SO(10) MSGUT: Spectra, Couplings and Threshold Effects

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ABSTRACT

We compute the complete gauge and chiral superheavy mass spectrum and couplings of the Minimal Susy GUT (based on the $210 - \overline{126} - 126 - 10$ irreps as the Higgs system) by decomposing SO(10) labels in terms of Pati Salam subgroup labels. The spectra are sensitive functions of the single complex parameter that controls MSGUT symmetry breaking. We scan for the dependence of the threshold corrections to the Weinberg angle and Unification scale as functions of this parameter. We find that for generic values of the GUT scale parameters the modifications are within 10% of the one loop values and can be much smaller for significant regions of the parameter space. This shows that contrary to longstanding conjectures, high precision calculations are not futile but rather necessary and feasible in the MSGUT. The couplings of the matter supermultiplets are made explicit and used to identify the channels for exotic ($\Delta B \neq 0$) processes and to write down the associated bare $d = 5$ operators (some of both are novel). The mass formulae for all matter fermions are derived. This sets the stage for a comprehensive RG based phenomenological analysis of the MSGUT.

1 Introduction

The Supersymmetric SO(10) GUT based on the $126, \overline{126}, 210$ Higgs multiplets \cite{1, 2, 3, 4} has, of late, enjoyed a much delayed bloom of interest motivated by its economy
and predictivity. Besides the traditional virtues of SO(10) this is the minimal renormalizable model which has shown itself capable of matching the observed fermion spectra, including the prima facie GUT repellent feature of maximal mixing in the neutrino sector \[5, 6, 7\]. Beyond the traditional scenario of perturbative unification of couplings due to the RG flow between \(M_S\) and \(M_X\) it also offers strong indications that the gauge coupling becomes strong above the GUT scale. We have argued \[8, 9\] that this necessarily leads to dynamical symmetry breaking of the GUT symmetry at a scale \(\Lambda_U\) (just above the perturbative unification scale \(M_U \sim 10^{16}\text{GeV}\)). Utilizing the quasi-exact supersymmetry at the GUT scale we made plausible \[8\] a scenario in which \(\Lambda_U\) is *calculated* determined by only the low energy data and structural features of the theory (such as the gauge symmetry group, supersymmetry and the very restricted Higgs multiplets available to generate fermion masses -particularly neutrino masses- in a renormalizable theory). This scenario offers interesting possibilities of a novel picture of elementarity and dual unification characterized by a new fundamental length scale \(\sim \Lambda_U^{-1}\) characterizing the “hearts of quarks.” \[8, 9\].

The MSGUT is thus the focus of multi-faceted interest and a detailed phenomenological analysis of the theory in terms of the structure dictated by its GUT scale minimality is thus called for. However such an analysis has been delayed by the computational difficulty of obtaining the GUT scale spectra and couplings and the effective Lagrangian describing the normal and exotic features (baryon and lepton violation etc) of the GUT derived MSSM (i.e extended by the leading \(d = 5\) exotic operators of the theory). The spectra and couplings are necessary to analyse threshold corrections to the gauge couplings near the GUT scale and are also a crucial input into deriving the lagrangian for exotic processes and parameters mediated by GUT scale massive fermions. In \[11\] we presented techniques for computing the decomposition of SO(10) invariants in terms of the unitary labels of its maximal (Pati-Salam) sub group \(SU(4) \times SU(2)_L \times SU(2)_R\). Once this decomposition is performed the computation of the complete spectrum and couplings is quite easy and the long standing vagueness regarding the “Clebsches” that arise can finally be banished. This allowed us to present, by way of illustration of the power of our method, the two most important mass matrices \((4 \times 4\) and \(5 \times 5\) respectively) affecting Electro weak symmetry breaking, fermion masses and nucleon decay: namely those for the MSSM type Higgs \(SU(2)_L\) doublets and baryon number violation mediating \(SU(3)_c\) colour triplets that mix with the the doublets and triplets of the fermion mass (FM) Higgs \((10, 1\overline{2}, 3)\). Moreover since our methods allow computation of the actual couplings of Higgs to spinors we could also obtain the \(d = 5\) operators for Baryon violation generated via exchange of triplet Higgsinos contained not only in the traditional \((6, 1, 1)\) submultiplets (of the \(10\) or those in the \(1\overline{2}6\) \[12\]) which had been noticed to provide a connection between neutrino masses and proton decay, but also in other channels arising from the exchange of colour triplets contained in \((10, 1, 3)\) submultiplets involved in neutrino mass generation \[11\]. A more complete calculation of these spectra
and effective lagrangians and an initial estimate of their effects is the subject of this paper.

While the calculations presented were in their final stages we were collaborating and cross checking with another parallel calculation [13] of mass spectra using a different [10] method which has since been published. Moreover another group [14, 15] has also recently published a calculation (using the same methods as [13]) of spectra and baryon decay effective potentials recently. As far as computation of chiral superfield spectra are concerned our results coincide (upto normalization and phase conventions) with those of [13]. However both our results diverge [11, 13] in certain details from the chiral spectra given in [14]. Moreover as already noted by us in (an update to) [11] we also disagreed with the results of [14] regarding the Higgsino channels available for baryon decay in this model. We found [11] that [14] obtained couplings between the $126$ multiplet and matter in the spinorial $16$ representation which were in contradic- tion with ours [11] not only as regards the numerical coefficients but also in the heavy Higgs channels to which matter fields couple in a baryon number violating way. In the revised version of [14] i.e [15] this defect has apparently been corrected at least modulo disagreement on values of clebsches. We shall try to settle these questions by tracing the reasons for the continuing discrepancy in explicit detail and confirm our previous assertions. We have also analyzed the gauge Dirac multiplet structure arising from the super-Higgs effect and the masses and vevs responsible for the Type I and Type II mechanisms [16, 17] of neutrino mass generation.

We emphasize that our method allows computation, not only of spectra but also of the couplings of all the multiplets in the theory (whether they are renormalizable or heavy-exchange induced effective couplings) without any ambiguity. Moreover our results are obtained by an analytic tensorial reprocessing of labels of fields in the Lagrangian. This approach might thus find preferment with field theorists in comparison with the more restricted capabilities of the approach of [10], which, so far, has not proved capable of generating all the Clebsches of the SO(10) theory and which relies on an explicit multiplet representative and computer based approach which is tedious to connect to the unitary group tensor methods so familiar to particle theorists.

It has long been held by some that $SO(10)$ GUTs specially Susy $SO(10)$ GUTs, are essentially self-contradictory [18] due to the apparently enormous threshold effects that might arise due to the large number of superheavy Higgs residuals in these theories. Thus the authors of [18] speak of the the “futility of high-precision calculations in SO(10)”. However these assertions have never been tested against any actual computations of mass spectra of a Susy SO(10) GUT and are only worst case estimates assuming that no cancellations occur. However we expect that cancellations will generically occur since the lepto-quark mass has no reason to lie at the edge of the mass spectrum nor are the coefficients all of the same sign. With the computed spectra now available, the threshold effects on observable quantities such as $\sin^2\theta_w, M_X$
etc become computable in terms of the relatively small number \[4\] of GUT scale parameters of the MSGUT. In fact \[4\] a single complex parameter controls the solutions of the cubic equation in terms of which all the superheavy vevs are defined. We have performed a preliminary scan of the parameter space of the MSGUT to see what is the typical size of these corrections. We find the striking result that such threshold corrections are generically \(\leq 10\%\) of the 1-loop results\[23, 24, 25, 26\] that underpin the the GUT scenario’s viability. Moreover, for significant and possibly interestingly restricted regions of parameter space these corrections can be much smaller i.e as small as \(.5\%\) of the one loop results. Thus far from indicating futility our results indicate that a thoroughgoing investigation of the compatibility of low energy precision data with the threshold corrections may significantly constrain the parameter space of the MSGUT. In any case we show that inclusion of threshold corrections is necessary and not futile. Such an analysis is now in progress \[27\].

In Section 2. we present a brief review of the principal features of the minimal Susy SO(10) theory \[1, 2, 3, 4\] and compute the gauge supermultiplet masses. In Section 3. we provide the PS reduction of the SO(10) Higgs superpotential. From this we computed the Chiral fermion mass terms and thus the supermultiplet spectra which we discuss here and list in an Appendix . In Section 4. we compute the threshold corrections to the 1-loop values of the unification scale \(M_X\) and \(\sin^2\theta_W(M_S)\). In Section 5. we present the couplings of the matter fields to FM Higgs fields in the superpotential as well as their couplings to the gauginos of the SO(10) model. This permits us to identify the possible channels for baryon violation in the low energy theory via exchange of Higgsinos or gauginos and compute the relevant effective lagrangians. Using the associated mass matrices we write down the \(d = 5\) effective lagrangians for baryon and lepton number violation which arise via exchange of superheavy fermions. In Section 6. we discuss the mass formulae for the matter fermions in this model. The majorana mass terms of the left and right handed neutrinos and the \(SU(2)_L\) triplet micro-vev responsible the Type II mechanism for neutrino mass is calculated along with the charged fermion mass matrices. In a final section we discuss our conclusions and results and plans for further investigations using the results derived here.

## 2 The Minimal Susy GUT

In accordance with our basic rationale we shall deal with a renormalizable globally supersymmetric \(SO(10)\) GUT whose chiral supermultiplets consist of “adjoint multiplet type” (or AM) totally antisymmetric tensors: \(210(\Phi_{ijkl}), \overline{126}(\Sigma_{ijklm}), 126(\Sigma_{ijklm})\) \((i,j = 1...10)\) which serve to break the GUT symmetry to the SM, together with Fermion mass (FM) Higgs \(10\)-plet(\(H_i\)). The \(\overline{126}\) plays a dual or AM-FM role since besides enabling Susy preserving GUT symmetry breaking, it also enables the generation of realistic charged fermion masses and neutrino masses and mixings (via the Type I and/or Type II mechanisms); three spinorial \(16\)-plets \(\Psi_A(A = 1, 2, 3)\)
contain the matter supermultiplets together with the three conjugate neutrinos \((\bar{\nu}^A_L)\).
The \(126(\Sigma), 126(\Sigma')\) pair is required to be present together to preserve Susy while
breaking \(U(1)_R \times U(1)_{B-L} \to U(1)_Y\) and is capable of generating realistic
neutrino masses and mixings via the type I or type II seesaw mechanisms. The complete superpotential in this theory is the sum of

\[
W_{AM} = \frac{1}{2} M_H H^2_i + \frac{m}{4!} \Phi_{ijkl} \Phi_{ijkl} + \frac{\lambda}{4!} \Phi_{ijkl} \Phi_{klmn} \Phi_{mnij} + M \frac{1}{5!} \Sigma_{ijklm} \Sigma_{ijklm}
\]
\[+ \frac{n}{4!} \Phi_{ijkl} \Sigma_{ijmn} \Sigma_{klmno} + \frac{1}{4!} H_i \Phi_{ijklm} (\gamma \Sigma_{ijklm} + \bar{\gamma} \Sigma_{ijklm})\]

(1)

and

\[
W_{FM} = h'_{AB} \psi_A^T \psi_B \Sigma_{ij} + \frac{1}{5!} f'_{AB} \psi_A^T \psi_B \Sigma_{ijklm} \Sigma_{ijklm} + \frac{1}{4!} \Phi_{ijkl} \Phi_{ijkl} + H_i^* H_i + \Psi_A^* \Psi_A
\]

(2)

Our notations and conventions for spinors can be found in [11]. The Yukawa couplings \(h'_{AB}, f'_{AB}\) are complex symmetric \(3 \times 3\) matrices and one of them, say \(f'\) can thus be
diagonalized (by an orthogonal transform \(UfU^T\) using a Unitary matrix \(U\) which leaves the matter kinetic terms invariant) to a real positive diagonal form \(f' = F \delta_{AB}\), thus leaving 15 residual real parameters in \(W_{FM}\). In addition the 7 complex parameters in \(W_{AM}\) can be reduced to 10 real ones by absorbing 4 phases by Higgs field
definitions. Then together with the gauge coupling one has in all exactly 26 non-soft parameters. Coincidentally, MSSM also has 26 non-soft couplings consisting of 9 quark and charged lepton masses, 3 majorana neutrino masses 3 quark (CKM) and 3 lepton (PMNS) mixing angles and 1 quark but 3 lepton CP phases together with 3 gauge couplings and a \(\mu\) parameter. Thus we see that the 15 parameters of \(W_{FM}\) must be essentially responsible for the 22 parameters describing fermion masses and mixings in the MSSM.

The kinetic terms are given by covariantizing in the standard way the global SO(10) invariant D-terms

\[
\left[\frac{1}{2.5!} (\Sigma^*_{ijklm} \Sigma_{ijklm} + \Sigma^*_{ijmn} \Sigma_{klmno}) + \frac{1}{4!} \Phi^*_{ijkl} \Phi_{ijkl} + H_i^* H_i + \Psi_A^* \Psi_A\right]_D
\]

(3)

Note that the extra factor of \((1/2)\) achieves canonical normalization for the 126 independent component fields of the self dual ( anti self-dual ) \(126(126^*)\) representa-
tions which would be otherwise be overcounted. We thus, unfortunately, differ from
the normalization used in the parallel computations of [13] with which our results are
nevertheless in agreement after appropriate rescalings and rephrasings of parameters
and fields. We emphasize that all our redefinitions of labels are unitary and thus
maintain unit norm relative to the above kinetic terms.1

1The relations between the quantities of [13] (denoted by primes) and ours are \(\Sigma' = \Sigma/\sqrt{2}, \Sigma = \Sigma/\sqrt{2}\) (also for vevs) \(\gamma' = \sqrt{2} \gamma, \bar{\gamma}' = \sqrt{2} \bar{\gamma}, \eta' = 2 \eta, M' = 2 M\).
The economy of the above superpotential eqns. (1, 2) is remarkable[4]. It’s few couplings together with the functional flexibility of the chosen Higgs multiplet set and its ability (in common with other renormalizable models using just $10 - \overline{126}$ FM Higgs) to fit all the fermion mass data[6, 7], justify its claim to being the MSGUT. The “small” number of non-soft parameters (26 as in the MSSM) implies that after fitting [5, 6, 7] the known quark, charged lepton masses and quark mixing angles together with the neutrino mass splittings very little play is left in the model and it becomes predictive and thus falsifiable. The nearest related model (NMSGUT?) (in some ways more logically complete since all the FM channels allowed by renormalizability would then be utilized) might be considered to be the one obtained by adding a 120-plet SO(10) FM Higgs. Alternatively one may consider SU(5) supplemented with right handed neutrinos or non-renormalizable terms [4]. Both models are far less economical and not so predictive. Therefore, as advocated in detail in [4], the first priority should be to pin down the predictions of this model. We began the development of a detailed framework for handling the group theoretic complexity of susy SO(10) models generally in [11] and this paper presents the results of calculations using the techniques developed there for computing couplings and spectra for MSSM fields from the MSGUT tree action by decomposing the fields according to the $SU(4) \times SU(2)_L \times SU(2)_R$ or Pati-Salam (PS) maximal subgroup.

We now specify how the symmetry is broken down to the MSSM gauge group [1, 2, 4] by superlarge vevs contained in the $210, 126, \overline{126}$-plet scalar vevs. Before doing so we introduce our submultiplet naming and indexing conventions. A host of further details related to the Pati Salam decomposition of SO(10) can be found in our earlier paper [11] where the foundation for the current program of computation of states, masses and couplings of this theory was laid and the spectrum of MSSM like $SU(2)_L$ doublets and $SU(3)_c$ “baryon decay” triplets first computed.

We denote quantum numbers w.r.t the SM gauge group by enclosing them in square brackets while those with respect to the PS group are denoted by round brackets. We have adopted the rule that any PS submultiplet of an SO(10) field is always denoted by the same symbol as its parent field, its identity being established by the indices it carries or by additional sub/superscripts $((a), (s), \pm, L, R)$ denoting (anti-)symmetry or (anti)-self duality, if necessary. On the other hand, since one often encounters several chiral MSSM multiplets of the same type arising from different SO(10) Higgs multiplets we will also introduce a naming convention using roman letters for these multiplets. If we need to denote the scalar component of a chiral superfield we use a tilde over the superfield symbol and sometimes use a superscript “F” to denote fermionic components of chiral superfields while gauginos are denoted by $\lambda$. Our notation for indices is as follows: The real indices of the vector representation of SO(10) are denoted by $i, j = 1..10$. The real vector index of the upper left block embedding (i.e. the embedding specified by the breakup of the vector multiplet $10 = 6 + 4$) of SO(6) in SO(10) are de-
noted $a, b = 1, 2 \ldots 6$ and of the lower right block embedding of SO(4) in SO(10) by $\hat{\alpha}, \hat{\beta} = 7, 8, 9, 10$. These indices are complexified via a Unitary transformation and denoted by $\hat{a}, \hat{b} = \hat{1}, \hat{2}, \hat{3}, \hat{4}, \hat{5}, \hat{6} \equiv \bar{\mu}, \bar{\nu} = \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}$ where $\hat{1} \equiv \bar{1}, \bar{1} \equiv \bar{1}$ etc. Similarly we denote the complexified versions of $\hat{\alpha}, \hat{\beta}$ by $\hat{\alpha}, \hat{\beta} = 7, 8, 9, \hat{10} \equiv \hat{0}$. Using this complexification we showed [11] how all SO(6) × SU(4) subinvariants of SO(10) tensor invariants could be systematically converted to SU(4) × SU(2)$_L$ × SU(2)$_R$ invariants whose indices are as follows: The indices of the doublet of SU(2)$_L$(SU(2)$_R$) are denoted $\alpha, \beta = 1, 2(\hat{\alpha}, \hat{\beta} = \hat{1}, \hat{2})$. Finally the index of the fundamental 4-plet of SU(4) is denoted by a (lower) $\mu, \nu = 1, 2, 3, 4$ and its upper-left block SU(3) subgroup indices are $\bar{\mu}, \bar{\nu} = 1, 2, 3$. The corresponding indices on the 4 are carried as superscripts. These doublets and quartets correspond to the chiral spinor representations of the SO(4) and SO(6) subgroups of SO(10). Details of the spinorial invariant decomposition techniques may be found in [11]. The component of the SU(4) adjoint in the direction of the Gell-Mann generator $\frac{i\lambda^{(15)}}{\sqrt{2}}$ is labelled with a superscript (15) or $(B - L)$.

Thus the PS decomposition of our SO(10) multiplets is

$$\phi = 210 = \phi^{\nu}(15, 1, 1) + \phi(1, 1, 1) + \tilde{\phi}^{\nu}(15, 3, 1) + \bar{\phi}^{\nu}(15, 1, 3)$$

$$+ \phi_{\mu\nu, \alpha\bar{\alpha}(\bar{s})}(6, 2, 2) + \phi_{\mu\nu, \alpha\bar{\alpha}(s)}(10, 2, 2) + \bar{\phi}^{\nu}(10, 2, 2)$$

(4)

$$\Sigma = \Sigma^+ = 126 = \Sigma^R(10, 1, 3) + \Sigma^L(10, 3, 1) + \Sigma^{\mu\nu}(6, 1, 1) + \Sigma_{\mu, \nu}^{\alpha\bar{\alpha}(15, 2, 2)}$$

(5)

$$\Sigma = \Sigma^- = 126 = \Sigma^R(10, 1, 3) + \Sigma^L(10, 3, 1) + \Sigma^{\mu\nu}(6, 1, 1) + \Sigma_{\mu, \nu}^{\alpha\bar{\alpha}(15, 2, 2)}$$

$$H = 10 = H_{\hat{\alpha}\bar{\alpha}}(1, 2, 2) + H_{\mu\nu}(6, 1, 1)$$

(7)

$$\Psi_A = 16 = 16_+ = (4_+, 2_+) + (4_-, 2_-) = F^a_{\bar{\alpha}}(\bar{\alpha}, 1, 2) + F_{\mu\alpha}(4, 2, 1)$$

(8)

$$F(4, 2, 1) = (Q_{\mu\alpha}, L_{\alpha})$$

$$\bar{F}(\bar{\alpha}, 1, 2) = (\bar{Q}^a_{\bar{\alpha}}, \bar{T}_\alpha)$$

(9)

with

$$Q = \left( \begin{array}{c} U \\ D \end{array} \right)$$

$$L = \left( \begin{array}{c} \nu \\ e \end{array} \right)$$

$$\bar{Q} = \left( \begin{array}{c} \bar{d} \\ \bar{u} \end{array} \right)$$

$$\bar{T} = \left( \begin{array}{c} \bar{e} \\ \bar{\nu} \end{array} \right)$$

(10)

The GUT scale vevs that break the gauge symmetry down to the SM symmetry are [12]:

$$\langle (15, 1, 1) \rangle_{210} : \langle \phi_{abcd} \rangle = \frac{a}{2} \epsilon_{abcdef} \epsilon_{ef}$$

(11)

where $[\epsilon_{ef}] = \text{Diag}(\epsilon_2, \epsilon_2, \epsilon_2)$. $\epsilon_2 = i\tau_2$. One dualizes

$$\phi_{ab} \equiv \frac{1}{4!} \epsilon_{abcdef} \phi_{cd}$$

(12)
Then in SU(4) notation $[\phi^\lambda]_\nu$ this vev is

$$\langle [\phi^\lambda]_\nu \rangle = \frac{ia}{2} \text{Diag}(I_3, -3) \equiv \frac{ia\Lambda}{2}$$

(13)

$$\langle (15, 1, 3) \rangle_{210} : \langle \phi_{ab\bar{\alpha}} \rangle = \omega \epsilon_{ab} \epsilon_{\bar{\alpha}{\bar{\beta}}}$$

(14)

where $[\epsilon_{\bar{\alpha}{\bar{\beta}}}]= \text{Diag}(\epsilon_2, \epsilon_2)$ which translates to

$$\langle (\phi^{(R)\nu}_\mu)^{12} \rangle = -\frac{\omega \Lambda}{\sqrt{2}} \equiv i\langle (\phi^{(R)\nu}_\mu)^0 \rangle$$

(15)

$$\langle (1, 1, 1) \rangle_{210} : \langle \phi_{\bar{\alpha}\bar{\beta}\bar{\gamma}} \rangle = p\epsilon_{\bar{\alpha}\bar{\beta}\bar{\gamma}}$$

(16)

$$\langle (10, 1, 3) \rangle_{126} : \langle \Sigma^{(R)}_{1\bar{5}3\bar{5}5\bar{6}} \rangle = \bar{\sigma} = -i\langle (\Sigma^{(R)}_{4\bar{4}4\bar{4}}) \rangle = \frac{\Sigma^{441}}{\sqrt{2}}$$

(17)

$$\langle (\bar{1}0, 1, 3) \rangle_{126} : \langle \Sigma^{(R)}_{2\bar{4}67\bar{7}} \rangle = \sigma = i\langle (\Sigma^{(R)}_{44}) \rangle = \frac{\Sigma^{44}}{\sqrt{2}}$$

(18)

Substituting these vevs into the superpotential one obtains

$$W = m(p^2 + 3a^2 + 6\omega^2) + 2\lambda(a^3 + 3p\omega^2 + 6a\omega^2)$$

$$+ (M + \eta(p + 3a - 6\omega))\sigma \bar{\sigma}$$

(19)

the nontrivial F term conditions are thus:

$$2mp + 6\lambda \omega^2 + \eta \sigma \bar{\sigma} = 0$$

(20)

$$2ma + 2\lambda(a^2 + 2\omega^2) + \eta \sigma \bar{\sigma} = 0$$

(21)

$$2m\omega + 2\omega \lambda(p + 2a) - \eta \sigma \bar{\sigma} = 0$$

(22)

$$(M + \eta(p + 3a - 6\omega))\sigma = 0$$

(23)

The vanishing of the D-terms of the SO(10) gauge sector potential imposes only the condition

$$|\sigma| = |\bar{\sigma}|$$

(24)

Except for degenerate cases corresponding to enhanced unbroken symmetry

(SU(5) × U(1), SU(5), G_{3,2,2,B-L}, G_{3,2,R,B-L}, etc) this system of equations is essentially cubic and can be reduced to the single cubic equation

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1 - x)^2$$

(25)
where $\xi = \frac{\lambda M}{m}$ and the other vevs can be expressed in terms of values of the variable $x$ which solve eqn (25). This parametrization of the MSGUT sb problem [4] is of great help computationally and clearly exhibits the crucial importance of the $\xi$ i.e of the ratio $M/m$. The important role played by a similar ratio in the other renormalizable SO(10) GUT based on the 45, 54, 126, 126 representations has already been noted [20].

When we measure vevs or masses in units of $\tilde{m} = m/\lambda$ we will put a tilde over the symbol. We also define the additional dimensionless parameters $\tilde{\eta} = \eta/\lambda$ and $\tilde{M}_H = M_H/\tilde{m}$.

Then the dimensionless vevs are $\tilde{\omega} = -x$ and

$$\tilde{ \dot{a} } = \frac{x^2 + 2x - 1}{(1 - x)} ; \quad \tilde{p} = \frac{x(5x^2 - 1)}{(1 - x)^2} ; \quad \tilde{\sigma} \tilde{\bar{\sigma}} = \frac{2x(1 - 3x)(1 + x^2)}{\tilde{\eta}} \frac{2x(1 - 3x)(1 + x^2)}{(1 - x)^2} \quad (26)$$

The solutions of the cubic equation (25) are generically complex. We will therefore nowhere assume hermiticity for our mass matrices, preferring to leave them undiagonalized for eventual numerical diagonalization so that all our results are applicable in the general case. We will not generate arrays of expressions in terms of the variable $x$, although it is easy to do so since, practically speaking, the substitutions are now handled via a computer anyway.

We conclude this section with a description of the super-Higgs effect for the breaking $SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_Y$ which is achieved by the above superheavy vevs. As is well known, as a consequence of gauge symmetry breaking, each massive gauge boson forms a massive supermultiplet together with its longitudinal goldstone pseudo scalar (and its real scalar partner) as the 4 bosonic degrees of freedom. It’s gaugino and the chiral fermion superpartner of the Goldstone scalar pair make up one Dirac fermion super-partner also with 4 degrees of freedom. This is the so called Dirac or massive vector gauge supermultiplet. These gauge boson/gaugino masses are the most fundamental thresholds of the GUT and it is appropriate to begin with a discussion of their values for this model. In Section 4. we follow [21, 22] to compute the threshold corrections using the spectra we compute. Then a Dirac gauge coset multiplet in representation $R$ of the MSSM gives rise to a RG mass threshold above which the gauge and chiral components of the Dirac multiplets separately contribute $-3T_i(R)$ and $T_i(R)$, respectively, to the beta function coefficients of the individual MSSM couplings (including the $U(1)_Y$ coupling!).

It is easiest to keep track of the gaugino masses and mixings. The combination of chiral fermions that forms a Dirac fermion together with a gaugino must, for consistency, be a zero mode of the mass matrix arising from the superpotential and this makes it easy to disentangle the gauge spectrum even in the case of complex vevs and parameters. For the symmetry breaking to the MSSM the gauginos of the coset $SO(10)/G_{321}$ lying in the PS representations $(6, 2, 2) \oplus (1, 1, 3)$ plus the triplets and
anti-triplets in \((15, 1, 1)\) (i.e 33 Dirac multiplets in all) obtain a mass by pairing with chiral AM Higgs fermions. One need only substitute the vevs given above into the PS decomposition of the gaugino Yukawa terms which have the form

\[
g\sqrt{2}\left\{ \frac{1}{3!} \left< \bar{\Phi}^*_{ijkl} \right> \lambda_{im} F^E_{mijkl} + \frac{1}{2 \cdot 4!} \left( < \bar{\Sigma}^*_{ijkl} > \lambda_{im} \Sigma^F_{mijkl} + < \bar{\Sigma}^*_{ijkl} > \lambda_{im} \Sigma^F_{mijkl} \right) \right\} + H.c
\]

One finds the following gaugino masses:

- (i) \(G[1,1,0] : m_{\lambda_G} = \sqrt{10g}\sigma\).

The mass term is

\[
\frac{g}{\sqrt{2}}(\sqrt{2}\lambda^{(R0)} - \sqrt{3}\lambda^{(15)})\sigma^*\Sigma^{R+} + \bar{\sigma}^*\Sigma^{44R+}) + H.c
\]

\[
\equiv m_{\lambda_G} \sqrt{2} G_6 (e^{-i\gamma_5} G_4 + e^{-i\gamma_5} G_5) + H.c.
\]

\[
G_6 \equiv \left( \frac{2}{5}\lambda^{(R0)} - \frac{3}{5}\lambda^{(15)} \right)
\]

\[
G_4 \equiv \frac{\Sigma^{R+}}{\sqrt{2}}; \quad G_5 = \frac{\Sigma^{44}}{\sqrt{2}}
\]

The naming conventions for the chiral states are given in Section IV and the Appendix. Here \(\gamma_\sigma, \gamma_\tilde{\sigma}\) are the phases of \(\sigma, \tilde{\sigma}\). Since the representation is real, the mass matrix \(G\) in this sector is symmetric. The complete \(G[1,1,0]\) sector mass matrix (including gauginos) \(G\) is \(6 \times 6\) while its pure chiral part \(G\) (which arises only from the superpotential) is \(5 \times 5\) and symmetric and the 5-tuple \((0,0,0,\sigma,\tilde{\sigma})\) is both a left and right null eigenvector of \(G\) - as will be obvious when it is presented further on (Section 2. and Appendix).

- (ii) \(J[3,1,-\frac{4}{3}] \oplus J[3,1,\frac{4}{3}] : m_{\lambda_J} = g\sqrt{8|a|^2 + 16|\omega|^2 + 2|\sigma|^2}

In this case \((J_4)^{\mu} = \lambda_4^{\mu}, (\bar{J}_4)^{\mu} = \tilde{\lambda}_4^{\mu}\) pair up with the combinations corresponding to the left and right null eigenvectors \(v_{0JL} = N_J(-\sigma, 2a, 2\sqrt{2}\omega), \quad v_{0JR}^T = N_J(\sigma, 2a, 2\sqrt{2}\omega)\) of the complex, non symmetric, upper left \(3 \times 3\) sub-matrix \(J\) of the \(4 \times 4\) mass matrix \(J\) in the J sector. The gaugino mass terms are

\[
ig\bar{J}_4(2\sqrt{2}a^*J_2 + 4a^*J_3 - 2\sqrt{2}\sigma^*J_1) - ig(2\sqrt{2}a^*J_2 + 4a^*J_3 + 2\sigma^*J_1)J_4 + H.c
\]

- (iii) \(\bar{F}[1,1,-2] \oplus F[1,1,2] : m_{\lambda_F} = g\sqrt{24|\omega|^2 + 2|\sigma|^2}

The chiral partners of the gauginos \(F_3 \equiv \lambda_{R+}, \bar{F}_3 \equiv \lambda_{R-}\) correspond to the right and left null eigenvectors \(v_{0FR} = (-\sigma, \sqrt{2i\omega})^T, \quad v_{0FL} = (\bar{\sigma}, \sqrt{2i\omega})\) of the \(2 \times 2\) \(\bar{F} - F\) chiral fermion mass matrix. The mass terms are
SU phenomenology becomes trivial since the embeddingizations without any ambiguity. Once this is done making contact with the MSSM although tedious, our approach allows one to keep track of all phases and normalizations of PS invariants using the translation techniques developed by us \[11\].

The simplicity of the SO(10) form of the GUT action is to rewrite SO(10) invariants as

\[ gE_3(-i\sqrt{2}\sigma^* E_2 + 2(a^* - \omega^*)E_3 + \sqrt{2}(\omega^* - p^*)E_4) \]

\[ + g((i\sqrt{2}\sigma^* E_2 + 2(a^* - \omega^*)E_3 + \sqrt{2}(\omega^* - p^*)E_4)E_5 \]

The chiral partners of the gauginos \( E_3 \equiv \lambda_\mu 41\alpha \), \( E_5 \equiv \lambda_\mu 42\alpha \) correspond to the null eigenvectors \( v_{0ER} = (i\sigma, \sqrt{2}(a-\omega), \omega - p)^T \); \( v_{0EL} = (-i\bar{\sigma}, \sqrt{2}(a-\omega), \omega - p) \) of the upper left 3 \times 3 corner \( E \) of the \( E \) sector 4 \times 4 chiral fermion mass matrix \( E \). \( E_1, \bar{E}_1 \) do not mix with other \( E \)-sector multiplets. The mass terms are

\[ gE_3(-i\sqrt{2}\sigma^* E_2 + 2(a^* - \omega^*)E_3 + \sqrt{2}(\omega^* - p^*)E_4) \]

\[ + g((i\sqrt{2}\sigma^* E_2 + 2(a^* - \omega^*)E_3 + \sqrt{2}(\omega^* - p^*)E_4)E_5 \]

The chiral partners of the gauginos \( X_3 \equiv \lambda_\mu 42a \), \( X_5 \equiv \lambda_\mu 41a \) correspond to the null eigenvectors \( v_{0XR} = (-\sqrt{2}(a + \omega), \omega + p) \); \( v_{0XL} = v_{0EL} \) of the upper left 2 \times 2 corner \( X \) of the 3 \times 3 X-sector chiral fermion mass matrix \( X \). The X-gaugino mass terms are

\[ gX_3(-2(a^* + \omega^*)X_1 + \sqrt{2}(p^* + \omega^*)X_2) + g(-2(a^* + \omega^*)X_1 + \sqrt{2}(p^* + \omega^*)X_2)X_3 \]

\[ 3 \quad AM \text{ Chiral masses via PS} \]

Our approach to opening up the maze of MSSM interactions coded in the deceptive simplicity of the SO(10) form of the GUT action is to rewrite SO(10) invariants as combinations of PS invariants using the translation techniques developed by us \[11\]. Although tedious, our approach allows one to keep track of all phases and normalizations without any ambiguity. Once this is done making contact with the MSSM phenomenology becomes trivial since the embedding \( SU(3) \times SU(2)_L \times U(1)_Y \subset SU(4) \times SU(2)_L \times SU(2)_R \) is trivial and transparent if one keeps in mind that

\[ Y = 2T_{3R} + B - L \]

We obtain for the PS form of the different terms in \( W_{AM} \)

\[ \frac{m}{4!} \phi_{ijkl}^2 = -m \{ \phi_{\mu} \phi_{\nu} + \phi_{\mu a} \phi_{\nu a} - \phi_{\mu a} \phi_{\nu a} + \phi_{\mu} \phi_{\nu} + \phi_{\mu} \phi_{\nu} + \phi_{\mu} \phi_{\nu} + \phi_{\mu} \phi_{\nu} + \phi_{\mu} \phi_{\nu} + \phi_{\mu} \phi_{\nu} + \phi_{\mu} \phi_{\nu} + \phi_{\mu} \phi_{\nu} \} \]

\[ + \frac{1}{2} \phi_{(a)} \phi_{(a)} - \phi^2 \]
\[
\frac{M}{5!} \Sigma_{ijklm} \delta_{ijklm} = M \{\overline{\Sigma}_{(a)}^{\mu \nu} \overline{\Sigma}_{(a)}^{\mu \nu} + 2 \Sigma_{\mu}^{\alpha \alpha} \Sigma_{\nu}^{\nu \alpha} + (\overline{\Sigma}_{(s)}^{\mu \nu} \overline{\Sigma}_{(s)}^{\mu \nu} + \Sigma_{(s)}^{\mu \nu} \overline{\Sigma}_{(s)}^{\mu \nu}) \} \quad (34)
\]

\[
\frac{1}{4!} \lambda \phi^3 = \lambda \left[ - \frac{2}{3} i \phi_{\mu}^{\nu} \phi_{\lambda}^{\mu} - 2 i (\phi_{\mu}^{\nu} \phi_{\nu(s)}^{a\alpha} \phi_{a\alpha}^{\lambda(s)}) \right] - 2 i \left\{ (\phi_{\mu}^{\nu} (\overline{\phi}_{\nu(R)}^{\lambda} \phi_{\lambda(R)}^{\mu}) + (\overline{\phi}_{\nu(L)}^{\lambda} \phi_{\lambda(L)}^{\mu}) \right\} + \left\{ (\phi_{\mu}^{\nu(a)} (\phi_{\nu(s)}^{(a) \alpha} \phi_{\nu(L)}^{(a) \beta} - \phi_{\mu(s)}^{(a) \alpha} \phi_{\mu(L)}^{(a) \beta})) + \phi_{\mu(s)}^{(a) \alpha} \phi_{\mu(L)}^{(a) \beta} \right\} \] 

\[
+ \sqrt{2} \left\{ \phi_{(a)}^{\alpha\alpha} (\phi_{(a)}^{(a) \alpha} \phi_{(a)}^{(a) \beta} + \phi_{(a)}^{(a) \beta} \phi_{(a)}^{(a) \alpha}) \right\} - \phi \left\{ (\overline{\phi}_{(R)}^{\lambda} \phi_{(R)}^{\mu} - \phi_{(L)}^{\lambda} \phi_{(L)}^{\mu}) \right\} - \frac{1}{\sqrt{2}} \left\{ \phi_{\mu(s)}^{(a) \alpha} \phi_{\nu(L)}^{(a) \beta} + \phi_{\mu(s)}^{(a) \beta} \phi_{\nu(L)}^{(a) \alpha} \right\} + \frac{1}{3} \left\{ \phi_{\mu(R)}^{\nu \lambda} \phi_{\nu(L)}^{\lambda \gamma} + \phi_{\nu(L)}^{\nu \gamma} \phi_{(a)}^{(a) \beta} \phi_{(a)}^{(a) \alpha} \right\} \quad (35)
\]

\[
\frac{1}{4!} \gamma \phi \Sigma H = \gamma \left[ i H_{\mu \nu(a)} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} + \phi_{\mu}^{\nu} \Sigma_{\nu}^{\mu \alpha} H_{\alpha \alpha} \right] - \frac{1}{2} H^{\alpha \alpha} \left( \phi_{\mu \nu(a)}^{(a) \beta} \Sigma_{\nu}^{(a) \alpha \beta} (R) + \phi_{\mu \nu(a)}^{(a) \beta} \Sigma_{\nu}^{(a) \alpha \beta} (L) \right) \] 

\[
- \frac{1}{\sqrt{2}} \left\{ \tilde{H}_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} \Sigma_{\nu}^{(a) \alpha \beta} (R) + \tilde{H}_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} \Sigma_{\nu}^{(a) \alpha \beta} (L) \right\} \] 

\[
+ \left( - \tilde{H}_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} \Sigma_{\nu}^{(a) \alpha \beta} (R) + \tilde{H}_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} \Sigma_{\nu}^{(a) \alpha \beta} (L) \right) \] 

\[
- \frac{1}{2} \Sigma_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} H_{\alpha \alpha} \] 

\[
- i H_{\mu \nu(a)} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} + \frac{1}{2} \phi \tilde{H}_{\mu \nu(a)} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} \quad (36)
\]

\[
\frac{1}{4!} \gamma \phi \Sigma H = \gamma \left[ - i H_{\mu \nu(a)} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} + \phi_{\mu}^{\nu} \Sigma_{\nu}^{\nu \alpha} H_{\alpha \alpha} \right] - \frac{1}{2} H^{\alpha \alpha} \left( \phi_{\mu \nu(a)}^{(a) \beta} \Sigma_{\nu}^{(a) \alpha \beta} (R) + \phi_{\mu \nu(a)}^{(a) \beta} \Sigma_{\nu}^{(a) \alpha \beta} (L) \right) \] 

\[
- \frac{1}{\sqrt{2}} \left\{ \tilde{H}_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} \Sigma_{\nu}^{(a) \alpha \beta} (R) + \tilde{H}_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} \Sigma_{\nu}^{(a) \alpha \beta} (L) \right\} \] 

\[
+ \left( - \tilde{H}_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} \Sigma_{\nu}^{(a) \alpha \beta} (R) + \tilde{H}_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} \Sigma_{\nu}^{(a) \alpha \beta} (L) \right) \] 

\[
- \frac{1}{2} \Sigma_{\mu \nu(a)}^{(a) \alpha \beta} \phi_{\mu \nu(s)}^{(a) \alpha \beta} H_{\alpha \alpha} \] 

\[
- i H_{\mu \nu(a)} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} + \frac{1}{2} \phi \tilde{H}_{\mu \nu(a)} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} \Sigma_{\nu}^{\nu \alpha} \phi_{\lambda}^{\mu} \quad (37)
\]
\[ - \frac{1}{2} \sum_{\mu \nu} (\phi_{\mu (a)} \phi_{\nu (a)}^* \Sigma_{\lambda \alpha} \Sigma_{\lambda \alpha}^* \Sigma_{\lambda \alpha}^* \Sigma_{\lambda \alpha}) + i \Delta H_{\mu (a)} \phi_{\nu (a)}^* \Sigma_{\lambda \alpha} A^\mu_{\alpha \alpha} + \frac{1}{2} \phi \Delta H_{\mu (a)} \Sigma_{\mu \nu} \\right\} \]  

(37)

\[ \frac{\eta}{4!} \phi \sum_{\lambda} = \eta \left[ 2i \phi_{\mu}^\nu \left( \sum_{\lambda} \phi_{\mu \lambda}^\nu \Sigma_{\lambda} + \sum_{\nu} \phi_{\mu \lambda}^\nu \Sigma_{\lambda} \right) + i \sqrt{2} \left( - \sum_{\mu \nu} \phi_{\mu \lambda(s)}^\nu a^\alpha \right) \right. \]

\[ \left. + i \sqrt{2} \left( - \sum_{\mu \nu} \phi_{\mu \lambda(s)}^\nu a^\alpha \right) \phi_{\mu \lambda(s)}^\nu a^\alpha - \sum_{\mu \nu} \phi_{\mu \lambda(s)}^\nu a^\alpha \phi_{\mu \lambda(s)}^\nu a^\alpha \right) \]

(38)

The purely chiral superheavy supermultiplet masses can be determined from these expressions simply by substituting in the AM Higgs vevs and breaking up the contributions according to MSSM labels.

It is again easiest to keep track of Chiral fermion masses since all others follow using supersymmetry and the organization provided by the gauge super Higgs effect.

There are three types of mass terms involving fermions from chiral supermultiplets in such models: (A) Unmixed Chiral (B) Mixed pure chiral (C) Mixed chiral and gaugino.

### 3.1 Unmixed Chiral

A pair of Chiral fermions transforming as \( SU(3) \times SU(2)_L \times U(1)_Y \) conjugates pairs up to form a massive Dirac fermion. For example for the properly normalized fields

\[ A[1, 1, -4] = \frac{\Sigma_{I 4(R-)}}{\sqrt{2}} \quad A[1, 1, 4] = \frac{\Sigma_{I 4(R+)}}{\sqrt{2}} \]  

(39)
one obtains the mass term

\[ 2(M + \eta(p + 3a + 6\omega))\tilde{A}A = m_A\tilde{A}A \]

The physical Dirac fermion mass is then \(|m_A|\) since the phase can be absorbed by a field redefinition. By supersymmetry this mass is shared by a pair of complex scalar fields with the same quantum numbers. If the representation is real rather than complex one obtains an extra factor of 2 in the masses. There are in fact 19 types of such multiplets and their (roman letter) labels are given along with their masses and SO(10) origins in Table I in the Appendix. The case of the sectors C[8, 2, ±1] and D[3, 2, ±1] bears special mention. The mass terms for these multiplets arise only between pairs drawn one each from \(\Sigma(15, 2, 2), \Sigma(15, 2, 2)\) and there is no mixing between a \(C, \bar{C}\) or \(D, \bar{D}\) drawn from the same SO(10) multiplet simply because the superpotential does not contain any term containing \(\Sigma^2\) or \(\Sigma^\pm\). This was the reason for the discrepancy in this sector between the results of [11, 13] and [14]: there simply is no such mixing.

### 3.2 Mixed Pure Chiral

In this case there are no contributions from the gaugino Yukawas or the D-terms to the supermultiplet masses, but there is a mixing among several multiplets of the same SM quantum numbers. There are only three such multiplet types:

- a) \([8, 1, 0](R_1, R_2) \equiv (\phi^\mu, \phi_{\mu(R)}^\mu)\)

These mix with mass matrix

\[ R = \begin{pmatrix} (m - \lambda a) & -\sqrt{2}\lambda \omega \\ -\sqrt{2}\lambda \omega & m + \lambda(p - a) \end{pmatrix} \]

with both rows and columns labelled by \((R_1, R_2)\). The masses are the magnitudes of the eigenvalues of the matrix \(R\).

\[ |R_{\pm}| = 2|m[1 + (\frac{\bar{p}}{2} - \bar{a}) \pm \sqrt{(\frac{\bar{p}^2}{4}) + 2\bar{\omega}^2}]| = m_{R\pm} \]

While the corresponding eigenvectors can be found by diagonalizing the matrix \(R^\dagger R\).

The mass matrices of the electroweak doublets \(h[1, 2, 1], \bar{h}[1, 2, -1]\) and colour triplets \(t[3, 1, -\frac{2}{3}], \bar{t}[3, 1, \frac{2}{3}]\) which mix with the multiplets contained in the 10plet FM Higgs are the most crucial ones for determining the phenomenology of the effective MSSM that arises from this GUT. These matrices were first calculated in [11](v2) and later, stimulated by a contradiction with a recent
paper [12], the $d = 5$ baryon violating operators induced by the exchange of heavy Higgsinos were computed and added to a revised version [1] (v4) by using the Clebsches for the $16 \cdot 16 \cdot \mathbf{126}$ and $16 \cdot 16 \cdot 10$ invariants calculated earlier by us. Thus one has:

- b) $[1, 2, -1](\tilde{h}_1, \tilde{h}_2, \tilde{h}_3, \tilde{h}_4) \oplus [1, 2, 1](h_1, h_2, h_3, h_4)$

$$\equiv (H, \Sigma^{(15)}_R, \Sigma^{(15)}_L, \phi^{(1)}_2) \oplus (H, \Sigma^{(15)}_R, \Sigma^{(15)}_L, \phi^{(1)}_2)$$

These multiplets label the 4 rows and columns of the $4 \times 4$ mass matrix $H$ which is given in the collection of mixing matrices in Appendix I. We note that we have redefined our mass parameters $m, M$ by a factor of 2 relative to those we used in [1]. To achieve the MSSM spectrum of one pair of light doublets, it is necessary to fine tune one of the parameters of the superpotential (e.g. $M_H$) so that $Det H = 0$. By extracting the null eigenvectors of $H^T H$ and $H H^T$ one can compute the coefficients of the various bi-doublets in the light doublet pair, and, in particular, we can find those for the doublets coming from the $10, \mathbf{126}$ multiplets which couple to the matter sector (see Section 6.). In this way the $SO(10)$ constraints on the fitting of the Yukawa coupling matrices $h^I, f^I$ can be brought into focus and the invalid assumption that the squares of these coefficients add up to 1 can be dispensed with.

- c) $[3, 1, \frac{2}{3}](\bar{t}_1, \bar{t}_2, \bar{t}_3, \bar{t}_4, \bar{t}_5) \oplus [3, 1, -\frac{2}{3}](t_1, t_2, t_3, t_4, t_5)$

$$\equiv (H^4, \Sigma^{4}_{(a)}, \Sigma^{4}_{(a)}, \Sigma^{4}_{(R)}, \phi^{(4)}_4) \oplus (H^4, \Sigma^{4}_{(a)}, \Sigma^{4}_{(a)}, \Sigma^{4}_{(R)}, \phi^{(4)}_4)$$

For generic values of the couplings all these particles are superheavy. These triplets and antitriplets participate in baryon violating process since the exchange of $(t_1, t_2, t_4) \oplus (\bar{t}_1, \bar{t}_2)$ Higgsinos generates $d = 5$ operators of type QQQL and $\bar{l}uudd$. The strength of the operator is controlled by the inverse of the $\bar{t} - t$ mass matrix $T$ which we computed in [1] and is given in the Appendix. We shall examine how $d = 5$ baryon and lepton number violating operators are generated in Section 5.

### 3.3 Mixed Chiral-Gauge

Finally we come to the mixing matrices for the chiral modes that mix with the gauge particles as well as among themselves. Apart from threshold effects, these are of some interest since one might ask whether the new types of coset gauginos present in in $SO(10)$ but not in $SU(5)$ namely $SO(10)/SU(5) \sim E[3, 2, \frac{1}{3}] \oplus E[3, 2, -\frac{1}{3}] \oplus F[1, 1, 2] \oplus F[1, 1, -2] \oplus G[1, 1, 0] \oplus J[3, 1\frac{1}{3}] \oplus J[3, 1, -\frac{1}{3}]$ (the $SU(5)/G_{321}$ leptoquarks are $X[3, 2, -\frac{5}{3}] \oplus X[3, 2, \frac{5}{3}]$) might not mediate interesting exotic processes by inducing $d = 5$ operators via mixed gaugino-chiral exchange. We have examined this question in some detail in Section 5.

These multiplet sets are:
a) \([1, 1, 0](G_1, G_2, G_3, G_4, G_5, G_6) \equiv (\phi, \phi^{(15)}, \phi^{(15)}, \frac{\Sigma^{44}}{\sqrt{2}}, \frac{\Sigma^{44}}{\sqrt{2}}, \frac{\sqrt{2}(\Sigma^{(R_0)} - \sqrt{3} \Sigma^{(15)})}{\sqrt{5}})\]

which mix via a 6 \times 6 mass matrix \(G\) given in the Appendix. The complex conjugates of the 6th row and column form left and right null eigenvectors \(v_{0GL}, v_{0GR}\) of the upper left 5 \times 5 block \(G\) of \(G\). The determinant of \(G\) is generically non zero although the determinant of the submatrix \(G\) vanishes. It will clearly not affect the evolution of the MSSM gauge couplings at one loop due to the singlet quantum numbers.

b) \([3, 2, -\frac{1}{3}](\bar{E}_2, \bar{E}_3, \bar{E}_4, \bar{E}_5) \oplus [3, 2, \frac{1}{3}](E_2, E_3, E_4, E_5) \equiv (\Sigma^{\mu \bar{a}}_{4a1}, \phi^{(a) \mu}_{\alpha 2}, \phi^{(a) \mu}_{\alpha 2}, \lambda^{(a) \mu}_{\alpha 2}) \oplus (\Sigma^{4}_{\mu \alpha 2}, \phi^{(a) \mu}_{\mu \alpha 1}, \phi^{(a) \mu}_{\mu \alpha 1}, \lambda^{(a) \mu}_{\mu \alpha 1})\]

The 4 \times 4 mass matrix \(E\) \(\equiv (\Sigma^{4}_{\mu \alpha 2}, \Sigma^{\mu \bar{a}}_{4a1})\) do not mix with the others) has the usual superhiggs structure: complex conjugates of the 4th row and column are left and right null eigenvectors of the upper left 3 \times 3 submatrix \(E\). \(E\) has non zero determinant although the determinant of \(E\) vanishes. As for the case of \(C[8, 2, \pm 1]\) and \(D[3, 2, \pm 7/3]\) type multiplets one finds that the conjugate types of \(E\) type multiplets drawn from the same \(SO(10)\) representation cannot mix. Furthermore explicit computation using the decomposition of the superpotential given in Section 3. shows that \(E_1[3, 2, \frac{1}{3}] = \Sigma^{\mu \bar{a}}_{4a1}\) and \(E_1[3, 2, -\frac{1}{3}] = \Sigma^{\mu \bar{a}}_{4a1}\) in fact decouple from the other members of the \(E\) sector so that the \(E\) sector mixing matrix is 4 \times 4 (including gauginos) and 3 \times 3 excluding gauginos. Note that our assertion is not that these couplings cancel but simply that they do not appear. To see why, for instance, there is no term mixing say \(E_1 = \Sigma^{\mu \bar{a}}_{4a1}\) with \(E_3 = \phi^{(s) \mu}_{\mu \alpha 1}\) coming from the \(\Phi(10, 2, 2)\) we observe that the terms mixing \(\Sigma(15, 2, 2)\) and \(\Phi\) (**decuplet**, 2, 2) via a righthanded vev could only come from the following two terms in eqn. (38):

\[
\frac{\eta}{4!} \phi \Sigma \Sigma = -2i \Sigma^{\mu \bar{a} \lambda \delta} (\Sigma^{\lambda(s)} \phi^{(s) \mu \lambda(s)} \delta) - 2i \Sigma^{\mu \bar{a} \lambda \delta} (\phi^{(s) \mu \lambda(s)} \delta) \Sigma^{(15, 2, 2)}
\]

We see that the pairs \(\Sigma(10, 1, 3)\) and \(\phi(10, 2, 2)\) and \(\Sigma(10, 1, 3)\) and \(\phi(10, 2, 2)\) simply do not mix. Now it is obvious that a \(\Sigma\) right handed vev will mix only \(\bar{E}_2\) coming from \(\Sigma(15, 2, 2)\) with \(E_3\) coming from \(\Phi(10, 2, 2)\) but not \(E_1\) coming from \(\Sigma(15, 2, 2)\) with \(E_3\) coming from \(\Phi(10, 2, 2)\). Similar considerations account for the other decouplings between \(E_1, \bar{E}_1\) and the rest of the \(E\) sector. Ultimately this correlation is accounted for by the correlation between the duality properties of \(SO(6)\) decuplets and \(SO(4)\) triplets within the \(SO(10)\) self-dual and anti-self-dual multiplets \(\Sigma, \bar{\Sigma}\).

c) \([1, 1, -2](\bar{F}_1, \bar{F}_2, \bar{F}_3) \oplus [1, 1, 2](F_1, F_2, F_3)\]

\(\equiv (\Sigma^{44}_{(R0)}, \phi^{(15)}_{(R_-)}, \lambda^{(R_-)}) \oplus (\Sigma^{44}_{(R0)}, \phi^{(15)}_{(R_+)}, \lambda^{(R_+)})\)
The mixing matrix $\mathcal{F}$ has the usual structure. The residual massive eigenstates after separating off the the two Dirac fermions of mass

$$m_{\lambda F} = g(24|\omega|^2 + 2|\sigma|^2)^{\frac{1}{2}}$$

is a Dirac fermion of mass

$$m_F = \left| \frac{g}{\omega} \sqrt{|\sigma|^2 + 12|\omega|^2} \right|$$

and the form of its chiral parts is

$$\mathcal{F} = N_F \left[ i \sqrt{12} \omega \mathcal{F}_1 + \sigma \mathcal{F}_2 \right] e^{i(\gamma_0 - \gamma_1 - \gamma_2)}$$

$$\bar{\mathcal{F}} = N_F \left[ -i \sqrt{12} \omega \bar{\mathcal{F}}_1 + \bar{\sigma} \bar{\mathcal{F}}_2 \right]$$

$$N_F^{-1} = \sqrt{12|\omega|^2 + |\sigma|^2}$$

\begin{itemize}
  \item d) $[3,1,-\frac{5}{2}] (\bar{J}_1, \bar{J}_2, \bar{J}_3, \bar{J}_4) \oplus [3,1,\frac{5}{2}] (J_1, J_2, J_3, J_4)
  \equiv (\sum_{(R-)} \phi_4^\mu, \phi_4^\mu(R_0), \lambda_4^\mu) \oplus (\sum_{(R+)} \phi_4^\mu, \phi_4^\mu(R_0), \lambda_4^\mu)$

The 4 \times 4 mass matrix $\mathcal{J}$ has the usual super-Higgs structure: complex conjugates of the 4th row and column are left and right null eigenvectors of the upper left 3 \times 3 submatrix $\mathcal{J}$. $\mathcal{J}$ has non zero determinant although the determinant of $J$ vanishes.

\item e) $[3,2,\frac{5}{3}] (\bar{X}_1, \bar{X}_2, \bar{X}_3) \oplus [3,2,-\frac{5}{3}] (X_1, X_2, X_3)$
  \equiv (\phi_{a1}^{\mu4(s)}, \phi_{a1}^{\mu4(a)}, \lambda_{a1}^{\mu4}) \oplus (\phi_{a2}^{\mu4}, \phi_{a2}^{\mu4(a)}, \lambda_{a2}^{\mu4})$

mix via a 3 \times 3 symmetric matrix $\mathcal{X}$ so the left and right null eigenvectors of the upper left 2 \times 2 submatrix $\mathcal{X}$, formed by the complex conjugates of the third row and column of $\mathcal{X}$, are the same. Separating off the two Dirac $[3,2,\pm\frac{5}{3}]$ gauge fermions of mass

$$m_{\lambda X} = g\sqrt{4|\alpha + \omega|^2 + 2|\rho + \omega|^2}$$

one is left with a Dirac fermion of mass

$$m_X = 2(2|m + \lambda(\alpha + \omega)| + |m + \lambda \omega|) = \frac{2|m|(2|x|^2 + |1 - x|^2)}{|1 - x|}$$

whose chiral parts are also neatly expressed in terms of $x$

$$(X, \bar{X}) = \frac{1}{\sqrt{2|x|^2 + |1 - x|^2}} (e^{i(\gamma_0 - \gamma_1 - x)}(\sqrt{2x}X_1 + (1-x)X_2), (\sqrt{2x}\bar{X}_1 + (1-x)\bar{X}_2))$$

(45)
This concludes our description of the superheavy mass spectrum of the minimal susy GUT. As mentioned earlier our results were calculated in collaboration with the authors of [13] and are in agreement with the chiral spectra calculated in [13]: whose results also confirm our earlier results [11] on the phenomenologically important matrices $\mathcal{H, T}$. Moreover we have evaluated the mixing of the gauginos with the chiral fermions explicitly and calculated the gauge spectra and eigenstates besides furnishing all the couplings in the superpotential sector explicitly. The gauge couplings and the
gauge Yukawa couplings to matter will be be given in the Section V. The gaugino mixing with the chiral fields will be useful to us when we examine $B + L$ violation mediated by gaugino exchange as well by Higgsino triplet exchange and when one wishes to examine the flow of gauge couplings past the gauge thresholds.

4 RG Analysis

The first phenomenological success of GUTs was the 1-loop calculation of the numerical value of Weinberg angle [23]. This was followed by the prediction [24] and then the verification [26] of an amazingly exact compatibility between UV gauge coupling convergence in the MSSM and the precision LEP data. The large mass at which the top quark was eventually discovered and the associated large value $\sin^2\theta_W \sim 0.23$ verified the originally somewhat far fetched conjecture of [24]: a historical fact that is still not always appreciated. The proposal of Weinberg [21] for calculating threshold effects within an effective field theory picture using mass independent renormalization schemes such as the standard $\overline{MS}$ renormalization scheme was taken up and developed in detail in [22]. Thereafter, using these results, it was argued [18] that high-precision calculations in SO(10), and particularly in supersymmetric SO(10) models which used large representations such as $210, 54, 126$ etc, were futile. This was due to the huge corrections to the one loop predictions that they expected in view of the large number of superheavy fields and the expected span in their masses. It should be remarked however that without an explicit calculation cancellations that might naturally occur would be overlooked. Such calculations were never done. These negative expectations were a motivation for the development [20] of a whole genre of SO(10) models that
eschewed large representations (and thus parameter counting minimality) in favour of models with a plethora of small representations and non-renormalizable interactions.

The other approach [1, 2, 3, 19, 20, 4] approach has all along been to retain renormalizability of the fundamental theory. We regard retention of Higgs multiplets just adequate to account for the gauge and fermion spectrum via renormalizable couplings as a sine qua non for even being clear as to what is testable about a given model. The inverse approach where representations ("hypotheses") are multiplied without necessity seems regressive to us. Thus the proposal of the Susy SO(10) GUT based on the $210 - 126 - 126 - 10$ [1, 2] Higgs system as being the Minimal Susy GUT [3, 4] must live or die by the criterion: Are the one loop values of
$\sin^2 \theta_W$ and $M_X$ generically stable against superheavy threshold calculations? By “generically” we mean: for a non-singular subset of the parameter space. So far this question could not be answered definitively since no complete mass spectrum was available in any Susy SO(10) model to settle the issue. Partly this was due to the lack of accessible techniques to calculate mass spectra and couplings in these models due to the difficulty in obtaining the relevant SO(10) “Clebsches”. Over the last few years we have developed [11] a complete technology for translating SO(10) tensor and spinor labels into those of the unitary labels of the Pati-Salam maximal subgroup $SU(4) \times SU(2)_L \times SU(2)_R$ of SO(10). This allowed us to compute first the mass matrices of SM type doublets and proton decay mediating triplets and then the complete spectrum and couplings reported in this paper. The partial technology of [10, 3] has also been used to compute [13, 14] this spectrum (but not the couplings). With the correct spectrum in hand we can apply the standard formulae of Hall [22] to compute the changes in the 1-loop GUT predictions as functions of the few MSGUT parameters $(\xi = \lambda M/\eta m, \lambda, \eta, \gamma, \bar{\gamma}, m, g, M_H)$ which are relevant at the GUT scale. Thereafter we can scan the parameter space to see how the corrections vary with these parameters.

A few remarks on the role of the parameters are in order. The parameter $\xi = \lambda M/\eta m$ is the only numerical parameter that enters into the cubic equation eqn. 25 that determines the parameter $x$ in terms of which all the superheavy vevs are given. It is thus the most crucial determinant of the mass spectrum. The dependence of the threshold corrections on the parameters $\lambda, \eta, \gamma, \bar{\gamma}$ seems quite mild (logarithmic) (this is especially obvious for the unmixed chiral multiplets) and is also suggested by our preliminary scans of the parameter space. Thus changing $\lambda, \eta$ by a factor of 100 each yields plots vs $\xi$ that seem indistinguishable from the ones presented below. From equations 26 we see that $m/\lambda$ can be extracted as the overall scale parameter of the vevs. Since the threshold corrections we calculate are dependent only on (logarithms of) ratios of masses the parameter $m$ does not play any crucial role in our scan of the parameter space: it is simply fixed in terms of the (threshold and two loop corrected) mass $M_V = M_X$ of the lightest superheavy vector particles mediating proton decay: which mass is chosen, in the approach of Hall, as the common “physical” matching point in the equations relating the running MSSM couplings to the SO(10) coupling [22]. Inasmuch as we take the parameters $\lambda, \eta$ as given, and the parameter $m$ is set by the overall mass scale, the freedom in the parameter $\xi$ is essentially that of choosing the $126 - \overline{126}$ mass parameter $M$ i.e the freedom in choosing the dimensionless parameter $\xi$, is essentially that of the ratio $M/m$: which ratio is already known to be a crucial control parameter of symmetry breaking in renormalizable models that utilize the $126, \overline{126}$ to complete and enforce the symmetry breaking down to the SM symmetry [20]. As for $M_H$ it is fine tuned to keep a pair of doublets light. The relation between the MSSM couplings at the susy breaking scale $M_S \sim 1$ TeV and the GUT coupling at the scale $M_X$ is given by
\[
\frac{1}{\alpha_i(M_S)} = \frac{1}{\alpha_G(M_X)} + 8\pi b_i \ln \frac{M_X}{M_S} + 4\pi \sum_j b_{ij} \ln X_j - 4\pi \lambda_i(M_X)
\]  
(46)

Here

\[
X_j = 1 + 8\pi b_j \alpha_G(M_X^0) \ln \frac{M_X^0}{M_S}
\]  
(47)

is understood to be evaluated at the values of \(M_X^0, \alpha_G(M_X^0)\) determined from the one loop calculations. In this equation the contribution of the Yukawa couplings has not been taken into account and this should also be done in a full investigation \[27\]. Here we will confine ourselves to estimating the corrections using the equations as given above, since these were already conjectured\[18\] to lead to a breakdown of the unification scenario. The coefficients

\[
\{b_1, b_2, b_3\} = (1/16\pi^2)\left\{\frac{33}{5}, 1, -3\right\}
\]  
(48)

\[
[b_{ij}] = \frac{1}{(16\pi^2)^2} \left( \begin{array}{ccc} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{array} \right)
\]  
(49)

are the standard one loop and two loop gauge evolution coefficients for the MSSM \[28\]. The term containing \(\lambda_i\) represents the leading contribution of the superheavy thresholds:

\[
\lambda_i(\mu) = -\frac{2}{21} (b_{iV} + b_{iGB}) + 2(b_{iV} + b_{iGB}) \ln \frac{M_V}{\mu} + 2b_{iS} \ln \frac{M_V}{\mu} + 2b_{iF} \ln \frac{M_F}{\mu}
\]  
(50)

where \(V, GB, S, F\) refer to vectors, Goldstone bosons, scalars and fermions respectively and a sum over heavy mass eigenstates is implicit. The formulae for the threshold corrections are

\[
\Delta^{(th)}(\ln M_X) = \frac{5\lambda_1(M_X^0) + 3\lambda_2(M_X^0) - 8\lambda_3(M_X^0)}{10b_1 + 6b_2 - 16b_3}
\]  
(51)

\[
\Delta^{(th)}(\log_{10} M_X) = 0.0217 + 0.0167(5b_1' + 3b_2' - 8b_3') \log_{10} \frac{M'}{M_X^0}
\]  
(52)

\[
\Delta^{(th)}(\sin^2 \theta_W(M_S)) = \frac{10\pi \alpha(M_S)}{5b_1 + 3b_2 - 8b_3} \sum_{ijk} \epsilon_{ijk}(b_i - b_j) \lambda_k(M_X^0)
\]  
(53)

\[
= 0.00004 - 0.00024(4b_1' - 9.6b_2' + 5.6b_3') \log_{10} \frac{M'}{M_X^0}
\]  
(54)
Where $\bar{b}_i = 16\pi^2 b_i'$ are the 1-loop beta function coefficients for multiplets with mass $M'$. To evaluate these formulae it is convenient to group the gaugino contributions along with the chiral fermions they mix with. The values of the indices $S_1, S_2, S_3$ combined as in eqns.[52, 53] i.e $S_W = 4S_1 - 9.6S_2 + 5.6S_3; \quad S_X = 5S_1 + 3S_2 - 8S_3$ are given in Table 2 in the appendix.

The two loop contributions

$$
\Delta^{(2\text{-loop})}(\ln M_X) = -\frac{1}{10b_1 + 6b_2 - 16b_3} \sum_j \left[ \frac{5b_{1j} + 3b_{2j} - 8b_{3j}}{b_j} \ln X_j \right]
$$

$$
\Delta^{(2\text{-loop})}(\sin^2 \theta_W(M_S)) = -\frac{10\pi \alpha(M_S)}{(5b_1 + 3b_2 - 8b_3)} \sum_{ijkl} \epsilon_{ijkl} (b_i - b_j) b_{kl} \ln X_l
$$

Using the values

$$
\alpha_G^0(M_X)^{-1} = 25.6 \quad ; \quad M_X^0 = 10^{16.25} \text{GeV} \quad ; \quad M_S = 1 \text{TeV}
$$

$$
\alpha_1^{-1}(M_S) = 57.45 \quad ; \quad \alpha_2^{-1}(M_S) = 30.8 \quad ; \quad \alpha_3^{-1}(M_S) = 11.04
$$

extrapolated from the global averages of current data, the two loops effects give

$$
\Delta^{2\text{-loop}}(\log_{10} \frac{M_X}{M_S}) = -.08 \quad ; \quad \Delta^{2\text{-loop}}(\sin^2 \theta_W(M_S)) = .0026
$$

The values of the 1-loop coefficients $\bar{b}_i = 16\pi^2 b_i$ corresponding to Vector, complex scalar and Weyl fermion fields are $-11S(R)/3, S(R)/3, 2S(R)/3$ where $S(R)$ is the index of the relevant representation. Note in particular that this implies that the nonzero hypercharge superheavy vector multiplets which are present in SO(10) models will contribute with negative coefficients to the evolution of even the $U(1)_Y$ coupling.

We have computed the threshold corrections for a range of values of $\xi$ keeping the other “insensitive” parameters fixed at randomly chosen representative values

$$
\lambda = 0.12 \quad ; \quad \eta = 0.21 \quad ; \quad \gamma = 0.23 \quad ; \quad \bar{\gamma} = 0.35
$$

The results for different values of these parameters (but with the same $\xi$) are very similar. We will therefore keep them fixed at these values throughout since here we only wish to illustrate the feasibility of precision RG calculations in the SO(10) MSGUT.

For real values of the superpotential parameters the cubic equation (24) that determines the vevs has one real and two complex (conjugate) solutions. The latter give essentially identical corrections. So for real $\xi$ we need to present plots for two solutions only. These are given as Figs. 1-6.

From Figs. 1, 3 we see that for most real values of $\xi$ the threshold effects on $\sin^2 \theta_W(M_S)$ are less than 10% of the 1-loop values. There are three exceptional values of $\xi$ very close to which this limit is breached but even then the change is only
about 25%. For large magnitudes of $\xi$ an asymptotic regime of around 10% change seems to supervene. Similarly Figs. 2, 4 show that the change in $M_X$ is also not drastic (though possibly phenomenologically interesting since the gauge contribution to the nucleon lifetime goes as $M_X^{-4}$) except at certain special points among which one recognizes certain known points of enhanced symmetry\cite{4, 13} such as $\xi = -5, 10$ (SU(5)), $\xi = 3$ ($G_{LR}$), $\xi = -2/3$ (flipped $SU(5) \times U(1)$). It is natural to expect that something similar accounts for the other sharp peaks and dips in these plots. Moreover their narrowness emphasizes that for generic values of the parameters one may expect the threshold corrections to be small for the real $\xi$ real $x$ cases. There are also regions in which the threshold corrections to $Log_{10} M_X$ are as large as $-5$ and these need special examination with regard to their phenomenological viability and consistency with the one scale breaking picture. It is interesting that in this way one can scan the parameter space of the MSGUT and obtain a global “tomograph” of the variation in its character with the ratio $M/m$.

Fig. 3., 4. we give a magnified view of the region $|\xi| < 2$. A comparison of the graphs shows clearly that the peaks in the threshold corrections coincide by either measure, obviously because some particles are becoming very light and enhancing the mass ratios that enter the formulae. It will be amusing to use these plots to identify and unravel the special regions of the MSGUT parameter space.

Figure 1: Plot of the threshold corrections to $Sin^2 \theta_w$ vs $\xi$ for real $\xi$: real solution for $x$. 
Let us turn next to the complex solutions of the cubic equation for $x$ but still with real values of $\xi$. We obtain the typical plots Fig. 5,6. The corrections to $\sin^2\theta_{W}(M_{S})$ are very small for small $|\xi| < 2$, with a minimum close to $\xi = 1$. From Fig. 6 we see that apart from the two peaks near $\xi = \pm 5$ the corrections to the unification scale are quite small for small $\xi$.

When we consider complex values of $\xi$ as shown in Fig.7-12 we see that the behaviour is quite regular (like the case of the complex solutions for real $\xi$) and once again there are large regions of parameter space where the corrections are less than 10% for $\sin^2\theta_{w}$ while $M_{X}$ changes by a factor of 10 or less. Thus even this cursory scan of the MSGUT parameter space shows that, quite contrary to expectations in the literature[18], precision RG analysis of the SO(10) MSGUT is far from being futile, since the hierarchy of magnitudes between MSSM one loop gauge coupling convergence values ($O(\alpha_{s}^{-1})$ effects) and the one loop threshold and two loop gauge coupling corrections ($O(1)$ effects [22]) is generically maintained at the level of 10% or less. Furthermore the RG analysis and parameter scan in terms of the single parameter $\xi$ can teach us much about the structure of the parameter space since it shows a sharp sensitivity to points of enhanced symmetry. We will return to these questions at length in the sequel [27].

Figure 2: Plot of the threshold corrected $\log_{10}M_{X}/M_{X}^{0}$ vs $\xi$ for real $\xi$: real solution for $x$. 

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5 \( d = 5 \) Operators for B,L violation

5.1 Higgsino exchange mediated violation

In SO(10) \( B - L \) is a gauge symmetry. Thus the vertices preserve this symmetry and the leading effects of the spontaneous violation of \( B - L \) by the superheavy \( \Sigma, \bar{\Sigma} \) vevs are just the neutrino mass phenomena. To examine the generation of effective (non-renormalizable) operators that violate \( B+L \) in the low energy theory via Higgsino and gaugino exchange we need the MSSM break-up of the SO(10) invariants \( 16 \cdot 16 \cdot 10 \) and \( 16 \cdot 16 \cdot 126 \). Since we had already presented the PS decomposition of these invaraints in [11] it is a trivial exercise to use that result to obtain the MSSM wise decompositions:

\[
W_{FM}^H = h'_{AB}\psi_A^T C_2(5) \gamma_i(5) \psi_B H_i
\]

\[
= \sqrt{2} h'_{AB} [H_{\mu\nu} \psi^\mu_A \psi^\nu_B \bar{\psi}_{B\dot{a}} + H_{\mu\nu} \psi^\mu_A \psi_{\nu\dot{a}B} - H^{\alpha\dot{\alpha}} (\bar{\psi}^\mu_{A\dot{\alpha}} \psi_{\alpha\muB} + \psi_{\alpha\muA} \bar{\psi}^\mu_{A\dot{\alpha}})]
\]

\[
= 2\sqrt{2} h'_{AB} [\bar{\tau_1}(\epsilon \bar{u}_A \bar{d}_B + Q_A L_B) + t_1 (\frac{\epsilon}{2} Q_A Q_B + \bar{u}_A \bar{e}_B - \bar{d}_A \bar{\nu}_B)]
\]

\[
- 2\sqrt{2} h'_{AB} h_1 [\bar{d}_A Q_B + \bar{e}_A L_B] + 2\sqrt{2} h'_{AB} h_1 [\bar{u}_A Q_B + \bar{\nu}_A L_B]
\]

(58)
Figure 4: Magnified Plot of the threshold corrected $\log_{10} M_X/M_X^0$ vs $\xi$ for real $\xi$: real solution for $x$.

\[ W_{\overline{\sum}}_{FM} = \frac{1}{5!} \psi^T \bar{C} (\gamma_1 \cdots \gamma_5 \chi \bar{\sum} \gamma_1 \cdots \gamma_5) \]
\[ = 2 \sqrt{2} (\bar{\sum}^{\mu \nu} \psi^{\alpha \mu} \chi_{\nu \alpha} - \bar{\sum}^{(a)} \psi^{\mu \alpha} \bar{\chi}^{(a)}_{\alpha}) + 4 \sqrt{2} \bar{\sum}^{\mu \alpha \dot{\beta}} (\bar{\psi}^{\alpha \mu} \chi_{\alpha \beta} + \psi_{\mu \alpha} \bar{\chi}^{(a)}_{\alpha}) + 4 (\bar{\sum}^{\dot{\beta} \dot{\alpha}} \bar{\psi}^{\dot{\mu} \dot{\alpha}} \bar{\chi}^{(a)}_{\dot{\alpha}} + \bar{\sum}^{\mu \alpha \dot{\beta}} \psi_{\mu \alpha} \chi_{\dot{\alpha} \dot{\beta}}) \] (59)

whence

\[ W_{\overline{\sum}}_{FM} = 4 \sqrt{2} f'_{AB} \left[ \bar{t}_2 \left( \frac{\epsilon}{2} \bar{Q}_A \bar{Q}_B - \bar{u}_A \bar{\epsilon}_B + \bar{v}_A \bar{d}_B \right) + \bar{t}_2 \left( \bar{Q}_A \bar{L}_B - \epsilon \bar{u}_A \bar{d}_B \right) \right] \]
\[ + 4 \sqrt{2} f'_{AB} \left[ \frac{i}{\sqrt{3}} \{ \bar{h}_2 (\bar{d}_A \bar{Q}_B - 3 \bar{\epsilon}_A \bar{L}_B) - \bar{h}_2 (\bar{u}_A \bar{Q}_B - 3 \bar{\psi}_A \bar{L}_B) \} \right] \]
\[ + 2 (\bar{C}_1 \bar{d}_A \bar{Q}_B - C_2 \bar{u}_A \bar{Q}_B) + 2 (E_1 \bar{d}_A \bar{L}_B - D_2 \bar{u}_A \bar{L}_B) \]
\[ + 2 (D_2 \bar{\epsilon}_A \bar{Q}_B - E_2 \bar{\psi}_A \bar{Q}_B) \]
\[ + 4 f'_{AB} \sum^{\dot{\beta} \dot{\alpha}} \bar{Q}_{A \dot{\beta}} \bar{Q}_{B \dot{\alpha}} + 2 i (\bar{A} \bar{\epsilon}_A \bar{e}_B - \bar{G}_5 \bar{\psi}_A \bar{\psi}_B) - 2 \sqrt{2} i \bar{F}_1 \bar{\epsilon}_A \bar{\psi}_B \]
\[ + (\bar{W} \bar{Q}_A \bar{Q}_B + 2 \bar{P} \bar{Q}_A \bar{L}_B + \sqrt{2} \bar{O} \bar{L}_A \bar{L}_B) \]
\[ - 2 i t_4 (\bar{d}_A \bar{\psi}_B + \bar{u}_A \bar{e}_B) + 2 i \sqrt{2} (K \bar{d}_A \bar{e}_B - J_1 \bar{u}_A \bar{\psi}_B) \] (60)
Figure 5: Plot of the threshold corrections to $\sin^2\theta_W$ vs $\xi$ for real $\xi$ : complex solution for $x$.

We have suppressed $G_{321}$ indices and used a sub multiplet naming convention specified in Section 2. and and Table I in the Appendix.

In order that the exchange of a Higgsino that couples to matter with a given $B + L$ lead to a $B + L$ violating $d = 5$ operator in the effective theory at sub GUT energies it is necessary that it have a nonzero contraction with a conjugate (MSSM) representation Higgsino that couples to a matter chiral bilinear with a $B + L$ different from the conjugate of the first $B + L$ value. Inspecting the above superpotentials one finds that only \{$\bar{t}_{(1)}, \bar{t}_{(2)}$\} and \{$t_{(1)}, t_{(2)}, t_{(4)}$\} satisfy this requirement. Terms containing the right handed neutrinos $\tilde{\nu}_A$ must be further processed to integrate out the heavy field $\tilde{\nu}_A$ in favour of the light neutrinos $\nu_A$. This will introduce an extra factor of $m^\nu_{Dirac}/M^\nu_{Majorana}$ and effectively lead to amplitudes suppressed like those of $d = 6$ operators. Thus on integrating out the heavy triplet Higgs supermultiplets one obtains the effective $d = 4$ Superpotential for Baryon Number violating processes:

$$W_{\epsilon_{eff}}^{\Delta B\neq 0} = L_{ABCD}(\frac{1}{2}\epsilon Q_A Q_B Q_C L_D) + R_{ABCD}(\epsilon\bar{\epsilon} A \bar{u}_B \bar{u}_C \bar{d}_D)$$

(61)

where the coefficients are
Figure 6: Plot of the threshold corrected $\log_{10} M_X/M_X^0$ vs $\xi$ for real $\xi$: complex solution for $x$.

\[ L_{ABCD} = S_1^1 h_{AB} h_{CD} + S_1^2 h_{AB} f_{CD} + S_2^1 f_{AB} h_{CD} + S_2^2 f_{AB} f_{CD} \]  

(62)

and

\[ R_{ABCD} = S_1^1 h_{AB} h_{CD} - S_1^2 h_{AB} f_{CD} - S_2^1 f_{AB} h_{CD} + S_2^2 f_{AB} f_{CD} \]

\[- i\sqrt{2} S_1^1 f_{AB} h_{CD} + i\sqrt{2} S_2^1 f_{AB} f_{CD} \]

(63)

here $S = T^{-1}$ and $T$ is the mass matrix for $[3,1,\pm 2/3]$-sector triplets: $W = \bar{t} T t + ...$, while

\[ h_{AB} = 2\sqrt{2} h'_{AB} \quad f_{AB} = 4\sqrt{2} f'_{AB} \]  

(64)

This expression and the “Clebsches” contained in it, as well as the new baryon decay channel mediated by the triplets $(t_{(4)})$ contained in $\Sigma_{126}(10,1,3)$ (the same PS multiplet that contains the Higgs field responsible for the right handed neutrino Majorana mass), were given in [11]. Previous work [12] on $\Sigma_{126}$ mediated decay focussed on the multiplets $t^{(2)}, \bar{t}^{(2)}$ and found that there was no contribution of $t^{(4)}, \bar{t}^{(4)}$
in their models. This new channel nominally strengthens the emergent link between neutrino mass and baryon decay. Note however that $t_{(4)}$ couples only to the RR combinations $\bar{d} \nu + \bar{u} \bar{e}$ and as such its exchange will contribute only to the RRRR channel which, at least in SO(10), seems \[12\] generically suppressed except at very large $\tan \beta$. However the mixing in the triplet mass matrix could also strengthen the effects of this channel.

5.2 Novel $d = 5, \Delta(B + L) \neq 0$ Operators via superheavy gaugino exchange?

A novel situation apparently arises in this GUT due to exchange of superheavy gaugino Dirac multiplets that couple to matter both via the gauge yukawa couplings of their gaugino part and the superpotential couplings of their ($\overline{126}$) chiral components to the matter sector. Such gauginos are not present in the case of $SU(5)$ As is evident from eqn.(60) the $\overline{126}$ submultiplet fields $\overline{E_2}[3, 2, -1/3], \overline{F_1}[1, 1, -2], \overline{J_1}[3, 1, 4/3]$ (which mix with the superheavy $SO(10)/SU(5)$ coset gauginos: see Section 2. and 3.) couple only to terms containing at least one superheavy neutrino field $\bar{\nu}_A$. Thus, to leading order in $M^{-1}_U$, the exchange of such gaugino dirac multiplets will not lead to any $d = 5$ operator with 4 light external fields. However a puzzle remains.
Figure 8: Plot of the threshold corrected $\log_{10} M_X/M_X^0$ vs $Re(\xi)$ for complex $\xi$: $Im\xi = 1.2$, first solution for $x$.

The superheavy neutrinos mix with the usual light neutrinos via Dirac masses. So in the effective theory one trades them for the light neutrinos by using their equations of motion to leading order in their (inverse) masses (effectively $\bar{\nu} = -2(m_{\nu}^{\text{Dirac}}/M_{\nu}^{\text{Majorana}})\nu + ...$). The chiral parts $\bar{E}_2, \bar{F}_1, J_1$ of the Gauge Dirac $E,F,J$ multiplets therefore couple to light neutrinos and another light matter field with a small coupling $\sim O(m_{\nu}^{\text{Dirac}}/M_{\nu}^{\text{Majorana}})$. Exchange of the gaugino dirac fermion between a gauge yukawa vertex and a $\frac{126}{1} \cdot 16 \cdot 16$ vertex can lead to effective operators involving 4 light matter fields of which at least one is a light neutrino and one is anti-chiral. This appears to violate the usual argument that in the effective MSSM arising from a Susy GUT, supersymmetric D terms involving 4 light (mixed chiral and anti-chiral) fields must be $d \geq 6$ or equivalently that the $d = 5$, $B, L$ violating terms are either of form $[QQQL]_F$ or $[\bar{e}u\bar{d}]_F$. Exchange of $SO(10)/SU(5)$ coset gauginos peculiar to $SO(10)$ however appears to lead to (admittedly suppressed) $d = 5$ chiral-anti-chiral operators with 4 light fields. These operators arise once the Electroweak scale vev that gives rise to neutrino Dirac masses is turned on. This vev is smaller than $M_S$ and arises after soft susy breaking terms are included. In this theory $B-L$ is spontaneously broken giving rise to the Majorana mass for conjugate neutrinos (which was used to eliminate them in favour of the SM neutrinos). Thus perhaps the contradiction is not as violent as it seems at first. We emphasize that there are no
Figure 9: Plot of the threshold corrections to $\sin^2 \theta_w$ vs $Re(\xi)$ for complex $\xi$: $Im \xi = 1.2$, Second solution for $x$.

analogous processes in SU(5) Susy GUTs since there the 12 coset gauginos acquire partners from the purely AM type 24plets which do not couple to the matter sector.

The couplings of the gauginos of SO(10) to the matter fields are easily computed by adapting the PS reduction of the SO(10) covariant derivative for the spinor [11]:

\[
\mathcal{L}_{g-Y} = 2ig [\tilde{\psi}_{\kappa\alpha} \lambda^A (\frac{t^A}{2})^\kappa_{\mu\alpha} + \tilde{\psi}^{\mu*}_{\dot{\alpha}} \lambda^A (\frac{-t^A}{2})_{\mu\kappa}^* \tilde{\psi}^\kappa_{\dot{\alpha}} + \tilde{\psi}^{\mu*}_{\beta} (\frac{\lambda_R \cdot \bar{\sigma}}{2})_{\beta}^\gamma \bar{\psi}^\gamma_{\mu\gamma} + \tilde{\psi}^{\mu*}_{\beta} (\frac{\lambda_L \cdot \bar{\sigma}}{2})_{\beta}^\gamma \bar{\psi}^\gamma_{\mu\gamma}] \\
+ \frac{g}{\sqrt{2}} [\tilde{\psi}_{\dot{\alpha}} \lambda_{\mu\nu}^\alpha \bar{\psi}_{\mu\alpha} + \tilde{\psi}^{\mu*}_{\nu} \lambda_{\mu\nu}^\alpha \tilde{\psi}^\mu_{\dot{\alpha}}] + H.c 
\]

The terms carrying $B, L$ are are

\[
\mathcal{L}_{\Delta(B+L)\neq 0} = \sqrt{2}g[\tilde{L}^* \tilde{J}_4 Q + \tilde{Q}^* J_4 L] - \sqrt{2}g[\tilde{d}^* \tilde{J}_4 \bar{e} + \tilde{u}^* \tilde{J}_4 \bar{\nu} + \tilde{e}^* J_4 \bar{d} + \tilde{\nu}^* J_4 \bar{u}] + (g/\sqrt{2})[-\tilde{d}^* X_3 \alpha L - \tilde{u}^* \bar{E}_{(5)} L + \tilde{e}^* \bar{X}_3 Q + \tilde{\nu}^* \bar{E}_5 Q + \epsilon \tilde{d}^* \bar{E}_5 Q + \epsilon \tilde{u}^* X_3 Q] \\
+ (g/\sqrt{2})[\tilde{L}^* (X_3 \bar{d} - E_5 \bar{u}) - \tilde{Q}^* (X_3 \bar{e} - E_5 \bar{\nu}) + \epsilon \tilde{Q}^* (E_5 \bar{d} - X_3 \bar{u})] + .... 
\]
Figure 10: Plot of the threshold corrected \( \log_{10} \frac{M_X}{M_X^0} \) vs \( \Re(\xi) \) for complex \( \xi \): \( \Im \xi = 1.2 \), Second solution for \( x \).

There are no \( X[3,1,\pm 5/3] \) sector submultiplets in the \( \mathbf{126} \). Thus we can focus on just the \( E[3,2,\pm 1/3] \) and the \( J[3,1,\pm 4/3] \) sectors here. As discussed in Section 2., the superheavy gauginos \( \mathcal{J}_4, E_5 \) mix with \( \mathbf{126} \) derived fermions \( J_1[3,1,4/3] \) and \( E_2[3,2,-1/3] \). Examining eqn\((60)\) we see that \( J_1, \bar{E}_2 \) couple only to operators involving at least one superheavy \( \bar{\nu} \) field (\( E_1 \in \mathbf{126} \) does not mix with \( E \)-gauginos):

\[
W_{\bar{\Sigma}} = -8\sqrt{2} f'_{AB} [\bar{E}_2 Q_B + i J_1 \bar{u}_B] \bar{\nu}_A = [\bar{E}_2 Q_A + i J_1 \bar{u}_A] (f' M^{-1}_\nu m^D)_{AB} \nu_B + ... \quad (68)
\]

Since \( J_1 \) couples to a \( B = -1/3, L = 1 \) operator while \( \bar{J}_4 \) couples only to \( B = 1/3, L = -1, J \) exchange does not lead to \( B + L \) violation. The \( E \) sector gaugino i.e \( E_5 \) couples as

\[
\frac{g}{\sqrt{2}} [\epsilon d^* E_5 Q + \tilde{Q}^* E_5 \bar{\nu} - \tilde{L}^* E_5 \bar{u}] \quad (69)
\]

Only the first terms in \((68,69)\) are relevant and thus we find the following effective lagrangian due to superheavy gaugino exchange:

\[
L_{\Delta(B+L)=2} = 4g (f'(M^0)^{-1}m^{(\nu D)})_{AB} \mathcal{E}^{-1} \frac{2}{5} [\epsilon d C Q C (Q_A \bar{u}_B + \tilde{Q}_A \nu_B)] \quad (70)
\]

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where $\mathcal{E}^{-12}_{3}$ is essentially the mass of the exchanged gaugino times mixing factors written compactly in terms of the inverse of the relevant fermion mass matrix (in the $E[3, 2, \pm \frac{1}{3}]$ sector). By dressing this with MSSM gauginos we obtain $\Delta B = \Delta L = 1$ violating 4-fermi vertices responsible for processes like

$$u_L d_L \rightarrow \bar{d}_L + \nu_L$$

(71)

This is a vertex quite distinct from the Higgsino mediated vertices since it involves exchange of massive gauginos between a chiral and an anti-chiral vertex. It requires non-zero external momenta for the fermions and vanishes in the limit of zero external momenta. Thus the coefficient of the corresponding 4-fermi operators for B violation in the effective lagrangian is $\sim M^{\nu D}m_{Nucl}/M_X^2 M_S^2$ where $M_S$ is the Susy breaking scale. This magnitude seems hopelessly suppressed (relative even to gauge boson exchange) to be observable. Nevertheless the contrast of its structure with that of the standard $QQQL$ and $\overline{u}u\overline{d}d$ operators perhaps warrants a more thorough investigation of the conditions for the possibility of its appearance in the effective theory.
Figure 12: Plot of the threshold corrected $\log_{10} M_X/M_X^0$ vs $\text{Re}(\xi)$ for complex $\xi$: $\text{Im}\xi = 1.2$, Third solution for $x$.

6 Fermion Mass Formulae

A vital issue for any SO(10) GUT is the type of predictions it makes for the relations among the parameters of the (Type I and Type II) seesaw mechanisms [16] by which Neutrino masses and mixings arise. From the coupling of neutrinos to the $\mathbf{126}$ we find that the Majorana mass matrix of the superheavy neutrinos $\bar{\nu}_A$ is (eqn.(60))

$$M_{\bar{\nu}A}^{\mu} = -4i\sqrt{2}f_{\bar{\nu}A}^{\prime} < \Sigma_{44}^{(R+)} > = 4\sqrt{2}f_{\bar{\nu}A}^{\prime}\bar{\sigma}$$

Similarly the Majorana mass matrix for the left neutrinos $\nu_A$ is (eqn.(60)).

$$M_{\nu A}^{\mu} = 4\sqrt{2}f_{\nu A}^{\prime} < \bar{O}^{11} > = 8if_{\nu A}^{\prime} < \bar{O}_- >$$

where $< \bar{O}_- >$ is the small vev of the $SU(2)_L$ triplet in the $(\mathbf{10}, 3, 1)_\Sigma$ induced by a tadpole that arises as a consequence of $SU(2)_L$ breaking (see below).

In addition to this there is the Dirac mass which mixes the left and right neutrinos:

$$m_{\nu A}^{D} = 2\sqrt{2}h_{\nu A}^{\prime} < h_2^{(1)} > + 4i\sqrt{6}f_{\nu A}^{\prime} < h_2^{(2)} >$$

$$M_{\bar{\nu}A}^{\mu} = -4i\sqrt{2}f_{\bar{\nu}A}^{\prime} < \Sigma_{44}^{(R+)} > = 4\sqrt{2}f_{\bar{\nu}A}^{\prime}\bar{\sigma}$$

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$$M_{\nu A}^{\mu} = 4\sqrt{2}f_{\nu A}^{\prime} < \bar{O}^{11} > = 8i$$
We must make the fine tuning $\text{Det} H = 0$ necessary to keep a pair of Higgs doublets $H_{(1)}, \bar{H}_{(1)}$ (which is to develop the EW scale vev) light. Then these doublets will be the left and right null eigenstates of the mass matrix $H$. If the bi-unitary transformations responsible for diagonalizing $H^T H$ and $H H^T$ are $U, \tilde{U}$ i.e

$$Diag(m_H^{(1)}, m_H^{(2)}, \ldots) = U^T H U$$

then writing

$$h^{(i)} = U_{ij} H^{(j)} \quad ; \quad \bar{h}^{(i)} = \tilde{U}_{ij} \bar{H}^{(j)}$$

where $H^{(j)}, \bar{H}^{(j)}$ are the mass eigenstate doublets, the contributions of any coupling in which $h^{(i)}, \bar{h}^{(i)}$ enter can be accounted for in the effective MSSM below the heavy thresholds of the GUT just by replacing $h^{(i)} \rightarrow \alpha_i H^{(1)}, \bar{h}^{(i)} \rightarrow \bar{\alpha}_i \bar{H}^{(1)}$ where $\alpha_i = U_{i1}, \bar{\alpha}_i = \bar{U}_{i1}$ and we have numbered the massless doublet pair “1”. These components are easily obtained from the normalized null eigenvectors $V, \bar{V}$ of $H^T H$ and $H H^T$ to be $U_{i1} = V_i, \bar{U}_{i1} = \bar{V}_i^*$. Thus the neutrino Dirac mass matrix becomes

$$M'^D_{AB} = (2\sqrt{2}h'_{AB}\alpha_1 + 4i\sqrt{6}f'_{AB}\alpha_2)v_u$$

To obtain the final formula for the neutrino masses and mixings we must eliminate the $\bar{\nu}$ fields which are superheavy and evaluate the tadpole that gives rise to the Type II seesaw. The first step is standard. As for the $O(10,3,1)_{126, \bar{O}(10,3,1)_{126}}$ vevs, inspection of the mass spectrum (Table I in Appendix) and eqns. \ref{eq:80}-\ref{eq:88} yields the relevant terms in the superpotential as

$$W_{\Sigma M} \Sigma \cdot \bar{\Sigma} = M_O \bar{\Sigma} O + \bar{O} \Sigma \sqrt{2}(i\gamma \bar{a}_1 + 2i\sqrt{3}\eta \bar{a}_3)\bar{a}_4 v_d^2 - O\Sigma \sqrt{2}(i\gamma \bar{a}_1 + i2\sqrt{3}\eta \bar{a}_2)\alpha_4 v_u^2$$

$$- 2\sqrt{2}i\eta(\Sigma_4 4^{a1} \Phi_{44a}^{\beta} \bar{O}_{a\beta} + \Sigma_4 4^{a2} \Phi_{44a}^{43} \bar{O}_{a\beta})$$

since $\Sigma_4 4^{a1} = \frac{-\sqrt{3}}{2} h^{(3)}_{(1)}, \Sigma_4 4^{a2} = \frac{\sqrt{3}}{2} h^{(2)}_{(1)}, \Phi_{44a} = -\sqrt{2}h^{(4)}_{a}, \Phi_{44a}^{44} = \sqrt{2}h^{(4)}_{a},$ one gets for the relevant terms:

$$M_O \bar{O} O + \bar{O} \sqrt{2}(i\gamma \bar{a}_1 + 2i\sqrt{3}\eta \bar{a}_3)\bar{a}_4 v_d^2 - O \sqrt{2}(i\gamma \alpha_1 + i2\sqrt{3}\eta \alpha_2)\alpha_4 v_u^2$$

Thus the vev we need i.e $< \bar{O}_- >$ is immediately determined to leading order in $M_W/M_U$ by by the equation for $O_+$ as

$$< \bar{O}_- > = \sqrt{2}(i\gamma \alpha_1 + 2i\sqrt{3}\eta \alpha_2)\alpha_4 \frac{v^2_u}{M_O}$$

and $M_O$ can be read off from Table I to be $M_O = 2(M + \eta(3a - p))$. The quark and charged lepton mass matrices are
\[ M^d = (2 \sqrt{2} h' \bar{\alpha}_1 - 4 \sqrt{\frac{2}{3}} i f' \bar{\alpha}_2) v_d \] (81)

\[ M^u = (2 \sqrt{2} h' \alpha_1 - 4 \sqrt{\frac{2}{3}} i f' \bar{\alpha}_2) v_u \] (82)

\[ M^l = (2 \sqrt{2} h' \bar{\alpha}_1 + 4 \sqrt{6} i f' \bar{\alpha}_2) v_d \] (83)

These formulae are now in a form ready to use for fitting the fermion mass and mixing data after lifting it via the RG equations of the MSSM to the GUT scale.

7 Discussion and Outlook

In this paper we have calculated the complete superheavy spectrum of the Minimal supersymmetric GUT along with the gauge and chiral couplings of all MSSM multiplets in a readily accessible form. Partial calculations of these spectra and couplings [3, 11, 13, 14] have been published earlier but our method is different from the computer based method of [3, 13, 14] and is more complete, especially regarding couplings. Being analytic and explicit it also allows us to trace and resolve discrepancies arising within the computer based approach. We used the calculated spectra to perform a preliminary scan of the parameter space of the MSGUT as regards the magnitude of the threshold corrections to two crucial phenomenological parameters of the MSGUT: the Weinberg angle at low energy and the mass of the X lepto-quark gauge supermultiplet. We obtained a result that is in sharp contrast with expectations in the literature[18] that precision RG calculations in SO(10) are futile. On the contrary we find that the 1-loop GUT threshold and gauge two loop contributions are modest but significant. Thus, on the one hand, the basic GUT picture suggested by the convergence of gauge couplings in the MSSM is in fact not destroyed by the contributions of the large number of superheavy fields. On the other hand extant precision calculations that ignore threshold effects in SO(10) GUTs seem to be of dubious validity. In particular the proper RG analysis of the MSGUT taking into account EWRSB, all fermion masses and GUT threshold effects still remains to be done. This calculation is now being performed[27]. In view of the other phenomenological successes of renormalizable Susy SO(10) [5, 6, 7] and the unforeseen correlations between disparate phenomena like neutrino oscillations and nucleon decay that have emerged[12, 11] the mildness and calculability of threshold effects in the MSGUT is a most welcome and promising development. Our preliminary scans of the MSGUT parameter space (whose very feasibility - based on there being just one “sensitive” control parameter (\( \xi \)) - is a matter of some astonishment) show that the threshold effects can potentially narrow down the allowed regions of the MSGUT parameter space.
and indicate correlations between the GUT scale and the $B - L$ violating scale which can be of crucial significance when cross checking the particle physics phenomenology against cosmology. We have also argued that above the perturbative unification scale realistic renormalizable SO(10) GUTs are necessarily strongly coupled\cite{8, 9}. We have recently reported the results of 2-loop calculations of MSGUT RG equations \textit{above} the SO(10) restoration scale which we used to show that the strong growth of the SO(10) coupling above $M_X$ cannot be evaded by taking shelter in a weakly coupled fixed point\cite{30}. On the other hand our work\cite{8, 9} has shown that a scenario of a \textit{calculable} dynamical symmetry breaking of the GUT symmetry which utilizes the nearly exact supersymmetry at the GUT scale offers rich possibilities for the significance of the new length scale associated with the condensation of $SO(10)/G_{321}$ coset gauginos implied by both holomorphic analysis and by the Konishi anomaly. The present calculations show the way for crossing the threshold and entering into the SO(10) regime in a controlled way. The emerging coherence of the low energy phenomenology, $B$ and $L$ violation, perturbative GUT structures (such as the natural R-parity preservation\cite{31, 20}, successful seesaw scenarios, leptogenesis etc) and exciting hints of deeper mysteries, perhaps unveilable\cite{8, 9}, carry to our hopeful nostrils the spoor of a grail perhaps within reach.

\textbf{Note Added}

After this paper was posted on the arXive as hep-ph/0405074 the authors of hep-ph/0405300 claimed that the mass spectra listed in Appendix A were not internally consistent with the requirements of $SU(5)$ or $SU(5) \times U(1)$ symmetry (at the special vevs where $p = a = \pm \omega$). However this is incorrect. The mass spectra we derived via a PS decomposition of SO(10) organize straightforwardly and termwise into appropriate $SU(5)$ invariants for SU(5) invariant vevs as given in the Appendix B (added as above to hep-ph/0205074v1). This term-wise reorganization of several hundred $G_{123}$ invariant mass terms into SU(5) invariant mass terms is a more stringent consistency test than the tests of hep-ph/0405300 which are based on traces and determinants and valid only for their conventions. The phase conventions and field normalizations of hep-ph/0405300 are quite different from our work. Thus the blind application of their trace and determinant consistency tests to our results cannot but fail. We maintain unit field normalizations throughout by using only unitary field redefinitions of fields with canonical kinetic terms. Finally our results for chiral spectra also coincide, up to minor convention related adjustments, with those obtained in a parallel computation reported in \cite{13}.

Finally we stress that our method\cite{11} yields \textit{all} coupling coefficients between both spinor and tensor irreps and not just the tensor irrep ones relevant for masses and symmetry breaking which were obtained using the method of \cite{10}. Moreover we note that the most complex of the mass matrices given here namely those of the
Higgs doublets and triplets relevant for proton decay were already derived by us in hep-ph/0204097v2(2003) [11].

Further Note Added

After version 2 of this paper (including the SU(5) reorganization given in Appendix B above) was accepted for publication the authors of hep-ph/0405030 issued yet another preprint (hep-ph/0412348v1) this time claiming that although our results pass the SU(5) reorganization test for $\sigma = \bar{\sigma} = 0$ they failed to do so for $\sigma = \bar{\sigma} \neq 0$ and that the counting of Goldstone modes and distinct mass eigenvalues was, in their opinion incorrect. Further they claimed that our results were inconsistent since they failed to pass certain ‘trace and hermiticity tests’ that they had applied successfully to their own results. All these claims are incorrect. Our results in fact pass all three tests. We have issued a preprint showing this explicitly [33]. Here we only remark that once the super-Higgs effect for $SO(10) \rightarrow MSSM$ has been verified it is scarcely feasible that the Goldstone-Higgs counting could fail for $SO(10) \rightarrow SU(5)$ since the latter is a special case of the same spectra! However the reader can easily check that the SU(5) singlet and 10plet mass matrices have zero determinant confirming that the required Goldstone supermultiplets $1 + 10 + \bar{10}$ are present. The demonstration that the trace constraints and ‘hermiticity’ tests of hep-ph/0205300 are also satisfied is also straightforward once proper account is taken of the difference in the phase conventions of the two calculations. Details may be found in [33]. Finally as this paper goes to press the authors of hep-ph/0412348v1 have reissued the preprint hep-ph/0412348v2 in which all the claims of the inconsistency of our results are totally retracted.

8 Acknowledgments

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Appendix A: Tables of masses and mixings

In this appendix we collect our results for the chiral fermion/gaugino states, masses and mixing matrices for the reader’s convenience. Apart from the discussion of gauge
multiplet masses our results have been obtained in parallel with and are compatible with those of [3], which, however, are computed with a different normalization for the $\Sigma_i, \Sigma$ fields resulting in a difference between the mass and yukawa coupling parameters $M, \eta$ of these multiplets in the two starting actions. Moreover certain minor phase differences also exist between the definitions of representative states used by them and our definitions for the same states (which follow directly from our consistent definitions of PS tensors from SO(10) submultiplets). Mixing matrix rows are labelled by barred irreps and columns by unbarred. Unmixed cases (i)) are given as Table I.

ii) Chiral Mixed states

a) $[8, 1, 0](R_1, R_2) \equiv (\hat{\phi}_R^\rho, \hat{\phi}_{R(0)}^\rho)$

$$\mathcal{R} = 2 \begin{pmatrix} (m - \lambda a) & -\sqrt{2} \lambda \omega \\ -\sqrt{2} \lambda \omega & m + \lambda(p - a) \end{pmatrix}$$

$$m_{R±} = |\mathcal{R}_±| = |2m[1 + (\frac{\tilde{p}}{2} - \tilde{a}) \pm \sqrt{(\frac{\tilde{p}}{2})^2 + 2\tilde{\omega}^2}]|$$

The corresponding eigenvectors can be found by diagonalizing the matrix $\mathcal{R} \mathcal{R}^\dagger$.

b) $[1, 2, -1](\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_4) \oplus [1, 2, 1](h_1, h_2, h_3, h_4)$

$$\equiv (H^\alpha_2, \Sigma^{(15)}_1, \Sigma^{(15)}_3, \frac{\phi_{2\alpha}}{\sqrt{2}}) \oplus (H_{\alpha 1}, \Sigma^{(15)}_{\alpha 1}, \Sigma^{(15)}_{\alpha 1}, \frac{\phi_{4\alpha i}}{\sqrt{2}})$$

$$\mathcal{H} = \begin{pmatrix} -M_H & \tau \sqrt{3}(\omega - a) & -\gamma \sqrt{3}(\omega + a) & -\gamma \sigma \\ -\gamma \sqrt{3}(\omega + a) & 0 & (2M + 4\eta(a + \omega)) & 0 \\ \gamma \sqrt{3}(\omega - a) & -(2M + 4\eta(a - \omega)) & 0 & -2\gamma \sigma \sqrt{3} \\ -\sigma \gamma & -2\eta \sigma \sqrt{3} & 0 & -2m + 6\lambda(\omega - a) \end{pmatrix}$$

The above matrix is to be diagonalized after imposing the fine tuning condition $Det\mathcal{H} = 0$ to keep one pair of doublets light.

c) $[3, 1, \frac{2}{3}](\bar{t}_1, \bar{t}_2, \bar{t}_3, \bar{t}_4) \oplus [3, 1, -\frac{2}{3}](t_1, t_2, t_3, t_4, t_5)$

$$\equiv (H^\mu_4, \Sigma^{(14)}_4, \Sigma^{(14)}_{\mu 4}, \Sigma^{(14)}_{R0}, \phi^{\mu}_{4(R+)}) \oplus (H_{\mu 4}, \Sigma_{(\mu a)}, \Sigma_{\mu 4(a)}, \Sigma_{\mu 4(R0)}, \phi^{4}_{(R-)})$$

$$\mathcal{T} = \begin{pmatrix} M_H & \tau (a + p) & \gamma (p - a) & 2\sqrt{2} i\omega \bar{\gamma} & i\bar{\sigma} \bar{\gamma} \\ \gamma (p - a) & 0 & 2M & 0 & 0 \\ \gamma (p + a) & 2M & 0 & 4\sqrt{2} i\omega \eta & 2i\eta \sigma \\ -2\sqrt{2} i\omega \gamma & -4\sqrt{2} i\omega \eta & 0 & 2M + 2\eta p + 2\eta a & -2\sqrt{2} \eta \sigma \\ i\sigma \gamma & 2i\eta \sigma & 0 & 2\sqrt{2} \eta \sigma & -2m - 2\lambda(a + p - 4\omega) \end{pmatrix}$$

iii) Mixed gauge chiral .

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Table 1: i) Masses of the unmixed states in terms of the superheavy vevs. The $SU(2)_L$ contraction order is always $\bar{F}^\alpha F_\alpha$. The primed fields defined for $SU(3)_c$ sextets maintain unit norm. The absolute value of the expressions in the column “Mass” is understood.
| Field$[SU(3), SU(2), Y]$ | $\{S_3, S_2, S_1\}$ | $S_W$ | $S_X$ |
|--------------------------|----------------------|-------|-------|
| $A[1,1,4]$               | $\{0,0,12/5\}$       | 9.6   | 12    |
| $B[6,2,5/3]$             | $\{5,3,5\}$          | 19.2  | -6    |
| $C[8,2,1]$               | $\{6,4,12/5\}$       | 4.8   | -24   |
| $D[3,2,7/3]$             | $\{1,3/2,49/10\}$    | 10.8  | 21    |
| $E[3,2,1/3]$             | $\{1,3/2,1/10\}$     | -8.4  | -3    |
| $F[1,1,2]$               | $\{0,0,3/5\}$        | 2.4   | 3     |
| $G[1,1,0]$               | $\{0,0,0\}$          | 0     | 0     |
| $h[1,2,1]$               | $\{0,1/2,3/10\}$     | -3.6  | 3     |
| $I[3,1,10/3]$            | $\{1/2,0,5\}$        | 22.8  | 21    |
| $J[3,1,4/3]$             | $\{1/2,0,4/5\}$      | 6     | 0     |
| $K[3,1,8/3]$             | $\{1/2,0,16/5\}$     | 15.6  | 12    |
| $L[6,1,2/3]$             | $\{5/2,0,2/5\}$      | 15.6  | -18   |
| $M[6,1,8/3]$             | $\{5/2,0,32/5\}$     | 39.6  | 12    |
| $N[6,1,4/3]$             | $\{5/2,0,8/5\}$      | 20.4  | -12   |
| $O[1,3,2]$               | $\{0,2,9/5\}$        | -12   | 15    |
| $P[3,3,2/3]$             | $\{3/2,6,3/5\}$      | -46.8 | 9     |
| $Q[8,3,0]$               | $\{9,16,0\}$         | -103.2| -24   |
| $R[8,1,0]$               | $\{3,0,0\}$          | 16.8  | -24   |
| $S[1,3,0]$               | $\{0,2,0\}$          | -19.2 | 6     |
| $t[3,1,2/3]$             | $\{1/2,0,1/5\}$      | 3.6   | -3    |
| $U[3,3,4/3]$             | $\{3/2,6,12/5\}$     | -39.6 | 18    |
| $V[1,2,3]$               | $\{0,1/2,27/10\}$    | 6     | 15    |
| $W[6,3,2/3]$             | $\{15/2,12,6/5\}$    | -68.4 | -18   |
| $X[3,2,5/3]$             | $\{1,3/2,5/2\}$      | 1.2   | 9     |
| $Y[6,2,1/3]$             | $\{5,3,1/5\}$        | 0     | -30   |
| $Z[8,1,2]$               | $\{3,0,24/5\}$       | 36    | 0     |

Table 2: Index values for the 26 different chiral multiplet types (used in the threshold corrections). Except for Q,R,S all other reps come in complex pairs. $S_W = 4S_1 - 9.6S_2 + 5.6S_3$, $S_X = 5S_1 + 3S_2 - 8S_3$ are the combinations that enter the threshold corrections to $\sin^2 \theta_W$ and to $\log_{10} M_X$.
\[ a) [1, 1, 0] (G_1, G_2, G_3, G_4, G_5, G_6) \equiv (\phi, \phi^{(15)}, \phi^{(15)}_{(R^0)}, \Sigma^{(15)}_{(R^0)}, \Sigma^{4(15)}_{44(15)}, \sqrt{2} \lambda_{(R^0)} - \sqrt{3} \lambda^{(15)}) \]

\[ G = 2 \begin{pmatrix}
    m & 0 & \sqrt{6} \lambda \omega & \frac{i \eta \sigma}{\sqrt{2}} & -\frac{i \eta \sigma}{\sqrt{2}} & 0 \\
    0 & m + 2 \lambda a & 2 \sqrt{2} \lambda \omega & \frac{i \eta \sigma}{\sqrt{3}} & -\frac{i \eta \sigma}{\sqrt{3}} & 0 \\
    \sqrt{6} \lambda \omega & 2 \sqrt{2} \lambda \omega & m + \lambda (p + 2a) & -i \eta \sqrt{3} \sigma & i \sqrt{3} \eta \sigma & 0 \\
    \frac{i \eta \sigma}{\sqrt{2}} & i \eta \sigma \sqrt{\frac{3}{2}} & -i \eta \sqrt{3} \sigma & M + \eta (p + 3a - 6 \omega) & 0 & \frac{\sqrt{5} \sigma^*}{2} \\
    -\frac{i \eta \sigma}{\sqrt{2}} & -i \eta \sigma \sqrt{\frac{3}{2}} & i \eta \sigma \sqrt{3} \sigma & \frac{\sqrt{5} \sigma^*}{2} & 0 \\
    0 & 0 & 0 & \frac{\sqrt{5} \sigma^*}{2} & 0 & 0
\end{pmatrix} \]

\[ b) [3, 2, -\frac{1}{3}] (\bar{E}_2, E_3, \bar{E}_4, E_5) \oplus [3, 2, \frac{1}{3}] (E_2, E_3, E_4, E_5) \]
\[ \equiv (\Sigma^{\mu_4}_{4a_1}, \phi^{(s)}_{4a_2}, \phi^{(s)}_{4a_2}, \lambda^{(s)}_{a_2}) \oplus (\Sigma^{4}_{\mu_4a_1}, \phi^{(s)}_{\mu_4a_1}, \phi^{(s)}_{\mu_4a_1}, \lambda^{(s)}_{\mu_4a_1}) \]

\[ E = \begin{pmatrix}
    -2(M + \eta (a - 3 \omega)) & -2 \sqrt{2} i \eta \sigma & 2 i \eta \sigma & i g \sqrt{2} \sigma^* \\
    2 i \sqrt{2} \eta \sigma & -2(m + \lambda (a - \omega)) & -2 \sqrt{2} \lambda \omega & 2 g(a^* - \omega^*) \\
    -2 i \eta \sigma & -2 \sqrt{2} \lambda \omega & -2(m - \lambda \omega) & \sqrt{2} g(\omega^* - p^*) \\
    -i g \sqrt{2} \sigma^* & 2 g(a^* - \omega^*) & \sqrt{2} g(\omega^* - p^*) & 0
\end{pmatrix} \]

\[ c) [1, 1, -2] (\bar{F}_1, F_2, F_3) \oplus [1, 1, 2] (F_1, F_2, F_3) \]
\[ \equiv (\Sigma^{44}_{(R^0)}, \phi^{(15)}_{(R^0)}, \lambda_{(R^-)} \oplus (\Sigma^{44}_{(R^0)}, \phi^{(15)}_{(R^0)}, \lambda_{(R^+)}). \]

\[ F = \begin{pmatrix}
    2(M + \eta (p + 3a)) & -2 i \sqrt{3} i \eta \sigma & -g \sqrt{2} \sigma^* \\
    2 i \sqrt{3} \eta \sigma & 2(m + \lambda (p + 2a)) & \sqrt{2} i g \omega^* \\
    -g \sqrt{2} \sigma^* & \sqrt{2} i g \omega^* & 0
\end{pmatrix} \]

\[ d) [3, 1, -\frac{1}{3}] (\bar{J}_1, J_2, \bar{J}_3, J_4) \oplus [3, 1, \frac{1}{3}] (J_1, J_2, J_3, J_4) \]
\[ \equiv (\Sigma^{4}_{\mu_4}, \phi^{(s)}_{4}, \phi^{(s)}_{4}, \lambda^{(s)}_{4}) \oplus (\Sigma^{4}_{\mu_4}, \phi^{(s)}_{\mu_4}, \phi^{(s)}_{\mu_4}, \lambda^{(s)}_{\mu_4}) \]

\[ J = \begin{pmatrix}
    2(M + \eta (a + p - 2 \omega)) & -2 \eta \sigma & 2 \sqrt{2} \eta \sigma & -i g \sqrt{2} \sigma^* \\
    2 \eta \sigma & -2(m + \lambda a) & -2 \sqrt{2} \lambda \omega & -2 i g \sqrt{2} a^* \\
    -2 \sqrt{2} \eta \sigma & -2 \sqrt{2} \lambda \omega & -2(m + \lambda (a + p)) & -4 i g \omega^* \\
    -i g \sqrt{2} \sigma^* & 2 \sqrt{2} i g a^* & 4 i g \omega^* & 0
\end{pmatrix} \]

\[ e) [3, 2, \frac{5}{3}] (\bar{X}_1, X_2, \bar{X}_3) \oplus [3, 2, -\frac{2}{3}] (X_1, X_2, X_3) \]
\[ \equiv (\phi^{(s)}_{4a_1}, \phi^{(s)}_{4a_1}, \lambda^{(s)}_{4a_1}) \oplus (\phi^{(s)}_{4a_2}, \phi^{(s)}_{4a_2}, \lambda^{(s)}_{4a_2}) \]

\[ X = \begin{pmatrix}
    2(m + \lambda (a + \omega)) & -2 \sqrt{2} \lambda \omega & -2 g(a^* + \omega^*) \\
    -2 \sqrt{2} \lambda \omega & 2(m + \lambda \omega) & \sqrt{2} g(\omega^* + p^*) \\
    -2 g(a^* + \omega^*) & \sqrt{2} g(\omega^* + p^*) & 0
\end{pmatrix} \]
Appendix B: $SU(5) \times U(1)$ Reassembly Crosscheck

Given the complexity of the spectra and couplings derived here it would be useful to have a method of cross checking the internal consistency of our results. A stringent check is provided by verifying that at special values of the vevs i.e.

\[ p = a = \pm \omega \]

where the unbroken symmetry includes $SU(5)$ the MSSM labelled mass spectra and couplings given in Appendix A do indeed reassemble into $SU(5)$ invariant form. For the mass spectra this is fairly straightforward to check and is reported explicitly below. A similar calculation \[32\] for the super potential couplings is much more tedious but furnishes an $SO(10) - SU(5) \times U(1)$ analog of the “$SO(10)$-PS Clebsches” reported here.

The decomposition of the chiral multiplets of the MSGUT into $SU(5) \times U(1)$ multiplets and of those into MSSM multiplets (named as per the alphabetic convention of Appendix A) is given below. The only complication is that certain MSSM multiplet types occur in several copies and (orthogonal) mixtures of these are present in the different $SU(5)$ multiplets. Thus, for instance, the 210 contains a 24 and a 75 of $SU(5)$ both of which contain mixtures of the $G_{123}$ multiplets $R_1(8,1,0)$ and $R_2(8,1,0)$. These mixtures must be orthogonal and must be precisely the eigenstates of the mass matrices in this $G_{123}$ sector which have the same masses as the rest of the $G_{123}$ submultiplet sets within the 24-plet and 75-plets as two wholes. The fact that this follows in every case from our results appears to confirm their reliability. The decompositions we need are:

\[
\begin{align*}
\mathbf{H} &= 10 = 5_1 + \bar{5}_{-1} \\
\mathbf{\Sigma} &= 126 = 1_{-5}(G_4) + \bar{5}_{-1} + 10_{-3} + \bar{15}_3 + 45_1 + \bar{50}_{-1} \\
5_{-1} &= \bar{h}_3(1,2,-1) + \bar{f}_{3,4}(\bar{3},1,\frac{2}{3}) \\
10_{-3} &= F_1(1,1,2) + \bar{J}_1(\bar{3},1,-\frac{4}{3}) + E_2(3,2,\frac{1}{3}) \\
\bar{15}_3 &= O(1,3,-2) + \bar{E}_1(\bar{3},2,-\frac{1}{3}) + \bar{N}(\bar{6},1,\frac{4}{3}) \\
45_1 &= h_3(1,2,1) + t_3(3,1,-\frac{2}{3}) + P(3,3,-\frac{2}{3}) + \bar{K}(\bar{3},1,\frac{8}{3}) + \bar{D}_1(\bar{3},2,-\frac{7}{3}) \\
&\quad + \bar{L}(\bar{6},1,-\frac{2}{3}) + C_1(8,2,1) \\
\bar{50}_{-1} &= A(1,1,4) + \bar{f}_{3,4}(\bar{3},1,\frac{2}{3}) + D_2(3,2,\frac{7}{3}) + W(6,3,\frac{2}{3}) + \bar{M}(\bar{6},1,-\frac{8}{3}) + \bar{C}_2(8,2,-1) \\
\mathbf{\Sigma} &= \mathbf{126} = 1_5(G_5) + 5_1 + \bar{10}_3 + 15_{-3} + 45_{-1} + 50_1
\end{align*}
\]
\[ 5_1 = h_2(1, 2, 1) + t_{2,4}(3, 1, -\frac{2}{3}) \]

\[ \overline{10}_3 = \bar{F}_1(1, 1, -2) + J_1(3, 1, \frac{4}{3}) + \bar{E}_2(3, 2, -\frac{1}{3}) \]

\[ 15_{-3} = \bar{O}(1, 3, 2) + E_1(3, 2, \frac{1}{3}) + N(6, 1, -\frac{4}{3}) \]

\[ 45_{-1} = \bar{h}_2(1, 2, -1) + \bar{r}_2(3, 1, \frac{2}{3}) + \bar{P}(3, 3, \frac{2}{3}) + K(3, 1, -\frac{8}{3}) + D_1(3, 2, \frac{7}{3}) \]

\[ + \quad L(6, 1, \frac{2}{3}) + \bar{C}_1(8, 2, -1) \]

\[ 50_1 = \bar{A}(1, 1, -4) + t_{2,4}(3, 1, -\frac{2}{3}) + \bar{D}_2(3, 2, -\frac{7}{3}) + \bar{W}(6, 3, -\frac{2}{3}) + M(6, 1, \frac{8}{3}) + C_2(8, 2, 1) \]

\[ \Phi = 210 = l_0 + 5_{-4} + 5_4 + 10_2 + \overline{10}_{-2} + 24_0 + 40_2 + \overline{40}_{-2} + 75_0 \]

\[ l_0 = G_{1,2,3} \]

\[ 5_{-4} = h_4(1, 2, 1) + t_5(3, 1, -\frac{2}{3}) \]

\[ 5_4 = \bar{h}_4(1, 2, -1) + \bar{t}_5(3, 1, \frac{2}{3}) \]

\[ 10_2 = F_2(1, 1, 2) + \bar{J}_{2,3}(3, 1, -\frac{4}{3}) + E_{3,4}(3, 2, \frac{1}{3}) \]

\[ \overline{10}_{-2} = \bar{F}_2(1, 1, -2) + J_{2,3}(3, 1, \frac{4}{3}) + \bar{E}_{3,4}(3, 2, -\frac{1}{3}) \]

\[ 24_0 = (1, 1, 0)G_{1,2,3} + S(1, 3, 0) + X_{1,2}(3, 2, -\frac{5}{3}) + \bar{X}_{1,2}(3, 2, \frac{5}{3}) + R_{1,2}(8, 1, 0) \]

\[ 40_2 = V(1, 2, -3) + E_{3,4}(3, 2, \frac{1}{3}) + \bar{J}_{2,3}(3, 1, -\frac{4}{3}) + \bar{U}(3, 3, -\frac{4}{3}) + Z(8, 1, 2) + \bar{Y}(6, 2, \frac{1}{3}) \]

\[ \overline{40}_{-2} = \bar{V}(1, 2, 3) + \bar{E}_{3,4}(3, 2, -\frac{1}{3}) + J_{2,3}(3, 1, \frac{4}{3}) + U(3, 3, \frac{4}{3}) + \bar{Z}(8, 1, -2) + Y(6, 2, -\frac{1}{3}) \]

\[ 75 = (1, 1, 0)G_{1,2,3} + I(3, 1, \frac{10}{3}) + \bar{I}(3, 1, -\frac{10}{3}) + X_{1,2}(3, 2, -\frac{5}{3}) + \bar{X}_{1,2}(3, 2, \frac{5}{3}) \]

\[ + \quad B(6, 2, \frac{5}{3}) + \bar{B}(6, 2, -\frac{5}{3}) + R_{1,2}(8, 1, 0) + Q(8, 3, 0) \quad (92) \]

If we insert \(a = -\omega = p\) in the mass matrices of Appendix A we find that, after diagonalizing the mass matrices of the submultiplets that mix, the resultant spectra group precisely as indicated by the decompositions above with all the subreps of a given SU(5) irrep obtaining the same mass. One obtains the SU(5) invariant mass terms:

\[ 2(M + 10\eta p) \Sigma \Sigma + 2(M + 4\eta p) \Sigma 5_2 5_2 + 2(M - 2\eta p) \Sigma 50_2 50_2 \]

\[ + \quad 2(M + 4\eta p) 10_2 \Sigma + 2(M + 2\eta p) 10_5 \Sigma + 2M 15_2 \Sigma + 2M 45_2 \Sigma \]

\[ + \quad M_H 5_H 5_H + (m + 6\lambda p)(1_\phi)^2 + 2(m + 6\lambda p) \Sigma 5_5 \phi + 2(m + 3\lambda p) 10_\phi + 10_\phi \]

\[ + \quad (m + \lambda p)(24_\phi)^2 + 2m 40_\phi 40_\phi + (m - 2\lambda p)(75_\phi)^2 \]

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\[
+ 2\sqrt{3}(\sigma(5\Sigma \Phi + 10\Sigma \Phi) + \sigma(5\Sigma \Phi + \Sigma 10\Phi))
\]
\[
+ 2\sqrt{3}(\gamma 5\Sigma \Phi + \gamma 5\Sigma \Phi) + 2\sigma(\Phi 1\Phi - \Phi 1\Phi) + \sigma 5\Phi \Phi 5\Phi + \gamma 5\Phi \Phi 5\Phi
\]

Where every $SU(5)$ invariant has been normalized so that the individual $G_{123}$ sub-rep masses can be read off directly from the coefficient of the invariant for complex $SU(5)$ representations which pair into Dirac supermultiplets and is 2 times the coefficient for the real representations which remain unpaired Majorana/Chiral supermultiplets. For $a = -\omega = p, \sigma = \bar{\sigma} = 0$ the 20 Goldstone supermultiplets $G, J, F, E, \bar{E}$ of the coset $SO(10)/SU(5) \times U(1)$ remain heavy (see the gauge-chiral super-Higgs mass formulae in Section 2) as they should since they are eaten in the spontaneous breaking $SO(10) \rightarrow SU(5) \times U(1)$ while the 12 fields in the \{X$_3$, \bar{X}$_3$\} multiplets lose their mass terms with \{X$_{1,2}$, \bar{X}$_{1,2}$\} since they form part of the unbroken $SU(5)$ gauge supermultiplet. When $a = \omega = p$ i.e for flipped $SU(5)$, the roles of the \{X$_3$, \bar{X}$_3$\} and \{E$_5$, \bar{E}$_5$\} gauge multiplets are interchanged, with the E’s remaining massless and the X’s becoming heavy, so that one obtains the $SU(5)$ invariant groupings corresponding to the “flipped” $SU(5) \times U(1) \subset SO(10)$ embedding. Note that this successful $SU(5)$ reassembly is a much more fine-grained consistency test than any overall trace or determinant test.

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