Analytical and numerical study of the autobalancing process of a dynamically unbalanced rigid rotor fixed in orthotropic viscoelastic supports

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Abstract. A dynamically unbalanced rotor in viscoelastic orthotropic supports equipped with a two-plane automatic ball balancer (ABB) is considered. Based on an analysis of the equations describing the stationary modes of motion of the system, the possibility of the existence of the balanced mode is shown and the necessary conditions for its existence are found. Critical rotor speeds are found for a case of a rotor without an ABB. Amplitude-frequency responses for rotors in isotropic and orthotropic supports are constructed. Autobalancing process for the cases of rotation at constant angular velocity and constant angular acceleration is investigated.

1. Introduction
The problem of autobalancing a dynamically unbalanced rotor was first considered in the article by Hedaya and Sharp [1], who proposed using a two-plane automatic ball balancer (ABB). A similar approach was considered in the works [2]-[3], where the existence and stability of modes that compensate for rotor oscillations were studied using the constructed linearised differential equations. In addition to analytical studies, these articles presented numerical results proving the possibility of stable compensation of imbalances, and also studied the effect of various system parameters on the time required for balancing. Another important result of these works was the demonstration of the impossibility of balancing the rotor in the frequency range not exceeding the second critical speed. A similar result was obtained using linear bifurcation analysis in the work [4].

It was found in the article [5] that under certain design conditions, the dynamic imbalance of the rotor can be compensated for by a single-plane ABB in the area of rotor angular velocities that exceed the first critical speed. The influence of various system parameters on the stability of a balanced mode for a dynamically unbalanced rotor in orthotropic supports was investigated using bifurcation analysis in [6]. The practical application of a two-plane ABB for the balancing of washing machines was considered in the work [7]. In [9] it was found that in the case of a dynamically unbalanced rotor with imperfect (eccentrically fixed) ABBs, it is impossible to completely compensate for the imbalance. In the works [8] and [10], the dynamics and stability of stationary modes of a statically unbalanced rotor in visco-elastic orthotropic supports were investigated. The stationary modes of rotor movement were investigated using the averaging method in [8]. In [10], analytical expressions for the residual amplitude of oscillations were obtained using the same method, and the stability of an almost balanced mode was also...
investigated. This work continues and develops the research carried out in [5], [8]-[10], for the case of a dynamically unbalanced rotor in viscoelastic orthotropic supports.

2. Mechanical model of a rotor with a two-plane ABB

We consider the dynamically unbalanced rigid rotor shown in Fig. 1, fixed in vertical hinged visco-elastic supports. Let us denote by \( O_1 \) and \( O_2 \) the centers of the bearings of the supports, \( G \) is the center of mass of the rotor, \( C \) is the point of intersection of the axis of rotation \( O_1O_2 \) with the horizontal plane, which is passing through the point \( G \), \( l_1 = |CO_1| \) and \( l_2 = |CO_2| \) are the distances from the point \( C \) to the supports. If \( l_1 = l_2 \), then we will name the rotor symmetrically fixed, otherwise, we will name it asymmetrically fixed.

\[
\begin{align*}
\text{Figure 1. Dynamically unbalanced rotor with a two-plane ABB}
\end{align*}
\]

We describe the general dynamic imbalance of the rotor using the following parameters: \( s = |CG| \) is the static eccentricity; \( \chi \) is the angle between the axis of rotation and the axis of dynamic symmetry of the rotor \( CP \) (moment eccentricity); \( \gamma \) is the angle between the plane of the moment eccentricity and the plane passing through the axis of rotation and the center of mass of the rotor (phase shift of the moment eccentricity with respect to the static eccentricity [11]). To compensate for the dynamic imbalance, the rotor is equipped with a two-plane ABB, which consists of two flat cages of radius \( r \) with balancing balls, that are located parallel to each other at the distances \( h_1 \) and \( h_2 \) from the point \( C \). Note that the position of the center of mass \( G \) takes into account the mass of the ABB’s cages, but does not take into account the mass of the balancing balls.

\[
\begin{align*}
\text{Figure 2. Reference frames.} & \quad \text{Figure 3. Angular coordinates of the rotor and balancing balls.}
\end{align*}
\]
To describe the movement of the rotor, we introduce the following reference frames (Fig. 2): $OXYZ$ is a fixed (absolute) frame, the $OZ$ axis is directed along the $O_1O_2$ axis in the undeformed state of the supports, and the $OX$ and $OY$ axes are directed along the axes of the yielding ellipse of the supports; $CX_1Y_1Z_1$ is a non-rotating moving frame with axes that are co-directed to the axes of the fixed frame; $GX_2Y_2Z_2$ is a frame where the axes are directed along the principal central axes of inertia of the rotor; $Cxyz$ is a frame, which is rigidly connected to the rotor; the $Cz$ axis is directed along the rotor rotation axis, and the $Cy$ axis is parallel to the equatorial plane of inertia of the rotor (i.e., $Cy \parallel GY_2$).

The angles of the deflection of the balancing balls $\psi_{ik}$, $(i = 1, 2; k = 1, n)$ in the planes of both ABB’s cages (Fig. 3b) as the generalized coordinates.

Then the potential energy of the supports and the Rayleigh dissipative function can be written (all balls are considered to be the same),

$$J = \sum_{i=1}^{n} \frac{m_i}{2} \dot{\psi}_{ik}^2,$$

where $m_i$ is the mass of the balancing ball $b_i$, $(i = 1, 2; k = 1, n)$.

Assuming further the angles $\chi, \alpha$ and $\beta$ to be small, we write expressions for the column vector $\mathbf{R}_G$ of the absolute coordinates of the point $G$, for the column vector $\mathbf{\Omega}$ of the projections of the absolute angular velocity vector the rotor on the $GX_2Y_2Z_2$ axes and for the column vectors $\mathbf{R}_{B_{ik}}$ of the absolute coordinates of the balancing balls $B_{ik}$ (5)

$$\mathbf{R}_G = \begin{pmatrix} X + s \cos(\theta + \gamma) \\ Y + s \sin(\theta + \gamma) \\ s(\alpha \sin(\theta + \gamma) - \beta \cos(\theta + \gamma)) \end{pmatrix}, \quad \mathbf{\Omega} = \begin{pmatrix} \dot{\chi} \cos \theta + \dot{\beta} \sin \theta - \dot{\theta} \chi \\ -\dot{\alpha} \cos \theta + \dot{\beta} \sin \theta + \dot{\theta} \chi \\ \chi(\dot{\alpha} \cos \theta + \dot{\beta} \sin \theta + \dot{\theta}) \end{pmatrix},$$

$$\mathbf{R}_{B_{ik}} = \begin{pmatrix} X + r \cos(\theta + \psi_{1k}) - h_1 \beta \\ Y + r \sin(\theta + \psi_{1k}) + h_1 \alpha \\ r(\alpha \sin(\theta + \psi_{1k}) - \beta \cos(\theta + \psi_{1k})) - h_1 \end{pmatrix}, \quad \mathbf{R}_{B_{2k}} = \begin{pmatrix} X + r \cos(\theta + \psi_{2k}) + h_2 \beta \\ Y + r \sin(\theta + \psi_{2k}) - h_2 \alpha \\ r(\alpha \cos(\theta + \psi_{2k}) - \beta \sin(\theta + \psi_{2k}))) + h_2 \end{pmatrix}.$$

The kinetic energy of the system has the following form:

$$T = \frac{1}{2}(m_0 \dot{\mathbf{R}}_G^2 + \mathbf{\Omega}^T(J_G \mathbf{\Omega})) + \frac{1}{2} \sum_{k=1}^{n}(\dot{\mathbf{R}}_{B_{1k}}^2 + \dot{\mathbf{R}}_{B_{2k}}^2),$$

where $m_0$ is the mass of the rotor with ABB’s cages (without balancing balls), $J_p$ and $J_t$ are the polar and equatorial moments of inertia of the rotor, $m_b$ is the mass of the balancing ball (all balls are considered to be the same), $J_G = \text{diag}\{J_1, J_t, J_p\}$.

Assuming that both supports have orthotropic elastic-viscous characteristics, we denote by $c_{ix}, c_{iy}$ and $d_{ix}, d_{iy}$, $(i = 1, 2)$ elastic and damping coefficients in the supports, respectively. Then the potential energy of the supports and the Rayleigh dissipative function can be written in the form

$$V = \frac{1}{2} \left( c_{ix}(X - l_1 \beta)^2 + c_{iy}(Y + l_1 \alpha)^2 + c_{2x}(X + l_2 \beta)^2 + c_{2y}(Y - l_2 \alpha)^2 \right)$$

$$D = \frac{1}{2} \left( d_{ix}\dot{X} - l_1 \dot{\beta})^2 + d_{iy}(\dot{Y} + l_1 \dot{\alpha})^2 + d_{2x}\dot{X} + l_2 \dot{\beta})^2 + d_{2y}(\dot{Y} - l_2 \dot{\alpha})^2 \right) +$$

$$+ \frac{d_\phi}{2} \dot{\phi}^2 + \frac{d_\psi}{2} \sum_{k=1}^{n}(\dot{\psi}_{1k}^2 + \dot{\psi}_{2k}^2),$$

where $d_\phi$ is the dissipation coefficient characterizing the force of resistance to the rotational motion of the rotor, and $d_\psi$ is the coefficient of viscous resistance to the movement of the balancing balls in the ABB’s cages.
Further, we shall assume that the angular velocity of rotation of the rotor is changed by a given law \( \dot{\theta} = \dot{\theta}(t) \). Using the expressions (1) – (3), we write down the system of \((4 + 2n)\) Lagrange equations of the second kind in matrix form:

\[
\begin{align*}
\{ (M_0 + nm_GM_1) \ddot{q} + (D - \dot{\theta}G)\dot{q} + Cq = F_0 + mbrF_1, \\
mbr^2 \ddot{\varphi}_{ik} + d_\psi \dot{\varphi}_{ik} = mbr((\dot{X} + \dot{h}_i\dot{\beta}) \sin \varphi_{ik} - (\dot{Y} - \dot{h}_i\dot{\alpha}) \cos \varphi_{ik}), \quad i = 1, 2, \ k = \overline{1, n},
\end{align*}
\]

where

\[
q = \begin{pmatrix} X \\ Y \\ \alpha \end{pmatrix}, \quad M_0 = \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & J_t \end{pmatrix}, \quad M_1 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & h_1 - h_2 \\ 0 & h_1 - h_2 & h_1^2 + h_2^2 \end{pmatrix}, \\
G = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -J_p \end{pmatrix}, \quad C = \begin{pmatrix} c_{11x} & 0 & -c_{12x} \\ 0 & c_{11y} & c_{12y} \\ -c_{12x} & 0 & c_{22x} \end{pmatrix}, \quad D = \begin{pmatrix} d_{11x} & 0 & -d_{12x} \\ 0 & d_{11y} & d_{12y} \\ 0 & d_{12y} & d_{22y} \end{pmatrix}, \\
F_0 = \begin{pmatrix} m_0(\dot{\theta}^2 \cos(\theta + \gamma) + \ddot{\theta} \sin(\theta + \gamma)) \\ m_0(\dot{\theta}^2 \sin(\theta + \gamma) - \ddot{\theta} \cos(\theta + \gamma)) \\ \chi(J_p - J_t)(\dot{\theta}^2 \sin \theta - \ddot{\theta} \cos \theta) \\ -\chi(J_p - J_t)(\dot{\theta}^2 \cos \theta + \ddot{\theta} \sin \theta) \end{pmatrix}, \quad F_1 = \begin{pmatrix} \sum_{k=1}^{n} \sum_{l=1}^{n} (\dot{\varphi}_{ik}^2 \cos \varphi_{ik} + \ddot{\varphi}_{ik} \sin \varphi_{ik}) \\ \sum_{k=1}^{n} \sum_{l=1}^{n} (\dot{\varphi}_{ik}^2 \sin \varphi_{ik} - \dot{\varphi}_{ik}^2 \cos \varphi_{ik}) \\ \sum_{l=1}^{n} \sum_{k=1}^{n} h_i(\dot{\varphi}_{ik}^2 \sin \varphi_{ik} - \dot{\varphi}_{ik}^2 \cos \varphi_{ik}) + R_1 \\ \sum_{l=1}^{n} \sum_{k=1}^{n} h_i(\dot{\varphi}_{ik}^2 \cos \varphi_{ij} + \ddot{\varphi}_{ij} \sin \varphi_{ij}) - R_2 \end{pmatrix},
\]

\[
R_1 = r \sum_{i=1}^{2} \sum_{k=1}^{n} \sin \varphi_{ik} \left( (\ddot{\beta} - 2\dot{\alpha} \dot{\varphi}_{ik} - \ddot{\varphi}_{ik} - \alpha \dot{\varphi}_{ik}) \cos \varphi_{ik} - (\ddot{\alpha} + 2\dot{\beta} \dot{\varphi}_{ik} - \alpha \dot{\varphi}_{ik} + \beta \dot{\varphi}_{ik}) \sin \varphi_{ik} \right),
\]

\[
R_2 = r \sum_{i=1}^{2} \sum_{k=1}^{n} \cos \varphi_{ik} \left( (\ddot{\beta} - 2\dot{\alpha} \dot{\varphi}_{ik} - \ddot{\varphi}_{ik} - \alpha \dot{\varphi}_{ik}) \cos \varphi_{ik} - (\ddot{\alpha} + 2\dot{\beta} \dot{\varphi}_{ik} - \alpha \dot{\varphi}_{ik} + \beta \dot{\varphi}_{ik}) \sin \varphi_{ik} \right),
\]

and

\[
\begin{align*}
d_{11x} & = d_{1x} + d_{2x}, & d_{12x} & = d_{1x}l_1 - d_{2x}l_2, & d_{22x} & = d_{1x}l_1^2 + d_{2x}l_2^2, & d_{11y} & = d_{1y} + d_{2y}, \\
d_{12y} & = d_{1y}l_1 - d_{2y}l_2, & d_{22y} & = d_{1y}l_1^2 + d_{2y}l_2^2, & c_{11x} & = c_{1x} + c_{2x}, & c_{12x} & = c_{1x}l_1 - c_{2x}l_2, \\
c_{22x} & = c_{1x}l_1^2 + c_{2x}l_2^2, & c_{11y} & = c_{1y} + c_{2y}, & c_{12y} & = c_{1y}l_1 - c_{2y}l_2, & c_{22y} & = c_{1y}l_1^2 + c_{2y}l_2^2, \\
\dot{h}_1 & = -h_1, & \dot{h}_2 & = h_2, & \varphi_{i,j} & = \theta(t) + \psi_{ij}, \quad (i = 1, 2; \ j = \overline{1, n}).
\end{align*}
\]

3. Stationary balanced mode of motion

In what follows, we will assume that each of the two-plane ABB’s cages contains only two balancing balls. Let us consider a fully balanced rotation mode of a rotor with constant angular velocity \( \dot{\theta} = \nu \). Substituting \( X = Y = \dot{X} = \dot{Y} = 0 \) and \( \alpha = \beta = \ddot{\alpha} = \ddot{\beta} = \dot{\beta} = 0 \) into equations (4), we obtain a system of four transcendental equations for the angles \( \psi_{11}, \ \psi_{12}, \ \psi_{21}, \ \psi_{22} \) and time. If we write the resulting system in complex form, then the terms depending on time will cancel, and we come to a system of two autonomous equations

\[
m_0se^{i\gamma} = -mbr \sum_{k,l=1}^{2} e^{i\psi_{kl}}, \quad \chi(J_p - J_t) = -mbr \sum_{k,l=1}^{2} \dot{h}_k e^{i\psi_{kl}},
\]

(5)
The real form of the equations (5) is split into two independent subsystems

\[
\begin{align*}
\cos \psi_{i1} + \cos \psi_{i2} &= -\frac{m_0\cos \chi(J_i - J_p)}{m_br(h_1 + h_2)}, \\
\sin \psi_{i1} + \sin \psi_{i2} &= -\frac{m_0\sin \chi(J_i - J_p)}{m_br(h_1 + h_2)},
\end{align*}
\] (6)

Equations (6) describe the stationary position of the balancing balls in each of the cages under conditions of balanced mode of rotor rotation. Solving them, we find

\[
\psi_{i1} = \arctan \frac{B_i}{A_i} \pm \arccos \sqrt{\frac{A_i^2 + B_i^2}{2}}, \quad \psi_{i2} = \arctan \frac{B_i}{A_i} \pm \arccos \sqrt{\frac{A_i^2 + B_i^2}{2}}, \quad i = 1, 2.
\] (7)

where

\[
A_i = -\frac{m_0\cos \chi(J_i - J_p)}{m_br(h_1 + h_2)}, \quad B_i = -\frac{m_0\sin \chi(J_i - J_p)}{m_br(h_1 + h_2)}, \quad i = 1, 2.
\]

We obtain the necessary conditions for the existence of the balanced mode from the formulas (7): \(A_i^2 + B_i^2 \leq 4\) for the first cage and \(A_i^2 + B_i^2 \leq 4\) for the second one.

4. Critical rotor speeds without an ABB. Campbell diagrams

It is known that passive autobalancing of the rotor is possible only in the area of angular velocities exceeding critical values. The analysis of the critical speeds of the balanced rotor will be carried out without taking into account the ABB. To do this, we put in the system (4) \(m_b = 0\), \(\vec{\theta} = \omega = \text{const}\) and write the first four equations

\[
M_0\ddot{q} + (D - \omega G)\dot{q} + Cq = F_0,
\] (8)

To find the critical angular velocities of a balanced rotor, we put in (8) \(s = 0\), \(\chi = 0\) and substitute in the homogeneous equation a particular solution of the form \(q = q_0 e^{\lambda t}\), where \(q_0 = \text{const}\). As a result, we obtain a system of linear homogeneous equations for \(q_0\), which has a nontrivial solution under the condition

\[
\det[M_0\lambda^2 + (D - \omega G)\lambda + C] = 0.
\] (9)

The characteristic equation (9) is an algebraic equation of the 8th order for \(\lambda\) and has 8 complex roots depending on the angular velocity \(\omega\). The critical rotor speeds corresponding to forward and reverse synchronous whirling are determined by the equations

\[
\operatorname{Im}[\lambda_k(\omega)] = \pm \omega, \quad k = 1, 8.
\] (10)

Fig. 4 shows a Campbell diagram showing the graphical solution of equations (10) for an asymmetrically fixed rotor in orthotropic supports. The calculation was carried out with the following parameter values: \(m_0 = 10\ \text{kg}\), \(s = 0.001\ \text{m}\), \(\gamma = 0.2\), \(\chi = 0.01\), \(l_1 = 0.4\ \text{m}\), \(l_2 = 0.6\ \text{m}\), \(J_p = 0.05\ \text{kgm}^2\), \(J_i = 0.325\ \text{kgm}^2\), \(c_{1x} = c_{2x} = 1.5 \cdot 10^5\ \text{N/m}\), \(c_{1y} = c_{2y} = 2.5 \cdot 10^5\ \text{N/m}\), \(d_{1x} = d_{2x} = 80\ \text{Ns/m}\), \(d_{1y} = d_{2y} = 120\ \text{Ns/m}\) and determines four critical speeds for direct synchronous whirling of the rotor: \(\omega_1 = 169.024\ \text{s}^{-1}\), \(\omega_2 = 218.129\ \text{s}^{-1}\), \(\omega_3 = 419.233\ \text{s}^{-1}\), \(\omega_4 = 553.79\ \text{s}^{-1}\).

A similar diagram calculated for the case of isotropic supports at \(d_{1x} = d_{1y} = d_{12x} = d_{12y} = 100\ \text{Ns/m}\), \(c_{11x} = c_{11y} = c_{12x} = c_{12y} = 2 \cdot 10^6\ \text{N/m}\), is shown in Fig. 4 by dashes. In this case, we have the following values of the critical speeds for the direct whirling of the rotor: \(\omega_1 = 195.036\ \text{s}^{-1}\), \(\omega_2 = 195.239\ \text{s}^{-1}\), \(\omega_3 = 462.354\ \text{s}^{-1}\), \(\omega_4 = 518.635\ \text{s}^{-1}\). Thus, we see that the orthotropy of elastic supports leads to a "divergence" of critical frequencies, i.e., the critical frequencies in the case of isotropic supports lie between the corresponding critical frequencies in the case of orthotropic supports.
5. Forced synchronous rotor whirling without an ABB

In the case when the rotor without an ABB is fixed in isotropic supports, the particular solution of the inhomogeneous system (8), corresponding to the regular synchronous whirling of the rotor, can be obtained explicitly. To do this, we introduce complex variables \( z = X + iY \), \( \phi = \beta - i\alpha \) and represent the system (8) in complex form

\[
\ddot{\tilde{q}} + (\tilde{D} - i\omega \tilde{G}) \dot{\tilde{q}} + \tilde{C} \tilde{q} = \omega^2 \tilde{F}(t)e^{i\nu t},
\]

where

\[
\tilde{q} = \begin{pmatrix} z \\ \phi \end{pmatrix}, \quad \tilde{M} = \begin{pmatrix} m_0 & 0 \\ 0 & J_t \end{pmatrix}, \quad \tilde{D} = \begin{pmatrix} d_{11} & -d_{12} \\ -d_{12} & d_{22} \end{pmatrix},
\]

\[
\tilde{C} = \begin{pmatrix} c_{11} & -c_{12} \\ -c_{12} & c_{22} \end{pmatrix}, \quad \tilde{G} = \begin{pmatrix} 0 & 0 \\ 0 & J_p \end{pmatrix}, \quad \tilde{F} = \begin{pmatrix} m_0 s \nu \gamma \\ \chi (J_t - J_p) \end{pmatrix}.
\]

Substituting a particular solution of the form \( \tilde{q} = \tilde{q}_0 e^{i\omega t} \), where \( \tilde{q}_0 = (z_0 \ \phi_0)^T \), \( z_0 = \text{const} \), \( \phi_0 = \text{const} \), into the system (11), we obtain the linear algebraic system

\[
(\tilde{C} - \omega^2 (\tilde{M} - \tilde{G})) \tilde{q}_0 = \omega^2 \tilde{F},
\]

the solution of which has the following form:

\[
z_0 = -\frac{\chi (J_t - J_p) P_{11} + m_0 s \nu \gamma P_{22}}{P_{11} P_{22} - P_{12}^2} \omega^2, \quad \phi_0 = \frac{\chi (J_t - J_p) P_{11} + m_0 s \nu \gamma P_{12}}{P_{11} P_{22} - P_{12}^2} \omega^2,
\]

where

\[
P_{11} = c_{11} - m_0 \omega^2 + i\omega d_{11}, \quad P_{12} = c_{12} + id_{12}, \quad P_{22} = c_{22} - (J_t - J_p) \omega^2 + i\omega d_{22}.
\]

Formulas (13) allow calculating the amplitudes \( a_1 = |z_0| \) and \( a_2 = |\phi_0| \) of the whirling motion of the rotor depending on the angular velocity \( \omega \) and constructing the corresponding amplitude-frequency responses (AFR).

In case of a rotor fixed in orthotropic supports, it is not possible to obtain an exact solution in the form of explicit formulas for the amplitudes of the whirling motion due to its more complex nature. Therefore, we will study the features of the whirling motion of the rotor in the critical frequency range by numerically integrating the original system of equations (4), assuming that the angular velocity of the rotor changes linearly \( \dot{\theta} = \omega(t) = \omega(0) + ut \), where \( u = 40 \text{ s}^{-2} \) is the constant angular acceleration of the rotor. The choice of such value of the angular acceleration provides a sufficiently "slow" passage of the resonance regions and guarantees the proximity of the amplitude curves to the stationary frequency response of the whirling of the rotor.
carrying out numerical calculations, the values of the model parameters introduced earlier in Section 4 were set.

The results of the numerical integration of the system (4) on the time interval providing the passage of the critical frequency region are shown in Fig. 5. The graphs of the amplitudes $a_1 = \sqrt{X^2 + Y^2}$ and $a_2 = \sqrt{\alpha^2 + \beta^2}$ of whirling motions of the rotor in orthotropic supports depending on the dimensionless angular velocity $\nu = \omega/\omega_0$, where $\omega_0 = \sqrt{\frac{c_{11x} + c_{11y}}{2m_0}}$, are shown in gray. For comparison, the AFRs of the isotropic rotor, calculated by the formulas (13), are highlighted in red. The graphs show that in the supercritical region the amplitude of the whirling motion of the point $C$ tends to the value of the static eccentricity $s$ (self-centering effect), and the amplitude of angular whirling tends to the value of the moment eccentricity $\chi$.

6. Autobalancing of a dynamically unbalanced rotor

Let us calculate the process of autobalancing of a dynamically unbalanced rotor in the case when its angular velocity $\omega$ is constant and exceeds the maximum critical velocity of the direct whirling. In the calculation, we will use the values of the rotor parameters introduced earlier, and the ABB parameters will be set as follows: $r = 0.1 \text{ m}$, $m_b = 0.1 \text{ kg}$, $n = 2$, $h_1 = h_2 = 0.35 \text{ m}$, $d_\psi = 0.6 \text{ Nms}$.

Figure 6 shows the results of numerical integration of the system (4) for rotors fixed in isotropic (a, b, c) and orthotropic (d, e, f) supports and rotating at an angular velocity of $\omega = 800 \text{ s}^{-1}$, which exceeds the maximum critical speed of the forward whirling $\omega_4$, calculated in Section 4. Curves 1 in Fig. 6a and d show the nature of the variation in the $a_1$ amplitude of
the whirling motion of the point C with time. Curves 1 in Fig. 6b and e show change in the \( a_2 \) amplitude of the angular whirling of the rotor. For comparison, the same figures show similar graphs calculated for a rotor without an ABB (highlighted in blue). Fig. 6c and f show graphs of changes in the relative angles of deflection of balancing balls \( \psi_k, k = 1, 4 \) depending on time. The analysis of the plotted graphs allows us to conclude that the anisotropy of the supports does not significantly affect the process of autocalibration of a dynamically unbalanced rotor.

**Figure 7.** Autocalibration of the rotor rotating at constant angular acceleration in isotropic (a, b, c) and orthotropic (d, e, f) supports.

Let us now investigate the process of automatic balancing of a rotor rotating with constant angular acceleration \( u = 400 \text{ s}^{-2} \). The results of numerical integration of the system (4) for \( \theta(t) = ut^2/2 \) are shown in Fig. 7. The process of passage of the critical region and subsequent autocalibration for rotors fixed in isotropic and orthotropic supports is shown in Fig. 7a, b, c and d, e, f respectively. Comparing the graphs for amplitudes \( a_1 \) and \( a_2 \) (shown in Fig. 7a, b and d, e), we can conclude that the orthotropy of the rotor has a significant effect on the oscillations during the passage of the critical region, but has practically no effect on the autocalibration process and does not affect the final position of the balancing balls (shown in Fig. 7c and e).

Thus the results of analytical and numerical study confirm that the complete autocalibration of a dynamically unbalanced rotor, fixed in viscoelastic orthotropic supports and rotating at an angular velocity exceeding the maximum critical speed of the rotor, is possible and the orthotropy of the supports insignificantly affects the autocalibration process, but has a significant effect on the amplitude of the rotor oscillation when passing through the critical speed region.

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