Strong Diffusion Effect of Charm Quarks on $J/\psi$ Production in Pb-Pb collisions at the LHC

Jiaxing Zhao$^1$ and Baoyi Chen$^2$

$^1$Department of Physics and Collaborative Innovation Center of Quantum Matter, Tsinghua University, Beijing 100084, China
$^2$Department of Physics, Tianjin University, Tianjin 300350, China

(Dated: March 2, 2018)

We study the $J/\psi$ production based on coalescence model at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV Pb-Pb collisions. With the colliding energy increasing from 2.76 TeV to 5.02 TeV, the number of charm pairs is enhanced by more than 50%. However, the ratio of $J/\psi$ inclusive nuclear modification factors $R_{AA}^{2.76\text{TeV}}/R_{AA}^{5.02\text{TeV}}$ is only about 1.1 ~ 1.2. We find that the regeneration of $J/\psi$ is proportional to the densities of charm and anti-charm quarks, instead of their total numbers. The charm quark density is diluted by the strong expansion of quark gluon plasma, which suppresses the combination probability of heavy quarks and $J/\psi$ regeneration. This effect is more important in higher colliding energies where QGP expansion is strong. We also propose the ratio $N_{J/\psi}/(N_D)^2$ as a measurement of $c$ and $\bar{c}$ coalescence probability, which is only affected by the heavy quark diffusions in QGP, and does not depend on the inputs such as cold nuclear matter effects and cross sections of charm quark production. Further more, we give the predictions at the energy of Future Circular Collider ($\sqrt{s_{NN}} = 39$ TeV).

PACS numbers: 25.75.-q, 12.40.Yx, 14.40.Pq, 14.65.Dw

A new kind of matter called “Quark Gluon Plasma” (QGP) is believed to be produced in the relativistic heavy ion collisions $^1$. $J/\psi$ has been considered as a probe of this deconfined matter for more than thirty years $^2$. The color screening and parton inelastic scatterings in QGP can result in the abnormal suppression of $J/\psi$ production in heavy ion collisions $^3$ $^4$. The nuclear modification factor $R_{AA}$ is a measurement of the cold and hot medium effects on charmonium production. Cold nuclear matter effects include the nuclear absorption $^5$, Cronin effect $^6$ and shadowing effect $^7$ $^8$. The first one means that primordially produced charmonium from parton hard scatterings $^9$ suffer the inelastic scatterings with surrounding nucleons before they move out of the nucleus. This nucleus suppression (“normal suppression”) can be neglected at the energies of Large Hadron Collider (LHC). Partons may scatter with other nucleons to obtain extra energy before they fuse into a charm pair (or charmonium). This energy will be inherited by the primordial $J/\psi$ and shift their transverse momentum distribution. Cronin effect increases with the number of participants, and can be included by the modification of charmonium initial production from pp collisions. Parton distributions may also be affected by the surrounding nucleons especially at the LHC energies. This will change the yields of primordial charmonium and charm quark pairs.

With more and more experimental data published from Relativistic Heavy Ion Collider (RHIC) $^{12}$ $^{13}$ and LHC $^{14}$ $^{15}$, the nuclear modification factor is enhanced at higher colliding energies. This is due to the recombination of $c$ and $\bar{c}$ quarks in the QGP. At LHC, most of primordially produced $J/\psi$ are melt by QGP. The recombination of uncorrelated $c$ and $\bar{c}$ quarks dominates the $J/\psi$ final yield in nucleus-nucleus collisions, especially at the low $p_T$ region. For the $p_T$-integrated observables in semi-central and central collisions, one can safely neglect the primordial production and focus on the $J/\psi$ regeneration $^{16}$ $^{19}$.

As final $J/\psi$ are mainly from the coalescence of $c$ and $\bar{c}$ quarks in QGP, $J/\psi$ production is closely connected with the heavy quark evolutions in the expanding QGP. The elliptic flows of D mesons at $\sqrt{s_{NN}} = 2.76$ TeV is close to the value of light hadrons, which indicate a kinetic equilibrium of charm quarks before hadronization $^{20}$ $^{21}$. Charm quarks expand outside with the QGP, which reduces its density in phase space. The effect of charm quark diffusion in coordinate space becomes more important in the higher colliding energies where QGP expansion is strong $^{22}$. With higher initial temperature, QGP takes longer time to cool down where charm quarks will be distributed in a larger volume $^{23}$. Therefore, the ratio of $J/\psi$ nuclear modification factors $R_{AA}^{2.76\text{TeV}}/R_{AA}^{5.02\text{TeV}}$ is only 1.1 ~ 1.2, even the total number of charm pairs is enhanced by more than 50% from 2.76 TeV to 5.02 TeV Pb-Pb collisions.

At the hadronization, partons in the deconfined phase are transformed into hadrons. Coalescence model has been widely used to describe the hadronization process of light hadrons and heavy quarkonium $^{24}$ $^{29}$. Whether $c$ and $\bar{c}$ quarks form into a quarkonium bound state depends on both their relative coordinate and momentum, and also the wave functions of charmonium. We employ the non-relativistic Schrödinger equation to obtain charmonium wave functions due to the large mass of charm quarks,

$$\left[ \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(r_1, r_2) \right] \Psi = E \Psi \quad (1)$$

$m_{1,2}$ and $p_{1,2}$ are the mass and momentum of charm (or anti-charm) quark. $V(r_1, r_2)$ is the heavy quark potential. In the coalescence model, $J/\psi$ is regenerated at $T = T_c$ just like light hadrons. On the hadronization hypersurface, one can neglect the parton color screening effect on heavy quark potential $^{30}$, and take $V(r_1, r_2)$ to be the form of Cornell potential, $V(r_1, r_2) = -\alpha/r + \sigma r$.
with \( r = | \mathbf{r}_1 - \mathbf{r}_2 | \) to be the relative distance between \( c \) and \( \bar{c} \) quarks. The charm quark mass \( m_1 = m_2 \) and \((\alpha, \sigma)\) in Cornell potential are taken as parameters here, which can be fixed by fitting the mass of \( J/\psi \) in vacuum. We obtain \( m_1 = m_2 = 1.25 \text{ GeV} \) and \((\alpha = \pi/12, \sigma = 0.2 \text{ GeV}^2)\).

The potential in above 2-body Schrödinger equation depends on the relative distance of two quarks, we can separate the two-body system into a motion of the mass center of two quarks which is just an equation of free motion, and their relative motion which is controlled by the potential \( V(r) \). After introducing a global coordinate \( \mathbf{R} = (r_1 + r_2)/2 \) and relative coordinate \( \mathbf{r} = r_1 - r_2 \), we write the total wave function of two body systems as \( \Psi(\mathbf{R}, \mathbf{r}) = \Theta(\mathbf{R}) \psi(\mathbf{r}) \). Further more, the Cornell potential is an isotropic potential. One can write the equations of \( \psi(\mathbf{r}) \) into a radial part and an angular part, \( \psi(\mathbf{r}) = \varphi(r)Y(\theta, \phi) \). Now the two-body system is simplified as a one dimensional problem,

\[
\left[ -\frac{1}{2m_\mu} \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + V(r) + \frac{L(L + 1)}{2m_\mu r^2} \right] \varphi(r) = \varepsilon \varphi(r)
\]

where \( m_\mu = m_c/2 \) is the reduced mass. \( \varphi(r) \) is the radial wave function. For \( J/\psi \), the angular momentum quantum number is \( L = 0 \). The radial wave function is normalized as \( \int_0^\infty |\varphi(r)|^2 r^2 dr = 1 \).

Including the dependence of both relative distance \( \mathbf{r} \) and relative momentum \( \mathbf{p} \), one can write the Wigner function for \( c \) and \( \bar{c} \) quarks hadronization into a charmonium as \( 31, 32 \),

\[
W(\mathbf{r}, \mathbf{p}) = \int d^3y e^{-i\mathbf{p}\cdot\mathbf{y}} \left( \mathbf{r} + \frac{\mathbf{y}}{2} \right)^* \psi^*(\mathbf{r} - \frac{\mathbf{y}}{2})
\]

where \( \psi(\mathbf{r}) \) is the wave function of \( J/\psi \) obtained in Eq.(2). The Wigner function is normalized as \( \int_0^\infty W(\mathbf{r}, \mathbf{p}) d^3r d^3p = 1 \) in the nonrelativistic limit.

Before doing dynamical evolutions of heavy quarks, we give the realistic evolutions of QGP produced in Pb-Pb collisions. The QGP turns out to be a very strong coupling system, which can be described well by hydrodynamic equations \( 33-35 \). With the assumption of Bjorken expansion for QGP longitudinal expansion, we employ the 2+1 dimensional hydrodynamic equations to simulate QGP transverse expansion in Pb-Pb collisions at LHC energies,

\[
\partial_t T^{\mu\nu} = 0
\]

Here \( T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - g^{\mu\nu} p \) is the energy-momentum tensor, and \( c, p, u^\mu \) are the energy density, pressure and four velocity of fluid cells, respectively. For the equation of state, the deconfined matter is taken as an ideal gas of \( u, d, s \) quarks and gluons \( 36 \). The hadron phase is an ideal gas of all known hadrons and resonances with mass up to 2 GeV \( 37 \). There is a first order phase transition between two phases. In the mixed phase, Maxwell construction is used to obtain the values of variables in Eq.[4].

From the charge multiplicity, one can obtain the initial maximum temperature of QGP at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) and 5.02 TeV Pb-Pb collisions to be \( T_0 = 485 \text{ MeV} \) and \( T_0 = 510 \text{ MeV} \) in central rapidity bin in the most central collisions (impact parameter \( b = 0 \) 38). \( \tau_0 \) is the time where QGP reaches local equilibrium and starts transverse expansion. Its value at RHIC 200 GeV Au-Au collisions and 2.76 TeV Pb-Pb collisions are both \( \tau_0 = 0.6 \text{ fm}/c \) 39, showing weak dependence of the colliding energy \( \sqrt{s_{NN}} \). Therefore, \( \tau_0 = 0.6 \text{ fm}/c \) \( \sqrt{s_{NN}} \). The initial maximum temperature of QGP at \( \sqrt{s_{NN}} = 39 \text{ TeV} \) Pb-Pb collisions in the Future Circular Collider is extracted as \( T_0 = 650 \text{ MeV} \) 40, 41.

\( J/\psi \) production from coalescence of \( c \) and \( \bar{c} \) quarks is proportional to the densities of charm and anti-charm quarks and Wigner function \( 32 \),

\[
dN_{J/\psi} \frac{d^2P_T}{d^2\eta} = C \int d^3r d^3p \int d^4p (2\pi)^4 \frac{(2\pi)^3}{(2\pi)^3} W(r,p) f_c(r_1, p_1) f_c(r_2, p_2)
\]

where \( P^\mu = (P^0, \mathbf{P}) \) is the momentum of \( J/\psi \), \( P^0 = \sqrt{m_{J/\psi}^2 + \mathbf{P}^2} \). The constant \( C \) comes from the intrinsic symmetry with \( C = 1/12 \) for vector mesons like \( J/\psi \). \( f_c(r, p) \) is the charm quark density in phase space. The coalescence of charm and antiquark quarks happens on the hadronization hypersurface \( \sigma_\mu(R) \), where the coordinates \( R_\mu = (t, \mathbf{r}) \) on the hypersurface is constrained by the hadronization condition: \( T(R_\mu) = T_c \).

In \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) Pb-Pb collisions, the large elliptic flow of D mesons supports the kinetic thermalization of charm quarks in QGP 20. However, how and when charm quarks reach kinetic thermalization still deserve more quantitative studies \( 43, 44 \). Instead of doing realistic evolutions of charm quarks which introduce additional parameters, for simplicity, we assume an instant kinetic thermalization of charm quarks at \( \tau = \tau_0 \) just like light partons. The situation of charm quark non-thermalization will be discussed in Fig.4 and does not change our main conclusions. The momentum distribution of charm quarks is

\[
f_c(r, p) = \rho_c(r) \frac{N_{\text{norm}}(r)}{e^{u^\mu(x)p_\mu/T(r)}} + 1
\]

where \( u^\mu(x) \) and \( T(r) \) are the four velocity and local temperature of QGP. \( N_{\text{norm}}(r) \) is the normalization factor. \( \rho_c(r) \) is charm quark spatial density. As charm quark mass is very large, it can hardly reach chemical equilibrium in the QGP with a typical temperature of 0.2 ~ 0.5 GeV. The total number of charm pairs is conserved. With the assumption of charm quark kinetic equilibrium in QGP, its diffusion in coordinate space can be expressed as a conservation equation,

\[
\partial_t (\rho_c u^\mu) = 0
\]

Obviously, the diffusion of charm quarks in coordinate space depends on the velocity \( u^\mu \) of QGP. For the input
of Eq.\,(7), as charm pairs are produced mainly from the parton hard scatterings, its initial distribution in coordinate space is

\[ \rho_c(\tau_0, x_T, \eta) = \frac{T_A(x_T)T_B(x_T - b) \cosh \eta d\sigma^{c\bar{c}}_{pp}}{\tau_0} \]

where \(T_A(B)\) is the thickness function of the nucleus A(B). \(b\) is the impact parameter, and \(d\sigma^{c\bar{c}}_{pp}/d\eta\) is the charm pair differential cross section with rapidity in proton-proton collisions. Note that in the colliding energies as LHC, the strong shadowing effect will suppress the initial number of charm pairs and change the Eq.\,(8). At \(\sqrt{s_{NN}} = 2.76\) TeV and 5.02 TeV, the shadowing effect will reduce around 20\% of total charm yields [47]. This will suppress the yield of regenerated \(J/\psi\) by 36\%. However, it does not affect the ratio of \(J/\psi\) yield over the square of D meson yield in Pb-Pb collisions, \(N_{J/\psi}/(N_D)^2\). We propose this observable as a mean probability of charm and anti-charm quark combination in the expanding QGP. It will be discussed in details later.

In order to quantitatively calculate the inclusive nuclear modification factor of \(J/\psi\) to compare with the experimental data, one need the cross sections of \(J/\psi\) and charm pairs in pp collisions. At \(\sqrt{s_{NN}} = 2.76\) TeV, ALICE Collaboration published the \(J/\psi\) inclusive cross section to be \(d\sigma_{J/\psi}^{c\bar{c}}/dy = 2.3 \, \mu$b at 2.5 < \(|y| < 4\) and \(4.1 \, \mu$b at \(|y| < 0.9\) [48]. At \(\sqrt{s_{NN}} = 5.02\) TeV, we take \(d\sigma_{J/\psi}^{c\bar{c}}/dy = 3.65 \, \mu$b at 2.5 < \(|y| < 4\) [49]. The \(p_T\)-dependent cross sections of inclusive \(J/\psi\) in pp collisions at \(\sqrt{s_{NN}} = 2.76\) TeV and 5.02 TeV are shown in Fig.1. It can be parameterized as [50],

\[ \frac{d\sigma_{J/\psi}^{c\bar{c}}}{2\pi d^2p_Tdp_T} = \frac{2(n - 1)}{2\pi(n - 2)(p_T^2)_{pp}} [1 + (n - 2)(p_T^2)_{pp}]^{\text{\(n\)}} \times \frac{d\sigma_{J/\psi}^{c\bar{c}}}{dy} \]

where \(n = 4.0\) and \((p_T^2)_{pp} = 7.8 \, (\text{GeV/c})^2\) at forward rapidity for \(\sqrt{s_{NN}} = 2.76\) TeV. At 5.02 TeV, The parameters are \(n = 3.9\) and \((p_T^2)_{pp} = 8.7 \, (\text{GeV/c})^2\) at forward rapidity.

For the charm pair production cross sections at \(\sqrt{s_{NN}} = 2.76\) TeV and 5.02 TeV pp collisions, they are taken as \(d\sigma^{c\bar{c}}_{pp}/dy(2.76\, \text{TeV}) = 0.33 \, \text{mb}\) and \(d\sigma^{c\bar{c}}_{pp}/dy(5.02\, \text{TeV}) = 0.57 \, \text{mb}\) at forward rapidities to explain the experimental data. Those values are consistent with the inputs of transport models [52, 53]. We also shift their values upward by 20\% to consider the uncertainties of charm production cross sections.

With the information of charm quark evolutions in QGP and also the coalescence probability of \(c\) and \(\bar{c}\) quarks which is connected with the wave function of produced particles (see Eq.\,(9)), we can calculate the \(J/\psi\) production at the temperature \(T = T_c\) of the phase transition with Eq.\,(5). The experimental data is for the \(J/\psi\) inclusive nuclear modification factor, which includes the decay contributions from B hadrons. The \(J/\psi\) from B hadron decays, labeled as “non-prompt \(J/\psi\)”, contribute around 10\% in the total inclusive yields. This fraction \(N_{B-J/\psi}/N_{\text{inclusive}}\) almost does not depend on the colliding energy [10]. With the prompt and non-prompt \(J/\psi\), one can write the inclusive nuclear modification factor as,

\[ R_{AA} = \frac{N_{c\bar{c} + J/\psi} + N_{B-J/\psi}}{N_{\text{coll}}N_{pp}} \]

where the first and second term in the numerator are \(J/\psi\) production from coalescence of \(c\) and \(\bar{c}\) quarks and B hadron decay. \(N_{\text{coll}}\) and \(N_{\text{inclusive}}\) are the number of binary collisions and \(J/\psi\) inclusive yield in pp collisions, which can be obtained from the integration of Eq.\,(9).

Besides the coalescence of heavy quarks and decay from B hadrons, \(J/\psi\) may also come from the parton hard scatterings just like charm quarks at the nucleus colliding time \(\tau = 0\). This is called “primordial production”. In the LHC colliding energies, the initial maximum temperature of QGP is around \(T \approx 3T_c\), which is far above the \(J/\psi\) maximum survival temperature \(T_d J/\psi \sim 1.2T_c - 2T_c\). The lower and upper limits of \(T_d \) correspond to the heavy quark potential to be \(V = F\) (free energy) and \(V = U\) (internal energy). In both situations, most of the primordially produced \(J/\psi\) will be dissociated in QGP. In the peripheral collisions, the initial temperature of the produced QGP becomes smaller. The primordial \(J/\psi\) may survive from hot medium and even dominate the final inclusive yields. Therefore, we perform our calculations at \(N_p \geq 100\) where the initial maximum temperature of QGP is \(T_d \text{QGP}(N_p = 100) > 2T_c \geq T_d \).

The \(J/\psi\) inclusive nuclear modification factors at 2.76 TeV and 5.02 TeV are plotted in Fig.2. From the definition of \(R_{AA}\), its value is proportional to the parameters of \((\sigma^{c\bar{c}}_{pp})^2/\sigma_{pp}^{J/\psi}\). One can expect a similar enhancement of \(J/\psi\) cross section from 2.76 TeV to 5.02 TeV just like \(ge^{c\bar{c}}_{pp}\), please see the cross sections in Section III. Therefore, the change of the \(J/\psi\) and \(cc\) production cross sections will make \(R_{AA}^{2.76\, \text{TeV}}/R_{AA}^{5.02\, \text{TeV}} \sim 1.8\) depending on the exact values of these cross sections (see the blue dashed line in Fig.3). It means, if we employ the same QGP
expansions in Pb-Pb collisions at 5.02 TeV as 2.76 TeV for \(J/\psi\) regeneration, \(J/\psi\) \(R_{AA}\) will be enhanced by 80\% at 5.02 TeV, due to the larger charm and \(J/\psi\) cross sections. However, with the realistic simulations of QGP expansion at 5.02 TeV, \(J/\psi\) \(R_{AA}\) is only enhanced by around 10\%, and the difference of 70\% is due to the stronger diffusions of charm quarks in QGP(5.02 TeV) than in QGP(2.76 TeV). As we point out above, the \(J/\psi\) production is the integration of charm quark density \(f_c(r,p)\), which is controlled by Eq.\(\text{[7]}\) and depends on the evolutions of QGP. The expansion of QGP will affect the dilution of charm quark density. Especially, the transverse expansion of QGP is an accelerating process. Charm quarks can be “blown” to a larger volume when QGP hadronize, which will strongly suppress the charm density at the hadronization hypersurface where \(J/\psi\) is produced from coalescence process. Therefore, the ratio of \(R_{AA}^{5.02\text{TeV}}/R_{AA}^{2.76\text{TeV}}\) is only 1.1 ~ 1.2 in semi-central and central collisions, see the solid line in Fig.\(\text{[3]}\) Note that in another philosophy, where \(J/\psi\) is believed to be produced in a temperature region \(T_c < T < T_{d/\psi}\), the strong diffusion of charm quarks in coordinate space still suppress the \(J/\psi\) regeneration.

**FIG. 2:** (Color online) \(J/\psi\) inclusive nuclear modification factor \(R_{AA}\) as a function of the number of participants \(N_p\) at the forward rapidity in Pb-Pb collisions at \(\sqrt{s_{NN}} = 2.76\text{TeV}\) and 5.02 TeV. Lower and upper limits of the bands correspond to the lower and upper limits of charm quark cross sections in pp collisions. Experimental data are from the ALICE Collaboration \[15, 19\].

At LHC colliding energies, the final \(J/\psi\) are mainly from the coalescence of \(c\) and \(\bar{c}\) quarks. The large uncertainty of charm quark production cross sections results in large uncertainty of theoretical calculations in transport models \[16, 17\] and coalescence models \[52\]. It would be better if we can find an observable to describe the effects of charm quark diffusion on charmonium production in the expanding QGP. In equation Eq.\(\text{[6]}\) we move the cross sections of charm quark production to the left hand side. The new observable \(N_{J/\psi}/(N_{D})^2\) does not depend on the cross sections of \(J/\psi\) and charm pairs in pp collisions. Further, it does not depend on the shadowing effect, which can reduce the number of charm quarks. Here D meson number equals to charm quark number \(N_D = N_c\), independent of coalescence or fragmentation for \(c \rightarrow D\). Note that \(J/\psi\) regeneration process reduces only < 1\% of total charm numbers and does not affect the relation of \(N_D = N_c\). From the formula of \(N_{J/\psi}/(N_{D})^2\), it is mainly determined by the \(J/\psi\) wavefunction and the evolutions of charm quarks which includes the information of bulk medium expansions. The combination probability of one \(c\) and \(\bar{c}\) quark becomes smaller if QGP expansion is stronger. The value of \(N_{J/\psi}/(N_{D})^2\) decreases with \(N_p\) and the colliding energy \(\sqrt{s_{NN}}\) in Pb-Pb collisions, see the lines in Fig.\(\text{[4]}\).

In the above calculations, we assume that charm quarks reach kinetic equilibrium at \(\tau_0\) for simplicity when QGP starts transverse expansion. However, from realistic dynamical evolutions of heavy quarks in the heavy ion collisions \[55\], charm quarks may be thermalized in a few fm/c at the LHC energies. We assume a free motion for charm quarks before the time scale \(\tau_c = 4\text{ fm/c}\). After this time scale, charm quark momentum reaches kinetic equilibrium and their motion is controlled by Eq.\(\text{[6]}\). Non-thermalization effect does not change our conclusions that \(J/\psi\) regeneration is suppressed by the spatial diffusion of charm quarks in the expanding QGP, see the lower panel of Fig.\(\text{[4]}\).

The strong diffusion of charm quarks also affects the shape of the transverse momentum distribution of \(J/\psi\) \(R_{AA}\). In Fig.\(\text{[5]}\) we calculate the \(J/\psi\) inclusive \(R_{AA}(p_T)\) in the centrality 0-20\% in forward rapidity. Absent of the primordial production which dominate the inclusive yield in high \(p_T\) region, we underestimate the \(R_{AA}\) at \(p_T > 4\text{ GeV/c}\). In the region of \(p_T < 4\text{ GeV/c}\), both experimental data and theoretical calculations show a “shift” behavior of \(R_{AA}\) toward larger \(p_T\) region from...
2.76 TeV to 5.02 TeV. In $\sqrt{s_{NN}} = 5.02$ TeV, the expansion of QGP is stronger, which pushes charm quarks to a larger transverse momentum region. $R_{AA}^{5.02 \text{TeV}}$ is almost the same with $R_{AA}^{2.76 \text{TeV}}$ at $p_T \approx 1$ GeV/c, but $R_{AA}^{5.02 \text{TeV}}$ is enhanced at $p_T \approx 3$ GeV/c due to the larger velocity of fluid cells in 5.02 TeV.

In summary, we employ the coalescence model to study $J/\psi$ production from the combination of charm quarks at the hadronization of QGP in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and 5.02 TeV. From the comparison of charm quark cross sections and $J/\psi$ inclusive $R_{AA}$ at these two colliding energies, we find that even the number of charm quarks is enhanced by more than 50% from 2.76 TeV to 5.02 TeV, the ratio $R_{AA}^{5.02 \text{TeV}} / R_{AA}^{2.76 \text{TeV}}$ is only 1.1 $\sim$ 1.2. $J/\psi$ production is connected with the number of charm pairs and also their diffusions in the expanding QGP. In higher colliding energies such as $\sqrt{s_{NN}} = 5.02$ TeV and 39 TeV, the initial energy density of QGP becomes larger, and QGP expansion lasts longer compared with the situation of 2.76 TeV. Strong diffusion of charm quarks in QGP reduces the probability of $c$ and $\bar{c}$ quark coalescence at the hadronization hypersurface ($T = T_c$). This effect also shifts the produced $J/\psi$ to a larger $p_T$ region, see $R_{AA}(p_T)$ in Fig. Further, We propose an observable of $N_{J/\psi}/(N_D)^2$ as a measurement of charm quark coalescence probability in QGP. It does not de-pend on the charm quark cross sections and cold nuclear matter effects (such as shadowing effect). It is dominated by the evolution history of charm quarks in QGP and also the wave function of the produced particle ($J/\psi$), which makes it a clean probe to study the charm quark evolutions and $J/\psi$ regeneration in heavy ion collisions.

Acknowledgement: B. Chen is supported by NSFC under Grant No. 11547043. J. Zhao is supported by NSFC and MOST under Grant Nos. 11335005, 11575093, 2013CB922000 and 2014CB8454000.

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