Romans type IIA theory and the heterotic strings

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Abstract: In this paper we study $T^2$ compactification of six-dimensional massive type IIA supergravity in presence of Ramond-Ramond background fluxes. The resulting theory in four dimensions is shown to possess $SL(2, R) \times SL(2, R) \times O(4, 20)$ duality symmetry. It is shown that specific elements of this symmetry relate massive type IIA compactified on $K3 \times T^2$ (fluxes along $K3$) to the ordinary type IIA compactified on $K3 \times T^2$ (fluxes along $T^2$). In turn, this relationship is exploited to relate Romans theory to heterotic strings. The D8-brane (domain-wall) wrapped on $K3 \times T^2$ is found to correspond to pure gravity heterotic solution which is a direct product of six-dimensional flat space and a four-dimensional Taub-NUT instanton.

Keywords: string, supergravity, compactification, duality

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1. Introduction

Recently a lot of attention is given to the studies of gauged and massive supergravity theories for the reason that these theories play a significant role in AdS/CFT analysis [1] and also because they might be useful in string phenomenology [2]. Massive supergravities are regarded as closed counterparts of gauged supergravities. In massive theories some of the vector or tensor fields become massive upon eating other fields in the spectrum, analogous to a Higgs type mechanism. In this procedure the total degrees of freedom remain unaltered and so do the number of supercharges, however it turns out that we need to sacrifice some of the global symmetries, like dualities. But it is not always so, certain fraction of these symmetries, at least the perturbative ones, could be restored. This is the goal of our investigation in this paper to look for these unbroken global symmetries.

A well studied and unique example of a massive theory of gravity is the massive type IIA supergravity in ten dimensions constructed by Romans [3]. In string theory, massive supergravities can typically be constructed through a Scherk-Schwarz reduction [4], in which some field strength $dA_{(p)}$ is given a non-trivial background value (flux) along the compact directions [5]. Established criterion for turning on such background fluxes with consistency is that the potential $A_{(p)}$ appears only through its field strength $dA_{(p)}$ in the action, or equivalently in the field equations. There has been several works along these lines in recent past [5–17], for more latest works see [18–23].

Amongst massive theories, massive type IIA supergravities can be easily characterised solely by there field content in which the NS-NS 2-rank tensor field is massive. In $D < 10$ massive type IIA theories, in which tensor fields are massive,
can also be obtained through generalised $K3$ and toroidal compactifications of the ordinary (massless) type IIA supergravities with Ramond-Ramond (RR) fluxes turned on [18, 21]. On the other hand massive heterotic supergravities with massive 1-form fields could be obtained by generalised Kaluza-Klein compactification [10, 16]. Thus there is no way out we can make a connection between massive type IIA theories and massive heterotic strings because different type of fields carry masses [18], though there is a duality relationship in $D = 6$ involving ordinary type II and heterotic supergravities, for review see [27]. Therefore unless there is a mechanism by which we can trade massive 2-rank tensor fields into a massive vector fields we cannot help us out. Fortunately there is such a relationship in four dimensions where a massive tensor field carries same degrees of freedom as a massive vector field carries [24]. So in four dimensions it would be possible to dualize a massive 2-rank tensor field into massive 1-form field. Hence the study of massive theories in $D = 4$ is very crucial for duality relationship between massive type IIA and heterotic strings. Though, we will not show this duality relationship between massive tensors and massive vector fields explicitly, we shall be adopting an equivalent tool of compactifications and use duality symmetries, which do this job implicitly.

Paper is organized in the following way. We work out a generalized $T^2$ compactification of the six-dimensional massive type IIA supergravity [18] with RR 2-form fluxes turned on. Our goal is to investigate the fate of the perturbative $SL(2, R) \times SL(2, R) \times O(4, 20)$ duality symmetry. We then are interested in relating this $6D N = 2$ massive type IIA compactified on $T^2$ to the ordinary $6D N = 2$ type IIA compactified on $T^2$ with fluxes and eventually relate it to heterotic string theory compactified on $T^4$. In section 2 we briefly recall $6D$ massive type IIA sugra and work out its compactification on $T^2$ with RR fluxes. We discover that provided RR 2-form fluxes are turned on the resulting four-dimensional theory can be presented in a manifestly $SL(2, R) \times SL(2, R) \times O(4, 20)$ covariant form. In section 3 we provide a mechanism to relate Romans theory compactified on $K3 \times T^2$ (with fluxes) to the ordinary type IIA compactified on $K3 \times T^2$ (with fluxes) which is in turn related to heterotic strings. In section 4 we study the vacuum solutions of this massive type IIA supergravity and relate them to the solutions of ordinary IIA by using the elements of duality group. We then use standard heterotic-type IIA duality in six dimensions to relate corresponding ordinary IIA vacua to heterotic solutions. Particularly the D8-brane solution is shown to correspond to a pure gravity heterotic vacua in ten dimensions which contains a domain-wall-type instanton line element. Thus perturbative duality symmetry relates the vacua of massive type IIA and ordinary type IIA theories in six dimensions provided we have $T^2$ isometries. This property also allows us to further relate massive type IIA vacua to six-dimensional heterotic solutions. We have summarized our results in the section 5.
2. Toroidal compactification

The compactification of Romans type IIA supergravity [3] on K3 with RR fluxes was provided in [18]. The resulting bosonic composition of $6D N = 2$ massive type II theory is given by

$$S_6 = \int \left[ e^{-2\hat{\phi} \hat{\lambda}} \left\{ \hat{R} \hat{\lambda} + 4 \hat{\phi} \hat{\lambda} \hat{\phi} - \frac{1}{2} \hat{H}_3 \hat{\lambda} \hat{H}_3 + \frac{1}{8} Tr \hat{M} \hat{\lambda} \hat{M}^{-1} \right\} - \frac{1}{2} \hat{F}_2^a \hat{\lambda} \hat{M}^{-1} \hat{F}_2^a \right] - \frac{1}{2} m_a \hat{\lambda} \hat{M}_{ab}^{-1} m_b + \frac{1}{2} \hat{B} \hat{F}_a \hat{F}_a - \frac{1}{2} \hat{B}^2 m_a \hat{F}_a + \frac{1}{3!} \hat{B}^3 m_a m_b m_c ,$$

(2.1)

where we have adopted the notation that every product of forms is understood as a wedge product.\(^1\) Here $\hat{\phi}$ is the dilaton field, $\hat{B}_{(2)}$ is NS-NS tensor field and $\hat{A}^a$ ($a = 1, \ldots , 24$) are RR vector fields which form a vectorial representation of the duality symmetry group $O(4,20)$. $m_a$ represents 24 mass (or flux) parameters which also form a vector of $O(4,20)$. The field strengths in the action (2.1) are given by

$$\hat{H}_3 = \hat{\lambda} \hat{B}_{(2)} , \quad \hat{F}_2^a = \hat{\lambda} \hat{A}_1^a + m^a \hat{B}_{(2)} ,$$

(2.2)

There are eighty scalar fields those parameterize $O(4,20)/O(4) \times O(20)$ coset matrix $M_{ab}$ which satisfies

$$\hat{M}^T L \hat{M} = L , \quad \hat{M}^{-1} = L \hat{M} \hat{L} , \quad \hat{M}^T = \hat{M}$$

(2.3)

where $L$ is $O(4,20)$ metric. Thus above six dimensional action has manifest $O(4,20)$ invariance [18]. Once $m_a$ is vanishing, above action represents ordinary type IIA compactified on $K3$.

We shall now work out the compactification of above theory on $T^2$ in presence of RR fluxes. To recall, $T^2$ compactification of Romans type IIA theory has been worked out in detail in [21], so we shall omit various fine details here. Thus for the 6-dimensional sechsbein we take the standard toroidal ansatz

$$\hat{e}_M^a (x,y) = \left( \begin{array}{cc} e_\mu^a (x) & e_m^K \mu \\ 0 & e_m^\mu \end{array} \right) ,$$

(2.4)

where coordinates $y^m$, $(m = 1, 2)$, are tangent to the tori. The internal metric on tori is given by $G_{mn}(x) = e_i^m \delta_{ij} e_i^n$ while the 4-dimensional spacetime metric is $g_{\mu\nu}(x) = e_\mu^a \delta_{ab} e_\nu^b$. $K^m_\mu$ are the Kaluza-Klein gauge fields and we define 1-forms $\eta^m = dy^m + K^m_\mu$. The standard toroidal ansätze for the dilaton, tensor field and the moduli matrix are taken to be

$$\hat{\phi}(x,y) = \hat{\phi}(x) , \quad \hat{B}_{(2)}(x,y) = \hat{B}_{(2)}(x) + \hat{A}_m(x) \eta^m + \frac{1}{2} b_{mn}(x) \eta^m \eta^n , \quad \hat{M}(x,y) = M(x) ,$$

(2.5)

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\(^1\)The signature of the metric is $(- + \ldots +)$ and for a $p$-form we use the convention $F_p = \frac{1}{p!} F_{M_1 \ldots M_p} dx^{M_1} \wedge \ldots \wedge dx^{M_p}$, while the Poincare dual is given by $\ast F_p = \frac{1}{p! (6-p)!} F_{M_1 \ldots M_p} \epsilon^{M_1 \ldots M_{p+1} \ldots M_6} dx^{M_{p+1}} \wedge \ldots \wedge dx^{M_6}$, $\ast \ast F_p = (-1)^{p(6-p)} F_p$, and $\ast 1 = \sqrt{-g} [dx^6]$. 

- 3 -
where \( \bar{B}_{(2)} = B_{(2)} - \frac{1}{2} \bar{A}_m K^m \) with \( B \) being a 2-form in four spacetime dimensions, \( \bar{A}_m \) are two vector fields and the \( b_{mn} \) represents scalar fields antisymmetric in \( m, n \) indices. With these ansätze the NS-NS part of the action (2.1) reduces to [25]

\[
\int e^{-2\phi} \left[ R \ast 1 + 4 d\phi \wedge d\phi - \frac{1}{2} H \star H + \frac{1}{4} dG^{mn} \wedge dG_{mn} - \frac{1}{4} db_{mn} \wedge db_{pq} G^{mp} G^{nq} - \frac{1}{2} dK^m \wedge dK^n G_{mn} - \frac{1}{2} (d\bar{A}_m - b_{np} dK^p) \ast (d\bar{A}_n - b_{nq} dK^q) G^{mn} + \frac{1}{8} Tr dM \ast dM^{-1} \right],
\]

(2.6)

with

\[
2\phi = 2\phi - \frac{1}{2} \ln \det(G_{mn}), \quad H = dB - \frac{1}{2}(\bar{A}_m dK^m + K^m d\bar{A}_m).
\]

(2.7)

Next, for twenty-four 1-forms \( \hat{A}^a \) we would consider a generalized Kaluza-Klein ansatz where RR fluxes along \( T^2 \) are included.\(^2\) This generalization is possible since \( \hat{A} \) appear in the action (2.1) only through derivatives, therefore an appropriate background value can be consistently turned on. We take an ansatz \( \hat{A}(x, y) = \bar{A}(1)(x) + \bar{A}(0)(x, y) dy^m \), where scalar fields \( \bar{A}^{(0)}(x, y) \) are allowed to retain a dependence on the coordinates of the torus. The consistency of toroidal reduction requires it can at most be a linear dependence on the torus coordinates, we define

\[
\bar{A}^{(0)}(x, y) = \bar{a}_m(x) - \frac{1}{2} \bar{w}_{mn} y^n,
\]

(2.8)

where new constants \( \bar{w}_{mn} \) are antisymmetric in indices. This gives us

\[
\hat{d}\hat{A} = \mathcal{D}\bar{A} + \mathcal{D}\bar{a}_m \eta^m + \frac{1}{2} \bar{w}_{mn} \eta^m \eta^n,
\]

(2.9)

where various \( \mathcal{D} \)-derivatives are defined as

\[
\mathcal{D}\bar{A} = d\bar{A} - d\bar{a}_m K^m, \quad \mathcal{D}\bar{a}_m = d\bar{a}_m + \bar{w}_{mn} K^n.
\]

(2.10)

Note that various forms are distinguishable by the symbols and the internal indices they carry. Thus through above generalized ansatz we have effectively introduced 24 new parameter \( \bar{w}_{12} \) in the form of fluxes. This generalization has been possible only because \( \hat{A} \)'s appears in the action covered with derivative and \( T^2 \) has 2-cycle along which an appropriate background flux could be turned on.

After compactification the bosonic spectrum of four-dimensional theory consists of the graviton \( g_{\mu\nu} \), dilaton \( \phi \), 2-form \( \bar{B} \), 4 scalars from the components of metric and tensor field, and 4 1-forms \( (\bar{A}_m, K^m) \) in the NS-NS sector. From the R-R

\(^2\)From here onwards we switch to a notation where an \( O(4, 20) \) vector like \( A^a \) is represented simply by an overhead vector, like \( \hat{A} \).
sector we have $2 \times 24$ scalars $\vec{a}_m$, and twenty-four 1-forms $\vec{A}$ whose field strength are (anti)self-dual in $D = 4$. Also we have two sets of 24 constant parameters in the form of fluxes $\vec{w}_{12}$ and masses $\vec{m}$. This is the precisely the bosonic field content of $N = 4$ type II supergravity theory in $D = 4$. In the massless case these fields fit in various representation of the T-duality group $SL(2, R) \times SL(2, R) \times O(4, 20)$. Here too various fields combine into the $SL(2, R) \times SL(2, R) \sim SO(2, 2)$ representations as

$$A_{(1)}^{ru} = (A^{r1}, A^{r2}), \quad A_{(2)}^{ru} = (\bar{A}_1, \bar{A}_2), \quad A_r^{u=2} = (K^2, -K^1),$$

$$\vec{a}_{(0)} = (\vec{a}_1, \vec{a}_2), \quad \vec{m}^u = (-\vec{w}_{12}, \vec{m}),$$

(2.11)

where indices $r = 1, 2$ belong to first $SL(2, R)$ while indices $u = 1, 2$ belong to the second $SL(2, R)$ group. Note that the mass and flux parameters also fit into a fundamental representation of $SL(2, R)$.

In order to obtain the action involving low energy modes of this theory we substitute the ansatz (2.4)-(2.9) into the action (2.1). The resulting four-dimensional bosonic action reads in the kinetic part

$$S_4 = \int \left[ e^{-2\phi} \left\{ R * 1 + 4 d\phi \wedge d\phi - \frac{1}{2} H \wedge H - \frac{1}{2} dA^{ru} \wedge dA^{su} N_{rs}^{-1} \mathcal{M}_{uv}^{-1} + \frac{1}{4} \text{Tr} d\mathcal{M}^{-1} \wedge d\mathcal{M} + \frac{1}{4} \text{Tr} dN^{-1} \wedge dN + \frac{1}{8} \text{Tr} dM \wedge dM^{-1} \right\} - \frac{1}{2} F(2) \wedge M^{-1} \bar{F}(2) \sqrt{G} - \frac{1}{2} \bar{S}_{(2)}^{r} M^{-1} \wedge \bar{S}_{(2)}^{l} N_{rs}^{-1} - \frac{1}{2} \vec{m}^u M^{-1} \wedge \vec{m}^u \mathcal{M}_{uv}^{-1} \right] + S_{CS},$$

(2.12)

where the Chern-Simon part of the action is

$$S_{CS} = \int \frac{\epsilon^{mn}}{2} \left[ b_{mn} \bar{F} L \bar{F} + \bar{F} L \left\{ B \vec{w}_{mn} - 2 \bar{A}_m \mathcal{D} \vec{a}_n + \vec{m} \bar{A}_m \bar{A}_n \right\} - \frac{1}{2} \bar{B}^2 \vec{m} L \vec{w}_{mn} - \bar{B} (d\vec{a}_m + \vec{w}_{mp} K^p) L (d\vec{a}_n + \vec{w}_{nq} K^q) \right].$$

(2.13)

Various field strengths in the above action are

$$H_{(3)} = dB + A^{ru} dA^{su} \eta_{rs} \mathcal{L}_{uv}, \quad \bar{S}_{(2)}^r = d\vec{a}^r + \vec{m}^u A^{ru}, \quad \bar{F}(2) = \mathcal{D} \bar{A} + \vec{m} (B - \frac{1}{2} \bar{A}_m K^m).$$

(2.14)

The indices $r$ and $u$ can be raised or lowered by the use of two $SL(2, R)$ metrics $\eta$ and $\mathcal{L}$, respectively, which are given by

$$\eta_{rs} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \equiv \mathcal{L}_{uv}.$$

The uni-modular matrices which belong to two $SL(2, R)/SO(2)$ cosets are given by

$$N^{-1} = \sqrt{G} \begin{pmatrix} G^{11} & G^{12} \\ G^{21} & G^{22} \end{pmatrix}, \quad \mathcal{M}^{-1} = \frac{1}{\sqrt{G}} \begin{pmatrix} 1 & -b \\ -b & (b)^2 + G \end{pmatrix},$$

(2.15)
and they satisfy $N^T \eta N = \eta, \ M^T \mathcal{L} M = \mathcal{L}$. We shall take $\varepsilon^{12} = 1$ and we have defined $b_{12} \equiv b, \ \bar{w}_{12} \equiv \bar{w}$. Under the $SL(2, R)$ transformations which act upon $u, v$ indices

$$
g_{\mu\nu} \rightarrow g_{\mu\nu}, \ \phi \rightarrow \phi, \ \ B \rightarrow B, \ \ M \rightarrow M, \ \ M \rightarrow \Lambda M \Lambda^T, \ \ A^u \rightarrow \Lambda^u A^v, \ \ \bar{m}^u \rightarrow \Lambda^u \bar{m}^v, \ \ (2.16)$$

$\Lambda^T \mathcal{L} \Lambda = \mathcal{L}$, alongwith the transformation for $\bar{A}$ which is yet to be determined. The second $SL(2, R)$ group acts only on $r, s$ indices in a similar way. The action (2.12) has manifest $O(4, 20)$ invariance from the beginning as all $O(4, 20)$ indices are contracted with metric $L$.

Note that the kinetic terms in the action (2.12) except the terms involving 24 two-form field strength $\bar{F}$ remain invariant under the action of above T-duality group. It remains to be seen if the field equations and the Bianchi identity for 1-form potential $\bar{A}$ transform covariantly. From the 4-dimensional action in (2.12) the field equations for $\bar{A}$ are

$$
d \left( -\sqrt{G} * M^{-1} \bar{F} + b \ L \bar{F} + \frac{1}{2} \epsilon^{mn} L[\bar{w}_{mn} B - 2 \bar{A}_m \bar{D} \bar{a}_n + \bar{m} \bar{A}_m \bar{A}_n] \right) = 0 \ (2.17)$$

while the Bianchi Identities are

$$
d \left( \bar{F} + d \bar{a}_m K^n - \frac{1}{2} \bar{w}_{mn} K^n K^m - \bar{m} \bar{B} \right) = 0 . \ (2.18)$$

We now define the dual field strengths as $\tilde{\bar{F}} = -\sqrt{G} * M L \bar{F} + b \bar{F}$, then field equations (2.17) and the Bianchi identities for $\tilde{\bar{A}}$ form a $SL(2, R)$ covariant set of equations provided $\bar{F}$ and its dual transform as a vector under $SL(2, R)$ transformations. Thus $SL(2, R)$ symmetry mixes $\bar{F}$ with its dual. One can also combine $\bar{F}$ and its dual into a $SL(2, R)$ field strength

$$
\tilde{\bar{F}}^u = d \tilde{\bar{A}}^u - d \bar{a}_r A^r u - \frac{1}{2} \bar{m}^u A^s u A^s + \bar{m}^u B , \ \ (2.19)
$$

where $\tilde{\bar{A}}^u = (\bar{\tilde{\bar{A}}}, \bar{\tilde{\bar{A}}})$.

This completes our analysis of the four-dimensional massive type II supergravity action which we have shown to possess an explicit $SL(2, R) \times SL(2, R) \times O(4, 20)$ duality symmetry at the level of field equations provided the fluxes on $T^2, \bar{w}$, and the masses, $\bar{m}$, which come from $K3$-compactification, transform as $SL(2, R)$ doublet.

The action (2.12) possesses Stueckelberg gauge invariances [3] which is obvious from the investigation of the field strengths in eq. (2.14). Through these gauge invariances, the vector fields $A^r u$ can eat the scalars $a^r$ and can become massive. Similarly the tensor field $B$ can eat one of the two vector fields $A^u$ and can become massive. However, this process of swallowing in of the fields will break the duality symmetry explicitly.
3. Massive IIA and heterotic strings

As we have seen in the previous section, the restoration of the T-duality symmetry of the massive II theories in $D = 4$ has been a direct consequence of our generalised flux-type ansatz in (2.8). This tells us that in this framework a wide class of type II theories with various RR fluxes on $K3$ and/or $T^2$, in fact, get unified. Specifically under the $SL(2, R)$ elements of this duality symmetry the $K3$-masses (or fluxes) and the $T^2$-fluxes are mixed up and rotated. We now discuss a particular case which is of interest in the rest of this paper. Consider a compactification of 6D $N = 2$ massive II theory (2.1) on $T^2$ without fluxes (i.e. $\vec{w} = 0$), the compactified four dimensional massive theory will then be characterised by the mass vector $m^u = (\vec{0}, \vec{m})$. Similarly if we compactify a 6D $N = 2$ ordinary type IIA theory (that means $\vec{m} = 0$ in (2.1)) on 2-torus with fluxes, the resulting four-dimensional theory will be characterised by a different mass vector $m^u = (-\vec{w}, \vec{0})$. These two four-dimensional theories obtained in two different ways can simply related by the following $SL(2, R)$ element

$$\Lambda = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$  

(3.1)

In many ways this is analogous to the identification between 2-form RR flux and the Romans’ mass parameter when 10D massive IIA is compactified on $T^2$ and a ordinary type IIA compactified on $T^2$ with RR flux [21]. Other elements of above $SL(2, R)$ group mix two types of fluxes which could be used to generate new background configurations. In the rest of this paper our aim is to relate massive type IIA backgrounds to heterotic string backgrounds.

Having achieved this relationship between 6D $N = 2$ massive IIA theory and the ordinary 6D $N = 2$ ordinary type IIA, both compactified on 2-torus, former without fluxes and the latter with fluxes, it is now straightforward to achieve the heterotic connection via following six-dimensional S-duality relations between type IIA compactified on $K3$ and heterotic string theory compactified on $T^4$ [27]

$$\phi_{het} = -\phi_{II}, \quad g^{het}_{\mu\nu} = e^{-\phi_{II}} g^{II}_{\mu\nu}, \quad M_{het} = M_{II}, \quad \vec{A}_{het} = \vec{A}_{II},$$

$$H^{het}_{(3)} = e^{-\phi_{II}} * H^{II}_{(3)}.$$  

(3.2)

In this approach Romans theory compactified on $K3(\text{fluxes}) \times T^2$ is mapped to the ordinary type IIA compactified on $K3 \times T^2(\text{fluxes})$ which in turn is related by duality (3.2) to heterotic string compactified on $T^4 \times T^2(\text{fluxes})$. In the next section we shall take an explicit examples of domain-wall solutions and display this duality chain.

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$^3$Here $\vec{0}$ represents 24-dimensional vector with zero entries.
4. D8-brane vs heterotic instanton

The ten-dimensional massive IIA supergravity theory has D8-brane (domain-wall) solutions which preserve sixteen supercharges [5]. In the string frame metric this solution is given by

\[ ds_{10}^2 = H^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_8^2) + H^{1/2}dz^2, \]
\[ 2\phi_{10} = -\frac{5}{2}\ln H, \]  
(4.1)

where \( H = 1 + m|z - z_0| \) is a harmonic function of only the transverse coordinate \( z \) and all other fields have vanishing background values, \( z_0 \) refers to the location of the domain-wall. We compactify this solution on \( K3 \) by wrapping four of its world-volume directions, say \( x_5, \ldots, x_8 \). The corresponding six-dimensional domain-wall solution of the action (2.1) can be written down as [18]

\[ ds_6^2 = H^{-1/2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2) + H^{1/2}dz^2, \]
\[ 2\hat{\phi}_6 = -\frac{3}{2}\ln H, \quad \hat{M} = \text{diag}(H^{-1}, 1, 1, \ldots, 1, 1, H), \]  
(4.2)

with mass vector \( \vec{m} = (0, \ldots, 0, m) \).

Clearly this vacuum configuration corresponds to the situation when there is no background flux along \( K3 \). The solution (4.2) is left with 8 unbroken supersymmetries. Further compactification of this on \( T^2 \) gives us a solution of the action (2.12)

\[ ds_4^2 = H^{-1/2}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/2}dz^2, \]
\[ 2\hat{\phi}_4 = -\ln H, \quad M = \text{diag}(H^{-1}, 1, 1, \ldots, 1, 1), \quad \hat{M} = \text{diag}(H^{-\frac{1}{2}}, H^{\frac{1}{2}}), \quad N = I_2. \]  
(4.3)

Now, by applying \( SL(2, R) \) transformations (2.10) on the fields in (4.3) the solutions with non-trivial R-R fluxes can be generated. Let us consider the specific case where \( SL(2, R) \) transformation is given by (3.1). Inserting \( \Lambda \) and the configuration (4.3) in (2.10) we get

\[ \vec{m}^u = \begin{pmatrix} 0 \\ \vec{m} \end{pmatrix} \to \vec{m}''^u = \begin{pmatrix} -\vec{m} \\ 0 \end{pmatrix}, \quad M \to M, \quad N \to N, \quad \mathcal{M} \to \mathcal{M}^{-1}, \]  
(4.4)

while four-dimensional metric and the dilaton remain invariant. The transformed mass vector \( \vec{m}''^u \) implies that the new configuration is a solution of a 6D ordinary IIA compactified on \( T^2 \) with 2-form fluxes given by \( \vec{w} = (0, \ldots, 0, m) \). Lifting the rotated solution (4.4) to six dimensions, we get the following ordinary type IIA

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\[ 4 \] The last entry in the mass vector \( \vec{m} \) represents the mass parameter of Romans theory.
configuration (we write new fields with a prime)
\begin{align*}
  d\hat{s}_6'^2 &= H^{-1/2}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/2}(dz^2 + dy_1^2 + dy_2^2), \\
  2\hat{\phi}_6' &= -\frac{1}{2}\ln H, \\
  \vec{A}' &= \vec{w} y_1 dy_2, \\
  \hat{M}' &= \hat{M} = \text{diag}(H^{-1}, 1, 1, \ldots, 1, 1, H).
\end{align*}
\hspace{1cm} (4.5)

This is in accordance with our ansatz in (2.8) and corresponds to switching on the flux. Since this solution is obtained by incorporating T-duality rotation (2.10) the number of preserved supersymmetries will remain unchanged. Thus by making an \(SL(2, R)\) transformation we have transformed domain-wall solution (4.2) of 6D massive IIA theory into a domain-wall solution (4.5) of 6D ordinary type IIA which is supported by a non-trivial 2-form flux. Thus, the four-dimensional perturbative duality interpolates between vacua of massive type IIA and ordinary type IIA. It is parallel to the situation encountered in the case of massive type II duality in \(D = 8\) [21].

**Heterotic instanton:** Since ordinary type IIA theory on \(K3\) is equivalent to heterotic theory on \(T^4\), in order to relate solutions of 6D massive IIA theory to six dimensional heterotic string vacua we need first to map them to the vacua of ordinary IIA by using the \(SL(2, R)\) element (3.1) and then use the relations (3.2). Let us consider for definiteness the configuration in eq. (4.5) which is already a 6D ordinary IIA background and can therefore be mapped to heterotic side using (3.2). After, some straightforward calculation we get the following six-dimensional ordinary heterotic solution (in string frame)
\begin{align*}
  ds^2_{6,\text{het}} &= -dt^2 + dx_1^2 + dx_2^2 + H(dz^2 + dy_1^2 + dy_2^2), \\
  2\phi_{6,\text{het}} &= -\frac{1}{2}\ln H, \\
  M_{\text{het}} &= M_{11} = \text{diag}(H^{-1}, 1, \ldots, 1, H), \\
  \vec{A}_{\text{het}} &= \vec{w} y_1 dy_2, \\
  \vec{w} &= (0, \ldots, 0, m),
\end{align*}
\hspace{1cm} (4.6)

where the harmonic function \(H = 1 + m|z - z_0|\). Note that heterotic string theory compactified on \(T^4\) has the T-duality group \(O(4, 20)\) and \(\vec{A}_{\text{het}}\) belongs to the vector representation of this group [25]. It could be easily seen that the vector field in (4.6) corresponds to a constant field strength in \(y_1, y_2\) directions which are along \(T^2\). A compactification of heterotic strings along these coordinates with such background fluxes gives rise to masses in four dimensions, see [10, 16]. On the other hand when (4.6) is oxidised to ten dimensions, as in ordinary toroidal cases, we obtain following 10D heterotic vacua
\begin{align*}
  ds^2_{10,\text{het}} &= H^{-1}(d\tau + \frac{m}{2}(y_1 dy_2 - y_2 dy_1))^2 + H(dz^2 + dy_1^2 + dy_2^2) - dt^2 + \sum_{i=1}^{5} dx_i^2,
\end{align*}
\hspace{1cm} (4.7)
where $\tau$ is one of the coordinates along $T^4$ on which heterotic string is compactified. $y_1$ and $y_2$ are also periodic but are along $T^2$. This pure gravity heterotic vacua preserves only 8 supersymmetries and has the geometry which is a product of a 4-dimensional Taub-NUT instanton, $\mathcal{E}^4$, and a 6-dimensional Minkowski space, $M_6$. Properties of these Taub-NUT type instanton line element $\mathcal{E}^4$ are discussed in detail [21, 28].

Thus D8-brane wrapped on $K3 \times T^2$ emerges from purely geometrical configuration of the heterotic strings such that it involves ‘domain-wall-instanton’ (4.7). Compare (4.7) with M-theory instanton which is also related via duality to the D8-brane wrapped on $T^2 \times S^1$ and is given by [21]\[^5\]

$$ ds_{11}^2 = H^{-1}[dx_{11} + \frac{m}{2} (y_1 dy_2 - y_2 dy_1)]^2 + H(dz^2 + dy_1^2 + dy_2^2) - dt^2 + \sum_{i=1}^6 dx_i^2, $$

$$ H = 1 + m|z - z_0|, \quad (4.8) $$

where $x_{11}$ is the coordinate of 11-dimensional circle $S^1$. This solution however preserves 16 supersymmetries [21]. This is quite consistent as Heterotic theory is obtained by orbifolding of M-theory. These triad of solutions (1.1), (4.7) and (4.8) thus represent the same duality web involving $Type\,IIA \longleftrightarrow M\,-\,theory \longleftrightarrow Heterotic$ chain. It has been shown [21] that M-theory compactifications on $M_7 \times \mathcal{E}_4$ correspond to 10D massive IIA compactification on 2-torus. Here we have presented an evidence that M-theory compactifications on $Z_2$ orbifold of $M_7 \times \mathcal{E}_4$ should correspond to heterotic compactification on $M_6 \times \mathcal{E}_4$.

5. Summary

To summarize in this work we have studied the $T^2$ compactification of six-dimensional massive type IIA theory [18] with Ramond-Ramond background fluxes corresponding to 2-form field strength. We have found that the resulting four-dimensional theory has $SL(2, R) \times SL(2, R) \times O(4, 20)$ global symmetries, same as the perturbative duality symmetries which appear in ordinary compactifications. The mass and flux parameters transform under $SL(2, R)$ accordingly. Thus the perturbative T-duality survives at the massive level, though in a different form that it requires appropriate masses and fluxes to be switched on. Next we have shown that the elements of this duality symmetry relate Romans theory compactified on $K3 \times T^2$ (with RR fluxes along $K3$) with ordinary type IIA compactified on $K3 \times T^2$ (with RR fluxes along the $T^2$). This relationship between ordinary and massive IIA theories compactified on $K3 \times T^2$ fluxes has led us to provide a heterotic string interpretation for massive IIA theory. As an example we have shown that the wrapped D8-brane solution of

\[^5\]11-dimensional solutions similar to (4.8) originally appear in [6, 8]. The line element in (4.8) differs only in the structure of the vector field from previous occasions.
massive type IIA turns out to be $SL(2, R)$ dual of the solution of ordinary type IIA theory with flux, which in turn is related to pure gravity vacua of heterotic string theory which is a direct product of 6D Minkowski spacetime and a 4D Ricci-flat instanton. The instanton line element is a domain-wall generalization of Taub-NUT instantons [26]. We recall that in [21] we have shown that D8-brane are also related to the compactifications of M-theory involving such instantons and these solutions have 16 supersymmetries intact. While the Heterotic solution (4.7) has only 8 supercharges intact. This is entirely consistent given the fact that heterotic strings are orbifolds of M-theory.

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