OSNR and dispersion tolerance of FWM based optical carrier recovery scheme

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Keywords: four wave mixing, optical carrier recovery, optical fiber communication, linewidth, signal to noise ratio, binary phase shift keying

Abstract
In this paper we demonstrate the Optical signal-to-noise ratio (OSNR) and Dispersion characteristics of Four-Wave Mixing (FWM) based optical carrier recovery scheme for a 10-Gb/s nonreturn-to-zero binary phase-shift-keyed input. The performance was evaluated by measuring the linewidth of the recovered carrier against different OSNR and accumulated dispersion values and recording the eye diagrams. The system characterization will enable new avenues of using the FWM based carrier recovery system in phase-sensitive amplification and heterodyne detection.

1. Introduction

Over the years, optical fiber network has been developed in such a way that the leading internet service providers are replacing their underlying copper cable networks with optical fiber. But the recent surveys show that existing bandwidth is not sufficient to overcome the end-user requirements [1]. The transport networks are upgrading its capacity continuously to provide the bandwidth requirement. As a result, the network capacities have moved from 40 Gbps to 100 Gbps data rate per single channel [2]. While increasing the data rate, the quality of the optical communication system should be preserved. One of the proven method is to change the modulation scheme from amplitude shift keying (ASK) to phase shift keying (PSK) [3]. The signal processing in ASK scheme will be no longer effective because, the currently developed optical communication systems are operating based on PSK scheme. Even though phase modulation is a better option for achieving high data rates, signal regeneration in the optical domain has become a challenge. The phase synchronization is a major requirement to achieve accurate and efficient signal reception and regeneration. Hence, it is necessary to extract the original carrier phase information from the incoming phase modulated signal for homodyne receiving process or signal regeneration and synchronization. However, in most of the PSK modulation scheme, a strong carrier is not available. Therefore, various optical carrier recovery methods have been experimented [4–6]. The optical carrier recovery method using four-wave mixing (FWM) in a highly nonlinear fiber (HNLF) was experimentally demonstrated [7]. However, in most of the demonstrated schemes have not done detail characterization.

In this research, it is expected to characterize the extracted recovered carrier with respect to the signal-to-noise ratio degradation and tolerance for accumulated dispersion of the incoming signal. This will help in process of homodyne receiving, signal regeneration, synchronization, format conversion, etc.

2. Principle of operation

In this research, FWM is used as a phase information extraction technique from an incoming signal. Here, we use the degenerate FWM in a HNLF. The noise influences from the signal laser for the recovered carrier was investigated. For that the linewidth of a recovered carrier was analyzed with various levels of optical signal to noise ratio (OSNR) and accumulated dispersion of a input modulated signal. Linewidth of a laser is often defined in terms of the Full Width at Half Maximum (FWHM) of the optical field power spectrum. Delayed self-heterodyne method is known to be one of the most sensitive method to analyze the laser linewidth. The FWHM
linewidth of a laser is often measured with respect to the assumed Lorentzian spectral shape. The Lorentzian optical power spectrum \( S(f) \) centered at \( f_c \) has a functional form given by \([8]\),

\[
S(f) = \frac{1}{1 + \left( \frac{f - f_c}{\Delta f / 2} \right)^2}
\]  

(1)

The Linewidth broadening of the recovered optical carrier due to the OSNR and dispersion of an incoming signal is investigated theoretically and experimentally.

As shown in figure 1, this experiment consists of 3 steps. These 3 steps were analyzed separately by directly connecting the laser signal as input and measuring the linewidth using delayed self-heterodyning linewidth measuring setup.

2.1. Linewidth broadening due to the modulation

The analysis starts with the optical field equation of upper and lower arms of the interferometer (figure 2). At the photodetector, which is assumed to have unity responsivity, after \( \tau \) delay time the output current produced is given as,

\[
I(t, \tau) = 2E_0^2 + E_0^2 e^{j(\Phi(t) - \omega_0(\tau) - \Phi(t + \tau)) e^{j\Omega_{RF} t} \sin(\Omega_{RF} t)} + E_0^2 e^{-j(\Phi(t) - \omega_0(\tau) - \Phi(t + \tau)) e^{j\Omega_{RF} t} \sin(-\Omega_{RF} t)}
\]  

(2)

Using the Bessel function \([9]\), rearranging the current produce by photo detector,

\[
I(t, \tau) = 2E_0^2 \left[ 1 + \sum_{m=\infty}^{\infty} J_m(D) \cos(\Phi(t) - \omega_0(\tau) - \Phi(t + \tau) + m\Omega_{RF} t) \right]
\]  

(3)

Let,

\[
\Phi(t) - \omega_0(\tau) - \Phi(t + \tau) = A
\]  

(4)

by substituting,

\[
I(t, \tau) = 2E_0^2 \left[ 1 + \sum_{m=\infty}^{\infty} J_m(D) \cos(A + m\Omega_{RF} t) \right]
\]  

(5)

Spectral density of the signal is given by taking the Fourier transformation of the output current in equation (5), Therefore, spectral density of the laser,
\[ S(f_0) = 2E_0^2 \left[ \sqrt{2\pi} \delta(f) + \sum_{m=-\infty}^{\infty} J_m(D) \left[ \frac{\pi}{\sqrt{2}} (e^{-i\delta f} + e^{i\delta f}) + \frac{D}{2} \frac{\pi}{\sqrt{2}} (e^{-i\delta f} - \Omega_{RF} + e^{i\delta f} + \Omega_{RF}) \right] \right] \]

where, \( \delta \) is the Dirac delta function. Here the zeroth and first kind Bessel functions are considered. For \( m = 0 \), \( J_0(D) = 1 \) and for \( m = 1, J_1(D) = D/2 \).

by substituting on equation (6),

\[ S(f_0) = 2E_0^2 \left[ \sqrt{2\pi} \delta(f) + \sqrt{\pi/2} J_0(D) \left[ \delta(f - \Omega_{RF}) + e^{iA_{R}\Omega_{RF}} \delta(f + \Omega_{RF}) \right] \right] \]

If the input optical field to the delayed self-heterodyne is a phase modulated signal, the optical field after phase modulation can be written as,

\[ E_{PM}(t) = E_0 e^{i\phi(t) + \Phi(t) + D \sin(\Omega_{RF} t)} \]

where, \( \Omega_{RF} \) is the phase noise of the phase modulator.

Output current of the photodetector (in the delayed self-heterodyne) corresponding to the phase modulated signal,

\[ I(t, \tau) = 2E_0^2 \left[ 1 + \sum_{m=-\infty}^{\infty} J_m(D) \cos(A + m\Omega_{RF} t - m\Omega_{RF} \tau) \right] \]

Spectral density of the signal is the Fourier transformation of the output current,

\[ S(f) = 2E_0^2 \left[ \sqrt{2\pi} \delta(f) + \sum_{m=-\infty}^{\infty} J_m(D) \left[ \sqrt{\pi/2} (e^{-i(A-m\Omega_{RF})}) \times \delta(f - m\Omega_{RF}) + e^{iA_{R}-m\Omega_{RF}} \delta(f + m\Omega_{RF}) \right] \right] \]

Similarly, lets consider only the zeroth and first kind of the Bessel functions for \( m = 0 \), \( J_0(D) = 1 \) and for \( m = 1, J_1(D) = D/2 \).

by substituting to equation (10),

\[ S(f) = 2E_0^2 \left[ \sqrt{2\pi} \delta(f) + \sqrt{\pi/2} (e^{-i\delta f} + e^{i\delta f}) + D/2 \sqrt{\pi/2} (e^{-i\Omega_{RF}} + e^{i\Omega_{RF}}) \right] \]

From equation (7) and equation (11),

Spectral density of phase modulated signal,

\[ S(f) = S(f_0) - D/2 \sqrt{\pi/2} \left[ e^{-i\delta f} (1 - e^{i\Omega_{RF}}) + e^{i\delta f} (1 - e^{-i\Omega_{RF}}) \right] \]

According to the above equation, the signal strength shifts to the modified Lorentzian pedestal from the delta function peak with the increase of a delay time. This happens until the power spectrum becomes exactly Lorentzian [8, 10].

Therefore, using equation (1) the linewidth at FWHM of the phase modulated signal can be written as,

\[ \Delta \nu = \frac{2(f - f_0)}{\sqrt{(1/S(f) - 1)}} \]

### 2.2. Linewidth broadening due to amplification

An EDFA is the most commonly used optical amplifier to compensate the loss of an optical fiber in long-distance optical communication. In an cascaded amplifier optical communication system, the spontaneous emission from optical amplifiers are accumulated and added to the transmitted signal. It deteriorates the signal to noise ratio (SNR) of the signal. The amplified spontaneous emission (ASE) noise is generated in the optical amplifier and it is considered as one of the main source of noise that limits the performance of the phase modulated transmission system.

The ASE noise field can be expressed in terms of in-phase and quadrature-phase noise components. The in-phase component becomes amplitude modulated (AM) noise of the signal and quadrature component serves as phase modulated (PM) noise. Such PM noise directly increases the spectral linewidth of the signal and AM noise is converted into PM noise due to the Kerr nonlinearity of the optical fiber. It also gives rise to spectral linewidth broadening of the signal.

The linewidth broadening due to ASE arises from optical amplification is given by, [11]

\[ \Delta \nu = (n_p h \nu B_0^2 (G - 1)) / (\alpha \pi G p_m) \]

where, \( n_p \) is a spontaneous emission factor, \( h \) is a planck’s constant, \( \nu \) is a frequency of signal, \( G \) and \( B \) are the gain and the bandwidth of the amplifier.
2.3. Linewidth broadening due to Four Wave Mixing (FWM)

The degenerate FWM process generates two idlers \([12–14]\) with phase \(\phi_{ix}\), frequency \(\omega_{ix}\), and amplitude \(E_{ix}\) relationship as shown below \([15–17]\):

\[
E_{i1} = E_s^2 \cdot E_p^* \cdot e^{i(2\omega_s - \omega_p) t - (2\Phi_s - \Phi_p)}
\]

\[
E_{i2} = E_s^2 \cdot E_p^* \cdot e^{i(2\omega_s - \omega_p) t - (2\Phi_s - \Phi_p)}
\]

The frequency and phase of new idlers closer to the signal and pump can be written as below:

\[
\omega_{i1} = 2\omega_s - \omega_p
\]

\[
\omega_{i2} = 2\omega_p - \omega_s
\]

\[
\Phi_{i1} = 2\Phi_s - \Phi_p
\]

\[
\Phi_{i2} = 2\Phi_p - \Phi_s
\]

Linewidth of the idler of our interest \([18]\),

\[
\Delta\nu_i = 2\Delta\nu_s - \Delta_p
\]

where, the \(\Delta\nu_s\) and \(\Delta_p\) are signal and pump linewidth respectively.

3. Simulation setup

The simulation setup for carrier information extraction from a BPSK modulated signal using FWM in a highly non-linear fiber is shown in figure 3 [7].

The BPSK modulated signal was produced by using a random incoming data sequence. A 10 Gbit/s BPSK signal (193.1 THz) was generated with a DFB laser (\(\text{LW} = 100\ \text{MHz}\)) and MZM modulator driven by a pseudo random bit sequence (PRBS) electrical signal from a pattern generator with a 1024 (PRBS 10) pattern length. This BPSK signal and a continuous wave laser pump signal (193.3 THz) was coupled, amplified and launched into a nonlinear media to participate in degenerate FWM process. The nonlinear media is a HNLF with a zero-dispersion wavelength (ZDW) of 1554 nm, a nonlinear coefficient of 10 (W/km), a dispersion slope of 0.08 ps/nm/km², an effective area of 12 \(\mu\text{m}^2\), and a fiber loss of 0.2 dB/km at 1550 nm [16]. The optical spectra after FWM process were observed using optical signal analyzer (OSA).

To set the OSNR to the system, a variable optical attenuator and an Erbium Doped Fiber Amplifier (EDFA) placed after the BPSK transmitter. Then the total optical power was restored to a constant level using a second EDFA and a variable attenuator. This signal was coupled with a continuous wave laser pump and launched into a HNLF to participate in degenerate FWM process. The polarization controllers in front of the signal and the pump were used to achieve a maximum efficiency for FWM process. Recovered idler was filtered out using a band-pass filter and the FWHM linewidth was measured using a delayed self-heterodyne technique [7, 17].

To characterize the system with dispersion the OSNR setup was replaced. To set the dispersion, variable lengths of fiber was placed after the BPSK transmitter to vary the dispersion at the input of the FWM setup [17]. The dispersed data signal was coupled with a continuous wave laser pump signal and launched into a HNLF to participate in degenerate FWM process. Similar to the OSNR setup, recovered idler was filtered using a band-pass filter and the FWHM linewidth was measured.
4. Result

The output spectrum of the highly nonlinear fiber after the FWM process is shown in figure 4. Output spectrum illustrate the newly generated idlers by confirming the FWM process. The recovered carrier at \((2\,fs - fp)\) was filtered using a 60 GHz Gaussian type filter [17].

Also figure 4 shows an enlarged image of idler 1 and it further confirmed that the modulation was stripped off from the incoming data signal and the phase information of the data laser was transferred to the idler 1.

4.1. System characterization for degrading OSNR

After successful carrier recovery, further experiments were carried out to confirm the OSNR tolerance of the system figure 3. The intended measurement were to degrade the input OSNR of the BPSK signal to the carrier recovery system. Figure 5 shows the captured eye diagram of the BPSK modulated signal for different level of OSNR. As shown in the figure 6, the height of eye opening reduces when decreasing the input OSNR value. The eye opening depicts the amplitude variation and OSNR degradation of the signal. The data laser was re-tuned from the pump laser to have frequency separations \(\Delta f\) from 175 GHz to 200 GHz with 5 GHz steps.

Figure 6 depicts the optical spectrum of recovered idler with the variation of input OSNR of BPSK signal. According to the figure, idler power (dBm) has been reduced with respect to the reduction of input OSNR value. Then the influence of the linewidth of the recovered carrier against different OSNR was investigated for a frequency separation of \((\Delta f = 190)\) GHz where \(\Delta f\) is the frequency separation between signal and pump. The
extracted carrier had a linewidth of 149.11 MHz. As shown in figure 7, the extracted carrier had a nearly constant linewidth ($LW = 149.11$ MHz) for an input OSNR greater than 14.32 dB.

The input OSNR of the BPSK signal of the carrier recovery setup was reduced and the effect of the linewidth for different $\Delta f$ values was measured. The linewidths measured for the OSNR values up to 16 dBm was between 175–200 GHz at 5 GHz intervals for all the $\Delta f$ variations. The laser linewidth was increased dramatically for the input OSNR value of less than 14.32 dB (figure 8). According to the equation (15), recovered carrier power and the signal power has a quadratic relationship, and the reduction in signal power through the degradation of OSNR, is the reason for the reduction of idler power. Decreasing of idler power causes the increase in idler linewidth.

It is obvious that the same carrier information with a slightly broadened linewidth can be obtained. The system can tolerate a constant recovered linewidth up to 14.32 dB. Hence, by combining this scheme with injection locking a more cleaned narrower linewidth can be generated [7]. This narrow linewidth laser is more suitable for achieving phase synchronization in higher performance in the PSA, optical synchronization schemes, Optical homodyne receivers, etc.

Linewidth broadening of a phase modulated signal was calculated using equations (7), (12) and (13) and linewidth broadening of an amplified signal is calculated using equation (14). For an initial input signal with 100 MHz linewidth, the percentage of theoretically calculated linewidth broadening and the percentage of simulated linewidth broadening is tabulated in table 1.

The FWM idler linewidth for BPSK modulated and pre-amplified signal is calculated using equation (21). According to the calculation the idler linewidth broadening value is 27.92 MHz and the measured linewidth broadening value is 34.34 MHz. Therefore, it can be concluded that the measured linewidth broadening values are in good agreement with the theoretically calculated linewidth broadening values. The calculated value of the
idler linewidth which underwent the phase modulation, amplification and the FWM process is, 127.92 MHz and the measured value is, 134.34 MHz.

4.2. System characterization for accumulated dispersion
The OSNR setup was replaced by dispersion setup. Different dispersion values was introduced by including different lengths of single mode fiber (SMF). The captured eye diagram of the input BPSK modulated signal is represent in figure 9 for different level of dispersion. According to the eye diagram, it can be seen that the noise both in amplitude and time has been increases with increased dispersion. Eye opening value is nearly equals to 152 mW at the zero dispersion tolerance.

Figure 10 depicts the linewidth variation of the recovered carrier with various dispersion values for $\Delta f$ of 185 GHz. As shown in figure, measured linewidth was started saturating after a higher dispersion value (greater than 70 ps nm$^{-1}$). The reason for this might be, even though the eye opening is fairly good (fairly better OSNR) an increase timing jitter will affect the carrier recovery and the system might collapse.

The linewidth measurement for the dispersion value up to 38.4 ps nm$^{-1}$ is shown in figure 11. The frequency separation between the pump and the data signal was changed between 180–200 GHz at 5 GHz intervals for all the $\Delta f$ variations. It is noted that with increased $\Delta f$ values the recovered linewidth is increased. The linewidth increment for zero dispersion value for different signal pump separation is mainly due to the reduction in the FWM conversion efficiency. This FWM efficiency reduction has an impact on modulation stripping system with increment of the frequency separation between pump and signal.

5. Conclusion
This paper presented the behaviour of the FWM based carrier recovery system for optical signal to noise ratio (OSNR) and dispersion degradation. Recovered carrier linewidth broadening due to input OSNR of the BPSK modulated signal was examined theoretically and experimentally. The obtained results show that the recovered carrier linewidth is degrading with more additive noise power and the theoretical value and measured values are in good agreement with each other. The system is working correctly for OSNR values greater than 14.32 dB for different pump signal separations. Furthermore, we have analyzed the linewidth of recovered carrier for different dispersion value of the input BPSK modulated signal and observed that linewidth is increasing with the higher dispersion value. Therefore, a proper dispersion compensation (up to 80 ps nm$^{-1}$) should be employed for better performance.

![Figure 8. Measured linewidths versus Input OSNR for different $\Delta f$.](image)

| Table 1. Linewidth broadening value. | Calculated value | Measured value |
|-----------------------------------|------------------|----------------|
| After phase modulation            | 13.92%           | 14.67%         |
| After amplification               | 0.036%           | 0.101%         |
Figure 9. BPSK signal eye diagrams for different input dispersion values: (a) 0 ps nm$^{-1}$, (b) 1.6 ps nm$^{-1}$, (c) 3.2 ps nm$^{-1}$, (d) 4.8 ps nm$^{-1}$, (e) 8 ps nm$^{-1}$, (f) 16 ps nm$^{-1}$, (g) 32 ps nm$^{-1}$, (h) 80 ps nm$^{-1}$, (i) 128 ps nm$^{-1}$.

Figure 10. Linewidth variation of idler with various dispersion values for $\Delta f$ of 185 GHz.
Acknowledgments

Authors would like to thank the National Research Council of Sri Lanka (NRC) (Grant No: 15-146) and Senate Research Committee (SRC) of University of Moratuwa for funding this research. Authors would also like to acknowledge Prof. V.K. Samaranayaka research grant from the LK Domain Registry for the given financial support for publication of this paper.

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