Abstract

We develop a model for γ-ray emission from the outer magnetosphere of pulsars (the outer-gap model). The charge depletion causes a large electric field which accelerates electrons and positrons. We solve the electric field with radiation and pair creation processes self-consistently, and calculate curvature spectrum and Inverse-Compton (IC) spectrum. We apply this theory to PSR B0833-45 (Vela) and B1706-44 for which their surface magnetic fields, observed thermal X-rays are similar to each other. We find that each observed cut-off energies of the γ-rays are well explained. By inclusion of emission outside the gap, the spectrum is in better agreement with the observations than the spectrum arising only from the inside of the gap. The expected TeV fluxes are much smaller than that observed by CANGAROO group in the direction of B1706-44.

1 Introduction

The Compton Gamma Ray Observatory has detected seven γ-ray pulsars. The observed light curves and energy spectra have been used to discriminate
possible radiation models. Moreover, the next-generation γ-ray space telescopes and the ground based Cherenkov telescopes will further constrain to the models.

The pulsed γ-rays imply that the electrons and positrons are accelerated up to about 10 TeV in the magnetosphere. For the outer-gap model (Chen, Ho & Ruderman, 1986), Hirotani & Shibata (1999, HS) solved the accelerating electric field with curvature radiation and pair creation processes self-consistently. They showed that the electric field along the magnetic field does not extend to the light cylinder, and calculated curvature spectra can explain the EGRET observations in the GeV bands. However, the gap emission could not explain the observations in the MeV band. Hence, to improve the HS model, we take account of the radiation from the outside of the gap and compare the corrected spectrum with the observations.

2 One dimensional model

In this section, we introduce the HS model and represent the Poisson eq. which describes the accelerating electric field, the continuity eqs. for particles and γ-rays, following HS.

2.1 Basic equations

We deal with the structure along the magnetic field lines. If the gap width (W) along the magnetic field is much less than the light radius (vlc), we can approximate the magnetic field lines as straight lines in the gap, and the electric field structures can be treated as one-dimensional, where the arc length from the surface along the last-closed line is denoted by s: we can write down the Poisson equation as

\[ \frac{dE_{\parallel}}{ds} = 4\pi e \left( N_+ - N_- - \frac{\rho_{GJ}}{e} \right), \tag{1} \]

where \( E_{\parallel} \) is the electric field along the magnetic field, \( N_+ (N_-) \) is the electron (positron) number density, and \( \rho_{GJ} \) is the Goldreich-Julian (GJ) charge density. Above equation describes that the charge depletion relative to \( \rho_{GJ} \) causes \( E_{\parallel} \).
By taking account of the electron-positron pair creation process, the continuity equations for particles and $\gamma$-rays are

$$\pm B \frac{d}{ds} \left( \frac{N_\pm}{B} \right) = \frac{1}{c \cos \Psi} \int_0^\infty d\epsilon [\eta_p G_+ + \eta_c G_-],$$

(2)

$$\pm B \frac{d}{ds} \left( \frac{G_\pm}{B} \right) = -\eta_{p\pm} G_\pm + \eta_c N\pm \frac{c}{c \cos \Psi},$$

(3)

where $G_+$ ($G_-$) is the distribution function of outward (inward) propagating $\gamma$-rays, $\epsilon$ refers to the photon energy in units of the electron’s rest mass energy, $B$ is the magnetic field strength, $\eta_{p\pm}$ is the pair-creation rate, $\eta_c$ is the emissivity of curvature radiation, and $\Psi$ is the angle between the particle’s motion and the meridional plane. We describe $G_\pm$ in several energy bins and represent then as $G_{\pm i}$ ($i = 1, 2, ...$). The particle’s Lorentz factor in the gap is obtained by assuming that particle’s motion immediately saturated at the balance between the electric and the radiation reaction forces, i.e.,

$$\Gamma_{sat} = \left( \frac{3 R_c^2}{2e E_{||}} \right)^{1/4},$$

(4)

where $R_c$ is the curvature radius of dipole magnetic field lines.

We impose some boundary conditions. The inner ($s_1$) and outer ($s_2$) boundaries are defined so that $E_{||}$ vanishes, i.e., $E_{||}(s_1) = E_{||}(s_2) = 0$. We assume that the $\gamma$-rays do not come into the gap through the boundaries, $G_{\pm i}(s_1) = G_{\pm i}(s_2) = 0$ ($i = 1, 2, ..., m$). We allow the particles to come into the gap. The particle flux is given by the non-dimensional parameters $j_1$ and $j_2$ as follows:

$$\frac{N_+(s_1)}{\Omega B(s_1)/2\pi c e} = j_1, \quad \frac{N_-(s_2)}{\Omega B(s_2)/2\pi c e} = j_2.$$  

(5)

The particle continuity equation (2) yields

$$\frac{N_+(s)}{\Omega B(s)/2\pi c e} + \frac{N_-(s)}{\Omega B(s)/2\pi c e} = \text{const along } s = j_{tot}.$$  

(6)

From eqs. (4) and (5), the current carriers created in the gap per unit flux tube is $j_{gap} = j_{tot} - j_1 - j_2$. The total current should be determined by the global condition which includes pulsar wind. Therefore, we use ($j_{tot}, j_1, j_2$) as free model parameters.
Table 1: Observed Parameters

| Pulsar     | Distance ($\Omega$) | $B_{12}$ | $kT_s$ | Ref.       |
|------------|---------------------|----------|--------|------------|
| Vela       | 0.5                 | 70.6     | 3.4    | 150        | Ogelman et. al. |
| B1706-44   | 1.8(DM)/2.5(HI)     | 61.6     | 3.1    | 143        | Gotthelf et. al. |

DM: Dispersion Measure, HI: HI absorption

2.2 X-ray & infrared (IR) field

Because the $\gamma\gamma$ pair-creation process is important in the outer magnetosphere, we need the X-rays for the target-photons in our case. In the present paper, we use the observed black body radiation from the pulsar surface for the X-rays (Table 1). We also need the IR field to calculate IC flux. The IR field is inferred from optical and radio observations for Vela, X-ray and radio observations for B1706-44 with a single power-low, because there are no available IR observations.

3 Radiation from outside of the gap

Near the boundaries, the real Lorentz factor of the accelerated particles must be larger than $\Gamma_{sat}$ given by eq. (4), because the particle’s cooling time for the radiation becomes larger than the crossing time for the gap. So, the particles come out from the gap with $\Gamma_{out} \sim a few times 10^7$ and emit $\gamma$-rays outside of the gap. Since the typical damping length for the curvature radiation is

$$l_{dam} = \frac{3m_ec^2R_c^2}{2e^2\Gamma^3} = 0.4\omega_{lc} \left( \frac{\Omega}{100\text{rads}^{-1}} \right)^{-1} \left( \frac{\Gamma}{10^7} \right)^{-3} \left( \frac{R_{oc}}{0.5\omega_{lc}} \right)^2,$$

(7)

the $\gamma$-ray radiation from the outside of the gap are also important unless $W \sim \omega_{lc}$.

We apply the our model to Vela and B1706-44 with the observed parameters in Table 1. We adopt $(j_{tot}, j_{1}, j_{2}) = (0.201, 0.191, 0.001)$ and $a_{inc} = 45^\circ$ for the angle between axes of rotation and magnetization for both pulsars.
Figure 1: The accelerating field for Vela (solid-line) and B1706-44 (dotted-line for dis=1.8kpc and dashed-line for dis=2.5kpc). \((j_{\text{tot}}, j_1, j_2) = (0.201, 0.191, 0.001)\) and \(a_{\text{inc}}=45\) deg.

4 Result

4.1 Electric field structure

The calculated \(E_\parallel\) for both pulsars are shown in Fig.1. The gap width \(W\) is shorter than \(\varpi_{lc}\). \(W\) is characterized by the pair-creation mean free path, which is given approximately by \(W \propto c / (\int \eta_p - \eta_c d\epsilon)^{1/2}\) by using the fact \(\eta_p \ll \eta_c\), due to the difference in the collision angle between \(\gamma\)-ray and X-ray. For Vela and B1706-44, one finds the mean free path to be shorter than \(\varpi_{lc}\).

We find that Vela has a nearer distance from the surface to the gap and larger calculated \(E_\parallel\) than B1706-44 when we adopt the same \((j_{\text{tot}}, j_1, j_2)\) and \(a_{\text{inc}}\). This is because Vela has the shorter rotation period and larger GJ density in the gap than B1706-44. The dependence of the assumed distance from the earth to B1706-44 is also shown in Fig.1. In general, if we adopt a nearer distance to the pulsar, the electric field becomes large, because the decrease in the estimated X-ray luminosity from the observations extends the gap width.
Figure 2: (a): γ-ray spectrum for Vela. The total spectrum (solid-line) includes the radiation from the outside of the gap (dots-line) as well as gap emission (dashed-line). (b): Total γ-ray spectrum for B1706-44. The dependence of distance is shown. The IC spectrum is also shown in figure.

4.2 Gamma-ray spectra

The calculated spectrum of outward propagating γ-rays radiated in the gap for Vela is shown in Fig.2(a) as dashed-line. The spectral cut-off around GeV is responsible for the acceleration limit. We find that this spectrum is in agreement with the EGRET observations (Thompson et.al. 1999) in the GeV bands. In the MeV bands, however, it is inconsistent with the observations. This is because the value of $\Gamma_{\text{sat}}$ makes the curvature spectrum with $E^2 F \propto E^\alpha, \alpha \sim 4/3$ in the MeV bands, although the observations have $\alpha \sim 1/3$.

In §3, we have pointed out that if $W \ll \omega_{lc}$, the curvature radiation from the outside of the gap is important. In the outside of the gap where $E_\parallel$ is vanished, the particles lose their energy by the radiation, the spectrum of which extends to the MeV band (dotted-line in Fig.2(a)). By inclusion of this emission, the total spectrum (solid-line in Fig.2(a)) is in good agreement with the EGRET observations.

The calculated total γ-ray spectrum for B1706-44 is shown in Fig.2 (b). We find that the calculated peak energy becomes slightly less than Vela because $E_{\parallel B1706} < E_{\parallel Vela}$ (§§4.1), and this peak energy also explains the observations. As mentioned in §§4.1, since the calculated $E_\parallel$ becomes large as
we assume the nearer distance to the pulsar, the spectrum becomes hard if we adopt a nearer distance. For 1.8 kpc, the calculated spectrum appears to be consistent with the observations. However, we must assume a large cross-section area of the gap, $A_{\perp} = (10W)^2 \sim (\varpi_{lc})^2$, to obtain the observed fluxes. Because the gap locates at $s \sim 0.5\varpi_{lc}$ (Fig.1), such $A_{\perp}$ should be unrealistic.

Fig.2 (b) also shows the calculated IC spectrum from the gap. The sharp-cut off in the spectrum which corresponds to the acceleration limit appears around 10TeV. Since IR flux might be overestimated, TeV flux would be less than $\sim 10^{-12}\text{erg cm}^{-2}\text{s}^{-1}$. Moreover, this calculated flux hardly depends on the distance to the pulsar. On these ground, we conclude that it is difficult to explain the unidentified TeV components observed by CANGAROO group (Kushida et.al., 2002) in the direction of B1706-44 with this model.

5 Discussion

In summary, we obtained the spectrum in good agreement with the EGRET observations for Vela by inclusion of the curvature radiation from the outside of the gap. We found that the observed peak energies for Vela and B1706-44 may imply that the almost the same currents in units of GJ value are running through the gap for both pulsars.

From Fig.2 (a), we recognize that the calculated spectrum is inconsistent with the COMPTEL observations. If we try to explain this observations, we need the very small curvature radius as compared with the dipole, which is unlikely. Therefore, this MeV emission will be obtained by inclusion of the synchrotron emission by pairs.

In §§4.2, we showed that we need very large $A_{\perp}$ to explain the observations of B1706-44. This may be due to the small $j_{tot}$, about 20% of the GJ current. However, if we adopt the nearly ($j_{tot} \sim 1$) or super ($j_{tot} > 1$) GJ current with this model, the gap width will be quenched.

Quite recently, Hirotani et.al. (2002) showed that the value of the $\Gamma_{sat}$ given by eq.(4) and also $\gamma$-ray fluxes calculated with this $\Gamma_{sat}$ are overestimated, because particles in the gap do not immediately saturate. But, the general features of the gap model and the result that we must take account of the radiation from the outside of the gap to explain the observations above 100MeV are not altered.

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