Clockwork for neutrino masses and lepton flavor violation

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A B S T R A C T

We investigate the generation of small neutrino masses in a clockwork framework which includes Dirac mass terms as well as Majorana mass terms for the new fermions. We derive analytic formulas for the masses of the new particles and for their Yukawa couplings to the lepton doublets, in the scenario where the clockwork parameters are universal. When the universal Majorana mass vanishes, the zero mode of the clockwork sector forms a Dirac pair with the active neutrino, with a mass which is in agreement with oscillations experiments for a sufficiently large number of clockwork gears. On the other hand, when it does not vanish, neutrino masses are generated via the seesaw mechanism. In this case, and due to the fact that the effective Yukawa couplings of the higher modes can be sizable, neutrino masses can only be suppressed by postulating a large Majorana mass scale. Finally, we discuss the constraints on the mass scale of the clockwork fermions from the non-observation of the rare leptonic decay $\mu \rightarrow e\gamma$.

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1. Introduction

The smallness of neutrino masses stands as one of the most puzzling open questions in Fundamental Physics. A plausible solution to this puzzle is provided by the seesaw mechanism, in which the smallness of neutrino masses is explained by the breaking of the lepton number at a very high energy scale [1–5]. Models with conserved lepton number, on the other hand, can also reproduce the observations, at the expense of postulating tiny Yukawa couplings of the neutrino to the Standard Model Higgs. Such small parameters are usually regarded as unnatural, however the existence of tiny Yukawa couplings is a phenomenologically viable possibility, and can be accomplished in further extensions of the model (for reviews and recent models, see e.g. in [6–19]).

Recently, a new mechanism of generating small couplings in theories coupled to the Standard Model has been introduced [20, 21]. The mechanism, reminiscent of deconstruction models [22,23], can be summarized as a linear quiver model with no large hierarchies in the theory parameters, that gives rise to site-dependent suppressed couplings to the zero-mode [24]. Originally, introduced for a quiver of Abelian Goldstone bosons (axions), it has been generalized to fermions, vectors and other fields [24,25] (See also [26]). Applications and generalizations of this mechanism have been discussed in [27–43], and specifically frameworks to explain the observed pattern of fermion masses in [44–46].

In this work we explore the application of the fermionic clockwork to the generation of small neutrino masses. Concretely, we identify the right-handed neutrinos with the zero modes of a clockwork sector [24], such that small couplings can be naturally generated and therefore small neutrino masses. We analyze in detail the framework where the Dirac masses, Majorana masses and nearest neighbor interactions are universal, complementing previous studies in [45,46] where the Majorana mass term is localized on just one of the modes. We derive analytical formulas for the masses of the new particles and for their couplings to the Standard Model fermions, for the cases when the Majorana mass terms are included in the Lagrangian and when they are vanishing. We show that the clockwork mechanism, i.e., the suppression of the Yukawa couplings by site dependent power factors, is not affected by the presence of the Majorana mass terms. In fact, while the zero mode contribution is a combination of the clockwork suppression and the Majorana seesaw, the scale is however set by the dominant contribution by the gears, which have $O(1)$ Yukawa couplings, through the standard seesaw mechanism. Furthermore, while the clockwork mechanism suppresses the couplings of the zero mode, the couplings of the higher modes can be sizable and induce, via loops, potentially large rates for the leptonic rare decays.

The rest of the paper is organized as follows. In section 2, we present the most general framework for clockwork neutrinos with...
Dirac and Majorana mass terms, and we discuss their phenomenology in subsections 2.1 and 2.2, respectively. In section 3, we discuss lepton flavor violation in the clockwork scenario and calculate limits on the gear masses. We close with a summary.

2. Neutrinos in clockwork

We extend the Standard Model with \( n \) left-handed and \( n + 1 \) right-handed chiral fermions, singlets under the Standard Model gauge group, which we denote as \( \psi_{L,i} (i = 0, \ldots, n - 1) \) and \( \psi_{R,i} (i = 0, \ldots, n) \) respectively. The Lagrangian of the model reads:

\[
\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\text{Clockwork}} + \mathcal{L}_{\text{int}} \tag{1}
\]

where \( \mathcal{L}_{SM} \) is the Standard Model Lagrangian, \( \mathcal{L}_{\text{Clockwork}} \) is the part of the Lagrangian involving only the new fermion singlets, and \( \mathcal{L}_{\text{int}} \) is the interaction term of the new fields with the Standard Model fields. Following [24], we assume that the Standard Model only couples to the last site of the fermionic clockwork, therefore,

\[
\mathcal{L}_{\text{int}} = -Y \tilde{H}_L \psi_{R,0} \tag{2}
\]

with \( \tilde{H} = i\tau_2 H^* \), \( H \) the Standard Model Higgs doublet and \( L_i \) the left handed lepton fields (we assume only one generation of fermions; the generalization to more than one generation will be discussed below).

In full generality, the clockwork Lagrangian can be cast as:

\[
\mathcal{L}_{\text{Clockwork}} = \mathcal{L}_{\text{kin}} - \sum_{i=0}^{n-1} (m_i \overline{\psi}_{L,i} \psi_{R,i} - m'_i \overline{\psi}_{L,i} \psi_{R,i+1} + \text{h.c.})
\]

\[
- \sum_{i=0}^{n-1} \frac{1}{2} M_{L,i} \overline{\psi}_{L,i} \psi_{L,i} - \sum_{i=0}^{n-1} \frac{1}{2} M_{R,i} \overline{\psi}_{R,i} \psi_{R,i} \tag{3}
\]

where \( \mathcal{L}_{\text{kin}} \) denotes the kinetic term for all fermions, and \( m, m' \) and \( M_{L,R} \) are mass parameters. Denoting \( \Psi = (\psi_{L,0}, \psi_{L,1}, \ldots, \psi_{L,n-1}, \psi_{R,0}, \psi_{R,1}, \ldots, \psi_{R,n}) \), the clockwork Lagrangian can be written in the compact form:

\[
\mathcal{L}_{\text{Clockwork}} = \mathcal{L}_{\text{kin}} - \frac{1}{2} (\overline{\Psi} \mathcal{M} \Psi + \text{h.c.}) \tag{4}
\]

with \( \mathcal{M} \) a \((2n + 1) \times (2n + 1)\) mass matrix. We note that \( \mathcal{L}_{\text{kin}} \) is invariant under the global group \( U(n) \times U(n + 1) \). The mass terms \( m_i \) break the global group \( U(n) \times U(n + 1) \) to \( \prod_{i=0}^{n-1} U(1)_i \), where \( U(1)_i \) acts as \( \psi_{L,i} \rightarrow e^{i\alpha} \psi_{L,i}, \psi_{R,i} \rightarrow e^{i\alpha} \psi_{R,i} \), and combined with the mass terms \( m'_i \), break the global symmetry to \( U(n + 1)_L \times U(n + 1)_R \). The interactions \( U(n+1)_L \rightarrow U(n+1)_R \) and \( U(n+1)_R \rightarrow U(n+1)_L \) are Majorana masses for the left and right handed singlet fields. It is sufficient that \( M_{L,i} \) or \( M_{R,i} \) is non-vanishing for one \( i \) to break the symmetry group \( U(n+1)_L \times U(n+1)_R \rightarrow \text{nothing} \).

We assume for simplicity universal Dirac masses, Majorana masses and nearest neighbor interactions, namely \( m_i = m, m'_i = m q, M_{L,i} = M_{R,i} = m \tilde{q} \) for all \( i \). Under this assumption, the mass matrix reads:

\[
\mathcal{M} = m \begin{pmatrix}
\tilde{q} & 0 & \cdots & 0 & -q & \cdots & 0 \\
0 & \tilde{q} & \cdots & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \tilde{q} & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & \tilde{q} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -q & 0 & \cdots & \tilde{q} \\
\end{pmatrix} \tag{5}
\]

which has eigenvalues \( M_k \) given by:

\[
M_0 = m \tilde{q} \\
M_k = m \tilde{q} - m \sqrt{k^2 - 2q^2}, \quad k = 1, \ldots, n \\
M_{n+k} = m \tilde{q} + m \sqrt{k^2 - 2q^2}, \quad k = 1, \ldots, n,
\]

with \( \lambda_k \) defined as

\[
\lambda_k \equiv q^2 + 1 - 2q \cos \frac{k\pi}{n+1} \tag{7}
\]

With our conventions, the eigenvalues can be positive or negative; the physical masses correspond to the moduli of the eigenvalues.

The mass eigenstates, which we denote as \( \chi_k \), are related to the interaction eigenstates \( \Psi_j \) by the unitary transformation \( U \), namely

\[
\Psi_j = \sum_k U_{jk} \chi_k
\]

The matrix \( U \) can be explicitly calculated, the result being:

\[
U = \begin{pmatrix}
\tilde{0} & \frac{1}{\sqrt{2}} U_L & -\frac{1}{\sqrt{2}} U_L \\
\tilde{0} & \frac{1}{\sqrt{2}} U_R & -\frac{1}{\sqrt{2}} U_R \\
\end{pmatrix}
\]

where \( \tilde{0} \) and \( \tilde{u}_k \) are \( n \)-dimensional vectors, with entries:

\[
\tilde{0}_j = 0, \quad j = 1, \ldots, n,
\]

\[
(u_k)_j = \frac{1}{q} \sqrt{\frac{q^2 - 1}{q^2 - q^2 - 2n}}, \quad j = 1, \ldots, n,
\]

while \( U_L \) and \( U_R \) are, respectively, \( n \times n \) and \( (n+1) \times n \) matrices with elements

\[
(U_L)_{jk} = \sqrt{\frac{2}{n+1}} \sin \frac{j k \pi}{n+1}, \quad j, k = 1, \ldots, n,
\]

\[
(U_R)_{jk} = \sqrt{\frac{2}{(n+1)\lambda_k}} \left[ q \sin \frac{j k \pi}{n+1} - \sin \frac{(j+1) k \pi}{n+1} \right], \tag{11}
\]

We note that the mixing matrix \( U \) does not depend on the parameter \( \tilde{q} \), which is a consequence of our assumption of universality of the Majorana masses \( M_{R,i} = M_{L,i} = m \tilde{q} \) for all \( i \).

The interaction Lagrangian of the clockwork fields to the Standard Model fields, Eq. (4), can now be recast in terms of mass eigenstates:

\[
\mathcal{L}_{\text{int}} = -Y \tilde{T}_L \tilde{H} \tilde{U} \tilde{W} \chi_k = \sum_{k=0}^{2n} Y_k \tilde{T}_L \tilde{H} \chi_k \tag{12}
\]

where

\[
Y_0 = Y (u_k)_n = \frac{Y}{q^2} \sqrt{\frac{q^2 - 1}{q^2 - q^2 - 2n}}
\]

\[
Y_k = Y_{k+n} = \frac{1}{\sqrt{n+1}} \frac{1}{\lambda_k} \left[ q \sin \frac{n k \pi}{n+1} \right], \quad k = 1, \ldots, n.
\]

The components \((u_k)_n\) and \((U_{R})_{np}\), which describe the fraction of the \( n \)-th “gear” in the zero mode, will play a major role in the phenomenology, as they parametrize the portal strength between the Standard Model sector and the clockwork sector.

After electroweak symmetry breaking, new mass terms arise which mix the Standard Model neutrino with the clockwork
fermions. The mass matrix of the $2n + 2$ electrically neutral fermion fields of the model reads:

$$
\begin{pmatrix}
  v_L & X_0 & X_1 & X_2 & \cdots & X_{2n}
  \\
  v_L & 0 & v_Y^0 & v_Y^1 & v_Y^2 & \cdots & v_Y^{2n}
  \\
  X_0 & v_Y^0 & M_0 & 0 & 0 & \cdots & 0
  \\
  X_1 & v_Y^1 & 0 & M_1 & 0 & \cdots & 0
  \\
  X_2 & v_Y^2 & 0 & 0 & M_2 & \cdots & 0
  \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots
  \\
  X_{2n} & v_Y^{2n} & 0 & 0 & 0 & \cdots & M_{2n}
\end{pmatrix},
$$

$$m_v = \begin{pmatrix}
  v_L^α & X_0^β & X_1^β & X_2^β & \cdots & X_{2n}^β
  \\
  v_L^{α\delta} & X_0^{β\delta} & X_1^{β\delta} & X_2^{β\delta} & \cdots & X_{2n}^{β\delta}
  \\
  v_Y^{α\delta} & v_Y^0 & v_Y^1 & v_Y^2 & \cdots & v_Y^{2n}
  \\
  v_Y^{α\delta} & M_0 & 0 & 0 & \cdots & 0
  \\
  v_Y^{α\delta} & M_1 & 0 & 0 & \cdots & 0
  \\
  v_Y^{α\delta} & M_2 & 0 & 0 & \cdots & 0
  \\
  \vdots & \vdots & \vdots & \vdots & \ddots & \vdots
  \\
  v_Y^{α\delta} & M_{2n} & 0 & 0 & \cdots & M_{2n}
\end{pmatrix}.
$$

This matrix has in general a non-trivial flavor structure and leads not only to mixing among the three active neutrinos, but also to potentially large lepton flavor violating charged current, neutral current and Higgs interactions, thus providing a possible test of this framework, as will be discussed in Section 3.

In what follows, we will consider separately the case when the universal Majorana mass is vanishing and when it is non-vanishing.

2.1. Vanishing universal Majorana mass

We consider first the case where the universal Majorana mass is equal to zero. In this case, the global symmetry of the Lagrangian is broken as $U(n) \times U(n+1) \rightarrow U(n)_{\text{EW}}$, which will be identified with total lepton number. The eigenstates and eigenvalues of the mass matrix can be determined using the results of Section 2, by setting $q = 0$.

It is useful to recast the clockwork Lagrangian as

$$
\mathcal{L}_{\text{clockwork}} = \mathcal{L}_{\text{Kin}} - \overline{N_L} m_P^D N_R + \text{h.c.}
$$

where we have defined new fields $N_L = (v_L, N_{L1}, \ldots, N_{Ln})$ and $N_R = (N_{R0}, N_{R1}, \ldots, N_{Rn})$, with

$$
N_{Rk} = \frac{1}{\sqrt{2}} (\chi_k + \chi_{k+n}) , \quad k = 0, \ldots, n
$$

$$
N_{Lk} = \frac{1}{\sqrt{2}} (\chi_k - \chi_{k+n}) , \quad k = 1, \ldots, n
$$

In this basis, the mass matrix has the form:

$$
\begin{pmatrix}
  N_{R0} & N_{R1} & N_{R2} & \cdots & N_{Rn}
  \\
  0 & 0 & 0 & \cdots & 0
  \\
  N_{L1} & 0 & M_1 & 0 & \cdots & 0
  \\
  0 & 0 & M_2 & 0 & \cdots & 0
  \\
  \vdots & \vdots & \vdots & \vdots & \ddots & \vdots
  \\
  0 & 0 & 0 & \cdots & M_n
\end{pmatrix}
$$

where $M_k = m_k \sqrt{\lambda_k}$, with $\lambda_k$ defined in Eq. (7). Namely, the fields $v_L$ and $N_{R0}$ form a massless Dirac pair, while the fields $N_{Rk}$ and $N_{Lk}$ form, for $k = 1, \ldots, n$, Dirac pairs with mass $M_k$. The overall scale of the massive pairs is determined by the parameter $m$, and the mass difference between pairs depends on $q$ and $n$. Assuming $q > 1$, one obtains that the masses of the modes with $k > 0$ increase monotonically with $n$, from $M_1 \approx m(q-1)$ to $M_n \approx m(q+1)$. In Fig. 1, left panel, we show for illustration the mass spectrum of the particles of the clockwork sector, labeled by $k$, taking for concreteness $n = 10$ and $q = 2$. The mass spectrum has been normalized to $m$.

The mass spectrum is modified after electroweak symmetry breaking by the interactions with the Higgs field. Expressed in terms of $N_{Rk}$, the interaction Lagrangian reads:
The Yukawa coupling of the massless mode $Y_0$ is suppressed by $q^a$, provided $q > 1$, whereas the couplings of the $k$th-mode are of the same order as $Y$. This is illustrated in Fig. 1, right panel, which shows the Yukawa couplings of the clockwork fermions to the Standard Model lepton doublets, normalized to $Y$, for the same values of $n$ and $q$ as in the left panel (in this case, $|Y_0|/Y \approx 8 \times 10^{-4}$ and is not visible from the figure).

The mass matrix of the electrically neutral fermion fields now reads:

\[
\begin{pmatrix}
N_{R0} & N_{R1} & N_{R2} & \cdots & N_{Rn} \\
\nu_L & \nu_Y0 & \nu_Y1 & \nu_Y2 & \cdots & \nu_Yn \\
N_{L1} & 0 & M_1 & 0 & \cdots & 0 \\
m_{\nu}^D & 0 & 0 & M_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
N_{Ln} & 0 & 0 & 0 & \cdots & M_n
\end{pmatrix}
\]

(28)

Concretely, a mass term for the active neutrinos is generated. Assuming that $M_k \gg Y_0v$, which as we will see below is justified from the current limits on rare leptonic decays, one can approximate the active neutrino mass by

\[m_\nu \approx vY_0\]

(29)

and can be made small by choosing appropriate values of $Y$, $q$ and $n$. For instance, assuming $Y = O(1)$, $q = 2$, one obtains $m_\nu = O(0.1)\text{eV}$ for $n \approx 40$.

The generalization of the above setup to three leptonic generations and $N$ clockwork generations is straightforward. The clockwork Lagrangian is:

\[
\mathcal{L}_{\text{clockwork}} = \mathcal{L}_{\text{kin}} - \sum_{k=0}^{n} \sum_{a=1}^{N} N_{Rk}^a N_{Lk}^a + \text{h.c.}
\]

(30)

with $N_{Lk} = (v_1^a, N_{L1}^a, \ldots, N_{Ln}^a)$ and $N_{Rk} = (N_{R0}^a, N_{R1}^a, \ldots, N_{Rn}^a)$, where

\[
N_{Rk}^a = \frac{1}{\sqrt{2}}(X_k^a + X_k^{a*}) \ , \quad k = 0, \ldots, n \quad \alpha = 1, \ldots, N
\]

(31)

\[
N_{lk}^a = \frac{1}{\sqrt{2}}(-X_k^a + X_k^{a*}) \ , \quad k = 1, \ldots, n \quad \alpha = 1, \ldots, N
\]

(32)

and the interaction Lagrangian,

\[
\mathcal{L}_{\text{int}} = -\sum_{k=0}^{n} \sum_{a=1}^{N} \bar{\nu}_{Rk}^a \nu_{Lk} \epsilon_{kL0} N_{Rk}^a
\]

(33)

with $\nu_{Rk}^a = Y_{\alpha k} U_{Lk}^\alpha$.

After electroweak symmetry breaking the neutrino mass matrix reads:

\[
\begin{pmatrix}
N_{R0}^\beta & N_{R1}^\beta & N_{R2}^\beta & \cdots & N_{Rn}^\beta \\
\nu_{L0}^\beta & \nu_{L1}^\beta & \nu_{L2}^\beta & \cdots & \nu_{Ln}^\beta \\
N_{L1}^\beta & 0 & M_1^\beta & 0 & \cdots & 0 \\
m_{\nu}^D & 0 & 0 & M_2^\beta & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
N_{Ln}^\beta & 0 & 0 & 0 & \cdots & M_n^\beta
\end{pmatrix}
\]

(34)

where $M_k^\beta$ is the mass of $k$-th clockwork gear for the Dirac pair $N_{Lk}^\beta, N_{Rk}^\beta$.

We analyze in detail the case where the clockwork consists of two generations with $n_1$ and $n_2$ gears, respectively. We scan $Y_{\alpha k}$ within the ranges $1/3 < |Y_{\alpha k}| < 4$, $q_{\alpha}$ between 1.5 and 6 and $n_\alpha$ between 15 and 55, and we select the points that reproduce the observed values of the solar and atmospheric mass splitting and mixing angles within $1\sigma$, as determined in Ref. [47]. In Fig. 2 (left panel) we show as green circles (yellow triangles) the values of $n_1$ ($n_2$) as a function of $q_1$ ($q_2$) that satisfy the experimental constraints. As apparent from the plot, larger $q_\alpha$ require a smaller number of gears to reproduce the small neutrino Yukawa coupling. Furthermore, the allowed values for $n_1$ and $n_2$ have a big overlap, which is a consequence of our assumption of comparable elements in the coupling $Y_{\alpha k}$ and the necessity of producing a mild hierarchy between the solar and the atmospheric neutrino mass scales.

In particular, we find that the scenario with $q_1 = q_2$ and $n_1 = n_2$, namely the scenario where the clockwork parameters are universal also among generations, is allowed by observations. This is illustrated in Fig. 2 (right panel), which shows the allowed values of $q_1 - q_2$ as a function of $n_1 - n_2$, and demonstrates the existence of viable points at the point $q_1 = q_2$ and $n_1 = n_2$. One concrete point which leads to the correct neutrino parameters is:...
The symmetry case two Yukawa couplings with the former scenario to be \( O(Y) \), the resulting neutrino mass can be orders of magnitude larger than the value inferred from oscillation experiments, unless \( Y \ll 1 \) and/or the gear masses are very large, in the same spirit as in the standard seesaw mechanism. A related analysis was also presented in [46].

3. Lepton flavor violation

The clockwork mechanism suppresses the Yukawa couplings for the zero mode, hence explaining the smallness of neutrino masses. However the Yukawa couplings for the higher modes are in general unsuppressed and can lead to observable effects at low energies. In particular, the lepton flavor violation generically present in the Yukawa couplings of the higher modes contributes, through quantum effects induced by clockwork fermions, to generate rare leptonic decays (such as \( l_i \rightarrow l_j \gamma \)) or \( \mu \rightarrow e \) conversion in nuclei, with rates that could be at the reach of current or future experiments if the gear masses are sufficiently low.

We calculate the rate for \( l_i \rightarrow l_j \gamma \) following [48–50]. For \( N \) clockwork generations, we obtain:

\[
B(\mu \rightarrow e \gamma) = \frac{3e_{\text{em}} v^4}{8\pi} \sum_{\alpha=1}^{N} \sum_{k=1}^{n_\alpha} \frac{1}{M_k^2} \left| F(x_k^\alpha) \right|^2.
\]

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\]

where \( e_{\text{em}} \) is the fine structure constant, \( n_\alpha \) is the number of gears in the \( \alpha \)-th generation, \( M_k^\alpha \) is the mass of the \( k \)-th mode in the
\( \alpha \)-th generation \((k = 1, \ldots, n_k)\), and \( \chi_k^a \equiv M_{k}^2 / M_W^2 \). The loop function \( F(x) \) is defined as
\[
F(x) \equiv \frac{1}{6(1-x)^4} \left( 10 - 43x + 78x^2 - 49x^3 + 4x^4 - 18x^3 \log x \right),
\]
and has limits \( F(0) = 5/3 \) and \( F(\infty) = 2/3 \).

The current upper bound \( \text{Br}(\mu \to e\gamma) \leq 4.2 \times 10^{-13} \) from the MEG experiment [51] (for a recent review, please see [52]) poses stringent constraints on the mass scale of the clockwork. In Fig. 4 we show the branching ratio expected for points reproducing the measured neutrino parameters, assuming two clockwork generations, as obtained in the scan presented in section 2.1, as a function of the mass of the first clockwork gear. It follows from the figure that the clockwork gears must be larger than \( \sim 40 \text{ TeV} \) in order to evade the experimental constraints, unless very fine cancellations among all contributions to this process exist. For a larger number of clockwork generations we expect even stronger lower limits on the lightest gear mass, due to the larger number of particles in the loop.

4. Summary

The origin of small neutrino masses remains a mystery to this day. The recently proposed clockwork mechanism provides new insights into this puzzle, as it naturally generates small parameters in the effective Lagrangian. In the present work, we have scrutinized the mechanism of neutrino mass generation within the clockwork framework. We have generalized the clockwork formalism to include, in addition to Dirac masses and nearest neighbor interactions, also Majorana mass terms in the clockwork sector; and we have derived analytical expressions for the masses and couplings of the new singlet fermions for the specific case where the Dirac masses, Majorana masses and nearest neighbor interactions are universal among all clockwork “gears”.

We have investigated in detail the impact of the Majorana masses in the clockwork sector in the generation of small neutrino masses. When the universal Majorana mass vanishes, the zero mode of the clockwork sector is strictly massless and forms a Dirac pair with the active neutrino. In this framework, small Dirac neutrino masses can be generated for a sufficiently large number of gears, depending on the hierarchy between the mass scales in the clockwork sector. On the other hand, when the universal neutrino mass is non-vanishing, the zero mode is no longer massless. However, in the corresponding Yukawa coupling still has the clockwork structure. In this case, the contribution from this particular mode is the result of the interplay between the standard seesaw mechanism and the “clockworked” Yukawa couplings. The contribution from the gears is typically proportional to their \( O(1) \) Yukawa couplings and they require a very large Majorana mass scale in order to reproduce the small neutrino masses inferred from oscillation experiments.

The Standard Model leptons couple to the fermions of the clockwork sector with a site dependent strength, giving rise to (possibly lepton flavor violating) charged current, neutral current and Higgs boson interactions. We have investigated the constraints on this framework from the non-observation of the rare leptonic decay \( \mu \to e\gamma \). Our results indicate that the lightest particle of the clockwork sector must have a mass \( \gtrsim 40 \text{ TeV} \), if the Yukawa couplings of the fundamental theory are \( O(1) \).

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References

[1] P. Minkowski, \( \mu \to e\gamma \) at a rate of one out of \( 10^9 \) muon decays?, Phys. Lett. B 67 (1977) 421–428.
[2] R.N. Mohapatra, G. Senjanovic, Neutrino mass and spontaneous parity violation, Phys. Rev. Lett. 44 (1980) 912.
[3] T. Yanagida, Horizontal symmetry and masses of neutrinos, Conf. Proc. C 7902131 (1979) 95–99.
[4] M. Gell-Mann, P. Ramond, R. Slansky, Complex spinors and unified theories, Conf. Proc. C 790927 (1979) 315–321.
[5] J. Schechter, J.W.F. Valle, Neutrino masses in SU(2) x U(1) theories, Phys. Rev. D 22 (1980) 2227.
[6] R.N. Mohapatra, et al., Theory of neutrinos: a white paper, Rep. Prog. Phys. 70 (2007) 1757–1867.
[7] W. Wang, R. Wang, Z-L. Han, J-Z. Han, The \( B \leftarrow I \) scotogenic models for dirac neutrino masses, 2017.
[8] L.M. Krauss, S. Nasri, M. Trodden, A model for neutrino masses and dark matter, Phys. Rev. D 67 (2003) 085002.
[9] W. Wang, Z-L. Han, Naturally small Dirac neutrino mass with intermediate SU(2)_L multiplet fields, J. High Energy Phys. 04 (2017) 166.
[10] E. Ma, O. Popov, Pathways to naturally small Dirac neutrino masses, Phys. Lett. B 764 (2017) 142–144.
[11] D. Borah, A. Dasgupta, Naturally light Dirac neutrino in left–right symmetric model, J. Cosmol. Astropart. Phys. 1706 (06) (2017) 003.
[12] D. Borah, A. Dasgupta, Common origin of neutrino mass, dark matter and Dirac leptonogenesis, J. Cosmol. Astropart. Phys. 1612 (12) (2016) 034.
[13] C. Bonilla, E. Ma, E. Peinado, J.W.F. Valle, Two-loop Dirac neutrino mass and WIMP dark matter, Phys. Lett. B 762 (2016) 214–218.
[14] S. Kanemura, K. Sakurai, H. Sugiyama, Probing models of dirac neutrino masses via the flavor structure of the mass matrix, Phys. Lett. B 758 (2016) 465–472.
[15] H. Okada, Two loop induced Dirac neutrino model and dark matters with global U(1)' symmetry, 2014.
[16] Y. Farzan, E. Ma, Dirac neutrino mass generation from dark matter, Phys. Rev. D 86 (2012) 033007.
[17] P.-H. Gu, U. Sarkar, Radiative neutrino mass, dark matter and leptonogenesis, Phys. Rev. D 77 (2008) 105031.
[18] D. Chang, R.N. Mohapatra, Small and calculable Dirac neutrino mass, Phys. Rev. Lett. 58 (Apr 1987) 1600–1603.
[19] M. Lindner, T. Ohlsson, G. Seidl, Seesaw mechanisms for Dirac and Majorana neutrino masses, Phys. Rev. D 65 (Feb 2002) 035014.
[20] K. Choi, S.H. Im, Realizing the relaxation from multiple axions and its UV completion with high scale supersymmetry, J. High Energy Phys. 01 (2016) 149.
[21] D.E. Kaplan, R. Rattazzi, Large field excursions and approximate discrete symmetries from a clockwork axion, Phys. Rev. D 93 (8) (2016) 085007.
[22] N. Arkani-Hamed, A.G. Cohen, H. Georgi, (De)constructing dimensions, Phys. Rev. Lett. 86 (2001) 4757–4761.
[23] C.T. Hill, S. Pokorski, J. Wang, Gauge invariant effective Lagrangian for Kaluza–Klein modes, Phys. Rev. D 64 (2001) 105005.
[24] G.F. Giudice, M. McCullough, A clockwork theory, J. High Energy Phys. 02 (2017) 036.
[25] N. Craig, I. Garcia Garcia, D. Sutherland, Disassembling the clockwork mechanism, 2017.
[26] G.F. Giudice, M. McCullough, Comment on “Disassembling the clockwork mechanism”, 2017.
[27] P. Saraswat, Weak gravity conjecture and effective field theory, Phys. Rev. D 95 (2) (2017) 025013.
[28] A. Kehagias, A. Riotto, Clockwork inflation, Phys. Lett. B 767 (2017) 73–80.
[29] M. Farina, D. Pappadopulo, F. Romanini, A. Tesi, The photo-phile QCD axion, J. High Energy Phys. 01 (2017) 095.
[30] A. Ahmed, B.M. Dillon, Clockwork Goldstone bosons, 2016.
[31] T. You, A dynamical weak scale from inflation, J. Cosmol. Astropart. Phys. 1709 (09) (2017) 019.
[32] A. Diez-Tejedor, D.J.E. Marsh, Cosmological production of ultralight dark matter axions, 2017.
[33] B. Batell, M.A. Fedderke, L.-T. Wang, Relaxation of the composite Higgs little hierarchy, 2017.
[34] W. Tangarife, K. Tobioka, L. Ubaldi, T. Volansky, Relaxed inflation, 2017.
[35] R. Coy, M. Frigerio, M. Ibe, Dynamical clockwork axions, 2017.
[36] I. Ben-Dayan, Generalized clockwork theory, 2017.
[37] D.K. Hong, D.H. Kim, C.S. Shin, Clockwork graviton contributions to muon g–2, 2017.
[38] H.M. Lee, Gauged U(1) clockwork theory, 2017.
[39] M. Carena, Y.-Y. Li, C.S. Machado, P.A.N. Machado, C.E.M. Wagner, Neutrinos in large extra dimensions and short-baseline ν, appearance, 2017.
[40] I. Antoniadis, A. Delgado, C. Markou, S. Pokorski, The effective supergravity of little string theory, 2017.
[41] L.E. Ibáñez, M. Montero, A note on the WGC, effective field theory and clockwork within string theory, 2017.
[42] A. Kehagias, A. Riotto, The clockwork supergravity, 2017.
[43] J. Kim, J. McDonald, A clockwork Higgs portal model for freeze-in dark matter, 2017.
[44] C. von Gersdorff, Natural fermion hierarchies from random Yukawa couplings, J. High Energy Phys. 09 (2017) 094.
[45] T. Hambye, D. Teresi, M.H.G. Tytgat, A clockwork WIMP, J. High Energy Phys. 07 (2017) 047.
[46] S.C. Park, C.S. Shin, Clockwork seesaw mechanisms, 2017.
[47] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo, Global constraints on absolute neutrino masses and their ordering, 2017.
[48] S.T. Petcov, The processes μ → e Gamma, μ → e e anti-e, neutrino’ → neutrino gamma in the Weinberg–Salam model with neutrino mixing, Sov. J. Nucl. Phys. 25 (1977) 340. Erratum: Yad. Fiz. 25 (1977) 1336.
[49] S.M. Bilenky, S.T. Petcov, B. Pontecorvo, Lepton mixing, μ → e + gamma decay and neutrino oscillations, Phys. Lett. B 67 (1977) 309.
[50] T.P. Cheng, L.-F. Li, μ → eγ in theories with Dirac and Majorana neutrino mass terms, Phys. Rev. Lett. 45 (1980) 1908.
[51] A.M. Baldini, et al., Search for the lepton flavour violating decay μ→eγ with the full dataset of the MEG experiment, Eur. Phys. J. C 76 (8) (2016) 434.
[52] L. Calibbi, G. Signorelli, Charged lepton flavour violation: an experimental and theoretical introduction, 2017.