On Irrelevance of Attributes in Flexible Prediction

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Abstract

This paper analyses properties of conceptual hierarchy obtained via incremental concept formation method called "flexible prediction" in order to determine what kind of "relevance" of participating attributes may be requested for meaningful conceptual hierarchy. The impact of selection of simple and combined attributes, of scaling and of distribution of individual attributes and of correlation strengths among them is investigated.

Paradoxically, both: attributes weakly and strongly related with other attributes have deteriorating impact onto the overall classification. Proper construction of derived attributes as well as selection of scaling of individual attributes strongly influences the obtained concept hierarchy. Attribute density of distribution seems to influence the classification weakly.

It seems also, that concept hierarchies (taxonomies) reflect a compromise between the data and our interests in some objective truth about the data.

To obtain classifications more suitable for one’s purposes, breaking the symmetry among attributes (by dividing them into dependent and independent and applying differing evaluation formulas for their contribution) is suggested. Both continuous and discrete variables are considered. Some methodologies for the former are considered.

Keywords: incremental concept formation; relevance of attributes; statistical and probabilistic aspects; asymmetric treatment of dependent and independent variables;

1 Introduction

Most knowledge discovery methods strive to obtain a kind of explicit (by dividing the set of observed objects into sets) or implicit (by deriving a dependence function between attributes of objects) "clustering" of objects based on a kind of "similarity" of them. There exist numerous statistical clustering methods (single linkage, k-means, ...) as well as ones elaborated within artificial intelligence community (conceptual clustering, incremental concept formation, ... and many others). Many seem to
share one common feature: complaints about impact of ”irrelevant” attributes which worsen the satisfaction with the resulting clustering.

This paper tries to shed some light into the question what that ”irrelevance” may mean. To make the presentation of our position more clear, we concentrate of the method of incremental concept formation proposed by Fisher and colleges, called ”flexible prediction”. In section 2 we recall basic concepts of this method. In section 3, recalling our own results, we demonstrate that ”relevance” means cheapness in achieving ones goals. In section 4 we demonstrate the impact of scaling of an attribute onto its ”relevance”. In sections 5,6 we show the impact of uniform probability distribution on classification process. Section 7 shows that pairs of strongly interrelated variables can dominate the overall classification. Section 8 summarizes our investigation and presents a proposal of modification of the approach to clustering tasks. The paper ends with some concluding remarks.

2 Purpose and Method of ”Flexible Prediction” Paradigm

Fisher and colleges [1, 2, 3, 4, 5] proposed a new type of clustering called ”flexible prediction”, which has been implemented in their COBWEB system (currently extended in ECOBWEB). The goal of clustering via ”flexible prediction” is to obtain such a hierarchy of concepts that the probability of prediction of sub-class membership from the value of a single parameter is maximized together with the probability of prediction of the value of the attribute from sub-class membership. The optimization criterion was: maximize the function [3]:

\[
\text{eval} = M^{-1} \left( \sum_{C_i} p(C_i) \sum_{A_j \in D} \sum_{V_{jk}} p(A_j = V_{jk} | C_i)^2 - \sum_{A_j \in D} \sum_{V_{jk}} p(A_j = V_{jk})^2 \right)
\]

with:
- \( D \) - set of discrete attributes
- \( p(C_i) \) - probability that an object belongs to Sub-category \( C_i \)
- \( p(A_j = V_{jk}) \) - probability that the discrete parameter \( A_j \) takes the value \( V_{jk} \).
- \( p(A_j = V_{jk} | C_i) \) - respective conditional probability within the sub-category \( C_i \)
- \( M \) - number of sub-categories.

The concept hierarchy is developed incrementally, with each new object in a sequence being classified on the top level into one of the existing classes or a separate one is created for it depending on which operation will maximize the above expression. The same is repeated on sublevels of the hierarchy.

Applicability for diagnosis of plant diseases was claimed [4].

Gennari et al. [5] extended this methodology to continuous attributes claiming success both for artificial and natural domain examples (”forming diagnostically useful categories”). The optimization criterion for concept hierarchy was [5]:

\[
\text{eval} = M^{-1} \left( \sum_{C_i} p(C_i) \sum_{A_j \in \mathbb{R}} \frac{1}{\sigma_{A_j | C_i}} - \sum_{A_j \in \mathbb{R}} \frac{1}{\sigma_{A_j}} \right)
\]
with: $K$ - set of continuous attributes  
$\sigma_{A_j}$ - standard deviation continuous parameter $A_j$  
$\sigma_{A_j C_i}$ - respective standard deviation within the sub-category $C_i$  
$M$ - number of sub-categories.

Gennari et al. [5] derived their formula - up to a constant factor depending on probability distribution considered - from Fisher’s one eqn(1) by substituting unconditional and conditional probabilities may be substituted by the respective probability density functions and summing with integration.

3 All Relevant versus Some Relevant

Developers of ”flexible prediction” paradigm noticed that irrelevant attributes negatively influence the quality of the obtained hierarchy. One shall however ask what does it mean that the attribute is irrelevant.

In a study, reported in [6] we assumed that a set of binary attributes $A_1,...,A_n$ (n less than 10) be relevant for a diagnostic task (previously studied using statistical methods). We made the naive assumption that if two attributes $A_i,A_j$ are relevant, then their logical conjunction $A_i \land A_j$, disjuction $A_i \lor A_j$ and negation $\neg A_j$ should also be relevant. In this way we built all the anywhere distinct boolean functions of the relevant attributes. Starting with the set of ”relevant” attributes we tried to construct the classification hierarchy of objects. It turned out that the optimization criterion eqn.(1) drove each object into a separate case unless two objects were characterized by exactly the same values of given attributes: any two objects proved to be equally similar to one another. The proof that this tendency is an intrinsic property of the flexible prediction algorithm is given in [6].

Clearly, our set of derived attributes contained such absurd functions as always true and always false ones, but even if one drops them, the situation does not improve in any way. The lesson from that study is that in order to obtain a concept hierarchy we need first to ponder some attributes from the space of primary and derived relevant attributes. One may do it on the grounds of e.g. importance for our target task (more weight for more important ones) and/or on costs of measurement of a given attribute (more weight for ”cheaper” attributes). But doing so we pre-specify the concept hierarchy we will derive from our data. And if we would like to get a different concept hierarchy, a different pondering of the same set of attributes for the very same set of data will do. We need only to outline our concept hierarchy in terms of class membership of clustered objects, and then appropriately ponder the (derived) attributes describing each class (assigning significantly larger weights to higher hierarchy levels). Hence we gain real knowledge from the data only in that case where we do attribute pondering prior to seeing the data.

By the way, this is exactly what statisticians always insist on: define statistical tests you want to perform before you start collecting the data.
4 Attributes Equally Pondered and the Effect of Rescaling

Usually we have no other choice but to decide which attributes are the basis of the clustering to be performed now. Within this set of attributes it is natural to assume the same pondering (weighting). We will consider both optimization criterion (1) and (2) to evaluate them from this point of view.

It seems that in formula (2) authors pretended to obtain only a formal and visual similarity with (1). To obtain scale-free property, however, the shape of the formula for continuous attributes should be a little bit different.

$$eval' = \mathcal{R}^{-1}\sum_{C_i} p(C_i) \sum_{A_j \in \mathcal{K}} \frac{\sigma_{A_j}}{\sigma_{A_j,C_i}} - 1 \quad (3)$$

This formula assures the same result for any linear transformation (i.e. scale) of attributes which were taken into account.

However, the need of "scale-freedom" effect is not bound to continuous attributes only.

The scaling effect for discrete attributes will be demonstrated by the following example: Let us consider the attribute $A$ taking 6 different values $v_1, v_2, v_3, v_4, v_5, v_6$. Then its contribution to the evaluation function (eqn.(1)) is:

$$eval_A = \sum_{C_i} p(C_i) \sum_{k=1}^{6} p(A = v_k|C_i)^2 - \sum_{k=1}^{6} p(A = v_k)^2 \quad (4)$$

Let us "rescale" this attribute as to obtain an attribute $B$ taking values $w_1, w_2, w_3$ as follows: $B = w_1$ iff $A = v_1$ or $A = v_6$, $B = w_2$ iff $A = v_2$ or $A = v_5$, $B = w_3$ iff $A = v_3$ or $A = v_4$. Then $B$ contributes:

$$eval_B = \sum_{C_i} p(C_i) \sum_{k=1}^{3} p(B = w_k|C_i)^2 - \sum_{k=1}^{3} p(B = w_k)^2 \quad (5)$$

$$= \sum_{C_i} p(C_i) \sum_{k=1}^{3} p(A = v_k|C_i) = v_{7-k}|C_i)^2 - \sum_{k=1}^{3} p(A = v_k|C_i) = v_{7-k})^2$$

Squaring out gives us (taking into account that probability of disjoint events equals sum of probabilities of the events):

$$eval_B = eval_A + \sum_{C_i} p(C_i) \sum_{k=1}^{3} 2p(A = v_k|C_i)p(A = v_{7-k}|C_i) \quad (6)$$

$$- \sum_{k=1}^{3} 2p(A = v_k)p(A = v_{7-k})$$

Since $p(A = v_k|C_i)$ for each $k=1,2,...,6$ can be treated as a function having as its domain the set of subcategories $\{C_i\}$, one can consider the following random variable:

$$Y_k = p(A = v_k|C_i) = y_k(I)$$

$I$ is the indicator random variable. One can check that expectation of $Y_k$

$$E(Y_k) = p(A = v_k), \quad k = 1, 2, ..., 6$$
It follows from (6) therefore that
\[ eval_A = eval_b - 2 \sum_{k=1}^{3} \text{cov}(y_k(I), y_{7-k}(I)) \] (7)

Let us consider now the reverse rescaling procedure: we start with the attribute B and afterwards we are trying to split its 3 values into 6. The intuition suggests us that if way of splitting does not take into account any information about membership of the record to certain subcategory \( C_i \) then no change should result in the evaluation function.

We model "non-informative splitting" in the following way.

First we randomly generate the record according to probabilities of subcategories (i.e. \( \{p(C_i)\} \)). Realization of the random variable \( I \) is given, therefore at this stage. From other point of view a subcategory is chosen first.

One can observe that evaluation function for discrete attributes does not depend on any one-to-one transformation of values of attributes. It follows therefore that "non-informative splitting" can have only the numeric effect if the probability
\[ p(B = w_k|C_i) \]
is split in a random way in two parts.

If the splitting is performed in the second stage of random generation procedure we always would have a negative correlation between the components
\[ p(A = v_k|C_i) + p(A = v_{7-k}|C_i) \]
of \( p(B = w_k|C_i) \).

It follows then from (7) that evaluation function for the attribute with more values is higher than for that with fewer values. All 3 covariances are negative.

Such result is difficult to take as rational. The attribute A add no classification information when comparing with B.

5 Unbounded Splitting of A Class For Uniformly Distributed Continuous Attributes

In this section we will consider somewhat ideal clustering situation in the case of continuous attributes.

Assume that each attribute \( A_j \) takes values in the interval of the length \( \Delta_j \). The behaviour of attributes is very positive and clear from the point of view of distinguishing subcategories \( C_i \). This behaviour is formalized in the following way.

Each attribute for the whole population has the uniform distribution within its interval. For the subcategories attributes take values only within intervals of the lengths which corresponds to probability of the subcategory. It means that attribute \( A_j \) for subcategory \( C_i \) has values within the interval of length:
\[ \Delta_j \cdot P(C_i) \]
For given attribute open subintervals which represent M subcategories are assumed to be disjoint. The value of attribute gives therefore the sure classification of the object (record) which corresponds to this value.

To conclude the description of the above ideal situation one can notice that distribution for subcategory (i.e., conditional distribution for records within $C_i$) must be uniform either.

The elementary calculations lead to the following simple expression of evaluation function (2) and (3):

$$eval_{in(2)} = \frac{(M - 1)2\sqrt{3}}{M} \sum_{A_j \in K} \frac{1}{\Delta_j}$$

$$eval_{in(3)} = M - 1$$

The second result for the idealized clustering situation seems to be more rational than for the first one. The proposed evaluation function depends neither on the domains of attributes nor on their number. Actually the entire clustering information is included in one attribute. All attributes are copies of each other from this point of view.

Of course the lengths $\Delta_j$ have nothing to do with precision of clustering. The precision is higher, however, for the attribute which classifies 3 subcategories than for attribute classifying 2 clusters. So it seems rational that the form (9) has stronger relationship with $M$ than (8).

## 6 Shift of Mass Between Classes of Different Density

Let us check one more aspect of attribute behavior. Let us approximate the overall attribute distribution with pieces of uniform density (e.g. a histogram). Let us examine the optimization tendency of the splitting criterion.

Let us consider the following experiment:

Let us take two "neighboring" classes $C_1, C_2$: two uniformly distributed neighboring intervals (densities $D_1$ and $D_2$ respectively) with lengths $b$ and $a$.

The contribution of these intervals to the evaluation is:

$$p(C_1) = D_1 \cdot b, p(C_2) = D_2 \cdot a, \sigma_{AC_1} = (2\sqrt{3})^{-1}b, \sigma_{AC_2} = (2\sqrt{3})^{-1}a,$$

hence:

$$M^{-1} \left( \frac{p(C_1)/\sigma_{AC_1} + p(C_2)/\sigma_{AC_2}}{1/\sigma_A} \right) =$$

$$M^{-1} \left( \left( D_1 \cdot b / ((2\sqrt{3})^{-1}b) + D_2 \cdot a / ((2\sqrt{3})^{-1}a) \right) - 1/\sigma_A \right) =$$

$$M^{-1} \left( (D_1 \cdot 2\sqrt{3} + D_2 \cdot 2\sqrt{3}) - 1/\sigma_A \right)$$

This actually means the following: the contribution of a attribute depends on the densities of intervals and not on their widths. We want next to optimize the criterion by shifting the boundary between the intervals increasing the class $C_1$ at the expense of the class $C_2$ so that $C_2$ consists only of an interval with width $a-x$ and density $D_2$, and $C_1$ consists of an interval with length $b$ and density $D_1$ and an interval $x$ with density $D_2$. It is obvious that the contribution of $C_2$ to the evaluation function will
remain the same so we need only to consider the class $C_1$. We will try to find out what value shall be taken by $x$.

For consideration of standard deviation within class $C_1$ we need to rescale the densities $D_1$, $D_2$ of subintervals $b$ and $x$ to $d_1, 2, 2$ so that $d_1 b + d_2 x = 1$. Variance of $C_1$ then equals to:

$$
\sigma^2_{AC_1} = \int_{-b}^{0} d_1 y^2 dy + \int_{0}^{x} d_2 y^2 dy - \left( \int_{-b}^{0} d_1 y dy + \int_{0}^{x} d_2 y dy - \right)^2 =
$$

$$
d_1 b^3/3 + d_2 x^3/3 - (-d_1 b^2/2 + d_2 x^2/2)^2
$$

We introduce the coefficient $q$ such that $d_2 = q d_1$, then $d_1 (b + q x) = 1$, Just $d_1 = \frac{1}{b+q x}, d_2 = \frac{q}{b+q x}$. With this substitution we get:

$$
\sigma^2_{AC_1} = 1 \frac{b^3}{b + q x^3} + q \frac{x^3}{b + q x^3} - \frac{1}{(b+q x)^2} 4 - \frac{q^2}{(b+q x)^2} 4 + 2 \frac{q}{(b+q x)^2} 4
$$

Probability of class $C_1$ is now $p_{C_1} = D_1 * b + D_2 * x$ with $D_2/D_1 = d_2/d_1 = q$, hence

$$
\frac{p_{C_1}}{\sigma_{AC_1}} = \sqrt{D_1}
$$

To maximize the quotient, the denominator, hence the squared denominator must be minimized. The derivative $d(\text{squared denominator})/dx = q * (1 - q) * (b + q x)^{-5} b^2 x (b + x)$. will be negative (hence steadily falling with increase of x) iff $q$ exceeds 1 which means $d_1 < d_2$, which means that lower density intervals will swallow higher density ones. We see that mutual prediction tends to isolate extremely narrow modes from a multimodal distribution. We have here again a lesson that discretization of the predicted attribute is essential.

7 Overfitting A Continuous Attribute

Let us consider again the discrete mutual prediction function of [4] for a selected attribute $A$ and the classifying variable $C$:

$$
\sum_{x} \sum_{y} p(A = y) * p(A = y|C = x) * p(C = x|A = y) = \sum_{x} \sum_{y} \frac{p(A = y \land C = x)^2}{p(C = x)}
$$

Let us generalize it to a continuous "classification" variable $C$ and continuous attribute $A$ connected by a bivariate normal distribution:

$$
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\pi \sqrt{1 - R^2} \sigma_x \sigma_y} \exp \left( \frac{1}{1 - R^2} \left( \left( \frac{x - \mu_x}{\sigma_x} \right)^2 - 2R \left( \frac{x - \mu_x}{\sigma_x} \right) \left( \frac{y - \mu_y}{\sigma_y} \right) + \left( \frac{y - \mu_y}{\sigma_y} \right)^2 \right) \right) dx dy =
$$

$$
= \left( 2\sigma_y \sqrt{\Pi (1 - 2R^2)} \right)^{-1}
$$

The resulting formula points at the following:
• Dependence on attribute scale \( \sigma \) - the larger the range of attribute values, the smaller attribute variance the higher its importance

• Dependence on correlation: the stronger the correlation (closer to 0.7!!!!) the higher the attribute importance.

REMARK: for correlations over 0.7 - nonsense values. \( 0.7 \times 0.7 = 0.49 \). - if C gets closer to A (higher correlation) then the term concerning A in Fisher’s evaluation function will absolutely dominate the whole expression, hence overfitting towards one of the attributes is guaranteed.

We shall conclude from this case study in "continuous classification" that strongly interrelated attributes can dominate the classification hierarchy so that they will disable it to predict values of other attributes of interest. Therefore when designing an evaluation function for a set of relevant attributes, if we want to have a balanced mutual prediction capability, correlation between the classification and each attribute of interest should negatively influence the weight of that attribute in the classification evaluation function.

8 Discussion

In this paper the flexible prediction (FP) paradigm for knowledge discovery from a set of cases has been studied. It has been demonstrated that:

• the optimization criterion of FP drives each object into a separate case unless two objects were characterized by exactly the same values of given attributes: any two objects proved to be equally similar to one another, if we take primary relevant attributes and all their derived attributes,

• for uniformly distributed continuous attributes the optimization function behaves in a not optimal way

• mutual prediction paradigm of COBWEB evaluation function tends to isolate extremely narrow modes from a multimodal distribution for continuous attributes

• the importance of an attribute in the optimization depends on its scale in an unwanted way

• the importance of an attribute in the optimization depends on its correlation with the classification criterion: the stronger the correlation the higher the attribute importance.

Therefore:

• in order to obtain a useful concept hierarchy we need first to ponder those attributes from the space of primary and derived relevant attributes, which are easily measurable and/or interesting ones
• if we want to have mutual prediction: classification \(<\rightarrow\) attribute value, we should treat both directions of ”implication” asymmetrically: On the one hand, implication classification \(<\rightarrow\) attribute value should exploit as high precision of attribute measurement as available, on the other hand the implication classification \(<\rightarrow\) attribute value should accept attribute value with as low precision as satisfactory.

8.1 A Possible Recovery

To meet the above requirements a redesign of the evaluation function for flexible prediction seems to be necessary. We shall handle below only the case of discrete attributes. First of all one shall not insists that all the attributes need to be predicted from the evaluation function. Hence the evaluation function should consist of two parts: one concerning the predicted attributes and one concerning predicted attributes.

As the contribution of predicting attributes to the evaluation function of the classification hierarchy is concerned, one shall construct a function \(f_A : A \rightarrow C\) (\(A - \) the attribute, \(C - \) the classification) in such a way as to ensure that \(p(C = f_A(A))\) (that is the probability that the attribute \(A\) predicts the classification \(C\) correctly) is maximized. Then the measure of contribution of the attribute \(A\) to the evaluation of classification would be \(p(C = f_A(A))\). Please pay attention to the fact that if we refine the attribute \(A\) (increase the precision of measurements), but this increase does not improve the quality of prediction of the class, then the value of \(p(C = f(A))\) will not change (it will not decrease). The only case when \(p(C = f(A))\) may decrease is when the number of classes in classification \(C\) is increased. In this way the symmetry, anchored in the correlation measures, between the predicting attribute and the classification variable is broken.

As the contribution of predicted attributes to the evaluation function of the classification hierarchy is concerned, one shall construct a function \(g_A : C \rightarrow A\) (\(A - \) the attribute, \(C - \) the classification) in such a way as to ensure that \(p(A = g_A(C))\) (that is the probability that the attribute \(A\) is predicted by classification \(C\) correctly) is maximized. Then the measure of contribution of the attribute \(A\) to the evaluation of classification would be \(p(A = g_A(C))\). Please pay attention to the fact that if we refine the classification \(C\) (increase the number of classes), but this increase does not improve the quality of prediction of the class, then the value of \(p(A = g_A(C))\) will not change (it will not decrease). The only case when \(p(A = g_A(C))\) may decrease is when the required precision of prediction of \(A\) is required. In this way the symmetry, anchored in the correlation measures, between the predicted attribute and the classification variable is broken.

Last not least the case of an attribute \(A\) shall be considered which shall both be predicted from the classification and if available predict the classification. We must then construct two attributes: \(A'\) and \(A''\) out of it constructing two functions: \(f_{A'} : A' \rightarrow C\) and \(g_{A''} : C \rightarrow A''\) according to the rules indicated above. The attribute \(A''\) should in general have a different precision (grid of values) that \(A'\): \(A'\) should be finer than \(A''\) so that \(A'\) is measured as precisely as possible and \(A''\) should be predicted only as precisely as necessary.
In the end, we shall consider the overall structure of the evaluation function. Please recall the fact that the flexible prediction paradigm is used in combination with incremental concept formation that is with each new incoming case the classification hierarchy is re-evaluated. We propose the following evaluation function for each step of the process:

\[
\sum_{A \in \text{Prtd}} w_A \frac{p(A = g_A(C))}{p'(A' = g'_A(C))} + w_C \sum_{A \in \text{Prting}} \frac{\max_A(p(A\text{available}) \cdot p(C = f_A(A)))}{\max_A(p(A\text{available}) \cdot p'(C' = f'_A(A)))}
\]

where \(p', C', f'\) and \(g'\) are probabilities, classification and prediction functions used on occasion of the previous case incorporated into the hierarchy, \(p(A \text{ available})\) is the probability that an attribute is available for observation, coefficients \(w\) are weights assigned to knowledge of predicted attributes and the classification, Prtd is the set of predicted attributes, Prting is the set of predicting attributes. The motivation behind the proposal is the following:

1. We maximize over probability of correct prediction of classification (weighed by probability of availability) in order to eliminate impact of irrelevant attributes: irrelevant attributes will simply not predict the classification strongly enough.
2. We relate predictability of each predicted attribute and of the classification to predictability of the previous step via quotients in order to balance the predictability among those attributes so that the classification does not tend to favor any of the predicted attributes. In this way a too strong correlation between classification variable and any of the predicted attributes at expense of low correlation with other predicted attributes is prevented.
3. By regulating proportions between weights of classification variable and predicted attributes we can refrain the classification from splitting into too much classes. This is because the \(p(C = f_A(A))\) gets lower if we split \(C\) beyond predictability from predicting attributes. On the other hand predicted attributes will always profit from split of classification because \(p(A = g_A(C))\) never gets lower with refinement of \(C\), so refinements of \(C\) are favored even if only prediction of few of predicted attributes can be improved from the refinement of \(C\).

Please notice that attachment of transformations of the space of predicting attributes (e.g. combining two attributes to one) will not deteriorate the performance of the evaluation criterion, we need only to calculate the probability of availability of the newly generated attribute.

The study carried out so far on this formula has been theoretic in nature and empirical tests on artificial data indicate necessity of development of stabilizing strategies for the starting stages of the concept formation process (e.g. replacement of maximum by "take n largest values" function). Furthermore, extensions to continuous case imply important questions of discretization strategies both for predicted and predicting attributes. However, we feel that the departure from symmetry of predicting and predicted attributes is a promising path of research, including split of attributes which are both predicted and predicting into two separately treated attributes. Also balancing of prediction of attributes from the classification variable seems to be important improvement of the flexible prediction paradigm.
We feel that usage of a kind of cost function instead of probability of availability may also be profitable.

9 Concluding Remarks

The flexible prediction paradigm for search of knowledge about relevant classification hierarchies has the following drawbacks:

- is negatively influenced by irrelevant attributes and may be badly influenced by abundance of relevant attributes,
- may be negatively influenced by increase of attribute precision,
- must be optimized for piecewise uniform continuous attributes,
- is sensitive to attribute scaling, variance and correlation

To achieve a balanced and useful classification, following modifications of the paradigm are necessary:

- attributes must be split into three categories: those which should be predicted from the hierarchy, those which are predicting the classification hierarchy and those having both roles
- all three categories need separate treatment:
  - predicted attributes should be carefully selected based on their utility, the domain of predicted attribute values should be as restricted as possible (precision as low as possible)
  - attributes only predicting the classification have no such restrictions - they may be as many as available and as precise as available, however they should be pondered depending on expenses of their measurement and possibility of their measurement.
- correlation between the classification and a predicted attribute should negatively influence the weight of that attribute in the classification evaluation function.

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