Accelerated Stochastic Gradient for Nonnegative Tensor Completion and Parallel Implementation

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Abstract—We consider the problem of nonnegative tensor completion. We adopt the alternating optimization framework and solve each nonnegative matrix completion problem via a stochastic variation of the accelerated gradient algorithm. We experimentally test the effectiveness and the efficiency of our algorithm using both real-world and synthetic data. We develop a shared-memory implementation of our algorithm using the multi-threaded API OpenMP, which attains significant speedup. We believe that our approach is a very competitive candidate for the solution of very large nonnegative tensor completion problems.

Index Terms—tensors, stochastic gradient, nonnegative tensor completion, optimal first-order optimization algorithms, parallel algorithms, OpenMP.

I. INTRODUCTION

Tensors have recently gained great popularity due to their ability to model multiway data dependencies [1], [2], [3], [4]. Tensor decomposition (TD) into latent factors is very important for numerous tasks, such as feature selection, dimensionality reduction, compression, data visualization and interpretation. The Canonical Polyadic Decomposition (CPD) is one of the most important tensor decomposition models. Tensor Completion (TC) arises in many modern applications such as machine learning, signal processing, and scientific computing.

We focus on the CPD model and consider the nonnegative tensor completion (NTC) problem, using as quality metric the Frobenius norm of the difference between the true and the estimated tensor. We adopt the Alternating Optimization (AO) framework, that is, we work in a circular manner and update each factor by keeping all other factors fixed. We update each factor by solving a nonnegative matrix completion (NMC) problem via a stochastic variant of the accelerated (Nesterov-type) gradient [5].

Recent tensor applications, such as social network analysis, recommendation systems, and targeted advertising, need to handle very large sparse tensors. We propose a shared-memory implementation of our algorithm that attains significant speedup and can efficiently handle very large problems.

A. Related Work

Most of the papers that consider sparse TD and TC focus on unconstrained problems. One of the earliest works is Gigatensor [6], which was followed by DFacTo [7]. In [8], two parallel algorithms for the unconstrained TC have been developed and results concerning the speedup attained by their MPI implementations on a linear processor array have been reported. In [9], the authors introduce fine- and medium-grained partitionings for the TC problem, while [10] incorporates dimension trees into the developed parallel algorithms. In [11] and [12], the AO-ADMM framework has been adopted for constrained matrix/tensor factorization and completion. In [13], the medium-grained approach of [9] was used for the solution of the nonnegative TD problem in distributed memory systems. The same problem has been considered in [14], where the authors incorporate the dimension trees and observe performance gains due to reduced computational load. The works in [15], [16], and [17] make use of either the MapReduce programming model or the Spark engine. In [18], a hypergraph model for general medium–grain partitioning has been presented.

Works that employ Stochastic Gradient Descent (SGD) on shared memory and distributed systems for sparse tensor factorization and completion include [19], [20], [21], [22]. In [19], the authors describe a TC approach which uses the CPD model and employs a proximal SGD algorithm, that can be implemented in a distributed environment. In [20], the authors examine three popular optimization algorithms:
alternating least squares (ALS), SGD, and coordinate descent (CCD++), implemented on shared- and distributed-memory systems. They conclude that SGD is most competitive in a serial environment, ALS is recommended for shared-memory systems, and both ALS and CCD++ are competitive on distributed systems. In [21], the authors propose a GPU-accelerated parallel TC scheme (GPU-TC) for accurate and fast recovery of missing data via SGD. Finally, [22] presents GentenMPI, a toolkit for sparse CPD that is designed to run effectively on distributed-memory high-performance computers. The authors use the Trilinos libraries and they present implementations of the CPD-ALS and an SGD method.

B. Notation

Vectors, matrices, and tensors are denoted by small, capital, and calligraphic capital letters, respectively; for example, $x$, $X$, and $\mathcal{X}$. $\mathbb{R}^{I_1 \times \cdots \times I_N}$ denotes the set of $(I_1 \times \cdots \times I_N)$ nonnegative tensors. The elements of tensor $\mathcal{X}$ are denoted as $\mathcal{X}(i_1, \ldots, i_N)$. In many cases, we use Matlab-like notation, for example, $A(j,:) \cdot \cdot \cdot$ denotes the $j$-th row of matrix $A$. The outer product of vectors $a$ and $b$ is defined as $a \circ b$. The Kronecker, Khatri-Rao, and Hadamard product of matrices $A$ and $B$, of compatible dimensions, are defined, respectively, as $A \otimes B$, $A \odot B$ and $A \boxplus B$; extensions to the cases with more than two arguments are obvious. $I_p$ denotes the $(P \times P)$ identity matrix, $\| \cdot \|_F$ denotes the Frobenius norm of the matrix or tensor argument, and $(X)_+$ denotes the matrix derived after the projection of the elements of $X$ onto $\mathbb{R}_+$.

II. NONNEGATIVE TENSOR COMPLETION

Let $\mathcal{X}^o \in \mathbb{R}^{I_1 \times \cdots \times I_N}_+$ be an $N$-th order tensor which admits the rank-$R$ CPD [3], [4]

$$\mathcal{X}^o = \sum_{r=1}^{R} u_r^{(1)} \circ \cdots \circ u_r^{(N)},$$

where $u_r^{(i)} \in \mathbb{R}^{I_i \times \cdots \times I_i}$. $\mathcal{X}^o$ is a rank-$R$ CPD of the matrix $X^o$. For row $i$ of matrix $X^o$, we sample $X(i)$, and $\mathcal{X}(i)$, is denoted as a tensor with $\mathcal{X}(i) = \mathcal{X}(i_1, i_2, \ldots, i_N)$ for $i = 1, \ldots, N$. We observe $\mathcal{X} = \mathcal{X}^o + \mathcal{E}$, where $\mathcal{E}$ is additive noise. Let $\Omega \subseteq \{1, \ldots, I_1\} \times \cdots \times \{1, \ldots, I_N\}$ be the set of indices of the observed entries of $\mathcal{X}$. Also, let $\mathcal{M}$ be a tensor with the same size as $\mathcal{X}$, with elements $\mathcal{M}(i_1, i_2, \ldots, i_N)$ equal to one or zero based on the availability of the corresponding element of $\mathcal{X}$. That is

$$\mathcal{M}(i_1, i_2, \ldots, i_N) = \begin{cases} 1, & \text{if } (i_1, i_2, \ldots, i_N) \in \Omega, \\ 0, & \text{otherwise}. \end{cases}$$

We consider the NTC problem

$$\min_{\{u(r) \in \mathbb{R}^{I_1 \times \cdots \times I_N}: r=1, \ldots, N\}} \sum_{i=1}^{N} \frac{\lambda}{2} \left\| u^{(i)} \right\|_F^2 + \frac{1}{2} \left\| \mathcal{X} - \sum_{r=1}^{R} u_r^{(1)} \circ \cdots \circ u_r^{(N)} \right\|_F^2.$$
The computation of each of the matrices $Y$ follows:

1. the computation of $W_l(p,:)$ requires $O(B_l R)$ arithmetic operations (in total, $O(B_l R)$);
2. the computation of $Z_l(p,:)$ requires $O(B_l R)$ arithmetic operations (in total, $O(B_l R)$);
3. the computation of $H_l,p$ requires $O(B_l R^2)$ arithmetic operations (in total, $O(B_l R^2)$);
4. the computation of $L_{p}$, via the power method, requires $O(R^2)$ arithmetic operations (in total, $O(PR^2)$).

The computation of each of the matrices $\nabla f_{\Omega_l}, A_{i+1}$ and $Y_{i+1}$ requires $O(PR)$ arithmetic operations.

For notational convenience, we denote Algorithm 1 as

\[ A_{\text{opt}} = S_{\text{NMC}}(X, M, B, A_*, \lambda) \]

We note that, in this work, $H_{l,p}$ is used only for the determination of $L_{p}$. A very interesting topic is the development of algorithms that fully exploit $H_{l,p}$. Initial efforts with a "projected Newton step" have not led to algorithms superior to the one presented in this paper, especially in the noisy cases. A related important topic is the development of more efficient methods for the estimation of $L_{p}$.

III. AO ACCELERATED STOCHASTIC NTC

In order to solve the NTC problem using our accelerated stochastic algorithm, we start from initial values $U_{0}^{(i)}$, $\ldots$, $U_{0}^{(N)}$ and solve, in a circular manner, NMC problems, based on the previous estimates. We define

\[ K_{k}^{(i)} = \left( U_{k}^{(1)} \otimes \cdots \otimes U_{k}^{(i+1)} \otimes U_{k+1}^{(i-1)} \otimes \cdots \otimes U_{k+1}^{(1)} \right), \]

where $k$ denotes the $k$-th AO iteration. The update of $U_{k}^{(i)}$ is attained by the function call $S_{\text{NMC}}(X_{(i)}, M_{(i)}, K_{k}^{(i)}, U_{k}^{(i)}, \lambda)$. The Stochastic NTC algorithm appears in Algorithm 2.

A. Parallel Implementation of AO accelerated stochastic NMC

In Algorithm 3 we provide a high level algorithmic sketch of the accelerated stochastic gradient for NMC. We employ the OpenMP API, which is suitable for our multi-threading approach. The update of each row can be done independently. Therefore, lines 5-13 of Algorithm 1 can be computed separately by each available thread.

IV. NUMERICAL EXPERIMENTS

In this section, we test the effectiveness of our algorithm in various test cases, using both real-world and synthetic data. We denote as epoch the number of iterations required to access all available tensor elements. In subsections IV-B and IV-C we compute averages over 5 Monte Carlo trials.
A. Stochastic NTC on corrupted image

We start by estimating missing data in images following the RGB model. In Fig. 1, we depict the results obtained from the application of the AO Stochastic NTC on a corrupted image of dimensions $1063 \times 1599 \times 3$ (90% sparse). We set $c = 0.02$, number of epochs 500, MAX_INNER = 1, and rank $R = 50$. We observe that the algorithm is able to reconstruct the image even for small values of $c$.

B. Convergence speed of Stochastic NTC

We experimentally evaluate the convergence speed of our algorithm using both real-world and synthetic data. Concerning the real-world data, we use the dataset “MovieLens 10M”[23] of dimensions $71567 \times 65133 \times 730$. Concerning the synthetic data, we generate a rank-10 nonnegative tensor $X^o$, of size equal to the real-world data, whose true latent factors have independent and identically distributed (i.i.d.) elements, drawn from $\mathcal{U}[0,1]$. The additive noise $E$ has i.i.d. elements $\mathcal{N}(0, \sigma^2_N)$. In order to create a tensor of the same sparsity level as the real-world dataset, we generate a tensor $M$ of the same size as $X^o$ according to (2). The observed incomplete tensor is expressed as $M \odot (X^o + E)$. We define the Signal-to-Noise ratio as

$$\text{SNR} := \frac{\|X^o\|_F^2}{\|X^o \odot E\|_F^2}. \quad (11)$$

In Fig. 2 we plot the average Relative Reconstruction Error for 100 epochs, MAX_INNER = 1 and rank $R = 10$. We observe that, in noisy environments, choosing small values of $c$ is not effective. On the contrary, in the high SNR cases, small values of $c$ are more suitable.

C. Execution time for parallel stochastic NTC

We assess the performance of our algorithm in a shared-memory environment using both real-world and synthetic data (of the same dimensions). Concerning the real-world data, we use the dataset “Uber Pickups”[24], which can be represented as a 4-th order tensor of dimensions $183 \times 24 \times 1140 \times 1717$ and $c = 0.5$, for various values of rank $R$, versus number of threads.

1Image can be found at https://images.freeimages.com/images/large-previews/7bb/building-1222550.jpg
Fig. 4: Speedup attained for synthetic (S) and real-world (T) dataset with dimensions $183 \times 24 \times 1140 \times 1717$ and $c = 0.5$, for various values of rank $R$, versus number of threads.

a rank-10 tensor $\mathbf{X}^o$, whose latent factors have i.i.d elements $\mathcal{U}[0,1]$. Similarly, we generate a tensor $\mathbf{M}$ of the same size with $\mathbf{X}^o$. Thus, the observed incomplete and noiseless tensor is $\mathbf{M} \odot \mathbf{X}^o$ with the same number of nonzeros as in the “Uber Pickups” dataset. We set $c = 0.5$, number of epochs $1$, MAX INNER $= 1$. In order to test the algorithm’s performance, we examine various values of the rank $R = 10, 30, 50$. In Fig. 4, we present the average execution time per epoch versus the number of threads. In Fig. 4, we present the average attained speedup. We observe that significant speedup is attained in all cases. We also observe that the speedup attained with synthetic data is somewhat higher than that attained with real-world data. This happens due to load imbalancing, caused by the nonuniform distribution of the nonzero elements of the real-world data, among the available threads.

V. CONCLUSION

We considered the NTC problem. First, we developed an accelerated stochastic algorithm for the NMC problem. A unique feature of our approach is that each row of the matrix variable is updated using a different step-size, specifically tailored to this row. Then, we used this algorithm and built an AO algorithm for the NTC problem. We tested the data reconstruction effectiveness as well as the convergence speed of our approach using both synthetic and real-world data. We implemented our algorithm using the OpenMP API, and observed significant speedup. Our method is an effective and efficient candidate for the solution of very large-scale NTC problems.

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