Analysis of proper fluctuations of technological systems

D Y Ershov, E G Zlotnikov and D Y Timofeev
Saint-Petersburg Mining University, 2, 21st Line, St. Petersburg 199106, Russia
E-mail: Ershov_DYu@pers.spmi.ru

Abstract. The article describes the principles of building dynamic models of technological systems with concentrated parameters with a finite number of degrees of freedom. Dynamic properties of technological system are considered taking into account elastic and inertial parameters of its elements. The model of three-mass torsional chain system and the model with progressively moving masses are used for estimation of vibrational processes in drives of the technological equipment. According to the proposed mathematical model the analysis is carried out and the forms of natural oscillatory movements in the system are revealed. Oscillations of zero, first and second eigenfrequencies are determined, for each of which sets of form of the coefficients are found that describe the system configuration in the process of free oscillations.

1. Introduction

In modern conditions, the intensification of production processes is observed, which eventually leads to an increase in the operating speeds of technological machines. There is a need for deeper consideration of dynamic factors affecting the accuracy of production equipment [1,2].

Any technological machine consists of several functional parts, namely, the engine, the mechanical system and the motion control system. The mechanical system of the machine carries out the transformation of relatively simple movements of the input links into complex movements of the actuators, which ensure the performance of given technological operations. Mechanical system components are primarily subject to different types of vibration. Occurring oscillations lead to essential distortions of the set functions of program movement of working elements, reduce accuracy in processing of products and worsen other conditions of normal operation of the equipment [5,6,8,14,15].

The study of the conditions of the emergence and analysis of vibrations during machining using modern high-performance CNC machines is of particular importance. Here, significant dynamic loads arise in the operation of main motion drives, as well as feed drives that provide the formation of a complex tool path along the surface to be machined. Dynamic errors caused by free oscillations in the unsteady modes of the machine tools many times exceed the static errors of positioning of actuators. The attenuation time of these vibrations can be commensurate with the time of program movements. In this case, accuracy is inevitably reduced, deviations from the given geometric shape appear, and the quality of the surface deteriorates [3,4,9,16].

The analysis of torsional oscillations in the main drive and feed drives reveals two areas of intense oscillation. The first is due to the dynamics of the main drive, and the second is due to the spectrum of the natural frequencies of the machine's support parts. Increase of rigidity of the mechanical system “machine-tool-detail”, reduction of friction forces in feed drives, selection of optimal cutting modes for areas with reduced vibration levels, application of software algorithms and design solutions for vibration...
damping are the main ways to solve the problem of improving the accuracy of machining on CNC machines [3, 5-7, 10].

2. Materials and methods
For engineering calculations dynamic models with concentrated (i.e. discretely set) parameters are often used, in which the number of degrees of freedom is finite. Construction of such models is based on the following principles [8, 9].

1. Inertial properties of a mechanical system are displayed by masses or moments of inertia, which are concentrated in separate points or sections.
2. These points or sections are connected by elastic, dissipative and geometric (kinematic) bonds without inertial properties. Dissipative bonds take into account the resistive forces that cause mechanical energy to dissipate and partially transfer it to other types of energy.

To simplify the dynamic model, the most massive elements and the most flexible (i.e. the least rigid) parts of the kinematic chain are singled out in the machine drive and its mechanisms [11, 12, 13].

The application of a sophisticated model can lead to errors due to inaccurate rigidity and dissipative factors in the system. To increase the reliability of the analysis it is necessary to choose the simplest dynamic models, which, nevertheless, are able to adequately reflect the vibrational processes under study in the technological system.

Many works related to the problem of limiting the elastic vibrations in technological systems take into account only the elastic properties of kinematic links and mechanical transmissions of motion. In this case, a two-mass design scheme is used to simulate the motion of the actuators by individual degrees of mobility. However, for the analysis of dynamic properties of high-performance equipment it is necessary to take into account also inertia of system elements and to use multi-mass design schemes, among which three-mass vibrating systems are of the greatest interest [6, 10, 11, 12, 13].

3. Mathematical description and analysis of the three-mass vibrating system
Let us consider the oscillating torsional chain system shown in Figure 1. There are three inertial discs on the elastic shaft with moments of inertia \( J_1, J_2 \) and \( J_3 \). The torsional stiffness coefficients of the shaft sections are indicated via \( C_1 \) and \( C_2 \). The mechanical drive system includes the motor rotor 1, gear 2 and implement 3. The inertia moments of the links shown in the diagram are calculated relative to the natural rotation axes. Motor torque \( M_1 \) is applied to the engine rotor, and the working body – technological resistance torque \( M_3 \). Viscous friction in the system is not considered in this article. The mechanical system has three degrees of freedom. Let us choose as generalized coordinates the rotation angles of disks \( \varphi_1, \varphi_2 \) and \( \varphi_3 \), counting them from any position of the system at the undeformed shaft.

![Figure 1. Mechanical system with three degrees of freedom.](image-url)

Let us record the kinetic energy of the system:

\[
T = \frac{1}{2} J_1 \dot{\varphi}_1^2 + \frac{1}{2} J_2 \dot{\varphi}_2^2 + \frac{1}{2} J_3 \dot{\varphi}_3^2. 
\]  

(1)

The potential energy is determined by the expression:

\[
\Pi = \frac{1}{2} C_1 (\varphi_1 - \varphi_2)^2 + \frac{1}{2} C_2 (\varphi_2 - \varphi_3)^2. 
\]  

(2)
Using the Lagrange equations of the second kind:
\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\varphi}_j} \right) - \frac{\partial T}{\partial \varphi_j} = -\frac{\partial \Pi}{\partial \dot{\varphi}_j}, \quad (j = 1, 2, 3).
\]  
(3)

Let us write down the differential equations of motion:
\[
\begin{align*}
J_1\ddot{\varphi}_1 + C_1\varphi_1 - C_1\varphi_2 &= 0, \\
J_2\ddot{\varphi}_2 + (C_1 + C_2)\varphi_2 - C_1\varphi_1 - C_2\varphi_3 &= 0, \\
J_3\ddot{\varphi}_3 + C_2\varphi_3 - C_2\varphi_2 &= 0.
\end{align*}
\]  
(4)

Before we begin to integrate the system (4), let us consider which movements are described by this system of equations. Having summed up all the differential equations, we obtain:
\[
J_1\ddot{\varphi}_1 + J_2\ddot{\varphi}_2 + J_3\ddot{\varphi}_3 = 0.
\]  
(5)

The obtained equation is convenient to write in the form of a theorem about the change in the kinetic moment of the system:
\[
\frac{d}{dt} \left( J_1\ddot{\varphi}_1 + J_2\ddot{\varphi}_2 + J_3\ddot{\varphi}_3 \right) = 0,
\]  
(6)

where we get:
\[
J_1\ddot{\varphi}_1 + J_2\ddot{\varphi}_2 + J_3\ddot{\varphi}_3 = \text{const}.
\]  
(7)

It is more convenient to estimate the possible system movements described by equation (7) using a model with progressively moving masses in the direction of the Ox axis (Figure 2). Let us write down an equation similar to equation (7):
\[
m_1\ddot{x}_1 + m_2\ddot{x}_2 + m_3\ddot{x}_3 = \text{const} = \dot{x}_c \left( m_1 + m_2 + m_3 \right),
\]  
(8)

where \(\dot{x}_1, \dot{x}_2, \dot{x}_3\) are the mass velocities, as well as the system's center of inertia.

\[\text{Figure 2. A model with progressively moving masses.}\]

Let us imagine the motion of the system bodies as complex and consisting of motion together with the center of inertia and relative to the center of inertia. Then the velocities of bodies can be written as follows:
\[
\begin{align*}
\dot{x}_1 &= \dot{x}_c + \dot{x}_{1r}, \\
\dot{x}_2 &= \dot{x}_c + \dot{x}_{2r}, \\
\dot{x}_3 &= \dot{x}_c + \dot{x}_{3r}.
\end{align*}
\]  
(9)

where \(\dot{x}_{1r}, \dot{x}_{2r}, \dot{x}_{3r}\) are the relative velocities of bodies relative to the center of inertia of the system.

Let us substitute (9) to (8):
\[
\left( m_1 + m_2 + m_3 \right) \dot{x}_c + m_1\dot{x}_{1r} + m_2\dot{x}_{2r} + m_3\dot{x}_{3r} = \text{const}.
\]  
(10)

The first term in (10) determines the progressive motion of the entire system as a rigid whole with the speed of the center of mass \(\dot{x}_c\). In this motion there is no deformation of elastic elements. The other three terms in (10) determine the motion of bodies relative to the system considered to be a rigid whole. In this case, the elastic elements undergo deformation. Consequently, the last three terms in (10) reflect the oscillations of the bodies superimposed on the motion of the system together with the center of inertia. From the comparison (8) and (10) we have:
\[
\begin{align*}
(m_1 + m_2 + m_3) \ddot{\chi} = \text{const} \\
m_1 \ddot{x}_1 + m_2 \ddot{x}_2 + m_3 \ddot{x}_3 = 0.
\end{align*}
\] (11)

From (11), it follows that:

1. If the initial conditions are such that \(\text{const} \neq 0\), the system moves as a rigid body with a speed equal to that of the body:

\[
\dot{\chi} = \frac{\text{const}}{(m_1 + m_2 + m_3)},
\]

and this movement can be accompanied by body oscillations.

2. If the initial conditions are such that \(\text{const} = 0\), there is no movement of the system as a rigid whole; only oscillations near the stationary center of inertia are observed.

3. The number of system movements in case of oscillations is zero.

The results obtained are fully applicable to the differential equation (7). In general, when \(\text{const} \neq 0\), this equation defines the rotation of the system as a rigid whole with a constant angular velocity \(\omega_0\):

\[
\phi_1 = \phi_2 = \phi_3 = \phi_0 + \omega_0 t,
\] (12)

where \(\phi_0\) and \(\omega_0\) are from the initial conditions.

In addition to solution (12), the system of equations (4) also allows for oscillations of the discs near the undeformed state of the shaft, described by the functions:

\[
\begin{align*}
\phi_1 &= A_1 \sin(kt + \alpha) \\
\phi_2 &= A_2 \sin(kt + \alpha) \\
\phi_3 &= A_3 \sin(kt + \alpha)
\end{align*}
\] (13)

The functions (13) define the motion of the system in which the discs oscillate at the same frequency \(k\) and with the same starting phase \(\alpha\), but with different amplitudes. Substitute (13) in equation (4) and after the dividing by \(\sin(kt + \alpha)\), we obtain a system of algebraic equations relative to the unknown amplitudes:

\[
\begin{align*}
\left( C_1 - J_1 k^2 \right) A_1 - C_1 A_2 &= 0 \\
- C_1 A_1 + \left( C_1 + C_2 - J_2 k^2 \right) A_2 - C_2 A_3 &= 0 \\
- C_2 A_1 + \left( C_2 - J_3 k^2 \right) A_3 &= 0
\end{align*}
\] (14)

The homogeneous system of equations (14) has non-zero solutions if the system determinant is equal to zero. Let us solve the system of equations (14) in a matrix way:

\[
\Delta(k^2) = \begin{vmatrix}
C_1 - J_1 k^2 & -C_1 & 0 \\
-C_1 & C_1 + C_2 - J_2 k^2 & -C_2 \\
0 & -C_2 & C_2 - J_3 k^2
\end{vmatrix} = 0.
\] (15)

Expanding the determinant, we obtain the frequency equation:

\[
J_1 J_2 J_3 k^2 \left[ k^2 - \left( C_1 J_1 J_2 + C_2 J_2 J_3 + J_3 \right) + \left( C_1 C_2 J_1 J_2 + J_3 \right) \right] = 0.
\] (16)

One of the roots of this equation is zero: \(k_0 = 0\). This root corresponds to the rotation of the system as a rigid whole. In addition to zero, equation (16) defines two more roots: the first \(k_1\) and the second \(k_2\) natural frequencies of the mechanical system.

Mechanical systems whose frequency equation has one or more zero-roots are called semi-defined. These roots correspond to a form of motion in which the deformation of the elastic links is absent and the system moves as a rigid whole. Thus, the torsional system shown in Figure 1 is semi-defined.
Since the determinant of the equation system (14) is zero, the equations themselves are not independent. This means that the system (14) allows only the amplitude ratios to be determined, not the amplitudes themselves. Therefore, let us introduce the form coefficients:

$$\mu_1 = \frac{A_1}{A_1}; \quad \mu_2 = \frac{A_2}{A_1}; \quad \mu_3 = \frac{A_3}{A_1}. \quad (17)$$

Using the first and second equations (14), the following is obtained:

$$\mu_2 = \frac{C_1 - J_2 k^2}{C_1}; \quad \mu_3 = -\frac{C_1 + C_2 - J_2 k^2}{C_2}. \quad (18)$$

One must note that $\mu_1=1$. Substituting the values of the roots $k_0$, $k_1$ and $k_2$ of the frequency equation into these equations, we will find a set of form coefficients corresponding to each eigenfrequency:

$$\mu_{10} = 1, \quad \mu_{20} = 1, \quad \mu_{30} = 1,$$

$$\mu_{11} = 1, \quad \mu_{21} = \frac{C_1 - J_2 k^2}{C_1}, \quad \mu_{31} = -\frac{C_1 + C_2 - J_2 k^2}{C_2},$$

$$\mu_{12} = 1, \quad \mu_{22} = \frac{C_1 - J_2 k^2}{C_1}, \quad \mu_{32} = -\frac{C_1 + C_2 - J_2 k^2}{C_2}. \quad (19)$$

Each set of form factors describes the configuration of the system at with amplitude deviations from the equilibrium position in the process of free oscillations with one of the natural frequencies. This configuration is called as its own form. Thus, the torsional system under consideration (Figure 1) has three own forms. The form of motion of the system with zero frequency $k_0 = 0$ is not oscillatory, but determines the motion in which the discs rotate at the same constant angular speed $\omega_0$. This movement can be conventionally called as the frequency $k_0$ oscillation.

**4. Conclusion**

Modern technological equipment can be considered as a complex oscillating system consisting of many elements with inertial and elastic parameters. An unlimited number of degrees of freedom in such system requires the construction of a relatively simple and adequate dynamic model to solve the problem of analyzing the natural oscillations in the system.

In the paper mathematical models of three-mass vibrating torsional system and system with progressively moving masses are considered. The solution of differential equations of motion revealed the forms of natural vibrational movements in the system. The frequency equation is obtained, three roots of which determine the oscillations of the zero, first and second natural frequencies of the mechanical system.

For each eigenfrequency, we find a set of form of the coefficients describing the configuration of the system in the process of free oscillations.

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