Naturally light Dirac neutrinos from SU(6)

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Abstract: A known mechanism for obtaining naturally light Dirac neutrinos is implemented in the context of SU(6) → SU(5) × U(1)_N.

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1 Introduction

Whether or not neutrinos are Majorana or Dirac is a fundamental issue in particle physics. Experimentally, there is still no evidence for one or the other, although it is known that at least two neutrinos must have masses, because of neutrino oscillations [1]. Theoretically, the standard model (SM) requires only neutrinos in the left-handed SU(2)\textsubscript{L} × U(1)\textsubscript{Y} doublets (ν, l)\textsubscript{L}. The singlet ν\textsubscript{R} is not necessary because it is trivial under the SM gauge group SU(3)\textsubscript{C} × SU(2)\textsubscript{L} × U(1)\textsubscript{Y}. To have a Dirac neutrino, ν\textsubscript{R} must exist. To justify its existence, a gauge extension of the SM is often considered, either U(1)\textsubscript{Y} → U(1)\textsubscript{Y} × U(1)\textsubscript{B−L} [2] or U(1)\textsubscript{Y} → SU(2)\textsubscript{R} × U(1)\textsubscript{B−L} [3], which may be incorporated into the grand unified structure of SO(10). The breaking of U(1)\textsubscript{B−L} [and SU(2)\textsubscript{R}] is usually assumed without hesitation to allow ν\textsubscript{R} to obtain a large Majorana mass, so that ν\textsubscript{L} gets a tiny seesaw Majorana mass, as is well-known.

There is actually another option. This breaking does not have to be $\Delta L = 2$. If it is $\Delta L = 3$ for example, then neutrinos are Dirac. This was first pointed out [4] for a general U(1)\textsubscript{X} symmetry and applied [5] to U(1)\textsubscript{L} for Dirac neutrinos. However, this mechanism does not by itself explain why the neutrino Higgs Yukawa couplings are so small.

To overcome this problem, the mechanism of ref. [6] is the simplest solution. The idea is to have at least two Higgs doublets, say $\Phi = (\phi^+, \phi^0)$ and $\eta = (\eta^+, \eta^0)$ which are distinguished by some symmetry, so that $\bar{\nu}_R \nu_L$ couples to $\eta^0$, but not $\phi^0$. This symmetry is then broken by the $\mu^2 \Phi^\dagger \eta$ term, under the condition that $m^2_\phi < 0$ and $m^2_\eta > 0$ and large. In that case, the vacuum expectation value $\langle \eta^0 \rangle$ is given by $-\mu^2 \langle \phi^0 \rangle / m^2_\eta$, which is naturally small, implying thus a very small Dirac neutrino mass. In the original application [6], $\nu_R$ is also allowed a large Majorana mass, hence the mass of $\nu_L$ is doubly suppressed. In that case, $m_\eta$ could well be of order 1 TeV. On the other hand, if the symmetry and the particle content are such that $\nu_R$ is prevented from having a Majorana mass, then a much larger $m_\eta$ works just as well for a tiny Dirac neutrino mass.

Recently this idea has been applied [7, 8] using a gauge U(1)\textsubscript{D} symmetry under which the SM particles do not transform, but $\nu_R$ and other fermion singlets do. The U(1)\textsubscript{D}
symmetry is broken by singlet scalars which transform only under \( U(1)_D \). The bridge between the SM and this new sector is a set of Higgs doublets which transform under both. The particle content is chosen such that global lepton number is conserved as well as a dark parity or dark number.

In this paper, instead of the \( ad \, hoc \) \( U(1)_D \) symmetry, \( \nu_R \) is identified as part of the fundamental representation of \( SU(6) \) which breaks to \( SU(5) \times U(1)_N \). Following a recent analysis [9], it is shown how naturally small Dirac neutrino masses occur in this context.

| fermion | SU(5) | SU(3)_C | SU(2)_L | U(1)_Y | U(1)_N | U(1)_X |
|---------|-------|---------|---------|--------|--------|--------|
| \( d^c \) | 5^*  | 3^*  | 1      | 1/3    | -1    | 1      |
| \( (\nu, e) \) | 5^*  | 1     | 2      | -1/2   | -1    | 1      |
| \( N \) | 1   | 1     | 2      | 0      | 5     | 1      |
| \( D^c \) | 5^*  | 3^*  | 1      | 1/3    | -1    | 2      |
| \( (E^0, E^-) \) | 5^*  | 1     | 2      | -1/2   | -1    | 2      |
| \( \nu^c \) | 1    | 1     | 1      | 0      | 5     | 2      |
| \( (u, d) \) | 10   | 3     | 2      | 1/6    | 2     | 0      |
| \( u^c \) | 10   | 3^*  | 1      | -2/3   | 2     | 0      |
| \( e^c \) | 10   | 1     | 1      | 1      | 2     | 0      |
| \( D \) | 5    | 3     | 1      | -1/3   | -4    | 0      |
| \( (E^+, \bar{E}^0) \) | 5    | 1     | 2      | 1/2    | -4    | 0      |

Table 1. Fermion content of \( SU(6) \rightarrow SU(5) \times U(1)_N \) model.

2 Description of model

Starting with the well-known \( SU(5) \) model [10] of grand unification, an extension to \( SU(6) \) is straightforward [11, 12]. Instead of having the anomaly-free combination of \( 5^* \) and 10 under \( SU(5) \) for each family, there should be now two \( 6^* = (5^*, -1) + (1, 5) \) and one \( 15 = (10, 2) + (5, -4) \) under \( SU(6) \rightarrow SU(5) \times U(1)_N \). Let

\[
6_{F1} = \begin{pmatrix} d^c \\ d^c \\ e^c \\ \nu \\ N \end{pmatrix}, \quad 6_{F2} = \begin{pmatrix} D^c \\ D^c \\ E^- \\ E^0 \\ \nu^c \end{pmatrix}, \quad 15_F = \begin{pmatrix} 0 & u^c & u^c & -u & -d & -D \\ -u^c & 0 & u^c & -u & -d & -D \\ u^c & -u^c & 0 & -u & -d & -D \\ u & u & u & 0 & -e^c & -E^+ \\ d & d & d & e^c & 0 & -E^0 \\ D & D & D & E^+ & E^0 & 0 \end{pmatrix}.
\]

(2.1)

Their \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N \) assignments are listed in table 1. Note that all are left-handed and \( \nu^c \) sits in a different multiplet from \( \nu \), whereas both are in the same multiplet in the recent proposal of ref. [13]. The added \( U(1)_X \) is a global symmetry imposed on the dimension-four couplings of the resulting Lagrangian, but softly broken by
bilinear and trilinear scalar terms. The scalars which break the SU(6) symmetry and allow these fermions to acquire masses are listed in table 2. The 35\textsubscript{S} breaks SU(6) at the grand unification scale \( u_1 \) to SU(3)\(_C\) \( \times \) SU(2)\(_L\) \( \times \) U(1)\(_Y\) \( \times \) U(1)\(_N\). The 6\textsubscript{S} breaks U(1)\(_X\) at a lower scale. Because it is charged under U(1)\(_X\), its only allowed coupling is 6\textsubscript{S} \( \times \) 6\textsubscript{F} \( \times \) 15\textsubscript{F}. Hence \( D^cD \) and \( E^-E^+ + E^0E^0 \) masses are proportional to \( u_2 \). The 21\textsubscript{S} also breaks U(1)\(_N\). The 21\textsubscript{S} \( \times \) 6\textsubscript{F}\(_1\) \( \times \) 6\textsubscript{F}\(_1\) term yields Majorana masses \( N\bar{N} \) proportional to \( u_3 \). The electroweak SU(2)\(_L\) \( \times \) U(1)\(_Y\) symmetry is broken by three Higgs doublets. The 84\textsubscript{S} \( \times \) 6\textsubscript{F}\(_1\) \( \times \) 15\textsubscript{F} term yields masses for \( d^c\bar{d}, e^c\bar{e}, N\bar{E}^0 \) which are proportional to \( v_1 \). The 15\textsubscript{S} \( \times \) 15\textsubscript{F} \( \times \) 15\textsubscript{F} term yields masses for \( u^c\bar{u} \) proportional to \( v_2 \), whereas the 15\textsubscript{S} \( \times \) 6\textsubscript{F}\(_1\) \( \times \) 6\textsubscript{F}\(_2\) term yields masses for \( \nu\bar{\nu}, N\bar{E}^0 \) proportional to \( v_3 \). Let these three Higgs doublets be named \( \Phi_{1,2,3} \) respectively.

To obtain a small \( v_3 \), it is clear that \( m_{\eta}^2 \) must be positive and large, as discussed in the introduction. Now both \( \eta \) and \( \Phi_2 \) come from 15\textsubscript{S} but are distinguished by U(1)\(_X\). Hence the soft term \( \mu^2\Phi_2^\dagger\Phi_2 \eta \) breaking U(1)\(_X\) by 3 units is available to make the Dirac neutrino masses very small. Note that \( \nu^c \) does not acquire a Majorana mass because of the absence of a scalar 21\textsuperscript{*} with 4 units of charge under U(1)\(_X\). The two soft trilinear terms 84\textsubscript{S} \( \times \) 6\textsubscript{S} \( \times \) 15\textsubscript{S} break U(1)\(_X\) by 3 and 6 units, whereas 6\textsubscript{S} \( \times \) 6\textsubscript{S} \( \times \) 21\textsubscript{S} does it by 6 units.

### 3 Residual symmetries

Because of the U(1)\(_X\) symmetry, Yukawa terms are restricted so that residual symmetries exist for the fermions of this model at the level of SU(3)\(_C\) \( \times \) SU(2)\(_L\) \( \times \) U(1)\(_Y\) \( \times \) U(1)\(_N\). The usual baryon number \( B \) and lepton number \( L \) are then conserved, together with a dark parity \( Z^D_2 \) which may be identified as \((-1)^{3B+L+2j}\), where \( j \) is the intrinsic spin of the particle, as shown in table 3. The 3 \( \times \) 3 mass matrix spanning \((N,E^0,\bar{E}^0)\) is of the form

\[
\mathcal{M}_{NE} = \begin{pmatrix}
    f_{Nu3} & f_{Nev3} & f_{Nev1} \\
    f_{Nev3} & 0 & f_{Eu2} \\
    f_{Nev1} & f_{Eu2} & 0
\end{pmatrix}.
\]

The lightest mass eigenstate is possible dark matter. However as shown in ref. [9], because of its SU(6) antecedent, \( N \) may decay through a superheavy gauge boson in analogy to proton decay. Note also that \( \nu_3 \) is very small. If \( u_3 \) is absent, then \( N \) gets a small seesaw

| \langle scalar \rangle | SU(6) | SU(5) | SU(3)\(_C\) | SU(2)\(_L\) | U(1)\(_Y\) | U(1)\(_N\) | U(1)\(_X\) |
|-----------------|------|------|---------|---------|--------|--------|--------|
| \( u_1 \)     | 35   | 24   | 1       | 1       | 0      | 0      | 0      |
| \( u_2 \)     | 6    | 1    | 1       | 1       | 0      | -5     | 2      |
| \( u_3 \)     | 21\textsuperscript{*} | 1    | 1       | 1       | 0      | 10     | 2      |
| \( v_1 \)     | 84   | 5    | 1       | 2       | 1/2    | 1      | 1      |
| \( v_2 \)     | 15\textsuperscript{*} | 5\textsuperscript{*} | 1       | 2       | -1/2   | 4      | 0      |
| \( v_3 \)     | 15\textsuperscript{*} | 5\textsuperscript{*} | 1       | 2       | -1/2   | 4      | 3      |

Table 2. Scalar content of SU(6) \( \rightarrow \) SU(5) \( \times \) U(1)\(_N\) model.
mass proportional to $v_1 v_3/u_2$. This makes it a candidate for very light freeze-in dark matter, as described in ref. [8]. In such a scenario, the $21^*$ scalar is not required.

In the $15_S \times 6^f_{F_1} \times 6^f_{F_2}$ coupling, the $(5, -4)$ component of $15_S$ contains $n^0$ which couples to $\nu \nu^c$ as well as $N E^0$. It also contains the scalar color triplet $\zeta = (3, 1, -1/3, -4)$ which couples to both $d^c \nu^c$ and $N D^c$. Hence $D^c$ must have $L = -1$ and $N$ must have $L = 0$. Note that $\zeta$ has then $B = 1/3$ and $L = 1$, so that its dark parity, i.e. $(-1)^{3B+L+2j}$, is even as expected. The dark quark $D$ decays through $\zeta$ to $d \nu N$. This process is a three-body decay with two invisible particles and not easy to observe.

### 4 Model characteristics

Below the breaking scale $u_{2,3}$ of $U(1)_N$, the particle content of the proposed model is that of the SM with the following changes. The neutrinos are Dirac particles. There are two Higgs doublets, one coupling to $\bar{u} u$ and the other to $\bar{d} d$ and $\bar{e} e$. A third Higgs doublet is very heavy and not observable, but it has a tiny vacuum expectation value and couples to $\bar{\nu} \nu$. There may also be a very light neutral Majorana fermion $N$ which is freeze-in dark matter in the case that $u_3 = 0$.

The above particles all interact with a new heavy gauge boson $Z_N$ coming from $U(1)_N$, according to their charges given in tables 1 and 2. The present collider limit [1] of $Z_N$ is estimated to be a few TeV. At or above the $Z_N$ mass scale, particles of the dark sector as well as one or more Higgs singlets should appear. The $D$ quark may be easily produced at the collider and decay to the $d$ quark plus a neutrino and the dark $N$. The doublet $(E^0, E^-)$ dark fermions also decay to $N$ plus a scalar or the $W^-$ gauge boson. More details are in ref. [9].

### 5 Concluding remarks

It has been shown that a framework exists for naturally small Dirac neutrino masses in the context of $SU(6) \to SU(5) \times U(1)_N$, where the right-handed neutrino singlet $\nu^c$ is embedded as shown in eq. (1). With the implementation of a softly broken global $U(1)_X$ symmetry as given in tables 1 and 2, the residual symmetries of baryon number $B$ and lepton number $L$ are preserved as given in table 3, with dark parity identified as $Z_2^D = (-1)^{3B+L+2j}$. The reason for a conserved lepton number parallels that of ref. [4], i.e. the interplay of $U(1)_X$ and the chosen $SU(6)$ representations makes it impossible for $\nu^c$ to be a Majorana fermion.

| fermion | $B$ | $L$ | $Z_2^D$ |
|---------|-----|-----|---------|
| $u, d$  | 1/3 | 0   | +       |
| $\nu, l$ | 0   | 1   | +       |
| $D$     | 1/3 | 1   | −       |
| $N, E^0, E^−$ | 0   | 0   | −       |

Table 3. Residual Symmetries of $SU(6) \to SU(5) \times U(1)_N$ model.
The dark sector consists of $D$ leptoquarks and $L = 0$ fermions $N$ and the vectorlike doublet $(E^0, E^-)$. Whereas $N$ mixes with $E^0$ and $E^0$, the former may be almost a mass eigensate and considered as dark matter. It is presumably of order a few TeV, but if the $21^*$ scalar is removed from table 2, then $u_3 = 0$ and eq. (2) yields a very small mass for $N$ which then becomes freeze-in dark matter.

Suppose the $U(1)_N$ breaking scale is much higher than a few TeV, then only the SM particles are observable, with the important difference that neutrinos are Dirac, and there are two Higgs doublets. The other possible addition is the Majorana fermion $N$ as freeze-in dark matter. If $u_3 = 0$, then its mass is proportional to $v_1v_3/u_2$ as remarked earlier, which is perhaps too small because $v_3$ is responsible for the Dirac neutrino mass and $u_2$ is now assumed to be very large. However, the $21^*$ scalar may be retained, and $u_3 \neq 0$ rendered small (but not too small) by the same mechanism [6] which makes $v_3$ small, i.e. of the form $-\mu_{23}u_2^2/m_3^2$ from the term $6_S \times 6_S \times 21^*_S$, where $m_3$ is the mass of the scalar $21^*$.

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