The solar neutrino spectrum derived from electron scattering and charged current interactions

G. Fiorentini and F.L. Villante

Dipartimento di Fisica, Università di Ferrara, I-44100, Ferrara (Italy)
Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara, I-44100, Ferrara (Italy)

Abstract

We provide a method for extracting information on the energy spectrum of solar neutrinos directly from the spectrum of scattered electrons. As an example, we apply it to the published Super-Kamiokande data. When combined with data from SNO on charged current interactions this method allows to derive separately the spectra of $\nu_e$ and of $\nu_\mu$ plus $\nu_\tau$. 

Corresponding author
F.L. Villante
Dipartimento di Fisica
Via Paradiso, 12 - 44100 Ferrara (Italy)
Telephone Number: +39 0532781879
Fax Number: +39 0532762057
1 Introduction

As a general rule, important information is contained in the spectrum of the radiation which is being used as a probe of a physical system. This holds for any radiation and thus also for neutrinos emitted from the Sun. Indeed, the reconstruction of the solar neutrino spectrum from experimental data is very interesting, as a way to establish neutrino properties and/or to study the stellar interior.

Solar neutrino experiments provide however indirect information about the neutrino spectrum. As an example, from radiochemical experiments [1, 2, 3], which detect the decay of nuclei produced by neutrino capture over the target atoms, one can derive an average flux of electron neutrinos arriving onto earth, where the average is weighted with the capture cross section and it generally involves neutrinos from different branches of the fusion chain.

Super-Kamiokande (SK) [4] is essentially sensitive to $^{8}$B neutrinos only. It studies Electron Scattering (ES),

$$\nu + e^- \rightarrow \nu + e^-,$$

by measuring the energy distribution of electrons for total energies $E_e > 5.0$ MeV. Electron scattering is being studied also by SNO, see [5], for kinetic energies $T_e > 6.75$ MeV.

ES measurements have given, so far, only indirect information on the distribution of the neutrino energy $E_\nu$. In the usual approach, one starts with a flux $\varphi_a(E_\nu)$, where the index $a$ corresponds to active neutrinos, and evaluates the scattered electron spectrum. If this is consistent (inconsistent) with experimental data then the input $\varphi_a$ is accepted (excluded) for describing the solar neutrino flux arriving onto Earth.

The main purpose of this letter is to provide an alternative method for extracting the energy spectrum of solar neutrinos directly from data on the spectrum of scattered electrons.

The basic idea is very simple. Electrons with total energy $E_e$ (which we assume much larger than $m_e$) are produced by neutrinos with $E_\nu > E_e - m_e/2$. Above this threshold, the cross sections $d\sigma_a/dE_e$ are practically independent of electron energy. Thus the difference of electron spectra centered at two neighbouring energies around $E_e$ gets contribution only from neutrinos with energy around $E_\nu = E_e - m_e/2$. Consequently, the neutrino spectrum can be calculated from the derivative of the measured electron energy spectrum.

First, we shall present this scheme by using simple analytical approximations. Next we shall confirm these estimates by means of a full numerical calculation.

As an example, we shall apply this method to the published SK data [4], see our results in fig. 5. We recommend however that a full analysis be done by the experimental group, since a detailed knowledge of the detector and of the data is important for extracting optimal information.

From ES experiments one derives information on a specific combination of the spec-
trum of \( \nu_e, \varphi_e \), with the spectrum of \( \nu_\mu \) plus \( \nu_\tau, \varphi_{\mu\tau} \):

\[
\varphi_{\text{ES}} = \varphi_e + \beta \varphi_{\mu\tau}
\] (2)

where \( \beta \) is a suitable ratio of \( \nu_\mu \) (\( \nu_\tau \)) to \( \nu_e \) cross sections. An important addition of SNO \cite{3}, by means of the Charged Current (CC) data on deuterium,

\[
\nu_e + d \rightarrow p + p + e^- ,
\] (3)

is the possibility of determining the \( \nu_e \) spectrum. We will show that, by combining ES and CC data, one can determine separately the spectrum of active neutrinos different from \( \nu_e \).

We remind that the comparison of the (energy integrated) electron signal from ES and CC data has already allowed to disentangle the presence of a \( \nu_{\mu\tau} \) component in the electron scattering data \cite{3, 3, 3}. By the method which we are proposing it becomes possible to determine the \( \nu_{\mu\tau} \) energy spectrum separately. This is not only interesting by itself, but it also offers a possibility to find a model-independent signature of sterile neutrinos in case they too are coming from the sun, as we shall comment at the end of the paper.

2 From electron spectrum to the neutrino spectrum

a) A simplified case

In the limit of infinite energy resolution and assuming full efficiency for electron detection, the electron energy spectrum from reaction (1) is given by:

\[
\frac{dN}{dE_e} = N t \int dE_\nu \left[ \varphi_e(E_\nu) \frac{d\sigma_e}{dE_e}(E_\nu, E_e) + \varphi_{\mu\tau}(E_\nu) \frac{d\sigma_{\mu\tau}}{dE_e}(E_\nu, E_e) \right] ,
\] (4)

where \( N \) is the number of target electrons, \( t \) is the measurement time, \( \varphi_a \) are the neutrino fluxes and \( d\sigma_a/dE_e \) are the differential cross sections for active neutrinos \( (a = e, \mu, \tau) \) \cite{3, 3, 3}. For energies much larger than \( m_e \), electrons with total energy \( E_e \) are produced by neutrinos with energy larger than \( E_{\min} = E_e - m_e/2 \). In addition, to a good approximation in the range of measured electron energies, the differential cross sections \( d\sigma_a/dE_e \) are independent of the energy of the scattered electron. In other words, we can write:

\[
\frac{d\sigma_a}{dE_e} = \sigma_a(E_\nu) \theta(E_\nu - E_{\min}) ,
\] (5)

where the factors \( \sigma_a \) are, moreover, weakly dependent on the neutrino energy \( E_\nu \). Therefore, by differentiating both sides of eq. (4) with respect to the electron energy we obtain:

\[
\varphi_e(E_\nu) + \beta \varphi_{\mu\tau}(E_\nu) = - \left( \frac{1}{N t \sigma_e} \right) \frac{d^2N}{dE_e^2} .
\] (6)
The factor $\beta = \sigma_\mu/\sigma_e$ is weakly dependent on $E_\nu$ in the energy of interest to us and the r.h.s. is calculated at:

$$E_e = E_\nu + m_e/2.$$  \hspace{1cm} (7)

Eq. (7), which is our basic result, is the direct information on the neutrino energy spectrum which can be derived from the electron spectrum. We remark that the r.h.s is purely defined in terms of measurable quantities.

Of course, there is no way at this stage to tell which are the separate contributions of $\nu_e$ and $\nu_\mu\tau$. This separation requires an additional information, such as it can be provided by the CC measurement of SNO, see below.

On the other hand equation (7) allows a direct test of oscillation models. Any proposed oscillation solution predicts a definite expression for the produced flux $\varphi(E_\nu)$ and for the oscillation probabilities $P_{ea}(E_\nu)$. In terms of these, the l.h.s of eq. (7) can be immediately calculated as $\varphi \left[ P_{ee} + \beta (P_{e\mu} + P_{e\tau}) \right]$ and compared with the observable quantity on the r.h.s.

b) The effect of the finite energy resolution

In practice, the situation is more complicated, since the experiment has a finite energy resolution. This implies that the observed electron energy $\epsilon_e$ is different from the true electron energy $E_e$. The observed spectrum $S(\epsilon_e)$ is related to the true energy spectrum $dN/dE_e$ by means of the following relation:

$$S(\epsilon_e) = \int dE_e \ r(\epsilon_e, E_e) \frac{dN}{dE_e}$$  \hspace{1cm} (8)

where the resolution function can be taken as a gaussian:

$$r(\epsilon_e, E_e) = \frac{1}{\sqrt{2\pi}\Delta} \exp \left( -\frac{(\epsilon_e - E_e)^2}{2\Delta^2} \right).$$  \hspace{1cm} (9)

By deriving both sides of eq. (8) with respect to the observed energy $\epsilon_e$ we obtain:

$$\frac{dS}{d\epsilon_e} = \int dE_e \ \frac{\partial r(\epsilon_e, E_e)}{\partial \epsilon_e} \frac{dN}{dE_e}.$$  \hspace{1cm} (10)

If we neglect the energy dependence of $\Delta$, from (9) we have $\partial r / \partial \epsilon_e = -\partial r / \partial E_e$. Integration by parts gives:

$$\frac{dS}{d\epsilon_e} = \int dE_e \ r(\epsilon_e, E_e) \frac{d^2N(E_e)}{dE_e^2}.$$  \hspace{1cm} (11)

By using eq. (8) we obtain:

$$\frac{dS}{d\epsilon_e} = -N \ t \sigma_e \ \int dE_\nu \ r(\epsilon_e, E_\nu + m_e/2) \left[ \varphi_e(E_\nu) + \beta \varphi_\mu\tau(E_\nu) \right],$$  \hspace{1cm} (12)

where we have taken advantage of the weak energy dependence of $\sigma_e(E_\nu)$ to take it out from the sign of integration.

Since experimental results are generally presented in terms of the ratios to the SSM prediction, it is convenient to write eq. (12) in a slightly different form.
We note that the electron spectrum predicted by Standard Solar Model (SSM) calculations, \( dS^{(SSM)}/d\epsilon_e \), satisfies an equation similar to (12):

\[
\frac{dS^{(SSM)}}{d\epsilon_e} = -N t \sigma_e \int dE_\nu \, r(\epsilon_e, E_\nu + m_e/2) \varphi^{SSM}(E_\nu) .
\]

(13)

Also, for each energy we can normalize the neutrino fluxes to the SSM prediction, \( \varphi^{SSM} \), by introducing the quantities:

\[
f_a(E_\nu) = \frac{\varphi_a(E_\nu)}{\varphi^{SSM}(E_\nu)} .
\]

(14)

With these definitions, from eqs (12) and (13) we have:

\[
\int dE_\nu \left[ f_e(E_\nu) + \beta f_{\mu\tau}(E_\nu) \right] \rho(\epsilon_e, E_\nu) = \frac{dS/d\epsilon_e}{dS^{(SSM)}/d\epsilon_e}
\]

(15)

where the response function \( \rho(\epsilon_e, E_\nu) \) is given by:

\[
\rho(\epsilon_e, E_\nu) = \frac{\int dE_e \, \frac{\partial r(\epsilon_e, E_e + m_e/2)}{\partial \epsilon_e} \varphi^{SSM}(E_e)}{\int dE_\nu \, r(\epsilon_e, E_\nu + m_e/2) \varphi^{SSM}(E_\nu)} .
\]

(16)

The function \( \rho(\epsilon_e, E_\nu) \) essentially measures the energy resolution which can be attained for determining the neutrino spectrum (or, more precisely, its deviation from the SSM prediction).

Eq. (15), which is our main result, is the extension of (6) for the case of finite energy resolution. It has a natural interpretation: due to the finite energy resolution, the derivative of the observed electron spectrum determines the neutrino spectrum, smeared over the energy resolution.

The relation between neutrino and electron energy, previously given by eq. (7), can now be derived as follows. For a fixed electron energy \( \epsilon_e \), the l.h.s of eq. (15) receives contribution from neutrino energies around \( E_\nu \) such that \( \partial \rho / \partial E_\nu = 0 \). Neglecting again the energy dependence of \( \Delta \) this gives:

\[
\epsilon_e = E_\nu + m_e/2 - \Delta^2 \frac{\partial \ln \varphi^{SSM}}{\partial E_\nu} .
\]

(17)

c) The general case

We remind that equations (13), (14) and (17) have been obtained by neglecting the electron energy dependence of \( d\sigma_a/dE_e \) and of \( \Delta \). If these approximations are released one obtains:

\[
\int dE_\nu \left[ f_e \rho_e + \beta f_{\mu\tau} \rho_\mu \right] = \frac{dS/d\epsilon_e}{dS^{(SSM)}/d\epsilon_e}
\]

(18)

where one has now two response functions:

\[
\rho_a(\epsilon_e, E_\nu) = \frac{\varphi^{SSM}(E_\nu)}{\Psi_a} \int dE_e \, \frac{\partial r(\epsilon_e, E_e)}{\partial \epsilon_e} \frac{d\sigma_a(E_\nu, E_e)}{dE_e} .
\]

(19)
The normalizations $\Psi_a$ are given by:

$$\Psi_a = \int dE\nu \varphi_{SSM}(E_{\nu}) \int dE_e \frac{\partial r(\epsilon_e, E_e)}{\partial \epsilon_e} \frac{d\sigma_a(E_{\nu}, E_e)}{dE_e}.$$  (20)

The factor $\beta$ is expressed as:

$$\beta(\epsilon_e) = \frac{\Psi_\mu}{\Psi_e}. $$  (21)

We can show that for SK eqs. (15-17) are quite accurate. In fact, in the case of SK, the energy resolution can be described by the expression (9) with $\Delta$ given by (11):

$$\Delta = 1.5 \text{ MeV} \sqrt{(E_e - m_e)/10\text{MeV}}.$$  (22)

By using this relation, the exact expression for the differential cross sections $d\sigma_a/dE_e$ and the SSM neutrino flux $\varphi_{SSM}$ one can calculate numerically the response functions $\rho_e$ and $\rho_\mu$ for Super-Kamiokande. In fig. 1 we present our results as a function of neutrino energy, for selected values of $\epsilon_e$. The functions $\rho_e$ and $\rho_\mu$ are bell shaped functions, with a full-width-half maximum of about 2.5 MeV. They are slightly narrower than the electron energy resolution $r$, due to the presence of $\varphi_{SSM}$ in their definition.

One sees that, a part for the smallest electron energies, the exact response functions $\rho_e$ and $\rho_\mu$ are close to the approximate expression $\rho$ given by eq.(16). For our purposes we can thus safely assume:

$$\rho_e(\epsilon_e, E_{\nu}) = \rho_\mu(\epsilon_e, E_{\nu}) = \rho(\epsilon_e, E_{\nu}).$$  (23)

In fig. 2 we show the relationship between neutrino and electron energies, calculated numerically by solving $\partial \rho_a/\partial E_{\nu} = 0$. One sees that the approximate solution given by eq. (17) is quite accurate.

Finally, in fig. 3 we present an estimate for $\beta$, calculated according to eq.(21), as a function of the electron energy. As the energy increases, it becomes approximatively constant, $\beta \simeq 0.15$.

### 3 Extraction of the $\nu$ energy spectrum from SK data

As an example, we outline here a numerical procedure for extracting physical information on $f_\alpha(E_{\nu}) = \varphi_\alpha/\varphi_{SSM}$ from the published SK data [1] by means of eqs. (15-17).

*We remark that the response functions depend only on the shape of the neutrino spectrum. SK is essentially sensitive to $^8\text{B}$ neutrinos only, with a small contribution from $\text{hep}$ neutrinos. We use the $^8\text{B}$ neutrino spectrum given in [10], and the ratio between $^8\text{B}$ and $\text{hep}$ neutrino flux predicted by [12]. Variations of the $\text{hep}$ neutrino flux within the current phenomenological limits do not affect our results.*
We remind that SK measures the electron spectrum $S(e_e)$ for electron energies in the range 5-20 MeV, see fig. [4]. Data are grouped into 18 bins, each 0.5 MeV wide, covering the range 5-14 MeV, with an additional bin extending from 14 to 20 MeV. The signal in the $i$-th bin, $S_i$, is associated with an error $\Delta S_i$, which we take (for the moment) as the sum in quadrature of the statistical and uncorrelated systematical errors given in [4]. Data correspond to 1258 days of exposure [4]. All quantities shown in fig. 4 are normalized to the predictions $S_i^{(SSM)}$ corresponding to the Standard Solar Model (SSM) of [12]†.

In order to evaluate the r.h.s. of eq.(15) and the associated error, it is convenient to divide the energy range probed by SK in a few intervals, each with a width comparable to the neutrino energy resolution, since the experiment is anyhow unsensitive to spectral deformations on smaller scales. On the other hand, by considering intervals larger than the original SK binning it is possible to decrease statistical fluctuations in the evaluation of the spectrum derivative.

We have considered five intervals, with energies in MeV between (5-7), (7-9), (9-11), (11-13) and (13-20) respectively. Inside each interval $S(e_e)$ and $S^{(SSM)}(e_e)$ can be approximated by straight lines and their slopes are determined by least squares fitting:

$$\frac{dS}{de_e} = \frac{\langle S(e_e) \rangle - \langle S \rangle \langle e_e \rangle}{\langle e_e^2 \rangle - \langle e_e \rangle^2}$$

and similarly for $dS^{(SSM)}/de_e$. In the average $\langle \rangle$ each point is weighted according to the experimental error $\Delta S_i$:

$$\langle X \rangle = \frac{\sum_i X_i / \Delta S_i^2}{\sum_i 1 / \Delta S_i^2}.$$  \hspace{1cm} (25)

For each interval we get

$$\frac{dS/d\epsilon_e}{dS^{(SSM)}/d\epsilon_e} = \frac{\langle S(e_e) \rangle - \langle S \rangle \langle e_e \rangle}{\langle S^{(SSM)}(e_e) \rangle - \langle S^{(SSM)} \rangle \langle e_e \rangle}$$

with an associated error:

$$\delta = \frac{[\langle e_e^2 \rangle - \langle e_e \rangle^2]^{1/2}}{\langle S^{(SSM)}(e_e) \rangle - \langle S^{(SSM)} \rangle \langle e_e \rangle} \cdot \sqrt{\frac{1}{\sum_i 1 / \Delta S_i^2}}.$$  \hspace{1cm} (27)

At this point we can calculate, by means of eqs. (15-17) the deviations of the neutrino spectrum from the SSM predictions by using the SK data.

Our results are presented in fig. 5 as a function of the neutrino energy. The quantity on the vertical axis represents $f_e + \beta f_{\mu\tau}$ averaged with the response function $\rho$, i.e.:

$$F_{ES}(E_\nu) = \int dE'_\nu \left[ f_e(E'_\nu) + \beta f_{\mu\tau}(E'_\nu) \right] \rho(\epsilon_e, E'_\nu)$$

† The SSM of [12] predicts a $^8$B neutrino fluxes $\Phi_B = 5.05 \cdot 10^6$ cm$^{-2}$ s$^{-1}$. Here and in the following, we use this value as a (convenient) normalization factor. We remark however that our results do not depend at all on solar models.
where $\epsilon_e(E_\nu)$ is given by eq. (17) and (22). The horizontal bar corresponds to the neutrino energy resolution calculated as the full width half maximum of the response function $\rho(\epsilon_e, E_\nu)$.

Concerning errors, the inner vertical bar takes into account statistical and energy uncorrelated systematical errors. The outer bar also accounts for energy correlated systematical errors, as given in [4]. Their effect has been evaluated by a simultaneous up and down shift of all the experimental points.

We remark that from SK we have been able to derive only a specific combination of $\nu_e$ and $\nu_{\mu\tau}$ fluxes, given by eq.(28). For extracting the individual contributions of $\nu_e$ and $\nu_{\mu\tau}$, i.e.: 

$$F_a(E_\nu) = \int dE'_\nu f_a(E'_\nu) \rho(\epsilon_e, E'_\nu) \quad (a = e, \mu, \tau)$$

(29)

additional information are needed.

4 The information from $\nu_e + d \rightarrow p + p + e^-$

SNO has recently presented [5] the energy spectrum of electrons from reaction (3). We show here that these data, which provide a direct determination of the electron neutrino spectrum, can be combined with ES data for determining the spectrum of $\nu_\mu$ plus $\nu_\tau$.

The SNO results are shown in the lower panel of fig. 4 as a function of the observed electron kinetic energy $T_e = \epsilon_e - m_e$. Data are grouped in 11 bins covering the energy range $T_e = 6.75 - 13$ MeV. The signal in each bin, $C_i$, is normalized to the predictions, $C_{SSM}^i$, corresponding to the SSM of [12] and the error bars take into account only statistical errors.

The measured electron spectrum $C(\epsilon_e)$ from reaction (3), normalized to the SSM expectation $C_{SSM}(\epsilon_e)$, is related to the the electron neutrino spectrum by:

$$\frac{C}{C_{SSM}} = \int dE_\nu f_e(E_\nu) \rho_{cc}(\epsilon_e, E_\nu)$$

(30)

where $\rho_{cc}$ is a suitable response function, defined in terms of the cross section for reaction (3), $d\sigma_{cc}/dE_e$ [3], and of the detector resolution function $r_{SNO}(\epsilon_e, E_e)$:

$$\rho_{cc}(\epsilon_e, E_\nu) = \frac{\varphi_{SSM}(E_\nu)}{\Psi_{cc}} \int dE_e r_{SNO}(\epsilon_e, E_e) \frac{d\sigma_{cc}(E_\nu, E_e)}{dE_e}.$$ 

(31)

The normalization factor $\Psi_{cc}$ is given by:

$$\Psi_{cc} = \int dE_\nu \varphi_{SSM}(E_\nu) \int dE_e r_{SNO}(\epsilon_e, E_e) \frac{d\sigma_{cc}(E_\nu, E_e)}{dE_e}.$$ 

(32)

The behaviour of the SNO response function is shown in fig. 6 as a function of neutrino energy, for representative $\epsilon_e$ values. One sees that for each measured electron energy $\epsilon_e$ the

\[^{\dagger}\] The SNO resolution function can be described by rel. (9) with $\Delta = (-0.4620 + 0.5470 \sqrt{E_e} + 0.008722 E_e) \text{ MeV}$ [5]. One can easily check that $r_{SNO} \simeq r$.
response function is peaked at a specific neutrino energy $E_\nu$. The rates $C_i/C_i^{SSM}$ can thus be interpreted as a determination of the deviations of the electron neutrino spectrum at $E_\nu$ averaged over the response function $\rho_{cc}$, i.e.:

$$F_{SNO}(E_\nu) = \int dE'_\nu f_e(E'_\nu) \rho_{cc}(\epsilon_e, E'_\nu)$$

(33)

where the numerical relation $\epsilon_e(E_\nu)$ is shown in fig. 2.

We remark that the width of the SNO response function, $\rho_{cc}$, is mainly determined by the SNO resolution $r_{SNO}$. In fact, if one neglects the recoil of the two protons in the final state, there is a fixed relation between the neutrino and electron energies, i.e. $d\sigma_{cc}/dE_e \propto \delta(E_e - E_\nu - m_d + 2m_p)$. As a consequence one expects, roughly:

$$\rho_{cc}(\epsilon_e, E_\nu) \propto \varphi^{SSM}(E_\nu) r_{SNO}(\epsilon_e, E_\nu + m_d - 2m_p).$$

(34)

We note that the structure of the previous relation is similar to that of rel. (16), which defines the response function $\rho$ associated to the SK spectrum derivative. If one considers that SK and SNO energy resolutions are almost equal, this suggests that SK and SNO response functions can be equalized with a proper choice of the detection energies.

The qualitative argument given above is clearly oversimplified. However, the possibility to equalize the SK and SNO response functions can be checked numerically. The results are shown in fig. 4. One sees that the response function $\rho_e$ and $\rho_{cc}$, calculated numerically according to rel. (19) and (31) are almost equal, i.e.

$$\rho_{cc}(\epsilon_e, E_\nu) = \rho_{cc}(\epsilon_{SNO}, E_\nu)$$

(35)

if one chooses the energies as follows:

$$\epsilon_{SNO} = 0.975 \epsilon_{SK} - 2.50 \text{ MeV}.$$  

(36)

From the equalities (23) and (35), it follows that one can identify the SNO charged current spectrum with the $\nu_e$ contribution to the SK spectrum derivative, i.e.:

$$F_e(E_\nu) = F_{SNO}(E_\nu).$$

(37)

As a consequence, SK and SNO-CC data can be combined to determine:

$$F_{\mu\tau}(E_\nu) = \frac{1}{\beta} (F_{ES}(E_\nu) - F_{SNO}(E_\nu))$$

(38)

and:

$$F_e(E_\nu) + F_{\mu\tau}(E_\nu) = \frac{1}{\beta} [F_{ES}(E_\nu) - (1 - \beta) F_{SNO}(E_\nu)].$$

(39)

The results obtained are shown in fig. 8 where, for convenience, the SNO-CC signals have been grouped into larger intervals so as to reduce statistical fluctuations and to take advantage of the SK-SNO correspondence, eq. (36).
We remark that, if there are no sterile neutrinos, then the sum of the spectra of active neutrinos has the same shape as the $^8$B spectrum in the laboratory, i.e.

$$\varphi_e(E_\nu) + \varphi_{\mu\tau}(E_\nu) = k \varphi^{SSM}(E_\nu)$$

where $k$ is a normalization constant, i.e. it does not depend on $E_\nu$. Violations of this sum rule, would clearly imply sterile neutrinos. Such deviations are not shown in the data, which are well consistent with a constant, see the lowest panel of fig.8.

5 Concluding remarks

We have provided a method for extracting information on the neutrino spectrum from the spectrum of scattered electrons, summarized in eq. (15-17). We summarize here a few points, concerning the possible applications and developments of our approach:

1) As an example we have applied our approach to the published SK data, see fig.5. We suggest that the analysis is performed by the experimental group, since a detailed knowledge of the detector is important for extracting optimal information.

2) We have shown that the information obtained by using our method can be directly combined with the information provided by SNO, so as to determine the spectrum of $\nu_\mu$ plus $\nu_\tau$ as well as the total spectrum of active neutrinos. Our results, obtained from eqs. (38) and (39), are shown in fig.8.

3) We remark that the SK-SNO comparison potentially allows for a model independent signature for sterile neutrinos. If there are no sterile neutrinos, in fact, the sum of the spectra of active neutrinos has the same shape as the $^8$B spectrum in the laboratory, i.e.

$$\varphi_e(E_\nu) + \varphi_{\mu\tau}(E_\nu) = k \varphi^{SSM}(E_\nu),$$

where $k$ is a normalization constant. Violations of this sum rule, would clearly imply sterile neutrinos. Such deviations are not shown in the data.

Acknowledgements

G.F. is grateful to the CERN theory division for generous hospitality.

References

[1] B. Cleveland et al., Ap. J. 496, 505 (1998).

[2] GNO collaboration Phys. Lett. B 490 (2000) 16-26.
[3] SAGE collaboration, Phys.Rev.Lett. 83 (1999) 4686-4689

[4] S. Fukuda et al. [SuperKamiokande Collaboration], Phys. Rev. Lett. 86, 5651 (2001) [arXiv:hep-ex/0103032].

[5] Q. R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 87, 071301 (2001) [arXiv:nucl-ex/0106013].

[6] F. L. Villante, G. Fiorentini and E. Lisi, Phys. Rev. D 59, 013006 (1999) [arXiv:hep-ph/9807360];
G. L. Fogli, E. Lisi, A. Palazzo and F. L. Villante, Phys. Rev. D 63 (2001) 113016 [arXiv:hep-ph/0102288];
G. Fiorentini, F. L. Villante and B. Ricci, arXiv:hep-ph/0109275.

[7] G. L. Fogli, E. Lisi, D. Montanino and A. Palazzo, Phys. Rev. D 64, 093007 (2001) [arXiv:hep-ph/0106247];
C. Giunti, arXiv:hep-ph/0107310;
V. Berezinsky, arXiv:hep-ph/0108166.

[8] J. N. Bahcall, Rev. Mod. Phys. 59, 505 (1987);

[9] We adopt the $\nu - e$ cross section calculations reported in J. N. Bahcall, M. Kamionkowski and A. Sirlin, Phys. Rev. D 51, 6146 (1995) [arXiv:astro-ph/9502003]. See also M. Passera, Phys. Rev. D 64, 113002 (2001) [arXiv:hep-ph/0011190], for recent refined $O(\alpha)$ QED corrections to $\nu - e$ scattering.

[10] C. E. Ortiz, A. Garcia, R. A. Waltz, M. Bhattacharya and A. K. Komives, Phys. Rev. Lett. 85 (2000) 2909 [arXiv:nucl-ex/0003006];

[11] Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 82, 2430 (1999) [arXiv:hep-ex/9812011]. For a parametrisation see J. N. Bahcall, P. I. Krastev and E. Lisi, Phys. Rev. C 55, 494 (1997) [arXiv:nucl-ex/9610010] and B. Faid, G. L. Fogli, E. Lisi and D. Montanino, Phys. Rev. D 55, 1353 (1997) [arXiv:hep-ph/9608311].

[12] J. N. Bahcall, M. H. Pinsonneault and S. Basu, Astrophys. J. 555, 990 (2001) [arXiv:astro-ph/0010346].

[13] K. Kubodera and S. Nozawa, Int. J. Mod. Phys. E 3, 101 (1994) [arXiv:nucl-th/9310014]. We use the computer programs described in J. N. Bahcall and E. Lisi, Phys. Rev. D 54, 5417 (1996) [arXiv:hep-ph/9607433] and available at URL http://www.sns.ias.edu/jnt (Neutrino Software and Data). See also S. Nakamura, T. Sato, V. Gudkov and K. Kubodera, Phys. Rev. C 63, 034617 (2001) [arXiv:nucl-th/0009012] and J. F. Beacom and S. J. Parke, Phys. Rev. D 64, 091302 (2001) [arXiv:hep-ph/0106128] for recent refined calculation of $\nu - d$ cross sections.
Figure 1: Response functions of Super-Kamiokande as a function of neutrino energy $E_\nu$, for selected values of the observed electron energy $\epsilon_e$. The exact response functions $\rho_e$ and $\rho_\mu$, are calculated according to eq.(19), whereas $\rho$ is the approximation given by eq.(18).
Figure 2: The relation between neutrino and electron energies for SK and SNO detectors. The upper lines refer to SK. They are obtained from numerical calculation (solid) and from the approximate relation ([7]) (dot-dashed). The dashed line refers to SNO.
Figure 3: The function $\beta$ from eq. (21).
Figure 4: Upper panel: The electron energy spectrum measured by SK [4] normalized to the SSM prediction [12]. Lower panel: The electron energy spectrum measured by charged current reaction (3) at SNO [5] normalized to the SSM prediction [12].
Figure 5: Deviations of the neutrino spectrum from the SSM prediction, as a function of neutrino energy $E_{\nu}$, from SK data. The horizontal bar is the neutrino energy resolution. The inner vertical bar denotes the uncorrelated error, the outer bar includes the effect of energy correlated systematical errors.
Figure 6: The SNO charged current response function $\rho_{cc}$, eq. (31), as a function of neutrino energy $E_\nu$, for selected values of the observed electron energy $\epsilon_e$. 

Figure 6: The SNO charged current response function $\rho_{cc}$, eq. (31), as a function of neutrino energy $E_\nu$, for selected values of the observed electron energy $\epsilon_e$. 

\[ \epsilon_e = 5 \text{ MeV} \]
Figure 7:  
a) The SK response function $\rho_e$, eq. (19), for the indicated electron energies $\epsilon_{SK}$ (dot-dashed lines). 
b) The SNO response function $\rho_{cc}$, eq. (31), for the electron energies $\epsilon_{SNO}$ given by eq. (36) (solid lines).
Figure 8: Deviations of the neutrino spectra from the SSM prediction as a function of the neutrino energy $E_{\nu}$. The vertical bars take into account only statistical errors. In the upper panel, the full circles correspond to $F_{\text{ES}}$, while the open circles correspond to $F_{\text{SNO}}$. 