Improved measurement of $CP$-violating parameters in $B^0 \rightarrow \rho^+ \rho^-$ decays

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We present a measurement of the CP-violating asymmetry in $B^0 \to \rho^+ \rho^-$ decays using 535 million $B\bar{B}$ pairs collected with the Belle detector at the KEKB $e^+ e^-$ collider. We measure CP-violating coefficients $A = 0.16 \pm 0.21 \text{(stat)} \pm 0.08 \text{(syst)}$ and $S = 0.19 \pm 0.30 \text{(stat)} \pm 0.08 \text{(syst)}$. These values are used to determine the unitarity triangle angle $\phi_2$ using an isospin analysis; the solution consistent with the Standard Model lies in the range $54^\circ < \phi_2 < 113^\circ$ at the 90% confidence level.

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$\text{CP}$ violation in the Standard Model is attributed to the presence of an irreducible complex phase in the Cabibbo-Kobayashi-Maskawa (KM) quark-mixing matrix. The unitarity of the CKM matrix leads to six triangles in the complex plane. One such triangle is given by the following relation among the matrix elements in the complex plane. One such triangle is

$$\begin{align*}
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\end{align*}$$
FIG. 1: Left: projections in $M_{bc}$ for events satisfying $-0.10 \text{ GeV} < \Delta E < 0.06 \text{ GeV}$. Right: projections in $\Delta E$ for events with $5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$. The top plots correspond to good quality tags ($0.75 < r < 1.0$), and the bottom plots correspond to lower quality tags ($r < 0.75$). The curves show fit projections: dashed is $\rho^+ \rho^- + \rho \pi \pi$, dotted is $q \bar{q}$, dot-dashed is $b \to c$, small solid is $b \to u$, and large solid is the total. For these plots the $R$ requirement has been tightened to increase the ratio of signal to background.

superconducting solenoid coil that provides a 1.5 T magnetic field.

We reconstruct $B^0 \to \rho^+ \rho^-$ decays by combining two oppositely charged pion tracks with two neutral pions. Each charged track is required to have a transverse momentum $p_T > 0.10 \text{ GeV}/c$ in the laboratory frame and originate within $dr < 0.2 \text{ cm}$ in the radial direction and within $|dz| < 4.0 \text{ cm}$ along the $z$-axis from the interaction point, which is determined run-by-run. A track is identified as a pion using information from the CDC, ACC and TOF systems. Tracks matched with clusters in the ECL that are consistent with an electron hypothesis are rejected.

The $\pi^0$ candidates are reconstructed from $\gamma \gamma$ pairs with an invariant mass in the range $117.8 \text{ MeV}/c^2 < M_{\gamma \gamma} < 150.2 \text{ MeV}/c^2$ (about $\pm 3\sigma$ in $m_{\pi^0}$ resolution). Photons are required to have energy $E_{\gamma} > 50 \text{ MeV}$ in the ECL barrel region ($32^\circ < \theta < 129^\circ$) and $E_{\gamma} > 90 \text{ MeV}$ in the endcap regions ($17^\circ < \theta < 32^\circ$ and $129^\circ < \theta < 150^\circ$), where $\theta$ denotes the polar angle with respect to the $z$ axis.

To reconstruct $\rho^\pm$ mesons, we combine $\pi^\pm$ candidates with $\pi^0$ candidates. The $\pi^\pm \pi^0$ combination must have an invariant mass in the range $0.62 \text{ GeV}/c^2 < M_{\pi^\pm \pi^0} < 0.92 \text{ GeV}/c^2$. To reduce combinatorial background, we reject $\pi^0$s with $p < 0.35 \text{ GeV}/c$ in the CM frame. We also require $-0.80 < \cos \theta_\rho < 0.98$, where $\theta_\rho$ is the angle between the direction of the $\pi^0$ from the $\rho^\pm$ and the negative of the $B^0$ momentum in the $\rho^\pm$ rest frame.

$B^0 \to \rho^+ \rho^-$ decays are identified using the beam-energy-constrained mass $M_{bc} = \sqrt{E_{\text{beam}}^2 - p_B^2}$ and energy difference $\Delta E = E_B - E_{\text{beam}}$, where $E_{\text{beam}}$ is the beam energy, and $E_B$ and $p_B$ are the energy and momentum of the reconstructed $B$ candidate, all evaluated in the CM frame.

The flavor of the $B$ meson accompanying the $B^0 \to \rho^+ \rho^-$ candidate is identified via its decay products: charged leptons, kaons, and $\Lambda$’s. A tagging algorithm yields the flavor of the tagged meson, $q$, and a quality factor, $r$. The parameter $r$ ranges from 0 for no flavor discrimination to 1 for unambiguous flavor assignment. We divide the data sample into six $r$ intervals (denoted $\ell=1,2,\ldots,6$). The wrong tag fractions $\Delta \ell_q$ for these intervals and the differences $\Delta \ell_{BG}$ in these fractions between $B^0$ and $\overline{B}^0$ decays are determined from data.

The dominant background originates from $e^+e^- \to q\bar{q}$ ($q = u, d, s, c$) continuum events. To separate $q\bar{q}$ jet-like events from more spherical $B\overline{B}$ events, we use event-shape variables, specifically, 16 modified Fox-Wolfram moments combined into a Fisher discriminant. We form signal and background likelihood functions $L_s$ and $L_{BG}$ by multiplying the PDF for the Fisher discriminant by a PDF for $\cos \theta_B$, where $\theta_B$ is the polar angle in the CM frame between the $B$ direction and the beam axis. The PDFs for signal and $q\bar{q}$ are obtained from Monte Carlo (MC) simulations and the data sideband $5.23 \text{ GeV}/c^2 < M_{bc} < 5.26 \text{ GeV}/c^2$, respectively. We calculate the ratio $R = L_s/(L_s + L_{BG})$ and make a loose requirement $R > 0.15$.

The decay vertices of a $\rho^+ \rho^-$ candidate and the tag-side $B$ meson are reconstructed using charged tracks that have a sufficient number of SVD hits and an interaction point constraint. The vertex reconstruction algorithm is described in Ref. 10.

The analysis is organized in two main steps. We first
determine the yields of signal and background components from a fit to the three-dimensional \((M_{\text{bc}}, \Delta E, R)\) distribution. Here, \(B^0\) candidates are required to satisfy 5.23 GeV/c^2 < \(M_{\text{bc}}\) < 5.29 GeV/c^2, \(-0.2\) GeV < \(\Delta E\) < 0.26 GeV, and \(R > 0.15\). We subsequently perform a fit to the \(\Delta E\) distribution to determine the \(CP\) parameters \(A\) and \(S\). The signal region used for the \(\Delta E\) fit is 5.27 GeV/c^2 < \(M_{\text{bc}}\) < 5.29 GeV/c^2, \(-0.12\) GeV < \(\Delta E\) < 0.08 GeV, and \(R > 0.15\).

About 12.6\% of events contain multiple \(B^0 \rightarrow \rho^+ \rho^-\) candidates, most of which arise from fake \(\pi^0\)'s combining with good tracks. We select the best candidate based on the \(\pi^0\) masses, i.e., minimizing \(\sum_{i=1,2} (m_{\gamma\gamma} - m_{\pi^0})^2\). For the small fraction (3\%) of multiple-candidate events that arise due to extraneous \(\pi^\pm\) tracks combining with a single \(\pi^0\), we select one randomly. Signal decays that have at least one \(\pi\) meson incorrectly identified are referred to as "self-cross-feed" (SCF) events.

The likelihood function used to determine the event yields is given by

\[
L = \exp \left( -\sum_j N_j \prod_{i=1}^{N_{\text{evt}}} \left[ \sum_j N_j \mathcal{P}_j(M_{\text{bc}}, \Delta E^i, R^i) \right] \right) \tag{2}
\]

where \(j\) indicates one of the following event categories: signal and \(\rho\rho\pi\) non-resonant decays, SCF events, continuum background \((q\bar{q})\), \(b \rightarrow c\) background, and charmless \((b \rightarrow u)\) background. \(N_j\) is the yield of each category, \(\mathcal{P}_j(M_{\text{bc}}, \Delta E^i, R^i)\) is the PDF for the \(i\)-th event for category \(j\), and \(N_{\text{evt}}\) is the total number of events in the fit. Except for the small contributions of \(b \rightarrow u\) background and SCF events, the yields \(N_j\) are determined from the fit. Due to the similar shapes of the \(M_{\text{bc}}, \Delta E, R\) distributions for signal and \(\rho\rho\pi\) events, we cannot distinguish these two components; the fraction of \(\rho\rho\pi\) events is measured in Ref. [4] and constitutes \((6.3 \pm 6.7)\%\) of the total \(N_{\rho\rho\pi\rightarrow\rho\rho\pi}\) signal. The fraction of SCF events is determined from MC simulation. The \(M_{\text{bc}}\) and \(\Delta E\) shapes for the signal and SCF components are modeled by a two-dimensional smoothed histogram obtained from a large MC sample. To take into account a small difference between the MC and data, the \(M_{\text{bc}} - \Delta E\) shapes are corrected according to calibration factors determined from a \(B^- \rightarrow D^0\rho^+, \bar{D}^0 \rightarrow K^+\pi^-\pi^0\) control sample. The \(R\) shapes are modeled by one-dimensional histograms, also obtained from MC simulation.

The PDF for \(b \rightarrow c\) background is the product of a threshold ARGUS function [13] for \(M_{\text{bc}}\), a quadratic polynomial for \(\Delta E\), and the sum of a Gaussian and a third-order polynomial for \(R\). The shapes of the \(\Delta E\) and \(R\) distributions depend on the tag quality bin \(\ell\). Parameters for all distributions are obtained from the MC.

The \(M_{\text{bc}}\) and \(\Delta E\) PDFs for \(q\bar{q}\) are modeled by an ARGUS function and a linear function, respectively. The \(\Delta E\) slope depends on \(R\) and the tag quality bin \(\ell\). The shape parameters for \(M_{\text{bc}}\) and \(\Delta E\) are floated in the fit. The \(R\) PDF for \(q\bar{q}\) background is taken to be an eighth-order polynomial function; the coefficients depend on the bin \(\ell\) and are determined from a data sample collected at a CM energy \(\sim 60\) MeV below the \(\Upsilon(4S)\).

The \(b \rightarrow u\) background is dominated by \(B \rightarrow (\rho \pi, a_1 \pi, a_1 \rho)\) decays. We estimate the \(B^\pm \rightarrow (a_1 \rho)\pm\) branching fractions (which are unmeasured) to be \((20 \pm 10) \times 10^{-6}\) using the measured value for \(B^0 \rightarrow a_1^\pm \pi^\mp\) [14]. For \(B^\pm \rightarrow (a_1 \rho)\pm\) we assume branching fractions of \((30 \pm 15) \times 10^{-6}\), consistent with the present upper limit for \(B^0 \rightarrow a_1^\pm \pi^\mp\) \((< 6 \times 10^{-5}\) [15]). The fraction of \(b \rightarrow u\) events is very small (0.37\%) and thus is fixed in the fit according to the prediction of MC simulation. A fit to 176843 events maximizing \(L\) yields \(N_{\rho\rho\pi\rightarrow\rho\rho\pi} = 576 \pm 53\). Figures 1 and 2 show the \(M_{\text{bc}}, \Delta E, R\) distributions along with projections of the fit result.

The \(CP\)-violating parameters \(A\) and \(S\) are obtained using an unbinned ML fit to the \(\Delta E\) distribution. The likelihood function for event \(i\) is given by

\[
L_i = \sum_n \int f_n(\hat{x}_i) \mathcal{P}_n(\Delta t') R_n(\Delta t') d\Delta t', \tag{3}
\]

where \(n\) is one of the six event categories: correctly reconstructed signal, SCF events, \(\rho\rho\pi\) non-resonant events, \(b \rightarrow u\) background, \(\Upsilon(4S)\) background, and \(b \rightarrow c\) background. The weights \(f_n\) are functions of \(\hat{x} \in (M_{\text{bc}}, \Delta E, R)\) and are normalized to the event fractions obtained from the \((M_{\text{bc}}, \Delta E, R)\) fit. The PDFs \(\mathcal{P}_n(\Delta t)\) are convolved with the corresponding \(\Delta t\) resolution functions \(R_n\). Both \(f_n\) and \(\mathcal{P}_n(\Delta t)\) depend on the tag quality bin \(\ell\).

The signal PDF is given by Eq. (1) modified to take into account the effect of incorrect flavor assignment: \(e^{-|\Delta t|/\tau}\mathcal{P}(4\Upsilon_{4S}) \times \{1 - q\Delta t\rho + q(1 - 2\Delta t)\mathcal{A} \cos(\Delta m \Delta t) + \mathcal{S} \sin(\Delta m \Delta t)\}\). As the fraction of longitudinal polarization \(f_L\) is close to 100\%, we assume that \(\mathcal{A} = \mathcal{A}_L, \mathcal{S} = \mathcal{S}_L\), and consider the potential contribution from a transversely polarized amplitude as a systematic uncertainty. The signal PDF is convolved with the same \(\Delta t\) resolution function as that used for Belle's sin2\(\phi_3\) measurement [10].

The fraction of SCF events with incorrectly reconstructed vertices is estimated from MC simulation to be \((6.5 \pm 0.1)\%\) of all signal events. The PDFs \(\mathcal{P}_{\rho\rho\pi}\) and \(\mathcal{P}_{\text{SCF}}\) are exponential with \(\tau = \tau_B\) and \(\tau \approx 0.96\) ps (from MC), respectively; these are smeared by a common resolution function.

The \(\Delta t\) PDFs for the backgrounds are modeled as a sum of prompt and exponential components: \(\mathcal{P}_k = f_k^p \delta(\Delta t) + (1 - f_k^p) e^{-|\Delta t|/\tau_k}/2\tau_k\), where \(k\) represents continuum, \(b \rightarrow c\), and \(b \rightarrow u\) backgrounds. \(f_k^p\) is the fraction of the prompt component, \(\delta(\Delta t)\) is the Dirac delta function, and \(\tau_k\) is an effective lifetime. These PDFs are convolved with a resolution-like function \(R_k\) parameterized
as a sum of two Gaussian functions. Parameters for $P_k$ and $R_k$ are determined from a data sideband for continuum background and from large MC samples for $b \rightarrow c$ and $b \rightarrow u$ backgrounds. To account for small correlations between the shape of the $\Delta t$ distribution and $R$ for $q\bar{q}$ background, the parameters are obtained separately for low ($0.15 < R < 0.75$) and high ($0.75 < R < 1.0$) $R$ regions.

We determine $A$ and $S$ by maximizing $\sum_i \log L_i$, where $i$ runs over the 18016 events in the $(M_{bc}, \Delta E, R)$ signal region. The results are $A = 0.16 \pm 0.21$ and $S = 0.19 \pm 0.30$, where the errors are statistical. The correlation coefficient is $-0.10$. These values are consistent with no $CP$ violation ($A = S = 0$); the errors are consistent with MC expectations. Figure 3 shows the data and projections of the fit result.

The sources of systematic error are listed in Table II. The error for most sources is evaluated by varying the corresponding parameters by $\pm 1$ standard deviation ($\sigma$). The effect of a possible asymmetry in $b \rightarrow c$ and $q\bar{q}$ is evaluated by adding such an asymmetry to the $b \rightarrow c$ and $q\bar{q}$ $\Delta t$ distributions. The uncertainty due to a possible asymmetry in $\rho \pi \pi$ non-resonant decays is estimated by varying $A_{\rho \pi \pi}$ and $S_{\rho \pi \pi}$ by 0.68, corresponding to a 68% confidence interval of a free distribution. We vary the branching fractions for $a_1 \rho$ and $a_1 \pi$ decays and also allow for a $CP$ asymmetry of up to 100% in these modes. The error due to transverse polarization is obtained by first setting $f_L$ equal to its central value [2] and varying $A_T$, $S_T$ from $-1$ to $+1$; then, conservatively assuming that the transversely polarized component (with fraction $f_T = 1 - f_L$) is pure $CP$-odd for which $A_T = A_L$, $S_T = -S_L$, and varying $f_L$ by its error. Summing up in quadrature all systematic uncertainties, we obtain overall systematic errors of $\pm 0.08$ for both $A$ and $S$. Thus,

$A = 0.16 \pm 0.21$ (stat) $\pm 0.08$ (syst),

$S = 0.19 \pm 0.30$ (stat) $\pm 0.08$ (syst).

These values are consistent with, and supersede, our previous measurement [4]. They are also consistent with results obtained by BaBar [5].

We constrain $\phi_2$ using an isospin analysis [17], which allows one to relate six observables to six underlying parameters: five decay amplitudes for $B \rightarrow \rho \rho$ and the angle $\phi_2$. The observables are the branching fractions for $B \rightarrow \rho^+ \rho^-$, $\rho^+ \rho^0$ [6], and $\rho^0 \rho^0$ [3]; the $CP$ parameters $A$ and $S$ (our results); and the parameter $A_{\rho \rho}$ for $B \rightarrow \rho^0 \rho^0$ decays. The last parameter is not yet measured, but nevertheless one can constrain $\phi_2$. The branching fractions must be multiplied by the corresponding longitudinal polarization fractions [6]. We neglect possible contributions from electroweak penguins and $I = 1$ amplitudes [18] and possible interference between signal and non-resonant components. We follow the statistical method of Ref. [19] and construct a $\chi^2(\phi_2)$ using the measured values and obtain a minimum $\chi^2$ (denoted $\chi^2_{\text{min}}$); we then scan $\phi_2$ from $0^\circ$ to $180^\circ$, calculating the difference $\Delta \chi^2 \equiv \chi^2(\phi_2) - \chi^2_{\text{min}}$. We insert $\Delta \chi^2$ into the cumulative distribution function for the $\chi^2$ distribution for one degree of freedom to obtain a confidence level (C.L.) for each $\phi_2$ value. The resulting function $1 - \text{C.L.}$ (Fig. 4) has more than one peak due to ambiguities that arise when solving for $\phi_2$. The “flat-top” regions in Fig. 4

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{The $\Delta t$ distribution and projections of the fit for events satisfying $0.5 < r < 1.0$: (a) $q = +1$ tags, (b) $q = -1$ tags. The hatched region shows signal events. The raw $CP$ asymmetry is shown in (c). For these plots the $R$ requirement has been tightened to increase the ratio of signal to background.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{1 - C.L. vs. $\phi_2$. The horizontal lines denote C.L. = 68.3% (solid) and C.L. = 90% (dashed).}
\end{figure}
TABLE I: Systematic errors for $CP$ coefficients $A$ and $S$.

| Type                                      | $\Delta A \times 10^{-2}$ | $\Delta S \times 10^{-2}$ |
|-------------------------------------------|-----------------------------|-----------------------------|
| Wrong tag fractions                       | $+\sigma$ 0.5 0.5 0.8 0.8   | $-\sigma$ 0.5 0.5 0.8 0.8   |
| Parameters $\Delta m_2$, $\tau_{\rho\phi}$ | $+\sigma$ 0.2 0.3 0.6 0.7   | $-\sigma$ 0.2 0.3 0.6 0.7   |
| Resolution function                        | $+\sigma$ 1.4 1.5 1.0 1.7   | $-\sigma$ 1.4 1.5 1.0 1.7   |
| Background $\Delta t$ distributions       | $+\sigma$ 0.5 0.5 1.0 1.1   | $-\sigma$ 0.5 0.5 1.0 1.1   |
| Component fractions                        | $+\sigma$ 1.5 1.9 3.9 3.7   | $-\sigma$ 1.5 1.9 3.9 3.7   |
| $\rho\pi\pi$ non-resonant fractions       | $+\sigma$ 1.2 1.0 1.5 1.2   | $-\sigma$ 1.2 1.0 1.5 1.2   |
| SCF fraction, $\Delta t$ PDF              | $+\sigma$ 0.2 0.2 0.1 0.1   | $-\sigma$ 0.2 0.2 0.1 0.1   |
| Shape of $R$ PDF                           | $+\sigma$ 0.8 0.7 1.2 1.3   | $-\sigma$ 0.8 0.7 1.2 1.3   |
| Vertexting                                 | $+\sigma$ 2.1 2.1 1.0 1.3   | $-\sigma$ 2.1 2.1 1.0 1.3   |
| Possible fitting bias                      | $+\sigma$ 0.3 0.0 0.3 0.0   | $-\sigma$ 0.3 0.0 0.3 0.0   |
| Background asymmetry                       | $+\sigma$ 1.1 0.0 0.0 0.4   | $-\sigma$ 1.1 0.0 0.0 0.4   |
| $b \to u$ asymmetry                        | $+\sigma$ 2.4 2.9 2.4 3.2   | $-\sigma$ 2.4 2.9 2.4 3.2   |
| $\rho\pi\pi$ asymmetry                     | $+\sigma$ 4.6 4.6 4.6 4.6   | $-\sigma$ 4.6 4.6 4.6 4.6   |
| Transverse polarization                    | $+\sigma$ 3.8 2.8 4.6 2.7   | $-\sigma$ 3.8 2.8 4.6 2.7   |
| Tag-side interference [16]                 | $+\sigma$ 3.7 3.7 0.1 0.1   | $-\sigma$ 3.7 3.7 0.1 0.1   |
| Total                                      | $+\sigma$ 8.3 8.0 8.4 7.9   | $-\sigma$ 8.3 8.0 8.4 7.9   |

arise because $A_{\rho\phi}$ is not measured. The solution consistent with the Standard Model is $62^\circ < \phi_2 < 106^\circ$ at 68% C.L. or $54^\circ < \phi_2 < 113^\circ$ at 90% C.L. Recently, an alternative model-dependent approach to extract $\phi_2$ using flavor $SU(3)$ symmetry has been proposed [20]. This method could potentially give more stringent constraints on $\phi_2$.

In summary, we present an improved measurement of the $CP$-violating coefficients $A$ and $S$ in $B^0 \to \rho^+ \rho^-$ decays using 492 fb$^{-1}$ of data, which corresponds to 535 million $B\bar{B}$ pairs. These measurements are used to constrain the angle $\phi_2$.

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