Investigation of the methods for optical wavefront parameter manipulation

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Abstract. Here we describe the efficiency of methods for obtaining optical wavefronts with the predetermined parameters, consider the main techniques of the wave field formation, and conduct the analysis of the applicability of these methods to specific problems. We also developed and analyzed the combination of wavefront formation methods. Among the results is the new method of the light beam synthesis based on the combination of the adaptive wavefront optimization approach and the direct prescription of the phase distribution via scalar diffraction theory.

1. Introduction

Formation of optical wave fields with the prescribed complex parameters using digital methods, which can effectively convert energy of laser radiation to the required form, are of particular interest for applications in micro-object manipulation (optical tweezers) [1], controllable thermonuclear synthesis with strong optical pulses [2], and aseptic surgery by the means of laser radiation [3]. Application of new digital methods for controlling light fields allows one to manipulate the field parameters such as amplitude, phase, and degree and kind of polarization. The distinct feature of digital methods is the possibility to create virtual optical devices, whose physical realization is impossible due the absence of manufacturing methods.

From the scalar diffraction theory it is known that the beam amplitude distribution in the arbitrary region of space can be governed by the boundary conditions describing the wave field. Therefore, in order to form the required light field in the given space area one has to find the corresponding amplitude and phase distribution in the wave field modulation plane. It can be implemented using such optical elements as the relief kinoforms (phase holographic platelets) [4], holographic optical elements [5], digital holograms recorded on films [6], and dynamic optical elements with the spatial light modulators (SLMs) [7]. The comprehensive review of the methods for the light field formation and their technological implementations is given in [8].

As of today, numerical calculations of optical fields with the prescribed parameters are conducted using the iteration algorithms, such as Gerchberg-Saxton algorithm [9], and scalar diffraction theory [10]. It should be noted that the employed methods are not suitable for all the existing problems, as their accuracy is not sufficient in some cases. For example, kinoform synthesis is a difficult technological task, and in the most cases, only the structures similar to the kinoforms are synthesized [4]. Suggestion of new methods for obtaining the determined wave field and their further development can potentially solve these problems.
2. Statement of the problem

Numerical simulation of optical wave fields is the common task for optical physics. As a rule, approaches based on geometric optics, iterative algorithms, and scalar diffraction theory are used to solve this problem. In this communication, we study and improve efficiency of the methods for obtaining optical wavefronts with the predetermined parameters. To achieve this, we propose the improvement and optimization of the existing methods for the optical field assignment, as well as the combination of the different methods. We consider and describe the main methods of the wavefront formation. Then we analyze the applicability of these methods to the specific problems.

3. Basic methods of optical field formation

In this section, we consider the basic approaches to formation of the optical fields with the given distribution parameters.

3.1. Direct assignment of the amplitude-phase distribution via scalar diffraction theory

Description of the evolution of the propagating wave field can be performed through the method of scalar diffraction theory. The angular spectrum (AS) determines the wave field amplitude through each component of the Fourier expansion:

\[ U(x,y) = \hat{F}^{-1}\{H(f_x,f_y,l) \cdot \hat{F}[U(x',y')]\}, \]  

where \( H(f_x,f_y,l) \) is the transfer function on the distance \( l \), given by:

\[
H(f_x,f_y,l) = \begin{cases} 
\exp \left[ i \frac{2\pi l}{\lambda} \sqrt{1-\lambda^2 \cdot (f_x^2 + f_y^2)} \right], & f_x^2 + f_y^2 < \frac{1}{\lambda^2}, \\
0, & f_x^2 + f_y^2 \geq \frac{1}{\lambda^2} 
\end{cases}
\]

Where \( \lambda \) is the wavelength of the propagated optical field, and \( f_x, f_y \) are the spatial frequencies along \( x, y \) axes, respectively. In practice, the idea of the spatial distribution of the wave field through the AS of plane waves is convenient due to the following reasons:

- There is no additional restrictions on the original description of the wave field through the Rayleigh-Sommerfeld integral;
- Calculation of the diffraction integral can be performed using the fast Fourier transform, which significantly shortens the simulation time.

3.2. Gerchberg-Saxton algorithm

The iterative Gerchberg-Saxton algorithm for the wavefront phase calculations consists of the following stages (see Fig. 1):

1. For the initial phase and intensity distributions defined in the input plane, the complex amplitude in the output plane is calculated;
2. The resulting intensity distribution is then replaced with the necessary intensity distribution, which is required to be formed in the output plane;
3. Numerical calculation of backward beam propagation to the input plane is performed;
4. The resulting intensity distribution is replaced with the necessary intensity distribution in the input plane. Calculated phase distribution is selected as the next approximation.
Then, the iterative process is repeated. The main disadvantage of this method is the inaccuracy of the resulting phase function, associated with the fact that the large number of iterations is required, and it is not always possible due to the time limits.

![Figure 1. Scheme of the phase distribution reconstruction according to the Gerchberg-Saxton algorithm; A and B are the input and output planes](image)

3.3. Adaptive optimization of the wavefront (AOWF algorithm)

Initially AOWF algorithm was created to solve the problem of the beam focusing through the scattering media [11]. Plane wavefront passed through the scattering sample, resulting in the randomly distributed intensity pattern on the registration plane. Thereafter, the AOWF algorithm was used converting the wavefront so that the incident beam from the source was focused on the registration plane. Previously, a simple case was considered when the scattering medium was absent [12]. In this case AOWF method formed an array of the optimal phase shift values, which was a discrete lens.

The principle of the algorithm operation can be summarized as follows (see Fig. 2):

1. Plane wavefront falls onto the SLM plane (input plane);
2. Separate segments of the wavefront acquire a predetermined phase delay on SLM pixels;
3. The intensity distribution captured on the registration plane is further numerically compared with the necessary distribution;
4. The computer selects the optimal phase delay in the current segment of SLM.

![Figure 2. Algorithm of AOWF operation without scattering medium](image)
This process is then repeated for the next segment of SLM. There are many algorithms for segment selection, for example, line-to-line processing of individual SLM segments, binary search, phase mask method, etc. Additional information about these algorithms can be found in [13].

4. Analysis of the applicability of the considered methods

In this section, we compare the proposed methods in the framework of experimental optics problems and tasks.

4.1. Focusing of the beam from the source in an output plane

Consider focusing of the radiation coming from the point source into a single point. This problem is trivial, and the fastest way to solve it is the straightforward usage of scalar diffraction theory, which means consideration of the backward emission of light from the point source to the given plane.

It is still of particular interest to consider the salvation of this problem with the suggested methods, namely the Gerchberg-Saxton algorithm and AOWF, and compare the results with the predictions of scalar diffraction theory. Such a comparison is given in Fig. 3. One can see that with the algorithm execution time being the same, the Gerchberg-Saxton algorithm is the most effective, as it transforms the phase integrally during each iteration, while AOWF consider each segment separately. However, AOWF method results in the closest approximation to the theoretical lens function.

Figure 3. Amplitude distributions ((a) – (c)) in the output plane and phase distributions ((d) – (e)) in the SLM (input) plane after application of AOWF (a, d), Gerchberg-Saxton algorithm (b, e), and using the scalar diffraction theory (c, f)

4.2. Obtaining of a Gaussian beam profile

Modulation of the complex parameters of a laser beam is an important application of the dynamic optical elements created with SLM. Beam manipulation are the major part of the engineering task, and it is of particular importance to be able to obtain an ideal intensity distribution in the beam for the most accurate measurements.

Let us consider an application of the two methods laser beam processing (AOWF method and Gerchberg-Saxton algorithm) and compare the obtained distributions. We assume here that the beam
initially have the plane phase distribution and the homogeneous intensity distribution. Results of the calculations are given in Fig. 4.

As is seen from the figure, the intensity distribution that is the closest to the required one is given by the Gerchberg-Saxton algorithm, which, however, results in the additional diffraction maxima, and AOWF algorithm is unable to provide the required distribution.

![Figure 4](image)

**Figure 4.** Required intensity distribution of the laser beam (a) and the result of the plane wavefront transformation after application of AOWF (b) and Gerchberg-Saxton algorithm (c)

### 5. Combination of AOWF and direct assignment of phase distribution

Let us consider the possibility of the combination of classical AOWF method and direct assignment of the phase distribution in a restricted area of SLM plane. A certain set of SLM pixels have a fixed phase shift. Therefore, with this conditions an optical element is synthesized in the SLM plane, consisting of two regions: predetermined phase distribution and optimized phase region. The second region focuses the preset amplitude configuration in the registration plane. The results of this method being applied are shown in Fig. 5.

![Figure 5](image)

**Figure 5.** The target image (a), the result of the plane wavefront transformation using combination method (b), and the phase (c) in the SLM plane. Parameters: size of the image $320 \times 320$ mm, $\lambda = 532$ nm, the phase gradation $l_p = 4$. The size of the fixed singular phase area $160 \times 160$ mm (topological charge $q = 5$)

Wavefront from the source has been redistributed into the three points [Fig. 5 (b)]. As a result of AOWF configuration we have the phase distribution with the singular condition [Fig. 5 (c)]. This method has the advantage over direct assignment of a phase distribution: in an unfixed region, it is possible to obtain an object of arbitrarily complex configuration without algorithm variation.

Using this combination method beams with unusual properties have been obtained. Consider the propagation of the beam after the registration plane, shown in Fig. 6. The initially focused beam [Fig. 6 (a), (e)] has the spherical phase distributions in the point locations after propagation [Fig. 6 (b), (f)]. Distributed beam acquires the form of a «donut», and the phase distribution loses its spherical components [Fig. 6 (c), (g)]. Subsequent propagation leads to a complete loss of spherical components and the formation of multiple phase singularities, while «donut» shaped intensity begins to increase in size [Fig. 6 (d), (h)]. It is worth noting that the phase distribution of the spiral beam, which was received during the propagation through the SLM, becomes chiral upon further propagation [Fig. 6 (h)]. This phenomenon is an intrinsic property [14] of spiral beams with a completed intensity distribution «donut» shaped intensity in Fig. 6 (d).
Figure 6. Amplitude [(a) - (d)] and phase [(e) - (h)] distributions of the generated beam after registration plane on a distance $l$

6. Conclusion

In this paper, we have studied the methods for obtaining optical wavefront with the predetermined parameters. Investigated methods have been theoretically described. These methods have been applied to the problems, the solution of which showed their applicability in the various scientific areas. Also, the combination method based on the AOWF algorithm and direct assignment of the phase distribution has been developed and investigated. This method opens up new possibilities for the optical beam synthesis and may result in the development of the new optical modulation algorithms.

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8. References

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