Suppression of FM-to-AM conversion in third-harmonic generation by tuning the ratio of modulation depth

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Issues of Frequency-to-Amplitude modulation (FM-to-AM) conversion occurred in phase-modulated third-harmonic generation (THG) process are investigated. An expression about group-velocity is theoretically derived to suppress the FM-to-AM conversion, which appears to be dependant on the ratio of modulation depth of fundamental to second-harmonic when given the same modulation frequencies of them. Simulation results indicate that the induced AM in THG process can be suppressed effectively when the expression about group-velocity is satisfied.

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Third-harmonic generation (THG) is a powerful technique to produce tunable-wavelength laser pulses, which have varieties of applications in fields such as inertial confinement fusion (ICF), photolithography, and biology [1, 2]. To meet the requirement of suppression of stimulated Brillouin scattering or beam smoothing, it is desirable to efficiently frequency-triple laser pulse that has a broad spectrum imposed by phase modulation [3]. Ideally, this phase modulation does not induce any variations in pulse intensity. However, as result of frequency-dependent effects like group-velocity dispersion, frequency modulation (FM) of input fields will be converted into amplitude modulation (AM) of output fields, which is named as FM-to-AM conversion [4]. Generally, the induced AM can lead to some higher-order nonlinear effects or may cause damages to optical elements due to instantaneous ultrahigh intensity in process, and thus needs to be prevented [3, 4]. Recently, the suppression of this FM-to-AM conversion has been attracting increasing interests, and many approaches, such as angular spectral dispersion, dual-tripler scheme, and pre-compensation with gratings or crystals, have been proposed and demonstrated [4-12].

In a previous paper, we reported that the induced AM in THG process could be suppressed at the retracing point of a crystal [13]. Here, the issue of FM-to-AM conversion in THG process is investigated from the viewpoint of modulation properties of laser pulses. An expression about group-velocity, which reveals the intrinsic group-velocity-matched relationship in phase-modulated THG process, is given to guide the suppression of FM-to-AM conversion.

Considering the effect of group-velocity mismatch, nonlinear coupling equations describing the THG process under the plane wave approximation can be written as [14]:

\[ \frac{\partial A_1(z, t)}{\partial z} + \frac{1}{v_{g1}} \frac{\partial A_1(z, t)}{\partial t} = \frac{i\omega_{1d_e}}{n_{1c}} A_3 A_2 e^{i\Delta k_0 z}, \quad (1) \]

\[ \frac{\partial A_2(z, t)}{\partial z} + \frac{1}{v_{g2}} \frac{\partial A_2(z, t)}{\partial t} = \frac{i\omega_{2d_e}}{n_{2c}} A_3 A_1 e^{i\Delta k_0 z}, \quad (2) \]

where subscripts 1, 2, and 3 refer to fundamental (FH), second-harmonic (SH), and third-harmonic (TH) pulse, respectively. By transforming the coordinate \((z, t)\) to local coordinates \((z, T = t - z/v_{g1})\) and \((z, T' = t - z/v_{g2})\), amplitude of the output TH field in frequency domain can be obtained under the pump undepletion approximation:

\[ \tilde{A}_3(z, \omega) = \frac{i\omega_{3d_e}}{n_{3c}} \int_0^z \Re \exp(-i\Delta k_0 \xi) d\xi, \quad (4) \]

where

\[ \Re = \int_{-\infty}^{\infty} \exp \left[ -a(t + \xi \nu_1)^2 - b(t + \xi \nu_2)^2 \right] \]

\[ \cdot \exp \left[ i\sigma_1 \sin \left( 2\pi \Omega_1(t + \xi \nu_1) \right) + i\sigma_2 \sin \left( 2\pi \Omega_2(t + \xi \nu_2) \right) \right] \cdot \exp(i\omega t) dt. \]

In Eq. (4), \(a = 1/2T_1^2\) and \(b = 1/2T_2^2\) are parameters determined by pulse-duration of FH and SH pulses, while \(\nu_1 = 1/v_{g3} - 1/v_{g1}\) and \(\nu_2 = 1/v_{g3} - 1/v_{g2}\) are so-called group-velocity mismatches. The pulse-duration term \(\exp[-a(t + \xi \nu_1)^2 - b(t + \xi \nu_2)^2]\), as analyzed in Ref. [15], is significantly crucial for ultrashort (e.g., picosecond, femtosecond, or even shorter) laser pulses. However, for phase-modulated broadband THG, pulse duration is generally around nanosecond [4], and the effect of that pulse-duration term is negligible.

Since the pulse-duration term can be neglected, Eq. (4) turns out to be the Fourier transformation of

\[ \exp \left[ i\sigma_1 \sin \left( 2\pi \Omega_1(t + \xi \nu_1) \right) + i\sigma_2 \sin \left( 2\pi \Omega_2(t + \xi \nu_2) \right) \right]. \quad (6) \]
For simplicity, we initially apply Fourier transforms on
\[ \exp \left[ i \sigma_1 \sin \left( 2 \pi \Omega_1 (t + \xi \nu_1) \right) \right], \]
and achieve
\[ \sum_{n=-\infty}^{\infty} J_n(\sigma_1) \delta(\omega - 2 \pi n \Omega_1) \exp(i \omega \xi \nu_1). \]  
(7)
Substituting (7) into (4), results show that the intensity of
TH pulse in frequency domain possesses the characteristic of
\[ |\tilde{A}_3(z, \omega)|^2 \propto \text{sinc}^2(\omega \nu_1 z), \]  
(8)
which implies that the output TH pulse will become
intensity-modulated if \( \nu_1 \neq 0 \). Since \( \nu_1 \neq 0 \) means no
group-velocity mismatch between FH and TH pulses, we
could conclude that the FM-to-AM conversion in process
is basically caused by group-velocity mismatches between
interacting phase-modulated pulses.

Similarly, for Eq.(6), we assume modulation frequencies
of FH and SH pulses to be the same (\( \Omega_1 = \Omega_2 = \Omega \)),
since random or unequal relations between \( \Omega_1 \) and \( \Omega_2 \)
makes analysis complicated. Let \( x = \sigma_2/\sigma_1 \), and reorganize
Eq.(6) as below
\[ \exp i \sigma_1 \left[ \sin(2 \pi \Omega_1 \nu_1) \cos(2 \pi \Omega_1 \nu_2) + \cos(2 \pi \Omega_1 \nu_1) \sin(2 \pi \Omega_1 \nu_2) \right]. \]  
(9)
Obviously, if group-velocity mismatches \( \nu_1 \) and \( \nu_2 \) satisfy
\[ \sin(2 \pi \Omega_1 \nu_1) + \sin(2 \pi \Omega_1 \nu_2) = 0, \]  
(10)
Eq.(9) could be simplified to \( \exp \left[ i \sigma' \sin(2 \pi \Omega t) \right] \), just with
the new modulation depth changing to \( \sigma' \) which is equivalent
to \( \sigma_1 \left[ \cos(2 \pi \Omega \nu_1) + \cos(2 \pi \Omega \nu_2) \right] \). Compared
with analysis ahead, this indicates that no amplitude
modulation will be induced on output TH pulse.

Under pump undepletion (\( \xi \to 0 \)) and accordingly
\( \sin \theta \approx \theta \) approximations, by substituting \( \nu_1 \) and \( \nu_2 \),
Eq.(10) transforms to
\[ \frac{1 + x}{v_{g3}} = \frac{1}{v_{g1}} + \frac{x}{v_{g2}} \quad (x = \sigma_2/\sigma_1, \Omega_2 = \Omega_1). \]  
(11)
This expression about group-velocity gives guidance for
suppression of FM-to-AM conversion occurred in phase-modulated
THG process. Coincidentally, Eq.(11) looks exactly the same with Eq.(19) in Ref.[12], the only difference
between them is the physical meaning of parameter \( x \). \( x \) here represents the ratio of modulation depth of
SH to FH pulse, while \( x \) in Ref.[12] represents the ratio
of pulse duration of FH to SH pulse. Meanwhile,
on the other hand, the two equations are consistent with
each other, as frequency bandwidth of sinusoidally phase-modulated pulse is determined by \( 2 \sigma \Omega \), while bandwidth of transform-limited ultrashort pulse is determined by the reciprocal of pulse duration.

Based on the split-step Fourier transform and the
fourth-order Runge-Kutta algorithm, the THG process

| Wavelength | Pulse Duration | Modulation Frequency | Group-velocity Peak |
|------------|----------------|----------------------|--------------------|
| 1\(\omega/1.053\mu m\) | 1 | 10 | 2.02\(\times 10^8\) |
| 2\(\omega/0.527\mu m\) | 1 | 10 | 1.94\(\times 10^8\) |

Table I. Basic Parameters of Input 1\(\omega\) and 2\(\omega\) pulses

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FIG. 1. Temporal profiles with different $x$. 

(a) $x=1$

(b) $x=2$

(c) $x=3$

(d) $x=4$
FIG. 2. Practical temporal profiles in KDP crystal

FIG. 3. Temporal profiles with $x = 1$ and $\sigma_1 = \sigma_2 = 15$

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