Diffraction plays an exceptional rôle in DIS off heavy nuclei. First, diffraction into hard dijets is an unique probe of the unintegrated glue in the target. Second, because diffraction makes 50 per cent of total DIS off a heavy target, understanding diffraction in a saturation regime is crucial for a definition of saturated nuclear parton densities. After brief comments on the Nikolaev-Zakharov (NZ) pomeron-splitting mechanism for diffractive hard dijet production, I review an extension of the Nikolaev-Schäfer-Schwiete (NSS) analysis of diffractive dijet production off nuclei to the definition of nuclear partons in the saturation regime. I emphasize the importance of intranuclear distortions of the parton momentum distributions.

1. The Dominance of the Pomeron-Splitting Mechanism for Diffractive Hard Dijets

The point that diffraction excitation probes the wave function of composite systems has been made some 50 years ago by Landau, Pomeranchuk, Feinberg and Glauber - it is very much relevant to QCD too!

The pQCD diagrams for production of diffractive dijets are shown in fig 1. In the Landau-Pomeranchuk diagram (b) the limited transverse momentum $p$ of the quark jet comes from the intrinsic momentum of quarks and/or antiquarks in the beam particle, whereas in the Pomeron splitting diagram (a) hard jets receive the transverse momentum from gluons in the pomeron. As shown by NSS the corresponding diffractive amplitude is proportional to the unintegrated gluon structure function of the target proton, $dG(x, p^2)/d\log p^2$, and the so-called lightcone distribution amplitude for the beam particle.

The NSS dominance of the pomeron-splitting contribution for hard di-
jets has fully been confirmed by the NLO order analysis of Chernyak et al. \(^5\) and Braun et al. \(^6\). The NLO correction to the NSS amplitude is found to be proportional to the asymptotic distribution amplitude and numerically quite substantial, so that the experimental data by E791 \(^7\) can not distinguish between the asymptotic and double-humped distribution amplitudes. According to NSS \(^4\) realistic model distributions do not differ much from the asymptotic one, though. To my view, the only caveat in the interpretation of the NLO results is that the issue of partial reabsorption of these corrections into the evolution/renormalization of the pion distribution amplitude has not yet been properly addressed. Anyway, there emerges a consistent pattern of diffraction of pions into hard dijets and in view of these findings the claims by Frankfurt et al. \(^8\) that the diffractive amplitude is proportional to the integrated gluon structure function of the target must be regarded null and void. Hopefully, some day the E791 collaboration shall report on the interpretation of their results within the correct formalism.

The current status of the theory has been comprehensively reviewed at this Workshop by Chernyak \(^9\) and Radyushkin en lieu of Dima Ivanov \(^10\) and there is no point in repeating the same the third time - the principal conclusions by NSS have been published some years ago and are found in \(^4\). I would rather report new results \(^11\) on the relevance of diffractive DIS to the hot issue of nuclear saturation of parton densities.

### 2. Diffractive and Truly Inelastic DIS off Free Nucleons and Heavy Nuclei

While the above cited NSS papers focused on diffraction on nuclei in the hard regime, in the rest of my talk I would like to discuss the opposite regime of nuclear saturation. Nuclear saturation is an opacity of heavy nuclei for color dipole states of the beam be it a hadron or real, and virtual, photons. The fundamental point about diffractive DIS is the counterin-
tuitive result by Nikolaev, Zakharov and Zoller \(^\text{12}\) that for a very heavy nucleus coherent diffractive DIS in which the target nucleus does not break and is retained in the ground state makes precisely 50 per cent of the total DIS events. Consequently, diffractive DIS is a key to an understanding of nuclear saturation. I note in passing that because of the very small fraction of DIS off free nucleons which is diffractive one, \(\eta_D \lesssim 6-10\%\), there is little room for a genuine saturation effects at HERA. Intuitively, such an importance of diffractive DIS which can not be treated in terms of parton densities in the target casts shadow on the interpretation of the saturation regime in terms of parton densities, which is one of the points from our analysis \(^\text{11}\).

The alternative interpretation of nuclear opacity in terms of a fusion and saturation of nuclear partons goes back to the 1975 papers by Nikolaev and Valentine Zakharov \(^\text{13}\): the Lorentz contraction of relativistic nuclei entail a spatial overlap of partons with \(x \lesssim x_A \approx 1/R_A m_N\) from different nucleons and the fusion of overlapping partons results in the saturation of parton densities per unit area in the impact parameter space. More recently this idea has been revived in the quantitative pQCD framework by McLerran et al. \(^\text{14}\).

We base our analysis on the color dipole formulation of DIS \(^\text{15,3,16,12}\). The total cross section for interaction of the color dipole \(r\) with the target nucleon equals

\[
\sigma(r) = \alpha_S(r)\sigma_0 \int d^2\kappa f(\kappa) \left[1 - \exp(i\kappa r)\right],
\]

where \(f(\kappa)\) is related to the unintegrated glue of the target by

\[
f(\kappa) = \frac{4\pi}{N_c\sigma_0} \cdot \frac{1}{\kappa^4} \cdot \frac{\partial G}{\partial \log \kappa^2}
\]

Figure 2. The pQCD diagrams for DIS off protons (a,b) and nuclei (c). Diagrams (a) and (b) show the 2-gluon tower approximation for the QCD pomeron. The diagram (c) shows the nuclear multiple scattering for virtual Compton scattering off nuclei; the diffractive unitarity cut is indicated.
and is normalized as \( \int d^2 \kappa f(\kappa) = 1 \). Here \( \sigma_0 \) describes the saturated total cross section for very large dipoles. The total virtual photoabsorption cross section for a free nucleon target equals

\[
\sigma_N(x, Q^2) = \langle \gamma^* | \sigma(\mathbf{r}) | \gamma^* \rangle = \int_0^1 dz \int d^2 \mathbf{r} \Psi_{\gamma^*}^*(z, \mathbf{r}) \sigma(\mathbf{r}) \Psi_{\gamma^*}(z, \mathbf{r}) \tag{3}
\]

\[
\frac{d\sigma_N}{d^2 \mathbf{p} dz} = \frac{\sigma_0}{2} \frac{\alpha_s(\mathbf{p}^2)}{(2\pi)^2} \int d^2 \kappa f(\kappa) |\langle \gamma^* | \mathbf{p} \rangle - \langle \gamma^* | \mathbf{p} - \kappa \rangle|^2 \tag{4}
\]

where \( \mathbf{p} \) is the transverse momentum, and \( z \) the Feynman variable, of the leading quark in the final state prior the hadronization, see figs. 2a, b.

Notice that the target nucleon is color-excited and there is no rapidity gap in the final state. This is a starting point for a definition of the small-x sea generated from the glue. The relevant wave functions of the photon are found in \(^2, ^{15, 3}\).

Because of the smallness of the electromagnetic coupling, the diffractive DIS of fig. 1 amounts to quasielastic scattering of CD states of the photon off the target proton \(^{15, 3, 16}\). In this case the target nucleon is left in the color singlet state and there is a rapidity gap in the final state. For the forward diffractive DIS, \( \gamma^* \mathbf{p} \rightarrow (q\bar{q}) + \mathbf{p}' \), with the vanishing \((\mathbf{p}, \mathbf{p}')\) momentum transfer, \( \Delta = 0 \),

\[
\frac{d\sigma_D}{d\Delta^2 dz d^2 \mathbf{p}} \bigg|_{\Delta^2=0} = \frac{1}{16\pi} \frac{1}{(2\pi)^2} |\langle \gamma^* | \mathbf{p} \rangle|^2 \tag{5}
\]

Because \( \eta_D \) for a free nucleon target is so small, in the parton model interpretation of the proton structure functions one customarily neglects diffractive absorption corrections, see however warnings in \(^{16}\).

Now consider DIS off nuclei at \( x \sim x_A \), when interaction of the \( q\bar{q} \) states dominates. The coherent diffractive cross section equals \(^{15, 12}\)

\[
\sigma_D = \int d^2 \mathbf{b} |\gamma^*| \left| 1 - \exp\left[-\frac{1}{2} \sigma(\mathbf{r}) T(\mathbf{b}) \right] \right|^2 |\gamma^*| \tag{6}
\]

Here \( T(\mathbf{b}) = \int dz n_A(z, \mathbf{b}) \) is the optical thickness of a nucleus at an impact parameter \( \mathbf{b} \). The \( \sigma_D \) sums all the unitarity cuts between the exchanged pomerons, so that none of the nucleons of the nucleus is color-excited and there is a rapidity gap in the final state, see fig. 2c.
The inelastic DIS describes all events in which one or more nucleons of
the nucleus are color-excited and there is no rapidity gap in the final state.
I omit a somewhat tricky derivation\(^1\)\(^1\) which is based on the technique
developed in\(^1\)\(^7\) and cite only the final result
\[
\frac{d\sigma_{in}}{d^2pdz} = \frac{1}{(2\pi)^2} \int d^2b \int d^2r'd^2r \exp[ip(r'-r)]\Psi^*(r')\Psi(r)
\left\{\exp[-\frac{1}{2}\sigma(r-r')T(b)] - \exp[-\frac{1}{2}[\sigma(r) + \sigma(r')]T(b)]\right\}.
\tag{7}
\]
The effect of nuclear distortions on the observed momentum distribution
of quarks is obvious: the dependence of nuclear attenuation factors on
\(r, r'\) shall affect strongly a computation of the Fourier transform (7).

Upon the integration over \(p\) one recovers the familiar color dipole
Glauber-Gribov formulas\(^1\)\(^5\),\(^3\),\(^12\) for the inelastic and total cross sections
\[
\sigma_{in} = \int d^2b \langle \gamma^* | 1 - \exp[-\sigma(r)T(b)]|\gamma^* \rangle\tag{8}
\]
\[
\sigma_A = \sigma_D + \sigma_{in} = 2 \int d^2b \langle \gamma^* | 1 - \exp[-\frac{1}{2}\sigma(r)T(b)]|\gamma^* \rangle\tag{9}
\]

### 3. Nuclear Parton Distributions as Defined by Diffraction

The next issue is whether nuclear DIS can be given the conventional parton
model interpretation or not. For the evaluation of the inclusive spectrum
of quarks in inelastic DIS we resort to the NSS representation\(^4\)
\[
\Gamma_A(b, r) = 1 - \exp\left[-\frac{1}{2}\sigma(r)T(b)\right] = \int d^2\kappa \phi_{WW}(\kappa)[1 - \exp(i\kappa r)].\tag{10}
\]

There is a close analogy to the representation (1),(2) in terms of \(f(\kappa)\) and
\[
\phi_{WW}(\kappa) = \sum_{j=1}^{\infty} \nu_A^j(b) \cdot \frac{1}{j!} f^{(j)}(\kappa) \exp\left[-\nu_A(b)\right]\tag{11}
\]
can be interpreted as the unintegrated nuclear Weizsäcker-Williams (WW)
glue per unit area in the impact parameter plane, normalized as
\[
\int d^2\kappa \phi_{WW}(\kappa) = 1 - \exp[-\nu_A(b)].\tag{12}
\]

Here
\[
\nu_A(b) = \frac{1}{2} \alpha_S(r)\sigma_0 T(b)
\]
defines the nuclear opacity and the \(j\)-fold convolutions
\[
f^{(j)}(\kappa) = \int \prod_i d^2\kappa_i f(\kappa_i)\delta(\kappa - \sum_i \kappa_i)
\]
describe the contribution to the diffractive amplitudes from the j split pomeron\textsuperscript{4}. The hard asymptotics of the WW glue has been analyzed by NSS, here I only mention that broadening of convolutions compensates completely the nuclear attenuation effects obvious in the expansion (11) and, furthermore, leads to a nuclear antishadowing for hard dijets\textsuperscript{4}.

A somewhat involved analysis of properties of convolutions in the soft region shows that they develop a plateau-like behaviour with the width of the plateau which expands \( \propto j \). Here I only point out that the gross features of WW glue in the soft region are well reproduced by

\[
\phi_{WW}(\kappa) \approx \frac{Q_A^2}{\pi (\kappa^2 + Q_A^2)^2},
\]

where the saturation scale \( Q_A^2 = \nu_A(b)Q_0^2 \propto A^{1/3} \). The soft parameters \( Q_0^2 \) and \( \sigma_0 \) are related to the integrated glue of the proton in soft region,

\[
Q_0^2\sigma_0 \sim \frac{2\pi^2}{N_c}G_{soft}, \quad G_{soft} \sim 1.
\]

Making use of the NSS representation (10) and the normalization (12), after some algebra one finds for the saturation domain of \( p^2 \sim Q^2 \lesssim Q_A^2 \)

\[
\frac{d\sigma_{in}}{d^2b d^2p dz} = \frac{1}{(2\pi)^2} \int d^2\kappa \phi_{WW}(\kappa) |\langle \gamma^*|p + \kappa \rangle|^2
\]

\[
\frac{d\sigma_D}{d^2b d^2p dz} = \frac{1}{(2\pi)^2} \left[ \int d^2\kappa \phi_{WW}(\kappa) (\langle \gamma^*|p\rangle - \langle \gamma^*|p - \kappa \rangle) \right]^2
\]

\[
\approx \frac{1}{(2\pi)^2} \left[ \int d^2\kappa \phi_{WW}(\kappa) \right] \left| \langle \gamma^*|p\rangle \right|^2 \approx \frac{1}{(2\pi)^2} \left| \langle \gamma^*|p\rangle \right|^2
\]

The last result is obvious from (6) because in this case all the color dipoles in the virtual photon meet the opacity criterion \( \sigma(r)T(b) \gtrsim 1 \), so that the nuclear attenuation terms can be neglected altogether.

4. The interpretation of the results

Following the conventional parton model wisdom, one may try defining the nuclear sea quark density per unit area in the impact parameter space

\[
\frac{d\tilde{q}}{d^2b d^2p dz} = \frac{1}{2} \cdot \frac{Q^2}{4\pi^2\alpha_{em}} \cdot \frac{d[\sigma_D + \sigma_{in}]}{d^2b d^2p dz} = \frac{1}{2} \cdot \frac{Q^2}{4\pi^2\alpha_{em}} \cdot \int dz
\]

\[
\times \left\{ \left[ \int d^2\kappa \phi_{WW}(\kappa) \right]^2 \left| \langle \gamma^*|p\rangle \right|^2 + \int d^2\kappa \phi_{WW}(\kappa) \left| \langle \gamma^*|p + \kappa \rangle \right|^2 \right\}
\]

It is a nonlinear functional of the NSS-defined WW glue of a nucleus. The quadratic term comes from diffractive DIS and measures the momentum
distribution of quarks and antiquarks in the $q\bar{q}$ Fock state of the photon. It has no counterpart in DIS off free nucleons because diffractive DIS off free nucleons is negligible small even at HERA, $\eta_D \lesssim 6-10\%$. The linear term comes from the truly inelastic DIS with color excitation of nucleons of the target nucleus. As such, it is a counterpart of standard DIS off free nucleons, but as a function of the photon wave function and nuclear WW gluon distribution it is completely different from (4) for free nucleons. This difference is entirely due to strong intranuclear distortions of the outgoing quark and antiquark waves in inelastic DIS off nuclei.

Up to now I specified neither the wave function of the photon nor the spin of charged partons - they could well have been scalar or spin-1 ones -, nor the color representation for charged partons. All our results would hold for any weakly interacting projectile such that elastic scattering is negligible small and diffraction excitation amounts to quasielastic scattering of Fock states of the projectile\textsuperscript{15,3}. Now take the conventional spin-$\frac{1}{2}$ partons and the photon’s virtuality $Q^2 \lesssim Q_A^2$ such that the opacity criterion is met for all color dipoles of the photon. Then upon the z-integration one finds for $p^2 \lesssim Q^2$ the plateau-like distribution from diffractive DIS,

$$\left. \frac{d \hat{q}}{d^2 \mathbf{b} d^2 \mathbf{p}} \right|_D = \frac{N_c}{4\pi^4}. \quad (17)$$

The inclusive spectrum of sea quarks from inelastic DIS also exhibits a plateau, but very different from (17):

$$\left. \frac{d \hat{q}}{d^2 \mathbf{b} d^2 \mathbf{p}} \right|_{\text{in}} = \frac{1}{2} \cdot \frac{Q^2}{4\pi^2 \alpha_{\text{em}}} \phi_{WW}(0) \int d^2 \kappa |\langle \gamma^* | \kappa \rangle|^2 = \frac{N_c}{4\pi^4} \cdot \frac{Q^2}{Q_A^2}. \quad (18)$$

The plateau for inelastic DIS extends up to $p^2 \sim Q_A^2$ and this nuclear broadening of momentum distributions of outgoing quarks is an obvious indicator of strong intranuclear distortions. Its height does explicitly depend on $Q^2$ and for $Q^2 \ll Q_A^2$ the inelastic plateau contributes little to the transverse momentum distribution of soft quarks. Still, the inelastic plateau extends way beyond $Q^2$ and its integral contribution to the spectrum of quarks is exactly equal to that from diffractive DIS. The two-plateau structure of the nuclear quark momentum distributions has not been discussed before. For $Q^2 \gtrsim Q_A^2$ the inelastic plateau coincides with the diffractive one, the both extend up to $p^2 \lesssim Q_A^2$. Here we agree with Mueller\textsuperscript{18}.

At this point I notice that after the formal mathematical manipulations with the NSS representation, the total nuclear cross section (9) can be cast in the form

$$\sigma_A = \int d^2 \mathbf{b} \int dz \int \frac{d^2 \mathbf{p}}{(2\pi)^2} \int d^2 \kappa \phi_{WW}(\kappa) |\langle \gamma^* | \kappa \rangle - \langle \gamma^* | \mathbf{p} - \kappa \rangle|^2 \quad (19)$$
which resembles (4): the $p$ distribution evolves from the WW nuclear glue in precisely the same manner as in DIS off free nucleons, which suggests the reinterpretation of the differential form of (19) in terms of the nuclear IS parton density. Furthermore, in the saturation regime the crossing terms can be neglected, while the remaining two terms would coincide with $\sigma_D$ and $\sigma_m$, respectively, giving some support to an extension of the parton model wisdom about the equality of the IS and FS parton densities to nuclear targets too. One should be aware of some caveats, though. First of all, the equality of IS and FS densities comes at the expense of a somewhat weird equating the diffractive FS spectrum to DIS spectrum from the spectator quark of fig. 2b and $\sigma_m$ with the contribution from the scattered quark, the both evaluated in terms of the WW nuclear glue. Second, as pointed out above, (19) implicitly includes the diffractive interactions, which make up 50% of the FS quark yield. Hence the thus defined parton density appears to be highly nonuniversal, recall that the diffractive final states are typical of DIS and would be quite irrelevant, e.g. in nuclear collisions. Furthermore, in sharp contrast to the situation on the proton target, in (19) in the saturation regime, the dominant contribution comes from the region of $p^2 \lesssim \kappa^2$, just opposite to the at not too small $x$ dominant strongly ordered DGLAP contribution from $\kappa^2 \ll p^2$.

One can go one step further and consider interactions with the opaque nucleus of the $q\bar{q}g$ Fock states of the photon. Then the above analysis can be extended to $x \ll x_A$ and the issue of the $x$-dependence of the saturation scale $Q_A^2$ can be addressed following the discussion in 16. I only mention here that as far as diffraction is concerned, the WW glue remains a useful concept and the close correspondence between $\phi_{WW}(\kappa)$ for the nucleus and $f(\kappa)$ for the nucleon is retained. The details of this analysis will be published elsewhere 11. For the shortage of space I didn’t report here the phenomenological consequences.

5. Summary and Conclusions

The NSS representation for nuclear profile function gives a convenient and unique definition of the WW gluon structure function of the nucleus from soft to hard region. The conclusion by NSS that diffraction into hard dijets off nucleons and nuclei is dominated by the pomeron splitting mechanism has been confirmed by NLO calculations.

Coherent diffractive DIS is shown to dominate the inclusive spectrum of leading quarks in DIS off nuclei. The observed spectrum of diffractive leading quarks measures precisely the momentum distribution of quarks in
the $q\bar{q}$ Fock state of the photon, the rôle of the target nucleus is simply to provide an opacity. It exhibits a saturation property and a universal plateau but its interpretation as a saturated density of sea quarks in a nucleus is questionable. The inelastic DIS also gives the plateau-like spectrum of observed quarks, but with the height that depends on $Q^2$. Nuclear broadening of the inelastic plateau is a clearcut evidence for an importance of intranuclear distortions of the spectrum of a struck quark.

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