Globular cluster formation with multiple stellar populations from hierarchical star cluster complexes

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ABSTRACT

Most old globular clusters (GCs) in the Galaxy are observed to have internal chemical abundance spreads in light elements. We discuss a new GC formation scenario based on hierarchical star formation within fractal molecular clouds. In the new scenario, a cluster of bound and unbound star clusters ('star cluster complex', SCC) that have a power-law cluster mass function with a slope ($\beta$) of 2 is first formed from a massive gas clump developed in a dwarf galaxy. Such cluster complexes and $\beta = 2$ are observed and expected from hierarchical star formation. The most massive star cluster ('main cluster'), which is the progenitor of a GC, can accrete gas ejected from asymptotic giant branch (AGB) stars initially in the cluster and other low-mass clusters before the clusters are tidally stripped or destroyed to become field stars in the dwarf. The SCC is initially embedded in a giant gas hole created by numerous supernovae of the SCC so that cold gas outside the hole can be accreted onto the main cluster later. New stars formed from the accreted gas have chemical abundances that are different from those of the original SCC. Using hydrodynamical simulations of GC formation based on this scenario, we show that the main cluster with the initial mass as large as $[2-5] \times 10^5 M_\odot$ can accrete more than $10^5 M_\odot$ gas from AGB stars of the SCC. We suggest that merging of hierarchical star cluster complexes can play key roles in stellar halo formation around GCs and self-enrichment processes in the early phase of GC formation.

Key words: galaxies: star clusters: general – galaxies: stellar content – galaxies:ISM – globular cluster: general – stars:formation

1 INTRODUCTION

One of remarkable discoveries in the field of globular clusters (GCs) is that old GCs in the Galaxy and intermediate-age ones in the Large Magellanic Cloud (LMC) have multiple stellar populations (e.g., Freeman & Rodgers 1975; Cohen 1981; Lee at al. 1999; Gratton et al. 2001; Bedin et al. 2004; Norris 2004; Piotto et al. 2005; Mackey & Broby Nielsen 2007; Lee et al. 2009; Da Costa et al. 2014; See Gratton et al. 2012 for a recent review). Most of the investigated GCs in the Galaxy are observed to show anti-correlations between light elements (e.g., C, N, and O) of cluster members stars (e.g., Carretta et al. 2009; C09) whereas only 8 GCs have been so far confirmed to have star-to-star abundance spreads in heavy elements (e.g., Yong et al. 2014; Marino et al. 2015). Extended main-sequence turn-offs (eMSTOs) and splits in main-sequence observed in the color magnitude diagrams (CMDs) of some LMC GCs (e.g., Mackey & Broby Nielsen 2007; Goudfrooij et al. 2014; Milone et al. 2016) can be possible evidence for the multiple stellar populations with different ages, though recent observations suggest that internal stellar rotation rather than age spreads could explain the physical properties of LMC clusters with eMSTOs (e.g., Bastian & De Mink 2009; Milone et al. 2016; Li et al. 2016). These new discoveries stimulated much discussion on the initial stellar mass function of stars in GCs, the formation processes of GCs, and the origin of the observed diversity in chemical and dynamical properties of GCs with multiple stellar populations (e.g., D’Antona & Caloi 2004; Bekki et al. 2007; Baumgardt et al. 2008; D’Ercole et al. 2008, D08; Vesperini et al. 2010; Renzini 2015; D’Antona et al. 2016).

A straightforward scenario for this multiple stellar population phenomenon in GCs is that gas ejected from the first generation (FG) of stars in a GC is converted into the second generation (SG) of stars with chemical abundances different from those of FG stars: multiple populations mean multiple generations of stars. In this scenario, chemical abundances between the two generations are different because chemical
abundances of gas ejected from fast-rotating massive stars (e.g., Decressin et al. 2007), supermassive stars (e.g., Denissenkov & Hartwick 2014), massive interacting binaries (e.g., Bastian et al. 2013), and AGB stars (e.g., D08) are quite different from the averaged ones of FG stars. This scenario has been suggested to have a number of serious problems in explaining the fundamental properties of GCs, for example, the larger fraction of SG stars in GCs with multiple stellar populations. For this scenario to explain the observed fraction of SG stars (∼70%; C09), the original mass of FG stars (\(M_{\text{FG}}\)) should be much more massive than the present-day GC mass (‘mass budget problem’).

This mass budget problem can be simply formulated as follows:

\[ M_{\text{FG}} = 4.7 \times 10^6 \left( \frac{M_{\text{sf}}}{0.3} \right)^{-1} \left( \frac{f_{\text{ej}}}{0.1} \right)^{-1} \left( \frac{M_{\text{SG},0}}{1.4 \times 10^5 M_\odot} \right) M_\odot, \] (1)

where \(M_{\text{SG},0}\) is the present-day total mass of SG stars in a GC, \(M_{\text{sf}}\) is the star formation efficiency in the SG star formation (\(M_{\text{SG}}/M_{\text{ej}}\), where \(M_{\text{ej}}\) is the total mass of gas ejected from ‘polluter’, such as AGB stars), and \(f_{\text{ej}}\) is the mass fraction of gas ejected from polluters in the FG stars. In this estimation of \(M_{\text{FG}}\), all SG stars are assumed to be long-lived low-mass stars that are alive in the present, which means that a very unique initial mass function (IMF) of stars (i.e., bottom-heavy and top-light IMF) is assumed just for clarity. Nevertheless, the required \(M_{\text{FG}}\) is much larger than both the present-day mass of FG stars (\(M_{\text{FG},0} = 6.0 \times 10^5 M_\odot\) for a typical present-day GC mass of \(M_{\text{gc}} = 2 \times 10^5 M_\odot\)) and the typical mass of GCs. Although the mass budget problem is more complicated than the above discussion, it is one of the most serious problems in previous GC formation scenarios. Smith & Norris (1982) first discussed this mass budget problem in the context of the origin of CN-weak and CN-strong populations in NGC 6752 and 47 Tuc, though they did not use the term ‘mass budget problem’.

There are three scenarios to solve the mass budget problem (e.g., D’Antona & Caloi 2004; Bekki & Norris 2006; Frantzos & Charbonnel 2006). First is that GCs were initially formed as very massive star clusters composed only of FG stars and then lost preferentially the large fraction of FG stars by some physical processes (‘VMSC’ scenario). Second is that GCs were stellar galactic nuclei of nucleated dwarfs that had been completely destroyed by their host galaxies’ tidal fields (‘SGN’ scenario). Third is that the IMF of FG stars is top-heavy (i.e., a larger mass fraction of massive stars and intermediate-mass stars that eject gas) whereas that of SG stars is bottom-heavy (i.e., a larger number of low-mass stars) for some physical reasons (‘top-heavy IMF’ scenario). Although GC formation processes based on the VMSC scenario have been extensively investigated by several authors (e.g., D08; Bekki 2011, B11), recent observations have suggested potentially serious problems of the VMSC scenario (e.g., Larsen et al. 2012). Although some massive GCs such as \(\omega\) Cen could have been formed from nucleated dwarf galaxies (e.g., Freeman 1993; Bekki & Freeman 2003), it is not clear whether typical GCs can be formed in the SGN scenario. The top-heavy IMF scenario has not clearly explained why the IMFs of FG stars can be top-heavy in GC formation.

The Tarantula Nebula (a.k.a ‘30 Dor’) in the LMC would provide a hint for the solution of the mass budget problem as follows. 30 Dor with a diameter of \(\sim 200\) pc contains a central main cluster ‘R136’, other low-mass clusters such as Hodge 301 and NGC 2060, numerous small stellar clumps and star-forming regions, and older pre-main sequence (PMS) stars (e.g., Grebel & Chu 2000; De Marchi et al. 2011 Sabbidi et al. 2013). De Marchi et al. (2011) analyzed the ages of PMS stars in 30 Dor and found that (i) there are several generations of stars with ages ranging from 1 to 30 Myr and (ii) older PMSs are mostly located in the outer (eastern) part of R136. These observations strongly suggest that a massive star cluster R136 can form with unbound stellar associations (that has become numerous field stars now) and low-mass clusters around R136 with a time scale of \(\sim 30\) Myr. These furthermore imply that massive young star clusters, which can be the progenitor of old GCs, can form with other numerous unbound stellar association and low-mass clusters within massive giant molecular clouds (GMCs).

Clustering of star clusters is observed in the star-forming regions of the Galaxy and the LMC, nearby star-forming galaxies such as M33 and M51, and galaxy mergers (e.g., Efremov 1995; Efremov & Elmegreen 1998; Zhang et al. 2001; Larsen 2004; Bastian et al. 2005; Elias et al. 2009; Adamo et al. 2012). These star cluster complexes (‘SCCs’) have been observationally investigated for some specific cases, and their physical properties have been derived. For example, Schepmaker et al. (2009) investigated the two-point autocorrelation function for three different groups of SCs with different age ranges in M51 and found that the projected fractal dimensions of 1.2 – 1.6 can well describe the observed slopes of the autocorrelation functions. Bastian et al. (2005) revealed that SCCs in M51 are all younger than 10 Myr and have sizes of 85 – 240pc and masses of \([3 – 30] \times 10^4 M_\odot\). The slopes of power-law cluster mass functions (i.e., \(N_{\text{sc}} \propto m_{\text{sc}}^{-\beta}\), where \(m_{\text{sc}}\) is a SC mass) have been also investigated by many authors, and \(\beta\) appears to be approximately 2 for SCs in different galaxies (e.g., Battinelli et al. 1994; Elmegreen & Efremov 1997; de Grijs et al. 2003). Such clustering of SCs with \(\beta = 2\) is a natural result of SC formation from hierarchical star formation (See Elmegreen 2008 for a review). It is therefore highly likely that the progenitor clusters of the present-day GCs were initially members of SCCs at their birth.

Such possibly more realistic GC-forming environments within SCCs were not considered in previous GC formation models (e.g., D08; B11) in which only AGB ejecta from GC progenitor massive single clusters can be converted into SG stars (‘self-accretion’). If gas ejected from massive AGB stars in unbound and bound low-mass clusters surrounding a very young massive SC (‘main cluster’) can be accreted onto the main cluster (‘external accretion’), then the total mass of these AGB ejecta can be a significant fraction of the total mass of the main cluster. Accordingly, the mass budget problem can be much less severe in the new GC formation from SCCs. However, low-mass clusters that form with the main cluster can be quickly stripped from the surrounding of the main cluster and subsequently destroyed by the tidal fields of their host dwarf galaxy and even by the main cluster itself. Therefore, it is possible that only a small fraction of their AGB ejecta could be accreted onto the main cluster before they are stripped or disintegrated to become field stars. It is thus worthwhile to investigate how much of gas
ejected from AGB stars born in low-mass clusters can be accreted onto the main cluster in the new SCC scenario.

The purpose of this paper and our forthcoming papers is to investigate the formation of GCs with multiple stellar populations from SCCs with \( \beta = 2 \) that are expected from hierarchical star formation within fractal molecular clouds. Using hydrodynamical simulations of dwarf galaxies with SCCs, we particularly investigate how much AGB ejecta from low-mass SCs that form with the main clusters can be accreted onto the main clusters within \( \sim 300 \) Myr. This possible gas accretion timescale of \( \sim 300 \) Myr is chosen, firstly because low-mass SCs in SCCs are expected to merge with one another or be stripped from the main clusters within \( \sim 300 \) Myr, and secondly because recent observations of the LMC GCs suggested that typical age spreads in the GCs are 100-400 Myr (e.g., Goudfroit et al. 2014). It should be noted here that it is still controversial whether the LMC clusters can contain stellar populations with different ages (e.g., Milone et al. 2016; Li et al. 2016). If the total mass of AGB ejecta accreted onto main clusters (\( M_{\text{acc}} \)) is significantly more than \( 10^4 M_\odot \) (corresponding to the total mass of SG stars in typical GCs), then the mass budget problem is much less serious in the SCC scenario. We investigate \( M_{\text{acc}} \) for different initial main cluster masses (\( m_{\text{acc}} \)) in SCCs, initial positions of the SCCs within their host dwarfs, and presence or absence of cold interstellar medium (ISM) in dwarf galaxies.

The plan of the paper is as follows. We outline the new scenario of GC formation from SCCs in dwarf galaxies in §2. We describe the models for dwarf galaxies, SCCs, and gas accretion onto main clusters in §3. We present the key results of the simulation, in particular, the time evolution of gas accretion rates and total masses of gas accreted onto main clusters in §4. We briefly discuss the important implication of the present results in the context of the observed physical properties of GCs with multiple stellar populations in §5. We summarize our conclusions in §6. It should be stressed here that this paper is the very first step toward the better understanding of GC formation in the context of the SCC scenario. Therefore, we investigate only one of the most important physical processes of GC formation in the present study. We will discuss other importance processes of GC formation in our forthcoming papers.

It is being hotly debated whether the observed eMSTOs and splits of main-sequence of the LMC clusters can result from age spreads or from stellar rotation (e.g., Milone et al. 2016; Li et al. 2016). Recently, For & Bekki (2016) have discovered direct evidence for ongoing star formation (i.e., the presence of young stellar objects) in the older LMC clusters with ages of 0.1 – 1 Gyr. Their results strongly suggest that at least some of the LMC clusters experienced secondary star formation after the main initial burst of star formation. They have also suggested that AGB ejecta needs to be accreted onto the older clusters by some physical mechanisms. They also have found that even some low-mass SCs with the masses less than \( 10^4 M_\odot \) can have ongoing star formation. The present results will be able to provide a new clue to the origin of these observations. Our recent simulations have found that (i) multiple stellar structures in FG stellar systems of the simulated GCs can be formed from massive gas clumps developed in gas-rich dwarf galaxies and (ii) some of the SG stars of the GCs can be formed from gas that are not

| Model ID | \( M_0 \) | \( M_c \) | \( M_s \) | \( c \) | \( R_e \) |
|----------|--------|--------|--------|------|------|
| DW1      | 1.0    | 6.0    | 0.0    | 16.0 | 1.75 |
| DW2      | 1.0    | 3.0    | 0.0    | 16.0 | 1.75 |
| DW3      | 1.0    | 6.0    | 0.6    | 16.0 | 1.75 |
| DW4      | 1.0    | 6.0    | 0.6    | 16.0 | 1.75 |
| DW5      | 1.0    | 3.0    | 0.0    | 5.4  | 1.0  |
| DW6      | 1.0    | 3.0    | 0.6    | 5.4  | 1.0  |
| DW7      | 1.0    | 6.0    | 0.6    | 5.4  | 1.0  |

- \( a \) The dwarf model used in the fiducial model is DW1. The models with \( c = 5.4 \) and \( r_e = 1 \) kpc correspond to high-z more compact dwarf models.
- \( b \) The initial total mass for dark matter halo in units of \( 10^{10} M_\odot \).
- \( c \) The initial total mass for stellar disk in units of \( 10^8 M_\odot \). The value of \( M_s \) in DW1 is referred to as \( M_{s,0} \) for convenience.
- \( d \) The initial total mass for gas disk in units of \( 10^8 M_\odot \).
- \( e \) The central concentration parameter in the in NFW dark matter profile.
- \( f \) The initial size of stellar disk. in units of kpc.

from AGB stars of the FG systems but from the surrounding field AGB stars that simultaneously form with the FG stars (Bekki 2015a, 2016). The present study is motivated by these recent results.

2 THE NEW SCENARIO

The new SCC scenario is based both on the observed properties of 30 Dor and SCCs in nearby galaxies (e.g., Efremov 1995; Bastian et al. 2005; Sabbi et al. 2013; Adamo et al. 2012) and on our recent numerical simulations of GC formation (Bekki 2015a; Bekki 2016). In the new scenario, a GC was initially the most massive star cluster (SC) that formed with other numerous stellar associations and low-mass clusters within a massive GMC (or a GMC association) with fractal structures in a gas-rich dwarf galaxy. Therefore, the formation processes of GCs in the SCC scenario, such as gas accretion onto existing stellar systems and the subsequent (secondary) star formation within them, could be significantly different from those described in previous models (e.g., D08; B11). The new scenario consists of the following seven stages, each of which needs to be investigated in this paper and forthcoming papers (See Fig 1 for the schematic representation of the new scenario).

2.1 Stage 1: Formation of a massive gaseous clump in a gas-rich dwarf

A massive gaseous clump with the initial masses as large as \( 10^2 – 10^5 M_\odot \) is formed from gravitational instability of the gaseous disk in a dwarf galaxy. Such a clump corresponds to a massive GMC or a GMC complex and has been demonstrated to be formed in luminous disk galaxies and dwarfs, if the gas mass fractions are higher (e.g., Shlosman & Nuguchi 1993; Nuguchi 1998; Bekki 2007). These massive clumps are highly likely to have fractal structures like the Galactic GMCs, and hierarchical cluster complexes can be formed from such fractal structures. These massive clumps
Figure 1. A brief illustration of the new scenario of GC formation from SCCs. The selected key four phase of GC formation processes are shown in the four panels, (A)-(D), in chronological order. After the formation of a SCC embedded in a giant gas hole, AGB stars in the main cluster and other low-mass clusters eject gas to the intra-cluster region through stellar winds (A). The AGB ejecta can be accreted onto the main cluster very efficiently while the cluster is in the gas hole (B). Cold ISM can be accreted onto the main cluster well after the cluster escapes from the hole (or the gas hole disappears through gaseous dissipation) so that star formation can become efficient in the inner region of the cluster (C). Most of the low-mass clusters are either stripped from the surrounding of the main cluster or disrupted by the tidal field of the host dwarf galaxy (D). Some low-mass clusters merge with the main cluster to be destroyed by the cluster. The remnants of the low-mass clusters become a diffuse stellar halo around the main cluster. The final cluster can appear as a GC that has a diffuse stellar halo and two stellar populations with different chemical abundances.

can be the progenitor of galactic bulges (e.g., Noguchi 1999; Elmegreen et al. 2008) and GCs (Shapiro et al. 2010; Adamo et al. 2013; Bekki 2015a, Bekki 2016). Star formation (i.e., FG star formation) starts to proceed very efficiently within the clumps, and consequently hierarchical stellar structures are developed.

2.2 Stage 2: Formation of a SCC from hierarchical stellar structure within the fractal clump

Numerous stellar associations and clusters are formed from dynamical relaxation processes of the hierarchical stellar structure within the clump with initial fractal structures. The most massive cluster (‘the main cluster’) in these stellar objects is regarded as the progenitor of a GC in the new scenario. The mass function of the SCC is likely to follow either the mass fraction of GMCs observed in nearby galaxies ($\beta$ of 1.71 – 2.49; Blitz et al. 2007) or that of young star clusters (e.g., de Grijs et al. 2003). Some very low-mass unbound clusters and stellar associations could have been already disintegrated owing to dynamical relaxation process to become field stars in the SCC at this stage. Since the mass fraction of such field stars in the SCC can not be estimated in the present study without further numerical simulation of the SCC formation, we simply assume that such a fraction is zero in the present simulation.

2.3 Stage 3: Explosions of multiple SNe and the formation of super-giant gaseous hole

Since the SCC has a large number of massive stars with $m_\odot \geq 8 M_\odot$, energetic feedback effects of massive stars and SNe can ionize cold gas left after star formation in the clump, blow out the gas from the host dwarf galaxy, and finally develop a giant gas hole with the diameter as large as 1 kpc (as LMC4 area in the LMC). The sizes of such giant holes depend on the masses of SCCs such that larger gaseous holes can be created in more massive SCCs owing to a larger number of SNe. Although there can be hot, tenuous gas left in the SCC, the total mass of such gas is negligible.
2.4 Stage 4: Gas ejection from AGB stars

When massive stars with \( m_s = 8 \, M_{\odot} \) enter into the AGB phases (roughly 30–40 Myr after their birth; \( t_{\text{agb}}(m_s = 8 \, M_{\odot}) \approx 30 – 40 \) Myr), stellar winds from the AGB stars can supply gas for further star formation in the SCC. There are two key questions here regarding the validity of the new scenario. One key question is whether the SCC is still a collection of clusters (i.e., whether the hierarchical structure can survive) when stars with \( m_s = 8 \, M_{\odot} \) become AGB stars. Merging of SCs during SCC formation from fractal gas clumps (GMCs or GMC associations) has partially or completely wiped out the initial hierarchical stellar structures of SCCs when massive AGB stars start to eject gas. A physical condition for survival of hierarchical structures in a SCC is described as follows:

\[
t_{\text{merge}} > t_{\text{agb}}(m_s = 8 \, M_{\odot}), \tag{2}
\]

where \( t_{\text{merge}} \) is merging timescale of SCs. This \( t_{\text{merge}} \) can be shorter than \( \sim 30 \) Myr in low-mass SCCs, which means that their initial hierarchical structures have been at least partially (or completely) lost in such low-mass SCCs. Our future simulations need to investigate to what extent initial hierarchical structures have been lost at the time of gas ejection from AGB stars with \( m_s = 8 \, M_{\odot} \) for a given SCC mass.

Bastian et al. (2005) estimated ages of stellar populations in low-mass SCCs with masses of \([3-30] \times 10^4 M_\odot\) and found that they are less than \( 10^7 \) yr old (typically a few Myr old). Their SCCs are quite low-mass ones from which GCs cannot be formed in the present scenario. Murray (2011) showed the lifetimes of the Galactic GMCs with star formation are only a bit shorter than 3 free-fall times, which means that the typical lifetime of the GMCs is \( \sim 30 \) Myr. If the ages (or age differences of stellar populations) of SCCs correspond to the lifetimes of their host GMCs, then it is expected that more massive SCCs (from more massive GMCs) can have older ages (or larger age spreads). Accordingly, if the ages of such low-mass SCCs are a few Myr (or at most 10 Myr), then the ages of high-mass SCCs (\( \sim 10^7 M_\odot \)) should be significantly older than a few Myr. It is thus highly likely that hierarchical structures can survive longer than 30 Myr in massive SCCs with their masses larger than \( 10^7 M_\odot \). It is our future work to investigate this issue using hydrodynamical simulations of star-forming GMCs with fractal structures.

The other related question is whether the SCC can keep its clustering status for an enough long time such that gas ejected from AGB stars evolving from intermediate-mass stars (\( 3 \leq m_s/M_\odot \leq 8 \)) can interact with the main cluster and can be subsequently captured by the cluster. Star clusters in a SCC can merge with one another to form a massive single cluster (Kroupa 1998; Fellhauere & Kroupa 2002; Bekki et al. 2004b) and the time scale for the completion of violent merging is as short as 10 \( t_{\text{c}} \), where \( t_{\text{c}} \) is the crossing time scale of the SCC (Fellhauere & Kroupa 2002; Bekki et al. 2004b). This merging timescale \( t_{\text{merge}} \) corresponds to 70 – 380 Myr for SCCs with a mass of \( 2 \times 10^7 M_\odot \) and sizes of 50 – 150 pc (Fellhauere & Kroupa 2002). In the present scenario, \( t_{\text{merge}} \) is as follows:

\[
t_{\text{merge}} = 3.9 \times 10^8 \left( \frac{\sigma}{10 \text{ km s}^{-1}} \right)^{-1} \left( \frac{r_{\text{sec}}}{200 \text{ pc}} \right), \tag{3}
\]

where \( \sigma \) is the stellar velocity dispersion of the SCC-host dwarf at the location of the SCC and \( r_{\text{sec}} \) is the radius of the SCC with a spherical cluster distribution. Here the relative velocity between the SCC and a member SC is assumed to be the same as \( \sigma \). The following relation is therefore ensured for very massive SCCs:

\[
t_{\text{merge}} > t_{\text{agb}}(m_s = [3 - 8] M_\odot). \tag{4}
\]

This means that gas from AGB stars that evolved from stars with \( m_s = [3 - 8] M_\odot \) can interact with the main cluster well before the SCC becomes dynamically relaxed through violent merging. It is likely that low-mass clusters are stripped before they merge with the main cluster owing to the tidal field of the host dwarf. Also, low-mass SCCs might have been well relaxed when gas from AGB stars is being accreted onto the central regions of SCCs. These points can be investigated in the present numerical simulations. Just for convenience, time \( T \) in a simulation is set to be 0 at this stage 4.

2.5 Stage 5: Accretion of AGB ejecta and cold gas onto the main cluster within the SCC

Intermediate-mass stars in stellar associations and low-mass clusters start to eject gas into ISM of the dwarf through stellar winds. A significant fraction of the AGB ejecta can be smoothly accreted onto the main cluster owing to the lack of hot gas in the hole. However, some fraction of low-mass clusters can be quickly stripped from the main cluster by the tidal field of the host dwarf so that most of their AGB ejecta can not be accreted onto the main cluster. Low-mass clusters that merge with the main cluster can provide more AGB ejecta for the main cluster. Cold gas initially outside the gas hole is not chemically polluted by the ejecta from SNe of the SCC and thus has chemical abundances similar to those of the main cluster. Accretion of the cold gas onto the main cluster comes later than that of the AGB ejecta in this scenario. Such cold gas accretion can dilute the AGB ejecta, and consequently, the chemical abundances of the mixed gas can be quite different from those of the AGB ejecta.

2.6 Stage 6: Secondary star formation from the accreted gas

Secondary star formation from the accreted gas starts whenever the physical properties of the gas satisfy the physical conditions required for star formation in dense stellar systems. Therefore, it is possible that AGB ejecta is converted into new stars without mixing with cold gas (i.e., well before the cold gas accretion onto the main cluster) in some cases. If this star formation lacks massive with \( m_s \geq 8 M_\odot \), then it can continue more than a few Myr without being influenced by energetic SN feedback effects. The IMF of the formation of SG stars is a key factor that determines the duration of secondary star formation (and gas accretion) thus the total mass of SG stars.

2.7 Stage 7: Complete disintegration of the SCC

Star formation can continue as long as accretion of AGB ejecta and cold gas onto the main cluster continues. Accretion of gas ejected from intermediate-mass AGB stars almost
SCs have been ongoing since the formation of SCs from a fractal 30-40 Myr after SC formation. However, this assumption can be integrated in a SCC when AGB stars start to eject gas (i.e., around the main cluster in the fiducial model). Each blue circle represents the location of a SC with the size indicating the mass of the SC. The red circle represents the size of the main cluster. Since there are too many very low-mass SCs in the SCC, only 10% of the SCs are shown for clarity. In the present study, it is assumed that most SCs still exist (i.e., without being disrupted). This assumption is adopted just for the purpose of estimating gas accretion rates onto forming GCs in the present GC formation scenario. Real GC formation processes can be more complicated than the present model. We focus exclusively on the Stage 4 and 5 in this paper and thereby investigate the total mass of gas accreted onto main clusters orbiting around their host dwarf galaxies. We will investigate physical processes in other stages of the new GC formation scenario in our future papers. In order to investigate the accretion processes of AGB ejecta and cold ISM, we use our original simulation code (Bekki 2013; Bekki 2015b, c) that can be run on GPU clusters. The code enables us to investigate chemical evolution, dust formation and evolution (Bekki 2013), formation of molecular hydrogen on dust grains (Bekki 2015b), photo-electric heating of gas by dust and star formation in galaxies (Bekki 2015c). Since the details of the code are already given in our previous papers, we just briefly explain the code in the present study. The time $t = 0$ in each simulation corresponds to when massive AGB stars ($m_\text{s} = 8M_\odot$) start to eject gas into ISM of dwarf galaxies through stellar winds.

### 3.1 Dwarf disk galaxy

We consider a gas-rich dwarf disk galaxy as a host of a SCC in the present study. The gas-rich dwarf consists of a dark matter halo, a stellar disk, a gas disk, and a SCC. The dark matter halo, the stellar disk, are all represented by collisionless N-body particles. Hydrodynamics of the gas disk, conversion from gas into new stars (‘star formation’), chemical evolution, and dust formation and evolution are all included in the present study. However, we do not include some dust-related physical processes, such as photo-electric heating and gas-dust drag in this study, because the main purpose of this paper is not to discuss such dust effects on galaxy evolution. The total masses of these components are denoted as $M_\text{h}$, $M_\text{d}$, $M_\text{g}$, and $m_{\text{acc}}$, respectively. The dark matter halo has the ‘NFW’ one (Navarro et al. 1996) density profile with a central cusp predicted by the Cold Dark Matter (CDM) model:

$$
\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2},
$$

where $r$, $r_0$, and $r_s$ are the distance from the center of the cluster, the central density, and the scale-length of the dark halo, respectively. We investigate only the models with $M_\text{h} = 10^{10}M_\odot$ in the present study. The virial radius ($r_{vir}$), the scale radius ($r_s$), and the ‘c’ parameter ($=r_{vir}/r_s$) are chosen such that the values are consistent with recent cosmological simulations for the adopted $M_\text{h}$ (Neto et al. 2007).

The radial ($R$) and vertical ($Z$) density profiles of the stellar and gaseous disks of a dwarf are assumed to be proportional to $\exp(-R/R_0)$ with scale length $R_0 = 0.2R_s$ and to $\sech^2(Z/Z_0)$ with scale length $Z_0 = 0.04R_s$ respectively. Although the gas mass fraction ($f_{\text{g}} = M_\text{g}/M_\text{h}$) is assumed to be a free parameter ($0 \leq f_{\text{g}} \leq 1$), we mainly investigate the models with $f_{\text{g}} = 0$ (i.e., ‘without cold ISM’), because we intend to understand the accretion of AGB ejecta onto main clusters in SCCs more clearly. In addition to the rotational motion caused by the gravitational field of disk and dark halo components, the initial radial and azimuthal velocity dispersions are assigned to the disc component according to the epicyclic theory with Toomre’s parameter $Q = 1.5$. The vertical velocity dispersion at a given radius is set to be 0.5 times as large as the radial velocity dispersion at that point.

Star formation from gas in a dwarf galaxy is included as follows. We assume that the following three physical conditions need to be met for each gas particle to be converted.
into a new star. First is that the local density ($\rho$) exceeds a threshold density ($\rho_{th}$) for star formation:

$$\rho_s > \rho_{th},$$

where $\rho_{th}$ is set to be 100 H atoms cm$^{-3}$. Second is that the local dynamical time scale is shorter than the sound crossing time scale, which mimics the Jeans instability in the gas disk. Third is that the local velocity field is consistent with that expected for dwarf galaxies with high-z more compact model. Thus we do not discuss much about secondary star formation within GCs in the present paper.

Initial gaseous metallicities of gas particles in a dwarf are set to be $-1.6$ for all models, and there is no radial metallicity gradient in the dwarf. Metallicity-dependent radiative cooling is self-consistently modeled according to the metallicities of gas particles. Chemical enrichment processes, dust formation and evolution in ISM, and SN feedback effects are included in the same way as done in our previous simulations (Bekki 2013; Bekki 15b,c). However, the details of these modeling are not so important in the present simulations, because we mainly investigate gas accretion onto massive SCs only for $\sim 300$ Myr.

We investigate dwarf galaxy models with a fixed $M_h = 10^{10} M_\odot$ yet different $M_c$ and $f_g$. The eight dwarf models are investigated, and the parameter values are given in Table 1. The total number of particles for dark matter, stars, gas (inclusive of AGB ejecta), and a main cluster in a model are 500000, 500000, 150000, and 10000, respectively. These four components have different initial gravitational softening lengths ($\epsilon$) according to their initial halo-number radius for each component. For example, $\epsilon$ is set to be 194pc for dark matter, 14.8 pc for stars and gas, and 0.47 pc for the main cluster in the dwarf model DW1. The high-z dwarf models (DW6, 7, and 8) have more compact distributions of baryon and dark matter, and the mean density of dark matter halo is consistent with that expected for dwarf galaxies with $M_h = 10^{10} M_\odot$ at $z = 2$.

### 3.2 SCC

A SCC is assumed to consist of one main cluster represented by the Plummer model with a size of $r_{mc}$ and a scale length of $0.2r_{mc}$ and numerous low-mass clusters by point-mass

### Table 2. The basic model parameters for SCCs.

| Model ID | Dwarf type | $m_{mc}$ | $r_{mc}$ | $R_{acc}$ | $f_{agg}$ | $r_{scc}$ | comments |
|----------|------------|----------|----------|-----------|-----------|-----------|----------|
| M1       | DW1        | 0.5      | 20       | 1.0       | 0.05      | 200       | fiducial |
| M2       | DW1        | 0.2      | 20       | 1.0       | 0.05      | 200       |          |
| M3       | DW1        | 1.0      | 20       | 1.0       | 0.05      | 200       |          |
| M4       | DW1        | 0.5      | 20       | 0.01      | 0.05      | 200       | nuclear SC model |
| M5       | DW1        | 0.5      | 20       | 0.3       | 0.05      | 200       |          |
| M6       | DW1        | 0.5      | 10       | 1.0       | 0.05      | 200       |          |
| M7       | DW1        | 0.5      | 50       | 1.0       | 0.05      | 200       |          |
| M8       | DW1        | 0.5      | 20       | 1.0       | 0.02      | 200       |          |
| M9       | DW1        | 0.5      | 20       | 1.0       | 0.1       | 200       | a more top-heavy IMF |
| M10      | DW2        | 0.5      | 20       | 1.0       | 0.05      | 200       |          |
| M11      | DW6        | 0.5      | 20       | 0.57      | 0.05      | 200       |          |
| M12      | DW1        | 0.5      | 20       | 1.0       | 0.05      | 200       | $1.2 \times 10^2 \leq m_{sc}/M_\odot \leq 3.6 \times 10^3$ |
| M13      | DW1        | 0.5      | 20       | 1.0       | 0.05      | 200       | $1.2 \times 10^2 \leq m_{sc}/M_\odot \leq 1.2 \times 10^5$ |
| M14      | DW1        | 0.5      | 20       | 1.0       | 0.05      | 200       | $\beta = 0$ ($m_{sc} = 1.2 \times 10^3 M_\odot$ for all SCs) |
| M15      | DW1        | 0.5      | 20       | 1.0       | 0.05      | 100       |          |
| M16      | DW1        | 0.1      | 10       | 1.0       | 0.05      | 200       |          |
| M17      | DW1        | 0.03     | 20       | 1.0       | 0.05      | 200       |          |
| M18      | DW1        | 0.03     | 20       | 1.0       | 0.05      | 200       | $3.6 \times 10 \leq m_{sc}/M_\odot \leq 1.2 \times 10^4$ |
| M19      | DW4        | 0.5      | 20       | 1.0       | 0.05      | 200       | Fiducial with cold ISM |
| M20      | DW4        | 0.2      | 20       | 1.0       | 0.05      | 200       |          |
| M21      | DW4        | 1.0      | 20       | 1.0       | 0.05      | 200       |          |
| M22      | DW4        | 0.5      | 20       | 1.0       | 0.05      | 200       | a smaller gas hole with $r_h = 200$ pc |
| M23      | DW5        | 0.5      | 20       | 1.0       | 0.05      | 200       | gas-poor host dwarf |
| M24      | DW7        | 0.5      | 20       | 0.57      | 0.05      | 200       | high-z more compact model |
| M25      | DW3        | 0.5      | 20       | 1.0       | 0.05      | 200       | low stellar density of the dwarf |
| M26      | DW4        | 0.5      | 20       | 1.0       | 0.05      | 200       |          |
| M27      | DW8        | 0.5      | 20       | 0.57      | 0.05      | 200       | high-Z, lower stellar density of the dwarf |

$^a$ The parameter values for each dwarf model is given in Table 1.

$^b$ The initial total mass for the main cluster in units of $10^6 M_\odot$.

$^c$ The initial size of the main cluster in units of pc. The scale length ($a_{mc}$) is roughly $0.2r_{mc}$.

$^d$ The initial distance of the SCC from the dwarf galaxy’s center in units of pc. For high-z dwarf models, $R_{acc}$ is chosen ($=0.57$) such that $R_{acc}/R_s$ is the same between high-z and other models with $R_{acc} = 1$ kpc.

$^e$ The mass fraction of AGB ejecta to the initial mass of a SC ($m_{sc}$).

$^f$ The initial size of a SCC in units of pc.
particles. It would be possible that one SCC has a few massive star clusters \((m_{\text{sc}} > 10^3 M_{\odot})\), where we investigate a case where one SCC has only one massive main cluster, because we can more clearly understand the roles of other clusters in the gas accretion on main clusters in SCCs without introducing other model parameters. The SCC has a power-law cluster mass function as follows,

\[ N(m_{\text{sc}}) = N_0 m_{\text{sc}}^{-\beta} \]  

where \(\beta = 2\), which is expected from hierarchical star formation \((e.g., \text{Elmegreen} 2008 \text{ for a review})\) and \(N_0\) is a constant. The mass of the main cluster is a free parameter denoted by \(m_{\text{sc}}\), and the lower and upper mass cut-offs are denoted by \(m_1\) and \(m_u\), respectively. The values of \(m_1\) and \(m_u\) are set to be \(1.2 \times 10^3 M_{\odot}\) and \(3.6 \times 10^4 M_{\odot}\), respectively, for most models in the present study. Low-mass clusters in the SCC are distributed within a sphere with a radius \(r_{\text{sc}}\) (\(=\) SCC size). The initial 3D velocities of each cluster is chosen such that they can be the same as those of a field star (of the host dwarf galaxy) that is closest to the cluster. The initial distribution of SCs within \(r_{\text{sc}} = 200\) pc is shown in Fig. 2.

The SCC initially embedded in a giant gas hole is assumed to orbit around the center of a dwarf galaxy. The SCC is initially located at \((x, y, z) = (R_{\text{sc}}, 0, 0)\), where the 3D coordinate of the host dwarf’s center is set to be \((0, 0, 0)\) and \(R_{\text{sc}}\) is a parameter that controls the initial distance of the SCC from the dwarf’s center. The SCC is assumed to have a circular orbit within the dwarf disk and the circular velocity is determined by the mass distribution of the dwarf. The gas hole is assumed to be created by energetic feedback effects of SNe of the SCC itself. Such a giant gas hole is observed in the LMC \((e.g., \text{LMC} 4)\) and could have been formed as a result of energetic feedback effects such as SNe explosion. The gas hole is assumed to have a circular shape and its radius is a free parameter defined by \(r_h\). No cold gas of the host dwarf galaxy is assumed to exist initially within \(r_h\).

We mainly investigate the evolution of SCCs with the initial stellar masses \((M_{\text{sc}})\) of \(\sim 10^7 M_{\odot}\), because more than \(10^3 M_{\odot}\) gas can be accumulated in such massive SCCs. The initial masses of gas clumps \((M_{\text{clump}})\) hosting such SCCs are described as follows:

\[ M_{\text{clump}} = 10^8 \left( \frac{\epsilon_{\text{sc}}}{0.1} \right)^{-1} \left( \frac{M_{\text{sc}}}{10^7 M_{\odot}} \right) M_{\odot}, \]  

where \(\epsilon_{\text{sc}}\) is the formation efficiency of a SCC within a gas clump. If \(\epsilon_{\text{sc}}\) corresponds to cluster formation efficiency (CFE), then the original gas mass of a GC with multiple stellar populations can be \(\sim 10^5 M_{\odot}\) for a reasonable CFE of 0.1 \((e.g., \text{Adamo} \ et \ al. \ 2015; \text{Johnson} \ et \ al. \ 2016)\). Although this means that the original gas clumps should be rather massive, the adopted low CFE of 0.1 does not meet the physical conditions \((i.e.,\ CFE \text{ significantly higher than } 0.1)\) required for the formation of bound SCs \((e.g., \text{Hills} 1980)\). Accordingly, the original clump masses of GCs can be significantly lower than the above \(\sim 10^5 M_{\odot}\).

The present SCC model is more realistic than previous GC formation models with a single massive cluster, and it allows us to investigate the roles of hierarchical star cluster complexes in GC formation. However, the model is still less realistic at some points \((i.e.,\ central \text{ massive clusters within SCCs})\) and needs to be improved in our future works. It should be stressed that the present model is chosen such that gas accretion processes onto proto-GCs can be investigated in a quantitative manner. Real cluster formation processes from SCCs are more complicated than the present model describes. For example, the observed large fraction of binary clusters in the LMC \((e.g., \text{Bhatia} \ & \text{Hatzidimitriou} 1988)\) cannot be simply explained by the present model, and it can be better explained by other cluster formation models based on GMC collisions \((e.g., \text{Bekki} \ et \ al. \ 2004a)\). Thus we need to discuss these other issues related to cluster formation in our future papers.

### 3.3 Ejection and accretion of gas from AGB stars

The wind velocity \((v_w)\) of gas ejected from AGB stars is assumed to be \(10 \text{ km s}^{-1}\), which is consistent with the observed value for low-metallicity AGB stars in the LMC \((e.g., \text{Marshall} \ et \ al. \ 2004)\). A gas particle \(\text{('AGB particle')}\) is ejected with \(v_w\) from each SC particle only once and the initial total mass of the gas particle is \(f_{\text{agb}} m_{\text{sc}}\), where \(f_{\text{agb}}\) is the mass fraction of AGB ejecta and \(m_{\text{sc}}\) is the initial total mass of the SC. This \(f_{\text{agb}}\) depends on the initial mass function (IMF) of stars \((B11)\). In the present study, the IMF for a stellar system is assumed to have a power-law function with a slope of \(\alpha\) as follows:

\[ \psi(m_s) = M_0 m_s^{-\alpha}, \]  

where \(M_0\) is a constant derived from the total mass of the stellar system and \(m_0\) is a stellar mass \((0.1 \leq m_0 \leq 100)\) and \(\alpha = 2.35\) corresponds to the canonical (Salpeter) IMF. We adopt the following relation between the initial stellar mass \((m_{\text{初}})\) and the total mass of gas ejected from the star \((m_{\text{ej}})\) \((\text{Weidemann} 2000; B11)\):

\[ m_{\text{ej}} = 0.916 m_{\text{初}} - 0.444. \]  

For the canonical IMF with \(\alpha = 2.35\), \(f_{\text{agb}}\) can be as large as 0.1 about 300 Myr after the initial burst of star formation in GC formation. Furthermore, \(f_{\text{agb}}\) can be larger for more top-heavy IMF with \(\alpha\) smaller than 2.35 \((\text{See Fig 1 in B11})\). Since we investigate the evolution of SCCs only for \(\sim 300\) Myr in the present study, it is appropriate to choose \(f_{\text{agb}}\) for stars that can commence their AGB phases within 300 Myr. Guided by the results of \(f_{\text{agb}}\) evolution shown in B11, we mainly investigate the models with \(f_{\text{agb}} = 0.05\). The results of other models with \(f_{\text{agb}} = 0.1\) and 0.03 are briefly discussed.

It is highly unrealistic to assume that only one SCC is formed in a dwarf galaxy at a given time. Accordingly, there should be many AGB stars evolving from field stars that form almost simultaneously with the SCC. We therefore assume that 10% of field stars are also ejecting gas through AGB winds. These AGB ejecta from field stellar populations can not contribute significantly to the gas accretion onto the main cluster of the SCC; we have confirmed this through a comparative experiment in which no field AGB stars are included. However, it should be stressed that AGB ejecta from field stellar populations can interact with AGB wind of the SCC. Therefore, the dynamical evolution of AGB ejecta from the SCC can be slightly influenced by the AGB ejecta from the field stellar populations of the SCC-hosting dwarf.

In order to estimate the time evolution of the accretion rate \((M_{\text{acc}})\) of AGB ejecta for the main cluster of a SCC in a...
onto the main cluster ($M_{\text{acc}}$, middle), and the accretion rate ($dM_{\text{acc}}/dt$; bottom) for the fiducial model M1. The filled square indicates the initial position of the main cluster and the two filled triangles indicate the location of the cluster at $T = 0.1$ and 0.2 Gyr in the top panel. The main cluster sinks into the inner region owing to dynamical friction of the cluster against the disk field stars. The dotted line in the middle panel indicates the maximum possible gas mass that can be accreted onto the main cluster from AGB stars of the cluster itself (‘self-accretion’).

simulation, we count the number of AGB particles that are within the cluster radius ($r_{\text{acc}}$) and have velocities less than the escape velocity of the cluster. Accordingly, we estimate the relative velocity of a AGB particle with respect to the main cluster ($v_{\text{rel}}$) at each time step. The AGB ejecta is regarded as being accreted onto the main cluster, if it meets the following condition:

\[ v_{\text{rel}} < v_{\text{esc}}(\phi(r, t)), \]

where $v_{\text{esc}}$ is the escape velocity of the main cluster, $\phi(r, t)$ is the gravitational potential of the main cluster at the distance $r$ from the cluster’s center at time $t$, $\phi$ is dependent on time and place owing to the mass loss by the tidal field of the host dwarf, and $r$ is the distance of the accreted AGB ejecta (i.e., $r < r_{\text{esc}}$) from the center of the main cluster. By estimating both $v_{\text{rel}}$ and $\phi$ at each time step, we avoid counting AGB particles that happen to be within $r_{\text{esc}}$ yet are not gravitationally trapped by the cluster.

We assume that these AGB particles gravitationally trapped by the main cluster can be finally used for secondary star formation within the cluster. The total mass of gas accreted on the main cluster ($M_{\text{acc}}$) is estimated as follows:

\[ M_{\text{acc}}(t) = \sum_{i=1}^{\text{nstep}} M_{\text{acc},i} dt_i \]

where $M_{\text{acc},i}$ ($dM_{\text{acc}}/dt$) is the gas accretion rate, $dt_i$ is the time step width, and nstep is the number of time steps for which $M_{\text{acc}}$ is estimated in each simulation. Once AGB particles are gravitationally trapped by the main cluster and its accretion rate is estimated at $T = t_i$, then these particles are not used for the estimation of gas accretion rates in the following time steps ($T > t_i$). The total mass of the accreted gas is either from AGB ($M_{\text{acc},\text{agb}}$) or from ISM ($M_{\text{acc},\text{ism}}$) in the present study, and they can be separately estimated:

\[ M_{\text{acc}} = M_{\text{acc},\text{agb}} + M_{\text{acc},\text{ism}}. \]

It is possible that not only AGB stars initially in the main cluster and other low-mass clusters of a SCC but also field AGB stars that form simultaneously with the SCC can contribute to $M_{\text{acc},\text{agb}}$. Therefore, $M_{\text{acc},\text{agb}}$ is further described by three terms as follows:

\[ M_{\text{acc},\text{agb}} = M_{\text{acc},\text{agb},\text{mc}} + M_{\text{acc},\text{agb},\text{sc}} + M_{\text{acc},\text{agb},f}, \]

where $M_{\text{acc},\text{agb},\text{mc}}$, $M_{\text{acc},\text{agb},\text{sc}}$, and $M_{\text{acc},\text{agb},f}$ are the accreted AGB ejecta from the main cluster of a SCC, low-mass clusters of the SCC, and field AGB stars, respectively. The contribution of field AGB stars is very minor in the present study. Bekki & Mackey (2009) and Pflamm-Altenburg & Kroupa (2009) investigated how star clusters can capture cold molecular gas or accrete ISM in galaxies using idealized modeling of the accretion process. The present work and theirs are therefore complementary to each other.

3.4 A parameter study

We first investigate the models without cold ISM of gas-rich dwarf galaxies in order to show more clearly the physical process of the accretion of AGB ejecta onto the main clusters. We then investigate the models with cold gas of dwarf galaxies in order to estimate the total gas accretion rates. Since the key parameters in the present study are $m_{\text{acc}}$, $R_{\text{acc}}$, and $f_{\text{agb}}$, we mainly discuss how the present results depend on these parameters. We only briefly discuss the importance of other parameters (e.g., $r_{\text{esc}}$ and dwarf structures) in the gas accretion processes in the present study. The values of the model parameters are summarized for each model in Table 2.
are accreted onto the main cluster at $T = 0$ (left) and $T = 0.28$ Gyr (right) in the fiducial model.

The derived short timescale of dynamical friction of the SCC itself against disk field stars of the dwarf. The SCC with a mass of $\sim 10^7 M_\odot$ can rapidly spiral into the nuclear region of the dwarf as long as it is not disintegrated by the dwarf’s tidal field. The main cluster can follow the orbit of the SCC so that the timescale of the main cluster’s spiraling-in can be quite short. Although AGB ejecta from low-mass clusters can not be gravitationally trapped by themselves owing to their low escape velocities ($< 10$ km s$^{-1}$), it can be trapped within the SCC with a much deeper gravitational potential. Therefore, the following condition is met in the SCC:

$$v_{\text{esc,sc}} > v_w > v_{\text{esc,sc}},$$

where $v_w$ is the wind velocity of AGB stars, $v_{\text{esc,sc}}$ and $v_{\text{esc,sc}}$ are the escape velocities of the SCC and its member SCs, respectively. A significant fraction of the AGB ejecta could be finally accreted on the main cluster in any SCC that meets the above condition in the new scenario.

The accretion rate is higher in the early evolution of the SCC with the maximum rate of $\sim 0.003 M_\odot$ yr$^{-1}$, mainly because AGB ejecta from numerous low-mass SCs that were born within 50pc from the main cluster can be efficiently and rapidly accreted onto the cluster. The total amount of AGB ejecta accreted onto the main cluster from other SCs can be as large as $8.0 \times 10^4 M_\odot$, which is significantly larger than the maximum possible mass of gas (0.05$m_{\text{esc}}$) that can be accreted onto the cluster from the cluster’s own AGB stars. If all AGB ejecta from this massive main cluster with $m_{\text{esc}} \geq 5 \times 10^5 M_\odot$ can be accreted onto the central region of the cluster (which is highly likely), the total mass of AGB ejecta ($M_{\text{acc}}$) accreted onto the main cluster within 0.28 Gyr can be as large as $1.3 \times 10^5 M_\odot$ in this model. This $M_{\text{acc}}$ is equivalent to the observed total mass of SG stars in typical GCs (Choi).

4 RESULTS

4.1 Without cold ISM

4.1.1 Gas accretion from other clusters

Fig. 3 shows that while the main cluster spirals into the central region of the host dwarf owing to dynamical friction, the main cluster can accrete AGB ejecta steadily from other low-mass SCs initially located in its surrounding in the fiducial model M1. The derived short timescale of dynamical friction of the main cluster appears to be at odds with the mass of just $m_{\text{esc}} = 5 \times 10^5 M_\odot$. However, this rapid spiraling-in can be understood in terms of much more efficient dynamical friction between the SCC and the host dwarf.

The SCC with a mass of $\sim 10^7 M_\odot$ can rapidly spiral into the nuclear region of the dwarf as long as it is not disintegrated by the dwarf’s tidal field. The main cluster can follow the orbit of the SCC so that the timescale of the main cluster’s spiraling-in can be quite short. Although AGB ejecta from low-mass clusters can not be gravitationally trapped by themselves owing to their low escape velocities ($< 10$ km s$^{-1}$), it can be trapped within the SCC with a much deeper gravitational potential. Therefore, the following condition is met in the SCC:

$$v_{\text{esc,sc}} > v_w > v_{\text{esc,sc}},$$

where $v_w$ is the wind velocity of AGB stars, $v_{\text{esc,sc}}$ and $v_{\text{esc,sc}}$ are the escape velocities of the SCC and its member SCs, respectively. A significant fraction of the AGB ejecta could be finally accreted on the main cluster in any SCC that meets the above condition in the new scenario.

The accretion rate is higher in the early evolution of the SCC with the maximum rate of $\sim 0.003 M_\odot$ yr$^{-1}$, mainly because AGB ejecta from numerous low-mass SCs that were born within 50pc from the main cluster can be efficiently and rapidly accreted onto the cluster. The total amount of AGB ejecta accreted onto the main cluster from other SCs can be as large as $8.0 \times 10^4 M_\odot$, which is significantly larger than the maximum possible mass of gas (0.05$m_{\text{esc}}$) that can be accreted onto the cluster from the cluster’s own AGB stars. If all AGB ejecta from this massive main cluster with $m_{\text{esc}} \geq 5 \times 10^5 M_\odot$ can be accreted onto the central region of the cluster (which is highly likely), the total mass of AGB ejecta ($M_{\text{acc}}$) accreted onto the main cluster within 0.28 Gyr can be as large as $1.3 \times 10^5 M_\odot$ in this model. This $M_{\text{acc}}$ is equivalent to the observed total mass of SG stars in typical GCs (Choi).

Fig. 4 shows the orbits of ten selected AGB particles that are accreted onto the main cluster by $T = 0.014$ Gyr in the fiducial model. Clearly, the accreted AGB stars originate from different directions with different velocities and angular momentum with respect to the main cluster. Accordingly, if these gas components collide with one another within the main cluster, then they can lose a large amount of their kinetic energy through gaseous dissipation owing to their large velocity differences. These gas accretion processes in the new scenario appear to be quite different from those described in previous simulations (Bekki 2010 and B11) in which AGB ejecta can rapidly form a rotating gas disk within existing massive SCs without much energy dissipation. Such a difference would end up with different histories of star formation (i.e., SG star formation) within SCs, which will need to be investigated in our forthcoming papers.

Fig. 5 demonstrates that most of the initial SCs (71%) can be stripped from the surrounding of the main cluster (i.e., outside 200 pc from the cluster) to become isolated SCs in the disk of the host dwarf within 0.28 Gyr. This means that although AGB ejecta from some SCs can be accreted onto the main cluster before the SCs are stripped, other SCs simply lose their AGB ejecta to the field of the host galaxy. Interestingly, SCs with the total mass of $3.3 \times 10^4 M_\odot$ can remain within 5 pc from the cluster at $T = 0.28$ Gyr. These
SCs merged with the cluster during the dynamical evolution of the SCC so that they can finally become a part of the cluster. The mass increase of the main cluster due to this merging is not so significant in this fiducial model.

Since SCs are represented by point-mass particles in the present simulation, the merging of the SCs with the main cluster and the subsequent destruction of them can not be investigated in detail. It is, however, very likely that most of the SCs are tidally destroyed to form diffuse stellar halo around the cluster because SCs follow the following mass-size relation (e.g., Zepf et al. 1999; Larsen 2004):

\[ r_{sc} = C_0 m_{sc}^{0.1}, \]

where \( C_0 \) is a constant. This means that the mass density of stars is lower for clusters with lower masses as follows:

\[ \rho \propto m_{sc}^{-7}. \]

Therefore, low-mass clusters in a SCC can be completely destroyed by the main cluster so that their stars can form diffuse stellar halos around the main cluster. These results are discussed in §5.3 in the context of the observed outer halos of GCs.

If all of the low-mass clusters within 5pc from the main cluster at \( T = 0.28 \) Gyr become the stellar halo in the fiducial model, then the mass fraction of the halo to the main cluster is \( \sim 0.06 \), which is much larger than the observed mass fractions (e.g., 0.001 for NGC 1851) of stellar halos in GCs (See Bekki & Yong 2012 for discussion on this issue). Therefore, the present study predicts that (i) stellar halos of the Galactic GCs were initially denser and more massive and (ii) they have lost the vast majority of their masses owing to tidal stripping by the Galaxy by now. It might be a formidable task for future observations to detect such a denser and more massive stellar halo around a GC at higher redshifts. If young massive clusters in nearby galaxies have such stellar halos, then they might have been formed as a results of cluster merging in their host cluster complexes.

The maximum possible mass of the accreted AGB ejecta (both from the main cluster and other low-mass ones) within 0.28 Gyr is \( 1.3 \times 10^5 M_\odot \) in the fiducial model. This is already similar to the present-day typical mass of SG stars in the Galactic GCs with Na-O anti-correlations (C09). In the present scenario, M acc of AGB ejecta and cold ISM can become as large as \([2-3] \times 10^5 M_\odot \) for a reasonable set of model parameters, as described in the following section (§4.2). The main cluster can lose a significant fraction of its stellar mass through (i) mass loss in AGB phases and (ii) stripping of stars by the tidal field of its host galaxy or other luminous galaxies (e.g., the Galaxy). The final (i.e., present-day) mass of the main cluster \( (m_{mc,f}) \) is therefore as follows:

\[ m_{mc,f} = (1 - f_{strip})(1 - f_{ej})m_{mc}. \]

where \( f_{strip} \) is the mass fraction of stars stripped from the main cluster and \( f_{ej} \) is the fraction of stellar mass that is lost through stellar winds in AGB phases. Since \( m_{mc} \) is defined as the total mass of the main cluster when high-mass AGB stars start to eject gas (i.e., not initial cluster mass before the loss of massive stars through SN explosions), \( f_{ej} \) is \( \sim 0.4 \) for \( \alpha = 2.35 \) and \( \sim 0.6 \) for \( \alpha = 2.05 \) (i.e., a top-heavy IMF). An appropriate value of \( f_{strip} \) is 0.4-0.5 for \( m_{mc} = 5 \times 10^5 M_\odot \) at \( R = 5 \) kpc from the center of the Galaxy. Thus, if we adopt \( f_{strip} = 0.5 \) and \( f_{ej} = 0.4 \), then

\[ m_{mc,f} \approx 0.3m_{mc}. \]

It should be noted here that since gaseous winds from all AGB stars with different masses are assumed to be ejected from SCs in this estimation of \( f_{ej} \), this \( f_{ej} \) is different from that in the equation (1).

This means that if almost all of the accreted gas can be converted into new stars (i.e., SG stars) and if the SG stars are not tidally stripped, then the mass fraction of SG stars can be significant depending on the IMF of the SG stars. For example, the present-day mass of a main cluster (FG stars) with \( m_{mc} = 5 \times 10^5 M_\odot \) is \( m_{mc,f} = 1.5 \times 10^5 M_\odot \) whereas the total mass of SG stars formed from the accreted gas can be as large as \( 2.0 \times 10^5 M_\odot \) for \( m_{mc,f} = 3.0 \times 10^5 M_\odot \); if only low-mass stars are formed owing to a bottom-heavy IMF. This means that the mass budget problem is much less severe in this scenario. Since this discussion is a bit qualitative, we will need to investigate the mass budget problem in this scenario more qualitatively using a model with different IMFs for FG and SG stars and a reasonable range of \( f_{strip} \) in our future papers.

### 4.1.2 Parameter dependence

Fig. 6 shows the following four dependences of \( M_{acc} \) on \( m_{mc} \) (main cluster’s mass), \( r_{acc} \) (SCC’s initial position), \( r_{mc} \) (main cluster’s size), and \( f_{sgb} \) (AGB ejecta mass fraction). First, more massive main clusters can accrete a larger amount of AGB eject (i.e., larger \( M_{acc} \)), though the ratios of \( M_{acc} \) to \( m_{mc} \) are not so different between the three models. This \( m_{mc} \)-dependent result is expected, because more massive main clusters have deeper gravitational potentials by which gas can be more efficiently trapped. Second, there is no significant difference in \( M_{acc} \) between the three models with different \( R_{acc} \). In the earlier phases of SCC evolution, AGB ejecta is more efficiently stripped from the SCC in the model with smaller \( R_{acc} \). However, \( M_{acc} \) can finally become the largest in the model in which the SCC is born in the nuclear region of the host dwarf. Third, main clusters with larger sizes are more likely to accrete more gas from surrounding SCs. This is mainly because the accretion radius for a main cluster is set to be the same as the cluster size (i.e., physical condition for gas accretion is less strict in the models with large \( r_{mc} \)). Fourth, the main cluster in the model with larger \( f_{sgb} (=0.1) \) expected from a more top-heavy IMF can accrete more gas from other SCs. This implies that the IMF of FG stars can determine the total mass of SG stars.

The mass distributions of SCC-host dwarfs, the lower and upper mass cut-offs of cluster mass function \( (m_1 \text{ and } m_u \text{, respectively}) \), the power-law slope of the cluster mass function \( (\beta) \), SCC sizes \( (r_{acc}) \) can determine \( M_{acc} \). Fig. 7 illustrates how \( M_{acc} \) depends on these parameters. First, the high-z dwarf model with a more compact disk shows larger \( M_{acc} \) whereas the model with a lower stellar density of the dwarf disk shows lower \( M_{acc} \). These are probably because SCCs are more strongly bound with the deeper gravitational potential well so that they can capture more gas from other low-mass SCs. These results imply that the mass densities of SCC-host dwarfs can determine \( M_{acc} \) of AGB ejecta in main clusters of SCCs. Second, the model with a larger number of very low-mass clusters (i.e., \( m_1 = 1.2 \times 10^2 M_\odot \) shows...
Figure 6. Total mass of AGB ejected accreted onto the main cluster ($M_{\text{acc}}$) for different models without cold ISM: different $m_{\text{mc}}$ (M1, M2, M3; upper left), $R_{\text{scc}}$ (M1, M4, M5; upper right), $r_{\text{mc}}$ (M1, M6, M7; lower left), and $f_{\text{agb}}$ (M1, M8, M9; lower right). Different colors and types are used to distinguish between the three models. For example, blue solid for M2, red dotted for M1, and green dashed for M3 in the upper left panel.

lower $M_{\text{acc}}$. This is firstly because the total mass of the SCs is lower (thus the total mass of AGB ejecta is lower), and secondly because these clusters can be quickly dispersed into the field of the host dwarf. The upper mass cut-off ($m_u$) does not influence the final $M_{\text{acc}}$ so much.

Third, if all SCs have the same masses of $1.2 \times 10^3 M_\odot$ (i.e., $\beta = 0$), then $M_{\text{acc}}$ becomes significantly smaller than the fiducial model with $\beta = 2$. This suggests that hierarchical cluster distribution can be important for the gas accretion process. Interestingly, the model with smaller $r_{\text{sc}}$ shows larger $M_{\text{acc}}$, which could be due to the initial more compact distribution of low-mass SCs that can donate gas to the main cluster. Fourth, very low-mass clusters with $m_{\text{mc}} = 10^4 M_\odot$ and $m_{\text{mc}} = 10^5 M_\odot$ can have small $M_{\text{acc}}$, as expected from their shallow gravitational potentials. However, it should be stressed here that the model with $m_{\text{mc}} = 10^4 M_\odot$ and $3.6 \times 10^4 M_\odot \leq m_{\text{sc}} \leq 1.2 \times 10^5 M_\odot$, can accrete AGB ejecta with $M_{\text{acc}} = 2.5 \times 10^4 M_\odot$. On the other hand, the model with $m_{\text{mc}} = 10^5 M_\odot$ and $1.2 \times 10^5 M_\odot \leq m_{\text{sc}} \leq 3.6 \times 10^5 M_\odot$ can not accrete AGB ejecta at all, because the main cluster is less massive than a significant fraction of SCs; the main cluster can not 'steal' gas from SCs more massive than the cluster. These result imply that even low-mass clusters can have multiple stellar populations, if they are embedded in clusters of very low-mass clusters.

4.2 With cold ISM

4.3 Later accretion of cold ISM

Fig. 8 shows that the gas accretion onto the main cluster is dominated by that of AGB ejecta in the early evolution of the SCC ($T < 0.1$ Gyr) in the fiducial model with cold ISM (M19). Owing to the large gas hole in the initial gas disk of the dwarf in this model, cold ISM, which was escaped from the energetic influence of multiple SNe and thus from chemical pollution by the ejecta of SNe, can start to interact with the SCC later ($T > 0.1$ Gyr). As a result of this, the rate of gas accretion of cold ISM onto the main cluster can dramatically increase around $T = 0.1$ Gyr and shows multiple peaks after $T = 0.1$ Gyr. The gas accretion rate of cold ISM can become almost always higher than that of AGB ejecta after $T = 0.1$ Gyr, and the total mass of the accreted gas can be $2.6 \times 10^5 M_\odot$ at $T = 0.28$ Gyr, which is by a factor of five larger than the maximum possible $M_{\text{acc}}$ expected from self-accretion of the main cluster itself.

If star formation is possible from the accreted gas in the early evolution of this SCC ($T < 0.1$ Gyr), then new (SG) stars can have chemical abundances determined by chemical yields of AGB ejecta, because no dilution of the ejecta by ISM is possible. Accordingly, the new stars formed earlier can have Na-enhanced chemical abundance patterns. For example, if the new SG stars are formed from ejecta of AGB stars with $m_s = 5 M_\odot$ and $Z = 3 \times 10^{-4}$, then $[\text{Na/Fe}]=+0.2$.
and [O/Fe] = −0.5 are expected for the SG stars for the AGB yield table provided by Ventura et al. (2013). However, Na-enhanced stars do not always show O-depletion, if they are formed from AGB ejecta (See Fig. 6 in Ventura et al. 2013). In the later evolution phase of this SCC, the total mass of the accreted ISM becomes significantly larger than that of the accreted AGB eject. Therefore, the chemical abundances of new stars formed later can be similar to those of ISM and FG stars (i.e., existing stars of the main cluster): these new stars should have Na-normal abundance patterns. Thus, the older SG stars have higher [Na/Fe] than the younger SG stars in this scenario (surely older and younger SG stars are younger than FG stars). Chemical abundances of cold ISM initially outside the gas hole of the SCC are assumed to be the same as those of FG stars in the main cluster in this scenario. However, it is possible that the chemical abundances of the ISM could have changed significantly over 0.2 Gyr (i.e., time lag between the formation of the SCC and the commencement of ISM accretion). If the metallicity (e.g., [Fe/H]) evolution due to star formation in ISM over 0.2 Gyr is well less than 0.05 dex, then this scenario would be still viable. This point will needs to be addressed in our future papers.

4.4 Parameter dependence

Fig. 9 demonstrates that $M_{\text{acc}}$ in the models M19-M24 can be significantly different depending on $m_{\text{acc}}$, $r_h$, $f_g$, and the stellar distributions of dwarfs: $M_{\text{acc}}$ depends on these parameters as follows. First, $M_{\text{acc}}$ can be larger in the model with larger $m_{\text{acc}}$, as the models without cold ISM, though the ratio of $M_{\text{acc}}$ to $m_{\text{acc}}$ is not so different between the three models with different $m_{\text{acc}}$ (M19, 20, and 21). It should be noted here that $M_{\text{acc}}$ can be $1.5 \times 10^5 M_\odot$ (corresponding to the total stellar mass of SG stars in a typical Galactic GC) in the main cluster of the model with $m_{\text{acc}} = 2 \times 10^5 M_\odot$. Second, the model with a smaller gas hole ($r_h = 200$ pc) shows large $M_{\text{acc}}$ and earlier commencement of ISM accretion onto the main cluster. The density of ISM around the main cluster can become higher from the early evolution phase of the SCC in this model so that the Bondi-type gas accretion (Bondi 1952) in the main cluster can become quite efficient in this model.

Third, $M_{\text{acc}}$ is lower in the model with a lower gas fraction (thus lower gas density) of the dwarf, because the efficiency of the Bondi-type gas accretion depends on the density of gas surrounding the accreting object. This result suggests that the gas mass fraction of a gas-rich dwarf galaxy can determine the mass fraction of SG stars in a GC formed in the galaxy. Fourth, the main cluster in the high-z more compact dwarf model can have $M_{\text{acc}}$ larger than that in the fiducial (less compact) model with cold ISM. It is confirmed that the models M25, M26, and M27, in which dynamical friction timescale is longer owing to the lower stellar mass densities of SCC-hosting dwarfs, show similar amount of $M_{\text{acc}}$: M27 model shows $M_{\text{acc}} = 2 \times 10^5 M_\odot$.

The large $M_{\text{acc}}$ ($\sim [0.2 - 0.3] \times m_{\text{acc}}$) derived in some models with ISM implies that the mass budget problem is
much less serious in the GC formation scenario from SCCs. However, the scenario has not yet provided a complete solution to the problem, because the physical origins for the required high star formation efficiency in SG star formation and preferential formation of low-mass SG stars are not clear in the scenario. Furthermore, it remains unclear whether $M_{\text{acc,agb}}$, $M_{\text{acc,ism}}$, and the time evolution of these derived in the present simulations can explain the observed distributions of [Na/Fe] and [O/Fe] in the Galactic GCs with multiple stellar populations, if SG stars can form from the mixed gas of AGB ejecta and ISM. These will need to be investigated extensively in our future studies.

5 DISCUSSION

5.1 The mass budget problem

In the present new GC formation scenario from SCCs, the main cluster within a SCC can accrete more than $10^5 M_\odot$ gas from AGB stars of the SCC, if $M_{\text{acc}}$ is as large as $10^7 M_\odot$. This means that if SG stars can be formed from the gas with a very high star formation efficiency, the mass budget problem can be much less severe in the scenario. However, the timescale of gas accretion onto forming GCs is quite long ($\sim 10^8$ yr), which is longer than the lifetimes of massive stars that explode as SNe. Therefore, it is possible that SNe from SG stars can truncate gas accretion onto forming GCs owing to their energetic feedback effects. This possible truncation of SG star formation by earlier SNe of SG stars themselves was already pointed by D’Ercole et al. (2010). Thus, the mass budget problem can be less severe in the new scenario only if the IMF of SG stars is top-light: the upper mass cutoff of the IMF should be quite low so as to suppress the formation of massive SNe. Our forthcoming papers based on the GC formation scenario will discuss this IMF issue in detail.

The new SCC scenario does not need to assume very massive single star clusters ($m_{\text{sc}} \gtrsim 4 \times 10^6 M_\odot$) as progenitor of GCs. Accordingly, efficient stripping of almost all (more than 90%) FG stars by GC host galaxies is not required either to explain the observed large fraction (70%) of SG stars in the Galactic GCs. FG stars can be more efficiently stripped by GC host galaxies than SG stars in the new scenario too, but the difference in the mass fractions of stripped stars between FG and SG stars is much smaller in the new scenario: only a factor of two decrease in mass is required to explain the present-day typical GC mass and the fraction of SG stars. Such a moderate mass decrease is consistent with theoretical predictions on the fraction of GC stars stripped by the Galaxy (e.g., Vesperini 1997; Rossi et al. 2016). The new scenario, however, still assumes that the initial total mass of a SCC is large ($M_{\text{acc}} = 10^6 - 10^7 M_\odot$) owing to its formation within a massive GMC or a GMC complex. Therefore, it appears to have a problem in explaining the observed small fraction of field stars with metallicities similar to those of GCs in the Fornax dwarf galaxy (Larsen et al. 2012).

The Fornax dwarf galaxy might have interacted violently with the Galaxy and even merged with other dwarf galaxies in the Galactic halo (e.g., Coleman et al. 2005; Yozin & Bekki 2012; del Pino et al. 2015). Therefore, it might have lost a large fraction of its initial dark matter and field stars through the past interaction and merging. Unlike field stars, GCs can sink into the central region of its host dwarf through dynamical friction against the host’s field stars so that they can become closer to the host’s center. It is thus possible that field stars of the Fornax dwarf have been preferentially lost to the Galactic halo through tidal stripping. Even if the Fornax dwarf initially had FG stars stripped from their GCs, such FG stars could have already stripped from the dwarf to form the Galactic halo field stars. Thus, the observed small fraction of low-metallicity field stars in the Fornax dwarf (Larsen et al. 2012) would not be a serious problem for the new scenario. Since the above discussion is only qualitative and slightly speculative, we will need to investigate the evolution of the mass fraction of low-metallicity field stars from

![Figure 8. The same as Fig.3 but for the model M19 for which the model parameters are exactly the same as the fiducial model M1 except $M_g = 0.1$ in this model. The total gas (blue solid), cold ISM (red dotted), and AGB ejecta (green dashed) that are accreted onto the main cluster are separately shown in this figure.](image)

![Figure 9. Evolution of $M_{\text{acc}}$ for different models with cold ISM, M20 (blue solid), M19 (red dotted), M21 (green short-dashed), M22 (cyan long-dashed), M23 (magenta dot-short-dashed), and M24 (dot-long-dashed).](image)
original GCs in the Fornax for the realistic 3D orbits of the Fornax around the Galaxy.

Niederhofer et al. (2016) have recently discovered chemical abundance spreads in intermediate-age GCs (Lindsay 1, NGC 416, and NGC 339) with ages ranging from 6 Gyr to 7.5 Gyr in the Magellanic Clouds. This discovery strongly suggests that multiple stellar populations are not limited to old (> 10 Gyr) Galactic GCs, though the mass fractions of SG stars (< 0.45) are lower in these GCs than in the Galactic GCs. The observed mass fractions however required large original masses of these GCs in any self-enrichment scenario. It is not clear, however, whether massive SCCs ($M_{sc} = 10^6 - 10^7 M_\odot$) can be formed within the Magellanic clouds about 7 Gyr in the present scenario. It is our future work to investigate how and why such intermediate-age clusters with multiple stellar populations could be formed within the Magellanic Clouds based on our improved SCC model.

5.2 Dilution of AGB ejecta by cold ISM initially outside giant gaseous holes

Time evolution of gas accretion from AGB stars and cold pristine gas are freely chosen so that the observed anti-correlations between light elements in GCs can be reproduced in previous chemical evolution models of GC formation (e.g., Bekki et al. 2007; D’Ercole et al. 2010). However, such evolution of gas accretion rates can not be so freely changed in the new scenario. SCCs are still embedded in giant gaseous holes when AGB stars start to eject gas through their stellar winds in the scenario. Accordingly, it takes longer time for cold gas initially outside the holes to be accreted onto the main clusters. The time lag between accretion of AGB ejecta and that of cold gas can be longer in more massive GCs, because bigger gaseous holes can form in more massive GCs with a larger number of energetic massive stars and SNe. The new scenario therefore provides the following predictions on the chemical abundances of GCs with multiple stellar populations. First, Na-enhanced stars are slightly older than Na-normal stars in GCs, because AGB ejecta with Na-enhanced abundances can be accreted onto the main clusters earlier than cold gas and then be converted into new stars. Na-normal stars can form only after a significant amount of cold gas is accreted onto the main clusters to dilute the existing AGB ejecta.

Second, a significantly larger amount of pure AGB ejecta accreted onto main clusters can be converted into star formation before the accretion of pristine cold gas in massive GCs, because accretion of cold gas on main clusters is much delayed in SCCs containing more massive main clusters (i.e., due to larger gaseous holes). Stars formed from AGB ejecta that is not mixed with pristine cold gas can have higher helium (He) abundances (Y), because AGB ejecta can have such Y (e.g., Ventura & D’Antona 2005). Therefore, the new scenario can explain the presence of subpopulations with large He abundances observed in NGC 2808 and omega Cen. Third, although low-mass GCs formed from low-mass SCCs can accrete only a small amount of AGB ejecta, they can still accrete some amount of cold gas from ISM in gas-rich dwarfs to form new stars within them. These low-mass GCs can therefore contain SG stars with less enhanced N. These predictions are based on the present hydrodynamical simulations of gas accretion in GCs, and accordingly will need to be confirmed in our future more sophisticated numerical simulations of GC formation with chemical evolution.

It should be noted that ISM diluting AGB ejecta is assumed to have metallicities almost the same as those of FG stars in the above discussion. Since the present study did not investigate the chemical evolution of ISM of a GC-hosting dwarf, it cannot predict the chemical abundances of ISM around forming GCs. It could be possible that cold ISM around a forming GC can have different metallicities owing to active formation of field stars around and within the ISM during the GC formation. It is our future study to investigate whether chemical abundances of ISM around forming GCs can be quite similar to those of FG stars of the GCs. If their chemical abundances are different from those of FG stars by more than 0.05 dex, then we will either discard the present GC formation scenario or need to consider other sources that can dilute AGB ejecta.

5.3 Stellar halos around GCs from destruction of low-mass clusters

Recent observations discovered diffuse stellar halos around a number of GCs, such as NGC 1851 (Olszewski et al. 2010) in Whiting 1 (Carraro et al. 2007), AM 4 (Carraro et al. 2009), NGC 5694 (Correnti et al. 2011), and NGC 1904 (Carballo-Bello & Martínez-Delgado 2011), and M2 (Kuzma et al. 2016). The stellar halos around NGC 1851 and M2 are rather large (200-500 pc) with spherical shapes and power-law density distributions, thought the mass fractions of them are quite minor (0.1% and 1.6%, respectively). It is not so clear whether only a fraction of GCs in the Galaxy can host such outer diffuse components or not. The physical process that forms the stellar halos is not understood yet. One of possible scenarios for the formation of the halos is that these GCs with halos were formed from nuclei of nucleated dwarfs that were destroyed by the Galaxy many Gyr ago (e.g., Bekki & Yong 2012; Bekki & Tsujimoto 2016; Kuzma et al. 2016). The stripped nuclei (=GCs) can still have old field stars that initially constituted the host dwarf galaxies. There could be some differences in chemical abundances between GCs and their stellar halos, because GCs and field stars could have formed in different areas of their host dwarfs.

The present scenario offers an alternative explanation for the origin of such stellar halos around GCs. A massive star cluster that is the progenitor of a GC should be a part of clustering of star clusters in hierarchical star formation: such a massive cluster should not be an isolated single cluster. During the dynamical evolution of the SCC, some low-mass clusters are stripped from the surrounding of the main massive cluster whereas some merge with the main cluster and consequently are destroyed by the cluster. Stars initially in the destroyed clusters can constitute the diffuse stellar halo around the main cluster. In this scenario, chemical abundances of the halo is almost exactly the same as those of the main cluster, which is in a striking contrast with the above stripped nuclei scenario. Thus, future observations on the chemical abundances of stars both for halos and main components of GCs will be able to discriminate between these two scenarios.
This model is exactly the same as the fiducial model M1 within dense clusters. However, it would be instructive for galaxy-wide star formation (not for SG formation) model adopted in the present study is appropriate of SCCs for various model in detail, because the star formation rate for SG stars is kept low (∼1 × 10^{-4} M⊙ yr^{-1}) for T < 0.1 Gyr, and it has a peak of 0.001 M⊙ yr^{-1} in a weak burst phase at T = 0.13 Gyr. These lower SFR might be difficult to be detected observationally in massive young star clusters with secondary star formation. The spatial resolution of the present simulation is 0.47 pc at most, which implies that the above results could be due partly to the simulations resolution that might not allow this study to investigate the subpc-scale SF physics in detail. We will address this important issue on the spatial distributions of SG stars in GCs in our future papers.

6 CONCLUSION

We have proposed a new GC formation scenario in which star cluster complexes (SCCs) can form GCs with multiple stellar populations in gas-rich dwarf galaxies. In the scenario, a present-day GC was initially the most massive star cluster ("main cluster") embedded in a loosely bound star cluster complex (SCC) consisting of numerous stellar associations and low-mass star clusters (SCs) that were formed almost simultaneously with the main cluster. The SCC resides in the central region of a giant gas hole (i.e., local region devoid of gas) that was created by energetic massive stars and SNe of the SCC itself. Gas ejected from AGB stars in low-mass SCs and stellar association of the SCC is accreted onto the main cluster and then converted into new stars (i.e., the second generation of stars; SG) in the scenario. The physical processes that have been/ will be investigated in the present or future studies are briefly summarized with some comments on the possible serious problems of the new scenario.

A crucial question in the scenario is whether an enough amount of AGB ejecta (∼10^5 M⊙) can be accreted onto the main cluster from individual low-mass SCs and stellar associations before they are tidally stripped from the main cluster and destroyed/disintegrated to become field stars in the GC host dwarf galaxy. We have investigated this question using hydrodynamical simulations of the evolution of AGB ejecta in gas-rich dwarfs for a wide range of model parameters. Key parameters are the initial mass of a main cluster (m_{mc}), the distance of the SCC from the center of its host dwarf galaxy (R_{scc}), the fraction of AGB ejecta in a star cluster (f_{eagb}), the total mass of gas initially in the dwarf disk (M_g), and structure parameters of the dwarf. The principal results are as follows.

(1) Main clusters can accrete a significant amount of AGB ejecta from other low-mass clusters in SCCs before...
Table 3. Description of the key physical processes of GC formation in the present GC formation scenario from star cluster complexes (SCCs).

| Physical process | Papers $^a$ | Comments $^b$ |
|------------------|-------------|---------------|
| Formation of massive gas clumps in dwarfs | - | Survival of hierarchical structure for more than $10^7$ yr is crucial. |
| Fractal structures in gas clumps | - | Future models need to investigate disintegration of small SCs. |
| Formation of SCCs within gas clumps | - | |
| Merging of SCs within SCCs | This work | |
| Ejection of AGB ejecta | This work | |
| Accretion of AGB ejecta onto forming GCs | This work | |
| Accretion of pristine ISM | This work | |
| Chemical mixing of AGB ejecta and ISM | - | Dilution of AGB ejecta by ISM needs to be modeled self-consistently. |
| Secondary star formation | - | SNe should be suppressed in SG star formation. |
| Chemical enrichment of ISM by field stars | - | Small metallicity difference between ISM and AGB ejecta is required. |
| Formation of giant HI holes by SNe | - | |

$^a$ If the listed physical process is investigated in our previous paper or in the present paper, then the reference of the paper is shown. If not, the mark ‘-’ is shown, which means that the physical process will be investigated in our forthcoming papers.

$^b$ The potential serious problems of the new scenario are listed if necessary.

most of the low-mass clusters are stripped from the surroundings of the main clusters in GC-host dwarfs. The total mass of the accreted AGB ejecta ($M_{\text{acc}}$) can be as large as $\sim 10^5 M_\odot$ for $m_{\text{acc}} = 5 \times 10^3 M_\odot$. This ‘donation’ of AGB ejecta from other member clusters in a SCC is significant, because there are numerous low-mass SCs in a SCC owing to the power-law cluster mass function with a power-law index of $\beta = 2$. This result suggests that hierarchical star formation, which is a main physical origin for the power-law cluster mass function, needs to be considered in discussing the origin of GCs with multiple stellar populations.

(2) The total masses of AGB ejecta accreted onto main clusters are larger in the models with larger $m_{\text{acc}}$ for a given set of other model parameters, though the ratio of $M_{\text{acc}}$ to $m_{\text{acc}}$ is not so different for $M_{\text{acc}} > 10^5 M_\odot$. Low-mass main clusters with $m_{\text{acc}} < 10^5 M_\odot$ can not accrete AGB ejecta so efficiently from other clusters as massive main clusters for a given cluster mass function. These results imply that $m_{\text{acc}}$ is a key parameter that controls the total mass of SG stars which can form from gas accreted onto the main clusters.

(3) The IMF-dependent parameter $f_{\text{agb}}$ can determine the time evolution of the accretion rates of AGB ejecta and thus the total amount of the accreted gas ($M_{\text{acc}}$). A larger amount of AGB ejecta can be accreted onto main clusters in the models with larger $f_{\text{agb}}$, which means that $M_{\text{acc}}$ is larger for SCCs with more top-heavy IMFs. The locations of SCCs within their host dwarfs can also influence the the accretion processes of AGB ejecta. Main clusters with smaller sizes can accrete a less amount of AGB ejecta, mainly because the accretion radius is smaller.

(4) Stellar structures of SCC-host dwarfs, cluster mass functions (e.g., $\beta$, $m_{\text{l}}$, and $m_{\text{u}}$), and SCC sizes can determine $M_{\text{acc}}$ of the main clusters in SCCs. The model with $\beta = 0$ (no hierarchy in SCC) shows significantly smaller $M_{\text{acc}}$ than the fiducial model with $\beta = 2$, which implies that hierarchical star formation, which determines $\beta$, is important for the gas accretion process. Low-mass main clusters with $m_{\text{acc}} = 10^5 M_\odot$ can accrete AGB ejecta from other low-mass clusters, if $m_{\text{l}} \sim 10^3 M_\odot$ and $m_{\text{u}} \sim 3 \times 10^3 M_\odot$.

(5) Cold gas initially outside large gas holes can be accreted onto the main clusters of SCCs later than AGB ejecta. This time lag between the accretion of AGB ejecta and that of cold ISM suggests that SG stars formed earlier in main clusters can show Na-enhanced abundance patterns. This results also implies that stars with Na-enhanced sub-populations in GCs should be younger than those with Na-normal stars. If AGB ejecta is helium-rich ($Y \sim 0.35$) and if secondary star formation occurs within 0.1 Gyr after the commencement of gas accretion of AGB ejecta onto main clusters (i.e., well before the accretion of cold ISM on the clusters), then the clusters can have sub-populations with rather high Y. If the host dwarf galaxies are gas-rich, then the total amount of AGB ejecta and cold gas accreted onto the main clusters can be as large as $[2 - 3] \times 10^5 M_\odot$ for $m_{\text{acc}} = 5 \times 10^5 M_\odot$.

(6) The total amount of AGB ejecta and cold ISM accreted onto the main clusters in the models with ISM depends on model parameters in a similar way as described for the models without ISM. The total mass of accreted ISM ($M_{\text{acc,ism}}$) is appreciably larger than that of accreted AGB ejecta ($M_{\text{acc,agb}}$) for most model in the present study. It is not clear whether the derived mass ratio of $M_{\text{acc,ism}}$ to $M_{\text{acc,agb}}$ is consistent with the chemical abundance distributions of light elements observed in the Galactic GCs with multiple stellar populations. This point will be investigated in our future studies.

(7) Low-mass SCs can merge with the main clusters in SCCs. Since the mass densities of the SCs is much lower than the more massive main clusters owing to the mass-size relation ($r_{\text{sc}} \propto n_{\text{sc}}^{-1/3}$), they can be destroyed by the main clusters to form diffuse stellar halos. Only a small fraction of the stars can add their masses to the main clusters. This result suggests that GCs formed in the SCC scenario might have diffuse stellar halos just after their formation (i.e., after the dispersal of SCCs). This result also implies that
the origin of the observed stellar halos in some GCs can be understood in terms of destruction of low-mass clusters in the GC formation from SCCs.

(8) The new SCC scenario with external gas accretion has predictions that are quite different from those of previous GC formation scenarios with self-accretion only (or ‘self-enrichment’ scenario). For example, the ‘donation’ of AGB ejecta from other member clusters in SCCs can potentially solve the mass budget problem of GCs with multiple stellar populations. Also, the scenario naturally predicts the later accretion of cold ISM onto main clusters and the formation of stellar populations. Also, the scenario naturally predicts the later accretion of cold ISM onto main clusters and the formation of stellar halos around GCs. Older SG stars are likely to have higher [Na/Fe] than younger SG stars in the present scenario, though such an age-[Na/Fe] relation in SG stars depends on the adopted AGB yields.

The present study is a very first step toward the better understanding of GC formation from SCCs with $\beta = 2$ that are expected to form in hierarchical star formation. Since the present study has investigated only the gas accretion processes in forming GCs, there are many other issues that remain to be investigated. In particular, the observed anti-correlations between light elements will need to be investigated in our future studies with more sophisticated simulations with chemical evolution in forming GCs.

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