Constraining CPT-odd nonminimal interactions in the Electroweak sector

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In this work, we propose two possibilities of CPT-odd and Lorentz-violating (LV) nonminimal couplings in the Electroweak sector. These terms are gauge-invariant and couple a fixed 4-vector to the physical fields of the theory. After determining the LV contributions to the electroweak currents, we reassert the evaluation of the decay rate for the vector mediators $W$ and $Z$. Using the experimental uncertainty in these decay rates, upper bounds of $10^{-15}$ (eV$^{-1}$) and $10^{-14}$ (eV$^{-1}$) are imposed on the magnitude of the proposed nonminimal interactions.

PACS numbers: 11.30.Cp, 12.60.Cn, 13.38.Dg, 13.38.Be

I. INTRODUCTION

Mechanisms of spontaneous Lorentz violation have been proposed in some candidate theories of quantum gravity. As a consequence, Lorentz-violating (LV) background tensors (generated as vacuum expectation values) are coupled to the physical fields of the Standard Model. The most general effective theory considering the explicit breaking of Lorentz and CPT symmetry is the minimal Standard Model Extension (mSME) [1], which is an extension of the Standard Model, $SU(3) \times SU(2) \times U(1)$, featuring terms breaking Lorentz and CPT symmetries in all of its sectors: lepton, quark, Yukawa, Higgs and gauge. Investigation of Lorentz symmetry violation is a rich line of research, embracing developments in the electromagnetic sector [2], [3], fermion sector [4], including photon-fermion interactions [5], [6], [7]. Such studies have detailed LV effects in very distinct physical systems, allowing to construct a precision programme to determine to what extent the Lorentz covariance is preserved in nature (by means of tight upper bounds on the LV coefficients). Nonminimal LV interactions have also been examined in an extension of the SME encompassing higher-order derivatives in both the gauge [8] and the fermion sector [9]. Some other models containing higher-dimension operators [10] and higher derivatives [11] have also been proposed and developed.

In the Electroweak sector of the mSME [1], the SU(2) and U(1) gauge fields are properly coupled to LV fixed tensors in renormalizable dimension four terms. The mSME lepton sector is composed of a CPT-even and a CPT-odd term, that is,

$$\mathcal{L}_{\text{lepton}}^{\text{even}} = i(cL)_{\mu AB} \tilde{L}_A \gamma^{(\mu} D^{\nu} L_B + i(cR)_{\mu AB} \bar{R}_A \gamma^{\mu} i D^{\nu} R_B,$$

$$\mathcal{L}_{\text{lepton}}^{\text{odd}} = -(aL)_{\mu AB} \tilde{L}_A \gamma^{\mu} L_B - (aR)_{\mu AB} \bar{R}_A \gamma^{\mu} R_B,$$

where $A, B = 1, 2, 3$ are the lepton flavor labels. At the same way, the SU(2) and U(1) gauge sectors are modified by the CPT-even terms:

$$\mathcal{L}_{\text{gauge}}^{\text{even}} = \frac{1}{2} (k_W)^{\mu\alpha\beta} (W^{\mu\nu} W_{\nu}^\alpha) - \frac{1}{4} (k_B)^{\mu\alpha\beta\gamma} B_{\mu\nu} B^{\alpha\beta\gamma},$$

while the CPT-odd generate instabilities in the theory, and are not considered. These coefficients are real, dimensionless, and possess the same symmetries of the Riemann tensor. The pure CPT-even Higgs sector is also modified by the following term:

$$\mathcal{L}_{\text{Higgs}}^{\text{even}} = \frac{1}{2} (k_{\phi B})_{\mu\nu} (D^\mu \phi^a) (D^n \phi^a) + h.c.$$

where the Higgs CPT-odd has the form $i(k_{\phi})_{\mu} (\phi^a) (D^\mu \phi^a)$, with $(k_{\phi})_{\mu}$ having dimension of mass.

In the Electroweak sector, LV studies were initially developed in connection with meson decays ($\pi^- \to \mu^- + \nu_\mu$), where the LV effects were considered at the level of the Feynman propagator of the W boson [12], $(W^{\mu\nu} W_{\nu}^\mu) = -i (g^{\mu\nu} + \chi^\mu\nu)/M_W^2$, with contributions coming from the Higgs ($\phi$) and the W sectors: $\chi^\mu\nu = k^{\mu\nu} - i \frac{1}{2g} L_{\mu}^{\nu} W_{\mu} + L^{\mu\nu}_{\phi} p_\alpha p_\beta$. Comparison with experimental data led to upper bounds of 1 part in 10$^4$. Contributions of the $k_W$ coefficients (3) to the W propagator, $(W^{\mu\nu} W_{\nu}^\mu) = -i (g^{\mu\nu} + \chi^\mu\nu)/M_W^2$, jointly with contributions stemming from the Higgs sector, $k_{\phi}, k_{W}$, see Eq. (4), were more explicitly considered in Ref. [13], with implications on the allowed nuclear decays and forbidden $\beta$ decays. This framework was also used: (i) to reinterpret experiments dedicated to searching for preferred directions in forbidden $\beta$-decays, implying upper bounds as tight as $10^{-8}$ on the LV parameters [14]; (ii) to constrain $\beta$ decay rate asymmetries to the level of 1 part in $10^6$ [15]; (iii) to study isotopes that undergo orbital electron capture [16]; (iv) to analyse LV effects on the kaon decay and evaluate asymmetries in the respective lifetime [17]. Another interesting study considered LV coefficients $(cL)_{\mu AB}$ of the lepton sector (1), with the
same flavor \((A = B)\),

\[
\mathcal{L}_{\text{lepton}} = i\epsilon_{\alpha\beta} [\bar{\psi} \gamma^\alpha \partial^\beta \psi + i \bar{\psi}_\nu \gamma^\alpha \partial^\beta \psi_\nu \\
+ \bar{\psi}_L (\gamma^\alpha W^{\alpha(-)}_\nu \psi_\nu(L) + i \bar{\psi}_\nu (L) \gamma^\alpha W^{\alpha(+)\nu}_\psi(L)],
\]

where \(\psi, \psi_\nu, \psi_\nu(L)\) represent leptons, neutrinos and left-handed leptons (of a given flavor), to examine effects on the pion-decay rate \([18]\), attaining upper bounds of the level of \(10^{-4}\). Some works also examined the possibility of LV electroweak terms to make feasible forbidden processes \((Z_0 \rightarrow \gamma + \gamma)\) \([19]\) or modify the reactions such as \(\gamma + e \rightarrow W + \nu_e, \gamma + \gamma \rightarrow W + W\) \([20]\). Lepton flavor violating decays triggered by renormalizable and nonrenormalizable (dimension five) terms belonging to the Higgs sector were recently considered as well \([21]\). Tree-level \(Z\)-boson contributions to the polarized Möller scattering were carried out, allowing to improve \(k_W\) upper bounds by two orders of magnitude \([22]\). Lorentz violation influence on neutrino oscillations was also probed using a distinct framework \([23]\).

A dimension five LV nonminimal coupling (NMC), representing unusual interactions between fermions and photons, \(g' \psi^\mu \gamma_\alpha \gamma_5 F^\mu_\alpha \gamma_\beta \psi\), was first introduced by means of the derivative, \(D_\mu = \partial_\mu + i e A_\mu + i \frac{g'}{2} \epsilon_{\mu\lambda\alpha\beta} V^\lambda F^{\alpha\beta}\), in the Dirac equation \([24]\), where \(V^\mu\) can be identified with the Carroll-Field-Jackiw four-vector. Such a coupling has been addressed in numerous aspects \([25]\), including the radiative generation of CPT-odd LV terms \([26]\), topological phases \([27], [28]\), and generation of electric dipole moment \([29]\). Dimension-five CPT-even NMCs were also proposed in the context of the Dirac equation \([30], [31], [32]\), with MDM and EDM experimental measurements being used to state upper bounds at the level of 1 part in \(10^{20}\) (eV\(^{-1}\)) and \(10^{24}\) (eV\(^{-1}\)), respectively. A systematic investigation on NMCs of dimension five and six was recently proposed in Ref. \([33]\).

Nonminimal interactions have been a topical issue in the latest years, mainly in the fermion and electromagnetic sectors. However, a NMC in the lepton electroweak sector of the Standard Model has not been proposed yet. In this work, we introduce two possibilities of CPT-odd LV nonminimal interactions in the Electroweak sector, the first one being proposed in the \(U(1)\) sector of the GSW model, while the second is considered in its \(SU(2)_L\) sector, both as extensions of the covariant derivative. After determining the terms engendered in the interaction Lagrangian, we evaluate the Lorentz-violating corrections to the decay rates of the following mediators: \(Z_0 \rightarrow l + l\) and \(W^- \rightarrow l + \bar{n}_l\). Using these results and the experimental uncertainty in the measurements, we impose upper limits on the magnitude of the LV parameters, achieving upper bounds as tight as \(10^{-15}\) (eV\(^{-1}\)).

### II. BASICS ABOUT THE GSW MODEL

In the Glashow-Salam-Weinberg electroweak model (GSW), with a \(SU(2)_L \times U(1)_Y\) gauge structure spontaneously broken via the Higgs mechanism, the vector bosons, \(W^\pm, Z^0\) and \(\gamma\) are mediator of the interactions, being introduced via minimal coupling to the matter fields. In this theory, left-handed leptons \((L_i)\) are represented by isodoublets

\[
L_i = \begin{pmatrix} \psi_\mu \\ \psi_\nu \end{pmatrix}_L = \frac{1 - \gamma_5}{2} \begin{pmatrix} \psi_\mu \\ \psi_\nu \end{pmatrix},
\]

while right-handed leptons \((R_i)\) are isosinglets,

\[
R_i = \begin{pmatrix} \psi_\mu \\ \psi_\nu \end{pmatrix}_R = \frac{1 + \gamma_5}{2} \psi_i,
\]

and \(l = 1, 2, 3\) is the lepton flavor label: \(\psi_l = (e, \mu, \tau)\). The part of the electroweak Lagrangian, in which the leptons interact directly with the gauge fields, is \(\mathcal{L}_{EW} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{lepton}}\), where

\[
\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W_{\mu\nu} \cdot W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},
\]

\[
\mathcal{L}_{\text{lepton}} = \bar{L} \gamma^\mu i D_\mu L + \bar{R} \gamma^\mu i D_\mu R,
\]

with \(W_{\mu} = (W_{\mu}^3, W_{\mu}^\gamma, W_{\mu}^Z)\) being a four-vector gauge field which is a three-vector in isospin space, and \(B_{\mu}\) a gauge four-vector field, whose field strengths are

\[
W_{\mu\nu} = \partial_\mu W_{\nu} - \partial_\nu W_{\mu} + g W_{\mu} \times W_{\nu},
\]

\[
B_{\mu\nu} = \partial_\mu B_{\nu} - \partial_\nu B_{\mu}.
\]

The covariant derivative involves both gauge fields,

\[
D_\mu = \partial_\mu - ig T \cdot W_{\mu} - \frac{g'}{2} Y B_{\mu}.
\]

Here, \(T = (T_1, T_2, T_3)\) stands for the generators of the group \(SU(2)_L\), and \(Y\) is the generator of the \(U(1)_Y\) group, fulfilling \([T_i, T_j] = i e \frac{g'}{2} T_k\) and \([T_i, Y] = 0\). Furthermore, \(Y = -1\) or \(Y = -2\) for left-handed and right-handed leptons, respectively. The lepton Lagrangian \((9)\) can be written as \(\mathcal{L} = i \bar{L} \gamma^\mu \partial_\mu L + i \bar{R} \gamma^\mu \partial_\mu R + \mathcal{L}_{\text{int}}^{(l)}\) with the interaction part given as

\[
\mathcal{L}_{\text{int}}^{(l)} = \frac{g}{2 \sqrt{2}} \left( J_+ (\gamma^\alpha W_{\alpha}^+ ) + J_- (\gamma^\alpha W_{\alpha}^- ) + J_0 (\gamma^\alpha Z_{\alpha}) \right) \\
- e J_{EM}^{(\alpha)} A_\alpha,
\]

where there appear charged currents, \(J_+^{(\alpha)}, J_-^{(\alpha)}, J_0^{(\alpha)}\), a neutral current, \(J_0^{(\alpha)}\), and the electromagnetic current, \(J_{EM}^{(\alpha)}\), given as

\[
J_+^{(\alpha)} = 2 \bar{L} L \gamma^\alpha T_+ L = \bar{\psi}_\mu \gamma^\alpha (1 - \gamma_5) \psi_l,
\]

\[
J_-^{(\alpha)} = 2 \bar{L} L \gamma^\alpha T_- L = \bar{\psi}_\mu \gamma^\alpha (1 - \gamma_5) \psi_{\nu l},
\]
\[ j^{(lv)}_0 = \left( \sqrt{2} \cos \theta \right)^{-1} \left[ \bar{\psi}_v \gamma^\alpha (1 - \gamma_5) \psi_v \right] - g' \gamma^\alpha (g V^\prime - g' A_\alpha) \psi^\prime, \]

\[ J^{(lv)}_{0, EM} = - \left[ L \gamma^\alpha \left( \frac{g' \cos \theta}{2} - g \sin \theta T_3 \right) L + g' \cos \theta R \gamma^\alpha R \right]. \]

Here, \( \theta \) is the Weinberg angle, and \( g, g' \) are the coupling constants, and the vector-axial interaction is controlled by \( g'_L = 1, \ g'_L = 1 - 4 \sin^2 \theta \). In the electroweak theory, the photon \((A_\mu)\) is a combination of the fields \( W^\pm_3 \) and \( B^\mu \), that is, \( A_\mu = B^\mu \cos \theta + W^3_\mu \sin \theta \), while the neutral intermediate boson is \( \zeta_\mu = -B^\mu \sin \theta + W^3_\mu \cos \theta \). The inverse relations are also well known, \( B^\mu = \cos \theta A_\mu - \sin \theta \zeta_\mu, W^3_\mu = \sin \theta A_\mu + \cos \theta \zeta_\mu \). The generators and isovector can also be written as \( T = (T_+, T_3, T_-) \), \( W_\alpha = (W^{(1)}_\alpha, 1, W^{(2)}_\alpha, 1) / \sqrt{2} \), where \( T_3 = \sigma_z/2 \), \( T_3 = \sigma_z/2 \), and \( W^{(2)}_\mu = \frac{1}{\sqrt{2}} \left( W^1_\mu \mp i W^2_\mu \right) \), and \( \alpha, \sigma_y, \sigma_z \) are the Pauli matrices.

### III. A NONMINIMAL COUPLING IN THE \( U(1)_Y \) SECTOR OF THE GSW MODEL

We have already mentioned how Lorentz-violating terms are inserted in the SME electroweak sector. Another route to consider it involves higher dimensional, nonrenormalizable NM operators. Gauge invariant NM interactions in the electroweak sector can be proposed in the context of the covariant derivative (12). A first possibility, in the \( U(1)_Y \) sector of the GSW model, is the NM derivative

\[ D_\mu = \partial_\mu - ig T \cdot W_\mu - i g'_L Y B^\mu + i g'_L Y B_\mu C_\nu, \]

where \( C_\nu \) is a fixed 4-vector that establishes a preferred direction in spacetime and violates Lorentz symmetry. Replacing such a derivative in Lagrangian (9), the non-minimal coupling yields additional electromagnetic and neutral Lorentz-violating interactions,

\[ \mathcal{L}_{LV(1)} = J^{(lv)}_{EM(LV)} A_\nu + J^{(lv)}_{0(LV)} Z_\nu, \]

given explicitly as

\[ J^{(lv)}_{EM(LV)} = g'_L \cos \theta \left[ \bar{\psi}_v \gamma^\mu (1 - \gamma_5) \psi_v \right] C^\nu \partial_\mu \]
\[ - g'_L \cos \theta \left[ \bar{\psi}_v \gamma^\nu (1 - \gamma_5) \psi_v \right] C^\mu \partial_\mu \]
\[ + g'_L \cos \theta \left[ \bar{\psi}_v \gamma^{(1)} C^\mu \partial_\mu \right] - g' \cos \theta \left[ \bar{\psi}_v \gamma^{(1)} C^\mu \partial_\mu \right], \]

\[ J^{(lv)}_{0, EM} = - g'_L \sin \theta \left[ \bar{\psi}_v \gamma^\mu (1 - \gamma_5) \psi_v \right] C^\nu \partial_\mu \]
\[ + g'_L \sin \theta \left[ \bar{\psi}_v \gamma^\nu (1 - \gamma_5) \psi_v \right] C^\mu \partial_\mu \]
\[ + g' \sin \theta \left[ \bar{\psi}_v \gamma^{(1)} C^\mu \partial_\mu \right] + g_0 \sin \theta \left[ \bar{\psi}_v \gamma^{(1)} C^\mu \partial_\mu \right]. \]

with \( j^{(lv)}_0 = \bar{\psi}_v (x) \gamma^\mu (3 + \gamma_5) \psi_v (x) / 2 \). These expressions are useful to show the processes that are directly affected, at tree-level, by the nonminimal derivative (18).

We now examine the effect of this nonminimal coupling on the decay of the \( Z_0 \) mediator in a pair lepton and antilepton,

\[ Z_0 \rightarrow \ell + \bar{\ell}, \]

evaluating the contributions implied to the decay rate. The total neutral current, \( \left( j^{(lv)}_0 + j^{(lv)}_{0(LV)} \right) Z_\mu \), that contributes for this process is

\[ = - \frac{g}{4 \cos \theta} \bar{\psi}_v (x) \gamma^\mu (g V^\prime - \gamma_5) \psi_v (x) Z_\mu (x) \]
\[ - g'_L \sin \theta \left[ \bar{\psi}_v \gamma^{(1)} C^\mu \partial_\mu \right] + g'_L \sin \theta \left[ \bar{\psi}_v \gamma^{(1)} C^\mu \partial_\mu \right], \]

where the first term is the usual Lorentz invariant contribution, the second and third terms stem from Eq. (21). Expression (23) shows how the NMC (18) affects the vertex of the neutral interaction. The scattering matrix for such a process is

\[ S = - i \int d^4 x \left( j^{(lv)}_0 + j^{(lv)}_{0(LV)} \right) Z_\mu = S_0 + S_{LV(1)} + S_{LV(2)}, \]

where the zero order and first order contributions in the LV parameters are

\[ S_0 = i \frac{g}{4 \cos \theta} \int d^4 x \bar{\psi}_v (x) \gamma^\mu (g V^\prime - \gamma_5) \psi_v (x) Z_\mu (x), \]

\[ S_{LV(1)} = i g'_L \sin \theta \int d^4 x \bar{\psi}_v (x) \gamma^{(1)} C^\mu \partial_\mu Z_\mu (x), \]

\[ S_{LV(2)} = - ig'_L \sin \theta \int d^4 x \bar{\psi}_v (x) \gamma^{(1)} C^\mu \partial_\mu Z_\mu (x). \]

In order to evaluate these elements, we propose plane wave expansions, \( Z_\mu (x) = N_k \bar{\epsilon}_\mu (k, \lambda) \exp (-i k \cdot x), \)
\( \bar{\psi}_v (x) = N_{q, s} u_l (q, s) \exp (-i q \cdot x), \) \( \bar{\psi}_v (x) = N_{q, s} u_v (q', s') \exp (i q' \cdot x) \), where \( k, q, q' \) stand for the 4-momentum of the \( Z_0 \) boson and the emerging leptons, respectively, and \( N_q = (2 V_0 q_0)^{-1/2} \). With these expressions, we obtain

\[ S_0 = i \frac{g}{4 \cos \theta} \left( 2 \pi \right)^4 \frac{\delta^4 (q + q' - k)}{[8 V_0^3 q_0 q_{0,0}]^{1/2}} M_0, \]

\[ S_{LV(1)} = i g'_L \sin \theta \left( 2 \pi \right)^4 \frac{\delta^4 (q + q' - k)}{[8 V_0^3 q_0 q_{0,0}]^{1/2}} M_{LV(1)}, \]

with \( a = 1, 2 \) representing the two LV contributions, which involves

\[ M_0 = \epsilon_\mu (k, \lambda) \bar{u}_l (q, s) \gamma^\mu (g V^\prime - \gamma_5) v (q', s'), \]

\[ M_{LV(1)} = C^\mu k_\epsilon \bar{u}_l (k, \lambda) j^{(lv)}_{0, q}, \]

\[ M_{LV(2)} = -C^\lambda k_\epsilon \bar{u}_l (k, \lambda) j^{(lv)}_{0, q}. \]
and \( j_{\nu} \) = \( \bar{u}_l (q, s) \gamma^\mu (3 + \gamma_5) v(q', s') \). The decay rate for the process (22) is given as usually evaluated, that is,

\[
\Gamma_{ll} = \frac{1}{T} \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3q'}{(2\pi)^3} 3 \sum_s \sum_{s'} |S|^2, \tag{33}\]

where \( S \) is given in (24), implying

\[
|S|^2 = S_0 S_0^\dagger + S_0 S_0^{LV(1)} + S_{LV(1)} S_0^\dagger + S_{LV(2)} S_0^\dagger, \tag{34}\]

in first order in the LV parameters. Substituting Eq. (34) in Eq. (33), we achieve

\[
\Gamma_{ll} = \Gamma_{S_0 S_0^\dagger} + \Gamma_{S_0 S_0^{LV(1)}} + \Gamma_{S_{LV(1)} S_0^\dagger} + \Gamma_{S_{LV(2)} S_0^\dagger}, \tag{35}\]

The first term, \( \Gamma_{S_0 S_0^\dagger} \), is the decay rate for the Lorentz invariant usual process \( Z_0 \to l + l \). In this evaluation, \( \Gamma_{S_0 S_0^{LV(1)}} = 0, \Gamma_{S_{LV(1)} S_0^\dagger} = 0 \), as a consequence of the current conservation, due to the presence of the momentum \( k_i \) in Eqs. (31, 32). The LV contribution is associated with \( \Gamma_{S_0 S_0^{LV(2)}} \) and \( \Gamma_{S_{LV(2)} S_0^\dagger} \), so that the total decay rate, \( \Gamma = \Gamma_{S_0 S_0^\dagger} + \Gamma_{S_0 S_0^{LV(2)}} + \Gamma_{S_{LV(2)} S_0^\dagger} \), is

\[
\Gamma_{ll} = \frac{g^2}{1536 \pi \cos^2 \theta} 8 M_Z \left[ (g_2^2 + 1) - 6g_0^2 \frac{m^2}{M_Z^2} \right] \nonumber
- \frac{g_0^2 \tan \theta}{768 \pi} 8 M_Z (C \cdot k) \left[ (3g_Y^2 - 2) - 27g_Y^2 \frac{m^2}{M_Z^2} \right] \nonumber
\times \Theta (M_Z - 2m_l). \tag{36}\]

We now use \( k^2 = M_Z^2 \) and \( C \cdot k = C_0 M_Z \). As the \( Z_0 \) mass \( (M_Z = 9.1 \times 10^{10} \text{eV}) \) is much larger than lepton masses, we can neglect the mass ratios for the electron, muon and tau \( (m_e^2/M_Z^2 \approx 2 \times 10^{-11}, m_\mu^2/M_Z^2 \approx 10^{-4}, m_\tau^2/M_Z^2 \approx 4 \times 10^{-4}) \), which are smaller than the experimental uncertainty in decay rate measurements. Thus, the result is written as

\[
\Gamma_{ll} = \frac{g^2}{192 \pi \cos^2 \theta} \left[ 1 - 8 \times |g_2^2 C_0| M_Z \right] \nonumber
\times \Theta (M_Z - 2m_l), \tag{37}\]

with the LV contribution appearing as a direct correction to the usual decay rate. We have used \( g = e/\sin \theta, \ g_Y^2 = 1 - 4 \sin^2 \theta, \ \sin^2 \theta = 0.23.\) In accordance with Ref. [35], the \( Z_0 \) decay rate (considering lepton universality) is \( \Gamma_{ll} = (83.985 \pm 0.086) \text{MeV}, \) or \( \Gamma_{ll} = 83.985 (1 \pm 0.001) \text{MeV}, \) so that the experimental uncertainty is of 1 part in \( 10^3.\) We thus impose \( |g_2^2 C_0| M_Z < 1.0 \times 10^{-3}, \) which leads to the following upper bound:

\[
|g_2^2 C_0| < 1.3 \times 10^{-15} \text{ (eV)}^{-1}. \tag{38}\]

**IV. A NONMINIMAL COUPLING IN THE SU(2)_L SECTOR OF THE GSW MODEL**

Analogously to the previous case, a gauge invariant nonminimal interaction in the SU(2)_L sector of the GSW model can be proposed as

\[
D_\mu = \partial_\mu - igT \cdot W_\mu - i \frac{g'}{2} Y B_\mu + ig_3 (T \cdot W_\mu) V^\nu. \tag{39}\]

where \( V^\nu \) is a fixed 4-vector that establishes a preferred direction in spacetime and violates Lorentz symmetry. The interaction term, \( L_{\mu} g^{\nu i} (ig_3 T \cdot W_\mu V^\nu) L_i \) embraces the following interactions at tree-level, \( L_{LV(2)} = J_{\mu}^{(LV)} W_\mu^{(+)} + J_{\mu}^{(LV)} W_\mu^{(-)} + J_{\mu}^{(LV)} Z_\nu, \) involving the vector bosons, in which the related currents read

\[
J_{\mu}^{(LV)} = -\frac{g_3}{2\sqrt{2}} \bar{\psi}_l \gamma^\mu (1 - \gamma_5) \psi_l V^\nu \partial_\mu, \tag{40}\]

\[
J_{\mu}^{(LV)} = -\frac{g_3}{2\sqrt{2}} \bar{\psi}_l \gamma^\mu (1 - \gamma_5) \psi_l V^\nu \partial_\mu, \tag{41}\]

\[
J_{\mu}^{(LV)} = -\frac{g_3 \cos \theta}{4} \left[ \bar{\psi}_l \gamma^\mu (1 - \gamma_5) \psi_l V^\nu \partial_\mu - \bar{\psi}_l \gamma^\mu (1 - \gamma_5) \psi_l V^\nu \partial_\mu \right] \nonumber
+ \bar{\psi}_l \gamma^\nu (1 - \gamma_5) \psi_l V^\mu \partial_\mu. \tag{42}\]

The current, \( J_{\mu}^{(LV)} \), given by Eq. (41), affects the processes mediated by the \( W^- \) particle, including the decay \( W^- \to l + \bar{\nu}_l \). The total electroweak current that contributes to this process is

\[
\left( J_{\mu}^{(LV)} + J_{\mu}^{(LV)} \right) W^\mu_{\mu} = \frac{1}{2\sqrt{2}} \left[ g_2^2 W^\mu_{\mu} (x) \right. \nonumber
- \frac{g_3}{2\sqrt{2}} \bar{\psi}_l \gamma^\mu (1 - \gamma_5) \psi_l V^\nu \partial_\nu W^\mu_{\mu} (x) \left. \right], \tag{43}\]

where \( j_{\mu} \) is the first term is the usual Lorentz invariant contribution. The scattering matrix for the process \( (W^- \to l + \bar{\nu}_l) \), at leading order, can be written as

\[
S = -i \int d^4x \left( J_{\mu}^{(LV)} + J_{\mu}^{(LV)} W^\mu_{\mu} \right), \tag{44}\]

that implies \( S = S_0 + S_{LV(1)} + S_{LV(2)}, \) with

\[
S_0 = -i \frac{g}{2\sqrt{2}} \int d^4x \left[ j_{\mu}^0 (x) W^\mu_{\mu} (x) \right], \tag{45}\]

\[
S_{LV(1)} = i \frac{g_3}{2\sqrt{2}} \int d^4x \left[ j_{\mu}^0 (x) V^\mu \partial_\mu W^\mu_{\mu} (x) \right], \tag{46}\]

\[
S_{LV(2)} = -i \frac{g_3}{2\sqrt{2}} \int d^4x \left[ j_{\mu}^0 (x) V^\lambda \partial_\lambda W^\mu_{\mu} (x) \right]. \tag{47}\]
Following the same steps of previous calculation, we obtain the decay rate for the usual Lorentz invariant process \((W^- \rightarrow l + \nu_l)\):

\[
\Gamma_{s_0 s_1 l} = \frac{g^2}{48\pi} M_W \left(1 - \frac{m_l^2}{M_W^2}\right)^2 \left(1 + \frac{m_l^2}{2M_W^2}\right) \Theta(M_W - m_l),
\]

where \(M_W, m_l\) label the \(W^-\) boson and lepton masses. As it occurs in the previous case, the quantities \(\Gamma_{s_0 s_1 l}^{(a)}\), \(\Gamma_{SLV(l)^{a}}\) also vanish, \(\Gamma_{s_0 s_1 l}^{(1)} = 0, \Gamma_{SLV(l)^{a}} = 0\). The terms, \(\Gamma_{s_0 s_1 l}^{(2)}\), \(\Gamma_{SLV(l)^2} = 0\), are computed, leading to the following decay rate:

\[
\Gamma = \frac{g^2}{48\pi} M_W \left(1 + (g_3'V_0) \frac{5M_W}{4g}\right) \Theta(M_W - m_l).
\]

where \(V \cdot k = V_0 M_W\) for the rest frame of the \(W^-\) mediator, and we have neglected the contributions in \(m_l^2/M_W^2, m_l^4/M_W^4\). This result can be also expressed as

\[
\Gamma = \frac{g^2}{48\pi} M_W \left(1 + (g_3'V_0) \frac{5M_W}{4g}\right) \Theta(M_W - m_l).
\]

Considering that the experimental uncertainty in the measures of this decay is at the level of \(\sim \Gamma\), as protocol to the current (the GSW theory. The NM interaction terms we found at \(g \neq 0\), \(M \neq 0\), \(\nu \neq 0\) as the current (50) frame [\(a\) way that they undergo sidereal variations in the Earth frame [31, 34]). It is necessary, therefore, to translate the bounds from the Earth-located Lab’s RF at the colatitude \(\chi\), rotating around the Earth’s axis with angular velocity \(\Omega\), to the Sun’s frame. For experiments up to a few weeks long, the transformation law for a rank-1 tensor, \(A_{\mu\nu}\), is merely a spatial rotation, \(A_{\mu\nu}^T = R_{\mu\nu} A_{\mu\nu}\), where the label \(T\) indicates the quantity measured in the Sun’s frame, and \(R_{01} = R_{00} = 0\) and \(R_{00} = 1\). Thus, four vector time-components are not modified, \(A_0^T = A_0\), so that the upper bounds (38), (51) could be equally written in the Sun’s frame. However, the situation is not so simple, as pointed out in Ref. [12] (for pion decays), once the decay rates (37), (50) were carried out in the rest frame of the decaying vector bosons, not in the Lab (Earth) frame, where the measurements are performed. In order to take into account this point, one option is to translate the upper bounds (38), associated with an evaluation at the vector boson rest frame, directly to the Sun’s frame, with the boost

\[
C^0 = \gamma_z (C_T^0 + \alpha^i C_T^i),
\]

where \(\gamma_z = \gamma(v_z)\) is the Lorentz factor, \(v_z\) is the boson velocity in the Sun’s frame, \(\alpha^i = v_z^i/c\). The data about width decays were attained in the LEP accelerator [35], constructed to work with centre-of-mass energy around 91 GeV, reaching 161 GeV in 2000. As the \(Z_0\) mass is close to the centre-of-mass energy, it happens that the Lorentz factor is nearly 1 (\(\gamma_z \simeq 1\)), and not larger than 2, which also implies a not (meaningful) relativistic velocity (\(v_z^i\)). In this case, the upper bounds (38), (51) can be read in the Sun’s frame as

\[
|g_3' (C_T^0 + \alpha^i C_T^i)| \lesssim 1 \times 10^{-15} \ (eV)^{-1},
\]

\[
|g_3' (V_T^0 + \alpha^i V_T^i)| \lesssim 1 \times 10^{-14} \ (eV)^{-1}.
\]

If the case the centre-of-mass energy is really close to the boson mass (\(\gamma_z \simeq 1\)), these bounds simplify to \(|g_3' C_T^0| < 1 \times 10^{-15} \ (eV)^{-1}\) and \(|g_3' V_T^0| < 1 \times 10^{-14} \ (eV)^{-1}\). Another possibility is to write the results (37), (50) in the Lab frame, in which \(C \cdot k = M(C_0 - \gamma_z \alpha^i C^i), \ V \cdot k = M(V_0 - \gamma_z \alpha^i V^i)\), procedure that recovers the bounds (53), (54) for \(\gamma_z \simeq 1\).

Finally, other impacts of these NMC can be investigated, with attention to the possible evaluation of differential decay rates of polarized processes, which could, in principle, yield improved upper bounds by two orders of magnitude.

**Acknowledgments**

The authors are grateful to J.A. Helayel-Neto and M. Schreck for useful comments. They also acknowledge CNPq, CAPES and FAPEMA (Brazilian research agencies) for invaluable financial support.
