Entanglement islands, fire walls and state paradox from quantum teleportation and entanglement swapping

Xuanhua Wang$^{1,2,*}$, Kun Zhang$^{3,4}$ and Jin Wang$^{2,3,*}$

1 Center for Theoretical Interdisciplinary Sciences, Wenzhou Institute, University of Chinese Academy of Sciences, Wenzhou, Zhejiang 325001, People’s Republic of China
2 Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794, United States of America
3 Department of Chemistry, Stony Brook University, Stony Brook, NY 11794, United States of America
4 School of Physics, Northwest University, Xi’an, Shaanxi 710069, People’s Republic of China

E-mail: wangxh@ucas.ac.cn and jin.wang.1@stonybrook.edu

Received 28 September 2022; revised 13 March 2023
Accepted for publication 28 March 2023
Published 18 April 2023

Abstract

Recent discovery of the fine-grained entropy formula in gravity succeeded in reconstructing the Page curves that are compatible with unitary evolution. The formula of generalized entropy derived from the gravitational path integration, nevertheless, does not provide a concrete insight on how information comes out from a black hole. In this paper, we start from a qubit model and provide a quantum informational interpretation of entanglement islands. We propose an identification of entanglement islands with quantum measurements and remark on the parallel between the black hole information problem and the old problem of quantum measurements. We show that the Page curve can still be realized even if information is lost so that the information paradox can be explained as one manifestation of measurement problem. We show that such interpretation is necessary for a quantum informational model if smooth horizons and bulk reconstruction are assumed, and demonstrate explicitly that Page curves of solvable 2D gravity can be obtained through teleportation and entanglement swapping. We argue that the similarities between the black hole information problem and the measurement problem suggest links in the origins of the two problems.

* Authors to whom any correspondence should be addressed.
1. Introduction

The information loss of black holes has been a long-standing problem that points to the incompatibility of several important fields of physics—quantum mechanics, thermodynamics and the theory of general relativity. It is remarked that this paradox may be of equal importance to the ultraviolet crisis in classical physics which was only resolved by the advent of quantum mechanics [1].

Recently, tremendous progress was made in producing a unitary Page curve of the black hole fine-grained entropy using gravitational path integration and replica wormholes [2–4]. The calculation not only returns unitary Page curves, but also suggests the peculiar property of information release on the boundary of an entanglement island past Page time. However, the calculation failed to present a clear picture regarding how such conservation of entropy is realized beyond the Page time. It was doubted that the quantum Ryu–Takayanagi (RT) prescription does not completely resolve the information problem in the bulk since it requires substantial failure of semi-classical gravity such as firewalls [5, 6]. Besides, ambiguities exist regarding the measure and the meaning of the gravitational path integral (GPI) [7, 8]. The Page-curve calculations show that the fine-grained entropy is dominant by the Hawking saddle (trivial RT surface) in the beginning of the evaporation. This confirms that the density matrix is consistent with that given by Hawking. At late times, the entropy of black hole has an $O(1)$ correction due to the dominance of the replica wormhole saddle. The decrease of entropy is sometimes interpreted as encoding the information inside the horizon being into the radiation. But one staggering problem is that no such mechanism is known and the then state of the radiation becomes ambiguous within this unitary description [6, 9].

Quantum mechanical models that reproduce the results of the replica wormhole calculations can provide insights in the understanding of black hole physics owing to their simplicity and clarity. One particular example is Page’s calculation of entanglement entropy of $N$-particle system, which generates the unitary Page curve and has been a strong motivation for seeing a similar curve for black holes [10, 11]. Besides, Horowitz and Maldacena proposed a final state boundary condition which acts like a measurement to collapse the state at black hole singularities to restore the unitary Page curve [12]. However, the model has various issues such as the violation of causality and information loss and does not completely agrees with the quantum RT calculation [13–15]. The endeavor of building quantum mechanical models has been undertaken by many studies (for a non-exhaustive list, see [12–14, 16–22]). However, new results from the last few years changed our understandings about black holes and the corresponding new quantum informational interpretation is needed.

The ambiguity of unitarity appears not only in black holes but also in quantum measurements and teleportations [23–26]. Nonlocal effects of measurements, which are exemplified by quantum teleportation and entanglement swapping protocols, appear to be at odd with information conservation. This has been demonstrated explicitly through quantum tomography. Manipulations of a distant quantum states through measurements are coined as quantum
steering [27, 28]. Its understanding, however, was long-debated in history and is still ongoing.5 Teleportation and entanglement swapping, which are based on the nonunitary measurements, do not violate the no-signaling principle (no-communication theorem) as signaling is prohibited by uncontrollable random measurement results, and both teleportation and entanglement swapping have been verified for space-like separated regions [28, 30–32]. If nonunitary measurements in the process are viewed as part of unitary operations in an open system, measurements in teleportation and entanglement swapping have to be a part of global unitary evolution acting on space-like regions. Such description avoids the conundrum of the wavefunction collapse but violates the classical causality and locality [33]. We will show that the difficulties concerning the interpretations of entanglement islands and Page transitions indeed have close ties with quantum measurements and teleportation.

In this study, we draw the parallel between the black hole information problem and the quantum measurement problem. Therein, an effective description of islands and Page curves by emergent measurements naturally show up with minimal assumptions. We show that the quantum measurement is the only possible interpretation within the framework of quantum mechanics under the following requirements: 1. black holes is macroscopic but evolves quantum mechanically; 2. the bulk reconstruction of the entanglement wedge is effective; 3. the smooth horizon; 4. quantum–classical transitions of quantum mechanics and the emergence of quantum measurements. If the assumption of quantum–classical transition is lifted, the paradox of black hole information paradox can be attributed to the quantum measurement paradox. We show that through this identification, we can restore the Page curves from a pure quantum mechanical argument. We discuss two possibilities which are indistinguishable from the outside that both give the same Page curve. The first possibility is a unitary evolution which conserves information and the second one loses information. The first possibility is what is expected from unitarity. In this proposal, the state of radiation is well-defined and the idea of gravity/ensemble duality has natural realization. However, as to how such measurement (projection) occurs, we do not pretend to have a definite answer and suggest mechanisms such as quantum-to-classical transitions or spontaneous wavefunction collapses [34–36]. The information loss in the second possibility is under the assumption that measurements are nonunitary. In this scenario, the final states are ensembles of pure states and not in one-to-one correspondence or bijection with initial data. Thus we cannot retrieve the initial information from a final state.

2. Wormholes and emergent measurements

Bekenstein–Hawking unitarity states that black holes evolve unitarily and can be properly modeled as a quantum system [37]. We dissect this principle and show how a black hole can be modeled as a quantum system yet does not evolve completely unitarily. In section 2.1, we explain why a quantum informational model is necessary for understanding black holes. In section 2.2, we give an account of Hawking’s and Penrose’s arguments on the interpretations of quantum measurements. Section 2.3 discusses the relation between the protocols and the black hole information. In the last subsection, we draw the link between concepts in gravity theory and those in quantum information which is summarized in an ad hoc dictionary. The related background on the state transfer protocols such as teleportation and entanglement swapping and implications of those protocols are given in appendixes ‘Background on

5 As phrased by Schrödinger [29]: ‘It is rather discomforing that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter’s mercy in spite of his having no access to it.’
quantum teleportation and entanglement swapping protocols’ and ‘Classical ensemble and quantum measurement’.

2.1. Purification of quantum states

It is believed that early Hawking radiation is purified due to the nonperturbative corrections to the density operator

\[ \rho_{\text{exact}} = \rho_{\text{thermal}} + \text{perturbative} + \mathcal{O}(e^{-\#S_{BH}}), \]

(2.1)

where the last term is the nonperturbative correction due to, for example, the appearance of the replica wormholes. The perturbative correction does not fix the information problem when considering the massive black holes [38]. The nonperturbative correction which looks like semi-random noise that is exponentially suppressed, gives \( \mathcal{O}(1) \) corrections to the entanglement entropy of the black hole after a sufficient amount of radiations have been emitted [39–41]. For a massive evaporating black hole, calculations using the generalized entropy formula suggest that after Page time, the outgoing Hawking particles have to be highly entangled with the early radiation so that the decrease rate of the radiation entropy is roughly the same order as the increase in the initial period. This requires \( \mathcal{O}(1) \) corrections to the density matrix of the micro-process which results in a ‘firewall’ at the horizon.

In the quantum RT prescription, the state of the late-time radiation is ambiguous when adopting the holographic arguments [6, 9]. The interpretation of island seems to obscure the radiation density matrix as the information within the island is believed to be encoded in the radiation while the experimental result can only give a yes or no answer. However, if a futuristic experiment were to be conducted to collect all Hawking radiations from a lab-made black hole, a pure state needs to be found in order to obey the Page curve. The density matrix can in principle be reconstructed through quantum tomography if sufficient identical copies of the states are given [42]. Therefore, no ambiguity is allowed.

The information loss in the purification process is also found in ordinary quantum systems [43]. In their study, a quantum system initially prepared in the pure state is deteriorated by the white noise of the environment. By allowing the system to interact with a prepared auxiliary state, the system is purified and the mixedness is transferred to the auxiliary system. What they found after investigating different unitary operations is that the increase of purity always comes at the price of reduced fidelity, i.e. the loss of information about the original state. The study shows that the purification of a random quantum state through unitary operations does not automatically guarantees the recovery of its information, but rather, it reduces the information content of the state and that an upper bound for the fidelity exists. Though the mechanism for the purification of Hawking radiation is different in that the radiation has to be purified by itself without an auxiliary system, the above result also has implications on quantum measurements. It suggests that ordinary unitary operations are fundamentally different from measurements since a measurement operation has a finite probability of recovering the original state with fidelity one.

2.2. Hawking–Penrose debate on quantum measurements

The idea that the black hole information and quantum measurements is closely related became more conspicuous after the Hawking–Penrose debate. Hawking and Penrose held different views on quantum measurements and other matters and the debate was documented in [23].

Penrose claimed that fundamental evolution processes of quantum systems are of two folds. One is the unitary evolution (‘U’ process) and the other is the reduction of state vectors upon
measurement (‘\(R\)’ process) where phase-space volume reduction and information loss arise. It aligns with Bell’s advocacy of objective collapse theory exemplified by Ghirardi–Rimini–Weber model [24]. Penrose’s primary argument is that the phase-space shrinking of an object falling into the singularity of a black hole violates the conservation of information and that the loss of phase-space volume is balanced by the spontaneous \(R\) process. In this description, information will be lost during the disappearance and reappearance of a black hole in a box and the time symmetry is broken. He further proposed to ‘gravitationalize quantum mechanics’ in which the superposition of the wavefunctions at different locations decays due to gravitational field with a lifetime \(\tau = \frac{\hbar}{\Delta E_G}\), where \(\Delta E_G\) is the difference in the gravitational potential.

On the other hand, Hawking opined that the unitary evolution according to the Schrödinger’s equation is a full description of quantum mechanics and no other evolution such as the ‘\(R\)’ process is needed. He argued that the reason Schrödinger’s cat problem exists is that one cannot isolate macroscopic objects as large as a cat from its environment or intermolecular electromagnetic fields. He reasoned that the whole Universe is supposed to be in a quantum state. We still detect decoherence and classical behaviors simply because we cannot observe the whole Universe but only parts of it. Both Hawking and Penrose had suggested the loss of information of black holes, but Hawking admitted a change of his view later due to the reasons related to the AdS/CFT conjecture.

The information problem of the black hole has been long intertwined with quantum measurements [44–46]. Recent studies on quantum measurements are also trying to realize objective wavefunction collapse (‘\(R\)’ process) by applying only the unitary evolution of Schrödinger’s equation [47]. Its central idea is that the measured system and the pointers together evolve unitarily but the measured system alone evolve nonunitarily and that information can be extracted from the measured system. The collapse of wavefunction can be derived from the Schrödinger’s equation of the total system. Though there are still issues in this study, it points to a potentially new way to interpret the Penrose’s \(R\) process. We shall see in this study that the Hawking–Penrose debate on quantum measurements is still relevant today despite the progress made since the debate, and that the ambiguity of black hole information puzzle, or its more recent incarnation—the state paradox, directly corresponds to the two different views in the Hawking–Penrose debate. To recapitulate, Hawking’s principal counterargument to Penrose’s \(U\) and \(R\) processes was that no measurement occurs inside black hole since no observer is present and that no natural collapse mechanism is known. Penrose’s claim was the phase space volume reduction when objects fall into a black hole, the breaking of time reversal symmetry during cycles of black hole appearance/disappearance, and the phase space volume gain due to natural collapses. Though the Page curve obtained from the island calculation is a sign of unitarity, we argue that this, however, can still be compatible with Penrose’s argument of spontaneous wavefunction collapse since the purification does not guarantee the information recovery. Therefore, the calculation of Page curve does not completely resolve the information puzzle of the black hole, but instead, points to the old debate of the interpretations of measurements.

2.3. Island-measurement identification

The crux of the correspondence is the following. In the quantum informational description, the island corresponds to where information is acausally transferred to the outside. For a black hole initially having a low entropy and a trivial quantum RT surface, an infalling Hawking state is maximally entangled with an outgoing state assuming the microcanonical description. On the other hand, for an ancient black hole that is sufficiently mixed, its entanglement island is the spacetime region where measurements or projections onto the Unruh state emerge.
Figure 1. The Hawking pair AB marked in blue created near the horizon (A and B are conventionally drawn parallel to the horizon as represented by the Unruh vacuum. Here, the picture is drawn to represent the particle-hole picture). The ingoing qubit A reaches the island represented by the slanted red line and gets measured. The Cauchy surface chosen at the instant of measurement is the green horizontal line. The early radiation D in black is maximally entangled with the black hole state C.

In this correspondence, the assumption that information will be teleported out is equivalent to that measurement results are predictable as long as one has a complete knowledge about the measuring apparatus and the state to be measured. When the fine-grained entropy of a black hole reaches Bekenstein–Hawking entropy, the infalling negative energy state and a black hole state are projected onto the vacuum state near the horizon, similar to an annihilation process seen from the outside (figure 1). The black hole state will be transferred to the radiation through an entanglement swapping protocol except that the classical information channel is replaced by a quantum measurement which allows one to calculate the measurement results. Under this protocol, the fine-grained entropy of the black hole decreases. This process starts to occur when the entanglement island forms inside the horizon.

On the other hand, one should note that the island, derived from the replica calculation of entanglement entropy, does not directly imply the transfer of information. Strictly speaking, it is only the region where the decrease of the entanglement entropy arises. If the requirement that information comes out from the black hole is lifted, then we only need to impose a Bell-state measurement on the island without the additional information transmission or the predetermined experimental results. The radiation is still purified and the black hole has the same Page curve as in the previous case. However, in this case the outgoing radiation is an ensemble of pure states so that the original information is still lost. The merit of this prescription is that no postselection or other postulates are needed.
The scrambling time has a simple explanation in terms of projection operations. The scrambling time was originally argued in a setup where a black hole directly spits out its information (qubits) as the Hawking radiation after Page time and information thrown into the black hole can be extracted out from radiation after a scrambling time [48, 49]. From the dictionary of anti de-Sitter (AdS)/conformal field theory (CFT), the bulk scattering processes in the causal wedge \(C_A\) of \(A\) are reconstructable in terms of certain nonlocal CFT operators in \(A\) through Hamilton–Kabat–Lifschytz–Lowe procedure [50]

\[
\langle \phi(x_1) \phi(x_2) \rangle_g = \int dX_1 dX_2 \ K(x_1;X_1)K(x_2;X_2) \langle 0 | \mathcal{O}(X_1) \mathcal{O}(X_2) | 0 \rangle_{\text{CFT}},
\]

where \(K(x_i;X_i)\) is the smearing operator which is dependent on the specifics of the spacetime. This causal wedge reconstruction was later proposed to extend into a larger entanglement wedge \(\mathcal{E}_A\) [51]. Intuitively, it suggests that when a qubit enters into the entanglement wedge of the quantum information language, when the qubit enters the island its state will be teleported out through the aforementioned protocols. One related proposal is the Horowitz–Maldacena (HM) model [12]. The difference is that in the HM model, one pre-determined projection into a chosen state is imposed for such measurements at the singularity. The model does not have the formation the entanglement wedge after the Page transition. Therefore, from the HM proposal a trivial Page curve is obtained. We will discuss this proposal in more detail in section 3.1.

A diagrammatic illustration of this mechanism in an asymptotically flat black hole background is given in figure 1. For example, we consider a Hawking pair \([R]_{\text{in}} \otimes [O]_{\text{out}}\) created near the horizon slightly after Page time. In the microcanonical picture, the black hole state \([m]\) is in maximal entanglement with early radiation \([i]\). Then the initial state in this micro-process is

\[
|\Psi\rangle_{\text{total initial}} = \frac{1}{\sqrt{N}} \sum_i N |m_i \rangle \otimes |i\rangle \otimes |R\rangle_{\text{in}} \otimes |O\rangle_{\text{out}} = \frac{1}{\sqrt{N}} \sum_{ij} |m_j \rangle \otimes |f_i \rangle_{\text{in}} \otimes |f_j \rangle_{\text{out}} \otimes |i\rangle.
\]

In the qubit description shown in figure 1, this process can be simplified (without loss of generality) to the following:

\[
|\Psi\rangle_{\text{initial}} = |\Psi^-\rangle_{AB} |\Psi^{-}\rangle_{CD} = \frac{1}{2} (|0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B) (|0\rangle_C |1\rangle_D - |1\rangle_C |0\rangle_D),
\]

where \(A\) and \(B\) are the infalling and outgoing Hawking states, and \(C\) is the black hole state that is maximally entangled with the early radiation state \(D\). When the qubit \(A\) reaches the island, for an external observer, it ‘annihilates’ with a black hole state \(C\). If we require the result of this annihilation to be a unique vacuum state, it can be simulated as a projection onto the maximally entangled vacuum state \(|\Psi^-\rangle_{AC}\). Notice that equation (2.4) can be rewritten as

\[
|\Psi\rangle_{\text{initial}} = \frac{1}{\sqrt{2}} |\Phi^+\rangle_{AC} |\Phi^+\rangle_{BD} - \frac{1}{\sqrt{2}} |\Phi^-\rangle_{AC} |\Phi^-\rangle_{BD} - \frac{1}{2} |\Psi^+\rangle_{AC} |\Psi^-\rangle_{BD} + \frac{1}{2} |\Psi^-\rangle_{AC} |\Psi^-\rangle_{BD},
\]

where the Bell states are defined as follows:

\[
|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle), \quad |\Phi^-\rangle = \frac{1}{\sqrt{2}} (|0\rangle \otimes |0\rangle - |1\rangle \otimes |1\rangle),
\]

\[
|\Psi^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle), \quad |\Psi^-\rangle = \frac{1}{\sqrt{2}} (|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle).
\]

\(^6\) The four Bell states form a basis for the Hilbert space \(\mathcal{H}_2 \otimes \mathcal{H}_2\), and cannot all correspond to the vacuum. Here, we pick the singlet state as the vacuum state and the other three states then correspond to the firewall states.
This is the same equation as the entanglement swapping protocol defined in equation (A.6) in the appendix. In the information-recovery model, only the last term is postselected and the state is renormalized. The infalling state corresponds bijectively to the radiation state and no information is lost. On the other hand, if we do not postselect the state, the operation on the AC state is an ordinary measurement following the Born rule. In this case, every effective measurement will cause a loss of information of \(-\text{tr}(\rho \log_2 \rho) = 2\), i.e. two bits of data will be lost per measurement in the Born measurement compared with that in the postselection model. On the other hand, regardless of the information conservation or not, the radiation state will change from a mixed state entangled with the black hole to an ensemble of pure states. Therefore, the fine-grained entropy decreases in either case. Mathematically, this is due to that the ensemble average is done over the entropy \(S_{vN}\) instead of over the density matrix, i.e. \(\langle S_{vN}(\rho) \rangle = 0\) while \(S_{vN}(\langle \rho \rangle) \neq 0\).

According to the Hayden–Preskill argument, any infalling information ‘thermalizes’ with the black hole information in a scrambling time [48]. The information can be deciphered if Hawking radiation contains data about the black hole state instead of being in thermal state. It has been shown in various models that the scrambling time is approximately what it takes for the state to reach the island. Consider a Hayden–Preskill process where a dictionary which is maximally entangled with the observer is thrown into the black hole. The dictionary will be measured once entering the island and the information encoded in the entanglement will be swapped into the outgoing qubit \(|B\rangle\) and the early radiation \(|D\rangle\). If we adopt the projection proposal which requires a unique measurement result, then the information can be retrieved after in a scrambling time, consistent with the quantum informational analyses given in [48, 49]. For a pure state thrown into the black hole, the projection generates quantum teleportation instead of entanglement swapping. If we release this requirement of a unique final state, information about initial conditions will be lost if the measurement results are not accessible. The information can be recovered if only the measurement result inside the black hole can be accessed from the outside. To conserve the information, certain violation of the randomness of the measurement is necessary assuming the horizon is smooth. Since the black hole does not conserve global symmetry charge, the measurement results may also disappear after the evaporation [5, 52, 53]. Alternatively, one can refer to the baby Universe model where the information is stored, but the result is the same—if we collect the Hawking radiation from the outside, information will be lost from our Universe.

In the measurement model, the uncertainty in the results of the measurements renders the density matrix of the outside as an ensemble. However, one should not confuse the coarse-grained density matrix where the ‘mixed state’ is the classical average due to our ignorance with the fine-grained density matrix where mixed states originate from entanglement with other systems. Classical ensembles do not change the vanishing fine-grained entropy of the radiation and the Page curve. A measurement process occurs when quantum coherence in the density matrix vanishes and diagonal terms in the measurement basis become different realities and are reinterpreted as classical probabilities in an ensemble. Though there are disputes on the interpretations of measurements\(^7\), we emphasize that issues regarding the states of Hawking radiation and the unitarity of quantum measurements are the same.

\(^7\) The decoherence program has helped in the understanding of environment-system interactions and can explain some types of measurements [54], but such mechanism does not resolve measurement problems related to the selection of one particular state and the violation of weak causality.
2.4. Dictionary from gravity theory to quantum information

The quantum informational interpretation of islands naturally bring in the correspondence between concepts in quantum information and those in black hole physics. Below we summarize the identification of concepts based on the interpretation.

**Entanglement island:** Bell-state measurement or projection.

The region inside the island, or the entanglement wedge of the radiation, is usually interpreted as where the information is encoded in the radiation. This understanding is borrowed from the bulk reconstruction in the AdS/CFT duality that bulk operators in AdS can be reconstructed as the CFT operators in spacial region $A$ provided that they lie in its entanglement wedge [55]. In the language of quantum information, it is the region where the state information is transferred to the outside (details provided in section ‘Background on quantum teleportation and entanglement swapping protocols’). One important remark we draw is that this process cannot be realized through local unitary processes. Naively, we expect that the nonunitarity is a result of tracing the uninterested degrees of freedom and the whole system evolves unitarily. However, this cannot be true considering the realization of entanglement swapping. It is unclear if measurements conserve information and therefore the black hole information paradox can be attributed to the old problem of measurement in quantum mechanics.

**Replica wormhole:** an entanglement swapping or a teleportation protocol.

It is unclear whether the island derived from the generalized entropy formula reflects anything physical or is due to the out of control of the GPI. The general picture is as follows. The island represents the mouth of a replica wormhole. Information entering the island goes through the replica wormhole and returns to the asymptotic region. One related argument is the ER = EPR provided by Maldacena and Susskind [56]. On the other hand, one interpretation of the island which seems more natural in the framework of standard quantum systems is that the information is teleported out via certain quantum teleportation protocols, i.e. ‘ER = QT’. More complicated multi-particle teleportation protocols can be identified as wormholes of higher genuses.

**Scrambling time:** the time before the measurement is completed.

This naturally follows from the above interpretation of the island and replica wormholes.

**Page transition:** quantum-to-classical transition or emergence of measurements.

Page transition occurs when island appears. In other words, it occurs when measurements begin to emerge and the entanglement breaks. Therefore, Page transition can correspond to certain quantum-to-classical transition. The transition point can be inferred through the GPI defined below. The partition function is

$$Z = \int DgD\phi \ e^{-S_{E}[g,\phi]}Z_{\text{matter}},$$

(2.7)

where

$$S_{E}[g] = -\frac{1}{16\pi G} \int \sqrt{-g} (R + \cdots) + \text{boundary}$$

(2.8)

is the Euclidean action of gravity. Semi-classically, the gravity part is only considered near the classical saddles of the general relativity $Z_{gr} \propto \sum e^{-S_{gr}}$. The transition occurs when the dominant contribution to the summation changes from the action of a trivial topology to that of a wormhole topology.

**Gravity/ensemble duality:** unitary interpretation/spontaneous wavefunction collapse indistinguishability.

The gravity/ensemble duality is a proposal that the GPI calculates an ensemble of quantum mechanical theories without gravity [6, 9, 41, 57, 58]. This idea was proposed to resolve the
state paradox of Hawking radiation and the factorization issue of the partition function in AdS/CFT and is supported by the results from random unitary dynamics. This duality can be directly translated to the interpretations of measurements. Unitarity can be restored in a quantum measurement assuming the certain superposition of classical realities. Such interpretation is indistinguishable from the nonunitary interpretation of the spontaneous collapse into a classical ensemble when the complexity of the system is large enough. The semi-classical gravity calculation suggests that a transition of dominant contributions to classical observables occurs when the fine-grained entropy of the measuring system is close to its thermodynamic entropy.

3. Comparison with other quantum informational models

In this section, we review some of the previous arguments that are related to our proposal and discuss the differences between these ideas and proposals. In section 3.1 we review the HM proposal. We discuss its relation to our proposal and its shortcomings. In section 3.2 we review the Osuga–Page (OP) model and comment on its strength and limitations. In section 3.3 we comment on the connection between our model and the Reeh–Schlieder (RS) theorem.

3.1. HM model: final-state projection of black holes

The HM proposal claims that the unitarity of black hole evaporation can be realized by imposing a final state boundary condition at the black hole singularity. The essential idea is similar to quantum teleportation, but instead of allowing uncertainties in the outcome of the measurement inside the black hole, the outcome (the final state) is required to be definite to avoid the information loss.

The gist of the argument is the following. In the semi-classical picture, the radiation is created in the form of Unruh state \( |\mathcal{R}\rangle \) (\( R \) stands for the radiation), given by
\[
|\mathcal{R}\rangle_{\text{in} \otimes \text{out}} = \frac{1}{\sqrt{N}} \sum_j |j\rangle_{\text{in}} \otimes |j\rangle_{\text{out}}.
\]

Here \( |\mathcal{R}\rangle \) is a maximally entangled two-qudit state where each qudit has dimension \( N \). Qudit generalizes qubit into a \( N \) level of state. The pure state of the black hole is denoted by \( |\psi\rangle_M \). The black hole plus the infalling radiation and the outgoing radiation together form the closed system, given by \( |\psi\rangle_M \otimes |\mathcal{R}\rangle_{\text{in} \otimes \text{out}} \). Horowitz and Maldacena argued that the information loss arises because the singularity is treated as another ‘asymptotic’ region. Therefore, no unitary operation can save the information from being lost. The key assumption in the HM proposal is that the singularity is not another asymptotic region, instead, it is forced to be a unique pure state with the form
\[
|\text{BH}\rangle_M \otimes |\text{in}\rangle = \frac{1}{\sqrt{N}} \sum_j |j\rangle_M \otimes |j\rangle_{\text{in}},
\]

which is a maximally entangled two-qudit state. Both the Hilbert spaces \( \mathcal{H}_M \) and \( \mathcal{H}_\text{in} \) are considered as inside the black hole. Analogous to equation (A.4), this result is equivalent to a teleportation
\[
|\text{BH}\rangle_M \otimes |\psi\rangle_M = \frac{1}{N} |\text{BH}\rangle_M \otimes |\psi\rangle_{\text{out}}.
\]

The information about the initial state of the black hole \( |\psi\rangle_M \) is teleported to the outside due to the known projection. After the black hole evaporates, the radiation becomes a pure state.
In the HM proposal, the information loss is circumvented by requiring a unique result of the measurement and the probability of the postselected state is removed by renormalizing the state. However, such deterministic projection is not a legitimate operation in the framework of quantum mechanics.

In the HM model, nonunitary operations are involved near the singularity to recover the unitarity outside the black hole. However, for HM model to conserve information more restrictions are needed. In the Comment \[13\], Gottesman and Preskill reasoned that since the infalling radiation interacts with the black hole as it falls to the singularity, there is a global evolution acting on $\mathcal{H}_M \otimes \mathcal{H}_{\text{in}}$. Then the boundary state $|\text{BH}\rangle_{\mathcal{M}\otimes\mathcal{\text{in}}}$ has to be changed accordingly (see equation (A.5) in the appendix). They pointed out that this cooperativity seemed to be an unnatural ‘conspiracy’ since different conditions and interactions need to be picked out accordingly for no other reasons but to conserve the unitarity. Otherwise, the information loss is almost unavoidable in the HM model \[13, 15\]. The model in our study is free from this ailment. The measurement occurs at the island which appears near the horizon after Page time. The infalling Hawking particles are measured immediately after being created at the horizon without further concerns about their interactions with the black hole when falling into the singularity. Therefore, no case-sensitive final-states are needed and it is free from such conspiracy. The initial period of the evaporation is the same as Hawking predicted until Page time when a transition occurs to the wormhole saddle with openings at the horizon.

The HM proposal is believed to reconcile the smoothness of the horizon with unitarity \[17\] but we argue that this viewpoint is also questionable. First of all, the projection is not a unitary operation even if we post-select the state. As remarked in \[13\], such measurement with predetermined results is at odds with quantum mechanics and violates causality. By renormalizing the state, the final-state projection is more properly understood as the boundary condition in the future which we can unitarily evolve backwards to obtain all the information. However, such boundary condition cannot be realized from the initial Unruh vacuum if we assume locality and unitarity. A simple argument is the following. Given a Hawking pair is formed near the horizon,

$$|\text{BH}\rangle \otimes |\text{R}_{\text{in}\otimes\text{out}}\rangle = \frac{1}{\sqrt{N}} \sum_j |\text{BH}\rangle \otimes |j\rangle_{\text{in}} \otimes |j\rangle_{\text{out}}.$$  \(3.4\)

Local unitary operations inside the black hole can mix the infalling states and the black hole states as follows,

$$|\text{BH}\rangle \otimes |\text{R}_{\text{in}\otimes\text{out}}\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_j (U_{\text{BH}\otimes\text{in}} \otimes I) |\text{BH}\rangle \otimes |j\rangle_{\text{in}} \otimes |j\rangle_{\text{out}},$$  \(3.5\)

where $U$ is the unitary operation on the black hole state $|\text{BH}\rangle$ and the infalling state $|j\rangle_{\text{in}}$. This operator does not change the entanglement between the outgoing states $|j\rangle_{\text{out}}$ and the total state inside the black hole $\sum_j (U \otimes I) |\text{BH}\rangle \otimes |j\rangle_{\text{in}}$ since the density matrix of the outgoing qubit is invariant under such unitary operations

$$\rho_{\text{out}} = \text{tr}_{\text{BH}\otimes\text{in}} |\text{BH}\rangle \otimes |\text{R}_{\text{in}\otimes\text{out}}\rangle \otimes |\text{R}\rangle \otimes (\rho_{\text{BH}}) \rightarrow \rho_{\text{out}}.$$  \(3.6\)

The fine-grained entropy inside the black hole does not change and therefore the initial state cannot be in the assumed state.

The appeal of the HM proposal is that the collapse of the wavefunction occurs at the singularity. However, once the state at the singularity is fixed, the infalling radiations must be subjected to the boundary condition throughout the history of the black hole from the very beginning. This gives a continuous information leaking picture which is different from what Page argued \[10, 11\]. The consequence of it is a constant trivial Page curve—the entropy of the
black hole or the radiation is always zero. In other words, the black hole just cannot entangle with anything!\footnote{To be precise, it can entangle only for a very brief time. Then the entanglement has to be broken and the black hole information is teleported out.}

On the other hand, the fast scrambling of black hole along with the bulk reconstruction suggests that such final state projection, if exists, already occurs when information enters the island near the horizon instead of waiting until it hits the singularity. This is also suggested by the bulk reconstruction from AdS/CFT that the states in the entanglement wedge can be retrieved from the boundary operators. It is more natural to introduce a measurement process than a post-selected measurement process. Then paradox of the information loss or conservation can be directly attributed to the same problem of unitarity and locality about measurements. In addition, in the island-measurement correspondence is free of firewalls (see section 5) and the entanglement wedge reconstruction has a natural correspondence. The connection with measurements may suggest that the Page transition can be understood as a quantum-to-classical transition at which the quantum states are effectively measured and wavefunctions are spontaneously collapsed.

3.2. Osuga–Page firewall-free qubit model

Osuga and Page considered a qubit model which allows information to escape without firewalls [19]. In their setup, the non-local gravitational degrees of freedom are assumed. The gist of their consideration is the following. The maximally entangled radiation qubits are created near the horizon in an ordinary Hawking process. As the outgoing qubit, representing the radiation, propagates outward from the horizon, its interaction with the nonlocal gravitational qubit induces a unitary transformation which turns the mixed state outgoing qubit into a pure state. An example of such the evolution operator is $e^{-i\pi(1-K/K_h)P_{ac}}$ such that it obeys

$$\lim_{r \to \infty} e^{-i\pi(1-K/K_h)P_{ac}}|q⟩_a|B⟩_{bc} = -|B⟩_{ab}|q⟩_c,$$

where $|B⟩_{ab}$ defined by

$$|B⟩_{ab} = \frac{1}{\sqrt{2}}(|0⟩_a|1⟩_b - |1⟩_a|0⟩_b)$$

is the singlet Bell state. $P_{ac} = |B⟩_{ab}⟨B|$ is the projection operator, and $K$ is some curvature invariant such as the Kretschmann scalar which decays from $K_h$ at horizon to zero at infinity. Here $a$ represents the black hole nonlocal qubits, $b$ is the infalling qubits and $c$ is the outgoing qubits.

Different from the usual wisdom of the nonlocal interactions between local qubits for resolving information issue of the black holes, the OP model suggests that information can come out due to the interactions between the Hawking radiation and the nonlocal black hole qubits. As the radiation propagates to the infinity, an teleportation-like process is effectively realized which switches the radiation qubits with the black hole qubits. However, this argument seems to be problematic if one collects the radiation at a finite distance from the black hole. When the radiations are collected at any positions with nonvanishing $K$, they will be found in a mixed state according to this prescription, as a consequence, unitarity and entropy will not preserved. On the other hand, even if the radiations are allowed to propagate to the infinity, the OP model appears to predict a trivial Page curve which is at odds with the recent results from the GPI. Such realization is similar to assuming that the effective measurements are nonlocal.
unitary operations in our proposal. Under this assumption, the teleportation protocol can be completed and the information is preserved.

3.3. Connection with Reeh–Schlieder theorem

It is widely believed that once the infalling matter has passed the event horizon, it can no longer influence the outside due to causality [59]. It is also assumed that either quantum mechanics breaks down or some new physics is required in order for the information inside the horizon to escape. However, axioms of quantum mechanics not only have the unitary evolution but also includes processes as measurements which may not be naively understood as just another unitary operation, otherwise teleportation will not be realized. The RS theorem states that the vacuum is a cyclic separating vector of the algebra of operators in any open region $\mathcal{U}$ [60].

It means that a dense set of states in the Hilbert space $\mathcal{H}$ can be produced by local operators acting on the vacuum. Intuitively, it suggest that one can ‘create the Moon’ from local operations inside the house. The requirement is that such operations have to be nonunitary and the vacuum is fully entangled\(^9\). For a nice review on RS theorem, see [61]. For a finite dimensional system, the RS theorem implies that for two maximally entangled subsystems that are spacelike separated, we can apply local operations like projections to manipulate the partner entangled subsystem. The set of such manipulations is dense in the space of operations. The quantum teleportation and entanglement swapping are examples of such operation provided that the measurement operator is nonunitary. For teleportation and entanglement swapping, classical communications are needed in order to have a complete control of what the states one can build at the other end since the results of the measurements are not known beforehand unlike the predetermined projections. If the measurements are some local unitary operators, such operation will not be possible. The RS theorem suggests a possible resolution of the seemingly acausal information transmission of black holes by nonunitary operators inside the black hole, which is similar to what we propose in this paper.

4. Regarding the Page curves in 2D asymptotically-flat gravity

The islands and Page curves of various 2D evaporating black holes have been explicitly studied in [62–64]. An example Page curve is shown in figure 3. However, a microscopic understanding of what happens in entangling process that results in the different rates of entropy change in the initial and final stage is far from clear. It is argued that this might be a simple artifact of nonequilibrium thermodynamics, in which the radiation process is irreversible and the time reversal symmetry is not valid [63, 64]. However, the irreversibility applies only to the coarse-grained quantities in the thermodynamic consideration of the radiation gas. The entanglement entropy of the gas and the black hole, on the other hand, is always the same for the two systems regardless of they being in equilibrium or not. The thermodynamic entropy of the radiation gas is dependent on quantities such as the spectrum density and its spatial size, and therefore may not be naively identified with the fine-grained entropy when a nonequilibrium process is involved [62]. Furthermore, despite many analogies between quantum entanglement and thermodynamics, entanglement behaves fundamentally different from a thermodynamic objects in that no analogous second law exists for the entanglement transformations [65]. Thus, arguments direct treating the entanglement entropy as the thermodynamic entropy.

\(^9\) The set of local Hermitian operators are also capable of such Moon construction.
can be problematic. We show that the informational interpretation of the islands proposed in this paper eases the tension between the different purification rates and will reproduce the right Page curves for the 2D asymptotically flat gravities.

The gist of our arguments is summarized as follows. Assuming the validity of Page’s theorem which treats the black hole and its radiation as two quantum systems, the radiation processes in the initial and final stages are equivalent up to a different identifications of the large and small systems. Therefore, the initial period leads to entanglement generation and the final period purification. Given the random Haar unitary, the emitted radiation at the beginning of the evaporation is maximally entangled with the black hole. This is due to that the black hole has larger degrees of freedom than the total radiation emitted (see figure 2). Consequently, the new emitted radiation in a short time interval is entangled with the large system, the black hole. This results to the increase in the entropy. In the final process, the roles of the black hole and the radiation switch and the total radiation is the larger system. In this case, the new emitted radiation is maximally entangled with the early radiation. This process is the purification of early radiation states which results in entropy reduction. Since the Hawking radiation is at a constant rate with temperature $T_{BH} = \frac{1}{2\pi}$ for the 2D dilaton gravity, the rate of entropy changes should be the same up to a minus sign according to this argument. There is a more succinct way of understanding the above argument. The evaporation process is assumed to be unitary, then one can simply reverse the time and run the evaporation backwards. The initial stage in the forward time and the final stage in the backward evolution are identical if we only keep track of the sizes of the systems. This argument using Page’s theorem in quantum informational picture produces a symmetrical Page curve which contradicts the Page curve calculated

![Figure 2. Assuming that the black hole is an ordinary quantum system, we can apply the Page theorem at the initial and final stages of the evaporation. The upper diagram represents the initial stage of evaporation where the black hole (QS 1) is viewed as the larger system. The lower diagram represents the late stage of evaporation where the radiation (QS 2) has larger degrees of freedom. The upper diagram and the lower graph are indistinguishable under time reversal if the evolution is unitary. Given that the black hole has a constant rate of evaporation, one should expect a symmetric Page curve as in [10].](image)
Figure 3. Page curves for 2D asymptotically flat BPP-RST models [62]. $S_{\text{BH}}$ is the Bekenstein–Hawking entropy of the initial black hole with ADM mass $M$. Page transition occurs at time $\tilde{\sigma}_{\text{Page}} = \frac{4}{3} M$. The solid blue line is the Page curve calculated from the island formula. The green line is from the Page’s theorem which is symmetric to the orange line.

From the quantum RT formula. From the quantum RT prescription, the entanglement entropy of the black hole is

$$S_{\text{ent}} = \min \left\{ \text{ext} \left[ \frac{\text{Area}(\partial I)}{4G_N} + S_{\text{matter}}(R \cup I) \right] \right\},$$

(4.1)

where $I$ represents the entanglement island. For 2D Russo-Susskind-Thorlacius (RST)-Bose-Parker-Peleg (BPP) black holes, the entanglement entropy is given by

$$S_{\text{ent}} = \min \left( N \frac{12}{12} \tilde{\sigma}^{-}, S_{\text{BH}} - N \frac{24}{24} \tilde{\sigma}^{-} \right),$$

(4.2)

where $\tilde{\sigma}^{-}$ is the conformal time and $N$ is the number of massless matter fields. Roughly speaking, before Page time the RT surface is trivial, the entropy is given by the semi-classical calculation

$$S_{\text{ent}} = S_{\text{matter}}(R),$$

(4.3)

which increases linearly with time. After Page time, the RT surface lies at the horizon of the black hole and is the dominant term. Therefore

$$S_{\text{ent}} \simeq \frac{\text{Area}(\partial I)}{4G_N},$$

(4.4)

which returns to the Bekenstein–Hawking entropy as $\text{Area}(\partial I)$ is the area of the horizon. The exact calculations give the Page curve as shown in figure 3.

In the 2D dilaton gravity, the Bekenstein–Hawking entropy is given by $S_{\text{BH}} = \frac{N}{2} M$, where $N$ is the number of scalar fields or the central charge of the CFT. In this case, the mass of the black hole is proportional to the area of the horizon, which counts the number of qubits in the unit.
of Planck area. For an evaporating black hole, island appears when the black hole has radiated a third of its mass. At this transition point, the remaining black hole mass $\frac{1}{3}M$ is twice of that radiated out $\frac{1}{M}$. We ignore the issues on the state counting of the Hilbert spaces of black holes which can be explained by nonorthogonality of semiclassical states. Adopting the quantum mechanical description where we imagine that we can only access to the region slightly outside the horizon, therefore we cannot be sure whether the inside is a black hole or the black hole has completely evaporated into a thermal gas. Since local unitary operations inside the horizon do not affect the entanglement structure between the black hole and the radiation. For simplicity, we can pick some unitary operator $U_{BH}$ on black hole states such that the post-operation system has the following property—a half of its qubits are pure states and the other half are maximally entangled with the radiation. For simplicity, we first consider the situation of no interactions between the black hole states. The island-measurement correspondence tells us that in the statistical sense, the teleportation and the entanglement swapping protocols are implemented with equal probability (the final step of the classical communication is not pertinent to the entanglement entropy). Therefore, half of the outgoing radiations are pure states and half are entangled with early radiations statistically. This gives the expected purification rate calculated from the fine-grained entropy formula (the blue curve in figure 3). Biases from this ratio will be automatically corrected in the evaporation and therefore the entropy has a stable decline at half the rate of the initial increase.

Considering the unitary evolution between the early infalling state and the initial black hole state, one gets a mixed teleportation and entanglement swapping when the measurements are performed. To be concrete, we can consider a micro-process where one such measurement is performed. Suppose that the pure state $|\psi\rangle_{C_1}$ represents the black hole initial state. The singlet state $|\Psi^-\rangle_{A_2B_1}$ represents the entangled state of early Hawking particle and the black hole state, where $A_2$ denotes the state inside the horizon. We introduce the scrambling evolution $U$ inside the black hole acting on $|\psi\rangle_{C_1} \otimes \rho_{A_1}$. After the page time, the island is formed and Bell measurements are evoked. Suppose that we have the new formed singlet states $|\Psi^-\rangle_{A_1B_1}$ and $|\Psi^-\rangle_{A_1B_1}$, where $B_1$ and $A_1$ denote the state inside the horizon (the region between the two dashed vertical lines in figure 4). Bell measurements are performed either on the recent infalling radiation state and the black hole state (teleportation) or on the recent and the early infalling radiation state (entanglement swapping). Combining equations (A.3) and (A.6), we have the teleportation equation for the mixed process,

$$
(\Pi_{A_1B_1} \otimes U_{C_1A_2} \otimes \Pi_{B_2A_3B_3}) |\Psi^-\rangle_{A_1B_1} \otimes |\psi\rangle_{C_1} \otimes |\Psi^-\rangle_{A_2B_1} \otimes |\Psi^-\rangle_{A_2B_1}
$$

$$
= \frac{1}{4} \sum_{i=1}^{4} \sum_{k=0}^{1} \hat{U}(k_1, k_2, k_3, k_4)_{B_1B_2} |\psi\rangle_{A_1} \otimes |\Psi^-\rangle_{B_1B_2}
$$

$$
\otimes \left( \Pi_2 \otimes \sigma^k_A \sigma^k_B \right) |\Psi^+\rangle_{B_1C_1} \otimes \left( \Pi_2 \otimes \sigma^k_A \sigma^k_B \right) |\Phi^+\rangle_{A_1A_2},
$$

where

$$
\hat{U}(k_1, k_2, k_3, k_4) = - (\sigma^k_A \sigma^k_B \otimes \sigma^k_A \sigma^k_B) \hat{U}(\Pi_2 \otimes \sigma^k_A \sigma^k_B).
$$

We can see that the scrambling evolution $U$ up to some Pauli matrices is also teleported out along with the states. The evolution $U$ can be decoded from outside if the measurement results $(k_1, k_2, k_3, k_4)$ are known. Therefore, such interaction processes will not influence the validity of the results of the simple analysis given in the last paragraph. Therefore, such mechanism...
Figure 4. Mixing of teleportation and entanglement swapping. The time direction goes up and the dashed vertical lines represent the horizon of the black hole. The solid vertical lines represent quantum states and the horizontal solid lines that link two quantum states represent the entanglement between the states. On the LHS, below \( t_1 \) is the initial state. From \( t_1 \) to \( t_2 \), the early infalling radiation \( \rho_A \) evolves unitarity with the initial black hole pure state \( |\psi\rangle_C \). Above \( t_2 \), the Bell states projections are applied. The RHS is the final state. The region between the two vertical lines for the left diagram are inside the horizon. For the right diagram, the region inside the horizon is to the right of the vertical line.

reproduces the Page curve in figure 3. The diagrammatic representation of this algebra is shown in figure 4 which we will describe in detail below.

Above description is similar to the concept of teleportation-based quantum computation \([66]\), in which there are some single- or two-qubit gates applied before the teleportation or entanglement swapping processes. The diagrammatic representation of teleportation-based quantum computation has been studied before in a different context \([67, 68]\). The mixing of teleportation and entanglement swapping process can also be presented diagrammatically as shown in figure 4. The Bell states \( |\Psi\rangle \) are represented by a cup \( \bigcap \). Correspondingly, the projection on Bell states are represented by joining a top cup \( \bigcap \) and a bottom cap \( \bigcup \). Local Pauli matrices are represented by the single dots, which can be moved along the connected lines. On the left, we have the black hole unentangled state \( C_1 \), the entangled state \( A_2 \) with the early radiation \( B_2, A_{1,3} \) and \( B_{1,3} \) are the new Hawking pairs generated near the horizon. After the unitary operation \( U \), the end state (on the top) is a pure state \( A_1 \) outside the horizon, two entangled pairs inside the horizon and an entangled pair outside horizon \( B_{2,3} \). The connected lines can be straightened, corresponding to the information flow. The bits \( k_1, k_2, k_3, k_4 \in \{0, 1\} \) correspond to the random measurement results. If final state projection has been applied, then those bits are fixed as 1. The LHS of figure 4 can be deformed into the RHS, where the factor \( 1/4 \) is required for normalization. The diagram on the right shows that the left diagram is equivalent to a unitary operation outside the horizon with a pure state and a pair of maximally entangled state outside. Figure 4 is equivalent to the teleportation equation given by equation (4.5). Such diagrammatic rules are based on the Temperley–Lieb algebra \([69]\), in which Bell states give a representation on \( \mathcal{H}_2 \otimes \mathcal{H}_2 \).

In addition, for eternal black holes in Hartle–Hawking states, the fine-grained entropy saturates at the Bekenstein–Hawking value \( S_{\text{BH}} \). The infalling thermal gas into the black hole, which is assumed to be in the thermal field double state, adds to the entanglement entropy of the black hole. Since the entanglement entropy of the black hole is fixed, the radiation process
has to reduce the entanglement entropy of the black hole equally fast. This is consistent with the entanglement swapping model discussed in this paper.

5. Firewalls

The information paradox of black holes triggered exuberant discussions after the proposal of Almheiri-Marolf-Polchinski-Sully (AMPS) firewalls, which is essential in the information conservation in black holes [70–74]. The logic for the firewall can be briefly summarized as follows. If the whole evaporation process is unitary, then information must either escape from the black hole or remain in other forms of remnants or baby universes. We constrain our discussion to the first scenario which is suggested by the quantum RT results. For a review of remnant models, see [77]. If information comes out from the black hole, the radiation will not be in the form given by Hawking. This is equivalent to that the vacuum near the horizon is not in the Unruh state which allows no outgoing information. Therefore, either unitarity or Hawking’s semi-classical treatment breaks down [78]. If the vacuum near the horizon is not the Unruh state, freely falling observers will not see vacuum but a wave of radiation at the horizon. Therefore, we will face the firewall scenario once requiring the unitary evolution of black holes and the validity of the effective field theories in the weak gravity limit. Even if one assumes that a certain quantum gravity theory can magically resolve all the conflicts, one still faces the ambiguity of the density matrix of Hawking radiations, which is termed as state paradox [6, 9].

We notice that there are two potential loopholes in this line of arguments. One is that the axioms of quantum mechanics not only include the unitary evolution but also quantum measurements following the Born rule. It came as a surprise that not all measurements in quantum mechanics can be understood as some local and unitary processes such as decoherence. The related problems are summarized as the measurement problem or Wigner’s friend paradox [54, 79–82]. To describe all measurements on any quantum systems, it seems that either unitarity or locality in the process needs to be abandoned. The long-standing problems of measurement and the black hole information are facing very similar conundrums. Certain measurement-like processes inside black holes have been given serious considerations in the [17, 82, 86]. However, these proposals have various issues and are not completely compatible with the current understanding of bulk reconstruction of black holes. The second potential loophole is that the outgoing radiations do not have to stay in the same state when propagating from the horizon to the future infinity. The entangled Hawking pairs created near the horizon can gradually disentangle as they propagate away from the black hole. Such process is achievable by either nonunitary operations inside the black hole which can be described through RS theorem or nonlocal interactions of gravity [1, 19, 83]. It is worth mentioning that both loopholes are related to the interpretations of quantum measurement.

---

10 It is worth noticing that classical solutions of general relativity which resemble firewalls were also constructed explicitly [75, 76]. However, those firewalls are classical objects representing the Planckian shell of energy density near the horizon and are not solutions of black holes.

11 It was believed that nonlocality is an intrinsic feature of strong gravitational dynamics [1, 83–85]. This is doubtful if one treats measurements on entangled quantum systems as certain projective operators. Local phases can disappear in various decoherence programs while global phases in entangled systems cannot be washed out from local interactions [54].

12 As is remarked in [1], high-energy scattering in string theory has not shown evidence of nonlocality on scales that would correspond to production of the requisite long strings despite that such nonlocality may resolve the information paradox of black holes.
6. Discussion

In this study, we give a quantum informational interpretation of the concepts in the black hole fine-grained entropy, and argue that the issues around the quantum measurement and the black hole information are identical in many aspects. The logic for this proposal is as below summarized. If the energetically singular horizon (firewall) is to be avoided, the initial Unruh state of the radiation must hold. But this initial radiation state, if propagates to the null infinity without any nonlocal effect, does not conserve the fine-grained entropy or carry any information. If the fine-grained entropy is required to return to zero for a pure-state black hole, the entanglement between the late Hawking radiation and the black hole has to be cut off to purify the early radiation. If entanglement is cut off after the creation, the operation cannot be described by a local unitary operator. Within the standard quantum mechanics, there exists but one such operation—the measurement, that effectively occurs inside the black hole and purifies the early Hawking radiation. Whether or not the measurement can be treated as an emergent operator from a soup of complex processes, the interpretation has been a long conundrum. If the additional information conservation is required besides the Page curve, the ‘measurement’ results cannot be completely random and it returns to the problem of information conservation in measurement process. If the interpretation of bulk reconstruction is accurate, such measurement processes have to occur at the surface of the island boundary which lies at the horizon up to Planck length. The exact position of the horizon is not well-defined quantum mechanically, so we do not differentiate the two. Hence, the issues of nonlocality and nonunitarity in the measurement problem are identified with similar issues in the black hole situation. The interpretation of the current understanding of black hole informational paradox in regular quantum language is what we aim for in this paper.

One should note that the cross-lightcone state manipulation (or quantum steering) is common in quantum information theory. Acausal information transfer would be possible if measurement results are not quantum random, such as that in the final-state projection. However, within the axioms of quantum mechanics, no such effective information transmission can be realized as measurements results are uncontrollable. This is under the assumption that the experimental results are fundamentally unpredictable even if we have all the initial data about the apparatus and the interactions. As required by unitarity, the information inside the black hole must somehow escape to the outside asymptotic region through a Hawking channel which does not carry initial data when leaving the horizon. In a standard quantum measurement of two entangled particles (A and B), $|\psi\rangle$ can change ‘acausally’ from a mixed state to a pure state when a local measurement is conducted on A regardless of whether or not the two events are within the light-cones of one another. The result is an ensemble of pure states. Similar nonunitary processes inside the black hole also purify the states outside viewing from outside. Similar phenomenon through nonunitary operations on vacuum is demonstrated by the RS theorem in quantum field theory. Therefore, if one requires that the radiation is purified and the initial Hawking particles carry no information, nonunitary operators are unavoidable inside the black hole. If we further require that the information inside the black hole comes out across the causal horizon (or unitarity in the entire process of evaporation), then viewing from the outside a deterministic measurement result is necessary once one gains a complete knowledge about the state of the black hole and the interactions. In this case, the measurement is a nonlocal but unitary operation. On the other hand, if measurements in quantum mechanics are fundamentally nonunitary processes, it should not be too surprising that information can be lost in the black hole. We reemphasize that although the two scenarios give the identical Page curve,
they do not all guarantee the conservation of information which depends on interpretations of measurements.

One can imagine two microscopically identical black holes, one formed from pure state matter and the other from maximally mixed matter entangled with a controlled sample. A scrambling time after the formation, we will observe that one black hole emits the expected thermal quanta entangled with it and the other emits quanta which are entangled with the controlled sample. In this case, the state of the radiation, which is derived from the spacetime geometry, is determined by the purity of the black hole. Assuming a smooth horizon leads to the conclusion that a sufficiently mixed black hole will purify the Hawking radiation through certain nonlocal effect and we provide one mechanism in the context of standard quantum information theory. If this conclusion can be extended beyond the background of this study, it may point to the quantum-to-classical transition which is determined by the purity of states, especially for the strong gravity systems. This is different from the gravitational wavefunction collapse or Diósi–Penrose model discussed in [35, 87–89] where the authors argued that gravitational potentials can destroy the superposition of quantum states.

Many studies suggest a resolution from the overcomplete basis of semi-classical states. This idea is also applied to resolve the state-counting issue in the black holes [41, 53]. This is related to the postulations in this study as ensemble averages of semi-random unitary theories or states is assumed to account for the nonvanishing off-diagonal terms in the double copies of density matrix, i.e. $|\langle \psi_i | \psi_j \rangle|^2$ but not in the single copy. Besides, studies on the islands in eternal black holes suggested certain topological changes based on the purity (or mixedness) of the black holes even if they have constant energies or masses and are in thermal equilibrium with radiations [90, 91]. This is beyond the classical gravity theory which has no hair, and it is unclear if this is a sheer artifact of the entropy calculations. The exotic islands which are not at the black-hole horizons in certain specific geometries are not considered in this study [92, 93]. Besides, one should note that there are many other completely different approaches to the information paradox. For a nonexhaustive list, the antipodal identification changes the horizon from a $S^2$ to a $PS^2$ to avoid the interior of the black hole [94]; the classical ‘firewall’ solution completely removes the trapped surface of black holes [75, 76]; the soft hair proposal requires black hole to carry a large amount of soft supertranslation modes [95].

**Data availability statement**

No new data were created or analyzed in this study.

**Acknowledgments**

X W wants to thank Ran Li and Tim Hollowood for the questions and comments, and thank Watse Sybesma and Hong Wang for helpful discussions.

**Appendix**

In this appendix, we demonstrate the basic reasoning on the insufficiency of interpreting measurements as local unitary operations as well as the background information on quantum teleportation and entanglement swapping.
Background on quantum teleportation and entanglement swapping protocols

Quantum teleportation and entanglement swapping protocols describe how to transmit quantum states with the help of quantum entanglements and measurements [96]. Suppose that the one qubit (pure state) has the generic form

\[ |\varphi\rangle = \alpha |0\rangle + \beta |1\rangle, \tag{A.1} \]

where the amplitudes \( \alpha \) and \( \beta \) represent the encoded information. The task is to transmit \( |\varphi\rangle \) state from Alice to Bob, where the state \( |\varphi\rangle \) is also unknown to Alice. Instead of physically sending the qubit to Bob, teleportation can send the information of state \( |\varphi\rangle \) without transmission any qubits.

The prerequisite for teleportation is the shared entangled state (Bell state) between Alice and Bob, denoted as

\[ |\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}} (|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B). \tag{A.2} \]

Then we can rewrite the initial state \( |\varphi\rangle_A \otimes |\Phi^+\rangle_{AB} \) in the Bell basis \( (\mathbb{I}_2 \otimes \sigma^j_z) |\Phi^+\rangle \) (with \( j,k = 0,1 \)) of \( \mathcal{H}_A \otimes \mathcal{H}_B \) as follows,

\[ |\varphi\rangle_A \otimes |\Phi^+\rangle_{AB} = \frac{1}{2} \sum_{j,k=0}^{1} (\mathbb{I}_2 \otimes \sigma^j_z) |\Phi^+\rangle_{AB} \otimes \sigma^k_z |\varphi\rangle_B, \tag{A.3} \]

where \( \sigma^j_z \) are the Pauli matrices. We also take the notations for the other Bell states: \( |\Phi^-\rangle = (\mathbb{I}_2 \otimes \sigma_z) |\Phi^+\rangle, \ |\Psi^+\rangle = (\mathbb{I}_2 \otimes \sigma_x) |\Psi^+\rangle, \ |\Psi^-\rangle = (\mathbb{I}_2 \otimes \sigma_y) |\Psi^+\rangle \). After the measurement, one can see that the information about the state \( |\varphi\rangle_A \) is transferred to \( |\varphi\rangle_B \). In fact, equation (A.3) does not represent the teleportation process, instead it is only the algebraic trick to rewrite the state \( |\varphi\rangle \) from the location \( A \) to the location \( B \). The RHS of equation (A.3) is in a superposition form. Measurement can eliminate the superpositions. Performing the projector \( |\Phi^+\rangle_{AB} \langle \Phi^+ | \) on the initial state, which gives

\[ |\Phi^+\rangle_{AB} \langle \Phi^+ | |\varphi\rangle_A \otimes |\Phi^+\rangle_{AB} = \frac{1}{2} |\Phi^+\rangle_{AB} \otimes |\varphi\rangle_B. \tag{A.4} \]

The normalization factor \( 1/2 \) suggests that the measurement result \( |\Phi^+\rangle_{AB} \) has the probability \( 1/4 \). The projector \( |\Phi^+\rangle_{AB} \langle \Phi^+ | \) ‘forces’ Bob’s state to be \( |\varphi\rangle \). Other measurement results (projection onto other Bell states) will transform Bob’s state to be \( |\varphi\rangle \) up to some Pauli operations. Note that the no-cloning theorem is not violated [97], since the measurement results does not reveal any original information of state \( |\varphi\rangle \).

The project measurement is the key in teleportation process, but it is not necessary to be the Bell project measurement. One can check that a more general teleportation operation than equation (A.3) that includes a unitary operation can be written as

\[ (U_A \otimes \mathbb{I}_B) |\varphi\rangle_A \otimes |\Phi^+\rangle_{AB} = \frac{1}{2} \sum_{j,k=0}^{1} |\psi(jk)\rangle \otimes \sigma^j_z |\varphi\rangle_B, \tag{A.5} \]

with \( |\psi(jk)\rangle = U_A (\mathbb{I}_2 \otimes \sigma^j_z) \langle \Phi^+ |_{AB} \). States \( |\psi(jk)\rangle \) also form an orthonormal basis, which can be the product state basis \( |jk\rangle \), if the corresponding unitary transformation \( U_A \) is applied.

Teleportation is not limited to the pure state transfer. In the original proposal of teleportation [96], the authors argue that Alice can also teleport a mixed state to Bob. Suppose that the qubit (given to Alice) prepared to be teleported is completely mixed, i.e. \( \rho_A = \mathbb{I}_2/2 \), which
has the purification of the maximal entangled state, such as $|\Phi^+\rangle_{CA}$. Then the teleportation equation (A.3) can be generalized to

$$|\Phi^+\rangle_{CA} \otimes |\Phi^+\rangle_{AB} = \frac{1}{2} \sum_{j,k=0}^{1,1} (\begin{array}{l} 1 \\ 1 \end{array} \otimes \sigma_j^x \sigma_k^z) |\Phi^+\rangle_{AA} \otimes (\begin{array}{l} 1 \\ 1 \end{array} \otimes \sigma_j^x \sigma_k^z) |\Phi^+\rangle_{CB}. \quad (A.6)$$

Similarly, to pick up one specific state on the RHS, we can perform the projector $|\Phi^+\rangle_{AA} \langle \Phi^+|$, which gives

$$|\Phi^+\rangle_{AA} (\Phi^+|_{CA} \otimes |\Phi^+\rangle_{AB} = \frac{1}{2} (\Phi^+|_{AA} \otimes |\Phi^+\rangle_{CB}. \quad (A.7)$$

In other words, the teleported mixed state preserves its correlation to the other state. Then the original correlation between $CA$ has swapped to $CB$. Any unitary evolution $U_{AA}$ can be applied before the teleportation, then the project two-qubit basis has to been changed accordingly. If the projection basis does not match the unitary evolution $U_{AA}$, the fidelity of the teleported state is less than 1 and we cannot fully recover the initial state $|\phi\rangle$.  

**Classical ensemble and quantum measurement**

If Bob does not know Alice’s measurement result, such as $|\Phi^+\rangle_{AA}$, Bob can only describe his state as an ensemble, which looks like a mixed state for him. However, in the viewpoint of Alice, her measurement purifies Bob’s state. We should distinguish the mixed state due to one’s ignorance and that due to entanglement. Mixed state due to classical ignorance, like Bob’s mixed state description, does not contribute to the entanglement entropy. Since entanglement entropy only counts the local ignorance due to the entanglement, which is beyond the classical description, such as the local hidden variable theory.

Although the teleportation and entanglement swapping protocols involve the nonunitary measurements, there is no information loss. Suppose that the information is encoded locally in the qubit $|\phi\rangle$ or nonlocality in the entanglement. The teleportation and entanglement swapping can always reconstruct the information if the measurement results are accessible. Therefore, teleportation and entanglement swapping are referred as the information transfer protocols, since they have no destructive effects on information. This is under the assumption that the initial state is in one of the maximally-entangled forms. Otherwise, the fidelity between the initial and final states is less than 1 and part of the information will be lost.

Teleportation and entanglement swapping take advantage of the state reduction (wave function collapse) in measurement processes, which is based on nonunitarity and is beyond the decoherence picture. Equations (A.3) and (A.6) show that one can always rewrite state $|\phi\rangle_A$ at location $A$ as superpositions of state $|\phi\rangle_B$ at location $B$ using entanglement. Quantum measurement picks up only one state from superpositions and teleportation can happen. If one views the measurement process as a unitary evolution between the measured system and the measurement apparatus with state $|m\rangle_M$ (like decoherence), then the teleportation process can be described as $(U_{MAA} \otimes \mathbb{I}_B) |m\rangle_M \otimes |\phi\rangle_A \otimes |\Phi^+\rangle_{AB}$ with the measurement evolution $U_{MAA}$. In this case, Bob’s state is given by

$$T_{MAA} \left[(U_{MAA} \otimes \mathbb{I}_B)|m\rangle_M \otimes |\phi\rangle_A \otimes |\Phi^+\rangle_{AB} (U_{MAA}^\dagger \otimes \mathbb{I}_B)\right] = \frac{1}{2} \mathbb{I}_B. \quad (A.8)$$

which cannot transfer to $|\phi\rangle_B$ in the view of Alice or Bob. Besides, Bob’s state is not purified and still entangled with Alice’s state (plus the measurement apparatus). To recapitulate, unitary evolutions between Alice’s state and the measurement apparatus cannot change Bob’s state and therefore does not support the information transfer.
The nonlocal effects of measurements in teleportation and entanglement swapping have been shown through quantum tomography. Its understanding has been long-debated in history and it is still ongoing. Teleportation and entanglement swapping does not violate the no-signaling principle (no-communication theorem) as signaling is prohibited by uncontrollable random measurement results. Both teleportation and entanglement swapping have been experimentally verified for space-like separated regions [30–32]. If the nonunitary measurement is viewed as a part of unitary operation in an open system, measurements in teleportation and entanglement swapping have to be a part of global unitary evolution acting on the space-like region. Such description violates the causality, which is not less bizarre than the wavefunction collapse.

Another striking feature about entanglement swapping is the ambiguous time when the entanglement is swapped [98, 99]. Recent experiments suggested that the entanglement may be swapped even before measurement occurs. In other words, we can test the entanglement properties of the system before the measurement operations. Such ‘steering-into-the-past’ behavior puts challenges on the understanding of the wavefunction if strict causality is posed. This may have further implications on the entanglement structure but the discussion is beyond the scope of this paper.

ORCID iDs

Xuanhua Wang https://orcid.org/0000-0002-7923-7670
Kun Zhang https://orcid.org/0000-0003-4819-5495

References

[1] Giddings S B 2006 Black hole information, unitarity and nonlocality Phys. Rev. D 74 106005
[2] Almheiri A, Mahajan R, Maldacena J and Zhao Y 2020 The Page curve of Hawking radiation from semiclassical geometry J. High Energy Phys. JHEP03(2020)149
[3] Almheiri A, Engelhardt N, Marolf D and Maxfield H 2019 The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole J. High Energy Phys. JHEP12(2019)063
[4] Penington G 2020 Entanglement wedge reconstruction and the information paradox J. High Energy Phys. JHEP09(2020)002
[5] Banks T and Seiberg N 2011 Symmetries and strings in field theory and gravity Phys. Rev. D 83 084019
[6] Bousso R and Tomášević M 2020 Unitarity from a smooth horizon? Phys. Rev. D 102 106019
[7] Gibbons G W and Hawking S W 1977 Action integrals and partition functions in quantum gravity Phys. Rev. D 15 2752–6
[8] Hawking S W 1978 Quantum gravity and path integrals Phys. Rev. D 18 1747–53
[9] Bousso R and Wildenhain E 2020 Gravity/ensemble duality Phys. Rev. D 102 066005
[10] Page D N 1993 Information in black hole radiation Phys. Rev. Lett. 71 3743–6
[11] Page D N 1993 Average entropy of a subsystem Phys. Rev. Lett. 71 1291–4
[12] Horowitz G T and Maldacena J M 2004 The black hole final state J. High Energy Phys. JHEP02(2004)008
[13] Gottesman D and Preskill J 2004 Comment on ‘The black hole final state’ J. High Energy Phys. JHEP03(2004)026
[14] Lloyd S 2006 Almost certain escape from black holes in final state projection models Phys. Rev. Lett. 96 061302
[15] Lee D and Yeom D-h 2021 Almost certain loss from black holes: critical comments on the black hole final-state proposal J. Korean Phys. Soc. 79 249–55
[16] Harlow D 2014 Aspects of the Papadodimas–Raju proposal for the black hole interior J. High Energy Phys. JHEP11(2014)055
[17] Lloyd S and Preskill J 2014 Unitarity of black hole evaporation in final-state projection models J. High Energy Phys. JHEP08(2014)126
[18] Bousso R 2014 Violations of the equivalence principle by a nonlocally reconstructed vacuum at the black hole horizon Phys. Rev. Lett. 112 041102
[19] Osuga K and Page D N 2018 Qubit transport model for unitary black hole evaporation without firewalls Phys. Rev. D 97 066023
[20] Chatwin-Davies A, Jermyn A S and Carroll S M 2015 How to recover a qubit that has fallen into a black hole Phys. Rev. Lett. 115 261302
[21] Borsten L, Duff M J and Levay P 2012 The black-hole/qubit correspondence: an up-to-date review Class. Quantum Grav. 29 224008
[22] Akil A, Dahlsten O and Modesto L 2021 Conditional entanglement transfer via black holes: restoring predictability New J. Phys. 23 113011
[23] Hawking S and Penrose R 2010 The Nature of Space and Time vol 3 (Princeton, NJ: Princeton University Press)
[24] Ghirardi G C, Rimini A and Weber T 1986 Unified dynamics for microscopic and macroscopic systems Phys. Rev. D 34 470
[25] Busch P, Cassinelli G, De Vito E, Lahti P and Leverro A 2001 Teleportation and measurement Phys. Lett. A 284 141–5
[26] Kirchmair G, Vlastakis B, Leghtas Z, Nigg S E, Paik H, Ginson E, Mirrahimi M, Frunzio L, Girvin S M and Schoelkopf R J 2013 Observation of quantum state collapse and revival due to the single-photon Kerr effect Nature 495 205–9
[27] Uola R, Costa A C, Nguyen H C and Gühne O 2020 Quantum steering Rev. Mod. Phys. 92 015001
[28] Fuwa M, Takeda S, Zwierz M, Wiseman H M and Furusawa A 2015 Experimental proof of nonlocal wavefunction collapse for a single particle using homodyne measurements Nat. Commun. 6 1–6
[29] Schrödinger E 1935 Discussion of probability relations between separated systems Math. Proc. Camb. Phil. Soc. 31 555–63
[30] Bouwmeester D, Pan J-W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 Experimental quantum teleportation Nature 390 575–9
[31] Pan J-W, Bouwmeester D, Weinfurter H and Zeilinger A 1998 Experimental entanglement swapping: entangling photons that never interacted Phys. Rev. Lett. 80 3891
[32] Herbst T, Scheidl T, Fink M, Handsteiner J, Wittmann B, Ursin R and Zeilinger A 2015 Teleportation of entanglement over 143 km Proc. Natl Acad. Sci. 112 14202–5
[33] Chiribella G, D’Ariano GM and Perinotti P 2011 Informational derivation of quantum theory Phys. Rev. A 84 012311
[34] Kofler J and Brukner Č 2007 Classical world arising out of quantum physics under the restriction of coarse-grained measurements Phys. Rev. Lett. 99 180403
[35] Penrose R 1996 On gravity’s role in quantum state reduction Gen. Relativ. Gravit. 28 581–600
[36] Bassi A, Lochan K, Satin S, Singh T P and Ulbricht H 2013 Models of wave-function collapse, underlying theories and experimental tests Rev. Mod. Phys. 85 471
[37] Marolf D and Maxfield H 2021 Observations of Hawking radiation: the Page curve and baby universes J. High Energy Phys. JHEP04(2021)272
[38] Harlow D 2016 Jerusalem lectures on black holes and quantum information Rev. Mod. Phys. 88 015002
[39] Papadodimas K and Raju S 2013 The unreasonable effectiveness of exponentially suppressed corrections in preserving information Int. J. Mod. Phys. D 22 1342030
[40] Papadodimas K and Raju S 2013 An infalling observer in AdS/CFT J. High Energy Phys. JHEP10(2013)212
[41] Stanford D 2020 More quantum noise from wormholes (arXiv:2008.08570)
[42] Nielsen M A and Chuang I L 2010 Quantum computation and quantum information
[43] Di Franco C and Paternostro M 2013 A no-go result on the purification of quantum states Sci. Rep. 3 1–5
[44] Brukner Č 2017 On the quantum measurement problem Quantum [Un] Speakables II: Half a Century of Bell’s Theorem (The Frontiers Collection) (Berlin: Springer) pp 95–117
[45] Page D N 1994 Black hole information Proc. 5th Canadian Conf. on General Relativity and Relativistic Astrophysics vol 1 (World Scientific) pp 1–41
[46] Preskill J 1992 Do black holes destroy information Proc. Int. Symp. on Black Holes, Membranes, Wormholes and Superstrings ed S Kalara and D V Nanopoulos (Singapore: World Scientific) pp 22–39
[47] Nag Chowdhury B and Chattopadhyay S 2021 On collapse of quantum state on measurement Research Square Preprint (https://doi.org/10.21203/rs.3.rs-169622/v2)
[48] Hayden P and Preskill J 2007 Black holes as mirrors: quantum information in random subsystems J. High Energy Phys. JHEP09(2007)120
[49] Sekino Y and Susskind L 2008 Fast scramblers J. High Energy Phys. JHEP10(2008)065
[50] Hamilton A, Kabat D N, Lifschytz G and Lowe D A 2006 Holographic representation of local bulk operators Phys. Rev. D 74 066009
[51] Czech B, Karczmarek J L, Nogueira F and Van Raamsdonk M 2012 The gravity dual of a dense matrix Class. Quantum Grav. 29 155009
[52] Kallosh R, Linde A D, Linde D A and Susskind L 1995 Gravity and global symmetries Phys. Rev. D 52 912–35
[53] Hsin P-S, Iliesiu L V and Yang Z 2020 A violation of global symmetries from replica wormholes and the fate of black hole remnants (arXiv:2011.09444)
[54] Schlosshauer M 2005 Decoherence, the measurement problem and interpretations of quantum mechanics Rev. Mod. Phys. 76 1267
[55] Dong X, Harlow D and Wall A C 2016 Reconstruction of bulk operators within the entanglement wedge in Gauge-gravity duality Phys. Rev. Lett. 117 021601
[56] Maldacena J and Susskind L 2013 Cool horizons for entangled black holes Fortschr. Phys. 61 781–811
[57] Penington G, Shenker S H, Stanford D and Yang Z 2022 Replica wormholes and the black hole interior J. High. Energy Phys. 3 1–87
[58] Pollack J, Rozali M, Sully J and Wakeham D 2020 Eigenstate thermalization and disorder averaging in gravity Phys. Rev. Lett. 125 021601
[59] Perry M J 2021 No future in black holes (arXiv:2106.03715)
[60] Reeh H and Schlieder S 1961 Bemerkungen zur unitäräquivalenz von lorentzinvarianten feldern Nuovo Cim. 22 1051–68
[61] Witten E 2018 APS medal for exceptional achievement in research: invited article on entanglement properties of quantum field theory Rev. Mod. Phys. 90 045003
[62] Wang X, Li R and Wang J 2021 Page curves for a family of exactly solvable evaporating black holes Phys. Rev. D 103 126026
[63] Hartman T, Shaghoulian E and Strominger A 2020 Islands in asymptotically flat 2D gravity J. High Energy Phys. JHEP05(2020)022
[64] Gautason F F, Schneiderbauer L, Sybesma W and Thorlacius L 2020 Page curve for an evaporating black hole J. High Energy Phys. JHEP05(2020)091
[65] Lami L and Regula B 2023 No second law of entanglement manipulation after all Nat. Phys. 19 184–9
[66] Gottesman D and Chuang I L 1999 Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations Nature 402 390–3
[67] Zhang Y and Pang J 2013 Space-time topology in teleportation-based quantum computation (arXiv:1309.0955)
[68] Zhang Y, Zhang K and Pang J 2016 Teleportation-based quantum computation, extended Temperley–Lieb diagrammatical approach and Yang–Baxter equation Quantum Inf. Process. 15 405–64
[69] Temperley H N and Lieb E H 1971 Relations between the ‘percolation’ and ‘colouring’ problem and other graph-theoretical problems associated with regular planar lattices: some exact results for the ‘percolation’ problem Proc. R. Soc. A 322 251–80
[70] Almheiri A, Marolf D, Polchinski J, Stanford D and Sully J 2013 An apologia for firewalls J. High Energy Phys. JHEP09(2013)018
[71] Almheiri A, Marolf D, Polchinski J and Sully J 2013 Black holes: complementarity or firewalls? J. High Energy Phys. JHEP02(2013)062
[72] Bousoo R 2013 Complementarity is not enough Phys. Rev. D 87 124023
[73] Abedi J, Dykaar H and Afshordi N 2017 Echoes from the Abyss: tentative evidence for Planck-scale structure at black hole horizons Phys. Rev. D 96 082004
[74] Polchinski J 2016 The black hole information problem Theoretical Advanced Study Institute in Elementary Particle Physics: New Frontiers in Fields and Strings (Singapore: World Scientific) p 9
[75] Kaplan D E and Rajendran S 2019 Firewalls in general relativity Phys. Rev. D 99 044033
[76] McManus R, Berti E, Kaplan D E and Rajendran S 2020 Quasinormal modes and stability of firewalls Phys. Rev. D 102 104031
[77] Chen P, Ong Y C and Yeom D-h 2015 Black hole remnants and the information loss paradox *Phys. Rept.* **603** 1–45
[78] Braunstein S L and Pati A K 2007 Quantum information cannot be completely hidden in correlations: implications for the black-hole information paradox *Phys. Rev. Lett.* **98** 080502
[79] Wigner E P 1995 Remarks on the mind-body question *Philosophical Reflections and Syntheses* (Berlin: Springer) pp 247–60
[80] Deutsch D 1985 Quantum theory as a universal physical theory *Int. J. Theor. Phys.* **24** 1–41
[81] Bong K-W, Utreras-Alarcón A, Ghafari F, Liang Y-C, Tischler N, Cavalcanti E G, Pryde G J and Wiseman H M 2020 A strong no-go theorem on the Wigner’s friend paradox *Nat. Phys.* **16** 1199–205
[82] Bousso R and Stanford D 2014 Measurements without probabilities in the final state proposal *Phys. Rev. D* **89** 044038
[83] Giddings S B 2006 Locality in quantum gravity and string theory *Phys. Rev. D* **74** 106006
[84] Giddings S B and Lippert M 2004 The information paradox and the locality bound *Phys. Rev. D* **69** 124019
[85] Giddings S B, Marolf D and Hartle J B 2006 Observables in effective gravity *Phys. Rev. D* **74** 064018
[86] Pasterski S and Verlinde H HPS meets AMPS: how soft hair dissolves the firewall (arXiv:2012.03850)
[87] Diosi L 1987 A universal master equation for the gravitational violation of quantum mechanics *Phys. Lett. A* **120** 377
[88] Diosi L 1989 Models for universal reduction of macroscopic quantum fluctuations *Phys. Rev. A* **40** 1165–74
[89] Donadi S, Piscicchia K, Cercu C, Diósi L, Laubenstein M and Bassi A 2021 Underground test of gravity-related wave function collapse *Nat. Phys.* **17** 74–78
[90] Wang X, Li R and Wang J 2021 Islands and page curves of Reissner-Nordström black holes *J. High Energy Phys.* **JHEP04(2021)103**
[91] Hashimoto K, Iizuka N and Matsuo Y 2020 Islands in Schwarzschild black holes *J. High Energy Phys.* **JHEP06(2020)085**
[92] Li R, Wang X and Wang J 2021 Island may not save the information paradox of Liouville black holes *Phys. Rev. D* **104** 106015
[93] Sybesma W 2021 Pure de Sitter space and the island moving back in time *Class. Quantum Grav.* **38** 145012
[94] ’t Hooft G 2018 What happens in a black hole when a particle meets its antipode (arXiv:1804.05744)
[95] Hawking S W, Perry M J and Strominger A 2016 Soft hair on black holes *Phys. Rev. Lett.* **116** 231301
[96] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels *Phys. Rev. Lett.* **70** 1895
[97] Wootters W K and Zurek W H 1982 A single quantum cannot be cloned *Nature* **299** 802–3
[98] Ma X-s, Zotter S, Kofler J, Ursin R, Jennewein T, Brukner C and Zeilinger A 2012 Experimental delayed-choice entanglement swapping *Nat. Phys.* **8** 479–84
[99] Ma X-s, Kofler J and Zeilinger A 2016 Delayed-choice gedanken experiments and their realizations *Rev. Mod. Phys.* **88** 015005