Baryon asymmetry from Barrow entropy: theoretical predictions and observational constraints

Giuseppe Gaetano Luciano\textsuperscript{1,2,a}, Emmanuel N. Saridakis\textsuperscript{3,4,5,b}

\textsuperscript{1} Dipartimento di Fisica, Università di Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano, SA, Italy
\textsuperscript{2} INFN, Sezione di Napoli, Gruppo Collegato di Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano, SA, Italy
\textsuperscript{3} National Observatory of Athens, Lofos Nymfon, 11852 Athens, Greece
\textsuperscript{4} Department of Astronomy, School of Physical Sciences, University of Science and Technology of China, Hefei 230026, Anhui, China
\textsuperscript{5} CAS Key Laboratory for Research in Galaxies and Cosmology, University of Science and Technology of China, Hefei 230026, Anhui, China

Received: 21 May 2022 / Accepted: 16 June 2022 / Published online: 24 June 2022
© The Author(s) 2022

Abstract We investigate the generation of baryon asymmetry from the corrections brought about in the Friedmann equations due to Barrow entropy. In particular, by applying the gravity-thermodynamics conjecture one obtains extra terms in the Friedmann equations that change the Hubble function evolution during the radiation-dominated epoch. Hence, even in the case of standard coupling between the Ricci scalar and baryon current they can lead to a non-zero baryon asymmetry. In order to match observations we find that the Barrow exponent should lie in the interval $0.005 \lesssim \Delta \lesssim 0.008$, which corresponds to a slight deviation from the standard Bekenstein–Hawking entropy. The upper bound is tighter than the one of other observational constraints, however the interesting feature is that in the present analysis we obtain a non-zero lower bound. Nevertheless this lower bound would disappear if the baryon asymmetry in Barrow-modified cosmology is generated by other mechanisms, not related to the Barrow modification.

1 Introduction

Since the first investigation of the thermodynamic features of black holes, the connection between gravity and thermodynamics has been widely explored, becoming even more central in the recent effort of developing a quantum theory of gravity [1–3]. This concept was later formalized by the so-called gravity-thermodynamics conjecture [4–6], which states that the field equations of General Relativity may arise from the laws of thermodynamics applied on spacetime itself [7]. On cosmological grounds, the investigation of the gravity-thermodynamics conjecture has revealed that Friedmann equations can be extracted by applying the first law of thermodynamics to the apparent horizon of the Universe [8–11], thus unveiling interesting scenarios in a plethora of contexts [12–18].

In the commonly adopted formulation, the gravity-thermodynamics conjecture makes use of the Bekenstein–Hawking (BH) area law $S_{BH} = A/(4G)$ for the black-hole horizon entropy ($A = 4\pi r_{hor}^2$ is the area with $r_{hor}$ the black-hole horizon) and applies it to the Universe apparent horizon. However, several extensions have appeared in the literature, by using various modified entropy relations, arising from non-extensive generalizations of the statistics of horizon degrees of freedom and/or quantum gravitational deformations of the horizon geometry. Among these, special focus has been placed on Tsallis [19] and Kaniadakis [20] entropies, whose cosmological applications have been addressed in [21–32].

Recently, a plausible generalized entropy which is based on a modified horizon endowed with a fractal structure, has been proposed by Barrow as [33]

\begin{equation}
S_{\Delta} = \left(\frac{A}{4G}\right)^{1+\Delta/2}.
\end{equation}

In particular, quantum effects are parameterized by the Barrow exponent $0 \leq \Delta \leq 1$, with $\Delta = 0$ giving the BH limit, while $\Delta = 1$ corresponds to the maximal deformation. Although Barrow entropy was formulated for black holes, in the lines of gravity-thermodynamic conjecture it can be applied in a cosmological framework. In this way, one acquires corrections to the Standard Model of Cosmology (SMC), namely on the Friedmann equations, brought about by the Barrow entropy [34]. Additionally, one can apply Barrow entropy to the holographic principle, obtaining Barrow holographic dark energy [35–39]. Hence, one can confront...
the above constructions with observational data end amongst others extract constraints on the Barrow exponent $\Delta$ [40–43]. As expected, in all these studies deviations from the BH entropy are found to be relatively small.

Up to now the phenomenology of Barrow-entropy-based cosmological models mostly deals with the impositions of constraints on Barrow exponent by comparison with confirmed predictions of cosmology. However, it would be interesting to examine whether these models can account for observational evidences which are not perfectly understood within standard cosmology. In this perspective, one of such puzzles is the origin of Baryon Asymmetry in the Universe (BAU). As it was found by Sakharov [44], in a particle physics theory three conditions have to be satisfied in order to produce BAU: (i) baryon number violation, (ii) $C$-symmetry and $CP$-symmetry violation and (iii) out-of-thermal-equilibrium interactions. However, the predicted BAU in the SMC is vanishing. Although several potential explanations have been hitherto suggested [45–62] (see also models with GUT interactions [63–66]), none of them offers a full and well-accepted solution [67].

Starting from the above premises, in this work we analyze baryogenesis in the framework of Barrow cosmology. Since (1) induces modifications in the Friedmann equations, it leads to modified energy density and pressure that can comply with all three Sakharov conditions. As a result, we obtain a non-vanishing ($\Delta$-dependent) expression for the baryon asymmetry parameter $\eta$, which allows us to (i) account for the origin of BAU and (ii) constrain Barrow parameter $\Delta$ by comparison with current observational bounds on $\eta$. We emphasize that this picture is physically motivated by the fact that quantum gravity corrections are expected to play a non-trivial role on the apparent horizon in the early Universe, with non-negligible effects on the subsequent cosmic evolution too. Therefore, while the research on Barrow-entropy-based cosmology should certainly be considered as preliminary, on the other hand it may provide valuable hints towards understanding the impact of quantum gravity on horizon properties and related phenomenology.

The present work is organized as follows: in Sect. 2 we implement the gravity-thermodynamics conjecture with Barrow entropy and we derive the modified Friedmann equations. Then in Sect. 3 we provide a detailed investigation of the baryogenesis procedure in Barrow cosmology, and we extract the constraints on Barrow exponent $\Delta$. Conclusions and outlook are finally discussed in Sect. 4. Throughout the manuscript, we work in natural units $\hbar = c = k_B = 1$, while we keep the gravitational constant $G$ explicitly.

### 2 Friedmann equations in Barrow cosmology

In this section we present a derivation of Friedmann equations for the Friedmann–Robertson–Walker (FRW) metric within the framework of Barrow cosmology. Towards this end, we consider the $(1 + 3)$-dimensional line element

$$ds^2 = h_{bc}dx^bdx^c + \tilde{r}^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

where $h_{bc} = \text{diag}[-1, a^2/(1 - kr^2)] (b, c = (0, 1))$ is the metric of a $(1 + 1)$-dimensional subspace of coordinates $x^b \equiv (t, r)$ and $\tilde{r} = a(t)r$, with $a(t)$ being the time-dependent scale factor. Here, we have denoted by $k$ the (constant) spatial curvature. Moreover, we assume that the Universe is bounded by the apparent horizon of radius $\tilde{r}_H = 1/\sqrt{H^2 + k/a^2}$, where $H = \dot{a}(t)/a(t)$ is the Hubble parameter (in the following dots denote time-derivatives). From the definition of surface gravity $\kappa$ on the apparent horizon, it is easy to show that the related temperature is [10]

$$T = \frac{\kappa}{2\pi} = -\frac{1}{2\pi \tilde{r}_H} \left(1 - \frac{\dot{\tilde{r}}}{2H\tilde{r}_H}\right).$$

We describe the content of the Universe as a perfect fluid of energy–momentum tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu},$$

where $\rho$ and $p$ are the energy density and pressure at equilibrium, while $u_\mu$ is the four-velocity of the fluid. The conservation equation $\nabla_\mu T^{\mu\nu} = 0$ for the FRW geometry then implies the continuity equation $\dot{\rho} = -3H(\rho + p)$. We mention that this fluid can be the total (matter plus radiation) one, the matter one or the radiation one, according to the application we are interested in each time.

Using the gravity-thermodynamics conjecture, the cosmological (Friedmann) equations can be derived by considering the Universe as a thermodynamic system bounded by the apparent horizon and applying the first law of thermodynamics

$$dE =TdS + WdV$$

on the horizon. Here, $E = \rho V$ is the total energy of the Universe of 3-dimensional volume $V = 4\pi \tilde{r}_H^3/3$ and surface area $A = 4\pi \tilde{r}_H^2$, while $S$ denotes the horizon entropy. Since the Universe is expanding, a work $W = -\frac{1}{2}T^{bc}h_{bc} = \frac{1}{2}(\rho - p)$ is associated to the change of volume $dV$. Concerning the entropy $S$, in general it is an extension of the Bekenstein-Hawking entropy, and focusing on modifying area laws it can be written as

$$S = \frac{f(A)}{4G},$$

where the function $f(A)$ quantifies the deviation from the standard BH relation (the latter is recovered for $f(A) = A$). Insertion of the above relations into (5) leads to the first Friedmann equation [34]

$$-4\pi G(\rho + p) = \left(\dot{\tilde{r}}^2 - \frac{k}{a^2}\right)f'(A),$$
where the prime denotes derivative with respect to $A$. Moreover, by utilizing the continuity equation and integrating we obtain the second Friedmann equation as

$$\frac{8\pi G}{3} \rho = -4\pi \int \frac{f'(A)}{A^2} dA. \tag{8}$$

Note that Eqs. (7) and (8) are of general validity, and the specific features of the adopted entropic model are quantified by the function $f(A)$ in (6). As expected in the case $f(A) = A$ of BH entropy, we recover the standard Friedmann equations.

In the case of Barrow entropy (1) we obtain the modified Friedmann equations [34]

$$-4\pi G (\rho + p) = \alpha_\Delta \left( H - \frac{k}{a^2} \right) \left[ G \left( H^2 + \frac{k}{a^2} \right) \right]^{-\Delta/2}, \tag{9}$$

$$\frac{8\pi G}{3} \rho = \frac{2\alpha_\Delta}{G(2-\Delta)} \left[ G \left( H^2 + \frac{k}{a^2} \right) \right]^{1-\Delta/2} + c, \tag{10}$$

with

$$\alpha_\Delta = \frac{\pi^{\Delta/2}(2+\Delta)}{2}. \tag{11}$$

Finally, note that the integration constant $c$ is identified with the cosmological through $c = 8\pi G A/3$. As mentioned above, in the limit $\Delta \to 0$, in which Barrow entropy recovers BH entropy, the above expressions reproduce the standard Friedmann equations.

In the following for simplicity we will focus on the flat case $k = 0$, and since we are interested in the early universe we consider that the Universe cosmic fluid corresponds to the radiation sector and we neglect the cosmological constant.

### 3 Baryon asymmetry from Barrow entropy

The origin of matter-antimatter asymmetry in the early Universe is one of the most debated problems in present day cosmology. Observations unambiguously indicate that the amount of matter prevails over antimatter, in contrast to the predictions of the Standard Model of Particle Physics [52]. As we mentioned in the Introduction, in order to generate dynamically the Baryon asymmetry in the universe (BAU), the three Sakharov conditions should be fulfilled.

The first Sakharov criterion ensures that the Universe evolves from an originally baryon-symmetric state into a configuration where the difference

$$\frac{n B - n \bar{B}}{s} \tag{12}$$

is no longer vanishing, with $n B$ ($n \bar{B}$) being the baryon (anti-baryon) number density, and $s$ the entropy density in the radiation-dominated era. The second Sakharov condition is required due to the fact that if $C$ and $CP$ were exact Hamiltonian symmetries then the total rate for any interaction producing an excess of baryons would be compensated by the complementary process producing an excess of anti-baryons. Finally, the last condition can be understood by calculating the equilibrium average of baryon number $B$ as [68]

$$(B)_\beta = \text{Tr} \left( e^{-\beta H} B \right) = \text{Tr} \left[ (CPT)^{-1} e^{-\beta H} B (CPT) \right] = -\text{Tr} \left( e^{-\beta H} B \right), \tag{13}$$

where $T$ is the time-reversal, and where we have exploited the fact that $H$ commutes with $CPT$. As a result, one obtains $(B)_\beta = 0$ at equilibrium, thus preventing net baryon number generation. While complying with the first two Sakharov conditions, standard cosmology fails to predict baryogenesis, since the last criterion is not satisfied during the whole radiation-dominated era.

One can satisfy the first Sakharov condition within certain supergravity theories, as highlighted in [45]. In this framework the $CP$-violating interaction in vacuum between the derivative of the Ricci scalar curvature $\mathcal{R}$ and the baryon number current $J^\mu$ takes the form [48,69]

$$\frac{1}{M^2} \int d^4x \sqrt{-g} \ J^\mu \partial_\mu \mathcal{R}, \tag{14}$$

where $M = (8\pi G)^{-1/2}$ is the characteristic cutoff scale (see also [70–75]). In order to create asymmetry one assumes that there exists some interaction violating the baryon number $B$. By noticing that the spatial part of $\mathcal{R}$ vanishes for the FRW metric, one has

$$\frac{1}{M^2} \ J^\mu \partial_\mu \mathcal{R} = \frac{1}{M^2} \left( n B - n \bar{B} \right) \dot{\mathcal{R}}. \tag{15}$$

Thus, in an expanding Universe where $\mathcal{R}$ and $\dot{\mathcal{R}}$ are non-zero, the interaction (14) can produce opposite energy contributions that differ for particles and antiparticles, i.e the above gravitational baryogenesis can generate the baryon-anti-baryon asymmetry. Hence, in this way one obtains a dynamical violation of $CPT$ symmetry, which affects thermal equilibrium distributions through an effective chemical potential $\mu_B = -\mu_{\bar{B}} = -\mathcal{R}/M^2$ [48].

Once the temperature drops below the decoupling value $T_D$, the Universe is driven towards a non-zero equilibrium asymmetry

$$n B - n \bar{B} = \frac{g_B}{6} \mu_B T^2, \tag{16}$$

where $g_B \sim O(1)$ is the number of the intrinsic degrees of freedom of baryons. Using (12), the baryon asymmetry in the standard notation then reads
indeed arise. Radiation era and by standard-cosmology energy density and pressure during the try can indeed occur as long as the Ricci scalar $R$ over time. However, since $\rho = \rho/3$, the standard Friedmann equations give $\dot{R} = 0$, and thus baryon asymmetry cannot arise. Nevertheless, as we will show in the following, this is not the case in Barrow cosmology, and baryon asymmetry through (17) can indeed arise.

Let us denote by $\bar{\rho} = 3H^2/(8\pi G)$ and $\bar{\rho} = \rho/3$ the standard-cosmology energy density and pressure during the radiation era and by $\delta\rho_\Delta$, $\delta p_\Delta$ the corresponding Barrow-entropy-induced extra terms in the Friedmann equations (9) and (10). Thus, we can write

$$\delta\rho_\Delta = \left[-1 + \beta_\Delta \left(G^2 \bar{\rho} \right)^{-\Delta/2}\right] \bar{\rho},$$

$$\delta p_\Delta = \left[-1 + \gamma_\Delta \left(G^2 \bar{\rho} \right)^{-\Delta/2}\right] \bar{\rho}/3,$$

where

$$\beta_\Delta = \frac{3^{\Delta/2} (2 + \Delta)}{2^{\Delta/2} (2 - \Delta)},$$

$$\gamma_\Delta = \beta_\Delta (1 - 2\Delta).$$

As expected, for $\Delta \to 0$ we acquire $\delta\rho_\Delta, \delta p_\Delta \to 0$, consistently with the recovery of standard cosmology in this limit. From the above we can clearly see that in the scenario at hand the Ricci scalar is non-zero during the radiation-dominated epoch, and in particular its time-derivative is given by

$$\dot{R}_\Delta = \frac{\pi^{3/2} 2^{(13-3\Delta)/2} \Delta (2 + \Delta)}{3^{(1-\Delta)/2}} G^{3/2-\Delta} \bar{\rho}^{(3-\Delta)/2},$$

where we have implemented the continuity equation at equilibrium. Substitution of (22) into (17) then gives

$$\eta_\Delta = \frac{35}{\pi^{1/2}} 2^{3(3-\Delta)/2} 3^{(1+3\Delta)/2} \Delta (2 + \Delta)$$

$$\frac{g_b}{g_* M_*^2 T_D} G^{3/2-\Delta} \bar{\rho}^{(3-\Delta)/2},$$

where $\bar{\rho}$ must be calculated at the decoupling point. This expression can be simplified by expressing the gravitational constant in terms of the Planck mass, $G = 1/M_P^2$. In our units, and replacing the equilibrium density by $\bar{\rho}|_{T=T_D} = \pi^2 g_* T_D^4/30$, obtaining

$$\eta_\Delta = \frac{\xi_\Delta g_b g_*^{(1-\Delta)/2}}{M_P} \left(\frac{T_D}{M_P}\right)^{3-2\Delta},$$

$$\xi_\Delta = \frac{7 \pi^{7/2-\Delta} 2^{6-\Delta} \Delta (2 + \Delta)}{3^{1-\Delta} \times 5^{(1-\Delta)/2}}.$$

Finally, since the Barrow exponent has been found by various studies to satisfy $\Delta \ll 1$ [40–43], which is expected since Barrow entropy should not deviate significantly from Bekenstein-Hawking one, we can expand the above expression resulting to

$$\eta_\Delta = \frac{896\pi^{7/2} \Delta}{3\sqrt{5}} g_b \sqrt{g_*} \left(\frac{T_D}{M_P}\right)^3 + \mathcal{O}(\Delta^2).$$

Hence, one can clearly see that the Barrow exponent can lead to a non-zero baryon asymmetry, due to te corrections in the Friedmann equations.

In order to proceed to quantitative calculations we consider as usual the decoupling temperature to be $T_D \simeq M_I$, where $M_I \sim 3.3 \times 10^{16}$ GeV is the upper bound on tensor mode fluctuations at inflationary scale [62]. In Fig. 1 we depict the prediction for the baryon asymmetry in the case of Barrow-entropy-based cosmology, as a function of the Barrow exponent $\Delta$, according to the exact expression (24). Additionally, in the same figure we also present the observational bounds on $\eta$ arising from baryogenesis, namely [62,76–80]:

$5.7 \times 10^{-11} \leq \eta \lesssim 9.9 \times 10^{-11}.$

Fig. 1 The baryon asymmetry $\eta_\Delta$ (blue solid curve) in the case of Barrow-entropy-based cosmology, as a function of the Barrow exponent $\Delta$, according to the exact expression (24). The red and green dashed lines mark the observational upper and lower bounds on $\eta$, respectively.
As we can observe, if we desire the baryon asymmetry to originate from the effects of the Barrow entropy in the Friedmann equations, we must require $0.005 \lesssim \Delta \lesssim 0.008$. This interval is tighter than the one arising from cosmological datasets from Supernovae (SNIa) Pantheon sample and cosmic chronometers, namely $\Delta = 0.0094^{+0.094}_{-0.101}$ [40,41], as well as from the one obtained from M87* and S2 star observations, namely $\Delta = 0.0036^{+0.015}_{-0.0145}$ [43], and it is slightly wider than the one from Big Bang Nucleosynthesis (BBN), i.e. $\Delta \lesssim 10^{-4}$ [42]. However, apart from obtaining a tight upper bound, the important feature of the present analysis, contrary to the other datasets, is that we obtain a non-zero lower bound. Definitely this lower bound would disappear if we do not require the baryon asymmetry to arise due to the extra terms in the Friedmann equations, even if we do have Barrow-modified cosmological equations (i.e. one could have the case of Barrow-modified cosmology in which baryon asymmetry is generated by other mechanisms that have been proposed in the literature, not related to Barrow-modified cosmology itself).

4 Discussion and conclusions

Observations reveal a baryon asymmetry that cannot be easily explained in the framework of standard cosmology, and thus offer an indication that some form of new physics might be needed. In this work we investigated the generation of baryon asymmetry due to the corrections brought about in the Friedmann equations due to Barrow entropy.

Barrow entropy is a modified entropy arising from quantum-gravitational effects on the Universe horizon, quantified by the new parameter $0 \leq \Delta \leq 1$. By applying the gravity-thermodynamics conjecture one obtains extra terms in the Friedmann equations. These extra terms change the Hubble function evolution during the radiation-dominated epoch, and hence even in the case of the standard coupling between the Ricci scalar and baryon current they can lead to a non-zero baryon asymmetry. We mention that a similar analysis of baryogenesis motivated by quantum gravity phenomenon has been carried out in the context of deformed uncertainty relations [62].

As we showed, if we desire the baryon asymmetry to be generated from the Barrow-entropy-based modified cosmology then we should have a Barrow exponent bounded in the interval $0.005 \lesssim \Delta \lesssim 0.008$, which corresponds to a slight deviation from the standard Bekenstein–Hawking entropy. The upper bound is tighter than the one of other observational constraints, however the interesting feature is that in the present analysis we obtain a non-zero lower bound. Nevertheless this lower bound would disappear if the baryon asymmetry in Barrow-modified cosmology is generated by other mechanisms, not related to the Barrow modification.

Finally, following the analysis of [81–84] for Tsallis thermodynamics, it would be interesting to investigate the extended model where the Barrow exponent has a running (i.e. time-dependent) behavior. Although not contemplated in the original formulation by Barrow, this feature might provide an interesting explanation on why datasets from different cosmological eras provide different bounds on $\Delta$. Such investigations lie beyond the scope of the present work, and are left for future projects.

Acknowledgements The authors acknowledge participation in the COST Association Action CA18108 “Quantum Gravity Phenomenology in the Multimessenger Approach”.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Funded by SCOAP3.

References

1. J.M. Bardeen, B. Carter, S.W. Hawking, Commun. Math. Phys. 31, 161 (1973)
2. J.D. Bekenstein, Phys. Rev. D 7, 2333 (1973)
3. S.W. Hawking, Commun. Math. Phys. 43, 199 (1975)
4. T. Padmanabhan, Phys. Rep. 406, 49 (2005)
5. C. Eling, R. Guedens, T. Jacobson, Phys. Rev. Lett. 96, 121301 (2006)
6. T. Padmanabhan, Rep. Prog. Phys. 73, 046901 (2010)
7. T. Jacobson, Phys. Rev. Lett. 75, 1260–1263 (1995)
8. A.V. Frolov, L. Kofman, JCAP 0305, 009 (2003)
9. R.G. Cai, S.P. Kim, JHEP 02, 050 (2005)
10. M. Akbar, R.G. Cai, Phys. Rev. D 75, 084003 (2007)
11. R.G. Cai, L.M. Cao, Phys. Rev. D 75, 064008 (2007)
12. M. Akbar, R.G. Cai, Phys. Lett. B 635, 7 (2006)
13. A. Paranjape, S. Sarkar, T. Padmanabhan, Phys. Rev. D 74, 104015 (2006)
14. G. Calcagni, JHEP 09, 060 (2005)
15. A. Sheykhi, B. Wang, R.G. Cai, Phys. Rev. D 76, 023515 (2007)
16. A. Sheykhi, B. Wang, Phys. Lett. B 678, 434 (2009)
17. E.N. Saridakis et al. [CANTATA], arXiv:2105.12582 [gr-qc]
18. A. Addazi, J. Alvarez-Muniz, R.A. Batista, G. Amelino-Camelia, V. Antonelli, M. Arzano, M. Asorey, J.L. Atteia, S. Bahamonde, F. Bajardi, et al., arXiv:2111.05659 [hep-ph]
19. C. Tsallis, J. Stat. Phys. 52, 479 (1988)
20. G. Kaniadakis, Phys. Rev. E 66, 056125 (2002)
