Spin pinning and spin-wave dispersion in nanoscopic ferromagnetic waveguides

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Abstract

Spin waves are investigated in magnetic nano-waveguides with aspect ratios (width/thickness) close to one. An analytic theory is developed and verified by numerical simulations and Brillouin Light Scattering spectroscopy of Yttrium Iron Garnet (YIG) waveguides with a thickness of 39 nm and widths ranging down to 50 nm. A critical width is found, below which the exchange interaction overcomes the dipolar pinning phenomenon. This changes the quantization criterion for the spin-wave eigenmodes and results in a pronounced modification of the spin-wave mode profiles and dispersion relations. These findings are of key importance for the growing field of nano-magnonics.
Spin waves and their quanta, magnons, typically feature frequencies in the GHz to THz range and wavelengths in the micrometer to nanometer range. They are envisioned for the design of faster and smaller next generational information processing devices where information is carried by magnons instead of electrons\textsuperscript{1-9}. The recent progress in fabrication technology leads to the development of nanoscopic magnetic devices where nano-structures with different shapes, such as rectangular waveguides\textsuperscript{9-15} and rhomboids\textsuperscript{16} or cylinders\textsuperscript{17-19} act as elements to carry\textsuperscript{9,10,12-14}, process\textsuperscript{9,10,15,20} or store\textsuperscript{17,18} information. In state-of-the-art magnonics, the feature sizes of magnonic circuits become compatible to CMOS. For rectangular strip waveguides, widths below 100 nm imply that the width and the thickness of the involved conduits become comparable. Thus, the classical thin-film approach to describe spin waves in strips becomes invalid\textsuperscript{9-11,21-24}.

Moreover, these feature sizes imply that the spin-wave modes bear a strong exchange character, since the widths of the structures are now comparable to the exchange length\textsuperscript{25} given by

$$\lambda = \sqrt{\frac{2A}{\mu_0 M_s^2}}$$

with the exchange constant $A$, the vacuum permeability $\mu_0$, and the saturation magnetization $M_s$. The proper description of the spin-wave eigenmodes in nanoscopic strips which considers the influence of exchange interaction as well as the shape of the structure is fundamental for the field of magnonics, which aims to encode information in the spin-wave amplitude and phase.

In this Letter, we discuss the evolution of the frequencies and profiles of the spin-wave modes in nanoscopic waveguides of width $w$ and thickness $h$ as the aspect ratio $h/w$ evolves from the thin-film case $h/w \rightarrow 0$ to a rectangular bar with $h/w \rightarrow 1$. A quasi-analytic theory is developed and its results are compared with the results of numerical simulations and experimental studies of Yttrium Iron Garnet (YIG) nano-structures by means of microfocused Brillouin Light Scattering (BLS) spectroscopy. It is shown that a critical waveguide width exists, below which the profiles of the waveguide modes are essentially uniform across the width of the waveguide. This is fundamentally different from the profiles in state-of-the-art waveguides of micrometer\textsuperscript{9,11,21-24} or millimeter sizes\textsuperscript{26,27}, where the profiles are non-uniform and pinned at the waveguide edges due to dipolar interaction. In nanoscopic waveguides, the exchange interaction suppresses this pinning due to its dominance over the dipolar interaction and, consequently, the exchange interaction defines the profiles of the spin-wave modes as well as the corresponding spin-wave dispersion characteristics.
Fig. 1 (a) Sketch of the sample and the experimental configuration: a set of Yttrium Iron Garnet (YIG) waveguides is placed on a microstrip line to excite the quasi-ferromagnetic resonance in the waveguides. Brillouin Light Scattering (BLS) spectroscopy is used to measure the local spin-wave dynamics. For these measurements a laser beam is focused through the transparent Gadolinium Gallium Garnet (GGG) substrate on the center of the respective waveguide. (b) and (c) Scanning electron microscopy (SEM) micrograph of a 1 µm and a 50 nm wide YIG waveguide of 39 nm thickness. The color code shows the simulated amplitudes of the fundamental mode at quasi-ferromagnetic resonance, i.e., $k_x = 0$, in the waveguides. The mode in the 50 nm waveguide is almost uniform across the width of the waveguide evidencing the unpinning directly.

In the experiment and the theoretical studies, we consider rectangular magnetic waveguides as shown schematically in Fig. 1(a). In the experiment, the spin-wave mode is excited by a stripline that provides a homogeneous excitation field over the sample containing various waveguides etched from a $h = 39$ nm thick YIG film grown by liquid phase epitaxy\textsuperscript{28} on a Gadolinium Gallium Garnet (GGG) substrate. The widths of the waveguides range from $w = 50$ nm to $w = 1$ µm and they all feature the same length of 60 µm. The waveguides are patterned by Ar$^+$ ion beam etching using a lithographically defined Cr/Ti hard mask and are well separated on the sample in order to avoid dipolar coupling between them\textsuperscript{9}. The waveguides are uniformly magnetized along their long axis by an external field $B$ (x-direction). Figure 1(b) and (c) show scanning electron microscopy (SEM) micrographs of the largest and the narrowest waveguide studied in the experiment, together with the simulated mode profile of the quasi-ferromagnetic resonance in a color-coded amplitude scale. From comparing the two profiles, one of the key features of nanoscopic waveguides can already be seen: While in the waveguide with microscopic width, the mode’s amplitude decreases towards the edges, i.e., the mode is partially pinned, it becomes unpinned and quasi-uniform in the nanoscopic case.
In the following, we will provide a semi-analytical theory to understand this phenomenon. In general, the lateral spin-wave mode profile \( m_k(y) \) and frequency can be found from the following equation \(^{29,30}\)

\[
-i\omega_k m_k(y) = \mathbf{\mu} \times \left( \hat{\Omega}_k \cdot m_k(y) \right),
\]

with appropriate exchange boundary conditions, which take into account the surface anisotropy at the edges (see Appendix). Here, \( \mathbf{\mu} \) is the unit vector in the static magnetization direction and \( \hat{\Omega}_k \) is a tensorial Hamilton operator, which is given by

\[
\hat{\Omega}_k \cdot m_k(y) = \left( \omega_H + \omega_M \lambda^2 \left( k_z^2 - \frac{d^2}{dy^2} \right) \right) m_k(y) + \omega_M \int \hat{G}_{y'}(y-y') \cdot m_k(y') dy'.
\]

Here, \( \omega_H = \gamma B \), \( B \) is the static internal magnetic field that is considered to be equal to the external field due to the negligible demagnetization along the \( x \)-direction, \( \omega_M = \gamma \mu_0 M_s \), \( \gamma \) is the gyromagnetic ratio. The Green’s function \( \hat{G}_{y'}(y) \) is given by:

\[
\hat{G}_{y'}(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{N}_k e^{ik'y} dk_y.
\]

Here,

\[
\hat{N}_k = \begin{pmatrix}
\frac{k^2}{k_z^2} f(kh) & \frac{k_z k_y}{k^2} f(kh) & 0 \\
\frac{k_z k_y}{k^2} f(kh) & \frac{k^2}{k_y^2} f(kh) & 0 \\
0 & 0 & 1 - f(kh)
\end{pmatrix},
\]

where \( f(kh) = 1 - (1 - \exp(-kh)) / (kh) \), \( k = \sqrt{k_x^2 + k_y^2} \) and it is assumed that the waveguides are infinitely long.

A numerical solution of Eq. (1) gives both, the spin-wave profiles \( m_{\ell\ell} \) and frequency \( \omega_{\ell\ell} \). Eq. (1) is a one-dimensional integro-differential equation, its numerical solution is specified in the appendix. In the following, we will regard the out-of-plane component \( m_z(y) \) to show the mode profiles representatively. The profiles of the spin-wave modes can be well approximated by sine and cosine functions. In the past, it was demonstrated that in microscopic waveguides, the lowest, fundamental waveguide mode is well fitted by the function \( m_z(y) = A_0 \cos(\pi y/w_{\text{eff}}) \) with the amplitude \( A_0 \) and the effective width \( w_{\text{eff}} \). This mode, as well as the higher modes, are referred to as ‘partially pinned’. Pinning hereby refers to the fact that the amplitude of the modes at the edges of the waveguides is reduced. This is a consequence of the dynamic dipolar stray fields created by the magnetization precession. Full pinning corresponds to the zero
precession angle at the edges, zero pinning corresponds to a free precession at the edges. Partial pinning corresponds to a reduced, but finite precession amplitude at the edge. In that case, the effective width \( w_{\text{eff}} \) determines where the amplitude of the modes would vanish outside the waveguide\(^9\). With this effective width, the spin-wave dispersion relation can also be calculated by the analytic formula\(^9\):

\[
\alpha_l(k_y) = \sqrt{\left(\alpha_{\text{hl}} + \alpha_{\text{hm}} \left(\lambda^2 K^2 + F_{kl}^\text{y} \right)\right) \left(\alpha_{\text{hl}} + \alpha_{\text{hm}} \left(\lambda^2 K^2 + F_{k,0}^\text{y} \right)\right)},
\]

where \( K = \sqrt{k_x^2 + k_y^2} \) and \( \kappa = \pi/w_{\text{eff}} \). The tensor \( \hat{F}_k = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sigma_i}{w} \mathbf{N}_y dk_i \) accounts for the dynamic demagnetization, \( \sigma_i = \int_{-w/2}^{w/2} m(y) e^{-\frac{y^2}{2}} dy \) is the Fourier-transform of the spin-wave profile across the width of the waveguide, \( \tilde{w} = \int_{-w/2}^{w/2} m(y)^2 dy \) is the normalization of the mode profile \( m(y) \). Comparing the classical dispersion relation from Ref. 31, we introduce the \( y \)-component demagnetizing factor \( F_{k,0}^\text{y} \) which is important for nanoscopic waveguides.

In the following, the theory is compared to micromagnetic simulations and to the experiment. The simulations are performed using the MuMax3 code\(^32\). The structure is schematically shown in Fig. 1(a). The following parameters were used: the saturation magnetization \( M_s = 1.37 \times 10^5 \text{ A/m} \) and the Gilbert damping \( \alpha = 6.41 \times 10^{-4} \) were extracted from stripline ferromagnetic resonance\(^33\) measurements of the plain YIG film. Moreover, values of a gyromagnetic ratio \( \gamma = 175.86 \text{ rad/(ns-T)} \) and an exchange constant \( A = 3.5 \text{ pJ/m} \) for a standard YIG film were assumed. The external filed \( B = 108.9 \text{ mT} \) is applied along the waveguide long axis. The simulated length of the waveguides is 30 \( \mu \text{m} \) and an increased damping at the long ends of the strips avoids the influence of reflections. A sinc field pulse \( b_\gamma = b_0 \text{sinc}(2\pi f_c t) \), with oscillation field \( b_0 = 1 \text{ mT} \) and cutoff frequency \( f_c = 10 \text{ GHz} \), was used to excite a wide range of spin waves. Then, the spin-wave dispersion relations were obtained by performing the two-dimensional Fast Fourier Transformation (FFT) of the time- and position-dependent data\(^9,10,34\). The spin-wave width profiles were extracted from the \( m_z \) component across the width of the waveguides using a single frequency excitation.

The central panel of Fig. 2 shows the spin-wave mode profile of the fundamental mode for \( k_y = 0 \), which corresponds to the quasi-ferromagnetic resonance, in a 1 \( \mu \text{m} \) (a2) and 50 \( \text{nm} \) (b2) wide waveguide which have been obtained by micromagnetic simulations (red dots) and by solving Eq. (1) numerically (black lines). The top panels (a1) and (b1) illustrate the mode profile and the local precession amplitude in the waveguide. As it can be seen, the two waveguides feature quite different profiles of their fundamental modes: in the 1 \( \mu \text{m} \) wide waveguide, the spins are partially pinned and the amplitude of \( m_z \) at the edges of the waveguide is reduced. This still resembles the cos-like profile of the lowest width mode \( n = 0 \) that has been well established in investigations of spin-wave dynamics in waveguides on the micron scale\(^11,13,24,35\)
and that can be well-described by the simple introduction of a finite effective width \( w_{\text{eff}} > w \) \( (w_{\text{eff}} = w \) for the case of full pinning). In contrast, the spins at the edges of the narrow waveguide are completely unpinned and the amplitude of the dynamic magnetization \( m_z \) of the lowest spin-wave mode \( n = 0 \) is almost constant across the width of waveguide, resulting in an infinite effective width \( w_{\text{eff}} \rightarrow \infty \).

![Fig. 2 Schematic of the precessing spins and simulated precession trajectories (ellipses in the second panel) and spin-wave profile \( m_z(y) \) of the quasi-ferromagnetic resonance. The profiles have been obtained by micromagnetic simulations (red dots) and using quasi-analytic approach (black lines) for an (a) 1 \( \mu \)m and a (b) 50 nm wide waveguide. Bottom panel: Normalized square of the spin-wave eigenfrequency \( \omega^2/\omega^2_M \) as a function of \( w/w_{\text{eff}} \) and the relative Dipolar and Exchange contributions. It is visible that the exchange energy is dominating in the case of nano-sized waveguide (panel b3) resulting in the minimum of the total energy at the point \( w/w_{\text{eff}} = 0 \) describing the unpinned state.](image)

To understand the nature of this depinning, it is instructive to consider the spin-wave energy as a function of the geometric width of the waveguide normalized by the effective width \( w/w_{\text{eff}} \). This ratio corresponds to some kind of pinning parameter taking values in between 1 for the fully pinned case and 0 for the fully unpinned case. The system will choose the mode profile which minimizes the total energy. This is equivalent to a variational minimization of the spin-wave eigenfrequencies as a function of \( w/w_{\text{eff}} \). To illustrate this, the lower panels of Figs. 2(a3) and (b3) show the square of the fictive spin-wave eigenfrequencies \( \omega^2 \) for the two different widths as a function of \( w/w_{\text{eff}} \). The minimum of \( \omega^2 \) is equivalent to the solution with the lowest energy which then corresponds to the effective width \( w_{\text{eff}} \) that minimizes the energy. Here, \( \omega^2 \) refers to a renormalized frequency square, not taking into account the Zeeman contribution, which only leads to an offset in frequency that is independent of \( w \) and \( w_{\text{eff}} \). In
addition to the total $\omega^2$ (black), also the individual contributions from the dipolar term (red) and the exchange term (blue) are shown, which can only be separated conveniently from each other if the square of Eq. 5 is considered for $k_z = 0$. The dipolar contribution is non-monotoneous and features a minimum at a finite effective width $w_{\text{eff}}$, which can clearly be observed for $w = 1 \mu$m. The appearance of this minimum, which leads to the effect known as “effective dipolar pinning”\cite{21,22}, is a result of the interplay of two tendencies: (i) an increase of the volume contribution with increasing $w/w_{\text{eff}}$, as for common Damon-Eshbach spin waves, and (ii) a decrease of the edge contribution when the spin-wave amplitude at the edges vanishes ($w/w_{\text{eff}} \rightarrow 1$). This minimum is also present in the case of a 50 nm wide waveguide (red line), even though this is hardly perceivable in Fig. 2(b3) due to the scale. In contrast, the exchange leads to a monotoneous increase of frequency as a function of $w/w_{\text{eff}}$, which is minimal for the unpinned case, i.e., $w/w_{\text{eff}} = 0$ implying $w_{\text{eff}} \rightarrow \infty$, when all spins are parallel. In the case of the 50 nm waveguide, the smaller width and the corresponding much larger quantized wavenumber in the case of pinned spins would lead to a much larger exchange contribution than this is the case for the 1 \mu m wide waveguide (please note the vertical scales). Consequently, the system avoids pinning and the solution with lowest energy is situated at $w/w_{\text{eff}} = 0$. In contrast, in the larger waveguide the interplay of dipolar and exchange energy implies that energy is minimized at a finite $w/w_{\text{eff}}$. The top panel Fig. 2(b1) shows an additional feature of the narrow waveguide: as the aspect ratio of the waveguides approaches unity, the ellipticity of precession, a well-known feature of micron-sized waveguides which still resemble a thin film\cite{24,36}, vanishes and the precession becomes nearly circular. Also, in nanoscale waveguides, the ellipticity is constant across the width, while in a large waveguide it can be different at the waveguide center and near its edges.

Fig 3: (a) Experimentally determined resonance frequencies (black squares) together with theoretical predictions and micromagnetic simulations. (b) Inverse effective width $w/w_{\text{eff}}$ as a function of the waveguide width.
width. The inset shows the critical width \( w_{\text{crit}} \) as a function of thickness \( h \). (c) Spin-wave dispersion relation of the first two width modes from micromagnetic simulations (color-code) and theory (dashed lines). (d) Inverse effective width \( w/w_{\text{eff}} \) as a function of the spin-wave wavenumber \( k_x \) for different thicknesses and waveguide widths, respectively.

As it is evident from the lower panel of Fig. 2, the pinning and the corresponding effective width have a large influence on the spin-wave frequency. This allows for an experimental verification of the presented theory, since the frequency of partially pinned spin-wave modes would be significantly higher than in the unpinned case. As mentioned above, for the experimental investigation, we study the quasi-ferromagnetic resonance in a series of YIG waveguides (fundamental mode \( n = 0 \) with \( k_x = 0 \)) due to the homogeneous excitation field. The intensity of the magnetization precession is measured by microfocused BLS spectroscopy\(^{37}\) (cf. Fig. 1(a)). For these measurements a laser beam of 457 nm wavelength and a power of 1.8 mW is focused through the transparent Gadolinium Gallium Garnet (GGG) substrate on the center of the respective waveguide using a compensating microscope objective. The effective spot-size is 350 nm. Black squares in Figure 3(a) summarize the dependence of the frequency of the quasi-ferromagnetic resonance measured for different widths of the YIG waveguides. The magenta line shows the expected frequencies assuming pinned spins, the blue (dashed) line gives the resonance frequencies extrapolating the formula conventionally used for micron-sized waveguides\(^{21,24}\) to the nanoscopic scenario, and the red line gives the result of the theory presented here, together with simulation results (green dashed line). As it can be seen, the experimentally observed frequencies can be well reproduced if the real pinning conditions are taken into account and the theory presented here is in excellent agreement with the experimental results as well as with the micromagnetic simulations. On the contrary, an assumption of fully or purely dipolarly pinned spins leads to a large overestimation of the resonance frequencies in nanoscopic waveguides on the order of several GHz, highlighting the crucial importance of the spin-wave mode profiles.

As has been discussed alongside with Fig. 2, the unpinning occurs when the exchange interaction contribution becomes so large that it compensates the minimum in the dipolar contribution to the spin-wave energy. Since the energy contributions and the demagnetization tensor change with the thickness of the investigated waveguide, the critical width below which the spins become unpinned is different for different waveguide thicknesses. This is shown in Fig. 3(b), where the inverse effective width \( w/w_{\text{eff}} \) is shown for different waveguide thicknesses. Symbols are the result of micromagnetic simulations, lines are calculated semi-analytically. As can be seen from the figure, for thicker films, the mode becomes unpinned at larger waveguide widths. This is summarized in the inset, which shows the critical width (i.e.
the width for which \( w/w_{\text{eff}} = 0 \) as a function of thickness. This critical width varies in the range from 140 nm for \( h = 10 \) nm YIG thickness up to 340 nm for \( h = 100 \) nm, and the spins can be considered unpinned below it. Please note that these results are obtained for YIG and the critical width values are different for materials with different values for the saturation magnetization and exchange constant.

Up to know, the discussion was limited to the special case of \( k_s = 0 \). In the following, the influence of finite wave vector will be addressed. The spin-wave dispersion relation of the fundamental \( (n = 0) \) mode obtained from micromagnetic simulations (color-code) together with the semi-analytical solution (white dashed line) are shown in Fig. 3(c) for the YIG waveguide of \( w = 50 \) nm width. The figure also shows the low-wavenumber part of the dispersion of the first width mode, which is pushed up significantly in frequency due to its large exchange contribution. Both modes are described accurately by the quasi-analytic theory. As it is described above, the spins are fully unpinned in this particular case. In order to demonstrate the influence of the pinning conditions on the spin-wave dispersion, a hypothetic dispersion relation for the case of partial pinning is shown in the figure with a dash-dotted white line (the case of \( w/w_{\text{eff}} = 0.63 \) is considered that would result from the usage of the thin-film approximation). One can clearly see that the spin-wave frequencies in this case are considerably higher and the separation between the dispersions decreases with an increase in the spin-wave wavenumber \( k_s \). This is explained by the fact that the relative contribution of the transversal standing wavenumber \( \kappa = \pi/w_{\text{eff}} \) to the total spin-wave wavenumber \( K = \sqrt{k_s^2 + \kappa^2} \) in Eq. (5) is decreasing with the increase in \( k_s \). Figure 3(d) shows the inverse effective width \( w/w_{\text{eff}} \) as a function of the wavenumber \( k_s \) for a waveguide thickness of \( h = 39 \) nm and three exemplary waveguide widths of \( w = 50 \) nm, 300 nm and 500 nm. As it can be seen from the figure, the effective width and, consequently, the ratio \( w/w_{\text{eff}} \) shows only a weak nonmonotonic dependence on the spin-wave wavenumber in the propagation direction. This dependence is a result of an increase of the dipolar fields inhomogeneity near the edges for larger \( k_s \), which increases pinning\(^{23} \), and of the simultaneous decrease of the overall strength of dynamic dipolar fields for shorter spin waves. Please note that the mode profiles are not only important for the spin-wave dispersion. They are also of crucial importance for the coupling efficiency between two waveguides\(^{9,38} \).

In conclusion, an analytic theory is developed and is employed together with micromagnetic simulations to study the spin-wave eigenmodes in nanoscopic waveguides with aspect ratio up to \( h/w = 1 \). The theoretical results are compared with measurements of the quasi-ferromagnetic resonance of individual wires with widths ranging from 1 \( \mu \)m down to 50 nm by means of Brillouin Light Scattering spectroscopy. It is found that the exchange interaction is getting dominant with respect to the dipolar interaction that is responsible for the phenomenon of dipolar pinning. This mediates an unpinning of the spin-wave modes if the width of the waveguide becomes smaller than a certain critical value. This exchange unpinning results in a quasi-uniform spin-wave mode profile in nanoscopic waveguides in
contrast to the cosine-like profiles in waveguides of micrometer widths. Our theory allows to calculate the spin pinning mode profiles as well as the spin-wave dispersion, and to identify a critical width below which fully unpinned spins need to be considered for a given thickness and choice of material. In addition, the presented results provide valuable guidelines for applications in nano-magnonics where spin waves propagate in nanoscopic waveguides with large aspect ratios and lateral sizes comparable to the sizes of modern CMOS technology.

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Appendix: Numerical solution of the eigenproblem

The eigenproblem Eq. (1) should be solved with proper boundary conditions at the lateral edges of the waveguide. Since we use a complete description of the dipolar interaction (via Green’s functions), the boundary conditions account for exchange interaction and surface anisotropy (if any) only, and read\(^{36}\):

\[
m \times \left( \mu_0 M_s \lambda^2 \frac{\partial m}{\partial n} - \nabla M E_a \right) = 0,
\]

where \(n\) is the unit vector defining an inward normal direction to the waveguide edge, and \(E_a(m)\) is the energy density of the surface anisotropy. In the studied case of a waveguide magnetized along its long axis, the conditions (A.1) for dynamic magnetization components can be simplified to:

\[
\pm \frac{\partial m}{\partial y} + \frac{d m}{\Delta w} \bigg|_{y=\pm \Delta w/2} = 0, \quad \frac{\partial m}{\partial y} \bigg|_{y=\pm \Delta w/2} = 0,
\]

where \(d = -2K_s/\left(M_s^2 l^2\right)\) is the pinning parameter\(^{34}\) and \(K_s\) is the constant of surface anisotropy at the waveguide lateral edges. More complex cases like, e.g., diffusive interfaces can be considered in the same manner\(^{39}\).

For the numerical solution of Eq. (1) it is convenient to use finite element methods and to discretize the waveguide into \(n\) elements of the width \(\Delta w = w/n\), where \(w\) is the width of waveguide. The discretization step should be at least several times smaller than the waveguide thickness and the spin-wave wavelength \(2\pi/k_s\) for a proper description of the magneto-dipolar fields. The discretization transforms Eq. (1) into a system of linear equations for magnetizations \(m_j, j = 1,2,3,\ldots n\):

\[
\left( \omega \left( \alpha_{y} + \alpha_{z} \kappa^2 \right) m_j - \alpha_{z} \kappa^2 \frac{m_{j+1} - 2m_j + m_{j-1}}{\Delta w^2} + \omega \kappa \sum_{j'} \mathbf{G}_{k_s,j'} \cdot m_{j'} \right) = \omega m_j,
\]

where dipolar interaction between the discretized elements is described by

\[
\mathbf{G}_{k_s,j}(y') = \frac{1}{\Delta w} \int_{-\Delta w/2}^{\Delta w/2} dy' \int_{-\Delta w/2}^{\Delta w/2} dy \mathbf{G}_{k_s}(y-y'-j\Delta w).
\]

The direct use of Eq. (A.4) is complicated since the Green’s \(\mathbf{G}_{k_s}(y)\) function is an integral itself. Using Fourier transform it can be derived as

\[
\mathbf{G}_{k_s,j}(y) = \frac{\Delta w}{2\pi} \text{sinc}(k_s \Delta w/2) \tilde{\mathbf{N}}_k e^{ik_s y} dk_s,
\]

which can be easily calculated, especially using FFT. Equation (A.3) is, in fact, a \(2n\)-dimensional linear algebraic eigenproblem (since \(m_j\) is a 2-component vector), which is solved by standard methods. The values \(m_0\) and \(m_{n+1}\) in Eq. (A.3) are determined from the boundary conditions (A.2). In particular, for negligible anisotropy at the waveguide edges one should set \(m_0 = m_1\) and \(m_{n+1} = m_n\).