Axial field induced chiral channels in an acoustic Weyl system

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Condensed-matter and other engineered systems, such as cold atoms, photonic, or phononic metamaterials, have proven to be versatile platforms for the observation of low-energy counterparts of elementary particles from relativistic field theories. These include the celebrated Majorana modes, as well as Dirac and Weyl fermions. An intriguing feature of the Weyl equation is the chiral symmetry, where the two chiral sectors have an independent gauge freedom. While this freedom leads to a quantum anomaly, there is no corresponding axial background field coupling differently to opposite chiralities in quantum electrodynamics. Here, we provide the experimental characterization of the effect of such an axial field in an acoustic metamaterial. We implement the axial field through an inhomogeneous potential and observe the induced chiral Landau levels. From the metamaterial perspective these chiral channels open the possibility for the observation of non-local Weyl orbits and might enable unidirectional bulk transport in a time-reversal invariant system.

Looking at Eq. (1), we observe that the effect of a magnetic field can be viewed as a space-dependent shift of the WP locations \( k^\text{WP}(x) \). This is most easily seen in the Landau gauge. For a magnetic field \( \mathbf{B} = B\hat{y} \), the momentum

\[
  k_z \rightarrow k_z + eBx
\]

is shifted in space as a function of \( x \) as a result of minimally coupling the corresponding gauge field. Taking this viewpoint of inhomogeneous WP positions, it has recently been realized that the application of other space dependent perturbations can lead to effects alike the ones induced by a magnetic field both for graphene\cite{19} as well as for Weyl semimetals\cite{18–24}. For example, in reaction to an inhomogeneous uniaxial strain, the locations of the WPs are moved in space\cite{21,22,25}. The main difference to a real magnetic field is the chirality dependence of the shift

\[
  k_z \rightarrow k_z + sB_zx,
\]

where \( B_z \) is the y-component of an axial magnetic field\cite{12,19} and moves WPs of opposite chiralities \( s \) in opposite directions, cf. Fig. 1a. Interestingly, the axial nature of the field, together with the chirality-dependence of Eq. (2) leads to co-propagating chiral channels in the presence of an axial magnetic field.

The axial field \( B_z \) is deeply connected to the chiral anomaly discussed in high-energy physics\cite{12,26–29}. Here, we set out to measure the effect of an axial field in an acoustic system, where a \( B_z \) is not a formal consequence of the chiral symmetry, but manifests itself in a set of chiral Landau levels. Moreover, the chosen “gauge” for \( B_z \) will have observable effects in the surface physics of our phononic crystal.

\[
\omega(k) = \text{sign}(B)sv_\parallel k_\parallel
\]

depends on the chirality \( s \) of the WPs and on the projection of the WP velocity onto the field direction \( v_\parallel \). This makes Weyl systems the momentum-space bulk analog of the quantum Hall effect\cite{15}. Unidirectional channels are separated in momentum space rather than in real space. Moreover, these chiral channels live in the three dimensional bulk as opposed to on the edge of a two-dimensional sample. Such unidirectional channels might harbor interesting physical effects or technological promises also for classical systems such as acoustic\cite{15,16} or electromagnetic metamaterials\cite{12}. However, as neither phonons nor photons carry an electromagnetic charge \( e \), it remained unclear if such chiral Landau levels can be observed in neutral metamaterials.
The starting point is the unit-cell shown in Fig. 1c, where we are interested in the pressure waves in air surrounding the shown structure. The pillars of radius $R$ and height $h$ create a hexagonal layer. The modes at the in-plane $K$ and $K'$-points are shown in Fig. 1b. We see that the low-frequency physics around these points is governed by modes localized at the corners of the unit cell, i.e., on a honeycomb lattice with nearest neighbor distance $d$. The width $r$ of the “ventilator holes” controls the strength of the interlayer coupling, while the turning angle $\vartheta$ of the ventilators determines the ratio $t_d/t_c$. Finally, the radius $n$ and depth $m$ of the holes below one sub-lattice detune the local mode and amount to the sub-lattice potential inducing the axial field.

We now optimize the parameters of this unit cell via finite-element simulations to find degenerate WPs at pre-determined locations $\delta k_x = \delta k_y = 0$ and $\delta k_z \in [-\pi/6\alpha_z, \pi/6\alpha_z]$ in the vicinity of the high-symmetry points $K$ and $H$. The location of the WPs around $K'$ and $H'$ are fixed by time-reversal symmetry. The final sample is then constructed by assembling unit-cells with the pre-determined locations of the WPs to obtain the sought after term $\propto B_3 x \sigma_z$. With this design strategy, we matched the low-frequency physics around the WPs of the acoustic sample to the low-frequency expansion of the tight-binding model. The details of the structure are given in the Appendix C. Note, that for a single WP, the field we apply corresponds to a magnetic flux equivalent to 1% of a flux quantum or a few hundred Teslas in typical electronic Weyl systems.

In summary, for our setup [see Fig. 1e] we expect four WPs, where the pair $K$ and $H$ sees an axial field $B_3$ and their time-reversed partners $K'$ and $H'$ the same with $-B_3$. Together with Eq. (2) this leads to the chiral Landau levels depicted in Fig. 1d, where the center of the chiral Landau levels are at $f = 7.7\, \text{kHz}$ with a group velocity of $v \approx 40\, \text{m/s}$. On the surfaces with surface normal $\pm\hat{x}$, we close the sample with hard walls, leading to well-defined zig-zag edges. The other four surfaces are left open, cf. App. C.

To characterize the properties of the three-dimensional sample we measure the spectral response of the acoustic field. We excite sound waves at different locations on the surface of the system. The response is then measured via a sub-wavelength microphone on the inside of the system. Using a lock-in measurement we measure phase and amplitude information of the acoustic field (App. D). This amounts to the measurement of the Greens function $G(r_i, r_j, \omega) = \langle \psi_i^\dagger(\omega) \psi_j(\omega) \rangle$, where $\psi_i(\omega)$ is the acoustic field at site $r_i$ and frequency $\omega$. By taking the spatial Fourier-transform of this signal, we obtain the spectral response shown in Fig. 2.

Owing to the gauge choice for the field $B_3$, the momentum in $x$-direction is not well defined. We therefore show the projection of the spectra onto $k_y$ and $k_z$ along the high-symmetry lines shown in Fig. 2a. The top and bottom panels of Fig 2d display the touching points of two bands at the $H'$, $H$ and $K'$ and $K$ points, respectively.
We further analyze the nature of these touching points through their associated surface physics below. The middle panel shows the evolution of the band-structure from $K$ to $H$, indicating a gap opening in $k_z$ direction assuring that we indeed couple different layers via the “ventilators”.

Around the frequency of the WPs, most of the acoustic response is concentrated around the $K$, $K'$, $H$, and $H'$ points. In Fig. 2b, we present the response for three selected frequencies in the $k_z = 0$ plane. The three columns show the response when excited from the surfaces with surface normal $\pm \hat{x}$ at the same time (bottom/top), from the surface $\hat{y}$ (right) and $-\hat{y}$ (left), respectively. Three observations can be made: (i) Only those modes which have a group-velocity that allows energy to be transported into the bulk are excited. (ii) Around 7.7 kHz, only modes around the $K$ and $K'$ are involved. (iii) When excited from the right (left), only the $K$ ($K'$) are excited. These three observations together essentially prove that we deal with a system with chiral channels around this frequency. This sensitivity on the excitation point is further quantified in Fig. 2c, where we show the integrated weight for all $x$ along $k_y$ at the $k_z = 0$ plane.

The open boundary conditions on the $\pm \hat{y}$ surfaces allow for a further analysis of the chiral channels. Radiation into free space essentially allows for any mode with arbitrary wave number to be supported inside our sample. In particular, no finite size quantization $k_\alpha = 2\pi m/L_\alpha$, with $\alpha = y, z$ and $m \in \mathbb{Z}$ occurs. We use the fact that the eigenstates are Bloch waves

$$\psi_\ell (\mathbf{k}) = e^{i \mathbf{k} \cdot \mathbf{r}} \begin{pmatrix} u_k \\ v_k \end{pmatrix},$$

(5)
where \( u_k \) and \( v_k \) are the sub-lattice weights. Using the above structure, we can fit the phase evolution from unit-cell to unit-cell to obtain \( k_y(\omega) \). In particular, we select the Fourier components \( \psi_{k_x, k_z}(y) \) of the measured fields corresponding to the high-symmetry lines. We then fit \( \arg[\psi_{k_x, k_z}(y)] \approx k_y(\omega)y \). This analysis is only possible owing to the fact that with the excitation-point selectivity shown in Fig. 2b/c, we can ensure that in the chiral channel region we fit the phase evolution of only one mode. The resulting dispersion curves \( \omega(k_y) \) are shown in Fig. 2e. The clearly visible four chiral channels in accordance with the expectations in Fig. 1d are the main result of this study.

A key property of Weyl systems manifests itself on the two-dimensional boundaries of a finite sample: Open equifrequency contours in the surface Brillouin zone end at the projections of the bulk WPs. These open contours (or Fermi surfaces called Fermi arcs for electronic systems) can be understood via layer Chern numbers, where each \( k_z \)-layer is seen as a two-dimensional system. When passing a layer with a WP, this Chern number is changing by \( \pm s \). The celebrated surface arcs are then nothing but the edge states induced by these layer Chern numbers. For our setup we expect an \( x \)-dependent \( k_x \)-Chern number as we move the location of the WPs with the axial field. In other words, we can expect the Fermi-arcs to terminate at different locations on the two \( \pm \hat{x} \) surfaces (called bottom/top before). This can be directly observed in the numerical simulations shown in Fig. 3a.

The experimentally measured Fermi arcs are shown in Fig. 3b. From the shown Green’s function at a given frequency on the left, we can extract \( \omega(k_y, k_z) \). Owing to our resolution in \( k_z \) and the dissipation broadened features, we cannot determine the end-points of this Fermi arc. However, we can observe the evolution of \( \omega(k_y, k_z) \) and hence determine the group velocity on the two opposing surfaces. In Fig 3c the \( +\hat{x} \) and \( -\hat{x} \) surfaces show indeed opposite group velocities, in line with the expectation of edges states induced by a Chern number. The upper half of the Brillouin zone is not shown and determined by time-reversal symmetry.

By directly observing chiral Landau levels in a Weyl system we have shown that axial fields rooted in the theory of high-energy physics can be implemented and observed in condensed matter systems. Many of the theoretically predicted phenomena, such as the chiral magnetic effect,\(^{31} \) the chiral vortical effect,\(^{12} \) strain induced quantum oscillations\(^{32} \) and many more seem now to be reachable in systems of classical metamaterials\(^{16,17,33,34} \) in cold-atoms, or in low-temperature electronic systems.

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### Appendix A: Tight-binding model

The dynamics governed by the tight-binding model of Fig. 1b can be written as

\[
\partial_t^2 \begin{pmatrix} u_k \\ v_k \end{pmatrix} = \begin{pmatrix} \gamma(\mathbf{k}) + \alpha(\mathbf{k}) & \beta(k_x, k_y) \\ \beta^*(k_x, k_y) & \gamma(\mathbf{k}) - \alpha(\mathbf{k}) \end{pmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix}, \quad \text{(A1)}
\]
with
\[
\gamma(k) = 2\cos(k_z a_z) \left[ t_d + t_c \cos(k_y a_n) + 2t_c \cos(\sqrt{3}k_z a_n/2) \cos(k_y a_n/2) \right],
\]
\[
\beta(k_x, k_y) = t_n \left[ e^{-i\sqrt{3}k_z a_n/3} + 2\cos(k_y a_n/2) e^{i\sqrt{3}k_z a_n/6} \right],
\]
\[
\alpha(k) = 2t_c \sin(k_z a_z) \left[ \sin(k_y a_n) - 2\cos(\sqrt{3}k_z a_n/2) \sin(k_y a_n/2) \right].
\]
Here, \(u_k\) and \(v_k\) describe the amplitudes of the Bloch waves of Eq. (5). The lattice constants are related to the sample geometry through \(a_n = \sqrt{3}d\) and \(a_z = L+h\). Note, however, that we only match the low-frequency physics around the WPs of model \((A1)\) to the corresponding low-frequency physics of the acoustic structure. In particular, we do not intend to match the full lattice model at all lattice momenta.

The model \((A1)\) features band touchings whenever \(\alpha(k) = \beta(k_x, k_y) = 0\). The function \(\beta(k_x, k_y)\) describes a simple honeycomb layer and hence has gap closings at the \(K_\pm = (0, 4\pi/3a_n, 0)\) and \(K'_\pm = (2\pi/\sqrt{3}a_n, 2\pi/3a_n, 0)\) points. A straight-forward analysis of \(\alpha(k)\) shows that the full model has WPs at the \(K = (0, 4\pi/3a_n, 0), \ K' = (2\pi/\sqrt{3}a_n, 2\pi/3a_n, 0), \ H = (0, 4\pi/3a_n, \pi/a_z), \) and \(H' = (2\pi/\sqrt{3}a_n, 2\pi/3a_n, \pi/a_z)\) points, respectively, cf. Fig. 1a.

The low-frequency physics around the Weyl points is fully described by the velocity tensors
\[
v_{\alpha\beta}^{K/K'} = \begin{pmatrix}
0 & 3t_n/2 & 0 \\
\mp3t_n/2 & 0 & 0 \\
0 & 0 & \mp3\sqrt{3}t_c
\end{pmatrix},
\]
\[
v_{\alpha\beta}^{H/H'} = \begin{pmatrix}
0 & 3t_n/2 & 0 \\
\mp3t_n/2 & 0 & 0 \\
0 & 0 & \mp3\sqrt{3}t_c
\end{pmatrix}.
\]

Going from \(K\) (\(H\)) to \(K'\) (\(H'\)) two rows change sign. When comparing \(K\) and \(H\), on the other hand, only one row differs in sign. This explains the distribution of chiralities (red and blue) in Fig. 1a.

From the above velocity tensors we see that only \(k_z\) couples to the \(\sigma_z\) matrix. If we now want to shift the WP in \(k_z\) direction as \(k_z \to k_z - sB_{3x}\), we need to couple a space dependent sub-lattice potential of the form
\[
V(x) = 3\sqrt{3}t_c B_3 x\sigma_z. \tag{A2}
\]
It is crucial to note that the axial nature of the field arises from the chirality dependent pre-factor of \(v_{zz}\). In other words, the above potential \(V(x)\) acquires the axial nature only in the low-energy theory.

So far, \(\gamma(k)\) in Eq. \((A1)\) has not been addressed. In order to avoid any frequency shift or tilt of the conical dispersion, we need \(\gamma(k) = 0\) for \(k = K, K', H, H'.\) A straight-forward analysis shows how this happens whenever \(2t_d = 3t_c\).

### Appendix B: Chirality and Berry curvature

For completeness, we present the derivation of the Berry-monopole represented by a WP. A WP is a conical touching of two bands, hence its general Hamiltonian can be written as
\[
H = \sum_\alpha d_\alpha(k) \sigma_\alpha, \tag{B1}
\]
where \(d(k)\) is a vector linear in \(k\). In polar coordinates \(d = |d| (\sin \vartheta \cos \varphi, \sin \vartheta \sin \varphi, \cos \vartheta)\). The eigenvalues of the Hamiltonian \((B1)\) are \(\epsilon(k) = \pm |d(k)|\). The eigenvectors are
\[
|\pm\rangle = \begin{pmatrix}
\sin \left(\frac{\vartheta}{2}\right) e^{i\varphi}
\
-\cos \left(\frac{\vartheta}{2}\right)
\end{pmatrix}, \quad \epsilon(k) = \pm |d(k)|. \tag{B2}
\]

To characterize the WP the lower band is relevant. Neglecting \(A_{|d|} = i(-|\partial_{|d|}|\langle - |\partial_{|d|}|\rangle)\) the remaining components of \(A\) are
\[
A_\vartheta = i(-|\partial_{|d|}|\langle - |\partial_{|d|}|\rangle) = 0, \tag{B3}
\]
\[
A_\varphi = i(-|\partial_{|\varphi}|\langle - |\partial_{|\varphi}|\rangle) = -\sin^2 \left(\frac{\vartheta}{2}\right). \tag{B4}
\]

From this the Berry curvature \(\mathcal{F} = \nabla \wedge A\) follows:
\[
\mathcal{F}_{|d|} = \frac{\sin \vartheta}{2}. \tag{B5}
\]
To get back to the original coordinates, this result needs to be multiplied by the Jacobian of the coordinate transformation. If \(d(k) = k\), the Jacobian is the one of a spherical coordinate transformation \((1/\sin \vartheta |k|^2)\) and the Berry curvature is
\[
\mathcal{F} = \frac{k}{2|k|^3}. \tag{B6}
\]
Integrated over a shell around WP, this amounts to a flux of \(2\pi\), i.e., the WP corresponds to a monopole charge. However, so far we have considered the case of isotropic WPs.

In the most general case the Weyl Hamiltonian is given by Eq. (1). In this case \(d(k) \neq k\). However, there is still a linear relation between \(d\) and \(k\). If the tensor \(v_{\alpha\beta}\) was diagonal it would amount to a simple rescaling. In the most general case, this transformation amounts to a permutation of the rows of the Jacobian and a rescaling. The parity of this permutation is captured by the determinant of the velocity tensor. Therefore the Berry curvature for the generic Weyl point described by Eq. (1), is
\[
\mathcal{F} = \text{sign}(|\text{det}(v_{\alpha\beta})|) \frac{k}{2|k|^3} = s \frac{k}{2|k|^3}. \tag{B7}
\]
In other words, the chirality of the WP defines the charge of the Berry monopole.
Appendix C: Sample details

The sample shown in Fig. 1c is printed on a Stratasys Connex Objet500 with PolyJet technology by Stratasys. The printed material is VeroWhitePlus. The 300 µm resolution in the xy-plane and a resolution of 30 µm in the z-direction assure a fine enough surface finish for good hard-wall boundary condition for the acoustic field. At least in the frequency/wavelength regime we are interested in. Each layer was printed in four parts and later assembled and stacked.

The full sample consists of $L_x \times L_y \times L_z = 20 \times 20 \times 12$ unit cells. The fixed dimensions of the sample are given by $\vartheta = 2.7$ rad, $d = 13$ mm, $a_z = 18.5$ mm, $R+r = 9$ mm. The depth $n = L/2 - 0.5$ mm is fixed to leave the separating wall between layers of constant thickness of 1 mm. The remaining parameters are optimized to seek the desired WP locations and varied between $0$ mm $\leq n \leq 3.0$ mm, $6.1$ mm $\leq R \leq 7.6$ mm, and $7.9$ mm $\leq h \leq 9.3$ mm. The detailed profile of these parameters are available from the authors.

The sample is terminated with open surfaces on the $\pm \hat{z}$ (ventilators) and $\pm \hat{y}$ (armchair) edges. These open boundary conditions allow for the phase fitting mentioned in the main text. Along the $\pm \hat{x}$ surface we close the sample with hard walls. In their absence the outermost mode localized on the corner of the unit cell in Fig. 1b is largely detuned. This leads effectively to a bearded edge, where spurious surface states appear around the $\Gamma$ and $\Lambda$ point. To avoid these states, we close the sample on the $\pm \hat{x}$ surfaces.

Appendix D: Measurement and signal analysis

The acoustic signals are generated with speakers SR-32453-000 from Knowles. The pressure fields are measured via a sub-wavelength microphone FG-23629-P16 from Knowles with a diameter of 2.6 mm which is mounted on a 2 mm steel rod to scan the inside of the acoustic crystal. We always measure 200 frequency points between 4 and 11 kHz to obtain phase and amplitude information using a lock-in amplifier. In unit-cell coordinates $(i_x, i_y, i_z)$ running from $i_x, i_y = 0, \ldots, 20$ and $i_z = 0, \ldots, 12$ we excited at $(0, 10, 6)$ [called bottom in the main text], $(20, 10, 6)$ [top], $(10, 0, 6)$ [left], and $(10, 20, 6)$ [right]. The crystal is scanned in a grid of $19 \times 20 \times 12 \times 2$ points corresponding to all the accessible sub-lattice sites of a set of 12 stacked layers of each $20 \times 20$ honeycomb unit-cells. In fact, the zig-zag terminations along the surfaces $\pm \hat{x}$ limit the number of points that can be measured at the surfaces. The data displayed in Fig. 2 are based on pure bulk measurement. The first two unit cells closest to the $\pm \hat{x}$ and the closest unit cells to $\pm \hat{\rho}$ surfaces have not been taken into consideration in these analysis to avoid spurious surface effects. On the other hand, the data of Fig 3 are based on surface measurements only, i.e., data taken on the first two unit-cells. Finally, for the density plots, the discrete spatial Fourier transforms are displayed with the Lanczos interpolation method for visual clarity, except in Fig 3b, where the discrete nature of the measurements is relevant. Note, that the chiral channel phase fitting in Fig 2e is not based on an interpolation of the Fourier-transform data.

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