DO QUARK MASSES RUN?

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The importance of measuring the b-quark mass at different scales is emphasized. The recent next-to-leading order calculation of three jet heavy quark production at LEP and its use to measure $m_b(m_Z)$ is discussed.

1 Introduction

The question of the origin of the masses of quarks and leptons is one of the unresolved puzzles in present high energy physics. To answer this question one needs to know precisely their value. However, quarks are not free and their mass has to be interpreted more like a coupling than an inertial parameter and it can run if measured at different scales. Moreover, in the standard model (SM) all fermion masses come from Yukawa couplings and those also run with the energy. To test fermion mass models one has to run masses extracted at quite different scales to the same scale and compare them with the same “ruler”. This way, for instance, one can check that in some unified models the $b$-quark mass and the $\tau$-lepton mass, although different at threshold energies they could be equal at the unification scale. For instance, in fig. 1 we plot the evolution of the $b$-quark and the $\tau$-lepton masses for both the standard model and the minimal supersymmetric standard model (MSSM). We see that the MSSM behaves much better than the SM since “unification” of masses could happen at much higher energies in the MSSM.

An important part of the running occurs at energies accessible in present experiments. However, the running of fermion masses, although predicted by quantum field theory, has not been tested experimentally until now. The

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The observable proposed as a means to extract the bottom-quark mass from LEP data was the ratio

$$R_{3}^{bd} = \frac{\Gamma_{3j}^{b}(y_{c})/\Gamma^{b}}{\Gamma_{3j}^{q}(y_{c})/\Gamma^{q}}.$$  

In this equation $\Gamma_{3j}^{q}(y_{c})/\Gamma^{q}$ is the three-jet fraction of $Z$-decays and $q$ denotes
the quark flavor. Obviously, the ratio \( R_{bd}^3 \) depends on the jet-clustering algorithm used to define the jets. In this ratio and at the leading order (LO) the quark mass effects can be as large as 1% to 6%, depending on the values of the mass and the jet-resolution parameter, \( y_c \).

Since the measurement of \( R_{bd}^3 \) is done far away from the threshold of \( b \)-quark production, it can, in principle, be used to test the running of a quark mass as predicted by QCD. However, the leading order calculation does not distinguish among the different definitions of the quark mass: perturbative pole mass, \( M_b \), running mass at \( M_b \)-scale, or running mass at \( m_Z \)-scale. Therefore, to distinguish them it is necessary to use a complete next-to-leading order (NLO) calculation of three-jet ratios including quark masses which was not available until very recently. Here we overview the calculation used by the DELPHI Collaboration to extract the \( b \)-quark mass at the \( m_Z \) scale.

2 Jet ratios with heavy quarks at NLO

The decay width of the \( Z \)-boson into three jets with a heavy quark can be written as follows

\[
\Gamma_{b}^{3j} = \frac{m_Z g^2 \alpha_s}{c_W^2 64 \pi^2} \left[ g_V^2 H_V(y_c, r_b) + g_A^2 H_A(y_c, r_b) \right],
\]

where \( g \) is the SU(2) gauge coupling constant, \( c_W \) and \( s_W \) are the cosine and sine of the weak mixing angle, \( g_V = -1 + 4/3 s_W^2 \) and \( g_A = 1 \) are the vector and axial-vector coupling of the \( Z \)-boson to the bottom quark and \( \alpha_s \) is the strong coupling constant. Functions \( H_V(A)(y_c, r_b) \) contain all the dependences on \( y_c \) and the quark mass, \( r_b = (M_b/m_Z)^2 \), for the different algorithms. These functions are computed perturbatively as an expansion in \( \alpha_s \) and can also be expanded in \( r_b \) for \( r_b \ll 1 \). At the NLO we have contributions to the three-jet cross section from three- and four-parton final states. One-loop three-parton amplitudes are both infrared (IR) and ultraviolet (UV) divergent. However, the UV divergences are removed after renormalization. The four-parton transition amplitudes are also IR divergent but the sum of both, four-parton and three-parton, contributions is IR finite. We use the so-called phase space slicing method to obtain the remaining finite results, that is, the functions \( H_V(A) \) in eq. (2) at order \( \alpha_s \). These results have been checked in different ways, in particular we checked that the massless result is nicely recovered when we take the limit \( M_b \rightarrow 0 \). These results have also been checked independently by two different groups.

Combining eq. (1), eq. (2) and using the known expression for \( \Gamma_b^{2j} \) we
write $R^{bd}_3$ as the following expansion in $\alpha_s$

$$R^{bd}_3 = 1 + r_b \left( b_0 + \frac{\alpha_s}{\pi} b_1 \right), \quad (3)$$

where the functions $b_0$ and $b_1$ are given by an average of the vector and axial-vector parts of the Z-widths, weighted by $c_V = g_2^2 / (g_2^2 + g_A^2)$ and $c_A = g_A^2 / (g_2^2 + g_A^2)$ respectively. It is important to note that, because of the particular normalization we have used in the definition of $R^{bd}_3$, most of the electroweak corrections cancel.

Although intermediate calculations have been performed using the pole mass, we can also re-express our results in terms of the running quark mass by using the known perturbative expression $M_0^2 = m_b^2(\mu)[1 + 2\alpha_s(\mu) / \pi \left(\frac{4}{3} - \log(m_b^2/\mu^2)\right)]$. We obtain

$$R^{bd}_3 = 1 + \bar{r}_b(\mu) \left( b_0 + \frac{\alpha_s(\mu)}{\pi} \left[ \bar{b}_1 - 2 b_0 \log \frac{m_Z^2}{\mu^2} \right] \right), \quad (4)$$

where $\bar{r}_b(\mu) = m_b^2(\mu)/m_Z^2$ and

$$\bar{b}_1 = b_1 + b_0 \left[ \frac{8}{3} - 2 \log(r_b) \right]. \quad (5)$$

$\bar{r}_b(\mu)$ can be expressed in terms of the running mass of the $b$-quark at $\mu = m_Z$ by using the renormalization group. At the order we are working $\bar{r}_b(\mu) = \bar{r}_b(m_Z) (\alpha_s(m_Z)/\alpha_s(\mu))^{-4\gamma_0/\beta_0}$ with $\alpha_s(\mu) = \alpha_s(m_Z)/(1 + \alpha_s(m_Z)\beta_0 t)$ and $t = \log(m^2/m_Z^2)/(4\pi)$, $\beta_0 = 11 - 2N_f/3$, $N_f = 5$ and $\gamma_0 = 2$.

At the perturbative level eq. (3) and eq. (4) are equivalent. However, they neglect different higher order terms and lead to different answers. The difference gives an estimate of the size of higher order corrections.

Here we present only the results for the DURHAM algorithm, which gives smaller radiative corrections and was the one used by the DELPHI Collaboration in its analysis.

The function $b_0$ describes the corrections due to the quark mass at LO and has an almost negligible residual mass dependence. A fit, $b_0 = \sum_{n=0}^{2} k_0^{(n)} \log^n y_c$, to the complete results for the DURHAM algorithm gives: $k_0^{(0)} = -10.521$, $k_0^{(1)} = 1.4352$, $k_0^{(2)} = -1.6629$.

The function $\bar{b}_1$ gives the NLO massive corrections to $R^{bd}_3$. It is important to note that $\bar{b}_1$ contains significant logarithmic corrections depending on the quark mass. We take them into account by using the form $\bar{b}_1 = k_1^{(0)} + k_1^{(1)} \log(y_c) + k_m^{(0)} \log(r_b)$ in the fit. For the DURHAM scheme we obtain: $k_1^{(0)} = 297.92$, $k_1^{(1)} = 59.358$, $k_m^{(0)} = 46.238$. 

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Figure 2: NLO results for $R^{bd}_3$ (DURHAM) written in terms of the pole mass, $M_b$ (dashed), or the running mass $m_b(m_Z)$ (solid) at LO (gray) and at the NLO (dark). The dotted line gives the NLO result in terms of the running mass at an intermediate scale $m_b(10 \text{ GeV})$. We use as a starting point $m_b(m_b) = 4.13$ to obtain both $M_b$ and $m_b(m_Z)$.

In fig. 2 we present $R^{bd}_3$ as a function of $y_c$ for DURHAM. To compute it, we use the low energy measurement of the b-quark mass \( m_b \), \( m_b(m_b) = 4.13 \). Note that choosing a low value for $\mu$ in the NLO predictions written in terms of the running mass makes it closer to the LO result written in terms of the pole mass, while choosing a large $\mu$ makes the result approach to the LO result written in terms of the running mass at the $m_Z$ scale.

### 3 $m_b(m_Z)$ from LEP data

Since $R^{bd}_3$ has been measured to good accuracy by DELPHI, one can use eq. (2) and the relationship between $m_b(\mu)$ and $m_b(m_Z)$ to extract $m_b(m_Z)$. On the other hand, one could use eq. (3) to obtain directly the pole mass, $M_b$, and then use the relationship between the pole mass and the running mass and the renormalization group to obtain also $m_b(m_Z)$. Both results are slightly different and have a residual dependence on the scale $\mu$. The difference gives an estimate of the errors due to the unknowledge of higher order corrections.

In fig. 3 we plot, as a function of $\mu$, the values of $m_b(m_Z)$ obtained from $R^{bd}_3_{\text{exp}} = 0.971$, by using the two methods. The most conservative estimate of the theoretical error is obtained by taking the spread of the results at $\mu = m_Z$. 


The final result, after including also statistical errors and the errors due to the uncertainties in the hadronization corrections has been presented by DELPHI in this conference. The obtained mass of the bottom-quark, $m_b(m_Z)$, measured from the three-jet decay of the Z-boson is fully compatible with the value obtained from low energy determinations after using the renormalization group. This provides, for the first time, a nice check of the quark mass sector of QCD in a very wide range of scales. These results can probably be improved by understanding better the scale dependence of the results, by resummation of large logs, by using other observables with a softer dependence on the scale, by reducing the hadronization uncertainties and finally by including all LEP data. We think that these studies can provide a rather precise value of $m_b(m_Z)$.

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