REGULAR BLACK HOLES AND BLACK UNIVERSES

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We give a comparative description of different types of regular static, spherically symmetric black holes (BHs) and discuss in more detail their particular type, which we suggest to call black universes. The latter have a Schwarzschild-like causal structure, but inside the horizon there is an expanding Kantowski-Sachs universe and a de Sitter infinity instead of a singularity. Thus a hypothetic BH explorer gets a chance to survive. Solutions of this kind are naturally obtained if one considers static, spherically symmetric distributions of various (but not all) kinds of phantom matter whose existence is favoured by cosmological observations. It also looks possible that our Universe has originated from phantom-dominated collapse in another universe and underwent isotropization after crossing the horizon. An explicit example of a black-universe solution with positive Schwarzschild mass is discussed.

1. Introduction

One of the long-standing problems in black hole (BH) physics is the existence of curvature singularities beyond the event horizon in the BH solutions obtained under the simplest and the most natural physical conditions (the Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman solutions of general relativity and their counterparts in many alternative theories of gravity). Singularities are places where general relativity (or another classical theory of gravity) does not work. Therefore, a full understanding of BH physics requires avoidance of singularities or/and modification of the corresponding classical theory and addressing quantum effects. There have been numerous attempts on this trend, many of them suggesting that a singularity inside the event horizon should be replaced with a kind of regular core.

In this paper, we discuss the possible geometry of classical nonsingular BHs, restricting ourselves to asymptotically flat static, spherically symmetric configurations. In Sec. 2, we enumerate and briefly describe different known types of regular BHs. The rest of the paper is devoted to their particular type which we suggest to call black universes, namely, those in which a possible BH explorer, after crossing the event horizon, gets into an expanding universe [1]. Such objects are naturally obtained if one considers local concentrations of dark energy in the form of phantom matter, favoured by modern cosmological observations [2]. Among theoretical reasons for considering phantom matter one may mention natural appearance of phantom fields in some models of string theory [3], supergravities [4] and theories in more than 11 dimensions like F-theory [5]. To avoid

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the obvious quantum instability, a phantom scalar may perhaps be regarded as an effective field description following from an underlying theory with positive energies \[6\]. Curiously, a classical massless phantom field even shows a more stable behaviour than its usual counterpart \[7, 8\].

In Sec. 3, we demonstrate the existence of black-universe solutions in general relativity with minimally coupled phantom fields, formulate the requirements to the field potential needed for obtaining them and present a specific example. In Sec. 4, we obtain similar requirements in a more general framework, general k-essence, and show that many (though not all) forms of phantom matter discussed in the recent cosmological literature can also lead to black universes. Sec. 5 is a conclusion.

2. Regular black hole geometries

We begin with the general static, spherically symmetric metric

$$ds^2 = A(\rho)dt^2 - \frac{d\rho^2}{A(\rho)} - r^2(\rho)d\Omega^2,$$

(1)

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the metric on a unit sphere. Our sign conventions are as follows: the metric signature $(+ - - -)$ and the curvature tensor $R^\sigma_{\mu \nu \rho} = \partial_\nu \Gamma^\sigma_{\mu \rho} - \ldots$, so that, e.g., the Ricci scalar $R = R^\mu_{\mu} > 0$ for de Sitter space-time.

The metric (1) is written in terms of the “quasiglobal” coordinate $\rho$, which is particularly convenient for dealing with Killing horizons where it behaves in the same way as the manifestly well-behaved Kruskal-like null coordinates. For this reason, in terms of $\rho$, one may consider regions on both sides of such a horizon remaining in a formally static framework.

To discuss BHs in the comparatively simple case of static spherical symmetry, we may leave aside more general and more rigorous definitions of horizons and black holes (see, e.g., \[9\]) and rely on the following working definition. A black hole is a space-time containing (i) a static region which may be regarded external (e.g., contains a flat asymptotic), (ii) another region invisible to an observer at rest residing in the static region, and (iii) a Killing horizon of nonzero area that separates the two regions and admits an analytical extension of the metric from one region to another.

The two functions, $A(\rho)$ (often called the redshift function) and $r(\rho)$ (the area function, equal to the radius of a coordinate sphere at given $\rho$) entirely determine the geometry under consideration. Asymptotic flatness is described, without loss of generality, by $A \to 1$ and $r \approx \rho$ as $\rho \to \infty$. A centre $\rho = \rho_c$ (if any) corresponds to $r = 0$ while Killing horizons (if any) are described by zeros of the function $A(\rho)$. Killing horizons, at which the timelike Killing vector becomes null, divide the whole space-time manifold into static (R) regions, in which $A > 0$, and nonstatic, homogeneous (T) regions whose geometry is that of a Kantowski-Sachs anisotropic cosmological model. The number, order and disposition of such zeros determine the global causal structure of space-time.

The following types of geometries of regular, 4-dimensional, asymptotically flat, static, spherically symmetric BHs are known in the literature:

1. BHs with a regular centre ($r \approx \text{const} \cdot \rho - \rho_c, A \to A_c > 0, A(dr/d\rho)^2 \approx 1 + O(r^2)$ as $\rho \to \rho_c$). Since a regular centre can only be located in an R region, such a BH must have at least two simple horizons or one double horizon, and its causal structure is then represented by the same Carter-Penrose diagram as that of the non-extreme or extreme Reissner-Nordström BH (diagrams 1b and 1c in Fig. 1), respectively. A larger number of horizons is not excluded, leading to more complex causal structures.
2. BHs without a centre, having second-order horizons of infinite area (so-called cold BHs because such horizons are always characterized by zero Hawking temperature) [8, 10]. They may have different causal structures. In one case, the Carter-Penrose diagram coincides with that of Kerr’s extreme BH, consisting of an infinite tower of R regions but certainly without a ring singularity which is present in Kerr’s solution (plots 2a and diagram 2b in Fig. 1). In another case, there are only four R regions (plots 2c and diagram 2d).

3. BHs whose causal structure coincides with that of a non-extreme Kerr BH, again without a singular ring [11, 12] (diagram 3b).

4. Regular BHs with a Schwarzschild-like causal structure [1] (diagram 4b) but with cosmological expansion instead of a singular centre.

Type 1 traces back to Bardeen’s work [13] which put forward the very idea of regular BHs instead of singular ones and suggested, as an example, a particular BH configuration with

\[ r \equiv \rho, \quad A(\rho) = 1 - \frac{M \rho^2}{(\rho^2 + q^2)^{3/2}}, \]  

(2)

where \( M, q = \text{const.} \) and two horizons exist provided \( q^2 < (16/27)M^2 \). Later on there appeared numerous examples ([14–16] and others) of regular BH solutions where, just as in Eq. (2), \( r \equiv \rho \).

This, by virtue of the Einstein equation, implies that the stress-energy tensor of the matter source satisfies the condition \( T^0_0 \equiv T^1_1 \), or, in other words, \( \varepsilon = -p_r \) (\( \varepsilon = T^0_0 \) is the energy density and \( p_r = -T^1_1 \) is the radial pressure). The latter condition is invariant under radial boosts, making it possible to ascribe the source to vacuum matter [14], and at a regular centre in this case the matter equation of state has necessarily the form of a cosmological constant [17], \( T^\nu_\mu \propto \delta^\nu_\mu \).

It was also shown [16] that regular BHs with \( r \equiv \rho \) and any \( A(\rho) \) satisfying the regular centre conditions may be obtained as magnetic monopole solutions of general relativity coupled to gauge-invariant nonlinear electrodynamics with the Lagrangian \( L(F) \), \( F := F_{\mu\nu}F^{\mu\nu} \) (\( F_{\mu\nu} \) is the electromagnetic field tensor): the arbitrariness in \( A(\rho) \) corresponds to the freedom of choosing the function \( L(F) \). Solutions with an electric charge were shown [16] to be impossible whatever be the choice of \( L(F) \) if \( L(F) \) is the same in the whole space; this theorem, however, may be circumvented by assuming different forms of \( L(F) \) near the centre and at large \( r \) [18], i.e., by requiring a sort of phase transition(s) at some value(s) of the radial coordinate.

Other examples of type 1 regular BHs have also been found and discussed, see, e.g., [19] and references therein.

Type 2 regular BHs, with and without an electric charge, have been obtained [8, 10] in the framework of the Brans-Dicke scalar-tensor theory (STT) with the coupling constant \( \omega < -3/2 \), when the theory is of anomalous, or phantom nature. Their existence requires fine tuning in the form of specific relations between \( \omega \) and the integration constants of the corresponding exact solutions. It is of interest to note that these regular cold Brans-Dicke BHs have singular counterparts in the Einstein frame, in other words, in general relativity with a minimally coupled phantom scalar field (the so-called anti-Fisher solution) [20].

Type 3 regular BHs have been found [11, 12] as static, spherically symmetric solutions to the effective equations [21] describing 4D gravity in an RS2 type brane world. It has been shown that such regular solutions are generic in a certain range of the integration constants [12] and that many of them are stable, at least under some kinds of perturbations [22]. It should be stressed, however, that these 4D equations do not form a closed set, to study the full 5D geometry of the
Figure 1: Plots showing the qualitative behaviour of the metric functions and Carter-Penrose diagrams for the four different types of regular static, spherically symmetric black holes. Diagrams 1b and 1d (like those for the non-extreme and extreme Reissner-Nordström metrics) refer to curves $A_1$ and $A_2$ in plot 1a, respectively. Diagram 2b, like that for the extreme Kerr metric, refers to plot 2a, diagram 2d to plot 2c, 3b to 3a and 4b to 4a. The R and T letters in the diagrams designate the R and T space-time regions. Diagrams 1b, 1c, 2b and 3b are infinitely extendible upward and downward. In all diagrams, all inner slanting lines depict horizons while all boundaries correspond to $r = \infty$, with the following exceptions: the verticals in diagrams 1b and 1d describe a regular centre, $r = 0$; the horizontals in diagram 4b correspond either to $r = \infty$ or to $r = r_0 > 0$, according to the curves $r_1(\rho)$ or $r_2(\rho)$ at large negative $\rho$.

bulk one should solve the corresponding 5D equations, and there are only tentative results in this direction [23].

Type 4 configurations have been obtained [1] as generic solutions to the Einstein-scalar equations for the case of minimally coupled phantom scalar fields with certain potentials. Such scalar fields have recently become popular in the cosmological context since they are able to provide an equation
of state with the pressure to density ratio $p/\varepsilon = w < -1$. It is this kind of equation of state that is probably required for the dark energy (DE) component of the material content of the Universe to account for its accelerated expansion (see, e.g., the reviews [24, 25] and references therein). Type 4 configurations (we suggest to call them black universes) are, in our view, of particular interest, and in what follows we will discuss them in some more detail.

3. Black universes with a minimally coupled scalar field

Consider the action for a self-gravitating phantom scalar field in general relativity

$$S = \int \sqrt{g} d^4x [R - (\partial \phi)^2 - 2V(\phi)],$$

where $g = |\det(g_{\mu\nu})|$, $(d\phi)^2 = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ and $V(\phi)$ is an arbitrary potential. With the metric (1) and $\phi = \phi(\rho)$, the scalar field equation and three independent combinations of the Einstein equations read

$$\left( A r^2 \phi' \right)' = -r^2 dV/d\phi,$$
$$\left( A' r^2 \right)' = -2r^2 V;$$
$$2r''/r = \phi'^2;$$
$$A(r^2)'' - r^2 A'' = 2,$$

where the prime denotes $d/d\rho$. The scalar field equation (4) is a consequence of Eqs. (5)–(7), which, given a potential $V(\phi)$, form a determined set of equations for the unknowns $r(\rho), A(\rho), \phi(\rho)$. Eq. (7) can be integrated giving

$$B' \equiv \left( A/r^2 \right)' = 2(\rho_0 - \rho)/r^4,$$

where $B(\rho) = A/r^2$ and $\rho_0$ is an integration constant.

Eq. (8) severely restricts the possible dispositions of zeros of the function $A(\rho)$ and hence the global causal structure of space-time [26].

Indeed, horizons are regular zeros of $A(\rho)$ and hence $B(\rho)$. By (8), $B(\rho)$ increases at $\rho < \rho_0$, has a maximum at $\rho = \rho_0$ and decreases at $\rho > \rho_0$. It can have at most two simple zeros, bounding a range $B > 0$ (R region), or one double zero and two T regions around. It can certainly have a single simple zero or no zeros at all.

So the choice of possible types of global causal structure is precisely the same as for the general Schwarzschild-de Sitter solution with arbitrary mass and cosmological constant. This result (the Global Structure Theorem [26]) equally applies to normal and phantom fields since Eqs. (7), (8) are the same for them. It holds for any sign and shape of $V(\phi)$ and under any assumptions on the asymptotics. BHs with scalar hair (respecting the existing no-hair theorems) are not excluded. Examples of (singular) BHs with both normal (e.g., [27–29]) and phantom [30] scalar hair are known. However, BHs with a regular centre (type 1 according to the previous section) are ruled out for our system since their existence requires a regular minimum of $B(\rho)$.

As shown in Ref. [1], the system (4)–(7) has as many as 16 types of regular solutions with flat, de Sitter and AdS asymptotic behaviour. Let us discuss asymptotically flat configurations, for which $A(\rho) \to 1$ and $r(\rho) \approx \rho$ as $\rho \to \infty$. Then, as $\rho$ decreases from infinity, the derivative $r'$ also decreases according to Eq. (6), the decrease rate depending on the details of the system. If the decrease is slow enough, $r(\rho)$ will reach zero at some finite $\rho$, which means that the system has a centre. The latter may be regular only if horizons are absent (otherwise there would be a type 1
regular BH which is ruled out here), and a particelike solution is then obtained instead of a BH one.

Assuming the absence of singularities, other opportunities are $r(\rho) \to r_0 = \text{const}$ and $r(\rho) \to \infty$ as $\rho \to -\infty$. (Note that any kind of oscillatory behaviour of $r(\rho)$ is ruled out by (6) according to which $r'' \geq 0$.)

In the first case, according to (8),

$$A' \approx -2\rho/r_0^2 \to +\infty, \quad A \approx -\rho^2/r_0^2 \quad \text{as} \quad \rho \to -\infty. $$

So this “$r_0$ asymptotic” is located in a T region. The radius $r_0$ is related to the limiting value of the potential: $V \to -1/r_0^2$. Changing the notations $-\rho \to T$ and $t \to x$ (since $\rho$ here becomes a temporal coordinate and the former time $t$ a spatial one), we can write the asymptotic form of the resulting Kantowski-Sachs metric as

$$ds^2 \approx \frac{r_0^2}{T^2}dT^2 - \frac{T^2}{r_0^2}dx^2 - r_0^2d\Omega^2 \quad \text{as} \quad T \to \infty. $$

(9)

It is a highly anisotropic universe, exhibiting no expansion in the two angular directions and an exponential (in terms of the physical time $\tau \sim \log T$) expansion in the third direction $x$. From the viewpoint of an observer at large positive $\rho$, this universe is located beyond the event horizon of a BH.

In case $r \to \infty$, the most interesting opportunity is that $r \sim |\rho|$ as $\rho \to -\infty$. Assuming $r \approx -a\rho$, $a = \text{const} > 0$, from (8) and (5) we obtain

$$A \approx 1/a^2 - Ca^2\rho^2, \quad V \approx 3Ca^2, \quad C = \text{const}. $$

(10)

It is easy to verify that $C = 0$ leads to a Minkowski metric at large negative $\rho$ (though the time rate will be different from that at large positive $\rho$ if $a \neq 1$), an anti-de Sitter metric if $C < 0$ and a de Sitter metric if $C > 0$. In the cases $C \leq 0$ horizons are absent since otherwise the function $B(\rho)$ would have a minimum at some finite $\rho$, which cannot happen due to (8). Thus possible solutions with $C \leq 0$ describe traversable wormholes, and the details of their geometry depend on the particular choice of $V(\phi)$.

Lastly, for $C > 0$ we obtain a de Sitter asymptotic behaviour of the solution, describing (since we are now in a T region) isotropic expansion or contraction. From the viewpoint of an external observer, located at large positive $\rho$, it is a BH, but a possible BH explorer now has a chance to survive for a new life in an expanding, gradually isotropizing Kantowski-Sachs universe. The specific BH profile and the isotropization regime after crossing the horizon depend on the choice of $V(\phi)$.

Since, according to the Global Structure Theorem (see above), an asymptotically flat configuration can have only one simple horizon, such a BH has a Schwarzschild-like causal structure, but the singularity $r = 0$ in the Carter-Penrose diagram is now replaced by the de Sitter infinity $r = \infty$.

A simple example may be obtained by putting [1]

$$r = (\rho^2 + b^2)^{1/2}, \quad b = \text{const} > 0. $$

(11)

and using the inverse problem scheme. Eq. (8) gives

$$B(\rho) = \frac{A(\rho)}{r^2(\rho)} = \frac{c}{b^2} + \frac{1}{b^2 + \rho^2} + \frac{\rho_0}{b^3} \left( \frac{bp}{b^2 + \rho^2} + \tan^{-1} \frac{\rho}{b} \right), $$

(12)
where \( c = \text{const.} \) Eqs. (6) and (5) then lead to expressions for \( \phi(\rho) \) and \( V(\rho) \):
\[
\phi = \pm \sqrt{2} \tan^{-1}(\rho/b) + \phi_0, \\
V = -\frac{c}{b^2} \frac{r^2 + 2\rho^2}{r^2} - \frac{\rho_0}{b^3} \left( \frac{3b\rho}{r^2} + \frac{r^2 + 2\rho^2}{r^2} \tan^{-1} \frac{\rho}{b} \right)
\]
with \( r = r(\rho) \) given by (11). In particular,
\[
B(\pm \infty) = -\frac{1}{3} V(\pm \infty) = \frac{2bc \pm \pi \rho_0}{2b^3}.
\]
Choosing in (13), without loss of generality, the plus sign and \( \phi_0 = 0 \), we obtain for \( V(\phi) \) (\( \psi := \phi/\sqrt{2} \)):
\[
V(\phi) = -\frac{c}{b^2} (3 - 2 \cos^2 \psi) - \frac{\rho_0}{b^3} [3 \sin \psi \cos \psi + \psi(3 - 2 \cos^2 \psi)].
\]

The solution behaviour is controlled by two integration constants: \( c \) that moves \( B(\rho) \) up and down, and \( \rho_0 \) showing the maximum of \( B(\rho) \). Both \( r(\rho) \) and \( B(\rho) \) are even functions if \( \rho_0 = 0 \), otherwise \( B(\rho) \) loses this symmetry. Asymptotic flatness at \( \rho = +\infty \) implies \( 2bc = -\pi \rho_0 \) while the Schwarzschild mass, defined in the usual way, is \( m = \rho_0/3 \).

Under this asymptotic flatness assumption, for \( \rho_0 = m = 0 \) we obtain the simplest symmetric configuration, the Ellis wormhole [31]: \( A \equiv 1, V \equiv 0 \). For \( \rho_0 < 0 \), according to (15), we obtain a wormhole with \( m < 0 \) and an AdS metric at the far end, corresponding to the cosmological constant \( \Lambda < 0 \). For \( \rho_0 > 0 \), when \( V_\omega > 0 \), there is a regular BH with \( m > 0 \) and a de Sitter asymptotic far beyond the horizon, precisely corresponding to the above description of a black universe.

The horizon radius \( r(\rho_h) \) may be obtained by solving the transcendental equation \( A(\rho_h) = 0 \), where \( A(\rho) \) is given by Eq. (12). It depends on both parameters \( m \) and \( b = \min r(\rho) \) and cannot be smaller than \( b \), which also plays the role of a scalar charge: \( \psi \approx \pi/2 - b/\rho \) at large \( \rho \). Since \( A(0) = 1 + c \), the throat \( \rho = 0 \) is located in the R region if \( c < -1 \), i.e., if \( 3\pi m < 2b \), at the horizon if \( 3\pi m = 2b \) and in the T region beyond it if \( 3\pi m > 2b \). The above relations between \( m \) and \( b \) show (and it is probably generically true) that if the BH mass dominates over the scalar charge, the throat is invisible to a distant observer, and the BH looks from the static region almost as usual in general relativity.

As follows from Eqs. (4) and (5), the potential \( V \) tends to a constant and, moreover, \( dV/d\phi \to 0 \) at each end of the \( \rho \) range. It is a general property of all classes of regular solutions indicated in [1]. More precisely, a regular scalar field configuration requires a potential with at least two zero-slope points (not necessarily extrema) at different values of \( \phi \).

Suitable potentials are, e.g., \( V = V_0 \cos^2(\phi/\phi_0) \) and the Mexican hat potential \( V = (\lambda/4)(\phi^2 - \eta^2)^2 \) where \( V_0, \phi_0, \lambda, \eta \) are constants. A flat infinity at \( \rho = +\infty \) certainly requires \( V_+ = 0 \), while a de Sitter asymptotic can correspond to a maximum of \( V \) since phantom fields tend to climbing up the slope of the potential rather than rolling down, as is evident from Eq. (4). Accordingly, Faraoni [32], considering spatially flat isotropic phantom cosmologies, has shown that if \( V(\phi) \) is bounded above by \( V_0 = \text{const} > 0 \), the de Sitter solution is a global attractor. Very probably this conclusion extends to Kantowski-Sachs cosmologies after isotropization.

One more point may be stressed: the late-time de Sitter expansion rate is entirely determined by the corresponding potential value \( V_\omega > 0 \) (which, in our notation according to (3), coincides with the effective cosmological constant \( \Lambda \) at late times) rather than by the details of the solution such as the Schwarzschild mass defined at the flat asymptotic.

We can conclude that black universes are a generic kind of solutions to the Einstein-scalar equations in the case of phantom scalars with proper potentials.
4. Black universes with other forms of phantom matter

Beside the minimally coupled scalar field (3), there are a number of other suggestions for modelling the possible phantom behaviour of dark energy, see, e.g., the review [24] and references therein.

Let us show that type 4 regular BHs, or black universes, are, under appropriate additional conditions, solutions to the field equations of at least two large classes of such theories: scalar-tensor theories (STT) of gravity, or theories with nonminimally coupled scalar fields, and the class of models called k-essence. We will show that black-universe solutions may be found in both these classes.

4.1. Scalar-tensor theories

Consider a general (Bergmann-Wagoner-Nordtvedt) STT, defined in a space-time manifold with the metric \( g_{\mu\nu} \) (called the Jordan conformal frame), for which the gravitational action is written of the form

\[
S_{\text{STT}} = \int d^4x \sqrt{|g|} \left[ f(\Phi) R + h(\Phi)(\partial\Phi)^2 - 2U(\Phi) \right],
\]

where \( g = |\det(g_{\mu\nu})| \), \( (d\Phi)^2 = g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \) and \( f, h, U \) are arbitrary functions of the scalar field \( \Phi \).

The action (17) is simplified by the well-known conformal mapping [33]

\[
g_{\mu\nu} = \bar{g}_{\mu\nu}/|f(\Phi)|, \quad \frac{d\phi}{d\Phi} = \pm \sqrt{\frac{|l(\Phi)|}{f(\Phi)}}, \quad l(\Phi) := fh + \frac{3}{2} \left( \frac{df}{d\Phi} \right)^2,
\]

removing the nonminimal scalar-tensor coupling expressed in the factor \( f(\Phi) \) before \( R \). The action (17) is now specified in a new manifold with the metric \( \bar{g}_{\mu\nu} \) (the Einstein frame) and the new scalar field \( \phi \):

\[
S_E = \int d^4x \sqrt{\bar{g}} \left\{ \text{sign} f[R + (\text{sign} l)(\partial\phi)^2] - 2V(\phi) \right\},
\]

where the determinant \( \bar{g} \), the scalar curvature \( \bar{R} \) and \( (\partial\phi)^2 \) are calculated using \( \bar{g}_{\mu\nu} \), and

\[
V(\phi) = |f(\Phi)|^{-2} U(\Phi).
\]

The action (20) is similar to that of GR with a minimally coupled scalar field \( \phi \) but contains two sign factors. The usual sign of gravitational coupling corresponds to \( f > 0 \), while \( l(\Phi) \) distinguishes normal \( (l > 0) \) and phantom \( (l < 0) \) fields. Of interest for us are phantom field theories with usual gravitational coupling, i.e., we will suppose \( f > 0, l < 0 \).

Then the vacuum field equations have precisely the form (4)–(7), with all the corresponding solutions. The main properties of these solutions are preserved after transformation back to the Jordan frame, provided the conformal factor \( 1/f \) is everywhere nonzero and regular. Indeed, in this case, a flat asymptotic transforms to a flat asymptotic (although maybe with a different Schwarzschild mass) and a horizon to a horizon of the same order. Thus one can guarantee that a black universe solution in the Einstein frame preserves its qualitative properties in the Jordan frame. However, some remarks are in order.

First, the asymptotic regimes in the Einstein frame occur at values of \( \phi \) where \( V(\phi) \) has zero slope and, moreover, asymptotic flatness requires \( V = 0 \). Assuming \( f \neq 0 \), we may reformulate
these requirements in terms of $U(\Phi)$: evidently, $U = 0$ at the flat asymptotic in the Jordan frame, and, at the other end, it is the function $V(\phi)$ given by Eq. (21) that must have zero slope, while nothing certain may be said about $U$ at the corresponding value of $\Phi$.

Second, a de Sitter asymptotic behaviour at $\rho \to -\infty$ is not necessarily preserved in Jordan’s frame even if it takes place in Einstein’s. In general, the expansion law will be different but isotropization must take place. Everything depends on the specific properties of $U$ and $f$.

In general, we see that black-universe solutions may be expected in a generic STT with a phantom scalar field provided the potential function $U(\phi)$ (or, as it is sometimes said, the scalar-dependent “cosmological constant”) satisfies some natural conditions which are best formulated in terms of the Einstein conformal frame.

### 4.2. k-essence

Another large class of scalar field ($\phi$) models invoked in order to describe the modern cosmological acceleration is characterized by a non-canonical dependence of the action on the derivatives $\partial_\mu \phi$. The most general form of such actions for scalar fields minimally coupled to space-time curvature may be written as

$$S = \int \partial^4 x \sqrt{|g|} R + F(\phi, X), \quad X := (\partial \phi)^2$$  \hspace{1cm} (22)

(see, e.g., [24]; references to numerous papers discussing particular cases of k-essence as well as other models of dark energy can also be found in this comprehensive review).

The stress-energy tensor of $\phi$ then has the form

$$T^\nu_\mu[\phi] = F_X \partial_\mu \phi \partial^\nu \phi - \frac{1}{2} F \delta^\nu_\mu,$$  \hspace{1cm} (23)

where $F_X \equiv \partial F/\partial X$. Obvious special cases of (22) are

(i) normal and phantom scalar fields with the usual kinetic term ($F = \epsilon X - 2V(\phi)$, $\epsilon = 1$ for a normal field and $\epsilon = -1$ for a phantom field);

(ii) the most frequently used forms of k-essence, with $F = V(\phi)U(X)$, and, in particular, the so-called tachyonic field, for which $F = -V(\phi)\sqrt{1 - X}$.

In the cosmological setting, the action (22) may also describe such forms of dark energy as

(iii) a perfect fluid with the barotropic equation of state $w = \epsilon/p = \text{const} < -1/3$, for which one should put

$$F(\phi, X) = V(\phi)X^{(w+1)/(2w)},$$  \hspace{1cm} (24)

with an arbitrary function $V(\phi)$;

(iv) a generalized Chaplygin gas, defined as a perfect fluid with the equation of state $p = -A/\varepsilon^\alpha$, with $A, \alpha = \text{const}$, for which one should choose $F(\phi, X)$ satisfying the equation

$$2X F_X = F + [-F/(2A)]^{-1/\alpha}.$$  \hspace{1cm} (25)

In particular, to obtain a “simple” Chaplygin gas, with the equation of state $p = -A/\varepsilon$, one can put

$$F = \pm 2\sqrt{A} \sqrt{1 + Xf(\phi)}$$  \hspace{1cm} (26)

with an arbitrary function $f(\phi)$.  

To verify this, it is sufficient to put \( \phi = \phi(t) \) and \( g_{00} = 1 \), where \( x^0 = t \) is the cosmological time, so that \( X = (d\phi/dt)^2 \); we have then \( \varepsilon = XF_X - F/2 \) and \( p = F/2 \).

Let us now return to static, spherically symmetric configurations with the metric (1). It is then straightforward to check that the field equations for the action (22) may be written in the following form similar to (4)–(7):

\[
\begin{align*}
2(F_X A r^2 \phi')' &= -r^2 F \phi; \\
(A' r^3)' &= r^2 (F - XF_X); \\
2 r''/r &= -\phi'^2 F_X, \\
A(r^2)'' - r^2 A'' &= 2.
\end{align*}
\]

Again, the scalar field equation (27) may be obtained from the Einstein equations (28)–(30), and so the latter three may be considered as a determined set of equations for \( \phi(\rho), A(\rho) \) and \( r(\rho) \) if the function \( F(\phi, X) \) has been prescribed.

Eq. (30) is the same as (7) due to the relation \( T_0^0 = T_2^2 \) valid for the tensor (23) just as for that of a usual minimally coupled scalar. Therefore, we again obtain the integral (8) and, as its consequence, the Global Structure Theorem [26] restricting the possible global properties of the solutions.

Since the action (22) is more general than (3), it is reasonable to expect that the set of possible solutions will also be richer. Let us try to formulate some necessary conditions for obtaining solutions of interest for us here, namely, black universes.

Above all, to obtain \( r \to \infty \) as \( \rho \to \pm \infty \) it is necessary to have \( r'' > 0 \) at least in some range of \( \rho \), hence \( F_X < 0 \) (the “phantom condition”).

Other conditions may be deduced by assuming \( r \sim \rho \) as \( \rho \to \pm \infty \) and requiring a Minkowski limit at large positive \( \rho \) and a de Sitter limit with some cosmological constant \( \Lambda = \Lambda_\pm \) at large negative \( \rho \). Let us also assume that \( F \) is a smooth function of both arguments while \( \phi \) tends to finite limits \( \phi_\pm \) and admits expansions in power series in \( 1/\rho \) as \( \rho \to \pm \infty \).

Substitution into the field equations then gives:

\[
\begin{align*}
F, F_\phi, XF_X &= O(\rho^{-4}) \quad \text{as} \quad \rho \to \infty, \\
F &\to \sqrt{\Lambda_0/3}, \quad XF_X, F_\phi = O(\rho^{-2}) \quad \text{as} \quad \rho \to -\infty.
\end{align*}
\]

The symbol \( O(x) \) means here a quantity of order \( x \) or smaller. In all cases \( X \to 0 \) at large \( |\rho| \), and so the function \( F(\phi, 0) \) has zero slopes at \( \phi = \phi_\pm \), just as the potential \( V(\phi) \) of a minimally coupled scalar field in Sec. 3..

The asymptotic regimes of \( F(\phi, X) \) are different in Minkowski and de Sitter limits. Indeed, if we assume \( \phi = \phi_+ + \text{const}/\rho \) and \( \phi' \sim \rho^{-2} \) as \( \rho \to \infty \), which is characteristic of a massless (long-range) scalar field, then (31) leads to \( F = O(\rho^{-4}) \), so that \( F(\phi, 0) \sim (\phi - \phi_+)^4 \) or is even smaller. This is not surprising since \( F(\phi, 0) \) behaves as a potential which must be smaller than quadratic for a massless behaviour of \( \phi \).

On the other hand, at the de Sitter asymptotic, under similar assumptions \( (\phi = \phi_- + \text{const}/\rho \) as \( \rho \to -\infty \)), we obtain a generic quadratic behaviour of \( F \): \( F(\phi, 0) \sim (\phi - \phi_-)^2 \), and the massless case is not distinguished in the behaviour of the “potential” \( F \).

These observations are confirmed by the asymptotic behaviour of the potential (16) in the above example (11)–(16): \( V \sim (\phi - \pi/\sqrt{2})^4 \) as \( \rho \to \infty \) and \( V - V_- \sim (\phi + \pi/\sqrt{2})^2 \) as \( \rho \to -\infty \) where \( V_- = \Lambda = V|_{\rho \to -\infty} = 3\pi \rho_0/b^3 \).

The conditions (31) are easily re-formulated for the frequently used kinds of k-essence: \( F = f(X) - 2V(\phi) \) and \( F = f(X)V(\phi) \).
It, however, can be easily shown that black-universe solutions are absent for the perfect fluid representations (24) and (26). Indeed, \( X = -A\phi'^2 \) changes its sign at the horizon, therefore \( F \) must make sense for any sign of \( X \). Meanwhile, in the function (24), the exponent \( (w + 1)/(2w) \) is fractional for any \( w < -1 \), and the whole expression loses its meaning at \( X < 0 \). As to the function (26), it is nonzero at \( X = 0 \), which is incompatible with asymptotic flatness.

Thus many, though not all, kinds of k-essence lead to black-universe solutions, under necessary conditions similar to those formulated for minimally coupled scalar fields.

## 5. Conclusion

In this paper, we have classified the possible geometries of static, spherically symmetric, asymptotically flat BHs and discussed in more detail their particular type, the black universes. Such hypothetical configurations combine the properties of a wormhole (absence of a centre, a regular minimum of the area function) and a black hole (a Killing horizon separating R and T regions).

Quite evidently, such unusual objects require unusual matter for their existence. It turns out that they are naturally obtained if one considers static, spherically symmetric distributions of phantom matter, which is a subject of vast discussion in modern cosmology. We have considered a sufficiently general frameworks for the description of phantom matter, namely, STT of gravity and the so-called k-essence, and found necessary conditions for the existence of black-universe solutions. A conclusion is that a great number of reasonable phantom matter models produce such solutions.

The latter lead to the idea that our Universe could appear from phantom-dominated collapse in another, “mother” universe and undergo isotropization (e.g., due to particle creation) soon after crossing the horizon. It is known that a Kantowski-Sachs nature of our Universe is not excluded observationally [34] if its isotropization had happened early enough, before the last scattering epoch (at redshifts \( z \gtrsim 1000 \)). One can notice that we are thus facing one more mechanism of universes multiplication, in addition to the well-known mechanism existing in the chaotic inflation scenario.

Somewhat similar ideas on possible appearance of baby universes inside BHs have already been discussed from different standpoints [35,36] (see also references therein), and some examples of two-dimensional regular BH metrics with properties more or less similar to ours are known [35,37,38]. It can be remarked, however, that in two dimensions, where space is simply a line, the notion of a centre, which plays an essential role in the four-dimensional picture, cannot be properly introduced. So it seems that the black-universe solutions considered here and in Ref. [1] are qualitatively new and (if certainly amended by adding realistic matter ingredients) may even lead to viable alternatives to the existing cosmological scenarios.

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