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Tracking the energies of one-dimensional sub-band edges in quantum point contacts using dc conductance measurements

A P Micolich¹ and U Zülicke²

¹ School of Physics, University of New South Wales, Sydney NSW 2052, Australia
² School of Chemical and Physical Sciences and MacDiarmid Institute for Advanced Materials and Nanotechnology, Victoria University of Wellington, Wellington 6140, New Zealand

E-mail: adam.micolich@nanoelectronics.physics.unsw.edu.au and Uli.Zuelicke@vuw.ac.nz

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Abstract

The semiconductor quantum point contact has long been a focal point for studies of one-dimensional (1D) electron transport. Their electrical properties are typically studied using ac conductance methods, but recent work has shown that the dc conductance can be used to obtain additional information, with a density-dependent Landé effective g-factor recently reported (Chen et al 2009 Phys. Rev. B 79 081301). We discuss previous dc conductance measurements of quantum point contacts, demonstrating how valuable additional information can be extracted from the data. We provide a comprehensive and general framework for dc conductance measurements that provides a path to improving the accuracy of existing data and obtaining useful additional data. A key aspect is that dc conductance measurements can be used to map the energy of the 1D sub-band edges directly, giving new insight into the physics that takes place as the spin-split 1D sub-bands populate. Through a re-analysis of the data obtained by Chen et al, we obtain two findings. The first is that the $2\downarrow$ sub-band edge closely tracks the source chemical potential when it first begins populating before dropping more rapidly in energy. The second is that the $2\uparrow$ sub-band populates more rapidly as the sub-band edge approaches the drain potential. This second finding suggests that the spin-gap may stop opening, or even begin to close again, as the $2\uparrow$ sub-band continues populating, consistent with recent theoretical calculations and experimental studies.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The quantum point contact (QPC) is a major landmark in the study of the electronic properties of nanoscale devices [1]. Advanced semiconductor production techniques such as molecular beam epitaxy [2] allow AlGaAs/GaAs heterostructures to be grown with monolayer precision. Such structures can support a buried two-dimensional electron gas (2DEG) that can be patterned electrostatically using metal ‘gates’ on the heterostructure surface. A QPC is typically defined by using a split-gate [3], a strip of metal with a $\sim 1 \mu m$ gap in the middle, which separates the 2DEG into source and drain reservoirs either side of an aperture of width comparable to the electron Fermi wavelength ($\sim 50$ nm). The width of the aperture can be tuned by adjusting the voltage $V_g$ applied to the gates, while the only requirement on length is that it is less...
than the elastic mean free path ($\sim 1-10 \mu m$) so that transport through the aperture is ballistic. At low temperature $T \lesssim 5 K$, the linear conductance $G$ reduces in quantized steps of $G_0 = 2e^2/h$ as the QPC is narrowed. This is due to depopulation of the 1D sub-bands within the QPC as they rise up above the Fermi energy in the reservoirs [4, 5], providing a striking demonstration of the importance of quantum effects in the operation of nanoscale devices.

A curious non-quantized plateau-like feature observed at $G = 0.7G_0$, first reported by Thomas et al. in 1996 [6], has drawn significant attention. The origin of this effect is still a matter of debate—while it is widely accepted as a many-body phenomenon, numerous microscopic mechanisms have been proposed including the presence of a static spin polarization due to the exchange interaction [6, 7] and a manifestation of the Kondo effect [8, 9], amongst others. There have also been several phenomenological models proposed based on the opening of an energy gap between the spin-up and spin-down components of the 1D sub-bands. These have successfully reproduced much of the essential behaviour observed experimentally for the 0.7 plateau [10–14]. Recent experiments by the Cambridge group have shed interesting new light on this problem. Studies of analogous non-quantized plateaus at $G > G_0$ called ‘0.7 analogs’ by Graham et al. suggest that the spin-down sub-bands drop rapidly in energy upon population [15] while the spin-up sub-band edges pin at the chemical potential [16], in general support of the spin-gap models.

Recently, Chen et al revealed that new information could be obtained by combining measurements of the dc conductance with the more commonly measured ac conductance [17–19]. Chen et al show additional evidence for differences in the population rates of spin-up and spin-down sub-bands based on plotting the ac and dc transconductance $dG/dV_g$ as a colour-map against gate voltage $V_g$ and the dc source–drain bias $V_{sd}$ [18]. Additionally, they report measurements of the effective Landé $g$-factor $g^*$ versus $V_g$, which exhibit a sawtooth appearance indicative of increasing $g$-factor with increasing electron density. This result seems counterintuitive considered alongside numerous preceding experiments showing instead that $g^*$ obtained from ac conductance measurements increases as the density is reduced in a QPC [6, 20–22, 24, 25].

The purpose of this paper is to take a further look at dc conductance measurements of QPCs and the information that can be gleaned from them. We will begin in section 2 with a brief discussion of the dc conductance technique. In section 3 we point out where the $g$-factor data presented by Chen et al have reduced accuracy, and suggest additional measurements that can provide both improved precision and new information that may have implications for the future interpretation of $g$-factor measurements obtained from the dc conductance. Finally, in section 4 we present a re-analysis of the data presented by Chen et al in [18] showing that the dc conductance can be used instead to map the evolution of the 1D sub-bands in energy with gate voltage. This re-analysis shows very clearly that the spin-down sub-bands drop rapidly in energy when they populate, in agreement with earlier data by Graham et al [15], and that the spin-up sub-bands populate much more slowly (a key conclusion of the paper by Chen et al [18]), at least initially. We find that the population rate of the spin-up sub-band increases as $V_g$ is made more positive, becoming comparable to that of the spin-down sub-band as the spin-up sub-band edge approaches the drain potential.

An interesting additional feature is observed: the spin-down sub-band edge appears to briefly track close to the source chemical potential, indicating delayed initial population for the spin-down sub-band also. This trend is observed for both the $2\downarrow$ and $3\downarrow$ sub-bands, albeit more strongly for the former. Our re-analysis and the limited data available in [18] demonstrate that more extensive measurements using this approach are warranted.

2. How the ac and dc conductances differ: a brief primer on dc conductance measurements

Experimental studies of electron transport in QPCs have traditionally relied on measurements of the ac conductance $G_{ac} = I_{sd}/V_{sd}^{ac}$, typically obtained by applying a 10–100 $\mu V$ ac bias $V_{sd}^{ac}$ at a frequency of $\sim 5–300$ Hz to the source, and measuring the resulting current $I_{sd}^{ac}$ at the drain using a lock-in amplifier. These measurements can be performed with the addition of a dc bias $V_{sd}^{dc}$ to the ac bias used to obtain $G_{ac}$ using a simple adder circuit. The dc bias separates the source $\mu_s$ and drain $\mu_d$ chemical potentials in energy by $\mu_s - \mu_d = eV_{sd}^{dc}$, allowing spectroscopic measurements of the 1D sub-band edges to be performed [10, 26] (see section 3). Very recently, measurements of the dc conductance $G_{dc} = I_{sd}^{dc}/V_{sd}^{dc}$ have been used to gain further insight into transport in QPCs [17–19]. The dc conductance can be measured simultaneously with the ac conductance by passing the output current from the device into a preamplifier to convert it to a voltage, and on into a lock-in amplifier and dc multimeter in a parallel circuit to ground [27].

The physical difference between the two conductivities is significant as they provide very different information about changes in the energy of the 1D sub-band edges. The ac conductance is a differential conductance, representing the gradient of the full dc $I$–$V$ curve for the QPC over a pair of narrow bias windows of width $V_{sd}^{dc}$, one centred on $\mu_s$ and the other on $\mu_d$. Because $V_{sd}^{ac}$ is usually kept small to minimize heating/broadening, $G_{ac}$ is only sensitive to a 1D sub-band edge passing through a chemical potential, giving a quantized jump in $G_{ac}$ of $G_0$ for a spin-degenerate sub-band in the zero dc bias limit where $\mu_s = \mu_d = \mu$, and a step of 0.25$G_0$ when a spin-polarized sub-band edge passes through $\mu_s$ or $\mu_d$ if $\mu_s - \mu_d > eV_{sd}^{dc}$. In contrast, the dc conductance is sensitive to sub-band edge motion through the entire window between $\mu_s$ and $\mu_d$. Consider a 1D sub-band that starts above $\mu_s$ which in turn is $eV_{sd}^{dc} \gg eV_{sd}^{ac}$ above $\mu_d$. As the 1D sub-band falls in energy, $G_{dc}$ remains constant until the sub-band edge

\[ \text{Note that a positive bias applied to one reservoir actually lowers it in energy with respect to the other. Hence there is an implicit sign reversal assumed here, such that the drain is held fixed at electrical ground and a positive } V_{sd}^{dc} \text{ raises } \mu_s \text{ above } \mu_d \text{ rather than lowering it.} \]
crosses $\mu_s$. The dc conductance then increases gradually as the sub-band edge lowers through the bias window, ultimately reaching $\mu_d$ where $G_d$ once again becomes constant, having increased by $G_0$ for a spin-degenerate sub-band and 0.5$G_0$ for a spin-polarized sub-band. The crucial aspect for this study is that $G_d$ provides information about the location and rate of movement of a 1D sub-band edge as a function of $V_g$ whenever it is located between $\mu_s$ and $\mu_d$, information that cannot be attained from ac conductance measurements alone [17].

3. Measurements of the $g$-factor using dc conductance

Measurements of $G_{ac}$ and $G_d$ versus $V_g$ as a function of $V_{sd}$ can be used as a spectroscopic tool for studying the physics of the 1D sub-bands [18, 19]. This is often achieved using a greyscale plot or colour-map of the transconductance $dG_{ac}/dV_g$ or $dG_d/dV_g$ versus $V_g$ (x-axis) and $V_{sd}$ (y-axis)—examples of such colour-maps for ac and dc transconductance appear in figures 3(a) and (b) of [18], respectively. Regions of low and high $dG_{ac}/dV_g$ (dark and bright in figure 3 of [18]) indicate conductance plateaus and rises in conductance between plateaus, respectively. In the ac case, if we start at $V_{sd} = 0$ and increase the dc bias, the high $dG_{ac}/dV_g$ regions evolve into V-shaped structures. The left- and right-moving branches correspond to a given 1D sub-band edge coinciding with $\mu_s$ and $\mu_d$, respectively. The area inside the V-shaped structure has $dG_{ac}/dV_g \approx 0$ as $G_{ac}$ is fixed when the sub-band edge is not within $V_{sd}$ of $\mu_s$ or $\mu_d$. In contrast, for a dc transconductance colour-map the V-shaped region is ‘filled’, with $dG_d/dV_g$, indicating the rate at which the sub-band edge moves in energy between $\mu_s$ and $\mu_d$.

To facilitate further discussion, we refer to the schematic transconductance map shown in figure 1(a), where the numbers correspond to the 1D sub-band index $n = 1, 2, 3, \ldots$, and $\uparrow$ and $\downarrow$ to spin up and down respectively. For direct comparison with [18], we consider the case where a strong in-plane magnetic field $B_z$ is applied, breaking the spin degeneracy of the 1D sub-bands. The left- and right-sloping dashed diagonal lines in figure 1(a) indicate the gate voltage settings where a given 1D sub-band edge coincides with $\mu_s$ and $\mu_d$, respectively, as the dc bias $V_{sd}$ is increased. Note that although these are presented as straight lines in our schematic, in reality they curve slightly due to the gate voltage dependent sub-band spacing caused by the self-consistent electrostatic potential of the QPC [28]. The $g$-factor measurements in [18] are obtained by following a very specific zig-zag path through the transconductance plot, indicated by the thick black line in figure 1(a) (compare with figure 3(a) of [18]). The path has six general ‘configurations’ from which an estimate of $g^s$ can be obtained. These configurations repeat cyclically moving right or left along the zig-zag path, and correspond to the six energy diagrams shown in figures 1(b)–(g) where we plot the positions of sub-band edges $n \uparrow$, $n \downarrow$, $n + 1 \uparrow$ and $n + 1 \downarrow$ relative to $\mu_s$ and $\mu_d$. These are labelled $\alpha_n, \alpha_{n+1}, \alpha_{n,n+1}, \delta_{n,n+1}, \delta_{n+1,n+1}$ and $\delta_{n,n+1}$ to correspond as directly as possible to the three configurations $\alpha$, $\beta$ and $\gamma$ identified for the first sub-band data in figure 4 of [18], and provide a more general framework for measurements of $g^s$.  

3.1. General framework for extracting $g^s$ from high-field source–drain bias spectroscopy

At the low $V_{sd}$ vertices, highlighted by six circles that alternate between blue and red in figure 1(a), 1D sub-band edges adjacent in energy align with $\mu_s$ and $\mu_d$, allowing their energy separation to be read directly as $E_{sd}$. These six points only require measurement of the ac conductance, and represent the method established by Patel et al [26] and used previously [6, 20–25] to determine the $g$-factor of the 1D sub-bands. The blue circles correspond to the $\alpha_{n,n+1}$ configuration (see figures 1(b) and 2(a)), where the $n \uparrow$ and $n \downarrow$ sub-band edges align with $\mu_s$ and $\mu_d$, respectively, and a pure, single sub-band $g^s$ value can be directly measured. The red circles correspond to the $\delta_{n,n+1}$ configuration (see figure 1(e)), here direct measurement using $eV_{sd}$ at the sub-band crossing gives instead an average of the $g$-factors for two adjacent sub-bands ($g_n^s + g_{n+1}^s$)/2 [22] unless additional information is known about the relative locations of various 1D sub-band edges, as applied by Chen et al [18] (see section 3.2).

Starting at $\alpha_{n,n+1}$ or $\delta_{n,n+1}$ has $\mu_s$ and $\mu_d \neq$ held at adjacent sub-band edges. Following the zig-zag in figure 1(a) involves first holding the lowest 1D sub-band edge at $\mu_d$ while raising $\mu_s$ up towards the next highest sub-band along the positive-gradient diagonal (see $\alpha_{n,n+1}$ in figure 1(c) and $\delta_{n,n+1}$ in figure 1(f)), and then holding $\mu_s$ at that sub-band edge while $\mu_d$ rises up to the sub-band directly below along the negative-gradient diagonal (see $\delta_{n,n+1}$ in figure 1(d) and $\delta_{n,n+1}$ in figure 1(g)). Note that moving right in figure 1(a) corresponds to all of the 1D sub-bands moving downwards together in energy. It is the vertical motion associated with the zig-zag that allows one of $\mu_s$ or $\mu_d$ to track one sub-band edge while keeping the adjacent 1D sub-band edge in the bias window to allow continuous measurement of $g^s$ via $G_d$ (e.g. see the horizontal purple bars in figure 3). For all positions on the $\alpha_{n,n+1}$, $\alpha_{n+1,n+1}$, $\delta_{n+1,n+1}$ and $\delta_{n,n+1}$ branches, the dc conductance $G_d = (n + \Delta V/V_{sd})/G_0$ is needed to establish the energy separation $e\Delta V$ between the $\mu_s$ and the edge of the sub-band edge sitting below it in the bias window in order to obtain information about $g^s$.

Finally, at the upper vertices of the zig-zag (yellow circles in figure 1(a)), $\mu_s$ and $\mu_d$ span two sub-band gaps, and these points allow direct local measurements of the energy separation between identical spin branches of adjacent sub-bands (i.e. $\Delta E_{n,n+1} = E_{n+1} - E_n$ or $\Delta E_{n+1,n+1}$). Although Chen et al do not mention this explicitly, a measurement of this gap is vital to their method for obtaining single sub-band $g^s$ values. It is the cause of significant problems for the accuracy of these $g^s$ values, as we now discuss.

3.2. A review of $g^s$ data obtained from dc conductance measurements

The $g^s$ data in [18] are obtained from five short segments from a zig-zag path such as that in figure 1(a). We start
Figure 1. (a) Schematic of an idealized transconductance map plotted against dc source–drain bias $V_{sd}$ (y-axis) and gate voltage $V_{g}$ (x-axis) for a large in-plane magnetic field such that the spin degeneracy of the 1D sub-bands is broken. The diagram contains a number of diagonal dashed lines forming V-shaped structures with a vertex on the $V_{g}$ axis. These vertices correspond to the $V_g$ at which a sub-band with index $n$ and spin up (↑) or down (↓) intercepts the chemical potential $\mu$. The left (right) branches of a given V-shaped structure correspond to that sub-band coinciding with the source $\mu_s$ (drain $\mu_d$) potentials. The blue and red circles indicate points where $\mu_s$ and $\mu_d$ coincide with adjacent 1D sub-band edges, these being the $n$↑ and $n$↓ ( $n$, $n+1$) sub-band edges for the blue (red) circles. The 1D sub-band spacing can be obtained at these points following Patel et al [26]. The yellow circles indicate points where $\mu_s$ and $\mu_d$ coincide with sub-band edges having the same spin but sub-band index differing by 1. The separations $\Delta E_{n,1}$ and $\Delta E_{n+1,1}$ are obtained at these points, which play an essential role in the method used by Chen et al to obtain $g^*$. The green circles indicate crossing points for spin-degenerate sub-band edges, where the separations $\Delta E_{n,1}$ would be obtained at $B_1 = 0$. The solid zig-zag line indicates the path taken in obtaining $g^*$ data, with the results in [18] obtained from short segments of such a path. (b)–(g) Sub-band edge energy diagrams corresponding to the six configurations in (a). The quantity $\epsilon \Delta V$ is the energy separation between $\mu_s$ and the next lowest sub-band edge, and is obtained using $G_k = (n + \Delta V/V_{sd})G_{0k}$. Other quantities are defined in the text. These are (b) $\alpha_{n+1}$ blue circles, (c) $\alpha_{n+1}$ red circles, (d) $\delta_{n+1}$ positive-gradient part of zig-zag, (e) $\alpha_{n+1}$ negative-gradient part of zig-zag, (f) $\delta_{n+1}$ positive-gradient part of zig-zag and (g) $\delta_{n+1}$ negative-gradient part of zig-zag.

3.2.1. Accuracy of the $g^*$ values. It is important to note that with the exception of the one point marked $\alpha$, the extraction of all of the first sub-band $g^*$ values in figure 4 of [18] relies on the implicit assumption that the sub-band separation $\Delta E_{1,2}$ is constant as a function of $V_g$. Significant caution needs to be exercised in making this assumption, because as figure 2(d) highlights, $\Delta E_{1,2}$ is not constant.

4 A careful analysis of figures 3 and 4 in [18] reveals that the fifth point on the $\gamma$ branch for the first sub-band lies beyond the crossing point for the $1^\uparrow$ and $2^\uparrow$ sub-bands, while the fourth, there is more than one 1D sub-band edge within the bias window, and these data points should thus be considered with caution.

this discussion by pointing out these data, and the three configurations $\alpha$, $\beta$ and $\gamma$ they identify for the first sub-band in figure 4 of [18], correspond to the more general framework discussed above and presented in figure 1.

The $\alpha$ configuration in [18] shown in figure 2(a) corresponds exactly to $\alpha_{1,2}$ (see figure 1(b)). In the data presented by Chen et al this configuration provides the lowest $g^*$ value at most negative $V_g$ for each sub-band (first, third and fifth open symbols in figure 4 of [18]). The next stretch of values, solid symbols denoted $\beta$ in [18], correspond to the energy diagram in figure 2(b) and to the negative-gradient $\alpha_{1,2}$ branch in figure 1(a) (i.e. the $\alpha_{1,2}$ branch is skipped in the measurements in [18]). The open symbol between the $\beta$ and $\gamma$ branches corresponds to $\delta_{1,2}$ in the nomenclature used in figure 1, and is the situation intermediate to figures 2(b) and (c) where $\mu_s$ ( $\mu_d$) coincides with 2↓ (1↑). Finally the $\gamma$ branch in figure 4 of [18] corresponds to the positive-gradient $\delta_{1,2}$ branch in figure 1 and the energy diagram in figure 2(c)4.
The gate voltage $V_g$ dependence of $\mu_\parallel$ (horizontal purple lines) and various 1D sub-band edges (triangles). Up (down) triangles correspond to spin-up (down) sub-bands and the colours black, red, blue and green correspond to sub-band indices $n = 1-4$. The drain reservoir $\mu_d$ has been set as the zero of energy. The corresponding configurations $\delta n_{\alpha+1}$, $\delta \beta_{\alpha,n+1}$, $\Delta \gamma_{\alpha+1}$ and $\alpha \gamma_{\alpha,n+1}$ are indicated to facilitate reference to figure 1. The circled regions A–C indicate the behaviours of the $2\uparrow$, $2\uparrow$ and $3\downarrow$ sub-bands as they populate, and are discussed in detail in the text.

Thus if we consider equation (1), it is clear that $\Delta E_{1\downarrow, 2\uparrow}$ cannot possibly be constant as a function of $V_g$.

Some simple estimates confirm the significance of this issue. The separation $\Delta E_{1\downarrow, 2\uparrow}$ can be directly and accurately measured at $V_g = -5.4785 \text{V}$, and using the data in figure 3(a) of [18], we obtain $\Delta E_{1\downarrow, 2\uparrow} = 3.33 \text{meV}$. If one considers the data in figure 4 in [18], the separation between $1\downarrow$ and $1\uparrow$ (i.e. $g_1^\parallel \mu_B B_1$) increases by a factor of over 3.5 from 0.8 meV at $V_g = -5.5431 \text{V}$ (ar peak) to 2.87 meV at $-5.37 \text{V}$ (right-most point of $\gamma$ branch). This corresponds to the full range over which $\Delta E_{1\downarrow, 2\uparrow}$ is assumed constant in determining $g^\parallel$. Clearly the effect of the change in $g_1^\parallel$ on $\Delta E_{1\downarrow, 2\uparrow}$ is impossible to neglect, and the gate voltage dependence of $\Delta E_{1\downarrow, 2\uparrow}$ will exacerbate this.

The $\Delta E_{n\uparrow, n+1\uparrow}$ or $\Delta E_{n\downarrow, n+1\downarrow}$ obtained at an upper sub-band vertex is a reasonable approximation on the branches running down either side from the corresponding vertex (i.e. $\alpha \gamma_{n,n+1}$ and $\alpha \beta_{n,n+1}$ for $\Delta E_{n\uparrow, n+1\uparrow}$, and $\delta \gamma_{n,n+1}$ and $\delta \beta_{n,n+1}$ for $\Delta E_{n\downarrow, n+1\downarrow}$). However, by the final $\gamma$ point in figure 4 in [18], which is obtained at the top of the $\delta \gamma_{1,2}$ branch using a $\Delta E_{1\downarrow, 2\uparrow}$ estimate obtained at the vertex of the $\alpha \gamma_{1,2}$ and $\alpha \beta_{1,2}$ branches, the $g^\parallel$ values become inaccurate. The accuracy may be improved by making full use of available sub-band spacing measurements. For example, at $B_|| = 0$ where the $\uparrow$ and $\downarrow$ branches are degenerate, the sub-band spacing $\Delta E_{1\downarrow, 2\uparrow}$ can be measured. The corresponding points are shown as green circles in figure 1(a), and the diagrams in figures 1(b)–(g) indicate how this can be used to obtain additional $g^\parallel$ estimates. Furthermore, the $\Delta E_{n\uparrow, n+1\uparrow}$ estimates can be used to obtain further measurements beyond

![Figure 2](image-url)

Figure 2. (a)–(c) Sub-band edge energy diagrams illustrating the three configurations $\alpha$, $\beta$ and $\gamma$ used in [18]. (d) Schematics illustrating why $\Delta E_{1\downarrow, 2\uparrow}$ cannot be safely assumed to be independent of $V_g$. For the sake of argument, schematic I is chosen to coincide with the left-most yellow circle in figure 1(a), where the first and second sub-band $g$-factors $g_1^\parallel$ and $g_2^\parallel$ are small in comparison to the spin-degenerate sub-band spacing $\Delta E_{1\downarrow, 2\uparrow}$. The separation $\Delta E_{1\downarrow, 2\uparrow}$ used by Chen et al to obtain $g_1^\parallel$ depends on $g_1^\parallel$ and $\Delta E_{1\downarrow, 2\uparrow}$ as given in equation (1). As $V_g$ is made more positive (i.e. moving right in figure 1(a) here and figure 4 of [18]), three changes occur: $g_1^\parallel$ increases dramatically, $\Delta E_{1\downarrow, 2\uparrow}$ decreases slightly due to weakening 1D confinement, and $g_2^\parallel$ may also increase, albeit to a much lesser extent than $g_1^\parallel$. Ultimately, this makes it impossible for $\Delta E_{1\downarrow, 2\uparrow}$ to remain constant, introducing significant systematic error into the $g^\parallel$ values presented by Chen et al, as discussed in the text.

Consider schematic I in figure 2(d); for the sake of argument, let us assume that this corresponds to the left-most yellow circle in figure 1(a), which is the only gate voltage at which a precise measured value for $\Delta E_{1\downarrow, 2\uparrow}$ can be obtained. The dashed horizontal lines indicate the spin-degenerate edges of the first and second sub-bands, their separation $\Delta E_{1\downarrow, 2\uparrow}$ is set by the QPC confinement potential. With a magnetic field $B_\parallel$ applied, the spin degeneracy is broken, and the $1\downarrow$ and $1\uparrow$ sub-band edges separate in energy by $g_1^\parallel \mu_B B_1$ while the $2\downarrow$ and $2\uparrow$ sub-band edges separate by $g_2^\parallel \mu_B B_1$. Note that $g_1^\parallel$ does not necessarily equal $g_2^\parallel$. As schematic I shows, this results in:

$$\Delta E_{1\downarrow, 2\uparrow} = \Delta E_{1\uparrow, 2\downarrow} + g_1^\parallel \mu_B B_1 - g_2^\parallel \mu_B B_1.$$  (1)

If we accept the premise that $g^\parallel$ is $V_g$ dependent in the manner suggested by figure 4 in [18], then making $V_g$ more positive would bring us to the scenario in schematic II in figure 2(d), where two changes will have occurred. First and foremost, $g_1^\parallel$ will have increased significantly, as we discuss below. Second, as $V_g$ becomes more positive the confinement is weakened, reducing the separation $\Delta E_{1\downarrow, 2\uparrow}$. There may also be an increase in $g_2^\parallel$, although this should be a much smaller contribution.
those presented in [18], as we discuss below. Putting all these measurements together and carefully considering the range of validity for each sub-band spacing, a more accurate picture can be obtained.

3.2.2. Additional data from $\alpha \gamma_{n,n+1}$ and $\delta \beta_{n,n+1}$ branches. Considering the data in [18] alongside figure 1 it is clear that while Chen et al have made an important contribution, the opportunity exists for a substantial amount of additional data to be obtained with their approach. This may point to behaviour more complex than the linear trend shown in figure 4 of [18]. The missing $\alpha \gamma_{1,2}$ branch for the first sub-band would provide information about what happens as the 1↑ sub-band begins to populate, and it would be interesting to compare this with the behaviour of the $\gamma$ branch in figure 4 of [18]. The $\gamma$ branch ($\delta \gamma_{1,2}$ in our nomenclature) corresponds to the initial population of 2↓, and the first point appears to deviate from the linear trend of the rest of the branch (we return to this in section 4). It is interesting to note that, in contrast to the first sub-band data, the $\alpha \gamma_{2,3}$ branch is measured for the second sub-band in [18], and the non-monotonicity (albeit with only three data points) may suggest that interesting behaviour occurs as 2↑ populates.

The missing $\delta \beta$ branch for both the first and second sub-bands would also be interesting, particularly combined with the additional data that could be obtained using the $\Delta E_{n1,n+n+1}$ measurements. In [18], the $\Delta E_{1,2}$ value obtained at the intersection of the $\alpha \gamma$ and $\alpha \beta$ branches is used to derive $g^*_1$ along the $\gamma$ branch (i.e. $\delta \gamma$ branch in figure 1(a)). However, $\Delta E_{1,2}$, which is obtained at the $\delta \gamma / \delta \beta$ intersection and is thus more accurate here, can be used to measure $g^*_2$ along the same $\gamma$ branch as well. With $g^*_2$ measured along both $\delta \gamma$ and $\delta \beta$ branches, it would be possible to establish precisely how the gap between 2↓ and 2↑ evolves as the 2↓ sub-band populates. It may also be possible to adapt this process to the left of the first $\alpha$ point in figure 4 of [18] to obtain useful information regarding the population of the 1↓ sub-band. However, since there is no sub-band separation information to the left of where $\Delta E_{1,2}$ is obtained, this data may be qualitative at best.

3.2.3. Interpretation of the g-factor data. The $g^*$ data presented in [18] are a surprising contrast to the previously accepted trend for $g^*$ to gradually and monotonically increase as the sub-band index $n$ is reduced [6, 20–22, 24, 25]. Chen et al note the remarkable similarity between these data and both the oscillatory $g^*$ in quantum Hall systems as consecutive Landau levels are filled [29] and the theoretical prediction by Wang and Berggren (see figure 2 of [30]). The latter is of particular interest, as a closer inspection reveals that the data and this prediction actually disagree. Firstly, we consider exactly what Wang and Berggren predict.

Figure 2 of Wang and Berggren’s paper [30] shows $g^*$, calculated using density functional theory, for the lowest spin-split sub-band as the 1D density within the QPC is increased such that the second, third and fourth 1D sub-bands populate. This calculated $g^*_1$ rises at the point where a spin-degenerate sub-band begins to populate as exchange effects lead to a spontaneous spin polarization and a finite spin-gap (i.e. separation between spin-up and spin-down components of that particular sub-band). This gap collapses once the opposite spin sub-band edge drops below the Fermi energy, driving $g^*$ back towards zero. The result is an ‘undulating’ $g^*_1$ versus $V_{sd}$ where the undulations get smaller as successively higher sub-bands fill. The behaviour Wang and Berggren calculate in figure 2 of [30] cannot be measured directly in the conductance. Looking at figure 1, the $\delta \gamma_{1,2}$ branch (i.e. $\gamma$ in figure 4 of [18]) is the last point where any direct information about $g^*_1$ can be obtained because from $\delta \beta_{1,2}$ onwards the edges of both 1↓ and 1↑ move below $\mu_d$ and away from the bias window.

However, from figure 1(a/b) of [30] it is clear that whenever an exchange induced spin-gap occurs in the first sub-band, it also occurs in the higher sub-bands. Thus in a measurement such as that presented by Chen et al where the focus needs to shift from one sub-band to the next as they fall below the dc bias window, the calculations by Wang and Berggren would still predict an oscillatory behaviour of $g^*$ (providing that there are no obscuring artefacts due to the change in the particular sub-band being measured). This can be determined by mapping the locations of the rises and falls in $g^*$ to where the sub-band edges pass into and out of the bias window, and this is where we find an important discrepancy.

Returning to Wang and Berggren’s calculation, $g^*_1$ only rises until the 1↑ sub-band falls below the chemical potential; it collapses back to almost zero thereafter. This would occur at point $\delta_{1,2}$, and so the continued rise in $g^*_1$ along the $\gamma$ branch in figure 4 of [18] contradicts the calculated behaviour in [30]. Note that the apparent precipitous drop in $g^*$ at the end of the $\gamma$ branch actually reflects a change in what is measured from $g^*_1$ to $g^*_2$, which is significantly smaller in magnitude. Indeed, the data in figure 4 of [18] are more consistent with the density-dependent spin-gap model developed by Reilly et al [12–14], and obtaining measurements of $g^*_2$ over the $\delta \gamma_{1,2}$ and $\delta \beta_{1,2}$ branches either side of the 1↑/2↑ sub-band crossing would be particularly enlightening in this regard.

4. Tracking the 1D sub-bands

We finish by considering an interesting question—why measure the g-factor at all? In obtaining the g-factor by the method used in [18], we take rather precise information about the location of two adjacent sub-band edges, one held at a chemical potential and the other measured relative to it using the dc conductance, and combine it with comparatively imprecise information about the energy separation between other sub-band edges. Why not confine our attention to the sub-band measurements since this should provide more precise and useful information?

To demonstrate that direct tracking of the 1D sub-band edges is possible, we have re-analysed the data presented in [18]. This was achieved by using the Datathief software package [31] to extract the $g^*$ versus $V_{sd}$ data in figure 4 of [18]. From figures 3(a, b) of [18] we similarly extracted the sub-band spacings $\Delta E_{n1,n+n+1}$ and $\Delta E_{n1,n+n+1}$, and the source–drain bias values $V_{sd}$ versus $V_g$ for each $g^*$ value.
shown in figure 4 of [18]. This made it possible to work backwards for each point to find, relative to \( \mu_d \), the energies of \( \mu_s \), the sub-band edge held at a reservoir potential and the sub-band edge in the bias window. These are plotted at their corresponding \( V_g \) values in figure 3. This figure is complex at first sight, we will therefore explain it step-by-step. The drain is at electrical ground (via the current input of the lock-in amplifier) in this measurement configuration, and hence we consider \( \mu_d \) as our zero of energy, in keeping with convention in related papers on QPCs. The drain potential is denoted by the black dashed horizontal line in figure 3. The horizontal purple bars show the position of \( \mu_s \) relative to \( \mu_d \) at each \( V_g \), which is obtained from the corresponding point on the zig-zag path shown in figure 1(a), when applied to figure 3(a) of [18]. The locations of the sub-band edges that are within the dc bias window are shown as solid symbols\(^5\), with upward (downward) triangles indicating spin up (down), and the colours black, red, blue and green indicating \( n = 1, 2, 3 \) and 4, respectively, to best match the colour scheme used in figure 4 of [18].

We are limited to the data available in [18], however it is possible to obtain a complete picture of both spin-up and spin-down components for the second sub-band, as well as the spin-down component of the third sub-band. The relevant data are circled and marked A–C and lead to some interesting conclusions. In region A, we start at \( \delta \gamma_2 \), where the \( 2 \uparrow \) edge coincides with \( \mu_s \) and the \( 1 \uparrow \) edge coincides with \( \mu_d \), and follow the \( \delta \gamma_2 \) branch where the \( 1 \uparrow \) edge is held at \( \mu_d \) and \( \mu_s \) is gradually raised in energy (see figure 1(f)). As the \( 2 \downarrow \) sub-band populates, it drops in energy, consistent with previous experimental findings by Graham et al. [15]. Similar behaviour is observed for the \( 3 \downarrow \) sub-band in region C. Interestingly, \( 2 \uparrow \) appears to briefly track \( \mu_s \) as \( V_d \) is initially increased. This initial delay in population is also apparent for \( 3 \downarrow \), albeit to a lesser extent. Delayed population of \( \downarrow \) sub-bands has not been previously reported to our knowledge, and would be worth further investigation in future studies.

Turning to region B, here we start at \( \delta \gamma_{1,3} \), where the \( 2 \uparrow \) edge coincides with \( \mu_s \) and the \( 2 \downarrow \) edge coincides with \( \mu_d \), and follow the \( \alpha \gamma_{2,3} \) branch where the \( 2 \downarrow \) edge is held at \( \mu_d \) and \( \mu_s \) is gradually raised in energy (see figure 1(c)). Here it is clear that \( 2 \uparrow \) initially populates more slowly than \( 2 \downarrow \), as pointed out by Chen et al. [18], and in general agreement with studies by Graham et al. [16]. A question of interest at present is whether the \( \uparrow \) sub-bands pin to \( \mu_s \) as they populate [16, 34] or merely appear due to a relatively slow population (see e.g. figure 1(b) of [14], or [35]). Our re-analysis in figure 3 shows no evidence that the \( 2 \uparrow \) edge pins to \( \mu_s \) after the \( 2 \downarrow \) sub-band edge reaches \( \mu_d \), and due to the nature of the method and limited available data, it is not possible to accurately comment on the behaviour of \( 2 \uparrow \) before \( 2 \downarrow \) reaches \( \mu_d \) at \( V_g = -5.34 \text{ V} \) (\( \alpha \gamma_{2,3} \) in figure 3). Note that although there is a missing branch between the \( \delta \gamma_{1,2} \) branch corresponding to region A and the \( \alpha \gamma_{2,3} \) branch corresponding to region B in figure 3, it would provide no new information about the behaviour of \( 2 \uparrow \) relative to \( \mu_s \). This is because \( 2 \uparrow \) is held at \( \mu_s \), throughout this \( \delta \gamma_{1,2} \) branch. However, it does provide new information about the motion of \( 2 \uparrow \) relative to \( 2 \downarrow \) that may be useful. By following other paths in the \( V_{sd} \) versus \( V_g \) space (e.g. vertical motion by changing \( V_{sd} \) at specifically chosen \( V_g \)) it may be possible to extract additional knowledge about sub-band edge motion and pinning without reliance on assumptions about sub-band spacing.

Another question of interest is how the gap between \( \uparrow \) and \( \downarrow \) sub-bands evolves after the \( \uparrow \) sub-band has dropped below \( \mu_s \)—does the gap keep opening [12], hold constant (see figure 5 of [18]), or start closing again [30, 34, 35]? The \( \alpha \beta \) branch data between regions B and C suggest that as \( 2 \uparrow \) approaches \( \mu_s \) it populates at a very similar rate to the \( 2 \downarrow \) sub-band under similar circumstances. Very similar behaviour is observed for \( 1 \uparrow \), and is consistent with recent measurements by Chen et al. [19]. There a study of plateau-like structures at 0.7–0.85\( G_0 \) in \( G_{ac} \) and \( G_{dc} \) at finite \( V_{sd} \) led Chen et al to conclude that there is an unusual population behaviour for the first spin-up sub-band as it moves between \( \mu_s \) and \( \mu_d \). The more rapid drop in the \( \uparrow \) sub-bands as \( V_g \) becomes less negative, as observed in figure 3, which suggests at least a stabilization of the spin-gap (i.e. it becomes constant in \( V_g \)), and perhaps that it may even close again. However, this latter possibility depends on how \( 2 \downarrow \) moves once it is below \( \mu_s \), something that is inaccessible to these measurements\(^6\). This behaviour, if it occurs, would be consistent with the Bruus, Cheianov and Flensberg model [11, 32] and recent calculations by both Jaksch et al. [33] and Lind et al. [35]. Clearly further measurements using this approach are warranted to look more closely at the evolution of the sub-bands as they are populated. It would be particularly interesting to use a device where independent control over the QPC width and the 2DEG density could be achieved, for example, by a top- or back-gate as in the devices studied by Reilly et al. [12] or Hamilton et al. [36], respectively.

5. Conclusions and outlook

In conclusion, we have provided a general framework for extracting 1D sub-band edge energies and \( g \)-factors using ac and dc conductance measurements of QPCs. This framework shows routes to improving the accuracy of measured \( g^* \) values and interesting opportunities for additional measurements, in particular, tracking of the second sub-band \( g \)-factor over the population of the \( 2 \downarrow \) sub-band. It also demonstrates that the measured data do not exhibit trends consistent with calculations by Wang and Berggren [30] but may instead point to a density-dependent spin-gap as predicted by Reilly et al. [12–14]. Finally, we show that the information extracted

\(^5\) For completeness, in supplementary figure 1 (available at stacks.iop.org/JPhysCM/23/362201/mmedia), we present a copy of figure 3 with an additional series of hollow data points that follow the same shape/colour convention as those presented in figure 3. These points correspond to sub-band edges that nominally fall outside the dc bias window. Their locations are estimated by assuming that \( \Delta E_{I_v,n+\frac{1}{2}} \) and \( \Delta E_{I_v,n+\frac{1}{2}} \) are \( V_g \) independent.

\(^6\) Note that at first this seems at odds with the data in figure 4 of [18] where \( g^* \) keeps increasing monotonically, however this is based on the assumption of constant \( \Delta E_{I_v,n+\frac{1}{2}} \), this is clear by examining supplementary figure 1 (available at stacks.iop.org/JPhysCM/23/362201/mmedia).
from dc conductance measurements can be used to map the evolution of the 1D sub-band edges with $V_g$ and may provide more useful knowledge about the physics occurring as the 1D sub-bands populate than conversion to a $g$-factor does. In particular, an analysis from a sub-band energy perspective shows that the $2\downarrow$ sub-band drops in energy as it populates, consistent with earlier measurements by Graham et al [15], but suggests that the $2\uparrow$ edge sub-band tracks $\mu_s$ closely at first, a feature not previously reported in the literature. The $2\uparrow$ sub-band initially populates more slowly, in general agreement with earlier work by Graham et al [16]. There is no evidence that the $2\uparrow$ edge pins to $\mu_s$, however it is not possible to measure this until $2\downarrow$ reaches $\mu_d$ using the data available in [18]. Our re-analysis also shows that the population rate for $2\uparrow$ eventually increases to become as rapid as that for $2\downarrow$. This suggests that the spin-gap may become independent of $V_g$, and perhaps even close again, as the $2\uparrow$ sub-band continues to populate. This behaviour would be in rough qualitative agreement with theoretical calculations [30, 33–35], and is consistent with the suggestion by Chen et al [19] that there is an unusual population behaviour of the first spin-up sub-band as it passes between the source and drain potentials. Our re-analysis highlights the opportunity for further measurements with this approach, particularly in devices where the QPC width and electron density can be tuned independently.

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References

[1] Berggren K-F and Pepper M 2002 Phys. World 15 (10) 37
[2] Cho A Y 1971 Appl. Phys. Lett. 19 467
[3] Thornton T J, Pepper M, Ahmed H, Andrews D and Davies G J 1986 Phys. Rev. Lett. 56 1198
[4] van Wees B J, van Houten H, Beenakker C W J, Williamson J G, Kouwenhoven L P, van der Marel D and Foxon C T 1988 Phys. Rev. Lett. 60 848
[5] Wharam D A, Thornton T J, Newbury R, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A and Jones G A C 1988 J. Phys. C 21 L209
[6] Thomas K J, Nicholls J T, Simmons M Y, Pepper M, Mace D R and Ritchie D A 1996 Phys. Rev. Lett. 77 135
[7] Wang C-K and Berggren K-F 1998 Phys. Rev. B 57 4552
[8] Meir Y, Hirose K and Wingreen N S 2002 Phys. Rev. Lett. 89 196802
[9] Cronenwett S M, Lynch H J, Goldhaber-Gordon D, Kouwenhoven L P, Marcus C M, Hirose K, Wingreen N S and Umansky V 2002 Phys. Rev. Lett. 88 226805
[10] Kristensen A, Bruus H, Hansen A E, Jensen J B, Lindelef P E, Markckmann C J, Nygård J, Sørenson C B, Beuscher F, Forchel A and Michel M 2000 Phys. Rev. B 62 10950
[11] Bruus H, Cheianov V V and Flensberg K 2001 Physica E 10 97
[12] Reilly D J, Buehler T M, O’Brien J L, Hamilton A R, Drurak A S, Clark R G, Kane B E, Pfeiffer L N and West K W 2002 Phys. Rev. Lett. 89 246801
[13] Reilly D J 2005 Phys. Rev. B 72 033309
[14] Reilly D J, Zhang Y and DiCarlo L 2006 Physica E 34 27
[15] Graham A C, Pepper M, Simmons M Y and Ritchie D A 2005 Phys. Rev. B 72 193305
[16] Graham A C, Sawley D L, Pepper M, Simmons M Y and Ritchie D A 2007 Phys. Rev. B 75 035331
[17] Chen T-M, Graham A C, Pepper M, Farrer I and Ritchie D A 2008 Appl. Phys. Lett. 93 021202
[18] Chen T-M, Graham A C, Pepper M, Sfigakis F, Farrer I and Ritchie D A 2009 Phys. Rev. B 79 081301
[19] Chen T-M, Graham A C, Pepper M, Farrer I, Anderson D, Jones G A C and Ritchie D A 2010 Nano Lett. 10 2330
[20] Patel N K, Nicholls J T, Martin-Moreno L, Pepper M, Frost J E F, Ritchie D A and Jones G A C 1991 Phys. Rev. B 44 10973
[21] Daneshvar A J, Ford C J B, Hamilton A R, Simmons M Y, Pepper M and Ritchie D A 1997 Phys. Rev. B 55 13409
[22] Danneau R, Klochan O, Clarke W R, Ho L H, Micolich A P, Simmons M Y, Hamilton A R, Pepper M, Ritchie D A and Züllicke U 2006 Phys. Rev. Lett. 97 026408
[23] Schäpers Th, Guzenko V A and Hardtdegen H 2007 Appl. Phys. Lett. 90 122107
[24] Martin T P, Szorkovszky A, Micolich A P, Hamilton A R, Marlow C A, Linke H, Taylor R P and Samuelson L 2008 Appl. Phys. Lett. 93 012105
[25] Martin T P, Szorkovszky A, Micolich A P, Hamilton A R, Marlow C A, Taylor R P, Linke H and Xu H Q 2010 Phys. Rev. B 81 041303
[26] Patel N K, Nicholls J T, Martin-Moreno L, Pepper M, Frost J E F, Ritchie D A and Jones G A C 1991 Phys. Rev. B 44 13549
[27] Hamilton A R and Chen T-M 2011 private communication
[28] Büttiker M 1990 Phys. Rev. B 41 7906
[29] Ando T and Uemura Y 1974 J. Phys. Soc. Japan 37 1044
[30] Wang C-K and Berggren K-F 1996 Phys. Rev. B 54 14257
[31] Tummers B, van der Lann J and Huyser K 2008 Physica E 41 448
[32] Kristensen A and Bruus H 2002 Phys. Scr. T101 151
[33] Jaksh P, Yakimenko I and Berggren K-F 2006 Phys. Rev. B 74 235320
[34] Laiss A, Schlagheck P and Richter K 2007 Phys. Rev. B 75 045346
[35] Lind H, Yakimenko I and Berggren K-F 2011 Phys. Rev. B 83 075308
[36] Hamilton A R et al 1992 Appl. Phys. Lett. 60 2782