Super-Penrose process for extremal rotating neutral white holes

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Abstract
We consider collision of two particles 1 and 2 near the horizon of the extremal rotating axially symmetric neutral generic black hole producing particles 3 and 4. We discuss the scenario in which both particles 3 and 4 fall into a black hole and move in a white hole region. If particle 1 is fine-tuned, the energy $E_{c.m.}$ in the centre of mass grows unbounded (the Bañados-Silk-West effect). Then, particle 3 can, in principle, reach a flat infinity in another universe. If not only $E_{c.m.}$ but also the corresponding Killing energy $E$ is unbounded, this gives a so-called super-Penrose process (SPP). We show that the SPP is indeed possible. Thus white holes turn out to be potential sources of high energy fluxes that transfers from one universe to another. This generalizes recent observations made by Patil and Harada for the Kerr metric. We analyze two different regimes of the process on different scales.

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1 Introduction

During last decade a lot of work has been made for investigation of properties of high energy processes near black holes. This was stimulated by the paper [1], where it was found that collision of two particles near rapidly rotating black holes can lead to unbounded energies $E_{c.m.}$ in the centre of mass frame. This was called the Bañados-Silk-West (BSW) effect, after the authors’ names. After its publication, it turned out, that there are also earlier works [2,3] in which near-horizon particle collisions in the Kerr metric were investigated. Meanwhile, a typical process considered there, includes head-on collision between two arbitrary particles, where particle 1 arrives from infinity while particle 2 comes from the horizon (see eq. 2.57 of [3]). But as far as particle 2 is concerned, this is nothing else than a typical behavior of a particle near a white hole. If such a region is allowed in the complete space-time, the effect of unbounded $E_{c.m.}$ for head-on collisions exists even in the Schwarzschild metric [4,5]. Thus white holes can be an alternative to black ones as a source of high energy collisions.

More important question is whether it is possible to gain not only unbounded $E_{c.m.}$ but also unbounded conserved Killing energies $E$ since it is the latter quantity which can be measured in the Earth laboratory, at least in principle. The collisions in the Schwarzschild background are useless for this purpose since energy cannot be extracted at all. For such an extraction, the existence of negative energies and ergoregion are required that makes it possible the Penrose process [6] or its collisional analogue [7]. If the energy gain is unbounded, this is called the super-Penrose process (SPP). For black holes, the energy gain is finite, so the SPP is impossible for them (see [8] and references therein).

In this context, there is a scenario with participation of white holes, different from those in [2–5]. Now, both particles collide near the black hole horizon in “our” part of Universe but afterwards the products of reaction leave it. Passing though the horizon, they appear inside a white whole region and, eventually, transfer energy to another universe. Or, vice verse, collision in another universe can give rise to high energy in our one. If such a process is possible, this would give astrophysical realization of high energy transfer with white holes as a source that was suggested earlier [9,10].

The concrete process of this kind in the Kerr background was considered recently in [11]. The authors showed that the conserved energy of produced particles can be as large as one like. In other words, the SPP is possible. Our aim is to extend consideration to generic rotating axially symmetric stationary white holes. In doing so, we exploit the approach that, in our view, is simpler and was already used for examination of the energy extraction from generic black holes of the aforementioned type including the Kerr metric [12,13]. In particular, we do not use transformation between the three frames (center of mass, locally non-rotating and stationary ones) and work in the original frame.

Although there are reasons to believe that white holes are unstable [14] (see also Sec. 15 of [15]), motivation for consideration of such objects stems from different roots. (i) High energy process, if they are confirmed, can themselves contribute to the instability of white holes, so they are important for elucidation of the fate of such objects. (ii) The complete theory of the BSW effect should take into consideration all possible configurations and scenarios, at least for better understanding the phenomenon.
We use the system of units in which the fundamental constants $G = c = 1$.

### 2 Basic equations

Let us consider the metric

$$ds^2 = -N^2 dt^2 + \frac{dr^2}{A} + g_\phi (d\phi - \omega dt)^2 + g_\theta d\theta^2,$$  

(1)

where $g_\phi \equiv g_{\phi\phi}$, $g_\theta \equiv g_{\phi\phi}$, all coefficients do not depend on $t$ and $\phi$. Before turning to the analysis of the collisions between two particles in the background of a white hole, we describe main features of motion inherent to an individual particle.

To simplify formulas, we assume that in the equatorial plane $A = N^2$. Otherwise, we can always achieve this equality by redefining the radial coordinate according to $r \rightarrow \tilde{r}$, where

$$\frac{dr}{\sqrt{A}} = \frac{d\tilde{r}}{N},$$  

(2)

so

$$\tilde{r} = \int r \frac{dr'}{\sqrt{A}}.$$  

(3)

In general, for an arbitrary $\theta$, this transformation does not work since $d\tilde{r}$ would not be a total differential, if $A$ and $N$ depend on $\theta$. However, for our purposes (for motion within the equatorial plane, so $\theta = \frac{\pi}{2}$ is fixed), this is a quite legitimate operation. It is valid for any metric of the type (1) including the Kerr one.

For a given energy $E$, angular momentum $L$ and mass $m$ the equation of motion in the equatorial plane read

$$m\dot{t} = \frac{X}{N^2},$$  

(4)

$$X = E - \omega L,$$  

(5)

$$m\dot{\phi} = \frac{L}{g_\phi},$$  

(6)

$$m\dot{r} = \sigma P, \sigma = \pm 1,$$  

(7)

$$P = \sqrt{X^2 - \tilde{m}^2 N^2},$$  

(8)

$$\tilde{m}^2 = m^2 + \frac{L^2}{g_\phi},$$  

(9)

dot denotes differentiation with respect to the proper time $\tau$. The forward-in-time condition requires

$$X \geq 0.$$  

(10)

In what follows, we will use the standard classification of particles. If $X_H = 0$, a particle is called critical. If $X_H = O(1)$, it is called usual. If $X_H = O(N_c)$, it is called near-critical. Here, subscripts “H” and “c” refer to the quantities calculated on the horizon and the point of collision, respectively.
For the near-critical particle, we use presentation

$$L = \frac{E}{\omega_H}(1 + \delta)$$  \hspace{1cm} (11)

exploited in [13]. Here,

$$\delta = C_1 N_c$$  \hspace{1cm} (12)

is a small quantity for collisions near the horizon, $C_1 = O(1)$ is a constant.

Near the horizon, we assume the Taylor expansion that for the extremal case reads [16]

$$\omega = \omega_H - B_1 N + O(N^2).$$  \hspace{1cm} (13)

Then, we have the following approximate expressions there.

The critical particle:

$$X = \frac{b}{h} E N + O(N^2),$$  \hspace{1cm} (14)

$$P = N \sqrt{E^2 \left( \frac{b^2 - 1}{h^2} \right) - \frac{1}{h^2} - m^2 + O(N^2)}.\hspace{1cm} (15)$$

A usual particle:

$$X = X_H + B_1 LN + O(N^2),$$  \hspace{1cm} (16)

$$X_H = E - \omega_H L,$$  \hspace{1cm} (17)

$$P = X + O(N^2).$$  \hspace{1cm} (18)

The near-critical particle:

$$X = E \left( \frac{b}{h} - C_1 \right) N + O(N^2)$$  \hspace{1cm} (19)

$$P = N \sqrt{E^2 \left[ \left( \frac{b}{h} - C_1 \right)^2 - \frac{1}{h^2} \right] - m^2 + O(N^2)}.\hspace{1cm} (20)$$

We introduced notations $b = B_1 \sqrt{g_H}, h = \omega_H \sqrt{g_H}$. Here, for shortness, we also use notation $g = g_\phi$, where $g_\phi$ is defined in (1).

To give an example, we list the concrete expressions for these relevant characteristic for the physically relevant case of the extremal Kerr–Newman metric:

$$g_H = \left( \frac{M^2 + a^2}{M^2} \right)^2,$$  \hspace{1cm} (21)

$$B_1 = \frac{2a}{M^2 + a^2},$$  \hspace{1cm} (22)

$$\omega_H = \frac{a}{M^2 + a^2}.\hspace{1cm} (23)$$
Here, $a$ is the standard parameter of the Kerr–Newman metric that characterizes its angular momentum, $M$ being the mass. It is interesting that the electric charge $Q$ drops out from the explicit expressions for $b$ and $h$. If $Q = 0$, we our extremal metric transforms to the Kerr one with $M = a$ and the values (24) return to $b = 2$ and $h = 1$ in accordance with eq. (46) of [13].

Meanwhile, below we operate with relevant quantities in a general form, without specifying the metric. This is justified by the fact that, as we will see, in the cases under discussion there is no concrete upper bound on the maximum possible energy, this result being model-independent.

It is worth noting that the coordinates in (1) generalize the Boyer–Lindquist ones for the Kerr metric. It is known that such coordinates do not cover the whole space-time. Therefore, one is led to introduce an infinite set of different coordinate patches, the space-time structure includes an infinite set of black hole and white hole regions. For the, say, extremal Kerr–Newman metric for the equatorial plane the Carter–Penrose diagram is similar to that for the Reissner–Nordström metric and is represented on Fig. 1 (see, e.g. detailed description of these metrics in [17], especially Ch. 11). We schematically showed a trajectory of a test particle Fig. 1. The concrete description of motion inside the horizon was done in [11] with the help of light-like coordinates for the Kerr metric. Although such details are of interest on their own right, for our goal (to elucidate the absence or presence of the upper bound on $E$) they are not necessary, so we use more simple and straightforward approach. It is based on already elaborated scheme exploited for the analysis of collisions in the black hole background [13,18].

### 3 Scenario of collision

In this section, we give brief set-up for the description of the process under consideration. We assume that particles 1 and 2 collide producing particles 3 and 4. We want to elucidate, whether the resulting energy in the center of mass frame $E_{\text{c.m.}}$ can grow unbounded, if we take into account the processes in the white hole region. Before elucidating this issue, we (i) describe basics of analysis of collision, (ii) possible scenarios of collision that can be realized near the horizon Afterwards, we discuss, which types of scenario correspond to the black hole region and which ones are relevant for the white hole one.

The general analysis of collisions relies on the restrictions that come from the conservation laws. We assume that such laws are valid in the point of collision. This includes the conservation laws for the energy and angular momentum:

\[
E_0 \equiv E_1 + E_2 = E_3 + E_4, \quad (25)
\]
\[
L_0 = L_1 + L_2 = L_3 + L_4. \quad (26)
\]
It follows from (25), (26) that
\[ X_0 = X_1 + X_2 = X_3 + X_4. \]  
(27)

There is also the conservation law for the radial momentum:
\[ \sigma_1 P_1 + \sigma_2 P_2 = \sigma_3 P_3 + \sigma_4 P_4. \]  
(28)

We assume that particles 1 and 2 fall from infinity, so \( \sigma_1 = \sigma_2 = -1 \). We are interested in high energy processes in which \( E_{c.m.} \) is unbounded since this is the necessary condition for \( E \) to be unbounded as well [19,20]. To this end, we choose particle 1 to be the critical, particle 2 being usual since this gives rise to the unbounded \( E_{c.m.} \) [1,21]. Then, one of particles (say, 3) is near-critical and the other one (4) is usual [12,13]. All possible scenarios can be described by two parameters—the sign of \( C_1 \) in (12) and the value of \( \sigma_3 \) immediately after collision (OUT for \( \sigma_3 = +1 \) and IN for \( \sigma_3 = -1 \)). As a result, we have 4 scenarios OUT+, OUT−, IN+, IN−.

The first three were already analyzed in [13]. In scenarios OUT+ and OUT− particle 3 after collision escapes immediately to infinity. In scenario IN+ particle 3 continues to move inward after collision, bounces back from the potential barrier and also returns to infinity. It turned out [12,13] that the later scenario is especially effective for the energy extraction. As far as IN− is concerned, both particles do not encounter a potential barrier and, therefore, fall into a black hole. For this reason, scenario IN−...
was rejected in [13] since no energy returns to infinity. However, now it is just this scenario which we focus on. It corresponds to high energy propagation in the white hole region (see below). Thus we have $\sigma_3 = \sigma_4 = -1$. We must analyze the process under discussion for $N_c \to 0$ on the basic of the conservation law (28). In doing so, we follow the lines of Ref. [13] applying the corresponding approach to the case that was not considered there.

4 Lower bounds on energy

If we collect the terms of the zeroth and first order in $N_c$ and take into account the approximate expressions (14)–(20), we obtain

$$ F = - \sqrt{E_3^2 \left[ \left( \frac{b}{h} - C_1 \right)^2 - \frac{1}{h^2} \right]} - m_3^2, \quad (29) $$

where

$$ F \equiv A + E_3 \left( C_1 - \frac{b}{h} \right), \quad (30) $$

$$ A = \frac{E_1 b - \sqrt{E_1^2 (b^2 - 1) - m_3^2 h^2}}{h}, \quad (31) $$

$$ C_1 = \frac{b}{h} - \frac{A^2 + m_3^2 + \frac{E_3^2}{h^2}}{2E_3 A}, \quad (32) $$

$$ F = \frac{A^2 - m_3^2 - \frac{E_3^2}{h^2}}{2A}. \quad (33) $$

We are interested in scenario IN$. Then, $C_1 < 0$ gives us

$$ E_3^2 - 2E_3 h A_1 b + h^2 (A^2 + m_3^2) > 0, \quad (34) $$

that can be rewritten as

$$ (E_3 - \lambda_+) (E_3 - \lambda_-) > 0, \quad (35) $$

$$ \lambda_{\pm} = h \left[ A_1 b \pm \sqrt{A^2 (b^2 - 1) - m_3^2} \right]. \quad (36) $$

The condition $F < 0$ gives us

$$ E_3^2 > h^2 (A^2 - m_3^2) \equiv \lambda_0^2, \quad (37) $$
If \( \lambda_\pm \) are real, both bounds \( E_3 > \lambda_+ \) and \( E_3 > \lambda_0 \) are quite compatible with each other. If \( \lambda_\pm \) are complex, (34) and (37) are mutually consistent as well. Thus there is no upper bound on \( E_3 \) and the SPP is possible.

As far as particle 4 is concerned, it has \( E_4 < 0 \). To obey the forward-in-time condition (10), it must have \( L_4 = |L_4| < 0 \). Then, \( X_4 = |L_4| \omega - |E_4| \). Assuming that there is a flat infinity, where \( \omega \to 0 \), we see that particle 4 either falls into singularity or oscillates between turning points \( r_1 \) and \( r_2 \). In doing so, it can intersect the horizons, thus appearing in new “universes” due to a potentially rich space-time structure inside similarly to what takes place for the Kerr metric [22]. However, under a rather weak and reasonable restrictions on the properties of the metric, it cannot have more than 1 turning point in the outer region, so the situation when \( r_+ < r_1 \leq r \leq r_2 \) is impossible. This was shown for the Kerr metric in [23] and generalized in [24]. For more information about trajectories of particles 3 and 4, the metric should be specified.

The above treatment changes only slightly if we consider the Schnittman process [25] when the critical particle 1 does not come from infinity but moves from the horizon. Then, instead of (31), we should take \( A = \frac{E_1 b + \sqrt{E_1^2 (b^2 - 1) - m_1^2 h^2}}{h} \).

### 5 Superenergetic particles

In the above treatment, we tacitly assumed that all energies and angular momenta are finite and do not grow unbounded when \( N_c \to 0 \). The only place where \( N_c \) appear in the relation between them are equalities (11), (12), where it gives only small corrections. The above approximate expressions for particle characteristics (14)–(20) take into account these circumstances. In particular, for a usual particle, \( X = O(1) \), the second term in the radical in (8) has the order \( N_c^2 \). For a near-critical one, both terms in \( P \) have the order \( O(N_c) \). Meanwhile, it turns out that there exists self-consistent scenario, in which

\[
L_3 = \frac{l_3}{\sqrt{N_c}},
\]

where \( l_3 \) is some coefficient not containing \( N_c \). For small \( N_c \), \( L_4 = L_0 - L_3 \approx -\frac{l_3}{\sqrt{N_c}} \).

It was found in [26], where it was pointed out that it corresponds to falling both particles in a black hole, so it was put aside since we were interested in particles returning to infinity. But now, it is this case that came into play. Therefore, we take advantage of formulas already derived in [26] but exploit them in a new context—see Eqs. (41), (42) below. If (38) is satisfied, the previous consideration fails and the conservation law (28) is to be analyzed anew. Now, for particles 3 and 4 the second term in (8) gives a small correction (whereas for finite \( L_3 \) both terms for particle 3 would have the same order), so

\[
P_{3,4} \approx \sqrt{X_{3,4}^2 - N_c \frac{l_{3,4}^2}{(g\phi)_H}} \approx X_{3,4} - \frac{N_c}{2X_{3,4}} \frac{l_3^2}{(g\phi)_H}.
\]
Then, taking into account (14)–(18) for particles 1 and 2, (25)–(27) and discarding the terms $O(N_c^2)$ and higher, one can show after algebraic manipulations that the following equation holds:

$$\frac{l_3^2}{2(g_\phi)_H} \left( \frac{1}{X_3} + \frac{1}{X_4} \right) = A.$$  \hspace{1cm} (40)

Using again (27), one can obtain

$$(X_{3,4})_c \approx \frac{(X_0)_c}{2} (1 \mp \sqrt{1-b}),$$  \hspace{1cm} (41)

where

$$b \equiv \frac{2l_3^2}{(g_\phi)_H X_0 A}.$$  \hspace{1cm} (42)

It is implied that $b < 1$.

It follows from definition (5) that now

$$E_3 = (X_3)_H + \omega_H L_3 \approx (X_3)_c + \omega_H \frac{l_3}{\sqrt{N_c}}.$$  \hspace{1cm} (43)

Thus we have two usual particles which come down into a black hole. This is contrasted with the standard case when particle 3 is near-critical and returns to infinity. We see that there are two energy scales for the SPP. On the first scale, $E_3$ can be as large as we like but with reservation that $E_3 \ll \text{const} \sqrt{N_c}$. On the second scale, $E_3 \sim \frac{1}{\sqrt{N_c}}$.

In doing so, $E_4 = E_0 - E_3$ is negative having the same order $N_c^{-1/2}$. Discussion about the properties of the trajectories of such particles from Sect. 4 applies now as well.

**6 Conclusions**

Thus we showed that particle collision on our side of Universe (near the black hole horizon) can lead to high energy fluxes on the other side. If, vice versa, collision occurs in “another world”, we can detect its consequences in our one. We did not resort to the transformation between the original stationary frame and the center of mass one. We would like to stress that high energy behavior is found for the energies $E_3$ that can be in principle detected in a laboratory. These results qualitatively agree with claims made in [11] for the Kerr metric.

It is instructive to compare the situation for static charged and neutral rotating black/white holes collecting the results of the present and previous works [18,27].

We see that for the Reissner–Nordström metric, if we compare collisions near black and white holes, the situation is partially complementary to each other. For finite $L_i$ (for example, with all $L_i = 0$), there is the SPP in the black hole case. However, it fails to exist near white holes. As shown in [27], only if particles with angular momenta...
Table 1  Conditions of the existence of the super-Penrose process

|                        | SPP | $q_3$  | $L_3$          |
|------------------------|-----|--------|----------------|
| Charged black holes    | Yes | Large  | Arbitrary       |
| Charged white holes    | Yes | Large  | $O(N_c^{-1/2})$|
| Neutral rotating black holes | No  | 0      | Arbitrary       |
| Neutral rotating white holes | Yes | 0      | Unbounded      |

$L_{3,4} = O(N_c^{-1/2})$ come into play, we obtain the SPP. Meanwhile, for the rotating case, the SPP does not exist for black holes at all. Instead, the white hole scenario opens new possibilities for the SPP, in which $E$ and $L$ of particles at infinity are unbounded. This happens on two scales: on the first one $E$ and $L$ do not contain the parameter $N_c^{-1}$, on the second scale they have the order $O(N_c^{-1/2})$ similarly to the static charged case. The concrete properties of collisions are described by somewhat different formulas and the type of energetic particles are different: in the first case particle 3 is near-critical, in the second one it is usual. However, in both cases the conserved energy $E_3$ is unbounded.

The phenomenon under discussion is two-faced. On one hand, it shows that high energy processes are indeed possible due to white holes and poses anew the question about their potential role in nature. From the other hand, it poses also a question about backreaction of such collisions on the metric itself, including the fate of white holes.

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References

1. Bañados, M., Silk, J., West, S.M.: Kerr black holes as particle accelerators to arbitrarily high energy. Phys. Rev. Lett. 103, 111102 (2009). arXiv:0909.0169
2. Piran, T., Shaham, J., Katz, J.: High efficiency of the Penrose mechanism for particle collisions. Astropophys. J. 196, L107 (1975)
3. Piran, T., Shaham, J.: Upper bounds on collisional Penrose processes near rotating black-hole horizons. Phys. Rev. D 16, 1615 (1977)
4. Grib, A.A., Pavlov, YuV: Are black holes totally black? Gravit. Cosmol. 21, 13 (2015). arXiv:1410.5736
5. Zaslavskii, O.B.: On white holes as particle accelerators. Gravit. Cosmol. 24, 92 (2018). arXiv:1707.07864
6. Penrose, R.: Gravitational Collapse: the role of general relativity. Rivista del Nuovo Cimento. Numero Speziale I, 252 (1969)
7. Schnittman, J.D.: The Collisional Penrose Process. Gen. Relativ. Gravit. 50, 77 (2018). arXiv:1910.02800
8. Zaslavskii, O.B.: Is the super-Penrose process possible near black holes? Phys. Rev. D 93, 024056 (2016). arXiv:1511.07501
9. Novikov, I.D.: Delayed explosion of a part of the Friedmann universe and quasars. Astron. Zh. 6, 1075 (1964). [Sov. Astronomy 8, 857 (1964)]
10. Narlikar, J.V., Appa Rao, K.M.V., Dadhich, N.: High energy radiation from white holes. Nature 251, 590 (1974)
11. Patil, M., Harada, T.: Extremal Kerr white holes as a source of ultra high energy particles. Phys. Rev. D 102, 024002 (2020). arXiv:2004.12874
12. Harada, T., Nemoto, H., Miyamoto, U.: Upper limits of particle emission from high-energy collision and reaction near a maximally rotating Kerr black hole. Phys. Rev. D 86, 024027 (2012). arXiv:1205.7088
13. Zaslavskii, O.B.: Energetics of particle collisions near dirty rotating extremal black holes: Banados-Silk-West effect versus Penrose process. Phys. Rev. D 86, 084030 (2012). arXiv:1205.4410
14. Eardley, D.M.: Death of white holes in the early universe. Phys. Rev. Lett. 33, 442 (1974)
15. Frolov, V.P., Novikov, I.D.: Physics of Black Holes. Kluwer Academic, Dordrecht (1998)
16. Tanatarov, I.V., Zaslavskii, O.B.: Dirty rotating black holes: regularity conditions on stationary horizons. Phys. Rev. D 86, 044019 (2012). arXiv:1206.2580
17. Griffiths, J.B., Podolsky, J.: Exact Space-Times in Einstein’s General Relativity. Cambridge University Press, Cambridge (2009)
18. Zaslavskii, O.B.: Energy extraction from extremal charged black holes due to the BSW effect. Phys. Rev. D 86, 124039 (2012). arXiv:1207.5209
19. Patil, M., Harada, T., Nakao, K., Joshi, P.S.: Kimura M (2016) Infinite efficiency of collisional Penrose process: can over-spinning Kerr geometry be the source of ultra-high-energy cosmic rays and neutrinos? Phys. Rev. D 93, 104015 (2016). arXiv:1510.08205
20. Tanatarov, I.V., Zaslavskii, O.B.: Collisional super-Penrose process and Wald inequalities. Gen. Relativ. Gravit. 49, 119 (2017). arXiv:1611.05912
21. Zaslavskii, O.B.: Acceleration of particles as universal property of rotating black holes. Phys. Rev. D 82, 083004 (2010). arXiv:1007.3678
22. Carter, B.: Global structure of the Kerr family of gravitational fields. Phys. Rev. 174, 1559 (1968)
23. Grib, A.A., Pavlov, YuV, Vertogradov, V.D.: Geodesics with negative energy in the ergosphere of rotating black holes. Mod. Phys. Lett. A 29, 1450110 (2014). arXiv:1304.7360
24. Zaslavskii, O.B.: On geodesics with negative energies in the ergoregions of dirty black holes. Mod. Phys. Lett. A 30, 1550055 (2015). arXiv:1412.1725
25. Schnittman, J.D.: Revised upper limit to energy extraction from a Kerr black hole. Phys. Rev. Lett. 113, 261102 (2014). arXiv:1410.6446
26. Zaslavskii, O.B.: Center of mass energy of colliding electrically neutral particles and super-Penrose process. Phys. Rev. D 100, 024050 (2019). arXiv:1904.04874
27. Zaslavskii, O.B.: Super-Penrose process for extremal charged white holes. arXiv:2005.11090

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