Damage localization in plates with use of the procedure based on Modal Filtration

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Abstract. Modal filtering has numerous applications in the analysis of object dynamics. One possible use of this technique, recently presented in the literature, is damage detection. The main idea behind it, is to compare system characteristics filtered with modal filter for damaged and undamaged state. If a structural change appears in the system the filter does not work perfectly and its output significantly differs from one obtained for the healthy structure. Method is based on the modal parameter variation due to damage but does not require the modal analysis for every diagnosis. Another advantages are: computational simplicity and robustness for ambient temperature changes. The method was extended in 2008 with the possibility of damage localization. The idea here was based on the fact that local damage disturbs mode shapes locally. Instead of one modal filter, set of local modal filters were identified and used for further processing. Area of the structure connected with the filter with the worst performance is suspected of containing the damage. The method was already presented and positively verified but only for the beam-like structures. In this paper, application for plate-like structures is shown.

1. Introduction
Structural Health Monitoring (SHM) is a field of growing importance. Autonomous detection and localization of damage in structures allows for considerable reduction of expenses related to their operation as well as increase in safety. Large amount of methods were proposed in this scope, including ultrasonics [1, 2], vision-based methods [3], acoustic emission [4] and many more. Among those, vibration-based methods play an important role, due to their global character and relative insensitivity to environmental changes [5]. Damage can be detected by observation of changes in natural frequencies [6], features calculated from signal spectrum or obtained by time-frequency methods [7]. In Modal Analysis, a modal model (MM) of an object under investigation is build out of vibration signals acquired in distinct points of its structure. Signals are usually acquired with sensors attached to the structure. However, several non-contact techniques were also proposed [8, 9]. MM consist of natural frequencies associated with damping factors and Modal Vector (MV). Damage detection with use of MM is performed by comparing model calculated for a known, intact state of a structure with one acquired in unknown conditions. Modal analysis has been successfully applied in many cases of SHM [9–11]. One of approaches employ Modal Filtration (MF) introduced by Meirovitch and Baruh [12]. It consists of calculation of a set of Reciprocal modal vectors (RMVs), out of which each is orthogonal to all but one Modal Vector. That feature can be used in decomposition of a system response to components related to natural frequencies. Occurrence of damage results usually
in local drop of stiffness, which, in turn, cause changes in modal parameters. Signal acquired in such a case cannot properly be decomposed with use of previously calculated RMVs. In contrast, global change of parameters does not modify the modal vectors and thus does not alter the output of decomposition with RMVs [13]. Modal filtration can successfully be used in damage detection under variable environmental conditions. However, its main disadvantage lies in inability to locate damage. That issue can be solved by dividing a structure into areas, for which modal filtration is performed separately. The extent of mode shapes change is dependent to the proximity of damage. This solution introduced in [14] was successfully tested for truss structures [15]. In this paper, the authors verified its efficiency on a plate-like structure.

2. Local modal filtration

The idea to extend the MF method by adding damage localization [13, 14] is based on the fact that damage, in most cases, only disturbs the mode shapes locally. That is why many methods of damage localization use mode shapes as input data [16–18]. It is then possible to divide an object into areas measured with the use of several sensors and build separate local modal filters for data coming only from these sensors. In areas without damage, the shape of modes does not change and the modal filter keeps working - there are no additional peaks on the filter output. When a group of sensors placed near the damage is considered, mode shape is disturbed locally due to damage and the MF does not perfectly filter the characteristics measured by these sensors.

It has been said [19] that the minimum number of sensors required to build an effective MF is equal to or greater than the number of modes in the frequency range of interest. Thus, by limiting the frequency range of analysis to the first two modes, it would be theoretically possible to construct MFs for object areas measured with only 2 sensors. This would significantly increase the damage location accuracy. In practice, however, to construct the modal filter for two modes, data from at least 4 sensors has to be used. The accuracy of damage localization depends on measuring net density. The graphical presentation of this idea is presented in Figure 1.

For each section tuning of local modal filter (LMF) is performed according to following procedure: The construction of the r-th modal filter, which corresponds to r-th pole of the transfer function $H(\omega)$, starts with the determination of the theoretical output of the considered modal filter $G_r(\omega)$. The function is calculated basing on the modal parameters of the system:

![Figure 1. Scheme of the proposed method of damage localization](image)
\[ G_r(\omega) = \frac{2\xi_r\omega_r^2}{\omega_r^2 - \omega^2 + 2j\xi_r\omega^2} \] (1)

Where \( \xi_r \) denotes \( r \)-th modal damping coefficient while \( \omega_r \) represents \( r \)-th natural frequency.

For the given frequency range, the filter output defined by eq. (1) is determined in the \( k \) values:

\[
H_{kN}(\omega) = \begin{bmatrix}
H_1(\omega_1) & H_2(\omega_1) & \ldots & H_N(\omega_1) \\
H_1(\omega_2) & H_2(\omega_2) & \ldots & H_N(\omega_2) \\
\vdots & \vdots & \ddots & \vdots \\
H_1(\omega_k) & H_2(\omega_k) & \ldots & H_N(\omega_k)
\end{bmatrix}
\] (2)

The FRF matrix formed in this way is used to determine the \( r \)-th reciprocal modal vector \( \Psi_r \):

\[
\Psi_r = H_{kN}^+ G_r
\] (3)

Which can be used to decompose acquired system response \( \{x(\omega)\} \) to modal coordinates \( \eta_r \):

\[
\eta_r(\omega) = \Psi_r^T \{x(\omega)\}
\] (4)

In the monitoring phase, some unknown signal \( \{x_u(\omega)\} \) is filtered by tuned RMVs according to the equation:

\[
\eta_{ru}(\omega) = \Psi_r^T \{x_u(\omega)\}
\] (5)

which, in turn, lead to FRFs. These can be compared with respective FRFs associated with a reference state. A measure of dissimilarity between both FRFs can be used as a damage index (DI). The authors decided to employ for this purpose a simple RMS of the signal calculated according to the equation:

\[
DI_{RMS} = 1 - \frac{\int_{t_1}^{t_2} [y(t) - x(t)]^2 \, dt}{\int_{t_1}^{t_2} x(t)^2 \, dt},
\] (6)

It is expected, that occurrence of damage would influence mainly those LMFs, which are placed in damage neighborhood. For practical applications it would be useful to set the threshold for the difference between damage index value for the regions (groups of sensors) with and without damage. Such a threshold would make it possible to detect damaged region more easily and even introduce some automation of diagnostic procedure. However, the threshold needs to be assessed individually depending on the object type and minimum damage size to be detected.

3. Experimental setup
The experiments were performed using a laser scanning Doppler vibrometer PSV-400, provided by Polytec, Germany. The device enabled contact-less measurement of out-of-plane plate velocity vibrations. The plate was fixed to the stand at one extreme. The signals were captured in 209 points distributed over the sample in rectangular-shape lattice (19x11 points), with horizontal spacing of 11 mm and vertical spacing of 16 mm. The acquisition was performed with sensitivity of the device was 50 mm/s/V and sampling frequency of 512Hz. Time duration of signal captured at each point was 32 s. The test sample was excited using white noise generated by a shaker K2004E01, provided by ModalShop. The excitation was transferred using a steel stinger to an
impedance head 288D01, provided by PCB, capable of force and acceleration measurements, however, only force signal was acquired for further processing.

Five measurements were performed: three for intact specimen, two for two damage cases. Two notches were introduced to the sample. Their location and size, along with specimen’s dimensions and measurement points placement, is depicted in Fig. 2(a). An image of the back side of the specimen after damage introduction is given in Fig. 2(b).

Figure 2. A scheme of specimen with measurement points’ locations marked by crosses. Circle refers to location of an excitation source (a). Image of the specimen is given in (b). Since frame of reference was defined on the face side of the beam, damage locations are shifted in X direction

4. Results
Signals acquired in first measurement step were used to perform modal analysis of a specimen and construct LMFs. For each measurement point its 3x3 neighborhood was used for creation of a local modal filter and corresponding FRFs. In following measurement steps, these filters were used for filtration of newly acquired data. FRFs obtained in that fashion were compared with corresponding reference FRFs with use of $DI_{RMS}$. Maximal DI acquired for all points under monitoring can be treated as a damage index for the whole specimen, thus damage is detected
Table 1. Results of damage localization with use of LMF. Percentage error given with respect to longest specimen dimension

| Damage location measured | Damage location detected | Error [mm] | Error [%] |
|--------------------------|--------------------------|------------|-----------|
| (30,140)                 | (88,150)                 | 58.85      | 14.71     |
| (35,250)                 | (36,246)                 | 4.12       | 1.03      |

by comparing value of DI with a given threshold. Once the threshold is exceeded, location of damage can be obtained by observation of DIs calculated for all LMFs.

Results of this procedure for 5th natural frequency are provided in Fig. 3. Detection and localization steps are both performed successfully: For undamaged state (Fig. 3(a) and Fig. 3(b)) maximum value of $D_{RMS}$ is significantly lower than for damaged states. Measured and detected damage location is given in table 1. Damage is not only located close to its actual position, but also correctly oriented and properly sized.

5. Conclusions

The method developed by authors was successfully applied in detection and localization of damage in plates. Good agreement between measured and detected location, size and orientation of damage reveals as well yet undiscovered potential for damage quantification. Since method has been tested both for truss structures and for plates, its wide potential of applicability include large amount structures, including bridges, cranes or high-voltage transmission lines. The main drawback of the developed solution is its vulnerability to selected mode shape. Selection of a proper mode is not easily achieved beforehand, since modes differ in their sensitiveness to damage in distinct areas of a specimen. Additional work should be performed to create a system that is able to perform diagnosis based on several modes at once.

6. Acknowledgements

This research was conducted under a GEKON1/02/214108/19/2014 project funded by NCBiR.

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Figure 3. Results of local modal filtration for intact state (a,b) and for damaged specimen (c,d) along with corresponding values of the $DI_{RMS}$. Since LMFs are constructed from 3x3 group of points, borders of the sample are excluded from the damage search. The light intensity refer to the value of $DI_{RMS}$ calculated for given LMF.