QCD at high Baryon Density and Temperature: Competing Condensates and the Tricritical Point

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The phase diagram of strongly interacting matter is explored as a function of temperature and baryon number density. We investigate the possible simultaneous formation of condensates in the conventional quark–anti-quark channel (breaking chiral symmetry) and in a quark–quark channel leading to color superconductivity: the spontaneous breaking of color symmetry via the formation of quark Cooper pairs. We point out that for two massless quark flavors a tricritical point in the phase diagram separates a chiral symmetry restoring first order transition at high densities from the second order transition at high temperatures. Away from the chiral limit this tricritical point becomes a second order phase transition with Ising model exponents, suggesting that a long correlation length may develop in heavy ion collisions in which the phase transition is traversed at the appropriate density.

1. Towards the phase diagram

The behavior of QCD at high temperature and baryon density is of fundamental interest and has applications in cosmology, in the astrophysics of neutron stars and in the physics of heavy ion collisions. Over the past years, considerable progress has been achieved in our understanding of high temperature QCD, where simulations on the lattice and universality arguments played an essential role. Recently, nonperturbative renormalization group methods have been established which account for both the low temperature chiral perturbation theory domain of validity and the domain of validity of universality associated with critical phenomena [1]. They may help to shed some light on the remaining pressing questions at finite temperature, like the nature of the phase transition for realistic values of the quark masses.

Our knowledge of the high density properties of strongly interacting matter is rudimentary so far. There are severe problems to use standard simulation algorithms at nonzero chemical potential on the lattice due to a complex fermion determinant. Different nonperturbative methods like the Exact Renormalization Group[1] or Schwinger–Dyson equations [3] seem to present promising alternatives. However, finding suitable nonperturbative approximation schemes often relies on some knowledge about propagators or the relevant

1First investigations in this direction done by D. Jungnickel, C. Wetterich and myself [2] have been presented by D. Jungnickel, these proceedings.
degrees of freedom. As a first step it seems well justified to consider models which allow us to describe likely patterns of symmetry breaking and to make rough quantitative estimates. In this talk, I will present an exploration of the phase diagram for strongly interacting matter, using a class of models for two flavor QCD in which the interaction between quarks is modelled by that induced by instantons. The model is discussed using a mean field approximation. The results are based on recent work [4] done in collaboration with Krishna Rajagopal.

QCD at high density is expected to have a rich phase structure. In addition to the nuclear and quark matter phases a number of possibilities like the formation of meson condensates or strange quark matter have been discussed. A particularly interesting possibility is the formation of Cooper pairs of quarks, where an arbitrarily weak attractive interaction between quarks renders the quark Fermi surface unstable and leads to the formation of a condensate. Since pairs of quarks cannot be color singlets, a diquark condensate breaks color symmetry. For recent reviews of this rapidly growing field of research see [5,6]. Different attractive channels may lead to a (simultaneous) formation of different condensates. In particular, the vacuum of QCD already has a condensate of a quark–anti-quark pair. An important ingredient for the understanding of the high density phase structure is the notion of competing condensates. One expects, and we find [4], that the breaking of color symmetry due to a $\langle \bar{\psi}\psi \rangle$ condensate is suppressed by the presence of a chiral condensate. Likewise, we find chiral symmetry restoration to be induced at lower densities by the presence of a color superconductor condensate. This behavior can be quantitatively understood from the effective potential which I discuss below. The competition between condensates may greatly simplify calculations. We find that previous treatments [7,8], which discuss the chiral and superconducting condensates separately and inspired this work, are a good approximation once the phase boundary is known. It is encouraging that this finding has also recently been confirmed in calculations in an instanton liquid model [9].

The phase diagram which we uncover has striking qualitative features, several of which we expect to generalize beyond the model which we consider. The phase diagram is shown in Fig. 1 for zero quark mass as a function of temperature $T$ and chemical potential $\mu$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{phase_diagram.png}
\caption{Phase diagram as a function of $T$ and $\mu$ in GeV. The solid curves are second order phase transitions; the dashed curve describes the first order transition.}
\end{figure}
For (net) quark number. At low temperatures, we observe chiral symmetry restoration via a first order transition between a phase with low baryon density and a high density phase with a condensate of quark–quark Cooper pairs in color antitriplet, Lorentz scalar, isospin singlet states. There are coexisting $\langle \psi \psi \rangle$ and $\langle \bar{\psi} \psi \rangle$ condensates in this phase in the presence of a current quark mass. We find color superconductivity at temperatures $T < T_c^{\Delta}$ where $T_c^{\Delta}$ is of order tens to almost one hundred MeV. The transition we find at $T_c^{\Delta}$ is second order for the present model, but the order of this transition may change once gauge field fluctuations are taken into account.

In the chiral limit, we find a second order chiral transition at zero chemical potential, and a treatment of the present model which went beyond mean field theory, e.g. along the lines presented in [1], would find critical exponents characteristic of the three dimensional $O(4)$ universality class. The point in the phase diagram where the second order transition meets the low temperature first order transition locates the tricritical point $T_{tc}$. The temperature is estimated in our model to be $T_{tc} \approx 101$ MeV and a chemical potential $\mu_{tc} \approx 211$ MeV. This point has been observed in different phenomenological models [14–16]. We note that the critical behavior in the vicinity of this point is governed by universality (pointed out independently in [13]). If two-flavor QCD has a second order transition at high temperatures and a first order transition at high densities, then it will have a tricritical point in the same universality class as our model. The physics near the tricritical point is described by a $\phi^6$ field theory and there are three independent critical exponents which are given quantitatively by our mean field analysis and by logarithmic corrections to scaling [17]. A nonzero quark mass $m$ explicitly breaks chiral symmetry and the second order phase transition above $T_{tc}$ turns into a smooth crossover. A small quark mass cannot eliminate the first order transition below $T_{tc}$. Therefore, whereas we previously had a line of first order transitions and a line of second order transitions meeting at a tricritical point, with $m \neq 0$ we now have a line of first order transitions ending at an ordinary critical point. The situation is precisely analogous to critical opalescence in a liquid gas system. At this critical point, one degree of freedom (that associated with the $\sigma$) becomes massless, while the pion degrees of freedom are massive since chiral symmetry is explicitly broken. Therefore, this transition is in the same universality class as the three dimensional Ising model.

From many studies of QCD at nonzero temperature, we are familiar with the possibility of a second order transition, with infinite correlation lengths, in an unphysical world in which there are two massless quarks. It is exciting to realize that if the finite density transition is first order at zero temperature (see also [2]) then there is a tricritical point in the chiral limit which becomes an Ising second order phase transition in a world with chiral symmetry explicitly broken. In a sufficiently energetic heavy ion collision, one may create conditions in approximate local thermal equilibrium in the phase in which spontaneous chiral symmetry breaking is lost. Depending on the initial density and temperature, when this plasma expands and cools it will traverse the phase transition at different points in the $(\mu, T)$ plane. Our results suggest that in heavy ion collisions in which the chiral symmetry

2There are also indications of a color 6, Lorentz axial vector, isospin singlet condensate which is many orders of magnitude smaller than the condensates we treat.

3Note that the presence of a tricritical point is precisely what is expected to occur at $\mu = 0$ at a particular value of the strange quark mass [14,12]. The two tricritical points are continuously connected [13].
breaking transition is traversed at baryon densities which are not too high and not too low, a very long correlation length in the $\sigma$ channel may be manifest even though the pion is massive $^{[14]}$. Recently, a number of distinctive signatures have been proposed $^{[13]}$ that could allow an identification of the predicted critical point.

2. Effective potential

Our results are obtained from a class of models for QCD where the fermions interact via the instanton induced interactions between light quarks. The interaction reflects the chiral symmetry of QCD: axial baryon number is broken, while chiral $SU(3)$ is respected. Color $SU(3)$ is realized as a global symmetry. We note that with the help of appropriate Fierz transformations the instanton interaction can be decomposed into two parts, where one part contains only color singlet fermion bilinears and the other part contains only color 3 bilinears: $S_I = S_I^{(1c)} + S_I^{(3c)}$ with (see $^{[4]}$)

$$
S_I^{(1c)} = G_1 T \sum_{n \in \mathbb{Z}} \int \frac{d^3 \vec{p}}{(2\pi)^3} \left\{ -O_{(\alpha)}[\psi, \bar{\psi}; -p]O_{(\sigma)}[\psi, \bar{\psi}; p] - O^{\alpha}_{(\pi)}[\psi, \bar{\psi}; -p]O_{(\pi)\alpha}[\psi, \bar{\psi}; p] + O^{\alpha}_{(a_0)}[\psi, \bar{\psi}; -p]O_{(a_0)\alpha}[\psi, \bar{\psi}; p] \right\},
$$

$$
S_I^{(3c)} = G_2 T \sum_{n \in \mathbb{Z}} \int \frac{d^3 \vec{p}}{(2\pi)^3} \left\{ -O^{\alpha}_{(\pi)}[\bar{\psi}, \psi; p]O_{(\alpha)}[\psi, \bar{\psi}; p] + O^{\alpha}_{(p)}[\bar{\psi}, \psi; p]O_{(p)\alpha}[\psi, \bar{\psi}; p] \right\}
$$

where we have generalized the interaction to allow $G_1$ and $G_2$ to take on independent values. Here $p \equiv (2n\pi T, \vec{p})$ due to the bosonic nature of the fermion bilinears. The bilinears $O_{(\sigma)}, O_{(\pi)}^{(\alpha)}, O_{(a_0)}^{(\alpha)}$ with $a = 1, 2, 3$ carry the quantum numbers associated with the scalar isosinglet ($\sigma$), the pseudo-scalar isotriplet ($\pi$), the pseudo-scalar isosinglet ($\eta'$) and the scalar isotriplet ($a_0$), respectively. Similarly, the bilinear $O_{(\sigma)}^{(\alpha)} (O_{(\pi)}^{(\alpha)})$, with the color index $\alpha = 1, 2$ or $3$, carries the quantum numbers of the color antitriplet scalar (pseudo-scalar) diquark. For the $\sigma$ and for the scalar diquark they read

$$
O_{(\sigma)}[\psi, \bar{\psi}; p] = -iT \sum_{n \in \mathbb{Z}} \int \frac{d^3 \vec{q}}{(2\pi)^3} F(\vec{q}) F(\vec{p} - \vec{q}) \bar{\psi}^i \alpha (-q) \psi^i \alpha (p - q) ,
$$

$$
O_{(\pi)}^{(\alpha)}[\psi, \bar{\psi}; p] = T \sum_{n \in \mathbb{Z}} \int \frac{d^3 \vec{q}}{(2\pi)^3} F(\vec{q}) F(\vec{p} + \vec{q}) (\psi^T)^i \beta (-p + q) C \gamma^5 \epsilon^{\alpha\beta\gamma} \epsilon_{ij} \psi^j \gamma (-q) (1)
$$

where $C$ denotes the charge conjugation matrix and we have supplemented the bilinears by suitable form factors $F(\vec{q})$ to mimic the effects of asymptotic freedom. The results we quote in our exploration of the phase diagram are obtained using the smooth form factor $F(\vec{q}) = \Lambda^2/(\vec{q}^2 + \Lambda^2)$ with $\Lambda = 0.8$ GeV and with $G_1$ fixed by requiring a constituent quark mass of $400$ MeV at $\mu = T = m = 0$ in order to obtain a reasonable, albeit qualitative, phenomenology and $G_2 = 3G_1/4$ which is motivated in $^{[4]}$. We note that the qualitative features which we address do not depend on this specific choice $^{[4]}$.

We note from the signs in $S_I$ that if we choose the sign of $G_1$ such that the interaction in the $\sigma$ channel is attractive, so that chiral symmetry breaking is favored, we may expect

$^{4}$There is a phenomenological upper bound which $G_2$ must satisfy. For $G_2 > 2G_1$ color would be spontaneously broken in the vacuum.
condensation in the $\pi$ and scalar diquark channels also. In the chiral limit, one can always make a rotation such that there is no $\pi$ condensate. A condensate in the pseudoscalar diquark channel would break parity spontaneously, but this seems not to be favored by the interaction (\cite{5}). We will use the model to explore condensates in the $\sigma$ and scalar diquark channels, i.e. we consider possible simultaneous condensates, $\langle O_{(\sigma)} \rangle \sim \langle \bar{\psi}\psi \rangle$ for chiral symmetry breaking and $\langle O_{(s)}^{\alpha} \rangle \sim \langle \bar{\psi}\psi \rangle$ for breaking of color symmetry $SU(3) \rightarrow SU(2)$.

The appropriate tool to study the phase structure of the model is to consider the effective potential $\Omega$ (generating functional of 1PI Green functions at zero momentum). Using standard techniques \cite{4} we compute the mean field effective potential $\Omega(\phi, \Delta; \mu, T)$ as a function of two order parameters $\phi$ and $\Delta$. Extremizing $\Omega$ leads to coupled gap equations, whose solutions $\phi_0$ and $\Delta_0$ are related to the chiral condensate and the condensate of Cooper pairs by $\phi_0 = 2G_1 \langle \bar{\psi}\psi \rangle$, $\Delta_0 = 2G_2 \langle \bar{\psi}\psi \rangle$. At its extrema it corresponds to the thermodynamic potential, related to the energy density $\epsilon$, the entropy density $s$, the quark number density $n$ and the pressure $P$ by

$$\Omega(\phi_0, \Delta_0; \mu, T) = \epsilon - Ts - \mu n = -P. \quad (3)$$

As the temperature, the chemical potential or the quark mass changes, the effective potential can have several local minima in the $(\phi, \Delta)$ plane. Only the global minimum corresponds to the lowest free energy state and is favored. The analysis yields the phase structure presented in Fig. 4.

To discuss the competition between condensates let us concentrate on $T = 0$. Fig. 2 shows ‘slices’ of $\Omega$ as a function of $\phi$ at $\Delta = 0$ and as a function of $\Delta$ at $\phi = 0$ for several chemical potentials. We find that $\Omega(\phi = 0, \Delta)$ shows a nonzero minimum $\Delta_0$ for arbitrarily small $\mu$: Since the instanton interaction provides an attractive interaction in the color $\bar{3}$ scalar diquark channel it renders the Fermi surface unstable and would

\footnotesize{\textsuperscript{5}We note that the two flavor diquark condensate is not a gauge invariant order parameter. In this respect it is analogous to the electroweak sector of the standard model. Likewise, there is the possibility of no sharp transition but rather a crossover with qualitatively different physical behavior and the same symmetries.}

\vspace{1cm}

Figure 2. The zero temperature thermodynamic potential $\Omega$ (in GeV$^4$) as a function of $\phi \sim \langle \bar{\psi}\psi \rangle$ at $\Delta = 0$ (left panel) and as a function of $\Delta \sim \langle \bar{\psi}\psi \rangle$ at $\phi = 0$ (right panel) for several chemical potentials. The curves correspond to (top to bottom) $\mu = 0, 0.292, 0.35$ GeV.
lead to the formation of a diquark condensate. However, one observes that the chiral condensate is favored for $\mu < 0.292$ GeV and the diquark condensate vanishes identically. For larger $\mu$ chiral symmetry restoration occurs via a first order transition, while the diquark condensate jumps to a nonzero value in the high density phase. We therefore have the situation that the competition with the chiral condensate leads to a first order color superconductor transition which would be continuous otherwise. One also notes from Figs. 2 that without considering the possibility of diquark condensation, i.e. for an identically vanishing $\Delta$, chiral symmetry restoration would have occurred at larger chemical potential when $\Omega(\phi, \Delta = 0)$ crosses the origin. In the chiral limit we observe a strong competition in the sense that where one condensate is nonzero the other vanishes (see also Fig. 3).

It is instructive to consider the physics away from the chiral limit. A nonzero quark mass breaks chiral symmetry explicitly even in the high density phase. Indeed, for an average current quark mass of 10 MeV one observes in Fig. 3 simultaneous condensates with comparable magnitude immediately upon the completion of the transition. Comparison with the chiral limit (left panel) shows that the diquark condensate is not significantly disturbed by the presence of $\phi_0$. This insensitivity can be understood by noting that although the Cooper pairs have low momentum, they are formed from quarks which have momenta close to the Fermi surface. Adding a quark mass $m \ll \mu$ does not significantly affect the density of states or the interactions of the quasiparticles with momenta of order $\mu$. The phenomenon of color superconductivity seems quite robust. Even away from the chiral limit it is not significantly disturbed by the simultaneous presence of a chiral condensate in the high density phase as long as $m + \phi_0$ is smaller than the chemical potential.

3. Outlook

We have presented here a first model calculation of the QCD phase diagram that takes into account the strong competition between the chiral and the color antitriplet scalar diquark condensate. The latter involves only the light $u$ and $d$ quarks of two colors. Though the strange quark is much heavier than the two light quarks, it may not be
neglected for chemical potentials much larger than $m_s$ or typical scales of a few hundred MeV. There is a compelling symmetry breaking scheme \cite{18}, involving three massless flavors, which locks color and flavor to a residual global $SU(3)$ symmetry. It is not clear what phase will be realized with physical mass $u$, $d$ and $s$ quarks. As pointed out in \cite{5}, one may think of having the two flavor condensate first and color–flavor locking at higher density. A first investigation may be performed along the lines presented here \cite{4} but finally more sophisticated methods than the mean field analysis will be needed to settle these questions. A promising possibility may be the use of truncated nonperturbative flow equations which have been successfully applied to finite temperature in the past.

The phase diagram which we uncover in our two flavor study has striking qualitative features, most remarkably the presence of a tricritical point. For nonzero quark masses a situation similar to the liquid–gas nuclear transition arises which has been much studied in low energy heavy ion collisions. We observe the (tri)critical point to emerge as a result of a second order transition/crossover in one region of phase space (high $T$, small $\mu$) and a first order transition for the same order parameter in another region (high $\mu$, low $T$). The question of whether the tricritical point is realized in QCD is closely connected with the still unsettled question of the order of the high temperature ($\mu = 0$) transition, and has to be addressed in a three flavor study. If the strange quark mass is too small, or if the axial $U(1)$ symmetry is effectively restored about the transition, then we may have a first order transition at high $T$ which is driven by fluctuations \cite{20}. In this case, a line of first order transitions connects the $T$ and $\mu$ axes. However, one should note that if the finding \cite{19} of a continuous high density crossover for physical mass $u$, $d$ and $s$ quarks is realized in nature, then again a critical point in the phase diagram may emerge: A high temperature first order transition line ending in an Ising endpoint in the high density region. The involved long–range correlations in any of these scenarios would be appealing both from the theoretical and the experimental point of view.

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