Weighted distributed consensus algorithm based on label propagation algorithm

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Abstract. In the practical application of multi-agent system, the convergence rate of the distributed consensus algorithm becomes slow with the expansion of the communication topology. In order to accelerate the convergence rate of distributed consensus under complex topology, in the paper, based on the fast linear iterations distributed consensus algorithm, a weighted distributed consensus algorithm based on label propagation algorithm was proposed. In the algorithm, firstly based on label propagation algorithm, the complex topology was composed into two layer of topologies, in the first layer of topology, the information was exchange each other within the same community, in the second layer of topology, the information was exchange within the different communities. The consensus firstly was reached in the first layer of topology, then the consensus was reached in the second of layer topology. The analysis and simulation of the convergence performance and the convergence speed were done. The results show that the average consensus can be reached in the weighted distributed consensus, and the convergence rate of the proposed algorithm was higher than that of the fast linear iterations distributed consensus algorithm.

1. Introduction

In past years, the distributed consensus problem has attracted much attention, the convergence rate of distributed consensus is one of the most fundamental problems in the multi-agent system. In order to improving the convergence rate, a large number of different methods were developed [1-9].

In [2], the fast linear iterations distributed consensus algorithm was proposed, the algorithm utilized the weighted values of nodes to accelerate the convergence rate of the distributed consensus problems. The fast linear iterations distributed algorithm improved dramatically the convergence rate of the consensus problem, but computing the weighted value of every node was difficult, and the computing volume increased dramatically with the expansion of the communication topology.

In a general way, the convergence rate of the distributed consensus algorithm become slow with the expansion of the communication topology. In the past years, the community detection algorithms were proposed, in the algorithms, the complex networks can be divided into many small communities. By community composition, the distributed consensus problem can be converted to the multi-layers of distributed consensus problem. In [10,11], the hierarchical consensus algorithm based on the community detection was applied to accelerate the convergence rate of average consensus problem, but the average consensus can't be reached in the hierarchical consensus algorithms. In [12,13], the label propagation algorithm (abbreviated to LPA) was proposed to community detection, the LPA was a simple and near linear time algorithm, so the algorithm had wide application in the community detection.
In the paper, based on the fast linear iterations distributed consensus algorithm, we proposed a distributed consensus algorithm with weighted value links based on community detection. In the paper, firstly, based on label propagation algorithm, the complex topology was composed into two layer of topologies, the first layer of topology was made of a few small communities, the information was exchanged within the same community. In the second layer of topology, the information was exchange among communities. The consensus firstly was reached in the first layer of topology, then the consensus was reached in the second layer of topology. The analysis and simulation of the convergence performance and the convergence speed were done.

2. Distributed consensus algorithm and label propagation algorithm

For a graph \( G(V,E) \), where \( V \) was a set of vertices, and \( E \) was a set of edges, where each edge \( \{i,j\} \in E \) was an unordered pair of distinct nodes, i.e. the pair of distinct nodes can communicated each other. The set of neighbors of node \( i \) was denoted \( N_i = \{j | \{i,j\} \in E \} \). For a graph \( G(V,E) \), the fast distributed consensus algorithm in [2] can be written as:

\[
x(k+1) = Wx(k)
\]

where the vector \( x(k) \) was the state vector the nodes of the graph \( G(V,E) \), \( d_i \) was the degree of the node \( i \), and the parameter \( \alpha \leq \frac{1}{1+d_{\text{max}}} \), \( d_{\text{max}} \) was the biggest degree of the nodes in communication topology.

Computing of the parameter \( d_{\text{max}} \) was difficult. The computing volume of the parameter \( d_{\text{max}} \) was in proportion to \( n^2 \), the \( n \) was the number of nodes in the communication topology, the computing volume increased dramatically with expansion of communication topology.

We introduced some notations and concepts about label propagation algorithm(LPA). the community detection can compose a large-scale network into many small communities, but the community detection algorithms need compute offline, obviously, the algorithms were not fit for the distributed consensus problem, until the label propagation algorithm was proposed.

The label propagation algorithm labeled every node of network with different numbers, every node updated the label of oneself based on the labels of neighbor nodes in every step, of all labels of neighbor nodes, the node select the label that the number of the label were the most as the new label of oneself, step by step, the nodes with the same label belong to the same community. The label propagation algorithm includes the synchronization algorithm and the asynchronous algorithm. In step \( k \), the synchronization algorithm utilized the labels of adjacent nodes in step \( k \) to update the label of oneself, the asynchronous algorithm utilized the labels of adjacent nodes in step \( k \) and step \( k-1 \) to update the label of oneself, and the stability of the asynchronous algorithm was better than that of the synchronization algorithm.

3. Weighted distributed consensus algorithm based on label propagation algorithm

In the paper, the asynchronous label propagation algorithm was selected because of stability. For sake of communication convenience, we updated the label of every node in step \( k+1 \) utilized the labels of adjacent nodes in step \( k \) and step \( k-1 \).

At first, we label every node with different number, the label of every node was updated based on the above-mentioned method, when the label of every node was no longer change, then the communication topology was composed into many communities. we considered as the communities as a layer of topology, the layer of topology was consisted of many communities, the communication among communities was considered as the other layer of topology, i.e., the graph \( G(V,E) \) was composed into two layer of topology \( G'(V',E') \) and \( G''(V'',E'') \) based on LPA, The first layer of topology was \( G'(V',E') \), there were \( m \) independent communities, i.e., \( G(V_1,E_1)\cdots G(V_m,E_m) \), and
$V_1 \cdots V_m \subset V' = V$, $E_1 \cdots E_m \subset E'$, $V_i \cap V_k = \emptyset$, $E_i \cap E_k = \emptyset$, $i, k = 1 \cdots m$. In step $k$, the consensus in every community can be written by:

$$x(k+1) = W_c x(k), \quad W_c = \{w_{ij}\}, \quad w_{ij} = \begin{cases} 0 & (i, j) \notin E_v \\ \alpha & (i, j) \in E_v \end{cases}, \quad w_a = 1 - d_a$$

(2)

where $v = 1 \cdots m$, $x_i = \{x_1 \cdots x_v\}$ was a vector, and $(x_1 \cdots x_v)$ was the state value of the community $G(V_v, E_v)$. The collective distributed consensus algorithm can be written by:

$$x(k+1) = W x(k), \quad W = \{w_{ij}\}, \quad w_{ij} = \begin{cases} 0 & (i, j) \notin E' \\ \alpha & (i, j) \in E' \end{cases}, \quad w_a = 1 - d_a$$

(3)

Where $W' = \text{diag}\{W_1 \cdots W_m\}$, $V' = V_1 \cup V_2 \cdots \cup V_m$.

In the second layer of topology, if the labels of neighbor's nodes of a node were different, then the node can exchange information with the nodes of different communities. In the second layer of topology $G''(V'', E'')$, $V'' = V$, $E'' \cup E'' = E$, the distributed consensus algorithm can be written as:

$$x(k+1) = W'' x(k+1), \quad W'' = \{w_{ij}\}'', \quad w_{ij} = \begin{cases} 0 & (i, j) \notin E'' \\ \alpha & (i, j) \in E'' \end{cases}, \quad w_a = 1 - d_a$$

(4)

According to equation (3) and equation (4), the weighted distributed consensus algorithm can be written as:

$$x(k+1) = W'' W' x(k)$$

(5)

Obviously, according to equation (1), equation (3) and equation (4), we can get

$$W' = W \Rightarrow W + I$$

(6)

For the weighted distributed consensus algorithm, we had the following result.

**Theorem 1.** for the weighted distributed consensus algorithm, when $a \leq \frac{1}{d_{\text{max}}}$, then consensus can be reached.

**Proof.** the matrix $W'$ and $W''$ was the matrix that the sum in every row and in every column was one, so $W' 1 = 1'$, $1' W' = 1'$, $W'' 1 = 1''$, $1'' W'' = 1''$, where $1$ was a vector that all element was 1, we can get $W' W 1 = 1''$, $1'' W' W'' = 1''$, then $\lim_{t \to \infty} x(t) = \frac{1}{n} 1'' x(0)$.

By the theorem 1, we proved that the weighted distributed consensus algorithm based on LPA can reach an average consensus, in the following, by simulation, we proved the convergence rate of the weighted distributed consensus algorithm based on LPA was faster than that in the fast linear iteration distributed consensus algorithm.

### 4. Simulation

In order to verify the performance and convergence rate of the weighted distributed consensus based on LPA proposed in the paper, simulation results were present in the paper.

There were 100 sensors scattered randomly in 100 square meter, there were only communication links among sensors within 20 meters, the communication topology was shown in Figure 1.

We assumed the weight $\alpha$ was the biggest value in the two algorithms. The state value of every node was respectively a number between 1 and 100. After running 1000 Monte-Carlo simulations, the simulating results were as shown in Figure 2. Figure 2 show the weighted distributed consensus algorithm can reach an average consensus, and the convergence rate of the weighted algorithm based on LPA was faster than that of the fast linear iteration distributed consensus in [2].
5. Conclusion
By analysis and simulation in the above section, the weighted distributed consensus algorithm can reach an average consensus, and the convergence rate of the weighted distributed consensus algorithm proposed in the paper was faster than that of the fast linear iteration distributed consensus algorithm in[2].
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