Study on the dependence of the force of shock interaction on the body size using a mechanorheological model

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Abstract. The issues of modeling processes and designs in various industries are crucial. Particular attention is paid to dynamic processes of movement, vibration and shock interaction of structural elements and mechanical systems. Using the viscoelastic plastic model, the influence of the size of a spherical body on the force of shock interaction was studied. With an increase in the diameter of a spherical body, the body mass and the radius of curvature increase. The impact of both factors and each separate factor was evaluated using mathematical models of shock interaction. With a simultaneous increase in the body mass and the radius of curvature due to an increase in the diameter of a spherical body, the force of shock interaction increases. It was identified that an increase in the body mass causes a more significant increase in the force of shock interaction. An increase in the radius of curvature causes a slight increase in the force of shock interaction. Simultaneous changes in two factors cause a more significant increase in the force of shock interaction. An increase in plasticity of the model decreases the force of shock interaction. Thus, the results of the study show that the force of shock interaction depends significantly on the size of an interacting body due to the influence of the body mass and the radius of curvature on the dynamics of movement and deformation of bodies.
1. Introduction

The issues of modeling processes and designs in various industries, including construction one, are crucial. Vibration protection of building structures (1, 2), optimization of structures by various criteria (3), processing of non-metallic building materials (4) are modeled and studied. Special attention is paid to dynamic processes of movement, vibration and shock interaction of structural elements and mechanical systems. Much attention is paid to key parameters of dynamic processes: forces of shock interaction, duration of shock interaction, speed deformation values, etc. Loads depend on accelerations during braking. They are determined by masses of interacting bodies. Accelerations and inertial forces often destroy machines and structures.

Studies on dynamic processes using mathematical models are important and relevant. When developing mathematical models, the mechanorheological approach is often used. A lot of researches deal with these issues [5-10].

The present article studies the influence of the size (diameter) of a spherical body on the force of shock interaction using the viscoelastic plastic mechanorheological model [11]. It consists of elastic, viscous and plastic (shear) elements which describe plastic deformations. To develop a model control method, some researches were carried out [12–16], including the researches on the influence of elastic, plastic and viscous parameters of the model on the force of shock interaction.

The model consists of two units: viscoelastic unit $K_1 - C$ and elastic plastic unit $K_2 - f_2$ (Fig. 1).
Figure 1. The scheme of the viscoelastic plastic model.

The mass of the spherical body is concentrated in inertia element $m_1$. The mass of element $m_2$ is negligibly small ($m_2 \to 0$). It was introduced for convenient development of the mathematical model with two second order differential equations.

At the initial interaction stage, only elastic deformations occur. Only the viscoelastic unit $K_1 - C$ becomes deformed. It describes elastic deformations and controls energy losses caused by energy dissipation. When the dynamic force has a target $F_{SF}$, value corresponding to intensive development of plastic (residual) deformations, the viscoelastic unit $K_2 - f_2$ comes into
operation. By modifying $F_{ST}$ values, one can model viscoelastic or viscoelastic deformation of the body. One or both units are working.

The maximum force of shock interaction $N_{MAX}$ corresponds to the unloading stage. The unloading process is described by the viscoelastic unit.

2. Purpose statement

The purpose of computer experiments was to study and analyze the dynamics of interaction of a spherical body with a flat surface using mathematical modeling of real shock interaction processes. The experiments aimed to study the influence of rheological properties of the surface material, the size of a spherical body on the force of shock interaction.

For this purpose, a more advanced viscoelastic plastic mechanorheological model was used. It helps identify the effect of material plasticity on the dynamics of shock interaction of bodies.

An important task is to adapt the model real shock interaction process. The dynamics of model movement was controlled by the developed methods [17, 18].

Viscoelastic plastic parameters of the model were changed in order to assess the effect of material plasticity on the dynamics of shock interaction. Spherical bodies of different diameters (9.5; 14.25; 19 mm) were used.

With an increase in the body size, the body mass and the radius of curvature increase. Both factors influence the dynamics of shock interaction. Therefore, the influence of both factors and the influence of each factor on the force of shock interaction were studied.

3. Theoretical background

The dynamics of the mechanorheological model is described using mathematical models. The differential equation of the motion of an elastic-viscous model can be written as:

$$m_0 \dddot{y}_i + C_i \ddot{y}_i + K_i y_i^{1/2} = -m_0 g$$

(1)

The motion of viscoelastic plastic model is described by two differential equations:

Viscoelastic plastic model movement can be described by two equations:

$$m_0 \dddot{y}_i + C_i (\dddot{y}_i - \dddot{y}_j) (\dddot{y}_i - \dddot{y}_j) + K_i (\dddot{y}_i - \dddot{y}_j)^{1/2} = -m_0 g$$

(2)
\[ m_1 y_1 + K_1 y_1^{\frac{3}{2}} + f_2 y_2 + C_1 (y_2 - y_1) (y_2 - y_1) + K_2 (y_2 - y_1)^{\frac{3}{2}} = -m_2 g + F_{ST} \quad (3) \]

where \( y_1, y_2, y_1', y_2' \) are the shift and speed of mass \( m_1 \) and mass \( m_2 \); \( K_1 \) is the ratio of stiffness of the elastic element of the viscoelastic model unit; \( C_1 \) is the ratio of viscosity of a viscous element of the viscoelastic model unit; \( F_{ST} \) is the force at the beginning of plastic deformation occurrence; \( K_2 \) is the ratio of stiffness of an elastic element of the elastic-plastic unit; \( f_2 \) is the ratio of shift of the elastic plastic unit or the ratio of ductility.

The shift from the viscoelastic model to the viscoelastic plastic model occurs when the force of normal response \( N = C_1 (y_1 - y_2) (y_1 - y_2) + K_1 (y_1 - y_2)^{\frac{3}{2}} \) has the target value of \( F_{ST} \).

The force of shock interaction can be determined from the viscoelastic or elastic plastic unit:

\[ N = C_1 (y_1 - y_2) (y_1 - y_2) + K_1 (y_1 - y_2)^{\frac{3}{2}} \quad \text{or} \quad N = K_2 y_2^{\frac{3}{2}} + f_2 y_2 + F_{ST}. \]

The elastic element of the model has non-linear characteristics [6-8]. Viscous resistances were assumed to be proportional to speed and elastic deformation values [12]. Equations (1) – (3) can be solved by the Runge-Kutta method. Based on the mathematical model, a special research program was developed.

4. Analysis of research results

The shock interaction process was modelled using steel spherical bodies of different diameters: 9.5; 14.25; 19 mm. The radii of curvature of the spherical surface were 4.75; 7.125; 9.5 mm respectively. Masses of the spherical body were 3.5; 11.82; 28.01 g. Several calculation variants were analyzed. Variant 1: \( F_{ST} = N_{\text{max}} \) – only elastic deformations occur. Variant 2: \( F_{ST} = 2N_{\text{max}}/3 \) Variant 3: \( F_{ST} = N_{\text{max}}/3 \). Variant 4: \( F_{ST} = 0 \) – plastic and elastic deformations occur since the beginning of the shock interaction process. The maximum value of the force of shock interaction \( N_{\text{MAX}} \) was used. Viscoelastic and plastic parameters \( K_1, K_2, f_2, C \) were modified within the target limits.

Figures 2 and 3 show the dependence of the force of shock interaction on diameters of the spherical body at different values of \( F_{ST} \) and simultaneous influence of two factors (body mass \( m_1 \) and radius of curvature \( R \)). Fig. 2 shows that elastic parameter \( K_1 \) had the minimum value. Fig. 3 shows that...
elastic parameter $K_1$ had the maximum value. Other parameters $K_2, f_2, C$ changed in a similar way.

Figures 4 shows the influence of the body mass $m_1$ on the force of shock interaction. Figure 5 shows the influence of the radius of curvature $R$ on the force of shock interaction.

The analysis allowed for the following conclusions. Under the simultaneous increase in two factors (body mass and radius of curvature),
force of shock interaction $N_{\text{MAX}}$ increases (Fig. 2, 3). An increase in the body mass at $R-\text{const}$ (Fig. 4) causes a more significant increase in $N_{\text{MAX}}$ than an increase in the radius of curvature (Fig. 5). The influence of the body mass on the force of shock interaction is due to increasing inertia forces which depend on the body mass and occur at the moment of interaction.

**Figure 4.** Dependence of the force of shock interaction $N_{\text{MAX}}$ on the body diameter $d$ at $R-\text{const}$.

An increase in the radius of curvature at $m_{j}-\text{const}$ causes a slight increase in $N_{\text{MAX}}$ (Fig. 5). The influence of the radius on the force of shock interaction is due to the elastic body rigidity increasing in the deformation zone of the
contact area which increases elastic resistance to deformation. Increasing resistance increases elastic parameter $K$. Changes in both factors cause a more significant increase in the force of shock interaction.

An increase in plasticity of the model ($F_{ST}=N_{max} \rightarrow F_{ST}=0$) decreases $N_{MAX}$ at various parameters of the model $K_1, K_2, f_2, C$.

5. Conclusion

Thus, the study on shock interaction of a spherical body with a surface carried out using a mechanorheological viscoelastic plastic model show that the force of shock interaction depends on the body size. This is due to the influence of the body mass and the radius of curvature on the dynamics of movement and deformation of bodies. The mass influencing inertia forces has a more significant effect on the force of shock interaction. Proper consideration of the factors influencing the dynamics of shock processes will improve modelling and research accuracy and reliability for various dynamic processes.

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