Tunneling into a Two-Dimensional Electron Liquid in a Weak Magnetic Field

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Abstract

We study the spectral density function of a two-dimensional electron liquid in a weak magnetic field, the filling factor $\nu \gg 1$. A hydrodynamic model for low-energy excitations of the liquid is developed. It is found that even at $\nu \gg 1$ the density of states exhibits a gap at low energies. Its width $2E_0$ depends on the strength of interaction only logarithmically, $2E_0 = (\hbar \omega_c/\nu) \ln(\nu e^2/\epsilon \hbar v_F)$. The effects of temperature and disorder on the density of states are discussed.

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Since the discovery of the integer and fractional quantum Hall effects \[1\], the properties of two-dimensional (2D) electron systems in a strong magnetic field have attracted persistent attention. For a long time, the magnetotransport coefficients were the main objects of study. Recently, however, experiments allowing a direct investigation of the one-electron density of states were performed \[2,3\]. In Ref. \[3\] the tunnel current between two 2D layers at small filling factors, $\nu$, was strongly suppressed at low bias, implying the existence of a gap in the spectral density at low energy.

This fact, which can only be caused by many-electron effects, motivated extensive theoretical studies \[4–9\] of tunneling at small filling factors $\nu < 1$. Despite rather different approaches, all these papers predicted that the width of the gap should be related to the single length scale in the lowest Landau level by $E_l \simeq \frac{e^2}{\varepsilon l}$ where $\varepsilon$ is the dielectric constant. The length scale $l$ can be taken to be either the interparticle spacing, the magnetic length, or the cyclotron radius since these are approximately equal in this regime. Such a value of the gap can be easily understood: $E_l$ equals the Coulomb energy of interaction of the extra electron with a “frozen” 2D electron system. If there were no relaxation in the electron system at all, the density of states would vanish at energies $|\hbar \omega| < \sim E_l$. The relaxation processes smear out the threshold in the density of states which therefore becomes sensitive to the details of these processes. However, the suppression is still strong at energies $|\hbar \omega| \lesssim E_l$ if the characteristic relaxation time exceeds $\hbar/E_l$.

In this Letter we study the energy dependence of the density of states at large non-integer filling factors, $\nu e^2/(\varepsilon \hbar v_F) \gg 1$ ($v_F$ is the Fermi velocity at zero magnetic field). In this regime the interparticle spacing, the cyclotron radius, and the magnetic length are distinctly different from each other. We will show that a gap followed by a sharp peak exists in the one-electron density of states. Furthermore, in contrast to the case $\nu \lesssim 1$ the width of the gap depends only logarithmically on the strength of interaction between electrons. At $\nu \simeq \varepsilon \hbar v_F/e^2$, which is the lower boundary of the applicability region, our result approaches the value $E_l \simeq e^2/\varepsilon l$ with the length scale $l$ equal to the cyclotron radius.

**Qualitative Discussion**—Our basic approximation is to treat the electrons as a charged continuous liquid. Immediately after the electron tunnels into the electron liquid, the system acquires extra energy of order $e^2 n_e^{1/2}/\varepsilon$ due to the interaction of this electron with the ones forming the liquid (here $n_e$ is the 2D electron concentration). The energy of repulsion is much greater than the energy of the final state which is determined by an external bias. Therefore, in order to reach the final state, the perturbation of the net electron density has to spread. The spreading occurs through the virtual emission of collective excitations, magnetoplasmons, which have a gap equal to the cyclotron frequency $\omega_c$ in their spectrum:

$$\omega(q) = \left(\omega_c^2 + 2\pi |q| \frac{e^2 n_e}{\varepsilon m}\right)^{1/2},$$

where $m$ is an effective electron mass. Because of the gap, the emission of magnetoplasmons occurs within a time of order $\omega_c^{-1}$. Therefore the density perturbation spreads over an area of radius $a_c \simeq v_g \omega_c^{-1}$ where $v_g$ is the group velocity of the magnetoplasmon \[1\]. Calculating the value of $v_g$ from Eq. \[1\], we find

$$a_c = \frac{2\pi e^2 n_e}{\varepsilon m \omega_c^2}.$$
As the charge spreads on the time scale $\sim \omega_c^{-1}$, the Lorentz force causes circular currents to appear. The Lorentz force due to these circulating currents compensates the excess pressure in the region of the density perturbation, blocking the further spreading of the charge and making this “vortex” configuration stable.

The total energy of the vortex consists of the Coulomb energy plus the kinetic energy of the circulating currents. The former part is of the order of $e^2(\varepsilon a_e)^{-1}$. The use of Eq. (2) yields the Coulomb energy of the vortex $\sim m\omega_c^2/n_e$. Remarkably, this expression does not contain the gas parameter at all. The kinetic energy can be estimated as $\sim m\nu_e^2(n_ea_e^2)$, where $\nu$ is the characteristic velocity of the liquid within the vortex. Using the fact that the Lorentz force compensates the force of electric repulsion $\sim e^2(\varepsilon a_e^2)^{-1}$, we can write down the characteristic velocity as $v \sim e^2/(m\omega_0a_e^2)$. Then, Eq. (2) shows that the kinetic energy approximately equals the Coulomb energy. The more rigorous treatment below reveals a logarithmic divergence of the kinetic energy at small distances. This divergence should be cut off at distances of order the Larmor radius $R_c = v_F/\omega_c$ since this is the scale of the spatial dispersion of the kinetic coefficients, and thus the length scale on which the hydrodynamic approximation fails. The final result for the energy of the vortex $E_0$ is

$$E_0 = \frac{m\omega_c^2}{4\pi n_e} \ln \left( \frac{a_e}{R_c} \right) \equiv \frac{\hbar \omega_c}{2\nu} \ln \left( \frac{\nu e^2}{\varepsilon h v_F} \right). \tag{3}$$

This qualitative argument requires the conditions $E_0 \ll \hbar \omega_c$ and $a_e \gg R_c$ which are both met provided the inequality $\nu e^2/(\varepsilon h v_F) \gg 1$ holds.

The value given by Eq. (3) sets a new energy scale associated with the tunneling electron. If the characteristic time of the further spreading of the vortex (caused, e.g., by dissipation) is much greater than $h/E_0$, a sharp peak appears in the spectral density at the energy $E_0$.

**Electron Liquid Model**— It is clear from the above discussion that the density of states at energies $E \lesssim E_0$ is determined by the long wave length excitations of the electron system. We describe these excitations by a hydrodynamic model defined by the velocity $\mathbf{v}(\mathbf{r}, t)$ and electron density $n(\mathbf{r}, t)$. The deviation of the concentration $\delta n(\mathbf{r}, t)$ from its equilibrium value $n_e(\mathbf{r})$ is related to $\mathbf{v}(\mathbf{r}, t)$ by the linearized continuity equation $\delta \dot{n} + \nabla_r (n_e \mathbf{v}) = 0$. The equilibrium electron density $n_e$ may depend on the coordinate because of a long range disorder potential produced by remote impurities. However, the spatial scale of the disorder potential is assumed to be larger than $R_c$, so that the hydrodynamic description still holds.

It is convenient to take account of the continuity constraint by introducing the field of liquid “displacements” $\mathbf{u}(\mathbf{r}, t)$, $\mathbf{v}(\mathbf{r}, t) = \dot{\mathbf{u}}(\mathbf{r}, t)$, $\delta n(\mathbf{r}, t) = -\nabla_r (n_e(\mathbf{r}) \mathbf{u}(\mathbf{r}, t)). \tag{4}$

The Hamiltonian of the liquid in a magnetic field is

$$H = \int d^3 r \left\{ \frac{1}{2mn_e(\mathbf{r})} \left( \mathbf{p} - \frac{mn_e(\mathbf{r}) \omega_c}{2} [\mathbf{z} \times \mathbf{u}] \right)^2 + \frac{e^2}{2} \int d^3 r_1 \frac{\nabla_r (n_e(\mathbf{r}) \mathbf{u}(\mathbf{r})) \nabla_{r_1} (n_e(\mathbf{r_1}) \mathbf{u}(\mathbf{r_1}))}{|\mathbf{r} - \mathbf{r_1}|} \right\}, \tag{5}$$

where $\mathbf{z}$ is the unit vector perpendicular to the plane of the liquid. The operator $\mathbf{p}$ is canonically conjugate to the operator $\mathbf{u}$, $[p_i(\mathbf{r}_1), u_j(\mathbf{r}_2)] = -i\hbar \delta_{ij} \delta(\mathbf{r}_1 - \mathbf{r}_2)$. We used the...
cylindrical gauge for the vector potential in the kinetic energy term in Eq. (5); the last term in the Hamiltonian is the density-density interaction.

**Tunneling Formalism**—For the quantitative description of tunneling we have to evaluate the spectral density function [10],

\[ A(\omega) = 2 \text{Re} \int_0^\infty dt e^{i\omega t} \langle \psi(t) \psi^\dagger(0) + \psi^\dagger(0) \psi(t) \rangle, \tag{6} \]

where \( \psi^\dagger, \psi \) are electron creation and annihilation operators, respectively, and \( \langle \ldots \rangle \) denotes averaging over the Gibbs distribution. We treat electrons as spinless fermions.

Now we have to establish the relation between the fermionic operators \( \psi^\dagger, \psi \) and the hydrodynamic variables \( \bm{u} \) and \( \bm{p} \). We consider only the local density of states, for which \( \psi^\dagger \) and \( \psi \) are taken at \( \bm{r} = 0 \), and adopt the following [11] approximation for the operator creating an electron at the origin:

\[ \psi^\dagger(t) \propto \exp[i\tilde{\Pi}(t)/\hbar], \tag{7} \]

with the “shift” operator \( \tilde{\Pi} \) defined as

\[ \tilde{\Pi}(t) = \int \frac{d^2 r}{2\pi r^2} \left( \frac{\bm{r} \cdot \bm{p}(\bm{r}, t)}{n_e(\bm{r})} + \frac{m\omega_c}{2} \frac{\bm{r} \times [\bm{z} \times \bm{u}(\bm{r}, t)]}{\hbar} \right). \tag{8} \]

The operator Eq. (7) creates an electron density perturbation localized near the origin and does not excite any motion of the electron liquid:

\[ \left[ \psi^\dagger(t), \delta n(\bm{r}, t) \right] = -\delta(\bm{r}) \psi^\dagger(t), \quad \left[ \psi^\dagger(t), \bm{v}(\bm{r}, t) \right] = 0, \]

where the velocity operator is \( \bm{v}(\bm{r}) \equiv i[H, \bm{u}(\bm{r})]/\hbar \). Because we are considering only the local density of states, our calculation is strictly applicable to the case of tunneling from a point contact to a 2D plane; however, because the magnetic field restricts the extent of the electron—a plane-wave basis is not suitable, for instance—we believe that this calculation is relevant to the experimentally interesting case of plane to plane tunneling. For the case of the local density of states, we expect the representation Eq. (7) to be an accurate description of the long-wavelength collective effects responsible for the gap structure in the density of states, while the effects of the single-particle degrees of freedom should not affect this structure.

The representation Eq. (7) of the creation operator \( \psi^\dagger \) in terms of the shift operator \( \tilde{\Pi}(t) \) allows us to relate the spectral density function \( A(\omega) \) to the response function of the electron liquid. Because the Hamiltonian Eq. (3) is quadratic, and operator (8) is linear in \( \bm{u} \) and \( \bm{p} \), the calculation may be performed in a standard way [10] and results in

\[ A(\omega) \propto \text{Re} \int_0^\infty dt e^{i\omega t} \left( e^{J(t)} + e^{J^*(t)} \right), \tag{9} \]

\[ J(t) = i[D(t) - D(0)]/\hbar. \]

Here we introduced the Green function of the electron liquid \( D(t) \equiv -i\langle \tilde{T}\tilde{\Pi}(t)\tilde{\Pi}(0) \rangle/\hbar \) where \( \tilde{T} \) stands for the time ordering. The most convenient way to proceed is to relate \( D(\omega) \) to the retarded Green function \( D^R(t) \equiv -i\theta(t)[\langle \tilde{\Pi}(t), \tilde{\Pi}(0) \rangle]/\hbar \) by the identity [10]
\[ D(\omega) = \text{Re} D^R(\omega) + i \coth \frac{\hbar \omega}{2k_B T} \text{Im} D^R(\omega). \] (10)

Using the equations of motion for the variables \( u \) and \( p \), one can check that \( \dot{\Pi} = \int d^2 r \delta n(r)(e^2/\epsilon |r|) \). This would allow one to express the Green functions \( D \) and \( D^R \) in terms of corresponding density-density correlation functions [10]. However, it is more convenient to obtain an equation that determines directly the Green function \( D^R \). To derive this equation, we notice that the retarded Green function coincides with the linear response of the average \( \langle \Pi(t) \rangle \) to an external perturbation of the form \( \delta(t)\Pi(t) \) added to the Hamiltonian Eq. (5). We calculate this linear response with the help of the equations of motion for \( u \) and \( p \). Because the Hamiltonian Eq. (5) is quadratic the equation for the Green function can be found in a closed form. A simple calculation yields

\[ D^R(t) = \lim_{r \to 0} \tilde{D}^R(t, r), \]

where \( \tilde{D}^R(t, r) \) is the solution of the equation

\[
\frac{e^2}{\epsilon m} \left\{ \left( \nabla \frac{\partial}{\partial t} + \omega_c [z \times \nabla] \right) \int d^2 r_1 \frac{n_e(r_1) \nabla_{r_1} \tilde{D}^R(r_1)}{|r - r_1|} \right\} \\
+ \frac{\partial}{\partial t} \left( \omega_c^2 + \frac{\partial^2}{\partial t^2} \right) \tilde{D}^R(r) = -[\omega_c^2 \theta(t) + \delta'(t)] \frac{e^2}{\epsilon |r|},
\]

(11)

at \( t \geq 0 \), and \( \tilde{D}^R(t, r) = 0 \) at \( t < 0 \).

**Homogeneous Liquid**— For a homogeneous liquid \( (n_e = \text{const}) \), Eq. (11) can be easily solved by Fourier transformation. Because \( E_0 < \hbar \omega_c \), only the asymptotic behavior at \( t \gg \omega_c^{-1} \) is needed. Using this solution in Eqs. (9) and (10), we obtain

\[ J(t) = -i \frac{E_t}{\hbar} - \frac{k_B T E_t^2}{\hbar^2} \int_{1/R_c}^{1/R_c} dk \frac{a_c}{1 + a_c k}, \]

(12)

where \( a_c \) is defined by Eq. (2), and

\[ E_t = \frac{m \omega^2}{4\pi n_e}. \]

(13)

The logarithmic divergence of the integral in Eq. (12) is cut off at wavevectors of order \( R_c^{-1} \) where the hydrodynamic approximation fails. Substituting Eq. (12) into Eq. (9) yields the spectral density for \( k_B T \ll E_0 \),

\[ A(\omega) \propto \frac{\hbar}{\sqrt{k_B T E_0}} \exp \left( -\frac{(|\hbar \omega| - E_0)^2}{4E_0 k_B T} \right), \]

(14)

with energy \( E_0 \) defined by Eq. (3). The lower the temperature, the sharper the peak in the density of states at \( \hbar \omega = E_0 \). The gap at smaller energies is due to the blockade of spreading of the electron liquid. As we already discussed, spreading results in the formation of a finite-size vortex with energy \( E_0 \). The peak at negative energy corresponds to an “antivortex” formed after an electron tunnels out from the electron liquid.

Finite temperature broadens the peak in the density of states by introducing some initial inhomogeneities in the electron liquid. For example, a vortex carrying some charge \( \delta q \) can be formed spontaneously by a thermal fluctuation; the probability of such a configuration is
proportional to \(\exp(-\epsilon_i/k_B T)\) with energy \(\epsilon_i = E_0(\delta q/e)^2\). Tunneling of an electron into the center of this configuration produces a vortex containing charge \(\delta q + e\), and therefore having energy \(\hbar \omega = E_0(1 + 2\delta q/e)\). These qualitative considerations lead directly to Eq. (14).

As we already mentioned, the sharp peak in the density of states at non-zero energy occurs because the charge spreading is blocked by the magnetic field applied to an ideal, homogeneous electron liquid. There are several ways in which this result should be modified in the non-ideal case. First, in a “viscous” liquid, a finite longitudinal dc-conductivity \(\sigma_{xx}\) allows charge spreading, and the gap is washed out. The microscopic theory [12] of conductivity for a special case \(\nu = 1/2\) allowed He et al. [6] to obtain the function \(A(\omega)\) in a wide range of energies. Second, at small filling factors the electron liquid may crystalize into a Wigner crystal. It was shown by Johansson and Kinaret [7] that the peak in the tunneling density of states in this case has a finite width. This occurs solely due to the existence of gapless magnetophonon modes in a crystal that are absent in a liquid. Neither of these approaches is suitable for the large filling factors considered here. Third, the charge may spread because of inhomogeneities [13] caused, e.g., by smooth disorder; we pursue this possibility below.

**Inhomogeneous Liquid**—It is well known that density inhomogeneities may give rise to gapless excitations of an electron liquid in a magnetic field. These gapless modes redistribute charge, and thus cause spreading of the vortex and broadening of the peak in the density of states. The origin of such excitations is analogous to that of the edge magnetoplasmon [14] propagating along the periphery of the system.

In order to estimate the possible effect of smooth disorder, we start with the simplest model of a constant gradient of the concentration \(n_e(r)\) in the region of the liquid into which the electron tunnels:

\[
n_e(r) = \bar{n}_e + |\nabla n_e| x.
\]

(15)

Here the \(x\) axis is in the direction of the gradient, and we assume the potential to be smooth so that the concentration changes slowly on the scale set by the vortex size, \(|\nabla n_e| a_c \ll \bar{n}_e\). In this case Eq. (11) for zero temperature and \(\omega_c t \gg 1\) yields

\[
J(t) = 2\pi E_t \int \frac{d^2 k}{(2\pi)^2} \frac{\alpha_e(\omega(k) t)}{|k|(1 + \alpha_e |k|)} \frac{e^{-i\omega(k)t} - 1}{|\hbar \omega(k)|}.
\]

(16)

where \(E_t\) is defined by Eq. (12), and the spectrum \(\omega(k)\) of the magnetoplasmon excitations which are associated with the inhomogeneity Eq. (15) is given by

\[
\omega(k) = \tau_d^{-1} \frac{k_y}{|k|(1 + \alpha_e |k|)}; \quad \tau_d \equiv \frac{\bar{n}_e}{\omega_c a_e |\nabla n_e|}.
\]

(17)

At \(t \ll \tau_d\) the exponent in Eq. (16) can be expanded, giving \(J(t) = -i E_0 t/\hbar - E_t t^2/(2\pi \tau_d \hbar)\), similar to Eq. (12). The corresponding result for the spectral density function at energies \(|\hbar \omega| - E_0| \lesssim E_t\) is:

\[
A(\omega) \propto \sqrt{\frac{\hbar \tau_d}{E_t}} \exp \left[ -\frac{\pi \tau_d E_t}{2\hbar} \left( \frac{|\hbar \omega| - E_0}{E_t} \right)^2 \right].
\]

(18)
Thus, the spreading of the charge through the low-frequency modes, Eq. (17), leads to a broadening of the peak similar in form to that caused by temperature in the homogeneous case, Eq. (14). As in the case of finite temperature, one can view this broadening as due to fluctuations in the initial conditions; however, the fluctuations now have a quantum origin in that they are produced by the zero-point motion of the low-frequency modes.

The region of validity of Eq. (18) corresponds to relatively short times \( t \lesssim \tau_d \) contributing to the integral in Eq. (9). On this time scale the vortex spreads on a distance \( l_d \simeq \tau_d (d\omega/dk_y) |_{|k|\simeq 1/a_c} \simeq a_c \). According to our assumption about the smoothness of the disorder, the corresponding spatial variation of electron density is small, which justifies the use of the gradient expansion Eq. (15).

We have found that the width of the peak in the local density of states is determined by the local concentration gradient in the case of smooth disorder. Turning now to large-area tunnel barriers, the differential conductance is related to the ensemble average of the density of states, and therefore the width of the peak of the average density of states is determined by the typical value of the concentration gradient. This yields a relation between the observable width of the peak and parameters of a sample.

For estimates we assume that the 2D electron liquid is created in a heterostructure, where the random potential is produced by a layer of the remote charged impurities with concentration \( N_D \), and that all the electrons in the 2D liquid originate from these impurities, \( \langle n_e \rangle = N_D \). Then, the characteristic spatial scale of the disorder potential is equal to the spacer width \( d \) and the typical deviation of the concentration from its average value \( \langle n_e \rangle \) is \( \sqrt{N_D/d} \). The density fluctuations are small if \( N_D d^2 \gg 1 \). We can determine the width of the peak \( \delta E \sim (E_\tau h/\tau_d)^{1/2} \) after estimating the characteristic values of \( E_\tau \) and \( \tau_d \):

\[
\frac{\delta E}{E_0} \sim \frac{a_B}{d} \left( \frac{\nu e^2}{\varepsilon h v_F} \right)^{3/2} \left[ \ln \left( \frac{\nu e^2}{\varepsilon h v_F} \right) \right]^{-1} \tag{19}
\]

where \( a_B = \varepsilon h^2/e^2 m \). As discussed above, this width is caused by quantum fluctuations of the low-frequency modes of the inhomogeneous electron liquid. Note that this width is significantly larger than that caused by the direct effect of static disorder on \( E_0 \) [i.e., by \( n_e(r) \) variation in Eq. (3)]. The constraint of smooth disorder on the scale set by the vortex size is satisfied if \( \nu \lesssim (\varepsilon h v_F/e^2)(d/a_B)^{1/2} \). The opposite case of a “short-range” potential \( R_c \ll d \ll a_c \) will be considered elsewhere.

In conclusion, we have demonstrated that even relatively weak magnetic field induces a gap in the tunneling density of states of an electron liquid. For a homogeneous liquid this gap is followed by a sharp peak at a finite energy. Low-frequency modes induced by disorder broaden this peak through quantum fluctuations. Eq. (19) shows that for a high-quality heterostructure \( (d/a_B \simeq 10) \) the peak should be observable for filling factors \( \nu \sim 3 \rightarrow 4 \).

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