Stability of the Bragg glass phase in a layered geometry

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Abstract

We study the stability of the dislocation-free Bragg glass phase in a layered geometry consisting of coupled parallel planes of d=1+1 vortex lines lying within each plane, in the presence of impurity disorder. Using renormalization group, replica variational calculations and physical arguments we show that at temperatures $T < T_G$ the 3D Bragg glass phase is always stable for weak disorder. It undergoes a weakly first order transition into a decoupled 2D vortex glass upon increase of disorder.
An important problem is to understand how impurity disorder affects the translational long range order of periodic media such as the Abrikosov vortex lattice [1], charge density waves (CDW) [2], Wigner crystals [3] and magnetic bubbles [4]. A purely elastic theory starting from the crystal, assuming the absence of topological defects, such as dislocations, predicts an algebraic decay of translational order [5–8] and that the resulting glass phase has divergent Bragg peaks [6]. This is in marked contrast with other models of glasses in superconductors, such as the “vortex glass” [9] or “hexatic glass” [10,11] where dislocations were argued to be present due to disorder beyond the Larkin length and translational order decays rapidly. It was therefore claimed [9] that the results of the elastic theory are unstable to topological defects. However, in [6] it was shown, using energy arguments, that as a consequence of the slow decay of correlations unbounded dislocations should not be present at equilibrium and at weak disorder. The resulting phase, called “Bragg glass”, is a glass with topological order. It has two important length scales, the Larkin-Ovchinnikov length $R_c$ which characterizes pinning, and the translational order correlation length $R_a$. $R_a$ can be much larger than $R_c$ at low temperature if the scale $r_f$ at which disorder varies (i.e the superconducting coherence length $\xi$) is small compared to the lattice spacing $a$. Since $R_c$ is renormalized upwards by thermal fluctuations, one expects these two lengths to be of the same order of magnitude at higher temperatures and near melting. Such a phase is compatible with present decoration and neutron experiments [11,13]. The random field 3D XY model being described by a very similar model, the same arguments in favor of a quasi-ordered vortex free phase apply at weak disorder. The existence of this topologically ordered phase in the 3D XY model now finds some support from numerical simulations [14]. Also, detailed scaling arguments and a variational calculation supporting the existence of the Bragg glass were constructed recently in [19].

In [6] the domain of validity of the elastic theory in presence of disorder was estimated to be $R_a \gtrsim a$. As we showed, this implies slow decay of correlations and, using energy arguments [6], the absence of dislocations. The Bragg glass is therefore a self-consistent solution until $R_a \gtrsim a$. Beyond, the Bragg glass must undergo a transition into an amorphous state containing topological defects, upon increase of disorder or field. The nature of this state is unknown, various possibilities are a strongly disordered vortex glass, an hexatic glass, or a strongly pinned liquid. A scenario suggested in [6], is that this transition occurs around the field $H_{tr}$ of the observed tricritical point [13] which separates, on the thermal melting line, first order melting at low field from second order vortex glass transition at higher fields. From usual 3D melting considerations, one expects that as long as $R_c \sim R_a \gg a$ the weakly disordered Bragg glass should melt through a first order transition upon raising temperature, while the amorphous state melts
through a second order transition. One can now find increasing experimental support for this scenario both in YBCO and BSCCO: the tricritical point can be lowered in field by controlled increase of disorder \[15\], the ”second peak” around \(H_{tr}\) appears as a sharp separation of two different types of behaviour in magnetic and transport properties \[16,17\] and neutrons \[18\] also indicate a sudden loss of translational order at about \(H_{tr}\).

The above self-consistency of the Bragg glass phase relies on arguments about dislocations. Also, to study analytically the transition between the two glass phases in \(d = 3\) is difficult since one must be able to describe the proliferation of topological defects. An interesting model geometry to study dislocation loops in presence of disorder was proposed by Kierfeld et al. \[19\], who also derived more detailed energy arguments in favour of the existence of the Bragg glass up to \(R_a \sim a\). The model consists in a model of coupled parallel layers of \(d = 1 + 1\) lines constrained to lie within the planes and allows to describe some topological defects. It is relevant to the case of a magnetic field directed along the \(ab\) plane. This model was previously studied in the absence of disorder \[20\], and found to exhibit a transition at \(T_c\) between a 3D coupled solid for \(T < T_c\) and a decoupled high temperature 2D phase. Since the flux lines are confined to the planes, an elastic description can still be used at weak disorder and therefore the methods of \[6\] to describe the Bragg glass can be extended to this particular geometry. In the present paper we give a detailed solution of this model using both the replica variational method and a renormalization group (RG) treatment. We show that at weak disorder the Bragg glass phase is stable, and that the criterion given in \[6\] properly estimates the domain of stability of the model. Similar conclusions were reached by Kierfeld et al. \[19\] using physical argument and a variational method.

Interacting flux lines confined to one plane in \(d = 1 + 1\), of average spacing \(a\) can be described in the continuum limit by introducing a smooth labelling phase field \(\phi = 2\pi u/a\), where \(u\) is the displacement field in the elastic limit. Since there are no topological defects within one plane the density of lines in each plane can be decomposed \[3,21\] as \(\rho(x) = \rho_0(1 - \nabla u(x) + \sum_{p \neq 0} \exp(ip(2\pi x + \phi(x))))\) where \(p\) are integers. The model studied here consist of \(N\) planes of vortices coupled via their local density and in the presence of weak impurity disorder modelled by a random potential \(V(x)\) with correlations \(\langle V_k V_{-k} \rangle = \Delta_k\). The model is defined by the following random field Hamiltonian \[22\]:

\[
H = \int d^2x \sum_{ij} \frac{1}{2} K_{ij}^{-1} \nabla \phi_i(x) \cdot \nabla \phi_j(x) - \sum_i \eta_i(x) \cdot \nabla \phi_i(x) - \mu_{ij} \cos(\phi_i(x) - \phi_j(x)) - \sum_{p,i} \text{Re}(\zeta_i^p e^{ip\phi_i(x)})
\]

(1)
with isotropic gaussian disorder $\tilde{\zeta}_i^a(x)\tilde{\zeta}_j^{a*}(x') = 4T g^a_{ij}\delta(x-x')$ and $\eta_i^a(x)\eta_j^{a*}(x') = T\Delta_{ij}\delta(x-x')$. The original model in its simplest isotropic version, is defined by a smaller number of parameters:

$$K^{-1}_{ij} = c\delta_{ij}; \quad \mu_{ij} = \frac{\mu}{2}(\delta_{i+1,j} + \delta_{i-1,j})$$

$$g^p_{ij} = g^p\delta_{ij}; \quad \Delta_{ij} = \Delta\delta_{ij}$$

$g^p = \rho_0^2\Delta_p/T$ is proportional to the disorder in plane $i$ with Fourier component close to $2\pi p/a$, $\Delta = \rho_0^2\Delta_0/T$ is the long wavelength disorder coupling to slow variations of density $\rho$. The extra terms present in $[1]$, i.e the longer range couplings $\mu_{ij}$, interplane disorder correlations $g^p_{ij}$, long wavelength couplings and disorder correlators between planes, are generated by renormalization. They must thus be added to the general model which includes all most relevant terms allowed by symmetry.

We start by applying the replica Gaussian Variational Method (GVM) to the replicated version of the starting Hamiltonian $[2]$:

$$H = \int d^2x \sum_{ai} \frac{1}{2}c(\nabla\phi^a_i)^2 - \mu \cos(\phi^a_i(x) - \phi^a_{i+1}(x))$$

$$- \sum_{abl} \Delta \nabla\phi^a_i \cdot \nabla\phi^b_l - g \cos(\phi^a_i(x) - \phi^b_l(x))$$

where $a = 1, \ldots, n$ is the replica index and one takes the limit $n \to 0$ at the end. For clarity have kept only the lowest harmonic $g = g^1$, but the general case will be discussed below. Note that for a single harmonic model $R_c \sim R_a [3]$. We study $N$ coupled planes with $N \to \infty$ and use periodic boundary conditions. All planes are then equivalent and it is convenient to introduce the Fourier transform along $z$, $\phi^a_j(q) = \int_{-\pi/l}^{\pi/l} dq e^{i(jq_z)}$. We denote $\int d^2q/(2\pi)^2$ by $\int_q$ and $l \int_{-\pi/l}^{\pi/l} dq_z/(2\pi)$ by $\int_{q_z}$ where $l$ is the interplane distance.

One can approximate $[4]$ by the variational Hamiltonian $[2]$ $H_0 = \frac{1}{2} \int_{qz} G(q,q_z)^{-1}_{ab} \phi^a(q,q_z) \phi^b(-q,-q_z)$. The propagator $G(q,q_z)^{-1}_{ab}$ is determined by optimizing the variational free energy $F_{var} = F_0 + \langle H - H_0 \rangle_{H_0}$. It is parametrized by a connected (thermal) part and a (disorder) self-energy part as $G_{ab}^{-1} = \sigma_{ab} G_c^{-1} - \Delta q^2$ with $\sigma_{ab} = 0$. The self-consistent saddle point equations $[2]$ read:

$$G_c(q,q_z) = 1/(c\varrho^2 + 2\tilde{\mu}(1 - \cos(q_z l)))$$

$$\tilde{\mu} = \mu e^{-\frac{1}{2}B^{aa}_{ii+1}} \sigma_{a\neq b} = 2g e^{-\frac{1}{2}B^{ab}}$$

$$B^{aa}_{ii+1} = 2T \int_{q,q_z} (1 - \cos(q_z l)) G_{aa}(q,q_z)$$

$$B^{ab}_{ii} = 2T \int_{q,q_z} G_{aa}(q,q_z) - G_{ab}(q,q_z)$$

4
the latter being the intraplane and off-diagonal interplane mean squared phase fluctuations, respectively. The quantity $\tilde{\mu}$ is the effective coupling between the planes and when it is non zero the problem is effectively 3D, whereas $\tilde{\mu} = 0$ is the signature of decoupling and corresponds to unbound dislocations being present between the planes.

These equations have several types of solutions, corresponding to the three phases of the model: high-temperature (replica symmetric), 3D coupled elastic glass with full Replica Symmetry Breaking (RSB) and 2D decoupled glass (one step RSB). Each of these solutions is qualitatively similar to the ones obtained in $[6]$, for $d = 3$ and $d = 2$. Here however the model naturally exhibits transitions between these solutions.

We first study the full RSB solution, as in $[6]$. Parametrizing $\sigma_{ab} \rightarrow \sigma(u)$ and $B_{ab} \rightarrow B(u)$, $0 < u < 1$, one has $\sigma(u) = 2g e^{-1/2B_{ii}(u)}$. The inversion formula for hierarchical matrices gives:

$$B_{ii}(u) = B_{ii}(u_c) + \int_{q,q_2}^{u_c} dv \left( \frac{\sigma'(v)}{c q^2 + 2\tilde{\mu}(1 - \cos(q_j l))} + [\sigma](u) \right)^2$$

(8)

and $B_{ii}(u_c) = 2T \int_{q,q_2} (cq^2 + 2\tilde{\mu}(1 - \cos(q_j l)))^{-1}$. Differentiating the saddle point equation with respect to $u$, integrating over momenta and using $[\sigma]' = u \sigma'$, one finds the full RSB solution for $u < u_c$:

$$\sigma(u) = \frac{2\tilde{\mu} v/\tilde{T}}{\sqrt{1 - v^2}} \ ; \quad [\sigma](u) = 2\tilde{\mu} \left( 1 + \frac{1}{\sqrt{1 - v^2}} - 1 \right)$$

(9)

where the variables $\tilde{T} = T/T_c$ and $v = u/\tilde{T}$ have been introduced for convenience, $T_c = 4\pi c$ being the transition temperature of the 2D glass. For $u > u_c$ one has $[\sigma](u) = \Sigma_1$. It remains to find the breakpoint $u_c$, which is determined together with the effective coupling $\tilde{\mu}$, by the equations $\sigma(u_c) = 2g e^{-B_{ii}(u_c)/2}$, and the equation for $\tilde{\mu}$. $B_{ii+1}$ and the diagonal correlations $G_{aa}(q,q_2)$ can be computed using the inversion formula $G_{aa} = G_c(1 + \int_0^1 dv [\sigma]/u^2(G_c^{-1} + [\sigma])$ and the above formulae. One finds two equations which determine $\tilde{v}$ and $\tilde{\mu}$:

$$\left( \frac{\tilde{\mu}}{\Lambda} \right)^{1-\tilde{T}} = g \tilde{T} \tilde{g}(v_c) \ ; \quad \left( \frac{\tilde{\mu}}{\Lambda} \right)^{1-\tilde{T}-\Delta} = \frac{\mu}{\Lambda} f(v_c)$$

(10)

where $\Lambda = cq_{max}^2 \sim (2\pi/a)^2$. The following functions have been defined, $g(v) = (1 + v)^{(\tilde{T}+1)/2}(1 - v)^{(1-\tilde{T})/2}v^{-1}$ and $f(v) = g(v)\phi(v)$ with $\phi(v) = 2\nu e^{(1+T)\sqrt{1-v^2}/(1+v^2)}/(1 + \sqrt{1-v^2})$. From the second equation in (10) one recovers the decoupling transition at $T_c$ for the pure system, and the corresponding value of $\tilde{\mu}$ for $T < T_c$ using that $v_c \rightarrow 0$ when $g \rightarrow 0$ and $f(0) = e^{\tilde{T}}$. 

5
For $\Delta = 0$ the equation for $v_c$ is remarkably simple and reads $g\bar{T}/\mu = \phi(v_c)$. In that case, and in the limit of very large cutoff, the transition from the 3D to the 2D vortex glass RSB solutions is continuous and happens when $v_c=1$, i.e $u_c = \bar{T}$. The transition line is thus determined by the equation:

$$\mu = \frac{e\bar{T}}{2} \, g$$

Indeed when $v_c \to 1$ one recovers smoothly the one step solution for the decoupled 2D glass obtained in Ref. [6] with $u_c = T/T_c$.

Several effects (such as a finite cutoff, a small non zero $\Delta$) transform this second order transition in a (weakly) first order transition (at weak disorder). When $\Delta > 0$ the above equation becomes:

$$\phi(v_c)(g(v_c))^\delta = \frac{\Lambda}{\mu} \left( \frac{g\bar{T}}{\Lambda} \right)^{1-\delta}$$

with $\delta = \Delta T/(T_c - T)$.

The function in the left hand side reaches its maximum before $v_c = 1$ and thus upon increasing $T$, the RSB solution ceases to exist before the point $v_c = 1$ is reached. Thus there is now a first order jump to the 2D decoupled phase. The value of $v_c$ at the jump is given by $v_c^* = (1 - \delta^2)/(1 + \delta^2)$. The equation of the transition line is then:

$$\frac{\Lambda}{\mu} \left( \frac{g\bar{T}}{\Lambda} \right)^{1-\delta} = 2 \frac{1 - \delta}{1 + \delta} \left( \frac{2\delta^{1-\bar{T}}}{1 - \delta^2} \right) e^{\delta(1+\bar{T})-1}$$

The value of $\bar{\mu}$ at the transition, i.e the size of the jump, is given by:

$$\left( \frac{\bar{\mu}^*}{\Lambda} \right)^{(1-\bar{T})(1-\delta)} = \frac{\mu}{\Lambda} \frac{4\delta^{1-\bar{T}}}{(1 + \delta)^2} e^{\delta(1+\bar{T})-1}$$

The above result (11) for the transition from the 2D to the 3D glass can be recovered in a simple way by considering the characteristic lengths of the problem. In the 3D phase (at relatively large $\mu$) one can expand the interplane coupling terms and obtain a manifold with bare elastic coefficients: $c_{11} = c(2\pi/a)^2/l$ isotropically in plane and $c_z = \mu(2\pi/a)^2$ along $z$. The 3D translational order correlation length $R_a$ along $z$ was computed in [6] as $R_{a,z}^{3d} = a^4 c_{11} c_z / \pi^3 \Delta$ with here $\Delta = 2T g/l$. One then finds that the above transition corresponds to the ratio $R_{a,z}^{3d}/l$ being a fixed number $R_{a,z}^{3d}/l \approx e = 2.7...$ This can be viewed as a "Lindemann criterion" and is in reasonable agreement with the one obtained in [19] by a different variational method. The physical interpretation is thus that the
transition happens when the 3D translational order correlation length becomes of order the interplane spacing. Note that in general it should be the translational order length \( R_a \) which controls the transition and should enter in a Lindemann type criterion, and not the Larkin length \( R_c \). Although for a single cosine model, valid at higher temperature or \( \xi \sim a \), one has \( R_a \sim R_c \), when one treats model (1) keeping all harmonics in order to describe the low temperature region, one finds indeed that it is \( R_a \) which is the relevant length.

To interpret further the above results in a simple way one can notice than in the equations for \( \tilde{\mu} \), relevant \( q_z \) momenta are the one close to the zone boundary \( (q_z \sim \pm \pi) \). The correlation function appearing in the equations for \( \tilde{\mu} \) are therefore analogous to two dimensional ones, but for the presence of a mass term of order \( \tilde{\mu} \) in the denominator. One can therefore crudely replace the equation for \( \tilde{\mu} \) by

\[
\tilde{\mu} = \mu e^{-\frac{1}{2}B_{2D}(r=1/\sqrt{\tilde{\mu}})}
\]  

(15)

where \( B_{2D}(r) \) is the correlation in the purely two dimensional (disordered) system. On this expression it becomes clear that if correlation functions in 2D decay too rapidly as a function of distance there will be no solution of (15) for small \( \tilde{\mu} \), and the transition will be first order. To get a continuous transition one need that the right hand side of (15) goes to zero with \( \tilde{\mu} \), at worst with a finite slope. This imposes at worst an algebraic decay of the correlation functions with an exponent smaller than one. This is the case for the pure system below \( T_c \), and for the disordered system with infinite cutoff and no \( \Delta \), where the exponent is frozen at the \( T_c \) value (i.e. one). In the presence of \( \Delta > 0 \), or if one takes solutions given by the Cardy-Ostlund RG, for which correlation functions decay as \( e^{-\ln(r)^2} \), one gets a discontinuous transition, as confirmed below. A similar discontinuous transition was found from the effect of a finite cutoff in [14]. Based on (15) one can construct an argument from the 2D translational order correlation length for a single plane. It was estimated in [4] as \( R_{2d} = a(2\pi \Lambda/Tg)^{1/2(1-T/T_c)} \). This length should be compared with another 2D length for the pure system, i.e the scale below which the coupled system is still 2D, \( R_{2d} = 2\pi \sqrt{c/\tilde{\mu}} = a(\mu \tilde{T}/\Lambda)^{1/2(1-T/T_c)} \). Remarkably the transition again occurs when these two lengths become comparable.

We now turn to the Renormalization Group (RG) method, which can be applied to this model near the decoupling transition of the pure system, i.e near \( T = T_c = 4\pi \), for small disorder \( g \) and couplings between planes. We study the model (1). Near \( T_c \) one needs to keep only the lowest harmonic \( p = 1 \), higher harmonics being irrelevant. We have obtained the (replica symmetric) RG recursion relations for \( K_{ij} \), \( \Delta_{ij} \), \( g_{ij} \) and \( \mu_{ij} \) using Coulomb gas techniques and fermion methods. This is a generalization of the Cardy Ostlund recursion relations [25] to a set of coupled planes. For simplicity we give here only the results for two planes [24]. The results for \( N \) planes will be presented in [23] but
FIG. 1. Schematic phase diagram for two coupled planes in presence of disorder

our preliminary studies show that they are qualitatively similar (see below). One defines, for two planes: $K_{ij}^{-1} = (T/T_c)\delta_{ij} + k_{ij}$, $k_c = k_{11} + k_{12}$, $k = k_{11} - k_{12}$, $\mu = \mu_{12} + g_{12}$, $g = g_{11}$ and $\delta = \Delta_{11} - \Delta_{12}$. One finds the following RG flow equations upon a change of cutoff $a \rightarrow a e^l$ [23]:

$$\frac{d}{dl} k = \frac{1}{2}(\mu^2 - g_{12}^2); \quad \frac{d}{dl} \delta = \frac{1}{4}(g^2 - g_{12}^2)$$  \hspace{1cm} (16)

$$\frac{d}{dl} g = (k + k_c)g + \mu g_{12} - g_{12}^2 - g^2$$  \hspace{1cm} (17)

$$\frac{d}{dl} g_{12} = (-2\delta + k + k_c)g_{12} + \mu g - 2g_{12}g$$  \hspace{1cm} (18)

$$\frac{d}{dl} \mu = 2(-\delta + k)\mu - g_{12}g$$  \hspace{1cm} (19)

and $\frac{d}{dl} k_c = 0$, $\frac{d}{dl} \Delta_{11} = g^2/4$. These equations are valid to second order in all coupling constants and in deviations from $T_c$. The parameter $k_c = (T_c - T)/T_c$ is the true reduced temperature and is not renormalized. $k$ controls the stiffness associated to the phase difference between the two planes, and is renormalized, indicating whether the planes are coupled or not.

The previously known limiting cases are the following. In the absence of disorder, i.e $g = g_{12} = \delta = 0$, one simply recovers a sine-Gordon model for the relative phase $\phi_1 - \phi_2$: there is a usual Kosterlitz-Thouless transition, with a separatrix $k = -\mu/2$ between a coupled phase $\mu(l) \rightarrow \infty$ and a decoupled high-temperature phase ($\mu(l) \rightarrow 0$). In the absence of coupling between planes, i.e $\mu = 0, g_{12} = 0, \Delta_{12} = 0, k = 0$, there is a a 2D glass phase for $k_c > 0$ - charachterized by the Cardy-Ostlund attractive fixed point $g^* = k_c$ and $\delta(l) \rightarrow \delta^*$ - as well as a high temperature phase for $k_c < 0$, where disorder is irrelevant and $g(l) \rightarrow 0$ and $\delta(l) \rightarrow \delta^*$.

In presence of both disorder and interplane coupling, we find, via numerical solution of the equations that there are three very different regimes of the flow
with marked separatrices between them (we start from the initial condition $k = k_c, g_{12} = 0, \Delta_{ij} = 0$ corresponding to model (4)). There is a decoupled 2D glass regime ($\mu(l) \to 0, g_{12}(l) \to 0, \delta \to \infty, k(l) \to k^*$), a 3D coupled glass phase (all constants become large) and a high temperature phase ($\mu(l), g_{12}(l), g(l) \to 0, \delta(l) \to \delta^*, k(l) \to k^*$). These regimes are very likely to correspond to three different phases, the uncertainty on the exact position of the separatrices being quite small [26]. We thus arrive at the phase diagram of Fig. 1. In particular the phase transition between the 2D and 3D glass phases occurs approximately for:

$$\mu \approx C \ g$$  (20)

with $C \approx 0.5 - 0.6$, and only a weak dependence on temperature. This result is in good agreement with the result for the transition line from the GVM. One also notices that, when approaching the 2D-3D glass separatrix from the 2D glass phase, the interplane coupling always becomes vanishingly small at a fixed characteristic length $l^*(g)$, which diverges as $g \to 0$ but remains fixed close to the transition. We find numerically that $l^*(g) \sim 1/(2k_c) \log(1/g)$ for small $g$ and thus we identify this length with the 2D Larkin length. This may be interpreted as the 2D to 3D glass transition being first-order, also in agreement with the GVM. Finally, the study for $N$ planes gives similar results for the phases. The separatrix remains roughly of the form (20), with $C \approx 0.7 - 1.0$, but deviations from the linear behaviour is observed [23] at small $g$ and near $T_c$.

In conclusion we have shown that in a layered geometry, the Bragg glass phase is stable at weak disorder. Upon increase of disorder it undergoes a transition into a topologically disordered glass. In this geometry this glass is in fact a decoupled 2D glass. The transition is found to occur roughly when the translational order length $R_a$ is of the order of the lattice spacing $a$ as suggested in [6]. This model has some peculiarities that make generalization to real vortex lattices difficult. However, the fact that it confirms the existence of a stable Bragg glass is an additional indication, besides the energy arguments of [6], that the Bragg glass occurs in real vortex lattices. It also confirms that the criterion $R_a \gtrsim a$ may indeed be used as stability criterion for the Bragg glass. Our conclusions agree with the one of [19].

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