CP Asymmetries in Charm Decays into Neutral Kaons

Di Wang,1 Fu-Sheng Yu,1,2 and Hsiang-nan Li2,3

1School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, People’s Republic of China
2Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China

We find a new CP-violation effect in charm decays into neutral kaons, which results from the interference between two tree (Cabibbo-favored and doubly Cabibbo-suppressed) amplitudes with the mixing of final-state mesons. This effect, estimated to be of an order of $10^{-3}$, is much larger than the direct CP asymmetries in these decays, but missed in the literature. It can be revealed by measuring the difference of the time-dependent CP asymmetries in the $D^+ \rightarrow \pi^+ K^0_S$ and $D^+_s \rightarrow K^+ K_S^0$ modes, which are accessible at the LHCb and Belle II experiments. If confirmed, the new effect has to be taken into account, as the above direct CP asymmetries are used to search for new physics.

CP violation plays an important role in interpreting the matter-antimatter asymmetry in the Universe and in searching for new physics beyond the Standard Model (SM). It has been well established in the kaon and $B$ meson systems, but not yet in the charm sector. Many theoretical and experimental efforts have been devoted to the study of CP violation in the singly Cabibbo-suppressed (SCS) $D$ meson decays, with the interests in flavor-changing-neutral currents from penguin amplitudes. The most precise individual measurements up to now are obtained for the time-integrated CP asymmetry by the LHCb Collaboration [1],

$$\Delta A_{CP} \equiv A_{CP}(D^0 \rightarrow K^+ K^-) - A_{CP}(D^0 \rightarrow \pi^+ \pi^-) = (-1.0 \pm 0.8 \pm 0.3) \times 10^{-3},$$

(1)

which is dominated by the direct CP violation $\Delta a_{CP}^{\text{dir}}$, and for the asymmetry in effective decay widths through time-dependent rates [2],

$$A_T(D^0 \rightarrow K^+ K^-) = (-0.30 \pm 0.32 \pm 0.10) \times 10^{-3},$$

which is sensitive to the indirect CP violation in the $D^0$-$\bar{D}^0$ mixing. With the precision lower than $10^{-3}$, there is still no evidence of CP violation in the charm system.

CP violation can also occur via the interference between the Cabibbo-favored (CF) and doubly Cabibbo-suppressed (DCS) channels of $D \rightarrow f K_S^0$, with $f$ being a final-state particle. These decays, with large branching fractions from the CF amplitudes, are more experimentally accessible. However, the CP asymmetries, such as

$$A_{CP}(D^+ \rightarrow \pi^+ K_S^0) = (-3.63 \pm 0.94 \pm 0.67) \times 10^{-3},$$

(2)

with $3.2\sigma$ from zero observed by the Belle Collaboration [3], are mainly attributed to the $K^0_S \bar{K}^0_L$ mixing. It has been claimed [3,7] that deviation from the kaon-mixing effect in a precise measurement of the above mode can be identified as the direct CP violation. Because of its smallness in the SM, the direct CP violation in these decays has been regarded as a promising observable for searching for new physics [8,9].

In this Letter, we will point out a new CP-violation effect in charm decays into neutral kaons, which results from the interference between the CF and DCS amplitudes with the mixing of final-state mesons. This new effect, estimated to be of the order of $10^{-3}$, turns out to be much larger than the direct CP asymmetry, but has been, to our surprise, missed in the literature [3,6]. We propose to measure the difference of the CP asymmetries in the decay chains $D^+ \rightarrow \pi^+ K(t) (\rightarrow \pi^+ \pi^-)$ and $D^+_s \rightarrow K^+ K(t) (\rightarrow \pi^+ \pi^-)$, where $K(t)$ represents a time-evolved neutral kaon $K^0(t)$ or $\bar{K}^0(t)$ with $t$ being the time difference between the charm decays and the neutral kaon decays in the kaon rest frame. It will be shown that the contributions to the above difference from the pure kaon mixing cancel, and the new effect can be clearly revealed. Only when this new effect has been well determined, can the direct CP asymmetries in charm decays into neutral kaons be extracted correctly and used to search for new physics.

A $K_S^0$ state is reconstructed via its decay into two charged pions at a time close to its lifetime $\tau_S$ in measurements of the $D \rightarrow f K_S^0$ processes. Hence, not only $K_S^0$, but also $K_L^0$ serve as the intermediate states in the $D \rightarrow f K(t) (\rightarrow \pi^+ \pi^-)$ chain decays through the $K_S^0$-$K_L^0$ oscillation, and to their CP asymmetries [3]. The $K_S^0$ and $K_L^0$ states are linear combinations of the flavor eigenstates

$$|R_{S,L}^0\rangle = p |K^0\rangle \mp q |\bar{K}^0\rangle,$$

(3)

where $q/p = (1 - \epsilon)/(1 + \epsilon)$, and $\epsilon$ is a small complex parameter characterizing the indirect CP violation in the kaon mixing with the magnitude $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$ and the phase $\phi_{\epsilon} = 43.52 \pm 0.05^\circ$ [10]. Let $m_{S,L}$, $\Gamma_{S,L}$, and $\tau_{S,L}$ denote the masses, widths, and lifetimes of $|R_{S,L}^0\rangle$, respectively. The average of widths is then given by $\Gamma = (\Gamma_S + \Gamma_L)/2$, and the differences of widths and masses are $\Delta \Gamma \equiv \Gamma_S - \Gamma_L$ and $\Delta m \equiv m_L - m_S$, respectively. We write the ratio between the DCS and CF amplitudes as

$$A(D \rightarrow f K^0)/A(D \rightarrow f \bar{K}^0) = r_f e^{i(\phi+\delta_f)},$$

(4)

with the magnitude $r_f \propto |V_{ud}^* V_{us}/V_{us}^* V_{ud}| \sim O(10^{-2})$, the relative strong phase $\delta_f$ that depends on final states,
and the weak phase \( \phi \equiv \text{Arg} \left[ -V_{cd}^* V_{us} / V_{cd}^* V_{ub} \right] = (-6.2 \pm 0.4) \times 10^{-4} \) in the SM.

We consider the time-dependent \( CP \) asymmetry

\[
ACP(t) = \frac{\Gamma_{\pi \pi}(t) - \Gamma_{\pi \bar{\pi}}(t)}{\Gamma_{\pi \pi}(t) + \Gamma_{\pi \bar{\pi}}(t)},
\]

where

\[
\Gamma_{\pi \pi}(t) \equiv \Gamma(D \to fK(t)\to \pi^+\pi^-), \quad \Gamma_{\pi \bar{\pi}}(t) \equiv \Gamma(\bar{D} \to \bar{f}K(t)\to \pi^+\pi^-).
\]

Neglecting the tiny direct \( CP \) asymmetry in the \( K \to \pi \pi \) decays, namely, assuming the equality of the amplitudes \( A(K^0 \to \pi^+\pi^-) = -A(K^0 \to \pi^+\pi^-) \), we derive from Eq. (5),

\[
ACP(t) \approx \frac{A_{\text{CP}}^0(t) + A_{\text{CP}}^\text{dir}(t) + A_{\text{CP}}^\text{int}(t)}{D(t)},
\]

with the denominator \( D(t) = e^{-\Gamma_{sl} t} (1-2r_f \cos \delta_f \cos \phi) + e^{-\Gamma_{sl} t} |t|^2 \). The first term corresponds to the known \( CP \) violation in the kaon mixing \[4\],

\[
A_{\text{CP}}^0(t) = 2e^{-\Gamma_{sl} t} \Re(e^{-i\phi}) - 2e^{-\Gamma_{sl} t} \left[ \Re(e) \cos(\Delta m t) + i \Im(e) \sin(\Delta m t) \right],
\]

which is independent of \( r_f \), i.e., of the DCS amplitude. The second term is the direct \( CP \) asymmetry originating from the interference between the CF and DCS amplitudes,

\[
A_{\text{CP}}^\text{dir}(t) = e^{-\Gamma_{sl} t} 2r_f \sin \delta_f \sin \phi.
\]

The third term in Eq. (7) represents the new \( CP \)-violation effect,

\[
A_{\text{CP}}^\text{int}(t) = -4r_f \cos \phi \sin \delta_f \left[ e^{-\Gamma_{sl} t} \Im(e) \right.
\]

\[
- \left. e^{-\Gamma_{sl} t} \left( \Im(e) \cos(\Delta m t) - \Re(e) \sin(\Delta m t) \right) \right],
\]

which is induced by the interference between the CF and DCS amplitudes of the decays \( D \to fK^0(t)\to \pi^+\pi^- \) and \( D \to fK^0(t)\to \pi^+\pi^- \) with the kaon mixing. The mechanism responsible for Eq. (10) is more complicated than for the ordinary mixing-induced \( CP \) asymmetry in, for example, the \( B^0(t) \to \pi^+\pi^- \) mode: both the oscillation and decay take place in the mother particle in the latter, while \( A_{\text{CP}}^\text{int} \) arises from the mother decay and the daughter mixing as depicted in Fig. 1. \( A_{\text{CP}}^\text{int} \) is not a direct \( CP \) asymmetry in charm decays, since it does not vanish as \( \phi = 0 \).

Compared to the SCS case, both the CF and DCS amplitudes, being of the tree level, can be extracted from data of branching fractions \[11\]-\[14\]. A global fit to the newest data in the factorization-assisted topological-amplitude (FAT) approach \[11\] gives the parameters \( r_{\pi+,K^+} \) and \( \delta_{\pi+,K^+} \) for the \( D^+ \to \pi^+K_S^0 \) and \( D^+_s \to K^+K_S^0 \) decays \[12\]

\[
r_{\pi+} = -0.073 \pm 0.004, \quad \delta_{\pi+} = -1.39 \pm 0.05, \quad r_{K^+} = -0.055 \pm 0.002, \quad \delta_{K^+} = +1.45 \pm 0.05.
\]

The solution with opposite signs of \( \delta_{\pi+,K^+} \) contributes equivalently to branching fractions, which depend only on the cosine of strong phases. The one presented above is preferred by the central value of the \( CP \)-asymmetry data in Eq. (2) in the global fit, to which the sign of strong phases is relevant.

The time-dependent \( CP \) asymmetries in the \( D^+ \to \pi^+K(t)\to \pi^+\pi^- \) decay as a function of \( t/\tau_S \) with the zoomed-in plot for the small \( t \) region in the lower plot. The gray bands represent the theoretical uncertainties.
the total \( CP \) asymmetry in \( D \rightarrow f K^0_S \) decays from \( A^\text{int}_{CPS} \) should be attributed to \( A^\text{int}_{CPS} \), instead of to the direct \( CP \) asymmetry. Figure 2 also indicates that the total \( CP \) asymmetry approaches to zero at \( t = 0 \), because both \( A^\text{dir}_{CPS} \) and \( A^\text{int}_{CPS} \) diminish at \( t = 0 \), and \( A^\text{int}_{CPS} \) is tiny. With the inputs in Eq. (11), the direct \( CP \) asymmetries are predicted to be

\[
\begin{aligned}
A^\text{dir}_{CPS}(D^+ \rightarrow \pi^+ K^0_S) &= (-8.6 \pm 0.4) \times 10^{-5}, \\
A^\text{dir}_{CPS}(D^+_s \rightarrow K^+ K^0_S) &= (6.6 \pm 0.3) \times 10^{-5}.
\end{aligned}
\]

Both the forthcoming experiments, Belle II and LHCb upgrade, cannot attain such a precision at an order of \( 10^{-5} \). However, a large weak phase difference between the CF and DCS amplitudes could exist in new physics models \([6–9, 16]\), resulting in a larger \( A^\text{dir}_{CPS} \). Therefore, an observation with nonvanishing \( A^\text{dir}_{CPS}(t = 0) \) at the Belle II and LHCb upgrade would be a signature of new physics.

Searching for new physics through the direct \( CP \) asymmetries in Eq. (12) might be more promising than through those in the SCS processes. For the latter, it is difficult to predict the \( CP \) asymmetries precisely due to the ambiguity in estimating the penguin contributions: the QCD-inspired approaches do not work at the charm scale, and the penguin topologies cannot be extracted from data of branching fractions. This is the reason why predictions for \( \Delta \pi^0_{CPS} \) in the SM vary from \( O(10^{-4}) \) to \( O(1\%\) \([11, 17–29]\), and cannot be used to discriminate new physics.

The denominator \( D(t) \) in Eq. (7) can be related to the \( K^0_S-K^0_L \) asymmetry,

\[
R \equiv \frac{\Gamma(D \rightarrow f K^0_S) - \Gamma(D \rightarrow f K^0_L)}{\Gamma(D \rightarrow f K^0_S) + \Gamma(D \rightarrow f K^0_L)} = -2r_f \cos(\phi + \delta_f) \approx -2r_f \cos \phi \cos \delta_f,
\]

in the limit \( \phi \to 0 \). The \( K^0_S-K^0_L \) asymmetry in \( D^+ \rightarrow \pi^+ K^0_{S,L} \) has been measured by the CLEO Collaboration with a value \( 0.022 \pm 0.024 \) \([30]\). The FAT approach leads to \( R(D^+ \rightarrow \pi^+ K^0_{S,L}) \) \( = 0.025 \pm 0.008 \), consistent with the data, and \( R(D^+_s \rightarrow K^+ K^0_{S,L}) \) \( = 0.012 \pm 0.006 \) \([15]\). The above small results, in agreement with those derived in the literature \([13, 31, 33]\), are due to \( \delta_{\pi^+ K^+} \approx \pm 2\pi/2 \) in Eq. (11). That is, the term \( -2r_f \cos \phi \cos \delta_f \) causes an effect at least one order of magnitude lower than \( A^\text{int}_{CPS} \).

Measurements of \( CP \) asymmetries depend on time intervals selected in experiments. To obtain a time-integrated \( CP \) asymmetry defined by

\[
A_{CP} = \frac{\int_0^\infty F(t)[\Gamma_{\pi^+}(t) - \Gamma_{\pi^0}(t)]dt}{\int_0^\infty F(t)[\Gamma_{\pi^+}(t) + \Gamma_{\pi^0}(t)]dt},
\]

a function of time, \( F(t) \), is introduced to take into account relevant experimental effects, such as detecting efficiencies and kaon energies. We adopt the approximation with \( F(t) = 1 \) in the interval \([t_1, t_2]\) and \( F(t) = 0 \) elsewhere \([4]\). Equation (14) then yields

\[
A_{CP}(t_1, t_2) = f_{t_1}^{t_2} \frac{[A^\text{dir}_{CPS}(t) + A^\text{int}_{CPS} + A^\text{int}_{CPS}]}{D(t)} dt
\]

\[
= \frac{2Re(x) - 4Im(\chi) r_f \cos \phi \sin \delta_f}{1 + 2r_f \cos \phi \cos \delta_f} \left[ 1 - \frac{c(t_1) - c(t_2)}{\tau \Gamma(1 + 2\mu)[e^{-t_1} - e^{-t_2}]} \right] + 2r_f \sin \delta_f \sin \phi,
\]

where \( x = \Delta m/\Gamma, c(t) = e^{-it}[\cos(\Delta mt) - x \sin(\Delta mt)], \) and \( s(t) = e^{-it}[x \cos(\Delta mt) + \sin(\Delta mt)] \). In the second and third lines the terms proportional to \( r_f \) stand for the new effect \( A^\text{int}_{CPS} \), and those without \( r_f \) for the \( CP \) violation in the neutral kaon system. The last term, being independent of \( t_1, t_2 \), corresponds to the direct \( CP \) asymmetry. The time-integrated \( CP \) asymmetries in the \( D^+ \rightarrow \pi^+ K^0_S \) decays are exhibited in Fig. 3 with the upper plot for the total \( CP \) asymmetry and the lower one

![Graph](image-url)
for the new effect. Both quantities are relatively large in some ranges of $t_1$ and $t_2$, suggesting the favorable time intervals for experimental investigations of these CP asymmetries.

With the same approximation as in [34] for the limit of $t_1 \ll \tau_S \ll t_2 \ll \tau_L$, we get

$$ A_{\text{CP}}(t_1 \ll \tau_S \ll t_2 \ll \tau_L) \approx -2\text{Re}(e) + 2r_f \sin \phi \sin \delta_f - 4\text{Im}(e) r_f \cos \phi \sin \delta_f \over 1 - 2r_f \cos \phi \cos \delta_f. $$

In the absence of the DCS contributions, i.e., $r_f = 0$, the above formula reduces to $-2\text{Re}(e)$ derived in [33–38].

The effect of $A_{\text{CP}}^{\text{int}}$, namely, the third term in the numerator of Eq. (16) was missed in the study of the CP asymmetry in $D^+ \to \pi^+ K^0_S$ by the Belle Collaboration [3]. The direct CP violation has to be extracted by subtracting the kaon-mixing and new effects from the total CP asymmetry. The sum of the latter two effects in Eq. (16) is predicted to be $(-3.57 \pm 0.05) \times 10^{-3}$. The direct CP violation $(-0.06 \pm 1.15) \times 10^{-3}$ is then obtained from the Belle data in Eq. (2), consistent with our prediction in Eq. (12). The $D^+ \to \pi^+ K^0_S$ and $D^+_s \to K^+ K^0_S$ decays have been employed to cancel the systematic asymmetries from the production and detection at the LHCb for the measurements of CP violation in the SCS processes [33–38]. The working assumption is that there is no sizable CP violation other than the one from the kaon mixing in the $D^+ \to \pi^+ K^0_S$ and $D^+_s \to K^+ K^0_S$ modes. However, $A_{\text{CP}}^{\text{int}}$ observed here is of the same order as the direct CP asymmetries in the SCS processes, which are expected to be $O(10^{-3})$ or $O(10^{-4})$. That is, the effect of $A_{\text{CP}}^{\text{int}}$ has to be considered in these measurements as well.

To verify the new CP-violation effect, we propose an observable, the difference of the time-integrated CP asymmetries in the $D^+ \to \pi^+ K^0_S$ and $D^+_s \to K^+ K^0_S$ modes,

$$ \Delta A_{\text{CP}}^{+,K^+} = A_{\text{CP}}^{D^+ \to \pi^+ K^0_S}(t_1, t_2) - A_{\text{CP}}^{D^+_s \to K^+ K^0_S}(t_1, t_2), $$

$$ \approx A_{\text{CP}}^{\text{int},D^+ \to \pi^+ K^0_S}(t_1, t_2) - A_{\text{CP}}^{\text{int},D^+_s \to K^+ K^0_S}(t_1, t_2). $$

Our global-fit analysis indicates that the new effect is the most significant in this observable. The CP violation in the kaon mixing, being mode-independent as implied by Eq. (3), is canceled in the above difference, and the direct CP violation is negligible. The new effect survives in $\Delta A_{\text{CP}}^{+,K^+}$ according to the following model-independent argument. The topological diagrams of the CP and DCS amplitudes in these two decays are exchanged to each other under the flavor SU(3) symmetry, $A(D^+ \to \pi^+ K^0_S)/V_{us} V_{cd} = A(D^+_s \to K^+ K^0_S)/V_{us} V_{cd} = C + A$ and $A(D^+ \to \pi^+ K^0_S)/V_{us} V_{cd} = A(D^+_s \to K^+ K^0_S)/V_{us} V_{cd} = T + C$, with the color-favored tree-emission diagram $T$, the color-suppressed tree-emission diagram $C$, and the W-annihilation diagram $A$. The relative strong phases $\delta_f$ in these two modes are, thus, opposite in sign, as shown in Eq. (11), so that the new effects are constructive in $\Delta A_{\text{CP}}^{+,K^+}$. The dependencies of $\Delta A_{\text{CP}}^{+,K^+}$ on $t_1$ and $t_2$ are displayed in Fig. 4. It is seen that this observable is of the order of $10^{-3}$ in most of the time intervals, and increases with $t_1$.

The effect of $A_{\text{CP}}^{\text{int}}$ are measurable in the forthcoming experiments. The precision of Belle II measurements on the CP asymmetry in $D^+ \to \pi^+ K^0_S$ can attain $3 \times 10^{-4}$ at 50 ab$^{-1}$ [12]. In the LHCb upgrade, the error bar of $\Delta A_{\text{CP}}$ defined in Eq. (1) would be reduced to $1.2 \times 10^{-4}$ at 50 fb$^{-1}$ [42]. The signal yields of $D^+ \to \pi^+ K^0_S$ and $D^+_s \to \pi^+ K^0_S$ are of the same order as of $D^0 \to \pi K^-$ and $\pi^+ \pi^-$. It is then expected that the precision of $\Delta A_{\text{CP}}^{+,K^+}$ can also reach $O(10^{-4})$ at the LHCb upgrade, and that $\Delta A_{\text{CP}}^{+,K^+} \sim O(10^{-3})$ is accessible at both the Belle II and LHCb upgrade.

In this Letter, we have studied the time-dependent and time-integrated CP asymmetries in the $D \to f K^0_S(\to \pi^+ \pi^-)$ chain decays. A new CP-violation effect was identified in these processes, which is induced by the interference between the CF and the DCS amplitudes with the $K^0_{S}-\bar{K}^0_{S}$ mixing. Compared to the SCS processes, both the CF and DCS amplitudes, occurring at the tree level, can be extracted from the data of branching fractions. Therefore, their CP asymmetries can be estimated more accurately, and have been shown to be as large as $10^{-3}$ in the $D^+ \to \pi^+ K^0_S$ and $D^+_s \to K^+ K^0_S$ modes. Nevertheless, its effect has been missed in the literature. To reveal this new CP-violation effect, we have proposed an observable, the difference of the CP asymmetries in the $D^+ \to \pi^+ K^0_S$ and $D^+_s \to K^+ K^0_S$ decays accessible at Belle II and LHCb. In addition, the direct CP asymmetries used to search for new physics can be determined either by subtracting the kaon-mixing and DCS.
interference effects from total $CP$ asymmetries, or by the
time-dependent measurements of $CP$ violation in these processes.

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* Electronic address: yufsh@lzu.edu.cn
dElectronic address: lmb@phys.sinica.edu.tw

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