Dark Matter in Models of String Cosmology

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Abstract.

The origin of dark matter in the universe may be weakly interacting scalar particles produced by amplification of quantum fluctuations during a period of dilaton-driven inflation. We present two interesting cases, the case of small fluctuations, and the resulting nonthermal spectrum, and the case of large fluctuations of a field with a periodic potential, the QCD axion.

We consider particle production in models of string cosmology which realize the pre-big-bang (PBB) scenario [1]. In this scenario the evolution of the universe starts from a state of very small curvature and coupling and then undergoes a long phase of dilaton-driven inflation (DDI) and at some later time joins smoothly standard radiation dominated (RD) cosmological evolution, thus giving rise to a singularity free inflationary cosmology. Particles are produced during DDI phase by the standard mechanism of amplification of quantum fluctuations [2]. Some debate about the naturalness of initial conditions and whether PBB models actually solve the cosmological problems has been taking place [3]. The smooth transition from DDI to RD is also not completely solved, for ideas about how this may come about see [4]. We will take a more phenomenological approach, and concentrate on interesting consequences of the models which do not depend on the detailed resolution of these issues, assuming that a resolution exists.

In the simplified model of background evolution we adopt, the evolution of the universe is divided into four distinct phases with specific (conformal) time dependence of the scale factor of the universe $a(\eta)$ and the dilaton $\phi(\eta)$. We assume throughout an isotropic and homogeneous four dimensional flat universe, described by a FRW metric. All other scalar fields are assumed to have a trivial vacuum expectation value during the inflationary phase.

We have computed spectrums of produced particles for the models described above [5]. We have solved a linear perturbation equation, $\chi''_k + (k^2 + M^2 a^2 - s''/s) \chi_k = 0$, where $s(\eta) \equiv a(\eta)^{m} e^{\iota \phi(\eta)/2} = a_s e^{\iota \phi_s /2} (\eta/\eta_s)^{1/2-n_s}$,
imposing initial conditions corresponding to normalized vacuum fluctuations. The parameter $m$ depends on the spin of the particle and $l$ depends on its coupling to the dilaton. Similar calculations have been performed by several groups and the results agree [6], whenever a comparison was possible.

We consider weakly interacting scalar particles, abundant in string theory and supergravity. For scalar fields $m = 1$, and we will consider for concreteness the values, $l = -1, 0, 1$ corresponding, respectively, to moduli, Ramond-Ramond axions, and Neveu-Schwartz axions. We will assume that the produced particles interact so weakly, that their interactions and decay are not sufficient to alter the primordial spectrum substantially. The particles we have in mind have typically gravitational strength interactions, which is definitely weak enough to satisfy our assumption, and their masses are below a fraction of an eV.

A typical spectrum of a light scalar may be divided, at a given time, into three physical momentum (PM) regions: i) The massless region, $\omega_S > \omega > M$, in this region particles are relativistic. ii) The “false” massive region, $M > \omega > \omega_m$, where $\omega_m = \omega_1 (M/M_s)^{1/2}$. In this region particles are NR, but have reentered the horizon as relativistic modes. iii) The “real” massive region $\omega_m > \omega$. In this region particles are non-relativistic (NR), and have reentered the horizon as NR modes. Note that PM redshift as the universe expands, and therefore boundaries of regions change in time,

$$\frac{d\Omega}{d\ln \omega} = N g_1^2 \left( \frac{g_1}{g_S} \right)^2 \left\{ \begin{array}{ll} \left( \frac{\omega}{\omega_S} \right)^x & \omega_S > \omega > M \\
\left( \frac{M}{\omega_S} \right)^{x-1} \left( \frac{\omega}{\omega_S} \right)^x & M > \omega > \omega_m \\
\left( \frac{M}{\omega_1} \right)^{2x} \left( \frac{\omega}{\omega_S} \right)^x & \omega_m > \omega, \end{array} \right.$$  

where $x \equiv 2 + 2\alpha + l\beta$, and $N$ is a numerical factor, estimated in [5], which we will set to unity in what follows. Parameters appearing in eq.(1) are, the string scale $M_s$, $z_S$ the total redshift during the string phase, $g_S$ and $g_1$, the string coupling at the beginning and end of the string phase, and $\omega_1$, the PM (today), corresponding to the end of the string phase, estimated in [7] to be $\omega_1 \sim 10^{-5}eV$, and the PM $\omega_S = \omega_1/z_S$, the PM (today) corresponding to the end of the DDI phase. In (1) we have assumed no substantial late entropy production has occurred. In general [7], the effect of late entropy production is to further redshift the physical frequencies as $\sim (1 - \delta s)^{1/3}$ where $\delta s$ is the fraction of produced entropy, and, more importantly, to dilute the contribution of modes which are already inside the horizon by a factor $(1 - \delta s)^{4/3}$. If a substantial amount of entropy is produced below $T < M_s/\sqrt{z_S}$, then spectrum (1) is no longer a good approximate spectrum.

The first example we look at is an example of small field fluctuations. The produced spectrum in this case is nonthermal, and may lead to an interesting case of cold and hot dark matter from the same source [8]. We look at generalized axions ($l = 1$) with masses below .1 eV in a cosmological model described in [1]. In this model, $d = 3$ spatial dimensions are expanding and $n = 6$ spatial dimensions are contracting, leading to $\alpha = -2/ (d + n + 3) = -1/6$ and $\beta = -4d/ (d + n + 3) =$
−1. For this specific model $x = 2/3$, $\Omega_{\text{REL}} \simeq g_1^2 \left( \frac{m_a}{g_{s}} \right)^2$, $\Omega_{\text{NR}} \simeq g_1^2 \left( \frac{m_a}{g_{s}} \right)^2 \frac{M}{M} \left( \frac{\omega_{\text{m}}}{\omega_{S}} \right)^{-1/3}$

Taking $10^{-10} \text{eV} < M < 10^{-2} \text{eV}$, for which the above condition is comfortably satisfied, we observe that the ratio $\Omega_{\text{REL}} : \Omega_{\text{NR}}$ at the start of structure formation era can vary in a range from well above unity to well below unity, corresponding to hot, mixed and cold dark matter. For example, choosing $g_1 = .1$ and $g_S = .01$ if the axion’s mass is $10^{-10} \text{eV}$, and for $z_S \sim 2 \times 10^4$ we get $\Omega_{\text{REL}} : \Omega_{\text{NR}} = 1 : 1$ with both energy densities being near critical, and if we choose $g_1 = .1$ and $g_S = .03$, making $\Omega_{\text{REL}} \simeq .1$, and if $z_S \sim 10^6$ we obtain $\Omega_{\text{REL}} : \Omega_{\text{NR}} = 1 : 10$, with $\Omega_{\text{NR}} \simeq 1$. Note that ratio depends on $z_S^{-2/3} M^{-5/6}$, so the previous examples correspond to a range of allowed values.

We now turn to the case of large field fluctuations. The spectrum in this case can be very different from that given by the naive result eq.(1). We look at the model independent axion [9], $l = 1$, assuming that it is the QCD axion [10] in a model of background evolution in which $d=3$ spatial dimensions expand, and $n = 6$ spatial dimensions are fixed, leading to $x = 3 - 2\sqrt{3} \sim -0.46$ [11].

Because of the negative exponent the spectrum is dominated by the lowest PM entering the horizon at a given time. The most interesting situation is when the axion potential turns on when the universe cools down to QCD temperatures. If we try to approximate the axion potential by a quadratic potential, leading to the result (1), we encounter a puzzle. The axion energy density becomes formally divergent as soon as the axion potential turns on! Once the potential is generated, all the low frequencies reenter the horizon at once, so to obtain the total energy density inside the horizon we need to integrate it from the minimal amplified frequency $\omega_{\text{min}}$, which is either zero, if the duration of the dilaton-driven phase is infinite, or exponentially small if the duration is finite but large. The lower frequency part of the spectrum yields a divergent contribution, proportional to $\omega_{\text{min}}^{3-2\sqrt{3}}$. This result does not make sense.

The resolution of the puzzle depends crucially on the periodic nature of axion potential $V(\psi) = \frac{1}{2} V_0 \left( 1 - \cos(\frac{\psi}{\psi_0}) \right)$. This point was first understood by Kofman and Linde [12], and we have adopted their ideas to our particular situation. First, the total potential energy is limited to $V_0$ and does not continue to increase indefinitely as the axion field increases, providing a “topological cutoff” on the total axionic energy density and as important, large fluctuations in the axion field are also “topologically cutoff”, producing exponentially small energy density perturbations. Large fluctuations lead to a uniform distribution of the axion field inside the horizon, with very small statistical fluctuations.

Using completely standard arguments [13], we may obtain a bound on the mass of the axion $m_a$ (or equivalently on $\psi_0$) by requiring that the energy density in the coherent axion oscillations be subcritical at the beginning of matter domination epoch. This requirement leads to the standard bounds on the axion mass, $\Omega_a h^2 \sim \frac{10^{-6} \text{eV}}{m_a}$, where $h$ is todays Hubble parameter in units of 100 km/Mpc/sec. Requiring subcritical $\Omega_a$ leads to the standard bound on $\psi_0$, $\psi_0 \lesssim 10^{12} \text{GeV}$ and $m_a \gtrsim 10^{-6} \text{eV}$. 
In string theory, natural values of $\psi_0$ are approximately $10^{16}\text{GeV}$, which, if taken at face value, would lead to overclosure of the universe with axions many times over. Two possible resolutions have been suggested [14] to allow our universe to reach its old age of today.

If dark matter in the universe is indeed made of light particles with gravitational strength interactions its detection in current direct searches is extremely difficult, and will probably require new methods and ideas.

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