Historical and futuristic perturbative and non-perturbative aspects of QCD jet physics

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A brief review of jet physics is presented with an emphasis upon open theoretical problems (non-perturbative domain; hadronization and confinement) and new phenomena (hadroproduction in heavy ion collisions).

1. Prehistory

Thirty years ago was the epoch of ISR and SPEAR — the first Jet labs. In the realm of hard interactions, R. Feynman invented his famous “plateau” — ln $E$ hadrons streaming from a single quark-parton that is struck out of the target proton in a Deep Inelastic Scattering process. Moving from the high energy side (soft interactions) V. Gribov, motivated by the Pomeron picture, drew an energetic hadron fluctuating into ln $E$ partons. Already then the key word duality was pronounced in the context of the inter-connection between partons and hadrons.

Those, however, were the times of pictures, of physical intuition, rather than theoretical expectations, let alone predictions. So that as late as 1976, three-jet pioneers still spoke about logarithmic multiplicity as being merely "fashionable".

The notion of hadron jets goes back to the dark times of the exclusive dominance of cosmic ray physics. On the theory side, the existence of jets was envisaged from “parton models” in the early 70’s. In 1974 J.B. Kogut and L. Susskind have remarked that hard gluon bremsstrahlung off the $q \bar{q}$ pair may be expected to give rise to three-jet events in the $e^+e^-$ annihilation into hadrons.

The first thorough analysis of three-jet events in the QCD context was due to John Ellis, Mary K.Gaillard & Graham G.Ross (1976). It is instructive, quoting [1], to recall the background knowledge the authors of this seminal study relied upon:
• “no direct experimental evidence yet exists for gluons” (except possibly the fact that not all the nucleon’s momentum is carried by known quark constituents);

• “there is no direct evidence for asymptotic freedom” (though there may be some deviations from scaling in DIS at high $Q^2$)

• “fashion sets” $\alpha_V(Q^2) \sim 0.2 - 1$ at $Q^2 \sim 10 \text{GeV}^2$.

It is amazing how far this brave youth managed to leap forward from such a shaky base! They professed coplanar structure of final states and cross section scaling in $x_T = 2p_T/Q$, and, having studied vector- versus scalar-gluon cases, managed to rightly guess the $\sim 10\%$ rate for three-jet events.

EGR also verified the asymptotic 2-Jetness of $e^+e^-$ annihilation events that had been advocated one year earlier by George Sterman and concluded

Present ideas about quark/gluon metamorphosis into hadrons suggest a third jet should exist in the direction of the large $p_T$ particle. [...] This prejudice is comforted by the observed jet structure in large $p_T$ $pp$ collisions.

Moreover, they drew a picture with two hadron chains stemming from gluon fragmentation and remarked, without much ado,

Looking at [this picture] one might naively expect more hadrons to be produced in gluon fragmentation than in quark fragmentation, and therefore that $f(x)$ for gluons should be more concentrated at low $x$.

2. Colour in hadroproduction and QCD jets

That naivete was to become the core of the colour driven picture of parton hadronization elaborated by Bo Andersson, Gösta Gustafson & Carsten Peterson in 1977.

Have we learned anything new about the EGR “metamorphosis”? Not much, to be honest. At the qualitative level we keep following “the fashion” — the classical Kogut–Susskind “vacuum breaking picture”.

In a deep inelastic scattering process a green quark in the proton is hit by a virtual photon. The quark leaves the stage and the Colour Field starts to build up. A green–anti-green quark pair pops up from the vacuum, splitting the system into two globally blanced sub-systems.
The Kogut–Susskind scenario had been realized by the phenomenologi-

cal Lund model of multiple hadroproduction [2]. The Lund hadronization

model embodied the key features of the Kogut–Susskind scenario namely,

the uniformity in rapidity, $dN_h \propto d\omega_h/\omega_h = d\Theta_h/\Theta_h$, and limited trans-

verse momenta of produced hadrons with respect to the jet direction.

Picturing a jet as a string of hadrons, the Lund model however made an

essential step beyond the naive parton model picture by putting a special

stress on the rôle of colour in hadronization of parton ensembles. This has

brought to life radiophysics of QCD jets.

Let me remind you that studying both Inter-Jet and Intra-Jet phenom-

ena fully revealed colour coherence in QCD parton multiplication [3]. Their

solid imprint upon the angular and energy spectra of relatively soft hadrons

has sent us a powerful message namely, that the confinement (=metamor-

phosis) is soft.

This is a free lunch that we have not yet found enzymes to digest. For the

time being, we are exploiting this gift: Hadron flow practitioners (who are

developing smart tools for triggering on new physics), Colour Glass brewers,

small-x BFKL lovers, — no-one would hesitate to put gluons and hadrons

into one-to-one correspondence as soon as final state particle production

issues come onto the stage.

There is nothing wrong with this. In so doing we simply follow the

opportunists’ motto “ain’t broken – don’t fix it”. It becomes mandatory,

however, that we start exploring the LPHD gift rather than simply exploiting

it. To set up the Quest, we have to turn now to the problematics of the

non-perturbative domain: what is it, what do we know about it, and, more

important still, what we don’t.

3. Non-perturbative effects in Event Shape observables

There is a specific (though not too narrow a) class of QCD observables

that taught us a thing or two about genuine non-perturbative effects in

multiple production of hadrons in hard processes. Among them — the

so-called event shapes which measure global properties of final states (jet

profiles) in an inclusive manner.

In $e^+e^-$, for example, one defines

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i,\vec{n}|}{\sum_i |\vec{p}_i|},$$

$$C_{\text{param.}} = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2},$$

left hemisphere \hspace{3cm} right hemisphere

$\vec{n}_T$
\[ \text{jet-mass } \rho = \left( \sum_{i \in \text{hemisphere}} p_i \right)^2 \left( \sum_{i} E_i \right)^2, \quad \text{broadening } B_T = \frac{\sum_i p_{ti}}{\sum_i |p_i|}. \]

(The two hemispheres in the jet-mass and transverse momentum component in broadening are defined with respect to the thrust axis.) These and similar event shape observables are formally calculable in pQCD (being collinear and infrared safe, CIS) but possess large non-perturbative \(1/Q\)-suppressed corrections. Being perturbatively calculable does not imply, however, being insensitive to non-perturbative physics \[4\].

Indeed to reconcile with the data the two-loop pQCD predictions for the means of two exemplary jet shapes (shown by dotted lines) one has to introduce, on the phenomenology side,

\[ \langle 1-T \rangle_{\text{hadron}} \approx \langle 1-T \rangle_{\text{parton}} + 1 \text{ GeV}/Q, \]
\[ \langle C \rangle_{\text{hadron}} \approx \langle C \rangle_{\text{parton}} + 4 \text{ GeV}/Q. \]

### 3.1. NP games

The pQCD motivated “theory” of genuine non-perturbative effects in jet shapes (about 8 years old and running) predicts the above ratio to be

\[ 3\pi/2 \approx 4/1. \]

The origin of power-suppressed corrections to CIS observables can be linked with the mathematical property of badly convergent perturbative series, typical for field theories, known under the name of “renormalons”. The renormalon-based analysis is perfectly capable of controlling the ratios of power terms, mentioned above, but can say next to nothing about the absolute magnitude of such a correction. To address the latter issue, an
additional hypothesis had to be invoked namely, that of the existence of an InfraRed-finite QCD coupling (whatever this might mean; a detailed discussion of the issue can be found in [5]).

3.2. NP power corrections and IR-finite $\alpha_s$

The “industry-standard” way of fitting event-shape power corrections [6] exploits the idea that the power correction is driven by the NP modification of the QCD coupling in the InfraRed. The leading NP contribution to a given observable $\mathcal{V}$ can be parametrized as

$$
\delta \mathcal{V}_p = \frac{2C_F}{\pi} \int_0^{\mu_I} \frac{dm}{m} \left( \frac{m}{Q} \right)^p \left( \alpha_s(m^2) - \alpha_s^{\text{PT}}(m^2) \right) \cdot c_V. \tag{1}
$$

The key features of this expression are as follows.

- It contains a PT controlled observable dependent coefficient $c_V$;
- The (integer) exponent $p$ determines sensitivity of a given observable to the IR momentum scales; for the vast majority of event shapes one has linear damping, $p = 1$;
- One subtracts off $\alpha_s^{\text{PT}}$, the fixed-order perturbative expansion of the coupling in order to avoid double counting.

The full answer for the case under interest, $p = 1$, takes the form

$$
\mathcal{V} = A\alpha_s + B\alpha_s^2 + c_V \frac{2C_F}{\pi} \frac{\mu_I}{Q} \left( \alpha_0 - \langle \alpha_s^{\text{PT}} \rangle_{\mu_I} \right) \tag{2}
$$

with $\alpha_0$ the first moment of the coupling in the InfraRed:

$$
\alpha_0 = \frac{1}{\mu_I} \int_0^{\mu_I} dm \alpha_s(m^2), \quad \langle \alpha_s^{\text{PT}} \rangle_{\mu_I} = \alpha_s(Q^2) + \beta_0 \frac{\alpha_s^2}{2\pi} \left( \ln \frac{Q}{\mu_I} + \frac{K}{\beta_0} + 1 \right).
$$

3.3. Heavy quark spectra

To the best of my knowledge, the first semi-rigorous attempt to put IR finite $\alpha_s$ at work in the pQCD context has been made in the pre-renormalon epoch, in the late 80s – early 90s, in the context of fragmentation of heavy quarks $H$. 
Thanks to $M_H \gg \Lambda_{\text{QCD}}$, the inclusive fragmentation function $H \rightarrow H(x) + \ldots$ becomes collinear finite and is formally PT-calculable. But only formally, since for large $x$-Feynman, $1 - x \lesssim \Lambda_{\text{QCD}}/M_H$, one hits the NP domain [7]. However, it is this – the most interesting – region where the fragmentation function is sitting! (the “leading heavy quark effect”).

Fragmentation Functions (FFs), as well as space-like parton distributions (SFs), are known to be IR safe: for soft gluons virtual corrections cancel against real radiation contributions. This cancellation, however, breaks down at the edge of the phase space, and the distributions acquire typical DL Sudakov suppression factors. Here the observable becomes sensitive to soft gluons — those very beasts that are the first to enter the NP domain. As a result, the FF gets a NP power suppressed $O(1/M_H)$ correction, quantifiable in terms of the IR coupling.

### 3.4. A brief time-line

Thus, one has to compare next-to-leading PT + NP predictions to data, fitting for $\alpha_s(Q^2)$ and $\alpha_0$ (IR-average coupling) in (2), in a hope to see that both $\alpha_s$ and $\alpha_0$ will turn out to be independent of the observable.

The power-corrections-to-event-shapes business underwent quite an evolution. Its dramatic element was largely due to impatience of experimenters who were too fast to feast on theoretical predictions before those could possibly reach a “well-done” cooking status. The new (PT-obtainable) coefficients $c_V$ in (1) evolved with time as shown by the following charts and figures elaborated by G.P. Salam [8].

| $\nu$ | $\tau = 1 - T$ | $C$ | $\rho$ | $B_T$ | $B_W$ |
|-------|----------------|-----|--------|--------|--------|
| $c_V$ | $2 (\frac{3.66}{2})$ ? | $3\pi$ | $1$ | $2$ | $\frac{4\ln Q}{4\ln Q}$ |

The fact that this table contains two competing predictions for the same variable, illustrates an intrinsic uncertainty of the then standard naive approach.

NB: It is fortunate that the comparison shown on the right insert was not available then (the $B$ and $\rho$ data appeared later). If it were, the whole business might have been abandoned as completely unsatisfactory.
In-depth analysis:
The situation has dramatically improved when the in-depth analysis had been carried out which included

- understanding the PT–NP interface,
- resummation of log-enhanced PT effects in NP contributions
- and, finally, introduction of the so-called “Milan factor” based on the two-loop analysis of NP terms:

\[
\begin{array}{c|ccc|c|c}
\nu & \tau & C & \rho & B_T & B_W \\
\hline
c \nu / M & 2 & 3 \pi & 1 & 1 & 2 \sqrt{C F \alpha_s} \\
\end{array}
\]

Mass Effects:
Finite hadron mass effects were recently taken care of by Salam and Wicke [9]. Their analysis called for special attention to be paid to the definition of observables at the hadron level, so as to ensure that hadron masses do not lead to trivial kinematic contributions that break universality.

3.5. Event shape distributions

The same technology turned out to be applicable to event shape distributions, dN/dV. Here the genuine NP physics manifests itself, basically, in shifting the corresponding PT spectra, in V variable, by an amount proportional to 1/Q [10]. Distributions turned out to be an important addition to the menu. Firstly, to study functions is more informative and revealing than numbers. Secondly, distributions are two-parameter objects and they don’t allow essential NP contributions to be hidden under the carpet by RG-improving PT series, as the means apparently do [11].

Last but not least, it was the studies of event shape distributions that allowed theorists to better understand what they have been doing, thanks to pedagogical lessons theorists were taught by those impatient colleagues experimenters. In particular, theoretical understanding of the physics of NP effects in jet broadening distributions which revealed a delicate NP–PT interplay was triggered by the deep study of the problem pioneered by the
JADE group \[12\].

NP effects aside, a purely perturbative part of the event shape distributions analysis is not peanuts either. All order resummation of logarithmically enhanced contributions was gradually carried out, at the next to leading order, for a vast number of observables — multi-jet cross sections, thrust, $C$ parameter, jet broadenings, various aplanarity characteristics of three-jet events (like $D$ parameter, $k_{\text{out}}$) etc. — first in $e^+e^-$ annihilation and then also in DIS, Drell–Yan processes and (partially) in the hadron–hadron collisions environment.

This laborious business (with an average of $\lesssim 1$ observable per paper) turned out to be a very error-prone one as well. Curiously, among “professional resummers”, only about $O(10\%)$ can say that all their final results were correct to the accuracy claimed. It should be noted that such a large yield of “wrong papers” (or rather of “authors who erred”) is mostly due to one singular reason: a new previously overlooked class of NLLO corrections that hit specifically the so-called non-global observables recently uncovered and baptized by Mrinal Dasgupta and Gavin Salam \[13\].

The good news is, one does not need theorists anymore anyway. The Computer Automated Expert Semi-Analytical Resummation package has been developed by Andrea Banfi, Gavin Salam and Giulia Zanderighi that will analyse for you any observable you might fancy to invent, find out whether this observable is legitimate (collinear and infrared safe\(^1\) and global) and spit out the NLLO resummed prediction \[14\]. Implementing the matching with exact low order matrix element calculations is underway \[15\] and incorporating leading power suppressed NP effects is planned.

### 3.6. Verifying Universality hypothesis

Theory & Phenomenology of $1/Q$ suppressed effects in event shape observables, both in $e^+e^-$ annihilation and DIS, pointed at the average value of the infrared coupling

$$\alpha_0 \equiv \frac{1}{2 \text{GeV}} \int_0^{2 \text{GeV}} dk \alpha_s(k^2) \sim 0.5.$$  

As the recent analysis by Dasgupta and Salam shows \[16\], the expected universality holds within a reasonable 15% margin.

\(^1\) there is a subtlety here: your observable has to be recursively CIS, see \[14\]. Fortunately, you have to be real wicked to invent one that isn’t.
The Universality Hypothesis is the key ingredient of the game: the new NP parameter $\alpha_0$ must inherit the universal nature of the QCD coupling itself.

Let us remark that the characteristic value of the typical PT expansion parameter $\alpha_s/\pi$ turns out to be numerically small, $\sim 0.17$ (which may open intriguing possibilities). Moreover, it is comfortably above the so-called critical value of the IR coupling that is necessary to trigger the super-critical light quark confinement mechanism suggested by V. Gribov [17] (for a recent review of, and around, the Gribov programme of attacking colour confinement see [18]).

It should be said that some recent analyses present a less positive picture of universality. Caution is needed in making strong claims. In $e^+e^-$, the majority of the analyses did not take finite hadron-mass effects into proper consideration; some fits look contradictory between different experiments (notably for $B_T$); two specific observables (those that select a wide/heavy hemisphere, $B_W$ and $\rho_h$) seem to misbehave, and probably require further theoretical insight; the potentially powerful technique of examining events with a hard final-state photon, so as to reduce the effective hadron cms energy and provide a lever arm in $Q$, is marred by the use of an incorrect assumption of factorisation of the QCD and QED matrix elements (this is especially of concern for mean values, since they are dominated by hard, non-factorising, emissions). In DIS, most mean values seem to lead to a highish value for $\alpha_s$, especially for $T_Z$ (which also has an uncomfortably low $\alpha_0$ value), for reasons that are not yet understood; at the same time the usually very tricky broadening measure seems to behave properly, even in the distribution.

From the above it is clear that the universality hypothesis still remains to be definitively (dis)proved, with open issues both experimentally and theoretically. To better understand the physics of non-perturbative effects in jets, it is mandatory to extend the studies to multi-jet final-states, not only in $e^+e^-$ and DIS, but also, just as importantly, in hadron-hadron scattering.\footnote{The above overview has been produced together with G.P. Salam.}

\section{Heavy Ions, small distances and Jets}

Ours are the times of \textit{A New Hope}. On the practice side, it is due to the full swing operation of the RHIC heavy ion facility at Brookhaven. It took off/over the CERN SPS programme which had already supplied us with a number of puzzles. Lead or gold, a bunch of intriguing phenomena have been observed in $pA$ and $AA$ high energy collisions. To name but a few,
• Large-\(p_t\) pion yield gets strongly suppressed in central collisions of heavy nuclei;
• Back flowing (recoiling) jets disappear — get washed away;
• Relative yields of strange mesons and baryons steadily increases with the number of collisions tending to “equilibrate” three quark flavours;
• Leading baryons disappear from the fragmentation region (“stopping”);
• Central production of secondary baryons catches up with (if not takes over) that of mesons at \(p_t \gtrsim 2\) GeV.

4.1. Nucleus as “hardener”

On the theory side it is becoming more and more clear that small distances emerge naturally in the multiple scattering environment. Treating physics that looks a priori soft, such as inelastic diffraction off nuclei \[19\], medium induced gluon radiation \[20\], various phenomena that one gathers under the banner of the colour glass condensate (CGC) picture \[21\], one observes that the hardness scale that characterizes these (and similar) process grows invariably as

\[
Q^2 \propto A^{1/3}.
\]  

(3)

A short sketch is due to illustrate this important point.

Consider for example a non-destructive high energy hadron–nucleus interaction known as diffraction. To muddle through a thick target without causing much damage (inelastic breakup of the target), our projectile should interact weakly, be rather transparent. Since the incident hadron is a composite object, this can be achieved by selecting compact configurations with relatively small separation between valence quarks in the impact parameter space,

\[
\sigma(b) \propto \alpha_s^2 \cdot |b|^2.
\]  

(4)

The “transparency” condition,

\[
\lambda \sim \frac{1}{\rho \sigma(b)} \gtrsim L,
\]

then gives

\[
Q^2 \equiv 1/|b|^2 \gtrsim \alpha_s^2 \rho \cdot L
\]  

(5)

(with \(L \propto A^{1/3}\) the target thickness and \(\rho\) the nuclear density). A pion in such a squeezed configuration will fragment in the final state into two quark jets with large transverse momenta \(k_T^2 \sim Q^2\) thus illuminating colour transparency \[22\]. Pion dissociation into two jets has been recently observed
by the Fermilab E-791 experiment \[23\] which verified jet energy, transverse momentum and $A$ dependences predicted by QCD.

A very similar structure of the characteristic scale emerges in the problem of medium induced \textit{transverse momentum broadening} and a closely related induced \textit{gluon radiation} (the LPM effect, see below). Here the relation mirroring (5) is expressed in terms of the so-called transport coefficient

$$Q^2 = \hat{q} \cdot L; \quad \hat{q} = \frac{\langle |b|^2 \rangle}{\lambda} = \frac{1}{\sigma \lambda} \int q_{\perp}^2 d\sigma(q^2_{\perp}) \propto \alpha_s \rho [xG(x, Q^2)]. \quad (6)$$

Finally, within the CGC approach to high energy phenomena in heavy ion interactions \textit{the same} characteristic parameter (6) appears \[24\] under the name of “saturation scale” $Q_s^2$.

To conclude, the gift (3) seems to be putting things under tighter pQCD control, shifting the emphasis towards underlying (perturbative) quark–gluon physics. A paradoxical situation emerges: on the other hand, the number of puzzles is steadily increasing in scattering of/ off nuclei; on the other hand, these phenomena have a good reason to be under the jurisdiction of pQCD.

Abundant puzzles and paradoxes constitute the best imaginable setup for the theory as they provide potential (or rather potential gradient — the field strength) for a revolutionary breakthrough. Jets — the subject of this talk — are to play a key rôle in this quest.

$4.2. \text{Collisions or Participants?}$

One of the difficult questions of the physics of heavy ion collisions is the question of \textit{scaling}. To be able to state that “new” physics manifests itself we better know what is to be expected if the physics were “old”? How to compare the quantity one measures in $AA$ (or $pA$) collisions with the one \textit{simply rescaled} from an elementary $pp$ interaction? It is in this harmlessly looking “simply rescaled” that the devil resides. Should a given observable in $AA$ interactions scale with the number of \textit{participating nucleons} (which may be as large as $n_p \leq 2A$) or instead as the number of elementary nucleon–nucleon collisions ($n_c \propto n_p \cdot A^{1/3}$)?

It is common wisdom to expect \textit{hard} interactions to scale as $n_c$ and \textit{soft} phenomena — as $n_p$. Nothing would be easier than to drown, without trace, in the discussion of what is soft and what is hard, how hard is hard etc. Since the purpose of this talk is to build up tension rather than help to release it, let me introduce more confusion into this (nuclear) matter.

This is easy to do by recalling the QCD pattern of gluon radiation induced by multiple scattering of a coloured projectile in the QCD medium.
The structure of the inclusive spectrum of medium-induced gluon radiation looks as follows:

\[
\frac{\omega dn}{d\omega} \simeq \frac{\alpha_s}{\pi} \cdot \left[ \frac{L}{\lambda} \right] \cdot \sqrt{\frac{\mu^2\lambda}{\omega}}, \quad \mu^2\lambda < \omega < \mu^2\lambda \left[ \frac{L}{\lambda} \right]^2.
\]  

(7)

Here \( \lambda \) is the mean free path of the projectile (quark, gluon, \ldots), \( L \) the size of the medium and \( \mu \) a typical transverse momentum transfer in a single scattering. The first factor on the r.h.s. of (7) corresponds to the Bethe–Heitler (BH) regime of independent emission of a gluon off each scattering centre. Taken together with the second factor (the total number of elementary collisions of the projectile, \( n_c = L/\lambda \)) this would correspond to secondary gluon production according to the \( n_c \)-scaling prescription. However, the third factor modifies the BH prediction by suppressing the yield of more energetic gluons. It is only the softest radiation with finite energies \( \omega \sim \mu^2\lambda \) that follows the BH pattern. This is what is going on, from the QCD point of view, in \( hA \) scattering. Quantum mechanical coherence suppresses production of more energetic gluons whose yield gradually turns into that of participant scaling (\( n_p = 1 \) in our example). As shown by the accompanying schematic picture of the particle yield in \( hA \) collisions, the transition is smooth and occupies a finite range in \( \eta = \ln \omega \) from the fragmentation region of the nucleus, \( \Delta \eta \simeq 2 \ln n_c \). This coherent suppression is similar in nature to that known under the name of the Landau–Pomeranchuk–Migdal (LPM) effect in QED.

The essence of the QCD LPM physics is easy to grasp. A number of scattering centres (\( N_{\text{coh.}} \)) that fall inside the formation length of the gluon act coherently as a single scatterer. At the same time, the gluon is subject to Brownian motion in the transverse momentum plane, so that

\[
k_{\perp}^2 \simeq N_{\text{coh.}} \cdot \mu^2, \quad N_{\text{coh.}} \simeq \frac{\ell_{\text{coh.}}}{\lambda} \simeq \frac{1}{\lambda} \cdot \frac{\omega}{k_{\perp}^2}.
\]  

(8)

Combining the two estimates results in

\[
N_{\text{coh.}} \simeq \sqrt{\frac{\omega}{\mu^2\lambda}} \quad \text{and} \quad k_{\perp}^2 \simeq \sqrt{\frac{\mu^2}{\lambda} \cdot \omega}.
\]  

(9)

It is the factor \( N_{\text{coh.}}^{-1} \) that describes the coherent LPM suppression in (7).
Now comes the confusion part. From (9) we observe that *more energetic* gluons have typically *larger transverse momenta*. This means, in turn, that the accompanying radiation corresponding to *larger hardness scales* (gluons with larger $k_\perp$) follows the *participant* scaling, while the *less hard* radiation (smaller $k_\perp$ and energies) obeys the *collisional scaling* pattern, in striking contradiction with the original expectation. It is quantum mechanics that is to be blamed for such a miserable failure of “common wisdom”.

4.3. Colour in multiple scattering

The situation appears even more confusing if we recall the rôle of colour in multiple hadroproduction and try to reconcile the underlying colour dynamics with the high energy hadron interaction phenomenology based on the multi-Pomeron scattering picture.

4.3.1. Single scattering, Pomeron and accompanying gluons.

From the QCD point of view, scattering of a pion due to one gluon exchange breaks coherence of the valence $q\bar{q}$ system and results in multiparticle production (inelastic process) via appearance of the two “quark chains” in the final state. These two Kogut–Susskind chains are nothing but an image of the Feynman plateau — the cut Pomeron. The same is true for inelastic scattering of a proton. Since one gluon exchange leaves a *diquark* unbroken, the same two chains develop, the only difference being that one of the chains starts off from a valence diquark and therefore gives birth, as a rule, to a *leading baryon* in the final state.

This two-quark-chain imagery of an universal Pomeron seems to be in a perfect accord with the LPHD view according to which secondary hadrons in the final state originate from accompanying gluon radiation. Indeed, consider gluon radiation (momentum $k$, colour $a$) in course of scattering of a projectile off the gluon field (momentum transfer $q$, colour $b$):
The sum of the three relevant amplitudes reduces to
\[
\frac{k_\perp T^b T^a}{k_\perp^2} + \frac{k_\perp T^a T^b}{k_\perp^2} + \frac{q_\perp - k_\perp}{(q_\perp - k_\perp)^2} i f_{abc} T^c = i f_{abc} T^c \cdot \left[ \frac{k_\perp}{k_\perp^2} + \frac{q_\perp - k_\perp}{(q_\perp - k_\perp)^2} \right].
\]
Since the colour group commutation relation \([T^a T^b] = i f_{abc} T^c\) is the same for any projectile (an arbitrary colour object with generator \(T\)), the accompanying radiation intensity turns out to be universal and proportional to the “colour charge” of the \(t\)-channel exchange, \((i f_{abc})^2 \propto N_c = 3\). This universality is a direct consequence of conservation of the colour current.

### 4.3.2. Multiple scattering, AFS and “colour capacity”

Problems start emerging when we turn to multiple scattering. Let us exchange two gluons and calculate the average colour charge of the \(t\)-channel two-gluon system (with bold numbers standing for colour factors):
\[
\frac{1}{64} \cdot 0 + \frac{8 + 8}{64} \cdot 3 + \frac{10 + 10}{64} \cdot 6 + \frac{27}{64} \cdot 8 = 6 = 2 \cdot 3.
\]
This looks satisfactory since the doubling of the radiation intensity will translate into the double density of produced hadrons (two cut Pomerons). However, validity of this result is questionable since in the above calculation we have completely ignored the nature of the projectile. Indeed, if we take a single quark or a colour-neutral \(q\bar{q}\) system (meson) as a projectile, the transition current can only be a colour octet (with a bit of a singlet), whatever the number of exchanged gluons! This means that multiple scattering of a \((q\bar{q})\) pion will never produce anything but a single-Pomeron particle density (unless we look specifically close to the target fragmentation, cf. the LPM discussion above).

In spite of the proton looking more capacious a projectile, the two-Pomeron exchange cannot be realized on a valence-built proton either. Indeed, since a 3-quark system can be repainted only into colour representations \(1 + 8 + 8 + 10\) (altogether 27 states), the average density of the gluon accompaniment will be
\[
\frac{1}{27} \cdot 0 + \frac{8 + 8}{27} \cdot 3 + \frac{10}{27} \cdot 6 = 4 = 1.5 \times \text{Pomeron (?!)}
\]
Let us remark that we should have expected this weird “1.5 Pomeron” yield from the naive chain consideration already, since it is three quark chains that develop in course of independent fragmentation of the valence quarks of the broken proton\(^3\) (while one would need four to picture two cut Pomerons).

\(^3\) By the way, the baryon number of a broken proton naturally sinks into the sea by 2–3 rapidity units (stopping).
QCD scenario of hadroproduction being due to coherent gluon accompaniment seems to invalidate the multi-Pomeron exchange picture. This striking statement is, however, as true as it is not new. Recall the good old Amati-Fubini-Stanghellini puzzle.

Successive scatterings of a single parton do not produce branch points in the complex angular momentum plane (Reggeon loops). Instead, it is the Mandelstam construction that generates “Reggeon cuts”, with Pomerons attached not to one but to separate — coexisting — partons.

Thus, to have \( n_c \)-gluon exchange produce (up to) \( n_c \) times enhanced density of the hadron plateau (\( n_c \) cut Pomerons), one must be able to find \( n_c \) independent (incoherent) partons inside the projectile.

The answer to whether the final hadron yield follows the collision or participant scaling depends on the question (as it does so often in quantum mechanics). It depends on what are we looking at and becomes the question of resolution.

To be able to absorb \( n_c \) gluons incoherently in order to give rise to \( n_c \) Pomeron chains (collision scaling), our projectile has to have sufficient “colour capacity”. We must compare the number of collisions \( n_c \) with the number of resolved partons inside the projectile,

\[
C(x_h, Q_{\text{res}}) \sim \int_{x_h}^{x_{\text{proj}}} \frac{dx}{x} \left[ xG_{\text{proj}}(x, Q_{\text{res}}^2) \right] + [\text{valence stuff}].
\]

By increasing \( Q_{\text{res}} \) (transverse momentum of the registered hadrons \( h \)) and/or by moving further away from the projectile in rapidity (increasing \( \ln(x_{\text{proj}}/x_h) \)) we will gradually get rid of colour coherence (\( n_p \) scaling) and approach the \( n_c \) scaling.

### 4.4. Confinement in new environment

In the framework of the standard multi-Pomeron picture\(^4\) one includes final state interactions to explain spectacular heavy ion phenomena like \( J/\psi \) suppression, enhancement of strangeness production and alike. “Final state interaction” is a synonym to “non-independent fragmentation” (one hears about cross-talking Pomerons, overlapping strings, “string ropes”, . . . , you name it).

From the point of view of the colour dynamics, in \( pA \) and \( AA \) environments we face an intrinsically new, unexplored, question: After the pancakes

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\(^4\) e.g., in the successful Dual Parton Model of Capella, Kaidalov et al. 25
separate, at each impact parameter we have the colour field strength corresponding to \( n_p/fm^2 \propto A^{1/3} \) “strings”. How does the vacuum break up in such a – stronger than usual – colour field? Imagine we stretch a high density field (pull apart over-charged capacitor plates). Will it go like

BOOOOM

or rather

TA-TA–TA—TA?

The first scenario corresponds to four cut Pomerons (quadruple multiplicity but standard particle abundances). In the second one vacuum break-up occurs at smaller distances and therefore will provide, in particular, a free strangeness lunch (together with other sweet cookies).

5. Conclusions

Jets as a PT instrument did the job they’ve been asked to perform: to verify the nature of the fundamental fields of the underlying QFT by measuring quark and gluon spins, to establish \( SU_c(3) \) as the true QCD gauge group.

Jets did more than that: studying inclusive energy spectra of (relatively soft) hadrons inside jets, and soft hadron multiplicity flows in-between jets taught us an important lesson, or rather have sent us a hint, about the non-violent nature of hadronization (“soft confinement”).

On the nuclear side, Jets stemming from diffractive hadron dissociation on nuclei reveal the internal small-distance structure of hadron projectiles; Jets that are produced in, and muddle through, the colour soup left behind head-on collisions of heavy nuclei bear information about new peculiar QCD media (“jet quenching”).

Jets — their appearance, disappearance, internal structure (particle abundances, shape observables, angular profiles, etc.) — are steadily becoming a Non-Perturbative tool for elucidating the physics of hadronization.

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