Lagrue–Gaussian vortex mode generation from astigmatic semiconductor microcavity

Kohki Nakagawa, Keisaku Yamane, Ryuji Morita, and Yasunori Toda

Department of Applied Physics, Hokkaido University, Kita-13, Nishi-8, Kita-ku, Sapporo 060-8628, Japan

E-mail: toda@eng.hokudai.ac.jp

Received February 9, 2020; revised March 2, 2020; accepted March 3, 2020; published online March 13, 2020

Lagrue–Gaussian (LG) vortex mode generations are demonstrated by employing the optical injection of a higher-order transverse mode into a vertical cavity surface emitting laser. In addition to the coherent LG injection, Hermite–Gaussian (HG) injection also enable LG mode generation, where the chirality is controllable by the HG mode angle of the injection beam. The result can be well understood when we consider the astigmatic Gouy phase shifts within the microcavity. HG induced vortex generation eases the symmetrical requirements of the cavity and thus extends flexibility as regards the design and fabrication of vortex lasers. © 2020 The Japan Society of Applied Physics

Supplementary material for this article is available online

Lagrue–Gaussian vortex modes (LG\(\ell p\)) are the transverse eigenmodes of laser cavities and are derived from the paraxial Helmholtz equation (PHE) in cylindrical coordinates, where \(\ell\) and \(p\) are azimuthal and radial indices, respectively. The \(\ell = 0\) LG beam, which is also known as an optical vortex, has a helical wavefront described by \(\exp(i\ell \varphi)\) as a function of the azimuthal coordinate \(\varphi\) around the beam axis. In addition, the azimuthal flow of the Poynting vector on the helical wavefront denotes the orbital angular momentum (OAM) carried by the beam, which is given by \(\ell h\) per photon. Thanks to these unique characteristics, studies of LG beams have led to the development of various applications. Accordingly, various types of vortex lasers have been developed that allow the direct generation of an LG beam. In particular, vortex lasers based on vertical cavity surface emitting lasers (VCSELs) have attracted considerable attention recently because of their utility for device applications. So far, modulating cavity structures such as photonic-crystal spiral phase plate, and metasurface have been demonstrated to generate optical vortices with a well-defined chiral symmetry. Controllable vortex generations have also been realized by external optical feedback using mode converter, variable phase plate, and spatial light modulator (SLM).

In contrast, Hermite–Gaussian modes (HG\(nm\)) are another set of eigenmodes in rectangular coordinates, where \(n\) and \(m\) are transversal indices (e.g. denoted by \(x\) and \(y\)). Because each of the LG and HG modes presents a complete basis set for the PHE, each LG mode can be produced by a superposition of HG modes and vice versa, thus allowing mode conversions between the modes. For example, the widely used astigmatic mode converter composed of a pair of cylindrical lenses is based on this relationship. However, for vortex lasers, the contributions of the HG modes need to be eliminated by preserving the cavity symmetry for the LG mode, which requires much more accurate and precise design and fabrication processes than those used for conventional lasers.

In this paper, we demonstrate that an alternative approach based on the astigmatism of the microcavity successfully generates LG\(61\) vortex modes from a conventional broad-area VCSEL. We utilize an optical injection technique, which is used to investigate the cavity resonances to investigate the transverse cavity modes of the VCSEL, where we promote the injection beams with various transverse modes for the excitation. In particular, we show that the HG beam injection enables LG mode generation, allowing a reduction of the symmetrical requirements of the cavity and thus extending the flexibility in design and fabrication of the vortex lasers. Throughout the paper, we focus only on the first-order LG (\(\equiv LG_6^1\)) and HG (\(\equiv HG_{6,10}\)) modes. For simplicity, we usually refer to these modes as the LG and HG modes, respectively.

Figure 1(a) shows a schematic view of the experimental setup. The transverse mode properties were investigated using an optical injection technique, where we employed commercially available broad-area VCSELs (single longitudinal mode operation at a wavelength of 780 nm) as both master and slave lasers. Each VCSEL consisted of 3 Al_{0.12}Ga_{0.88}As/Al_{0.30}Ga_{0.70}As quantum wells between two distributed Bragg reflectors for \(\lambda\)-cavity and had a large aperture size (~10 \(\mu\)m in diameter) that was responsible for generating higher-order transverse modes. Throughout the measurements described in this paper, the injection current of the slave VCSEL was set at 1.6 mA, which is just below the threshold current.

To prepare an injection beam with a well-defined LG or HG mode, the output of the master laser was set at a fundamental Gaussian beam by propagation through a single-mode fiber (SMF), and then converted into a linearly polarized LG or HG beam, where the polarization was set parallel to the lasing polarization of the slave VCSEL by a half-wave plate (HWP) or linear polarizer (LP). The LG mode is generated from a zero-order vortex half-wave retarder (VHWR) if we introduce a circularly polarized Gaussian beam. Here, the azimuthally rotated fast axis in the VHWR provides an azimuthal rotating phase difference and generates an LG beam. In this scheme, we can switch the chirality of the vortex by changing the orientation of the quarter-wave plate (QWP), i.e. by changing the direction of the circular polarization. We can also obtain an axisymmetrically polarized beam from the VHWR when we...
introduce a linearly polarized Gaussian beam. This beam is converted into the HG beam after passing through a LP. With the HG mode, we can select the direction of the phase variation (HG angle, $\theta$) by rotating the HWP. Although the injection power is as weak as less than 0.1 mW, we observed lasing from the slave laser at the resonances [Fig. 1(b)].

For the resonance spectrum analysis, we swept the laser frequency ($\nu$) by changing the temperature of the slave VCSEL. The transverse mode analysis of the output beam was carried out with OAM-resolved spectroscopy using a SLM. The output of the slave VCSEL was introduced onto a computer generated hologram grating on the SLM and resolved into the OAM components with a topological charge $\ell$. We also evaluated the output beam intensity profile with a CCD camera.

A typical cavity resonance spectrum of the VCSEL is shown in Fig. 1(b). The resonance condition is determined by the total optical phase within one round trip of the cavity ($2L$), which should be equal to an integral ($N$) multiples of $2\pi$: $\Delta \nu = k \cdot 2L + 2\Phi_G^{\text{mode}}$. Here, $k = 2\pi \nu / c$ is the scalar wave number, and the Gouy phase $\Phi_G^{\text{mode}}$ is typically expressed by $\Phi_G^{\text{mode}} = -(n + m + 1)\phi_G$ for HG$_{nm}$ mode using the Gouy phase of the fundamental Gaussian mode $\phi_G$.

The resonance frequency is thus defined by

$$\Delta \nu = \frac{k \cdot 2L}{c} + 2\pi \nu / c,$$

where $k$ is the scalar wave number and $\nu$ is the laser frequency. The resonance frequency is thus defined by the relationship between the cavity length and the optical path difference caused by the Gouy phase.

Fig. 1. (Color online) (a) Schematic illustration of the experimental setup. The transverse mode of the (slave) VCSEL is analyzed by using the optical injection of LG or HG mode derived from another (master) VCSEL of the same type; SMF: single-mode fiber, ISO: isolator, HWP: half-wave plate, QWP: quarter-wave plate, GTP: Glan-Thompson polarizer, VHWR: vortex half-wave retarder, LP: linear polarizer. We employ OAM-resolved spectroscopy provided by a SLM and a charge coupled device (CCD) camera to analyze the OAM mode property of the output beam from the slave VCSEL. (b) Cavity resonances and their transverse mode profiles obtained from a frequency scan of the slave VCSEL. Comparable results are also obtained by the frequency scan of the master VCSEL. The detuning frequency ($\Delta \nu$) is defined as the difference from the fundamental Gaussian resonance.

Fig. 2. (Color online) (a), (b) The results of (right) resonance spectra of the OAM-resolved output beam intensities together with (left) the transverse intensity profile of the injection beam. $\Delta \nu$ covers the HG resonances and the dominant 3 OAM components ($\ell = 0, \pm 1$) are plotted. The pure OAM signal located outside of the resonance is due to the reflection of the injection LG beam. (c)–(h) OAM spectra of the output beam and the transverse intensity profiles (inset) at $\Delta \nu$ labeled in (a) and (b), where (c) and (f) are at the HG$_x$ resonance while (e) and (h) at the HG$_y$ resonance.

© 2020 The Japan Society of Applied Physics
\[ \nu_{N\text{um}} = \frac{c}{2L} \left[ N + (n + m + 1) \frac{\phi_G}{\pi} \right]. \]  

The upper images in Fig. 1(b) show the transverse mode profiles at the resonances, from which we can attribute the resonances to the fundamental Gaussian and HG modes, respectively. In the HG modes, the lift of the degeneracy is clearly visible, indicating the presence of the residual astigmatism of the VCSEL cavity. \(^2\) In this case, the HG resonances are assigned to the orthogonal HG modes (HG\(_x\) and HG\(_y\)) associated with the principal axes of the astigmatism. By using corresponding astigmatic Gouy phases \(\phi_{Gx}\) and \(\phi_{Gy}\), Eq. (1) can be rewritten as

\[ \nu_{N\text{um}} = \frac{c}{2L} \left[ N + \left( n + \frac{1}{2} \right) \frac{\phi_{Gx}}{\pi} + \left( m + \frac{1}{2} \right) \frac{\phi_{Gy}}{\pi} \right]. \]  

Each component is thus characterized by the orthogonal transverse mode index and hereafter they are referred to as HG\(_x\) and HG\(_y\), respectively. The frequency shifts from the fundamental Gaussian resonance are given by \(\Delta \nu_{(1\ell)} = \nu_{\text{FSR}} \phi_{Gx,y} \pi\), where \(\nu_{\text{FSR}}\) denotes the free spectral range (FSR) of the cavity. The splitting of HG\(_{x,y}\) is thus proportional to the difference between the astigmatic Gouy phase shifts.

Figure 2 shows the LG beam injection results, where (a) and (b) are resonance spectra of the output beam induced by the LG beams with \(\ell = \pm 1\), respectively. The cross-sectional intensity profiles of the injection beams are also shown in each figure. In the frequency scans of the slave VCSEL, the transverse output mode intensity is resolved into the dominant 3 OAM components (\(\ell = 0, \pm 1\)), where the \(\ell = 0\) component indicates the background level. Figures 2(c)–2(e) are OAM spectra excited by the LG beam with \(\ell = +1\) and the corresponding output mode profiles obtained at three typical detuning frequencies labeled (c)–(e) in (a), where (c) and (e) correspond to the HG\(_x\) and HG\(_y\) resonances, respectively. The identifications of the orthogonal HG resonances are confirmed by the intensity distributions [insets in (c) and (e)]. Figures 2(f)–2(h) show the results obtained by the LG injection with \(\ell = -1\). In both LG injections with \(\ell = \pm 1\), the doughnut-like LG vortex mode generation emerges in between the HG\(_x\) and HG\(_y\) resonances [Figs. 2(d) and 2(g)].

The LG mode generation at the intersection of the orthogonal HG resonances is reasonable if we remember the relationship between the LG and HG modes \(^{2,3}\) given by

\[ \nu_{N\text{um}} = \frac{c}{2L} \left[ N + (n + m + 1) \frac{\phi_G}{\pi} \right]. \]  



Fig. 3. (Color online) (a), (b) Results for (left) transverse intensity profile of the y-like HG injection beam and (right) frequency scan of the OAM-resolved output beam, where the HG mode angle (\(\theta\)) of the injection beam is slightly rotated around the y-axis between (a) and (b). (c)–(h) OAM spectra and (inset) transverse intensity profiles of the output beam at \(\Delta \nu\) labeled in (a) and (b). The LG vortex modes with \(\ell = \pm 1\) are observed at the HG\(_x\) resonances (c) and (f), respectively, while the inherent HG\(_y\) mode is generated at the HG\(_y\) resonance in both (e) and (h). (i), (j) As in (a), (b) but using a z-like HG beam for the injection. The LG vortex modes with \(\ell = \pm 1\) emerge at the HG\(_y\) resonance.
Indeed, the LG mode with opposite chirality is generated by changing the chirality of the injected LG beam [Figs. 2(d) and 2(g)]. In each case, the LG\(^{\pm 1}\) mode dominance \((L^\pm)\) greater than 99\% is achieved. Here, we estimate \(L^\pm = I^\pm/(I^+ + I^-)\) using the LG\(^{\pm 1}\) mode intensities \((I^\pm)\) after subtracting the \(l = \pm 1\) mode component as a background of the first-order resonance. It is important to note that with \(H_G\) and \(H_G\) in a degenerate condition, LG modes with \(l = \pm 1\) are also degenerate. Therefore, the chirality (sign of the phase difference \(\pi/2\)) must be selected to generate the LG mode with a distinct sign, and in our system this is realized by the coherent OAM transfer from the injection LG to the VCSEL cavity mode. Also note that the signs of the output LG are opposite to those of the injection LG. This is due to a \(\pi/2\) phase shift of the radiation within the cavity relative to the injection beam.

Next, we consider the HG beam injection. For the first-order HG mode, the orientation of the phase variation (HG angle, \(\theta\)) is selectable (see the mode converter of the HG in Fig. 1). We first show the results of \(y\)-like HG mode excitation in Figs. 3(a) and 3(b). Surprisingly, the LG mode generations occur at the HG resonance labeled (c) and (f). In Fig. 3(a), the \(l = -1\) component is enhanced while the \(l = +1\) component is suppressed at the HG resonance. The generation of the LG mode with \(l = -1\) (\(LG^-\)) is also confirmed in Fig. 3(c). Moreover, as shown in Figs. 3(b) and 3(f), the LG mode with opposite chirality \(l = +1\) (\(LG^+\)) is generated by slightly rotating \(\theta\) around the \(y\)-axis as shown in the inset in Fig. 3(b). In contrast, the HG resonance shows the inherent HG mode generation, which remains virtually unchanged when \(\theta\) is rotated in (c) and (h).

The frequency scans of the OAM-resolved intensities obtained by \(x\)-like HG beam excitation are shown in Figs. 3(i) and 3(j). In contrast to the case of \(y\)-like HG beam excitation [Figs. 3(a) and 3(b)], LG mode generation occurs at the HG resonance, the chirality of which is selected by rotating \(\theta\) around the \(x\)-axis; Fig. 3(i) for \(LG^+\) and (j) for \(LG^-\), respectively. The HG resonance at this time shows the inherent HG mode generation.

At the cavity resonance, the modal gain is given by the transverse overlap between the injection beam \([U(x, y)]^2\) and resonance mode \(g_r(x, y)\), which is expressed by

\[
g(x, y) = \int g_r(x, y) |U(x, y)|^2 \, dx \, dy.
\]  

It is therefore reasonable that the \(x(y)\)-like HG beam excitation efficiently generates the HG\(_{xy}\) mode at the HG\(_{xy}\) resonance. On the other hand, owing to the transverse orthogonality of the HG\(_{xy}\) modes, the \(x(y)\)-like HG beam induced HG\(_{xy}\) mode generation is less efficient around the HG\(_{xy}\) resonance (anti-resonance). Instead, since HG\(_{xy}\) resonance originally includes the LG\(^{\pm 1}\) modes as mentioned by Eq. (3), the HG injection beam stimulates the LG mode generation around the anti-resonance axis. Here, the chirality must be chosen to complete the LG mode generation, which can be realized by astigmatic cavity Gouy phase shifts, as discussed in detail later.

Figures 4(a) and 4(b) show the polar plots of the LG\(^{\pm 1}\) mode dominances \(L^\pm\) (black squares for \(LG^+\) and magenta circle for \(LG^-\)) as a function of the \(\theta\) value of the injection HG beam at the HG\(_x\) and HG\(_y\), respectively. The solid lines indicate the resonance \([x\text{-axis in (a) and } y\text{-axis in (b)}]\) and anti-resonance \([y\text{-axis in (a) and } x\text{-axis in (b)}]\). The \(L^\pm(\theta)\) in (a) and (b) show individual uniaxial anisotropies, which are inclined nearly parallel (perpendicular) to the anti-resonance (resonance) axes. The LG\(^{\pm 1}\) mode generations are realized at the peak directions indicated by the dashed lines. In all cases, the mode dominance values better than 96\% are achieved. The angle between the dashed lines around the anti-resonance axes are (a) 36\(^\circ\) and (b) 45\(^\circ\).

It should be noted that distinct LG modes are obtained at specific \(\theta\)\(_{\text{HG}}\). In contrast, in the region around the resonance, the HG injection mostly generates inherent resonance HG modes. Here, the output HG mode profile remains unchanged when \(\theta\) is rotated around the resonance axis (see also the supplementary materials Visualization 1 and Visualization 2, available online at stacks.iop.org/APEX/13/042001/mmedia). On the other hand, the resonance HG mode virtually disappears around the anti-resonance (near-orthogonal HG condition between the injection and the resonance), where the gain of the LG becomes comparable to that of the HG.

LG mode generation is realized by tilting \(\theta\) from the anti-resonance axis \(y\)-axis in Fig. 4(a) and \(x\)-axis in (b)), and its chirality can be selected by changing the direction of \(\theta\) around these axes. This is reminiscent of the astigmatic mode conversion using a pair of cylindrical lenses,\(^{23}\) where the difference in the Rayleigh range between the orthogonal HG modes provides a Gouy phase difference of \(\pm \pi/2\) for generating the LG mode, and its chirality can be selected by changing the orientation of the HG angle about the lens axis. In our experiment, the astigmatic cavity Gouy phase shifts can provide a phase difference between the orthogonal HG components. Moreover, we now consider the anti-resonance condition for the HG mode, where the gain of the LG modes becomes efficient. A small tilt from the anti-resonance axis introduces a superposition of the orthogonal HG components that can satisfy the phase difference of \(\pm \pi/2\) and thus stimulate the LG mode generation with selected chirality, where the chirality of the LG mode \(\text{i.e., the sign of the phase difference } \pm \pi/2 \text{ in Eq. (3)}\) can be selected by the direction (sign) of the tilt angle.

Note that, the absolute astigmatic phase difference between the orthogonal HG modes is much smaller than \(\pi/2\) for one round trip of the cavity. The phase difference for the LG mode generation is thus provided by the accumulation of the phase difference in the cavity.

The LG mode generation enabled by the HG beam injection makes it possible to reduce the severe requirements regarding the cavity formation and symmetry of the vortex lasers, and thus extends the flexibility in relation to design and fabrication. In other words, LG mode generation based on an astigmatic cavity is promising for practical use because conventional lasers usually include astigmatism. For example, in our previous study, we demonstrated LG mode generation from a VCSEL with external optical feedback using an SLM.\(^{22}\) By optimizing the phase modulation on the SLM, we found that HG-like modulation superimposed on a fork grating is responsible for generating the LG mode, where we were not able to clarify the exact contribution of HG modulation. However, on the basis of the generation
and (b) HG lasers. Leads to an easing of the symmetry requirements for vortex HG to LG mode conversion within the VCSEL cavity and is thus essential for LG mode generation in a conventional VCSEL with an astigmatic cavity.

In conclusion, we demonstrated the LG mode generation of a VCSEL by using optical injection with LG and HG beams. The results of the LG injection showed that the vortex mode generation is induced in orthogonal HG resonance, where the chirality of the LG is controllable by the HG mode angle of the injection beam, indicating a coherent OAM transfer from the LG beam to the VCSEL cavity mode. Different from LG injection, HG injection enables LG mode generation at HG resonances, where the chirality of the LG is controllable by the HG mode angle of the injection beam. The mechanism is well understood in terms of the astigmatic cavity and is thus essential for LG mode generation in a conventional VCSEL with an astigmatic cavity.

Acknowledgments This work is supported by Japan Society for the Promotion of Science. KAKENHI (No. JP19H05826, 19H02621, 19K22138).

Fig. 4. (Color online) LG±1 mode dominances (L±(θ)) of the output transverse modes as a function of the HG mode angle (θ) at (a) HGx (Δν = 61.8 GHz) and (b) HGy (Δν = 79 GHz) resonances. The black squares and magenta circles indicate L±(θ) and L±(0), respectively.

1) L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A 45, 8185 (1992).
2) M. J. Padgett, Opt. Express 25, 11265 (2017).
3) G. Gibson, J. Courtial, M. J. Padgett, M. Vasnetsov, V. Pas’ko, S. M. Barnett, and S. Franke-Arnold, Opt. Express 12, 5448 (2004).
4) J. Wang et al., Nat. Photonics 6, 488 (2012).
5) N. Bozinovic, Y. Yue, Y. Ren, M. Tur, P. Kristensen, H. Huang, A. E. Willner, and S. Ramachandran, Science 340, 1545 (2013).
6) T. Omatsu, K. Chuo, K. Miyamoto, M. Okida, K. Nakamura, N. Aoki, and R. Morita, Opt. Express 18, 17967 (2010).