Dynamics of axialized laser-cooled ions in a Penning trap

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We report the experimental characterization of axialization — a method of reducing the magnetron motion of a small number of ions stored in a Penning trap. This is an important step in the investigation of the suitability of Penning traps for quantum information processing. The magnetron motion was coupled to the laser-cooled modified cyclotron motion by the application of a near-resonant oscillating quadrupole potential (the “axialization drive”). Measurement of cooling rates of the radial motions of the ions showed an order-of-magnitude increase in the damping rate of the magnetron motion with the axialization drive applied. The experimental results are in good qualitative agreement with a recent theoretical study. In particular, a classical avoided crossing was observed in the motional frequencies as the axialization drive frequency was swept through the optimum value, proving that axialization is indeed a resonant effect.

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I. INTRODUCTION

The first proposal [1] for the use of trapped ions for quantum information processing (QIP) has been realized in its original and in other forms in several groups around the world [2 3 4 5]. Recent work on large-scale entanglement and scalability [6 7] of ion-trap quantum computing and the demonstration of the essential feature of quantum error correction [8] mean that trapped ions offer the prospects of QIP. The availability of an all-solid-state laser system for addressing the ground state and the metastable D5/2 level of 40Ca+ [19] have already been shown to be a viable system for this purpose [20]. For this reason, and because of the advantages of Penning traps for quantum information processing, the ground state and the metastable D5/2 level of 40Ca+ have already been shown to be a viable system for this purpose [20]. For this reason, and because of the availability of an all-solid-state laser system for addressing the cooling and qubit transitions, we have chosen to investigate the laser cooling of 40Ca+ ions. Doppler cooling of this ion in a Penning trap is reported in Ref. [9]. We present here the first demonstration of axialization of this ion in the presence of laser cooling, and the measurement of the damping rates of small numbers of ions, both axialized and unaxialized.

II. ION MOTION IN THE PENNING TRAP

The ideal Penning trap has the same geometry as a Paul trap, namely a pair of end-caps and a ring electrode, with all three electrodes having hyperbolic cross-sections. Traps with non-hyperbolic electrodes can give a reasonable approximation to a pure quadrupole potential and are often employed. A DC voltage applied between the end-caps and the ring provides confinement in the axial direction but repels the ion in the radial direction. Radial confinement is provided by adding an axial magnetic field, forcing the ion into cyclotron-like loops.
The motion in the axial direction is simple harmonic motion in a potential well. The motion in the radial plane is an epicycle, composed of a high-frequency cyclotron motion and a lower frequency drift arising from the cross product of the electric and magnetic fields. The cyclotron motion is modified from that which would occur in the presence of a magnetic field alone, due to the electric field. The high- and low-frequency motions are referred to as the modified cyclotron and magnetron motions respectively.

A detailed description of ion motion in a Penning trap may be found in [21, 22]. In summary, the potential from the hyperbolic electrodes is given by,

$$\phi = U_0 (2z^2 - x^2 - y^2)/R^2,$$

where $U_0$ is the DC voltage and $R^2 = r_0^2 + 2z_0^2$ is a geometrical factor depending on the trap radius ($r_0$) and the end-cap separation ($2z_0$). The magnetic field, $B$, is in the $-z$ direction. The axial motion is independent of the magnetic field, with frequency,

$$\omega_z = \sqrt{\frac{4eU_0}{mR^2}}.$$

The radial component of the ion motion may most naturally be examined in a frame rotating with the ions at half the true cyclotron frequency (i.e. at $eB/2m$) [23]. In this frame the force due to the magnetic field is exactly canceled by the Coriolis force. The two motions are then found to be circular orbits with opposite directions and the same frequency $\omega_1$, where,

$$\omega_1 = \omega_z^2/4 - \omega_z^2/2.$$ 

Superpositions of the two motions give rise to elliptical and (for equal amplitudes of the two motions) linear trajectories in the rotating frame. This is analogous to the way in which circular polarizations of light may be decomposed into linear polarizations and vice versa.

Rotation in the same direction as that of the frame is found to correspond to modified cyclotron motion in the laboratory frame (at $\omega_c/2 + \omega_1$) while rotation in the opposite direction corresponds to magnetron motion (at $\omega_c/2 - 1$).

III. LASER COOLING IN A PENNING TRAP

Although laser cooling of bound atoms was first demonstrated in a Penning trap [24], it is rather more difficult than in a Paul trap. A relatively modest magnetic field of 1T can give rise to Zeeman splittings of many gigahertz (see Fig. 1). The splittings can be so large that it is impossible to use a single, modulated, laser to address all transitions from a given level. The set-up and maintenance of many narrow-linewidth, drift-free laser systems is a significant complication.

As well as the large number of lasers required, the choice of detuning from resonance and the (radial) spatial offset of the cooling beam from the center of the trap is also important. This was investigated in an analytical model in Ref. [25] and further developed in Ref. [26].

To model the laser cooling process in a Penning trap, it is important to take account of both the lineshape of the cooling transition (Lorentzian for a cold ion). The laser scattering rate depends on the instantaneous slopes of the tangents to these two curves and is varied by changing the spatial offset of the beam from trap center and by detuning the laser from resonance.

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To model the laser cooling process in a Penning trap, it is important to take account of both the lineshape of the cooling transition and also the spatial profile of the cooling laser (Fig. 2). The ion position in the laser’s spatial profile gives rise to a force varying approximately linearly with position along the axis perpendicular to the laser,

$$F_\alpha = -2\alpha m y.$$

The ion’s velocity along the direction of the laser beam leads to a variation of the force on the ion as the ions are Doppler shifted into or out of resonance with the laser frequency, $\nu_L$,

$$F_\beta = -2\beta m \dot{x}.$$

Adding these two forces to the equations of motion and making the assumptions that $\alpha \ll \omega_z^2$ and $\beta \ll \omega_c$, leads
to new solutions with frequencies \[26\],
\[\begin{align*}
\omega_c' + i \frac{\beta \omega_c' - \alpha}{2\omega_1}, \\
\omega_m + i \frac{\alpha - \beta \omega_m}{2\omega_1}.
\end{align*}\]

That is, the motional frequencies remain unchanged by the cooling. The cooling rates (the imaginary parts) are determined by \(\alpha\) and \(\beta\). It can be seen that efficient cooling of both motions is not possible due to conflicting requirements: positive \(\beta\) (i.e. a red-detuned laser) for the modified cyclotron motion and negative \(\beta\) (blue-detuned laser) for the magnetron motion. A compromise may be found where both motions are cooled, but not efficiently \[27\]:
\[\omega_m < \alpha/\beta < \omega_c'.\]

This corresponds to a red-detuned laser and a small spatial offset such that the laser is along the direction of the magnetron motion.

IV. AXIALIZATION

Since it is relatively easy to cool the modified cyclotron motion, it would be desirable to deterministically couple it to the magnetron motion. In this way the amplitudes of both motions could be reduced without direct damping of the magnetron motion. This is the essence of the axialization technique. We have described how the two modes of motion may be thought of as counter-propagating circular motions in a rotating frame. In order to couple two such modes in a potential well, it is necessary to compress the well along one axis. This will result in an initially circular motion evolving into an elliptical motion which becomes more and more eccentric until it is linear. The motion will then become elliptical once again until a circular motion reappears, with equal amplitude but opposite direction to the original motion.

The required compression of the potential well (in the rotating frame) is achieved by adding a static quadrupole field. To obtain such a quadrupole field in the rotating frame a rotating field must be applied in the laboratory frame. A quadrupole field oscillating at \(\omega_c\) can be decomposed into two counter-rotating quadrupole fields each at \(\omega_c/2\). In the frame rotating at \(\omega_c/2\), one of these rotates at \(\omega_c\) and has no effect on the motion. The other is the desired static quadrupole.

An oscillating azimuthal quadrupole field to couple the motions was first proposed by Dehmelt and Wineland \[28\,29\]. Although the motion described is classical, their description of the effect of the coupling field is in quantum mechanical terms. Such a drive has been applied in the presence of resistive cooling \[28\] and buffer-gas cooling \[19\] where it is referred to as “sideband cooling” \[33\] or “radial centering”.

A. Quadrupole drive in the presence of laser cooling

We have recently performed a detailed calculation examining the effect of the quadrupole (axialization) drive when one of the radial motions is laser cooled \[26\]. In the rotating frame, the additional force on the ion due to the static component of the axialization field is given by
\[\begin{align*}
F_x &= -\frac{eV_0}{2\nu_0} x, \\
F_y &= +\frac{eV_0}{2\nu_0} y,
\end{align*}\]

where \(V_0\) is the amplitude of the axialization drive on the ring segments and \(r_0\) is the inner radius of the ring electrode. We introduce a parameter, \(\epsilon\), to describe the axialization strength,
\[\epsilon = \frac{eV_0}{2mv^0},\]
and write \(v = x + iy\). Thus, we can re-write the force on the ion as
\[F_v = F_x + iF_y = -\epsilon mv^*\cdot\]

The effect of laser cooling is described in the laboratory frame, with the co-ordinate, \(u\):
\[\ddot{u} + (\beta - i\omega_c)\dot{u} - \left(\frac{\omega_c^2}{2} + i\alpha\right) u = 0.\]

In the rotating frame moving at \(\omega_r\), with \(v = u e^{i\omega_r t}\),
\[\ddot{v} + (\beta - i\omega_c + 2i\omega_r)\dot{v} - \left(\frac{\omega_c^2}{2} + i\alpha + \omega_r^2 - \omega_r(\omega_c + i\beta)\right) v + \epsilon v^* = 0.\]

where we have included the axialization term. Rotational symmetry is broken by the \(v^*\) term and the solution is in the form of elliptical motion with uniform decay. This can be written as a superposition of two circular motions in opposite senses and with different amplitudes:
\[v = Ae^{i\omega_1 t} + Be^{-i\omega_2 t}\cdot\]

The complete solution is described in \[26\]. We define \(\omega_0\), the axialization-drive frequency and \(\Delta\), half the detuning from the true cyclotron frequency, i.e. \(\omega_0 = \omega_c + 2\Delta\). The solutions to Equation \[14\] have motional frequencies similar to the unperturbed frequencies, \(\omega_1\), with a frequency shift \(\delta_0\). The damping rates are obtained from the imaginary part of the solution, \(\gamma_0\).
\[\omega = \omega_1 + \delta_0 + i\gamma_0.\]

The solutions give,
\[\delta_0 = \pm \frac{1}{\sqrt{2}} \sqrt{N + \sqrt{N^2 + \Delta^2M^2}},\]

where \(N\) and \(M\) are determined by \(|\alpha|\) and \(\beta\).
important experimentally, where \((\epsilon/\omega_1)^2 \gg M^2\). In this case the axialization rate is faster than the rate of damping due to laser cooling. Both the magnetron and modified cyclotron motions are cooled and the sensitivity of the damping rates to the axialization-drive frequency is low compared to the other regimes (Fig. 3). This means that very precise tuning of the axialization drive is unnecessary and that the damping rates are insensitive to magnetic field fluctuations. An interesting feature of this regime is that an avoided crossing of the motional frequencies measured in the laboratory frame is predicted.

V. EXPERIMENTAL SETUP

The setup has been described in some detail in Refs. [9, 30]. We perform laser cooling of small clouds of calcium ions in a Penning trap using diode lasers. An important feature of the system is that the ring electrode of the trap is divided into quadrants to allow the application of the axialization drive.

A. Laser systems

A pair of laser diodes in commercial systems (Toptica DL100) provides light on the cooling transition \(S_{1/2}^-\ P_{1/2},\ 397\ nm\) while home-built extended cavity diode lasers at 866 nm and 854 nm (\(D_{3/2}\ P_{1/2}\) and \(D_{5/2}\ P_{3/2}\) respectively) close the cooling cycle. Repumping from the \(D_{5/2}\) level is necessary due to a small amount of population of the \(P_{3/2}\) level by the cooling lasers (ASE light at 393 nm) and subsequent decay to the long-lived D state.

Two lasers are used on the 397 nm transition on account of the large ground-state Zeeman splitting of 27 GHz, making a single, modulated source difficult to produce. The two lasers at 866 nm each have sidebands imposed on them via an RF source added to the diode current. The two pairs of sidebands then cover the four \(\sigma\) transitions. A single 854-nm laser is found to be sufficient to repump the small amount of population found in the \(D_{5/2}\) level.

All of the diode lasers are locked with side-of-fringe locks to low-finesse cavities. The cavities are constructed from Zerodur and are tunable via piezo transducers arranged in a re-entrant design to minimize thermal drift. The cavities are kept in insulated containers which are temperature-stabilized. Roughly linear drifts of between 15 and 75 MHz per hour have been measured. (The cooling transition natural linewidth is 23 MHz for comparison.)

B. Trap with segmented ring electrode

The trap itself (Figure 1) was first described in [9] and is housed in a vacuum chamber kept at ultra-high vacuum \((10^{-8}\ mbar\ or\ lower)\). It approximates an ideal Pen-
The magnetron damping rate was measured as a function of the axialization drive amplitude. Measurements were made with small clouds of roughly 10 to 20 ions. Between measurements, the laser frequencies were re-adjusted to return the fluorescence rate to that recorded.
at the beginning of the measurement. This reduced the changes in the damping rate due to laser frequency drifts. A series of damping rate measurements is shown in Fig. 6.

The unaxialized magnetron damping rate was typically 2–3 Hz. This is lower than expected from theory, in part due to the size of the ion cloud. With more than one ion, space charge effects increase the size of the cloud and it spends less time in the laser beam.

The magnetron damping rate was seen to increase with increasing axialization-drive amplitude. The error bars are derived from the fit to the phase plot. It is difficult to infer the exact relationship between damping rate and axialization drive from the graph since the damping rate is also dependent on the laser frequencies. Drifts of the four laser-frequency reference cavities were measured to be in the region of tens of megahertz per hour. A typical damping rate measurement required twenty minutes of data acquisition.

As well as a measured increase in the damping rate, the ion cloud’s radius, dominated by the magnetron amplitude, is seen to decrease. This effect was less marked than in previous experiments with a single magnesium ion. Again, this is due to space-charge effects that come into play when more than one ion is trapped. The aspect ratio of the ion cloud gives a clearer indication of the effect of the drive for larger numbers of ions (Fig. 7).

A. Phase measurements

In the unaxialized case there is no coupling between the two radial motions, and two $\pi$ phase changes are observed, at the magnetron and modified cyclotron frequencies respectively. When the coupling is introduced, the frequencies of the resonances are shifted and additional phase changes appear, so that there are observed phase changes at two frequencies near the magnetron frequency and two near the modified cyclotron frequency (Equation 17).

It is expected that a phase change will occur at each of these “dressed states”. An example of the two phase changes near the magnetron motional frequency are shown in Fig. 8. A similar pair could also be observed near the modified cyclotron frequency.

B. Classical avoided crossing

The frequencies of the phase changes are predicted by the analytical model summarized in Section IV A, taking...
account of the laser cooling rates in a Penning trap and the strength of the axialization field. We operated in a regime where \((\epsilon/\omega_1)^2 \gg M^2\), i.e. the coupling rate due to the axialization field was large in comparison to the laser cooling rate.

In this case of a strong axializing field (200 mV), we expect to see an avoided crossing of the two measured frequencies as we scan the frequency of the axializing field across resonance with the true cyclotron frequency. We confirmed this by making measurements of the frequencies from plots such as those in Fig. 8 and plotting them against the axialization drive frequency (Figure 9). From a fit to the plot, we can infer that the true cyclotron frequency was \(379.5 \text{ kHz}\), an unaxialised magnetron frequency of \(23.9 \text{ kHz}\), and \((\epsilon/\omega_1) = 5.6 \text{ kHz}\).

From a simulation of the trap geometry in the SIMION package, we estimate that the theoretical value for \((\epsilon/\omega_1)\) should be \(8.2 \pm 0.8 \text{ kHz}\) for a drive of 200 mV. The discrepancy between the measured and calculated values is in part due to the measurement being made on a cloud rather than a single ion.

**VII. CONCLUSIONS**

We have demonstrated the axialization of calcium ions. The use of RF excitation coupled with time-correlated photon detection allowed the measurement of the damping rates and motional frequencies. The magnetron damping rate could be increased by an order of magnitude using this technique. We have also measured the predicted avoided crossing of the magnetron frequency.

The results are in excellent qualitative agreement with theory. The error in damping rate measurements was relatively large due to laser-frequency drifts. Improvements to the laser systems should allow quantitative investigation of the relationship between axialization drive amplitude and motional damping rates in the future.

The axialization of laser-cooled calcium ions is an important step in the investigation of the viability of a Penning trap quantum computer. Future plans include the measurement of heating rates and spectroscopy of the \(S_{1/2}\) to \(D_{3/2}\) quadrupole transition. If the expected low heating rates are confirmed, early experiments investigating arrays of Penning traps will allow for scalable quantum information processing with Penning traps [12, 13].

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**FIG. 9**: An avoided crossing is observed when the axialization drive frequency is scanned across the true cyclotron frequency.

The fit parameters correspond to a true cyclotron frequency of \(379.5 \text{ kHz}\), an unaxialised magnetron frequency of \(23.9 \text{ kHz}\), and \((\epsilon/\omega_1) = 5.6 \text{ kHz}\).
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