Analytical investigation of gas production from methane hydrates and the associated heat and mass transfer upon thermal stimulation employing a coaxial wellbore

M. Roostaie¹ and Y. Leonenko¹,²,*

¹Department of Earth & Environmental Sciences
²Department of Geography and Environmental Management

University of Waterloo
200 University Ave. W
Waterloo, Ontario, Canada N2L 3G1

*corresponding author, leonenko@uwaterloo.ca, 519-888-4567 (Ex. 32160)

Keywords: methane hydrate, dissociation, thermal stimulation, coaxial wellbore heat source, modeling, wellbore layers

Abstract

In this study, a radial 2D analytical approach has been developed to couple the wellbore heating process and the associated methane hydrate dissociation in the reservoir. A coaxial wellbore is assumed as the heat source where both conduction and convection heat transfers are considered. It consists of an inner tube and an outer structure of casing, gravel, and cement layers. In the reservoir, a similarity solution employing a moving boundary separating the dissociated and undissociated zones is employed to build the analytical solution. Two different operating schemes for water supply into wellbore heat source have been studied: i) from the inner tube; and ii) from the annulus section of the wellbore. Temperature distribution along the wellbore, temperature and pressure distributions in the reservoir, hydrate dissociation rate, and energy efficiency considering various initial and boundary conditions and reservoir properties are evaluated. The two different operating schemes have almost the same results with slightly higher gas production in the case of hot water entry into annulus, which is in direct contact with the reservoir. Increasing the inlet water temperature or decreasing the wellbore pressure increases gas production. Applying them simultaneously results in a greater gas production and energy efficiency. Some of the reservoir’s properties, such as porosity, thermal diffusivity, thermal conductivity, and reservoir thickness,
have direct relation with the dissociation rate, but the reservoir’s permeability and gas viscosity have almost no impact on the process. The wellbore parameters, such as flow rate of hot water, inlet temperature, and wellbore radius except the inner tube radius, have direct impact on the wellbore mean temperature and the associated results in the dissociation process.

Introduction

Natural gas hydrates are cage-like substances composed of gas molecules trapped inside water molecules by high pressure and low temperature conditions underground in the presence of sufficient water and gas [1-3]. These substances represent a significant source of energy due to their unique structure and arrangement. More than 160 volumes of gas can be produced from 1 volume of natural gas hydrate, which is about 150 to 180 times greater than the amount of methane present in an equal volume free gas reservoir [4, 5]. However, an accurate estimation of worldwide natural gas hydrate reservoirs or their specific locations is difficult. Methane hydrates (MH), which are reported to be the most common gas hydrate type in nature, can be found mostly in Canada under thick permafrost in Polar Regions or in deep sediments of the continental shelf regions. The total estimated amount of methane is more than 10^{14} m^3, which is substantially more than the conventional gas reservoirs in Canada [4]. Gas production from MH reservoirs may become necessary in the near future due to higher gas demand (i.e., local residential gas demand in the impassable Northern regions of Canada), or due to the environmental or industrial hazards. In addition, other environment-friendly sources of energy are renewable energy sources, such as compressed air energy storage (CAES) and geothermal energy storage, which have been suggested in recent researches to have the potential as an alternative energy source [6-9]. However, this research area is not well-developed yet because some drawbacks have been reported on the practicability of such resources that need to be addressed, for instance the variable and irregular power generation of CAES systems when employing in grids scale and induced seismicity of the geothermal systems during hydraulic fracturing [10-13]. On the other hand, MH dissociation can release methane to the environment if its temperature and pressure are shifted from equilibrium induced by the negative effects of global warming and climate change on the regions containing hydrates [14]. Thus, recently, MH reservoirs have attracted researchers’ interest as a potential
contributor to global warming [15, 16]. The presence of MH reservoirs is also taken into account as a concern in the oil and gas industry during the exploration process [17].

MH reservoirs found in permafrost regions comprises only a small portion of the world’s total reservoirs, most of which are in off-shore regions (approximately 3%), but they are accessible and affordable sites for research activities [18]. The field tests that have been performed on the MH reservoirs located in the Malik site, Mackenzie Delta, Northwest Territories, Canada showed the maximum stability depth of MH around 1.4 km, under which the specific temperature and pressure equilibrium conditions of MH cannot be met [19, 20]. MH occurrence is not consistent vertically through a specific range of depth, but, the total overall thickness of MH in the Northwest Territories of Canada is estimated to be around 80 m, with the total area of approximately 125000 km² [4]. But, more investigations are also required before the practical gas production and extraction occur from such reservoirs.

Main methods applied to dissociate MH are as follows [1, 21]: i) depressurizing MH reservoirs under the equilibrium pressure [22-26]; ii) thermally stimulating the reservoirs by increasing the MH temperature beyond its equilibrium [27-29]; iii) coupling depressurization with thermal stimulation, an approach reported to be more efficient than one of the above methods alone [30-32]; iv) injecting inhibitors, such as methanol to induce instability in MH [33-35]; and v) replacing methane by CO₂ in MH reservoirs [36-41].

Many mathematical studies of hydrate dissociation employing the above methods have been performed so far and can be categorized into the following: i) analytical solutions, which provide fast answers with a good mechanistic prospective of the phenomena; and ii) numerical solutions, which are comprehensive but complicated by a number of assumptions. For instance, a three dimensional (3D) numerical study on MH dissociation upon depressurization was performed in 1982 assuming an MH layer with a free gas zone and involving the effect of conduction heat transfer and gas fluid flow [42]. A few years later, in 1986, this work was extended by taking into account the effect of water flow that is produced during the dissociation [43]. In 1991, a numerical study modelled MH dissociation by depressurization, considering three phases, water, gas, and hydrate, in porous media with gas-water flow and without the effect of heat transfer [44]. Then, other research built on this study considering water-gas flow and convective-conductive heat transfer [45, 46]. Thermal stimulation method was first modelled through numerical works by
assuming different media permeabilities and an impermeable moving dissociation boundary that separates the dissociated and undissociated MH zones as dissociation progresses [47]. The dissociation continues until the pressure and temperature falls to the equilibrium. A numerical work, in which finite difference method was employed, considering the effect of heat transfer during depressurization reported how the wellbore pressure affects MH dissociation [48]. Different numerical simulators have been developed to investigate MH dissociation, for example TOUGH2 [49]. New modules have since been introduced, enabling the study of different dissociation methods with different components and up to nine phases in both kinetic and equilibrium models [50]. A numerical work employing TOUGH2 to investigate depressurization and thermal stimulation in determining the gas production potential from MH reservoirs located in the Mackenzie Delta, Northwest Territories, Canada, found higher efficiencies when both methods are used together [50, 51]. The kinetic reaction models avoid under-prediction of recoverable MH, but require more computational effort compared to equilibrium reaction models [52]. Researches using the TOUGH-Fx/HYDRATE simulator reported low gas production from dispersed oceanic MH reservoirs with low hydrate saturation upon depressurization, accompanied by high water production, suggesting that gas production from such reservoirs is not economically feasible [53]. Increasingly, MH reservoirs are attracting researchers, who study them deeply through mathematical tools as well as real field or experimental work [54-61].

In 1990, Selim and Sloan [62] employed an analytical one-dimensional (1D) model to investigate MH dissociation upon thermal stimulation. They employed a moving dissociation boundary to separate the dissociated zone from the undissociated one. Although, the produced water was assumed to remain motionless in the pores, the effect of gas convection heat transfer and its flow was taken into account. In 1982, hydrate dissociation upon both thermal stimulation using hot water circulation into reservoir and depressurization was investigated by employing two models (i.e., the frontal-sweep model and the fracture-flow model). The authors reported higher efficiencies with depressurization than with thermal stimulation [63]. Makogon [64] analytically investigated the temperature and pressure distributions in MH reservoirs upon depressurization, considering the throttling effect in the energy equation, and assuming a moving dissociation boundary as in the previous work. This work was continued by including similarity solutions in temperature and pressure calculations as well as the water and gas movement effects [65]. In 2001,
Makogon’s model [23] was extended by including heat conduction. An analytical study on depressurization method showed that the effect of gas-water (two-phase) flow on MH dissociation is smaller than the effect of heat transfer and the intrinsic kinetics of MH decomposition [66].

Apart from the mathematical investigations, several experiments have also been designed and performed on MH dissociation upon different methods. One of the major challenges in those experiments, which significantly affects the outcome of tests, is the size of the setup [67]. For example, the main involving mechanism in the hydrate dissociation in porous media depends on the scale of experiments [67]. This mechanism could be one of the followings: i) heat transfer in the decomposing zone; ii) intrinsic kinetics of hydrate decomposition; or iii) multiphase flow (i.e., gas-water flow) during gas production [66]. Tang et al. [68] reported that the determining factor in core-scale and larger scale experiments (or field works) is respectively the intrinsic kinetics of hydrate decomposition and heat transfer in the decomposing zone. An experimental work by employing a 3D cubic hydrate simulator (CHS) showed that MH dissociation upon thermal stimulation progresses by a moving boundary [69]. Li et al. [70] designed experiments by employing two hydrate simulators with different scales to investigate the MH dissociation upon depressurization. They reported that the heat transfer from ambient is the main factor for MH dissociation. They also showed that the size of the reservoir affects the gas production rate and time. Another experimental work revealed that the conduction heat transfer is the main heat transfer mechanism to the dissociating zone [71]. A 3D Pilot-Scale Hydrate Simulator (PHS) was designed by Wang et al. [67] to investigate MH dissociation upon depressurization. They simulated MH dissociation below the quadruple point in the sandy sediment using PHS and reported ice formation in pores of the media, which increased the dissociation rate. Another experimental work employing a pilot-scale hydrate simulator reported that depressurization in conjunction with thermal stimulation is the most efficient method for the dissociation of water-saturated MH [31].

Recently, an experimental work studied depressurization, thermal stimulation, and depressurization in conjunction with thermal stimulation methods in MH dissociation this work, which was supported by an analytical model, also showed that employing depressurization and thermal stimulation simultaneously provided higher efficiency [72].

During the previous decades, several investigations have been performed on gas production from MH dissociation and the associated changes in the reservoir. However, no analytical study has
considered the effect of wellbore geometry and associated structure—generally consisting of casing, cement, and gravel [73-75]—and wellbore heating (i.e., heat pump) on MH dissociation upon thermal stimulation so far [58, 76-78]. Recently, Roostaie and Leonenko et al. [79] designed an analytical semi-infinite 1D model and addressed this gap of knowledge. Their results, which were verified against previous numerical and experimental works, showed that the wellbore structure (casing, gravel, and cement) affects the dissociation upon thermal stimulation. In investigations related to geothermal energy, ground-source heat pumps are employed as a heat source/sink to derive heat transfer process to/from underground [80-84]. Heat pumps come in two types [85]: i) Coaxial boreholes heat exchangers; and ii) U-tube heat exchangers. These heat exchangers do not inject water directly into reservoirs, but rather they indirectly transfer heat through both their own and the wellbore’s structure. This study aims to expand the previous work [79] by considering an infinite 2D radial geometry and cylindrical coaxial wellbore as the heat source. The present work focuses on the coaxial boreholes heat exchangers because their associated design has been used and investigated since decades ago [86-88]. They also have higher efficiencies in terms of heat transfer to the reservoir compared to those of U-tube heat exchangers [89-91]. It should also be noted that employing radial coordinates and the mechanism of wellbore heating in the previous analytical studies of MH dissociation upon thermal stimulation have not been treated. These assumptions make the outcomes closer to the real operational conditions.

The purpose of the present work is to develop two-dimensional analytical models of MH dissociation upon thermal stimulation, assuming an infinite hydrate reservoir in the radial coordinates and considering a wellbore heat source with three main completion layers—the casing, cement, and gravel—as its external structure. A coaxial heat exchanger with an inside tube and a surrounding annulus is assumed to model hot water injection in the wellbore and heat transfer to the reservoir. The energy efficiency, gas production, and temperature and pressure distributions assuming different heat exchanger and reservoir characteristics are calculated and compared with the previous studies. The results of this study, for the first time, provides an important opportunity to advance the understanding of the effect of wellbore structure, composition, and its associated heating mechanism on the gas production from MH reservoirs upon thermal stimulation.
Materials and methods

A schematic of the dissociation process inside the reservoir is presented in Figure 1a. Zone II is composed of undissociated hydrate at a temperature of $T_0$ at infinity ($r \to \infty$), which increases to converge with the dissociation temperature ($T_s(t)$) as it approaches the moving dissociation boundary ($r=S(t)$), denoted by the dashed circle. Zone I represents the dissociated region, with its different temperatures and pressures at various locations ($r$) and times ($t$). The grey ring in the middle of Figure 1a shows the supporting structure of the wellbore. A top view of the wellbore is shown in Figure 1b. The wellbore considered in this study has coaxial configuration with pipe-in-pipe geometry [86, 87]. Hot water is supplied to the wellbore either through the inner tube or the annulus between the inner and outer tubes, it is then extracted from the other path (Figure 1b). Heat transfer to the reservoir occurs through the borehole’s external wall. The borehole’s external layers, shown in Figure 1c, appear in cross section area of the borehole wall in Figure 1b. These layers (from inner to outer) are the 1) inner tube wall; 2) inside casing; 3) gravel; 4) outside casing; and 5) cement, and the associated radii are shown in Figure 1c. The following basic steps represent the MH dissociation process in this modelling: i) before dissociation begins, MH is assumed to be in equilibrium condition at a temperature $T_0$ in the reservoir pores; ii) at time $t = 0$, hot water is injected into wellbore with a constant flow rate and temperature, inducing heat transfer to the reservoir; iii) when the temperature at the external wall of the borehole reaches the dissociation temperature of MH, dissociation begins, followed by a sharp moving boundary surface separating the dissociated zone (the water and gas produced in Zone I) from the undissociated zone (the undissociated MH in Zone II).
As mentioned previously, two operation conditions are considered for the heat source: i) hot water injection into the inner tube and extraction from the annulus; and ii) hot water injection into the annulus and extraction from the inner tube. The flow rate and the inlet temperature are constant in models. However, the temperature distribution of the fluid along the inner tube ($T_{fi}$) and the annulus ($T_{fa}$) is not constant, and changes over time due to the heat transfer to the outside via conduction and the convection heat transfer of the fluid inside the wellbore.

The heat transfer rate inside the wellbore (e.g., between the annulus and inner tube) and to the reservoir are represented respectively by $q_i$ and $q_a$ (Figure 1c). The former ($q_i$) consists of three heat transfer components: i) convection heat transfer in the inner tube between the water flow and the inner surface of the tube; ii) conduction heat transfer through the inner tube thickness to the
annulus; and iii) convection heat transfer between the outer surface of the inner tube and the water flow in the annulus. The latter \( q_a \) consists of: i) convection heat transfer between the water flow and the outer surface of the annulus; and ii) conduction heat transfer through the external wall of the wellbore (cement, casing, and gravel layers). For each of the heat transfer components of \( q_i \) and \( q_a \), the thermal resistance is defined in the following based on the heat transfer calculation of the wellbores presented by Hellstrom [92] in 1992.

Convective heat transfer resistance \((\text{m.K/W})\) between the inner tube surface \((r=r_1)\) and the water flow inside the tube is defined as follows:

\[
R_{ci} = \frac{1}{\pi k_f Nu_{ii}} \tag{1}
\]

where \( R_{ci} \) is the convective heat transfer resistance \((\text{m.K/W})\), \( k_f \) is the fluid thermal conductivity \((\text{W/m.K})\), and \( Nu_{ii} \) is the Nusselt’s number of the fluid flow in the inner tube. Different formulas have been suggested for Nusselt’s number in previous literature with various degrees of accuracy [93-100]. Among them, Gnielinski [100] reported a suitable and accurate correlation for coaxial heat exchangers considering fluid flow in the annulus [94, 101-103]. Nusselt’s number of fluid flow in the inner tube is defined as follows based on the Gnielinski formula [100]:

\[
Nu_{ii} = \frac{(f/2)(Re_i-1000)Pr}{1+12.7(f/2)^{1/2}(Pr^{3/2}-1)} \tag{2}
\]

where \( Re_i \) is the Reynolds number in the inner tube, \( Pr \) is the Prandtl number, and \( f \) is the friction factor for smooth pipes. \( Re_i, Pr, \) and \( f \) are calculated respectively as follows:

\[
Re_i = \frac{v_f D}{v_f} = \frac{2vy_f \rho_f}{\mu_f} = \frac{2\rho_f V_f}{\pi \mu_f r_i} \tag{3}
\]

\[
Pr = \frac{\mu_f C_f}{\rho_f k_f} \tag{4}
\]

\[
f = (1.58 \ln(Re_i) - 3.28)^2 \tag{5}
\]
where $D$ is the pipe diameter (m), $v_f$ is the water flow velocity (m/s), $f_{v_f}$ is the kinematic viscosity (m$^2$/s), $\rho_f$ is the water density (kg/m$^3$), $V_f$ is the water flow rate (m$^3$/s), $r_i$ is the radial distance (m) as shown in Figure 1c, $C_f$ is the fluid volumetric heat capacity (J/(m$^3$.K)), and $\mu_f$ is the water dynamic viscosity (Pa.s).

The conduction thermal resistance (m.K/W) of the tube is calculated as follows:

$$R_p = \frac{\ln(r_2 / r_i)}{2\pi k_p}$$

(6)

where $r_2$ and $r_i$ are inner tube radii as shown in Figure 1c, and $k_p$ is the inner tube’s thermal conductivity (W/(m.k)).

The convection thermal resistance (mK/W) of the heat transfer from the outer surface of the inner tube ($r=r_2$) to the fluid flow in the annulus can be represented as follows [92]:

$$R_{coi} = \frac{1}{\pi k_f Nu_{oi} \left( \frac{1}{r} - 1 \right)}$$

(7)

where $Nu_{oi}$ is the Nusselt’s number of the fluid flow close to the outer surface of the inner tube, and $r^*$ is the ratio between the inner and outer radii of the annulus ($r_2 / r_3$). Petuhkov and Roizen [104] derived an expression for the Nusselt’s number in the annulus close to the inner tube based on experimental data,

$$Nu_{oi} = \zeta Nu_{pipe} 0.86(r^*)^{-0.16}$$

(8)

where $\zeta$ is a constant, equal to 1 in this case, and $Nu_{pipe}$ is the Nusselt’s number of the water flow in the annulus close to the inner pipe, calculated based on equation 2, using the Prandtl number and friction factor formulas from equations 4 and 5, and a Reynolds number as follows:

$$Re_a = \frac{v_f d_h}{\nu_f} = \frac{2(r_3 - r_2) v_f \rho_f}{\mu_f} = \frac{2(r_3 - r_2) \rho_f V_f}{\pi (r_3^2 - r_2^2) \mu_f}$$

(9)
where $r_3$ and $r_2$ are the radii (m) of the annulus, shown in Figure 1c, and $d_h$ is the hydraulic diameter (m) of the annulus, which can be calculated as follows:

\[ d_h = 2(r_3 - r_2) \]  

(10)

The convection thermal resistance (m.K/W) of water flow and the outer surface of the annulus ($r=r_3$) can be calculated based on the following formula [92]:

\[ R_{cia} = \frac{1}{\pi k_J N_{u_{oa}}} \left(1 - r^*\right) \]  

(11)

where $N_{u_{oa}}$ is the Nusselt’s number of the water flow close to the outer surface of the annulus, which is defined as follows based on the work performed by Petuhkov and Roizen [104]:

\[ N_{u_{oa}} = N_{u_{pipe}} (1 - 0.14(r^*)^{0.6}) \]  

(12)

And finally, in order to calculate the conduction thermal resistance of the external wall of the wellbore, the wall can be considered as a homogenous layer by summing the thermal resistances of the cement, casing, and gravel layers together as stated in equation 13:

\[ R_w = \frac{\ln(r_5 / r_3)}{2\pi k_p} + \frac{\ln(r_5 / r_4)}{2\pi k_g} + \frac{\ln(r_6 / r_3)}{2\pi k_s} + \frac{\ln(r_7 / r_6)}{2\pi k_c} \]  

(13)

where $k_z$ stands for thermal conductivity (W/(m.K)) with the $s$, $c$, $p$, and $g$ subscripts respectively referring to the steel (casing), cement, inner pipe, and gravel. Figure 1 also shows $r_3$-$r_7$.

Therefore, the total thermal resistances (m.K/W) associated with $q_i$ and $q_a$ are respectively as follows:

\[ R_1 = R_{cia} + R_p + R_{coi} \]  

(14)

\[ R_2 = R_{cia} + R_w \]  

(15)
At steady-state condition, the convection heat transfer in the water flow should be equal to the transverse heat transfer between the inner tube and annulus \((q_i)\) and between the annulus and wellbore \((q_a)\). This expression is presented in equations 16 and 17:

\[
\pm C_f V_f \frac{\partial T_{fi}(z,t)}{\partial z} = \frac{T_{fi}(z,t) - T_{fa}(z,t)}{R_i} q_i(z,t) \\
\pm C_f V_f \frac{\partial T_{fa}(z,t)}{\partial z} = \frac{T_{fa}(z,t) - T_f(z,t)}{R_2} + \frac{T_{fa}(z,t) - T_{fi}(z,t)}{R_1} q_a(z,t)
\]

where \(T_{fi}\) is the water flow temperature (K) in the inner tube, and \(T_{fa}\) is the water flow temperature in the annulus (K), and \(z\) represents the axial distance along the wellbore, which is in the range of 0 (at the base of the wellbore) to \(h\) (top border of the hydrate zone). Some notations in equations 16 and 17 should be taken into account: i) the \(\pm\) sign refers to the direction of the water flow, with + standing for the water flow in the direction of the \(z\)-axis; and ii) in \(q_a\) calculation, an average temperature along the outer surface of the external wall of the wellbore \((T_f(r_i))\) is considered.

The temperature distribution resulted from the two operation models \((T_{fi} \text{ and } T_{fa})\) can be obtained by solving equations 16 and 17 and considering two boundary conditions: i) the inlet temperature \((T_i)\) is constant; and ii) no heat flux occurs at the base of the wellbore. More details are provided in the supplementary file. Then, the heat transfer rate to the reservoir can be obtained, after which, the subsequent hydrate dissociation will be calculated as described next.

Basically, the heat transfer from the heat source to the reservoir is consumed in two ways: i) by increasing the temperature of the sediments and the water-gas produced in the dissociated zone; and ii) hydrate dissociation at the moving boundary and increasing the temperature of the matrix materials in Zone II close to the moving boundary. The input heat should transfer through the dissociated region, during which, a large part of the input heat will be consumed in the first way mentioned above (i), reducing the rate of hydrate dissociation and the speed of the moving interface as the process continues.
Figure 2 shows the trends of temperature and pressure distribution in the reservoir during dissociation. The temperature and pressure ranges for a specific time in Zone I are respectively $T_s < T_1 < T_i(r_7)$ and $P_i < P < P_s$, and in Zone II, the temperature range will be $T_0 < T_{II} < T_s$ with a constant pressure equal the equilibrium pressure of MH. The temperature at the outer surface of the wellbore ($r=r_7$) changes with time due to the heat transfer from the wellbore and temperature changes of the inside fluid, but it is always lower than the temperature on the other side of the wellbore ($T_{fa}$). The temperature and pressure at the moving dissociation boundary are not also constant.

![Figure 2. Schematic of pressure and temperature distribution in the reservoir during MH dissociation.](image)

The produced gas will stream towards the heat source according to Darcy’s Law, inducing a sudden change in density at the dissociation front due to the gas production. Assumptions made in the dissociation models of the current work are provided in the following in accordance with previous analytical works [29, 62]: i) MH completely fills the porous media; ii) thermodynamic equilibrium is applied at the dissociation interface for temperature and pressure; iii) the water produced during the dissociation remains motionless in the porous media; iv) thermophysical properties of the phases remain constant during dissociation; v) the produced gas is modelled as an ideal gas; vi) the produced gas instantaneously reaches the same temperature as that of local sediments; vii) no viscous dissipation or inertial effect exists.
The following formulas represent the basic equations and solution procedure used in exploring dissociation while considering heat transfer from the wellbore.

The continuity equation of gas in Zone I is:

\[ \phi \left( \frac{\partial \rho_g}{\partial t} \right) + \left( \frac{\partial \rho_g v_g}{\partial r} \right) = 0, \quad t > 0 \]

(18)

where \( \rho_g \) is gas density (kg/m\(^3\)), \( \phi \) is reservoir porosity, \( r \) is the radial distance (m), and \( v_g \) is gas velocity (m/s). The gas velocity \( v_g \) in Zone I is calculated as follows by employing Darcy’s Law:

\[ v_g = -\left( \frac{k}{\mu} \right) \left( \frac{\partial P}{\partial r} \right), \quad t > 0 \]

(19)

where \( k \) is gas relative permeability (md), \( \mu \) is gas dynamic viscosity (Pa.s), and \( P \) is gas pressure (Pa).

Equations 20 and 21 show the energy balance in Zones I and II, respectively:

\[ \rho_I C_{pl} \frac{\partial T_I}{\partial t} + \rho_g C_{pg} v_g T_I = k_I \frac{\partial}{\partial r} \left( r \frac{\partial T_I}{\partial r} \right), \quad t > 0, \quad r_I < r < S \]

(20)

\[ \frac{\partial T_{II}}{\partial t} = \alpha_{II} \frac{\partial}{\partial r} \left( r \frac{\partial T_{II}}{\partial r} \right), \quad t > 0, \quad S < r \]

(21)

where \( \rho_I \) is the density (kg/m\(^3\)) of the matrix in Zone I, \( \rho_g \) is gas density (kg/m\(^3\)), \( C_{pl} \) is the specific heat capacity (J/(kg.K)) of the matrix in Zone I, \( C_{pg} \) is the specific heat capacity of gas (J/(kg.K)), \( T_I \) is the temperature (K) of the matrix in Zone I, \( T_{II} \) is the temperature (K) of the matrix in Zone II, \( \alpha_{II} \) is the thermal diffusivity (m\(^2\)/s) of the matrix in Zone II, and \( k_I \) is the thermal conductivity (W/(m.K)) of the matrix in Zone I.

The gas density in Zone I, based on the ideal gas law, can be calculated by the following equation:

\[ \rho_g = \frac{mP}{RT_I}, \quad r_I < r < S, \quad t > 0 \]

(22)

where \( m \) is the gas’s molecular mass (kg/mol), and \( R \) is the universal gas constant (J/(mol.K)). The above equations are representative of the fundamental concept of the MH dissociation process.
employed in the present work. The initial and boundary conditions are: i) temperature and pressure at the inner wall of the wellbore \((r_3)\) are known at each time step due to the calculations mentioned earlier for temperature distribution inside the wellbore; and ii) no pressure drop is assumed in the wellbore, and the pressure at the outer surface of the well \((r_7)\) is \(P_i\).

The heat transfer from the wellbore’s external wall to the reservoir is stated by Equation 6:

\[
-k_f A_w \frac{\partial T_I}{\partial r} = \frac{(T_{Ia} - T_I)}{R_2}, \quad r = r_7, t > 0
\]

(23)

where \(A_w\) is the wellbore surface area \((\text{m}^2)\), \(k_f\) is the thermal conductivity of Zone I \((\text{W/(m.K)})\), \(r\) is the radius \((\text{m})\). It should be noted that various structures and geometries for wellbores exist \([75, 105, 106]\), and the wellbore structure proposed in the present work is assumed as a general model based on previous works \([73, 74, 81, 107]\).

There is thermodynamic connection between the temperature and pressure at the moving dissociation interface, represented by the Antoine Equation as follows:

\[
P_s = \exp(A_a - B_a / T_s), \quad r = S, t > 0
\]

(24)

where \(P_s\) and \(T_s\) are respectively the pressure \((\text{Pa})\) and temperature \((\text{K})\) at the moving dissociation interface, \(A_a\) and \(B_a\) are constants.

Equations 25, 26, and 27 respectively represent the mass and energy balances at the dissociation interface and the MH dissociation heat \([62]\) with the associated initial and boundary conditions represented through equations 28-30:

\[
F_{gh} \sigma_{HII} \left( \frac{dS}{dt} \right) + \rho_g v_g = 0, \quad r = S, t > 0
\]

(25)

\[
k_{II} \frac{\partial T_{II}}{\partial r} - k_f \frac{\partial T_I}{\partial r} = \phi \rho H Q_{HII} \frac{dS}{dt}, \quad r = S, t > 0
\]

(26)

\[
Q_{HII} = c + d T_s
\]

(27)

\[
T_I = T_{II} = T_s(t), \quad r = S, t > 0
\]

(28)

\[
T_{II} = T_0, \quad r \to \infty, t > 0
\]

(29)
\[ T_{II} = T_0, \ r_\gamma < r < \infty, t = 0, \ S = 0 \]  

(30)

where \( F_{gH} \) is the mass ratio of the methane gas trapped inside the MH to the total mass of hydrate and is assumed to be equal to 0.1265 kg CH\(_4\)/kg hydrate [62], \( \rho_H \) is the hydrate density (kg/m\(^3\)), \( k_H \) is the thermal conductivity (W/(m.K)) of Zone II, \( Q_{Hd} \) is MH dissociation heat (J/kg), and \( c \) and \( d \) are constants.

Equations 18, 20, and 24 are simplified by employing equations 19 and 22 to eliminate the gas velocity and density, provided through the following equations 31-33:

\[
\phi \frac{\partial}{\partial t} \left( \frac{P}{T_i} \right) - k \frac{\partial}{\partial r} \left( \frac{P}{\mu} \frac{\partial P}{\partial r} \right) = 0 \tag{31}
\]

\[
\rho_i C_{pl} \frac{\partial T_L}{\partial t} + \frac{kmC_{tg}}{\mu R} \frac{\partial}{\partial r} \left( \frac{P}{\mu} \frac{\partial P}{\partial r} \right) = \frac{k_i}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_L}{\partial r} \right), \ t > 0, \text{ in Zone I} \tag{32}
\]

\[
F_{gH} \phi \sigma_H \left( \frac{dS}{dt} \right) - \frac{kmP}{\mu RT_i} \frac{\partial P}{\partial r} = 0, \ r = S, t > 0 \tag{33}
\]

To solve the abovementioned equations, the similarity solution is employed, using a dimensionless parameter relating the movement of the dissociation interface to the square root of time \((t^{1/2})\), as represented by a non-dimensional parameter shown in equation 34. This method, first introduced by Neumann [108, 109], also satisfies the initial and boundary conditions.

\[
\lambda = \frac{r}{\sqrt{4\alpha_H t}} \tag{34}
\]

On the moving dissociation interface, equation 20 becomes:

\[
\beta = \frac{S}{\sqrt{4\alpha_H t}}, \ r = S, t > 0 \tag{35}
\]

And, on the outer surface of the wellbore:

\[
\lambda_{os} = \frac{r_\gamma}{\sqrt{4\alpha_H t}}, \ r = r_\gamma, t > 0 \tag{36}
\]

Therefore, by using equations 34-36, the previous equations are simplified and transformed, as presented in the supplementary information in detail.
An exponential integral (Ei) function is employed to find the solution to the temperature distribution in the reservoir during dissociation as described in equations 37 and 38. This solution has also been recommended in previous works [108, 109].

\[ T_I = -AEi(-(a\lambda + b)^2) + AEi(-b^2) + B \]  \hspace{1cm} (37)

\[ T_{II} = T_0 + CEi(-\lambda^2) \]  \hspace{1cm} (38)

The \( A, B, C, a, \) and \( b \) constants are defined in the supplementary information.

Then, the pressure distribution in Zone I can be calculated from equation S12 as follows:

\[ P(\lambda) = \left( P_0^2 + \frac{4F_{shl} \phi \rho_m \alpha_m \mu R \beta}{km} \int T_d d\lambda \right)^{1/2} \]  \hspace{1cm} (39)

By replacing \( T_I \) according to equation 37, the pressure distributions in zone I will be calculated as shown in equation 40:

\[ P = \left( P_0^2 + L(\beta)(K(\beta)\lambda - AN(\lambda) - (K(\beta)\lambda_{as} - AN(\lambda_{as}))) \right)^{1/2}, \]  \hspace{1cm} (40)

where \( L(\beta), N(\lambda), \) and \( K(\beta) \) are defined in the supplementary information. It should be noted that the obtained expressions for temperature and pressure distributions can satisfy the basic equations and boundary conditions, mentioned earlier, by direct substitution.

Heat flux (J/sm\(^2\)) from the wellbore to the reservoir as a function of time can be obtained from the following formula:

\[ u_r = -k_j \frac{\partial T_I}{\partial r} , \ r = r_j \]  \hspace{1cm} (41)

Equation 41 can be transformed into an equal equation according to equation 20 (supplementary information).

By integrating Equation 41, the total heat input into the reservoir from the wellbore (J/m\(^2\)) up to time \( t \) can be calculated as in the following equation:

\[ Q_n = -k_j \int_0^t \frac{\partial T_I}{\partial r} dt \ , \ r = r_j \]  \hspace{1cm} (42)

The total volume of gas produced up to time \( t \) can be calculated as follows at the standard temperature and pressure (STP) of dry gas:
\[ V_p = \frac{n_n RT_{\text{STP}}}{P_{\text{STP}}} \]  

(43)

where \( V_p \) and \( n_n \) are respectively the total volume (m\(^3\)/m\(^2\)) and total moles (mole/m\(^2\)) of produced gas per surface area of the moving interface up to time \( t \) with \( T_{\text{STP}} \) and \( P_{\text{STP}} \) respectively as the temperature and pressure of dry gas at STP conditions. Further details of the calculation of the total volume of produced gas are provided in the supplementary information.

In order to assess the efficiency of gas production during a dissociation process that uses thermal stimulation method, the ratio of the amount of energy that can be produced from combustion of the total produced gas to the amount of input energy to the system during dissociation is introduced as the energy efficiency ratio (equation 44) [110]:

\[ \eta_f = \frac{V_p Q_g}{Q_n (2\pi r_t h)} \]  

(31)

where \( \eta_f \) is the energy efficiency ratio, and \( Q_g \) is the heating value of the gas at STP conditions (J/m\(^3\)).

**Results and discussion**

Pressure and temperature at the dissociation interface (\( P_s \) and \( T_s \)) are coupled to the pressure and temperature at the wellbore (outer surface of the wellbore) due to their associated equations, mentioned earlier in the previous section. Thus, The dimensionless position and dimensionless velocity (\( v_s / (4\alpha_\mu t)^{1/2} \)) of the dissociation interface, both represented by \( \beta \) (equation 35), which are connected to \( P_s \) and \( T_s \) (equation S27), are related to the pressure and temperature at the wellbore. The temperature at the outer surface of the wellbore is dependent on the temperature of the annulus section and time. The temperature inside the wellbore changes due to time and the temperature at the wellbore, which changes over time as the dissociation progresses. Table 1 shows
the proposed properties and parameters for the base model according to previous studies [4, 62, 80, 82, 111-114].

As shown in Figure 3a, $\beta$ decreases at the beginning of the process because the thickness of Zone I increases over time, and the associated matrix in this zone absorbs a higher amount of transferred heat. Thus, dissociation is responsible for most of the energy consumption in the early stages of the process. But the energy consumption converges to a fixed value because the temperature at the wellbore surface is also converging slightly to that of inside of the well ($T_{fa}$). Figure 3b shows the location of the interface ($S$) during the process. The slope in the diagram is the highest at the beginning, but it decreases over time, which supports the observations from previous studies [30, 50, 110]. This behavior is in line with $\beta$, which decreases significantly at the beginning but slightly converges to a constant value and can be justified for the same reasons. Wang et al. [27] and Li et al. [115, 116], who experimentally investigated the hydrate dissociation upon thermal stimulation, also proved that at the beginning of the dissociation process, the rate of dissociation close to the wellbore is the highest. Roostaie and Leonenko [79], who designed a 1D analytical model for MH dissociation upon wellbore heating with a specific structure for wellbore, reported variable dissociation rates ($\beta$) due to the variable temperature at the wellbore. When hot water is injected through the annulus, temperature distribution through this section, from which the heat transfer to the reservoir takes place, is higher, as shown in Figures 3c and 3d, inducing slightly higher values for $\beta$ and $S$ compared to those of the other operating scheme. The temperature difference becomes significant through the extraction part especially at the beginning of the process (Figures 3c and 3d). However, the associated differences decrease as the process continues to 100 days. This behavior was also reported by Beier et al. [82] who analytically and experimentally investigated the temperature distribution in the coaxial heat exchangers as ground source heat pumps.
| Parameter                                                                 | Value          |
|--------------------------------------------------------------------------|----------------|
| Table 1. Parameters used for the base model.                              |                |
| $r_1, \text{ m}$                                                        | 0.043          |
| $r_2, \text{ m}$                                                        | 0.05           |
| $r_3, \text{ m}$                                                        | 0.07           |
| $r_4, \text{ m}$                                                        | 0.077          |
| $r_5, \text{ m}$                                                        | 0.092          |
| $r_6, \text{ m}$                                                        | 0.099          |
| $r_7, \text{ m}$                                                        | 0.124          |
| Methane hydrate thickness, $\text{m}$                                    | 80             |
| Thermal conductivity of inner tube (polyethylene tube), $k_p$, W/(m.K)   | 0.4            |
| Thermal conductivity of cement, $k_c$, W/(m.K)                           | 0.933          |
| Thermal conductivity of gravel, $k_g$, W/(m.K)                           | 0.4            |
| Thermal conductivity of casing (steel), $k_s$, W/(m.K)                   | 43.3           |
| Thermal conductivity of water, $k_f$, W/(m.K)                            | 0.667          |
| Density of water, $\rho_f$, kg/m$^3$                                     | 971.79         |
| Dynamic viscosity of water, $\mu_f$, Pa.s                               | $3.54 \times 10^{-4}$ |
| Specific heat capacity of water, $C_f$, J/(kg.K)                        | 3.89           |
| Porosity, $\phi$                                                        | 0.3            |
| Water flow rate, $V_f$, L/s                                             | 0.56           |
| Permeability, $k$, $\mu$m$^2$                                           | 1              |
| Thermal diffusivity of Zone I, $\alpha_I$, $\mu$m$^2$/s                | $2.89 \times 10^6$ |
| Thermal conductivity of Zone I, $k_I$, W/(m.K)                           | 5.57           |
| Thermal diffusivity of Zone II, $\alpha_{II}$, $\mu$m$^2$/s             | $6.97 \times 10^5$ |
| Thermal conductivity of Zone II, $k_{II}$, W/(m.K)                       | 2.73           |
| Hydrate density, $\rho_H$, kg/m$^3$                                      | 913            |
| Heat of dissociation of hydrate, $Q_{Hd}$, J/kg                         | $446.12 \times 10^3 - 132.638 T_s$ |
| Gas heat capacity, $C_{pg}$, J/(kg.K)                                   | 8766           |
| Dynamic viscosity of gas, $\mu$, Pa.s                                   | $10^{-4}$      |
| Heating value of the gas at STP conditions, $Q_g$, MJ/m$^3$              | 37.6           |
| Molecular mass of methane, $m$, g/mol                                    | 16.04          |
| Mass ratio of the methane gas trapped inside the hydrate to the mass of hydrate, $F_{gH}$ | 0.1265 |
| Universal gas constant, $R$, J/(mol.K)                                  | 8.314          |
Figure 3. Variation of a) $\beta$ and b) interface location versus time for the two operation schemes. Temperature distribution in the wellbore versus depth when hot water is injected into c) inner tube and d) annulus. The conditions are: $T_0=280$ K and $T_i=365$ K, $P_i=7.6$ MPa, and a water flow rate of 0.56 L/s.

Figure 4a shows that decreasing the pressure at the wellbore and increasing the temperature of injected water decrease the value of $\beta$ after 100 days operation. Further information on how the injected water temperature affects dissociation is provided in the following results. The initial temperature of MH also has a direct effect on $\beta$ as shown in Figure 4b. These results are consistent with those of the previous work by Selim and Sloan [62] with the same trend, but $\beta$ is lower (approximately 48%) in the present model. Roostaie and Leonenko [79] also reported the same behavior for $\beta$ against various initial pressures and temperatures, but with higher values.
(approximately 66%). These differences may be due to the following reasons: i) Selim and Sloan [62] considered a 1D flat heat source without wellbore thickness was considered; and ii) both of the previous works [62, 79] assumed a very high and constant heat-source temperature.
Furthermore, the locus at which the dissociation temperature is equal to 280 K (MH equilibrium temperature) is also shown on Figure 4. At the points on the left and right sides of this locus, the dissociation temperature decreases and increases, respectively. The dissociation temperature may depend on heat source pressure, whereas it is almost independent from the temperature of the heat source and MH. This result is in agreement with those of previous works [62, 79], which also reported that at heat-source pressures lower than 6 MPa, $T_s$ may reduce to the freezing point of water, and the ice generation can halt dissociation. For the situations, in which $T_s$ is higher than the MH’s temperature, some part of input heat from the heat source will be consumed to bring the temperature of MH zone close to the dissociation front up to $T_s$. If $T_s$ approaches $T_0$, all heat from the heat source is consumed for dissociation and no heat will be transferred to or from the MH zone near the dissociation interface, because the temperature of the hydrate zone remains constant. On the other hand, if $T_s$ falls below $T_0$, some part of the heat required for dissociation will be provided from the hydrate zone, causing in the temperature of the hydrate zone near the dissociation interface to drop.
Temperature and pressure distributions calculated for the two operation schemes are respectively depicted in Figures 5 and 6 for different time frames, with the following two boundary conditions (BCs): BC 1) \( T_i = 340 \) K, \( P_i = 10 \) MPa, and \( T_0 = 280 \) K, and BC 2) \( T_i = 365 \) K, \( P_i = 7.6 \) MPa, and \( T_0 = 275 \) K. It should be noted that the temperature of the injected water is below the saturation temperature of water in the present work. Black dashed lines in Figure 5 represent the temperature at the dissociation interface. Although, as mentioned earlier, \( T_i \) depends on the temperature at the wellbore and does not change significantly (Figure 5) because the temperature increase at the wellbore becomes smaller in longer time frames for the reasons stated earlier for the trend of \( \beta \) during the process. Figures 5 and 6 along with Figure 4 show that the interface moves further if the inlet temperature is increased and the pressure at the wellbore is decreased, again reflecting reports by Selim and Sloan [62]. The interface pressure (Figure 6) is not constant and increases due to temperature changes at the wellbore surface. The pressure increment does not follow a constant slope and decreases as the temperature at the wellbore surface gets closer to that inside the well. Thus, it does not change significantly, resulting in convergence of the interface pressure. These results are in line with those reported by Roostaie and Leonenko [79], mainly because the temperature at the wellbore was not constant in their work due to the heat conduction through the wellbore structure. In accordance with the present results, a study by Tsimpanogiannis and Lichter [29], who built a semi-analytical model for hydrate dissociation, demonstrated that increasing the temperature of the wellbore raised the pressure at the interface. The temperature distribution differences between the two operation schemes is negligible, with only slightly higher temperature at the wellbore upon injection into the annulus, causing higher dissociation pressures and farther interface location from the wellbore.
Figure 5. Temperature distribution at different time frames for two initial and boundary conditions (i.e., BC 1: a and c, BC 2: b and d). a) and b) are for hot water injection into annulus, and c) and d) are for hot water injection into the inner tube. The black dashed line specifies the temperature at the dissociation interface.
Figure 6. Pressure distribution in the dissociated zone at different time frames for two initial and boundary conditions (i.e., BC 1: a and c, BC 2: b and d). a) and b) are for hot water injection into annulus, and c) and d) are for hot water injection into the inner tube.

Figure S1 shows the total volume of produced gas (m$^3$) under STP conditions, and input heat (MJ/m$^2$) in two models with their specific operation schemes, considering the two initial and boundary conditions over 100 days. The produced gas volume is almost the same for both models, with slightly higher values for the case with hot water injection into the annulus, due to the direct heat transfer from heat source to reservoir. Beier et al. [82], who analytically investigated borehole heat exchangers and verified their results against experiments, reported the same heat transfer to the earth following injection into the inner tube or annulus. Holmberg et al. [80] performed and validated a set of numerical investigations on underground coaxial borehole heat exchangers and
reported that different flow directions have almost the same efficiency. Figures S1c and S1d also show a negligible difference between the amount of heat transfer to the reservoir following injection into the inner tube or annulus. The produced gas and the input heat are higher under BC 2 than BC 1. The energy efficiency is higher in the model with BC 2 than that with BC 1 as shown in Figure 7, demonstrating that lowering the pressure at the wellbore and increasing the inlet temperature simultaneously result in more efficient gas production during dissociation. Over time, the sediment matrix of Zone I becomes wider and absorbs a larger part of the input heat, causing reduction of the energy efficiency slope.

Experimental researches investigated MH dissociation upon thermal stimulation by hot water circulation in a reservoir. For example, Song et al. [110] by using this method reported the same trend for energy efficiency in their experimental work. The energy efficiency in their work was between 18 and 40. Comparison of the results of energy efficiency and gas production with those of Wang et al. [30], who performed an experimental work employing the same approach by huff and puff method, also shows the same trend. The present results are consistent with the experimental data of Li et al. [115, 116], who also employed huff and puff method in a 5.8 L cubic reactor. The calculated energy efficiency of the process was approximately 20.6 and the produced gas had the similar trend as the present work. Wang et al. [27] also reported an energy efficiency of between 6 and 20 during their experimental investigations on MH dissociation using the same method. The associated difference with the present results could be due to some experimental conditions, such as direct hot water circulation into the reservoir and not considering the effect of wellbore structure and geometry as a noteworthy factor.

Selim and Sloan [62] in their analytical work reported an energy efficiency between 6.4-11.2, which remained constant during the process. Some of the assumptions in their work, which are mentioned earlier, are the reasons of the difference between the results of the present study from theirs.
A parametric study has been designed based on the parameters listed in Table 2, to investigate the effect of various characteristics of reservoir and wellbore on the dissociation process. From Figures 8 and 9: i) the higher the thermal diffusivities and conductivities of Zone I, the higher the dissociation rate and gas production will be; ii) the lower the thermal diffusivities and conductivities of Zone II, the higher the dissociation rate will be, on the other hand, higher thermal diffusivities and lower thermal conductivities induce more gas production; iii) the dissociation rate and gas production have almost no connection to the permeability and gas viscosity; and iv) the porosity of the media has a direct relation to the dissociation rate, in contrast, it has an inverse relation with the gas production.

Figure S3 in the supplementary file displays the input heat from the wellbore to the reservoir during the parametric study of the reservoir’s characteristics. Higher thermal conductivity in Zone I induces significantly higher amounts of input heat from the reservoir (Figure S3a). Higher thermal diffusivity (lower heat capacity while the density is constant) induces less heat storage in the media (lower input heat), which in turn, causes more heat transfer to the dissociation front. Thus, increasing the thermal diffusivity and thermal conductivity of Zone I increases the dissociation rate (Figure 8a) and gas production level (Figure S2a). Lower thermal diffusivity of Zone II increases the dissociation rate (Figure 8b) due to greater storage of the transferred heat to this zone from the dissociation interface, which in the end, will be released and consumed for dissociation.
Increasing the thermal conductivity of Zone II causes faster heat transfer to this zone from the moving interface, reducing the dissociation rate and gas production. Increasing the thermal diffusivity of Zone II reduces heat storage in this zone, increasing heat consumption for dissociation and raising gas production (Figure S2b). Figure S3b shows that decreasing the thermal diffusivity and increasing the thermal conductivity of Zone II, resulting in more input heat for the same reason stated for input heat increment induced by the same change in thermal conductivity and diffusivity in Zone I (Figure S3a). However, the amount of input heat increment is much lower for the case of Zone II than that of Zone I due to the direct contact of Zone I with the wellbore. Conduction heat transfer and input heat from the wellbore decrease for higher porosities (Figure S3c), ultimately causing a reduction in the dissociation rate (Figures 8c and 8d). Furthermore, higher porosity increases the amount of MH trapped in the pores, eventually increasing the amount of gas produced (Figures S2c and S2d).

Figures 9a and 9b clearly show the direct relation between energy efficiency and thermal diffusivity in Zones I and II, caused by the rise in gas production and drop in input heat induced by increasing the thermal diffusivity. On the other hand, the energy efficiency has an inverse relation with the thermal conductivity of Zones I and II, perhaps due to: i) higher input heat induced by the higher thermal conductivities of Zone I (Figure S3a); and ii) the lower gas production caused by the higher thermal conductivities of Zone II (Figure S2b).

Selim and Sloan [62] and Roostaie and Leonenko [79] performed a similar parametric study and reported similar results for the rate of dissociation (Figure 8). Zhao et al. [117] mathematically showed that increasing the thermal conductivity had a direct positive effect on a dissociation process based on thermal stimulation. They also reported that almost no change occurred in the dissociation by changing the relative permeability of water and gas, due to the negligible impact that the convection heat transfer of water and gas has on the process. Zhao et al. [118] in another numerical work showed that increasing the sediments’ thermal conductivity caused a higher gas generation rate at the beginning of dissociation employing depressurization. Both of their works were verified by the experimental data of Masuda’s work [45]. Tsimpanogiannis and Lichtner [29], who performed a similar parametric study on MH dissociation upon thermal stimulation, showed that the higher the thermal conductivity of the porous media, the higher the MH dissociation will be. Moridis et al. [50] numerically showed that a higher initial formation temperature, well
temperature, and formation thermal conductivity increased the amount of gas production at the Mallik site. They also reported that the dissociation is not affected by the formation permeability and the specific heat of the rock and MH. It is also shown in Figures 8 and S2 (the dissociation rate ($\beta$) has a direct relation to the amount of produced gas). It should be noted that some of different working conditions, such as direct hot water circulation into the reservoir, the duration of experiments, and model parameters (i.e., hydrate saturation) may have caused the differences between the results of the experimental works and those of the present work.

| Reservoir parameters                      |                  |
|------------------------------------------|------------------|
| Porosity, $\phi$                          | 0.1 to 0.5       |
| Permeability, $k$, $\mu$m$^2$             | 0.1 to 5         |
| Thermal diffusivity of Zone I, $\alpha_I$, $\mu$m$^2$/s | $1\times10^6$ to $5\times10^6$ |
| Thermal conductivity of Zone I, $k_I$, W/(m.K) | 3 to 7          |
| Thermal diffusivity of Zone II, $\alpha_{II}$, $\mu$m$^2$/s | $4\times10^5$ to $8\times10^5$ |
| Thermal conductivity of Zone II, $k_{II}$, W/(m.K) | 1 to 5         |
| Gas viscosity, $\mu$, Pa.s                | $10^{-4}$ to $10^{-6}$ |

| Wellbore parameters                       |                  |
| Water flow rate, $V_f$, m$^3$/s            | 0.0004 to 0.006  |
| Inlet temperature, $T_i$, K                | 330 to 365       |
| Thickness of MH reservoir, $h$, m          | 15 to 80         |
| Outer radius of the wellbore, $r_7$, m     | 0.114 to 0.174   |
| Only the annulus radius, $r_7$, m          | 0.114 to 0.164   |
| Only the inner tube radius, $r_2$, m       | 0.013 to 0.053   |

Table 2. Range of parameters assumed in the parametric study.
Figure 8. The effect of various parameters on the interface movement after 100 days dissociation considering both operating schemes: a) thermal diffusivity and thermal conductivity of Zone I, b) thermal diffusivity and thermal conductivity of Zone II, c) porosity with various permeabilities, and d) porosity with various gas viscosities.

Solid lines and dashed lines respectively represent the operating schemes of hot water injection into annulus and into the inner tube.
Figure 9. Energy efficiency after 100 days dissociation considering both heat sources and various parameters: a) thermal diffusivity and thermal conductivity of Zone I, b) thermal diffusivity and thermal conductivity of Zone II, c) porosity with various permeabilities, and d) porosity with various gas viscosities.

Solid lines and dashed lines respectively represent the operating schemes of hot water injection into annulus and into the inner tube.

Figures 10, 11, S4, and S5 show the results of the parametric study examining how the wellbore parameters (Table 2) affect dissociation. They highlight the following points: i) a higher wellbore...
radius (annulus and inner tube with the same amount) increases the dissociation rate (Figure 10a) and gas production level (Figure S4a) because the contact surface enlargement affects the process significantly; ii) the dissociation rate decreases in reservoirs with larger thickness (Figure 10b), whilst, the gas production increases due to the contact surface enlargement (Figure S4b); iii) enlarging the annulus radius, while the inner tube’s radius is fixed, has a direct relation on the dissociation rate (Figure 10c) and produced gas (Figure S4c) due to the contact surface enlargement; iv) higher inner tube’s radii, while keeping the annulus’s radius fixed, has almost no effect on the dissociation process. Raymond et al. [119] through analytical investigations, revealed that increasing the inner tube’s radius has little effect on the thermal resistance of coaxial borehole heat exchangers; v) higher inlet temperatures has a direct and noteworthy impact on both the dissociation rate (Figure 10e) and produced gas (Figure S4e); vi) the dissociation rate and gas production increase initially in response to a transition to higher water flow rates, but they tend to remain almost unchanged after a certain water flow rate.

Increasing the wellbore radius reduces the velocity of the hot water flow inside the wellbore, which in turn reduces the mean temperature inside the wellbore. As a consequence, the input heat from the wellbore to the reservoir drops as well (Figure S5a). The energy efficiency of the process decreases as the wellbore radius increases (Figure 11a) because the ratio of produced gas to the input heat also decreases.

A higher reservoir thickness decreases the input heat (Figure S5b), due to the reduction in the mean temperature inside the wellbore induced by longer wellbores. Holmberg et al. [80] also reported that the deeper coaxial borehole heat exchangers are not good at heat transfer into the reservoir, but are useful for heat extraction from the reservoir. Raymond et al. [119] analytically proved that longer coaxial boreholes have higher thermal resistances. On the other hand, increasing the reservoir’s thickness boosts energy efficiency slightly (Figure 11b) because there is a greater rise in gas production than in input heat. The differences in results between injection into the inner tube and injection into the annulus increases with an increase in the reservoir thickness. Actually, in longer wellbores, hot water injection into the inner tube makes the drop in the flow mean temperature in the annulus become more pronounced because the flow enters the annulus with a lower temperature, while in the other case, the hot water inlet is the annulus, which is in direct contact with the reservoir.
The input heat decreases by increasing only the annulus radius (Figure S5c) due to the same reason stated previously for the effect of the wellbore radius on the input heat. This observation also accords with another analytical study on ground coaxial heat exchangers conducted by Raymond et al. [119]. They reported that increasing only the annulus radius reduced the wellbore thermal resistance. In contrast to the higher produced gas induced by bigger annulus radius, the energy efficiency decreases (Figure 11c). Bigger annulus radii while keeping the inner tube’s radius fixed will reduce the flow rate in the annulus significantly, reducing the mean temperature in annulus, and it causes more input heat reduction compared to the case of increasing the radius of both annulus and inner tube together.

The input heat and the energy efficiency increase by increasing the inlet temperature (Figures S5e and 11e). The difference of energy efficiency and input heat results between the two operating schemes tends to get slightly bigger as the inlet temperature increases. Increasing the inlet temperature in the case of hot water injection into annulus directly affects the input heat, but in the other case it causes higher mean temperature in the inner tube, and the associated temperature increase in annulus is not significant.

The input heat increases by increasing the flow rate (Figure S5f), but it converges to almost a fixed value. It can be concluded that the mean temperature inside the wellbore initially increases but increasing the flow rate more than 0.0016 m³/s almost has no effect on the mean temperature inside the wellbore and the associated input heat to the reservoir. Zanchini et al. [120], who carried out numerical analysis on the performance of coaxial borehole heat exchangers, showed that the heat transfer capacity of the wellbore would be increased by increasing the flow rate of the wellbore. This is also in line with the results of an analytical work performed by Raymond et al. [119], who showed that the higher flow rates decreased the thermal resistance of coaxial ground heat exchangers. Despite the produced gas increment due to the higher flow rates, energy efficiency of the process reduces (Figure 11f) because the input heat increment is slightly higher than that of the produced gas.

The amount of produced gas and input heat are always higher for the case of injection into annulus than those of the other operating scheme, but, the resulted energy efficiency is always slightly higher for the case of injection into the inner tube except for the cases of studying different flow
rates. In fact, by increasing the flow rate in the annulus or in the inner tube, the flow rate would be increased through the wellbore by the same ratio, and it does not depend on hot water inlet.
Figure 10. The effect of various parameters on the interface movement after 100 days dissociation considering both operating schemes: a) the wellbore radius, b) the reservoir thickness, c) the annulus radius, d) the inner tube radius, e) the injection temperature, and f) the flow rate.

Dashed lines and solid lines are respectively representative of the operating schemes of hot water injection into annulus and into the inner tube.
Figure 11. The effect of various parameters on the energy efficiency after 100 days dissociation considering both operating schemes: a) the wellbore radius, b) the reservoir thickness, c) the annulus radius, d) the inner tube radius, e) the injection temperature, f) the flow rate.

Dashed lines and solid lines respectively represent the operating schemes of hot water injection into annulus and into the inner tube.

Conclusions

The present study was designed to determine different aspects of MH dissociation upon thermal stimulation employing a coaxial wellbore structure as the heat source. For the first time, 2D radial analytical models have been developed to couple operational conditions of the heat source and the associated MH dissociation in the reservoir. Heat and mass transfer in both wellbore and reservoir
as well as the convection heat transfer in the wellbore are taken into account. Two operating schemes for wellbore heating are considered: i) hot water supply into the inner tube; and ii) hot water supply into the annulus section. The effects of various parameters of wellbore and reservoir as well as different boundary conditions on dissociation are also evaluated. Two main factors are considered to assess the production process performance: i) dissociation rate ($\beta$), and ii) energy efficiency of the process ($\eta$). Taken together, the most important findings to emerge from the results are as follows:

- The dissociation process has direct relation to the temperature of the reservoir and the inlet water, but it has an inverse relation to the wellbore pressure.
- The different operating schemes have approximately the same outcome with negligible differences due to the model conditions.
- Increasing the inlet water temperature while decreasing wellbore’s pressure increases the dissociation rate and the produced gas. It also increases the energy efficiency despite some of the previous works’ reports. This could be due to the inconstant wellbore temperature in the present work. Hence, more information about the heat source operating conditions can help increasing the accuracy on this matter.
- Different wellbore radii, water inlet temperatures and flow rates along with various properties, such as thermal diffusivities, porosities, thermal conductivities, and reservoir thicknesses have significant effects on the process. On the other hand, some of the reservoir and wellbore parameters, such as inner tube radius, reservoirs permeability, and gas viscosity, have almost no impact on the process. Thus, a thorough investigation on the reservoir properties and applicable heat source characteristics is necessary before performing a field work.
- Temperature at the wellbore surface changes over time due to the different temperatures inside the wellbore induced by convection and conduction heat transfer of the hot water flow inside the wellbore. It is also affected by the heat conduction from wellbore to reservoir, which is dependent on the wellbore outer structure.
- Temperature at the wellbore surface affects both the pressure and temperature at the dissociation front.
- The results of this study are in good agreement with those of the previous experimental and numerical studies. It also validates the assumptions made during the model development.
The findings of the present study contribute in several ways to our understanding of MH dissociation upon wellbore heating method and provide a basis for future investigations. The presented analytical approach takes into account: i) coupling the heat transfer process inside the coaxial wellbore and the associated thermal response in the reservoir which governs the dissociation and methane production; ii) heat transfer through the wellbore structure, consisting of multiple layers with different thermal properties; and iii) a more comprehensive model compared to the previous study [79]. Consequently, the models are more reliable and closer to the real practical conditions.

Acknowledgement

Financial support for this work provided by Natural Sciences and Engineering Research Council of Canada (NSERC)

| Nomenclature | Definition |
|---------------|------------|
| $A$ | Dimensionless variable in equation 37 |
| $A_{di}$ | Dimensionless constant in equation 24 |
| $A_s$ | Average area of the dissociation front |
| $A_w$ | Wellbore area |
| $a$ | Dimensionless constant in equation 37 |
| $B$ | Dimensionless variable in equation 37 |
| $B_{di}$ | Dimensionless constant in equation 24 |
| $BC 1$ | Boundary conditions |
| $BC 2$ | Boundary conditions |
| $b$ | Dimensionless constant in equation 37 |
| $C$ | Dimensionless variable in equation 38 |
| $C_f$ | Fluid volumetric heat capacity |
| $q_i$ | Heat transfer rate from/to the inner tube to/from the annulus |
| $R$ | Universal gas constant |
| $R_1$ | Thermal resistivity for heat transfer through inner tube |
| $R_2$ | Thermal resistivity for heat transfer through annulus |
| $R_{w}$ | Thermal resistivity of the wellbore |
| $R_{coi}$ | Convection thermal resistance between water flow and outer surface of the annulus |
| $R_{cii}$ | Convective heat transfer resistance between the inner tube surface and the fluid flow in the tube |
| $R_p$ | Conduction thermal resistance of the inner tube |
| $r$ | Radial distance |
| $r_1$ | Inside radius of inner tube |
| $r_2$ | Outside radius of inner tube |
| Symbol | Definition |
|--------|------------|
| $C_{pl}$ | Specific heat capacity of Zone I |
| $C_{pg}$ | Specific heat capacity of gas |
| $c$ | Dimensionless constant in equation 27 |
| $D$ | Dimensionless variable in equation S14 |
| $d$ | Dimensionless constant in equation 27 |
| $d_h$ | Hydraulic diameter of the annulus |
| $E$ | Dimensionless variable in equation S14 |
| $F$ | Dimensionless variable in equation S14 |
| $F_{gH}$ | Ratio of mass of the methane gas trapped inside the MH to the mass of hydrate |
| $f$ | Friction factor |
| $G(\beta)$ | Dimensionless constant in equation S24 |
| $H(\beta)$ | Dimensionless constant in equation S24 |
| $h$ | Thickness of MH reservoir |
| $I(\beta)$ | Dimensionless constant in equation S24 |
| $K(\beta)$ | Function in equation 40 |
| $k$ | Permeability |
| $k_f$ | Fluid thermal conductivity |
| $k_p$ | Inner tube’s thermal conductivity |
| $k_I$ | Thermal conductivity of Zone I |
| $k_{II}$ | Thermal conductivity of Zone II |
| $k_c$ | Thermal conductivity of cement |
| $k_g$ | Thermal conductivity of gravel |
| $k_{pI}$ | Thermal conductivity of inner tube |
| $r_3$ | Inside radius of wellbore/ Inside radius of casing 1 |
| $r_4$ | Outside radius of casing 1/ Inside radius of gravel part |
| $r_5$ | Outside radius of gravel part/ Inside radius of casing 2 |
| $r_6$ | Outside radius of casing 2/ Inside radius of cement part |
| $r_7$ | Outside radius of cement part/ Outside radius of wellbore |
| $r^*$ | Ratio of the inner and outer radii of the annulus $(r_2 / r_3)$ |
| $Re_i$ | Reynolds number in the inner tube |
| $S$ | Interface position |
| $S_i$ | Interface position at time $t$ |
| $T_{fi}$ | Water flow temperature in the inner tube |
| $T_{fa}$ | Water flow temperature in the annulus |
| $T_i$ | Hot water injection temperature |
| $T_I$ | Temperature in Zone I |
| $T_{II}$ | Temperature in Zone II |
| $T_0$ | Initial temperature of hydrate |
| $T_{fi}$ | Fluid temperature in the inner tube |
| $T_{fa}$ | Fluid temperature in the annulus |
| $T_s$ | Temperature at the interface |
| $T_{STP}$ | Temperature of gas at STP conditions |
| $t$ | Time |
| $u_r$ | Heat flux from the well |
| $V_f$ | Water flow rate |
| $V_g$ | Volume of produced gas per surface area of the moving interface in the time fraction of “$t,t-1$” |
| Symbol | Description |
|--------|-------------|
| $k_i$  | Thermal conductivity of casing |
| $L(\beta)$ | Function in equation 40 |
| $M(\beta)$ | Function in equation S19 |
| MH   | Methane hydrate |
| $m$  | Gas molecular mass |
| $N(\lambda)$ | Function in equation 40 |
| $N_{u_i}$ | Nusselt’s number of the fluid flow in the inner tube |
| $N_{u_{oa}}$ | Nusselt’s number of the water flow close to the outer surface of annulus |
| $N_{u_{oi}}$ | Nusselt’s number of the fluid flow in the annulus close to the inner tube |
| $N_{u_{pipe}}$ | Nusselt’s number of the fluid flow in the annulus close to the inner pipe |
| $n_r$ | Total moles of produced gas per surface area of the moving interface in the time fraction of “$t, t-1$” |
| $n_{ri}$ | Total moles of produced gas per surface area of the moving interface up to time $t$ |
| $P$  | Pressure in Zone I |
| $P_s$ | Pressure at the interface |
| $P_i$ | Pressure of the heat source |
| $P_{STP}$ | Pressure of gas at STP conditions |
| Pr   | Prandtl number |
| $Q_{Hd}$ | Heat of MH dissociation |
| $Q_f$ | Heating value of the gas at STP conditions |
| $Q_{in}$ | Total input heat to the reservoir from the heat source |
| $q_a$ | Heat transfer rate from annulus to the reservoir |

$V_p$ Total volume of produced gas per surface area of the moving interface up to time $t$

$v_s$ Interface velocity

$v_f$ Water flow velocity

$v_g$ Gas velocity

$z$ Axial distance along the wellbore

$\rho_i$ Density of the matrix in Zone I

$\rho_f$ Fluid density

$\rho_g$ Gas density

$\rho_{H}$ Hydrate density

$\mu$ Gas viscosity

$\lambda$ Dimensionless variable in equation 34

$\lambda_{oa}$ Dimensionless variable in equation 36

$\beta$ Dimensionless constant in equation 35

$\phi$ Porosity

$\alpha_i$ Thermal diffusivity of Zone I

$\alpha_{II}$ Thermal diffusivity of Zone II

$\mu_f$ Fluid dynamic viscosity

$\nu_f$ Water kinematic viscosity

$\eta_r$ energy efficiency ratio

$\zeta$ Constant in equation 8
References

[1] J.-C. Feng, Y. Wang, X.-S. Li, G. Li, Z.-Y. Chen. Production behaviors and heat transfer characteristics of methane hydrate dissociation by depressurization in conjunction with warm water stimulation with dual horizontal wells. Energy. 79 (2015) 315-24.

[2] M.K. Davie, B.A. Buffett. A numerical model for the formation of gas hydrate below the seafloor. Journal of Geophysical Research: Solid Earth. 106 (2001) 497-514.

[3] T.S. Collett. Assessment of gas hydrate resources on the North Slope, Alaska, 2008. AGU Fall Meeting Abstracts2008.

[4] J. Majorowicz, K. Osadetz. Gas hydrate distribution and volume in Canada. AAPG bulletin. 85 (2001) 1211-30.

[5] M. Max, W.P. Dillon. Oceanic methane hydrate: the character of the Blake Ridge hydrate stability zone, and the potential for methane extraction. Journal of Petroleum Geology. 21 (1998) 343-58.

[6] A. Razmi, M. Soltani, M. Tayefeh, M. Torabi, M. Dusseault. Thermodynamic analysis of compressed air energy storage (CAES) hybridized with a multi-effect desalination (MED) system. Energy Conversion and Management. 199 (2019) 112047.

[7] A. Razmi, M. Soltani, F.M. Kashkooli, L.G. Farshi. Energy and exergy analysis of an environmentally-friendly hybrid absorption/recompression refrigeration system. Energy conversion and management. 164 (2018) 59-69.

[8] M. Soltani, F. Moradi Kashkooli, A. Dehghani-Sanj, A. Nokehosteen, A. Ahmadi-Joughi, K. Gharali, et al. A comprehensive review of geothermal energy evolution and development. International Journal of Green Energy. 16 (2019) 971-1009.

[9] M. Soltani, F.M. Kashkooli, A. Dehghani-Sanj, A. Kazemi, N. Bordbar, M. Farshchi, et al. A comprehensive study of geothermal heating and cooling systems. Sustainable Cities and Society. 44 (2019) 793-818.

[10] X. Hu, S.E. Li, Y. Yang. Advanced machine learning approach for lithium-ion battery state estimation in electric vehicles. IEEE Transactions on Transportation Electrification. 2 (2015) 140-9.

[11] A. Razmi, M. Soltani, C. Aghanajafi, M. Torabi. Thermodynamic and economic investigation of a novel integration of the absorption-recompression refrigeration system with compressed air energy storage (CAES). Energy Conversion and Management. 187 (2019) 262-73.

[12] E.L. Majer, R. Baria, M. Stark, S. Oates, J. Bommer, B. Smith, et al. Induced seismicity associated with enhanced geothermal systems. Geothermics. 36 (2007) 185-222.

[13] R.J. Skoumal, M.R. Brudzinski, B.S. Currie. Earthquakes induced by hydraulic fracturing in Poland Township, Ohio. Bulletin of the Seismological Society of America. 105 (2015) 189-97.

[14] N. Shakhova, I. Semiletov, O. Gustafsson, V. Sergienko, L. Lobkovsky, O. Dudarev, et al. Current rates and mechanisms of subsea permafrost degradation in the East Siberian Arctic Shelf. Nature communications. 8 (2017) 15872.

[15] S. Dallimore, T. Collett. Intrapermafrost gas hydrates from a deep core hole in the Mackenzie Delta, Northwest Territories, Canada. Geology. 23 (1995) 527-30.

[16] K.A. Kvenvolden. Gas hydrates—geological perspective and global change. Reviews of geophysics. 31 (1993) 173-87.

[17] V. Yakushev, T. Collett. Gas hydrates in Arctic regions: risk to drilling and production. In: The Second International Offshore and Polar Engineering Conference. International Society of Offshore and Polar Engineers; 1992. Francisco, California, USA.

[18] Y.F. Makogon. Natural gas hydrates—A promising source of energy. Journal of Natural Gas Science and Engineering. 2 (2010) 49-59.

[19] S. Dallimore, T. Collett. Summary and implications of the Mallik 2002 gas hydrate production research well program. Scientific results from the Mallik. (2002) 1-36.
[20] K. Yamamoto, S. Dallimore. Aurora-JOGMEC-NRCAN Mallik 2006-2008 gas hydrate research project progress. Natural Gas & Oil. 304 (2008) 285-4541.
[21] X.-S. Li, C.-G. Xu, Y. Zhang, X.-K. Ruan, G. Li, Y. Wang. Investigation into gas production from natural gas hydrate: A review. Applied Energy. 172 (2016) 286-322.
[22] M. Yousif, P. Li, M. Selim, E. Sloan. Depressurization of natural gas hydrates in Berea sandstone cores. Journal of inclusion phenomena and molecular recognition in chemistry. 8 (1990) 71-88.
[23] C. Ji, G. Ahmadi, D.H. Smith. Natural gas production from hydrate decomposition by depressurization. Chemical Engineering Science. 56 (2001) 5801-14.
[24] Y. Konno, Y. Masuda, K. Akamine, M. Naiki, J. Nagao. Sustainable gas production from methane hydrate reservoirs by the cyclic depressurization method. Energy conversion and management. 108 (2016) 439-45.
[25] A. Chejara, B. Kvamme, M.T. Vafaei, K. Jemai. Simulations of long term methane hydrate dissociation by pressure reduction using an extended RetrasoCodeBright simulator. Energy conversion and management. 68 (2013) 313-23.
[26] X. Yang, C.-Y. Sun, K.-H. Su, Q. Yuan, Q.-P. Li, G.-J. Chen. A three-dimensional study on the formation and dissociation of methane hydrate in porous sediment by depressurization. Energy conversion and management. 56 (2012) 1-7.
[27] Y. Wang, X.-S. Li, G. Li, Y. Zhang, B. Li, Z.-Y. Chen. Experimental investigation into methane hydrate production during three-dimensional thermal stimulation with five-spot well system. Applied energy. 110 (2013) 90-7.
[28] G. Li, X.-S. Li, Y. Wang, Y. Zhang. Production behavior of methane hydrate in porous media using huff and puff method in a novel three-dimensional simulator. Energy. 36 (2011) 3170-8.
[29] I.N. Tsimpanogiannis, P.C. Lichtner. Parametric study of methane hydrate dissociation in oceanic sediments driven by thermal stimulation. Journal of Petroleum Science and Engineering. 56 (2007) 165-75.
[30] Y. Wang, X.-S. Li, G. Li, N.-S. Huang, J.-C. Feng. Experimental study on the hydrate dissociation in porous media by five-spot thermal huff and puff method. Fuel. 117 (2014) 688-96.
[31] Y. Wang, J.-C. Feng, X.-S. Li, Y. Zhang, Z.-Y. Chen. Fluid flow mechanisms and heat transfer characteristics of gas recovery from gas-saturated and water-saturated hydrate reservoirs. International Journal of Heat and Mass Transfer. 118 (2018) 1115-27.
[32] S. Yang, X. Lang, Y. Wang, Y. Wen, S. Fan. Numerical simulation of Class 3 hydrate reservoirs exploiting using horizontal well by depressurization and thermal co-stimulation. Energy conversion and management. 77 (2014) 298-305.
[33] G. Li, X.-S. Li, L.-G. Tang, Y. Zhang. Experimental investigation of production behavior of methane hydrate under ethylene glycol injection in unconsolidated sediment. Energy & fuels. 21 (2007) 3388-93.
[34] W. Sung, H. Lee, H. Lee, C. Lee. Numerical study for production performances of a methane hydrate reservoir stimulated by inhibitor injection. Energy Sources. 24 (2002) 499-512.
[35] M.H. Yousif. Effect of under-inhibition with methanol and ethylene glycol on the hydrate control process. In: Offshore Technology Conference; 1996. Houston, Texas, USA.
[36] A. Saji, H. Yoshida, M. Sakai, T. Tanii, T. Kamata, H. Kitamura. Fixation of carbon dioxide by clathrate-hydrate. Energy Conversion and Management. 33 (1992) 643-9.
[37] N. Nishikawa, M. Morishita, M. Uchiyama, F. Yamaguchi, K. Ohtsubo, H. Kimuro, et al. CO2 clathrate formation and its properties in the simulated deep ocean. Energy Conversion and Management. 33 (1992) 651-7.
[38] V. Khlebnikov, S. Antonov, A. Mishin, D. Bakulin, I. Khamidullina, M. Liang, et al. A new method for the replacement of CH4 with CO2 in natural gas hydrate production. Natural Gas Industry B. 3 (2016) 445-51.
[39] Q. Yuan, C.-Y. Sun, X. Yang, P.-C. Ma, Z.-W. Ma, B. Liu, et al. Recovery of methane from hydrate reservoir with gaseous carbon dioxide using a three-dimensional middle-size reactor. Energy. 40 (2012) 47-58.

[40] B. Li, T. Xu, G. Zhang, W. Guo, H. Liu, Q. Wang, et al. An experimental study on gas production from fracture-filled hydrate by CO2 and CO2/N2 replacement. Energy conversion and management. 165 (2018) 738-47.

[41] Y. Lee, W. Choi, K. Shin, Y. Seo. CH4-CO2 replacement occurring in sII natural gas hydrates for CH4 recovery and CO2 sequestration. Energy conversion and management. 150 (2017) 356-64.

[42] G.D. Holder, P.F. Angert. Simulation of gas production from a reservoir containing both gas hydrates and free natural gas. In: SPE annual technical conference and exhibition; 1982. New Orleans, Louisiana, USA.

[43] M. Burshears, T. O'brien, R. Malone. A multi-phase, multi-dimensional, variable composition simulation of gas production from a conventional gas reservoir in contact with hydrates. SPE Unconventional Gas Technology Symposium. Society of Petroleum Engineers 1986.

[44] M. Yousif, H. Abass, M. Selim, E. Sloan. Experimental and theoretical investigation of methane-gas-hydrate dissociation in porous media. SPE reservoir Engineering. 6 (1991) 69-76.

[45] Y. MASUDA. Modeling and experimental studies on dissociation of methane gas hydrates in Berea sandstone cores. In: Proceedings of the third International Gas Hydrate Conference; 1999. Salt Lake City, Utah, USA.

[46] Y. Masuda. Numerical calculation of gas production performance from reservoirs containing natural gas hydrates. In: Annual Technical Conference, Soc of Petrol Eng; 1997. San Antonio, Texas, USA.

[47] G.G. Tsypkin. Mathematical models of gas hydrates dissociation in porous media. Annals of the New York academy of sciences. 912 (2000) 428-36.

[48] G. Ahmadi, C. Ji, D.H. Smith. Numerical solution for natural gas production from methane hydrate dissociation. Journal of petroleum science and engineering. 41 (2004) 269-85.

[49] K. Pruess. TOUGH2-A general-purpose numerical simulator for multiphase fluid and heat flow. (1991).

[50] G.J. Moridis, T.S. Collett, S.R. Dallimore, T. Satoh, S. Hancock, B. Weatherill. Numerical studies of gas production from several CH4 hydrate zones at the Mallik site, Mackenzie Delta, Canada. Journal of petroleum science and engineering. 43 (2004) 219-38.

[51] G.J. Moridis. Numerical studies of gas production from methane hydrates. SPE Gas Technology Symposium. Society of Petroleum Engineers 2002.

[52] M.B. Kowalsky, G.J. Moridis. Comparison of kinetic and equilibrium reaction models in simulating gas hydrate behavior in porous media. Energy conversion and management. 48 (2007) 1850-63.

[53] G.J. Moridis, E.D. Sloan. Gas production potential of disperse low-saturation hydrate accumulations in oceanic sediments. Energy conversion and management. 48 (2007) 1834-49.

[54] Y. Konno, T. Fujii, A. Sato, K. Akamine, M. Naiki, Y. Masuda, et al. Key findings of the world’s first offshore methane hydrate production test off the coast of Japan: Toward future commercial production. Energy & Fuels. 31 (2017) 2607-16.

[55] W. Chen, R.L. Hartman. Methane Hydrate Intrinsic Dissociation Kinetics Measured in a Microfluidic System by Means of in Situ Raman Spectroscopy. Energy & Fuels. 32 (2018) 11761-71.

[56] M. Mardani, A. Azimi, J. Javanmardi, A.H. Mohammadi. Effect of EMIM-BF4 Ionic Liquid on Dissociation Temperature of Methane Hydrate in the Presence of PVCap: Experimental and Modeling Studies. Energy & Fuels. 33 (2018) 50-7.

[57] Y. Wang, J.-C. Feng, X.-S. Li, Y. Zhang. Experimental and modeling analyses of scaling criteria for methane hydrate dissociation in sediment by depressurization. Applied energy. 181 (2016) 299-309.

[58] J. Zhao, Z. Fan, H. Dong, Z. Yang, Y. Song. Influence of reservoir permeability on methane hydrate dissociation by depressurization. International Journal of Heat and Mass Transfer. 103 (2016) 265-76.
[59] X. Wang, B. Dong, F. Wang, W. Li, Y. Song. Pore-scale investigations on the effects of ice formation/melting on methane hydrate dissociation using depressurization. International Journal of Heat and Mass Transfer. 131 (2019) 737-49.
[60] Y. Feng, L. Chen, A. Suzuki, T. Kogawa, J. Okajima, A. Komiya, et al. Enhancement of gas production from methane hydrate reservoirs by the combination of hydraulic fracturing and depressurization method. Energy conversion and management. 184 (2019) 194-204.
[61] J. Chen, J. Liu, G.-J. Chen, C.-Y. Sun, M.-L. Jia, B. Liu, et al. Insights into methane hydrate formation, agglomeration, and dissociation in water+ diesel oil dispersed system. Energy conversion and management. 86 (2014) 886-91.
[62] M. Selim, E. Sloan. Hydrate dissociation in sediment. SPE Reservoir Engineering. 5 (1990) 245-51.
[63] P.L. McGuire. Recovery of gas from hydrate deposits using conventional technology. SPE Unconventional Gas Recovery Symposium. Society of Petroleum Engineers 1982.
[64] Y.F. Makogon. Hydrates of hydrocarbons: Pennwell Pub. Comp Tulsa, Oklahoma. (1997).
[65] G. Tsypkin. Regimes of dissociation of gas hydrates coexisting with a gas in natural strata. Journal of engineering physics and thermophysics. 74 (2001) 1083-9.
[66] H. Hong, M. Pooladidarvish, P. Bishnoi. Analytical modelling of gas production from hydrates in porous media. Canadian Journal of Petroleum Technology. 42 (2003) 45-56.
[67] Y. Wang, J.-C. Feng, X.-S. Li, Y. Zhang, G. Li. Large scale experimental evaluation to methane hydrate dissociation below quadruple point in sandy sediment. Applied energy. 162 (2016) 372-81.
[68] L.-G. Tang, X.-S. Li, Z.-P. Feng, G. Li, S.-S. Fan. Control mechanisms for gas hydrate production by depressurization in different scale hydrate reservoirs. Energy & Fuels. 21 (2007) 227-33.
[69] X.-S. Li, Y. Wang, G. Li, Y. Zhang, Z.-Y. Chen. Experimental investigation into methane hydrate decomposition during three-dimensional thermal huff and puff. Energy & Fuels. 25 (2011) 1650-8.
[70] X.-S. Li, B. Yang, Y. Zhang, G. Li, L.-P. Duan, Y. Wang, et al. Experimental investigation into gas production from methane hydrate in sediment by depressurization in a novel pilot-scale hydrate simulator. Applied energy. 93 (2012) 722-32.
[71] J. Zhao, C. Cheng, Y. Song, W. Liu, Y. Liu, K. Xue, et al. Heat transfer analysis of methane hydrate sediment dissociation in a closed reactor by a thermal method. Energies. 5 (2012) 1292-308.
[72] Y. Wang, J.-C. Feng, X.-S. Li, Y. Zhang, G. Li. Analytic modeling and large-scale experimental study of mass and heat transfer during hydrate dissociation in sediment with different dissociation methods. Energy. 90 (2015) 1931-48.
[73] J. Pucknell, J. Mason. Predicting the Pressure Drop in a Cased-Hole Gravel Pack Completion. In: European Petroleum Conference. Society of Petroleum Engineers; 1992. Cannes, France.
[74] E. Xu, K. Soga, M. Zhou, S. Uchida, K. Yamamoto. Numerical analysis of wellbore behaviour during methane gas recovery from hydrate bearing sediments. In: Offshore Technology Conference; 2014. Houston, Texas.
[75] W. Xiong, M. Bahonar, Z. Chen. Development of a thermal wellbore simulator with focus on improving heat loss calculations for SAGD steam injection. In: SPE Canada Heavy Oil Technical Conference; 2015. Calgary, Alberta, Canada.
[76] G. Li, G.J. Moridis, K. Zhang, X.-S. Li. Evaluation of gas production potential from marine gas hydrate deposits in Shenhu area of South China Sea. Energy & Fuels. 24 (2010) 6018-33.
[77] X.-S. Li, B. Li, G. Li, B. Yang. Numerical simulation of gas production potential from permafrost hydrate deposits by huff and puff method in a single horizontal well in Qilian Mountain, Qinghai province. Energy. 40 (2012) 59-75.
[78] Q.-C. Wan, H. Si, B. Li, G. Li. Heat transfer analysis of methane hydrate dissociation by depressurization and thermal stimulation. International Journal of Heat and Mass Transfer. 127 (2018) 206-17.
[79] M. Roostaie, Y. Leonenko. Analytical modeling of methane hydrate dissociation under thermal stimulation. Journal of Petroleum Science and Engineering. (2019) 106505.

[80] H. Holmberg, J. Acuña, E. Næss, O.K. Sønju. Thermal evaluation of coaxial deep borehole heat exchangers. Renewable energy. 97 (2016) 65-76.

[81] L. Wang, H. Liu, Z. Pang, X. Lv. Overall heat transfer coefficient with considering thermal contact resistance in thermal recovery wells. International Journal of Heat and Mass Transfer. 103 (2016) 486-500.

[82] R.A. Beier, J. Acuña, P. Mogensen, B. Palm. Borehole resistance and vertical temperature profiles in coaxial borehole heat exchangers. Applied energy. 102 (2013) 665-75.

[83] F. Sun, Y. Yao, X. Li, J. Tian, G. Zhu, Z. Chen. The flow and heat transfer characteristics of superheated steam in concentric dual-tubing wells. International Journal of Heat and Mass Transfer. 115 (2017) 1099-108.

[84] J. Phirani, K.K. Mohanty, G.J. Hirasaki. Warm water flooding of unconfined gas hydrate reservoirs. Energy & Fuels. 23 (2009) 4507-14.

[85] L. Lamarche, S. Kajil, B. Beauchamp. A review of methods to evaluate borehole thermal resistances in geothermal heat-pump systems. Geothermics. 39 (2010) 187-200.

[86] H.J. Braud, H. Klimkowski, J. Oliver. Earth-source heat exchanger for heat pumps. Transactions of the ASAE. 26 (1983) 1818-22.

[87] V. Mei, S. Fischer. Vertical concentric tube ground-coupled heat exchangers. ASHRAE transactions. 89 (1983) 391-406.

[88] K. Morita, W.S. Bollmeier, H. Mizogami. Analysis of the results from the downhole coaxial heat exchanger (DCHE) experiment in Hawaii. (1992).

[89] V. Yavuzturk, A.D. Chiasson. Performance analysis of U-tube, concentric tube, and standing column well ground heat exchangers using a system simulation approach. ASHRAE Transactions. 108 (2002) 925.

[90] G. Hellström. Thermal performance of borehold heat exchangers. In: Stockton International Geothermal Conference; 1998. Stockton, California, USA.

[91] G. Hellström. Borehole heat exchangers: state of the art 2001. (2002).

[92] G. Hellström. Ground heat storage: Thermal analyses of duct storage systems. I. Theory. (1992).

[93] F. Dittus, L. Boelter. Publications on Engineering, vol. 2, 443 Berkeley. CA: University of California Press1930.

[94] W.M. Rohsenow, J.P. Hartnett, E.N. Ganic. Handbook of heat transfer fundamentals. New York, McGraw-Hill Book Co, 1985, 1440 p No individual items are abstracted in this volume. (1985).

[95] H. Kraussold. Wärmeabgabe von zylindrischen Flüssigkeitschichten bei natürlicher Konvektion. Forschung im Ingenieurwesen. 5 (1934) 186-91.

[96] H. Hausen. Neue Gleichungen fur die Wärmeubertragung bei freier oder erzwungener Stromung. Allg Wärmetech. 9 (1959) 75-9.

[97] E.N. Sieder, G.E. Tate. Heat transfer and pressure drop of liquids in tubes. Industrial & Engineering Chemistry. 28 (1936) 1429-35.

[98] B. Petukhov. Heat transfer and friction in turbulent pipe flow with variable physical properties. Advances in heat transfer. Elsevier1970. pp. 503-64.

[99] R. Notter, C. Sleicher. A solution to the turbulent Graetz problem—III Fully developed and entry region heat transfer rates. Chemical Engineering Science. 27 (1972) 2073-93.

[100] V. Gnielinski. New equations for heat and mass transfer in the turbulent flow in pipes and channels. NASA STI/recon technical report A. 75 (1975) 8-16.

[101] M. Ntuli, J. Dirker, J. Meyer. Heat transfer and pressure drop coefficients for turbulent flow in concentric annular ducts. 19th International Congress of Chemical and Process Engineering2010.

[102] V. Gnielinski. Heat transfer coefficients for turbulent flow in concentric annular ducts. Heat transfer engineering. 30 (2009) 431-6.
[103] V. Wärmeatlas. Berechnungsblätter für den Wärmeübergang. VDI-Verlag, Düsseldorf. (1988).
[104] B. Petukhov, L. Roizen. Heat Exchange During Gas Flow in Pipes with an Annular Cross Section. Heat and Mass Transfer. 1967. p. 56.
[105] H. Wang, X. Li, K. Sepehrnoori, Y. Zheng, W. Yan. Calculation of the wellbore temperature and pressure distribution during supercritical CO2 fracturing flowback process. International Journal of Heat and Mass Transfer. 139 (2019) 10-6.
[106] F. Sun, Y. Yao, X. Li, G. Li, Z. Sun. A numerical model for predicting distributions of pressure and temperature of superheated steam in multi-point injection horizontal wells. International Journal of Heat and Mass Transfer. 121 (2018) 282-9.
[107] A. Florez Anaya, M.A. Osorio. A Successful Gravel-Packing Technique in Vertical and Deviated Wells with Enlarged Open Hole in Cased Completions: A Case Study, Rubiales and Quifa Fields. In: SPE Heavy and Extra Heavy Oil Conference: Latin America. Society of Petroleum Engineers; 2014. Medellín, Colombia.
[108] H. Carslaw, J. Jaeger. Conduction of heat in solids: Oxford Science Publications. Oxford, England 1959.
[109] M.N. Â–zisik, M.N. Özisik, M.N. Özışık. Heat conduction. John Wiley & Sons 1993.
[110] Y. Song, C. Cheng, J. Zhao, Z. Zhu, W. Liu, M. Yang, et al. Evaluation of gas production from methane hydrates using depressurization, thermal stimulation and combined methods. Applied Energy. 145 (2015) 265-77.
[111] C.P. Remund. Borehole thermal resistance: laboratory and field studies. ASHRAE transactions. 105 (1999) 439.
[112] G. Dalla Santa, F. Peron, A. Galgaro, M. Cultrera, D. Bertermann, J. Mueller, et al. Laboratory measurements of gravel thermal conductivity: An update methodological approach. Energy Procedia. 125 (2017) 671-7.
[113] W.-L. Cheng, Y.-H. Huang, D.-T. Lu, H.-R. Yin. A novel analytical transient heat-conduction time function for heat transfer in steam injection wells considering the wellbore heat capacity. Energy. 36 (2011) 4080-8.
[114] J. Acuña, B. Palm. A novel coaxial borehole heat exchanger: description and first distributed thermal response test measurements. Proceedings of the World Geothermal Congress 2010. p. 7.
[115] X.-S. Li, Y. Wang, G. Li, Y. Zhang. Experimental investigations into gas production behaviors from methane hydrate with different methods in a cubic hydrate simulator. Energy & Fuels. 26 (2011) 1124-34.
[116] G. Li, X.-S. Li, B. Li, Y. Wang. Methane hydrate dissociation using inverted five-spot water flooding method in cubic hydrate simulator. Energy. 64 (2014) 298-306.
[117] J. Zhao, J. Wang, W. Liu, Y. Song. Analysis of heat transfer effects on gas production from methane hydrate by thermal stimulation. International Journal of Heat and Mass Transfer. 87 (2015) 145-50.
[118] J. Zhao, D. Liu, M. Yang, Y. Song. Analysis of heat transfer effects on gas production from methane hydrate by depressurization. International Journal of Heat and Mass Transfer. 77 (2014) 529-41.
[119] J. Raymond, S. Mercier, L. Nguyen. Designing coaxial ground heat exchangers with a thermally enhanced outer pipe. Geothermal Energy. 3 (2015) 7.
[120] E. Zanchini, S. Lazzari, A. Priarone. Improving the thermal performance of coaxial borehole heat exchangers. Energy. 35 (2010) 657-66.
Supplementary information

- **Materials and methods**
In the following, the solution process of equations 16 and 17 is provided. This process is a little bit different for the two operation models.

For the model in which the hot water is injected to the inner tube, \( T_{fa} \) is calculated first based on \( T_{fi} \) as follows:

\[
T_{fa}(z,t) = R_i C_f V_f \frac{\partial T_{fi}(z,t)}{\partial z} + T_{fi}(z,t) \quad (S1)
\]

Then, by inserting equation S1 to equation 17, the resulted expression is:

\[
R_i R_2 (C_f V_f)^2 \frac{\partial^2 T_{fi}(z,t)}{\partial z^2} - R_i C_f V_f \frac{\partial T_{fi}(z,t)}{\partial z} - T_{fi}(z,t) + T_i(r_f,t) \quad (S2)
\]

Equation S2 is a second-order nonhomogeneous differential equation, which can be solved to obtain \( T_{fi} \) due to the following initial and boundary conditions: i) injection temperature is fixed and constant through the process \((T_{fi}(h,t))\); and ii) no heat flow occurs at the base of the wellbore \((T_{fi}(0,t) = T_{fa}(0,t) \rightarrow \frac{\partial T_{fi}(0,t)}{\partial z} = \frac{\partial T_{fa}(0,t)}{\partial z} = 0)\).

For the other model of how water injection into the annulus, \( T_{fa} \) based on \( T_{fi} \) is as follows:

\[
T_{fa}(z,t) = R_i C_f V_f \frac{\partial T_{fi}(z,t)}{\partial z} + \frac{R_i}{R_2} (T_{fi}(z,t) - T_i(r_f,t)) + T_{fi}(z,t) \quad (S3)
\]

Then, by inserting equation S3 to equation 17, the new expression based on \( T_{fi} \) is:

\[
R_i R_2 (C_f V_f)^2 \frac{\partial^2 T_{fi}(z,t)}{\partial z^2} + R_i C_f V_f \frac{\partial T_{fi}(z,t)}{\partial z} - T_{fi}(z,t) + T_i(r_f,t) \quad (S4)
\]

Equation S4 is also a second-order nonhomogeneous differential equation, and can be solved by the same procedure with the same initial and boundary conditions as those of equation S2.
In the following, the transformation of the fundamental equations of dissociation process with the initial and boundary conditions in terms of $\lambda$ is provided.

Equation 21: \[
\frac{d^2T_{II}}{d\lambda^2} + \left(\frac{2\lambda^2 + 1}{\lambda}\right)\frac{dT_{II}}{d\lambda} = 0, \quad \beta < \lambda < \infty \tag{S5}
\]

Equation 23: \[
\frac{-k_i A_w \frac{dT_I}{d\lambda}}{\sqrt{A_\alpha t}} = \frac{(T_{f_0}(t) - T_I)}{R_w}, \quad \lambda = \lambda_{os} \tag{S6}
\]

Equation 24: \[
P_s(t) = \exp(A_a - B_a / T_s(t)), \quad \lambda = \beta \tag{S7}
\]

Equation 26: \[
k_{II} \frac{dT_{II}}{d\lambda} - k_{I} \frac{dT_I}{d\lambda} = 2\phi \rho_H \alpha_{II} Q_{II} \beta, \quad \lambda = \beta \tag{S8}
\]

Equation 29: \[
T_{II} = T_0, \quad \lambda \rightarrow \infty \tag{S9}
\]

Equation 31: \[
\frac{d}{d\lambda} \left( \frac{P}{T_I} \frac{dP}{d\lambda} \right) + \frac{2\phi \alpha_{II} \mu \lambda}{k} \frac{d}{d\lambda} \left( \frac{P}{T_I} \right) = 0, \quad \lambda_{os} < \lambda < \beta \tag{S10}
\]

Equation 32: \[
\frac{d^2T_I}{d\lambda^2} + \left( \frac{2\alpha_I \lambda}{\alpha_I} + \frac{1}{\lambda} \right)\frac{dT_I}{d\lambda} + \frac{C_{pe} km \frac{d^2P^2}{d\lambda^2}}{2k_I \mu R} = 0, \quad \lambda_{os} < \lambda < \beta \tag{S11}
\]

And, from equation 33: \[
\frac{P}{T_I} \frac{dP}{d\lambda} = \frac{2F_{gH} \phi \rho_H \alpha_{II} \mu R \beta}{km}, \quad \lambda = \beta \tag{S12}
\]

Selim et al. [62] reported that the very small ratio of $\frac{\rho_s}{\rho_H}$ indicates the very slow rate of hydrate dissociation (dS/dt), which neglects the transient term in the continuity equation (second term in equation S5).

In the equations 37 and 38, $A, A_1, B, B_1,$ and $C$ constants are defined using equations S2 and 28 as follows:

\[
A = \frac{(T_s - T_{f_0}(t))F}{-FEi (-a\beta + b^2) + FEi (-b^2) + D + E} \tag{S13}
\]

\[
B = \frac{A(D + E)}{F} + T_{f_0}(t) \tag{S14}
\]
\[ C = \frac{T_s(t) - T_0}{Ei(-\beta^2)} \]  

(S15)

where \(a, b, D, E,\) and \(F\) are as follows:

\[ a = \left(\frac{\alpha_{II}}{\alpha_I}\right)^{1/2} \]  

(S16)

\[ b = \frac{C_{pg} F_{gH} \phi_{H} \alpha_{II} \beta}{ak_I} \]  

(S17)

\[ D = -r_2 \left(2a \exp(- (a \lambda_{os} + b)^2) \right) \sqrt{\frac{\pi}{e}} \]  

(S18)

\[ E = R_w (-Ei(-(a \lambda_{os} + b)^2) + Ei(-b^2)) \]  

(S19)

\[ F = \frac{1}{R_w} \]  

(S20)

The \(L(\beta), M(\beta), N(\lambda),\) and \(K(\beta)\) functions used in equation 40 (pressure distribution in zone I) are as follows:

\[ L(\beta) = \frac{4F_{gH} \phi_{H} \alpha_{II} \mu R \beta}{km} \]  

(S21)

\[ N(\lambda) = \frac{Ei(-(a \lambda + b)^2)(a \lambda + b) - \sqrt{\pi}erf(a \lambda + b)}{a} \]  

(S22)

\[ K(\beta) = B + AEi(-b^2) \]  

(S23)

Now, equation S8 by insertion of the resulted \(T_I\) and \(T_{II}\) formulas and considering equation 27 becomes:

\[ \frac{ak_I(T_{II}(t) - T_I(t))}{k_g(T_{II}(t) - T_o)} \left(-F \exp(-(a \beta + b)^2) \right) \frac{\exp(-\beta^2)}{\beta Ei(-\beta^2)} \]  

(S25)

\[ \sqrt{\pi} \phi_{H} \alpha_{II} \beta \frac{(c + dT_s(t))}{k_y(T_s(t) - T_o)} \]

So, the temperature at the dissociation interface based on equation S22 can be calculated as follows:
\[ T_s(t) = \frac{k_u T_0 H(\beta) - ak_i \overline{T_m(t)} G(\beta) + cI(\beta)}{-ak_i G(\beta) - dI(\beta) + k_u H(\beta)} \quad (S26) \]

The pressure at the dissociation interface \( P_s \) would be calculated from the equation 40:

\[ P_s(t) = \left( P_i^2 + L(\beta)(K(\beta)\beta - AN(\beta) - (K(\beta)\lambda_{oa} - AN(\lambda_{oa}))) \right)^{1/2} \quad (S27) \]

The following equation is a transformation of equation 24 (Antoine equation):

\[ \frac{B_a}{A_a - \ln(P_s)} - T_s = 0 \quad (S28) \]

Then, by replacing \( T_s \) and \( P_s \) in equation S27 with the equations S25 and S26, and solving the resulted expression to find \( \beta \), the exact solution of temperature and pressure distributions will be achieved. Actually, at each time step, equation S27 has only one unknown, which is \( \beta \), and MATLAB programming software is used to find this unknown as well as performing all other calculations in this study.

The transformed from of equation of heat flux from the wellbore (equation 41) based on the transformation term of equation 34 is:

\[ u_r = \frac{-k_i}{\sqrt{4\alpha_{ht} t}} \cdot dT_i, \quad \lambda = \lambda_{oa} \quad (S29) \]

By inserting \( T_i \) from equation 37 into equation S28:

\[ u_r = \frac{2ak_iA \exp(-a\lambda_{oa} + b)^2)}{(a\lambda_{oa} + b)\sqrt{4\alpha_{ht} t}} \quad (S30) \]

In order to obtain the total volume of gas produced at STP conditions, the produced gas moles at each time step should be calculated. Equations S31 and S32 show respectively the volume of dissociated hydrate and the moles of produced gas per average surface area of the dissociation front at each time step \( (t-1, t) \) as shown in equation S30.

\[ A_s = 2\pi \left( \frac{S_t + S_{t-1}}{2} \right) \quad (S31) \]

\[ V_{np} = \phi h \pi (S_t^2 - S_{t-1}^2) \quad (S32) \]
\[ n_r = \frac{F_{gh} \rho_h V_{wp}}{(A_s m)} \quad \text{(S33)} \]

where \( h \) is the methane hydrate thickness (m). By summing the moles of produced gas at each time step from the beginning of dissociation to time \( t \) as shown in equation S33, the total number of moles of produced gas up to that time can be calculated.

\[ n_{rt} = \sum_{t=0}^{t} n_r \quad \text{(S34)} \]

Finally, by using equations S33 and 36 the total volume of produced gas per average area of the dissociation front at STP conditions up to time \( t \) can be achieved as presented in equation 43.
Figure S1. Volume of produced gas in the model with hot water injection into the a) annulus and b) inner tube, and the amount of input heat in the model with hot water injection into the c) annulus and d) inner tube during hydrate dissociation for two BCs.
Figure S2. The effect of various parameters on the produced gas after 100 days dissociation considering both operating schemes: a) thermal diffusivity and thermal conductivity of Zone I, b) thermal diffusivity and thermal conductivity of Zone II, c) porosity with various permeabilities, and d) porosity with various gas viscosities.

Dashed lines and solid lines respectively represent the operating schemes of hot water injection into annulus and into the inner tube.
Figure S3. The effect of various parameters on the input heat from the wellbore after 100 days dissociation considering both operating schemes: a) thermal diffusivity and thermal conductivity of Zone I, b) thermal diffusivity and thermal conductivity of Zone II, c) porosity with various permeabilities, and d) porosity with various gas viscosities.

Dashed lines and solid lines are respectively representative of the operating schemes of hot water injection into annulus and into the inner tube.
Figure S4. The effect of various parameters on the produced gas after 100 days dissociation considering both operating schemes: a) the wellbore radius, b) the reservoir thickness, c) the annulus radius, d) the inner tube radius, e) the injection temperature, and f) the flow rate.

Dashed lines and solid lines respectively represent the operating schemes of hot water injection into annulus and into the inner tube.
Figure S5. The effect of various parameters on the input heat from the wellbore after 100 days dissociation considering both operating schemes: a) the wellbore radius, b) the reservoir thickness, c) the annulus radius, d) the inner tube radius, e) the injection temperature, and f) the flow rate.

Dashed lines and solid lines respectively represent the operating schemes of hot water injection into annulus and into the inner tube.