Snaking states on a cylindrical surface in a perpendicular magnetic field

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We calculate electronic states on a closed cylindrical surface as a model of a core-shell nanowire. The length of the cylinder can be infinite or finite. We define cardinal points on the circumference of the cylinder and consider a spatially uniform magnetic field perpendicular to the cylinder axis, in the direction South-North. The orbital motion of the electrons depends on the radial component of the field which is not uniform around the circumference: it is equal to the total field at North and South, but vanishes at the West and East sides. For a strong field, when the magnetic length is comparable to the radius of the cylinder, the electronic states at North and South become localized cyclotron orbits, whereas at East and West the states become long and narrow snaking orbits propagating along the cylinder. The energy of the cyclotron states increases with the magnetic field whereas the energy of the snaking states is stable. Consequently, at high magnetic fields the electron density vanishes at North and South and concentrates at East and West. We include the spin-orbit interaction and show that if the linear Rashba and Dresselhaus types coexist the charge and spin densities around a cylinder of finite length become skewed.

PACS numbers: 73.20.At, 71.70.Ej, 73.21.Hb.

I. INTRODUCTION

In the beginning of the 90’s nanomagnets of 30-100 nm diameter were fabricated using a scanning tunnelling microscope\textsuperscript{1}. Soon after that electronic transport in the presence of nonuniform magnetic fields like magnetic steps, magnetic barriers or magnetic wells have been studied and interesting conductance properties have been predicted\textsuperscript{2}. For example, the tunneling probability of an electron through a magnetic barrier depends not only on the momentum component perpendicular to the barrier (like for an electric barrier), but also on the component parallel to the barrier. The nonuniform magnetic field is in fact equivalent to an effective potential which depends on the momentum\textsuperscript{3}. This is in fact the idea of magnetic lenses used to focus the electron beam, for example in cathode tubes.

Then, periodic magnetic fields over hundreds of nm were created in GaAs heterostructures. One design was with superconducting stripes placed on the surface of the semiconductor heterostructure, but which produced only weak variations of the magnetic field at the level of the two-dimensional electron gas\textsuperscript{4}. Another way was with ferromagnetic stripes grown at the surface, which produced fields with much stronger gradients, and even changing sign\textsuperscript{5}. In some regions the field component perpendicular to the two-dimensional electron system was “up” and in other regions was “down”. In such a situation, for an appropriate orientation of the electron velocity along the zero-field line, the Lorentz force on both sides tends to bend the trajectory back towards the line such that the electron remains trapped in an open snaking orbit\textsuperscript{6}. For this reason a strong positive magnetoresistance was observed in the experiments\textsuperscript{5}. For a large amplitude of the periodic magnetic field (like half a tesla) the electronic states were a superposition of two dimensional cyclotron orbits and one-dimensional free electron states along the snaking paths\textsuperscript{7,8}. Another interesting example of electrons in an inhomogeneous magnetic field is when the electrons are situated on a cylindrical surface. Such an electron system has been built by several groups using the strain architecture technology\textsuperscript{9}. By etching at a certain depth underneath an InGaAs heterostructure the upper layers automatically bend, due to the strain relaxation, until becoming a cylinder with radius of 10-20 μm\textsuperscript{10,11}. Of course the curved surface is never closed, but it can also be rolled up many times until it looks like a carpet roll\textsuperscript{12}. In the presence of a magnetic field perpendicular to the axis of the cylinder the Lorentz force depends only on the radial component and vanishes at right angles relatively to the external field.

Four-terminal transport measurements on a small region defined on such a curved surface, with the zero-field line inside the transport region, show oscillations of the magnetoresistance indicating that the electrons scatter into the snaking orbits and then deviate either towards the positive side or towards the negative side of the magnetic field. Although the two-dimensional electron gas has high mobility, it is not clear whether the electronic transport is truly ballistic in this case, due to the relatively rough boundaries of the transport region. An interesting hypothesis is that the Dresselhaus spin-orbit effect is responsible for the preferential deviation of the electrons\textsuperscript{13}. With a totally different technology, thin nanowires of diameters down to 100 nm or less can now be grown\textsuperscript{14}. Field effect transistors with high electron mobility have been realized with InAs/InP core-shell cylinders\textsuperscript{15}. The core-shell cylinders consist of an insulating core cov-
ered by a shell of a conductive material, such that the carriers are located within the thin outer tube. The materials used may be InN\textsuperscript{16,17} or InAs\textsuperscript{18} and more recently GaAs/InAs\textsuperscript{19} Transport properties of phase-coherent type in GaAs/InAs core-shell nanowires with a magnetic field in different directions, and also temperature dependent measurements, have been recently reported\textsuperscript{20,21} Conductance oscillations are observed as functions of a longitudinal magnetic field and a gate voltage, and also an antilocalization effect due to the electron spin. Hall effect measurements are performed on InAs nanowires\textsuperscript{22} However, to our knowledge, systematic magnetotransport measurements with the magnetic field perpendicular to the cylinder are not reported, and the role or the presence of the snaking orbits is not clear.

Several theoretical papers addressed the quantum states and some transport properties of electrons situated on a closed cylindrical surface in the presence of an external magnetic field. For example the oscillations of magnetococonductance in the case of ballistic transport were described as interference effects between electron propagation through different channels along the cylinder\textsuperscript{23} Oscillations of Aharonov-Bohm type are also predicted for the magnetizations of core-shell nanowires\textsuperscript{24} In particular, an interesting paper by Ferrari et al\textsuperscript{24} describes states in transverse magnetic field, possibly perpendicularly oriented along the length of the cylinder. The authors give a hint of the snaking orbits, but they do not focus on them.

In more recent papers from the same group\textsuperscript{25,26} core-shell nanowires with hexagonal cross section are considered, whereas the top and the bottom regions become totally depleted. This result may possibly be guessed from previous theoretical papers, but it has not been explicitly shown. In addition we include the spin and the spin-orbit interaction (SOI) within standard models incorporating a Rashba effect\textsuperscript{27} and a Dresselhaus contribution\textsuperscript{28} and we calculate the spin polarization around the cylinder.

In this paper we reconsider the quantum mechanics of the electrons on the cylindrical surface in perpendicular magnetic field. We show that for a sufficiently strong magnetic field, but realistic, perpendicular to the axis of the cylinder, the electron density concentrates on the lateral sides of the cylinder, along the snaking orbits, whereas the top and the bottom regions become totally depleted. This result may possibly be guessed from previous theoretical papers, but it has not been explicitly shown. In addition we include the spin and the spin-orbit interaction (SOI) within standard models incorporating a Rashba effect\textsuperscript{29} and a Dresselhaus contribution\textsuperscript{30} and we calculate the spin polarization around the cylinder.

In Sec. \textsuperscript{II} we describe the model and we build the quantum mechanical states. In Sec. \textsuperscript{III} we discuss the cyclotron and the snaking states. In Sections \textsuperscript{IV} and \textsuperscript{V} we show our results for cylinders of an infinite and finite length, respectively. Finally in Sec. \textsuperscript{VI} we summarize the conclusions.

\section{II. Quantum Mechanical Model for the Infinite Cylinder}

We consider electrons on a cylindrical surface of zero thickness, radius $R$, and infinite length, which is illustrated in Fig. \textsuperscript{I} The length of the cylinder is in the $z$ direction, the vertical axis is $x$, and the horizontal axis is $y$. For convenience we also name the “cardinal” points as North (N), South (S), East (E), and West (W). In order to calculate the quantum mechanical states electrons situated on this surface we describe the corresponding Hilbert space with the basis set $\{mks\} \equiv \langle \varphi |m\rangle \langle z |k\rangle |s\rangle$, where

$$\langle \varphi |m\rangle = \frac{1}{\sqrt{2\pi}} e^{im\varphi}, \langle z |k\rangle = \frac{1}{\sqrt{L}} e^{ikz}, |s\rangle = |\pm 1\rangle. \quad (1)$$

In this basis set the quantum number $m = 0, \pm 1, \pm 2,...$ describe the $z$-projection of the angular momentum, $k$ is the wave vector in the $z$ direction and $L$ is the unspecified length of the cylinder. The spin states are associated to an abstract binary vector $|s\rangle$, conventionally chosen as eigenvectors of the spin angular momentum in the same direction $z$.

The basis set are the eigenvectors of the Hamiltonian without magnetic field and spin,

$$H_0 = -\frac{\hbar^2}{2m_{\text{eff}} R^2} \frac{\partial^2}{\partial \varphi^2} + \frac{p_z^2}{2m_{\text{eff}}}, \quad (2)$$

with corresponding eigenenergies

$$\varepsilon_{mk} = \frac{\hbar^2}{2m_{\text{eff}} R^2} \left[ m^2 + (kR)^2 \right]. \quad (3)$$

The total single particle Hamiltonian of the problem, $H$, is $H_0$ plus the magnetic and spin contributions shown below.

In the presence of a magnetic field along the $x$ direction, $\mathbf{B} = (B,0,0)$, the vector potential can be chosen as $\mathbf{A} = (0,0,By) = (0,0,BR\sin \varphi)$. With the momentum transformation $p_z \to p_z + eA_z = p_z + eBR \sin \varphi$ we obtain the orbital contribution of the magnetic field,

$$H_B = \frac{eBR}{m_{\text{eff}}} p_z \sin \varphi + \frac{(eBR)^2}{2m_{\text{eff}}} (\sin \varphi)^2. \quad (4)$$

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{An infinitely long cylinder in perpendicular magnetic field. Electrons are situated on the cylinder surface. The “cardinal” points (N,S,E,W) are indicated and used suggestively.}
\end{figure}
The spin Zeeman term is then given by
\[ H_Z = -\frac{1}{2} \gamma_{\text{eff}} \mu_B B \sigma_x , \] (5)
where the Zeeman energy is included through the effective \( \gamma \)-factor (\( \gamma_{\text{eff}} \)) of the electron in the semiconductor host material and where \( \mu_B \) is Bohr’s magneton (defined with the mass of the free electron and the absolute value of the electron charge).

The spin-orbit interaction is described with the linear Rashba \((H_R)\) and Dresselhaus \((H_D)\) models as follows.
\[ H_R = \frac{\alpha}{\hbar} (\sigma_x p_z + e BR \sin \varphi) - \sigma_z p_\varphi , \] (6)
which corresponds to the radial confinement of the electrons on the cylindrical surface analog to the planar confinement of the two-dimensional electron gas in an asymmetric quantum well. Here \( \sigma_\varphi = \sigma_y \cos \varphi - \sigma_x \sin \varphi \) and \( p_\varphi = -i \hbar / R \partial / \partial \varphi \) are the azimuthal Pauli matrix and momentum, respectively.

The second type of SOI is the Dresselhaus coupling in zincblende structures, due to the bulk inversion asymmetry, which is of the form
\[ H_D = \frac{\beta}{\hbar} \left[ (\sigma_x p_\varphi + p_\varphi \sigma_\varphi) - \sigma_z (p_z + e BR \sin \varphi) \right] , \] (7)
which is the standard model for the two-dimensional semiconductor system, but here symmetrized in the azimuthal direction \( \varphi \). Nanowires grown from materials that are zincblende in the bulk can occur with a wurtzite configuration due to the structural changes in the lattice. A nanowire with a wurtzite lattice has a linear SOI term, which is different than for a zincblende. Nevertheless, here we consider nanowires that still have a zincblende structure, which is possible since the crystallographic phase can be controlled at growth.

The total Hamiltonian is now
\[ H = H_0 + H_B + H_Z + H_R + H_D , \] (9)
whose eigenstates \( |ak\rangle \) are expanded in the basis \( |\rangle \):
\[ H|ak\rangle = E_{ak}|ak\rangle , \]
\[ |ak\rangle = \sum_{m,s} c_{a,m,s}(k)|mks\rangle \equiv \sum_{q} c_{aq}(k)|qk\rangle . \] (10)
The wave vector \( k \) is a good quantum number and it is therefore conserved. For any fixed \( k \) the label \( a = 1, 2, 3, \ldots \) denotes the eigenstates in the increasing order of the energies \( E_{ak} \) and incorporates the mixed orbital \((m)\) and spin \((s)\) states. Obviously the spin is not conserved due to the presence of SOI. For simplicity a composite discrete label \( q = \{m,s\} \) has been introduced. The coefficients \( c_{aq}(k) \) are found by a numerical diagonalization of the matrix elements of \( H \) calculated in the basis \( |\rangle \), for independent values of \( k \) on a discrete, but sufficiently dense grid, with 200–400 points. In the numerical calculations, for each \( k \) value the basis is truncated to \( |m| \leq 50 \). All the matrix elements of the Hamiltonian are straightforward integrals of trigonometric functions which are calculated analytically, without any special function involved, and are shown in the Appendix.

Once the single particle states are known the chemical potential \( \mu \) can be calculated by fixing the number of electrons \( N \) or the average electron density \( \langle \rho \rangle \), and by solving the equation for the occupation numbers,
\[ \langle n \rangle \equiv \frac{N}{2\pi RL} = \frac{1}{2\pi RL} \sum_{ak} \mathcal{F}_{ak} , \] (11)
where \( \mathcal{F}_{ak} = F[(E_{ak} - \mu)/k_B T] \) is the Fermi function. For the infinite cylinder the standard treatment of the summation over the continuous variable \( k \) is to transform it into an integral and absorb the unspecified length of the cylinder \( L \) into the differential element \( dk \), i.e. \( \int \Sigma_k \rightarrow (1/2\pi) \int dk \). The integration is then carried out numerically on the grid of \( k \) values using the simple trapezoidal rule.

The electron density is further calculated as
\[ n(\varphi) = \sum_{ak} \mathcal{F}_{ak} \psi^\dagger_{ak}(\varphi,z) \psi_{ak}(\varphi,z) , \] (12)
where \( \psi_{ak}(\varphi,z) \equiv \langle r|ak \rangle \) are the wave functions in the position representation, but seen as two-component vectors in the spin space (i.e. spinors). Hence the scalar product in the spin space is implicitly assumed in Eq. (12). The spin density is calculated in a similar way,
\[ s_{\alpha}(\varphi) = \sum_{ak} \mathcal{F}_{ak} \psi^\dagger_{ak}(\varphi,z) \left( \frac{\hbar}{2} \sigma_\alpha \right) \psi_{ak}(\varphi,z) , \] (13)
where \( \alpha = x,y,z \) denotes the spatial coordinate. In the numerical implementation of the particle and spin densities the wave functions are expanded in the basis \( |\rangle \) and the \( k \) integrals are stored in the matrix
\[ G_{q_1,q_2} = \frac{1}{2\pi} \sum_{a} \int dk \mathcal{F}_{ak} c^\dagger_{aq_1}(k) c_{aq_2}(k) . \] (14)
With the help of this matrix the spin density \( s_{\alpha}(\varphi) \) can be written as
\[ s_{\alpha}(\varphi) = \frac{\hbar}{2\pi R} \sum_{q_1,q_2} e^{i(m_2-m_1)\varphi} G_{q_1,q_2} (\sigma_\alpha)_{s_1s_2} , \] (15)
where \( q_1 = \{m_1,s_1\} \) and \( q_2 = \{m_2,s_2\} \). The electron density \( \langle \rho \rangle \) can be obtained by substituting the Pauli matrix elements $$\langle \sigma_\alpha \rangle_{s_1s_2}$$ with \((2/\hbar) \delta_{s_1s_2}$$.
III. SNAKING ORBITS AND THE EFFECTIVE POTENTIAL

In our setup the orbital motion of the electrons is determined by the component of the magnetic field perpendicular to the surface, if we neglect some additional (small) effects due to the SOI. The electrons experience a nonuniform radial magnetic field at different points on the cylinder, which changes orientation and vanishes at right angles relatively to the external field, i.e. at the W and E points, as shown in Fig. 2. The motion of the electrons along the zero-field line has the form of a snaking orbit. It is a channeled trajectory laterally confined by the Lorentz force which from both sides of the zero-field line tends to bend the trajectory back towards the line. Obviously only one orientation of the velocity is compatible with a snaking state. In the regions where the radial field does not vanish the classical trajectory of the electrons correspond to closed cyclotron orbits, possibly distorted by the field gradient. For a weak magnetic field or for a cylinder with a small radius $R$ the two types of states (snaking and cyclotron) cannot be really distinguished. But for a strong field, such that the magnetic length $\ell_B \equiv \sqrt{\hbar/eB} < R$ the two type a states may become spatially separated. The cyclotron orbits may occupy the N and S regions (and the surroundings), whereas the snaking states may concentrate around the W and E regions.

The confined snaking states can be easily explained by isolating an effective “magnetic” potential in the orbital Hamiltonian. Since the momentum in the $z$ direction $p_z = \hbar k$ is conserved we can look at the matrix elements of the orbital terms of the Hamiltonian $H_O \equiv H_0 + H_B$ and write them in the form

$$H_O(k) = -\frac{\hbar^2}{2m_{\text{eff}}R^2} \frac{\partial^2}{\partial \varphi^2} + V_{\text{mag}}(k, \varphi),$$

where

$$V_{\text{mag}}(k, \varphi) = \frac{(eBR)^2}{2m_{\text{eff}}} \left( \frac{k\ell_B^2}{R} + \sin \varphi \right)^2$$

is an effective potential which depends on the wave vector, or in fact on the dimensionless parameter $\lambda = k\ell_B^2/R$, which can bind (or localize) states with low energies. For $|\lambda| < 1$ the potential has two minima, as seen in Fig. 3 and hence the probability densities are expected to show two maxima. In particular, for $\lambda = 0$ (or $k = 0$), the states are confined on at $\varphi = 0$ and $\varphi = \pi$, i.e. at the N and S points. For a $|\lambda| \geq 1$ the potential has only one minimum, at $\varphi = 3\pi/2$ for $k > 0$ and at $\varphi = \pi/2$ for $k < 0$, where the snaking orbits are captured. The snaking states with the lowest energy will thus have the wave vectors $k_m = \pm R/\ell_B$.

IV. NUMERICAL RESULTS FOR THE INFINITE CYLINDER

In the following examples we consider a cylindrical surface with parameters as for InAs comparable to those used by Bringer and Schäpers. The effective electron mass and g-factor are $m_{\text{eff}} = 0.023m_0$ and $g_{\text{eff}} = -14.9$, respectively. The average electron density on the surface is $\langle n \rangle = 1.17 \times 10^{11}$ cm$^{-2}$. We use a Rashba SOI strength $\alpha = 20$ meV nm, corresponding to a strong radial electric field which confines the electrons on the surface, and a Dresselhaus SOI parameter $\beta = 3$ meV nm, both inspired from flat heterostructures. Indeed, in a real sample, there is a lot of a uncertainty of these parameters, but in the present work we are aiming at qualitative results. In the calculations of the spin and particle densities we use a finite temperature $T = 1$ K, sufficiently low to have no real role, but only to avoid possible numerical artefacts produced by a pure step-like Fermi function. In Fig. 4(a) we show energy spectra together with the corresponding charge and spin densities for a cylinder of
radius \( R = 35 \text{ nm} \) at a low magnetic field \( B = 0.16 \text{ T} \). The energy spectrum is organized in discrete energy bands (determined by the finite circumference \( 2\pi R \)) which are even functions of \( k, E_{\alpha k} = E_{\alpha-\pi k}, \alpha = 1, 2, \ldots \). The spin splitting of the energy bands is very small in this case, corresponding to a Zeeman energy of 0.14 meV, and thus indistinguishable in the figure. We observe a very weak variation of the electron density, relatively to the mean value, around the entire circumference, Fig. 4b. This result is consistent with the classical mechanics where according to the Bohr-van Leeuwen theorem a magnetic field (homogeneous or not) should have no effect on the density of electrons. The magnetic field only translates the momenta in the statistical distribution functions and therefore has no influence on the equilibrium states \( \text{mean} \). In our case the classical limit corresponds to \( \ell_B \gg R \) or to a high electron density such that the number of occupied subbands is much larger than one. Therefore the charge oscillations around the circumference have to be interpreted as quantum effects. The only difference between the electrons situated on the upper and lower halves of the cylinder is the orientation of the cyclotron motion on the surface relatively to the normal direction. Consequently the electron density has both up-down (or N-S) and left-right (or E-W) symmetries, i.e. \( n(\varphi) = n(\pi - \varphi) \) and \( n(\varphi) = n(2\pi - \varphi) \equiv n(-\varphi) \).

The weak oscillations have thus two identical minima and two identical maxima. In the present example the minima occur at the \( \varphi = 0 \) (N) and \( \varphi = \pi \) (S) regions and the maxima at \( \varphi = \pi/2 \) (E) and \( \varphi = 3\pi/2 \) (W) sides, but they may interchange for another position of the chemical potential over the energy spectrum.

The calculated spin density corresponding to the spin direction along the magnetic field, i.e. \( S_\varphi(\varphi) \), follows the weak charge oscillations, with maxima and minima at the same locations, but indeed with a larger relative amplitude. In this case the \( S_\varphi(\varphi) \) is proportional to the electron density, like in a (quasi) homogeneous system. The spin density perpendicular to the magnetic field, in the \( y \) direction, i.e. transverse to the net motion, is determined by the Rashba SOI, and it is an odd function of angle, \( S_y(\varphi) = -S_y(-\varphi) \), and also antisymmetric on the lateral sides of the cylinder relatively to \( \varphi = \pm \pi/2 \). \( S_y(\varphi) = -S_y(\pi - \varphi) \). The effect of the Dresselhaus SOI is to tilt the spin in the \( z \) direction, i.e. along the net motion, but with the symmetries \( S_z(\varphi) = -S_z(-\varphi) = S_z(\pi - \varphi) \). Indeed, the amplitudes of the \( y \) and \( z \) spin densities are much smaller than for the \( x \) spin projection as long as the spin-orbit interactions are much smaller than the Zeeman interaction.

In the next example we increase the magnetic field to \( B = 7 \text{ T} \) and show the results in Fig. 5b. In this case the magnetic length \( \ell_B = 9.7 \text{ nm} \ll R = 35 \text{ nm} \) and therefore the electrons situated at the N or S regions have almost localized wave functions around \( \varphi = 0 \) and \( \varphi = \pi \), respectively, where they experience a nearly constant magnetic field. Such states are in fact Landau levels which can be seen in the center of the energy bands, around \( k = 0 \). In Fig. 5a) the chemical potential is slightly below the lowest Landau level. Each Landau level is actually split in two sublevels because of the two equivalent regions N and S. For such a large magnetic field the two sublevels almost merge for \( k \approx 0 \) and the Landau level becomes almost degenerate. The Zeeman energy is now 6 meV and the next band top above the chemical potential coincides with the same ground Landau level, but with the opposite spin. The states which
are not around the center of the energy bands have wave functions on the lateral sides of the cylinder, possibly in the snaking areas. The snaking states with the lowest energy correspond to the lowest side minima of the lowest band and have the wave vectors which minimize the effective potential \( \mu = \pm R/\ell_B^2 = \pm 0.37 \, \text{nm}^{-1} \).

Before we go into a more detailed analysis of electron states let us observe another interesting feature of the energy spectrum. The Landau states increase in energy proportionally with the strength of the magnetic field, whereas the minima of the magnetic potential are always zero. Which means that with increasing the magnetic field the tops of the bands at \( k = 0 \) rise whereas the side minima are relatively stable. Consequently the electron density develops minima at the N and S poles and maxima at the E and W sides, as can be seen in Fig. 7(b).

Concerning the spin density, because the Zeeman gap is now large, the \( x \) projection does not follow the electron density like in the low field limit, Fig. 7(c).

Next we show in Fig. 6 results for the same magnetic field strength as before, \( B = 7 \, \text{T} \), but for a cylinder with radius \( R = 100 \, \text{nm} \). In this case the Landau states extend over wider surfaces on the top and bottom of the cylinders, or in other words on a larger interval of center coordinates \( k \ell_B \). However, with the chemical potential below the lowest Landau state the electron density totally vanishes at the N and S poles. Obviously, with increasing the magnetic field, or reducing the electron density, the depleted regions will expand, and eventually the electrons will concentrate only along thin lateral channels.

The spin-\( x \) density has pronounced peaks matching the lateral maxima of the charge distribution which correspond to electrons in the lowest pair of (almost degenerate) energy bands, and thus all with spin “up”. The secondary maxima at \( \varphi = \pi/2 \) and \( \varphi = 3\pi/2 \) are given by the snaking electrons. But due to the heavy bending of the energy bands, they now reside in the lowest three pairs of energy bands, the first two (more or less) with compensated spin, but the third again with spin “up”.

The probability density for some selected single particle states around the circular contour is shown further in Fig. 7. The selected single particle states are marked with point symbols in the energy spectra, and each point symbol is reproduced in the center of each circle. The state marked with the symbol + belongs to the band \( E_{1k} \), having energy \( 13.5 \, \text{meV} \) and wave vector \( k = 0.3 \, \text{nm}^{-1} \). It is a state of cyclotronic type. The probability density has two symmetric peaks slightly tilt towards the W side as shown in the top-left example of Fig. 4. The state \( E_{2k} \) is the next on the energy scale, but practically with the same energy as \( E_{1k} \), and with the same probability density (not shown). Going along the lowest band we show the state indicated by the symbol \( \times \) with \( k = 1.05 \, \text{nm}^{-1} \), which is very close to the band minimum which is at \( k_m = R/\ell_B^2 = 1.06 \, \text{nm}^{-1} \). Here the two probability-density peaks, like those shown for the previous state, merge into one single peak, and the state becomes of a snaking type. Increasing further \( k \) within the same energy band we find only snaking states with one distribution peak, like for example the state marked with the point symbol \( * \) which has \( k = 1.11 \, \text{nm}^{-1} \).

![Figure 6](image6.png)

**FIG. 6.** (Color online.) (a) Energy spectra for a cylindrical surface of radius \( R = 100 \, \text{nm} \) vs. the wave vector \( k \). In this case only the energies with \( k > 0 \) are shown. Selected energies are marked with various point symbols to be correlated with Fig. 7. The dashed horizontal line indicates the chemical potential. Right: (b) The electron density \( n \), (c) the spin density \( S_x \), and (d) the spin densities \( S_y \) and \( S_z \) as functions of the polar angle on the surface \( \varphi \). The magnetic field strength \( B = 7 \, \text{T} \).

![Figure 7](image7.png)

**FIG. 7.** (Color online.) The probability density for selected single particle states is drawn with the continuous red line. The dashed blue line indicates the contour of the cylinder. The selected single particle states are marked with point symbols in the energy spectra, and each point symbol is reproduced in the center of each circle, respectively. The states with maxima towards N and S behave like cyclotron orbits. The states concentrated at W are snaking orbits. For each energy band \( E_a(k), a = 1, 2, \ldots \), the states become snaking orbits for a sufficiently large \( k \). The number of peaks of a snaking orbit is related to the energy quantization in the effective potential well created around the W point, along the line of zero normal magnetic field component. The wave vectors (in units of \( \text{nm}^{-1} \)) for the selected states are: \( k = 0.3 \) for +, \( \varnothing \); \( k = 0.5 \) for \( \times \), \( \bullet \); \( k = 1.008 \) for \( \square \); \( k = 1.11 \) for \( * \), \( \blacksquare \), \( \triangle \).
In the second energy band we show the state □, which is in transition from cyclotronic to snaking type, and further the state ■ which has again \( k = 1.11 \text{ nm}^{-1} \) and it is snaking. In this region of the spectrum this later state differs from the state * by the spin orientation. In the absence of the SOI a band crossing would occur close to the state ● with \( k = 1.05 \text{ nm}^{-1} \). Here, i. e. on the third energy band \( E_{3k} \), the snaking states have a double peak distribution which can be seen all the way to higher \( k \), e. g. for the state △ having again \( k = 1.11 \text{ nm}^{-1} \).

The double peak shows that the snaking state corresponds to a bound state in the effective magnetic potential which in no longer the ground state, but the first excited state, i. e. with one extra oscillation. The same type of state is the one marked with ▲, though it is different from the former by the spin state.

V. CYLINDER OF FINITE LENGTH

As a second model of a cylinder, perhaps more realistic for the experimental research, we consider one with a finite length \( L \). We impose hard wall boundaries in the \( z \) direction at \( z = 0 \) and \( z = L \) and hence the previous axial plane waves \( \langle z | k \rangle \) must be replaced by the discrete set

\[
\langle z | p \rangle = \sqrt{\frac{2}{L}} \sin \frac{p\pi z}{L}, \quad p = 1, 2, \ldots \quad (17)
\]

Obviously, in this case the matrix elements of the total Hamiltonian couple together all degrees of freedom, i. e. the angular, the axial, and the spin motions. However, for regions far from the edge we expect results comparable with the infinite cylinder if \( R \ll L \). An effective potential for the snaking states cannot be easily defined, but the presence of such states can still be identified.

The matrix elements of the Hamiltonian with boundary conditions are shown in the Appendix. We performed numerical calculations for a cylinder of length \( L = 336 \text{ nm} \) and a ten times smaller radius \( R = 33.6 \text{ nm} \). The number of electrons on the surface is \( N = 90 \) or \( N = 10 \), as further specified. \( N = 90 \) corresponds to a surface density of \( 1.27 \times 10^{11} \text{ cm}^{-2} \), which is realistic. The number of electrons can actually be controlled by gates attached to the surface of the cylinder.\(^12\) The Rashba and Dresselhaus SOI strengths are the same as for the infinite cylinder, 20 and 3 meV nm, respectively. The numerical results were convergent for a number of states in the basis up to 2738.

We first show in Fig. 8(a) the density of electrons around the circumference, in a point situated somewhere in the middle of the length, at \( z = 160 \text{ nm} \), for \( N = 90 \) electrons and three values of the magnetic field strength, \( B = 0.05, 0.16, \) and \( 2 \text{ T} \). Indeed, the fluctuations of the charge density increase with the magnetic field. At \( B = 0.16 \text{ T} \) the density profile is very similar to the result for the infinite cylinder shown in Fig. (b). Increasing the strength of the magnetic field to \( B = 2 \text{ T} \) we notice

the density dips at angles corresponding to the N and S poles (\( \varphi = 0, \pi, 2\pi \)). If we want to reach the conditions for total depletions of those regions we need to increase the magnetic field or lower the Fermi energy by reducing the electron density. In order to maintain a moderate computational effort we reduced the number of electrons to \( N = 10 \), and show in Fig. 8(b) the resulting density profiles for \( B = 2 \) and \( 4 \text{ T} \).

In Fig. 8(a) we show the particle density in a location closer to the end of the cylinder, at \( z = 59 \text{ nm} \). Obviously in this case the density profile may look totally different from the results in the bulk, because of the interference of the edge states, and because \( \ell_B \) becomes comparable to the distance to the edge. But what is in particular interesting in this case is the left-right asymmetry of the electron density relatively to the NS (or \( x \)) axis, or around \( \varphi = \pi \) and \( \varphi = 0 \). The reason for this is the presence of both types of SOI. A similar effect has been discussed in the recent literature for quantum rings in a magnetic field perpendicular to the plane of the ring.\(^35–38\) The combination of the two types of SOI breaks the circular symmetry in the spin space and leads to the deformation of both charge and spin densities. In

![Graph showing electron density](image-url)

FIG. 8. (Color online.) Electron density calculated somewhere in the middle of the cylinder, at \( z = 160 \text{ nm} \) from one edge. (a) With \( N = 90 \) electrons maxima are observed in the East and West regions where snaking orbits are formed and become more pronounced when the magnetic field increases. (b) With \( N = 10 \) electrons the Fermi energy decreases and so the N and S regions may become totally unpopulated for a magnetic field strength like \( B = 4 \text{ T} \).
FIG. 9. (Color online.) Electron density at \( z = 59 \) nm from the edge of a 350 nm long cylinder. The number of electrons is \( N = 90 \). (a) With both types of SOI the charge distribution is not E-W symmetric. (b) A similar or even more pronounced asymmetry is obtained for the spin projection along the cylinder, \( S_z \). At \( B = 2 \) T almost all spins are polarized along the field direction and so the spin distribution is almost flat.

In Fig. 9(b) we also show the spin density corresponding to the spin projection along the wire \( S_z \). As already mentioned in Sec. IV for the infinite wire the spin may tilt in the \( z \) direction only because of the Dresselhaus SOI. The situation is different for the finite cylinder where the electrons have a mixed circular and longitudinal motion. Rashba SOI tends to tilt the spin lateral to the direction of motion and therefore both in the \( y \) and (especially at the edges) in the \( z \) directions. In addition the asymmetry induced by the double SOI is even more pronounced for \( S_z \) than for the electron density.

Finally, in Fig. 10 we illustrate the equilibrium current density on the surface of the cylinder without SOI (left) and with SOI (right) \( B = 2 \) T. Cyclotron and snaking orbits coexist.

FIG. 10. (Color online.) Equilibrium current density on the surface of the cylinder without SOI (left) and with SOI (right) \( B = 2 \) T. Cyclotron and snaking orbits coexist.

VI. CONCLUSIONS

We calculated electronic quantum states on a surface of a cylinder of radius of 30-100 nm having in mind a core-shell nanowire. We used a uniform magnetic field transverse to the cylinder and we emphasized on the snaking orbits created on the lateral sides of the surface. To our knowledge other studies of such a system did not focus on this aspect. For a strong magnetic field, like a few tesla, when the magnetic length is smaller than the radius of the cylinder, the energy spectrum can be considered a combination of locally closed cyclotron orbits and long, by skewing up the closed loops. The snaking orbits may also be seen as very large loops of current surrounding the whole cylinder, which would correspond to large, special cyclotron orbits associated with a very low net magnetic field experienced on the average by the electrons trapped along the zero-radial-field lines.
open, snaking states. If the Fermi energy is sufficiently low, corresponding to realistic electron densities of the order of $10^{11}$ cm$^{-2}$, the distribution of the electrons around the circumference of the cylinder may totally vanish at the top and bottom regions and the electrons will concentrate on the sides of the surface. This highly anisotropic distribution may have interesting effects on the magnetoresistance and possibly on other transport or optical properties of core-shell nanowires whenever the local electron density or the local current density plays a role. For example if such a wire can function as a nano antenna then the directivity may be easily controlled by an external magnetic field.

We also calculated the anisotropic spin distribution around the cylinder at these high fields in all spatial directions, using standard models for the Rashba and Dresselhaus spin-orbit interactions. In particular, for a cylinder of finite length, their combined effect may break the left-right symmetry of the charge and spin distributions, relatively to the direction of external magnetic field, an effect known for quantum rings in perpendicular magnetic field.

For the finite cylinder, with hard-wall boundary conditions in the $z$ direction, the basis is $|m\ell_p\rangle$ with integer $p$. In this case we obtain:

\begin{equation}
\left\{ \begin{array}{l}
\frac{E_0}{2} \left[ m^2 + \left( kR \right)^2 \right] \delta_{m,m'} - \frac{i \hbar \omega_c k R}{2 \ell_B} \delta_{m,m'+1} + \frac{\hbar \omega_c R^2}{8 \ell_B^2} \left( \delta_{m,m'} - \delta_{m,m'+2} \right) \\
-2i \hbar \omega_c \frac{R^2}{L} \delta_{m,m'+1} \delta_{s,s'} + H.c.
\end{array} \right. \quad \text{if } p = p', \]
\end{equation}

\begin{equation}
\left\{ \begin{array}{l}
\frac{E_0}{2} \left[ m^2 + \left( kR \right)^2 \right] \delta_{m,m'} \delta_{s,s'} + \frac{\alpha R}{4 \ell_B} \left( (1-s) \delta_{m,m'+2} - \delta_{m,m'} \right) \delta_{s,s'} + H.c.
\end{array} \right. \quad \text{if } p + p' \text{ odd , otherwise .}
\end{equation}

The matrix elements of the Zeeman Hamiltonian are now

\begin{equation}
\langle m' \ell_p' s' | H_Z | m \ell_p s \rangle = -\frac{1}{2} g_{\text{eff}} \mu_B B \delta_{m,m'} \delta_{s,s'} , \quad \text{if } m' = m \text{ and } \ell_p = \ell_p' ,
\end{equation}

and the SOI terms become

\begin{equation}
\langle m' \ell_p' s' | H_R | m \ell_p s \rangle = \left\{ \begin{array}{l}
-\frac{\alpha R}{4 \ell_B} \left[ m^2 + \left( \frac{p \ell_p R_s^2}{L} \right)^2 \right] \delta_{m,m'} \delta_{s,s'} + \frac{\alpha R}{4 \ell_B} \left( (1-s) \delta_{m,m'+2} - \delta_{m,m'} \right) \delta_{s,s'} + H.c. \quad \text{if } p = p', \\
\frac{\alpha L}{p \ell_p' - p \ell_p} (1-s) \delta_{m,m'+1} \delta_{s,s'} + H.c. \quad \text{if } p + p' \text{ odd , otherwise .}
\end{array} \right.
\end{equation}
\begin{equation}
\langle \text{mps} \mid H_D \mid m'p's' \rangle = \begin{cases} 
\frac{i}{2} \frac{\alpha B}{\epsilon} (1 - s)(2m - 1) \delta_{m,m'+1}\delta_{s,s'} + \frac{i\alpha B}{2}\delta_{m,m'+1}s\delta_{s,s'} + \text{H.c.} & \text{if } p = p', \\
2i \frac{\alpha B}{\epsilon^2 - p^2} \delta_{m,m'} \delta_{s,s'} + \text{H.c.} & \text{if } p + p' \text{ odd}, \\
0 & \text{otherwise}.
\end{cases}
\tag{A.8}
\end{equation}

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