Mimetic $F(R)$ gravity: inflation, dark energy and bounce

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We propose the mimetic $F(R)$ theory and investigate the early-time and late-time acceleration in such theory. It is demonstrated that inflation consistent with observable data may be realized in such theory. The reconstruction of realistic $\Lambda$CDM era is also possible as well as the unification of early-time inflation with late-time acceleration or bounce universe. It is stressed that specific universe evolution is governed by mimetic $F(R)$ theory which is different from convenient $F(R)$ gravity. The corresponding examples are presented. Mimetic $F(R)$ gravity is generalized by the addition of the scalar potential in the formulation of convenient $F(R)$ gravity with specific Lagrange multiplier constraint. It is demonstrated that such theory may admit the arbitrary universe evolution via the corresponding choice of the scalar potential and/or function $F(R)$.

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I. INTRODUCTION

During last years it has been proven that modified gravity (for a recent review, see [1]) is quite successful in description of the early-time and late-time acceleration of the universe. Number of viable modified gravity theories has been elaborated. One of the most interesting models of that sort is $F(R)$ gravity which is known to be ghost-free theory like much more complicated ghost-free massive gravity (for recent review, see [2]).

Recently, new approach to GR has been developed in such a way that it respects conformal symmetry as internal degree of freedom. Usually, the parametrization is invariant under the Weyl transformation $\hat{g}_{\mu\nu} \rightarrow e^{\xi(x)} \hat{g}_{\mu\nu}$, the variation over $\hat{g}_{\mu\nu}$ gives the traceless part of the equation. In fact, in case of the Einstein gravity, whose action is given by

$$S = \int d^4x \sqrt{-g} \left( R(\hat{g}_{\mu\nu}, \phi) + \mathcal{L}_{\text{matter}} \right),$$

the variation over $\hat{g}_{\mu\nu}$ gives the traceless part of the Einstein equation:

$$0 = -R(\hat{g}_{\mu\nu}, \phi)_{\mu\nu} + \frac{1}{2} g(\hat{g}_{\mu\nu}, \phi)_{\mu\nu} R(\hat{g}_{\mu\nu}, \phi) + \kappa^2 T_{\mu\nu} + \partial_{\mu} \phi \partial_{\nu} \phi \left( R(\hat{g}_{\mu\nu}, \phi) + \kappa^2 T \right).$$

Here $T$ is the trace of the matter energy-momentum tensor $T_{\mu\nu} = g(\hat{g}_{\mu\nu}, \phi)_{\mu\nu} T_{\mu\nu}$. Eq. (1) shows

$$g(\hat{g}_{\mu\nu}, \phi)_{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi = -1.$$  

freedom. Usually, the variation over the Weyl factor in the metric gives the equation for the trace part, that is, in case of the Einstein gravity, $R + \kappa^2 T = 0$ but due to the parametrization in (1), the variation over $\phi$ gives

$$0 = \nabla \left( g(\hat{g}_{\mu\nu}, \phi)_{\mu\nu} \partial_{\mu} \phi \left( R(\hat{g}_{\mu\nu}, \phi) + \kappa^2 T \right) \right).$$  

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Note that $\nabla_\mu$ is the covariant derivative with respect to $g_{\mu\nu}$. Eq. (5) shows that there can be a wider class of the solutions in the mimetic model if compare with the Einstein gravity. In fact, Eq. (5) effectively induces dark matter \cite{3}, which will be discussed in the following section in case of the $F(R)$ extension of the mimetic model. We should note that in Eqs. (3) and (4), $\hat{g}_{\mu\nu}$ appears only in the combination of $g_{\mu\nu}$ in \cite{1} and therefore $\hat{g}_{\mu\nu}$ does not appear explicitly.

In the present letter we propose new model: mimetic $F(R)$ gravity. This theory seems to be ghost-free like the conventional $F(R)$ gravity and conformally-invariant one. We investigate the early-time and late-time accelerated universe in mimetic $F(R)$ gravity. It is demonstrated that inflation consistent with observable data may be realized within such approach. Furthermore, the reconstruction of LCDM model is also possible as well as unification of early-time inflation with late-time acceleration in the spirit of first proposal in the original $F(R)$ gravity \cite{4}. The example of bounce universe is also constructed. It is important to note that same cosmological evolution is realized by different form of the function $F(R)$ in mimetic theory if compare with convenient $F(R)$ gravity. The explicit examples of such different forms of $F(R)$ are given. Like usual mimetic gravity, the theory under consideration admits the generalization by adding the scalar potential in the formulation with Lagrange multiplier. The accelerating cosmology for such theory is briefly discussed.

II. ACCELERATING UNIVERSE FROM MIMETIC $F(R)$ GRAVITY

Let us start from $F(R)$ gravity, whose action is given by

$$S = \int d^4x \sqrt{-g} \left( F(R) + \mathcal{L}_{\text{matter}} \right).$$

(6)

Here $F(R)$ is some function of the Ricci scalar $R$ and $\mathcal{L}_{\text{matter}}$ is matter Lagrangian. If we parametrize the metric as in \cite{1}, the variation of the metric is given by

$$\delta g_{\mu\nu} = \hat{g}^{\rho\tau} \hat{g}_{\tau\omega} \hat{g}^{\omega\sigma} \partial_\rho \phi \partial_\sigma \phi \hat{g}_{\mu\nu} - \hat{g}^{\rho\tau} \partial_\rho \phi \hat{g}^{\omega\sigma} \hat{g}_{\tau\omega} \hat{g}^{\omega\mu} \partial_\sigma \phi - 2\hat{g}^{\rho\tau} \partial_\rho \phi \delta \hat{g}_{\mu\nu} \hat{g}^{\omega\nu}.$$

(7)

Then in case of $F(R)$ gravity, by using the parametrization of the metric as in \cite{1},

$$S = \int d^4x \sqrt{-g} \left( \hat{g}_{\mu\nu}, \phi \right) \left( F(R(\hat{g}_{\mu\nu}, \phi)) + \mathcal{L}_{\text{matter}} \right),$$

(8)

the variation of the action is given by

$$\delta S = \int d^4x \sqrt{-g} \left( \hat{g}_{\mu\nu}, \phi \right) - g^{\mu\tau} \hat{g}_{\tau\omega} \hat{g}^{\omega\nu} \partial_\rho \phi \partial_\sigma \phi \hat{g}_{\mu\nu} - \hat{g}^{\rho\tau} \partial_\rho \phi \hat{g}^{\omega\sigma} \hat{g}_{\tau\omega} \hat{g}^{\omega\mu} \partial_\sigma \phi - 2\hat{g}^{\rho\tau} \partial_\rho \phi \delta \hat{g}_{\mu\nu} \hat{g}^{\omega\nu}$$

$$\times \left( \frac{1}{2} g_{\mu\nu} F(R(\hat{g}_{\mu\nu}, \phi)) - R(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} F'(R(\hat{g}_{\mu\nu}, \phi)) \right) + \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) F'(R(\hat{g}_{\mu\nu}, \phi))$$

$$- g(\hat{g}_{\mu\nu}, \phi) \nabla(\hat{g}_{\mu\nu}, \phi) F'(R(\hat{g}_{\mu\nu}, \phi)) + \frac{1}{2} T_{\mu\nu} \right)$$

$$= \int d^4x \sqrt{-g} \left( \hat{g}_{\mu\nu}, \phi \right) \left( -g^{\mu\tau} \hat{g}_{\tau\omega} g^{\omega\nu}$$

$$\times \left( \partial_\mu \phi \partial_\nu \phi \left( 2F(R(\hat{g}_{\mu\nu}, \phi)) - R(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) + 3 \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) F'(R(\hat{g}_{\mu\nu}, \phi)) + \frac{1}{2} T \right)$$

$$+ g_{\mu\nu} F(R(\hat{g}_{\mu\nu}, \phi)) - R(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} F'(R(\hat{g}_{\mu\nu}, \phi)) + \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) F'(R(\hat{g}_{\mu\nu}, \phi))$$

$$- g(\hat{g}_{\mu\nu}, \phi) \nabla(\hat{g}_{\mu\nu}, \phi) F'(R(\hat{g}_{\mu\nu}, \phi)) + \frac{1}{2} T_{\mu\nu} \right)$$

$$+ 2 \delta \phi \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) \left( \partial_\mu \phi \left( 2F(R(\hat{g}_{\mu\nu}, \phi)) - R(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) - R(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} F'(R(\hat{g}_{\mu\nu}, \phi))$$

$$- 3 \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) F'(R(\hat{g}_{\mu\nu}, \phi)) + \frac{1}{2} T \right) \right) \right).$$

(9)

Hence, the $F(R)$ equation corresponding to \cite{3} has the following form:

$$0 = \frac{1}{2} g_{\mu\nu} F(R(\hat{g}_{\mu\nu}, \phi)) - R(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} F'(R(\hat{g}_{\mu\nu}, \phi)) + \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) \nabla \left( g(\hat{g}_{\mu\nu}, \phi), \mu_{\nu} \right) F'(R(\hat{g}_{\mu\nu}, \phi))$$
We now assume the FRW space-time with flat spatial part,
\[ ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} dx_i^2, \]
where \( R = 6\dot{H} + 12H^2 \) and \( \phi \) only depends on time \( t \). Due to Eq. (4), we find \( \phi = t \). Then Eq. (11) gives
\[ \frac{C_\phi}{a^3} = 2F(R) - RF'(R) - 3\Box F'(R) + \frac{1}{2} T = 2F(R) - 6\left(\dot{H} + 2H^2\right)F'(R) + 3\frac{d^2 F'(R)}{dt^2} + 9H \frac{dF'(R)}{dt} + \frac{1}{2} (-\rho + 3p). \] (13)
Here \( C_\phi \) is a constant. Then in the second line of Eq. (10), only \((t,t)\) component does not vanish and behaves as \( a^{-3} \) and therefore the solution of Eq. (13) with \( C_\phi \neq 0 \) plays a role of the mimetic dark matter as in [3].

On the other hand the \((t,t)\) and \((i,j)\)-components in (10) give the identical equation:
\[ 0 = \frac{d^2 F'(R)}{dt^2} + 2H \frac{dF'(R)}{dt} - \left(\dot{H} + 3H^2\right) F'(R) + \frac{1}{2} F(R) + \frac{1}{2} p. \] (14)
By combining (13) and (14), we obtain
\[ 0 = \frac{d^2 F'(R)}{dt^2} - H \frac{dF'(R)}{dt} + 2\dot{H} F'(R) + \frac{1}{2} (\rho + \rho) + \frac{4C_\phi}{a^3}. \] (15)
When \( C_\phi = 0 \), the above equations reduce to those in the standard \( F(R) \) gravity, or in other words, when \( C_\phi \neq 0 \), the equation and therefore the solutions are different from those in the standard \( F(R) \) gravity. Note that when \( \rho = p = C_\phi = 0 \), the de Sitter or anti-de Sitter solution, where the scalar curvature \( R \) is a constant \( R = R_0 = 12H_0^2 \) (\( H = H_0 \)), is always a solution if the following equation given by (14) is satisfied,
\[ 0 = -6H_0^2 F'(12H_0^2) + F'(12H_0^2). \] (16)
This situation is not changed from the standard \( F(R) \) gravity.

Now one may study an arbitrary evolution of the scale factor \( a(t) \) and assume the explicit \( a(t) \) dependence of \( \rho \) and \( p \) as for usual perfect fluid. Let us consider the following differential equation:
\[ 0 = \frac{d^2 f(t)}{dt^2} - H(t) \frac{df(t)}{dt} + 2\dot{H}(t)f(t). \] (17)
Let the two solutions of (17) be \( f_1(t) \) and \( f_2(t) \). Then \( F'(R) \), which is the solution of (15), is given by
\[ F'(R(t)) = -f_1(t) \int^t dt' \frac{\gamma(t')f_2(t')}{W(f_1(t'), f_2(t'))} + f_2(t) \int^t dt' \frac{\gamma(t')f_1(t')}{W(f_1(t'), f_2(t'))}, \]
\[ \gamma(t) = -\frac{1}{2} (\rho + \rho) - \frac{4C_\phi}{a^3}. \]
\[ W (f_1, f_2) \equiv f_1(t) f'_2(t) - f_2(t) f'_1(t). \]  

By using the \( t \) dependence of the scalar curvature \( R = R(t) \), we find \( t \) as a function of \( R, t = t(R) \). Hence, one gets the explicit form of \( F'(R) \).

As an explicit example, we consider
\[ H = \frac{H_0}{1 + \epsilon H_0 t}, \]
which gives a constant slow roll parameter \( \epsilon = -\frac{\dot{H}}{H^2} \). Then
\[ f_{1,2}(t) = f_{\pm}(t) = f_{\pm}^{(0)} (1 + \epsilon H_0 t)^{\alpha_{\pm}}, \quad \alpha_{\pm} \equiv \frac{1 + \frac{1}{2} \pm \sqrt{(1 + \frac{1}{2})^2 + \frac{2}{r}}}{2}. \]

Here \( f_{\pm}^{(0)} \) are constants. We now assume \( p \) and \( \rho \) are given by the perfect fluid whose equation of state parameter is \( w \), that is, \( p = w\rho \). Then by using (18), we find
\[ F(R) = A R^{-\frac{3(1+w)}{2}} + B R^{-\frac{3}{2}} + C_+ R^{-\frac{3}{2}} + C_- R^{-\frac{3}{2}}. \]

Here \( A, B, \) and \( C_{\pm} \) are constant. The constant \( A \) is proportional to the energy density and \( B \) is proportional to \( C_{\phi} \) but \( C_{\pm} \) can be arbitrary and as a special case \( C_{\pm} \) may vanish. Therefore in the standard \( F(R) \) gravity \( B = 0 \) but in the mimetic \( F(R) \) theory, \( B \) should not vanish. Then if we do not include the matter, that is, \( A = 0 \), the simplest action in the standard \( F(R) \) gravity could be
\[ F(R) = C_+ R^{-\frac{3}{2}} \quad \text{or} \quad F(R) = C_- R^{-\frac{3}{2}}. \]

but the simplest action in the mimetic \( F(R) \) gravity could be
\[ F(R) = B R^{-\frac{3}{2}}. \]

Therefore for the identical evolution of the universe, the corresponding forms of \( F(R) \) can be different in the standard \( F(R) \) gravity and the mimetic \( F(R) \) gravity, in general.

Other slow roll parameters are given by
\[ \eta \sim \frac{3\dot{H}}{2H^2} = \frac{3}{2} \epsilon, \quad \xi^2 \sim \frac{3\dot{H}^2}{2H^4} = \frac{3}{2} \epsilon^2, \]
and also
\[ n_s - 1 \sim -3\epsilon, \quad \alpha_s \sim -3\epsilon^2. \]

The tensor-to-scalar ratio is given by
\[ r = 16\epsilon, \]

The Planck analysis \[5, 6\] gives \( n_s = 0.9603 \pm 0.0073 (68\% \text{ CL}) \) and \( \alpha_s = -0.0134 \pm 0.0090 (68\% \text{ CL}) \) by using the data of the Planck and WMAP \[7, 10\]. We should note that the sign of \( \alpha_s \) is negative at 1.5\( \sigma \) level. We also find \( r < 0.11 (95\% \text{ CL}) \). The result of the BICEP2 experiment, however, gives \( r = 0.20^{+0.07}_{-0.05} (68\% \text{ CL}) \) \[11\] (see, e.g., Refs. \[12, 13\] for recent discussions). If \( n_s = 0.9603 \), Eq. (25) gives \( \epsilon = 0.0132 \) and therefore \( r = 0.2117 \), which might be consistent with the observed data.

In case of the late-time \( \Lambda \)CDM era, the scale factor is
\[ a(t) = A \sinh^{\frac{3}{2}} (\alpha t), \]
with constants \( A \) and \( \alpha \), Eq. (17) has the following form:
\[ 0 = \frac{d^2 f}{dt^2} - \frac{3}{2} \alpha \coth (\alpha t) \frac{df}{dt} - \frac{3\alpha^2}{\sinh^2 (\alpha t)} f. \]
By changing the variables by

\[ x \equiv -\frac{1}{\sinh^2(\alpha t)}, \]  

(29)

Eq. (28) becomes hypergeometric differential equation,

\[ 0 = x (1 - x) \frac{d^2 f}{dx^2} + \left\{ \gamma - (\alpha + \beta + 1) x \right\} \frac{df}{dx} - \alpha \beta f, \]

\[ \gamma = 7, \quad \alpha + \beta + 1 = 9, \quad \alpha \beta = -\frac{3}{4}, \]  

(30)

whose solutions are given by the hypergeometric functions:

\[ f_1 = {}_2F_1(\alpha, \beta; \gamma; x), \quad f_2 = (1 - x)^{1-\gamma} {}_2F_1(1 + \alpha - \gamma, 1 + \beta - \gamma, 2 - \gamma; x). \]  

(31)

We should note that the scalar curvature is given by

\[ R = 9\alpha^2 \left(3 + \frac{2}{\sinh^2(\alpha t)}\right), \]  

(32)

and therefore

\[ x = \frac{3}{2} - \frac{R}{18\alpha^2}. \]  

(33)

Because

\[ \int dx {}_2F_1(\alpha, \beta, \gamma; x) = \frac{\gamma - 1}{(\alpha - 1)(\beta - 1)} {}_2F_1(\alpha - 1, \beta - 1, \gamma - 1; x), \]  

(34)

when \( \rho = \rho = C_{\phi} = 0 \) as a special case, by using the first solution in (31), we find

\[ F(R) = F_0 {}_2F_1(\alpha - 1, \beta - 1, \gamma - 1; \frac{3R}{18\alpha^2}). \]  

(35)

with a constant \( F_0 \). The obtained expression (35) is not changed from that in [14] because we have considered the case that \( \rho = \rho = C_{\phi} = 0 \). When \( C_{\phi} \neq 0 \), the form of \( F(R) \) is different from that of the standard \( F(R) \) gravity although the explicit form becomes very complicated. The explicit form is given in Appendix. Thus, we demonstrated that realistic dark energy epoch may be obtained from the mimetic \( F(R) \) gravity different from convenient \( F(R) \).

We may reconstruct the evolution, which describes both of inflation and the recent accelerating expansion of the universe:

\[ H = H_0 \left(1 + \epsilon \left(\frac{t}{t_0}\right)^2\right). \]  

(36)

When \( t \to 0 \), \( H \) behaves as

\[ H \sim H_0 \left(1 - (1 - \epsilon) \left(\frac{t}{t_0}\right)^2 + \mathcal{O}\left(\left(\frac{t}{t_0}\right)^4\right)\right), \]  

(37)

and \( H \) goes to a constant \( H_0 \) and e-foldings number \( N \) is given by

\[ N \sim H_0 t_0. \]  

On the other hand, when \( t \) is large, we find

\[ H \sim \epsilon H_0 \left(1 + \left(\frac{1}{\epsilon} - 1\right) \left(\frac{t_0}{t}\right)^2 + \mathcal{O}\left(\left(\frac{t_0}{t}\right)^4\right)\right), \]  

(38)

and therefore \( H \) goes to a constant \( \epsilon H_0 \). It is rather difficult to solve (17) explicitly. One can, however, solve (17) when \( t \to 0 \) or \( t \to \infty \). When \( t \to 0 \), a solution of (17) is given by

\[ f(t) = f_0 \left(1 - \frac{4(1 - \epsilon)}{H_0 t_0} \left(\frac{t}{t_0}\right) - 2(1 - \epsilon) \left(\frac{t}{t_0}\right)^2 + \mathcal{O}\left(\left(\frac{t}{t_0}\right)^3\right)\right). \]  

(39)
On the other hand, when \( t \to \infty \), we find
\[
f(t) = f_\infty \left( 1 + 2 \left( \frac{1}{\epsilon} - 1 \right) \left( \frac{t_0}{t} \right)^2 + \mathcal{O} \left( \left( \frac{t_0}{t} \right)^3 \right) \right).
\]
(40)

When \( t \to 0 \), \( R \) is given by
\[
R = 12H_0^2 - \frac{12H_0 (1 - \epsilon) t}{t_0^3} + \mathcal{O} \left( \left( \frac{t}{t_0} \right)^3 \right),
\]
and \( f(t) \) in (39) has the following form
\[
f(t) \sim f_0 \left( 1 - \frac{R - 12H_0^2}{3H_0^2} \right).
\]
(42)

Then in case that \( \rho = p = C_\phi = 0 \), we find
\[
F(R) \sim f_0 \left( R - 12H_0^2 - \frac{(R - 12H_0^2)^2}{6H_0^2} \right) + \text{const}.
\]
(43)

On the other hand, when \( t \to \infty \), we find
\[
R = 12H_0^2 \left( 1 + 2 \left( \frac{1}{\epsilon} - 1 \right) \left( \frac{t_0}{t} \right)^2 \right) + \mathcal{O} \left( \left( \frac{t}{t_0} \right)^4 \right),
\]
and therefore Eq. (40) gives
\[
f(t) = f_\infty R \frac{e^{-\alpha t^2}}{12\epsilon^2 H_0^2},
\]
(45)
and
\[
F(R) \sim f_\infty R^2 \frac{e^{-\alpha t^2}}{24\epsilon^2 H_0^2} + \text{const}.
\]
(46)

The obtained expressions in (43) and (46) are not changed from those in the standard \( F(R) \) gravity because we are considering the case that \( \rho = p = C_\phi = 0 \). When \( C_\phi \neq 0 \), the form of \( F(R) \) is different from that if the standard \( F(R) \) gravity. We do not present it explicitly because it has very complicated form. Thus, the possibility to unify inflation with dark energy in the mimetic \( F(R) \) gravity is possible following the first proposal of Ref. [6].

We finally consider the bouncing universe (for review, see [15]), whose scale factor is given by
\[
a(t) = e^{\alpha t^2},
\]
(47)
with a constant \( \alpha \). Then Eq. (17) has the following form:
\[
0 = \frac{d^2 f(t)}{dt^2} - 2\alpha t \frac{df(t)}{dt} + 4\alpha f(t),
\]
whose solutions are given by
\[
f_1(t) = f_1^{(0)} \left( t^2 - \frac{1}{2\alpha} \right), 
\quad f_2(t) = f_2^{(0)} \left( t^2 - \frac{1}{2\alpha} \right) \int t^4 dt' \frac{e^{-\alpha t'^2}}{(2\alpha t'^2 - 1)},
\]
(49)

Here \( f_1^{(0)} \) and \( f_2^{(0)} \) are constants. Because it is difficult to give the explicit expression for (48), we again consider the simple case where \( \rho = p = C_\phi = 0 \). Then we find
\[
F(R) = F_0 \left( \frac{R^2}{\alpha} - 72R + 144\alpha \right),
\]
with a constant \( F_0 \), which reproduces the expression in [10]. Thus, the possible occurrence of bouncing universe is also demonstrated.
III. DISCUSSION.

In summary, following the idea of Ref. 3 we presented mimetic \( F(R) \) gravity. It is demonstrated that such theory being conformally invariant one, admits the inflation, dark energy, unification of inflation with dark energy as well as bounce much in the same way as convenient \( F(R) \) gravity. Note that interpretation of mimetic dark matter is the same as in usual mimetic gravity. Note also that convenient \( F(R) \) gravity is known to be ghost-free. Usually, the addition of Lagrange multiplier constraint does not violate the ghost-free property. Hence, we expect that mimetic as in usual mimetic gravity. Note also that convenient \( F(R) \) gravity under consideration is ghost-free. Nevertheless, this conjecture should be verified in hamiltonian formulation.

It might be interesting if we add a potential term for \( \phi \) to the action \( 2 \)

\[
S = \int d^4x \sqrt{-g} \left( \frac{R(g_{\mu\nu}, \phi)}{2\kappa^2} - V(\phi) + \mathcal{L}_{\text{matter}} \right).
\]

Due to the constraint in \( 1 \), one can identify \( \phi \) with time in the FRW space-time \( 12 \). Therefore, with the action \( 3 \) one effectively obtains time-dependent energy density, which could give quite an interesting cosmology.

In \( 4 \), it has been proposed that instead of parameterizing the metric as in \( 1 \), we may impose the condition \( 4 \) via the Lagrange multiplier field \( \lambda \) addition:

\[
S = \int d^4x \sqrt{-g} \left( \frac{R(g_{\mu\nu})}{2\kappa^2} - V(\phi) + \lambda (g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + 1) + \mathcal{L}_{\text{matter}} \right).
\]

Anyway, one can identify \( \phi \) with time in the FRW space-time \( 12 \), and the potential \( V(\phi) = V(t) \) gives a time-dependent energy density. Hence, one may realize arbitrary evolution of the expansion of the universe by adjusting \( V(\phi) \).

Then instead of \( 52 \), we may consider the following action of mimetic \( F(R) \) gravity with scalar potential:

\[
S = \int d^4x \sqrt{-g} \left( F(R(g_{\mu\nu})) - V(\phi) + \lambda (g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + 1) + \mathcal{L}_{\text{matter}} \right).
\]

This action is of the sort of modified gravity with Lagrange multiplier constraint \( 17 \). By the variation of the action \( 53 \) with respect to the metric, one obtains

\[
0 = \frac{1}{2} g_{\mu\nu} F'(R) - R_{\mu\nu} F'(R) + \nabla_{\mu} \nabla_{\nu} F'(R) - g_{\mu\nu} \Box F'(R)
+ \frac{1}{2} g_{\mu\nu} \left( -V(\phi) + \lambda (g^{\sigma\tau} \partial_{\sigma} \phi \partial_{\tau} \phi + 1) \right) - \lambda \partial_{\mu} \phi \partial_{\nu} \phi + \frac{1}{2} T_{\mu\nu}.
\]

On the other hand, by the variation with respect to \( \phi \), we get

\[
0 = -2 \nabla^\mu (\lambda \partial_\mu \phi) - V'(\phi).
\]

By construction, the variation with respect to \( \lambda \) gives,

\[
0 = g^{\mu\sigma} \partial_\mu \phi \partial_\sigma \phi + 1.
\]

In the FRW space-time, assuming that \( \phi \) depends only on time coordinate \( t \), Eqs. \( 55 \), \( 56 \), and \( 57 \) are:

\[
0 = -F(R) + 6 \left( \dot{H} + H^2 \right) F'(R) - 6H \frac{dF'(R)}{dt} - \lambda \left( \dot{\phi}^2 + 1 \right) + V(\phi) + \rho,
\]

\[
0 = F(R) - 2 \left( \dot{H} + 3H^2 \right) + 2 \frac{dF'(R)}{dt^2} + 4H \frac{dF'(R)}{dt} - \lambda \left( \dot{\phi}^2 - 1 \right) - V(\phi) + p,
\]

\[
0 = \frac{d}{dt} \left( \lambda \dot{\phi} \right) + 6H \lambda \dot{\phi} - V'(\phi),
\]

\[
0 = \ddot{\phi}^2 - 1.
\]

Eq. \( 59 \) shows that \( \phi \) can be identified as the time coordinate: \( \phi = t \). Then Eq. \( 58 \) can be rewritten as

\[
0 = F(R) - 2 \left( \dot{H} + 3H^2 \right) + 2 \frac{d^2F'(R)}{dt^2} + 4H \frac{dF'(R)}{dt} - V(\phi = t) + p.
\]
Therefore in case that matter contribution can be neglected \((p = \rho = 0)\), we find
\[
V(t) = F(R) - 2 \left( \dot{H} + 3H^2 \right) + 2 \frac{d^2 F'(R)}{dt^2} + 4H \frac{dF'(R)}{dt} .
\] (62)

Taking \(\rho = 0\), Eq. (57) can be solved with respect to \(\lambda\):
\[
\lambda(t) = -\frac{1}{2} F(R) + 3 \left( \dot{H} + H^2 \right) F'(R) - 3H \frac{dF'(R)}{dt} .
\] (63)

Hence, Eq. (59) is automatically satisfied. Then by choosing the potential \(V(\phi)\), we can construct a model which reproduces arbitrarily given evolution \(H\). Doing this one can work with viable \(F(R)\) gravities [1]. For instance, we can easily get the same accelerating behavior as in the previous section. Note that for given scalar potential one can reconstruct function \(F(R)\) to reproduce the arbitrarily given universe evolution. Of course, also using the arbitrary form of the function \(V(\phi)\) one can get accelerating universe for specific \(F(R)\) gravity which basically does not admit such evolution or admits totally different accelerating expansion. The detailed study of accelerating cosmology in mimetic \(F(R)\) gravity with scalar potential will be done elsewhere.

Final remark is in order. The method to make the conformal symmetry to be the internal property of theory via the corresponding parametrization of the metric seems to be quite wide. It would be interesting to apply it to higher-derivative (quantum) dilaton gravity of Ref. [18] what may make the bridge between quantum gravity and mimetic dark matter.

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Appendix A: Explicit form of \(F(R)\) when \(C_\phi \neq 0\)

In this appendix, we give an explicit form of \(F(R)\) when \(C_\phi \neq 0\) for the model in [27]. First we rewrite Eq. (18) in terms of \(x\) in [29] instead of \(t\) as follows,
\[
F'(R(t)) = -f_1(x) \int^{\gamma} dt' \frac{\gamma(x') f_2(x')}{\tilde{W}(f_1(s'), f_2(x'))} + f_2(x) \int^{\gamma} dx' \frac{\gamma(x') f_1(x')}{\tilde{W}(f_1(x'), f_2(x'))}.
\] (A1)

The scale factor \(a\) is also expressed in terms of \(x\),
\[
a = A \left( \frac{1}{x} \right)^{\frac{3}{4}} .
\] (A2)

For simplicity, we consider the case \(\rho = p = 0\). Then by using (31) and formula
\[
\frac{d_2 F_1 (\alpha, \beta, \gamma; x)}{dx} = \frac{\alpha\beta}{\gamma} 2 F_1 (\alpha + 1, \beta + 1, \gamma + 1; x) ,
\] (A3)

we find more explicit form of \(F(R)\) corresponding to [27] as follows,
\[
F(R) = C_\phi A^3 \int^{x} dx' \frac{\gamma(x')}{1 + \frac{R}{18\alpha^2}} \left[ -2 F_1 \left( \alpha, \beta; 3 \frac{1}{2} - \frac{R}{18\alpha^2} \right) (1 - x')^{1 - \gamma} 2 F_1 (1 + \alpha - \gamma, 1 + \beta - \gamma, 2 - \gamma; x') 
+ \left( 1 - 3 \frac{1}{2} + \frac{R}{18\alpha^2} \right)^{1 - \gamma} 2 F_1 \left( 1 + \alpha - \gamma, 1 + \beta - \gamma, 2 - \gamma; 3 \frac{1}{2} - \frac{R}{18\alpha^2} \right) 2 F_1 (\alpha, \beta; x') \right]
\times \left[ 2 F_1 (\alpha, \beta; x') \left( (1 - \gamma) (1 - x')^{-\gamma} 2 F_1 (1 + \alpha - \gamma, 1 + \beta - \gamma, 2 - \gamma; x') \right) \right]
\]
\begin{align}
&\frac{(1 + \alpha - \gamma)}{2 - \gamma} (1 + x')^{1-\gamma} \, _2F_1 (2 + \alpha - \gamma, 2 + \beta - \gamma, 3 - \gamma; x') \\
&- \frac{\alpha \beta}{\gamma} \, _2F_1 (\alpha + 1, \beta + 1, \gamma + 1; x') (1 - x')^{1-\gamma} \, _2F_1 (1 + \alpha - \gamma, 1 + \beta - \gamma, 2 - \gamma; x') \bigg]^{-1}.
\end{align}

(A4)

Here we have used (33). Then the obtained mimetic $F(R)$ (A4) is completely different from that in case $C_\phi = 0$ in (35).

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