On the D–wave state component of the deuteron in the Nambu–Jona–Lasinio model of light nuclei

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Abstract

The D–wave state component of the neutron–proton bound state in the deuteron is calculated in the Nambu–Jona–Lasinio model of light nuclei. For the ratio of D–to S–state deuteron wave functions we obtain equal to \( \eta_d = 0.0238 \). This agrees well with the phenomenological value \( \eta_d = 0.0256 \pm 0.0004 \) quoted by Kamionkowski and Bahcall (ApJ. 420, 884 (1994)).

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1 Introduction

The Nambu–Jona–Lasinio model of light nuclei or differently the nuclear Nambu–Jona–Lasinio (NNJL) model suggested in [1–3] represents a relativistically covariant quantum field theoretic approach to the description of low–energy properties and interactions of the deuteron and light nuclei. The NNJL model is fully motivated by QCD [1]. The deuteron appears in the nuclear phase of QCD as a neutron–proton collective excitation, the Cooper np–pair, induced by a phenomenological local four–nucleon interaction. The NNJL model describes low–energy nuclear forces in terms of one–nucleon loop exchanges providing a minimal transfer of nucleon flavours from initial to final nuclear states and accounting for contributions of nucleon–loop anomalies which are completely determined by one–nucleon loop diagrams. The dominance of contributions of nucleon–loop anomalies to effective Lagrangians of low–energy nuclear interactions is justified in the large $N_C$ expansion, where $N_C$ is the number of quark colours.

Nowadays there is a consensus concerning the existence of non–nucleonic degrees of freedom in nuclei [4]. The non–nucleonic degrees of freedom can be described either within QCD in terms of quarks and gluons [5] or in terms of mesons and nucleon resonances [6]. In the NNJL model the non–nucleonic degrees of freedom of nuclei have been investigated in terms of the $\Delta(1232)$ resonance and calculated the contribution of the $\Delta\Delta$ component to the deuteron [2]. The obtained result $P(\Delta\Delta) = 0.3 \%$ agrees well with the experimental upper bound $P(\Delta\Delta) < 0.4 \%$ [7] and other theoretical estimates [4].

As has been shown in [3] the NNJL model describes well low–energy nuclear forces for electromagnetic and weak nuclear reactions with the deuteron of astrophysical interest such as the neutron–proton radiative capture $n + p \rightarrow D + \gamma$, the solar proton burning $p + p \rightarrow D + e^+ + \nu_e$, the pep–process $p + e^- + p \rightarrow D + \nu_e$ and reactions of the disintegration of the deuteron by neutrinos and anti–neutrinos caused by charged $\nu_e + D \rightarrow e^- + p + p$, $\bar{\nu}_e + D \rightarrow e^+ + n + n$ and neutral $\nu_e(\bar{\nu}_e) + D \rightarrow \nu_e(\bar{\nu}_e) + n + p$ weak currents.

The important problem which has not been jet clarified in the NNJL model is related to the value of the contribution of the $D$–wave state to the wave function of the deuteron. In this paper we fill this blank. In section 2 we calculate the contribution of the D–wave state to the wave function of the deuteron. We use a relativistically covariant partial–wave analysis developed by Anisovich et al. [8] for the description of nucleon–nucleon scattering. The fraction of the D–wave state of the deuteron wave function relative to the S–wave one we obtain equal to $\eta_d = 0.0238$. This agrees well with the value $\eta_d = 0.0256 \pm 0.0004$ quoted by Kamionkowski and Bahcall [9] who used this parameter for the phenomenological description of the realistic wave function of the deuteron in connection with the calculation of the astrophysical factor $S_{pp}(0)$ for the solar proton burning $p + p \rightarrow D + e^+ + \nu_e$ in the potential model approach. In the Conclusion we discuss the obtained result.

\footnote{The value $\eta_d = 0.0256 \pm 0.0004$ was taken by Kamionkowski and Bahcall from Ref. [10].}
2 The D–wave state component of the deuteron

The calculation of the value of the D–wave state contribution to the wave function of the deuteron we would carry out in terms of the amplitude of the transition \( n + p \rightarrow D \). We show that the neutron–proton pair couples to the deuteron in both the S–wave state and the D–wave state with the fraction of the D–wave state agreeing with low–energy nuclear phenomenology.

In the NNJL model the phenomenological Lagrangian of the npD interaction is defined by [1]

\[
\mathcal{L}_{\text{npD}}(x) = -ig_V \bar{\rho}(x) \gamma^\mu n(x) - \bar{\nu}(x) \gamma^\mu p(x) D^\dagger_\mu(x) + \frac{g_T}{2M_N}[\bar{\rho}(x)\sigma^{\mu\nu} n(x) - \bar{\nu}(x)\sigma^{\mu\nu} p(x)] D^\dagger_{\mu\nu}(x) + \text{h.c.} \tag{2.1}
\]

where \( D^\dagger_\mu(x) \), \( n(x) \) and \( p(x) \) are the interpolating fields of the deuteron, the neutron and the proton, \( D^\dagger_\mu(x) = \partial_\mu D^\dagger(x) - \partial_\mu D^\dagger_\mu(x) \) is the deuteron field strength. The phenomenological coupling constant \( g_V \) is related to the electric quadrupole moment of the deuteron \( Q_D = 0.286 \text{ fm}^2 \), \( g_V^2 = 2\pi^2 Q_D M_N^2 \) [1], where \( M_N = 940 \text{ MeV} \) is the nucleon mass. The coupling constants \( g_V \) and \( g_T \) are connected by the relation [1]

\[
g_T = \sqrt{\frac{3}{8}} g_V, \tag{2.2}
\]

which is valid at leading order in the large \( N_C \) expansion [1].

The amplitude of the transition \( n + p \rightarrow D \) is determined by

\[
\langle k_D, \lambda_D | \mathcal{L}_{\text{npD}}(0) | k_p, \sigma_p; k_n, \sigma_n \rangle = \frac{M(n(k_n, \sigma_n) + p(k_p, \sigma_p) \rightarrow D(k_D, \lambda_D))}{\sqrt{2E_D V 2E_n V 2E_p V}}, \tag{2.3}
\]

where \( (E_D, k_D, \lambda_D) \), \( (E_p, k_p, \sigma_p) \) and \( (E_n, k_n, \sigma_n) \) are energies, 4–momenta and polarizations of the deuteron, the proton and the neutron, respectively, \( V \) is a normalization space volume. The wave functions of the initial and the final states of the transition \( n + p \rightarrow D \) are given by

\[
|k_p, \sigma_p; k_n, \sigma_n\rangle = a^\dagger_p(k_p, \sigma_p) a^\dagger_n(k_n, \sigma_n)|0\rangle,
\]

\[
\langle k_D, \lambda_D| = \langle 0| a_{D}(k_D, \lambda_D), \tag{2.4}
\]

where \( a^\dagger_p(k_p, \sigma_p) \) and \( a^\dagger_n(k_n, \sigma_n) \) are creation operators of the proton and the neutron, \( a_{D}(k_D, \lambda_D) \) is the annihilation operator of the deuteron and \( |0\rangle \) is a vacuum wave function. The relativistically invariant amplitude \( M(n(k_n, \sigma_n) + p(k_p, \sigma_p) \rightarrow D(k_D, \lambda_D)) \) reads

\[
M(n(k_n, \sigma_n) + p(k_p, \sigma_p) \rightarrow D(k_D, \lambda_D)) = e^{*\nu}(k_D, \lambda_D)
\times \left\{ 2ig_V[\bar{\nu}^\dagger(k_n, \sigma_n)\gamma_\nu u(k_p, \sigma_p)] - \frac{2ig_T}{M_N}[\bar{\nu}^\dagger(k_n, \sigma_n)\sigma_{\mu\nu} u(k_p, \sigma_p)](k_n + k_p)^\mu \right\}, \tag{2.5}
\]

where \( \bar{\nu}^\dagger(k_n, \sigma_n) \) and \( u(k_p, \sigma_p) \) are bispinorial wave functions of the neutron and the proton with 4–momenta \( k_n, k_p \) and polarizations \( \sigma_n, \sigma_p; e^{*\nu}(k_D, \lambda_D) \) is a 4–vector of polarization.
of the deuteron with a 4–momentum \( k_D \) and polarization \( \lambda_D \). The 4–momenta \( k_D, k_n \) and \( k_p \) are related by \( k_D = k_n + k_p \) due to conservation of energy and momentum.

As has been shown by Anisovich et. al. [8] for neutron–proton scattering the neutron–proton densities describing the S– and D–wave states of a neutron–proton pair are equal to

\[
\Psi_{\nu}(^3S_1; \sigma_n, \sigma_p) = [\bar{u}(k_n, \sigma_n)S_{\nu}u(k_p, \sigma_p)], \\
\Psi_{\nu}(^3D_1; \sigma_n, \sigma_p) = [\bar{u}(k_n, \sigma_n)D_{\nu}u(k_p, \sigma_p)],
\]

where \( S_{\nu} \) and \( D_{\nu} \) are relativistically covariant operators of the projection onto the S–wave and the D–wave state, respectively [8]:

\[
S_{\nu} = \frac{1}{\sqrt{2}s} \left[ \frac{\gamma_{\nu} + 2k_{\nu}}{2M_D + \sqrt{s}} \right], \\
D_{\nu} = \frac{2}{s^{3/2}} \left[ \frac{1}{4} (4M_N^2 - s) \gamma_{\nu} - (M_N + \sqrt{s}) k_{\nu} \right].
\]

Here \( P = k_p + k_n, k = \frac{1}{2} (k_p - k_n), s = P^2, P \cdot k = 0 \) and

\[
\gamma_{\nu} = \gamma_{\nu} - \hat{P} \frac{P_{\nu}}{s}.
\]

The neutron–proton densities Eq.(2.6) are normalized by the condition [8]

\[
\frac{1}{3} \int \text{tr} \{L_{\mu}(\hat{k}_p + M_N)L_{\mu}(-\hat{k}_n + M_N)\} (2\pi)^4 \delta^{(4)}(P - k_p - k_n) \frac{d^3k_p}{(2\pi)^3 2E_p} \frac{d^3k_n}{(2\pi)^3 2E_n} = \rho_L(s),
\]

where \( L_{\mu} = S_{\mu} \) or \( D_{\mu} \), the factor 3 in the denominator of the l.h.s. describes the number of the states of a neutron–proton density with a total momentum \( J = 1, 2J + 1 = 3 \), and \( \rho_S(s) \) and \( \rho_D(s) \) amount to

\[
\rho_S(s) = \frac{1}{8\pi} \left( \frac{s - 4M_N^2}{s} \right)^{1/2}, \\
\rho_D(s) = \frac{1}{8\pi} \left( \frac{s - 4M_N^2}{s} \right)^{5/2}.
\]

In the center of mass frame of the neutron–proton pair the densities Eq.(2.6) are equal to

\[
\Psi_{0}(^3S_1; \sigma_n, \sigma_p) = [\bar{u}(k_n, \sigma_n)S_{0}u(k_p, \sigma_p)] = 0, \\
\Psi(3S_1; \sigma_n, \sigma_p) = [\bar{u}(k_n, \sigma_n)S_{u}u(k_p, \sigma_p)] = \frac{1}{\sqrt{2}} \varphi_{\sigma_n}^1(\sigma_n) \hat{\sigma} \varphi_{\sigma_p}(\sigma_p), \\
\Psi_{0}(^3D_1; \sigma_n, \sigma_p) = [\bar{u}(k_n, \sigma_n)D_{0}u(k_p, \sigma_p)] = 0, \\
\Psi(3D_1; \sigma_n, \sigma_p) = [\bar{u}(k_n, \sigma_n)D_{u}u(k_p, \sigma_p)] = -\frac{1}{2} \varphi_{\sigma_n}^1(\sigma_n) [3 (\hat{\sigma} \cdot \vec{v}) \vec{v} - \vec{v}^2 \hat{\sigma}] \varphi_{\sigma_p}(\sigma_p),
\]

where \( \vec{v} = \vec{k}/\sqrt{\vec{k}^2 + M_N^2} = \sqrt{1 - 4M_N^2/s} \) and \( \vec{k} \) are a relative velocity and a 3–momentum of the neutron–proton pair, \( \varphi_{\sigma_n}(\sigma_n) \) and \( \varphi_{\sigma_p}(\sigma_p) \) are spinorial wave functions of the neutron and the proton, respectively. It is obvious that the densities Eq.(2.11) describe the
neutron–proton pair in the S– and D–wave states with a total spin \( S = 1 \) and a total momentum \( J = 1 \).

The neutron–proton densities Eq.(2.11) are normalized by

\[
\frac{1}{3} \sum_{\sigma_n = \pm 1/2} \sum_{\sigma_p = \pm 1/2} \bar{\Psi}^\dagger(3S_1; \sigma_n, \sigma_p) \cdot \bar{\Psi}^\dagger(3S_1; \sigma_n, \sigma_p) = 1,
\]

\[
\frac{1}{3} \sum_{\sigma_n = \pm 1/2} \sum_{\sigma_p = \pm 1/2} \bar{\Psi}^\dagger(3D_1; \sigma_n, \sigma_p) \cdot \bar{\Psi}^\dagger(3D_1; \sigma_n, \sigma_p) = v^4 = \left(1 - \frac{4M_n^2}{s}\right)^2. \tag{2.12}
\]

The decomposition of the neutron–proton densities in the amplitude Eq.(2.3) into the densities with a certain orbital momentum we would carry out at leading order in the large \( N_C \) expansion [1–3]. This would allow to consider the neutron and the proton as free particles obeying free equations of motion

\[
\bar{u}^c(k_n, \sigma_n)(\hat{k}_n + M_N) = 0,
\]

\[
(\hat{k}_p - M_N) u(k_p, \sigma_p) = 0. \tag{2.13}
\]

In order to express the neutron–proton densities in the amplitude Eq.(2.3) in terms of the projection operators Eq.(2.7), first, we have to exclude the term containing \( \sigma_{\mu\nu} \). This can be carried out by using Gordon’s identity

\[
[u^c(k_n, \sigma_n)\sigma_{\mu\nu}u(k_p, \sigma_p)] \frac{(k_n + k_p)^\mu}{2M_N} = -[u^c(k_n, \sigma_n)\gamma^\nu u(k_p, \sigma_p)] + \frac{k_\nu}{M_N} [u^c(k_n, \sigma_n)u(k_p, \sigma_p)]. \tag{2.14}
\]

Substituting Eq.(2.14) in Eq.(2.3) we get

\[
M(n(k_n, \sigma_n) + p(k_p, \sigma_p) \rightarrow D(k_D, \lambda_D)) = 2i(g_V + 2g_T) e^{*\nu}(k_D, \lambda_D) \times \left\{ [u^c(k_n, \sigma_n)\gamma^\nu u(k_p, \sigma_p)] - \frac{2g_T}{g_V + 2g_T} \frac{k_\nu}{M_N} [u^c(k_n, \sigma_n)u(k_p, \sigma_p)] \right\}. \tag{2.15}
\]

In terms of \( S_\nu \) and \( D_\nu \) vectors \( \gamma^\nu \) and \( k_\nu \) are determined by

\[
\gamma^\nu = \frac{2\sqrt{2}}{3} (M_N + \sqrt{s}) S_\nu - \frac{2}{3} \frac{s}{2M_N + \sqrt{s}} D_\nu,
\]

\[
k_\nu = \frac{1}{3\sqrt{2}} (4M_N^2 - s)s \cdot \frac{s}{1}{3} D_\nu. \tag{2.16}
\]

Substituting Eq.(2.16) in Eq.(2.15) and taking into account that \( [u^c(k_n, \sigma_n)\hat{P} u(k_p, \sigma_p)] = 0 \) we obtain

\[
M(n(k_n, \sigma_n) + p(k_p, \sigma_p) \rightarrow D(k_D, \lambda_D)) = 4\sqrt{2}i(g_V + 2g_T) M_N e^{*\nu}(k_D, \lambda_D) \times \left\{ [u^c(k_n, \sigma_n)S_\nu u(k_p, \sigma_p)] + \eta_d [u^c(k_n, \sigma_n)D_\nu u(k_p, \sigma_p)] \right\} =
\]

\[
= 4\sqrt{2}i(g_V + 2g_T) M_N e^{*\nu}(k_D, \lambda_D) \left[\Psi_\nu(3S_1; \sigma_n, \sigma_p) + \eta_d \Psi_\nu(3D_1; \sigma_n, \sigma_p)\right], \tag{2.17}
\]

where \( \eta_d \) describes the fraction of the D–wave state in the wave function of the deuteron. It is equal to

\[
\eta_d = \frac{1}{3\sqrt{2}} \frac{2g_T - g_V}{2g_T + g_V}. \tag{2.18}
\]
For the derivation of Eqs. (2.17) and (2.18) we have set \( s = M_D^2 \) and neglected the contribution of the binding energy of the deuteron in comparison with a nucleon mass \( M_N \). This means that \( 4M_N^2 - s = 0 \) when compared with \( M_N^2 \).

Using the relation Eq. (2.2) the parameter \( \eta_d \) takes the value

\[
\eta_d = \frac{1}{3\sqrt{2}} \left( \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \right) = 0.0238. \tag{2.19}
\]

This agrees well with the value \( \eta_d = 0.0256 \pm 0.0004 \) that was used in low–energy nuclear phenomenology for the description of the realistic wave function of the deuteron within the potential model approach [9, 10].

## 3 Conclusion

We have shown that the NNJL model describes well in agreement with low–energy nuclear phenomenology [10] such a fine structure of the deuteron as a contribution of the D–wave state. The calculation of the fraction of the D–wave state to the wave function of the deuteron we have carried out at leading order in the large \( N_C \) expansion [1]. This has allowed to treat the neutron and the proton as free particles on–mass shell [2] obeying free equations of motion. To the decomposition of the amplitude of the transition \( n + p \rightarrow D \) into the neutron–proton quantum field configurations having certain orbital momenta and corresponding to the S– and D–wave states, respectively, we have applied a relativistically covariant partial–wave analysis invented by Anisovich et al. [8] for the description of nucleon–nucleon scattering with nucleon–nucleon pairs coupled in the states with certain orbital momenta.

The theoretical value of the D–wave state fraction in the wave function of the deuteron \( \eta_d = 0.0238 \) calculated in the NNJL model agrees well with low–energy nuclear phenomenology giving \( \eta_d = 0.0256 \pm 0.0004 \) [10]. The former was quoted by Kamionkowski and Bahcall [9] for the parameterization of the realistic wave function of the deuteron in connection with the calculation of the astrophysical factor \( S_{pp}(0) \) for the solar proton burning \( p + p \rightarrow D + e^+ + \nu_e \). The calculation of the contribution of the D–wave state fraction of the wave function of the deuteron in agreement with low–energy nuclear phenomenology testifies that the NNJL model describes to full extent low–energy tensor nuclear forces playing an important role in low–energy nuclear physics on the whole and for the existence of the deuteron, in particular [11].

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