The Variant-Rule
Another Logically Universal Rule

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Abstract
The Variant-rule derives from the Precursor-rule[6] by interchanging two
classes of its 28 isotropic mappings. Although this small mutation con-
serves most glider types and stable blocks, glider-gun engines are changed,
as are most large scale pattern behaviors, illustrating both the robustness
and fragility of evolution. We demonstrate these newly discovered struc-
tures and dynamics, and utilising two different glider types, build the
logical gates required for universality in the logical sence.

keywords: universality, cellular automata, glider-guns, logical gates.

1 Introduction

The idea of Cellular Automata (CA) was conceived by von Neumann in the
1940s, applying the earliest computers to construct an abstract self-reproducing
machine. That a CA itself might become a computer by its dynamic patterns can
be traced to back to 1970 with Conway’s Game-of-Life[1] and the first glider-gun,
discovered by Gosper, which lead to a demonstration of universal computation
based on memory, transmission and processing[1], and the proof[10] was based
on the Turing Machine.

Here we consider one aspect of universal computation, “universality in the
logical sense” — that logical gates can be built within the CA dynamics by
means of a glider-gun. By this definition, several logically universal CA have
been created, some called “Life-like” because they are variations on the Game-
of-Life birth-survival scheme[2], and others were built on schemes different
from birth/survival. Under this last approach, Sapin found a universal CA
in 2004 [11], and Gómez-Soto/Wuensche published three more: the the X-Rule
in 2015[5], the Precursor-Rule in 2017[6], and the Sayab-Rule in 2018[7].

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In this paper we present the “Variant-rule”, another logically universal CA, made from a chance mutation of the Precursor-rule. The Precursor-rule is part of the family of CA discovered by Gómez-Soto/Wuensche using the input-entropy search method. The rule is isotropic — patterns and mechanisms operate equivalently in any direction. A 2D isotropic CA with a $3 \times 3$ neighborhood can be defined by 102 symmetric groups that map to either of two states, 0 or 1. The Precursor-rule has 28 symmetric groups mapping to 1. The Variant-rule has the same number, but one of these groups has been interchanged for another.

Essential properties to support CA logical universality are dynamic periodic patterns: gliders and glider-guns, and stable blocks that can destroy gliders called “eaters”. Useful dynamical interaction between these and other objects include reflection, transformation and oscillation.

The glider-gun, a periodic structure ejecting gliders into space, is the key and most elusive mechanism. Figure 1 compares the three very different GGa glider-guns, of the Variant, Precursor, and Sayab rules, all ejecting the same 4-phase diagonal glider Ga (figure 2), which is used to construct logical gates. However, the ejected GGa gliders-streams differ in spacing and mix of phases.

![Figure 1: Comparing the three GGa glider-guns of the Variant, Precursor, and Sayab rules firing Ga gliders, with the period $p$ (firing frequency) indicated. Note the different mix of glider-stream phases, where Sayab has just one phase per time-step, the other rules have two, but a different mix. Gliders streams are stopped by eaters. Green denotes motion.](image)

![Figure 2: The 4 phases of the diagonal glider Ga and the orthogonal glider Gc, moving as indicated by arrows. The speed of Ga=$c/4$, Gc=$c/2$.](image)
Figure 3: The Variant-rule glider-gun GGa attractor cycle incorporates the sub-glider-gun GGc. The period is 22 time-steps showing all phases/patterns of the GGa. The direction of time is clockwise. Inset: A glider-gun phase shown at a larger scale (green denotes motion) alongside the same phase on the attractor cycle. Note the the glider Gc is shot to the East then reflected/transformed to glider Ga travelling NW, which is stopped by an Eater.

The Precursor and Variant rules both also feature GGc glider-guns (figure) which act as the initial, intermediate, components for GGa glider-guns, where Gc gliders bounce/transform to make Ga gliders, whereas the Sayab-rule GGa glider-gun ejects Ga gliders directly.

Given the genetic closeness of the Precursor and Variant rules, it is not unexpected that both share glider types, small oscillators, and small stable blocks which act as eaters or reflectors. However, despite this closeness, their glider-guns and larger scale pattern behaviors are very different. This perhaps illustrates both the robustness and fragility of evolution, and suggests a direction for further study.
Figure 4: A typical evolution emerging after 61 time-steps from a 30x30 30% density random zone. Gliders and still-lives have emerged spontaneously. Green denotes dynamics or motion for a given number of time-steps, and shows glider direction by their trails — this applies in all similar images in the paper.

The Variant-rule features spontaneously emergent gliders, stable blocks and oscillators (figure 4), but the complex patterns of its glider-guns GGc and GGa, shown in figure 3 as an attractor cycle1 of all 22 phases, are unlikely to emerge spontaneously. Glider-guns need to be constructed, and this has been achieved from the interaction of two independent oscillating structures. There are other glider-guns types built from combinations of these basic glider-guns, including glider-guns with variable periods, and surely glider-guns yet to be discovered. These structures can combine with each other and with eaters, reflectors, oscillators and collisions to build ever increasing complexity2 by multiple assemblies of sub-components, including the logical gates, NOT, AND and OR, by GGa or GGc glider-guns, demonstrating logical universality.

The paper is structured in the following further sections: (2) the Variant-rule definition, (3) a description of gliders, (4) collisions, (5) glider-guns, oscillators and reflectors, (6) glider stream circuits, (7) variable period glider-guns, (8) logical universality, (9) spaceships, puffers and rakes, and (10) concluding remarks.

2 The Variant-rule definition

Definitions of CA can be found from many sources, so we will skip the details here. We just note that this paper deals with binary 2D classical synchronous CA, comparable to the Game-of-Life (GoL) with a Moore neighborhood, but not based on birth/survival, and with periodic (or null) boundary conditions.

1 A “bare” attractor cycle, free of transients as in figure 3 and similar figures, can be generated in DDLab 17 from a seed state by setting the number of transient levels to zero 16.

2 Some of these, oscillators, glider-guns, puffers, rakes, etc. owe their discovery to contributions of the ConwayLife forum 19.
The Moore neighborhood has $3 \times 3 = 9$ neighbors giving a full lookup-table with $2^9$ outputs, a rule-space of 512 (figure 5), but we consider isotropic rules only, equal outputs for any neighborhood rotation, reflection, or vertical flip. If rules are classified by isotropy the number of effective outputs, one for each symmetry class, reduces rule-space to 102 (figure 6). Within this isotropic rules-space, both the Precursor and Variant rules have 28 symmetry classes with an output of 1 (figure 6). When the Precursor-rule was announced in the ConwayLife forum, an active member with the handle “Wildmyron”, mistranscribed the rule into Golly’s software format. The symmetry class

56: \[\begin{array}{|c|c|c|}
\hline
1 & 1 & 1 \\
\hline
\end{array}\] was replaced by the symmetry class

85: \[\begin{array}{|c|c|c|}
\hline
0 & 0 & 0 \\
\hline
\end{array}\]

with the happy consequence that the forum was able to discover many interesting dynamical properties of the mutated rule, which we named the “Variant rule”. The rule, and seed files for DDLab and Golly, as well as other rules in this family, can be found in the “Logical Universality in 2D Cellular Automata” website, so experiments can be repeated or new ones initiated.

Figure 5: Top: The Variant rule-table based on to all 512 neighborhoods, and Below: expanded to show each neighborhood pattern. 136 black neighborhoods map to 1, 386 blue neighborhoods map to 0. Because the rule is isotropic, only 102 symmetry classes are significant (figure 6).
3 Gliders

Travelling patterns, and their collisions with each other and with stationary patterns, can be used to simulate logical processing in CA. A travelling pattern is often periodic, translating through space via a number of phases — the distance travelled by a particular phase to a new position gives the velocity measured in time-steps. Because of the nearest neighbor Moore neighborhood, the fastest velocity, the “speed of light” $c$, is one lattice cell, orthogonal or diagonal, per time-step, but there may be slower velocities — a stationary pattern has zero velocity. In the lexicon of the Game-of-Life played on an orthogonal lattice, such a pattern moving diagonally is a “glider” whereas a pattern moving orthogonally is a “space-ship”, but in this paper we use the term “glider” for both.

We have seen in figure 2 all 4 phases of the two gliders, Ga and Gc — Variant (and Precursor) glider-guns have been found for these and also for G2a. Figure 7 is a summary showing just one phase of these other gliders in the Variant-rule. All have 4 phases, and the velocity is $c/4$ for diagonal gliders, and $c/2$ for orthogonal. All but Gd and Ge gliders also operate in the Precursor rule.
Glider types in the Variant-rule — one representative phase for each glider. Glider-guns have been created for Ga, Gc and G2a in both the Variant and Precursor rules. All gliders except for Gd and Ge also operate in the Precursor rule. All have 4 phases, and the velocity is \( c/4 \) for diagonal gliders, and \( c/2 \) for orthogonal.

### 4 Collisions

Collisions are fundamental in the research of logical universality in CA to manipulate gliders streams shot from a glider-gun, and control other logical artifacts. There are many possible collision scenarios between gliders, stable blocks, and oscillators, where collision outcomes depend on the exact point of impact and phase. As these are deterministic systems, a theory of collision behavior should be possible but is beyond the present state of the art, so for a given CA that supports complex glider dynamics one must resort to experiment and compile a catalogue of useful collisions that may serve as logical data transmission components. The Variant rule is rich in useful collision outcomes, with a degree of overlap with the Precursor rule.

Among necessary collision behaviors for logical universality are,

- The destruction of a glider by a stable block (an “eater”) which survives intact to destroy subsequent gliders in a glider stream (figures 8, 16, 18).
- Mutual destruction when two gliders collide (figure 9).

These are present in the Variant rule, but other collisions, also involving oscillators and reflectors, enrich the behavior of the dynamical system in unexpected ways — transforming glider types, transforming oscillators, changing glider direction — which would be significant to achieve universality in its full sense, to include other functionality such as memory by data storage. Figures 8 – 11 provide a selection of collision examples.
Figure 8: Destruction of gliders Ga and Gc by an Eater stable block, which survives the collision, showing phases/time-steps approaching the Eater as well as the impact. These dynamics apply to both the Variant and Precursor rules. The Gc eater shape can also be \[ \text{Gc Eater in 7 steps} \] and the Ga eater \[ \text{Ga Eater in 6 steps} \].

Figure 9: Mutual destruction by colliding gliders, Ga and Gc, showing phases/time-steps on the approach as well as the impact. These dynamics apply to both the Variant and Precursor rules.
Figure 10: Examples of collisions outcomes which transform and change the direction of gliders. Top: collision with a stable block. Below: collisions with the oscillator P22 described in figure 13.
Figure 11: Collisions resulting in oscillator P15. Top: Ga collides with oscillator P22 and transforms it to oscillator P15 — Ga is destroyed. Below: Gc collides with a stable block resulting in oscillator P15 — both Gc and the block are destroyed.

5 Glider-Guns, oscillators and reflectors

Glider-guns can be built from two types of oscillator, P22 and P15, and also from related interacting reflectors. If these sub-components are juxtaposed to interact precisely, gliders are ejected with a rhythm related to the oscillation or reflection period. Several different glider-guns are built by these methods, shooting glider types Ga, Gc, and G2a (a double Ga). It’s quite possible that other glider-guns are out there in the Variant rule, to be discovered.

5.1 Glider-Guns from oscillator P22

The P22 oscillator, named for its 22 time-step frequency, but which divides into two sets of 11 reflected patterns, is detailed in figure and is used to build Gc and Ga glider-guns in figures and.
Figure 13: The P22 oscillator showing all 22 phases (time-steps) as an attractor cycle\cite{13, 16} where the direction of time is clockwise. Inset: An oscillator phase shown at a larger scale alongside the same phase on the attractor cycle.

Figure 14: Two GGc glider-guns, their centers offset, shoot Gc gliders at each other. The Gc collisions create two Ga gliders, thus the combination creates a double GGa glider-gun. The Ga glider streams are stopped by eaters.
5.2 Glider-Guns from oscillator P15

The P15 oscillator, named for its 15 time-step frequency, is detailed in figure 15 and is used to build G2a and Ga glider-guns in figure 16.

Figure 15: The P15 oscillator showing all 15 phases (time-steps) as an attractor cycle where the direction of time is clockwise. Inset: An oscillator phase shown at a larger scale alongside the same phase on the attractor cycle.

Figure 16: Double Ga (G2a) gliders can be shot by a GG2a glider-gun constructed from two (same phase) P15 oscillators correctly juxtaposed at 90°. Left: The glider stream is stopped by a G2a eater. Right: Two GG2a glider-guns at 90° create a Ga glider stream, stopped by an eater.
5.3 Glider-Guns from reflector

A Gc glider is able to bounce off a stable reflector as in figure 17. Two such reflectors correctly juxtaposed at 90° create a G2a glider-gun with a frequency of 27 time-steps, and two of these correctly juxtaposed at 90° build a Ga glider-gun (figure 18). The glider spacing is wider than the glider-guns in sections 5.1 and 5.2.

Figure 17: A Gc glider bounces of a stable reflector, showing 26 consecutive time-steps. The reflector can take up any of these shapes, and the corner shapes can be mixed.

Figure 18: Using the the Gc reflection property in figure 17 double Ga (G2a) gliders, with a frequency of 27 time-steps, can be shot by a GG2aR glider-gun constructed from two Gc reflectors (same phase) correctly juxtaposed at 90° 22. Left: The glider stream is stopped by a G2a eater. Right: Two GG2a glider-guns at 90° create a Ga glider stream, stopped by an eater.
5.4 Small oscillators

A variety of small oscillators exist in the Variant rule, some where the period is related to the size of an extendable pattern between reflectors with a bouncing interior. There are significant overlaps with small oscillators in the Precursor rule[6]. Figures 19, 20 and 21 give examples.

Figure 19: Simple reflecting oscillators (SROs) — a Gc glider bouncing between stable reflectors. The period depends on the gap between reflectors. These SROs are also present in the Precursor rule.

Figure 20: Examples of extendable trapped oscillators (ETOs) between reflectors where the dynamics is more complicated than simple bouncing. The periods are 2, 43, 46 and 50 respectively.

Figure 21: Top: Oscillators with periods P2 (\(g\) and \(h\) are extendable). Below: Oscillators with periods P3, P6, P8, and P15.
6 Glider stream circuits

The various glider-guns already described can themselves become sub-components, that together with eaters, collisions, blocks, reflectors and oscillators, can build super-glider-guns, super-oscillators, and glider stream circuits of ever increasing complexity. Figures 22 and 23 give examples.

Figure 22: A glider stream (GS) circuit where a Gc-GS is transformed to double spaced Ga-GS by interaction with a block and two P22 oscillators. The circuit sub-components: 1) GGc glider-gun, 2) Gc-GS collision with a block, 3) transform to Ga-GS, 4) interaction with P22 oscillator, 4) transform to double spaced Gc-GS by eliminating alternate gliders, 5) interaction with P22 oscillator to create two doubly spaced Ga-GS, stopped by eaters.

Figure 23: A glider stream (GS) circuit folds into a spiral, where a Gc-GS is transformed to double spaced Ga-GS by interaction with two P22 oscillators and three blocks. The circuit repeats steps 1) to 4), then 5) collision with a block, 6) transform to Ga-GS, 7) interaction with P22 oscillator, 8) transform to Gc-GS, 9) collision with a block, 10) transform to Ga-GS, stopped by an eater.
7 Variable period glider-gun

Variant-rule features an interesting collision where a Gc glider brushes past a P15 oscillator resulting in two Gc gliders moving in opposite directions (figure 24), from which a variable period Gc glider-gun (GGcV) is built by introducing a second P15 separated from the first by 27 cells (figure 25). Gc gliders are shot in opposite directions at a frequency of 120 time-steps, and can be stopped by eaters. The P15 separation (S) can be increases by intervals of 30, which increases the period (P) by intervals of 120, giving S/P of 57/240, 87/360, and so on.

The Gc glider stream can be transformed to a Ga glider stream by an appropriate block or another P15 oscillator as in figure 26, making a variable Ga glider-gun (GGaV), with the same variability as GGcV.

It’s easy see that other permutations are possible by adapting the same mechanism, for example a combined Ga and Gc glider-gun.

Figure 24: A Gc glider precisely brushes past a P12 oscillator resulting in two Gc gliders in opposite directions. In the sequence above, the Westward glider continues, the new Eastward glider is displaced South by one cell.

Figure 25: A GGcV27 variable glider-gun constructed from two P15 oscillator separated by 27 time-steps, shooting gliders with a frequency of 120 time-steps. Increasing the separation by 30 increases the frequency by 120. The Gc glider streams are stopped by eaters.
8 Logical Universality

Post’s Functional Completeness Theorem\cite{9, 3} established a disjunctive (or conjunctive) normal form formula using the logical gates NOT, AND and OR to satisfy negation, conjunction and disjunction, and we apply the term “logical universality” to a CA if these gates can be demonstrated.

However, for a CA to be universal in the full sense according to Conway\cite{1}, two further conditions (1 and 2 below), are required, giving a full list as follows,

1. Data storage or memory.
2. Data transmission requiring the equivalent of wires and an internal clock.
3. Data processing requiring a universal set of logic gates NOT, AND, OR.

The Variant rule probably has a sufficient variety of logical components to establish all three conditions following similar methods for the Game-of-Life\cite{1, 4}, but we will postpone that investigation and confine our demonstration to item 3, logical universality only. To achieve this we will need the following basic ingredients\cite{6}:

1. A glider-gun or “pulse generator”, sending a stream of gliders into space. So far 11 glider-guns have been discovered in the Variant-rule, shooting Gc, Ga and G2a gliders, all moving in 4 phases.
(a) Three Gc glider-guns (figures 12, 22, 25).
(b) Six Ga glider-guns (figures 12R, 14, 16R, 22, 23, 26).
(c) Two GG2a glider-guns (figures 16, 18).

2. A stable eater, based on a block, oscillator or another glider-gun. The eater must destroy each incoming glider and survive the collision to destroy the next, so capable of stopping a glider stream.

3. Complete self-destruction when two gliders collide at an angle. Any debris must quickly dissipate, and the gap between gliders must be sufficient so as not to interfere with the next incoming glider.

Both GGa and GGc glider-guns meet these conditions, and possible any of the other glider-guns. In sections 8.1 and 8.2 we demonstrate the logical gates NOT, AND, OR for GGa and GGc, following Conway’s method[1] where a data stream of 1s/0s is implemented by gliders/gaps. Here the gaps are marked as grey discs.

8.1 GGa NOT, AND, OR

Figure 27: An example of the GGa NOT gate: \((-1,1 \rightarrow 0 \text{ and } 0 \rightarrow 1)\) or inverter, which transforms a stream of data to its complement, represented by gliders and gaps (grey discs). Left: The 5-bit input string A (10001) moving SE is about to interact with a GGa glider-stream moving NE. Right: The outcome is NOT-A (01110) moving NE, shown after 134 time-steps.
Figure 28: An example of the AND gate \((1 \land 1 \rightarrow 1, \text{else } \rightarrow 0)\) making a conjunction between two streams of data, represented by gliders and gaps (grey discs). \textit{Left:} The 5-bit input strings \(A\) (10001) and \(B\) (10100) both moving SE are about to interact with a GGa glider-stream moving NE. \textit{Right:} The outcome is A-AND-B (10000) moving SE shown after 184 time-steps.

The dynamics making this AND gate first makes an intermediate NOT-A (NE 01110 – figure 27) which interacts with input B to simultaneously produce both A-AND-B (SE 10000), and the A-NOR-B (NE 01010) which will be required to make the OR gate in figure 29.
Figure 29: An example of the OR gate ($1 \lor 1 \to 1$, else $\to 0$) making a disjunction between two stream of data represented by two streams of gliders and gaps (grey discs). Left: The 5-bit input strings $A$ (10001) and $B$ (10100) both moving SE are about to interact with two GGa glider-streams, the lower GGa shooting NE, and the upper GGa shooting SE. Right: The outcome is $A \lor B$ (10101) moving SE shown after 264 time-steps.

The dynamics making this OR gate first makes an intermediate NOT-A (NE 01110 – figure 27) which interacts with input $B$ to make $A \lor \neg B$ (NE 01010, as in figure 28) which interacts with the upper GGa shooting SE to make $A \lor B$ (SE 10101). A residual bi-product is $A \land B$ (SE 10000 – figure 28).
8.2 GGc NOT, AND, OR

Figure 30: An example of the GGc NOT gate: \((\neg 1, 1 \rightarrow 0 \text{ and } 0 \rightarrow 1)\) or inverter, which transforms a stream of data to its complement, represented by gliders and gaps (grey discs). *Left:* The 5-bit input string A (10001) moving East is about to interact with a GGc glider-stream moving North. *Right:* The outcome is NOT-A (01110) moving North, shown after 134 time-steps.
Figure 31: An example of the AND gate \((1 \land 1 \rightarrow 1, \text{else} \rightarrow 0)\) making a conjunction between two streams of data, represented by gliders and gaps (grey discs). Left: The 5-bit input strings \(A\) (10001) and \(B\) (10100) both moving East are about to interact with a GGc glider-stream moving North. Right: The outcome is A-AND-B (10000) moving East shown after 179 time-steps.

The dynamics making this AND gate first makes an intermediate NOT-A (North 01110 – figure 30) which interacts with input B to simultaneously produce both A-AND-B (East 10000), and the A-NOR-B (North 01010) which will be required to make the OR gate in figure 32.
Figure 32: An example of the OR gate (\(1 \lor 1 \rightarrow 1\), else \(\rightarrow 0\)) making a disjunction between two stream of data represented by two streams of gliders and gaps (grey discs). Top: The 5-bit input strings A (10001) and B (10100) both moving East are about to interact with two GGc glider-streams, the lower GGc shooting North, and the upper GGc shooting East. Below: The outcome is A-OR-B (10101) moving East shown after 220 time-steps.

The dynamics making this OR gate first makes an intermediate NOT-A (North 01110 – figure 30) which interacts with input B to make A-NOR-B (North 01010 – figure 31) which interacts with the upper GGc shooting East to make A-OR-B (SE 10101). A residual bi-product is A-AND-B (East 10000 – figure 31).

9 Spaceships, puffers and rakes

The Variant rule features many other interesting larger scale moving patterns, named from the Game-of-Life lexicon. A spaceship is a compound-glider built from subunits, a puffer-train is a spaceship leaving debris in its wake, a rake ejects gliders as it moves, and there are intermediate or ambiguous structures
such as puffer-rakes. Most of these patterns were discovered by members of the ConwayLife forum[19]. Figures 33 to 37 give examples.

Figure 33: Left: Spaceships built from Gc glider subunits separated by one cell, and Right: dragging a periodic tag[20]. More units can be added.

Figure 34: Periodic puffer-trains and a rake. Right: initial states are made from increasing numbers (puf2[22] to puf5) of touching Gc+block subunits, with period p shown. Left: the pattern fronts advance in 4 phases with speed c/2, and are shown after 149 time-steps with trailing debris. From the debris in puf3, which could also be called a puffer-rake, bursts of Gc and Ga gliders emerge every 96 time-steps. These and further figures have dynamic trails of 88 time-steps.
Figure 35: Periodic puffer-rake. Right: the initial states. Left: the pattern front advances in 4 phases with speed $c/2$, and is shown after 274 time-steps with trailing debris, from which bursts of Ga and Gc gliders emerge every 24 time-steps.

Figure 36: Three periodic rakes[22]. Right: initial states. Left: Gc gliders/patterns move West/East in 4 phases with speed $c/2$, shown after 149 time-steps. (a) the central zone has a period of 50. (b) Ga gliders emerge every 24 time-steps. (c) Gb gliders (figure[7]) are shot East every 48 time-steps.
Figure 37: Two periodic rakes[22]. Right: initial states based on 3 leading Gc’s separated by 2 cells, and other structures. Left(a): Gc travel West, and North/South every 144 time-steps. Ga gliders travel NW. The pattern front moves East in 4 phases with speed \(c/2\). Shown after 243 time-steps. Left(b) Gc gliders travelling North/South emerge every 24 time-steps, as the pattern front moves East in 4 phases with speed \(c/2\). Shown after 138 time-steps.


10 Concluding remarks

A chance mutation while the ConwayLife Forum [19] scrutinised the Precursor rule [6], revealed a new “Variant” rule with an interesting diversity of different patterns, despite the small divergence from the Precursor. Members of the forum applied their considerable know-how in Game-of-Life pattern search to discover glider-guns and other patterns in the Variant rule — a selection are now documented and elaborated in this paper. It’s very possible that other significant patterns exist, to be discovered.

The Variant-rule has gliders, glider-guns, eaters and convenient collision from which we are able to construct the logical gates in at least two distinct ways, and demonstrate universality in the logical sense. More work would be necessary to show universality in the Turing sense [10] and in terms of Conway [1]. The Variant-rule enriches the family of cellular automata with glider-gun complex properties that are not based on Life-like birth/survival schemes.

It would be interesting in the future to study which patterns are common or not, between the Variant, Precursor [6] and X-rule [5], and also the Sayab-rule [7], as well as the Game-of-Life. The Variant and Precursor rules have very different glider-guns and larger scale pattern behaviors despite their genetic closeness, though they share glider types and small scale features, illustrating both the robustness and fragility of evolution. The discovery of new complex glider-gun rules based on small mutations would be a promising approach towards a general theory of glider-gun dynamics.

11 Experiments and Acknowledgements

Experiments were done with Discrete Dynamics Lab (DDLab) [16, 17], Mathematica and Golly, and can be repeated and extended from initial states detailed at the “Logical Universality in 2D Cellular Automata” web page [8].

The Precursor-Rule was found during a collaboration at a workshop in June 2017 at the DDLab Complex Systems Institute in Ariège, France, and in London, UK, and also at the Universidad Autónoma de Zacatecas, México in 2018, 2019, Later patterns were discovered during interactions with the ConwayLife Forum [19] where many people made important contributions. J.M. Gómez Soto also acknowledges his residency at Discrete Dynamics Lab, and financial support from the Research Council of Mexico (CONACyT).

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