PROBING ELECTROWEAK SYMMETRY BREAKING MECHANISM AT THE LHC: A GUIDELINE FROM POWER COUNTING ANALYSIS

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Abstract

We formulate the equivalence theorem as a theoretical criterion for sensitively probing the electroweak symmetry breaking mechanism, and develop a precise power counting method for the chiral Lagrangian formulated electroweak theories. Armed with these, we perform a systematic analysis on the sensitivities of the scattering processes $W^\pm W^\pm \rightarrow W^\pm W^\pm$ and $q\bar{q} \rightarrow W^\pm Z$ for testing all possible effective bosonic operators in the chiral Lagrangian formulated electroweak theories at the CERN Large Hadron Collider (LHC). The analysis shows that these two kinds of processes are complementary in probing the electroweak symmetry breaking sector.

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Recent LEP/SLC experiments support the spontaneously broken $SU(2)_L \otimes U(1)_Y$ gauge theory as the correct description of electroweak interactions. However, a light standard model (SM) Higgs boson has not been found. The current experiments, allowing the Higgs mass to range from 65 GeV to about $O(1)$ TeV, are not very sensitive to the electroweak symmetry breaking (EWSB) sector. Therefore, the EWSB mechanism remains a great mystery, and the probe of it has to include both weakly and strongly interacting cases. If there is a relatively light resonance originated from the EWSB mechanism, the probe would be easier. However, even if such a resonance is detected at the future colliders, it is still crucial to further test if it is associated with a strong dynamics, because it is unknown a priori whether such a resonance trivially serves as the SM Higgs boson or comes from a more complicated mechanism. If the EWSB is driven by a strong dynamics with no new resonance much below the TeV scale, the probe becomes more difficult. In this paper, we study the latter case concerning the test at the CERN Large Hadron Collider (LHC).

The most economical description of the EWSB sector below the related new resonance scale is given by the electroweak chiral Lagrangian (EWCL) which can reflect both the heavy Higgs SM and other types of new strong dynamics. This general effective field theory approach is complementary to those specific model buildings. Following Refs. [3, 4], the EWCL can be formulated as

$$L_{\text{eff}} = L_G + L_F + L^{(2)} + L^{(2)'} + \sum_{n=1}^{14} L_n = \sum_n \ell_n \frac{f_\pi}{\Lambda_{\alpha_n}} \mathcal{O}_n(W_{\mu\nu}, B_{\mu\nu}, D_\mu U, U, f, \bar{f}),$$  \hspace{1cm} (1)

where $L_G = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$ and $L_F$ denotes the fermionic part. Here we concentrate on probing the new physics from all possible bosonic effective operators so that we do not include the next-to-leading order fermionic operators in $L_F$. In (1), $U = \exp[\text{i} \tau^a \pi^a / f_\pi]$ and $\pi^a$ is the would-be Goldstone boson (GB) field. $f_\pi = 246$ GeV is the vacuum expectation value breaking the electroweak gauge symmetry, and the effective cut-off $\Lambda \approx 4\pi f_\pi \approx 3.1$ TeV is the highest energy scale below which (1) is valid. The explicit expressions for nonlinear bosonic operators in $L_{\text{eff}}$ have been given by Refs. [3, 4], in which the leading order operator $L^{(2)} = \frac{1}{2} f_\pi^2 \text{Tr}[(D^\mu U)(D^\nu U)^\dagger]$ is universal, and the next-to-leading order operators $L^{(2)'}$, $L_{11-11}$ ($CP$-conserving) and $L_{12-14}$ ($CP$-violating) are model-dependent. Here, the dimensionless coefficients $\ell_n$’s for these next-to-leading order operators are related to the corresponding notations $\alpha_n$’s in Ref. [3] by definition $\alpha_n \equiv \left( \frac{f_\pi}{\Lambda} \right)^2 \ell_n$. From the consistency requirement of the chiral perturbation theory, these coefficients $\ell_n$’s can be naturally around of $O(1)$ [3].

We know that only the longitudinal component $V^a_L$ of the weak-boson $V^a (W^\pm, Z^0)$, arising from “eating” the would-be Goldstone boson $\pi^a (\pi^\pm, \pi^0)$, is sensitive to the EWSB sector, while the transverse component $V^a_T$ is not. For the strongly coupled EWSB sector, the longitudinal $V^a_L$-scattering cross-sections are measurable at the LHC and can thus probe the EWSB mechanism. The physical $V^a_L$-scattering amplitude is quantitatively related to the corresponding GB-amplitude by the electroweak Equivalence Theorem (ET) [1-3]. In Ref. [3], we precisely formulate the ET as follows

$$T[V^a_{L1}, \cdots, V^a_{Ln}; \Phi_\alpha] = C \cdot T[-i\pi^{a1}, \cdots, -i\pi^{an}; \Phi_\alpha] + B, \hspace{1cm} (2a)$$

$$E_j \sim k_j \gg M_W, \hspace{1cm} (j = 1, 2, \cdots, n) , \hspace{1cm} (2b)$$

$$C \cdot T[-i\pi^{a1}, \cdots, -i\pi^{an}; \Phi_\alpha] \gg B , \hspace{1cm} (2c)$$

1
where $\pi^a$'s are GB fields, $\Phi_\alpha$ denotes other possible physical in/out states, $C \equiv C_{\text{mod}}^{a_1} \cdots C_{\text{mod}}^{a_n}$ with $C_{\text{mod}}^{n} = 1 + O(\text{loop})$ a renormalization scheme-dependent constant, $B = \sum_{j=1}^{\infty} \left( C_{\text{mod}}^{a_{j+1}} \cdots C_{\text{mod}}^{a_n} T[a^{a_1}, \ldots, v^{a_1}, \ldots, \pi^{a_{j+1}}, \ldots, -i\pi^{a_n}; \Phi_\alpha] \right)$ with $v^{a_1} \equiv v^\mu V_\mu^a$ and $v^{\mu} \equiv v^\mu - k^\mu / M_V = O(M_W/E)$, $(M_V = M_Y, M_Z)$, and $E_j$ is the energy of the $j$-th external line. The modification factor $C_{\text{mod}}^{n}$ has been generally studied in Refs. [7]-[8], and can be exactly simplified to unity in certain convenient renormalization schemes [8]. It is clear that the term $B$ in (2a) is $O(M_W/E)$-suppressed relative to the GB-amplitude $C \cdot T[-i\pi^{a_1}, \ldots]$ , but this does not mean that $B$ itself is necessarily of $O(M_W/E)$ since the GB-amplitude contains positive powers of $E$ in the CLEWT. In fact, our power counting rule [cf. (5)] shows that, in the CLEWT, the leading term in $B$ for $V^a V^b$ scatterings is of $O(g^2)$ which is model-independent and of the same order as the leading pure $V_T$-amplitudes [9]. Hence $B$ is insensitive to the EWSB mechanism, and it serves as an intrinsic background to sensitively probing the EWSB mechanism by the $V_L$-amplitude. Therefore, a sensitive probe at least requires the GB-amplitude dominates over $B$ to validate the equivalence between the $V_L$ and GB amplitudes in (2a). (2b)$^1$ and (2c) are the precise conditions for this equivalence, and thus serve as the necessary conditions for sensitively probing the EWSB mechanism via $V_L$-scattering experiments. Hence, we see the profound physical content of the ET: it provides a necessary theoretical criterion for sensitively probing the EWSB mechanism, and is much more than just a technical tool for simplifying explicit calculations.

To see the precise meaning of (2c), we consider a certain perturbative expansion of the GB-amplitude. To a given order $N$ in the expansion, the amplitude $T$ can be written as $T = \sum_{\ell=0}^{N} T_\ell$ with $T_0 > T_1, \ldots, T_N$. Let $T_{\text{min}} = \{ T_0, \ldots, T_N \}_{\text{min}}$. Then, to the precision of $T_{\text{min}}$, condition (2c) precisely implies

$$T_{\text{min}}[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] \gg B \ .$$

For the CLEWT, the leading amplitude $T_0$ in (2a) is of $O(E^2)$ and is model-independent. Thus, for distinguishing different strongly interacting EWSB mechanisms, we have to consider the model-dependent next-to-leading order amplitude $T_1$ which can be of $O(E^4)$ . Hence, we take $T_{\text{min}} = T_1$ in (3). From (3), we can now theoretically define various levels of the sensitivity for probing $T_1$ as follows. The probe is classified to be sensitive if $T_1 \gg B$ , marginally sensitive if $T_1 > B$ (but $T_1 \gg B$) , and insensitive if $T_1 \leq B$ . Note that in the following power counting analysis (cf. Table 1 and 2) both the GB-amplitude and the $B$-term are explicitly estimated by our counting rule (5). The issue of numerically including/ignoring $B$ in an explicit calculation is essentially irrelevant here. If $T_1 \leq B$ , this means that the sensitivity is poor so that the probe of $T_1$ is experimentally harder and requires a higher experimental precision of at least $O(B)$ to test $T_1$.

To make a systematic global analysis on the sensitivity of each physical scattering process for probing the new physics operators in the EWCL (1), we need a convenient method to obtain the scattering amplitudes contributed by all these operators. For this purpose, we generalize Weinberg’s power counting rule for the ungauged nonlinear sigma model (NLSM) [1] to the EWCL (1) and develop a precise power counting method for the CLEWT to separately count the power dependences on the energy $E$ and all relevant mass scales. The original Weinberg’s counting rule is to count the $E$-power dependence ($D_E$) for a given $L$-loop level S-matrix element

$^1$ Condition (2b) is different from the usual condition $E \gg M_W$ for the total center of mass energy $E$. An illustrating example is given in Ref. [8].
where \( V_n \) is equal to the number of type-\( n \) vertices in \( T \), \( d_n \) and \( f_n \) are the numbers of derivatives and fermion-lines at a type-\( n \) vertex, respectively, and \( e_v \) is the number of possible external \( v^\mu \)-factors [cf. (2a) and below for the \( B \)-term]. Note that the counting rule (4) only holds for amplitudes without any external \( V_L \)-line. Since there is non-trivial cancellation of the \( E \)-power factors from the external \( V_L \)-polarizations in the \( V_L \)-amplitude due to gauge-invariance, the \( V_L \)-amplitude cannot be directly counted by applying (4). However, there are no such \( E \)-power cancellations on the RHS of (2a). Therefore (4) can be applied to amplitudes with external \( V_L \)-lines by counting the RHS of the ET relation (2a).

Besides counting the power of \( E \), it is also crucial to separately count the power dependences on the two typical mass scales in the EWCL, namely the vacuum expectation value \( f_\pi \) and the effective cut-off \( \Lambda \), otherwise the result will be off by orders of magnitudes since \( \Lambda/f_\pi \approx 4\pi > 12 \). The \( \Lambda \)-dependence comes from two sources: (i). from tree vertices: \( T \) contains \( V = \sum_n V_n \) vertices, each of which contributes a factor of \( 1/\Lambda^{a_n} \) [cf. (1)] so that the total factor from \( V \)-vertices is \( 1/\Lambda^{\sum_n a_n} \); (ii). from loop-level: Since each loop brings in a factor of \((1/4\pi)^2 \approx (f_\pi/\Lambda)^2\), the \( \Lambda \)-dependence from \( L \)-loop contribution is \( 1/\Lambda^{2L} \). Hence the total \( \Lambda \)-dependence should be \( 1/\Lambda^{\sum_n a_n + 2L} \). Let us denote the total dimension of \( T \) as \( D_T \), then \( T \) can always be written as \( f_\pi^{D_T} \) times some dimensionless function of \( E, \Lambda, \) and \( f_\pi \) since the vacuum expectation value \( f_\pi \) is generic to any spontaneously broken gauge theories. With these ready, we can generally construct the following precise counting rule for \( T \):\(^2\)

\[
T = c_T f_\pi^{D_T} \left( \frac{f_\pi}{\Lambda} \right)^{N_0} \left( \frac{E}{f_\pi} \right)^{D_{E0}} \left( \frac{E}{4\pi f_\pi} \right)^{D_{EL}} \left( \frac{M_W}{E} \right)^{e_v} H(\ln E/\mu) ,
\]

\[
N_0 = \sum_n a_n , \quad D_{E0} = 2 + \sum_n V_n \left( d_n + \frac{1}{2} f_n - 2 \right) , \quad D_{EL} = 2L ,
\]

where the dimensionless coefficient \( c_T \) contains possible powers of gauge couplings \( (g, e) \) and Yukawa couplings \( (y_f) \) from the vertices of \( T \), which can be directly counted. \( H \) is a dimensionless function of \( \ln(E/\mu) \) coming from loop corrections in the standard dimensional regularization \( \{1, 2\} \), \( \mu \) is the relevant renormalization scale, and is thus insensitive to \( E \). (Here we note that the dimensional regularization supplemented by the minimal subtraction renormalization is particularly clean and convenient for effective theory calculations, as emphasized in Ref. \{12\}.)

Neglecting the insensitive factor \( H(\ln E/\mu) \), we can extract the main feature of scattering amplitudes by simply applying (5) to the corresponding Feynman diagrams.

\(^2\)In (5) we still explicitly keep the loop factor \( (1/4\pi)^{D_{EL}} \) for generality, since the effective cut-off \( \Lambda \) denotes the lowest new resonance scale and could be somehow lower than the theoretical upper bound \( 4\pi f_\pi \approx 3.1 \text{ TeV} \) for strongly coupled EWSB sector, as indicated by some model buildings. For the case \( \Lambda \approx 4\pi f_\pi \), this loop factor reduces to \( (f_\pi/\Lambda)^{D_{EL}} \) as mentioned above.
Based upon the basic features of the chiral perturbation expansion, we further build the following electroweak power counting hierarchy for the S-matrix elements,

\[
\frac{E^2}{f_\pi^2} \gg \frac{E^2}{f_\pi^2} \lambda^2, \quad g \frac{E}{f_\pi} \gg \frac{E}{f_\pi} \lambda^2, \quad g^2 \gg g^2 \frac{E^2}{f_\pi^2}, \quad g^3 \frac{f_\pi}{E} \gg g^3 \frac{f_\pi}{E} \lambda^2, \quad g^4 \frac{f_\pi^2}{E} \gg g^4 \frac{f_\pi^2}{E} \lambda^2,
\]

which, in the typical high energy region \( E \in (750 \text{ GeV}, 1.5 \text{ TeV}) \) for instance, numerically gives (for \( \Lambda \approx 4\pi f_\pi \approx 3.1 \text{ TeV} \)):

\[
(9.3, 37) \gg (0.55, 8.8), (2.0, 4.0) \gg (0.12, 0.93), (0.42, 0.42) \gg (0.025, 0.099), (0.089, 0.045) \gg (5.3, 10.5) \times 10^{-3}, (19.0, 4.7) \times 10^{-3} \gg (1.1, 1.1) \times 10^{-3}.
\]

The power counting hierarchy (6) provides a useful theoretical base for our global classifications of various high energy scattering amplitudes.

In the literature (cf. Ref. [10]), what usually done is to study only a small subset of all effective operators in the EWCL (1) for simplicity. But, to have a complete test of the EWSB sector by distinguishing different kinds of dynamical models, it is necessary to know how to best measure all these operators through various high energy \( VV \)-fusion and \( q\bar{q}^{(l)} \)-annihilation processes. For this purpose, our global power counting analysis provides a simple and convenient way to quickly grasp the overall physical picture and guides us to perform further elaborate numerical calculations. In the following, we shall make the classifications for both the \( S \)-matrix elements and the LHC event rates.

We first analyze the contributions of the fifteen effective operators in (1) to all \( V^a - V^b \) scatterings, which are dominated by the 4-GB-vertices [cf. the power counting rule (5)]. According to the hierarchy (6) and at the level of \( S \)-matrix elements, Table 1 gives a complete sensitivity classification, which shows the relevant effective operators and the corresponding physical processes for probing the EWSB mechanism when calculating the scattering amplitude to the desired accuracy. In Table 1, MI and MD stand for model-independent and model-dependent operators, respectively. Here, for simplicity we have taken \( \Lambda \approx 4\pi f_\pi \) whenever the one-loop MI contributions from \( L^{(2)} \) is concerned. It is easy to change the one-loop factor back to \( (1/4\pi)^2 \) [cf. (5)] when \( \Lambda < 4\pi f_\pi \). Also, we have explicitly estimated all relevant contributions from the \( B \)-term. Here, \( B^{(i)}_\ell \) (\( i = 0, 1, \ldots; \ell = 0, 1, \ldots \)) denotes the \( B \)-term from the \( \ell \)-loop level \( V_L \)-amplitude containing \( i \) external \( V_T \)-lines. From Table 1, we first see that the MI operator \( L^{(2)} \) contained in \( L_{\text{MI}} \equiv L^{(2)} + L_G + L_F \) mainly discriminating between the strongly and weakly interacting mechanisms, can be sensitively probed in the \( 4V_L(\neq 4Z_L) \) channel to the level of \( O(E^2/f_\pi^2) \).

For the MD operators, the \( 4V_L \) channel can probe \( L_{4,5} \) most sensitively. The contributions of \( L^{(2)'} \) and \( L_{2,3,9} \) to this channel lose the \( E \)-power dependence by a factor of 2. Hence this channel is less sensitive to these operators. The \( 4V_L \) channel cannot probe \( L_{1,8,11-14} \) (which can only be probed via channels with \( V_T \)'s). Among \( L_{1,8,11-14} \), the contributions from \( L_{11,12} \) to channels with \( V_T \)'s are most important though they are still suppressed by a factor of \( g f_\pi/E \) relative to the leading contributions from \( L_{4,5} \) to the \( 4V_L \) channel. \( L_{1,8,13,14} \) are generally suppressed by higher powers of \( g f_\pi/E \) and are thus the least sensitive.

Table 2 classifies all \( q\bar{q}^{(l)} \)-annihilation processes. The operator \( L_{\text{MI}} \) can be probed via tree-level constant \( O(g^2) \) amplitude through either \( V_LV_L \) or \( V_TV_T \) final states, which are not enhanced by high energy \( E \)-powers. Among all next-to-leading order operators, the probe
of $\mathcal{L}_{2,3,9}$ is most sensitive via $q\bar{q}^{(l)} \to W^+_L W^-_L$ amplitude and the probe of $\mathcal{L}_{3,11,12}$ is best via $q\bar{q} \to W^+_L Z_L$ amplitude, to the precision of $g^2 E^2 / \Lambda^2$. For operators $\mathcal{L}_{1,8,13,14}$, the largest amplitudes are $T_1[q\bar{q}; W^+_L W^-_T / W^+_T W^-_L]$ and $T_1[q\bar{q}; W^+_L Z_T / W^+_T Z_L]$, which are at most of $O\left( g^3 E^2 / \Lambda^2 \right)$. The contributions to total cross sections from above amplitudes can exceed that from the corresponding $B = O\left( g^2 M_Z^2 / \Lambda^2 \right)$ (for $V_L V_L$) or $O\left( g^2 M_W^2 / \Lambda^2 \right)$ (for $V_L V_T$) in the high energy region when polarizations are summed up. The next-to-leading order operators $\mathcal{L}_{4,5,6,7,10}$ do not contribute to $q\bar{q}^{(l)}$-annihilations at the $1/\Lambda^2$-order and thus will be best probed via $VV$-fusions (cf. Table 1).

Before further classifying the various contributions to a given process at the event rate level for the LHC, we have compared some of our results with those available in Ref. [13] from precise calculations, to test the above power counting method. The authors of Ref. [16] and we have used the effective-W approximation (EWA) [14, 15] for computing the event rates. Two typical processes for $W W$-fusion and $q\bar{q}$-annihilation are compared in Fig. 1a and 1b, respectively. The event rates $R_{\alpha,\beta,\gamma,\delta}$ and $R_{\alpha,\beta,\ell}$ are calculated up to one-loop level for the two processes. (Here $\alpha, \beta, \gamma, \delta = L, T$ specify the polarizations of the incoming/out-going $W^\pm$ or $Z^0$ gauge bosons, and $\ell = 0, 1$ denote the tree and one-loop level contributions, respectively.) The comparison in Fig. 1 shows that the agreements are within a factor of 2 or even better. So, our simple power counting rule (5) does conveniently give reasonable systematic estimates and is thus useful for making global analyses on probing the EWSB mechanisms at the LHC and future linear colliders.

Then, we calculate the number of events per [100 fb$^{-1}$ GeV] contributed from each next-to-leading order effective operator at the LHC by the power counting rule (5) combined with the EWA.$^3$ In the following numerical analysis we typically take $\Lambda \approx 4\pi f_\pi \approx 3.1$ TeV. But we keep in mind that our estimates for the number of events contributed by the next-to-leading order operators will be increased by a factor of $(3.1$ TeV$/\Lambda)^2$ for $\Lambda < 3.1$ TeV in the energy region below $\Lambda$. We first consider the $W^+ W^+$ channel which is most important for the non-resonance scenario [16, 17]. In Fig. 2, the event rate $|R_1|$ contributed from each next-to-leading order operator is separately shown for the $W^+ W^+$ channel for $\ell_n \approx O(1)$ with the polarizations of the initial and final states summed over. We note that the experiments actually contain the contributions from all operators and are thus more complicated. For simplicity and clearness, one can make the well-known naturalness assumption (i.e., contributions from different operators do not accidentally cancel each other), as widely adopted in the literature [17], and estimate the bounds on each single operator. We can clearly see, from Fig. 2, the sensitivities for probing these new physics operators by comparing the event rates $|R_1|$ contributed from these operators with the rate $|R_B|$ from the $B$-term (which serves as a necessary criterion as defined above). In Fig. 2a, the event rates from $\mathcal{L}_{4,5}$ are larger than that from the $B$-term when $E > 600$ GeV, while those from $\mathcal{L}_{3,9,11,12}$ can exceed the rate from $B$ only if $E > 860$ GeV. In Fig. 2b, the rates from $\mathcal{L}_{(2)^7}$ and $\mathcal{L}_{1,2,8,13,14}$ are all below the rate from $B$ for a wide range of energy up to about

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$^3$ We clarify that the theoretical criterion (3) is necessary but not sufficient at the event rate level, since the leading $B$-term, as an intrinsic background to any strong $V_L-V_L$ scattering process, denotes a universal part of the full backgrounds [14, 15]. The sufficiency will of course require detailed numerical analyses on the detection efficiency for suppressing the full backgrounds to observe the specific decay mode of the final state (as discussed in Ref. [15]). This is beyond our present first step global analysis and will be left to a future detailed numerical study with this work as a useful guideline.
It is well-known that the validity of the EW A requires $\ell_n \simeq O(1)$, the probe of $L_{4,5}$ is most sensitive, that of $L_{3,9,11,12}$ is marginally sensitive, and that of $L^{(2)}_{1,2,8,13,14}$ is insensitive. In this case, a precise test of the marginal operators $L_{3,9,11,12}$ via the $W^+W^+$ channel requires including the $B$-term in calculating the weak-boson scattering amplitudes which have to be obtained from a full calculation beyond the EWA $^3$ It also implies that a higher luminosity of the collider is needed for probing these operators via the $W^+W^+$ productions. For the case with $\ell_n \simeq O(5 \sim 10)$, the probe of $L_{3,9,11,12}$ can become sensitive, while $L^{(2)}$ and $L_{1,2,8,13,14}$ still cannot be sensitively probed in the $W^+W^+$ channel. A similar conclusion holds for $W^-W^-$ channel except that its event rate is lower by about a factor of $3 \sim 5$ in the TeV region since the quark luminosity for producing $W^-W^-$ pairs is smaller than that for $W^+W^+$ pairs in pp collisions.

Next we compute the event rates for the important $q\bar{q}' \to W^+Z$ process. Fig. 3 shows that for $\ell_n \simeq O(1)$, the probe of $L_{3,11,12}$ are sensitive when $E > 750$ GeV, while that of $L_{8,9,14}$ are marginally sensitive when $E > 950$ GeV. The probe of $L^{(2)}$ becomes marginally sensitive when $E > 1.4$ TeV, and that of $L_{1,2,13}$ is insensitive for $E < 1.9$ TeV. (We note that $L_1$ and $L^{(2)}$ can be better measured at the low energy experiments through $S$ and $T$ parameters, respectively, while $L_{13,14}$ can be more sensitively probed via $e^-\gamma \to \nu eW_L^{-}Z_{0}^{0}$, $e^-W_{L}^{-}W_{L}^{+}$ processes at the future TeV linear collider $^{[15]}$.) The event rate for $q\bar{q}' \to W^+Z$ is slightly higher than that of $q\bar{q}' \to W^-Z$ by about a factor of 1.5 (or smaller) due to the higher quark luminosity for producing $W^+$ bosons in pp collisions. Hence, the similar conclusion holds for the $q\bar{q}' \to W^-Z$ process. Comparing the above $W^\pm W^\pm \to W^\pm W^\pm$ fusions and $q\bar{q}' \to W^\pm Z$ annihilations, we see that these two kinds of processes are complementary to each other in probing the effective operators of the EWCL (1).

In summary, the analyses presented in this paper are consistently performed based upon the electroweak power counting rule (5) combined with the effective-W method. In Table 1 and 2, the sensitivity classifications are summarized at the level of the $S$-matrix elements and for all $V^aV^b \to V^cV^d$ and $q\bar{q}' \to V^aV^b$ processes according to the electroweak power counting hierarchy (6). Estimates $^5$ on the event rates at the 14 TeV LHC with an integrated luminosity of 100 fb$^{-1}$ are given for both $W^+W^+ \to W^+W^+$ (cf. Fig. 2) and $q\bar{q}' \to W^+Z^0$ (cf. Fig. 3) channels, which are shown to be complementary in probing the operators in (1). By these, we give a clear physical picture for globally classifying all bosonic effective operators to probing the underlying EWSB mechanism. This provides a useful guideline for future detailed numerical computations and analyses. The extension of our analysis to future linear colliders are given in Ref. $^{[18]}$.

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$^4$ It is well-known that the validity of the EWA requires $M_{VV} \gg 2M_W$ and the scattering angles big enough to be away from kinematic singularities $^{[14]}$ $^{[15]}$. The condition $M_{VV} \gg 2M_W$ shows that the EWA has a precision similar to that of the ET. The $V_T-V_L$ interference term ignored in the usual EWA $^{[4]}$ is also of the same order as the $B$-term in (2a) $^{[4]}$. The sole purpose of a recent paper (hep-ph/9502309) by A. Dobado et al was to avoid the ET, but still within the usual EWA, for increasing the calculation precision and extending the results to lower energy regions. This approach is, however, inconsistent because both the ET and EWA are valid only in the high energy regime and have similar precisions as explained above.

$^5$ cf. footnote-3.
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Table Captions

Table 1. Global classification for probing direct and indirect EWSB information at the level of $S$-matrix elements (A). (a)

Notes:
(a) The contributions from $L_{1,2,13}$ are always associated with a factor of $\sin^2 \theta_W$, unless specified otherwise. Also, for contributions to the $B$-term in a given $V_L$-amplitude, we list them separately with the $B$-term specified.
(b) MI = model-independent, MD = model-dependent.
(c) There is no contribution when all the external lines are electrically neutral.
(d) $B_0^{(1)} \simeq T_0[2\pi, v, V_T]$ ($\neq T_0[2\pi^0, v^0, Z_T]$), $B_0^{(3)} \simeq T_0[v, 3V_T]$ ($\neq T_0[v^0, 3Z_T]$).
(e) $T_1[2V_L, 2V_T] = T_1[2Z_L, 2W_T]$, $T_1[2W_L, 2Z_T]$, or $T_1[Z_L, W_L, Z_T, W_T]$. 
(f) $L_2$ only contributes to $T_1[2\pi^\pm, \pi^0, v^0]$ and $T_1[2\pi^\pm, \pi^\pm, v^\pm]$ at this order; $L_{6,7}$ do not contribute to $T_1[3\pi^\pm, v^\pm]$.
(g) $L_{10}$ contributes only to $T_1[\cdots]$ with all the external lines being electrically neutral.
(h) $B_0^{(2)}$ is dominated by $T_0[2V_T, 2v]$ since $T_0[\pi, 2V_T, v]$ contains a suppressing factor $\sin^2 \theta_W$ as can be deduced from $T_0[\pi, 3V_T]$ times the factor $\sin \alpha = O \left( \frac{\Lambda}{M_W} \right)$.
(i) Here, $T_1[2W_L, 2W_T]$ contains a coupling $e^4 = g^4 \sin^4 \theta_W$.
(j) $L_2$ only contributes to $T_1[3\pi^\pm, v^\pm]$.
(k) $L_{1,13}$ do not contribute to $T_1[2\pi^\pm, 2v^\pm]$.

Table 2. Global classification for probing direct and indirect EWSB information at the level of $S$-matrix elements (B). (a)

Figure Captions

Fig. 1. Comparison with the calculations of Ref. [13] up to 1-loop for $\sqrt{S} = 40$ TeV. The solid and dashed lines are given by our power counting analysis and Ref. [13], respectively. ($R_{\alpha\beta\gamma\delta}(\pm) = R_{\alpha\beta\gamma\delta}(0) \pm |R_{\alpha\beta\gamma\delta}(1)|$ and $R_{\alpha\beta}(\pm) = R_{\alpha\beta}(0) \pm |R_{\alpha\beta}(1)|$, where $R_{\alpha\beta\gamma\delta}(\ell)$ and $R_{\alpha\beta}(\ell)$ are explained in the text.)

Fig. 2. Sensitivities of probing $\mathcal{L}^{(2)\prime}$ and $\mathcal{L}_{1\sim14}$ in the $W^+W^+$ channel at the 14 TeV LHC.

Fig. 3. Sensitivities of probing $\mathcal{L}^{(2)\prime}$ and $\mathcal{L}_{1\sim14}$ in the $q\bar{q}' \rightarrow W^+Z^0$ channel at the 14 TeV LHC.
Table 1. Global classification for probing direct and indirect EWSB information at the level of S-matrix elements (A). (a)

| Required Precision | Relevant Operators | Relevant Amplitudes | MI or MD (b) |
|--------------------|--------------------|---------------------|--------------|
| $O \left( \frac{E^2}{f^2} \right)$ | $L_{\mi} \equiv L_G + L_F + L^{(2)}$ | $T_0[4V_L](\neq T_0[4Z_L])$ | MI |
| $O \left( \frac{E^2 g^2}{f^2}, g \frac{f}{F} \right)$ | $L_{\mathcal{E}4,5}, L_{\mathcal{E}6,7}, L_{\mathcal{E}10}, L_{\mathcal{E}MI}$ | $T_1[2Z_L, 2W_L], T_1[4Z_L]$ | MD |
| $O \left( \frac{E^2 g^2}{f^2}, g^2 \right)$ | $L_{\mathcal{E}1,2,3,4,5,6,7,9,11,12}, L_{\mathcal{E}1,3,11,12}, L_{\mathcal{E}1,9,1,11,14}, L_{\mathcal{E}4,5,6,7,10}, L_{\mathcal{E}MI}$ | $T_1[3W_L, Z_T], T_1[2Z_L, W_L, W_T]$ | MD |
| $O \left( \frac{E^2 g^2}{f^2}, g^4 \frac{E}{f} \right)$ | $L_{\mathcal{E}MI,1,2,3,4,5,6,7,9,11,12}, L_{\mathcal{E}4,5}, L_{\mathcal{E}6,7,10}, L_{\mathcal{E}2,5,8,9,11,12,14}, L_{\mathcal{E}MI}$ | $T_1[2W_L, 2W_T], T_1[2W_L, Z_T]$ | MD |
| $O \left( \frac{g^2 E}{f^2}, g^4 \frac{f}{g} \right)$ | $L_{\mathcal{E}1,1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}$ | $T_1[4V_L, 3V_T], T_1[2z_L, 2V_T], B^{(0)}_L(3 \pi, v)$ (c,d) | MD |
| $O \left( \frac{g^2 E}{f^2}, g^4 \frac{E}{f} \right)$ | $L_{\mathcal{E}1,1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}$ | $T_1[4V_L, 3V_T], T_1[4V_L], T_1[2Z_L, 2W_L], T_1[2V_L, 2W_T], T_1[4V_L, 2V_T]$ | MD |
| $O \left( \frac{g^2 E}{f^2}, g^4 \frac{E}{f} \right)$ | $L_{\mathcal{E}1,1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}$ | $T_1[3W_L, 2Z_T], T_1[2Z_L, Z_T]$ | MD |
| $O \left( \frac{g^2 E}{f^2}, g^4 \frac{E}{f} \right)$ | $L_{\mathcal{E}1,1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}$ | $T_1[2W_L, 2Z_T], T_1[2W_L, 2Z_T], T_1[4V_L, 2V_T], B^{(0)}_T(3 \pi, v)$ (c,d) | MD |
| $O \left( \frac{g^2 E}{f^2}, g^4 \frac{E}{f} \right)$ | $L_{\mathcal{E}1,1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}$ | $T_1[2W_L, 2Z_T], T_1[2W_L, 2Z_T], T_1[4V_L, 2V_T], B^{(0)}_T(3 \pi, v)$ (c,d) | MD |
| $O \left( \frac{g^2 E}{f^2}, g^4 \frac{E}{f} \right)$ | $L_{\mathcal{E}1,1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}$ | $T_1[2W_L, 2Z_T], T_1[2W_L, 2Z_T], T_1[4V_L, 2V_T], B^{(0)}_T(3 \pi, v)$ (c,d) | MD |
| $O \left( \frac{g^2 E}{f^2}, g^4 \frac{E}{f} \right)$ | $L_{\mathcal{E}1,1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}, L_{\mathcal{E}1,2,3,4,5,6,7,10}$ | $T_1[2W_L, 2Z_T], T_1[2W_L, 2Z_T], T_1[4V_L, 2V_T], B^{(0)}_T(3 \pi, v)$ (c,d) | MD |

(a) See text for details.
Table 2. Global classification for probing direct and indirect EWSB information at the level of $S$-matrix elements (B). (a)

| Required Precision | Relevant Operators | Relevant Amplitudes | MI or MD (b) |
|--------------------|--------------------|---------------------|--------------|
| $O(g^2)$           | $\mathcal{L}_{MI}$ $=$ $\mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}$ | $T_0[q\bar{q}; V_L V_L]$, $T_0[q\bar{q}; V_T V_T]$ | MI           |
| $O \left( g^2 \frac{E^2}{M_W^2}, g^3 \frac{E^2}{M_T^2} \right)$ | $\mathcal{L}_{2,3,9}$, $\mathcal{L}_{3,11,12}$, $\mathcal{L}_{MI}$, $\mathcal{L}_{MI}$, $\mathcal{L}_{MI}$ | $T_1[q\bar{q}; W_L W_L]$, $T_1[q\bar{q}; W_L Z_L]$, $T_0[q\bar{q}; V_L V_T]$, $T_1[q\bar{q}; V_L V_L]$, $B_0^{(1)} \simeq T_0[q\bar{q}; V_T, v]$ | MD, MI, MI |
| $O \left( g^3 \frac{E^2}{M_T^2}, g^4 \frac{E^2}{M_T^2} \right)$ | $\mathcal{L}_{1,2,3,8,9,11\sim 14}$, $\mathcal{L}_{MI}$, $\mathcal{L}_{MI}$ | $T_1[q\bar{q}; V_L V_T]$, $T_1[q\bar{q}; V_L V_T]$, $B_0^{(0)} \simeq T_0[q\bar{q}; \pi, v]$ (c) | MD, MI |
| $O \left( g^2, g^4 \frac{L^2}{M_T^2} \right)$ | $\mathcal{L}^{(2)'}$, $\mathcal{L}_{1,2,3,8,9,11\sim 14}$, $\mathcal{L}_{MI}$ | $T_1[q\bar{q}; V_L V_L]$, $T_1[q\bar{q}; V_T V_T]$, $B_1^{(1)} \simeq T_1[q\bar{q}; \pi, v]$ | MD, MD, MI |

(a) The contributions from $\mathcal{L}_{1,2,13}$ are always associated with a factor of $\sin^2 \theta_W$, unless specified otherwise.

$\mathcal{L}_{4,5,6,7,10}$ do not contribute to the processes considered in this table. Also, for contributions to the $B$-term in a given $V_L$-amplitude, we list them separately with the $B$-term specified.

(b) MI = model-independent, MD = model-dependent.

(c) Here, $B_0^{(0)}$ is dominated by $T_0[q\bar{q}; 2v]$ since $T_0[q\bar{q}; \pi, v]$ contains a suppressing factor $\sin^2 \theta_W$ as can be deduced from $T_0[q\bar{q}; \pi V_T]$ times the factor $v^0 = O \left( \frac{M_W}{M_T} \right)$.  

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\[ \text{Fig. 1.} \]
Fig. 2.

\[ W^*W^* \rightarrow W^*W^* \]

\( (l_i = O(1)) \)
$q\bar{q}' \rightarrow W^*Z^0$

($\mathcal{L}=O(1)$)

Fig. 3.