New Covariant Gauges in String Field Theory

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A single-parameter family of covariant gauge fixing conditions in bosonic string field theory is proposed. This is a natural string field counterpart to the covariant gauge in the conventional gauge theory, which includes the Landau gauge, as well as the Feynman (Siegel) gauge as special cases. The action in the Landau gauge is greatly simplified in such a manner that many of the component fields have no derivatives in their kinetic terms and appear in at most quadratic forms in the vertex.

§1. Introduction

In contrast to ordinary local gauge field theories, the covariant string field theory action was first constructed in gauge fixed form by Siegel$^1)$. After the gauge unfixed action was found$^{2)-4)}$ (see Refs. 5) and 6) for the interacting case), it was realized that the former is fixed with the simple gauge condition $b_0 \Phi = 0$, called the Siegel gauge, where $\Phi$ is a string field and $b_0$ is the zero mode of the worldsheet anti-ghost. Twenty years later, we still have no other choice for the covariant gauge in a practical sense. Formally, we know a general method$^7)$ of fixing the gauge symmetry with an infinitely reducible hierarchy, like that in string field theory. An explicit gauge fixing condition, however, with sufficient simplicity and consistency, like that of the Siegel gauge, is not known to date.

In an intensive study of the tachyon condensation problem, in particular in level truncation approach, the Siegel gauge has been used extensively and certainly good results have been obtained.$^{*}$ In course of these studies, pathological behavior of this gauge was also encountered. For example, the branch points in the tachyon potential prevent us from going beyond a small region of the tachyon field value. Therefore, an alternative useful gauge choice is needed for confirming the known results and extending the analysis.

Also, a recent proposal for an analytic solution of the tachyon vacuum made by Schnabl$^9)$ utilizes the analogue of Siegel gauge in a special conformal frame adapted to the star product of cubic open string field theory.$^5)$ There, a modification of the original Siegel gauge provides a clue to obtaining a closed subset of the space of the string field. To further obtain analytic solutions of lower-dimensional D-branes or time-dependent ones, a more efficient gauge choice may be necessary.

In the present paper, we propose a new covariant gauge in bosonic open string field theory which has a simple expression and has, in some respects, an advantage

$^{*}$ See Ref. 8) for a pedagogical review.
over the existing one. Actually, our new gauge contains a single parameter $a$. The Siegel gauge corresponds to the special value $a = 0$. A particularly interesting case is $a = \infty$, which is considered below. Varying $a$, we are now able to study the gauge (in-)dependence of various kinds of quantities which were computed previously only in the Siegel gauge, and in this way we are able to determine which behavior is physical or an artifact of the gauge.

The new gauge condition we propose is

$$ (b_0 M + a b_0 c_0 \tilde{Q}) \Phi_1 = 0, \quad (1.1) $$

for the string field $\Phi_1$ of ghost number one, where $a$ is a real parameter, and $\tilde{Q}$ and $M$ are defined through the decomposition of the BRST operator $Q$ with respect to the zero mode of the worldsheet ghost and anti-ghost:

$$ Q = c_0 L_0 + b_0 M + \tilde{Q}. \quad (1.2) $$

We have to take our gauge parameter $a$ to satisfy the condition $a \neq 1$ for free theory, since for $a = 1$ the left-hand side of Eq. (1.1) is invariant under inhomogeneous gauge transformations, and hence this condition does not fix the gauge for the free theory. A gauge fixing procedure including the gauge condition for sectors of other ghost numbers (i.e., ghost, ghost for ghost, ...) is described in a subsequent section.

One of the special cases for the above family of gauge conditions is that with $a = 0$, because, due to the basic property of the operator $M$, Eq. (1.1) with $a = 0$ is equivalent to the Siegel gauge, $b_0 \Phi_1 = 0$.

Another notable case is that with $a = \infty$, in which the gauge condition reduces to

$$ b_0 c_0 \tilde{Q} \Phi_1 = 0. \quad (1.3) $$

Remarkably, this gauge condition corresponds to the Landau gauge for the massless vector gauge field $A_\mu(x)$ contained in the string field, while the Siegel gauge corresponds to the Feynman gauge. Thus, the one parameter extrapolation between them is expected to give a general covariant gauge with a gauge parameter, say $\alpha$, in the gauge field sector which is widely used in the literature. In order to be definite, let us consider free abelian gauge theory. The gauge fixed action under consideration is

$$ S_{\text{gauge}} = \int d^2 x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + B \partial_\mu A^\mu + \frac{\alpha}{2} B^2 + i \bar{c} \partial_\mu \partial^\mu c \right], \quad (1.4) $$

where $B$ is the Nakanishi-Lautrup field, $c$ and $\bar{c}$ are the Faddeev-Popov ghost and anti-ghost fields, respectively. Then, we can indeed show that the gauge fixing condition (1.1) leads to $\alpha = 1/(a - 1)^2$.

Another characteristic feature of the $a = \infty$ gauge (1.3) is that in this case a large number of fields become merely auxiliary fields; i.e., the kinetic term has no derivative. This can be an advantage in the study of tachyon condensation, especially for space-time dependent solutions. Also, these auxiliary fields appear in at most quadratic forms in the interaction vertex. These facts lead to simplifications of the computation in comparison with those for other gauges.
We applied new gauges to the level truncation analysis of tachyon condensation and found remarkably smooth behavior of the tachyon potential near the Landau point, \( a = \infty \) (or sufficiently far away from the Siegel point, \( a = 0 \)). This will be reported in a forthcoming paper.\(^{11} \)

In the following, after introducing the necessary notation and tools in §2, we recall the action and the gauge transformation of bosonic open string field theory in §3. There we also show that the action can be decomposed into two parts each of which is independently gauge invariant and has a quite suggestive form. In §4, we propose new gauge conditions, including all ghost number sectors, prove the BRST invariance of the gauge-fixed action, and determine the relationship to the covariant gauges in ordinary gauge theory. Finally, §5 is devoted to discussion. Some useful relations and technical points are given in the Appendices.

§2. Properties of state space

In this section, we summarize the properties of the state space of open bosonic string theory on \( D = 26 \) as preparation for defining our new gauge fixing conditions. We first introduce our notation and then prove some basic facts about the properties of the BRST operator \( Q \) (and its constituents \( \bar{Q} \) and \( M \)) and their action on the state space.

The Fock space \( \mathcal{F}(p) \) for the bosonic open string is spanned by states of the form

\[
|f\rangle = \alpha_{-n_1}^{\mu_1} \cdots \alpha_{-n_i}^{\mu_i} c_{-l_1} \cdots c_{-l_j} b_{-m_1} \cdots b_{-m_k} |0, p; \downarrow\rangle, \tag{2.1}
\]

with

\[
0 < n_1 \leq n_2 \leq \cdots \leq n_i, \ 0 < l_1 < \cdots < l_j, \ 0 < m_1 < \cdots < m_k,
\]

or the states obtained by operating on \( |f\rangle \) with \( c_0 \):

\[
c_0|f\rangle = \alpha_{-n_1}^{\mu_1} \cdots \alpha_{-n_i}^{\mu_i} c_0 c_{-l_1} \cdots c_{-l_j} b_{-m_1} \cdots b_{-m_k} |0, p; \downarrow\rangle. \tag{2.2}
\]

Here, \( \alpha_n^{\mu} \) is a matter oscillator, and \( c_n \) and \( b_n \) are the worldsheet ghost and anti-ghost modes, respectively. The state \( |0, p; \downarrow\rangle = |0, p\rangle \otimes |\downarrow\rangle \) is annihilated by \( \alpha_n^{\mu} \), \( c_n \) and \( b_n \) (\( n > 0 \)). The Fock ground state \( |0, p\rangle \) is a momentum eigenstate, and the ghost ground state \( |\downarrow\rangle \) is related to the \( SL(2, R) \) invariant vacuum \( |0\rangle \) as \( |\downarrow\rangle = c_1 |0\rangle \). We fix the definition of the ghost number operator \( N^g \) as \( N^g = 0 \) for \( |0\rangle \), and thus \( N^g |\downarrow\rangle = |\downarrow\rangle \). Then, we have \( N^g |f\rangle = (j-k+1) |f\rangle \) and \( N^g c_0 |f\rangle = (j-k+2) c_0 |f\rangle \) for the states (2.1) and (2.2), respectively. We define the operator \( \tilde{N}^g = \sum_{n>0} (c_n b_n - b_n c_n) \), which counts the ghost number of the non-zero frequency modes, and divide the space \( \mathcal{F}(p) \) by \( \tilde{N}^g \) as

\[
\mathcal{F} = \bigoplus_{\tilde{N}^g = -\infty}^{\infty} \left( \mathcal{F}^{\tilde{N}^g} + c_0 \mathcal{F}^{\tilde{N}^g} \right). \tag{2.3}
\]

Here \( \mathcal{F}^{\tilde{N}^g} \) (or \( c_0 \mathcal{F}^{\tilde{N}^g} \)) consists of the states (2.1) (or (2.2)) with \( j - k = \tilde{N}^g \). Each space \( \mathcal{F}^{\tilde{N}^g} \) or \( c_0 \mathcal{F}^{\tilde{N}^g} \) can be further divided by the level \( N \) as \( \mathcal{F} = \bigoplus_{N \geq 0} \mathcal{F}_N \). The level \( N \) of the state \( |f\rangle \) or \( c_0 |f\rangle \) above is \( N = n_1 + \cdots + n_i + l_1 + \cdots + l_j + m_1 + \cdots + m_k \).
The BRST operator for the theory is given by\textsuperscript{10)

\begin{equation}
Q = \tilde{Q} + c_0 L_0 + b_0 M, \tag{2.4}
\end{equation}

where

\begin{equation}
M = -2 \sum_{n>0} nc_n c_n \tag{2.5}
\end{equation}

and

\begin{equation}
\tilde{Q} = \sum_{n \neq 0} c_n L_n^{(m)} - \frac{1}{2} \sum_{m, n \neq 0} (m - n) : c_{-m} c_{-n} b_{n+m} :. \tag{2.6}
\end{equation}

Note that \( L_0 = L_0^{(m)} + L_0^{(g)} \) gives \( L_0 |f⟩ = (\alpha' p^2 + N - 1) |f⟩ \). The operators \( M \) and \( \tilde{Q} \) act on \( \mathcal{F}^{\tilde{N}g} \) and map it to \( \mathcal{F}^{\tilde{N}g+2} \) and \( \mathcal{F}^{\tilde{N}g+1} \), respectively:

\begin{equation}
M : \mathcal{F}^{\tilde{N}g} \rightarrow \mathcal{F}^{\tilde{N}g+2}, \tag{2.7}
\end{equation}

\begin{equation}
\tilde{Q} : \mathcal{F}^{\tilde{N}g} \rightarrow \mathcal{F}^{\tilde{N}g+1}. \tag{2.8}
\end{equation}

They act on the space \( c_0 \mathcal{F}^{\tilde{N}g} \) similarly, as we have \([M, c_0] = [\tilde{Q}, c_0] = 0\). We also have the relation

\begin{equation}
\tilde{Q}^2 = -L_0 M, \tag{2.9}
\end{equation}

owing to the equalities \( Q^2 = 0 \) and \([\tilde{Q}, L_0] = [\tilde{Q}, M] = 0\).

The operators \( M, M^- \) and \( M_z \), where

\begin{equation}
M^- = -\sum_{n>0} \frac{1}{2n} b_{-n} b_n \tag{2.10}
\end{equation}

and

\begin{equation}
M_z = \frac{1}{2} \tilde{N}^g = \frac{1}{2} \sum_{n>0} (c_{-n} b_n - b_{-n} c_n), \tag{2.11}
\end{equation}

constitute the \( SU(1, 1) \) algebra:

\begin{equation}
[M, M^-] = 2M_z, \quad [M_z, M] = M, \quad [M_z, M^-] = -M^-; \tag{2.12}
\end{equation}

Now we analyze the properties of the state space \( \mathcal{F}^{\tilde{N}g} \) concerning this \( SU(1, 1) \) algebra and the operator \( \tilde{Q} \). We can treat the states with and without \( c_0 \) separately in this analysis, since the algebra and \( \tilde{Q} \) both (anti-)commute with \( c_0 \). In the remaining of this section, we only employ the space \( \oplus_{N_g} \mathcal{F}^{\tilde{N}g} \).

**Isomorphism between \( \mathcal{F}^n \) and \( \mathcal{F}^{-n} \), and the ‘inverse’ operator \( W_n \)**

Every state in the space \( \oplus_{N_g} \mathcal{F}^{N_g} \) can be decomposed into finite-dimensional irreducible representations of \( SU(1, 1) \), which are classified as being of integer or half-integer spin \( s \): States belonging to a certain spin \( s = k/2 \) representation form a \((k + 1)\)-dimensional vector of states as

\begin{equation}
|f^{(-k)}⟩_{s=\frac{k}{2}} \xrightarrow{M} |f^{(-k+2)}⟩_{s=\frac{k}{2}} \xrightarrow{M} \cdots \xrightarrow{M} |f^{(k-2)}⟩_{s=\frac{k}{2}} \xrightarrow{M} |f^{(k)}⟩_{s=\frac{k}{2}}, \tag{2.13}
\end{equation}
where \(|f^{(n)}\rangle_{s=\frac{k}{2}} \in \mathcal{F}^n\) and \(M^{-}|f^{(-k)}\rangle_{s=\frac{k}{2}} = M|f^{(k)}\rangle_{s=\frac{k}{2}} = 0\). Note also that
\[
(M^{-})^j M^{i} |f^{(-k)}\rangle_{s=\frac{k}{2}} \propto M^{i-j} |f^{(-k)}\rangle_{s=\frac{k}{2}}
\]
for \(1 \leq i \leq k\) and \(j \leq i\). The spaces \(\mathcal{F}^n\) and \(\mathcal{F}^{-n}\) both consist only of the states with spin greater than or equal to \(n/2\), i.e., \(s = n/2 + i (i \geq 0)\). From the relation (2.13), each state \(|f^{(-n)}\rangle \in \mathcal{F}^{-n}\) is transformed into a state in \(\mathcal{F}^n\) by operating on it with \(M^n\) and, conversely, any \(|f^{(n)}\rangle \in \mathcal{F}^n\) can be written as \(|f^{(n)}\rangle = M^n|g\rangle\) for some \(|g\rangle \in \mathcal{F}^{-n}\). We also have
\[
M^n|f^{(-n)}\rangle = 0 \Rightarrow |f^{(-n)}\rangle = 0. \tag{2.14}
\]
Thus, \(\mathcal{F}^{-n}\) is isomorphic to \(\mathcal{F}^n\), and there exists an inverse \(W_n : \mathcal{F}^n \rightarrow \mathcal{F}^{-n} (n > 0)\) which for any \(|f^{(-n)}\rangle \in \mathcal{F}^{-n}\) satisfies
\[
W_n M^n |f^{(-n)}\rangle = |f^{(n)}\rangle. \tag{2.15}
\]
For example, \(W_1\) can be explicitly written as
\[
W_1 = \sum_{i=0}^{\infty} (-1)^i \left[ \frac{1}{(i+1)!} \right]^2 M^i (M^{-})^{i+1}. \tag{2.16}
\]
For \(|f^{(n)}\rangle \in \mathcal{F}^n (n > 0)\), we have
\[
W_n|f^{(n)}\rangle = 0 \Rightarrow |f^{(n)}\rangle = 0 \tag{2.17}
\]
and
\[
M^n W_n |f^{(n)}\rangle = |f^{(n)}\rangle, \tag{2.18}
\]
which are useful in the gauge fixing procedure presented below.

**Properties of \(\hat{Q}\) for \(L_0 \neq 0\)**

- We also use the following properties of \(\hat{Q}\).
  - For a state \(|f^{(-n)}\rangle \in \mathcal{F}^{-n}(p) (n > 0)\), we have
    \[
    \hat{Q}|f^{(-n)}\rangle = 0 \Rightarrow |f^{(-n)}\rangle = 0 \quad \text{if} \quad L_0|f^{(-n)}\rangle \neq 0. \tag{2.19}
    \]
    **Proof:** If \(\hat{Q}|f^{(-n)}\rangle = 0\), then \(0 = \hat{Q}^2|f^{(-n)}\rangle = -ML_0|f^{(-n)}\rangle\).
    Thus, \(M^n|f^{(-n)}\rangle = 0\), and hence \(|f^{(-n)}\rangle = 0\) from (2.14).
  - We also have
    \[
    \hat{Q} (\mathcal{F}^n(p))_{L_0 \neq 0} (p) = (\mathcal{F}^{n+1}(p))_{L_0 \neq 0} \quad \text{for} \quad n \geq 0. \tag{2.20}
    \]
    Here \((\mathcal{F}(p))_{L_0 \neq 0}\) denotes the space \(\{|f\rangle \in \mathcal{F}(p) \mid L_0|f\rangle \neq 0\}\).
    **Proof:** The relation \(\hat{Q} \mathcal{F}^n \subset \mathcal{F}^{n+1}\) is trivial. By contrast, if \(|f^{(n+1)}\rangle \in \mathcal{F}^{n+1}\), then \(|f^{(n+1)}\rangle = M^{n+1}|g\rangle\) for some \(|g\rangle \in \mathcal{F}^{-n+1}\) from the isomorphism of \(\mathcal{F}^{n+1}\)
and \(\mathcal{F}^{-n+1}\). Thus, if \(L_0|f^{(n+1)}\rangle \neq 0\), we have \(|f^{(n+1)}\rangle = \hat{Q} \left( -\frac{\hat{Q}}{L_0} M^n |g\rangle \right)\) for
\(n \geq 0\). This completes the proof.
§3. Gauge invariant action

In this section, we first recall the general form of the gauge invariant free action of covariant bosonic string field theory\(^{2)–4)}\) and then we show that the action can be written as a sum of two gauge invariant combinations that are quite suggestive. We also briefly describe the action and the gauge invariance in the interacting case of cubic open string field theory\(^{5)}\).

3.1. Free action

The gauge invariant free action of covariant bosonic open string field theory is defined through use of the BRST operator \(Q\) as\(^{2)–4)}\)

\[
S_{\text{quad}} = -\frac{1}{2} \langle \Phi_1, Q\Phi_1 \rangle .
\] (3.1)

In terms of the usual inner product of states, \(\langle A, B \rangle = \langle \text{bpz}(A)|B\rangle\), where \(\text{bpz}(A)\) denotes the BPZ conjugate of \(A\). Some properties of the inner product and BPZ conjugation are given in Appendix B. The string field \(\Phi_1 = \Phi_1(x^\mu(\sigma), c(\sigma), b(\sigma))\) \((0 \leq \sigma \leq \pi)\) has ghost number \(N^g = 1\) (which we explicitly express with the subscript) and is Grassmann odd. It is expanded in Fock space states as

\[
\Phi_1 = \phi^{(0)} + c_0 \omega^{(-1)} ,
\] (3.2)

with

\[
\phi^{(0)} = \int \frac{d^{26} p}{(2\pi)^{26}} \left[ \sum |f^{(0)}\rangle \psi_{|f^{(0)}\rangle}(p) \right], \quad (|f^{(0)}\rangle \in \mathcal{F}^0(p)) \tag{3.3}
\]

and

\[
\omega^{(-1)} = \int \frac{d^{26} p}{(2\pi)^{26}} \left[ \sum |g^{(-1)}\rangle \psi_{|g^{(-1)}\rangle}(p) \right], \quad (|g^{(-1)}\rangle \in \mathcal{F}^{-1}(p)) \tag{3.4}
\]

where the coefficient \(\psi_{|f^{(0)}\rangle}(p)\) or \(\psi_{|g^{(-1)}\rangle}(p)\) of each state represents the corresponding space-time field. The Hermitian property of the field (\(\psi^\dagger = \psi\) or \(= -\psi\)) is determined by that of \(S\). In terms of \(\phi^{(0)}\) and \(\omega^{(-1)}\), the action is given by

\[
S_{\text{quad}} = -\frac{1}{2} \left( \langle \phi^{(0)}, c_0 L_0 \phi^{(0)} \rangle + 2 \langle \tilde{Q}\phi^{(0)}, c_0 \omega^{(-1)} \rangle + \langle M\omega^{(-1)}, c_0 \omega^{(-1)} \rangle \right) .
\] (3.5)

It can also be written in a simpler form as

\[
S_{\text{quad}} = -\frac{1}{2} \left( \left( \phi^{(0)} - \frac{1}{L_0} \tilde{Q}\omega^{(-1)} \right) , c_0 L_0 \left( \phi^{(0)} - \frac{1}{L_0} \tilde{Q}\omega^{(-1)} \right) \right)
\] (3.6)

if \(L_0 \neq 0\).

This action is invariant under the gauge transformation

\[
\delta \Phi_1 = Q\Lambda_0 .
\] (3.7)

The gauge parameter \(\Lambda_0\) is a Grassmann-even string field with \(N^g = 0\) and is decomposed as

\[
\Lambda_0 = \lambda^{(-1)} + c_0 \rho^{(-2)} ,
\] (3.8)
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where \(\lambda^{(-1)}\) and \(\rho^{(-2)}\) consist of states in \(\mathcal{F}^{-1}\) and \(\mathcal{F}^{-2}\), respectively. In terms of \(\phi^{(0)}\) and \(\omega^{(-1)}\), the transformation is written

\[
\delta \phi^{(0)} = \tilde{Q} \lambda^{(-1)} + M \rho^{(-2)}, \quad \delta \omega^{(-1)} = L_0 \lambda^{(-1)} - \tilde{Q} \rho^{(-2)}.
\] (3.9)

Note that if \(L_0 \neq 0\), it can also be written as

\[
\delta (\phi^{(0)} + c_0 \omega^{(-1)}) = \left( \frac{\tilde{Q}}{L_0} + c_0 \right) (L_0 \lambda^{(-1)} - \tilde{Q} \rho^{(-2)}).
\] (3.10)

We see that the combination

\[
\zeta^{(1)} = \tilde{Q} \phi^{(0)} + M \omega^{(-1)}
\] (3.11)

is gauge invariant. Thus, from (2.15), we represent \(\omega^{(-1)}\) in terms of \(\zeta^{(1)}\) and \(\phi^{(0)}\) as

\[
\omega^{(-1)} = W_1 (\zeta^{(1)} - \tilde{Q} \phi^{(0)}),
\] (3.12)

and the action (3.5) is rewritten using \(\zeta^{(1)}\) instead of \(\omega^{(-1)}\) as

\[
S^{\text{quad}} = -\frac{1}{2} \left( \langle \phi^{(0)}, c_0 L_0 \phi^{(0)} \rangle - \langle \tilde{Q} \phi^{(0)}, c_0 W_1 (\tilde{Q} \phi^{(0)}) \rangle + \langle \zeta^{(1)}, c_0 W_1 \zeta^{(1)} \rangle \right). \tag{3.13}
\]

Remarkably the terms \(\langle \phi^{(0)}, c_0 L_0 \phi^{(0)} \rangle - \langle \tilde{Q} \phi^{(0)}, c_0 W_1 (\tilde{Q} \phi^{(0)}) \rangle\) and \(\langle \zeta^{(1)}, c_0 W_1 \zeta^{(1)} \rangle\) are both gauge invariant independently. The former represents the kinetic terms for each field in \(\phi^{(0)}\). The latter is a sum of the terms that are purely quadratic in each field in \(\zeta^{(1)}\), which means that \(\zeta^{(1)}\) contains only auxiliary fields.

For example, up to level \(N = 1\), the string field is expanded in the tachyon \(\phi\), massless gauge field \(A_\mu\) and a scalar \(\chi\) as*

\[
\phi^{(0)}_{N \leq 1} = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{1}{\sqrt{\alpha'}} \left( \phi(p)|0, p; \downarrow \rangle + A_\mu(p) \alpha_{-1}^\mu |0, p; \downarrow \rangle \right),
\] (3.14)

\[
\omega^{(-1)}_{N \leq 1} = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{i}{\sqrt{2}} \chi(p) b_{-1} |0, p; \downarrow \rangle.
\] (3.15)

The gauge invariant \(\zeta^{(1)}\) is written explicitly as

\[
\zeta^{(1)}_{N \leq 1} = \int \frac{d^{26}p}{(2\pi)^{26}} \sqrt{2} (-i \chi(p) + A_\mu(p) p^\mu) c_{-1} |0, p; \downarrow \rangle,
\] (3.16)

and the action up to this order is a sum of two gauge invariant combinations:

\[
-\frac{1}{2} \left( \langle \phi^{(0)}, c_0 L_0 \phi^{(0)} \rangle - \langle \tilde{Q} \phi^{(0)}, c_0 W_1 (\tilde{Q} \phi^{(0)}) \rangle \right) \bigg|_{N \leq 1}
= -\frac{1}{2} \int \frac{d^{26}p}{(2\pi)^{26}} \phi(p) \left( p^2 - \frac{1}{\alpha'} \right) \phi(-p) - \frac{1}{2} \int \frac{d^{26}p}{(2\pi)^{26}} A_\mu(p) (\gamma^\mu \gamma^\nu p^2 - p^\mu p^\nu) A_\nu(-p)
\] (3.17)

* Here, we make the component fields real (hermitian) by appropriately inserting \(i\) in the coefficients. (See Appendix C for the general rule concerning the hermiticity assignment.)
and  
\[-\frac{1}{2}\langle \zeta^{(1)}, c_0 W_1 \zeta^{(1)} \rangle \bigg|_{N \leq 1} = -\frac{1}{2} \int \frac{d^{26}p}{(2\pi)^{26}} (\chi(p) + ip^\mu A_\mu(p)) (\chi(-p) + (-ip^\nu) A_\nu(-p)).\]  

(3.18)

Note that the second term on the r.h.s. of Eq. (3.17) coincides with the gauge invariant action of the massless vector field, $-\frac{1}{4} F_\mu \nu F^{\mu \nu}$, where $F_\mu \nu = \partial_\mu A_\nu - \partial_\nu A_\mu$. The gauge transformation up to level $N = 1$ is written in terms of the gauge parameter $\lambda$ as  
\[\delta A_\mu(p) = ip_\mu \lambda, \quad \delta \chi(p) = p^2 \lambda.\]  

(3.19)

### 3.2. Cubic action

The action for the cubic open string field theory is given by  
\[S = -\frac{1}{2} \langle \Phi_1, Q \Phi_1 \rangle - \frac{g}{3} \langle \Phi_1, \Phi_1 * \Phi_1 \rangle.\]  

(3.20)

Here $*$ denotes the star product and $g$ the coupling constant. This action is invariant under the gauge transformations  
\[\delta \Phi_1 = Q A_0 + g(\Phi_1 * A_0 - A_0 * \Phi_1).\]  

(3.21)

The action can be rewritten in terms of $\phi^{(0)}$ and $\omega^{(-1)}$ as  
\[S = -\frac{1}{2} \left( \langle \phi^{(0)}, c_0 L_0 \phi^{(0)} \rangle - \langle \tilde{Q} \phi^{(0)}, c_0 W_1 (\tilde{Q} \phi^{(0)}) \rangle + \langle \zeta^{(1)}, c_0 W_1 \zeta^{(1)} \rangle \right) \]
\[-\frac{g}{3} \left( \langle \phi^{(0)}, \phi^{(0)} * \phi^{(0)} \rangle + 3 \langle \phi^{(0)}, \phi^{(0)} * c_0 \omega^{(-1)} \rangle + 3 \langle \phi^{(0)}, c_0 \omega^{(-1)} * c_0 \omega^{(-1)} \rangle \right)\].  

(3.22)

Note that there is no term cubic in $\omega^{(-1)}$ in this action. For this reason, together with the observation concerning the quadratic term, we expect that a simpler gauge-fixed action can be obtained by gauge fixing $\phi^{(0)}$ to a greater degree than $\omega^{(-1)}$.

### §4. Gauge fixing

Now we consider the gauge fixing of the action $S$. The most commonly used gauge condition (and in a practical sense, essentially the only one known and used gauge condition) is the Siegel gauge condition, $b_0 \Phi_1 = 0$ (or $\omega^{(-1)} = 0$). It is known that this gauge condition exactly fixes the gauge invariance of the quadratic $g = 0$ part of the action if we assume $L_0 \neq 0$.

The gauge fixed action for the Siegel gauge is obtained in the literature as  
\[S_{\text{Siegel}} = -\frac{1}{2} \langle \Phi, Q \Phi \rangle - \frac{g}{3} \langle \Phi, \Phi * \Phi \rangle + \langle b_0 B, \Phi \rangle\]
\[= -\frac{1}{2} \sum_{n=-\infty}^{\infty} \langle \Phi_n, Q \Phi_{-n+2} \rangle - \frac{g}{3} \sum_{l+m+n=3} \langle \Phi_l, \Phi_m * \Phi_n \rangle + \sum_{n=-\infty}^{\infty} \langle b_0 B_{-n+4}, \Phi_n \rangle.\]  

(4.1)
New Covariant Gauges in String Field Theory

Here $\Phi$ and $B$ consist of the string fields of all the ghost numbers as $\Phi = \sum_{n=-\infty}^{\infty} \Phi_n$ and $B = \sum_{n=-\infty}^{\infty} B_n$. Note that component fields are taken to be Grassmann odd (even) for $\Phi_n = \text{even}$ ($\Phi_n = \text{odd}$), and thus $\Phi$ is always Grassmann odd.

In principle, this action is obtained from the gauge invariant action (3.20) by adding Faddeev-Popov ghost terms and gauge fixing terms repeatedly until no gauge symmetry remains in the action. Here, we instead obtain the same action by first extending the action to include the string fields with all ghost numbers as

$$\tilde{S} = -\frac{1}{2} \langle \Phi, Q\Phi \rangle - \frac{g}{3} \langle \Phi, \Phi \ast \Phi \rangle, \quad \Phi = \sum_{n=-\infty}^{\infty} \Phi_n, \quad (4.2)$$

and then adding the gauge fixing term $\langle b_0 B, \Phi \rangle$ so as to completely fix the gauge invariance of the extended action $\tilde{S}$ for $g = 0$.

$$\delta \Phi = QA + g(\Phi \ast A - A \ast \Phi), \quad A = \sum_{n=-\infty}^{\infty} A_n. \quad (4.3)$$

In order to obtain the gauge fixed action for another gauge condition that we propose below, we employ this latter procedure. That is, for each gauge condition we take, we show that we can choose an appropriate operator $O$ so that the condition $b_{pz}(O)\Phi = 0$ exactly fixes the gauge invariance of the extended action $\tilde{S}$ for $g = 0$. We then add the term $\langle OB, \Phi \rangle$ to $\tilde{S}$ and regard the result as the gauge fixed action. Finally, as a confirmation, we check that the action obtained in this way possesses BRST invariance instead of gauge invariance.

As stated in the Introduction, we propose a one-parameter family of new gauge fixing conditions in this section. However, we first discuss a special value (corresponding to the Landau-type gauge) of the parameter, because there is a technically subtle point in the case of this value.

4.1. Landau-type gauge

4.1.1. Gauge condition

The new gauge condition we propose in this subsection is

$$b_0 c_0 \tilde{Q}\Phi_1 = 0. \quad (\Leftrightarrow \tilde{Q}\phi^{(0)} = 0) \quad (4.4)$$

For the level $N = 1$ gauge field, the condition (4.4) implies $p^\mu A_\mu(p) = 0$. Thus, this gauge condition is an extension of the Landau gauge of ordinary gauge theory, just as the Siegel gauge is that of the Feynman gauge. We now show that this new condition exactly fixes the gauge invariance of the quadratic action $S^{\text{quad}}$ when $L_0 \neq 0$.

First, any string field $\Phi_1 = \phi^{(0)} + c_0 \omega^{(-1)}$ with $L_0 \neq 0$ can be transformed so as to satisfy this gauge condition through the gauge transformation (3.7) [or (3.9) in terms of $\phi^{(0)}$ and $\omega^{(-1)}$] with the gauge parameter

$$A_0(= \lambda^{(-1)} + c_0 \rho^{(-2)}) = \frac{1}{L_0} W_1(\tilde{Q}\phi^{(0)}), \quad (4.5)$$

since we have the relation

$$\tilde{Q}(\phi^{(0)} + \delta\phi^{(0)}) = \tilde{Q} \left( \phi^{(0)} + \tilde{Q} \frac{1}{L_0} W_1(\tilde{Q}\phi^{(0)}) \right) = 0. \quad (4.6)$$
In the last equation, we have used Eqs. (2.9) and (2.18) for \( n = 1 \). Furthermore, we can show that there remains no residual gauge transformation within the condition (4.4). This can be seen by first noting that if there were such a transformation, it would be given by the gauge parameter \( \Lambda_0 = \lambda^{(-1)} + c_0 \rho^{(-2)} \) that satisfies \( \tilde{Q}(\delta \Lambda_0 \phi^{(0)}) = 0 \). This implies that \( 0 = \tilde{Q}(\tilde{Q} \lambda^{(-1)} + M \rho^{(-2)}) = M(-L_0 \lambda^{(-1)} + \tilde{Q} \rho^{(-2)}) \), and from Eq. (2.14), we have \(-L_0 \lambda^{(-1)} + \tilde{Q} \rho^{(-2)} = 0\). If \( L_0 \neq 0 \), this leads to \( \delta \Phi_1 = 0 \) by Eq. (3.10).

### 4.1.2. Gauge fixed action

The gauge fixed action for this gauge condition is obtained by using the method explained in the beginning of this section. The resulting action is given by

\[
S_L = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \langle \Phi_n, \tilde{Q} \Phi_{-n+2} \rangle - \frac{g}{3} \sum_{l+m+n=3} \langle \Phi_l, \Phi_m \Phi_n \rangle \\
+ \sum_{n=2}^{\infty} \left( \langle (O_L \mathcal{B})_{-n+3}, \Phi_n \rangle + \langle (O_L \mathcal{B})_n, \Phi_{-n+3} \rangle \right),
\]

with

\[
(O_L \mathcal{B})_n = c_0 b_0 M^{n-2} \tilde{Q} \mathcal{B}_{3-n} ,
\]

\[
(O_L \mathcal{B})_{-n+3} = c_0 b_0 W_{n-1} \tilde{Q} \mathcal{B}_n + b_0 b \text{pz}(1 - P \tilde{Q} M^{n-2}) \mathcal{B}'_{4-n}.
\]

Here, \( \mathcal{B}_{-n+3} \), \( \mathcal{B}_n \) and \( \mathcal{B}'_{4-n} \) \( (n > 1) \) are Grassmann odd string fields and \( P \tilde{Q} M^{n-2} \) denotes the projection operator which restricts the fields in \( \mathcal{F}^{n-2} \) \( (n > 1) \) as

\[
P \tilde{Q} M^{n-2} |f^{(n-2)}\rangle \in \tilde{Q} M^{n-2} \mathcal{F}^{n-1}.
\]

We have \((1 - P \tilde{Q} M^{n-2})b_0 M^{n-2} \tilde{Q} \mathcal{B}_{3-n} = 0\), and it follows that

\[
\langle (O_L \mathcal{B})_n, (O_L \mathcal{B})_{-n+3} \rangle = 0.
\]

From this, it can be shown that the action \( S_L \) is invariant under the BRST transformation. We actually show this fact as the following general proposition. An arbitrary operator \( \mathcal{O} \) below is just \( O_L \) for the gauge under consideration. The proposition is sufficiently general, and it can be applied to a wide class of operators \( \mathcal{O} \), including those in the next subsection.

**Proposition 1** The general gauge fixed action of the form

\[
S_{GF} = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \langle \Phi_n, \tilde{Q} \Phi_{-n+2} \rangle - \frac{g}{3} \sum_{l+m+n=3} \langle \Phi_l, \Phi_m \Phi_n \rangle \\
+ \sum_{n=2}^{\infty} \left( \langle (\mathcal{O} \mathcal{B})_{-n+3}, \Phi_n \rangle + \langle (\mathcal{O} \mathcal{B})_n, \Phi_{-n+3} \rangle \right)
\]

is invariant under the BRST transformation with a Grassmann odd parameter \( \eta \)

\[
\delta_B \Phi_n = \eta (\mathcal{O} \mathcal{B})_n, \quad (n > 1)
\]
\[ \delta_B \Phi_n = \eta \left( Q \Phi_{n-1} + g \sum_{k=-\infty}^{\infty} (\Phi_{n-k} \ast \Phi_k) \right), \quad (n \leq 1) \]  
(4.14)

\[ \delta_B \mathcal{B}_n = 0, \]  
(4.15)

if \( \langle (\mathcal{O}\mathcal{B})_n, (\mathcal{O}\mathcal{B})_{-n+3} \rangle = 0 \).

This can be proved straightforwardly by using the relations (B.1) and (B.2) and noting that \( \Phi \) is Grassmann odd and \( \mathcal{O}\mathcal{B} \) is even. ■

The conditions \( \text{bpz}(O_L)\Phi = 0 \) given by the action \( S_L \) restrict each string field \( \Phi_n(= \phi^{(n-1)} + c_0\omega^{(n-2)}) \) or \( \Phi_{3-n}(= \phi^{(2-n)} + c_0\omega^{(1-n)}) \) for \( n > 1 \) as

\[ \text{bpz}(c_0b_0M^{n-2}\tilde{Q})\Phi_{-n+3} = 0, \quad (\Leftrightarrow \tilde{Q}M^{n-2}\phi^{(-n+2)} = 0) \]  
(4.16)

\[ \text{bpz}(c_0b_0W_{n-1}\tilde{Q})\Phi_n = 0, \quad (\Leftrightarrow \phi^{(n-1)} = 0) \]  
(4.17)

\[ b_0(1 - \mathcal{P}_Q M^{n-2})\Phi_n = 0. \quad (\Leftrightarrow \omega^{(n-2)} \in \tilde{Q}M^{n-2}\mathcal{F}^{-n+1}) \]  
(4.18)

Here, we have used Eqs. (2.17) and (2.19) to obtain the expression in the parentheses of Eq. (4.17). Note also that from the first condition (4.16), \( \phi^{(-n+2)} \) is restricted to include only spin \( (n - 2)/2 \) states.

The proof that the above conditions fix exactly the gauge symmetry \( \tilde{\delta}\Phi = QA \) of the extended action (4.2) for \( g = 0 \) in the case \( L_0 \neq 0 \) is given in Appendix A.

### 4.2. One-parameter extension

As we have a one-parameter family of gauge conditions that interpolate between the Feynman gauge and the Landau gauge in the gauge theory, we also should have intermediate gauge conditions between the Feynman-Siegel gauge and the Landau-type gauge for the string field theory. In this section, we propose such conditions and give the corresponding gauge fixed action.

#### 4.2.1. Gauge condition

The intermediate gauge condition we propose here is

\[ b_0(M + ac_0\tilde{Q})\Phi_1 = 0, \quad \left( \Leftrightarrow M\omega^{(-1)} + a\tilde{Q}\phi^{(0)} = 0 \right) \]  
(4.19)

where \( a \) is a real parameter. Though it may not be clear at a glance, this condition reduces to the Siegel gauge for \( a = 0 \), since \( M\omega^{(-1)} = 0 \) is equivalent to \( \omega^{(-1)} = 0 \), as seen from Eq. (2.14). The Landau-type gauge condition is also obtained by taking \( a \rightarrow \pm \infty \). We cannot take \( a = 1 \) as a gauge condition for \( S^\text{quad} \), because the condition takes the form \( M\omega^{(-1)} + \tilde{Q}\phi^{(0)} = 0 \), and the left-hand side coincides with the gauge invariant combination \( \zeta^{(1)} \) of Eq. (3.11). For each \( a \neq 1 \), by choosing the gauge parameter of the gauge transformation \( \delta\Phi_1 = QA_0 \) as

\[ A_0(= \lambda^{(-1)} + c_0\rho^{(-2)}) = \frac{1}{a - 1} \frac{1}{L_0} W_1(M\omega^{(-1)} + a\tilde{Q}\phi^{(0)}), \]  
(4.20)

any string field \( \Phi_1 = \phi^{(0)} + c_0\omega^{(-1)} \) with \( L_0 \neq 0 \) is transformed so as to satisfy the condition (4.19). This can be shown by using the relations (2.9) and (2.18). If there is a gauge transformation \( A_0 \) under which \( \Phi_1 \) continues to satisfy the gauge
condition, then we have
\[ 0 = M(\delta\omega^{(-1)}) + a\bar{Q}(\delta\phi^{(0)}) = M(1-a)(L_0\lambda^{(-1)} - \tilde{Q}\rho^{(-2)}). \] (4.21)

For \( a \neq 1 \) and \( L_0 \neq 0 \), this leads to \( L_0\lambda^{(-1)} - \tilde{Q}\rho^{(-2)} = 0 \) and \( \delta\phi_1 = 0 \). Thus we have shown the validity of the gauge condition for \( a \neq 1 \).

4.2.2. Gauge fixed action

The gauge fixed action for this case should be taken as
\[
S_a = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \langle \Phi_n, Q\Phi_{-n+2} \rangle - \frac{g}{3} \sum_{l+m+n=3} \langle \Phi_l, \Phi_m \ast \Phi_n \rangle 
+ \sum_{n=2}^{\infty} \left( \langle (O_aB)_{-n+3}, \Phi_n \rangle + \langle (O_aB)_n, \Phi_{n-3} \rangle \right),
\] (4.22)

where
\[
(O_aB)_n = \left( b_0 M^{n-1} + ac_0 b_0 M^{n-2} \tilde{Q} \right) B_{3-n},
\] (4.23)
\[
(O_aB)_{-n+3} = \left( b_0 W_{-n-2} + ac_0 b_0 W_{n-1} \tilde{Q} \right) B_n.
\] (4.24)

If \( a = 0 \) is taken, this action consistently reduces to \( S_{\text{Siegel}} \) of Eq. (4.1), because the replacement \( b_0 M^{n-1} B_{3-n} \rightarrow b_0 B_{n+1} \) and \( b_0 W_{n-2} B_n \rightarrow b_0 B_{-n+4} \) can be performed through the isomorphism represented by \( M \) and \( W_n \) explained in § 2.

We can check that this action is invariant under the BRST transformation given by Eqs. (4.13)–(4.15) with \( O = O_a \), since for \( n > 1 \) we have
\[
\langle (O_aB)_n, (O_aB)_{-n+3} \rangle 
= -\langle B_{-n+3}, \text{bpz} \left( b_0 M^{n-1} + ac_0 b_0 M^{n-2} \tilde{Q} \right) B_n \rangle 
= (-)^n a \left( B_{-n+3}, \left( b_0 M^{n-1} W_{n-1} \tilde{Q} - b_0 \tilde{Q} M^{n-2} W_{n-2} \right) B_n \right) 
= 0.
\] (4.25)

In the above equation, we have used Eq. (2.18) and
\[
\text{bpz} \left( b_0 M^{n-1} + ac_0 b_0 M^{n-2} \tilde{Q} \right) = (-)^{n-1} \left( b_0 M^{n-1} + ab_0 c_0 \tilde{Q} M^{n-2} \right).
\]

The condition \( \text{bpz}(O_a)\Phi_n = 0 \) is explicitly written for each \( \Phi_n \) or \( \Phi_{-n+3} \) \( (n > 1) \) as
\[
b_0 \left( M^{n-1} + ac_0 \tilde{Q} M^{n-2} \right) \Phi_{-n+3} = 0 \leftrightarrow M^{n-1} \omega^{(-n+1)} + a\tilde{Q} M^{n-2} \phi^{(-n+2)} = 0, \] (4.26)
\[
b_0 \left( W_{n-2} + ac_0 \tilde{Q} W_{n-1} \right) \Phi_n = 0 \leftrightarrow W_{n-2} \omega^{(n-2)} + a\tilde{Q} W_{n-1} \phi^{(n-1)} = 0. \] (4.27)

Note that the conditions (4.26) and (4.27) imply that any \( \omega \) can be written in terms of \( \phi \) as
\[
\omega^{(-n+1)} = -a W_{n-1} \tilde{Q} M^{n-2} \phi^{(-n+2)}, \quad \omega^{(n-2)} = -a M^{n-2} \tilde{Q} W_{n-1} \phi^{(n-1)}.
\] (4.28)

The validity of the above conditions as a gauge fixing condition for the extended action (4.2) in the case \( g = 0 \) is analyzed in Appendix A.
4.3. Relation to the gauge in ordinary gauge theory

Let us consider the relation of the above covariant gauge to that in ordinary gauge theory by explicitly choosing the level-one fields as follows:
\[
\phi^{N=1} = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{1}{\sqrt{\alpha'}} \left( \gamma(p)b_{-1} + A_{\mu}(p)\alpha^\mu_{-1} + i\tilde{\gamma}(p)c_{-1} \right) |0, p; \downarrow\),
\]
(4.29)
\[
\omega^{N=1} = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{1}{\sqrt{2}} \left( i\chi(p)b_{-1} + u_{\mu}(p)\alpha^\mu_{-1} + v(p)c_{-1} \right) |0, p; \downarrow\).
\]
(4.30)
We also expand the field \( B = c_0 B_\omega \) in the same way as above:
\[
B_\omega^{N=1} = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{1}{\sqrt{2}} \left( i\beta_\chi(p)b_{-1} + \beta_{u_{\mu}}(p)\alpha^\mu_{-1} + \beta_{v}(p)c_{-1} \right) |0, p; \downarrow\).
\]
(4.31)
Then we obtain the \( a \)-gauge fixed quadratic action for them in the coordinate representation as
\[
S_{a, N=1}^{\text{quad}} = \int d^26x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\chi + \partial_\mu A^\mu)^2 - i\tilde{\gamma} \partial_\mu \bar{\gamma} - iu_{\mu} \partial^\mu \bar{\gamma} \\
+ \beta_\chi(\chi + a \partial_\mu A^\mu) + \frac{1}{2} \beta_{u_{\mu}}(u^\mu - a \partial^\mu \bar{\gamma}) + \frac{1}{4} \beta_{v} v \right],
\]
(4.32)
where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). By use of the field redefinitions
\[
B = (a - 1)\beta_\chi, \quad \tilde{\chi} = \chi + \partial_\mu A^\mu - \beta_\chi, \\
\bar{c} = (a - 1)\bar{\gamma}, \quad \tilde{u}^\mu = u^\mu - a \partial^\mu \bar{\gamma}, \\
c = \gamma, \quad \tilde{\beta}_{u_{\mu}} = \beta_{u_{\mu}} + 2i \partial_\mu \bar{\gamma},
\]
the above action can be written in the well-known piece plus a decoupled auxiliary field term:
\[
S_{a(\alpha), N=1}^{\text{quad}} = \int d^26x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + B \partial_\mu A^\mu + \alpha \frac{\alpha}{2} B^2 + i\bar{c} \partial_\mu \bar{\gamma}c \\
- \frac{1}{2} \tilde{\chi}^2 + \frac{1}{2} \tilde{\beta}_{u_{\mu}} \tilde{u}^\mu + \frac{1}{4} \beta_{v} v \right].
\]
(4.33)
The first four terms here constitute the usual abelian gauge theory action in the covariant gauge with the gauge parameter \( \alpha \), which is related to \( a \) as
\[
\alpha = \frac{1}{(a - 1)^2}.
\]
(4.34)
Here, \( B \) is the Nakanishi-Lautrup field, and \( c \) and \( \bar{c} \) are the Faddeev-Popov ghost and anti-ghost fields, respectively. We can read from the above argument that \( a = 0 \) (\( \alpha = 1 \)) corresponds to the Feynman gauge and \( a = \infty \) (\( \alpha = 0 \)) corresponds to the Landau gauge. Also, we recognize that \( a = 1 \) (\( \alpha = \infty \)) is the value at which the gauge is not fixed for the free theory.
For the Landau-type gauge, we can obtain the level-one part of the action similarly to that for the above $a$-gauge. In this case, however, we need the $B_\omega' = c_0 B_\omega'$ field in addition to the above $B_{\omega}^{N=1}$. Noting that the projection operator $\mathcal{P}_Q^{N=1}$ on $B_\omega'$ can be written as

$$bpz(1 - \mathcal{P}_Q^{N=1})B_\omega' = \left(1 - \frac{L_1 L_{-1}}{2 L_0}\right)B_\omega', $$

we obtain the explicit form of $S_{L, N=1}^{1\text{quad}}$, which coincides with $S_{a(\alpha), N=1}^{\text{quad}}$ for $\alpha = 0$ ($a \to \infty$), after performing the appropriate field redefinitions.

Note that the relation (4.34) is valid for the free theory. In the interacting case, the forms of the actions for the string field level and the effective field theory level are not identical because the latter is obtained by integrating out the higher mass level fields. In fact, the free action in general has the symmetry $S_a^{\text{quad}} = S_{2-a}^{\text{quad}}$, while the interacting action does not. In addition, the form of the gauge transformation is also different from that in the free case, due to the homogeneous term, and hence the gauge non-fixed point is shifted in accordance with the value of the fields. This is indeed seen in the level truncated scalar effective potential.\textsuperscript{11)}

4.4. Properties of the gauge conditions

To clarify the properties of each gauge condition we have defined, we choose the two distinctive types of conditions, that of the Siegel gauge ($a = 0$) and that of the Landau-type gauge ($a \to \infty$) and compare their properties. Other gauge conditions realized for finite $a$ have properties similar to those of the Siegel gauge.

For the Siegel gauge, all the component fields in $\phi^{(0)}$ of $\Phi_1 = \phi^{(0)} + c_0 \omega^{(-1)}$, remain in the gauge fixed action, while $\omega^{(-1)}$ is completely gauged away. In the action $S_{a=0}$, each field $\psi_\omega(p)$ in $\phi^{(0)}$ similarly has a kinetic term through $-\frac{1}{2}(\phi^{(0)}, c_0 L_0 \phi^{(0)}).

For the Landau gauge, all the fields in $\omega^{(-1)}$ and the part of the $\phi^{(0)}$ field ($\phi^{(0)}$) satisfying $\tilde{Q} \phi^{(0)} = 0$ for $L_0 \neq 0$ remain in the gauge fixed action.\textsuperscript{*} The $\Phi_1$ part of the quadratic action for this gauge is $-\frac{1}{2}(\langle \phi^{(0)}, c_0 L_0 \phi^{(0)} \rangle + \langle M \omega^{(-1)}, c_0 \omega^{(-1)} \rangle)$, which means that all fields $\psi_\omega(p)$ in $\omega^{(-1)}$ are auxiliary fields, since none of the $\psi_\omega(p)$ contains a derivative in the action. In addition, we see from Eq. (3.22) that even in the full action, there are no terms that are of greater than quadratic order in $\omega^{(-1)}$. Thus, by choosing the Landau-type gauge condition, the number of dynamical fields is much smaller than for the other gauges, and we expect that the analysis of the theory is thus much simpler.

§5. Discussion

We have proposed a one-parameter family of covariant gauge conditions for open bosonic string field theory that includes the Siegel gauge as a special case ($a = 0$). For the massless gauge field $A_\mu$ contained in the string field $\Phi_1$, these gauge conditions precisely correspond to a family of conventional covariant gauges parameterized by

\textsuperscript{*} We can check that the degrees of freedom of $\phi^{(0)}$ and $(\phi^{(0)}, \omega^{(-1)})$ are the same, since for $L_0 \neq 0$ we have the relation $\mathcal{F}^0 = \tilde{Q} \mathcal{F}^{-1} \oplus (\mathcal{F}^0) \tilde{Q}$ from (2.20). (Here $(\mathcal{F}^0) \tilde{Q} = \{|f^{(0)}| \tilde{Q}|f^{(0)}\rangle = 0\}$.)

In the family of gauge conditions, the $a = \infty$ gauge (which corresponds to the Landau gauge in gauge theory) in particular has several striking features, as we mentioned in the previous section. These features result from the facts that the $\omega^{(-1)}$ part of the string field $\Phi_1$ has no derivatives in the free action and that it is at most quadratic in the interaction terms, since the $a = \infty$ gauge condition only restricts the $\phi^{(0)}$ part of $\Phi_1$ and leaves $\omega^{(-1)}$ unaltered. (The situation for the Siegel gauge is the opposite, since in this case, the gauge condition completely eliminates $\omega^{(-1)}$ and leaves $\phi^{(0)}$ unaltered.) In the ordinary gauge theory, we know that gauge parameter is not renormalized under the Landau gauge. It might be possible to find a similar property for the $a = \infty$ gauge counterpart of string field theory. From these advantages of this gauge, we expect that the quantum analysis is much simpler in this gauge than in other gauges.

The form of the quadratic action for the $a = \infty$ gauge is reminiscent of that of the action for the string field based on the old covariant quantization,

$$S = \frac{1}{2} \Phi (L_0 - 1) \Phi, \quad \text{with} \quad L_n \Phi = 0, \quad n \geq 1$$

(5.1)

because the condition $\tilde{Q}\phi^{(0)} = 0$ reduces to $L_n \phi^{(0)} = 0$ ($n \geq 1$) if there are no ghost fields ($c_{-n}$ or $b_{-n}$) in $\phi^{(0)}$. It should be possible to find an explicit basis for the set of states satisfying the condition $\tilde{Q}\mid f\rangle = 0$ as we have done in the old covariant theory for the states satisfying $L_n \mid f\rangle = 0$ ($n \geq 1$). With such a basis of states, analysis of the $a = \infty$ gauge should become easier in various situations.

As an application of our new gauge conditions, we can analyze the problem of tachyon condensation. This problem has been mostly analyzed in the Siegel gauge by using the level truncation method, and various significant results have been obtained. However, the limitations of this analysis have also been recognized. Our one-parameter family of new gauge conditions makes it possible to analyze the problem in particular from the viewpoint of gauge dependence (or independence). In fact, we have analyzed properties of the tachyon potential under various gauges using the level truncation method and have obtained some interesting results, which will be reported in a separate paper.

We have also presented a rather general method to obtain the proper gauge fixed action for a given gauge fixing condition. Using the technique, we should be able to further extend our analysis to more general (covariant or non-covariant) gauge fixing conditions. For example, consider the condition $B_0 \Phi_1 = 0$, which is used to obtain an analytic solution for tachyon condensation in Ref. 9). Here, $B_0$ is the zero mode of the worldsheet $b(\tilde{z})$ field in the coordinate $\tilde{z}$, which is taken to be different from the canonical one. Thus, this condition is the counterpart to the Siegel gauge in this conformal frame. It should be possible to find the exact gauge fixed action for this gauge condition, or to further extend this condition to a one-parameter family. Such an analysis may provide some new insight into the problem related to tachyon condensation.

We have only analyzed open string field theory in the present paper. Extensions
of our new gauge conditions to closed string field theory and superstring field theory is left to future works.

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Appendix A

The Gauge Fixing Condition $bpz(O)\Phi = 0$ for the Extended Action $\tilde{S}$

We now show that the conditions $bpz(O_L)\Phi = 0 \leftrightarrow \text{Eqs. (4.16)-(4.18)}$ and $bpz(O_a)\Phi = 0 \leftrightarrow \text{Eqs. (4.26) and (4.27)}$ are both valid for fixing the gauge symmetry $\tilde{\delta}\Phi = QA$ of the quadratic ($g = 0$) part of the action $\tilde{S}$ if we assume $L_0 \neq 0$. We first consider the condition $bpz(O_a)\Phi = 0$ and confirm that (a) any $\Phi$ can be transformed so as to satisfy $bpz(O_a)\Phi = 0$ and that (b) there is no residual gauge symmetry within the condition. Then we show the same for the condition $bpz(O_L)\Phi = 0$ by recognizing this condition as the $a \to \infty$ limit of the condition $bpz(O_a)\Phi = 0$.

The condition $bpz(O_a)\Phi = 0$ ($a \neq 1$)

(a) For $n \leq 1$, we can show that any $\Phi_n$ is transformed so as to satisfy the condition (4.26) by the extended gauge transformation $\tilde{\delta}\Phi_n = QA_{n-1}$, with the gauge parameters

$$A_{n-1} = \frac{1}{a-1} \frac{1}{L_0} W_{n+2} \left( M^{-n+2}b_0 + a\tilde{Q}M^{-n+1}b_0c_0 \right) \Phi_n. \quad (A.1)$$

To analyze the $n > 1$ case, we first divide $\mathcal{F}^n$ for $n \geq 0$ as

$$\mathcal{F}^n = (\mathcal{F}^n)\tilde{Q} \oplus \mathcal{G}^n, \quad (A.2)$$

where $(\mathcal{F}^n)\tilde{Q}$ consists of states $f^{(n)} \in \mathcal{F}^n$ satisfying $\tilde{Q}f^{(n)} = 0$ and $\mathcal{G}^n$ is the complement of $(\mathcal{F}^n)\tilde{Q}$. Also, we can divide the space $\mathcal{F}^{-n}$ for $n \geq 0$ into $\tilde{Q}\mathcal{F}^{-n-1}$ and its complement, $\mathcal{K}^{-n}$, as

$$\mathcal{F}^{-n} = \left( \tilde{Q}\mathcal{F}^{-n-1} \right) \oplus \mathcal{K}^{-n}. \quad (A.3)$$

If we limit our consideration to states with $L_0 \neq 0$, there is the isomorphism $(\tilde{Q}\mathcal{F}^{-n-1})_{L_0 \neq 0} \sim (\mathcal{G}^n)_{L_0 \neq 0}$, and thus $(\mathcal{K}^{-n})_{L_0 \neq 0} \sim ((\mathcal{F}^n)\tilde{Q})_{L_0 \neq 0}$. The former can be shown using $M^{n+1}\mathcal{F}^{-n-1} = \mathcal{F}^{n+1}$ and $(\mathcal{F}^{n+1})_{L_0 \neq 0} = (\tilde{Q}\mathcal{F}^n)_{L_0 \neq 0} = (\tilde{Q}\mathcal{G}^n)_{L_0 \neq 0}$. Explicit relations between these spaces are as follows:

$$ (M^n\tilde{Q}\mathcal{F}^{-n-1})_{L_0 \neq 0} = (\mathcal{G}^n)_{L_0 \neq 0}, \quad W_n((\mathcal{F}^n)\tilde{Q})_{L_0 \neq 0} = (\mathcal{K}^{-n})_{L_0 \neq 0}. \quad (A.4)$$
From the above discussion, for \( \Phi_n = \phi^{(n-1)} + c_0 \omega^{(n-2)} \) with \( n > 1 \), we can rewrite \( \omega^{(n-2)} \) as

\[
\omega^{(n-2)} = M^{-2} \tilde{Q} f^{(-n+1)} + \bar{\omega}^{(n-2)}, \quad \left( f^{(-n+1)} \in \mathcal{F}^{-n+1}, \ \bar{\omega}^{(n-2)} \in (\mathcal{F}^{-n})^\Lambda \right)
\]

according to Eqs. (A.2) and (A.4). Then, we see that any \( \Phi_n \) is transformed so as to satisfy the condition (4.27) by the gauge transformation if we choose the gauge parameter as

\[
\Lambda_{n-1} = \frac{1}{a - 1} \frac{1}{L_0} M^{-2} \tilde{Q} \left( f^{(-n+1)} + a W_{n-1} \phi^{(n-1)} \right) - \frac{1}{L_0} \bar{\omega}^{(n-2)}. \quad (A.6)
\]

(b) For \( \Phi_n \) (\( n \leq 1 \)), the gauge symmetry under the condition (4.26) is represented by the gauge parameter \( \Lambda_{n-1} = \lambda^{(n-2)} + c_0 \rho^{(n-3)} \) satisfying \((1 - a)M^{-2} \rho^{(n-3)} = 0\), if such exists. This reduces to \( L_0 \lambda^{(n-2)} - \tilde{Q} \rho^{(n-3)} = 0 \) from Eq. (2.14), and thus \( QA_{n-1} = (L_0 + c_0)(L_0 \lambda^{(n-2)} - \tilde{Q} \rho^{(n-3)}) = 0 \), which means that there is no residual symmetry.

For \( \Phi_n \) (\( n > 1 \)), the gauge symmetry under the condition (4.27) is represented by \( \Lambda_{n-1} \) with

\[
\left( W_{n-2} + a \frac{1}{L_0} \tilde{Q} W_{n-1} \tilde{Q} \right) (L_0 \lambda^{(n-2)} - \tilde{Q} \rho^{(n-3)}) = 0. \quad (A.7)
\]

We represent \( L_0 \lambda^{(n-2)} - \tilde{Q} \rho^{(n-3)} \in \mathcal{F}^{n-2} \) by \( h^{(-n+1)} \in \mathcal{F}^{-n+1} \) and \( \tilde{\eta}^{(n-2)} \in (\mathcal{F}^{n-2})^\Lambda \) as

\[
L_0 \lambda^{(n-2)} - \tilde{Q} \rho^{(n-3)} = M^{-2} \tilde{Q} h^{(-n+1)} + \tilde{\eta}^{(n-2)}.
\]

Then, Eq. (A.7) can be rewritten as

\[
(1 - a) \tilde{Q} h^{(-n+1)} + W_{n-2} \tilde{\eta}^{(n-2)} = 0.
\]

When \( a \neq 1 \), this leads to \( h^{(-n+1)} = \tilde{\eta}^{(n-2)} = 0 \). Thus, we conclude that there remains no residual symmetry.

The condition \( \text{bpz}(O_L) \Phi = 0 \) (the \( a \to \infty \) limit of \( \text{bpz}(O_a) \Phi = 0 \))

(a) We can show that any \( \Phi_n \) is transformed so as to satisfy the conditions (4.16) and (4.17) by \( \tilde{\delta} \Phi_n = QA_{n-1} \) with

\[
\Lambda_{n-1} = \begin{cases} 
\frac{1}{L_0} W_{n+2} \tilde{Q} M^{-1} \phi^{(n-1)}, & (n \leq 1) \\
\frac{1}{L_0} M^{-2} \tilde{Q} W_{n-1} \phi^{(n-1)} - \frac{1}{L_0} \bar{\omega}^{(n-2)}, & (n > 1)
\end{cases} \quad (A.8)
\]

Note that the above \( \Lambda_{n-1} \) coincides with \( \lim_{a \to \infty} A_{n-1} \) of Eqs. (A.1) and (A.6), respectively, for \( n \leq 1 \) and \( n > 1 \).

(b) We can prove that there exists no residual gauge symmetry by simply considering the \( a \to \infty \) limit of the analysis for the \( |a| < \infty \) case.
The BPZ conjugation of states is defined by the following equations:

\[ \text{bpz}(|0, p\rangle) = \langle 0, -p|, \quad \text{bpz}(|0\rangle) = |0|; \]

\[ \text{bpz}(b_n) = (-1)^n b_n, \quad \text{bpz}(c_n) = (-1)^{n-1} c_n, \quad \text{bpz}(\alpha^\mu_n) = (-1)^{n-1} \alpha_n^\mu, \]

\[ \text{bpz}(\alpha_\beta) = (-1)^{|\alpha||\beta|} \text{bpz}(\beta) \text{bpz}(\alpha), \]

where \(|\alpha\rangle \) and \(|\beta\rangle\) are the Grassmann parity of \(\alpha\) and \(\beta\), respectively, and \(|\alpha\rangle = 1\) (\(|\alpha\rangle = 0\)) for the Grassmann odd (even) operator \(\alpha\). Note that \(|0\rangle\) is Grassmann even. The (anti-)commutation relations among \(\alpha^\mu_n\), \(c_n\) and \(b_n\) are given by

\[ [\alpha^\mu_n, \alpha^\nu_m] = m \eta^{\mu\nu} \delta_{m+n,0}; \quad \{b_m, c_n\} = \delta_{m+n,0}; \quad \{b_m, b_n\} = \{c_m, c_n\} = 0. \]

The normalization of states that we use is

\[ \langle 0, p; \downarrow |c_0|0, p'; \downarrow \rangle = (2\pi)^{26} \delta(p - p'). \]

For string fields \(A\) and \(B\), we have the relations

\[ Q(A * B) = (QA) * B + (-1)^A A * (QB) \quad \text{(B.1)} \]

and

\[ \langle A, B \rangle = (-1)^{|A||B|} \langle B, A \rangle, \quad \text{(B.2)} \]

which we use for the proof of the gauge or BRST invariance of the action.

**Appendix C**

**Hermiticity of Fields**

Here we explain the assignment of the hermiticity of each field in the string field. Any string field \(\Phi\) can be expanded in terms of the first-quantized states \(|s(p)\rangle\) as

\[ \Phi = \int \frac{d^{26}p}{(2\pi)^{26}} \sum_s |s(p)\rangle \psi^s(p), \quad \text{(C.1)} \]

where \(\psi^s(p)\) is a component field in the momentum representation. Using the fact that the BPZ conjugate of \(|s(p)\rangle\) is related to the hermitian conjugate of \(|s(-p)\rangle\) by a sign factor \(\varepsilon_s\), i.e., \(\langle \text{bpz}(s(p))| \equiv \text{bpz}(|s(p)\rangle) = \varepsilon_s |s(-p)\rangle\), let us consider

\[ \langle \Phi, \mathcal{H}\Phi \rangle = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{d^{26}p'}{(2\pi)^{26}} \sum_{s,s'} \psi^s(p') \langle \text{bpz}(s'(p'))|\mathcal{H}|s(p)\rangle \psi^s(p). \quad \text{(C.2)} \]

Then, for any hermitian operator \(\mathcal{H}(= \mathcal{H}^\dagger)\) diagonal in \(p^\mu\), we have

\[ \langle \Phi, \mathcal{H}\Phi \rangle = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{d^{26}p'}{(2\pi)^{26}} \sum_{s,s'} \psi^s(p') \varepsilon_s \langle s'(-p')|\mathcal{H}|s(p)\rangle \psi^s(p) \quad \text{(C.3)} \]
\[ \langle \Phi, \mathcal{H}\Phi \rangle^* = \int \frac{d^{26}p}{(2\pi)^{26}} \sum_{s,s'} (\psi^s(p))^* (\varepsilon_{s'} \langle s'(p)|\mathcal{H}(p)|s(p)\rangle \psi^s(p)), \] (C.4)

\[ \langle \Phi, \mathcal{H}\Phi \rangle^* = \int \frac{d^{26}p}{(2\pi)^{26}} \sum_{s,s'} (\psi^s(p))^* (\varepsilon_{s'} \langle s'(p)|\mathcal{H}(p)|s(p)\rangle)^* (\psi^s(-p))^* \] (C.5)

\[ = \int \frac{d^{26}p}{(2\pi)^{26}} \sum_{s,s'} (\psi^s(p))^* \langle s(p)|\mathcal{H}(p)|s'(p)\rangle \varepsilon_{s'} (\psi^s(-p))^* \] (C.6)

\[ = \int \frac{d^{26}p}{(2\pi)^{26}} \sum_{s,s'} (\psi^s(p))^* \varepsilon_{s} \langle \text{bpz}(s(-p))|\mathcal{H}(p)|s'(p)\rangle \varepsilon_{s'} (\psi^s(-p))^*. \] (C.7)

Thus, we have \( \langle \Phi, \mathcal{H}\Phi \rangle^* = \langle \Phi, \mathcal{H}\Phi \rangle \) if we assign \( (\psi^s(p))^* = \varepsilon_{s} \psi^s(-p) \) or equivalently \( (\psi^s(x))^* = \varepsilon_{s} \psi^s(x) \) in the coordinate representation. Also, this assignment is equivalent to the relation \( \text{bpz}(\Phi) = \Phi^* \). This can be immediately applied to the kinetic term of the string field by taking \( Q \) as \( \mathcal{H} \).

The sign factor \( \varepsilon_s \) for a state

\[ |s(p)\rangle = \alpha_{-n_1} \cdots \alpha_{-n_i} c_{-l_1} \cdots c_{-l_j} b_{-m_1} \cdots b_{-m_k} |0,p; \downarrow \] (C.8)

with

\[ 0 < n_1 \leq n_2 \leq \cdots \leq n_i, \ 0 \leq l_1 < \cdots < l_j, \ 0 < m_1 < \cdots < m_k \]

is explicitly computed as

\[ \varepsilon_s = (-)^{N+i-k+\frac{(j+k)^2}{2}}, \] (C.9)

where \( N = \sum a_n + \sum b_l + \sum c_m \) is the level of the state. Note that if the state is twist even \((N = \text{even})\), scalar \((i = \text{even})\) and of ghost number 1 \((j = k, (j+k)^2/2 = \text{even})\), then it is always the case that \( \varepsilon_s = 1 \). This is used in the analysis of tachyon condensation in the level truncation.

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