**R-symmetric NMSSM from Unification**

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We study the NMSSM with the knowledge about unification. While this model is a simple and effective extension to avoid the dilemmas the MSSM confronts in the infrared energy region, it is challenging to seclude the singlet $N$ in the renormalization group trajectory from unification scale to weak scale. We propose for the first time identifying the singlet as the goldstino supermultiplet of supersymmetry breaking. Following this proposal, we derive the constraints from naturalness, show simple examples of general O’Raifeartaigh models, and discuss distinctive features in the R-symmetric NMSSM with $N$ as the goldstino supermultiplet.

I. INTRODUCTION

On the realm of supersymmetry, the next-to-minimal supersymmetric model (NMSSM) is a well motivated scenario in diverse aspects of new physics. Not only it naturally explains the 125 GeV Higgs mass [1–6] observed at the LHC [7, 8], but also reduces the tension on the MSSM neutralino dark matter [9, 10] set by dark matter direct detection experiments [11–13]; as well as addresses the baryon asymmetry of Universe by the means of strong first-order phase transition [14–16]. Apart from these phenomenological features in the infrared energy region, it also retains the unification in the ultraviolet energy region similar to the MSSM due to the fact that $N$ is a Standard Model (SM) singlet.

These two requirements are the main obstacle to obtain realistic unification of NMSSM, as they are easily violated by two classes of sources as shown in Fig. 1. As we will explain, in the first class there are a large amount of mixings between $N$ and the Higgs fields $\rho_i$ responsible for the breaking of GUT group $G$; while the second one excluding $N$ from the SUSY-breaking sector is a highly nontrivial task.

II. CLASSIFICATION

Before the issues as mentioned above are settled, it is too early to say that the NMSSM can be successfully secluded in the renormalization group trajectory from unification scale to weak scale. There are a few different considerations for $N$ as follows.

- $N$ is a singlet representation of $G$.
- $N$ is a component field of some high-dimensional representation of $G$, where $N$ is a SM singlet but charged under a subgroup of $G$.
- $N$ is a goldstino supermultiplet of SUSY-breaking sector.

The first two classes confront the problem of how to seclude $N$ from the GUT-breaking sector as shown in Fig. 1. We restrict to the conventional settings, where the Higgs doublets $H_u, d$ are embedded into $5 + \bar{5}$ and 10 for $G = SU(5)$ and $SO(10)$, respectively. The first class was excluded as shown in [17], since a number of mixings between $N$ and Higgs fields $\rho_i$ cannot be eliminated.

The situation changes when $N$ is a component field of some high-dimensional representation of $G$. For $G = SU(5)$, the Yukawa interaction in Eq. (1) implies that $N$ is a component field of 24. However, such 24 always mixies with the 24 that breaks $SU(5)$ into $G_{SM}$, where $G_{SM}$ refers to the SM gauge group. For $G = SO(10)$, $N$ is a component field of 54. Unfortunately, there is no suitable component in 54 to accommodate the Yukawa interaction in Eq. (1) for the well-known breaking patterns such as $SO(10) \rightarrow SU(5) \times U(1) \rightarrow \cdots G_{SM}$ or $SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R \rightarrow \cdots G_{SM}$. 

FIG. 1. A sketch of the problem when NMSSM faces unification, where two sources - the sectors of grand unification theory (GUT) and supersymmetry (SUSY) breaking - can feed the singlet $N$ a mass or a vev far above the weak scale.

However, the implications of unification are far more than what we naively expect due to the singlet nature of $N$. We can understand the main point from the superpotential

\[ W_{\text{NMSSM}} \supset \lambda N H_u H_d + \frac{\kappa}{3} N^3, \tag{1} \]

In order to be natural, there are at least two requirements on the singlet mass parameters [17]:

- The mass of $N$ should be of order weak scale.
- The vacuum expectation value (vev) $\langle N \rangle$ should be beneath the weak scale.
Even though the Yukawa interaction in Eq. (1) is reproduced, it is rather difficult to split $N$ from the other components in $24$ ($54$) in $G = SU(5)(SO(10))$ without affecting the unification.

III. GOLDSTINO SUPERMULTIPELT

Since the first two classes above have been excluded, we discuss $N$ as the goldstino supermultiplet field of SUSY-breaking sector in the third one. Let us use a general O’ Raifeartaigh (OR) model [13] for illustration, which represents a large class of SUSY-breaking models in the literature (For a review on SUSY breaking, see ref. [19]). The general OR superpotential reads as

$$W_{OR} = fX + (M_{ij} + N_{ij}X) \varphi_i \varphi_j,$$ (2)

for which the moduli space of SUSY-breaking vacuum is given by

$$\langle \varphi_i \rangle = 0, \quad X \text{ arbitrary,} \quad V_{\text{tree}} = f^2,$$ (3)

If we link $N$ with the goldstino supermultiplet $X$, the seclusion of $N$ from the characteristic scale of $\varphi_i$ can be understood as a result of SUSY breaking.

However, such behavior is not always respected by the radioactive correction, which in certain situation produces $m_X$ far larger than the weak scale. With the one-loop corrections taken into account, the effective potential for the goldstino supermultiplet field $X$ is given by,

$$V_{\text{eff}} \simeq f^2 + m_X^2 \mid X \mid^2 + \frac{\lambda_X}{4} \mid X \mid^4 + O(\mid X \mid^6).$$ (4)

From Eq. (4), one finds that $\kappa \sim \lambda_X/4$. The sign of $\lambda_X$ is strictly positive. Otherwise, it would contradict with the fact the potential must be bounded below in the region $X \to \infty$. On the other hand, the sign of $m_X^2$ depends on the $R$ charge assignments on $\varphi_i$ [20, 21]. If the sign($m_X^2$) < 0, the vev squared $\mid X \mid^2 \sim m_X^2/|\lambda_X$. Since $\lambda_X$ is always far less than $O(1)$ in the parameter region where $m_X^2$ is of order the weak scale squared, there is a large mass splitting between $\mid X \mid$ and the weak scale. In contrast, given sign($m_X^2$) > 0, the vev $\mid X \mid \sim 0$, which is consistent with the two requirements as mentioned in the Introduction.

In order to compare with the weak scale, we expand $m_X^2$ in terms of the small SUSY-breaking parameter $y = f/M^2$:

$$m_X^2 = \frac{M^2}{16\pi^2} \left[ y^2 \delta_1 + y^4 \delta_2 + O(y^6) \right],$$ (5)

then we should ensure that [22]

$$y^2 \delta_1 + y^4 \delta_2 \sim \frac{y^2}{16\pi^2}$$ (6)

so as to obtain $m_X$ of order weak scale in the context of gauge mediation.

Following Eq. (3), we reformulate the mass squared $m_X^2$ in terms of $\delta_1$ and $\delta_2$ by using the expression in ref. [20]. With the small SUSY breaking $y$ and mass matrix $M_{ij}$ with the characteristic mass $M$, they are given as,

$$\delta_1 = \int_0^\infty d\hat{\mu} \hat{\mu}^3 \text{Tr} \left[ \hat{\mathcal{F}}^4 \hat{\mu}^2 - 2\hat{\mathcal{F}}^2 \hat{\mu}^2 \right],$$

$$\delta_2 = \int_0^\infty d\hat{\mu} \hat{\mu}^3 \text{Tr} \left[ \hat{\mathcal{F}}^4 \hat{\mu}^2 - 2 \left( \hat{\mathcal{F}}^4 \hat{\mu}^2 \hat{\mu}^2 + \hat{\mathcal{F}}^2 \hat{\mu}^2 \hat{\mu}^4 \right) \right].$$ (7)

where dimensionless function $\hat{\mathcal{F}}(\hat{\mu}) = \hat{N}/(\hat{\mu}^2 + \hat{M}^2)$ with $\hat{\mu} = \mu/M$, and dimensionless matrixes $\hat{M}$ and $\hat{N}$ are defined as [21]

$$\hat{M} = \frac{1}{M} \left( \begin{array}{cc} 0 & M_{ij} \\ M_{ij} & 0 \end{array} \right), \quad \hat{N} = \left( \begin{array}{cc} 0 & N_{ij} \\ N_{ij} & 0 \end{array} \right).$$ (8)

Comparing Eq. (7) with Eq. (6) we have a few observations as follows. First, in order to fulfill Eq. (6), $\delta_1$ has to be of order $1/16\pi^2$ if one doesn’t want to bother large fine tunings. Second, $\delta_1$ and $\delta_2$ are proportional to $\lambda^4$ and $\lambda^6$ respectively, with $\lambda$ referring to the Yukawa coupling(s) in $\hat{N}$. Thus, their magnitudes are sensitive to $\lambda$.

IV. EXAMPLES

The requirement in Eq. (5) imposes strong constraint on the structures of matrixes $M$ and $N$ or equivalently the OR superpotential $W_{OR}$. We will explore this class of SUSY-breaking models from low- to high-dimensional $D$ - the number of $\varphi_i$.

![Graph](image.png)

FIG. 2. The magnitudes of $\log(\lambda_X)$ and $\log(|y^2 \delta_1 + y^4 \delta_2|)$ as function of $M_2/M_1$ for $\lambda = 1.0$, $y = 0.1, 0.01$ in the case $D = 3$. The sign of $y^2 \delta_1 + y^4 \delta_2$ flips at the critical value $r_c \sim 2.1$, while that of $\lambda_X$ is always positive. The horizontal dotted lines represent the constraint in Eq. (6) with $y = 0.1$ (dotted) and $y = 0.01$ (solid), respectively.
In the simplest case $D = 3$, there is only a nontrivial candidate with the set of $R$-charges $\{-1, 1, 3\}$:

$$M_{ij} = \begin{pmatrix} 0 & 0 & M_1 \\ 0 & M_2 & 0 \\ M_1 & 0 & 0 \end{pmatrix}, \quad N_{ij} = \begin{pmatrix} 0 & \lambda_1 & \lambda_2 \\ \lambda_1 & 0 & 0 \\ \lambda_2 & 0 & 0 \end{pmatrix}. \quad (9)$$

Substituting Eq. (9) into Eq. (7) yields the values of $\delta_{1,2}$ as shown in Fig. 2. We find that the sign of $y^{2}\delta_1 + y^{1}\delta_2$ flips at the critical ratio $r_c = M_2/M_1 \approx 2.1$ regardless of the value of $\lambda$. As shown by the green curves in the figure, for $\lambda = 1.0$ the total magnitude of $y^{2}\delta_1 + y^{1}\delta_2$ satisfies the requirement in Eq. (8) with $r \sim 2.05$. The required value of $r$ moves towards to the left side when one chooses smaller value of Yukawa coupling $\lambda$. In this sense, a large portion of parameter region of $r$ is covered by tuning $\lambda$.

Similar results are expected in the case of $D \geq 4$. In the case $D = 4$, there is a set of nontrivial choices on the matrices $M$ and $N$ given as,

$$M_{ij} = \begin{pmatrix} 0 & 0 & M_1 \\ 0 & M_3 & M_2 \\ M_1 & M_2 & M_4 \end{pmatrix}, \quad N_{ij} = \begin{pmatrix} 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1 & 0 & 0 & 0 \\ \lambda_2 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 \end{pmatrix}. \quad (10)$$

which correspond to a set of $R$ charges: $\{-1, 1, 1, 3\}$. We divide the study into three different patterns.

- **i).** $M_2 = 0$. This case is reduced to that of $D = 3$ for $M_3 = M_4$, except that $\lambda$ is now replaced by $\sqrt{\lambda_1^2 + \lambda_2^2}$.

- **ii).** $M_3 = M_4 = 0$ and $\lambda_1 \neq \lambda_2$. In Fig. 3 we show the magnitudes the same as in Fig. 2 with $\lambda_1 = 0.3$ and $\lambda_2 = 1.0$. The most difference between Fig. 2 and Fig. 3 is the number of critical values $r_c$, around which sign($m^2_{X}$) flips.

- **iii).** $M_3 = 0$ or $M_4 = 0$. In this case, the integrals in Eq. (7) are rather complicated. They can be simplified by reducing the number of free parameters. In simplified cases such as $M_3 = 0$ and $M_4 = 2M_2$ similar patterns of the plots in Fig. 3 are verified.

In summary, we have studied how to obtain a SM singlet with mass of order weak scale in the context of supersymmetry. Either the GUT-breaking or SUSY-breaking effects on the singlet mass are dynamically suppressed due to the $R$ symmetry and identifying the singlet as the goldstino supermultiplet of general OR models. We have shown explicit examples with the number of messengers $D = 3 - 4$, in which by adjusting the Yukawa coupling constants in the general OR models of SUSY breaking, there is rational parameter space composed of mass parameters in the OR superpotential. Since identifying $N$ as the goldstino supermultiplet of SUSY-breaking sector is a natural choice, we should pay more attention to the $R$-symmetric version of NMSSM than the conventional one.

If we stick to naturalness, it is unlikely to break the $R$ symmetry simultaneously. As shown in Fig. 2, $\langle X \rangle \sim 10^{2} \mid m_X \mid$ for $y = 0.1$, where $\mid m_X \mid$ is of the weak scale. The mass splitting between $\langle X \rangle$ and $\mid m_X \mid$ is expected to be larger for smaller $y$. In this sense, the small SUSY breaking doesn’t favor a spontaneous breaking of $R$ symmetry in a natural way.

V. $R$-SYMMETRIC NMSSM

After identifying $N$ as the goldstino supermultiplet $X$ of SUSY-breaking sector and coupling it to the Higgs doublets through the Yukawa interaction in Eq. (1), we obtain $R$-symmetric NMSSM, in which the $R$ charges of Higgs doublets are zeros [23]. Note, this Yukawa interaction doesn’t affect our previous discussions about $X$ because of the large mass hierarchy between the weak scale and the characteristic mass $M$.

There are rich phenomenologies in the $R$-symmetric NMSSM with $N$ as the goldstino supermultiplet, some of which obviously differ from either the case of $R$-symmetric MSSM or conventional NMSSM. In the literature, there are discussions about the phenomenologies of $R$-symmetric MSSM [23]. Model building along this line can be found e.g. in refs. [24, 25]. The first important difference is that unlike $R$-symmetric MSSM, the constraints from Higgs mass, flavor violation and naturalness can be simultaneously satisfied in the $R$-symmetric NMSSM. Because of $R$ symmetry, the holomorphic soft masses such as $A$ terms and gaugino masses can’t arise.
from effective operators such as
\[
\frac{m_\lambda}{f} \int d^2 \theta X W^a_i W^{ia} + \frac{A_f}{f} \int d^2 \theta X Q u + \cdots \text{H.c.}
\]
(11)

Therefore, the scalar \( N \) differs from the conventional sgoldstino in the literature \[26\]–\[31\] based on the interactions in Eq. (11). On the contrary, it mainly plays a role similar to the singlet of NMSSM through the Yukawa interaction in Eq. (2). Meanwhile, vanishing \( A \) terms such as \( A_f \) from the second class of operators in Eq. (11) suggest that the Higgs mass can’t be explained in the \( R \)-symmetric MSSM without violating naturalness, but they are consistent with each other in the \( R \)-symmetric NMSSM.

Gauginos can’t be Majorana fermions if the \( R \) symmetry is unbroken, but they can be Dirac fermions \[23\]–\[32\] by introducing chiral superfields in the adjoint representation of SM gauge group, i.e. a SM singlet \( \phi_g \), a \( SU(2)_L \) triplet \( \phi_3 \), and \( SU(3) \) octet \( \phi_8 \) \[23\]. These fields expand the contents of both neutral and charged fermions. The Dirac mass matrices \( M_{N,C} \) for both of them in the \( R \)-symmetric NMSSM satisfy
\[
\text{Det} M_{N,C} = 0,
\]
(12)
which implies that there is at least one massless fermion in each sector. In order to avoid this, one has to further enlarge the content of matters e.g. by adding another two doublets \[23\] or a triplet \[32\] to the electroweak sector. In this process one should be cautious, as not all of them are consistent with unification.

The second important difference is that unlike in the conventional NMSSM, in the \( R \)-symmetric NMSSM the neutralinos are Dirac \[34\]–\[37\] rather than Majorana fermions. Dirac neutralinos can be natural realization of leptophilic dark matter \[38\], because the annihilation of Dirac neutralinos into SM fermion pairs \( \chi \chi \rightarrow f \bar{f} \) are not suppressed by the SM fermion masses as in the case of Majorana neutralinos. They can yield signals of \( e^+ e^- \) or \( \gamma \) rays saturating the limits of upcoming astrophysical experiments. For earlier discussions, see e.g. \[39\]–\[40\]. What is even more interesting is that the idea \[41\] of sgoldstino (or gravitino) as the main component of dark matter is naturally accommodated in our model.

The final important difference is about the thermal production \[41\] of gravitino \( G \). With the mixing effects between the goldstino and the Higgsinos \( H_{u,d} \) through the NMSSM superpotential in Eq. (1), one can obtain the goldstino (gravitino) mass of order \( \mathcal{O}(10 - 100) \) \( \text{GeV} \) larger than \( \sim f/M_P \), where \( M_P \) refers to the Planck mass. Uplifting the gravitino mass can help suppress the thermal production of gravitino from the decays of sparticles, and thus avoid the strong constraint \[43\] on the reheating temperature in the context of gauge mediation.

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24. S741 (2007), [hep-ph/0702069].
[20] D. Shih, JHEP 0802, 091 (2008), [hep-th/0703196].
[21] A. E. Nelson and N. Seiberg, Nucl. Phys. B 416, 46 (1994), [hep-ph/9309299].
[22] G. F. Giudice and R. Rattazzi, Phys. Rept. 322, 419 (1999), [hep-ph/9801271].
[23] G. D. Kribs, E. Poppitz and N. Weiner, Phys. Rev. D 78, 055010 (2008), [arXiv:0712.2039 [hep-ph]].
[24] S. D. L. Amigo, A. E. Blechman, P. J. Fox and E. Poppitz, JHEP 0901, 018 (2009), [arXiv:0809.1112 [hep-ph]].
[25] M. Luo and S. Zheng, JHEP 0901, 004 (2009), [arXiv:0812.4600 [hep-ph]].
[26] E. Perazzi, G. Ridolfi and F. Zwirner, Nucl. Phys. B 574, 3 (2000), [hep-ph/0001025].
[27] E. Perazzi, G. Ridolfi and F. Zwirner, Nucl. Phys. B 590, 287 (2000), [hep-ph/0005076].
[28] D. S. Gorbunov and N. V. Krasnikov, JHEP 0207, 043 (2002), [hep-ph/0203078].
[29] C. Petersson and A. Romagnoni, JHEP 1202, 142 (2012), [arXiv:1111.3368 [hep-ph]].
[30] B. Bellazzini, C. Petersson and R. Torre, Phys. Rev. D 86, 033016 (2012), [arXiv:1207.0803 [hep-ph]].
[31] C. Petersson and R. Torre, JHEP 1601, 099 (2016), [arXiv:1508.05632 [hep-ph]].
[32] A. E. Nelson, N. Rius, V. Sanz and M. Unsal, JHEP 0208, 039 (2002), [hep-ph/0206102].
[33] G. Grilli di Cortona, E. Hardy and A. J. Powell, JHEP 1608, 014 (2016), [arXiv:1606.07090 [hep-ph]].
[34] S. Y. Choi, M. Drees, A. Freitas and P. M. Zerwas, Phys. Rev. D 78, 095007 (2008), [arXiv:0808.2410 [hep-ph]].
[35] G. Belanger, K. Benakli, M. Goodsell, C. Moura and A. Pukhov, JCAP 0908, 027 (2009), [arXiv:0905.1043 [hep-ph]].
[36] E. J. Chun, J. C. Park and S. Scopel, JCAP 1002, 015 (2010), [arXiv:0911.5273 [hep-ph]].
[37] M. R. Buckley, D. Hooper and J. Kumar, Phys. Rev. D 88, 065032 (2013), [arXiv:1307.3561 [hep-ph]].
[38] E. A. Baltz and L. Bergstrom, Phys. Rev. D 67, 043516 (2003), [hep-ph/0211325].
[39] C. R. Chen and F. Takahashi, JCAP 0902, 004 (2009), [arXiv:0810.4110 [hep-ph]].
[40] P. J. Fox and E. Poppitz, Phys. Rev. D 79, 083528 (2009), [arXiv:0811.0399 [hep-ph]].
[41] M. Bolz, A. Brandenburg and W. Buchmuller, Nucl. Phys. B 606, 518 (2001), Erratum: [Nucl. Phys. B 790, 336 (2008)], [hep-ph/0012052].
[42] T. Moroi, H. Murayama and M. Yamaguchi, Phys. Lett. B 303, 289 (1993).
[43] A. de Gouvea, T. Moroi and H. Murayama, Phys. Rev. D 56, 1281 (1997), [hep-ph/9701244].