On Generalized $N^*$-Closed Set in NANO-$N^*$Topological Spaces With Some Properties

Nabila I. Aziz$^1$ and Taha H. Jasim$^2$

$^1$Department of Mathematics, College of Computer Science and Mathematics, Tikrit University, Iraq
$^2$Department of Mathematics, College of Computer Science and Mathematics, Tikrit University, Iraq

1nabila.be@tu.edu.iq
2tahahameed@tu.edu.iq

Abstract: In this paper a class of sets called generalized $N^*$-closed set introduced and its properties were studied. Further the notation of relationships among them were discussed.

Keywords: $N^*R\alpha -o(x)$, $N^*R\beta -o(x)$, $N^*g\alpha -c(x)$, $N^*g\beta -c(x)$, $N^*Rg\alpha -c(x)$, $N^*g\beta -c(x)$, $N^*\alpha g-c(X)$, $N^*g\beta -c(X)$, $N^*\alpha - c(X)$.

1. Introduction

At 1970, Levine N. [4] presented the Concept of Generalized Closed sets as a Generalization of closed sets in Topological Spaces. Thereafter In 2011, Bhattacharya, S. [1] have presented the notation of Generalized Regular Closed Sets in Topological Space. In 2014 R.T. Nachiyar and K. Bhuvaneswari [8] introduced Nano generalized $\alpha$-closed sets and Nano $\alpha$-generalized closed sets in Nano topological spaces. In 2016 Qays Hatem, Murtadha M. Abdulkadhim and Mustafa H. Hadi, [7] introduced Nano Generalized Alpha Generalized Closed Sets in Nano Topological Spaces. After that in 2017 Rajasekaran, M. Mearin and O. Nethaji [9], are introduced nano $g\beta$-closed sets. In 2019 S. Sathyapriya, S. Kamali and M. Kousalya [11] introduced $N^*g\alpha$-closed sets in Nano Top s. After that in 2018 M. Hosny [5], introduced Nano $\delta\beta$-open sets and studied the Nano $\delta\beta$-continuity. Lellis Thivagar [12] introduced the notation of Nano Topology. Which was defined in terms of approximations, Boundary region of a subset of an Universe using an Equivalence Relation on it as well defined $N-c(x)$, $\text{nint}(B)$ and $\text{ncl}(B)$. Nasir A. Murad, Taha H. Jasim [6] present the concept of $N^*$-open set by means of in Nano Top s.. Moreover In this Paper presented and defined a new Class of Sets Called Generalized $N^*$-Closed set We also introduce the notation of relation among them are discussed and study its properties.
2. Preliminaries

We recall some important definition and some basic concept which our work needed it.

**Definition 2.1** [5]: Let $U$ be a non-empty finite set of elements, said to be the universe and $R$ be an equivalence relation on $U$ named as the indiscernibility relation. The elements in the same class, are called indiscernible with one another. Then $(U, R)$ is said to be the approximation space. Let $X \subseteq U$.

A) The lower approximation of $X$ with respect to $R$ is a collection of all elements which can be for certain classified as $X$ with respect to $R$ and denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{R(X) \subseteq X} R(X)$.

B) The upper approximation of $X$ with respect to $R$ is a collection of all elements that can be possibly classified as $X$ with respect to $R$ and denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{R(X) \cap X \neq \emptyset} R(X)$.

C) The boundary region of $X$ with respect to $R$ is a collection of all elements that can be classified as $X$ nor as not $X$ with respect to $R$ denoted by $B_R(X)$.

**Definition 2.2** [12]: Let $U$ be the universe, $R$ be an equivalence relation on $U$ and $X \subseteq U$. The Nano topology on $U$ with respect to $X$ is defined by $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ and $(U, \tau_R(X))$ is called the Nano topological space. The elements of $\tau_R(X)$ are called Nano open sets and the complement of any Nano open set $V$ is denoted by $n\beta(V)$.

3- The intersection of the elements of any finite Sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

**Definition 2.3** [12]: Let $(U, \tau_R(X))$ be a Nano topological space with respect to $X$, $X \subseteq U$. A subset $P \subseteq U$:

i. The Nano interior of $P$ is defined as the Union of all Nano open subsets contained in $P$ and is denoted by $n\text{int}(P)$.

ii. The Nano closure of $P$ is defined as the Intersection of all Nano closed subsets containing $P$, and is denoted by $n\text{cl}(P)$.

**Definition 2.4** [12, 10]: Let $(U, \tau_R(X))$ be a Nano topological space, $K \subseteq U$. The set $K$ is called:

a) Nano Regular open, if $K = n\text{int}(n\text{cl}(K))$.

b) Nano $\alpha$-open, if $K \subseteq n\text{int}(n\text{cl}(K))$.

c) Nano $\beta$-open if $K \subseteq n\text{cl}(n\text{int}(n\text{cl}(K)))$. The family of all Nano regular open (respectively, Nano $\alpha$-open and Nano $\beta$-open) sets in a Nano topological space $(U, \tau_R(X))$ is denoted by $nRO(U, X)$ (respectively, $n\alpha(U, X)$ and $n\beta(U, X)$).

**Definition 2.5** [6]: So be it $(U, \tau_R(X))$ be a Nano topological space, then all $M \subseteq U$ is said to be Nano-$N^*$-open sets if $M$ satisfies the conditions

i. $\forall s \in M, \exists N\alpha$-open set $F$ such that $s \in F \subseteq N\text{Cl}(M)$.

ii. If $M \cap \{U_R(X)\} \neq \emptyset$ then $U_R(X) \subseteq M$ and the set of all Nano-$N^*$-open (briefly $N^*$-open) of $U$ with respect to $X$ denoted by $\tau^*_R(X)$.

**Example 2.6** [6]: $\omega = \{r, s, p\}$ with $\omega / R = \{\{r\}, \{s, p\}\}$ and $X = \{s, p\}$. Then $\tau_R(X) = \{\omega, \emptyset, \{s, p\}\}$.

**Definition 2.7** [2]: Let $(U, \tau_R(X))$ be a Nano topological space, a subset $\mu$ of $(\omega, \tau_R(X))$ is said to be Nano generalized closed set (briefly Ng-closed) if $n\text{cl}(\mu) \subseteq V$ where $\mu \subseteq V$ where $V$ is Nano open set.
3. On Generalized N*-Closed Set in NANO-N*Topological Spaces With Some Properties

Now we introduced a new concepts in a Nano N*-topological space and some of their properties with many examples.

**Definition 3.1:** \((\omega, \tau_{N*}(X))\) is a Nano-N*-space, \(\kappa \subseteq \omega\) then \(\kappa\) is called a N\(^*\)Regular \(\beta\)-open (briefly, N\(^*\)R\(\beta\)-open) set. If \(\exists\) a N Regular open set \(G\) such that \(G \subseteq \kappa \subseteq N\^*\)cl\((G)\). The Family of all N\(^*\)Regular \(\beta\)-open sets of \(\omega\) symbolized by N\(^*\)R\(\beta\)o(x).

**Example 3.2:** Let \(\varphi = \{a, b, c, d\}\), \(\varphi / R = \{\{b, d\}, \{c\}, \{a\}\}\), \(X = \{b, d\}\)

\(\tau_{N}(\omega) = \{\emptyset, \varphi, \{b, d\}, \{\{b, d\}\}, \{c\}, \{a\}\}\), \(\tau_{N*}(X) = \{\emptyset, \varphi, \{b, d\}, \{\emptyset, \varphi, \{b, d\}\}, \{c\}, \{a\}\}\)

N\(^*\)\(\beta\)-oc(x) = \(\{\emptyset, \varphi, \{b, d\}, \{\emptyset, \varphi, \{b, d\}\}, \{c\}, \{a\}\}\), \(N\^*\)\(\alpha\)oc(x) = \(\{\emptyset, \varphi, \{b, d\}, \{\emptyset, \varphi, \{b, d\}\}, \{c\}, \{a\}\}\)

Thus A = \(\{b, d\}\), \(G = \{b, d\}\), N\(^*\)cl\((G)\) = \(G \subseteq \kappa \subseteq N\^*\)cl\((G)\), \(\kappa = \{\emptyset\}\) is N\(^*\)R\(\beta\) -open set.

**Proposition 3.5:** All N\(^*\) regular \(\alpha\)-open set is N\(^*\) regular \(\beta\)-open set and the converse is not true.

**Example 3.3:** Let \(\varphi = \{a, b, c, d\}\), \(\varphi / R = \{\{b, d\}, \{c\}, \{a\}\}\), \(X = \{b, d\}\)

\(\tau_{N}(\omega) = \{\emptyset, \varphi, \{b, d\}, \{\{b, d\}\}, \{c\}, \{a\}\}\), \(\tau_{N*}(X) = \{\emptyset, \varphi, \{b, d\}, \{\emptyset, \varphi, \{b, d\}\}, \{c\}, \{a\}\}\)

N\(^*\)\(\alpha\)oc(x) = \(\{\emptyset, \varphi, \{b, d\}, \{\emptyset, \varphi, \{b, d\}\}, \{c\}, \{a\}\}\), \(N\^*\)\(\beta\)oc(x) = \(\{\emptyset, \varphi, \{b, d\}, \{\emptyset, \varphi, \{b, d\}\}, \{c\}, \{a\}\}\)

Then A = \(\{b, d\}\) is N\(^*\)R\(\beta\)-open set and A is not N\(^*\)R\(\alpha\)-open set.

**Proposition 3.6:** All N\(^*\) regular \(\alpha\)-open set is N\(^*\) regular \(\alpha\)-open set and the converse is not true.

**Example 3.8:** Let \(\varphi = \{r, h, j\}\) with \(\varphi / R = \{\{r\}, \{h, j\}\}\) and X = \(\{h, j\}\), then \(\tau_{R}(\omega) = \{\emptyset, \varphi, \{h, j\}\}\) and N\(^*\)-Closed sets are \(\{\varphi, \{r\}\}\). N\(^*\)\(\alpha\)oc(x) = \(\{\varphi, \{r\}\}\), \(T_{N*}(X) = \{\{r\}, \{h, j\}\}\)

Hence W is N\(^*\)\(\alpha\)oc(x).

**Proposition 3.9:** All N\(^*\) Regular \(\alpha\)-Open set is N\(^*\)\(\beta\)-Open set. And the converse is not true.

Since every N\(^*\)R\(\beta\)oc(x) is N\(^*\)\(\alpha\)oc(x) And \(\nabla\) N\(^*\)\(\beta\)oc(x) \(\nabla\) N\(^*\)\(\alpha\)oc(x).
Now we introduce the diagram to explain the relations among them.

**Proposition 3.10** [6]: Every $N^*\alpha$ -open set is $N^*\beta$ -open set.

**Proposition 3.11**: Every $N^*$ Regular $\beta$ - open set is $N^*\beta$ - open set. But the converse not be true.

**Proof**: Let $M$ is $N^* R\beta$ -open(x). To prove $M$ is $N^*\beta$-open(x).

Since every $N^* R\beta$ O(x) is $N^*\alpha$(x) and every $N^*\alpha$-o(x) is $N^*\beta$-o(x) [6].

Hence $M$ is $N^*\beta$ -open set.

Recall Example 3.6: Let $\varphi=\{a, b, c, d\}$, $\varphi \cap R=\{b, d\}, \{c\}, \{a\}$, $X=\{b, d\}$

$\tau_{R(X)}=\{\emptyset, \varphi, \{b, d\}, \{c\}, \{a\}\}$, $N^*\alpha(x)=$ $\{\emptyset, \varphi, \{b, d\}, \{c\}, \{a\}\}$

$N^*\beta$-o(x)=$\{\varphi, \emptyset, \{b, d\}, \{c\}, \{a\}\}$, $N^*\alpha$-o(x)=$\{\emptyset, \varphi, \{b, d\}, \{c\}, \{a\}\}$

**Proposition 3.12**: A subset $B$ of $(U, \tau_{R(X)})$ is called a $N^*$ Regular $\alpha$ -closed (briefly $N^*R\alpha$-c(x)) set in a $N^*$ nano Top. s iff $G \subseteq N^*\alpha Cl \kappa(\alpha)$ Whenever $\kappa \subseteq G$ and $G$ is a $N^*\alpha$-o(x).

Recall Example 3.8: $\varphi=\{r, h, j\}$ with $\varphi \cap R=\{\{r\}, \{h, j\}\}$ and $X=\{h, j\}$, then $\tau_{R(X)}=\{\emptyset, \varphi, \{h, j\}\}$

$\tau_{R(X)}=\{\varphi, \emptyset, \{h, j\}, \{h\}\}$, $N^*\alpha(x)=\{\varphi, \emptyset, \{h, j\}, \{h\}\}$

$N^*\beta$-c(x)=$\{\varphi, \emptyset, \{h, j\}, \{h\}\}$, $N^*\alpha$-c(x)=$\{\varphi, \emptyset, \{h, j\}, \{h\}\}$.

**Definition 3.13**: A subset $B$ of $(U, \tau_{R(X)})$ is called a $N^*$ Regular Generalized $\alpha$ -closed (Briefly $N^*Rg\alpha$-c(x)) set in a $N^*$ nano Top. s iff $\forall V \subseteq N^*\alpha Cl \kappa(\alpha)$ Whenever $\forall \subseteq V$, $V$ is a $N^*\alpha$-open set.

Recall Example 3.8: $\varphi=\{r, h, j\}$ with $\varphi \cap R=\{\{r\}, \{h, j\}\}$ and $X=\{h, j\}$, then $\tau_{R(X)}=\{\emptyset, \varphi, \{h, j\}\}$

$\tau_{R(X)}=\{\varphi, \emptyset, \{h, j\}, \{h\}\}$, $N^*\alpha(x)=\{\varphi, \emptyset, \{h, j\}, \{h\}\}$

$N^*\alpha$-c(x)=$\{\varphi, \emptyset, \{h, j\}, \{h\}\}$, $N^*\beta$-c(x)=$\{\varphi, \emptyset, \{h, j\}, \{h\}\}$.

**Definition 3.14**: A subset $B$ of $(U, \tau_{R(X)})$ is called a $N^*$ Generalized $\beta$ -Closed (Briefly $N^*g\beta$-c(x)) set in a $N^*$ nano Top. s iff $\forall V \subseteq N^*\beta cl(B)$ Whenever $\forall \subseteq V$, $V$ is a $N^*\beta$-open set.

Recall Example 3.8: $\varphi=\{r, h, j\}$ with $\varphi \cap R=\{\{r\}, \{h, j\}\}$ and $X=\{h, j\}$, then $\tau_{R(X)}=\{\emptyset, \varphi, \{h, j\}\}$

$\tau_{R(X)}=\{\varphi, \emptyset, \{h, j\}, \{h\}\}$, $N^*\beta$-c(x)=$\{\varphi, \emptyset, \{h, j\}, \{h\}\}$

$N^*\alpha$-c(x)=$\{\varphi, \emptyset, \{h, j\}, \{h\}\}$, $B=\{r, j\}$, $V=\{r, j\}$
Definition 3.15: A subset B of \( \varphi, \tau_{R(X)} \) is said to be N*Regular Generalized \( \beta \) -Closed (Briefly \( N*\text{RG}_\beta(c(x)) \)) set in a N* nano topo. s iff \( V \subseteq N*\text{cl}(B) \). Whenever \( B \subseteq V \), \( V \) is a N*R\( \beta \)-open set.

Recall Example 3.8: \( \varphi = \{ r, h, j \} \) with \( \varphi/R = \{ \{ r \}, \{ h, j \} \} \) and \( X = \{ h, j \} \), \( \tau_{R(X)} = \{ \varphi, \{ h \}, \{ j \}, \{ h, j \} \} \). \( N*\text{RG} \beta (x) = \{ \varphi, \{ h \}, \{ j \}, \{ h, j \}, \{ r, h, j \} \} \).

\[ R(X) = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \} \} = N*\text{co}(x), \quad N*\text{RG} \beta (x) = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \}, \{ r, h, j \} \} \]

\[ N*\text{RG} \beta -o(x) = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \} \} \]

Proposition 3.16: Every \( N* \alpha -c(x) \) is \( N* \beta -c(x) \). The opposite is not true.

Proof: Let \( B \) be \( N* \alpha -c(x) \) \( \Rightarrow V \subseteq N*\text{cl}(B) \), \( B \subseteq V \) and \( V \) is a \( N* \alpha -\) open set. Since \( N*\text{cl}(B) \subseteq N*\text{cl}(B) \) \( \Rightarrow G \subseteq N*\text{cl}(B) \). And since every \( N* \alpha -\) open set is \( N* \)-open set [6]. Thus \( B \) is \( N* \beta -c(x) \).

Recall example 3.6: Let \( \varphi = \{ a, b, c, d \} \), \( \varphi/R = \{ \{ b, d \}, \{ c \}, \{ a \} \} \) and \( X = \{ b, d \} \).

\[ \tau_{R(X)} = \{ \varphi, \emptyset, \{ b, d \}, \{ c \}, \{ a \}, \{ a, b, c, d \}, \{ a, b, d \} \} \]

\[ N*\text{RG} \beta (x) = \{ \varphi, \emptyset, \{ b, d \}, \{ c \}, \{ a \}, \{ a, b, c, d \}, \{ a, b, d \} \} \]

Proposition 3.17: Every \( N* \text{RG} \beta -c(x) \) is \( N* \alpha -c(x) \). The opposite is not true.

Proof: Let \( B \) be \( N* \text{RG} \beta -c(x) \) \( \Rightarrow V \subseteq N*\text{cl}(B) \), \( B \subseteq V \) and \( V \) is a \( N* \text{RG} \beta -o(x) \).

Recall Example 3.8: \( \varphi = \{ r, h, j \} \) with \( \varphi/R = \{ \{ r \}, \{ h, j \} \} \) and \( X = \{ h, j \} \).

\[ \tau_{R(X)} = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \} \} = N*\text{co}(x), \quad N*\text{RG} \alpha (x) = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \} \} \]

\[ N*\text{RG} \beta -o(x) = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \} \} \]

Proposition 3.18: Every \( N* \text{RG} \alpha -c(x) \) is \( N* \text{RG} \beta -c(x) \). The opposite is not true.

Proof: Let \( M \) be \( N* \text{RG} \alpha -c(x) \) \( \Rightarrow V \subseteq N*\text{cl}(M) \), \( M \subseteq V \) and \( V \) is a \( N* \text{RG} \alpha -o(x) \).

Recall Example 3.8: \( \varphi = \{ r, h, j \} \) with \( \varphi/R = \{ \{ r \}, \{ h, j \} \} \) and \( X = \{ h, j \} \).

\[ \tau_{R(X)} = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \} \} = N*\text{co}(x), \quad N*\text{RG} \alpha (x) = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \} \} \]

\[ N*\text{RG} \beta -o(x) = \{ \varphi, \emptyset, \{ h \}, \{ j \}, \{ h, j \} \} \]

Proposition 3.19: All \( N* \text{RG} \alpha -c(x) \) is \( N* \text{RG} \beta -c(x) \). The opposite is not true.

Proof: Let \( M \) be \( N* \text{RG} \alpha -c(x) \) \( \Rightarrow V \subseteq N*\text{cl}(M) \), \( M \subseteq V \) and \( V \) is a \( N* \text{RG} \alpha -o(x) \).

Recall Example 3.6: Let \( \varphi = \{ a, b, d \} \), \( \varphi/R = \{ \{ b, d \}, \{ c \}, \{ a \} \} \) and \( X = \{ b, d \} \).

\[ \tau_{R(X)} = \{ \varphi, \emptyset, \{ b, d \}, \{ c \}, \{ a \} \} \]

\[ N*\text{RG} \beta -o(x) = \{ \varphi, \emptyset, \{ b, d \}, \{ c \}, \{ a \}, \{ a, b, c \}, \{ a, a, d \}, \{ a, b, d \} \}

Proposition 3.20: Every \( N* \text{RG} \beta -c(x) \) is \( N* \text{RG} \beta -c(x) \). The opposite is not true.

Proof: Let \( S \) be a \( N* \text{RG} \beta -c(x) \) \( \Rightarrow V \subseteq N*\text{cl}(S) \), \( S \subseteq V \) and \( V \) is a \( N* \text{RG} \beta -o(x) \). The opposite is not true.
That is $S$ is $N^\ast g\beta -c(x).$

Recall Example 3.8: $\phi = \{ r, h, j \}$ with $\phi / R = \{ [r], [h, j] \}$ and $X = [h, j], \tau_{R(x)} = \{ \phi, \sigma, [h, j] \}, \tau_{R(X)} = \{ \phi, \sigma, [h, j] \} \Rightarrow V = [h, j] \Rightarrow V \subseteq N^\ast \beta -c(x)$ but $S$ is not $N^\ast Rg\beta -c(x)$ because $\{ h, j \}$ is not $N^\ast R -o(x).$

Now we introduce the diagram to explain the relations among them

**Diagram 2.2**

![Diagram 2.2](image)

**Definition 3.21:** A subset $B$ of $(\phi, \tau_{R(X)})$ is said to be $N^\ast$Generalized-Closed (Briefly $N^\ast g-c(x)$) set in a $N^\ast$ nano Top. $s$ iff $N^\ast cl(B) \subseteq V.$ Whenever $B \subseteq V$ and $V$ is a $N^\ast$-open set.

**Proposition 3.25:** All $N^\ast g\alpha -c(x)$ is $N^\ast g\beta -c(x).$

Proof: Let $B$ is $N^\ast g\alpha -c(x)$ iff $B \subseteq N^\ast cl(B),$ $B \subseteq V$ and $V$ is a $N^\ast$-open.

Since $N^\ast cl(B) \subseteq N^\ast cl(B),$ $V \subseteq N^\ast cl(B),$ that is $B$ is $N^\ast g\beta -c(x).$

The opposite of this proposition is not true. The counter Example shows this.

**Example 3.26:** Let $\phi = \{ r, p, q, s \}$ with $\phi \setminus R = \{ [r], [p], [q, s] \}$ and $X = \{ s, q, r \}, \tau_{R(X)} = \{ \phi, \sigma, [q, s], [r, q], \{ r, q, s \}, \{ r, q, s, [r, q], [r, q], [r, s], [q, s], [p, s], [r, p], [q, p], [p, q], [r, q], \} \} = N^\ast o(x).$

**Example 3.27:** Let $\phi = \{ r, p, q, s \}$ with $\phi \setminus R = \{ [r], [p], [q, s] \}$ and $X = \{ s, q, r \}, \tau_{R(X)} = \{ \phi, \sigma, [q, s], [r, q], \{ r, q, s \}, \{ r, q, s, [r, q], [r, q], [r, s], [q, s], [p, s], [r, p], [q, p], [p, q], [r, q], \} \} = N^\ast c(x).$
Proposition 3.27: \( N^{*}g-c(x) \) with \( N^{*}g^c(x) \). The relationship among them is independent.

Recall Example 3.8: \( \varphi = \{ r, h, j \} \) with \( \varphi / R = \{ \{ r \}, \{ h \}, \{ a, c \} \} \) and \( X = \{ b, d \} \), then \( \tau_{R_{\varphi}} = \{ \{ \phi, \varphi, \{ b, d \}, \{ a, c \} \} \} \).

\( N^{*}_{\varphi} - c(x) \) and \( B \) is Not \( N^{*}g^c(x) \). But \( N^{*}_{\varphi} cl(B) \) is independent.

Example 3.28: Let \( \varphi = \{ a, b, c, d \} \), \( \varphi / R = \{ \{ b, d \}, \{ a, c \} \} \), \( X = \{ b, d \} \).

\( N^{*}_{\varphi} - o(x) = \{ \{ b, d \}, \{ a, c \} \} \).

\( N^{*}_{\varphi} cl(B) \) is not \( N^{*}_{\varphi} g^c(x) \). The relationship among them is independent.

Proposition 3.29: All \( N^{*}_{\varphi} g^c(x) \) is \( N^{*}_{\varphi} g^c(x) \). Proof: Let \( B \) is \( N^{*}_{\varphi} g^c(x) \).

The opposite of this proposition is not true. The counter Example shows this.

Recall Example 3.43: Let \( \varphi = \{ a, b, c, d \} \), \( \varphi / R = \{ \{ b, d \}, \{ a, c \} \} \).

\( N^{*}_{\varphi} - o(x) = \{ \{ b, d \}, \{ a, c \} \} \).

\( N^{*}_{\varphi} cl(B) \) is not \( N^{*}_{\varphi} g^c(x) \). But \( B \) is not \( N^{*}_{\varphi} g^c(x) \).

Theorem 3.30: \( N^{*}_{\varphi} g^c(x) \) and \( N^{*}_{\varphi} g^c(x) \) are independent.

Proof: Let \( B \) is \( N^{*}_{\varphi} g^c(x) \).

Conversely: Let \( B \) is \( N^{*}_{\varphi} g^c(x) \).

Proposition 3.31: \( N^{*}_{\varphi} g^c(x) \) with \( N^{*}_{\varphi} g^c(x) \). The relationship among them is independent.

Recall Example 3.43: Let \( \varphi = \{ a, b, c, d \} \), \( \varphi / R = \{ \{ b, d \}, \{ a, c \} \} \).

\( N^{*}_{\varphi} - o(x) = \{ \{ b, d \}, \{ a, c \} \} \).

\( N^{*}_{\varphi} cl(F) = F \) is not \( N^{*}_{\varphi} g^c(x) \), because \( N^{*}_{\varphi} cl(F) \) is not \( N^{*}_{\varphi} g^c(x) \).
References

[1] Bhattacharya S 2011 Study On Generalized Regular Open Sets in Ordinary Topological Space, India, Department of Mathematics, Tripura University, Suryamaninagar, 99130.

[2] K Bhuvaneswari and K Mythili Gnanapriya 2014 Nano Generalized Closed sets in Nano topological spaces, International Journal of Scientific and Research Publications, Volume 4, Issue 5.

[3] K Bhuvaneswari and K Mythili Gnanapriya 2014 Nano Generalized pre Closed sets and Nano Pre Generalized Closed Sets in Nano topological spaces, International Journal of Innovative Research in Science, Engineering And Technology, Volume 3 Issue 10.

[4] Levine N 1970 Generalized closed sets in Topology, Rend. Circ. Math. Palermo, 19, 89-96.

[5] M Hosny 2018 Nano $\delta\beta$-open sets and nano $\delta\beta$-continuity, J. Egypt. Math. Soc. 26(2), 365-375.

[6] Nasir A Murad and Taha H Jasim 2020 N* Open Sets Via Nano Topological Spaces, A Thesis Submitted to The Council of the College of Computer Science and Mathematics University of Tikrit In Partial Fulfillment of the Requirement for the Degree of Master of Science in Mathematics.

[7] Qays Hatem Imran, Murtadha M. ABDULLAKDHIEM and Mustafa H. HADI 2016 On Nano Generalized Alpha Generalized Closed Sets in Nano Topological Spaces, Gen. Math. Notes, ICSRS Publication, Vol. 34, No. 2, pp. 39-51.

[8] R T Nachiyar and K Bhuvaneswari 2014 On nano generalized $\alpha$-closed sets and nano $\alpha$-generalized closed sets in nano topological spaces, International Journal of Engineering Trends and Technology, 13(6), 257-260.

[9] Rajasekaran M. Meharin and O. Nethaji 2017 On nano $g\beta$-closed sets, International Journal of Mathematics And its Applications, 5(4-C), 377–382.

[10] Revathy A. and Ilango G 2015 On nano $\beta$-open sets, Int. J. Eng. Contemp. Math. Sci. 1(2), 1–6.

[11] S Sathyapriya, S. Kamali and M. Kousalya 2019 On Nano $Ag^s$-Closed Sets in Nano Topological Spaces International Journal of Innovative Research in Science, Engineering and Technology Vol. 8, Issue 2.

[12] Thivagar M L, Richard C 2013 On Nano forms of weakly open sets, Int. J. Math. Stat. Inven. 1(1), 31–37.