K-essential Leptogenesis

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K-essence is a possible candidate for dark energy of the Universe. In this paper we consider couplings of k-essence to the matter fields of the standard electroweak theory and study the effects of the cosmological CPT violation induced by the CPT violating Ether during the evolution of the k-essence scalar field on the laboratory experiments and baryogenesis. Our results show that the matter and antimatter asymmetry can be naturally explained via leptogenesis without conflicting with the experimental limits on CPT violation test. The mechanism for baryogenesis proposed in this paper provides a unified picture for dark energy and baryon matter of our Universe and allows an almost degenerate neutrino mass pattern with a predicted rate on the neutrinoless double beta decays accessible to the experimental sensitivity in the near future.

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There are strong evidences that the Universe is spatially flat and accelerating at the present time $\Omega$. The simplest account of this cosmic acceleration seems to be a remnant small cosmological constant, however many physicists are attracted by the idea that a new form of matter, usually called dark energy [2] is causing the cosmic accelerating. A simple candidate for dark energy is a scalar field (or multi scalar fields), quintessence [3, 4, 5, 6] which includes a canonical kinetic term and a potential term in the Lagrangian. Another one is a scalar field with non-canonical kinetic terms which is called in the literature as k-essence [7, 8]. Differing from quintessence, for k-essence the accelerating expansion of the Universe is driven by its kinetic rather than potential energy.

Being a dynamical component, the scalar field dark energy is expected to interact with the ordinary matters. There are many discussions on the explicit couplings of quintessence to baryons, dark matter and photons [9, 10, 11, 12], however as argued in Refs. [9, 10] for most of the cases the couplings are strongly constrained. But there are exceptions. For example, Carroll [2] has considered an interaction of form $Q F_{\mu \nu} F^{\mu \nu}$ with $F_{\mu \nu}$ being the electromagnetic field strength tensor which has interesting implication on the rotation of the plane of polarization of light coming from distant sources. In addition, if the interaction is not universal, as argued in Ref. [13], a sizable coupling is possible. And the authors of Ref. [13] have also studied the quintessence non-minimally coupled to gravity. Recent data on the possible variation of the electromagnetic fine structure constant reported in [14] has triggered interests in studies related to the interactions between quintessence and the matter fields.

In a recent paper [16] we introduced a type of interaction between quintessence and the ordinary matters, then studied its implication in the generation of the baryon number asymmetry of the universe. Specifically, we have considered a derivative coupling of the quintessence scalar $Q$ to the matter fields,

$$\mathcal{L}_{\text{int}} = \frac{c}{M} \partial_\mu Q J^\mu,$$  \hspace{1cm} (1)

where $M$ is a cut-off scale which, for example could be the scale of Planck or Grand Unified Theory (GUT), $c$ is a coupling constant which characterizes the strength of the quintessence interaction with the ordinary matter fields in the Standard Model of the electroweak theory. $J^\mu$ is the current of the matter fields, which in Ref. [16] we take to be the baryon current or the current of the baryon number minus lepton number for the purposes of baryogenesis and leptogenesis. The Lagrangian (1) involves a derivative and obeys the symmetry $\phi \rightarrow \phi + \text{constant}$, so it will not change the quintessence potential by the quantum corrections. As shown explicitly in Ref. [16], with a modified exponential quintessence potential given in Ref. [18], the ratio of the baryon number to entropy is given by

$$\frac{n_B}{s}|_{T_D} \sim \frac{0.01 c T_D}{M},$$  \hspace{1cm} (2)

where $T_D$ denotes the epoch when the B-violating interactions freeze out.

One silent feature of this scenario for baryogenesis is that the present accelerating expansion and the generation of the matter and antimatter asymmetry of our universe is described in a unified way. Furthermore in this scenario the baryon number asymmetry is generated in thermal equilibrium [19], which violates one of the conditions by Sakharov [20]. This is due to the existence of the CPT violating Ether during the evolution of the quintessence scalar field.

One may wonder if this type of CPT violation will affect the laboratory experiments. At present time the quintessence field is slowly rolling and $\dot{Q}$ is bounded from above. To get the maximal value, $\dot{Q}$, note that $\frac{1}{2} Q^2$ is bounded from below. To get the maximal value, $\dot{Q}$, note that $\frac{1}{2} Q^2 \leq \rho Q \leq \rho_c \sim 10^{-47} [\text{GeV}]^4$. So we have $\dot{Q} \leq 10^{-23} [\text{GeV}]^2$. The experiment of CPT test with a spin-polarized torsion pendulum [21] puts strong limits on the axial vector
background \( b_\mu \) which is defined by \( \mathcal{L} = b_\mu \tilde{e} \gamma^\mu \gamma_5 e \) \cite{22}:

\[
|\tilde{b}| \leq 10^{-28} \text{ GeV} .
\]  

(3)

For the time component \( b_0 \), the bound is relaxed to be at the level of \( 10^{-25} \text{ GeV} \) \cite{22}. Taking the current \( J^\mu \) in Eq. (1) to be \( \tilde{e} \gamma^\mu \gamma_5 e \), \( b_0 \) here corresponds to \( c \frac{Q}{M} \) and it requires that \( b_0 \sim c \times 10^{-23} \text{GeV}^2 \leq 10^{-25} \text{ GeV} \). This puts a constraint on the cutoff scale \( M \), however, if taking \( M \) to be around the Planck or GUT scale the CPT violating effects at the present time is much below the current experimental sensitivity.

In this paper, we will investigate the possibilities of CPT violation and its implications in baryogenesis with k-essence field \( \phi \). We will firstly show that the current experimental limits on the CPT violation rule out the interaction in Eq. (1) for k-essence. Then we propose one type of coupling (see Eq. (19)) and show that the baryon number asymmetry can be explained naturally via leptogenesis. We also study the neutrino mass limits imposed by the leptogenesis of our model.

We start with a brief review on the properties of k-essence:

\[
\mathcal{L}_0 = p(\phi, X) = \frac{1}{\sqrt{2}} \tilde{p}(X) ,
\]  

(4)

where

\[
X = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi .
\]  

(5)

The pressure of the k-essence field is given by \( p(\phi, X) \). The energy momentum tensor has the form of the perfect fluid:

\[
T^\mu_\nu = (\rho + p) U^\mu U_\nu - p \delta^\mu_\nu ,
\]  

(6)

where the energy density and 4-velocity are,

\[
\rho = 2X p, X - p = \frac{1}{\sqrt{2}} (2X \tilde{p} \partial_X (X - \tilde{p}(X))
\]

\[
\equiv \frac{1}{\sqrt{2}} \tilde{p}(X) ,
\]  

(7)

\[
U_\nu = \frac{\partial_\mu \phi}{\sqrt{2} X} ,
\]  

(8)

where \( p, X \) represents the derivative of \( p(\phi, X) \) with respect to \( X \). We will not go into the details on the k-essence model, rather simply point out a notable feature of k-essence, i.e., the attractor-like behavior \cite{22, 24}. In the attractor regimes \( X \approx \text{const.} \). For example, in the radiation dominated era, one has \cite{27},

\[
w(X) = \frac{\tilde{p}(X)}{\tilde{\rho}(X)} = \frac{1}{3} ,
\]

\[
X = \frac{1}{2} \phi^2 = \text{const.} .
\]  

(9)

Furthermore, the stability of the attractor solution requires \cite{7}:

\[
\epsilon^2 = \frac{\tilde{p}_X}{\tilde{\rho}_X} > \frac{1}{3} .
\]  

(10)

For the purpose of baryogenesis, we consider the k-essence field coupled to the matter in the following way:

\[
\mathcal{L}_\text{int} = f(\phi) \partial_\mu \phi J^\mu ,
\]  

(11)

where \( f(\phi) \) is a function of \( \phi \). Taking \( f(\phi) = c/M \), the interaction above is identical to Eq. (1) for quintessence. And to study baryogenesis, we take \( J^\mu \) to be baryon current, then we calculate \( \tilde{\phi} \) and the baryon number asymmetry for a given specific model. For example, we consider the model proposed in Ref. \cite{7}.

\[
\tilde{p}(X) = M_p^6 [ -2.01 + 2 \sqrt{1 + \frac{X}{M_p^4} + 3 \times 10^{-17} \left( \frac{X}{M_p^4} \right)^3 - 10^{-24} \left( \frac{X}{M_p^4} \right)^4} ] ,
\]  

(12)

where \( M_p \) is the reduced Planck mass \((M_p^2 \equiv \frac{3m_{pl}^2}{8\pi}, \) and the Planck mass \( m_{pl} \equiv \sqrt{\frac{8\pi}{3m_{pl}}} = 1.22 \times 10^{19} \text{ GeV} \). In the radiation dominated era, the positive real solutions to Eq. (9) are \( X/M_p^4 \approx 0.02, 9.3 \times 10^9, 1.2 \times 10^9 \). When considering the stability condition \cite{10} and the requirement that the energy density of k-essence is smaller than the critical density of the Universe \( \Omega_\phi < 1 \) (the representation of \( \Omega_\phi \) in the radiation epoch is derived blow, see Eq. (22)), the solution \( X/M_p^4 = 1.2 \times 10^7 \) is obtained.

Using \( X = \frac{1}{2} \tilde{\phi}^2 \), we have

\[
\tilde{\phi} \sim 580m_{pl}^2 .
\]  

(13)

And the ratio of the baryon number density to entropy density is given by

\[
\frac{n_B}{s} \sim 0.01 \left( \frac{\tilde{\phi}}{MT} \right) .
\]  

(14)

Since \( \tilde{\phi} \) is as large as 580\( m_{pl}^2 \), it is quite easy to have \( n_B/s \sim 10^{-10} \) for both \( M \) and \( T \) in the range of 100 GeV to the Planck scale. However we will show below that the large value of \( \tilde{\phi} \) at the present time induces CPT violation at the level which has conflicted already with the experimental limits.

At the present time \( \tilde{\phi} \) is smaller than the value in the radiation epoch given in Eq. \cite{13}, however, it is still quite big. Given the specific model in Eq. \cite{12}, the authors of Ref. \cite{27} have numerically studied the evolution of the equation of state \( w(z) \), and obtained that at the present epoch \( w \approx -0.77 \). Using the relation \( w = \frac{p}{\rho} \), we can evaluate that \( \tilde{\phi} \approx 0.006m_{pl}^2 \). As argued above in the fifth paragraph of this paper, the non-zero value of \( \tilde{\phi} \) will induce CPT violating effect in the electron system if the k-essence field coupled to the electron current.
\( J^\mu = \bar{c} \gamma^\mu \gamma_5 \bar{c} \) through the same way as \( \frac{1}{M} \partial_\mu \phi J^\mu_c \). Since \( \dot{\phi} \sim 0.0069 \xi_{pl}^2 \), even taking the cut-off scale \( M \) to be the Planck mass scale, we get \( b_0 = c \dot{\phi}/m_{pl} \) which is as large as order of \( 0.7 \times 10^{12} \) GeV and will be \( 10^{42} \) times bigger than the current experimental limit unless \( c \) is fine tuned to be smaller than the order of \( 10^{-42} \). With such a small \( c \sim 10^{-42} \), the baryon number asymmetry generated by Eq. (14) will be much smaller than the observational data. In the arguments above we have assumed that the coefficients \( c \) are equal in magnitude for k-essence couplings to the baryon as well as the electron current. This assumption is reasonable and natural in the sense of without fine tuning.

Before considering a different possibility of the function \( f(\phi) \), we note that in the small-\( X \) regime, the Lagrangian in Eq. (4) can be transformed to a Lagrangian of a scalar field with a canonical kinetic energy and an exponential potential. To the leading order of \( X/M^4 \), where \( M \) is the Planck scale \( m_{pl} \) (or the reduced Planck scale \( M_p \) in Eq. (12)), \( \hat{p}(X) \) can be expanded in general as

\[
\hat{p}(X) = aM^6 + bM^6 \frac{X}{M^4},
\]

where \( a, b \) are dimensionless parameters. For the model we quoted above in Eq. (12), \( a = -0.01 \) and \( b = 1 \). As long as \( b > 0 \), Eq. (4) can be changed to

\[
\mathcal{L}_0 = \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - V_0 \exp \left[ -\frac{2\phi}{\sqrt{bM}} \right],
\]

with the help of the field redefinition given below:

\[
\partial_\mu \psi = \sqrt{b} M_\phi \partial_\mu \phi \\
\psi = \sqrt{b} M_\phi \int d\ln \phi.
\]

Since \( \mathcal{L}_0 \) in Eq. (18) describes a scalar field \( \psi \) with an canonical kinetic term, one expects the form of the scalar \( \psi \) coupled to the matter field to be:

\[
\mathcal{L}_{int} = \frac{c}{\bar{c}} \bar{c} \gamma_5 \gamma_\mu J_\mu.
\]

Given Eq. (17), in terms of the scalar \( \phi \), Eq. (18) becomes

\[
\mathcal{L}_{int} = \frac{c}{\phi} \partial_\mu \phi J^\mu,
\]

which corresponds to

\[
f(\phi) = \frac{c}{\bar{c}} \frac{1}{\phi} = \frac{c'}{\phi}.
\]

The arguments above show that if we choose \( f(\phi) \) in the form given in Eq. (20), \( \mathcal{L}_{int} \) in Eq. (14) will coincide with Eq. (11) in the limit of small \( X \) when the kinetic term normalized canonically. We note that in k-essence models, \( X \) is actually large. Since it remains a question how to derive a realistic k-essence model (for example, the one given in [14]) from a fundamental physics, we consider the coupling [19] for a study only in the sense of phenomenology. In the following we will focus on the phenomenological implication of \( \mathcal{L}_{int} \) in Eq. (14) in baryogenesis. Our results show that with \( f(\phi) \) in Eq. (20), the current experimental limits on CPT test can be satisfied and the enough baryon number asymmetry can be generated. For the simplicity of notation, we will drop the prime of \( c \) in the rest of the paper.

Taking \( J^\mu = J^\mu_p \), during the evolution of the spatial flat Friedmann-Robertson-Walker Universe, \( \mathcal{L}_{int} \) in Eq. (14) generates an effective chemical potential \( \mu_b \) for baryons:

\[
f(\phi) = \frac{c}{\bar{c}} \frac{1}{\phi} = \frac{c'}{\phi}.
\]

In thermal equilibrium, the net baryon number doesn’t vanish as long as \( \mu_b \neq 0 \) (when \( T \gg m_b \) [22]):

\[
n_B = \frac{9bT^3}{6\pi^2} \left( \frac{3m_b}{H_0} \right) \left( \frac{H_0}{T} \right)^3,
\]

where \( g_b \) is the number of intrinsic degrees of freedom of baryon. The final ratio of the baryon number to entropy is

\[
\frac{n_B}{s} \mid_{T_D} \approx \frac{15g_b}{4\pi^2g_\star \psi T_D}.
\]

where the cosmic entropy density is \( s = \frac{2\pi^2}{45} g_\star T^3 \) and \( g_\star \) counts the total degrees of freedom of the relativistic particles in the Universe at \( T_D \) when the B-violating interactions freeze out.

The value of the effective chemical potential can be obtained by solving the equation of motion for k-essence field which in general is given by:

\[
(\ddot{p}_{,X} + 2X\dddot{p}_{,XX})\ddot{\phi} + 3H\dddot{p}_{,X}\dot{\phi} - \frac{2}{\phi} (2X\dddot{p}_{,X} - \dddot{p}) = -c\phi(\dot{n}_B + 3Hn_B),
\]

where \( \dddot{p}_{,XX} \) represent the second derivative of \( \dddot{p}(X) \) with respect to \( X \). Given Eqs. (21), (24) and the Hubble constant during the radiation dominated epoch,

\[
H = \frac{1}{2t} = 1.66g_{\star}^{1/2} \frac{1}{\xi_{pl}^2},
\]

the right-hand side of Eq. (24) can be rewritten as

\[
-c\phi(\dot{n}_B + 3Hn_B) = \frac{c^2 g_b T^4}{6} (\ddot{\phi} + H\dot{\phi} - \frac{2}{\phi} X).\]

So if \( T^2 \) is much smaller than \( \dddot{p}_{,X} \) we can drop off the right-hand side of Eq. (24). This means that under this
condition the interaction between baryons and k-essence will not change the main properties of k-essence described in Ref. 7. Since k-essence has the feature of attractor behaviour shown in Eq. 9, we obtain from Eq. 24 that
\[
\frac{\dot{\phi}}{\phi} = 2H = 3.32g^{1/2} \frac{T^2}{m_{pl}} ,
\]
and the fraction of the energy density of k-essence is
\[
\Omega_\phi \equiv \frac{8\pi (\phi, X)}{3H^2 m_{pl}^2} = \frac{8\pi \phi}{H^2 m_{pl}^2} \phi^2 = \frac{8\pi \phi, X}{m_{pl}^2} .
\]
The constraint on \(\Omega_\phi\) by BBN 20 is \(\Omega_\phi < 0.045\). Taking \(\Omega_\phi \sim 10^{-2}\), we have \(\bar{p},X \sim 10^{-4} - 10^{-3}\). In obtaining the equations above, we have assumed that \(\frac{T^2}{\bar{p},X} \ll 1\). To verify this inequality, note that
\[
\frac{T^2}{\bar{p},X} = \frac{T^2}{m_{pl}^2} \frac{n_{pl}}{p,X} \sim \frac{T^2}{m_{pl}^2} (10^3 - 10^4) ,
\]
and one can see that \(T^2\) will be less than \(\bar{p},X\) for \(T < 10^{-9} m_{pl}\). Moreover, for the temperature \(T\) in the range which we are interested in for Baryogenesis, \(T \sim 10^{10}\) GeV (see Eq. 31 below), \(\bar{p},X\) is much smaller than \(\bar{p},X\) and we can safely neglect the right-hand side in solving Eq. 24.

With the value of the effective chemical potential, we arrive at a final expression of the baryon number asymmetry:
\[
\left. \frac{n_B}{s} \right|_{T_D} = \frac{15}{2\pi^2} \frac{g_0 H(T_D)}{g_0 T_D} \sim 1.26 \times 10^{-4} \frac{T_D}{m_{pl}} .
\]
In the calculations above, we have used \(g_0 \sim \mathcal{O}(1)\) and \(g_0 \sim \mathcal{O}(100)\). Taking \(c \sim \mathcal{O}(1)\), \(n_B/s \sim 10^{-10}\) requires the decoupling temperature \(T_D\) to be in the order of:
\[
T_D \sim 10^{-9} m_{pl} \sim 10^{10} \text{GeV} .
\]

A value of \(T_D\) at or larger than \(10^{10}\) GeV can be achieved in GUT easily. However, if the B-violating interactions conserve \(B - L\), the asymmetry generated will be erased by the electroweak Sphaleron 27. In this case \(T_D\) is as low as around 100 GeV and \(n_B/s\) generated will be of the order of \(10^{-18}\). So now we turn to leptogenesis 27, 29. We take \(J^L\) in Eq. 19 to be \(J^{B-L}\). Doing the calculations with the same procedure as above for \(J^L\) we have the final asymmetry of the baryon number minus lepton number
\[
\left. \frac{n_{B-L}}{s} \right|_{T_D} \sim 0.1 c \frac{T_D}{M} .
\]

The asymmetry \(n_{B-L}\) in 32 will be converted to baryon number asymmetry when electroweak Sphaleron \(B + L\) interaction is in thermal equilibrium which happens for temperature in the range of \(10^2\) GeV \(~ 10^{12}\) GeV. \(T_D\) in 32 is the temperature below which the \(B - L\) interactions freeze out.

Now we study the limits on the neutrino masses in our model. In the scenario of leptogenesis studied in this paper, the \(B - L\) number asymmetry is generated in the thermal equilibrium. This requires that the rate of \(B-L\) violating interactions be larger than the Hubble expanding rate for the temperature above \(T_D\).

In the Standard Model of the electroweak theory, \(B - L\) symmetry is exactly conserved, however many models beyond the standard model, such as Left-Right symmetric model predict the violation of the \(B - L\) symmetry. In this paper we take an effective Lagrangian approach and parameterize the \(B - L\) violation by higher dimensional operators. There are many operators which violate \(B - L\) symmetry, however at dimension 5 there is only one operator,
\[
\mathcal{L}_E = \frac{2}{f} l_L l_L \chi + \text{H.c.} ,
\]
where \(f\) is a scale of new physics beyond the Standard Model which generates the \(B - L\) violations, \(l_L\) and \(\chi\) are the left-handed lepton and Higgs doublets respectively.

When the Higgs field gets a vacuum expectation value \(< \chi >\sim v\), the left-handed neutrino receives a Majorana mass \(m_\nu \sim \frac{v}{T_D}^2\).

In the early universe the lepton number violating rate induced by the interaction in 33 is
\[
\Gamma_E \sim 0.04 \frac{T^3}{f^2} .
\]
Since \(\Gamma_E\) is proportional to \(T^3\), for a given \(f\), namely the neutrino mass, \(B - L\) violation will be more efficient at high temperature than at low temperature. Requiring this rate be larger than the Universe expansion rate \(\sim 1.66g_0^{1/2} T^2/m_{pl}\) until the temperature \(T_D\), we obtain a \(T_D\)-dependent lower limit on the neutrino mass:
\[
\sum_i m_i^2 = (0.2 \text{ eV}(\frac{10^{12} \text{GeV}}{T_D})^{1/2})^2 .
\]

Taking three neutrino masses to be approximately degenerated, i.e., \(m_1 \sim m_2 \sim m_3 \sim \bar{m}\) and defining \(\Sigma = 3\bar{m}\), in Fig. 1 we plot the freezing out temperature \(T_D\) as a function of \(\Sigma\). One can see that for \(T_D \sim 10^{10}\) GeV, three neutrinos are expected to have masses \(\bar{m}\) around \(\mathcal{O}(1)\) eV. Numerically, taking \(T_D = 1.0 \times 10^{10}\) GeV, we have \(\bar{m} = 1.2\) eV, for \(T_D = 5.0 \times 10^{10}\) GeV, \(\bar{m} = 0.52\) eV. The current cosmological limit comes from WMAP 31. The analysis of Ref. 31 gives \(\Sigma < 0.69\) eV. Another analysis shows, however that \(\Sigma < 1.0\) eV 32. These limits on the neutrino masses requires \(T_D\) be larger than
2.5 \times 10^{11} \text{ GeV} or 1.2 \times 10^{11} \text{ GeV}. The almost degenerate neutrino masses required by the leptogenesis of this model will induce a rate of the neutrinoless double beta decays accessible for the experimental sensitivity in the near future \cite{15}. Interestingly, a recent study \cite{14} on the cosmological data showed a preference for neutrinos with degenerate masses in this range.

Assuming the k-essence scalar couples to the electron axial current the same as Eq. \cite{19}, we can estimate the CPT-violation effect on the laboratory experiments. From the studies of Ref. \cite{6}, at the present time, our Universe is approaching the k-attractor phase (labeled by the subscript $k$), where $\kappa_{\phi_k} \to 1$ and $w(X_k) = \text{const.} < -\frac{1}{3}$.

So in the near future, one has $\frac{\phi_0}{\phi_k} = \frac{3}{2} (1 + w_k) H_k$. Thus the current value of $\frac{\phi_0}{\phi_k}$ is about $\sim H_0$, and the induced CPT-violating $b_0$ is

$$ b_0 \sim c \frac{\phi_0}{\phi_k} \sim c H_0 \leq 10^{-42} \text{ GeV}, \quad (36) $$

which is much below the current experimental limits.

In summary we have studied in this paper the possibility of baryogenesis in the framework of k-essence as dark energy, and explicitly showed that the baryon number asymmetry, $n_b/s \sim 10^{-10}$ can be generated via leptogenesis. Our scenario provides a unified description for the present accelerating and the generation of the baryon number asymmetry of our universe by the dynamics of k-essence.

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