Influence of ubiquitiform complexity on fracture energy of concrete

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Abstract. The crack extension path of concrete materials was simulated numerically based on the non-uniform numerical model of Weibull distribution, and the complexities of various ubiquitiform fracture surfaces were calculated to explore the relationship between complexity and fracture energy. To study the influence of specific crack extension law and fracture mode on the relationship between the complexity and fracture energy, numerical simulation of the crack extension was carried out based on the numerical model with random aggregate distribution, and the reason for the change of the relationship between complexity and fracture energy were discussed.

1. Introduction

Following the pioneering work of Mandelbrot et al. [1], it is believed that fracture surfaces in many kinds of materials should be fractal in nature [2-5]. Over the past decades, the mechanics of the emergent fractal fracture has been generally accepted in describing the irregularities and roughness of fractured surfaces in heterogeneous quasi-brittle materials [6-10]. However, with the development of fractal application, some inherent difficulties gradually appear, especially when it comes to the fractal measure of the object under consideration. To overcome the difficulties in fractal applications, recently, Ou et al. [11] proposes a concept of ubiquitiform, that is, the geometry of nature should be ubiquitiformal rather than fractal, and it is better for a geometric or a physical object in nature to be characterized by a ubiquitiform rather than by a fractal, especially when we consider the integral scale of the object under consideration. According to Ou et al. [11], a ubiquitiform is defined as a physical structure of finite order self-similarity (or self-affine), usually constructed by a finite iterative process, so that it has the same Hausdorff dimension as the initial element, which in practice is always an integer dimension. In addition, for certain ubiquitiform, there is always a associated fractal generated by the corresponding infinite limit process. The fractal dimension is defined as the complexity of the ubiquitiform to represent its non-fractal characteristics.

Over the past decades, many researchers have done a lot of experimental and theoretical research on the relationship between the fractal dimension of the fracture surface of a material and its mechanics parameters, but no unified conclusion has been reached, including concrete materials. Saouma and Barton [5] performed splitting experiments on specimens of different types and sizes of aggregate, which confirmed that the fractal dimensions decrease with increasing fracture toughness. Issa et al. [12] found that the fractal dimension increase with fracture energy by studying the tensile specimen of concrete and analyzing their fractal dimension. Mecholsky et al. [13] found that the fractal dimension of the fractal profile increase with the fracture energy of the materials in the study of the fractal characteristic of vermiculite and ceramic materials.
Studies have proved that the strength of aggregates has a great influence on the crack extension path when the boundary between aggregates and matrix is in an ideal bounding state, which indicates that there is no defect at the boundary. For example, the fracture mode of composite materials with lower aggregate strength tends to be a transgranular fracture, which makes the fracture surface more smooth. The fracture mode of composite materials with higher aggregate strength tends to be an intergranular fracture, which makes the fracture surface more tortuous [12]. Crack extension path directly determines the complexity of the fracture surface. Generally, the more tortuous the fracture surface, the greater the complexity.

In this study, the crack extension path of concrete materials was simulated numerically based on the non-uniform numerical model of Weibull distribution. Also, the complexities of various ubiquitiform fracture surfaces were calculated to explore the relationship between complexity and fracture energy. Furthermore, to study the influence of specific crack extension law and fracture mode on the relationship between the complexity and fracture energy, numerical simulation of crack extension was carried out based on the numerical model with random aggregate distribution, and the reason for the change of the relationship between complexity and fracture energy were discussed.

2. The method to establish the crack extension model

In the conception of ubiquitiform, a specific crack extension path or a fractured surface should be of ubiquitiform rather than fractal. Principally, there is not any singularity for a ubiquitiformal crack. Therefore, unlike that for a fractal crack, the boundary value problem of a ubiquitiform crack is defined and used to describe the propagation path or crack surface of the ubiquitiform crack. The authors of this study believe that the ubiquitiform crack extension path or the ubiquitiform fracture surface is derived from the heterogeneity of material properties. In fact, recently, assume that the heterogeneity of material obeys Weibull distribution [14,15], which has been widely used in statistical fracture mechanics, the present authors proposed a simply statistical model to describe the ubiquitiformal crack extension [16]. The calculated results agree well with the experimental data.

In this study it is assumed that the tensile strength of concrete material obeys weibull distribution in the following form:

\[
F(x) = 1 - \exp \left[ - \left( \frac{x}{\beta} \right)^m \right]
\]  

(1)

where the distribution parameters \( \beta \) and \( m \) are the homogeneity indexes characterizing the scaling parameter and the degree of homogeneity of the material properties, respectively. For simplicity, we assume that the concrete is composed of the circle aggregates and the mortar matrix [17,18], and a 2D rectangular linear elastic plate model in unit thickness is adopted. The aggregates distribution model with the maximum size of aggregate \( D_{\text{max}} \) is established by using the Monte Carlo method. In this study, the aggregates can be stochastically put into a square computational domain sized in 150mm \( \times \) 150mm, which is then equally divided into \( N = 225 \) characteristic elements, each in size of 10mm \( \times \) 10mm. The tensile strength of the aggregate particles and the mortar matrix are taken to be \( f_{ta} = 6 \text{MPa} \) and \( f_{tm} = 2 \text{MPa} \), respectively.

Based on the meso-element equivalent method, when the volume fractions of the mortar matrix and the aggregates within each characteristic element are determined, the effective tensile strength \( f_{eq}^t \) of each characteristic element is

\[
f_{eq}^t = C_1 f_{ta} + C_0 f_{tm}
\]

(2)

where \( C_0 \) and \( C_1 \) are the volume fractions of the mortar matrix and the aggregates, respectively, within a characteristic element. Moreover, according to [19], arranging the values of the effective tensile strength in ascending order, a statistical distribution function \( F(x_n) \) can then be defined as

\[
F(x_n) = \frac{x_n - 0.5}{M} \quad (1 \leq n \leq N)
\]

(3)

where \( x_n \) represents the effective tensile strength value of the \( n \)th characteristic element, and \( M \) is the sum of all the effective tensile strengths.
The distribution function Eq.(3) can be used to determine approximately the Weibull distribution function \( F(x) \) in Eq.(1). For this purpose, taking twice the logarithm of Eq.(1) and then substituting Eq.(3) into it reaches:

\[
\ln \left\{ \ln \left[ 1 - \left( \frac{x_m - 0.5}{M} \right) \right] \right\} = m \ln x + m \ln \beta 
\]

(4)

Let \( X = \ln x \) and \( Y = \ln \left\{ -\ln \left[ 1 - \left( \frac{x_m - 0.5}{M} \right) \right] \right\} \), one can rewritten Eq.(4) into a linear equation as \( Y = AX + B \), where \( A = m \) and \( B = -m \ln \beta \), which can be obtained by a linear fitting method, and then the homogeneity index \( m \) of the heterogeneous materials property as well as the scale parameter \( \beta \) can also be determined. By using the ABAQUS software, with the displacement boundary condition, the numerical simulations for the ubiquitiformal crack extension under uniaxial tension on a two-dimensional rectangular linear elastic plate in-unit thickness are carried out. For the crack extension path obtained numerically, the box-counting dimension method is used to determine its ubiquitiformal complexity.

3. The numerical results and discussion

To verify the availability of the above described method, the wedge-splitting specimen test presented by Issa et al. [20] is examined. In their test, the maximum aggregate sizes of the specimens are 4.75mm, 19mm, and 38mm, respectively, and the calculated numerical results of corresponding Weibull distribution parameters, together with the material properties, are listed in Table 1.

| Model | \( D_{\text{max}} \) (mm) | \( m \) | \( \beta \) | \( E \) (GPa) | \( \nu \) |
|-------|-----------------|------|------|------|------|
| 1     | 4.75            | 10.47| 3.96 | 30.9 | 0.2  |
| 2     | 19              | 8.62 | 4.02 | 32.4 | 0.2  |
| 3     | 38              | 6.16 | 4.11 | 30.9 | 0.2  |

Hence, the corresponding heterogeneous numerical models can be built by the above described method, which can be used to simulate the ubiquitiformal crack configurations and then the corresponding complexities can be obtained, as listed in Table 2.

| Model | \( D_{\text{max}} \) (mm) | Complexity | Deviation (\%) |
|-------|-----------------|------------|---------------|
|       |                  | Experiment | Simulation    |
| 1     | 4.75            | 1.150      | 1.163         | 1.13          |
| 2     | 19              | 1.170      | 1.185         | 1.28          |
| 3     | 38              | 1.198      | 1.217         | 1.58          |

It can be seen from the table 2 that the complexity are agree well with previous experimental data [12]. The complexity \( D \) of the ubiquitiform crack increase with the decrease of the homogeneity index \( m \). It can also be concluded that the complexity increases with the maximum aggregate particle size, namely, the more heterogeneous the material, the higher the complexity of the crack profile or the fracture surface will be, the larger the maximum aggregate size the more tortuous the fracture surface, so the higher the complexity.

The cohesive fracture energy \( W_{uf} \) consumed by the structure when the material is completely broken were 55N/mm, 78.3N/mm and 116.8N/mm, the generalized fracture energy \( G_{uf} \) of this 2D rectangular linear elastic plate model in unit thickness can be calculated as

\[
G_{uf} = \frac{W_{uf}}{A_{uf}} = \frac{W_{uf}}{L_{uf} \times 1}
\]

(5)

and the results are 0.049N/mm, 0.061N/mm and 0.075N/mm as shown in Table 3, respectively, where \( L_{uf} \) is the measure of the ubiquitiform crack.
Table 3 The calculated fracture energy

| $D_{\text{max}}$ (mm) | $L_{uf}$ (mm) | $W_{uf}$ (N/mm) | $G_{F}$ (N/mm) |
|-----------------------|---------------|----------------|----------------|
| 4.75                  | 1124.19       | 55             | 0.049          |
| 19                    | 1283.96       | 78.3           | 0.061          |
| 38                    | 1557.74       | 116.8          | 0.075          |

As shown in Figure 1, the linear relationship between fracture energy and fractal dimension proposed by Issa et al. [12] as Eq. (6), where $G_F^0$ represents the fracture energy of smooth fracture surface and constant $k$ is the slope of the line fitting the fracture energy and the fractal dimension, which indicates that the complexity increase with the fracture energy.

$$G_F = G_F^0 \left[ 1 + k(C - 1) \right]$$

(6)

It can be concluded that the complexity can not only quantitatively describe the fracture surface, but also have a good correlation with fracture energy.

From the statistical calculation process of the Weibull distribution parameters mentioned above, the influence of Weibull distribution on the complexity reflects the influence of the aggregate size and the aggregate volume fraction on the complexity.

In order to simulate internal crack propagation corresponding to different aggregate fracture energies and calculate its complexity. We have established a random aggregate numerical model with maximum aggregate size $D_{\text{max}} = 30$ mm. Figure 2 shows the crack propagation for aggregate fracture energy $G_{fa} = 0.05$ N/mm and $G_{fa} = 0.08$ N/mm respectively. It can be seen that under a given load, the crack will extend in the conclusion when the aggregate fracture energy is low, and the crack will extend in the matrix when the aggregate fracture energy is high. This result is consistent with the calculation results of Lopez et al. [21]. Calculate the complexity C of two ubiquitiform crack paths in Figure 2 by using the box-counting dimension method, as shown in Figure 3.

![Figure 1](image1.png)

**Figure 1.** The relationship between the complexity and fracture energy

![Figure 2](image2.png)

**Figure 2.** Ubiquitiform crack extension (a) $G_{fa} = 0.05$ N/mm (b) $G_{fa} = 0.08$ N/mm
Figure 3. The complexities of the ubiquitiform cracks with $G_{Fa} = 0.05\text{N/mm}$ and $G_{Fa} = 0.08\text{N/mm}$

It can be seen from the above results that the fracture energy of aggregates has an influence on the ubiquitiform crack and its complexity. Figure 4 shows the relationship between the complexity and fracture energy of aggregate. With the increase of the fracture energy of aggregate, the relationship between the complexity and the fracture energy of the aggregate has an inflection point, the complexity first increases with fracture energy of the aggregate, then remains basically unchanged after the reflection point. This indicates that the fracture mode of the material with low aggregate fracture energy is mainly transgranular fracture. At this time, the fracture energy of aggregate has a greater influence on the complexity. With the increase of the strength of aggregate, the fracture mode changes from transgranular fracture to intergranular fracture, which leads to the decrease of the influence of the aggregate strength on the complexity. The reason for this change can be seen from the stress-displacement curve of aggregates with different strength in Figure 5. When the intergranular fracture occurs, the tensile response of aggregates with different fracture energy is basically the same. When the transgranular fracture occurs, the response of softening section is different.

Figure 4. The relationship between the complexity and the fracture energy of aggregates
4. Conclusion

Based on the Weibull distribution model, numerical simulation of crack extension in aggregate composites is carried out, then the relationship between the complexity and fracture energy is preliminarily discussed. We establish the numerical calculation of crack propagation based on the random aggregate model. The influence of the stochastic spatial distribution of aggregates, different aggregates sizes, and different aggregate fracture energy on the complexity is studied. Further discussed the influence of specific crack extension law and fracture mechanism on the relationship between the complexity and concrete fracture energy. The results indicate that for concrete materials, the relationship between the complexity and fracture energy is related to the fracture mode of materials: When the fracture mode of material is dominated by transgranular fracture, the complexity is negatively correlated with fracture energy. When the fracture mode of material is dominated by intergranular fracture, the complexity is positively correlated with fracture energy.

5. Acknowledgments

This work was supported by The National Nature Science Foundation of China under Grant 11772056

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