Regression models for intrinsic constants of reconstituted clays

Farzad Habibbeygi1*, Hamid Nikraz1 and Bill K Koul2

Abstract: In this study, four models were developed to predict intrinsic constants based on some simple physical parameters as well as clay mineralogy of a reconstituted clay sample. The effect of each predictor on the response was evaluated for each individual clay mineralogy. According to the results, it appears that the void ratio at liquid limit has the greatest effect on clays with a considerable amount of smectite, while the effect of the initial void ratio of such clays is the least amongst other clay minerals. The accuracy of the predictive model increases with the inclusion of clay mineralogy as an input parameter. \( R^2 \) increases from 0.978 to 0.99 and from 0.831 to 0.896 for intrinsic parameters of \( e_{100} \) and \( e_{1000} \), respectively. A simplified method is also presented to determine the virgin compression line of reconstituted clays using the initial void ratio, the void ratio at the liquid limit, and its clay mineralogy.

1. Introduction
Dredged slurry sedimentation is currently used frequently in land reclamations as a sustainable solution to manage the huge quantities of slurries dredged from lakes and rivers every year. The deposited slurries can be found all over the world where reclamation is considered to be an essential step in developing new urban areas. Over the past seven decades, researchers and geotechnical engineers across the world have been interested in better comprehending the compression behaviour of dredged slurries at high initial water content due to the potential effect of their settlement on future coastal development projects.

ABOUT THE AUTHOR
Farzad Habibbeygi is a PhD student at School of Civil and Mechanical Engineering, Faculty of Science and Engineering, Curtin university and a member of Engineers of Australia. His research areas of interest are mechanical and compression behaviour of expansive clays, numerical modelling and artificial neural network. He has also published several research papers in international journals in his area of interest.

PUBLIC INTEREST STATEMENT
This paper introduces four predictive models for intrinsic constants based on some simple geotechnical parameters as well as predominant clay mineral of a reconstituted clay sample. The effect of each input parameter on the response of the models was evaluated for each individual clay mineralogy. According to the results, the void ratio at liquid limit has the greatest effect on clays with a considerable amount of smectite, while the effect of the initial void ratio of such clays is the least amongst other clay minerals.
During the dredging process, the structure of a clayey material is broken down such that the sedimented dredged clay can be considered as a reconstituted soil. A reconstituted soil is defined as a type of remoulded sample which is prepared at a water content equal to or greater than its liquid limit following Burland’s (1990) procedure. During the past seven decades, many researchers have performed a considerably large number of consolidation tests on remoulded/reconstituted samples to investigate the compressibility of clays (Butterfield, 1979; Habibbeygi, Nikraz, & Verheyde, 2017; Hong & Tsuchida, 1999; Hong, 2006; Mesri & Olson, 1971; Mitchell & Soga, 1976; Nagaraj & Miura, 2001; Nagaraj & Murthy, 1983; Sridharan & Gurtug, 2005; Sridharan & Nagaraj, 2000). However, a relatively accurate prediction of the compressibility behaviour of dredged slurries can be derived by performing modified oedometer (consolidation) tests on these types of materials; unfortunately, consolidation tests on reconstituted clays are always time-consuming, costly and cumbersome. The determination of compression index is considered to be a relatively expensive and time-consuming test in most geotechnical projects, especially when undertaken on clays with high initial water content. Considering all these difficulties and limitations in assessing the compressibility of reconstituted clays at high initial water content, geotechnical engineers estimating the volumetric behaviour of reconstituted clays often tend to use empirical correlations instead of performing one-dimensional consolidation tests. Accordingly, having some reliable empirical equations to predict the compression behaviour of such soils is beneficial. In this case, compressibility can be estimated by understanding some simple physical characteristics of the soil sample under study (i.e. initial water content, liquid and plastic limits, natural void ratio), which can be determined conveniently in the laboratory.

To the authors’ best knowledge, some research has previously been conducted to consider the effect of initial water content and soil properties on the compression behaviour of reconstituted clays (Cerato & Lutenegger, 2004; Kootahi & Moradi, 2016; Lee, Hong, Kim, & Lee, 2015; Lei, Wang, Chen, Huang, & Han, 2015; Takashi, 2015; Xu & Yin, 2015; Zeng, Hong, Cai, & Han, 2011), but only a few studies have taken into account the influence of clay mineralogy (Habibbeygi, Nikraz, & Chegenizadeh, 2017; Xu & Yin, 2015). In this paper, the intrinsic concept was used as a basic frame of reference for interpreting and evaluating the compressibility of reconstituted clays. Furthermore, the effect of clay mineralogy on compression behaviour has been assessed using a broad range of geotechnical data from the literature. Eventually, four series of practical relationships were developed to estimate the intrinsic constants and compressibility of a reconstituted clay based on some simple physical parameters, as well as the clay’s mineralogy.

2. Existing correlations for predicting intrinsic constants

Burland (1990) proposed a unique framework for normalising the compression behaviour of reconstituted clays by introducing the intrinsic concept in his 40th Rankine lecture. The intrinsic concept has since been consistently used worldwide to explain the behaviour of clays (Al Haj & Standing, 2015; Habibbeygi et al., 2017; Hong, Lin, Zeng, Cui, & Cai, 2012; Hong, Yin, & Cui, 2010; Horpibulsuk, Liu, Zhuang, & Hong, 2016). Intrinsic compression behaviour can be interpreted by two constants of compressibility—compression index ($C_c$) and $e_{100}^*$—and a void ratio invariant, named the void index ($I_v$). According to Burland, compression index ($C_c$) can be defined as the subtraction of two definite void ratios (Equation 1). The void index at each state ($I_v$) can also be expressed with the related void ratio and the intrinsic constants as follows (Burland, 1990):

$$C_c = e_{100}^* - e_{100}^*$$

$$I_v = \frac{e - e_{100}^*}{C_c}$$

where $e_{100}$ and $e_{1000}$ are the void ratios of the reconstituted clay at the effective vertical consolidation stresses ($\sigma_v^*$) at 100 kPa and 1000 kPa, respectively. To distinguish the reconstituted invariants from those of natural clays, the symbol (*) has been used with compression parameters in the equations related to a reconstituted clay.
Burland (1990) stated that the compression behaviour of reconstituted clays can be generalised by using the normalised invariant of the void index and, in fact, a unique line can interpret the compressibility of all different clays independent of their natural state. However, it has been recently documented that the initial water content ($w_0$) of reconstituted clays has a considerable effect on the intrinsic compression line (ICL) at low stress levels (Habibbeygi et al., 2017; Hong et al., 2010; Xu, Gao, Yin, Yang, & Ni, 2014), and the ICL is almost unique for a broad range of initial water content for a “medium to high” stress range when $I_v$ is plotted against $\sigma_0^{iv}$ in a semi-log scale.

Burland (1990) also proposed an empirical equation for calculating the ICL indirectly. The value of void index ($I_v$) to estimate the inherent compressibility of reconstituted clays can be calculated as follows:

$$I_v = 2.45 - 1.285(\log \sigma_0^{iv}) + 0.015(\log \sigma_0^{iv})^3$$

(3)

Burland (1990) also suggested a method for calculating the ICL indirectly in the absence of one-dimensional consolidometer test results, based on the regression analyses on a various range of clays (Equations 4 and 5). The use of the following equations must be limited to the liquid limit ($w_L$) from 25% to 160% (i.e. $e_L$ in the range of 0.6–4.5) (Burland, 1990):

$$e_{100}^* = 0.109 + 0.679e_L - 0.089e_L^2 + 0.016e_L^3$$

(4)

$$C_z^* = 0.256e_L - 0.04$$

(5)

where $e_L$ is the void ratio of reconstituted clays at liquid limit.

Yin and Miao (2013) proposed a modification to Burland’s relationships by considering the influence of initial water content on the intrinsic constants of reconstituted clays. Forty-two samples from three different sampling sites in China were collected in their study. The geotechnical parameters of the investigated soils can be summarised as follows:

The liquid limit varied between 61% and 91%; The plastic limit was limited to a narrow range of 30% to 38%; and the initial water content varied between 43% and 180%.

The modified equations for estimating the intrinsic constants ($C_z^*$ and $e_{100}^*$) in terms of $w_L$, and $w_0$ are as follows:

$$e_{100}^* = 1.13w_L + 0.39w_0/w_L - 0.084$$

(6)

$$C_z^* = 0.91w_L + 0.25w_0/w_L - 0.461$$

(7)

where $w_0$ and $w_L$ are the initial water content and the liquid limit of the studied reconstituted clay, respectively.

Zeng, Hong, and Cui (2015) performed 48 consolidometer tests on natural clays, as well as reconstituted kaolinite clays, with different initial water content to assess the inherent compression behaviour of such soils. They used the consolidation test results of their study, as well as the data of earlier research, to suggest equations for estimating the intrinsic constants. They suggested two groups of equations based on the range of $e_L$. Equations (8) and (9) were developed for the $e_L$ range of 0.76–2.7, while Equations (10) and (11) were suggested to be used for a broader range of $e_L$ of 0.66 to 5.72. Density of solid particles of the studied soils varied from 2.65 g/cm$^3$ to
2.75 g/cm³. The initial water content ($w_0$) ranged from 22.3% to 163.3% and the liquid limit from 28.1% to 100%:

$$e_{100} = 0.357 + 0.171e_0 + 0.223e_L \quad (For \ e_L = 0.76 - 2.7) \quad (8)$$

$$C_c^* = -0.069 + 0.109e_0 + 0.152e_L \quad (9)$$

$$e_{100} = 0.223 + 0.261e_0 + 0.282e_L - 0.018e_0^2 - 0.05e_L^2 + 0.015e_L \quad (For \ e_L = 0.66 - 5.72) \quad (10)$$

$$C_c^* = -0.064 + 0.153e_0 + 0.11e_L - 0.006e_0^2 \quad (11)$$

where $e_0$ and $e_L$ are the void ratios at initial state and the liquid limit of the investigated reconstituted clay, respectively.

Xu and Yin (2015) investigated the intrinsic compression curves of clays with three different minerals (kaolinite, illite and smectite). The consolidation tests results of their work on the reconstituted samples, with initial water content ranging from 1 to 2 times their respective liquid limits, revealed that the effects of the initial water content on the intrinsic constants of reconstituted clays depended on the type of the clay minerals. For example, their study demonstrated that $w_0$ has less impact on $e_{100}$ of the predominant montmorillonite clay than on other secondary clay minerals.

Habibbeygi et al. (2017) performed eight series of consolidation tests on reconstituted samples to investigate the impact of mineralogy on the compressibility of expansive clays. The initial water content of their study ranged from 0.67 $w_L$ to 1.33 $w_L$, and vertical consolidation stress varied from as low as 1 kPa to as high as 1,600 kPa to consider a broad range of consolidation stress. Their studies depicted that the initial water content had a considerable impact on the intrinsic constants for the studied soil. In fact, $e_{100}$ and $C_c^*$ increased with increases in the initial water content. Furthermore, the intrinsic constants of the studied clay with smectite as the predominant clay mineral were higher than the estimated values from existing empirical equations.

In summary, it appears that there is a considerable influence of clay mineralogy on the intrinsic constants of a reconstituted clay. While some experimental equations have been proposed to estimate the intrinsic constants of reconstituted clays ($e_{100}$ and $C_c^*$) by past researchers (Burland, 1990; Yin & Miao, 2013; Zeng et al., 2015), none of these equations has taken into account the effect of clay mineralogy on the intrinsic constants. Given past research, the main objective of this paper has been to investigate the effect of clay mineralogy on the value of intrinsic constants, and to suggest some equations for estimating these constants and compressibility based on simple, measureable, physical geotechnical parameters considering the influence of clay mineralogy.

3. Estimating the intrinsic constants

To develop some empirical equations that are able to predict values of the intrinsic constants under different circumstances, a large series of 1D consolidation test data was collected in this study to include not only diverse initial states and physical properties, but also the variety of clay mineralogy in the predicting relationships. The collected data were then used to derive some regression models based on the availability of the input data. Finally, the proposed model, including the clay mineralogy, was compared to the existing models, and the influence of each parameter in the model on the response was evaluated for various clay minerals.

4. Experimental data of reconstituted clays

A broad range of experimental data of 94 consolidation tests on various reconstituted/remoulded clays, with different mineralogy and initial water contents, was used in this study. A summary of the geotechnical properties, initial state, and references to these tests is tabulated in Table 1.
| Soil description          | $w_0$ | $G_s$ | $w_L$ | $w_p$ | Chief clay mineral | Clay % | Stress range          | No. of tests | Reference                          |
|---------------------------|-------|-------|-------|-------|--------------------|--------|-----------------------|--------------|------------------------------------|
| Black soil                | 88    | 2.72  | 60    | 30    | Illite-Smectite    | 48     | Up to 8 MPa           | 1            | Al Haj and Standing (2015)         |
| Red soil                  | 138   | 2.78  | 87    | 33    | Illite-Smectite    | 50     | Up to 8 MPa           | 1            |                                   |
| Baldwin clay              | 55-109| 2.6   | 82    | 35    | Smectite           | 68     | 1-1600 kPa            | 8            | Habibbeygi et al. (2017)          |
| Atchafalaya              | 101-177| 2.8  | 101   | 35    | Montmorillonite    | 66.6   | Up to 2 MPa           | 8            | Cerato and Lutenegger (2004)       |
| Boston blue clay          | 45-79 | 2.8   | 45    | 23    | Illite             | 48.4   |                       |              |                                   |
| Kaolinite                 | 42-74 | 2.68  | 42    | 26    | Kaolinite          | 36.2   |                       |              |                                   |
| Kaolin                    | 45-113| 2.69  | 55.3  | 25.2  | Kaolinite          | 62     | 0.5-1600 kPa          | 9            | Xu and Yin (2015)                 |
| Illite                    | 45-104| 2.7   | 52.7  | 29.6  | Illite             | 40     | 0.5-1600 kPa          | 10           | Xu and Yin (2015)                 |
| Montmorillonite           | 190-529| 2.72 | 258.9 | 42.8  | Smectite           | 84     | 0.5-1600 kPa          | 8            | Xu and Yin (2015)                 |
| Lianyungang clay          | 50-146| 2.71  | 74    | 33    |                    | 23     | 0.5-1600 kPa          | 14           | Hong et al. (2010)                |
| Baimahu clay              | 64-180| 2.65  | 91    | 38    |                    | 20     | 0.5-1600 kPa          | 14           | Hong et al. (2010)                |
| Kemen clay                | 43-122| 2.67  | 61    | 30    |                    | 19     | 0.5-1600 kPa          | 14           | Hong et al. (2010)                |
| Lin-Gang Clay1 (undisturbed) | 46-124| 2.8  | 56.4  | 26.5  | Illite             | 55.4   | 2-827 kPa             | 4            | Lei et al. (2015)                 |
| Central Fishing Port Clay (undisturbed) | 22.4  | 2.8  | 39.8  | 15    | Illite             | 46     |                       | 1            |                                   |
| Qing-Fang Clay1 (undisturbed) | 43.4 | 2.8  | 40.8  | 22.1  | Illite             | 38.1   |                       | 1            |                                   |
| Qing-Fang Clay2 (remoulded) | 43.3 |       |       |       |                    |        |                       | 1            |                                   |
Density of soil particles ranged from 2.57 g/cm³ to 2.80 g/cm³ and the initial water content varied from 22.4% to 52.8%. The liquid and plastic limits of the studied soils were in the range of 39.8% to 258.9% and 15% to 42.8%, respectively. As can be seen from Table 1, the data include three different clay minerals—kaolinite, illite and smectite.

Figure 1 illustrates the compression curves of all these clays in the form of e vs. log σ₀ relationship. The initial consolidation stress in the tests was as low as 0.5 kPa and reached high stress levels of 1,600 kPa to 4,000 kPa. As expected, most curves are inverse S-shaped, with a distinct remoulded yield stress. Figure 1 shows two sets of curves, separated approximately at e = 5, as
there were no samples available at this void ratio. Remoulded yield stress of a reconstituted clay, which is similar to pre-consolidation stress of a natural clay, is a particular stress beyond which the inherent structure of a reconstituted clay breaks down, and the compressibility (the slope of virgin compression line) increases abruptly.

The compression curves of the studied soils are replotted in a normalised plane, using the void index in Figure 2 ($I_v$ vs. $\log \sigma_0^v$). However, there is a considerable disparity between normalised compression curves of reconstituted clays at low stress level; Burland's equation can express these well when the vertical consolidation stress is higher than the vertical consolidation stress at the remoulded yield stress. A new polynomial equation (Equation 12)
has also been fitted to the data with a reasonable correlation coefficient, as high as 0.93, to refine the existing ICL equations:

\[ I_v = 1.906 - 0.392 \left( \log \sigma_v \right) - 0.457 \left( \log \sigma_v \right)^2 + 0.088 \left( \log \sigma_v \right)^3 \]  

(12)

The refined equation, as well as the Burland's equation, is plotted in Figure 2 for comparison. It is noted that Burland's empirical equation has been developed for the reconstituted soils, with \( e_L \) lower than 4.44, while the proposed equation used data with \( e_L \) up to 7.04. However, Burland's equation and the refined equation are close to each other for the medium level of effective vertical stress (50–100 kPa), and even overlap for some stresses in this range. The modified equation becomes slightly more concave upwards than Burland's equation for consolidation stresses higher than 100 kPa. As shown in Figure 2, there is a disparity between the results of this modified equation (i.e. central fitting of scatter plot) and the experimental data, especially for the low-stress level (i.e. less than 20 kPa). The initial soil sample condition, clay mineralogy, non-linear behaviour of clays, change in the soil permeability, the coefficient of consolidation during consolidation procedure and secondary consolidation are some of the factors that affect the compression curves.

5. Regression models

Following the introduction of the intrinsic concept by Burland (1990), the void index has been used herein to normalise the compression curves and to interpret the compressibility of reconstituted clays. Based on Burland's work, the compressibility of a reconstituted clay can also be estimated by knowing the values of \( e_{100} \) and \( e_{1000} \). These two void ratios can also be used to determine the intrinsic compression index (\( I_v \)) and to predict the compression behaviour of a reconstituted clay.

Two simple geotechnical parameters, i.e. the void ratio at initial condition (\( e_0 \)) and the void ratio at liquid limit (\( e_L \)), along with the type of clay mineralogy (CLM), were adopted as predictors to obtain the empirical equations for estimating the intrinsic constants in this study. As all input parameters may not be available, the following four regression models have been developed based on the degree of availability of geotechnical parameters of the investigated soil:

Model 1: If only the void ratio at the liquid limit (\( e_L \)) is available.

Model 2: If both the void ratio at the liquid limit (\( e_L \)) and the initial void ratio (\( e_0 \)) are known.

Model 3: If, in addition to the void ratios of (\( e_0 \) and \( e_L \)), clay mineralogy (CLM) is also available.

Model 4: A simplified form of Model 3, which is easier to use without losing much accuracy from Model 3.

5.1. Model 1

Three different regression models were developed to estimate \( e_{100} \) and \( e_{1000} \) depending on the information obtainable for the studied reconstituted clay. The first model, which is the simplest one, uses only one parameter, \( e_L \), to estimate the intrinsic parameters. The void ratio at the liquid limit can be calculated by assuming saturated condition for the clayey soil, using the relationship \( e_L = G_s w_L \), where \( G_s \) is the specific gravity, and \( w_L \) is the liquid limit of the studied clay. The equations for this model can be summarised as follows:

\[ e_{100} = 1.167 - 0.334 e_L - 0.242 e_L^2 - 0.016 e_L^3, \quad (R^2 = 0.978) \]  

(13)

\[ e_{1000} = 2.093 - 1.712 e_L - 0.637 e_L^2 - 0.057 e_L^3, \quad (R^2 = 0.831) \]  

(14)

\[ C_v = -0.926 - 2.046 e_L - 0.879 e_L^2 - 0.073 e_L^3 \]  

(15)
The relationship of $e_L$ and the intrinsic constants predicted from this model are illustrated in Figure 3. The values of intrinsic constants increase continuously when $e_L$ increases in this model, which is in good agreement with the observed data and the results of past research (Burland, 1990; Hong et al., 2010).

5.2. Model 2
In Model 2, the effect of initial state has been applied to the model by adding the initial void ratio into the predicting relationships. The following equations estimate the intrinsic parameters of a reconstituted clay when both initial state ($e_0$) and $e_L$ are accessible:

$$e_{100}^* = 0.813 + 0.182e_0 - 0.112e_L - 0.001e_0^2 + 0.075e_L^2, \quad (R^2 = 0.987)$$  \hspace{1cm} (16)

$$e_{1000}^* = 0.777 + 0.101e_0 - 0.166e_L - 0.003e_0^2 + 0.033e_L^2, \quad (R^2 = 0.877)$$  \hspace{1cm} (17)

$$C_c^* = 0.036 + 0.081e_0 + 0.054e_L + 0.002e_0^2 + 0.042e_L^2$$  \hspace{1cm} (18)

5.3. Model 3
In the third model, which is the most sophisticated model to predict intrinsic constants, in addition to initial state parameters, $e_0$ and $e_L$, the clay mineralogy is also included. Clay mineralogy of the investigated soil was assumed to be represented by its most predominant clay mineral. Clay was implied into the regression analysis by a dummy parameter named CLM. Intrinsic equations and the values of dummy parameters of CLM\textsubscript{1} to CLM\textsubscript{3}, depending on the type of principal clay mineral, are presented as follows:

$$e_{100} = CLM_1 + 0.2e_0 - 0.208e_L - 0.002e_0^2 + 0.081e_L^2, \quad (R^2 = 0.990)$$  \hspace{1cm} (19)

$$e_{1000} = CLM_2 + 0.111e_0 - 0.209e_L - 0.003e_0^2 + 0.035e_L^2, \quad (R^2 = 0.896)$$  \hspace{1cm} (20)

$$C_c = CLM_3 + 0.081e_0 + 0.054e_L + 0.002e_0^2 + 0.042e_L^2$$  \hspace{1cm} (21)
It is much easier to use the Wilkinson notation to express the regression equations (Wilkinson & Rogers, 1973), as it is a very powerful notation in presenting a complex regression model. The notation is used to present a model in terms of a response and predictors in the form of a simple equation with a table of predictor coefficients. In this notation, ‘1’ stands for constants and each predictor is separated with a symbol of “+”. If there is a powered predictor in the equation, it means that all lower order terms of the predictor are included in the equation, unless otherwise noted. Thus, substituting Equations (19)–(21) yields

$$e_{100} = 1 + e_0 + e_L + e_0^2 + e_L^2$$

where CLM is the categorical invariant of clay mineralogy and can be expressed in this model by two dummy variables, CLMK and CLMM. The coefficients of predictors ($e_0$, $e_L$, $e_0^2$, $e_L^2$, and CLM) for this relationship are tabulated in Table 2.

### 5.4. Model 4

Model 4 is a simplified version of Model 3. The model still considers the clay mineralogy, initial state, and properties of the studied soil, yet it retains its simplicity. Despite the fact the model is simple, it still has a good accuracy in predicting the intrinsic parameters of $e_{100}$ and $e_{1000}$. The correlation coefficients of $R^2$ are 0.99 and 0.85 for $e_{100}$ and $e_{1000}$, respectively,

$$e_{100} = 1 + e_0 + e_L + e_0^2 + e_L^2. \quad (R^2 = 0.990)$$

$$e_{1000} = 1 + e_L + e_L : CLM. \quad (R^2 = 0.840)$$

Wilkinson notation has been used to express Equations (23) and (24). A symbol of “:” means only the product of two predictors without the lower order terms of the predictor. The coefficient of each predictor and dummy variables are presented in Table 3.

### Table 2. Predictor coefficients of Model 3

| Coefficients | $e_{100}$ | $e_{1000}$ | $C_L^2$ |
|--------------|-----------|------------|--------|
| Constant     | 0.887     | 0.796      | 0.092  |
| $e_0$        | 0.200     | 0.111      | 0.088  |
| $e_L$        | -0.208    | -0.209     | 0.001  |
| CLMK         | 0.069     | 0.066      | 0.003  |
| CLMM         | 0.207     | 0.128      | 0.080  |
| $e_0^2$      | -0.002    | -0.003     | 0.002  |
| $e_L^2$      | 0.081     | 0.035      | 0.046  |

$CLMK$ and $CLMM$ are 1.0 when the principal clay of the studied soil is Kaolinite and Smectite, respectively, otherwise they are zero.


6. Comparison and discussion

A comparison of the ability of the proposed models to predict intrinsic constants has been presented in this section. Figure 4 demonstrates the values of predicted $\varepsilon_{100}^i$ and $C_i$ against the related measured values obtained from the experimental data. It can be seen that the estimated values of $\varepsilon_{100}^i$ and $C_i$ are in good agreement with the observed values. Moreover, Model 4 can estimate the parameters accurately even though it has relatively lesser order of terms of predictors than Model 3. A summary of the results is presented in Table 4. As can be noted, the accuracy of the models increases when the clay mineralogy is considered as an input parameter (Models 3 and 4).

The dataset reported by Habibbeygi et al. (2017) were used to compare the different empirical equations existing in the literature. The $\varepsilon_{100}^i$ and $C_i$ values of Baldivis clay, estimated for different conditions of initial water content, are plotted in Figure 5. The geotechnical and physical parameters for the studied clay are presented in Table 1. According to Burland’s equation, intrinsic constants are independent of the initial state (for example, $e_0$ or $w_0$). Zeng et al. (2015) modified Burland’s equation to consider the effect of the initial state. The values estimated by equations suggested by Zeng et al. (2015) increase with an increase in the value of $w_0$, in a trend similar to the observed values. However, both these equations underestimate the intrinsic constants of the investigated clays with a considerable amount of smectite in its mineralogy. On the other hand, as depicted in Figure 5, the estimated values of intrinsic constants using the proposed model of this study are higher than the predicted values from other methods, and are close to the observed values. Thus, the predicted intrinsic constants from the proposed model are in good agreement with the measured constants for the investigated clay.

The effect of each predictor on the response of the proposed model has been examined for Model 4. Figure 6 presents the influence of change in each predictor on $\varepsilon_{100}^i$. As can be seen from Figure 6(a), the change in $e_0$ has the greatest effect on $\varepsilon_{100}^i$ among other predictors. It shows that changing $e_0$ from 0.63 to about 14 increases $\varepsilon_{100}^i$ by about 3. Moreover, it also shows that changing $e_1$ from 1.11 to 7 raises $\varepsilon_{100}^i$ by about 2. Figure 6(b) shows the interaction plot for observing the effect of changing one predictor while keeping others fixed. For example, the increase in $e_1$ (from 1.11 to 7.0) is much more effective on the growth of $\varepsilon_{100}^i$ when the principal mineral of clay is smectite, with an increase of 3, whereas the increase of $\varepsilon_{100}^i$ is just one (1) when the principal clay mineral is illite. Interaction graphs for $e_0$ and $e_1$ are plotted in Figure 6(c,d) for comparison. The increase in both $e_0$ and $e_1$ lead to an increase in $\varepsilon_{100}^i$, but with different rates for different clay minerals. For example, the rate of increase of $\varepsilon_{100}^i$ with $e_0$ for smectite is the lowest. On the other hand, the rate of increase of $\varepsilon_{100}^i$ with the increase of $e_1$ is the highest in smectite. In Model 4, $\varepsilon_{100}^i$ is only a function of $e_1$ and its mineralogy. The effects of the variation of these predictors are plotted in Figure 7. While the increase in $e_1$ leads to an increase in $\varepsilon_{1000}^i$, the variation is the least for illite, with a growth of about 0.8.

The non-linear compressibility can be explained by the rheological behaviour. Rheological behaviour accounts for the secondary consolidation of clays. The rheological behaviour exists in the primary consolidation and cannot be neglected.

### Table 3. Predictor coefficients of Model 4

| Coefficients | $e_{100}^i$ | Coefficients | $e_{1000}^i$ |
|--------------|-------------|--------------|--------------|
| Constants    | 0.696       | Constants    | 0.502        |
| $e_0$        | 0.157       | $e_0$        | 0.147        |
| $e_0 : CLM_k$| 0.139       | $e_0 : CLM_k$| 0.092        |
| $e_0 : CLM_{MM}$| −0.096   | $e_0 : CLM_{MM}$| 0.059        |
| $e_1 : CLM_k$| −0.110      | $e_1 : CLM_{MM}$| 0.215        |
| $e_0^2$      | 0.005       |              |              |
| $e_1^2$      | 0.040       |              |              |
Figure 4. Comparison of predicted intrinsic constants and measured ones: (a) $\varepsilon_{100}$ (b) $C$.
Table 4. $R^2$ of the proposed models for intrinsic parameters ($e_{100}$ and $e_{1000}$)

| Model                | $R^2$ for $e_{100}$ | $R^2$ for $e_{1000}$ | Input parameters                |
|----------------------|---------------------|----------------------|---------------------------------|
| Model 1              | 0.978               | 0.831                | $e_L$                           |
| Model 2              | 0.987               | 0.877                | $e_0$ and $e_L$                 |
| Model 3              | 0.990               | 0.896                | $e_0$, $e_L$, and CLM           |
| Model 4 (simplified) | 0.990               | 0.840                | $e_0$, $e_L$, and CLM           |

Figure 5. Comparison of predicted intrinsic constants by different methods for Baldivis clay: (a) $e_{100}$ (b) $C'_c$. 

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7. Virgin compression line estimation

An advantage of using the void index for normalisation is that it allows estimation of compression curves without performing consolidation tests. In other words, the virgin compression line (VCL) of a reconstituted clay can be determined by substituting the modified equation of ICL (Equation 12) into the void index definition equation (Equation 2) as follows:

\[
\varepsilon = C_c \left( 1.906 - 0.392x - 0.457x^2 + 0.088x^3 \right) + \varepsilon_{100} \quad \text{For } \sigma'_v \geq \sigma'_{yr}
\]  

(25)

where \(x = \log \sigma'_v\); \(\sigma'_v\) is in kPa, and \(\sigma'_{yr}\) is the remoulded yield stress of a reconstituted clay. This equation can be used for a broad range of \(e_L\) (1.1–7.0) and the initial state of \(\varepsilon_0\) (0.63 to 14.38).

Depending on the availability of information about the studied soil, intrinsic constants of \(\varepsilon_{100}\) and \(\varepsilon_{1000}\) can be calculated from any of the four models—Models 1 to 4. In cases where the three main physical parameters of clay mineralogy, the initial void ratio and the void ratio at liquid limit are available, Equations (19) and (21) can be used. Otherwise, if clay mineralogy is not known with any degree of accuracy, either groups of equations—Equations (13)–(15) or Equations (16)–(18)—can be used for two different conditions of accessibility to the initial state. Moreover, the void ratio at liquid limit can always be computed by knowing the specific gravity of the investigated soil and the liquid limit. Therefore, the VCL of a reconstituted clay can be computed indirectly, without performing time-consuming laboratory 1D consolidation tests, by knowing simple physical geological parameters.
To sum up, the steps are as follows:

- First determine the initial state, including clay mineralogy and the liquid limit of the studied soil;
- Secondly, the intrinsic parameters should be estimated by using one of the four proposed models—Models 1 to 4—based on the degree of understanding of the initial state; and
- Finally, the VCL can be determined by Equation (25).
The ICL calculated by this method has been plotted for Baldivis clay in Figure 8 for consolidation stress greater than 100 kPa (i.e. stress range higher than the remoulded yield stress of Baldivis clay).

Figure 8 identifies that the refined equation of the void index can satisfactorily predict the void index of the studied reconstituted clay for stresses higher than the remoulded yield stress. Predicted values of the void ratio of Baldivis clay using this method are illustrated in Figure 9 for models 3 and 4. As can be seen, there is good agreement between the measured values and predicted values for both models. The estimated void ratios are in the range of ± 10% of the observed value for the studied soil.

8. Conclusions
Based on the intrinsic concept, the compression behaviour of a reconstituted clay can be estimated if the intrinsic constants are known. Four regression models are proposed for estimating the intrinsic constants based on the degree of available data for the studied clay. The proposed models use the maximum advantage of the available data so that when enough information is obtainable, accuracy increases significantly. The results show that the clay mineralogy of a reconstituted clay has a considerable impact on the values of intrinsic constants. The effect of each predictor on the response of the suggested model is also investigated in this paper. The results show that, however, $\varepsilon_1^{100}$ increases with an increase in $\varepsilon_1$ for all types of clay minerals, but the increase is greatest in clays with smectite in their mineralogy. On the other hand, it seems that $\varepsilon_0$ has a lesser effect on $\varepsilon_1^{100}$ for a clay with smectite as its principal mineral as compared to other clay minerals. The refined equation of the void index is also used to predict the compression behaviour of...
a reconstituted clay from Baldivis, Western Australia. The results show that the predicted void ratios are in good agreement with measured values from consolidation tests for a wide range of initial water content. It is recommended that further investigation on the natural clays be carried out through a laboratory testing programme targeted to study the accuracy of the proposed

Figure 9. Comparison of estimated and observed void ratios for Baldivis clay (a) Model 3 and (b) Model 4.
models for a broader range of initial conditions and the clay mineralogy. Remote sensing can be used as a more accurate method for identifying the soil characteristics and the models can accordingly be modified based on the test results.

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Author details
Farzad Habibbeygi
E-mail: farzad.habibbeygi@postgrad.curtin.edu.au
ORCID ID: http://orcid.org/0000-0002-5231-3397
Hamid Nikraz
E-mail: H.Nikraz@curtin.edu.au
Bill K Koul
E-mail: bkk@4dg.com.au

1 Department of Civil Engineering, Science and Engineering, Curtin University, Perth, Australia.
2 4DGeotechnics Pty Ltd, West Leederville, Perth, WA, Australia.

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