Contact fatigue analysis of gear transmission system of drive axle final drive of truck

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Abstract. The function of the final drive of a truck axle is to reduce the speed and increase the torque. The main transmission system is composed of a spiral bevel gear mechanism and a helical gear mechanism. After fully considering powertrain mass, moment of inertia and dedendum transition fillet, the dynamic calculation model of the meshing collision of the gear teeth was established. The speed of the gear and the contact force of the tooth surface in a moving cycle are calculated and analysed. Based on the results of the dynamic calculation as the load data, the finite element computation model of frictional contact between spiral bevel gear mechanism and helical gear mechanism is established. Using the augmented Lagrange multiplier method, the contact stress distribution of the tooth surface under the transient impact load is calculated and analysis in a motion cycle when the teeth are engagement-in and engagement-out. In order to avoid the damage of gear tooth contact fatigue, the more realistic surface contact fatigue strength of the gears was obtained and compared with the theoretical calculation according to the finite element calculation results.

1. Introduction

The final drive is an important part of the axle. Its function is to increase the input torque and reduce the speed, as well as to change the direction of torque rotation when the engine is mounted longitudinally[2]. In order to transport more goods to reduce transport costs, the engine of truck are generally in pursuit of large torque. So improve the transmission capacity is important. By studying the dynamic characteristics of the gears in the final drive, the collision force between the teeth of the gears is obtained. In the process of meshing, the analysis of the contact stress and the bending stress of the tooth root, as well as the fatigue characteristics of the gear, can provide a reference for the improvement and design of the structure, and greatly improve the reliability of the design[3].

In this paper, the three-dimensional software Solidworks is used for modeling, Adams and Ansys are used for dynamic and static analysis of gears.
2. Simulation calculation of virtual prototype

2.1. The establishment of dynamic model
The imported 3D geometry is simulated and analyzed in Adams. The established dynamic model is shown in figure 1.

![Dynamic model of drive axle](image)

**Figure 1.** Dynamical model of drive axle

2.2. The contact impact model of gear
The distance between the gears in the process of being engaged is constantly changing. In order to simplify the calculation, it is equivalent to a nonlinear spring damping system. At the same time, considering the influence of friction and ignoring the influence of other constraints, the equation of normal contact force in the kinetic calculation is[4]:

\[
F_n = \begin{cases} 
0 & d > d_o \\
K(d - d) - D_{\text{max}} & 0 \leq d \leq d_o 
\end{cases}
\]

(1)

In this formula: \(d_0\) is the initial distance and displacement variable of component contact point; \(K\) is the contact stiffness coefficient; \(e\) is contact force index; \(d'\) is deformation increment; \(D\) is damping coefficient; \(s(\cdot)\) is the step function. Where, both the contact force index and the contact stiffness are constant, then the contact force can be determined according to Johnson's theory[5]:

\[
F = \left( \frac{16}{9} R^* E^{*\eta} \right)^{\frac{1}{2}} \delta^{\eta/2} = K \delta^e
\]

(2)

In this formula: \(R^*\), \(E^*\) are equivalent contact radius and equivalent elastic modulus of the contact body, \(\delta\) is the relative displacement of the contact body.

The contact of gear transmission system is rigid contact, and the damping of component can be assumed to be 0, so it can be found that the theoretical contact force is equal to the actual contact force. \(F_n = F\).

So \(e = 1.5\), \(K = \left( \frac{16}{9} R^* E^{*\eta} \right)^{\frac{1}{2}} = \ldots \text{N/mm}\)

Calculation of stiffness coefficient of helical gear:

\[
K = \frac{4}{3} \left( \frac{u(d \sin \alpha)}{(1 + u)2 \cos \beta} \right)^{\frac{1}{3}} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)^{-1}
\]

(3)
In this formula: \( \beta \) are base spiral angle; \( \alpha \) is reference circle meshing pressure angle.

Calculation of stiffness coefficient of spiral bevel gear:

\[
K = \frac{4}{3} \left( \frac{d_1 \cos \alpha \tan \alpha' - u}{2 \cos \beta_1 (u + 1)} \right) \left( \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2} \right) \tag{4}
\]

In this formula: \( \beta_1 \) is Position angle of base circular tooth line; \( u \) is gear ratio; \( \alpha_1 \) pressure angle of base circle; \( \alpha_1' \) is engagement angle of reference circle

2.3. Dynamic simulation

As shown in figure 1, the constraint relationship between the components is added to the dynamic model. The mass, moment of inertia and root transition Angle (planetary gear and half-axle gear is 1mm, the rest of the gear is 2mm) of each component are fully considered, and the influence of other constraints in the transmission process is ignored.

The helical gear stiffness is \( 1.47E+005 \) N/mm, spiral bevel gear stiffness is \( 1.52E+005 \) N/mm, Exponent is 1.5, Damping is 50 N - sec/mm, Penetration Depth is 0.1 mm, Static Coefficient is 0.08, Dynamic Coefficient is 0.05.

The input speed on the input axis is \( 627d/s \). The simulation time is 0.57s for the movement of the transmission mechanism. And add a workload of 2103.8 Nm. The results are shown in figures 2 through 5.

As can be seen from figure 5. The average transmission of helical gear ratio is 1. Careful observation of figures 4 and 5 shows that the average speed of the driving gear is \( 627d/s \). The average speed of the driven bevel gears is approximately \( 326d/s \). According to the speed, the transmission ratio is 1.923. It is basically consistent with the transmission ratio of 1.933 at the time of calculation.

At this point, it is the movement state of the drive axle when the truck is climbing gear, and at this time, the force of each transmission mechanism reaches the maximum value. So that the force meets the requirements, the other (except the start and stop) movement state should meet.

It can be seen from figure 2 and 3 that the average value is 15213.1 N and 10734.3 N.

It can be calculated that the allowable circumferential force of helical gear is 19066 N, and that of spiral bevel gear is 20939 N. Is greater than the average value shown in the image. That is, the two pairs of teeth meet the requirements of the truck in the climbing gear in the dynamic simulation.
When the truck starts, the active helical gear has the drive torque, but the driven bevel gear not move. The second is to stop when the active helical gear does not move, but the passive spiral bevel gear will have a load torque. Load torque $2103.8\, \text{N} \cdot \text{m}$ is applied to the passive spiral bevel gear. As shown in figures 6 and 7. The average value of the curve in figure 6 is 19329 N, and the average value of the curve in figure 7 is 11801 N.

![Figure 6. Contact force of spiral bevel gears under load](image1)

![Figure 7. Contact force of helical gears under load](image2)

Equivalent torque $2002\, \text{N} \cdot \text{m}$ is applied to the active helical gear as shown in figure 8 and 9. The average values are 18878 N and 15245 N respectively.

![Figure 8. Contact force of spiral bevel gears add the actuation](image3)

![Figure 9. Contact force of helical gears add the actuation](image4)

Through the above dynamic analysis, it can be seen that the truck in climbing gear, the start and stop. The two gears of the final drive do not exceed the allowable circumferential force. Its force is fully meet the design requirements.
3. Finite element analysis of helical gear
Adopts the regular tetrahedron grid division method, set the boundary condition of the global grid is 6mm. As a result of the analysis is mainly tooth contact stress, set up the mesh boundary conditions is 3mm on the contact surface. The final partition result is: the number of nodes is 927959, and the number of cells is 534699.

![Figure 10. Grid partitioning](image)

3.1. Establishment of contact finite element model of meshing gear pair
The material properties of the new material 20CrMnTi were established, including young's modulus was $2.12 \times 10^5 \text{ MPa}$, Poisson’s ratio was 0.25 and Density was $7.8 \times 10^3 \text{ kg/m}^3$.

In actual work, there are only three gear teeth in contact with each other. In order to reduce the computation and save the calculation time, a part of the total model is intercepted for analysis and calculation.

3.2. Transient dynamic finite element analysis
Augmented Lagrange has been set to Formulation, that is Lagrange algorithm has been improved.

$$F_{\text{normal}} = k_{\text{normal}} x + \lambda$$  \hspace{1cm} (5)

The above is the formula of the enhanced Lagrange algorithm, which becomes relatively insensitive to the value of the penalty stiffness $k_{\text{normal}}$ because additional factor $\lambda$ is added.

After fully considering the mass of each component, the moment of inertia and the transition fillet of the gear root. The circumferential force of helical gear teeth in one cycle from the process of engaging-in to engaging-out is extracted from the results of dynamic calculation. And determine the calculated torque according to the circumferential force. The time from engaging-in to engaging-out of helical gear is 0.167s.

| Time (s) | 0.002 | 0.004 | 0.006 | 0.008 | 0.010 | 0.012 | 0.014 | 0.016 | 0.018 | 0.020 |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Circumferential force (N) | 1203 | 1084 | 1155 | 1049 | 1096 | 1111 | 1140 | 1042 | 1031 | 1087 |
| Torque (N·m) | 1263 | 1138 | 1213 | 1102 | 1151 | 1167 | 1197 | 1094 | 1084 | 1142 |

To ensure the precision and accuracy of the calculation, ten circumferential forces of equal time intervals within 0.2s were calculated by dynamics as the basis for calculation. As shown in table 1.

Set the time varying Moment of the driven gear rotating about the Z axis. As shown in table 1.
Figure 11. Cloud diagram of helical gear transient dynamics calculation

Perform transient dynamics calculation from the gear engaging-in to engaging-out of one cycle. Get the maximum and minimum contact stress clouds as shown in figure 11. And the figure 12.

By substituting each parameter into the equation contact stress of helical gear tooth surface, the contact stress can be obtained as: $987.8169 \text{ MPa}$.

The circumferential forces extracted from the kinetic calculations in Table 1 and subjected to transient dynamics calculations were calculated in the form of theoretical calculations for their allowable contact stresses, and the results are shown in figure 12.

Figure 12. Theoretical and finite element calculation of helical gear contact stress

As shown in figure 12. By comparing the allowable theoretical contact stress calculated by various theories with the contact stress obtained by finite element calculation, it can be known that the results of transient dynamic calculation are all smaller than the results of theoretical calculation. The actual work requirements is met.

4. Finite element analysis of spiral bevel gear

4.1. Transient dynamic finite element analysis of spiral bevel gears

From the results of dynamic calculation, ten circumferential forces of equal spacing were extracted within 0.367s, which is the time from engagement-in to the engagement-out. The calculated torque was shown in table 2 and then the transient dynamic calculation was carried out. The calculation results are shown in figure 13, and the figure 14.

Table 2. Circumferential force and torque of spiral bevel gears dynamics computation

| Time( s ) | 0.004  | 0.008  | 0.012  | 0.016  | 0.020  | 0.024  | 0.028  | 0.032  | 0.036  | 0.040  |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Circumferential force( N ) | 1545   | 1499   | 1516   | 1441   | 1508   | 1493   | 1463   | 1480   | 1575   | 1546   |
| Torque( N•m ) | 1391   | 1350   | 1365   | 1298   | 1358   | 1344   | 1317   | 1333   | 1418   | 1392   |
Calculation of contact stress on tooth surface of spiral bevel gear[1]:

$$\sigma_j = \frac{C_p}{d_j} \sqrt{\frac{27TK_JK_J^2}{K_hbJ} \times 10^4}$$  \hspace{1cm} (6)

In this formula:

- $C_p$ —— Material elasticity coefficient, 232.6N/mm
- $T$ —— Torque of drive gear
- $K_o$ —— The overload coefficient, 1
- $K_v$ —— weight coefficient, 1.0;
- $K_a$ —— load sharing ratio, 1.10
- $K_s$ —— size factor, 1.0
- $K_f$ —— superficial mass factor, 1.0
- $J$ —— Comprehensive coefficient of contact stress, 0.105

Get the $T_j = \min[T_{j_a}, T_{j_b}]$, $T_{j_a} = 2103.8 \text{ N\cdot m}$

Result is: $\delta_j = 1167.3 \text{ MPa}$

The theoretical calculations were made from the forces taken in the kinetic calculations, and the results of the allowable contact stresses are shown in figure 14.

![Figure 13. Cloud diagram of bevel gear transient dynamics calculation](image)

![Figure 14. Theoretical and finite element calculation of bevel gear contact stress](image)

The comparison between the allowable contact stress obtained by theoretical calculation after full consideration of the powertrain mass, moment of inertia and the transition fillet and the contact stress obtained by finite element transient dynamics calculation shows. The stress calculated by finite element method is less than the allowable contact stress, which satisfies the actual working condition.
5. Conclusion

1. Driving axle of the final drive gear in the start, stop, movement of the force is not the same, so the contact strength of the gear put forward higher requirements. By means of virtual prototype technology, the dynamic simulation and static mechanics simulation in two ways of the gear are carried out respectively with considering powertrain mass, moment of inertia and transition fillet. In this way, the reliability of the simulation calculation is better guaranteed by comparing the simulation results.

2. The dynamic analysis of the transmission system is carried out by co-simulation of virtual prototype and finite element analysis. The dynamic data of dynamic calculation of virtual prototype is extracted for the transient dynamic analysis of finite element, which is combined with the theoretical calculation and compared with the calculation results. The multi-direction and multi-data comparative analysis not only ensures more accurate calculation but also determines whether the transmission system has the possibility of practical use.

3. This paper provides a variety of methods for contact force analysis of the drive axle final drive gear, which can be used together to better determine whether the designed mechanism is practical feasibility. In this way, the design cycle can be reduced and the design efficiency can be improved.

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