ON THE RATIO OF CIRCUMFERENCE TO DIAMETER FOR THE LARGEST OBSERVABLE CIRCLES: AN EMPIRICAL APPROACH

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ABSTRACT

I present here a measurement of \( \pi \) as determined for the largest observable circles. Intriguingly, the value of 16/5 asserted by the House of Representatives of the State of Indiana in 1897 is still viable, although strongly disfavored relative to 22/7, another popular value. The oft-used ‘small-circle’ value of 3 is ruled out at greater than 5\( \sigma \). We discuss connections with string theory, sterile neutrinos, and possibilities for (very large) lower limits to the size of the Universe.

Subject headings: cosmology – cosmology:cosmic microwave background – cosmology: observations – large-scale structure of universe

1. INTRODUCTION

The ratio of the circumference of a circle to its diameter is commonly referred to as \( \pi \) and is one of the most important constants, with many applications across a great many fields of science. It is the only mathematical constant that has its own widely-recognized day of celebration \(^1\).

The value of \( \pi \) has been controversial at least since the time Pythagoras drowned a man for asserting its irrationality. This controversy persists despite theoretical calculation of its value to ridiculously high precision and proofs of its irrationality.

Given the importance of \( \pi \) it is surprising that there is not much of a literature on its measurement since Gauss effectively measured it by testing that the sums of the interior angles of a triangle formed by 3 mountain tops added up to 180 degrees \(^2\). I provide a measurement here.

This work is inspired by the Indiana House of Representatives’ unanimous vote for a bill that asserted the value of \( \pi = 3.2 \). Since this was an establishment of the value of \( \pi \) by legislative means we will refer to it as \( \pi_{\text{Leg}} \).

Most people react with astonishment to this vote, immediately jumping to the conclusion that it is self-evidently false. Even at the time the Senate did not go along, so it never became law. But what if the basis for the House Bill, and indeed its actual meaning, has been misunderstood all this time? What if they were on to something?

It is especially troubling that the Senate did not even allow a vote on the measure. How can we know the truth when this bill never even received the open give and take of parliamentary debate? Was the Senate trying to hide something?

Fortunately, there are other ways to know. We can measure it! An immediate question is what kind of circle we should use. Circles around spherical mass distributions have \( \pi < \pi_{\text{flat}} \) where \( \pi_{\text{flat}} \) is the ratio of circumference to diameter in Euclidean geometry. Whereas \( \pi_{\text{Leg}} > \pi_{\text{flat}} \). Such circles will not help us to see how 3.2 can be the correct value.

Thus we turn to cosmological circles. The larger a circle we consider, the greater the effect of curvature and so we wish to study the largest possible circles. Here we conservatively limit ourselves to our own horizon, though consider larger circles in the Discussion.

2. RESULT

I assumed a \( \Lambda \)CDM cosmology with the usual six vanilla parameters, plus one parameter for the mean curvature. Using a chain available on the LAMBDA archive that included WMAP7 \(^3\) \cite{Bent2011}, BAO \(^4\) \cite{Percival2010} and \( H_0 \) measurements \(^5\) \cite{Riess2009}, as described in \cite{Komatsu2011}, I calculated the comoving angular-diameter distance to the horizon, \( D_A \) and also the comoving distance to the horizon, \( r_H \). Then I defined

\[
\pi_{\text{Hor}} = 2\pi_{\text{flat}} D_A / (2r_H) = \pi_{\text{flat}} D_A / r_H.
\]

Histogramming the chain results in the probability distribution displayed in Fig. 1.

3. DISCUSSION

Note that in a \( \Lambda \)-dominated Universe with a small amount of mean curvature the value of \( \pi \) varies with time. We define a “Legislative Universe” (LU) as one for which \( \pi_{\text{Hor}} = 3.2 \) at some point in time. In future work we will determine the boundary in the \( \Omega_K-\Omega_V \) plane dividing LUs from non-LUs.

We note that LUs require a negative spatial curvature (positive \( \Omega_K \)). Perhaps coincidentally, from the string theory landscape we also expect our Universe to be one with negative spatial curvature, since tunneling events result in such a geometry. Or perhaps this “coincidence” points toward some deep connection between string theory and the theoretical underpinnings of the Legislative value. We tentatively posit that while \( \pi_{\text{Hor}} > \pi_{\text{flat}} \) would be evidence for string theory, \( \pi_{\text{Hor}} = 3.2 \) would be evidence of this connection. It might instead be evidence of a connection with the de Sitter equilibrium cosmology \(^6\) \cite{Albrecht2011}, for which one also expects a small negative curvature.

\(^1\) That day is, of course, 3/14. In North America this is March 14th, while in much of the rest of the world the interpretation is not quite as straightforward but usually taken to be the 3rd of February using the convention that months numbered greater than 12 are to be understood mod 12.

\(^2\) Zoltan Haiman, personal communication
If one adopts a prior that the Legislative value must hold for some circle in the Universe, then we can turn the arguments used here around to place a lower limit on the size of the Universe. As long as mean curvature is less than zero (no matter how close it is to zero), if the Universe is big enough, the Legislative value will hold for a big enough circle.

We have explicitly assumed the standard cosmological model extended to include non-zero mean curvature. Departures from this model would change our interpretation of the data. In particular, if we consider a model with fewer species of neutrinos then this increases the sound horizon at last scattering (Hou et al. 2011). In order to keep the angular size of the sound horizon fixed to the observed value, the increased sound horizon means the model would have to adjust in a way to increase the angular-diameter distance to last scattering. One way to do this is to increase $\Omega_K$ (Smith et al. 2011). Since the data prefer not fewer species, but an excess of species, allowing $N_{\text{eff}}$ to vary would put further pressure on $\pi_{\text{Hor}} = \pi_{\text{Leg}}$. Conversely, adopting a “Legislative prior” would increase the posterior probability for the standard model value of $N_{\text{eff}}$ relative to $N_{\text{eff}} = 4$.

We look forward to new data from the South Pole Telescope and Planck, combined with new BAO measurements, from which we can place tighter constraints on $\pi_{\text{Hor}}$, or, by adopting the Legislative prior, interesting lower limits to the size of the Universe.

I thank Ryan Foley for encouraging the communication of this work and Zoltan Haiman for further encouragement as the final hours of April 1, 2012 were suddenly upon me.

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