Λ(1405) IN THE BOUND STATE SOLITON MODEL

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ABSTRACT

The strong and electromagnetic properties of the Λ(1405) hyperon are studied in the framework of the bound state soliton model. We explicitly evaluate the strong coupling constant $g_{Λ^∗NK}$, the Λ* magnetic moment, mean square radii and radiative decay amplitudes. The results are shown to be in general agreement with available empirical data. A comparison with results of other models is also presented.

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1 Introduction

The Λ(1405) resonance is one of the most poorly understood states amongst low-mass baryons. Originally treated as a \(KN\) bound system \([1]\) it was later argued to have a more natural description in terms of a conventional 3-quark state. However, in most of the quark model calculations its rather light mass has been quite hard to describe \([2]\). Within this picture one expects the mass of the Λ(1405) and of the Λ(1520) to be very close since they are “LS–partners” and LS splittings are known to be small, at least in the non–strange sector. Only by introducing further assumptions, like e.g. three–quark interactions or “ad hoc” meson–quark interactions \([3]\) the Λ(1405) mass can be brought into agreement with the empirical value. Similar problems are found within bag model calculations \([4]\). In fact, a cloudy bag model analysis \([5]\) seems to indicate that the Λ(1405) is mostly a meson-baryon bound state.

Unfortunately, up to now only very little empirical information is available about the Λ(1405) properties. This situation is very likely to change soon with the anticipated completion of CEBAF. In fact, one experiment to study the electromagnetic decays of the Λ(1405) using that facility has been already approved \([6]\). Proposals to use other planned experimental facilities for the study of low-lying excited hyperon properties have been recently put forward as well \([7]\).

Given this renewed interest in the understanding of the structure of the Λ(1405) (in what follows we will also use the notation \(Λ^*\)) a comprehensive study of its properties in a soliton model is highly desirable. The aim of this paper is to calculate the Λ\(^*\) strong and electromagnetic properties in the context of the bound-state soliton model \([8, 9]\). In this model, the Λ(1405) resonance has a natural explanation as a bound state of a kaon in the background potential of the soliton. Because of the particular form of the effective interaction potential the \(l = 1\) partial wave has a lower bound state energy than the \(l = 0\) wave. In fact, positive parity low-lying hyperons (e.g. the Λ hyperon) are obtained by populating the \(P\)–wave bound state while the \(S\)–wave soliton-kaon bound system describes the Λ(1405). The present work also complements previous bound state model studies of strong coupling constants \([10]\) and electromagnetic properties \([11, 12, 13]\) where only ground state hyperons were considered. This will allow a more detailed comparison with empirical data as well as with other model calculations.

The paper is organized as follows: in Sec. 2 we briefly describe the main features of the bound state soliton model; in Sec. 3 we discuss the calculation of the strong coupling constant. Sec. 4 is devoted to the electromagnetic properties (magnetic moments, mean square radii, radiative decays). Finally, the conclusions are given in Sec. 5.
2 General Formalism

In this section we outline the necessary formalism for the description of the Λ* properties. As the bound state soliton model is rather well known by now, we will refer to previous publications whenever possible.

The effective $SU(3)$ chiral action with an appropriate symmetry breaking term can be written as
\[
\Gamma = \int d^4x \left\{ -\frac{f^2}{4} \text{Tr}(L_\mu L^\mu) + \frac{1}{32\pi^2} \text{Tr}[L_\mu L_\nu]^2 \right\} + \Gamma_{\text{WZ}} + \Gamma_{\text{sb}}. \tag{1}
\]
Here $\Gamma_{\text{WZ}}$ is the non–local Wess-Zumino action and $\Gamma_{\text{sb}}$ is the symmetry breaking term. Their explicit form can be found, for instance, in Refs. [14, 15]. In Eq.(1) the left current $L_\mu$ is expressed in terms of the chiral field $U$ as $L_\mu = U^\dagger \partial_\mu U$.

In the spirit of the bound state soliton model, we introduce the Callan–Klebanov ansatz [8]
\[
U = \sqrt{U_\pi} U_K \sqrt{U_\pi}, \tag{2}
\]
where
\[
U_K = \exp \left[ i \frac{\sqrt{2}}{f_K} \begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix} \right], \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \tag{3}
\]
and $U_\pi$ is the soliton background field written as a direct extension to $SU(3)$ of the $SU(2)$ field $u_\pi$, i.e.,
\[
U_\pi = \begin{pmatrix} u_\pi & 0 \\ 0 & 1 \end{pmatrix}, \tag{4}
\]
with $u_\pi$ being the conventional hedgehog solution $u_\pi = \exp[i\tau \cdot \hat{r} F(r)]$.

According to the usual procedure, one expands up to the second order in the kaon field. The Lagrangian density can therefore be rewritten as a pure $SU(2)$ Lagrangian depending on the chiral field only and an effective Lagrangian, describing the interaction between the soliton and the kaon fields
\[
\mathcal{L} = \mathcal{L}_{SU(2)}(u_\pi) + \mathcal{L}(K, u_\pi). \tag{5}
\]
The explicit form of $\mathcal{L}(K, u_\pi)$ can be found in Ref. [12]. Therefore in the bound state approach, the net effect of the harmonic expansion is the reduction to an effective Lagrangian describing a kaon moving in the background field of the soliton. The problem now consists in looking for possible bound states, i.e. in solving the eigenvalue equation for the meson field $K$ in the potential field of the $SU(2)$ soliton. The bound state solutions to this wave equation represent stable hyperon states. Upon a
mode decomposition of the kaon field in terms of the grand spin $\Lambda = L + T$ (where $L$ represents the angular momentum operator and $T$ is the isospin operator), the Lagrangian $\mathcal{L}(K, u_\pi)$ yields a wave equation of the form:

$$\left[-\frac{1}{r^2} \frac{d}{dr}(r^2 h \frac{d}{dr}) + m_K^2 + V_{\text{eff}}^{\Lambda,l} - f \omega_{\Lambda,l}^2 - 2 \lambda \omega_{\Lambda,l}\right] k_{\Lambda,l}(r) = 0. \quad (6)$$

Here $\omega_{\Lambda,l}$ is the bound state energy for given $(\Lambda, l)$. The radial functions $h, f, \lambda$ and $V_{\text{eff}}^{\Lambda,l}$ are functions of the chiral angle $F(r)$ only. The explicit form of these functions is given in the Appendix. As customary the chiral angle is determined by minimization of the $\mathcal{L}_{SU(2)}$ Lagrangian density in Eq.(5).

It has been shown [9, 15] that for a typical set of lagrangian parameters there are only two bound states in the strange sector. One bound state lies in the $\Lambda = 1/2, l = 1$ channel and the other in the $\Lambda = 1/2, l = 0$ channel. Since both states have $\Lambda = 1/2$, in what follows we will label any quantity that depends on the channel quantum numbers using only the corresponding value of the angular momentum $l$. From the behaviour of the effective potential at short distances the $l = 0$ eigenstate is expected to lie at higher energy than the $l = 1$ state. Numerical calculations confirm this fact. Therefore, by populating the $l = 1$ state one obtains the positive parity octet and decuplet hyperons. On the other hand the $l = 0$ state describes the $\Lambda(1405)$ resonance. Table I shows some typical results for the eigenenergies together with the definition of Set I and Set II parameters which will be adopted throughout the paper.

By naively adding once the value of the bound state energy $\omega_l$ to the soliton mass one obtains only the centroid mass of the $S = -1$ hyperons. The splittings among hyperons with different spin and/or isospin are given by the rotational corrections, introduced according to the time–dependent rotations:

$$u_\pi \rightarrow Au_\pi A^\dagger, \quad K \rightarrow AK. \quad (7)$$

This transformation adds an extra term to the Lagrangian

$$\delta \mathcal{L} = \mathcal{L}_{\text{rot}}(u_\pi, K, K^\dagger, A, A^\dagger), \quad (8)$$

which is of order $1/N_c$. For the particular case of $\Lambda$ and $\Lambda^*$ hyperons the mass formula which takes into account these rotational corrections can be written as

$$M = M_{\text{sol}} + \omega_l + \frac{3}{8\Omega} c_l^2. \quad (9)$$

Here, $M_{\text{sol}}$ and $\Omega$ are the soliton mass and moment of inertia, respectively. $c_l$ is the hyperfine splitting constant. Its explicit form for the cases of interest in this paper
can be easily obtained from the general form given in Ref. [15]. The corresponding numerical values are given in Table I.

As reported in Ref. [15], positive parity hyperons are well described in the present model. For example, the calculated mass of the ground state Λ(1116) is 1086 MeV using Set I parameters and 1105 MeV with Set II parameters. On the other hand the Λ(1405) predicted mass is 1297 MeV using Set I parameters, and 1325 MeV with Set II parameters. This results are roughly 100 MeV too low with respect to the experimental masses. The situation contrasts with typical quark model predictions, where the Λ* mass comes out too high in energy.

3 The Strong Coupling Constant

Due to its vicinity to the KN system mass, the Λ(1405) resonance plays a dominant role in the analysis of processes such as K−p → Λγ and K−p → Σ0γ. In these analyses the coupling constant gKpΛ* is usually considered an adjustable parameter [16].

In the bound state soliton model, the KpΛ* coupling constant can be explicitly calculated from the effective interaction Lagrangian upon projection of the hedgehog solution and of the kaon field onto states of proper spin and isospin. Since we are interested in the matrix elements between a Λ* state (rotating soliton–kaon bound system) and a final state composed by a nucleon (rotating soliton) and a free kaon, the adiabatic rotation is performed only on the SU(2) hedgehog and the bound kaon. Following the general guidelines given in Ref. [10] and introducing the identities that relate operators in the “collective” representation to those expressed in terms of the conventional spin and isospin representation

\[ \langle \Lambda^*| A^\dagger| NK \rangle = \frac{-i}{\sqrt{8\pi}} \langle \Lambda^*| - \mathbf{1}| NK \rangle, \]

\[ \langle \Lambda^*| \tau A^\dagger| NK \rangle = \frac{-i}{\sqrt{8\pi}} \langle \Lambda^*| - \mathbf{\sigma}| NK \rangle, \]

one obtains the corresponding interaction vertex at threshold:

\[ \frac{g_{\Lambda^*KN}}{\sqrt{4\pi}} = \frac{1}{\sqrt{2}} \int dr r^2 k_0 \left[ f m_K \omega_0 + \lambda (m_K + \omega_0) - m_K^2 - V_{eff}^0 \right]. \]

As already mentioned, the functions f and λ are defined in Appendix. V_{eff}^0 can be obtained from the general expression given in Appendix, replacing the values of \( \Lambda = 1/2 \) and \( l = 0 \).

At this stage it should be noticed that there is a certain ambiguity in the definition of the KpΛ* coupling within our model. In general, for a negative parity
resonance the pseudoscalar and pseudovector couplings have the form

\[ \mathcal{L}_{PS} = g_{\Lambda^*N K} \, \bar{u}_{\Lambda^*} \, u_{N} \, K, \]  
(12)

\[ \mathcal{L}_{PV} = -i \, \frac{g_{\Lambda^*N K}}{M_{\Lambda^*} - M_N} \, \bar{u}_{\Lambda^*} \gamma_\mu \, u_{N} \, \partial^\mu K. \]  
(13)

Integrating by parts Eq.(13) and using the free Dirac equation of the $\Lambda^*$ and $N$ one would obtain Eq.(12). Therefore both forms of the coupling are the same for free baryons. However, if we now perform the non-relativistic reduction of these interaction Lagrangians at threshold we get expressions which are somewhat different. From the pseudoscalar interaction we obtain

\[ \mathcal{L}_{PS} = g_{\Lambda^*N K} \, \chi^\dagger_{\Lambda^*} \, \chi_N \, K, \]  
(14)

where $\chi$ are the baryon spinors. On the other hand the non-relativistic reduction of the pseudovector interaction reads

\[ \mathcal{L}_{PV} = g_{\Lambda^*N K} \, \frac{m_K}{M_{\Lambda^*} - M_N} \, \chi^\dagger_{\Lambda^*} \, \chi_N \, K. \]  
(15)

Similar differences are known to happen in the case of the strong couplings of the ground state $\Lambda$ \[17\]. Therefore if only the non-relativistic form of the interaction Lagrangian is known (as it is the case in the bound state soliton model) there is no unique definition of the $g_{\Lambda^*N K}$. In writing Eq.(11) we have assumed a pseudoscalar coupling, corresponding to the reduction (14). Had we assumed a pseudovector form for the interaction lagrangian, the r.h.s. of Eq.(11) would have had to be multiplied by $\frac{M_{\Lambda^*} - M_N}{m_K} \simeq 0.94$. Since this factor is quite close to unity the difference in the numerical results is not very significant.

Numerical evaluation of Eq.(11) gives $g_{\Lambda^*N K} = 1.6$ for Set I parameters and $g_{\Lambda^*N K} = 2.2$ for Set II. These values are within the range of typical empirical results quoted in the literature \[18\]. Moreover, in a very recent analysis of the empirical $KN$ scattering lengths \[19\] the value $g_{\Lambda^*N K} \simeq 1.9$ has been obtained. On the other hand, chiral bag model calculations \[20, 21\] yield a smaller value $g_{\Lambda^*N K} = 0.46$.

4 The Electromagnetic Properties

The electromagnetic properties can be derived entirely from the electromagnetic current

\[ J^{\text{e.m.}}_\mu = J^3_\mu + \frac{1}{\sqrt{3}} J^8_\mu \]  
(16)

obtained from the effective Lagrangian Eq.(1) by means of the Noether theorem. Once the Callan–Klebanov ansatz is used, the current is naturally divided into isovector
and isoscalar parts. The first one contributes to the Λ∗ magnetic moment and mean square radii. It also describes the Λ∗ → Λγ decay (ΔI = 0). The isovector component is responsible for the Λ∗ → Σ0γ decay (|ΔI| = 1).

4.1 Magnetic Moments

The standard expression for the magnetic moment operator reads:

$$\mu = \frac{1}{2} \int d^3r \mathbf{r} \times \mathbf{J}_{\text{em}}.$$  \hspace{1cm} (17)

Since the Λ∗ is an isoscalar resonance, there is no purely solitonic contribution. Namely, only the term in the isoscalar piece of the current which is quadratic in the kaon field contributes. A straightforward calculation leads to an expression for the Λ∗ magnetic moment of the form:

$$\mu_{\Lambda^*} = \frac{1}{2} \left( c_0 \mu_{s,sol} - a(k_0) \right), \hspace{1cm} (18)$$

where the isoscalar soliton magnetic moment $\mu_{s,sol}$ and $a(k_0)$ are

$$\mu_{s,sol} = -\frac{2M_N}{3\pi \Omega} \int dr r^2 \sin^2 \mathbf{F} \mathbf{F}', \hspace{1cm} \text{Eq. (19)}$$

$$a(k_0) = \frac{4}{3} M_N \int dr r^2 \left\{ k_0^2 \sin^2 \frac{F}{2} + \frac{1}{4e^2 f_K^2} \left[ k_0^2 \sin^2 \frac{F}{2} (F'^2 + \frac{4 \sin^2 \frac{F}{2}}{r^2}) - 3k_0 k'_0 \sin \mathbf{F} \mathbf{F}' \right] \right\}. \hspace{1cm} \text{Eq. (20)}$$

Here, $M_N$ is the nucleon mass (the magnetic moment is expressed in Bohr magnetons). In comparing these contributions with those corresponding to the ground state Λ (see Eqs.(27-32) in Ref. [12]) we notice that the fact that the kaon is bound in the $l = 0$ channel modifies not only the expression of the hyperfine splitting constant but also the explicit form of $a(k_i)$. This is because although both states are bound in the same grand-spin channel their isospin and spatial structure depend on the angular momentum $l$.

In Table II, the Λ and Λ∗ magnetic moments as calculated in our model are presented. As usual results are given with respect to the calculated proton magnetic moment. This is due to the well known fact that although soliton models predict a somewhat small absolute value for the proton magnetic moment, they describe magnetic moments ratios quite accurately [11, 22]. The predictions for the Λ magnetic moment have already been given elsewhere [11, 12] and are quoted here only to serve as reference. We note that results for the Λ∗ are somewhat less dependent on the parameter sets than in the Λ case. We also observe that in contrast to the case of the
ground state $\Lambda$, we predict a small and positive magnetic moment for the $\Lambda^*$. This is the result of two effects. On one hand the hyperfine splitting constant is larger in the $l = 0$ channel than in the $l = 1$ channel. On the other hand, the value of $a(k_0)$ is smaller than that of $a(k_1)$. This second effect can be understood by noting that although the quadratic contributions (i.e. first line in Eq. (20)) to $a(k_l)$ have roughly the same value in both channels the quartic contributions are much smaller in the $l = 0$ case. Since in the integrand of the quartic term contributions the functions that depend on the chiral angle are peaked at short distances and the $l = 0$ wave function is peaked at a larger radius than the $l = 1$ one, this kind of behaviour is to be expected.

**4.2 Magnetic and Electric Radii**

The magnetic mean square radius of the $\Lambda^*$ can be obtained by integrating the magnetization density of Eqs. (19, 20) weighted with an extra $r^2$ and normalized with a proper coefficient [11, 13]. The magnetic radius then reads

$$\langle r^2 \rangle_{M, \Lambda^*} = \frac{1}{2\mu_{\Lambda^*}} \left( c_0 \langle r^2 \rangle_{s,sol} - \langle r^2 \rangle_a \right),$$

where

$$\langle r^2 \rangle_{s,sol} = -\frac{2M_N}{5\pi\Omega} \int dr r^4 \sin^2 FF',$$

$$\langle r^2 \rangle_a = \frac{4}{5} M_N \int dr r^4 \left\{ k_0^2 \sin^2 \frac{F}{2} ight.$$

$$+ \frac{1}{4\epsilon^2 f_K} \left[ k_0^2 \sin^2 \frac{F}{2} \left( F'^2 + \frac{4 \sin^2 F}{r^2} \right) - 3k_0k_0' \sin FF' \right] \}.$$

The factor $\frac{1}{5}$ instead of the factor $\frac{1}{3}$ in Eqs. (22–23) derives naturally from the normalization of the magnetic form factor in the limit of zero momentum transfer.

For the isoscalar $S = -1$ hyperons the electric mean square radii can be simply written as [11, 13]

$$\langle r^2 \rangle_E = \frac{1}{2} \left[ \langle r^2 \rangle_B - \langle r^2 \rangle_S \right],$$

where the elementary radii are defined as

$$\langle r^2 \rangle_B = -\frac{2}{\pi} \int dr r^2 \sin^2 FF',$$

$$\langle r^2 \rangle_S = 2 \int dr k_l^2 r^4 [f \omega_l + \lambda].$$

Numerical values of the calculated magnetic and electric radii are given in Table II. We observe that the predicted $\Lambda^*$ magnetic radii are much larger that in the $\Lambda$
case. This can be understood as follows. In the case of the ground state \( \Lambda \) there is a partial cancellation between the \( c_1 \langle r^2 \rangle_{s,sol} \) contribution and the \( \langle r^2 \rangle_a \) term. As a result of this the corresponding magnetic radius is small. In the \( l = 0 \) channel, however, the extra \( r^2 \) factor in the integrand of Eq.(23) makes the \( \langle r^2 \rangle_a \) almost to vanish. Therefore there is no partial cancellation in the numerator of Eq.(21). This fact together with the small value of \( \Lambda^* \) magnetic moment conspires to give a rather large \( \Lambda^* \) magnetic radius.

The results for the electric radii are easier to understand. The only difference between the \( \Lambda \) and the \( \Lambda^* \) cases appears in the strangeness radii \( \langle r^2 \rangle_S \). Being composed by soliton-kaon system in an excited state the strangeness radius is expected to be larger for the \( \Lambda^* \). Since this contribution has to be subtracted from the soliton baryon radius to get the electric radius, it is reasonable to get a smaller (and even negative) value for the \( \Lambda^* \).

### 4.3 Radiative Decay Amplitudes

As already mentioned, the \( \Lambda(1405) \) has two electromagnetic decay modes, namely \( \Lambda(1405) \rightarrow \Lambda\gamma \) and \( \Lambda(1405) \rightarrow \Sigma^0\gamma \). They are related to the isoscalar and to the isovector part of the e.m. current, respectively. The decay amplitude corresponding to these processes can be written as

\[
\Gamma = k \sum_{m_i, m_f} \sum_{\lambda=\pm 1} |\langle J_f, m_f | \hat{\epsilon}_\lambda(\hat{k}) \cdot \mathbf{J}(\hat{k})|J_i, m_i\rangle|^2. \tag{27}
\]

Here \( \mathbf{k} \) is the momentum of the emitted photon, \( \hat{\epsilon}_\lambda \) its polarization tensor and \( \mathbf{J}(\hat{k}) \) the Fourier transform of the e.m. current \( \mathbf{J}(\mathbf{r}) \). Also, \( k = |\mathbf{k}| \). As usual, we sum and average over final and initial spin states.

Explicit calculation shows that the relevant matrix elements of the e.m. current \( \mathbf{J}(\mathbf{r}) \) can be written as

\[
\mathbf{J}^{\Lambda^*H}(\mathbf{r}) = <\Lambda^*|\mathbf{J}|H> = \frac{i}{4\pi} \left[ g_1^{\Lambda^*H}(r) \mathbf{T} + g_2^{\Lambda^*H}(r) \mathbf{T} \cdot \hat{\mathbf{r}} \hat{\mathbf{r}} \right]. \tag{28}
\]

Here \( H \) represents the final hyperon state, namely \( H = \Lambda \) for the isoscalar decay and \( H = \Sigma^0 \) for the isovector one. The explicit forms of the radial functions \( g_1^{\Lambda^*H} \) and \( g_2^{\Lambda^*H} \) are as follows

\[
g_1^{\Lambda^*\Lambda} = \cos F \left[ 1 + \frac{1}{e^2 f_K^2} \left( F^2 + \frac{\sin^2 F}{r^2} \right) \right] \frac{k_0 k_1}{r} + \frac{3}{4e^2 f_K^2} \left[ F^2 \frac{\sin F}{r} (k_0' k_1 + k_0 k_1') - F^2 \cos F \frac{k_0 k_1}{r} \right], \tag{29}
\]
Given the form of $g^\Lambda F_2$ where $j_0(k_0 k_1 - k_0 k'_1)$

$$g^\Lambda F_2 = -N_c \left[ \frac{\sin 2F}{2r k_0 k_1} \right] \left[ \int dr \ r^2 \left( g^\Lambda F_1 (r) j_0 (k r) + \frac{g^\Lambda F_2 (r)}{3} (j_0 (k r) + j_2 (k r)) \right) \right]$$

and

$$g^\Lambda F_3 = \frac{2 \cos F - 1}{3} g^\Lambda F_2 + \frac{2 \sin^2 F}{3 e^2 f^2 K} \left( k_0 k_1 - \frac{7}{4} F^2 + 2 \frac{\sin^2 F}{r^2} \right) k_0 k_1$$

From these expressions, we get:

$$g^\Lambda F_2 = -N_c \left[ \frac{\sin 2F}{2r k_0 k_1} \right] \left[ \int dr \ r^2 \left( g^\Lambda F_1 (r) j_0 (k r) + \frac{g^\Lambda F_2 (r)}{3} (j_0 (k r) + j_2 (k r)) \right) \right]$$

and

$$g^\Lambda F_3 = \frac{2 \cos F - 1}{3} g^\Lambda F_2 + \frac{2 \sin^2 F}{3 e^2 f^2 K} \left( k_0 k_1 - \frac{7}{4} F^2 + 2 \frac{\sin^2 F}{r^2} \right) k_0 k_1$$

from which it is easy to get its Fourier transform $J^\Lambda F_H (k)$. We get:

$$J^\Lambda F_H (k) = \int \left[ J^\Lambda F_1 (k) \mathbf{T} + J^\Lambda F_2 (k) \mathbf{T} \cdot \mathbf{k} \hat{\mathbf{k}} \right]$$

where

$$f^\Lambda F_1 (k) = \int dr \ r^2 \left[ g^\Lambda F_1 (r) j_0 (k r) + \frac{g^\Lambda F_2 (r)}{3} (j_0 (k r) + j_2 (k r)) \right]$$

and

$$f^\Lambda F_2 (k) = -\int dr \ r^2 \ g^\Lambda F_2 (r) j_2 (k r)$$

Here, $j_0$ and $j_2$ are spherical Bessel functions of zeroth and second order. Combining all these expressions we get

$$\Gamma(\Lambda^* \to \Lambda \gamma) = k |f^\Lambda F_1 (k)|^2$$

$$\Gamma(\Lambda^* \to \Sigma^0 \gamma) = k |f^\Lambda F_2 (k)|^2$$

Following the standard prescription, we take the $k$ of the emitted photon to be the energy difference between the initial and final hyperon state. In should be noticed...
that since in our model the $\Lambda^*$ mass turns out to be rather small such splittings are somewhat underestimated. For this reason in Table III we list the results for both the calculated $k_{\text{calc}}$ (obtained as the difference between the calculated initial and final hyperon masses) and the empirical one $k_{\text{emp}}$. This last one is of course the difference between the empirical hyperon masses. Since $f_1^{N\pi}$ is basically constant in the relevant range of $k$-values the decay amplitudes turn out to be roughly proportional to $k$. Also given in Table III are the results of a quark model (QM) calculation [23], an MIT bag model (BM) calculation [24] and cloudy bag model (CBM) calculation [20]. Finally, we have also listed some empirical values obtained from an analysis of kaonic atoms decays (KA) [25]. We observe that our results are smaller than those of the quark model and in reasonable agreement with the bag model predictions. The main discrepancy with the CBM results appears in the $\Gamma(\Lambda^* \rightarrow \Sigma^0\gamma)$ decay width which is predicted to be very small in that model. In general our results are somewhat larger than those obtained from the empirical analysis of Ref. [25]. It should be noticed however that in that analysis the poorly known strong coupling constant $g_{\Lambda^*NK}$ appears as input parameter. In Ref. [25] the value $g_{\Lambda^*NK} = 3.2$ has been taken. A smaller value of this coupling constant (as obtained in our model) would lead to decay widths in closer agreement with our predicted values.

5 Conclusions

In this work we have studied the strong and electromagnetic properties of the $\Lambda(1405)$ resonance using the bound state soliton model. Within this model such hyperon is composed by an $SU(2)$ soliton and a kaon bound in a $S$-wave. We have found that the predictions for the strong coupling constant $g_{\Lambda(1405)NK}$ are within the range of the empirically (not very well) known values. This contrasts with the situation in models based on a dominant 3 quark description of the $\Lambda(1405)$ where much smaller values are obtained [21]. It is interesting to note that in the case of the ground state hyperons both type of models predict similar values for the corresponding coupling constants [10, 21].

We have made predictions for the $\Lambda(1405)$ magnetic moment and electromagnetic radii. Unfortunately, these magnitudes are quite difficult to determine empirically. In any case, our values for the $\mu_{\Lambda^*}/\mu_p$ are in qualitative agreement with calculations quoted in the literature [20]. Perhaps more interesting are the predictions for the $\Lambda(1405)$ electromagnetic decay amplitudes. We have computed both the decay amplitudes corresponding to the isoscalar process $\Lambda^* \rightarrow \Lambda\gamma$ and to the isovector one $\Lambda^* \rightarrow \Sigma^0\gamma$. Our results are much smaller than those of the quark model [23].
and in reasonable agreement with the bag model results of Ref. [24]. On the other hand, they are somewhat larger than the values extracted from kaonic atom decays [25]. This might be due to the value of $g_{\Lambda(1405)NK}$ used in such analysis. In any case, it is clear that better empirical information about the $\Lambda(1405)$ properties is needed. For this reason, we hope that the results of the planned experiments for the study of hyperon properties at CEBAF and other facilities will soon be available.

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**Appendix**

In this Appendix we write down the explicit expressions of the radial functions appearing in the kaon equation of motion Eq. (38). These expressions have been derived elsewhere. The effective potential for a general $(\Lambda, l)$ partial wave is given by

$$V^{\Lambda l}_{\text{eff}} = 2 \left( \frac{\sin^2 F/2}{r} \right)^2 \left( 1 + \frac{1}{4e^2 f_K^2} \left[ F'^2 + \frac{\sin^2 F}{r^2} \right] \right) - \frac{1}{4} \left[ F'^2 + \frac{2 \sin^2 F}{r^2} \right]$$

$$- \frac{1}{4e^2 f_K^2} \left[ \frac{2 \sin^2 F}{r^2} \left( 2F'^2 + \frac{\sin^2 F}{r^2} \right) \right]$$

$$- \frac{6}{r^2} \left( \frac{\sin^2 F}{r^2} \sin^4 F/2 + \frac{d}{dr} \left( F' \sin F \sin^2 F/2 \right) \right)$$

$$+ \left[ 1 + \frac{1}{4e^2 f_K^2} \left( F'^2 + \frac{\sin^2 F}{r^2} \right) \right] \frac{l(l+1)}{r^2}$$

$$+ \left\{ \frac{4 \sin^2 F}{2e^2 f_K^2} \left[ 1 + \frac{1}{4e^2 f_K^2} \left( F'^2 + \frac{\sin^2 F}{r^2} \right) \right] \right\}$$

$$- \frac{3}{2e^2 f_K^2} \frac{1}{r^2} \left[ \frac{\sin^2 F}{r^2} \cos F - \frac{d}{dr} (F' \sin F) \right]$$

$$- \frac{f_\pi^2 m_\pi^2}{2f_K^2} (1 - \cos F).$$

The explicit form for the radial functions $h$, $f$ and $\lambda$ is:

$$h = 1 + \frac{1}{2e^2 f_K^2} \frac{\sin^2 F}{r^2},$$

$$f = 1 + \frac{1}{4e^2 f_K^2} \left( F'^2 + \frac{\sin^2 F}{r^2} \right),$$

$$\lambda = -\frac{N_c}{8\pi^2 f_K^2} \frac{\sin^2 F}{r^2} F'.$$
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Table and Figure Captions

Table 1 : Bound state energies $\omega_l$ and hyperfine splitting constants $c_l$ for $l = 0$ and $l = 1$ bound states. The pion mass and decay constant ($m_\pi$ and $f_\pi$) and the Skyrme parameter $e$ are chosen in order to reproduce the phenomenological values of the $N$ and $\Delta$ baryon masses in the $SU(2)$ sector. The ratio of the kaon to the pion decay constant is set to its phenomenological value $f_K/f_\pi \sim 1.22$. For a discussion of the parameters, see e.g. Ref. [15] and references therein.

Table II : Magnetic moments and electric and magnetic mean square radii (in $fm^2$) of the $\Lambda(1405)$ resonance. We report also the elementary contributions together with the corresponding quantities for the $\Lambda$ hyperon. The results are given for both Set I and Set II parameters.

Table III : Radiative decay amplitudes of the $\Lambda^*$ (in $keV$). We quote the results for the empirical and for the calculated photon momentum. Also listed are predictions of the quark model (QM) [23], MIT bag model (BM) [24], cloudy bag model (CBM) [20] and of an analysis of kaonic atoms decays (KA) [25].
Table I

|                  | SET I          | SET II         |
|------------------|----------------|----------------|
| $m_\pi$ (input)  | 138 MeV        | 0              |
| $f_\pi$ (input)  | 54 MeV         | 64.5 MeV       |
| $e$ (input)      | 4.84           | 5.45           |
| $\omega_1$ (MeV)| 209            | 221            |
| $\omega_0$ (MeV)| 388            | 415            |
| $c_1$            | 0.39           | 0.50           |
| $c_0$            | 0.78           | 0.77           |

Table II

|                  | $\Lambda^*$  | $\Lambda$ |
|------------------|--------------|-----------|
|                  | Set I | Set II | Set I | Set II |
| $\mu_{s,sol}$    | 0.73   | 0.56   | 0.73  | 0.56   |
| $a(k_l)$         | 0.27   | 0.26   | 1.35  | 1.06   |
| $\mu/\mu_p$      | 0.08   | 0.09   | -0.27 | -0.21  |
| $\langle r^2 \rangle_{s,sol}$ | 0.40 | 0.28 | 0.40 | 0.28 |
| $\langle r^2 \rangle_a$ | -0.02 | 0.01 | 0.37 | 0.22 |
| $\langle r^2 \rangle_M$ | 1.14 | 1.21 | 0.20 | 0.11 |
| $\langle r^2 \rangle_B$ | 0.47 | 0.35 | 0.47 | 0.35 |
| $\langle r^2 \rangle_S$ | 0.64 | 0.60 | 0.27 | 0.18 |
| $\langle r^2 \rangle_E$ | -0.09 | -0.12 | 0.10 | 0.09 |

Table III

|                  | Set I         | Set II        | QM     | BM     | CBM    | KA     |
|------------------|---------------|---------------|--------|--------|--------|--------|
| $k_{emp}$        | 67            | 44            | 56     | 40     | 143    | 60     |
| $k_{calc}$       | 40            | 143           | 44     | 60     | 75     | 27 ± 8 |
| $\Gamma(\Lambda^* \rightarrow \Lambda\gamma)$ | 29            | 13            | 29     | 17     | 91     | 18     |
| $\Gamma(\Lambda^* \rightarrow \Sigma^0\gamma)$ | 75            | 2.4           | 10 ± 4 | 23 ± 7 |        |        |