Stability control of in-wheel motor driven vehicle based on extension pattern recognition

Wang Hongbo¹,²,³, Sun Youding¹, Tan Hongliang¹ and Lu Yongjie³
¹School of Automotive and Transportation Engineering, Hefei University of Technology, Hefei, China
²Anhui Intelligent Vehicle Engineering Laboratory, Hefei, China
³State Key Laboratory of Mechanical Behavior and System Safety of Traffic Engineering Structures, Shijiazhuang Tiedao University, Shijiazhuang, China

Abstract
According to the characteristics that the torque of each wheel of the in-wheel motor driven vehicle is independent and controllable, the stability control of in-wheel motor driven vehicle based on extension pattern recognition method is proposed in this paper. The dynamic model of the vehicle is established by Matlab/Simulink and Carsim. Taking two-degree-of-freedom (2-DOF) vehicle model as reference model, the vehicle yaw rate and the sideslip angle as the control objectives. The differences between the actual values and the reference values of the yaw rate and the actual sideslip angle are used to define the vehicle stability status. The vehicle stability status is divided into four stability control patterns, which are the no control pattern, the yaw rate control pattern, the yaw rate and sideslip angle joint control pattern, and the sideslip angle control pattern, respectively. The extension pattern recognition algorithm is used to determine the vehicle control pattern. The fuzzy controllers of yaw rate and sideslip angle are designed to obtain the additional yaw moment. Besides, the optimal torque distribution method is proposed by taking the lowest total energy loss of four motors as the objective function. The feasibility and effectiveness of the proposed control strategy are verified by Matlab/Simulink and Carsim joint simulation platform and hardware-in-the-loop (HIL) test.

Keywords
In-wheel motor driven vehicle, extension pattern recognition, direct yaw moment control, torque distribution, stability control

Corresponding author:
Lu Yongjie, State Key Laboratory of Mechanical Behavior and System Safety of Traffic Engineering Structures, Shijiazhuang Tiedao University, 17 Northeast, Second Inner Ring, Shijiazhuang, Hebei Province 050043, China.
Email: lu-yongjie@163.com

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Introduction

In order to alleviate energy shortage and reduce pollution, the new energy vehicle is a new direction of vehicle development. The in-wheel motor driven vehicle, which is a significant developing direction of new energy vehicle, plays a significant role in energy saving and emission reduction. Its vehicle structure is simplified and the way of driving is more flexible.\(^1\) Since the torque of each wheel is independent and controllable, the additional yaw moment can be generated by the difference of the driving/braking torque of each wheel, which greatly improves the motion control effect and energy efficiency of the vehicle, especially in the dynamic stability control of the vehicle.\(^4\) In recent years, research on the control of motor to enhance the motor performance has also promoted the further development of electric vehicles.\(^7\) Thus, the research on the in-wheel motor driven vehicle attracts the attention from many enterprises and scientific research institutions.

In Li et al.,\(^10\) the vehicle lateral stability was analyzed based on the response of sideslip angle. Then the PID control was designed for the active steering system to improve the yaw plane stability, and the simulation results showing that the vehicle lateral stability was enhanced. However, the simulation results demonstrated that this method could not guarantee the lateral stability when the sideslip angle was larger. In Zhang and Wu,\(^11\) the path tracking method combining the front-wheel steering and differential braking was proposed, which could compensate for the tracking deviation caused by the front-wheel steering under the complex condition. The simulation results showed that the combined control method could significantly improve the robustness of path tracking and vehicle stability. The in-wheel motor driven vehicle rollover stability was studied in Jin et al.,\(^12\) a hierarchical anti-rollover control strategy was proposed which could effectively improve the anti-rollover stability when the vehicle driving on the uneven road. A driver model was involved in the upper controller to predict the driver’s intention in Chen et al.,\(^13\) where the intention of driver and vehicle states were obtained and the required yaw moment was determined by adopting the sliding-mode control in the upper-layer controller, and the longitudinal forces were distributed according to the required yaw moment in the lower-layer controller. In references,\(^14\)\(^−\)\(^16\) the stability control of the four-wheel independent drive vehicle was studied through the development of the four-wheel independent motor driven electric vehicle. Different yaw rate control strategies were used to achieve stability control and optimal traction distribution.

In order to further improve the vehicle lateral stability, some scholars divided the stability region by vehicle states. In references,\(^17,18\) the phase plane method was used to determine the stability control state. In,\(^19,20\) through the division of vehicle stability status, different objectives were designed to be controlled under different stability patterns, and the desired additional yaw moment was obtained according to the designed yaw moment controller, so as to ensure vehicle stability. However, the irregularity and ambiguity of the stability boundary were not considered in the division of the stability pattern.

It can be seen that the four-wheel motor driven vehicle stability control is a very important research point due to its chassis structure. Whereas, the stability control
should not be the same when the vehicle is in different stability degrees. When the vehicle is stable or far from the stability domain, the adopted vehicle control should be different. Therefore, before the corresponding vehicle stability control is adopted, it is necessary to make clear the vehicle’s stability degree and use the proper method to divide the stability domain.

Extension pattern recognition is a method based on Extenics, which is proposed by Wen Cai in 1983. Nowadays, Extenics has been widely used in engineering control. In Zhao et al., to solve the contradiction between the control output and the control effect under extreme condition, the authors introduced the extension control into $\mathbb{H}_\infty$ mixed sensitivity control about sensitivity function $S/T$ problem. Extenics was also applied to the decision-making problem in lane departure assistant system. In order to solve the low accuracy of lane keeping control for intelligent vehicles on high curvature roads, a lane keeping control system based on extension switching control is proposed in Cai et al. Extension pattern recognition is a method using the matter element, relationship function and the calculation to determine which category the matter is belonging to. The stability degrees can be divided by the extension pattern recognition method by using the different patterns to represent the corresponding stability degrees. After the stability patterns are divided, the corresponding control is designed to enhance the vehicle stability.

When controlling the vehicle yaw stability, the wheel drive torque changes greatly, it will cause great energy loss. Therefore, we also considered the energy consumption of vehicle in this paper. Combined with the previous research results, on the basis of ensuring the vehicle yaw stability, we try to make the loss power of the motor driven system as low as possible, so that the motor is in the high efficiency working range to reduce the vehicle driving energy consumption and improve the endurance mileage.

In this paper, aiming at the yaw stability control of four-in-wheel motor driven electric vehicle, the vehicle dynamics model is established by Carsim and Matlab/Simulink, and the vehicle controller is established by Simulink. The vehicle stability is divided into four patterns with taking the difference between the actual value and the desired value of yaw rate and the actual sideslip angle as the character variables of the stability pattern division. Different objectives are controlled in different stability patterns and the real-time states of the vehicle are collected, then the extension pattern recognition algorithm is used to determine the stability pattern of the vehicle. The yaw rate controller and the sideslip angle controller are designed to determine the additional yaw moment needed to maintain the stability of the vehicle. The objective function is to minimize the total energy loss of the motor driven systems, and the additional yaw moment, the motor torque/speed output and the maximum adhesion provided by the road surface are set as the constraints to optimize the torque distribution. Finally, the simulation and HIL test results show that the proposed strategy can effectively guarantee the yaw stability of vehicle and reduce the energy consumption.
Dynamic modeling of in-wheel motor driven vehicle

Vehicle model

Using Matlab/Simulink and Carsim to establish the in-wheel motor driven vehicle model, the vehicle model b-class is selected, the four-wheel drive mode is chosen and the power transmission part is canceled. Due to the permanent magnet brushless motor’s torque has the rapid closed-loop control response characteristics, the in-wheel motor model is simplified as follows.

\[
\frac{T_{mi}}{T_{wi}} = \frac{\rho s R_m}{(L_m - L_s)s + R_m} \cdot \frac{1}{T_p s + 1} = \frac{1}{2\xi^2 s^2 + 2\xi s + 1}
\]

where \( T_{mi} \) is the motor torque output, \( T_{wi} \) is the target torque, and the motor parameter \( \xi \) is determined by the motor magnetic logarithm \( \rho \), motor resistance \( R_m \), rotor flux \( \Phi \), self-inductance \( L_m \), mutual inductance \( L_s \), motor period \( T_p \) and other parameters.

Vehicle reference model

The 2-DOF model is used as the reference model to obtain the reference values of vehicle yaw rate and sideslip angle, as shown in Figure 1. The simplified 2-DOF model is as follows.

The differential equations of 2-DOF vehicle dynamics are as follows.

\[
\begin{align*}
\dot{\beta} &= \frac{k_1 + k_2}{mv_x} \beta + \left( \frac{l_f k_1 - l_r k_2}{mv_x} - 1 \right) \omega - \frac{k_1}{mv_x} \delta_f \\
\dot{\omega} &= \frac{l_f k_1 - l_r k_2}{I_z} \beta + \frac{l_f^2 k_1 + l_r^2 k_2}{I_z v_x} \omega - \frac{l_f k_1}{I_z} \delta_f
\end{align*}
\]

where \( \beta \) is the sideslip angle of the vehicle, \( I_z \) is the moment of inertia of the vehicle around the z axle, and \( \omega \) is the vehicle yaw rate. \( k_1 \) and \( k_2 \) are the cornering stiffness of the front and rear tires, \( v_x \) is the vehicle longitudinal speed, \( \delta_f \) is the front wheel angle, \( m \) is the vehicle mass, and \( l_f \) and \( l_r \) are the distances from the centroid to the front, rear axles respectively.

Figure 1. 2-DOF vehicle dynamics model.
When the vehicle enters the steady state, the sideslip angle and yaw rate of the vehicle are constant. At this time, $\beta = 0$, $\dot{\omega} = 0$, and the desired sideslip angle and yaw rate of the vehicle can be obtained from equation (3).

$$\begin{align*}
\omega^* &= \frac{v_x/l}{1 + Kv_x^2} \delta_f \\
\beta^* &= \frac{b/l + ml_f v_x^2/(l^2 k_2)}{1 + Kv_x^2} \delta_f
\end{align*}$$  \hspace{1cm} (3)

where $\beta^*$ is desired sideslip angle, $\omega^*$ is desired yaw rate, $l$ is wheel base, $K$ is stability coefficient and $K = \frac{m}{l^2} \left( \frac{l_f}{k_1} - \frac{l_r}{k_2} \right)$.

**Tire model**

In this paper, Pacejka’s magic formula is used to describe the dynamics of tires because of its high precision and wide range of application, and it is widely used in the field of vehicle dynamics simulation and analysis. Its general form is as shown in equation (4).

$$Y(x) = D \sin \left\{ C \tan^{-1} \left[ B(x + s_h) - E \left( B(x + s_h) - \tan^{-1}(B(x + s_h)) \right) \right] \right\} + s_v$$  \hspace{1cm} (4)

where $Y(x)$ could represent longitudinal tire forces $F_x$, lateral tire forces $F_y$, and aligning torques $M_z$, and $x$ can be longitudinal slip ratio $\lambda$ and wheel side-slip angle $\alpha$.

**Extension recognition of vehicle stability**

The control framework of this paper is shown in Figure 2. By extracting the character variables of the vehicle, we can first determine whether the vehicle is unstable. If the vehicle has been unstable, we can obtain the desired additional yaw moment by taking the sideslip angle as the control objective. If the vehicle is stable, we can use the extension pattern recognition algorithm to determine the stability pattern of the vehicle status, so as to obtain the desired additional yaw moment.

**Establishment of matter element model**

_Matter element model._ Matter element model is to use the extension theory to build the mathematical model by the vehicle stability pattern and the vehicle status, so as to the extension pattern recognition algorithm can be utilized.

Firstly, the matter element model of stability pattern is established.

An object has multiple features. If an object is described by multiple features $c_k(k = 1, 2, \cdots, n)$ and corresponding values $v_k(k = 1, 2, \cdots, n)$, it can be expressed as follows.
In order to establish the matter element model, the boundary of stability level should be divided.

**Pattern division of yaw rate deviation.** The tolerance zone division method\(^\text{30}\) is adopted for yaw rate deviation, and the absolute stability boundary satisfy \(|\omega - \omega^*| \leq |C_1 \omega^*|\), that is to say the boundary is \(|\Delta \omega| = |C_1 \omega^*|\), and the corresponding yaw rate deviation is defined as \(\Delta \omega_1\), that means when \(|\Delta \omega| \leq |\Delta \omega_1|\), we consider the vehicle is absolutely stable. When controlling the yaw rate, the vehicle stability is improved obviously when the yaw rate deviation satisfies \(|C_1 \omega^*| \leq |\omega - \omega^*| \leq |C_2 \omega^*|\), that is to say the boundary is \(|\Delta \omega| = |C_2 \omega^*|\) and the corresponding yaw rate deviation is defined as \(\Delta \omega_2\). The constant coefficients \(C_1\) and \(C_2\) are determined to be set as 0.05 and 0.12 respectively. When the control effect of the single yaw rate control is poor but the vehicle have not lose stability, its boundary is \(|\Delta \omega| = |C_3 \omega^*|\), \(C_3\) is set as 0.145, the corresponding yaw rate deviation is defined as \(\Delta \omega_3\). When dividing the vehicle instability boundary, according
to the tolerance zone method, the instability boundary is determined as $|\Delta \omega| = |C \omega^*|$, where $C$ is 0.165, and the corresponding yaw rate deviation is defined as $\Delta \omega_4$.

The yaw rate deviation meets the following relationship.

$$\Delta \omega_1 < \Delta \omega_2 < \Delta \omega_3 < \Delta \omega_4$$  \(6\)

To sum up, it is considered that the absolute stability boundary of vehicle yaw rate is $|\Delta \omega| = |\Delta \omega_1|$, the instability boundary of vehicle is $|\Delta \omega| = |\Delta \omega_3|$, and the boundary when vehicle stability can be significantly improved by yaw rate control is $|\Delta \omega| = \omega_2$. When the yaw rate deviation is in $\langle \Delta \omega_1, \Delta \omega_2 \rangle$, it is difficult to determine whether the vehicle needs yaw rate control. When the yaw rate deviation is in $\langle \Delta \omega_1, \Delta \omega_2 \rangle$, it is difficult to determine whether to use yaw rate control or yaw rate and sideslip angle joint control.

**Pattern division of sideslip angle.** The absolute stability boundary of the sideslip angle is determined by whether the yaw rate gain is in the linear range.\(^3\) When no control is applied, the front wheel angle input is set to gradually increase at different vehicle speeds, and the relationship between the extreme value of the front wheel angle in the linear range and the vehicle speed is fitted, and the relationship is input into the 2-DOF model, the stability boundary is $\beta_1$. By a large number of simulations, it is concluded that when the sideslip angle is $1.3 \beta_1$, the vehicle stability can be significantly improved by the yaw rate control, and the corresponding sideslip angle is $\beta_2$. When the effect of the single yaw rate control is poor and the vehicle is stable, the vehicle sideslip angle boundary is $1.45 \beta_1$, the corresponding sideslip angle is $\beta_3$.

When the vehicle is in the unstable status, the limit value of the sideslip angle is $|\beta| = |\tan^{-1}(0.02 \mu g)|$, $\mu$ is road adhesion coefficient, then the vehicle sideslip angle is set as $\beta_4$.

The sideslip angle satisfies the following relationship.

$$\beta_1 < \beta_2 < \beta_3 < \beta_4$$  \(7\)

It is considered that the absolute stability boundary of the sideslip angle is $|\beta| = \beta_1$, and the instability boundary is $|\beta| = \beta_3$. The yaw rate control can significantly improve the vehicle stability when the boundary is $|\beta| = \beta_2$. When the sideslip angle is in $\langle \beta_1, \beta_2 \rangle$, it is difficult for the vehicle to determine whether to control the yaw rate, and when the sideslip angle is in $\langle \beta_2, \beta_3 \rangle$, it is difficult to determine whether to use the yaw rate control or the yaw rate and sideslip angle joint control.

Therefore, the extension pattern recognition algorithm is needed to determine the area and divide the stability pattern so as to apply the corresponding control for enhancing vehicle stability.

According to the above analysis, when dividing the stability patterns, if the character variables of vehicle status are in the region $\{ \langle \Delta \omega_1, \Delta \omega_2 \rangle \}$ and $\{ \langle \Delta \omega_2, \Delta \omega_3 \rangle \}$, it is difficult to determine the stability status of the vehicle directly.
In this paper, the stability levels are divided as follows.

Level 0 stability area is

\[
\begin{aligned}
0 & < |\Delta \omega| < \Delta \omega_2 \\
0 & < |\beta| < \beta_2
\end{aligned}
\] (8)

Level 1 stability area is

\[
\begin{aligned}
\Delta \omega_1 & < |\Delta \omega| < \Delta \omega_3 \\
\beta_1 & < |\beta| < \beta_3
\end{aligned}
\] (9)

Level 2 stability area is

\[
\begin{aligned}
\Delta \omega_2 & < |\Delta \omega| < \Delta \omega_4 \\
\beta_2 & < |\beta| < \beta_4
\end{aligned}
\] (10)

Level 3 stability area is

\[
\begin{aligned}
\Delta \omega_4 & < |\Delta \omega| \\
\beta_4 & < |\beta|
\end{aligned}
\] (11)

Finally, the extension pattern recognition algorithm is used to judge which stability level the vehicle is in when the vehicle state is within \( \{ (\Delta \omega_1, \Delta \omega_2) \} \) and \( \{ (\beta_1, \beta_2) \} \) and \( \{ (\Delta \omega_2, \Delta \omega_3) \} \) and \( \{ (\beta_2, \beta_3) \} \). When any of the two character variables of the vehicle exceeds the limit value, namely \(|\Delta \omega| > \Delta \omega_3\) or \(|\beta| > \beta_3\), the vehicle has lost the handling stability, the vehicle is in level 3, and the sideslip angle control will be carried out. Then the matter element model can be expressed as follows.

\[
R_{00} = (N_{00}, C, X_{00}) = \begin{bmatrix}
N_{00} \\
N_{01} \\
N_{02}
\end{bmatrix} \begin{bmatrix}
c_1 \\
c_2
\end{bmatrix} \begin{bmatrix}
0, \beta_2 \\
0, \Delta \omega_2
\end{bmatrix}
\] (12)

\[
R_{01} = (N_{01}, C, X_{01}) = \begin{bmatrix}
N_{01} \\
N_{02}
\end{bmatrix} \begin{bmatrix}
c_1 \\
c_2
\end{bmatrix} \begin{bmatrix}
\langle \beta_1, \beta_3 \rangle \\
\langle \Delta \omega_2, \Delta \omega_3 \rangle
\end{bmatrix}
\] (13)

\[
R_{02} = (N_{02}, C, X_{02}) = \begin{bmatrix}
N_{02} \\
N_{01}
\end{bmatrix} \begin{bmatrix}
c_1 \\
c_2
\end{bmatrix} \begin{bmatrix}
\langle \beta_2, \beta_4 \rangle \\
\langle \Delta \omega_2, \Delta \omega_4 \rangle
\end{bmatrix}
\] (14)

**Segment field matter elements.** The segment field refers to the limit value of the vehicle states when the vehicle is about to lose stability. Therefore, according to the pattern division method, the segment field matter element is established as follows.

\[
R_p = (P, C, X_p) = \begin{bmatrix}
P \\
P
\end{bmatrix} \begin{bmatrix}
c_1 \\
c_2
\end{bmatrix} \begin{bmatrix}
0, \beta_3 \\
0, \Delta \omega_3
\end{bmatrix}
\] (15)
**Unidentified matter element.** The unidentified matter element refers to the matter element to be identified formed by the real-time value of each character variable, that is, the yaw rate deviation and the sideslip angle.

Establishing the unidentified matter element as

\[
R_x = \begin{bmatrix} P & c_1 & x_1 \\ c_2 & x_2 \end{bmatrix}
\]

where \( c_1 \) represents the sideslip angle, \( c_2 \) represents the yaw rate deviation, \( x_1 \) represents the value of sideslip angle, and \( x_2 \) represents the value of yaw rate deviation.

**Vehicle stability pattern recognition algorithm**

According to the matter element model of vehicle stability pattern, the matter element model of segment field and the unidentified matter element, the stability pattern of vehicle actual status is determined by using the pattern recognition algorithm.

Relationship function is a function used to describe the degree of objects belonging to a certain pattern, and the relationship degree is calculated by relationship function. It can be used in vehicle stability pattern recognition to quantitatively reflect the possibility that vehicle status belongs to a certain level of stability.

To calculate the relationship function, the first step is to calculate the extension distance, which refers to the relative position of the unidentified status and the boundary of each stability pattern. If the classic domain is \( X_0 = (a_0, b_0) \), segment field is \( X_p = (a_p, b_p) \), the extension distance can be expressed as

\[
D(x, X_0, X_p) = \begin{cases} 
\rho(x, X_p) - \rho(x, X_0) & x \notin X_0 \\
-|X_0| & x \in X_0
\end{cases}
\]

\[
\rho(x, X_0) = |x - \frac{1}{2}(a_0 + b_0)| - \frac{1}{2}(b_0 - a_0)
\]

\[
\rho(x, X_p) = |x - \frac{1}{2}(a_p + b_p)| - \frac{1}{2}(b_p - a_p)
\]

\[
|X_0| = |b_0 - a_0|
\]

Then the relationship degree is as follows.

\[
K(x, X_0, X_p) = \begin{cases} 
\frac{-\rho(x, X_0)}{|X_0|} & x \in X_0 \\
\frac{\rho(x, X_0)}{\rho(x, X_p) - \rho(x, X_0)} & x \notin X_0
\end{cases}
\]
Stability pattern determination

After the calculation of relationship function, the membership degree of vehicle status and stability level can be further calculated. The product of the relationship function and the weight coefficient of a character variable in classic domain is called as the membership degree of the character variable in this pattern. The stability pattern is determined by the sum of membership degree of each character variable calculated in the classic domain of each pattern. According to the principle of maximum membership, the stability pattern of vehicle is obtained.

To calculate the relationship degree of the matter element on the level $j$, we should firstly calculate the weight coefficient of the character variable.

The weight coefficient of each character variable is calculated by equation (22).

$$a_{ij}(m) = \frac{x_i/b_{ij}}{\sum_{i=1}^{3}(x_i/b_{ij})}$$  \hspace{1cm} (22)

where the symbol $i$ ($i=1, 2$) represents the character variable, $j$ represents the level, and $x_i$ represents the value of the character variable to be identified. Then the relationship degree of unidentified matter element with respect to the level $j$ is as follows.

$$K_{ji}(N_x) = \sum_{i=1}^{3} a_{ij}k_j(x_i)$$  \hspace{1cm} (23)

The stability pattern level of the vehicle is determined by the principle of maximum membership.

The judgment rule is shown in equation (24).

$$K_{j0}(N_x) = \max\{K_j(N_x)\}; j = 1, 2, 3$$  \hspace{1cm} (24)

If equation (24) is satisfied, then $N_x$ is determined to be of the level $j_0$.

After the stability pattern of the vehicle is determined, the corresponding control strategy can be adopted to enhance the vehicle’s stability.

Stability control system design

Design of yaw rate controller

Fuzzy control is used in this paper which has many advantages and is easy to be applied. The yaw rate deviation and its differential are set as the input of fuzzy controller, and the output is the additional yaw moment $\Delta M_\omega$ needed in yaw rate control.

After a large number of simulations, the yaw rate deviation and its differential are set as the input and output variables with $[-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]$. The fuzzy set is $[NB, NM, NS, ZO, PS, PM, PB]$, in which $NB, NM, NS$ are negative large, negative medium and negative small, $ZO$ is zero, $PS, PM, PB$
are positive small, positive medium, and positive large. The table of control rules is shown in Table 1.

### Design of sideslip angle controller

The deviation and its differential of the sideslip angle are set as the input of the fuzzy controller, and the output is the yaw moment $\Delta M_\beta$ needed in the control of the sideslip angle.

After a large number of simulations, the input and output variables of yaw rate deviation and its differential are $[-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]$. The fuzzy set is $[NB, NM, NS, ZO, PS, PM, PB]$, in which $NB, NM, NS$ are negative large, negative medium and negative small, $ZO$ is zero, $PS, PM, PB$ are positive small, positive medium, and positive large. The membership functions of output and input are all triangle functions. The table of control rules is shown in Table 2.

Thus, the additional yaw moment required to maintain vehicle stability is summed up as follows.
\[
\Delta M = \begin{cases} 
0 & \text{level0} \\
\Delta M_\omega & \text{level1} \\
\eta_\omega \cdot \Delta M_\omega + \eta_\beta \cdot \Delta M_\beta & \text{level2} \\
\Delta M_\beta & \text{level3}
\end{cases}
\] (25)

where \(\eta_\omega\) and \(\eta_\beta\) are the weighting coefficients of yaw rate and sideslip angle respectively.

**Design of optimal torque distribution strategy**

The working process of electric motor is a process of energy conversion, which can optimize the energy management of electric vehicle by improving the energy efficiency. That is, the conversion efficiency when the electric energy is converted into the kinetic energy of the car. The higher the conversion efficiency, the better the energy saving effect of the car. The optimization management of power and electrical system and the lightweight development of the vehicle are the traditional methods of vehicle energy optimization management. Because the transmission system, differential mechanism and clutch mechanism are removed, the mechanical loss of the four-wheel independent drive electric vehicle is reduced, and the efficiency map of the motor also exists. We can control the output torque of the four electric motors to keep the total drive efficiency of the four drive motors in a higher range under the condition that the total drive torque during the driving process of the vehicle meets the desired torque demand. When the motor works, because of the heating and friction of the motor itself and the energy loss of its driving system, its efficiency will change when the speed and output torque are different. The efficiency map of the motor is shown in Figure 3.
Objective function design

According to the characteristics of the motor, on the premise of meeting the total driving torque demand of the vehicle, the motor torque is optimally distributed by an objective function, so as to minimize the total energy loss of the vehicle. When the output torque $T_{out}$ and speed $\omega_{out}$ of the drive motor are fixed, the efficiency $\eta$ of the motor drive system can be obtained through the motor efficiency map. It can be seen from $P_{out} = T_{out} \times \omega_{out}$ that when the output power of the motor is fixed, so the power loss of a single motor can be expressed by the output power and the motor efficiency.

$$P_{loss} = P_{in} - P_{out} = \frac{P_{out}}{\eta} - P_{out} = P_{out} \left(1 - \frac{\eta}{\eta}ight)$$  \hspace{1cm} (26)$$

According to equation (26), the total power loss of four motors is as follows.

$$P = P_{fr} \left(\frac{1 - \eta_{fr}}{\eta_{fr}}\right) + P_{fl} \left(\frac{1 - \eta_{fl}}{\eta_{fl}}\right) + P_{rr} \left(\frac{1 - \eta_{rr}}{\eta_{rr}}\right) + P_{rl} \left(\frac{1 - \eta_{rl}}{\eta_{rl}}\right)$$  \hspace{1cm} (27)$$

where $P_{fr}$, $P_{fl}$, $P_{rr}$, and $P_{rl}$ represent the output power of the front-left, front-right, rear-left, and rear-right motors of the vehicle respectively. $\eta_{fr}$, $\eta_{fl}$, $\eta_{rr}$, and $\eta_{rl}$ represent the efficiency of the corresponding motor, and the efficiency can be obtained by querying the map. The total power loss can be written as follows.

$$P = \sum_{j} P_{j} \left(\frac{1 - \eta_{j}}{\eta_{j}}\right); j = fr, fl, rr, rl$$  \hspace{1cm} (28)$$

The objective optimization function of motor drive system can be expressed as

$$J = \min \left(\sum_{j} P_{j} \left(\frac{1 - \eta_{j}}{\eta_{j}}\right)\right)$$  \hspace{1cm} (29)$$

Constraints

In the design of constraints, it is necessary to maintain the handling stability of the vehicle when optimizing the torque distribution based on the energy-efficient driving mode, so it is expected that the additional yaw moment $\Delta M$ will be one of the constraints, the sum of the driving torques of four motors should meet the total driving torque demand of the vehicle, and the output torque of each motor should be less than the maximum output torque of the motor. The output torque/speed of each motor should be less than the maximum torque/speed that the motor can output, and the longitudinal force of the wheel should be less than maximum longitudinal force provided by the road.

To sum up, the constraints of the torque distribution should be as follows.
\[
\frac{d}{2R} \left( -T_{dfl} + T_{dfr} - T_{drl} + T_{drr} \right) = \Delta M_z
\]
\[
T_{dfl} + T_{dfr} + T_{drl} + T_{drr} = T_t
\]
\[
\left| \frac{T_{di}}{R} \right| \leq \mu F_{zi}, \quad i = fl, fr, rr, rl
\]
\[
n \leq n_{\text{max}}
\]

where, \( d \) represents the wheel track, \( R \) represents the tire radius. \( T_{dfl}, T_{dfr}, T_{drl}, \) and \( T_{drr} \) represent the torque of front-left, front-right, rear-left, and rear-right wheel respectively, \( T_t \) represents the total torque of four wheels. \( T_{di} \) represents the wheel torque, \( T_{d\text{imax}} \) represents the maximum wheel torque, \( \mu \) represents the road adhesion coefficient, \( F_{zi} \) represents the tire vertical load, \( n \) represents the wheel speed, and \( n_{\text{max}} \) represents the maximum wheel speed.

### Simulation analysis

The vehicle parameters used in this paper are shown in Table 3.

#### Table 3. Vehicle parameters.

| Parameters                     | Value  |
|-------------------------------|--------|
| Vehicle mass \( m \) (kg)     | 1200   |
| Vehicle moment of inertia \( l_z \) (kg·m²) | 3100   |
| Wheel moment of inertia \( l_w \) (kg·m²) | 1.1    |
| Distance from centroid to front axle \( l_f \) (m) | 1.1    |
| Distance from centroid to rear axle \( l_r \) (m) | 1.7    |
| Cornering stiffness of front tires \( k_1 \) (N/rad) | 80,000 |
| Cornering stiffness of rear tires \( k_2 \) (N/rad) | 80,000 |
| Wheel track \( l_w \) (m) | 1.7    |
| Effective tire radius \( R_w \) (m) | 0.29   |

### Simulation of stability pattern recognition and division

In order to show the advantage of stability pattern recognition and division, the simulation is carried out when the four-wheel torque average distribution is adopted under the different conditions and controllers.

Steering wheel angle is set as sinusoidal input, the maximum steering wheel angle of sinusoidal input is 60°. The road adhesion coefficient is set as 0.8 and the vehicle speed is 90 km/h. The steering wheel angle and simulation results are shown in Figure 4.

It can be seen from Figure 4 that when the sinusoidal input of steering wheel angle is set, the control effect by yaw rate control is close to that by the control strategy. The yaw rate changes within ±0.24 rad/s, and the sideslip angle changes within ±0.018 rad. When the sideslip angle is controlled only, the sideslip speed
changes within ±0.35 rad/s, and the sideslip angle changes within ±0.025 rad/s. The results show that the results by the proposed control strategy and the yaw rate control are better than that by the sideslip angle control.

For the simulation under double lane change condition, the road adhesion coefficient is set as 0.3 and the vehicle speed is 60 km/h, and the simulation results are shown in Figure 5.

It can be seen from Figure 5(a) that when only yaw rate control is adopted, the vehicle has serious understeer and deviates from the desired path when turning. When the sideslip angle control is adopted, the control effect is greatly improved, but compared with the control strategy in this paper, the vehicle can track the desired path more effectively under the proposed control. In conclusion, the control strategy in this paper can deal with the complex driving environment and improve the vehicle handling stability more effectively. From the analysis of the simulation results, it can be concluded from Figure 5(b) and (c) that under this working condition, the road adhesion coefficient is low, and the vehicle is easy to lose the handling stability. When the vehicle only controls the yaw rate, the control effect is limited, and the vehicle loses the handling stability. The main reason is that the sideslip angle is relatively large at this time, and the sideslip angle mainly

Figure 4. Simulation results under the sinusoidal input. (a) Steering wheel angle. (b) Yaw rate. (c) Sideslip angle.
represents the stable state of the vehicle in this condition. It is difficult to maintain the stable driving of the vehicle when the yaw rate is controlled only. The effect of sideslip angle control is greatly improved compared with the yaw rate control, the yaw rate changes within \(\pm 0.15\) rad/s, and the sideslip angle changes within \(\pm 0.03\) rad, but the curve fluctuation of the two is smaller when the proposed control strategy is adopted.

Simulation and analysis of optimal torque distribution

Simulation under the double lane change condition. The vehicle speed is set as 90 km/h, and the road adhesion coefficient is set as 0.8. The simulation results are shown in Figure 6.

The simulation is carried out under the motors’ torque is distributed evenly and optimally respectively. It can be seen from Figure 6(a) to (c) that the vehicle stable driving can be guaranteed by the two torque distribution methods, but from the aspect of path tracking and the aspect of yaw rate and sideslip angle tracking effect, the torque optimization distribution obtains better effect, and the overshoot and fluctuation following the desired value are lower by the torque optimization.
distribution. Figure 6(d) shows the four-wheel torque which is optimally distributed. In terms of energy consumption, it can be seen from Figure 6(d) and (e) that when the vehicle starts to drive, the output torque of the two front wheels is greater than the torque of the two rear wheels. By the analysis of energy consumption under the whole working condition, the energy loss is 7830 kJ when the torque is not optimized. When the torque is optimized, the energy loss under the whole working condition is 6550 kJ, the energy consumption of the vehicle is reduced, and the energy is saved by 16.3%.
Simulation under Fishhook condition. Set the vehicle speed to 60 km/h and the road adhesion coefficient to 0.3. The steering wheel angle and simulation results are shown in Figure 7.

According to the simulation results, when the vehicle is turning on the low adhesion road, it can be concluded from Figure 7(b) and (c) that the vehicle is very easy to lose the handling stability due to low road adhesion and large steering wheel angle. At this time, the direct yaw moment control is mainly based on the sideslip angle control, and the optimal torque distribution can better reduce the overshoot.
and oscillation during turning, which can track the desired yaw rate and the side-slip angle more effectively than the torque average distribution. Figure 7(d) shows the torque of four wheels when the torque is optimally distributed. Figure 7(e) shows that when the torque is evenly distributed, the energy consumption of the vehicle in the whole simulation process is 5298 kJ, while when the torque is optimally distributed, the energy consumption of the vehicle is 4690 kJ. The energy is saved by up to 11.5%. Due to the poor driving conditions of the vehicle, the main purpose is to ensure the vehicle stability under this condition, the energy-saving effect has declined compared with that under the double lane change condition.

**HIL test**

*Structure of in-wheel motor test bench*

The in-wheel motor HIL test bench includes in-wheel motor and its controller, torque/speed sensor, coupling, magnetic powder brake and its controller, battery pack, computer, and PXI. The peak output torque of the used in-wheel motor is 200 Nm, the rated power is 5 kW, the peak speed is 1000 r/min, and the rated voltage of the motor is 72 V. The torque and speed sensor is used to collect the torque and speed of the in-wheel motor in real time. The torque range is 0 to 200 Nm, and the speed range is 0 to 1000 r/min. The main function of the magnetic powder brake is to simulate the load. During the hardware in the loop test, in order to simulate the load, resistance moment, and other factors of the vehicle, the output braking moment of the magnetic powder brake is used as the simulated load. PXI is mainly used for information collection and transmission. The simulation results are displayed on the computer. The structure of in-wheel motor bench is as shown in Figure 8.

The HIL test is realized by using CarSim and LabVIEW, and the Simulink model of two motors in the joint simulation model is replaced by the in-wheel motors on the test-bed. The motor speed signal collected by computer, which is different from the target speed, is converted into PWM voltage signal and sent to the motor controller, so as to adjust the motor speed, make the motor speed track the target speed and realize the speed control. At the same time, the computer collects the motor torque signal, then vehicle controller solves the current desired torque, converts it into PWM voltage signal, sends it to the magnetic powder brake controller by the acquisition card, adjusts the magnetic powder brake tension, controls the output torque of the in-wheel motor.

**HIL test**

In the test, the in-wheel motors on the test-bed is used to replace the two wheels on the right side of the vehicle, and the HIL test is carried out under the double lane change condition with the vehicle speed 90 km/h and the road adhesion coefficient 0.3. The stability control strategy based on the extension pattern recognition is verified. In the simulation results, the torque of the two wheels on the right side is the
collected through test data, and the others are output from the LabVIEW model program. The test results are shown in Figure 9.

It can be concluded from Figure 9(a) that although there is a certain deviation in the vehicle path tracking effect, the overall tracking effect is good with a maximum displacement deviation of 0.5m. It can be concluded from Figure 9(b), there is a certain deviation in the sideslip angle with a maximum deviation of 0.6°, but the overall trend of the curve is the same with the desired value which does not affect the vehicle yaw stability. As shown in Figure 9(c), the tracking effect of vehicle yaw rate is good, and the maximum deviation is about 2°/s. Figure 9(d) shows the actual torque of two motors during HIL test. The results of HIL test show that although there is a certain gap between the results of HIL test and the simulation results, the overall trend is the same, which shows that the control strategy is effective for vehicle yaw stability control.

Figure 8. Structure of in-wheel motor bench. (a) HIL bench structure. (b) Bench picture.
Conclusions

The vehicle dynamics model and the 2-DOF reference model are established. The vehicle stability level is divided by setting the yaw rate deviation and the sideslip angle as the character variables. The extension theory is used to express the mathematical model of the divided vehicle stability levels. First, it is judged whether the vehicle is unstable. Then, the stability pattern of the vehicle is determined by the extension pattern recognition algorithm, and the yaw rate and the sideslip angle controllers are designed to obtaining the desired additional yaw moment. The four-wheel torque is optimally distributed. The simulation verification of the stability pattern division method is carried out. The results show that the proposed method is feasible and effective. The HIL test is also carried out to prove the effectiveness of the control strategy.

Taking the lowest energy loss of the motor as the objective function, the additional yaw moment needed to maintain the vehicle stability, motor performance, road adhesion conditions are used to optimize the torque distribution, and the simulation results under different conditions are carried out. The simulation results show that the optimal torque distribution strategy can not only enhance the vehicle stability, but also save the vehicle energy efficiency.

Figure 9. HIL results under adhesion double lane change condition with low adhesion coefficient. (a) Trajectory of vehicle. (b) Sideslip angle. (c) Yaw rate. (d) Right wheel torque.
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ORCID iDs
Wang Hongbo https://orcid.org/0000-0002-8083-6427
Sun Youding https://orcid.org/0000-0001-8012-5601

References
1. Sato M, Yamamoto G, Gunji D, et al. Development of wireless in-wheel motor using magnetic resonance coupling. IEEE Trans Power Electron 2016; 31(7): 5270–5278.
2. Xizheng Z and Yongqi T. Hierarchically coordinated vehicle dynamics control for in-wheel-driven electric vehicles. Int J Adv Comput Technol 2012; 4(20): 582–590.
3. Li G, Hong W, Liang H, et al. Four-wheel independently driven in-wheel motors electric vehicle AFS and DYC integrated control. SAE technical paper 2012-01-0258, 2012.
4. Lu Q, Sorniotti A, Gruber P, et al. $H_{\infty}$ loop shaping for the torque-vectoring control of electric vehicles: theoretical design and experimental assessment. Mechatronics 2016; 35: 32–43.
5. Zhao H, Gao B, Ren B, et al. Integrated control of in-wheel motor electric vehicles using a triple-step nonlinear method. J Franklin Inst 2015; 352(2): 519–540.
6. Shuai Z, Zhang H, Wang J, et al. Lateral motion control for four-wheel-independent-drive electric vehicles using optimal torque allocation and dynamic message priority scheduling. Contr Eng Pract 2014; 24: 55–66.
7. Luo S and Gao R. Chaos control of the permanent magnet synchronous motor with time-varying delay by using adaptive sliding mode control based on DSC. J Franklin Inst 2018; 335(10): 4147–4163.
8. Liang J, Yang H, Wu J, et al. Power-on shifting in dual input clutchless power-shifting transmission for electric vehicles. Mech Mach Theory 2018; 121: 487–501.
9. Sun X, Hu C, Lei G, et al. State feedback control for a PM hub motor based on grey wolf optimization algorithm. IEEE Trans Power Electron 2020; 35(1): 1136–1146.
10. Li B, Li W, Kennedy O, et al. Dynamics analysis is of an omni-directional vehicle. Int J Automot Technol 2014; 15(3): 387–398.
11. Zhang L and Wu G. Combination of front steering and differential braking control for the path tracking of autonomous vehicle. SAE technical paper 2016-01-1627, 2016.
12. Jin Z, Chen G and Zhao W. Rollover stability and control of in-wheel motor drive electric vehicles. Chin Mech Eng 2018; 29(15): 1772–1779.
13. Chen Y, Karl HJ and Guo K. A novel direct yaw moment controller for in-wheel motor electric vehicles. *Veh Syst Dyn* 2013; 51(6): 924–942.

14. Hori Y. Future vehicle driven by electricity and control-research on four-wheel-motored “UOT electric march II”. *IEEE Trans Ind Electron* 2004; 51(5): 954–962.

15. He P and Hori Y. Optimum traction force distribution for stability improvement of 4WD EV in critical driving condition. In: *9th IEEE international workshop on advanced motion control*, Istanbul, 27–29 March 2006, pp. 596–601. New York: IEEE.

16. He P, Hori Y, Kamachi M, et al. Future motion control to be realized by in-wheel motored electric vehicle. In: *31st annual conference of IEEE industrial electronics society, Raleigh, NC*, 6–10 November 2005, pp. 2632–2637. New York: IEEE.

17. Zhai L, Sun T and Wang J. Electronic stability control based on motor driving and braking torque distribution for a four in-wheel motor drive electric vehicle. *IEEE Trans Veh Technol* 2016; 65(6): 4726–4739.

18. Wu X, Zhou B, Wen G, et al. Intervention criterion and control research for active front steering with consideration of road adhesion. *Veh Syst Dyn* 2018; 56(4): 553–578.

19. Liu Z and Liu G. Simulation and test of stability control for distributed drive electric vehicles. *Automot Eng* 2019; 41(7): 792–799.

20. Chen W, Wang X, Tan D, et al. Study on the grey predictive extension control of yaw stability of electric vehicle based on the minimum energy consumption. *J Mech Eng* 2019; 55(2): 156–167.

21. Cai W. Extension set and non-compatible problems. *Sci Explor* 1983, 1: 83–97.

22. Zhao W, Fan M, Wang C, et al. $H_{\infty}$/extension stability control of automotive active front steering system. *Mech Syst Signal Process* 2019; 115: 621–636.

23. Chen W, Hu Z, Wang H, et al. Study on extension decision and artificial potential field based lane departure assistance system. *J Mech Eng* 2018; 54(16): 134–143.

24. Cai Y, Zang Y, Sun X, et al. Lane-keeping system of intelligent vehicles based on extension switching control method. *China J Highw Transp* 2019; 32(06): 43–52.

25. Wang J, Wang Q, Jin L, et al. Independent wheel torque control of 4WD electric vehicle for differential drive assisted steering. *Mechatronics* 2011; 21(1): 63–76.

26. Wang J. *Study on differential drive assist steering technology for electric vehicle with independent-motorized-wheel-drive*. Changchun: Jilin University, 2009.

27. Wang Q, Liu W, Chen W, et al. Sliding mode control of vehicle electronic stability program based on road identification. *Automot Eng* 2018; 40(1): 82–90.

28. Cheng S, Li L, Mei M, et al. Multiple-objective adaptive cruise control system integrated with DYC. *IEEE Trans Veh Technol* 2019; 68(5): 4550–4559.

29. Cai W and Yang C. Basic theory and methodology on Extenics (in Chinese). *Chin Sci Bull* (Chin Ver) 2013; 58: 1190–1199.

30. Gao Y. *Stability control study based on the stability boundary of phase plane for light vehicle*. Changchun: Jilin University, 2013.

31. Liu X. *Study of vehicle stability based on direct yaw moment control*. Hefei: Hefei University of Technology, 2010.

32. Tao T and Su SF. Moment adaptive fuzzy control and residue compensation. *IEEE Trans Fuzzy Syst* 2014; 22(4): 803–816.
Author biographies

Wang Hongbo, received the B.Tech., M.S., and Ph.D. degrees from Hefei University of Technology, Hefei, China, in 2003, 2006, and 2014, respectively. He is currently an Associate Professor with the School of Automotive and Transportation Engineering, Hefei University of Technology. He has authored more than 30 journal papers. His current research interests include vehicle dynamics and control and sensor information fusion of intelligent vehicle.

Sun Youding received the B.S. degree from Shandong University of Technology, Zibo, China, in 2018. He is currently working toward the M.S. degree with the Hefei University of Technology, Hefei, China. His research interest includes vehicle dynamics and control, particularly the design of driver assistance system.

Tan Hongliang received his M.S. degree from Hefei University of Technology, Hefei, China in 2018. He is now working in Weichai Group, China. His research interests include vehicle dynamics and control, and vehicle motor control.

Lu Yongjie received her Ph.D degree from Beijing Jiaotong University, Beijing, China in 2011. She is a professor with School of Mechanical Engineering, Shijiazhuang Tiedao University now, and has authored more than 30 journal papers. Her research interests include vehicle system dynamics and control, multibody dynamics, and vehicle-road coupling dynamics.