Investigation on axial-lateral-torsion nonlinear coupling vibration characteristic of drilling string in ultra-HPHT curved wells

X Q Guo1,2,3, J Liu1,2∗, J X Wang1, L M Dai2
1. School of Mechanical and Electrical Engineering, Southwest Petroleum University, Chengdu, 610500, China
2. School of Mechanical Engineering, Chengdu University, Chengdu, 610106, China
3. Industrial Systems Engineering, University of Regina, Regina, SK S4S 0A2, Canada

∗Corresponding author, Email: 201031010081@swpu.edu.cn.

Abstract. In view of the vibration failure of drilling string in ultra-high temperature and high pressure (ultra-HPHT) curved wells, an axial-lateral-torsion coupling (ALTC) nonlinear vibration model of drilling string system was established using energy method and Hamiltonian principle, in which, the influence of wellbore trajectory change, wellbore constraint, interaction between bit and rock and ultra-HPHT of wellbore on elastic modulus and viscosity of drilling fluid were taken into account. The finite element method (FEM) is used to realize the numerical solution of the nonlinear vibration model. The correctness and validity of the ALTC nonlinear vibration model was verified by comparing the measured data of four ultra-HPHT wells with the theoretical calculation results of the proposed model. The research results provide a theoretically sound guidance for designing and practically sound approach for effectively improving rate of penetration (ROP) and the service life of drilling string in ultra-HPHT curved wells.

1. Introduction
With the dwindling of shallow oil and gas resources, well drilling processes are being implemented on deep formations with ultra-high-pressure, ultra-high-temperature (ultra-HPHT) and complex structures, and the well type changes from vertical well to directional well (Figure 1(a)), highly deviated well and horizontal well (Figure 1(b)). According to the US Code of Federal Regulations 30 CFR 250 804 (b) (1), the pressure rating greater than 15,000 psi (103 MPa) with the temperature rating greater than 350 °F (175 °C) is considered ultra-HPHT [1]. Drilling string system is the core equipment of oil&gas well drilling technology, and its safety determines the safety and efficiency of the whole drilling technology. Compared with conventional gas well drilling, the ultra-HPHT drilling string system is subjected to greater risks. Firstly, with the increase of temperature, the stiffness of the drilling string system will be declined, and the drilling string is more prone to buckling deformation, which increases the probability of contact with the casing, and the wear of the drilling string is more serious [2]. Secondly, when the stiffness of drilling string decreases, it is more prone to large deformation
vibration, and the ALTC vibration effect is more obvious. Meanwhile, the drilling string is prone to non-periodic violent vibration, which will increase its axial load and easily lead to strength fracture failure and fatigue failure of drilling string [3]. Thirdly, with the complex vibration of drill string system, it is prone to stick slip, jump, whirl and other harmful movements for the drilling string system, which reduce the drilling efficiency [4].

Drilling string vibration has been concerned by scholars in all the world. In the early years, the lumped mass method is used to establish torsional vibration model of the drilling string in vertical wells [5], in which BHA is assumed to be a rigid body and the moment of inertia of drilling pipe is ignored. After that, Challamel [6] constructed a torsional dynamic model of the drilling string system when it was simplified as a continuous beam. Based on that, the torsional vibration characteristics of drilling string was investigated, and the linear stability of the torsional steady-state solution was analyzed using Liapunov and linearization methods. In order to improve the operation efficiency of drilling string, the dynamic response characteristic of drill string systems was analyzed [7-9], in which the stick slip vibration and its bifurcation characteristics were studied, and the causes and vibration mechanism of stick slip vibration of drilling string were revealed. On this basis, the method to control stick slip vibration of drilling string was proposed.

The earliest research on the longitudinal vibration of drilling string can be traced back to the 1960s [10]. The initial research mainly focused on the longitudinal natural frequency of drilling string and the influence of different boundary conditions on the longitudinal vibration and the stability of the longitudinal vibration of drilling string. Then, the decoupled the longitudinal resonance from the excitation frequency of drill bit using shock absorber and changing the length and rotary speed of BHA [11]. Based on the longitudinal vibration model of drilling string, Dareing [12] studied the influence of longitudinal vibration on ROP using energy method, and found that the greater the longitudinal force is, the higher the ROP of drill bit can be effectively improved. Based on this, Wiercigroch [13, 14] proposed resonance enhanced drilling (RED) technology for the first time. By adding a tool stub and applying a longitudinal impact load on the bit, rock resonance is driven to accelerate rock fracture and improve drilling speed. A lumped mass model is established to analyze the nonlinear characteristics of longitudinal vibration of drilling string during RED drilling [15].

In the actual drilling process, the vibration forms (such as longitudinal vibration, lateral vibration and torsional vibration) of drilling string are coupled with each other. Although the single torsional and longitudinal vibration model research can reveal the rule of drilling string vibration as a certain extent, the multi-directional vibration coupling dynamic model has more important significance to guide the actual drilling engineering, and it can more accurately predict the dynamic behavior of drilling string [16, 17]. For this purpose, some scholars [18, 19] have constructed the longitudinal-torsional coupling vibration model (RGD) of drilling string. The research results show that the self-excited vibration of drilling string can degenerate into stick slip vibration or skip drilling under certain conditions, and the reduction of torque with the angular velocity of drilling head is not due to self-excited vibration. At the same time, some scholars [20] have carried out the research on the transverse-torsional coupling vibration model of drilling string system. The model includes four independent degrees of freedom as radial displacement of drilling string, whirl of cross section, bending and torsion along tangent direction. The influence of bending and torsion, whirl of drilling string and friction
between drilling string and well were considered. The relationship between stick slip and whirl of drill string is mainly analyzed. It is found that the stick slip phenomenon disappears with the vortex. Then, in order to more accurately analyze the vibration characteristics of the actual drilling string system, some scholars [21, 22] devoted themselves to establishing the three-dimensional vibration model of the drilling string in vertical wells.

Thus, the existing researches mainly focus on the vibration problem of drilling string in conventional oil&gas wells, in which, the interaction between the drilling bit and rock was ignored. However, with the wide application of ultra-HPHT curved wells, the actual drilling string vibration cannot be effectively and accurately analyzed using the existing vibration model. Therefore, in this study, an ALTC nonlinear coupling vibration model of drilling string system is established using finite element method, energy method and Hamiltonian principle. Based on PDC bit type used on site, the mechanical model of interaction between drill bit and rock is established. The incremental form Newmark and Newton Raphson joint iteration methods is proposed. The correctness of the model is verified by the field test data.

2. ALTC nonlinear vibration model

2.1. Vibration control equation

Some text. In this section, the ALTC vibration control equations of infinitesimal drilling string were established through the energy method and Hamilton variational principle. A coordinate system is established in which the depth direction set z-axis, the horizontal direction set x-axis and the y-axis satisfies the right-hand rule (Figure 2).

![Figure 2. 3D coordinates and cross section diagram of drill string system](image)

In the Figure 2, the point o , o’ , and c are the coordinate origin, centroid of drilling string arbitrary section, and barycenter of drilling string arbitrary section, respectively. If the displacement of centroid of drilling string arbitrary section in the fixed coordinate system (oxyz) can be expressed as \((u,v,w)\), the displacement of barycenter of drilling string arbitrary section can be expressed as:

\[
\hat{v} = \left[u + e \cos(\phi + \alpha)\right] \hat{i} + \left[v + e \sin(\phi + \alpha)\right] \hat{j} + w \hat{k}
\]  

(1)

where, \(e\) is the eccentricity between centroid and barycenter (m); \(\alpha\) is initial eccentricity angle (rad); \(\hat{i}\), \(\hat{j}\) and \(\hat{k}\) are unit vectors along \(x\), \(y\) and \(z\) directions; \(\phi\) is angle of torsion. If the first derivative of time of the above formula, the velocity of drilling string centroid can be obtained as follows:

\[
V_e = \frac{\partial v}{\partial t} = \left[\hat{k} - e \sin(\phi + \alpha)\right] \hat{i} + \left[\hat{v} + e \cos(\phi + \alpha)\right] \hat{j} + w \hat{k}
\]  

(2)

According to elastic-plastic mechanics, the total kinetic energy \(T\) of drilling string including translational and rotational kinetic energy can be expressed as:

\[
T = E_w + E_i = \frac{1}{2} \int_0^l \rho A \hat{v}^2 dz + \frac{1}{2} \int_0^l \rho I_{\phi \phi} \hat{\phi}^2 dz
\]

\[
= \frac{1}{2} \int_0^l \left\{ \rho A \left[ (\hat{v}^2 + \hat{w}^2 + \hat{k}^2) + e^2 \hat{\phi}^2 + 2e^2 \hat{\phi} \left( \cos(\phi + \alpha) - \hat{v} \sin(\phi + \alpha) \right) \right] + \rho I_{\phi \phi} \hat{\phi}^2 \right\} dz
\]  

(3)
where, $E_{k}$ is the translational kinetic energy of drilling string (J), $E_{r}$ is rotational kinetic energy of drilling string (J), $\rho$ is density of drilling string (kg/m$^3$), $I_{\rho}$ is the polar moment of inertia of drilling string (m$^4$). In order to derive the potential energy of drilling string, the nonlinear geometric equation of spatial beam element should be obtained. As shown in Fig. 2(b), if the drilling string does not twist, the displacement of any point is the same as that of the section centroid. Therefore, the displacement of any point on the section is equal to the displacement of the section centroid minus the displacement caused by torsion, which can be expressed as follows:
\[
\begin{align*}
\dot{u} &= u - \left[ r \cos \alpha - r \cos (\alpha + \phi) \right] = u - x \cos \phi - y \sin \phi \\
\dot{v} &= v + \left[ r \sin (\alpha + \phi) - r \sin \alpha \right] = v + y \cos \phi + x \sin \phi
\end{align*}
\]  
(4)

where, $\dot{u}$ and $\dot{v}$ are respectively the change of any point on the section along $x$ and $y$ directions (m), $u$ and $v$ are respectively the displacement of the section centroid along $x$ and $y$ directions (m), $x$ and $y$ are the coordinate components of the centroid respectively. Thus, when $\phi$ is very small, the equations can be expressed as $\cos \phi \approx 1$ and $\sin \phi \approx \phi$. By substituting it into Eq. (4), which can be simplified as follows:
\[
\begin{align*}
\dot{u} &= u - y \phi \\
\dot{v} &= v + x \phi
\end{align*}
\]  
(5)

According to the expression above, the infinitesimal segment can be expressed as:
\[
ds \approx \left[ 1 + \frac{1}{2} \left( \frac{\partial u}{\partial z} \right)^2 + \frac{1}{2} \left( \frac{\partial v}{\partial z} \right)^2 \right] dz
\]  
(6)

The total strain can be expressed as the sum of the corresponding linear strain and nonlinear strain:
\[

\varepsilon = \frac{\partial w}{\partial z} - x \frac{\partial^2 \dot{u}}{\partial z^2} - y \frac{\partial^2 \dot{v}}{\partial z^2} + \frac{1}{2} \left( \frac{\partial \dot{u}}{\partial z} \right)^2 + \frac{1}{2} \left( \frac{\partial \dot{v}}{\partial z} \right)^2
\]  
(7)

By substituting equation (5) into equation (7), which can be further written as following:
\[

\varepsilon = w' - x(u'' - y\phi') - y(v'' + x\phi') + \frac{1}{2}(u'' - y\phi')^2 + \frac{1}{2}(v'' + x\phi')^2
\]  
(8)

According to elastic-plastic mechanics, the strain energy $U$ of drilling string can be expressed as:
\[
U = \frac{1}{2} \int_{0}^{l} \left( E \varepsilon^2 + G \gamma^2 \right) dl = \frac{1}{2} \int_{0}^{l} \left( E (u'' + v'')^2 + G (\phi')^2 \right) dl
\]  
(9)

where, $E$, $G$ are Young's modulus and Shear modulus respectively (Pa), $\gamma$ is shear strain (in this study, geometric nonlinearity of drilling string is taken into account), which can be expressed as:
\[
\gamma = \rho_{e} \frac{\partial \phi}{\partial z}
\]  
(10)

where, $\rho_{e}$ is the distance from the infinitesimal segment to the intersection of $x$-axis and $y$-axis (m). By substituting equations (8) and (10) into equation (9), we obtain
\[
U = \frac{1}{2} \int_{0}^{l} \left[ EA (w'' + w'u'' + v'v'') \right] dl + \frac{1}{2} \int_{0}^{l} \left[ EI (u'' + v'') \right] dl

+ \int_{0}^{l} \left[ EI (w'' + v'') \right] dl + \int_{0}^{l} \left[ EI \phi' (-u'' + u'v'' + v') \right] dl + \frac{1}{2} \int_{0}^{l} G (\phi')^2 dl
\]  
(11)

where, $\frac{1}{2} \int_{0}^{l} \left[ EA (w'' + w'u'' + v'v'') + GI (\phi')^2 \right] dl$ represent strain energy of tension/compression, bending and torsion deformation, respectively (J), $I$ ($= \int_{A} x^2 dA = \int_{A} y^2 dA$) is the moment of inertia (m$^4$), $I_{\rho}$ ($= \int_{A} \rho_{e}^2 dA$) is the polar moment of inertia (m$^4$), the other items correspond to the strain energy of tension bending coupling, tension torsion coupling and bending torsion coupling.
The external force work of drilling string mainly consists of three parts: axial force work, lateral force work and torque work, which can be written as following:

\[
W_p = \int_0^l p(z,t)w(z)\,dz \\
W_r = \int_0^l \left[ F_r(z,t)u(z) + F_s(z,t)v(z) \right] dz \\
W_m = \int_0^l M(z,t)\left[ \phi(z) + (u'\delta v'' - v'\delta u'') \right] dz
\]  

(12)

where, \( p(z,t) \) is Longitudinal external load on drilling string (N), \( F_r(z,t) \) and \( F_s(z,t) \) are component of lateral external load of drilling string along \( x \) axis and \( y \) axis (N), \( M(z,t) \) is the torque on drill string (N·m).

According to Hamilton variational principle \( \delta \int_0^l (T - U + W)dt = 0 \), the three-dimensional vibration control equations of drilling string system were obtained.

\[
Ehi'' + \rho A\phi + M(x,t)v'' - EA(w''u' + w'u'') = F_r(x,t) \\
Elv'' + \rho A\phi - M(x,t)u'' - EA(w''v' + w'v'') = F_s(x,t) \\
-\rho A\phi' + EAu'' + EA(u'u'' + vv'') + 2EI\phi'' = p(x,t) \\
\rho I_p\phi'' - 2EI(w''\phi' + w'\phi'') - GI_p\phi'' = M(x,t)
\]  

(13)

The upper end of the drilling string is connected with the hook and rotary table, in which, the hook provides axial load \( P_0 (=F_0 - WOB) \), \( F_0 \) is the floating weight of drilling string (N), and the rotary table provides constant rotary speed \( \Omega_0 \). The lower end of the drilling string is connected with the drill bit and belongs to the free end. The drilling string system does not vibrate at the initial time. Therefore, the boundary and initial conditions of the system can be expressed as follows:

\[
\begin{align*}
  u(0,t) &= 0, v(0,t) = 0, P(0,t) = P_0, \frac{\partial \phi(0,t)}{\partial t} = \Omega_0 \\
  u(z,0) &= 0, v(z,0) = 0, w(z,0) = 0, \phi(z,0) = 0
\end{align*}
\]  

(14)

2.2. Vibration control equation

2.2.1. Contact load between drilling string and wellbore

When the radial displacement of the drilling string is greater than the clearance between the drilling string and the wellbore, the movement of the drilling string will be constrained by the wellbore. Then, the forces on the drilling string includes radial contact force, tangential friction force and friction torque (as shown in Figure 3). In this paper, the Hertz contact theory [23] is used to establish the contact model of the drilling string. The normal contact force, the tangential friction force and the friction torque can be expressed as follows:

\[
F_r = \begin{cases} 
-K_h(S - c_a) & S \geq c_a \\
0 & S < c_a 
\end{cases} \quad F_i = -\mu F_r \quad T_c = F_tr_o
\]  

(15)

where, \( S = \sqrt{u^2 + v^2} \) is radial displacement of drilling string (m), \( c_a \) is interval between drilling string and wellbore (m), \( K_h \) is the shaft stiffness, \( \mu \) is friction coefficient of drilling string.

2.2.2. Interaction force between drill bit and rock

In the actual drilling process, the drill bit contacts with rock at all times, and the lower boundary conditions of the drilling string system can be determined by interaction forces. Therefore, it is necessary to establish the interaction force calculation model of drill bit. The PDC bit is widely used in
ultra-HPHT gas wells drilling because of its good rock breaking performance, high ROP, long service life, greatly improving drilling efficiency and significant comprehensive economic benefits. The total interaction of PDC bit and rock can be obtained by accumulating the interaction between a single bit and rock. According to the interaction diagram of single bit (as shown in Figure 4), the interaction force of single PDC bit can be obtained by the experiment research in Liang’s work[24].

\[
\begin{align*}
F_t &= \mu f_t K_d A_t^{0.5472} ; p_t = 4.832e^{3.1724\sin \phi} ; q_t = 1.8196(\cos \phi)^{1.6419} \\
F_a &= \mu f_a K_d A_a^{0.7564} ; p_a = 0.48 e^{6.9291\sin \phi} ; q_a = 2.4154(\cos \phi)^{4.8154}
\end{align*}
\]

where, \( F_t \) and \( F_a \) are respectively tangential force and axial force of single PDC bit (N), \( \phi \) is caster angle of PDC bit (\( \phi \geq 10^\circ \)), \( A_t \) is cutting area (m²), \( f \) is arc length coefficient, \( d \) is diameter of PDC bit composite (m), \( \mu \) is friction coefficient of cutting teeth, \( K_d \) is rock drillability extremum. On the basis of the interaction force acting on the single PDC bit, the tangential force and axial force should be further decomposed into longitudinal, transverse and torsional directions, as shown in Figure 5. The center coordinate of any cutting tooth can be expressed as \( (R_c, H_c, \theta_c) \). According to the force analysis, the formula of longitudinal force, transverse force and torque of single PDC tooth can be obtained as follows:

\[
\begin{align*}
F_x &= F_t \sin \theta - (F_a \sin \gamma + F_t \tan \beta) \cos \theta \\
F_y &= F_t \cos \theta + (F_a \sin \gamma + F_t \tan \beta) \sin \theta \\
F_z &= F_a \cos \gamma \quad M = F_t R_c
\end{align*}
\]

where, \( F_x \) and \( F_y \) are respectively the component forces of the transverse force on the contact teeth along the \( x \) axial and \( y \) axial directions (N), \( F_z \) and \( M \) are the longitudinal force (N) and torque along the two directions (N/m) of the contact teeth, \( R_c \) and \( H_c \) are the radial coordinate of contact tooth on drill bit (m), \( \theta \) is the position angle of cutting teeth in the direction of drilling circumference (°), \( \gamma \) is the normal angle (°), \( \beta \) is lateral-rotation angle (°).

![Figure 3. Contact diagram between drilling string and wellbore](image)

![Figure 4. Diagram of interaction between single tooth and rock](image)

![Figure 5. Force analysis diagram of single tooth of PDC bit](image)

### 2.2.3. Influence of temperature and pressure

(1) In ultra-HPHT gas wells, the pressure rates greater than 15,000 psi (103 MPa) with the temperature rating greater than 350 °F (175 °C), which affected the material properties of drilling string, density and dynamic viscosity of drilling fluid. Based on the investigation of ultra-HPHT gas wells in Ledong block, South China Sea, the formation temperature is 212.5 °C and the formation pressure is 103.5 MPa. With the increase of temperature, the elastic modulus of drill string will be affected, and its expression can be obtained according to the thermoeelastic theory [25]:

\[
E = E_o + E_t \Delta T + E_e (\Delta T)^2
\]

where \( \Delta T \) is temperature difference between any interface of wellbore and wellhead (°C), \( E_o \) (\( = 2.1 \times 10^{11} \)) is the elastic modulus of drilling string at normal temperature (\( T=25 \) °C), \( E_t \)
\[ ( = -6.373476 \times 10^{-4}) \text{ and } E_2 \ ( -1.2638 \times 10^{-6}) \] are the coefficients. It can be seen that the elastic modulus varies nonlinearly with temperature, and the elastic modulus of drill string at arbitrary well depth can be calculated using Eq. (18).

(2) The density of drilling fluid will decrease with the increase of downhole temperature, so the influence of temperature on the density of drilling fluid must be considered in ultra-HPHT curved wells, and the expression of which can be obtained according to the literature [26]:

\[ \rho_f = \rho_{f0} e^{a(P_f - P_0) + b(T_f - T_0) + c(T_f - T_0)^2} \]

(19)

where, \( \rho_f \) and \( \rho_{f0} \) are the densities of drilling fluid at arbitrary well depth and wellhead (kg/m\(^3\)), \( P_0 \) and \( T_0 \) are the normal atmospheric (\( P=1 \times 10^5 \) Pa) and temperature (\( T=25 \) °C), \( a \), \( b \) and \( c \) are the characteristic constants for density of drilling fluid which are taken as \( 3.0296 \times 10^{-6} \) (1/Pa), \( 2.4385 \times 10^{-4} \) (1/°C) and \( -1.3482 \times 10^{-6} \) (1/(°C^2)).

In this study according to the literature, \( P_f \) and \( T_f \) are the pressure and temperature of drilling fluid at arbitrary well depth, which can be expressed as follows:

\[ P_f = 0.023 \times H + P_{f0} ; \quad T_f = 0.0398 \times H + T_{f0} \]

(20)

where \( H \) is the vertical depth (m), \( P_{f0} \) and \( T_{f0} \) are the outlet temperature (°C) and pressure of drilling fluid (Pa).

(3) In the process of ultra-HPHT drilling, the dynamic viscosity of drilling fluid is affected by high temperature and high pressure, while the pressure and pressure change have little influence on the drilling fluid rheology. Therefore, the influence of temperature is taken as the key point in this study, and the dynamic viscosity under actual temperature can be predicted according to the reference [27], which can be expressed as follows:

\[ \mu_f = \mu_{f0} e^{(A(T_f - T_{f0}) + B)} \]

(21)

where, \( \mu_f \) is dynamic viscosity of drilling fluid at arbitrary well depth (mPa·s), \( \mu_{f0} \) dynamic viscosity of drilling fluid at wellhead (mPa·s), \( A \) and \( B \) are the characteristic constants for dynamic viscosity of drilling fluid which are taken as \( 74.1068 \) (°C) and \( -0.1667 \) (1/Pa) in this study.

2.3. Model solution

This study used the linear Lagrange and cubic Hermitian functions to express the longitudinal displacement, transverse displacement and torsion of the drilling string; the finite element discrete forms can be expressed as follows:

\[ u = \phi_1^T \mathbf{d}, \quad v = \phi_2^T \mathbf{d}, \quad w = \phi_3^T \mathbf{d}, \quad \phi = \phi_4^T \mathbf{d} \]

(22)

\[
\begin{bmatrix}
\phi_1
\phi_2
\phi_3
\phi_4
\end{bmatrix} = \begin{bmatrix}
u_1 & \frac{dv_1}{dx} & v_1 & \frac{du_1}{dx} & u_1 & \phi_1 & u_2 & \frac{dv_2}{dx} & v_2 & \frac{du_2}{dx} & u_2 & \phi_2 & u_3 & \frac{dv_3}{dx} & v_3 & \frac{du_3}{dx} & u_3 & \phi_3 & u_4 & \frac{dv_4}{dx} & v_4 & \frac{du_4}{dx} & u_4 & \phi_4
\end{bmatrix}
\]

(23)

By substituting the displacement obtained from Eqs. (22) and (23) into the energy functional, the standard forms of the strain energy function \( U \), kinetic energy function \( T \) and energy function with external force \( W \) expressed by the node displacement vectors can be obtained. After assembling the structural elements, the discrete dynamic equation of the system can be obtained according to the
variational principle:
\[ M(t)\ddot{\mathbf{D}} + C(t)\dot{\mathbf{D}} + K(t)\mathbf{D} = \mathbf{F}(t) \]  \hfill (24)

where \( \mathbf{D} \) represents the matrix of overall displacement, which is given by Eq. (23), and \( \mathbf{F} \) represents the load column vector of the structure, which accounts for contact load between drilling string and wellbore as well as interaction force between drill bit and rock, and the expressions of them are shown in section 2.2. \( K, M, \) and \( C \) represent the matrices of the overall stiffness, mass, and damping, respectively. The dynamic responses of the tubing string can be solved via a Newmark-\( \beta \) step-by-step integration method. The procedure shown in Figure 6 is used to solve Eq. (24), with the numerical calculation program of the vibration model written in the FORTRAN language.

![Figure 6. Flow chart of solving vibration model of drilling string](image)

3. Model validation

3.1. Correctness verification

In order to verify the correctness of the proposed nonlinear model, the numerical simulation results using the proposed model are compared with the measured results on-site in Teng’s work[28]. In the process of numerical calculation, the field parameters are same as those in reference (as shown in Table 1), and the drilling string is divided into 1000 units. The simulation time is 120s, and the time step is 0.002. Finally, the rotation speed and acceleration of the drilling string are calculated, and which are compared with the field test data were measured using ESM measurement sub for an ultra-deep vertical well in KeS operation area of Tarim Oilfield to verify the correctness of the ALTC nonlinear vibration model.

| Parameter | Value |
|-----------|-------|
| Well depth \( r \) (m) | Inner/outer diameter of drill pipe |
| Bit type | Outer diameter of DC |
| Outer diameter of drill | Length of DC |
| Ground rotary B | Drilling fluid density |

Table 1. Model calculation parameters [28]
Table 2. Calculation parameters of four ultra-HPHT wells

| Value | bit (m) | collar (m) | (r/min) | (kg/m^3) |
|-------|--------|------------|---------|----------|
| 6200  | 0.1086/0.12 PD 7 0.3334 | 0.2032 | 178.7 | 120.0 | 140 | 1800 |

Figure 7 shows the calculation result of bit speed varying with time at 5165m depth, and it can be observed in which, there are viscous phase and slippage phase both in test date and numerical simulation data for drilling bit rotation speed. In the numerical simulation data, the rotational speed of viscous phase is 0.0 r/min, and the maximum rotational speed of slippage phase is 273.8 r/min. In the test data, the maximum rotational speed of slippage phase is 330.0 r/min, which is obviously higher than the peak speed in other measured data. Therefore, it can be judged that the maximum speed of 330.0 r/min is accidental, which cannot reflect the general regular pattern of stick slip vibration of drilling bit. The maximum speed should be removed in the data comparison, and the maximum speed of 302.7 r/min is obtained. Then, by comparing the drilling bit speed amplitude, the accuracy of the model is 90.5%. On the other hand, the average periods of stick slip vibration and viscous phase are 9.4 s and 2.4 s in the theoretical model calculation results, and 9.7 s and 2.2 s in the measured results, which indicated that the accuracy of the model is 96.9% and 91.7%. Thus, the comparison results effectively verifies the correctness of the proposed nonlinear model. Because the vibration of drill string is affected by the contact friction with wellbore and the buoyancy of drilling fluid, the vibration is very complex. In addition, the friction coefficient between tubing and wellbore changes greatly on-site, which has a great impact on the triaxial acceleration, but the theoretical model cannot consider the change of friction coefficient. Therefore, this work mainly compares the overall change trend. It can be seen that the triaxial acceleration simulation results have the same trend as the triaxial acceleration results measured on-site, which can reflect the synchronous periodic characteristics of "strong vibration-weak vibration-weak vibration" when the stick slip vibration of the drilling string occurs.

3.2. Validity verification

Stick slip vibration of drill string is a kind of harmful movement in the drilling process, which seriously affects the effective drilling of bit. That is to say, the longer the time of viscous phase in stick slip vibration, the smaller the ROP is. In order to further verify that the proposed model can effectively analyze the stick slip characteristics of the drilling string system, according to the parameters (as shown in Table 2) of four ultra-HPHT wells (10-A, 10-B, 10-C and 10-D) in Ledong block, South China Sea, the viscous time of the drill string system were calculated (as shown in red curve of Figure 8), which were compared with the measured ROP data (as shown in black curve of Figure 8). It can be observed from these Figs. that the variation trend of viscosity time of drill string system was opposite to that of ROP, which can verify that the proposed model can effectively predict the ROP change trend of drilling string system by calculating its viscosity time, and lay the foundation for improving the ROP of drilling string system on-site.

Figure 7. Vibration response of drilling string system at 5165m depth

Table 2. Calculation parameters of four ultra-HPHT wells
4. Model validation

An ALTC nonlinear vibration model of drilling string system in ultra-HPHT curved wells was established using energy method, and Hamilton principle, in which, the influence of wellbore trajectory change, wellbore constraint, interaction between bit and rock and ultra-HPHT of wellbore on elastic modulus of drill string and viscosity of drilling fluid were taken into account. Because the nonlinear factors considered in the model are complex and its solution was extremely difficult, the authors use Lagrange function and Cubic-Hermite function to discrete the vibration control equation, and uses the incremental form of Newmark-β method and Newton Raphson method to solve the discrete control equation, so as to realize the numerical solution of the nonlinear vibration model. Finally, the correctness and validity of the ALTC nonlinear vibration model was verified by comparing the measured data with the theoretical calculation results of the model.

References
[1] Shadravan A and Amani M 2012 HPHT 101: What every engineer or geoscientist should know about high pressure high temperature wells SPE-163376-MS
[2] Jardine S, Malone D and Sheppard M 1994 Putting a damper on drilling's bad vibrations Oilfield Review 6(1) 15-20
[3] Bailey J R, Biediger E, Sundararaman S, Carson A, Elks W and Dupriest F 2008 Development and Application of a BHA Vibrations Model IPTC-12737-MS
[4] Brett J F 1992 The genesis of torsional drill string vibrations SPE-21943-PA
[5] Tucker W R and Wang C 1999 An integrated model for drill-string dynamics J. Sound Vib. 224(1) 123-165
[6] Challamel N 2000 Rock destruction effect on the stability of a drilling structure J. Sound Vib. 233(2) 235-254
[7] Liu Y, Lin W, Páez C J and De Sa R 2019 Torsional stick-slip vibrations and multistability in drill-strings. Appl. Math. Model. 76 545-557
[8] Fu M, Zhang P, Li J and Wu Y 2019 Observer and reference governor based control strategy to suppress stick-slip vibrations in oil well drill-string J. Sound Vib. 457 37-50
[9] Lin W, Páez C J, Liu Y, Yang Y and Kuang Y 2020 Stick-slip suppression and speed tuning for a drill-string system via proportional-derivative control Appl. Math. Model. 82 487-502

[10] Bailey J and Finnie I 1960 An analytical study of drill-string vibration J. Manuf. Sci. Eng.-Trans. ASME 82(2) 122-127

[11] Skaugen E, Kyllingstad A, Ararestad T V and Tonnesen H A 1987 Experimental and theoretical studies of vibrations in drill strings incorporating shock absorbers WPC-22217

[12] Dareing D W 1985 Vibrations increase available power at the bit J. Energy Resour. Technol.-Trans. ASME 107(1), 138-141

[13] Wiercigroch M, Vaziri V and Kapitaniak M 2017 RED: Revolutionary drilling technology for hard rock formations SPE-184665-MS

[14] Li S, Vaziri V, Kapitaniak M, Millett J and Wiercigroch M 2020 Application of resonance enhanced drilling to coring J. Pet. Sci. Eng. 188 106866

[15] Liao M, Liu Y, Páez C J, Chong A and Wiercigroch M 2018 Dynamics of vibro-impact drilling with linear and nonlinear rock models Int. J. Mech. Sci. 146-147 200-210

[16] Ghasemloonia A, Geoff R D and Butt S D 2015 A review of drillstring vibration modeling and suppression methods J. Pet. Sci. Eng. 131 150-164

[17] Li Z, Zhang C and Song G 2017 Research advances and debates on tubular mechanics in oil and gas wells J. Pet. Sci. Eng. 151 194-212

[18] Zheng X, Agarwal V, Liu X and Balachandran B 2020 Nonlinear instabilities and control of drill-string stick-slip vibrations with consideration of state-dependent delay J. Sound Vibr. 473 115235

[19] Liu X, Long X, Zheng X, Meng G and Balachandran B 2020 Spatial-temporal dynamics of a drill string with complex time-delay effects: bit bounce and stick-slip oscillations Int. J. Mech. Sci. 170, 105338

[20] Kapitaniak M, Vaziri V, Páez C and Wiercigroch M 2018 Experimental studies of forward and backward whirls of drill-string Mech. Syst. Signal Proc. 100 454-465

[21] Tran Q, Nguyen K, Manin L, Andrianoely M, Dufour R, Mahjoub M and Menand S 2019 Nonlinear dynamics of directional drilling with fluid and borehole interactions J. Sound Vibr. 462 114924

[22] De M and Savi M A 2019 Drill-string vibration analysis considering an axial-torsional-lateral nonsmooth model J. Sound Vibr. 438 220-237

[23] Zhang, H., 2019. Research on the mechanism of vibration excitation and the drillstring dynamics in ultra-deep wells. Shanghai University

[24] Liang E, Li Z and Zou D 2009 Experimental research on integrated mechanical model of PDC bit Rock and oil Mechanics 30(4) 938-942

[25] Wang H 1989 Introduction to thermoelasticity (Beijing: Tsinghua University Press)

[26] Wang H, Liu Y and Yang L 2000 Effect of temperature and pressure on drilling fluid density in HTHP wells Drilling & Production Technology 01 58-62

[27] Bu H, Sun J, Wang C, Yang Z and Yang Y 2012 Study on the rheological properties of ultra-high temperature drilling fluids Journal of Southwest Petroleum University (Science & Technology Edition) 34(4) 122-126

[28] Teng X, Di Q, Li N, Chen F, Zhou B and Wang M 2017 Measurement and analysis of stick-slip characteristics of drill string in ultra-deep wells. Petroleum Drilling Techniques 45(2) 32-39

Acknowledgments
This work was partially supported by the China Postdoctoral Science Foundation (Grant No. 2021TQ0273), the National Natural Science Foundation of China (Grant No. 51875489) and the China Scholarship Council (Award No. 201908510191).