The core density of dark matter halos: a critical challenge to the \(\Lambda\)CDM paradigm?

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ABSTRACT

We compare the central mass concentration of Cold Dark Matter halos found in cosmological N-body simulations with constraints derived from the Milky Way disk dynamics and from the Tully-Fisher relation. For currently favored values of the cosmological parameters ($\Omega_0 \sim 0.3; \Lambda_0 = 1 - \Omega_0 \sim 0.7; h \sim 0.7; \text{COBE- and cluster abundance-normalized } \sigma_8; \text{Big-Bang nucleosynthesis } \Omega_b$), we find that halos with circular velocities comparable to the rotation speed of the Galaxy have typically three times more dark matter inside the solar circle than inferred from observations of Galactic dynamics. Such high central concentrations of dark matter on the scale of galaxy disks also imply that stellar mass-to-light ratios much lower than expected from population synthesis models must be assumed in order to reproduce the zero-point of the Tully-Fisher relation. Indeed, even under the extreme assumption that all baryons in a dark halo are turned into stars, disks with conventional $I$-band stellar mass-to-light ratios ($M/L_I \sim 2 \pm 1(M/L_I)_\odot$) are about two magnitudes fainter than observed at a given rotation speed. We examine several modifications to the $\Lambda$CDM model that may account for these discrepancies and conclude that agreement can only be accomplished at the expense of renouncing other major successes of the model. Reproducing the observed properties of disk galaxies thus appears to demand substantial revision to the currently most successful model of structure formation.

Subject headings: cosmology: theory – galaxies: formation, evolution – methods: numerical
1. Introduction

Over the past few years, cosmological models based on the paradigm of an inflationary universe dominated by cold dark matter (CDM) have proved remarkably successful at explaining the origin and evolution of structure in the Universe. The free parameters of astrophysical relevance in this modeling are surprisingly few: the current rate of universal expansion, $H_0$; the mass density parameter, $\Omega_0$; the primordial baryon abundance, $\Omega_b$; and the overall normalization of the power spectrum of initial density fluctuations, $\sigma_8$. Over the past few years, limits on the values allowed for these parameters have been consistently refined by improved observational techniques and theoretical insight, and it is widely accepted that a new “standard” model has emerged as the clear front-runner amongst competing models of structure formation.

This model, which we shall call “standard” $\Lambda$CDM, or s$\Lambda$CDM for short, envisions an eternally expanding universe with the following properties (Bahcall et al 1999): (i) matter makes up at present less than about a third of the critical density for closure ($\Omega_0 \sim 0.3$); (ii) a non-zero cosmological constant restores the flat geometry predicted by most inflationary models of the early universe ($\Lambda_0 = 1 - \Omega_0 \sim 0.7$); (iii) the present rate of universal expansion is $H_0 \sim 70$ km s$^{-1}$ Mpc$^{-1}$ ($h = H_0/100$ km s$^{-1}$ Mpc$^{-1} \sim 0.7$); (iv) baryons make up a very small fraction of the mass of the universe ($\Omega_b \approx 0.0125 h^{-2} \sim 0.0255 \ll \Omega_0$); and (v) the present-day rms mass fluctuations on spheres of radius $8 h^{-1}$ Mpc is of order unity ($\sigma_8 \sim 1.1$). The s$\Lambda$CDM model is consistent with an impressive array of well-established fundamental observations: the age of the universe as measured from the oldest stars (e.g., Chaboyer et al 1998), the extragalactic distance scale as measured by distant Cepheids (e.g., Madore et al 1998); the primordial abundance of the light elements (e.g., Schramm & Turner 1998), the baryonic mass fraction of galaxy clusters (e.g., White et al 1993), the amplitude of the Cosmic Microwave Background fluctuations measured by COBE (e.g., Lawrence, Scott & White 1999), the present-day abundance of massive galaxy clusters (e.g., Eke, Cole & Frenk 1996), the shape and amplitude of galaxy clustering patterns (e.g., Wu, Lahav & Rees 1999), the magnitude of large-scale coherent motions of galaxy systems (e.g., Strauss & Willick 1995, Zaroubi et al 1997), and the world geometry inferred from observations of distant
Because its major parameters are fixed, $\Lambda$CDM is an eminently falsifiable model whose predictive power may be used to ascertain its validity on scales different from those used to tune the model. One scale of particular interest is that of individual galaxies, since few observational constraints on these small scales have been used to adjust the parameters of $\Lambda$CDM. This exercise is especially compelling because the dark halo structure found in cosmological N-body simulations of Cold Dark Matter universes seems at odds with dynamical studies of disk galaxies that assign a substantial gravitational role to the disk component (see, e.g., the “maximal disk” solutions of Debattista & Sellwood 1998, and references therein) as well as with rotation curve studies of dark matter-dominated galaxies (Moore 1994, Flores & Primack 1994, McGaugh & De Block 1998, Moore et al 1999, Navarro 1999). These claims are based largely on comparisons of the detailed shape of the rotation curve of very low surface brightness dwarfs with the innermost density profile of simulated dark halos. Unfortunately, the scales where deviations are most pronounced (the inner few kpc) are also the most compromised by numerical uncertainties (most simulations relevant to this problem published to date have gravitational softening scales of order 1-2 kpc). The comparison is thus rather uncertain. For example, Kravtsov et al (1998) have argued, on the basis of simulations similar to those used by the other authors, that CDM halos are actually consistent with the rotation curves of dark matter-dominated disks, a somewhat surprising result that illustrates well, nonetheless, the vulnerability of numerical techniques on scales close to the numerical resolution of the simulations. From a strictly pragmatic numerical standpoint, it would be desirable to circumvent these uncertainties by adopting comparison criteria that are less sensitive to the effects of numerical shortcomings.

One possible choice is to use, rather than the dark matter density profile near the center, the total amount of dark mass within the main body of individual galaxies. For spiral galaxies, this criterion would imply that simulations that can estimate reliably the amount of dark mass within a couple of exponential scalelengths may be safely used for comparison with observations. For bright spirals like the Milky Way this corresponds to radii of about 5-10 kpc, well outside type Ia supernovae (e.g., Perlmutter et al 1999, Garnavich et al 1998), among others.
the region that may be compromised by numerical artifacts in the current generation of N-body experiments.

In this paper we follow this proposal and compare the results of recent very high-resolution simulations of the formation of dark halos in the sΛCDM model with observational constraints on the total dark mass within spiral disks derived from the Tully-Fisher relation and from observations of Galactic dynamics. The numerical setup of the simulations is identical to that described by Navarro, Frenk & White (1997, hereafter NFW97) but the number of particles has been increased more than tenfold. As a result, each simulated sΛCDM halo has of order 250,000 dark matter particles within the virial radius and several thousands within radii comparable to the Sun’s distance from the Galactic center (the “solar circle” $R_o$) so our numerical uncertainties are for all practical purposes negligible.

We show below that comparison between these simulations and observational constraints reveals a severe inconsistency: sΛCDM halos have substantially more mass near the center than the maximum inferred from observations. We argue that this presents a serious challenge to the sΛCDM cosmogony and to many of its likely variants and that possible solutions involve fundamental revision of the basic premises or parameters of the model.

2. Observational Constraints on Dark Mass in Individual Galaxies

2.1. The Milky Way

Kinematic observations of stars and gas in the Galaxy provide tight constraints on the total amount of dark matter within the solar circle, $R_o$. A direct estimate can be made by assuming that the halo is spherically symmetric, $M_{\text{dark}}(r < R_o) = V_{\text{dark}}^2(R_o) R_o / G$, where $V_{\text{dark}}(R_o)$ is the contribution of the dark halo to the circular velocity at $R_o$. This may be obtained by subtracting the disk contribution from the total circular velocity, $V_{\text{dark}}^2(R_o) = V_c^2(R_o) - V_{\text{disk}}^2(R_o)$. For the IAU-sanctioned values of $R_o = 8.5$ kpc and $V_c(R_o) = 220$ km s$^{-1}$, and assuming that the disk potential is well approximated by an exponential disk with scalelength $r_{\text{disk}} = 3.5$ kpc and total
mass $M_{\text{disk}} = 6 \times 10^{10} M_{\odot}$ (Binney & Tremaine 1987), we find

$$M_{\text{dark}}(r < R_{\odot}) = 5.2 \times 10^{10} M_{\odot}. \quad (1)$$

The uncertainty in this determination is hard to assess, although the evidence suggests that the mass in eq. 1 is actually an upper limit to the dark mass inside the solar circle. This is in good agreement with the recent Milky Way mass models of Dehnen & Binney (1998), who find that, within the slightly larger radius of 10 kpc the dark halo accounts for less than about $5-6 \times 10^{10} M_{\odot}$, and perhaps as little as $2.5 \times 10^{10} M_{\odot}$.

The disk contribution to the circular velocity increases with our estimated value of $M_{\text{disk}}$, which in turn depends on: (i) the local density of the disk derived from the vertical kinematics of stars in the solar neighborhood (i.e., from “Oort limit” analysis, $\Sigma_{\odot} \sim 70 M_{\odot} \text{pc}^{-2}$), (ii) on the exponential scalelength of the disk, and (iii) on the solar circle itself, through $M_{\text{disk}} \propto \Sigma_{\odot} r_{\text{disk}}^2 e^{R_{\odot}/r_{\text{disk}}}$. Recent reviews (Reid 1993, Sackett 1997) of available data suggest that the exponential scalelength assumed above may need to be revised downwards by up to 20%, but otherwise leave $\Sigma_{\odot}$ and $R_{\odot}$ largely unchanged from the values assumed above. Such revision would increase the disk mass, leading to values of $M_{\text{dark}}(r < R_{\odot})$ lower than quoted in eq. 1. Indeed, Sackett (1997) concludes that the Milky Way disk may very well be “maximal” once this revision is taken into account. Finally, we note that our procedure neglects the contribution of the Galaxy’s bulge, lending further support to our interpretation of eq. 1 as an upper limit to the dark mass inside $R_{\odot}$.

Figure 1 compares, as a function of halo mass, the dark mass estimate in eq. 1 with the results of simulations of several $\Lambda$CDM halos. Halo masses ($M_{200}$) are measured inside the radius, $r_{200}$, of a sphere of mean density 200 times the critical density for closure, and are typically characterized by the circular velocity at that radius, $V_{200} = (G M_{200}/r_{200})^{1/2} = (10 G H_0 M_{200})^{1/3}$. The reason for this choice is that at $r_{200}$ the circular orbit timescale is approximately equal to the

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Note, however, that Olling & Merrifield (1998), however, argue for $R_{\odot} \sim 7.1$ kpc upon analysis of the role of interstellar gas on the local values of Oort’s constants. Since the circular velocity at $R_{\odot}$ decreases correspondingly, this modification has little effect on our conclusions.
age of the universe; $r_{200}$ thus separates the “virialized” region of the halo from the region where mass shells are infalling into the system for the first time. The relevance of this definition stems from the latter property: only baryons inside $r_{200}$ may contribute to the baryonic mass of the galaxy, since those beyond this radius have yet to reach the center of the halo (White et al 1993).

This property may be used to derive a firm lower limit to the mass of the halo that surrounds the Milky Way, corresponding to the case where the baryonic mass of the disk equals the total baryonic mass inside $r_{200}$: i.e., $M_{\text{disk}} \leq M_{\text{disk}}^{\text{max}} = (\Omega_b/\Omega_0)M_{200}$, implying that $V_{200} \gtrsim [10G H_0 (\Omega_0/\Omega_b) M_{\text{disk}}]^{1/3}$. For the disk mass adopted above and the cosmological parameters appropriate to sΛCDM discussed in §1, we find that

$$V_{200} \gtrsim 130 \text{ km s}^{-1}. \quad (2)$$

We emphasize that this is a strict lower limit to the mass of the halo that surrounds the Milky Way, since experiments show that typically not more than 80% of the baryons within $r_{200}$ are actually accreted into the central disk (Navarro & White 1994, Navarro & Steinmetz 1997). A similar, albeit more stringent, constraint may be derived by comparing the angular momentum of the Milky Way with those of sΛCDM halos. Halos with $V_{200} \leq 150$ km s$^{-1}$ typically have specific angular momenta lower than that of the Milky Way disk (Mo, Mao & White 1998, Syer et al 1999) and are therefore unlikely hosts of the Galaxy. Indeed, baryons typically lose a significant fraction of their angular momentum as they collapse to the disk (Navarro & White 1994, Navarro & Steinmetz 1997) so we conclude that almost certainly the circular velocity of the Milky Way halo must exceed 130-150 km s$^{-1}$.

Figure 1 compares the constraints from eqs. 1 and 2 with the results of our sΛCDM numerical simulations. The comparison shows clearly a major discrepancy between the maximum dark matter inside $R_o$ allowed by observations and the results of the numerical experiments. For example, sΛCDM halos with circular velocities similar to that of the Milky Way disk ($V_{200} \approx V_c(R_o) = 220$ km s$^{-1}$) have about three times more dark mass inside the solar circle than inferred from observations. Even for the extreme case where the halo has the strict minimum circular velocity allowed by eq. 2, the simulations indicate an excess of more than 50% in the dark mass within $R_o$. 
This serious discrepancy only worsens if we take into account that some extra dark material may be drawn inside $R_o$ by the formation of the disk. A rough estimate of the magnitude of this correction can be made by assuming that the halo responds adiabatically to the assembly of the disk; the discrepancy then increases from 50% to almost 80% for the least massive halo allowed by eq. 2. We conclude that halos formed in the sΛCDM scenario are too centrally concentrated to be consistent with observations of the dynamics of the Galaxy.

2.2. The Tully-Fisher relation

It is possible to extend the analysis of the previous subsection to a large fraction of disk galaxies by examining the tight correlation between the total luminosity of galaxy disks and the rotation speed of their gas and stars (the Tully-Fisher relation, Tully & Fisher 1977). Provided that disk mass-to-light ratios and exponential scalelengths can be estimated reliably, it is possible to evaluate the disk contribution to the circular velocity and to apply the same analysis of the previous subsection to derive constraints on the total dark mass contained within the optical radius of the galaxy.

We choose to carry out the analysis at 2.2 exponential scalelengths ($2.2 r_{\text{disk}}$) from the center, since the contribution of exponential disks to the circular velocity peaks there and it is at that radius that Tully-Fisher velocities are typically measured (Courteau 1997). (We note that $R_o \approx 2.4 r_{\text{disk}}$ in the case of the Milky Way, so this choice of radius is similar to that adopted in §2.1.) Constraints on dark masses inside $2.2 r_{\text{disk}}$ depend sensitively on estimates of the total mass associated with galaxy disks. Because of the lack of an Oort-limit analog in external galaxies, we resort to broad-band colors as estimators of the mass-to-light ratio of the stellar disk. Late type spirals such as those that make up the majority of galaxies in Tully-Fisher samples have $B - R$ colors in the range (0.2, 1.0) (Courteau 1999) which, for a galaxy that has been steadily forming stars for $\sim 13$ Gyrs, imply $I$-band mass-to-light ratios of order $(M_{\text{disk}}/L_I) \approx 2 \pm 1(M/L_I)_\odot$. This estimate assumes a Salpeter initial mass function and exponential star formation histories with timescales which vary from $\tau_{\text{SF}} \sim 1$ Gyr to star formation rates that are constant over the age of
the universe. Similar values are obtained using the GISSEL96 models of Bruzual & Charlot (1996) and the PEGASE models of Fioc & Rocca-Volmerange (1997). Uncertainties in this estimate of $(M_{\text{disk}}/L_I)$ are hard to assess, but are unlikely to be larger than about a factor of two, which is the value of the “error” we assume here.\footnote{We note that the same procedure gives $V$-band mass-to-light ratios of about $2(M/L_V)_{\odot}$ for galaxies in the same color range, more than a factor of 2 lower than that derived for the Milky Way disk from the Oort limit (Binney & Tremaine 1987). This reflects the well known Galactic “disk dark matter” content. Assuming that Tully-Fisher galaxies have similar amounts of (presumably baryonic) “disk dark matter” would make all the arguments we develop here even stronger.}

With this caveat, we compute the rotation speeds ($V_{\text{rot}} = V_c(2.2 r_{\text{disk}})$) of hypothetical exponential disk galaxies with $M_{\text{disk}}/L_I = 2(M/L_I)_{\odot}$ assembled at the center of the simulated dark halos shown in Figure 1. Disk radii are chosen so as to satisfy the empirical relation, $r_{\text{disk}} \approx 3(V_{\text{rot}}/200 \text{ km s}^{-1}) h^{-1} \text{ kpc}$ (Courteau 1999, Mo, Mao & White, 1998, Navarro 1999). Since disk radii depend on $V_{\text{rot}}$, an iterative procedure is needed, which we implement as follows. Given a halo of circular velocity $V_{200}$ we assign to it a disk with exponential scalelength derived assuming that $V_{\text{rot}} = V_{200}$. The simulated halo structure and the disk potential are then used to compute a new $V_{\text{rot}}$ estimate, taking into account the “adiabatic” response of the halo to the disk potential (see Navarro, Frenk & White 1996, hereafter NFW96, for details). This new velocity estimate is used to recompute $r_{\text{disk}}$ and the procedure is then iterated until convergence.

The solid-line curves in Figure 2 illustrate the result of applying this procedure to three representative dark halos of different mass (or luminosity, since we assume a constant mass-to-light ratio), as a function of the total mass adopted for the disk of the galaxy. In each case we vary the total disk mass from zero to $M_{\text{disk}}^{\text{max}}$, the maximum value compatible with the baryonic content of the halo. As the disk mass increases, each hypothetical galaxy moves from left to right across the plot. When the disk mass becomes comparable to the dark mass inside $2.2 r_{\text{disk}}$ the curve inches upwards and becomes essentially parallel to the observed Tully-Fisher relation. The increased
potential due to the disk mass and the extra dark mass drawn inside $2.2 \, r_{\text{disk}}$ under the assumption that the halo responds adiabatically to the assembly of the disk contribute similarly to the overall increase in rotation speed over and above that of the original dark halo. The rightmost point of each curve is reached when the mass of the disk equals $M_{\text{disk}}^{\text{max}}$.

It is clear from Figure 2 that, even under the extreme assumption that galaxies contain all available baryons in each halo, simulated disks are almost two magnitudes fainter than observed. Increasing the baryonic mass of a halo has virtually no effect on this conclusion, since in this case model galaxies would just move further along paths approximately parallel to the Tully-Fisher relation, as shown in Figure 2. Disk galaxies assembled inside sΛCDM halos therefore cannot match the observed Tully-Fisher relation, unless one or more of the assumptions in our procedure are grossly in error.

Perhaps the most uncertain step in our argument is the stellar mass-to-light ratio adopted for the analysis. The horizontal “error bar” shown on the starred symbols in Figure 2 indicates the effect on our results of varying the $I$-band mass-to-light ratio by a factor of two from our fiducial value of 2 in solar units. This is not enough to restore agreement with observations, which would require $(M_{\text{disk}}/L_I) \sim 0.4$, a value much too low to be consistent with standard population synthesis models. The vertical “error bars” illustrate the effect of varying the “concentration” of each halo by a factor of two. Even with this large variation in halo structure, our hypothetical disks fail to reproduce the observations.

A second uncertainty comes from the “adiabatic contraction” correction applied to dark halos in order to mimic the halo response to the disk assembly, and one may wonder whether our “adiabatic” contraction assumption is at all correct. This possibility may be checked, however, through direct numerical simulation. We have therefore included gas in the simulations and have

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5The NFW “concentration parameter” is defined as $c = r_{200}/r_s$, where the scale radius, $r_s$ is one of the parameters of the density profile model proposed by NFW96 and NFW97, $\rho(r) \propto (r/r_s)^{-1}(1 + r/r_s)^{-2}$, see those papers for details.
evolved them again using with the simplified treatment of radiative cooling and star formation described in detail in Steinmetz & Navarro (1999). The results are shown as open squares in Figure 2, and are in good agreement with the results of the simple modeling proposed above.

As in the case of the Milky Way discussed in §2.1, the problem can be traced to the large central concentration of sΛCDM halos. Indeed, one way to solve the problem would be to reduce substantially (by a factor of two or three) the dark mass in the innermost few kpc of galaxy-sized dark halos. This would significantly reduce $V_{\text{rot}}$, bridging the gap between model and observations. In terms of the halo density profile model proposed by NFW96 and NFW97, this would be equivalent to reducing the “concentration parameter”, $c$, by a factor of about five. This is the same conclusion reached by Navarro (1999), who advocated that halos with very low “concentration parameters”, $c \lesssim 3$, and stellar mass-to-light ratios as low as $\sim 0.5 \, h (M/L_\odot)$ in low surface brightness galaxies were required in order to match the shapes of disk galaxy rotation curves. We emphasize, however, that the present conclusion is independent of assumptions about the detailed shape of the dark matter density profile, and depends largely on the total dark mass on scales of individual galaxies.

3. Discussion and Conclusions

The analysis of the previous subsection demonstrates conclusively that the central mass concentration of sΛCDM halos is inconsistent with observations of the dynamics of spiral galaxies. Does this argument rule out all models based on the CDM+inflation paradigm or are there plausible modifications to the sΛCDM parameters that may bring the model into agreement with observations?

In principle, there are parameter choices that may help make ΛCDM models consistent with observations of the internal dynamics of galaxies, but the magnitude of the modifications required are, however, uncomfortably large, and come at the expense of other major successes of the model. As discussed by NFW97, the central concentration of dark halos is directly proportional to the
mean matter density of the universe at a suitably defined collapse time. For halos of fixed mass, these collapse times may depend sensitively on the values adopted for the cosmological parameters. However, as discussed by Navarro (1999, see his Figure 8), the combination of parameters needed to reproduce the present-day abundance of galaxy clusters is such that the characteristic densities of galaxy-sized dark halos is approximately independent of $\Omega_0$ and of the value of $H_0$. Indeed, halos in the former “standard” CDM model ($\Omega_0 = 1$, $h = 0.5$, $\sigma_8 \sim 0.6$) have very similar concentrations as the s$\Lambda$CDM halos we discuss here since both models are normalized to match the abundance of massive clusters at $z = 0$. This implies that all CDM models that match the abundance of clusters are likely to have difficulty reproducing the dynamics of spiral galaxies.

Even if cluster normalization is dropped from the list of relevant constraints and only the COBE measurements are used to normalize the power spectrum of initial density fluctuations, reducing substantially the central concentrations of dark halos still implies uncomfortable parameter choices. For example, if $\Omega_0$ is taken as a free parameter, reducing the dark mass of a $V_{200} \sim 200$ km s$^{-1}$ halo inside $R_o \sim 8.5$ kpc by a factor of three relative to s$\Lambda$CDM requires $\Omega_0 \sim 0.05$, a very low number that would call into question the need for a dominant non-baryonic dark matter component in the Universe. Similarly, if rather than $\Omega_0$, $H_0$ is allowed to vary, values as low as 30 km s$^{-1}$ Mpc$^{-1}$ are required to obtain the desired effect, in gross disagreement with current observational estimates.

If we are to preserve the successes of the s$\Lambda$CDM model, what seems to be required is a substantial change in the shape of the power spectrum relative to that predicted by the CDM+inflation paradigm. A “designer” power spectrum that would go some way towards reconciling the model with observations would suppress power on galactic and subgalactic scales while keeping the large scale properties of the model virtually unchanged, as envisioned, for example, in models that introduce a “tilt” in the primordial power spectrum relative to the standard Harrison-Zel’dovich value. This would in principle allow galaxy-sized dark halos to collapse later and thus become less centrally concentrated, although the magnitude of the tilt required is still unclear. Other alternative modifications that may in principle reproduce the
desired trend would involve the existence of a sizeable “hot dark matter” component (although much higher than derived from current estimates of the neutrino mass, see, e.g., Primack & Gross 1998), or possibly even dark matter candidates that may annihilate without trace in dense regions such as the centers of galaxies. One problem that afflicts all these proposed modifications is that they may hinder the formation of massive galaxies at high redshift, at odds with the mounting evidence that such galaxies are fairly common at $z \gtrsim 3$ (see, e.g., Steidel et al 1998). A thorough investigation of these possibilities will be needed in order to assess just how radically the new “standard” model of structure formation must be revised in order to bring spiral galaxies into the realm of observations that are consistent with the current paradigm of structure formation.

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REFERENCES

Bruzual, G., & Charlot, S. 1996, unpublished.

Bahcall, N., Ostriker, J.P., Perlmutter, S., Steinhardt, P. 1999, Science, in press (astro-ph/9906463).

Binney, J., Tremaine, S. 1987, Galactic Dynamics, Princeton University Press.

Chaboyer, B., Demarque, P., Kernan, P.J., Krauss, L.M. 1998, ApJ, 494, 96.

Courteau, S. 1997, AJ, 114, 2402

Courteau, S., 1999, astro-ph/9903297.

Debattista, V.P., Sellwood, J.A. 1998, ApJ, 493, L5.

Dehnen, W., Binney, J. 1998, MNRAS, 294, 429.

Eke, V.R., Cole, S., Frenk, C.S. 1996, MNRAS, 282, 263.

Fioc, M., Rocca-Volmerange, B., 1997, å, 326, 950.

Flores, R.A., Primack, J.R. 1994, ApJ, 427, L1

Giovanelli, R., Haynes, M.P., Herter, T., & Vogt, N.P. 1997, AJ, 113, 22

Han, M., & Mould, J.R. 1992, ApJ, 396, 453

Garnavich, P.M., et al 1998, ApJ, 509, 74.

Kravtsov, A.V., Klypin, A.A., Bullock, J.S., Primack, J.R. 1998, ApJ, 502, 48

Lawrence, C.R., Scott, D., White, M. 1999, PASP, 111, 525.

Madore, B.F.,Freedman, W. L.; Silbermann, N.; Harding, P.; Huchra, J.; Mould, J. R.; Graham, J. A.; Ferrarese, L.; Gibson, B. K.; Han, M.; Hoessel, J. G.; Hughes, S. M.; Illingworth, G. D.; Phelps, R.; Sakai, S.; Stetson, P. 1998, Nature, 395, 47.
Mathewson, D.S., Ford, V.L., & Buchhorn, M. 1992, ApJS, 81, 413

McGaugh, S.S., De Block, W.J.G. 1998, ApJ, 499, 41.

Mo, H.J., Mao, S., & White, S.D.M. 1998, MNRAS, 295, 319

Moore, B. 1994, Nature, 370, 629

Moore, B., Quinn, T., Governato, F., Stadel, J., Lake, G., 1999, MNRAS, submitted (astro-ph 9903164)

Navarro, J.F. 1999, ApJ, submitted (astro-ph 9807084)

Navarro, J.F., Frenk, C.S., & White, S.D.M. 1996, ApJ, 462, 563 (NFW96).

Navarro, J.F., Frenk, C.S., & White, S.D.M. 1997, ApJ, 490, 493 (NFW97).

Navarro, J.F., & Steinmetz, M. 1997, ApJ, 478, 13

Navarro, J.F., & White, S.D.M. 1994, MNRAS, 267, 401

Olling, R.N., Merrifield, M.R. 1998, MNRAS, 297, 943

Perlmutter, S. et al 1999, ApJ, 517, 565.

Primack, J.R., Gross, M.A.K., 1998, in “The Birth of Galaxies”, Xth Rencontres de Blois, astro-ph/9810204.

Reid, M.J. 1993, ARA&A, 31, 345

Sackett, P.D. 1997, ApJ, 483, 103

Schramm, D.N., Turner, M.S. 1998, Rev.Mod.Ph. 70, 303.

Steidel, C.C., Adelberger, K.L., Dickinson, M., Giavalisco, M., Pettini, M., Kellogg, M. 1998, ApJ, 492, 428.
Steinmetz, M., & Navarro, J.F. 1999, ApJ, 513, 555

Strauss, M.A., Willick, J.A., 1995, Phys. Rep. 261, 271.

Syer, D., Mao, S., & Mo, H.J. 1999, MNRAS, 405, 357

Tully, R.B., & Fisher, J.R. 1977, A&A, 54, 661

White S. D. M., Navarro J. F., Evrard A. E., & Frenk C. S. 1993, Nature, 366, 429

Wu, K., Lahav, O., Rees, M., 1999, Nature, 397, 225.

Zaroubi, S., Zehavi, I., Dekel, A., Hoffman, Y., Kolatt, T., 1997, ApJ, 486, 21.
Fig. 1.— Dark mass enclosed within a radius $R_o = 8.5$ kpc, the Sun’s distance from the center of the Milky Way, versus the circular velocities of $\Lambda$CDM halos. The shaded region highlights the allowed parameters of the dark halo surrounding the Milky Way, as derived from observations of Galactic dynamics and by assuming that the disk mass cannot exceed the total baryonic content of the halo. The filled circles show the loci of $\Lambda$CDM halos as determined from high-resolution N-body simulations. The solid line is the circular velocity dependence of the dark mass expected inside $R_o$ for halos that follow the density profile proposed by NFW96 and NFW97. The circular velocity dependence of the NFW “concentration” parameter of the simulated halos is well approximated on these scales by $c \approx 20 (V_{200}/100 \text{ km s}^{-1})^{-1/3}$ (dotted line). This is slightly higher than predicted by the approximate formula proposed by NFW97 but consistent with their published results.
Fig. 2.— The $I$-band Tully Fisher relation compared with the loci of hypothetical exponential disk galaxies assumed to assemble at the center of three representative sΛCDM halos. Dots are a compilation of the data by Giovanelli et al. (1997), Mathewson, Ford & Buchhorn (1992) and Han & Mould (1992). The solid line is the best fit to the data advocated by Giovanelli et al. The hypothetical galaxies have radii consistent with observations and move from left to right along each curve (labeled by the circular velocity of the halo at the virial radius) as the disk mass increases, under the assumption of a constant stellar mass-to-light ratio, $M/L_I = 2$ in solar units. The starred symbols correspond to the maximum disk mass, $M_{\text{max, disk}}$, allowed by the universal baryon fraction of the sΛCDM model. Open squares are N-body gasdynamical simulations of the formation of galaxies within these halos. Error bars correspond to two different choices of IMF, as discussed in...