Reduction of the finite element model of a gas turbine engine rotor using CMS technique

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Abstract. This paper is devoted to the problems of modeling the vibration behavior of rotors of gas turbine engines. The necessity of modeling their dynamic behavior is caused by their strong influence on the strength characteristics of the engine and its resource. The complexity of solving this problem lies in the fact that the design of modern gas turbine engines has many features that do not allow the use of only one-dimensional models, and the application of three-dimensional models requires a lot of computing resources. Thus, it is suggested to use methods of reduction of finite element models for solving the problems in three-dimensional statement. This paper uses the mode component synthesis (CMS) method to reduce the dimensionality of the rotor model. It also compares the speed of solving modal analysis tasks and determining critical frequencies using complete and reduced models. The quality of reduction is determined. Conclusions about the efficiency of the reduction technique aimed at solving rotor dynamics problems are made.

1. Introduction

Developing a new generation of aircraft engines requires a comprehensive solution to the problems associated with the exclusion of dangerous vibrations. The main sources of GTE vibrations are rotors [1]. World experience of GTE development shows that rotor vibration problems should be solved as early as possible during the engine development stages, because otherwise they will require significant design changes, which will lead to substantial time and material costs.

Methods of mathematically modeling rotor systems’ dynamic behavior are widely used to exclude their dangerous vibrations. For a long time, due to low cost and simplicity, one-dimensional beam elements with lumped masses were used to solve most rotor dynamics problems [1,2]. Today three-dimensional finite element models are more widely used. They are seen as more accurate and convenient modeling tools, especially when used in conjunction with modern CAD/CAE systems. However, the overwhelming majority of 3D models is used in linear and axisymmetric statement [3]. Their accuracy in this case is comparable with that of one-dimensional models [4].

However, in the case of complex rotary systems, such as the rotary systems of aircraft gas turbine engines, there are some difficulties in using one-dimensional models. The tendency to reduce the mass of engines has led to a decrease in the stiffness of shafts and discs, the appearance of anisotropy of stiffness of stator parts and the displacement of frequencies of nonaxisymmetric forms of oscillations in the operating frequency range [5, 6]. All this made it necessary to abandon the traditional one-dimensional models and proceed to calculations using three-dimensional finite element models without cyclic symmetry. The decision to avoid the above mentioned problems by using this method is
extremely popular, however, it always leads to a significant increase in the dimensionality of finite element models and increases the calculation time.

One of the most effective ways to reduce the dimension of a finite element model is to apply reduction methods [7]. The following methods are used to simulate the behaviour of rotary systems: the Guyan method, the modal analysis method (MA), method of Component Mode Synthesis (CMS), Balance Implementation Method (BT), Structural Transformation Method (SPT), Systemically Equivalent Reduction and Recovery Process (SEREP) and modified SEREP [7]. The most developed and used in rotary dynamics methods are the Gayane method and the method of modal component synthesis (CMS) [8].

The purpose of this paper is to demonstrate the CMS method for solving problems of modal analysis and determining critical frequencies through the example of a high pressure rotor model of the gas turbine engine BR700, as well as to check the effect of the reduced model application on the speed of calculation. Modelling was performed in Samcef Desktop package.

2. Reduction method

For our purposes we decided to use Component Mode Synthesis (CMS) condensation method. In the framework of this method, the whole rotor model has to be divided into substructures. Some model nodes are used as boundary nodes (constraint nodes) whereas some of the others are used as internal nodes. Usually boundary nodes become master nodes which are retained after condensation and used as interface nodes. In this case all internal nodes become slave nodes which will be dropped after condensation.

This method’s main difference from another popular condensation method – Guyan reduction method – is that in CMS some modal data is added to the reduced matrix to increase the accuracy of condensation. Modal data includes vibration modes and eigen frequencies. If all modal data is included in to the reduced matrix the condensed model will have the same accuracy as original model but the same size either. However, in most cases keeping only a few vibration modes is sufficient to obtain quite accurate results. The more modes are retained the bigger the size of the condensed model. Thus, the full matrix of a primary model has m+n nodes where m is the number of master nodes and n is the number of slave nodes. After the reduction is complete the reduced matrix has m+k nodes where k is the number of vibration modes included in reduction.

The procedure of condensation implies splitting the full matrix according to the choice of master and slave nodes. The example of stiffness matrix:

\[
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\]

The corresponding DOF vector is defined by \((x_1, x_2)\) where:

\[
x_2 = x_2^\prime + x_2^\ddprime
\]

(2)

\[
x_2^\ddprime = -[K_{22}]^{-1} \cdot [K_{21}] \cdot x_1
\]

(3)

\(x_2^\ddprime\) is obtained by solving the following equation:

\[
([K_{22}] - \lambda \cdot [M_{22}]) \cdot x_2^\ddprime = 0
\]

(4)

The modal transformation is conducted as follows: \(x_2^\ddprime = \phi \cdot \eta_2\)

(5)

The generalized coordinates will be:

\[
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix}
x_1 \\
x_2^\prime + x_2^\ddprime
\end{bmatrix} = \begin{bmatrix}
x_1 \\
-[K_{22}]^{-1} \cdot [K_{21}] \cdot x_1 + \phi \cdot \eta_2
\end{bmatrix}
\]

(6)
or can be written as:

\[
\begin{bmatrix}
  x_1 \\
  x_2 
\end{bmatrix} = \begin{bmatrix} I & 0 \\ -[K_{22}]^{-1} \cdot [K_{21}] & \phi \end{bmatrix} \begin{bmatrix} \psi \\ \eta_2 \end{bmatrix}
\]

where \([\psi]\) is called transformation matrix and given by:

\[
[\psi] = \begin{bmatrix} I & 0 \\ -[K_{22}]^{-1} \cdot [K_{21}] & \phi \end{bmatrix}
\]

By using the transformation matrix \([\psi]\), reduced matrices can be expressed as:

\[
[K_{\text{red}}] = [\psi]^T [K] [\psi]
\]

The same expression can be used for the gyroscopic matrix \([G_{\text{red}}]\)

\[
[G_{\text{red}}] = [\psi]^T [G] [\psi]
\]

and other matrices [9].

3. Reduction of the finite element rotor model and evaluation of the reduction quality

Based on the motor cross-section [10], a three-dimensional geometric model of the high-pressure rotor GTE was constructed (see Figure 1a). The modulus of elasticity and density of materials were selected according to the design and taking into account the degradation of properties under the influence of temperature. The blades were taken into account as a mass distributed in the area of the lock, which has the appropriate moment characteristics.
A finite element model, consisting of 15 675 elements (9801 hexahedra and 5874 pentahedra) and 18447 nodes, was built based on the geometric model (see Fig. 1b). This model was reduced using the CMS method. The attachment points to the supports were chosen as master nodes (see Figure 2). The frequency range was 0-1500 Hz. The number of vibration modes and frequencies included in the frequency range was 30.

After condensation, the model consisted of one element and 67 nodes. This element included reduced matrices of masses, stiffnesses and gyroscopic moments. The quality of condensation was checked by carrying out modal analysis for the full and reduced models and comparing their natural frequencies in the same vibration modes (see Table 1). In the modal analysis, the first 10 natural frequencies were determined with the exclusion of vibration modes associated with motion as a rigid body. Boundary conditions are free-free.

| # Mode | Full model | Reduced model | Discrepancy, % |
|--------|------------|---------------|----------------|
| 1      | 346.52     | 347.04        | 0.15           |
| 2      | 346.52     | 347.13        | 0.18           |
| 3      | 443.08     | 443.13        | 0.01           |
| 4      | 469.82     | 469.83        | 0.002          |
| 5      | 490.69     | 490.70        | 0.0008         |
| 6      | 497.59     | 497.65        | 0.01           |
| 7      | 509.83     | 509.83        | 0.0002         |
| 8      | 527.37     | 527.64        | 0.05           |
| 9      | 606.51     | 607.44        | 0.15           |
| 10     | 606.51     | 607.49        | 0.16           |
As you can see from Table 1, the largest frequency discrepancy was 0.15%, which may indicate a high degree of the models’ correspondence to each other. In addition, it should be noted that the calculations were performed in different time periods. Determining modal characteristics of the full model took 13 seconds on a single 2.8 GHz core of PC, and 0.56 seconds for the reduced model.

4. Analysis of the rotor’s critical speeds

The elastic elements with a power stiffness of $10^7$ N/m and a bending stiffness of $10^{10}$ N·m/rad were specified as supports. The design of the front support involves using a ball bearing, so the force stiffness was specified in both radial and axial directions. The rear support uses a roller bearing for which the rigidity was only set in the radial direction. The bending stiffnesses for bending moments around the rotational axis were not considered. The rotor speed range is 8400 rpm (140 Hz) to 16200 rpm (270 Hz). A design range of 0 Hz to 300 Hz in 1 Hz steps was used in the calculation. The number of defined forms is 10. Figure 3 shows Campbell diagrams obtained for complete (a) and reduced (b) rotor models.

![Campbell diagrams for complete (a) and reduced (b) rotor models](image)

The Campbell diagrams show that complete and reduced models have similar distribution of critical modes. Table 2 shows a comparison of the obtained critical frequencies for the complete and reduced rotor models. 5 critical modes are defined in the examined range, none of which falls into the operating speed range.
Table 2 Comparison of critical frequencies of the full and reduced model with respect to the stiffness of the supports

| # Mode | Full model | Reduced model | Discrepancy, % |
|--------|------------|---------------|----------------|
| 1      | 27.69      | 27.69         | 0.0001         |
| 2      | 38.98      | 38.97         | 0.0006         |
| 3      | 38.99      | 38.99         | 0.0006         |
| 4      | 74.52      | 74.52         | 0.0026         |
| 5      | 91.52      | 91.52         | 0.0083         |

The table shows that the largest difference was 0.0083%, which indicates the adequacy of replacing the full model with the reduced one. At the same time, the models showed a significant difference in calculation time: calculation on the full model took 4 minutes 36 seconds, while for the reduced model it took 2 seconds. This may indicate the reduced model’s superiority in performance over the full model when solving iteration problems.

Figure 4 shows the times for solving various problems using the complete and reduced models.

Figure 4. Time of solving the problems of modal analysis (10 eigenforms) and determining critical frequencies (n is the number of calculated forms) using the full (blue) and reduced models (red)

5. Conclusion

The paper demonstrates the technique of reduction of finite element models of GTE rotors through an example of a high-pressure rotor of the double-circuit aviation engine from a foreign manufacturer. The quality of the reduction has been estimated by comparing natural frequencies. It is shown that the method of the component mode synthesis (CMS) allows to carry out qualitative reduction of finite element models of GTE rotors, allowing to considerably accelerate the solving of such problems, as defining modal characteristics of models and calculating critical modes of a rotor. The shown technique can be recommended for use at the designing stage of gas turbine engines manufacturing.

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