Stability and stabilisation for switched impulsive positive systems

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Abstract
The current study is dedicated to addressing the stability and weighted $L_1$-gain performance analysis of switched impulsive positive systems (SIPSs). Firstly, by designing a novel multiple piecewise-continuous copositive linear Lyapunov function and using the mode-dependent average dwell time (MDADT) switching method, improved stability conditions that are able to achieve a tighter dwell time bound are developed. It is shown that the system under study is stable and possesses an attenuation property under the designed switching signals. Secondly, with the above stability conditions, a more effective controller design strategy has been proposed. The solved controllers are both quasi-time dependent and mode-dependent. Moreover, the constraint on the rank of the controller and the computation burden caused by iteration algorithm used in literature are relaxed. Finally, the feasibility and superiority of the proposed strategies are verified by some simulations.

1 | INTRODUCTION

In the past few decades, considerable attentions have been paid to switched systems owing to their potential use in the fields of engineering such as networked control systems and switched flow network. Recently, stability and stabilisation analysis for such a class of systems has received a lot of attentions and fruitful results have been obtained [1—4]. However, it should be noted that the states of these systems can be arbitrary values and may not be effective to express the dynamical behavior of some systems. In many research communities such as electrical design, economics, communication network and medical treatment, the state (output) and control input are usually restricted to positive orthant for all the time. For instance, the height of liquids, the population of animals (plant) and the mass of objects are clearly not less than zero. Systems with the property that for any nonnegative initial condition, their state/output are nonnegative, are called positive systems [5—9]. Due to their huge application potential, positive systems attract a lot of focuses and until now many excellent results for such a class of systems have been reported [10—13].

With the development of positive systems and the switched systems, switched positive systems (SPSs) have received great attentions from many researchers. SPSs are usually made up of a switching signal and a bunch of positive subsystems. The SPSs can be discovered in plenty of engineering areas such as networks utilising TCP, image processing and formation control of unmanned aerial systems [14—17]. Note that owning to the required positive property of SPSs, the analysis and stabilisation strategies pertained to switched systems are not suitable for the former systems any more. It is well known that the quadratic Lyapunov function is an effective method for investigating the stability and the stabilisation problems of switched systems, but it is a useless tool for the SPSs. Consequently, the copositive Lyapunov function (CLF) has been designed to analyse SPSs, and up to now, abundant achievements have been obtained [18—24]. It is required that the derivative/difference of the CLF
has to be nonpositive along the system trajectory in the positive orthant.

The abrupt changes of the states are ubiquitous in practical engineering systems owning to some reasons, such as communication failures. For example, in some circuit systems, the circuit switching usually causes system state to change abruptly. Systems with abrupt changes are usually referred as impulsive systems. The impulsive positive systems, which consider the impulsive characters and the positive property simultaneously, exhibit complex dynamical behaviour. The switched impulsive positive systems (SIPSs) have numerous applications in fields like mechanical systems, automotive industry, aircraft, air traffic control and so on. Therefore, investigating the stability of SIPSs is of both theoretical and practical importance. Recently, investigations on such a class of systems have become active and some interesting achievements have been reported [25–33]. However, compared with switched impulsive systems, the research for SIPSs is still at a relatively early stage. The stability and stabilisation for linear impulsive positive systems under several types of dwell time are studied in [26], where stability conditions are expressed by linear programming. In [27], by using a linear CLF, it solves a series of stability problems of impulsive positive systems. In [28], by using Lyapunov-like function method, this paper studies the stability analysis problem of switched impulsive nonlinear systems. In [33], by introducing multilinear CLF and the average dwell time (ADT) approaches, exponential stabilisation analysis of a discrete time SIPS is studied.

Though some research findings have been developed about switched impulsive systems, the controller design problem has not been well addressed till now. From [34], it can be seen that multiple CLF is an effective technology to solve the stability problem of SPSs under time-limited switching signals. However, the adopted CLF in the literature is usually continuous in each activation time interval, which will result in conservative results. In [35], the influences of impulse on the stability and stabilisation of positive systems with time delays are studied, and the design method of the controller is given. However, the rank of controller gain is required to be 1. Moreover, stability of many SIPSs may not be assured under arbitrary switching, but it can be guaranteed under restricted switching signals. Several time-restricted switching signals are given based on dwell time, ADT and mode-dependent average dwell time (MDADT), respectively. Among these switching signals, the dwell time can be regarded as an exceptional case of ADT, and the latter further is regarded as an exceptional case of MDADT. In [36], the stability analysis and delay control problem of the switched positive linear system are studied and a complex iterative algorithm is adopted to solve the desired controller. How to eliminate the rank constraint and to reduce the computation burden caused by the iterative algorithm is still unsolved. Moreover, there is still space for improving the design of dwell time. Therefore, proposing an efficacious stability and controller synthesis method to improve the dwell time design as well as to remove the requirement of controller gain rank and to simplify the iterative algorithm is of importance.

Inspired by the above analysis, this paper is concerned with the stability and stabilisation problem of SIPSs by using the MDADT technique. Specifically, in order to solve the system’s stability and stabilisation problem, we define a multiple piecewise-continuous copositive linear Lyapunov function, which is both mode-dependent and quasi-time dependent. Improved stability conditions are exploited for the studied SIPSs. Compared with existing results, the developed stability conditions can provide lower dwell time. Moreover, a novel controller design strategy has been provided to reduce the complexity of the iteration algorithm and also the limitation of the rank requirement.

The organisation of this paper is as follows. Some preliminary knowledge of SIPSs and the description of problem are provided in Section 2. In Section 3, stability conditions for the studied SIPSs are established, and in Section 4, performance analysis and controller design are presented on the basis of the above conditions. In Section 5, two examples are given to prove the validity of the presented method. In the last section, some conclusions are given.

Notation. The sets of real numbers, positive real numbers and nonnegative integers are represented by $\mathbb{R}$, $\mathbb{R}_{>0}$ and $\mathbb{Z}_{\geq 0}$. The notation $S < 0(\leq 0)$ signifies that every element of vector $S$ is negative (nonpositive). If $T$ is a positive (nonnegative) matrix, then it is denoted as $T > 0(\geq 0)$. $I$ represents a unit matrix with compatible dimensions unless there is an explicit statement. $I_p$ represents a vector whose elements are all 1. $\| \cdot \|$ stands for Euclidean norm of a vector. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space. $A'$ represents the transposition matrix of $A$. $1_m$ represents a $m$-dimensional column vector whose element $s$ is one while the rest are zeros.

## 2 Preliminaries and Problem Statement

In paper, we will study a class of SIPSs with the following form

$$\dot{x}(t) = A_{\varphi(t)}x(t) + B_{\varphi(t)}u(t) + F_{\varphi(t)}\gamma(t), t \neq t_{s},$$

$$\Gamma x(t) = Qx(t^-), t = t_{s},$$

$$\zeta(t) = C_{\varphi(t)}x(t) + D_{\varphi(t)}\gamma(t),$$

(1)

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, w(t) \in \mathbb{R}^m, \gamma(t) \in \mathbb{R}^r$ stand for state, input of system (1), external disturbances and output of system (1), respectively, $\varphi(t)$ stands for switching signal belonging to a finite set $\mathcal{S}_q := \{1, \ldots, N\}$. The switching and impulsive instants of system (1), defined as $\{t_i\}, i \in \mathbb{Z}_{\geq 0}$, are supposed to be strictly increasing. $\Gamma x(t) := x(t) - x(t^-)$ is the difference after and before the instant $t$. The system matrices $A_{\varphi}, B_{\varphi}, C_{\varphi}, D_{\varphi}, F_{\varphi}, u = 1, \ldots, N$ and $Q$ are constant ones, and are given before-hand. Next, some basic knowledge for developing the main results will be provided.

Remark 1. In many research communities such as electrical design, economics and communication network, the state and
control input are usually restricted to positive orthant. Moreover, abrupt changes of the states are ubiquitous in practical engineering systems. For example, in some circuit systems, the circuit switching usually causes system state to change abruptly. The switched impulsive positive systems have numerous applications in fields like mechanical systems, automotive industry, aircraft, air traffic control and so on. Therefore, investigating the stability of switched impulsive positive systems is of both theoretical and practical importance.

**Definition 1** [37]. If there exists a scalar $\varepsilon > 0$ to ensure $S + \varepsilon I \succeq 0$, then matrix $S$ is called a Metzler matrix.

**Definition 2** [38]. For a given bounded time interval $[R_a, R_b]$ and a switching signal $\varphi(t)$, let $N_{\varphi}(R_a, R_b)$ and $\mathcal{T}_\varphi(R_a, R_b)$ be the total switching numbers and the total operating time of the $r^{th}$ activated subsystem over such a bounded time interval. If there are two positive parameters $\mathbb{N}_{\varphi r}$ and $\varphi_{\varphi r}$ such that

$$\mathbb{N}_{\varphi r}(R_a, R_b) \leq \mathbb{N}_{\varphi r} + \mathcal{T}_\varphi(R_a, R_b)/\varphi_{\varphi r}, \forall R_b > R_a \geq 0,$$

then it is said that $\varphi(t)$ has a MDADT $\varphi_{\varphi r}$ and $\mathbb{N}_{\varphi r}$ is referred as the chatting bound.

**Definition 3.** For any given switching signal $\varphi(t)$, if for given initial condition $x(t_0) \succeq 0$ and $w(t) \succeq 0$, there is a $u(t) \succeq 0$ to ensure that the state trajectory $x(t) \succeq 0$ and the system output $z(t) \succeq 0$ hold true for all $t \in [0, +\infty)$, then system (1) is positive.

According to the above definition, we propose the following lemma.

**Lemma 1.** For any given switching signal $\varphi(t)$, system (1) is positive iff $A_p, p \in \mathbb{S}_q$, is a Metzler matrix and $B_{ps}, F_{ps}, C_{ps}, D_{ps}, (I + Q)$ are nonnegative matrices for all $u \in \mathbb{S}_q$.

**Proof.** We first show the sufficiency. □

**Sufficiency:** For any bounded time interval $[t_0, T]$, we let $0 = t_0 < t_1 < t_2 < \cdots < t_i < \cdots < t_n(0, T) = T$ be the switching and impulsive instants, where $\mathbb{N}_\varphi = \sum_{t \in \mathbb{T}_\varphi} \mathbb{N}_{\varphi r}(0, T)$. Then it follows from (1) that

$$x(t) = e^{A_p(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A_p(t-t')}B_{ps}(t)w(t')dt' + F_{ps}(t)w(t),$$

for any time $t \in [t_0, t_1)$. By using Theorem 1.18 of [39], one has that $e^{A_p(t-t_0)} \succeq 0$, whenever $A_p$ is a Metzler matrix. Therefore, if $B_{ps} \succeq 0, F_{ps} \succeq 0$, then one can obtain $x(t) \succeq 0$, for any time $t \in [t_0, t_1)$ and $x(t) \succeq 0$. According to (1), it holds that $x(t^+) = (I + Q)x(t^-) \succeq 0$ as $(I + Q) \succeq 0$. Similarly, for any time $t$ belongs to $[t_1, t_2)$, the state is able to calculated as

$$x(t) = e^{A_p(t-t_1)}x(t_1) + \int_{t_1}^{t} e^{A_p(t-t')}B_{ps}(t)w(t')dt' + F_{ps}(t)w(t),$$

which indicates $x(t) \succeq 0$ for any time $t \in [t_1, t_2)$ and $x(t^-) \succeq 0$. Calculating the state $x(t)$ recursively, then it can be proved that $x(t) \succeq 0$. Moreover, if $C_{ps} \succeq 0$ and $D_{ps} \succeq 0$, then the positive property of $z(t)$ can be ensured.

**Necessity:** We first consider the case in which system (1) not evolves at the switching instants. Conversely, assume that one of the off-diagonal elements, such as $a_{pq}$, where $a_{pq}$ denotes the entry of matrix $A_p$ in the $p^{th}$ row and $q^{th}$ column. Using (1), one has that

$$\dot{x}_p(t) = \sum_{i=1, i \neq p} a_{pq}x_i(t) + a_{pq}x_q(t) + a_{pq}x_p(t) + \sum_{i=1}^n b_{pq}u_i(t),$$

In the same way, when $x_i(t) = 0$, we can acquire that $\dot{x}_p(t) < 0$ is possible. Clearly, $x_p(t^+)<0$ if system (1) is positive. Furthermore, one can draw the same conclusion for the similar case of matrix $F_p, \forall p \in \mathbb{S}_q$. Now, we consider the dynamical behaviour at the switching instants. From (1), it is clear that the following equation holds at the switching instants.

$$x(t^+) = (I + Q)x(t^-),$$

Assume there is an element $q_{pq} < 0$, where $q_{pq}$ denotes the entry of matrix $(I + Q)$ in the $p$-th row and $q$-th column. From (7), we have that

$$x_p(t^+) = q_{pq}x_q(t^-) + \sum_{i=1, i \neq p} q_{pq}x_q(t^-).$$

Therefore, according to the above proposition, we have the following result.
Then, for $x(t) \neq 0$, it is possible that $x(t^+ \tau') < 0$. This also produces a contradiction with positive property of system (1).

Therefore, system (1) is positive iff $A_u$ is a Metzler matrix and $B_u, F_u, C_u, D_u, (I + Q)$ are nonnegative matrices. The proof is thus completed.

**Definition 4.** System (1) is said to be globally exponentially stable (GES), if there are two positive scalars $\omega_1 \geq 1, \omega_2$ such that the following inequality

$$\|x(t)\| \leq \omega_1 \exp(-\omega_2(t - t_0))\|x(t_0)\|,$$

subject to the condition $w(t) = 0$.

**Definition 5.** For a given $\gamma > 0$, system (1) can achieve a weighted $L_1$-gain no greater than $\gamma$ if system (1) is GES and moreover the following inequality holds

$$\int_{t_0}^{\infty} \exp(-\omega_3 u)\|z(u)\|_1 du \leq \gamma_0 \int_{t_0}^{\infty} \|w(u)\|_1 du,$$

for a scalar $\omega_3 > 0$.

**Lemma 2[40].** Given several matrices $U, V, Y$ with compatible dimensions, it holds that $\text{vec}(UY^\top) = (Y \otimes U)\text{vec}(V^\top)$, where $\text{vec}(V)$ denotes the stacked column of this matrix from left to right.

Note that in existing results, continuous Lyapunov function is usually adopted during each activated interval for the stability analysis, which leads to conservative results. For the aim of reducing the conservatism of existing methods, we will provide piecewise continuous Lyapunov function during each activated time interval. Suppose the $u^{th}$ subsystem is activated during the time interval $[t_i, t_{i+1})$. Divide such an activated time interval into $G + 1$ segments, then it is clear that the length of each segment is $\ell_i = (t_{i+1} - t_i)/(G + 1)$. Define $\ell_i = t_{i+1} - t_i$ for $i = 0, \ldots, G$ with $t_i = t_0$ and $t_{G+1} = t_{G+1}$. Clearly, the $u^{th}$ activated time interval $[t_i, t_{i+1}) = \bigcup_{s=0}^{G}[t_i, t_{i+1})$ for all $s \in \mathbb{Z}_{\geq 0}$. Now, we introduce an indicator $\tau_i$ that will be used for Lyapunov function design, where $\tau_i = i_s$ for $t \in [t_i, t_{i+1})$.

For system (1), we put forward the following control law

$$u(t) = K_i(x(t), t),$$

for the activated time interval $[t_i, t_{i+1})$, where $u$ denotes the $u$-th activated subsystem and $\tau_i \in \mathbb{G}$, where $\mathbb{G} := \{0, 1, \ldots, G\}$.

**Remark 2.** Note that different from existing results where the controller is continuous in an activated time interval $[t_i, t_{i+1})$, $s \in \mathbb{Z}_{\geq 0}$, the designed controller (11) is a piecewise continuous one relying on both subsystem mode and the indicator $\tau_i$, thus, it is said that the controller is both mode-dependent and quasi-time dependent.

### 3 | EXPONENTIAL STABILITY AND WEIGHTED $L_1$-GAIN PERFORMANCE ANALYSIS

Now, without considering the system input $u(t)$, we first provide a result for system (1). Under such a circumstance, the autonomous system is given as

$$\dot{x}(t) = A_{q(t)}x(t) + B_{q(t)}w(t), t \neq t_i,$$

and

$$\Gamma x(t) = Q x(t), t = t_i,$$

where

$$\zeta(t) = C_{q(t)}x(t) + D_{q(t)}w(t).$$

The sufficient conditions for the stability of the system (12) and the expected $L_1$-gain performance are given in the following theorem.

**Theorem 1.** For system (12), for any given scalars $\kappa_\xi > 1, \xi > 0, \xi > 0 < \xi_i \leq 1, \gamma \in \mathbb{S}_{\gamma}$, suppose there is a set of positive vectors $\xi_i(\sigma) \in \mathbb{R}_{\geq 0}$, $\gamma \in \mathbb{S}_{\gamma}, \sigma = 0, \ldots, G$ such that $\forall (u, v) \in \mathbb{S}_{\gamma} \times \mathbb{S}_{\gamma}$

$$A_i \xi_i(\sigma) + B_i \xi_i(\sigma) + C_i 1_\xi \leq 0,$$

$$F_i \xi_i(\sigma) + D_i 1_\xi - \gamma 1_\xi \leq 0,$$

$$\zeta_i(\sigma + 1) - \xi_i(\sigma) \leq 0, \sigma \neq G,$$

and

$$(I + Q) \xi_i(0) - \kappa_i \xi_i(G) \leq 0, (16)$$

hold for a Metzler matrix $A_u$ and nonnegative matrices $B_u, C_u, D_u, F_u, (I + Q)$, furthermore if the switching signal $q(t)$ has the following property

$$\phi_{\text{act}} > \phi^*_{\text{act}} = \frac{\ln(\kappa_i + G \ln \xi_{\text{act}})}{\xi_{\text{act}}},$$

with $\kappa_\xi \xi_{\text{act}} \geq 1$, then system (12) is GES. Moreover, the anticipated $L_1$-gain performance can be acquired.

**Proof.** For system (12), we design the following novel CLF, which relies on both the mode and the introduced indicator $\tau_i$,

$$V(t) = V_{q(t)}(\delta, \tau_i) = x(t) \xi_{q(t)}(\tau_i).$$

Calculating the derivative of (18) along system (12) yields

$$\dot{V}(t) = \left(A_{q(t)}x(t) + B_{q(t)}w(t)\right) \xi_{q(t)}(\tau_i).$$
Let $\Gamma(t) = \|z(t)\|_1 - \gamma\|w(t)\|_1$. Then, one has that

$$V(t) = \dot{z}(t) + \xi \dot{\xi}(t) + \Gamma(t)$$

$$= \left(-\dot{z}(t) + \xi \dot{\xi}(t) + \Gamma(t)\right) + \|z(t)\|_1 - \gamma\|w(t)\|_1$$

$$= \dot{z}(t) + \xi \dot{\xi}(t) + \Gamma(t)$$

which further can get

$$V(t) \leq e^{-\delta_0(t-t'_i)} V_{\phi(t'_i)}(t'_i) - \int_{t'_i}^{t} e^{-\delta_0(t'-t)} \Gamma(t') \, \text{d}t'.$$

Condition (15) indicates that at the time instant $t'_i$

$$\dot{x}(t'_i)^T \xi_a(\sigma + 1) \leq \xi \cdot x(t'_i)^T \xi_a(\sigma), \sigma = 1, \ldots, G - 1.$$ (23)

Combining (22) and (23) together yields that

$$V(t_{i+1}) = V_{\phi(t'_i)}(t_{i+1}) \leq e^{-\delta_0(t_{i+1}-t)} V_{\phi(t'_i)}(t'_i)$$

$$- \int_{t'_i}^{t} e^{-\delta_0(t''-t)} \Gamma(t'') \, \text{d}t''$$

$$\leq e^{-\delta_0(t_{i+1}-t)} \xi V_{\phi(t'_i)}(t'_i)$$

$$- \xi \int_{t'_i}^{t} e^{-\delta_0(t''-t)} \Gamma(t') \, \text{d}t'$$

$$\leq e^{-\delta_0(t_{i+1}-t)} \xi V_{\phi(t'_i)}(t'_i)$$

$$- \xi \int_{t'_i}^{t} e^{-\delta_0(t''-t)} \Gamma(t'') \, \text{d}t''$$

$$\leq e^{-\delta_0(t_{i+1}-t)} \xi V_{\phi(t'_i)}(t'_i)$$

$$- \xi \int_{t'_i}^{t} e^{-\delta_0(t''-t)} \Gamma(t'') \, \text{d}t''$$

where $N_{\phi}(\tau, t)$ stands for the numbers of the occurrence of $\xi$.

In addition, one concludes from condition (16) at the impulsive instants $t_t$

$$V(t_t, G) = \dot{x}(t_t)^T \xi_a(G), \dot{V}(t_t, 0) = \dot{x}(t_t)^T (I + Q)^T \xi_a(0).$$ (25)

By virtue of (25), one has

$$V(t_{i+1}) \leq e^{-\delta_0(t_{i+1}-t)} \xi \int_{t'_i}^{t} e^{-\delta_0(t''-t)} \Gamma(t'') \, \text{d}t''$$

$$\leq e^{-\delta_0(t_{i+1}-t)} \xi V_{\phi(t'_i)}(t'_i)$$

$$- \xi \int_{t'_i}^{t} e^{-\delta_0(t''-t)} \Gamma(t') \, \text{d}t'$$

$$\leq e^{-\delta_0(t_{i+1}-t)} \xi V_{\phi(t'_i)}(t'_i)$$

$$- \xi \int_{t'_i}^{t} e^{-\delta_0(t''-t)} \Gamma(t'') \, \text{d}t''$$

where $N_{\phi}(\tau, t)$ stands for the numbers of the occurrence of $\xi$.
Based on (17) and (27), we have that

\[
V(t) \leq \prod_{r=1}^{N} g_{r}(\xi)^{(N_{r}+1)} \exp \left\{ - \sum_{r=1}^{N} g_{r} \sum_{j \in \Omega(r)} (t_{j+1} - t_{j}) \right. \\
- g_{r}(t_{j})(t - t_{j}) \left. \right\} V_{\Phi}(h_{0}) \left( h_{0} \right) \\
- \int_{0}^{t} \exp \left\{ - \sum_{r=1}^{N} g_{r} T_{r}(\nu, t) \right. \\
+ \sum_{r=1}^{N} N_{r} T_{r}(\nu, t) \left( \ln \kappa_{r} + G \ln \xi \right) \left\} \xi^{N_{r}(\nu, t)} \Gamma(\nu) d\nu \\
\leq \exp \left\{ \sum_{r=1}^{N} N_{r} G \ln \xi \right\} \\
\times \exp \left\{ \sum_{r=1}^{N} \left( \ln \kappa_{r} + G \ln \xi \right) - g_{r} \right\} T_{r}(h_{0}, t) \\
- \int_{0}^{t} \exp \left\{ - \sum_{r=1}^{N} g_{r} T_{r}(\nu, t) \right. \\
+ \sum_{r=1}^{N} N_{r} T_{r}(\nu, t) \left( \ln \kappa_{r} + G \ln \xi \right) \left\} \xi^{N_{r}(\nu, t)} \Gamma(\nu) d\nu. \\
\right.
\] (28)

We first show that the exponential stability is ensured without considering the external disturbances \( w(t) \). In such a case, one has that

\[
V'(t) \leq \omega_{1} e^{-\omega_{2}(t-h_{0})} V_{\Phi}(h_{0}),
\] (29)

where \( \omega_{1} = \exp \left\{ \sum_{r=1}^{N} N_{r} G \ln \xi \right\}, \omega_{2} = \max_{r \in S_{\Phi}} \left( g_{r} - (\ln \kappa_{r} + G \ln \xi) / \phi_{ar} \right) \). By using the definition of the Lyapunov function (18), it holds that

\[
x(t)'^{\Phi}(\nu)(\tau_{i}) \leq \omega_{1} e^{-\omega_{2}(t-h_{0})} x(h_{0})^{\Phi}(0) .
\] (30)

Using the positive property of \( x(t) \) and \( x^{\Phi}(\nu)(\tau_{i}) \), one has

\[
x(t)'^{\Phi}(\nu)(\tau_{i}) \geq \omega_{3} \| x(t) \| ,
\] (31)

and

\[
x(h_{0})^{\Phi}(0) \leq \sqrt{\omega_{4}} \| x(h_{0}) \| ,
\] (32)

where \( \omega_{3} = \min_{r=0, \ldots, G_{1} \leq \nu \leq \xi} \left\{ x^{\Phi}(\nu)(\tau_{i}) [i] \right\}, \omega_{4} = \max_{r=0, \ldots, G_{1}} \left\{ \right. \right. \\
1 \leq i \leq N_{r}(\Phi)(h_{0}) [i] \right\} \right. \right. \text{ and the property } \sqrt{\sum_{r=1}^{G_{1}} x^{2}[r]} \leq \sqrt{g} \sqrt{\sum_{r=1}^{G_{1}} x^{2}[r]} .
\]

Taking (30)–(32) together, we can achieve that

\[
\| x(t) \| \leq \frac{\sqrt{\sum_{r=1}^{G} a_{r}}}{\omega_{3}} e^{-\omega_{2}(t-h_{0})} \| x(h_{0}) \|. 
\] (33)

Therefore, the stability proof is thus completed.

Next, the analysis for the assurance of the weighted \( L_{2} \)-gain will be presented. Since \( V(t) \geq 0 \), then under the condition \( x(h_{0}) = 0 \), it can be inferred that

\[
0 \leq - \int_{0}^{t} \exp \left\{ - \sum_{r=1}^{N} g_{r} T_{r}(\nu, t) + \sum_{r=1}^{N} N_{r} T_{r}(\nu, t) \right. \\
\times \left( \ln \kappa_{r} + G \ln \xi \right) \left\} \xi^{N_{r}(\nu, t)} \Gamma(\nu) d\nu. \] (34)

One has that \( G \sum_{r=1}^{N} N_{r} T_{r}(\nu, t) \leq N_{\xi} T_{r}(\nu, t) \leq (\sum_{r=1}^{N} N_{r} T_{r}(\nu, t) + 1)G \) from the above definition of \( N_{\xi}(\nu, t) \). Then, inequality (34) further shows that

\[
\xi^{G} \int_{0}^{t} \exp \left\{ - \sum_{r=1}^{N} g_{r} T_{r}(\nu, t) \\
+ \sum_{r=1}^{N} N_{r} T_{r}(\nu, t) \left( \ln \kappa_{r} + 2G \ln \xi \right) \right\} \| z(\nu) \|_{1} d\nu \\
\leq \int_{0}^{t} \exp \left\{ - \sum_{r=1}^{N} g_{r} T_{r}(\nu, t) + \\
\sum_{r=1}^{N} N_{r} T_{r}(\nu, t) \left( \ln \kappa_{r} + 2G \ln \xi \right) \right\} \| w(\nu) \|_{1} d\nu. 
\] (35)

Multiplying this inequality by \( \exp \left\{ - \sum_{r=1}^{N} N_{r} T_{r}(0, t) \right\} \left( \ln \kappa_{r} + G \ln \xi \right) \) leads to

\[
\xi^{G} \int_{0}^{t} \exp \left\{ - \sum_{r=1}^{N} g_{r} T_{r}(\nu, t) \\
- \sum_{r=1}^{N} N_{r} T_{r}(0, t) \left( \ln \kappa_{r} + 2G \ln \xi \right) \right\} \| z(\nu) \|_{1} d\nu \\
\leq \int_{0}^{t} \exp \left\{ - \sum_{r=1}^{N} g_{r} T_{r}(\nu, t) + \\
- \sum_{r=1}^{N} N_{r} T_{r}(0, t) \left( \ln \kappa_{r} + 2G \ln \xi \right) \right\} \| w(\nu) \|_{1} d\nu. \] (36)

Let \( \delta_{\max} = \max_{r \in S_{\Phi}} \{ \delta_{r} \}, \delta_{\min} = \min_{r \in S_{\Phi}} \{ \delta_{r} \} \). Since \( \phi_{ar} > (\ln \kappa_{r} + G \ln \xi) / g_{r} > (\ln \kappa_{r} + 2G \ln \xi) / g_{r} \), then one has \( 1 / \phi_{ar} < g_{r} / (\ln \kappa_{r} + 2G \ln \xi) \). Therefore, \( \sum_{r=1}^{N} N_{\xi}(0, t) \left( \ln \kappa_{r} + 2G \ln \xi \right) \leq \delta_{\max} (v - \gamma_{0}) \). Then, it
follows from (36)
\[ \xi^G \int_0^t e^{-\delta_{\max}(\nu)} \|z(\nu)\| d\nu. \]
Integrating this inequality from zero to +∞, we have that
\[ \xi^G \int_0^t e^{-\delta_{\max}(\nu)} d\nu \leq \int_0^t e^{-\delta_{\max}(\nu)} \|z(\nu)\| d\nu. \]
By changing the order of integration, we can get
\[ \xi^G \int_0^t e^{-\delta_{\max}(\nu)} dt \leq \int_0^t e^{-\delta_{\max}(\nu)} \|z(\nu)\| d\nu. \]
From inequality (39), one has
\[ \int_0^t e^{-\delta_{\max}(\nu)} d\nu \leq \gamma_\star \int_0^t \|z(\nu)\| d\nu, \]
where \( \gamma_\star = \frac{\delta_{\max}}{\delta_{\max} G} \), and the initial condition \( t_0 = 0 \) is adopted.

Note that the MDADT switching means is used for stability analysis in Theorem 1. If the mode-dependent property is not considered, the MDADT technique becomes the corresponding ADT technique, and the corresponding corollary of Theorem 1 is presented.

**Corollary 1.** Consider system (12). For any given scalars \( 0 < \xi \leq 1, \eta > 1, \gamma > 0 \), suppose that there is a set of positive vectors \( \xi_u(\sigma) \in \mathbb{R}_+, u \in \mathbb{S}_G, \sigma = 0, \ldots, G \) such that \( \forall(u, v) \in \mathbb{S}_G \times \mathbb{S}_G \),
\[
A_u^T \xi_u(\sigma) + B_u^T \xi_u(\sigma) + C_u^T 1_\eta + \eta_u \leq 0,
\]
\[
F_u^T \xi_u(\sigma) + D_u^T 1_\gamma - \gamma_1_\eta \leq 0,
\]
\[
\xi_u(\sigma + 1) - \xi_u(\sigma) \leq 0, \sigma \neq G,
\]
\[
(I + Q)^T \xi_u(0) - \kappa_\xi_u(G) \leq 0,
\]
hold for a Metzler matrix \( A_u \) and nonnegative matrices \( B_u, C_u, D_u, F_u, (I + Q) \). Moreover, if the switching signal \( \phi(t) \) has the following property
\[
\phi \geq \phi_\star = \frac{\ln x + \ln \xi}{\xi},
\]
with \( \kappa_\xi G \geq 1 \), then system (12) is GES. In addition, expected L2-gain performance can be achieved.

### 4 CONTROLLER DESIGN

In the above section, sufficient conditions are developed for system stability analysis and expected performance synthesis. However, the controller design problem has not been addressed successfully. The design scheme of controller will be provided in this section. The design procedure is illustrated in Theorem 2.

**Theorem 2.** Consider system (1) with control input (11). For any given scalars \( 0 < \xi \leq 1, \kappa_\xi > 1, 0 < t_\star, u \in \mathbb{S}_G, \sigma > 0, u \in \mathbb{S}_G, \sigma = 0, \ldots, G, \tau, \sigma = 1, \ldots, m \) such that \( \forall(u, v) \in \mathbb{S}_G \times \mathbb{S}_G \), and if (14)–(16) hold true for nonnegative matrices \( B_u, C_u, D_u, F_u, (I + Q) \), then system (1) is GES and meanwhile the desired system performance can also be ensured if the switching signal \( \phi(t) \) satisfies (17). Moreover, the controller can be calculated as
\[
K_\phi(\tau_i) = \frac{\sum_{i=1}^m v_i(\eta_u)}{\xi_u(\tau_i)B_u 1_m}, u \in \mathbb{S}_G, \tau, \in \mathbb{G}.
\]

**Proof.** By invoking Lemma 2, it can be inferred from (47) that
\[
1^T_B \xi_u(\sigma) A_u^T + \sum_{i=1}^m \eta_u(v_i) B_u^T + \gamma_1_\eta \leq 0.
\]
Such an inequality implies that
\[
A_u + B_u \sum_{i=1}^m v_i(\eta_u) + \gamma_1_\eta \geq 0.
\]
Following Definition 1, it can be easily validated that matrix $A_s + B_sK_s(\tau)$, $s \in \mathbb{S}_p$, $\tau \in G$ is a Metzler matrix. Hence, the positive property has been ensured. Now, we will show that the GES and also the expected system performance can be attained. For any time $t \in [\tau^i, \tau^{i+1})$, we can get that

$$V(t) + g_0V(t) + \Gamma(t)$$

$$= x(t)'\begin{bmatrix} A_s + \sum_{m=1}^{m} \eta_m(V_k)B_k' \end{bmatrix}_x + g_0x + C_s1$$

$$\leq x(t)'\begin{bmatrix} A_s + \sum_{m=1}^{m} \eta_m(V_k)B_k' \end{bmatrix}_x + g_0x + C_s1$$

Taking the similar proof procedure as Theorem 1, then it can be acquired that system (1) is GES under the designed controller and also possesses a weighted $L_1$-gain performance if the Theorem 2 is satisfied. This completes the proof.

**Remark 4.** In [36], an iterative algorithm is adopted to solve the desired controller, while our proposed method relaxes such an iteration requirement. Thus, the computation burden is reduced. Compared with [41], our proposed MDADT introduce a parameter $\sigma$, which will require extra decision variables than [41], however, it can provide a lower MDADT bound. Therefore, one should choose $\sigma$ carefully to balance the computation burden and the lower MDADT bound. A larger $\sigma$ will result in a lower MDADT bound while consumes more computation resource.

**Remark 5.** For the autonomous system (12), it is necessary that matrix $A_s$ is a Metzler one, while such a condition is relaxed in Theorem 2 for system (1). There is no such restriction on the matrix $A_s$. Theorem 2 shows that as long as the conditions are satisfied under the designed switching signal, then one can always find a set of controller $K_s(\tau)$ to ensure the stability and positive property.

**Remark 6.** Compared with [36], the proposed controller design method eliminates the requirement of gain rank of the controller and reduces the iteration in solving the gain $K_s(\tau)$. It is shown that the proposed method is more efficient.

| TABLE 1 Comparison of our proposed Theorem 1 and Corollary 1 of [41] |
|-----------------|-----------------|-----------------|
| $k_1 = 2, k_2 = 3$ | $k_1 = 4, k_2 = 5$ | $k_1 = 6, k_2 = 7$ |
| Term | Value | Value | Value |
| Corollary 1 of [41] $s = 2$ | $\phi_{\star1} = 1.1552$ | $\phi_{\star2} = 2.3105$ | $\phi_{\star3} = 2.9863$ |
| $\phi_{\star1}$ | 1.8310 | 2.6824 | 3.2432 |
| Theorem 1 $G = 0$ | $\phi_{\star1} = 1.1385$ | $\phi_{\star2} = 2.2937$ | $\phi_{\star3} = 2.9695$ |
| $\phi_{\star1}$ | 1.8143 | 2.6656 | 3.2264 |
| $G = 1$ | $\phi_{\star1} = 1.1217$ | $\phi_{\star2} = 2.2770$ | $\phi_{\star3} = 2.9528$ |
| $\phi_{\star1}$ | 1.7975 | 2.6489 | 3.2097 |
| $G = 2$ | $\phi_{\star1} = 1.1050$ | $\phi_{\star2} = 2.2602$ | $\phi_{\star3} = 2.9360$ |
| $\phi_{\star1}$ | 1.7808 | 2.6327 | 3.1929 |

### 5 | NUMERICAL SIMULATIONS

We will provide two examples to demonstrate the validity of the proposed theorems in this section. Theorem 1 is verified by the first example, and Theorem 2 is verified by the second example.

**Example 1.** In our paper, we will study a SIPS with the parameters given below:

$$A_1 = \begin{bmatrix} -3 & 0.5 \\ 1 & -2 \end{bmatrix}, F_1 = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -4 & 1 \\ 1 & -2 \end{bmatrix}, F_2 = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix}$$

$$D_1 = 0.3, D_2 = 0.2, Q = \begin{bmatrix} -0.25 & 0 \\ 0 & -0.5 \end{bmatrix}$$

In this example, for simulation, the parameters are chosen as $G = 2$, $k_1 = 1.1$, $k_2 = 1.5$, $\xi = 0.99$, $g_1 = 0.1$, $g_2 = 0.3$, then we can get the MDADT $\phi_{\star1} = 0.7521$ and $\phi_{\star2} = 1.2845$, respectively. Under the above conditions, we can get the value of the $L_1$-gain is 0.3854, and corresponding vectors $\xi_s(\sigma), s \in \mathbb{S}_p$, $\sigma \in G$ are obtained as follows:

$$\xi(0) = \begin{bmatrix} 0.1179 \\ 0.1815 \end{bmatrix}, \xi(1) = \begin{bmatrix} 0.1060 \\ 0.1606 \end{bmatrix}$$

$$\xi(2) = \begin{bmatrix} 0.1090 \\ 0.3047 \end{bmatrix}, \xi(0) = \begin{bmatrix} 0.1088 \\ 0.3017 \end{bmatrix}, \xi(1) = \begin{bmatrix} 0.1078 \\ 0.2987 \end{bmatrix}$$

For comparison purpose, the MDADT under our presented method and the existing MDADT of [41] are provided in Table 1. In [41], the proposed MDADT is $\phi_{\star1} > \phi_{\star2} = \ln k_s/g_0$.
in Corollary 1, however, our MDADT is \( \phi_{\ast \ast} > \phi_{\ast} = (\ln \kappa_{2} + G \ln \xi) / \xi_{\ast}, 0 < \xi \leq 1 \).

The comparative results are conducted under \( g_{1} = g_{2} = 0.6 \). It can be seen from Table 1 that the bound of MDADT by our proposed method is smaller and therefore more effective. If \( G = 0 \), the results are in accord with Corollary 1 of [41]. In addition, it is shown that as \( G \) increases, a smaller bound of MDADT can be obtained.

**Example 2.** Consider a SIPS consisting of two subsystems with following parameters

\[
A_{1} = \begin{bmatrix} -5 & 0.5 \\ 1 & 4 \end{bmatrix}, B_{1} = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}, F_{1} = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix},
\]

\[
A_{2} = \begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix}, B_{2} = \begin{bmatrix} 0.2 & 0.2 \\ 0.3 & 0.3 \end{bmatrix}, F_{2} = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix},
\]

\[
C_{1} = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, C_{2} = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}, D_{1} = 0.3,
\]

\[
D_{1} = 0.2, Q = \begin{bmatrix} -0.7 & 0 \\ 0.1 & -0.2 \end{bmatrix}.
\]

With the above parameters, we use Theorem 2 to design the controllers. The parameters are chosen as \( G = 2, \kappa_{1} = 1.1, \kappa_{2} = 1.15, \xi = 0.99, g_{1} = g_{2} = 0.05, t_{1} = 0.3, t_{2} = 1.1 \). For these two subsystems, then, one has that the MDADTs are \( \phi_{\ast \ast 1} = 1.504 \) and \( \phi_{\ast \ast 2} = 2.39 \), respectively. Choose \( \phi_{\ast 1} = 1.6, \phi_{\ast 2} = 2.5, x(0) = [3 \ 1]' \), one can solve the controllers as follows.

\[
K_{1}(0) = \begin{bmatrix} -1.7949 & -0.8423 \\ -1.7340 & -0.8085 \end{bmatrix},
\]

\[
K_{1}(1) = \begin{bmatrix} -1.8131 & -0.8508 \\ -1.7515 & -0.8167 \end{bmatrix}.
\]
In such a case, the value of the $L_1$-gain is $\gamma = 0.3284$. In the simulation, the external interference $w(t)$ is supposed to be $w(t) = \exp(-0.2t)$, and we complete the simulation over time interval $[0, 18\delta]$. The designed switching signal $\varphi(t)$ is displayed in Figure 1. We can see from the middle part of Figure 1, the switching signal $\varphi(t)$ satisfies the MDADT condition. At the same time, the corresponding state trajectories are also shown in Figure 1. The output of the considered SIPS is displayed in Figure 2.

It can be seen that the proposed scheme can effectively resolve the stabilisation problem and is used to design the expected controllers. In addition, it can be seen that the states and output of the designed controllers tend to zero over time.

6 CONCLUDING REMARKS

Stability and weighted $L_1$-gain performance analysis problem of SIPs have been investigated in this paper. By designing a new multiple piecewise-continuous copositive linear Lyapunov function and using the MDADT switching, improved stability conditions which can attain a tighter dwell time bound have been provided for the studied SIPs. Moreover, the controllers which are also both quasi-time dependent and mode-dependent are designed. The accuracy and effectiveness of the proposed method are verified by two examples. In this paper, the subsystems of the considered switched positive systems are all stable. The stability and stabilisation problem of SIPs with part or all unstable subsystems will be our future research topic. Moreover, since the time delays are often encountered in practical engineering, investigating the stabilisation problem for SIPs with time delays will be another research topic.

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REFERENCES
1. Phat, V. N., Ratchagit, K.: Stability and stabilization of switched linear discrete-time systems with interval time-varying delay. Nonlinear Anal. Hybrid Syst. 5(4), 605–612 (2011)
2. Rajchakit, G., et al.: Robust stability and stabilization of uncertain switched discrete-time systems. Adv. Differ. Equ. 2012(1), 134 (2012)
3. Yang, D., et al.: Output tracking control of delayed switched systems via state-dependent switching and dynamic output feedback. Nonlinear Anal. Hybrid Syst. 32, 294–305 (2019)
4. Rajchakit, G.: Switching design for the robust stability of nonlinear uncertain stochastic switched discrete-time systems with interval time-varying delay. J. Comput. Anal. Appl. 16(1), 10–19 (2014)
5. Farina, L., Rinaldi, S.: Positive Linear Systems. John Wiley & Sons, Inc., New York (2011)
6. Benvenuti, L., Farina, L.: A tutorial on the positive realization problem. IEEE Trans. Autom. Control 49(5), 651–664 (2004)
7. Khong, S. Z., et al.: Positive systems analysis via integral linear constraints. In: 2015 54th IEEE Conference on Decision and Control (CDC), pp. 6373–6378. IEEE, Piscataway (2015)
8. Rajchakit, G.: Robust stability and stabilization of nonlinear uncertain switched stochastic discrete-time systems with time interval time-varying delays. Appl. Math. Inf. Sci. 6, 555–565 (2012)
9. Liu, X., et al.: Stability analysis for continuous-time positive systems with time-varying delays. IEEE Trans. Autom. Control 55(4), 1024–1028 (2010)
10. Briat, C.: Stability analysis and stabilization of stochastic linear impulsive, switched and sampled-data systems under dwell-time constraints. Automatica 74279–287 (2016)
11. Margaliot, M.: Stability analysis of switched systems using variational principles: An introduction. Automatica 42(12), 2059–2077 (2006)
12. Du, S., et al.: Stability and $H_\infty$ analysis for switched positive $T$–$S$ fuzzy systems under asynchronous switching. J. Franklin Inst. 355(13), 5912–5927 (2018)
13. Du, S., Qiao, J.: Stability analysis and $H_\infty$ controller synthesis of switched positive $T$–$S$ fuzzy systems with time-varying delays. Neurocomputing 275, 2616–2623 (2018)
14. Jadabhaee, A., et al.: Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans. Autom. Control 48(6), 988–1001 (2003)
15. Niamsup, P., et al.: Guaranteed cost control for switched recurrent neural networks with interval time-varying delay. J. Inequalities Appl. 2013(1), 292 (2013)
16. Mason, O., Shorten, R.: On linear copositive Lyapunov functions and the stability of switched positive linear systems. IEEE Trans. Autom. Control 52(7), 1346–1349 (2007)
17. Sun, X., et al.: Stabilization of positive switched linear systems and its application in consensus of multiagent systems. IEEE Trans. Autom. Control 62(12), 6608–6613 (2017)
18. Zhao, X., et al.: Stability of switched positive linear systems with average dwell time switch. Automatica 48(6), 1132–1137 (2012)
19. Sun, Y.: Stability analysis of positive switched systems via joint linear copositive Lyapunov functions. Nonlinear Anal. Hybrid Syst. 19, 146–152 (2016)
20. Rajchakit, M., et al.: A switching rule for exponential stability of switched recurrent neural networks with interval time-varying delay. Adv. Differ. Equ. 2013(1), 44 (2013)
21. Meng, Z., et al.: Stability of positive switched linear systems: Weak excitation and robustness to time-varying delay. IEEE Trans. Autom. Control 62(1), 399–405 (2016)
22. Liu, X., Dang, C.: Stability analysis of positive switched linear systems with delays. IEEE Trans. Autom. Control 56(7), 1684–1690 (2011)
23. Liu, X.: Stability analysis of switched positive systems: a switched linear copositive lyapunov function method. IEEE Trans. Circuits Syst. II Exp. Briefs 56(5), 414–418 (2009)
24. Blanchini, F., et al.: Co-positive Lyapunov functions for the stabilization of positive switched systems. IEEE Trans. Autom. Control 57(12), 3038–3050 (2012)
25. Hu, M-J., et al.: Stability analysis of switched positive T–S fuzzy systems with time-varying delays. J. Franklin Inst. 355(13), 5912–5927 (2018)
26. Briat, C.: Stability analysis and stabilization conditions for linear positive impulsive and switched systems. Nonlinear Anal. Hybrid Syst. 24, 198–226 (2017)
27. Zhang, J-S., et al.: Stability analysis of impulsive positive systems. IFAC Proc. Vol. 47(3), 5987–5991 (2014)
28. Xiang, W., Xiao, J.: Stability analysis and control synthesis of switched positive systems. Int. J. Robust Nonlinear Control 22(13), 1440–1459 (2012)
29. Liu, T., et al.: Asynchronously finite-time control of discrete impulsive switched positive time-delay systems. J. Franklin Inst. 352(10), 4503–4514 (2015)

30. Rajchakit, G., et al.: Impulsive effects on stability and passivity analysis of memristor-based fractional-order competitive neural networks. Neurocomputing 417, 290–301 (2020)

31. Zhu, B., et al.: Stability analysis and l1-gain characterization for impulsive positive systems with time-varying delay. J. Franklin Inst. 357(13), 8703–8725 (2020)

32. Wang, Y.-W., et al.: Exponential stability of impulsive positive systems with mixed time-varying delays. IET Control Theory Appl. 8(15), 1537–1542 (2014)

33. Liu, L., et al.: Exponential stability of discrete-time positive impulsive switched systems. In: 2018 IEEE International Conference on Information and Automation (ICIA), pp. 517–520.IEEE, Piscataway (2018)

34. Zhang, J., et al.: Stability and stabilization of positive switched systems with mode-dependent average dwell time. Nonlinear Anal. Hybrid Syst. 9(1), 42–55 (2013)

35. Hu, M.-J., et al.: Impulsive effects on the stability and stabilization of positive systems with delays. J. Franklin Inst. 354(10), 4034–4054 (2017)

36. Zhao, X., et al.: Stability analysis and delay control for switched positive linear systems. IEEE Trans. Autom. Control 63(7), 2184–2190 (2018)

37. Horn, R. A., Johnson, C. R: Topics in Matrix Analysis. Cambridge University Press, Cambridge, UK (1991)

38. Zhang, J., et al.: Robust finite-time stability and stabilisation of switched positive systems. IET Control Theory Appl. 8(1), 67–75 (2014)

39. Kaczorek, T.: Positive 1D and 2D Systems, pp. 173–240. Springer Science & Business Media, London (2002)

40. Rami, M. A.: Solvability of static output-feedback stabilization for LTI positive systems. Syst. Control Lett. 60(9), 704–708 (2011)

41. You, L., et al.: Stability of switched positive linear systems with actuator saturation under mode-dependent average dwell time. Int. J. Control Autom. Syst. 18, 817–823 (2020)

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