Uncertainties in Relic Density Calculations in mSUGRA

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Abstract: We compare the relic density of neutralino dark matter within the minimal supergravity model (mSUGRA) using four different public codes for supersymmetric spectra evaluation. While the predictions for the relic density of neutralinos are rather stable in most of the mSUGRA space, it is in the most physically interesting regions that large discrepancies can be observed, in particular the focus point, large tan $\beta$ and coannihilation regions.
1. INTRODUCTION

One of the most stringent constraints on supersymmetric models with R-parity conservation arises from the upper limit on the relic density of dark matter. This is particularly true with the recent precise measurements of the cosmological parameters realised by WMAP. It is therefore crucial to quantify the theoretical uncertainties that enter the calculation of the relic density of the lightest supersymmetric particle (LSP) and to see how they reflect on the allowed parameter space. We do not attempt to answer this question fully here. We will only consider one aspect: the uncertainty introduced by the calculation of the weak scale SUSY parameters using renormalization group equations (RGE) within the context of the mSUGRA model. As a measure of the theoretical uncertainty on the mSUGRA parameters, we use the four public state-of-the-art RGE codes: Isajet7.69 [1], SOFTSUSY1.8.3 [2], SPHENO2.20 [3] and Suspect2.2 [4], link them to micrOMEGAs1.2 [5] and compare estimates for the relic density. At this point no attempt is made to estimate the uncertainties that could arise directly in the calculation of the relic density itself.

2. RGE CODES AND RELIC DENSITY CALCULATIONS

A detailed study of theoretical uncertainties on the supersymmetric spectra as obtained by RGE codes was presented in [6]. It was shown that differences in masses less than a few percent are usually found, although some corners of parameter space are still difficult to tackle and can display much larger differences. The discrepancies can be traced back to the level of approximation used in the weak-scale boundary conditions. The large tan β
region and the focus point region (large $M_0$) are still subject to large theoretical errors. Both of these regions are precisely where one can find cosmologically interesting values for the relic density, $\Omega h^2 < .128$. In the focus point region, the LSP is mainly a Higgsino and annihilates efficiently into gauge bosons. At large tan $\beta$, even rather heavy neutralinos can annihilate into $b\bar{b}$ pairs via s-channel exchange of a heavy Higgs. The coannihilation region where the Next-to-Lightest supersymmetric particle (NLSP) is nearly degenerate in mass with the LSP, is another cosmologically relevant region. Although it is a priori not difficult to handle by the RGE codes, the value of the relic density depends sensitively on the mass difference between the NLSP and the LSP and even shifts of $\mathcal{O}(1)$ GeV can cause large shifts in the relic density. The other cosmologically viable mSUGRA region, the bulk region, shows a much smaller induced sensitivity upon the MSSM mass spectrum.

The link between micrOMEGAs1.2 and the RGE codes is done within the spirit of the SUSY Les Houches Accord [7]: common input values are chosen and pole masses, mixing matrices, the $\mu$ parameter and the trilinear couplings are calculated by the RGE codes. All parameters are read by micrOMEGAs1.2. The annihilation cross-sections are then evaluated at tree-level. Important radiative corrections to the Higgs widths and in particular the $\Delta m_b$ correction are taken into account.

3. RESULTS

For the numerical results as default values we have fixed $m_t = 175$ GeV, $\alpha_s(M_Z)^{\overline{MS}} = .1172$ and $m_b(m_b)^{\overline{MS}} = 4.16$ GeV. This corresponds to $m_b(M_Z)^{\overline{DR}} = 2.83$ GeV. We concentrate on the three regions where the relic density is within the WMAP range and where potentially large discrepancies can be observed: the focus point region, the large tan $\beta$ region and the coannihilation region.

3.1 Coannihilation

$M_0 = 150$ GeV, $A_0 = 0$, $\tan \beta = 10$, $\mu > 0$

The small $M_{1/2}$ region corresponds to the so-called bulk region where the bino-LSP annihilates into lepton pairs via s-channel $Z$ or Higgs exchange or t-channel slepton exchange. Here one finds very good agreement between the values of $\Omega h^2$ using the different RGE codes (see Fig. 1a) since the predicted values for slepton and neutralino masses are in good agreement (within a few GeV). The exact position of the $Z$ pole (corresponding to the big dip in $\Omega h^2$) is slightly shifted for SPHENO2.20 but the range of values of $M_{1/2}$ for which $\Omega h^2 < .128$ are basically identical. Note that the $Z$ pole region is ruled out by the LEP constraints on neutralinos within the context of mSUGRA models.

As one moves up in $M_{1/2}$, one reaches the so-called coannihilation region where the $\tilde{\tau}$ is the NLSP and is nearly degenerate with the neutralino, as in Fig. 1b. Coannihilation with the $\tilde{\tau}$, and to a lesser extent the selectron and smuon, brings the relic density in the desired range. For a given value of $M_{1/2}$, differences between the codes can reach a factor 2, the largest differences are found between SPHENO2.20 and SOFTSUSY1.8.3. However very good agreement is found between all codes when the relic density is plotted as a function of the mass difference between the LSP and the NLSP (here the $\tilde{\tau}$). All codes obtain values
of $\Omega h^2$ compatible with WMAP for mass differences $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0} \approx 4$ GeV (at the extreme left of Fig. 1b), even though the corresponding value of the neutralino mass can differ. The value of $M_{1/2}$ for which the relic density becomes compatible with WMAP varies from 670 GeV (SPHENO2.20) to 790 GeV (SOFTSUSY1.8.3), a 12% difference on $M_{1/2}$.

3.2 Focus point

$M_{1/2} = 300$ GeV, $A_0 = 0$, $\tan \beta = 10$, $\mu > 0$

In addition to the small $M_0$ (bulk/coannihilation region) where annihilation into leptons is important, the cosmologically relevant region is found at values of $M_0$ well above 1 TeV. As one approaches the region where electroweak symmetry breaking is forbidden, the $\mu$ parameter approaches zero. This means that the LSP is mainly Higgsino. This LSP can then annihilate very efficiently into gauge bosons (WW/ZZ) and to a lesser extent into $Zh$. The parameter $\mu$ is however very sensitive to the top Yukawa coupling, $h_t$ (which is also reflected in a sensitivity to the value of the top quark mass) and huge differences between codes were observed. The impact on the relic density and on the exclusion region is likewise very significant.

As can be seen in Fig. 2, all codes agree very well for $M_0 < 1$ TeV but as one gets to large values of $M_0$, more than one order of magnitude differences in $\Omega h^2$ can be found. For $m_t = 175$ GeV, only Isajet finds a large drop in the $\mu$ parameter as one moves to $M_0 \approx 3000$ GeV, this is when $\Omega h^2$ drops below the upper limit from WMAP. The other codes do not find this drop in $\mu$ and do not obtain a cosmologically interesting region for $M_0 < 4000$ GeV. These large differences between codes however are just a reflection of the
sensitivity to the top Yukawa, $h_t(M_{SUSY})$ which is proportional to $m_t$. We show in Fig. 2b, the variation of $\Omega h^2$ with $m_t$ using SOFTSUSY1.8.3 and SPHENO2.20 for $M_0 = 3000$ GeV. The value $\Omega h^2 = .128$ found in Isajet7.69 for $m_t = 175$ GeV can be reproduced in SOFTSUSY1.8.3 (SPHENO) by changing the input to $m_t = 172.2(172.5)$ GeV.

3.3 Large $\tan \beta$

$m_{1/2} = 1500$ GeV, $A_0 = 0, \tan \beta = 52, \mu > 0$

At large $\tan \beta$ the new feature is the annihilation of neutralinos into $b\bar{b}$ via heavy Higgs exchange. With the current version of the RGE codes, this is observed only for very large values of $\tan \beta$. The crucial parameter here is $M_A/2m_{\tilde{\chi}_1^0}$ which must be close to unity to provide sufficient annihilation of neutralinos. Large differences in the value of $M_A$ between the different RGE codes occur because of the sensitivity of the RGE to the bottom Yukawa as well as from taking into account higher loop effects.

As Fig. 3a shows, all 4 programs predict a large drop in the relic density when the neutralino mass gets close to $M_A/2$ although this drop occurs at much lower values of $M_{1/2}$ for SPHENO, $M_{1/2} \approx 1250$ GeV than for Isajet7.69, $M_{1/2} \approx 1750$ GeV. However, here again the results are very sensitive to the input parameters, in this case the value of the b-quark mass. For $M_{1/2} = 1300$ GeV, we find an order of magnitude shift in $\Omega h^2$ for $m_b(m_b) = 4 - 4.4$ GeV with the program SOFTSUSY1.8.3. By a slight shift of the b-quark mass we can find perfect agreement between SPHENO2.20 and SOFTSUSY1.8.3, as shown in Fig. 3b.
4. CONCLUSION

While the predictions for the relic density of neutralinos are rather stable in most of the mSUGRA space, it is in the most physically interesting regions that large discrepancies can be observed, in particular the focus point, large $\tan \beta$ and coannihilation regions. It is however reassuring to find that with the newer versions of the codes, the discrepancies in the sparticle spectra tend to be reduced. More details on the theoretical uncertainties in the evaluation of the relic density arising from the standard model parameters, $\alpha_s, m_b, m_t$, used as input in a RGE code can be found in [9].

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