MONTE CARLO SIMULATIONS OF THE PHOTOSPHERIC EMISSION IN GAMMA-RAY BURSTS

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Abstract

We studied the decoupling of photons from ultra-relativistic spherically symmetric outflows expanding with constant velocity by means of Monte Carlo simulations. For outflows with finite widths we confirm the existence of two regimes: photon-thick and photon-thin, introduced recently by Ruffini et al. (RSV). The probability density function of the last scattering of photons is shown to be very different in these two cases. We also obtained spectra as well as light curves. In the photon-thick case, the time-integrated spectrum is much broader than the Planck function and its shape is well described by the fuzzy photosphere approximation introduced by RSV. In the photon-thin case, we confirm the crucial role of photon diffusion, hence the probability density of decoupling has a maximum near the diffusion radius well below the photosphere. The time-integrated spectrum of the photon-thin case has a Band shape that is produced when the outflow is optically thick and its peak is formed at the diffusion radius.

Key words: gamma-ray burst: general – radiation mechanisms: thermal

1. INTRODUCTION

Photospheric emission is considered to be among the leading mechanisms generating gamma rays from relativistically expanding plasma in cosmic gamma-ray bursts (GRBs; Goodman 1986). In both fireball (Piran 1999) and fireshell (Ruffini et al. 2009) models, photospheric emission is naturally produced when expanding initially optically thick plasma becomes transparent for photons.

While within the fireball model the basic approximation involved is that of a (infinitely long) steady wind, in the fireshell model the geometry of the outflow is a thin shell. It is shown by Ruffini et al. (2011; hereafter RSV) that in both wind and shell models, when outflows with finite duration are considered, some important aspects of the optical depth behavior are overlooked. These authors introduced a new physically motivated classification: photon-thick and photon-thin outflows with, respectively, \( R_p^h < R_t \) and \( R_p^h > R_t \), with \( R_p^h \) being the photospheric radius, \( R_t = 2\Gamma^2 l \) the radius of transition, \( \Gamma \) the Lorentz factor, and \( l \) the laboratory width of the outflow. Light curves and spectra from finite outflows were obtained using two different approximations: sharp and fuzzy photospheres. These approximations were derived from the transport equation for radiation.

In this work we examine the validity of these approximations using Monte Carlo (MC) simulations of photon scattering. We compute the probability density function of the position of the last scattering in finite relativistic outflows. Then we obtain light curves and spectra from both photon-thick and photon-thin outflows, and compare them with the results obtained previously.

2. THE MODEL

Following RSV we consider a spherical wind of finite width \( l \) with the laboratory number density of electrons being zero everywhere except in the region \( R(t) < r < R(t) + l \) where it is

\[
n_t = n_0 \left( \frac{R_0}{r} \right)^2, \quad \Gamma(r) = \text{const},
\]

with \( R_0 \) and \( n_0 \) being, respectively, the radius and the density at the base of the outflow. This outflow expands with a constant Lorentz factor \( \Gamma \), so that \( R(t) = R_0 + \beta ct \), where \( \beta = \sqrt{1 - \Gamma^{-2}} \). Accelerating outflows are not considered here. Note that when there is strong relative difference of Lorentz factors within the outflow with \( \Delta \Gamma \sim \Gamma \), a radial spreading phenomenon occurs, as discussed in Piran et al. (1993) and Meszaros et al. (1993). The outflow width starts to increase linearly with \( R(t) \) at the radius

\[
R_s = \frac{\Gamma^2 l}{\Delta \Gamma}. \tag{2}
\]

It is clear from this formula that such a phenomenon occurs at radii much larger than \( \Gamma l \) when \( \Delta \Gamma < \Gamma \). In what follows we consider only such a case. Our results are valid for an arbitrary constant \( l \). The expression for the optical depth in the model defined by Equation (1) is found in RSV and it reads

\[
\tau(r, \theta, t) = \tau_0 R_0 \left[ \frac{\theta - \tan^{-1} \left( \frac{r \sin \theta}{\sqrt{c^2 t + r^2 \sin^2 \theta}} \right)}{r \sin \theta} \right] - \beta \left( \frac{1}{r} - \frac{1}{\sqrt{(c t + r \cos \theta)^2 + (r \sin \theta)^2}} \right), \tag{3}
\]

where

\[
\tau_0 = \sigma n_0 R_0 = \frac{\sigma E_0}{4 \pi m_p c^2 R_0 \beta}, \tag{4}
\]

and \( r \) is the position of photon emission, \( \theta \) is the angle between the momentum of a photon and the radius vector, \( \sigma \) is the Thomson cross section, \( t \) is the time the photon remains within the outflow, \( E_0 \) is the total energy release in the GRB, \( m_p \) is the proton mass, and \( c \) is the speed of light. Time \( t \) is found via the equations of motion of the outflow and of the photon.

The optical depth in finite relativistic outflows has two different asymptotes, depending on initial conditions: in the photon-thick case \( \tau \) is almost constant within the outflow while in the photon-thin case it linearly increases with depth from the outer boundary at \( R(t) + l \).

In RSV, diffusion of photons within the outflow is found to play a crucial role in the photon-thin case: the radius at
which photons effectively diffuse out of the outflow is smaller than the photospheric radius $R_{ph}$. The latter is defined by equating expression (3) to unity for emission with $\theta = 0$ and $t$ corresponding to the inner boundary. Hence, when the photon-thin outflow reaches the photospheric radius there are few photons left in it.

3. MONTE CARLO SIMULATIONS

We have used spherically symmetric MC simulations of photon scattering inside the outflow described by Equation (1). For more details on interactions in relativistic expanding plasma, see D. Bégúé et al. (in preparation). In this simulation each photon is followed as it interacts with electrons until it decouples from the outflow. Photons are injected into the outflow when the optical depth of the inner boundary is $\tau_i$, given in Table 1. It is also assumed that their initial distribution is isotropic and thermal. The radial position of each photon inside the outflow is chosen randomly.

The code consists mainly of a loop computing each scattering. We proceed with two steps. First, an infinite and steady wind with $l \to \infty$ and $R \to 0$ (already treated by Beloborodov 2010) is considered. For a given position characterized by $r$ and $\theta$ of a photon in the outflow, we compute a maximal value for the optical depth $\tau_{max}$ using Equation (3), with $r \to \infty$. The probability of the photon decoupling the outflow is $\exp(-\tau_{max})$. Then a random number $X \in [0,1]$ is chosen. On the one hand, if $X < \exp(-\tau_{max})$ the photon is considered decoupled. Afterward, the photon remains in the outflow but does not scatter. This case corresponds to decoupling in the photon-thick case, for which the presence of boundaries is not essential. On the other hand, if $X \geq \exp(-\tau_{max})$ the position $r_s$ of the next scattering is computed from the optical depth

$$\tau(r, r_s, \theta) = \tau_0 R_0 \left[ \frac{\theta - \theta_s}{r \sin(\theta)} - \frac{1}{r} \right] - \frac{1}{r_s}, \quad (5)$$

where $\theta_s$ is the angle between the photon momentum and the radius vector at the position of scattering. The new radial position $r_s$ such that $\exp[-\tau(r, r_s, \theta)] \equiv X$ is found by iterations.

The second step is to take into account the finite width of the outflow. To do so, $r_s$ is compared with the radii of the inner and outer boundaries of the outflow. If $r_s < R(t_s)$ or $r_s > R(t_s) + l$, scattering does not take place and the photon is considered decoupled. Such decoupling occurring at the boundaries corresponds to the photon-thin case. In the opposite case, $R(t_s) \leq r_s \leq R(t_s) + l$, scattering is assumed to occur and the loop is repeated until the photon decouples.

We consider two models of scattering: coherent isotropic scattering and Compton scattering. The former is tested for comparisons with results in the literature, e.g., Beloborodov (2010). This model is also interesting per se because it preserves the Planck spectrum and it traces only geometrical effects. By contrast, when Compton scattering is considered, the equilibrium spectrum is the Wien spectrum because stimulated emission is not taken into account by an MC simulation. For an initially large optical depth, the spectrum indeed first acquires the Wien shape and changes the shape again only at the photosphere.

The coherent scattering is computed by Lorentz transformation to the reference frame comoving with the outflow. In addition, when Compton scattering is considered, another Lorentz transformation to the rest frame of the electron is performed. The electron is chosen randomly from the Boltzmann distribution at a given comoving temperature defined as

$$T = T_0 \Gamma^{-1/3} \left( \frac{R_0}{r} \right)^{2/3}, \quad T_0 \approx \left( \frac{3E_0}{4\pi a R_0^3} \right)^{1/4}, \quad (6)$$

where $a = 4\sigma_{SB}/c$, $\sigma_{SB}$ is the Stefan–Boltzmann constant; see, e.g., Ruffini & Vereshchagin (2012).

Various tests of the code are performed. Before dealing with finite outflows we also reproduced the results of Beloborodov (2010). In the next section, we present the results for two specific cases. The set of parameters for these simulations is given in Table 1. The left (right) column corresponds to the photon-thick (thin) case.

| Table 1 | Parameters for the Simulations |
|----------|-------------------------------|
|          | Photon-thick | Photon-thin |
| $R_0$ (cm) | $10^8$ | $10^8$ |
| $l$ (cm)    | $4.5 \times 10^8$ | $10^8$ |
| $\tau_0$    | $2.3 \times 10^{10}$ | $1.2 \times 10^{13}$ |
| $E_0$ (erg) | $1.5 \times 10^{52}$ | $10^{54}$ |
| $\Gamma$    | 500 | 300 |
| $R_{ph}$ (cm) | $4.6 \times 10^{12}$ | $3.3 \times 10^{14}$ |
| $R_D$ (cm)  | $3.5 \times 10^{13}$ | $1.0 \times 10^{14}$ |
| $\tau_i$    | $10^2$ | $1.3 \times 10^3$ |

4. RESULTS

4.1. Probability Density Function of the Position of the Last Scattering

Consider first the probability density of the last photon scattering position as a function of depth (top panel of Figure 1). Photon decoupling from photon-thick outflows is expected to be local and the presence of boundaries should not change this probability substantially; indeed, the probability distribution function of the last scattering is found to be almost independent of the depth. On the contrary, in photon-thin outflows there is enough time for photons to be transported by diffusion to the boundaries as discussed in RSV. As a result, the probability density is peaked at the boundaries.

The difference between photon-thin and photon-thick outflows is also reflected in the probability density of the last scattering as a function of the radius, shown at the bottom of Figure 1. In the photon-thin outflow most photons escape from the outflow well before the photospheric radius, namely, near the diffusion radius

$$R_D = (\tau_0 \Gamma^2 R_0^2)^{1/3}. \quad (7)$$

By contrast, for photon-thick outflows, the probability density function of the last scattering is found to be close to that of an infinite and steady wind, found by Beloborodov (2010). The finite extension of the outflow results in the exponential cutoff for the probability density function at radii larger than $R_c$. Let us stress how different the probability densities for the photon-thick and photon-thin cases are. Not only the positions of the maximum, but also the shapes, are different. In the photon-thin case, at small radii the number of photons that diffuse out is determined by the change in the diffusion coefficient, i.e., the electron density, which follows a power law. In expanding...
plasma, while the density decreases, the mean free path for photons increases which makes the diffusion faster. At large radii, almost all photons have already diffused out, and the probability has an exponential cutoff. As for the photon-thick case, the probability density dependence on the radius is the opposite. For small radii, the probability for the last scattering is exponentially suppressed because of large optical depth, while at large radii it follows a power law, as discussed in Pe’er (2008). At larger radii, we find an exponential decrease due to the finiteness of the outflow; see Figure 1.

We also found a difference between photon-thick and photon-thin cases when we computed the average number of scatterings as a function of initial optical depth, shown in Figure 2. The derivation of analytic results is reported in the Appendix.

4.2. Spectra and Light Curves from Photon-thick Outflows

The average comoving energy of photons for a steady wind was considered by Pe’er (2008) and Beloborodov (2011). For a finite outflow we show this quantity in Figure 3 and compare it with the \( r^{-2/3} \) dependence characteristic of adiabatic cooling. The difference between Compton and coherent scattering comes from the fact that the average energy in the optically thick regime is \( 3kT \) for the Wien spectrum and \( 2.82kT \) for the Planck spectrum. In agreement with previous studies we find that the average energy of photons at the photosphere is higher than adiabatic cooling predicts. It is 1.58 in the case of Compton scattering and 1.4 in the case of coherent scattering.

In Figure 4 we compare time-integrated spectra of a photon-thick outflow obtained with different models. Each involves \( 5 \times 10^7 \) photons. In order to compare the results of RSV with our MC simulations we shifted the spectra for sharp and fuzzy approximations, as well as the Plank spectrum by the factor 1.58, which takes into account the effect discussed above. The results obtained in fuzzy approximations are in good agreement with MC simulations. The latter gives a low energy spectra index \( \alpha = 0.24 \) for Compton scattering and \( \alpha = 0.19 \) for coherent scattering compared with \( \alpha = 0 \) found in RSV.

We also show in Figure 5 time-resolved spectra at selected arrival times which are clearly far from being Planckian. The effect of spectral flattening at late times discovered by Pe’er & Ryde (2011) is visible.
Finally, light curves are presented in Figure 6, where we defined $t_{\text{ph}} = l/c$ and $t_a = t_e - \mu R_e/c$, with $r_e$ and $t_e$ being the laboratory radius and time of emission. We chose $t_a = 0$ for a photon emitted at $R_e = R_0$. The results of the MC simulation are in qualitative agreement with both approximations, slightly favoring the sharp photosphere case.

### 4.3. Spectra and Light Curves from Photon-thin Outflows

In the photon-thin case diffusion has been shown to be important. Both the light curve and spectrum are characterized by the radius of diffusion $R_D$ and the associated arrival time $t_D = R_D/(2\Gamma^2c)$. It should be noted that the photospheric emission carries only a fraction of the total energy of a GRB. In the photon-thin case the duration of the photospheric emission is relatively short (see the formula above) which implies that the luminosity of this component can be comparable to the luminosity of the rest of the prompt emission (Bernardini et al. 2007). Such short and intense photospheric emission preceding the main burst has been reported in many GRBs (Ryde & Pe’er 2009).

Since photons are still strongly coupled to the outflow, local thermodynamic equilibrium is maintained everywhere but at the very boundary. Indeed, this part of the outflow loses photons via random walks. For MC simulations of the photon-thin outflows we used the coherent scattering model. The results are compared with RSV where the light curve and spectrum are obtained using the sharp photosphere approximation after solving the radiative diffusion equation. It has been demonstrated that the temperature at the boundary of the outflow follows a different law from that of adiabatic cooling: $T(r) \propto r^{-13/24}$ with both temperatures nearly coinciding at the radius of diffusion. So we expect the time-integrated spectrum obtained by MC simulations has more power at energies above the peak.

Finally, in both MC simulations and in the work of RSV the photons emitted outside the relativistic beaming cone from the photon-thin outflows are neglected. This is a good approximation since such photons have much lower energy due to the Doppler effect. Besides this, their arrival time is $2\Gamma^2$ longer than the arrival time of diffusion.

Spectra and light curves are shown, respectively, in Figures 7 and 8. The simulation involves $10^6$ photons. We see a good agreement between both methods of computation. The high energy tail with a power law in the time-integrated spectrum is clearly visible. Our MC simulation shows the exponential cutoff at the energy corresponding to the temperature of photons at the...
The power law above the peak energy should extend up to an initial temperature \( T_0 \); see RSV. This also results in the missing initial part of the light curve (see Figure 8).

The power-law index is \( \beta = -3.5 \) for the model of the outflow given by Equation (1). Such a power law is steeper than typical high energy power laws observed in GRBs. It remains to be analyzed how \( \beta \) changes for different hydrodynamic models of the outflow. Besides, high energy GeV emission cannot be explained with this model since an exponential cutoff exists that corresponds to the initial temperature in the source of a GRB, typically being in the MeV region.

5. CONCLUSIONS

To summarize, in this work we considered the photospheric emission from a finite relativistic outflow by MC simulations of photon scattering. The validity of the assumptions made in RSV is verified and it is found that the fuzzy photosphere approximation used in that paper is adequate for describing both the light curves and the shape of the spectra of photospheric emission from photon-thick outflows. The sharp photosphere approximation produces qualitatively acceptable results for photon-thick outflows, but underestimates the high energy part of the spectrum and the decaying part of the light curve.

As expected, diffusion is found to dominate the photospheric emission of photon-thin outflows. The probability density of the last scattering is obtained in this case both as a function of outflow depth and its radial position. Both these functions differ from their photon-thick counterparts, once more pointing out the essential difference between these two cases. Good agreement is obtained between the MC simulations and the sharp photosphere approximation for both the light curve and spectrum of a photon-thin outflow.

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APPENDIX

AVERAGE NUMBER OF SCATTERINGS

We derive here the average number of scatterings expected for photons in both photon-thin and photon-thick outflows.

From RSV we recall the expressions of the optical depth for these two cases:

\[
\tau = \begin{cases} 
\frac{\tau_{i}}{\tau_{\text{c}}} & \text{if } r r_{D}, \end{cases} \quad \Gamma R_{0} \ll r \ll \Gamma^{2}l
\]

The average number of scatterings is defined as

\[
\langle N \rangle = \int_{t_{i}}^{t_{f}} \frac{cdt_{c}}{\Gamma} = \int_{t_{i}}^{t_{f}} \sigma n_{c}cdt_{c},
\]

where \( t_{c} \) is the comoving time, \( \lambda = 1/(\sigma T n_{c}) \) is the comoving mean free path of photons, \( n_{c} \) is the comoving density, \( r_{i} \) is the initial comoving time, and \( t_{f} \) is the final comoving time when the photon leaves the outflow. The integral (A2) should be taken along the average photon path. In the case of an optically thick medium this path is given by the equation of motion of the outflow

\[
r = r_{i} + \beta c t_{c},
\]

where \( r_{i} \) is the laboratory radial position of the photon at an initial laboratory time.

In the photon-thick case the photons stay in the outflow long after decoupling, so \( t_{f} \to \infty \). Then, taking into account the relations \( n_{c} = n_{l}/\Gamma, \ dr = \beta cdt, \) and \( t_{c} = t/\Gamma \) along the world-line (A3), we obtain

\[
\langle N \rangle \approx \int_{t_{i}}^{t_{f}} \frac{cdt_{c}}{\Gamma} = 2\tau_{i},
\]

where \( \tau_{i} \) is the optical depth of the outflow at \( r_{i} \). This result is in agreement with Vurm et al. (2011).

In the photon-thin case the photons leave the outflow from its boundaries, and the time interval needed for the photon to reach them by random walk can be estimated as \( t_{f}^{\text{th}} = l_{c}^{2}/D_{c}, \) where \( l_{c} = \Gamma l \) is the comoving radial thickness of the outflow and the diffusion coefficient is \( D_{c} = (c\lambda_{c})^{2} = c/(3\sigma n_{c}). \) When this time is much less than the characteristic time of expansion, equal to \( t_{r} \), which is the case when the initial radius \( r_{i} \) is much larger than the radius of diffusion \( R_{D} \), we have

\[
\langle N \rangle \approx 3\tau_{i}^{2},
\]

In the opposite case, when the initial radius \( r_{i} \) is much smaller than the radius of diffusion \( R_{D} \), we have

\[
\langle N \rangle \approx \frac{1}{\Gamma^{2}} \sqrt{\tau_{i} \tau_{0} R_{0}} / l.
\]

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Figure 8. Light curves from a photon-thin outflow obtained by MC simulation (crosses) and sharp photosphere approximation (thick curve).