Study on the Expansion of Primary Flow for the Mixing Uniformity in the Two-Phase Ejector

Lixing Zheng, Hongwei Hu and Changning Mi
School of Electric Power, Civil Engineering and Architecture, Shanxi University, Taiyuan, Shanxi, China.
Email: lxzheng@sxu.edu.cn

Abstract. The expansion of primary flow in the suction chamber of the CO\textsubscript{2} two-phase ejector is investigated and its influences on the mixing characteristics are analyzed. An ejector model is developed, by constructing differential equations for mass, momentum and energy then get the governing equation. In the suction chamber, the expansion of primary flow and the compression of secondary flow are modeled along the flow path. Based on the constant-pressure mixing theory, the pressure equilibrium positions of two stream (namely at the inlet and inside of mixing chamber, respectively) are considered. The mass and energy transfer in the mixing chamber were analyzed by using the double-flow model formulation. The ejector performance parameters are obtained for the different operation conditions, and the distributions of temperature and velocity of two streams in the mixing chamber are presented. The simulation results showed the influence of primary flow expansion on the pressure lift ratio was relatively obvious, and the larger expansion distance was helpful to improve the mixing efficiency and decrease the thermodynamic entropy change during the mixing. Moreover, the temperature of secondary flow for lower primary flow pressure presented larger descent rates at the initial of mixing. This work is helpful for the improvement of ejector theoretical model and the optimization design.

Keywords. Suction chamber, CO\textsubscript{2} ejector, expansion, mixing uniformity.

1. Introduction
Nowadays, energy and environmental problems are becoming increasingly prominent. In the field of energy, research is developing towards energy conservation and efficiency improvement. Using CO\textsubscript{2} fluid as working medium can improve system performance and is widely used [1]. CO\textsubscript{2} is considered as a very promising working fluid substitute because of its low price, zero ozone depletion potential, almost zero GWP effect, non-toxic and nonflammability advantages [2]. At the same time, due to the special thermal and physical properties of CO\textsubscript{2}, CO\textsubscript{2} gradually becomes the working fluid that people are keen to study.

In most applications of transcritical carbon dioxide refrigeration systems, the CO\textsubscript{2} two-phase ejector has significant advantages. It can recover the expansion work lost during the throttling process, so it has great potential as an alternative method to improve cycle efficiency [3]. Although the structure of the ejector is simple, its internal flow characteristics are very complex, including supersonic flow, phase change, and mass transfer, momentum transfer and energy transfer between primary and secondary flows. Therefore, it is necessary to study its flow characteristics for the ejector design and optimization.
Many studies were carried out by establishing theoretical models and experiments tests to several related performance parameters of the ejector were studied. Haida et al. [4-5] reported a series of researches on the CO\textsubscript{2} two-phase ejector of the homogeneous equilibrium model and establishment the homogeneous relaxation model. After that, the model was continuously improved to better describe the internal flow field of the CO\textsubscript{2} ejector, thereby improving of ejector efficiency and other related performance. Zhu et al. [6] studied the specific flow state mechanism of supersonic two-phase flow of CO\textsubscript{2} ejector and the influence law of ejector performance through visual experiments. Through photography could intuitively see the characteristics of primary flow. At the same time, the influence of primary flow expansion angle on ejector performance was also studied. Li et al. [7] also used the visualization experiment in studying the two-phase flow characteristics of the trans-critical CO\textsubscript{2} ejector. For the isentropic expansion process of primary flow, the change of phase transition position was obtained on the basis of experiment and theoretical analysis. In these literatures, there are few studies on the fluid mixing between the suction chamber and the mixing chamber in the CO\textsubscript{2} ejector. Zheng and Deng [8] used the lumped parameter method and took the isentropic flow as the energy conservation equation to analyze the suction chamber. Wang and Yu [9] used the lumped parameter method as the control equation of the suction chamber and developed a thermodynamic model of the ejector.

In terms of ejector design theory, Keenan et al. proposed mixing theory with the same pressure and mixing theory with the same area. For the same pressure mixing theory, the equal pressure was kept and mixed the primary flow and secondary flow; the constant-area mixing theory is to mix the primary flow and the secondary flow with a constant cross area in the ejector mixing chamber. In specific applications, the constant-pressure mixing theory is better than the constant-area mixing theory. Taleghani et al. [10] came up with a newly ejector model where it was assumed that the two flows reached the same pressure before the inlet of the mixing chamber. Zhu [11] assumed that the pressure of the primary flow at the inlet of the mixing chamber was the same as the inlet pressure of the suction, and then the two flows maintained a constant pressure. However, for some working conditions, the primary flow may continue to expand in the mixing chamber after passing through the suction chamber. As a consequence, it is indispensable to study the primary flows expansion inside the suction chamber and mixing section to analyze the mixing characteristics.

This paper mainly studies the pre-mixing and mixing characteristics of CO\textsubscript{2} two-phase ejector. The governing equations are expressed using differential equations for mass, energy, and momentum. The mixing chamber is modeled by using a double-flow model, and the mass and energy transfer during mixing are considered. Two mixing conditions (I two streams reach the constant pressure at the mixing inlet, II two streams reach the constant pressure inside the mixing chamber) are discussed, and the effect for the mixing performance of ejector efficiency is analyzed. The entrainment ratio, pressure lift ratio, mixing efficiency and thermodynamic entropy change under different working conditions are obtained. The temperature and velocity distribution of the primary flow and the secondary flow in the mixing chamber are described in order to study the mixing characteristics of the ejector.

2. Mathematical Model Formulation
An ejector model was presented in which the circulation path is divided into many elements, and the governing equation were expressed through the differential equations of mass transfer equation, momentum equation and energy exchange equation. In the suction chamber, the differential equations of the ejector fluid primary flow and the secondary flow were formulated separately. In the model, there are different assumptions, the mixing principle and the equation solving method are different, so the assumptions used to facilitate the problem in the presented CO\textsubscript{2} two-phase ejector fluid mixing model are as follows:

- Flow of the entire ejector was one dimensional and steady.
- The two-phase flow was assumed to be a homogeneous equilibrium model.
- The influence of thermal diffusion and the turbulent viscous heating was not considered for two flows of fluid in the ejector.
The motive nozzle was the type of converging and the fluid was chock at the nozzle throat position.

The non-isentropic oblique wave and shock wave were neglected and the two streams were assumed no interaction along the flow path.

Coefficient $\psi$ was assumed to represent the primary flow expansion and the secondary flow acceleration.

The primary flow and secondary flow reached the uniform pressure and then mixed with the same pressure.

The mass transfer between the two streams was calculated by the condensation and entrainment mechanisms [12].

The momentum transfer was determined by the resistance of interface between the primary flow and the secondary flow these two fluids and the mass transfer is the main cause of momentum gain or loss.

The energy transfer was denoted by the enthalpy and the mass transfer leaded to the kinetic energy gain or loss.

The friction between the two fluids and the wall causes a pressure drop inside the ejector.

The heat transfer process between the fluid medium and the wall was ignored.

2.1. Single-Flow (Motive Nozzle, Suction Chamber)

Based on the previous set of assumptions, the transfer equation between masses, momentum change equation and governing conservation equation of energy, along with the state equation of primary flow and secondary flow these two fluids in the motive nozzle and suction chamber represented a system of ordinary differential equations, the specific equation is as follows:

\[
\frac{1}{V} \frac{dV}{dl} + \frac{1}{A} \frac{dA}{dl} + \frac{1}{\rho} \frac{d\rho}{dl} = 0
\]

\[
A \frac{dp}{dl} + \rho VA \frac{dV}{dl} + f \rho \frac{V^2}{2} \frac{dF}{dl} = 0
\]

\[
\frac{dh}{dl} + V \frac{dV}{dl} = 0
\]

\[
\frac{d\rho}{dl} - \frac{\partial \rho}{\partial s} \left|_{\rho} \right. \frac{ds}{dl} - \frac{\partial \rho}{\partial \rho} \left|_{\rho} \right. \frac{dp}{dl} = 0
\]

The corresponding integral of the established differential equation is obtained in matrix form was shown in equation 5.

\[
\begin{bmatrix}
0 & \frac{1}{V} & 0 & \frac{1}{\rho} \\
A & A \rho V & 0 & 0 \\
0 & V & 1 & 0 \\
\frac{\partial \rho}{\partial P_{ix}} - \frac{1}{T \rho} \frac{\partial \rho}{\partial s} & 0 & \frac{1}{T} \frac{\partial \rho}{\partial s} & -1 \\
\end{bmatrix}
\begin{bmatrix}
\frac{dp}{dl} \\
\frac{dV}{dl} \\
\frac{dh}{dl} \\
\frac{d\rho}{dl} \\
\end{bmatrix}
= \begin{bmatrix}
-\frac{1}{A} \frac{dA}{dl} \\
-f \rho \frac{V^2}{2} \frac{dF}{dl} \\
0 \\
0 \\
\end{bmatrix}
\]

Among them, the derivative of pressure, velocity, enthalpy and density are the matrix of the unknowns, $X=[dp/dl \ dV/dl \ dh/dl \ d\rho/dl]^T$. During the calculation of primary flow expansion, we firstly
assumed the velocity of primary flow inlet \( V_{p,in} \), and the inlet parameter including the primary flow pressure \( p_p \), temperature \( T_p \), inlet area \( A_{n,in} \), throat area \( A_t \) and axis length \( L_n \) of the motive nozzle were the known parameters. The partial derivatives of \( \frac{\partial p}{\partial s} \) and \( \frac{\partial p}{\partial s} \), according to the state parameters and geometric structure of each element, the changeable cross-sectional area along the flow path \( dA/dl \) and the derivative of lateral surface area \( dF/dl \) were obtained. Owing to the expansion of the flow in the nozzle, two-phase flow will occur and develop, so the sound velocity was dealt with each case on its merits referring to the literature [13]. In addition, the friction factor \( f \) was predicted by literature [14]. The system differential equations were resolved through the fourth order Runge-Kutta method, and the fluid pressure \( p_n \), specific enthalpy \( h_n \), velocity \( V_n \) and density \( \rho_n \) were obtained. When an element was completed, the next continued to compute until the nozzle throat position, and the Mach number was equal to 1.0 as the iteration condition.

For the calculation in the suction chamber of CO\textsubscript{2} ejector, which was similar that in the motive nozzle. Herein, the expansion angle of primary flow at the motive nozzle outlet \( \gamma_n \) was assumed. The inlet parameter of the secondary flow including pressure \( p_s \), temperature \( T_s \), inlet area \( A_{s,in} \), velocity \( V_{s,in} \) as well as the outlet parameters of motive nozzle were the known parameters. The exit position (NXP) of the motive nozzle with respect to the mixing chamber was utilized to calculated the cross area of two streams along the passage. For this paper, we discussed the performance parameters and flow distribution when the position of two streams reaching the constant pressure was at the inlet and inside of mixing chamber, respectively. To be more specific, the separation from the motive nozzle outlet position to the position of two streams reaching constant pressure \( L_s \) was 8.4mm and 9.4 mm, respectively. The former was equal to the value of NXP which meant the pressure equivalent position was at the inlet of mixing chamber while the later was larger than NXP and the position inside the mixing chamber. To combine the known structure parameters of suction chamber, the flow parameters are determined and then iterated the expansion angle \( \gamma_n \). Then the entrainment ratio \( \mu \) (\( \mu = \dot{m}_s / \dot{m}_p \)) of ejector performance parameter and the expansion coefficient of primary flow (\( \psi = A_{p,mix}/A_n \)) was obtained, where \( A_{p,mix} \) was the cross area of primary flow occupied at the pressure equivalent position for the two streams. Figure 1 shows the iteration of the primary flow and secondary flow in the suction chamber.
2.2. Double-Flow (Mixing chamber, Diffuser)

The molding of mixing chamber and diffuser are created by using a double-flow model formulation. The coaxial double-fluid flow was formed through the primary flow ($\alpha$) and secondary flow ($\beta$), where the primary flowed in the middle and secondary flow in a loop. The equations of mass, momentum as good as energy for mixing chamber and diffuser were similar, while the geometric relationship was different. The detailed governing equations and the variable parameter could refer to the previous literature [8].

The nine-order system variation was obtained: \[ X = \begin{bmatrix}
\frac{dp}{dl} & \frac{dV_\alpha}{dl} & \frac{dV_\beta}{dl} & \frac{dh_\alpha}{dl} & \frac{dh_\beta}{dl} & \frac{dp_\alpha}{dl} & \frac{dp_\beta}{dl} & \frac{dA_\alpha}{dl} & \frac{dA_\beta}{dl}
\end{bmatrix}^T. \]

The inlet conditions were the results of previous calculations, which included pressure $p$, velocities $V_\alpha, V_\beta$, specific enthalpies $h_\alpha, h_\beta$, densities $\rho_\alpha, \rho_\beta$, and the cross-sectional area $A_\alpha, A_\beta$ of the flow passage. Figure 2 shows the calculation procedure of the mixing chamber.
The main losses in the mixing chamber were from the influence of friction factors and the change in the transfer of momentum. Therefore, the mixing chamber efficiency $\eta_{\text{mix}}$ was proposed for the loss, which involves the momentum transfer inefficiency of primary flow and secondary flow, the friction impact at the duct wall and the secondary flow as well as the imaginary mixing layer between two streams of fluid. Therein, we referred to the expression proposed by the literature [8]. In addition, due to the above mix efficiency describing the momentum conservation, lacking the representation of energy loss, while the thermodynamic entropy change more effective to represent the irreversibility generated in the fluid core. Thus the entropy change in the mixing chamber was also calculated:

$$\Delta S_{\text{mix}} = n_p \left( s_{p, \text{out}} - s_{p, \text{in}} \right) + n_s \left( s_{s, \text{out}} - s_{s, \text{in}} \right)$$

After calculation of the mixing chamber, the similar procedure was applied to the diffuser and the pressure lift ratio ($\lambda = p_{\text{out}}/p_s$) could be obtained.

3. Simulation Conditions

3.1. CO$_2$ Ejector Structure

In this paper, the ejector structure referred to the previous published article [8], which proposed an adjustable ejector and carried out the experiment test to analyze the ejector performance. Herein on this basis the simulation study was conducted. The basic structure and dimensions of the ejector used in this model are shown in figure 3, and the corresponding size values of the ejector are listed in table 1.
3.2. Working Conditions
This paper studies the ejector performance under various working conditions. The primary flow parameters: pressure $p_p$ are 9.0 MPa, 10.0 MPa, and 11.0 MPa respectively; temperature $T_p$ is equal to 313.5 K. The secondary flow parameters: pressure $p_s$ increases from 3.2 MPa to 3.9 MPa with an interval of 0.1 MPa; temperature $T_s = T_{sat}(p_s) + 5.0$ K. The diameter of the nozzle throat is 0.546 mm. In order to compare the difference of mixing performance between the expansion distance $L_s$ equaling to NXP (8.4mm) and $L_s$ larger than NXP, the simulation calculations were performed. When the two streams reached the constant pressure inside the mixing chamber, the position was located at the one fifth of mixing chamber length, namely $L_s$ was equal to 9.4mm. In addition, during the formulation mentioned in section 1.2, the friction factor $\varepsilon$ referenced for the object of two-phase flow was 0.2 for mixing section and diffuser, respectively. The scale coefficient $a$ in the interface drag coefficient presented in the differential of the interface momentum transfer rate, which was 0.5. The flow process of primary flow and secondary flow is always accompanied by the mutual transfer of heat, and the heat transfer coefficient between the two flow was referred to literature by Hwang [15].

4. Results and Discussions
Figure 4 indicates how the entrainment ratio $\mu$ changes as the secondary flow inlet available pressure $p_s$ under different primary pressures $p_p$ corresponding to the two expansion distances. It can be seen from the curve variation trend in this figure, as the secondary flow pressure of the ejector increase, the entrainment ratio $\mu$ also gradually increases; the results show that, under the same working conditions, the entrainment ratio $\mu$ corresponding to the larger expansion distance is also larger, which means that the fluid is the expansion inside the mixing chamber is larger than the entrainment ratio $\mu$ produced by the expansion at the inlet; the magnitude of the primary flow pressure will also produce the same result. This is because an increase in the expansion distance will cause the expansion
coefficient of the primary flow to increase, which in turn increases the secondary flow velocity. Even if the cross-sectional area occupied by the secondary flow in the ejector is gradually reduced, the velocity increases faster under this working condition, which ultimately leads to an increase in the entrainment ratio $\mu$.

From figure 5, we can see the relationship between the pressure lift ratio $\lambda$ and the secondary flow available pressure, and at the same time, it can also reflect the corresponding pressure lift ratio $\lambda$ relationship between the expansion distance and the primary flow change of pressure level. The results show that the pressure lift ratio $\lambda$ decreases as the fluid flows increase of secondary flow pressure. It could be found that when the expansion distance of primary flow $L_s$ was larger than NXP (8.4 mm), $\lambda$ decreased and it was evident when the $p_p$ was higher value. This is mainly because higher secondary flow pressure is more conducive to carrying more mass secondary flow, and for the primary flow, the higher the pressure, the more energy the primary flow has, and the more mass secondary flow can be sucked up. Expansion in the mixing chamber is beneficial to improve the ejector coefficient.

Figure 4 and figure 7 show the simulation results of variation of efficiency and entropy change respectively in the mixing chamber. As we can see, with the build up secondary flow inlet pressure $p_s$, the mixing efficiency increases gradually, while the entropy change shows the opposite change. The mixing efficiency of $L_s$ equal to 9.4 mm is obviously higher than that of $L_s$ equal to 8.4 mm, this is because the friction and momentum transfer between primary flow and secondary flow is less than $L_s$ equal to 8.4 mm, which leads to the increase of mixing efficiency, but for entropy change, the simulation result of expansion distance of 8.4 mm is higher than that of 9.4 mm. When the expansion distance is 8.4 mm, under the same secondary flow inlet pressure, the mixing efficiency with low primary flow pressure is relatively higher, but the entropy change is opposite. On the whole, the expansion distance can affect the efficiency of the mixing chamber in the ejector, and the efficiency is also closely related to the pressure of the two fluids.
Figure 6. $\eta_{\text{mix}}$ vs $p_s$ for different $p_p$ and $L_s$.

Figure 7. $\Delta S$ vs $p_s$ for different $p_p$ and $L_s$.

Figure 8 and figure 9 show the temperature and velocity distributions of the two streams of fluid in the mixing chamber when the evaporation pressure is 3.9 MPa. Among them, the parameters of the primary flow were represented by solid lines, and the parameters of the secondary flow were represented by dashed lines. It can be seen from figure 8 that with the extension of the flow path, the temperature of the primary flow increases and the temperature of the secondary flow decreases, but the temperatures of the two flows are finally close. When the primary flow pressure was low, the temperature of the leading secondary flow decreases rapidly at the beginning of mixing. As can be seen in figure 9, along the flow path, the primary flow velocity gradually decreases while the secondary flow velocity increases, which is mainly due to the mass and energy transfer between the two streams.

Figure 8. Temperature distribution along the axis of mixing chamber ($p_e$=3.9MPa).

Figure 9. Velocity distribution along the axis of mixing chamber ($p_e$=3.9MPa).

5. Conclusions
The effect of primary flow expansion on mixing performance in the suction chamber of CO$_2$ two-phase ejector was studied in this study. The ejector model was established by using the distributed parameter method, especially for the suction chamber, the expansion of the primary flow and the compression of the secondary flow are established by using the differential equations of mass, momentum and energy conversion. The conditions under which the two kinds of flow reach equal
压力在混合室的入口和内部混合室被分析，并且它们对不同工作条件下的喷射器性能差异的影响进行了比较。

当主流和次级流达到相同的内部压力时，进气比$\mu$大于在两个流体压力等同位置位于入口时，而进气提升比$\lambda$则相反。此外，混合室效率$\eta_{\text{mix}}$对扩张距离$L_s$大于NXP时更大，当$L_s$等于NXP时。由于质量、动量和能量的转移，这些不可逆损失导致混合室的熵变，当$L_s$大于NXP时，它小于$L_s$等于NXP时。这一研究帮助理解了喷射器内部的流场分布，了解了喷射器的高效设计。当模拟吸入室时，分布参数方法被使用，期间，两流体在流过程中，非等熵偏斜波和激波波的相互作用被忽略，所以模型应继续改进在未来的研究。

**Acknowledgments**

这项工作得到了中国国家自然科学基金（项目号51806132），山西省应用基础研究项目基金（项目号201801D221353）和山西省科学技术创新高校和大学（项目号201802011）的资助。

**Reference**

[1] Liu Y X, Zhao Y Y, Yang Q C, Liu G B and Li L S 2020 *Case Studies in Thermal Engineering* 21
[2] Dai B M, Zhao P, Liu S C, Su M Q, Zhong D, Qian J B, Hu X W and Hao Y 2020 *Energy Conversion and Management* 222.
[3] Taleghani S T, Sorin M, Poncet S and Nesreddine H 2019 *Energy Conversion and Management* 185 442–54
[4] Haida M, Smolka J, Hafner A, Ostrowski Z, Palacz M, Madsen K B, Forsterling S, Nowak A J and Banasiak K 2018 *Energy Conversion and Management* 153 933-948
[5] Haida M, Smolka J, Hafner A, Palacz M, Banasiak K and Nowak A J 2018 *International Journal of Refrigeration* 85 314-33
[6] Zhu Y H, Wang Z C, Yang Y P and Jiang P X 2017 *International Journal of Refrigeration* 74 354-61
[7] Li Y F, Deng J Q, Ma L and Zhang Y Z 2018 *Energy Conversion and Management* 71 729-41
[8] Zheng L X and Deng J Q 2017 *Energy Conversion and Management* 142 244-56
[9] Wang X and Yu J L 2016 *International Journal of Refrigeration* 71 26-38
[10] Taleghani S T, Sorin M and Poncet S 2018 *International Journal of Refrigeration* 87 91-105
[11] Zhu Y H and Jiang P X 2018 *International Journal of Refrigeration* 86 218-27
[12] Banasiak K and Hafner A 2011 *International Journal of Thermal Sciences* 50 2235-47
[13] Cardemil M J and Colle S 2012 *Energy Conversion and Management* 64 79-86
[14] Revellin R and Thome J R 2007 *Experimental Thermal and Fluid Science* 31 673-85
[15] Hwang Y and Rademacher R 1998 *HVAC&R Research* 4 245-63