A Brane World Solution to the Cosmological Constant Problem

S.-H. Henry Tye* and Ira Wasserman†

Laboratory for Nuclear Studies and Center for Radiophysics and Space Research
Cornell University
Ithaca, NY 14853
(March 27, 2022)

Abstract

We consider a model with two parallel (positive tension) 3-branes separated by a distance $L$ in 5-dimensional spacetime. If the interbrane space is anti-deSitter, or is not precisely anti-deSitter but contains no event horizons, the effective 4-dimensional cosmological constant seen by observers on one of the branes (chosen to be the visible brane) becomes exponentially small as $L$ grows large.

11.10.Kk, 04.50.+h, 98.80.-k, 98.80.Hw
Recent observational data \cite{1} indicate that there is a positive cosmological constant in the universe, which, compared to the Planck or the electroweak scale, is many orders of magnitude smaller than expected within the context of ordinary gravity and quantum field theory. This is the well-known cosmological constant problem \cite{2}. Here we propose a solution to this problem in which the cosmological constant becomes exponentially small compared to all the other scales in the model.

The scenario is a variant of the Randall-Sundrum model \cite{3}. Let us consider two parallel 3-branes (or two stacks of 3-branes) in 5-D spacetime separated by a distance \( L \) in the fifth dimension which is not necessarily compactified. Let the brane on the left be the visible brane, with positive brane tension \( \sigma_0 \). The brane on the right has positive brane tension \( \sigma_L \neq \sigma_0 \), generally. Assume that the bulks outside the branes are anti-deSitter(AdS) spaces, with cosmological constants \( -\Lambda_l \) and \( -\Lambda_r \), respectively. There may be numerous solutions of the 5-D Einstein equations for the behavior of the bulk between the branes, but let us focus attention on the simplest, an AdS space with bulk cosmological constant \( -\Lambda_m \). (We discuss other possibilities briefly below, and in more detail elsewhere.) We shall see that \( \Lambda_m \) may be expressed as a function of \( L, \Lambda_l, \Lambda_r, \sigma_0, \sigma_L \) and the 5-D gravitational coupling constant \( \kappa^2 = 8\pi G \). Thus, we regard \( \Lambda_m \) as a derived parameter; more generally, in a dynamical 5-D spacetime, \( \Lambda_m \) may evolve with time as branes move together or apart. For this simplest model for the interbrane spacetime, we can determine the effective 4-D cosmological constant \( \Lambda_{\text{eff}} \) in terms of \( \Lambda_l, \Lambda_r, \sigma_0, \sigma_L, L \) and \( \kappa^2 \). We show that for large \( L \),

\[
\Lambda_{\text{eff}} \approx F(\kappa, \Lambda_l, \Lambda_r, \sigma_0, \sigma_L) e^{-\alpha_0 L} \tag{1}
\]

where \( F \) is independent of \( L \) and the positivity of the 4-D Newton constant \( G_N \) requires \( \alpha_0 = \kappa^2 \sigma_0/3 - \sqrt{\kappa^2 \Lambda_l/6} > 0 \). For \( \alpha_0 L \gg 1 \), the effective 4-D cosmological constant becomes exponentially small in this simple model without any fine tuning of the parameters. In a \( S^1/Z_2 \) orbifold version of the model, the only bulk is between the branes, and

\[
\Lambda_{\text{eff}} \approx 2\sigma_0 \frac{(\sigma_L + \sigma_0)}{(\sigma_L - \sigma_0)} e^{-\kappa^2 \sigma_0 L/3}. \tag{2}
\]
These results remain true even if $\sigma_0$, which is simply the vacuum energy density of the 4-D quantum field theory (which includes the standard model of strong and electroweak interactions) on the visible brane, changes due to phase transitions or other dynamics on the brane. We shall also comment briefly on solutions where the bulk between branes is not a pure AdS space. Demanding the absence of an event horizon between the branes, we again find that $\Lambda_{eff}$ to be vanishing small for large $L$. This suggests that an exponentially small effective cosmological constant may be a robust property of two-brane models with large interbrane separation. One is tempted to speculate that the pure AdS solution is a stable fixed point as the branes move apart and the 3-brane universe expands.

When $\alpha_0L$ is not large, $\Lambda_{eff}(L)$ is more complicated than eqs. (1) and (2). Furthermore, at small separations, we expect additional nongravitational brane-brane interactions, but for large separations, it is reasonable to assume that the brane dynamics is dominated by pure gravity as described here. In a more realistic situation, the matter density on the visible brane (and dark matter on the other brane) should be included, and the separation distance $L$ should be treated as a dynamical variable. For slow-moving branes, a Born-Oppenheimer-like approach is valid, and the spacetime evolves quasistatically from one nearly time-independent solution to another along the sequence of two-brane models. The result resembles the quintessence picture \[\text{[4]}\] and will be discussed elsewhere. Below, we derive eqs. (1) and (2).

Consider a pair of 3-branes at $y = 0$ and $y = L$ with different positive brane tensions $\sigma_0$ and $\sigma_L$. Let the bulk cosmological constant be $-\Lambda_l$ for $y < 0$, $-\Lambda_r$ for $y > L$. Choose the energy-momentum tensor for the bulk between the branes to have the diagonal form

$$T_{AB} = (-\lambda_0, \lambda, \lambda, \lambda, \psi),$$

where $\lambda_0(y)$, $\lambda(y)$ and $\psi(y)$ are functions of $y$. Choosing $\lambda_0 = \lambda$ allows us to use the metric ansatz

$$ds^2 = dy^2 + A(y)[-dt^2 + e^{2Ht}dx^idx^j].$$

The $G_{05}$ component of the Einstein’s equation $G_{AB} = \kappa^2 T_{AB} = 8\pi GT_{AB}$ is satisfied trivially, while the $G_{00}$ and the $G_{55}$ components give, respectively,
\[
\frac{A''}{A} = \frac{2H^2}{A} + \frac{2\kappa^2}{3} \left[ \Lambda_l \Theta(-y) + \lambda(y)\Theta(y)\Theta(L - y) + \Lambda_r \Theta(y - L) - \sigma_0 \delta(y) - \sigma_L \delta(y - L) \right]
\]
\[
\left( \frac{A'}{A} \right)^2 = \frac{4H^2}{A} + \frac{2\kappa^2}{3} \left[ \Lambda_l \Theta(-y) + \psi(y)\Theta(y)\Theta(L - y) + \Lambda_r \Theta(y - L) \right].
\]  

(4)

The \( G_{ij} \) component does not yield additional equations (alternatively, with the above metric, this component requires \( \lambda_0 = \lambda \)). In the bulk between the branes, we have (differentiate \( \psi(y) \) in eq. [4] and manipulate)

\[
\psi' + \frac{2A'}{A}(\psi - \lambda) = 0.
\]

(5)

It is clear that if \( \psi = \lambda \) then \( \lambda(y) = \psi(y) = \Lambda_m \), independent of \( y \).

Let us first consider this particularly simple case, in which the space between the branes \((0 < y < L)\) is AdS, with a cosmological constant \(-\Lambda_m\) that will be determined from the solution of the 5-D Einstein equations. Define \( k_l = \sqrt{\kappa^2 \Lambda_l/6} \), \( k_m = \sqrt{\kappa^2 \Lambda_m/6} \) and \( k_r = \sqrt{\kappa^2 \Lambda_r/6} \). Outside the branes, the solution for the bulks is

\[
A(y) = \frac{H^2 \sinh^2[k_l(y + y_l)]}{k_l^2} \quad (y < 0)
\]
\[
A(y) = \frac{H^2 \sinh^2[k_m(y + y_m)]}{k_m^2} \quad (0 < y < L)
\]
\[
A(y) = \frac{H^2 \sinh^2[k_r(y + y_r)]}{k_r^2} \quad (y > L),
\]  

(6)

where \( y_l, y_m \) and \( y_r \) are constants. These bulk solutions are similar to those in Ref [5]. Continuity of the metric at the branes implies that

\[
\frac{\sinh^2(k_l y_l)}{k_l^2} = \frac{\sinh^2(k_m y_m)}{k_m^2} \quad \frac{\sinh^2[k_m(L + y_m)]}{k_m^2} = \frac{\sinh^2[k_r(L + y_r)]}{k_r^2}.
\]

(7)

The jump conditions at the two branes are

\[
\frac{k_l \cosh(k_l y_l)}{\sinh(k_l y_l)} - \frac{k_m \cosh(k_m y_m)}{\sinh(k_m y_m)} = q_0
\]
\[
\frac{k_m \cosh[k_m(L + y_m)]}{\sinh[k_m(L + y_m)]} - \frac{k_r \cosh[k_r(L + y_r)]}{\sinh[k_r(L + y_r)]} = q_L.
\]

(8)
where \( q_0 \equiv \kappa^2 \sigma_0/3 \) and \( q_L \equiv \kappa^2 \sigma_L/3 \).

The expansion rate seen by observers on the brane at \( y = 0 \) is \( H(0) = H/\sqrt{A(0)} \), where

\[
\frac{H^2}{A(0)} = \frac{k_i^2}{\sinh^2(k_i y_l)} = \frac{k_m^2}{\sinh^2(k_m y_m)} = \frac{[k_m^2 - (k_l + q_0)^2][k_m^2 - (k_l - q_0)^2]}{4q_0^2},
\]

(9)

with \( k_m^2 - (k_l \pm q_0)^2 > 0 \) or \( < 0 \), in agreement with Ref \([6]\), which uses a slightly different approach. Similarly, the expansion rate seen by observers on the brane at \( y = L \) is \( H(L) = H/\sqrt{A(L)} \), where

\[
\frac{H^2}{A(L)} = \frac{k_r^2}{\sinh^2[k_r(L + y_r)]} = \frac{k_m^2}{\sinh^2[k_m(L + y_m)]} = \frac{[k_m^2 - (k_r + q_L)^2][k_m^2 - (k_r - q_L)^2]}{4q_L^2},
\]

(10)

with \( k_m^2 - (k_r \pm q_L)^2 > 0 \) or \( < 0 \). We can rescale \( t \) so that \( A(0) = 1 \), and the Hubble constants on the two branes are, respectively, \( H(0) = H \) and \( H(L) = H/\sqrt{A(L)} \). Note that although eqs. (9) and (10) appear to determine the expansion rates on the two branes completely in terms of local quantities (i.e., the local brane tensions, and bulk cosmological constants just outside each brane), the values of these quantities on/near the two branes are connected via \( k_m \) and \( y_m \).

The 4-D Newton’s constant \( G_N \) can be determined by introducing a small matter density \( \rho \) to the visible brane, that is, \( q_0 \rightarrow q_0 + \kappa^2 \rho/3 \). Requiring the Hubble constant \( H \) to have the standard form

\[
H^2 \approx \frac{8\pi G_N}{3} (\Lambda_{\text{eff}} + \rho + ...)
\]

(11)
yields \( 4\pi q_0 G_N = \kappa^2 \alpha_0 k_l \) \([3][4]\]. Positivity of \( G_N \) requires \( \alpha_0 = q_0 - k_l > 0 \); to be specific, let us consider

\[
0 \leq k_m \leq q_0 - k_l.
\]

(12)

Since the expansion rates \( H \) and \( H(L) \) given in eqs. (9) and (10) depend on \( k_m \), our goal is to express \( H \) and \( H(L) \) as functions of \( L \) and the parameters \( k_l, k_r, q_0 \) and \( q_L \). This requires an expression relating \( L \) and \( k_m \); from eqs. (7) and (8) we find
\[
\begin{align*}
k_m L &= \sinh^{-1} \left( \frac{k_m}{H(0)} \right) + \sinh^{-1} \left( \frac{k_m}{H(L)} \right) \\
&= \sinh^{-1} \left( \frac{2k_m q_0}{\sqrt{[k_m^2 - (k_l + q_0)^2][k_m^2 - (k_l - q_0)^2]}} \right) \\
&+ \sinh^{-1} \left( \frac{2k_m q_L}{\sqrt{[k_m^2 - (k_r + q_L)^2][k_m^2 - (k_r - q_L)^2]}} \right)
\end{align*}
\]

Here, eq.(13) is regarded as a relation that determines \( k_m \) in terms of \( L, q_0, q_L, k_l \) and \( k_r \).

In general, \( H(0) \neq H(L) \). Because of the condition (12), \( H \to 0 \) as \( k_m \to \alpha_0 = q_0 - k_l \) from below. This means that the expansion rate as seen by observers on the visible brane becomes exponentially small as \( L \) increases,

\[
H^2 \approx 4\alpha_0^2e^{-2(\alpha_0 L - C)}
\]  

where \( \alpha_0 > 0 \), and \( \sinh(C) = k_m/H(L) \). As \( k_m \to \alpha_0 \) from below, \( H(L) \) approaches a \( L \)-independent constant given by eq.(10) and so does \( C \). This implies that \( \Lambda_{\text{eff}} \) becomes exponentially small as \( L \) increases.

In the symmetric case, where \( k_l = k_r = k \) and \( q_0 = q_L = q \) (and let \( \alpha = q - k = \alpha_0 \)), we have \( H(L) = H(0) = H \), and \( y_m = -y_l = y_r = -L/2 \). The constant \( C \) in eq.(14) becomes \( C = \alpha L/2 \), so

\[
H^2 \approx 4\alpha_0^2e^{-\alpha L} + \frac{2\kappa^2 \alpha k}{3q \rho}.
\]

Thus, \( \Lambda_{\text{eff}} \) still decreases exponentially with \( L \), but slower than in the nonsymmetric case.

To this point, we have concentrated on spacetimes that are noncompact in \( y \), but similar results can be derived for the compactified case. First, we may choose to identify \( k_l = k_r = k_m \) and derive \( k_m \) and \( H \) in terms of \( L \). Next, we can compactify the \( y \) direction and further perform a \( Z_2 \) orbifold, with one brane sitting at each of the two fixed points \( (y = 0, L) \). This \( S^1/Z_2 \) orbifold model is particularly simple, since there is only one bulk space between the branes sitting at the two end points. This is an expanding (non-supersymmetric) version of the Horava-Witten model [10], with branes with tension \( \sigma_0 = 3q_0/\kappa^2 \) at \( y = 0 \) and \( \sigma_L = 3q_L/\kappa^2 \) at \( y = L \), separated by AdS space with bulk cosmological constant \( -6k^2/\kappa^2 \).

The solution is
\[ A(y) = \frac{H^2}{k^2} \sinh^2[k(y - sy_0)] \] (16)

where \( s = +1 \) for \( y > 0 \) and \( s = -1 \) for \( y < 0 \). Because of the symmetry of the model, we only need to consider the jump conditions at \( y = 0 \) and \( y = L \), which are \( 2k/q_0 = \tanh ky_0 \) and \( 2k/q_L = \tanh[k(L - y_0)] \), respectively. Combining the jump conditions implies

\[ \frac{q_0}{2k} = \frac{\tanh kL - q_L/2k}{1 - (q_L/2k) \tanh kL}; \] (17)

if \( q_L/2k = \pm 1 \), then \( q_0/2k = \mp 1 \) irrespective of \( kL \), but for \( q_L/2k \neq 1 \), \( q_0/2k \rightarrow 1 \) as \( kL \rightarrow \infty \). According to our viewpoint, eq.(17) determines \( k \) given \( q_0, q_L \) and \( L \).

The expansion rates on the branes are \( H(0) = H/\sqrt{A(0)} \) and \( H(L) = H/\sqrt{A(L)} \), where \( H^2(0) = k^2[(q_0/2k)^2 - 1] \) and \( H^2(L) = k^2[(q_L/2k)^2 - 1] \). For large values of \( q_0L \) we find that

\[ H^2(0) \approx \frac{q_0^2(q_L + q_0)}{(q_L - q_0)} e^{-q_0L} \] (18)

Thus, for \( q_0L \gg 1 \) and \( q_L > q_0 \), the cosmological constant on the \( y = 0 \) brane is exponentially small. Moreover, although eq.(18) may appear singular as \( q_L \rightarrow q_0 \), in fact \( H^2(0) \approx q_0^2 e^{-q_0L/2} \) in that case. If \( q_L = -q_0 \), as considered in Ref [3], then \( y_0 = \pm \infty \), so \( H(0) = H(L) = 0 \) and therefore \( |q_0| = 2k \) for any finite non-zero \( L \). Our model is qualitatively different, since it involves two positive tension branes, and interprets the bulk cosmological constant as a parameter derived from the brane tensions and separation.

Notice that \( A(y) \) given by eq.(18) may vanish at certain values of \( y \). We view points where \( A(y) = 0 \) as event horizons since it takes a test particle leaving either brane an infinite time to reach them according to observers on the brane from which it is launched. (There are no spacetime singularities at these points, since all derivatives of \( A(y) \) are finite.) We do not worry about event horizons outside the branes \((y < 0\) and \(y > L\)), which recede to large distances from the branes as \( L \) increases, but an event horizon between the branes might be worrisome. To avoid an event horizon between branes, we need \( y_m < L + y_m < 0 \), which can be true if \( k_r^2 - q_r^2 - \alpha_0^2 > 0 \). This condition can be satisfied when the two branes and the outside bulks are not identical, but in the symmetric case, avoiding a horizon between
the branes requires \( \alpha < 0 \), which violates the positivity of \( G_N \). So, in the symmetric case, we will have an event horizon right between the branes even when they are close together \((L \to 0)\). (Non-gravitational interbrane interaction could alter this conclusion for very small \( L \), but not in general.) In the orbifold case, although we can allow arbitrary values of \( q_0 \) and \( q_L \), horizons are only avoided in the model if \( L < y_0 \), which implies \( q_L < 0 \). Fortunately, horizons can be avoided altogether if the space between the branes is not pure AdS.

There is nothing sacred about AdS space between the branes (see [11]), and we might expect non-AdS behavior to be the rule rather than the exception, particularly at different stages in the evolution of a slowly-changing 5-D spacetime. Generally speaking, we can find infinitely many families of solutions with different \( \lambda(y) \) and \( \psi(y) \) in the regions between branes. As a first stab at a more complicated two-brane model than we have presented here, we have studied the case where \( \lambda = \Lambda_m \) is a constant, but \( \psi(y) = \Lambda_m - K/A^2(y) \), which satisfies eq. (3) for \( y \)-independent \( K \). In this case, we do not get a relation between \( k_m \) and \( L \) (because \( \psi(y) \neq \Lambda_m \)). The question of whether \( H^2 \) is small is related to horizon formation between the branes. If horizons are absent, then \( H^2 \) must be exponentially small for large \( L \). If a horizon may only appear between branes at large \( L \), then \( H^2 \) must also be exponentially small. In a dynamical solution with separating branes, we might imagine that \( H^2 \to 0 \) as \( t \to \infty \), and no horizon forms.

To be specific, let us consider the orbifold model in some detail. Here, we find (for \( K > 0 \) and \( A(0) \equiv 1 \))

\[
A(y) = -\frac{H^2}{2k^2} + \left(1 + \frac{H^2}{2k^2}\right) \frac{\cosh[2k(y - sy_0)]}{\cosh 2ky_0}
\]

with \( s \) chosen as for AdS. In this case the jump conditions at \( y = 0 \) and \( y = L \) are

\[
\frac{q_0}{2k} = \left(1 + \frac{H^2}{2k^2}\right) \tanh 2ky_0
\]

\[
\frac{q_L}{2k} = \left(1 + \frac{H^2}{2k^2}\right) \frac{\sinh[2k(L - y_0)]}{A(L) \cosh 2ky_0};
\]

using the first of these relations in the second implies
\[
\frac{H^2}{k} = \frac{[\left(q_0 + q_L \right) \cosh 2kL - (1 + q_0q_L/2k) \sinh 2kL]}{\sinh 2kL - (q_L/2k)(1 - \cosh 2kL)} \\
\approx \left(q_0 - 2k\right) \left(1 + \frac{2qL e^{-2kL}}{q_L - 2k}\right).
\]

Horizons are guaranteed to be absent in this model if

\[
\frac{H^2}{2k^2} < \frac{1}{\cosh 2k y_0 - 1} \approx 2e^{-2k y_0}.
\]  \hspace{1cm} (22)

Using eq.(21) in the jump condition at \(y = 0\) implies \(e^{-2k y_0} \approx e^{-k L} \sqrt{q_L(q_0 - 2k)/q_0(q_L - 2k)}\), and so horizons are absent if

\[
\frac{q_0}{2k} - 1 < \frac{4qL e^{-2kL}}{q_L - 2k}
\]  \hspace{1cm} (23)

in the general case, \(q_L \neq q_0\). If \(q_0 = q_L\), then \(e^{-2k y_0} \approx e^{-k L}\) and horizons are avoided if \(q_0/2k - 1 < 2e^{-k L}\). In either case, the expansion rate is exponentially small if there are no horizons in the solution.

The dynamical treatment of such a separation modulus may follow that discussed in Ref [12] and is under investigation. For large separations, it is reasonable to assume that the brane-brane interaction is dominated by pure gravity as described above. At small separations, we expect brane-brane interactions beyond pure gravity. In Ref [8] where an AdS-Schwarzschild solution is considered, the deviation from the pure AdS space red-shifts away rapidly as the universe expands. This leads one to conjecture that the pure AdS solution between the branes is a stable fixed point. If so, as branes move apart and the universe expands, the bulk solution remains horizon-free, and approaches pure AdS space asymptotically. Clearly this issue needs careful investigation.

One may even envision a multi-brane scenario, where the separation distances between branes play the roles of various scalar fields, as suggested by string/M theory. In particular, the separation of two nearby branes may play the role of an inflaton [13], while the separation distance between two far-apart branes may play the role of a scalar field similar to quintessence [4]. Immediately after inflation, the separation of two nearby branes is stabilized by short-range brane-brane interactions [12,13], when the effective cosmological
constant is small compared to the radiation density. Then a third brane starts to move away while the 3-brane universe is expanding, yielding an exponentially small $\Lambda_{\text{eff}}$.

We thank Eanna Flanagan and Horace Stoica for useful discussions. This research is partially supported by NSF (S.-H.T.) and NASA (I.W.).
REFERENCES

[1] A. Balbi et al., astro-ph/0005124 (MAXIMA-1); A.E. Lange et al., astro-ph/000504 (BOOMERANG); A.G. Riess et al., Astron. J., 116, 1009 (1998); S. Perlmutter et al., Astrophys. J., 517, 565 (1999); See also N.A. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, Science 284, 1481 (1999), astro-ph/9906463.

[2] For some useful reviews, see e.g., S. Weinberg, Rev. Mod. Phys. 61, 1 (1989); S.M. Carroll, W.H. Press and E.L. Turner, Ann. Rev. Astron. Astrophys., 30, 499 (1992); S. Weinberg, astro-ph/0005265; E. Witten, hep-ph/0002297; S.M. Carroll, hep-th/0004073.

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), hep-ph/9905221; Phys. Rev. Lett. 83, 4690 (1999), hep-th/9906064.

[4] B. Ratra and P.J.E. Peebles, Phys. Rev. D37, 3406 (1988); C. Wetterich, Nucl. Phys. B302, 668 (1988); I. Zlatev, L. Wang, and P.J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999).

[5] N. Kaloper, Phys. Rev. D60, 123506 (1999), hep-th/9905210; T. Nihei, Phys. Lett. B465, 81 (1999), hep-ph/9905487.

[6] H. Stoica, S.-H.H. Tye and I. Wasserman, hep-ph/0004112.

[7] P. Binétruy, C. Deffayet and D. Langlois, hep-th/9905012; C. Csaki, M. Graesser, C. Kolda and J. Terning, Phys. Lett. B462, 34 (1999), hep-ph/9906513; J.M. Cline, C. Grojean and G. Servant, Phys. Rev. Lett. 83, 4245 (1999), hep-ph/9906523; T. Shiromizu, K. Maeda and M. Sasaki, gr-qc/9910076; S. Mukohyama, T. Shiromizu and K. Maeda, hep-th/9912287.

[8] P. Kraus, JHEP 9912, 011 (1999), hep-th/9910419; E.E. Flanagan, S.-H.H. Tye and I. Wasserman, hep-ph/9910498; P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, hep-th/9910213.
More precisely, \(4\pi G_N q_0 = \kappa^2 \alpha_0 k_l[1 + 2\kappa^2 \Lambda_{\text{eff}} (2\alpha_0 + k_l)/3q_0k_l]\). Although \(L\)-dependent, the correction is small if \(G_N\Lambda_{\text{eff}}/k_l^2 \ll 1\).

[10] P. Horava and E. Witten, Nucl. Phys. B460 506 (1996); B475 94 (1996)

[11] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, [hep-th/9909134]; P. Kanti, I.I. Kogan, K.A. Olive and M. Pospelov, Phys. Lett. B468, 31 (1999), [hep-ph/9909481].

[12] W.D. Goldberger and M.B. Wise, Phys. Rev. Lett. 83, 4922 (1999), [hep-ph/9907447]; hep-ph/9911457; C. Csaki, M. Graesser, L. Randall and J. Terning, hep-ph/9911406.

[13] G. Dvali and S.-H.H. Tye, Phys. Lett. B450, 72 (1999), [hep-ph/9812483]. E.E. Flanagan, S.-H.H. Tye and I. Wasserman, hep-ph/9909373.