What evidences the mass distribution in halo nucleus?

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Abstract

Detailed analysis of the distribution of nuclear matter in the halo nucleus $^{11}$Be is performed within the microscopic approach based on the flexible variational basis of the functions of the "polarized" orbitals model. It is shown that the proposed approach reproduces, in agreement with the experiment, the most distinguishing features of the mass distribution in this system. Taking into account the results of specific calculations and the exceptionally high reliability of the experimental data on the nuclear density distribution in the "tail" of $^{11}$Be, we make a conclusion about the existence of new structures with the sizes considerably exceeding the size of the nucleus (up to tens of fm) and the density being thousands times less that the typical nuclear one.

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I. INTRODUCTION

As is known, during the last 10-15 years, a number of nuclides with the anomalous N/Z ratio has been discovered (see, for instance, [1], [2], [3], [4], [5], [6]). These nuclei are far from the beta-stability area and may closely approach the boundary of the nuclear or nucleon stability. In most of the cases, these are the strongly neutron-excessive isotopes of light and middle elements, which include a number of exotic nuclei with interesting and unusual properties and structure. Naturally, these nuclides attracted considerable attention of both experimentalists and theorists.

Let us say a few words about the term "halo nuclei". The latter has become very popular in recent years and is also used for the well known nuclides (like $^6$He, $^8$He, etc.). For this reason, in order to bring, at least partially, the terminology and the physical meaning into agreement, there has been one more extravagant term, "Borromean nuclei", introduced [7]. Borromean or, alternatively, halo nuclei of II-nd kind, represent weakly coupled and very much fragile nucleon systems being at the same time rather compact in size. They should be more exactly referred to as "neutron skin" rather than "halo" nuclei.

Quite different situation is observed in the case of halo nuclei of I-st kind (the one-neutron halo nuclei $^{11}$Be and $^{19}$C). These nuclides are unusual from every point of view. Let us mention the anomalously high cross-sections of the interaction with various nuclei, the unexpected inverted spectra of the lower energy levels, and the most unexpected mass distribution with the compact core nucleus in the center accompanied by a long sharply directed "tail" of very thinned nuclear matter [1], [8].

We are not going to consider in detail the basic features of halo nuclei, since there is a number of exhaustive reviews on the subject in the literature (see, for instance, [2], [3], [4], [5], [6]). Let us cite only the experimental results on the distribution of the nuclear matter in $^{11}$Be keeping in mind that these latter will be the major subject of our further theoretical consideration. These results have been obtained by a large team of Japanese physicists in RIKEN [2], where the dependence of the density distribution for the nuclear matter in the nucleus $^{11}$Be on the distance from the nucleus center has been studied. The experimental points obtained there have been put within some stripe area bounded by a bold line, in order to make allowance for the possible inaccuracies in the determination of density (see Fig.2 in [8]). The authors’ very convincing and rather strong statements are cited below.
In first place, the existence of the halo tail in $^{11}\text{Be}$ nucleus is associated with the last weakly coupled neutron only, and, in addition, this valence neutron is the main material provider for the "tail" density distribution of nuclear matter. The contributions of the rest of nucleons to the "tail" are negligible. The magnitude of the density in the halo tail is more than three orders less than the typical density of nuclear matter.

In second place, while in determination of the central and intermediate (with respect to $r$) regions there is a possibility of effects of additional factors, which, in principle, may bring the magnitude of density beyond the area of experimental data obtained by the authors, the slope and the magnitude in the "tail" distribution are determined very reliably and accurately. The question is, what are the reasons for such confidence of the authors? The point is that the most sensitive process with respect to the distribution of the matter in the "tail" is that of $^{11}\text{Be}$ and $^{10}\text{Be}$ fragmentation, and the cross section $\sigma_F$ of this process is anomalously high. As a result, the experimental determination of the above mass distribution in $^{11}\text{Be}$ can be made with such an impressive accuracy.

One more point important for the theory, which has been emphasized in the paper [8], can be concisely formulated as the indifference of the halo tail in relation to the core nucleus. In other words, the presence of the halo tail in $^{11}\text{Be}$ almost does not affect the magnitude of the basic parameters of the core nucleus $^{10}\text{Be}$.

All the above mentioned relates to the experimental situation on halo nuclei. As regards the theory, the mainstream of creative ideas here is focused on the cluster model. There are hundreds of papers in the literature, where the successful description of exotic halo nuclei is based on the employing one or another of cluster modes, e.g., the core plus one valence neutron, or the core plus two valence neutrons. At first sight, such interpretation appears very attractive. However, a closer examination reveals here serious difficulties. Let us point out some of them. To begin with recall the size parameters of $^{11}\text{Be}$. The root-mean-square radius of the nuclear matter in $^{11}\text{Be}$ is $R_{\text{rms}}^{m} = (2, 90 \pm 0, 05) \text{ fm}$ [9], [10], whereas the radius of the core nucleus $^{10}\text{Be}$ is $R_{\text{rms}}^{m} = (2, 30 \pm 0, 02) \text{ fm}$. In order to explain the experimental spectrum of the Coulomb decay, the value of the root-mean-square radius of the valence nucleon and the diameter of the halo tail must be, in accordance with [11], $R_{\text{rms}}^{m} \sim 7 \text{ fm}$, and $D_{\text{halo}} \approx 30 \text{ fm}$, respectively. If one assumes that the valence nucleon moves outside the core (the concept underlying the cluster representation), a serious question arises as to the origin of the long-range interaction between the valence neutron and the nucleons.
of the core. To treat this problem, one may introduce the so-called "shallow neutron-core potential" [12], and, by appropriately adjusting the parameters for this potential, achieve success in calculating specific physical quantities. It is clear that such an interpretation is rather phenomenological, and the situation here could be hardly said to be satisfactory.

The situation with halo nuclei could be clarified within the consistent microscopic approach [13] based on the many-particle Schrödinger equation for the wave function $\Psi(1, 2, ..., A)$ with some particular choice of the "fundamental" pair NN-potential. Actually, the dependence of the wave function $\Psi(1, 2, ..., A)$ on the coordinates of all A nucleons enables one to calculate almost arbitrary nuclear property and to reveal the underlying physical picture by means of the analysis of relevant theoretical and experimental quantities.

Taking into account all the above mentioned, we perform in this work exactly this analysis for the case of one-neutron halo nucleus $^{11}$Be. In Section II of the paper, we give a brief presentation of the algorithm for calculations within the framework of the polarized orbitals model [14], taking an accurate account of the hierarchy of basic principles of the theory. In Section III, we present the results of calculations of the nuclear matter density in the nucleus $^{11}$Be. The discussion of theoretical data, along with comparison between the known experimental data and similar results of other authors, is given in Section IV.

II. MANY-PARTICLE WAVE FUNCTION AND THE VARIATIONAL PRINCIPLE

In a microscopic model of interacting particles, the starting point is the relevant many-particle Hamiltonian, where, as a rule, only pair interactions are taken into account, i.e.

$$H_A \Psi = E \Psi; \quad H_A = -\sum_{j=1}^{A} \frac{\hbar^2}{2m} \nabla_j^2 - T_{\text{c.m.}} + \sum_{i>j=1}^{A} V_{i,j} + U_{\text{Coulomb}}.$$

(1)

As regards the work [14], which provides the basis for our further consideration, the pair inter-nucleon interaction employed therein is taken in the form of effective central exchange NN-potential with the radial dependence specified by the superposition of five Gaussian terms [15]

$$V_{ij} = \sum_{S,T=0,1}^{5} \sum_{\nu=1}^{2S+1,2T+1} V_{ij}^{[\nu]} \exp\left\{-\frac{(r_i - r_j)^2}{\mu_{\nu}^2}\right\} \cdot P_{S,T}(i, j),$$

(2)
where $V_{\nu}^{[\nu]}_{2S+1,2T+1}$ and $\mu_\nu$ are the intensity and the radius of the $\nu$-th component of the interaction potential between the $i$-th and $j$-th nucleons, respectively; $\mathbf{P}_{S,T}$ is the known projection operator (see, for instance, [16]), which separates out the two-nucleon states with the spin $S$ and isospin $T$ in the wave function $\Psi$.

Along with the nuclear interaction, the Hamiltonian (1) includes also the Coulomb repulsion between the protons in the nucleus, which can be conveniently written in the form of the integral representation

$$U_{\text{Coulomb}} = \sum_{i>j=1}^{Z} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} = \frac{2e^2}{\sqrt{\mu}} \sum_{i>j=1}^{Z} \int_{0}^{1} e^{-\frac{(r_i-r_j)^2}{\mu^2}} \frac{d\tau}{(1 - \tau^2)^{3/2}}, \quad \mu_c \equiv \frac{1}{\tau} \sqrt{1 - \tau^2}. \quad (3)$$

Thus, formally, we deal with the conventional many-particle problem for bound states. In view of the possibilities of modern computers, it is worth to seek the solution to the problem (1)-(3) on the basis of the well known variational Ritz' principle (see, for instance, [17]). In doing so, it is natural to describe the nuclear system in terms of individual particle states with the total wave function of the nucleus being approximated by the antisymmetrized product of one-particle functions

$$\Psi_E(1, 2, \ldots, A) = \frac{1}{\sqrt{A!}} \det[\varphi_i(j)]; \quad \varphi_j(i) = \psi_{n_i}(\mathbf{r}_j; a_i, b_i, c_i) \cdot \chi_{m_{s_j}}^i(j) \eta_{m_{t_j}}^i(j), \quad (4)$$

where $\chi_{m_{s_j}}$ and $\eta_{m_{t_j}}$ are, respectively, the spin and isospin wave functions for $j$-th nucleon [18], and $\psi_{n_i}$ are the one-particle orbitals. It is clear that the success of such treatment will mainly depend on the appropriate choice of variational basis for one-particle orbitals.

Conventionally, for the functions $\psi_{n_i}$, one employs the functions of harmonic oscillator. In our model, which is also called in Ref. [14] as "the model of polarized orbitals", each single orbital is represented by an oscillator function as well. However, in contrast to other authors, we use for every $i$-th one-particle state its own independent variational parameters $(a_i, b_i, c_i)$.

Note that, as distinct to the cluster models, all the nucleons equally participate here in forming the dynamics of the system.

The optimal values for the variational parameters $\{a_i, b_i, c_i\}$ as well as the optimal configuration (determined by the set of quantum numbers $\{n_i = (n_{x_i}^i, n_{y_i}^i, n_{z_i}^i)\}$) are found as a result of minimizing the functional $f_E(\{n_i; a_i, b_i, c_i\})$ for the total energy of the nucleus under consideration. If the chosen variational basis of the functions $\psi_{n_i}$ is flexible enough,
we can expect that the solution $\Psi^{[\ell]}_{IM,\alpha}(1,2,...,A)$ to the many-particle problem found in such way would be sufficiently realistic.

We are not going to dwell upon the so-called "technical" issues, although, exactly these latter frequently determine the success of a work. Let us only list the steps made for evaluating the functional $f_E(\{n_i; a_i, b_i, c_i\})$: i) matrix elements for the operators of all physical quantities entering the Hamiltonian (1) have been obtained; ii) projection of the internal wave function (4) onto the energy state $J^\pi = \frac{1}{2}^+$, associated with the ground state of $^{11}$Be has been carried out on the basis of the Hill-Wheeler integral; iii) search for the global minimum of the functional $f_E(\{n_i; a_i, b_i, c_i\})$ for the total energy of the nucleus in the hyperspace of variational parameters $\{a_i, b_i, c_i\}$ has been performed.

III. RESULTS OF NUMERICAL COMPUTATIONS

As mentioned above, the calculations of the needed wave function $\Psi^{[\ell]}_{IM,\alpha}(1,2,...,A)$ and some basic features of the nucleus $^{11}$Be have been performed in previous works. Therefore, we can use here the results of [14] in order to evaluate the sought-for distribution of the nuclear matter in $^{11}$Be.

Recall that, according to [14], the ground state of the nucleus $^{11}$Be is associated with the configuration:

$$\text{protons } Z = 4 \to (000)^2(001)^2; \quad \text{neutrons } N = 7 \to (000)^2(001)^2(010)^2(002)^1,$$

(5)
i.e., the protons have the regular subsequently filled configuration, whereas the neutron configuration is unusual. The optimal values for variational parameters of the orbitals $\psi_{n_i}$ are given in Table 1.

Note that the physical meaning of the variational parameters $(a_i, b_i, c_i)$ is that they determine the extent of spreading for the i-th one-particle state along $x, y, z$ axis, respectively. The neutron configuration indicated in (5) turned out to be energetically favorable only in our model of "polarized" orbitals, where for the first time in calculations of such kind the variational parameters $\{a_i, b_i, c_i\}$ were independent for any of the orbitals $\psi_{n_i}$ and in any of the directions $x, y, z$. Unproportionally large value of the variational parameter $c_n^4 = 5.38 \, fm$ (see Table 1) evidences the anomalous spreading of the valence neutron along one of the coordinate axis.
The information on the explicit expression for the wave function enables us to evaluate the sought-for mass distribution $\varrho_A(r)$ in the nucleus $^{11}\text{Be}$. To this end, we have to average the operator for the one-particle spatial density with the wave functions $\Psi_{IM,\alpha}^{[f]}(1,2,...,A)$, i.e.

$$\varrho_A(r) = \langle \Psi_{IM,\alpha}^{[f]} | \sum_{i=1}^{A} \delta(r-r_i) | \Psi_{IM,\alpha}^{[f]} \rangle.$$  \hspace{1cm} (6)

Here the coordinates $r_i$ are written in center mass system $R_{c.m.} = 0$.

Note that in experiments, only the radial dependence for the nuclear density distribution averaged over the angles $\Omega$

$$\varrho_A(r) = \frac{1}{4\pi r^2} \int_{0}^{2\pi} d\vartheta \int_{0}^{\pi} \varrho_A(r) \sin \vartheta d\vartheta; \quad r > 1\text{fm}$$  \hspace{1cm} (7)

is measured. In the vicinity of the point $r = 0$, the averaging over the angles $\Omega$ has to be performed as

$$\varrho_A(r) \approx \frac{1}{N_{points}} \sum_{i=1}^{N_{points}} \varrho_A(r_i); \quad r < 2\text{fm},$$  \hspace{1cm} (8)

where the points $r_i$ are taken on the sphere of the radius $r = |r_i|$.

The distribution of nuclear matter for the wave function of the ground $^{1/2^-}$ state of the nucleus calculated in the above way is shown in Fig.1. Actually, this figure represents an almost identical copy of Fig.2 of Ref. [8], where, in addition, our theoretical curve (the line with dots) is displayed. The three curves with $\beta = 0$, $\beta = 0.5$ and $\beta = 0.7$ in Fig.1 are the attempts of other authors to describe the experimental situation within the framework of the deformed shell model (see the references in [8]). As is seen from Fig.1, the theoretical data (the line with dots) and the experimental estimates for the nuclear density distribution in the nucleus $^{11}\text{Be}$ are in good agreement. It is particularly true for the magnitude and the slope in the "tail", where the experimental estimates are very reliable and a complete coincidence between the theory and experiment is observed.

It is worth mentioning that in theory, in contrast to experimental studies, there exists a possibility to investigate the above mass distribution in the internal coordinate system rigidly coupled to the nucleus. In this case we deal with the spatial structure $\varrho_A(r)$, which is the function of three coordinates $x,y,z$. In Fig.2, we give one of the sections (in $(x,z)$ plane) for the function $\varrho_A(r)$ for the nucleus $^{11}\text{Be}$ under consideration. The figure in the plane $(y,z)$ would be similar, as far as the size properties are concerned. The choice of the values for the lines of constant density in Fig.2 has been made in agreement with the relevant
experimental data within the interval \([5\text{fm} < r < 12\text{fm}]\). Finally, in Fig.3 we present the distribution of nuclear matter \(g_{\text{neutron}}(r)\) for the valence nucleon, or, more exactly, the more exactly, the lines of constant density in the plane \((x,z)\). In this figure, we clearly observe three structures, the two lateral ones, symmetrically placed at the distance of 10\(\text{fm}\), and the central one being considerably smaller in size than the lateral structures. All these three structures have approximately equal density with the magnitude \((2 - 5) \cdot 10^{-5} \text{fm}^{-3}\).

IV. ANALYSIS AND CONCLUSIONS

After introduction to both experimental and theoretical results on the mass distribution in halo nucleus \(^{11}\text{Be}\), let us try to make analysis in order to obtain a physical interpretation. Let us start with Fig.1. The radial distribution for the density of nuclear matter presented here could be hardly brought into agreement with the notion of "atomic nucleus" conventionally given in reference books. Specifically, the nucleon density is said therein to be nearly constant in the nucleus center, and to decrease exponentially at periphery. As is seen from Fig.1, the nucleus \(^{11}\text{Be}\) is not the case. Other features of nucleon system under consideration are in contrast with the classical concept of "atomic nucleus" as well, let alone the size properties.

Fig.2 and Fig.3 strongly evidence that in the case of the halo nucleus \(^{11}\text{Be}\) we deal, in fact, with two distinctly different physical systems, the well known nucleus \(^{10}\text{Be}\) referred here to as the "core nucleus", and the structure of intermediate, between atomic and nuclear, nature. The latter is presented by three separate objects. This structure is, first of all, a result of the action of the Pauli principle with respect to all 11 nucleons constituting the \(^{11}\text{Be}\) system. It is coupled to the core nucleus by its central part, which completely envelopes the core nucleus. Two lateral much more massive parts are outside the nucleus \(^{10}\text{Be}\), i.e., beyond the action of nuclear forces. These three parts-objects are held together only due to the fact that they originate from a single quantum of nuclear matter, the neutron.

To put it in the other way, the nuclear processes give rise to the conditions, which make it energetically favorable for the system of 4 protons and 7 neutrons to form the compact nucleus \(^{10}\text{Be}\) and an unusual structure with the mass and the quantum numbers being exactly equal to the valence neutron. According to the experimental estimates, this energy gain is equal to only \(\delta E = 0.32 \text{MeV}\). Exactly at this distance, on energy scale, the first
excited $\frac{1}{2}^-$ state of the nucleon system under consideration is observed.

Thus, the analysis of the experimental situation on the distribution of nuclear matter in the exotic nucleus $^{11}\text{Be}$ performed in this work has shown that the nucleon system $^{11}\text{Be}$ should be considered as two different in nature subsystems, the compact nucleus $^{10}\text{Be}$, and some structure emerging in the vicinity of this nucleus as a result of Pauli principle, which is distinctly different in its features from an atomic nucleus. Therefore, we should regard the discovery of the halo nuclei of I-st kind as the discovery of new structures of microcosm with the sizes ranging up to tens of fm and the density being thousand times less than the nuclear one.
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TABLE I: The optimal values of the variational parameters for one-particle orbitals $\psi_n$ in $^{11}$Be.

| Parameters $\{a_i, b_i, c_i\}$ | The optimal values, $fm.$ |
|---------------------------------|--------------------------|
| $a_1^p$                         | 1.355                    |
| $a_2^p$                         | 1.49                     |
| $b_1^p$                         | 1.55                     |
| $b_2^p$                         | 1.685                    |
| $c_1^p$                         | 1.84                     |
| $c_2^p$                         | 1.92                     |
| $a_1^n$                         | 1.375                    |
| $a_2^n$                         | 1.51                     |
| $a_3^n$                         | 1.61                     |
| $a_4^n$                         | 1.725                    |
| $b_1^n$                         | 1.52                     |
| $b_2^n$                         | 1.64                     |
| $b_3^n$                         | 1.80                     |
| $b_4^n$                         | 1.90                     |
| $c_1^n$                         | 1.89                     |
| $c_2^n$                         | 1.96                     |
| $c_3^n$                         | 2.18                     |
| $c_4^n$                         | 5.38                     |
FIG. 1: Density distribution of nucleons on logarithmic scale for the ground $\frac{1}{2}^+$ state of $^{11}$Be. The results of RIKEN [8] are shown by thick lines; the results of our calculation are shown by thin line with filled circles. The thin curves with $\beta = 0$, $\beta = 0.5$ and $\beta = 0.7$ are the results of the deformed model (see [8]).
FIG. 2: Intrinsic structure of \( ^{1+} \) state of \(^{11}\)Be. The figure shows sections of the matter density \( \varrho_A(x, y = 0, z) \) in the plane \((x, z)\).
FIG. 3: The density distribution for the valence neutron in the plane \((x, z)\).