Finite Temperature Closed Superstring Theory: Infrared Stability and a Minimum Temperature

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Abstract

We find that the gas of IIA strings undergoes a phase transition into a gas of IIB strings at the self-dual temperature. A gas of free heterotic strings undergoes a Kosterlitz-Thouless duality transition with positive free energy and positive specific heat but vanishing internal energy at criticality. We examine the consequences of requiring a tachyon-free thermal string spectrum. We show that in the absence of Ramond-Ramond fluxes the IIA and IIB string ensembles are thermodynamically ill-defined. The 10D heterotic superstrings have nonabelian gauge fields and in the presence of a temperature dependent Wilson line background are found to share a stable and tachyon-free ground state at all temperatures starting from zero with gauge group $SO(16) \times SO(16)$. The internal energy of the heterotic string is a monotonically increasing function of temperature with a stable and supersymmetric zero temperature limit. Our results point to the necessity of gauge fields in a viable weakly coupled superstring theory.

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1 Introduction

It is generally assumed to be the case [1, 2] that in a quantum field theory known to be perturbatively renormalizable at zero temperature, the new infrared divergences introduced by a small variation in the background temperature can be self-consistently regulated by a suitable extension of the renormalization conditions, at the cost of introducing a finite number of additional counter-terms. The zero temperature renormalization conditions on 1PI Greens functions are conveniently applied at zero momentum, or at fixed spacelike momentum in the case of massless fields. Consider a theory with one or more scalar fields. Then the renormalization conditions must be supplemented by stability constraints on the finite temperature effective potential [1]:

\[ \frac{\partial V_{\text{eff}}(\phi_{\text{cl}})}{\partial \phi_{\text{cl}}} = 0, \quad \frac{\partial^2 V_{\text{eff}}(\phi_{\text{cl}})}{\partial \phi_{\text{cl}}^2} \equiv G_{\phi}^{-1}(k)|_{k=0} \geq 0. \] (1)

Here, \( V_{\text{eff}}(\phi_{\text{cl}}) = \Gamma(\phi_{\text{cl}})/V\beta \), where \( \Gamma \) is the effective action functional, or sum of connected 1PI vacuum diagrams at finite temperature. In the absence of nonlinear field configurations, and for perturbation theory in a small coupling about the free field vacuum, \( |0> \), \( \phi_{\text{cl}} \) is simply the expectation value of the scalar field: \( \phi_{\text{cl}} = <0|\phi(x)|0> \). The first condition holds in the absence of an external source at every extremum of the effective potential. The second condition states that the renormalized masses of physical fields must not be driven to imaginary values at any non-pathological and stable minimum of the effective potential.

The conditions in Eq. (1) are rather familiar to string theorists. Weakly coupled superstring theories at zero temperature are replete with scalar fields and their vacuum expectation values, or moduli, parameterize a multi-dimensional space of degenerate vacua. Consider the effect of an infinitesimal variation in the background temperature. Such an effect will necessarily break supersymmetry, and it is well-known that in the presence of a small spontaneous breaking of supersymmetry the dilaton potential will generically develop a runaway direction, signaling an instability of precisely the kind forbidden by the conditions that must be met by a non-pathological ground state [4]. Namely, while the zero temperature effective potential is correctly minimized with respect to the renormalizable couplings in the potential and, in fact, vanishes in a spacetime supersymmetric ground state, one or more of the scalar masses is driven to imaginary values in the presence of an infinitesimal variation in background temperature. Such a quantum field theory is simply unacceptable both as a self-consistent effective field theory in the Wilsonian sense [14], and also as a phenomenological model for a physical system [1]. The same conclusion must hold for a weakly coupled superstring theory with these pathological properties.

In the absence of gauge fields, the low temperature behavior of the ten-dimensional type II superstrings is pathological in the sense described above [5, 6]. While the zero temperature free string Fock space is supersymmetric and tachyon-free, an equilibrium thermal ensemble of either closed superstring, type IIA or type IIB, contains a tachyonic physical state at infinitesimal temperature [5]. It follows that, \textit{in the absence of gauge fields, flat spacetime is an unstable ground state for the ten-dimensional weakly coupled type II string theories under a small variation in the background temperature}. This result, first pointed out by us in [6], can be stated within the framework of the renormalization conditions for a consistent finite temperature closed string perturbation theory: there are no non-tachyonic solutions to the renormalization conditions in the vicinity of the zero
temperature ground state, in the absence of gauge fields that can potentially arise in a non-trivial Ramond-Ramond sector.

On the other hand, in the presence of temperature dependent Wilson lines, the 10D heterotic superstrings have a stable and tachyon–free finite temperature ground state at all temperatures starting from zero, but with gauge group $SO(16) \times SO(16)$. The low energy effective theory is a finite temperature gauge-gravity theory, formulated in modified axial gauge with the Euclidean time component of the vector potential, $A_0$, set equal to a temperature dependent constant. The additional massless vector bosons that lead to an enhancement of the gauge group in the supersymmetric zero temperature limit to either $E_8 \times E_8$, or $SO(32)$, are massive at finite temperature. As was shown in [6], an analogous result holds for the finite temperature type I string. As explained in the conclusions [6], at least for infinitesimal temperatures, it should be possible to interpret the type I thermal string as the strong coupling dual of the thermal $SO(16) \times SO(16)$ heterotic string. We defer further discussion of finite temperature open and closed string theory to [50].

In this paper we will compute the generating functional of connected one-loop vacuum string graphs in the embedding space $R^9 \times \mathbb{S}^1$ (a compact one-dimensional target space with Euclidean signature) for each of the 10D fermionic closed string theories. As pointed out in [6, 19], there is an ambiguity in the Euclidean time prescription that becomes apparent when we try to apply it to finite temperature string theory: a one-dimensional compact space can have the topology of a circle or that of an interval, which assumption yields the correct answer for the thermal spectrum of a gas of free strings? In [19] we resolve this ambiguity by showing that a $Z_2$ twist in Euclidean time is necessary for the absence of spurious massless vector bosons in the finite temperature spectrum. Given the residual $d$-dimensional gauge invariance, the number of propagating degrees of freedom in a finite temperature gauge theory in $d$ spatial dimensions correspond precisely to those in an axial gauge quantization, $A^0 = 0$, of the Euclidean $d+1$ dimensional gauge theory [3]. Thus, by requiring that the massless states in the thermal string spectrum match with the physical degrees of freedom in a consistent finite temperature gauge-gravity the ambiguity in the Euclidean time prescription is resolved.

In the fermionic strings: IIA, IIB, or heterotic, the $Z_2$ orbifold twist in Euclidean time naturally extends to a $Z_2$ superorbifold twist, identifying an appropriate fixed line of $\hat{\phi} = 1$ superconformal field theories parameterized by interval length, or inverse temperature: $\beta = \pi r_{\text{circ}}$. The super-orbifold action must be consistent with the appropriate superconformal invariance on the worldsheet. As in the bosonic string [6, 19], we find that the Hagedorn radius $(2\alpha')^{1/2}$ of a gas of fermionic strings always coincides with the Kosterlitz-Thouless point on a fixed line of superorbifolds with interval length $\pi \alpha'^{1/2}$: the $\hat{\chi} = 1$ orbifold of the circle compactified superconformal field theory at the self-dual radius. The nature of the duality transition at the Kosterlitz-Thouless point differs: the IIA and IIB thermal strings are related by a T-duality transformation in Euclidean time, while the heterotic string is self-dual.

Following the approach taken by Polchinski [9], our starting point for a discussion of string thermodynamics is the world-sheet path integral representation of the generating functional of connected one-loop vacuum string graphs: $W(\beta) \equiv \ln Z(\beta)$. But, as explained above [6, 19], we introduce a $Z_2$ twist in Euclidean time in order to match with the physical degrees of freedom in the low energy finite temperature gauge-gravity theory: $W(\beta)$ is computed in the embedding space $R^9 \times S^1 / Z_2$ with interval length identified as the inverse temperature $\beta$. The thermodynamic
relations for the free string gas take the form:

\[ W(\beta) \equiv \ln Z, \quad F(\beta) = -W/\beta, \quad \rho(\beta) = -W/V\beta, \quad U(\beta) = -T^2 \left( \frac{\partial W}{\partial T} \right)_V = \left( \frac{\partial W}{\partial \beta} \right)_V \quad . \quad (2) \]

\( F \) is the Helmholtz free energy of the ensemble of free strings, \( U \) is the internal energy, and \( \rho \) is the finite temperature effective potential. Notice that \( \rho \) is the finite temperature analog of the one-loop vacuum energy density or cosmological constant, \( \rho_0 \). \( \beta \) is a continuously-varying background parameter describing a one-parameter family of consistent ground states of string theory, characterized by the \( \beta \) dependent effective potential. In practise, we will find it impossible to satisfy the requirement of a tachyon-free spectrum over the entire temperature range in the case of the type II strings, at least in the absence of a Ramond-Ramond sector. For the heterotic string, the presence of gauge fields permits a one-parameter family of tachyon-free ground states over the entire temperature axis upon inclusion of a temperature dependent Wilson line. As in the bosonic string [19], the thermodynamic potentials of the free string ensemble are obtained directly from the generating functional of connected vacuum graphs: \( \ln Z(\beta) \), as opposed to \( Z(\beta) \). Thus, we never need address the troubling issue of defining the thermodynamic limit of the canonical ensemble, since we never compute the canonical partition function for string states directly in the path integral framework.

An important new issue is the determination of modular invariant but supersymmetry breaking temperature dependent phases in the fermionic closed string path integral. We will find that the type II string theories display an unusual phenomenon: unlike the bosonic string, a stable and tachyon-free thermal ensemble exists only beyond a string scale minimum temperature. The minimum temperature is the temperature at which the leading tachyonic momentum mode turns massless. Thermal duality interchanges the momentum and winding modes, respectively, of IIA and IIB string theories. Consequently, each type II string theory also exhibits a characteristic maximum temperature, at which the first winding mode turns tachyonic. The necessity for both winding and momentum modes in either type II string follows from modular invariance. Finally, the inclusion of a duality invariant phase ensures that upon mapping IIA winding modes to IIB momentum modes, and vice versa, the expression for the vacuum functional of the IIA theory is likewise mapped to that for the IIB theory. Thus, in the absence of background fields, either type II free string ensemble is stable, but only within a string scale temperature regime containing the self-dual point, \( T_C = 1/\pi\alpha'^{1/2} \). The Kosterlitz-Thouless phase transition at \( T_C \) should now be viewed as a continuous phase transition mapping the thermal IIA string to the thermal IIB string. A similar result holds for the type I string theory and its thermal dual, the type \( \tilde{1} \) string [6, 50].

The 10D \( E_8 \times E_8 \) and \( SO(32) \) heterotic strings share a common finite temperature ground state, and the vacuum functional of this nonsupersymmetric and tachyon-free string theory is self-dual under thermal duality transformations. For either 10D heterotic superstring, we will show that turning on a temperature dependent Wilson line background permits a stable and tachyon-free nonsupersymmetric ground state at all temperatures starting from zero, but with non-abelian gauge group \( SO(16) \times SO(16) \). The expressions for the Helmholtz and Gibbs free energies are finite at the Hagedorn temperature, confirming the absence of any exponential divergences in the free energy of a tachyon-free string theory and, consequently, the absence of a Hagedorn phase transition as anticipated in [6, 19].

The self-dual heterotic string gas displays the Kosterlitz-Thouless duality transition described
in section 4 of [19]. The Helmholtz and Gibbs free energies are minimized at criticality, and the internal energy vanishes. As explained in [19], the cancelation is simply understood as a shift in balance between energy and entropy: winding and momentum modes contribute equally to both the free energy and internal energy at criticality, and the gas of free strings transitions from a phase of bound vortex pairs to the high temperature long string phase dominated by winding modes [11]. Similar arguments explain the zeroes of the one-loop effective potential found in the numerical computations of Ginsparg and Vafa [22, 15], and of Itoyama and Taylor [16].

The free energy of the gas of SO(16)×SO(16) heterotic strings is positive at criticality, a consequence of an excess of spacetime fermions over spacetime bosons in the physical state spectrum. As in the bosonic string [9, 19], this aspect of the $R^9\times S^1/\mathbb{Z}_2$ Euclidean time ground state simply mirrors an analogous result for the corresponding Lorentzian signature, nonsupersymmetric and tachyon-free, 10D heterotic string ground state found in [23, 24, 25, 26]. As noted in both [24] and [25], unlike the tachyonic ground states, the 10D nonsupersymmetric and tachyon-free $O(16)\times\bar{O}(16)$ heterotic ground state has positive cosmological constant. As anticipated in [19], we can verify that the Gibbs and Helmholtz free energies are minimized at the self-dual temperature, and the vanishing internal energy results in positive specific heat at the transition temperature. This is an encouraging indication of the thermodynamic stability of the finite temperature ground state.

The plan of this paper is as follows. In section 2, we begin with a brief overview of the stability conditions at finite temperature on $W(\beta)$, the generating functional of connected vacuum graphs in string theory. This is followed by a discussion of the implementation of thermal boundary conditions on fermions in the superstring path integral in section 2.1. The Polyakov path integral representation of the string vacuum functional for an equilibrium ensemble of type IIA or type IIB strings at finite temperature, including determination of the supersymmetry breaking phases, is presented in Section 3. In Section 3.1, we obtain the minimum temperature for tachyon-free IIA or IIB thermal ensembles, at which the leading tachyonic momentum mode turns massless. We clarify the existence of a continuous Kosterlitz-Thouless phase transition between the thermal type IIA and thermal type IIB strings at the self-dual Hagedorn point. The source of the tachyonic instability at $T_{\text{min}}$, and its possible resolution, are discussed briefly.

Section 4 contains a description of the nonsupersymmetric 9D SO(16)×SO(16) heterotic string, the tachyon-free finite temperature ground state common to both 10D heterotic superstrings. We explain how the 9D SO(16)×SO(16) heterotic string is obtained by turning on a temperature-dependent timelike Wilson line in either zero temperature supersymmetric 10D heterotic string [6]. An infinitesimal variation in temperature away from the zero temperature limit of the heterotic superstring results in the breaking of both the supersymmetry, and a partial breaking of the non-abelian gauge symmetry, with inverse temperature playing the role of a small control parameter. The mechanism for symmetry breaking in the vicinity of the zero temperature ground state appears to have no field theoretic correspondence that I am aware of, but it can be seen to be a consequence of requiring a modular invariant and tachyon-free thermal string spectrum. From the viewpoint of the low-energy, effective, finite temperature gauge-gravity theory, the timelike Wilson line background is interpreted as the imposition of a particular axial gauge condition. We explain why the twist II orbifold construction of [16] suggests the reverse, namely, the spontaneous restoration of symmetries at high temperature. For comparison, we also clarify the finite temperature behavior of the nonsupersymmetric and tachyon-free 10D SO(16)×SO(16) heterotic string theory, first discovered in
We conclude with a brief discussion of the Helmholtz free energy and, more generally, of the hierarchy of thermal duality relations satisfied by the thermodynamic potentials of the free heterotic string ensemble. The conclusions explain the implications of these results, both for the further elucidation of string thermodynamics, and also for the clarified understanding of dualities in the moduli space of String/M theory. We discuss some open questions for future work.

2 Thermal Fermionic String Theories

In this section, we will compute the generating functional of connected one-loop vacuum string graphs in the Euclidean embedding space $R^9 \times S^1 / Z_2$ for both the type II and heterotic closed strings. As was shown in [19], the expression for the string vacuum functional is the starting point for a discussion of string thermodynamics. The Helmholtz free energy and remaining thermodynamic potentials are obtained by simply taking appropriate derivatives with respect to inverse temperature, or volume, using the usual thermodynamic definitions for a canonical ensemble.

As in finite temperature field theory [1], a small variation in the background temperature can generically result in tachyonic instabilities in the thermal free string ensemble if the mass of a physical, or unphysical, state in the mass spectrum is driven imaginary [9, 5, 18]. It was pointed out in [6] that the generating functional of vacuum graphs in a fully consistent string theory at finite temperature—heterotic, type I, type IIA or type IIB, should have no infrared instabilities in the low temperature regime. In other words, while it is reasonable to allow for the possibility of a genuine phase transition where the Helmholtz free energy is divergent for all temperatures beyond some finite transition temperature, the zero temperature free string Fock space of a consistent supersymmetric string theory should not contain physical states whose masses are driven to imaginary values at infinitesimal temperature. As mentioned in the introduction, in the absence of background fields, all of the weakly coupled ten-dimensional supersymmetric strings are pathological in this respect.

As described in [19], the starting point for a discussion of string thermodynamics is $W = \ln Z$, the generating functional of connected one-loop vacuum string graphs in the Euclidean embedding space $R^9 \times S^1 / Z_2$, where the interval in the imaginary time direction has length equal to the inverse temperature $\beta = \pi r_{\text{circ}}$. This yields the one-loop contribution to the finite temperature effective potential in string theory via the relation: $\Gamma_{\text{eff.}} = -W/\beta V = \rho(\beta)$ [9]. Successive partial derivatives of $W$ with respect to $\beta$ at fixed spatial volume will yield the thermodynamic potentials:

\[
F = -W/\beta, \quad U = T^2 \frac{\partial}{\partial T} W, \quad P = -\left( \frac{\partial F}{\partial V} \right)_T, \quad S = -\left( \frac{\partial F}{\partial T} \right)_V, \quad C_V = T \left( \frac{\partial S}{\partial T} \right)_V, \quad (3)
\]

where $F$ and $U$ denote, respectively, the Helmholtz free energy and the internal energy, $S$ denotes the entropy, and $C_V$ denotes the specific heat at constant volume, of the canonical ensemble. For the gas of free strings, the finite temperature effective potential is the extensive vacuum energy density: $\rho = F/V$ [9]. It is easy to see that the pressure of the free string gas vanishes, so that the enthalpy equals the internal energy: $H = U$, and, consequently, that the Gibbs and Helmholtz free energies are identical: $G = H - TS = U - TS = F$. We emphasize that this holds in the absence of string interactions.

We will require that the finite temperature effective potential or, equivalently, the Helmholtz free energy, are free from thermal tachyons. Remarkably, for the heterotic string we will find that a
tachyon-free ensemble exists at all temperatures starting from zero, in the presence of a temperature dependent Wilson line background, a result that could have been anticipated from the earlier works [23, 24, 25, 26, 22].

2.1 Thermal Boundary Conditions in Fermionic Strings

In this subsection, we give a discussion of thermal boundary conditions and the precise action of the orbifold group on world-sheet fermions. Recall that in the imaginary time formalism, spacetime fields obeying Fermi statistics are to be expanded in a Fourier basis of antiperiodic modes alone. This implies antiperiodic boundary conditions for the spacetime fermionic modes of the string path integral in the imaginary time direction. The spacetime spin and statistics of fields in the Euclidean embedding space can be determined in string theory by suitably adapting the GSO prescription, given by a specification of the world-sheet spin and statistics of every state in the free string spectrum [23, 27, 25, 24, 26, 12]. We will use the Ramond-Neveu-Schwarz (RNS) formulation to compute the spectrum of physical states in a fermionic string theory. Fermions on the worldsheet are simultaneously spacetime vectors and worldsheet spinors.

There is no spin-statistics theorem in two dimensions and we are therefore free to choose arbitrary boundary conditions on the world-sheet fermions [27]. We emphasize that such a generalized sum is necessitated by modular invariance and the thermal boundary conditions. In practise, it is often easier to arrive at the required one-loop amplitude by manipulating modular invariant blocks of Jacobi theta functions which preserve spacetime Lorentz invariance, and consequently, the world-sheet superconformal invariances. Thus, we identify modular invariant blocks of eight spin structures pertaining to the eight transverse world-sheet fermions. These blocks are weighted by, a priori, undetermined temperature-dependent phases. The form of the temperature dependent phases is guided by requiring a modular invariant partition function which interpolates continuously between the supersymmetric zero temperature result, and a finite temperature expression which transforms correctly under a thermal duality transformation.

To motivate our result, it is helpful to consider the generalization of the thermal closed bosonic string given in [6, 19]. The action of a fermionic R-orbifold group on \( \psi^0, X^0 \), consistent with the preservation of an N=1 world-sheet superconformal invariance, satisfying thermal boundary conditions, and transforming correctly under a thermal duality transformation, remains to be specified. To begin with, we recall the analysis in [20] where the distinct possibilities for fixed lines of N=1 superconformal theories with superconformal central charge \( \hat{c} = \frac{2}{3}c = 1 \) were classified. We can identify points under the reflection \( R, X^0 \approx -X^0 \), with fundamental region the half-line, \( X^0 \geq 0 \), supplementing with the periodic identification, \( t^w : X^0 \approx X^0 + 2\pi w r_{\text{circ}}, w \in \mathbb{Z} \). The resulting fundamental region is the interval, \( 0 \leq X^0 < \pi r_{\text{circ}} \). An N=1 world-sheet supersymmetry requires that \( \psi^0 \approx -\psi^0 \) under \( R \), and the periodic identification \( t^w \) ordinarily preserves the superconformal generator, \( \psi^0 \partial_z X^0 \), leaving the fermions invariant. The partition function of an Ising fermion is the sum over spin structures \( (A,A), (A,P), (P,A), \) and \( (P,P) \), manifestly invariant under \( \psi \rightarrow -\psi \) [20]. Thus, two fixed lines of \( \hat{c}=1 \) theories are obtained by simply tensoring the Ising partition function with either \( Z_{\text{circ}} \), or \( Z_{\text{orb}} \), from the previous section [20]:

\[
\hat{Z}_{\text{circ}} = Z_{\text{circ}} Z_{\text{Ising}}, \quad \hat{Z}_{\text{orb}} = Z_{\text{orb}} Z_{\text{Ising}}, \quad Z_{\text{Ising}} = \frac{1}{2} \left( \frac{|\Theta_3|}{|\eta|} + \frac{|\Theta_4|}{|\eta|} + \frac{|\Theta_2|}{|\eta|} \right). \quad (4)
\]
Accompanying $R$, $t^w$ with the action of the $Z_2$-moded generator, $(-1)^{N_F}$, where $N_F$ is spacetime fermion number, one can define additional super-orbifold theories, $\hat{Z}_{sa}$, $\hat{Z}_{so}$, and $\hat{Z}_{\text{orb}}$, giving a total of five fixed lines of $\hat{c} = 1$ theories [20]:

$$\hat{Z}_{\text{orb}} = R(-1)^{N_F} \hat{Z}_{\text{circ}}, \quad \hat{Z}_{sa} = (-1)^{N_F} e^{2\pi i \delta(p)} \hat{Z}_{\text{circ}}, \quad \hat{Z}_{so} = R \hat{Z}_{sa} = R(-1)^{N_F} e^{2\pi i \delta(p)} \hat{Z}_{\text{circ}},$$  \hspace{1cm} (5)

where $\delta(p)$ acts on the momentum lattice as an order two shift in the momentum eigenvalue, $n \rightarrow n + \frac{1}{2}$. Note that this shift vector is not thermal duality invariant. We will adapt the left-right symmetric analysis of [20] for the different ten-dimensional fermionic string theories and taking into account the thermal boundary conditions. Thus, the action of the super-orbifold group will be required to preserve either one, or both, of the holomorphic N=1 superconformal invariances in, respectively, the heterotic, or type II, string theories, consistent with modular invariance and the correct thermal duality transformations.

It is convenient to introduce lattice partition functions that transform straightforwardly under a thermal duality transformation [20]. A Poisson resummation makes their transformation properties under modular transformations manifest. As in [19], we introduce a dimensionless inverse temperature (radius) defining $x \equiv r(2/\alpha')^{1/2}$, with $\beta = \pi(\alpha'/2)^{1/2}x$. The dimensionless quantized momenta live in a (1, 1) dimensional Lorentzian self-dual lattice [21, 20, 12]:

$$\Lambda^{(1,1)} : \quad \left(\frac{\alpha'}{2}\right)^{1/2}(p_L, p_R) \equiv (l_L, l_R) = \left(\frac{n}{x} + \frac{wx}{2}, \frac{n}{x} - \frac{wx}{2}\right),$$  \hspace{1cm} (6)

with a natural decomposition into even and odd integer momentum (winding) sums. Thus, we define:

$$\Gamma^{++}(x) \equiv \sum_{w \in 2Z, n \in 2Z} q^{\frac{1}{2}(\frac{n}{x} + \frac{wx}{2})^2} q^{\frac{1}{2}(\frac{n}{x} - \frac{wx}{2})^2}, \quad \Gamma^{--}(x) \equiv \sum_{w \in 2Z} q^{\frac{1}{2}(\frac{n}{x} + \frac{wx}{2})^2} q^{\frac{1}{2}(\frac{n}{x} - \frac{wx}{2})^2},$$

$$\Gamma^{+-}(x) \equiv \sum_{w \in 2Z, n \in 2Z+1} q^{\frac{1}{2}(\frac{n}{x} + \frac{wx}{2})^2} q^{\frac{1}{2}(\frac{n}{x} - \frac{wx}{2})^2}, \quad \Gamma^{-+}(x) \equiv \sum_{w \in 2Z+1, n \in 2Z} q^{\frac{1}{2}(\frac{n}{x} + \frac{wx}{2})^2} q^{\frac{1}{2}(\frac{n}{x} - \frac{wx}{2})^2}.$$

It is evident that the functions $\Gamma^{++}$ and $\Gamma^{--}$ are invariant while $\Gamma^{+-}$ is mapped to $\Gamma^{-+}$, and vice versa, under both modular and thermal duality transformations. For example, the following shift by half a lattice vector, $\delta = \frac{1}{2}(\frac{1}{x} + \frac{1}{x}, \frac{1}{x} - \frac{1}{x})$, giving shifted lattice partition functions:

$$\Gamma_\delta(x) \equiv \sum_{w \in Z + \frac{1}{2}, n \in Z + \frac{1}{2}} q^{\frac{1}{2}(\frac{n}{x} + \frac{wx}{2})^2} q^{\frac{1}{2}(\frac{n}{x} - \frac{wx}{2})^2},$$  \hspace{1cm} (7)

is invariant under a thermal duality transformation. The shifted lattice can be further decomposed into even and odd lattices as above. All of these considerations extend to lattices of higher dimension. This generalizes the construction given in [20].

### 3 Type IIA-IIB Strings at Finite Temperature

We will now show that the vacuum functional for an ensemble of IIA or IIB strings takes identical form in the absence of gauge fields, as a consequence of the thermal duality transformation mapping...
the thermal IIA string to the thermal IIB string. Although the thermal type II partition functions we derive will turn out to be pathological due to the appearance of tachyons—over the entire finite temperature range as below, or above a string-scale minimum temperature as in the next subsection, it is a useful warm-up exercise prior to the discussion of the tachyon-free heterotic string ensemble that follows. The behavior of the thermal type II ensemble at the critical temperatures when, respectively, the leading tachyonic, momentum or winding, modes turn massless, might be of intrinsic interest for the understanding of the moduli space of string/M theory \[9, 5, 12, 30\].

The normalized generating functional of connected one-loop vacuum graphs for either type II string theory in the embedding space \(R^9 \times S^1 / \mathbb{Z}_2\) is given by a straightforward extension of \([19]\). For either type II string, we have an expression of the form:

\[
W_{II} = \int F \frac{d^2 \tau}{4 \tau^2} (2\pi \tau)^{-4} |\eta(\tau)|^{-14} Z_{II}(\beta),
\]

where the spatial volume, \(V = L^9 (2\pi \alpha')^9 / 2\), and the inverse temperature is given by \(\beta = \pi r_{\text{circ}}\).

The function \(Z_{II \text{ orb.}}\) is the product of appropriate fermion and boson partition functions: \(Z_F Z_B\), to be defined below. As in the bosonic string, the \(n=w=0\) sector is required to be a subspace of the full thermal spectrum due to the necessity for separate left and right-moving conserved charges on the world-sheet: namely, the momentum modes labeled \(+n\) and \(-n\), with \(n\) non-zero, are both in the thermal spectrum, and consequently, so is the \(n=0\) mode. Likewise for winding states. The spectrum of thermal modes is unambiguously determined by modular invariance and by requiring that IIA winding modes are mapped to IIB momentum modes, and vice versa, under a thermal duality transformation. Thus, the vacuum functional of the IIA string, being an intensive thermodynamic variable, is required to precisely map into the vacuum functional of the IIB string under a thermal duality transformation.

We will consider the action of the super-orbifold group \(R(-1)^{N_F}\), either with or an accompanying half-momentum or half-winding shift, \(\delta\), in the one-dimensional Lorentzian momentum lattice. Modular invariance requires that we include both momentum and winding modes in the type IIA, IIB, thermal partition functions. The phase in the path integral is determined by the thermal duality transformation: since IIA winding modes are mapped to IIB momentum modes, and vice versa, under thermal duality, the phase factor in either type II string path integral is required to be thermal duality invariant. We emphasize that this should not be misinterpreted as requiring self-duality of the IIA or IIB ensemble; it follows as a consequence of requiring that the vacuum functional for IIA maps to that for IIB. In the presence of a Ramond-Ramond sector, the mapping between the vacuum functionals is less straightforward.

The purpose of the shift in the momentum lattice is to introduce a positive, temperature-dependent, shift in the spectrum of thermal masses. The spacetime supersymmetry breaking orbifold action \((-1)^{N_F}\) is modified by the introduction of modular invariant, and temperature-dependent, phases. The phase factor must also be compatible with both the thermal duality transformations, as explained above, and also the requirement that spacetime supersymmetry is restored in the zero temperature limit of the vacuum functional, an idea originally proposed in \([5]\). Note that, for the thermal type II strings, due to the appearance of a tachyonic state at a characteristic minimum temperature, continuation of our final expression for the vacuum functional down to the
supersymmetric zero temperature limit is formal, at best. This issue will however be resolved upon introduction of gauge fields in the type II strings via a non-trivial Ramond-Ramond sector.

The action of \( R \) on \( X^0 \), and the computation for the bosonic \( R \)-orbifold partition function, is reviewed in [12, 19]. The result is:

\[
Z_{\text{orb.}} = \frac{1}{2} \frac{1}{\eta^2} \sum_{n,w \in \mathbb{Z}} \left[ q^{\frac{1}{2}(\frac{n}{2} + \frac{w}{2})^2} \bar{q}^{\frac{1}{2}(\frac{n}{2} - \frac{w}{2})^2} + |\Theta_3\Theta_4| + |\Theta_2\Theta_3| + |\Theta_2\Theta_4| \right] .
\]

The world-sheet fermions can be conveniently complexified into left- and right-moving Weyl fermions. As in the superstring, the spin structures for all ten left- and right-moving fermions, \( \psi^\mu, \bar{\psi}^\mu \), \( \mu = 0, \cdots, 9 \), are determined by those for the world-sheet gravitino associated with left- and right-moving \( N=1 \) superconformal invariances. Begin by recalling the spacetime supersymmetric sum over spin structures familiar from the partition function of the type II superstring [12]. We set \( Z_F = Z_{SS} \), with:

\[
Z_{SS} = \frac{1}{4} \frac{1}{\eta^4 \bar{\eta}^4} \left[ (\Theta_3^4 - \Theta_1^4 - \Theta_2^4)(\bar{\Theta}_3^4 - \bar{\Theta}_1^4 - \bar{\Theta}_2^4) \right] .
\]

A factor of \( \frac{1}{4} \) arises from the average of spin structures in the type II theory [12]. Notice that the first of the relative signs in each round bracket preserves the tachyon-free condition. The second relative sign determines whether spacetime supersymmetry is preserved in the zero temperature spectrum. Under the action of the orbifold group \( R \cdot (-1)^{N_F} \), ordinarily acting as +1 on spacetime bosons and as (-1) on spacetime fermions, the phase of the contribution from spacetime fermions must be reversed as required by thermal boundary conditions. There is a unique choice of phases which implements this constraint consistent with both modular invariance and broken supersymmetry at arbitrary temperature: we replace the theta functions with their absolute magnitudes, summing over the different spin structures. This gives \( Z_F = Z_{NS} \), with:

\[
Z_{NS} = \frac{1}{4} \frac{1}{\eta^4 \bar{\eta}^4} \left[ (|\Theta_3|^4 + |\Theta_4|^4 + |\Theta_2|^4)(|\bar{\Theta}_3|^4 + |\bar{\Theta}_4|^4 + |\bar{\Theta}_2|^4) \right] .
\]

We now wish to generalize to an expression in which the spacetime supersymmetric and tachyon-free combination of spin structures given in Eq. (10) is recovered at zero temperature [5], consistent with modular invariance and the thermal duality transformation interchanging IIA with IIB. Combining with the bosonic contributions, the required orbifold partition function describing either the thermal IIA or IIB ensemble is a simple modification of that given in [5]:

\[
Z_{II} = \frac{1}{8} \frac{1}{|\eta|^10} \sum_{n,w \in \mathbb{Z}} \{(|\Theta_3|^8 + |\Theta_4|^8 + |\Theta_2|^8)
\]

\[+e^{\pi i(n+w)}[\Theta_2^4 \Theta_3^4 - \Theta_3^4 \Theta_2^4 + \Theta_1^4 \Theta_3^4 - \Theta_3^4 \Theta_1^4 + \Theta_2^4 \Theta_4^4 - \Theta_4^4 \Theta_2^4]q^{\frac{1}{2}n^2} \bar{q}^{\frac{1}{2}n^2} \cdots ,
\]

where the ellipses denote the temperature independent states of the \( R \)-orbifold. This expression is modular invariant and transforms as desired under a thermal duality transformation: taking \( \beta_{IIA} \rightarrow \beta_{IIB} = \beta_{IIA}^2 / \beta_{IIA} \), and simultaneously interchanging the identification of winding and momentum modes, \( (n, w)_{IIA} \rightarrow (n' = w, w' = n)_{IIB} \). The expression also has the desired zero temperature
limits of the IIA and IIB strings: for large $\beta_{\text{IIA}}$, terms with $w_{\text{IIA}} \neq 0$ decouple in the double summation since they are exponentially damped. The remaining terms are resummed by a Poisson resummation, thereby inverting the $\beta_{\text{IIA}}$ dependence in the exponent, and absorbing the phase factor in a shift of argument. The large $\beta_{\text{IIA}}$ limit can then be taken smoothly providing the usual momentum integration for a non-compact dimension, plus a factor of $\tau_{2}^{-1/2}$ from the Poisson resummation. This recovers the supersymmetric zero temperature partition function. A thermal duality transformation maps small $\beta_{\text{IIA}}$ to large $\beta_{\text{IIB}}=\beta_{\text{C}}^{2}/\beta_{\text{IIA}}$, also interchanging the identification of momentum and winding modes. We can analyze the limit of large dual inverse temperature as before, obtaining the zero temperature limit of the dual IIB theory. At any intermediate temperature, all of the thermal modes contribute to the vacuum functional with a phase that takes values ($\pm 1$) only. Note that the spacetime fermions of the zero temperature spectrum now contribute with a reversed phase, evident in the first term in Eq. (12), as required by the thermal boundary conditions. The non-trivial winding and momentum modes in the thermal spectrum have no zero temperature counterpart; they are necessitated by modular invariance of the string spectrum.

Note that while this expression has the expected benign spacetime supersymmetric zero temperature limits, the physical state spectrum is replete with thermal tachyons over the entire finite temperature range as in the closed bosonic string [19]. It describes pathological, and physically unacceptable, thermal type II string theories. The pathological behavior over the entire finite temperature range of the expression in Eq. (12) is also true of the expression for the vacuum functional of the thermal type II string proposed in Eq. (5.20) of reference [5], An important difference is that due to the momentum and winding number dependent phases introduced in the sum over spin structures, the expression for the type II vacuum functional given in [5] does not transform correctly under a thermal duality transformation. Namely, while spacetime supersymmetry holds in the zero temperature limit, a thermal duality transformation mapping $\beta_{\text{IIA}} \leftrightarrow \beta_{\text{IIB}}=\beta_{\text{C}}^{2}/\beta_{\text{IIA}}$ gives a function that is tachyonic and nonsupersymmetric in the (dual) zero temperature limit. This behavior was explicit in Eq. (5.20) of [5].

Although we have presented this discussion for pedagogical purposes, we emphasize that the tachyonic type II partition functions discussed in this subsection describe inherently unstable thermal ground states. We will dismiss them as pathological, and modify our type II orbifold construction as described next.

### 3.1 Minimum Temperature for the Thermal Type II Strings

A non-pathological thermal type II vacuum functional describing a tachyon-free spectrum within a finite, albeit string-scale, temperature range, and transforming correctly under a thermal duality transformation, is obtained as follows. Consider the duality invariant (1,1) lattice partition function obtained by shifting each $(n,w)$ vector by the constant vector, $\delta$, composed from a half-winding plus a half-momentum shift. The following expression is modular invariant. It also transforms correctly under thermal duality. We replace the function $Z_{\text{II}}$ appearing in Eqs. (8), (12) with:

$$Z_{\text{II}} = \frac{1}{4} \frac{1}{|\eta|^10} \sum_{n,w \in \mathbb{Z}^+} (|\Theta_{3}|^8 + |\Theta_{4}|^8 + |\Theta_{2}|^8)$$

$$+ e^{\pi i (n+w)} \left\{ (\Theta_{1}^{4} \bar{\Theta}_{1}^{4} + \Theta_{2}^{4} \bar{\Theta}_{2}^{4}) - (\Theta_{3}^{4} \bar{\Theta}_{3}^{4} + \Theta_{4}^{4} \bar{\Theta}_{4}^{4} + \Theta_{2}^{4} \bar{\Theta}_{2}^{4} + \Theta_{3}^{4} \bar{\Theta}_{3}^{4}) \right\} q^{\frac{1}{2}L_{I}} q^{\frac{1}{2}R} + \cdots$$

10
describing either type IIA or type IIB thermal strings. The IIA and IIB descriptions are interchangeable, related by a thermal duality transformation: $\beta_{IIA} \leftrightarrow \beta_{IIB} = \beta_2^C / \beta_{IIB}$, simultaneously interchanging momentum and winding modes. This partition function describes non-supersymmetric theories, except in the zero temperature limit of large radius.

The expression in Eq. (13) describes a viable candidate world-sheet superconformal field theory satisfying the required consistency conditions for a weakly coupled ground state of the type IIA, or IIB, string theories. There is an obvious underlying worldsheet representation of the lattice partition function by a pair of chiral bosons. We will now show that Eq. (13) gives a physically meaningful vacuum functional describing the equilibrium thermal behavior of a stable ensemble of type II strings. To check for potential tachyonic instabilities, consider the mass formula in the (NS,NS) sector for world-sheet fermions, with $l_L^2 = l_R^2$, and $N = \bar{N} = 0$:

$$\text{(mass)}^2_{nw} = \frac{2}{\alpha'} \left[ -1 + \frac{\alpha' \pi^2 (n + \frac{1}{2})^2}{2 \beta^2} + \frac{\beta^2 (w + \frac{1}{2})^2}{2 \pi^2 \alpha'} \right]. \quad (14)$$

This is the only sector that can contribute tachyons to the thermal spectrum. A nice check is that the momentum dependent phase factor introduced above does not enter the NS-NS sector; all potentially tachyonic states are spacetime scalars as expected. Notice that the $n=w=0$ sector common to both type II string theories contains a potentially tachyonic state whose mass is now temperature dependent. However, it corresponds to an unphysical tachyon. The potentially tachyonic physical states are the pure momentum and pure winding states, $(n,0)$ and $(0,w)$, with $N = \bar{N} = 0$. The R-orbifold projects to the symmetric linear combinations of net zero momentum and net zero winding number states as explained in [19].

The effect of the shift in the momentum lattice is to open up a window on the temperature axis, for which the physical state spectrum is tachyon-free. Analogous to the bosonic string analysis, we can compute the temperatures below, and beyond, which these modes become tachyonic in the absence of oscillator excitations. Each momentum mode, $(\pm n,0)$, is tachyonic \textit{upto} some critical temperature, $T_n^c = (2/\alpha')^{1/2}/\pi(n + \frac{1}{2})$, after which it becomes stable. Conversely, each winding mode $(0,\pm w)$, is tachyonic \textit{beyond} some critical temperature, $T_w^c = (w + \frac{1}{2})/\pi(2\alpha')^{1/2}$. It is evident that the $(1,0)$ and $(0,1)$ states determine the upper and lower critical temperatures for a tachyon-free physical state spectrum. We refer to these, respectively, as the minimum and maximum temperatures for the given type II string. A thermal duality transformation maps IIA to IIB, mapping $T_{\text{min}}(IIA)$ to $T_{\text{max}}(IIB)$, and vice versa, since the identification of momentum and winding modes is switched in the mapping. Expressed in terms of the dual temperature variable, the minimum and maximum temperatures will, of course, coincide. As a consequence, the IIA and IIB thermal strings have common minimum and maximum temperatures. The thermal mass spectrum of either type II string is tachyon-free for temperatures in the range:

$$T_{\text{min}} < T < T_{\text{max}}, \quad \text{where} \quad T_{\text{min}} = 2\sqrt{2}/3\pi\alpha'^{1/2}, \quad T_{\text{max}} = 3/2\sqrt{2}\pi\alpha'^{1/2}. \quad (15)$$

Notice that there are no additional massless thermal modes at the self-dual temperature, $\beta_2^C = \pi^2 \alpha'$, from which we can infer that only one fixed line of the world-sheet superconformal field theory passes through the self-dual point, parameterized by the inverse temperature. The thermal type II strings are stable at least in the vicinity of the self-dual temperature.
The Kosterlitz-Thouless transition at $T_C$ is a continuous phase transition mapping the thermal type IIA string to the thermal IIB string. On the other hand, at the minimum and maximum temperatures, respectively, the $(1,0)$ and $(0,1)$ modes which are massive within the allowed temperature regime, turn tachyonic. They represent instabilities of the thermal type II string phase. Consequently, the tachyon-free thermal type II strings described here are inherently finite temperature non-supersymmetric ground states of string/M theory. The thermal ground state is physically sensible only for a background temperature of order the string scale, $T \approx T_C = 1/\pi \alpha'^{1/2}$. This is unambiguous indication that flat supersymmetric 10D spacetime without gauge fields is an unstable perturbative ground state for both type IIA and type IIB string theories [6]. In the conclusion, we point out that a rather simple resolution of this sickness of type II string theory does exist, but it requires the inclusion of potential gauge fields from a non-trivial Ramond-Ramond sector.

It is interesting that the source of this instability is distinct from that found in the finite temperature field theoretic analysis of [31]. The problem addressed here is that of the internal consistency required of a non-pathological, and modular invariant, thermal free string spectrum: the thermal strings do not gravitate at this order in string perturbation theory, and the origin of our no-go result should not be confused with the gravitational instability of flat space at finite temperature.

4 The 10D Heterotic Superstrings at Finite Temperature

Unlike the type II string which has no Yang-Mills fields in the absence of a Ramond-Ramond sector and, consequently, no weak coupling tachyon-free non-supersymmetric ground states at low temperatures, flat and tachyon-free non-supersymmetric ground states are known to exist in the perturbative heterotic string theory [23, 24, 25, 26, 52]. The dilaton one-point function is likely to be non-vanishing in such a vacuum, and it will be important to verify that the constant dilaton one-point function can consistently be absorbed in a renormalization of the bare string tension [28, 14]. This is discussed further in the conclusions. In the event of a non-constant one-point function for dilaton, or graviton, an inescapable flow to strong coupling would be inevitable. However, since our interest is in the equilibrium behavior of free heterotic strings, we will ignore such loop effects for the moment and consider the possibilities for a tachyon-free finite temperature ground state in the heterotic string theory.

We will find that the 10D $E_8 \times E_8$ and $SO(32)$ heterotic superstrings share a stable and tachyon-
free ground state at all temperatures starting from zero in the presence of a temperature-dependent Wilson line background, with non-abelian gauge group $SO(16) \times SO(16)$. The crucial difference from type II string theory is the availability of gauge fields at weak coupling [21, 22, 12]. It is interesting that even an infinitesimal change in the background temperature results in some of the massless vector bosons of the 10D gauge theory acquiring masses of order the supersymmetry breaking scale which, in this case, is the string scale, $\alpha'^{−1/2}$. This unexpected result is a consequence of the temperature-dependent Wilson line, necessitated by requiring a stable, and tachyon-free, thermal ensemble of heterotic strings.

We consider an equilibrium ensemble of free heterotic strings in nine noncompact dimensions occupying the box-regulated spatial volume $V = L^9(2\pi \alpha')^{9/2}$. The normalized vacuum functional, or the generating functional of connected one-loop vacuum string graphs, is given by:

$$W_{\text{het.}} = \int \frac{d^2 \tau}{4 \tau_2^2} (2\pi \tau_2)^{-9/2} |\eta(\tau)|^{-14} Z_{\text{het.}}(\beta) ,$$

(16)

where the precise action of the super-orbifold group on both world-sheet, and gauge, fermions that determines the form of $Z_{\text{het.}}$ remains to be specified. The free energy of the free heterotic string gas can be obtained from the usual relation: $F = -W_{\text{het.}}/\beta$. We begin with the modular invariant and spacetime supersymmetric vacuum functional of the ten-dimensional $E_8 \times E_8$ heterotic string:

$$W_{\text{het.}}|_{\beta=\infty} = \int \frac{d^2 \tau}{4 \tau_2^2} (2\pi \tau_2)^{-5} |\eta(\tau)|^{-16} \frac{1}{8 \eta^4} \left( \Theta_3 - \Theta_4 - \Theta_2 \right) \left[ \left( \frac{\Theta_3}{\eta} \right)^8 + \left( \frac{\Theta_4}{\eta} \right)^8 + \left( \frac{\Theta_2}{\eta} \right)^8 \right]^2 .$$

(17)

The partition function in Eq. (17) is of the form $Z_S(Z_8)^2$, where $Z_S$ is the holomorphic partition function given by the spacetime supersymmetric sum over chiral spin structures, and $(Z_8)^2$ is the contribution of the 32 gauge fermions. In bosonic form, $Z_8^2$ may be expressed as the lattice partition function for the 16D $E_8 \oplus E_8$ Euclidean self-dual lattice [12]. As in the thermal type II case, we wish to identify a a nonsupersymmetric and tachyon-free partition function at finite temperature, $Z_{\text{het}}(\beta)$, appropriate for substitution in Eq. (16). $Z_{\text{het}}$ describes the mass spectrum of free $E_8 \times E_8$ strings at finite temperature. It is required to be both modular invariant, and also self-dual under thermal duality transformations, reverting to the supersymmetric partition function, $Z_S(Z_8)^2$, in the limit of zero temperature. Finally, as evident from the analysis in [22], the vacuum functional of the finite temperature $E_8 \times E_8$ heterotic string is expected to map precisely into the vacuum functional of the finite temperature $SO(32)$ heterotic string under appropriate $SO(17, 1)$ transformations.

We will consider the action of the super-orbifold group $R(-1)^{N_F}$ accompanied by a temperature dependent Wilson line in the $E_8 \oplus E_8$ lattice. The reflection, $R$, has been considered at length in both the bosonic case [19], and in earlier sections. Let us focus on understanding the Wilson line background. Since $(-1)^{N_F}$ acts trivially on the timelike coordinate, $X^0$, and the 16 current algebra bosons, the result of a temperature-dependent timelike Wilson line follows from the analysis of supersymmetric 9D heterotic strings in [22]. Namely, we compactify the 10D $E_8 \times E_8$ supersymmetric string on a circle of radius $r_{\text{circ.}}$. Introduction of a radius-dependent Wilson line background, $A = \frac{\pi}{4}(1,0^7,-1,0^7)$, $x=(\frac{2}{\alpha'})^{1/2}r_{\text{circ.}}$, interpolates continuously between a 9D supersymmetric $SO(16) \times SO(16)$ string at generic radii and the non-compact limit where the gauge group is enhanced to $E_8 \times E_8$. Note that the states in the spinor lattices of $SO(16) \times SO(16)$ provide additional massless vector bosons only in the limit of infinite $x$. Consider the $(17, 1)$-dimensional
Wilson line can be understood as imposing a modified axial gauge condition:

\[ (p; l_L, l_R) \rightarrow (p'; l'_L, l'_R) = (p + wxA; u_L - p \cdot A - \frac{wx}{2}A \cdot A, u_R - p \cdot A - \frac{wx}{2}A \cdot A) \]  \quad (18)

\( p \) is a 16-dimensional lattice vector in \( E_8 \oplus E_8 \). The vacuum functional of the 9D supersymmetric theory, with generic radius and generic Wilson line in the compact spatial direction, can accordingly be written in terms of a sum over vectors in the boosted lattice:

\[ W_{\text{het.}}(r_{\text{circ}}; A) = \int \frac{d^2 \tau}{4\tau^2} (2\pi \tau)^{-5} |\eta(\tau)|^{-16} \frac{1}{8} \frac{1}{\eta^4} (\Theta_3^4 - \Theta_4^4 - \Theta_2^4) \left[ \frac{1}{\eta^{16}} \sum_{(p' , l'_L , l'_R)} q^{\frac{1}{2} (p'^2 + l'_L^2 + l'_R^2)} q^{\frac{1}{2} u'^2} \right] \]  \quad (19)

The corresponding compactification of the 10D \( E_8 \times E_8 \) string on the spacetime \( R^9 \times S^1 / Z_2 \) with Euclidean signature, describes the equilibrium thermal spectrum of \( E_8 \times E_8 \) strings at finite temperature. The interval length of Euclidean time is the inverse temperature, with \( \tau \) = \( \frac{1}{T} \). Turn on the temperature dependent Wilson line: \( A = \frac{2}{\pi} \text{diag}(1, 0^7, -1, 0^7) \) [22], where \( A \) is the timelike component of the vector potential, accompanied by \( R(-1)^{N_F} \) acting on the ten right-moving worldsheet fermions. The result will be a nonsupersymmetric, but tachyon-free, 9d \( SO(16) \times SO(16) \) heterotic string. We should clarify that, although it is not apparent in the form of the Wilson line, the Lorentzian self-dual boosted lattice is in fact invariant under a thermal duality transformation.

Notice that, from the viewpoint of the low-energy finite temperature gauge theory, the timelike Wilson line can be understood as imposing a modified axial gauge condition: \( A^0 = \text{const.} \). The dependence of the constant on background temperature is chosen so as to provide a shift in the string mass spectrum that precisely compensates for potential low temperature \( (n, 0) \) tachyonic modes. Such a special gauge choice may surprise the reader. We note that from the viewpoint of dynamics in the finite temperature gauge theory, the value of this constant is of no consequence. On the other hand, from the viewpoint of heterotic string theory, \( A = \frac{2}{\pi} \text{diag}(1, 0^7, -1, 0^7) \), \( 0 \leq x \leq \infty \), describes a one-parameter direction in field space along which supersymmetry is spontaneously broken at the string scale. While possibly not of phenomenological interest, it is remarkable that the mass spectrum of the resulting line of nonsupersymmetric string theories is \textit{tachyon-free at all temperatures starting from zero}. This is clear evidence that 10D N=1 supersymmetric flat space with either \( E_8 \times E_8 \) or \( SO(32) \) Yang-Mills gauge fields is a tachyon-free heterotic string ground state at weak coupling, under both infinitesimal, and finite, variation in the background temperature. At finite temperature, we have a spontaneous breaking of both supersymmetry and the non-abelian gauge symmetry. It would be of great interest to understand the dynamics underlying this phenomenon.

It remains to identify the form of the interpolating function \( Z_{\text{het.}}(\beta) \) that describes the modular invariant partition function of the \( SO(16) \times SO(16) \) theory, such that the \( E_8 \times E_8 \) partition function given in eq. (17) is recovered in the zero temperature limit. In addition, the vacuum functional is required to be self-dual under thermal duality transformations, with simultaneous interchange of thermal momentum and thermal winding modes. Consider the modular invariant function:

\[ Z_{\text{het.}} = \frac{1}{4} \sum_{n, w} \left[ \frac{\Theta_2}{\eta} (\frac{\Theta_4}{\eta})^8 (\frac{\Theta_3}{\eta})^4 - \frac{\Theta_2}{\eta} (\frac{\Theta_3}{\eta})^8 (\frac{\Theta_4}{\eta})^4 - \frac{\Theta_3}{\eta} (\frac{\Theta_4}{\eta})^8 (\frac{\Theta_2}{\eta})^4 \right] \frac{1}{q^{2u_L^2} q^{2u_R^2}} \]
\[ -\frac{1}{2} \sum_{n,w} e^{\pi i (n+w)} \left[ \left( \frac{\Theta_2}{\eta} \right)^4 \left( \frac{\Theta_3}{\eta} \right)^8 \left\{ \left( \frac{\Theta_2}{\eta} \right)^8 + \left( \frac{\Theta_4}{\eta} \right)^8 + \cdots \right\} \right] q^{2u^2_1} \bar{q}^{2u^2_2} + \text{other} \]  

(20)

The ellipses denote symmetrization among the three Jacobi theta functions. "Other" denotes the temperature independent contributions from the fixed points of the R-orbifold discussed in [19]. Note that taking the \( x \to \infty \) limit, by similar manipulations as in the type II case, yields the partition function of the supersymmetric 10D \( E_8 \times E_8 \) string. The Wilson line becomes irrelevant in the noncompact limit. We can follow the \( SO(17,1) \) transformation described above with a lattice boost decreasing the size of the interval [22]:

\[ e^{-\alpha_{00}} = \frac{1}{1 + |A|^2/4} \]  

(21)

This recovers the Spin(32)/\( \mathbb{Z}_2 \) theory compactified on an interval of size \( 2/x \), but with Wilson line \( A = x \text{diag}(1^8, 0^8) \) [22]. Thus, taking the large radius limit in the dual variable, with dual Wilson line background, gives instead the 10D Spin(32)/\( \mathbb{Z}_2 \) supersymmetric heterotic string. The thermal \( E_8 \times E_8 \) and \( SO(32) \) heterotic string are found to share the same tachyon-free finite temperature ground state. The Kosterlitz-Thouless transformation at \( T_C = 1/\pi \alpha'^1/2 \) is a self-dual continuous phase transition in this theory. We comment that combining the \( R(-1)^{N_F} \) action with modding out by the outer automorphism exchanging the two \( E_8 \) lattices gives a distinct tachyon-free ground state with gauge group \( E_8 \) which is discussed elsewhere [15, 36, 6, 50].

It is interesting to compare with a different noncompact limit of the nonsupersymmetric and tachyon-free 9d \( SO(16) \times SO(16) \) theory: consider directly taking \( \beta \to \infty \) with no corresponding action on the world-sheet fermions. Notice that although the Wilson line above becomes trivial in the noncompact limit, the approach to 10D occurs by a distinct sequence of continuously connected \( SO(17,1) \) transformations: in this case, simple radius change. As a consequence, the 10D theory is nonsupersymmetric, tachyon-free, and with unchanged gauge group, defined on a Euclidean spacetime \( R^9 \times S^1/\mathbb{Z}_2 \), where the \( \mathbb{Z}_2 \) denotes the bosonic R-orbifold. This theory describes the finite temperature behavior of equilibrium nonsupersymmetric \( SO(16) \times SO(16) \) heterotic strings living in a 10D flat spacetime with Lorentzian signature. This Lorentzian solution was first discovered in [24], and gives the unique modular invariant, nonsupersymmetric, and tachyon-free, 10D heterotic string partition function. The thermal partition function of the 9D theory takes the simple form:

\[ Z_{\text{NS}} = \frac{1}{4} \left[ \left( \frac{\Theta_2}{\eta} \right)^8 \left( \frac{\Theta_4}{\eta} \right)^4 \left( \frac{\Theta_3}{\eta} \right)^4 - \left( \frac{\Theta_2}{\eta} \right)^8 \left( \frac{\Theta_3}{\eta} \right)^8 \left( \frac{\Theta_4}{\eta} \right)^4 - \left( \frac{\Theta_3}{\eta} \right)^8 \left( \frac{\Theta_4}{\eta} \right)^8 \left( \frac{\Theta_2}{\eta} \right)^4 \right] \sum_{n,w} q^{2u^2_1} \bar{q}^{2u^2_2} + \cdots , \]  

(22)

where the ellipses denote temperature independent contributions from the fixed points of the R-orbifold. The term in square brackets is the partition function of the 10D nonsupersymmetric and tachyon-free string found in [24]. The \((1,1)-d\) lattice partition function containing thermal momentum and winding modes is completely decoupled. The thermal self-duality transition at the Kosterlitz-Thouless point in this theory is identical to that in the closed bosonic string. The only distinction is that the string vacuum functional, the Helmholtz and Gibbs free energies, the internal energy, and all subsequent thermodynamic potentials, are both finite and tachyon-free. Notice that the low energy finite temperature gauge theory is now quantized in ordinary axial
Consider instead an orbifold compactification of the 10D nonsupersymmetric \(SO(16) \times SO(16)\) string, acting with the order two twist, \(e^{2\pi i \delta \cdot p}\), where \(\delta\) is a half-winding lattice vector. This was referred to as the twist I orbifold in [16]. It was shown to have a small radius limit in which spacetime supersymmetry is asymptotically restored with simultaneous enhancement of the gauge group to \(E_8 \times E_8\). This is the reverse of the phenomenon we considered above. It may be interpreted as the spontaneous restoration of symmetries at high temperatures, with asymptotic approach to zero radius. We should clarify that the result of a thermal duality transformation on the vacuum functional of the self-dual twist I orbifold, will give the vacuum functional of the dual twist II orbifold of [16], where twist II refers to \(\delta\) a half-momentum lattice vector. Written in the dual variable, the vacuum functional demonstrates a spontaneous restoration of supersymmetry with enhancement of gauge group to \(E_8 \times E_8\) in the large dual radius, or zero temperature, limit. Presumably, the 9D nonsupersymmetric and tachyon-free \(SO(16) \times SO(16)\) string can also be obtained as an orbifold compactification of the 10D \(SO(32)\) heterotic superstring. This would complete the correspondence with the circle of dualities described above. What underlies this correspondence is the well-known equivalence of quantized Wilson line backgrounds with a shift in the periodicity, or world-sheet boundary condition, of the chiral bosons in the current algebra underlying the gauge lattice of the heterotic string. The description in terms of Wilson line backgrounds [22] gives a clearer picture of the connectivity of the moduli space of the finite temperature theory.

The Helmholtz free energy of the tachyon-free gas of free heterotic strings follows from the definition below Eq. (16), and is clearly finite at \(T_H\), with no evidence for a Hagedorn divergence and, consequently, no Hagedorn phase transition. The internal energy of the ensemble of free \(SO(16) \times SO(16)\) heterotic strings can be computed as follows. Recalling the thermodynamic relations and definitions given in [19], we obtain the following expression for the internal energy:

\[
U = - \left( \frac{\partial W}{\partial \beta} \right)_V = \frac{1}{2} \int \frac{|d\tau|^2}{4\tau^2} (2\pi \tau_2)^{-9/2} |\eta(\tau)|^{-16} \frac{4\pi \tau_2}{\beta} \sum_{n,w \in \mathbb{Z}} \left( \frac{w^2 x^2}{4} - \frac{n^2}{x^2} \right) \cdot q^{\frac{w^2 l^2}{2}} \cdot \bar{q}^{\frac{w^2 l^2}{2}} \cdot Z_{[SO(16)]^2},
\]

where \(Z_{[SO(16)]^2}\) denotes the factor in square brackets appearing in, respectively, Eq. (20), or Eq. (22). The two results differ in the noncompact zero temperature limit giving, respectively, the supersymmetric \(E_8 \times E_8\), or nonsupersymmetric \(SO(16) \times SO(16)\), 10D string theories. We emphasize that an unambiguous specification of the zero temperature limit of the finite temperature heterotic string requires specifying a possible Wilson line background in the timelike direction. From the viewpoint of the low energy gauge-gravity theory, this is the assertion that recovering the supersymmetric zero temperature limit requires imposing a particular axial gauge choice for the finite temperature gauge-gravity theory.

The thermodynamic potentials can be computed using the same methods as shown for the self-dual— but tachyonic, ensemble of free closed bosonic strings in [19]. It is evident that taking successive partial derivatives of the string effective action functional yields an infinite hierarchy of analytic functions of the inverse temperature. Each of the thermodynamic potentials displays a continuous phase transition at the self-dual temperature, unambiguously identifying a phase transition of the Kosterlitz-Thouless type [11, 19]. Of course, unlike the ensemble of closed bosonic
strings, the thermodynamic potentials of the heterotic ensemble are finite normalizable functions at all temperatures starting from zero. We emphasize that, unlike the self-dual vacuum functional, the expressions for the Helmholtz free energy and generic thermodynamic potentials are not invariant under thermal duality transformations. The vacuum functional of the $SO(16) \times SO(16)$ string is known to be negative [24], due to the preponderance of spacetime fermionic modes over spacetime bosonic modes. The internal energy of free heterotic strings is negative at low temperatures, vanishing precisely at the self-dual temperature $x_c=2^{1/2}$, $T_c=1/\pi \alpha'^{1/2}$. The Helmholtz, or Gibbs, free energies are minimized at the self-dual temperature. As is physically reasonable for a stable thermodynamic ensemble at equilibrium [5], the internal energy is found to be a monotonically increasing function of temperature.

5 Conclusions

We have shown that the clarification of the connectivity of the moduli space of nine dimensional heterotic string ground states given in [21, 22] enables interpretation of the tachyon-free, and nonsupersymmetric, 9D $SO(16) \times SO(16)$ heterotic theory as the finite temperature ground state shared by the 10D $E_8 \times E_8$ and $SO(32)$ heterotic superstrings [6]. The crucial difference in the type II strings is the fact that, in the absence of gauge fields, even an infinitesimal variation in background temperature away from the zero temperature vacua—the 10D supersymmetric flat spacetime IIA or IIB strings, results in low temperature tachyonic modes in the string spectrum. For the 10D type I and heterotic string theories, on the other hand, the presence of gauge fields enables turning on a temperature dependent timelike Wilson line, resulting in a tachyon-free and nonsupersymmetric string spectrum at all temperatures starting from zero. The tachyon-free finite temperature string theories have nonabelian gauge group $SO(16) \times SO(16)$ [6, 50].

We should emphasize that the absence of tachyonic modes in the string mass spectrum resulting from an infinitesimal change in the background temperature away from zero, is a crucial low temperature consistency condition for any phenomenologically viable supersymmetric String/M theory ground state describing our Universe. Surprisingly, this concern has not received much attention in the String/M phenomenology literature. Finite temperature necessarily breaks supersymmetry. Not surprisingly, one or other massless modulus, including the dilaton field, could develop a potential with subsequent evolution to imaginary eigenvalues in the mass matrix. But our analysis of free, noninteracting, closed strings shows that the tachyonic thermal instabilities have a different origin—with no obvious correspondence in the low energy effective field theory. They are a consequence of a well-established, ultraviolet-infrared (UV-IR), correspondence in the amplitudes of perturbative string theories. The UV-IR correspondence manifests itself as modular invariance of the amplitudes of any closed string theory or, equivalently, as the property of open-closed worldsheet duality in the amplitudes of a perturbative open and closed string theory [9, 12, 14]. Unlike field theory, where finite temperature alters the infrared behavior of the theory, leaving its renormalizability and short distance properties untouched—at least at low temperatures, in perturbative string theory it is simply not possible to alter the IR limit, without simultaneously altering the UV behavior.

To understand why, consider exciting only the lowest-lying $n=1$ momentum mode in the zero temperature vacuum. At low temperatures, the energy of the $n=1$ thermal vacuum is an apparently negligible shift from the energy of the zero temperature vacuum: $+\frac{1}{\beta^2}$, $\beta>>1$. Nevertheless, modu-
lar invariance of the one-loop closed string amplitude has the consequence of introducing a Virasoro tower of states with string-scale masses contributing to the intensive vacuum functional and, consequently, to the Helmholtz free energy and the remaining thermodynamic potentials. The Virasoro tower arises from excitations of the infinity of allowed string oscillators in the n=1 thermal vacuum. Moreover, thermal self-duality of the heterotic theory or, in the case of type II, invariance under the thermal duality transformation mapping IIA to IIB, implies that, for any momentum mode in the spectrum, we must include a corresponding winding mode, w=1. Further, since the tower of oscillator excitations of the n=1 and w=1 vacua have comparable mass to states in the Virasoro tower of the n=2, and w=2, thermal vacua, it would be inconsistent to leave these out. Iterating this argument, we see that even an infinitesimal shift in temperature away from zero temperature in the closed string vacuum, results in a vacuum functional, and free energy, with contributions from an infinite sum over momentum and winding thermal vacua, each contributing a Virasoro tower of excited string states.

In the Introduction, we have explained why the huge multiplicity of states in the string spectrum accounts for both the exponentially diverging density of states function in the UV limit and, as a consequence of modular invariance, the absence of any signal of this exponential degeneracy in the behavior of the free energy. That this is only true of a tachyon-free string theory is a consequence of the UV-IR relations: as was first noted in [9], the decidedly IR phenomenon of a tachyonic state in the string spectrum can manifest itself as an apparent UV divergence in the free energy. This can be seen by a Poisson resummation of the modular invariant partition function, which switches the UV and IR asymptotics [12]. In the superstring theories, the UV-IR consistency conditions have dramatic consequence. The absence of the zero temperature tachyon in the supersymmetric spectrum, and the vanishing free energy, are a consequence of the GSO projection: the cancellation follows from the existence of modular invariant fermionic string partition functions which also respect the global $SO(8)$ rotational symmetry of the transverse degrees of freedom on the worldsheet in a Lorentz invariant 10D ground state. The requirement of partition functions that are both invariant under the one-loop modular group, and which also respect the global $SO(8)$ symmetry, is remarkably restrictive. As a consequence, there are no viable alternatives to the supersymmetric combination of spin structures other than those considered in [7, 5], and also in Sections 3 and 4. Thus, there are no “small perturbations” of the supersymmetric GSO projection: modular invariance of the global $SO(8)$ invariant thermal partition functions destroys the tachyon cancellation mechanism in entirety, even for an infinitesimal variation in temperature away from zero temperature.

Our results are striking confirmation of the necessity to extend one’s purvey of type II string theory to include gauge fields arising due to the presence of Dbrane or NSbrane fluxes. Whether this can be achieved at weak string coupling remains an interesting question. We note that it should be possible to use the extensive exploration of tachyon-free type 0 modular invariants in the recent literature, whether by the introduction of a Ramond-Ramond flux [34], or by suitable orbifold or orientation-reversal action on the type 0 partition function [35]. A Ramond-Ramond flux will shift the mass squared of the zero temperature tachyonic state to positive values. In this case, it should be straightforward to construct a modular invariant type II vacuum functional which also transforms correctly under thermal duality: namely, undergoing a Kosterlitz-Thouless continuous phase transition at $T=T_C$ which maps the thermal IIA string to the thermal IIB string, and with a tachyon-free spectrum at all temperatures above zero. An important issue is the clarification of
a natural choice of RR or NS fluxes in keeping with the interpretation of the tachyon-free finite temperature ground state as a one-parameter fixed line describing a stable IIA or IIB thermal ensemble. This, of course, assumes a resolution at weak string coupling.

In the event that the resolution of this puzzle involves a flow to strong type II string coupling, strong-weak coupling dualities in the moduli space of String/M theory will be helpful. To begin with, recall that IIA/M theory orbifolds with Wilson line background for the Ramond-Ramond one-form potential have been widely studied in the duality literature [43]. There is a close correspondence between the heterotic self-dual momentum lattice and the cohomology lattice describing the moduli space of orbifolds of M/IIA theory compactified on quantum K3 surfaces with Wilson line background for the RR one-form potential, extensively explored in [43]. It is possible that a Wilson line background for the RR one-form potential gives a simple, and universal, resolution of the tachyonic instability in the thermal IIA string theory. Thermal duality can then be invoked to infer a similar resolution for the thermal IIB string.

In this context, notice that the shift in momentum eigenvalue: \((n, w) \rightarrow (n + \frac{1}{2}, w + \frac{1}{2})\), introduced in Eq. (13) in order to obtain a stable thermal ensemble at temperatures of order the string scale, results in a temperature-dependent shift in the vacuum energy of the \((n, w)\) thermal vacuum. This is reminiscent of the 10D cosmological constant in the massive IIA string [44], the low energy effective limit of which is Roman’s massive 10D IIA supergravity [45]. Dimensional reduction of the massive IIA supergravity gives a 9D massive supergravity which coincides with a Scherk-Schwarz dimensional reduction of the IIB theory [46, 47]. Finally, while the massive IIA supergravity cannot be obtained from 11D supergravity, or any covariant massive deformation thereof, Hull [48] has pointed out that the massive IIA superstring nevertheless has a consistent M theory origin. In previous work [49], we have noted that the mass parameter appears to be quantized in string soliton backgrounds of the 9D massive supergravity with a consistent type II world-sheet description. Nevertheless, in the absence of gauge fields, the tachyonic instability in the thermal type II string at a temperature of \(O(\alpha'^{1/2})\), remains a puzzle. We remark that it is possible that the low energy description of the putative stable and tachyon-free IIA or IIB thermal ensemble, of necessity, is always a ground state of the massive 9D type II supergravity: a finite temperature vacuum of the massive IIA string. As emphasized above, since IIA and IIB are related by a thermal duality transformation in 9D, we may equivalently interpret this as a thermal IIB vacuum. If true, this would give important insight into M theory beyond its low energy 11D supergravity effective limit [47, 48, 49].

It is also of interest to examine the full generality of the tachyonic instability at the string-scale temperature, \(T_{\text{min}}\), found in the thermal IIA and IIB strings described in Section 3.1. Consider compactifying this theory on an additional circle: namely, the IIA or IIB string compactified on the Euclidean space \(R^8 \times S^1 \times S^1 / \mathbb{Z}_2\). Upon compactification, the arguments given in section 3.1 for determining the supersymmetry breaking phases, and the general form of the modular invariant combination of Jacobi theta functions in the fermionic partition function, are unchanged. The mass formula gains a new contribution from the nontrivial momentum and winding modes in the compact spatial direction, \(X^9\). Under a spatial \(T_9\)-duality mapping IIA to IIB, we find that spatial IIA momentum modes are switched with spatial IIB winding modes, and vice versa, so that, from momentum conservation, the \(n_0 = w_0 = 0\) sector is retained in the physical state spectrum. This sector of the theory has thermal tachyons obeying the mass formula given in Eq. (14). Thus, by the same reasoning as given in section 3.1, the tachyonic thermal modes of the thermal flat space type II
string are retained in the zero spatial momentum sector of the compactified thermal type II string.

Notice that further compactification, or orbifolding, of additional spatial coordinates has no impact whatsoever on the thermal tachyonic instability in the untwisted zero spatial momentum sector of the compactified thermal type II string. We comment that taking the large radius limit of the Euclidean space, \( \mathbb{R}^n \times T^{0-m} \times S^1 / \mathbb{Z}_2 \), or any \( \mathbb{Z}_N \) orbifold thereof, where the \( \mathbb{Z}_N \)'s act as spatial rotations and the orbifold singularities are appropriately repaired in the noncompact limit, will give an ALE space: an asymptotically locally Euclidean space of the general form \( R^k / \Gamma \). In our example, taking the large radius limit for all ten coordinates describes the asymptotic approach to a noncompact type II vacuum in the vicinity of the zero temperature supersymmetric vacuum. However, in the absence of a timelike Wilson line, the thermal tachyonic instability expected at the string-scale minimum temperature would appear to be an obstruction to obtaining a noncompact ALE space in the timelike direction. This appears to be a counter-example to the cases discussed in [30].

The broader question of whether the formation of a condensate in a tachyonic closed string ground state can drive a phase transition to a tachyon-free vacuum—non-supersymmetric, or supersymmetric, is of considerable interest both in the context of finite temperature instabilities, but also for a deeper understanding of the configuration space of String/M theory. This question has been addressed in recent work [30] in the context of the conjectured IIA-type 0 dualities [32, 34, 33]. The \( O(\alpha') \) analysis of the fate of closed string tachyons in nonsupersymmetric ALE spacetimes in [30] suggests that a chain of such phase transitions will always terminate in a stable supersymmetric ALE spacetime. We note that a more detailed analysis of this issue using worldsheet methods, and in compact flat spacetime geometries, may be viable in the heterotic string theory. Recall that tachyonic vertex operators correspond to relevant perturbations of the worldsheet superconformal field theory and, from Zamolodchikov's c theorem, the RG flow is towards a fixed point of lower matter central charge. In the manifestly Weyl invariant framework of string theory, this implies nontrivial dynamics for the Liouville field along the flow to the new fixed point.

Ordinarily, this would be considered an incalculable problem because of the famed \( \hat{c} = 1 \) barrier [37] to an analytic treatment of the Liouville superconformal field theory. But in the context of the finite temperature induced tachyonic instability it may be possible to address relevant flows: the \( (X^0_E, \phi_L) \) dynamics is a 2D superconformal field theory coupled to 2D supergravity, which fits nicely within the realm of solvable 2D field theories [37, 39]. We should point out that the modular invariant 10D nonsupersymmetric and tachyonic heterotic partition functions have been classified, both in an analysis of abelian orbifolds by Dixon and Harvey [25], and also in the free fermionic spin structure construction by Kawai, Lewellen, and Tye [26]. Thus, all of the potential tachyonic noncompact limits of the stable and tachyon-free finite temperature heterotic ground state are known. It is natural to conjecture that any unstable 10D heterotic vacuum would of necessity flow to the unique tachyon-free 9D nonsupersymmetric heterotic string ground state, and with gauge group \( SO(16) \times SO(16) \). Furthermore, we should note that there already exists a nonperturbative framework for 2D bosonic string theory in the form of the \( c=1 \) matrix model [42], with extensive studies

\[ ^3 \text{As an aside, note that it is perfectly consistent with the c-theorem that the number of noncompact spacetime dimensions at the new fixed point is lower: a dynamical mechanism for choosing a stable ground state with noncompact spacetime dimension } D<10, \text{ similar in spirit to the dynamical approach pursued in the nonperturbative matrix formulation [40, 41].} \]
of tachyon dynamics, although the $\hat{c}=1$ super-matrix model remains puzzling in many respects. Nevertheless, we find it intriguing that an exactly solvable 2D theory could provide insight into the nonperturbative finite coupling behavior of realistic supersymmetric heterotic string theories at finite temperature.

In [6], motivated by a knowledge of the phase structure of the simplest lattice field theories, and the combined conjectural and numerical evidence for the two-parameter $(g_{YM}, \beta)$ phase diagram of continuum Yang-Mills theory at finite temperature and finite gauge coupling, we noted that it is natural to ask whether analogous questions can be addressed in String/M theory—- even in limited regions of the moduli space. We emphasize that our goal here is not further elucidation of finite temperature gauge theory at strong coupling— although this would be an added plus. Instead, the general theme of the work in [49, 6, 50, 14] has been to invoke analogy with gauge theory in order to motivate world-sheet computations probing sub-string scale distances which might give insight into nonperturbative aspects of String/M theory, possibly even at strong coupling. The spontaneous breaking of thermal duality by thermal Dbranes in the type I' string, and a worldsheet computation of the pair correlator of timelike Wilson loops with fixed loop separation $r<\alpha'^1/2$ in the tachyon-free thermal type I' ground state, are explored at length in [6, 50]. As in ordinary gauge theory, the correlator serves as an order parameter for a continuous phase transition in the thermal type I' string at a string-scale transition temperature computed in [6, 50]: above the transition temperature, we find that the order parameter vanishes asymptotically in the $\beta \to 0$ high temperature limit, signaling deconfinement.

We conclude with the encouraging observation that our results are evidence for the limited validity of strong-weak coupling duality conjectures in the absence of spacetime supersymmetry [43, 51, 52, 53, 54, 33]: an infinitesimal background temperature cannot change the nature of the weak-strong coupling dualities holding in the vicinity of a non-pathological weakly-coupled supersymmetric ground state. Thus, the strong-weak coupling duality relations valid for the supersymmetric dual string pair, are expected to hold for the tachyon-free finite temperature dual string pair, at least at infinitesimal temperature. Of necessity, such dual string pairs will exhibit broken supersymmetry with inverse radius, $1/\beta$, playing the role of a small susy-breaking control parameter.

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Note Added (Sep 2005): Many of the points made in this paper are either extraneous, or incorrect in the details, although the broad conclusions summarized in the abstract stand. Namely, that there is no self-consistent type II superstring ensemble, in the absence of a Yang-Mills gauge sector. And that both heterotic and type I theory have equilibrium canonical ensembles free of
thermal tachyons; a crucial role is played by the temperature dependent Wilson line wrapping Euclidean time. I refer the reader to hep-th/0506143.

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