Piezoelectric energy harvesting from a curved plate subjected to time-dependent loads using finite elements

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Abstract. The present paper focuses on the energy harvesting based piezoelectric material from curved plate (CP). Thus, a FEM Model of CP is equipped with piezoelectric patches act as harvesters and connected in parallel with load resistor. The harvester’s positions have been obtained using new technique based on modal analysis maximum voltage output. The idea is to distribute patches all over the plate and then performing modal analysis. The harvesters position are then chosen based on the maximum output voltage extracted from the results. This new approach not just save time by avoiding iterative optimizations but also helps to use the needed harvester according to the applied load and its frequency. The methodology are check in time and frequency domain.

1. Introduction

The present paper not just is the first contribution focusing on piezoelectric harvesting energy from curved plates but also propose new and easy method for choosing the optimal harvester positions. Modal analysis is performed to choose the optimal position of energy harvester. Energy harvesting from smart PZT sensors/actuators have been explored by investigators in the recent years as a substitute to conventional power sources. Sufficient amount of energy remains available in the form of vibration energy which can be derived from piezolaminated composites. This paper presents a energy-harvesting techniques derived from a curved plate with the use of piezoelectric patches. The piezoelectricity phenomenon is mostly used to transform mechanical energy to electricity, because of its direct and converse piezeoelectric effect, easy to implement. From the implementation point of view, we have developed model of PZT energy harvester to create adequate power under a dynamic excitation. Various designs and techniques of optimization for harvesting materials are studied and reviewed in literature. These piezoelectric transducers have intrinsic electromechanical coupling and much power density in comparison to that of electrostatic and electromagnetic transducers now a day’s widely employed to derive power from such sources of vibration energy.

Along the years, different solution techniques which derives ample amount of energy from sensors have emerged. From these, ambient renewable energy sources such as wind, sun and water have gain a quick growth in the last decade. Other available energy sources like ambient vibrations became a subject of vigorous research also. Indeed, structures such as beams and plates are subjected to loads of different nature that vibrates them. This energy can be harvested and converted into needful electrical power. The available main known mechanisms among the physical viz. electromagnetic, electrostatic
and piezoelectric are generally used to transform such released ambient energy into electrical power. The alternative in terms of PZT has an intrinsic preference over the two other mechanisms not only for its huge power densities but also for its ease of implementation. Cottone et al [1] emphasized on nonlinear energy harvesting and gave some information. Stanton et al [2] performed dynamic nonlinear analysis for broadband energy harvesting with its investigation of a piezoelectric inertial generator. Adhikari et al [3] derived energy from broadband random vibrations using piezoelectric energy harvester. Daqaq [4] studied response of harvesters comprised of uni-modal duffing-type undergone to random excitations due to forces. Wankhade [5] emphasized on geometric nonlinear analysis. Litak et al [6] used magneto-piezoelastic energy harvester driven by arbitrary excitations. Bajoria and Wankhade [7,8] performed free vibration of SSSS piezolaminated composites by the employment of FEM. Deshpande and Wankhade [9] provided solution for thick beams. Wankhade and Bajoria [10-11, 15, 18-19] worked on vibration and Stability of smart piezolaminated composite plates using higher order FEM. Wankhade et al [12-14] focused on buckling of composites with different parameters. Bendine et al [16-17] employed higher order theory for active vibration control of FG beams with piezoelectric substrates. Bendine and Wankhade[20] developed vibration control equations for FGM piezoelastic plate based on LQR genetic search. Bendine et al [21] used structural modelling for active vibration attenuation of smart FGM composites using ANSYS. Daraji et al [22] worked on doubly curved shells. Wankhade et al [23 and 26] put forth dynamics/buckling characteristic of laminated as well as piezolaminated composites. Bendine et al [24 and 25] proposed some part of energy harvesting using PZT. In this work the energy harvesting based on piezoelectric material from curved plate (CP) is developed. A FEM Model of CP is equipped with piezoelectric patches act as harvesters and connected in parallel with load resistor are shown with its response.

2. Problem Formulation and Modelling

2.1 Electro-Mechanical Coupling

Due to the direct and converse piezoelectric effect an electro-mechanical coupling between electrical and mechanical loading can be expressed by the virtue of following equations.

\[
\{D\} = \varepsilon \{\varepsilon\} + \sigma \{E^p\} \\
\{\sigma\} = \mathcal{C} \{\varepsilon\} - \varepsilon \{E^p\}
\]

(1)

In the above set of equations, \{D\} represents the electric displacement vector, \{\varepsilon\} shows dielectric permittivity matrix, \{\varepsilon\} is the strain vector, \{E^p\} is the dielectric matrix. \{\sigma\} is the stress vector and \mathcal{C} is the elastic matrix for constant electric field.

\[
[M]\{\dot{U}\} + [C_d]\{\ddot{U}\} + [K]\{U\} = \{F\}
\]

(2)

where \{U\} represents the structural displacement vector, and[M], [K], [C_d] and \{F\} represents the mass, the elastic stiffness, the damping matrices and the vector of external forces, respectively
2.2 Piezoelectric Harvester Modelling

\[
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
D_3
\end{pmatrix} = \begin{pmatrix}
\tilde{C}_{11} & \tilde{C}_{12} & 0 & e_{31} \\
\tilde{C}_{21} & \tilde{C}_{22} & 0 & e_{32} \\
0 & 0 & \tilde{C}_{66} & 0 \\
e_{31} & e_{32} & 0 & e_{33}
\end{pmatrix} \begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
E_3
\end{pmatrix}
\]  

(3)

In above \( \varepsilon_i \), \( \sigma_i \) represents the strain and stress tensors and \( \tilde{C}_{ij} \), \( e_{ij} \) are the elastic, the piezoelectric coupling and the dielectric permittivity constants, respectively. It shall be noted that we have neglected electric field along the x and y directions (i.e. \( E_x = E_y = 0 \)) and thus only D3 is taken into account. Now, the vector of electric field ‘E’ constant along the thickness \( t_p \) can further be given as:

\[
E = \begin{pmatrix}
0 \\
0 \\
-1/t_p
\end{pmatrix}^T \quad \text{and} \quad V = B_p u_p
\]

(4)

For which \( V \) is the applied voltage. The GE equations for the FE electromechanical laminate can be extracted using Hamilton’s principle, which is now written as below:

\[
\int_{-t}^{t} \left[ (k - \psi + We + W) \right] dt = 0
\]

(5)

In which, \( \kappa \) is the kinetic energy, \( \psi \) represents the strain energy, \( We \) gives the dielectric energy, and \( W \) shows the work of the applied loads. The Hamilton principle can also be reconstructed.

3. Problem description

The problem under consideration consists of a fully clamped curved plate aluminium made equipped with piezoelectric patches (CPP). The CPP is with dimension of 500×500×3 mm and an inner radius of 1500 mm. The searching CPP model has been decomposed into one hundred sub-areas each representing the piezoelectric patches possible locations with principal role to harvest the wasted vibration energy. All the harvesters used are with 1mm thickness as shown in Figure 2. Table 1 shows the properties of the piezoelectric harvesters used in the present study.

| Parameters | Value          |
|------------|----------------|
| C11, C22 (GPa) | 120.3          |
| C12 (GPa)     | 75.2           |
| C13, C23 (GPa) | 75.1           |
| C33 (GPa)     | 110.9          |
| C66 (GPa)     | 22.7           |
| Resistive value, R(Ω) | 1e6            |
| \( \alpha \) (rad/s) | 4.886          |
| \( \beta \) (s/rad)   | 1.2433 × 10^{-5} |
| Parameters                                      | Value |
|------------------------------------------------|-------|
| Young’s Modulus of the PZT (GPa)              | 66    |
| Mass density of the PZT (Kg/m$^3$)           | 7800  |
| Piezoelectric constant $e_{31}$, $e_{12}$ (C/m$^2$) | -5.2  |
| Piezoelectric constant $e_{33}$ (C/m$^2$)    | 15.9  |
| Permittivity $\varepsilon_{33}$ (nF/m)       | 15    |

3.1 Harvester’s locations decision

In order to choose the suitable positions for the harvesters, the CP was equipped with 25 piezoelectric patches regularly distributed. Then, a modal analysis has been performed. The three first mode with the corresponding potential output are presented in figure 1. The figure reveals clearly the distribution of the potential for each patch ranging from the best performing one (maximum) to the worst (Minimum). Thus, and based on, the best position were choosing for the harvesters. It is worth to noted here that this method presents high accuracy with low time consuming.

![Figure 1](image-url). FEM model of the proposed curved plate bounded with six piezoelectric harvesters coupling with a $10^6 \, \Omega$ resistor.
4. Results and discussions

The present section is used to check the results obtained by the previous section. Consequently, a CPP is equipped with six harvesters optimally positioned according to the previous Model based optimisation. The model with the harvesters is shown in figure 2.

![Figure 2](image)

Figure 2. FEM model of the proposed curved plate bounded with six piezoelectric harvesters coupling with a $10^5 \, \Omega$ resistor.

The model is then, subject to harmonic time and frequency based. First a harmonic analysis is performed in the range of $[0, 280]$ Hz to evaluate the voltage output for four different modes. In the present work the force has been on the model all nodes as a unit value equal 1. Figure 3 presents the corresponding results and as it can be noticed the maximum harvester voltage with the value of nearly 2V has been reached when the applied load frequency matches the model second natural frequency, this can be justified by the fact that the second mode has common harvesters with the other modes, consequently more harvesters are involved in this case.

![Figure 3](image)

Figure 3. Voltage output in frequency domain.
The next simulation is performed in time domain with holding the same conditions from the previous analysis. The applied load is taken as the following expression:

\[
\begin{align*}
F &= 0 & 0 < t < 0.3 \\
F &= A \sin(\omega_1 t) & 0.3 < t < 1 \\
F &= 0 & 1 < t < 1.3 \\
F &= A \sin(\omega_2 t) & 1.3 < t < 2 \\
F &= 0 & 2 < t < 2.3 \\
F &= A \sin(\omega_3 t) & 2.3 < t < 3
\end{align*}
\]

where \(A\) is the force amplitude, \(\omega_1, \omega_2, \omega_3\) are the first three natural frequencies of the model, \(t\) is the time step.

The loading and unloading have been divided into ranges to allow better distinguishing between the applied forces and the corresponding voltage output results. Three frequency has been investigated and the results are depicted in figure 4. It is noticed from the figure that more voltage can be harvested when the force taking the second frequency and that can be justify similarly as the previous analysis.

![Figure 4. Voltage output in Time domain.](image)

5. Conclusion

Hence the energy harvesting based piezoelectric material from curved plate (CP) is discussed in the present work. A FEM Model of CP is equipped with piezoelectric patches act as harvester which is connected in parallel with load resistor. To obtain the harvesters positions new technique based on modal analysis maximum voltage output is adopted. Firstly piezoelectric patches are distributed all over the plate and modal analysis is performed. The harvester’s position is then obtained based on the maximum output voltage received from the results. This new approach is effective in saving time by avoiding iterative optimizations. Also it helps to use the needed harvester according to the applied load and its frequency. The methodology is verified in time and frequency domain.
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