Quantum gates by coupled quantum dots and measurement procedure in Si MOSFET

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(February 4, 2022)

We investigated the quantum gates of coupled quantum dots, theoretically, when charging effects can be observed. We have shown that the charged states in the qubits can be observed by the channel current of the MOSFET structure.

I. INTRODUCTION

Since Shor’s factorization program was proposed, many studies have been carried out in order to realize the quantum computer [1,4]. Nakamura succeeded in the control of the macroscopic quantum state as the solid devices of Josephson junctions [3]. Recently, we have proposed the quantum computer in the asymmetric coupled dot in Si nanocrystals [5]. The advantage of using the coupled dots attached by the gate electrode is that a quantum state can be controlled by the gate voltage in the solid circuits. Moreover, when the coupled Si quantum dots are embedded in the gate insulator of the MOSFET [6,7], the quantum state in the two coupled quantum dots are expected to be detected by the channel current of the MOSFET structure.

II. CAPACITIVELY COUPLED QUANTUM DOTS AS TWO QUBITS

The configuration of the quantum dots and capacitances are shown in Fig. 1. We assume that the coupling between qubits is weaker than that in a qubit (the full formation of the electrostatic energy is shown in the Appendix), and we take

\[ C_1 = C_2 = \sqrt{2} C_3 = \sqrt{2} C_4 \ll C_5 (= C_8), C_6 (= C_9), C_7 (= C_{10}). \]

(1)

By this assumption, we can expand the charging energy as a function of \( n_a \equiv N_A - N_B \) and \( n_b \equiv N_C - N_D \) \((N = N_A + N_B = N_C + N_D)\), by a small \( C_3 \), and obtain the electrostatic energy:

\[ U(n_a, n_b) = E_c \left[ n_a + \frac{\eta}{2 E_c} n_b + \frac{C_7 - C_5}{C_5 + C_7} N - \frac{2 C_5 C_7}{C_5 + C_7} V_a \right]^2 \]

\[ + E_c \left[ n_b + \frac{C_7 - C_5}{C_5 + C_7} N - \frac{2 C_5 C_7}{C_5 + C_7} V_b \right]^2 \]

(2)

where \( E_c \equiv (C_5 + C_7)/(8(C_5 C_6 + C_6 C_7 + C_7 C_5)) \) and

\[ \eta = \frac{\sqrt{2}(C_5^2 + C_7^2 - \sqrt{2} C_5 C_7)}{4(C_5 C_6 + C_6 C_7 + C_7 C_5)^2} C_3 \]

(3)

If we consider the gate voltage region where the electrostatic energy of \( n_j = 0(j = a, b) \) state crosses that of \( n_j = 1 \) state described by Shnirman [3] and Averin [9], the electrostatic energy and the tunneling amplitude, \( \Omega_j \), constitute the Hamiltonian of the two-state system as

\[ H = \sum_{j=a,b} \left( \epsilon_j \sigma_z + \Omega_j \sigma_x \right) - (\eta/4) \sigma_x \sigma_y \]

(4)

where \( \epsilon_a = E_c(1/2 + [(C_7 - C_5) N - 2 C_5 C_7 V_a]/(C_5 + C_7)) \) and \( \epsilon_b = E_c(1/2 + [(C_7 - C_5) N - 2 C_5 C_7 V_b]/(C_5 + C_7)) \). \( \sigma_x \) and \( \sigma_z \) are Pauli matrices. When the capacitances are approximated as \( C_i = 2 \pi \epsilon_{ox} d_i / (\epsilon_{ox} / \epsilon_{si}) r \) where \( \epsilon_{ox} = 4, \epsilon_{si} = 12, d_i \) is the size of the capacitance and \( r \) is the radius of each quantum dot, then \( V_a, V_b \) is of the order of tens of meV.

The mechanism of the controlled NOT operation is similar to that of the Josephson junction [3]. Whether \( n_a = 0 \) or \( n_b = 1 \), the level-crossing gate voltage, \( V_a \), shifts (see Eq. (4)) and the controlled NOT operation is realized.

The dynamical motion of the excess charge in the two-state system can be easily considered by solving
the time-dependent Schrödinger equation \[ \dot{\mathbf{I}} = -i \mathbf{H} \mathbf{I} \], and the excess charge shows the oscillating behavior depending on the energy of the two states. The time period of the oscillation, \( \tau_\delta \), can be simply approximated as \( \tau_\delta \sim \hbar/\sqrt{2\Omega^2 + (\epsilon_a - \epsilon_b)/4} \).

III. DISSIPATION BY ENVIRONMENTAL PHONONS

The polarized charged state of a qubit (coupled dots) behaves as the dipole moment under the electric field generated by the gate electrode. By the coupling of the dipole with the electric field, the two-state system can be described by the Bloch equation and the quantum calculation is realized similar to the NMR quantum computer. It is well known that the one of the attractive characteristics of the NMR quantum computer is its long decoherence time. Here, we comment on the decoherence time of the semiconductor dot array. The decoherence time in semiconductor dot array has been considered to be short. This is the reason why Shnirman and Averin used the Josephson effects in quantum gates. The decoherence in this case is considered mainly to originate from the phonon environments, where the interaction between the two-state system and the phonon bath is largely given by a deformation potential. The estimated decoherence time is not so short and of the order of \( 10^{-7} \) sec from the analysis based on the model of Leggett. This relatively long decoherence time will be related to the ‘phonon bottleneck’ derived by Zanardi. We will have to include the effects of the higher excited energy-levels and temperature for more detailed estimation.

IV. MEASUREMENT MECHANISM IN MOSFET

The qubit which changes the charge distribution can be detected by the MOSFET structure. MOSFET structure is considered to be a more efficient detecting devise in semiconductor quantum dots compared to the SET structure. This is due to the change in the charge distribution in the qubit being detected by the capacitance effects similar to those of the quantum point contact. In this section, we show the detailed detecting mechanism of the MOSFET based on the long-channel MOSFET model in the case of two qubits. The qubit system in the MOSFET proposed here can be seen as series of single coupled-dot MOSFETs. When bias, \( V_D \), is applied between the source and drain, the depletion region expands from the source and drain such that the width of the depletion region increases toward the drain. Thus, the channel current differs depending on the positions of the qubits which change the charge distribution. The channel current between the \( i \)-th qubit and \((i-1)\)-th qubit is given for a small \( V_D \) region as \[ I_D^{(i)} \sim \beta_0 \left( (V^G_i - V^G_{i-1})(V_i - V_{i-1}) - \frac{1}{2} \alpha (V^2_i - V^2_{i-1}) \right), \]
where \( \beta_0 \equiv Z \mu_0 C_0 / L_i \) (\( Z \) is the channel width, \( \mu_0 \) is the mobility, \( L_i \) is the channel length of \( i \)-th qubit where we set \( L_1 = L_2 = \cdots = L_N \), and \( C_0 \) is the capacitance of the SiO\(_2\)) and \( \alpha \equiv 1 + \frac{1}{4 L_B} \frac{Q_B}{C_0} \) where \( Q_B \) is the charge within the surface depletion region, \( V_i \) is the gate voltage, and the threshold voltage, \( V_{th}^{(i)} \), is given by \( V_{th}^{(i)} = V_{th} + \Delta V_{th}^{(i)} \) where \( V_{th} \equiv V_{FB} + 2 \varphi_B + \frac{2}{C_0} \) (\( V_{FB} \) is a flat band voltage, \( \varphi_B \) is the potential difference between the Fermi level and the intrinsic Fermi level of substrate), and the shift by the change of the charge distribution, \( \Delta V_{th}^{(i)} \) in the \( i \)-th qubit. Then, we have the following conditions:

\[ V_N = V_{DS} \quad \text{and} \quad I_D^{(1)} = I_D^{(2)} = \cdots = I_D^{(N)} \]

In the case of two qubits, with \( V_{G1} = V_{G1}^{(i)} - V_{th}^{(i)} \) \( (V_{G1} \gg V_D \) is assumed),

\[ I_D = \frac{\beta_0}{(V_{G1} + V_{G2})} (V_{G1} V_{G2} V_D - \alpha \frac{V_{G1}}{2} V_D^2) \]

Thus, whether \( (V_{G1} = V_g - \Delta V_{th} \) and \( V_{G2} = V_g \) ) or \( (V_{G1} = V_g \) and \( V_{G2} = V_g - \Delta V_{th} \)), the difference of the corresponding currents, \( \Delta I_{D}^{(12)} \), is given as

\[ \Delta I_{D}^{(12)} \approx \frac{\beta_0 \alpha}{2(2V_g - \Delta V_{th})} \Delta V_{th} V_B^2. \]

This difference can be observed in the nonlinear \( I_D \)-\( V_D \) region and, in the pure linear region where the terms which include \( \alpha \) disappear, the changed qubits cannot be distinguished.

V. CONCLUSIONS

We have investigated the quantum gates of the capacitively coupled quantum dot array in the MOSFET structure, and have derived the two-state Hamiltonian by analyzing the channel current based on the conventional MOSFET model.

ACKNOWLEDGMENTS

The author is grateful to N. Gemma, S. Fujita, K. Ichimura, and J. Koga for fruitful discussions.

APPENDIX A: THE ELECTROSTATIC ENERGY OF THE TWO QUBITS

In this section we show the electrostatic energy of the two qubits in terms of the capacitances of dots and the
By minimizing the energy Eq. (A1) at the fixed values of \( V_A \), \( V_B \) and \( N_{A...N_D} \), we have

\[
U = \sum_{i=1,...,10} \frac{q_i^2}{2C_i} - q_1 V_a - q_{10} V_b,
\]

where \( q_i \) shows the charge of the \( i \)-th capacitance and we have the relation between \( q_i (i = 1, 2...10) \) and the total charge of the fore dots, \( N_A, ..., N_D \) as

\[
-N_A = q_1 + q_3 + q_5 + q_6, \quad -N_B = q_2 + q_4 + q_5 + q_7,
-N_C = -q_1 - q_4 + q_8 + q_9, \quad -N_D = q_2 - q_3 - q_9 + q_{10}.
\]

By minimizing the energy Eq. (A1) at the fixed values of \( V_A, V_B \) and \( N_{A...N_D} \), we have

\[
U(a_n, n_b) = \frac{1}{16} \left( \frac{1}{C_a} + \frac{1}{C_b} \right) n_a^2 + \frac{1}{4} \left[ \frac{C_7 - C_5}{C_A C_B - (C_3 + C_6)^2} N - \frac{C_7 C_5}{C_A C_B - (C_3 + C_6)^2} V^+ \right] n_a
\]

\[
+ \frac{1}{16} \left( \frac{1}{C_a} + \frac{1}{C_b} \right) n_b^2 + \frac{1}{4} \left[ \frac{C_7 - C_5}{C_A C_B - (C_3 + C_6)^2} N - \frac{C_7 C_5}{C_A C_B - (C_3 + C_6)^2} V^+ \right] n_b
\]

\[
+ \frac{1}{8} \left( \frac{1}{C_a} - \frac{1}{C_b} \right) n_a n_b + \frac{1}{4 C_A} N^2 + \frac{1}{4 C_A} \left[ \frac{C_7 + C_5}{C_A} \right] \left[ \frac{C_7 + C_5}{C_A} \right] N + C_7 V^+ \right]^2 \left[ \frac{C_7 + C_5}{C_A} \right] \left( C_7 V^+ \right)^2
\]

where

\[
C_A = C_3 + C_5 + C_6, \quad C_B = C_3 + C_6 + C_7,
C_C = \varrho C_3 + C_5 + C_6, \quad C_D = \varrho C_3 + C_6 + C_7,
\]

and \( V^\pm = V_A \pm V_B, \) \( 1/C_A = (C_5 + C_7)/(C_5 + C_7)(C_3 + C_6 + C_5 + C_7), \) \( 1/C_B = [C_5 + C_7 + 2(\varrho + 1)C_3]/[C_5 + C_6 + \eta C_3](C_6 + C_7 + \varrho C_3) - (C_6 - C_3)^2 \) and \( \varrho = 2\sqrt{2} + 1. \)

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