Qubit-environment entanglement outside of pure decoherence: hyperfine interaction

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In spin-based architectures of quantum devices, the hyperfine interaction between the electron spin qubit and the nuclear spin environment remains one of the main sources of decoherence. This paper provides a short review of the current advances in the theoretical description of the qubit decoherence dynamics. Next, we study the qubit-environment entanglement using negativity as its measure. For an initial maximally mixed state of the environment, we study negativity dynamics as a function of environment size, changing the numbers of environmental nuclei and the total spin of the nuclei. Furthermore, we study the effect of the magnetic field on qubit-environment disentangling time scales.

I. INTRODUCTION

Non-classical correlations, such as entanglement are crucial resources for future quantum technologies [1-6]. Entanglement and coherent superpositions of states of a given, well controlled quantum system are the key ingredients of future quantum computers. However, beside entanglement within the controlled quantum system, uncontrolled processes such as interactions with the environment which lead to decoherence, can also be a result of the formation of quantum correlations. The effect of decoherence on the operation of a quantum algorithm is very detrimental, since it leads to the quantum superposition behaving as a statistical mixture of states [7,8]. The result is that the gain from the use of quantum resources during computation is diminished. As such, finding methods to minimize decoherence and restore quantumness is very important for applicative quantum technologies.

One of the critical directions in modern quantum sciences is research on entanglement generation between the controlled quantum system and its environment in order to understand how to overcome limitations of quantum devices due to decoherence processes. Understanding decoherence lies at the heart of measurement, quantum information processing, and, more fundamentally, the transition from the quantum to the classical world. One of the important aspects of system-environment entanglement generation is a situation when the entanglement is generated during the joint qubit-environment evolution with the sole exception of the case when the initial state reduces the evolution to pure decoherence. There is a qualitative change between a small environment regime and the situation when its Hilbert space is larger either by means of many nuclei or large nuclear spin. The exception here is a single nuclear spin environment with a large spin, which displays much more structured behaviors. The application of the magnetic field, which significantly changes the free Hamiltonian of the qubit, has a strong impact on the evolution of entanglement, leading to much more complex time-dependencies.

In this paper we present current status in the field of entanglement generation between controlled spin-qubit and its environment, and we present qualitative predictions on disentanglement times as a function of environment size.

The paper is structured as follows. In Sec. III we present a brief review of achievements in the field of decoherence processes between quantum systems and their environments. Next, in Sec. IV we present the simplest Hamiltonian of a spin-qubit interacting with a spin-bath. In Sec. V we concisely introduce the entanglement measure used for qubit-bath entanglement characterization. In Sec. VI we present the results, analyzing the evolution of Negativity for different parameters of the environment, initial states of the qubit, and values of applied magnetic field. Finally, we conclude in Sec. VII.

II. STATE OF THE ART

The simplest system-environment models consist of a central spin or qubit coupled to the environment modeled by an ensemble of 1/2-spins denoted as a spin-bath [8,9]. This model is a very good theoretical platform allowing studies on system-environment entanglement for one of the most promising candidates for quantum computation, solid-state spin systems [20-22], which are inevitably coupled to their surrounding environment, usually through interactions with neighboring nuclear spins and are vulnerable to decoherence processes. Over the last years much research was focused on spin-bath entanglement [23-26], qubit decoherence reduction during dynamical decoupling [27-38], decoherence suppression due to spin coupling to the self-interacting bath [39-42], entanglement sharing [43], coupling to decoherence free subspaces [44-46], and disentanglement [47].

For pure states of both the qubits and the environment, decoherence can only be a result of entanglement being formed [7]. As the states of most environments are mixed, there is a distinction between decoherence resulting from classical noise [48-55] and the more quantum processes, for which decoherence is a direct consequence of the build-up of quantum correlations [56-57]. Both types are abundant in nature and for the most part, decoherence processes have never been qualified in this context. This is because the study of the amount of quantum correlations in systems which are large and mixed is very taxing numerically.

Considering the recent progress in the control and operation
of quantum systems, the distinction between classical and entangling decoherence processes becomes important. This is because the presence of entanglement with the environment is likely to lead to measurable changes in the evolution of a system of qubits which is interspaced by gate operation and measurements, in the same way, as desired quantum correlations between the qubits can lead to the increased capabilities of quantum devices. Uncontrolled entanglement can in turn lead to unpredicted errors in the operation of quantum algorithms.

In the case of pure decoherence, quantification of entanglement with the environment can be grossly simplified \[58, 61\]. This allowed for entanglement to be unambiguously connected with the transfer of information about the state of the qubits into the environment and consequently, for methods for the direct detection of such entanglement to be proposed \[61–64\] and entanglement to be measured \[65\]. These methods involve basic operations and measurements performed on the qubit subsystem, and rely on the fact that decoherence which is the result of entanglement generation alters the state of the environment in a distinctly different manner than classical noise. Since entanglement in pure decoherence can now be studied on a very general level \[58, 59\], the effect of system-environment entanglement on quantum teleportation has been studied \[66\], yielding an effect when decoherence can counter-intuitively be used for purification of the qubit state \[67\].

In this paper, we will focus on the study of quantum correlations between a qubit defined on a localized electron spin interacting with an environment of nuclear spins via the hyperfine interaction. Such a coupling does not lead to pure decoherence unless the magnetic field yields a very large splitting between the qubit states or the initial qubit state is an equal superposition state, while it is a very fundamental decoherence process for solid state spin qubits. It can be used to describe decoherence for charge carrier spins confined in quantum dots \[68–72\], NV-center spin qubits \[73–77\], and spin qubits on donor atoms \[78–81\].

Outside of pure decoherence the means of the study of quantum correlations with an environment remain highly numerical regardless of the chosen entanglement measure. This means that it is necessary to study a concrete ensemble and set of initial states, as general the behavior of a given form of interaction remains out of reach. We will study quantum correlations generated between the spin of an electron confined in a GaAs quantum dot interacting with an environment composed of \(F = 3/2\) nuclear spins of the gallium and arsenite atoms \[82, 84\], but as our aim is the more general study of entanglement temperatures \[83, 85\]. It is also an interesting limit from the point of view of entanglement theory, since the environment is as classical as possible. In case of pure decoherence, no quantum correlations with a qubit can be formed if the environment is fully mixed \[58, 61\], but entanglement is possible for a qutrit and larger systems \[59\]. The limitation on qubit-environment entanglement for infinite temperature environments does not hold outside of pure-decoherence (an example with an “environment” composed of a single qubit can be found in the appendix of Ref. \[58\]).

III. THE QUBIT AND THE ENVIRONMENT

The qubit-environment system under study consists of an electron confined in a lateral GaAs quantum dot. The qubit is defined on the spin states of the electron and is initially in a pure state. The environment consists of nuclear spins of the surrounding atoms, initially in a maximally mixed state due to the very small Zeeman splitting characteristic for such nuclear spins. They interact via the hyperfine coupling, which is the dominant spin-spin coupling here.

Let us start with a general Hamiltonian for the single qubit interacting with the spin-bath, i.e.

\[
\hat{H} = \hat{H}_S + \hat{H}_B + \hat{H}_{BB} + \hat{H}_{SB},
\]

where \(\hat{H}_S\) is the Hamiltonian for a central spin, \(\hat{H}_B\) is a Hamiltonian for the spin-bath (also denoted as environment) consisting of many spins, \(\hat{H}_{BB}\) describes interaction between spins in the bath, and \(\hat{H}_{SB}\) is the qubit-bath coupling term. The qubit spin operator \(\hat{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)\) is defined on an electron confined in a quantum dot (QD) interacting with an ensemble of environmental nucleus via the hyperfine interaction, where \(k\)-th nuclei is described by the spin operator \(\hat{F}_k = (\hat{F}_{x,k}, \hat{F}_{y,k}, \hat{F}_{z,k})\). The respective parts of the total Hamiltonian \(\hat{H}\) read

\[
\begin{align*}
\hat{H}_S &= -g\mu_B B S_z, \\
\hat{H}_B &= E_I \sum_{k=1}^n \hat{F}_{z,k}, \\
\hat{H}_{BB} &= \sum_{k \neq l}^n d_{kl} \hat{F}_k \hat{F}_l, \\
\hat{H}_{SB} &= \sum_{k=1}^n A_k \hat{S} \hat{F}_k.
\end{align*}
\]

The \(\hat{H}_S\) term is the Zeeman splitting of the central spin (qubit) under the assumption that the magnetic field \(B\) was applied in the \(z\) direction (perpendicular to the plane of the QD). Here, \(\mu_B\) is the Bohr magneton, and \(g = -0.44\) is the electron g-factor. The Zeeman term for the environment, \(\hat{H}_B\), can be omitted due to the small magnitudes of the splitting per Tesla of magnetic field compared to the thermal energy at temperatures characteristic for experiments on QD spin qubits \[82, 87, 88\]. The \(\hat{H}_{BB}\) term describing dipolar interaction between nuclear spins in the system under study is also known to be negligible and has been omitted \[82\]. The \(\hat{H}_{SB}\) term is the hyperfine interaction with the spins of the environment.
Finally, the Hamiltonian describing the qubit and environment is given by

$$\hat{H} = \hat{H}_S + \hat{H}_{SB} = -g\mu_B B\hat{S}_z + \sum_{k=1}^{n} A_k \hat{S}_k \hat{F}_k,$$

where the coupling constants $A_k$ describe the contact hyperfine interaction between the electronic spin and $k$-th nucleus. In general they are defined as $A_k = A_k^0 V_0 |\psi(r_k)|^2$, with $A_k^0 = \frac{2}{\hbar} \gamma_k \alpha_k \mu_0$. Here, $V_0$ is the unit cell volume, $\psi(r_k)$ is a value of the electronic wave function at the position of the $k$-th nucleus and $\gamma_k$ denote the nuclear and electronic giro-magnetic ratios, respectively [39]. We use parameters characteristic for lateral GaAs QDs which are presented in Table 1. The envelope of the electron wave function $\psi(r_k)$ is modeled by an anisotropic Gaussian reflecting the shape of the QD, with in-plane width $l_\perp = 20$ nm and $l = 2$ nm in the $z$-direction.

| $Ga^{69}$ | $Ga^{71}$ | $As^{75}$ |
|-----------|-----------|-----------|
| Abundance | 60.4%     | 59.6%     | 100%      |
| Spin moment $F$ | 3/2 | 3/2 | 3/2 |
| $A_k^0 [\mu eV]$ | 36 | 46 | 43 |
| $\gamma_k [10^7 \text{rad}^{-1} \text{s}^{-1}]$ | 6.44 | 8.18 | 7.29 |
| $\gamma_0 = 0.176 \times 10^7 [\text{rad}^{-1} \text{s}^{-1}]$ | | |
| $V_0 = 800 [\text{nm}^3]$ | | |

Table 1. Material parameters of the atomic components of GaAs quantum dots used to calculate the coupling constants between the nuclear bath and the spin qubit.

In the following we will use the common approximation used for Hamiltonian (3), namely the box model [24, 91–92]. This entails replacing all coupling constants $A_k$ in the interaction by their average value, which for the QD under study yields $A_k = A \approx 83 \mu eV$ [24]. The second term in Hamiltonian (4) can now be written as

$$\hat{H}_{SB} \approx A \hat{S}\hat{K},$$

where $\hat{K}$ denotes the total bath spin operator, $\hat{K} = \sum_{k=1}^{n} \hat{F}_k$. The approximation allows the Hamiltonian to be diagonalized analytically, since it can be represented in a block-diagonal $2 \times 2$ matrix form.

Let us denote the qubit states as $|\uparrow\rangle$ and $|\downarrow\rangle$ and the environmental basis states, which are the eigenstates of the total bath spin operator and its $z$ component, $|K,m\rangle$. Using these bases we can write the eigenstates of the Hamiltonian (3) in the box model approximation as

$$|+K,m\rangle = \cos \theta_{K,m} |\uparrow\rangle |K,m\rangle + \sin \theta_{K,m} |\downarrow\rangle |K,m + 1\rangle,$$

and

$$|-K,m\rangle = -\sin \theta_{K,m} |\uparrow\rangle |K,m\rangle + \cos \theta_{K,m} |\downarrow\rangle |K,m + 1\rangle,$$

with

$$\sin \theta_{K,m} = \frac{M_{K,m}}{\sqrt{(E_m + E_{m+1})^2 + M_{K,m}^2}}$$

where the elements within each block of the matrix are given by

$$E_m = \hbar^2 (-g\mu_B B/h + A_m),$$

$$M_{K,m} = h A/2 \sqrt{K(K + 1) - m(m + 1)},$$

and the eigenvalues corresponding to eigenvectors (5) have the form

$$E_{K,m}^\pm = \frac{1}{2} (-h A \pm 2 \sqrt{(E_m + E_{m+1})^2 + 4M_{K,m}^2}).$$

Since the eigenvectors and eigenvalues of the Hamiltonian are all known, regardless of the size of the environment, the evolution of any initial qubit-environment state can be found at any time. In the following, we will always be studying a product initial state, where the state of the qubit is given by

$$|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$$

and the density matrix of the environment is a product of nuclear spin states, in which the state of each nucleus is a maximally mixed state (infinite-temperature Gibbs state [93]).

Transforming the state of the environment into the total nuclear spin basis $|K,m\rangle$ has to take into account the multiple occurrences of certain states due to the rules of addition for angular momentum, so the density matrix can be written as

$$\hat{\rho}_E(0) = \sum_m P_{K,m} |K,m\rangle \langle K,m|,$$

with probabilities given by [94, 95]

$$P_{K,m} = \frac{1}{Z} \sum_k (-1)^k \binom{n}{k} \binom{m + n - 2mk + k - K - 2}{n - 2},$$

where $Z$ is the normalization constant.

IV. ENTANGLEMENT MEASURES - NEGATIVITY

Since the system is bipartite and built of qubit and environment having in principle many degrees of freedom we use the Negativity $N$ [96, 97] as an entanglement measure, which is based on Positive Partial Transpose (PPT) criterion [98, 99]. However, one cannot exclude a possibility of a bound entanglement formation [100]. This in principle could be problematic, since Negativity $N$ is blind to this kind of entanglement, but since the qubit is always initially in a pure state while the environment is maximally mixed, the purity of the whole system throughout the evolution is twice the minimum purity for a given system size, and it has been shown in Ref. [101] that bound entangled states cannot occur in this case.
Based on Peres-Horodecki criterion of separability, Negativity is defined as the sum of the negative eigenvalues of the partially transposed density matrix. It is irrelevant with respect to which subsystem the partial transposition is being made. Negativity can be written in the general form

\[
N(\rho) = \sum_{i} \left| \lambda_i - \lambda_i^{\Gamma_A} \right| / 2,
\]

where \(\rho\) is the density matrix of the system and \(\lambda_i\) are eigenvalues of \(\rho^{\Gamma_A}\), where \(\Gamma_A\) denotes partial transposition with respect to subsystem \(A\) (which in our case is the qubit). The structure of the density matrix obtained during the evolution allows for simple partial transposition with respect to the qubit subspace by flipping two off diagonal quarters of the total density matrix.

V. RESULTS

In the following we study the evolution of qubit-environment entanglement as a function of different factors, such as the magnetic field, which impacts the splitting between the energy levels of the qubit. The initial state of the qubit is also a factor, because it influences the nature of the evolution, and for an equal superposition state, the hyperfine interaction only leads to pure decoherence. A relevant factor is also the number of nuclear spins \(n\). Since the calculation of negativity requires diagonalization of matrices of the same dimension as the qubit-environment density matrix, there is a limit on the size of the environment that can be studied. Nevertheless, it is possible to model the decoherence of large environments with the help of a manageable number of nuclei. Since the study of entanglement is much more dependent on the specific parameters of the qubit-environment density matrix, it is a reasonable expectation that the convergence of results to the infinite-reservoir limit will be much slower than in case of decoherence and may not emerge at all.

Another parameter is the spin of each nucleus \(F\); this changes the size of the single nucleus density matrix and modifies the nature of the interaction with the qubit (allowing different transitions within the subspace of each nucleus and affecting the coupling strengths). Although in GaAs, all nuclei have spin \(F = 3/2\), we are interested in more general qualities of entanglement evolution driven by the hyperfine interaction and change this parameter freely. It is relevant to note that the same decoherence can be modeled by environments with different total spin of each nucleus, but the evolution of entanglement shows qualitative differences for different species of nuclei.

Negativity \(N(\hat{\rho})\) is bounded from above by the negativity of the isotropic state

\[
\hat{\rho}_{iso} = \kappa |\Phi^+\rangle \langle \Phi^+| + (1 - \kappa) \frac{1}{N} \mathbf{1},
\]

parametrized by a scalar \(\kappa\), where \(|\Phi^+\rangle\) is a maximally entangled state, denoted as \(N_{iso}\). Keeping in mind, that for any two-qubit state \(\hat{\rho}\) for which \(\text{tr}[\hat{\rho}^2] = \text{tr}[\hat{\rho}_{iso}^2]\), the following holds \(N_{iso} \geq N(\hat{\rho})\). Next, noticing that the purity of the smallest system considered here is \(\text{tr}[\hat{\rho}^2] = 1/2\), which equals to the purity of the isotropic state with \(\kappa \sim 0.5774\) and finally, noticing that entanglement between the qubit and the large spin-bath cannot be larger than between two qubits, the maximal negativity in the considered qubit-bath system is given by \(N_{iso} \approx 0.3661\).

Besides the evolution of the negativity \(N\) we focus on time scale \(\tau\) on which qubit-bath disentanglement appear, and its scaling with the total bath spin number \(K = nF\).

A. Single nucleus bath

Figure 1. Negativity dynamics for a single nucleus bath with \(F = 1/2\) and four different initial states of the spin-qubit with \(\alpha = \{(1, 0.9, 0.8, 1\sqrt{2})\}\), eq. (9). Maximal value of the negativity corresponds to \(\alpha = 1\) and vanishes for equal superposition \(\alpha = \beta = 1/\sqrt{2}\), in such case the system is separable during the whole evolution. The negativity period remains constant for each \(\alpha \neq 1/\sqrt{2}\).

Let us start with the situation when the environment consists of only one nuclear spin with \(F = 1/2\). This means that we are dealing with a two-qubit scenario, in which one qubit is initially in a pure state, while the other is maximally mixed, so the initial purity of the whole state is half of the maximum two-qubit purity. The negativity \(N\) has a time-periodic structure and vanishes with period \(\tau\), where

\[
\tau \approx \frac{3}{\hbar \sqrt{A^2 + g^2 \mu_B^2 B^2}},
\]

indicating disentanglement of the spin-qubit from its single spin \(F = 1/2\) bath.

Fig. 1 presents negativity evolution at zero magnetic field for different initial states of the central qubit, eq. (9). The interaction is symmetric with respect to the spin-flip of the qubit, so only parameters \(\alpha\) between \(\frac{1}{\sqrt{2}}\) and \(1\) are of interest. Maximal value of the negativity corresponds to \(\alpha = 1\), while entanglement is never generated for the equal superposition initial qubit state \(\alpha = \beta = 1/\sqrt{2}\); in such case the system is separable during the whole evolution. In these two cases, negativity can be found exactly. For \(\alpha = 1\) this is because the two-qubit system is an X-state throughout the evolution and entanglement is generated through transitions into the \(|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle\)
subspace. For $\alpha = 1/\sqrt{2}$ the hyperfine interaction leads to fundamentally different processes and leads to pure decoherence for the central spin. The evolution is separable because it is not possible for entanglement to be generated, if the environment is initially in a maximally mixed state regardless of environment size [58]. For intermediate values of $\alpha$, the two-qubit density matrix is not sparse enough for analytical calculation of negativity, but the interaction does not lead to pure decoherence and transitions between different two-spin states which lead to entanglement generation do occur at a lesser degree than for the spin-up or spin-down states.

We further study the dependence of entanglement evolution as a function of the growing Hilbert space of the environment, which is taken into account in two distinct ways. Firstly, the total spin number $F$ is varied as before, but also the number of nuclei $n$ is increased. In Fig. 3(a), negativity evolution is plotted for a fixed value of the spin of each nucleus $F = 2$ for different values of the number of environmental spins, $n = 5, 7, 9$, while in Fig. 3(b), the number of spins is constant $n = 10$, while the spin of each nucleus is varied, $F = 1/2, 3/2, 5/2$. One can observe that the evolution is qualitatively different from the situation when there is only one nucleus of the environment, which is to be expected, since the nature of the initial state changes drastically when a second nucleus is added. This is because of the rules for addition of angular momenta in quantum mechanics, which result in multiple occurrences of the same total angular momentum $|K, m\rangle$ states, which is reflected in the probabilities (11), while for a single spin, $K = F$ and the probabilities for each state of the environment are equal in the infinite-temperature Gibbs state.

What is more surprising is that when the Hilbert space of the environment is large (outside of $n = 1$), there is no qualitative difference in the evolution for different parameter choices. A bigger environment (obtained by either a

In Fig. 2(a) we show the same evolution of entanglement while varying the total spin of the single environmental nucleus, $F$. The initial state of the qubit is always $|\uparrow\rangle$, yielding the maximum possible negativity of all initial states. The nuclear spin varies from $F = 1/2$ to $F = 17/2$ with a step of $1/2$. Interestingly, there is a large qualitative change in the behavior of entanglement when the spin is small (between $F = 1/2$ and $F = 5/2$), but for larger spins the only effect of the increased size of the Hilbert space of the environment, is that negativity is smaller, while growth and decay are qualitatively the same and occur at the same time-intervals for $F \geq 5/2$. This is reflected well on Fig. 2(b) where maximum negativity is plotted as a function of $F$. At larger values, the decay is well fitted by an exponential function.

B. Bath of many nuclei

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larger \( n \) or \( F \) results in a longer time until the whole qubit-environment system returns to its initial state (and then the evolution continues, which is not plotted in the figures for the sake of clarity) and a smaller maximum negativity which is reached, but the curves can be easily and with a reasonable degree of accuracy transformed into one another by rescaling of the time and varying the amplitude. Fig. 3(c,d) present time \( \tau \) at which a separable qubit-bath state is obtained as a function of the bath size. Fig. 3(c) presents \( \tau \) as a function of bath spin number \( F = 1/2, 3/2, 2, 5/2 \) for fixed total nuclei in bath \( n = 4, 6, 10 \), which has linear scaling \( \tau \propto F \). Panel Fig. 3(d) shows \( \tau \) as a function of \( n \) for fixed \( F = 1/2, 1, 2 \) which scales as \( \tau \propto \log(n) \). For the total spin of the bath, \( K = nF \) with \( F = 1/2 \), the disentanglement time scale \( \tau \) depends on fermionic/bosonic nature of the bath. In particular, the bosonic bath (even number \( n \)) manifest larger entangling times than fermionic counterparts (odd \( n \)). Such a behavior is related to existence of the zero \( z \)-component of the spin, i.e. element \( m = 0 \), in the absence of the magnetic field

\[
\sin \theta_{K,0} = \frac{\sqrt{K}}{\sqrt{2K+1}}, \tag{15}
\]

which contributes to the evolution of the off-diagonal density matrix elements. For large bath spins, and high \( K \), \( \sin \theta_{K,0} \approx 1/\sqrt{3} \) leading to equal superposition in this spin sector and as a consequence - longer entangling times.

C. External magnetic field

Applying a finite magnetic field causes a splitting of the qubit eigenstates and leads to a qualitative difference in the evolution of entanglement already for the two-qubit case (when the environment is a single nucleus with spin \( 1/2 \)). This is plotted in Fig. 4(a), where we observe a departure from sine behavior and a larger value of negativity at mid-range times, proportional to the value of the magnetic field. Much more complex evolutions are observed in the presence of the magnetic field for moderately larger Hilbert spaces of the environment, as plotted in Figs. 4(b,c) for nuclear spin \( 1/2 \) and \( n = 4 \) nuclei, and relatively small (a) and strong (c) magnetic fields. These plots show distinct points of non-differentiability which are characteristic to evolutions of negativity. Applying the magnetic field yields curves which are much less structured and resemble the curves characteristic for the bigger environments more.

VI. CONCLUSIONS

We have studied the generation of entanglement in a qubit-environment setup where the interaction is driven by the hyperfine coupling, which under most circumstances does not lead to pure decoherence. The initial state of the qubit is pure while the environment is maximally mixed. We have shown that when the decoherence does involve transitions between the qubit states, it is accompanied by the creation of entanglement, even though the environment is as classical as possible for its size. This cannot be the case for pure decoherence and when the evolution reduces to pure decoherence (for an equal superposition state of the qubit), we show that entanglement was not generated.

We used material and structure parameters characteristic for lateral GaAs quantum dots to calculate the strength of the interaction and worked in the box model approximation which is known to reproduce hyperfine-induced decoherence well. On the other hand, we treated the number of environmental spins as well as the total spin of each nucleus as a parameter, in order to be able to describe and understand transitions from small to (relatively) large environments. We have observed that the case of a single nuclear spin is special in the sense that increasing the Hilbert space of the environment by enlarging the total spin of a single nucleus does not lead to the emergence of more complicated entanglement dynamics characteristic for big environments an above some threshold spin, there is not qualitative difference connected with growing total nuclear spin. Otherwise, these behaviors emerge similarly regardless if the Hilbert space is increased by the increase of the total spin of a given number of particles, or by increasing the number of nuclei. It is relevant to note, that a bigger environment leads to less entanglement and slower evolution, in accordance to expectations.

Applying a finite magnetic field, which makes the free
Hamiltonian of the qubit nontrivial, results in much more complicated evolutions of entanglement regardless of all other parameters. Even for the effectively two-qubit case, there is a non-trivial effect resulting from the splitting of the qubit states, and the larger the environment, the more complex features are displayed.

We have shown that the evolution of qubit-environment entanglement driven by the hyperfine Hamiltonian yields qualitatively different results than are possible due to any Hamiltonian that leads to pure decoherence. In opposition to pure decoherence, maximum entanglement is obtained when the initial qubit state is either spin-up or spin-down, while the equal superposition state does not entangle during evolution (even though decoherence is observed). The choice of initial state also has a greater influence on the decay that follows, whereas for pure decoherence, it only changes the amplitude of entanglement.

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