Numerical analysis of space charge in a point cathode thermionic emission gun *

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Space charge in a point cathode thermionic emission gun has been studied with numerical method. The method is an iterative one, consisting of the field determination using the integral form of Poisson’s equation, the direct ray tracing in 3D, and the estimation of space charge from the traced rays. The rays are traced for limited emitting conditions. The estimated space charge is treated as a number of the coaxial charged rings with different radii and positions. Using the method, the potential distribution around the cathode tip was examined at different cathode temperatures. The numerical results at the cathode temperatures of 2800 K and 2950 K are given, and the influence of space charge is discussed. [DOI: 10.1380/ejssnt.2003.147]

Keywords: Electron emission; Thermionic emission; Space charge; Computer simulations; Poisson equation; Electron Microscopy

I. INTRODUCTION

The emission property of thermionic cathode is influenced by space charge. In the conventional hairpin cathode gun, the emission current density or the beam brightness is limited by the space charge when the cathode temperature is raised above 2700 K. The space charge is caused by low electric field in front of the cathode. The field can be increased by the use of the point cathode having a sharply etched tip. The point cathode gun provides the space charge free emission up to higher temperatures [1]. Measurements showed that the emission current density or the beam brightness of the point cathode gun is maximized at the wehnelt bias shift of several volts from the cutoff [2]. Such a small bias shift means that the emission is restricted in a narrow area of the cathode tip. There is the space charge limited region near the cathode tip. So, the space charge can modify the potential distribution around the cathode tip in the high brightness operation.

Numerical method is needed to study the space charge effects in practical gun geometry. Because the space charge distribution is initially unknown, it has to be determined by tracing electron rays. Then, a self-consistent result of Poisson’s equation has to be obtained with iterative method. A rigorous estimation requires a large number of the traced rays, considering the energy and angular distributions of thermally emitted electrons [3]. This leads to a very long computing time, when the integral equation method, such as the surface charge method or the boundary element method (BEM) based on Green’s function, is used for the field determination.

This paper describes the numerical method developed for the analysis of the space charge in a tungsten point cathode gun. The method uses the integral form of Poisson’s equation for the field determination. The electron rays are traced for limited emitting conditions. The space charge is estimated from the rays and treated as a number of the coaxial charged rings. The method and results are described.

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II. NUMERICAL METHOD

A. Electrode geometry

The electrode geometry analyzed is shown in Fig. 1. The geometry is that for the gun we have developed. In this gun, the tip part of the cathode is locally heated by a focused electron beam from the heater assembly placed inside the wehnelt cylinder [2]. The heater assembly is not considered in the numerical calculation. The cathode is a tungsten filament of 0.1 mm in diameter. It has the tip radius of 0.4 µm. The tip is placed at a height of 0.3 mm from the wehnelt aperture. The aperture size is 1.2 mm in diameter. The distance between the cathode tip and the anode is 9.2 mm. The gun is operated at the anode voltage of 50 kV. For the field determination, each electrode surface is divided into small sections. The tip part of the cathode is divided into smaller sections to keep accuracy in the field determination near the tip. The number of the divided surface sections is 200 in total.

FIG. 1: Electron gun. (a) Three electrodes and (b) the enlarged view of the tip part of the cathode. The cathode is a tungsten wire of 0.1 mm in diameter. The cathode tip radius is 0.4 µm. The tip is placed at a height of 0.3 mm from the wehnelt aperture. The aperture size is 1.2 mm in diameter.
The integral form for Poisson’s equation is expressed as

\[
\Phi(r) = \frac{1}{4\pi \varepsilon_0} \int_S \frac{\sigma(r')}{R'} dS' + \frac{1}{4\pi \varepsilon_0} \int \int_V \frac{\rho(r'')}{R''} dV'',
\]

(1)

where \( \Phi(r) \) is the potential at a position of \( r \), \( \sigma(r') \) is the surface charge density distribution on the electrode \( S' \), \( R' = |r' - r| \), \( \rho(r'') \) is the space charge density distribution, and \( R'' = |r'' - r| \). The electric field is given by the following equation:

\[
E(r) = -\frac{1}{4\pi \varepsilon_0} \int \int_S \sigma(r') \nabla \left( \frac{1}{R'} \right) dS - \frac{1}{4\pi \varepsilon_0} \int \int_V \rho(r'') \nabla \left( \frac{1}{R''} \right) dV''.
\]

(2)

When the surface and space charge density distributions are obtained, the potential and the field at any position can be determined by numerical integration of them. The numerical calculation is carried out using the integral forms for rotational symmetric electrode system.

Fig. 2 represents the iterative method for solving Poisson’s equation. The method consists of the estimation of the surface charge, the ray tracing and the estimation of the space charge. The space charge distribution is initially unknown, so the iteration is started with no space charge, \( \rho(r'') = 0 \). Then, the electron rays are traced, and the space charge distribution is estimated from the rays. In the next iteration, the surface charge is recalculated considering the space charge potential, the last term of Eq. (1). The iteration is continued until it gives a self-consistent result.

1. Estimation of surface charge

The surface charge density in the absent of the space charge is calculated with the surface charge method [4,5]. In the method, the surface charge density on each divided section of the electrode surfaces is assumed to be constant and it is determined by using the relation,

\[
\Phi_i = \sum_{j=1}^{j_{\text{max}}} F_{ij} \sigma_j,
\]

(3)

Where \( \Phi_i \) is the potential of the \( i \)-th divided section, \( j_{\text{max}} \) is the total number of the surface sections, and \( F_{ij} \) is the integration of the electrode geometry function as

\[
F_{ij} = \frac{1}{4\pi \varepsilon_0} \int \int_S \frac{1}{r' - r_j} dS',
\]

(4)

and \( \sigma_j \) is the surface charge density of the \( j \)-th section. Eq. (3) is derived putting the position vector \( r \) on the midpoint of \( i \)-th section. Applying Eq. (3) to all the divided sections, we obtain linear equations for the surface charge densities. The surface charge densities are determined by solving the linear equations.

When the space charge potential is taken into account, Eq. (3) is modified as follows,

\[
\Phi_i - \frac{1}{4\pi \varepsilon_0} \int \int_V \frac{\rho(r'')}{|r''' - r_i|} dV'' = \sum_{j=1}^{j_{\text{max}}} F_{ij} \sigma_j.
\]

(5)

The second term is the space charge potential on the \( i \)-th divided section, which can be calculated from the space charge distribution and the electrode geometry. The surface charge densities in the presence of the space charge are determined by solving linear equations derived from Eq. (5).
FIG. 3: Potential distribution and the traced rays for the estimation of the space charge. (a) Potential distribution at the wehnelt bias $V_B = -184$ V in the absence of space charge. Equipotential lines are at 0.2 V step. (b) The traced rays from the cathode area between $z = 0$ and -35 $\mu$m. The half of the traced rays is given. The view of 50 $\mu$m square area. The cutoff bias $V_{B\text{cutoff}} = -190$ V.

2. Ray tracing

The rays are traced from the cathode area from $z = 0$ (the tip apex) to $z = -35\mu$m. The area is chosen to be larger than the emission area: we use the term of the emission area as a meaning that the electrons emitted from this area arrive at the anode. The cathode area is divided into 183 sections with different widths of $\Delta z$. From a middle point of each section, four rays are started into different directions with the emitting energy of $kT$ ($k$ Boltzmann’s constant, $T$ the cathode temperature); the energy is the most probable value of Maxwell-Boltzmann distribution. The emitting angles are put to $\alpha = 0^\circ$ (normal) and $45^\circ$. At $\alpha = 45^\circ$, the electrons are started into the azimuths of $\beta = 0^\circ$ and $180^\circ$ (in the meridional plane), and $\beta = 90^\circ$. The ray of $\beta = 270^\circ$ is not traced, because it has the same values of $(z(t), r(t))$ as the ray emitting into the direction of $\beta = 90^\circ$. The total number of the traced rays is 732 in the present study. The equation of motion in 3D [6] is solved with Adams-Moulton’s method.

3. Estimation of space charge

The distribution of space charge is estimated from the rays. The space around the cathode, in the range from $z = -40\mu$m to $z = 10\mu$m, is divided into small volume elements of $\Delta V'' = 2\pi r \Delta r \Delta z$ with the widths $\Delta r = \Delta z = 0.2\mu$m. The charge in each volume element is obtained by using the relation

$$\Delta Q = -\sum i_e \Delta t,$$  \hspace{1cm} (6)

where $i_e$ the current of the ray passing through the element, $\Delta t$ is the time of flight within the element. The current $i_e$ is estimated from the Richardson-Dushman equation, putting the Richardson constant $A = 80$ A/cm$^2$K$^2$ and the work function $\phi = 4.5$eV.

In the potential and field calculations, the space charge $\Delta Q$ of each volume element is treated as a coaxial charged ring. The ring is placed at the center of each volume element. The integration of the space charge in Eqs. (1) and (2) is evaluated using an analytical potential for a ring charge [7] and its derivatives.

The space charge is overestimated in the first iteration. This causes the fluctuation of the space charge distribution in the subsequent steps of iteration. To reduce the fluctuation, the charge of each ring is replaced with its mean value after fourth step of iteration. The space charge estimated from the rays of only the emitting energy $kT$ is smaller than the expected value. So, the iteration gives the emission current larger than the measured value. To avoid an additional ray tracing and a long computing time, the ring charges are multiplied by a weight factor 2 after fourth step of iteration. The iterative method provides the emission current near the measured one after about eight repetitions. The number of iteration depends on the bias voltage or the emission area of the cathode.

III. RESULTS AND DISCUSSION

The calculation was done at different wehnelt bias conditions and at different cathode temperatures. The cutoff bias $V_{B\text{cutoff}}$ is found to be around -190 V for the given electrode geometry. Figs. 3 (a) and (b) show the potential distribution (0.2 V step) around the cathode tip and the traced rays in the absence of space charge: the bias voltage $V_B = -184$ V (the bias shift is 6 V from the cutoff). The rays are those traced with the emitting energy...
0.241 eV, which corresponds to the most probable energy $kT$ at $T=2800$ K. Some of the rays pass over the tip portion of the cathode. They are the electrons emitted into $\beta = 90^\circ$ or $270^\circ$ at $\alpha = 45^\circ$. At the conical shank part, most of the rays are repelled due to negative potential. These rays cause a dense space charge. Figs. 4 (a) and (b) represent the potential distribution (0.05 V step) and the traced rays near the cathode tip and the positions of the coaxial rings for the space charge estimation. The total number of the rings amounts to about 7000 in this case, which depends on the bias voltage and the cathode temperature (the emitting energy).

The comparison of the potential distributions at different cathode temperatures are shown in Fig. 5: (a) the potential distribution in the absence of space charge ($T=0$), (b) and (c) the potential distributions when the space charges at the cathode temperature of 2800 K and 2950 K are taken into account. The wehnelt bias voltage $V_B = -184$ V. The equipotential lines are at intervals of 0.2 V. The zero-volt line intersects the cathode surface. As increasing the cathode temperature, the zero-volt line moves toward the cathode tip, and the potentials at the conical shank part becomes more negative. The variations indicate the space charge effects on the emission area. The emission current density is 5.0 A/cm$^2$ at 2800 K, which increases to 14.3 A/cm$^2$ at 2950 K.

Table 1 shows the emission area $z$ of the cathode (the tip $z = 0$) at different operating conditions. The area was estimated from the emitting positions of the rays that pass through the wehnelt aperture and arrive the anode space. At the bias condition of $V_B - V_{B \text{ cutoff}} = 5$ V, the area is 11.5 $\mu$m when the space charge effect is not considered. The area decreases to 9.1 $\mu$m at 2800 K and to 7.9 $\mu$m at 2950 K, when the space charge is taken into account. The numerical results show that the space charge effects on the emission area become important at the cathode temperatures above 2800 K.

Different methods utilized for the numerical analysis of the space charge in a triode gun are described in ref. [3], together with some difficulties that appear in a rigorous estimation of the space charge from the traced rays. In the present study, the space charge was treated as many coaxial charged rings, and the charges were estimated from the rays traced for limited emitting conditions. The validity of the method was examined by using the total emission current. The total emission current was obtained by summing up the currents of the rays passing through the wehnelt aperture. The current was compared to the measured value by plotting these values as a function of $(V_B - V_{B \text{ cutoff}})$. Comparison showed that the method provides the total emission current in good agreement with the measured value.

### IV. CONCLUSIONS

The space charge in the point cathode thermionic emission gun has been analyzed with the numerical method. The method uses the integral form of Poisson’ equation for the field determination. The space charge is estimated from the rays traced for limited emitting conditions, and
FIG. 5: Influence of the space charge on the potential distribution near the cathode tip. (a) T=0 K (no space charge), (b) T=2800 K and (c) 2950 K at the wehnelt bias $V_B=-184$ V. Equipotential lines are at intervals of 0.2 V. The view of 40 µm square area. The cutoff bias $V_{B\text{ cutoff}} = -190$ V.

treated as a number of coaxial charged rings. Using the method, the potential distribution around the cathode tip was examined at different bias conditions and at different cathode temperatures. The results show that the emission area is influenced by space charge at the cathode temperatures above 2800 K. The method provides useful information about the space charge effects in the gun.

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