Model independent analysis of the trilinear gauge boson couplings at LC: role of polarized cross sections

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Abstract

By means of a model-independent analysis, we discuss the constraints on anomalous trilinear gauge-boson couplings that can be obtained from the study of electron-positron annihilation into $W$ pairs at LC with $\sqrt{s} = 0.5\, TeV$ and $1\, TeV$. We consider the general $CP$ conserving anomalous effective Lagrangian, as well as some specific models with reduced number of independent couplings. The analysis is based on combinations of observables with initial and final state polarizations, that allow to separately constrain the different couplings and to improve the corresponding numerical bounds.

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The values of the $WW\gamma$ and $WWZ$ couplings, and the corresponding non abelian
gauge structure of the Standard Model (SM), have not been tested yet. In this regard,
the reaction

$$e^+ + e^- \rightarrow W^+ + W^-$$

(1)
at high energy $e^+e^-$ colliders is particularly important because, for such process, de-
viations from the SM predictions due to anomalous values of the trilinear coupling
constants are significantly enhanced by increasing the CM energy, and the related
sensitivity is improved. The general anomalous trilinear gauge boson Lagrangian
has a complicated structure, containing both CP violating and CP conserving inter-
actions. The set of cross-section measurements for process (1), relevant to the CP
violating couplings and their separation, was discussed in Ref.[1].

In what follows, we examine the possibility of constraining the CP conserving
effective Lagrangian, which can be expressed as follows: [3, 4]

$$\mathcal{L} = -ie \left[ A_\mu \left(W^{-\mu}W^+_{\mu} - W^{+\mu}W^-_{\mu}\right) + F_{\mu\nu}W^{+\mu}W^{-\nu}\right] - ie x_\gamma F_{\mu\nu}W^{+\mu}W^{-\nu}
- ie (\cot \theta_W + \delta_Z) \left[Z_\mu \left(W^{-\mu}W^+_{\mu} - W^{+\mu}W^-_{\mu}\right) + Z_{\mu\nu}W^{+\mu}W^{-\nu}\right]
- ie x_Z Z_{\mu\nu}W^{+\mu}W^{-\nu} + ie \frac{y_\gamma}{M_W^2} F^{\nu\lambda}W^+_{\lambda\mu}W^{-\nu}_{\mu} + ie \frac{y_Z}{M_W^2} Z^{\nu\lambda}W^+_{\lambda\mu}W^{-\nu}_{\mu},$$

(2)

where $W^{\pm}_{\mu\nu} = \partial_{\mu}W^\pm_{\nu} - \partial_{\nu}W^\pm_{\mu}$ and $Z_{\mu\nu} = \partial_{\mu}Z_{\nu} - \partial_{\nu}Z_{\mu}$. Clearly, in the SM: $\delta_Z = x_\gamma = x_Z = y_\gamma = y_Z = 0$. As Eq.(2) shows, in general we have five independent couplings.

Models with smaller number of anomalous couplings naturally obtain in the frame-
work of the effective theory, where the existence of some new interaction, acting at
a mass scale $\Lambda$ much higher than the Fermi scale, is assumed. In this case, anom-
alous couplings originate as remnants of such interaction at lower energy scales, in the
form of corrections to the SM suppressed by inverse powers of $\Lambda$ [5]. Specifically, the
effective weak interaction Lagrangian is expanded as:

$$\mathcal{L}_W = \mathcal{L}_{SM} + \sum_d \sum_k \frac{f^{(d)}_k}{\Lambda^{d-4}_k} O^{(d)}_k,$$

(3)

where $\mathcal{L}_{SM}$ is the SM interaction and the second term, representing the ‘low-energy’
new interaction effects, is expressed in terms of $SU(2) \times U(1)$ gauge invariant opera-

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4A discussion of constraints on the C and P violating (but CP conserving) anapole coupling from
measurements of process (1) with longitudinally polarised leptons was given in Ref.[4].
tors $O_k^{(d)}$ with dimension $d$ made of $\gamma$, $W$, $Z$ and Higgs fields times their respective coupling constants $f_k^{(d)}$ not fixed by the symmetry. Truncation of the sum in Eq. (3) to the lowest significant dimension, $d = 6$, limits the number of allowed independent operators (and their corresponding constants) to three [5]-[7]:

\[
O_{WW}^{(6)} = Tr \left[ \hat{W}_{\mu\nu} \hat{W}^{\nu\rho} \hat{W}^{\rho}_{\mu} \right], \\
O_{W}^{(6)} = (D_\mu \Phi)^\dagger \hat{W}_{\mu\nu} (D_\nu \Phi), \\
O_{B}^{(6)} = (D_\mu \Phi)^\dagger \hat{B}_{\mu\nu} (D_\nu \Phi),
\]

where $\Phi$ is the Higgs doublet and, in terms of the $B$ and $W$ field strengths: $\hat{B}_{\mu\nu} = i(g'/2)B_{\mu\nu}$, $\hat{W}_{\mu\nu} = i(g/2)\vec{\tau} \cdot \vec{W}_{\mu\nu}$ with $\vec{\tau}$ the Pauli matrices. Eq. (4) implies the relations among the anomalous coupling constants:

\[
x_\gamma = \cos^2 \theta_W \left( f_B^{(6)} + f_W^{(6)} \right) \frac{M_Z^2}{2\Lambda^2}; \quad y_\gamma = f_{WW}^{(6)} \frac{3M_W^2g^2}{2\Lambda^2}; \\
\delta_Z = \cot \theta_W f_W^{(6)} \frac{M_Z^2}{2\Lambda^2}; \quad x_Z = -\tan \theta_W x_\gamma; \quad y_Z = \cot \theta_W y_\gamma,
\]

so that only three (e.g., $x_\gamma$, $y_\gamma$ and $\delta_Z$) are independent.

More relations occur from further specializations. An example is represented by the two-parameter ‘HISZ scenario’ [3], that assumes the relation $f_B^{(6)} = f_W^{(6)}$ in the above equations, such that

\[
\delta_Z = \frac{1}{2\sin \theta_W \cos \theta_W} x_\gamma, \quad x_Z = -\tan \theta_W x_\gamma, \quad y_Z = \cot \theta_W y_\gamma.
\]

As an alternative to the above formulation, the number of independent trilinear anomalous can be reduced by the assumption of global $SU(2)_L$ symmetry of Eq. (2), which directly implies the relation $x_Z = -\tan \theta_W x_\gamma$ [4] plus the neglect dimension 6 quadrupole operators leading to $y_\gamma = y_Z = 0$, and imposing the cancellation at order $s^2$ of the tree-level unitarity violating contributions to $WW$ scattering, which in turn implies the relation $\delta_Z = x_\gamma / \sin \theta_W \cos \theta_W$ [8, 9]. In this case, therefore, only one independent parameter remains.

As previously noticed, in the general case of Eq. (2) the five independent trilinear constant cannot be separately studied and constrained by the unpolarized cross

\(^5\)Such spontaneously broken gauge invariance requirement for the new interaction is naturally justified on phenomenological grounds. Similarly, lepton couplings are assumed to be unaffected by the new interaction.

\(^6\)The same as in Eq. (3).
section alone, which depends on all the couplings. Separate measurements of the cross sections for specific initial and final states polarizations, depending on independent combinations of the trilinear coupling constants, give the necessary additional information that allows to disentangle the couplings in a model independent way. Ideally, to that purpose, the three the possible $W^+W^−$ polarizations ($LL$, $TL$ and $TT$), combined with the two longitudinal $e^−e^+$ ones ($RL$ and $LR$) should determine a sufficient set of observable cross sections.

The basic objects to be studied are the potential deviations of the polarized cross sections from the SM predictions due to finite values of anomalous couplings in Eq. (2):

$$\Delta \sigma = \sigma - \sigma_{SM}. \quad (8)$$

Limiting to the Born level $\gamma-, Z-$ and $\nu$-exchange amplitudes:

$$d\sigma \propto |A(\gamma) + \Delta A(\gamma) + A(Z) + \Delta A(Z) + A_1(\nu)|^2 + |A_2(\nu)|^2,$$

$$d\sigma_{SM} \propto |A(\gamma) + A(Z) + A_1(\nu)|^2 + |A_2(\nu)|^2. \quad (9)$$

In Eq. (9), it is convenient to distinguish the $\nu$- exchange amplitudes with $|\lambda - \bar{\lambda}| \leq 1$ from $|\lambda - \bar{\lambda}| = 2$ ones, with $\lambda$ and $\bar{\lambda}$ the $W^−$ and $W^+$ helicities. Using the explicit helicity amplitudes given, e.g., in Ref.[4], and the Lagrangian Eq. (2), the amplitudes deviations $\Delta A$'s with initial beams and final $W$'s specific polarizations have the following dependence on the anomalous couplings:

$$\Delta A_{LL}^{ab}(\gamma) \propto x_\gamma$$

$$\Delta A_{LL}^{ab}(Z) \propto \left(x_Z + \delta_Z \frac{3 - \beta_W^2}{2}\right) g_e^a, \quad (10)$$

$$\Delta A_{TL}^{ab}(\gamma) \propto x_\gamma + y_\gamma$$

$$\Delta A_{TL}^{ab}(Z) \propto (x_Z + y_Z + 2\delta_Z) g_e^a, \quad (11)$$

and

$$\Delta A_{TT}^{ab}(\gamma) \propto y_\gamma$$

$$\Delta A_{TT}^{ab}(Z) \propto \left(y_Z + \delta_Z \frac{1 - \beta_W^2}{2}\right) g_e^a. \quad (12)$$
In Eqs. (10)-(12): $\beta_W = \sqrt{1 - 4M_W^2/s}$, the indices $LL, TL$ and $TT$ refer to the final $W^-W^+$ polarizations $LL, TL + LT$ and $TT$ respectively, while the upper indices $a$ and $b$ refer to the $e^- e^+ RL$ or $LR$ polarization. Furthermore, $g_e^R = \tan \theta_W$ and $g_e^L = g_e^R \left(1 - 1/2 \sin^2 \theta_W\right)$ represent the corresponding electron couplings. One can notice that $\sigma_{LL}, \sigma_{TL}$ and $\sigma_{TT}$ depend on the combinations $(x_{\gamma}, x_Z + \delta_Z (3 - \beta_W^2)/2)$, $(x_{\gamma} + y_{\gamma}, x_Z + y_Z + 2\delta_Z)$ and $(y_{\gamma}, y_Z + \delta_Z (1 - \beta_W^2)/2)$ respectively.

As a procedure to quantitatively assess the sensitivity of the different cross sections to the gauge boson couplings, we divide the experimentally significant range of the production angle $\cos \theta$ (assumed to be $|\cos \theta| \leq 0.98$) into 10 ‘bins’, and define the $\chi^2$ function:

$$\chi^2 = \sum_{i}^{\text{bins}} \left[ \frac{N_{SM}(i) - N_{anom}(i)}{\delta N_{SM}(i)} \right]^2,$$

where $N(i) = L_{int} \sigma_i \varepsilon_W$ represents the expected number of events in the $i$-th bin with $\sigma_i$ the corresponding cross section (either the SM or the anomalous one):

$$\sigma_i \equiv \sigma(z_i, z_{i+1}) = \int_{z_i}^{z_{i+1}} \left( \frac{d\sigma}{dz} \right) dz, \quad z = \cos \theta.$$ (14)

The efficiency $\varepsilon_W$ for $W^+W^-$ reconstruction in the various polarization states is taken as $\varepsilon_W \approx 0.3$ [10]-[13] from the channel of lepton pairs ($e\nu + \mu\nu$) plus two hadronic jets and the corresponding branching ratios. In fact, the actual value of $\varepsilon_W$ for polarized final states might be considerably smaller, depending on experimental details [10], but definite estimates are not available at present. As a compensation, for the time-integrated luminosity which is multiplied by $\varepsilon_W$ everywhere, we assume the rather conservative value (compared with recent findings [14]): $L_{int} = 20 fb^{-1}$ for the NLC(500) and $L_{int} = 50 fb^{-1}$ for the NLC (1000).

In the case no deviations were observed in the cross sections under consideration, allowed regions for the anomalous coupling constants can be obtained by adopting, as a criterion, that $\chi^2 \leq \chi^2_{\text{crit}}$, where $\chi^2_{\text{crit}}$ is a number that corresponds to the chosen confidence level. Since each polarized cross section involves two well-defined combinations of anomalous couplings at a time, as Eqs. (10)-(12) show, with two independent degrees of freedom $95\%$ CL bounds in each separate case are obtained by choosing $\chi^2_{\text{crit}} = 6$.

Since, in practice, initial beams polarization will not be perfect, to adapt the
analysis to the possible experimental situation we should consider the cross section
\[ \frac{d\sigma}{dz} = \frac{1}{4} \left[ (1 + P_1) \cdot (1 - P_2) \frac{d\sigma^{RL}}{dz} + (1 - P_1) \cdot (1 + P_2) \frac{d\sigma^{LR}}{dz} \right], \quad (15) \]
where $P_1$ ($P_2$) are less than unity, and represent the actual degrees of longitudinal polarization of $e^-$ ($e^+$). Reasonable possibilities seems to be: $d\sigma^R/dz$ ($P_1 = 0.9, P_2 = 0$) and $d\sigma^L/dz$ ($P_1 = -0.9, P_2 = 0$).

We present numerical results case by case, starting from longitudinally polarized $e^- e^+ \rightarrow W^- W^+$ production, for both possibilities of the electron beam longitudinal polarization. The typical resulting area, allowed to the combinations of anomalous couplings in Eq. (10) at the 95% CL for both $\sqrt{s} = 0.5$ and $1 TeV$, can be directly read from Fig. 2 of Ref.[13]. It implies the pair of inequalities
\[ -\alpha_{1,2}^{LL} < x_\gamma < \alpha_{1,2}^{LL}, \quad (16) \]
\[ -\beta_{1,2}^{LL} < x_Z + \delta_Z \frac{3 - \beta_W^2}{2} < \beta_{1,2}^{LL}, \quad (17) \]
so that only $x_\gamma$ is separately constrained at this stage. Here, $\alpha_{1,2}^{LL}$ and $\beta_{1,2}^{LL}$ are the projections of the combined allowed area on the horizontal and vertical axes.

The same kind of analysis can be applied to the other polarized cross sections. From $e^+ e^- \rightarrow W_T^+ W_T^- + W_L^+ W_L^-$ we obtain the allowed region for the combinations of coupling constants in Eq. (11), that implies the analogous inequalities:
\[ -\alpha_{1,2}^{TL} < x_\gamma + y_\gamma < \alpha_{1,2}^{TL}, \quad (18) \]
\[ -\beta_{1,2}^{TL} < x_Z + y_Z + 2\delta_Z < \beta_{1,2}^{TL}. \quad (19) \]
Finally, from $e^+ e^- \rightarrow W_T^+ W_T^-$ one obtains the corresponding inequalities:
\[ -\alpha_{1,2}^{TT} < y_\gamma < \alpha_{1,2}^{TT}, \quad (20) \]
\[ -\beta_{1,2}^{TT} < y_Z + \frac{1 - \beta_W^2}{2} \delta_Z < \beta_{1,2}^{TT}. \quad (21) \]
By combining Eqs. (17)-(21), one can very simply disentangle the bounds for $\delta_Z, x_Z$ and $y_Z$:
\[ -\frac{1}{\beta_W^2} B_2 < \delta_Z < \frac{1}{\beta_W^2} B_1, \quad (22) \]
\[ -\left( \beta_{1,2}^{LL} + \frac{3 - \beta_W^2}{2\beta_W^2} B_1 \right) < x_Z < \beta_{2}^{LL} + \frac{3 - \beta_W^2}{2\beta_W^2} B_2, \quad (23) \]
Table 1: Model independent limits on the five $CP$ even nonstandard gauge boson couplings at the 95% CL.

| $\sqrt{s}$ (TeV) | $x_\gamma$ (10$^{-3}$) | $y_\gamma$ (10$^{-3}$) | $\delta_Z$ (10$^{-3}$) | $x_Z$ (10$^{-3}$) | $y_Z$ (10$^{-3}$) |
|------------------|---------------------|---------------------|---------------------|------------------|------------------|
| 0.5              | $-2.0 \div 2.2$    | $-11.0 \div 10.6$  | $-52 \div 45$      | $-51 \div 59$   | $-22 \div 30$   |
| 1                | $-0.6 \div 0.6$    | $-3.2 \div 3.4$    | $-19 \div 16$      | $-18 \div 20$   | $-5.7 \div 6.2$ |

Table 2: Limits on anomalous gauge boson couplings at the 95% CL for the models with three, two and one independent parameters.

| Model with three independent anomalous constants | $x_\gamma$, $y_\gamma$, $\delta_Z$; $x_Z = -\tan \theta_W x_\gamma$, $y_Z = \cot \theta_W y_\gamma$. |
|------------------------------------------------|------------------------------------------------------------------|
| $\sqrt{s}$ (TeV) | $x_\gamma$ (10$^{-3}$) | $\delta_Z$ (10$^{-3}$) | $x_Z$ (10$^{-3}$) | $y_\gamma$ (10$^{-3}$) | $y_Z$ (10$^{-3}$) |
| 0.5              | $-2.0 \div 2.2$    | $-3.8 \div 3.8$    | $-1.2 \div 1.1$  | $-7.0 \div 7.5$   | $-12.8 \div 13.7$ |
| 1                | $-0.6 \div 0.6$    | $-1.1 \div 1.1$    | $-0.3 \div 0.3$  | $-4.0 \div 4.5$   | $-7.3 \div 8.2$  |

| Model with two independent anomalous constants | $x_\gamma$, $y_\gamma$, $\delta_Z = x_\gamma/2 \sin \theta_W \cos \theta_W$, $x_Z = -\tan \theta_W x_\gamma$, $y_Z = \cot \theta_W y_\gamma$. |
|------------------------------------------------|------------------------------------------------------------------|
| $\sqrt{s}$ (TeV) | $x_\gamma$ (10$^{-3}$) | $\delta_Z$ (10$^{-3}$) | $x_Z$ (10$^{-3}$) | $y_\gamma$ (10$^{-3}$) | $y_Z$ (10$^{-3}$) |
| 0.5              | $-1.8 \div 1.8$    | $-2.1 \div 2.1$    | $-1.0 \div 1.0$  | $-6.6 \div 6.8$   | $-12.1 \div 12.4$ |
| 1                | $-0.5 \div 0.5$    | $-0.6 \div 0.6$    | $-0.3 \div 0.3$  | $-3.0 \div 2.4$   | $-5.5 \div 4.4$  |

| Model with one independent anomalous constant | $x_\gamma$; $x_Z = -\tan \theta_W x_\gamma$, $y_Z = -\sin^2 \theta_W \delta_Z$. |
|------------------------------------------------|------------------------------------------------------------------|
| $\sqrt{s}$ (TeV) | $x_\gamma$ (10$^{-3}$) | $\delta_Z$ (10$^{-3}$) | $x_Z$ (10$^{-3}$) | $y_\gamma$ | $y_Z$ |
| 0.5              | $-1.1 \div 1.1$    | $-2.6 \div 2.6$    | $-0.6 \div 0.6$  | 0          | 0      |
| 1                | $-0.3 \div 0.3$    | $-0.8 \div 0.8$    | $-0.2 \div 0.2$  | 0          | 0      |

$$- \left( \beta_1^{TT} + \frac{1 - \beta^2_W}{2\beta^2_W} B_1 \right) < y_Z < \beta_2^{TT} + \frac{1 - \beta^2_W}{2\beta^2_W} B_2,$$ (24)

where $B_1 = \beta_1^{LL} + \beta_1^{TT} + \beta_2^{TL}$ and $B_2 = \beta_2^{LL} + \beta_2^{TT} + \beta_1^{TL}$. Adding these constraints to those in Eqs. (16) and (20) for $x_\gamma$ and $y_\gamma$, we finally obtain, by this simple procedure, separate bounds for the five anomalous couplings.

With the chosen inputs for the luminosity and the beam polarization quoted previously, numerical results are as reported in Tab. [4]

Tab. [3] summarizes the numerical bounds that can be obtained from our analysis of the models with smaller number of independent anomalous couplings introduced previously. Comparing to the results in Tab. [1], one can observe that $\delta_Z$ can be more tightly constrained in this case than in the general one. Concerning $y_\gamma$, the most stringent constraints are obtained from the combination of $W_L W_L$ and $W_L W_T$ production channels. In the case of the two-parameter model of Ref. [3], the bounds
on $x_\gamma$ and $\delta_\gamma$ are obtained in the same way as above, and are numerically identical. Finally, in the two-parameter model of Ref. [5], due the relation (7) among the couplings, $\sigma^L$ numerically proves to be more sensitive than $\sigma^R$. Concerning final state polarizations, the bound on $x_\gamma$ is obtained from $W_L W_L$ production, while that on $y_\gamma$ involves the combination of both $LL$ and $TL + LT$ polarized cross sections.

In summary, the obtained results indicate that the analysis of the cross sections of process (1) with definite initial and final polarizations potentially allows to derive separate, and model dependent, contraints $CP$ conserving couplings in Eq. (2) with considerable sensitivity, typically of the order of $10^{-3}$ or better, depending on $E_{CM}$. Particularly stringent bounds can be expected for dynamical models beyond the SM with reduced number of independent couplings.
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