Constraints on Dark Energy Models in Cosmology from Double-Source Plane Strong Lensing System

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Abstract. We performed an investigation of the possibility of constraining the dark energy models such as w-cold dark matter, holographic dark energy, and Ricci dark energy, using a special type of Einstein ring system, called as double-source plane (DSP) lensing system. We found that DSP lensing system only gives a good constraint of the model parameters. We also found that the method highly complimentary to cosmic microwave background measurement for each model.

1. Introduction

One of the important problems in the current cosmology is to reveal the origin of the accelerated expansion observed today [12, 13]. If Einstein theory of gravity describes correctly the global behavior of the universe, the accelerated expansion is only possible if the universe is dominated by a component with the equation of state parameter \( w < -1/3 \). Such component is called the dark energy. Until now, Λ-cold dark matter (ΛCDM) is the leading candidate of the dark energy and consistent with the observational data. However, this model confronts with the severe theoretical problems, i.e. "fine tuning" and "cosmic coincidence" [2]. Various types of dark energy have been proposed, in terms of physics different from ΛCDM, i.e. w-cold dark matter (wCDM), holographic dark energy (HDE), Ricci dark energy (RDE) model, etc. These models are attracting lots of studies and good alternative candidate for dark energy [8, 17]. Each model must be tested with many observational data in all types and scales.

In order to obtain tighter constraints on parameters each dark energy models, we need more cosmological probes, for example, strong gravitational lensing. Strong gravitational lensing is potentially a powerful astrophysical tool to test cosmological models [4, 9, 10], especially for special case, double-source plane lensing system. The DSP lensing can provide more information than single-source plane lensing system about the proper angular diameter distances between lens and first source, lens and second source, observer and first source, and observer and second source. Since these distances depend on cosmological geometry, we can use its ratio to constrain the parameter in dark energy models.

In this paper, we investigated the possibility of constraining some dark energy models using DSP lensing system. Collet et al [4, 5] showed that a single DSP system give a good constraint.
Figure 1. Double-source plane lensing system scheme. For special case under a singular isothermal sphere assumption, the dimensionless distance ratio $\eta$ is the product of $D_{LS1}$ and $D_{S2}$ divided by the product of $D_{LS2}$ and $D_{S1}$.

on $w - \Omega_m$. Here, we follow their method for HDE and RDE, and wCDM as comparison to see how good the constraint from DSP lensing system for each model. We also assumed speed of light, $c = 1$.

2. Double-Source Plane Lens And Dark Energy Models

2.1. Double-Source Plane Lensing System

The DSP lensing system is a rare strong gravitational lensing system when two sources at different redshifts are on the line-of-sight with a lens and the observer (see Figure 1). In the lensing analysis, the Einstein radius ($\theta_E$) is a basic quantity, which under the singular isothermal sphere (SIS) model assumption can be expressed as

$$\theta_E = 4\pi \sigma^2_{SIS} \frac{D_{LS}}{D_S}$$  \hspace{1cm} (1)

where $\sigma_{SIS}$ is velocity dispersion of SIS model.

In a spatially flat Friedmann-Robertson-Walker (FRW) universe, the evolution of angular diameter distances as a function of redshift

$$D_A(z) = \frac{1}{H_0(1 + z)} \int_0^z \frac{dz'}{E(z')}$$  \hspace{1cm} (2)

where $E(z) \equiv H(z)/H_0$ is the normalized Hubble parameter. We define the quantity $\eta$ as the ratio of the two Einstein radii (for simplicity we neglect first source mass),

$$\eta = \frac{\theta_{E,1}}{\theta_{E,2}} = \frac{D_{LS1}D_{S2}}{D_{LS2}D_{S1}} = \eta_{SIS}$$  \hspace{1cm} (3)

Here we can see that the ratio $\eta$, is independent of the Hubble constant and mass (in the case of an SIS model), so it is a function only of unique parameters of each model and the redshifts of the lens and sources.

2.2. Dark Energy Model

The HDE and RDE model are dynamical dark energy models through the consideration of the holographic principle. In general dark energy density is described as follow

$$\rho_{de} \propto M_p^2 L^{-2}$$  \hspace{1cm} (4)
Figure 2. The redshift evolution of $E(z)$ of HDE, wCDM, and RDE model. In this figure $\Omega_{m0} = 0.27$ is adopted. We varied the unique parameters for each model that are shown by dashed, dashed-dot, and lines.

where $M_p$ is the reduced Planck mass defined by $M_p^2 = (8\pi G)^{-1}$ and $L$ is the infrared (IR) cutoff length scale in the effective quantum field theory. For HDE, the total energy of a system with size $L$ should not exceed the mass of a black hole with the same size. In the RDE case, the IR cutoff $L$ is connected to Ricci scalar curvature, $R = -6(\dot{H} + 2H^2)$ (See [3, 17] for details).

We can get $E(z)$ for the HDE

$$\frac{1}{E(z)} \frac{dE(z)}{dz} = -\frac{\Omega_{de}}{1+z} \left( \frac{1}{2} + \frac{\sqrt{\Omega_{de}}}{C} - \frac{\Omega_r + 3}{2\Omega_{de}} \right)$$

(5)

$$\frac{d\Omega_{de}}{dz} = -2\Omega_{de}(1 - \Omega_{de}) \left( \frac{1}{2} + \frac{\sqrt{\Omega_{de}}}{C} \right) \frac{\Omega_r}{2(1 - \Omega_{de})}$$

(6)

where $\Omega_{de}$ is the fractional density of dark energy, the fractional density of radiation $\Omega_r = \Omega_{r0}(1 + z)^4/E(z)^2$ with $\Omega_{r0} = 2.469 \times 10^{-5} h^{-2} (1 + 0.2271 N_{\text{eff}})$ with the effective number of neutrino species $N_{\text{eff}} = 3.046$, and $\Omega_{m0}$ is the fractional density of matter at the present. We can solve equation 5 and 6 numerically to obtain $E(z)$ of HDE with initial conditions $E_0 = 1$ and $\Omega_{de0} = 1 - \Omega_{m0} - \Omega_{r0}$ at $z = 0$.

For the RDE,

$$E(z) = \sqrt{\frac{2\Omega_{m0}}{2 - \alpha}(1 + z)^3 + \Omega_{r0}(1 + z)^4 + \left( 1 - \frac{2\Omega_{m0}}{2 - \alpha} - \Omega_{r0} \right)(1 + z)^{4 - \frac{2}{\alpha}}}. \quad (7)$$

For the wCDM model ($w$ is a free parameter), $E(z)$ is given by

$$E(z) = \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{r0}(1 + z)^4 + (1 - \Omega_{m0} - \Omega_{r0})(1 + z)^{3(1+w)}}. \quad (8)$$

We can see from Figure 2 that each dark energy model will mimic certain model, for example, $\Lambda$CDM for certain value of model parameter.
Figure 3. The unique parameters of each model—$\Omega_m$ plane: blue contours are the constraint from our fiducial three DSP lensing system (left to right panel: wCDM, HDE, RDE) and black lines show CMB results [1, 11, 14], respectively. All dark energy models panel show the orthogonality between DSP and CMB constraint.

Table 1. Constraint results for the wCDM, HDE, and RDE models by using the DSP lensing system.

| Model | Parameter | wCDM | HDE | RDE |
|-------|-----------|------|-----|-----|
|       | $\Omega_{m0}$ | $w$  | $\Omega_{m0}$ | $C$  | $\Omega_{m0}$ | $\alpha$ |
| DSP   | $0.41^{+0.17}_{-0.21}$ | $-1.19^{+0.30}_{-0.48}$ | $0.33^{+0.17}_{-0.18}$ | $0.50^{+0.25}_{-0.29}$ | $0.30^{+0.13}_{-0.15}$ | $0.39^{+0.11}_{-0.15}$ |

2.3. Method

We followed Collet et al. [4] to investigate the unique parameters of each model and $\Omega_m$. We first chose some DSP lensing system, with $z_L = (0.222, 0.656, 0.795)$, $z_{S1} = (0.609, 1.847, 1.302)$, $z_{S2} = (2.400, 2.900, 1.998)$ from [9, 16, 15] respectively. We used a uniform prior of $0 \leq \Omega_m \leq 1$ and $-2 \leq w \leq -1/3$ (wCDM), $0 \leq C \leq 2$ (HDE), $0 \leq \alpha \leq 1$ (RDE) and assumed the observational precision on $\eta$ to be 1 per cent to perform a Markov Chain Monte Carlo (MCMC) [7] analysis of the unique parameters of each model and $\Omega_m$ plane. For our MCMC analysis, we assumed Gaussian likelihoods given by

$$L \propto \exp \left[ -\frac{1}{2} \frac{(\eta(\Omega_m, \text{par}) - \eta_{\text{fid}})^2}{\sigma^2} \right]$$  \hspace{1cm} (9)

where $\eta_{\text{fid}}$ is the value of $\eta$ with redshifts from observation and assume fiducial cosmology with $\Omega_m = 0.27$, $w = -1$, $C = 0.5$, $\alpha = 0.4$, $\text{par}=\{w, C, \alpha\}$.

3. Results

We present our investigation results of constraining the dark energy model on wCDM, HDE and RDE. We also can reproduced $w - \Omega_m$ plane on wCDM model as well as result from [4]. For each model, the use of DSP lensing data solely results in significantly less tight constraint on the parameters than that from CMB, however, the best-fit of $\Omega_{m0}$ is still close to our fiducial cosmology (see Figure 3 and Table 1). It is better than the result from single-source plane strong
gravitational lensing only which give the best-fit of $\Omega_{m0}$ which is one order smaller than current observations [6].

4. Conclusion
In this paper, we studied the possibility using DSP lensing system to constraint the dark energy models. We found that DSP is highly complimentary and breaking degeneracy between $\Omega_m$ and model parameter of HDE and RDE model which are almost orthogonal to CMB constraint. This result based on a simple assumptions of lens mass model, however this method gives an alternative. In future, large galaxy surveys will discover more DSP lensing systems that can improve this study.

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