Constraining DHOST theories with linear growth of matter density fluctuations

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We investigate the potential of cosmological observations, such as galaxy surveys, for constraining degenerate higher-order scalar-tensor (DHOST) theories, focusing in particular on the linear growth of the matter density fluctuations. We develop a formalism to describe the evolution of the matter density fluctuations during the matter dominated era and in the early stage of the dark energy dominated era in DHOST theories, and give an approximate expression for the gravitational growth index in terms of several parameters characterizing the theory and the background solution under consideration. By employing the current observational constraints on the growth index, we obtain a new constraint on a parameter space of DHOST theories. Combining our result with other constraints obtained from the Newtonian stellar structure, we show that the degeneracy between the effective parameters of DHOST theories can be broken without using the Hulse-Taylor pulsar constraint.

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I. INTRODUCTION

In the era of precision cosmology, one of the most significant problems is the elucidation of the origin of the late-time acceleration of the Universe [1, 2]. Modified gravity theories have been widely studied as an alternative to the most straightforward candidate of this origin, i.e., a cosmological constant. Among them, scalar-tensor theories with higher-order derivatives are receiving increasing attention. In general, such theories would have pathological extra ghost degrees of freedom because Ostrogradski’s theorem [3, 4] requires that the equations of motion for a metric and a scalar field should be of second order to avoid such a ghost degree of freedom. A new wider class of healthy scalar-tensor theories with higher derivatives has been proposed recently and is called the degenerate higher-order scalar-tensor (DHOST) theories [5–8], in which the equations of motion are of higher order, but one can reduce them to a second-order system due to the degeneracy (for a review, see [9–11]). DHOST theories include previously known scalar-tensor theories such as the Horndeski theory [12–14], its disformal relatives [15], and (a subclass of) the Gleyzes-Langlois-Piazza-Vernizzi (GLPV) theory [16, 17]. Then, it is intriguing to investigate observational and experimental constraints on these theories.

One of the most stringent constraints on gravity theories is obtained from the gravitational wave event GW170817 [18] and its optical counterpart GRB 170817A [19], which gave the constraint on the speed of gravitational waves, $c_{GW}$, as $|c_{GW}/c - 1| \lesssim 10^{-15}$ with $c$ being the speed of light (hereafter we use units in which $c = 1$). This observation can be used to rule out scalar-tensor theories which predict a variable gravitational-wave speed at low redshifts [20–26]. One finds that there still is a broad class of viable scalar-tensor theories. In particular, a certain subclass of quadratic DHOST theories [5–7] survived after this event.

Of course, even before GW170817 lots of stringent constraints on local gravity had been obtained, implying that gravity must be consistent with general relativity at least on small scales and in the weak gravity regime. Therefore, viable scalar-tensor theories are required to have a mechanism that suppresses the fifth force mediated by the scalar field on small scales, and Vainshtein screening is a typical one of such mechanisms in the Horndeski and related theories. Interestingly, DHOST theories generically exhibit Vainshtein screening outside matter, whereas its partial breaking occurs inside [26–29]. As the gravitational laws inside an astrophysical body differ from the standard ones, this phenomenon leads to a modification of its internal structure, which can be used to constrain DHOST theories [30–35]. The authors of Ref. [29] applied this idea to the DHOST theories satisfying $c_{GW}^2 = 1$ and obtained constraints on the parameters which characterize the theories.

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In this paper, in addition to the above constraints, we investigate the possibility of constraining DHOST theories from the current/future precise cosmological observations. In particular, we focus on the linear evolution of the matter density fluctuations, which can be measured by observations of large scale structure. Measuring the linear growth rate of large-scale structure, \( f(a) \), is known to be a powerful tool to test modifications of gravity responsible for the present cosmic acceleration. To compare the observational data with theoretical predictions, the simplest approach is to introduce an additional parameter called gravitational growth index, \( \gamma \), defined in terms of the linear growth rate and the fraction parameter of non-relativistic matter \( \Omega_m \) as [36]

\[
\gamma := \frac{d \ln f}{d \ln \Omega_m}.
\] (1)

The purpose of this paper is to obtain a novel constraint on DHOST theories with \( c^2_{GW} = 1 \) from the observations of the linear growth rate. To do so, we develop a formalism to describe DHOST cosmology during the matter dominated era and the early stage of the dark energy dominated era, and evaluate the growth index at high redshifts. We expect that the current observations of the growth index yield new constraints on DHOST theories which are complementary to the existing bounds.

The paper is organized as follows. In Sec. II, we derive cosmological background equations in class I quadratic DHOST theories. Then we consider linear cosmological perturbations and derive the evolution equation of the density fluctuations. In Sec. III, we introduce our formalism to model DHOST cosmology and evaluate the growth index as a probe of modifications gravity. We thereby give novel constraints on DHOST theories from current observations. Finally, we discuss our results and future prospects in Sec. V.

II. DHOST THEORIES: BACKGROUND AND PERTURBATION EQUATIONS

A. Action

The action of the quadratic DHOST theories [5, 6] is given by

\[
S = \int d^4x \sqrt{-g} \left[ G_2(\phi, X) - G_3(\phi, X) \nabla \phi + G_4(\phi, X) R + \sum_{i=1}^{5} a_i(\phi, X) L_i \right],
\] (2)

where we have several functions of the scalar field \( \phi \) and its kinetic term \( X := (-1/2)\phi_{\mu} \phi^{\mu} \). The Lagrangians \( L_i \) are quadratic in the second derivatives of \( \phi \) and are given by

\[
L_1 = \phi_{\mu \nu} \phi^{\mu \nu}, \quad L_2 = (\nabla \phi)^2, \quad L_3 = (\nabla \phi) \phi^\mu \phi_{\mu \nu} \phi^{\nu}, \quad L_4 = \phi^\mu \phi_{\mu \nu} \phi^\nu \phi_{\nu}, \quad L_5 = (\phi^\mu \phi_{\mu \nu} \phi^{\nu})^2,
\] (3)

where \( \phi_{\mu} := \nabla_{\mu} \phi \) and \( \phi_{\mu \nu} := \nabla_{\nu} \nabla_{\mu} \phi \).

In order for this higher-derivative theory to be free of Ostrogradsky ghosts, we must impose the degeneracy conditions that relate \( G_4 \) and \( a_i \). The quadratic DHOST theories are classified in several subclasses [5, 6], among which we are interested in the so-called class I theories, because theories in other subclasses exhibit some pathologies in a cosmological setup [37, 38]. (The class I DHOST theories are conformally/disformally related to the Horndeski theory [6, 7].) The class I degeneracy conditions are summarized as

\[
a_1 + a_2 = 0, \quad \beta_2 = -6\beta_1^2, \quad \beta_3 = -2\beta_1 [2(1 + \alpha_H) + \beta_1 (1 + \alpha_T)],
\] (4)

where

\[
M^2 = 2(G_4 + 2X a_1), \quad M^2 \alpha_T = -4X a_1, \quad M^2 \alpha_H = -4X(G_{4X} + a_1),
\]

\[
M^2 \beta_1 = 2X(G_{4X} - a_2 + Xa_3), \quad M^2 \beta_2 = 4X[a_1 + a_2 - 2X(a_3 + a_4) + 4X^2a_5],
\]

\[
M^2 \beta_3 = -8X(G_{4X} + a_1 - Xa_4).
\] (5)

Here we write the derivative of a function \( f(X) \) with respect to \( X \) as \( f_X \). We thus have 3 constraints among 6 functions \( (G_4 \text{ and } a_i) \), leaving 3 free functions in addition to \( G_2 \) and \( G_3 \).

Note that the propagation speed of gravitational waves is given by \( c^2_{GW} = 1 + \alpha_T \). The gravitational wave event GW170817 [18] and its optical counterpart GRB 170817A [19] have placed a tight bound \( c^2_{GW} \simeq 1 \). We therefore have \( \alpha_T \simeq 0 \), provided that this constraint is valid at low energies where dark energy/modified gravity models are used [39]. Imposing \( \alpha_T = 0 \) amounts to taking \( a_1 = a_2 = 0 \), but for the moment we do not require this.
B. Background equations in shift-symmetric DHOST theories

In the rest of the paper we focus on the shift-symmetric subclass of DHOST theories, in which the Lagrangian is invariant under a constant shift of the scalar field, namely \( \phi \rightarrow \phi + \text{const} \). This means that the free functions contained in the Lagrangian are dependent only on the scalar field kinetic term \( X \).

As a matter component we only consider pressureless dust and assume that it is minimally coupled to gravity. For a homogeneous and isotropic background, \( ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j \), \( \phi = \phi(t) \), with the matter energy density \( \rho_m \), the gravitational field equations read

\[
3M^2H^2 = \rho_m + \rho_\phi, \tag{6}
\]
\[
-M^2 \left( 2\dot{H} + 3H^2 \right) = p_\phi, \tag{7}
\]

where \( H = \dot{a}/a \) (a dot denotes differentiation with respect to \( t \)), and

\[
\rho_\phi := \dot{\phi}J - G_2 - M^2H^2 \left( 6\beta_1 y - \frac{1}{2} \beta_2 y^2 \right), \tag{8}
\]
\[
p_\phi := G_2 + 2M^2H^2 \left[ (\alpha_B + 3\beta_1)y - \left( \beta_1 + \frac{\beta_2}{4} \right) y^2 \right] + 2M^2\beta_1 \frac{d}{dt}(yH), \tag{9}
\]

with \( J \) being the shift current defined shortly. Here we defined \( y := \frac{\ddot{\phi}}{H\dot{\phi}} \) and

\[
\alpha_M := \frac{1}{M^2H} \frac{dM^2}{dt}, \tag{10}
\]
\[
\alpha_B := -\frac{\dot{\phi}XG_3X}{M^2H} + \frac{\alpha_H}{y} - (3 - \alpha_M) \beta_1 + \frac{\beta_1}{H} + \left( \beta_1 + \frac{\beta_2}{2} \right) y. \tag{11}
\]

The scalar field equation can be written using the shift current as

\[
\dot{J} + 3HJ = 0, \tag{12}
\]

where

\[
\dot{J} = 2XG_2X + M^2H^2 \left[ \frac{3\alpha_M}{y} - 6\alpha_B + 6 \left( \alpha_M \beta_1 + \frac{\beta_1}{H} \right) + 6\beta_1 y - \frac{1}{2} \left( \alpha_M \beta_2 + \frac{\beta_2}{H} \right) y \right]
+ 6M^2\beta_1 \dot{H} - M^2\beta_2 \frac{d}{dt}(yH). \tag{13}
\]

Equation (12) implies that in the expanding Universe \( J = \text{const}/a^3 \rightarrow 0 \) and hence attractor solutions are characterized by \( J = 0 \).

The background equations (6), (7), and (12) contain the higher derivatives \( \dddot{\phi} \), \( \ddddot{\phi} \), and \( \dddot{H} \). However, the degeneracy conditions (4) allow us to reduce the system to the second-order one. It is not so obvious to demonstrate this explicitly, but one can follow Refs. [40, 41] to see that it is indeed possible to do so.

C. Density perturbations

Let us study matter density fluctuations in the Newtonian gauge. The perturbed metric in the Newtonian gauge is given by

\[
ds^2 = -[1 + 2\Phi(t, x)]dt^2 + a^2(t) [1 - 2\Psi(t, x)] \delta_{ij} dx^i dx^j. \tag{14}
\]

We write the perturbation of the scalar field as

\[
\phi(t, x) = \phi(t) + \pi(t, x). \tag{15}
\]

It is convenient to introduce a dimensionless variable \( Q := H\pi/\dot{\phi} \), and we will use this instead of \( \pi \). The density perturbation is defined by

\[
\rho_m(t, x) = \bar{\rho}_m(t)[1 + \delta(t, x)]. \tag{16}
\]
We study the quasi-static evolution of the perturbations inside the sound horizon scale. The quasi-static approximation indicates that $\dot{\epsilon} \sim H\epsilon \ll \partial_\epsilon$, where $\epsilon$ is any of perturbation variables. Expanding the action to second order in perturbations under the quasi-static approximation, we obtain the following effective action:

$$S_{\text{eff}} = \int d^4x \, \mathcal{L}_{\text{eff}},$$  \hspace{1cm} (17)

with

$$\mathcal{L}_{\text{eff}} = \frac{M^2 a}{2} \left\{ (c_1 \Phi + c_2 \Psi + c_3 Q) \partial^2 Q + 4(1 + \alpha_H) \Psi \partial^2 \Phi - 2(1 + \alpha_T) \Psi \partial^2 \Psi 
- \beta_3 \Phi \partial^2 \Phi + \left[ 4 \alpha_H \frac{\dot{\Psi}}{H} - 2(2\beta_1 + \beta_3) \frac{\dot{\Phi}}{H} + (4\beta_1 + \beta_3) \frac{\ddot{Q}}{H^2} \right] \partial^2 Q \right\} - a^3 \rho_m \Phi \delta,$$  \hspace{1cm} (18)

where

$$c_1 := -4 \left\{ \alpha_B - \alpha_H + \frac{\beta_3}{2} (1 + \alpha_M) + \frac{\beta_3}{2} \right\},$$  \hspace{1cm} (19)

$$c_2 := 4 \left\{ \alpha_H (1 + \alpha_M) + \alpha_M - \alpha_T + \frac{\dot{\alpha}_H}{H} \right\},$$  \hspace{1cm} (20)

$$c_3 := -2 \left\{ \left( 1 + \alpha_M + \frac{\dot{H}}{H^2} \right) \left( \alpha_B - \alpha_H \right) + \frac{\dot{\alpha}_B - \dot{\alpha}_H}{H} + \frac{3\Omega_m}{2} + \frac{\dot{H}}{H^2} + \alpha_T - \alpha_M 
+ \left[ -2 \frac{\dot{H}}{H^2} \beta_1 + \frac{\beta_3}{4} (1 + \alpha_M) + \frac{\beta_3}{2} \right] \left( 1 + \alpha_M - \frac{\dot{H}}{H^2} \right) - 2 \frac{\dot{H}^2}{H} \beta_1 + \frac{\dot{H}}{H^2} \frac{2 \beta_3}{2} + \frac{\dot{\alpha}_M}{H} \beta_3 + \frac{\ddot{\beta}_3}{4 H^2} \right\},$$  \hspace{1cm} (21)

and

$$\Omega_m := \frac{\rho_m}{3M^2 H^2}. \hspace{1cm} (22)$$

We have three terms whose coefficients are written solely in terms of $\beta_1$ and $\beta_3$. (The latter can be expressed in terms of $\alpha_H$, $\alpha_T$, and $\beta_1$ using the degeneracy condition given by Eq. (4).) These are the new terms in DHOST theories. The other terms are present in the Horndeski and GLPV theories, but as $c_1$ and $c_3$ are dependent on $\beta_1$ and $\beta_3$ one can see implicitly the contributions of these parameters characterizing DHOST theories.

The field equations are derived by varying the effective action with respect to $\Phi$, $\Psi$, and $Q$. Going to Fourier space, they are given by

$$(1 + \alpha_H) \Psi - \frac{\beta_3}{2} \Phi + b_1 Q + \frac{2 \beta_1 + \beta_3}{2} \frac{\dot{Q}}{H} + \frac{a^2}{2M^2 k^2} \delta \rho_m = 0,$$  \hspace{1cm} (23)

$$(1 + \alpha_T) \Psi - (1 + \alpha_H) \Phi + b_2 Q + a_H \frac{\dot{Q}}{H} = 0,$$  \hspace{1cm} (24)

$$c_2 \Psi + c_1 \Phi + b_3 Q + 4 \alpha_H \frac{\dot{\Psi}}{H} - 2(2\beta_1 + \beta_3) \frac{\dot{\Phi}}{H} + b_4 \frac{\ddot{Q}}{H} + 2(4\beta_1 + \beta_3) \frac{\ddot{Q}}{H^2} = 0,$$  \hspace{1cm} (25)

where $k$ denotes a comoving wavenumber in Fourier space and $\Phi$, $\Psi$, and $Q$ are now understood as the Fourier components. Here, the coefficients $b_i$ ($i = 1, 2, 3, 4$) are defined as

$$b_1 := \frac{c_1}{4} + \frac{1}{2} (1 + \alpha_M)(2\beta_1 + \beta_3) + \frac{1}{2} \frac{d}{dt} \left( \frac{2\beta_1 + \beta_3}{H} \right),$$  \hspace{1cm} (26)

$$b_2 := -\frac{c_2}{4} + (1 + \alpha_M) \alpha_H + \frac{d}{dt} \left( \frac{\alpha_H}{H} \right),$$  \hspace{1cm} (27)

$$b_3 := 2c_3 + \left[ \left( 1 + \alpha_M - \frac{\dot{H}}{H^2} \right) (1 + \alpha_M) + \frac{\dot{\alpha}_M}{H} \right] (4\beta_1 + \beta_3)$$

$$+ 2(1 + \alpha_M) \frac{d}{dt} \left( \frac{4\beta_1 + \beta_3}{H} \right) + \frac{d^2}{dt^2} \left( \frac{2\beta_1 + \beta_3}{H^2} \right),$$  \hspace{1cm} (28)

$$b_4 := 2 \left[ \left( 1 + \alpha_M - \frac{\dot{H}}{H^2} \right) (4\beta_1 + \beta_3) + \frac{d}{dt} \left( \frac{4\beta_1 + \beta_3}{H} \right) \right].$$  \hspace{1cm} (29)
Since matter is assumed to be minimally coupled to gravity, the fluid equations are the same as the standard ones, and hence under the quasi-static approximation the matter density fluctuations $\delta(t, \mathbf{x})$ and the velocity field $u^i(t, \mathbf{x})$ obey
\begin{align}
\dot{\delta} + \frac{1}{a} \partial_i [(1 + \delta) u^i] &= 0, \\
\dot{u}^i + Hu^i + \frac{1}{a} \partial_j u^i &= - \frac{1}{a} \partial^i \Phi.
\end{align}
At linear order, these equations are combined to give
\begin{equation}
\ddot{\delta} + 2H \dot{\delta} + \frac{k^2}{a^2} \Phi = 0,
\end{equation}
where we moved to Fourier space. The effects of modified gravity come into play through the gravitational potential $\Phi$ which is determined by solving Eqs. (23)–(25).

Let us then solve Eqs. (23)–(25) to express $\Phi$, $\Psi$, and $Q$ in terms of $\delta$ and its time derivatives. We will follow the same procedure as that used in [42]. This procedure is feasible thanks to the degeneracy of the theory. Solving Eqs. (23) and (24) for $\Phi$ and $\Psi$ and substituting these solutions into Eq. (25), one finds that $\dot{Q}$ and $\dot{Q}$ terms are canceled due to the degeneracy, and hence $Q$ can be expressed in the form
\begin{equation}
- \frac{k^2}{a^2 H^2} Q = \kappa_Q \delta + \nu_Q \frac{\dot{\delta}}{H},
\end{equation}
where the explicit expressions for the time-dependent coefficients $\kappa_Q$ and $\nu_Q$ are presented in Appendix A. Finally, substituting this back into Eqs. (23) and (24), the gravitational potentials $\Phi$ and $\Psi$ can be expressed in terms of $\delta$, $\dot{\delta}$, and $\ddot{\delta}$ as
\begin{equation}
- \frac{k^2}{a^2 H^2} \Phi = \kappa_\Phi \delta + \nu_\Phi \frac{\dot{\delta}}{H} + \mu_\Phi \frac{\ddot{\delta}}{H^2},
\end{equation}
\begin{equation}
- \frac{k^2}{a^2 H^2} \Psi = \kappa_\Psi \delta + \nu_\Psi \frac{\dot{\delta}}{H} + \mu_\Psi \frac{\ddot{\delta}}{H^2}.
\end{equation}
The explicit expressions for the time-dependent coefficients $\mu_i$, $\nu_i$, and $\kappa_i$ ($i = \Phi, \Psi$) are also shown in Appendix A. Within the Horndeski theory we have $\mu_1 = \nu_2 = 0$ and in the GLPV theory we still have $\mu_\Psi = 0$. That is, $\mu_\Psi$ first appears in DHOST theories beyond GLPV. Equation (34) allows us to eliminate $\Phi$ from Eq. (32) and we obtain the closed-form equation for $\delta$ as
\begin{equation}
\ddot{\delta} + (2 + \varsigma) H \dot{\delta} - \frac{3}{2} \Omega_m \Xi_\Phi H^2 \delta = 0,
\end{equation}
where the additional friction $\varsigma$ and the effective gravitational coupling (multiplied by $8\pi M^2$) $\Xi_\Phi$ are written in terms of $\mu_\Phi$, $\nu_\Phi$, and $\kappa_\Phi$ as
\begin{align}
\varsigma &= \frac{2 \mu_\Phi - \nu_\Phi}{1 - \mu_\Phi}, \\
\Xi_\Phi &= \frac{2}{3} \Omega_m \frac{\kappa_\Phi}{1 - \mu_\Phi}.
\end{align}
These two functions characterize modification of gravity. The evolution equation (36) has essentially the same form as that in DHHOST theories with $c_{\text{GW}}^2 = 1$ [40] and in the GLPV theory [17, 43]. Whether or not $c_{\text{GW}}^2 = 1$ does not play an important role in determining the qualitative form of Eq. (36). In the case of the Horndeski theory ($\alpha_H = \beta_1 = 0$), the additional friction term vanishes, $\varsigma = 0$, and the result of Ref. [44] is recovered.

Equation (36) tells us that, even in DHOST theories under the quasi-static approximation, the evolution of the matter density fluctuations is independent of the wavenumber, so that as usual we can write the growing solution to Eq. (36) as
\begin{equation}
\delta(t, \mathbf{k}) = D_+(t) \delta_L(\mathbf{k}),
\end{equation}
where $\delta_L(\mathbf{k})$ represents the initial density field. The effect of the modified evolution of the density perturbations is thus imprinted in the growth factor, $D_+(t)$. Introducing the linear growth rate, $f := \text{d} \ln D_+/\text{d} \ln a$, the evolution equation can be written as
\begin{equation}
\frac{\text{d} f}{\text{d} \ln a} + \left(2 + \varsigma + \frac{\text{d} \ln H}{\text{d} \ln a}\right) f + f^2 - \frac{3}{2} \Omega_m \Xi_\Phi = 0.
\end{equation}
Thus, we rate by solving the above equation.

**III. MODELING DHOST COSMOLOGY IN THE MATTER DOMINATED ERA**

We consider possible cosmological constraints on DHOST theories from observables during the matter dominated era and in the early stage of the dark energy dominated era. To do so, we assume that during these stages \( y, G_2, \alpha_i (i = H, M, B, T), \) and \( \beta_1 \) can be expressed as a series expansion form in terms of \( \varepsilon := 1 - \Omega_m (\ll 1) \)

\[
\begin{align*}
  y &= y_0 + \mathcal{O}(\varepsilon), \\
  G_2 &= g_2 M^2 H^2 \varepsilon + \mathcal{O}(\varepsilon^2), \\
  \alpha_i &= c_i \varepsilon + \mathcal{O}(\varepsilon^2), \\
  \beta_1 &= \beta \varepsilon + \mathcal{O}(\varepsilon^2),
\end{align*}
\]

where \( y_0, g_2, c_i, \) and \( \beta \) are constants. Note that the expansion of \( \alpha_i \) and \( \beta_1 \) starts at \( \mathcal{O}(\varepsilon) \), as modifications of gravity are supposed not to be significant at early times. As seen below, the background equations are consistent with Eqs. (41)–(44). The expansion coefficients \( (y_0, g_2, c_i, \beta) \) are not all independent parameters. We will express some of them in terms of the other coefficients and the parameters of an underlying model.

Substituting Eqs. (41)–(44) to Eqs. (8) and (9), one finds, for the attractor solutions \( (\mathcal{J} = 0) \), that

\[
\begin{align*}
  \rho_\phi &= -(g_2 + 6\beta y_0) M^2 H^2 \varepsilon + \mathcal{O}(\varepsilon^2), \\
  p_\phi &= \left[ g_2 + 2(c_B + 3\beta) y_0 - 2\beta y_0^2 \right] M^2 H^2 \varepsilon + 2M^2 \beta y_0 \dot{H} \varepsilon + \mathcal{O}(\varepsilon^2).
\end{align*}
\]

Noting that \( 3M^2 H^2 - \bar{\rho}_m = 3M^2 H^2 \varepsilon = \rho_\phi, \) we have

\[
  g_2 = -3(1 + 2\beta y_0).
\]

The effective dark energy equation of state parameter, \( w_\phi := p_\phi/\rho_\phi, \) can be expanded as

\[
w_\phi = w^{(0)} + \mathcal{O}(\varepsilon).
\]

From Eqs. (45)–(47) and \( \dot{H}/H^2 = -3/2 + \mathcal{O}(\varepsilon) \) we obtain

\[
w^{(0)} = -1 + \frac{2}{3} \left( c_B y_0 - \frac{3}{2} \beta y_0 - \frac{3}{2} \beta y_0^2 \right).
\]

Using the above expression for \( w^{(0)} \), one has the following useful formulas valid up to \( \mathcal{O}(\varepsilon) \):

\[
\frac{\dot{H}}{H^2} = -\frac{3}{2} \left( 1 + w^{(0)} \varepsilon \right) + \mathcal{O}(\varepsilon^2), \quad \frac{\varepsilon}{H} = \left( c_M - 3w^{(0)} \right) \varepsilon + \mathcal{O}(\varepsilon^2).
\]

To proceed further, let us assume that \( G_2 \propto X^p \), where \( p \) is a constant model parameter. This assumption leads to the relation \( XG_2X = pG_2 \). Using this assumption and Eq. (50), we find

\[
0 = \dot{\mathcal{J}} = \left[ 2p g_2 + 3(y_0^{-1} c_M - 2c_B) - 9\beta + 6\beta (c_M - 3w^{(0)}) + 6\beta y_0 \right] M^2 H^2 \varepsilon + \mathcal{O}(\varepsilon^2).
\]

Equations (49) and (51) give

\[
\begin{align*}
  w^{(0)} &= -1 - \frac{y_0}{3} (c_H + 2p), \\
  c_B &= -p - \frac{c_H}{2} + \beta \left( y_0 + \frac{3}{2} \right).
\end{align*}
\]

Thus, \( w^{(0)} \) and \( c_B \) are expressed in terms of the model parameter \( p \) and the other coefficients.

In the following we consider tracker solutions characterized by the condition

\[
H \phi^{2q} = \text{const}.
\]
where \( q \) is a constant. Such tracker solutions have been studied in the context of the Horndeski theory [45, 46] and its extensions [40, 41, 47]. For instance, the cosmological solution discussed in [40, 41] corresponds to the case with \( p = 2 \) and \( q = 1 \). In this paper we regard \( q \) as another model parameter. For the solutions satisfying Eq. (54), it is easy to see that

\[
y_0 = \frac{3}{4q}.
\]  

(55)

In what follows we will use \( q \) instead of \( y_0 \).

So far we have not imposed \( c^2_{GW} = 1 \) \( (\Leftrightarrow \alpha_T = 0) \), as \( \alpha_T \) does not appear explicitly in the background equations. Upon imposing \( \alpha_T = 0 \), it follows from the definitions that \( M^2 = 2G_4 \) and \( M^2 \alpha_H = -4XG_{4x} \), which implies another relation between the parameters:

\[
\alpha_M = -y_0 \alpha_H \quad \Rightarrow \quad c_M = -y_0 c_H.
\]

(56)

Thus, under the assumption of \( c^2_{GW} = 1 \), we have four independent parameters, \((p, q, c_H, \beta)\), in terms of which \( g_2, c_M, c_B \), as well as \( w^{(0)} \), can be expressed.

### IV. CONSTRAINING DHOST COSMOLOGY

#### A. Growth index

Let us derive the solution to (40) in a series expansion form in terms of \( \varepsilon \). We start with expanding \( \varsigma \) and \( \Xi_\Phi \) in terms of \( \varepsilon \). Since \( \alpha_i = \mathcal{O}(\varepsilon) \) and \( \beta_1 = \mathcal{O}(\varepsilon) \), we have \( \varsigma \to 0 \) and \( \Xi_\Phi \to 1 \) for \( \varepsilon \to 0 \), so that, to \( \mathcal{O}(\varepsilon) \), \( \varsigma \) and \( \Xi_\Phi \) can be written as

\[
\varsigma = \varsigma^{(1)} \varepsilon + \mathcal{O}(\varepsilon^2), \quad \Xi_\Phi = 1 + \Xi_\Phi^{(1)} \varepsilon + \mathcal{O}(\varepsilon^2),
\]

(57)

where \( \varsigma^{(1)} \) and \( \Xi_\Phi^{(1)} \) can be written in terms of the parameters introduced in the previous section. See Appendix A for their explicit expressions. Then, Eq. (40) reduces to

\[
(c_M - 3w^{(0)})\varepsilon \frac{df}{d\varepsilon} + \left[ \frac{1}{2} + \left( \varsigma^{(1)} - \frac{3}{2}w^{(0)} \right) \varepsilon \right] f + f^2 - \frac{1}{2} \left[ 1 - \left( 1 - \Xi_\Phi^{(1)} \right) \varepsilon \right] + \mathcal{O}(\varepsilon^2) = 0,
\]

(58)

where we used Eq. (50). The solution to this equation is given by

\[
f = 1 - \left[ \frac{3(1 - w^{(0)}) + 2\varsigma^{(1)} - 3\Xi_\Phi^{(1)}}{5 - 6w^{(0)} + 2c_M} \right] \varepsilon + \mathcal{O}(\varepsilon^2).
\]

(59)

From the solution (59) we immediately obtain

\[
\gamma = \frac{3(1 - w^{(0)}) + 2\varsigma^{(1)} - 3\Xi_\Phi^{(1)}}{5 - 6w^{(0)} + 2c_M} + \mathcal{O}(\varepsilon).
\]

(60)

It is easy to see that the standard result \( \gamma = 6/11 \) is recovered for \( w^{(0)} = -1 \), \( c_M = \varsigma^{(1)} = \Xi_\Phi^{(1)} = 0 \). Substituting the explicit expressions for \( \Xi_\Phi^{(1)} \) and \( \varsigma^{(1)} \) [Eqs. (A14) and (A13)] into Eq. (60), one can evaluate an approximate form of the growth index \( \gamma \) during the matter dominated era and the early stage of the dark energy dominated era satisfying \( \varepsilon \ll 1 \):

\[
\gamma = \frac{3(1 - w^{(0)}) - c_T}{5 - 6w^{(0)} + 2c_M} - \frac{2}{\Sigma} \left[ c_B - c_M + c_T - \beta(c_M - 3w^{(0)}) \right]^2
+ \frac{c_H + \beta}{\Sigma} \left\{ 6(1 + w^{(0)}) + (c_M - c_T)[1 - 2(c_M - 3w^{(0)})]ight. \\
+ \left. \left[ c_B - \beta(c_M - 3w^{(0)}) \right] [5 - 2(c_M - 3w^{(0)})] + 5(c_H + \beta)(c_M - 3w^{(0)}) \right\} + \mathcal{O}(\varepsilon),
\]

(61)
In the current observations as shown above can be roughly estimated as \( \lesssim \gamma \) in Ref. [52] (by adding tomographic analysis). Since the typical value of the deviation from the central value of configuration space). The constraints from BOSS DR14 are given as \( \gamma = 0.52 \pm 0.10 \) in Ref. [49] (based on the analysis in Fourier space) and \( \gamma = 0.609 \pm 0.079 \) in Ref. [50] (based on the analysis in configuration space). The constraints from BOSS DR14 are given as \( \gamma = 0.55 \pm 0.19 \) in Ref. [51] and \( \gamma = 0.580 \pm 0.082 \) in Ref. [52] (by adding tomographic analysis). Since the typical value of the deviation from the central value of \( \gamma \) in the current observations as shown above can be roughly estimated as \( \lesssim \mathcal{O}(0.1) \), let us employ \( \gamma = 6/11 \pm 0.1 \) as a conservative constraint. For a given set of the model parameters \((p,q)\), this can be translated into constraints on \((\beta,c_H)\) using Eq. (63). The parameter regions in the \(\beta-c_H\) plane allowed by the constraint \( \gamma = 6/11 \pm 0.1 \) are plotted in Fig. 1 for \((p,q) = (1,1/2)\) (red), \((1,3/2)\) (green), and \((3,1/2)\) (blue). One finds from Fig. 1 that a constant-\(\gamma\) curve for fixed \(p\) and \(q\) is a hyperbola in the \(\beta-c_H\) plane for \((p,q)\) and \(\gamma\) that we are considering. This means that we have degeneracy between \(c_H\) and \(\beta\) in the observations of the growth index. In contrast, in the GLPV theory we have \(\beta = 0\), and hence we can obtain for instance the following constraints on \(c_H\): \(-0.4 \leq c_H \leq 0.4\) for \((p,q) = (1,1/2)\), \(-0.4 \leq c_H \leq 0.5\) for \((1,3/2)\), and \(-1.1 \leq c_H \leq 0.7\) for \((3,1/2)\). Deriving the constraints for other values of \((p,q)\) is straightforward. It should be emphasized that the constraints we have obtained in Fig. 1 are those at high redshifts satisfying \(\Omega_m \simeq 1\).

To compare our results with previously known constraints, it is necessary to make further assumptions that connect the series expansion of \(\alpha_H\) and \(\beta_1\) to their present values. Specifically, we assume that \(\alpha_H = c_H (1 - \Omega_m)\), \(\beta_1 = \gamma \), and \(c_H \neq 0\).
\( \beta (1 - \Omega_m) \), and the leading order expression of \( \gamma \) [Eq. (63)] are valid all the way up to the present time. Hereafter we focus on the specific parameter values \((p, q) = (1, 1/2)\), which corresponds to the model discussed in [40, 41], and demonstrate the allowed parameter region. Though details of constraints will be different for different choices of \((p, q)\), we expect that the order of the bounds is approximately the same.

Existing constraints on DHOST theories mainly come from the Newtonian stellar structure modified due to the partial breaking of the Vainshtein mechanism, which is characterized by a single parameter \( \Upsilon_1 := -2(\alpha_H + \beta_1)^2 / (\alpha_H + 2\beta_1) \) (the definition here is for theories with \( c_{\text{SW}} = 1 \)) [26, 27, 29]. The lower bound on \( \Upsilon_1 \) has been obtained from the requirement that gravity is attractive at the stellar center: \( \Upsilon_1 > -2/3 \) [30]. The upper bound is given by comparing the minimum mass of stars with the hydrogen burning with the minimum mass of observed red dwarfs: \( \Upsilon_1 < 1.6 \) [31].

There are several attempts for improving the above bounds [32–34], including the one concerning the speed of sound in the atmosphere of the Earth [35]. Aside from the constraints from the Newtonian stellar structure, another constraint has been proposed, which comes from precise observations of the Hulse-Taylor pulsar. This can severely constrain the effective parameters through the coupling of gravitational waves to matter [29, 53] : \(-7.5 \times 10^{-3} \leq \alpha_H + 2\beta_1 \leq 2.5 \times 10^{-3} \). However, when deriving this result, several assumptions have been made and the resultant constraint would depend on the details of how the screening mechanism operates in a binary system. In this paper, we try to constrain the effective parameters without taking into account these potentially more stringent bounds, and use the most conservative constraint: \(-2/3 < \Upsilon_1 < 1.6 \).

We plot in Fig. 2 the allowed parameter region in the \( \beta - c_H \) plane obtained from the constraints on the growth index (red) and stellar structure (black). As shown in Fig. 2, combining our results and the conservative constraints discussed above can break the degeneracy between \( c_H \) and \( \beta \) without using the Hulse-Taylor pulsar bound. The overlap region between these gives the constraints on both parameters: \(-1.0 \leq c_H \leq 1.7 \) and \(-4.7 \leq \beta \leq 1.8 \).

Note that recently it was pointed out in Ref. [54] that the absence of gravitational wave decay into scalar modes requires \( \alpha_H + 2\beta_1 = 0 \). As seen from the fact that the denominator of \( \Upsilon_1 \) vanishes when this is satisfied, this is a special case which has not been explored so far. It would be interesting to investigate the behavior of gravity in this limiting case in detail, but it is beyond the scope of this paper, and we do not consider this constraint.

V. SUMMARY

In this paper, we have considered a possibility to constrain degenerate higher-order scalar-tensor (DHOST) theories by using the information about the linear growth of matter density fluctuations. In DHOST theories, the evolution equation for the linear matter density fluctuations is modified in such a way that the effective gravitational coupling is changed by the factor \( \Xi_f \) and the friction term has an additional contribution \( \varsigma H \), both of which can be expressed in terms of the effective parameters \( \alpha_i \) and \( \beta_i \) used in the literature.

We have constructed cosmological models in DHOST theories as a series expansion in terms of \( 1 - \Omega_m \). In doing
so, we have assumed for simplicity that cosmological solutions under consideration are attractors in shift-symmetric theories and subject to the tracker ansatz. The resultant cosmology is characterized by two model parameters \((p, q)\) and four independent effective parameters in general (i.e., six parameters in total), and upon imposing \(c_{GW}^2 = 1\) the number of independent parameters reduces to four in total. Our construction thus provides a concise description of DHOST cosmology during the matter dominated era and the early stage of the dark energy dominated era.

We have then explicitly expressed the gravitational growth index \(\gamma\) in terms of \((p, q)\) and the effective parameters. We have found that the constant-\(\gamma\) curve in the \(\beta-c_H\) plane generically is a hyperbola for \(c_{GW}^2 = 1\) and fixed \((p, q)\). One can thus obtain constraints on a certain combination of the effective parameters at high redshifts by using the observations of the growth index alone.

Under the additional assumption that our leading order results in \(1 - \Omega_m\) expansion can be extrapolated all the way to the present time, we have compared the constraints from the growth index with the previously known bounds. Combining our results and the constraints from modifications of the gravitational law inside stellar objects, we have shown that the parameter degeneracy between \(\alpha_H/(1 - \Omega_m)\) and \(\beta_1/(1 - \Omega_m)\) could be broken without using the Hulse-Taylor pulsar constraint, though our results slightly depend on the model parameters. Future-planned observations for large-scale structure would exclude the currently allowed region of the parameter space and serve as tests of the viability of DHOST theories.

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Appendix A: Explicit expressions for some coefficients

Let us write down explicitly the coefficients in Eqs. (33)–(34). The coefficients in Eq. (33) are given by

\[
\nu_Q = \frac{3\Omega_m}{2Z} N_\Phi, \quad (A1)
\]

\[
\kappa_Q = \frac{3\Omega_m}{8Z} \left\{ \left[ c_1 + 2(2\beta_1 + \beta_3) \right] F_\Phi + (c_2 - 4\alpha_H) F_\Psi - M^2 H \left[ 2(2\beta_1 + \beta_3) \frac{d}{dt} \left( \frac{F_\Phi}{M^2} \right) - 4\alpha_H \frac{d}{dt} \left( \frac{F_\Psi}{M^2} \right) \right] \right\}, \quad (A2)
\]

where we have defined the some dimensionless parameters as

\[
SN_\Phi = \alpha_H (1 + \alpha_H) - \frac{1}{2} (1 + \alpha_T) (2\beta_1 + \beta_3), \quad (A3)
\]

\[
SF_\Phi = 1 + \alpha_T, \quad SF_\Psi = 1 + \alpha_H, \quad (A4)
\]

\[
S = (1 + \alpha_H)^2 - \frac{1}{2} (1 + \alpha_T) \beta_3, \quad (A5)
\]

The denominator \(Z\) can be written as

\[
Z = \frac{1}{4} \left\{ \mathcal{E}_\Phi c_1 + \mathcal{E}_\Psi c_2 - \left[ b_3 + \frac{2(2\beta_1 + \beta_3)}{H} \dot{\mathcal{E}}_\Phi - \frac{4\alpha_H}{H} \dot{\mathcal{E}}_\Psi \right] \right\}, \quad (A6)
\]

where \(c_1, c_2, \text{ and } b_3\) were defined in Eqs (19), (20), and (28). We have also defined

\[
SE_\Phi = b_1 (1 + \alpha_T) - b_2 (1 + \alpha_H), \quad (A7)
\]

\[
SE_\Psi = b_1 (1 + \alpha_H) - \frac{1}{2} b_2 \beta_3. \quad (A8)
\]
The coefficients in Eqs. (35) and (35) are
\[ \mu_a = N_a \nu Q, \]  
\[ \nu_a = -E_a \nu Q + N_a \left[ \kappa Q + \frac{1}{2 a^2 H^2} \frac{d}{d t} (a^2 H \nu Q) \right], \]  
\[ \kappa_a = \frac{3}{2} \Omega_m \zeta_a - E_a \kappa Q + \frac{N_a}{2 a^2 H^2} \frac{d}{d t} (a^2 H^2 \kappa Q), \]  
for \( a = \Psi, \Phi \), where
\[ S \nu^\Psi = -(1 + \alpha_H) \beta_1 - \frac{1}{2} \beta_3. \]

The coefficients in Eq. (57) are given by
\[ \xi^{(1)} = \frac{3}{Z} \left( c_H + \beta \right)^2 \left( c_M - 3 w^{(0)} \right), \]  
\[ \Xi^{(1)} = c_T + \frac{2}{Z} \left[ c_B - c_M + c_T - \beta \left( c_M - 3 w^{(0)} \right) \right]^2 \]  
\[ - \frac{c_H + \beta}{Z} \left\{ 6 \left( 1 + w^{(0)} \right) + \left( c_M - c_T \right) \left[ 1 - 2 \left( c_M - 3 w^{(0)} \right) \right] \right. \]  
\[ + \left. \left[ c_B - \beta \left( c_M - 3 w^{(0)} \right) \right] \left[ 5 - 2 \left( c_M - 3 w^{(0)} \right) \right] + 3 \left( c_H + \beta \right) \left( c_M - 3 w^{(0)} \right) \right\}, \]  
where
\[ Z = 3 \left( 1 + w^{(0)} \right) + 2 c_M + \left[ 1 - 2 \left( c_M - 3 w^{(0)} \right) \right] \left[ c_B - c_H + \beta \left( c_M - 3 w^{(0)} \right) + 1 \right]. \]

We can then finally obtain the explicit expression \( \gamma \) in the main text by substituting Eqs. (A13) and (A14) into Eq. (60).
Rev. D 96 (2017) no.12, 123516 [arXiv:1709.03243 [astro-ph.CO]].

[49] J. N. Grieb et al. [BOSS Collaboration], “The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: Cosmological implications of the Fourier space wedges of the final sample,” Mon. Not. Roy. Astron. Soc. 467, no. 2, 2085 (2017) [arXiv:1607.03143 [astro-ph.CO]].

[50] A. G. Sanchez et al. [BOSS Collaboration], “The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological implications of the configuration-space clustering wedges,” Mon. Not. Roy. Astron. Soc. 464, no. 2, 1640 (2017) [arXiv:1607.03147 [astro-ph.CO]].

[51] H. Gil-Marin et al., “The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: structure growth rate measurement from the anisotropic quasar power spectrum in the redshift range 0.8 < z < 2.2,” Mon. Not. Roy. Astron. Soc. 477, no. 2, 1604 (2018).

[52] G. B. Zhao et al., “The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: a tomographic measurement of cosmic structure growth and expansion rate based on optimal redshift weights,” arXiv:1801.03043 [astro-ph.CO].

[53] J. Beltran Jimenez, F. Piazza and H. Velten, “Evading the Vainshtein Mechanism with Anomalous Gravitational Wave Speed: Constraints on Modified Gravity from Binary Pulsars,” Phys. Rev. Lett. 116 (2016) no.6, 061101 [arXiv:1507.05047 [gr-qc]].

[54] P. Creminelli, M. Lewandowski, G. Tambalo and F. Vernizzi, “Gravitational Wave Decay into Dark Energy,” JCAP 1812, no. 12, 025 (2018) [arXiv:1809.03484 [astro-ph.CO]].