FORWARD-BACKWARD MULTIPLICITY CORRELATIONS IN $e^+e^-$ ANNIHILATION AND $\bar{p}p$ COLLISIONS AND THE WEIGHTED SUPERPOSITION MECHANISM

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Abstract

Forward-backward multiplicity correlations in symmetric collisions are calculated independently of the detailed form of the corresponding multiplicity distribution. Applications of these calculations to $e^+e^-$ annihilation and $\bar{p}p$ collisions confirm the existence of the weighted superposition mechanism of different classes of substructures or components. When applied to $\bar{p}p$ collisions in particular, clan concept and its particle leakage from one hemisphere to the opposite one become of fundamental importance. The increase with c.m. energy of the correlation strength as well as the behaviour of the average number of backward particles vs. the number of forward particles are correctly reproduced.

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1 Essentials on forward-backward multiplicity correlation in symmetric collisions

The average number of charged particles generated in different events in the backward hemisphere (B), \( \bar{n}_B \), is a function of the number of particles occurring in the forward hemisphere (F), \( n_F \), controlled by the correlation strength \( b_{FB} \)

\[
b_{FB} = \frac{\text{Cov}[n_B, n_F]}{\sqrt{\text{Var}[n_B]\text{Var}[n_F]}}. \tag{1}
\]

In hadron-hadron collisions\[1, 2, 3, 4\] the correlation strength parameter is rather large with respect to \( e^+e^- \) annihilation\[5, 6\] and is growing with c.m. energy in the total sample of events as shown in Table 1.

In addition in \( e^+e^- \) annihilation at LEP energies it has been found\[7\] that \( b_{FB} \approx 0 \) in the separate two- and three-jet sample of events. No information is available on the correlation strength in the separate samples of soft (no minijets) and semihard (with minijets) events in hadron-hadron collisions.

2 The problem

We want to calculate the parameter \( b_{FB} \) for the multiplicity distribution

\[
P(n) = \sum_{n_B+n_F=n} P_{\text{total}}(n_F, n_B), \tag{2}
\]

where \( n_F \) and \( n_B \) are random variables and \( P_{\text{total}}(n_F, n_B) \) is the joint distribution for the weighted superposition of different classes of events,\[7\] i.e.,

\[
P_{\text{total}}(n_F, n_B) = \alpha P_1(n_F, n_B) + (1-\alpha)P_2(n_F, n_B), \tag{3}
\]

\( \alpha \) being the weight of class 1 events with respect to the total.

3 The general solution

\[
b_{FB} = \frac{\alpha b_1 D_{n,1}^2(1+b_2) + (1-\alpha)b_2 D_{n,2}^2(1+b_1) + \frac{1}{2}\alpha(1-\alpha)(\bar{n}_2 - \bar{n}_1)^2(1+b_1)(1+b_2)}{\alpha D_{n,1}^2(1+b_2) + (1-\alpha)D_{n,2}^2(1+b_1) + \frac{1}{2}\alpha(1-\alpha)(\bar{n}_2 - \bar{n}_1)^2(1+b_1)(1+b_2)}, \tag{4}
\]

### Table 1: Experimental results on forward-backward correlation strength.

| \( b_{FB} \) | 
| --- | --- | --- |
| pp UA5 | 0.43 ± 0.01 (1 < |\( \eta \)| < 4) | 546 GeV c.m. energy |
| pp ISR | 0.58 ± 0.01 (0 < |\( \eta \)| < 4) | |
| \( e^+e^- \) OPAL | 0.103 ± 0.007 | LEP |
| TASSO | 0.080 ± 0.016 | 22 GeV c.m. energy |
where $b_i$ are the correlation strengths of class 1 ($i = 1$) and class 2 ($i = 2$) events, $D_{n,i}$ are the multiplicity distribution dispersions of class 1 ($i = 1$) and class 2 ($i = 2$) events and $\bar{n}_i$ the corresponding average charged multiplicity for class 1 ($i = 1$) and class 2 ($i = 2$) events.

In case $b_1$ and $b_2$ are zero (as in the separate two samples of events in $e^+e^-$ annihilation) one finds

$$b_{FB} = \frac{\frac{1}{2}\alpha(1 - \alpha)(\bar{n}_2 - \bar{n}_1)^2}{\alpha D^2_{n,1} + (1 - \alpha) D^2_{n,2} + \frac{1}{2}\alpha(1 - \alpha)(\bar{n}_2 - \bar{n}_1)^2}.$$  

(5)

It should be pointed out that above formulas are independent from any specific form of the multiplicity distributions $P_1$ and $P_2$. They depend only on the weight alpha and average charged multiplicities and dispersions of the two classes of events.

4 Applications of Eqs. (4) and (5)

4.1 An intriguing application of Eq. (5) to $e^+e^-$ annihilation

Opal collaboration has found that forward backward multiplicity correlations are non-existent in the separate two- and three-jet samples of events i.e. $b_1$ and $b_2$ in the first general formula are zero and the correlation strength of the total sample of 2-jet and 3-jet events is equal to $0.103 \pm 0.007$.

Using a fit to OPAL data with similar conditions to the jet finder algorithm for the separate samples of events we can determine all parameters in formula (5) and test its prediction with the experimental finding.

It turns out that the values of the parameters$^8$ needed in (5) are $\alpha = 0.463$, $\bar{n}_1 = 18.4$, $\bar{n}_2 = 24.0$, $D^2_1 = 25.6$, $D^2_2 = 44.6$ and the predicted value of $b_{FB}$ is 0.101, in extraordinary agreement with experimental data!

4.2 A suggestive application of Eq. (4) to $p\bar{p}$ collisions

The application of (5) to $p\bar{p}$ collisions leads to unsatisfactory results but opens a new perspective: forward-backward multiplicity correlations cannot be neglected in the separate components. Accordingly Equation (4) and not (5) should be used.

Repeating the same approach done in $e^+e^-$ annihilation for calculating $b_{FB}$ (i.e., assuming that in the separate samples of events FB multiplicity correlations are absent, $b_1 = b_2 = 0$) in the case of $p\bar{p}$ collisions at 546 GeV c.m. energy and using Fuglesang’s fit$^9$ to soft and semihard events (accordingly $\alpha = 0.75$, $\bar{n}_1 = 24.0$, $\bar{n}_2 = 47.6$, $D^2_{n,1} = 106$, $D^2_{n,2} = 209$) one finds $b_{FB} = 0.28$ ($b_{FB}^{(exp)} = 0.58$). The theoretical prediction in this case is too small! It is clear that our working hypothesis was not correct in this case. In conclusion forward-backward multiplicity correlations are needed in each class of events, i.e., $b_1$ and $b_2$ should be different from zero, and after their determination general formula (4) and not formula (5) should be used!

Results in 4.1 and 4.2 are a striking test of the existence of the weighted superposition effect, only a guess up to now.
5 A new theoretical problem

Following above conclusions the next problem is how to determine \( b_1 \) and \( b_2 \) when explicit data on forward-backward multiplicity correlations in the two separate samples of events are lacking and \( b_{FB} \) of the total sample is known from experiments.

The generality of Equation (4) should be limited by introducing additive assumptions inspired by our phenomenological knowledge of the particle emission process in the collision under examination.

Assuming for instance that

a. particles are independently produced in the collision,

b. binomially distributed in the forward and backward hemispheres,

it is found that

\[
b_i = \frac{D^2_{n,i} - \bar{n}_i}{D^2_{n,i} + \bar{n}_i},
\]

where \( D_{n,i} \) and \( \bar{n}_i \) are the dispersion and the average charged multiplicity of the overall multiplicity distribution of each component being as usual \( i = 1, 2 \).

Assuming next that

c. the multiplicity distribution in each \( i \)-component is NB(Pascal) with parameters \( \bar{n}_i \) and \( k_i \) (an assumption which is suggested by the success of the weighted superposition mechanism of NB(Pascal)MD’s in describing shoulder effect in charged particle multiplicity distributions and \( H_q \) vs \( q \) oscillations and which we hardly would like to abandon), we find

\[
b_i = \frac{\bar{n}_i}{\bar{n}_i + 2k_i}.
\]

Accordingly \( b_i \) values can be calculated by using again Fuglesang’s fit parameters on the two components at 546 GeV c.m. energy. After inserting in the general formula (4) these parameters we find \( b_{FB} = 0.78 \).

A too large value with respect to the experimental one (\( b_{FB} = 0.58 \))! This result leads to the following question: Which one of above mentioned apparently quite reasonable assumptions should be modified?

Our guess is that charged particle FB multiplicity correlation is not compatible with independent particle emission but is compatible with the production in cluster, i.e., clan within a NB(Pascal)MD framework. An idea which we propose to develop and to explore in the following.

6 Clan concept is of fundamental importance

Successive steps of our argument are

i) the joint distribution \( P_{total}(n_F, n_B) \) is written as the convolution over the number of produced clans and over the partitions of forward and backward produced particles among clans:

\[
P(n_F, n_B) = \sum_{N_F, N_B} P(N_F, N_B) \sum_{m'_F, m'_B, n_F, m''_F, m''_B, n_B} p_F(m'_F, m'_B | N_F) p_B(m''_F, m''_B | N_B).
\]

(8)
ii) forward backward hemispheres symmetry property is used

\[ p_F(n, m|N) = p_B(m, n|N). \]  

(9)

iii) leakage parameter \( p \) is introduced: it controls the probability that a binomially distributed particle generated by one clan lying in one hemisphere has to leak in the opposite hemisphere, \( q \) is the leakage parameter working in the symmetric direction, \( p + q = 1 \) (notice that \( p = 1 \) or \( q = 0 \) means no leakage, the variation domain of \( p \) is \( 0.5 \leq p < 1 \) and when \( p < 0.5 \) the clan is classified in the wrong domain).

iv) covariance \( \gamma \equiv \langle (\mu_F - \bar{\mu}_F)(\mu_B - \bar{\mu}_B) \rangle \) of \( \mu_F \) forward and \( \mu_B \) backward particles within a clan for forward and backward binomially distributed particles generated by clans is also introduced.

v) clans are binomially produced in the forward and backward hemispheres with the same probability and particles within a clan are independently distributed in the two hemispheres.

It follows for each \( i \)-component

\[
b = \frac{D_N^2 - 4\langle d_{N_F}^2(N) \rangle (p-q)^2 + 4\bar{N}\gamma/\bar{n}_c^2}{D_N^2 + 4\langle d_{N_F}^2(N) \rangle (p-q)^2 - 4N\gamma/\bar{n}_c^2 + 2ND_c^2/\bar{n}_c^2} \\
= \frac{D_N^2/\bar{n} - D_c^2/\bar{n}_c - 4\langle d_{N_F}^2(N) \rangle (p-q)^2\bar{n}_c/\bar{N} + 4\gamma/\bar{n}_c}{D_N^2/\bar{n} + D_c^2/\bar{n}_c + 4\langle d_{N_F}^2(N) \rangle (p-q)^2\bar{n}_c/\bar{N} - 4\gamma/\bar{n}_c}. \tag{10}
\]

Eq. (10) assuming NB (Pascal) behavior with characteristic \( \bar{n}_i \) and \( k_i \) parameters for each component, binomial clan distribution in the two hemispheres, binomial distribution in the two hemispheres of logarithmically produced particles from each clan according to clan structure analysis gives

\[
b_i = \frac{2\bar{n}_i p_i q_i}{\bar{n}_i + k_i - 2\bar{n}_i p_i q_i}. \tag{11}
\]

Accordingly the problem is therefore reduced to determine leakage parameters \( p_i \) in the two classes of events!

Notice that in the limit \( \bar{n}_i \to \infty \), for decreasing \( k_i \), \( b_i \) depends on \( p_i \) only.

7 A phenomenological argument for determining leakage parameters \( p_i \)

By assuming that the semihard component is negligible at 63 GeV c.m. energy and knowing \( b_{FB} \) from experiment at such energy, equation (11) allows to determine \( p_{\text{soft}} \) (0.78); the relatively small variation of \( \bar{n}_{c,\text{soft}} \) from 63 GeV to 900 GeV (it goes from \( \approx 2 \) to \( \approx 2.44 \)) leads to the conclusion that the leakage parameter for the soft component \( p_{\text{soft}} \) can be considered in the GeV domain nearly constant, i.e., \( p_{\text{soft}} = 0.78 \); therefore the correlation strength for the soft component at 546 GeV c.m. energy, \( b_{\text{soft}}(546 \text{ GeV}) \), can easily be determined.

The germane equation for \( b_{\text{semihard}}(546 \text{ GeV}) \) contains of course the unknown parameter \( p_{\text{semihard}} \) at the c.m. energy of 546 GeV. By inserting in equation (4) for \( b_{FB} \) (total) \( b_{\text{soft}}(546 \text{ GeV}) = 0.78 \) and \( b_{\text{semihard}}(546 \text{ GeV}) \) as given by equation (11) with unknown
Figure 1: Predictions for the correlation coefficients for each component (soft and semi-hard) and for the total distribution in pp collisions in scenario 2. Three cases are illustrated, corresponding to the three numbered branches: leakage increasing with $\sqrt{s}$ (upper branch, ➀), constant leakage (middle branch, ➁) and leakage decreasing with $\sqrt{s}$ (lower branch, ➂). Leakage for the soft component is assumed constant at all energies. The dotted line is a fit to experimental values.

$p_{\text{semi hard}}$ parameter, $p_{\text{semi hard}}$ at 546 GeV can be calculated from the experimental value of $b_{\text{FB}}$ (total) = 0.58. It is found $p_{\text{semi hard}}$(546 GeV) = 0.77.

Since $\bar{n}_{c,\text{semi hard}}$ does not vary too much in the GeV region (it goes from 1.64 at 200 GeV c.m. energy to 2.63 at 900 GeV c.m. energy, a relatively small variation which will hardly affect the corresponding leakage parameter in this domain) it is not hazardous to take $p_{\text{semi hard}} \approx$ constant in the same region.

Under just mentioned assumptions

a. the correlation strength c.m. energy dependence is correctly reproduced in the GeV energy range from ISR up to UA5 top c.m. energy and follows the phenomenological formula $b_{\text{FB}} = -0.019 + 0.061 \ln s$ (see Fig. 1).

b. when extrapolated to the TeV energy domain in the scenarios discussed in Ref. 7 with the same values of $p_{\text{soft}}$ obtained in the GeV region ($\bar{n}_{c,\text{soft}}$(14 TeV) being \approx 2.98 makes this guess acceptable) and $p_{\text{semi hard}}$ also constant (a too strong assumption of course), a clean bending effect in $b_{\text{FB}}$ vs. $\ln s$ is predicted. Bending effect is enhanced or reduced by allowing $p_{\text{semi hard}}$ to increase (less leakage from clans and more bending) or to decrease logarithmically with c.m. energy (more leakage from clans and less bending). Energy dependence of leakage parameter for the semihard component is clearly expected in the TeV region in a scenario with strong KNO scaling violation in view of the quite large average number of particles per clan with respect to that found at 900 GeV c.m. energy (it goes from 2.63 at 900 GeV up to 7.36 at 14 TeV). See again Fig. 1.

c. in addition $\bar{n}_B(n_F)$ behavior at 63 GeV c.m. energy (ISR data) is quite well described in terms of the soft component (single NB) only and at 900 GeV c.m. energy (UA5 data) in terms of the weighted superposition of soft and semihard components, i.e., of the superposition of two NB(Pascal)MD’s. (See Fig. 2, where the second case is shown).
Figure 2: Results of our model for $\bar{n}_B(n_F)$ vs. $n_F$ compared to experimental data in the pseudo-rapidity interval $|\eta| < 4$ at 900 GeV.

8 Conclusions

Weighted superposition mechanism of two samples of events describes forward backward multiplicity correlations in $e^+e^-$ annihilation independently of the specific form of the charged particle MD in the different classes of events: only the average numbers of particles and related dispersions in addition to the weight factor are needed.

In order to describe forward backward multiplicity correlations in pp collisions lack of information on FB multiplicity correlations in the separate components is demanding to specify the form of particle multiplicity distributions of the two components.

The choice of NB(P)MD for each component (supported by its success in describing shoulder effect and $H_q$ vs $q$ oscillations) outlines the role of clan properties in this framework and allows to determine correctly $b_{FB}$ energy dependence for the total sample of events in the GeV region. Its bending in the TeV region within possible scenarios discussed in the literature is predicted.

$\bar{n}_B(n_F)$ vs $n_F$ trend is also nicely reproduced at 63 GeV (only soft component is assumed to contribute) and at 900 GeV (superposition of soft and semihard components is used), and its behavior in the TeV energy range predicted.

Last but not least we have found that our study on FB multiplicity correlations in pp collisions when extended to the TeV energy region assuming KNO scaling violation for the semihard component enhances the intriguing connection already shyly anticipated in the GeV region between particle populations within clans, particle leakage from clans in one hemisphere to the opposite one and the superposition effect between different components. Clan concept appears in this framework as a powerful tool which goes far beyond its simple statistical interpretation and raises the question on its real physical
significance: an interesting but also compulsory question for future experimental work in pp collisions and not only.

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