Radiative Corrections for H1 $F_2(x, Q^2)$ Measurement

A. A. Akhundov

DESY - Institut für Hochenergiephysik Zeuthen,
Platanenallee 6, D-15738 Zeuthen, Germany

and

Institute of Physics, Azerbaijan Academy of Sciences
pr. Azizbekova 33, 370143 Baku, Azerbaijan

ABSTRACT

The numerical study of QED radiative corrections in the low $x$ region for the H1 experiment at HERA was performed. The analytical program TERAD91 was used to get the size of the radiative corrections for the measurement of the proton structure function $F_2(x, Q^2)$ in the range $x = 10^{-2} - 10^{-4}$ and $5 \text{ GeV}^2 < Q^2 < 80 \text{ GeV}^2$. It was found that radiative corrections can be not small in some points of the phase space of the mixed variables.

In the long period from the SLAC experiments till the operation of HERA, the experiments with neutral current deep inelastic $ep$-scattering

$$e(k_1) + p(p_1) \rightarrow e(k_2) + X(p_2) \quad (1)$$

had a relatively simple structure, and the measurements completely rested on the registration of energy and angle of the scattered electron. The new detector generation, the HERA detectors H1 and ZEUS, allows to measure both the scattered electron and hadronic final state.

For the physical analysis of the $ep$-collisions at HERA one can use not only the familiar electron variables

$$Q_e^2 = -(k_1 - k_2)^2, \quad y_e = p_1(k_1 - k_2)/p_1k_1, \quad x_e = Q_e^2/(sy_e), \quad (2)$$

where

$$s = (k_1 + p_1)^2 = 4E_eE_p, \quad (3)$$

but also the kinematical variables from the hadron measurement

$$Q_h^2 = -(p_2 - p_1)^2, \quad y_h = p_1(p_2 - p_1)/p_1k_1, \quad x_h = Q_h^2/(sy_h), \quad (4)$$
or some mixture of both. Here $E_e$ and $E_p$ are the energies of incident electron and proton.

In order to extract from experimental data the inclusive cross section of deep inelastic $ep$-scattering in one-boson-exchange approximation one must to take into account contributions from higher order QED and electroweak processes. For example the bremsstrahlung process:

\[ e(k_1) + p(p_1) \rightarrow e(k_2) + X(p_2) + \gamma(k), \quad (5) \]

and also the elastic radiative tail :

\[ e(k_1) + p(p_1) \rightarrow e(k_2) + p(p_2) + \gamma(k). \quad (6) \]

contribute to the observed cross section of the reaction (1).

For different choices of the variables there are differences in the predictions for the contribution of the higher order processes.

The theoretical investigations of these radiative effects, or radiative corrections (RC), for experiments at HERA were summarized in Proceedings of the Workshop "Physics at HERA". Different programs to calculate the radiative corrections have been cross-checked.

The analysis of neutral current deep inelastic scattering data in the H1 experiment at HERA in 1992 was based on two independent approaches:

I) "detector oriented" approach ("Brussels-Paris-Saclay analysis")

II) "physics oriented" approach ("Zeuthen analysis")

Within the analysis I the kinematics of the inclusive deep inelastic scattering process (1) were determined from the energy $E'_e$ of the scattered electron, and its polar angle $\theta_e$, measured relative to the proton beam direction

\[ Q_e^2 = 4E_eE'_e \cos^2(\theta_e/2), \quad (7) \]
\[ y_e = 1 - (E'_e/E_e) \sin^2(\theta_e/2). \quad (8) \]

The cross sections, acceptances of detector, radiative corrections and resolution effects were obtained in $(\sqrt{E'_e}, \theta_e)$ bins. The cross sections, that have been measured using this binning, were transformed into cross sections in $(x_e, Q_e^2)$-bins.

In the analysis II the kinematics of the process (1) were obtained from a measurement of the electron variables $E'_e$, $\theta_e$ and scaling variable $y_h$ from the hadrons. The hadronic variable $y_h$ (4) can be determined using the Jacquet-Blondel method

\[ y_h = \sum_{\text{hadrons}} \frac{E_h - p_{z,h}}{2E_e}, \quad (9) \]
where $E_h$ is the energy of a hadron and $p_{z,h}$ its momentum component along the incident proton direction.

The momentum transfer $Q^2$ was determined from the electron variables $\theta_e$ and $E'_e$ as in (7) and Bjorken $x$ by using a mixture of the electron and hadron measurements \(^9\)

$$x_m = Q^2_e/(s y_h). \quad (10)$$

In the analysis II cross sections, radiative corrections and efficiencies were calculated in $(x_m, Q^2_e)$-bins.

A further difference between the two methods of the analysis of the data was the use of two different types of programs — the Monte Carlo event generators HERACLES\(^{10}\) in the analysis I and the analytical program TERAD91\(^{11}\) in the analysis II — for the calculations of the radiative correction factor $\delta(x, Q^2)$. The radiative correction factor $\delta(x, Q^2)$ is defined by the equation

$$\delta(x, Q^2) = \frac{d^2\sigma^{\text{theor}}}{dxdQ^2} \frac{d^2\sigma^{\text{Born}}}{dxdQ^2} - 1, \quad (11)$$

where $d^2\sigma^{\text{theor}}/dxdQ^2$ is the theoretical approximation to the measured cross section $d^2\sigma^{\text{meas}}/dxdQ^2$, which contains the contributions from higher order electroweak processes, and $d^2\sigma^{\text{Born}}/dxdQ^2$ is the Born cross section of the process (1).

In this note we present the numerical calculations of the RC factor $\delta(x, Q^2)$ for the ”Zeuthen analysis” of the proton structure function $F_2(x, Q^2)$ in the range $x = 10^{-2} - 10^{-4}$ and $5 \text{GeV}^2 < Q^2 < 80 \text{GeV}^2$ in the H1 experiment at HERA.\(^7\)

The Monte Carlo HERACLES is suited for the calculation of RC to the differential cross sections in terms of the electron variables because the program calculates cross sections and generates events in terms of the variables $x_e$ and $Q^2_e$. Therefore most of the generated radiative events in HERACLES have low $Q^2_h$ and the calculation of the radiative corrections in the terms of other variables ( hadron, mixed ) can be performed with less efficiency only. For this reason in the analysis II was used the analytical program TERAD91.

TERAD91 - a program package to calculate radiative corrections for deep inelastic $ep$-scattering - was created in 1991 during Workshop ”Physics at HERA” and accumulates about 15 years experience in the field of calculation of the radiative corrections for deep inelastic $lN$-scattering.\(^{12}\) The program TERAD91 can be applied to calculate the RC factor of the order $O(\alpha)$ to the measured differential cross section, $d^2\sigma^{\text{meas}}/dxdQ^2$, after the acceptance and smearing corrections.

The program package TERAD91 contains one part, TERAD, which is based on the model-independent treatment \(^{13}\) of the leptonic QED corrections for neutral current $ep$-scattering, including bremsstrahlung process ( 5) and QED vertex corrections. TERAD does not necessarily rely on the quark parton model and uses the phenomenological structure functions $F_1, F_2$ and $F_3$, which are defined in the Born approximation.
The program TERAD91 can be used for the model-independent calculations of QED corrections for neutral current deep inelastic ep-scattering in the terms of several variables - electron, hadron, mixed variables and in the new version TERAD93 also in terms of the Jacquet-Blondel variables. The leptonic QED corrections are dominated and must be calculated carefully.

In the low $Q^2$ region the Born cross section of the deep inelastic scattering (1) is determined by two structure function $F_2$ and $2xF_1 = F_2/(1+R)$:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{Q^4x} \left( 2(1-y) + \frac{y^2}{1+R} \right) F_2(x, Q^2).$$ (12)

In order to calculate the RC one has to use a realistic parametrization of the structure functions $F_i(x, Q^2)$ over the full range of $x$ and $Q^2$. We have used several different parameterizations of the deep inelastic proton structure functions, including MRS D- and MRS D0 parameterizations. For region $Q^2 < 5 \text{ GeV}^2$ the parameterizations were taken at $Q^2 = 5 \text{ GeV}^2$ and multiplied by factor $(1 - \exp(-aQ^2))$, $a = 3.37 \text{ GeV}^{-2}$. In our calculations we have assumed $R = 0$.

In the "Zeuthen analysis" the following $x, Q^2$-binning was used. There were four $Q^2$-bins:

$$Q^2 = 5 - 10, 10 - 20, 20 - 40, 40 - 80 \text{ GeV}^2$$ (13)

and four bins per $x$-decade:

$$\log x = -4.0, -3.75, -3.50, -3.25, ..., -1.0.$$ (14)

The kinematical region of the experimental measurement was defined by:

$$157.5^\circ < \theta_e < 172.5^\circ, \quad 0.05 < y_e < 0.60$$ (15)

for electron variables and

$$157.5^\circ < \theta_e < 172.5^\circ, \quad y_h < 0.50, \quad \theta_{jet} > 10^\circ$$ (16)

for mixed variables.

Figs.1,2 show the results for the radiative corrections in the terms of the mixed variables $x_m, Q^2_e$, which are calculated by the integrations over bins. The RC in the terms of the mixed variables, $\delta(x_m, Q^2_e)$, are not small and can be reach 7% at $x = 10^{-2}$.

The "Zeuthen analysis" of the deep inelastic scattering has included also the analysis of the experimental data in the terms of the electron variables $x_e, Q^2_e$ corresponding to the binning (13),(14). Fig.3,4 show the results of the calculations of the RC in the bins $x_e, Q^2_e$, that have been obtained with the program TERAD91.

The corrections determined by the electron variables, $\delta(x_e, Q^2_e)$, are known to be large at low $x$ and dominated by hard photon emission from the incoming electron.
From the comparison of Fig.3 and Fig.4 it is seen that the sensitivity of the RC to the choice of the structure functions is not small.

The comparison of the results for the radiative corrections in the terms of the electron variables $E'_e, \theta_e$, that have been obtained \(^{19}\) with event generator HERACLES (interface DJANGO\(^{20}\)) and analytical program TERAD93, is shown on Fig.5 for the bin $162.5^\circ < \theta_e < 167.5^\circ$. Here the variable $u$ is defined by the equation

$$u = 40\sqrt{E'_e/E_e},$$

and for the structure function $F_2$ the parametrization MRS D- was used and $R=0$. The results of the both programs are in good agreement.

Now we will discuss the influence of the calorimetric measurement of the energy of the scattered electron on the size of the radiative corrections.

Because the collinear radiation of photons in the final state of the reaction \((5)\) is not resolved in the electromagnetic calorimeter, one must calculate the RC in terms of the calorimetric variables including the total energy of the electromagnetic cluster

$$E_{vis} = E'_e + E_\gamma.$$ \(18\)

Fig.6 shows the results for the radiative corrections including the effect of the calorimetric measurement of the final state photon radiation obtained \(^{19}\) with the Monte Carlo HERACLES. For comparison the RC in the terms of the pure electron variables are shown also.

In the program TERAD93 the possibility to calculate the RC in the terms of calorimetric variables, $E_{vis}, \theta_e$, has not been included yet, but with the help of the leading log calculations \(^{21}\) one can estimate the size of the influence of the calorimetric measurement on the radiative corrections.

The calorimetric measurement cannot distinguish between a single electron and an electron with accompanying photons. Therefore the final state radiation has experimentally no effect on the measurement of the kinematics of deep inelastic scattering and can be absorbed in the definition of the final electron energy of the process \((1)\).

This leads to the following way \(^{19}\) to take into account the calorimetric measurement. We can calculate the contribution of the final state radiation in the leading logarithmic approximation and subtract from the total RC, that we obtain with the program TERAD93.

The more detailed investigation of the calorimetry problem will be present in the separate publication.

Acknowledgements

I would like to thank D. Bardin and M. Klein for numerous helpful discussion. It is a pleasure to thank the DESY–IfH Zerthen for the good opportunities to work at DESY.
References

1. E. D. Bloom et al., Phys. Rev. Lett. 23 (1969) 930.
   M. Breidenbach et al., Phys. Rev. Lett. 23 (1969) 935.
2. I. Abt et al., The H1 Detector at HERA, DESY preprint 93-103 (1993).
3. M. Derrick et al., Phys. Lett. B293 (1992) 465.
4. L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41 (1969) 205;
   Y. S. Tsai, preprint SLAC-Pub-848 (1971).
5. Proceedings of the Workshop “Physics at HERA”, W. Buchm"uller and G. Ingelman (eds.), Hamburg (1991).
6. H. Spiesberger et al., in Proceedings of the Workshop “Physics at HERA”, ed. W. Buchm"uller and G. Ingelman, Hamburg, (1991) p. 798.
7. A. De Roeck, Deep Inelastic Scattering at Low-x. Results from the H1 Experiment, DESY preprint 93-087 (1993);
   I. Abt et al., Measurement of the Proton Structure Function $F_2(x,Q^2)$ in the Low $x$ Region at HERA, DESY preprint 93-117 (1993).
8. A. Blondel and F. Jacquet, in Proceedings of the Study of an ep Facility for Europe, ed. U. Amaldi, DESY preprint 79-48 (1979) 391.
9. M. Klein, in Proceedings of 4th San Miniato Topical Seminar, ed. P. Giusti et al, Singapore (1990) p. 31
10. A. Kwiatkowski, H. Spiesberger, and H.-J. M"ohring, Comp. Phys. Commun. 69 (1992) 155.
11. A. Akhundov et al., in Proceedings of the Workshop “Physics at HERA”, ed. W. Buchm"uller and G. Ingelman, Hamburg, (1991) p. 1285.
12. A. A. Akhundov, D. Yu. Bardin, N. M. Shumeiko, JINR Dubna preprint E2-10147 and E2-10205 (1976);
    A. Akhundov, D. Bardin, N. Shumeiko, Sov. J. Nucl. Phys. 26 (1977) 660;
    N. Shumeiko, Sov. J. Nucl. Phys. 29 (1979) 807;
    A. Akhundov, D. Bardin, W. Lohmann, JINR Dubna preprint E2-86-104 (1986) and program TERAD86;
    A. A. Akhundov, D. Yu. Bardin, N. M. Shumeiko, Sov. J. Nucl. Phys. 44 (1986) 988;
    D. Bardin, C. Burdik, P. Christova and T. Riemann, Z. Phys. C42 (1989) 679;
    D. Bardin, C. Burdik, P. Christova and T. Riemann, Z. Phys. C44 (1989) 149;
    A.A. Akhundov et al., Z. Phys. C45 (1990) 645.
13. A. Akhundov, D. Bardin, L. Kalinovskaya and T. Riemann, in preparation.
14. D. Bardin, A. Akhundov, L. Kalinovskaya and T. Riemann, in Proceedings of the Zeuthen Workshop on Elementary Particle Theory – Deep Inelastic Scattering, Tepetiz, 1992, ed. J. Bl"umlein and T. Riemann, Nucl. Phys. B (Proc. Suppl.) 29A (1992) 209
15. A. Akhundov et al., DESY preprint, in preparation.
16. A. Akhundov, D. Bardin, L. Kalinovskaya and T. Riemann, Phys. Lett. 301B 447.
17. A. D. Martin, W. J. Stirling, R. G. Roberts, Phys. Rev. D47 (1993) 867.
18. N. Yu. Volkonsky and L. V. Prokhorov, Sov. J. Exp. Theor. Phys. Lett. 21 (1975) 389.
19. A. Akhundov et al., H1-Note, in preparation.
20. G. Schuler and H. Spiesberger, in *Proceedings of the Workshop “Physics at HERA”*, ed. W. Buchmüller and G. Ingelman, Hamburg, (1991) p.1419.
21. J. Blümlein, *Z. Phys.* C47 (1990) 89; *Phys. Lett.* B271 (1991).
Fig. 1. Radiative correction factor $\delta(x_m, Q^2) \ (%)$ in $(x, Q^2)$-bins for MRS D-parametrization.
Fig. 2. Radiative correction factor $\delta(x_m, Q^2) \ (%)$ in $(x, Q^2)$-bins for MRS D0 parametrization.
Fig. 3. Radiative correction factor \( \delta(x_e, Q^2) \) (%) in \((x, Q^2)\)-bins for MRS D-parametrisation.
Fig. 4. Radiative correction factor $\delta(x_e,Q^2_e)\%$ in $(x,Q^2)$-bins for MRS D0 parametrization.
Fig. 5. Radiative correction $\delta(u, \theta_e)$ in $u$-bins for $162.5^\circ < \theta_e < 167.5^\circ$. 
Fig. 6. Effect of calorimetric measurement on size of radiative correction \( \delta(u, \theta_e) \) for \( 167.5^\circ < \theta_e < 172.5^\circ \).