Quantum-conformal Field Theory

Daniel C. Galehouse

Physics Department, University of Akron, Akron, Ohio 44325

(April 14, 2018)

Abstract

A theoretical study is made of conformal factors in certain types of physical theories based on classical differential geometry. Analysis of quantum versions of Weyl’s theory suggest that similar field equations should be available in four, five and more dimensions. Various conformal factors are associated with the wave functions of source and test particles. This allows for certain quantum field equations to be developed. The curvature tensors are calculated and separated into gravitational, electromagnetic and quantum components. Both four and five dimensional covariant theories are studied. Nullity of the invariant five scalar of curvature leads to the Klein-Gordon equation. The mass is associated with an eigenvalue of the differential operator of the fifth dimension. Different concepts of interaction are possible and may apply in a quantum gravitational theory.

MS: , PACS 03, 04.50

Typeset using REVTEX
I. INTRODUCTION

This article is the second in a sequence that discusses how differential geometry is capable of describing quantum mechanical effects. The objective is to obtain compatible quantum field equations that complement the geodesics of the preceding article. Important results for a single particle field equation are presented. Results for the a complete set of source equations is not contained herein but some of the initial problems are addressed. The usual methods of developing field equations are not effective for this type of construction and are counterindicated by the considerations of [1]. The standard methods fail and more elementary arguments are used to guide the developments. Progress is slow because the basic equations must be found by physical principles, such as equivalence, or by the mathematical study of limiting forms of the curvature tensor. It is a process of discovery in elementary geometrical systems.

The issue of the particle mass is crucial and some of the characteristic roles are resolved in a synthesis that includes quantum effects. The mass as a constant must be introduced in a way that is compatible with the characteristic properties of all fields. One would wish to develop, as in the Weyl theory [2–5], a Klein-Gordon equation [6,7] in some form. Some guidance is available as there are various theories in the literature [8–16,18] that apply in an approximation or in a limiting case.

Particle motion as discussed in the preceding paper [17] "Quantum Geodesics", (QG) has not been assigned a particular particle mass. This defect is to be remedied here. Five dimensional field equations are developed in which the mass appears as a measure of inertia. An appropriate construction is suggested by a version of the Weyl theory. An identified relationship with quantum phenomenology has been known for some time. These associations provide a way to identify quantum structures in other types of geometry. To do this, the Weyl theory is reviewed and rewritten in a form that is appropriate. This is applied to Riemannian theories and in five dimensions, the field equations can be combined with the geodesics of the previous article.
II. QUANTUM EXTENSION OF WEYL GEOMETRY

Although the mathematics of Weyl’s theory has been known since 1918, the physical interpretation has been in dispute. Any non-Riemannian geometry can be interpreted as a modification to the derivatives as they are applied microscopically to particle motion. The usual relation between the coefficients of connection and the Christoffel symbols is changed. An arbitrary tensor $D^\beta_{\mu\nu}$ is added to give a new connection.

$$\Gamma^\beta_{\mu\nu} = \left\{ \begin{array}{c} \beta \\ \mu\nu \end{array} \right\} - D^\beta_{\mu\nu}$$  \hspace{1cm} (1)

The tensor $D^\beta_{\mu\nu}$ is chosen symmetrical in $(\mu\gamma)$. Infinitesimal displacements, calculated from these new values $\Gamma^\beta_{\mu\gamma}$, may now cause a change in the length of a displaced vector besides the usual change in direction. Previous studies show that quantum and electromagnetic effects are correctly included if

$$D^\beta_{\mu\nu} = \delta^\beta_{\mu}(\phi_{\nu} - \ln |\psi_{\nu}|) + \delta^\beta_{\nu}(\phi_{\mu} - \ln |\psi_{\mu}|) - g_{\mu\nu}(\phi^\beta - \ln |\psi|^\beta)$$  \hspace{1cm} (2)

where $\psi$ is the wave function and $\phi_{\mu} = i e A_{\mu}$. The effect of a displacement around a closed loop can be used to define a more general Riemann tensor.

$$\delta V^\mu = R^\mu_{\nu\lambda\beta} V^\nu \delta x^\lambda x^\beta$$  \hspace{1cm} (3)

with the definition

$$R^\mu_{\nu\lambda\beta} = \frac{\partial \Gamma^\mu_{\nu\beta}}{\partial x^\lambda} - \frac{\partial \Gamma^\mu_{\nu\lambda}}{\partial x^\beta} + \Gamma^\gamma_{\nu\beta} \Gamma^\mu_{\gamma\lambda} - \Gamma^\gamma_{\nu\lambda} \Gamma^\mu_{\gamma\beta}.$$  \hspace{1cm} (4)

With the implied complex factors in the connections given by (1) and (2), the usual non-integrable part of the Weyl displacement is associated with the phase. The part of Weyl’s connections which would affect the scale of $g_{\mu\nu}$ are integrable. For this real conformal factor, and in this gauge, Einstein’s objection to Weyl’s theory does not hold \cite{2,5}. The integrable part of the length change seems to be mathematically related to the gravitational red shift, although the quantum interpretation is not equivalent to the original Weyl interpretation.
There are a number of theories that study invariance under conformal transformations \[19–21\]. This point of view may apply in the classical limit. The quantum geometrical system used here departs from this interpretation and supports well defined physical meanings for microscopic conformal transformations. Invariance under conformal transformations, especially in classical theories, is identified as a means to suppress the intrinsic quantum terms.

Conformal transformations of the four metric can be identified with variations of probability density. This applies to single particle states. Consider an observer’s space that, for the argument, is flat and euclidean. As represented in the diagram, the probability density is measured by counting the number \(N\) of particles in a small region of dimensions \((\Delta x, \Delta y, \Delta z)\) having volume \(\Delta V\). The predicted probability density, \(P^0\), is taken as the first component of the quantum conserved current density. Let the particle be described by the metric \(g_{\mu\nu}\), which varies from the observers’ metric \(\dot{g}_{\mu\nu}\) by a point conformal transformation, \(g_{\mu\nu} = \lambda \dot{g}_{\mu\nu}\). The number of particles in the region, as determined by the neutral observer is given by

\[
N = P_0 \Delta x \Delta y \Delta z \sim \psi^* \frac{\partial}{\partial t} \psi \Delta x \Delta y \Delta z
\]  
(5)

Of course the geodesics of \(g_{\mu\nu}\) are not straight in the observers frame, but in a very small region where both coordinate systems are flat, they can be chosen euclidean and parallel. The fixed volume \((\Delta x, \Delta y, \Delta z, dt)\) has new dimensions \((\Delta x', \Delta y', \Delta z', dt') = (\Delta x, \Delta y, \Delta z, dt)\lambda^{(1/2)}\) while the number of counts stays constant.

The time dependence of the derivative under conformal transformation must be compensated in the same way as the spatial coordinates. The implied relation between \(g_{\mu\nu}\) and \(\psi\) during a gauge transformation can be used to show that the number of counts in a fixed region is constant. Transforming

\[
g_{\mu\nu} \rightarrow g'_{\mu\nu} = \lambda' \dot{g}_{\mu\nu}
\]  
(6)

\[
\psi \rightarrow \psi' = \frac{\psi}{\sqrt{N}}
\]  
(7)
it follows that

\[ N \rightarrow N' = \psi \frac{\partial}{\partial t'} \psi' \Delta x' \Delta y' \Delta z' = \psi \frac{\partial}{\partial t} \psi \Delta x \Delta y \Delta z \] (8)

Of special interest is that if for a given solution \( \psi \), the value of \( \lambda \) is chosen equal to the value of \(|\psi|^2\) then after the gauge of \( g_{\mu\nu} = \dot{g}_{\mu\nu} \) is changed to \( g_{\mu\nu} = \lambda \dot{g}_{\mu\nu} \), the magnitude of the wave function is forced to a constant over the whole space. Probability density information can be transferred to the metric. The observable quantities are invariant although the magnitude of the wave function has been transformed away and is no longer an independent field.

### III. QUANTUM-GEOMETRIC FIELD EQUATIONS

The field equations of quantum mechanics can be made part of this geometrical structure. It is easiest to first consider a four dimensional example that ignores electrodynamics. For this relatively simple geometry, the Klein-Gordon equation can be identified with a modified type of curvature scalar. To do this, geometrical invariants may be formed from the two metrics and derived tensors. An appropriate scalar is formed by using the tensor \( R^\beta_{\mu\lambda\nu} \) calculated from \( g_{\mu\nu} \) and contracting it with the inverse of the observer’s metric, \( \dot{g}^{\mu\nu} \).

This imposes a external length standard on the quantum equations that is based on the observer’s frame. It also implies that phenomenological measurements performed to determine the observer’s metric, \( \dot{g}_{\mu\nu} \), must be made in a way that is consistent with quantum mechanics. In particular, the clocks must all be based on fundamental quantum processes. Direct calculation gives

\[ R^\mu_{\nu\lambda\rho} \dot{g}^{\lambda\rho} = \frac{6 \Box \psi}{\psi} \] (9)

where \( g_{\mu\nu} = \psi^2 \dot{g}_{\mu\nu} \) with \( \dot{g}_{\mu\nu} \) still assumed euclidean. This means that the identification

\[ R^\mu_{\nu\lambda\rho} \dot{g}^{\nu\rho} = -6m^2 \] (10)

is equivalent to the Klein-Gordon equation.
The field equation for $\psi$ is demonstrated to be part of the geometry along with the probabilistic interpretation.

Thus, single particle relativistic quantum mechanics, can be described by the distortions of a kind of space-time. A larger system is needed to include a complex wave function and practical interactions. In particular the five dimensional theory, helps resolve some of the problems. To develop a useful interpretation, the Weyl theory must be rewritten without an explicit wave function. To do this, the gauges must be adjusted. Starting with some combination $\psi, A_\mu$ and $g_{\mu\nu}$, the gauge is transformed so that $\psi$ is equal to unity everywhere. At the same time both $A_\mu$ and $g_{\mu\nu}$ are kept real. This leaves the metric unique up to a constant multiplicative factor and the vector potential completely specified. These new field variables, $g_{\mu\nu}$ and $A_\mu$ are applicable to other types of geometrical theories.

In the case where $A_\mu$ is integrable, the corresponding congruences are mathematically geodesics. In this situation $\phi_\mu$ can be combined into a complex Christoffel symbol. Notwithstanding the interpretational problems, the trajectories are geodesics for a complex path parameter. The difficulties of defining geodetic motion for non-integrable $A_\mu$ are relieved in the five dimensional theories. The significant mathematical quantities are more accessible. The imaginary parts that naturally appear in the Weyl theory are often related to the variation with proper time rather than with coordinate time. Because the fifth signature element in the five metric is of sign opposite to that for the coordinate time, this dependency can appear with the factor $i$. The problems of changing the integrability of the vector potential is not easily resolved in a Weyl theory.

The additional transformations involving $\tau$ must be considered point transformations that may not leave physical quantities invariant. The effect on a Weyl connection can be calculated in a way that is similar to the construction in [1]. If it is applied to the four dimensional neutral space connections, the Weyl connection is generated. The electromagnetic term in the Weyl connection can be thought of as generated by derivatives with respect
to \( \tau \) which persist after a series of conformal and cut transformations. The terms in \( \Lambda_\mu \) are multiplied by the quantity \( im \) which produces the imaginary factor. The \( \tau \) derivative must be allowed to annihilate on an exponential term. The coefficients in \( \Lambda_\mu \) are multiplied by the quantity \( im \). If a conformal transformation is included, an electromagnetic field is generated. The Weyl theory can be thought of as a study of conformal waves in which the five dimensional coordinate transformations generate apparent non-Riemannian effects. In this context, the Weyl theories are a phenomenological expression of higher geometries.

Calculation of the analogous invariant \( R^\mu_\nu_{\gamma\beta} \) using the full connection (1) includes additional terms in the vector potential and metric. These reduce to the usual Klein-Gordon equation as shown in [3]. These identified quantum-Weyl field equations can be rewritten by recalculating from the invariant \( (10) \). Keeping \( \lambda \) explicit, the contraction of the metric tensor is first written:

\[
R^\nu_\mu_{\nu\beta} g^{\nu\beta} = \lambda R^\nu_\mu_{\nu\beta} g^{\mu\beta}
\]

(12)

Calculation of the “pure” invariant \( R^\nu_\mu_{\nu\beta} g^{\mu\beta} \) follows from equation (4), (1), and (2). The Weyl curvature can be computed as

\[
R^\beta_{\mu\nu\rho} = Q^\beta_{\mu\nu\rho} - D^\beta_{\mu\nu}|\rho + D^\beta_{\mu\rho}|\nu - D^\beta_{\rho\nu}|\mu + D^\beta_{\nu\tau} D^\tau_{\mu\rho} + D^\beta_{\nu\rho} D^\tau_{\mu\nu}
\]

(13)

where

\[
Q^\beta_{\mu\nu\rho} = \frac{\partial}{\partial x^\rho} \left\{ \beta_{\mu\nu} \right\} - \frac{\partial}{\partial x^\rho} \left\{ \beta_{\mu\rho} \right\} + \left\{ \beta_{\tau\rho} \right\} \left\{ \nu_\mu \right\} - \left\{ \beta_{\tau\nu} \right\} \left\{ \rho_\mu \right\}
\]

(14)

is the Riemannian tensor of the undotted metric \( g_{\mu\nu} \) and \( D^\beta_{\mu\rho} |\rho \) is the non-Riemannian covariant derivative of the tensor \( D^\beta_{\mu\nu} \) with respect to the full connection \( \Gamma^\beta_{\mu\nu} \).

Continuing with

\[
g_{\mu\nu}|\rho = 2g_{\mu\nu} \phi_\rho
\]

(15)

and the derived expression

\[
\phi_\tau|\rho = (\phi^\tau g_{\tau\mu})|\rho = \phi^\tau|\rho g_{\mu\tau} + 2\phi_\mu \phi_\rho
\]

(16)
used to eliminate terms in $\phi^\tau|_\rho$, there results

$$
R^\beta_{\mu\nu\rho} = Q^\beta_{\mu\nu\rho} - \delta^\beta_\rho \phi_\nu|_\rho + \delta^\beta_\rho \phi_\mu|_\nu - \delta^\beta_\rho \phi_{\mu\rho} - \delta^\beta_\rho \phi_{\mu\nu} + g_{\mu\nu} g^\lambda\beta \phi_\lambda|_\rho - g_{\mu\rho} g^\lambda\beta \phi_\lambda|_\nu
$$

$$
- g_{\mu\nu} \phi^\beta_\rho + g_{\mu\nu} \phi^\beta_\phi + g_{\mu\nu} \phi^\beta_\phi + g_{\mu\nu} \phi^\beta_\phi - g_{\mu\rho} \phi^\beta_\phi - g_{\mu\rho} \phi^\beta_\phi - g_{\mu\rho} \phi^\beta_\phi + g_{\mu\rho} \phi^\beta_\phi + \phi^\beta_\phi + \phi^\beta_\phi
$$

$$
(17)
$$

The contraction with respect to first and third indices is

$$
R^\nu_{\mu\rho} \equiv R_{\mu\rho} = Q_{\mu\rho} - 3\phi_\mu|_\rho + \phi_\rho|_\mu - g_{\mu\rho} \phi_\lambda|_\lambda + 2\phi_\mu \phi_\rho - 2g_{\mu\rho} \phi^\tau \phi^\tau
$$

$$
(18)
$$

and contracting with the undotted inverse, $g^{\mu\rho}$, gives

$$
R^\nu_{\mu\rho} g^{\mu\rho} = Q(g_{\mu\nu}) - 6\phi_\lambda|_\lambda - 6\phi_\lambda \phi^\lambda
$$

$$
(19)
$$

Reduction of the Weyl derivative can be made by

$$
\phi_\lambda|_\nu g^{\mu\nu} = \phi_{\mu\nu} - 2\phi_\lambda \phi^\lambda
$$

$$
(20)
$$

which comes from equations (14), (1), and (2) and gives

$$
R^\nu_{\mu\rho} g^{\mu\rho} = Q(g_{\mu\nu}) - 6\phi_\lambda|_\lambda + 6\phi_\lambda \phi^\lambda
$$

$$
(21)
$$

The semicolon denotes the Riemannian covariant derivative using the undotted christoffel symbols. It is necessary to eliminate the undotted metric in favor of the observers dotted metric and the conformal ratio $\lambda$. The fundamental quantity for the electromagnetic field is $\phi_\mu$ with the index lowered. The index of $\phi_\mu$ in equation (21) has been raised with the undotted metric. The conventional physical quantity is actually

$$
\phi_\mu = \lambda \phi^\rho \dot{g}_{\mu\rho}
$$

$$
(22)
$$

The curvature scalar $Q(g_{\mu\nu})$ of the dotted metric can be related to the curvature scalar $Q(g_{\mu\nu})$ of the undotted metric by a calculation entirely analogous to those already done. The complete expression becomes

$$
- 6m^2 = R_{\mu\nu} \dot{g}^{\mu\nu} = \lambda R_{\mu\nu} g^{\mu\nu} = Q(\dot{g}_{\mu\nu}) + \frac{3}{\lambda \sqrt{-\dot{g}}} \frac{\partial}{\partial x^\beta} \left( \sqrt{-\dot{g}} \dot{g}^{\beta\alpha} \frac{\partial \lambda}{\partial x^\alpha} \right)
$$

$$
- \frac{3}{2\lambda^2} \dot{g}^{\alpha\beta} \frac{\partial \lambda}{\partial x^\alpha} \frac{\partial \lambda}{\partial x^\beta} - \frac{6}{\sqrt{-\dot{g}}} \frac{\partial}{\partial x^\beta} \left( \sqrt{-\dot{g}} \phi^\beta \right) + 6\lambda \phi_\beta \phi^\beta
$$

$$
(23)
$$
Since $\phi_\beta$ is pure imaginary in this gauge, the linear term separates and after substituting for $\phi_\beta$ with $A_\beta$, there results two real equations:

$$\frac{\partial}{\partial x^\beta}(\sqrt{-\mathring{g}} \mathring{g}^{\beta\alpha} \lambda A_\alpha) = 0. \quad (24)$$

and

$$m^2 - e^2 \mathring{g}^{\mu\nu} A_\mu A_\nu = Q(\mathring{g}_{\mu\nu}) - \frac{1}{2\sqrt{-\mathring{g}} \lambda} \frac{\partial}{\partial x^\beta} \left(\sqrt{-\mathring{g}} \mathring{g}^{\beta\alpha} \frac{\partial \lambda}{\partial x^\alpha} \right) + \frac{\mathring{g}^{\alpha\beta}}{4\lambda^2} \frac{\partial \lambda}{\partial x^\beta} \frac{\partial \lambda}{\partial x^\alpha} \quad (25)$$

The first of these expresses the conservation of the quantum probability current as a congruence of trajectories. In the second equation, the left side includes all classical terms from the Hamilton-Jacobi equation for a general relativistic particle in an electromagnetic field. This can be seen by reversing the gauge transformations and identifying the wave function with the action by $\psi = \exp(iS)$. The classical limit corresponds to neglecting derivatives of the magnitude of the wave function. The terms on the right side represent curvature corrections due to either the Riemannian curvature of the observer’s space or the quantum curvature derived from the conformal ratio $\lambda$. The conformal contributions to the curvature create the expected quantum effects.

Note that equations (24) and (25) are entirely real and do not contain any explicit reference to the wave function $\psi$. This form is useful because many previous five dimensional theories are known that have two real fields $A_\mu$ and $g_{\mu\nu}$. An attempt can be made to quantize some of these by the substitution of the fixed gauge fields. This process often seems to generate quantum terms and interpretations. In other cases it is completely unworkable. However, an approach of this type to first quantization avoids the serious problems that have been discussed.

IV. CONFORMAL FACTORS

It is now essential to understand how the conformal factors developed in (QG) can be used to form a field equation. Only the most elementary arguments are available because
the standard methods that use lagrangians fail. Moreover the conformal factors do not have any established interpretations. The characteristic equations that contains these factors are studied and then interpreted according to physical principles or experiments.

It is by way of the quantum field equation that the mass must be introduced. This external constant characterizes wave aspects of particle motion. The introduction of the associated length scales must be compatible with the scale sizes of other physical effects. Boundaries, slits and shutters are scaled according to the masses of the particles of which they are made. Electromagnetic and gravitational source currents are scale sensitive and require particles at measured positions. The extrinsic nature of mass is even suggested by particle creation.

It is known that free space electromagnetic theory is invariant under conformal gauge change and can contain no intrinsic scale size. Scale dependence can enter only through the source terms. Thus any distance dependency found in electrodynamics must derive ultimately from the intrinsic scale sizes of quantum source currents and must be related to the masses of the source particles. If the electric source equations can be combined with the quantum field equations in a geometrical theory, the electric interaction constant becomes geometrical and should have a geometrical interpretation. This allows the conversion of $e^2/mc$ to $e^2/\hbar c$. It may or may not be calculable, but one can anticipate that some theories will have favored values.

Complex wave functions are used in the accepted description of quantum mechanics [22]. They can be used legitimately in a geometrical theory in so far as the phase and amplitude are combined into a linear wave equation. And while the quantum geodesics are completely real, it seems reasonable to allow the use of complex quantities in the search for fundamental invariants [23, 24]. The success of this approach is not assured, but if it happens that such equations are identified, rearrangements to a real theory should be possible.

Following (QG), source currents manifest their effects on the metric through the conformal factors. In constructing a theory of interaction, the three factors $\omega, \lambda, \chi$ must depend on either the test particle wave function $\psi_1$ or on one or more source wave functions $\psi_2 \cdots \psi_N$. 
It is by no means obvious which combinations of fields \( \psi_1 \cdots \psi_N \) are important in giving values to each of \( \omega, \lambda, \) and \( \chi \). Various geometrical quantities must be evaluated and compared with known quantum equations. Since none of these factors affect the congruence of a specific quantum state, the physical meaning must be inferred from the effect that they have on the field equations. The motion, as described by a particular congruence, may be explained by different combinations of source currents. A simple form for interactions should be obtained by allowing the source currents to affect the five metric through the conformal factors.

A number of conformally covariant theories have used the factors \( \omega \) and \( \chi \), in addition to \( \lambda \). These quantities should all be identified with the physics of interaction. Applied as contact transformations, the effect is to change the five-gauge of \( \gamma_{mn} \), the one-gauge of the fifth dimension \( \tau \) or the four-gauge of \( g_{\mu \nu} \). This includes, implicitly, the integrating factor (or dis-integrating factor) of the vector potential.

A preliminary investigation \( \omega \) shows that it may generate five-covariant interactions. If, following reference [25], the metric is conformally transformed, so that the new Riemann tensor is

\[
\Gamma_{mn} = e^{2\sigma} \gamma_{mn},
\]  

then the new curvature is

\[
\Theta_{mnab} = e^{2\sigma} \left[ \Theta_{mnab} + \gamma_{ma} \sigma_{nb} + \gamma_{mb} \sigma_{na} - \gamma_{na} \sigma_{mb} + (\gamma_{ma} \gamma_{nb} - \gamma_{mb} \gamma_{na}) \gamma^{lt} \sigma_{.l} \sigma_{.t} \right]
\]

and the Ricci tensor \( \Theta_{mn} \) is given by

\[
\Theta_{mn} = \Theta_{mn} + (n - 2)(\sigma_{;mn} - \sigma_{;m} \sigma_{;n}) + \gamma_{mn}[\gamma^{ab} \sigma_{;ab} + (n - 2)\gamma^{ab} \sigma_{;a} \sigma_{;b}].
\]  

If the lowest order dependence, perhaps as expressed in a local euclidean region, is exponential, possibly of the form \( e^{\iota(\kappa x - \omega t + m\tau)} \), the lowest order linear terms in velocity occur in the \( (5\mu) \) positions and the quadratic terms in the \( (\mu\nu) \) positions. The additivity is
correct for the source terms of classical equations. A product of external factors $e^{2\sigma_1}e^{2\sigma_2}$ will have linear additivity with $(5\mu)$ terms of the form $\sigma_{1\mu} + \sigma_{2\mu}$. This is appropriate for electromagnetic source currents. The quadratic additivity of the $\mu\nu$ terms will be of the form $(\sigma_{1\mu} + \sigma_{2\mu})^2$ which is appropriate for gravitational effects. This suggests that, at least in the classical limit, $\sigma_i$ must be part of the source current. Probably in the quantum limit, it must also be some part of the quantum source current. This is apparently a primitive type of interaction. The internal conformal factors may also participate but may reduce the invariance to four dimensions.

The association of these factors with real source currents is still under study. A full understanding should give a quantum version of the Maxwell-Einstein equations. A prerequisite to such a derivation will require an understanding of how to interpret the coherent quantum terms geometrically. Because, for this formalism, there is no classical foundation, the problem of quantum understanding must be addressed first. The source currents are postponed and the quantum field equations are taken up instead.

V. CONFORMAL WAVES

It seems reasonable to attempt to construct a quantum equation in five space from an assumption about geometrical invariants [26]. Since the coordinate $\tau$ is not accessible, there is no direct way to observe the actual system $(x^\mu, \tau)$. Nevertheless, there is some indication of the importance of this formal coordinate. It is that the collection of non-inertial systems that transform among themselves by a cut transformation remain physically equivalent until $\tau$ becomes a universal coordinate. Otherwise stated, defining a fixed $\tau$ coordinate specifies a particular gauge and the associated motion for each particle. This specification represents the beginning of a systemwide inertial structure. A similar idea has been identified by Schouten [27].

It is in practice the ratio of rest masses, that can be measured by quantum diffraction. One should therefore demand a theory for which individual particles have constant rest mass.
ratios at points taken where trajectories intersect. Given any two distinct particles, $A$ and $B$, whose trajectories intersect at a set of points. $N_1, N_2, \cdots, N_k$, then there must be a way to arrange the theory so that they have equal effective mass ratios.

\[
\left( \frac{M_A}{M_B} \right)_1 = \left( \frac{M_A}{M_B} \right)_2 = \cdots = \left( \frac{M_A}{M_B} \right)_k
\]  

(29)

Such unvarying mass ratios of discreet particles measured quantum mechanically is taken as an experimental fact. To reproduce this observation with an extrinsic definition of mass, the properties of each of the two particles must be referred to a common geometrical system. The neutral observer must be able to choose the rest masses of particles so that they will not vary over space-time. Otherwise the clocks cannot be systematically calibrated. In so far as the neutral observer can uniformly calibrate a system of quantum clocks, the absolute mass of a particle can become a viable global concept.

The concepts of mass and quantum time come together consistently if the particle fields have a proper time dependence that appears always through the product combination $m\tau$. Now while the $\tau$ coordinate must be universal, the fields observed in neutral space-time must also be $\tau$ independent. For five dimensional invariance, field equations containing $\tau$ derivatives are unavoidable. These derivatives must operate on some residual $\tau$ dependence which is then replaced by factors of $m$. This departs from Klein’s assumption of the charge as an eigenvalue [28] and does not support a sequence of quantized values for the charge.

Qualitatively, one expects a Klein-Gordon equation to appear as

\[
\left( \frac{\partial}{\partial t^2} - \frac{\partial}{\partial x^2} - \frac{\partial}{\partial y^2} - \frac{\partial}{\partial z^2} \right) \Psi = -m^2 \Psi \equiv \frac{\partial}{\partial \tau^2} \Psi
\]  

(30)

with

\[
\Psi = e^{im\tau} \psi(x^\mu)
\]  

(31)

where the factor $e^{im\tau}$ applies only in a coordinate system that has a fixed preferred alignment to the observers’ neutral frame. The introduction of the mass reduces the five dimensional dispersion free form and creates quantum dispersion [29]. This reduction is essential and
ultimately produces classical inertia. Once this is done, the cut transformations as an
arbitrary coordinate transformation must be relinquished as it will cause contact variations
of physical parameters.

In this way, the constancy of the mass ratios at each point, can be assured from the
requirement of using a single fifth coordinate axis for all particles. The universality of the
five space also is inferred. By using a real particle for the construction of clocks, and by
choosing a fixed numerical value of the mass, a uniform quantum clock is generated. At
this point, coordinate transformations that affect the scale size of the fifth coordinate axis
correspond to a continuous rescaling of all masses (rather than individual masses) over
space-time. The quantum behavior references the fifth dimension, which must be assumed
common.

From this construction in five dimensions it is seen that the mass spectrum consists of
a single value. A more complicated spectrum, might come from other geometries. The
elementary particles have distinct masses but they are also all distinguished by additional
interactions. Perhaps a geometrical theory that includes such fundamental interactions will
produce a more realistic mass spectrum. Since the mass appears here as a property that is
extrinsic to the particle, calculation of the mass from a field equation is possible in principle.

The conformal waves can now be calculated for other cases. An equivalent structure
should occur in five dimensions. Let a metric in \( n \) dimensional space be of the form \( \omega \eta_{ma} \)
where \( \eta_{ma} \) is a unit diagonal tensor and \( \omega \) is a manifold variable conformal factor. The
contribution of \( \omega \) to the curvature scalar is calculated directly. It must be linear in the
second derivatives of \( \omega \) and quadratic in the first derivatives of \( \omega \). The numerical factors
depend on the number of dimensions \( n \).

\[
R = (n - 1) \frac{1}{\omega^2} \frac{\partial^2 \omega}{\partial x^a \partial x_a} + \frac{(n - 1)(n - 6)}{4} \frac{1}{\omega^3} \frac{\partial \omega}{\partial x^a} \frac{\partial \omega}{\partial x_a} \tag{32}
\]

A transformation of the form \( \omega = \psi^p \) can be used to eliminate the terms quadratic in
the first derivative.

\[
R \psi^p = (n - 1)p \left\{ \frac{1}{\psi} \frac{\partial^2 \psi}{\partial x^2} + \left[ \frac{(n - 6)}{4} p + (p - 1) \right] \frac{1}{\psi^2} \frac{\partial \psi}{\partial x^a} \frac{\partial \psi}{\partial x^a} \right\} \tag{33}
\]
Taking the term in brackets as zero gives

\[ p = \frac{4}{n-2} \]  

(34)

If \( R = 0 \) be chosen as a fundamental invariant equation, a linear wave equation for the conformal factor is generated except for the cases \( n = 1 \) and \( n = 2 \). The extended Weyl theory corresponds to \( n = 4, p = 2 \) with the additional condition that \( R\psi^p \) is constant. This works physically as long as a properly scaled base metric is available to form the modified curvature scalar that absorbs the nonlinear factor \( \lambda = \psi^p = |\psi|^2 \). This is automatically accomplished if quantum objects, transported from region to region are used to make local measurements. In the five dimensional case \( p = 4/3 \) and a term \( R\psi^{4/3} \) is not linear. The scalar \( R \) must be chosen zero in agreement with the concept of null five vectors. The remaining terms give a wave equation of the expected form. In almost any number of dimensions, conformal variations can be used to generate a linear wave equation.

The conformal waves of different dimensionality may relate to each other nonlinearly. A derived invariant linear equation for one value of \( n \) will not necessarily be linear as observed in a different number of dimensions. Because the fifth coordinate is not directly observable from space-time, the internal conformal factor \( \lambda \), (and indirectly \( \chi \)) might also participate in various types of wave behavior. With both factors involved together, it is necessary to consider curvature scalars for a metric of the form

\[
\begin{pmatrix}
\lambda \omega & & & & \\
& \lambda \omega & & & \\
& & \lambda \omega & & \\
& & & \lambda \omega & \\
& & & & \omega
\end{pmatrix}
\]

(35)

Here both \( \lambda \) and \( \omega \) are possibly dependent on five coordinates.

The curvature derivation is carried out with all diagonal terms formally positive. The result for signature \((1,-1,-1,-1,-1)\), can be found by appropriate sign changes. Let \( g_{mn} \) be diagonal but with possibly different values for each element. A long but elementary calculation gives the Riemann tensor as
\[ R_{abcd} = \frac{1}{2} \left( -g_{aa,be} \delta_{ae} + g_{bb,ae} \delta_{be} + g_{aa,bd} \delta_{ac} - g_{bb,ad} \delta_{bc} \right) + \frac{\delta_{ad}}{4} \left[ g_{aa,g_{aa,b}g_{aa,c}} + g_{bb,g_{aa,b}g_{bb,c}} + g_{cc,g_{aa,c}g_{cc,b}} \right] - \frac{\delta_{bd}}{4} \left[ g_{aa,g_{aa,c}g_{bb,a}} + g_{bb,g_{aa,b}g_{bb,c}} + g_{cc,g_{aa,c}g_{bb,b}} \right] - \frac{\delta_{ac}}{4} \left[ g_{aa,dg_{aa,b}} + g_{bb,g_{aa,b}g_{bb,d}} + g_{dd,g_{aa,d}g_{bb,d}} \right] + \frac{\delta_{bc}}{4} \left[ g_{aa, dg_{aa,d}g_{bb,a}} + g_{bb, g_{aa,d}g_{bb,d}} + g_{dd, g_{dd,a}g_{bb,d}} \right] - \frac{1}{4} \left( \delta_{ad} \delta_{bc} - \delta_{ac} \delta_{bd} \right) \sum_m g_{aa,m}g_{bb,m}g_{mm}^m \] (36)

in which all sums are written explicitly.

The Ricci tensor, given by the first contraction with the inverse metric \( g^{ac} = \delta^{ac}/g_{aa} \), is

\[ R_{bd} = \frac{1}{2} \left[ \frac{\delta_{bd}}{m} \sum_{m} g_{mm} g_{bb,mm} + \sum_{m} g_{mm} g_{mm, bd} - g_{dd, g_{dd,bd}} - g_{bb, g_{bb,bd}} \right] + \frac{1}{2} \left[ (g^{dd})^2 g_{dd,bd} + (g^{bb})^2 g_{bb,bd} + g^{dd} g_{dd, g_{dd,bd}} \right] - \frac{1}{4} \sum_{m} \left[ (g_{mm})^2 g_{mm, g_{mm,b}} + g_{mm, g_{bb,mm} g_{bb,bd}} + g_{mm, g_{dd, g_{mm,b}} g_{dd,bd}} \right] - \frac{\delta_{bd}}{2} \sum_{m} \left[ (g_{mm})^2 g_{mm, g_{bb,m} g_{bb,m}} + g_{mm, g_{bb,bb,m} g_{bb,m}} \right] + \frac{\delta_{bd}}{4} \sum_{m,n} g_{mm, n} g_{bb, n} g_{mm} g_{mn} \] (37)

again with all sums explicit. The scalar, given by another contraction with the inverse metric

\[ R = \frac{3}{2} \sum_{m} (g_{mm})^3 (g_{mm,m})^2 - \sum_{m} (g_{mm})^2 g_{mm,mm} + \sum_{m,n} g_{mm} g_{mn} g_{mm,nn} \]

\[ - \sum_{m,n} \left[ (g_{mm})^2 g_{mm, g_{mm,m} g_{mm,m}} + \frac{3}{4} (g_{mm})^2 g_{mm, g_{mm,n} g_{mm,n}} \right] + \frac{1}{4} \sum_{m,n,p} g_{mm, p} g_{nn, p} g_{mm} g_{nn} g_{pp} \] (38)

with all sums explicit.

This can be used to evaluate the diagonal case of equation (35) by choosing \( g_{\mu \mu} = \omega \lambda \), and \( g_{55} = \omega \) and rearranging, using \( x^a = x^\alpha \) for \( a = \alpha = 1, 2, 3, 4 \) and \( x^5 = \tau \).

\[ R = \frac{3}{\lambda^2 \omega} \frac{\partial^2 \lambda}{(\partial x^\beta)^2} - \frac{3}{2 \lambda^3 \omega} \left( \frac{\partial \lambda}{\partial x^\beta} \right)^2 + \frac{4}{\lambda \omega^2} \frac{\partial^2 \omega}{(\partial x^\beta)^2} - \frac{1}{\lambda \omega^3} \left( \frac{\partial \omega}{\partial x^\beta} \right)^2 + \frac{4}{\omega^2} \frac{\partial^2 \omega}{\partial \tau^2} - \frac{1}{\omega^3} \left( \frac{\partial \omega}{\partial \tau} \right)^2 \]
\[ + \frac{4}{\lambda^2 \omega^2} \frac{\partial \omega}{\partial x^3} \frac{\partial \lambda}{\partial x^3} + \frac{8}{\lambda \omega} \frac{\partial \omega}{\partial \tau} \frac{\partial \lambda}{\partial \tau} + 4 \frac{\partial^2 \lambda}{\partial \tau^2} + \frac{1}{\lambda^2 \omega} \left( \frac{\partial \lambda}{\partial \tau} \right)^2 \] 

(39)

It is easy to see that \( \lambda \) and \( \omega \) both have wave properties which can be coupled to each other by an imposed constraint on the curvature scalar. It is suggestive to loosely associate \( \omega \) and \( \lambda \) with wave functions. From this expression, the coupling terms are of the form \( A_{\mu} J^\mu \). Considering first just the \( \lambda \) wave, and removing the coupling by setting \( \omega = 1 \), the expression for the curvature scalar becomes

\[ R = -\frac{3}{\lambda^2} \left( \frac{\partial^2 \lambda}{\partial x^\mu} \right)^2 + \frac{3}{2 \lambda^3} \left( \frac{\partial \lambda}{\partial x^\mu} \right)^2 - \frac{4}{\lambda} \frac{\partial^2 \lambda}{\partial \tau^2} - \frac{1}{\lambda^2} \left( \frac{\partial \lambda}{\partial \tau} \right)^2 \] 

(40)

Because the first two terms will reduce to the form \( \Box \psi \psi \) after the substitution \( \lambda = \psi^2 \), a four dimensional covariant equation is possible for the four dimensional conformal waves that are internal to the five dimensional metric.

Alternatively, the same sort of calculation of scalar curvature dependence on \( \omega \) with \( \lambda = 1 \) gives the result predicted by equation (32). Both types of waves may be important to define a form of unified interaction. The following sections consider these two cases with some care to evaluate gravitational and electromagnetic effects.

**VI. INTERNAL KLEIN-GORDON EQUATION**

Consider first the four dimensional conformal factor. Because in equation (40), the terms in \( \tau \) do not have the same degree of homogeneity in \( \lambda \) as the terms in \( x^\mu \), the \( \tau \) dependence cannot be used to generate a mass term. Apparently, this structure corresponds more to the Weyl theory in which the mass is introduced as an external constant. The resulting equation is not a five dimensional invariant and the mass must scale against the observer’s metric. As with the Weyl theory, the observers’ metric must be set up so that a quantum object, perhaps an hydrogen atom, is spherically symmetric and of constant size. Relative to this particular system, conformal changes must be executed with care and with consideration for the gauge system. Measurements of the gauge of the observed metric, \( \dot{g}_{\mu \nu} \) in separated regions of space-time must match up when expanded to overlap.
The gauge factor $\lambda$ is a neutral space four-scalar but might also be a function of $x^5 = \tau$. This dependence is presumed to be of the form $e^{i\eta x^5 \tau}$. Any other proper time dependencies might show effects that would be explicitly observable by measuring quantum probability densities. Most physical interpretations support the interpretation of $\lambda$ as a function only of $x^\mu$. This transformations does leave the trajectories invariant. With this factor included, it is the most general factor that is allowed for the conditions imposed.

The calculation of curvature, including electromagnetic and gravitational effects, is somewhat involved. By transforming the coordinate system at a fixed but arbitrary point $P$, it is possible to reduce $\gamma_{mn}$ to a locally pseudo-euclidean system with diagonal$(1, -1, -1, -1)$ and with derivatives $\gamma_{mn,a} = 0$. This is the local co-moving frame that, as required by five space equivalence, removes all interactions. The electromagnetic field becomes zero since then $A_{\mu,\nu} = 0$ and $H_{\mu\nu} \equiv A_{\mu,\nu} - A_{\nu,\mu} = 0$. This transformation would require the observed macroscopic fields $\dot{g}_{\mu\nu}$ and $A_{\mu}$ to be explicitly $\tau$ dependent. A milder transformation that looks like it is four dimensional to the observer is more useful.

Let the four-space coordinates $x^\mu$ be transformed at a fixed point $P$ so that $\partial \dot{g}_{\mu\nu} / \partial x^\beta = 0$ making the neutral space locally geodesic. Now perform a cut transformation

$$\tau' = \tau + \Pi(x^\mu)$$

and choose $\Pi$ so that $A'_\mu\big|_P = 0$ and $(A'_{\mu,\nu} + A'_{\nu,\mu})\big|_P = 0$. This value can be calculated explicitly if

$$A'_\mu = A_\mu + \Pi_\mu$$

and

$$A'_{\mu,\nu} = A_{\mu,\nu} + \Pi_{\mu\nu}.$$  

Assuming a polynomial expansion gives

$$\Pi = -A_\mu\big|_P x^\mu - \frac{1}{4}(A_{\mu,\nu} + A_{\nu,\mu})\big|_P x^\mu x^\nu$$
The first condition (42) simplifies the metric at $P$ so that it is diagonal while the second is a local Killing condition for $A'_\mu$. Of course $H'_\mu\nu = H_{\mu\nu}$ is gauge invariant and four covariant as it should be. The reverse transformation to (44) must be performed in order to return to the coordinate system of the original metric.

The calculation of the Riemann tensor can be carried out at point $P$ and the full tensor regenerated later. The result is unwieldy in some cases. This requires values of $\gamma^{mn}$, the Christoffel symbols $[m, np]$, and their derivatives $[m, np], q$. Evaluations at point $P$ are simpler because many quantities are zero. In particular,

$$\dot{g}_{\mu\nu,\sigma}\big|_P = \Sigma_{\mu\nu}\big|_P = A_\mu\big|_P = A_{\mu,5} = \dot{g}_{\mu\nu,5} = 0$$

$$\gamma_{mn}\big|_P = \begin{pmatrix} \lambda \dot{g}_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}$$

$$2\{\gamma_{\alpha,\beta,\gamma}\} = g_{\alpha\beta,\gamma} + g_{\gamma\alpha,\beta} - g_{\beta\gamma,\alpha} = \dot{g}_{\alpha\beta}\lambda_{,\gamma} + \dot{g}_{\gamma\alpha}\lambda_{,\beta} - \dot{g}_{\beta\gamma}\lambda_{,\alpha}$$

$$2[5, \beta\gamma] = \Sigma_{\beta\gamma} = g_{\beta\gamma,5}$$

$$2[\alpha, \gamma] = g_{\alpha\gamma,5} + H_{\alpha\gamma}$$

$$2[\alpha, 5\gamma] = 2[5, \alpha\gamma] = 2[5, 5\gamma] = 0.$$  

Where $g_{\mu\nu} = \lambda \dot{g}_{\mu\nu}$, $H_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$, and $\Sigma_{\mu\nu} = A_{\mu,\nu} + A_{\nu,\mu}$. The Christoffel symbols are

$$2[\alpha, \beta, \gamma] = g_{\alpha\beta,\gamma} + g_{\gamma\alpha,\beta} - g_{\beta\gamma,\alpha} = \dot{g}_{\alpha\beta}\lambda_{,\gamma} + \dot{g}_{\gamma\alpha}\lambda_{,\beta} - \dot{g}_{\beta\gamma}\lambda_{,\alpha}$$

$$2[5, \beta, \gamma] = g_{\beta\gamma,5}$$

$$2[\alpha, 5\gamma] = g_{\alpha\gamma,5} + H_{\alpha\gamma}$$

These lead to the calculated values,

$$2[\alpha, \beta, \gamma]\big|_P = g_{\alpha\beta,\gamma} + g_{\gamma\alpha,\beta} - g_{\beta\gamma,\alpha} = \dot{g}_{\alpha\beta}\lambda_{,\gamma} + \dot{g}_{\gamma\alpha}\lambda_{,\beta} - \dot{g}_{\beta\gamma}\lambda_{,\alpha}$$

$$2[5, \beta, \gamma]\big|_P = -g_{\beta\gamma,5} = -\dot{g}_{\beta\gamma}\lambda_{,5}$$

$$2[\alpha, 5\gamma]\big|_P = g_{\alpha\gamma,5} + H_{\alpha\gamma} = g_{\alpha\gamma}\lambda_{,5} + H_{\alpha\gamma}$$

and

$$2 \{\frac{\mu}{\beta, \gamma}\} \big|_P = \delta_{\gamma}^{\mu} \frac{\lambda_{,\beta}}{\lambda} + \delta_{\beta}^{\mu} \frac{\lambda_{,\gamma}}{\lambda} - \dot{g}_{\beta,\gamma} \gamma_{\mu,\lambda} \frac{\lambda_{,\alpha}}{\lambda} - \frac{(A_{\mu} A_{\beta})_{,\gamma}}{\lambda} + \frac{(A_{\mu} A_{\gamma})_{,\beta}}{\lambda} + \frac{(A_{\beta} A_{\gamma})_{,\alpha}}{\lambda} \dot{g}_{\alpha,\mu}$$
\[ 2 \left\{ \frac{5}{\beta\gamma} \right\}_P = -\sigma_{\beta\gamma} + \dot{g}_{\beta\gamma}\lambda,5 \]  
(56)

\[ 2 \left\{ \frac{\alpha}{5\gamma} \right\}_P = \frac{\delta^\alpha_\lambda}{\lambda} + \frac{H^\alpha_\gamma}{\lambda}. \]  
(57)

And for the derivatives at point \( P \)

\[ 2[\alpha, \beta\gamma]_\epsilon = g_{\alpha\beta\gamma\epsilon} + g_{\alpha\gamma\beta\epsilon} - g_{\beta\gamma\alpha\epsilon} - H_{\alpha\gamma\lambda\beta\epsilon} - H_{\alpha\beta\lambda\gamma\epsilon} = \]

\[ \dot{g}_{\alpha\beta\gamma\epsilon} + \dot{g}_{\alpha\gamma\beta\epsilon} + \dot{g}_{\alpha\beta\lambda\gamma\epsilon} - \dot{g}_{\beta\gamma\lambda\alpha\epsilon} - H_{\alpha\gamma\lambda\beta\epsilon} - H_{\alpha\beta\lambda\gamma\epsilon} \]  
(58)

\[ 2[5, \beta\gamma]_\epsilon = \Sigma_{\beta\gamma\epsilon} - g_{\beta\gamma,5\epsilon} \]  
(59)

\[ 2[\alpha, 5\gamma]_\epsilon = \dot{g}_{\alpha\gamma,5\epsilon} + H_{\alpha\gamma,\epsilon} \]  
(60)

\[ 2[\alpha, \beta\gamma]_5 = \dot{g}_{\alpha\beta\lambda\gamma5} + \dot{g}_{\alpha\gamma\lambda\beta5} - \dot{g}_{\beta\gamma\lambda\alpha5} \]  
(61)

\[ 2[5, \beta\gamma]_5 = -\dot{g}_{\beta\gamma\lambda,55} \]  
(62)

\[ 2[\alpha, 5\gamma]_5 = \dot{g}_{\alpha\gamma\lambda,55}. \]  
(63)

For the Riemann tensor calculation, terms can be grouped by whether they are quantum mechanical \( Q_{abcd} \) with factors of \( \lambda \), electromagnetic \( E_{abcd} \) with factors of \( H_{\mu\nu} \) or gravitational \( R_{abcd} \) with factors of \( \dot{g}_{\mu\nu\alpha\beta} \). Depending on the type of theory, these terms may transform into each other. In five dimensions the full tensor can be written

\[ \Theta_{abcd} \equiv Q_{abcd} + R_{abcd} + E_{abcd} = [a, bc], d - [a, bd], c + \gamma^t_u [t, bd] [u, ac] - \gamma^t_u [t, bc] [u, ad] \]  
(64)

Three classes of terms can be considered and calculated separately depending on whether there is no index equal to 5, one index equal to 5 or two indices equal to 5. For the first case,

\[ R_{\alpha\beta\gamma\epsilon} = \frac{1}{2} \left[ \dot{g}_{\alpha\gamma\lambda,\beta\epsilon} - \dot{g}_{\beta\gamma,\alpha\epsilon} - \dot{g}_{\alpha\epsilon,\beta\gamma} + \dot{g}_{\beta\epsilon,\alpha\gamma} \right], \]  
(65)

\[ Q_{\alpha\beta\gamma\epsilon} = \frac{1}{2} \left[ \dot{g}_{\alpha\gamma\lambda,\beta\epsilon} - \dot{g}_{\beta\gamma,\alpha\epsilon} - \dot{g}_{\alpha\epsilon,\beta\gamma} + \dot{g}_{\beta\epsilon,\alpha\gamma} \right] 
+ \frac{1}{4} \left[ g_{\tau\beta,\lambda,\epsilon} + g_{\tau\epsilon,\lambda,\beta} - g_{\beta\epsilon,\lambda,\tau} \right] g^{\tau\mu} \left[ g_{\mu\alpha,\gamma,\epsilon} + g_{\mu\epsilon,\gamma,\alpha} - g_{\alpha\gamma,\lambda,\mu} \right] 
- \frac{1}{4} \left[ g_{\tau\beta,\lambda,\gamma} + g_{\tau\gamma,\lambda,\beta} - g_{\beta\gamma,\lambda,\tau} \right] g^{\tau\mu} \left[ g_{\mu\alpha,\lambda,\epsilon} + g_{\mu\epsilon,\lambda,\alpha} - g_{\alpha\lambda,\mu,\epsilon} \right] \]

\[- \frac{1}{4} g_{\beta\epsilon,\gamma,\alpha}(\lambda,5)^2 + \frac{1}{4} g_{\beta\gamma,\alpha\epsilon}(\lambda,5)^2 \]  
(66)
and

\[ E_{\alpha\beta\gamma\epsilon} = \Lambda_{\alpha,\epsilon} \Lambda_{\beta,\gamma} - \Lambda_{\alpha,\gamma} \Lambda_{\beta,\epsilon} - 2 \Lambda_{\alpha,\beta} \Lambda_{\gamma,\epsilon} \]  \hspace{1cm} (67)

For the terms containing one 5

\[ R_{\alpha\beta5} = 0, \]  \hspace{1cm} (68)

\[ Q_{\alpha\beta5} = \frac{1}{2} (\dot{g}_{\alpha\gamma} \lambda_{\beta5} - \dot{g}_{\beta\gamma} \lambda_{\alpha5}) + \frac{1}{2\lambda} (g_{\beta\gamma} \lambda_{5,\alpha} - g_{\alpha\gamma} \lambda_{5,\beta}), \]  \hspace{1cm} (69)

and

\[ E_{\alpha\beta\gamma5} = \frac{1}{2} (H_{\alpha\beta} \lambda_{\gamma} - H_{a\beta,\gamma}) + \frac{1}{4} (H_{\beta\gamma} \lambda_{\alpha} - H_{\gamma\alpha} \lambda_{\beta} + g_{\beta\gamma} \lambda_{5}^{\tau} H_{\tau\alpha} - g_{\alpha\gamma} \lambda_{5}^{\tau} H_{\tau\beta}). \]  \hspace{1cm} (70)

For terms containing two 5's,

\[ R_{\alpha55} = 0, \]  \hspace{1cm} (71)

\[ Q_{\alpha55} = \frac{1}{2} \dot{g}_{\alpha\beta} \lambda_{55} - \frac{1}{4\lambda} \dot{g}_{\alpha\beta} \lambda_{5,5}, \]  \hspace{1cm} (72)

and

\[ E_{\alpha55} = -\frac{1}{4\lambda} H_{\alpha}^{\tau} H_{\tau\beta}. \]  \hspace{1cm} (73)

All other four index terms are zero.

The Ricci tensor \( \Theta_{ac} = \Theta_{abce} \hat{\gamma}^e \) can be split up the same way,

\[ R_{\alpha\gamma} = \frac{1}{2\lambda} [\dot{g}_{\alpha\gamma,\beta\epsilon} \hat{\gamma}^\beta \epsilon - 2 \dot{g}_{\beta\gamma,\alpha} \hat{\gamma}^\beta \epsilon + \dot{g}_{\beta\epsilon,\alpha} \hat{\gamma}^\beta \epsilon], \]  \hspace{1cm} (74)

\[ Q_{\alpha\gamma} = \frac{1}{2\lambda} (\dot{g}_{\alpha\gamma} \lambda_{\mu\tau} \hat{\gamma}^{\mu\tau} + 2 \lambda_{\alpha\gamma}) - \frac{3}{2\lambda^2} \lambda_{\alpha} \lambda_{\gamma} + \frac{1}{2} \dot{g}_{\alpha\gamma} \left( -\frac{\lambda_{5,5}}{\lambda} + \lambda_{55} \right) \]  \hspace{1cm} (75)

and

\[ E_{\alpha\gamma} = -\frac{1}{2\lambda} H_{\alpha\mu} H_{\gamma\tau} \hat{\gamma}^{\mu\tau}. \]  \hspace{1cm} (76)
Also

\[ Q_{\beta 5} = \frac{3}{2} \left( \frac{\lambda_{\beta 5}}{\lambda} - \frac{\lambda_{\beta} \lambda_{5}}{\lambda^2} \right) \]  

(77)

and

\[ E_{\beta 5} = -\frac{1}{2\lambda} H_{\alpha\beta,\gamma} \dot{g}^{\alpha\gamma}. \]  

(78)

Then

\[ R_{55} = 0 \]  

(79)

\[ Q_{55} = \frac{2\lambda_{55}}{\lambda} - \frac{(\lambda_{5})^2}{\lambda^2} \]  

(80)

and

\[ E_{55} = -\frac{1}{4\lambda^2} H_{\alpha\beta} H_{\mu\tau} \dot{g}^{\alpha\mu} \dot{g}^{\beta\tau} \]  

(81)

Finally, the five scalar curvature can be calculated

\[ R = \frac{\dot{R}}{\lambda}, \]  

(82)

\[ Q = \frac{3}{\lambda^2} \lambda_{,\mu\tau} \dot{g}^{\mu\tau} - \frac{3}{2\lambda^2} \lambda_{,\mu} \lambda_{,\tau} \dot{g}^{\mu\tau} - \frac{4}{\lambda} \lambda_{55} - \frac{(\lambda_{5})^2}{\lambda^2} \]  

(83)

and

\[ E = -\frac{1}{4\lambda^2} H_{\alpha\beta} H_{\mu\tau} \dot{g}^{\alpha\mu} \dot{g}^{\beta\tau} \]  

(84)

These quantities are basic four-space invariants of the metric \( \gamma_{ab} \). The term \( Q \) has the form of a Klein-Gordon equation, including correct electromagnetic effects when retransformed back to the original coordinate system using the inverse of equations (41) and (44). Even with the electromagnetic and gravitational terms intact, this result is entirely similar to the Weyl theory. The five dimensional effects take the place of the non-Riemannian terms.

There are two characteristics of this construction which make it useful. Presumably any interaction which can be described by a congruence has a Weyl like form. And as expected,
the calculation of a four dimensional conformal variation within a five dimensional space generates a reasonable and expected result. The more important point is that whatever structure quantum mechanics has, if it is to be described in a four dimensional space-time, it must satisfy certain covariance properties. This derivation defines some of the combinations of field quantities that can be used while allowing for the known, accepted, covariance of space-time measurements.

VII. INVARIANT KLEIN-GORDON EQUATION

The five dimensional conformal factor $\omega$ generates a different aspect of a quantum system. If, as suggested by the condition of nullity, $R$ is set equal to zero, then equation (33) for $n=5$ is apparently also equivalent to the Klein-Gordon equation. The terms in equation (39) $\partial^2 \omega/({\partial x^\mu})^2$ and $(\partial \omega/\partial x^\mu)^2$ combine, as suggested into a linear equation of the proper form and in this case the terms in $\partial^2 \omega/\partial \tau^2$ and $(\partial \omega/\partial \tau)^2$ can be included. An exponential $\tau$ dependence for $\omega$ is acceptable and provides a means to introduce the mass.

Following section (V), the curvature scalar, $\Theta$, should be set to zero. Conformal waves in $\gamma_{mn}$ are defined by

$$\gamma_{mn} = \omega \dot{\gamma}_{mn}. \quad (85)$$

Where here $\dot{\gamma}_{mn}$ is a representation of local electromagnetic and gravitational effects. The derivation follows reference [25] with $\omega = e^{2\sigma}$. The scalar curvature depends on $\omega$ and the untransformed metric $\dot{\gamma}_{jk}$ according to

$$\Theta = \frac{1}{\omega} \left[ \dot{\Theta} + \frac{4}{\sqrt{\gamma}} \frac{\partial}{\partial x^j} \left( \sqrt{\gamma} \dot{\gamma}^{jk} \frac{\partial \omega}{\partial x^k} \right) + \frac{3\dot{\gamma}^{jk}}{\omega^2} \frac{\partial \omega}{\partial x^j} \frac{\partial \omega}{\partial x^k} \right] \quad (86)$$

As suggested by the calculation of section (V), a rearrangement of terms that are powers of $\omega$ gives with $\Theta = 0$ and $\Psi = \omega^{3/4}$,

$$0 = \dot{\Theta} + \frac{16}{3\Psi \sqrt{\gamma}} \frac{\partial}{\partial x^j} \left( \sqrt{\gamma} \dot{\gamma}^{jk} \frac{\partial \Psi}{\partial x^k} \right). \quad (87)$$

This can be compared to Klein’s paper where it is shown that the appropriate equation is
\[ \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial x^m} \left( \sqrt{\gamma} \gamma^{mn} \frac{\partial \Psi}{\partial x^n} \right) = 0 \] (88)

with

\[ \Psi = \psi(x^\mu) e^{im\tau}. \] (89)

They are the same except for the term \( \dot{\Theta} = R - \frac{1}{4} H_{\mu\nu} H_{\beta\lambda} \dot{g}^{\mu\beta} \dot{g}^{\nu\lambda} \) which is small unless extremely high fields are present. It appears additive to \( m^2 \). In the unusual situation where \( \dot{\Theta} \) may be large, the effect is to cause the particle to propagate as if it had a different rest mass. The gravitational part of \( \dot{\Theta} \) is equal to the local gravitational scalar curvature. As discussed in [3], it is very small on the atomic scale and it is conventional to choose the laboratory coordinates so that it is zero. There may be some extraterrestrial circumstances where this term is important. The electromagnetic component is very small, of order \( \sim 10^{-40} \) for the Klein normalization of \( \dot{g}_{\mu\nu} \). It is an electrodynamic correction to \( \Theta \) that can become important if the electromagnetic potential changes significantly over a Planck length. Such a process could be related to particle creation or other violent effects at high energy [30].

Since the term is additive to the mass, particles in very strong electromagnetic fields will propagate as if their effective rest mass is altered. In this case, because of the sign, the effect is to reduce the rest mass in regions near a localized point charge, allowing for the possibility of solutions having space like trajectories for very short distances. The fundamental concept of space can be retained but the notion of time like motion fails. The uncertainty principle, being derived from the properties of this differential equation, can also fail in regions of high field. In an extended theory, this type of effect may be important for describing the motion of light particles, such as electrons, inside the nucleus. The particle mass for the Klein normalization of \( \dot{g}_{\mu\nu} \) must be a large value numerically. To what extent this affects the possible relationship between the Planck clock and the quantum clock depends on the choice of the remaining field equations.

At this point, many of the questions of inertia discussed in (QG) are resolved. The mass, as an external constant, reproduces quantum diffraction and classical inertia. It is also...
apparent that a quantum clock can be standardized relative to the assumed mass cofactor in the product $m\tau$. The five dimensional space thus combines the concepts of Mach, Newton and Einstein. It is absolute, as Newton would have assumed. It has no overall intrinsic inertial structure as Mach would have assumed. It provides a mathematical substrate for four dimensional covariance as Einstein would have insisted.

As is well known, measurements of the gauge of the observed metric, $\dot{g}_{\mu\nu}$ in separated regions of space-time match up when the separated space-time regions are expanded to overlap. This is automatically accomplished if quantum objects, transported from region to region are used to make local measurements during determination of $\dot{g}_{\mu\nu}$. The quantum behavior references the fifth dimension, which must be assumed common to the entire space-time, and guarantees the length scales.

Because this equation is based on an invariant, it can be used to calculate the results of quantum mechanics in arbitrary gravitational fields. In the classical limit, the Klein-Gordon equation reduces to the Hamilton-Jacobi equation, and the macroscopic manifestations of this field equation will match the known properties of a classical charged particle. The gravitational mass remains positive and unchanged for antiparticles, which should be in agreement with experiment [31]. This limit seems also to agree with the question of coordinate conditions in reference [32].

Once a local metric is established, quantum effects can be derived. For instance, in a general case one could start with $j_\mu$ and $T_{\alpha\beta}$ as classical source currents and get in the classical limit $A_\mu$ and $g_{\mu\nu}$. In other cases, the calculation of fields is not part of the experiment and they are determined by direct local measurements. The assumption of a universal value for the metric $\dot{g}_{\mu\nu}$ can be the basis for a calculation of isolated quantum effects. Either way, the terms in $\dot{g}_{\mu\nu}$ and the vector potential provide a starting point and give a fundamental five dimensional tensor $\gamma_{mn}$. This method might not work if essential quantum interactions occur between the source and test particle. The result should hold for a separated quantum experiment with external classical interactions.

The theory can be applied to diffraction of neutrons [33] or atoms [34] in situations
where only semiclassical theories have previously been used. As the nuclear phase shifts are not part of geometrical theories so far developed, the phenomenological values need to be inserted. The theoretical situation is better for atomic diffraction where, excepting spin, the capabilities of this analysis are adequate. See also reference [35].

Earthbound experiments may not be useful for testing some of the predictions. Large gravitational fields in which gravitational non-linearities are significant would be required to observe differences from the classical relativistic calculations. Significant experiments are difficult to find and an evaluation may be inconclusive until a more complete theory is known. The best type of test might involve more complicated theories that include nuclear or particle properties.

A set of field equations that apply to coherent sources is needed. The simple substitution of coherent for incoherent terms cannot be correct because the coherent effects are not always additive in the classical limit. Second order terms in the source fields are required to reproduce the known dependence of the gravitational potential on the quantum state. The effect is required by the classical principle of equivalence applied to internal state changes of the sources [36]. The gravitational potential of a superimposed beam of electrons and protons will depend on whether they have combined to form hydrogen. The effective mass density depends on the quantum state and hence cannot be calculated classically. The second order terms analogous to those in the Schrödinger or Klein-Gordon equation are required to extract the correction. Some purely gravitational theories [37] have suggestive second order factors.

The problem of coherence is more apparent for photon correlation experiments. If the detectors are treated as classical source currents, the correlation cannot be predicted because there are no absorbing quantum states that can be used to identify the properties of the participating photons. The correlations are in principle still present because the advanced potentials must be used to generate the radiative forces which cause the absorption and emission. The photon correlations however, cannot be observed without accounting for the inherently non-linear quantum effects. For this reason, a correct mathematical theory of
interaction must be sophisticated enough to include quantum electrodynamics in that limit.

Finally, the question of the establishment of a coordinate system from quantum measurements can be addressed. Specifically, the null geodesics of (QG) reduce to the classical geodesics that form the epistemological basis of covariant measurements. They are conceptually sufficient for the construction of a general coordinate system. By choosing experiments in which $\hbar/m$ is effectively small compared to other distances, the simplified motion of classical particles results directly. This is a critically important process because it indicates how a quantum general relativistic theory can reduce to classical general relativity. Moreover, it generates the accepted scheme for constructing the coordinate system of the neutral observer. The probability density trajectories match the classical solutions of the Hamilton-Jacobi equation to lowest order in $\hbar$. In the classical limit, the fixed gauge condition for the vector potential can be relaxed. The magnitude of the wave function is effectively constant and the phase or action can transform with the gauge of the vector potential. The arbitrary motion associated with diffraction and interference is gone because of the assumed simplification of $\psi$. Thus, from the quantum geodesics and the concept of null five-curvature, comes a metaphysical basis for the operational demonstration of space-time metrics in arbitrary coordinate systems.

VIII. DISCUSSION

Some of the probable consequences of the geometrical approach are worth noting. Studies are continuing with regard to interaction theory and constants. Of immediate interest is some set of quantum Einstein-Maxwell equations. It appears that this will use a Klein like ex-

$^1$ The classical limit can also be studied as a limit of wave packet motion. This also works since without the higher order terms, the packets becomes non-dispersive and follow the Hamilton-Jacobi trajectories. The wave packet case is, perhaps more realistic while the DeBroglie-Bohm like trajectory formalism may be more relevant to some situations.
pression for the gravitational constant and that the fine structure constant will be an internal geometric ratio. It is a complicated problem because the interaction mechanism between five dimensional representations of quantum particles needs to be observed from neutral space-time. At least three metrical structures are needed. In addition, the known allowed solutions for two particles without background interactions are restricted \[38\]. Additional distant interacting particles, that participate in the quantum-gravitational-electrodynamic boundary conditions, may be important, at least to generate the equivalent of free fields. The problem is related to the arguments of Renninger \[39\]. It is necessary to include all possible emitting and absorbing particles in the system if normal radiative behavior is to be expected.

Because of the analysis of conformal waves, it is easy to see that the number of theories that can generate a conformal interpretation for quantum effects is limited. Probably in a five dimensional theory, only the four and five dimensional waves are realistic candidates. Since the quantum structure must be introduced integrally, other possibilities may not be available.

The mathematical differences between this theory and the conventional ones may allow new predictions for quantum electrodynamics. Few such effects are expected to be observable in the laboratory. For instance, one could consider a photon correlation experiment in which the photon energies or polarizations are changed by a gravitational field between the emitter and the detector. One would expect an appropriate correlation to be present. The laboratory optical correlation experiments do not differentiate between quantum and geometrical theories. Further experiments of this type seem unnecessary. Prediction of the correlations seems to be accomplished by time symmetric potentials interacting nonlinearly with geometrical quantum states.

A greater concern is the inhomogeneous quantum-Maxwell equations that are required for the calculation of the electrodynamic field. These appear to be similar to the classical Maxwell equations except for the substitution of quantum source currents. This part of the structure is not yet a unified geometrical reality. A definitive resolution of the photon
correlation experiments must await a complete set of source equations. A five dimensional theory would also reasonably include quantum gravitational effects. In the meanwhile, it is necessary to argue physically that the accepted quantum-Maxwell equations are adequate. Nevertheless, it is easy to see that once the form of the electromagnetic interactions is chosen to give the correct results for common electromagnetic experiments, the photon correlations will be predicted correctly.

Because any motion, possibly even non-inertial, can be described by a five-coordinate transformation, it appears that an absolute global five space can be chosen flat for some types of interaction theory. A particular quantum-gravitational-electromagnetic state $(\psi, A_\mu, g_{\mu\nu})$ may then be embedded in some sense. The congruence of trajectories becomes part of a curvilinear coordinate system. This embedment is not the same as the usual general relativistic problem of embedding [40] because only one quantum state is embedded at a time. The conventional approach seeks to embed the entire classical dynamics at each point. Thus it must include all classically allowed initial conditions. The quantum case, as approached by the geometrical method, is much simpler because at each point the congruence has only one direction and magnitude. Each velocity at each given point is part of a different physical problem. For changed boundary or initial conditions, new fields are chosen and a new embedding calculation is to be performed. It is in part because of this simplification that an elementary five dimensional theory of interaction may be possible.

It is expected that the external invariant Klein-Gordon equation applies as a primitive constraint on the motion of the test particle while the internal four dimensional equation is to be applied to the source particles as a condition on the description of the source current. The condition that the source terms have the proper dependence on the wave function means that the two interpretations of $\psi$, as either an electric source current or as a probability density can be identified with each other. It is this equality that allows the definition of the electric charge. For the primitive geometrical Maxwell’s equations, the concept of stream electricity must be used [41] which has the interpretation here of mutually interpenetrating geometrical congruences. The exact relation of the internal and external Klein-Gordon equations must
be part of an interaction theory.

This relationship between currents and particle motion forms a hidden variable theory of the crypto-deterministic type. That is, while it is technically possible to describe particles as moving points on trajectories, the experimental elucidation of those trajectories is not possible. For a true, physically detectable, hidden variable theory, one would imagine that the unity of the congruence is broken and that separate parts of it could interact with different observers. That this is not the case is manifested by having the interacting fields depend on source currents represented only as complete congruences without identified specific trajectories. Other forms of interaction could conceivably destroy the particle quantum coherence. There is no experimental evidence for such a failure of this “stream electricity”. It is appropriate to assume that geometrical theories should have congruence based interactions and remain crypto-deterministic. This is only possible if one rejects the notion of a classical basis for quantum theory. Apparently, the semiclassical radiation theories produce incorrect results because a classical point particle is placed on a calculated trajectory. The source of the field must be the quantum current as a whole.

Thus the trajectories are not required to have a separate existence. They may however be treated as a conceptual aid to the visualization and calculation of congruences. One may also choose to ignore the crypto-determinism and maintain a version of wave-particle duality. The establishment of quantum particle trajectories as real ontological objects is not necessary. Like most fields, they have been invented for mathematical convenience and metaphysical comfort.

The difference between incoherent and coherent sources for electromagnetism should be noted. The experimental inverse square law for point particles appears only in the classical limit when the wave function has been sufficiently contracted. It does not apply to a discrete quantum particle traveling on the congruence. The classical limit must come when the congruence itself is bundled into a small space. In this limit each coherent source term can be treated as one of a collection of incoherent particles. The classical electrodynamics obtains. It is this inverse square law of a well localized packet, that is universally accepted.
The kernel used in quantum electrodynamics is quite different metaphysically and must be applied only to the quantum current to determine the interaction of the geometrical streams. By making this distinction explicit, problems of semi-classical radiation theory are avoided. The construction of a classical inverse square interaction must not be supposed instantaneous, but is the sum of effects, propagated at advanced or retarded times in accord with the requirements of relativistic invariance.

One looks to Kaluza’s theory to supply the quantum source terms that are necessary. It fails in detail because the coupling to the quantum density cannot readily be specified in a theory that is constructed on a classical basis. The Kaluza results should be interpreted as a sort of classical limit. Unfortunately, the extended principle of equivalence plus the concept of implicit quantization means that quantum mechanics cannot readily be separated from this electrodynamics. There is no obvious way to quantize Kaluza’s theory for the same reasons that there is no way to quantize general relativity. One can only hope to derive these as the limit of some higher construction. The standard Einstein-Maxwell construction demonstrates some of the same difficulties. It has no source terms and it may not be possible to give it five-covariant source terms except in the quantum case. As derived, these equations are quantum suppressed and can at most apply to a classical system.

Second quantization is not directly addressed here, but a few comments should indicate the approach. The complicated formalism of equal time commutators is not needed. These have the apparent function of introducing the additional derivatives (to make second order field equations) in a way that can keep them separated during multiparticle interactions. Basically, the second order terms implied by the commutation relations must each be identified with a separate particle. The field quantities of distinct particles must always commute. In a geometrical theory, differential invariants generate these second order terms from the curvature calculations. The construction of multiple metrics that is presented in (QG) replaces the equal time commutation relations. Each particle has an associated metric, and from it, the second order terms are generated from the invariant tensors. The commutation between distinct particle fields is automatic.
If the source density can be integrated into such a theory, the fine structure constant may be implicitly defined. Curvature terms suggest factors of the form $(n_1/4\pi n_2)$ for small integers $n_1, n_2$. Such a value is approximately ten times larger than the measurements. Any final relation awaits a firmer theory. A value that agrees with experiment is probably not to be expected since the issues of spin and weak or strong interactions are not addressed. The process of renormalization may also be important, but this construction takes a very different form in an exact quantum-gravitational theory. Any further consideration of these matters is intended to be taken up at a later time.

While the literature on spin is extensive, there are several studies that may be useful to mention. The problem has been discussed for both five dimensional theories \(^{43}\), and for Weyl theories \(^{44}\). Any of these must yet be adapted to this approach with a fixed gauge and geometrical interpretation. The fundamentality of fermions and their universal description in terms of complex four by four spinors suggests that the Dirac spinors, five in number, may be used to define an extension to five dimensional geometry. It is anticipated that the vector congruence should be replaced by a pentad (funfbein) congruence. A better understanding of spin in this context is required. Because of the change in the nature of spinor representations beyond $n = 5$, further extension may be limited. These issues may also be considered in more detail later.

The justification for associating the conformal machinery of a geometrical theory with quantum mechanics is crucial to the validity of this approach. It is an interesting hypothesis that a wholly geometrical theory of quantum mechanics can be found. So far in the literature, very little has been done, especially for theories that do not derive from a classical model. The conformal factors are studied here because they seem to be the simplest possibility and demonstrate a characteristic quantum behavior.

To evaluate this approach, it is reasonable to ask whether the results of experiments are correctly explained and whether the mathematical system is free of damaging inconsistencies. These results appear to be better than the standard formalism on both accounts. They allow for the discussion of quantum effects in the presence of gravitational fields and they avoid
the serious problems due to an inconsistent use of derivatives. While there is much more to do, the possibility of further predictions of physical importance is a realistic claim.

IX. SUMMARY

A five dimensional covariant theory is studied as a mechanism to discover how quantum fields can be made integral to a system based on classical differential geometry. The basic mechanism is to understand how internal and external conformal factors might be related to the source fields that can produce a given metric.

The non-Riemannian Weyl theories demonstrate how the quantum structure can be interpreted. This calculation, repeated in fixed gauge form, produces a five dimensional derivation. A study of the conformal factors indicate that the wave behavior of the five metric is related to the wave functions of the test and source particles. It becomes possible to view quantum field equations as a set of conditions on the curvature tensors.

Some of the metaphysical problems of combining general relativity and quantum mechanics are resolved. Quantum mechanics has been given a geometrical origin. General relativity has a space time structure that is accurately represented by the trajectories of a quantum particle in the classical limit. Both are to be developed from a more fundamental and primitive geometry. A primitive electrodynamics is also present. Reference to classical theory is avoided.

The introduction of the mass as an extrinsic quantum constant generates a unified theory of inertia that reproduces classical and quantum observations. As an eigenvalue, it produces the correct mass dependence and the correct quantum field equation. The fifth dimension is necessary. The resulting interaction theory appears to be reasonable if applied by using classical equivalence. It is also possible to calculate quantum effects in gravitational fields that are classically generated. The geodesics can be used to construct the perceived classical coordinate system if combined with fundamental quantum clocks.

The combined field system predicts diffraction and interference effects while ascribing
the actual motion to the null geodesics. The Klein-Gordon equation gains a term that may be important for high field densities and for situations where field derivatives are significant over particle scales. Further studies are needed to establish a mathematically closed system of equations. The quantum source currents for the calculation of the fields must yet be defined. Some of the problems of doing this are discussed.

Beyond new understanding, the important result is that there seems to be a way to combine gravitation with other interactions into a covariant theory. It is at least successful in that it shows that the geometrical perspective can be extended to include the basic quantum processes.
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FIGURES

Under a continuous conformal transformation in four dimensions, the measured value of the probability density is modified by the local conformal parameters. The actual number of counts for a fixed region of space-time is invariant because of the adjustment to the numerical volume.
Fig. 1  galehouse q-conformal field theory