Planck 2015 constraints on spatially-flat dynamical dark energy models

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Abstract We determine constraints on spatially-flat tilted dynamical dark energy XCDM and φCDM inflation models by analyzing Planck 2015 cosmic microwave background (CMB) anisotropy data and baryon acoustic oscillation (BAO) distance measurements. XCDM is a simple and widely used but physically inconsistent parameterization of dynamical dark energy, while the φCDM model is a physically consistent one in which a scalar field φ with an inverse power-law potential energy density powers the currently accelerating cosmological expansion. Both these models have one additional parameter compared to standard ΛCDM and both better fit the TT + lowP + lensing + BAO data than does the standard tilted flat-ΛCDM model, with Δχ² = −1.26 (−1.60) for the XCDM (φCDM) model relative to the ΛCDM model. While this is a 1.1σ (1.3σ) improvement over standard ΛCDM and so not significant, dynamical dark energy models cannot be ruled out. In addition, both dynamical dark energy models reduce the tension between the Planck 2015 CMB anisotropy and the weak lensing σ₈ constraints.

Keywords Cosmic background radiation · Cosmological parameters · Large-scale structure of universe · Observations

1 Introduction

The standard cosmological model, spatially-flat ΛCDM (Peebles 1984), is parameterized by six cosmological parameters conventionally taken to be: Ωb h² and Ωc h², the current values of the baryonic and cold dark matter (CDM) density parameters multiplied by h² [where h = H₀/(100 km s⁻¹ Mpc⁻¹) and H₀ is the Hubble constant]; θ, the angular diameter distance as a multiple of the sound horizon at recombination; τ, the reionization optical depth; and Aₘ and nₘ, the amplitude and spectral index of the (assumed) power-law primordial scalar energy density inhomogeneity power spectrum (Ade et al. 2016). In this model, the currently accelerating cosmological expansion is powered by the cosmological constant Λ which is equivalent to a dark energy ideal fluid with equation of state parameter \( w₀ = -1 \). For reviews of this model see Ratra and Vogeley (2008), Martin (2012), and Brax (2018). This model assumes flat spatial hypersurfaces, which is largely consistent with most available observational constraints (Ade et al. 2016, and references therein).1

1Using a physically consistent non-flat inflation model (Gott 1982; Hawking 1984; Ratra 1985) power spectrum of energy density inhomogeneities (Ratra and Peebles 1995; Ratra 2017) to analyse the Planck 2015 cosmic microwave background (CMB) anisotropy measurements (Ade et al. 2016), Ooba et al. (2018a) find that these data do not require flat spatial hypersurfaces in the six parameter non-flat ΛCDM model (also see Park and Ratra 2019a,b,d). In the non-flat ΛCDM model, compared to the standard flat-ΛCDM model, there is no simple tilt option so nₘ is no longer a free parameter and it is instead replaced by the current value of the spatial curvature energy density parameter Ωk. CMB anisotropy data also do not require flat spatial hypersurfaces in the seven parameter non-flat XCDM and φCDM inflation models (Ooba et al. 2018b,c; Park and Ratra 2018, 2019b,d). In both these models nₘ is again replaced by Ωk. These models differ from the seven parameter spatially-flat XCDM and φCDM inflation models we study in this paper, in which nₘ is a parameter but Ωk is not.
However, there also are suggestions that flat-$\Lambda$CDM might not be as compatible with different or larger compilations of cosmological measurements (Sahni et al. 2014; Ding et al. 2015; Zheng et al. 2016; Solà et al. 2015, 2017a,b,c, 2018; Zhang et al. 2017; Zhao et al. 2017; Solà and de Cruz Pérez 2017; Gómez-Valent and Solà 2017; Cao et al. 2018) that might be more consistent with dynamical dark energy models.2 The simplest, but physically inconsistent and widely used, dynamical dark energy parameterization is the seven parameter XCDM model in which the equation of state relating the pressure and energy density of the dark energy fluid is $p_X = w_0 \rho_X$ and $w_0$ is the additional, seventh, parameter. The simplest physically consistent dynamical dark energy model is the seven parameter $\phi$CDM model, in which a scalar field $\phi$ with potential energy density $V(\phi) \propto \phi^{-\alpha}$ is the dynamical dark energy (Peebles and Ratra 1988; Ratra and Peebles 1988) and $\alpha > 0$ is the seventh parameter that governs dark energy evolution.3 In this paper we use the Planck 2015 CMB anisotropy data to constrain the seven parameter spatially-flat XCDM and $\phi$CDM models. Ooba et al. (2018c) were the first to derive proper (non-approximate) CMB anisotropy data constraints on the physically consistent (non-flat) dynamical dark energy $\phi$CDM model.4 In this paper we present results from the first complete (non-approximate) analyses of CMB anisotropy data using the spatially-flat tilted $\phi$CDM model.

The structure of our paper is as follows. In Sec. 2 we summarize the methods we use in our analyses here. Our parameter constraints are tabulated, plotted, and discussed in Sec. 3, where we also comment on the goodness-of-fit of the best-fit XCDM and $\phi$CDM models. We conclude in Sec. 4.

### 2 Methods

In the XCDM parameterization the equation of state of the dark energy fluid is $p_X = w_0 \rho_X$. In this parameterization, to render it physically sensible, we make the additional (somewhat arbitrary) assumption that spatial inhomogeneities in the dark energy fluid propagate at the speed of light.

In the $\phi$CDM model the equations of motion are

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} - \kappa a m_p^2 \phi^{-\alpha - 1} = 0,$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_p^2}(\rho + \rho_\phi),$$

$$\rho_\phi = \frac{m_p^2}{32\pi}(\phi^2 + 2\kappa m_p^2 \phi^{-\alpha}).$$

Here the scalar field potential energy density $V(\phi) = \kappa m_p^2 \phi^{-\alpha}$, $m_p$ is the Planck mass, and $\kappa$ is determined in terms of the other parameters. $\alpha$ is the cosmological scale factor and an overdot represents a derivative with respect to time.

$\rho$ and $\rho_\phi$ are the energy densities excluding the scalar field and that of the scalar field, respectively. The $\phi$CDM model equations of motion has a time-dependent attractor or tracker solution and so predictions in this model do not depend on initial conditions (Peebles and Ratra 1988; Ratra and Peebles 1988; Pavlov et al. 2013). On this solution, the initially subdominant scalar field energy density evolves in a manner to attempt to become the dominant energy density; this mechanism could partially alleviate the fine-tuning associated with the currently accelerating cosmological expansion.

Figure 1 shows the dynamical evolution of the equation of state parameter (the ratio of pressure to energy density) of dark energy in some $\phi$CDM and XCDM models and the effects of dynamical dark energy on the CMB temperature anisotropy spectrum.

In this study we compute the angular power spectra of the CMB anisotropy by using CLASS (Blas et al. 2011)5 and perform the Markov chain Monte Carlo analyses with Monte Python (Audren et al. 2013). In both spatially-flat dynamical dark energy models the primordial power spectrum of energy density inhomogeneities is taken to be that generated by quantum-mechanical fluctuations in the spatially-flat tilted inflation model (Lucchin and Matarrese 1985; Ratra 1992, 1989)

$$P(k) = A_s \left(\frac{k}{k_0}\right)^{n_s},$$

where $k$ is wavenumber and $A_s$ is the amplitude at the pivot scale $k_0 = 0.05$ Mpc$^{-1}$.

2Amongst these analyses that also make use of CMB anisotropy data, those that have used a physically consistent dynamical dark energy model such as $\phi$CDM (Solà et al. 2017b,c; Solà and de Cruz Pérez 2017; Gómez-Valent and Solà 2017) have performed only an approximate CMB anisotropy analysis.

3While XCDM is often used to model dynamical dark energy, it is not a physically consistent model as it cannot describe the evolution of energy density inhomogeneities. Also, XCDM does not accurately model $\phi$CDM dark energy dynamics (Podariu and Ratra 2001).

4Aside from CMB anisotropy measurements, many other observations have been used to constrain the $\phi$CDM model (see, e.g., Chen and Ratra 2004, 2011b; Samushia et al. 2007; Yashar et al. 2009; Samushia and Ratra 2010; Farooq and Ratra 2013; Pavlov et al. 2014; Avsajanishvili et al. 2015; Farooq et al. 2017; Solà et al. 2017b,c; Solà and de Cruz Pérez 2017; Zhai et al. 2017; Gómez-Valent and Solà 2017; Avsajanishvili et al. 2017; Ryan et al. 2018, 2019; Park and Ratra 2019c,d; Khadka and Ratra 2019).

5Our flat space $\phi$CDM CMB anisotropy angular power spectra differ somewhat from earlier results in Brax et al. (2000) and Mukherjee et al. (2003). We have verified that our results are accurate.
We consider a flat prior with the ranges of the cosmological parameters chosen to be

\[
100\theta \in (0.5, 10), \quad \Omega_b h^2 \in (0.005, 0.04), \\
\Omega_c h^2 \in (0.01, 0.5), \quad \tau \in (0.005, 0.5), \\
\ln(10^{10} A_s) \in (0.5, 10), \quad n_s \in (0.5, 1.5), \\
\omega_0 \in (-3, 0.2), \quad \alpha \in (0, 8).
\]

The CMB temperature and the effective number of neutrinos were set to \( T_{\text{CMB}} = 2.7255 \, \text{K} \) from COBE (Fixsen 2009) and \( N_{\text{eff}} = 3.046 \) with one massive (0.06 eV) and two massless neutrino species in a normal hierarchy. The primordial helium fraction \( Y_{\text{He}} \) is inferred from standard Big Bang nucleosynthesis, as a function of the baryon density.

We constrain model parameters by comparing our results to the CMB angular power spectrum data from the Planck 2015 release (Ade et al. 2016) and the baryon acoustic oscillation (BAO) distance measurements from the matter power spectra obtained by the 6dF Galaxy Survey (Beutler et al. 2011), the Baryon Oscillation Spectroscopic Survey (LOWZ and CMASS) (Anderson et al. 2014), and the Sloan Digital Sky Survey main galaxy sample (MGS) (Ross et al. 2015).

3 Results

In this section we tabulate, plot, and discuss the resulting constraints on the spatially-flat tilted XCDM and \( \phi \)CDM inflation models. Table 1 lists mean values and 68.27% limits on the cosmological parameters for the XCDM parameterization, and Table 2 lists those for the \( \phi \)CDM model (95.45% upper limits on \( \alpha \)).

Fig. 2 shows two-dimensional constraint contours and one-dimensional likelihoods from the 4 different CMB and BAO data set combinations used in this study. Here all other parameters are marginalized. CMB temperature anisotropy spectra for the best-fit XCDM and \( \phi \)CDM models are shown in Fig. 3, compared to that of the standard spatially-flat tilted \( \Lambda \)CDM model. Contours at 68.27% and 95.45% confidence level in the \( \sigma_B - \Omega_m \) plane are shown in Fig. 4, with other parameters marginalized.

Our Table 1 column 2 and 3 results for XCDM are in good agreement with the Planck 2015 results in Sect. 6.3 of Ade et al. (2016) and Tables 21.1 and 21.3 of “Planck 2015 Results: Cosmological Parameter Tables” at wiki.cosmos.esa.int/planckpla2015/images/f/f7/Baseline_params_table_2015_limit68.pdf for most variables (and our Table 1 column 4 and 5 results for XCDM are in good agreement with those of Tables 21.20 and 21.21 of this compilation). However, our \( \omega_0 \) (and derived \( H_0, \Omega_m, \) and \( \sigma_8 \)) values differ somewhat from the Planck 2015 ones because of the different \( H_0 \) flat prior ranges used (we use 0.2 ≤ \( h \) ≤ 1.3 here while Planck 2015 used \( h \leq 1 \)).

Comparing the Table 1 column 5 TT + lowP + lensing + BAO results for the spatially-flat tilted XCDM model here to those for the non-flat XCDM model in column 5 of Table 1 in Ooba et al. (2018a), we see that \( \Omega_c h^2, \Omega_b h^2, \tau, \ln(10^{10} A_s), \theta, \Omega_m, \omega_0, H_0, \) and \( \sigma_8 \) differ by 4.2σ, 2.5σ, 2.2σ, 1.8σ, 1.6σ, 0.66σ, 0.25σ, 0.21σ, and 0.20σ (of the quadrature sum of the two error bars). Similarly for the \( \phi \)CDM case in column 5 of Table 2 here and Table 2 of Ooba et al. (2018c), we find that \( \Omega_c h^2, \Omega_b h^2, \ln(10^{10} A_s), \theta, \Omega_m, H_0, \) and \( \sigma_8 \) differ by 4.2σ, 2.4σ, 2.1σ, 2.0σ, 1.4σ, 1.2σ, and 1.0σ (of the quadrature sum of the two error bars).

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We thank C.-G. Park for pointing out a numerical error in our initial CMB only \( \phi \)CDM analyses. Our corrected results here are in very good agreement with those of Park and Narita (2018).
Table 1 68.27% (95.45%) confidence limits on cosmological parameters of the XCDM parameterization from CMB and BAO data

| Parameter                  | TT+lowP            | TT+lowP+lensing | TT+lowP+BAO        | TT+lowP+lensing+BAO |
|----------------------------|--------------------|----------------|--------------------|---------------------|
| $\Omega_b h^2$             | 0.02231 ± 0.00024  | 0.02229 ± 0.00024 | 0.02227 ± 0.00022  | 0.02227 ± 0.00022   |
| $\Omega_c h^2$             | 0.1191 ± 0.0023    | 0.1182 ± 0.0021  | 0.1189 ± 0.0019    | 0.1183 ± 0.0018     |
| $100\theta$                | 1.04195 ± 0.00046  | 1.04210 ± 0.00045 | 1.04196 ± 0.00044  | 1.04205 ± 0.00042   |
| $\tau$                     | 0.076 ± 0.020      | 0.060 ± 0.018    | 0.080 ± 0.020      | 0.071 ± 0.017       |
| $\ln(10^{10} A_s)$         | 3.085 ± 0.038      | 3.049 ± 0.032    | 3.092 ± 0.038      | 3.071 ± 0.031       |
| $n_s$                      | 0.9662 ± 0.0065    | 0.9678 ± 0.0064  | 0.9669 ± 0.0058    | 0.9679 ± 0.0056     |
| $w_0$                      | $-1.95^{+0.61}_{-0.31}$ | $-1.77^{+0.42}_{-0.66}$ | $-1.06^{+0.35}_{-0.08}$ | $-1.03^{+0.35}_{-0.07}$ |
| $H_0$ [km/s/Mpc]           | >71.26 [2σ limit] | 96.08 ± 21.63    | 69.35 ± 1.84       | 68.91 ± 1.78        |
| $\Omega_m$                 | 0.150$^{+0.014}_{-0.069}$ | 0.172$^{+0.023}_{-0.092}$ | 0.294 ± 0.014       | 0.296 ± 0.014        |
| $\sigma_8$                 | 1.096$^{+0.17}_{-0.090}$ | 1.025$^{+0.10}_{-0.11}$ | 0.843 ± 0.027       | 0.826 ± 0.020        |

Table 2 68.27% (95.45%) confidence limits on cosmological parameters of the ϕCDM model from CMB and BAO data

| Parameter                  | TT+lowP            | TT+lowP+lensing | TT+lowP+BAO        | TT+lowP+lensing+BAO |
|----------------------------|--------------------|----------------|--------------------|---------------------|
| $\Omega_b h^2$             | 0.02218 ± 0.00024  | 0.02220 ± 0.00024 | 0.02239 ± 0.00021  | 0.02238 ± 0.00021   |
| $\Omega_c h^2$             | 0.1199 ± 0.0023    | 0.1192 ± 0.0021  | 0.1171 ± 0.0015    | 0.1169 ± 0.0014     |
| $100\theta$                | 1.04184 ± 0.00045  | 1.04193 ± 0.00044 | 1.04215 ± 0.00042  | 1.04219 ± 0.00041   |
| $\tau$                     | 0.077 ± 0.019      | 0.073 ± 0.017    | 0.088 ± 0.019      | 0.082 ± 0.015       |
| $\ln(10^{10} A_s)$         | 3.089 ± 0.037      | 3.078 ± 0.030    | 3.104 ± 0.037      | 3.092 ± 0.028       |
| $n_s$                      | 0.9643 ± 0.0064    | 0.9657 ± 0.0060  | 0.9714 ± 0.0051    | 0.9716 ± 0.0050     |
| $\alpha$ [2σ limit]        | < 1.46             | < 1.19          | < 0.28             | < 0.28              |
| $H_0$ [km/s/Mpc]           | 63.37 ± 3.00       | 63.69 ± 3.11    | 67.32 ± 0.89       | 67.33 ± 0.90        |
| $\Omega_m$                 | 0.357 ± 0.035      | 0.352 ± 0.036   | 0.308 ± 0.009      | 0.307 ± 0.009       |
| $\sigma_8$                 | 0.789 ± 0.031      | 0.783 ± 0.028   | 0.815 ± 0.018      | 0.809 ± 0.012       |

Fig. 2 68.27% and 95.45% confidence level contours for the XCDM and ϕCDM models using various data sets, with the other parameters marginalized
Fig. 3 The best-fit $C_\ell$’s for the XCDM parameterization (left panels (a), (c) and (e)) and the $\phi$CDM model (right panels (b), (d) and (f)) compared to the spatially-flat tilted $\Lambda$CDM model (gray solid line). Linestyle information are in the boxes in the two lowest panels. Planck 2015 data are shown as black points with error bars. The top panels show the all-$\ell$ region. The middle panels show the low-$\ell$ region $C_\ell$ and residuals. The bottom panels show the high-$\ell$ region $C_\ell$ and residuals.

$1.3\sigma$, $0.17\sigma$, and $0.16\sigma$ (of the quadrature sum of the two error bars). On the other hand, comparing the spatially-flat tilted XCDM and $\phi$CDM TT + lowP + lensing + BAO results we have derived here (and listed in columns 5 of Tables 1 and 2) we see that $H_0$, $\sigma_8$, $\Omega_m$, $\Omega_\Lambda h^2$, $\ln(10^{10} A_s)$, $n_s$, $\tau$, $\Omega_b h^2$, and $\theta$ differ by $0.79\sigma$, $0.73\sigma$, $0.66\sigma$, $0.61\sigma$, $0.50\sigma$, $0.49\sigma$, $0.36\sigma$, and $0.24\sigma$ (of the quadrature sum of the two error bars). We note however that XCDM is not a physical model and so it might not be very meaningful to compare cosmological parameter values measured using the XCDM parameterization and the $\phi$CDM model.

In agreement with Park and Ratra (2019a), who compared cosmological parameter measurements made from cosmological observations by using the spatially-flat tilted $\Lambda$CDM model and the non-flat $\Lambda$CDM model, we also find that when space curvature is allowed to vary many cosmo-
logical parameters cannot be determined in a model independent way from cosmological data, with the possible exceptions of $\sigma_8$ and $H_0$ (and $w_0$) in the XCDM parameterization. We emphasize that the somewhat widely held belief that the baryonic matter density $\Omega_b h^2$ can be pinned down in a model independent manner by CMB anisotropy and other cosmological observations is not true.\footnote{See Penton et al. (2018) for a discussion of how observed deuterium abundances can be used to constrain spatial curvature.} When spatial curvature vanishes it appears that cosmological parameters can be determined in a more model independent fashion, although, again, this is based on using the somewhat arbitrary XCDM parameterization. It is interesting that in this case $H_0$ and $\sigma_8$ are the most model dependent parameters.

Focusing again on the TT + lowP + lensing + BAO data, we measure $H_0 = 68.91 \pm 1.78$ (67.33 $\pm$ 0.90) km s$^{-1}$ Mpc$^{-1}$ for XCDM ($\phi$CDM), both of which are consistent with the most recent median statistics estimate of $H_0 = 68 \pm 2.8$ km s$^{-1}$ Mpc$^{-1}$ \cite{Chen2011}. They also are consistent with many other recent estimates \cite{Calabrese2012, Sievers2013, Aubourg2015, Chen2017, Lin2017, Abbott2017b, Yu2018, Haridasu2018}, although both are lower than the recent local expansion rate determination of $H_0 = 73.45 \pm 1.66$ km s$^{-1}$ Mpc$^{-1}$ \cite{Riess2018}.

The TT + lowP + lensing + BAO values of $\tau = 0.071 \pm 0.017$ (0.082 $\pm$ 0.015) for XCDM ($\phi$CDM) measured here are a bit larger than the value of $\tau = 0.066 \pm 0.013$ measured using TT + lowP + lensing + NewBAO data in the tilted flat-$\Lambda$CDM model \cite{Park2019a}, but not as large as the values of $\tau$ found in the non-flat models, for TT + lowP + lensing + NewBAO in non-flat $\Lambda$CDM $\tau = 0.115 \pm 0.011$ \cite{Park2019a}, and for TT + lowP + lensing + BAO in non-flat XCDM ($\phi$CDM) $\tau = 0.121 \pm 0.015$ (0.129 $\pm$ 0.013) \cite{Ooba2018b,c, Tables 1}. The larger value for $\tau$ in the non-flat $\Lambda$CDM case has very interesting implications for reionization \cite{Mitra2018,2019}.

In both dynamical dark energy models, XCDM and $\phi$CDM, the data favor non-evolving dark energy, although they are not yet good enough to rule out the possibility of mild dark energy time evolution. More and better-quality data will be needed to resolve this issue. The situation in the non-flat models is quite different, where the data favor mildly closed models in which the curvature energy density contributes about a per cent to the current cosmological energy budget \cite{Ooba2018b,c, Park2018,2019a,b,d, Tables 1}. The larger value for $\tau$ in the non-flat $\Lambda$CDM case has very interesting implications for reionization \cite{Mitra2018,2019}.

The Dark Energy Survey \cite{Abbott2017a} measures $\Omega_m = 0.264^{+0.032}_{-0.019}$ and $\sigma_8 = 0.807^{+0.062}_{-0.041}$ (DES Y1 All, both 68.27% confidence limits). Our XCDM and $\phi$CDM TT + lowP + lensing + BAO results are consistent with these limits (with our $\phi$CDM $\Omega_m$ value being the most deviant, high by 1.3$\sigma$ of the quadrature sum of the two error bars). The Dark Energy Survey constraints are also consistent with the XCDM and $\phi$CDM confidence level contours in the $\sigma_8$–$\Omega_m$ plane shown in Fig. 4, but are a little more difficult to reconcile with the standard tilted flat-$\Lambda$CDM model results. Gómez-Valent and Solà \cite{2017} draw a similar conclusion for these models. The non-flat models are also more consistent with the weak lensing constraints \cite{Ooba2018a,b,c, Park2018,2019a,b,d} than is the standard $\Lambda$CDM model.

Fig. 4 68.27% and 95.45% confidence level contours in the $\sigma_8$–$\Omega_m$ plane.
As can be seen from Fig. 3 here, and the corresponding ones for the non-flat models (Ooba et al. 2018a,b,c; Park and Ratra 2018, 2019a,b), the spatially-flat tilted XCDM and φCDM models do not do as well at fitting the lower-ℓ Cℓ temperature data as do the non-flat models, but the flat models better fit the higher-ℓ Cℓ’s than do the non-flat ones.

While both spatially-flat dynamical dark energy models considered here are more consistent with the weak lensing constraints than is tilted flat-ΛCDM, the XCDM parameterization and the φCDM model both have one extra parameter so it is necessary to quantify how well these models fit the totality of data. Table 3 shows Δχ^2 values for the spatially-flat XCDM and φCDM models relative to the flat-ΛCDM model. Here χ^2 is determined from the maximum value of the likelihood, L_{max}. Unlike the non-flat ΛCDM, XCDM, and φCDM models which are not straightforwardly related to the standard ΛCDM model (Ooba et al. 2018a,b,c; Park and Ratra 2018, 2019a,b), the tilted spatially-flat XCDM and φCDM models here are single parameter extensions of the tilted flat-ΛCDM model and so we are comparing nested models here. In this case we can work around the ambiguity in the number of Planck 2015 data points and translate the Δχ^2 values of Table 3 to relative probabilities. From Table 3, for the TT + lowP + lensing + BAO case, the XCDM parameterization and the φCDM model, from \sqrt{−Δχ^2} for one additional free parameter, are 1.1σ and 1.3σ better fits to the data than is tilted flat-ΛCDM. The corresponding p values are 0.26 and 0.21 with one additional degree of freedom. These results indicate that the improvement in fit, in going from tilted flat-ΛCDM to one of the tilted spatially-flat dynamical dark energy models, is not significant. On the other hand, the dynamical dark energy models cannot be ruled out and continue to be of interest, especially φCDM which is a physically consistent model. This means that more and better-quality data will be needed to determine whether the dark energy density is constant or decreases slowly with time. For goodness-of-fit, one may also consider the AIC, which will penalize the dynamical dark energy models for the additional parameter. From ΔAIC the XCDM (φCDM) model is only 69% (82%) as likely as the standard tilted flat-ΛCDM model, again not a strong result either way.

### 4 Conclusion

We present constraints on the tilted spatially-flat XCDM and φCDM inflation models determined by analyzing Planck 2015 CMB anisotropy data as well as BAO distance measurements. XCDM is a simply parameterized dynamical dark energy model, and φCDM is a physically consistent one in which a scalar field φ with an inverse power-law potential energy density acts as dynamical dark energy and powers the currently accelerating cosmological expansion.

Both of these dynamical dark energy models better fit, although not significantly so, the TT + lowP + lensing + BAO data combination than does the tilted flat-ΛCDM model. Perhaps more interestingly, the dynamical dark energy models reduce the tension between the Planck 2015 CMB anisotropy and the weak lensing σ8 constraints. More and better data, which should soon be available, is needed to determine if dynamical dark energy can be ruled out, or if dark energy is dynamical.

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