Extraction of the neutron structure function $F_2^n$ from inclusive scattering data on composite nuclei

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(February 10, 2020)

Abstract

We consider a generalized convolution, linking Structure Functions (SF) $F_2^N$ for nucleons, $F_2^A$ for a physical nucleus and $f^{P.N,A}$ for a nucleus, composed of point-nucleons. In order to extract $F_2^n$ we employ data on $F_2^{p,A}$ and the computed $f^{P.N,A}$. Only for $Q^2 \approx 3.5\text{ GeV}^2$ do data permit the extraction of $F_2^A(x,3.5)$ over a sufficiently wide $x$-range. Applying Mellin transforms, the above relation between SF turns into an algebraic one, which one solves for the Mellin transform of the unknown $F_2^n$. We present inversion methods leading to the desired $F_2^n$, all using a parametrization for $C(x,Q^2) = F_2^n(x,Q^2)/F_2^p(x,Q^2)$. Imposing motivated constraints, the simplest parametrization leaves one free parameter $C(x = 1,Q^2)$. For $Q^2 = 3.5\text{ GeV}^2$ its average over several targets and different methods is $\langle C(1,3.5) \rangle = 0.54\pm0.03$. We argue that for the investigated $Q^2$, $C(x \to 1,3.5)$ is determined by the nucleon-elastic ($NE$) part of SF. A calculation of the latter comes close to the extracted value. Both are close to the SU(6) limit $u_V(x,3.5) = 2d_V(x,3.5)$ for parton distribution functions.

It has been recently proposed to extract the ratio $C(x,Q^2) = F_2^n(x,Q^2)/F_2^p(x,Q^2)$ of the
structure functions (SF) from data on ratios $\sigma(^3\text{H})/\sigma(^3\text{He})$ of inclusive cross sections. An electron beam with $E \approx 11$-12 GeV is expected to provide the required wide, continuous kinematic ranges. The proposal has already elicited discussions, mainly on the stability of the suggested extraction techniques for $C(x, Q^2)$.

An experiment, involving the SF of the lightest isobars with minimal I- spin symmetry breaking is presumably best suited to obtain $C(x, Q^2)$. However, even when an upgrading of the beam will be approved, results on the $A=3$ proposal are not expected before 2006. One therefore wonders whether existing data on $F^{A}_{2}$ may furnish similar information. Below we explore this possibility and present first results for $C(x, Q^2)$ and $F^{n}_{2}(x, Q^2)$ from inclusive scattering data on D, C and Fe.

In the following we limit ourselves to inclusive scattering of unpolarized electrons from randomly oriented targets $A$. The cross section per nucleon for beam energy $E$ reads

$$
\frac{d^2\sigma^{eA}(E; \theta, \nu)}{d\Omega d\nu} = \frac{2}{M} \sigma_{M}(E; \theta, \nu) F^{A}_{2}(x, Q^2) \left[ \frac{xM^2}{Q^2} + \tan^2(\theta/2) F^{A}_{1}(x, Q^2) \right],
$$

(1)

where $\theta$ and $\nu$ are the scattering angle and the energy loss. $F^{A}_{1,2}(x, Q^2)$ are nuclear structure functions (SF) per nucleon, expressed in terms of the squared 4-momentum transfer $Q^2 = q^2 - \nu^2$ and the Bjorken variable $x = Q^2/2M\nu$ ($M$ is the nucleon mass) with range $0 \leq x \leq A$.

Henceforth nuclear and nucleon SF are assumed to be related by a generalized convolution $F^{A} = f \ast F^{N}$, for instance

$$
F^{A}_{k}(x, Q^2) = \int_{x}^{A} \frac{dz}{z^2-k} \left[ \frac{Z}{A} f^{P_{N,A}}(z, Q^2) F^{p}_{k}(x/z, Q^2) + \frac{N}{A} f^{n_{N,A}}(z, Q^2) F^{n}_{k}(x/z, Q^2) \right] \quad (2a)
$$

$$
\approx \int_{x}^{A} \frac{dz}{z^2-k} f^{P_{N,A}}(z, Q^2) F^{(N)}_{k}(x/z, Q^2) \quad (2b)
$$

In Eq. (2) $f^{P_{N,A}}$ is the SF for a nucleus, composed of point-nucleons, where initially a $p$ or a $n$ absorbs the virtual photon; $f^{P_{N,A}}$ is their average over the number of protons and neutrons in the target $A(Z, N)$. $F^{(N)}_{k}$ stands for the similarly averaged nucleon ($N$) SF (We often drop arguments of functions when there is no danger of confusion).

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1* In Eqs. (2) appear admixtures of nucleon SF, which are negligible for the involved $Q^2$. 

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\[ f^{P.N,A} = \frac{Z}{A} f_{p}^{P.N,A} + \frac{N}{A} f_{n}^{P.N,A} \]

\[ F^{(N)}_k = \frac{Z}{A} F_{k}^{p} + \frac{N}{A} F_{k}^{n} \] (3)

Eq. (2) describes partons from nucleons in a nucleus, but not those from other sources (virtual bosons) neither does it account for anti-screening. Both limit the use of (2) to \( x \gtrsim 0.15-0.20 \) [9]. Eq. (2) is estimated to be valid for \( Q^2 \gtrsim Q^2_c \approx 2 - 2.5 \text{ GeV}^2 \) [10,11].

We briefly mention previously suggested extraction methods for \( F^{n}_2 \). Those dealt mostly with D data and use \( f^{PN} \) in the Impulse Approximation (IA) [12,13]. It has for instance been emphasized that Eq. (2) is a Fredholm integral equation for the unknown \( F^{(N)}_k \). Discretization in \( x \) and \( z \) produces a set of linear equations with a solution, tending to the exact answer for \( \Delta z \to 0 \) [14]. The strong variation of the 'kernel' \( f^{PN} \) apparently hampers an actual application of the above. We also mention iteration methods to deconvolute nuclear effects for a D target [12,13].

A different deconvolution has been suggested for the proposed precision data, relating \( F^{A}_k \) [1]. For those one may define super-ratios [2,4]

\[ \mathcal{R}^{A_1,A_2}(x) = \frac{\rho^{A_1}(x)}{\rho^{A_2}(x)} \] (4a)

\[ \rho^A(x) = \frac{[F^A_2(x)/F^p_2(x)]}{a[F^n_2(x)/F^n_2(x)] + b} \] (4b)

with, in principle, arbitrary \( a \) and \( b \). For given \( F^A_2 \) those ratios contain \( C(x) \) explicitly, and are from Eq. (2) seen to depend implicitly on \( C(x/z) \). Iteration determines \( C(x) \).

Finally we recall the expansion of \( F^{(N)}(x/z) \) in Eq. (3) around the maximum \( z \approx 1 \) of \( f(z) \) [16]

\[ F^A_2(x) = F^{(N)}_2(x) + \mathcal{M}_1(x)[x F^{(N)(1)}_2(x)] + \mathcal{M}_2(x)[x F^{(N)(1)}_2(x) + \frac{x^2}{2} F^{(N)(2)}_2(x)] + \ldots , \] (5)

with \( \mathcal{M}_n(Q^2) = \int_0^A dz f^{P.N,A}(z,Q^2)(1-z)^n \). The first \( A \)-independent term \( F^{(N)}_2(x) \) adequately represents the series for \( x \lesssim 0.4 \). Insufficient knowledge of \( F^{(N)(l)}_2(x) \), the \( l \)-th derivative of \( F^{(N)}_2(x) \), prevents the use of the expansion (4) beyond that range. Although not the region of prime interest, the low-\( x \) result will guide the extraction of \( F^{n}_2 \).
We first recall calculations of $F_A^k$ from Eq. (2) with $F_{\kappa}^{p,n}$ as input. For nucleons in vacuum there are abundant data on $F_2^p$ \cite{17} and less accurate ones for $F_1^p$ \cite{18}. Inclusive data do not reach the elastic point $x = 1$, but the used parametrizations do. With no direct information on $F_k^n$, one frequently uses the 'primitive' assumption

$$F_k^{n,pr} = 2F_k^D - F_k^p,$$

which is the first term in Eq. (5) for the D and thus holds only for small $x$. The question of interest is how modifications of Eq. (6) for $x \gtrsim 0.4$ influence $F_A^k(x, Q^2)$, Eq. (2).

The SF $f_{P,N,A}$ is a many-body property and requires a model for a calculation, such as the perturbative IA, which to lowest order contains the single-hole spectral function \cite{19,20}. There nucleons appear off their mass shell and a prescription is needed to handle those. We prefer a non-perturbative version, based on a generalized Gersch-Rodriguez-Smith (GRS) theory, where the SF of nucleons $F_N$ are reasoned to be on their mass shell. That method moreover allows a computation of $f_{P,N,A}$ beyond its lowest order \cite{7,11,21–23}.

The above program has initially been realized for $A \geq 12$. It has been shown that $f_{P,N,A}$ is nearly independent of $A$, and using Eq. (2) one proves the same for $F_A^k$. Data indeed show that for $x \lesssim 0.8$, the ratios $\mu_{A,A'} = [d^2\sigma_{eA}/A]/[d^2\sigma_{eA'}/A'] \approx F_2^A/F_2^{A'}$, $A, A' \geq 12$ differ from 1 by less than 2-3\% \cite{21,23,24}.

The above does not hold for the lightest nuclei, in particular not for $A' \to D$, when

$$\mu^A(x, Q^2) = \mu^{A,D}(x, Q^2) = \frac{d^2\sigma_{eA}(x, Q^2)/A}{d^2\sigma_{eD}(x, Q^2)/2} \approx \frac{F_2^A(x, Q^2)}{F_2^D(x, Q^2)},$$

are the EMC ratios, with their characteristic deviations from 1 in the range $x \lesssim 0.9$ \cite{25}.

For $A \leq 4$ and given $NN$ interaction, one may presently compute with great precision nuclear ground states \cite{26}, as well as $f_{P,N,A}$ in Eq. (3). Using those, GRS calculations of inclusive cross sections have recently been completed for $D$ \cite{27} and $^4$He \cite{28}.

We now address the inverse problem of trying to extract $F_n^k$ from data on $F_2^{p,A}$ and the above-mentioned computed $f$. First one needs to obtain nuclear SF and we shall use JLab data for D \cite{29,30} and for C, Fe \cite{31}, supplemented by some older NE3 data of more restricted kinematics \cite{32}. We mention two approaches:
i) $R$-ratios: Ideally one performs a Rosenbluth extraction of $F_k^A$ from cross sections for fixed $x,Q^2$ and varying $\theta$ or $E$, which provides
\[ R^A(x,Q^2) = \left(1 + \frac{4M^2x^2}{Q^2}\right)\left(\frac{F_2^A(x,Q^2)}{2xF_1^A(x,Q^2)}\right) - 1 \] (8)
Unfortunately, the set of $(x,Q^2)$ points in the JLab and NE3 data does not enable the above Rosenbluth extractions. Instead one frequently invokes an empirical expression such as $R^A(x,Q^2) \approx 0.32/Q^2$. It assumes $A$, as well as $x$-independence and prescribes the dependence on $Q^2$ [33]. The above empirical $R$ actually contradicts theoretical predictions [11]. We shall return to this point below.

ii) **Theoretical information**: Using the primitive $F^{n,pr}$, computed cross sections as function of $\theta,\nu$ generally agree with data in the deep-inelastic regions, $x \lesssim 1$, $(Q^2 \gtrsim Q^2_c)$. There we select data, for which the relative deviation $\alpha(= \alpha(x,Q^2)) = (F_2^{A,th} - F_2^{A,exp})/F_2^{A,exp}$ satisfies $|1 - \alpha| \lesssim 0.2$ and which, for given $\theta$ appear to change smoothly with $x$. We suggest to attribute the above deviations in equal measure to both structure functions $F_k^A$ in [4], which enables the definition of quasi-data
\[ F_k^{A,qd}(x,Q^2) \equiv \alpha(x,Q^2)F_k^{A,th}(x,Q^2) \] (9)
By construction, the $F_k^{A,qd}$ produce an exact fit to cross sections.

Use of Eq. (2) for the purpose of inversion requires input for fixed $Q^2$ and running $x$, whereas cross section data are for essentially one beam energy $E$ and cover separated $x$ bins. $Q^2$ varies mildly within those $x$-bins, but not when going from one bin to a neighbouring one. It is thus necessary to make cuts for fixed $Q^2$, containing a maximal number of $x$-points with $x \lesssim 1.0 - 1.1$. After careful interpolation between quasi-data, the above appears only fulfilled for $Q^2$ between 3-4 GeV$^2$ and then only for $0.5 \lesssim x \lesssim 1.1$. It is clearly impossible to reliably extrapolate the manipulated quasi-data into the crucial range $x \lesssim 0.5$.

At this point we invoke the empirical observation that for $x_0 \approx 0.16 - 0.18$, $F_2^p(x_0,Q^2)$ and $F_2^D(x_0,Q^2)$ are practically constant as function of $Q^2$ [17]. We exploit this by considering Eq. (2) for small $x \leq x_0$. Upon substitution of the small-$x$ part of Eq. (5) into (2) and use of $\int_0^1 dz f_{P,N,A}(z) = 1$, one easily shows that for any target
\[ F_{2}^{A,\text{nucl}}(0.18) \approx F_{2}^{N}(0.18) \approx 0.34 \quad (10a) \]
\[ F_{2}^{A,\text{nucl}}(0) = F_{2}^{(N)}(0) = F_{2}^{N}(0) \neq 0 \quad (10b) \]

The latter extension to \( x = 0 \) is in agreement with Eq. (3) and will be needed below.

We recall that \( f^{P,N,A} \) in Eq. (2) relates only to nucleons in the target and as a result, Eqs. \( (10a) \) hold only for the nucleonic component \( F_{2}^{A,\text{nucl}} \). That component may now be interpolated for \( x \lesssim 0.55 \), completing knowledge of \( F_{2}^{A,\text{nucl}}(x) \) over the entire relevant range \( x \lesssim 1.1 \).

After the critical remarks on the empirical form of the \( R \)-ratio it comes as a surprise that in the range \( 0.5 \lesssim x \lesssim 1 \), \( F_{2}^{Fe,qd}(x, 3.5) \) and the empirically extracted \( F_{2}^{Fe(R)}(x, 3.5) \) agree to within \( \pm 5\% \), and even for \( 1 \lesssim x \lesssim 1.5 \) to within \( \pm 12\% \)! It vindicates Arrington’s claim that even a 100\% uncertainty in the phenomenological \( R \) incurs only a 5\% uncertainty in \( F_{2}^{A(R)} \).

We proceed as follows. For \( Fe \), for which the number of available deep-inelastic data points is largest, we use \( F_{2}^{Fe,qd} \) from method ii). With insufficient \( C, D \) data for application of that method, we exploit Eq. (7) and find
\[
F_{2}^{C,qd} = F_{2}^{C,R} F_{2}^{Fe,qd}
\]
\[
F_{2}^{D,qd} = \left[ \mu^{C} \right]^{-1} F_{2}^{C,qd} \quad (11)
\]

EMC data for \( \mu^{C} \) are in the desired range \( 0.3 \lesssim x \lesssim 0.80 \) where they require some smoothing. For \( x \lesssim 0.30 \) we interpolated the purely nucleonic component as in ii) and added for \( 0.80 < x < 0.88 \) averages of Be and Al data.

We now introduce Mellin transforms (MT) and their inverses, which for real \( g(x) \) are defined as
\[
\tilde{g}(u) = \int_{0}^{\infty} dx x^{u-1} g(x) \quad (12a)
\]
\[
g(x) = \frac{1}{\pi} \int_{0}^{\infty} dt \text{Re} \left[ \frac{g(a + it)}{x^{a+it}} \right] , \quad (12b)
\]

\(^2\) The above explains why for all \( A \) and \( Q^{2} \), EMC ratios \( \mu^{A}(x, Q^{2}) \) cross 1 for \( x \approx 0.18 \).
The constant $a$ is chosen such, that $g(u) = g(a + it)$ is free of singularities in the complex $u$ plane to the right of the imaginary $u$ axis shifted by $a$.

Application of Eq. (12a) to Eq. (2), turns the generalized convolution into a linear relation, which can be solved for $\tilde{F}_2^n(u, Q^2)$. For fixed $Q^2$

$$\tilde{F}_2^n(A) = \mathcal{F}_2^n(u = 0) = \sum_{k \geq 0} d_k(Q^2)(1 - x)^k$$

with $G^{N,NE}$, combinations of the standard nucleon static formfactors, making up the $NE$ parts of $F_2^{N,NE}$ (see for instance Ref. [11]). Above we denote by $\tilde{F}_2^n(A)$ the $A(N, Z)$-dependent right-hand side of Eq. (13a). For exact input $F_2^n$, it should coincide with $\tilde{F}_2^n(u)$, the MT of $F_2^n(x)$.

Next we consider $u = 0$ in Eq. (13b). Since the normalization of $f^{P,N,A}$ implies $\tilde{F}_2^n(u = 0) = 1$, one finds

$$\tilde{F}_2^n(A)(u = 0) = \frac{A}{N} \tilde{F}_2^{A,nucl}(u = 0) - \frac{Z}{N} \tilde{F}_2^p(u = 0) - \left[ \frac{Z}{N} G^p + G^n \right]$$

Consequently, Eq. (12b), when used in (10a) implies that none of the MT of SF are defined for $u = 0$. This has numerical consequences also in the immediate neighbourhood of $u = 0$.

Unfortunately, the direct inversion Eq. (12b) of $\tilde{F}(u)$ runs into serious numerical problems. We therefore take recourse to indirect methods, all featuring a parametrization of the ratio $C$ in

$$F_2^n(x, Q^2) = F_2^n(x, Q^2; d_k) = C(x, Q^2; d_k) F_2^p(x, Q^2)$$

$$C(x, Q^2; d_k) = \sum_{k \geq 0} d_k(Q^2)(1 - x)^k$$

Eq. (10a) implies a first constraint $C(0) = F_2^p(0)/F_2^p(0)) = \sum_{k \geq 0} d_k(Q^2) = 1$ Next, one may exploit $F_2^n(x) = F_2^{n,pr}(x)$, Eq. (4), for small $x \lesssim 0.35$. We use only $F_2^n(0.2, 3.5) = 0.75$ which for $k_{max} = 2$ leaves one free parameter $d_0 = C(1)$.

It is convenient to re-parametrize $F_2^p(x, 3.5)$ as follows
\[ F_2^p(x, Q^2) = x^{-a^2} \sum_{m \geq 1} c_m (1 - x)^m; x \geq 0.02 \]
\[ = 0.42 \quad ; x \leq 0.02 \] (16a)

A small \( a^2 \ll 1 \) in Eq. (16a) produces the observed rise of \( F_2^p(x) \) for small \( x \), while the cut-off in Eq. (16b) avoids the singularity in (16a). Eq. (10a) again implies the same for \( F_2^{A,n} \).

In the region \( 0.02 \lesssim x \lesssim 0.9 \), \( F_2^p \), Eq. (16a), practically coincides with the parametrization in Ref. [17], both agreeing with data averaged over resonances.

We now explore some extraction methods, comparing relevant computed or extracted expressions \( A \), related to \( F_n^p \) and parametrized forms \( B \):

I) The extracted \( \tilde{F}_2^{n(A)}(u) \), Eq. (13b) and the MT of \( F_2^p \), Eq. (13).

II) \( F_2^{A,qd}(x) \), Eq. (13), with the nuclear SF \( F_2^A(x) \), computed from (2), now using Eq. (15).

The parameters \( d_k \), are determined by minimization of variances \( w(d_0) = \sum_i |A_i - B_i|^2 \) (or of relative variances). Table I summarizes our results. The spread of the individual entries \( C(1) \) from I) reflects variations in the smallest and largest \( u \), retained in the above sum (see remark after Eq. (14)). An empty entry indicates the absence of a minimum in the studied intervals. We note that in those cases the slope of the variance is very small and that the range of \( C(1) \) is actually compatible with values from real minima. The results for the two methods and for the three targets produces a well-determined average \( \langle C(1,3.5) \rangle = 0.54 \pm 0.03 \).

In Fig. 1 we show the full \( C(x,3.5) \), as well as \( F_2^{p,n}(x,3.5) \). On the right abscissa are marked the values 2/3, 3/7, 1/4 for \( C(1) \), corresponding to exact SU(6) symmetry, dominance of \( S = 0 \) over \( S = 1 \) di-quark coupling and the same for the \( z \)-component \( S_z \) for \( x \to 1 \) (see for instance Ref. [15]).

We now argue that for our purposes the regarded \( Q^2 \approx 3.5 \text{ GeV}^2 \) is not 'large'. First

\[ ^3 \text{* The extracted } F_2^n \text{ allows an evaluation of the Gottfried sumrule } S_G(3.5) = \int_0^1 (dx/x)[F_2^p(x,Q^2) - F_2^n(x,Q^2)] = 0.251. \text{ This is close to the recent value } S_G(4.0) = 0.256 \pm 0.026 \]
we note that $F_{2}^{p,n}(x,Q^2)$ has a first inelastic threshold $N + m$, $m$ being the pion mass, or $x\text{thr}(Q^2) = [1 + 2Mm/Q^2]^{-1}$, i.e. a finite $x$-distance from the elastic point $x = 1$. That interval shrinks with increasing $Q^2$, but for the case at hand $x\text{thr}(3.5) \approx 0.93$, which is marked by a vertical line in Fig. 1. Data show that $F_{2}^{p}(x,3.5)$ is negligibly small beyond $x \approx 0.9$ [12,29,34].

The above implies that $C(x \to 1,3.5)$ can be ascribed to the $NE$ part $F^{N,NE}$, i.e. to static $N$ form factors (cf. Eq. (13)). Disregarding the electric form factor $G_{E}^{n}$ of the $n$, one finds ($\mu_{N}$ are the magnetic moments of the nucleons)

$$\lim_{x \to 1} C(x,Q^2) = \left[\frac{\mu_{n}\alpha_{n}(Q^2)}{\mu_{p}\alpha_{p}(Q^2)}\right] \left[1 + \frac{4M^2}{Q^2} \left(\frac{\gamma(Q^2)}{\mu_{p}}\right)^2\right]^{-1}$$

(17)

with

$$\gamma(Q^2) = \frac{\mu_{p}G_{E}^{p}(Q^2)}{G_{M}^{p}(Q^2)} ; \quad \frac{\alpha_{n}(Q^2)}{\alpha_{p}(Q^2)} = \frac{G_{M}^{n}(Q^2)/\mu_{n}}{G_{M}^{p}(Q^2)/\mu_{p}}$$

(18)

Recent data show that $\gamma, \alpha_{p}, \alpha_{n}$ deviate from 1 and equal for $Q^2 = 3.5, 5.0$ GeV$^2$: $\gamma=0.552$, 0.349 and $\alpha_{n}/\alpha_{p} \approx 1.2, 1.1$ [33,36]. Eqs. (17), (18) then yield

$$C(x = 1,3.5) \approx 0.61 ; \quad C(x = 1,5.0) \approx 0.56$$

(19)

The large $Q^2$ limit essentially depends on the same for $\alpha_{n}/\alpha_{p}$. For a value 1, $C(x \to 1,Q^2 \to \infty) = 0.469$, which shows that in the above sense $C(1,3.5)$ is still far from a scaling limit.

The above value 0.61 is reasonably close to the extracted one and has also been entered on the abscissa in Fig.1. Either one definitely exceeds previously cited values $C(1) \approx 0.42$ from D data (cf. Ref. [13]). The above seems to indicate that SU(6) symmetry $u_{v}(x,3.5) = 2d_{v}(x,3.5)$ for the up and down quark parton distribution functions is only mildly broken. It is moreover in agreement with globally extracted distribution functions [38,39].
The above reasoning rests on forms for the nucleon SF, averageKd over resonances. Retaining those in detail, the sharp first inelastic threshold will become blurred, but it seems reasonable that $C(1, Q^2)$ for relatively low $Q^2$ remains determined by static elastic form factors. That point is presently under study. A similar large-$Q^2$ result has been found from quark-hadron duality considerations, pushed to the extreme for the part $F_2^{N(NE)}$ [10].

Summarizing, we have described methods to extract the neutron Structure Function from existing inclusive scattering data on various targets. The experimental material from several targets provided consistent values for $C(1, 3.5)$. It is very desirable to plan the forthcoming inclusive scattering experiments on $D$ and $^4$He with the available 6 GeV beam [11], such that there be coverage of continuous $x \lesssim 1$-range for at least one $Q^2$. Confrontation of those data with accurate calculations, possible for those nuclei [27,28] will sharpen the present outcome for $F_2^n(x, Q^2), C(x, Q^2)$.

The authors thank G. Petratos for giving detailed information on the proposed $A=3$ experiment and J. Arrington for putting at our disposal $D$ and Coulomb-corrected NE3 data for C and Fe. ASR profited from discussions with G. Salmè, E. Pace, several experimentalists at JLab and in particular with W. Melnitchuk.
REFERENCES

[1] G.G. Petratos, Draft version PAC18, Jefferson Lab, July 2000.

[2] I.R. Afnan, F. Bissey, J. Gomez, A.T. Katramatou, W. Menitchouk, G.G. Petratos and A.W. Thomas, Phys. Lett. B 493 (2000) 36; F. Bissey, A.W. Thomas and I.R. Afnan, Phys. Rev. C 64 (2001) 024004.

[3] K. Saito, C. Boros, K. Tsushima, F. Bissey, I.R. Afnan and A.W. Thomas, Phys. Lett. B 493 (2000) 288.

[4] E. Pace, G. Salmè, S. Scopetta and A. Kievsky, Phys. Rev. C64 (2000)05523; Nucl. Phys. A 689 (2001) 453.

[5] M.M. Sargassian, S. Simula and M.I. Strickman, nucl-th/0105052

[6] S.V. Akulinichev, S.A. Kulagin and V.M. Vagradov, Phys. Lett. 158 B (1985) 485; G.V. Dunne, Nucl. Phys. A455 (1986) 701.

[7] S.A. Gurvitz and A.S. Rinat, TR-PR-93-77/ WIS-93/97/Oct-PH; Progress in Nuclear and Particle Physics, Vol. 34 (1995) 245.

[8] G.B. West, Ann. of Phys. (NY) 74, (1972) 464; W.B. Atwood and G.B. West, Phys. Rev. D 7, (1973) 773.

[9] C.H. Llewelyn Smith, Phys. Lett B 128 (1983) 107; M. Ericson and A.W. Thomas, ibid 112.

[10] A.S. Rinat, Proceedings of Meeting 'Prospects on Hadron and Nuclear Physics', Trieste, IT (1999) World Scientific.

[11] A.S. Rinat and M.F. Taragin, Phys. Rev. C 62 (2000) 034602.

[12] A. Bodek et al, Phys. Rev. D 20 (1979) 1471.

[13] S. Liuti and Franz Gross, Report TH-95-06
[14] A.Yu. Umnikov, F.C. Khanna and L.P. Kaptari, Z. F. Physik A 348 (1994) 211.

[15] W. Melnitchouk and A.W. Thomas, Phys. Lett. B 377 (1996) 11.

[16] S.V. Akulinitchev, S.A. Kulagin and G.M. Vagradov, Phys. Lett. B158 (1985) 485.

[17] P. Amadrauz et al, Phys. Lett B295 (1992) 159; M. Arneodo et al, ibid B364 (1995) 107.

[18] A. Bodek and J. Ritchie, Phys. Rev. D 23 (1981) 1070.

[19] See for instance: C. Ciofi degli Atti, E. Pace and G. Salmè, Phys. Rev. C 43 (1991) 11275; C. Ciofi degli Atti, D.B. Day and S. Liuti, ibid C 46 (1994) 1045.

[20] O. Benhar, A. Fabrocini, S. Fantoni, G.A. Miller, V.R. Pandharipande and I. Sick, Phys. Rev. C44 (1991) 2328; Phys. Lett. B359 (1995) 8.

[21] A.S. Rinat and M.F. Taragin, Nucl. Phys. A598 (1996); 349 ibid A620, 412 (1997); Erratum ibid A623 (1997) 773.

[22] S.A. Gurvitz and A.S. Rinat, [nucl-th/0106032](#), Phys. Rev. C 65, to be published.

[23] A.S. Rinat and M.F. Taragin, Phys Rev. C 60 (1999) 044601.

[24] J.Arrington et al, Phys. Rev. C 53 (1996) 224

[25] J. Gomez et al, Phys. Rev. D 49 (1994) 4348.

[26] H. Kamada et al, Phys. Rev. C 64 (2001) 044001.

[27] A.S. Rinat and M.F. Taragin, Phys. Rev. C 65 (2002) 041610.

[28] M. Viviani, A. Kievsky and A.S. Rinat, [nucl-th/0111048](#), submitted to Phys. Rev. C

[29] I. Niculescu et al, Phys. Rev. Lett. 85 (2000) 1182.

[30] J. Arrington, private communication.

[31] J. Arrington et al, Phys. Rev. Lett. 82 (1999) 2056; CalTech PhD thesis 1998.
Fig. 1. The ratio $C(x, 3.5) = F_{2}^{n}(x, 3.5)/F_{2}^{p}(x, 3.5)$ for $Q = 3.5$ GeV$^2$ from data on D, C, Fe. The drawn line corresponds to $C(1) = 0.54$ and the band represents the spread in the result, from averaging over different targets and methods. The vertical line for $x = 0.93$ marks the pion threshold $x_{\text{thr}}(3.5)$. The numbers on the right abscissa are quark model and QCD predictions for $C(1)$, while 0.61 is the NE limit [18]. Also entered are $F_{2}^{p}$ and $F_{2}^{n}$, corresponding to $C$. The band for the latter is hardly noticeable in $F_{2}^{n}$. 

[32] D.B. Day et al, Phys. Rev. C 40 (1993) 1849.

[33] S. Dasu et al Phys. Rev. Lett. 61 (1988), 1061; Phys. Rev. D 49 (1994) 5641; P.E. Bosted et al, Phys. Rev. C 46 (1992) 2505; B.W. Fillipone et al, Phys. Rev. C 45 (1992) 1582.

[34] I. Niculescu et al, Phys. Rev. Lett. 85 (2000) 1186.

[35] M. Jones et al, Phys. Rev. Lett. 84 (2000) 1398; Third Workshop on 'Perspective in Hadronic Physics' Trieste 2001, IT; to be published.

[36] A.F Sill et al, Phys. Rev. D 48 (1993) 29.

[37] M. Arneodo et al, Phys. Rev. D 50 (1994) R1.

[38] H.L. Lai et al, Phys. Rev. D 51 (1995) 4763; ibid D 55 (1997) 1280; Eur. Phys. J. C 12 (2000) 375.

[39] V. Barone, C. Pascaud and F. Zomer, Eur. Phys. J. C 12 (2000) 243.

[40] W. Melnitchouk, Phys. Rev. Lett. 86 (2001) 35.

[41] J. Arrington et al, Spokesman proposal JLab, May 2000.
TABLES

TABLE I. Values of $C(1, 3.5)$ from minimizing variances $w(d_0)$. Results are from data on D,C,Fe, using extraction methods I),II). The spread in results for I correspond to varying $\omega$-intervals in the variances. No entry corresponds to cases for which there is no minimum within the above intervals.

|    | D       | C       | Fe       |
|----|---------|---------|----------|
| I  | 0.55 ± 0.05 | -       | 0.55 ± 0.03 |
| II | 0.55    | 0.50    | -        |
