ULTIMATE LIMIT OF FUTURE COLLIDERS*

M.Bai †, SLAC National Accelerator Laboratory, Menlo Park, CA
V.Shiltsev†, FermiLab, Batavia, IL, 60510, USA
G. White †, SLAC National Accelerator Laboratory, Menlo Park, CA
F. Zimmermann †, CERN, Geneva, SW

Abstract

With seven operational colliders in the world and two under construction, the international particle physics community not only actively explores options for the next facilities for detailed studies of the Higgs/electroweak physics and beyond-the-LHC energy frontier, but also seeks a clear picture of the limits of the colliding beams method. In this paper, we try to consolidate various recent efforts in identifying physics limits of colliders in conjunction with societal sustainability, and share our thoughts about the perspective of reaching the ultimate collider that is at the quantum limit.

THE LANDSCAPE OF COLLIDERS

The development of accelerators and beams in the past century has led to incredible discoveries in physics, chemistry, biology, etc. Up to date, about 25 Nobel Prizes in Physics and 7 in Chemistry were due to the significant contribution of the development in accelerator and beams [1,3]. Among the family of accelerator, collider has been the engine of discovery for particle physicists to discover new particles and understand the fundamental laws that govern the subatomic structure. Fig. 1 [4] provides an overview of most of particle colliders from the past, currently in operation as well as the proposed ones for future. It is clear that the energy of the colliders has increased orders of magnitudes over the past half century. Table 1 shows the colliders that are currently in operation and under construction, i.e. NICA at Russia and Electron Ion Collider at BNL in the USA. Despite the pace of constructing high energy collider has also been noticeably become slower over the past couple of decades, the quest of continuing pushing the collider physics to the next energy frontier has never quenched. The latest discovery of Higgs at the LHC has not only brought the triumph to the Standard model, but also inspired the desire of further pushing the energy frontier of the next colliders. At the latest Snowmass community meeting, the high energy physics community has proposed the road map for the next 100-year of following the ongoing LHC luminosity upgrade with realizing Higgs factory of electron position collider to study the properties of Higgs at high precision, multi TeV lepton collider or 100 TeV proton collider for discovery new physics. These requirements clearly pushes the future colliders into unprecedented scale as shown in Fig. ??, and face many performance limits.

ULTIMATE LIMITS OF FUTURE ENERGY FRONTIER COLLIDERS

Energy

With the conventional RF acceleration technology, the beam energy of a collider is ultimately determined by the limit of acceleration gradient for linear collider, bending magnetic field for circular collider, and sustainable size for construction and operation. For electron-positron collider, due to the synchrotron radiation, the energy of the circular collider is ultimately limited by $E_{e^+e^-} < 500 GeV (\frac{R}{100 m})^{1/3}$. For protons, the energy collider is ultimately determined by the limits of bending magnetic field. The current proposed 100 TeV center-of-mass energy proton proton colliders such as FCC at CERN requires dipole magnet of 16 Tesla, which

---

* Work supported by DOE
† mbai@slac.stanford.edu

---

**Table 1: Margin Specifications**

| Collider   | Location | Species | Max Beam Energy |
|------------|----------|---------|-----------------|
| LHC        | CERN, Europe | p       | 6.8 TeV         |
| RHIC       | Upton, NY | Pb, Xe  | 2.56 TeV        |
|             |          | p↑      | 255 GeV         |
| SuperKEKB  | KEK, Japan | e^+e^-  |                 |
| BEPCII     | IHEP, China | e^+e^-  |                 |
| DAΦNE      | INFN,      | e^+e^-  |                 |
| VEPP-2000. | Russia     | e^+e^-  |                 |
| VEP-4M.    | Russia     | e^+e^-  |                 |
| NICA       | JINR, Russia | p      |                 |
| EIC        | BNL, USA.  | e^+p↑   |                 |
|            |           | Au, U, etc. |               |

---

Figure 1: The so-called Livingston plot represents the evolution of the colliders from the past to the future.
is currently under research and development. To reach 1 PeV center-of-mass energy in the same 100 km tunnel will require a bending magnet of 160 Tesla! To go further beyond, the circular proton proton collider beam energy is also ultimately limited by \(E_{pp} < 10 \text{PeV} \left( \frac{R}{10 \text{km}} \right)^{1/3} \) due to the synchrotron radiation.

For electron-positron linear collider, the ultimate beam energy is determined by the limit of acceleration gradient. For CLIC with 100 MV/m acceleration gradient normal conducting technology, 50km long laser straight tunnel is required to reach 3 TeV. The newly developed cryogenic copper c-band technology has shown the path to push the acceleration gradient towards 150 MV/m 

Cool copper collider 

a newly proposed linear collider based on the high gradient cryogenic copper c-band technology, could push the center-of-mass energy to 4.5 GeV within the same 50km tunnel. To reach center-of-mass energy of 10 TeV and beyond, will require quite a few substantial developments in

- extreme high gradient RF acceleration structure
- laser straight long tunnel construction to avoid vertical emittance growth due to earth magnetic field
- cost-effective high efficient RF power source

The concept of colliding muons has been developed for several decades. Despite the advantage of less susceptible to synchrotron radiation in comparison to electron or positron, the muon collider energy is ultimately by the available acceleration within its lifetime.

**luminosity**

The discovery of new fundamental constituents with the lepton collider requires the luminosity scaled with the centre-of-mass energy \(E_{cm} \) as \(( \frac{E_{cm}}{1 \text{TeV}})^2 \times 10^{35} \text{cm}^{-2} \text{s}^{-1} \) [liantao, 3]. This means the minimum luminosity for a direct search of new physics at a lepton collider of 10 TeV \(E_{cm} = 2 \times 10^{21} \text{cm}^{-2} \text{s}^{-1} \) and minimum luminosity is \(E_{cm} = 3 \times 10^{35} \text{cm}^{-2} \text{s}^{-1} \) for precision measurement. For hadron colliders, the mass of the particle that can be created scales with \(E_{cm}^{2/3}L^{1/3} \) [liantao, 3], and the ultimate beam in this case is to reach collision at \(>= 100 \text{ TeV} \) centre-of-mass energy with \(10^{35} 10^{36} \text{cm}^{-2} \text{s}^{-1} \) luminosity 

In general, the luminosity in a collider is given in Eq. 1.

\[
L = \frac{1}{4\pi} \frac{P_{wall} \eta}{mC^2} \frac{N_p}{\beta' e_n} F_{geom}
\]

where \(P_{wall} \) is the wall-plug power, \(\eta \) is the ratio of wall-plug power to beam power, \(N_p \) is the number of particles per bunch, \(\beta' \) is the beta function at the IP, \(e_n \) is the normalised beam emittance and \(F_{geom} \) describes the geometric factors such as hour-glass, pinch effect, etc. Eq. 1 shows the brighter the beam, the higher luminosity. And, for a given optics at IP, the luminosity scales linearly with energy. Hence, to meet the required quadratic scaling of luminosity w.r.t. beam energy for the lepton colliders, the beam brightness needs to be scaled with energy accordingly. This requires one to overcome many limiting factors starting from obtaining and preserving low emittance bright beam through acceleration and final focusing ?? . Thanks for the advancements in beam dynamics and beam techniques, such as beam-beam effect, crab crossing, etc, over the past decade, the luminosity performance of current operating colliders has been very impressive as shown in Fig. 3. Despite their low energy and compact size, both DAΦNE and BEPC-II were able to reach much higher peak luminosity than the first generation high energy e+e- linear collider SLC. Nevertheless, to reach similar peak luminosity performance, future multi-TeV e+e- faces additional unique limiting factors due to beamstrahlung, Oide effects, and coherent pair production. R&Ds in mitigating these effects such as ultra short bunch [?] are ongoing.

In principle, the ultimate achievable normalized beam emittance is its quantum limit, i.e. \(\epsilon_{\beta' e_{n}}^{QM} = \bar{\epsilon}_{\beta' e_{n}}/2 \) ?? . In such scenario without taking into account of other limiting factors such as beamstrahlung and Oide effect, the achievable luminosity is orders of magnitude above desired luminosity for the multi-TeV e+e- colliders [3]. Nevertheless, to obtain beam emittance at quantum limit at the collision point requires the final beam delivery system being able to reach similar level of precision, which is beyond the current conventional technology.

**Advanced acceleration concept based collider**

It is evident that so far the colliders based on conventional RF technology have not yet reached the physics limits, neither in energy nor in luminosity. Nevertheless, the physical limits, both engineering and societal sustainability, have been the main limiting factor of the proposed future high energy colliders. Fig. ?? shows the power consumption of energy frontier colliders, both current in operation and proposed for future. For the case of LHC, the current power consumption of LHC operation including detectors is 600
clear that disruptive technologies for acceleration as well as beam manipulation are required to find the path of realizing the future energy frontier colliders within the societal sustainability and continue pushing the energy frontier far beyond the currently proposed multi-TeV colliders.

Both laser driven and beam driven plasma wakefield acceleration, aka, LWFA and PWFA, have been pursued and intensified worldwide [2]. While unprecedented acceleration gradient has been demonstrated with both PWFA and LWFA [3], the achieved beam quality, both intensity and brightness, has not yet reached the comparable level as what conventional RF technology based colliders have delivered. The recent progress of AWAKE has shown very encouraging steps towards a possibly of very high energy electron-proton collider based on beam-driven PWFA. Nevertheless, the projected luminosity is on the order of $10^{28}$ to $10^{29} \text{cm}^{-2} \text{s}^{-1}$. Overall, the path towards next generation TeV collider still requires numerous marvelous in beam physics as well as engineering to meet the repetition rate, staging requirement and ultimately the beam performance that conventional RF technology based accelerator has achieved.

Nevertheless, as rapid development in the advanced concept acceleration field, it is not appropriate to estimate the performance limit at this point of time.

In addition to the LWFA and PWFA, new ideas about how to reach quantum limit of beam energy and luminosity are emerging. One example is nanotube acceleration.

CONCLUSION

The past half century has seen the rise of colliders that led to many discoveries. While the pace for new collider at energy frontier has slowed down, the quest for the next discovery collider still remains strong. The realization of currently proposed future colliders, $e^+e^-$ linear colliders or 100 TeV pp colliders, is likely to push the energy frontier by another order of magnitude. Nevertheless, they are still far below the ultimate frontier, both energy and luminosity, at the quantum limit.

Hence, new paradigm for acceleration and beam manipulation is required to overcome the barriers that seem to be formidable with current available technologies. Such a break through will not only benefit the energy frontier particle physics but also can be game changer for other accelerator based scientific field such as X-FEL.

ACKNOWLEDGEMENTS

REFERENCES

[1] https://indico.fnal.gov/event/46742/
[2] V. Shiltsev, https://lss.fnal.gov/archive/2021/conf/fermilab-conf-21-257-ad-apc.pdf
[3] F. Zimmermann et al, Proceedings of IPAC21, 2021 qm F. Zimmermann, “Quantum Mechanical Limits on Beam Demagnification and Luminosity,” in Proc. Nanobeam’03, Lausanne, Switzerland, 2003, p. 69
[4] W. Barletta. et al, Planning the Future of U.S. Particle Physics, Snowmass 2013: Chapter 6: Accelerator Capabilities, 2014
[5] P. Lebrun, Energy Efficiency of high energy particle accelerators. https://indico.eli-np.ro/event/1/contributions/65/attachments/92/131/4_Talk_Energy_Efficiency_of_Accelerators_and_Colliders.pdf
[6] https://home.cern/resources/faqs/facts-and-figures-about-lhc#:~:text=What\%20is\%20the\%20LHC\%20power,is\%20750\%20GWh\%20per\%20year.