Particle Heating in Advection-Dominated Accretion Flows

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**Abstract.** I review particle heating by MHD turbulence in collisionless plasmas appropriate to advection-dominated accretion flows. These considerations suggest that the preferential turbulent heating of protons assumed by theoretical models is only achieved for relatively subthermal magnetic fields.

1. Introduction

A number of authors have argued that, at sub-Eddington accretion rates, the gravitational potential energy released by turbulent stresses in an accretion flow may be stored as thermal energy, rather than radiated as in thin accretion disks (Ichimaru 1977; Rees et al. 1982; Narayan & Yi 1994, 1995; Abramowicz et al. 1995; see Narayan, Mahadevan, & Quataert 1998 and Kato, Fukue, & Mineshige 1998 for reviews). In this case, the gas heats up to nearly virial temperatures and is assumed to adopt a two-temperature configuration, with the protons significantly hotter than the radiating electrons.

There are two crucial microphysical issues relevant to these accretion models, which are called either ion tori (Rees et al. 1982) or advection-dominated accretion flows (ADAFs; Narayan & Yi 1994). The first is the value of \( \delta \), the fraction of the turbulent energy in the plasma which heats the electrons (a fraction \( 1 - \delta \) heats the protons). The second is whether or not the dominant process for exchanging energy between electrons and protons is Coulomb collisions. ADAF models typically assume that \( \delta \ll 1 \) (preferential proton heating) and that Coulomb collisions represent the only thermal coupling. In this case, the plasma is unable to cool because it is nearly collisionless. The protons, which by assumption receive the turbulent energy, are (1) unable to radiate and (2) unable to transfer their thermal energy to the electrons, the more efficient radiators.

The purpose of this review is to present some rather simple, but general, theoretical considerations regarding particle heating in ADAFs, which aim to address the value of \( \delta \).
2. Physical Picture

A fundamental assumption of my analysis is that the particle distribution functions, while not necessarily strictly thermal, are not pathologically nonthermal; that is, I assume that the plasma does not reach an equilibrium configuration with, e.g., highly anisotropic or bump-in-tail distribution functions.

For the simple considerations presented here, only two properties of the plasma are important. The first is the proton to electron temperature ratio, $T_p/T_e$, taken to be $\gg 1$. Note that it is consistent to assume $T_p \gg T_e$ while simultaneously investigating particle heating. This is because the two temperature nature of the plasma is in large part due to the efficiency with which relativistic electrons cool; it is rather insensitive to $\delta$. The second property of importance is the magnetic field strength in the plasma, parameterized by $\beta \equiv P_{\text{gas}}/P_{\text{mag}}$. The quantity $\beta$ must be $\geq 1$ for the gravitationally confined plasmas of interest.

Like thin disks, ADAFs are turbulent magnetized plasmas. On scales much larger than the Larmor radius of the thermal protons ($\equiv \rho$), we can treat the turbulence using MHD, i.e., as a superposition of the magnetosonic and Alfvén waves. Particles are heated when the turbulence cascades to sufficiently small scales that it is dissipated.

3. Collisionless Damping

Let a wave in the plasma have wave vector $k$ and frequency $\omega$ and define $\perp$ and $\parallel$ to be directions perpendicular and parallel to the local magnetic field.

In collisionless plasmas, the wave dissipation mechanisms of interest are wave-particle resonances (molecular viscosity, thermal conductivity, electrical resistivity, etc. are entirely unimportant). In the next section, I will argue that the turbulence always has frequencies well below the proton cyclotron frequency. In this case, resonance occurs when the wave’s phase speed along the field line, $v = \omega/k\parallel$, equals $v\parallel$, the velocity of a particle along the field line. A necessary (but not sufficient) condition for strong damping is that $v$ be comparable to the thermal speed of the particles, so that there are a large number of resonant particles.

This resonance actually corresponds to two physically distinct wave-particle interactions. In Landau damping (LD), particle acceleration is due to the wave’s longitudinal electric field perturbation (i.e., the usual electrostatic force, $E\|)$.

For a wave damped solely by LD, $\delta \sim 1$ since the electrons are preferentially heated by a factor of $\approx (m_e T_p^3/m_p T_e^3)^{1/2}$, which is $\gg 1$ for $T_p \sim 10^2 - 10^3 T_e$. In transit-time damping (TTD), the magnetic analogue of LD (and the collisionless analogue of Fermi acceleration; Achterberg 1981), the interaction is between the particle’s effective magnetic moment ($\mu = mv^2/2B$) and the wave’s longitudinal magnetic field perturbation, $B\parallel$ (Stix 1992). For a wave (with $k\perp \rho < 1$) damped solely by TTD, the protons are preferentially heated by a factor of $\approx (m_p T_p/m_e T_e)^{1/2} \gg 1$; the contribution to $\delta$ is therefore always $\ll 1$. This is because in plasmas with $T_p \gg T_e$, the protons have the larger magnetic moment and so couple better to a wave’s magnetic field perturbation.
4. MHD Turbulence

In a plasma with $\beta > 1$, one can (roughly speaking) decompose MHD turbulence into three modes, the Alfvén wave and the fast and slow magnetosonic modes. The fast mode has the dispersion relation $\omega \approx v_p|k|\lambda_p$ ($v_p$ is the proton thermal speed) and is essentially a sound wave. The slow mode has $\omega \approx v_A|k|\lambda_p$ ($v_A \approx v_p\beta^{-1/2}$ is the Alfvén speed) and, for $k_\perp \neq 0$, has a parallel magnetic field perturbation ($B_\parallel$). The Alfvén wave also has $\omega \approx v_A|k|\lambda_p$, but is incompressible, with $E_\parallel = B_\parallel = 0$ in the limit that the wavelength is long compared to the Larmor radius of thermal protons ($|k|\rho \ll 1$).

4.1. Sound Waves (Fast Modes)

Sound waves in an unmagnetized collisionless plasma are strongly damped by LD. For $T_p > T_e$, the dissipation time is formally $\ll$ the mode period, implying that the modes are non-propagating. These modes are unlikely to be efficiently excited since, quite generally, strongly damped oscillators dissipate less energy than weakly damped oscillators (consider, e.g., the analogous problem of two resistors in parallel; the resistor with the smallest resistance dissipates the most power). The same holds true for sound waves in a $\beta > 1$ plasma, provided $k_\perp \sim k_\parallel$. For $k_\perp \gg k_\parallel$, i.e., nearly perpendicular sound waves, the parallel phase speed is $v \approx v_p k_\perp / k_\parallel \gg v_p, v_e$ (since $v_e \sim v_p$ by $T_p > T_e$). Since the wave is highly suprathermal, there are very few particles for the wave to resonate with and it is consequently undamped. These waves likely dissipate by steepening and forming weak shock waves.

In what follows I will neglect quasi-perpendicular sound waves, assuming that they are energetically unimportant. The reason is that I believe that the turbulence in ADAFs is intrinsically magnetic in origin, being initiated by some combination of Balbus-Hawley and convective instabilities. The turbulent speed ($v_t$) is therefore $\sim v_A < v_p$. Moreover, I will argue that $v_A$ is likely to be $\ll v_p$, in which case, if $v_t \sim v_A$, excitation of sound waves will be strongly suppressed (subsonic instabilities do not easily excite sound waves).

4.2. Slow Modes

In a $\beta \gg 1$ collisionless plasma, slow modes are strongly damped and non-propagating for $k_\perp \neq 0$ (Foote & Kulsrud 1979). The reason is that, even in the limit of wavelengths long compared to the Larmor radius of thermal protons ($|k|\rho \ll 1$), slow modes have a parallel magnetic field perturbation ($B_\parallel$) and are damped by TTD. They are, as with sound waves, unlikely to be excited. For $k_\perp \to 0$, these considerations fail, since $B_\parallel \to 0$. Quasi-parallel slow modes are, however, likely to cascade in $k_\perp$, develop $B_\parallel \neq 0$, and damp by TTD (heating the protons).

4.3. Alfvén Waves

The Alfvén wave has a parallel phase speed $|v| = v_A$; in plasmas appropriate to ADAFs, $v_A$ is comparable to the electron and proton thermal speeds and so there are a large number of particles available to resonate with the wave. In the MHD limit, however, the Alfvén wave is undamped by linear collisionless
effects. This is because \( E_\parallel = 0 \) and \( B_\parallel = 0 \), i.e., there is no electric/magnetic field perturbation associated with the wave which can accelerate particles.

This lack of dissipation on large scales implies that understanding heating of particles by Alfvénic turbulence requires understanding how the energy cascades to small scales, where dissipation finally becomes important.

**Alfvénic Turbulence**  The nature of Alfvénic turbulence in a collisionless, \( \beta > 1 \), plasma is not fully understood. A plausible scenario is due to Goldreich and Sridhar (1995; GS). They argue that Alfvénic turbulence naturally evolves into a critically balanced state in which the timescale for nonlinear effects to transfer energy from a wavevector \( \sim k \) to a wavevector \( \sim 2k \) (≡ the cascade time, \( T_c \)) is comparable to the linear wave period at that scale, \( T = 2\pi \omega^{-1} \); this determines how rapid the dissipation must be to halt the cascade. It also implies that the cascade is highly anisotropic, with the energy cascading primarily perpendicular to the local magnetic field; the parallel and perpendicular sizes of a wave at any scale are correlated, with \( k_\parallel \sim k_\perp^{2/3} L^{-1/3} \ll k_\perp \), where \( L \) is the outer scale of the turbulence.

Quasi-perpendicular Alfvénic turbulence has the important property that its frequencies are always much less than the proton cyclotron frequency. This is because the cascade occurs primarily in \( k_\perp \) while the Alfvén frequency is \( \propto k_\parallel \). This justifies my claim in §3 that low frequency wave-particle resonances are of primary importance.

**Alfvén Wave Damping**  The character of an Alfvén wave changes when \( k_\perp \rho \sim 1 \); at this point the wave develops a non-zero parallel magnetic field perturbation (in contrast to long wavelength perturbations, for which \( B_\parallel = 0 \)). The waves can therefore be dissipated by TTD (Quataert 1998, Gruzinov 1998, Quataert & Gruzinov 1998; QG).

Figure 1 shows kinetic theory calculations of the dissipation of nearly perpendicular Alfvén waves. For \( k_\perp \rho < 1 \) (where the waves can still be called Alfvén waves), the dissipation is independent of \( T_p/T_e \), but is a strong function of \( \beta \). The electron temperature is unimportant since the electrons do not participate significantly in the damping. The dissipation is sensitive to \( \beta \approx (v_p/v_A)^2 \), since this determines the number of resonant protons and the protons’ magnetic moment (which is the effective coupling coefficient between the protons and the wave).

For large \( \beta \gg 1 \), Alfvén waves are strongly damped for \( k_\perp \rho \sim 1 \). Most of the turbulent energy heats the protons, since they are responsible for TTD (§3). For \( \beta = 1 \), however, the maximal dissipation rate for an Alfvén wave in the GS cascade (obtained at \( k_\perp \rho = 1 \)) is \( \gamma T \approx 0.1 \) (see Figure 1). Since the timescale for energy to cascade through the inertial range is \( T_c \approx T \), this suggests that, for plasmas with equipartition magnetic fields (\( \beta \approx 1 \)), very little of the turbulent energy is dissipated on scales comparable to or greater than \( \rho \).

Alfvén waves only exist for \( k_\perp \rho < 1 \). For \( k_\perp \rho > 1 \), the same mode is called the whistler. As emphasized by Gruzinov (1998; see also QG), whistlers are unlikely to heat the protons. For \( k_\perp \rho > 1 \), but \( k_\perp \gg k_\parallel \), whistlers have \( \omega \ll \Omega_p \). Thus, in a mode period, a particle undergoes many Larmor orbits. Since the mode’s perpendicular wavelength is smaller than the proton Larmor
radius \( k_{\perp} \rho > 1 \), the protons (but not the electrons) sample a rapidly varying electro-magnetic field in the course of a Larmor orbit. As a result, they are “frozen out” and become dynamically unimportant; the protons effectively no longer “see” the wave. They therefore cannot contribute to damping the whistler energy, which cascades to smaller length scales until it is damped by the electrons (this is the origin of the strong damping at \( k_{\perp} \rho \gg 1 \) in Figure 1).

The Physics of \( \delta \) The above arguments suggest that the crucial physics determining the electron heating rate in ADAFs is what fraction of the turbulent energy is dissipated as Alfvén waves (heating the protons) and what fraction cascades to small scales, becoming whistlers and heating the electrons. This depends on \( \beta \), but not on \( T_p/T_e \). Unfortunately, estimates of the fraction of the turbulent energy that becomes whistlers are highly uncertain, as they are exponentially sensitive to the details of turbulence on the scale of the proton Larmor radius, where both nonlinear and kinetic effects are important. For example, we do not accurately know \( T_c \), the nonlinear timescale of the turbulence. Nor is the calculation of the damping of the turbulence that secure (because it is based on a theory of linear Alfvén waves damped by thermal particles; both linearity and thermal protons are crude approximations).

Parameterizing the uncertainty in the details of the turbulence, and using numerical simulations of MHD turbulence as a guide, QG estimated that the critical \( \beta \) above which proton heating dominates (say \( \delta < 0.1 \)) is \( \sim 10 \), with an uncertainty of \( \sim 10! \)

5. Summary and Future Work

The physics of particle heating in a collisionless plasma suggests that, in ADAFs and ion-tori, proton heating may be favored for relatively subthermal magnetic fields, but not for strictly equipartition ones (\( \beta \equiv 1 \)).
In this paper, I have focused primarily on heating of particles by small scale, incompressible, Alfvénic turbulence. For $\beta \gg 1$, it should be quite accurate to neglect sound wave and slow mode excitation. As emphasized by Blackman (1998), however, one expects the importance of compressibility to be an increasing function of decreasing $\beta$. For example, at $\beta = 1$, slow mode excitation cannot be neglected (these modes are damped by TTD on relatively large scales, heating the protons; e.g., Blackman 1998). Compressibility (and hence dissipation) of large scale Alfvén waves is also potentially important. My belief is that, while these effects are undoubtedly present, it is unlikely that the majority of the turbulent energy is dissipated on large scales; thus, to order of magnitude, the analysis presented in this review is appropriate. More quantitative estimates are, however, clearly needed.

The unknown importance of magnetic reconnection, and its (presumed) electron heating, remains a significant uncertainty for ADAF models (Bisnovatyi-Kogan & Lovelace 1997; see also QG).

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References

Abramowicz, M. et al., 1995, ApJ, 438, L37
Achterberg, A. 1981, A&A, 97, 259
Bisnovatyi-Kogan, G. S., & Lovelace, R. V. E., 1997, ApJ, 486, L43
Blackman, E. 1998, MNRAS, in press [astro-ph/9710137]
Foote, E. A. & Kulsrud, R. M. 1979, ApJ, 233, 302
Goldreich, P. & Sridhar, S. 1995, ApJ, 438, 763 (GS)
Gruzinov, A. 1998, ApJ, 501, 787
Ichimaru, S. 1977, ApJ, 214, 840
Kato, S., Fukue, J., Mineshige, S., 1998, Black-Hole Accretion Disks (Japan: Kyoto University Press)
Narayan, R., Mahadevan, R., & Quataert, E., 1998, in The Theory of Black Hole Accretion Disks, eds. M.A. Abramowicz, G. Bjornsson, and J.E. Pringle (Cambridge: Cambridge University Press) [astro-ph/9803131]
Narayan, R., & Yi, I., 1994, ApJ, 428, L13
Narayan, R., & Yi, I., 1995, ApJ, 452, 710
Quataert, E. 1998, ApJ, 500, 978
Quataert, E., & Gruzinov, A., 1998, ApJ submitted [astro-ph/9803112] (QG)
Rees, M. J., Begelman, M. C., Blandford, R. D., & Phinney, E. S., 1982, Nature, 295, 17
Stix, T.H. 1992, Waves in Plasmas (New York: AIP)