Second Order Corrections to Large Scale Structure Weak Lensing: Background Source Clustering

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ABSTRACT

We discuss the second order contributions to lensing statistics resulting from the clustering of background sources from which galaxy shape measurements are made in weak lensing experiments. In addition to a previously discussed contribution to the lensing skewness, background source clustering also contributes to the two-point correlation function, such as the angular power spectrum of convergence or shear. At arcminute scales or above, the second order contribution to the angular power spectrum of convergence due to source clustering is below the level of a few percent. The background clustering of sources also results in a non-Gaussian contribution to the power spectrum covariance of weak lensing convergence through a four-point correlation function or a trispectrum in Fourier space. The increase in variance is, at most, a few percent relative to the Gaussian contribution while the band powers are also correlated at the few percent level. The non-Gaussian contributions due to background source clustering is at least an order of magnitude smaller than those resulting from non-Gaussian aspects of the large scale structure due to the non-linear evolution of gravitational perturbations. We suggest that the background source clustering is unlikely to affect the precision measurements of cosmology from upcoming weak lensing surveys.

Key words: cosmology: observations — gravitational lensing

1 INTRODUCTION

Weak gravitational lensing of faint galaxies probes the distribution of matter along the line of sight. Lensing by large-scale structure (LSS) induces correlation in the galaxy ellipticities at the few percent level and can be detected through challenging statistical studies of galaxy shapes in wide-field surveys (e.g., Blandford et al. 1991; Miralda-Escudé 1991; Kaiser 1992). An important aspect of weak lensing is that these ellipticity correlations, or associated statistics, provide cosmological information that is, in some cases, complementary to those supplied by the cosmic microwave background data at the same level of precision or better (e.g., Jain & Seljak 1997; Bernardeau et al. 1997; Kaiser 1998; Schneider et al. 1998; Hu & Tegmark 1999; Cooray 1999; van Waerbeke 1999; see Mellier 1999 and Bartelmann & Schneider 2000 for recent reviews). Indeed, a wide number of recent studies have provided the clear evidence for weak lensing due to large scale structure (van Waerbeke et al. 2000; Bacon et al. 2000; Wittman et al. 2000; Kaiser et al. 2000), though more work is clearly needed to understand even the statistical errors and biases. While biases can result from certain aspects related to the selection of observational fields (e.g., Cooray et al. 2000), statistical errors include those that are fundamentally present due to the non-Gaussian nature of the large scale structure (e.g., Cooray & Hu 2001b).

Current predictions on statistics related to large scale structure weak lensing and their ability to measure cosmological parameters are based on several assumptions; for example, it is implicitly assumed that background galaxies, from which galaxy shape measurements are made, are uniformly distributed. Another assumption is the use of so-called Born approximation where one integrates along unperturbed photon geodesics instead of perturbed photon paths. Here, we discuss the former assumption while Cooray & Hu (2002) presents a discussion of the corrections resulting from dropping the Born approximation and including the so-called lens-lens coupling between two lenses at different redshifts.

The effect of background source clustering was first discussed with respect to the lensing three point statistics, such as convergence skewness (Bernardeau 1998). Hamana et al. (2001) includes an extended discussion of this contribution. Recently, the same effect was revisited by Schneider
et al. (2002) as a possible source of curl-like modes in lensing statistics. Here, we present a general discussion of the corrections resulting from clustering of background sources and consider effects two, three and four-point correlation functions. We suggest that the contributions are negligible and are unlikely to affect the current and upcoming weak lensing observations as a probe of cosmological parameters by comparing to predictions that neglect background source clustering.

2 CALCULATIONAL METHOD

We discuss our statistics in terms of lensing convergence,

\[ \kappa(\chi) = \frac{3}{2} \Omega_m H_0^2 \int_0^\infty d\chi n_s(\chi) \int_0^\chi d\chi' g(\chi', \chi) \delta(\chi - \chi') \cdot (1) \]

where \( n_s(\chi) \) is the normalized radial distribution of background sources such that \( \int d\chi n_s(\chi) = 1 \) and \( g(\chi', \chi) \) is the lensing weight function when a background source is at a radial distance of \( \chi \):

\[ g(\chi', \chi) = \frac{d_A(\chi')d_A(\chi - \chi')}{d_A(\chi)} \cdot (2) \]

Here, \( \chi \) is the radial distance, or lookback time, from the observer, given by

\[ \chi(z) = \int_0^z \frac{dz'}{H(z')} \cdot (3) \]

and the analogous angular diameter distance

\[ d_A(\chi) = H_0^{-1} \Omega_K^{-1/2} \sinh(H_0 \Omega_K^{1/2}/\chi) \cdot (4) \]

with the expansion rate for adiabatic CDM cosmological models with a cosmological constant given by

\[ H^2 = H_0^2 \left[ \Omega_m (1 + z)^3 + \Omega_K (1 + z)^2 + \Omega_\Lambda \right] \cdot (5) \]

Here, \( H_0 \) can be written as the inverse Hubble distance today \( cH_0^{-1} = 2997.96^{-1}\text{Mpc} \). We follow the conventions that in units of the critical density \( 3H_0^2/8\pi G \), the contribution of each component is denoted \( \Omega_i \), \( i = c \) for the CDM, \( b \) for the baryons, \( A \) for the cosmological constant. We also define the auxiliary quantities \( \Omega_m = \Omega_c + \Omega_b \) and \( \Omega_K = 1 - \sum \Omega_i \), which represent the matter density and the contribution of spatial curvature to the expansion rate respectively. Note that as \( \Omega_K \to 0 \), \( d_A(\chi) \to \chi \) and we define \( \chi(z = \infty) = \chi_0 \). Though we present a general derivation of second order contributions to weak lensing due to background source clustering, we show results for the currently favorable ΛCDM cosmology with \( \Omega_m = 0.05 \), \( \Omega_m = 0.35 \), \( \Omega_\Lambda = 0.65 \) and \( h = 0.65 \).

We assume that background sources are clustered in radial space such that \( n_s(\chi) \approx n_s(\chi)[1 + b n_s(\chi)] \) where fluctuations in the number counts of background sources are related to that of the density field via a time, and possibly a scale dependent, bias

\[ \delta n_s(\chi) = b_1(\chi) \delta(\chi) + \frac{1}{2} b_2(\chi) \delta^2(\chi) \cdot (6) \]

Here we have considered, perturbatively, contributions up to the second order in density perturbations. It is important that we consider contributions up to the \( \delta^2 \) term since these terms can contribute at the same second order level in the power spectrum. Though we discuss the effect in terms of lensing convergence, it should be noted that our calculation equally well apply for, correlations of shear, such as the gradient, or electric-like, modes (see, Schneider et al. 2002).

In Fourier space, we can write the first, second and third order contribution to convergence as

\[ \kappa^{(1)}(l) = \frac{3}{2} \Omega_m H_0^2 \int d^2\theta e^{-i\theta_0} \int_0^{\chi_0} d\chi n_s(\chi) \]

\[ \times \int_0^{\chi} d\chi' g(\chi', \chi) \int \frac{d^4k}{(2\pi)^4} \delta(k, \chi') e^{ik \cdot \chi'}, \]

\[ \kappa^{(2)}(l) = \frac{2}{2} \Omega_m H_0^2 \int d^2\theta e^{-i\theta_0} \int_0^{\chi_0} d\chi n_s(\chi) b_1(\chi) \]

\[ \times \int \frac{d^4k_1}{(2\pi)^3} \delta(k_1, \chi) \int_0^{\chi} d\chi' g(\chi', \chi) \int \frac{d^4k_2}{(2\pi)^3} \delta(k_2, \chi') \]

\[ e^{i\theta \cdot (k_1 + k_2)} \cdot (7) \]

and

\[ \kappa^{(3)}(l) = \frac{3}{2} \Omega_m H_0^2 \int d^2\theta e^{-i\theta_0} \int_0^{\chi_0} d\chi n_s(\chi) b_2(\chi) \]

\[ \times \int \frac{d^4k_1}{(2\pi)^3} \delta(k_1, \chi) \int \frac{d^4k_2}{(2\pi)^3} \delta(k_2, \chi) \int_0^{\chi} d\chi' g(\chi', \chi) \]

\[ \times \int \frac{d^4k_3}{(2\pi)^3} \delta(k_3, \chi') e^{i\theta \cdot (k_1 + k_2 + k_3)}, \]

(8)

respectively.

We define the associated angular power spectrum, bispectrum and trispectrum of weak lensing convergence, in flat-sky as appropriate for current and upcoming experiments as

\[ \langle \kappa(l) \kappa(l') \rangle = (2\pi)^2 \delta_D(l + l') C_{\kappa}^{\kappa \kappa} \]

\[ \langle \kappa(l) \kappa(l') \kappa(l''') \rangle = (2\pi)^2 \delta_D(l + l' + l'') B_{\kappa}(l, l', l'') \]

\[ \langle \kappa(l) \kappa(l') \kappa(l'') \rangle = (2\pi)^2 \delta_D(l + l' + l'') T_{\kappa}(l, l', l''), \]

(9)

Here, \( k_{i-j} = k_i + \ldots + k_j \) and \( \delta_D \) is the delta function not to be confused with the density perturbation. Note that the subscript \( _c \) denotes the connected piece, i.e., the trispectrum is defined to be identically zero for a Gaussian field. Here and throughout, we occasionally suppress the redshift dependence where no confusion will arise.

We define the power spectrum of density fluctuations as

\[ \langle \delta(k) \delta(k') \rangle = (2\pi)^3 \delta(k + k') P_\delta(k), \]

(10)

where

\[ \frac{k^3 P(k)}{2\pi^2} = \delta_H \left( \frac{k}{H_0} \right)^{\alpha + 3} T^2(k), \]

(11)

in linear perturbation theory. We use the fitting formulae of Eisenstein & Hu (1999) in evaluating the transfer function.
$T(k)$ for CDM models. Here, $\delta_H$ is the amplitude of present-day density fluctuations at the Hubble scale; with $n \simeq 1$, we adopt the COBE normalization for $\delta_H$ (Bunn & White 1997) of $4.2 \times 10^{-5}$, consistent with galaxy cluster abundance (Viana & Liddle 1999), with $\sigma_8 \simeq 0.86$. To capture the non-linear aspects of the power spectrum, we use the prescription by Peacock & Dodds (1996).

We will now discuss contributions to the angular power spectrum, bispectrum and trispectrum of convergence from clustering of background sources. To illustrate results, we take a redshift distribution for the background sources of the form

$$n_s(z) = \left(\frac{z}{z_0}\right)^{\alpha} \exp\left(-\left(\frac{z}{z_0}\right)^{\beta}\right),$$ \hspace{1cm} (12)

where $(\alpha, \beta)$ denote the slope of the distribution at low and high $z$’s, respectively around a mean-like parameter given by $\sim z_0$. For the purpose of this calculation we take $\alpha = \beta = 1.5$ and take $z_0$ to be 1.0 so as to mimic the expected background sources from current and upcoming lensing catalogs. In general, since clustering evolves to low redshifts, with a decrease in the background source redshift distribution to a lower redshift, we expect an increase in the importance of second order effects associated with clustering of background sources.

### 2.1 Power Spectrum

To the first order, using $(\zeta^{(1)}(l) \zeta^{(1)}(l'))$, we simplify with the Limber approximation (Limber 1954) in Fourier space following Kaiser (1992; Kaiser 1998) to obtain the well-known result that

$$C_l^{\kappa\kappa} = \frac{9}{4} \Omega_m^2 H_0^2 \left[\int_0^{\chi_0} d\chi \bar{n}_s(\chi)\right]^2 \times \int_0^\chi d\chi' g^2(\chi', \chi) \frac{d\chi}{dA(\chi')},$$ \hspace{1cm} (13)

The second order convergence power spectrum resulting from source clustering involves two terms: $(\zeta^{(2)}(l) \zeta^{(2)}(l')$ and $(\zeta^{(3)}(l) \zeta^{(3)}(l')$. The latter includes an additional, and equal, term per a permutation. We simplify these contributions again using the Limber approximation (Limber 1954) such that

$$\frac{9}{4} \Omega_m^2 H_0^4 \int_0^{\chi_0} d\chi \frac{[b_1 \bar{n}_s(\chi)]^2}{dA(\chi')} \int_0^\chi d\chi' \frac{g^2(\chi', \chi)}{dA(\chi')},$$ \hspace{1cm} (14)

and

$$\frac{9}{8} \Omega_m^2 H_0^4 \int_0^{\chi_0} d\chi \frac{[b_2 \bar{n}_s(\chi)]^2}{dA(\chi')} \int_0^\chi d\chi' \frac{g^2(\chi', \chi)}{dA(\chi')}.$$ \hspace{1cm} (15)

Note that the total contribution to the convergence power spectrum follows as $(\zeta^{(2)}(l) \zeta^{(2)}(l'))+(\zeta^{(3)}(l) \zeta^{(3)}(l'))$. We denote the first contribution by $C_l^{22}$ and the latter two terms by $C_l^{31}$. Thus, the total contribution to the power spectrum due to source clustering is

$$C_l^{\kappa\kappa} = C_l^{22} + C_l^{31}.$$ \hspace{1cm} (16)

In figure we show the second order correction to the angular power spectrum of convergence. The solid line is the well known first order result, while the dashed line is the contribution from the $C_l^{22}$ term. Note that $C_l^{22} \propto b_1^2$ and we have taken the value of $b_1 = 1$ for illustration purposes. The long-dashed line is the $C_l^{31}$ contribution, where $C_l^{31} \propto b_2$ and we have taken $b_2 = 1$ for simplicity. Since we do not have detailed information on the galaxy bias, to illustrate our results, we have taken the bias to be redshift and scale independent. Adding a redshift dependent bias of the form $b(z) \propto (1 + z)^{\gamma}$, however, did not lead to a significantly different result from the one suggested in figure when $|\gamma| < 2$.

There is one aspect of bias that should be kept in mind when interpreting figure. In general, quadratic bias is expected to be negative, such that the two terms, $C_l^{22}$ and $C_l^{31}$, added together will give a contribution which is lower than what one would naively expect if simply added together. In the IRAS PSCz catalog, Feldman et al. (2001) finds $1/b_1 = 1.20^{+0.18}_{-0.15}$ and $b_2/b_1^2 = -0.42 \pm 0.19$. Similar results we obtained from the 2dF survey by Verde et al. (2002): $b_1 = 1.04 \pm 0.11$ and $b_2 = -0.054 \pm 0.08$. Since there is no conclusive evidence for a non-zero value for $b_2$, the first approximation that $b_2 = 0$ and $b_1 = 1$ leads to the dashed line with the conclusion that second order effects are generally below the few percent level for multipole less than $10^5$ corresponding to angular scales less than a few arcminutes.

On the other hand, if $b_2 = -0.5$ and $b_1 = 1$, we obtain the dot-dashed line as the total contribution to the angular power spectrum of convergence due to source clustering; the shape of the power spectrum is due to the fine cancellation of $C_l^{22}$ and $C_l^{31}$ terms. We suggest that, in addition to the linear bias, the extent to which background source clustering affects statistics such as power spectra or correlations depends on the detail aspects of galaxy biasing such as the quadratic bias. In any case, we find that source clustering effects are unlikely to be a strong contaminant for current lensing experiments.
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presented here. As we do not have a reliable method to predict
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power spectrum lies between the linear and non-linear cases
considering terms such as

2.2 Bispectrum

Figure 1. The angular power spectrum of convergence. The solid
line shows the first order contribution while corrections due to
background source clustering is shown with dashed \((C_l^{22})\) and
long-dashed \((C_l^{33})\) lines. We have assumed \(b_1 = 1\) and \(b_2 = 1\)
in these two cases, respectively. The dot-dashed line is the total
second order power spectrum when \(b_1 = 1\) and \(b_2 = -0.5\) con-
sistent with suggestions in the literature for galaxy bias (see text
for details). The dotted line is the \(C_l^{33}\) contribution when galaxies
trace the linear density field instead of the fully non-linear power
spectrum.

There is also another important aspect related to \(C_l^{33}\). The
integral over \(l_1\) denotes the power spectrum traced by
galaxies and \(C_l^{33}\) effectively scales with this integral as an
overall normalization. If galaxies do not fully trace the non-
linear power spectrum, as predicted by the Peacock & Dodds
(1996) formulae for the dark matter, then the contribution
would be lower than what we have predicted. We can bracket
the expected range of variation by replacing the power spec-
trum involved with the integral over \(l_1\) with that of the linear
power spectrum, as it is generally expected that the galaxy
power spectrum lies between the linear and non-linear cases
of dark matter. Since the contribution to the integral here
comes from all angular scales, the behavior of either the linear
or non-linear power spectrum at small angular scales becomes
to some extent important for the calculation pre-
sented here. As we do not have a reliable method to predict
the non-linear power spectrum at small scales, we safely cut
off the calculation at \(k \sim 10^9\) h Mpc\(^{-1}\).

2.2 Bispectrum

We can write the resuluting contribution to the bispectrum by
considering terms such as \((\kappa^{(2)}(l_1) \kappa^{(1)}(l_2) \kappa^{(1)}(l_3))\) and add
the necessary permutations. We write one of these terms as

\[
\kappa^{(2)}(l_1) \kappa^{(1)}(l_2) \kappa^{(1)}(l_3) = \left(2 \pi \right)^2 \delta_D(1 + l' + l'') \left(\frac{3}{2} \Omega_m H^2_0\right)^3 \times 
\int_0^{\chi_0} d\chi \bar{n_s}(\chi) \int_0^{\chi} d\chi' \frac{g^2(\chi', \chi) b_1(\chi) \bar{n_s}(\chi')}{d_A^2(\chi')} P\left(\frac{\ell_1}{d_A(\chi')}, \chi'\right)
\]

so that the bispectrum is

\[
B_n(l_1, l_2, l_3) = \left(\frac{3}{2} \Omega_m H^2_0\right)^3 \times 
\int_0^{\chi_0} d\chi \bar{n_s}(\chi) \int_0^{\chi} d\chi' \frac{g^2(\chi', \chi) b_1(\chi) \bar{n_s}(\chi')}{d_A^2(\chi')} P\left(\frac{\ell_1}{d_A(\chi')}, \chi'\right) + \text{Perm.}
\]

where the permutations are with respect to the ordering of
\((l_1, l_2, l_3)\) and involves five additional terms.

Following Cooray & Hu (2001a), we can write the third
moment of convergence using the bispectrum as

\[
\langle \kappa^3(\sigma) \rangle = \frac{1}{4\pi} \sum_{i,j,k\neq l} \frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi} \times \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right) B_{l_1 l_2 l_3} \kappa(l_1) \kappa(l_2) \kappa(l_3),
\]

where the quantity within parantheses is the Wigner-3j sym-

bolic, which in the case of no angular dependence can be written

\[
\left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right) = (-1)^{L/2} \left( \frac{L}{2} - l_1 \right)! \left( \frac{L}{2} - l_2 \right)! \left( \frac{L}{2} - l_3 \right)! \times \left( \frac{L - 2l_1!}{(L - 2l_1)!} \frac{(L - 2l_2)!}{(L - 2l_2)!} \frac{(L - 2l_3)!}{(L - 2l_3)!} \right)^{1/2} \frac{1}{(L + 1)!}
\]

for even \(L\); it vanishes for odd \(L\). We refer the reader to
Cooray & Hu (2000) for additional details on the Wigner
3-j symbol.

Using the third moment, we construct the skewness as

\[
S_3(\sigma) = \frac{\langle \kappa^3(\sigma) \rangle}{\langle \kappa^2(\sigma) \rangle^3/2},
\]

where the all-sky expression for bispectrum, in terms of the
flat-sky derivation, is

\[
B_{\kappa}^{l_1 l_2 l_3} = \frac{1}{4\pi} \sum_l \frac{(2l + 1)}{C_l} W^2_l(\sigma) \times \kappa(\sigma).
\]

Similarly, the second moment is defined as

\[
\langle \kappa^2(\sigma) \rangle = \frac{1}{4\pi} \sum_l (2l + 1) C_l^\kappa W^2_l(\sigma).
\]
In prior publications, the contribution to skewness is at the level of few tens of percent of the total expected skewness based on the halo model calculation following Cooray & Hu (2001b). For comparison, we show results from N-body particle-mesh simulations by White & Hu (1999). The higher order correction to skewness resulting from background source clustering is shown with a dot-dashed line. The second order contribution is at the level of few tens of percent of the total expected from non-linear evolution of gravitational perturbations.

In figure 2, we summarize our results on the expected contribution to the convergence skewness. We refer the reader to Bernardeau (1998) and Hamana et al. (2002) for an extended discussion on the effects of background source clustering on skewness. As shown in figure 2 and discussed in prior publications, the contribution to skewness is at the level of few tens of percent and depends strongly on parameters such as the mean redshift of background sources and the width of the redshift distribution. A higher mean redshift and a smaller width result in a smaller contribution to skewness while a lower mean redshift and a broader distribution can contribute up to 30% or more of the skewness expected from non-linear evolution of gravitational perturbations.

2.3 Trispectrum

We can write the resulting contribution to the trispectrum by considering terms such as \( \langle \kappa^{(2)}(1)\kappa^{(2)}(1')\kappa^{(1)}(1'')\kappa^{(1)}(1''') \rangle \) and add the necessary permutations. We write one of these terms as

\[
\langle \kappa^{(2)}(1)\kappa^{(2)}(1')\kappa^{(1)}(1'')\kappa^{(1)}(1''') \rangle = (2\pi)^2 \delta_D (1 + 1' + 1'' + 1''') \left( \frac{3}{2} \Omega_m H_0^2 \right)^4
\]

\[
\times \int_0^{\infty} d\chi \frac{b_n b_n'}{d_A(\chi)} \left[ P \left( \frac{l_1 + 1}{d_A(\chi)}, \chi \right) + P \left( \frac{l_1 + 1}{d_A(\chi)}, \chi \right) \right] \times \int_0^{\infty} d\chi' \frac{g(\chi, \chi') g(\chi', \chi')}{d_A(\chi')} \left( \frac{l_2}{d_A(\chi')}, \chi' \right),
\]

so that the trispectrum is

\[
T_\kappa(l_1, l_2, l_3, l_4) = \left( \frac{3}{2} \Omega_m H_0^2 \right)^4
\]

\[
\times \int_0^{\infty} d\chi \frac{b_n b_n'}{d_A(\chi)} \left[ P \left( \frac{l_1 + 1}{d_A(\chi)}, \chi \right) + P \left( \frac{l_1 + 1}{d_A(\chi)}, \chi \right) \right] \times \int_0^{\infty} d\chi' \frac{g(\chi, \chi') g(\chi', \chi')}{d_A(\chi')} \left( \frac{l_2}{d_A(\chi')}, \chi' \right),
\]

where the permutations are with respect to the ordering of \((l_1, l_2, l_3, l_4)\).

For the purpose of this calculation, we assume that upcoming weak lensing convergence power spectrum will measure binned logarithmic band powers at several \( l_i \)'s in multipole space with bins of thickness \( \delta l_i \).

\[
C_l = \int d^2l \frac{l^2}{A_{\delta l}} \kappa(l) \kappa(-l),
\]

where \( A_{\delta l} = \int d^2l \) is the area of 2D shell in multipole and can be written as \( A_{\delta l} = 2\pi l_i \delta l_i + \pi(\delta l_i)^2 \).
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We can now write the signal covariance matrix as

\[ C_{ij} = \frac{1}{A} \left[ \frac{(2\pi)^2}{A_{\text{xi}}} 2C_{\text{G}}^2 + T_{ij}^{\text{G}} \right], \tag{27} \]

\[ T_{ij}^{\text{G}} = \int \frac{d^2 l_i}{A_{\text{xi}}} \int \frac{d^2 l_j}{A_{\text{sj}}} \frac{P_{ij}(\ell_i)}{(2\pi)^2} T^n(\ell_i, -\ell_i, 1, 1, -1, -1), \tag{28} \]

where \( A \) is the area of the survey in steradians. Again the first term is the Gaussian contribution to the sample variance and the second the non-Gaussian contribution. A realistic survey will also have shot noise variance due to the finite number of source galaxies in the survey. For a comparison of previous calculations, we take the same binning scheme as the one used in Cooray & Hu (2001b) and used in White & Hu (1999).

In figure 3, we show the ratio of \( R \equiv C_{ij}/C_{ij}^{\text{Gaus}} \) where \( C_{ij}^{\text{Gaus}} \) is the contribution with simply the Gaussian variance. This ratio can also be written as

\[ R \equiv 1 + \frac{A_{\text{xi}} T_{ij}^{\text{G}}}{(2\pi)^3 2C_{ij}^{\text{G}}}, \tag{29} \]

and we plot \( R - 1 \) to highlight the difference between source clustering and non-Gaussian aspect of large scale structure. As shown, the clustering only leads to a few percent contribution, at \( l \sim 10^3 \), beyond the Gaussian variance while the non-Gaussianities due to large scale structure clustering contributes at the level of 10% or more. One can safely ignore the relative increase in the variance of power spectrum measurements due to background source clustering.

An additional aspect of the covariance resulting from non-Gaussianities is that band power estimates are correlated. These correlations can be written as

\[ \tilde{C}_{ij} \equiv \frac{C_{ij}}{\sqrt{C_{ii} C_{jj}}}. \tag{30} \]

In figure 3 we show the behavior of the correlation coefficient between a fixed \( l_i \) as a function of three \( l_j \)'s. When \( l_i = l_j \) the coefficient is 1 by definition. Due to the presence of the dominant Gaussian contribution at \( l_i = l_j \), the coefficient has an apparent discontinuity between \( l_i = l_j \) and \( l_i \approx l_{j-1} \) that decreases as \( l_j \) increases and non-Gaussian effects dominate. As shown, however, the correlation coefficients due to the non-Gaussian nature of the large scale structure is over an order of magnitude larger than than the correlations resulting from the clustering of background sources. The results related to the covariance suggests that non-Gaussian effects resulting from the clustering of background sources is not expected to strongly influence the abilities of weak lensing experiment to obtain precision measurements of cosmology.

3 SUMMARY & CONCLUSIONS

We have discussed the second order contributions to weak gravitational lensing convergence resulting from the clustering of background sources from which galaxy shape measurements are made in weak lensing experiments. The clustering of source galaxies induce a second order contribution to the two point statistics such as weak lensing convergence angular power spectrum. For the angular scales of interest, we have shown that this contribution is at the level of few percent; to some extent, however, the exact contribution is uncertain due to unknown aspects associated with galaxy biasing such as the quadratic bias.

Our calculations related to the skewness generated by background source clustering is consistent with previous calculations by Bernard et al. (1998) and Hamana et al. (2001). We have discussed a new non-Gaussian aspect of the background source clustering involving a contribution to the four point correlation function of shear or the trispectrum in Fourier space. The non-Gaussian four-point function is of interest since it determines the covariance of power spectrum measurements. The background source clustering increases the Gaussian covariance at the level of few percent when \( l \sim 10^3 \). This increase, however, is an order of magnitude or more below the increase resulting from the intrinsic non-Gaussian nature of the large scale structure due to the non-linear evolution of gravitational perturbations. The trispectrum contribution to the covariance also leads to correlations between band power estimates, though, these are again at the few percent level or below and are unlikely to be a significant source of error for current and upcoming weak lensing experiments.

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REFERENCES

Bacon, D., Refregier, A., Ellis R. 2000, MNRAS, 318, 625
Bartelmann, M., Schneider, P. 2000, Physics Reports in press, astro-ph/9912508
Bernaldeau, F., van Waerbeke, L., Mellier, Y. 1997, A&A, 322, 1
Blandford, R. D., Saust, A. B., Brainerd, T. G., Villumsen, J. V. 1991, MNRAS 251, 60
Bunn, E. F., White, M. 1997, ApJ, 480, 6
Cooray, A. R. 1999, A&A, 348, 31
Cooray, A., Hu, W., Miralda-Escudé, J. 2000, ApJ, 536, L9
Cooray, A., Hu, W. 2000, ApJ, 534, 533
Cooray, A., Hu, W. 2001a, ApJ, 548, 7
Cooray, A., Hu, W. 2001b, ApJ, 554, 56
Eisenstein, D.J., & Hu, W. 1997, ApJ, 480, 6
Feldman, H. A., Frieman, J. A., Fry, J. N., & Scoccimarro, R. 2001, Phys. Rev. Lett. 86, 1434
Hu W., Tegmark M. 1999, ApJ, 514, L65
Jain B., Seljak U. 1997, ApJ, 484, 560
Kaiser, N. 1992, ApJ, 388, 286
Kaiser, N. 1998, ApJ, 498, 26
Kaiser, N., Wilson, G. & Luppino, G. A. 2000, ApJ, astro-ph/0003338.
Limber, D. 1954, ApJ, 119, 655
Mellier, Y. 1999, ARA&A, 37, 127.
Miralda-Escudé J. 1991, ApJ, 380, 1
Peacock, J.A., Dodds, S.J. 1996, MNRAS, 280, L19
Schneider P., van Waerbeke, L., Jain, B., Guido, K. 1998, MNRAS, 296, 873
Schneider P., van Waerbeke, L., Mellier, Y. 2002, A&A submitted astro-ph/0112441.
Scoccimarro, R. & Frieman, J. 1999, ApJ, 520, 35
van Waerbeke, L., Bernardeau, F., Mellier, Y. 1999, A&A, 342, 15
van Waerbeke, L., Mellier, Y., Erben, T. et al. 2000, A&A, 358, 30
Verde, L., Heavens, A. F., Percival, W. J. et al. MNRAS submitted astro-ph/0112161.
Viana, P. T. P., Liddle, A. R. 1999, MNRAS, 303, 535
White, M., Hu, W. 1999, ApJ, 537, 1
Wittman, D. M., Tyson, J. A., Kirkman, D., Dell’Antonio, I., Bernstein, G. 2000, Nature, 405, 143