ON THE STRUCTURE OF DARK MATTER HALOS AT THE DAMPING SCALE OF THE POWER SPECTRUM WITH AND WITHOUT RELICT VELOCITIES

PEDRO COLÍN
Centro de Radioastronomía y Astrofísica, Universidad Nacional Autónoma de México, 58089 Morelia, Michoacán, Mexico

AND

OCTAVIO VALENZUELA AND VLADIMIR AVILA-REESE
Instituto de Astronomía, Universidad Nacional Autónoma de México, 04510 D.F., Mexico

Received 2007 July 2, accepted 2007 September 25

ABSTRACT

We report a series of high-resolution cosmological N-body simulations designed to explore the formation and properties of dark matter halos with masses close to the damping scale of the primordial power spectrum of density fluctuations. We further investigate the effect that the addition of a random component, \(v_{\text{rms}}\), into the particle velocity field has on the structure of halos. We adopted as a fiducial model a WDM cosmology with a nonthermal sterile neutrino mass of 0.5 keV. The filtering mass corresponds then to \(M_f = 2.6 \times 10^{12} \, h^{-1} \, M_\odot\). Halos of masses close to \(M_f\) were simulated with several million particles. The results show that, on the one hand, the inner density slope of these halos (at radii \(\leq 0.02 \) the virial radius \(R_v\)) is systematically steeper than the one corresponding to the NFW fit or to the CDM counterpart. On the other hand, the overall density profile (radii \(>0.02R_v\)) is less curved and less concentrated than the NFW fit, with an outer slope shallower than \(-3\). For simulations with \(v_{\text{rms}}\), the inner halo density profiles flatten significantly at radii smaller than \(2-3 \, h^{-1} \, \text{kpc} \left(\leq 0.010R_v - 0.015R_v\right)\). A constant density core is not detected in our simulations, with the exception of one halo for which the flat core radius is \(\approx 1 \, h^{-1} \, \text{kpc}\). Nevertheless, if “cored” density profiles are used to fit the halo profiles, the inferred core radii are \(\approx (0.1-0.8) \, h^{-1} \, \text{kpc}\), in rough agreement with theoretical predictions based on phase-space constraints and on dynamical models of warm gravitational collapse. A reduction of \(v_{\text{rms}}\) by a factor of 3 produces a modest decrease in core radii, by less than a factor of 1.5. We discuss the extension of our results into several contexts, for example, to the structure of the cold DM microhalos at the damping scale of this model.

Subject headings: dark matter — galaxies: halos — methods: n-body simulations

Online material: color figures

1. INTRODUCTION

The nature of dark matter (DM) is one of the most intriguing and fundamental problems in cosmology and particle physics. The standard hypothesis assumes that dark matter is made of nonbaryonic collisionless elemental particles that become nonrelativistic very early in the history of the universe (cold). This minimal scenario, named cold dark matter (CDM), has successfully explained the observed structure of the universe at large scales, including the two-point correlation function of galaxies and the cosmic microwave background radiation (CMBR) anisotropies (for recent results see Springel et al. 2006; Spergel et al. 2007). Confrontation of model predictions with observations turns out to be more complicated at galactic scales, because nonlinear dynamics and baryonic processes may distort considerably the underlying DM distribution. Thus, the predictions of the CDM scenario and its variants at the scale of galaxies are an active subject of study.

Two of the most controversial CDM predictions are (1) the large abundance of subhalos in galaxy-sized halos (Kauffmann et al. 1993) and (2) the cuspy inner density profile of dark halos (e.g., Navarro et al. 2004; Diemand et al. 2004). Based on comparison with observations, it has been suggested that both predictions may indicate a flaw of the CDM scenario (Klypin et al. 1999b; Moore et al. 1999a; Moore 1994; de Blok et al. 2001; Gentile et al. 2005, 2007). However, these comparisons might be biased by astrophysical processes that are active during galaxy assembly and evolution, such as, for example, the inhibition of star formation in small subhalos (Bullock et al. 2000; Benson et al. 2002; Governato et al. 2007) or the halo core expansion due to energy or angular momentum transfer from dark/baryonic structures (Ma & Boylan-Kolchin 2004; El-Zant et al. 2004; Weinberg & Katz 2007; but see also Colín et al. 2006; Ceverino & Klypin 2007; Sellwood 2007). It has also been shown that the disagreements may be a consequence of systematics in the observational inferences (e.g., Simon & Geha 2007; Rhee et al. 2004; Hayashi & Navarro 2006; Valenzuela et al. 2007).

It is also possible that slight modifications to the CDM particle properties could solve or ameliorate these potential problems if they persist (e.g., Spergel & Steinhardt 2000; Colín et al. 2002). As the precision of observations and the control of systematics improve, comparison with model predictions will open valuable possibilities for constraining the dark matter properties. Among the “slight” modifications of the CDM scenario is the introduction of warm particles (warm dark matter [WDM]) instead of cold ones. WDM implies two more degrees of freedom in comparison to CDM: (1) a damping (filtering) of the power spectrum at some intermediate scale, \(\lambda_f\), due to the relativistic free streaming and (2) some primordial random velocity in the dark particles, \(v_{\text{rms}}\) (for CDM, the mass corresponding to \(\lambda_f\), \(M_f\) is comparable to planet masses [e.g., Diemand et al. 2005a; Profumo et al. 2006], and \(v_{\text{rms}} \approx 0\)).

Previous numerical studies have shown that WDM can be very effective in reducing the amount of (sub)structure below the filtering mass, \(M_f\) (e.g., Colín et al. 2000; Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002). In addition, both the peculiar...
dynamical formation history of these halos and its random \( v_{\text{rms}} \) can also impose an upper limit on the phase-space density, potentially producing an observable core of constant density (Hogan & Dalcanton 2000; Avila-Reese et al. 2001). If \( v_{\text{rms}} \) has a thermal origin, its amplitude is linked directly to the mass of the WDM particle (Hogan & Dalcanton 2000); notice, however, that the amplitude of the random velocities may depend on other physical factors not directly related to the particle mass. This is the case, for instance, with gravitinos produced nonthermally by late decays of the next-to-lightest supersymmetric particle (NLSP; see for a recent review Steffen 2006; Feng et al. 2003; Strigari et al. 2007).

Thus, an exploration of the effect of a random velocity component independently of the dark matter particle mass seems to be necessary.

Predictions of the core radius in WDM halos have been computed assuming a King profile (Hogan & Dalcanton 2000) or a subclass of the Zhao (1996) profiles (Strigari et al. 2006, hereafter S2006). It is not yet known which estimates give the more accurate value, yet these predictions are necessary for comparison with observations (see, e.g., S2006; Gilmore et al. 2007). Unfortunately, the predicted core radii for masses of the most popular WDM particle allowed by observational constraints, the sterile neutrino (\( \gtrsim 2 \text{ keV} \); see § 5 for references), are below the resolved scales in current simulations.

The structure of WDM halos of scales below the filtering radius \( \rho \) might be different from their CDM counterparts not only in the central parts but also in their overall mass distribution. This discrepancy is somehow expected because the assembly history of these halos is different from the hierarchical one. Besides, they form later and have concentrations lower than the ones derived for CDM halos (Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002). However, it is controversial whether the shape of the density profile differs systematically from the corresponding CDM profile (for different results, see, e.g., Huss et al. 1999; Moore et al. 1999b; Knebe et al. 2003). On the other hand, WDM halos of mass close to \( M_f \) (with \( v_{\text{rms}} \) set to 0) can be thought of as scaled-up versions of the first microhalos in a CDM cosmology. The earliest collapse of CDM microhalos is a subject of considerable current interest (e.g., Diemand et al. 2005a; Gao et al. 2005).

In this paper we use numerical simulations to explore the two questions mentioned above: the overall structure of dark halos with masses close to or just below the cutoff mass in the power spectrum of fluctuations; and the inner density profile of these halos when a random velocity is added to the particles. For our numerical study we use the truncated power spectrum corresponding to a nonthermal sterile neutrino of \( m_{\nu} = 0.5 \text{ keV} \) (\( M_f = 2.6 \times 10^{12} \text{ h}^{-1} M_\odot \)). We initially neglect the particle random velocity (\( v_{\text{rms}} = 0 \)), and later we consider two values of \( v_{\text{rms}} \) that cover the range of velocities corresponding to thermal and nonthermal \( m_{\nu} = 0.5 \text{ keV} \) WDM particles. It is important to remark that our goal is not to study a specific WDM model but to explore in general the effects on halos of the power spectrum truncation and the addition of random velocities to the particles.

The structure of the paper is as follows. In § 2 we describe the cosmological model that we use for our investigation: a WDM model with a filtering radius at the scale of Milky Way–sized halos. Halos of these scales were resimulated with higher resolutions, first without adding the corresponding random velocity and then with the addition of this velocity component to the particles. Details of the numerical simulations carried out in this paper are given in § 3. The results from our different simulations are presented in § 4. Section § 5 is devoted to a discussion of the results and their implications. Finally, in § 6 we present the main conclusions of the paper.

2. THE COSMOLOGICAL MODEL

The general cosmological background that we use for our numerical simulations corresponds to the popular flat low-density model, with \( \Omega_0 = 0.3, \Omega_{\Lambda} = 0.7, \) and \( h = 0.7 \) (the Hubble constant in units of \( 100 \text{ km} \text{s}\text{ }^{-1} \text{ Mpc}^{-1} \)).

For the experiments designed to explore the structure of dark matter halos with masses close to \( M_f \), we adopt an initial power spectrum corresponding to a nonthermal sterile neutrino of 0.5 keV. Even if this WDM model is ruled out by observations (see § 5 for references), it still remains adequate for the purposes stated in § 1. As is shown below, the filtering mass, \( M_f \), corresponding to this WDM particle is of the order of Milky Way–sized halos, namely, the halos that we are able to follow with high-resolution in a cosmological simulation. The high resolution of the simulations avoids the formation of halos with a mass scale near \( M_f \) being dominated by discreteness effects (see § 3.1). Moreover, for the resolution we attain, we expect to resolve the inner regions where a flattening in the inner halo density is predicted for the case when \( v_{\text{rms}} \) is introduced.

Here we use the transfer function \( T_s \) for the nonthermal sterile neutrino derived in Abazajian (2006a). The WDM power spectrum is then given by

\[
P_{\text{WDM}}(k) = T_s^2(k) P_{\text{CDM}}(k),
\]

where

\[
T_s(k) = [1 + (\alpha k)^2]^{-\mu}
\]

and \( P_{\text{CDM}} \) is the CDM power spectrum given by Klypin & Holtzman (1997). This fit is in excellent agreement at the scales of interest with the power spectrum obtained with \texttt{lingzer}, which is contained in the \texttt{cosmics} package.\(^1\) The parameter \( \alpha \) is related to the mass of the sterile neutrino, \( \Omega_{\nu \text{DM}} \), and \( h \) through

\[
\alpha = a \left( \frac{m_{\nu}}{1 \text{ keV}} \right)^b \left( \frac{\Omega_{\nu \text{DM}}}{0.26} \right)^c \left( \frac{h}{0.7} \right)^d \text{h}^{-1} \text{ Mpc},
\]

where \( a = 0.189, b = -0.858, c = -0.136, d = 0.692, \nu = 2.25, \) and \( \mu = 3.08 \). The power spectrum is normalized to \( \sigma_8 = 0.8 \), a value close to that estimated from the third release of data from the Wilkinson Microwave Anisotropy Probe (WMAP; Spergel et al. 2007). Here \( \sigma_8 \) is the rms of mass fluctuations estimated with the top-hat window of radius 8 h\(^{-1}\) Mpc.

As in Avila-Reese et al. (2001), we have defined the free-streaming (damping) wavenumber, \( k_f \), as the k for which the WDM transfer function \( T_s^2(k) \) decreased to 0.5, and we compute the corresponding filtering mass in the linear power spectrum as

\[
M_f = \frac{4\pi}{3} \rho \left( \frac{k_f}{2} \right)^3,
\]

where \( \rho \) is the present-day mean density of the universe. The filtering wavelength \( \lambda_f = 2\pi/k_f \) of a nonthermal sterile neutrino of 0.5 keV is 3.9 h\(^{-1}\) Mpc, which corresponds to a filtering mass \( M_f = 2.6 \times 10^{12} \text{ h}^{-1} M_\odot \). The random component was linearly added to the velocities calculated with the Zel’dovich approximation at the onset of the simulation.

A number of Milky Way–sized halos are simulated with the WDM power spectrum given by equation (1) and with \( v_{\text{rms}} = 0 \).

\(^1\) See http://web.mit.edu/edbert/cosmics-1.04.tar.gz.
In this way we isolate the effects of the power spectrum filtering on the structure of the halos. We resimulate the same halos later, but with the addition of random velocities to the particles. We approximate the shape of the particle phase-space distribution function (DF) with the corresponding thermal one, i.e., we use a Fermi-Dirac DF. This is a good approximation for a nonthermal function (DF) with the corresponding thermal one, i.e., we use a but with the addition of random velocities to the particles. We on the structure of the halos. We resimulate the same halos later,

### Table 1

| Name Tag | \(v_{\text{rms}}\) \((\text{km s}^{-1})\) | \(\Delta a_0\) \((10^{-3})\) | \(h_{\text{fit}}\) \((\text{kpc h}^{-1})\) | \(m_p\) \((10^4 \text{ M}_\odot \text{ h}^{-1})\) | \(M_\ast\) \((10^{12} \text{ M}_\odot \text{ h}^{-1})\) | \(V_{\text{max}}\) \((\text{km s}^{-1})\) | \(R_s\) \((\text{kpc h}^{-1})\) | \(c_{\text{LS}}\) | \(c_{\text{NFW}}\) \((\text{M}_\odot \text{ pc}^{-3})\) | \(r_v\) \((\text{kpc h}^{-1})\) | \(r_{v_{\text{rms}}}^{1/3}\) | \(r_{v_{\text{rms}}}^{1/2}\) |
|----------|----------------|------------------|----------------|----------------|----------------|----------------|----------------|-----------|----------------|---------------|----------------|----------------|
| A2560.0... | 0.0 | 0.305 | 0.96 | 2.84 | 224.1 | 286 | 5.6 | 8.3 | -1.44 | ... | ... | ... |
| A2560.1... | 0.1 | 0.305 | 0.96 | 2.37 | 218.2 | 228 | 5.0 | 5.9 | -1.44 | 0.082 | 1.151 | 0.572 |
| B2560.0... | 0.0 | 0.5 | 0.96 | 4.36 | 253.2 | 330 | 5.1 | 7.7 | -1.46 | ... | ... | ... |
| B2560.1... | 0.1 | 0.5 | 0.96 | 4.35 | 250.8 | 330 | 5.0 | 6.7 | -1.54 | 0.098 | 1.380 | 0.565 |
| C5120.3... | 0.3 | 0.5 | 0.96 | 4.35 | 158.5 | 221 | 4.3 | 3.6 | -1.99 | 0.121 | 1.527 | 0.827 |
| C5120.1... | 0.1 | 0.5 | 0.96 | 4.35 | 160.1 | 221 | 4.3 | 3.6 | -1.54 | ... | ... | ... |
| D5120.0... | 0.0 | 0.152 | 0.62 | 1.29 | 160.7 | 222 | 4.2 | 6.4 | -1.53 | ... | ... | ... |
| D5120.1... | 0.1 | 0.152 | 0.62 | 1.26 | 158.9 | 220 | 4.3 | 4.7 | -1.79 | 0.093 | 1.265 | 0.689 |
| D5120.0... | 0.0 | 0.152 | 0.62 | 1.28 | 158.5 | 221 | 4.3 | 3.6 | -1.99 | 0.121 | 1.527 | 0.827 |
| D5120.1... | 0.1 | 0.152 | 0.62 | 1.28 | 160.1 | 221 | 4.3 | 3.6 | -1.54 | ... | ... | ... |
| E5120.0... | 0.0 | 1.0 | 0.152 | 2.13 | 197.3 | 262 | 4.9 | 6.4 | -1.57 | ... | ... | ... |
| E5120.1... | 0.1 | 1.0 | 0.152 | 2.13 | 197.3 | 262 | 4.9 | 6.4 | -1.57 | ... | ... | ... |
| E5120.3... | 0.3 | 1.0 | 0.152 | 2.13 | 197.3 | 262 | 4.9 | 6.4 | -1.71 | 0.101 | 1.380 | 0.519 |

Note.—All the halos presented in this table were resimulated from a \(L_{\text{box}} = 10 \text{ h}^{-1} \text{ Mpc} \) box simulation.

2 This radius is defined as the radius at which the mean overdensity is equal to \(337 \text{ for our selected cosmological model.}\)

3 The random velocity component was linearly added to the velocities corresponding to the Zel'dovich approximation at the onset of the simulation.
The longest plotted wavelength is sizes, namely, 15 and 20.

Concern may arise about the structure of halos simulated in a cosmology, where there is a scale below which the power spectrum is concerned, it does not matter if the simulation is started at $z = 40$ or 20. Unlike the previous case, no differences between the power spectra were detected. In other words, as far as the initial power spectrum is concerned, it does not matter if the simulation is started at $z = 40$ or 20.

The bound density maxima (BDM) group-finding algorithm (Klypin et al. 1999a), or a variant of it (Kravtsov et al. 2004), is used to locate the halos in the simulations and to generate their density profiles. The BDM finds positions of local maxima in the density field smoothed at the scale of interest and applies physically motivated criteria to test whether a group of particles is a gravitationally bound halo.

Aside from those halos shown in Table 1, for the halo D with $v_{\text{rms}} = 0$, we have also run a very high resolution simulation with about 31 million particles in the high-resolution zone. This halo was taken from the same 10 $h^{-1}$ Mpc box run, but it has 128$^3$ particles in the LMR mode. As far as we know, this is the highest resolution simulation of a halo run in a WDM cosmology. The same halo was also simulated with lower resolution for a convergence test. The parameters of the sequence of halos D are listed in Table 2.

The simulations presented here differ in several aspects from previous WDM simulations. First, it should be emphasized that our aim, rather than to discuss a specific WDM model, is to explore the influence of the truncation of the power spectrum and/or the addition of random velocities on the structure of dark halos of masses close to the truncation scale. For this aim we need to simulate (1) halos with very high resolution and (2) halos with masses close to $M_f$. The halos simulated in Avila-Reese et al. (2001) had several times fewer particles than the best resolved halos presented here, and the aims in that paper were to explore general halo properties for a concrete WDM model. Other papers aimed to study the properties of WDM halos (Bode et al. 2001; Knebe et al. 2002; Busha et al. 2007), focused more in the statistical aspects than in details of the inner halo structure; therefore, the halos in these papers had resolutions much lower than those attained here. The properties of the WDM halos simulated here are in general agreement with previous findings; for example, their concentrations are systematically lower (Avila-Reese et al. 2001; Eke et al. 2001; Bode et al. 2001) and they form later (Knebe et al. 2002; Busha et al. 2007) than the corresponding CDM halos.

### 3.1. Discreteness Effects

One of the motivations of this paper is to investigate the structure of well-resolved halos with masses close to or below the damping (truncation) scale in the power spectrum, $M_f$. The origin of these halos is controversial. Halos with masses close to $M_f$ (truncation halos) could be formed by a quasi-monolithic collapse of filaments of size $\sim \lambda_f$ (e.g., Avila-Reese et al. 2001). They could also just be the result of an incomplete collapse, highly deviated from the spherical-symmetric case, of originally larger structures assembled hierarchically (Busha et al. 2007). On the other hand, it has been suggested that halos with masses considerably less than $M_f$ form by fragmentation of the shrinking filaments of size $\sim \lambda_f$ (e.g., Valinia et al. 1997; Avila-Reese et al. 2001; Bode et al. 2001; Götz & Sommer-Larsen 2003; Knebe et al. 2003). However, it is also known that the filaments in hot dark matter simulations that start from a cubic lattice break up into regularly spaced clumps, which reflect the initial grid pattern. Therefore, some of these halos seen in WDM simulations could be spurious, the product of discreteness effects. Recently, Wang & White (2007) have shown that this artifact is present even for a glasslike initial particle load (White 1996).

As Wang & White (2007) show, halos of masses smaller than a given effective fraction of $M_f$, which depends on the resolution of the simulation, will be spurious. We selected the WDM model (§ 2) and the number of particles in the simulations in such a way

### Table 2

| $L_{\text{box}}$ ($h^{-1}$ Mpc) | Name Tag | $v_{\text{rms}}$ | Time Step ($10^{-3}$) | Resolution ($h^{-1}$ kpc) | $m_p$ ($h^{-1}$ $M_\odot$) | $M_{\text{vir}}$ ($10^{12}$ $h^{-1}$ $M_\odot$) |
|-----------------|---------|----------------|---------------------|-----------------|----------------|-----------------|
| 10.................. | D1024a,0 | off | 0.5 | 0.040 | 7.75 $\times$ 10$^4$ | 1.28 |
| 10.................. | D512a,0 | off | 0.5 | 0.152 | 6.20 $\times$ 10$^4$ | 1.27 |
| 10.................. | D256a,0 | off | 0.5 | 0.305 | 4.96 $\times$ 10$^4$ | 1.27 |
| 10.................. | D128a,0 | off | 0.5 | 0.610 | 3.97 $\times$ 10$^4$ | 1.25 |
that the halos studied here can neither be fake nor affected by
discreteness effects. Following Wang & White (2007), in our model
with 10243 effective number of particles, only structures with
masses lower than $10^{12} h^{-1} M_{\odot}$ are candidates to be spu-
rious. For the models with 5123 and 2563 effective number of
particles, the masses of structures triggered by the initial grid
spacing are $< 2 \times 10^{10} h^{-1} M_{\odot}$ and $4 \times 10^{10} h^{-1} M_{\odot}$,
respectively. For our WDM model, the first halos to form have masses
$\approx 10^{12} h^{-1} M_{\odot}$, which is far from the noted effective resolution
limits.

4. RESULTS

We first present the results of our WDM simulations with $v_{\text{rms}} = 0$. In this way we explore the inner structure of halos close
to the truncation mass $M_f$, which could be scaled-up versions of
the first CDM microhalos (of masses $\sim 10^{-5} M_{\odot}$ for a neutralino
mass of 100 GeV). Afterward we present the results of the same
simulations while introducing random velocities to the particles
with two different amplitudes, $v_{\text{rms}}(0) = 0.1$ and 0.3 km s$^{-1}$ (see
§ 2). The main goal of the latter simulations is to explore the
predicted flattening in the halo inner density profile produced by
the addition of a random component to the particle velocities
(see § 1). Table 1 resumes the main properties of the resimulated
halos, which were selected to have masses close to the truncation
mass of the initial power spectrum.

4.1. The Structure of Halos at the Scale of Damping

Figures 2 and 3 show the spherically averaged density profiles
measured for the halos of masses around the power-spectrum
filtering mass, $M_f$, and with $v_{\text{rms}} = 0$. The left panel of Figure 2
shows halos A2560.0 and B2560.0 (the latter was shifted by $-1$ in
the log); the middle and right panels show halos C5120.0 and
E5120.0, respectively. Figure 3 shows only halo D, but now simu-
lated with four different resolutions. In both figures the thin
dashed lines are the best Navarro-White-Frenk (NFW; Navarro
et al. 1997) fits to the illustrated density profiles. In the bottom
panels of each figure the residuals of the measured density profile
and the NFW fit are plotted with the same line coding as in the
corresponding top panels.

Halos A2560.0 and B2560.0 in the left panel and C5120.0 and
E5120.0 in the middle and right panels of Figure 2 were simul-
ated with formal spatial resolutions $h_{\text{for}} = 0.305 h^{-1}$ kpc and
0.152 $h^{-1}$ kpc, respectively (see Table 1). Previous convergence
studies for CDM halos simulated with the ART code have shown
that the innermost halo density is reliable only for radii larger than
4 times $h_{\text{for}}$ and containing more than 200 particles (Klypin et al.
2001). For all the density profiles shown in Figures 2 and 3, the
innermost plotted point corresponds to radii larger than $h_4 = 4 \times
h_{\text{for}}$ by $\approx 30\%$, and they contain more than 200 particles. The
convergence analysis that we have carried out for our WDM halos suggests that instead of \( h_4 \), the innermost radius should be close to \( 8 \times h_{80} (h_8) \).

Figure 3 compares the density profiles of halo D, which was resimulated with four different resolutions, each separated by a factor of 8 in the particle mass (see Table 2). As can be seen, convergence is achieved at about \( h_8 \). The arrows in Figures 2 and 3 indicate \( h_8 \) for the corresponding simulations. In Figure 3, the solid-line arrow is for the highest resolution simulation (1024 \(^3\) effective number of particles), while the dashed-line arrow is for the 512 \(^3\) simulation; the NFW fit is shown only for these two cases.

The inner density profiles of all halos simulated here are systematically steeper than the corresponding fitted NFW law. The slopes of the profiles at \( r \approx 1\% \) the virial radii, \( R_v \), span a range from \(-1.4 \) to \(-1.6 \). For comparison, the slope of the density profile of a typical LCDM halo of \( 2 \times 10^{12} h^{-1} M_\odot \) at \( 0.01 R_v \) is \(-1.2 \) (a NFW profile was used with the corresponding concentration given by Bullock et al. [2001] and rescaled to \( \sigma_s \approx 0.8 \)). At radii smaller than 0.01\( R_v \), the slopes tend to become shallower, but the halos are still denser and slopes steeper than the corresponding NFW fit up to the resolution limits (\( \approx h_8 \); see Figs. 2 and 3).

The overall density profile shapes of our halos are also somehow different from the NFW function. For radii larger than 0.02\( R_v \), the profiles tend to be in general slightly less curved than in the NFW model. This is why the residuals shown in Figures 2 and 3 indicate a systematical defect at intermediate radii and then an excess at the outer radii. The outer slopes are \( \approx -3 \).

In summary, the density profiles of the halos simulated here, with masses close to or below the filtering mass, have a shape slightly flatter than the NFW law for \( \approx 0.02 R_v \), and slopes significantly steeper at \( \approx 0.02 R_v \). Nevertheless, each profile is different. The profile of halo E512\(_{0.0}\) has minimal deviations from the NFW function, while the profile of halo D512\(_{0.0}\) significantly deviates from this function.

Table 1 details the main properties of the halos studied here. Reported in columns (6) to (14) are, respectively, the virial mass, \( M_v \), the maximum circular velocity, \( v_{\text{max}} \), the \( c_{1/3} \) and NFW concentration parameters, the average density within 1% the virial radius, and three core radii estimated by different criteria (the latter quantities apply only to halos simulated with \( v_{\text{rms}} \); see § 4.2). The NFW and \( c_{1/3} \) concentrations are defined, respectively, as the ratios between \( R_v \) and the NFW scale radius and those between \( R_v \) and the radius where one-third of \( M_v \) is contained (Avila-Reese et al. 1999). As found in previous results, halos with masses below the truncation mass in the power spectrum tend to be less concentrated than LCDM halos of similar masses (Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002). We have simulated some LCDM (\( \sigma_s = 0.8 \)) halos of masses \( \approx 2 \times 10^{12} h^{-1} M_\odot \) and measured NFW and \( c_{1/3} \) concentrations around 8–15 and 6.0–11.0, respectively, to be compared with the values given in Table 1 for the \( v_{\text{rms}} = 0 \) cases.

### 4.2. The Inner Structure of Halos Simulated with \( v_{\text{rms}} \) Velocity Added to the Particles

We reran the halos presented in the previous subsection, now introducing a random velocity component, \( v_{\text{rms}} \). As explained in § 2, the particle DF used corresponds to a Fermi-Dirac function. Regarding the \( v_{\text{rms}} \) amplitude, we use two values: \( v_{\text{rms}}(0) = 0.1 \) and 0.3 km s\(^{-1}\). These values are for WDM particles of thermal origin and for nonthermal sterile neutrinos, respectively, in both cases with \( m_{\nu} = 0.5 \) keV. Our goal is to explore whether the inner structure of the halos is significantly affected or not by adding \( v_{\text{rms}} \).

We first present results for the case \( v_{\text{rms}}(0) = 0.1 \) km s\(^{-1}\), and then explore how the inner halo structures change when \( v_{\text{rms}}(0) \) is increased from 0.1 to 0.3 km s\(^{-1}\), a more appropriate value for the 0.5 keV sterile neutrino model used to generate the initial power spectrum of the simulations (see § 2). For halo D, we simulated the 512\(^3\) case with \( v_{\text{rms}} > 0 \). Unfortunately, due to limitations in our computational resources, it was not possible to run the simulation with 1024\(^3\) particles for \( v_{\text{rms}} > 0 \).

In Figure 4 we present spherically averaged profiles of different properties for all the simulated halos without and with \( v_{\text{rms}} \) added. In the first and third columns, two halos (A and B, and Db and D, respectively) are presented, but the latter ones are offset for clarity. The top panels of Figure 4 show the density profiles of our halos without (black solid lines) and with \( v_{\text{rms}}(0) = 0.1 \) km s\(^{-1}\) (dot-dashed line) and 0.3 km s\(^{-1}\) (dashed line). Plotted in the middle panels the corresponding three-dimensional (3D) velocity dispersion profiles, \( \sigma_{3D}(r) \), and the bottom panels show the corresponding coarse-grained phase-space density profiles, \( Q(r) = \rho(r)/\sigma_{3D}^2 \). As in Figure 2, the arrows indicate the strong resolution limit radius \( h_8 \) of the simulations. For halo D (third column), the simulations with \( v_{\text{rms}} > 0 \) were carried out with 512\(^3\) particles and using two different random seeds for the particle DF calculation (see below).

To some degree, all the simulated halos have been affected in their inner regions by the injection of initial random velocities to their particles. For the less resolved halos A and B (left panels), the deviations at \( \approx h_8 \) of the inner density profiles from the profiles obtained in the simulations with \( v_{\text{rms}} = 0 \) are still marginal, but in the expected direction. For the halos C, D, Db, and E simulated with 512\(^3\) particles, the deviations down to \( h_8 \) are significant: the density profiles systematically flatten with respect to the corresponding \( v_{\text{rms}} = 0 \) cases. The radii at which the density profiles of halos with \( v_{\text{rms}} \) start to deviate (flatten) from the ones without \( v_{\text{rms}} \), are \( \approx 0.01 R_v \), well above the resolution limit of the simulations.

In a second series of experiments, we have resimulated halos C, Db, and E (512\(^3\) particles) with \( v_{\text{rms}} \) 3 times larger, i.e., \( v_{\text{rms}}(0) = 0.3 \) km s\(^{-1}\). The profiles corresponding to these simulations are plotted in Figure 4 with dashed lines. The flattening of the inner density profiles is clearly more pronounced than in the simulations with \( v_{\text{rms}}(0) = 0.1 \) km s\(^{-1}\).

The innermost density profiles actually vary from halo to halo. Again, halo E is the least affected, not only by the damping in the power spectrum but also by the injection of \( v_{\text{rms}} \), and halo D is the most affected by both effects (bottom dot-dashed curve in the corresponding panel). The latter actually shows a "true" flat core already at \( h_8 \). Since the core of this halo is also different with respect to the other ones, we decided to explore whether the large difference can be explained by a rare fluctuation in the random procedure of particle velocity assignment. Thus, the same halo D512\(_{0.1}\) was resimulated with a different seed in the random number generator used to draw the particle velocities. The top curves in the third-column panels of Figure 4 correspond to the profiles for this halo, called D512\(_{0.1}\).b. The inner density profile of this halo is not too different from the profiles of the other halos, although it remains the flattest among all the simulated halos, with \( v_{\text{rms}}(0) = 0.1 \) km s\(^{-1}\).

In Figure 5 we attempt to fit different functions to the density profiles of halos C, D, and E (512\(^3\) particles) with \( v_{\text{rms}}(0) = 0.1 \) and 0.3 km s\(^{-1}\). A general function to describe density profiles of cosmic objects was proposed by Zhao (1996):

\[
\rho(r) = \frac{\rho_0}{(r/r_0)^{\alpha} [1 + (r/r_0)^{\alpha} ]^{(\beta - \gamma)/\alpha}}.
\]
The NFW profile corresponds to \((\alpha, \beta, \gamma) = (1, 3, 1)\). This function does not provide a good description of the profiles of our halos with \(v_{\text{rms}} > 0\), in particular in the inner regions. We have fitted the halo profiles to the NFW in order to obtain an estimate of the concentration \(c_{\text{NFW}}\) given in Table 1.

Strigari et al. (2006) suggested a “cored” density profile in order to derive constraints on the size of a possible shallow core in the halo of the Fornax dwarf spheroidal galaxy (Goerdt et al. 2006; Sánchez-Salcedo et al. 2006). This profile is described by equation (5), with \((\alpha, \beta, \gamma) = (1.5, 3, 0)\), and Strigari et al. define the core radius, \(r_c\), as the radius where the inner log slope, \(g\), reaches the value of \(-0.1\). Thus, \(r_c = r_0/(-3/\gamma - 1)^{\alpha} \approx 0.1r_0\). The dot-dashed curves in Figure 5 show the best fits using the S2006 profile. As can be seen, this profile does not describe well the density profiles of the WDM halos in the simulations. Note that \(\alpha\) characterizes the sharpness of the change in logarithmic slope. As already seen in Figures 2 and 4, the profiles of our halos tend to be less curved than the usual NFW profile. Therefore, values of \(\alpha\) smaller than 1 should be used instead of values larger than 1. We have obtained a reasonable description of our WDM profiles with \((\alpha, \beta, \gamma) = (0.7, 3, 0)\). The dashed curves in Figure 5 are the best fits with these profiles. The core radius, as defined above, is in this case \(r_c \approx 0.0064r_0\). Finally, we also tried fits to the S2006 function while taking into account only the central halo regions.

![Figure 4](https://example.com/figure4.png)

**Fig. 4.** — Top row: Comparing the density profiles of simulated halos without (solid lines) and with (dashed and dot-dashed lines) adding \(v_{\text{rms}}\) to the particles. Left to right (first and third panels): Halos B and D have been shifted by \(-1\) in the log. The innermost plotted radii are at \(\approx 1.3h_4\) and the arrows indicate the corresponding \(h_8\) radii. Middle row: The corresponding 3D velocity dispersion profiles of the halos shown in the top row. Halos B and D have been shifted in the log by \(-0.5\) and \(-0.15\), respectively. Bottom row: Coarse-grained phase-space density profiles corresponding to the halos shown in the top row. As in the top row, halos B and D here were also shifted by \(-1\) in the log. [See the electronic edition of the Journal for a color version of this figure.]
Fig. 5.—Density profiles of simulated ($512^3$ particles) halos with added $v_{\text{rms}}$ and fitted to different model profiles. Long-dashed line: $(\alpha, \beta, \gamma) = (0.7, 3, 0)$; dot-dashed line: profile proposed in S2006, $(\alpha, \beta, \gamma) = (1.5, 3, 0)$; dotted line: same S2006 profile but fitted only to the inner 30 kpc. [See the electronic edition of the Journal for a color version of this figure.]
up to \( \approx 30 \; h^{-1} \text{kpc} \). The fits are shown in Figure 5 with dotted curves. The fit is especially good for halo D5120.1 and those with \( \sigma_{v_{\text{rms}}} = 0.3 \; \text{km} \; \text{s}^{-1} \). Columns (12) to (14) in Table 1 report the values of \( r_c \) obtained with the three different fits.

As to the 3D velocity dispersion profiles of the simulated halos, they do not differ significantly between the cases with and without \( v_{\text{rms}} \), the exception being halo D. In the innermost regions, \( \sigma_{3D}(r) \) is similar to or higher for halos with \( v_{\text{rms}} \) than for those without \( v_{\text{rms}} \). The largest differences are for halo D, for which the introduction of random velocities to the particles produces a relatively hot core.

Finally, from the measured density and dispersion velocity profiles, we calculate the coarse-grained phase-space density profiles, \( Q(r) \). As seen in the bottom panels of Figure 4, excepting the innermost regions, the \( Q(r) \) profiles are well described by a power law \( Q \propto r^{-\alpha} \) with \( \alpha \approx -1.9 \), close to that obtained for CDM halos by Taylor & Navarro (2001). For the inner regions the \( Q(r) \) profile of the halos simulated without \( v_{\text{rms}} \) tends to steepen, especially in halos D5120.1 and C5120.0. The opposite happens for the halos simulated with the addition of \( v_{\text{rms}} \): the inner \( Q(r) \) profile tends to be flatter as \( v_{\text{rms}} \) is increased.

5. DISCUSSION

5.1. Robustness of the Results

The halos studied here are, on one hand, among the first virialized structures to form in our simulations, while, on the other, their assembling process started relatively late in the universe, between \( z \approx 0.6 \) and 1 (for discussions of the mass assembling process of halos with masses close to the damping scale in the power spectrum, see, e.g., Moore et al. 2001; Avila-Reese et al. 2001; Bode et al. 2001; Knebe et al. 2002; Busha et al. 2007). Because of the late collapse, one might argue that the halos studied here are not relaxed, and it is therefore not surprising to see deviations from the NFW profile, as reported in § 4.1 for the experiments with \( v_{\text{rms}} = 0.0 \). We have followed the evolution of some of our halos for more than a Hubble time (scale factor \( a > 1 \)), finding a negligible evolution in the density profiles since \( a = 1 \). For example, halo D5120.1 was run until \( a = 1.4 \) (18.4 Gyr). The density profile of the halo at this epoch is practically the same as at \( a = 1 \) (13.7 Gyr; see Fig. 4). As discussed in previous studies (see the references above), the collapse of halos of scales close to the damping scale seems to be quasi-monolithic (although highly nonspherical). Thus, in regions that remain relatively isolated, as in the case of the halos selected for our study, halos suffer low-mass accretion and their structures remain almost unaltered since the initial collapse.

We checked that the effects on the structure of halos reported in § 4 are systematic by running for a WDM model—not shown in Table 1—the corresponding CDM simulation, using the same random phases and changing only the initial power spectrum. We found that the density profile of the CDM halo is well fitted by the NFW function, while the corresponding WDM halo presents the systematic deviation already seen in Figure 2. Figure 6 compares the density and circular velocity profiles for the halo in question in its two versions, CDM and WDM without random velocities. Since the WDM halo ends up at \( z = 0 \) with a slightly lower mass than the CDM halo, we correct the profiles so as to make the comparison at a fixed mass \( = 3.0 \times 10^{12} \; h^{-1} M_\odot \). Notice, in particular, that the inner density profile of the WDM halo is indeed steeper than its CDM counterpart.

Concerning the resolution limit in our simulations, based on the convergence study carried out for halo D (see Fig. 2), we find a strong limit at \( h_8 = 8h_{\text{los}} \), although resolution might be still acceptable for radii slightly larger than \( h_8 = 4h_{\text{los}} \) (a value suggested previously for CDM halos in “equilibrium” simulated with the ART code; Klypin et al. 2001). With a resolution limit at \( h_8 \), our simulations allow us to resolve the inner structure of halos down to \( 1.2 \; h^{-1} \text{kpc} \) for the 512\(^3 \) runs and down to \( 0.61 \; h^{-1} \text{kpc} \) for the 1024\(^3 \) run. These radii correspond, respectively, to \( \sim 0.5\% \) and 0.25\% of the virial radius in our halos.

Finally, it is important to recall that the masses of the WDM halos analyzed in our simulations are well above the mass scale affected by discreteness effects, such as the spurious formation of structures and substructures due to the initial grid pattern (see § 3.1).

5.2. Do Soft Cores Form in WDM Halos?

Early structure formation studies based on a WDM cosmology considered particles that originated in thermal equilibrium. For this case both \( v_{\text{rms}} \) and \( \lambda_f \) depend only on the particle mass \( m_W \). The smaller the value of \( m_W \), the larger is that of \( \lambda_f \) and \( r_{\text{rms}} \). Controlled numerical simulations of isolated halos showed that in order to produce “observable” soft cores, the amplitude of \( v_{\text{rms}} \) should be several times higher than the values corresponding to thermal WDM particles of masses \( m_W \geq 1 \text{ keV} \) (Avila-Reese et al. 2001). Particle masses smaller than \( \sim 1 \text{ keV} \) are not allowed by the constraints on satellite galaxy abundances or by the Ly\( \alpha \) power spectrum alone or combined with CMBR and large-scale structure data. The Ly\( \alpha \) power spectrum is the strongest of the constraints. For the nonthermal sterile neutrino, it places a limit on it mass at \( m_W \geq 2 \text{ keV} \) (Seljak et al. 2005; Viel et al. 2006; Abazajian 2006b). For thermal WDM particles, the observational constraints give a limit of \( m_W \geq 0.5 \text{ keV} \) (Narayanan et al. 2000; Viel et al. 2005; Abazajian 2006b), while a different analysis, using different simulations, provides a stricter limit, \( m_W \geq 2.5 \text{ keV} \) (Seljak et al. 2006).
We can estimate the expected flat core radii of \(2 \times 10^{12} h^{-1} M_{\odot}\) WDM halos for thermal particles in the mass range \(m_w = (2 - 0.5)\) keV by using the approximation given in Avila-Reese et al. (2001; their eq. [13]). This approximation is based on the monolithic collapse of halos with nonnegligible particle random velocities before the collapse. For thermal WDM particles of masses 2 and 0.5 keV, the \(z_M\) values of a \(2 \times 10^{12} h^{-1} M_{\odot}\) perturbation are \(\approx 3.4\) and 1.9, respectively, while the \(v_{\text{rms}}(0)\) values corresponding to these masses are 0.015 and 0.1 km s\(^{-1}\). Therefore, the expected core radii are \(r_c \approx 30\) and 210 pc.

The value of \(v_{\text{rms}}\) for a nonthermal sterile neutrino of 0.5 keV is approximately 3 times larger than that corresponding to the thermal particle of the same mass. Therefore, for this case \(r_c \approx 630\) pc. Thus, the resolutions that we can attain in our simulations of WDM halos for the \(m_w = 0.5\) keV sterile neutrino are already close to these estimates of the flat core radii.

Recently, alternative particle models, such as super-WIMPS, were proposed. The dark particles in these models may acquire random velocities nonthermally, for example, through the decay process of NLSP particles (e.g., charged sleptons into gravitinos; Feng et al. 2003, and for more references see Steffen 2006). In these cases \(v_{\text{rms}}\) does not depend directly on the damping scale of the linear power spectrum. However, an extra parameter, the decaying epoch, is introduced. Strategies to constraint this parameter using astrophysical observations have been proposed (Feng et al. 2003; Strigari et al. 2007).

Some cosmological models with super-WIMP particles may be in agreement with constraints based on structure formation, especially the Ly\(\alpha\) forest, and still allow for relatively large velocity dispersions, adequate to ameliorate the potential problems of \(\Lambda\)CDM at small scales. This is the case with the so-called meta—dark matter models that consider the late decay of neutralinos into gravitinos. These models preserve a power spectrum similar to that of \(\Lambda\)CDM models and at the same time set a phase-space limit in the innermost structure of dark halos due to the injection of random velocities to the particles (Strigari et al. 2007; Kaplinghat 2005). However, as mentioned in \(\S\) 1, it remained an open question how efficient the introduction of \(v_{\text{rms}}\) is for producing significant effects on the inner structure of simulated dark halos. One of the goals of the present paper was just to explore this question by means of numerical simulations able to resolve the WDM halos down to \(\sim 0.005R_c\).

The results presented in \(\S\) 4.2 show definitively that the addition of \(v_{\text{rms}}\) flattens the inner density profiles of WDM halos, and do so more as \(v_{\text{rms}}\) increases. Differences in density profiles of halos simulated with \(v_{\text{rms}} > 0\) and those with \(v_{\text{rms}} = 0\) start to be evident at radii of 2–3 \(h^{-1}\) kpc \((\sim 0.01R_c - 0.015R_c)\) for our \(\approx 2-4 \times 10^{12} h^{-1} M_{\odot}\) halos. The halo average inner density measured at 0.01R\(_c\), \(\rho_{\gamma}\), decreases on average by factors of 1.5 and 2.5 for the models with \(v_{\text{rms}}(0) = 0.1\) and 0.3 km s\(^{-1}\), respectively (col. [11] in Table 1). Nevertheless, we notice that the concentration of the halos seems not to be changed significantly by the introduction of \(v_{\text{rms}}\).

In general, the NFW function does not describe well the inner density profiles of our simulated halos; their profiles are much shallower than \(\gamma = -1\), as can be seen in Figure 5. However, does the size of the random velocity effect match theoretical expectations? The theoretical predictions are based on the existence of an upper limit in the fine-grained phase-space density due to the collisionless nature and finite relict velocity dispersion of particles. This upper limit, \(Q_{0,\text{max}}\), implies that the halo density profile must saturate and form a constant-density core (Hogan & Dalcanton 2000).

For the random velocities used in this study \([v_{\text{rms}}(0) = 0.1\) and 0.3 km s\(^{-1}\), corresponding respectively to thermal and non-thermal 0.5 keV sterile neutrinos], \(Q_{0,\text{max}} = 3 \times 10^{-5}\) and \(1.1 \times 10^{-6}\) \(M_{\odot}\) pc\(^{-3/2}\)(km s\(^{-1}\))^\(3\). According to Hogan & Dalcanton (2000) the core radius produced by the phase-space packing scales as \(R_c \approx v_{\text{rms}}^{-1/2}\), where \(v_{\text{rms}}\) is the asymptotic circular velocity for the assumed nonsingular isothermal sphere (their eq. [18]). This implies that more massive halos have smaller, more tightly bound cores. Applying the same equation for \(m_w = 0.5\) keV and \(v_{\text{ rms}} = 200\) km s\(^{-1}\), the core radius for the thermal particle \([Q_{0,\text{max}} = 3 \times 10^{-5} M_{\odot}\) pc\(^{-3/2}\)(km s\(^{-1}\))^\(3\)] is 85 pc, while for the sterile neutrino \([Q = 1.1 \times 10^{-6} M_{\odot}\) pc\(^{-3/2}\)(km s\(^{-1}\))^\(3\)] is 450 pc.

In any case, our results can only marginally test these estimates. The resolution limit in our simulations with nonzero \(v_{\text{rms}}\) is in between \(\approx 1\) and 1.7 kpc. If the inner density profile response to the introduction of \(v_{\text{rms}}\) is gradual, one expects to see yet some effects at these radii, and this happen to be the case.

A way to attempt to infer (extrapolate) the sizes of possible flat cores in the simulated halos is to fit the measured density profiles to an analytical function that implies a flat core. Results of these fits were presented in \(\S\) 4.2 using the Zhao (1996) profile with \((\alpha, \beta, \gamma) = (0.7, 3, 0)\) as well as the one suggested by S2006. The latter function gives a poor description of the overall measured density profiles, which tend to be significantly less curved than the analytical model (see Fig. 5). The obtained (overestimated) values for \(r_c = (0.1r_{\text{S}}; \text{see}\;\S\;4.2)\) are reported in column (13) of Table 1. When the S2006 function is fitted to only the inner 30 kpc, the fits improve and the estimated core radii become smaller by roughly a factor of 2 (col. [14]). However, even for this case the core radii seems to be upper limits, with the exception of halo D512\(_{1,1}\).

We have found that the WDM profiles are better described by the Zhao function with \((\alpha, \beta, \gamma) = (0.7, 3, 0)\). The best fits to the measured profiles gave extrapolated core radii \(r_c \approx 5-8\) times smaller than the S2006 profile fitted to only the inner 30 kpc. If we had sufficient resolution to resolve the flat cores, their radii would lie in between \(r_c\) and \(r_S\). The only halo for which the flat core is present at our resolution limit is D512\(_{1,1}\); a visual inspection shows that the core radius is close to 1 \(h^{-1}\) kpc.

Our results show that the different estimates of the core radius increase by less than a factor of 1.5 from the simulations with \(v_{\text{rms}}(0) = 0.1\) km s\(^{-1}\) to the ones with \(v_{\text{rms}}(0) = 0.3\) km s\(^{-1}\). The amount of this increase is less than the one we would predict using the monolithic collapse approximation of Avila-Reese et al. (2001); according to this approximation, the core radius of halos formed at the same time depends linearly on the injected \(v_{\text{rms}}\) at the maximum expansion of the perturbation, \(r_c \propto v_{\text{rms}} z_{\text{max}} = v_{\text{rms}}(0)(1 + z_{\text{max}})\). We have estimated the predicted values of \(r_c\) for our halos by using this approximation. From the simulations, we find that the redshifts of maximum expansion of halos C, D, and E are roughly \(z_{\text{max}} = 1.6, 1.3,\) and 1.8, respectively; these redshifts are practically the same for the different values of \(v_{\text{rms}}\). The calculated \(r_c\) for \(v_{\text{rms}}(0) = 0.1\) (0.3) km s\(^{-1}\) are then 221 (663), 240 (720), and 209 (627) pc, respectively. Thus, in general these predictions give core radii just in between \(r_c\) and \(r_S\) (see Table 1), although the dependence on \(v_{\text{rms}}\) is much more pronounced than it is for the (extrapolated) core radii estimated from the fits to our halos.

5.3. The Structure of Halos at the Damping Scale

The simulations carried out in this paper also allowed us to explore the structure of dark matter halos formed from perturbations at the scale of damping of the linear power spectrum. The cutoff in the power spectrum used here corresponds to a relatively large
mass, $M_f = 2.6 \times 10^{12} \, h^{-1} \, M_\odot$. Therefore, the formation of the first structures in this model, namely, those structures with masses close to $M_f$, happens relatively late. We speculate that the formation process of the truncation halos is generic. If this is true, then the structure of the late-formed truncation halos simulated here with several million particles ($v_{\text{rms}} = 0$) should be similar to the structure of truncation (micro)halos formed early in models with much smaller filtering masses than the one used here, for example, in the CDM models. If this is the case, then our results may enrich the discussion about the formation and structure of the first microhalos in CDM models (on Earth-mass scales).

We have found a clear systematic trend in the density profiles of the simulated truncation halos: they are significantly steeper than $r^{-1}$ in the inner regions, $r \leq 0.02 R_c - 0.03 R_c$, and lie below the best NFW fits in the intermediate region. CDM halos with an inner slope steeper than $r^{-1}$ have been reported in other contexts: recently merged group- and cluster-sized halos (Knebe et al. 2002; Tasitsiomi et al. 2004) or microhalos formed at the scale of CDM power-spectrum damping (Diemand et al. 2005b). It has been argued that the recent major merger is to blame for the steepening of the density profile, while subsequent secondary infall modifies the external region. For the halos at the damping scale, the dynamical situation corresponds to a fast (quasi-monolithic) collapse rather than a major merger. However, in both cases, the process is dynamically violent. We have checked that the obtained density profiles do not correspond to a transient configuration. As mentioned in \S 5.1, for halo D the profile remains almost unchanged until $a = 1.4$.

Extrapolating our results to the damping scale of CDM, we can speculate that CDM microhalos may be significantly steeper than the NFW profile. This implies that the possible contribution of surviving microhalos to the $\gamma$-ray flux originated by neutralino annihilation might be comparable to the central flux from host halos (Diemand et al. 2006). Another implication of our results could be related to the buildup of the inner density profile of dark halos in general. Dehnen (2005) and Kazantzidis et al. (2006) argued that the assembly of halo inner density profiles happens very early in the history of the universe and specifically that the inner slope of the cuspy progenitor survives up to the final halo. If the density profiles in our simulated halos are representative of objects formed at the damping of power scales in general, then, according to these studies, the central slope of present-day dark matter halos should be much steeper than the NFW profile.

In order to verify our results, a more systematic study halo structure at the scale of damping is required, which explores any possible dependence with the shape of the cutoff and the power spectrum slope at the scale of damping. Currently the only studies discussing a similar situation report different results on the scale of galaxy clusters (Moore et al. 1999b) and microhalos (Diemand et al. 2005a). It is unclear whether this suggests a slope dependence on the profile of the smallest dark matter halos with the power spectrum slope.

6. CONCLUSIONS

We have used cosmological $N$-body simulations to study the structure of dark halos formed in the context of a WDM model corresponding to a nonthermal sterile neutrino particle of mass $m_{\nu_W} = 0.5$ keV. The first series of simulations did not include the injection of a random velocity component, $v_{\text{rms}}$, to the particles and was aimed at exploring the structure of the halos formed from perturbations at the damping scale in the linear power spectrum ($M_f = 2.6 \times 10^{12} \, h^{-1} \, M_\odot$) for the concrete WDM model studied here. The second series of simulations included a $v_{\text{rms}}$ component of (1) a $m_{\nu_W} = 0.5$ keV thermal neutrino [$v_{\text{rms}}(z = 0) = 0.1 \, \text{km} \, \text{s}^{-1}$] and one that roughly corresponds (2) to a $m_{\nu_W} = 0.5$ keV nonthermal sterile neutrino [$v_{\text{rms}}(z = 0) \approx 0.3 \, \text{km} \, \text{s}^{-1}$]. This latter model was used to generate the initial power spectrum. These simulations were aimed at exploring the effect that a random velocity component has on the inner structure of halos; in particular, the aim was to elucidate whether or not constant-density cores are produced in the simulations. The results of our study lead us to the following two main conclusions:

1. The structure of halos formed from perturbations of scales close to $M_f$, and resolved with up to more than 16 million particles (with $v_{\text{rms}} = 0$), is peculiar: the inner density profile ($r < 0.02 R_c$) is systematically steeper than the best corresponding NFW fit (and the respective CDM counterpart), and the overall density profile ($r > 0.02 R_c$) tends to be less curved than the best NFW fit; the outer profile slope is never steeper than $-3$. According to our tests, these differences with respect to the structure of halos assembled hierarchically, can hardly be attributed to a peculiar dynamical state of the halos simulated here.

2. The effect of adding $v_{\text{rms}}$ to the particles produces a significant flattening of the inner density profile ($r \leq 2 - 3 \, h^{-1} \, \text{kpc}$ corresponding to $\approx 0.10 R_c - 0.15 R_c$) of the simulated halos. The different estimated (extrapolated) sizes of the nearly constant-density cores are of the order of the theoretical predictions, which give values below our resolution limit. For the halo masses simulated here, $M_c \approx (2 - 4) \times 10^{12} \, h^{-1} \, M_\odot$, the flat core radii estimated from different fittings are between $\approx 0.1$ and $0.8 \, h^{-1} \, \text{kpc}$. An increase in $v_{\text{rms}}(0)$ from 0.1 to 0.3 km s$^{-1}$ produces an increase in the extrapolated core radii of a factor 1.5 or less. For one of our simulations (halo DS1201), the presence of a nearly constant-density core, of radius $\approx 1 \, h^{-1} \, \text{kpc}$, is already revealed at the resolution limit; the same halo simulated with a different random velocity seed is less flattened.

Although the simulations presented here refer to a concrete WDM model, they can be interpreted in a wide range of contexts. The density profile of dark halos with masses close to the truncation mass in the linear power spectrum is systematically different from the NFW profile; in particular, the inner regions tend to be steeper. These findings could have important implications in the context of CDM models if a significant fraction of microhalos formed at the free-streaming CDM scales ($\approx 10^{-3} \, h \, \text{Mpc}$) have survived until the present epoch. In that case, the predicted $\gamma$-ray flux from the neutralino annihilation in the center of these cuspy microhalos might be comparable to the central flux from host halos. On the other hand, the fact that the microhalos are so cuspy could have some interesting implications for the formation of the next hierarchies in the assembly of halos, as well as in the inner structure of the larger halos.

Regarding the effects of injecting random velocity to the particles, our results show evidence of significant inner flattening of the halo density profile at our resolution radii. These resolutions are not sufficient to directly test the predicted core radii by phase-space constraints (e.g., Hogan & Dalcanton 2000) or by dynamical models of gravitational collapse with initial random velocities (e.g., Avila-Reese et al. 2001; Bode et al. 2001). However, the inner extrapolations of the best-fit models to our simulated halos are consistent with such predictions.

This work was supported by PAPIIT-UNAM grants IN112806-2 and IN107706-3 and by a bilateral CONACyT-DFG grant.
The authors gratefully acknowledge the hospitality extended by the Astrophysikalisches Institut Potsdam, where this paper was performed. O. V. acknowledges support from the NSF grant 02-05413 assigned to the University of Washington during the initial stage of the project, and a CONACyT Repatriación fellowship. Some of the simulations presented in this paper were performed using the HP CF 4000 cluster (Kan-Balam) at DGSCA-UNAM. We acknowledge the anonymous referee, whose helpful comments and suggestions improved some aspects of this paper.

REFERENCES

Abazajian, K. 2006a, Phys. Rev. D, 73, 063506
———. 2006b, Phys. Rev. D, 73, 063513
Avila-Reese, V. Colin, P., Valenzuela, O., D’Onghia, E., & Firmani, C. 2001, ApJ, 559, 516
Avila-Reese, V., Firmani, C., Klypin, A., & Kravtsov, A. V. 1999, MNRAS, 310, 527
Benson, A. J., Frenk, C. S., Lacey, C. G., Baugh, C. M., & Cole, S. 2002, MNRAS, 333, 177
Bode, P., Ostriker, J. P., & Turok, N. 2001, ApJ, 556, 93
Bullock, J. S., Kolatt, T. S., Sigad, Y., Somerville, R. S., Kravtsov, A. V., Klypin, A., Primack, J. R., & Dekel, A. 2001, MNRAS, 321, 559
Bullock, J. S., Kravtsov, A. V., & Weinberg, D. H. 2000, ApJ, 539, 517
Busha, M. T., Evrard, A. E., & Adams, F. C. 2007, ApJ, 665, 1
Ceverino, D., & Klypin, A. 2007, MNRAS, 379, 1155
Colin, P., Avila-Reese, V., & Valenzuela, O. 2000, ApJ, 542, 622
Colin, P., Avila-Reese, V., Valenzuela, O., & Firmani, C. 2002, ApJ, 581, 777
Colin, P., Valenzuela, O., & Klypin, A. 2006, ApJ, 644, 687
de Blok, W. J. G., McGaugh, S. S., Bosma, A., & Rubin, V. C. 2001, ApJ, 552, L23
Dehnen, W. 2005, MNRAS, 360, 892
Dierand, J., Kuhlen, M., & Madan, P. 2006, ApJ, 649, 1
Dierand, J., Moore, B., & Stadel, J. 2004, MNRAS, 353, 624
———. 2005a, Nature, 433, 389
Dierand, J., Zemp, M., Moore, B., Stadel, J., & Carollo, C. M. 2005b, MNRAS, 364, 665
El-Zant, A. A., Hoffman, Y., Primack, J., Combes, F., & Shlosman, I. 2004, ApJ, 607, L75
Feng, J. L., Rajaraman, A., & Takayama, F. 2003, Phys. Rev. Lett., 91, 011102
Gao, L., White, S. D. M., Jenkins, A., Frenk, C. S., & Springel, V. 2005, MNRAS, 363, 379
Gentile, G., Burkert, A., Salucci, P., Klein, U., & Walter, F. 2005, ApJ, 634, L145
Gentile, G., Tonini, C., & Salucci, P. 2007, A&A, 467, 925
Gilmore, G., Wilkinson, M. I., Wyse, R., Kleyna, J. T., Koch, A., & Evans, N. W. 2007, ApJ, 663, 948
Goerdt, T., Moore, B., Read, J. I., Stadel, J., & Zemp, M. 2006, MNRAS, 368, 1073
Gotz, M., & Sommer-Larsen, J. 2003, Ap&SS, 284, 341
Governato, F., Willman, B., Mayer, L., Brooks, A., Stinson, G., Valenzuela, O., Wadsley, J., & Quinn, T. 2007, MNRAS, 374, 1479
Hayashi, E., & Navarro, J. F. 2006, MNRAS, 373, 1117
Hogan, C. J., & Dalcanton, J. J. 2000, Phys. Rev. D, 62, 063511
Huss, A., Jain, B., & Steinmetz, M. 1999, ApJ, 517, 64
Kaplinghat, M. 2005, Phys. Rev. D, 72, 063510
Kauffmann, G., White, S. D. M., & Guideroni, B. 1993, MNRAS, 264, 201
Kazantzidis, S., Zentner, A. R., & Kravtsov, A. V. 2006, ApJ, 641, 647
Klypin, A., Gottlöber, S., Kravtsov, A. V., & Khokhlov, A. M. 1999a, ApJ, 516, 530
Klypin, A., & Holtzman, J. 1997, preprint (astro-ph/9712217)
Klypin, A., Kravtsov, A. V., Bullock, J. S., & Primack, J. R. 2001, ApJ, 554, 903