Numerical Investigations into the Effect of Confinement on the Stability of an Oscillating Planar Liquid Jet

Ashish Arote*, Mukund Bade, Jyotirmay Banerjee
Department of Mechanical Engineering, SV National Institute of Technology, Surat, India-395007
E-mail: ashisharote4@gmail.com

Abstract. The liquid jet under certain conditions when perturbed will lead to instability. The understanding about an effect of confinement on these jets is still obscure. Hence, numerical investigations are reported in the present study for two phase spatially oscillating planar jet in a quiescent air under the confined conditions. Simulations are performed by solving the Navier-Stokes equations using volume of fluid (VOF) method to track the air-water interface. In the present study an oscillating jet is subjected to various confinement ratios ($CR$) to understand the jet behaviour under such conditions. It is demonstrated that the amplitude of oscillation increases as the confinement ratio decreases. This behaviour is a result of increased intensity of vortex rotation at each jet peaks as the confinement ratio increases. Moreover, it is also demonstrated that the vortex structure interact with the side-walls and disturb the incoming flow causing non-linear growth of oscillations leading to primary breakup in these jets.

1. Introduction
Oscillating jet is a spatially perturbed periodic jet which is produced using a fluidic oscillator. These jets undergo an oscillation instability in which its oscillation amplitude increases as the jet propagates into the domain. These jets are found to be of significant importance in the applications involving mixing, atomization and dispersion. The typical applications involve cooling of electronic circuits [1][2], flow control strategies [3], mixing and separation of chemical components [4].

Initial study of an instability associated with planar jet was performed by Squire [5] and, Hagerty and Shea [6]. They identified the as symmetric and asymmetric modes of instability in their study. The limitation of these studies was the assumption of uniform sheet thickness which is unable to predict jet breakup. Therefore, Clark and Dombrowski [7] performed a second order non-linear analysis to predict the instability in these jets. The results of these studies were based on temporal analysis which assumes that the perturbations grow only with time. Thus, spatio-temporal analysis was needed to understand the instability associated with these jets. This issue was resolved by Asare [8], Li [9], Chuech [10] and, Sirignano and Mehring [11] in their work and highlighted the effect of non-linearity in the growth of induced perturbations. Thus, it was demonstrated that the sinuous instability of jet is well predicted in the near-field whereas as the aero-dynamic effect increases the jet takes a saw-tooth shape due to non-linear growth.
The effect of confinement on the inviscid planar jets was studied by Juniper [12] and demonstrated that the jet are most unstable at confinement of $h = 1$. The similar observation was made by Juniper [13] for circular jets. Rees and Juniper [14] demonstrated that for viscous flows also the flow destabilizes when the confinement is increased. Healey [15] showed that semi-confinement (confining only one side of the faster stream) can also lead to instability in the flow. Wang et al. [16] studied the instability of confined non-Newtonian liquid jet in an annular gas layer and demonstrated that an increase in thickness of gas layer destabilizes the jet. Mosavati et al. [17] studied the jet instability and vortex structures in a confined cavity where the jet undergoes self oscillations. Hao et al. [18] have reported the numerical results of the confined impinging jet mixers, where the flow structures in the shear layer are studied to understand the mixing characteristics of the jets.

The fundamental understanding about the effect of confinement on the stability of oscillating jets is vital in improving the efficacy of industrial applications. Thus, the numerical study can provide better insights into the cause of instability and the breakup mechanisms in these jets. Moreover, to the best of authors’ knowledge there are no numerical studies in the previous literature that investigate influence of confinement on the stability of these jets.

Structure of the present paper is as follows. Section 2 discusses the governing equations and their solution methodology. The results are discussed for the different confinement ratios in Section 4. The present study is summarized in Section 5.

2. Numerical Methodology

The numerical methodology adapts the one fluid model where both phases of the fluid share one set of governing equations [19]. Both phases are assumed to in a continuum and are incompressible. The interface between both the phases is considered to be sharp and under the influence of forces such as surface tension and gravity. The present study is carried out using an in-house high resolution solver [23].

2.1. Governing Equations

The fluid flow for the present problem is governed by Navier-Stokes equations. Non-dimensional integral form of these equations is given as,

\[
\iiint_{CS} u_j^* d\vec{A} = 0
\]

\[
\iiint_{CV} \frac{\partial u_i^*}{\partial t} + \iiint_{CS} (u_i^* u_j^*) d\vec{A} = -\frac{1}{\rho^*} \iiint_{CV} \frac{\partial p^*}{\partial x_i^*} d\vec{\gamma} + \frac{1}{\rho^* Re} \iiint_{CS} (\mu^* \nabla^2 u_i^*) d\vec{A} + \frac{1}{\rho^* We} \iiint_{CV} F_{ST} d\vec{\gamma} - \frac{1}{Fr^2} \iiint_{CV} \tilde{d}\vec{\gamma}
\]

such that the variable with the superscript * is non-dimensional. Term $u_i$ and $u_j$ represent the velocity vector components wherein $j = 1, 2, 3$ and $i = 1, 2, 3$ denote the repeating and non-repeating indices respectively. Variables $p$, $\rho$ and $\mu$ denote the pressure, density and viscosity of the fluid with a computational cell having a control surface ($CS$) area $d\vec{A}$ which encloses a cell volume of $d\vec{\gamma}$ inside a control volume ($CV$). Moreover, $Re = \rho_u h_0 / \mu_t$, $Fr = U/(g h_0)$ and $We = \rho U^2 h_0 / \sigma$ denote the Reynolds number, Froude number and Weber number respectively; such that $U$, $g$ and $\sigma$ are the characteristic velocity (unity), acceleration due to gravity and surface tension coefficient, respectively. On the right hand side of Equation 2 the term $F_{ST}$ represents the surface tension force whereas, the last term on the right denotes the force due
to gravity. The mixed cell properties such as density and viscosity are obtained using the fluid volume fraction ($\phi$) as

$$
\begin{align*}
\rho^* &= \phi \rho_g + (1 - \phi) \rho_l \\
\mu^* &= \phi \mu_g + (1 - \phi) \mu_l
\end{align*}
$$

(3)

where the variables with subscript $g$ and $l$ represent the mixture cell properties of gas and liquid phase respectively. Equation 1 and Equation 2 is scaled with the help of half jet width as presented in [23].

The equations governing the flow field are discretized using the finite volume methodology with the flow properties situated at the cell centroid. Temporal terms are discretized using Runge-Kutta (RK2) method [20] which is explicit and second order accurate. Advection term is discretized with the help of total variation diminishing (TVD) scheme [21] respectively.

Navier-Stokes equations are solved using the semi-explicit formulation. The solution for pressure Poisson equation is obtained with the help of bi-conjugate gradient stabilized (Bi-CGSTAB) method [22] and the residue of $1 \times 10^{-6}$ is maintained for the present computation. Courant number limit is set as 0.1 using the average time stepping of $10^{-4}$. The present computations are carried out in parallel using OpenMP. The details of compiler directives followed by the code is discussed in detail previously by [23].

2.2. Surface Tension Modelling

Surface tension force $F_{ST}$ in Equation 2 is discretized using continuum surface force (CSF) method [24]. In the study, spurious velocities at an interface are minimized with the help of balanced CSF method [25]. An interface curvature is evaluated using the height function (HF) methodology [26].

2.3. Interface Tracking

An interface between two phases is tracked with the help of volume of fluid (VOF) method [27]. In the present study, the advection equation for scalar volume fraction ($\phi$) is solved by second order accurate operator split method, where the fluid is advected consecutively in $x$ and $y$ directions during a single time step. Fluxes at the cell interface are evaluated using the case-wise geometric PLIC [28] [29] method. Therefore, interface reconstruction is performed on the basis of Rudman algorithm [28] that identifies the fluid orientation cases in each computational cell using the interface normals. This methodology is adopted due to its computational efficiency.

3. Simulation Setup and Validations

The domain considered for the present study and the boundary conditions used in it is demonstrated in Fig. 1. Here, an oscillating planar water jet having a width of $2h_0$ enters the quiescent surrounding of air at a velocity of $V_{inlet}$. An inlet section is positioned at centre of the top face and the jet develops into the negative $y$ direction. An inlet velocity vector of the jet consist of the two velocity components, out of which, one provides the jet the required inlet velocity ($V_{inlet}$) whereas, the other provides the periodic perturbations to it. The $u^*$ and $v^*$ velocity components for this inlet vector are given as,

$$
\begin{align*}
u^* &= A_0 \sin(2\pi S \times t^*) \\
v^* &= V_{inlet}
\end{align*}
$$

(4)

where, $S$ represents the Strouhal number that accounts for the perturbation frequency ($f$) and an initial amplitude of oscillation $A_0$ such that, $S = fU/h_0$. The vertical side walls are treated as with the free slip boundary condition whereas; the bottom face of the domain acts as a
typical pressure outlet. The values for physical properties of air and water used for the study with the non-dimensional numbers are provided in the previous study by the authors [23]. The flow conditions used for the present study are provided in Tab. 1.

| Fr  | We  | Re   | $V_{in}$ (m/s) | $w_0$ (m) | $h_0$ (m) | S   | $A_0$ |
|-----|-----|------|----------------|----------|----------|-----|-------|
| 5.08| 54.72| 4426.96 | 2.05           | 0.075    | 0.00395  | 0.495| 0.2531|

The confinement ratio ($CR$) for the present study is defined as the ratio of domain width to the jet width. The jet width in the computational domain is maintained as unity and the domain width is varied as $50h_0$, $70h_0$, $90h_0$ and $130$ to change the confinement ratio.

![Figure 1. Details of the computational domain and flow conditions.](image)

3.1. Grid Refinement Study
The grid refinement studies based on Richardson’s extrapolation [30] were previously carried out in [23]. It is demonstrated that when the spatial refinement in all three directions is performed using three grid levels, the Grid Convergence Index (GCI) between medium and the finest grid was within the convergence limits. Therefore, for the present study, grid resolution of $250 \times 700$, $350 \times 700$, $450 \times 700$ and $550 \times 700$ are selected for the confinement ratio of $CR_1 = 50$, $CR_2 = 70$, $CR_3 = 90$ and $CR_4 = 110$ respectively.

3.2. Validations of the Numerical Model
Validations for the present numerical model with the experimental results of Asare et al. [8] were previously discussed by Arote et al. in [23]. These validations are carried out for the growth envelope ($A_e/A_0$) that evolve due to the instability associated with the oscillations of these jets and were found to be in agreement with the experimental results.

4. Results and Discussion
The present section provides the discussion on the effect of confinement ratio on the growth of oscillation amplitude. The growth envelope ($A_e/A_0$) is tracked as demonstrated in Fig. 1. The
bulges at the oscillation peaks are neglected to avoid any ambiguity in the measurement of an amplitude.

Fig. 2 demonstrates the development of oscillating jet for different \( CR \) values. It is observed that at the near field development of the jet is independent of the confinement effects. However, at the downstream, as the aerodynamic interaction between the gas-liquid phases increase the confinement is having significant impact on the development of jet. Here, the oscillation

![Figure 2. Development of oscillating jets for a) \( CR_1 = 50 \), b) \( CR_2 = 70 \), c) \( CR_3 = 90 \) and d) \( CR_4 = 110 \).](image-url)
amplitude tends to increase at the downstream due to the low pressure region created at every oscillation peak [23]. Moreover, it is observed from Fig. 2 that as the confinement ratio increases the gas interactions become dominant and oscillations grow non-linearly which results in primary breakup of the jet.

The comparison of growth rates for oscillation amplitude for various confinement ratios is depicted in Fig. 3. It is observed that the instability associated with the amplitude of oscillation increases with increase in the confinement ratio. A reason behind the destabilization of oscillating jets can be understood through the analysis of enstrophy ($\omega^2$) values at the typical oscillation peak where $\omega$ is the vorticity in flow. Fig. 4 demonstrates the comparison of enstrophy values at a oscillation peak between different $CR$ values. It is observed that the intensity of vortex rotation at the oscillation peaks increases as the confinement ratio increase. As this rotation intensity increases the pressure at the eye of the vortex reduces and leads to the jet being drawn towards it which promotes instability [23].

![Figure 3](image3.png)  
Figure 3. Comparison of the instability growth rates in terms of the growth envelope $A_e/A_0$ between various confinement ratios.

![Figure 4](image4.png)  
Figure 4. Comparison of enstrophy values along $x^*$ at $y^* = 152$ for various confinement ratios.

Fig. 5 demonstrates the vorticity contours for various $CR$ values. It is observed that the vortices interact with the side-walls and are directed towards the incoming jet oscillations. These vortices affect the development of these oscillations more when the $CR$ values are less. Moreover, the momentum transfer into the surrounding fluid is observed to increase as the $CR$ value increases.

5. Conclusions

The effect of confinement on the growth envelop of the oscillation amplitude is studied. The development oscillating jets is studied. The reason behind the associated instability in the oscillating jet is also identified in the present study. It is demonstrated that the jet development in the near filed is not affected by the confinement. Moreover, at the downstream, the primary atomization is accelerated if the $CR$ values increase. Furthermore, it demonstrated that an increase in the confinement ratio destabilizes the oscillating jet; which is caused by the increased intensity of vortex rotation at the jet peaks. Stability is also affected by the vortex structures disturbing the incoming flow of jet.
Figure 5. Vorticity plot for various confinement ratios at $t^* = 70$.

Acknowledgement
Authors would like to acknowledge the financial assistance provided by the Science and Engineering Research Board, India (Grant No. MTR/2019/000941).

References
[1] Mon G. Pulsatile Impinging Cooling and Systems Using Fluidic System for Electronic Modules Oscillators. US Patent; 5,190,099, 1993.
[2] Sutton TG. Fluidic Feedback-Controlled Liquid Cooling Module. US Patent. 5,815,370, 1998.
[3] Raghu S. Fluidic oscillators for flow control. Exp Fluids. 2013;54(2).
[4] Tesar V. Oscillator micromixer. Chem Eng J. 2009;155(3):78999.
[5] Squire HB. Investigation of the instability of a moving liquid film. Br J Appl Phys. 1953;4(6):1679.
[6] Shea WW. Hagerty J. F. 1955 A study of the instability of plane fluid sheets. J. Appl. Mech.; 22:509-14.
[7] Clark CJ, Dombrowski N. Aerodynamic Instability and Disintegration of Inviscid Liquid Sheets. Proc R Soc A Math Phys Eng Sci. 1972;329(1579):46778.
[8] Asare HR, Takahashi RK, Hoffman MA. Liquid Sheet Jet Experiments: Comparison With Linear Theory. J Fluids Eng. 1981;103(4):595.
[9] Li X. On the instability of plane liquid sheets in two gas streams of unequal velocities. Acta Mech. 1994;106(34):13756.
[10] Chueh SG. Spatial instability of a viscous liquid sheet. Int J Numer Methods Fluids. 2006;50(12):146174.
[11] Sirignano WA, Mehring C. Review of theory of distortion and disintegration of liquid streams. Prog Energy Combust Sci. 2000;26(4):60955.
[12] Juniper MP. The effect of confinement on the stability of two-dimensional shear flows. J Fluid Mech. 2006;565:17195.
[13] Juniper MP. The effect of confinement on the stability of non-swirling round jet/wake flows. J Fluid Mech. 2008: 0565:22752.
[14] Rees SJ. Juniper MP. The effect of confinement on the stability of viscous planar jets and wakes. J Fluid Mech. 2010;656:30936.
[15] Healey JJ. Destabilizing effects of confinement on homogeneous mixing layers. J Fluid Mech. 2009;623:241717.
[16] Wang XT, Ning Z, L M. Temporal instability analysis of a confined non-Newtonian liquid jet with heat and mass transfer. Eur J Mech B/Fluids. 2020:84:3506. Available from: https://doi.org/10.1016/j.euromechflu.2020.07.005
[17] Mosavati M, Barron RM, Balachandar R. Characteristics of self-oscillating jets in a confined cavity. Phys Fluids. 2020;32(11). Available from: https://doi.org/10.1063/5.0023833
[18] Hao Y, Seo KH, Hu Y, Mao HQ, Mittal R. Flow physics and mixing quality in a confined impinging jet mixer. AIP Adv [Internet]. 2020;10(4):110. Available from: https://doi.org/10.1063/5.0002125
[19] Tryggvason G, Scardovelli R, Zaleski S. Direct Numerical Simulations of Gas-Liquid Multiphase Flows. Cambridge: Cambridge University Press; 2011.
[20] Sigal Gottlieb, David Ketcheson, Chi-Wang S. Strong Stability Preserving Runge-Kutta and Multistep Time Discretizations. 1st Edition. World Scientific; 2011.

[21] Harten A. High Resolution Schemes for Hyperbolic Conservation Laws. J Comput Phys. 1983;49(3):35793.

[22] 1. Vorst HA van der. Bi-CGSTAB: A Fast and Smoothly Converging Variant of Bi-CG for the Solution of Non-Symmetric Linear Systems. SIAM J Sci Stat Comput. 1992;13(2):63144.

[23] Arote A, Bade M, Banerjee J. Numerical investigations on stability of the spatially oscillating planar two-phase liquid jet in a quiescent atmosphere. Phys Fluids. 2019 Nov 1;31(11):112103.

[24] 1. Brackbill JU, Kothe DB. ScienceDirect - Journal of Computational Physics : A continuum method for modeling surface tension*. J Comput Phys. 1992;335354.

[25] Francois MM, Cummins SJ, Dendy ED, Kothe DB, Sicilian JM, Williams MW. A balanced-force algorithm for continuous and sharp interfacial surface tension models within a volume tracking framework. Vol. 213, Journal of Computational Physics. 2006. p. 14173.

[26] Lopez J, Zanzi C, Gomez P, Zamora R, Faura F, Hernandez J. An improved height function technique for computing interface curvature from volume fractions. Comput Methods Appl Mech Eng [Internet]. 2009;198(3336):255564.

[27] C. W., Hirt; B. D. N. Volume of Fluid (VOF) Method for the Dynamics of Free Boundaries. Int J Comput Phys. 1979;39(1):20125.

[28] Murray Rudaman. Volume-Tracking Methods for Interfacial Flow Calculations. Int J Numer Methods Fluids. 1997;24(7):67191.

[29] Saincher S, Banerjee J. A redistribution-based volume-preserving PLIC-VOF technique. Numer Heat Transf Part B Fundam. 2015;67(4):33862.

[30] Roache PJ. Perspective: A method for uniform reporting of grid refinement studies. Journal of Fluid Engineering. 1994; 116(3):40513.