Finiteness of Soft Terms in
Finite N=1 SUSY Gauge Theories

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Abstract

Recently proposed relations between the renormalization group functions of the soft supersymmetry breaking terms and those of a rigid theory allow one to consider a possibility of constructing a totally all loop finite N=1 SUSY gauge theory, including the soft SUSY breaking terms. The requirement of finiteness, which can be satisfied in previously constructed finite SUSY GUTs, imposes some constraints on the SUSY breaking parameters which, in the leading order, coincide with those originating from the supergravity and superstring-inspired models. Explicit relations between the soft terms, which lead to a completely finite theory in any loop order, are given.

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1 Introduction

In the recent papers \cite{1, 2}, the relations between the soft term renormalizations and those of an unbroken SUSY gauge theory have been derived. It has been shown that the soft term renormalizations are not independent, but can be calculated from the known renormalizations of a rigid theory with the help of the differential operators. The explicit form of these operators has been found in a general case and in some particular models like SUSY GUTs or the MSSM.

As an application of the above mentioned relations we consider a possibility of constructing totally finite supersymmetric theories including the soft breaking terms. This problem has already been discussed several times \cite{3, 4}. In particular, it has been found that imposing some condition on the soft terms (see below) one can reach complete one and two-loop finiteness. This is similar to the finiteness of the rigid SUSY theories themselves where imposing conditions on the particle content and the Yukawa couplings one can reach finiteness in one and two loops \cite{3}. To go further, one has to fine-tune the Yukawa couplings order by order in perturbation theory \cite{3, 4, 5}. We will show below that the same procedure works for the softly broken theory as well. By choosing the soft terms in a proper way one can reach complete all loop finiteness. Moreover, there is no new fine-tuning. The soft terms are fine-tuned in exactly the same way as the corresponding Yukawa couplings.

2 Soft SUSY Breaking and Renormalization

Consider an arbitrary $N = 1$ SUSY gauge theory with unbroken SUSY. The Lagrangian of a rigid theory is given by

$$
L_{\text{rigid}} = \int d^2 \theta \frac{1}{4g^2} \text{Tr} W^\alpha W_\alpha + \int d^2 \bar{\theta} \frac{1}{4g^2} \text{Tr} \bar{W}^\alpha W_\alpha.
$$

(1)

$$
+ \int d^2 \theta d^2 \bar{\theta} \bar{\Phi}^i (e^V)^i_j \Phi_j + \int d^2 \theta \ W + \int d^2 \bar{\theta} \ \bar{W},
$$

where $W^\alpha$ is the field strength chiral superfield defined by

$$
W^\alpha = D^2 \left(e^{-V} D^\alpha e^V\right),
$$

and the superpotential $W$ has the form

$$
W = \frac{1}{6} \lambda^i_j k \Phi_i \Phi_j + \frac{1}{2} M^{ij} \Phi_i \Phi_j.
$$

(2)

To perform the SUSY breaking, which satisfies the requirement of ”softness”, one can introduce a gaugino mass term as well as cubic and quadratic interactions of the scalar superpartners of the matter fields. They should not break a gauge invariance.

The soft SUSY breaking term satisfying these requirements can be written as \cite{9}

$$
- L_{\text{soft-breaking}} = \frac{m_A}{2} \lambda \lambda + \frac{m_A}{2} \bar{\lambda} \bar{\lambda},
$$

$$
+ \left[ \frac{1}{6} A^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} B^{ij} \phi_i \phi_j + h.c. \right] + (m^2)^{ij} \phi_i^* \phi^j,
$$

(3)
where $\lambda$ is the gaugino field and $\phi_i$ is the lower component of the chiral matter superfield.

The key point in establishing the relation between the soft term renormalizations and those of a rigid theory is the possibility to rewrite the soft terms in terms of superfields \[9\]. To do this, let us introduce the external spurion superfields $\eta = \theta^2$ and $\bar{\eta} = \bar{\theta}^2$, where $\theta$ and $\bar{\theta}$ are the Grassmannian parameters \[10\]. The softly broken Lagrangian can then be written as \[11\]

$$L_{soft} = \int d^2\theta \left[ \frac{1}{4g^2} (1 - 2\mu\theta^2) \text{Tr} W^\alpha W_\alpha + \int d^2\bar{\theta} \left( \frac{1}{4g^2} (1 - 2\bar{\mu}\bar{\theta}^2) \text{Tr} \bar{W}^\alpha \bar{W}_\alpha \right) + \int d^2\theta \left[ \frac{1}{6} (\lambda_{ijk} - A_{ijk} \eta) \Phi_i \Phi_j \Phi_k + \frac{1}{2} (M^{ij} - B^{ij} \eta) \Phi_i \Phi_j \right] + \text{h.c.} \right]$$

The external spurion superfield can be considered as a vacuum expectation value of a dilaton superfield emerging from supergravity, however, we will not explore this fact in further discussion.

As has been shown in Ref. \[2\] the following statement is valid:

**Statement 1** Let a rigid theory \[1\, 2\] be renormalized via introduction of the renormalization constants $Z_i$, defined within some minimal subtraction massless scheme. Then, a softly broken theory \[4\] is renormalized via introduction of the renormalization superfields $\tilde{Z}_i$ which are related to $Z_i$ by the coupling constants redefinition

$$\tilde{Z}_i(g, \lambda, \bar{\lambda}) = Z_i(g^2, \bar{\lambda}, \bar{\lambda}),$$

where the redefined couplings are

$$g^2 = g^2 (1 + \mu \eta + \bar{\mu} \bar{\eta} + 2\mu \bar{\mu} \eta \bar{\eta}), \quad \eta = \theta^2, \quad \bar{\eta} = \bar{\theta}^2,$$

$$\tilde{\lambda}_{ijk} = \lambda_{ijk} - A_{ijk} \eta + \frac{1}{2} (\lambda_{njk} (m^2)_n^i + \lambda_{ink} (m^2)_n^j + \lambda_{ijn} (m^2)_n^k) \eta \bar{\eta},$$

$$\tilde{\bar{\lambda}}_{ijk} = \bar{\lambda}_{ijk} - \bar{A}_{ijk} \bar{\eta} + \frac{1}{2} (\bar{\lambda}_{njk} (m^2)_j^n + \bar{\lambda}_{ink} (m^2)_j^n + \bar{\lambda}_{ijn} (m^2)_k^n) \eta \bar{\eta}.$$
The Rigid Terms & The Soft Terms

| \( \beta_{\alpha_i} = \alpha_i \gamma_{\alpha_i} \) | \( \beta_{m_{\alpha_i}} = D_1 \gamma_{\alpha_i} \) |
| \( \beta^i_M = \frac{1}{2} (M^{i\dot{u}} \gamma_{\dot{u}}^j + M^{i\dot{j}} \gamma_{\dot{j}}^l) \) | \( \beta_{m_{M_i}} = \frac{1}{2} (B^{i\dot{u}} \gamma_{\dot{u}}^j + B^{i\dot{j}} \gamma_{\dot{j}}^l) \) |
| \( \beta^{ijk} = \frac{1}{2} (y^{ijk} \gamma_{\dot{k}}^l + y^{ik\dot{k}} \gamma_{\dot{l}}^j + y^{ijk} \gamma_{\dot{j}}^l) \) | \( \beta^{ijk} = \frac{1}{2} (A^{ijk} \gamma_{\dot{k}}^l + A^{i\dot{k}j} \gamma_{\dot{j}}^l + A^{ik\dot{j}} \gamma_{\dot{j}}^l) \) |

\[
D_1 = m_{A_i} \alpha_i \frac{\partial}{\partial \alpha_i} - A^{ijk} \frac{\partial}{\partial y^{ijk}}, \quad \tilde{D}_1 = m_{A_i} \alpha_i \frac{\partial}{\partial \alpha_i} - A^{ijk} \frac{\partial}{\partial \tilde{y}^{ijk}}
\]

\[
D_2 = \tilde{D}_1 D_1 + m_{A_i} \alpha_i \frac{\partial}{\partial \alpha_i} \left( \frac{1}{2} (m^2)^2 \left( y^{nbc} \frac{\partial}{\partial y^{abc}} + y^{nbc} \frac{\partial}{\partial y^{bac}} + y^{nbc} \frac{\partial}{\partial y^{bca}} + y^{bca} \frac{\partial}{\partial y^{bca}} \right) \right)
\]

where to simplify the formulae, we use the following notation:

\[ \alpha_i = \frac{g_i^2}{16 \pi^2}, \quad y^{ijk} = \lambda^{i\dot{j}k}/4\pi, \quad y_{ijk} = \bar{\lambda}_{i\dot{j}k}/4\pi, \quad A^{ijk} = \tilde{A}^{i\dot{j}k}/4\pi, \quad A_{ijk} = \tilde{A}_{ij\dot{k}}/4\pi. \]

### 3 Renormalization of the Soft Terms in SUSY GUTs

The general rules described in the previous section can be applied to any model, in particular to a SUSY GUT. In the case when the field content and the Yukawa interactions are fixed, it is more useful to deal with numerical rather than with tensor couplings. Rewriting the superpotential \([2]\) and the soft terms \([3]\) in terms of group invariants, one has

\[
\mathcal{W}_{SUSY} = \frac{1}{6} \sum_a y_a \lambda^{i\dot{j}k} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \sum_b M_b h^{i\dot{j}} \Phi_i \Phi_j,
\]

and

\[
-\mathcal{L}_{soft} = \left[ \frac{1}{6} \sum_a \mathcal{A}_a \lambda^{i\dot{j}k} \phi_i \phi_j \phi_k + \frac{1}{2} \sum_b \mathcal{B}_b h^{i\dot{j}} \phi_i \phi_j + \frac{1}{2} m_{A_i} \lambda_j \lambda_j \Phi - \text{h.c.} \right] + (m^2)^2 \phi_{i\dot{i}} \phi_{j\dot{j}}.
\]

where we have introduced numerical couplings \(y_a, M_b, \mathcal{A}_a\) and \(\mathcal{B}_b\).

Usually, it is assumed that the soft terms obey the universality hypothesis, i.e. they repeat the structure of a superpotential, namely

\[
\mathcal{A}_a = y_a A_a, \quad \mathcal{B}_b = M_b B_b, \quad (m^2)^2 = m_i^2 \delta^i_j.
\]

Thus, we have the following set of couplings and soft parameters:

\[ g_j, y_a, M_b, A_a, B_b, m_i^2, m_{A_i}. \]
Then, the renormalization group \( \beta \) functions of a rigid theory \(^{[12]}\) look like (we assume the diagonal renormalization of matter superfields)

\[
\begin{align*}
\beta_{\alpha_j} &= \beta_j \equiv \alpha_j \gamma_{\alpha_j}, \\
\beta_{y_a} &= \frac{1}{2} y_a \sum_i K_{ai} \gamma_i, \quad \sum_i K_{ai} = 3, \\
\beta_{M_b} &= \frac{1}{2} M_b \sum_i T_{bi} \gamma_i, \quad \sum_i T_{bi} = 2,
\end{align*}
\]

where \( \gamma_i \) is the anomalous dimension of the superfield \( \Phi_i \), \( \gamma_{\alpha_j} \) is the anomalous dimension of the gauge superfield (in some gauges) and numerical matrices \( K \) and \( T \) specify which particular fields contribute to a given term in eq.(9).

To get the renormalization of the soft terms, one has to apply the formulae of the previous section. In terms of numerical couplings they are simplified.

The renormalizations of the soft terms are expressed through those of a rigid theory in the following way:

\[
\begin{align*}
\beta_{m_{A_j}} &= D_1 \gamma_{\alpha_j}, \\
\beta_{A_a} &= -D_1 \sum_i K_{ai} \gamma_i, \\
\beta_{B_b} &= -D_1 \sum_i T_{bi} \gamma_i, \\
\beta_{m_{i}^2} &= D_2 \gamma_i,
\end{align*}
\]

and the operators \( D_1 \) and \( D_2 \) now take the form

\[
\begin{align*}
D_1 &= m_{A_i} \alpha_i \frac{\partial}{\partial \alpha_i} - A_a Y_a \frac{\partial}{\partial Y_a}, \\
D_2 &= (m_{A_i} \alpha_i \frac{\partial}{\partial \alpha_i} - A_a Y_a \frac{\partial}{\partial Y_a})^2 + m_{A_i}^2 \alpha_i \frac{\partial}{\partial \alpha_i} + m_i^2 K_{ai} Y_a \frac{\partial}{\partial Y_a}.
\end{align*}
\]

where \( Y_a = y_a^2 \).

4 Finiteness of Soft Parameters in a Finite SUSY GUT

Consider now the application of the proposed formulae to construct totally finite softly broken theories.

For rigid N=1 SUSY theories there exists a general method of constructing totally all loop finite gauge theories proposed in refs. \(^{[3] \ [4] \ [8]}\). The key issue of the method is the one-loop finiteness. If the theory is one-loop finite and satisfies some criterion verified in one loop \(^{[12]}\), it can be made finite in any loop order by fine-tuning of the Yukawa couplings order by order in PT. In case of a simple gauge group the Yukawa couplings have to be chosen in the form

\[
Y_a(\alpha) = c_1^a \alpha + c_2^a \alpha^2 + ...,
\]
where the finite coefficients $c_n^a$ are calculated algebraically in the $n$-th order of perturbation theory. Since the one-loop finite theory is automatically two-loop finite \(^{[3]}\), the coefficients $c_2^a = 0$.

Suppose now that a rigid theory is made finite to all orders by the choice of the Yukawa couplings as in eq.\((21)\). This means that all the anomalous dimensions and the $\beta$ functions on the curve $Y_a = Y_a(\alpha)$ are identically equal to zero.

Consider the renormalization of the soft terms. According to eqs.\((15-18)\) and \((19,20)\) the renormalizations of the soft terms are not independent but are given by the differential operators acting on the same anomalous dimensions. One has either

$$\beta_{\text{soft}} \sim D_1 \gamma(Y, \alpha) = \left(m_A \alpha \frac{\partial}{\partial \alpha} - A_a Y_a \frac{\partial}{\partial Y_a}\right) \gamma(Y, \alpha), \quad (22)$$

or

$$\beta_{\text{soft}} \sim D_2 \gamma(Y, \alpha) = \left((m_A \alpha \frac{\partial}{\partial \alpha} - A_a Y_a \frac{\partial}{\partial Y_a})^2 + m_A^2 \alpha \frac{\partial}{\partial \alpha} + m_i^2 K_{ia} Y_a \frac{\partial}{\partial Y_a}\right) \gamma(Y, \alpha), \quad (23)$$

where $\gamma(Y, \alpha)$ is some anomalous dimension.

From the requirement of finiteness

$$\gamma_i(Y_a(\alpha), \alpha) = 0, \quad (24)$$

the Yukawa couplings $Y_a(\alpha)$ are found in the form \((21)\).

To reach the finiteness of all the soft terms in all loop orders, one has to choose the soft parameters $A_a$ and $m_i^2$ in a proper way. The following statement is valid:

**Statement 2** The soft term $\beta$ functions become equal to zero if the parameters $A_a$ and $m_i^2$ are chosen in the following form:

$$A_a(\alpha) = -m_A \alpha \frac{\partial}{\partial \alpha} \ln Y_a(\alpha), \quad (25)$$

$$m_i^2 = -m_A K_{ia}^{-1} \frac{\partial}{\partial \alpha} \alpha A_a(\alpha)$$

$$= m_A^2 K_{ia}^{-1} \frac{\partial}{\partial \alpha} \alpha^2 \frac{\partial}{\partial \alpha} \ln Y_a(\alpha), \quad (26)$$

where the matrix $K_{ia}^{-1}$ is the inverse of the matrix $K_{ai}$.

This statement follows from the form of the operators $D_1$ and $D_2$. After substitution of solutions \((25)\) and \((26)\) into $D_1$ and $D_2$, $D_1$ becomes a total derivative over $\alpha$ and $D_2$ becomes a second total derivative.

Indeed, consider eq.\((22)\). For $A_a$ chosen as in eq.\((25)\) the differential operator $D_1$ takes the form

$$D_1 = m_A \alpha \frac{\partial}{\partial \alpha} - A_a Y_a \frac{\partial}{\partial Y_a} = m_A \left( \frac{\partial}{\partial \alpha} \ln Y_a + \frac{\partial}{\partial \alpha} \ln Y_a \right) = m_A \frac{d}{d \ln \alpha}.$$
Hence, since on the curve \( Y_a = Y_a(\alpha) \) the anomalous dimension \( \gamma(Y_a, \alpha) \) identically vanishes, so does its derivative

\[
\frac{d}{d\ln \alpha} \gamma(Y_a(\alpha), \alpha) = 0.
\]

The situation with the operator \( D_2 \) is a bit more complicated. In this case, one has the second derivative. However, using eq.(26) one has

\[
D_2 = (m_A \alpha \frac{\partial}{\partial \alpha} - A_a Y_a \frac{\partial}{\partial Y_a})^2 + m^2_A \alpha \frac{\partial}{\partial \alpha} + m^2_i K_a Y_a \frac{\partial}{\partial Y_a}
\]

\[
= (m_A \alpha \frac{\partial}{\partial \alpha} - A_a Y_a \frac{\partial}{\partial Y_a})^2 - m_A \frac{\partial A_a}{\partial \ln \alpha} Y_a \frac{\partial}{\partial Y_a} + m_A (m_A \alpha \frac{\partial}{\partial \alpha} - A_a Y_a \frac{\partial}{\partial Y_a})
\]

\[
= m_A \frac{d}{d\ln \alpha} + m^2_A \frac{d^2}{d\ln^2 \alpha}.
\]

The term with the derivative of \( A_a \) is essential to get the total second derivative over \( \alpha \), since in the bracket the derivative \( \alpha \partial / \partial \alpha \) does not act on \( A_a \) by construction.

Like in the previous case the total derivatives identically vanish on the curve \( Y_a = Y_a(\alpha) \)

\[
(m_A \frac{d}{d\ln \alpha} + m^2_A \frac{d^2}{d\ln^2 \alpha}) \gamma(Y_a(\alpha), \alpha) = 0.
\]

The solutions (25,26) can be checked perturbatively order by order. It is not difficult to see the general combinatoric law and to check their validity. We have done this in two particular cases: a general finite SUSY theory with one Yukawa coupling and the finite SUSY SU(5) GUT with 5 Yukawa couplings considered in ref. [6]. In both cases the calculation has been performed up to three loops.

In the leading order eqs.(25, 26) give

\[
A_a = -m_A, \quad m_i^2 = \frac{1}{3} m^2_A,
\]

since \( \sum_a K^{-1}_{ia} = 1/3 \). These relations coincide with the already known ones [3] and with those coming from supergravity [13] and superstring-inspired models [14]. There they usually follow from the requirement of finiteness of the cosmological constant and probably have the same origin. Note that since the one-loop finiteness of a rigid theory automatically leads to the two-loop one and hence the coefficients \( c^2_a = 0 \), the same statement is valid due to eqs.(25,26) for a softly broken theory. Namely, relations (27) are valid up to two-loop order in accordance with [4]. In higher orders, however, they have to be modified.

The parameters \( M_b \) and \( B_b \) are not specified and are finite provided eqs.(25), (26) are satisfied. In particular GUT models, some of parameters \( B_b \) may be equal to \( A_b \) for the reason of fine-tuning during spontaneous breaking of a GUT symmetry group.
5 Conclusion

Thus, we have demonstrated how one can construct all loop finite N=1 SUSY GUT including the soft SUSY breaking terms. Remarkably that in the leading order the relations between the soft terms coincide with those of supergravity and superstring theory. In higher orders, however, one needs some fine-tuning which is given by exactly the same functions as in a rigid theory.

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