A photometric study of the high-mass-ratio contact binary AV Puppis

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Abstract The multi-color photometric light curves for contact binary AV Pup (AV Pup) in $V R_e I_e$ bandpasses are presented and analyzed by using the 2013 version of the Wilson-Devinney (W-D) code. The solutions suggest that AV Pup is a peculiar A-subtype W UMa contact binary with a high mass ratio ($q = m_2/m_1 = 0.896$) and a low fill-out factor ($f = 10\%$). Combining our newly determined times of minimum with those collected from literatures, the orbital period changes of this system are investigated. The $O - C$ analysis indicates that the orbital period of AV Pup is increasing at a rate of $dP/dt = 4.83 \times 10^{-7}$ days yr$^{-1}$, which can be explained by mass transfer from the less massive component to the more massive one.

Key words: techniques: photometric — stars: magnetic field — stars: individual: AV Pupis

1 INTRODUCTION

W Ursae Majoris (W UMa) contact binaries are the most common eclipsing systems in the vicinity of the solar system (Shapley 1948). They are short-period binary systems with both components filling their inner Roche lobes and sharing a common envelope. The formation and evolutionary ending of contact binaries are still open questions in astrophysics. The most plausible scenario is that they are formed from detached binaries via angular momentum loss (AML) (Vilhu 1982) or evolutionary expansion of the components (Webbink 1976). Model calculations suggest that this type of system will ultimately evolve into a contact binary with an extreme mass ratio or even into a fast-rotating single star under the influence of AML (Li et al. 2004b).

AV Pup (AV Pup, GSC 05998–02010, $\alpha_{2000} = 08^h24^m32.30^s$, $\delta_{2000} = -16^\circ24'11.24''$) was firstly reported by Hoffmeister (1930) as a variable star. Brancewicz & Dworak (1980) classified it as a semi-detached binary star with light curves of W UMa type. Wadhwa (2005) analyzed this system by using the V band photometric data of the All Sky Automated Survey (ASAS). He found this system is a contact binary with a high mass ratio of 0.80 and a low fill-out factor of 10\%. This system has since been barely studied.

In this work, we present two years of photometric observations in $V R_e I_e$ bandpasses targeting AV Pup. We also obtain new photometric solutions and perform a period analysis for this system. The rest of this article is organized as follows: in Section 2, the new observations for AV Pup are presented; in Section 3, a period analysis is conducted for AV Pup; in Section 4, the photometric solutions for the system are presented; finally, the summary and some discussions are given in Section 5.

2 OBSERVATIONS

New $V R_e I_e$ photometric observations of AV Pup were obtained in 2014 and 2015 using the 1 m Cassegrain reflector telescope operated by Yunnan Observatories. An Andor DW436 2048 $\times$ 2048 CCD camera with an effective field of view of $7.3 \times 7.3$ arcmin$^2$ is mounted on the telescope. The aperture photometry package in IRAF was applied for the data reduction. In our observations, the nearby stars TYC 5998–1820–1 and TYC 5998–2135–1 were employed as the comparison star and the check star, respectively. Their coordinates are listed in Table 1. The new light curves observed in two recent observing seasons are displayed in Figure 1. It is found in Figure 1 that the two sets of light curves are quite similar, and there is only a slight difference between phases 0.25 and 0.75.

Six new times of minimum were determined by using the Kwee-van Woerden (K-W) method (Kwee & van Woerden 1956) based on our observations. We then took

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Fig. 1 The observed light curves in \( VRc \) and \( Ic \) bandpasses for AV Pup. The bottom panel is the magnitude differences between comparison and check stars. Open circles and open squares represent the data from 2014 and 2015, respectively.

Table 1 Coordinates of AV Pup, the Comparison Star and the Check Star

| Target          | Name     | \( \alpha_{2000} \) | \( \delta_{2000} \) | \( V \) mag |
|-----------------|----------|----------------------|----------------------|-------------|
| Variable        | AV Pup   | 08 24 32.30          | –16 24 11.24         | 10.68       |
| The comparison  | TYC 5998–1820–1 | 08 24 15.30          | –16 23 13.43         | 11.56       |
| The check       | TYC 5998–2135–1 | 08 24 22.63          | –16 25 15.75         | 11.53       |

The average value of three bandpasses as one minimum, which are listed in Table 2.

Table 2 New Times of Minimum for AV Pup

| HJD            | Error | p/s | Filter          |
|----------------|-------|-----|-----------------|
| 2456715.2366   | 0.0001| p   | \( VRc,Ic \)    |
| 2456716.1067   | 0.0001| p   | \( VRc,Ic \)    |
| 2456717.1950   | 0.0001| s   | \( VRc,Ic \)    |
| 2457094.1343   | 0.0002| p   | \( VRc,Ic \)    |
| 2457096.0923   | 0.0002| s   | \( VRc,Ic \)    |
| 2457098.0484   | 0.0001| p   | \( VRc,Ic \)    |

the orbital period of AV Pup as 0.435010 d, which is different from 0.556339 d obtained by Brancewicz & Dworak (1980). We find that the former seems to be more reliable based on our new observations. Utilizing the orbital period 0.435010 d and a primary minimum at HJD 2456715.2366, we can express the following linear ephemeris

\[
\text{Min.I} = \text{HJD} 2456715.2366 + 0.435010 \times E. \quad (1)
\]

The \( O-C \) values are calculated based on Equation (1) and are listed in the fourth column of Table 3. As seen in Table 3, the minimum at HJD 2453109.6840 evidently deviates from the other minima, so we neglected this value. The \( O-C \) values are shown in Figure 2. A clear parabolic trend can be seen from this figure. We use a least-squares solution to fit all available times of minima and arrive at the following quadratic ephemeris

\[
\text{Min.I} = \text{HJD} 2456715.2369(3) + 0.4350142(1) \times E + 2.88(16) \times 10^{-10} \times E^2. \quad (2)
\]

The period of AV Pup exhibits a secular increase. The rate of increase is derived as \( \frac{dP}{dt} = 4.83 \times 10^{-7} \text{ d yr}^{-1} \) based on Equation (2). Since the CCD times of minimum only last less than 20 yr, there is no clear cyclic variation apparent in the \( O-C \) diagram.

4 LIGHT CURVE SOLUTION

We analyzed our new light curves based on the 2013 version of the Wilson-Devinney (W-D) code (Wilson &...
Devinney 1971; Wilson 1979, 1990). Wadhwa (2005) assigned the surface temperature of AV Pup as 6255 K, which corresponds to a spectral type of F8 in the General Catalog of Variable Stars. We took this value as the effective temperature of star 1 (eclipsed at phase 0.0) in our calculation. This effective temperature means that AV Pup should have a common convective envelope, so the bolometric albedos and gravity-darkening coefficients were set as $A_1 = A_2 = 0.5$ (Ruciński 1969) and $g_1 = g_2 = 0.32$ (Lucy 1967), respectively. We applied the logarithmic law format of limb-darkening coefficients (van Hamme 1993), which were computed internally by the DC program of the W-D code.

Due to the lack of spectroscopic mass ratio, a $q$-search procedure was performed to determine the initial mass ratio of AV Pup. A series of fixed mass ratios, ranging from 0.1 to 5.0 with a step of 0.05, was applied. In the $q$-search procedure, the adjustable parameters in our calculation were: the orbital inclination ($i$), the effective temperature of star 2 ($T_2$), the surface potential ($\Omega_1$ and $\Omega_2$) and the bandpass luminosity of star 1 ($L_1$). Due to the confusion between the semi-detached configuration and contact configuration for AV Pup, we ran the DC subroutine of the W-D code beginning with Mode 2 (a detached configuration) for each fixed $q$. However, the program quickly converged to Mode 3 (a contact configuration) at last, which means AV Pup should be a contact binary, as derived by Wadhwa (2005). For a clear view, Figure 3 only shows the relation between $\Sigma$ (the mean residuals for input data) and $q$ in the range of $q = 0.3$ to 3.5. As ascertained from Figure 3, the results of two $q$-search procedures based on the light curves observed in 2014 and 2015 are quite similar. They have a flat pattern from $q = 0.5$ to $q = 2.0$ and a minimum at $q = 0.85$. We took $q = 0.85$ as an initial mass ratio and assigned $q$ to be an adjustable parameter for the later calculation.

### Table 3 All CCD Times of Minimum for AV Pup

| HD (2400000+) | Error | Epoch | $O - C$ | Min | Filter | Reference |
|---------------|-------|-------|---------|-----|--------|-----------|
| 52002.5424    | -10833.5 | -0.0134 | II | CCD | AAVSO |           |
| 52237.2300    | -10294.0 | -0.0137 | I  | $R_e$ | Nagai (2001) |
| 52655.0563    | -9333.5 | -0.0145 | II | $R_e$ | Nagai (2004) |
| 53020.0929    | -8494.5 | -0.0143 | II | $R_e$ | Nagai (2005) |
| 53021.7705    | -8490.5 | -0.0137 | II | CCD | AAVSO |           |
| 53035.6907    | -8458.5 | -0.0138 | II | CCD | AAVSO |           |
| 53043.0860    | -8441.5 | -0.0137 | II | $R_e$ | Nagai (2005) |
| 53074.6232    | -8369.0 | -0.0147 | I  | CCD | AAVSO |           |
| 53109.6840    | -8288.5 | 0.0278 | II | CCD | AAVSO |           |
| 53403.0552    | -7614.0 | -0.0153 | I  | $R_e$ | Nagai (2006) |
| 53405.0126    | -7609.5 | -0.0154 | II | $R_e$ | Nagai (2006) |
| 53426.9828    | -7559.0 | -0.0132 | I  | V   | Nagai (2006) |
| 53743.2333    | -6832.0 | -0.0150 | I  | V   | Nagai (2007) |
| 53761.0689    | -6791.0 | -0.0148 | I  | V   | Nagai (2007) |
| 54119.0815    | -5968.0 | -0.0154 | I  | V   | Nagai (2008) |
| 54126.0425    | -5952.0 | -0.0146 | I  | V   | Nagai (2008) |
| 54127.7828    | 0.0003 | -5948.0 | I  | $R_e$ | Samolyk (2008) |
| 54526.6873    | 0.0001 | -5031.0 | I  | $R_e$ | Samolyk (2008) |
| 54526.6874    | 0.0002 | -5031.0 | I  | $R_e$ | Samolyk (2008) |
| 54545.6101    | 0.0002 | -4987.5 | I  | $R_e$ | Samolyk (2008) |
| 54548.0023    | -4982.0 | -0.0145 | I  | V   | Nagai (2009) |
| 54877.7380    | 0.0004 | -4224.0 | I  | $R_e$ | Samolyk (2009) |
| 54901.6674    | 0.0002 | -4169.0 | I  | $R_e$ | Samolyk (2010b) |
| 55127.3443    | 0.0002 | -3583.5 | I  | $R_e$ | Samolyk (2010a) |
| 55232.7119    | 0.0003 | -3408.0 | I  | $R_e$ | Samolyk (2010a) |
| 55271.6457    | 0.0001 | -3318.5 | II | $R_e$ | Samolyk (2011a) |
| 55612.6958    | 0.0002 | -2534.5 | II | $R_e$ | Samolyk (2011b) |
| 55622.7003    | 0.0002 | -2511.5 | II | $R_e$ | Samolyk (2011b) |
| 55630.7481    | 0.0002 | -2493.0 | II | $R_e$ | Diethelm (2011) |
| 56290.8767    | 0.0015 | -975.5  | II  | $R_e$ | Diethelm (2013) |
| 56737.6404    | 0.0001 | 51.5    | II  | $R_e$ | Samolyk (2014) |
| 56715.2366    | 0.0001 | 0.0     | II  | $R_e$ | Samolyk (2014) |
| 56716.1067    | 0.0001 | 2.0     | II  | $R_e$ | Samolyk (2014) |
| 56717.1950    | 0.0001 | 4.5     | II  | $R_e$ | Samolyk (2014) |
| 57034.5430    | 0.0040 | 734.0   | I   | $R_e$ | Paschke (2015) |
| 57094.1343    | 0.0002 | 871.0   | I   | $R_e$ | Paschke (2015) |
| 57096.0923    | 0.0002 | 875.5   | I   | $R_e$ | Paschke (2015) |
| 57098.0494    | 0.0001 | 880.0   | I   | $R_e$ | Paschke (2015) |
| 58135.6908    | 0.0003 | -2595.5 | I   | $R_e$ | Nagai (2015) |
| 58135.7783    | 0.0001 | 3265.5  | II  | $RV$ | AAVSO |           |
| 58203.6405    | 0.0002 | 3421.5  | II  | $RV$ | AAVSO |           |
As seen from Figure 1, a slight O’Connell effect is manifested in the light curves of AV Pup, so we attempted to model the light curves with spots. During the calculation, a third light was also taken into account. After some attempts, we finally got the best convergent solutions with a cool spot located on star 1 for the data spanning two years. The results are given in Table 4 and the comparison between observed and computed light curves is highlighted in Figure 4.

Our solutions suggest that AV Pup is an A-subtype contact binary. It has a high mass ratio of 0.896 and an inclination around 81°. The system is a shallow contact system with a fill-out factor around 10%, which coincides with the result of Wadhwa (2005). The slight difference between light curves from the two years that we analyzed can be explained by spot activity. Because there are no radial velocity curves observed for this system, we cannot obtain the precise absolute parameters for the components of AV Pup. Therefore, we utilized the mass-temperature relation (Harmanec 1988) to derive the mass of the primary star. The primary’s mass can be estimated as $M_1 = 1.27 M_\odot$, then the mass of the secondary star is determined to be $M_2 = 1.14 \pm 0.01 M_\odot$ based on our photometric solution. The radii and luminosities for the two components can also be obtained as: $R_1 = 1.29 \pm 0.01 R_\odot$, $L_1 = 2.29 \pm 0.02 L_\odot$, $R_2 = 1.23 \pm 0.02 R_\odot$ and $L_2 = 1.94 \pm 0.10 L_\odot$.

5 SUMMARY AND DISCUSSION

In this paper, we presented two years of CCD photometric observations targeting AV Pup. We derived the photomet-
Fig. 4 Comparison of observed (open circles) and computed (black lines) light curves for AV Pup. The upper and lower panels highlight the comparison of data from 2014 and 2015, respectively.

### Table 4 Photometric Solutions of AV Pup

| Parameter          | 2014          | 2015          |
|--------------------|---------------|---------------|
| $g_1 = g_2$        | 0.32 (fixed)  | 0.32 (fixed)  |
| $A_1 = A_2$        | 0.5 (fixed)   | 0.5 (fixed)   |
| $T_1$ (K)          | 6255 (fixed)  | 6255 (fixed)  |
| $T_2$ (K)          | 6150 ± 9      | 6150 ± 9      |
| $q(M_2/M_1)$       | 0.896 ± 0.003 | 0.896 ± 0.003 |
| $\Omega_1 = \Omega_2$ | 3.525 ± 0.005 | 3.525 ± 0.005 |
| $r_1$ (pole)       | 0.3720 ± 0.0004 | 0.3720 ± 0.0004 |
| $r_1$ (side)       | 0.3924 ± 0.0005 | 0.3924 ± 0.0005 |
| $r_1$ (back)       | 0.4263 ± 0.0006 | 0.4263 ± 0.0006 |
| $r_2$ (pole)       | 0.3538 ± 0.0011 | 0.3538 ± 0.0011 |
| $r_2$ (side)       | 0.3723 ± 0.0014 | 0.3723 ± 0.0014 |
| $f(\%)$            | 10.9 ± 1.6    | 10.2 ± 1.1    |

Spot parameters:

- Latitude (deg): 24.4 ± 0.5
- Longitude (deg): 144.0 ± 0.5
- Radius (deg): 31.0 ± 0.6
- $T/T_1$: 0.9 ± 0.1

| Parameter          | 2014          | 2015          |
|--------------------|---------------|---------------|
| $A_1 = A_2$        | 0.5 (fixed)   | 0.5 (fixed)   |
| $T_1$ (K)          | 6255 (fixed)  | 6255 (fixed)  |
| $T_2$ (K)          | 6150 ± 9      | 6150 ± 9      |
| $q(M_2/M_1)$       | 0.896 ± 0.003 | 0.896 ± 0.003 |
| $\Omega_1 = \Omega_2$ | 3.525 ± 0.005 | 3.525 ± 0.005 |
| $r_1$ (pole)       | 0.3720 ± 0.0004 | 0.3720 ± 0.0004 |
| $r_1$ (side)       | 0.3924 ± 0.0005 | 0.3924 ± 0.0005 |
| $r_1$ (back)       | 0.4263 ± 0.0006 | 0.4263 ± 0.0006 |
| $r_2$ (pole)       | 0.3538 ± 0.0011 | 0.3538 ± 0.0011 |
| $r_2$ (side)       | 0.3723 ± 0.0014 | 0.3723 ± 0.0014 |
| $f(\%)$            | 10.9 ± 1.6    | 10.2 ± 1.1    |

### Table 5 A-subtype Contact Binaries with a High Mass Ratio

| Name        | $T_1$ (K) | $T_2$ (K) | $q$  | $f$ (%) | Reference                  |
|-------------|-----------|-----------|------|---------|---------------------------|
| OO Aql      | 5700      | 5472      | 0.844 | 21.4    | İcli et al. (2013)         |
| V2150 Cyg   | 8000      | 7920      | 0.802 | 19.0    | Kreiner et al. (2003)      |
| V1101 Her   | 5920      | 5690      | 0.800 | 14.2    | Pi et al. (2017)           |
| AU Ser      | 5495      | 5153      | 0.710 | 19.8    | Gürol (2005)               |

The two years of observations manifest a slight difference, which can be explained by spot variation. Contact binaries tend to have a low mass ratio (Rucinski 2001), consequently only a few contact binaries with high mass ratio have been discovered, such as WZ And ($q = 1.0$, Zhang & Zhang 2006), HT Vir ($q = 0.9$, Bensch et al. 2014) and so on. Meanwhile, A-subtype contact binaries have a relatively low mass ratio and a relatively high contact degree in general (Hilditch et al. 1988; Jiang et al. 2009). However, a few A-subtype contact binaries were found to have a high mass ratio and a shallow common envelope (listed in Table 5). AV Pup seems to exhibit the same characteristics as these peculiar contact binaries.

We also conducted the first period change analysis for AV Pup. A least-squares fitting indicates that the orbital ratio of $q = m_2/m_1 = 0.896$. The system is a shallow contact binary with a low fill-out factor of around 10%.
period of AV Pup is experiencing a secular increase at a rate of \(dP/dt = 4.83 \times 10^{-7} \text{ d yr}^{-1}\), which may be caused by mass transfer from the less massive component to the more massive one. Assuming the mass transfer is conservative, we can calculate the mass transfer rate from the formula derived from Kepler’s third law

\[
\frac{dM_2}{dt} = \dot{M}_2 = \frac{\dot{P} M_1 M_2}{3P(M_2 - M_1)},
\]

The estimated mass transfer rate is \(dM_2/dt = -4.12 \times 10^{-6} M_\odot \text{ yr}^{-1}\). This mass transfer rate seems quite high for contact binaries, but it coincides with the mass transfer properties of contact binaries in the Kepler Eclipsing Binary Catalog (Kouzuma 2018). The time scale of mass transfer for the donor star can be estimated as \(\tau_{\text{MT}} = 2.77 \times 10^7 \text{ yr}\), which is much shorter than the thermal time scale \(\tau_{\text{th}} \sim (GM^2)/(RL) \sim 1.70 \times 10^7 \text{ yr}\) (Paczyński 1971). This suggests that the donor star cannot maintain its thermal equilibrium. This system might be evolving from a contact configuration to a semi-detached configuration caused by a third body, which is reflected by the third light appearing in the photometric solutions. We should also notice that these results were achieved according to the absolute parameters only derived from the photometric observations and, therefore, spectroscopic observations are urgently needed.

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