Research on suspender force measurement of arch bridge based on finite element model

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Abstract. The frequency method is the most commonly used method for arch bridge suspender force measurement. However, due to the complex boundary conditions of the suspender, it may bring error when the suspender is simply considered hinged. In this paper, the boundary condition of the suspender is identified by numerical simulation method. Also, the damping rubber and the flexural rigidity of the suspender are considered in the finite element model. The suspender force is identified by the least squares method based on the measured frequency. And the results show that the method has high precision and can meet the requirements of practical projects.

1. Introduction

The suspender is an important load transmission element in the arch bridge structure and plays a decisive role in the overall structural safety control. The magnitude of the suspender force reflects the stress state of the entire structure. Therefore, accurate measurement of the suspender force is required during the construction and service stage. At present, the main methods for arch suspender force measurement include [1]: pressure sensor method, hydraulic method, magnetic flux method and frequency method. The pressure sensor method is by installing a pressure sensor at the connecting rod of the suspender. The method is only applicable during the construction stage, while not be suitable for the service stage; the hydraulic method is used to estimate the force by reading the oil pump and it is only suitable during the suspender construction tension stage; the magnetic flux method is greatly affected by the environmental vibration and there are only a few applications abroad at present; the frequency method picks up the vibration frequency of the suspender, and identify the magnitude of the suspender force based on the relationship between the frequency and the force. The frequency method can be used during the service stage without affecting the use of the structure and this is a significant advantage over other methods [2].

The accuracy of the frequency method depends mainly on the test accuracy of the suspender frequency and the accuracy of the suspender force analysis model [3]. At present, the test accuracy of the suspender frequency can fully meet the engineering requirements, and then the test accuracy of the frequency method mainly depends on the accuracy of the analytical model. The precision of the suspender force analysis model is affected by multiple factors, such as the suspender flexural rigidity, boundary conditions, temperature and damping devices [4]. The analytical basis of the frequency method which is widely used in engineering is that the two ends of suspender are considered being hinged, the difference between assumption and reality is large, and the impact of the damping device cannot be considered.

In this paper, the elastic stiffness of the suspenders is solved based on the integral finite element model of arch bridge structure; then the finite element model of each suspender is established in which
the elastic stiffness of the suspenders and the damping rubber stiffness are both considered. According to the measured frequency, the suspender force is identified based on a numerical fitting algorithm. Finally the method was applied in an actual project to analyse the test accuracy.

2. General engineering situation
This paper takes a rigid tied arch bridge in Shanghai as the background to study the measurement of suspender force. The main span of the bridge is 88m, the rise-span ratio is 1/5.667, and the arch axis adopts the second parabola. The upper structure of the bridge contains 3 steel pipe concrete arch, and each steel pipe concrete arch contains 15 suspender witch’s space interval is 5m. The middle arch suspender specification is 127Φ7, the side arch suspender specification is 91Φ7, and the standard strength of the prestressed parallel steel wire is 1670MPa. The schematic view of the elevation of the bridge is shown in Figure 1.

3. Determination of suspender boundary conditions
3.1. Beam vibration model
The suspender of arch bridge is generally short in length, and its vibration state is greatly affected by the boundary conditions and the flexural rigidity. The vibration characteristics of the suspender are similar to those of the beam vibration. Assuming that the mass per unit length of the suspender is \(m\), the flexural rigidity of the suspender is \(EI\), and the suspender is tensioned under the effect of the internal force of the \(T\), then the equivalent elastic boundary model of the suspender is shown in Figure 2. In Figure 2, \(K_1\) and \(K_3\) represents vertical support stiffness of the arch end of the suspender and the beam end of the suspender respectively, \(K_2\) and \(K_4\) represents rotational restraint stiffness of the arch end of the suspender and the beam end of the suspender respectively, \(K_5\) and \(K_6\) represents vertical support stiffness of the arch end of the damping rubber and the beam end of the damping rubber respectively.

3.2. Calculation of elastic boundary conditions of the suspender
Elastic boundary conditions can be solved by the static method. The solution steps are: ①build the integral finite element model of the arch bridge; ②delete the suspender unit in the overall model; ③apply a unit displacement in one direction or a unit force to the corresponding position of the
suspender, and maintain the remaining three directions’ displacement or external force is 0; ④ run the whole finite element model to obtain the corresponding displacement of the unit displacement or the displacement response under the unit force, then the elastic boundary conditions can be calculated based on displacement response results. The overall finite element model of this bridge is shown in Figure 3. The calculation results of the elastic boundary conditions of the suspenders are shown in Table 1. The numbering sequence of the suspender is from north to south, and D, Z, X represent the east arch, the middle arch and the west arch respectively.

Figure 3. The overall finite element model of this bridge.

Table 1. Calculation results of elastic boundary condition of the suspender.

| serial number | K₁/kN·m⁻¹ | K₂/kN·m⁻¹ | K₃/kN·m⁻¹ | K₄/kN·m⁻¹ |
|---------------|------------|------------|------------|------------|
| D1/X1         | 3669207    | 50460511   | 7620485    | 1382687    |
| D2/X2         | 1514000    | 45061254   | 16944386   | 10501706   |
| D3/X3         | 1028836    | 39593930   | 10988270   | 9797656    |
| D4/X4         | 982895     | 37474244   | 8718698    | 9176863    |
| D5/X5         | 1217420    | 37673499   | 7561384    | 8776064    |
| D6/X6         | 1748235    | 39972974   | 6922498    | 8568634    |
| D7/X7         | 2519517    | 43625775   | 6593419    | 8490753    |
| D8/X8         | 3020608    | 45727891   | 6490674    | 8476191    |
| Z1            | 4707344    | 69317170   | 67032357   | 19292551   |
| Z2            | 2086142    | 62869236   | 20561247   | 15041914   |
| Z3            | 1474876    | 56027496   | 13843017   | 13605971   |
| Z4            | 1432643    | 53356995   | 11147471   | 12381901   |
| Z5            | 1774842    | 53693922   | 9767883    | 11687013   |
| Z6            | 2517761    | 56871557   | 9009520    | 11345665   |
| Z7            | 3568304    | 61776909   | 8619025    | 11206657   |
| Z8            | 4234879    | 64627436   | 8494399    | 11177116   |

3.3. Calculation of vertical support stiffness of the damping rubber
The vertical support stiffness of the damping rubber is determined by the rigidity of the high-damping rubber itself and the stiffness of the outer rubber sleeve. Sleeve stiffness $K_{ss}$ and rubber stiffness $K_{rp}$ can be calculated as following (5):

$$K_{ss} = \frac{3E_S I_S}{L_S^3}$$

(1)

Amongst: $E_S$ --- elastic modulus of the steel sleeve;

$I_S$ --- sectional moment of inertia of the steel sleeve;

$L_S$ --- distance between the damping rubber center and suspender anchor plate

$$K_{rp} = \frac{\pi (E_r + G_r) l}{\ln(D_r/D_1)}$$

(2)

Amongst: $E_r$ --- elastic modulus of the damping rubber;
$G_{sp}$ ---shear modulus of the damping rubber;
$l_s$ ---the length of the damping rubber;
$D_s$ ---the outside diameter of the damping rubber;
$D_i$ ---the inside diameter of the damping rubber.

The vertical support stiffness of the damping rubber can be calculated as following:

$$K = \frac{K_{ss} K_p}{K_{ss} + K_p}$$  \hspace{1cm} (3)

The vertical support stiffness of two ends of the damping rubber are shown in Table 2.

| serial number                  | vertical support stiffness of beam end/kN·m$^{-1}$ | vertical support stiffness of arch end/kN·m$^{-1}$ |
|--------------------------------|---------------------------------------------------|---------------------------------------------------|
| suspender of the side arch     | 2197                                              | 2197                                              |
| suspender of the middle arch   | 2038                                              | 2038                                              |

4. **Research on suspender force measurement**

4.1. **The establishment of a suspender finite element model**

To analyse the beam vibration model, it is necessary to satisfy the assumptions that the material of the suspender is homogeneous, the material of the suspender can satisfy Hooke's law, only the in-plane vibration of the suspender can be considered and the suspender vibration does not account for the influence of damping. Based on the above assumptions and only when the beam boundary is hinged, the beam vibration model can be derived from the explicit expression as following:

$$T = 4 m L^2 \left( \frac{f_n}{n} \right)^2 \frac{n^3 E I \pi^2}{L^2}$$  \hspace{1cm} (4)

Amongst: $T$——suspender force;

$n$——order number of natural frequency of the suspender;

$f_n$——$n$th order natural frequency;

$L$——calculation length of the suspender;

$I$——sectional moment of inertia of the suspender.

Obviously, the above formula result has a large error if the boundary of the suspender is complex. In order to consider the influence of the complicated boundary of the suspender effectively, the finite element method can be adopted to identify the suspender force.

Use beam elements to perform finite element model for every suspender in order to consider the effect of bending stiffness, and the model parameters are shown in Table 3, the suspender finite element model is shown in Figure 4; the values of the boundary conditions of the suspender and the vertical support stiffness of the damping rubber are taken from the values in Table 1 and Table 2.

| serial number                  | length of the suspender/m | mass per unit length/kg·m$^{-1}$ | elastic modulus /N·m$^{-2}$ | moment of inertia/ m$^4$ |
|--------------------------------|---------------------------|---------------------------------|-----------------------------|--------------------------|
| D1/X1                          | 5.818                      | 30.7                            | 2.05E+11                    | 3.99808E-06              |
Figure 4. Finite element model of the suspender

4.2. Suspender force identification based on least squares method
During the measurement of the suspender force, collect acceleration versus time data of each suspender and pick up the first $m$ order natural frequencies through the FFT analysis of the acceleration versus time data. And the order natural frequency is recorded as $f^i_1 \sim f^i_m$.

Select a set of possible suspender forces as $T_i \sim T_n$; load the forces into finite element model to identify the first $m$ order natural frequencies as $f_i \sim f_m$.

$$V_i = \sum_{j=1}^{m} (f_j - f^i_j)^2$$  \hspace{1cm} (5)

Then the suspender forces are the values that make $V_i$ the smallest value.

4.3. Results and analysis of the suspender force measurement
During the replacement of the suspender of this project, the force value of the replaced suspender can be read by the hydraulic pressure, thus the actual replaced suspender can be used to identify the force value and error analysis.

Each suspender was tested for vibration, and both the equation 4 and the finite element model method were used to identify the force value of the suspender. Compare the suspender force identifications to the hydraulic pressures, the comparison results are shown in Table 4. The error 1 in the table represents the comparison between the finite element method and hydraulic pressures while the error 2 in the table represents the comparison between the formula method and hydraulic pressures. From Table 4, we can see that the error of the identification by the formula method which is commonly used is very large, and the maximum error is 51.8%, this is mainly due to the inaccurate calculation of the suspender length. The error range of the identification by finite element method is between -8.4% and 8.6%, and the error is within ±10%, which can meet the requirements of arch-bridge short suspender measurement accuracy.
5. Conclusions
Through the research on the measurement method of suspender force based on the finite element model, the following conclusions are drawn:

(1) The vibration characteristics of the short suspender are closer to the vibration of the beam. For the short suspender force measurement, it is not suitable to identify the suspender force on the theory of the formula method directly.

(2) The suspender force can be measured via firstly building the integral finite element model of the arch bridge, secondly calculating the boundary conditions of each suspender, thirdly using beam elements to perform finite element model of each suspender and finally identifying the suspender force with the measured natural frequencies through least squares method. The method can improve the accuracy of the suspender force measurement and meet the actual engineering requirements.

(3) The influence of the variation of the stiffness of the existing bridge structure and the variation of the rigidity at the end of the suspender on the accuracy of suspender force measurement is worth further study.

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