Abstract

Masses of the lowest multiplets of $c, b$-mesons in $S$- and $P$-wave states with quantum numbers $J^{PC}: 0^{-+}, 1^{--}, 0^{++}, 1^{-+}, 1^{++}, 2^{++}$ are obtained with the help of dispersion $N/D$ method of heavy quark effective theory. The results are in a good agreement with experimental data. Radiative decay widths for some states mentioned above were calculated.

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1. Introduction

The mass spectrum of the charmed and beauty mesons is the subject of investigation of many theoretical papers using the different approaches. The unquestionable advantage of the potential approach to the description of heavy mesons is its simplicity and clarity. One can calculate the masses, the radiative and the annihilation decay widths using a chosen potential [1-7].
This allows to determine the meson wave functions in a wide range of distances and to describe the quark interaction dynamics. The method of QCD sum rules has been successfully used to determine the masses of the light and heavy mesons [8,9]. Owing to a small relative difference of the resonance masses the bottomonium contribution of $\Upsilon$-meson and ground states in other channels dominates only for the momenta with $n \geq 20$. The most difficulty of applying the sum rule method for bottomonium - is to take into account the relativistic corrections, i.e. the next term in the expansion in $n^{-1}$. For this it is necessary to sum up all the terms of the type of $(\alpha_s n^{1/2})^k n^{-1}$, or $\alpha_s^2 (\alpha_s n^{1/2})^k$ (since $\alpha_s n^{1/2} \sim 1$), which side by side with the pure relativistic effects (described by the Breit-Fermi Hamiltonian) contain the radiation corrections of the order of $\alpha_s^2$. This difficulty reveals, for example, in the calculating of masses $1P$-bottomonium levels. In this case the quark model can be used, which takes into account the relativistic effects of the quark interaction dynamics.

After the discovery of the $D_{sJ}^*(2317)$ [10] and $D_{sJ}^*(2460)$ [11] mesons a number of experimental investigations of these narrow resonances was carried out [10-14]. Their masses are 100 MeV smaller than those predicted by the quark model. A large number of theoretical papers was devoted to the study of structure of these mesons. The interpretation of the $D_{sJ}^*(2317)$, $D_{sJ}^*(2460)$ as the lowest $P$-wave $\bar{c}s$-states [15, 16] is natural. However another nature of these states is also supposed: $DK$-molecule [17], tetraquark states [18, 19], $D\pi$-atom [20] and others. In our study these states are considered as the $\bar{c}s$-mesons. The masses of all states with the open charm and the charmonium states with the quantum numbers $J^{PC} : 0^{--}, 1^{+-}, 0^{++}, 1^{++}, 1^{++}$ have been calculated (Table 1). The analogous computations have been carried out to the $b$-mesons including the $u, d, s, c, b$-quarks (Table 2). We have used the dispersion $N/D$ method of heavy quark effective theory.

Section II is devoted to the computation of the mass spectrum of $c, b$-
mesons with the quantum numbers $J^{PC} = 0^{-+}, 1^{--}, 0^{++,} 1^{+-}, 1^{++}, 2^{++}$ with the help of the dispersion N/D method of heavy quark effective theory (Tables 1, 2).

The radiative decay widths of heavy mesons are calculated in Section III with the help of the dispersion integration method.

The results of the $c, b$-meson characteristics computation and the problems, which remain open, are discussed in the Conclusion.

2. N/D method in heavy quark effective theory

The amplitude of the $N/D$ method is defined as:

$$A(s) = \frac{N(s)}{D(s)}, \quad D(s) = 1 - B(s),$$

$$B(s) = \int_{(m_1 + m_2)^2}^\Lambda ds' \rho(s') \frac{G(s')}{s' - s}.$$  \hspace{1cm} (1)

Here $m_1$ is the heavy quark $Q$ ($Q = c, b$) mass, $m_2$ is the light quark $q$ ($q = u, d, s; q\bar{Q}$–meson) mass or the heavy quark $Q$ ($Q = c, b; Q\bar{Q}$–meson; $m_2 \leq m_1$) mass, $\Lambda$ is the cutoff parameter, $\rho(s)$ is the two-particle phase space, which is given by formula [21-23]:

$$\rho(s) = \left( \alpha \frac{s}{(m_1 + m_2)^2} + \beta + \frac{1}{s\delta} \right) \sqrt{\frac{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}{s}},$$

$$\alpha, \beta, \delta$$ are the coefficients, which are different for different states (Table 3), $G(s)$ are smooth functions of $s$, which depend on the quantum numbers $J^{PC}$.
of the states (Table 4). The amplitude $A(s)$ has only one singularity point $s = (m_1 + m_2)^2$, therefore using the small value of ratio $\frac{m_2}{m_1}$ ($m_2$ is the light quark mass) Eq.(3) can be rewritten in the form:

$$\rho(s) = \left( \alpha \frac{s}{(m_1 + m_2)^2} + \beta + \frac{1}{s} \delta \right) \sqrt{\frac{s - (m_1 + m_2)^2}{s}}. \quad (4)$$

We take into account, that the expression $\sqrt{\frac{s - (m_1 - m_2)^2}{s}} \sim 1$ in the middle point of the physical region $(m_1 + m_2)^2 + \Lambda$. We obtain an expression for the function $B(s)$, which describes accurately the heavy meson spectrum and has also the same structure as the explicit expression for $B$-function [21-23]:

$$B(s) = G(s) \left[ \left( \alpha \frac{s}{(m_1 + m_2)^2} + \beta + \frac{1}{s} \delta \right) \sqrt{\frac{s - (m_1 + m_2)^2}{s}} \times \right.$$

$$\times \left( \ln \frac{\Lambda - (m_1 + m_2)^2}{\Lambda} - \sqrt{\frac{s - (m_1 + m_2)^2}{s}} \left( \frac{1}{\sqrt{\Lambda - (m_1 + m_2)^2}} + i \pi \right) + \alpha \frac{\Lambda (\Lambda - (m_1 + m_2)^2)}{(m_1 + m_2)^2} + \right. \right.$$

$$+ 2 \delta \frac{\sqrt{\Lambda - (m_1 + m_2)^2}}{\Lambda} + \left( \beta + \alpha \left( \frac{s}{(m_1 + m_2)^2} - \frac{1}{2} \right) \ln \frac{1 + \sqrt{\frac{\Lambda - (m_1 + m_2)^2}{\Lambda}}}{1 - \sqrt{\frac{\Lambda - (m_1 + m_2)^2}{\Lambda}}} \right) \left. \right.$$

$$\left. \right] \quad (5)$$

Upon explicitly calculating the function $B$ and analytically continuing it in $s$ to the region $(m_1 + m_2)^2 < s < \Lambda$, we arrive at an ultimate expression (5) for $B(s)$.

By substituting the calculated function $B(s)$ into (1), we find the roots $s_0$ of Eq (1). Equation (1) can be solved numerically, but it is more convenient to use a graphical method to find all roots $s_0$ in the complex-plane region under investigation. The imaginary and the real part of the expression $1 - B(\text{Re}s, \text{Im}s)$ correspond individually to a certain three-dimensional surface above the $s$ plane. A section of each such surface by the $s$ plane gives a
certain set of planar curves in the $z=0$ plane. The intersection of these sets of planar curves gives the set of roots $s_0$. Upon plotting the imaginary and the real part calculated by a computer individually, one finds their intersection and visually seeks the roots in the complex-plane region under consideration. The accuracy of the search may be improved by enlarging the scale of the graph. Considering that $s_0 = M^2 - iM\Gamma$, we can obtain the mass $M$ and the decay width $\Gamma$ as:

$$M^2 = \text{Re} s_0, \quad M\Gamma = -\text{Im} s_0. \quad (6)$$

The calculations show that there is only one root $s_0$ in the complex-plane region of interest; at a given value of $\Lambda$ the mass and the width of the state corresponding to this root depend only on $g$ that is a dimensionless constant from the expression for $G(s)$.

Four parameters were used for the calculation of the $q\bar{c}$-meson masses: $g_{qc}, \Lambda_{qc}, \Delta_q, \Delta_c$, where $g_{qc}$ determines the constant $G(s)$, $\Lambda_{qc}$ - a parameter giving the cutoff; $\Delta_q, \Delta_c$ - $P$-wave mass shifts of the $q$- and $c$-quarks. The effective parameters $\Delta_q$ and $\Delta_c$ of quark mass shifts $m_{\text{eff}} = m + \Delta$ [24] lead to the scale change of quark potential and allow to obtain bound states in $P$-wave.

The values of the light quark mass ($m_{u,d} = 0.385$ GeV, $m_s = 0.510$ GeV) and the heavy quark masses ($m_c = 1.645$ GeV, $m_b = 4.94$ GeV) were taken from [25]. Four parameters mentioned above were determined by using the experimental data on the masses of $D$, $D^*$, $D_1$, $D^*_2$-mesons. For the computation of $c\bar{c}$-mesons two more parameters were used: $g_c, \Lambda_c$, where $g_c$ determines $G(s)$, $\Lambda_c$ - a parameter giving the cutoff. These two parameters were determined by using the experimental data on masses of the $\eta_c$, $J/\psi$-mesons. Analogous two parameters were used for the $q\bar{b}$-mesons: $g_{qb}, \Lambda_{qb}$, where $g_{qb}$
determines $G(s)$, $\Lambda_{qb}$ — a parameter giving the cutoff. The parameters were determined by the experimental values of the $B$, $B^*$-meson masses.

For the $c\bar{b}$-mesons one parameter — $g_{cb}$ was used, which is determined by the experimental value of the $B_c$-meson mass [26].

$\Lambda_{cb}$ is derived from $\Lambda_c$ and $\Lambda_b$:

$$\Lambda_{cb} = \frac{1}{4}(\sqrt{\Lambda_c} + \sqrt{\Lambda_b})^2.$$  

Three parameters were used for the $b\bar{b}$-mesons: $g_b$, $\Lambda_b$, $\Delta_b$, where $g_b$ determines $G(s)$, $\Lambda_b$ — a parameter giving the cutoff. $\Delta_b$ determines the $p$-wave mass shift of the $b$-quark. These parameters are fitted on the experimental values of the $\eta_b(1S)$, $\Upsilon(1S)$, $\chi_{b2}(1P)$-meson masses. The results of the computation are shown in Tables 1, 2. It is seen from Table 1 that for $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ states the calculation results on masses are 100 MeV lower than experimental values. It seems necessary to take into account not only $\bar{c}s$, but tetraquark $\bar{c}sn\bar{n}$ states also. Such consideration makes possible to increase the masses of the $D_{sJ}^*$ and $D_{sJ}$ states.

3. Radiative decays of heavy mesons

Calculations of radiative decay widths are carried out with the masses, which are taken from the experimental tables [26].

$$V \to \gamma P$$

The decay width $V \to \gamma P$: ($V$ corresponds to the notation of a vector meson, $P$ - pseudoscalar meson)

$$\Gamma_{V \to \gamma P} = \frac{\alpha}{24} \left( 1 - \left( \frac{M_P}{M_V} \right)^2 \right) \frac{1}{M_V} \frac{|F_{V \to \gamma P}(0)|^2}{|F_V(0)||F_P(0)|^2},$$  

(7)
where $\alpha = e^2/4\pi = 1/137$, $M_V$ is the vector meson mass, $M_P$ is the pseudoscalar meson mass. We are using the dispersion integration method [27]

$$F_{V \rightarrow \gamma P}(q^2) = \frac{\pi}{(m_1 + m_2)^2} \int \frac{ds}{\pi} \int \frac{ds'}{\pi} \frac{disc_{\gamma}disc_{\gamma'} F_{V \rightarrow \gamma P}(s, s', q^2)}{(s - M_V^2)(s' - M_P^2)}. \quad (8)$$

In the center-of-mass frame [28,29]:

$$F_{V \rightarrow \gamma P}(q^2) \sim k_{\text{max}} \int_0^{k_{\text{max}}} dk \int_{-1}^1 dy \cdot k^2 \cdot \left( D_1(s, s', q^2) S_{V \rightarrow \gamma P}(s, s', q^2) \cdot e_1 f_1(q^2) \times \frac{G_P G_V}{(s - M_V^2)(s' - M_P^2)} + (1 \leftrightarrow 2) \right), \quad (9)$$

where $S_{V \rightarrow \gamma P}(s, s', q^2)$ is a spin factor, $G_P$ and $G_V$ are the vertices, which practically do not depend on energy, $e_1$ is the charge of quark 1, $f_1$ is its form factor,

$$k_{\text{max}} = \sqrt{\frac{\Lambda}{4} + \frac{(m_1^2 - m_2^2)^2}{4\Lambda}} - \frac{1}{2}(m_1^2 + m_2^2), \quad \Lambda = \Lambda_{qQ} \frac{(m_1 + m_2)^2}{4},$$

where $k_{\text{max}}$ is the quark momentum cutoff,

$$s = (\sqrt{m_1^2 + k^2} + \sqrt{m_2^2 + k^2})^2, \quad s' = s - d_1 q^2 - \frac{|k y|}{c_1} \sqrt{q^2 s(q^2 - 4c_1)},$$

$$d_1 = \frac{1}{c_1}(\sqrt{(m_1^2 + k^2)(m_2^2 + k^2)}k^2 y^2), \quad c_1 = m_1^2 + k^2(1 - y^2),$$

$$D_1 = \frac{s}{2c_1 \sqrt{m_2^2 + k^2}} \left( 1 + \frac{k y \sqrt{-q^2}}{\sqrt{(m_1^2 + k^2)(4c_1 - q^2)}} \right).$$
\[ F_V(q^2) \sim \int_{k_{\text{max}}} \int_0^1 dy \int_{-1}^1 dk \cdot k^2 \cdot \left( D_1(s, s', q^2)S_{V}^{\text{tr}}(s, s', q^2) \cdot e_1 f_1(q^2) \times \frac{G_V^2}{(s - M_V^2)(s' - M_V^2)} + (1 \leftrightarrow 2) \right), \quad (10) \]

\[ F_P(q^2) \sim \int_{k_{\text{max}}} \int_0^1 dy \int_{-1}^1 dk \cdot k^2 \cdot \left( D_1(s, s', q^2)S_{P}^{\text{tr}}(s, s', q^2) \cdot e_1 f_1(q^2) \times \frac{G_P^2}{(s - M_P^2)(s' - M_P^2)} + (1 \leftrightarrow 2) \right), \quad (11) \]

The expressions for the spin factors [28, 29]:

\[ S_{V \rightarrow P}(s, s', q^2) = 8 \left\{ m_1 \frac{(s' - s)q^2}{(s' - s)^2 - 2q^2(s' + s) + q^4} - \frac{m_2^2 q^2}{(s' - s)^2 - 2q^2(s' + s) + q^4} \right\}, \]

\[ S_{P}^{\text{tr}}(s, s', q^2) = -q^2 \left\{ \frac{(s' + s - q^2 + 2(m_1^2 - m_2^2))(s + s' - 2(m_1 - m_2))}{(s' - s)^2 - 2q^2(s' + s) + q^4} + \frac{s' + s - \frac{(s' - s)^2}{q^2} + 2(m_2^2 - m_1^2)}{(s' - s)^2 - 2q^2(s' + s) + q^4} \right\}, \quad (12) \]
\[ S_V^{tr}(s, s', q^2) = -\frac{2}{3} q^2 \left( 1 + \frac{s + s' - q^2 + 2(m_1^2 - m_2^2)}{4q^2s - (s' - s - q^2)^2} \right) \left[ \frac{s+s'-q^2+4m_1m_2+\frac{1}{2}m_2-m_1}{ss'} \times \right. \\
\left. \times \left\{ (s+s'-q^2) \left( (m_2+m_1)(s+s') - \frac{1}{2}(m_2-m_1)(s+s'-q^2-2(m_2+m_1)^2) \right) - \\
- 2(m_2+m_1)(2ss'+(m_2^2-m_1^2)(s+s')) \right\} + \\
\right. \\
\left. + \frac{1}{2} \frac{m_2-m_1}{ss'} \left[ m_1(s+s')+m_2q^2+\frac{1}{2}(m_2+m_1)(s+s'-q^2) \right] \right] \]

The numerical results:

\[ B^*(5325) \rightarrow \gamma B(5279) \]

\[ k_{max} = 1.163 \text{ GeV}, \Lambda = 39.70 \text{ GeV}^2 \]

\[ \Gamma_{B^*(5325) \rightarrow \gamma B(5279)} \approx 0.8 \text{ keV} \]

The decay width \( A \rightarrow \gamma V \): (\( V \) corresponds to the notation of a vector meson, \( A \)– the axial meson)

\[ \Gamma_{A \rightarrow \gamma V} = \frac{\alpha}{12} \left( 1 - \left( \frac{M_V}{M_A} \right)^2 \right) \frac{1}{M_A} M_A^3 + 6M_A^2M_V^2 + M_V^4 \frac{|F_{A \rightarrow \gamma V}(0)|^2}{2M_V^2 |F_V(0)||F_A(0)|}, \]

where \( M_V \) is the vector meson mass, \( M_A \) is the axial meson mass.

\[ F_{A \rightarrow \gamma V}(q^2) = \int_{(m_1+m_2)^2}^{\infty} \frac{ds}{\pi} \int_{(m_1+m_2)^2}^{\infty} \frac{ds' \; disc_s disc_s' F_{A \rightarrow \gamma V}(s, s', q^2)}{\pi (s - M_A^2)(s' - M_V^2)}, \]

In the center-of-mass frame [28,29]:

\[ F_{A \rightarrow \gamma V}(q^2) \sim \int_0^{k_{max}} dk \int_{-1}^1 dy \cdot k^2 \cdot \left( D_1(s, s', q^2)S_{A \rightarrow \gamma V}(s, s', q^2) \cdot e_1 f_1(q^2) \times \right. \\
\left. \times \frac{G_A G_V}{(s - M_A^2)(s' - M_V^2)} + (1 \leftrightarrow 2) \right), \]
where \( S_{A \rightarrow \gamma V}(s, s', q^2) \) is the spin factor, \( G_A \) and \( G_V \) are the vertices, which practically do not depend on energy, \( e_1 \) is the charge of quark 1, \( f_1 \) is its form factor,

\[
F_A(q^2) \sim \int_0^{k_{\text{max}}} dk \int_{-1}^1 dy \cdot k^2 \cdot D_1(s, s', q^2) S_A^{tr}(s, s', q^2) \cdot e_1 f_1(q^2) \times
\]

\[
\times \frac{G_A^2}{(s - M_A^2)(s_1' - M_A^2)} (1 \equiv 2), \tag{16}
\]

The expressions for the spin factors [28, 29]:

\[
S_{A \rightarrow \gamma V}(s, s', q^2) = \frac{1}{z_{11}(s, s', q^2)} \cdot 4i \cdot \sqrt{2 \over 3s} \left\{ m_1^2 \left( \frac{(s' - s)}{2} - m_2^2 q^2 \right) + \frac{q^2}{2} \left( m_2^2 q^2 + \right. \right.
\]

\[
+ \left. \left. 2 \left( \frac{s'}{2} - \frac{m_1^2 + m_2^2}{2} \right) - \frac{m_1^2 + m_2^2}{2} \right\} \right\} \left\{ \frac{(m_1 + m_2)(s - s' + q^2)(s + s' - q^2 + 2m_1^2 - 2m_2^2)}{4q^2 s' - (s - s' + q^2)^2} + \right.
\]

\[
+ \frac{m_1 - m_2}{s'} \left( \frac{s + s' - q^2}{2} - (m_1 + m_2)^2 \right) \right\}, \tag{17}
\]

where

\[
z_{11}(s, s', q^2) = \frac{-(s^2 + 6ss' + s'^2) + q^2(2s + 2s' - q^2)}{2s'},
\]

\[
S_A^{tr}(s, s', q^2) = \frac{4}{3q^2 s' - (s - s' + q^2)^2} \left\{ m_1^2 \left( \frac{(s' - s)}{2} - m_2^2 q^2 \right) + \right.
\]

\[
+ \frac{q^2}{2} \left( m_2^2 q^2 + 2 \left( \frac{s'}{2} - \frac{m_1^2 + m_2^2}{2} \right) \left( \frac{s}{2} - \frac{m_1^2 + m_2^2}{2} \right) \right) \right\}. \tag{18}
\]
The numerical results:

\[ D_{sJ}(2460) \to \gamma D^*_s(2112) \]

\[ k_{\text{max}} = 1.139 \text{ GeV}, \Lambda = 12.97 \text{ GeV}^2 \ (m_1 = 2030.5 \text{ MeV}, \ m_2 = 565 \text{ MeV}) \]
\[ \Gamma_{D_{sJ}(2460) \to \gamma D^*_s(2112)} \approx 5.73 \text{ keV}. \]

\[ S \to \gamma V \]

The decay width \( S \to \gamma V \): (\( V \) corresponds to the notation of a vector meson, \( S \)— the scalar meson)

\[
\Gamma_{S \to \gamma V} = \frac{\alpha}{2} \left( 1 - \left( \frac{M_V}{M_S} \right)^2 \right) \frac{1}{M_S} \frac{|F_{S \to \gamma V}(0)|^2}{|F_V(0)||F_S(0)|},
\]

(19)

where \( M_V \) is the vector meson mass, \( M_S \) is the scalar meson mass, which is above threshold, i.e. \( M_S > (m_1 + m_2)^2 \). The Lehmann representation [30] is used here

\[
F_{S \to \gamma V}(q^2) = \int_{(m_1 + m_2)^2}^{\infty} ds \int_{(m_1 + m_2)^2}^{\infty} ds' \frac{1}{\pi} \frac{disc'disc's'F_{S \to \gamma V}(s, s', q^2)}{D_R(s)D_R(s')}.
\]

(20)

The inverse propagator

\[
D_R(m^2) = m^2_R - m^2 + Re\left( \Pi_R(m^2) \right) - \Pi_R(m^2),
\]

(21)

where \( Re(\Pi_R(m^2_R) - \Pi_R(m^2) \) takes into account the finite width corrections of the scalar \( R \) resonance, which take into account the contribution of the two-particle virtual intermediate \( ab \) states to self-energy of the \( R \) resonance,

\[
\Pi_R(m^2) = \sum_{ab} \Pi_R^{ab}(m^2).
\]

(22)
In real axis of $m^2$

$$Im(\Pi_R(m^2)) = m\Gamma_R(m) = m\sum_{ab} \Gamma(R \rightarrow ab, m)\theta(m - m_a - m_b), \quad (23)$$

where

$$\Gamma(R \rightarrow ab, m) = \frac{g_{R_{ab}}^2}{16\pi m} \rho_{ab}(m^2), \quad (24)$$

is the width of the $R \rightarrow ab$ decay, $m = m_{ab}$ is the invariant mass of the $ab$ state, $g_{R_{ab}}$ is the coupling constant of the $R$ resonance with the two-particle $ab$ state, and

$$\rho_{ab}(m^2) = \sqrt{\left(1 - \frac{m_+^2}{m^2}\right)\left(1 - \frac{m_-^2}{m^2}\right)}, \quad m_\pm = m_a \pm m_b. \quad (25)$$

Propagators under discussion satisfy the Lehmann representation

$$\frac{1}{D_R(m^2)} = \frac{1}{\pi} \int_{(m_a + m_b)^2}^{\infty} \frac{Im(D_R^{-1}(m^2))}{m^2 - \bar{m}^2 - i\epsilon} \bar{m}^2 \, d\bar{m}^2 =$$

$$= \frac{1}{\pi} \int_{(m_a + m_b)^2}^{\infty} \frac{\bar{m}\Gamma_R(\bar{m})}{D_R(\bar{m}^2)|^2(\bar{m}^2 - m^2 - i\epsilon)} \, d\bar{m}^2 \quad (26)$$

in the wide domain of coupling constants of the scalar $R$ resonance with the two-particle $ab$ states.

In the one-channel case when the decay threshold is lower than the $R$ resonance one has:

$$\Pi_R(m^2) = \Pi_R^{ab}(m^2) = \frac{g_{R_{ab}}^2}{16\pi^2} \left\{ \frac{(m^2 - m_+^2)m_-}{m^2} \ln \frac{m_a}{m_b} + \right.$$

$$\left. + \rho_{ab}(m^2) \left[i\pi + \ln \frac{\sqrt{m^2 - m_-^2} - \sqrt{m^2 - m_+^2}}{\sqrt{m^2 - m_-^2} + \sqrt{m^2 - m_+^2}} \right] \right\} \quad (27)$$
at \( m \geq m_a + m_b, \ (m_a \geq m_b) \).

In the center-of-mass frame:

\[
F_{S \rightarrow \gamma V}(q^2) \sim \int_{0}^{k_{max}} dk \int_{-1}^{1} dy \frac{k^2}{D_R(s)} \times \left( D_1(s, s', q^2)S_{S \rightarrow \gamma V}(s, s', q^2) \cdot e_1 f_1(q^2) \cdot \frac{G_S G_V}{s' - M_V^2} + (1 \leftrightarrow 2) \right),
\]

(28)

\[
F_S(q^2) \sim \int_{0}^{k_{max}} dk \int_{-1}^{1} dy \frac{k^2}{D_R(s)} \times \left( D_1(s, s', q^2)S_{tr}^{s}(s, s', q^2) \cdot e_1 f_1(q^2) \cdot \frac{G_S^2}{s' - M_S^2} + (1 \leftrightarrow 2) \right),
\]

(29)

The expressions for the spin factors:

\[
S_{S \rightarrow \gamma V}(s, s', q^2) = \frac{1}{s'}(m_2 - m_1)(s + m_1^2 - m_2^2 - q^2)(s' - (m_1 + m_2)^2) - \]

\[
- m_1 \left( 4m_1m_2 - \frac{5}{2}m_1^2 - \frac{3}{2}m_2^2 + 2s' \right) + \frac{1}{s'} \frac{2s'q^2 + (s' - s + q^2)(s + m_1^2 - m_2^2 - q^2) - s'(s' - s + q^2)^2}{4s'q^2 - (s' - s + q^2)^2} \times \]

\[
\times \{ 2s'(m_1(s' - s) - m_2q^2) + (m_2 - m_1)(s' - s + q^2)(s' - (m_1 + m_2)^2) \},
\]

(30)

\[
S_{tr}^{s}(s, s', q^2) = 4q^2(s's' - m_2(m_1 + m_2)(s + s' - q^2) - (m_1 + m_2)^2(m_1^2 - m_2^2)) \]

\[
4q^2s - (s - s' + q^2)^2
\]

The numerical results:

\[
D_{sJ}^*(2317) \rightarrow \gamma D_s^*(2112)
\]

\[
k_{max} = 1.135 \text{ GeV}, \ \Lambda = 8.94 \text{ GeV}^2, \ g_{R \rightarrow ab} \approx 1.5 \text{ GeV}^2.
\]

\[
\Gamma_{D_{sJ}^*(2317) \rightarrow \gamma D_s^*(2112)} \approx 2.2 \text{ keV}.
\]

For comparison the calculation results on these radiative decays in other models are presented in table 5.
4. Conclusion

The relativistic quark model for the study of the heavy meson spectroscopy is constructed in the framework of the dispersion approach. The N/D method of heavy quark effective theory was used for the computation of the mass spectrum of the charmed and beauty mesons: the multiplets $J^{PC} = 0^{-}, 1^{--}$ ($S$-wave) and $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$ ($P$-wave). The $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ states attract a particular attention. The masses of these states are obtained to be 100 MeV smaller as compared to the experimental data (Table 1). It seems not enough to account the contribution of the $\bar{c}s$ configuration only, but it is necessary to take into account a composite of tetraquark state [31] to obtain the masses close to the experimental data. The radiative decay widths have been calculated for these mesons using the experimental data on their masses. The radiative decay widths of these states are in good agreement with the results of other quark models (Table 5) [32, 33]. Table 5 contains also the calculation results in the framework of the Model of QCD Sum Rules [34] and of the Vector Dominance Model [35]. In the present work we managed to describe all $S$- and $P$-wave multiplets of the charmed and beauty mesons using a small number of parameters. The proposed method allows to satisfy in the case of $b$-mesons the qualitative ratio for gluon coupling constants: $g_{qb} < g_b$. The use of the dispersion N/D method in heavy quark effective theory allows to consider the spectroscopy of exotic heavy mesons and baryons taking into account the threshold singularity.

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References

1. J. M. Richard, Phys. Lett. B 139, 408 (1984).
2. A. A. Martin, Phys. Lett. B 100, 511 (1981).
3. J. L. Basdevant, and S. Boukraa, Z. Phys. C 30, 103 (1986).
4. H. W. Crater, and P. V. Alstine, Phys. Rev. D 37, 1982 (1988).
5. K. G. Boreskov and A. B. Kaidalov, Yad. Fiz. 37, 174 (1983).
6. C. Quigg, J. L. Rosner, Phys. Rep. C 56, 167 (1979).
7. A. A. Bykov, I. M. Dremin and A. V. Leonidov, Usp. Fiz. Nauk 143, 3 (1984).
8. M. A. Shifman, Usp. Fiz. Nauk 151, 193 (1987).
9. M. B. Voloshin, Yad. Fiz. 29, 1368 (1979).
10. Babar Collaboration, B. Aubert et al., Phys. Rev. Lett. 90, 242001 (2003).
11. Belle Collaboration, P. Krokovny et al., Phys. Rev. Lett. 91, 262002 (2003).
12. Belle Collaboration, Y. Mikami et al., Phys. Rev. Lett. 92, 012002 (2004).
13. Belle Collaboration, A. Drutskoy et al., Phys. Rev. Lett. 94, 061802 (2005).
14. Babar Collaboration, G. Calderini et al., hep-ex/0405081.
15. P. Colangelo et al., hep-ph/0512083.
16. W. Wei, P-Z. Huang and S.-L. Zhu, hep-ph/0510039.
17. T. Barnes, F. E. Close, and H. J. Lipkin, Phys. Rev. D 68, 054006 (2003).
18. H.-J. Cheng and W.-S. Hou, Phys. Lett. B 566, 193 (2003).
19. K. Terasaki, Phys. Rev. D 68, 011501 (2003).
20. A. P. Szszepaniak, Phys. Lett. B 567, 23 (2003).
21. V. V. Anisovich, S. M. Gerasyuta, I. V. Keltuyala, Yad. Fiz. 38, 200.
22. V. V. Anisovich, S. M. Gerasyuta, Yad. Fiz. 44, 174 (1986)
23. V. V. Anisovich, S. M. Gerasyuta and A. V. Sarantsev, Int. J. Mod. Phys. A 6, 625 (1991).
24. S. M. Gerasyuta, I. V. Keltuyala, Yad. Fiz. 54, 793 (1991)
25. S. M. Gerasyuta, A. V. Sarantsev, Yad. Fiz. 52, 1483 (1990)
26. W.-M. Yao et al. (Particle Data Group), J. Phys. G 33, 1 (2006).
27. V. V. Anisovich, A. V. Sarantsev, Yad. Fiz. 45, 1479 (1987)
28. A. V. Anisovich, V. V. Anisovich and V. A. Nikonov, hep-ph/0108186.
29. A. V. Anisovich, V. V. Anisovich, V. N. Markov, M. A. Matveev, N. A. Nikonov, and A. V. Sarantsev, hep-ph/0509042.
30. N. N. Achasov and A. V. Kiselev, hep-ph/0405128.
31. S. M. Gerasyuta, V. I. Kochkin, Yad. Fiz. 61, 1504 (1998)
32. S. Godfrey, Phys. Lett. B 568, 254 (2003).
33. W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D 68, 054024 (2003).
34. P. Colangelo, F. De Fazio and A. Ozpineci, Phys. Rev. D 72, 074004 (2005).
35. P. Colangelo, F. De Fazio, and R. Ferrandes, Mod. Phys. Lett. A 19, 2083 (2004).
Table 1. The lowest states of the charmonium and states with open charm masses.

|       | $m(0^{−+})$, GeV | $m(1^{−−})$, GeV | $m(0^{++})$, GeV |
|-------|------------------|------------------|------------------|
| $uar{c}$ $dar{c}$ | $D$ 1.867 (1.867) | $D^*$ 2.010 (2.010) | $D_0^*$ 2.103 (2.352±0.05) |
| $sar{c}$ | $D_s$ 1.916 (1.969) | $D_s^*$ 2.105 (2.112) | $D_{sJ}^*$ 2.223 (2.317) |
| $car{c}$ | $\eta_c(1S)$ 2.980 (2.980) | $J/\psi$ 3.097 (3.097) | $\chi_{c0}(1P)$ 3.393 (3.415) |

|       | $m(1^{−−})$, GeV | $m(1^{++})$, GeV | $m(2^{++})$, GeV |
|-------|------------------|------------------|------------------|
| $uar{c}$ $dar{c}$ | $D_1$ 2.302 (2.420) | $D_1(2430)$ 2.430 (2.430) | $D_2^*$ 2.460 (2.460) |
| $sar{c}$ | $D_{sJ}$ 2.350 (2.460) | $D_{s1}$ 2.514 (2.536) | $D_{s2}$ 2.559 (2.573) |
| $car{c}$ | 3.726 (-) | $\chi_{c1}(1P)$ 3.824 (3.511) | $\chi_{c2}(1P)$ 3.863 (3.556) |

The model parameters: $g_{qc} = 1.26$, $\Lambda_{qc} = 7.90$, $\Delta_q = 0.035$ GeV, $\Delta_c = 0.404$ GeV, $g_c = 2.92$, $\Lambda_c = 5.49$.

The experimental data are given in parentheses [26].
Table 2. The lowest states of the bottomonium and states with open bottom masses.

|        | \( m(0^{-+}), \text{GeV} \) | \( m(1^{--}), \text{GeV} \) | \( m(0^{++}), \text{GeV} \) |
|--------|-----------------------------|-----------------------------|-----------------------------|
| \( u\bar{b} \, d\bar{b} \) | \( B \) 5.279 (5.279) | \( B^* \) 5.325 (5.325) | 5.508 (-) |
| \( s\bar{b} \) | \( B_s \) 5.339 (5.370) | \( B_s^* \) 5.419 (5.417) | 5.629 (-) |
| \( c\bar{b} \) | \( B_c \) 6.400 (6.4±0.39±0.13) | 6.477 (-) | 6.818 (-) |
| \( b\bar{b} \) | \( \eta_b(1S) \) 9.330 (9.300±0.040) | \( \Upsilon(1S) \) 9.460 (9.460) | \( \chi_{b0}(1P) \) 10.139 (9.859) |

|        | \( m(1^{++}), \text{GeV} \) | \( m(2^{++}), \text{GeV} \) |
|--------|-----------------------------|-----------------------------|
| \( u\bar{b} \, d\bar{b} \) | 5.534 (-) | 5.579 (-) |
| \( s\bar{b} \) | 5.595 (-) | 5.664 (-) |
| \( c\bar{b} \) | 6.973 (-) | 7.046 (-) |
| \( b\bar{b} \) | 9.767 (-) | \( \chi_{b1}(1P) \) 9.870 (9.893) | \( \chi_{b2}(1P) \) 9.912 (9.912) |

The model parameters: \( g_{qb} = 3.36, \quad \Lambda_{qb} = 5.70, \quad g_{cb} = 2.92, \quad g_b = 4.07, \quad \Lambda_b = 4.9, \quad \Delta_b = 0.232 \text{ GeV}. \)

The experimental data are given in parentheses [26].
Table 3. Coefficients $\alpha$, $\beta$, $\delta$.

| $J^{PC}$ | $\alpha$ | $\beta$ | $\delta$ |
|----------|----------|---------|---------|
| 0--      | $\frac{1}{2}$ | $-\frac{1}{2} (m_1 - m_2)^2$ | 0 |
| 1--      | $\frac{1}{3}$ | $\frac{1}{6} - \frac{1}{3} (m_1 - m_2)^2$ | $-\frac{1}{6} (m_1 - m_2)^2$ |
| 0++      | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| 1+-      | $\frac{1}{2}$ | $-\frac{1}{2} (m_1 - m_2)^2$ | 0 |
| 1++      | $\frac{1}{2}$ | $-\frac{1}{2} (m_1 - m_2)^2$ | 0 |
| 2++      | $\frac{3}{10}$ | $\frac{1}{5} \left( 1 - \frac{3}{2} (m_1 - m_2)^2 \right)$ | $-\frac{1}{5} (m_1 - m_2)^2$ |
Table 4. Functions $G(s)$.

| $J^{PC}$ | $G(s)$ |
|----------|--------|
| 0$^-$ $+$ | $\frac{8g}{3} - \frac{4g}{3} (m_1 + m_2)^2$ $s$ |
| 1$^-$ $-$ | $\frac{4g}{3}$ |
| 0$^+$ $+$ | $-\frac{8g}{3} + \frac{4g}{3} (m_1 - m_2)^2$ $s$ |
| 1$^+$ $-$ | $\frac{8g}{3} - \frac{4g}{3} (m_1 + m_2)^2$ $s$ |
| 1$^+$ $+$ | $\frac{4g}{3}$ |
| 2$^+$ $+$ | $\frac{4g}{3}$ |
Table 5. The radiative decay widths of the charmed mesons (in keV)

| Decay                  | LCSR [34] | VMD [35] | QM [32] | QM [33] | [*] |
|------------------------|-----------|----------|---------|---------|-----|
| $D_{sJ}^*(2317) \to \gamma D_s^*$ | 4-6       | 0.85     | 1.9     | 1.74    | 2.2 |
| $D_{sJ}(2460) \to \gamma D_s^*$      | 0.6-1.1   | 1.5      | 5.5     | 4.66    | 5.73|

The symbol [*] denotes the results of this study.