Polarisation as a Tool for Gravitational Microlensing Surveys

John F.L. Simmons\textsuperscript{1}, Jon P. Willis\textsuperscript{1}, and Andrew M. Newsam\textsuperscript{1}

Department of Physics and Astronomy, University of Glasgow

Abstract. Much interest has been generated recently by the ongoing MACHO, EROS and OGLE projects to identify gravitationally lensed stars from the Large Magellanic Cloud and Galactic bulge, and the positive identification of several events (Alcock et al., 1993, Aubourg et al., 1993, and Udalski et al., 1993). The rate at which such events are found should provide considerable information about the distribution of the low mass compact objects, brown dwarfs etc. responsible for the lensing, and their relative importance as a component of the cosmological ‘dark matter’. We show measurement of the polarisation of starlight during these events can yield considerably more information about the lensing objects than was previously considered possible. Furthermore, the consideration of extended sources is shown to have a significant effect on the interpretation of the profiles and statistics of the events.

More detailed modelling of the gravitational microlensing of stars by low mass objects can yield considerable information about the lens and the lensing geometry. In particular, the measurement of the variable polarisation produced by gravitational lensing of stars could in principle provide further information about the mass, distance and velocity of the lensing objects. This possibility appears to have been overlooked in both the MACHOs and EROS programmes.

Although the idea that polarisation could be produced by gravitational lensing was raised and investigated in relation to supernovae (Schneider and Wagoner, 1987), it seems that not to have been considered for stars. Polarisation produced by lensing of stars is not large, but should be measurable for sufficiently bright stars.

Normally the light received from stars is unpolarised, unless the star is rotationally distorted, or has an asymmetric envelope or wind. However the light that emerges from the limb of a star can be expected to be polarised (up to 10%). This effect, predicted by Chandrasekhar (1960), who calculated it for an electron scattering photosphere, depends on the scattering mechanism and on the direction of the emergent radiation from the normal to the stellar photosphere. It has been observed in the Sun. However, for a star, the polarised light flux observed at the Earth is obtained by integrating the polarised intensity over the stellar disc. The polarisation of the light at the north/south points is in an opposite direction to that at the east/west points (see figure 1). If the star is rotationally symmetric, no net polarisation should be observed owing to cancellation. In the case of a gravitationally lensed star, the amplification at different points on the stellar disc will be different, so the net polarisation differs from zero by an amount depending on the distance of the lensing object projected onto the source (star’s) plane. The direction of this polarisation will also vary with as angle of the line of centres of source and lensing object changes.

One can obtain an estimate of the degree of polarisation for a Schwarzschild lens as follows. Distances are most conveniently written in units of the Einstein radius projected onto the source plane, $\eta_0$, given by

$$\eta_0^2 = \left( \frac{1}{a_L} - \frac{1}{a_S} \right) 2 R_S a_S^{-2}$$

(1)

where $a_L$ and $a_S$ are the distances to the lens and source respectively, and $R_S = 2GM/c^2$ is the Schwarzschild radius of the lensing object. The Amplification at distance $x$ is given by $A(z) = \frac{1}{2} \left( z + \frac{1}{z} \right)$ where $z = \left( 1 + \frac{R_S}{x} \right)^{1/2}$. For small $x$, the amplification is given by $A(z) \approx z + \frac{1}{2} z^2$.

Fig. 1. Two stages during an event. In (a), the event is about to begin. There is no noticeable difference in the amplification of the N/S and E/W regions so the star remains unpolarised (although a flux increase might already have been seen). In (b), however, the lens is at closest approach and the significant amplification of South w.r.t. East and West gives a net polarisation in the direction at S.
$A(x) \sim 1/x$. For polarisation greater that 1%, say, this amplification must vary by 10% over one stellar radius, $R$ (assuming a limb polarisation of around 10%). This immediately yields the condition that $R > 0.1d$, where $d$ is the distance of approach to the centre of the stellar disc.

Exact expressions for the polarised and unpolarised flux, $F_U, F_Q$ and $F_I$ can readily be obtained from the form of the polarised and unpolarised intensity (stokes parameters) as functions of the angle of emergence to the normal, $\theta$, at the surface of the star. If we assume these to take the form

$$I = u_0 + i_1 \mu$$

$$U = u_0(1 - \mu)$$

where $\mu = \cos \theta$, we obtain on integrating over the stellar disc the expressions

$$F_I = \frac{R^2}{a_s^2} \int_0^{2\pi} \int_0^1 (i_0 + i_1 \mu) A(x(\mu, \phi)) \mu \, d\mu d\phi$$

$$F_U = \frac{R^2}{a_s^2} \int_0^{2\pi} \int_0^1 u_0(1 - \mu) A(x(\mu, \phi)) \mu \cos 2\phi \, d\mu d\phi$$

$$F_Q = \frac{R^2}{a_s^2} \int_0^{2\pi} \int_0^1 u_0(1 - \mu) A(x(\mu, \phi)) \mu \sin 2\phi \, d\mu d\phi$$

where

$$x^2 = d^2 + R^2(1 - \mu^2) - 2(1 - \mu^2)^{1/2} R d \cos \phi$$

$a_s$ is the distance to the source and $\phi$ the position angle.

The observed degree of polarisation is given by

$$p = \frac{(F_Q^2 + F_U^2)^{1/2}}{F_I}$$

and is a function of $R$ and $d$. (In fact $F_Q = 0$ in the coordinate system chosen). The latter is easily expressed in terms of the transit velocity of the lensing object and the distance of closest approach or impact parameter, $d_0$ and time (all in the source plane).

Thus simultaneous measurement of the time variable polarisation and unpolarised flux yields a lot of information about the lensing set-up. If one assumes the radius of the star is known, then one immediately obtains the value of $\eta_0$. Modelling of the stellar atmosphere should not even be necessary, as the parameters $i_0$, $i_1$ and $u_0$ could be obtained from the time profile of $p$, $F_I$ and position angle. If one assumes additionally, though less reasonably, a transit velocity for the lensing object one can obtain the distance to the lens, and indeed its Schwarzschild radius (mass). (Of course one does need to assume a distance to the source).

The statistics of the events are potentially more important. We may consider three types of event

(i) flux variation

(ii) polarisation variation

(iii) transit events (i.e. where some part of the source is directly in line with the lens centre and observer) which should appear as sudden increases in flux.

An event (i) will be recorded when the flux variation is greater than some chosen value. In the case of a point source, this will simply be when the impact parameter is less than some constant times the Einstein radius, $\eta_0$, i.e. $d_0 < \alpha \eta_0$. The behaviour of the time profile of the flux for an extended source is significantly different to that of a point source (Simmons et al., 1994). If $R \sim d_0$ then the amplification is smaller in the wings but higher at closest approach, giving a sharper profile. This is possibly the explanation of the outlier at maximum amplification observed by the MACHOs group for the first event reported (Alcock et al., 1993). On the other hand if $R > 3$ the amplification is almost unobservable regardless of the impact parameter. Since the Einstein radius, $\eta_0$ depends on $M^{1/2}$, this would seriously impede the detection of low mass lenses.

Cases (ii) and (iii) will only occur for extended sources. The polarisation time profile in (iii) will be quite specific, with two equal maxima, and a central minimum. The relative frequency of the different types of events will indicate the spatial distribution of lensing objects.

The domain of values in the $R$ and $d$ plane is given in Figure 2 (see also Simmons et al., 1994), the unit of distance being $\eta_0$. Thus for lenses of the same mass near the galactic centre $R$ and $d$ take small values, and for lenses near the LMC $R$ and $d$ are large. If the stars in the source plane are taken to be uniformly and randomly distributed, the fraction of events of any one type for a fixed radius of star is simply proportional to the length of the interval in $d$. For example, for $R = 1$, more than half of flux events also show variable polarisation (see figure).

From the rate of events, and the relative frequency of types (i), (ii) and (iii) it should be possible, given a sufficiently large sample, determine the spatial distribution of lenses.

Two further points should be stressed. The rise in polarisation takes place later than the rise in total flux, by approximately a factor of 2. Thus an event suspected because of an observed amplification in flux, could be monitored and confirmed polarimetrically. Secondly, the polarisation effects arise from the limb dependency of polarisation in an extended source.

Fig. 2. Contours of 0.1% polarisation and a flux amplification of 1.34 (corresponding to amplification of a point source at the Einstein radius) as a function of source radius and impact parameter. For values of $R$ and $d_0$ inside the polarisation “ellipse” variable polarisation will be seen. Similarly, points in the bottom left corner enclosed by the flux contour are flux events. The points with $R > d_0$ are transit events.
Even for an unpolarised source the parameters occurring in expression (3) for the intensity should depend on wavelength both in continuum and lines, so one might expect that for an extended source where limb darkening is present the relative amplitude of the flux variation would depend on frequency. For a point source, or for a star with no limb darkening, this effect would not be seen, and the profile would be achromatic.

Polarisation measurements are certainly feasible. Interstellar polarisation might make interpretation more difficult, but as we are dealing here with variable polarisation, this could be overcome. This should also provide valuable information about the lensing, and the importance of MACHOs as a dark matter component.

1. Acknowledgements

JPW performed this work as part of a Cormack Research Scholarship. AMN was funded by a PPARC Postgraduate Studentship.

References

Alcock, C et al, Nature, 365, 621, (1993)
Aubourg, E. et al, Nature, 365, 623, (1993)
Chandrasekhar, S., Radiative Transfer, Dover, (1960)
Schneider, P. and Wagoner, R., ApJ, 314, 154, (1987)
Simmons, J., Newsam, A., Willis, J., In preparation
Udalski, A. et al Ann. N.Y. Acad. Sci. 688, 626, (1993)
Direction of Polarisation

- Source
- Lens
- Flux (1.34)
- Polarisation (0.1%)