Nucleon axial radius and muonic hydrogen—a new analysis and review

Richard J Hill\textsuperscript{1,2,3}, Peter Kammel\textsuperscript{4}, William J Marciano\textsuperscript{5} and Alberto Sirlin\textsuperscript{6}

1 Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, United States of America
2 Fermilab, Batavia, IL 60510, United States of America
3 Perimeter Institute for Theoretical Physics, Waterloo, ON N2L 2Y5, Canada
4 Center for Experimental Nuclear Physics and Astrophysics and Department of Physics, University of Washington, Seattle, WA 98195, United States of America
5 Department of Physics, Brookhaven National Laboratory, Upton, NY 11973, United States of America
6 Department of Physics, New York University, New York, NY 10003, United States of America

E-mail: pkammel@uw.edu

Received 27 September 2017, revised 19 April 2018
Accepted for publication 1 May 2018
Published 30 July 2018

Abstract

Weak capture in muonic hydrogen ($\mu$H) as a probe of the chiral properties and nucleon structure predictions of quantum chromodynamics (QCD) is reviewed. A recent determination of the axial-vector charge radius squared, $r_A^2(z \text{ exp.}) = 0.46(22) \text{ fm}^2$, from a model independent $z$ expansion analysis of neutrino-nucleon scattering data is employed in conjunction with the MuCap measurement of the singlet muonic hydrogen capture rate, $\Lambda_{\text{MuCap}}^{\text{singlet}} = 715.6(7.4) \text{ s}^{-1}$, to update the induced pseudoscalar nucleon coupling $g_{\text{P}}^{\text{MuCap}} = 8.23(83)$ derived from experiment, and $g_{\text{P}}^{\text{theory}} = 8.25(25)$ predicted by chiral perturbation theory. Accounting for correlated errors this implies $g_{\text{P}}^{\text{theory}} / g_{\text{P}}^{\text{MuCap}} = 1.00(8)$, confirming theory at the 8% level. If instead, the predicted expression for $g_{\text{P}}^{\text{theory}}$ is employed as input, then the capture rate alone determines $r_A^2(\mu\text{H}) = 0.46(24) \text{ fm}^2$, or together with the independent $z$ expansion neutrino scattering result, a weighted average $r_A^2(\text{ave.}) = 0.46(16) \text{ fm}^2$. Sources of theoretical uncertainty are critically examined and potential experimental improvements are described that can reduce the capture rate error by about a factor of 3. Muonic hydrogen can thus provide a precise and independent $r_A^2$ value which may be compared with other determinations, such as ongoing lattice gauge theory calculations. The importance of an improved $r_A^2$ determination for phenomenology is illustrated by considering the impact on critical neutrino-nucleus cross sections at neutrino oscillation experiments.

Keywords: nucleon form factors, weak axial current, muon capture, neutrino scattering, neutrino oscillation, radiative corrections

(Some figures may appear in colour only in the online journal)
1. Introduction

Muonic hydrogen, the electromagnetic bound state of a muon and proton, is a theoretically pristine atomic system. As far as we know, it is governed by the same interactions as ordinary hydrogen, but with the electron of mass 0.511 MeV replaced by the heavier muon of mass 106 MeV, an example of electron-muon universality. That mass enhancement (\( \frac{106}{0.511} \approx 207 \)) manifests itself in much larger atomic energy spacings and a proton-muon universality. That mass enhancement (\( \frac{106}{0.511} \approx 207 \)) had, for some time, appeared problematic \([8]\). All \( g_\rho \) extractions from ordinary muon capture in hydrogen suffered from limited precision, while the more sensitive extraction from radiative muon capture \([9]\) disagreed with ordinary muon capture and the solid prediction of chiral perturbation theory (\( \chiPT \)) \([10-14]\). An important underlying contribution to this problem was the chemical activity of muonic hydrogen, which like its electronic sibling, can form molecular ions, \( (pp\mu)^+ \). The highly spin dependent weak interaction leads to very different capture rates from various muonic atomic and molecular states. Thus, atomic physics processes like ortho-para transitions in the muonic molecule, which flip the proton spins, significantly change the observed weak capture rates and often clouded the interpretation of experimental results in the 55-year history of this field. Unfortunately, the uncertainty induced by molecular transitions was particularly severe for the most precise measurements which were performed with high density liquid hydrogen targets, where, because of rapid \( pp\mu \) formation, essentially capture from the molecule, not the \( \mu p \) atom, is observed. This problem was resolved by the MuCap Collaboration at the Paul Scherrer Institute (PSI) which introduced an active, \textit{in situ}, target, where ultra-pure hydrogen gas served both as the target as well as the muon detector, thus enabling a measurement of the muonic hydrogen capture rate at low density, where \( pp\mu \) formation is suppressed. MuCap unambiguously determined the spin singlet muonic hydrogen capture rate \( \Lambda_{\text{singlet}} = 715.6(7.4) \text{ s}^{-1} \) \([15, 16]\) to 1% accuracy which, when corrected for an enhancement from radiative corrections \([17]\), and using prevailing form factor values at the time implied \( g_\rho \text{MuCap} = 8.06(55) \), in excellent agreement with \( g_\rho \text{theory} = 8.26(23) \), the predicted value.

We note, however, that the determination of \( g_\rho \) from both experiment and theory required the input of the axial charge radius squared, traditionally taken from dipole form factor fits to neutrino-nucleon quasielastic charged current scattering \( (\nu_\mu n \rightarrow \nu_\mu p) \) and pion electroproduction \( (eN \rightarrow eN'\pi) \) data, which at the time implied the very precise \( r_A^2(\text{dipole}) = 0.454(13) \text{ fm}^2 \) \([18]\). Recently, that small (~3%) uncertainty in \( r_A^2 \) has been called into question, since it derives from the highly model dependent dipole form factor assumption\(^7\). The axial radius, which is central to this paper, governs mass kinematically allows the weak muon capture process depicted in figure 1,

\[
\mu^- + p \rightarrow \nu_\mu + n ,
\]

to proceed, while ordinary hydrogen is (fortunately for our existence) stable.

Weak muon capture in nuclei has provided a historically important probe of weak interactions and a window for studying nuclear structure. In particular, weak capture in muonic hydrogen is a sensitive probe of the induced pseudoscalar component of the axial current \( p \rightarrow n \) matrix element which is well predicted from the chiral properties of QCD. However, early experimental determinations of that pseudoscalar coupling, \( g_\rho \), had, for some time, appeared problematic \([8]\). All \( g_\rho \) extractions from ordinary muon capture in hydrogen suffered from limited precision, while the more sensitive extraction from radiative muon capture \([9]\) disagreed with ordinary muon capture and the solid prediction of chiral perturbation theory (\( \chiPT \)) \([10-14]\). An important underlying contribution to this problem was the chemical activity of muonic hydrogen, which like its electronic sibling, can form molecular ions, \( (pp\mu)^+ \). The highly spin dependent weak interaction leads to very different capture rates from various muonic atomic and molecular states. Thus, atomic physics processes like ortho-para transitions in the muonic molecule, which flip the proton spins, significantly change the observed weak capture rates and often clouded the interpretation of experimental results in the 55-year history of this field. Unfortunately, the uncertainty induced by molecular transitions was particularly severe for the most precise measurements which were performed with high density liquid hydrogen targets, where, because of rapid \( pp\mu \) formation, essentially capture from the molecule, not the \( \mu p \) atom, is observed. This problem was resolved by the MuCap Collaboration at the Paul Scherrer Institute (PSI) which introduced an active, \textit{in situ}, target, where ultra-pure hydrogen gas served both as the target as well as the muon detector, thus enabling a measurement of the muonic hydrogen capture rate at low density, where \( pp\mu \) formation is suppressed. MuCap unambiguously determined the spin singlet muonic hydrogen capture rate \( \Lambda_{\text{singlet}} = 715.6(7.4) \text{ s}^{-1} \) \([15, 16]\) to 1% accuracy which, when corrected for an enhancement from radiative corrections \([17]\), and using prevailing form factor values at the time implied \( g_\rho \text{MuCap} = 8.06(55) \), in excellent agreement with \( g_\rho \text{theory} = 8.26(23) \), the predicted value.

We note, however, that the determination of \( g_\rho \) from both experiment and theory required the input of the axial charge radius squared, traditionally taken from dipole form factor fits to neutrino-nucleon quasielastic charged current scattering \( (\nu_\mu n \rightarrow \nu_\mu p) \) and pion electroproduction \( (eN \rightarrow eN'\pi) \) data, which at the time implied the very precise \( r_A^2(\text{dipole}) = 0.454(13) \text{ fm}^2 \) \([18]\). Recently, that small (~3%) uncertainty in \( r_A^2 \) has been called into question, since it derives from the highly model dependent dipole form factor assumption\(^7\). The axial radius, which is central to this paper, governs mass kinematically allows the weak muon capture process depicted in figure 1,

\[
\mu^- + p \rightarrow \nu_\mu + n ,
\]

1. Introduction

Muonic hydrogen, the electromagnetic bound state of a muon and proton, is a theoretically pristine atomic system. As far as we know, it is governed by the same interactions as ordinary hydrogen, but with the electron of mass 0.511 MeV replaced by the heavier muon of mass 106 MeV, an example of electron-muon universality. That mass enhancement (~207) manifests itself in much larger atomic energy spacings and a smaller Bohr radius of 2.56 × 10^{-3} Å. This places the muonic hydrogen size about halfway (logarithmically) between the atomic angstrom and the nuclear fermi (1 fm = 10^{-5} Å scale.

Those differences make muonic hydrogen very sensitive to otherwise tiny effects such as those due to proton size and nucleon structure parameters governing weak interaction phenomenology. Indeed, muonic hydrogen Lamb shift spectroscopy \([1, 2]\) has provided a spectacularly improved measurement of the proton charge radius that differs by about 7 standard deviations from the previously accepted value inferred from ordinary hydrogen and electron–proton scattering \([3]\). (The final verdict on this so called proton radius puzzle \([4-6]\) is still out. For recent hydrogen spectroscopy measurements, see \([7]\)). Similarly, the larger muon
the momentum dependence of the axial-vector form factor, by means of the expansion at small $q^2$,

$$F_A(q^2) = F_A(0) \left( 1 + \frac{1}{6}r_A^2 q^2 + \ldots \right).$$  
(2)

In the one-parameter dipole model, the terms denoted by the ellipsis in equation (2) are completely specified in terms of $r_A^2$. However, the true functional form of $F_A(q^2)$ is unknown, and the dipole constraint represents an uncontrolled systematic error. We may instead employ the $z$ expansion formalism, a convenient method for enforcing the known complex-analytic structure of the form factor inherited from QCD, while avoiding poorly controlled model assumptions. This method replaces the dipole $F_A(q^2)$ with $F_A(z(q^2))$, which in terms of the conformal mapping variable $z(q^2)$, has a convergent Taylor expansion for all spacelike $q^2$. The size of the expansion parameter, and the truncation order of the expansion necessary to describe data of a given precision in a specified kinematic range, are determined a priori. This representation helps ensure that observables extracted from data are not influenced by implicit form factor shape assumptions. Using the $z$ expansion [19] to fit the neutrino data alone leads to [20] $r_A^2(\text{exp., } \nu) = 0.46(22)\text{ fm}^2$ with a larger ($\sim 50\%$), more conservative but better justified error.

As we will discuss below, traditional analyses of pion electroproduction data have also used a dipole assumption to extract $r_A^2$ from $F_A(q^2)$, and in addition required the a priori step of phenomenological modeling to extract $F_A(q^2)$ from data. Since these model uncertainties have not been quantified, we refrain from including pion electroproduction determinations of $r_A^2$ in our analysis. Similarly, we do not include extractions from neutrino-nucleus scattering on nuclei larger than the deuteron, in order to avoid poorly quantified nuclear model uncertainties. In this context, we note that dipole fits to recent $\nu$-C scattering data suggest a smaller $r_A^2 \approx 0.26\text{ fm}^2$ [21], compared to historical dipole values $r_A^2 \sim 0.45\text{ fm}^2$ [18]. This discrepancy may be due to form factor shape biases [19] (i.e. the dipole assumption), mismodeling of nuclear effects [22–30], or something else. Independent determination of $r_A^2$ is a necessary ingredient for resolving this discrepancy. Finally, we do not include recent interesting lattice QCD results [31–35], some of which suggest considerably smaller $r_A^2$ values.

As we shall discuss below in section 5, future improvements on these lattice QCD results could provide an independent $r_A^2$ value with controlled systematics, that would open new opportunities for interpreting muon capture. To illustrate the broad range of possible $r_A^2$ values, we provide in table 1 some representative values considered in the recent literature.

Table 1. Illustrative values obtained for $r_A^2$ from neutrino-deuteron quasi elastic scattering ($\nu-d$), pion electroproduction ($e^+N \rightarrow e^+n\pi^+$), neutrino-carbon quasielastic scattering ($\nu-C$), muon capture (MuCap) and lattice QCD. Values labeled ‘dipole’ enforce the dipole shape ansatz. The value labeled ‘$z$ exp.’ uses the model independent $z$ expansion.

| Description | $r_A^2$ (fm$^2$) | Source/Reference |
|-------------|------------------|------------------|
| $\nu d$ (dipole) | 0.453(23) | [18] |
| $e^+N \rightarrow e^+n\pi^+$ (dipole) | 0.454(14) | [18] |
| Average | 0.454(13) | |
| $\nu C$ (dipole) | 0.26(7) | [21] |
| $\nu d$ (z exp.) | 0.46(22) | [20] |
| MuCap | 0.46(24) | This work |
| Average | 0.46(16) | |
| Lattice QCD | 0.213(6)(13)(3)(0) | [31] |
| | 0.266(17)(7) | [32] |
| | 0.360(36)$^{+80}_{-38}$ | [33] |
| | 0.24(6) | [34] |
The remainder of this paper is organized as follows: in section 2 we give an overview and update regarding the theory of $\mu$-$p$ capture in muonic hydrogen. After describing the lowest order formalism, we discuss the magnitude and uncertainty of radiative corrections (RC) to muon capture. Normalizing relative to superallowed and neutron beta decays, we argue that the uncertainty in the singlet muon capture rate from radiative corrections is much smaller than the conservative estimate of $\pm 0.4\%$ originally given in [17]. Based on further considerations, we now estimate it to be $\pm 0.1\%$. Uncertainties in the input parameters are described, with particular emphasis on the numerical analysis of the axial charge radius squared and its potential extraction from the singlet $1S$ capture rate in $\mu$H. Then, in section 3, we describe the experimental situation. After reviewing the MuCap result, we discuss possible improvements for a next generation experiment that would aim for a further factor of $\sim 3$ error reduction. In section 4, we discuss what can be learned from the present MuCap result and an improved experiment. We update the determination of $\bar{g}_P$ from the MuCap measurement using the more conservative $z$ expansion value of $r_2^A$ obtained from neutrino-nucleon scattering. Then, as a change in strategy, using the theoretical expression for $\bar{g}_P$ obtained from $\chi$PT as input, $r_2^A$ is extracted from the MuCap capture rate and averaged with the $z$ expansion value. Other utilization of MuCap results are also discussed. In section 5, we illustrate the impact of an improved $r_2^A$ determination on quasielastic neutrino scattering cross sections and discuss the status of, and prospects for, improving alternative $r_2^A$ determinations. Section 6 concludes with a summary of our results and an outlook for the future.

2. Muon capture theory update

The weak capture process, equation (1), from a muonic hydrogen bound state is a multi-scale field theory calculational problem, involving electroweak, hadronic and atomic mass scales. In this section, we review the essential ingredients of this problem before discussing the status of phenomenological inputs and the numerical evaluation of the capture rate $\Lambda$. The calculation can be viewed as an expansion in small parameters, $\alpha \sim m_\mu^2/m_N^2 \sim \epsilon^2$, and will result in a structure

$$\Lambda \sim F_1 F_A + \epsilon (F_2 g_P + \epsilon^2 r_1^A r_2^A) + \mathcal{O}(\epsilon^3). \quad (3)$$

where in this formula $F_i$ denotes a form factor at $q^2 = 0$, $r_i^A$ is the corresponding radius defined as in equation (2) and $g_P$ the pseudoscalar coupling at $q^2_0$. Thus to achieve permille accuracy on the capture rate (where $10^{-3} \sim \epsilon^2$), we require all $\mathcal{O}(\epsilon^2)$ corrections with $\pm 10\%$ precision, and $\mathcal{O}(\epsilon^3)$ corrections with $\pm 100\%$ precision.

2.1. Preliminaries

For processes at low energy, $E \ll m_W$, where $m_W \approx 80$ GeV is the weak charged vector boson mass, the influence of heavy particles and other physics at the weak scale is rigorously encoded in the parameters of an effective Lagrangian containing four-fermion operators. For muon capture the relevant effective Lagrangian is

$$\mathcal{L} = \frac{G_F V_{td}}{\sqrt{2}} \bar{\mu} \gamma^\mu (1 - \gamma^5) \mu \bar{d} r_\mu (1 - \gamma^5) u + \text{H.c.} + \ldots, \quad (4)$$

where $G_F$ and $V_{td}$ are the Fermi constant and the CKM up-down quark mixing parameter respectively (see table 2), and the ellipsis denotes effects of radiative corrections. Atomic physics of the muonic hydrogen system is described by the effective Hamiltonian, valid for momenta satisfying $|p| \ll m_\mu$:

$$H = \frac{p^2}{2m_\mu} - \frac{\alpha}{r} + \delta V_{\text{np}} - \frac{G_F^2 |V_{td}|^2}{2} \left[ c_0 + c_1 (s_\mu + s_p)^2 \right] \delta^3(r), \quad (5)$$

where $m_\mu = m_\mu m_p/(m_\mu + m_p)$ is the reduced mass, $\delta V_{\text{np}}$ accounts for electron vacuum polarization as discussed below, and $s_\mu, s_p$ are muon and proton spins. The annihilation process is described by an anti-Hermitian component of $H$ [36]. Since the weak annihilation is a short-distance process compared to...
atomic length scales, this anti-Hermitian component can be expanded as a series of local operators. At the current level of precision terms beyond the leading one, \( \delta^2(\rho) \), are irrelevant [36]. Relativistic corrections to the Coulomb interaction in equation (5) are similarly irrelevant [37]. In both cases, neglected operators contribute at relative order \( v^2/c^2 \sim \alpha^2 \), where \( v \) is the nonrelativistic bound state velocity. Electron vacuum polarization enters formally at order \( \alpha^2 \), but is enhanced by a factor \( m_\mu/m_e \) making it effectively a first order correction [38, 39].

Having determined the structure of the effective Hamiltonian (5), the coefficients \( c_i \) are determined by a matching condition with the quark level theory (4). The annihilation rate in the 1S state is then computed from \( H \) to be

\[
\Lambda = G_\text{F}^2 |V_{ud}|^2 \times |c_0 + c_1 F(F + 1)| \times |\psi_{1S}(0)|^2 \ldots (6)
\]

where \( |\psi_{1S}(0)|^2 = m_\pi^4/\pi \) is the ground state wavefunction at the origin squared and \( F = 0 \) for singlet, \( F = 1 \) for triplet. Equation (6), with \( c_i \) expressed in terms of hadronic form factors (see equation (8) below), exhibits the factorization of the process into contributions arising from weak, hadronic and atomic scales.

2.2. Tree level calculation

Hadronic physics in the nucleon matrix elements of the vector and axial-vector quark currents of equation (4) is parameterized as:

\[
\langle n | (V^\mu - A^\mu) | p \rangle = \bar{u}_n \left[ F_1(q^2) \frac{q^\mu}{2m_N} + i F_2(q^2) \sigma^{\mu\nu} q_\nu + i F_T(q^2) \sigma^{\mu\nu} q_\nu \right] u_p + \ldots
\]

\[
= F_A(q^2) \gamma^\mu \gamma^5 - \frac{F_P(q^2)}{m_N} q^\mu \gamma^5 + \frac{F_S(q^2)}{m_N} q^\mu - \frac{i F_T(q^2)}{2m_N} \sigma^{\mu\nu} q_\nu \gamma^5 (7)
\]

where \( V^\mu - A^\mu = \partial^\mu \gamma^\nu u - \partial^\nu \gamma^\mu u \), and the ellipsis again denotes effects of radiative corrections. For definiteness we employ the average nucleon mass \( m_N \equiv (m_u + m_d)/2 \). The form factors \( F_S \) and \( F_T \) are so-called second class amplitudes that violate G parity and are suppressed by isospin violating quark masses or electromagnetic couplings [41–44]. They would appear in the capture rate, equation (8) below, accompanied by an additional factor \( m_\mu/m_N \) relative to \( F_1 \) and \( F_A \). Similar to isospin violating effects in \( F_2(0) \), discussed below in section 2.4, power counting predicts negligible impact of \( F_S \) and \( F_T \) at the permille level; we thus ignore them in the following discussion.

The \( c_i \) in equation (6) are determined by matching the quark level theory (4) to the nucleon level theory (5), using the hadronic matrix elements (7). This matching is accomplished by enforcing, e.g. equality of the annihilation rate for \( \mu^+ \to \nu_\mu n \) computed in both theories for the limit of free particles, with

---

\[ Q^2 \sim \text{weak, hadronic and atomic scales.} \]

---

\[ \text{Figure 2. Example of an } \mathcal{O}(a) \gamma W \text{ exchange box diagram radiative correction to muon capture.} \]

The proton and muon at rest. For the coefficients corresponding to singlet and triplet decay rates, this yields [17, 45]

\[
c_0 = \frac{G_F^2}{2\pi M} (M - m_\mu)^2 \left[ \frac{2M - m_\mu}{M - m_e} F_1(q_0^2) + \frac{2M + m_\mu}{M - m_e} F_2(q_0^2) \right] + 2F_T(q_0^2) \nonumber \]

\[
+ 2 \frac{m_\mu}{m_N} F_1(q_0^2) \left( F_A(q_0^2) - F_S(q_0^2) \right) - \frac{2m_\mu}{m_N} F_2(q_0^2) \right] \nonumber \]

\[
\frac{2M + m_\mu}{M - m_e} F_2(q_0^2) \right] \nonumber \]

\[
(8)
\]

where the initial state mass is \( M \equiv m_\mu + m_p \), the neutrino energy is \( E_\nu \equiv (M^2 - m_\mu^2)/2M = 99.1482 \text{ MeV} \), and the invariant momentum transfer is

\[
q_0^2 \equiv m_\mu^2 - 2m_\mu E_\nu = -0.8768 m_\mu^2.
\]

Since the matching is performed with free particle states, the quantities \( M, E_\nu \) and \( q_0^2 \) are defined independent of the atomic binding energy, as necessary for determination of the state-independent coefficients \( c_i \) of the effective Hamiltonian (5). \(^{10}\)

The amplitudes (8) can also be expressed as an expansion in \( \chi \text{PT} \) [13, 46–48]. However, the general formulas in equation (8) allow us to more directly implement and interpret experimental constraints on the form factors and do not carry the intrinsic truncation error of NNLO \( \chi \text{PT} \) derivations (estimated in [48] as \( \pm 1\% \)). For example, we may take the vector form factors \( F_1, F_2 \) directly from experimental data, rather than attempting to compute them as part of an expansion in \( \chi \text{PT} \). We investigate below the restricted application of \( \chi \text{PT} \) to express \( F_2(q_0^2) \) in terms of \( r_\rho^2 \) and other experimentally measured quantities.

2.3. Radiative corrections

The electroweak radiative corrections to muon capture in muonic hydrogen, depicted in figure 2, were first calculated in [17]. Here, we briefly describe the origin of such quantum loop effects and take this opportunity to update and reduce

\[^{10}\text{In particular, a binding energy is not included in the initial-state mass } M, \text{ but would anyways correspond to a relative order } a^2 \text{ correction that is beyond the current level of precision.} \]
their estimated uncertainty. The computational strategy relies on the well known electroweak corrections to (i) the muon lifetime [49, 50], (ii) super-allowed $0^+ \rightarrow 0^+$ $\beta$ decays [49, 51, 52], and (iii) the neutron lifetime [53–55].

Radiative corrections to weak decay processes in the Standard Model involve ultraviolet divergences that can be renormalized, yielding finite phenomenological parameters such as the Fermi constant $G_F$ obtained from the measured muon lifetime [50] and the CKM matrix element $|V_{ud}|$ obtained from super-allowed $\beta$ decays (see Table 2). In terms of those parameters, the radiative corrections to the neutron lifetime and the muon capture rate are rendered finite and calculable. We note that the matrix element of the vector current is absolutely normalized at $\tau^\mu = 0$, corresponding to a conserved vector current (CVC): $F_1(0) = 1$, up to second order corrections in small isospin violating parameters [56–58]. On the other hand, the normalization of the remaining form factors appearing in equation (8) requires a conventional definition in the presence of radiative corrections. This definition is specified at $q^2 = 0$ by a factorization requirement that expresses the total process as a tree level expression times an overall radiative correction. For example, the neutron decay rate in this scheme involves the factor $(1 + 3g_A^2)(1 + RC)$, where $(1 + 3g_A^2)$ is the tree level expression with $F_A(0) = g_A$, and RC denotes the radiative corrections. With that definition of $g_A$, the factorized radiative corrections are taken to be the same for vector and axial-vector contributions but actually computed for the vector part which is cleaner theoretically because of CVC. Radiative corrections that are different for the axial current amplitude are then, by definition, absorbed into $g_A$. That well defined $g_A$ can be precisely obtained from the neutron lifetime, $\tau_n$, in conjunction with $V_{ud}$ determined from superallowed beta decays via the relationship [52–55]

$$\left(1 + 3g_A^2\right) |V_{ud}|^2 \tau_n = 4908.6(1.9) \text{ s},$$

where the uncertainty comes primarily from radiative corrections to the neutron lifetime. Alternatively, that same $g_A$ can be directly obtained from neutron decay final state asymmetries after accounting for small QED corrections [59] and residual Coulomb, recoil, and weak magnetism effects [60]. We employ the lifetime method here, because it is currently more precise.

In the case of muon capture, we have four form factors all evaluated at $q_0^2$: vector ($F_1$), induced weak magnetism ($F_2$), axial-vector ($F_A$) and induced pseudoscalar ($F_P$). We define these form factors to all have the same electroweak radiative corrections and explicitly compute those corrections for $F_1$.

Short-distance corrections (which dominate) correspond to a renormalization of the relevant quark-level four-fermion operator, and are automatically the same for all form factors. Long distance corrections are smaller but significant. They include contributions from the $g$-function computed for neutron decay [61] which contains lepton energy dependent corrections not affected by strong interactions that are the same for the vector and axial-vector form factors. The $g$-function is of the same form for both neutron decay and muon capture but differs significantly in magnitude for the two cases because of the different charged lepton masses and kinematics involved in the two processes. In addition to the $g$-function, there are strong interaction dependent vector and axial-vector radiative corrections that can be estimated (for the vector amplitude) and factored into the RC. Differences in the axial amplitude can then be absorbed into a redefinition of $g_A$. Invoking G-parity and charge-symmetry of the strong interactions, they can be shown to be the same in neutron decay and muon capture, up to small $\mathcal{O}(\alpha m_e/m_p)$ corrections. Hence, the $g_A$ definition is essentially the same for both processes. That feature is important for reducing the radiative correction uncertainty in muon capture.

Modulo the radiative corrections absorbed into the form factor definitions, the $1 + RC$ common radiative correction factor applicable to muon capture can be written as the sum of three terms

$$\text{RC} = \text{RC(electroweak)} + \text{RC( finite size)} + \text{RC(electron VP)},$$

where $\text{RC(electroweak)}$, where $E_0$ is the charged lepton energy and $q$ the momentum transfer\textsuperscript{11}, the radiative corrections to the vector parts of neutron decay and muon capture are of the same form, but evaluated at different $q^2$ and with different lepton mass. The RC (electroweak) radiative corrections to muon capture [17] were obtained from the original neutron decay calculation, but including higher-order leading log effects denoted by ellipse in the following equation (12):

$$\text{RC(electroweak)} = \frac{\alpha}{2\pi} \left[ 3 \ln \frac{m_Z}{m_p} - 0.595 + 2C + g(m_\mu, \beta_\mu = 0) \right] + \cdots + 0.0237(4),$$

where $m_Z = 91.1876$ GeV, $m_p = 0.9383$ GeV, $C = 0.829$ [52], and $g(m_\mu, \beta_\mu = 0) = 3 \log(m_p/m_\mu) - 27/4 = -0.199$ was obtained from equation (20b) in [61] by replacing $m_e \rightarrow m_\mu$, ignoring bremsstrahlung and taking the $\beta_\mu = 0$ limit. The higher order (in $\alpha$) corrections enhanced by large logarithms [53] have been added to the $+2.23\%$ order $\alpha$ correction to obtain the total $+2.57\%$ electroweak radiative correction.

In equation (12) the uncertainty in the electroweak radiative corrections has been reduced by an order of magnitude to about $4 \times 10^{-4}$ compared with the very conservative $4 \times 10^{-3}$ value in [17]. The source of this uncertainty is the axial-vector induced $-0.595 + 2C$ terms in equation (12) contributing to the RC in Figure 2. In [52] an assumed 10% uncertainty in C and 100% uncertainty in the matching of long and short distance radiative corrections were evaluated to contribute $\pm 2 \times 10^{-4}$ and $\pm 3 \times 10^{-4}$, respectively and an error of $\pm 1 \times 10^{-4}$ was assigned to possible unaccounted for 2 loop effects. Taken together in quadrature they give an overall error of $4 \times 10^{-4}$ for the electroweak radiative corrections to muon capture. That estimate is essentially the same as the error found for the universal radiative corrections to neutron decay and superallowed beta decays [52]. As a result, the $\pm 4 \times 10^{-4}$ error manifests itself also in $|V_{ud}|$ when extracted from superallowed beta decays where it is

\textsuperscript{11}For the kinematics of muon capture, $E_0/m_\mu \sim q/m_p \sim m_\mu/m_p$. 


anti-correlated with the corresponding errors in neutron decay and muon capture. Consequently, when $|V_{ud}|^2$ obtained from superallowed beta decays is used in neutron decay and muon capture calculations, the $±4 × 10^{-4}$ uncertainties cancel and one is left with essentially no electroweak RC uncertainty at least to $O(10^{-6})$. Indeed, even if we were more conservative by increasing these uncertainties by a common factor, they would continue to cancel. That cancellation and the common $g_\alpha$ definition in muon capture and neutron decay together help to keep the electroweak RC uncertainty small. There are, however, contributions to muon capture of $O(\alpha m_N/m_p)$ in the box diagram of figure 2 that could potentially be significant. However, a recent calculation$^{12}$, that ignores nucleon structure, found such contributions to either cancel among themselves or have small coefficients that render them negligible.

Next, we assume that corrections of $O(\alpha m_N/m_p)$ due to nucleon structure are parametrized by the nucleon finite size reduction factor$^{[62]}$

$$|\psi_{1S}(0)|^2 \rightarrow m_n^2 \frac{\alpha^3}{\pi} (1 - 2 \alpha m_n \langle r \rangle),$$  

(13)

where $\langle r \rangle$ denotes the first moment of the proton charge distribution. Based on a range of model forms for this distribution, the correction (13) evaluates to

$$\text{RC}(\text{finite size}) = -0.005(1),$$  

(14)

where the error, which is the current dominant overall uncertainty in the RC, spans the central values $-0.0044$ $[^{42}]$, $-0.005$ $[^{17}]$, and $-0.0055$ $[^{46}]$ given in the literature. We note that the quoted uncertainty may not fully account for possible additional effects of nucleon structure which could be estimated using a relativistic evaluation of the $\gamma$-W box diagrams including structure dependence, but are beyond the scope of this article$^{13}$.

The corrections RC(electroweak) and RC(finite size) modify the coefficients $c_i$ of the effective Hamiltonian (5). The remaining radiative correction, from the electron vacuum polarization modification to the muonic atom Coulomb potential, is described by $\delta V_{VP}$. This contribution amounts to

$$\text{RC}(\text{electron VP}) = +0.0040(2),$$  

(15)

where the very small uncertainty 0.02% is estimated by the difference between 1.73 $\alpha/\pi$ of $[^{17},^{38}]$ and 1.654 $\alpha/\pi$ of $[^{46}]$.

In equation (11), we have defined the total radiative correction to include electroweak, finite size and electron vacuum polarization contributions. In $[^{17}]$, the finite size correction was treated separately, and ‘radiative correction’ referred to the sum of our RC(electroweak) and RC(electron VP), amounting to 2.77%. That central value of the total RC to muon capture derived in $[^{17}]$ and applied here are in agreement, but its overall uncertainty has been reduced by a factor of 4 in the present paper.

2.4. Inputs

The relevant inputs used to compute the capture rate are displayed in table 2. The Fermi constant $G_F$ is determined from the muon lifetime$[^{50}]$ and its uncertainty is negligible in determining the muon capture rate. The CKM matrix element $|V_{ud}|$ is determined from superallowed $\beta$ decays$[^{51}]$. The uncertainty in table 2 is divided into a nucleus-independent radiative correction term, 0.000 18, and a second term 0.000 10 representing the sum in quadrature of other theoretical-nuclear and experimental uncertainties. The former radiative correction is strongly correlated with RC(electroweak) in equation (12), and the corresponding uncertainty largely cancels when the muon capture rate is expressed in terms of $|V_{ud}|$. This cancellation has been accounted for in our discussion of radiative corrections; in the numerical analysis the uncertainty contribution 0.000 18 to $|V_{ud}|$ is dropped$^{14}$.

The charged current isovector form factors are obtained from the isovector combination of electromagnetic form factors. Deviations from $F_1(q^2) = 1$ occur at second order in small isospin violating quantities. At the quark level these quantities may be identified with the quark mass difference $m_u - m_d$ and the electromagnetic coupling $\alpha$. At the hadron level, isospin violation manifests itself as mass splittings within multiplets, such as isodoublet $m_u - m_d$ and isotriplet $m_d - m_s$ $[^{56}-^{58}]$. As shown in $[^{56}]$, first-order isodoubt mass splitting corrections vanish in $F_1(q^2)$ and $F_2(q^2)$, for general $q^2$, while first order isotriplet ones cancel in $F_1(q^2)$ but contribute in $F_1(q^2)$ for $q^2 \neq 0$ and in $F_2(q^2)$ for all values of $q^2$. Estimating these corrections to be of $O(|m_\rho^2 - m_\pi^2|/m_N^2) = 2.1 × 10^{-3}$, where $m_\rho \approx 770$ MeV is the $\rho$ meson mass (representing a typical hadronic mass scale), we note that in $F_1(q^2)$ they are accompanied by the further suppression factor $q^2 r_1^2/6 = -2.4 × 10^{-2}$, so they amount to $-5 × 10^{-5}$. Corrections to the isospin limit in $F_1(q^2)$ are thus negligible at the required permille level. In the case of $F_2(q^2)$, we note that in the expression for the singlet capture rate (equation (22) below), a $2.1 × 10^{-3}$ correction to the $F_2$ term within square brackets amounts to $6.67 × 10^{-4}$, while the total contribution from the four form factors is 4.217$^{15}$. Thus, a $2.1 × 10^{-3}$ isotriplet mass splitting correction to $F_2(q^2)$ induces a $2 × 6.67 × 10^{-4}/4.217 = 3.2 × 10^{-4}$ correction to the singlet capture rate, which is also negligible at the permille level.

Neglecting these small corrections, the Dirac form factor is thus normalized to $F_1(0) = 1$. The Pauli form factor at zero momentum transfer is given by the difference of the proton and neutron anomalous magnetic moments: $F_2(0) = \kappa_p - \kappa_n$, where $\kappa_p = 1.794 08$ and $\kappa_n = -1.914 36$ are measured in units of $e/2m_N$. This leads to $F_2(0) = 3.708 44$. Note that since the PDG $[^{54}]$ expresses both proton and neutron magnetic

---

$^{12}$ A Sirlin, unpublished.

$^{13}$ The finite size ansatz (13) becomes exact in the large-nucleus limit, $r_{\text{nucleus}} \gg r_{\text{weak}}$, where $r_{\text{nucleus}} \sim r_{\text{pp}}$ is the nuclear (proton) charge radius and $r_{\text{weak}} \in \{r_1, r_2, r_a\}$ denotes a weak vector or axial radius.

$^{14}$ The muon capture rate could be expressed directly in terms of $\beta$ decay observables, such as the neutron lifetime and superallowed beta decay fit values, where $|V_{ud}|$ does not appear explicitly.

$^{15}$ The additional suppression may be traced to a factor $m_u/m_N$ appearing in the coefficients of $F_2$ relative to $F_1$ in equation (8). A similar power counting applies to the second class form factors, $F_3$ and $F_4$ in equation (7), that we have neglected in our analysis.
moments in units of e/2m_p, our value for F_3(0) differs from a simple difference of magnetic moments quoted there by a factor m_N/m_p = 1.00069.

The q^2 dependence of the form factors is encoded by the corresponding radii, defined in terms of the form factor slopes:
\[ \frac{1}{F_1(0)} \frac{dF_1}{dq^2} \bigg|_{q^2=0} \equiv \frac{1}{6} r_1^2. \]

Curvature and higher-order corrections to this linear approximation enter at second order in small parameters \( q_0^2 / \Lambda^2 \sim m^2 / m_p^2 \), where \( \Lambda \) is a hadronic scale characterizing the form factor. These corrections may be safely neglected at the permille level. Isospin violating effects in the determination of the radii may be similarly neglected. The Dirac–Pauli basis \( F_1, F_2 \) is related to the Sachs electric–magnetic basis \( G_E, G_M \) by \( G_E = F_1 + (q^2 / 4m^2) F_2, G_M = F_1 + F_2 \). In terms of the corresponding electric and magnetic radii\(^{16}\),
\[ r_i^2 = r_{E,i}^2 - r_{M,i}^2 = \frac{3}{2m^2} F_i(0), \quad r_i^2 = \frac{1}{F_i(0)} \left( r_{E,M,i}^2 - r_{E,A}^2 - r_{M,i}^2 - r_{E}^2 \right). \]

The neutron electric radius is determined from neutron-electron scattering length measurements, \( r_{E,n}^2 = -0.1161(22) \) fm\(^2\) \([54]\). The proton electric radius is precisely determined from muonic hydrogen spectroscopy, \( r_{E,p} = 0.84087(39) \) fm \([2]\); this result remains controversial, and is 5.6σ discrepant with the value \( r_{E,p} = 0.8751(61) \) fm obtained in the CODATA 2014 adjustment \([3]\) of constants using electron scattering and ordinary hydrogen spectroscopy. We take as default the more precise muonic hydrogen value, but verify that this \( r_{E,p} \) puzzle does not impact the capture rate at the projected 0.33% level. The magnetic radii are less well constrained. We adopt the values \( r_{M,p} = 0.776(38) \) fm \([54]\) and \( r_{M,n} = 0.893(3) \) fm \([65]\).\(^{17}\)

Currently, the most precise determination of \( g_A \) comes indirectly via the neutron lifetime, \( \tau_n \), used in conjunction with \( V_{ud} = 0.97420(18) \) \([10]\) obtained from super-allowed nuclear \( \beta \) decays \([51–53]\). Correlating theoretical uncertainties in the electroweak radiative corrections to \( \tau_n \) and \( V_{ud} \),\(^{18}\) reduces the uncertainty in equation (10) to

\[ 1 + 3g_A^2 = 5172.0(1.1) \text{ s/} \tau_n \]

In this review we use the average lifetime \( \tau_n = 879.4(6) \) s \([55]\) from the UCN storage experiments (applying a scale factor of \( \tau_n = 879.4(6) \) s, we adopt the value from the expansion reanalysis \([65]\) of A1 collaboration electron–proton scattering data \([67]\). A similar reanalysis of this experimental data \([66]\) obtained \( r_{M,p} = 0.914(35) \) fm. We verify that this \( r_{M,p} \) discrepancy does not impact the captured τ at the projected 0.33% level. For \( r_{M,n} \), we adopt the value from the expansion reanalysis \([65]\) of \( G_{M,n} \) extractions, combined with dispersive constraints (see also \([68]\)). The larger uncertainty encompasses the PDG value, \( 0.864^{+0.009}_{-0.008} \) fm, obtained by averaging with the dispersion analysis of \([69]\).

The first, 1.8 \times 10^{-4}, uncertainty on \( |V_{ud}| \) in table 2 is correlated with the 1.9 s uncertainty on the right hand side of equation (10). These uncertainties cancel.

\(^{16}\) The isovector form factors can be written in the form \( F_i = F_{i,p} - F_{i,n} \) \((i = 1, 2, 3) \). \( G_{E} = G_{E,p} - G_{E,n} \), \( G_{M} = G_{M,p} - G_{M,n} \), where the subscripts \( p \) and \( n \) refer to the proton and neutron contributions. The electric and magnetic radii are defined analogously to equation (16) in terms of the slopes of \( G_{E}, G_{E}, G_{M}, G_{M} \) and \( G_{M} \). For the neutron, with \( G_{E} = 0 \), \( r_{E,n}^2 = 6 G_{E,n} \).

\(^{17}\) This PDG value for \( r_{E,p} \) represents the \( z \) expansion reanalysis \([66]\) of A1 collaboration electron–proton scattering data \([67]\). A similar reanalysis of other world data in \([66]\) obtained \( r_{M,p} = 0.914(35) \) fm. We verify that this \( r_{M,p} \) discrepancy does not impact the captured \( \tau \) at the projected 0.33% level. For \( r_{M,n} \), we adopt the value from the expansion reanalysis \([65]\) of \( G_{M,n} \) extractions, combined with dispersive constraints (see also \([68]\)). The larger uncertainty encompasses the PDG value, \( 0.864^{+0.009}_{-0.008} \) fm, obtained by averaging with the dispersion analysis of \([69]\).

\(^{18}\) The first, 1.8 \times 10^{-4}, uncertainty on \( |V_{ud}| \) in table 2 is correlated with the 1.9 s uncertainty on the right hand side of equation (10). These uncertainties cancel.

\( S = 1.5 \) according to the PDG convention \([54]\) and do not include the larger lifetime measured in beam experiments. This choice is motivated by two additional and rather precise \( \tau_n \) measurements with trapped neutrons \([70, 71]\) since the last PDG evaluation, and the excellent agreement of the average UCN storage lifetime with the more recent neutron decay asymmetry measurements \([55, 72]\). Employing equation (18) yields \( g_A = 1.2756(5) \), which we use throughout this paper. The current \( g_A = 1.2731(23) \) from neutron decay asymmetry measurements, the PDG value updated with the new UCNA result \([72]\), is lower and has nearly 5-times larger uncertainty, partially due to the large error scaling factor \( S = 2.3 \) caused by the inconsistency between earlier and post-2002 results. On the other hand, if one includes only PERKEO and UCNA values, in particular the recent preliminary PERKEO III result\(^{19}\), the asymmetry measurements provide a consistent \( g_A \) determination of equal precision to the one used in this paper.

Our knowledge about the functional form of \( F_A(q^2) \) relies primarily on neutrino-deuteron scattering data from bubble chamber experiments in the 1970’s and 1980’s: the ANL 12-foot deuterium bubble chamber experiment \([73–75]\), the BNL 7-foot deuterium bubble chamber experiment \([76]\), and the FNAL 15-foot deuterium bubble chamber experiment \([77, 78]\). As mentioned in the Introduction, the original analyses and most follow-up analyses employed the one-parameter dipole model of the axial form factor. A more realistic assessment of uncertainty allows for a more general functional form. Using a \( z \) expansion analysis \([20]\), the uncertainty on the axial radius is found to be significantly larger than from dipole fits,

\[ r_A(z \exp., \nu) = 0.46(22) \text{ fm}^2. \]

That analysis also investigated the dependence of the extracted axial radius on theoretical statistical priors (different orders of truncation for the \( z \) expansion, the range over which expansion coefficients were allowed to vary, and choice of the parameter \( t_0 \) defined by \( z(t_0) = 0 \)), finding that these variations are subdominant in the error budget. Radiative corrections were not incorporated in the original experimental analyses. Radiative corrections and isospin violation would contribute percent level modifications to the kinematic distributions from which \( r_A \) is extracted, well below the statistical and systematic uncertainties of the existing datasets. The value in equation (19) may be compared to a fit of scattering data to the dipole form, \( r_A(z \exp., \nu) = 0.453(23) \text{ fm}^2 \) \([18]\). Note that the value \( r_A(z \exp., \nu) = 0.454(13) \text{ fm}^2 \) quoted in the Introduction is obtained by averaging this neutrino scattering result with an extraction from pion electroproduction \([18]\).

The first, 1.8 \times 10^{-4}, uncertainty on \( |V_{ud}| \) in table 2 is correlated with the 1.9 s uncertainty on the right hand side of equation (10). These uncertainties cancel.

\(^{19}\) H.Saul, International Workshop on Particle Physics at Neutron Sources 2018, ILL, 2018.
where \( m_\pi = 139.571 \text{ MeV} \) is the charged pion mass. Two loop \( \chi PT \) corrections, indicated by the ellipsis in equation (20), were estimated to be negligible, as long as the low energy constants involved remain at natural size [14]. \( f_\pi \) is determined from the measured rate for \( \pi^- \to \mu^- \bar{\nu}_\mu (\gamma) \), and its uncertainty is dominated by hadronic structure dependent radiative corrections. For \( g_{\pi NN} \) we take as default the value \( g_{\pi NN} = 13.12(6)(7)(3) = 13.12(10) \) [63, 64], where the first two errors are attributed to pion-nucleon scattering phase shifts and integrated cross sections, respectively, entering the Goldberger–Miyazawa–Oehme (GMO) sum rule for \( g_{\pi NN} \). The third error is designed to account for isospin violation and was motivated by evaluating a subset of \( \chi PT \) diagrams. Other values include \( g_{\pi NN} = 13.06(8) \) [79], \( g_{\pi NN} = 13.25(5) \) [80] from partial wave analysis of nucleon-nucleon scattering data; and \( g_{\pi NN} = 13.14(5) \) [81], \( g_{\pi NN} = 13.150(5) \) [82] from partial wave analysis of pion-nucleon scattering data. That range of values is covered by the error given in table 2.

2.5. Numerical results

Employing the radiative corrections given above, the full capture rates become

\[
\Lambda = [1 + RC] \Lambda_{\text{tree}} = [1 + 0.0277(4)(2) - 0.005(1)] \Lambda_{\text{tree}},
\]

(21)

where \( \Lambda_{\text{tree}} \) is the tree level expression for the chosen spin state. We have displayed a conventional separation of the radiative corrections in equation (21), where the first +2.77% includes the electroweak and electron vacuum polarization corrections, and the second −0.5% is the finite size correction. Inserting the relevant quantities from table 2, the singlet 1S capture rate is given by

\[
\Lambda_{\text{singlet}} = 40.229(41) [F_1(q_0^2) + 0.0883 F_2(q_0^2) + 2.63645 \Lambda \bar{\gamma}_A - 0.0454 44 \bar{g}_P^2] s^{-1},
\]

(22)

where the quantities \( \bar{g}_P \) and \( \bar{g}_A \) are defined below and the relative uncertainty \( u_1 = 1.0 \times 10^{-3} \) in the prefactor of equation (22) quadratically sums the relative uncertainties \( u_1(\text{RC}) = (1.0, 0.4 \text{ and } 0.2) \times 10^{-3} \) and \( (0.4 \text{ and } 0.2) \times 10^{-3} \) resulting from \( u_1(V_{ud}^2) \), taking into account that the two \( 0.4 \times 10^{-3} \) uncertainties are anticorrelated and cancel each other.

In the discussion above, we define \( u_1 = \delta X/X \) as the relative uncertainty in the considered quantity \( X \) having an uncertainty \( \delta X \). The relative uncertainty in \( X \) induced by parameter \( p \) with uncertainty \( \delta p \) is \( u_1(p) = X^{-1}(\partial X/\partial p)\delta p \).

As a next step, we evaluate the form factors at the momentum transfer \( q_0^2 \) relevant for muon capture. For the vector form factors, we expand to linear order using equation (16),

\[
F_1(q_0^2) = 0.97578(8), \quad F_2(q_0^2) = 3.5986(82).
\]

(23)

For the axial form factor we have \( g_A \equiv F_A(q_0^2) = 1.2510 \times 10^{-11}(\delta q^2) / 5.2 = 1.2510(118) \), with the uncertainty dominated by \( u_1(\delta q^2) = 9.4 \times 10^{-3} \). Finally, the pseudoscalar form factor predicted by \( \chi PT \) is

\[
\bar{g}_P = \frac{m_N}{m_P} F_P(q_0^2) = 8.743(67) \epsilon_{\pi NN}(9) \epsilon_{\pi NN} - 0.498(238) \delta q^2 = 8.25(25),
\]

(25)

where the contribution from the pole and higher order term in equation (22) are shown separately. While the pole term dominates the value for \( \bar{g}_P \), the uncertainty is actually dominated by the non-pole term, due to the rather dramatically increased uncertainty in \( \delta q^2 \).

We exhibit the sensitivity to the axial form factors by inserting the relatively well known vector form factors in equation (22) to obtain

\[
\Lambda_{\text{singlet}} = 67.323(70) \times 1.00000(56) - 2.03801 \bar{g}_A - 0.03513 \bar{g}_P^2 \text{ s}^{-1}.
\]

(26)

At the central values for \( \bar{g}_A \) and \( \bar{g}_P \), the uncertainty in this equation from the remaining inputs is \( \delta \Lambda_{\text{singlet}} = 1.03 \text{ s}^{-1} \), corresponding to a relative error \( u_1 = 1.0 \times 10^{-3} \), which is still dominated by RC, with a minor contribution from \( u_1(F_2) = 0.3 \times 10^{-3} \). At this point the traditional approach would be to insert \( \bar{g}_A \) and \( \bar{g}_P \) in the equation above and to specify the uncertainties in \( \Lambda_{\text{singlet}} \) arising from these two axial form factors. However, as both \( \bar{g}_A \) and \( \bar{g}_P \) depend on the axial radius squared \( \Lambda_{\text{singlet}} \), which is not well known, they cannot be treated as independent input quantities. To avoid their correlation, we express \( \Lambda_{\text{singlet}} \) in terms of the independent input parameters (\( g_{A}, \Lambda_{\text{singlet}} \), \( \Lambda_{\text{triplet}} \), \( g_{\pi NN} \)):

\[
\Lambda_{\text{singlet}} = 67.323(70) \times 1.00000(56) - 0.02341(3) g_{\pi NN} + (2.03801 - 0.05556 \Lambda_{\text{singlet}}^2) g_{A} \frac{1}{s},
\]

(27)

in units of \( \text{fm}^{-2} \). Using the current knowledge of these independent input quantities from table 2, we obtain our best prediction for the muon capture rate in the singlet and triplet hyperfine states of muonic hydrogen as

\[
\Lambda_{\text{singlet}} = 715.4(6.9) \text{ s}^{-1}.
\]

(28)

\[
\Lambda_{\text{triplet}} = 12.10(52) \text{ s}^{-1}.
\]

(29)

We have employed the same methodology as above for \( \Lambda_{\text{singlet}} \) to obtain \( \Lambda_{\text{triplet}} \). The total relative uncertainty for \( \Lambda_{\text{singlet}} \), \( u_1 = 9.7 \times 10^{-3} \), is calculated as the quadratic sum of \( u_1(\text{RC}) = 1.0 \times 10^{-3} \), \( u_1(g_{\pi NN}) = 1.4 \times 10^{-3} \), \( u_1(\Lambda_{\text{singlet}}) = 0.6 \times 10^{-3} \), \( u_1(\Lambda_{\text{triplet}}) = 9.5 \times 10^{-3} \) and a negligible uncertainty from \( f_\pi \). Assuming no uncertainty in \( \Lambda_{\text{singlet}} \), the predictions for \( \Lambda_{\text{singlet}} \) would have a more than 5 times smaller error of 1.3 s\(^{-1}\).

3. Muon capture experiment update

Precise measurements of muon capture in hydrogen are challenging, for the following reasons [8, 83]. (i) Nuclear capture
Figure 3. Reaction sequence after muons stop in hydrogen. Triplet states are quickly quenched to the singlet $\mu H$ ground state. In collisions, $(pp\mu)$ ortho-molecules are formed proportional to the hydrogen density $\phi$ and the formation rate $\lambda_{pp}$. Ortho-molecules can convert to para-molecules with the poorly-known rate $\lambda_{op}$. Reproduced with permission from [8] © Copyright 2010 Annual Reviews.

...takes place after muons come to rest in matter and have cascaded down to the ground state of muonic atoms. As exemplified for the case of $\mu H$ in equation (6), the capture rate is proportional to the square of the muonic wavefunction at the origin $|\psi(0)|^2$, which, after summing over the number of protons in a nucleus of charge $Z$, leads to a steep increase of the capture rate with $\sim Z^2$, such that the muon capture and decay rates are comparable for $Z \sim 13$. For $\mu H$, where $Z = 1$, this amounts to a small capture rate of order $10^{-3}$ compared to muon decay, as well as dangerous background from muon stops in other higher $Z$ materials, where the capture rate far exceeds the one in $\mu H$. (ii) On the normal atomic scale, $\mu H$ atoms are small and can easily penetrate the electronic cloud to transfer to impurities in the hydrogen target gas, or to form muonic molecular ions $(pp\mu)^+$. The former issue requires target purities at the part-per-billion level. The latter problem, depicted in figure 3, has been a primary source of confusion in the past, as the helicity dependence of weak interactions implies large differences in the capture rates from the possible states. The rates for the two atomic hyperfine $\mu H$ states are given in equations (28) and (29), while the molecular rates can be calculated as

$$ \Lambda_{\text{ortho}} = 544 \text{ s}^{-1}, \quad \Lambda_{\text{para}} = 215 \text{ s}^{-1}, \quad (30) $$

using the molecular overlap factors given in equation (11) of [8]21. To interpret a specific experimental capture rate, the fractional population of states for the given experimental conditions has to be precisely known, which is especially problematic for high density targets. (iii) Finally, muon capture in hydrogen leads to an all neutral final state, $n + \nu$, where the $5.2$ MeV neutron is hard to detect with well-determined efficiency.

### 3.1. MuCap experiment: strategy and results

Over the past two decades, the $^3\mu$He experiment, the MuCap and later the MuSun collaboration have developed a novel active target method based on high pressure time projection chambers (TPC) filled with pure $^3$He, ultra-pure hydrogen (1% of liquid hydrogen (LH$_2$) density) or cryogenic deuterium gas (6% of LH$_2$ density), respectively, to overcome the above challenges. The first experiment [86], benefiting from the charged final state, determined the rate for $\mu + ^3$He $\to t + \nu$ with an unprecedented precision of 0.3% as $1496.0 \pm 4.0 \text{ s}^{-1}$. The most recent extraction [87] of $g_\mu$ from this result gives $g_\mu = 8.2(7)$, with uncertainties due to nuclear structure theory. Additional uncertainties would enter if the new $r_A^2(z \exp., \nu)$ is taken into account22.

**MuCap measured $\Lambda_{\text{singlet}}$ in the theoretically clean $\mu H$ system to extract $g_\mu$ more directly.** The original publication [15] gave $\Lambda_{\text{MuCap}} \approx (714.9 \pm 5.4_{\text{stat}} \pm 5.1_{\text{syst}}) \text{ s}^{-1}$, which was slightly updated based on an improved determination of the $(pp\mu)$ molecular formation rate $\lambda_{pp}$ [16] to its final value

$$ \Lambda_{\text{MuCap}} = (715.6 \pm 5.4_{\text{stat}} \pm 5.1_{\text{syst}}) \text{ s}^{-1}. \quad (32) $$

The scientific goal of MuSun [8, 88] is the determination of an important low energy constant (LEC), which characterizes the strength of the axial-vector coupling to the two-nucleon system and enters the calculation of fundamental neutrino astrophysics reactions, like $pp$ fusion in the sun and $\nu d$ scattering in the Sudbury neutrino observatory [89].

As muon capture involves a characteristic momentum transfer of the order of the muon mass, extractions of form factors and LECs from all of these experiments are sensitive to the modified theoretical capture rate predictions or uncertainties implied by the use of the new $r_A^2(z \exp., \nu)$. In view of potential further improvements, let us analyze in some detail how MuCap achieved its high precision 1% measurement. Figure 4 illustrates the basic concept. Muons are detected by entrance detectors, a $500 \mu$m thick scintillator ($\mu$SC) and a wire chamber ($\mu$PC), and pass through a $500 \mu$m-thick hemispherical beryllium pressure window to stop in the TPC, which is filled with ultrapure, deuterium-depleted hydrogen gas at a pressure of 1.00 MPa and at ambient room temperature. Electrons from muon decay are tracked in two cylindrical wire chambers ($\epsilon$PC, green) and a 16-fold segmented scintillator array ($\epsilon$SC, blue). The experimental strategy involves the following key features.

**Low density and suppressed $pp\mu$ formation:** As the target has only 1% of liquid hydrogen density, molecule formation is suppressed and 97% of muon capture occurs in the $\mu H$ singlet atom, providing unambiguous interpretation of the signal.

**Lifetime method [90]:** the observable is the disappearance rate $\lambda_-$ of negative muons in hydrogen, given by the time between muon entrance and decay electron signal. The capture rate is extracted as the difference $\Lambda_{\text{MuSun}} \approx \lambda_- - \lambda_+$, where $\lambda_+$ is the precisely known positive muon decay rate [50]. Contrary to the traditional method of detecting capture neutrons from process (1) which requires absolute efficiencies, only precise time measurements are needed, albeit at large statistics.

---

20 Compare [84] for corrections relevant for capture and muon-electron conversion in heavy nuclei.

21 We do not estimate uncertainties for the molecular rates, as a reliable error evaluation at the permille level should include a modern confirmation of the original calculation of the $(pp\mu)^+$ space and spin structure [85].
Selection of muon stops in hydrogen by tracking: The TPC [91] tracks the incident muons in three dimensions to accept only hydrogen stops sufficiently far away from wall materials with higher capture rate. Its sensitive volume is $15 \times 12 \times 28$ cm$^3$, with an electron drift velocity of 5.5 mm $\mu$s$^{-1}$ at a field of 2 kV cm$^{-1}$ in vertical y-direction. The proportional region at the bottom of the chamber was operated at a gas gain of 125–320, with anode (in x-direction) and cathode (in z-direction) wires read out by time-to-digital converters using three different discriminator levels.

Ultra-pure target gas: target purity of $\sim$10 ppb was maintained with a continuous circulation and filter system [92]. The TPC allowed in situ monitoring of impurities by observing charged nuclear recoils from $\mu^- + O \rightarrow \nu_\mu + N^*$, in the rare cases of muon transfer to impurities, occurring at the $10^{-5}$ level. Isotopically pure protium was produced onsite [93] and verified by accelerator mass spectroscopy [94]. In total, $1.2 \times 10^{10}$ decay events were accepted with muons stopping in the selected restricted fiducial volume.

### 3.2. Conceptual ideas towards a 3-fold improved muon capture experiment

The 3-fold uncertainty reduction over MuCap implies a precision goal of $\delta \Lambda_{\text{singlet}} \sim 2.4$ s$^{-1}$ which, for definitiveness, we assume equally shared between $\delta \Lambda_{\text{singlet}}(\text{stat}) = \delta \Lambda_{\text{singlet}}(\text{syst}) = 1.7$ s$^{-1}$. Achieving this goal is no small feat, and we hope that the motivation for a low-q$^2$ measurement of the axial form factor outlined in this paper will stimulate further innovative experimental ideas. However, in the remainder of this section we follow the more conservative approach to consider incremental improvements to the MuCap strategy only. We note that MuCap was a pioneering experiment not only because of its innovative ideas but also because of its engineering success.

#### 3.2.1. Statistics

A reduction of the 5.4 s$^{-1}$ MuCap statistical uncertainty to 1.7 s$^{-1}$ requires about a 10-fold increase in statistics. Typically, such order of magnitude advances in nuclear/particle physics experiments require concerted upgrades in detector and beam performance.

MuCap accepted only events with a single muon entering the TPC separated from neighboring muons by at least $T_{\text{obs}} = 25$ $\mu$s. This eliminated combinatorial distortions to the measured time spectra. In the PSI continuous beam with a rate $R_{\text{mu}}$, the rate for those single muon events would be $R_{\mu} e^{-R_{\mu} T_{\text{obs}}}$ only, which is limited to roughly 7 kHz. Thus a muon-on-request scheme was developed (see figure 4): a muon in the beam scintillator $\mu$SC triggered a fast kicker [95], which deflects the beam for the measuring period $T_{\text{obs}}$ to avoid muon pile-up. With $R_{\mu} = 65$ kHz, $\mu$SC had a pile-up free rate of 22 kHz, of which a fraction $\epsilon_{\text{fid}} \approx 0.3$ stopped in the fiducial volume of the TPC selected for physics analysis. Including the electron detection efficiency of $\epsilon_e = 0.5$ and deadtime losses, the rate for accepted events was $R_{\text{acc}} \sim 2$ kHz.

To increase this rate, it is certainly worthwhile to explore whether the experiment could run with multiple muons in the TPC. If this idea leads to unacceptable systematic complications, the single muon concept could still be preserved by increasing the muon stopping efficiency from $\epsilon_{\text{fid}} = 0.3 \rightarrow 0.9$ and the electron detection efficiency $\epsilon_e = 0.5 \rightarrow 0.7$, together with supplemental improvements in data taking efficiency. The resulting rate increase of about 4.5 yields $R_{\text{acc}} \sim 9$ kHz, so that $12 \times 10^{10}$ events can be collected in 6 months of data taking (including typical up-time fractions for beam and experiment).

We consider 3 main upgrades to reach this goal. (i) Minimize any material traversed by the muon beam, so that the beam momentum can be decreased from 34 MeV c$^{-1}$ to 29 MeV c$^{-1}$, which reduces the muon range by nearly a factor 2$^{23}$. Since longitudinal range straggling, as well as part of the transverse expansion of the beam, scales with the total range, a much more compact stopping volume can be realized. The evacuated muon beam pipe should be directly connected to

---

23This strong impact of the muon momentum $p$ on its range $R$ follows from the approximate relation $R \propto p^3$, which can be understood from integrating the Bethe-Bloch energy loss equation, see III.20 of [96].
the TPC entrance flange, with the beam detectors reduced to a 200 \( \mu \text{m-thin} \) \( \mu \)SC operating in vacuum with modern silicon photomultiplier (SiPM) technology and the Be window diameter reduced, so that it can be made thinner. The beam to air windows and the wire chamber are eliminated, the latter replaced by a retractable beam spot monitor, which is only used during beam tuning and for systematic studies, but does not add material during production data taking. The beam pipe should be designed as a safety containment volume in case of a breach of the Be window, by placing an additional thin window or fast interlock in an upstream focus. In the first focus downstream of the kicker, another thin scintillator might serve as the beam trigger to minimize kicker delay. (ii) With a collimated beam impinging on \( \mu \)SC and the detector itself positioned as close as possible to the TPC, a stopping efficiency \( \epsilon_{\text{st}} \) approaching unity can be expected for muons seen by this detector. (iii) As the beam rate for negative muons drops steeply with momentum, more powerful PSI beams, existing or under development, should be considered.

3.2.2. Systematics. An uncertainty goal of \( \delta \Lambda_{\text{singlet}}(\text{syst}) = 1.7 \text{ s}^{-1} \) implies that the negative muon decay rate \( \Lambda_\mu \) in hydrogen has to be measured at least at a precision of 3.7 \( \text{s} \) relative to muon decay. This poses unprecedented requirements on the TPC track reconstruction, as no early to late effects over the measuring interval \( T_{\text{obs}} \) distorting the decay spectrum are allowed at this level. The main systematic corrections enumerated in table II of [15] can be grouped into 4 distinct classes:

(i) Boundary and interference effects: by definition, for an infinite TPC no boundary effects (like wall stops, scattering and diffusion) would occur. Interference effects, on the other hand, are generated by decay electrons, affecting the muon stop reconstruction in a time dependent manner. The experiment has to balance these two competing systematic effects, by carefully selecting muons within a clean fiducial volume, without introducing interference distortions. For the final, best MuCap run R07 their total uncertainty added up to \( \delta \Lambda_{\text{singlet}} = 3.3 \text{ s}^{-1} \). The obvious remedy for boundary effects is a larger TPC volume, coupled with potential geometry improvements, as well as the reduction in the beam stopping volume. While the dimensions perpendicular to the TPC drift field are only constrained by practical considerations, the drift time cannot be much longer than \( T_{\text{obs}} \) in order to avoid reducing the acceptable beam rate. We expect that the TPC can be operated with drift fields up to 10kV cm\(^{-1}\), as demonstrated at higher density in MuSun [97], which would double the drift velocity and allow drift distances of 20 cm. To improve the tracking quality, a geometry of independent pads like MuSun has proven advantageous, as the MuCap cathode wires strung in the direction of the muon tracks provide very limited information. As MuSun demonstrated, full digitization of all signals instead of simple threshold timing information, adds powerful tracking capabilities.

(ii) Gas impurities: For the R07 run, transfer to gas impurities amounted to \( \delta \Lambda_{\text{singlet}} \sim 1 \text{ s}^{-1} \). Gold coating of inner vessel surfaces, improved gas chromatography (already achieved in MuSun) and/or spectroscopy and, most importantly, full digital readout of the TPC signals [97], for \textit{in situ} detection of capture recoils, should reduce this uncertainty to below 0.1 \( \text{s}^{-1} \).

(iii) Electron detector effects: a significant uncertainty of \( \delta \Lambda_{\text{singlet}} \approx 1.8 \text{ s}^{-1} \) was included in the MuCap error budget, because of incompletely understood discrepancies between alternative electron track definitions. Because diffusion processes in hydrogen introduce systematic problems when applying tight vertex cuts, MuCap concluded that precision tracking is not essential. Thus a new experiment should use scintillators or scintillating fibers with SiPM readout, which are simple, robust and more stable than the wire chambers used in MuCap. Then an instrumental uncertainty below 1 ppm similar to MuLan [50] can be expected.

(iv) \( p\bar{p} \) molecular effects: although capture from \( p\bar{p} \) molecules amounts only to 3% in 1MPa hydrogen gas, the uncertainty introduced by the inconsistent determinations of the ortho-para rate \( \lambda_{\text{op}} \) [8] shown in figure 3, introduces a \( \delta \Lambda_{\text{singlet}} \sim 1.8 \text{ s}^{-1} \) uncertainty [16]. As shown in figure 2 of [15], the poor knowledge of \( \lambda_{\text{op}} \) also leaves unresolved the question whether the previous measurement of ordinary muon capture in liquid hydrogen [90] or, alternatively, the measurement of radiative muon capture [9] strikingly deviates from theory. The high density cryogenic TPC developed for the MuSun \( \mu \)D experiment, could settle both issues with a first precise measurement of \( \lambda_{\text{op}} \) when filled with protium gas of about 10% liquid density.

Finally, it should be mentioned that the MuCap TPC occasionally suffered sparking issues, which required running with reduced voltage. Better stability and higher gain should be achieved by starting some R&D efforts with smaller prototypes, with improvements to the classical proportional wire chamber technique used by MuCap as well as tests of—now mature—micro-pattern chamber alternatives, like GEMS and micro-megas.

4. Results and opportunities

Having reviewed the status of theory and explored the reach for experiment, in this section we evaluate how well the nucleon form factors and coupling constants can be determined by the present MuCap experiment at 1\% precision, and by a potential new experiment at the 0.33\% level.

4.1. Updated value for the pseudoscalar coupling \( g_P \) and extraction of \( g_{\text{NN}} \)

We begin our applications by using the final MuCap experimental result, \( \Lambda_{\text{MuCap}} = 715.6(7.4) \text{ s}^{-1} \), together with our updated \( \Lambda_{\text{singlet}} \) in equation (26), to extract a value for \( g_P \) that
can be compared with the prediction of $\chi$PT. Both the experimental value and theoretical prediction depend on $r_A^2$. To illustrate that dependence, we start with the traditional value of $r_A^2$ (dipole, $\nu/\Lambda = 0.453(23)$ fm$^2$ obtained from dipole fits to neutrino scattering data with a very small ($\sim 5\%$) uncertainty. It leads to:

$$g_p^\text{MuCap} = 8.26(48)_{\text{exp}}(8)_{\text{EA}}(4)_{\text{RC}} = 8.26(49),$$

$$g_p^\text{theory} = 8.25(7).$$

(33)

For comparison, we take the ratio and find $g_p^\text{theory}/g_p^\text{MuCap} = 1.00(6)$, which exhibits very good agreement at the $\pm6\%$ level. Alternatively, employing the more conservative $z$ expansion value obtained from neutrino scattering, $r_A^2(z, \nu, \nu/\Lambda = 0.46(22)$ fm$^2$, with its nearly 50$\%$ uncertainty, one finds:

$$g_p^\text{MuCap} = 8.23(48)_{\text{exp}}(68)_{\text{EA}}(4)_{\text{RC}} = 8.23(83),$$

$$g_p^\text{theory} = 8.25(25).$$

(34)

where $g_p^\text{theory}$ is obtained from equation (25). The uncertainties are considerably larger. However, taking the ratio and accounting for correlated errors, $g_p^\text{theory}/g_p^\text{MuCap} = 1.00(8)$. Agreement is still very good and theory is tested at about $\pm8\%$, not a significant loss of sensitivity. If $r_A^2$ could be independently determined with high precision (for example, using lattice-gauge theory techniques), then a new MuCap experiment with a factor of 3 improvement would test $\chi$PT at about the 2$\%$ level.

Alternatively, the measured capture rate in conjunction with the theoretical formalism can be used to determine the pion-nucleon coupling $g_{\pi NN}$ from the $\mu$H atom. This approach is closely related to the extraction of the pseudoscalar form factor, as $g_{\pi NN}$ appears as the least well known parameter in the PCAC pole term of equation (20). For this purpose equation (26) was recast in terms of the independent parameters ($g_{\pi NN}$, $r_A^2$ and $g_p^\text{theory}$) into equation (27), avoiding the correlation between the axial form factors introduced by $r_A^2$. That prescription gives, for $r_A^2 = 0.46(22)$ fm$^2$:

$$g_{\pi NN}^\text{MuCap} = 13.11(72)_{\text{exp}}(4)_{\text{EA}}(67)_{\text{RC}}(7)_{\text{RC}} = 13.11(99),$$

$$g_{\pi NN}^\text{exp} = 13.12(10).$$

(35)

The result is in very good agreement with the external $g_{\pi NN}$ obtained from pion-nucleon phase shift and scattering cross section data, such as the value given in table 2. It provides a direct 8$\%$ test of $\chi$PT essentially the same as indirectly obtained from the $g_p$ analysis given above. As in the case of $g_p$, a future factor of 3 improvement in the capture rate combined with an independent precise determination of $r_A^2$ would determine $g_{\pi NN}$ to 2$\%$.

4.2. Determination of $r_A^2$ from muon capture

The basic premise of this paper has been that the error on $r_A^2$ extracted from neutrino scattering data is much larger (by about an order of magnitude) than generally assumed. Indeed, the value $r_A^2(z, \exp, \nu, \nu/\Lambda = 0.46(22)$ fm$^2$, based on the $z$ expansion method, that we employed, has a nearly 50$\%$ uncertainty. As we shall see in section 5, this is problematic for predicting quasi-elastic neutrino scattering cross sections needed for next-generation neutrino oscillation studies. For that reason, it is timely and useful to consider alternative ways of determining $r_A^2$. Various possibilities are discussed in section 5; however, first we consider existing and possible future implications from the MuCap experiment.

Muon capture provides a unique opportunity to determine $r_A^2$, highly complementary to neutrino charged-current scattering. The momentum transfer $q^2_0$ is small and well defined, rendering higher terms in the $q^2_0$ Taylor expansion negligible. However, the effect of $r_A^2$ is small, with $F_A(q_0^2)$ being only $r_A^2 q_0^4/6 \approx 2\%$ smaller than $F_A(0)$. Thus precision experiments at the sub-percent level are called for.

The change in $\Lambda_{\text{singlet}}$ due to a change in $r_A^2$ is given in equations (26), (27), and can be quantified as

$$\frac{\partial \Lambda_{\text{singlet}}}{\partial r_A^2} = \frac{\partial \Lambda_{\text{singlet}}}{\partial g_A} \frac{\partial g_A}{\partial r_A^2} + \frac{\partial \Lambda_{\text{singlet}}}{\partial g_p} \frac{\partial g_p}{\partial r_A^2} = -47.8 + 16.7 = -31.1 \times 10^{-4} \text{ fm}^2. $$

(36)

Thus, a one sigma step of 0.22 fm$^2$ in $r_A^2$ changes $\Lambda_{\text{singlet}}$ by 6.8 $\text{ s}^{-1}$ or about 1$\%$. Unfortunately, for the present purpose, the sensitivity to the axial radius is reduced, as the contributions from $g_A$ and $g_p$ counteract.

Employing equation (27) with the input from table 2 we find

$$r_A^2(\text{MuCap}) = 0.46(24)_{\text{exp}}(2)_{\text{EA}}(3)_{\text{exp}}(3)_{\text{RC}} = 0.46(24) \text{ fm}^2. $$

(37)

This result is comparable in uncertainty to the $z$ expansion fit to the pioneering neutrino scattering experiments [20]. To compare the two processes at higher precision would require inclusion of electroweak radiative corrections for the neutrino data at a level comparable to what has been done for muon capture. Making the reasonable assumption that the two approaches are uncorrelated, we can compute the weighted average

$$r_A^2(\text{ave.}) = 0.46(16) \text{ fm}^2. $$

(38)

The averaged uncertainty has been reduced to about 35$\%$. A future experiment, assumed to reduce the overall MuCap error from 1$\%$ to 0.33$\%$ would reduce the error in $r_A^2$ to

$$\delta r_A^2(\text{future exp.}) = (0.08)_{\text{exp}}(0.02)_{\text{EA}}(0.03)_{\text{RC}} = 0.09 \text{ fm}^2. $$

(39)

The muon capture squared axial radius determination, when averaged with the neutrino scattering $z$ expansion result, would then have about a 20$\%$ uncertainty. This precision level is important, as it would be sufficient to reduce the $r_A^2$ dependent theoretical uncertainty in neutrino quasielastic cross sections to a subdominant contribution, as we demonstrate below in section 5.1.

If we would have used the updated average from the asymmetry measurements, $g_A = 1.273(23)$, the extracted $r_A^2$ from MuCap would be lower, $r_A^2(\text{MuCap}) = 0.38(25) \text{ fm}^2$, but still
easily within the current large error\textsuperscript{24}. For a 3-fold improved MuCap experiment, the uncertainty introduced from that $g_A$ would be more serious, about the same as the experimental contribution to $r_A^2$. However, we expect that in the near future $g_A$ values will have converged, which we anticipate by using the more precise $g_A = 1.2756(5)$ in this review.

4.3. Determination of $g_A$ and electron-muon universality

The axial coupling governing neutron $\beta$ decay, $g_A = F_A(0)$, is a critically important QCD induced physics parameter [98]. Taken together with the neutron lifetime, $\tau_n$, it can provide a clean determination of $V_{ud}$ free of nuclear physics uncertainties, via equation (10). In addition, $g_A$ is needed for constraining the number of effective neutrino species from primordial nucleosynthesis; computing reactor and solar neutrino fluxes and cross-sections; parametrizing the proton spin content and testing the Goldberger–Treiman relation [99]. In this paper we use the value $g_A = 1.2756(5)$, derived from the chosen average $\tau_n$ of neutron bottle experiments and $V_{ud}$ given in table 2 according to equation (18). The error on $g_A$ is expected to be further reduced to about $\pm 0.01\%$, by future $\tau_n$ and direct neutron decay asymmetry measurements. It will be interesting to see if the two methods agree at that level of precision.

For now, the value of $r_A^2$ obtained from the $z$ expansion fit to neutrino-nucleon quasi-elastic scattering together with the MuCap singlet muonic Hydrogen capture rate $A_{\text{singlet}}^{\text{MuCap}}$ can be used in equation (27) to obtain a muon based value, $g_A = 1.276(8)r_A^2(8)_{\text{MuCap}} = 1.276(11)$. That overall roughly $\pm 1\%$ sensitivity is to be compared with the current, better than $\pm 0.04\%$, determination of $g_A$ from the electron based neutron lifetime that we have been using in our text. The good agreement can be viewed as a test of electron-muon universality in semileptonic charged current interactions at roughly the $1\%$ level by determining $g_A'/g_A = 0.9996(86)$. A similar, but not identical test is provided by the ratio $(A_{\text{singlet}}^{\text{theory}}/A_{\text{singlet}}^{\text{MuCap}})^{1/2} = 0.9998(70)$, since the theory prediction uses primarily electron based parameters. The latter test is closely related to the ratio of electroweak coupling constants $g_e/g_\mu = 0.9996(12)$ determined from the $\pi \rightarrow e\nu(\gamma)/\pi \rightarrow \mu\nu(\gamma)$ branching ratio [100]. Leptonic pion decays are considered to be one of the best experimental tests of weak charged current $e$-$\mu$ universality. Somewhat surprisingly, they are only a factor of 6 better than the comparison of theory with experiment in muon capture. We have described how a factor of 3 improvement in the MuCap capture rate may be experimentally feasible. A similar factor of 3 or even much better improvement in $r_A^2$ seems possible from lattice QCD first principles calculations. Together, such advances would provide a muon based determination of $g_A$ to about $\pm 0.2$–$0.3\%$ and improve the capture based test of electron-muon universality by about a factor of 4. Such a comparison is graphically illustrated in figure 5 where the current electron determination of $g_A$ from

\textsuperscript{24}The sensitivity of $r_A^2$ to $g_A$ is given by $r_A^2(\text{MuCap}) = 0.46(24) + 28.4(g_A - 1.2756)\text{ fm}^2$.

5. Towards a more precise $r_A^2$

The momentum dependence of nucleon form factors is critical in many physical processes. The various nucleon radii, defined for each form factor analogously to equation (2), parameterize this momentum dependence at low $q^2$. In fact, for momentum transfers $|q^2| \lesssim \text{few GeV}^2$, the form factors become approximately linear functions in the $z$ expansion: even in the high-statistics datasets for electromagnetic form factors, the curvature and higher order coefficients of the $z$ expansion are only marginally different from zero [68]. This emphasizes the prominence of the form factor charges (i.e. normalizations at $q^2 = 0$) and radii (i.e. slopes at $q^2 = 0$)\textsuperscript{25}. In a nonrelativistic picture, the form factor radii can be interpreted in terms of nucleon structure. For example, the electric charge form factor of the proton represents the Fourier transform of the proton charge distribution, and $r^2$ is readily identified as a mean-square radius in this picture. Similarly, the isovector axial form factor can be interpreted as the Fourier transform of a spin-isospin distribution within the nucleon. Independent

\textsuperscript{25}The $|q^2| \lesssim \text{few GeV}^2$ regime encompasses many physics applications. At larger momentum transfers, inelastic processes compete with the elastic process that is determined by the form factors. For a discussion of $F_A(q^2)$ at large $|q^2|$ see [101].
of intuitive nonrelativistic models, the form factor charges and radii systematically describe the response of nucleons to weak and electromagnetic probes.

Since the form factors are approximately linear over a broad $q^2$ range, the radii constrain (and can be probed by) a variety of processes. For example, the proton charge radius that is probed at $\sim$eV energy scales using hydrogen spectroscopy can be compared with measurements at $\sim$GeV energy scales using elastic electron–proton scattering\textsuperscript{26}. Similarly, constraints on the axial radius from low-energy muon capture translate to constraints on higher-energy neutrino scattering processes. In section 5.1 below, we highlight an important application to quasielastic neutrino scattering and the precision neutrino oscillation program.

The current uncertainty on $r_A^2$ from the $z$ expansion fit to neutrino-deuteron scattering data is about 50%. In this paper, we have shown that the MuCap experiment already provides similar sensitivity and a future factor of 3 improvement in a MuCap like experiment could lead to a roughly 20% determination of $r_A^2$. In section 5.2, we address the capability of other approaches to the precise determination of $r_A^2$. As we shall see, currently, it seems that dedicated lattice studies and neutrino scattering experiments offer the best opportunities.

5.1. Impact of improved $r_A^2$ on accelerator neutrino cross sections

The discovery of neutrino masses, mixing and oscillations provides our first real indication of ‘new physics’, beyond Standard Model expectations. The source of those effects is likely to arise from very short-distance phenomena that may require new technologies and high energy colliders to unveil. However, in the meantime, improvements in neutrino oscillation measurements can still provide important new discoveries. In that regard, ongoing and proposed neutrino oscillation experiments will address the following questions: Is CP violated in neutrino mixing? Are the neutrino masses ordered in magnitude in the same way or a different way than their charged lepton counterparts? Do neutrinos have additional interactions with matter that can be explored through neutrino oscillation interferometry? Answers to those questions could help explain the source of the matter-antimatter asymmetry of our universe, a deep fundamental mystery tied to our very existence.

Neutrino-nucleus interaction cross sections at GeV energies are critical to extracting fundamental neutrino properties and parameters from long baseline oscillation experiments\textsuperscript{103–105}. Uncertainties in these cross sections arise from the elementary nucleon level scattering amplitudes, and from data-driven nuclear modeling for detectors consisting of carbon, water, argon, etc\textsuperscript{22, 106, 107}. A typical oscillation experiment employs a ‘near’ detector, close to the production source of neutrinos, and a ‘far’ detector, located at a sufficiently large distance to allow for observable oscillations. Naively, the near-far detector comparison can be used to avoid reliance on neutrino interaction cross sections. However, a number of effects do not cancel in this comparison: flux differences between near and far (e.g. due to oscillation effects and neutrino beam divergence); flavor dependence of cross sections (e.g. the near detector may constrain $\nu_e$ cross sections, whereas the far detector may search for $\nu_\mu$ appearance signal); and degeneracies between errors in neutrino energy reconstruction from undetected particles (such as neutrons, and other sub-threshold particles) and errors from neutrino interaction uncertainties\textsuperscript{22}.

At the nucleon level, the $q^2$ dependence of the axial form factor is an important source of uncertainty. This uncertainty directly impacts the final cross section, but also complicates the validation of nuclear, flux, and detector modeling, all of which are predicated on quantitatively understanding the simplest quasielastic process. As an example, the MiniBooNE\textsuperscript{21} analysis of quasielastic neutrino-carbon scattering data yielded $r_A^2(\text{MiniBooNE}) = 0.26(7)$ fm$^2$, in tension with historical values obtained from neutrino-deuteron scattering data. Without quantitative control over the nucleon-level amplitudes it is not possible to unambiguously identify the source of the discrepancy.

As a proxy for the relevant class of neutrino observables, let us consider the quasielastic neutrino-neutron cross section at neutrino energy $E_\nu = 1$ GeV. Assuming the dipole ansatz for $F_\Lambda(q^2)$, with\textsuperscript{18} $r_A^2(\text{dipole}) = 0.454(13)$ fm$^2$, this cross section may be evaluated as\textsuperscript{27}

\[
\sigma_{\nu n \rightarrow \mu p}(E_\nu = 1 \text{ GeV}, \text{ dipole}) = 10.57(14) \times 10^{-39} \text{ cm}^2,
\]

(40)

where for the present illustration, we neglect uncertainties from sources other than $F_\Lambda(q^2)$, such as radiative corrections and vector form factors. Using instead the $z$ expansion representation of $F_\Lambda(q^2)$ in\textsuperscript{20}, the result is

\[
\sigma_{\nu n \rightarrow \mu p}(E_\nu = 1 \text{ GeV}, z \exp.) = 10.19(9) \times 10^{-39} \text{ cm}^2,
\]

(41)

i.e. an uncertainty of order 10%, an order of magnitude larger than the uncertainty obtained from the corresponding dipole prediction.

In order to illustrate the impact of improved constraints on $r_A^2$, we begin by reproducing the fits of\textsuperscript{20}, using in addition to the neutrino-deuteron scattering data, an external constraint on $r_A^2$ (coming, e.g. from muon capture). The results are displayed in figure 6. Here we first compare the reference fit to a fit where the slope $(\propto r_A^2)$ is constrained to a particular value (chosen for illustration as the central value $r_A^2 = 0.46$ fm$^2$ of the reference fit). The yellow band in the figure represents the cross section uncertainty that would result from an external radius constraint with negligible error. As we noted above, $F_\Lambda(q^2)$ becomes approximately linear when expressed as a Taylor expansion in $z$, in the sense that curvature in $z$ and higher order $z$ expansion coefficients, are consistent with zero, within errors\textsuperscript{20}. However, these coefficients, when varied over their allowed range, contribute to the error budget, represented by the width of the yellow band in the figure.

\textsuperscript{26} For a discussion of scheme conventions relevant to this comparison, see\textsuperscript{102}.

\textsuperscript{27} For definiteness we employ the remaining parameter and form factor choices of\textsuperscript{20}.
5.2. Other constraints and applications

Given the importance of $r_A^2$, and more generally $F_A(q^2)$, let us understand what complementary information exists from other approaches. This information comes from theoretical approaches to determine $F_A(q^2)$ from the QCD Lagrangian; and from experimental measurements using weak and electromagnetic probes of the nucleon.

5.2.1. Lattice QCD. Lattice QCD is a computational method for determining low energy properties of hadrons based on first principles starting from the QCD Lagrangian. This method has reached a mature state for meson properties. Nucleons present an additional challenge for lattice simulations, owing to a well-known noise problem. A variety of approaches are being taken to explore and address the simultaneous challenges of excited states, lattice size, finite volume, as well as statistical noise. In many cases, the need to extrapolate from unphysically large light quark masses is overcome by performing the lattice calculation at (or near) the physical masses. Background field and correlator derivative techniques are being explored to optimize the isolation of nucleon properties.

Recent computations of the isovector axial charge with a complete stated error budget include: $g_A = 1.195(20)(33)$ [115], where the first error is due to extrapolating in lattice spacing, lattice volume and light quark masses, and the second error is statistical and other systematics; and $g_A = 1.2711(103)(39)(15)(19)(04)(55)$, where the uncertainty contributions refer to statistical, chiral, continuum, finite volume, isospin breaking and model selection (see [113] for details). Other recent preliminary results and discussions may be found in [31, 32, 116–121]. We remark that QED radiative corrections are below the current lattice QCD sensitivity, and the details of the $g_A$ definition in the presence of radiative corrections are thus not yet relevant for this comparison. Note also that the isovector quark current is scale independent in the usual $\overline{MS}$ scheme used to present lattice results.

Lattice QCD is approaching the few percent level for $g_A$. A complete calculation of $r_A^2$ that would rival the precision of neutrino-nucleon scattering and muon capture is not yet available from lattice QCD. However, illustrative values have been obtained, typically using simplified functional forms for the $q^2$ behavior, unphysically large light quark masses, and/ or neglect of strange and charm quarks. A dipole form factor ansatz fit to two-flavor lattice QCD extractions of $F_A(q^2)$ [32] found a result, $r_A^2 = 0.266(17)(7)$ fm$^2$, where the first error is statistical and the second is systematic due to excited states; this result lies closer to the “large $m_A$” MiniBooNE dipole result [21] than to the “small $m_A$” historical dipole average [18, 122]. A $z$ expansion fit to $F_A(q^2)$ obtained using three-flavor QCD with physical strange quark mass, and heavier-than-physical up and down quark masses (corresponding to pion mass 317 MeV) [31], yielded $r_A^2 = 0.213(6)(13)(3)(0)$ fm$^2$, where the uncertainties are from statistics, excited states, fitting and renormalization. A first order $z$ expansion fit to $F_A(q^2)$ using two-flavor QCD, extrapolated to physical pion mass [33] yielded $r_A^2 = 0.360(36)+80 \, \text{fm}^2$, where the first error is statistical and the second error is systematic. Finally, a $z$ expansion fit to four-flavor lattice QCD data using a range of lattice parameters [34] yielded $r_A^2 = 0.24(6)$ fm$^2$. Some

For a brief introduction and references see the lattice QCD review of Hashimoto, Laho and Sharpe in [54].

For a review and further references, see [108].

For recent examples and further references, see [110–114].
of these $r_A^2$ values are well below the historical dipole value and even disagree somewhat with our conservative average of $r_A^2(\text{avg.}) = 0.46(16)$ fm$^2$ in equation (38). This situation suggests that either remaining lattice corrections will involve large corrections that will significantly shift the lattice determination of the radius, or perhaps more exciting that a disagreement may persist as further lattice progress is made, leading to a new paradigm in our understanding of $r_A^2$. However, at this point, further work is needed to obtain precise lattice results with more complete error budgets.

5.2.2. Pion electroproduction. Fits to pion electroproduction data have historically contributed to the determination of the axial radius, with a small quoted uncertainty that can be traced to the assumed dipole form factor constraint. The statistical power of available data would be comparable to the neutrino-deuteron scattering determination, but relies on extrapolations beyond the regime of low energies where chiral corrections are controlled. The axial form factor appears in a low energy theorem for the $S$-wave electric dipole amplitude of threshold charged pion electroproduction $(e^{-} p \rightarrow e^{-} n^+ \pi^{-})$ [123, 124],

$$E_{0^+}(z) \bigg|_{m_s=0} = \sqrt{1 - \frac{q^2}{4m_N^2} \frac{g_A}{8\pi f_\pi} \left[ F_A(q^2) + \frac{q^2}{4m_N^2} \frac{e}{2m_N} F_M(q^2) \right]}.$$  

(43)

This low energy theorem is strictly valid in the chiral limit $(m_\pi = 0)$ for threshold production (invariant mass $W = m_N + m_\pi$) in final state hadronic system. The chiral and threshold limits do not commute, but corrections to the low energy theorem may be calculated within $\chi$PT [125]. Two complications enter. First, experimental measurement is difficult, involving the detection of either a recoiling neutron or a low energy pion. Most of the statistical power of available data involves energies and momentum transfers outside of the regime where a chiral expansion is reliable. The data have been interpreted in terms of a phenomenological framework, whose associated systematic uncertainty is difficult to assess. Second, taking at face value the phenomenological extraction of $F_A(q^2)$ at certain kinematic points from the experimental data, the interpretation as a measurement of the radius has assumed a dipole shape that strongly influences the result.

Using the extracted form factor values at particular kinematic points from [126–130], but replacing dipole with $z$ expansion, [19] obtained $r_A^2 = 0.55(17)$ fm$^2$, compared to the dipole analysis of [122] which gave $r_A^2 = 0.467(18)$ fm$^2$. The datasets were selected to coincide with those that appear in the compilation [122] in order to make a direct comparison with their dipole fit (see figure 1 of that reference). These datasets explicitly list inferred values of $F_A(q^2)$ (see also [131–135]). Reference [135] provides a value $r_A^2 = 0.449(28)$ fm$^2$ based on data at $W = 1125$ MeV and $Q^2 = 0.117, 0.195$ and 0.273 GeV$^2$, and a phenomenological Lagrangian analysis [31]. Reference [136] presents data at $W = 1094$ MeV and $Q^2 = 0.078$ GeV$^2$. Regardless of the precise choice of dataset, the error is significantly larger when the strict dipole assumption is relaxed, even when systematics associated with extrapolations outside of the chiral Lagrangian framework are neglected. Further effort is needed before pion electroproduction provides a robust answer for $r_A^2$.

5.2.3. Lepton scattering. Since the most direct constraints on $F_A(q^2)$ come from neutrino scattering data, it is natural to ask whether improved measurements are feasible. The world dataset for neutrino deuteron scattering consists of a few thousand quasielastic events from bubble chamber data of the 1970s and 1980s. Systematic uncertainties from hand-scanning of photographs and from nuclear modeling are comparable to statistical errors, contributing to the total quoted uncertainty on $r_A^2$ of 0.22 fm$^2$ in [20]. Note that the flux is determined self consistently from the quasielastic events, so that flux errors associated with neutrino production are not relevant. Although nuclear corrections for deuteron targets are relatively small compared to heavier nuclei, errors are difficult to quantify at the desired few percent level, for accelerator neutrino beams of GeV energies. Antineutrino data on hydrogen would eliminate even these relatively small corrections. Existing antineutrino quasielastic data is very sparse, owing to the combined penalties of smaller production cross section for creating antineutrino versus neutrino beams, and smaller scattering cross section for antineutrons versus neutrons. Thus most data were taken in neutrino mode versus antineutrino mode. Reference [137] reported 13 ± 6 events. References [138, 139] reported results for antineutrino-proton scattering inferred from data taken on nuclear (carbon) targets. Currently available analysis techniques with an active target detector should reduce or eliminate scanning and efficiency systematic corrections. Modern neutrino beams have much higher flux compared to the beams used for the existing datasets which would enable either a much smaller detector or a much larger dataset over a given timescale. Technical, cost and safety considerations must be addressed in order to make such a new measurement feasible.

The capture process $\mu^+ p \rightarrow \nu_\mu n$ in muonic hydrogen, and the time reversed process $\nu_\mu n \rightarrow \mu^- p$ measured in neutrino scattering, both probe the charged-current component of the isovector axial vector nucleon matrix element. By isospin symmetry, this isovector matrix element can also be accessed via the neutral component. Parity violating electron–nucleon elastic scattering [140, 141], induced by weak $Z^0$ exchange, is a probe of this matrix element, but simultaneously involves also isoscalar and strange quark contributions that must be independently constrained. Available data do not have discriminating power to reliably extract axial radius or form factor shape information. For example, the G0 experiment [141] analyzed electron–proton and electron-deuteron scattering data to perform a simultaneous fit of the isovector axial form factor, and the strange vector form factors, taking the remaining form factors from other sources. An amplitude was measured for $F_A(q^2)$ at $Q^2 = -q^2 = 0.22$ and 0.63 GeV$^2$, but with insufficient precision to extract shape information. The process $e^- d \rightarrow \nu e p p$ is another possibility to access the charged current nucleon interaction, $e^- n \rightarrow \nu e p$ using electron (positron) beams. No measurements of this process currently exist.
5.2.4. Summary of complementary constraints. A range of processes and techniques have potential to help constrain the nucleon axial radius. Some of these, such as pion electroproduction and parity violating electron–proton scattering, access the form factor and radius indirectly and suffer significant model-dependent corrections that need to be further addressed to achieve ~10% accuracy on $r_A^2$. Lattice QCD and elementary target neutrino scattering are potentially pristine theoretical or experimental approaches. However, lattice QCD has not yet achieved the requisite accuracy, and hydrogen or deuterium active target neutrino experiments are fraught with surmountable but difficult technical and safety issues. Figure 7 displays the range of values for $r_A^2$ as tabulated in Table 1, including the MuCap determination presented in this paper. Our average, equation (38), is obtained from the $z$ expansion $\nu d$ and MuCap results, which have complete error budgets. The future is sure to witness an interesting complementarity between different approaches to axial nucleon structure, with a wide range of constraints and applications.

6. Summary and outlook

In this paper we considered the status and prospects of constraints on the isovector axial nucleon form factor, $F_A(q^2)$, which describes a range of lepton-nucleon reactions. We focused in particular on its prominent role in neutrino-nucleus scattering cross sections underlying neutrino oscillation experiments at accelerator energies. A precise knowledge of these cross sections, for momentum transfers $|q^2| \lesssim$ few GeV$^2$, is required for the next generation of precision studies of neutrino properties in long baseline oscillation experiments. Fully utilizing the oscillation data will require better knowledge of the structure of $F_A(q^2)$, in concert with data-driven improvements in heavy nuclear target modeling.

For many processes, the first two terms in the $q^2$ expansion of $F_A(q^2)$ can be shown to dominate. These terms are parameterized by $g_A \equiv F_A(0)$, and $r_A^2$, which is proportional to the slope of $F_A(q^2)$ at $q^2 \to 0$. The axial nucleon coupling, $g_A$, is precisely determined from neutron beta decay; we use $g_A = 1.2756(5)$ in this work. The nucleon axial radius squared, $r_A^2$, was considered well-determined and uncontroversial for a long time, with $r_A^2 \text{(dipole)} = 0.454(13)$ fm$^2$, derived from a dipole fit to neutrino scattering and pion electro-production data. A recent analysis, however, eliminated the dipole shape constraint for $F_A(q^2)$, as not justifiable from first principles. Using instead the $z$ expansion as a model independent formalism to enforce properties inherited from the underlying QCD structure, a value $r_A^2 (z \exp., \nu) = 0.46(22)$ fm$^2$ was derived [121] from a fit to the $\nu d$ scattering data. The more conservative, but better justified error is an order of magnitude larger than that from the dipole fit, with nearly 50% uncertainty. In this work we assessed some ramifications of this new development, and in particular reviewed and suggested opportunities to reduce the uncertainty in $r_A^2$.

We started from the vantage point of muon capture, in particular muon capture in the theoretically pristine atom of muonic hydrogen, $\mu H$. Muon capture is a charged-current reaction with a small momentum transfer, $q_0^2 \approx -0.9 m_{\mu}^2$, so that $F_A(q_0^2)$ is only ~2% smaller than $F_A(0)$. In the past, the uncertainty introduced by the error in $r_A^2 \text{(dipole)}$ was considered negligible, and capture in $\mu H$ was used to determine the nucleon pseudoscalar coupling $g_P$. The recent 1% MuCap measurement of the spin singlet muonic hydrogen capture rate, $\Lambda_{\text{singlet}} = 715.6(7.4)$ s$^{-1}$, determined $g_P^{\text{MuCap}} = 8.06(55)$, using theory and form factors available at the time. The agreement of this result with the precise prediction of $\chi\text{PT}$ is considered an important test of the chiral structure of QCD. Given the dramatically increased uncertainty in $r_A^2$, we addressed the following questions, answering both in the affirmative: Does the comparison of $g_P$ between experiment and theory still provide a robust test of $\chi\text{PT}$? And, in a reversal of strategy, can muon capture be used to determine a competitive value of $r_A^2$?

High precision is required both in theory and experiment to utilize the small effect of $r_A^2$ on muon capture. In this paper, we have reduced the uncertainty in the electroweak radiative corrections to muon capture to the 0.10% level and extracted an updated value, $g_P^{\text{MuCap}} = 8.23(83)$, from the MuCap experiment. Agreement with the updated theoretical prediction from $\chi\text{PT}$, $g_P^{\text{theory}} = 8.25(25)$, remains excellent. It confirms expectations at a sensitivity level of ±8%, weakened only slightly (from ±6%) by the larger uncertainty in $r_A^2 (z \exp., \nu)$ compared to $r_A^2 \text{(dipole)}$. The MuCap result was also used to provide a self-consistent test of the pion-nucleon coupling process and techniques have potential to help constrain the nucleon axial radius. Some of these, such as pion electroproduction and parity violating electron–proton scattering, access the form factor and radius indirectly and suffer significant model-dependent corrections that need to be further addressed to achieve ~10% accuracy on $r_A^2$. Lattice QCD and elementary target neutrino scattering are potentially pristine theoretical or experimental approaches. However, lattice QCD has not yet achieved the requisite accuracy, and hydrogen or deuterium active target neutrino experiments are fraught with surmountable but difficult technical and safety issues. Figure 7 displays the range of values for $r_A^2$ as tabulated in Table 1, including the MuCap determination presented in this paper. Our average, equation (38), is obtained from the $z$ expansion $\nu d$ and MuCap results, which have complete error budgets. The future is sure to witness an interesting complementarity between different approaches to axial nucleon structure, with a wide range of constraints and applications.

6. Summary and outlook

In this paper we considered the status and prospects of constraints on the isovector axial nucleon form factor, $F_A(q^2)$, which describes a range of lepton-nucleon reactions. We focused in particular on its prominent role in neutrino-nucleus scattering cross sections underlying neutrino oscillation experiments at accelerator energies. A precise knowledge of these cross sections, for momentum transfers $|q^2| \lesssim$ few GeV$^2$, is required for the next generation of precision studies of neutrino properties in long baseline oscillation experiments. Fully utilizing the oscillation data will require better knowledge of the structure of $F_A(q^2)$, in concert with data-driven improvements in heavy nuclear target modeling.

For many processes, the first two terms in the $q^2$ expansion of $F_A(q^2)$ can be shown to dominate. These terms are parameterized by $g_A \equiv F_A(0)$, and $r_A^2$, which is proportional to the slope of $F_A(q^2)$ at $q^2 \to 0$. The axial nucleon coupling, $g_A$, is precisely determined from neutron beta decay; we use $g_A = 1.2756(5)$ in this work. The nucleon axial radius squared, $r_A^2$, was considered well-determined and uncontroversial for a long time, with $r_A^2 \text{(dipole)} = 0.454(13)$ fm$^2$, derived from a dipole fit to neutrino scattering and pion electro-production data. A recent analysis, however, eliminated the dipole shape constraint for $F_A(q^2)$, as not justifiable from first principles. Using instead the $z$ expansion as a model independent formalism to enforce properties inherited from the underlying QCD structure, a value $r_A^2 (z \exp., \nu) = 0.46(22)$ fm$^2$ was derived [121] from a fit to the $\nu d$ scattering data. The more conservative, but better justified error is an order of magnitude larger than that from the dipole fit, with nearly 50% uncertainty. In this work we assessed some ramifications of this new development, and in particular reviewed and suggested opportunities to reduce the uncertainty in $r_A^2$.

We started from the vantage point of muon capture, in particular muon capture in the theoretically pristine atom of muonic hydrogen, $\mu H$. Muon capture is a charged-current reaction with a small momentum transfer, $q_0^2 \approx -0.9 m_{\mu}^2$, so that $F_A(q_0^2)$ is only ~2% smaller than $F_A(0)$. In the past, the uncertainty introduced by the error in $r_A^2 \text{(dipole)}$ was considered negligible, and capture in $\mu H$ was used to determine the nucleon pseudoscalar coupling $g_P$. The recent 1% MuCap measurement of the spin singlet muonic hydrogen capture rate, $\Lambda_{\text{singlet}} = 715.6(7.4)$ s$^{-1}$, determined $g_P^{\text{MuCap}} = 8.06(55)$, using theory and form factors available at the time. The agreement of this result with the precise prediction of $\chi\text{PT}$ is considered an important test of the chiral structure of QCD. Given the dramatically increased uncertainty in $r_A^2$, we addressed the following questions, answering both in the affirmative: Does the comparison of $g_P$ between experiment and theory still provide a robust test of $\chi\text{PT}$? And, in a reversal of strategy, can muon capture be used to determine a competitive value of $r_A^2$?

High precision is required both in theory and experiment to utilize the small effect of $r_A^2$ on muon capture. In this paper, we have reduced the uncertainty in the electroweak radiative corrections to muon capture to the 0.10% level and extracted an updated value, $g_P^{\text{MuCap}} = 8.23(83)$, from the MuCap experiment. Agreement with the updated theoretical prediction from $\chi\text{PT}$, $g_P^{\text{theory}} = 8.25(25)$, remains excellent. It confirms expectations at a sensitivity level of ±8%, weakened only slightly (from ±6%) by the larger uncertainty in $r_A^2 (z \exp., \nu)$ compared to $r_A^2 \text{(dipole)}$. The MuCap result was also used to provide a self-consistent test of the pion-nucleon coupling
and to obtain a roughly $\pm 1\%$ muon-based value of $g_A$, which was found to be in agreement with the electron-based value traditionally extracted from neutron decay (thereby, testing electron-muon universality). Of course, all such tests would be improved by a better independent determination of $r_A^2$ and the factor of 3 improvement in a next generation muon capture experiment advocated here.

As a novel application of the muonic Hydrogen capture rate, we explored its use as an alternative method for determining the nucleon axial radius squared. Using the rather precise rate, we explored its use as an alternative method for determining $r_A^2$ (ave.) = 0.46(16) fm$^2$. We also examined the possibility of improving the MuCap experiment by roughly a factor of 3 and thereby determining $r_A^2$ to about $\pm 20\%$. As demonstrated, that level of accuracy would be sufficient to reduce the $r_A^2$ induced uncertainties in neutrino-nucleon scattering to a subdominant level. Moreover, it would start to become a standard for comparison with other methods of $r_A^2$ determination, several of which were discussed. For such comparisons, lattice gauge theory calculations appear to hold the most promise. Although that Monte Carlo approach to QCD is still not fully mature as applied to $r_A^2$, it promises a first principles strong coupling limit of $2 r_A^2 \Lambda$ compared to historical dipole averages, but it is still too early to scrutinize or average the lattice results in a meaningful way. Future confrontation between experiment and lattice QCD will be interesting to watch and could provide surprises.

The nucleon axial radius has reached an exciting new stage. Until recently, it was thought to be well determined by dipole form factor fits to neutrino-nucleon scattering and electroproduction measurement. However, driven especially by the need for better neutrino cross section predictions, that common lore has been replaced by more conservative healthy skepticism. The axial vector form factor is now being approached from many directions, with the potential to challenge conventional dogma as it enters a new precision era.

**Acknowledgments**

We thank M Hoferichter for helpful discussion of the pion-nucleon coupling. We acknowledge the Institute for Nuclear Theory at the University of Washington, where the idea for this review was conceived. RJH thanks TRIUMF for hospitality where a part of this work was performed. Research of RJH was supported by a NIST Precision Measurement Grant. Research at Perimeter Institute is supported by the Government of Canada through the Department of Innovation, Science and Economic Development and by the Province of Ontario through the Ministry of Research and Innovation. Fermilab is operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States Department of Energy. The work of PK was supported by the US Department of Energy Office of Science, Office of Nuclear Physics under Award No. DE-FG02-97ER41020. The work of WJM was supported by the US Department of Energy under grant DE-SC0012704. The work of AS was supported in part by the National Science Foundation under Grant PHY-1620039.

**ORCID IDs**

Richard J Hill [https://orcid.org/0000-0003-1982-589X](https://orcid.org/0000-0003-1982-589X)
Peter Kammel [https://orcid.org/0000-0003-4730-4274](https://orcid.org/0000-0003-4730-4274)

**References**

[1] Pohl R et al 2010 The size of the proton Nature 466 213–6
[2] Antognini A et al 2013 Proton structure from the measurement of 2S − 2P transition frequencies of muonic hydrogen Science 339 417–20
[3] Mohr P J, Newell D B and Taylor B N 2016 CODATA recommended values of the fundamental physical constants: 2014 Rev. Mod. Phys. 88 035009
[4] Pohl R, Gilman R, Miller G A and Pachucki K 2013 Muonic hydrogen and the proton radius puzzle Ann. Rev. Nucl. Part. Sci. 63 175–204
[5] Carlson C E 2015 The proton radius puzzle Prog. Part. Nucl. Phys. 82 59–77
[6] Hill R J 2017 Review of experimental and theoretical status of the proton radius puzzle EPJ Web Conf. 137 01023
[7] Fleurbaey H et al 2018 New measurement of the 1S − 3S transition frequency of hydrogen: contribution to the proton charge radius puzzle Phys. Rev. Lett. 120 183001
[8] Beyer A et al 2017 The Rydberg constant and proton size from atomic hydrogen Science 358 79–85
[9] Kammel P and Kubodera K 2010 Precision muon capture Ann. Rev. Nucl. Part. Sci. 60 327–53
[10] Wright D H et al 1998 Measurement of the induced pseudoscalar coupling using radiative muon capture on hydrogen Phys. Rev. C 57 373–90
[11] Adler S L and Dothan Y 1966 Low-energy theorem for the weak axial-vector vertex Phys. Rev. 151 1267–77
[12] Wolfenstein L 1970 High Energy Physics and Nuclear Structure (New York: Plenum)
[13] Bernard V, Kaiser N and Meissner U G 1994 QCD accurately predicts the induced pseudoscalar coupling constant Phys. Rev. D 50 6899–901
[14] Bernard V, Hemmert T R and Meissner U-G 2001 Ordinary and radiative muonic capture on the proton and the pseudoscalar form-factor of the nucleon Nucl. Phys. A 686 290–316
[15] Kaiser N 2003 Induced pseudoscalar form-factor of the nucleon at two loop order in chiral perturbation theory Phys. Rev. C 67 027002
[16] Andreev V et al 2013 Measurement of muon capture on the proton to 1% precision and determination of the pseudoscalar coupling $g_P$ Phys. Rev. Lett. 110 012504
[17] Andreev V A et al 2015 Measurement of the formation rate of muonic hydrogen molecules Phys. Rev. C 91 055502
[18] Czarnecki A, Marciano W J and Sirlin A 2007 Electroweak radiative corrections to muon capture Phys. Rev. Lett. 99 032003
[19] Bodek A, Avvakumov S, Bradford R and Budd H S 2008 Vector and axial nucleon form factors: a duality constrained parameterization Eur. Phys. J. C 53 349–54
[19] Bhattacharya B, Hill R J and Paz G 2011 Model independent determination of the axial mass parameter in quasiclassical neutrino-nucleon scattering Phys. Rev. D 84 073006

[20] Meyer A S, Betancourt M, Gran R and Hill R J 2016 Deuteron target data for precision neutrino-nucleon cross sections Phys. Rev. D 93 113015

[21] Aguilar-Arevalo A A et al 2010 First measurement of the muon neutrino charged current quasiclassical double differential cross section Phys. Rev. D 81 092005

[22] Alvarez-Ruso L, et al 2018 NuSTEC white paper: status and challenges of neutrino-nucleus scattering Prog. Part. Nucl. Phys. 100 1–68

[23] Benhar O, Farina N, Nakamura H, Sakuda M and Seki R 2005 Electron- and neutrino-nucleus scattering in the impulse approximation regime Phys. Rev. D 72 053005

[24] Ankowski A M and Sobczyk J T 2006 Argon spectral function and neutrino interactions Phys. Rev. C 74 045816

[25] Ankowski A M, Benhar O and Farina N 2010 Analysis of the Q2-dependence of charged-current quasiparticle processes in neutrino-nucleon interactions Phys. Rev. D 82 013002

[26] Justczak C, Sobczyk J T and Zmuda J 2010 On extraction of value of axial mass from MiniBooNE neutrino quasielastic double differential cross section data Phys. Rev. C 82 045502

[27] Amaro J E, Barbaro M B, Caballero J A, Donnelly T W and Williamson C F 2011 Meson-exchange currents and quasielastic neutrino-nucleon cross sections in the SuperScalings Approximation model Phys. Lett. B 696 151–5

[28] Nieves J, Ruiz Simo I and Vicente Vacas M J 2012 The nucleon axial mass and the MiniBooNE quasielastic neutrino-nucleon scattering problem Phys. Lett. B 707 72–5

[29] Martini M, Ericson M and Chanfray G 2012 Neutrino energy reconstruction problems and neutrino oscillations Phys. Rev. D 85 093012

[30] Nieves J, Sanchez F, Ruiz Simo I and Vicente Vacas M J 2012 Neutrino energy reconstruction and the shape of the CCQE-like total cross section Phys. Rev. D 85 113008

[31] Green J, Hasan N, Meinel S, Engelhardt M, Krieg S, Laeuchli J, Negele J, Orginos K, Pochinsky A and Williamson C F 2011 Meson-exchange currents and neutrino-nucleon scattering Phys. Rev. D 84 053005

[32] Alexandrou C, Constantinou M, Hadjiyiannakou K, Jansen K, Kallidonis C, Koutsou G and Vaqueiro Aviles-Casco A 2017 Nucleon axial vector form factors using Nf = 2 twisted mass fermions with a physical value of the pion mass Phys. Rev. D 96 054507

[33] Capitani S, Della Morte M, Djukanovic D, von Hippel G M, Friar J L, Shann R T 1971 Electromagnetic effects in the decay of neutrons Nucl. Phys. A 69 116

[34] Gupta R, Jang Y-C, Lin H-W, Yoon B and Bhattacharya T 2017 Axial vector form factors of the nucleon from lattice QCD Phys. Rev. D 96 114503

[35] Yao D-L, Alvarez-Ruso L and Vicente-Vacas M J 2017 Extraction of nucleon axial charge and radius from lattice QCD results using baryon chiral perturbation theory Phys. Rev. D 96 116022

[36] Hill R J and Lepage G P 2000 Order (αβ) binding effects in the deuteronium nucleus decay Phys. Rev. D 62 113013

[37] Lepage G P 1997 How to renormalize the Schrodinger equation Nuclear physics. Proc., 8th Jorge Andre Swieca Summer School (Sao Jose dos Campos, Campos do Jordao, Brazil, 26 January–7 February 1997) pp 135–80

[38] Eides M I, Groth H and Shelyuto V A 2001 Theory of light hydrogen—like atoms Phys. Rept. 342 63–261

[39] Eiras D and Soto J 2000 Light fermion finite mass effects in non-relativistic bound states Phys. Lett. B 491 101–10

[40] Andrev V A et al 2007 Measurement of the rate of muon capture in hydrogen gas and determination of the proton’s pseudoscalar coupling g8 Phys. Rev. Lett. 99 032002

[41] Shiomi H 1996 Second class current in QCD sum rules Nucl. Phys. A 603 291–302

[42] Govaerts J and Lucio-Martinez J-L 2000 Nuclear muon capture on the proton and He-3 within the standard model and beyond Nucl. Phys. A 678 110–46

[43] Minamisono K et al 2002 New limit of the G parity irregular weak nucleon current detected in beta decays of spin aligned B-12 and N-12 Phys. Rev. C 65 015501

[44] Holstein B R 2014 Precision frontier in semileptonic weak interactions: theory J. Phys. G: Nucl. Part. Phys. 41 114001

[45] Santisteban A and Pascual R 1976 Muon capture by hydrogen and He-3 Nucl. Phys. A 260 392–400

[46] Raha U, Myhrer F and Kubodera K 2013 Ordinary muon capture in hydrogen reexamined Phys. Rev. C 87 055501

[47] Pastore S, Myhrer F and Kubodera K 2013 The muon capture rate on hydrogen and the values of gA and g(5/2) Nucl. Phys. Rev. C 88 058501

[48] Pastore S, Myhrer F and Kubodera K 2014 An update of muon capture on hydrogen Int. J. Mod. Phys. E 23 1430010

[49] Sirlin A 1978 Current algebra formulation of radiative corrections in gauge theories and the universality of the weak interactions Rev. Mod. Phys. 50 573

[50] Sirlin A 1978 Rev. Mod. Phys. 50 905

[51] Tishchenko V et al 2013 Detailed report of the MuLaN measurement of the positive muon lifetime and determination of the Fermi constant Phys. Rev. D 87 052003

[52] Hardy J C and Towner I S 2015 Superallowed 0+ → 0+ nuclear β decays: 2014 critical survey, with precise results for Vud and CKM unitarity Phys. Rev. C 91 025501

[53] Marciano W J and Sirlin A 2006 Improved calculation of electroweak radiative corrections and the value of Vud Phys. Rev. D 90 093002

[54] Czarnecki A, Marciano W J and Sirlin A 2004 Precision measurements and CKM unitarity Phys. Rev. D 70 093006

[55] Patrignani C et al 2016 Review of particle physics Chin. Phys. C 40 100001

[56] Czarnecki A, Marciano W J and Sirlin A 2018 The neutron lifetime and axial coupling connection Phys. Rev. Lett. 120 202002

[57] Behrends R E and Sirlin A 1960 Effect of mass splittings on the conserved vector current Phys. Rev. Lett. 4 186–7

[58] Terentev M 1963 Conservation of vector current in weak interactions and the mass difference in isotopic multiplets: the π+ → e+νe0 decay Zh. Eksp. i Teor. Fiz. 44 1320

[59] Terentev M 1963 Sov. Phys.—JETP 17 890

[60] Ademollo M and Gatto R 1964 Nonrenormalization theorem for the strangeness violating vector currents Phys. Rev. Lett. 13 264–5

[61] Shahn R T 1971 Electromagnetic effects in the decay of polarized neutrinos Nuovo Cimento A 5 591–6

[62] Wilkinson D H 1982 Analysis of neutron beta decay Nucl. Phys. A 377 474–504

[63] Sirlin A 1967 General properties of the electromagnetic corrections to the beta decay of a physical nucleus Phys. Rev. 164 1767–75

[64] Friar J L 1979 Nuclear finite size effects in light muonic atoms Ann. Phys. 122 151

[65] Baru V, Hanhart C, Hoferichter M, Kubis B, Nogga A and Phillips D R 2011 Precision calculation of the π– deuteron scattering length and its impact on threshold π N scattering Phys. Lett. B 694 473–7

[66] Baru V, Hanhart C, Hoferichter M, Kubis B, Nogga A and Phillips D R 2011 Precision calculation of threshold π– d scattering, πN scattering lengths and the GMR sum rule Nucl. Phys. A 872 69–116

[67] Epstein Z, Paz G and Rev J 2014 Model independent extraction of the proton magnetic radius from electron scattering Phys. Rev. D 90 074027
in quantum field theory and the calculation of matrix elements. Phys. Rev. D 96 014504.

[113] Chang C C et al 2018 A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics. Nature 558 91.

[114] Alexandrou C 2017 Novel applications of lattice QCD: parton distribution functions, proton charge radius and neutron electric dipole moment. EPJ Web Conf. 137 01004.

[115] Bhattacharya T, Cirigliano V, Cohen S, Gupta R, Lin H-W and Yoon B 2016 Axial, scalar and tensor charges of the nucleon from $2 + 1 + 1$-flavor lattice QCD. Phys. Rev. D 94 054508.

[116] Abramczyk M, Lin M, Lylle A and Ohta S 2016 Nucleon structure from $2 + 1$-flavor dynamical DWF ensembles. PoS LATTICE2016 150.

[117] Yamazaki T 2016 Light nuclei and nucleon form factors in $N_{I} = 2 + 1$ lattice QCD. PoS LATTICE2015 081.

[118] Liang J, Yang Y-B, Liu K-F, Alexandru A, Draper T and Sufian R S 2017 Lattice calculation of nucleon isovector axial charge with improved currents. Phys. Rev. D 96 034519.

[119] Abdel-Rehim A et al 2015 Nucleon and pion structure with lattice QCD simulations at physical value of the pion mass. Phys. Rev. D 92 114513.

Abdel-Rehim A et al 2016 Phys. Rev. D 93 039904 (erratum).

[120] Djukanovic D, Harris T, von Hippel G, Junnarkar P, Meyer H B and Wittig H 2017 Nucleon form factors and couplings with $N_{I} = 2 + 1$ Wilson fermions. PoS LATTICE2016 167.

[121] Meyer H B and Wittig H 2017 Nucleon form factors and couplings with $N_{I} = 2 + 1$ Wilson fermions. PoS LATTICE2016 167.

[122] Bernard V, Elouadrhiri L and Meissner U-G 2002 Axial structure of the nucleon: topical review. J. Phys. G: Nucl. Part. Phys. 28 R1–35.

[123] Nambu Y and Urlic D 1962 Chirality conservation and soft pion production. Phys. Rev. 125 1429–36.

[124] Nambu Y and Shraen E 1962 Soft pion emission induced by electromagnetic and weak interactions. Phys. Rev. 128 862–8.

[125] Bernard V, Kaiser N and Meissner U G 1992 Measuring the axial radius of the nucleon in pion electroproduction. Phys. Rev. Lett. 69 1877–9.

[126] Amaldec E, Benevanto M, Borgia B, De Notaristefani F, Fronzaroli A, Pistilli P, Sestili I and Severi M 1972 Axial-vector form-factor of the nucleon from a coincidence experiment on electroproduction at threshold. Phys. Lett. 41B 216–20.

[127] Brauel P et al 1973 $\pi^{+}$ electroproduction on hydrogen near threshold at four-momentum transfers of 0.2, 0.4 and 0.6 GeV2. Phys. Lett. 45B 389–93.

[128] Del Guerra A, Giazotto A, Giorgi M A, Stefanini A, Botterill D R, Brubek D W, Clarke D and Norton P R 1975 Measurements of threshold $\pi^{+}$ electroproduction at low momentum transfer. Nucl. Phys. B 99 253–86.

[129] Del Guerra A, Giazotto A, Giorgi M A, Stefanini A, Botterill D R, Montgomery H E, Norton P R and Matone G 1976 Threshold $\pi^{+}$ electroproduction at high momentum transfer: a determination of the nucleon axial vector form-factor. Nucl. Phys. B 107 65–81.

[130] Esaulov A S, Pilipenko A M and Titov Yu I 1978 Longitudinal and transverse contributions to the threshold cross-section slope of single pion electroproduction by a proton. Nucl. Phys. B 136 511–32.

[131] Amaldec E, Borgia B, Pistilli P, Balla M, Di Giorgio G V, Giazotto A, Serbassi S and Stoppini G 1970 On pion electroproduction at 5 fm-squared near threshold. Nuovo Cimento A 65 377–96.

[132] Bloom E D, Cottrell R L, DeStaebler H C, Jordan C L, Piel H, Prescott C Y, Siemann R, Stein S and Taylor R E 1973 Measurements of inelastic electron scattering cross-sections near one pion threshold. Phys. Rev. Lett. 30 1186.

[133] Joos P et al 1976 Determination of the nucleon axial vector form-factor from $\pi^{+}$ electroproduction near threshold. Phys. Lett. B 62 230–2.

[134] Choi S et al 1993 Axial and pseudoscalar nucleon form-factors from low-energy pion electroproduction. Phys. Rev. Lett. 71 3927–30.

[135] Liesenfeld A et al 1999 A measurement of the axial form-factor of the nucleon by the $p(e, e'\pi^{+})n$ reaction at $W = 1125$ MeV. Phys. Lett. B 468 20.

[136] Friešić I et al 2017 Measurement of the $p(e, e'\pi^{+})n$ reaction close to threshold and at low $Q^{2}$. Phys. Lett. B 766 301–5.

[137] Fanourakis G, Resvanis L K, Grammatikakis G, šč Išč Išč Esaulov A S, Pilipenko A M and Titov Yu I 1978 Determination of the nucleon axial vector form-factor from $\pi^{+}$ electroproduction at low $Q^{2}$. Phys. Rev. Lett. 30 1186.

[138] Ahrens L A et al 1987 Measurement of neutrino—proton and anti-neutrino—proton elastic scattering. Phys. Rev. D 35 785.

[139] Ahrens L A et al 1988 A study of the axial vector form-factor and second class currents in anti-neutrino quasielastic scattering. Phys. Lett. B 202 284–8.

[140] Beise E J, Pitt M L and Spadeo D T 2005 The SAMPLE experiment and weak nucleon structure. Prog. Part. Nucl. Phys. 54 289–350.

[141] Andrei D et al 2010 Strange quark contributions to parity-violating asymmetries in the backward angle $\ell$ electron scattering experiment. Phys. Rev. Lett. 104 012001.
Richard J. Hill is Associate Professor of Physics at University of Kentucky and Adjunct Scientist at Fermilab. Since completing his doctorate in 2002 at Cornell University, he was a postdoctoral researcher at SLAC and Fermilab, Assistant Professor at University of Chicago, and Distinguished Visiting Scientist at TRIUMF and Perimeter Institute.

Peter Kammel was born in Vienna, Austria and obtained his Ph.D. at the University of Vienna. After holding several positions as Research Scientist/Professor at the Austrian Academy of Sciences, the University of California, and the University of Illinois at Urbana-Champaign, he is now a Res. Professor at the Univ. of Washington and the Center for Experimental Nuclear Physics and Astrophysics, Seattle. He is a fellow of the APS and co-spokesperson of several muon physics experiments at the Paul Scherrer Institute, Switzerland, including the MuCap, MuSun and AlCap experiments.

William J. Marciano is a Senior Physicist at Brookhaven National Laboratory where he has worked since 1981. Before that, he held faculty appointments at Northwestern University and The Rockefeller University. He received his PhD from New York University in 1974 under the supervision of Professor Alberto Sirlin with whom he shared the 2002 APS J.J. Sakurai Theoretical Particle Physics prize for their work on radiative corrections.

Alberto Sirlin was born in Buenos Aires, Argentina. After obtaining his Ph.D. in Theoretical Particle Physics at Cornell Univ. and a Res. Assoc. position at Columbia Univ., he joined the faculty of New York Univ. in 1959, where he became full Professor in 1968. Since 2008 he is Prof. Emeritus. Prof. Sirlin received numerous rewards during his distinguished career including: Fellow, Amer. Phys. Soc.(APS); Guggenheim Fellow; Humboldt Prize; Member, Nat. Acad. Exact, Physical and Natural Sciences of Argentina; J.J. Sakurai Prize for Theoretical Particle Physics, shared with W.J. Marciano, (APS), 2002.