Photon and Axion Splitting in an Inhomogeneous Magnetic Field

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Abstract

The axion photon system in an external magnetic field, when the direction of propagation of axions and photons is orthogonal to the direction of the external magnetic field, displays a continuous axion-photon duality symmetry in the limit the axion mass is neglected. The conservation law that follow in this effective 2 + 1 dimensional theory from this symmetry is obtained. The magnetic field interaction is seen to be equivalent to first order to the interaction of a complex charged field with an external electric potential, where this ficticious "electric potential" is proportional to the external magnetic field. This allows one to solve for the scattering amplitudes using already known scalar QED results. From the scalar QED analog the axion and the photon are symmetric and antisymmetric combinations of particle and antiparticle. If one considers therefore scattering experiments in which the two spatial dimensions of the effective theory are involved non trivially, one observes that both particle and antiparticle components of photons and axions are preferentially scattered in different directions, thus producing the splitting or decomposition of the photon and axion into their particle and antiparticle components in an inhomogeneous magnetic field. This observable in principle effect is of first order in the axion photon coupling, unlike the "light shining through a wall phenomena ", which is second order.

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I. INTRODUCTION

The possible existence of a light pseudo scalar particle is a very interesting possibility. For example the axion [1], [2], [3] which was introduced in order to solve the strong CP problem has since then also been postulated as a candidate for the dark matter. A great number of ideas and experiments for the search this particle have been proposed [4], [5].

Here we are going to focus on a particular feature of the axion field $\phi$, which is its coupling to the photon through an interaction term of the form $g\phi \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} F_{\alpha \beta}$. In fact a coupling of this sort is natural for any pseudoscalar interacting with electromagnetism, as is the case of the neutral pion coupling to photons (which as a consequence of this interaction decays into two photons).

A way to explore for observable consequences of the coupling of a light scalar to the photon in this way is to subject a beam of photons to a very strong magnetic field.

This affects the optical properties of light which could lead to testable consequences[6], also a magnetic field in the early universe can lead to interesting photon-axion conversion effects [7] and in the laboratory photon-axion conversion effects could be responsible for the "light shining through a wall phenomena", which are is obtained by first producing axions out of photons in a strong magnetic field region, then subjecting the mixed beam of photons and axions to an absorbing wall for photons, but almost totally transparent to axions due to their weak interacting properties which can then go through behind this "wall", applying then another magnetic field one can recover once again some photons from the produced axions [8], [9]. Notice however that the "light shining through a wall phenomena" involves two interactions, once to produce the axions and then to obtain photons once again from the produced axions. Since the axion photon coupling is so small, the amplitude for such effect is highly suppressed.

In this paper we will show that photons and axions split in the presence of an external magnetic field, in a way that we will make more precise. By this we mean that from beam of photons we will get two different kind of scattered components (plus the photons that do not suffer any interactions), each of the scattered beams has also an axion component, but each of the beams is directly observable due to its photon component and an observable process is obtained to first order in the axion photon interaction, unlike the "light shining through a wall phenomena". Although we cannot claim yet that this could provide a more favorable
experimental set up, since such subject involves many practical questions in addition to the existence of the first order process. Beyond the question of observability, the existence of this kind of effects highlights many basic features of the axion photon system.

II. ACTION AND EQUATIONS OF MOTION

The action principle describing the relevant light pseudoscalar coupling to the photon is

$$S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{8} \phi \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \right]$$  \hspace{1cm} (1)

We now specialize to the case where we consider an electromagnetic field with propagation along the y and z directions and where a strong magnetic field pointing in the x-direction is present. This field may have an arbitrary space dependence in y and z, but it is assumed to be time independent. In the case the magnetic field is constant, see for example [10] for general solutions.

For the small perturbations we consider only small quadratic terms in the action for the axion fields and the electromagnetic field, following the method of for example Ref. [10], but now considering a static magnetic field pointing in the x direction having arbitrary y and z dependence and specializing to y and z dependent electromagnetic field perturbations and axion fields. This means that the interaction between the background field, the axion and photon fields reduces to

$$S_I = -\int d^4x \left[ \beta \phi E_x \right]$$  \hspace{1cm} (2)

where $\beta = gB(y, z)$. Choosing the temporal gauge for the photon excitations and considering only the x-polarization for the electromagnetic waves, since only this polarization couples to the axion, we get the following 2+1 effective dimensional action (A being the x-polarization of the photon, so that $E_x = -\partial_t A$)

$$S_2 = \int dydzdt \left[ \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + \beta \phi \partial_t A \right]$$  \hspace{1cm} (3)

Since we consider only $A = A(t, y, z)$, $\phi = \phi(t, y, z)$, we have avoided the integration over $x$, for the same reason, in (3) $\mu$ runs over $t$, $y$ and $z$ only. This leads to the equations

$$\partial_\mu \partial^\mu \phi + m^2 \phi = \beta \partial_t A$$  \hspace{1cm} (4)
\[ \partial_\mu \partial^\mu A = -\beta \partial_t \phi \]  

(5)

As is well known, in temporal gauge, the action principle cannot reproduce the Gauss constraint (here with a charge density obtained from the axion photon coupling) and has to be imposed as a complementary condition. However this constraint is automatically satisfied here just because of the type of dynamical reduction employed and does not need to be considered anymore.

III. THE CONTINUOUS AXION PHOTON DUALITY SYMMETRY AND THE SCALAR QED ANALOGY

Without assuming any particular \( y \) and \( z \)-dependence for \( \beta \), but still insisting that it will be static, we see that in the case \( m = 0 \), we discover a continuous axion photon duality symmetry (these results were discussed previously in the 1+1 dimensional case, when only \( z \) dependence was considered in [11]), since,

1. The kinetic terms of the photon and axion allow for a rotational \( O(2) \) symmetry in the axion-photon field space.

2. The interaction term, after dropping a total time derivative can also be expressed in an \( O(2) \) symmetric way as follows

\[
S_I = \frac{1}{2} \int dydzdt \beta \left[ \phi \partial_t A - A \partial_t \phi \right]
\]  

(6)

The axion photon symmetry is in the infinitesimal limit

\[ \delta A = \epsilon \phi, \delta \phi = -\epsilon A \]  

(7)

where \( \epsilon \) is a small number. Using Noether's theorem, this leads to the conserved current \( j_\mu \), with components given by

\[ j_0 = A \partial_t \phi - \phi \partial_t A + \frac{\beta}{2} (A^2 + \phi^2) \]  

(8)

and

\[ j_i = A \partial_i \phi - \phi \partial_i A \]  

(9)
Here \( i = y, z \) coordinates. We define now the complex field \( \psi \) as
\[
\psi = \frac{1}{\sqrt{2}} (\phi + iA)
\]
we see that in terms of this complex field, the axion photon density takes the form
\[
j_0 = i(\psi^* \partial_t \psi - \psi \partial_t \psi^*) + \beta \psi^* \psi
\]

We observe that to first order in \( \beta \), (6) represents the interaction of the magnetic field with the "axion photon density" (8), (11) and also this interaction has the same form as that of scalar QED with an external "electric" field to first order. In fact the magnetic field or more precisely \( \beta/2 \) appears to play the role of external electric potential that couples to the axion photon density (8), (11) which appears then to play the role of an electric charge density. From this analogy one can obtain without effort the scattering amplitudes, just using the known results from the scattering of charged scalar particles under the influence of an external static electric potential, see for example [12].

One should notice however that the natural initial states used in a real experiment, like an initial photon and no axion involved, is not going to have a well defined axion photon charge in the second quantized theory (although its average value appears zero), so the S matrix has to be presented in a different basis than that of normal QED. This is similar to the difference between working with linear polarizations as opposed to circular polarizations in ordinary optics, except that here we talk about polarizations in the axion photon space. In fact pure axion and pure photon initial states correspond to symmetric and antisymmetric linear combinations of particle and antiparticle in the analog QED language. The reason these linear combinations are not going to be maintained in the presence on \( B \) in the analog QED language, is that the analog external electric potential breaks the symmetry between particle and antiparticle and therefore will not maintain in time the symmetric or antisymmetric combinations.

From the point of view of the axion-photon conversion experiments, the symmetry (7) and its finite form, which is just a rotation in the axion-photon space, implies a corresponding symmetry of the axion-photon conversion amplitudes, for the limit \( \omega >> m \).

In terms of the complex field, the axion photon current takes the form
\[
\tilde{j}_k = i(\psi^* \partial_k \psi - \psi \partial_k \psi^*)
\]
IV. THE PARTICLE ANTI-PARTICLE REPRESENTATION OF AXIONS AND PHOTONS AND THEIR SPLITTING IN AN EXTERNAL MAGNETIC FIELD

Now let us introduce the charge conjugation \([13]\), that is,

\[
\psi \rightarrow \psi^* \quad (13)
\]

We see then that the free part of the action is indeed invariant under \([13]\). The \(A\) and \(\phi\) fields when acting on the free vacuum give rise to a photon and an axion respectively, but in terms of the particles and antiparticles defined in terms of \(\psi\), we see that a photon is an antisymmetric combination of particle and antiparticle and an axion a symmetric combination, since

\[
\phi = \frac{1}{\sqrt{2}}(\psi^* + \psi), \quad A = \frac{1}{i\sqrt{2}}(\psi - \psi^*) \quad (14)
\]

So that the axion is even under charge conjugation, while the photon is odd. These two eigenstates of charge conjugation will propagate without mixing as long as no external magnetic field is applied. The interaction with the external magnetic field is not invariant under \([13]\), in fact under \([13]\) we can see that

\[
S_I \rightarrow -S_I \quad (15)
\]

Therefore these symmetric and antisymmetric combinations, corresponding to axion and photon are not going to be maintained in the presence on \(B\) in the analog QED language, since the ”analog external electric potential” breaks the symmetry between particle and antiparticle and therefore will not maintain in time the symmetric or antisymmetric combinations. In fact if the analog external electric potential is taken to be a repulsive potential for particles, it will be an attractive potential for antiparticles, so the symmetry breaking is maximal.

Even at the classical level these two components suffer opposite forces, so both a photon or an axion under the influence of an inhomogeneous magnetic field will be decomposed through scattering into their particle and antiparticle components, each of which is scattered in a different direction, since the analog electric force is related to the gradient of the effective electric potential, i.e., the gradient of the magnetic field, times the \(U(1)\) charge which is opposite for particles and antiparticles.
For this effect to have meaning, we have to work at least in a 2+1 formalism, the 1+1 reduction \[11, 13\] which allows motion only in a single spacial direction is unable to produce such separation, since in order to separate particle and antiparticle components we need at least two dimensions to obtain a final state with particles and antiparticles going in slightly different directions.

This is in a way similar to the Stern Gerlach experiment in atomic physics \[15\], where different spin orientations suffer a different force proportional to the gradient of the magnetic field in the direction of the spin. Here instead of spin we have that the photon is a combination of two states with different $U(1)$ charge and each of these components will suffer opposite force under the influence of the external inhomogeneous magnetic field. Notice also that since particle and antiparticles are distinguishable, there are no interference effect between the two processes.

Therefore an original beam of photons will be decomposed through scattering into two different elementary particle and antiparticle components plus the photons that have not undergone scattering. These two beams are observable, since they have both photon components, so the observable consequence of the axion photon coupling will be the splitting by a magnetic field of a photon beam. This effect being however an effect of first order in the axion photon coupling, unlike the "light shining through a wall phenomena".

V. CONCLUSIONS

The limit of zero axion mass when considering the scattering of axions and photons with the geometry relevant to the axion-photon mixing experiments reveals a continuous axion photon duality symmetry. This symmetry leads to a conserved current and then one observes that the interaction of the external magnetic field with the axion and photon is, to first order in the magnetic field, of the form of the first order in coupling constant interaction of charged scalars with an external electric scalar potential. Here the role of this fictitious external electric scalar potential is played (up to a constant) by the external magnetic field.

Pure axion and pure photon initial states correspond to symmetric and antisymmetric linear combinations of particle and antiparticle in the analog QED language. Notice in this respect that charge conjugation of \[10\] corresponds to sign reversal of the photon field. The reason these linear combinations are not going to be maintained in the presence on
a nontrivial $B$ in the analog QED language, is that the analog external electric potential breaks the symmetry between particle and antiparticle and therefore will not maintain in time the symmetric or antisymmetric combinations.

In this paper we present the 2+1 dimensional generalization of our previous work that allowed only 1+1 reductions [11] [13]. One possible application of this that has not been discussed here could be the generalization of the soliton solutions found in [14].

We have focused now on the implications of representing a photon (and also the axion) as a linear combination of particle and antiparticles. Even at the classical level these two components suffer opposite forces, so both a photon or an axion under the influence of an inhomogeneous magnetic field (since the analog electric force is related to the gradient of the effective electric potential, i.e., the gradient of the magnetic field) will be decomposed through scattering into its particle and antiparticle components, each of which is scattered in a different direction. For this effect to have meaning, we have to work at least in a 2+1 formalism, since the 1+1 reduction [11], [13], which allows motion only in a single spatial direction, is unable to describe such separation, since in order to separate particle and antiparticle components we need at least two dimensions (in order to obtain a final state with particles and antiparticles in slightly different directions). Notice also that since particle and antiparticles are distinguishable, there are no interference effect between the two processes.

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under the influence of the external inhomogeneous magnetic field.

Our analysis will apply also to neutral pions and photons, for example a very energetic gamma ray scattering from an inhomogeneous magnetic field could give rise to two scattered beams (each of them containing both pions and photons) if the scattering takes place in the plane orthogonal to the magnetic field. Possible observable effects of photon splitting out of cosmic magnetic fields and not just laboratory ones could also be considered as a new source of multiple images for example.

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[1] R.D. Peccei and H.R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977). 
[2] S. Weinberg, *Phys. Rev. Lett.* **40**, 223 (1978). F. Wilczek, *Phys. Rev. Lett.* **40**, 279 (1978). 
[3] F. Wilczek, *Phys. Rev. Lett.* **40**, 279 (1978). 
[4] For a very early proposal see J.T. Goldman and C.M. Hoffman, *Phys. Rev. Lett.* **40**, 220 (1978). 
[5] For a review see G.G. Raffelt, [hep-ph/0611118](http://arxiv.org/abs/hep-ph/0611118). 
[6] E. Zavattini, et. al. (PVLAS collaboration), arXiv:hep/0706.3419. 
[7] T. Yanagida and M. Yoshimura, *Phys. Lett.* **B202**, 301 (1988). 
[8] See for example K. Van Bibber et. al., *Phys. Rev. Lett.* **59**, 759 (1987). 
[9] R. Rabdan, A. Ringwald and K. Sigurson, *Phys. Rev. Lett.* **96**, 110407 (2006) and references here; several techniques are studied in S. L. Adler et. al., [arXiv:0801.4739](http://arxiv.org/abs/0801.4739) [hep-ph]. 
[10] S. Ansoldi, E.I. Guendelman and E. Spallucci, *JHEP* **0309**, 044 (2003). 
[11] E.I. Guendelman, *Mod. Phys. Lett.* **A23**, 191 (2008); [arXiv:0711.3685](http://arxiv.org/abs/0711.3685) [hep-th]. 
[12] J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics, N.Y., McGraw-Hill, 1964. 
[13] E.I. Guendelman, [arXiv:0711.3961](http://arxiv.org/abs/0711.3961) [hep-ph]. 
[14] E.I. Guendelman, [arXiv:0801.0503](http://arxiv.org/abs/0801.0503) [hep-th]. 
[15] W. Gerlach, O. Stern, *Z.Phys.* **8**, 110 (1922); W. Gerlach, O. Stern, *Z.Phys.* **9**, 349 (1922).