Mass shift of charmonium states in $\bar{p}A$ collision

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Abstract

The masses of the low lying charmonium states, namely, the $J/\Psi$, $\Psi(3686)$, and $\Psi(3770)$ are shifted downwards due to the second order Stark effect. In $\bar{p} + Au$ collisions at $6 - 10$ GeV we study their in-medium propagation. The time evolution of the spectral functions of these charmonium states is studied with a Boltzmann-Uehling-Uhlenbeck (BUU) type transport model. We show that their in-medium mass shift can be observed in the dilepton spectrum. Therefore, by observing the dileptonic decay channel of these low lying charmonium states, especially for $\Psi(3686)$, we can gain information about the magnitude of the gluon condensate in nuclear matter. This measurement could be performed at the upcoming PANDA experiment at FAIR.

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Introduction. — In Quantum Chromo Dynamics (QCD) the condensates, like the quark condensate \( m_q \langle \bar{q}q \rangle \) \([1, 2]\) and the gluon condensate \( \langle \alpha_s G^2 \rangle \) \([1]\) are fundamental quantities, which are important to understand hadron phenomenology. Their values in vacuum are quite well known \([1, 3–8]\). However, in matter we do not have this information, we only know their first nonzero coefficients in the density expansion \([9–11]\). The observation of in-medium modifications of hadrons may provide us valuable information about these condensates in matter. While the masses of hadrons consisting of light quarks changes mainly because of the (partial) restoration of the chiral symmetry – through their dependence on the chiral order parameter \( m_q \langle \bar{q}q \rangle \rho \) –, those made of heavy quarks are sensitive mainly to the changes of the non-perturbative gluon dynamics manifested through the changes in the gluon condensates \([12, 13]\). In the low density approximation the gluon condensate is expected to be reduced by \( 5–7\% \) at normal nuclear density \([14, 15]\). Therefore, the masses of the charmonium states – which can be considered as a color dipole in color electric field – are shifted downwards because of the second order Stark effect \([15–17]\). Moreover, since the \( D \) meson loops contribute to the charmonium self-energies and they are slightly modified in-medium, these modifications generate further minor contributions to the charmonium in-medium mass shifts \([16]\).

In this paper, our aim is to propose a way to “measure” the gluon condensate in nuclear matter via studying the mass shifts of the charmonium states by observing their dileptonic decays.

Antiproton induced reactions are the most prominent candidates to observe charmed particles in nuclear matter, since the medium is much simpler in this case than the one created in heavy ion collisions or even in proton induced reactions. Furthermore, the two main background contribution to the dilepton yield in the charmonium region, namely the Drell-Yan and the “open charm decay” are expected to be small. There is only a few energetic hadron-hadron collisions that can produce heavy dileptons via the Drell-Yan process. In the open charm decay the \( D \) mesons decay weakly. The \( c (\text{or } \bar{c}) \) quark in \( D (\text{or } \bar{D}) \) decays dominantly to leptons and \( s (\text{or } \bar{s}) \) quark. Consequently, the \( e \) and \( \bar{e} \) are usually accompanied by the \( K \) and \( \bar{K} \) mesons. Therefore, not very far above the threshold, the production of electron-positron pairs via the open charm decay with large invariant mass is energetically suppressed. The observation of charmonium in vacuum and in medium in antiproton induced reactions is an important goal of the AntiProton Annihilation at Darmstadt (PANDA) collaboration at the
soon to be built Facility for Antiproton and Ion Research (FAIR) accelerator complex.

The dynamics of the antiproton-nucleus reactions are described with a Boltzmann-Uehling-Uhlenbeck (BUU) type transport model. The spectral functions of the \( J/\Psi \), \( \Psi(3686) \), and \( \Psi(3770) \) vector mesons are expected to be modified in a strongly interacting environment according to [15–17]. Therefore, one has to propagate the spectral functions of these charmonium states properly.

Similar investigation has been carried out in [18], however, they did not consider substantial in-medium mass shift for the charmonium states, i.e. that work misses the essence of this investigation. An initial version of this approach is to be published in [19], where important ingredients such as the background contributions and the charmonium absorption were missing and the collisional broadening was only taken into account approximately.

**Transport model.** — Our original model was developed for the energy range of the Heavy-Ion-Synchrotron (SIS18) experiment at GSI in Germany. It contained 27 baryons and 6 mesons. The details of this transport model can be found in [20–22].

Recently, we improved the model in order to be applicable for higher energies. The relevant changes concern on the built-in elementary cross sections of the model, namely, we included cross sections for the production of charmonium and D-mesons states. We calculated these unknown cross sections, such as \( \bar{p}p \rightarrow J/\psi \pi \), or \( \bar{p}p \rightarrow D\bar{D} \) with the help of a statistical bootstrap model developed by some of us [23]. The antiproton-nucleon cross section was set to 20 mb taken from [24]. We apply energy independent charmonium absorption cross section for every hadrons as 4.18 mb for \( J/\Psi \) and 7.6 mb for \( \Psi(3686) \) and \( \Psi(3770) \) according to Ref. [25]. In \( \bar{p}A \) collisions at relativistic energies charmonium absorption does not play such an important role as at ultrarelativistic energies, since the hadron density is much less here. It should be noted that the charmonium states in the transport model are produced perturbatively. That is, after they are created with some probability through the antiproton annihilation process, in every time step it could decay with some probability, however, we do not let it decay, instead, we use this probability to add its contribution to the dilepton spectra.

If we create a particle in a medium with an in-medium mass, through its evolution, it should regain its vacuum mass, when it leaves the collision zone. If a local density approximation is used for changing its mass, the energy conservation will be clearly violated. For the propagation of off-shell particles a more sophisticated method is needed. One
can describe the in-medium properties of particles with a so-called “off-shell transport”. These equations are derived by starting from the Kadanoff-Baym equations [26] for the Green’s functions of the particles. Applying first-order gradient expansion after a Wigner transformation [27, 28] one arrives at a transport equation for the retarded Green’s function. To solve numerically the Kadanoff-Baym equations one may exploit the test-particle ansatz for the retarded Green’s function [27, 28].

The equations of motion of the test-particles have to be supplemented by a collision term, which creates couplings among the equations for the various particle species. It can be shown [28] that the collision term has the same form as in the standard BUU approach. The same model was used to study the propagation of low mass vector meson spectral functions at lower energies [20, 29].

The explicit form of the “off-shell transport” equations can be found in [19, 27]. To solve those equations an explicit expression for the real and imaginary part of the self-energy of the charmonium particle at hand is needed. In our calculations the following simple, density dependent form is assumed for each charmonium state – indexed by $V$,

\[ \Re \Sigma_{V}^{\text{ret}} = 2m_{V} \Delta m_{V} \frac{\rho}{\rho_{0}}, \]  
\[ \Im \Sigma_{V}^{\text{ret}} = -m_{V}(\Gamma_{\text{vac}}^{V} + \Gamma_{\text{coll}}). \]

Eq. (1) results in a “mass shift” of the form $\Delta m_{V}^{\text{shift}} = \sqrt{m_{V}^{2} + \Re \Sigma_{V}^{\text{ret}} - m_{V}} \approx \Delta m_{V} \frac{\rho}{\rho_{0}}$, where $\rho_{0} = 0.16 \frac{\text{fm}^{3}}{\text{fm}^{3}}$ stands for the normal nuclear density. The imaginary part incorporates a vacuum width $\Gamma_{\text{vac}}^{V}$ term and a collisional broadening term having the form

\[ \Gamma_{\text{coll}} = \frac{v \sigma \rho}{\sqrt{1 - v^{2}}}, \]

where $v$ is the relative velocity of the particle in the local rest frame, $\sigma$ is the total cross section of the particle colliding with nucleons and $\rho$ is the local density. The parameters ($\Delta m_{V}$) are taken from [16] and are given in Table I. The first values in Table I come from the second order Stark-effect (which depends on the gluon condensate), while the second ones emanate from the D-meson loops.

The electromagnetic branching ratios $\text{Br}^{\text{el}}$ of the charmonium states decaying into dileptons may change due to the broadening. In our approximation the electromagnetic in-medium widths are kept at the same value as in vacuum ($\Gamma_{\rho}^{\text{el}} = \Gamma_{\text{vac}}^{\text{el}}$), i.e. they are not increased by the collisional broadening. Consequently, the electromagnetic in-medium branching ratio will change in line with the changes of the total decay width, $\text{Br}_{\rho}^{\text{el}} = \Gamma_{\text{vac}}^{\text{el}}/\Gamma_{\rho}^{\text{tot}}$. The
TABLE I. Charmonium mass shift parameter values taken from [16]. In $\Delta m_V$ the first terms result from the second order Stark-effect, while the second ones emanate from the D-meson loops.

| Charmonium type ($V$) | $\Delta m_V$       |
|----------------------|--------------------|
| $J/\Psi$             | $-8 + 3$ MeV       |
| $\Psi(3686)$         | $-100 - 30$ MeV    |
| $\Psi(3770)$         | $-140 + 15$ MeV    |

in-medium electromagnetic widths are probably larger than the vacuum ones, but to be on the safe side the minimal value is chosen not to overestimate the resulting dilepton invariant mass spectrum in comparison with the background, that is, the genuine change in $\Gamma^\rho_\ell$ could only increase the spectrum not decrease.

If a meson is generated at a given density, its mass is distributed in accordance with its in-medium spectral function. If the given meson propagates into a region of higher (or lower) density, then its mass will decrease (or increase). This method, which is based on the "off-shell" transport equations, is energy conserving. We note that the propagation of $\omega$ and $\rho$ mesons in the energy range of the HADES experiment at GSI ($\sim 1 - 2$ AGeV) have been investigated in [20] with the same method.

Results.— In the top panel of Fig. 1 the evolution of the masses of randomly chosen test particles representing charmonium mesons is shown in case of $\bar{p} + \text{Au}$ collisions at 6 GeV bombarding energy. It should be noted that for getting a better overview in the figure the mass of $\Psi(3686)$ was shifted downwards by 300 MeV. It can be seen that at the end of the collision process, where the density is very low, the masses reach the vacuum value as expected. However, the mass of the $\Psi(3770)$ state spreads even at the end of the collision due to its substantial vacuum width. Most of the time the masses of these mesons are either shifted downwards to the mass corresponding to a density around $\rho_0$ or at their vacuum value, i.e. there are only relatively short transition periods in between. This is because the surface layer is quite narrow compared to the diameter of the gold nucleus. The evolution of the average density felt by the charmonium states are shown in the bottom panel of Fig. 1. The same conclusion can be drawn based on this figure too, namely, the transition period from the dense medium to the vacuum is small, approximately 4 fm/c long.

In Fig. 2 the antiproton penetration into the nucleus is shown. In the top figure we show...
FIG. 1. Top: Evolution of the masses of the $J/\Psi$, $\Psi(3686)$, and $\Psi(3770)$ charmonium states. For the better visibility the masses of the $\Psi(3686)$ are shifted downwards artificially by 300 MeV. Each line belonging to a given charmonium state represents a different individual particle evolution through the medium. Bottom: The average density felt by the c charmonium states as a function of time.
FIG. 2. Top: Distribution of the $z$ coordinate of the annihilation position of the antiproton, where the $z$ axis is in parallel with the beam momentum. The vertical line shows the surface of the nucleus on the side of the projectile. Bottom: The density distribution of the charmonium creation points.
that there are some annihilation even before the $\bar{p}$ reaches the surface of the nucleus due to the size of the projectile and to the surface thickness of the target ($\approx \pm 0.5$ fm). Most of the antiprotons annihilate close to the surface but a substantial amount can reach the center of the target, since at these energies the annihilation cross section is not too large ($\approx 20$ mb). In the bottom figure we show the density distribution of the annihilation points. Most of the charmoniums are created at densities close to $\rho_0$.

To summarize the dynamics of charmonium production the following plausible picture can be given: Most of the antiprotons annihilate on, or close to the surface of the heavy nucleus creating a charmonium (with some probability). The charmonium travels through the interior of the nucleus giving some contribution to the dilepton yield. That is the charmoniums are treated perturbatively (see also the 'Transport model' section). Traversing the thin surface again on the other side of the nucleus, it arrives to the vacuum, where most of the charmonium actually decays.

In Fig. 3 the charmonium contributions to the dilepton spectrum in a central $\bar{p} +$ Au is shown in an 6 GeV collision. The impact parameter is integrated out from zero to 4.5 fm, which gives approximately the central one third of the total cross section (radius of the Au nucleus is about 6.6 fm).

The dilepton invariant mass spectra for the three mentioned charmonium states are shown along with the background contribution, which is the sum of Drell- Yan and open charm decays. Since in our calculations detector resolution is not included, the vacuum contributions for the $J/\Psi$ and for the $\Psi(3686)$ result in very sharp peaks, almost discrete lines. A two-peak structure can be seen for each meson, corresponding to its vacuum and the in-medium mass from the interior of the nucleus. However, for the $J/\Psi$ the effect is negligible, since its mass shift is very small. For the $\Psi(3770)$ there are two peaks, however, its yield is covered by the contribution of $\Psi(3686)$, thus its observation could be very difficult. On the other hand, in the $\Psi(3686)$ dilepton mass spectra, the two peaks are clearly observable. The peak from the medium contribution is around two times higher than the deepest point in the valley between the peaks (note the logarithmic scale). Consequently, the two peaks should be experimentally easily separated and observed. The shift between the peaks corresponds approximately to a mass shift at 0.9$\rho_0$ density. We repeated the calculation with double and half mass shifts, too. The qualitative picture has not changed. Considering $\Psi(3686)$ the two peak structure has remained, and the distance between the peaks has also been cor-
FIG. 3. Charmonium contribution to the dilepton spectra in a $\bar{p}Au$ collision at 6 GeV bombarding energies, where the in-medium modifications are accounted for. The background is the sum of the Drell-Yan and the open charm contributions, while the impact parameter is integrated from zero to 4.5 fm. The contribution of $J/\Psi$ is divided by 100 to get a better overview.

responded to the mass shift at the same $0.9\rho_0$ density. This gives us a chance to determine the value of the gluon condensate in nuclear matter, at around $0.9\rho_0$ density, by measuring the distance between the two peaks for $\Psi(3686)$, since we know the dependence of the mass shift of the charmonium on the gluon condensate. We performed the same calculation for antiproton bombarding energies 8 and 10 GeV as well, which gave qualitatively the same result. At even lower energies, the peak structures are distorted strongly, thus the effect is
not cleanly visible. Probably, 6 GeV is the best bombarding energy, since at higher energies the annihilation cross section is lower, so more antiprotons penetrate deep inside the target, giving less contribution from the dense region to the dilepton yield. At higher energies the background is also higher.

The double peak structure of charmonium contributions to the dilepton invariant mass spectrum is a novel feature of our model in contrast to the work of Ref. [18] where the mass shift of the charmonium states was not considered.

Summary. — We calculated the charmonium contribution to the dilepton invariant mass spectra. We have shown that via their dileptonic decay there is a good chance to observe the in-medium modification of the higher charmonium state \( \Psi(3686) \) in a central \( \bar{p} + \text{Au} 6 \text{ GeV} \) collision. This opens up the unique possibility to “measure” the gluon condensate in nuclear matter. The distance of the two peaks corresponds to a mass shift at approximately \( 0.9 \rho_0 \) density. The D-meson loop contributes only by \( 25 - 30 \text{ MeV} \) to the mass shift. The rest (which is expected to be the major part) is the result of the second order Stark effect, thus we can determine the gluon condensate that has resulted in such a mass shift. Therefore \( \bar{p}A \) collision could provide us valuable information on the in-medium properties of the strong interaction.

The considered energy regime will be available by the forthcoming PANDA experiment at FAIR.

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