Some Theoretical Estimations of Spatial Distribution
of Compton Backscattered Laser Photons Beam.

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Abstract
Spatial distribution of intensity, degree and direction of linear polarization of
tagged photon beam, which is obtained due to Compton backscattering of laser
light on high-energy electron beam, are calculated. Effects of angular dispersion
and spatial spread of electron beam are taken into account. Calculations have been
carried out for the example of LEGS facility.

Method of obtaining monochromatic and polarized photon beams of high energy and
intensity by the Compton backscattering of laser photon on high-energy electron beam
is widely used in many active and planned facilities. The most attractive feature of
such photon beams is the circumstance that obtained beam completely inherits polarizing
characteristics of an initial laser beam in a vicinity of the top edge of a spectrum (Compton
edge), and these characteristics can be controlled well in this region. But in process of
deviation from Compton edge of spectrum the polarization of outgoing photons decreases
and shape of spatial distribution of intensity and polarization becomes rather complicated.
The photon beam could be effectively split on two ones, linear polarization in the center
of photon spot is less than at its periphery. This moment is important for the analysis
of experimental results, but it is usually discussed at a level of scientific folklore, and the
purpose of the present note is - to discuss this effects in more details.

Let’s consider kinematics of the Compton scattering process

$$\gamma(k_1) + e^-(p_1) \rightarrow \gamma(k_2) + e^-(p_2)$$

in the laboratory frame where axis $z$ coincides with a beam’s axis and $x$ axis belongs to
horizontal plane of accelerator’s beam. From 4–momentum conservation law $k_1 + p_1 =
k_2 + p_2$ it follows

$$(\alpha_x - \theta_x)^2 + (\alpha_y - \theta_y)^2 = r^2$$

(1)

where

$$\alpha_x = \theta_1 \cos \varphi_1, \ \alpha_y = \theta_1 \sin \varphi_1$$

are plane angles in $x$ and $y$ directions of initial electron ($\theta_1, \varphi_1$ are its polar and azimuthal angles),

$$\theta_x = \theta_2 \cos \varphi_2, \ \theta_y = \theta_2 \sin \varphi_2$$

are plane angles in $x$ and $y$ directions of final photon ($\theta_2, \varphi_2$ are its polar and azimuthal angles).

$$r^2 = \gamma^{-2} \left[ \lambda \left( \frac{\varepsilon_1 - \omega_2}{\omega_2} \right) - 1 \right]; \ \ \gamma = \frac{\varepsilon_1}{m}; \ \ \lambda = \frac{2 \langle k_1 \cdot p_1 \rangle}{m^2} \approx \frac{4 \omega_1 \varepsilon_1}{m^2};$$


\( \varepsilon_1, m \) are energy and mass of initial electron, \( \omega_1, \omega_2 \) are energies of initial and final photons, correspondingly. Equation (1) is valid with an accuracy up to terms

\[
\left[ \frac{\omega_1}{\varepsilon_1}, \theta_2^2, \theta_x^2, \theta_y^2, \gamma^{-2} \right] \ll 1
\]

(2)

Thus we can see from (1) that photons with a definite energy \( \omega_2 \) are emitted at the surface of the circle cone with an axis along direction of initial electron motion and the opening angle \( r \). Allowable energy of outgoing photon is determined from a condition of positive definiteness of \( r^2 \), and maximal possible energy of secondary photon is

\[
\omega_{2,\text{max}} = \frac{\varepsilon_1 \lambda}{1 + \lambda};
\]

(3)

The cross section of the Compton scattering of the polarized photon by the unpolarized electron when one detects final photon with a Stoke’s parameters \( \xi^{(2)} \) is [1, 2]

\[
\frac{d^2 \sigma}{d\omega_2 d\varphi} = S p \{ \hat{\rho}_c \cdot \hat{\rho}^{(2)} \} = \frac{r_o^2}{2 \varepsilon_1 \lambda (1 + u^2)^2 (1 + \lambda + u^2)} \left\{ \Phi_0 + \xi_1^{(2)} \Phi_1 + \xi_2^{(2)} \Phi_2 + \xi_3^{(2)} \Phi_3 \right\}
\]

(4)

Here \( \hat{\rho}^{(2)} \) is a density matrix of detected final photon:

\[
\hat{\rho}^{(2)} = \frac{1}{2} \left( \begin{array}{cc}
1 + \xi_3^{(2)} & \xi_1^{(2)} - i \xi_2^{(2)} \\
\xi_1^{(2)} + i \xi_2^{(2)} & 1 - \xi_3^{(2)}
\end{array} \right);
\]

and matrix \( \hat{\rho}_c \) can be considered as product of cross section for unpolarized photon and proper density matrix of final photon:

\[
\hat{\rho}_c = \frac{r_o^2}{2 \varepsilon_1 \lambda (1 + u^2)^2 (1 + \lambda + u^2)} \left( \begin{array}{cc}
\Phi_0 + \Phi_3 & \Phi_1 + i \Phi_2 \\
\Phi_1 - i \Phi_2 & \Phi_0 - \Phi_3
\end{array} \right);
\]

(5)

where

\[
\Phi_0 = 2 + 2 \lambda + \lambda^2 + u^2 (2 + \lambda^2) + 2u^4 (1 + \lambda) + 2u^6 - 4 \left( \xi_3^{(1)} \cos 2 \varphi - \xi_1^{(1)} \sin 2 \varphi \right) u^2 (1 + \lambda + u^2)
\]

(6)

\[
\Phi_1 = 2 (1 + \lambda + u^2) \left( \xi_3^{(1)} (-1 + u^4 \cos 4 \varphi) + u^4 \xi_3^{(1)} \sin 4 \varphi - 2u^2 \sin 2 \varphi \right);
\]

(7)

\[
\Phi_2 = -\xi_2^{(1)} (1 - u^2) (2 + 2 \lambda + \lambda^2 + 2u^2 (2 + \lambda + u^2));
\]

(8)

\[
\Phi_3 = 2 \left( 1 + \lambda + u^2 \right) \left( \xi_3^{(1)} (1 + u^4 \cos 4 \varphi) - \xi_1^{(1)} u^4 \sin 4 \varphi - 2u^2 \cos 2 \varphi \right);
\]

(9)

In the expressions (4 – 9) \( r_o \) is classical radius of electron, \( \varphi \) is azimuthal angle of outgoing photon, which is counted from \( x \) direction in the spherical reference frame with polar axis along initial electron momentum, parameter \( \lambda = 2(k_1 \cdot p_1)/m^2 \approx 4\omega_1 \varepsilon_1 \), and \( u = \gamma r \).

Stoke’s parameters of initial laser photon \( \xi^{(1)} \) and of final photon \( \xi^{(2)} \) are defined relatively to the axes \{\( x, y \)\}, which are horizontal and vertical axes of electron beam. According to general theory [3, 4] from (4-5) follows that proper Stoke’s parameters of outgoing photon determined relatively to the laboratory frame axes \{\( x, y, z \)\} are:

\[
\xi^{(f)}_1 = \frac{\Phi_1}{\Phi_0}, \quad \xi^{(f)}_2 = \frac{\Phi_2}{\Phi_0}, \quad \xi^{(f)}_3 = \frac{\Phi_3}{\Phi_0};
\]

(10)
We shall use following parameterization of the photon’s polarization characteristics \( \xi \):

\[
\begin{align*}
\xi_1 &= P \cdot \cos 2\beta \cdot \sin 2\phi; \\
\xi_3 &= P \cdot \cos 2\beta \cdot \cos 2\phi; \\
\xi_2 &= P \cdot \sin 2\beta
\end{align*}
\] (11)

where \( P = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2} \) is degree of the total polarization,
\( P_1 = \sqrt{\xi_1^2 + \xi_3^2} \) is degree of the linear photon polarization,
\( \phi \) is the angle between axis \( x \) and direction of maximal linear polarization of photon that is reading counter clockwise when one looks from the end of photon momentum vector.

Let us consider what spatial distribution of tagged photons with defined energy we shall obtain at the plane of target situated at distance \( L \) from Compton interaction region. We shall suppose that (due to the large value of \( L \) in comparison with longitudinal size of laser and electron beams crossing region) the Compton interaction points are distributed at one plane according to Gaussian low with dispersions \( \delta_x \) and \( \delta_y \). The plane angles \( \theta_{1x}, \theta_{1y} \) of electrons relatively to beam axes also have Gaussian distribution with dispersion \( \sigma_x \) and \( \sigma_y \). So, spatial - angular density of probability of initial electron states is:

\[
F(x_1, y_1, \theta_{1x}, \theta_{1y}) = \frac{1}{2\pi \sqrt{\delta_x \delta_y}} \exp -\frac{x_1^2}{2\delta_x^2} \exp -\frac{y_1^2}{2\delta_y^2} \frac{1}{2\pi \sqrt{\sigma_x \sigma_y}} \exp -\frac{\theta_{1x}^2}{2\sigma_x^2} \exp -\frac{\theta_{1y}^2}{2\sigma_y^2}
\] (12)

There is one-to-one correspondence between variables \( (\omega_2, \varphi) \) and point \( (x, y) \) of photon arriving at the target if set of variables \( (x_1, y_1, \theta_{1x}, \theta_{1y}) \), which describe state of initial electron, are fixed:

\[
x = x_1 + L (\theta_{1x} + r \cos \varphi); \\
y = y_1 + L (\theta_{1y} + r \sin \varphi);
\] (13)

The tagging system allows to select events with defined photon energy, \( \omega_2 \). To take into account influence of tagging system, we have to integrate differential cross section (11) over photon energy with some appropriate distribution which describes this system. Since accuracy of modern tagging systems is rather high, near \( (1 - 2)\% \), we can use as such distribution a \( \delta \)-function. Then distribution at the target plane of arrival points \( \{x, y\} \) of tagged photons with energy \( \omega^0 \), which have been produced at point \( \{x_1, y_1\} \) by Compton interaction laser photon and electron with plane angles \( \{\theta_{1x}, \theta_{1y}\} \), is described by matrix:

\[
\hat{\rho_c}^{(tag)} d\omega_2 d\varphi = \hat{\rho_c} \delta(\omega_2 - \omega^0) d\omega_2 d\varphi = \hat{\rho_c} \delta(r^2 - r_0^2) \frac{dx dy}{2L^2};
\] (14)

where \( \hat{\rho_c} \) is defined in (10),

\[
r^2 = \left( \frac{x - x_1}{L} - \theta_{1x} \right)^2 + \left( \frac{y - y_1}{L} - \theta_{1y} \right)^2, \\
r_0^2 = \gamma^{-2} \left[ \lambda \left( \frac{\omega_1 - \omega_2}{\omega_2} \right) - 1 \right]_{\omega_2 = \omega^0};
\]

To obtain desired final distribution we have to integrate distribution (14) over states of initial electron with distribution (12). In result we have:

\[
\langle \hat{\rho_c}^{(tag)} \rangle dxdy = \frac{dxdy}{2\pi \sqrt{\Delta_x \Delta_y}} \int_0^{2\pi} d\varphi \ \exp -\frac{(x - r_0L \cos \varphi)^2}{2\Delta_x^2} \exp -\frac{(y - r_0L \sin \varphi)^2}{2\Delta_y^2} \hat{\rho_c}
\] (15)

where \( \hat{\rho_c} \) is defined in (10), and

\[
\Delta_x = \sqrt{(L \sigma_x)^2 + \delta_x^2}; \quad \Delta_y = \sqrt{(L \sigma_y)^2 + \delta_y^2};
\]
In other words, contribution to events, when at the point \( \{x, y\} \) comes photon with defined energy \( \omega_2 \), can give whole set of initial electron directions which belongs to circle cone with opening angle \( r_0 \). Therefore, to obtain polarization characteristics of photon beam we have to use integrated matrix \( \langle \hat{\rho}_{c(\text{tag})} \rangle \). Thus, we have for distribution \( I(x, y) \) of final photons with energy \( \omega_2 \) intensity following value:

\[
I(x, y) = \frac{J r_o^2 \langle \Phi_0 \rangle}{2\epsilon_1 \lambda \left( 1 + u^2 \right)^2 \left( 1 + \lambda + u^2 \right)^2};
\]

where \( J \) is appropriate current which describes intensity of electron and laser beams.

For distribution of degree of final photons linear polarization \( P(x, y) \) we have:

\[
P(x, y) = \sqrt{\left( \frac{\langle \Phi_1 \rangle}{\langle \Phi_0 \rangle} \right)^2 + \left( \frac{\langle \Phi_3 \rangle}{\langle \Phi_0 \rangle} \right)^2};
\]

and distribution of angles \( \phi(x, y) \) between direction of linear polarization and axis \( x \) can be defined from conditions:

\[
\frac{\langle \Phi_1 \rangle}{\langle \Phi_0 \rangle} = P(x, y) \sin 2\phi(x, y); \quad \frac{\langle \Phi_3 \rangle}{\langle \Phi_0 \rangle} = P(x, y) \cos 2\phi(x, y);
\]

Operation \( \langle ... \rangle \) in (16 - 18) means integration (15).

At the figures (1 - 13) results of calculations on the base of formulae (16 - 18) are depicted. This calculations have been carried out for example of LEGS facility. In this case energy of electron beam \( \epsilon_1 = 2580 \text{ MeV} \), UV laser photons have energy \( \omega_1 = 3.7 \text{ eV} \), so \( \lambda = 4\epsilon_1 \omega_1 = 0.146 \) and upper limit of backscattered photons’ energy is \( \omega_{2\text{max}} = 330 \text{ MeV} \). Dispersion of electron beam in horizontal direction is \( \sigma_x = 240 \text{ } \mu \text{rad} \), and in vertical direction \( \sigma_y = 80 \text{ } \mu \text{rad} \). Dispersion of \( \{x_1, y_1\} \) distribution of Compton interaction points is \( \delta_x = \delta_y = 0.1 \text{ cm} \). The distance from Compton interaction region to the target is \( L = 45 \text{ m} \). It is supposed that laser photon beam has 100\% linear polarization in horizontal plane \( (\xi_3^1 = +1) \) or in vertical plane \( (\xi_3^1 = -1) \).

![Figure 1: Spatial distribution of photon beam intensity. (Arbitrary units.) Photon energy \( \omega_2 = 330 \text{ MeV}, \xi_3^1 = +1 \).](image)

At the figure (1) a spatial distribution of intensity of photons near upper edge of spectrum is shown. One can see that this distribution practically repeats Gaussian distribution of initial electron beam. At the figures (2 - 3) distributions of linear polarization degree for photon energy \( \omega_2 = 320 \text{ MeV} \) are shown. Figure (2) corresponds to the case...
when initial laser photons are polarized in horizontal plane ($\xi_3^1 = +1$), and figure (3) — to the case when laser photons are polarized in vertical plane ($\xi_3^1 = -1$). One can see that polarization of final photons is near 98.9% for this energy but in the first case polarization in the center of photon beam spot is slightly less.

The essentially other situation arises when photon energy decreases below 280 MeV. Then in the case of horizontal polarization of laser ($\xi_3^1 = +1$) secondary photon beam is split into two ones and degree of polarization has deep minimum at line $y = 0$. (See figures 4 - 5).

After change of laser polarization direction this splitting of secondary beam disappeared and distribution of polarization degree has a saddle shape. (See figures 6 - 7). Let’s note that these effects have been observed and reported in [6].

In some types of experiment with the purpose of statistics increasing the events of secondary photons in some interval of photon energy $\omega_2^{(2)} > \omega_2 > \omega_2^{(1)}$ are taken into account. In this case to obtain resulting intensity and polarization we must integrate matrix $\hat{\rho}_e$ not only over spatial and angular distribution of initial electrons but also over appropriate photon energies interval. Then we shall have for spatial distributions of intensity $I(x, y)$, degree of polarization $P(x, y)$ and for angle of polarization direction
Figure 6: Spatial distribution of photon beam intensity. (Arbitrary units.) Photon energy \( \omega_2 = 220 \text{ MeV}, \xi_3^1 = -1 \).

\( \phi(x, y) \) the same formulae as (16 - 18) but averaging operation \( \langle ... \rangle \) have to be replaced into \( \langle ... \rangle_{\omega} \) where

\[
\langle \hat{\rho}_{c(tag)} \rangle_{\omega} = \int_{\omega_2^{(1)}}^{\omega_2^{(2)}} d\omega \langle \hat{\rho}_{c(tag)} \rangle;
\]

At the figures (8 - 9) such summarized distribution of intensity and degree of linear polarization for the case of horizontal polarization of laser photons is shown. At the figures (10 - 11) analogous distribution for the case of vertical polarization of laser photons are shown.

At last, at the figures (12 - 13) the averaged over interval of photon energies \( 220 \text{ MeV} < \omega_2 < 330 \text{ MeV} \) spatial distribution of the angle between axis \( x \) and direction of linear polarization of final photons \( \phi(x, y) \) are shown for the cases of horizontal (fig. 12) and vertical (fig. 13) polarization of initial laser photons.

From these figures one can see that \( \phi(x, y) = 0 \) at the lines \( x = 0 \) and \( y = 0 \), but at periphery of photon spot it can reach \( 5^\circ \) relatively to direction of initial laser polarization in the case of horizontal and \( 10^\circ \) in the case of vertical polarization of laser.

Figure 7: Degree of photons’ linear polarization, \( P, \% \). Photon energy \( \omega_2 = 220 \text{ MeV}, \xi_3^1 = -1 \).

\( \phi(x, y) \) the same formulae as (16 - 18) but averaging operation \( \langle ... \rangle \) have to be replaced into \( \langle ... \rangle_{\omega} \) where

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\langle \hat{\rho}_{c(tag)} \rangle_{\omega} = \int_{\omega_2^{(1)}}^{\omega_2^{(2)}} d\omega \langle \hat{\rho}_{c(tag)} \rangle;
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Figure 8: Spatial distribution of photon beam intensity. (Arbitrary units.) Photon energy \( 220 \text{ MeV} < \omega_2 < 330 \text{ MeV}, \xi_3^1 = +1 \).

Figure 9: Degree of photons’ linear polarization, \( P, \% \). Photon energy \( 220 \text{ MeV} < \omega_2 < 330 \text{ MeV}, \xi_3^1 = +1 \).
Figure 10: Spatial distribution of photon beam intensity. (Arbitrary units.) Photon energy $220 \text{ MeV} < \omega_2 < 330 \text{ MeV}$, $\xi_3 = -1$.

Figure 11: Degree of photons’ linear polarization, $P$, %. Photon energy $220 \text{ MeV} < \omega_2 < 330 \text{ MeV}$, $\xi_3 = -1$.

Figure 12: Spatial distribution of angle $\phi(x,y)$ between axis $x$ and direction of photon linear polarization Photon energy $220 \text{ MeV} < \omega_2 < 330 \text{ MeV}$, $\xi_3 = +1$.

Figure 13: Spatial distribution of angle $\phi(x,y)$ between axis $x$ and direction of photon linear polarization Photon energy $220 \text{ MeV} < \omega_2 < 330 \text{ MeV}$, $\xi_3 = -1$.

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