SCET, Light-Cone Gauge and the T-Wilson Lines

Miguel García-Echevarría,1,* Ahmad Idilbi,1,† and Ignazio Scimemi1,‡

1Departamento de Física Teórica II, Universidad Complutense de Madrid (UCM), 28040 Madrid, Spain

Soft-Collinear Effective Theory (SCET) has been formulated since a decade now in covariant gauges. In this work we derive a modified SCET Lagrangian applicable in both classes of gauges: regular and singular ones. This extends the range of applicability of SCET. The new Lagrangian must be used to obtain factorization theorems in cases where the transverse momenta of the particles in the final states are not integrated over, such as semi-inclusive deep inelastic scattering, Drell-Yan and the Higgs production cross-section at low transverse momentum. By doing so all non-perturbative matrix elements appearing in the factorized cross-sections are gauge invariant.

In recent years Soft-Collinear Effective Theory (SCET) [1, 2] has emerged as an important tool to describe jet-like events ranging from heavy quark hadronic decays to Large Hadron Collider (LHC) physics. The advantage of this effective theory of QCD is that it incorporates, at the Lagrangian level, all the kinetic symmetries of a particular jet-like event. The fields in SCET are either collinear, anti-collinear or soft (low energetic) depending whether they carry most of their energy along a light-like vector \( (n, n^2 = 0) \), an anti-light-like vector \( (\bar{n}, \bar{n}^2 = 0, \bar{n} \bar{n} = 2) \) or if the energy is soft and radiated isotropically. The current formulation of SCET is rather limited to a class of regular gauges. In this class of gauges the gauge boson fields vanish at infinity in coordinate space, thus no gauge transformation can be performed at that point. This limitation has a rather important implications as we discuss below. Moreover, using a singular gauge like the light-cone gauge (LCG), it is possible to improve the symmetry of the effective Lagrangian(s) because the gauge fixing conditions also respect the symmetry of the jet-like event.

In a previous work [3] it was argued that by extending the formulation of SCET to the class of singular gauges, a new Wilson line, the T-Wilson line, has to be invoked as a basic SCET building block. The T-Wilson line discussed in [3]—which is exactly 1 in covariant gauge—is built using LCG ghost field \( A_{n\perp}(x^+, \infty^-, x_{\perp}) \). This transverse Wilson line allows for a complete gauge invariant definitions of the non-perturbative matrix elements of collinear particles to be obtained from first principles, it allows to properly factorize high-energy processes with explicit transverse-momentum dependence and it reads \(^1\),

\[
T_n = \hat{P} \exp \left[ i g \int_{0}^{\infty} d\tau l_{\perp} \cdot A_{n\perp}(x^+, \infty^-, x_{\perp} - l_\perp \tau) \right],
\]

where \( \hat{P} \) stands for anti-path ordering. Below we discuss also the transverse Wilson lines built from soft gluons which allows for a gauge invariant definitions of soft matrix elements with transverse space separation.

In this work we show how to implement the T-Wilson line at the level of the soft-collinear Lagrangians. The results obtained (see below) are Lagrangians applicable in covariant gauges as well as in light-cone gauge (LCG) both in SCET-I and in SCET-II. This in turn will enable us to explicitly invoke the transverse-momentum dependence to the collinear quark and gluon jets. Those latter quantities form the fundamental blocks for the non-perturbative matrix elements like transverse-momentum dependent parton distribution functions (TMDPDFs), beam-functions (BFs) [4] and the like [5, 6].

We start the discussion with the LCG: \( \bar{n}A = A^+ = 0, \bar{n}\bar{n} = 0 \). QCD can be canonically quantized in this gauge [7] and the quantization fixes the Feynman rules for the gauge bosons with the Mandelstam-Leibbrandt (ML) prescription [8]. Passing from QCD to SCET however some new issues arise. For instance, in QCD one needs to specify one gauge fixing condition. In the effective theory however every light-cone (LC) direction defines a collinear gauge sector and it is not clear, beforehand, how the gauge fixing conditions respect the power counting of the different collinear sectors.

\( ^{\dagger} \) In this paper we have adapted our convention for Wilson lines to the one of Ref. [1]. This is consistent with the results of Ref. [3].
In the following we show how LCG can be implemented both in collinear and soft sectors of SCET. In particular we study in which cases LCG is compatible with power counting and “multipole expansion” in SCET-I and SCET-II. First (in sec. I) we illustrate the origin of the T-Wilson line from full QCD. Then we explicitly show how the T-Wilson line appears in the SCET Lagrangians. This result is the main result of this paper: For the (leading order) SCET Lagrangians and the basic building-blocks to be gauge invariant in covariant as well as in LCG, transverse gauge links have to be introduced in SCET-I and in SCET-II. In sec. II we discuss the applications of those results then we conclude.

I. THE QCD AND SCET LAGRANGIANS IN LC GAUGE: T-WILSON LINES

In this chapter we want to write the SCET matter Lagrangian in LCG outlining the role of ghost fields in LCG. First we recall some of the features of the gluon fields in QCD in LCG from Ref. [7]. To fix matters, we work in QCD with the gauge fixing condition \( nA = 0 \). The canonical quantization of the gluon field proceeds by inserting in the Lagrangian the gauge fixing term \( L_{gf} = \Lambda^a(nA^a) \). The \( \Lambda^a \) is a field whose value on the Hilbert space of physical states is equal to zero. It is possible to write the most general solution of the equation of motion of the boson field \( A^a_\mu \) via decomposing it into

\[
A^a_\mu(k) = T^a_\mu(k)\delta(k^2) + \tilde{n}_\mu \delta(\tilde{n}k)\Lambda^a(nk, k_\perp) + \frac{ik_\mu}{k_\perp} \delta(\tilde{n}k)U^a(nk, k_\perp),
\]

where the field \( T^a_\mu \) is such that \( \tilde{n}^a T^a_\mu(k) = 0 \) and \( k^\mu T^a_\mu(k) = 0 \). Fourier transforming this expression we see that in general the field \( A^a_\mu(x) \) has non-vanishing “-n” and “⊥” (respectively \( nA^a(x) \) and \( A^a_\mu(x) \)) components when \( x^- \to \pm \infty \). We define

\[
A^{(\infty)}(x^+, x_-) \overset{\text{def}}{=} A(x^+, \infty^-, x_-)
\]

\[
\tilde{A}(x^+, x^-, x_-) \overset{\text{def}}{=} A(x^+, x^-, x_-) - A^{(\infty)}(x^+, x_-)
\]

Then we have the following relation:

\[
iD_\perp = i\partial_\perp + gA_\perp = i\tilde{\partial}_\perp + g\tilde{A}_\perp + gA^{(\infty)}_\perp \overset{\text{def}}{=} i\tilde{D}_\perp + gA^{(\infty)}_\perp
\]

and we can show that

\[
iD_\perp = T i\tilde{D}_\perp T^\dagger,
\]

where

\[
T^\dagger = P \exp \left[ -ig \int_0^\infty d\tau l_\perp \cdot A^{(\infty)}_\perp(x^+, x_- - l_\perp \tau) \right]
\]

and we use that in Eq. (5) the fields \( \tilde{A}_\perp \) and \( A^{(\infty)}_\perp \) are evaluated at space-like separated points. The proof of this equation leads us automatically to the inclusion of the T-Wilson line in the Lagrangian. We write

\[
i\partial_\perp T^\dagger = iT^\dagger l_\perp \partial_\perp \left[ -ig \int_0^\infty d\tau l_\perp A^{(\infty)}_\perp(x^+, x_- - \tau l_\perp) \right]
\]

\[
= gT^\dagger \int_0^\infty d\tau \int \frac{d^4k}{(2\pi)^4} (i\partial_\perp k_\perp) l_\perp A^{(\infty)}_\perp(k) \times e^{ik(x^-l_\perp \tau)} = gT^\dagger l_\perp A^{(\infty)}_\perp(x^+, x_-),
\]

which proves Eq. (5), as \( l_\perp \) is an arbitrary vector. Moreover \( 1/\tilde{n}\partial_\perp \) and T commute because the T-Wilson line does not depend on \( x^- \). Under gauge transformation \( \delta A^{(\infty)}_\perp = D_\perp \omega \) one has \( T(x^+, x_-) \to U(x^+, x_-)T(x^+, x_-)U^\dagger(x^+, x_- - l_\perp \infty) = U(x^+, x_-)T(x^+, x_-) \) since \( A^{(\infty)}_\perp(x^+, \infty) = 0 \). Notice also that the T-Wilson lines are independent of \( l_\perp \).

Now we split the fermion field into big and small components using the usual projectors \( \notn \not\bar{\psi} \) and \( \not\bar{\psi} \) and eliminate the small components using the equations of motion [1]. The result of this is

\[
\mathcal{L} = \tilde{\xi}_n \left( inD + iD_\perp \frac{1}{inD} i\tilde{D}_\perp \right) \frac{\not\bar{\psi}}{2} \xi_n.
\]

In QCD in LCG with the gauge condition \( nA = 0 \),

\[
\mathcal{L} = \tilde{\xi}_n \left( inD + Ti\tilde{D}_\perp \frac{1}{i\tilde{n}\partial_\perp} i\tilde{D}_\perp T^\dagger \right) \frac{\not\bar{\psi}}{2} \xi_n.
\]

In order to get the SCET Lagrangian we must implement multipole expansion and power counting on the fields that appear in Eq. (8). In SCET we have also the freedom to choose a different gauge in the different sectors of the theory. We distinguish the cases of SCET-I and SCET-II. The two formulations differ essentially in the scaling of the soft sector of the theory. In SCET-I, collinear fields describe particles whose momentum \( k \) scales like \( (\tilde{n}k, nk, k_\perp) \sim Q(1, \lambda^2, \lambda) \) where \( \lambda \ll 1 \) and \( Q \) is a hard energy scale. Also the components of the collinear gluons, \( A^a_\mu \), in SCET-I (\( nA_n, nA_n, A_{\perp n} \)) scale like \( (1, \lambda^2, \lambda^2) \). The scaling of ultra-soft (u-soft) momenta in SCET-I is \( \sim Q(\lambda^2, \lambda^2, \lambda^2) \) and u-soft gluon fields (\( \bar{n}A_{us}, nA_{us}, A_{\perp us} \)) scale as \( (\lambda^2, \lambda^2, \lambda^2) \). In SCET-II we have for collinear field: \( (\tilde{n}k, nk, k_\perp) \sim Q(1, \eta^2, \eta) \) where \( \eta \ll 1 \). For soft momentum: \( (\tilde{n}k, nk, k_\perp) \sim Q(\eta, \eta, \eta) \) and collinear (soft) gluons field components scale accordingly. The main difference is that in SCET-I all the components of the soft momentum scale like the small component of the collinear fields, while in SCET-II the soft modes scale like the transverse component of the collinear fields.

SCET-I. Consider first the case of SCET-I where the u-soft sector is treated in covariant gauge while the LCG
is imposed on the collinear gauge fields. In LCG: \( \hat{n}A_n = 0 \) and with the multipole expansion [1] of Eq. (8) gives

\[
\mathcal{L}_\xi = \bar{\xi}_n \left( i n D_n + g n A_{us}(x^+) + T_n i \nabla_\perp W_n \frac{1}{i n D_n} i \nabla_\perp T_n \frac{\bar{g}}{2} \xi_n \right),
\]

where \( i D_\mu^\perp = i \partial_\mu + g A_\mu \), the u-soft field depends on \( x^+ \) (transverse and collinear variations of the u-soft field are power suppressed) and the \( T \)-Wilson line is given in Eq. (1). The presence of the \( T \)-Wilson line is essential to have gauge invariance as was shown in Ref. [3]. Let us decide now to impose LCG also on the u-soft sector of the theory, \( nA_{us} = 0 \). The corresponding \( T \)-Wilson line that would arise, following Eq. (8) disappears due to multipole expansion and power counting. In fact u-soft fields cannot depend on transverse coordinates in the leading order Lagrangian of SCET-I. In other words the \( T \)-Wilson line breaks the power counting of SCET-I and so the u-soft part of SCET-I cannot be written in LCG. The other possible choice \( \hat{n}A_n = 0 \) has no impact on the leading order SCET Lagrangian. Thus the most general formula for the SCET-I Lagrangian is

\[
\mathcal{L}_\xi = \bar{\xi}_n \left( i n D_n + g n A_{us}(x^+) + T_n i \nabla_\perp W_n \frac{1}{i n D_n} i \nabla_\perp T_n \frac{\bar{g}}{2} \xi_n \right). \tag{11}
\]

**SCET-II.** The analysis of the collinear sector in SCET-II is the same as for SCET-I. In the soft sector however one has new features. In regular gauges soft particles do not interact with collinear particles because the interactions knock the collinear fields off-shell. This is also true in LCG except when one makes the choice \( nA_n = 0 \) (take here a gauge for collinear fields for fixing ideas). It is easy to be convinced that interactions like \( \prod \phi_\perp(x) A_\perp \phi_\perp(x^-) \), where here \( \phi_\perp \) refers to the + direction and \( \phi_\perp(x) \) are generic collinear fields, preserve the off-shellness of the collinear particles. Using multipole expansion the vertex becomes \( \prod \phi_\perp(x) A_\perp \phi_\perp(0^-) \) (because for collinear fields \( x^- \sim 1 \) and for the soft field \( x^- \sim 1/\eta \)). In this gauge the covariant derivative for collinear particles becomes

\[
i D_\mu^\perp = i \partial_\mu + g A_\mu \phi_\perp + g A_\perp A_\perp^{(0)}(0^-, x_\perp) \cdot (0^-, x_\perp). \tag{12}
\]

The gauge ghost \( A_\perp^{(0)} \) however can be decoupled from collinear gluons defining a “soft free” collinear gluon \( A_\perp^{(0)}(x^\perp) = T_{sn}(x_\perp) A_\perp(x^\perp) T_{sn}(x_\perp) \) where

\[
T_{sn} = \tilde{P} \exp \left[ ig \int_0^\infty d\tau \cdot A_\perp^{(0)}(0^-, x_\perp - \tau) \right].
\]

Defining \( D_\perp^{(0)} = i \partial_\perp + g A_\perp^{(0)} \), we have

\[
D_\perp^{(0)} = T_{sn}(x_\perp) i D_\perp^{(0)} T_{sn}(x_\perp) \tag{13}
\]

where \( \xi_n = T_{sn}(x_\perp) \bar{\xi}_n(x) \) and \( W_\perp^{(0)} = W_\perp^{(0)}(0) \) are made out of soft free gluons. Thus thanks to \( T_{sn} \), Wilson lines the soft particles are completely decoupled from collinear particles. The details of the contemporary gauge choice \( nA_n = 0 \) and \( nA_s = 0 \) is left for future publication [13].

Alternatively the Wilson lines in SCET are obtained through “auxiliary fields” as illustrated, in covariant gauge, in the second work of [1]. Through a two-step of matching of the full QCD heavy-to-light current onto the SCET-II effective one, it was shown that the collinear Wilson line and the soft one can be obtained after integrating out all off-shellnesses larger than \( Q^2 \eta^2 \) by introducing auxiliary fields for each field that goes off-shell due to soft-collinear interactions. In LCG however the situation is a bit different. In the case of \( T_{sn} \) the auxiliary field method would contain a sub-leading (\( O(\lambda) \)) Lagrangian (including only the transverse component of the collinear gluon field.) Moreover the resulting fermion (in LCG \( \bar{n}A_n(nA_n) = 0 \)) is still on-shell due to \( \delta(nk) \) so the auxiliary field method breaks down. In SCET-II however all soft gluon field components have the same scaling and in LCG \( nA_n(nA_n) = 0 \) the resulting fermion is off-shell of \( O(Q^2 \eta^2) \) thus the auxiliary field method might be applicable. More on this in Ref. [13].

**II. APPLICATIONS AND CONCLUSIONS**

The above derived Lagrangians extend the formulation of SCET valid in covariant gauges to singular gauges as well. As it is the case in SCET in covariant gauges, the most important application of these Lagrangians is establishing factorization theorems in下一篇

energy processes. This is especially true for differential cross-sections with \( p_T \) dependence where one expects that the non-perturbative matrix elements entering those factorization theorems to be un-integrated with respect to the transverse momentum. In such cases those matrix elements need a gauge link in the transverse space so as to obtain complete gauge invariance. This is implemented naturally with the \( T \)-Wilson lines that should be invoked at the level of the basic building blocks of SCET. The above discussion allows us to obtain gauge invariant expressions for any non-perturbative matrix elements involving quantum fields separated in the transverse direction. The gauge invariant quark jet was given

first in [3] and is obtained by simply replacing $W_\alpha$ with $W^T_\alpha = T_\alpha W_\alpha$, $\chi_\alpha(x) = W^T_\alpha \xi_\alpha(x)$. Similarly the gluon jet [9] reads

$$g B^\mu_{n,\perp} = [W^T_\alpha \xi_\alpha W^T_{\perp}],$$

where the derivative operator acts only within the square brackets. These jets enter the different beam functions introduced in Ref. [5] and in [6]. In both of these works a low transverse-momentum dependent cross-sections is considered respectively for the Drell-Yan production, Ref. [5], and for the Higgs boson production, Ref. [6], and the factorization theorems are obtained within the SCET formalism. However the non-perturbative matrix elements, the so-called “beam functions”, entering those factorization theorems (see Eq. (9) in [5] and Eqs. (32) and (33) in [6]) are gauge invariant only in the class of regular gauges. As in the case of the TMDPDF in Ref. [3], the introduction of the $T$-Wilson line at the level of the SCET Lagrangian and the quark and gluon jets allows us to obtain, from first principles of the SCET, a well-defined and gauge invariant physical quantities that allows us to obtain, from first principles of the SCET, a well-defined and gauge invariant physical quantities that are relevant for such important LHC processes and cross-sections. The gluon jet functions in Ref. [6] should be written in terms of $B^\mu_{n,\perp}$ as given in Eq. (14).

We remark that the dependence of the soft functions on transverse fields and transverse coordinates is sensible only in the framework of SCET-II. The proper definition of the TMDPDF has to include a subtracted soft function to avoid double counting. This subtraction has long been argued by Collins and Hautmann [10, 11] and more recently verified in [12] by considering the renormalization group (RG) properties of the TMDPDF. This soft function includes a transverse gauge link so as to obtain gauge invariance. The analysis of [10–12] was performed in QCD. For SCET to generate the results of QCD for the TMDPDF (and for the generalized quantities thereof such as beam functions) the effective theory formalism has to include quantities like transverse “soft” Wilson lines to properly account the gauge invariance of the soft function and the RG properties of TMDPDFs and beam functions alike. In SCET-II the typical regular gauge matrix element of soft jets $< S_n(x) S^\dagger_n(0) S^\dagger_n(0) >$ should be replaced by $< S^T_n(x) S^T_n(0) S^T_n(0) S^T_n(0) >$ where $S^T_n(x) = S_n(x) T_{sn(sn)}(x_{\perp})$. $T_{sn(sn)}(x_{\perp})$ are the soft Wilson lines that arise with the gauge fixing $n A_x(\vec{n} A_x) = 0$ and are 1 otherwise. For instance fixing $n A_x = 0$ one gets $< T_{sn}(x_{\perp}) S^\dagger_n(0) T^\dagger_{sn}(0) >$.

Concluding, we have explained the origin of transverse gauge links $T$ within SCET. We have studied which light-cone conditions are compatible with the power counting of the effective theory. The relevance of the $T$-Wilson lines in SCET was studied in Ref. [3] for TMDPDF. We have provided a gauge invariant definition of both quark and gluon jets. The $T$-Wilson lines enter the definitions of non-perturbative matrix elements in some processes relevant for LHC, like Drell-Yan and Higgs production. We have finally commented on the transverse momentum dependence of the soft functions. Further study in this direction is expected in the near future [13].

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