Free and forced vibrations of a wavy cylinder in cross flow at low Reynolds numbers

Kai Zhang\textsuperscript{1,2}, Dai Zhou\textsuperscript{1,3†}, Hiroshi Katsuchi\textsuperscript{4}, Hitoshi Yamada\textsuperscript{4}, Zhaolong Han\textsuperscript{1,3} and Yan Bao\textsuperscript{1}

\textsuperscript{1}School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{2}Department of Mechanical Engineering, Florida State University, Tallahassee, FL 32310, USA
\textsuperscript{3}State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{4}Department of Civil Engineering, Yokohama National University, Yokohama, Kanagawa 2408501, Japan

Wavy circular cylinders have been reported to be effective in suppressing the Kármán vortex shedding and reducing hydrodynamic forces. However, their vibrational characteristics in cross flow have not been well understood. By means of direct numerical simulations, the current paper investigates the free and forced vibrations of a wavy cylinder at low Reynolds numbers. The wavy cylinder is optimally designed so that it annihilates the vortex shedding in the fixed configuration for $Re \geq 120$. Nevertheless, it is disclosed that by flexible-mounting the coupled fluid-structure system could still be destabilized, leading to large-amplitude vibrations. The forced vibration reveals that for a fixed amplitude, a critical forcing frequency exists, below which the oscillating wavy cylinder preserves the flow control efficacy, and above which the inherent shedding resurrects in the wake, further leading to the lock-in phenomenon. More interestingly, the developed vortex shedding could persist even without the sustained forcing, implying the existence of the bistable states in the wavy cylinder wake. The observations in the current work suggest that the application of wavy cylinder as drag/vibration-mitigation device in realistic engineering structures should not be encouraged.

1. Introduction

The 3-D forcing technique, which applies varying controls along the spanwise direction, has been recognized as an effective approach to control the wakes of nominally 2-D bluff bodies (Choi \textit{et al.} 2008). Circular cylinders with spanwise sinusoidally varying diameter, referred to as wavy cylinders hereafter, present an omni-directional realization pertaining to this category. Owing to its potential application in engineering structures such as bridge cables and deep-water risers, the flow past wavy cylinders in the fixed configuration has been subjected to extensive investigations (Ahmed \& Bays-Muchmore 1992; Lam \& Lin 2009; Xu \textit{et al.} 2010; Jung \& Yoon 2014; Lin \textit{et al.} 2016). It is agreed that the spanwise periodic undulations could give rise to the counter-rotating streamwise vortices, which inhibit the formation of the Kármán vortex shedding (Lam \& Lin 2009; Hwang \textit{et al.} 2013). When optimally designed, such wavy cylinders could even suppress the vortex shedding completely, leading to maximum reduction of the hydrodynamic forces (Lam \& Lin 2009).

The superior performance of the wavy cylinder instigates its use as a vortex-induced

\† Email address for correspondence: zhoudai@sjtu.edu.cn
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vibration (VIV) suppression device. Such conjecture is established based on the conventional viewpoint that VIV, as its name suggests, is "induced" by the vortex shedding. Thus, in the extreme event of complete suppression of vortex shedding, there is no obvious reason to expect the vibrations (Lam & Lin 2009). The assertion is also espoused by the recent studies on the harbour seal whisker, which features spanwise waviness on an elliptic base cylinder. It is revealed that this specialized undulated morphology is able to repel the self-excited vibrations, allowing the harbour seal to track the prey with higher sensitivity (Hanke et al. 2010; Beem & Triantafyllou 2015). Moreover, wavy tubercles at the leading edge of the flippers of humpback whales have also been shown to delay stall and increase hydrodynamic efficiency (Fish & Battle 1995). All these evidences together, the wavy cylinder seems promising as a low-drag alternative to the conventional VIV-suppression apparatuses such as helical strakes (Triantafyllou et al. 2016).

The above conjecture, however, are confounded by several unsuccessful attempts to achieve control for both drag and vibration at the same time. Owen et al. (2001) studied the flow around a cylinder with spiraling hemispheric bumps. VIV still develops even though periodic shedding cannot be detected in the wake of the fixed body. Similarly, by additive of surface roughness, Pastò (2008) forced the flow to enter the drag crisis regime where the flow is characterized by cessation of the Kármán vortex shedding. Nevertheless, vibrations were still found to persist. The above examples attest to the statement by Dong et al. (2008) that some flow control techniques that are effective for the fixed configuration may not be valid for the dynamic cases. Essentially, being a typical fluid-structure interaction (FSI) problem, VIV could not be mitigated simply by reducing the hydrodynamic forces.

The contradiction mentioned above entails a detailed investigation into the fluid mechanics of the oscillating wavy cylinder in cross flow, which is attempted in the current work by means of numerical simulations. Specifically, to clarify the VIV mitigation efficacy, the free vibration is examined for the flexibly-mounted wavy cylinder, which is optimally designed to fully suppress the vortex shedding in the fixed configuration. The forced vibration of the same wavy cylinder is then inspected to provide interpretations for the phenomenon observed in the free vibrations. Novel discoveries will be unveiled in the course of this study, offering renewed perception of the fluid dynamics of the wavy cylinder as well as its flow control mechanism.

2. Computational setup and method

2.1. Problem description

The geometry of the wavy cylinder is schematically depicted in figure 1. The diameter of the wavy cylinder varies sinusoidally along the spanwise direction $z$ according to

$$D(z) = D_m + 2a \cos(2\pi z/\lambda),$$  

(2.1)

where $D_m$ is the averaged diameter, $\lambda$ and $a$ are termed the geometric wavelength and amplitude, respectively. In the current paper, we assign $\lambda = 2.5D_m$ and $a = 0.175D_m$, which will be shown to fully suppress the Kármán vortex shedding at certain Reynolds numbers. This wavy cylinder is subjected to an uniform incoming flow $U_\infty$ in the $x$ direction. In the study of vortex-induced vibration, the displacement of the cylinder is restricted to the cross flow direction ($y$), where a spring-damper system with stiffness $k$ and damping $c$ is attached. The mass of the cylinder is denoted $m$.

It will prove useful to introduce some non-dimensional parameters. The Reynolds number for the wavy cylinder is defined as $Re = U_\infty D_m/\nu$, where $\nu$ is the kinematic
viscosity of the fluid. The reduced velocity is defined as $U_r = U_\infty / (f_n D_m)$, in which $f_n = \sqrt{k/m(2\pi)}$ is the natural frequency of the mass-spring system. The mass ratio is written as $m^* = 4m/(\rho \pi D_m^2 \lambda)$, where $\rho$ is the density of the fluid. The damping ratio is defined as $\zeta = 2c/\sqrt{km}$.

It is also worthwhile to differentiate several frequencies that will be frequently encountered in the texts that follow. Apart from the natural frequency of the spring-mass system $f_n$, we use $f_0$ to indicate the vortex shedding frequency for the fixed cylinders. $f_d$ stands for the frequency of displacement in the free vibrations. In forced vibration, $f_e$ is the forcing frequency and $f_s$ represents the inherent shedding frequency that is revealed outside the lock-in regime. All the mentioned frequencies have been made dimensionless by scaling themselves with $U_\infty/D_m$.

2.2. Governing equations and numerical method

The flow is governed by the incompressible Navier-Stokes equation, which, in the Arbitrary Lagrangian-Eulerian formulation, reads as

$$\nabla \cdot u = 0,$$  \hspace{1cm} (2.2a)

$$\frac{\partial u}{\partial t} + (u - c) \cdot \nabla u = -\nabla p + \frac{1}{Re} \Delta u,$$ \hspace{1cm} (2.2b)

where $u$ is the velocity vector and $p$ the pressure. $c$ represents the velocity vector of the grid motion. The free vibration of the cylinder is governed by the equation of motion written as

$$\ddot{Y} + \frac{2\pi \zeta}{U_r} \dot{Y} + \frac{4\pi^2}{U_r^2} Y = \frac{2C_L}{m^*},$$  \hspace{1cm} (2.3)

where $Y$ denotes the transverse displacement of the cylinder. $C_L = 2F_L/(\rho U_\infty^2 D_m)$ is the lift coefficients, in which $F_L$ denotes the integrated fluid-induced lift force over the cylinder surface. In a similar vein, the drag coefficient is defined as $C_D = 2F_D/(\rho U_\infty^2 D_m)$, where $F_D$ is the drag force acting on the cylinder. The coupling between the flow and the structure is achieved by the imposition of the no-slip boundary condition on the cylinder surface.

The finite-volume based open-source CFD software OpenFoam is used to carry out the simulations in this work. The PIMPLE algorithm, which is an combination of the SIMPLE and PISO algorithms is employed for the pressure-velocity coupling (Issa 1986). The fluid-structure interaction problem is solved through a partitioned, weakly coupled algorithm in which the two systems are temporally advanced in a staggered fashion. This numerical setup will be shown to generate accurate results compared with the literature.

The cylinder is placed in the centre of a circular computational domain with radius of $30D_m$ and height of $H = 2.5D_m$, with periodic boundary condition specified at the
spanwise ends. The spatial mesh resolution of $N_c \times N_r \times N_z = 140 \times 140 \times 40$ (where $N_c$, $N_r$, $N_z$ represent the grids in the circumferential, radial and spanwise directions) is used. The time-step is set to be $\Delta t U_\infty / D_m = 0.02$. Such mesh resolutions have been chosen based on a careful mesh dependency test, which is omitted here for the sake of brevity. It should be stressed here that, unless otherwise stated, all the simulations are started from the uniform initial condition, i.e., $u = (1, 0, 0)$ at $t = 0^+$.

3. Results and discussion

3.1. Static configuration revisit

Firstly, the flow around the static wavy cylinder is revisited at $Re = 30 \sim 160$ to locate the control-effective regime. The mean drag, rms lift coefficients and the shedding frequencies of the 2-D and wavy cylinders are summarized in figure 2, together with the vortical structures at some representative Reynolds numbers. For $30 \leq Re \leq 110$, the hydrodynamic performance of the wavy cylinder is almost indistinguishable from its 2-D peer. Both cylinders undergo the Hopf bifurcation from the steady state to the periodic shedding state, although the critical $Re$ for bifurcation is slightly delayed for the wavy cylinder. Great control of wake is achieved as $Re$ is increase to 120. The drag of the wavy cylinder suffers from a drastic decrease compared to its 2-D peer and more notably, the lift coefficient drops to zero. Accordingly, the wake at this regime features a pair of steady counter-rotating vortices, which is similar to that at $Re = 30$. It should be noted that the steady state achieved at higher end of $Re$ should be attributed to the prevalence of the stabilizing streamwise vortices over the Kármán vortex shedding, as has been elucidated in previous works (Lam & Lin 2009; Lin et al. 2016; Hwang et al. 2013).
3.2. Free vibration

Now that the Kármán vortex shedding has been completely annihilated in the wake of a fixed wavy cylinder at \( Re \geq 120 \), we proceed to inspect whether under such condition VIV would develop. For the sake of completeness, three representative Reynolds numbers, i.e., \( Re = 30, 100 \) and 150 are examined to delineate a whole picture of VIV at different regimes. We employ relatively low mass ratio of \( m^* = 2.0 \) and low damping ratio \( \zeta = 0.007 \) to encourage large vibrations. The normalized vibration amplitude \( Y_{\max}/D_m \), reduced frequency \( f_d/f_n \) and the phase difference \( \phi \) between the lift and displacement are presented in figure 3. The results from Carmo et al. (2011), who employed the spectral/hp code to study the VIV of a 2-D cylinder with the same structural parameters, are included as an evidence of the accuracy of our numerical approach. At \( Re = 30 \), free vibrations of the 2-D and wavy cylinders alike are observed at a limited range of reduced velocities, although the breadth and amplitude of VIV for the wavy cylinder are relatively smaller. As the Reynolds number increases to \( Re = 100 \), since the flow in the static configuration already breeds vortex shedding, VIV could naturally be expected. Except for the slightly smaller vibration amplitude of the wavy cylinder, not much difference could be discerned in the response curves between the two cylinders at \( Re = 100 \).

The situation at \( Re = 150 \) is interesting since this is the regime where the flow control mechanism is active. However, in the flexibly-mounted cases, large-amplitude vibrations comparable to the 2-D cylinder are still observed for the wavy cylinder at a wide range of \( U_r \). The response curves of the wavy cylinder are shifted slightly to the higher reduced velocity, nevertheless, the major features of VIV such as the initial-lower branches, frequency lock-in, phase jump, etc., are all manifested. The numerical evidence presented here clearly suggests that the satisfactory flow control efficacy of the fixed wavy cylinder does not carry over to the flexibly-mounted cases, at least not at low mass and
Figure 4. Lift spectrum of the 2-D cylinder undergoing forced vibration at (a) $Re = 30$ and (b) $Re = 150$

damping. The validity of this conclusion holds true for higher $Re$ of $O(10^3)$, as reported in our previous work (Zhang et al. 2017).

The above numerical results have also been supported by the recent experimental investigation by Assi & Bearman (2018), who examined the VIV of an undulated elliptical cylinder and asserted that vibration occurs as a result of the re-correlated spanwise shedding. In view of the above, the spanwise waviness presents another typical case testifying to the statement that the flow control techniques designed for the static configuration are not necessarily effective for flexibly mounted structures (Dong et al. 2008). As for the vibration-suppression reported in the whisker of the harbour seals (Hanke et al. 2010), it is suspected that the mechanism might be ascribed to the more streamlined elliptic base cylinder. This is partially supported by observation in Beem & Triantafyllou (2015) that both the elliptical cylinder and the whisker model exhibited negligible vibrations when positioned with their streamlined direction. The spanwise waviness might only play a side role in reducing the hydrodynamic forces.

It is compelling to relate the destabilization of the wavy cylinder at $Re = 150$ to that at $Re = 30$, since in both cases the vibrations develop out of steady flow. Recently, linear stability analysis (Meliga & Chomaz 2011; Zhang et al. 2015; Mittal 2016; Yao & Jaiman 2017) on the coupled FSI systems have identified two modes of interest, i.e., the wake mode responsible for the Kármán vortex shedding, and the structural mode for the vibration of the spring-mass system. At sub-critical $Re$, although the wake mode is damped, the structural mode could be destabilized at a certain range of reduced velocities, providing the initial impetus for the vibration. With this mechanism, it is not surprising that wavy cylinder could develop large-amplitude vibration in the event of full suppression of vortex shedding. In what follows, we employ forced vibration to confirm the existence of the wake mode and further inspect its property.

3.3. Forced vibration

In this section, the cylinder is given a prescribed transverse sinusoidal motion with fixed amplitude $A_e = 0.2D_m$, and forcing frequencies ranging from $f_e = 0.05$ to $0.28$. The lift spectrum for the 2-D cylinder at $Re = 30$ and 150 are firstly presented in figure 4 to facilitate the discussion of the wavy cylinders. For $Re = 150$, the spectrum is featured by two distinctive peaks at the forcing frequency $f_e$ and the inherent shedding frequency $f_s \approx 0.17 \sim 0.18$. A lock-in regime, in which $f_s$ submits to $f_e$, is observed as the forcing frequency approaches $f_0 = 0.185$. On the other hand, the spectrum at the sub-critical Reynolds number of 30 appears manipulative, as only a single peak exactly at the forcing frequency is illuminated. As a matter of fact, although the wake mode is damped at such low $Re$, it is still able to lock itself onto the forcing and exhibits periodic shedding at
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Figure 5. Lift spectrum of the wavy cylinder undergoing forced vibration. Instantaneous vortical structures are presented at several forcing frequencies, with transparent gray standing for iso-surface of $\omega_z = \pm 0.5$, and red and blue for $\omega_x = 0.3$ and -0.3, respectively.

a range of $f_c$ (Buffoni 2003). For the wavy cylinder at the sub-critical ($Re \lesssim 50$) and periodic shedding ($60 \lesssim Re \lesssim 110$) regimes, the responses under the same forcing are found to be qualitatively similar with the 2-D cylinder, thus will not be discussed here.

The lift spectrum of the wavy cylinder under forced vibration at $Re = 150$ is presented in figure 5, along with the vortical structures represented by iso-surfaces of $\omega_x$ and $\omega_z$ at selected forcing frequencies. At small forcing frequencies, the flow control efficacy of the wavy cylinder is well preserved. The wake vortical structure at $f_c = 0.05$ features a pair of elongated spanwise vortex loops, although they sway gently owing to the periodic excitation of the cylinder. The streamwise vortices remain in the near wake and play its role in suppressing the roll-up of the spanwise vortices. As a result, only a single peak at the forcing frequency is illuminated in the lift spectrum, which is similar to the situation of 2-D cylinder at $Re = 30$. The vortical structure at $f_c = 0.08$ appears more unsteady, nevertheless, the spanwise vorticity sheets extend long into the wake and still a single peak is observed in the spectrum. The situation is significantly different when it comes to $f_c \geq 0.09$. In the lift spectrum, apart from the forcing frequency, another peak reminiscent of the inherent shedding frequency emerges at $f_s = 0.15 \sim 0.18$, signaling the resurrection of the Kármán vortex shedding. This is also manifested in the corresponding wake vortical structures, where the roll-up of the free shear layers occurs much closer to the cylinder compared with $f_c = 0.08$. Along with the spanwise rollers, the periodic shedding of the streamwise vortices are also observed. With the revival of the inherent shedding, the wake of the wavy cylinder is able to lock onto the forcing at $f_c = 0.14 \sim 0.18$. Further increasing the forcing frequency again reveals $f_s$, although its value is slightly smaller than that at smaller $f_c$. Similar to the free vibration, we have confirmed that the resurrection of the wake mode in forced vibration is also a feature at much higher Reynolds number (Zhang et al. 2018).

The observations described herein clearly suggest the existence of the wake mode in the wake of the wavy cylinder at $Re = 150$. Fundamentally different from the situation at the sub-critical Reynolds numbers, where the wake mode is absolutely damped, at $Re = 150$ the wake mode is conditionally stable, as it could be provoked once the forcing exceeds criticality. With the revival of such wake mode, the wavy cylinder exhibits similar hydrodynamic performance with the 2-D cylinder. Despite the peculiar feature of the wake mode, we note that at the incipient stage, the VIV of the wavy cylinder observed in
the previous section should still be imputed to the destabilization of the structural mode in the coupled FSI system, as is the case for the VIV at sub-critical Reynolds numbers. This is because the revival of the wake mode requires relatively high forcing, which is not available in the flow if the vibrations stem from uniform initial condition.

3.4. Bistability in the fixed wake

More interestingly, with the resurrection of wake instability, the developed shedding vortices could persist even without sustained forcing. To prove this, we manually terminate the forced vibration when the cylinder reaches its maximum displacement, at which the velocity of the cylinder becomes zero, so that a smooth transition from the dynamic simulation to a static one is achieved. The wake then develops by itself starting from the initial condition dictated by the forced vibration. The time histories of $C_D$ obtained by this procedure are presented in figure 6(a) for selected forcing frequencies. Below $f_e = 0.09$, the drag coefficients eventually converge to a fixed value of $C_D = 1$ (state I) as reported in §3.1. On the other hand, for $f_e \geq 0.09$, for which the inherent shedding in the forced wake has revived, $C_D$ arrives at the oscillation state with mean value of around 1.31 (state II), signaling the persistence of the periodic shedding and loss of flow control effectiveness. The oscillatory state II could also be reproduced at $Re = 150$ by starting the simulation with initial conditions attained from the unsteady shedding states at $Re = 60 \sim 110$. Such strong dependence of the long-term flow states on the initial condition is in glaring contrast to the conventional 2-D bluff body flows, for which the effect of initial condition is usually limited in time, and the long-term state depends only on the Reynolds number (Laroussi et al. 2014). This discovery further casts shadow on the applicability of the wavy cylinder in realistic engineering structures, even in the sense of drag reduction, as the cables or risers are inevitably affected by the omnipresent external disturbances such as turbulence buffeting, anchorage movement, etc., and may easily cease to be control-effective.

Based on the above observation, a schematic diagram for the bistable states of the wavy cylinder wake at $Re = 150$ is conceivable. As shown in figure 6(b), both the steady state I and oscillatory state II are stable so that there exists a barrier between the two states. For weakly disturbed flow, the oscillations in the flow are damped and the flow eventually converges to the steady state I. However, the barrier is easily overcome when the initial condition is sufficiently perturbed, causing the flow to overshoot to the oscillatory state II.

Bistability in bluff body flow has also been reported in other configurations, such as the hysteresis loops in the transitional regimes in free and forced vibrations (Khalak...
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& Williamson 1999; Blackburn & Henderson 1999) and the flip-flop phenomenon in flow around two side-by-side cylinders (Mizushima & Ino 2008). In both examples, the flow involves multiple mechanisms or modes that compete with each other, rendering it highly sensitive to the initial condition or prone to mode switching. Equivalently, in the case of the wavy cylinder, the characteristics of the wake is determined by the two competing mechanisms, i.e., the streamwise vortices that try to stabilize the flow, and the spanwise vortex shedding (absolute instability) that destabilizes the flow. While the latter mechanism is ever-present at super-critical Reynolds numbers, the streamwise vortices are susceptible to external disturbances. It could be imagined that once the inherent shedding is triggered, the steady streamwise vortices that are responsible for the wake stabilization are compelled to oscillate by the spanwise rollers and could no longer return to its initial position. As a result, the wake surrenders to the periodic Kármán vortex shedding and the flow control efficacy is lost. In such an event, since the wake mode has already been brought alive, the driving mechanism of VIV does not need to invoke the sub-critical Re analogy as discussed in §3.3.

4. Concluding remarks

Direct numerical simulations have been conducted to study the vibrational characteristics of the wavy cylinder in cross flow at low Reynolds numbers. Having recognized the extraordinary flow control efficacy of the wavy cylinder in the fixed configuration, the current work provided numerical evidence that the flow control mechanism does not carry over the the flexibly mounted cases. Deeper insights are obtained by perturbing the flow with sinusoidal structural oscillations with varying frequencies. It is disclosed that the control mechanism of the wavy cylinder could only be preserved with weak forcing. As the forcing frequency exceeds criticality, the inherent shedding frequency that has been concealed in the fixed configuration revives, further leading the flow to lock-in. The resurrected inherent shedding vortices could persist even without sustained forcing, implying the existence of the bistable states in the wavy cylinder wake. The insights obtained from the current work suggest that the flow control efficacy of the wavy cylinder requires delicate flow condition and is not robust. Its application in engineering structures as a drag/vibration mitigation device should not be encouraged.

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