Effects of non-uniform acceptance in anisotropic flow measurement

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Applicability of anisotropic flow measurement techniques and their extension for detectors with non-uniform azimuthal acceptance are discussed. Considering anisotropic flow measurement with two and three (mixed harmonic) azimuthal correlations we introduce a set of observables based on the $x$ and $y$ components of the event flow vector. These observables provide independent measures of anisotropic flow, and can be used to test self-consistency of the analysis. Based on these observables we propose a technique that explicitly takes into account the effects of non-uniform detector acceptance. Within this approach the acceptance corrections, as well as parameters which define the method applicability, can be determined directly from experimental data. For practical purposes a brief summary of the method is provided at the end.

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I. INTRODUCTION

One of the most important observable in the study of ultra-relativistic nucleus-nucleus collisions is the anisotropic transverse flow $[1, 2, 3, 4]$. It is quantified by coefficients in Fourier decomposition of particle azimuthal distributions relative to the collision reaction plane $[5, 6]$, the latter is defined by the beam direction and the impact parameter. The second harmonic in such a decomposition is called the elliptic flow. Observation of strong in-plane elliptic flow increasing with collision energy from top AGS (Alternating Gradient Synchrotron) energies $[7, 8]$, then at CERN SPS (Super Proton Synchrotron) $[9, 10]$, and recently at RHIC (Relativistic Heavy Ion Collider) $[11, 12, 13, 14, 15, 16, 17]$ shows that the bulk matter created in the high energy heavy ion collision strongly interacts and behaves collectively. Taken together with a key feature of anisotropic flow to be sensitive to the early stage of the collision, this indicates rapid attainment of local thermal equilibrium in the created system. For central collisions, RHIC results are in agreement with ideal (zero viscosity) hydrodynamic predictions $[18, 19, 20]$. These observations have played an important role in the discovery of the strongly interacting Quark Gluon Plasma (sQGP), the new form of matter formed in heavy ion collisions.

There exist a set of different techniques for anisotropic flow measurement, which have been successfully applied at a variety of experimental setups worldwide $[7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]$. The reaction plane is not known experimentally, various methods exploit azimuthal correlations between particles as all of the particles are correlated to the same reaction plane. The most widely used are the event plane $[7, 8]$, the scalar product $[21, 22]$, and mixed harmonic $[6, 23, 24, 25]$ methods. Analysis of anisotropic flow with azimuthal correlations requires to examine contribution from effects not related to orientation of the reaction plane, such as resonance decays, jets, as well as effects of conservation laws (charge, momentum, etc). Most of these so called non-flow correlations are due to few particle interactions, and their relative contribution scales inversely proportional to particle multiplicity in the event. For that reason in multi-particle correlations these effects are suppressed compared to collective effects such as anisotropic flow. The complete consideration of methods to study non-flow effects is beyond the scope of this paper (for more discussions see, for example, $[22]$ and references therein). In this paper we assume that the azimuthal distributions of particles produced in the collision depends only on the orientation relative to the reaction plane.

High statistics experimental data collected in recent years at RHIC allow to perform very precise measurements $[7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]$. With the new data, the systematic uncertainty in the analysis becomes dominant compared to the statistical errors. It is vital to carefully investigate the systematic errors, in particular due to non-perfect azimuthal acceptance, as well as to review the applicability of different methods in this respect. In addition, the availability of different experimental setups with complicated azimuthal acceptance, such as central arms of PHENIX (A Physics Experiment at RHIC) detector $[26, 27]$, NA49 TPC (Time Projection Chamber) $[28]$, and PHENIX/STAR (Solenoideal Tracker At RHIC) ZDC SMD (Zero Degree Calorimeter Shower Maximum Detector) $[29]$, requires further development of new and/or generalization of known methods for use with detectors with significant acceptance non-uniformity. Such generalization would allow to enrich already available data with anisotropic flow measurement results from a wider range of experimental setups. This further provides an opportunity for a comprehensive comparison of available theoretical predictions against the experimental data.

In this paper we describe a procedure to broaden the applicability of known methods to measure anisotropic flow for a range of detectors with non-uniform azimuthal acceptance. Distinctive feature of the suggested approach is that the acceptance corrections can be determined directly from experimental data. This eliminates the need to perform time-consuming and model dependent Monte-Carlo simulations of the detector acceptance.
and efficiency. The main idea is demonstrated on an example of two-particle correlations, but for completeness we also provide formulae for the case of three particle correlations (mixed harmonic) technique, which, as discussed above are less susceptible to non-flow effects. We keep the discussion of more complicated three particle correlation case in separate subsections, such that they can be easily skipped if the reader is interested only in the main idea.

The paper is organized as follows. In section III we review notations and definitions of basic quantities used in anisotropic flow analysis. We formulate them in a way that later helps us to introduce new observables based on different components of the event flow vector. These observables provide independent measures of anisotropic flow, and can be used to test self-consistency of the results. In section III we discuss the effects of non-uniform detector acceptance, and describe the procedure of correcting the single particle and event flow vector such that observables derived for the perfect detector can be used. We provide the smallness parameters that can be used to quantify the range of applicability of the proposed method. These parameters can be estimated directly from experimental data. Finally, we summarize the method.

A similar problem of flow analysis with non-uniform acceptance detector was discussed in [30, 31] in the context of cumulant and Lee-Yang zeros analyses based on the use of generating functions. Though in some cases these techniques might yield to similar results, our independent approach clarifies the nature of the acceptance effects in flow studies, and further provides the required tools for the most often used analyses based on correlations with the event flow vector.

II. METHODS

A. Definition and notations

Anisotropic transverse flow of particles produced in heavy ion collision is quantified by coefficients in Fourier decomposition of particle azimuthal distribution [32, 33]. In this paper we use particle azimuthal spectra normalized to unity (particle production probability density):

$$\rho(\phi - \Psi_{RP}) = \frac{1}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos \{n(\phi - \Psi_{RP})\} \right).$$  

(1)

Here $\phi$ is the particle azimuthal angle, and $v_n$ is the $n$-th harmonic anisotropic flow. The reaction plane angle $\Psi_{RP}$ in equation (1) is an azimuthal angle of the impact parameter. In general, coefficients $v_n$ are functions of particle transverse momentum $p_t$ and pseudorapidity $\eta$. Such dependences are hereafter assumed implicitly and not indicated in the notation for the sake of brevity and simplicity.

We introduce a unit vector $u_n (n > 0)$, defined for each particle:

$$u_n = x_n + iy_n \equiv \cos n\phi + i \sin n\phi = \exp\{in\phi\}. \quad (2)$$

In this equation $u_n$ is given as a complex number with angle $n\phi$ and absolute value of unity. Throughout this paper we use complex number representation, but all equations can be re-written in terms of normally used algebra of 2-dimensional vectors. An estimate of the reaction plane orientation is usually obtained with the so called $n$-harmonic event flow vector $Q_n$, which is defined as a sum of $u_n$-vectors over a specific subset "EP" of particles produced in the collision:

$$Q_n = \sum_{EP} u_n = \sum_{EP} (\cos n\phi + i \sin n\phi) \equiv X_n + i Y_n = |Q_n| \exp\{in\Psi_{EP}\}. \quad (3)$$

Here $X_n$ and $Y_n$ are the event vector components, and $\Psi_{EP}$ is $n$-th harmonic event plane angle. For each $n$ the event flow vector $Q_n$ (or $\Psi_{EP}$) provides an independent estimate of the reaction plane orientation. Note that $u_n$ itself can serve as a $Q_n$-vector. However this is not very practical, since the more particles used to define the event plane the closer its orientation will be to that of the reaction plane.

To derive the main formulae of proposed technique we find it very useful to introduce, though experimentally unobservable, $u_n$ averaged over events with fixed orientation of the reaction plane:

$$\langle u_n \rangle_{\Psi_{RP}} = \langle x_n \rangle_{\Psi_{RP}} + i \langle y_n \rangle_{\Psi_{RP}} = \int d\phi \ u_n \ \rho(\phi - \Psi_{RP})$$

$$= \int \frac{d\phi}{2\pi} \ u_n \left( 1 + 2 \sum_{m=1}^{\infty} v_m \cos \{m(\phi - \Psi_{RP})\} \right)$$

$$= v_n (\cos n\Psi_{RP} + i \sin n\Psi_{RP}). \quad (4)$$

In this section we consider the case of a detector with perfect azimuthal acceptance. This implies that an integration over azimuthal angle $\phi$ goes over $2\pi$ without any weight. We will relax this assumption in section III when considering detector acceptance effects. Similarly:

$$\langle Q_n \rangle_{\Psi_{RP}} = \langle X_n \rangle_{\Psi_{RP}} + i \langle Y_n \rangle_{\Psi_{RP}} = \langle M v_n \rangle_{EP} (\cos n\Psi_{RP} + i \sin n\Psi_{RP})$$

$$= V_n (\cos n\Psi_{RP} + i \sin n\Psi_{RP}). \quad (5)$$

Here $V_n \equiv \langle M v_n \rangle_{EP}$ is an average $n$-harmonic anisotropic flow $v_n$ convoluted with multiplicity $M$ of particles from a subset "EP" used to calculate the event flow vector $Q_n$.

From equation (3) it follows that $Q_n$ is defined as a vector in transverse plane, which on average has an orientation of that of the reaction plane. This feature can be used to define the event flow vector with detectors without tracking that are only sensitive to the shape of the particle distribution in the transverse plane (for example, calorimeters). The only, but very important, requirement to be fulfilled is that the $Q_n$ components, $X_n$ and $Y_n$, should be on average proportional to $\cos n\Psi_{RP}$ and $\sin n\Psi_{RP}$, respectively.
B. Anisotropic flow from different components

1. Two particle correlations

Anisotropic flow via two particle correlations can be obtained with the so-called scalar product method $^{22}$. According to the scalar product technique, one considers the average of the product of $u_n$ and $Q_n$ vectors over all events. With the help of Eqs. (4) and (5) this average can be written as an average over all events with fixed reaction plane with further average over all reaction plane orientations:

$$\langle u_n Q_n^\ast \rangle = \langle x_n X_n \rangle + \langle y_n Y_n \rangle$$

(6)

$$= \int_0^{2\pi} \frac{d\Psi_{RP}}{2\pi} \langle u_n \rangle_{\Psi_{RP}} \langle Q_n^\ast \rangle_{\Psi_{RP}} = v_n V_n.$$  

Here, angle brackets with subscripts $\Psi_{RP}$, $\langle \ldots \rangle_{\Psi_{RP}}$, denote the average over events with fixed $\Psi_{RP}$; angle brackets without subscripts, $\langle \ldots \rangle$, correspond to the average over entire ensemble of events with all possible orientations of the reaction plane.

The left hand side of Eq. (6) can be measured from experimental data. To obtain $v_n$ one needs to evaluate $V_n$. This can be done by using random sub-events, i.e. randomly assigning particles to construct the event flow vector into two subsets $a$ and $b$ $^{3}$:

$$\langle Q_n^{ab} \rangle = \langle X_n^a X_n^b \rangle + \langle Y_n^a Y_n^b \rangle = \frac{1}{4} V_n^2.$$ (7)

The factor of 1/4 here takes into account the multiplicity difference between the full event and that of sub-events $a$ and $b$.

From Eqs. (6) and (7) we obtain:

$$v_n = \frac{\langle u_n Q_n^\ast \rangle}{2\sqrt{\langle Q_n^{ab} \rangle}}.$$ (8)

We further note that the two terms in Eqs. (6) and (7) are statistically independent, which allows to consider them separately:

$$\langle x_n X_n \rangle = \langle y_n Y_n \rangle = \frac{1}{2} v_n V_n,$$ (9)

$$\langle X_n^a X_n^b \rangle = \langle Y_n^a Y_n^b \rangle = \frac{1}{8} V_n^2,$$ (10)

thus providing two independent measures of anisotropic flow:

$$v_n = \frac{\langle x_n X_n \rangle}{\sqrt{2 \langle X_n^a X_n^b \rangle}} = \frac{\langle y_n Y_n \rangle}{\sqrt{2 \langle Y_n^a Y_n^b \rangle}}.$$ (11)

Independent observables (11) can be used to check the self-consistency of the results.

Note, that with normalization of the $Q_n$-vector to unity, $Q_n \rightarrow Q_n/|Q_n|$, the average $\langle u_n Q_n^\ast \rangle$ in (6) reduces to $\langle \cos \{ n(\phi - \Psi_{EP}^n) \} \rangle$, and Eq. (3) leads to the main observable of the event plane method $^{3}$:

$$v_n = \sqrt{\frac{\langle \cos \{ n(\phi - \Psi_{EP}^{na,b}) \} \rangle}{\langle \cos \{ n(\Psi_{EP}^{na} - \Psi_{EP}^{nb}) \} \rangle}}.$$ (12)

Similarly, the second equality in formula (11) gives an observable used by the NA49 Collaboration $^{10}$:

$$v_n = \sqrt{\frac{\langle \sin n\phi \cdot \sin n\Psi_{EP}^{na,b} \rangle}{\langle \sin n\Psi_{EP}^{na} \cdot \sin n\Psi_{EP}^{nb} \rangle}}.$$ (13)

Here and in Eq. (12) we use the event plane angle defined for the subevents, which resulted in an extra factor of two compared to Eqs. (8, 11).

2. Three particle correlations

In the case of three particle correlations one considers:

$$\langle u_n^a u_n^b Q_n^{2n} \rangle = \langle x_n^a x_n^b X_2 - y_n^a y_n^b X_2 + x_n^a y_n^b Y_2 + y_n^a x_n^b Y_2 \rangle$$ (14)

$$= \int_0^{2\pi} \frac{d\Psi_{RP}}{2\pi} \langle u_n^a \rangle_{\Psi_{RP}} \langle u_n^b \rangle_{\Psi_{RP}} \langle Q_n^{ab} \rangle_{\Psi_{RP}} = v_n^2 V_n.$$  

where:

$$V_n = \langle M v_n \rangle_{EP} = 2 \sqrt{\langle Q_n^{ab} \rangle}.$$ (15)

Then, the anisotropic flow $v_n$ is given by:

$$|v_n| = \frac{\langle u_n^a u_n^b Q_n^{2n} \rangle}{2 \sqrt{\langle Q_n^{ab} \rangle}}.$$ (16)

If one normalizes $Q_n$-vector to unity, formula (16) leads to an observable of the mixed harmonic method $^{23,25}$:

$$|v_n| = \left| \frac{\langle \cos \{ n(\phi_a + \phi_b - 2\Psi_{2n,a,b}) \} \rangle}{\langle \cos \{ 2n(\Psi_{2n,a} - \Psi_{2n,b}) \} \rangle} \right|.$$ (17)

All terms in formula (17) are statistically independent, which leads to a set of equalities:

$$\langle x_n^a x_n^b X_2 \rangle = - \langle y_n^a y_n^b X_2 \rangle = \langle x_n^a y_n^b Y_2 \rangle = \langle y_n^a x_n^b Y_2 \rangle = \frac{1}{4} v_n^2 V_n.$$ (18)

Thus one obtains four independent observables to measure anisotropic flow from three particle correlations:

$$|v_n| = \left| \frac{\langle x_n^a x_n^b X_2 \rangle}{\sqrt{\langle X_n^a X_n^b \rangle}} \right| = \left| \frac{\langle y_n^a y_n^b X_2 \rangle}{\sqrt{\langle Y_n^a Y_n^b \rangle}} \right| = \left| \frac{\langle x_n^a y_n^b Y_2 \rangle}{\sqrt{\langle Y_n^a Y_n^b \rangle}} \right| = \left| \frac{\langle y_n^a x_n^b Y_2 \rangle}{\sqrt{\langle Y_n^a Y_n^b \rangle}} \right|.$$ (19)
As in the case of the two particle correlations, each of the four terms in Equation (19) provides an independent measure of anisotropic flow, and can be used to check the self-consistency of the results.

Note, that one can construct three particle correlation function from \( u_n \) and \( Q_m \) vector components other than that defined by formula (14). Some examples are \( \langle u_n Q_n^b Q_m^b \rangle \) or \( Q_n^b u_m^b u_n^b \). Derivation of observables based on these correlators are similar, but in this paper we only consider combination (14), which in the case of \( n = 1 \) leads to the known observable for directed flow [24, 25].

### III. Effects of Non-Uniform Acceptance

In order to generalize our consideration for the case of imperfect acceptance we introduce the acceptance function \( A(\phi) \) which we normalize to unity (similar to [30, 31]):

\[
\int \frac{d\phi}{2\pi} A(\phi) = 1.
\]

Then the average of some function \( f(\phi) \), which depends on particle azimuthal angle \( \phi \), at fixed reaction plane orientation is given by the integral:

\[
\langle f \rangle_{\Psi_{\text{RP}}} = \int \frac{d\phi}{2\pi} A(\phi) f(\phi) \rho(\phi - \Psi_{\text{RP}})
\]

\[
= \overline{f} + \sum_{m=1}^{\infty} v_m \left[ \overline{F}_m \cos m\Psi_{\text{RP}} + \overline{F}_m \sin m\Psi_{\text{RP}} \right].
\]

Here for brevity we introduce notation \( c_m = \cos m\phi \) and \( s_m = \sin m\phi \), and denote by \( \overline{f} \), the average over the detector acceptance:

\[
\overline{f} = \int \frac{d\phi}{2\pi} A(\phi) f(\phi).
\]

One might note that \( \overline{v}_n \) and \( \overline{s}_n \) represent \( n \)-th harmonic coefficients in the Fourier expansion of the acceptance function \( A(\phi) \). An important observation is that the acceptance average of \( f, \overline{f} \), coincides with the event average, \( \langle f \rangle \):

\[
\langle f \rangle = \frac{\int d\Psi_{\text{RP}} d\phi A(\phi) f(\phi) \rho(\phi - \Psi_{\text{RP}})}{\int d\Psi_{\text{RP}} d\phi A(\phi) \rho(\phi - \Psi_{\text{RP}})} = \overline{f}.
\]

We assume here that the distribution of the reaction plane angle, \( \Psi_{\text{RP}} \), is uniform within a given centrality event sample. Experimentally, this can be achieved by using for the collision centrality determination the independent detector with a good azimuthal coverage. Consequently, all acceptance average quantities can be extracted directly from the experimental data by the corresponding average over all particles in the event sample.

Formula (21) allows to re-write the expressions for \( \langle x_n \rangle_{\Psi_{\text{RP}}} \) and \( \langle y_n \rangle_{\Psi_{\text{RP}}} \) taking into account the effects of the non-uniform detector acceptance. For clarity of comparison with Eq. (4) we separate the term with \( m = n \), which is the only non-vanishing term in case of perfect acceptance:

\[
\langle x_n \rangle_{\Psi_{\text{RP}}} = \overline{r}_n + v_n a_{2n}^+ \left\{ \cos n\Psi_{\text{RP}} + \lambda_{2n}^+ \sin n\Psi_{\text{RP}} \right\}
\]

\[
+ \sum_{m \neq n} (\lambda_{n-m}^+ + \lambda_{n+m}^+ \cos m\Psi_{\text{RP}} + \lambda_{n+m}^- \sin m\Psi_{\text{RP}})
\]

\[
\langle y_n \rangle_{\Psi_{\text{RP}}} = \overline{y}_n + v_n a_{2n}^- \left\{ \cos n\Psi_{\text{RP}} + \lambda_{2n}^- \sin n\Psi_{\text{RP}} \right\}
\]

\[
+ \sum_{m \neq n} (\lambda_{n+m}^- \cos m\Psi_{\text{RP}} + \lambda_{n-m}^- \sin m\Psi_{\text{RP}})
\]

Here we have introduced the acceptance coefficient \( a_{2n}^\pm \):

\[
a_{2n}^\pm = 1 \pm r_{2n} = 1 \pm \cos 2n\phi,
\]

and the following smallness parameters:

\[
\lambda_{m \mp n}^\pm = \frac{v_m r_{m \mp n}}{v_n a_{2n}^\pm}, \lambda_{m \pm n}^\pm = \frac{v_m r_{m \pm n}}{v_n a_{2n}^\pm}.
\]

These parameters define the relative contribution of different terms in Eq. (24) and (25). For a particular case of \( m = n \), values of \( \lambda_{m \pm n}^\pm \) are defined only by detector acceptance, while in general they also depend on the ratio of anisotropic flow \( v_m \) and \( v_n \). For a perfect detector \( r_m = r_n = 0 \) (and consequently all parameters \( \lambda_{m \pm n}^\pm = 0 \)), and Eqs. (24) and (25) are reduced to Eq. (4).

Eqs. (24) and (25) show that acceptance effects result in coupling of equations for flow of different harmonics, and in general a simultaneous analysis of all harmonics is required. However we proceed below neglecting \( m \neq n \) terms. The relative contribution of \( m \neq n \) terms is of the order of \( \lambda_{m \pm n}^\pm \), and this case can be understood as either when the \( n \)-th harmonic flow is dominant: \( v_n \gg v_m \) \((m \neq n)\) (such an assumption, for example, is made in [31] when discussing acceptance effects), or the acceptance effects for higher harmonics are small: \( r_{m \pm n}/a_{2n}^\pm \ll 1 \), or both.

In the following, we distinguish three types of acceptance effects:

1. **Shift of the \( u_n \)-vector** due to non-zero values of \( \overline{r}_n \) and \( \overline{s}_n \) in Eqs. (24) and (25). This effect can be corrected for by subtracting from the \( u_n \)-vector components their corresponding average values:

\[
x_n' = x_n - \overline{r}_n, \quad y_n' = y_n - \overline{s}_n.
\]

This procedure is called re-centering.
2. *Twist of the uₙ-vector* that results in appearance of \( \sin n \Psi_{RP} \) terms in \( x_n \) (\( y_n \)) components of \( u_n \)-vector. Determined by non-zero values of \( \lambda^{\pm}_2 \) in Eqs. (24) and (25), this effect can be corrected for by the diagonalization procedure (after re-centering has been applied):

\[
x''_n = \frac{x' - \lambda^{\pm}_2 y'}{1 - \lambda^{\pm}_2 \lambda^{\pm}_2}, \quad y''_n = \frac{y' - \lambda^{\pm}_2 x'}{1 - \lambda^{\pm}_2 \lambda^{\pm}_2}.
\] (29)

Twist effect is zero if \( \sin 2\pi n = 0 \), e.g., in case \( n = 1 \) for detectors symmetric in x or y (such as of rectangular shape).

3. *Rescaling of the uₙ-vector*, which is defined by the coefficients \( a^{\pm}_{2n} \) in Eqs. (24) and (25). This effect is the most important one next to the shift of the \( u_n \)-vector, and it can be corrected for by rescaling the \( u_n \)-vector components with acceptance coefficients \( a^{\pm}_{2n} \) (after the re-centering and twist corrections have been applied):

\[
x'''_n = \frac{x''}{a^{\pm}_{2n}}, \quad y'''_n = \frac{y''}{a^{\pm}_{2n}}.
\] (30)

The acceptance corrected \( u'''_n \)-vector has the same average \( \langle u'''_n \rangle_{\Psi_{RP}} \) as in the case of a detector with perfect acceptance (compare with Eq. 11):

\[
\langle u'''_n \rangle_{\Psi_{RP}} = v_n (\cos n \Psi_{RP} + i \sin n \Psi_{RP}).
\] (31)

Similar corrections can be applied for the \( Q_n \)-vector components, which we write in the following form (contributions from \( m \neq n \) terms have been neglected):

\[
\langle X_n \rangle_{\Psi_{RP}} = \overline{X}_n + A^{\pm}_{2n} (\cos n \Psi_{RP} + \Lambda^{\pm}_{2n} \sin n \Psi_{RP}),
\]

\[
\langle Y_n \rangle_{\Psi_{RP}} = \overline{Y}_n + A^{-\pm}_{2n} (\sin n \Psi_{RP} + \Lambda^{\pm}_{2n} \cos n \Psi_{RP}).
\]

The symmetry requires \( A^{\pm}_{2n} \Lambda^{\pm}_{2n} = A^{-\pm}_{2n} \Lambda^{\pm}_{2n} \). Applying corrections (28-30) for the \( Q_n \)-vector (\( a^{\pm}_{2n}, \lambda^{\pm}_{2n} \) have to be replaced with \( A^{\pm}_{2n}, \Lambda^{\pm}_{2n} \)), one gets:

\[
\langle Q_n \rangle_{\Psi_{RP}} = \cos n \Psi_{RP} + i \sin n \Psi_{RP}.
\] (32)

From Eqs. (31) and (32) it follows that all equations given in section 11D can be applied to \( u'''_n \) and \( Q'''_n \) vectors, and the same observables \( 11_{'''} \), \( 11_{'''} \), and \( 11_{'''} \) can be used for acceptance corrected \( u'''_n \) and \( Q'''_n \) vector components.

To clarify better how the above described corrections work we consider below the correlations between uncorrected \( u_n \) and \( Q_n \) vector components, and discuss what kind of effects are removed by each of the corrections 28-30.

### A. Two particle correlations

Acceptance effects in conjunction with anisotropic flow may lead to various spurious correlations, such as correlations in multiplicity and/or transverse momentum. In particular, multiplicity correlations in two kinematic regions \( a \) and \( b \) are given by the following equation:

\[
\rho_{a,b} = \int \frac{d\Psi_{RP}}{2\pi} d\phi_{a} d\phi_{b} \Lambda(\phi_{a}) \Lambda(\phi_{b}) \rho_{a,b} = 1 + 2 \sum_{m=1}^{\infty} \langle \pi_{m}\pi_{m} \rangle_{a} \langle \pi_{m}\pi_{m} \rangle_{b},
\] (33)

where \( \pi_{m} = \pi_{m} + \pi_{m} \), and \( \langle \cdots \rangle_{a,b} \) denotes the average over kinematic regions \( a \) and \( b \). According to Eqs. 24-33, the (event) average of the product of two functions \( f \) and \( g \) defined in regions \( a \) and \( b \) can be written as:

\[
\langle f_{a}g_{b} \rangle = \frac{1}{|\rho_{a,b}|} \int \frac{d\Psi_{RP}}{2\pi} \langle f_{a} \rangle_{\Psi_{RP}} \langle g_{b} \rangle_{\Psi_{RP}}.
\] (34)

Deviation of the denominator from unity (which is the value for no multiplicity correlation) is defined by non-zero terms in the sum in Eq. 33 over \( m \)-harmonics.

Taking into account that the measured anisotropic flow at RHIC is \( v_{m} \leq 10 \% \), in the most pessimistic estimate, using \( \langle \pi_{m} \rangle_{a,b} \sim 1 \), we obtain \( |\rho_{a,b} - 1| \leq 2 \% \). In practice, \( \langle \pi_{m} \rangle_{a,b} \leq 0.2 \pm 0.3 \), which reduces the acceptance effects on multiplicity correlations to the level of a tenth of a percent. In principle, such effects can be consistently taken into account, but for the sake of simplicity, below we proceed neglecting these multiplicity correlations.

In analogy to 9 we consider the correlations between uncorrected \( u_n \) and \( Q_n \) vector components:

\[
\langle x_{n}X_{n} \rangle = \langle x_{n} \rangle \langle X_{n} \rangle + \frac{v_{n}}{2} A^{\pm}_{2n} A^{\pm}_{2n} (1 + \Lambda^{\pm}_{2n} \Lambda^{\pm}_{2n}),
\]

\[
\langle y_{n}Y_{n} \rangle = \langle y_{n} \rangle \langle Y_{n} \rangle + \frac{v_{n}}{2} A^{-\pm}_{2n} A^{-\pm}_{2n} (1 + \Lambda^{-\pm}_{2n} \Lambda^{-\pm}_{2n}).
\] (35)

The first terms, \( \langle x_{n} \rangle \langle X_{n} \rangle \) and \( \langle y_{n} \rangle \langle Y_{n} \rangle \), can be removed by the re-centering procedure 28. Note, that \( \overline{x}_{n} \) and \( \overline{X}_{n} \) enter as a product, what allows to re-center only the event vector components. Similar, to remove the second order terms, \( \Lambda^{\pm}_{2n} \Lambda^{\pm}_{2n} \), it is sufficient to apply the twist correction only for the \( Q_{n} \)-vector components. Parameters \( A^{\pm}_{2n} \), and \( \Lambda^{\pm}_{2n} \) can be obtained with the random subevent technique. In that case, they are defined by a set of coupled equations:

\[
8 \langle X_{n}^{\prime \prime}X_{n}^{\prime \prime} \rangle = A^{\pm}_{2n}^{2} (1 + \Lambda^{\pm}_{2n}^{2}),
\]

\[
8 \langle Y_{n}^{\prime \prime}Y_{n}^{\prime \prime} \rangle = A^{-\pm}_{2n}^{2} (1 + \Lambda^{-\pm}_{2n}^{2}),
\]

\[
8 \langle X_{n}^{\prime \prime}Y_{n}^{\prime \prime} \rangle = A^{\pm}_{2n} A^{-\pm}_{2n} (\Lambda^{\pm}_{2n} + \Lambda^{-\pm}_{2n}).
\] (36)

After the re-centering and twist corrections have been applied Eqs. 36 leads to the following observable for the anisotropic flow \( v_{n} \):

\[
v_{n} = \frac{1}{a^{\pm}_{2n} \sqrt{2 \langle X_{n}X_{n}^{\prime \prime} \rangle}} = \frac{1}{a^{\pm}_{2n} \sqrt{2 \langle Y_{n}Y_{n}^{\prime \prime} \rangle}}.
\] (37)

Rescaling of the \( u_{n} \)-vector reduces this to Eq. 11, which should be written for rescaled \( u_{n} \) and shifted
and twisted $Q''_n$ vectors. Note, that correction factors
\[ \sqrt{(X'^a_n X'^b_n)} \text{ and } \sqrt{(Y'^a_n Y'^b_n)} \] correspond to rescaling of the $Q''_n$-vector.

Eq. (37) shows that in case of two particle correlations re-centering and twist corrections of the $u_n$-vector are not required. This equation can be also used for the $Q''_n$-vector, if the second order corrections defined by the terms $\lambda^{2\pm 4}_n$ are small, and can be neglected (twist correction is not required).

B. Three particle correlations

In the case of three particle correlations we consider:
\[
\langle x'^a_n x'^b_n x'^c_n \rangle = \langle x'^a_n x'^b_n \rangle \langle x'^c_n \rangle + \langle x'^a_n x'^c_n \rangle \langle x'^b_n \rangle + \langle x'^b_n x'^c_n \rangle \langle x'^a_n \rangle \\
+ \frac{\alpha^2 + \tilde{\alpha}^2}{4} a_{2n} X_n \left( 1 - \lambda^{2+2}_n + 2 \lambda^{2+4}_n \right) \tag{38}
\]

Similar expressions can be written for other terms in Eq. (13). In contrast to the case of two particle correlations, re-centering procedure (28) is required for all three vectors $u'_n$, $u''_n$, and $Q''_n$. Acceptance coefficients $A_{2n}^{+\pm}$ and $A_{4n}^{+\pm}$ are given by a set of coupled equations (36) written for the $Q''_n$-vector. Twist corrections applied to $Q''_n$ and $u''_n$ vectors removes the terms $\lambda^{2\pm 4}_n$ and $2 \lambda^{2\pm 4}_n$, and Eq. (38) leads to observable (19) written for $u''_n$ and $Q''_n$ vector components.

IV. METHOD SUMMARY AND CONCLUSION

In this paper we discuss new observables for anisotropic flow measurement based on correlations of $x$ and $y$ components of the flow vectors. Providing independent measures of anisotropic flow they can be used to check self-consistency of the analysis. Moreover, these observables allow direct accounting for acceptance effects, which we discuss in detail for two particular cases of anisotropic flow measurement with two and three (mixed harmonic) particle correlations. Importantly, acceptance corrections and parameters, which define applicability of these observables, can be determined directly from experimental data.

Non-uniformity of the detector acceptance is quantified with coefficients $\tau_n = \cos n \phi$ and $\tau_n = \sin n \phi$, which further define coefficients $\alpha^{c,\pm}_{2n}$ and $\lambda^{c,\pm}_{m,2n}$ given by Eqs. (26) and (27). Though accounting for the acceptance effects in general might be difficult as it requires a solution of a set of coupled equations with different harmonics involved, in the case when the contribution of $m \neq n$ harmonics can be neglected, the problem significantly simplifies. It becomes possible to correct the single particle $u_n$ and event flow $Q_n$ vectors such that the conventional observables (derived for the perfect detector) can be used. Note, that both, the acceptance coefficients $\tau_n$ and $\tau_n$, and the correlators between $u_n$ and $Q_n$ vectors can be obtained during a single pass over the data. This can significantly reduce the amount of time needed for the calculation. At the same time, due to variation of detector acceptance in time, with collision centrality, vertex position, etc., it may be important to apply the acceptance corrections separately run-by-run, for different vertex position, etc. In that case it might be more convenient to split the procedure into a few steps with two passes over the data. During the first pass the acceptance coefficients $\tau_n$ and $\tau_n$ are extracted as a function of different centrality, time, etc. and all coefficients needed for acceptance corrections presented in Eqs. (28-30), both, for $u_n$ and $Q_n$ vectors, are calculated. During the second pass over the data the correlators of the standard procedure given by Eq. (18) (Eq. (11) in the case of three particle correlations) are calculated. Finally, the flow values are extracted as given in Eqs. (11), (19).

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[1] B. B. Back et al., Nucl. Phys. A757, 28 (2005), nucl-ex/0410022.
[2] J. Adams et al. (STAR), Nucl. Phys. A757, 102 (2005), nucl-ex/0501099.
[3] K. Adcox et al. (PHENIX), Nucl. Phys. A757, 184 (2005), nucl-ex/0410003.
[4] S. A. Voloshin, Nucl. Phys. A715, 379 (2003), nucl-ex/0210014.
[5] S. Voloshin and Y. Zhang, Z. Phys. C70, 665 (1996), hep-ph/9407282.
[6] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C58, 16 (1998), nucl-ex/9805001.
[7] J. Barrette et al. (E877), Phys. Rev. Lett. 73, 2532 (1994), hep-ex/9405003.
[8] J. Barrette et al. (E877), Phys. Rev. C55, 1420 (1997), nucl-ex/9610006.
[9] H. Appelshausen et al. (NA49), Phys. Rev. Lett. 80, 4136 (1998), nucl-ex/9711001.
[10] C. Alt et al. (NA49), Phys. Rev. C68, 034903 (2003), nucl-ex/0303001.
[11] K. H. Ackermann et al. (STAR), Phys. Rev. Lett. 86, 402 (2001), nucl-ex/0009011.
[12] C. Adler et al. (STAR), Phys. Rev. Lett. 87, 182301 (2001), nucl-ex/0107003.
[13] S. S. Adler et al. (PHENIX), Phys. Rev. Lett. 91, 182301 (2003), nucl-ex/0305013.
[14] K. Adcox et al. (PHENIX), Phys. Rev. Lett. 89, 212301 (2002), nucl-ex/0204005.
[15] B. B. Back et al. (PHOBOS), Phys. Rev. Lett. 89, 222301 (2002), nucl-ex/0205021.
[16] B. B. Back et al. (PHOBOS), Phys. Rev. C72, 051901 (2005), nucl-ex/0407012.
[17] B. B. Back et al. (PHOBOS), Phys. Rev. Lett. 94, 122303 (2005), nucl-ex/0406021.
[18] J.-Y. Ollitrault, Phys. Rev. D46, 229 (1992).
[19] D. Teaney, J. Lauret, and E. V. Shuryak, Nucl. Phys. A698, 479 (2002), nucl-th/0104041.
[20] D. Teaney, J. Lauret, and E. V. Shuryak (2001), nucl-th/0110037.
[21] A. Poskanzer and S. Voloshin, LBNL Annual Report (1998), http://ie.lbl.gov/nsd1999/rnc/RNC.htm.
[22] C. Adler et al. (STAR), Phys. Rev. C66, 034904 (2002), nucl-ex/0206001.
[23] N. Borghini, P. M. Dinh, and J.-Y. Ollitrault, Phys. Rev. C66, 014905 (2002), nucl-th/0204017.
[24] J. Adams et al. (STAR), Phys. Rev. Lett. 92, 062301 (2004), nucl-ex/0310029.
[25] J. Adams et al. (STAR), Phys. Rev. C72, 014904 (2005), nucl-ex/0409033.
[26] M. Aizawa et al. (PHENIX), Nucl. Instrum. Meth. A499, 508 (2003).
[27] K. Adcox et al. (PHENIX), Nucl. Instrum. Meth. A499, 489 (2003).
[28] S. Afanasev et al. (NA49), Nucl. Instrum. Meth. A430, 210 (1999).
[29] STAR ZDC-SMD proposal (STAR), STAR Note SN0448 (2003).
[30] N. Borghini, P. M. Dinh, and J.-Y. Ollitrault, Phys. Rev. C64, 054901 (2001), nucl-th/0105040.
[31] R. S. Bhalerao, N. Borghini, and J. Y. Ollitrault, Nucl. Phys. A727, 373 (2003), nucl-th/0310016.