Low-energy spectral features of supernova (anti)neutrinos in inverted hierarchy

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In the dense supernova core, self-interactions may align the flavor polarization vectors of $\nu$ and $\bar{\nu}$, and induce collective flavor transformations. Different alignment ansatzes are known to describe approximately the phenomena of synchronized or bipolar oscillations, and the split of $\nu$ energy spectra. We discuss another phenomenon observed in some numerical experiments in inverted hierarchy, showing features akin to a low-energy split of $\nu$ spectra. The phenomenon appears to be approximately described by another alignment ansatz which, in the considered scenario, reduces the (nonadiabatic) dynamics of all energy modes to only two $\nu$ plus two $\bar{\nu}$ modes. The associated spectral features, however, appear to be fragile when passing from single- to multi-angle simulations.

I. INTRODUCTION

Supernova (SN) neutrinos continue to be a subject of great interest in astroparticle physics [1]. In particular, renewed attention is being paid to collective features of flavor transformations induced by $\nu$ ($\bar{\nu}$) self-interactions (see [2] and refs. therein). The observed collective phenomena of synchronized [3] and bipolar [4, 5] oscillations, and the split of $\nu$ spectra [6, 7] can be largely understood, in flavor space, in terms of various alignment (or antialignment) ansatzes for the Bloch polarization vectors of $\nu$ ($\bar{\nu}$) and $\bar{\nu}$ ($\bar{\nu}$) at different energy $E$. Conversely, lack of alignment indicates flavor decoherence [8].

In this work we deal with a minor—yet interesting—phenomenon previously observed in the numerical experiments of [9] and then of [10, 11], akin to a low-energy “antineutrino spectral split” in inverted hierarchy. Different from the neutrino case, this feature appears to have a nonadiabatic origin. We show that the $\bar{\nu}$ spectral split observed in [9] can be approximately described in terms of $\mathbf{P}$ and $\mathbf{P}$ alignment along four global modes (at low and high energy, for $\nu$ and $\bar{\nu}$). We also discuss a scenario with different input spectra, where the $\bar{\nu}$ split is slightly more evident, at least in the approximation of averaged trajectories. The effects described herein might play a role, in principle, in future low-energy SN neutrino observations via $\bar{\nu}_e + p \rightarrow n + e^+$ scattering.

II. EQUATIONS AND NUMERICAL RESULTS

As in [9], we consider an effective two-flavor ($\nu_e, \nu_x$) scenario with mass-mixing parameters ($\Delta m^2, \theta_{13}$). After trajectory averaging (single-angle approximation), the $\mathbf{P}$ and $\mathbf{P}$ modes obey the equations of motion (EOM)

$$\dot{\mathbf{P}} = \mathbf{H} \times \mathbf{P} \equiv (+\omega \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathbf{P},$$

$$\dot{\mathbf{P}} = \mathbf{H} \times \mathbf{P} \equiv (-\omega \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathbf{P},$$

where $\omega = \Delta m^2 / 2E$ is the vacuum oscillation frequency, $\lambda = \sqrt{2}G_F N_e$ is the matter potential related to the $e^-$ density ($N_e$), $\mu = \sqrt{2}G_F (N + \bar{N})$ is the neutrino-neutrino potential related to the effective $\nu$ and $\bar{\nu}$ densities ($N$ and $\bar{N}$), and $\mathbf{B} = (\sin 2\theta_{13}, 0, \mp \cos 2\theta_{13})$, where the upper (lower) sign refers to normal (inverted) hierarchy. Only the latter case will be considered hereafter.

Global polarization vectors are defined as integrals weighted by $\nu$ and $\bar{\nu}$ spectral densities ($n$ and $\bar{n}$) [9]

$$\mathbf{J} = \frac{1}{N + N} \int_0^\infty dE n \mathbf{P},$$

$$\mathbf{J} = \frac{1}{N + N} \int_0^\infty dE \bar{n} \mathbf{P}$$

and their difference provides the vector $\mathbf{D} = \mathbf{J} - \mathbf{J}$.

Numerically, we take $\Delta m^2 = 2 \times 10^{-3}$ eV$^2$, so that

$$\omega [\text{km}^{-1}] = \frac{5.07}{E [\text{MeV}]}.$$  

We also assume $\sin^2 \theta_{13} = s_{13}^2 = 10^{-4}$ as reference value. It is then $\mathbf{B} \simeq \mathbf{z}$, with $D_x = \mathbf{D} \cdot \mathbf{z} \simeq \mathbf{D} \cdot \mathbf{B}$ being a constant of motion [3]. Concerning the potentials $\lambda$ and $\mu$, their radial profile above the neutrinosphere ($r > 10$ km) are taken as in [3] and shown in Fig. 1.

![Fig. 1: Radial profiles adopted for the matter ($\lambda$) and self-interaction ($\mu$) potentials, in the range $r \in [10, 250]$ km.](image-url)
Figure 2 shows typical results at the end of collective flavor transformations in inverted hierarchy (not much happens in normal hierarchy), as obtained by solving Eqs. (1, 2) with the same initial conditions as in [3]. The most prominent feature in Fig. 2 is the $\nu$ spectral split in the left panel at a critical energy $E_c \approx 7\text{ MeV}$, but a “minor” $\tau$ spectral split is also visible in the right panel of Fig. 2 at $E_c$, as observed in [3]. A similar minor feature has been later observed as a small low-energy “shoulder” [10] or “bump” [11] in the final $\tau$ spectra.

III. A DESCRIPTION WITH FOUR VECTORS

In general, numerical solutions to Eqs. (1, 2) can be partly understood via appropriate “alignment” ansatzes for $P$ and $\bar{P}$ modes. In particular, for large $\mu$, one can posit that they are separately pinned to their sum (i.e., $P \parallel J$ and $\bar{P} \parallel \bar{J}$) [3]. The EOM for $J$ and $\bar{J}$ can then be cast in a closed form, formally equivalent to the EOM of a gyroscopic pendulum [3], which elegantly explains the so-called synchronized and bipolar oscillations observed in numerical experiments [4, 5]. Alternatively, for slowly varying (adiabatic) $\mu$, one can posit alignment with the Hamiltonians in Eqs. (1, 2): $P \parallel H$ and $\bar{P} \parallel \bar{H}$, explaining nicely [3] the so-called $\nu$ spectral split [7, 9, 13, 14].

The above two ansatzes are, in part, exclusive. The “pendulum” solution describes well the synchronized and bipolar oscillation phases, but not the transient, nonadiabatic bipolar oscillations. However, neither describe the low-energy $\tau$ spectral split of Fig. 2. This phenomenon must thus be related to the breaking of global alignment and of adiabaticity.

Adiabaticity breaking is expected in our scenario, especially at low $E$. According to the criterion proposed in [3] (which is independent of both $\theta_{13}$ and $\lambda$), the evolution becomes nonadiabatic when the value of $\mu$ approaches a typical vacuum frequency $\omega$. For $E \lesssim 40\text{ MeV}$ (Fig. 2), the nonadiabaticity criterion is roughly $\mu \sim \omega \gtrsim 0.13 \text{ km}^{-1}$, and is satisfied at any radius $r$ in Fig. 1. Moreover, since $\mu$ is a decreasing function of $r$, adiabaticity is violated more at high $\omega$ (low $E$). However, we have not been able to sharpen this qualitative expectation and to “predict,” for instance, the observed value of the $\tau$ critical energy $E_c$ in terms of a specific transition to nonadiabaticity. For instance, we find that $E_c$ depends also on $\theta_{13}$ and $\lambda$; e.g., it decreases by $\sim 2\text{ MeV}$ by switching off the matter term $\lambda$, but is restored to $E_c \sim 4\text{ MeV}$ for $s_{13}^2 = 10^{-6}$ at $\lambda = 0$. Numerical observations thus suggest that the origin of the $E_c$ value is linked to the complex dynamics close to the $z$ axis, which can be rather subtle.

Here we adopt a pragmatic approach, and take $E_c$ and $E_c$ from the numerical experiment at face value. Our main observation is that these energies naturally separate “low” ($l$) and “high” ($h$) energy modes, and that these modes appear to be separately pinned to their global sum. More precisely, we define four new global vectors by splitting the energy intervals in Eqs. (6, 7) as $[0, E_c] \cup [E_c, \infty]$ for $\nu$ and $[0, E_c] \cup [E_c, \infty]$ for $\bar{\nu}$,

$$J = J_l + J_h , \quad \bar{J} = \bar{J}_l + \bar{J}_h .$$

We find that the moduli of the above vectors ($J_l = |J_l|$) etc.) are approximately conserved during the evolution.

Figure 3 shows, for each of the four vectors defined above, both the modulus and the $z$-component as a function of $r$. The initial moduli, calculated for $E_c \approx 7.6\text{ MeV}$ and $E_c \approx 3.9\text{ MeV}$, are:

$$J_l \approx 0.149 , \quad J_h \approx 0.096 , \quad \bar{J}_l \approx 0.008 , \quad \bar{J}_h \approx 0.101 .$$
Such values do not change much with $r$, despite the fact that the four vectors, initially aligned in the synchronized phase ($r \lesssim 70$ km), spread apart and mutate during the bipolar phase ($70 \lesssim r \lesssim 120$ km) and finally split up with low-energy (high-energy) modes aligned (antialigned) with the $z$ axis, corresponding to no (full) flavor conversion.

A posteriori, this behavior is equivalent to posit the (approximate) alignment ansatz: $\mathbf{P} \parallel \mathbf{J}_f$ for $E < E_c$, $\mathbf{P} \parallel \mathbf{J}_h$ for $E > E_c$, and similarly for antineutrinos. Consistency of this ansatz with conservation of $D_z$ from the initial (aligned) state to the final (split) state requires

$$(J_l + J_h) - (\mathcal{J}_l + \mathcal{J}_h) = (J_l - J_h) - (\mathcal{J}_l - \mathcal{J}_h) ,$$

namely,

$$J_h = \mathcal{J}_h ,$$

which, in our case, is fulfilled within a few %. The fact that the flavor evolution in Fig. 3 is reasonably described by such an ansatz is a clear signal of deviations from the purely adiabatic solution, which would instead predict an alignment of the $\mathbf{P}$’s and $\mathbf{P}$’s to linear combinations of $\mathbf{B}$ and $\mathbf{D}$ [6,13], which we do not observe. On the other hand, the fact that $E_c$ may change somewhat with $r$ in the adiabatic solution [13] (while we have assumed $r$-independent $E_c$ and $\overline{E}_c$ values a priori) suggests that our ansatz, despite its effectiveness, may be further refined.

Summarizing, the global flavor evolution in our scenario appears to be effectively described in terms of four vectors with nearly conserved lengths, collecting $\nu$ energy modes below and above a critical value $E_c$, and $\overline{\nu}$ modes below and above a (lower) critical $\overline{E}_c$. It is then useful to check if the behavior in Fig. 3 is also captured by the solutions of EOM reduced to exactly four modes, namely,

$$\dot{J}_l = (+\omega_l \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times J_l ,$$

$$\dot{J}_h = (+\omega_h \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times J_h ,$$

$$\dot{\mathcal{J}}_l = (-\overline{\omega}_l \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathcal{J}_l ,$$

$$\dot{\mathcal{J}}_h = (-\overline{\omega}_h \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathcal{J}_h ,$$

where the four frequencies are defined as averages of $\omega$ (with weights $n_e - n_x$ and $\overline{\mathbf{P}}_e - \overline{\mathbf{P}}_x$ for $\nu$ and $\overline{\nu}$, respectively [8]) in the corresponding energy ranges ($0$, $\overline{E}_c$ for $\omega_l$, etc.). Numerically, in our case: $\omega_l \simeq 1.21$, $\omega_h \simeq 0.69$, $\overline{\omega}_l \simeq 2.04$, and $\overline{\omega}_h \simeq 0.64$ (all in km$^{-1}$). The initial conditions correspond to alignment of all vectors to $+\mathbf{z}$, with moduli given by Eqs. [3,4]. Such moduli are trivially conserved by construction. The $z$-components are more interesting, and evolve in a way rather similar to those in Fig. 3 (not shown). In particular, since conservation of $D_z$ prevails over exact alignment, the vectors $\mathbf{J}_l$ and $\overline{\mathbf{J}}_f$ ($\mathbf{J}_h$ and $\overline{\mathbf{J}}_h$) are almost—but not exactly—aligned (antialigned) to $+\mathbf{z}$. Therefore, even this simplified (4-mode) case shows an “antineutrino spectral split” to a good approximation, via the opposite behavior of $\overline{\mathbf{J}}_l$ and $\overline{\mathbf{J}}_h$, similar to the results of the complete (100-mode) spectral case shown in Figs. 2 and 3.

We conclude by discussing the tiny effect of ordinary Mikheev-Smirnov-Wolfenstein (MSW) nonadiabatic resonances in matter for very low energy $\overline{\nu}$ modes. In our scenario ($s_{13}^2 = 10^{-4}$ and $\mu D_z \ll \lambda$) the $\overline{\nu}$ MSW resonance condition $\mathcal{H} \cdot \mathbf{z} = 0$ (i.e., $\omega \cos 2\theta_{13} = \lambda + \mu D_z$) approximately reads

$$\omega \simeq \lambda .$$

This condition is met before the end of collective effects ($r \gtrsim 250$ km) for $\omega \gtrsim 5$ km$^{-1}$ ($E \lesssim 1$ MeV), see Fig. 1. When passing from $\lambda_1/\omega \gg 1$ (initial state $i$) to $\lambda_f/\omega \ll 1$ (final state $f$) through the resonance, the $\overline{\nu}$ flavor survives with probability $P_{ee} \simeq P_c$, where $P_c = \exp(-2\pi \omega s_{13}^2 \lambda/\lambda)$ is the level crossing probability [16]. The relation between $P_{ee}$ and the $z$-components of the $\overline{\nu}$ polarization vectors $\overline{\mathbf{P}}$ [6] provides the depth of the MSW resonance as

$$\overline{P}_{fz} \simeq \overline{P}_{ez}(2P_c - 1) .$$

Figure 4 shows the radial profile of $\overline{P}_{fz}$, as obtained in our numerical experiment for three representative (very low) energies. The radius and depth of the MSW resonance agree with Eqs. [15] and [17], respectively. The lower the energy, the deeper the resonance, the more inverted is the final polarization vector $\overline{\mathbf{P}}$. However, the non-collective ($\omega > \mu$) MSW effect is limited to such low energies to be practically unobservable in Fig. 2, and is numerically irrelevant for the 4-mode approximation described above. Finally, we mention that, in normal hierarchy, we find that the spectra of $\nu$ and $\overline{\nu}$ are basically unaffected by self-interaction effects, the only tiny change being a spectral swap of neutrinos for $E \lesssim 1$ MeV, induced by MSW effects (not shown).
IV. SINGLE- VERSUS MULTI-ANGLE

So far we have examined in detail the low \( E \) features of the same scenario of [9] in single-angle approximation. It was shown in [8], however, that the features in Fig. 2 (most notably the \( \nu \) spectral split) may be smeared out in more realistic (“multi-angle” [2]) numerical experiments (multi-angle) shows, in comparison with Fig. 2 (single-angle), that: (1) the \( \nu \) split is broadened, and (2) the \( \overline{\nu} \) one is largely suppressed, and survives as a slight low-energy “shoulder.” At present, this (unfavorable) numerical observation [17] remains analytically unexplained. One cannot exclude, however, that the \( \overline{\nu} \) spectral split feature may be less suppressed in other multi-angle cases, whose investigation is left to future work.

V. SUMMARY

We have studied some low-energy features of the \( \overline{\nu} \) energy spectrum, focusing on the spectral split phenomenon emerging (in inverted hierarchy) in single-angle simulations. We have related its origin to nonadiabatic aspects of the \( \nu \) and \( \overline{\nu} \) evolution, which can be simply modeled through an alignment ansatz with four energy modes. However, the \( \overline{\nu} \) spectral split feature appears to be fragile when passing to multi-angle simulations. Further studies are required to deepen its analytical understanding, as well as the conditions for its observability.

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