Research Article

Three-Dimensional Stability Characteristics of Finite Electrified Conducting Fluids Streaming through a Porous Medium

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1. Introduction

The Kelvin–Helmholtz instability (KHI) appears if two superposed fluids streaming with a relative horizontal velocity. This instability is very important in describing many kinds of astrophysical and space phenomena involving sheared stream. As a consequence of its connection to geophysical, astrophysical, and laboratory areas [1], it has been examined by numerous contributors. The linear KHI is treated in Chandrasekhar’s monograph [2]. The impact of streaming is destabilized in the linear access in the KHI problem.

Electrohydrodynamics (EHD) can be treated as the branch of fluid mechanics interested in electrical force effects or as the part of EHD that is concerned with the influence of fluid motion on electric fields. Consequently, it is concerned with (electric fields polarized or free charges) interactions in fluids. The fluids may be very good insulators, slight conductors, or even high conductors. A survey on EHD subject with specific reference to several of the advancements in this area is provided by Melcher [3].

In the surface wave phenomena through dielectric fluids when both surface tension and gravity are essential, the capillary waves and the gravity couple to create one wave distinguished by the two influences. By contrast, the effect of an external electric field during conducting fluid flows has many several characteristics and various valuable engineering and physical applications. When surface charges share in the motion of fluids, the arising waves are mentioned as EHD surface waves. The effect of an electric field on the linear stability of a sharp interface dividing two nonconducting dielectric fluids of unlimited range has been investigated by Melcher and Schwarz [4]. A review of the role of interfacial stresses on EHD has been presented by Melcher and Taylor [5] and later the same characteristics have been reviewed by many authors (see Baygents and Baldessare [6] and Rudraiah et al. [7]).

In the linear EHD stability theory, it is well known that the horizontal electric field has a stabilizing effect while the vertical one always has a destabilizing influence [8–12]. EHD stability problems of the interface between conducting fluids in planar and cylindrical geometries have been as of late examined by many authors [13–18].

Flows through a porous medium have been a topic of high interest for the latest many decades. This interest was...
driven by many engineering and geophysical applications in various majors [19]. Many recent works on this topic are investigated by many authors [20–23]. In most earlier investigations on porous media, many researchers considered the treatments based on Darcy’s law and Forchheimer-extended Darcy’s law models. Darcy’s law is an experimental formula relating the bulk viscous resistance, the gravitational force, and the pressure gradient in a porous medium.

The KHI for flow in porous media has received great attention in the scientific literature. Sharma and Spanos [24] investigated the instability of the plane interface separating two uniform superposed fluids motion through porous media. El-Sayed [25] analyzed the instability of two uniform superposed fluids motion through porous media. El-Sayed investigated the instability of the plane interfaces separating two conducting incompressible fluids that have a uniform thickness moving through a porous medium under a uniform electric field. For recent surveys regarding the developments of linear and nonlinear electromagnetic flows in porous media, see refs. [26–36].

The goal of the present paper is to investigate the linear stability of two electrically conducting fluids of finite thickness moving through a porous medium under a uniform horizontal electric field. In addition, the uniform flow is taken through three dimensions to be more general and then using the normal modes analysis to obtain the solutions of system parameters. This problem, as far as we know, has not been studied yet. The stability results are presented in figures. All these flows are new and are presented for the first time in the literature.

2. Problem Description

We consider the three-dimensional finite amplitude surface waves propagating at the interface \( z = 0 \) that represent the balance situation. The perturbed interface \( z = \zeta (x, y, t) \) separates the following two conducting incompressible fluids that have a uniform thickness. Fluid (1) occupies the lower region \( -h_1 < z < \zeta (x, y, t) \) while fluid (2) occupies the upper region \( \zeta (x, y, t) < z < h_2 \). The two fluids are streaming with uniform velocities \( U_1 \) and \( U_2 \) along the \( x \)-axis through a porous medium. The system is influenced by a constant horizontal electric field \( E_0 \) in the \( x \)-direction. The gravitational acceleration \( g \) directed normal to the interface. The surface tension forces \( T \) are taken into account between the two fluids. We shall denote \( \rho_j, \mu_j, \varepsilon_j, \alpha_j, E_j, \lambda_1, \) and \( m, (j = 1, 2) \) to the fluids densities, viscosities coefficients, dielectric constants, electrical conductivities, electric field elements, permeability, and porosity of the medium. A sketch of the physical problem is given in Figure 1. The disturbed interface may be written as

\[
F(x, y, z, t) = z - \zeta (x, y, t) = 0. \tag{1}
\]

The unit outer perpendicular to the interface is

\[
\mathbf{n} = \frac{\nabla F}{|\nabla F|} = \left[1 + \zeta_x^2 + \zeta_y^2\right]^{-1/2} (-\zeta_x, -\zeta_y, 1). \tag{2}
\]

The equation of motion for an incompressible viscous fluid in a porous medium is

\[
\frac{\rho}{m} \left[ \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{m} (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p - \rho \mathbf{g} \varepsilon_z - \frac{\mu}{\lambda_1} \mathbf{v}. \tag{3}
\]

The continuity equation is

\[
\nabla \cdot \mathbf{v} = 0, \tag{4}
\]

where \( p \) and \( \mathbf{v} \) are the pressure and the velocity, respectively. Considering irrotational fluid flows, then the velocity potentials is \( \Phi_j (x, y, z, t) \) such that

\[
\mathbf{v}_j = U_j \mathbf{e}_x + \nabla \Phi_j, \quad (j = 1, 2), \tag{5}
\]

where \( \mathbf{e}_x \) is the unit vector in the \( x \)-direction. The potentials \( \Phi_j \) satisfy the Laplace’s equations:

\[
\nabla^2 \Phi_1 = 0, \quad \text{in} \ -h_1 < z < \zeta (x, y, t), \tag{6}
\]

\[
\nabla^2 \Phi_2 = 0, \quad \text{in} \ \zeta (x, y, t) < z < h_2. \tag{7}
\]

It is supposed that, in electrodynamics, the quasi-static approximation is valid. Hence, the electrical equations are

\[
\nabla \times \mathbf{E} = 0, \tag{8}
\]

\[
\nabla \cdot (\varepsilon \mathbf{E}) = q. \tag{9}
\]

The charge conservation equation is

\[
\nabla \cdot \mathbf{J} + \frac{\partial q}{\partial t} = 0, \tag{10}
\]

where \( \mathbf{J} = \alpha \mathbf{E} + q \mathbf{v} \) is the free current density (stationary current), \( \alpha \) is the electrical conductivity, and \( q \) is the free charge density. Therefore, the electric field can be represented as the gradient of electrostatic potentials \( \Psi_j (x, y, z, t) \):

\[
E_j = E_0 \mathbf{e}_x - \nabla \Psi_j, \quad (j = 1, 2). \tag{11}
\]

From equations (8) and (10), the scalar electric potentials \( \Psi_j \) to satisfy the Laplace’s equations are as follows:

\[
\nabla^2 \Psi_1 = 0, \quad \text{in} \ -h_1 < z < \zeta (x, y, t), \tag{12}
\]

\[
\nabla^2 \Psi_2 = 0, \quad \text{in} \ \zeta (x, y, t) < z < h_2. \tag{13}
\]

3. The Boundary Conditions

(1) The vanishing of the normal fluid velocities on both the lower and upper boundaries is

\[
\frac{\partial \Phi_j}{\partial z} = 0, \quad \text{at} \ z = (-1)^j h_j, \quad (j = 1, 2). \tag{14}
\]

(2) The vanishing of the normal components of the electric fields also on these boundaries is

\[
\frac{\partial \Psi_j}{\partial z} = 0, \quad \text{at} \ z = (-1)^j h_j, \quad (j = 1, 2). \tag{15}
\]
(3) The kinematic boundary condition at the interface 
\( z = \xi(x, y, t) \) is given by
\[
m\frac{\partial \xi}{\partial t} - \frac{\partial \Phi_j}{\partial z} + \frac{\partial \xi}{\partial x} \left( U_j + \frac{\partial \Phi_j}{\partial x} \right) + \frac{\partial \xi}{\partial y} \frac{\partial \Phi_j}{\partial y} = 0,
\]
where \( j = 1, 2 \).

(4) The tangential electric field component is supposed continuous at the interface \( z = \xi(x, y, t) \):
\[
\|\Psi_x\| + \xi \|\Psi_y\| = 0,
\]
where \( \|f\| = f_2 - f_1 \) describes the jump of the quantity \( f \) over the interface.

(5) When uniform conductivity of fluids is considered, the problem became more attractive yet difficult. Surface charges play an essential role in such cases; the conduction process in the interface region is a significant factor in many electrical systems. The continuity of stationary current normal to the interface \( z = \xi(x, y, t) \) must lead to charge accumulation on the interface [17]:
\[
\xi_x \|\Psi_x\| + \xi_y \|\Psi_y\| - \xi_x E_0 \|\sigma\| - \xi_x E_0 \|\sigma\| = 0.
\]

(6) The interfacial normal stress component must be continuous at the interface \( z = \xi(x, y, t) \):
\[
\frac{1}{m^2} \|\rho (\sigma \Phi_1 + U \Phi_2)\| + \frac{1}{\Lambda_2} \|\mu \Psi_1\|
+ \xi_x E_0 \|\sigma\| + 2 \xi_x E_0 \|\sigma\| + 2 \xi_x E_0 \|\sigma\|
\]
\[
- \xi_x \|\Psi_x\| - \xi_y \|\Psi_y\| + g(\rho_1 + T(\xi_x + \xi_y)) = 0.
\]

4. Linear Theory and Characteristic Equation

To study the stability of the problem, first we shall analyze the perturbation quantities into normal modes, and we suppose that all the perturbed quantities take the form
\[
f(r) \exp(i \theta),
\]
where \( f(r) \) is any function of \( r, \theta = \omega t \) is the carrier wave phase, \( K = \sqrt{k^2 + l^2} \), and \( k \) and \( l \) represent the wave number elements in \( x \)– and \( y \)– directions, respectively, while \( \omega, A, i, \) and \( c.c \) denote the angular frequency, the complex amplitude of the surface wave, the imaginary unit, and the complex conjugate of the previous terms, respectively.

Here, using equation (19) together with equations (6), (7), (11), and (12) with the suitable boundary conditions, the potential function solutions may be written as
\[
\zeta = A \exp(i \theta) + c.c,
\]
\[
\Phi_1 = \frac{i(k U_1 - m \omega)}{K \sinh Kh_1} \cos K(z + h_1) A \exp(i \theta) + c.c,
\]
\[
\Phi_2 = \frac{i(k U_2 - m \omega)}{K \sinh Kh_2} \cos K(z - h_2) A \exp(i \theta) + c.c,
\]
\[
\Psi_1 = \frac{i k E_0 (\alpha_2 - \alpha_1)}{K (\alpha_1 \sigma_1 + \alpha_2 \sigma_2)} \cos K(z + h_1) A \exp(i \theta) + c.c,
\]
\[
\Psi_2 = \frac{i k E_0 (\alpha_2 - \alpha_1)}{K (\alpha_1 \sigma_1 + \alpha_2 \sigma_2)} \cos K(z - h_2) A \exp(i \theta) + c.c.
\]

Now, \( \sigma_j = \tanh Kh_j \) \((j = 1, 2)\), and to obtain a nontrivial starting solution, the frequency \( \omega \) and the wave number \( K \) should satisfy the next characteristic equation:
\[
S(\omega, K) = \frac{1}{m^2 K} \left\{ \frac{\rho f (m \omega - K U_1)^2}{\sigma_1} + \frac{\rho f (m \omega - K U_2)^2}{\sigma_2} \right\}
\]
\[
\frac{i}{K \Lambda_1} \left\{ \frac{\mu f (m \omega - K U_1)}{\sigma_1} + \frac{\mu f (m \omega - K U_2)}{\sigma_2} \right\}
\]
\[
- \frac{k^2 E_0^2 (\epsilon_2 - \epsilon_1) (\alpha_2 - \alpha_1)}{K (\alpha_1 \sigma_1 + \alpha_2 \sigma_2)} + g(\rho_2 - \rho_1) - TK^2 = 0.
\]

Equation (21) is simplified in the dispersion relation
\[
a_0 \omega^2 + (a_1 + ib_1) \omega + (a_2 + ib_2) = 0,
\]
\[ a_0 = \left( \frac{\rho_1}{\sigma_1} + \frac{\rho_2}{\sigma_2} \right), \]

\[ a_1 = -\frac{2k}{m} \left( \frac{\rho_1 U_1}{\sigma_1} + \frac{\rho_2 U_2}{\sigma_2} \right), \]

\[ b_1 = \frac{m}{\lambda_1} \left( \frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2} \right), \]

\[ a_2 = \frac{k^2}{m} \left( \frac{\rho_1 U_1^2}{\sigma_1} + \frac{\rho_2 U_2^2}{\sigma_2} \right) - \frac{k^2 E_0^2 (\epsilon_2 - \epsilon_1) (\sigma_2 - \sigma_1)}{(\sigma_1 \sigma_1 + \sigma_2 \sigma_2)}, \]

\[ + gK (\rho_2 - \rho_1) - TK^3, \]

\[ b_2 = -\frac{k}{\lambda_1} \left( \frac{\mu_1 U_1}{\sigma_1} + \frac{\mu_2 U_2}{\sigma_2} \right). \]

We have the following special cases:

(1) In two semiinfinite nonconducting fluids, equation (22) reduces to the corresponding dispersion relation obtained first by El-Sayed [25]

(2) In two-dimensional nonporous case, equation (22) reduces to the same equation derived by Melcher [8]

(3) In the absence of the electric field and the porosity, equation (22) reduces to the corresponding relation presented by Chandrasekhar [2]

5. Stability Analysis and Discussion

Applying here the Routh–Hurwitz stability criterion [37] to equation (22), the system will be linearly stable according to

\[ b_1 > 0, \]

\[ a_2 b_1^2 - a_1 b_1 b_2 + a_1 b_2^2 \leq 0. \]  \hspace{1cm} (24)

Since \( m \) and \( \lambda_1 \) are positive parameters, the first condition in (24) is trivially satisfied. The second inequality is satisfied if

\[ E_0^2 \geq E_c, \]  \hspace{1cm} (25)

where

\[ E_0^2 = \left( \frac{a_1 \sigma_1 + a_2 \sigma_2}{k^2(\epsilon_2 - \epsilon_1)(\sigma_2 - \sigma_1)} \right) \left\{ \frac{k^2 (U_2 - U_1)^2 \left( \rho_2 \sigma_2 \mu_1^2 + \rho_1 \sigma_1 \mu_2^2 \right)}{m^2 (\mu_1 \sigma_2 + \mu_2 \sigma_1)^2} \right\}, \]

\[ + gK (\rho_2 - \rho_1) - TK^3 \right\}. \]  \hspace{1cm} (26)

From the previous stability criteria, it is obvious that the permeability \( \lambda_1 \) of the medium has no effect on the linear stability of the system. The horizontal electric field has a dual role according to the sign of the term \( (\epsilon_2 - \epsilon_1)(\sigma_2 - \sigma_1) \) in contrast with the pure dielectric fluids case, and this was well studied by Moatimid [38]. The surface tension has stabilizing effects, while the uniform streaming velocity is strictly destabilizing. Next, we shall make a numerical discussion to clarify the stability of the problem by graphing the transition curves represented by equation (26) in \( E_0^2 - k \) plane when \( E_0^2 = E_c^2 \). These transition curves that represent the marginal stability state separate the upper stable S regions from the lower \( U \) unstable ones. The calculations are made in that case, for a system having \( \rho_1 = 0.000365 \) gr/cm\(^3\), \( \rho_2 = 0.597 \) gr/cm\(^3\), \( \epsilon_1 = 1.007 \) farad/cm, \( \epsilon_2 = 1.7 \) farad/cm, \( \alpha_1 = 0.2 \) mho/cm, \( \sigma_2 = 0.6 \) mho/cm, \( h_1 = 1.5 \) cm, \( h_2 = 3.5 \) cm, \( \mu_1 = 0.03 \) cm\(^2\)/sec, \( \mu_2 = 0.7 \) cm\(^2\)/sec, \( U_1 = 4 \) cm/sec, \( U_2 = 20 \) cm/sec, \( g = 981 \) cm/sec\(^2\), \( T = 0.05 \) dyn/cm, and \( m = 0.06 \) sec/cm. In Figures 2–8, the first dimension expresses disturbance wave number \( k \), the second expresses the critical electric field \( E_0^2 \), and the third expresses one of the previous parameters.

Figure 2 shows the variation of the electric field \( E_0^2 \) versus \( k \) for different electrical conductivity values \( \alpha_1 \) and \( \alpha_2 \) in the two-dimension \((l = 0)\) and three-dimension \((l = 1, 2)\) cases. It was clear that, for \( l = 0 \), the stable region increases by increasing \( \alpha_1 \) and \( \alpha_2 \), while it decreases by increasing the dimension \( l \) \((l = 1, 2)\) for small wave number range \((k \leq 6)\) after that the stability does not depend on \( l \) since the curves coincide. Also, for any value \( l \) and small \( E_0^2 \) values, the system is always unstable, while for high \( E_0^2 \), the electric field has a stabilizing effect since stable region increases by increasing the electric field \( E_0^2 \) and wave number value \( k \). Then, both the electrical conductivities \( \alpha_1 \) and \( \alpha_2 \) and the electric field \( E_0^2 \) have stabilizing effects, while the dimension \( l \) has a destabilizing effect only for small wave number value \( k \).

Also the effects of the porosity \( m \) of the medium (see Figure 3) as well as the fluid viscosities \( \mu_1 \) and \( \mu_2 \) (see Figure 4) on the stability are found to be similar to the effect of the electrical conductivities \( \alpha_1 \) and \( \alpha_2 \) given in Figure 2. Thus, each of the porosity \( m \) of the medium and the fluid viscosities \( \mu_1 \) and \( \mu_2 \) has stabilizing effects on the system.

In Figure 5, we draw \( E_0^2 \) versus \( k \) for different surface tension values \( T \) in two- and three-dimension disturbances cases. It was clear that, when \( l = 0 \) and for small values of \( T \), the stability increases by increasing \( T \) while it decreases by increasing the dimension \( l \) for small wave numbers \((k \leq 6)\) just like the previous figures. In contrast, for high values of \( T \), it is seen that the stability increases by increasing \( T \) and the dimension \( l \) for wave numbers range \((k \geq 1)\). We conclude that, the dimension \( l \) plays a dual role (stabilizing and destabilizing) when there are large variation in surface tension force between the fluids.

Figure 6 shows the variation of \( E_0^2 \) against \( k \) for different fluid velocities values \( U_1 \) and \( U_2 \) in two- and three-dimension cases. It is shown that, when \((l = 0, 1, 2)\), the instability increases by increasing the fluid velocities \( U_1 \) and \( U_2 \), and for any fixed values of \( U_1 \) and \( U_2 \), the instability also increases by increasing \( l \) for small wave number \((k \leq 6)\) values after which \( l \) has no effect on the stability. Therefore, both the fluid velocities \( U_1 \) and \( U_2 \) and the dimension \( l \) have destabilizing effects, while for the high wave number value \( k \), the dimension \( l \) has no effect on the stability.

Finally, Figures 7 and 8 show the effects of the fluid depths \( h_1 \) and \( h_2 \) accompanied with the dimension \( l \) on the stability, stabilizing the system for small values of \( h_1 \) and \( h_2 \) (see Figure 7) while destabilizing it for large values of \( h_1 \) and \( h_2 \) (see Figure 8). Thus, the fluid depths and the dimension have a dual role in the system.
Figure 2: Variation of $E_0^2$ with $k$ for different values of the electrical conductivities $\alpha_1$ and $\alpha_2$.

Figure 3: Variation of $E_0^2$ with $k$ for different values of the porosity of a porous medium $m$. 
Critical electric field

\( S \)

\( \mu_1 = 0.1, \mu_2 = 0.9 \)
\( \mu_1 = 0.05, \mu_2 = 0.2 \)
\( \mu_1 = 0.07, \mu_2 = 0.35 \)

**Figure 4:** Variation of with \( k \) for different values of the fluid viscosities \( \mu_1 \) and \( \mu_2 \).

Critical electric field

\( U \)

**Figure 5:** Variation of \( E_0^2 \) with \( k \) for different values of the surface tension \( T \).
Figure 6: Variation of $E_0^2$ with $k$ for different values of the fluid velocities $U_1$ and $U_2$.

Figure 7: Variation of $E_0^2$ with $k$ for small values of the fluid velocities $h_1$ and $h_2$. 
6. Conclusions

In this paper, new results of bounded flows through porous medium have been presented and analyzed for weakly electrically conducting fluids. Exact analytical solutions have been given for all parameters.

Using the normal mode analysis, we obtain quadratic dispersion relation of complex coefficients characterizing the behaviour of the disturbed system. Based on appropriate data selections, we conclude the following:

1. The permeability $\lambda_1$ of the medium has no effect on the linear stability of the system.
2. The horizontal electric field $E_2^0$, porosity of the medium $m$, surface tension $T$, fluid viscosities $\mu_1$ and $\mu_2$, and electrical conductivities $\alpha_1$ and $\alpha_2$ have stabilizing effects or enhance the stability.
3. The fluid velocities $U_1$ and $U_2$ (including the case of absence of fluid velocities $U_1 = 0$ and $U_2 = 0$ or Rayleigh–Taylor instability) have destabilizing effects or tend to reduce the stability.
4. Both the fluid depth $h_1$ and $h_2$ and the dimension $l$ have a dual role in the system, i.e., stabilizing the system for small values of $h_1$ and $h_2$, while destabilizing it for large values of $h_1$ and $h_2$ (including the case of two semiinfinite fluids).

According to the importance of nonlinear effects in EHD phenomena, the describing equations of EHD flow are nonlinear. Therefore, we will discuss the nonlinear stability characteristics for this problem in a subsequent article.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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