Uplink User Capacity in a CDMA System with Hotspot Microcells: Effects of Finite Transmit Power and Dispersion

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Abstract—This paper examines the uplink user capacity in a two-tier code division multiple access (CDMA) system with hotspot microcells when user terminal power is limited and the wireless channel is finitely-dispersive. A finitely-dispersive channel causes variable fading of the signal power at the output of the RAKE receiver. First, a two-cell system composed of one macrocell and one embedded microcell is studied and analytical methods are developed to estimate the user capacity as a function of a dimensionless parameter that depends on the transmit power constraint and cell radius. Next, novel analytical methods are developed to study the effect of variable fading, both with and without transmit power constraints. Finally, the analytical methods are extended to estimate uplink user capacity for multicell CDMA systems, composed of multiple macrocells and multiple embedded microcells. In all cases, the analysis-based estimates are compared with and confirmed by simulation results.

Index Terms—Cellular systems, code division multiple access, microcells.

I. INTRODUCTION

Wireless operators often install low-power hotspot microcell base stations to provide coverage to small high-traffic regions within a larger coverage area. These microcells enhance the user capacity and coverage area supported by the existing high-power macrocell base stations. In this paper, we study these gains for a two-tier code division multiple access (CDMA) network in which both the macrocells and microcells use the same set of frequencies. Specifically, we study the effects, on the uplink capacity and coverage area, of transmit power constraints and variable power fading due to multipath.

Earlier studies have examined the uplink performance of these single-frequency, two-tier CDMA systems, e.g., [1]-[6]. In [4], we showed how to compute the uplink user capacity for a single-macrocell/single-microcell (two-cell) system using exact and approximate analytical methods that accounted for random user locations, propagation effects, signal-to-interference-plus-noise ratio (SINR)-based power control, and various methods of base station selection. In [5], we calculated capacity gains under similar assumptions for a system composed of multiple macrocells and/or multiple microcells; the results pointed to a roughly linear growth in capacity as the number of these base stations increase, where the constants of the linear curve were derivable solely from the analysis for the two-cell case. This linear approximation was conjectured based on the trends observed from several simulations. More recently, we have developed an analytical approximation to the user capacity of a multicell system which is much tighter than the empirical result of [5]; and is valid over a larger number of embedded microcells. Interestingly, this new analytical approach also depends on constants obtained from a far-simpler two-cell analysis. These studies therefore highlight the importance of understanding two-cell performance in computing the performance of larger multicell systems.

The capacity gains demonstrated in these earlier works were based on assumptions that 1) user terminals have unlimited power; and 2) that the wireless channel is so wideband that user signals have constant output power after RAKE receiver processing. This latter condition is equivalent to assuming that user signals go through an infinitely-dispersive channel before reception. In this paper, we remove these conditions. Portions of this work were presented in [7] and a companion paper on downlink capacity [8] showed that this type of system is uplink-limited. Here, we improve the presentation in [7] and present several new results. Specifically, we show how the capacity/coverage tradeoff under finite power constraints can vary significantly with shadow fading. Further, we bridge our study of finite power constraints and finite channel dispersion by presenting a new analysis of user capacity in two-cell systems under both maximum transmit power constraints and variable power fading. Finally and most significantly, we develop and verify analytical expressions for the total user capacity of multicell two-tier CDMA systems under finite dispersion.

Section III describes the basic two-cell system and the model used to capture the power gain between a user and a base; and it summarizes our previous results for total uplink capacity. Section IV assumes a limit on terminal transmit

1By “infinitely dispersive,” we mean that the channel has an infinitude of significant paths so that, after RAKE processing, the receiver signal output has constant power.
power and approximates total capacity as a function of this power and cell size. Section [V] relaxes the condition on infinite dispersion and approximates the total capacity, for both limited and unlimited terminal power, as a function of the multipath delay profile. Section [V] extends the results for finite dispersion and unlimited terminal power to the case of multiple macrocells and multiple microcells. Simulations confirm the accuracy of the approximation methods over a wide range of practical conditions and assumptions.

II. SYSTEM AND CHANNEL DESCRIPTION

The System: We first consider a system with a macrocell and an embedded microcell which together contain \( N \) total users, comprising \( N_M \) in the macrocell and \( N_\mu = N - N_M \) in the microcell. We assume users have random codes of length \( W/R \), where \( W \) is the system bandwidth and \( R \) is the user rate. Each user is power-controlled by its base so as to achieve a required uplink power level. It was shown in [4] that the required received power levels to meet a minimum SINR requirement of \( \Gamma \) are

\[
S_M = \eta W \frac{K - N_\mu + I_M}{(K - N_\mu)(K - N_M) - I_M I_\mu} \tag{1}
\]

and

\[
S_\mu = \eta W \frac{K - N_M + I_\mu}{(K - N_\mu)(K - N_M) - I_M I_\mu} \tag{2}
\]

at the macrocell and microcell base stations, respectively. Here, \( K = 1 + (W/R)/\Gamma \) denotes the single-cell pole capacity [9], \( \eta \) is the power spectral density of the additive white Gaussian noise, and \( I_M \) and \( I_\mu \) are normalized cross-tier interference terms. Specifically, \( S_M I_M \) is the total interference power at the macrocell due to microcell users and \( S_\mu I_\mu \) is the total interference power at the microcell due to macrocell users. Thus,

\[
I_M = \sum_{j \in U_\mu} \frac{T_M j}{T_{Mj}}; \quad I_\mu = \sum_{j \in U_M} \frac{T_\mu j}{T_{Mj}}, \tag{3}
\]

where \( U_\mu \) represents the set of microcell users, \( U_M \) represents the set of macrocell users, and \( T_{Mk} \) and \( T_{\mu k} \) are the path gains from user \( k \) to the macrocell and microcell base stations, respectively. We say that a given arrangement of user locations and path gains is feasible if and only if the common denominator in (1) and (2) is positive. The fraction of all possible cases (i.e., all combinations of \( I_M \) and \( I_\mu \)) for which this occurs is called the probability of feasibility. The cross-tier interference terms depend on the path gains for all users to both base stations and on the method by which users select base stations, leading to the sets \( U_\mu \) and \( U_M \).

Path Gain Model: We model the instantaneous power gain (i.e., the sum of the power gains of the resolvable multipaths) between user \( k \) and base station \( l \) as

\[
T_{lk} = \begin{cases} 
H_l \left( \frac{b_l}{\delta_{lk}} \right)^2 10^{\kappa/10} \rho & d_{lk} < b_l \\
H_l \left( \frac{b_l}{\delta_{lk}} \right)^4 10^{\kappa/10} \rho & d_{lk} \geq b_l
\end{cases}
\]

where \( d_{lk} \) is the distance between user \( k \) and base station \( l \); \( b_l \) is the breakpoint distance of base station \( l \) at which the slope of the decibel path gain versus distance changes; \( H_l \) is a gain factor that captures the effects of antenna height and gain at base station \( l \); \( 10^{\kappa/10} \rho \) represents shadow fading; and \( \rho \) is the variable fading due to multipath [10]-[11]. Note that \( \kappa \) and \( \rho \) are different for each user-base pair \((l,k)\) but, for convenience, we suppress the subscripts for these terms. Since the antenna height and gain are greater at the macrocell, we can assume \( H_M > H_\mu \). We model \( \kappa \) as a zero mean Gaussian random variable with variance \( \sigma_\kappa^2 \) for base station \( l \). The fading due to multipath, \( \rho \), is scaled to be a unit-mean random variable (\( \mathbb{E} \{\rho\} = 1 \)). In an infinitely dispersive channel, \( \rho = 1 \). Let \( T_{lk} \) denote the path gain between user \( k \) and base \( l \) in an infinitely dispersive channel. Thus, \( T_{lk} \) represents the local spatial average of \( T_{lk} \), and \( T_{lk} = \tilde{T}_{lk} \rho \).

Base-Selection Scheme: We assume a user \( k \) chooses to communicate with the macrocell base station if \( \tilde{T}_{Mk} \) exceeds \( \tilde{T}_{\mu k} \) by a fraction \( \delta \), called the desensitivity parameter. Thus, in both finitely-dispersive and infinitely dispersive channels, the user selects base stations according to the local mean path gains.\(^2\) With no loss in generality, we assume \( \delta = 1 \) in this study.

Capacity: The path gain model and the selection scheme described above imply that the cross-tier interference terms are random variables; they depend on the random variations due to shadow and multipath fading and on the random user locations. In [4], we demonstrated how to compute the cumulative distribution functions (CDF’s) of the two cross-tier interference terms in (3) under the condition of infinite dispersion. Let \( \tilde{I}_M \) and \( \tilde{I}_\mu \) denote these cross-tier terms under that condition. The CDF’s of \( \tilde{I}_M \) and \( \tilde{I}_\mu \) were used to compute the exact uplink user capacity in a two-cell system with unlimited terminal power. In addition, the mean values of \( \tilde{I}_M \) and \( \tilde{I}_\mu \) were used to reliably approximate attainable user capacity for the two-cell system as

\[
\hat{N} = \frac{2K}{1 + \sqrt{\tilde{v}_M \tilde{v}_\mu}}, \tag{5}
\]

where \( \tilde{v}_M \) and \( \tilde{v}_\mu \) are the mean values (over all user locations and shadowing fadings) of single terms in the sums \( \tilde{I}_M \) and \( \tilde{I}_\mu \), respectively.\(^3\) The total user capacity of the two-cell system thus depends on \( K \) (a system parameter) and the product \( \tilde{v}_M \tilde{v}_\mu \). We computed this product under various conditions and observed that it (and therefore \( \hat{N} \)) is robust to variations in propagation parameters, separation between the two base stations, and user distribution. Its value under all conditions examined was about 0.09, leading to the robust result \( \hat{N} \approx 1.5K \).

\(^2\)In [4], we studied this path gain-based selection method as well as the selection scheme in which users are assigned to base stations such that each terminal requires the least transmit power. The result show that the latter scheme out-performs the path-gain-based method by roughly 15% in capacity.

\(^3\)In computing \( \hat{N} \), we used the fact, proved in [5], that maximal total capacity results when \( N_M = N_\mu \).
III. USER CAPACITY WITH LIMITED TERMINAL POWER

A. Analysis

In this section, we assume an infinitely dispersive channel and study a system where each user terminal has a maximum transmit power constraint of $P_{\text{max}}$. The two-cell system is unable to support a given set of $N$ users if the users are infeasible (i.e., unsupportable even with unlimited terminal power) or if any one of the $N$ users requires a transmit power higher than $P_{\text{max}}$. We have already computed the probability of infeasibility of $N$ total users in [4]; we now determine the probability that any one of the $N$ feasible users exceeds the terminal power limit of $P_{\text{max}}$. The combined event, when the system is either infeasible or feasible but a user terminal exceeds the transmit power requirements, is referred to as outage. The probability of outage for $N$ users can then be written as

$$P_{\text{out}}(N) = P_{\text{inf}}(N) + (1 - P_{\text{inf}}(N)) \cdot \Pr \left[ P > P_{\text{max}} \mid N \right]$$

(6)

where $P_{\text{inf}}(N)$ is the probability of infeasibility of $N$ users and $\Pr \left[ P > P_{\text{max}} \mid N \right]$ is the probability that the transmit power of any one of the $N$ feasible users exceeds $P_{\text{max}}$. We seek the values of $N$ that a two-cell system can support for a specified probability of outage, i.e., we desire the largest $N$ such that $P_{\text{out}}(N) \leq \alpha_{\text{out}}$.

We assume $b_M = b_\mu = b$. For analytical purposes, it is convenient to write the path gain between user $k$ and base station $l$ as:

$$\tilde{T}_{lk} = H_l \left( \frac{b}{d_{\text{max}}} \right)^4 \tilde{T}_{lk},$$

(7)

where $d_{\text{max}}$ is the desired maximum distance from the macrocell base station to a system user (it can be regarded as the macrocell radius), and

$$\tilde{T}_{lk} = \begin{cases} \left( \frac{d_{\text{max}}}{d_{\text{lk}}} \right)^2 \left( \frac{d_{\text{max}}}{d_{\text{lk}}} \right)^2 10^{\zeta/10}, & d_{\text{lk}} < b \\ \left( \frac{d_{\text{max}}}{d_{\text{lk}}} \right)^4 10^{\zeta/10}, & d_{\text{lk}} \geq b \end{cases}.$$  

(8)

Microcell user $j$ transmits at power $P = S_{\mu j}/\tilde{T}_{\mu j}$; thus $P$ exceeds $P_{\text{max}}$ if

$$\frac{\tilde{S}_\mu}{T_{\mu j}} > \frac{P_{\text{max}} H_{\mu}}{\eta W} \left( \frac{b}{d_{\text{max}}} \right)^4 \equiv F$$

(9)

where $\tilde{S}_\mu = S_\mu/\eta W$. Similarly, the required transmit power of macrocell user $i$ exceeds $P_{\text{max}}$ if

$$\frac{\tilde{S}_M}{T_{Mi}} > \frac{P_{\text{max}} H_M}{\eta W} \left( \frac{b}{d_{\text{max}}} \right)^4 = FH,$$

(10)

where $\tilde{S}_M = S_M/\eta W$ and $H = H_M/H_\mu$.

Note that $\tilde{S}_M$ and $\tilde{S}_\mu$ and all path gains are random variables, related to the randomness of user locations and shadow fading. Following the steps outlined in [4] to obtain the probability distributions of $\tilde{I}_M$ and $\tilde{I}_\mu$, we can extend the analysis to compute the distributions of $\tilde{S}_M$ and $\tilde{S}_\mu$. Next, we can determine the distributions of $\tilde{T}_M$ and $\tilde{T}_\mu$ for a random macrocell user and a random microcell user, respectively.

With the distributions of $\tilde{S}_M$, $\tilde{S}_\mu$, $\tilde{T}_M$ and $\tilde{T}_\mu$, we can then determine $\Pr \left[ P > P_{\text{max}} \mid N \right]$ as a function of $F$:

$$\Pr \left[ P > P_{\text{max}} \mid N \right] = \sum_{n=0}^{N} \binom{N}{n} p^n q^{N-n} (1 - p)^n (1 - p)^{N-n},$$

(11)

where $p$ is the probability of a user assignment to the macrocell; $q = 1 - p$.

$$p_M = P[\tilde{S}_M/\tilde{T}_M \leq FH]; \quad p_\mu = P[\tilde{S}_\mu/\tilde{T}_\mu \leq F].$$

(12)

Finally, we can determine the largest $N$ for which $P_{\text{out}}(N) \leq \alpha_{\text{out}}$ for a given value of $F$. Instead of computing the distributions of $\tilde{S}_M$ and $\tilde{S}_\mu$, we propose an approximate method that works quite well, namely, using the means of $\tilde{S}_M$ and $\tilde{S}_\mu$. This approach retains (4) and (11) but uses the following modified values for $p_M$ and $p_\mu$:

$$\tilde{p}_M = \Pr \left[ \tilde{S}_M/\tilde{T}_M \leq FH \right]; \quad \tilde{p}_\mu = \Pr \left[ \tilde{S}_\mu/\tilde{T}_\mu \leq F \right].$$

(13)

(14)

This greatly simplifies calculating the probability of outage, with minor loss in accuracy.

B. Numerical Results

We use simulation to study the two-cell system in Figure 1. Specifically, we assume a square geographic region, with sides of length $S$. At a distance $D$ from the center of this square is a smaller square region, with sides of length $s$. A portion of the total user population $N$ is uniformly distributed over the larger square region and the remaining users are uniformly distributed over the smaller square region. The smaller square thus represents a traffic hotspot within the larger coverage region. A macrocell base with antenna height $h_M$ is at the center of the larger square, while a microcell base with antenna height $h_\mu$ is at the center of the smaller square. We assume each user terminal has antenna height $h_m$. These antenna heights are used to compute the distance between transmit antenna at each user terminal and the receive antenna at the two base stations. The breakpoint of the path loss to the microcell base is engineered to be $s/2$, to help ensure that the microcells provide strong signals to those users contained within the high density region, and sharply diminishing signals to users outside it. Finally, we assume that on average half of the $N$ users are uniformly distributed over the hotspot region surrounding the microcell base. This is done to obtain a roughly equal number of users for each base, i.e., to ensure that the maximal user capacity occurs with high probability.

Using the parameter values listed in Table 1, we determine the total user capacity that can be sustained with 5% outage probability for various values of $P_{\text{max}}$. Figure 2 shows the resulting user capacity as a function of $F$ as defined in (9). We observe the close correspondence between the approximate and simulation results. We also note that the user capacity goes to an asymptotic value as $F$ gets large and that, below a critical value of $F$ (denoted by $F^*$), the user

$p$ can be computed using the methods in [4].
capacity decreases sharply as $F$ decreases. (From the figure, $F^* \approx 1$.) In other words, there are critical combinations of $P_{\text{max}}$ and $d_{\text{max}}$ such that, if we either reduce the transmit power constraint or increase the desired coverage area, the overall user capacity noticeably drops; and, for values of $F > F^*$, the system operates as if there is unlimited terminal power.

In an earlier study [12], we found that total user capacity is nearly invariant to $\sigma_M$ and $\sigma_\mu$, increasing very slightly as $\sigma_M$ and $\sigma_\mu$ increase. This finding was for unlimited terminal power. We now consider the uplink user capacity as a function of $F$ for various pairs of $(\sigma_M, \sigma_\mu)$. The results, obtained via simulation, are in Figure 3. Even though the capacity for unlimited transmit power ($F$ infinite) is slightly larger for $\sigma_M = 12, \sigma_\mu = 6$ [12], the curves increase much faster for smaller values of $\sigma_M$ and $\sigma_\mu$. Thus, although for unlimited transmit power the total user capacity is roughly invariant to $\sigma_M$ and $\sigma_\mu$, it can vary significantly with these parameters under transmit power constraints.

### IV. User Capacity with Variable Fading: Two-Cell System

#### A. Analysis

We now consider finite dispersion (i.e., a finite number of significant multipaths), which causes the sum of the multipath powers to fade with time as a user moves in the environment. We assume that users still select bases according to the slowly varying path gains, $\tilde{T}_{lk}$, but the fluctuations of signals and interferences due to multipath lead to instantaneous occurrences of outage.

1) **Channel Delay Profile:** The instantaneous path gain from base $l$ to user $k$ is $T_{lk} = \rho \tilde{T}_{lk}$, where $\rho$ is independent and identically distributed (i.i.d.) over all $(l, k)$. Let $r_n$ represent the instantaneous power gain of the $n$-th multipath component of a particular uplink channel. Then, for that channel,

$$\rho = \sum_{n=1}^{L_p} r_n,$$

where $L_p$ is the total number of multipaths and we assume a scaling such that $E\{\rho\} = 1$. We consider four possible multipath delay profiles, for each of which $\rho$ has a different statistical nature. One is for the so-called *uniform channel*, in which the $L_p$ multipaths are i.i.d. and Rayleigh-fading, i.e., each has a power gain that is exponentially distributed with a
mean of $1/L_p$.

The other three delay profiles studied here are based on cellular channel models for the typical urban (TU), rural area (RA), and hilly terrain (HT) environments provided in third-generation standards. See, for example, [13], which tabulates $E\{r_n\}$ in dB for these environments.

2) Methodology: Base selections (and thus, the sets $U_\mu$ and $U_M$) are made according to the local average path gains, $T_{lk}$.

The instantaneous cross-tier interference powers are

$$I_M = \sum_{i \in U_M} \frac{T_{M_i}}{T_{M_i}} = \sum_{i \in U_M} \frac{\rho_{M_i}}{T_{M_i}}, \quad (16)$$

$$I_\mu = \sum_{i \in U_\mu} \frac{T_{\mu_i}}{T_{M_i}} = \sum_{i \in U_\mu} \frac{\rho_{\mu_i}}{T_{M_i}}, \quad (17)$$

where $\rho_{M_i}$ and $\rho_{\mu_i}$ are i.i.d. random variables, each distributed as $\rho$ in (15). We see from the definitions of $I_M$ and $I_\mu$ that, at any one instant, the system could become infeasible or, more generally, experience outage, depending on the values of $\rho_{M_i}$ and $\rho_{\mu_i}$. Here again, we seek the largest number of users, $N$, that can be supported such that $P_{\text{out}}(N) \leq \alpha_{\text{out}}$.

The exact method for calculating $N$ requires that the distributions of $I_M$ and $I_\mu$ be computed. The procedure is even more complex than before since we must incorporate the effects of the additional random quantities, $\rho_{M_i}$ and $\rho_{\mu_i}$. Thus, we resort to two analytical approximations. The first estimates total user capacity under finite dispersion but for large $F$ (i.e., $F > F^*$); and the second estimates capacity under both finite dispersion and finite transmit power requirements.

3) Total User Capacity for $F$ Large:

a) Estimated User Capacity in a Uniform Channel: The total user capacity in a uniform channel, $N_u$, can be approximated by modifying the mean method presented in (5).

Specifically, we postulate that

$$N_u = \frac{2K}{1 + \sqrt{\bar{F} \bar{\nu}}}, \quad (18)$$

where $\bar{F} = E\{\kappa\} \bar{v}_M$ and $\bar{\nu} = E\{\kappa\} \bar{v}_\mu$. The mean values $\bar{v}_M$ and $\bar{v}_\mu$ were defined earlier; and $\kappa = \rho_1/\rho_2$, where $\rho_1$ and $\rho_2$ are i.i.d. Gamma random variables with unit mean and $L_p$ degrees of freedom. In other words, $\rho_1$ represents either $\rho_{M_i}$ or $\rho_{\mu_i}$, and $\rho_2$ represents the other. For a uniform channel with $L_p$ paths, the probability density of $\rho$ (and therefore of $\rho_1$ or $\rho_2$) is

$$f_\rho(x) = \frac{L_p}{(L_p-1)!} e^{-x/L_p}, \quad x > 0. \quad (19)$$

We can compute the mean of $\kappa$ as $E\{\kappa\} = E\{\rho_1\} \cdot E\{\rho_2^{-1}\}$, where $E\{\rho_2\} = 1$ and

$$E\left\{\frac{1}{\rho_2}\right\} = \frac{L_p}{L_p-1} \int_0^\infty \frac{1}{(L_p-2)!} y^{L_p-2} e^{-y} dy \quad (20)$$

$$= \frac{L_p}{L_p-1}, \quad L_p > 1. \quad (21)$$

We can therefore relate $N_u$ to $L_p$, as follows:

$$N_u(L_p) = \frac{2K}{1 + \frac{L_p}{L_p-1} \sqrt{\frac{\bar{F} \bar{\nu}}{\rho}}}. \quad (22)$$

Clearly, this approximation breaks down as $L_p$ approaches 1, as it predicts zero capacity in that case.\footnote{The true capacity for a single-path Rayleigh fading channel is in fact quite poor, though not zero.} For $L_p > 1$, we obtain a simple relationship between user capacity and the degree of multipath dispersion. As $L_p$ becomes infinite, user capacity converges to the estimate given by (5).

b) Estimated User Capacity in a Non-Uniform Channel: The result in (22) can be used to approximate user capacity for any delay profile. Consider a channel delay profile having some arbitrary variation of $E\{r_n\}$ with $n$ over the $L_p$ paths.

We can compute a diversity factor, $DF$, defined as the ratio of the square of the mean to the variance of $\rho$ in (15). For independent Rayleigh paths, $DF$ can be shown to be [14]

$$DF = \left(\frac{\sum_{n=1}^{L_p} E\{r_n\}}{\sum_{n=1}^{L_p} E\{r_n^2\}}\right)^2. \quad (23)$$

From this definition, we see that the diversity factor for a uniform channel is precisely $L_p$.\footnote{Furthermore, via a simple implementation of the Schwarz inequality, it can be shown that the diversity factor for a non-uniform channel with $L_p$ paths is less than $L_p$.}

The true capacity for a single-path Rayleigh fading channel is in fact quite poor, though not zero.
For a uniform channel with \( L_p \) paths, we note that \( v_M = \kappa \tilde{v}_M \), where \( \kappa = \rho_1 / \rho_2 \), and \( \rho_1 \) \((\ell \in 1, 2)\) has probability density given in \( [19] \). Thus, the density of \( v_j \) \((j \in \{M, \mu\})\) can be computed as

\[
f_{v_j}(v) = \int_0^{\infty} f_{v_j}(v/x) \cdot f_\kappa(x) \, dx, \quad v_j \in \{M, \mu\},
\]

where the probability density of \( \tilde{v}_j \) is given in \([4]\) and

\[
f_\kappa(x) = \frac{1}{(L_p - 1)!} \sum_{i=0}^{L_p-1} \frac{(L_p - 1 + i)!}{i!} \cdot \frac{x^{i-1}(L_p x - i)}{(x+1)^{L_p+1+i}}.
\]

We obtain \( f_\kappa(x) \) from its CDF, which can be computed as

\[
F_\kappa(x) = P[\kappa \leq x] = P[\rho_1 \leq \rho_2 x] = 1 - \frac{\rho_2}{\rho_1} + \frac{1}{(L_p - 1)!} \sum_{i=0}^{L_p-1} \frac{x^i}{i!} \cdot \frac{(L_p - 1 + i)!}{(x+1)^{L_p+1+i}}.
\]

With the density of \( v_M \) and \( v_\mu \) at hand, we use the analysis in \([4]\) to compute the densities of \( I_{\kappa} \) and \( I_\mu \) for a given value of \( N \). We then use the densities of these cross-tier interferences to compute the densities of \( S^M_\kappa \) and \( S^\mu_\kappa \), which in turn give us the mean values needed in \( [25] \). The outage probability for this capacity \( N \) is then determined using \( [6] \) and \( [11] \) with the substitutions for \( p_M \) and \( p_\mu \) indicated in \( [25] \). Finally, we can approximate the maximum number of users supported for a desired outage level \( (\alpha_{out}) \) for the uniform multipath channel.

b) Estimated User Capacity in a Non-Uniform Channel:
Given a non-uniform channel, we propose an approximation to user capacity for a given value of \( F \) that uses the above technique for the uniform channel. Specifically, we use the results of Section \( [V-A,4] \) to claim that a system in a non-uniform channel with diversity factor \( DF \) supports roughly the same number of users as the same system in a uniform channel with \( DF \) paths. Thus, for a given non-uniform delay profile, we first compute \( DF \) from \( [23] \). We then set \( L_p \) as the integer closest to \( DF \) and use the analysis above to estimate the total user capacity with outage \( \alpha_{out} \) in a uniform channel with \( L_p \) paths. This value approximates user capacity in the given non-uniform channel.

B. Numerical Results
We use the two-cell system described in Figure \( \Box \) and Table 1 to obtain simulation results for finitely-dispersive channels. For a given configuration of \( N \) users, i.e., a fixed set of user locations and shadow fadings, we generate 1000 values of \( \rho_M \) and \( \rho_\mu \) for each user (based on the given channel delay profile) and record the instances of outage. We then calculate this outage measurement over 999 other random user locations and shadow fadings (assuming a fixed value of \( N \)). The total instances of outage over all these trials determine the probability of outage for \( N \).

Using this simulation method, we first calculate the user capacity with 5\% outage probability when there are no transmit power constraints \((F = \infty)\). This is done for the uniform channel and the RA, HT, and TU environments. For the uniform channel, we determine user capacity as a function of \( L_p \). Figure \( \Box \) shows results for both the simulation and approximation methods. The approximation curve follows the simulation curve closely, especially for smaller values of \( L_p \). Figure \( \Box \) also shows the infinite-\( F \) capacity for the RA, HT, and TU environments (from simulation). We show each of these capacities at a value of \( L_p \) equal to its diversity factor. The diversity factors for the RA, HT, and TU environments are 1.6, 3.3, and 4.0, respectively. We see that, using the diversity factor, the capacity of a non-uniform channel can be mapped to the computed curve for the uniform channel, yielding a simple and reliable analytical approximation to user capacity. For channels with \( DF > 5 \), the total user capacity appears to be within 5\% of that for an infinitely-dispersive channel.

We also obtained user capacity (with 5\% outage probability) as a function of \( F \) for the 2-path and 4-path uniform channels. This was done via both simulation and analytical approximation, and the results are presented in Figure \( \Box \). We observe, first, that the approximation matches the simulation results for the 2-path and 4-path channels in the region of interest, i.e., for \( N \geq K \). Overall, the approximations improve as \( L_p \) increases. Furthermore, the user capacities noticeably decrease for \( F < F^* \), where \( F^* \approx 1 \) for all three channels studied here, a result which is consistent with the infinitely-dispersive case (Figure \( \Box \)).

In Figure \( \Box \) we plot user capacity versus \( F \) using the approximation method of Section \( [V-A,4] \) for \( L_p = 2, 3, 4 \).
and 4 and simulation results for the RA, HT, and TU environments. The RA results closely match those for \( L_p = 2 \); the HT results closely match those for \( L_p = 3 \); and the TU results closely match those for \( L_p = 4 \). Thus, the diversity factor method of Section IV-A.4 allows us to reliably estimate user capacity versus \( F \) for any realistic delay profile.

Finally, we note that as in Figure 5, the critical value \( F^* \) is around 1 for all cases in Figure 6. Thus, we find that, regardless of the delay profile, \( F^* \approx 1 \). For the remainder of the paper, we assume practical values of \( F_{\text{max}} \) and \( d_{\text{max}} \) such that \( F > 1 \).

V. USER CAPACITY WITH VARIABLE FADING: MULTICELL SYSTEMS

A. Estimated User Capacity in Infinitely-Dispersive Channels

In computing the approximate user capacity in an infinitely dispersive channel, we use a result in [5] which states that maximal user capacity results when each cell in a multicell system supports an equal number of users. Let us denote \( N_p \) as the (equal) number of users supported in each cell and \( N_\infty(L,M) \) as the total number of users supported in a system with \( L \) microcells, \( M \) macrocells, and an infinitely dispersive channel.9 We assume initially that all \( L \) microcells are embedded within one of the \( M \) macrocells (macrocell 1) and that the remaining \( M - 1 \) macrocells surround this macrocell. We first write \( L + M \) inequalities representing the minimum SINR requirements at the \( L + M \) bases. The received SINR’s are determined using average interference terms. For example, the SINR requirement at macrocell base station 1 (which contains embedded microcells) is

\[
\frac{\eta W}{\pi} \sum_{l=1}^{L} S_l N_p \tilde{v}_M + \sum_{k=2}^{M} S_{M_k} N_p \tilde{v}_{M_M} + \eta W \geq \Gamma,
\]

where \( N_p = N_p - 1 \), \( S_{M_j} \) is the required received power at macrocell \( j \), \( N_p = N(L,M)/(L + M) \) (as per the optimal equal distribution of users), and the same-tier interference between macrocells is represented by its average value, i.e., \( N_p \tilde{v}_{M_M} \), which we assume to be the same for all macrocells. Here \( \tilde{v}_{M_M} \) is the interference into a macrocell caused by a user communicating with a neighboring macrocell, averaged over all possible user locations in \( R \) and over shadow fading. In our calculations, we simplify the SINR requirement in (30) by assuming that \( \tilde{v}_{M_M} \approx \tilde{v}_M \), i.e., interference due to two users in neighboring macrocells is roughly equal to the interference at a macrocell due to an embedded microcell user. Although macrocells transmit at higher powers than microcells, they are further apart from each other than is a macrocell base from an embedded microcell user. This approximation assumes that the larger distance between macrocells balances the impact of higher transmit powers.

Next, we determine the SINR requirement at macrocell \( j \) (\( j \geq 2 \)). To do so, we assume (as above) that \( \tilde{v}_{M_M} \approx \tilde{v}_M \). We further assume that the interference from neighboring macrocells is much larger than the interference caused by transmissions in a few isolated microcells in a nearby macrocell, i.e., \( \sum_{k \neq j} S_{M_k} \tilde{v}_{M_M} N_p \gg \sum_{l=1}^{L} S_l \tilde{v}_M N_p \), where \( \tilde{v}_{M_M} \) is the interference at any macrocell base due to a microcell user embedded in a neighboring macrocell averaged over all possible user locations and shadow fadings. Under these two assumptions, the SINR requirement at macrocell \( j \) (\( j \geq 2 \)) is approximately

\[
\frac{\eta W}{\pi} S_{M_j} + \sum_{k \neq j} S_{M_k} \eta W \geq \Gamma.
\]

Finally, we calculate the SINR requirements at the \( L \) microcell bases. Here, we assume first that \( \tilde{v}_{M_M} \approx \tilde{v}_M \), where \( \tilde{v}_M \)

9Note that \( N_\infty(L,M) = (L + M)N_p \).
represents the interference into any microcell base due to a
user in a neighboring macrocell averaged over user location
and shadow fading. In other words, we assume the interference
at a microcell from neighboring macrocells is comparable to
interference caused by transmission to the umbrella macrocell.
Note that this is a somewhat pessimistic assumption as \( \bar{v}_{\mu_\text{M}} \)
will typically be less than \( \bar{v}_\mu \). This pessimistic approach
is balanced by assuming further that the microcell-to-microcell
interference \( \bar{v}_{\mu_\mu} \) is negligible. Here \( \bar{v}_{\mu_\mu} \) is the average inter-
ference into any microcell base due to a user communicating
with any other microcell. Thus, the required SINR expression at
microcell \( l \) is

\[
\frac{W}{\sigma^2} S_{\mu l} + S_{Ml} N_p \bar{v}_\mu + \sum_{k=2}^{M} S_{MK} N_p \bar{v}_\mu + \eta W \geq \Gamma. \tag{32}
\]

These \( L + M \) inequalities can be used to show \( [6] \) that \( S_{Mj} \geq 0 \) and \( S_{\mu l} \geq 0 \) for all \( j \in \{1, 2, \ldots, M\} \) and \( l \in \{1, 2, \ldots, L\} \) if and only if

\[
N_\infty(L, M) = (L + M)N_p \leq \frac{K(L + M)}{1 + \sqrt{(L + M - 1)\bar{v}_M \bar{v}_\mu}}. \tag{33}
\]

A benefit of this simplified capacity condition is that it
allows for capacity calculation using \( \bar{v}_M \) and \( \bar{v}_\mu \) alone. As a
result, it is no more complex to determine than is \( N_T \), the
user capacity when \( M = 1 \) and \( L = 1 \), Section 2. Further,
as the product \( \bar{v}_M \bar{v}_\mu \) is robust to variations in propagation
parameters and user distributions, so is the approximation in
\( (33) \). Despite its simplified form, this approximation yields a
fairly accurate estimate to the attainable user capacity.

A simple extension of the capacity expression in \( (33) \)
can now be used to find the user capacity in a general
multicell system, i.e., a system in which the \( L \) microcells can
be embedded anywhere within the coverage areas of the \( M \)
macrocells. We first note that, in the general system, each
macrocell contains on average \( L/M \) microcells. Next, we use
\( (33) \) to determine the user capacity for a multicell system
in which there are \( M \) macrocells but only the center macrocell
contains \( L/M \) microcells. In this case, each cell contains, \( N_p \)
users, where

\[
N_p = \frac{K}{1 + \sqrt{\frac{L}{M} + M - 1)\bar{v}_M \bar{v}_\mu}}. \tag{34}
\]

Next, we assume that this value of \( N_p \) hardly changes when
some other macrocell contains (on average) \( L/M \) embedded
microcells. Thus, we can finally approximate the total attain-
able capacity for the general multicell system as \( (L + M) \)
times the result in \( (34) \), i.e.,

\[
N_\infty(L, M) \approx \frac{K(L + M)}{1 + \sqrt{\frac{L}{M} + M - 1)\bar{v}_M \bar{v}_\mu}}. \tag{35}
\]

B. Estimated User Capacity with Variable Fading

The goal here is to develop a calculation for the multi-
cell user capacity in finitely dispersive channels which is as
straightforward to calculate as the capacity expression in \( (35) \).

Although there are several approaches to this problem, we pro-
pose an extremely simple and highly accurate approximation
method. First, we compute the diversity factor for the given
multipath channel. Next, we make the assumption that systems
with the same diversity factor support equivalent numbers of
users and we seek to find \( N_{DF}(L, M) \), the user capacity in
a system with \( L \) microcells, \( M \) macrocells, and a multipath
channel with diversity factor \( DF \). We then use the results of
Section IV to compute the total user capacity for a two-cell
system, i.e,

\[
N_{DF}(1, 1) = \frac{2K}{1 + \sqrt{\bar{v}_M \bar{v}_\mu}}. \tag{36}
\]

We also compute the total user capacity for a two-cell system
with infinite dispersion, i.e.,

\[
N_{\infty}(1, 1) = \frac{2K}{1 + \sqrt{\bar{v}_M \bar{v}_\mu}}. \tag{37}
\]

We use these two capacities to determine the capacity percent-
age loss due to dispersion, \( p_{\text{loss}}(DF) \), for the two-cell system.
This loss is given as

\[
p_{\text{loss}} = \frac{N_{DF}(1, 1)}{N_{\infty}(1, 1)} = \frac{1 + \sqrt{\bar{v}_M \bar{v}_\mu}}{1 + \frac{DF}{DF-1} \sqrt{\bar{v}_M \bar{v}_\mu}}. \tag{38}
\]

Finally, we claim and demonstrate through extensive numerical
results that the total attainable user capacity in a multicell
system with \( L \) microcells, \( M \) macrocells, and finite dispersion
\( DF \) is simply \( p_{\text{loss}}(DF) \) times the total attainable user
capacity of a system with \( L \) microcells, \( M \) macrocells, and
infinite dispersion, i.e.,

\[
N_{DF}(L, M) \approx p_{\text{loss}}(DF) \cdot N_{\infty}(L, M) \tag{39}
\]

\[
= \frac{1 + \sqrt{\bar{v}_M \bar{v}_\mu}}{1 + \frac{DF}{DF-1} \sqrt{\bar{v}_M \bar{v}_\mu}} \cdot \frac{K(L + M)}{1 + \sqrt{\frac{L}{M} + M - 1)\bar{v}_M \bar{v}_\mu}}. \tag{40}
\]

C. Numerical Results

1) Single-Macrocell/Multiple-Microcell System: We begin
with a large square region having side \( S \) and a macrocell
base at its center. A fraction of the total system users are
uniformly distributed over this region. This larger square
is divided into \( n^2 \) squares with side \( s \), as shown in Figure 7 for
\( n = 5 \). The smaller squares represent potential high-density
regions. In each simulation trial, we randomly select \( L \) high
density regions (excluding the center square which contains
the macrocell base) and place a microcell base at the center.
Finally, we assume that the \( L \) microcells have identical values
for \( \bar{v}_\mu \), \( H_\mu \), and \( \sigma_\mu \).

Our simulations assume \( n = 5 \), meaning 24 possible
candidate high density regions and the parameters in Table 1.
For a given value of \( L \), the simulation randomly selects
\( L \) of the 24 high density regions. A portion of the total
user population placed in each region is \( \frac{1}{24} \). As before,
this was done to ensure that maximal user capacity occurs
with high probability [5]-[6]. The simulation then determines the total number of users supported with 5% outage. This is done for 23 other random selections of \( L \) high density regions, and the average over these selections is computed as a function of \( L \).

We performed the above simulations for uniform channels with \( L_p = 2 \) and \( L_p = 4 \), and for the infinitely dispersive channel. The results are in Figure 8. In addition, we show the attainable user capacities predicted by (35) and (40). The approximations closely match simulation points up to at least \( L = 72 \), which translates to roughly eight microcells per macrocell (fill factor of about one third). Thus, the approximation method is reliable in the domain of hotspot microcells.

2) Multiple-Macrocell/Multiple-Microcell System: We simulated a multiple-macrocell/multiple-microcell system by extending the system studied above. Specifically (Figure 9), there are \( m^2 \) macrocells and \( m^2(n^2 - 1) \) candidate high density regions (locations for microcell bases). We randomly select \( L \) high density regions from this pool. We again accrue the average total capacity and its one-standard-deviation spread for the two extreme cases of dispersion, i.e., infinite dispersion and \( L_p = 2 \). The results for \( m = 3 \), \( n = 5 \) are in Figure 10. Also shown are the capacities predicted by (35) and (40) for \( M = 9 \) and \( DF = 2 \). The approximations closely match simulation points up to at least \( L = 72 \), which translates to roughly eight microcells per macrocell (fill factor of about one third). Thus, the approximation method is reliable in the domain of hotspot microcells.

VI. CONCLUSION

We first examined the effect of transmit power constraints and cell size on the uplink performance of a two-cell two-tier CDMA system. We then investigated the effect of finite channel dispersion (which causes variable fading of the output signal power) on this capacity. Analysis was presented to approximate the uplink capacity of any channel delay profile using the uniform multipath channel, both with and without terminal power constraints. Finally, we presented an approximation to the uplink user capacity in larger multicell systems with finite dispersion and demonstrated its accuracy using extensive simulation.

While a particular set of system parameters was used in obtaining our numerical results, the analytical approximation
methods, and their confirmation via simulations, are quite general. What we have shown here, first, is that two-tier uplink user capacity can be reliably estimated in a two-cell system for arbitrary pole capacity, channel delay profile, shadow fading variances, transmit power limit, and cell size. Moreover, under realistic conditions on transmit power and cell size ($F > 1$), the method can be extended to the case of multiple macrocells with multiple microcells. Thus, we have developed simple, general and accurate approximation methods for treating a long-standing problem in two-tier CDMA systems.

ACKNOWLEDGMENT

We thank the editor, Dr. Halim Yanikomeroglu, for his helpful suggestions on expanding the scope of our study.

REFERENCES

[1] J. Shapira, “Microcell engineering in cdma cellular networks,” *IEEE Trans. Vehic. Technol.*, vol. 43, no. 4, pp. 817–825; Nov. 1994.

[2] J. S. Wu, et al., “Performance study for a microcell hot spot embedded in cdma macrocell systems,” *IEEE Trans. on Vehic. Technol.*, vol. 48, no. 1, pp. 47–59; Jan. 1999.

[3] J. J. Gaytán and D. Mu noz Rodríguez, “Analysis of capacity gain and her performance for cdma systems with desensitized embedded microcells,” in *Proc. of International Conf. on Universal Personal Commun.* ’98, 1998, vol. 2, pp. 887–891; Oct. 1998.

[4] S. Kishore, et al., “Uplink capacity in a cdma macrocell with a hotspot microcell: exact and approximate analyses,” *IEEE Trans. on Wireless Comm.*, vol. 2, no. 2, pp. 364–374; Mar. 2003.

[5] S. Kishore, et al., “Uplink user capacity of a multi-cell cdma system with hotspot microcells,” in *Proc. of Vehic. Techn. Conf. S-02*, vol. 2, pp. 992–996; May 2002.

[6] S. Kishore, et al., “Uplink user capacity of multi-cell cdma system with hotspot microcells,” submitted to *IEEE Trans. on Wireless Comm.*, Sept. 2003, [http://www.eecs.lehigh.edu/~skishore/research/inprogress.htm](http://www.eecs.lehigh.edu/~skishore/research/inprogress.htm).

[7] S. Kishore, et al., “Uplink user capacity in a cdma macrocell with a hotspot microcell: Effects of finite power constraints and channel dispersion,” in *Proc. of IEEE Globecom*, vol. 3, pp. 1558–1562, Dec. 2003.

[8] S. Kishore, et al., “Downlink user capacity in a cdma macrocell with a hotspot microcell,” in *Proc. of IEEE Globecom*, vol. 3, pp. 1573–1577, Dec. 2003.

[9] K. S. Gilhousen, et al., “On the capacity of a cellular cdma system,” *IEEE Trans. on Vehic. Technol.*, vol. 40, no. 2, pp. 303–312; May 1991.

[10] V. Erceg, et al., “An empirically based path loss model for wireless channels in suburban environments,” *IEEE J. on Sel. Areas in Comm.*, vol. 17, no. 7, pp. 1205–1211; July 1999.

[11] T. S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice Hall, 1996; Chapter 3.

[12] S. Kishore, *Capacity and Coverage in Two-tier Cellular CDMA Networks*, Ph.D. Thesis, Department of Electrical Engineering, Princeton University; Jan. 2003.

[13] “Deployment Aspects,” 3GPP Specifications, TR 25.943 V4.0.0, (2001–2006), 2001.

[14] A. Awoniyi, et al., “Characterizing the orthogonality factor in wcdma downlink,” *IEEE Trans. on Wireless Comm.*, vol. 2, no. 4, pp. 611–615; July 2003.

### Table I

| TABLE I | SYSTEM PARAMETERS USED IN SIMULATION. |
|---------|--------------------------------------|
| $W/R$   | 128                                  |
| $\Gamma_M$ | 7 dB                                  |
| $b_M$   | 100 m                                 |
| $b_H$   | 100 m                                 |
| $H_M$   | 10 $b_H$                              |
| $\sigma_M$ | 8 dB                                  |
| $s$     | 200 m                                 |
| $\mu_H$ | 1.5 m                                 |
| $\Gamma_M$ | 7 dB                                  |
| $b_H$   | 9 m                                   |
| $b_M$   | 100 m                                 |
| $H_M$   | 300 m                                 |
| $\sigma_M$ | 4 dB                                  |
| $s$     | 1 km                                  |

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