Diffractive $\eta_c$ and $\eta_b$ productions by neutrinos via neutral currents

A. Hayashigaki(1), K. Suzuki(2) and K. Tanaka(3)

1. Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
2. Division of Liberal Arts, Numazu College of Technology, Shizuoka 410-8501, Japan
3. Department of Physics, Juntendo University, Inaba-gun, Chiba 270-1695, Japan

(Dated: December 3, 2018)

We report a first theoretical study for neutrino-induced diffractive productions of heavy pseudoscalar mesons, $\eta_c$ and $\eta_b$, off a nucleon. Based on factorization formalism for exclusive processes, we evaluate the forward diffractive production cross section in perturbative QCD in terms of the light-cone $Q\bar{Q}$ wave functions (WFs) of $\eta_c, b$ mesons and the gluon distribution of the nucleon. The light-cone WFs of the $\eta_c$ ($\eta_b$) meson are constructed to satisfy the spin symmetry relations with those of the $J/\psi$ (T) meson. The diffractive $\eta_c$ production is governed by the axial-vector coupling of the longitudinally polarized $Z$ boson to $Q\bar{Q}$ pair, and the resulting $\eta_c$ production rate is larger than the $J/\psi$ one by one order of magnitude. We also discuss the production of bottomonium $\eta_b$, which shows up for higher beam energy.

PACS numbers: 12.38.Bx, 12.39.St, 13.60.Le, 14.70.Hp

Exclusive diffractive leptonproductions of neutral vector mesons provide unique insight into an interplay between nonperturbative and perturbative effects in QCD. The diffractive processes are mediated by the exchange of a Pomeron with the vacuum quantum numbers, whose QCD description is directly related to the gluon distributions inside the nucleons for small Bjorken-$x$. The processes also allow us to probe the light-cone wave functions (WFs) of the vector mesons. Relating to the latter point, however, the applicability is apparently limited to probing the neutral vector mesons.

In this Letter, we propose the exclusive diffractive productions of mesons in terms of the neutrino beam. The weak currents allow us to observe both neutral ($M^0$) and charged mesons ($M^\pm$) as $\nu_{\mu} + N \rightarrow \nu_{\mu}/\bar{\nu}_{\mu} + N + M^0/ M^\pm$ by $Z/W$ boson exchange. The mesons can be not only vector but also other types of mesons including pseudoscalar mesons. Thus, such processes may reveal structure of various kinds of mesons, the coupling of the QCD Pomeron to quark-antiquark pair with various spin-flavor quantum numbers, and information on the CKM matrix elements. There already exist some experimental data for $\pi, \rho, D^+_s, D_s^0, D_s^{*0}$ and $J/\psi$ production, but there are only a few theoretical calculations, e.g., for the $J/\psi$ production in a vector meson dominance model and for $D_s^-$ production with the generalized parton density. Our work gives a first QCD calculation for diffractive meson productions via the weak neutral current. Here, our interest will be directed to productions of heavy pseudoscalar mesons, especially $\eta_c$ and $\eta_b$ as in Fig. 1. So far $\eta_c$ has been observed via the decays of $J/\psi$ or $B$ mesons produced by $p\bar{p}$ and $e^+e^-$ reactions, while $\eta_b$ has not been observed. The diffractive productions via the weak neutral current will give a direct access to $\eta_c$ as well as a new experimental method to identify $\eta_b$ by e.g. measuring the two photon decay.

![Diagram of exclusive diffractive $\eta_Q$ production](image)

FIG. 1: A typical diagram for the exclusive diffractive $\eta_Q$ ($Q = c, b$) productions induced by neutrino ($\nu$) through the $Z$ boson exchange. There are other diagrams by interchanging the vertices on the heavy-quark lines.

We treat the $\eta_c$ and $\eta_b$ productions by generalizing the approach in the leading logarithmic order of perturbative QCD, which has been developed successfully for the vector meson electroproductions. We consider the near-forward diffractive productions $Z^* (q) + N(P) \rightarrow \eta_Q(q + \Delta) + N'(P - \Delta)$, where $Q = c, b$, and $q, q + \Delta, P$ and $P - \Delta$ denote the momenta of the virtual $Z$ boson, $\eta_Q$ meson, initial and final nucleons, respectively. The total center-of-mass energy $W = \sqrt{(P + q)^2}$ is much larger than any other mass scales involved, i.e., $W^2 \gg K^2$ with $K^2 = Q^2, -t, m_\eta^2, \Lambda_{QCD}^2, \ldots$, where $Q^2 = -q^2$, $t = \Delta^2$, and $m_\eta$ is the heavy-quark mass. We also suppose $-t \ll m_\eta^2$ and $\Lambda_{QCD}^2 \ll m_\eta^2$. The heavy quark mass $m_\eta$ ensures that perturbative QCD can be applied even for $Q^2 = 0$ to calculate the creation of the $Q\bar{Q}$ pair as well as its time development before the nonperturbative effects to form a quarkonium state $\eta_Q$ set in. The crucial point is that at high $W$ the scattering of the $Q\bar{Q}$ pair on the nucleon occurs over a much shorter timescale than the $Z^* ightarrow Q\bar{Q}$ fluctuation or the $Q\bar{Q} \rightarrow \eta_Q$ formation times (see Fig. 1). As a result, the production amplitudes obey...
factorization in terms of the $Z$ and $\eta_Q$ light-cone WFs. The $QQ$-$N$ elastic scattering amplitude, sandwiched between the $Z$ and $\eta_Q$ WFs, is further factorized into the $QQ$-gluon hard scattering amplitude and the nucleon matrix element corresponding to the (unintegrated) gluon density distribution. This latter step in the factorization to get the gluon distribution can be carried out in the same manner as in the previous works \[3,4,5\]. The participation of the “new players” $Z$ and $\eta_Q$ in the former step requires an extension of the previous works by introducing the corresponding light-cone WFs.

First of all, we discuss the extension due to the participation of the $Z$ boson. The $ZQ$ weak vertex of Fig. 3 is given by $(g_W/2\cos\theta_W)\gamma_\mu(c_V - c_A\gamma_5)$, where $(g_W^2/2\cos\theta_W)^2 = 2G_F M_Z^2$ with $G_F$ the Fermi constant and $M_Z$ the $Z$ mass. $c_V = 1/2 - (4/3)\sin^2\theta_W$, $c_A = 1/2$ for the $c$-quark and similarly for the $b$-quark. As usual, we introduce the two light-like vectors $q'$ and $p'$ by the relations $q = q' - (Q^2/s)p'$, $P = p' + (M_N^2/s)q'$, $q'^2 = p'^2 = 0$, $s = 2q' \cdot p'$ with $M_N$ the nucleon mass and $s \equiv W^2 + Q^2$, and the Sudakov decomposition of all momenta, e.g., $k = \alpha q' + \beta p' + k_\perp$. We also introduce the longitudinal momentum fraction $\alpha$ and the transverse momentum $k_\perp$ as $k = \alpha q' + \beta p' + k_\perp$, $k_\perp^2 = -k_\perp^2$. Obviously, the light-cone WFs for the virtual $Z$ boson can be obtained from eq. (2) by the replacement $e_Q\gamma_\mu \rightarrow (g_W/2\cos\theta_W)\gamma_\mu(c_V - c_A\gamma_5)$ as

$$
\Psi^{Z(\xi)}(\alpha, k_\perp) = \frac{g_W}{2\cos\theta_W} \sqrt{\frac{\alpha(1-\alpha)\bar{u}_\lambda(k)\delta(\xi) (c_V - c_A\gamma_5) v_\lambda(q-k)}{\alpha(1-\alpha)Q^2 + k_\perp^2 + m_Q^2}}.
$$

Next we proceed to the light-cone WFs for the $\eta_Q$ meson, corresponding to Fig. 2(b). Again, it is convenient to exploit the correspondence with the $J/\psi$ electroproductions. The light-cone WFs for the heavy vector mesons $V = J/\psi$, $\Upsilon$ have been discussed in many works, but are still controversial in the treatment of subleading effects like the Fermi motion corrections \[6,7,8\], corrections to ensure the pure $S$-wave $QQ$ state \[9,10\], corrections via the Melosh rotation \[11\], etc. Here we employ the light-cone WFs for the vector meson given by (see Fig. 2(b))

$$
\Psi^{\Upsilon(\xi)}(\alpha, k_\perp) = \frac{\bar{u}_\lambda(\tilde{q} - \tilde{k})}{\sqrt{1 - \alpha}} \epsilon_0^\mu \epsilon_1^\nu R \gamma_\lambda(k) \phi^\star(\alpha, k_\perp) \frac{M_V}{M_V},
$$

where $\tilde{q} = q + \Delta$, $M_V$, and $\epsilon_0^\mu(\omega = 0, \pm 1)$ are momentum, mass, and the polarization vector of the vector meson with $q^2 = M_V^2$, $\epsilon_0(\omega = 0, \pm 1)$, and $\epsilon_0^\mu\epsilon_1^\nu = -\delta_{\mu\nu}$. $R = (1 + \tilde{q}/M_V)/2$ denotes the projection operator, $R^2 = R$, to ensure the $S$-wave $QQ$ state in the heavy-quark limit \[12,13,14\]. (This projection operator coincides with that discussed in Ref. \[15\], up to the binding-energy effects of the quarkonia.) Note that eq. (4) reduces to the vector-meson WFs of Ref. \[12\] by the replacement $R \rightarrow 1$. The physical interpretation of eq. (4) is similar to the “perturbative” WFs \[6,8,9\] and \[12\], but the scalar function $\phi(\alpha, k_\perp)$ contains nonperturbative dynamics between $Q$ and $\bar{Q}$. Now, the light-cone WFs for the $\eta_Q$ meson can be derived from eq. (1) utilizing spin symmetry, which is exact in the heavy-quark limit. This symmetry relates the $S$-wave states, $\eta_Q$ and the three spin states of the vector meson \[16\]. Namely, $\eta_{Q0} = M_V$, and the pseudoscalar state is related to the vector state with longitudinal polarization as $|\eta_Q\rangle = 2S_Q^3 |V(\omega = 0)\rangle$, where $S_Q^3$ is the third component of the hermitean spin operator $\hat{S}_Q$ which acts on the spin of the heavy quark $Q$ but does not act on $\bar{Q}$. For the WFs we get (see eq. (1))

$$
\Psi^{\eta_Q(\xi)}(\alpha, k_\perp) = \frac{\bar{u}_\lambda(\tilde{q} - \tilde{k})}{\sqrt{1 - \alpha}} \epsilon_0^\mu \epsilon_1^\nu R (2S_Q^3) \frac{u_\lambda(k) \phi^\star(\alpha, k_\perp) M_V}{\sqrt{\alpha}}.
$$

Here $S^3$ is a matrix repesentation of $S_Q^3$ as $S^3 = \gamma_5 \sigma^0 (2M_V)$, which is related to a spin matrix $\sigma^0/2 = \gamma_5 \gamma^0 \gamma^3/2$ in the meson rest frame. Thus the $\eta_Q$ is described by the
same nonperturbative WF $\phi(\alpha, k_\perp)$ as the vector meson. Note that, due to the presence of $R$, the “$Q\bar{Q}q\bar{q}$ vertex” involves pseudovector as well as pseudoscalar coupling.

Combining our $Z$ and $q\bar{q}$ WFs with the $Q\bar{Q}-N$ elastic amplitude which was obtained in Ref. [3], we get the total amplitude for the polarization $\varepsilon^{(\xi)}$ of the $Z$ boson as

$$\text{Im}M^{(\xi)} = \frac{\sqrt{2} g_s}{2\sqrt{\alpha}} \sum_{\lambda_\Lambda} \int d^2 k_\perp \frac{\alpha_s(t^2_\perp)}{t^2_\perp} \int d^2 l_\perp \psi^{(\xi)}(\alpha, k_\perp) f(x, l^2_\perp) \left[ 2\psi^{(\xi)}(\alpha, k_\perp - l_\perp) \right]$$

up to the terms suppressed for high $W$ and $\Delta^2 \approx -t \to 0$. Here $\alpha_s(t^2_\perp)$ denotes the running coupling constant for $N_c$ colors. As usual, we have explicitly dealt with the imaginary part of the production amplitude, because the small real part can be reconstructed perturbatively (see eq. 5) below. $f(x, l^2_\perp)$ denotes the unintegrated gluon density, and $x = (Q^2 + M^2_{q\bar{q}})/s$ and $l_\perp$ are the longitudinal momentum fraction and the transverse momentum, respectively, which are carried by a gluon inside the nucleon [3]. We substitute eqs. 4 and 5 into eq. 6, and go over to the “$b$-space” conjugate to the $k_\perp$-space via the Fourier transformation

$$\phi(\alpha, k_\perp) = \pi^2 \int d^2 k_\perp e^{-i k_\perp \cdot b} \phi(\alpha, b); \quad b \text{ denotes the transverse separation between } Q \text{ and } \bar{Q}. \quad \text{Then, it is straightforward to see } \text{Im}M^{(\pm 1)} = 0 \text{ which reflects helicity conservation in the high energy limit}. \quad \text{Im}M^{(0)} \text{ can be calculated in parallel with previous works for the vector meson production [3, 6, 7, 13]. In the integrand, there appear the terms } e^{1/2} b \text{ and } e^{-1/2} b \text{. The leading } \ln(Q^2/\Lambda^2_{QCD}) \text{ contribution comes from the region } \Lambda^2_{QCD} \ll l^2_\perp \ll Q^2_{\text{eff}}(\equiv [Q^2 + M^2_{q\bar{q}}]/4) \text{ of the } l^2_\perp \text{-integral [3, 6, 13], where } l_\perp \cdot b \ll 1 \text{ is satisfied because } |b| \sim 1/\Lambda_{QCD}. \text{ We retain only the leading nonzero term in the power series in } l_\perp \cdot b \text{, which corresponds to the “color-dipole picture” [3]. Then, using }

$$iM^{(0)} = \frac{-\sqrt{2} g_s}{\sqrt{\Lambda^2_{QCD}}} \sum_{\lambda_\Lambda} \frac{M_{q\bar{q}}}{Q} \cos \theta_W \alpha_s(Q^2_{\text{eff}}) \left[ 1 + \frac{i \pi}{2 \Lambda_{QCD}} \right] \int dG(x, Q^2_{\text{eff}})$$

where $Q_m = [\alpha(1 - \alpha)]^{1/2}, K_1$ is a modified Bessel function, and we have included the real part of the amplitude as a perturbation [3, 6, 13]. As expected, the result 6 is proportional to $c_A$, so that the $q\bar{q}$ meson is generated by the axial-vector part of the weak current.

For comparison, we also calculate the diffractive vector meson production via the weak neutral current, by the replacement $\psi^{(q\bar{q})}_{\lambda\lambda'} \rightarrow \psi^{(V,\pi)}_{\lambda\lambda'}$ in eq. 4. Substituting eq. 4, we find that $\xi = \pi$, i.e., $M^{(0)}$ and $M^{(\pm 1)}$ give the production of the longitudinally and transversely polarized vector mesons, respectively, and that all these amplitudes are proportional to the vector coupling $c_V$. (When we replace the factor $g_s c_V/(2 \cos \theta_W)$ by $c_V = 2 \epsilon/3$ in these results, we obtain the amplitudes $M^{(\xi)}$ identical to those for the diffractive electroproduction of $J/\psi$ [4, 5, 19], up to the corrections due to $R$ of eq. 4.)

Combining eq. 4 with the $Z$ boson propagator and the weak neutral current by a neutrino, the forward differential cross section for the $q\bar{q}$ production is given by

$$d^3\sigma(\nu N \rightarrow \nu' N'q\bar{q}) = \frac{1}{4(\pi)^3 E^2_{\nu} M^2_{Z} \cos \theta_W}$$

where $E_\nu$ is the neutrino beam energy in the lab system, and $\epsilon = [4(1 - y) - Q^2/E^2_\nu]/[2\{1 + (1 - y)^2 + Q^2/E^2_\nu\}]$ with $y = s/(2M_N E_\nu)$ is the polarization parameter of the virtual $Z$ boson [4, 19]. In order to evaluate the corresponding elastic $q\bar{q}$ production rate, we assume the $t$-dependence as $d^3\sigma/dsdQ^2dt = d^3\sigma/dsdQ^2dt|_{t=0} \exp(B_{\eta}t)$ with a constant diffractive slope, as in the case of the vector meson production. Integrating $d^3\sigma/dsdQ^2 = (1/B_{\eta})d^3\sigma/dsdQ^2dt|_{t=0}$ over $Q^2$ and $s$, we get the elastic production rate $\sigma(\nu N \rightarrow \nu' N'q\bar{q})$ as a function of $E_\nu$. For numerical computation of $\sigma(\nu N \rightarrow \nu' N'q\bar{q})$, we need explicit form of $\phi(\alpha, b)$ of eq. 4. From eqs. 3 and 4, we can use the corresponding nonperturbative part of the vector-meson WF which was constructed in the previous works based on, e.g., non-relativistic potential model for heavy quarkonia [4, 6, 13]. In this work, we adopt the WF from the Cornell potential model with the corresponding quark masses $m_c = 1.5$ GeV, $m_b = 4.9$ GeV [20]. A procedure to construct $\phi(\alpha, k_\perp)$ with the light-cone variables from the usual Schrödinger WF $\phi_{NR}(k)$ has been given in Refs. [4, 6, 13]. Also, we use the empirical values for the masses $M_{c\bar{c}} = 2.98$ GeV, $M_{b\bar{b}} = 0.94$ GeV and $M_{Z} = 91.2$ GeV. For $\eta_0$, we use $M_{\eta_0} = 9.45$ GeV estimated in Ref. [21]. Because the slope $B_{q\bar{q}}$ introduced above is unknown, we assume that $B_{q\bar{q}}$ has the same value as that for the corresponding vector meson: $B_{c\bar{c}} = 4.5$ GeV$^{-2}$ from the experimental value for $J/\psi$ [22] and $B_{b\bar{b}} = 3.9$ GeV$^{-2}$ following the value for $\Upsilon$ in Ref. [23]. For the gluon distribution $G(x, Q^2_{\text{eff}})$ of eq. 4, we employ GRV95 NLO parameterization [24].

We show the elastic $\eta_0$ production rate, $\sigma(\nu N \rightarrow \nu' N'\eta_0)$, by the solid curve in Fig. 3. The result monotonically increases as a function of the beam energy $E_\nu$. Such behavior is similar with that observed in the $\pi$-production data by the neutral [6] and charged currents [3]. For comparison, we show the elastic $J/\psi$ production rate $\sigma(\nu N \rightarrow \nu' N'J/\psi)$ by the dashed curve (see the discussion below eq. 4) [24]. The rate for $\eta_0$ production is much larger than that for $J/\psi$ by a factor $\sim 20$. This is mainly due to the relevant weak couplings, $c_A$ for $\eta_0$ and $c_V$ for $J/\psi$, as $(c_A/C V)^2 \approx 7$. Another important effect comes from the different be-
behavior of the $Z$ light-cone WFs \cite{5b} between the axial-vector and vector channels: It is straightforward to see that the $\eta_c$ diffractive amplitude \cite{5} as well as the corresponding amplitudes for the longitudinally and transversely polarized $J/\psi$ decreases rather rapidly for high $Q^2$ region, $Q^2 \gtrsim 4m_c^2$, so that the elastic production rates are in principle dominated by the low $Q^2$ contribution of the differential cross sections (similar behavior is observed also for $J/\psi$ electroproductions \cite{5b}). For weak neutral current processes, however, a typical factor $Q^2/(Q^2 + M_Z^2)^2$ appears in the differential cross sections as in eq. \cite{5b}. The low $Q^2$ contribution of the differential cross section for $J/\psi$ is suppressed by this factor, while that for $\eta_c$ avoids the suppression due to $\sim 1/Q$ behavior of eq. \cite{5a}. This particular behavior of the $\eta_c$ diffractive amplitude comes from the spinor matrix element of the $Z$ light-cone WFs \cite{5b} as $\bar{u}_\lambda(k)\gamma_\nu\gamma_\lambda(q-k)\sim 1/Q$ for $Q \to 0$. On the other hand, $\bar{u}_\lambda(k)\gamma_\nu\gamma_\lambda(q-k)\sim Q$ and $\bar{u}_\lambda(k)\gamma_\nu\gamma_\lambda(q-k)\sim \text{const}$, for the vector channels relevant for $J/\psi$.

In Fig. 3 we also show the $\eta_b$ production rate $\sigma(\nu N \rightarrow \nu' N' \eta_b)$ by the dotted curve. Although the rate for $\eta_b$ is generally much smaller than that for $\eta_c$, the former increases more rapidly than the latter for increasing $E_\nu$. Therefore, the $\eta_b$ production rate could become comparable with the $\eta_c$ production for higher beam energy, suggesting a possibility to observe $\eta_b$ through the diffractive productions by high-intensity neutrino beams available in ongoing or forthcoming neutrino facilities.

In conclusion, we have computed the diffractive production cross sections of $\eta_{c,b}$ mesons via the weak neutral current, using the new results of the light-cone WFs for $Z$ and $\eta_{c,b}$. Our results demonstrate that neutrino-induced productions will open a new window to measure $\eta_{c,b}$.

Acknowledgments

We are pleased to acknowledge useful discussions with T. Hatsuda and H. Mineo in the early stage of this work. One of the authors (A.H.) is grateful to T. Hirano for his support in numerical calculation. A.H. is supported by JSPS Research Fellowship for Young Scientists.

[1] H. Abramowicz, Int. J. Mod. Phys. A15S1, 495 (2000).
[2] M. G. Ryskin, Z. Phys. C57, 89 (1993).
[3] S.J. Brodsky, L. Frankfurt, J.F.Gunion, A.H. Mueller and M. Strikman, Phys. Rev. D50, 5134 (1994).
[4] L. Frankfurt, W. Koepf and M. Strikman, Phys. Rev. D57, 512 (1998).
[5] M.G.Ryskin, R.G. Roberts, A.D. Martin and E.M. Levin, Z. Phys. C76, 231 (1997).
[6] E.M. Levin, A.D. Martin, M.G. Ryskin and T. Teubner, Z. Phys. C74, 671 (1997).
[7] B.Z. Kopeliovich and P. Marage, Int. J. Mod. Phys. A8, 1513 (1993).
[8] E815 Collaboration, T. Adams et al., Phys. Rev. D61, 092001 (2000).
[9] E632 Collaboration, S. Willocq et al., Phys. Rev. D47, 2661 (1993).
[10] CHORUS Collaboration, P. Annis et al., Phys. Lett. B435, 458 (1998).
[11] J.H. Kühn and R. Rückl, Phys. Lett. B295, 431 (1980).
[12] B. Lehnmann-Dronke and A. Schäfer, Phys. Lett. B521, 55 (2001).
[13] S. Gieseke and C.-F. Qiao, Phys. Rev. D61, 074028 (2000).
[14] P. Hoodbhoy, Phys. Rev. D56, 388 (1997).
[15] I. P. Ivanov and N. N. Nikolaev, JETP Lett. 69, 294 (1999).
[16] J. Hüfner, Yu.P. Ivanov, B.Z. Kopeliovich and A.V. Tarasov, Phys. Rev. D62, 094022 (2000).
[17] E. L. Berger and D. L. Jones, Phys. Rev. D23, 1521 (1981).
[18] H. Georgi, Phys. Lett. B240, 447 (1990); T. Mannel and G. A. Schuler, Phys. Lett. B349, 181 (1995).
[19] K. Suzuki, T. Hayashi, A. Hayashigaki, J. Alam and T. Hatsuda, Phys. Rev. D62, 031501(R) (2000).
[20] E.J. Eichten and C. Quigg, Phys. Rev. D52, 1726 (1995).
[21] D.S. Hwang and G.-H. Kim, Z. Phys. C76, 1726 (1997).
[22] L. Frankfurt, M. McDermott and M. Strikman, JHEP 02, 002 (1999).
[23] M. Glück, E. Reya and A. Vogt, Z. Phys. C67, 433 (1995).
[24] A vector dominance model \cite{5b} gives $\sigma(\nu N \rightarrow \nu' N' J/\psi) \sim 10^{-2}$ fb for $E_\nu = 100$ GeV with similar $E_\nu$ dependence as our QCD result (dashed curve). At present, we do not have enough experimental data \cite{5b} to be compared with these theoretical predictions.