Quantitative analysis for affecting factors of firing dispersion of tank

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Abstract. To improve the firing dispersion of the tank, the influences of random factors on firing dispersion are analyzed quantitatively. Based on the six-degree-of-freedom rigid-body ballistic equation, considering the influences of processing and assembly errors, initial disturbances and the wind on flight, using the Sobol’ global sensitivity analysis method, the weights of affecting factor are obtained. The analysis results show that, under the current manufacturing and measuring conditions, the initial disturbance of projectile is the premier factor that takes effect on the firing dispersion of the straight shooting weapon (such as a tank) with small elevation angle.

1. Indroduction
The tank is the main assault force of the armored mechanized units in the modern ground warfare. The advanced main battle tank has become a symbol of military strength and conventional deterrence capabilities of the country [1]. As a vast country with vast land, neighboring countries and territorial disputes with many of them, the role of tanks is even more prominent. Firing precision is an important technical indicator of the tank, including firing accuracy and firing dispersion. Tank firing dispersion is the basis of firing accuracy [2]. Therefore, the current research on tank firing precision is mainly focused on solving the problem of firing dispersion.

The traditional firing dispersion error analysis is based on the assumption that each error term is independent [3]. The research shows that there are coupling effects between the factors affecting the fire dispersion and the error terms caused by each factor are not independent. In this sense, the traditional fire dispersion error analysis method is insufficient. Sobol' global sensitivity analysis method [4-7] is a Monte Carlo method based on variance analysis. Compared with the traditional fire dispersion error analysis method, the core ideas of the two methods are based on variance decomposition. The definitions of indicators both are based on the ratio of variance. The difference is that this method can evaluate the sensitivity of interactions of single factor and multiple factor to the indexes, and it is very suitable for strongly nonlinear and non-monotonic systems. In recent years, Sobol's method has been widely used in many fields.
This paper comprehensively considers the processing and assembly errors such as projectile mass eccentricity, dynamic unbalance, and weight deviation, initial disturbance such as declination angle, angle of attack and angular velocity of attack, and the influence of random factors such as random wind on the flight process of the projectile. Based on the exterior ballistic equation [8] considering the factors such as initial disturbance and random wind, using Sobol's global sensitivity analysis method, the sensitivity of each factor to the impact point can be obtained. Using the idea of the references [9,10], the sensitivity obtained by each factor is extended to the influence weight of fire dispersion of standing target. The intensity of the influence of the intensity. The research results provide an important basis for improving the firing dispersion of tank by controlling the initial disturbance.

2. Main factors affecting tank fire dispersion

Projectile launching and flight processes are affected by a variety of random factors. From the perspective of exterior ballistics, the main factors affecting the fire dispersion of the projectile can be divided into three categories: processing assembly errors, initial disturbance and meteorological conditions. The influence mechanism of these random factors on the firing dispersion is complicated, and there are also interactions between the factors.

2.1. Processing assembly error

Due to the existence of various errors in the crafting process system, such as the geometric errors of the process system, positioning errors, the force of the process system and the machining errors caused by the thermal deformation, adjustment errors and measurement errors, the projectile will inevitably form errors during the processing and assembly process. According to the actual measurement results and historical data statistics, the probability distribution of the projectile characteristic parameters is a normal distribution. The statistical characteristics of the characteristic parameters of a batch of projectiles are shown in table 1. The mass eccentricity and dynamic unbalance angles in table 1 are the components of the mass eccentricity, \( L_{m1} \) and \( L_{m2} \), and the dynamic unbalance angle, \( \beta_{D1} \) and \( \beta_{D2} \), in the two directions of the projectile coordinate system [11].

| Parameter name                  | Standard Deviation |
|---------------------------------|--------------------|
| Mass eccentricity \( L_{m1} \)/mm | 0.01               |
| Mass eccentricity \( L_{m2} \)/mm | 0.01               |
| Dynamic imbalance angle \( \beta_{D1} \)/mrad | 0.01               |
| Dynamic imbalance angle \( \beta_{D2} \)/mrad | 0.01               |
| Projectile quality \( m \)/kg   | 1.7×10^{-3}        |

2.2. Initial disturbance

The initial disturbance of the projectile is the angle of attack \( \delta \) and the speed/(rate) of angle of attack \( \dot{\delta} \) and the off-angle \( \psi \) of the projectile at the end of the launching process (end point of the post-effect period). They are used as the starting conditions for the free flight phase of the projectile, and are the results of the combined effects of factors such as of projectiles, guns, medicines and environment during the launching process. Since the factors that cause the initial disturbance are divided into random factors and non-random factors, so the initial disturbances are also random and non-random. Studying the initial perturbation caused by non-random factors is to provide a basis for the ballistic correction theory to improve the shooting hit rate; to study the initial disturbance generated by random factors, is to control the projectile scattering [12]. In view of the characteristics
of the completion of the fire process, in the process of tracking a fixed or moving target during the modern tank battle, the references [9,11,13] based on the new multi-body system transfer matrix method, deducing the launch dynamics equation of the tank. Using the Monte Carlo stochastic simulation technology, the random launch dynamics simulation system for tanks with inter-vehicle firing was compiled, and the dynamic response of the tank, the internal ballistics, the force, the initial disturbance of the projectile, the exterior ballistics, and the vertical target density were obtained. The simulation results were verified by experiments. The initial disturbance of the projectile obtained by the simulation system approximates the normal distribution.

Table 2 gives the simulation results of the statistical characteristics of the initial disturbance of a tank shooting at 0°/5 mil direction angle/shot angle.

| Parameter name                  | Mean   | Standard Deviation |
|---------------------------------|--------|--------------------|
| Vertical deviation angle $\psi_1$/mrad | -0.963 | 0.304              |
| Lateral declination angle $\psi_2$/mrad | -0.022 | 0.297              |
| Vertical angle of attack $\delta_1$/mrad | -0.083 | 0.581              |
| Lateral angle of attack $\delta_2$/mrad | 0.039  | 0.555              |
| vertical angular velocity of attack $\dot{\delta}_1$/(rad/s) | -0.249 | 0.803              |
| Lateral angular velocity of attack $\dot{\delta}_2$/(rad/s) | 0.069  | 0.819              |

2.3. Meteorological conditions
Meteorological parameters mainly include atmospheric sound velocity, atmospheric density and winds. This paper studies the influence of various factors on the fire intensity of tank. The maximum fire height of the projectile is only a few meters. The flight time of each projectile is less than 1 second. Therefore, the temperature, sound velocity and density of the atmosphere can be neglected with height. Only the wind effect is considered. The effect of the fire dispersion, assuming that the wind speed follows a normal distribution, and the standard Deviation of the wind speed is 2 m/s.

For unlisted factors, such as atmospheric temperature, sound velocity and density, the displacement, velocity, rotation angle and angular velocity of the projectile when the projectile exits the muzzle are constant in the calculation.

3. Exterior ballistics equation
In order to consider the effects of projectile dynamic imbalance, mass eccentricity, winds and other factors on the flight process of the projectile, the 6D rigid-body ballistic equation [8] is established as follows:
\[
\frac{dx}{dt} = v_x, \quad \frac{dy}{dt} = v_y, \quad \frac{dz}{dt} = v_z, \quad \frac{d\varphi_x}{dt} = \dot{\varphi}_x, \quad \frac{d\varphi_y}{dt} = \dot{\varphi}_y, \quad \frac{d\varphi_z}{dt} = \dot{\varphi}_z
\]
\[
\frac{dv_x}{dt} = -\frac{R_z}{m} v_z - \frac{R_y}{m \sin \delta_r} (\sin \delta_v \cos \delta_r \sin \theta_r + \sin \delta_v \sin \psi_r \cos \theta_r) + \frac{R_z}{m \sin \delta_r} (\sin \psi_r \cos \theta_r \cos \delta_r - \sin \theta_r \sin \delta_r)
\]
\[
\frac{dv_y}{dt} = -\frac{R_z}{m} v_z + \frac{R_y}{m \sin \delta_r} (\sin \delta_v \cos \delta_r \cos \theta_r - \sin \delta_v \sin \psi_r \sin \theta_r)
\]
\[
\frac{dv_z}{dt} = -\frac{R_z}{m} v_z - \frac{R_y}{m \sin \delta_r} \sin \delta_r \cos \psi_r - \frac{R_z}{m \sin \delta_r} \cos \psi_r \sin \delta_r
\]
\[
\frac{d\dot{\gamma}}{dt} = -\dot{\varphi}_z \sin \varphi_v - \dot{\varphi}_y \cos \varphi_z \cos \varphi_v - k_{\alpha_d} (\dot{\gamma} + \dot{\varphi}_z \sin \varphi_v) v_x
\]
\[
\frac{d\dot{\varphi}_z}{dt} = \frac{M_x (\cos \delta_z \sin \alpha_v + \sin \delta_v \cos \alpha_v) - M_y (\cos \delta_v \sin \alpha_v \cos \theta_r - \sin \delta_v \cos \alpha_v)}{A \cos \varphi_z \sin \delta_r} + \frac{(2A-C)\dot{\varphi}_z \sin \varphi_v - C \dot{\varphi}_z \cos \varphi_v - k_{\alpha_d} \dot{\varphi}_z \cos \varphi_v + \frac{A-C}{A} \dot{\gamma} \beta_{D_z} + \frac{mL_m}{A} \dot{\gamma}}{A}
\]
\[
\frac{d\dot{\varphi}_y}{dt} = \frac{M_x (\cos \delta_v \sin \alpha_v - \sin \delta_v \sin \alpha_v) + M_y (\cos \delta_v \sin \alpha_v \cos \theta_r + \sin \delta_v \cos \alpha_v)}{A \sin \delta_r} - \frac{(A-C)\dot{\varphi}_y \cos \varphi_v \sin \varphi_v + C \dot{\varphi}_y \cos \varphi_v \cos \varphi_v - k_{\alpha_d} \dot{\phi}_y \cos \varphi_v + \frac{A-C}{A} \dot{\gamma} \beta_{D_y} + \frac{mL_m}{A} \dot{\gamma}}{A}
\]

Where, \( \psi_r, \theta_r \) are the angles relative to the atmospheric wind, \( w_x \) is the range wind velocity, \( w_z \) is the crosswind velocity, \( \dot{v}_r \) is the relative acceleration; \( R_x \) is the oncoming resistance force, \( R_y \) is the lift force, \( R_z \) is the Magnus force, \( \psi_r \) is the trajectory inclination angle, \( \theta_r \) is the relative ballistic inclination, \( \delta_r \) is the projection of the relative angle of attack on the vertical plane, \( \delta_z \) is the projection of the relative angle of attack on the side plane, \( \delta_r \) is the relative angle of attack, \( \dot{\psi}_r \) is the relative angular velocity of the declination angle, \( \dot{\theta}_r \) is the angular velocity of the ballistic inclination, \( \alpha_v \) is the relative aiming angle, \( k_{\alpha_d} \) is the Polar damping moment coefficient, \( k_{\alpha_d} \) is the equatorial polar damping moment coefficient, \( M_x \) is the Magnus torque, and \( M_z \) is the stable torque. The meaning of the unsigned symbols and the
calculation method of each parameter are shown in reference [8].

4. **Sobol' global sensitivity analysis principle**

The core idea of Sobol' sensitivity analysis method is variance decomposition, which decomposes the model into single parameters or a function of combining parameters. The importance of the parameters is analyzed by calculating the influence of the variance of the single input parameters or the input parameters set on the total output variances and the interaction between the parameters.

It is assumed that the domain of the model \( Y = f(x) \) is a unit \( I^k \) of dimension \( k \) which includes \( x = [x_1, x_2, x_3, \cdots, x_k]^T \), \( f(x) \) can be square integrable, and \( f(x) \) can be decomposed into functions of subdivision parameters and mutual combination of parameters, that is

\[
f(x) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum_{1 \leq i < j \leq k} f_{i,j}(x_i, x_j) + \cdots
\]

(2)

Where, \( f_0 \) is a constant, and the other items have zero integral for any variable they contain, that is

\[
\int f_{i_1,i_2,\cdots,i_s}(x_{i_1}, x_{i_2}, \cdots, x_{i_s}) \, dx_{i_j} = 0 \quad (1 \leq j \leq s)
\]

(3)

It can be seen from equations (2) and (3) that all addends in equation (2) are orthogonal, that is if

\[
(i_1, i_2, i_3, \cdots, i_s) \neq (j_1, j_2, j_3, \cdots, j_p)
\]

(4)

\[
\int f_{i_1,i_2,\cdots,i_s} \cdot f_{j_1,j_2,\cdots,j_p} \, dx = 0
\]

(5)

Thus, the equations of equation (2) are unique and can be obtained by the integration shown below [4]

\[
f_0 = \int f_0 \, dx
\]

(6)

\[
f_i(x_i) = \int f(x) \prod_{s \neq i} dx_s - f_0
\]

(7)

\[
f_{i,j}(x_i, x_j) = \int f(x) \prod_{s \neq i, j} dx_s - f_0 - f_i(x_i) - f_j(x_j)
\]

(8)

The formula \( f_{i_1,i_2,\cdots,i_s}(x_{i_1}, x_{i_2}, \cdots, x_{i_s}) \) can be calculated.

The variance of \( f(x) \) is \( D \).

\[
D = \int f^2(x) \, dx - f_0^2
\]

(9)

The variance of formula \( f_{i_1,i_2,\cdots,i_s}(x_{i_1}, x_{i_2}, \cdots, x_{i_s}) \) is \( D_{i_1,i_2,\cdots,i_s} \),

\[
D_{i_1,i_2,\cdots,i_s} = \int \cdots \int f^2_{i_1,i_2,\cdots,i_s}(x_{i_1}, x_{i_2}, \cdots, x_{i_s}) \, dx_{i_1} \cdots dx_{i_s}
\]

(10)

that is

\[
D = \sum_{i=1}^{k} D_i + \sum_{1 \leq i < j \leq k} D_{i,j} + \cdots + D_{1,2,3,\cdots,k}
\]

(11)

Define the ratio of variance
The random parameters sensitivities of parameter solution, the sample obtained is of i numerical output.

In the substituted factors influence s of the parameter

\[ S_{i_1, i_2, \ldots, i_k} = \frac{D_{i_1, i_2, \ldots, i_k}}{D} \quad (1 \leq i_1 < \cdots < i_k \leq k) \] (12)

To measure the sensitivity of the input parameters (independent variables), we can see from equation (10)

\[ \sum_{i=1}^{k} S_i + \sum_{1 \leq i < j \leq k} S_{i,j} + \cdots + S_{1,2,3,\ldots,k} = 1 \] (13)

Where, \( S_i \) is the “main effect” index of the parameter \( x_i \), or called first-order sensitivity, describes the contribution of the random factor \( x_i \) “individual” to the total variance of the output; \( S_{i_1, i_2, \ldots, i_k} \) is the \( s \) order sensitivity, which describes the contribution of the interaction of the random factors \( x_{i_1}, x_{i_2}, \ldots, x_{i_k} \) on the total variance of output.

The total sensitivity coefficient \( S_i^T \) is the sum of the sensitivity coefficients of each parameter \( x_i \), expressed as

\[ S_i^T = 1 - S_{-i} \] (14)

Where, \( S_{-i} \) is the influence of all factors except the influencing factors \( x_i \) on the output.

5. Quantitative analysis for affecting factors of firing dispersion of tank

5.1. Simulation of impact point sensitivity

The Latin hypercube sampling method [14] is used to sample the parameters in table 1 (statistical characteristics of projectile characteristic parameters) and table 2 (statistical characteristics of initial disturbances) and random wind according to their respective probability distributions; In the method [7], the sample is substituted into the exterior ballistic equation (1) to obtain a numerical solution, and the impact point is obtained, thereby obtaining the sensitivity of each parameter. The calculation results of the first-order sensitivity and global sensitivity of each random factor to the impact point \( (Y, Z) \) (see table 3). For the sake of simplicity, the data in table 3 is represented by the histogram shown in figure 1.

Table 3. The sensitivities of affecting factors relative to impact points.

| Random factor       | First-order sensitivity of the impact point \( Y \) | Global sensitivity of the impact point \( Y \) | First-order sensitivity of the impact point \( Z \) | Global sensitivity of the impact point \( Z \) |
|---------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Mass eccentricity   | 0.0065                                        | 0.0185                                        | 0.0084                                        | 0.0307                                        |
| \( L_{m_1} \)       |                                               |                                               |                                               |                                               |
| Mass eccentricity   | 0.0052                                        | 0.0176                                        | 0.0092                                        | 0.0354                                        |
| \( L_{m_2} \)       |                                               |                                               |                                               |                                               |
| Dynamic imbalance   | 0.0047                                        | 0.0130                                        | 0.0078                                        | 0.0272                                        |
| angle \( \beta_{D_1} \) |                                               |                                               |                                               |                                               |
| Dynamic imbalance   | 0.0038                                        | 0.0107                                        | 0.0094                                        | 0.0387                                        |
| angle \( \beta_{D_2} \) |                                               |                                               |                                               |                                               |
| Projectile mass \( m \) | 0.0011                                        | 0.0075                                        | 0.0013                                        | 0.0087                                        |
| Factor                                | Value 1 | Value 2 | Value 3 | Value 4 |
|---------------------------------------|---------|---------|---------|---------|
| Vertical declination angle $\psi_1$   | 0.8278  | 0.8341  | 0.0067  | 0.0105  |
| Lateral declination angle $\psi_2$    | 0.0081  | 0.0095  | 0.7815  | 0.8019  |
| Vertical angle of attack $\delta_1$   | 0.0365  | 0.0589  | 0.0014  | 0.0045  |
| Lateral angle of attack $\delta_2$    | 0.0029  | 0.0237  | 0.0332  | 0.0571  |
| Vertical angular velocity of attack $\dot{\delta}_1$ | 0.0762 | 0.0878  | 0.0016  | 0.0085  |
| Lateral angular velocity of attack $\dot{\delta}_2$ | 0.0034 | 0.0251  | 0.0811  | 0.1084  |
| Random wind $w$                       | 0.0044  | 0.0141  | 0.0301  | 0.0418  |

Figure 1. The sensitivities of affecting factors relative to impact points.

5.2. **Weights analysis for affecting factors of firing dispersion of tank**

The core ideas of Sobol' sensitivity analysis method and traditional fire dispersion error analysis method are based on variance decomposition. The definition of indicators is based on the ratio of variance. Considering the coupling between various factors affecting the fire dispersion, the error terms caused by each factor are not independent. In this paper, the sensitivity of each factor obtained by Sobol' sensitivity analysis method is used to extend the sensitivity of the impact point to the weight of the fire dispersion of standing target. The first-order sensitivity is the weight of the independent role of factor of fire dispersion of standing target, and second-order and other high-order sensitivities are the weight influences of the interaction between the corresponding factors on the target concentration. From table 3, the weights of the three types of random factors (projectile processing error, initial disturbance and meteorological conditions) on the fire dispersion of standing target of a tank can be obtained, as shown in figure 2. Combined with the simulation results in Section 4.1, the following conclusions can be obtained:
(1) It can be seen from figure 1 that there is a difference between the first-order sensitivity and the global sensitivity of each random factor, that is, there are interaction couplings between the factors; as can be seen from figure 2, the independent action of each factor is the main factor affecting the fire dispersion. The interaction weight between the various factors affects the fire dispersion by 2% to 3%.

(2) It can be seen from figure 2 that under the current firing conditions (angle/direction angle is 5 mil/0°), the initial disturbance of the projectile is the main influencing factor of height and direction fire dispersion; the influence weight of the initial disturbance accounts for 90%-95%, which is far greater than the processing errors and the influence weight of random winds on the fire dispersion; the processing error and the influence of random wind on the fire dispersion are very small, respectively 2%-4% and 1%-3%, the main reason is that the flight time of the projectile from the muzzle to the target is very short.

(3) It can be seen from table 3 that the main influencing factors of vertical fire dispersion are vertical declination angle $\psi_1$, the comprehensive weight reaches 0.8341, the vertical angle of attack is second, and the comprehensive weight is 0.0878; the main influencing factor of direction fire dispersion is lateral declination angle $\psi_2$. The combined weight is 0.8019, and the lateral angle of attack $\delta_2$ speed is second.

6. Conclusion

Based on the six-degree-of-freedom rigid-body ballistic equation, considering the effects of processing assembly error, initial disturbance and meteorological conditions on the flight process of the projectile, Sobol's global sensitivity analysis method is used to quantitatively analyze the weights of influence of random factors on the fire dispersion. The analysis results show that under the current processing assembly and measurement error level, for the tank with direct firing as the main firing mode, the initial disturbance of the projectile is the most important factor affecting the fire dispersion, and the initial declination angle is the main factor which affects fire dispersion. The work of this paper can provide support and reference for the measures to improve the fire dispersion of tank firing from the perspective of controlling the initial disturbance.

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