Microscopic origin of de Sitter entropy

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(Dated: June 22, 2018)

It has been argued recently that the entropy of black holes might be associated with soft scalar, graviton and photon states at the event horizon, as number of such possible soft states is proportional to the horizon area. However, the coefficient of proportionality between the number of soft states and the horizon area of a black hole has not been established. Here, similar arguments are applied to de Sitter spacetime and it is shown that soft scalar gravitational modes account for the full de Sitter entropy \( S = \frac{1}{4}M_p^2 A \) with the correct numerical prefactor in front of the horizon area. We also find how the value of de Sitter temperature naturally emerges in the treatment of a scalar quantum field theory on the planar patch of \( dS_4 \).

I. INTRODUCTION

Recently, Hawking, Perry and Strominger [1] have argued that the physical origin of black hole entropy [2–4] should be associated with soft scalar, graviton and photon supertranslation hair, carried by a black hole. The associated soft modes are first excited in the process of gravitational collapse thus carrying information about particulars of the collapse process. Also, for a formed black hole, the flux of infalling matter leads to excitation of the soft modes present on the event horizon, which thus also encode information about the matter falling into the black hole post collapse phase, after the event horizon is formed. Presumably, this information is not entirely lost in the subsequent process of black hole evaporation being encoded in the phases (and perhaps amplitudes) of outgoing infrared modes, and the presence of soft supertranslation degrees of freedom can thus lead to the resolution of the celebrated black hole information loss paradox [5, 6].

Earlier we have also established [7] using the first quantization picture of gravitational collapse [8, 9] how exactly such information encoding can happen in the process of black hole formation: the particle production during the gravitational collapse leads to a restructuring of the outgoing vacuum state as observed at spatial infinity; as this particle production is unbounded in the sense that the occupation numbers for the modes with comoving frequencies \( \omega_k < R_S \) blow up reaching the asymptotic behavior \( n_k \sim \frac{1}{\omega_k^2} \) at \( t \gg R_S \), and the integral \( \int n_k d\omega_k \) is IR divergent, they contribute to the infinite renormalization of the vacuum state of the asymptotic observer. When one reduces physics at infinity to observable quantities only, one subtracts this infinite contribution, but the price paid is the fact that the spatio-temporal distribution of the phases and amplitudes of the modes with \( \omega_k < R_S \), which become strongly redshifted at \( t \to \infty \) (measured by the clock of an observer at asymptotic spatial infinity), can be rather involved. Our practical inability to probe this spatio-temporal structure is what generates Bekenstein-Hawking entropy: as an observer at spatial infinity has a very hard time discriminating between vacua with different IR phase structures, there exists an associated entropy which, as the authors of [1] argued, is proportional to horizon area. We note that it was impossible to establish the coefficient proportionality between the entropy and the area using the arguments of [1] alone, and thus it remains unclear whether scalar degrees of freedom are the only ones which contribute to the Bekenstein-Hawking entropy or other degrees of freedom (vector, tensor, etc.) can contribute, too.

Here we would like demonstrate that a logic similar to [1], when applied to the case of de Sitter spacetime, allows one to correctly estimate its entropy

\[
S_{ds} = \frac{\pi M_p^2}{H^2},
\]

where \( H \) is its Hubble scale. As it turns out, de Sitter entropy is entirely associated with soft gravitational scalar modes, as counting of the latter gives the correct numerical value for the ratio of de Sitter entropy and the horizon area of de Sitter spacetime. In what follows, we shall work in planar patch of 4-dimensional de Sitter spacetime, although the generalization to higher dimensions is straightforward.

II. LANGEVIN AND FOKKER-PLANCK EQUATIONS IN PLANAR PATCH

As usual, the gravitational perturbation modes in the de Sitter spacetime can be represented using the ADM split [10, 11]

\[
ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt)(dx^j + N^j dt),
\]

\[
h_{ij} = a^2 (1 + 2\zeta) \delta_{ij} + \gamma_{ij}, \quad a = a_0 e^{\int H dt} \approx a_0 e^{H_0 t}.
\]

We assume that the only relevant matter degree of freedom is a single scalar field \( \phi \) with the potential \( V(\phi) \), so that anisotropic stress is entirely absent; the perturbations of the scalar field are denoted by \( \delta \phi \). It is well known that in the gauge

\[
\delta \phi = 0, \quad h_{ij} = a^2 (1 + 2\zeta) \delta_{ij} + \gamma_{ij}, \quad \partial_t \gamma_{ij} = 0, \quad \gamma_{ii} = 0
\]
and in the case of exact de Sitter symmetry $H = H_0$ the scalar mode $\zeta$ remains a purely gauge degree of freedom \[^{12}\]. In the quasi-de Sitter case $H \neq H_0$, $|H| \ll H^2$ it becomes physical and observable (if de Sitter and post-inflationary reheating stages \[^{13,19}\] come to the end, and superhorizon modes of the scalar degree of freedom $\zeta$ start to reenter the horizon \[^{11}\]); this quasi-de Sitter case is assumed below.

Here, we shall instead consider the gauge $\zeta = 0$ such that the scalar gravitational degrees of freedom are entirely determined by the perturbations $\delta \phi$ of the scalar field $\phi$. Following \[^{21,22}\], one can extract the soft gravitational scalar mode by decomposing the scalar field as

$$
\phi (\log a, x) = \Phi (\log a, x) + \int \frac{d^3k}{(2\pi)^3/2} \Theta (k - eaH) \times (4)
\times \left( \hat{a}_k \phi_k (\log a) e^{-i k x} + \hat{a}^+_k \phi^*_k (\log a) e^{i k x} \right) + \Delta \phi (\log a, x),
$$

where $\Phi$ is the soft (superhorizon) component of the scalar field, the second term in the r.h.s. of (4) determines the subhorizon modes of the scalar field ($e \lesssim 1$ is assumed), and the third term includes corrections suppressed by powers of the slow roll parameters, which are not accounted for by the first two terms. The component $\Phi$ can be associated with the soft scalar gravitational mode, because (a) it is physically impossible for any observer in a given Hubble patch to discriminate $\phi$ from the classical inflaton component and thus from the vacuum contribution, (b) the inflaton $\phi$ is the only physical scalar gravitational degree of freedom in the chosen gauge.

It can be shown straightforwardly by substituting the representation (4) into the equation of motion for the inflaton that the soft mode $\Phi$ satisfies the Langevin equation

$$
\frac{\partial \Phi}{\partial \log a} = -\frac{1}{3H^2} \frac{\partial V}{\partial \Phi} + f (\log a),
$$

where the “random force” term $f$ is subject to the relation

$$
\langle f (\log a_1) f (\log a_2) \rangle = \frac{H^2}{4\pi^2} \delta (\log a_2 - \log a_1),
$$

and average is taken over the product of Bunch-Davies vacuum states $\prod_k |0_k \rangle$ for the modes $\phi_k$ of the scalar field; the operator $f$ is also quasi-classical in the sense that it commutes with itself \[^{22}\]. Following the usual prescription, the Fokker-Planck equation

$$
\frac{\partial P}{\partial \log a} = \frac{1}{3\pi M_p^2} \frac{\partial^2}{\partial \Phi^2} \left( \frac{1}{V (\Phi)} \partial \frac{\partial P}{\partial \Phi} \right) + \frac{M_p^2}{8\pi} \frac{\partial}{\partial \Phi} \left( \frac{1}{V (\Phi)} \frac{\partial V}{\partial \Phi} \right)\left( V (\Phi) P \right) +
$$

(7)

for the probability to measure a given value $\Phi$ in a given Hubble patch directly follows from the Langevin equation \[^{5}\]. We shall only be interested in the stationary solution of the Fokker-Planck equation (7)

$$
P (\Phi, \log a \to \infty) = \text{Const.} \frac{\exp \left( \frac{3M_p^2}{8V (\Phi)} \right)}{V (\Phi)} ,
$$

(8)

which is asymptotically approached as $a \to \infty$; the stationary solution exists if the integral $\int P (\Phi, \log a \to \infty) d\Phi$ is finite.

III. CALCULATING ENTROPY

It is possible to directly extract the de Sitter entropy from the distribution \[^{8}\]. To do this, consider a spacetime with asymptotic de Sitter symmetries; the de Sitter causal structure (with important corrections discussed below) is realized for a spacetime filled with a scalar field with the potential of the form

$$
V (\Phi) \approx V_0 + \delta V (\Phi), \quad |\delta V (\Phi)| \ll V_0 ,
$$

(9)

where $V_0$ is constant. The Hubble expansion rate is given in the leading slow roll approximation by

$$
H^2 = \frac{8\pi}{3M_p^2} V (\Phi) \approx H_0^2 + \frac{8\pi}{3M_p^2} \delta V (\Phi),
$$

(10)

and one finds for the exponent in \[^{8}\]

$$
\frac{3M_p^2}{8(V_0 + \delta V (\Phi))} \approx \frac{3M_p^2}{8V_0} \frac{V_0}{8V_0} = \frac{\pi M_p^2}{H_0^2} \frac{3M_p^4}{V_0} \frac{\delta V (\Phi)}{V_0} = \pi M_p^2 \frac{\delta V (\Phi)}{V_0}.
$$

(11)

The probability to measure a given background field $\Phi$ in a given Hubble patch is thus given by

$$
P (\Phi, \log a \to \infty) \sim \text{Const.} \exp \left( \frac{\pi M_p^2}{H_0^2} - \frac{3M_p^2}{8V_0} \frac{\delta V (\Phi)}{V_0} \right).
$$

(12)

The first term in the exponent coincides with the entropy of 4-dimensional de Sitter space. Indeed, for the $dS_4$ entropy one has $S = \frac{M_p^4}{4 \pi A}$, where $A = \frac{4\pi\pi H_0^2}{\hbar}$ is the horizon area of de Sitter spacetime with the Hubble expansion rate $H_0$. We see that the standard prefactor $1/4$ for the gravitational entropy is recovered correctly. Then, expanding in small $\delta V$ one can also write

$$
P (\Phi, \log a \to \infty) \sim \text{Const.} \exp \left( S - \frac{8\pi^2 \delta V (\Phi)}{3H_0} \right).
$$

(13)

Let us take a closer look at the second term in the exponent in \[^{13}\]. It is well known that $T = \frac{M_p^4}{4\pi^2}$ is the temperature of radiation as experienced by the observer in the static patch of de Sitter spacetime \[^{22}\]. Thus, it is convenient to rewrite the second term in the exponent \[^{13}\] as $\frac{4\pi \delta V}{3H_0 T}$. The factor $\frac{4\pi}{3H_0 T}$ in turn coincides with a
volume of the 3-dimensional sphere with the radius $H_0^{-1}$, i.e., the comoving 3-volume cross-section $v_{3S}$ of a single Hubble patch. We finally conclude that

$$P(\Phi) \sim \text{Const.} \exp \left( S_{3S} - v_{3S} \frac{\delta V(\Phi)}{T} \right) = \text{Const.} \exp \left( S_{3S} - \frac{F(\Phi)}{T} \right),$$

where the $F = v_{3S} \delta V(\phi)$ is the “free energy” of the scalar field slowly rolling towards the minimum of its effective potential as perceived by a physical observer living within the given Hubble patch.

We note in passing that the expression similar to (13) can be derived for arbitrary matter content introduced in the de Sitter universe given that the energy density of the added degrees of freedom coarse-grained at the Hubble scale is sufficiently small ($\epsilon \ll V_0$); such coarse-grained energy density should again be understood as a correction to the vacuum energy density as perceived by an observer living within a given Hubble patch. In that case, the “free energy” will acquire an additive correction $\sim \epsilon \cdot v_{3S}$.

For completeness, we shall also derive here the expression for de Sitter entropy in an arbitrary number of dimensions. Using stochastic formalism for a $D$-dimensional quasi-de Sitter spacetime [24], we find for the asymptotic probability to measure a given value of the inflaton field $\Phi$ in a given Hubble patch

$$P(\Phi, \log a \to \infty) \sim V^{-1} \exp \left( \frac{\pi^{D-1}}{\Gamma \left( \frac{D-1}{2} \right)} \left( \frac{M_p}{H} \right)^{D-2} \right).$$

Applying the same arguments as presented above for the case of 4-dimensional de Sitter spacetime and recalling the expressions for the volume of a $d$-dimensional ball as well as the area of a sphere serving as its boundary, we also conclude that (a) de Sitter entropy is universally given by

$$S_{3S} = \frac{1}{4} M_p^{D-2} A$$

with the same numerical prefactor $1/4$ in all dimensions as should have been expected, while the de Sitter temperature is

$$T_{3S} = \frac{(D-2) H_0}{4 \pi}.$$ 

### IV. DE SITTER ENTROPY IN THE PRESENCE OF DISORDER

Before proceeding to the discussion of the physical implications of the formulae (13) and (14), let us consider how the expression (14) changes in the presence of disorder in the potential $V(\Phi)$. The main motivation to ask this question is the fact that within the framework of string theory one expects to find a very large landscape of de Sitter vacua for moduli and other scalars able to support eternal inflation [23–22].

Considering a particular Hubble patch (that is, focusing on physical questions asked by an observer living in it), it is also natural to expect that in the limit $\log a \to \infty$ most such vacua will be visited, and the resulting potential of the effective inflaton field will be strongly disordered. (Here by the inflaton one can understand a master scalar field, obtained from the effective potential of scalars present in string theory by model reduction [22].) Moreover, it will be possible to obtain the effective probability $P(\Phi)$ to measure a given value of the effective inflaton field $\Phi$ by averaging over disorder (exactly because in the limit $\log a \to \infty$ most quasi-de Sitter vacua on the landscape are already visited).

Consider for simplicity a random potential of the form

$$V(\Phi) \approx V_0 + \delta V(\Phi), \quad |\delta V(\Phi)| \ll V_0$$

distributed according to a Gaussian measure

$$\int d\delta V \exp \left( - \int dx \sqrt{\frac{\delta V^2}{2\Delta}} \right) \sim$$

$$\int d\delta V \exp \left( -\frac{\delta V^2}{2\Delta} \right) = \int d\delta V \exp \left( -\frac{\delta V^2}{2\Delta} \right),$$

where integration in the first expression is taken over the spatial cross-section (comoving 3-volume) of the planar patch, and $\Delta = \frac{\Delta_0}{V_0} = \frac{3 M_p^2}{4 \pi V_0}$. (The distribution function for disorder will be of course of a generic non-Gaussian form, but non-Gaussian corrections to the integral (17) are expected to be suppressed in the ultralocal approximation effectively realized for a given observer in the quasi-de Sitter universe.)

As we already mentioned, from the point of view of an observer in a given Hubble patch the effective value of the cosmological constant $\sim V_0$ is determined by the fact that the field $\Phi$ visits all possible configurations of disorder realized on the landscape. We thus find

$$\langle P(\Phi) \rangle_{\text{disorder}} \sim \int d\delta V \frac{1}{V_0 + \delta V} \times$$

$$\times \exp \left( \frac{3 M_p^4}{8 V_0} \right) \exp \left( -\frac{\delta V^2}{2\Delta} \right).$$

The integral in (18) can be calculated using the saddle point approximation, and one finds that

$$\langle P(\Phi) \rangle_{\text{disorder}} \sim \exp \left( \frac{3 M_p^4}{8 V_0 \left( 1 - \frac{3 M_p^2 \Delta}{8 V_0} \right)} \right),$$

i.e., a weak disorder present in the scalar field potential leads to a slight increase in de Sitter entropy, which is of the order $\delta S_{3S} \approx S_{3S}^2 V_0^{-1}$, where $\frac{\Delta}{V_0} S_{3S} \ll 1$ and $S_{3S} = \frac{3 M_p^4}{8 V_0}$ — the expression (19) holds as long as disorder is sufficiently weak (we naturally expect $S_{3S} \gg 1$).
V. DISCUSSION

We have seen above that the stochastic formalism describing IR dynamics of a scalar field in quasi-de Sitter universe can serve as a window into gravitational de Sitter thermodynamics. In particular, it allows to calculate de Sitter entropy with the correct numerical prefactor and obtain a correct value for the temperature of de Sitter radiation despite the fact that the spectra of scalar field modes in the planar patch of de Sitter spacetime are manifestly non-thermal [11]. It also makes it possible to finally address an old question how the thermal physics of the static patch (a string theorist’s favorite) of de Sitter spacetime [33] is related to the physics of planar patch (a cosmologist’s favorite) [34]. This question is important as the actual causal structure of self-reproducing quasi-de Sitter inflationary universe is very different from the one of static patch: one of the diamonds is entirely absent from the resulting Penrose diagram (and thus tracing out the corresponding degrees of freedom in order to obtain thermal flux is less than straightforward), while the other is modified into an infinite self-replicating set. Where does the “thermal flux” and de Sitter entropy then come from and is it even physical? The correct answer is that it is physical, and the de Sitter entropy comes from effective averaging over superhorizon modes constantly generated in the quasi-de Sitter universe (i.e., the Fokker-Planck distribution [27]): if accelerated expansion never comes to the end, a physical observer living in a given Hubble patch is unable to discriminate between the true vacuum state (the direct product of Fock vacuum states for individual modes) and the state containing arbitrary modes reentering the horizon in the asymptotic future.

1) “What does de Sitter spacetime evaporate into?” If a black hole emitting thermal flux from the event horizon inevitably evaporates [5], and the physical picture of the static patch of de Sitter spacetime is also the one supporting a thermal flux from the horizon [33], what is the final outcome of thermal evaporation process in this case? If $V_0 > 0$, the answer is eternally inflating universe with self-reproducing causal structure; the pure de Sitter spacetime is indeed unstable [35], but the physical origin of this instability is with respect to changing the matter content of the theory (or even introducing perturbations to $V(\Phi)$, so that $V(\Phi) \neq V_0$): theories with a slightly different matter content will lead to a widely different self-replicating causal structure of spacetime at $\log a \to \infty$.

2) Is the physical picture of gravitational thermodynamics universal? Is it thermal picture applicable to the global causal structure of the resulting spacetime? While the common opinion is that it absolutely is [37–43], we would like to take an opposite point of view motivated by two facts. Namely, (a) if $V_0 = 0$ and inflation comes to the end being replaced by a decelerating FRW expansion, the Fokker-Planck equation [7] does not admit a normalizable time-independent asymptotic solution of the form [12]; instead, the general solution of the Fokker-Planck equation decays to zero as $\log a \to \infty$. As such, there is no thermal flux and no associated entropy hard-wired into the theory, which is in a sense natural: once inflation ends and is replaced by a decelerated expansion continued ad infinitum, all inflationary modes eventually reenter the cosmological horizon, and the whole inflationary history can be potentially recovered by a sufficiently long-living and patient observer. Naturally, it is impossible to argue for the thermal equilibrium (and thermal flux) when the characteristic temperature behaves non-adiabatically, $T \lesssim T^2$ (such is the case for a decelerating FRW universe). Also, (b) the “Hawking-Moss instanton”-like distribution [12] is manifestly non-Boltzmann [36], and while the leading terms in $V(\Phi)$ (or $\frac{1}{a}$) expansion do correspond to a quasi-Boltzmann form [12], higher order expansion terms spoil it.

3) Existence of a non-trivial gravitational entropy is determined by accessibility of information. As we just argued, information encoded in amplitudes and phases of inflationary modes can become accessible if inflation comes to the end, universe passes through reheating stage and inflationary modes which reenter the cosmological horizon become available to probe. A counterpart of the Fokker-Planck equation [7] for cosmology including a decelerating FRW branch does not admit a stationary solution at $\log a \to \infty$, because all inflationary modes reenter the horizon in the asymptotic future, and there is no entropy associated with inaccessible information (even if we prescribe entropy to a decaying de Sitter spacetime, complete recovery of all information encoded in inflationary modes implies zero final entropy thus violating the second law of thermodynamics). Note however that if a decelerating FRW expansion stage is followed up by another quasi-de Sitter stage with a Hubble rate $H_A < H_0$, a fraction of inflationary modes with $k < aH_A$ will never reenter the horizon, and there will be an associated remaining entropy $\sim H^2$ much higher than the entropy $\sim H_0^{-2}$ of the quasi-de Sitter spacetime realized during primordial inflation. Ignoring the matching FRW stage of decelerating expansion, one can simply describe behavior of the universe following the the dynamics of the effective inflaton field between the minima of its potential $V(\Phi)$. In the limit $\log a \to \infty$ the low-
est among such minima will be the ones dominating the asymptotic form of the distribution function $P(\Phi)$. Does not it look like the Universe we are currently living in? [44–47] Of course, it depends on the particular form of the effective equation of state for dark energy whether the current quasi-de Sitter stage also comes to the end [48, 49].

4) Do gravitational vector/tensor modes contribute to the gravitational entropy? We have just seen that the scalar gravitational degrees of freedom produce de Sitter entropy with a correct numerical prefactor $1/4$, and thus the contribution of gravitational tensor and vector modes to this entropy amounts to zero. The physical reason for this conclusion is probably that tensors and vectors are unable to produce a non-trivial large scale superhorizon field structure. We believe the answer to be naturally the same for a Schwarzschild black hole, although only deriving and using an analogue of stochastic formalism for a Schwarzschild spacetime would allow to set the record straight, which is a task worth investing some time into. We expect the answer to change for an electrically charged or a rotating black hole.

5) Arrow of time. Personally, the most physically interesting consequence of the arguments presented above is related to the nature of the arrow of time. An entire matter content of our Hubble patch has the entropy $\sim 10^{104}k$, while the gravitational entropy associated with $dS$ horizon (corresponding the present regime of accelerated expansion) is about $10^{122}k$. In the early Universe, the gap between the gravitational and matter content entropies was even worse, and we would like to conclude that the analysis of the emergence of the arrow of time should be performed within the context of quantum gravity (at least, in the WKB approximation) - inflation does explain why the matter entropy was so low at Big Bang, but does not explain why the gravitational entropy was also extremely low at the beginning of inflation. On the other hand, a huge $dS$ entropy is itself generated only if the expanding branch of the general solution of the Wheeler-deWitt equation is picked, as we can see from [7]. So, something else than matter decoherence/gravitational decoherence due to scattering of gravitational degrees of freedom against matter ones should be responsible for the arrow of time [50].

6) IR vacuum restructuring in different theories. In principle, the situation with IR vacuum restructuring is not uncommon in physics: consider even an extremely well studied 4-dimensional quantum electrodynamics, which is famously plagued with the IR catastrophe problem [53]. It is well-known that the latter can be resolved once one takes the bath of infrared photons into account; photons with very large wavelengths cannot be practically detected by any physical detector with a finite bandwidth, and the bandwidth cutoff also serves as an effective IR cutoff $\Lambda_{IR}$ for the theory. In a sense, the true vacuum state of QED is a formal Fock vacuum state plus the bath of deep infrared photons. It is interesting to check if there is an entropy associated with this vacuum degeneracy in QED (naively, there is since the infrared cutoff in the theory implies tracing out all degrees of freedom with $\lambda > \Lambda_{IR}^{-1}$ and an associated entropy $\sim \Lambda_{IR}^{-2}$ [54]). In both cases of interest for us (black holes and de Sitter space) the physical situation is largely similar: there exist modes with sufficiently large IR wavelengths such that an observer performing observations of the physical state during a finite interval of time is unable to discriminate between the formal vacuum state of the field theory and the state containing an arbitrary ensemble of such IR modes. For $dS$ spacetime, the modes are the superhorizon modes. In the case of a spacetime containing an evaporating black hole, such deep infrared (as perceived by observer at asymptotic spatial infinity) modes are the ones associated with black hole soft hair.

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