Joint Location and Power Optimization for THz-enabled UAV Communications

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Abstract—In this paper, the problem of unmanned aerial vehicle (UAV) deployment and power allocation is investigated for a UAV-assisted wireless system operating at terahertz (THz) frequencies. In the studied model, one UAV can service ground users using the THz frequency band. However, the highly uncertain THz channel will introduce new challenges to the UAV location and user power allocation optimization problems. Therefore, it is necessary to design a novel framework to deploy UAVs in the THz wireless systems. This problem is formally posed as an optimization problem whose goal is to minimize the sum uplink and downlink transmission delays between the UAV and the ground users by jointly optimizing the deployment of the UAV and the transmit power of the users. To tackle this nonconvex delay minimization problem, an alternating algorithm is proposed while iteratively solving two subproblems: location optimization subproblem and power control subproblem. Simulation results show that the proposed algorithm can reduce the transmission delay by up to 67.3% and 52.5% respectively compared to baseline algorithms that fix transmit power control or UAV location.

Index Terms—UAV communication, THz communication, resource allocation.

I. INTRODUCTION

Unmanned aerial vehicles (UAVs) are an effective solution to provide temporary wireless connectivity due to their flexible maneuverability and convenient deployment. Compared to terrestrial base stations, UAVs can provide higher energy efficiency, improved coverage, and higher wireless capacity. However, deploying UAVs for wireless communications also faces several challenges in terms of deployment, trajectory design, and system optimization.

A number of existing works in studied the use of UAVs for wireless communications. The authors in maximized the number of users that are served by one UAV. In , the altitude and location of a UAV were jointly optimized so as to minimize the UAV transmit power used to service ground users. The problem of user-UAV association is studied in with the goal of minimizing transmit power. The authors in studied the problem of optimal UAV trajectory design so as to service ground users while minimizing energy consumption. In , the optimal deployment of multiple UAVs was analyzed to maximize the downlink coverage. Meanwhile, the work in studied the problem of three-dimensional (3D) coverage maximization for UAV networks. Moreover, for an UAV enabled mobile edge computing network, the authors in jointly optimized the location and transmit power of UAVs to minimize the energy consumption. However, none of these existing works such as in considered the use of the terahertz (THz) band for UAVs. The THz band (0.1-10 THz) is considered to be one of the 6G driving trends owing to its ultra-wide bandwidth . Nevertheless, given the high uncertainty at higher frequency bands such as THz, it is important to provide more degrees of freedom and control in network management. As such, UAVs can provide line-of-sight (LoS) transmission links to the ground users given their ability to fly. To reap the benefits of UAV deployment in THz systems, it is necessary to optimize the position of the UAVs in a way to provide continual LoS links to the ground users . In this respect, THz-operated UAV deployment analysis is inherently different from lower frequency bands (i.e., sub-6 GHz) given how fast the THz channel varies, especially in highly mobile scenarios. As such, existing solutions in do not take into consideration phenomena like the molecular absorption, short communication range, or sudden deep fades that are a main byproduct of the unnature of the THz channel.

The main contribution of this paper is a first fundamental analysis of the optimal deployment of a UAV that operates at THz frequencies. We consider the downlink and uplink of a wireless network in which THz-enabled UAVs can serve the ground users. Our goal is to minimize the downlink and uplink sum delays between the UAV and the users, by jointly optimizing the location of the operating UAV as well as minimizing the transmit power of the users. To solve this optimization problem, we propose an alternating algorithm that divides the original optimization problem into two subproblems: A location optimization subproblem and a power control subproblem that are solved iteratively. Both subproblems are shown to be convex, and hence, they can be effectively solved using standard optimization methods. Numerical results show that the proposed algorithm can yield up to 67.3% and 52.5% reduction in terms of the transmission delay compared to, respectively, a baseline with fixed transmit power control and a baseline with fixed UAV location.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a UAV-assisted wireless communication system that consists of one UAV and users. The coordinates of the UAV is denoted by , while the coordinates of each user is given by . Here, the altitude of the UAV is assumed to be constant.
The THz channel gain between the UAV and user $n$ can be modeled as:

$$h_n = d_n^{-2} e^{-a(f)d_n},$$

(1)

where $d_n = ||u_n - u_0||$ is the distance between user $n$ and the UAV, $e^{-a(f)d_n}$ is the path loss caused by the molecular absorption, and $a(f)$ is a molecular absorption coefficient that depends on the operating frequency $f$ and concentration of water vapor molecules. For notational simplicity, hereinafter, we use $a$ to represent $a(f)$.

Based on (1), the achievable rate for the uplink transmission from user $n$ to the UAV is:

$$t_n^U = w_n \log_2 \left( 1 + \frac{p_n h_0}{w_n d_n^2 e^{-a} \sigma_n^2} \right),$$

(2)

where $w_n$ is the bandwidth allocated to user $n$, $p_n$ is the transmit power of user $n$, $\sigma_n^2$ is the Gaussian noise power, and $h_0$ is the channel gain at a reference distance $d_0 = 1$ m. According to (2), the uplink transmission delay of user $n$ is:

$$t_n^U(p_n, X, Y) = \frac{D_n}{w_n \log_2 \left( 1 + \frac{q_n h_0}{w_n d_n^2 e^{-a} \sigma_n^2} \right)},$$

(3)

where $D_n$ is the size of the data that user $n$ needs to transmit to the UAV. Similarly, the achievable rate for the downlink transmission from the UAV to user $n$ is:

$$r_n^D = w_n \log_2 \left( 1 + \frac{q_n h_0}{w_n d_n^2 e^{-a} \sigma_n^2} \right),$$

(4)

where $q$ is the transmit power of the UAV. Based on (4), the downlink transmission delay from the UAV to user $n$ is:

$$t_n^D(q, X, Y) = \frac{E_n}{w_n \log_2 \left( 1 + \frac{q_n h_0}{w_n d_n^2 e^{-a} \sigma_n^2} \right)},$$

(5)

where $E_n$ is the data that the UAV needs to transmit to user $n$.

Our goal is to minimize the total communication time between the UAV and its served users while satisfying the energy constraints of all users. Mathematically, this optimization problem can be posed as follows:

$$\min_{P, p_n, X, Y} \sum_{n=1}^{N} \left( t_n^U(p_n, x, y) + t_n^D(q, x, y) \right),$$

(6)

s.t. $0 \leq t_n^U(p_n, x, y) p_n \leq Q_n$, $n = 1, \ldots, N$,  

(6a)

where $Q_n$ is the maximum total energy of user $n$ and $P = [p_1, \ldots, p_n]$. Constraint (6a) indicates that each user’s energy used for data transmission is limited. Problem (6) is novel due to the following two reasons. First, the channel model in (1) takes the molecular absorption into account. Second, the molecular absorption coefficient is a function of distance, which further makes problem (6) different from previous UAV location optimization problems in [5]-[11] without THz channel.

### III. Proposed Algorithm

Problem (6) is a non-convex problem because of the non-convex optimization function and constraints. It is generally challenging to obtain the globally optimal solution for problem (6). To obtain a suboptimal solution of problem (6), we propose an algorithm which optimizes $p_n, x$, and $y$ in an alternating manner.

### A. User Transmission Power Optimization

With fixed $x$ and $y$, problem (6) is formulated as:

$$\min_{P} \sum_{n=1}^{N} \left( \frac{D_n}{w_n \log_2 \left( 1 + k_n(x, y)p_n \right)} + A_n \right),$$

(7)

s.t. $\log_2 \left( 1 + k_n(x, y)p_n \right) \geq l_n$, $n = 1, \ldots, N$,  

(7a)

where $k_n(x, y) = \frac{h_0}{w_n d_n^2 e^{-a} \sigma_n^2}$, $l_n = \frac{D_n}{w_n q_n}$, and $A_n = \frac{E_n}{w_n \log_2 \left( 1 + k_n(x, y)p_n \right)}$. To solve problem (7), we need to prove that constraint (7a) and the optimization function in (7) are convex so as to show that problem (7) is convex.

First, since $\log_2 \left( 1 + k_n(x, y)p_n \right)$ is concave with respect to (w.r.t) $p_n$, constraint $\log_2 \left( 1 + k_n(x, y)p_n \right) \geq l_n$ is convex w.r.t. $p_n$. As a result, constraint (7a) is convex.

To prove that the optimization function in (7) is convex, we only need to show that the Hessian matrix in objective function (7) is positive semi-definite. We now define:

$$T = \sum_{n=1}^{N} \left( \frac{D_n}{w_n \log_2 \left( 1 + k_n(x, y)p_n \right)} + A_n \right).$$

(8)

From (8), we can see that $\frac{\partial^2 T}{\partial p_n^2} = 0$ for any $m \neq n$, with $1 \leq m, n \leq N$. When $m = n$, the second-order derivative of $T$ can be given by:

$$\frac{\partial^2 T}{\partial p_n^2} = \frac{D_n k_n(x, y) q_n^2 (2 + \ln (k_n(x, y)p_n + 1)) \ln 2}{w_n (k_n(x, y)p_n + 1)^2 \ln^2 (k_n(x, y)p_n + 1)}.$$  

(9)

From (9), we have $\frac{\partial^2 T}{\partial p_n^2} > 0$ for any $n$. Let $q_n^* = \frac{\partial^2 T}{\partial p_n^2}$, then the Hessian matrix $H$ of the objective function (7) is:

$$H = \begin{bmatrix}
q_1 & 0 & \cdots & 0 \\
0 & q_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & q_N
\end{bmatrix}.$$  

(10)

From (10), we can see that $H$ is positive semi-definite since all eigenvalues are positive. Thus, the objective function in (7) is convex. Since both the objective function and constraint are convex, problem (7) is convex.

Next, we find the optimal transmit power of each user given the location of the UAV, as shown in the following proposition.

**Proposition 1:** Given the UAV location $(x, y, H)$, the optimal transmit power of each user $n$ is given by:

$$p_n^* = \left( W \left( -\frac{l_n \ln 2}{k_n(x, y)} - \frac{\ln \left( \frac{\ln 2}{k_n(x, y)} \right)}{\ln \left( \frac{\ln 2}{k_n(x, y)} \right)} \right) - \frac{1}{k_n(x, y)} \right)^{\frac{1}{l_n \ln 2}} - 1,$$

(11)

where function $W(x)$ is the inverse function of $xe^x$.

**Proof:** Since the objective function of problem (7) is a decreasing function w.r.t. $p_n$, the optimal $p_n^*$ is the maximum value in the feasible region. According to (7a), we find that $\frac{\ln 2}{k_n(x, y)p_n}$ is a decreasing function w.r.t. $p_n$. Hence, the optimal transmit power $p_n^*$ satisfies the equation $\frac{\ln 2}{k_n(x, y)p_n^*} = l_n$. This completes the proof. \hfill $\square$

From Proposition 1, we can see that the optimal transmit power of each user $n$ depends on the molecular absorption coefficient, the bandwidth of each user, and the location of the UAV.
As a result, the total complexity of solving problem (6) is to run Algorithm 1.

B. UAV Location Optimization

Algorithm 1 Proposed Algorithm for Problem (6)
1: Initialize $\{(x^{(1)}, y^{(1)})\}$, the iteration number $k = 1$.
2: repeat
3: With given $(x^{(k)}, y^{(k)})$, obtain $p_n^{(k+1)}$ by solving problem (7).
4: With given $p_n^{(k+1)}$, obtain $(x^{(k+1)}, y^{(k+1)})$ by solving problem (12).
5: Set $k = k + 1$.
6: until objective function converges
7: return $x, y, p_n, n = 1, 2, \ldots, N$

TABLE I: System Parameters

| Parameters                  | Values       |
|-----------------------------|--------------|
| Gaussian noise power $\sigma^2$ | -17.4 dBm    |
| Channel gain at $d_0 = 1$ m $h_0$ | -40 dB       |
| Molecular absorption coefficient $a$ | 0.005 m$^{-1}$ |
| Operating frequency $f$      | 0.8 THz      |
| Data that user $n$ transmits to the UAV $D_n$ | [10,8,6,4] Tbits |
| Data that the UAV transmits to user $n$ $E_n$ | [8,6,4,8,3.2] Tbits |
| The transmit power of the UAV $q$ | 2 Watts       |
| Bandwidth of each user $w_n$ | $w_1 = \ldots = w_N = W$ |
| Maximum total energy of each user $Q_n$ | $Q_1 = \ldots = Q_N = Q$ |

C. Algorithm Complexity

The overall algorithm used to solve problem (9) is summarized in Algorithm 1. From Algorithm 1, the main complexity of solving problem (6) lies in solving the power optimization problem in (7). The optimal solution of problem (7) is obtained in closed-form equation (11), of which the complexity is $O(N)$. As a result, the total complexity of solving problem (6) is $O(T_0N)$, where $T_0$ is the total number of iterations needed to run Algorithm 1.

IV. SIMULATION RESULTS

For our simulations, a 50 m $\times$ 50 m square area is considered with $N = 4$ randomly distributed users (unless stated otherwise) and one UAV. Other parameters are listed in Table I. All statistical results are averaged over a large number of simulation runs.

Fig. 1 shows how the transmission delay changes as the molecular absorption coefficient $a$ varies when the bandwidth of each user $W$ is 20 GHz and the maximum total energy of each user $Q$ is 8 J. From Fig. 1, we can see that the transmission delay increases with the molecular absorption coefficient $a$. This is due to the fact that as $a$ increases, the molecular absorption loss increases. From Fig. 1, we can also see that the proposed algorithm can reduce the transmission delay by up to 52.5% and 67.3% compared to a first baseline (labeled as ‘FUL’) that optimizes the users’ transmit powers under a fixed UAV location and a second baseline (labeled as ‘FP’) that optimizes the UAV deployment with fixed transmit powers. Besides, we use the exhaustive search method (labelled as ‘EXH’) to obtain a near global optimal solution, which uses interior point method to jointly optimize transmit power and location with multiple initial solutions. From Fig. 1, the EXH algorithm achieves the best performance at the cost of computation capacity. The gap between the proposed algorithm and the EXH algorithm is small, which indicates that the proposed algorithm can approach the optimal solution.

Fig. 2 shows how the transmission delay changes as the bandwidth of each user varies when the maximum total energy of each user $Q$ is 2 J. From Fig. 2, we can see that the transmission delay decreases as the bandwidth of each user increases. This is because as $W$ increases, the achievable rate between the user and UAV increases. We can also see that the transmission delay increases as the UAV’s altitude increases under a fixed bandwidth. This is due to the fact that, as the distance between the UAV and users increases, the channel gain decreases.

Fig. 3 shows how the transmission delay changes as the number of users vary when the bandwidth of each user $W$ is 20 GHz and the maximum total energy of each user $Q$ is 2 J. From Fig. 3, we can see that the transmission delay increases as the number of users increases and the larger the number of users, the faster the growth rate. This is because as $N$ increases, the distances between the UAV and individual users will be very far.
significantly reduce the transmission delay compared to two power. Simulation results show that the proposed algorithm can explicitly consider the molecular absorption effect in the THz-

For notational simplicity, hereinafter, we use $B$ to represent $B(e_n)$. The Hessian matrix $G$ of (13) can be expressed as

$$G = \begin{bmatrix}
\frac{\partial^2 B}{\partial x^2} & \frac{\partial^2 B}{\partial x \partial y} \\
\frac{\partial^2 B}{\partial y \partial x} & \frac{\partial^2 B}{\partial y^2}
\end{bmatrix}.$$ (14)

Then, to prove that the function $B$ is convex, we need to prove that the Hessian matrix $G$ is positive semi-definite. Hence, we first prove that the first-order determinant of $G$ is positive and then we prove that the second-order determinant of $G$ is positive.

Based on (14), the first order determinant of $G$ is expressed as follows:

$$\frac{\partial B}{\partial x^2} = \frac{\partial^2 B}{\partial d_n^2} - \frac{\partial B}{\partial d_n} \frac{\partial^2 d_n}{\partial x^2} + \frac{\partial B}{\partial d_n} \frac{\partial^2 d_n}{\partial x \partial y},$$ (15)

where $\frac{\partial B}{\partial d_n} = -\frac{D_n \rho_a}{w_n e_n \ln^2 e_n}$ and $\frac{\partial^2 B}{\partial d_n^2} = \frac{h_0 \rho_a (a + 2)}{w_n \sigma_n^2 e_n \ln^2 e_n}$. Based on (15), we rewrite $\frac{\partial^2 B}{\partial x^2}$ as

$$\frac{\partial^2 B}{\partial x^2} = \frac{D_n \rho_a \ln (e_n - 1)}{w_n e_n^2 \ln^3 e_n},$$ (16)

where $I = I_1 (a + 2) d_n^2 e_n \ln e_n, z = (x - x_n)^2$ and $I_1 = (2e_n - \ln e_n - 2) (a + 2) - e_n \ln e_n (a + 2) - e_n \ln e_n$.

To prove that $\frac{\partial^2 B}{\partial x^2}$ is positive, we only need to prove that $I > 0$.

First, from the expression of $I$, we can treat it as a linear function with slope $I_1$. To determine if $I$ is greater than 0, we consider the following two cases:

**Case 1:** If $I_1 \geq 0$, we can directly obtain $I > 0$.

**Case 2:** If $I_1 < 0$, function $I$ is a decreasing function w.r.t $z$. Since $z = (x - x_n)^2 < d_n^2$, we have $0 < z < z_{\text{max}}$, where $z_{\text{max}} = \frac{\rho_a \ln (a + 2)}{e_n \ln e_n}$. Thus, to prove that $I > 0$ in its feasible region, we only need to prove that $z_{\text{max}} < z^*$, where $z^*$ is the zero point of $I$, and the expression of $z^*$ is given by:

$$z^* = \frac{(a + 2) d_n^2 e_n \ln e_n}{-I_1}.$$ (17)

From (17), we find that the minimum value of $z^*$ is $z^*_{\text{min}} = \frac{(a + 2) d_n^2 e_n \ln e_n}{-I_1_{\text{min}}}$, where $I_{\text{min}}$ is the minimum value of $I$. If $z_{\text{min}} > z_{\text{max}}$, then we have $z^* > z_{\text{min}} > z_{\text{max}}$ so that $I > 0$. Thus, we first find $I_{\text{min}}$, then compare $z_{\text{min}}$ with $z_{\text{max}}$.

From the expression of $I_1$, we can see that $I_1$ is a function of $(a + 2)$. When $e_n > 1$, we find that $(2e_n - \ln e_n - 2)$ is positive. Hence, the function $I_1$ has two solutions, i.e.,

$$\frac{e_n \ln e_n}{2(2e_n - \ln e_n - 2)} - 2.$$ (18)

Fig. 2: Transmission delay (s) versus the bandwidth (GHz) of users ($Q = 2$ J).

**V. CONCLUSION**

In this letter, we have proposed a novel approach to deploy a UAV for providing wireless THz connectivity. We have explicitly considered the molecular absorption effect in the THz-enabled UAV channel gain model. Then, we have formulated an optimization problem that seeks to minimize the transmission delay between the UAV and the ground users while meeting the communication requirements of each user. To solve this problem, we have proposed an alternating algorithm that divides the original optimization problem into two subproblems to optimize two variables: the location of the UAV and the users' transmit power. Simulation results show that the proposed algorithm can significantly reduce the transmission delay compared to two baselines.

**APPENDIX**

We rewrite the function $D_n \rho_a \ln e_n \ln e_n$ as

$$B(e_n) = \frac{D_n \rho_a}{w_n \ln e_n},$$ (13)
we only need to prove that 
\[ g(\sigma) < e \]
that 
\[ I_1 \]  
increases as \((ad_n + 2)\) increases in its feasible region. Then, we have:

\[
z_{\text{min}}^{*} = \frac{(ad_n + 2) d^2 e_n \ln e_n}{-I_1} 
\[
> \frac{2w_n\sigma^2 (e_n - 1) (2 + e_n \ln e_n - 2e_n)}{\partial \partial x \partial \partial y}
\]

To compare the value of \(z_{\text{min}}^{*}\) with \(z_{\text{max}}\), we have:

\[
z_{\text{min}}^{*} - z_{\text{max}} > \frac{\partial \partial x \partial \partial y}{w_n h_0 p_n g(e)}
\]

where \(g(e_n) = 4e_n - 4 - e_n\ln e_n - 2\ln e_n\). From (19), we can see that the values of \(z_{\text{min}}^{*}\) and \(z_{\text{max}}\) depend on the value of \(g(e_n)\). Therefore, we next analyze the function \(g(e_n)\).

By finding the first and second derivatives of \(g(e_n)\), we obtain the trend of \(g(e_n)\) in Fig. 4(c). As shown in Fig. 4(c), we define \(e_n = 41.4125\) as the non-zero point of function \(g(e_n)\) and \(e_n^{\max}\) as the maximum value of \(e_n\). Since \(e_n < 41\), we have \(e_n < e_n^{\max}\). As a result, we have \(g(e_n) > 0\). Since \(g(e_n) > 0\), then \(z_{\text{min}}^{*} > z_{\text{max}}\) and hence we have \(I_1 > 0\). Finally, based on Fig. 4(c), we obtain that the first order determinant of \(G\) is positive.

Next, we prove that the second-order determinant of \(G\) is positive. The second order determinant of \(G\) is expressed as

\[
G_1 = \left| \begin{array}{cc} \frac{\partial^2 B}{\partial x^2} & \frac{\partial^2 B}{\partial x \partial y} \\ \frac{\partial^2 B}{\partial y \partial x} & \frac{\partial^2 B}{\partial y^2} \end{array} \right| = \left( D_n p_n (e_n - 1) \ln 2 \right)^2 (2 + ad_n) L
\]

where \(L = (d_n^2 - H^2) S + H^2 e_n \ln e_n (2 + ad_n) - S = (2e_n - 2 - \ln e_n) (2 + ad_n)^2 - 2e_n \ln e_n\). To prove \(G_1 > 0\), we only need to prove that \(L\) is positive. Since \(S\) is an increasing function w.r.t. \(a\), we have \(L > (d_n^2 - H^2) S_1 + H^2 e_n \ln e_n (2 + ad_n)\) where \(S_1 = S|_{a=0} = 2g(e_n) > 0\). As a result, we have \(G_1 > 0\).

Since both the first and second order determinants of \(G\) are positive, the Hessian matrix \(G\) is positive semi-definite. As a result, function \(D_n p_n / w_n \ln (1 + e_k(n - y/p_n))\) is convex w.r.t. \(x, y\). This completes the proof.

1 For the typical value, \(d_n \geq 10 \text{ m}, a = 0.005 \text{ m}^{-1}\) at \(f = 0.8 \text{ THz}, h_0 = -40 \text{ dB}, \sigma^2 = -174 \text{ dBm}, w_n = 10 \text{ GHz} \text{ and } p_n = 0.001 \text{ W}, \text{ we always have } e_n \leq 23.781 < 41\).