Two fluid anisotropic dark energy models in a scale invariant theory

S. K. Tripathy*, B. Mishra† and P. K. Sahoo‡

Abstract

We have investigated some anisotropic dark energy models in a simple scale invariant theory of gravity. The anisotropic nature of the universe is considered through a spatially homogeneous and anisotropic Bianchi type V space-time. The matter field is considered to be composed of two non interacting fluids namely the usual bulk viscous fluid and that of the dark energy fluid. Pressure anisotropy is considered along different spatial directions. From the constructed cosmological models, we found a dynamic pressure anisotropy which continues along with the cosmic expansion. The models are found to be mostly dominated by phantom behaviour. The presence of bulk viscous fluid does not affect substantially the general nature of the cosmic dynamics. The scale invariant theory of gravity is found to have a dominant role in the cosmic dynamics and help the de Sitter universe to exit from a catastrophic situation.

Keywords: Scale invariant theory; Dirac gauge; Dark energy; Anisotropic pressure; Bulk Viscosity

1 Introduction

Observations have confirmed that presently the universe is undergoing an accelerated phase of expansion [1, 2, 3, 4, 5]. General relativity predicts a cosmic fluid with a static or almost static density to cause such an acceleration[6]. The late time cosmic acceleration is attributed to an exotic dark energy form that refers to such a hypothetical fluid. Dark energy is believed to provide a strong negative pressure leading to an anti gravity effect that drives the acceleration (for recent reviews see [6, 7, 8, 9] and references therein). Observations of large scale structure and cosmic microwave background (CMB) [10, 11], X-ray clusters [12] and Baryon Acoustic Oscillations (BAO) [13] provide additional evidences in support of dark energy. The dark energy equation of state defined through the ratio of the dark energy pressure to dark energy density has been constrained in a tighter range by Sullivan et al. [14] and Suzuki et al. [15]. Betoule et al. from an analysis of SDSS-II and SNLS supernova samples have measured...
a constant dark-energy equation of state parameter $\omega_d = -1.018 \pm 0.057$ for a flat universe [16]. Recent observational data suggest that dark energy has a lion share of 68.3% in the mass-energy budget of the universe. Dark matter and baryonic matter comprise only 26.8% and 4.9% respectively [17, 18, 19, 20]. The simplest and natural candidate for dark energy is a cosmological constant in classical Friedmann-Robertson-Walker (FRW) model or the $\Lambda$ dominated Cold Dark Matter ($\Lambda$CDM) model since it has a constant density $\rho_\Lambda$ and a negative pressure $p_\Lambda = -\rho_\Lambda$. But the cosmological constant is known for long for various problems it faces (see [21, 22] for reviews on cosmological constant problem). Therefore, dynamical behaviour of the cosmological constant has been suggested in many works to account for the huge difference in the predicted and observed value of cosmological constant. A dynamical cosmological constant rolls down from a very high value at the beginning of the universe to a very small positive value at the present epoch. Besides the consideration of a cosmological constant two different approaches have been used in recent times to account for the dark energy driven cosmic acceleration. In the first approach, alternative dark energy candidates such as a canonical scalar field like quintessence models [23], a phantom field, a scalar field with negative kinetic term [24], ghost condensate [25] or k-essence [26] have been considered. The scalar field is assumed to roll dynamically over a potential. In the second approach, dynamical dark energy is described by a modification of the geometrical part of Einstein-Hilbert action by using functions of curvature scalar ($f(R)$ gravity models [27, 28], $f(R, T)$ gravity model [29, 30], $f(R, L_m)$ model [31]), of Gauss-Bonnet invariant [32] or higher derivatives of the action [33], holographic properties [34] etc. Even though the dark energy and cosmic acceleration has triggered a good deal of research interest in recent time, the nature and origin of dark energy is still unclear. The only thing known about dark energy so far is that it violates the strong energy condition and clusters only at the largest accessible states.

Besides the issue of late time cosmic acceleration, the observed anisotropy in temperature power spectrum has triggered much attention in recent times. According to the the standard cosmological model (ACDM model), the universe is mostly flat and spatially isotropic. Also, the precise measurements of the CMB temperature anisotropy from Wilkinson Microwave Anisotropy Probe (WMAP) are consistent with the standard cosmological model [35]. However, there remains some anomaly at large scale such as (i) observed large scale velocity flows than prediction, (ii) a statistically significant alignment and planarity of the CMB quadrupole and octupole modes and (iii) the observed large scale alignment in the quasar polarization vectors [36]. Recent data from Planck collaboration show a slight red-shift of the primordial power spectrum from the exact scale invariance [17, 18, 19, 20]. Planck data clearly shows that, at least at low multipoles ACDM model does not fit well to the temperature power spectrum [18]. Measurements from WMAP also predict an asymmetric expansion with one direction expanding differently from the other two transverse direction at equatorial plane which signals a non trivial topology of the large scale geometry of the universe [37, 38, 39]. Recently, some anisotropic plane symmetric models have been proposed to address such issues [40, 41, 42, 43]. In this context, Bianchi type cosmological models play an important role in observational cosmology. Bianchi type models are the generalization of the open universe in FRW cosmology and can be able to handle the smallness in the angular power spectrum of temperature anisotropy.
In the context of late time cosmic dynamics dominated by dark energy and the anisotropy in CMB temperature power spectrum, many authors have investigated different models considering pressure anisotropies along different spatial directions in the framework of Einstein’s relativity (see for example [44, 45]). In many such earlier works, the dark energy pressure is considered to be different in different directions which result into directional dark energy equation of states. The source of pressure anisotropy in all those works are not specifically mentioned. In a recent work [46], we have investigated Bianchi V dark energy models with pressure anisotropy considering only the contribution coming from dark energy in the framework of a scale invariant theory as proposed by Wesson [47, 48]. Scale invariant theory is one of the prominent alternative theory to general relativity. The reason behind the formulation of such a theory is that matter in the universe represented by the galaxies appears to be describable in a scale free manner. In other words, the cluster of galaxies and the distribution of galaxies over large distances can be described mathematically through functions which should be free of any fixed space or time scales [47]. Another reason behind the proposition of a scale invariant theory of gravity is that, basically it is a gauge theory and gauge theories in particle physics provide better explanation for interaction between particles. Even though several scale invariant theories of gravity have been proposed with varying degrees of success in explaining cosmological observations, Wesson in his work [47] has claimed that, his theory is a simple one and may be superior to others. In this theory of gravitation, Einstein field equations have been written in a scale-independent way by performing the conformal or scale transformation

\[ g_{ij} = \beta^2 (x^k) g_{ij}, \]  

where the gauge function \( \beta \), in its most general formulation, is a function of all space-time coordinates. Through such a conformal transformation, Wesson [47, 48] transformed the usual Einstein field equations into

\[ G_{ij} + 2 \frac{\beta_{ij}}{\beta} - \frac{4}{\beta^2} (g^{ab} \frac{\beta_a}{\beta} \frac{\beta_b}{\beta} - 2 g^{ab} \frac{\beta_{ij}}{\beta}) g_{ij} + \Lambda_0 \beta^2 g_{ij} = -T_{ij}, \]  

where

\[ G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}. \]  

Here, \( G_{ij} \) is the conventional Einstein tensor involving the metric tensor \( g_{ij} \). Semicolon and comma after the gauge function respectively denote covariant differentiation with respect to \( g_{ij} \) and partial differentiation with respect to coordinates. \( R_{ij} \) is the Ricci tensor, and \( R \) is the Ricci scalar. The cosmological term \( \Lambda_0 g_{ij} \) of Einstein theory is now transformed to \( \Lambda_0 \beta^2 g_{ij} \) in scale invariant theory with dimensionless cosmological constant \( \Lambda_0 \). \( T_{ij} \) is the energy momentum tensor of the matter field.

The role played by scale invariant theory of gravity concerned with the elementary particle physics and cosmology, has been discussed by some authors viz., Dirac [49, 50], Canuto et al. [51], Beesham [52, 53, 54], Mohanty and Mishra [55], Mishra et al. [56, 57, 58, 59] have investigated several cosmological aspects of scale invariant theory. Recently, several other investigations
have been made with different propositions of scale invariant theory of gravity [60, 61, 62, 63, 64, 65].

In the present work, we have extended our earlier work [46] and constructed some anisotropic dark energy models at the background of Bianchi type V (BV) universe in a simple scale invariant theory of gravity. In the present investigation, we have considered two different fluids contributing to the matter field. The first one is the usual bulk viscous cosmic fluid and the second one is due to the dark energy. The two fluids are considered to be non interacting. As in the previous work [46], pressure anisotropies for dark energy contributions are assumed along different spatial directions. The paper is organised as follows: In Section 2, the field equations for an anisotropic and spatially homogeneous BV metric are set up along with the formulations of the physical and kinematical parameters. The anisotropy in the cosmic fluid is thought to be due to different dark energy pressures along different spatial directions. In Section 3, the formulations for the skewness parameters and pressure anisotropies are derived in a model independent manner. Considering a constant deceleration parameter we have constructed two different cosmological models corresponding to power law of expansion and de Sitter kind of expansion in Section 4. The dynamics of the model along with the pressure anisotropies in the form of skewness parameters are discussed. At the end, the conclusions of the work are presented in section 5. Throughout the work, we have used the gravitational units $8\pi G = c = 1$, where $G$ is the Newtonian gravitational constant and $c$ is the speed of light in vacuum.

2 Basic Formalism

We consider the BV space-time with a Dirac gauge function $\beta = \beta(t)$ of the form

$$ds^2_W = \beta^2 ds^2_E. \tag{4}$$

with

$$ds^2_E = -dt^2 + A^2 dx^2 + e^{2\alpha x} (B^2 dy^2 + C^2 dz^2). \tag{5}$$

The metric potentials $A$, $B$ and $C$ are functions of $t$ only. $ds^2_W$ and $ds^2_E$ respectively represent the intervals in Wesson and Einstein theory.

The average scale factor $a$ and volume scale factor $V$ for the BV model are $a = (ABC)^{\frac{1}{3}}$ and $V = a^3 = ABC$. The generalized mean Hubble’s parameter $H$ is $H = \frac{\dot{a}}{a} = \frac{1}{3}(H_x + H_y + H_z)$, where $H_x = \frac{1}{A}, H_y = \frac{1}{B}, H_z = \frac{1}{C}$ are the directional Hubble parameters in the direction of $x$, $y$ and $z$ respectively.

The energy momentum tensor for an environment with two non interacting fluids can be written as

$$T_{ij} = T_{ij}^{(vis)} + T_{ij}^{(de)} \tag{6}$$

where $T_{ij}^{(vis)}$ and $T_{ij}^{(de)}$ respectively denote the contribution to the energy momentum tensor from bulk viscous cosmic fluid and dark energy respectively.

The energy momentum tensor for viscous fluid is given by

$$T_{ij}^{(vis)} = (\rho + \bar{p}) u_i u_j + \bar{p} g_{ij}. \tag{7}$$
where $\bar{p} = p - \xi u^i_i$, so that $T^{(\text{vis})}_{ij} = \text{diag}[-\rho, p, p, p]$. In comoving coordinates, $u^i = \delta^i_0$ is the four velocity vector and $\xi$ is the coefficient of bulk viscosity. $\xi u^i_i = 3\xi H$ is usually referred to as the bulk viscous pressure. The energy-momentum tensor for dark energy is considered in the form

$$T^{(de)}_{ij} = \text{diag}[-\rho_d, p_{dx}, p_{dy}, p_{dz}] = \text{diag}[-1, \omega_d, \omega_{dy}, \omega_{dz}]\rho_d = \text{diag}[-1, (\omega_d + \delta), (\omega_d + \gamma), (\omega_d + \eta)]\rho_d. \quad (8)$$

where $\omega_d$ is the dark energy equation of state parameter (EoS) and $\rho_d$ is the dark energy density. The skewness parameters $\delta$, $\gamma$, and $\eta$ are the deviations from $\omega_d$ on $x$, $y$ and $z$ axes respectively.

The field equations for a two fluid dark energy model in the framework of scale invariant theory become

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{B}C}{BC} - \frac{\alpha^2}{A^2} + 2\frac{\beta}{\beta} \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + 2\frac{\beta^2}{\beta^2} \beta^2 + \Lambda_0\beta^2 = -p + 3\xi H - (\omega_d + \delta)\rho_d. \quad (9)$$

$$\frac{\dot{A}}{A} + \frac{\dot{C}}{C} + \frac{\dot{A}C}{AC} - \frac{\alpha^2}{A^2} + 2\frac{\beta}{\beta} \left( \frac{\dot{A}}{A} + \frac{\dot{C}}{C} \right) + 2\frac{\beta^2}{\beta^2} \beta^2 + \Lambda_0\beta^2 = -p + 3\xi H - (\omega_d + \gamma)\rho_d. \quad (10)$$

$$\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{A}B}{AB} - \frac{\alpha^2}{A^2} + 2\frac{\beta}{\beta} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) + 2\frac{\beta^2}{\beta^2} \beta^2 + \Lambda_0\beta^2 = -p + 3\xi H - (\omega_d + \eta)\rho_d. \quad (11)$$

$$\frac{\dot{A}B}{AB} + \frac{\dot{B}C}{BC} + \frac{\dot{C}A}{CA} - 3\frac{\alpha^2}{A^2} + 2\frac{\beta}{\beta} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + 3\frac{\beta^2}{\beta^2} \beta^2 + \Lambda_0\beta^2 = \rho + \rho_d. \quad (12)$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \quad (13)$$

The overhead dot on a field variable denotes differentiation with respect to time $t$. One should note that, in the absence of bulk viscous cosmic fluid, the above equations reduce to the corresponding field equations in Ref. [46]. Integrating eqn. (13) and absorbing the integration constant into $B$ or $C$, we obtain $A = (BC)^{1/2}$. The scalar expansion $\theta$ and shear scalar $\sigma^2$ in the model are defined as $\theta = 3H = \frac{\dot{V}}{V} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}$ and $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2} \left( \Sigma H^2 - \frac{1}{3}\theta^2 \right)$, where $H_i$ with $i = 1, 2, 3$ are the directional Hubble parameters along $x$, $y$ and $z$ axes respectively. Also, $\sigma_{ij} = \frac{1}{2}(u_{i,k}h^k_j + u_{j,k}h^k_i - \frac{1}{3}\theta h_{ij})$ and $h_{ij} = g_{ij} - u_iu_j$ is the projection tensor.

The average anisotropy parameter $A$ is defined as

$$A = \frac{1}{3} \Sigma \left( \Delta H_i \right)^2, \quad (14)$$
where $\Delta H_i = H_i - H; i = 1, 2, 3$. $A$ is a measure of deviation from isotropic expansion. One can get the isotropic behaviour of the model for $A = 0$. A model is considered to isotropize at late times if the volume scale factor increases to infinitely large value and the average anisotropic parameter reduces to zero for infinitely large cosmic time. For a spatially homogeneous metric, the scalar expansion is considered to be proportional to the shear scalar which leads to an anisotropic relationship among the directional scale factors $B$ and $C$ as $B = C^m$, $m \neq 1$. $m$ is a positive constant and takes care of the anisotropic nature of the model. For $m = 1$, the average anisotropic parameter $A$ vanishes and the model reduces to the isotropic one. The directional scale factors can be expressed as $A = a$, $B = C^m = a^{\frac{m}{m+1}}$. Consequently, $H_x = H$, $H_y = \left(\frac{2m}{m+1}\right)H$ and $H_z = \left(\frac{m}{m+1}\right)H$. It may be noted here that, along the $x$–axis, the directional Hubble parameter is the same as that of the mean Hubble parameter which implies that along $x$–axis, the deviation of the anisotropic dark energy equation of state from mean equation of state is minimum or zero i.e $\delta$ should vanish.

The average anisotropic parameter is obtained for this model as

$$A = \frac{2}{3} \left(\frac{m-1}{m+1}\right)^2.$$  \hspace{1cm} (15)

The average anisotropic parameter is a constant quantity and depends only on the exponent $m$. It vanishes for $m = 1$ so that the model reduces to an isotropic case. From recent WMAP data, the present value of the average anisotropic parameter is obtained to be $|\sqrt{A}| = 10^{-5}$ which constrains the anisotropic parameter in terms of the exponent $m$ as $m = 1.0001633$ in our present model. One may note that, the universe is observed to be mostly flat and isotropic. The observations regarding the spatial anisotropy of the universe should imply a sort of small perturbation to the usual isotropic nature.

### 3 Skewness parameters and Pressure Anisotropy

The energy conservation of the anisotropic fluid, $T^{ij}_{\text{fluid}} = 0$, yields

$$\dot{\rho} + 3(\dot{\rho} + \rho)H + \rho_d + 3\rho_d(\omega_d + 1)H + \rho_d(\delta H_x + \gamma H_y + \eta H_z) = 0.$$  \hspace{1cm} (16)

We consider here a non interacting dark energy in presence of a viscous cosmic fluid which leads to the splitting of the above equation into two parts: one corresponding to the usual viscous fluid with equal pressure along all the spatial directions and the continuity equation for dark energy i.e

$$\dot{\rho} + 3(\dot{\rho} + \rho)H = 0.$$  \hspace{1cm} (17)

and

$$\dot{\rho_d} + 3\rho_d(\omega_d + 1)H + \rho_d(\delta H_x + \gamma H_y + \eta H_z) = 0.$$  \hspace{1cm} (18)

The energy momentum conservation equation of the dark energy can further be split into the isotropic part and the anisotropic part.
\[ \dot{\rho}_d + 3\rho_d(\omega_d + 1)H = 0, \]  
\[ \rho_d(\delta H_x + \gamma H_y + \eta H_z) = 0. \]

The second equation deals with the deviations from the total dark energy pressure along different spatial directions. According to eqn. (19), the behaviour of \( \rho_d \) is controlled by the deviation free part of EoS parameter of dark energy i.e \( \rho_d = \rho_{d0}a^{-3(\omega_d+1)} \), where \( \rho_{d0} \) is the dark energy density at the present epoch.

Algebraic manipulation of the equations (9), (10) and (11) yield
\[ (m-1)F(H,\beta) = (\gamma - \eta)\rho_d \]  
and
\[ \left( \frac{m-1}{2} \right) F(H,\beta) = (\gamma - \delta)\rho_d, \]
where
\[ F(H,\beta) = \frac{2}{m+1} \left( \dot{H} + 3H^2 + 2H \ddot{\beta} \beta \right) \]  
and \( \gamma = 2\delta - \eta \).

The skewness parameters can now be obtained from equations (20), (21) and (22) as
\[ \delta = - \left( \frac{m-1}{3\rho_d} \right) \chi(m)F(H,\beta), \]  
\[ \gamma = \left( \frac{5 + m}{6\rho_d} \right) \chi(m)F(H,\beta), \]  
\[ \eta = - \left( \frac{5m + 1}{6\rho_d} \right) \chi(m)F(H,\beta), \]
where, \( \chi(m) = \frac{m-1}{m+1} \). Here the functional \( \chi(m) \) is a measure of deviation from isotropic behaviour. For an isotropic model, \( \chi(m) \) vanishes and consequently, the skewness parameters all vanish.

The dark energy density \( \rho_d \) and the EoS parameter \( \omega_d \) are obtained as
\[ \rho_d = \left( \frac{2(m^2 + 4m + 1)^2}{(m+1)^2} \right) H^2 - \frac{3\alpha^2}{a^2} + 6H \beta \beta \]  
\[ + 3 \left( \frac{\dot{\beta}}{\beta} \right)^2 \Lambda_0 \beta^2 - \rho, \]  
\[ \omega_d\rho_d = \dot{\rho} - \left( \frac{2}{m+1} \right)^2 m(m-1)H^2 + \alpha^2 \]  
\[ - 2(2m+1) \left( \dot{H} + 2H \beta \beta \right) - 2\beta \beta \left( \frac{\dot{\beta}}{\beta} \right)^2 \Lambda_0 \beta^2. \]

The behaviour of the skewness parameters are controlled by the behaviour of the functional \( F(H,\beta) \) and the dark energy density \( \rho_d \) and more precisely, the skewness parameters depend on the ratio \( \frac{F(H,\beta)}{\rho_d} \). The functional \( F(H,\beta) \) depends upon the exponent \( m \), the time dependence of mean Hubble parameter \( H \) and the gauge function \( \beta \). The behaviours of dark energy density and dark energy
EoS parameter are decided by the anisotropic nature of the model through the exponent $m$, the time dependence of mean Hubble parameter $H$, the gauge function $\beta$ and the rest energy density $\rho$. It is interesting to note that, the dark energy density and the dark energy EoS parameter depend on the usual cosmic fluid. In the absence of usual cosmic fluid, the above equations (26) and (27) reduce to the earlier expressions of Ref. [46].

For a barotropic bulk viscous pressure $\bar{p} = \varepsilon \rho$ [67, 68], the rest energy density of universe can be expressed as

$$\rho = \rho_0 e^{-3(1+\varepsilon) \int H dt},$$  

(28)

where $\rho_0$ is the rest energy density at the present epoch. Bulk viscosity has already been recognised as a dissipative phenomena which plays a crucial role in getting accelerated phase of expansion. It is worth to mention here that the barotropic bulk viscous pressure includes the contributions from the usual cosmic fluid and that of the bulk viscosity so that $\varepsilon$ can take values both from the positive and negative domain. Here it is assumed that the contribution from bulk viscosity to pressure is proportional to the rest energy density of the universe [67, 68]. If the contribution from the bulk viscosity is greater than the usual perfect fluid pressure than the net pressure becomes negative with a negative $\varepsilon$ and otherwise $\varepsilon$ becomes positive. In case, the usual pressure from perfect cosmic fluid equals to the contribution coming from the bulk viscosity, then the cosmic fluid behaves like a pressureless dusty system. The accelerated expansion in the present epoch is usually attributed to a fluid with negative pressure and hence, it can be thought that the contribution coming from the bulk viscosity is greater than the usual pressure. However, the presence of an exotic dark energy form leads to a negative pressure of the universe which simulates an anti-gravity effect that drives the acceleration. DE density and DE EoS can be calculated if the rest energy density of the universe is known. The rest energy density of the universe can be calculated from (28) for a given value of $\varepsilon$, if the time variation of the mean Hubble rate is known.

Deceleration parameter $q = -\frac{a''}{a'}$ is an important quantity in the description of cosmic dynamics concerning the late time acceleration. A positive deceleration parameter implies a decelerating universe whereas its negative value describes a universe with acceleration expansion. A null value of this parameter shows a universe expanding with constant rate. With the advent of a lot of observational data favouring an accelerating universe, models with late time cosmic dynamics have gained much importance in recent times. In the present work, we are particularly interested in the kinematics of late time accelerating universe when the deceleration parameter is believed to vary slowly with cosmic time or becoming a constant. It is worth to mention here that, determination of deceleration parameter has remained a tough task of the observation of high redshift supernovae. Observations from type Ia Supernovae predict an accelerating universe with deceleration parameter $q = -0.81 \pm 0.14$ in the present time [70]. Type Ia Supernovae data in combination with BAO and CMB observations constrain the deceleration parameter as $q = -0.53^{+0.17}_{-0.14}$ [71]. However, the time dependence of deceleration parameter is still elusive even though a transition from positive to negative value is more favourable at late times. In view of the present scenario of the universe, one can choose the behaviour of the volume scale factor with all plausible assumptions. Common choices for the
scale factor are the de Sitter solution and the power law expansion. In de Sitter solution, the scale factor increases exponentially with cosmic time whereas in the power law expansion, the scale factor is chosen to vary as certain power of cosmic time. Obviously, a power law behaviour and an exponential behaviour of the scale factor lead to a constant deceleration parameter. Specific choices of the scale factor will consequently decide the time dependence of the directional scale factor \( C \) and hence the mean and directional Hubble rates. In the present work, we have considered these two choices of the scale factor which result into a constant deceleration parameter.

4 Some Cosmological Models

4.1 Power law expansion

In this model, we choose to have power law functional form for the scale factor in the form \( a = t^{(\frac{m+1}{2})n} \), where the exponent \( n \) is an arbitrary positive constant. If we define a constant \( k \) as \( k = (\frac{m+1}{2})n \), the scale factor can be expressed as \( a = t^k \). The volume scale factor will now behave like \( V = t^{3k} \) and the Hubble parameter is \( H = \frac{k}{t} \). It is obvious that, for \( k > 1 \), the model will be an accelerating one. The directional Hubble parameters are \( H_x = \left( \frac{m+1}{2} \right)^n t^n \), \( H_y = \frac{mn}{2} t^n \) and \( H_z = \frac{n}{2} t^n \) and consequently, the mean deceleration parameter \( q = -1 - \frac{\dot{H}}{H^2} \) becomes \( q = -1 + \frac{2}{m(m+1)} \). In terms of redshift \( z \), the mean Hubble rate is \( H(z) = k(1+z)^{\frac{1}{2}} \). Here we consider the relationship between the scale factor and redshift as \( 1 + z = \frac{1}{t} \) and the scale factor in the present epoch is 1. The parameter \( k \) is related to the value of Hubble parameter in the present epoch. The deceleration parameter is a negative constant quantity for \( n > \frac{2}{m+1} \) since \( m \) and \( n \) are positive constants and is in conformity with the present observational data predicting an accelerating universe. In order to get an accelerating model with this power law scale factor, the exponent \( n \) should always be greater than one i.e. \( n > 1 \) if \( m < 1 \) otherwise it has to be decided from \( n > \frac{2}{m+1} \). The universe is in general isotropic but recent observations from CMB temperature anisotropy, the anisotropic nature is favoured. However, any anisotropy in spatial expansion must be considered as a little perturbation of the isotropic behaviour which suggests that the exponent \( m \) must be very close to 1. In fact, in the present model, from the analysis of anisotropy as predicted from WMAP data, \( m = 1.0001633 \). This further restricts the value of the parameter \( n \) to be very close to 1 or more. It is worth to mention here that, cosmologies with power law scale factor are widely discussed in literature \[72, 73, 76, 77, 78, 79, 80\]. The power law model is quite successful in the sense that it does not encounter the horizon problem and do not witness flatness problem with \( n > \frac{2}{m+1} \). Recently, from the analysis of observational constraints from \( H(z) \) and SNIa data, power law cosmology is shown to be viable in the context of the present day cosmic acceleration even though it fails to produce primordial nucleosynthesis \[72\].

The gauge function \( \beta \) may be chosen suitably for getting a viable model. It can be a function of cosmic time or a function of cosmic scale factor. In the present work, we use a gauge function which behaves reciprocally with the cosmic time, so that at late time of cosmic evolution, the function will have a little contribution. It is necessary to mention here that, such a choice retains
the dimensional consistency of the gauge function $\beta$ as has been pointed out by Wesson [47].

The energy density contribution coming from the usual cosmic fluid for the power law model reduce to

$$\rho = \frac{\rho_0}{t^{\frac{3}{2}(1+\varepsilon)(m+1)n}}. \quad (29)$$

Now, with the choice of $\beta = \frac{1}{t}$, the functional $F(H, \beta) = F(t)$ and the dark energy density for this model are expressed as

$$F(t) = \left[ \frac{n^2(m + 1)}{2} - n \right] \frac{3}{t^2}, \quad (30)$$

$$\rho_d = \left[ \frac{(m^2 + 4m + 1)n^2}{2} - 3n(m + 1) + (3 + \Lambda_0) \right] \frac{1}{t^2} \quad (31)$$

$$- \frac{3\alpha^2}{t^{\frac{2}{n}(m+1)}} - \frac{\rho_0}{t^{\frac{3}{2}(1+\varepsilon)(m+1)n}}. \quad (32)$$

The dark energy EoS parameter for the model is

$$\omega_d = \frac{\rho_d}{\rho_d} \left[ \left( 2m + 1 \right)n - \frac{m(m - 1)n^2}{2} - (3 + \Lambda_0) \right] \frac{1}{t^2} + \frac{\alpha^2}{t^{\frac{2}{n}(m+1)}} + \frac{\varepsilon \rho_0}{t^{\frac{3}{2}(1+\varepsilon)(m+1)n}} \right]. \quad (33)$$

The functional $F(t)$ has the same form as that of Ref. [46] and is not affected by the presence of a usual bulk viscous cosmic fluid. As usual, it behaves as $t^{-2}$. The signature of $F(t)$ is positive for $n > \frac{2}{m+1}$ and negative if $n < \frac{2}{m+1}$. Hence, it decreases from a large positive value in early times to small values at late times for $n > \frac{2}{m+1}$ and increases from a large negative value to small negative values at later epochs for $n < \frac{2}{m+1}$. The functional $F(t)$ vanishes for the critical relationship $n(m + 1) = 2$ and consequently all the skewness parameters vanish implying an isotropic dark energy pressure in all spatial directions. Since, we are interested in the late time acceleration of the universe, for the power law expansion, $n > \frac{2}{m+1}$ and hence $F(t)$ remains in the positive domain. The dark energy density also decreases with the growth of time. The decrement in $\rho_d$ is decided by three different factors i.e. $t^{-2}$ in the first term, $t^{-n(m+1)}$ in the second term and $t^{-\frac{3}{2}(1+\varepsilon)(m+1)n}$ in the third term. The role of bulk viscous cosmic fluid comes through the third term. One may note that, if $\varepsilon = -1$, even though the contribution coming from the usual cosmic fluid does not vanish, it does not contribute to the time variation of the dark energy density. For $\varepsilon = -\frac{1}{3}$, the time variation of second and third terms can be clubbed together. For the specific choice of the parameters, $n(m + 1) = 2$ and $\varepsilon = -\frac{1}{3}$, the dark energy density behaves as $t^{-2}$ and hence the ratio $\frac{F(t)}{\rho_d}$ becomes independent of time. Consequently, for this choice the skewness parameters becomes a constant quantity appearing as simple time independent deviations from usual isotropic pressure. This situation can also arise for $n = \frac{2}{m+1}$ and $\varepsilon = -1$. If $n(m + 1) > 2$ and $\varepsilon > -\frac{1}{3}$, the magnitude of decrement in the dark energy density becomes more rapid compared to the decrement in $F(t)$. Accordingly, the ratio $\frac{F(t)}{\rho_d}$ for the particular case will increase with cosmic time. However, for large value of $n$, the ratio $\frac{F(t)}{\rho_d}$ behaves as unity. Therefore, for large value of $n$, the skewness
parameters depend only on the parameter $m$ and become independent of cosmic time.

Figure 1: Evolution of the functional $\frac{F(t)}{\rho_d}$ with cosmic time for the power law model of expansion for three representative value of the parameter $\varepsilon$.

In Figure 1, we have plotted the ratio $\frac{F(t)}{\rho_d}$ as a function of cosmic time for three representative values of the parameter $\varepsilon$, namely $\varepsilon = -\frac{1}{3}, -\frac{2}{3}$ and $-1$. As discussed above, except for the specific choice $\varepsilon = -\frac{1}{3}$, the ratio $\frac{F(t)}{\rho_d}$ increases with the increase in cosmic time. The slope of the curves increases with the decrease in the value of $\varepsilon$. In other words, the curve for $\varepsilon = -1$ becomes more stiff than the curve with $\varepsilon = -\frac{2}{3}$. $\frac{F(t)}{\rho_d}$ is almost time independent for $\varepsilon = -\frac{1}{3}$. It can be observed from the figure that, for all considered values of $\varepsilon$, $\frac{F(t)}{\rho_d}$ becomes equal at the present cosmic time $t = 1$. Since the behaviour of the skewness parameters depends mostly on the behaviour of the ratio $\frac{F(t)}{\rho_d}$, we can easily assess them from Fig.1.

In Figure-2, the dark energy equation of state is plotted as a function of redshift for four representative values of $\varepsilon$, namely $\varepsilon = 0, -\frac{1}{3}, -\frac{2}{3}$ and $-1$. The case $\varepsilon = 0$ corresponds to the cosmic fluid with no usual matter but of only dark fluid. For this particular case, the dark fluid behaves like a cosmological constant with $\omega_d = -1$. The dark energy equation of state, for all the cases considered here, remains in the phantom region. As expected, for $\varepsilon = -\frac{1}{3}$, the dark energy equation of state is independent of time but it remains in the phantom region below the phantom divide. In the remaining two cases, $\omega_d$ decreases with the growth of cosmic time. At early phase of cosmic evolution, the model behaves like a cosmological constant for these cases of $\varepsilon$. At late phase of cosmic evolution, $\omega_d$ decreases to acquire larger negative value. However, the
Figure 2: Dynamics of the dark energy equation of state parameter $\omega_d$ for the power law model of expansion for four representative value of the parameter $\varepsilon$.

$\varepsilon = 0$ represents a universe with no usual bulk viscous matter but of only dark fluid.

present model with two non interacting fluids favour phantom field to dominate the cosmic dynamics. One can note from the figure that, the dark energy equation of state decreases more rapidly for lower values of $\varepsilon$ after $\varepsilon = -\frac{1}{3}$. It is also interesting to note that at certain redshift around $z = 0.25$, $\omega_d$ is the same $-1.07$ for all the three cases of $\varepsilon$ except $\varepsilon = 0$.

The equation of state parameter is sensitive to the choice of the parameters $\alpha$ and $n$. In Figure-3, we have plotted the variation of $\omega_d$ with respect to $\alpha$ and in Figure-4, the response of $\omega_d$ for a variation of $n$ is shown. For all the choices of $\alpha$, the general trend in $\omega_d$ is the same eventhough, $\omega_d$ remains in the positive domain for some higher values of $\alpha$. $\omega_d$ shows an increasing trend with the increase in $n$. For some higher values of $n$, $\omega_d$ becomes positive all through the cosmic evolution.

The pressure anisotropies in the form of skewness parameters, $\delta, \eta$ and $\gamma$, normalised to the functional $\chi(m)$ are shown in Figure-5 as function of redshift. Since the behaviour of the functional $\frac{E}{P_d}$ is the same for all the cases of $\varepsilon$, we have only plotted the skewness parameters for a representative $\varepsilon = -\frac{2}{3}$. The skewness parameters closely depend on the behaviour of $\frac{E}{P_d}$. Therefore, we have shown $\frac{E}{P_d}$ for $\varepsilon = -\frac{2}{3}$ as a function of $z$ alongside of the skewness parameters for a quick reference. The behaviour of $\frac{E}{P_d}$ is clearly reflected in the behaviour of skewness parameters. As expected, the skewness parameter along x-axis , $\delta$, vanishes implying that along x-axis, the cosmic fluid has a pressure equal to the
total pressure. The pressure anisotropies maintain almost a constant value from some early times to considerably late phase of cosmic evolution. At large cosmic time, the magnitudes of pressure anisotropies tend to increase. It is certain that, the behaviour of \( \eta \) is just opposite to that of \( \gamma \). \( \gamma \) is positive whereas \( \eta \) is negative all through the cosmic evolution. It is interesting to note that, for \( n = \frac{2}{m+1} \), the skewness parameters \( \delta, \gamma, \eta \) along with the equation of state parameter \( \omega_{ld} \) become independent of time. In other words, for this particular choice of the exponent \( n \), the anisotropic pressures along different directions maintain a constant barotropic relationship with the rest energy density and throughout the cosmic evolution, the pressure anisotropy is maintained.

In the present work, we have also investigated the role of scale invariance on the dark energy equation of state. In the absence of scale invariance, the dark energy equation of state remains completely in the phantom region with much larger negative values than the scale invariant model. Also, the cosmological constant in the field equations plays a greater role. In its absence, the dark energy equation of state parameter is found to remain in the positive domain throughout the cosmic expansion history.

4.2 de Sitter expansion

Recently results from the measurements of B mode (curl component) power spectrum of CMB polarisation around \( l \sim 80 \) in BICEP2 (Background Imaging
of Cosmic Extragalactic Polarization) experiments predict a tensor-to-scalar ratio $r = 0.2^{+0.07}_{-0.05}$ providing a convincing evidence for standard inflationary scenario even though it contradicts the limits from Planck data. The inflationary scenario predicts the generation of gravitational waves during de Sitter kind of expansion. In de Sitter model, the scale factor $a$ is taken as $a = e^{\frac{(m+1)}{2} \xi t}$ and the volume scale factor behaves as $V = e^{\frac{3(m+1)}{2} \xi}$. $\xi$ is a positive constant and is related to the Hubble parameter through the relation $\xi = \frac{2H}{m+1}$. In this model, the Hubble parameter is a constant quantity and remains the same throughout the cosmic evolution. Consequently, the deceleration parameter $q = -1$. The directional Hubble rates along different spatial directions are given by $H_x = (\frac{m+1}{2}) \xi$, $H_y = m \xi$ and $H_z = \xi$. These directional Hubble rates are also constant quantities.

The energy density contribution coming from the usual viscous cosmic fluid for the de Sitter model reduce to

$$\rho = \frac{\rho_0}{e^{\frac{(1+\varepsilon)(m+1)}{2} \xi t}}. \tag{34}$$

The energy density depends on the value of $\varepsilon$. $\rho$ decreases with the increase in $\varepsilon$ and the other way around for a decrement in $\varepsilon$. For the particular choice $\varepsilon = -1$, $\rho$ becomes independent of time and assumes a constant value $\rho_0$ throughout the cosmic evolution. As earlier, we restrict to the same choice for the Dirac gauge function i.e. $\beta = \frac{1}{7}$. The functional $F(t)$ and the rest energy density for this model can now be obtained as

Figure 4: Variation of $\omega_d$ with the parameter $n$ of the exponent in the power law model of expansion. Since the presence of a bulk viscous fluid does not affect the dynamics substantially we have shown only the case for $\varepsilon = -2/3$. 

\[n=1.05\] \[n=1.04\] \[n=1.03\] \[n=1.01\] \[n=1.00\] \[n=0.99\]
Figure 5: (a)(upper panel) Dynamical evolution of skewness parameters along three spatial directions with respect to redshift for the power law model of expansion only for the case $\varepsilon = -2/3$. (b) (lower panel) Since the behaviour of $F(t)$ controls the nature of the pressure anisotropies in dark energy fluid, its evolution as function of redshift is shown for $\varepsilon = -2/3$ for reference.

\[ F(t) = \frac{3\xi^2(m+1)}{2} - \frac{2\xi}{t}, \quad (35) \]
\[ \rho_d = \frac{(m^2 + 4m + 1)\xi^2}{2} - \frac{3\xi(m+1)}{t} + \frac{(3 + \Lambda_0)}{t^2}, \quad (36) \]
\[ -\frac{3\alpha^2}{e^{\xi(m+1)t}} - \frac{\rho_0}{e^{\frac{1}{2}(1+\varepsilon)(m+1)t}}, \quad (37) \]

The dark energy EoS parameter for the model is

\[ \omega_d = \frac{1}{\rho_d} \left[ -\frac{m(m-1)\xi^2}{2} + \frac{2\xi(2m+1)}{3t} - \frac{(3 + \Lambda_0)}{t^2} + \frac{\alpha^2}{e^{\xi(m+1)t}} + \frac{\varepsilon\rho_0}{e^{\frac{1}{2}(1+\varepsilon)(m+1)t}} \right]. \quad (38) \]

In the de Sitter model, the functional $F(t)$ increases from some large negative value in early times to certain asymptotic constant positive value at a later epoch. The rate of growth of the functional is governed by the factor $t^{-1}$. The presence of bulk viscous cosmic fluid does not affect this functional and hence for all the choices of $\varepsilon$, its behaviour remains the same. The dark energy density decreases with the growth of cosmic time and asymptotically reduces to a positive constant value. The decrement in $\rho_d$ is decided by four different factors.
i.e $t^{-1}$ in the first term, $t^{-2}$ in the second term, $t^{-n(m+1)}$ in the third term and $t^{-\frac{2}{3}(1+\varepsilon)(m+1)n}$ in the fourth term. The role of bulk viscous cosmic fluid comes through the fourth term. The contribution from the bulk viscous cosmic fluid becomes time independent for $\varepsilon = -1$. The pressure anisotropies along different spatial directions depend on the behaviour of $F(t)$ and $\rho_d$ and more specifically on $\frac{F}{\rho_d}$. One should note that, $F(t)$ is not affected by the presence of the bulk viscous cosmic fluid but the dark energy density $\rho_d$ depends on the barotropic equation of state $\varepsilon$ and hence the ratio $\frac{F}{\rho_d}$ depends on $\varepsilon$. We have shown the time variation of this functional $\frac{F}{\rho_d}$ in Figure 6 for three representative values of $\varepsilon$ namely $\varepsilon = -\frac{1}{3}, -\frac{2}{3}$ and $-1$. Barring for some cosmic time in the early phase, $\frac{F}{\rho_d}$ remains in the positive domain all through the cosmic evolution. It increases with time, peaks at around $t = 1.5$ and then decreases with the cosmic dynamics. At large cosmic time, $\frac{F}{\rho_d}$ becomes time independent and asymptotically reduces to $\frac{3(m+1)}{m^2+4m+1}$. For all the cases of bulk viscous barotropic cosmic fluid considered here, the functional $\frac{F}{\rho_d}$ behaves alike throughout the cosmic evolution except near the peak. The peaks for different $\varepsilon$ are different. The peak is higher for $\varepsilon = -1$ and lower for $\varepsilon = -\frac{1}{3}$.

![Figure 6: Evolution of the functional $\frac{F(t)}{\rho_d}$ with cosmic time for the de Sitter model of expansion for three representative value of the parameter $\varepsilon$.](image)

In Figure-7, the dark energy equation of state is plotted as a function of redshift for four different values of $\varepsilon$, namely $\varepsilon = 0, -\frac{1}{3}, -\frac{2}{3}$ and $-1$. The case $\varepsilon = 0$ corresponds to the cosmic fluid with no usual matter but of only dark fluid. The dark energy EoS, for all the cases considered here, decreases from some constant value at early cosmic phase becomes the lowest for some particular redshift $z = -0.16$ and then increases with cosmic time. At a redshift
Figure 7: Dynamics of the dark energy equation of state parameter $\omega_d$ for the de Sitter model of expansion for four representative value of the parameter $\varepsilon$.

$z = -0.525$ it vanishes and becomes positive at some future cosmic time. The behavioural switching over of $\omega_d$ is predicted to occur at a future time. At the present epoch, it maintains the same decreasing trend and lies in the phantom region. The bulk viscous cosmic fluid has a very little impact on the general behaviour of the EoS parameter. For all the values of $\varepsilon$ considered in the work, it maintains the same evolutionary trend. However, near the well (negative peak), the value of $\omega_d$ is the lowest for $\varepsilon = -1$ and highest for $\varepsilon = 0$.

The evolution of $\omega_d$ with time is similar to the case of phantom inflation in little rip behaviour with a monotonically increasing potential $V(\phi) \sim \phi^N$ of the phantom field $\phi$ satisfying the continuity equation $\ddot{\phi} + 3H\dot{\phi} - V(\phi) = 0$ \cite{82}. In little rip scenario \cite{83}, the cosmic acceleration is driven by a phantom field where the energy density increases without bound but the EoS parameters tend to behave as a cosmological constant with $\omega_d = -1$ asymptotically and rapidly thereby avoiding the rip singularity within a finite time.

The equation of state parameter is very much sensitive to the choice of $\xi$. In Figure 8, we have shown the EoS parameter for the present de Sitter model for different choices of $\xi$. With the increase in the value of $\xi$, the behavioural switching over time of the universe shifts towards the past. However, at very early time and at late cosmic time, $\omega_d$ becomes independent of the choice of $\xi$. In Figure 9 we have shown $\omega_d$ for different choices of the parameter $\alpha$. $\omega_d$ remains the same for all values of $\alpha$ both at early and late cosmic evolutionary phases but differs in the middle phase of cosmic evolution. With the increase in the value of $\alpha$, the decrement in $\omega_d$ is more rapid. Also, the value of $\omega_d$ near the well (negative peak) is more for higher value of $\alpha$. 

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The evolution of skewness parameters with cosmic dynamics is shown in Figure 10. In the figure, the time evolution of the functional $F(t)$ for a particular choice of $\varepsilon = \frac{-2}{3}$ is also shown for reference. It can be noted from the figure that the behaviour of the skewness parameters are totally controlled by the behaviour of the functional $F(t)$. At early cosmic phase $\eta$ starts with a negative value close to zero and switches over to positive values at a redshift of $z = 0.14$. $\eta$ increases with the cosmic time to become maximum at redshift $z = 0.23$ and then again decreases with further increase in cosmic time. The evolutionary behaviour of $\gamma$ is just the mirror image of $\eta$. The pressure anisotropy along the x-axis vanishes as expected earlier. It is interesting to note that, at the switching over redshift $z = 0.14$, all the skewness parameter vanish.

In Figure 11, the cosmic evolution of the equation of state parameter in the absence of scale invariance is shown to understand the role played by it. In Figure 11(a), the role of bulk viscous fluid on the cosmic dynamics in the absence of scale invariance is shown. It is obvious that, the presence of a bulk viscous fluid does not affect appreciably to the equation of state parameter. In order to assess the role played by scale invariance of the field equations, in Figure 11(b) we have compared the evolution of $\omega_d$ with that when scale invariance is included. It is interesting to note that, the scale invariance substantially affects the dynamics of the cosmic evolution. In the absence of scale invariance, $\omega_d$ is mostly dominated by phantom field and remains in the negative domain. It evolves from above the phantom divide in the early times to enter into the phantom region at late time of cosmic evolution. $\omega_d$ appears to cross the phantom...
divide at a future time with redshift $z$ around $-0.39$.

5 Summary and Conclusions

There has been a continuous trial to understand the dark driven late time cosmic acceleration. Different dark energy models have been proposed in this context. Also, it is required further that, many other dark energy models should also be constructed to extend our understanding regarding the dark side of the universe. In the present work, keeping in view of the late time acceleration phenomena, we reconstructed some cosmological models considering an anisotropic Bianchi $V$ universe in the framework of a simple scale invariant gravity theory as proposed by Wesson. The scale invariant theory we have adopted has already been claimed by Wesson to be a simple one that has passed some tests and may be considered superior to others proposed earlier. The gauge function is considered to depend only on cosmic time in a reciprocal manner. The matter field is considered to be composed of two non interacting fluids namely the usual bulk viscous fluid and that of the dark energy fluid. Pressure anisotropy is considered along different spatial directions. At late times, the deceleration parameter is considered to be negative and is slowly time varying or practically a constant. A negative value of the deceleration parameter simulates an accelerating universe. Alternately, this constant deceleration parameter leads to two different laws for the volumetric expansion namely power law of expansion and de Sitter solution. From the reconstructed models, we have studied the dynamics of the universe

through the dark energy EoS parameter. The skewness parameter or pressure anisotropies are also calculated. The role of a bulk viscous fluid in addition to dark components is also investigated.

We found that the skewness parameters dynamically evolve with the cosmic expansion. Along the x-axis, the dark energy pressure is the same as that of the total dark pressure contribution. In the two other spatial directions, i.e. y- and z- direction, the pressure anisotropies behaves just like the mirror image of one another. In the power law model, the pressure anisotropies almost remain constant through out the cosmic evolution. However, at future phase, they tend to increase. In the de Sitter model, at early times, the universe is predicted to have almost isotropic fluid which becomes anisotropic with the growth of cosmic time. At late phase of cosmic evolution, the pressure anisotropy again tend to decrease. Over all, the pressure anisotropy can not be removed altogether even at late times and there remains a residual pressure anisotropy at late phase of evolution. The presence of a non interacting bulk viscous cosmic fluid along with the dark fluid does not substantially affect the dynamics of pressure anisotropy.

The present model mostly favours a phantom energy dominated universe with the dark energy equation of state lying primarily below the phantom divide i.e. $\omega_d < -1$. In the power law model, the dark energy EoS parameter becomes time independent for two specific choices of $\varepsilon$ namely $\varepsilon = 0$ and $-1/3$. For $\varepsilon = 0$, the model behaves like a cosmological constant with $\omega_d = -1$. With
a decrease in the value of $\varepsilon$ in the negative domain, the DE EoS decrease to acquire more phantom energy. In the de Sitter model, we get interesting results for the dynamic evolution of the DE EoS parameter. $\omega_d$ decreases initially and after attaining a negative peak at certain cosmic time it again increases to exit from a catastrophic situation. At late phase of time it becomes constant. This behaviour is similar to a phantom inflation in little rip model with the phantom field monotonically rolling over a power law potential. It is interesting to note that, this situation only occurs when we considered a scale invariant theory of gravity but in the absence of scale invariance in the field equations, no such behaviour is seen. In the absence of scale invariance, the DE equation of state parameter decreases rapidly in the phantom region with $\omega_d < -1$. The presence of a non interacting bulk viscous fluid does not affect a lot to the dynamics of the DE EoS in the de Sitter model.

In the present work, we observed that the scale invariant theory used to reconstruct anisotropic dark models, present more interesting results favouring phantom kind of behaviour and is consistent with the recent observations concerning the late time acceleration of the universe where phantom field is believed to play greater role in the cosmic dynamics. From our work, it is certain that pressure anisotropies in the dark energy fluid play some important and interesting roles in dark energy models. However, more involved investigation of the role of the pressure anisotropy should be carried out for further understanding.

Figure 11: (a) Dynamical evolution of the dark energy equation of state parameter with respect to redshift is shown for two representative values of $\varepsilon$ in the absence of scale invariance in the field equations. (b) The same for the case of $\varepsilon = -2/3$. The evolution of $\omega_d$ in scale invariant theory is also shown in the plot for comparison.
of the cosmic mechanism.

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