Discharge coefficient for rectangular notch using a dimensional analysis technique

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Abstract
The rectangular weir (notch) is a common device used to regulate and measure discharge in irrigation projects. The current research was based mainly on laboratory experiments studying the hydraulic characteristics of rectangular notches. Four rectangular notches were used in this research in different models. Notches for all models were designed with the same shape, arrangement, and width (4 cm), but differed in height, with examples at 6, 8, 10, and 12 cm. The main objective of this research was to study the influence of rectangular notch dimensions and upstream water depth on discharge coefficients.

The results obtained from this research indicate that the relationship between the discharge coefficient and the upstream water depth is a power function. The values of the discharge coefficient increase with increases in the values of the upstream water depth. The relationship between the discharge coefficient and the Reynolds number is also a power function, and an increase in the Reynolds number leads to a decreased discharge coefficient. In addition, when the value of the Reynolds number is high (turbulent flow), the values of the discharge coefficient converge to an approximately constant value. The flow in all runs was subcritical and the relationship between discharge coefficient and Froude number was also found to be a power function. An increase in Froude number thus leads to a decrease in discharge coefficient. The slope of the discharge coefficient-Froude number curve values gradually decreases until the value the discharge coefficient reaches an approximately constant value. A dimensional analysis technique was used to estimate the values of the discharge coefficient for various rectangular notch dimensions, and an empirical equation for discharge coefficient estimating was derived using regression procedure. This equation has a coefficient of determination R² of 0.955.

Keywords: rectangular weir; notch: broad-crested weir; discharge coefficient; relative height of weir.

1. Introduction
A weir is a type of human-made structure used mainly to regulate and measure the discharge through streams. Weirs can be classified into several types according to their shape and the construction of their crests. Side and labyrinth weirs, for example, are typical types used for diverting flow at maximized flow rate. Physical models can be constructed to simulate any type of weir, such as the = 4 mm-rectangular notch used in the current work; the resulting laboratory results can then be extrapolated from the prototype to a scale structure [1].

The discharge coefficient is the main parameter in discharge equations, and it is ascertained in various ways by researchers. A suitable review of coefficients was documented by [2]. For the widely used sharp crested weir [3], more than forty research projects about flow behaviour have been undertaken, including weirs with rectangular and triangular crests. The discharge coefficients were thus obtained for different widths and heights of this configuration, in as these are the parameters that most affect flow rate. CFD numerical models have been used to discover the relationships that allow estimation of the discharge
coefficient [4], [5] and [6], with laboratory measurements for head, velocity, and discharge carried out over different shapes of a sharp crest weir for calibration purposes.

A solution to the modelling of flow through sharp crest weirs based on the potential flow approach was conducted by [7]. Discussion in [8] related to applying critical flow theory to curved streamlines, and the compound sharp crest weir is one of suggested shapes with a reasonable sensitivity flow in a wide range [9]. Discharge coefficients for many geometrical specifications were thus found. Local materials should be used to construct weirs, and the effects of height and width using such materials were studied and reviewed by [10] and by [11]. A thin plate weir made of different materials was, however, used for experimental study work by [12] to calibrate the relationship between discharge and geometrical specifications. Skew weirs, such as labyrinth and curved weirs, have also been calibrated by many researchers, such as [13] and [14].

Detailed experimental work was undertaken by [15] and [16] for a sharp crest weir of 1 to 2 mm crest width and a 45° to 60° downstream inclined edge. A brief comparison was made in [17] between the discharge through a sharp-crested weir and a finite crest weir. A side contraction weir with a 4-mm thick crest and a 90° downstream edge was chosen in this research to estimate the flow rate passing through it and to find the coefficient of discharge for different weir heights. Dimensional analysis was used to investigate the relationships between the discharge and the corresponding geometric and hydraulic parameters.

2. Experimental Work

All tests were carried out in the hydraulic laboratory of the College of Engineering of Kerbala University. A channel was built with the hydraulic bench with a length of 70 cm, width of 25 cm, and the height of the side-walls set at 16.6 cm, as shown in Figures (1) and (2). The bench had a closed-loop water system, and the main tank, of 1.2 m³ capacity, was located below the bench flume. Water was conveyed from the main tank to an inlet tank of 0.2 m³ capacity located at the upstream. The bed of the bench was maintained at a horizontal slope during all of the tests. A centrifugal pump with a rated capacity of 40 l/s was used to deliver flow to the flume. Movable carriages with electronic verniers were mounted on a brass rail at the top of the channel sides; these devices had an accuracy of 0.01 mm.

![Figure 1. Side view of the hydraulic bench.](image-url)
2.1. Rectangular notch model

All rectangular notch models were fabricated from acrylic glass sheets of 4 mm thickness. Figure (3) shows the form for each model, where $b$ is the weir (notch) width, $h$ is a notch height, $H$ is a depth of water over the crest, and $P$ is the crest height. Four rectangular weirs (notches), known as RW, were used in this study on different models. All models of RW were created in the same shape and arrangement, and with a width of 4 cm, but they differed in height of notch. The details, dimensions, and water depths of these models are indicated in Table (1).
**Table 1.** Dimensions and upstream water depth of the models used.

| Model No. | b (cm) | P (cm) | h (cm) | H (cm) |
|-----------|--------|--------|--------|--------|
| 1         | 4      | 4.6    | 12     | From 6 to 12 |
| 2         | 4      | 6.6    | 10     | From 2 to 6.5 |
| 3         | 4      | 8.6    | 8      | From 1.5 to 5.5 |
| 4         | 4      | 10.6   | 6      | From 2 to 5   |

2.2. **Tests Procedure**

Experiments were conducted for various ranges of flow depth and flow discharge. Discharge values used to conduct the experiments ranged between 0.0149 and 0.255 \( l/s \). Tests for each rectangular notch model were carried out based on the following steps:

1. Installing the model of notch in the dedicated position at the channel of a hydraulic bench.
2. Establishing a flow rate by adjusting a control valve in the bench supply line.
3. After developing a stable flow, taking measurements of depth at the notch upstream face such that the measurement location was 40 cm from the notch, then measuring the upstream water head at a position one to three times the maximum water depth above the notch crest [18].
4. Recording the volume of water accumulated in the bench tank over time for each case.
5. Repeating the process from points 2 to 5 for further runs.
6. Repeating steps 2 to 6 for other notch models, as shown in Table (2).

To investigate the discharge coefficient values for all models, twenty-seven laboratory test runs were carried out. Table (2) presents a summary of the test runs carried out with different models.

3. **Results and Discussions**

3.1. **Estimation of Cd**

Figure (4) shows the variation of \( \ln(Q) \) values with \( \ln(H^{1.5}) \) for all test runs carried out on RW models. In these figures, the \( \ln(Q) \) value is directly and linearly proportional with \( \ln(H^{1.5}) \) for all notch models. For the purpose of calculating the values of the theoretical discharge coefficient for all notch models, the traditional rectangular weir equation was used:

\[
Q_{th} = \frac{2}{3} \sqrt{2ghH^{1.5}} \tag{1}
\]

where \( Q_{th} \) is the theoretical discharge. The effects of drawdown and model geometry [18] mean that the actual value of \( H \) is less than the measured value, and thus the actual discharge, \( Q \), is less than \( Q_{th} \). Therefore, it is necessary to correct equation (1) by adding a discharge coefficient, \( C_d \). Thus, the equation takes the form

\[
Q = C_d \frac{2}{3} \sqrt{2ghH^{1.5}} \tag{2}
\]
Table 2. Details of the test runs for all models

| Model No. | P cm | h cm | Run No. | Time s | Volume m³ | H m | Q act. m³/s |
|-----------|------|------|---------|--------|-----------|-----|-------------|
| No.1      | 4.6  | 12   | 1       | 83.84  | 0.01      | 0.014| 0.000119    |
|           |      |      | 2       | 32.05  | 0.01      | 0.0266| 0.000312    |
|           |      |      | 3       | 17.56  | 0.01      | 0.0381| 0.000569    |
|           |      |      | 4       | 12.67  | 0.01      | 0.0462| 0.000789    |
|           |      |      | 5       | 10.36  | 0.01      | 0.0535| 0.000966    |
|           |      |      | 6       | 8.302  | 0.01      | 0.063 | 0.001205    |
|           |      |      | 7       | 7.56   | 0.01      | 0.0699| 0.001323    |
|           |      |      | 8       | 6.336  | 0.01      | 0.0743| 0.001578    |
|           |      |      | 9       | 53.32  | 0.01      | 0.0212| 0.000188    |
|           |      |      | 10      | 28.96  | 0.01      | 0.0284| 0.000345    |
|           |      |      | 11      | 23.49  | 0.01      | 0.0326| 0.000426    |
|           |      |      | 12      | 21.3   | 0.01      | 0.0352| 0.000469    |
|           |      |      | 13      | 15.76  | 0.01      | 0.0434| 0.000635    |
|           |      |      | 14      | 13.58  | 0.01      | 0.0459| 0.000736    |
|           |      |      | 15      | 11.02  | 0.01      | 0.0533| 0.000907    |
|           |      |      | 16      | 8.35   | 0.01      | 0.0656| 0.001198    |
|           |      |      | 17      | 74.6   | 0.01      | 0.0156| 0.000134    |
|           |      |      | 18      | 34.11  | 0.01      | 0.0261| 0.000293    |
|           |      |      | 19      | 22.38  | 0.01      | 0.0338| 0.000447    |
|           |      |      | 20      | 17.76  | 0.01      | 0.0389| 0.000563    |
|           |      |      | 21      | 13.24  | 0.01      | 0.0461| 0.000755    |
|           |      |      | 22      | 10.31  | 0.01      | 0.056 | 0.00097     |
|           |      |      | 23      | 14.81  | 0.01      | 0.044 | 0.000675    |
|           |      |      | 24      | 18.76  | 0.01      | 0.0378| 0.000533    |
|           |      |      | 25      | 24.02  | 0.01      | 0.0329| 0.000416    |
|           |      |      | 26      | 28.67  | 0.01      | 0.0281| 0.000349    |
|           |      |      | 27      | 11.75  | 0.01      | 0.0487| 0.000851    |
The value of Cd was estimated for each case by equating the discharge for Equation (2) with the discharge equation for each case. For example, the Cd for model-1 is derived using the following steps,

1- The relationship between the discharge and the water depth upstream of the notch is

\[ \ln Q = 1.0237 \ln H^{1.5} - 2.4722 \]  

\[ R^2 = 0.9982 \]  \[ (3) \]

2- Equation (3) gives

\[ Q = 0.0844H^{1.5356} \]  

\[ (4) \]

3- After using equations 2 and 4, this becomes

\[ \ln Q = 1.0782 \ln H^{1.5} - 2.2677 \]  

\[ R^2 = 0.9932 \]  

\[ (4) \]

\[ \ln Q = 1.047 \ln H^{1.5} - 2.3893 \]  

\[ R^2 = 0.9992 \]  

\[ (4) \]

\[ \ln Q = 1.0812 \ln H^{1.5} - 2.2076 \]  

\[ R^2 = 0.9892 \]  

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\[ (4) \]

\[ \ln Q = 1.0812 \ln H^{1.5} - 2.2076 \]  

\[ R^2 = 0.9892 \]  

\[ (4) \]
\[ \frac{2}{3} \sqrt{2gCdbH^{1.5}} = 0.0844H^{1.5356} \]  

(5)

4- Therefore, the relationship between the discharge coefficient and the upstream water depth is a power function:

\[ Cd = 0.7145H^{0.0356} \]  

(6)

In the same way, discharge and discharge coefficients equations were obtained for the other models, as shown in Table (3). As explained, the values of the discharge coefficient increase with increases in the values of the upstream water depth, as shown in Fig. (5). This result agrees with [19].

**Table 3.** Formulas for estimating Q and Cd of all notch models.

| Model No. | Discharge equation | \( R^2 \) | Discharge coefficient equation |
|-----------|--------------------|---------|-------------------------------|
| 1         | \( Q = 0.0844H^{1.5356} \) | 0.998  | \( Cd = 0.7145H^{0.0356} \) |
| 2         | \( Q = 0.1036H^{1.6173} \) | 0.993  | \( Cd = 0.8771H^{0.1173} \) |
| 3         | \( Q = 0.0917H^{1.5705} \) | 0.999  | \( Cd = 0.7763H^{0.0705} \) |
| 4         | \( Q = 0.10994H^{1.6218} \) | 0.989  | \( Cd = 0.931H^{0.1218} \) |

**Figure 5.** Relationship between discharge coefficient and upstream water depth for all models.

3.2. **Variation of discharge coefficient with Reynolds number**

The Reynolds number, \( Re \), serves as a criterion to distinguish between laminar and turbulent flow. Here \( Re \) is defined as

\[ Re = \frac{\rho VR}{\mu} \]  

(7)
where \( \rho \) is the mass density of fluid, (ML\(^{-3}\)); \( R \) is the hydraulic radius at the location of head measurement of bench channel, (L); and \( \mu \) is the dynamic viscosity (ML\(^{-1}\)T\(^{-1}\)) of the fluid. Figure 6 represents the relationship between the discharge coefficient and the Reynolds number. As illustrated, the relationship between discharge coefficient and Reynolds number is a power function, and an increase of \( Re \) leads to decrease in discharge coefficient. In addition, it is clear that when the value of the Reynolds number becomes greater than 400, where the flow becomes fully rough and turbulent [1], the values of the discharge coefficient reach an approximately constant value.

Figure 6. Variation of the discharge coefficient with the Reynolds number in test runs with all models.
3.3. Variation of discharge coefficient with Froude number

The Froude number is a ratio of inertia force to gravity force. The Froude number serves as a criterion to distinguish between critical, subcritical, and supercritical flows, where F is defined in the following way:

\[ F = \frac{V}{\sqrt{gD}} \]  \hspace{1cm} (8)

where D is the hydraulic depth of the bench channel (L).

Figure (7) represents the relationship between discharge coefficient and Froude number. The flow in all runs is subcritical flow (Fr<1) and the relationship between discharge coefficient and Froude number is a power function; an increase of Fr leads to a decrease in discharge coefficient. In addition, as illustrated, when the values of the Froude number are high, the slope of the Cd-Fr curve values gradually decreases until the value of the discharge coefficient reaches an approximately constant value.

Figure 7. Variation of the discharge coefficient with the Froude number in test runs with all models.

\[ a - Cd - Fr \text{ for model No. 1.} \]

\[ b - Cd - Fr \text{ for model No. 2.} \]

\[ c - Cd - Fr \text{ for model No. 3.} \]

\[ d - Cd - Fr \text{ for model No. 4.} \]
3.4. Discharge Coefficient Based on Dimensional Analysis technique

Generally, the discharge coefficients of rectangular notches depended on geometric variables for all RW models. These variables were expressed functionally as

\[ f(C_d, H, P, g, h, \rho) = 0 \]  

where

- \( C_d \) = discharge coefficient, (dimensionless),
- \( g \) = gravitational acceleration (LT\(^{-2}\)),
- \( \rho \) = mass density (ML\(^{-3}\)), and

As shown above, only three fundamental units are involved (\( m = 3 \)), which are L, M, and T). The total number of variables is 6, i.e. \( n = 6 \).

According to Buckingham’s \( \pi \) theorem, there should thus be \( (6 - 3) = 3 \) terms, giving

\[ f = (\pi_1, \pi_2, \pi_3) \]  

The number of variables in each term is \( (m+1) \) so \( (3+1) = 4 \), taking \((g, H, \rho)\) as repeating variables, so:

- \( \pi_1 = g^a H^b \rho^c h \)
- \( \pi_2 = g^a H^b \rho^c P \)
- \( \pi_3 = g^a H^b \rho^c C_d \)

Expressing these in dimension terms for each model, we have:

- \( \pi_1 = \frac{h}{H} \)
- \( \pi_2 = \frac{P}{H} \)
- \( \pi_3 = C_d \)

After the simplification of the equations above, the dimensional relationship can be simplified to:

\[ C_d = f \left[ \frac{P}{H}, \frac{h}{H} \right] \]  

Applying a non-linear regression analysis to the data using the computer package STATISTICA produces the following formula:

\[ C_d = 0.5767 + 0.0152 \frac{P}{H} + 0.01112 \frac{h}{H} \]  

The coefficient of determination, \( R^2 \), of this formula is 0.955. Figure (8) thus provides good agreement between the experimental observed results and the data predicted by the empirical formula.
4. Conclusions
A series of experiments was carried out to investigate the effect of upstream water depth and rectangular weir (notch) geometry on discharge coefficient values. According to the results obtained from laboratory experiments, several conclusions were obtained as given below:

1. Discharge coefficient is directly proportional to the discharge of flow passing through a rectangular weir.
2. A strong linear relationship was found between the ln discharge coefficient and ln discharge for all four notch models; the average $R^2$ was 0.99.
3. The relationship between the discharge coefficient and the upstream water depth is a power function.
4. The values of the discharge coefficient increased with increases in the values of the upstream water depth.
5. The relationship between discharge coefficient and Reynolds number is a power function. The effect of the Reynolds number reduces gradually, vanishing when the flow reaches a fully turbulent state.
6. An increase of Re leads to decrease in discharge coefficient.
7. When the value of the Reynolds number is high (Turbulent flow), the value of the discharge coefficient reaches an approximately constant value.
8. The relationship between discharge coefficient and Froude number is a power function.
9. An increase in Fr leads to a decrease in discharge coefficient.
10. When the values of the Froude number become higher, the slope of the discharge coefficient-Froude number curve values will gradually decrease until the value the discharge coefficient reaches an approximately constant value.
11. The use of the dimensional analysis technique demonstrated consistency between the values of the calculated and estimated discharge coefficients, with $R^2=0.955$. 

Figure 8. Comparison of formula (13) with the experimental results of discharge coefficient.
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