Signals of the Abelian $Z'$ boson within the analysis of the LEP2 data

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Abstract

The preliminary LEP data on the $e^+e^- \rightarrow l^+l^-$ scattering are analysed to establish a model-independent search for the signals of virtual states of the Abelian $Z'$ boson. The recently introduced observables give a possibility to pick up uniquely the Abelian $Z'$ signals in these processes. The mean values of the observables are in accordance with the $Z'$ existence. However, the accuracy of the experimental data is deficient to detect the signal at more than the 1σ confidence level. The results of other model-independent fits and further prospects are discussed.

1 Introduction

The recently stopped LEP2 experiments have accumulated a huge amount of data on four-fermion processes at the center-of-mass energies $\sqrt{s} \sim 130 - 207$ GeV [1, 2]. Besides the precision tests of the Standard Model (SM) of elementary particles these data allow the estimation of the energy scale of a new physics beyond the SM.

Various approaches to detect manifestations of physics beyond the SM have been proposed in the literature. They could be subdivided into model-dependent and model-independent methods. The former usually means the comparison of experimental data with the predictions of some specific models which extend the SM at high energies. In this way some popular Grand Unified theories, the supersymmetry models as well as the theories of technicolor or extra dimensions are intensively discussed and the values of couplings, mixing angles, and particle masses are constrained. In particular, model-dependent bounds are widely presented in the LEP reports [1, 2].

In the model-independent approach one fits some effective low-energy parameters such as four-fermion contact couplings. Below the scale of the heavy

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particle decoupling various theories beyond the SM can be described by the same set of effective contact interactions between the SM particles. The only difference is in the values of the corresponding coefficients, which can be fitted by experimental data. This idea is elaborated in the Effective Lagrangian method [3] as well as in the helicity ‘models’ introduced by the LEP Collaborations (LL, RR, . . . ) [1, 2]. An advantage of the approach is the restricted set of parameters which describe the low-energy phenomenology of any model beyond the SM for a specified scattering process. Unfortunately, each effective model-independent parameter conceals a number of different scenarios of new physics. As a consequence, the model-independent approach gives a possibility to detect a signal of new physics, but it cannot distinguish the particle (defined by specific quantum numbers) responsible for the signal.

It seems to us that it is reasonable to develop the model-independent searches for the manifestations of heavy particles with specific quantum numbers. Such an approach is intended to detect the signal of some heavy particle (for example, the heavy neutral vector boson) by means of the data of the LEP2 or other experiments without specifying a model beyond the SM. In this way, it is also possible to derive model-independent constraints on the mass and the couplings of the considered heavy particle. To develop this approach one has to take into account some model-independent relations between the couplings of the heavy particle as well as some features of the kinematics of the considered scattering processes.

In the present paper we focus on the problem of model-independent searches for signals of the heavy Abelian $Z'$ boson [4] by means of the analysis of the LEP2 data on the lepton processes $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$. This particle is a necessary element of different models extending the SM. The low limits on its mass estimated for a variety of popular models ($\chi$, $\psi$, $\eta$, L–R models [5] and the Sequential Standard Model (SSM) [6]) are found to be in the wide energy interval 600–2000 GeV [1, 2]. In what follows we assume that the $Z'$ boson is heavy enough to be decoupled at the LEP2 energies.

In the previous papers [7] we argued that the low-energy $Z'$ couplings to the SM particles satisfy some model-independent relations, which are the consequences of renormalizability of a theory beyond the SM remaining in other respects unspecified. These relations, called the renormalization group (RG) relations, predict two possible types of the low-energy $Z'$ interactions with the SM fields, namely, the chiral and the Abelian $Z'$ bosons. Each $Z'$ type is described by a few couplings to the SM fields. Therefore, it is possible to introduce observables which uniquely pick up the $Z'$ virtual state [7].

The $Z'$ signal in the four-lepton scattering process $e^+e^- \rightarrow l^+l^-$ can be detected with a sign-definite observable, which is ruled by the center-of-mass energy and an additional kinematic parameter. The incorporation of the next-to-leading terms in $m_{Z'}^{-2}$ allows to consider the $Z'$ effects beyond the approach of four-fermion contact interactions [8]. As a consequence, the four-fermion contact couplings and the $Z'$ mass can be fitted separately.

The introduced observable can be computed directly from the differential cross-sections. However, the statistical errors of the available differential cross-
sections of LEP2 experiments are of one order larger than the accuracy of the corresponding total cross-sections and the forward-backward asymmetries. Fortunately, the differential cross-sections of the $e^+e^- \rightarrow l^+l^-$ processes at the LEP2 energies (including the one-loop radiative corrections) can be successfully approximated by a two-parametric polynomial of the cosine of the scattering angle. It gives a possibility to recalculate the observable from the total cross-sections and the forward-backward asymmetries reducing noticeably the experimental uncertainty.

Thus, the outlined analysis has to answer whether or not one could detect the model-independent signal of the Abelian $Z'$ boson by treating the LEP2 data. As it will be shown, the LEP2 data on the scattering into $\mu$ and $\tau$ pairs lead to the Abelian $Z'$ signal at about 1\,$\sigma$ confidence level.

The paper is organized as follows. In sect. 2 the necessary information on the model-independent description of the $Z'$ interactions at low energies and the RG relations are given. In sect. 3 the observables to pick up uniquely the $Z'$ boson are introduced. In the last section the results on the LEP data fit and the conclusions as well as further prospects are discussed.

2 $Z'$ couplings to the SM particles

The Abelian $Z'$ boson can be introduced in a model-independent (phenomenological) way by defining its effective low-energy couplings to the SM particles. Such a parameterization is well known in the literature [4]. Since we are going to account of the $Z'$ effects in the $e^+e^- \rightarrow l^+l^-$ process at LEP2 energies $\sqrt{s} \ll m_{Z'}$, we parameterize the $Z'$ interactions induced at the tree-level, only. As the decoupling theorem [9] guarantees, they are of renormalizable type, since the non-renormalizable interactions are generated at higher energies due to radiation corrections and suppressed by the inverse heavy mass ($m_{Z'}$ in our case). The SM gauge group $SU(2)_L \times U(1)_Y$ is considered as a subgroup of the underlying theory group. So, the mixing interactions of the types $Z'W^+W^-$, $Z'ZZ$, ... are absent at the tree level. Under these assumptions the $Z'$ couplings to the fermion and scalar fields are described by the Lagrangian:

$$\mathcal{L} = \left| \left( D^{\text{ew},\phi}_\mu - \frac{ig}{2} \tilde{Y}(\phi) \tilde{B}_\mu \right) \phi \right|^2 + i \sum_{f=f_L,f_R} \tilde{f}_\gamma^\mu \left( D^{\text{ew},f}_\mu - \frac{ig}{2} \tilde{Y}(f) \tilde{B}_\mu \right) f, \tag{1}$$

where $\phi$ is the SM scalar doublet, $\tilde{B}_\mu$ denotes the massive $Z'$ field before the spontaneous breaking of the electroweak symmetry, and the summation over the all SM left-handed fermion doublets, $f_L = \{(f_u)_L,(f_d)_L\}$, and the right-handed singlets, $f_R = (f_u)_R,(f_d)_R$, is understood. The notation $\tilde{g}$ stands for the charge corresponding to the $Z'$ gauge group, $D^{\text{ew},\phi}_\mu$ and $D^{\text{ew},f}_\mu$ are the electroweak covariant derivatives. Diagonal $2 \times 2$ matrices $\tilde{Y}(\phi) = \text{diag}(\tilde{Y}_{\phi,1},\tilde{Y}_{\phi,2}), \tilde{Y}(f_L) =$
diag(\tilde{Y}_{L,f}, \tilde{Y}_{L,f}) and numbers \tilde{Y}(f_R) = \tilde{Y}_{R,f} mean the unknown \(Z'\) generators characterizing the model beyond the SM.

In particular, the Lagrangian (1) generally leads to the \(Z-Z'\) mixing of order \(m_Z^2/m_{Z'}^2\), which is proportional to \(\tilde{Y}_{\phi,2}\) and originated from the diagonalization of the neutral vector boson states. The mixing contributes to the scattering amplitudes and cannot be neglected at the LEP2 energies [10].

The \(Z'\) couplings to a fermion \(f\) are parameterized by two numbers \(\tilde{Y}_{L,f}\) and \(\tilde{Y}_{R,f}\). Alternatively, the couplings to the axial-vector and vector fermion currents, \(a_{Z'} \equiv (\tilde{Y}_{R,l} - \tilde{Y}_{L,l})/2\) and \(v_{Z'} \equiv (\tilde{Y}_{L,l} + \tilde{Y}_{R,l})/2\), can be used. Their values are determined by the unknown model beyond the SM. Assuming an arbitrary underlying theory one usually supposes that the parameters \(a_f\) and \(v_f\) are independent numbers. However, if a theory beyond the SM is renormalizable these parameters satisfy some relations. For \(Z'\) boson this is reflected in the correlations between \(a_f\) and \(v_f\) [11]. These correlations are model-independent in a sense that they do not depend on the underlying model. The detailed discussion of this point and the derivation of the RG relations are given in Ref. [7]. Therein it is shown that two types of the \(Z'\) boson interactions are admitted – the chiral and the Abelian ones. In the present paper we are interested in the Abelian \(Z'\) couplings which are described by the relations:

\[v_f - a_f = v_{f^*} - a_{f^*}, \quad a_f = T_{3,f}Y_{\phi}, \quad \tilde{Y}_{\phi,1} = \tilde{Y}_{\phi,2} \equiv \tilde{Y}_{\phi} \equiv \tilde{Y}_{\phi}, \quad (2)\]

where \(T_{3,f}^*\) is the third component of the fermion weak isospin, and \(f^*\) means the isopartner of \(f\) (namely, \(l^* = \nu_l, \nu^* = l, \ldots\)).

The relations (2) ensure, in particular, the invariance of the Yukawa terms with respect to the effective low-energy \(U(1)\) subgroup corresponding to the Abelian \(Z'\) boson. As it follows from the relations, the couplings of the Abelian \(Z'\) to the axial-vector fermion currents have the universal absolute value proportional to the \(Z'\) coupling to the scalar doublet. So, in what follows we will use the short notation \(a = a_l = -Y_{\phi}/2\). Note also that the \(Z-Z'\) mixing is expressed in terms of the axial-vector coupling \(a\).

An important benefit of the relations (2) is the possibility to reduce the number of independent parameters of new physics. For example, they can be used to relate the coefficients of the effective Lagrangians [11]. Due to a fewer number of independent \(Z'\) couplings the amplitudes and cross-sections of different scattering processes are also related. As a result, one is able to pick up the characteristic signal of the Abelian \(Z'\)-boson in these processes and to fit successfully the corresponding \(Z'\) couplings. In the present paper we take into account the RG relations (2) in order to introduce the observables convenient for detecting uniquely the Abelian \(Z'\) signals in LEP experiments and to obtain the corresponding experimental constraints on the signal.
3 Observables

3.1 The differential cross section

Let us consider the processes $e^+e^- \to l^+l^-$ ($l = \mu, \tau$) with the non-polarized initial- and final-state fermions. In order to introduce the observable which selects the signal of the Abelian $Z'$ boson we need to compute the differential cross-sections of the processes up to the one-loop level.

The lower-order diagrams describe the neutral vector boson exchange in the $s$-channel ($e^+e^- \to V^* \to l^+l^-$, $V = A, Z, Z'$). As for the one-loop corrections, two classes of diagrams are taken into account. The first one includes the pure SM graphs (the mass operators, the vertex corrections, and the boxes). The second set of the one-loop diagrams improves the Born-level $Z'$-exchange amplitude by “dressing” the $Z'$ propagator and the $Z'$-fermion vertices. We assume that $Z'$ states are not excited inside loops. Such an approximation means that the $Z'$-boson is completely decoupled. Then, the differential cross-section consists of the squared tree-level amplitude and the term from the interference of the tree-level and the one-loop amplitudes. To obtain an infrared-finite result, we also take into account the processes with the soft-photon emission in the initial and final states.

Various computational software for calculation of amplitudes and cross-sections has been developed. For example, the SM cross-sections in the LEP fits are usually generated with ZFITTER. However, ZFITTER requires severe modifications to incorporate the effects of heavy particles beyond the SM. Therefore, we perform the necessary calculations with other software. The Feynman diagrams and the amplitudes are generated with FEYNARTS. The algebraic reduction of the one-loop tensor integrals to the scalar integrals as well as the cross-section construction are carried out with FORMCALC. The scalar one-loop integrals are evaluated with LOOPTOOLS library within the $\overline{\text{MS}}$ renormalization scheme. The unknown Higgs boson mass is set to 125GeV in accordance with the present day bounds.

In the lower order in $m_{Z'}^{-2}$ the $Z'$ contributions to the differential cross-section of the process $e^+e^- \to l^+l^-$ are expressed in terms of four-fermion contact couplings, only. If one takes into consideration the higher-order corrections in $m_{Z'}^{-2}$, it becomes possible to estimate separately the $Z'$-induced contact couplings and the $Z'$ mass [8]. In the present analysis we keep the terms of order $O(m_{Z'})^{-4}$ to fit both of these parameters.

Expanding the differential cross-section in the inverse $Z'$ mass and neglecting the terms of order $O(m_{Z'}^{-6})$, we have

$$\frac{d\sigma_1(s)}{dz} = \frac{d\sigma_{1}^{\text{SM}}(s)}{dz} + \sum_{i=1}^{7} \sum_{j=1}^{7} [A_{ij}(s, z) + B_{ij}(s, z)\zeta] a_i a_j$$

$$+ \sum_{i=1}^{7} \sum_{j=1}^{7} \sum_{k=1}^{7} C_{ijkn}(s, z)a_i a_j a_k a_n,$$  \hspace{1cm} (3)
where the dimensionless quantities
\[ \zeta = \frac{m_2^2}{m_{Z'}^2}, \quad \epsilon = \frac{\tilde{g}^2 m_2^2 a^2}{4\pi m_{Z'}^2}, \]
\[ (a_1, a_2, a_3, a_4, a_5, a_6, a_7) = \sqrt{\frac{\tilde{g}^2 m_2^2}{4\pi m_{Z'}^2}}(a, v_e, v_\mu, v_\tau, v_d, v_s, v_b) \] (4)
are introduced. In what follows the index \( l = \mu, \tau \) denotes the final-state lepton.

The coefficients \( A, B, C \) are determined by the SM couplings and masses. Each factor may include the tree-level contribution, the one-loop correction and the term describing the soft-photon emission. The factors \( A \) describe the leading-order contribution, whereas others correspond to the higher order corrections in \( m_{Z'}^{-2} \).

3.2 The observable

To take into consideration the correlations (2) we introduce the observable \( \sigma_l(z) \) defined as the difference of cross sections integrated in some ranges of the scattering angle \( \theta \) [12]:
\[ \sigma_l(z) \equiv \int_z^1 \frac{d\sigma_l}{d\cos\theta} d\cos\theta - \int_{-1}^z \frac{d\sigma_l}{d\cos\theta} d\cos\theta, \] (5)
where \( z \) stands for the cosine of the boundary angle. The idea of introducing the \( z \)-dependent observable (5) is to choose the value of the kinematic parameter \( z \) in such a way that to pick up the characteristic features of the Abelian \( Z' \) signals.

The deviation of the observable from its SM value can be derived by the angular integration of the differential cross-section and has the form:
\[ \Delta \sigma_l(z) = \sigma_l(z) - \sigma_l^{SM}(z) = \sum_{i=1}^{7} \sum_{j=1}^{i} \left[ \tilde{A}_{ij}^l(s, z) + \tilde{B}_{ij}^l(s, z) \zeta \right] a_i a_j \]
\[ + \sum_{i=1}^{7} \sum_{j=1}^{i} \sum_{k=1}^{j} \sum_{n=1}^{k} \tilde{C}_{ijkn}^l(s, z) a_i a_j a_k a_n. \] (6)

There is an interval of values of the boundary angle, at which the factors \( \tilde{A}_{111}, \tilde{B}_{111}, \) and \( \tilde{C}_{111111} \) at the sign-definite parameters \( \epsilon, \epsilon \zeta, \) and \( \epsilon^2 \) contribute more than 95% of the observable value. It gives a possibility to construct the sign-definite observable \( \Delta \sigma_l(z^*) < 0 \) by specifying the proper value of \( z^* \).

In general, one could choose the boundary angle \( z^* \) in different schemes. In the previous papers [12, 10] we considered just a few number of tree-level four-fermion contact couplings and specified \( z^* \) in order to cancel the factor at the vector-vector coupling. However, if one-loop corrections are taken into account there are a large number of additional contact couplings. So, we have to define some quantitative criterion \( F(z) \) to estimate the contributions from
sign-definite factors at a given value of the boundary angle \( z \). Maximizing the criterion, one could derive the value \( z^* \), which corresponds to the sign-definite observable \( \Delta \sigma_l(z^*) \). Since the observable is linear in the coefficients \( A, B, \) and \( C \), we introduce the following criterion,

\[
F = \frac{|\tilde{A}_{11}| + \omega_B |\tilde{B}_{11}| + \omega_C |\tilde{C}_{1111}|}{\sum_{\text{all } \tilde{A}} |\tilde{A}_{ij}| + \omega_B \sum_{\text{all } \tilde{B}} |\tilde{B}_{ij}| + \omega_C \sum_{\text{all } \tilde{C}} |\tilde{C}_{ijkn}|}, \tag{7}
\]

where the positive ‘weights’ \( \omega_B \sim \zeta \) and \( \omega_C \sim \epsilon \) take into account the order of each term in the inverse \( Z^* \) mass.

The numeric values of the ‘weights’ \( \omega_B \) and \( \omega_C \) can be taken from the present day bounds on the contact couplings [1, 2] or [13]. As the computation shows, the value of \( z^* \) with the accuracy \( 10^{-3} \) depends on the order of the ‘weight’ magnitudes, only. So, in what follows we take \( \omega_B \sim 0.004 \) and \( \omega_C \sim 0.00004 \).

The function \( z^*(s) \) is the decreasing function of the center-of-mass energy. It is tabulated for the LEP2 energies in Tables 1-2. The corresponding values of the maximized function \( F \) are within the interval \( 0.95 < F < 0.96 \).

Since \( A_{11}^* (s, z^*) < 0 \), \( B_{11}^* (s, z^*) < 0 \) and \( C_{1111}^* (s, z^*) < 0 \), the observable

\[
\Delta \sigma_l(z^*) = \left[ \tilde{A}_{11}^* (s, z^*) + \zeta \tilde{B}_{11}^* (s, z^*) \right] \epsilon + \tilde{C}_{1111}^* (s, z^*) \epsilon^2 \tag{8}
\]

is negative with the accuracy 4-5%. Since this property follows from the RG relations [2] for the Abelian \( Z^* \) boson, the observable \( \Delta \sigma_l(z^*) \) selects the model-independent signal of this particle in the processes \( e^+ e^- \rightarrow l^+ l^- \). It allows to use the data on scattering into \( \mu \mu \) and \( \tau \tau \) pairs in order to estimate the Abelian \( Z^* \) coupling to the axial-vector lepton currents.

Although the observable can be computed from the differential cross-sections directly, it is also possible to recalculate it from the total cross-sections and the forward-backward asymmetries. The recalculation procedure has the proper theoretical accuracy. Nevertheless, it allows to reduce the experimental errors on the observable, since the published data on the total cross-sections and the forward-backward asymmetries are still more precise than the data on the differential cross-sections.

The recalculation is based on the fact that the differential cross-section can be approximated with a good accuracy by the two-parametric polynomial in the cosine of the scattering angle \( z \):

\[
\frac{d\sigma_l(s)}{dz} = \frac{d\sigma_l^{\text{SM}}(s)}{dz} + (1 + z^2)\beta_l + z\eta_l + \delta_l(z), \tag{9}
\]

where \( \delta_l(z) \) measures the difference between the exact and the approximated cross-sections. The approximated cross-section reproduces the exact one in the limit of the massless initial- and final-state leptons and if one neglects the contributions of the box diagrams.

Performing the angular integration, it is easy to obtain the expression for the observable:

\[
\Delta \sigma_l(z^*) = \sigma_l(z^*) - \sigma_l^{\text{SM}}(z^*) = (1 - z^*^2)\eta_l - \frac{2\beta_l}{g} z^*^3(3 + z^*^2) + \tilde{\delta}_l(z^*), \tag{10}
\]
### Table 1: The boundary angle and the observable for the scattering into $\mu$ pairs at the one-loop level.

| $\sqrt{s}$, GeV | $z^*$ | $F_{\text{max}}$ | $\Delta \sigma_\mu(z^*)$          |
|-----------------|------|-----------------|----------------------------------|
| 130             | 0.450| 0.89            | $-729\epsilon - 1792\epsilon \zeta - 19636\epsilon^2$ |
| 136             | 0.439| 0.91            | $-709\epsilon - 1859\epsilon \zeta - 16880\epsilon^2$ |
| 161             | 0.400| 0.94            | $-643\epsilon - 2183\epsilon \zeta - 6890\epsilon^2$ |
| 172             | 0.390| 0.95            | $-619\epsilon - 4099\epsilon \zeta - 4099\epsilon^2$ |
| 183             | 0.383| 0.95            | $-599\epsilon - 2545\epsilon \zeta - 1334\epsilon^2$ |
| 189             | 0.380| 0.96            | $-586\epsilon - 2635\epsilon \zeta - 495\epsilon^2$  |
| 192             | 0.380| 0.96            | $-579\epsilon - 2681\epsilon \zeta - 63\epsilon^2$   |
| 196             | 0.380| 0.96            | $-571\epsilon - 2745\epsilon \zeta - 528\epsilon^2$  |
| 200             | 0.378| 0.95            | $-564\epsilon - 2811\epsilon \zeta - 1137\epsilon^2$ |
| 202             | 0.376| 0.96            | $-560\epsilon - 2845\epsilon \zeta - 1448\epsilon^2$ |
| 205             | 0.374| 0.96            | $-555\epsilon - 2897\epsilon \zeta - 1923\epsilon^2$ |
| 207             | 0.372| 0.96            | $-552\epsilon - 2932\epsilon \zeta - 2245\epsilon^2$ |

### Table 2: The boundary angle and the observable for the scattering into $\tau$ pairs at the one-loop level.

| $\sqrt{s}$, GeV | $z^*$ | $\Delta \sigma_\tau(z^*)$          |
|-----------------|------|----------------------------------|
| 130             | 0.460| $-687\epsilon - 1664\epsilon \zeta - 25782\epsilon^2$ |
| 136             | 0.442| $-688\epsilon - 1779\epsilon \zeta - 20784\epsilon^2$ |
| 161             | 0.400| $-625\epsilon - 2097\epsilon \zeta - 10993\epsilon^2$ |
| 172             | 0.391| $-601\epsilon - 2263\epsilon \zeta - 8382\epsilon^2$ |
| 183             | 0.385| $-571\epsilon - 2402\epsilon \zeta - 7580\epsilon^2$ |
| 189             | 0.380| $-568\epsilon - 2533\epsilon \zeta - 5135\epsilon^2$ |
| 192             | 0.380| $-562\epsilon - 2578\epsilon \zeta - 4769\epsilon^2$ |
| 196             | 0.379| $-554\epsilon - 2640\epsilon \zeta - 4272\epsilon^2$ |
| 200             | 0.378| $-547\epsilon - 2704\epsilon \zeta - 3761\epsilon^2$ |
| 202             | 0.377| $-543\epsilon - 2736\epsilon \zeta - 3501\epsilon^2$ |
| 205             | 0.374| $-548\epsilon - 2834\epsilon \zeta - 1292\epsilon^2$ |
| 207             | 0.372| $-544\epsilon - 2868\epsilon \zeta - 1010\epsilon^2$ |
and for the total and the forward-backward cross-sections:

\[
\Delta \sigma^T_l = \sigma^T_l - \sigma^{T,SM}_l = \frac{8\beta_l}{9} + \tilde{\delta}_l(-1), \\
\Delta \sigma^{FB}_l = \sigma^{FB}_l - \sigma^{FB,SM}_l = \eta_l + \tilde{\delta}_l(0). \tag{11}
\]

Then, the factors $\beta_l$ and $\eta_l$ can be eliminated from the observable:

\[
\Delta \sigma_l(z^*) = (1 - z^{*2})\Delta \sigma^{FB}_l - \frac{3}{12}z^{*3}(3 + z^{*2})\Delta \sigma^T_l + \xi_l. \tag{12}
\]

The quantity $\xi_l$,

\[
\xi_l = \tilde{\delta}_l(z^*) - (1 - z^{*2})\tilde{\delta}_l(0) + \frac{3}{12}z^{*3}(3 + z^{*2})\tilde{\delta}_l(-1), \tag{13}
\]

measures the theoretical accuracy of the approximation.

The forward-backward cross-section is related to the total one and the forward-backward asymmetry by means of the following expression

\[
\Delta \sigma^{FB}_l = \Delta \sigma^T_l A^{FB}_l + \sigma^{T,SM}_l \Delta A^{FB}_l. \tag{14}
\]

As the computation shows, $\tilde{\delta}_l(z^*) \simeq 0.01\Delta \sigma_l(z^*)$, $\tilde{\delta}_l(0) \simeq 0.007\Delta \sigma^{FB}_l$, and $\tilde{\delta}_l(-1) \simeq -0.07\Delta \sigma^T_l$ at the LEP2 energies. Taking into account the experimental values of the total cross-sections and the forward-backward asymmetries at the LEP2 energies ($\Delta \sigma^T_l \simeq 0.1pb$, $\sigma^{T,SM}_l \simeq 2.7pb$, $\Delta A^{FB}_l \simeq 0.04$, $A^{FB}_l \simeq 0.5$), one can estimate the theoretical error as $\xi_l \simeq 0.003pb$. At the same time, the corresponding statistical uncertainties on the observable are larger than 0.06pb. Thus, the proposed approximation is quite good and can be successfully used to obtain more accurate experimental values of the observable.

### 4 Data fit and Conclusions

To search for the model-independent signals of the Abelian $Z'$-boson we will analyze the introduced observable $\Delta \sigma_l(z^*)$ on the base of the LEP2 data set. In the lower order in $m_{Z'}^2$ the observable depends on one flavor-independent parameter $\epsilon$,

\[
\Delta \sigma_l^{th}(z^*) = \tilde{A}_{111}(s, z^*)\epsilon + \tilde{C}_{11111}(s, z^*)\epsilon^2, \tag{15}
\]

which can be fitted from the experimental values of $\Delta \sigma_\mu(z^*)$ and $\Delta \sigma_\tau(z^*)$. As we noted above, the sign of the fitted parameter ($\epsilon > 0$) is a characteristic feature of the Abelian $Z'$ signal.

In what follows we will apply the usual fit method based on the likelihood function. The central value of $\epsilon$ is obtained by the minimization of the $\chi^2$-function:

\[
\chi^2(\epsilon) = \sum_n \frac{[\Delta \sigma_{\mu,n}^{ex}(z^*) - \Delta \sigma_{\mu,n}^{th}(z^*)]^2}{\delta \sigma_{\mu,n}^{ex}(z^*)^2}, \tag{16}
\]
where the sum runs over the experimental points entering a data set chosen. The 1σ confidence-level interval \((b_1, b_2)\) for the fitted parameter is derived by means of the likelihood function \(L(\epsilon) \propto \exp[-\chi^2(\epsilon)/2]\). It is determined by the equations:

\[
\int_{b_1}^{b_2} L(\epsilon') d\epsilon' = 0.68, \quad L(b_1) = L(b_2).
\]

(17)

To compare our results with those of Refs. [1, 2] we introduce the contact interaction scale

\[\Lambda^2 = 4m_Z^2 \epsilon^{-1}.\]

(18)

This normalization of contact couplings is admitted in Refs. [1, 2]. We use again the likelihood method to determine a one-sided lower limit on the scale \(\Lambda\) at the 95% confidence level. It is derived by the integration of the likelihood function over the physically allowed region \(\epsilon > 0\). The exact definition is

\[
\Lambda = 2m_Z(\epsilon^*)^{-1/2}, \quad \int_{\epsilon^*}^{\infty} L(\epsilon') d\epsilon' = 0.95 \int_0^{\infty} L(\epsilon') d\epsilon'.
\]

(19)

We also introduce the probability of the Abelian \(Z'\) signal as the integral of the likelihood function over the positive values of \(\epsilon\):

\[P = \int_0^{\infty} L(\epsilon') d\epsilon'.\]

(20)

Actually, the fitted value of the contact coupling \(\epsilon\) originates mainly from the leading-order term in the inverse \(Z'\) mass in Eq. (8). The analysis of the higher-order terms allows to estimate the constraints on the \(Z'\) mass alone. Substituting \(\epsilon\) in the observable (8) by its fitted central value, \(\bar{\epsilon}\), one obtains the expression

\[
\Delta\sigma_l(z^*) = \left[ A_{11}^T(s, z^*) + \zeta B_{11}^l(s, z^*) \right] \bar{\epsilon} + \tilde{C}_{1111}^l(s, z^*) \bar{\epsilon}^2,
\]

(21)

which depends on the parameter \(\zeta = m_Z^2/m_{Z'}^2\). Then, the central value on this parameter and the corresponding 1σ confidence level interval are derived in the same way as those for \(\epsilon\).

To fit the parameters \(\epsilon\) and \(\zeta\) we start with the LEP2 data on the total cross-sections and the forward-backward asymmetries [1, 2]. Those data are converted into the experimental values of the observable \(\Delta\sigma_l(z^*)\) with the corresponding errors \(\delta\sigma_l(z^*)\) by means of the following relations:

\[
\Delta\sigma_l(z^*) = \left[ A_{11}^{\text{FB}} (1 - z^{*2}) - \frac{z^*}{4} (3 + z^{*2}) \right] \Delta\sigma_l^T + (1 - z^{*2}) \sigma_l^{T,\text{SM}} \Delta A_l^{\text{FB}},
\]

\[
\delta\sigma_l(z^*)^2 = \left[ A_{11}^{\text{FB}} (1 - z^{*2}) - \frac{z^*}{4} (3 + z^{*2}) \right]^2 (\Delta\sigma_l^T)^2 + \left[ (1 - z^{*2}) \sigma_l^{T,\text{SM}} \right]^2 (\Delta A_l^{\text{FB}})^2.
\]

(22)
Table 3: The contact coupling $\epsilon$ with the 68% confidence-level uncertainty, the 95% confidence-level lower limit on the scale $\Lambda$, the probability of the $Z'$ signal, $P$, and the value of $\zeta = m_{Z'}^2/m_{Z}^2$ as a result of the fit of the observable recalculated from the total cross-sections and the forward-backward asymmetries.

| Data set        | $\epsilon$               | $\Lambda$, TeV | $P$         | $\zeta$          |
|-----------------|--------------------------|-----------------|-------------|------------------|
| Winter 2002     |                          |                 |             |                  |
| $\mu\mu$       | $0.0000482^{+0.0000436}_{-0.0000493}$ | 15.7            | 0.83        | $0.007 \pm 0.215$ |
| $\tau\tau$     | $0.0000016^{+0.0000056}_{-0.00000396}$ | 16.0            | 0.51        | $-0.052 \pm 8.463$|
| $\mu\mu$ and $\tau\tau$ | $0.0000313^{+0.0000395}_{-0.0000395}$ | 18.1            | 0.78        | $0.006 \pm 0.264$ |
| Summer 2002     |                          |                 |             |                  |
| $\mu\mu$       | $0.0000366^{+0.0000489}_{-0.0000436}$ | 16.4            | 0.77        | $0.009 \pm 0.278$ |
| $\tau\tau$     | $0.0000266^{+0.0000643}_{-0.0000039}$ | 17.4            | 0.34        | $-0.001 \pm 0.501$|
| $\mu\mu$ and $\tau\tau$ | $0.0000133^{+0.0000389}_{-0.0000387}$ | 19.7            | 0.63        | $0.017 \pm 0.609$ |

We perform the fits assuming several data sets, including the $\mu\mu$, $\tau\tau$, and the complete $\mu\mu$ and $\tau\tau$ data, respectively. The results are presented in Table 3. As is seen, the more precise $\mu\mu$ data demonstrate the signal of about 1$\sigma$ level. It corresponds to the Abelian $Z'$-boson with the mass of order 1.2–1.5 TeV if one assumes the value of $\tilde{\alpha} = g^2/4\pi$ to be in the interval 0.01–0.02. No signal is found by the analysis of the $\tau\tau$ cross-sections. The combined fit of the $\mu\mu$ and $\tau\tau$ data leads to the signal below the 1$\sigma$ confidence level.

Note that the mean values of $\epsilon$ have changed by 20% in comparison with the Winter 2002 data, whereas the uncertainties remain approximately the same. So, the Abelian $Z'$ signal will be probably picked up at no more than 1$\sigma$ confidence level when the final LEP2 data on $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$ will be completed.

Being governed by the next-to-leading contributions in $m_{Z'}^2$, the fitted values of $\zeta$ are characterized by significant errors. The $\mu\mu$ data set gives the central value which corresponds to $m_{Z'} \sim 1.1$ TeV.

We also perform a separate fit of the parameters based on the direct calculation of the observable from the differential cross-sections. The complete set of the available data [14] is used (see Table 4). The results are given in Table 5. As is seen, the experimental uncertainties of the data on the differential cross-sections are of one order larger than the corresponding errors of the total cross-sections and the forward-backward asymmetries. These data also provide the larger values of the contact coupling $\epsilon$. As for the more precise $\mu\mu$ data, three of the LEP2 Collaborations demonstrate positive values of $\epsilon$. The combined $\epsilon$ is also positive and remains practically unchanged by the incorporation of the $\tau\tau$ data.

Now, we compare the fits based on the differential cross-sections and the total cross-sections. As it was mentioned in the previous section, the indirect computation of the observable from the total cross-sections and the forward-backward asymmetries inspires some insufficient theoretical uncertainty about 2% of the statistical one. It also increases the statistical error because of the
Table 4: The differential cross-sections used for fitting. The letters F and P mark the final and the preliminary data, respectively.

| √s, GeV | ALEPH | DELPHI | L3 | OPAL |
|---------|-------|--------|----|------|
| 130     | F     |        |    |      |
| 136     | F     |        |    |      |
| 161     | F     |        |    |      |
| 172     | F     |        |    |      |
| 183     | F     | F      | F  | F    |
| 189     | P     | F      | F  | F    |
| 192     | P     | P      | P  | P    |
| 196     | P     | P      | P  | P    |
| 200     | P     | P      | P  | P    |
| 202     | P     | P      | P  | P    |
| 205     | P     | P      | P  | P    |
| 207     | P     |        | P  |      |

Table 5: The contact coupling $\epsilon$ with the 68% confidence-level uncertainties, computed from the differential cross-sections.

|       | $\mu$ data | $\tau$ data | $\mu$ and $\tau$ data |
|-------|------------|-------------|------------------------|
| ALEPH | 0.00014$^{+0.00006}_{-0.00006}$ | -0.00007$^{+0.0010}_{-0.0012}$ | 0.00009$^{+0.00059}_{-0.00060}$ |
| DELPHI| -0.00010$^{+0.00070}_{-0.00070}$ | 0.00000$^{+0.00140}_{-0.00140}$ | -0.00008$^{+0.00062}_{-0.00063}$ |
| L3    | 0.00015$^{+0.00043}_{-0.00043}$ | 0.00024$^{+0.00053}_{-0.00054}$ | 0.00018$^{+0.00033}_{-0.00033}$ |
| OPAL  | 0.00028$^{+0.00074}_{-0.00075}$ | -0.00017$^{+0.00120}_{-0.00120}$ | 0.00015$^{+0.00063}_{-0.00063}$ |
| Combined | 0.00012$^{+0.00028}_{-0.00030}$ | 0.00012$^{+0.00043}_{-0.00043}$ | 0.00012$^{+0.00024}_{-0.00024}$ |
recalculation procedure. Nevertheless, the uncertainty of the fitted parameter \( \epsilon \) within the recalculation scheme is of one order less than that for the direct computation from the differential cross-sections. This difference is explained by the different accuracy of the available experimental data on the differential and the total cross-sections. If the final LEP2 differential cross-sections will be so accurate as the present data on the total cross-sections, the direct computation of the observable can reduce, in principle, the uncertainty of the fitted coupling \( \epsilon \).

To compare our results with the fits of the contact couplings presented by the LEP Collaborations in Refs. \[1, 2\] let us briefly describe the approach used therein. Since only one parameter of new physics can be successfully fitted, the LEP Collaborations usually discuss eight ‘models’ (LL, RR, LR, RL, VV, AA, A0, V0) which assume specific helicity couplings between the initial-state and the final-state fermion currents. Each model is described by only one non-zero four-fermion coupling, while others are set to zero. For example, in the LL model the non-zero coupling of left-handed fermions is taken into account. The signal of new physics is fitted by considering the interference of the SM amplitude with the contact four-fermion term. Whatever physics beyond the SM exists, it has to manifest itself in some contact coupling mentioned. Hence, it is possible to find the low limit on the masses of the states responsible for the interactions considered. In principle, a number of states may contribute into each of the models. Therefore, the purpose of the fit described by these models is to find any signal of new physics. No specific types of new particles are considered in this analysis.

As it was shown, the characteristic signal of the Abelian \( Z' \) boson is related to the flavor-independent couplings to the axial-vector currents. To pick up the signal we construct the observable which is dominated by the axial-vector couplings. The contributions of the rest couplings are suppressed in the observable by the special choice of the kinematic parameters. In this regard, let us turn to the helicity ‘models’ of Refs. \[1, 2\] and compare our results with the fit for the axial model (AA). As it follows from the present analysis, this model could be sensitive to the signals of the Abelian \( Z' \) boson. Of course, the parameters \( \epsilon \) in Refs. \[1, 2\] (in what follows we will mark it as \( \epsilon_{\text{EWKG}} \)) and \( \epsilon \) in the present paper are not the same quantity. First, they are normalized by different factors and related as \( \epsilon_{\text{EWKG}} = -\epsilon m_{Z'}^2/4 \). Second, as we already noted, in the AA model the \( Z' \) couplings to the vector fermion currents are set to zero, therefore it is able to describe only some particular case of the Abelian \( Z' \) boson. Moreover, in this model both the positive and the negative values of \( \epsilon_{\text{EWKG}} \) are considered, whereas in our approach only the positive \( \epsilon \) values (which correspond to the negative \( \epsilon_{\text{EWKG}} \)) are permissible. As the value of the four-fermion contact coupling in the AA model is dependent on the lepton flavor, the Abelian \( Z' \) induces the axial-vector coupling which is universal for all lepton types. Considering the Winter 2002 data \[1\], it is interesting to note that the fitted value of \( \epsilon_{\text{EWKG}} \) in the AA model for the \( \mu^+ \mu^- \) final states \((-0.0025^{+0.0018}_{-0.0023} \text{ TeV}^{-2})\) as well as the value derived under the assumption of the
lepton universality ($-0.0018_{-0.0019}^{+0.0016}$ TeV$^{-2}$) are similar to our results which correspond to $-0.0015 \pm 0.0015$ TeV$^{-2}$ and $-0.0009 \pm 0.0012$ TeV$^{-2}$, respectively. As for the Summer 2002 data [2], the value of $\epsilon_{\text{EWWG}}$ under the assumption of the lepton universality is available only ($-0.0013 \pm 0.0017$ TeV$^{-2}$). It is close to our result for the $\mu\mu$ process ($-0.0011 \pm 0.0015$ TeV$^{-2}$). However, the central value of $\epsilon_{\text{EWWG}}$ is about three times greater than the corresponding one for the combined $\mu$ and $\tau$ data ($-0.0004 \pm 0.0012$ TeV$^{-2}$). Thus, the signs of the central values in the AA model agree with our results, and the uncertainties are of the same order. The fitted values of the 95% confidence-level lower limit on the scale $\Lambda$ are again in a good accordance with the corresponding values of $\Lambda^-$ for the AA model of [2].

So, we come to a conclusion that the AA model is mainly responsible for signals of the Abelian $Z'$ gauge boson although a lot of details concerning its interactions is not accounted for within this fit.

It worth to mention the recent paper by Chivukula and Simmons [15] who derived model-dependent bounds on the mass of the $Z'$ boson for flavor-changing technicolor models. It has been obtained that in these models $m_{Z'}$ is heavier than about 1 TeV. It is interesting to note that this value is very close to our model-independent result which corresponds to the flavor-conserved case.

As it follows from the present analysis, the Abelian $Z'$ boson has to be light enough to be discovered at the LHC. On the other hand, the LEP2 data on the processes $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$ do not provide the necessary statistics for the detection of the model-independent signal of the Abelian $Z'$ boson at more than 1$\sigma$ confidence level. So, it is of interest to find the observables for other scattering processes in order to increase the data set. In this regard, let us note the paper [16] where the helicity ‘models’ were applied to the Bhabha scattering $e^+e^- \rightarrow e^+e^- (\gamma)$. As it was shown therein, the AA model demonstrates the 2$\sigma$-level deviation from the SM. However, these deviations could not be interpreted directly as the signal of the Abelian $Z'$ boson because of the reasons mentioned above. Therefore, it seems to us perspective to find the observable for the Abelian $Z'$ search in the process $e^+e^- \rightarrow e^+e^-$. 

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