Bounded excursion stable gravastars and black holes

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\textbf{Abstract.} Dynamical models of prototype gravastars were constructed in order to study their stability. The models are the Visser–Wiltshire three-layer gravastars, in which an infinitely thin spherical shell of stiff fluid divides the whole spacetime into two regions, where the internal region is de Sitter, and the external one is Schwarzschild. It is found that in some cases the models represent the ‘bounded excursion’ stable gravastars, where the thin shell is oscillating between two finite radii, while in other cases they collapse until the formation of black holes occurs. In the phase space, the region for the ‘bounded excursion’ gravastars is very small in comparison to that of black holes, but not empty. Therefore,
although the possibility of the existence of gravastars cannot be excluded from such dynamical models, our results indicate that, even if gravastars do indeed exist, that does not exclude the possibility of the existence of black holes.

**Keywords:** dark energy theory, semi-analytic modelling, massive black holes, star formation

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### Contents

1. Introduction 2

2. Formation of gravastars from gravitational collapse 5  
    2.1. $m = 0$ and $\Lambda \neq 0$ 6  
    2.2. $\Lambda = 0$ and $m \neq 0$ 7  
    2.3. $m \neq 0$ and $\Lambda \neq 0$ 9  
        2.3.1. $m = 10^{-0.286}, L = 2.5 \times 10^5$ 9  
        2.3.2. $m = 10^{-4}, L = 2.5 \times 10^5$ 9

3. Conclusions 11

Acknowledgments 12

References 12

### 1. Introduction

Black holes are well known and accepted objects not only in the scientific community but also by the general public. This is, for example, attested to partially by the phenomenal success of the well-known film, *Black Holes* (New River Media, 1998), directed by Pappi Corsicato, and by several best selling popular books, such as Kip S Thorne, *Black Holes and Time Warps: Einstein’s Outrageous Legacy* (W W Norton and Company, 1995), and Stephen Hawking, *A Brief History of Time* (Bantam Books publisher, 10th edition, 1998). However, the real detection of black holes strongly demands their detailed explorations, including the forms of gravitational waves emitted by them. In principle, such a detection can never be conclusive and is fundamentally impossible [1], although there are many strong astronomical evidences for their existence.

The simplest example of black holes is the spherically symmetric vacuum solution of the Einstein field equations,

\[
ds^2 = -f(r) \, dt^2 + \frac{1}{f(r)} \, dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2),
\]  

(1.1)
where the function $f(r)$ is given by

$$f(r) = 1 - \frac{2M}{r}, \tag{1.2}$$

in the units where $c = 1 = G$. The metric is singular at both $r = 0$ and $r = r_g \equiv 2M$. However, the natures of these singularities are different. In particular, the one at $r = 0$ is generic and the spacetime curvature diverges there, while the one at $r = r_g$ is a coordinate one and can be made to disappear after proper coordinate transformations. A classical point test particle will freely fall through $r = r_g$ without experiencing anything special. Such a surface is usually called an event horizon. The studies of the properties of such horizons are fundamentally important, and it is found that there may exist a deep connection between gravity and thermodynamics [2]. The discovery of the quantum Hawking radiation [3] and the black hole entropy which is proportional to the area of the event horizon of the black hole [4] further supports this idea. This interesting relation was first manifested when Jacobson derived the Einstein field equations from the first law of thermodynamics (FLT) by assuming the proportionality of the entropy and the horizon area for all local acceleration horizons [5]. For static spherically symmetric and stationary axisymmetric spacetimes, Padmanabhan et al showed that Einstein’s equations at the horizon give rise to the FLT [6, 7]. Such considerations were further generalized to the Lovelock gravity [8]. In [9], on the other hand, the gravitational field equations (GFEs) for the non-linear $f(R)$ theory were derived from the FLT by adding some non-equilibrium corrections.

For the de Sitter space there also exist the Hawking temperature and entropy associated with the cosmological event horizon and its thermodynamic laws [10]. In this space, the event horizon coincides with the apparent horizon. For more general cosmological models, event horizons may not exist, but apparent horizons always do, so it is possible to have a Hawking temperature and entropy associated with apparent horizons. Along these lines, connections between the FLT of apparent horizons and the Friedmann equations in Einstein’s theory with/without the Gauss–Bonnet term, as well as in the Lovelock theory, were found [11]. More recently, it was found that this is also true for the brane world cosmology [12]. In [13], with the help of a new mass-like function, it was shown that the FLT of apparent horizons in equilibrium can be derived from the Friedmann equations in various theories of gravity, including the Einstein, scalar–tensor, non-linear $f(R)$, and Lovelock.

Despite all of these theoretical and observational successes, a number of paradoxical problems regarding black holes also exist [14], which frequently motivate authors to look for other alternatives, in which the endpoints of gravitational collapse are massive stars without horizons. Examples of such models include gravastars [15], Bose superfluid [16], and black stars [17], to name only a few of them. Among these models, gravastars have received particular attention recently [18], partially due to the tight connection between the cosmological constant and a currently accelerating universe [19], although very strict observational constraints on the existence of such stars may exist [20]. In the original model of Mazur and Mottola (MM) [15], gravastars consist of five layers: an internal core $0 < r < r_1$, described by the de Sitter universe, an intermediate thin layer of stiff fluid $r_1 < r < r_2$, an external region $r > r_2$, described by the Schwarzschild solution (1.1) and (1.2), and two infinitely thin shells, appearing, respectively, on the hypersurfaces $r = r_1$ and $r_2$. By properly choosing the free parameters involved, one can show that the two shells can have only tensions but with opposite signs [15]. Visser and Wiltshire (VW)
Figure 1. (a) The Penrose diagram for the Schwarzschild vacuum solution. Region II is the region where $r > a(\tau)$. (b) The Penrose diagram for the de Sitter solution. Region I is the region where $r < a(\tau)$. An infinitely thin shell located at $r = a(\tau)$ connects these two regions to form a dynamical spacetime of a prototype gravastar, as shown in figure 2.

Figure 2. The Penrose diagram for the solution given by equation (1.3). The infinitely thin shell at $r = a(\tau)$ connects the two regions I and II shown in figure 1 to form a spacetime of a prototype gravastar.

argued that such five-layer models can be simplified to three-layer ones [21], in which the two infinitely thin shells and the intermediate region are replaced by one infinitely thin shell, so that the function $f(r)$ in the metric (1.1) is given by

$$f(r) = \begin{cases} 
1 - \frac{2M}{r}, & r > a(\tau), \\
1 - \left(\frac{r}{l}\right)^2, & r < a(\tau), 
\end{cases}$$

(1.3)

where $r = a(\tau)$ is a time-like hypersurface, at which the infinitely thin shell is located, and $\tau$ denotes the proper time of the thin shell. The constant $l \equiv \sqrt{3/\Lambda}$ denotes the de Sitter radius. On the hypersurface $r = a(\tau)$ Israel junction conditions yield

$$\frac{1}{2}a^2 + V(a) = 0,$$

(1.4)

where an overdot denotes the derivative with respect to the proper time $\tau$ of the thin shell. Therefore, in the region $r > a(\tau)$ the spacetime is locally Schwarzschild, while in the region $r < a(\tau)$ it is locally de Sitter. These two different regions are connected through a dynamical infinitely thin shell located at $r = a(\tau)$ to form a new spacetime of the gravastar (cf figures 1 and 2).
To study the dynamics of equation (1.4), one can follow two different approaches: one is to prescribe a potential $V(a)$ and leave the equation of state of the shell as derived, and the other is to prescribe an equation of state of the shell and leave the potential $V(a)$ as derived. VW followed the first approach, and studied in detail the case where

$$V(a_0) = 0, \quad V'(a_0) = 0, \quad V''(a_0) > 0,$$

where a prime denotes the ordinary differentiation with respect to the indicated argument. If and only if there exists such an $a_0$ for which the above conditions are satisfied, the model is said to be stable. Among other things, VW found that there are many equations of state for which the gravastar configurations are stable, while others are not [21]. Carter studied the same problem and found new equations of state for which the gravastar is stable [22], while DeBenedictis et al [23] and Chirenti and Rezzolla [24] investigated the stability of the original model of Mazur and Mottola against axial perturbations, and found that gravastars are stable to these perturbations. Chirenti and Rezzolla also showed that their quasi-normal modes differ from those of a black hole of the same mass, and thus can be used to discern a gravastar from a black hole.

As VW noticed, there is a less stringent notion of stability, the so-called ‘bounded excursion’ models, in which there exist two radii $a_1$ and $a_2$ such that

$$V(a_1) = 0, \quad V'(a_1) \leq 0, \quad V(a_2) = 0, \quad V'(a_2) \geq 0,$$

with $V(a) < 0$ for $a \in (a_1, a_2)$, where $a_2 > a_1$.

In this paper, our purpose is twofold. (a) First, we construct three-layer VW dynamical models, some of which represent ‘bounded excursion’ stable gravastars, and some represent the collapse of a prototype gravastar, where the final fate of the collapse is the formation of a black hole. (b) Second, in the phase space we compare the region of such stable gravastars with the one of black holes, and show explicitly that both of them are non-zero, although the former is much smaller than the latter. The rest of the paper is organized as follows. In section 2 we shall study various cases in which all the possibilities of forming black holes, gravastars, and de Sitter and Minkowski spacetime exist. In section 3 we present our main conclusions.

Before turning to the next section, we note some relevant work. In particular, the gravitational collapse of dark energy in the background of dark matter was studied in [25], and it was found that when only dark energy is present, black holes are never formed. When both of them are present, black holes can be formed, due to the condensation of the dark matter. Similar results were obtained in [26]–[28]. Recently, such studies were further generalized to a homogeneous and isotropic expanding Friedmann–Robertson–Walker universe dominated by dark energy [29].

### 2. Formation of gravastars from gravitational collapse

To keep the ideas of MM as much as possible, we consider the thin shell as consisting of a stiff fluid, $\sigma = -\vartheta$, where $\sigma$ and $\vartheta$ denote, respectively, the surface energy density and
tension of the shell. Then, we find that [21]
\[ \sigma = \sigma_0 \left( \frac{a_0}{a} \right)^4, \]
where \( \sigma_0 \) and \( a_0 \) are integration constants, and have dimensions of surface energy density and length, respectively. It can be shown that the potential appearing in equation (1.4) now can be cast in the form
\[ V(R) = -\frac{1}{2} \left( 1 + \frac{m}{R} + \frac{1}{4R^6} + m^2R^4 + \frac{R^2}{2L^2} - \frac{mR^7}{L^2} + \frac{R^{10}}{4L^4} \right), \]
where
\[ m \equiv \frac{M}{k^{1/3}}, \quad R \equiv \frac{a}{k^{1/3}}, \quad L \equiv \frac{l}{k^{1/3}}, \]
with \( k \equiv 4\pi a_0^4 \sigma_0 \). Therefore, for any given constants \( m \) and \( L \), equation (1.4) uniquely determines the collapse of the prototype gravastar. Depending on the initial value \( R_0 \), the collapse can form either a black hole, or a gravastar, or a Minkowski or de Sitter space. In the last case, the thin shell first collapses to a finite non-zero minimal radius and then expands to infinity. To guarantee that initially the spacetime does not have any kind of horizon, cosmological or event, we must restrict \( R_0 \) to the range
\[ 2m < R_0 < L, \]
and correspondingly \( r_0 \in (r_g, l) \). When \( m = 0 = \Lambda \), the thin shell disappears, and the whole spacetime is Minkowski. So, in the following we shall not consider this case, and begin with the one with \( m = 0 \) and \( \Lambda \neq 0 \).

### 2.1. \( m = 0 \) and \( \Lambda \neq 0 \)

In this case, the spacetime outside the thin shell is flat, and the mass of the shell completely screens the mass of the internal de Sitter spacetime. From equation (2.2) we find that
\[ V(R) = -\frac{1}{2} \left( 1 + \frac{m}{R} + \frac{1}{4R^6} - \frac{R^2}{2L^2} - \frac{R^{10}}{4L^4} \right). \]
Then, it can be shown that the equations \( V(R) = 0 \) and \( V'(R) = 0 \) have the explicit solution
\[ L = L_c \equiv \left[ \frac{5}{3} \left( \frac{1}{2} \right)^{8/3} \right]^{1/2} \simeq 0.9588, \quad R = R_{min} \equiv \left( \frac{5}{3} \right)^{1/3}. \]
For \( L < L_c \) the potential \( V(R) \) is strictly negative as shown in figure 3. As a result, if the star starts to collapse at \( R = R_0 \), it will collapse continuously until \( R = 0 \), whereby a Minkowski spacetime is formed, as shown by the bottom line in figure 4. When \( L = L_c \simeq 0.9588 \), since \( R_0 < L_c \), we can see that, like in the last case, the star will collapse until the center \( R = 0 \) and turns the whole spacetime into a Minkowski one, as shown by the middle line in figure 4. For \( L > L_c \), the potential \( V(R) \) is positive between \( R_1 \) and \( R_2 \), where \( R_{1,2} \) are the two real roots of the equation \( V(R, L > L_c) = 0 \) with \( R_2 > R_1 > 0 \). In this case, if the star starts to collapse with \( R_0 < R_1 \), as can be seen from figure 3, it will collapse to \( R = 0 \), whereby a Minkowski spacetime is finally formed. If it starts to collapse with \( R_0 > R_2 \), it will first collapse to \( R = R_2 \) and then starts to expand until \( R = \infty \), and the whole spacetime is finally de Sitter, as shown by the top line in figure 4.
2.2. $\Lambda = 0$ and $m \neq 0$

In this case, equation (2.2) yields

$$V(R) = \frac{1}{2} \left( 1 - \frac{m}{R} - \frac{1}{4R^6} - m^2 R^4 \right), \quad (2.7)$$
Figure 5. The potential \( V(R) \) for \( \Lambda = 0 \) (or \( L = \infty \)). The top line is for \( m < m_c \simeq 10^{-0.286} \), the middle line is for \( m = m_c \), and the bottom line is for \( m > m_c \).

from which we find that the equations \( V(R) = 0 \) and \( V'(R) = 0 \) have the explicit solution

\[
m = m_c \equiv \frac{3}{4} \left( \frac{4}{5} \right)^{5/3} \approx 10^{-0.286}, \quad R = R_c \equiv \left( \frac{3}{4m_c} \right)^{1/5} = \left( \frac{5}{4} \right)^{1/3}.
\] (2.8)

For \( m > m_c \) the potential \( V(R) \) is strictly negative as shown in figure 5. Then, the collapse always forms black holes, as shown clearly by the bottom line in figure 6. For \( m = m_c \), there are two different possibilities, depending on the choice of the initial radius \( R_0 \). In particular, if the star begins to collapse with \( R_0 > R_c \), the collapse will asymptotically approach the minimal radius \( R_c \). Once it collapses to this point, the shell will stop collapsing and remains there for ever, as can be seen from the middle line in figure 6. However, in this case this point is unstable, and any small perturbations will lead the star either to expand for ever and leave behind a flat spacetime, or to collapse until \( R = 0 \), whereby a Schwarzschild black hole is finally formed. On the other hand, if the star begins to collapse with \( 2m_c < R_0 < R_c \), as shown by figure 5, the star will collapse until a black hole is formed. For \( m < m_c \), the potential \( V(R) \) has a positive maximal, and the equation \( V(R, m < m_c) = 0 \) has two positive roots \( R_{1,2} \) with \( R_2 > R_1 > 0 \). As in the last case, now there are also two possibilities, depending on the choice of the initial radius \( R_0 \). If \( R_0 > R_2 \), the star will first collapse to its minimal radius \( R = R_2 \) and then expand to infinity, whereby a Minkowski spacetime is finally formed, as shown by the top line in figure 6. If \( 2m < R_0 < R_1 \), the star will collapse continuously until \( R = 0 \), and a black hole will be finally formed.
Bounded excursion stable gravastars and black holes

Figure 6. The motion of the shell $R(\tau)$ versus its proper time $\tau$ for $\Lambda = 0$. The top line is for $m < m_c \simeq 10^{-0.286}$, the middle line is for $m = m_c$, and the bottom line is for $m > m_c$.

2.3. $m \neq 0$ and $\Lambda \neq 0$

In this case, from equation (2.2) we find that the equations $V(R) = 0$ and $V'(R) = 0$ have a solution of the form $m = m_c(L)$ for any given $L$. The exact dependence of $m_c$ on $L$ cannot be given explicitly. Instead, in the following we consider some representative cases.

2.3.1. $m = 10^{-0.286}, L = 2.5 \times 10^5$. If we set $m = m_c \simeq 10^{-0.286}$ and gradually turn on the cosmological constant, we find that the potential $V(R)$ becomes completely negative, as shown by figure 7. Then, for any given $R_0$ with $2m < R_0 < L$, the star will always collapse to form a black hole. Comparing it with the case of $m = m_c$ and $\Lambda = 0$, we find that the presence of the cosmological constant makes the collapse more likely to form black holes than gravastars, or any of the other cases.

2.3.2. $m = 10^{-4}, L = 2.5 \times 10^5$. If we keep $L$ fixed, i.e., $L = 2.5 \times 10^5$, and tune $m$ downward, we find that, for $m = 10^{-4}$, the potential takes the shape given by figure 8, from which we can see that $V(R) = 0$ now has four real roots, say, $R_i$, where $R_{i+1} > R_i$. If we choose $R_0 > R_4$, then the star will first collapse to $R = R_4$, and then expand to infinity, whereby a de Sitter space is finally formed. However, if we choose $R_2 < R_0 < R_3$, the collapse will bounce back and forth between $R = R_2$ and $R = R_3$. Such a possibility is shown in figure 9. This is exactly the so-called ‘bounded excursion’ model mentioned in [21], but not studied there or anywhere else. Of course, in a realistic situation, the star will emit both gravitational waves and particles, and the potential will be self-adjusted to produce a minimum at $R = R_{\text{static}}$ where $V(R = R_{\text{static}}) = 0 = V'(R = R_{\text{static}})$ (cf figure 10), whereby a gravastar is finally formed [21].
Figure 7. The potential $V(R)$ for $m = 10^{-0.286}$ and $L = 2.5 \times 10^5$, which is always negative.

Figure 8. The potential $V(R)$ for $m = 10^{-4}$ and $L = 2.5 \times 10^5$.

Although such a shape of potential is very difficult to find in the phase space of $m$ and $L$, its measurement is not zero. So, one cannot completely exclude the possibility of the existence of gravastars. However, we do find that it is easy to find potentials that lead to the formation of black holes. Figures 11 and 12 are for $m = 1.0$ and $L = 3.0$ and for $m = 10^{-4}$ and $L = 3.0$, respectively. From these figures we can see that, properly choosing the initial radius $R_0$, the collapse always forms black holes. In contrast, in these cases gravastars cannot be formed no matter how one tunes $R_0$. On the other hand, we are not able to find values of $m$ and $L$ for which only gravastars can be formed.
3. Conclusions

In this paper, we have studied the problem of the stability of gravastars by constructing dynamical three-layer models of VW [21], which consists of an internal de Sitter space, a dynamical infinitely thin shell of stiff fluid, and an external Schwarzschild spacetime. We have shown explicitly that the final output can be a black hole, a 'bounded excursion' stable gravastar, or a Minkowski or de Sitter spacetime, depending on the total mass $m$ of the system, the cosmological constant $\Lambda$, and the initial position $R_0$ of the dynamical shell. All these possibilities have non-zero measurements in the phase space of $m$, $\Lambda$ and $R_0$, although the region of gravastars is very small in comparison with that of black holes. Therefore, even though the possibility of the existence of gravastars cannot be completely excluded in these dynamical models, our results do indicate that, even if gravastars exist, they do not exclude the possibility of the existence of black holes.
Figure 11. The potential $V(R)$ for $m = 1.0$ and $L = 3.0$.

Figure 12. The potential $V(R)$ for $m = 10^{-4}$ and $L = 3.0$.

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Bounded excursion stable gravastars and black holes

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