Innovative Seismic Simulation Methodology Using Fourier Phase Matrix

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Abstract. In dynamic analysis for seismic design, frequency domain methodology has been utilized for input ground motion generation since the 1960s. An innovative scheme is proposed in the article for seismic simulation based on the matrix of Fourier phase spectra. Compared to the conventional methodology, the proposed scheme enables stability of the simulation results for each trial with different random phase spectra. In the certain scheme, manual modification and screening for random results are avoided and computational efficiency is improved. The scheme is evaluated with a representative response spectrum for seismic design of the nuclear equipment. The simulation result shows that the stability for each trial of simulation is maintained by the modified methodology and the computational efficiency is improved considerably. It is approved that the scheme manages to generate design ground motions reliably and efficiently.

1. Introduction
Input ground motion is essential to structural dynamic analysis for seismic design and study. In the common situation, recorded natural earthquakes are utilized as the input ground motions[3]. However, the number of previous time-histories is limited, and it is hard for their frequency domain characteristics to adjust to the demand in seismic design codes, so that seismic simulation is often conducted to generate corresponding artificial ground motions. Frequency domain methodology has been widely accepted for seismic simulation since the 1960s[6,10], although simulation result by such method depends on the selection of the random Fourier phase spectrum which cannot be manually controlled. The simulation precision of spectrum acceleration of artificial ground motions varies a lot on different trials of random Fourier phase spectra, leading to the extra costs and workloads.

In the paper, an innovative seismic simulation methodology is proposed based on the matrix formed by Fourier phase spectra, which ensures the stability of the simulation results and improves the computational efficiency of the procedure.

2. Conventional Methodology
Random vibration theory suggests that the ground motion consists of simple harmonic motions of different frequencies with certain amplitudes and phases, based on which frequency domain method is conducted to generate artificial ground motions[1,5]. In the scheme, Fourier Transform and Monte Carlo simulation are both utilized for signal processing with Kaul’s model[8] to transform response spectrum
to power spectrum density, which is generally derived as:

$$S_x(\omega) = \frac{\xi}{\pi \omega} \cdot \left[ S_a(\omega) \right]^2 \cdot \frac{1}{\ln(\frac{1}{\gamma})} \cdot \frac{1}{\ln(\omega)}$$

(1)

where $S_a(\omega)$ denotes power spectrum density of the target signal, transformed from response spectrum $S_a(\omega)$, and $\xi$ denotes damping ratio with $\gamma$ representing exceedance probability. Afterwards, Fourier amplitude is accessed by calculated power spectrum density as:

$$A(\omega) = [S_x(\omega) \cdot \Delta \omega]^{1/2}$$

(2)

where $A(\omega)$ represents Fourier amplitude, and $\Delta \omega$ denotes sampling interval for discrete Fourier coefficients. Then, inverse Fourier Transform is conducted with an enveloping function $I(t)$ to control the shape of acceleration time-series.

$$x(t) = FT^{-1}(A(\omega) \cdot e^{j\phi(\omega)}) \cdot I(t)$$

(3)

where $x(t)$ denotes the artificial time-series generated by inverse Fourier Transform operator FT, with a random phase spectrum $\phi(\omega)$.

The scheme works in common situations, although in some certain cases the artificial response spectrum does not converge to the design response spectrum and the error between them even diverges in the iteration process.

3. Methodology Using Phase Matrix

With regards to the issues mentioned above, the scheme is presented in the chapter. Basically, the scheme utilizes $(R \times 10)$ series of random phase spectra, based on which $(R \times 10)$ series of artificial ground motions are generated in $R$th iteration for Fourier amplitude. The iteration is derived from the error between artificial response spectra and design response spectrum as the conventional methodology.

First, power spectrum density (PSD) function is attained from design response spectrum by Equation (1). The Fourier amplitude function is transformed from PSD by Equation (2). The sampling frequencies are represented by $\omega_k$, and the sum of sampling points is denoted as $K$.

Then, a random phase matrix of $(10 \times K)$ is given as:

$$\Theta^1 = \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1K} \\ \theta_{21} & \theta_{22} & \cdots & \theta_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{101} & \theta_{102} & \cdots & \theta_{10K} \end{bmatrix}$$

(4)

$$\theta_{nk} = [0, 2\pi]$$

(5)
\[ n = 1, 2, \cdots, 10 \]
\[ k = 1, 2, \cdots, K \]

where \( \theta_{nk} \) denotes a random phase from 0 to \( 2\pi \), randomly generated by algorithm of Mersenne Twister[7].

Random phase matrix is added to Fourier amplitude obtained at first, and 10 artificial ground motions \( G_{1n}(t_m) \) are generated. Duhamel’s Integral[2] is conducted to calculate 10 series of response spectra:

\[
G_{1n}(t_m) = \sum_{k=1}^{K} A(\omega_k) \cdot \cos(t_m + \theta_{nk}) \cdot e^{-\xi \omega_k(t_{m-1} - t_m)} \sin[\omega_k \sqrt{1 - \xi^2} (t_{m-1} - t_m)] \Delta t \tag{8}
\]

\[
S_{1n}(\omega_k) = \frac{1}{\omega_k \sqrt{1 - \xi^2}} \sum_{m=0}^{M-1} G_{1n}(t_m) e^{-\xi \omega_k(t_{m-1} - t_m)} \Delta t \tag{9}
\]

where \( t_m \) denotes sampling points for ground motion and sampling sum equals to \( M \), with \( \xi \) as damping ratio and \( \Delta t \) as sampling interval of artificial ground motions in time domain. For \( S_{1n}(\omega_k) \), \( n = 1, 2, \cdots, 10 \), the mean value is obtained, and the corresponding error is utilized for iteration on Fourier amplitude:

\[
A_{1}(\omega_k) = A(\omega_k) \cdot \left( \frac{S_p(\omega_k)^{10}}{S_{11}(\omega_k) + S_{12}(\omega_k) + \cdots + S_{110}(\omega_k)} \right)^2 \tag{10}
\]

Equation (10) is substituted into Equation (8), and \( 2 \times 10 \) series of time histories are generated. The steps above are the first iteration process.

In the scheme for generating artificial ground motions, for the \( r \)th iteration process, \( 10 \times r \) series of artificial ground motions are required, and Equation (4), (6) are modified as:

\[
\Theta^r = \begin{bmatrix}
\theta_{11} & \theta_{12} & \cdots & \theta_{1K} \\
\theta_{21} & \theta_{22} & \cdots & \theta_{2K} \\
\vdots & \vdots & \ddots & \vdots \\
\theta_{(10 \times r)1} & \theta_{(10 \times r)2} & \cdots & \theta_{(10 \times r)K}
\end{bmatrix} \tag{11}
\]

\[
G_{rn}(t_m) = \sum_{k=1}^{K} A_r(\omega_k) \cdot \cos(t_m + \theta_{nk}) \cdot e^{-\xi \omega_k(t_{m-1} - t_m)} \sin[\omega_k \sqrt{1 - \xi^2} (t_{m-1} - t_m)] \Delta t \tag{12}
\]

\[
S_{rn}(\omega_k) = \frac{1}{\omega_k \sqrt{1 - \xi^2}} \sum_{m=0}^{M-1} G_{rn}(t_m) e^{-\xi \omega_k(t_{m-1} - t_m)} \Delta t \tag{13}
\]

\[
A_r(\omega_k) = A_{r-1}(\omega_k) \cdot \left( \frac{S_p(\omega_k)^{10 \times r}}{S_{r1}(\omega_k) + S_{r2}(\omega_k) + \cdots + S_{r(10 \times r)\omega_k}} \right)^2 \tag{14}
\]

10 times of iterations are considered enough for convergency for frequency domain in most cases. In 10th iteration, the final ground motion is accessed by phase matrix \( \Theta^{10} \cdot \Theta^{100-K} \):

\[
G(t_m) = \sum_{n=1}^{100} \sum_{k=1}^{K} A_{10}(\omega_k) \cdot \cos(t_m + \theta_{100nk}^{100}) \tag{16}
\]

Stubborn points in frequency domain become relatively less with this methodology and the error between response spectra is barely affected by the random phase matrix selected. In the scheme, 550 phase spectra are required for 10 times iteration, and fluctuation by the random phase spectrum to convergence result is reduced considerably.

### 4. Numerical Simulation

A representative response spectrum for seismic design of nuclear equipment is selected for numerical simulation in the chapter[4,9]. The target spectrum is relatively more complicated to regular design spectrum of building structures and bridge structures, in which there is more than 13 stages for spectral amplitude, enhancing the confidence and creditability of the simulation result.

10 trials of simulation with the proposed methodology are conducted to generate 10 artificial ground motions with response spectrum compatible. The duration of artificial time-series is 40.96 seconds, and the sampling frequency is 100Hz. The comparison of representative response spectrum and response spectra of artificial ground motions is shown in Fig 2, and the one of acceleration time-series is shown in Fig 2.

In Fig 2, it could be found that simulation results by the methodology are in good quality and the stability of the scheme is as we supposed that appear consistency with different trials of Fourier phase spectra.
5. Conclusion

Artificial ground motion simulation is a classic research topic in the field of seismic engineering and structural dynamics, although conventional methods have a variety of defects to be solved. In the paper, an innovative seismic simulation methodology using Fourier phase matrix is proposed in order to enhance the consistency of the simulation result of artificial ground motions. Numerical study is conducted with a representative response spectrum for seismic design of nuclear equipment, and the result shows that the stability for each trial of simulation is maintained by the modified methodology and the computational efficiency is improved considerably.

For prospective study in the future, further applications of the scheme on more design response
spectra of other structures are supposed to have more research and practice.

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