function is defined by:

\[ f = \bar{\sigma}(\sigma) - \sigma_\gamma(\varepsilon^p) \leq 0 \] (1)

where \( \bar{\sigma}(\sigma) \) is a function of stress components, defined in Equation (2) and \( \sigma_\gamma(\varepsilon^p) \) is the flow curve of specific material as a function of the equivalent plastic strain. Yield function defines the yielding process onset when \( f = 0 \).

\[ \bar{\sigma}(\sigma) = \sqrt{F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2La_{23}^2 + 2M\sigma_{13}^2 + 2N\sigma_{12}^2} \] (2)

where \( F, G, H, L, M \) and \( N \) are material coefficients controlling the anisotropy of the yield surface. In the Hill48 plasticity model, the flow curve is obtained from the experimental true stress-true strain curve, normally the uniaxial tensile test along the rolling direction, and extrapolated to large strains. For the determination of the anisotropic coefficients of the Hill48 model, there are various possible ways, and among them two methods, the stress based method and r-value based method are the often used approaches. For details, readers are referred to Ref. [11].

One practical motivation for the non-associated Hill48 (nHill48) development is the inconsistent anisotropic prediction by the parameters calibrated based on the stress method or r-value method. For example, the model calibrated from the stress based method could properly describe the yield stress directionality but result in an inaccurate prediction on the r-value, or vice versa. The basic idea for the nHill48 formulation is that the yield function and flow potential both take the form of Hill48 model, but using the stress based method to calibrate the yield function and the r-value based method to calibrate the flow potential. To account for the anisotropic hardening, it basically requires that the yield surface/function is changing with respect to the evolution of the plastic strain. As the advantage of the nHill48 model is the straightforward anisotropic coefficient calibration without any iterative numerical procedure, the extension of it to account for the anisotropic hardening is simply to change the constant material parameters \( F \sim N \) to a plastic strain dependent function \( F(\varepsilon^p) \sim N(\varepsilon^p) \).

For the prediction of FLC, the pioneering work was developed by Hill [12] for the localized necking and by Swift [13] for the diffuse necking. Based on the maximum force criterion by Swift [13], Hora [10] developed the MMFC, motivated by the fact that the critical phase in the metal forming processes is not the occurrence of diffuse necking, but the localized necking, i.e. the plastic deformation is still allowed to a certain extent till a plane strain tension mode after the maximum force is reached. Therefore, the stress \( \sigma_{11} \) is not only a function of strain hardening but also the strain ratio \( \beta \): 

\[ \frac{\partial \sigma_{11}}{\partial \varepsilon_{11}} + \frac{\partial \sigma_{11}}{\partial \beta} \cdot \frac{\partial \beta}{\partial \varepsilon_{11}} = \sigma_{11} \] (3)

where \( \sigma_{11} \) and \( \varepsilon_{11} \) are, respectively, the major principal stress and strain; \( \beta \) is defined as the ratio between the major and minor principal strain rate, \( \beta \equiv \dot{\varepsilon}_{22}/\dot{\varepsilon}_{11} = \text{const} \).

In this study, the plane stress condition is assumed. The non-associated Hill48 model is implemented together with the MMFC in an iterative manner to determine the FLC for various loading conditions. Due to the space limit, the detailed equations are not shown here.

3. Materials

In this study, a ferritic stainless steel sheet (AISI 439) with 1 mm thickness was investigated. It is a titanium stabilized, 17-18% chromium alloyed, low carbon steel, designed to resist corrosion in a variety of oxidizing environments from fresh water to boiling acids. The steel owns completely ferritic microstructure, i.e. body-centred cubic (bcc) crystal structure. Owning to its previous manufacturing processes, such as rolling and annealing, the steel shows strong anisotropy and off-\( \gamma \)-fibre texture component \(~\{554\}<225>\) texture. For detailed texture information, readers are referred to Ref. [14].
4. Results

Tensile tests were conducted to characterise the mechanical properties according to the European standard EN 10002-1. To study the plastic anisotropy and to calibrate the material parameters of the anisotropic plasticity model, tensile tests along three typical directions, rolling direction (0°), diagonal direction (45°) and transverse direction (90°) were conducted. The engineering and true stress–strain curves for 0°, 45° and 90° are shown in Figure 1 (a) and (b), respectively. The intention of this study is to evaluate the anisotropic hardening behaviour, and therefore, the r-value results are not given due to the space limit. For more complete information, readers are referred to Ref. [11, 14].

Figure 1. (a) The engineering stress–strain curves of AISI439 for different loading directions with respect to rolling direction (0°, 45°, 90°); (b) The true stress–strain curves and the anisotropic hardening behaviour along different loading directions (0°, 45°, 90°).

In Figure 1 (b), one can see that compared to the scaled isotropic hardening curve (dashed lines) calculated from 0° based on the initial anisotropic factor, the experimental flow curves for 45° and 90° are much lower at larger strains, although the agreement is very good at the beginning of the deformation. It is clearly noted that using the scaled initial anisotropic ratio results in substantial errors when the plastic deformation is getting large. Therefore, in this study, the yield surface evolution is calibrated for the non-associated Hill48 model, so that very accurate description for the anisotropic stress–strain curves at large deformation can be obtained in the subsequent formability prediction.

For the cold formability characterisation, the Nakajima tests were performed according to the standard (DIN EN ISO 12004-2). More details of the testing setup can be found in Ref. [15]. The final forming limit curves (FLCs) are presented in Figure 2 with two evaluation methods of the FLC at necking, the visual method and position dependent method.

Figure 2. The FLCs measured by Nakajima tests at the instant of necking based on visual and position dependent method and the FLCs predicted by the MMFC for AISI 439.
Both the conventional associated Hill48 and the evolving nHill48 models are coupled with MMFC to perform the FLC prediction and the results are shown in Figure 2. It is noted that the parameters for the associated Hill48 are calibrated based on the stress method and only isotropic hardening is assumed. Clearly, the prediction by the associated Hill48 model overestimates the forming limit strains, especially in the biaxial tension region. On the other hand, the evolving nHill48 significantly improves the results in this region and, interestingly, it also shows a platform near the plane strain region that gives a very similar trend of the experimental results.

5. Conclusions
The aim of the study is to perform a formability prediction study by a non-associated Hill48 model accounting for the anisotropic hardening in conjunction with the modified maximum force criterion (MMFC). The following conclusions can be drawn:

- For the investigated bcc steel, significant anisotropic hardening is observed from the tensile tests along rolling, diagonal and transverse directions.
- With the consideration of the anisotropic hardening feature, the forming limit prediction results are significantly improved, especially in the biaxial tension loading region, compared to the conventional associated Hill48 model with isotropic hardening.

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