An Experimental Realization of Quantum-vacuum Geometric Phases by Using the Gyrotropic-medium Optical Fiber

Jian Qi Shen 1,2

1 Centre for Optical and Electromagnetic Research, State Key Laboratory of Modern Optical Instrumentation, Zhejiang University, Hangzhou Spring Jade 310027, P.R. China
2 Zhejiang Institute of Modern Physics and Department of Physics, Zhejiang University, Hangzhou 310027, P.R. China

Abstract

The connection between the quantum-vacuum geometric phases (which originates from the vacuum zero-point electromagnetic fluctuation) and the non-normal product procedure is considered in the present Letter. In order to investigate this physically interesting geometric phases at quantum-vacuum level, we suggest an experimentally feasible scheme to test it by means of a noncoplanarly curved fiber made of gyrotropic media. A remarkable feature of the present experimental realization is that one can easily extract the nonvanishing and nontrivial quantum-vacuum geometric phases of left- and/or right-handed circularly polarized light from the vanishing and trivial total quantum-vacuum geometric phases.

PACS numbers: 03.65.Vf, 03.70.+k, 42.70.-a

Since Berry discovered that a topological (geometric) phase exists in quantum mechanical wavefunction of time-dependent systems, geometric phase problems have captured considerable attention of researchers in various fields such as quantum mechanics [1], differential geometry [2], gravity theory [3,4], atomic and molecular physics [5–7], nuclear physics [8], quantum optics [9], condensed matter physics [10,11], molecular systems and chemical reaction [5] as well. More recently, many authors concentrated their particular attention on the potential applications of geometric phases to the geometric quantum computation, quantum decoherence and related topics [10,12–15]. One of the most important physical realizations of Berry’s phase (i.e., cyclic adiabatic geometric phase) is the model describing the propagation of photons inside a helically curved optical fiber, which was proposed by Chiao and Wu [16], and later performed experimentally by Tomita and Chiao [17]. Afterwards, a large number of investigators treated this photon geometric phases by making use of the classical Maxwell’s electrodynamics, differential geometry method (parallel transport) and quantum adiabatic theory both theoretically and experimentally [18–21]. Based on the above investigations, we studied the nonadiabatic noncyclic geometric phases of photons propagating inside a noncoplanarly curved optical fiber [22,23] by means of the Lewis-Riesenfeld invariant theory [24] and the invariant-related unitary transformation formulation [25]. By using the obtained results [22,23], we considered the photon helicity inversion in the curved fiber and its potential applications to information science [26] and proposed a second-quantized spin model to describe the coiled light in a curved fiber, where the vacuum zero-point fluctuation is involved. As was stated by Fuentes-Guridi et al., in a strict sense, the Berry phase has been studied only in a semiclassical context until now [27]. Thus the effects of the vacuum field on the geometric evolution are still unknown [27]. In their paper [27], Fuentes-Guridi et al. considered the time evolution of a spin-1/2 particle interacting with a second-quantized external magnetic field and proposed a vacuum-induced spin-1/2 Berry’s phase, which they regarded as the effect of vacuum photon fluctuation. In this Letter, we will propose a new nontrivial vacuum effect, i.e., the quantum-vacuum geometric phases, and study its novel properties (particularly its connection with normal-order procedure in quantum field theory), and then suggest an experimental realization of this geometric phases at quantum-vacuum level by using the gyrotropic-medium fiber.

Note that here the quantum-vacuum geometric phases of photons results from the zero-point energy of vacuum quantum fluctuation. This, therefore, means that this geometric phases is quantal in character and, moreover, has no classical counterpart, namely, it cannot survive the correspondence-principle limit into the classical level. It is well known that in the conventional quantum field theory, both infinite vacuum fluctuation energy and divergent vacuum electric charge density are removed by the so-called normal-order procedure and new vacuum backgrounds of quantum fields, in which the vacuum expectation values of both charge density and Hamiltonian vanish, are therefore re-defined. Since in the time-independent quantum field theory, the infinite constant is harmless and easily removed, the normal-order procedure applied to these time-independent cases is reliable and valid indeed. However, in the time-dependent quantum field theory (such as quantum field theory in curved space-time, e.g., time-dependent gravitational backgrounds and expanding universe), the time-dependent vacuum zero-point fields may also participate in the time evolution process and therefore cannot be regarded merely as an inactive onlooker. We think that in these time-dependent cases, the non-normal or-
nder for operator product in quantum field theory should be taken into account and one can therefore predict some physically interesting quantum-vacuum effects. In what follows by analyzing the time evolution of photon wavefunction we consider such vacuum effect in a time-dependent quantum system, i.e., the Chiao-Wu case [16,17,22] where the rotation of photon polarization planes, which gives rise to photon geometric phases, in a noncoplanar fiber takes place.

According to the Liouville-Von Neumann equation [22], where the Lewis-Riesenfeld invariant [24] is the photon helicity $h = \mathbf{k} \cdot \mathbf{S}/k$, the effective Hamiltonian that describes the propagation of coiled light in a curved fiber is of the form [22,23] $H_{\text{eff}}(t) = \left[\mathbf{k}(t) \times \dot{\mathbf{k}}(t)\right] \cdot \mathbf{S}/k^2$ with $\mathbf{k}$, $\mathbf{S}$ representing the wave vector, the magnitude of $\mathbf{k}$ and the spin operator of photons. The dot denotes the derivative of $\mathbf{k}$ with respect to time. Thus the time-dependent Schrödinger equation governing the time evolution of photon wavefunction inside the noncoplanar curved fiber is written in the form [22,23] (in the unit $\hbar = c = 1$)

$$i \frac{\partial |\sigma, \mathbf{k}(t)\rangle}{\partial t} = \frac{\mathbf{k}(t) \times \dot{\mathbf{k}}(t)}{k^2} \cdot \mathbf{S} |\sigma, \mathbf{k}(t)\rangle,$$

where $\sigma = \pm 1$ is the eigenvalues of photon helicity $h$ corresponding to the right- and left-handed circularly polarized photons. The photon wave vector $\mathbf{k}$ inside the fiber can be written in the spherical polar coordinate system, i.e., $\mathbf{k} = k(\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$, which is always along the tangent to the curved fiber at each point at arbitrary time. By making use of the Lewis-Riesenfeld invariant theory and the invariant-related unitary transformation formulation [24,25], we obtain the exact solutions $|\sigma, \mathbf{k}(t)\rangle = \exp \left[\frac{i}{\hbar}\phi^{(S)}(t)\right] V(t) |\sigma, k\rangle$ to Eq. (1), where $|\sigma, k\rangle \equiv |\sigma, \mathbf{k}(t = 0)\rangle$ is the initial photon polarized state, $V(t) = \exp[\beta(t)\mathbf{S}_z - \beta^*(t)\mathbf{S}_z]$ with $\beta(t) = -[\theta(t)/2]\exp[i\varphi(t)]$, $\beta^*(t) = -[\theta(t)/2]\exp[-i\varphi(t)]$ [22] and $\mathbf{S}_\pm = \mathbf{S}_1 \pm i\mathbf{S}_2$. The noncyclic nonadiabatic geometric phase is given as follows

$$\phi^{(S)}_{\sigma}(t) = \left\{ \int_0^t \dot{\varphi}(t') [1 - \cos \theta(t')] dt' \right\} |\sigma, k\rangle [S_3 |\sigma, k\rangle].$$

It is apparent that in the adiabatic cyclic case for the Chiao-Wu’s coiled light in a helically curved fiber, where $\dot{\varphi} = 0$ and $\varphi$ (expressed by $\Omega$ that is constant) is the rotating frequency of photon moving on the fiber helicoid, the geometric phase in a cycle ($T = 2\pi/\Omega$) over the photon momentum space takes the form $\phi^{(S)}_{\sigma}(T) = 2\pi(1 - \cos \theta)(\sigma, k) [S_3 |\sigma, k\rangle$, where the expression $2\pi(1 - \cos \theta)$ denotes the solid angle subtended by a curve traced by the wave vector at the center of photon momentum space. This fact demonstrates the topological and global properties of geometric phases. This shows that the above calculation is self-consistent.

Now we consider the expectation value, $\langle \sigma, k | S_3 |\sigma, k\rangle$, of the third component of photon spin operator in Eq. (2). Substitution of the Fourier expansion series of three-dimensional magnetic vector potentials $\mathbf{A}(\mathbf{x}, t)$ into the expression $S_{ij} = -\int (\dot{A}_i A_j - \dot{A}_j A_i) d^3x$ for the spin operator of photon fields yields [22]

$$S_3 = \frac{i}{2} [a(k, 1)a^\dagger(k, 2) - a^\dagger(k, 1)a(k, 2)] - a(k, 2)a^\dagger(k, 1) + a^\dagger(k, 2)a(k, 1)]$$

with $a(k, \lambda)$ and $a(k, \lambda)$ ($\lambda = 1, 2$) being the creation and annihilation operators of polarized photons corresponding to the two mutually perpendicular real unit polarization vectors. Note that here $S_3$ is of the non-normal-order form.

In what follows we define the creation and annihilation operators, $a^\dagger_R(k, \lambda)$, $a_L(k, \lambda)$, of right- and left-handed circularly polarized light [28], which are expressed in terms of $a^\dagger(k, \lambda)$ and $a(k, \lambda)$, i.e., $a^\dagger_R(k, \lambda) = 1/\sqrt{2}[a^\dagger(k, 1) + ia(k, 2)]$, $a_R(k, \lambda) = 1/\sqrt{2}[a(k, 1) - ia(k, 2)]$, $a^\dagger_L(k, \lambda) = 1/\sqrt{2}[a^\dagger(k, 1) - ia(k, 2)]$ and $a_L(k, \lambda)$ = $1/\sqrt{2}[a(k, 1) + ia(k, 2)]$. Thus Eq.(3) can be rewritten in terms of the creation and annihilation operators of right- and left-handed polarized photons, namely,

$$S_3 = \frac{1}{2} [a_R(k)a^\dagger_R(k) + a_L(k)a_R(k)] - \frac{1}{2} [a_L(k)a^\dagger_L(k) + a_L^\dagger(k)a_L(k)],$$

which can also be rewritten as $S_3 = [a^\dagger_R(k)a_R(k) + 1/2] - [a^\dagger_L(k)a_L(k) + 1/2]$.

The monomode multiphoton states of left- and right-handed (LRH) circularly polarized light (at $t = 0$) can be defined as $|\sigma = -1, k, n_L\rangle = (n_L!)^{-1/2}a^\dagger_L(k)^{n_L}|0_L\rangle$ and $|\sigma = +1, k, n_R\rangle = (n_R!)^{-1/2}a^\dagger_R(k)^{n_R}|0_R\rangle$ with $n_L$ and $n_R$ being the LRH polarized photon occupation numbers, respectively. In the following we will calculate the geometric phases of multiphoton states $|\sigma = +1, k, n_R; \sigma = -1, k, n_L\rangle$, which is the direct product of LRH multiphoton states, i.e., $|\sigma = +1, k, n_R\rangle \otimes |\sigma = -1, k, n_L\rangle$. Insertion of the expression for the monomode multipolarized states into Eq.(2) yields the geometric phases of multiphoton polarized states, $\phi^{(S)}(t) = (n_R - n_L) \left\{ \int_0^t \dot{\varphi}(t') [1 - \cos \theta(t')] dt' \right\}$, which is independent of $k$ but dependent on the geometric nature of the pathway (expressed in terms of $\theta$ and $\varphi$) along which the light wave propagates. Although the phases $\phi^{(S)}(t)$ associated with the photonic occupation numbers $n_R$ and $n_L$ are quantal geometric phases of photons [23], they do not belong to the geometric phases at quantum-vacuum level which arises, however, from the zero-point electromagnetic energy of vacuum quantum fluctuation. It follows from the expression (4) for $S_3$ that both the geometric phases of left- and right-handed circu-
larly polarized photon states, \( i.e., |\sigma = -1, k, n_L \rangle \) and \( |\sigma = +1, k, n_R \rangle \), are respectively as follows

\[
\phi_L^{(g)}(t) = -(n_L + \frac{1}{2}) \left\{ \int_0^t \dot{\phi}(t') [1 - \cos \theta(t')] dt' \right\},
\]

\[
\phi_R^{(g)}(t) = +(n_R + \frac{1}{2}) \left\{ \int_0^t \dot{\phi}(t') [1 - \cos \theta(t')] dt' \right\}. \tag{5}
\]

According to Eq. (5), it is readily verified that the time-dependent zero-point energy possesses physical meanings and therefore contributes to geometric phases of photon fields. Thus the noncyclic nonadiabatic geometric phases of left- and right-handed polarized states at quantum-vacuum level are of the form

\[
\phi_{\sigma=\pm 1}^{(vac)}(t) = \pm \frac{1}{2} \left\{ \int_0^t \dot{\phi}(t') [1 - \cos \theta(t')] dt' \right\}. \tag{6}
\]

However, it should be pointed out that, unfortunately, even at the quantum level, this quantum-vacuum geometric phases \( \phi_{\sigma=\pm 1}^{(vac)}(t) \) that is observable in principle is absent in the previous fiber experiments [17–20], since it follows from (5) and (6) that the signs of quantal geometric phases are only those associated with the creation operators \( a_L^\dagger \) and \( a_R^\dagger \) of LRH polarized photons, the cyclic adiabatic case of which has been measured in the optical fiber experiments performed by Tomita and Chiao et al. [17–20]. Although the total of LRH quantum-vacuum geometric phases (6) is trivial, the vacuum geometric phase of separate circularly polarized field is nontrivial, which deserves investigation experimentally. The troublesome problem left to us now is how can we detect the above quantum-vacuum geometric phases of left- and/or right-handed polarized fields that has been cancelled by each other?

More recently, we suggest a new scheme to test the existence of this vacuum effect, the idea of which is to extract the nonvanishing cyclic quantum-vacuum geometric phases \( \phi^{(vac)}_{\sigma=+1}(T) \) or \( \phi^{(vac)}_{\sigma=-1}(T) \) by changing the mode distribution structures of vacuum photon field (or inhibiting vacuum photon fluctuation of certain propagation constant). This is not strange to us. For example, it is well known that in Casimir's effect the vacuum-fluctuation electromagnetic field in a finitely large space (\( i.e., \) the space between two parallel metallic plates) will alter its mode structures, namely, the zero-point field with wave vector \( k \) less than \( \frac{2\pi}{a} \) does not exist in this space with a finite scale length \( a \). Another illustrative example is the inhibition of spontaneous emission in photonic crystals [29] and cavity resonator [30].

For this aim, we take into account the peculiar wave propagation inside a kind of anisotropic materials (gyrotropic media), the electric permittivity and magnetic permeability of which are tensors taking the following form [31]

\[
\hat{\varepsilon} = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \quad \hat{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \tag{7}
\]

Assuming that the direction of the electromagnetic wave vector \( \mathbf{k} \) is parallel to the third component of the Cartesian coordinate system, with the help of Maxwell’s Equations, one can arrive at

\[
n_\perp^2 = (\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2), \tag{8}
\]

where \( n_+ \) and \( n_- \) are the optical refractive indices squared of such gyrotropic media corresponding to the right- and left-handed circularly polarized light, respectively [31,32]. Since in such gyrotropic media, one is positive and the other negative for the optical refractive index squared \( n^2 \) corresponding to the two directions of polarization of the electromagnetic wave, only one wave can propagate in gyrotrropic media. So, the quantum-vacuum geometric phases of LRH polarized photons cannot be eliminated by each other and it is therefore possible for physicists to easily test the remainder of quantum-vacuum geometric phases experimentally. If, for example, by taking some certain values of \( \epsilon_1, \epsilon_2, \mu_1 \) and \( \mu_2 \), then \( n_\perp^2 < 0 \) while \( n_\parallel^2 > 0 \) and consequently the left-handed polarized light cannot be propagated in this medium, and in the meanwhile the quantum vacuum fluctuation corresponding to the left-handed polarized light will also be inhibited (\( e.g., \) the wave amplitude exponentially decreases because of the imaginary part of the refractive index \( n_L \)) in this anisotropic absorptive medium. Thus the vacuum-fluctuation electromagnetic field alters its mode structures in the absorptive medium. For this reason, the only retained geometric phases is that of right-handed polarized light, which we can detect experimentally.

As an illustrative example, we now discuss the light propagation inside an optical fiber made of gyrotrropic media. We only consider the condition under which \( |\epsilon_2| > |\epsilon_1| = -\epsilon_1 \) and \( \mu_1 \pm \mu_2 > 0 \). If, for instance, \( \epsilon_2 \) is positive, then the right-handed polarized light can be propagated while the left-handed polarized light cannot be propagated in the fiber (because of the negative \( n_-^2 \) and the consequent imaginary propagation constant \( k_- \), which is expressed by \( n_-\omega/c \)); conversely, if \( \epsilon_2 \) is negative, then the left-handed polarized light can be propagated while the right-handed polarized light is inhibited from being propagated (due to the imaginary propagation constant \( k_+ \), which equals \( n_+\omega/c \)). Thus in the former case the phase \( \phi_R^{(vac)}(T) \) of right-handed polarized light, and in the latter case the phase \( \phi_L^{(vac)}(T) \) of left-handed polarized light instead, may respectively be detected in this gyrotrropic-medium fiber experiment.
Since the vacuum photon fluctuation with \( k \) less than \( \pi/a \) will be inhibited in the space between two parallel conducting plates whose separation is \( a \), we can suggest another scheme to detect \( \phi_{\sigma=\pm1}(T) \): specifically, if \( \epsilon_1 \) and \( \epsilon_2 \) (or \( \mu_1 \) and \( \mu_2 \)) of gyrotropic medium are chosen to be \( \epsilon_1 = \epsilon_2 \) (or \( \mu_1 = \mu_2 \)), then the vacuum fluctuation corresponding to the left-handed polarized light is inhibited since its propagation constant \( k_- \) tends to zero (and hence the wavelength is larger than the space scale \( a \)). Thus the only retained vacuum geometric phase is that of right-handed polarized light.

The physical significance of the subject presented in this Letter may be as follows:

(i) the quantum-vacuum geometric phases found here possesses interesting properties, for it has an important connection with the topological nature of time evolution of quantum vacuum fluctuation. To the best of our knowledge, in the literature, less attention was paid to such geometric phases at purely quantum-vacuum level. According to Fuentes-Guridi’s statement, such vacuum geometric phases may open up a new areas to the study of the consequences of field quantization in the geometric evolution of states [27]. It is emphasized that this vacuum effect deserves detailed consideration both theoretically and experimentally;

(ii) in order to extract the nontrivial quantum-vacuum geometric phases of polarized light from the trivial total quantum-vacuum geometric phases (which has been cancelled by each other and therefore vanishing), a new scheme, which is somewhat ingenious, by using the gyrotropic-medium optical fiber is proposed;

(iii) we think that the detection of quantum-vacuum geometric phases may be essential for the investigation of the time-dependent quantum field theory. As was discussed above, the quantum-vacuum geometric phases is related close to the non-normal order for operator product in second quantization. If the existence of quantum-vacuum geometric phases is truly demonstrated in experiments, then we should consider the validity problem of normal product technique in the time-dependent field theory. This, therefore, means that the experimental study of quantum-vacuum geometric phases would be a fundamental and important subject.

Recently, many authors applied geometric phases to some areas such as quantum decoherence and geometric (topological) quantum computation [10,12–15]. It may be believed that the quantum-vacuum geometric phases in the fiber would have some possible interesting applications to these subjects. We hope all the effects and phenomena presented in this Letter would be investigated experimentally in the near future.

Acknowledgements I thank X.C. Gao for his helpful proposals. This project was supported in part by the National Natural Science Foundation of China under the project No. 90101024.

J.-Q. Shen’s electronic address: jqshen@coer.zju.edu.cn

[1] M.V. Berry, Proc. Roy. Soc. London, Ser. A 392, 45 (1984).
[2] B. Simon, Phys. Rev. Lett. 51, 2167 (1983).
[3] C. Furtado and V.B. Bezerra, Phys. Rev. D 62, 045003 (2000).
[4] J.Q. Shen, H.Y. Zhu, S.L. Shi, and J. Li, Phys. Scr. 65, 465 (2002).
[5] Y.S. Wu and A. Kuppermann, Chem. Phys. Lett. 201, 178 (1993).
[6] Y.S. Wu and A. Kuppermann, Chem. Phys. Lett. 186, 319 (1991).
[7] B.G. Levi, Phys. Today (March), 17 (1993).
[8] A. Tomita and R.Y. Chiao, Phys. Lett. A 268, 209 (2000).
[9] L. F. Gong, Q. Li, Y. L. Chen, Phys. Lett. A 251, 387 (1999).
[10] Y. Taguchi et al., Science 291, 2573 (2001).
[11] G. Falci et al., Nature 407, 355 (2000).
[12] J.Q. Shen, S.S. Xiao, and Q. Wu, Chin. Opt. Lett. 1, 183 (2003).
[13] J.A. Jones, Nature 403, 869 (2000).
[14] X.B. Wang and M. Keiji, Phys. Rev. Lett. 87, 097901 (2001).
[15] S.L. Zhu and Z.D. Wang, Phys. Rev. Lett. 89, 097902 (2002).
[16] R.Y. Chiao and Y.S. Wu, Phys. Rev. Lett. 57, 933 (1986).
[17] A. Tomita and R.Y. Chiao, Phys. Rev. Lett. 57, 937 (1986).
[18] P.G. Kwiat and R.Y. Chiao, Phys. Rev. Lett. 66, 588 (1991).
[19] A.L. Robinson, Science 234, 424 (1986).
[20] F.D.M. Haldane, Opt. Lett. 11, 730 (1986).
[21] F.D.M. Haldane, Phys. Rev. Lett. 15, 1788 (1987).
[22] J.Q. Shen and H.Y. Zhu, Ann. Phys. (Leipzig) 12 (2003) 131.
[23] X.C. Gao, Chin. Phys. Lett. 19, 613 (2002).
[24] H.R. Lewis and W.B. Riesenfeld, J. Math. Phys. 10, 1458 (1969).
[25] X.C. Gao, J.B. Xu, and T.Z. Qian, Phys. Rev. A 44, 7016 (1991).
[26] J.Q. Shen and L.H. Ma, Phys. Lett. A 308, 355 (2003).
[27] I. Fuentes-Guridi, A. Carollo, S. Bose, and V. Vedral, Phys. Rev. Lett. 89, (2002) 220404.
[28] J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields (Mc Graw-Hill Company, New York, 1965) Chap. 14.
[29] E. Yablonovitch, Phys. Rev. Lett. 58, (1987) 2059.
[30] R.G. Hulet, E.S. Hilfer, and D. Kleppner, Phys. Rev. Lett. 55, 2137 (1985).
[31] V.G. Veselago, Sov. Phys. Usp. 10, 509 (1968).
[32] J.Q. Shen, arXiv: cond-mat/0305414 (2003).