Robust Multi-robot Formations via Sliding Mode Controller and Fuzzy Compensator

To form up a multiple-robot system, a robust adaptive control scheme is addressed. The control scheme is based on the methodology of sliding mode control (SMC). The formation system is leader-follower-based, whose dynamics are subject to uncertainties. A fuzzy compensator is adopted to approximate the uncertainties. To attenuate the approximation error, a robust adaptive law of the fuzzy compensator is introduced. In the sense of Lyapunov, not only such a control scheme can asymptotically stabilize the whole formation system, but also the convergence of the approximation error can be guaranteed. Compared with the sole sliding mode controller without compensator, some numerical simulations verify the feasibility and effectiveness of the control scheme for the multiple-robot system in the presence of uncertainties.

Key words: Multiple-robot system, Sliding mode, Leader-follower formation, Uncertainties, Fuzzy compensator

1 INTRODUCTION

A multi-robot system composed of several mobile robots can perform collaborative tasks in manufacturing, surveillance and space exploration. Such coordination among the robots is an important and promising research field [1–3]. Compared to a complex single-robot system, the ability of coordinating the robots is superior, including but not limited to economy, effectiveness and efficiency [4, 5]. There are many cooperative tasks like formation, coverage and target aggregation, where the formation control problem stands out because the robots can efficiently accomplish cooperative tasks by forming up and maintaining some formations [6, 7].

Up to now, various formation schemes have been presented and carried out. They can roughly be classified as behavior-based methods, virtual structure techniques and leader-follower schemes [4, 8]. Among them, the leader-follower approaches have become the most popular ones because the idea is simple enough to permit complete analyses and experiments, but the internal formation stability can be theoretically guaranteed [9, 10].

The basic idea of the leader-follower formations [8, 11] is that one robot in a multi-robot system is selected as leader and the other robots are called followers. The leader is responsible for guiding the formation and the followers are required to track the leader in the desired distance \( l \) and the desired relative angle \( \psi \). Accordingly, there are two kinds of control frameworks, i.e., \( l - l \) and \( l - \psi \). Considering the leader-follower approaches, a characteristic is that the formation problem of the multi-robot system is transformed into the independent trajectory tracking problem of each follower robot [4, 8–11]. Although the basic idea is in dispute over its 'single point of failure', such characteristic benefits the formation problem a lot.

Concerning the leader-follower formation maneuvers, various control methods have been reported, i.e., model
predictive control [10], feedback linearization control [12], backstepping control [13], decentralized control [14], intelligent control [15, 16] and sliding mode control (SMC) [17–20], to name but a few. Since SMC is of invariance to matched uncertainties [21, 22], the SMC method has been paid more and more attention. This paper investigates the application of SMC for the leader-follower formation control problem of multiple robots.

Uncertainties are inevitable in practice [23]. The leader-follower formation maneuvers are subject to uncertainties [24]. The uncertainties include parameter fluctuations, model uncertainties and random external disturbances, which turn out to be mismatched [11]. Since the mismatched uncertainties cannot be suppressed by the invariance of SMC, a critical assumption in the aforementioned reports [17–20] is that the uncertainties must have a known boundary in order to guarantee the formation stability. However, the assumption is not mild because the boundary value is hard to exactly know in advance. The lack of the important information may cause several drawbacks such as deficiency of the system stability, decrease of the system robustness and deterioration of the system performance. In order to take SMC into account for the formation control problem, it is necessary to approximate and compensate the uncertainties.

The methodology of fuzzy logic can effectively handle complex nonlinear systems with uncertainties [25, 26]. Some real applications have been reported, such as MEMS triaxial gyroscope system [27], mechanical manipulator [28] and vector voltmeter [29]. In order to attack the uncertainties of the SMC-based formation control problem, this paper considers the fuzzy logic method as an alternative choice. So far, how to overcome the adverse effects triggered by such mismatched uncertainties still remains unsolved and problematic for the SMC-based formation maneuvers.

Since an adaptive fuzzy inference system can approximate a wide class of nonlinear systems via arbitrary closeness [27], this paper proposes a multiple-input multiple-output fuzzy inference system to compensate the uncertainties. The fuzzy system has to adaptively approximate the uncertainties because the assumption that the uncertainties have an unknown boundary holds true. To attenuate the approximation error, a robust adaptive law is employed. The fuzzy compensator and the sliding mode controller work in parallel to achieve the robust formation maneuvers. The SMC law and the robust adaptive law are derived from the Lyapunov’s direct method. The feasibility, validity and robustness of the proposed approach are illustrated by a multi-robot system. Compared with the results via the sole SMC method without compensator, the proposed approach can improve the tracking performance. The benefit we can earn from the proposed approach is that the approach is only subject to the mild assumption that the uncertainties have an unknown boundary.

2 MATHEMATICAL MODEL

2.1 Modeling a single robot

The robot in Fig. 1 is unicycle-like. It is round with 2r in diameter and has two parallel driven wheels. The wheels are independently controlled by two DC motors and they are with the same axis. One wheel is mounted on each side of the robot’s center. Since the robot is capable of simultaneous arbitrary rotation and translation in the horizontal plane, it can be described by a 3-dimensional vector \( \tau = [x \ y \ \theta]^T \). The translational coordinates can be depicted by \( (x, y) \), which are the midpoint between the two driven wheels. The midpoint is also the center of the round robot. Shown by the dash lines in Fig. 1, another coordinate system fixed on the robot’s body is assigned. The rotational coordinate \( \theta \), meaning the orientation angle with respect to the fixed frame, can be calculated via the two coordinate systems. To capture the robot’s position, a position sensor is located at the front castor.

Consider a multiple robot system composed of \( N \) robots. The robots in the group are identical (See in Fig. 1). Assuming that pure rolling and non-slipping motion, the ideal kinematic equations of the \( n \)th robot [17] are described by

\[
\tau_n = \begin{bmatrix}
\dot{x}_n \\
\dot{y}_n \\
\dot{\theta}_n
\end{bmatrix} = \begin{bmatrix}
\cos \theta_n & 0 \\
\sin \theta_n & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
v_n \\
w_n
\end{bmatrix} \quad (1)
\]

where \( v_n \) and \( w_n \) are the linear and angular velocities of the \( n \)th robot, respectively. The assumption means (1) is subject to the following nonholonomic constraint.

\[
[-\sin \theta_n & \cos \theta_n & 0] \cdot \dot{\tau}_n = 0 \quad (2)
\]

Consider the variations on the mass and the moment of inertia and the external uncertainties like the slipping
or skidding effects. The dynamic equations to depict the motion of the ith robot [17] are defined by
\[
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{\theta}_i
\end{bmatrix} = \begin{bmatrix}
-y_i \theta_i \\
x_i \theta_i \\
0
\end{bmatrix} + \begin{bmatrix}
\cos \theta_i & 0 & 0 \\
\sin \theta_i & 0 & 0
\end{bmatrix} \begin{bmatrix}
\alpha_i \\
\beta_i \\
\gamma_i
\end{bmatrix} \Delta_i + \pi_i (\tau_i, \tau_i)
\]

(3)

where the control input is \( u_i \), defined by \( u_i = [\alpha_i \beta_i]^T \). \( \alpha = F_n/m_n \) is the robot acceleration and \( \beta = \sigma_n/J_n \) is the angular acceleration of the robot, where \( m_n, J_n, F_n \) and \( \sigma_n \) denote the nominal mass, the nominal moment of inertia, the force and the torque applied to the robot, respectively. \( \Delta_i \) is the parameter variations, written by
\[
\Delta_i = \begin{bmatrix}
\varepsilon_i & 0 \\
0 & \varepsilon_i'
\end{bmatrix}
\]

here \( \varepsilon_i \) and \( \varepsilon_i' \) represent the variations on the mass and the moment of inertia, respectively. In (3), \( \pi_i (\tau_i, \tau_i) \) written by \( [\pi_{\alpha\theta}, \pi_{\theta\theta}, \pi_{\theta\theta\theta}]^T \) means the mixed model uncertainties and external disturbances.

Remark 1: (3) formulates the dynamic model of the single robot with uncertainties in Fig. 1. There are two terms to depict the system uncertainties. Mathemetic definitions about the category of uncertainties can be found in [23]. Since \( \Delta_i \) enters the robot system by the control channel, the parameter variations in (3) belong to matched uncertainties without doubt. Since \( \pi_i (\tau_i, \tau_i) \) mixes model uncertainties and external disturbances, it is usually matched. The two uncertain terms will result in the uncertainties of the following formation dynamics.

2.2 Leader-Follower Formation Scheme

In the group of robots, each robot is with its own kinematic and dynamic models as (1) and (3), respectively. To realize their formation maneuvers, it is necessary to define a control mechanism. The leader-follower approach is such a mechanism to coordinate the robots. The approach is hierarchical. One robot in the group is assigned to be the leader robot. The other robots are named followers. The relationship among the followers is equal to each other. Each follower is cascaded to the leader so that \( N - 1 \) leader-follower pairs exist. Take a leader-follower pair as an example. The follower’s motion is decided by the leader. The decision is made in light of the desired distance and the desired relative angle between the follower and the leader.

Without loss of generalization, the ith robot in the group is picked as the leader robot. One robot among all the followers is also selected and index \( k \) represents the follower. The pair in Fig. 2 is employed to demonstrate the leader-follower mechanism. Provided that a receiver is located at the centre of the leader robot \( i \), Fig. 2 illustrates the definitions of the distance \( l_{ik} \) and the relative angle \( \psi_{ik} \) between the leader and the follower. \( l_{ik} \) is defined by the distance between the centre of the leader robot \( i \) and the front caster of the follower robot \( k \), formulated by
\[
l_{ik} = \sqrt{(x_i - \bar{x}_k)^2 + (y_i - \bar{y}_k)^2}
\]

(4)

In (4), the coordinates \( (x_i, y_i) \) denote the centre position of the leader and the coordinates \( (\bar{x}_k, \bar{y}_k) \) denote the caster position of the follower, where \( \bar{x}_k = x_k + r \cos \theta_k, \bar{y}_k = y_k + r \sin \theta_k \) and \( r \) is the radius of the robots.

According to the geometric relationship, \( \psi_{ik} \) in Fig. 2 is determined by
\[
\psi_{ik} = \pi + \zeta_{ik} - \theta_i
\]

(5)

here \( \theta_i \) is the orientation angle of the leader robot \( i \) and \( \zeta_{ik} \) is calculated by
\[
\zeta_{ik} = \arctan \frac{y_i - y_k - r \sin \theta_k}{x_i - x_k - r \cos \theta_k}
\]

(6)

In order to achieve the formation maneuvers, each follower needs to keep at the desired distance and the desired relative angle with respect to the leader. Since the robots are subject to uncertainties, their formation dynamics inherently become uncertain. The motivation of the paper is to develop a control scheme that can form up the robots while resist the adverse effects of uncertainties on the formation stability. To concentrate on the motivation, The following ideal conditions are considered, 1) there are no collisions between the robots, 2) there is no communication delay between the leader and all the followers, 3) each follower robot knows not only its own position and velocity but also the corresponding information of the leader.
Differentiate (4) and (5) twice with respect to time $t$ and substitute (2) into the derivatives. Then, the second derivatives of $l_{ik}$ and $\psi_{ik}$ can be obtained. Define $T_{ik} = [l_{ik} \quad \psi_{ik}]^T$.

The dynamic equations depicting the leader-follower formation maneuvers [11] are described by

$$\dot{T}_{ik} = G_{ik}(I_2 + \Delta_l)u_k + L_{ik}(I_2 + \Delta_l)u_l + F_{ik} + P_{ik}$$  \hspace{1cm} (7)

where $I_2$ is a $2 \times 2$ identity matrix, $G_{ik}$ is written by

$$G_{ik} = \begin{bmatrix} \cos \phi_{ik} & r \sin \phi_{ik} \\ -\sin \phi_{ik} & \cos \phi_{ik} \end{bmatrix}$$

$L_{ik}$ is described by

$$L_{ik} = \begin{bmatrix} -\cos \psi_{ik} & 0 \\ \sin \psi_{ik} & -1 \end{bmatrix}$$

here $\phi_{ik} = \psi_{ik} + \theta_{ik}$ and $\theta_{ik} = \phi_{ik} - \phi_{ik}$. Further, $F_{ik} = [F_1 \quad F_2]^T$ and $P_{ik} = [P_1 \quad P_2]^T$ are given as follows.

$$F_1 = \frac{1}{\Delta_k}l_{ik}^2 + 2\psi_{ik}\theta_{ik}l_{ik} + (\theta_k^2 + G_{ik}^2) - \cos \phi_{ik} \dot{\theta}_k^2 - (\dot{x}_k \dot{\theta}_k - x_k \dot{\theta}_k) \cos(\psi_{ik} + \theta_k) - (\dot{x}_k \dot{\theta}_k - \dot{x}_k \dot{\theta}_k) \sin(\psi_{ik} + \theta_k)$$

$$F_2 = \frac{1}{\Delta_k}l_{ik}^2 \dot{x}_k^2 \cos(\psi_{ik} + \theta_k) - (\dot{x}_k \dot{\theta}_k - \dot{x}_k \dot{\theta}_k) \sin(\psi_{ik} + \theta_k) - \dot{\theta}_k \dot{\theta}_k \sin \phi_{ik}$$

$$P_1 = -(\pi_{i_k} - \pi_{b_k}) \cos(\psi_{ik} + \theta_k) - (\pi_{i_k} - \pi_{b_k}) \sin(\psi_{ik} + \theta_k) + r \pi_{i_k} \sin \phi_{ik}$$

$$P_2 = \frac{(\pi_{x_k} - \pi_{b_k}) \sin(\psi_{ik} + \theta_k)}{l_{ik}} - (\pi_{x_k} - \pi_{b_k}) \cos(\psi_{ik} + \theta_k) - r \pi_{x_k} \cos \phi_{ik} - l_{ik} \pi_{i_k} \phi_{ik}$$

Define $x_{ik} = [x_{ik} \quad \Delta_{ik}]^T = [l_{ik} \quad \psi_{ik}]^T$. Then, the following state equations are deduced from (7).

$$\dot{x}_{ik} = G_{ik}u_k + F_{ik}(x_{ik}, \Delta_{ik}, t) \hspace{1cm} (8)$$

where $F_{ik}(x_{ik}, \Delta_{ik}, t) = G_{ik}\Delta_{ik}u_k + L_{ik}(I_2 + \Delta_l)u_l + F_{ik} + P_{ik}$.

In the mentioned literature about the SMC-based formation maneuvers [17–20], one common assumption is that the uncertain term $d_{ik}$ has a known boundary. However, the assumption is too strict to be satisfied in reality. Owing to the invariance of SMC, this paper considers that these uncertainties are mismatched with an unknown boundary. To achieve the formation maneuvers under the challenging assumption, a fuzzy compensator will be designed to attack the issue.

$$R = \bigcup_{l=1}^{m} R_l$$  \hspace{1cm} (9)

In (9), $R$ indicates a collection of fuzzy rules, $M$ denotes the total number of rules and the $l$th rule is described by

$$R_l : \text{IF } a_1 \text{ is } A_{1l}^p \text{ and } a_2 \text{ is } A_{2l}^q \text{ ... } a_m \text{ is } A_{ml}^m \hspace{1cm} (10)$$

$$\text{THEN } b_1 \text{ is } B_{1l}^p \text{ and } b_n \text{ is } B_{nl}^q$$

$A_{jl}^p$ and $B_{ql}^q$ in (10) are the linguistic variables of the fuzzy sets, where $p = 1, 2, \ldots, m$ and $q = 1, 2, \ldots, n$. Both the linguistic variables are determined by their
membership functions $\mu_{A_l'}(a_p)$ and $\mu_{b_l}(b_q)$, here $l = 1, 2, \ldots, M$.

Adopting the singleton fuzzifier mapping, the product
inference engine and the center-average defuzzifier mapping, the $q$th output of the FIS has a form of

$$b_q = \sum_{i=1}^{M} \tilde{b}_{iq} \xi_q(a) = \Theta_q^T \tilde{\xi}_q(a) \quad (11)$$

In (11), $\tilde{\xi}_q(a) = [\xi_1(a), \ldots, \xi_M(a)]^T \in \mathbb{R}^M$ is the fuzzy basis functions vector, where $\xi_q(a)$ is calculated by

$$\xi_q(a) = \frac{\prod_{i=1}^{M} \mu_{A_{q'i}}(a_p)}{\sum_{i=1}^{M} (\prod_{i=1}^{M} \mu_{A_{q'i}}(a_p))}$$

$\Theta_q = [\tilde{b}_{q1}, \ldots, \tilde{b}_{qM}]^T \in \mathbb{R}^M$ is named the parameter vector, where $\tilde{b}_{q}$ is an adaptively adjustable parameter. The vector is the $q$th column of an $M \times n$ parameter matrix $\Theta$.

Associated with the application of the FIS, the purpose is to approximate the system uncertainties in (8) so that the approximate values are definitely selected as the FIS outputs, i.e., $b = \tilde{d}_k(x_k, x_k, t)$. Moreover, the system uncertainties in (8) are functions of $x_k$ and $x_k$ so that $l_k, \varphi_k, \hat{l}_k$ and $\hat{\varphi}_k$ are picked up as the FIS inputs, i.e., $a = [l_k, \varphi_k, \hat{l}_k, \hat{\varphi}_k]^T$.

The linguistic labels of $A_l'$ are chosen as five levels, i.e., NB, NS, ZO, PS and PB. The labels denote negative big, negative small, zero, positive small and positive big, respectively. The membership function of $A_l'$ is Gaussian, defined by

$$\mu_{A_{l'}}(a_p) = \exp \left(-\frac{(a_p - c_p)^2}{2\sigma_p^2}\right) \quad (12)$$

where $c_p$ and $\sigma_p$ are the center and width of the Gaussian membership function to describe the linguistic variable $A_{l'}$ and $p = 1, 2, 3, 4$.

From (11), the approximate values $\tilde{d}_k(x_k, x_k, \hat{x}_k, t)$ can be calculated by $\tilde{d}_k(x_k, x_k, \hat{x}_k, t)$, written by

$$\tilde{d}_k(x_k, x_k, \hat{x}_k, t) = \begin{bmatrix} \tilde{d}_{l,k,1}(x_k, x_k, \hat{x}_k, \hat{x}_k) \\ \tilde{d}_{l,k,2}(x_k, x_k, \hat{x}_k, \hat{x}_k) \end{bmatrix} = \Theta^T \xi(x_k, x_k) \quad (13)$$

here $\Theta \in \mathbb{R}^{625 \times 2}$ and $\xi(x_k, x_k) \in \mathbb{R}^{625 \times 1}$.

Assuming that an optimal parameter matrix $\Theta^*$ exists [25], the matrix $\Theta^*$ has a form of

$$\Theta^* = \arg\min_{\Theta \in Z_0} \sup |\tilde{d}_k(x_k, x_k, \hat{x}_k, t) - d_k(x_k, x_k, \hat{x}_k, t)|$$

where $\chi_0$ is a proper compact set. Considering the assumption, the minimum approximate errors vector can be determined by

$$\rho = \hat{d}(x_k, \hat{x}_k)|_{\Theta^*} - \hat{d}(x_k, \hat{x}_k, t) \quad (15)$$

where $\rho = [\rho_1, \rho_2]^T \in \mathbb{R}^2$.

The designed FIS is a 4-input–2-output system and the linguistic variables of the inputs are divided into 5 levels. Consequently, the FIS covers $5^4$ fuzzy rules in its knowledge base. There are two ways to reduce the number of fuzzy rules. One is to cut the number of inputs. The other is to reduce the number of levels. From (8), the uncertain term $\hat{d}_k(x_k, \hat{x}_k, t)$ is functions of $x_k$ and $\hat{x}_k$. Once the number of inputs is decreased, the FIS may not work because it cannot obtain enough information. Once the number of levels is reduced, the accuracy of the FIS will definitely decrease. The accuracy of the FIS may increase if the number of levels is increased. But the increase will result in computational burden. Here, the number of levels is selected as 5 by a trade-off between accuracy and computational burden.

**Remark 3:** It has been proven that fuzzy systems in the form of (11) can approximate continuous function over a compact set to an arbitrary degree of accuracy provided that enough number of rules is given [25]. Here the ability of approximation of the FIS is employed to approximate the unknown uncertainties.

### 3.2 Design of Sliding Mode Controller

To coordinate the leader-follower pair in Fig. 2 by a sliding mode controller, the sliding surfaces vector is defined by

$$s_k(t) = \dot{x}_k + \lambda x_k$$

where $\lambda$ is the sliding mode controller gain. The sliding mode controller gain $\lambda$ is selected as 5 by a trade-off between accuracy and computational burden.

Pointed out by V.I. Utkin [23], the SMC method covers two stages. One is sliding mode stage. The other is reaching mode stage. On the reaching mode stage, the system states in (8) tend to the sliding surfaces vector (16) in the state space. On the sliding mode stage, not only the SMC states in (8) are decreasing. The accuracy of the FIS may increase if the number of levels is increased. But the increase will result in computational burden. Here, the number of levels is selected as 5 by a trade-off between accuracy and computational burden.

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mode stage. However, neither (17) nor (18) holds true in the reaching mode stage. For the sake of formula expression, a reference vector is introduced as

\[ \delta(t) = \delta^T_k(t) - \lambda x^T_k(t) \]  

(19)

Finally, the SMC-based control law is designed as

\[ u_k = -G^{-1}_{ik} \left[ \delta^T_k(x_k, \hat{x}_k(\Theta)) \right] - \delta + \kappa x_k + \eta \text{sign}(s_k) \]  

(20)

where \( \kappa = \text{diag}\{ \kappa_1, \kappa_2 \} \), \( \eta = \text{diag}\{ \eta_1, \eta_2 \} \), \( \kappa_1, \kappa_2, \eta_1, \) and \( \eta_2 \) are predefined parameters. Further, \( s_k \) is defined by \( \text{sign}(s_k) = [\text{sign}(s_{ik,1}(t)), \text{sign}(s_{ik,2}(t))]^T \).

From (20), the control signal \( u_k \) of the leader \( i \) actually exists as disturbances for the follower \( k \). Since the task of the leader \( i \) is to track a predefined trajectory, the leader's control problem can be considered as the trajectory tracking of a single robot. There has been a great deal of work in this field and different control methods have been developed. Usually, it is directly assumed that the leader robot is well controlled by a developed technology [9].

### 3.3 Stability Analysis

**Theorem 1:** Define the sliding surfaces vector (16), design the fuzzy compensator (13) and adopt the control law (20). The SMC-based formation control system with fuzzy compensator is of asymptotical stability if

\[ \dot{\theta}_q = \Gamma_q^{-1} s_{ik,q} \tilde{\zeta}(x_k, \hat{x}_k) \]  

(21)

where \( \Gamma_q > 0 \in \mathbb{R}^1, \kappa_q > 0, \eta_q > |\rho_q| \) and \( q = 1, 2 \).

**Proof:** Take the following positive definite function into account as a Lyapunov candidate function.

\[ V(t) = \frac{1}{2} (s_{ik}^T s_{ik} + \sum_{q=1}^{2} \Theta_q^T \Gamma_q \Theta_q) \]  

(22)

where \( \Theta_q = \Theta_q^* - \theta_q \) and \( \Theta_q^* \) is the \( q \)-th column vector of the optimal parameter matrix \( \Theta^* \).

Differentiating (22) with respect to \( t \) gives

\[ \dot{V}(t) = s_{ik}^T \dot{s}_{ik} + \sum_{q=1}^{2} \Theta_q^T \Gamma_q \dot{\Theta}_q \]  

(23)

From (16) and (19), (24) can be obtained.

\[ s_{ik}(t) = \hat{s}_{ik}(t) - \hat{\delta}(t) \]  

(24)

Differentiating (24) with respect to \( t \) and substituting the first derivative of \( s_{ik} \) into (23) yields

\[ \dot{V}(t) = s_{ik}^T \dot{s}_{ik} - \delta \]  

(25)

From (8), (26) can be formulated by

\[ \dot{V}(t) = s_{ik}^T \left[ \hat{d}_{ik} u_k + d_{ik}(x_k, \hat{x}_k, t) - \delta \right] + \sum_{q=1}^{2} \Theta_q^T \Gamma_q \dot{\theta}_q \]  

(26)

Substitute (15) and (20) into (26). Then, (27) can be drawn

\[ \dot{V}(t) = -s_{ik}^T [\hat{d}_{ik}(x_k, \hat{x}_k(\Theta)) - d_{ik}(x_k, \hat{x}_k, t) + \kappa x_k] + \eta \text{sign}(s_k) \]  

\[ + \sum_{q=1}^{2} \Theta_q^T \Gamma_q \dot{\theta}_q \]  

(27)

Adopting the robust adaptive law (21), (27) becomes

\[ \dot{V}(t) = -s_{ik}^T \rho - s_{ik}^T \kappa x_k - s_{ik}^T \eta \text{sign}(s_k) \]  

(28)

Substituting \( \eta_q > |\rho_q| \) into (28) yields

\[ V(t) \leq -s_{ik}^T \kappa x_k < 0 \]  

(29)

(29) means that the Lyapunov function (22) satisfies \( \dot{V}(t) < 0 \). Consequently, the SMC-based formation control system with fuzzy compensator is asymptotically stable in the sense of Lyapunov once the robust adaptive law (21) is adopted.

\[ d_{ik}(x_k, \hat{x}_k, t) \]  

(30)

is mismatched and it has an unknown boundary. Owing to its adverse effects, the stability of the formation system cannot be guaranteed by the invariance of SMC. In order to guarantee the system stability, the designed FIS is employed to approximate \( d_{ik}(x_k, \hat{x}_k, t) \) so that the SMC-based formation controller has the ability against the adverse effects of mismatched uncertainties.

**Remark 4:** \( G_{ik} \) is invertible. \( G_{ik} \) is formulated by (7), \( \det(G_{ik}) = \frac{1}{m} < 1 \). Note that \( l_{ik} \) is formulated by the distance between the center of the leader \( i \) and the front caster of the follower \( k \), that is, \( l_{ik} \) is always larger than \( r \).

### 4 SIMULATION RESULTS

The designed robust adaptive control scheme is composed of one sliding mode controller and one fuzzy compensator. The structure of the presented control scheme is illustrated in Fig. 4. Concerning the control structure,
the fuzzy compensator is located at the feedforward channel and cooperates with the sliding mode controller placed at the feedback channel. The compensator and controller work in parallel to achieve the formation maneuvers. To demonstrate the feasibility, validity and robustness of such a control scheme, some simulations are performed via a multi-robot system and the results are discussed in this section.

The multi-robot system consists of one leader and two followers. The three mobile robots are identical (See in Fig. 1). Each robot is 0.05m in radius. The leader is controlled by the kinematic based sliding mode controller to track its predefined trajectories [30]. The two followers each have the presented robust adaptive control scheme (20) to track their leader, independently. The formation objective is to make the two followers maintain their desired distances and relative angles with respect to the leader in the presence of uncertainties.

The parameter variation matrices of the three robots are considered by

$$\Delta_l = \Delta_k = \begin{bmatrix} 0.3 \times \text{rand}() - 0.15 & 0 \\ 0 & 0.3 \times \text{rand}() - 0.15 \end{bmatrix}$$

(30)

here $i = 1$ denotes the leader and $k = 2$, 3 denotes the two followers. rand() is a MATLAB command and it generates a uniformly distributed random number on the open interval ($0, 1$).

From (3), $\Delta_n$ indicates the parameter variations such that (30) means that the payload variations on the mass and the moment of inertia are as much as maximum $\pm 15\%$ of the nominal parameters. The reason that we consider the $15\%$ variation is that each robot is driven by two DC motors. The $15\%$ parameter variation is enough to cover the regular parameter perturbation. Excessive variation may result in motor overload in practice.

Furthermore, $P_{ik}$ in (8) is related to $\pi_e$ in (3). Note that $\pi_1 = \pi_2$ and $\pi_3 = \pi_2$, exist in $P_{kk}$, meaning that it is not representative to define $\pi_1 = \pi_k$. Here, the following different functions are assigned to the leader and the two followers, defined by

$$\begin{align*}
\pi_{\alpha_l} &= \pi_{\alpha_2} = \pi_{\alpha_3} = 0.5 \sin(2\pi t) \\
\pi_{\beta_l} &= \pi_{\beta_2} = \pi_{\beta_3} = 0.2 \sin(\pi t)
\end{align*}$$

(31)

All the widths in (12) are set by $\frac{\sqrt{3\pi}}{8}$. Since the linguistic variables are divided into 5 levels, the centers of $c_p$ are set by $-\frac{\pi}{5}, -\frac{2\pi}{5}, 0, \frac{2\pi}{5}, \frac{\pi}{5}$, respectively. Other parameters in (21) are selected by $\gamma_1 = \gamma_2 = 0.00005$. Additionally, the parameter matrix $\lambda$ in (16) is chosen as $\lambda = \begin{bmatrix} 2.3 & 0 \\ 0 & 2 \end{bmatrix}$.

$\kappa$ and $\eta$ in (20) are determined by $\kappa = \begin{bmatrix} 5 & 0 \\ 0 & 15 \end{bmatrix}$ and $\eta = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$, respectively.

4.1 Line Formation Moving in Circular Trajectories

Consider a set of concentrically circular trajectories. The initial posture vector of the leader is located at $\tau_1 = [0.5m 0m \frac{\pi}{2}\text{rad}]^T$. The initial posture vectors of the two followers are set by $\tau_2 = [0.8m -0.4m 0\text{rad}]^T$ and $\tau_3 = [1m 0.5m \frac{\pi}{2}\text{rad}]^T$, respectively.

According to the above initial posture vectors, the initial distances and relative angles of the two followers with respect to the leader can be calculated as $x_{12}^0 = [0.5m 0.3\frac{\pi}{2}\text{rad}]^T$ and $x_{13}^0 = [0.707m 0.5\frac{\pi}{2}\text{rad}]^T$. The desired system states of the two followers are set by $x_{12}^f = [0.13m 1\frac{\pi}{2}\text{rad}]^T$ and $x_{13}^f = [0.26m 1\frac{\pi}{2}\text{rad}]^T$. The desired linear and angular velocities of the leader robot are set by $v_1^f = 0.5m/s$ and $w_1^f = 1\text{rad/s}$.

Fig. 5 demonstrates the group of robots forms up a line formation while moving in the concentrically circular trajectories, where the colorful squares denote the initial positions of the three robots and the colorful circles indicate the robots in the dynamic process. In each square or circle, the arrow means the orientation angle of the robot. To demonstrate their postures and formation, the black dash in Fig. 5 bonds the robots together at the same moment. From Fig. 5, the two followers follow the leader and keep the desired distances and the desired relative angles.

To demonstrate the formation performance of the presented control scheme, the comparisons between the designed robust adaptive SMC method (R-A SMC) and the sole SMC method are shown in Figs. 6-9, where the parameters of the sole sliding mode controller are kept unchanged from the presented control scheme.

The curves in Fig. 6a and b illustrate the distance $l_{12}$ and the relative angle $\psi_{12}$ between Leader 1 and Follower 2, respectively. In Fig. 6c and d, the distance $l_{13}$ and the relative angle $\psi_{13}$ between Leader 1 and Follower 3 are displayed. Fig. 7a and b shows the control inputs $\alpha_2$ and $\beta_2$ of Follower 2. The curves of the control inputs $\alpha_1$ and...
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The uncertainties in (31) are random. To overcome the phenomenon of chattering, a drawback of the SMC methodology, the jumping points of the sliding surface vectors in Fig. 12, the initial posture vector of the leader is set by $x^0_{12} = [0m 0m 0rad]^T$. The initial posture vectors of the two followers are set by $x^0_{13} = [0.4m -0.3rad]^T$ and $x^0_{14} = [0.3m -0.5rad]^T$. The desired states of the two followers are calculated as $x^d_{12} = [\sqrt{2}m -0.2\pi]^T$ and $x^d_{13} = [\sqrt{2}m -0.2\pi]^T$. The controller and compensator parameters are kept unchanged from the aforementioned straight line formation in the concentric circles.

Fig. 11 illustrates the multi-robot system forms up a straight line when moving along sinusoidal trajectories. In Fig. 11, the initial posture vector of the leader is set by $\tau_1 = [0m 0m 0rad]^T$ and the initial posture vectors of the two followers are set by $\tau_2 = [1m -1m -0.2\pi]^T$ and $\tau_3 = [-1m 1m -0.1\pi]^T$. The desired system states of the two followers are set by $x^d_{12} = [0.5m -0.2\pi]^T$ and $x^d_{13} = [0.26m -0.1\pi]^T$. The controller and compensator parameters are kept unchanged from the aforementioned straight line formation in the concentric circles.

4.2 More Formation Maneuvers

Fig. 9 is the outputs of the designed FIS. The sliding-surface vectors in Fig. 11 illustrate the multi-robot system forms up a straight line when moving along the concentric circles. In Fig. 11, the initial posture vector of the leader is set by $\tau_1 = [0m 0m 0rad]^T$ and the initial posture vectors of the two followers are set by $\tau_2 = [1m -1m -0.2\pi]^T$, and $\tau_3 = [-1m 1m -0.1\pi]^T$, respectively. According to the above three initial posture vectors, the initial distances and relative angles of the two followers with respect to the leader are calculated as $x^d_{23} = [0.13m -0.1\pi]^T$ and $x^d_{13} = [0.26m -0.1\pi]^T$. The controller and compensator parameters are kept unchanged from the aforementioned two formations.

4.3 Extensions

To demonstrate the scalability of the presented control scheme, two more robots are complemented to the aforementioned three-robot simulation platform. The five robots are identical. The presented control scheme is carried out by such an augmented multi-robot system. Displayed in Fig. 12, the five uncertain robots form up a pentagon when moving along straight line trajectories. In Fig. 12, the initial posture vector of the leader is set by $\tau_1 = [0m 0m 0rad]^T$. The initial posture vectors of the followers are set by $\tau_2 = [0m 0.3m 0rad]^T$, $\tau_3 = [0m 0.6m 0rad]^T$, $\tau_4 = [0m -0.3m 0rad]^T$ and $\tau_5 = [0m -0.6m 0rad]^T$, respectively. According to these initial posture vectors, the initial distances and relative angles of the four followers with respect to the leader can be calculated as $x^0_{12} = [0.3m -0.2\pi]^T$, $x^0_{13} = [0.6m -0.5\pi]^T$, $x^0_{14} = [0.3m -0.5\pi]^T$ and $x^0_{15} = [0.6m -0.5\pi]^T$. To form up and maintain the pentagonal formation, the desired system states of the followers are set by $x^d_{12} = [0.2m -0.3\pi]^T$, $x^d_{13} = [0.32m -0.2\pi]^T$, $x^d_{14} = [0.32m -0.4\pi]^T$ and $x^d_{15} = [0.2m -0.4\pi]^T$. The desired linear and angular velocities of the leader robot are set by $v^d_1 = 0.5m/s$ and $\omega^d_1 = 0rad/s$. The parameters of the controller and compensator for the
Fig. 6. Distances and relative angles of the two followers with respect to the leader. a. Distance $l_{12}$, b. Relative angle $\psi_{12}$, c. Distance $l_{13}$, d. Relative angle $\psi_{13}$.

Fig. 7. Accelerations and angular accelerations of the two follower robots. a. Acceleration of Follower 2 $\alpha_2$, b. Angular acceleration of Follower 2 $\beta_2$, c. Acceleration of Follower 3 $\alpha_3$, d. Angular acceleration of Follower 3 $\beta_3$. 
Fig. 8. Sliding-surface vectors and their elements. a. $s_{12,1}$, b. $s_{12,2}$, c three-dimensional plot of the sliding-surface vector $s_{12}$, d. $s_{13,1}$, e. $s_{13,2}$ and f. three-dimensional plot of the sliding-surface vector $s_{13}$.

Fig. 9. Uncertain term $d_{ik}$ and its approximation $\hat{d}_{ik}$. a. $d_{12,1}$ and $\hat{d}_{12,1}$, b. $d_{12,2}$ and $\hat{d}_{12,2}$, c. $d_{13,1}$ and $\hat{d}_{13,1}$, d. $d_{13,2}$ and $\hat{d}_{13,2}$. 
5 CONCLUSIONS

This paper has addressed the formation control problem for swarms of uncertain mobile robots. The formation mechanism is based on the leader-follower scheme. To suppress the mismatched uncertainties when forming up the robots, the SMC-based control method with fuzzy compensator is employed, where the SMC controller and the fuzzy compensator are located at the feedback and feedforward channels, respectively. The theoretical analysis proves that the coordinated formation control system with mismatched uncertainties is of asymptotic stability in the sense of Lyapunov. The presented control scheme has achieved the formation maneuvers of a multi-robot system composed of three mobile robots. The simulation results have demonstrated the effectiveness, feasibility and scalability of the presented control scheme.

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