On exotic hybrid meson production in $\gamma^*\gamma$ collisions

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We present a theoretical study of exotic hybrid meson ($J^{PC} = 1^{-+}$) production in photon-photon collisions where one of the photons is deeply virtual, including twist 2 and twist 3 contributions. We calculate the cross section of this process, which turns out to be large enough to imply sizeable counting rates in the present high luminosity electron-positron colliders. We emphasize the importance of the $\pi\eta$ decay channel for the detection of the hybrid meson candidate $\pi_1(1400)$ and calculate the cross section and the angular distribution for $\pi\eta$ pair production in the unpolarized case. This angular distribution is a useful tool for disentangling the hybrid meson signal from the background. Finally, we calculate the single spin asymmetry associated with one initial longitudinally polarized lepton.

I. INTRODUCTION

Photon-photon collisions, with one deeply virtual photon, is an excellent tool for the study of different aspects of QCD. The main feature of such processes is that a QCD factorization theorem holds, which separates a hard partonic subprocess involving scattered photons from a distribution amplitude describing a transition of a quark–antiquark pair to a meson or a generalized distribution amplitude describing the transition of a quark–antiquark pair to two- or three-meson states. The success of this description with respect to recent LEP data is indeed striking enough to propose such processes to be used as a tool for the discovery of new hadronic states. In this paper, we focus on the process where an exotic $J^{PC} = 1^{-+}$ is triplet hybrid meson $H$ (which may be the $\pi_1(1400)$ state) is created in photon-photon collisions and then decays into $\pi^0\eta$. This exotic particle has been the subject of intensive studies for many years.

In previous papers, we have shown that, contrary to naive expectations, the longitudinally polarized hybrid meson possesses a leading twist distribution amplitude (DA) related to the usual non-local quark-antiquark correlators. We were also able to estimate the normalization of this DA, which is by no means small, with respect to the one for usual non-exotic mesons. We thus advocated that exclusive deep electroproduction processes will be able to copiously produce these exotic states. In this paper, we extend our analysis of to $\gamma^*\gamma$ collisions with production of both longitudinally and transversely polarised hybrid meson. We calculate the hard amplitude up to the level of twist 3 and thus ignore the contributions of mass terms. Indeed, the case of hybrid production and its decay products, $\pi\eta$ pair in electron-photon collisions is similar to the electron-proton...
Let us form the light-cone basis adapted for character of such correlators leads to the gluonic degrees of freedom included in the gauge-invariant link. Leading contribution to a hard amplitude comes from the non-local quark-antiquark correlators. The non-local quark-antiquark model. We have shown \cite{9} that in the case of the longitudinal hybrid meson polarization the meson with such quantum numbers should be built of quarks and gluon, that is one should work beyond the $\rho$ symmetry properties of DAs are manifested in the following relations: 

\begin{equation}
\langle H(p, \lambda) | \bar{\psi}(z) \gamma_{\mu} [z; -z] \psi(-z) | 0 \rangle = \int_0^1 du e^{i(u-\bar{u})p \cdot z} \phi_T^H(u) + e^{(\lambda)} \int_0^1 du e^{i(u-\bar{u})p \cdot z} \phi_3^H(u),
\end{equation}

respectively, and express the photons and hybrid meson momenta via the Sudakov decomposition:

\begin{equation}
q = n^* - Q^2 n, \quad q' = M_H^2 + Q^2 n, \quad p = n^* + \frac{M_H^2}{2} n.
\end{equation}

We define the transverse tensor $g_{\mu \nu}^T = g_{\mu \nu} - n^*_\mu n_\nu - n_\mu n^*_\nu$.

Using the results of seminal studies on the DAs of vector mesons \cite{10}, we write ($\bar{u} = 1 - u$):

\begin{equation}
\langle H(p, \lambda) | \bar{\psi}(z) \gamma_{\mu} [z; -z] \psi(-z) | 0 \rangle = \int_0^1 du e^{i(u-\bar{u})p \cdot z} \phi_T^H(u) + e^{(\lambda)} \phi_3^H(u),
\end{equation}

for the vector correlator, and

\begin{equation}
\langle H(p, \lambda) | \bar{\psi}(z) \gamma_5 \gamma_{\mu} [z; -z] \psi(-z) | 0 \rangle = i \int_0^1 du e^{i(u-\bar{u})p \cdot z} \phi_A^H(u),
\end{equation}

for the axial correlator. We use the following short notation: $\varepsilon_{skml} = \varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} s_{\mu_1} k_{\mu_2} m_{\mu_3} l_{\mu_4}$. In eqns \cite{4} and \cite{4}, the polarization vector $\varepsilon^{(\lambda)}_{\mu}$ describes the spin state of the hybrid meson. Due to C-charge invariance, the symmetry properties of DAs are manifested in the following relations:

\begin{equation}
\phi_T^H(u) = -\phi_T^H(1 - u), \quad \phi_3^H(u) = -\phi_3^H(1 - u), \quad \phi_A^H(u) = \phi_A^H(1 - u).
\end{equation}

Compared to the $\rho$ meson matrix elements \cite{10}, one can see that the corresponding DAs for the exotic hybrid meson has different symmetry properties.

II. VACUUM–TO–HYBRID MESON, VACUUM–TO–HADRONS MATRIX ELEMENTS AND THEIR PROPERTIES

The hybrid meson Distribution Amplitude

Let us first consider relevant vacuum–to–hybrid meson matrix elements which enter in exclusive hard amplitudes. We suppose that the hybrid meson is in the isotriplet state with $J^{PC} = 1^{-+}$ quantum numbers. A meson with such quantum numbers should be built of quarks and gluon, that is one should work beyond the quark-antiquark model. We have shown \cite{9} that in the case of the longitudinal hybrid meson polarization the leading contribution to a hard amplitude comes from the non-local quark-antiquark correlators. The non-local character of such correlators leads to the gluonic degrees of freedom included in the gauge-invariant link.

Let us form the light-cone basis adapted for $\gamma^* \gamma \to H$ and $\gamma^* \gamma \to \pi \eta$ processes. We introduce the ”large” and ”small” vectors as

\begin{equation}
n^* = (\Lambda, 0_T, \Lambda), \quad n = (\frac{1}{2\Lambda}, 0_T, -\frac{1}{2\Lambda}), \quad n^* \cdot n = 1,
\end{equation}

and express the photons and hybrid meson momenta via the Sudakov decomposition:

\begin{equation}
q = n^* - Q^2 n, \quad q' = M_H^2 + Q^2 n, \quad p = n^* + \frac{M_H^2}{2} n.
\end{equation}

We define the transverse tensor $g_{\mu \nu}^T = g_{\mu \nu} - n^*_\mu n_\nu - n_\mu n^*_\nu$.

Using the results of seminal studies on the DAs of vector mesons \cite{10}, we write ($\bar{u} = 1 - u$):

\begin{equation}
\langle H(p, \lambda) | \bar{\psi}(z) \gamma_{\mu} [z; -z] \psi(-z) | 0 \rangle = \int_0^1 du e^{i(u-\bar{u})p \cdot z} \phi_T^H(u) + e^{(\lambda)} \int_0^1 du e^{i(u-\bar{u})p \cdot z} \phi_3^H(u),
\end{equation}

for the vector correlator, and

\begin{equation}
\langle H(p, \lambda) | \bar{\psi}(z) \gamma_5 \gamma_{\mu} [z; -z] \psi(-z) | 0 \rangle = i \int_0^1 du e^{i(u-\bar{u})p \cdot z} \phi_A^H(u),
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\begin{equation}
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\end{equation}

Compared to the $\rho$ meson matrix elements \cite{10}, one can see that the corresponding DAs for the exotic hybrid meson has different symmetry properties.
The leading twist longitudinally polarized hybrid meson DA has been discussed in [12] and shown to be asymptotically equal to

$$\phi^H_1(u) = 30u(1-u)(1-2u)$$  \hspace{1cm} (6)$$
and $f_H$ to be of the order of 50 MeV. In the present work, we will not include any QCD evolution effects in GDA.

In order to be able to estimate twist 3 contributions, we have to build a model for the twist 3 DAs $\phi^H_3$ and $\phi^H_A$. We will not innovate on this point but restrict to the Wandzura-Wilczek parts [11], which read [10]:

$$\phi^W_3(u) = \frac{1}{2} \left[ \int^u_0 dv \frac{\phi_1(v)}{v} - \int^1_u dv \frac{\phi_1(v)}{v} \right]$$

$$\phi^W_A(u) = \frac{1}{2} \left[ \int^u_0 dv \frac{\phi_1(v)}{v} + \int^1_u dv \frac{\phi_1(v)}{v} \right]$$

yielding for our choice of $\phi^H_1$:

$$\phi^W_3(u) = -\frac{5}{2} (1-2u)^3$$

$$\phi^W_A(u) = \frac{5}{2} [1 + 6u(u-1)]$$

(7)

The $\pi\eta$ Generalized Distribution Amplitude

We now come to a discussion of the $\pi\eta$ GDA which enters the amplitude, if we try to detect the hybrid meson through its $\pi\eta$ decay mode, which may be the dominant one for the $\pi_1(1400)$ state. Using the results of Ref. [12] we have

$$\langle \pi^0(p_\pi)\eta(p_\eta)|\bar{\psi}(-z)\gamma^\mu[z,-z]|\psi(z)|0\rangle =$$

$$P_{\pi\eta}^\mu \int_0^1 du \epsilon^{(\bar{u}-u)P_{\pi\eta}z} \Phi^\pi_1(u,\zeta, m^2_{\pi\eta}) + \Delta_{\pi\eta} \int_0^1 du \epsilon^{(\bar{u}-u)P_{\pi\eta}z} \phi^W_3(u,\zeta, m^2_{\pi\eta})$$

(9)

for the vector correlator, and

$$\langle \pi^0(p_\pi)\eta(p_\eta)|\bar{\psi}(-z)\gamma^\mu_5[-z,z]|\psi(z)|0\rangle = \zeta^{\mu\alpha\beta\gamma} \Delta_{\pi\eta} \int_0^1 du \epsilon^{(\bar{u}-u)P_{\pi\eta}z} \Phi^\pi_A(u,\zeta, m^2_{\pi\eta})$$

(10)

for the axial correlator. In [9] and [10] the total momentum and relative momentum of $\pi\eta$ pair are

$$P_{\pi\eta} = p_\pi + p_\eta = n^* + m^2_{\pi\eta}/2, \quad \Delta_{\pi\eta} = p_\pi - p_\eta = (2\zeta - 1 + \frac{m^2_{\pi\eta} - m^2_{\eta}}{m^2_{\pi\eta}})P_{\pi\eta} = (1 - 2\zeta)m^2_{\pi\eta}n + \Delta_{\pi\eta}.$$ 

\hspace{1cm} (11)

The $\pi\eta$ leading twist GDA has been discussed in [12] in the $J^{PC} = 1^{-+}$ channel as

$$\check{\Phi}^{\pi\eta}_1(u,\zeta, m^2_{\pi\eta}) = 30u(1-u)(1-2u) B_{11}(m^2_{\pi\eta}) P_1(\cos\theta),$$

(12)

with $\cos\theta = (2\zeta - 1)/\beta$, $\beta = \lambda(m^2_{\pi\eta}, m^2_\eta, m^2\eta')/m^2_{\pi\eta}$, and the coefficient function $B_{11}(m^2_{\pi\eta})$ related to the Breit-Wigner amplitude when $m^2_{\eta'}$ is in the vicinity of $M^2_H$ as

$$B_{11}(m^2_{\pi\eta}) \bigg|_{m^2_{\eta'} = M^2_H} = \frac{5}{3} \frac{g_{H\pi\eta}f_H M_H \beta}{M^4_H - m^2_{\pi\eta} - 4\Gamma_H M_H}.$$ 

(13)
The formalism of GDA includes a background in a natural way, i.e., the contribution of final states uncorrelated with the main signal of hybrid meson exchange. We will model this background as a \( J = 0 \) contribution (i.e., \( \zeta \)-independent) without structure in \( m_{\pi \eta}^2 \), and with the asymptotic \( u \)-dependence (i.e., \( u(1-u)(2u-1) \)). We know nothing of its phase but that it should be quite constant in the considered region. For simplicity, and contrary to the analysis of Ref. [9], we do not include any \( J = 2 \) component to the GDA. Thus our model for the leading twist \( \pi \eta \) GDA reads

\[
\Phi_1^{(\pi \eta)}(u, \zeta, m_{\pi \eta}^2) = 30u(1-u)(1-2u)(K e^{i\alpha} + B_{11}(m_{\pi \eta}^2) \cos \theta) .
\]  

(14)

Note that the GDAs \( \Phi_3^{(\pi \eta)}(u, \zeta, m_{\pi \eta}^2), \Phi_A^{(\pi \eta)}(u, \zeta, m_{\pi \eta}^2) \) are twist 3 functions; they have a part which comes from the Wandzura-Wilczek (WW or kinematical) twist 3 and another part which is usually called genuinely twist 3 (see, for instance [12]). The WW part reads

\[
\Phi_{WW}^3(u, \zeta) = -\frac{1}{4} \int_0^u dy \frac{\partial \Phi_1(y, \zeta)}{y-1} + \frac{1}{4} \int_0^1 dy \frac{\partial \Phi_1(y, \zeta)}{y} ,
\]

\[
\Phi_A^{WW}(u, \zeta) = -\frac{1}{4} \int_0^u dy \frac{\partial \Phi_1(y, \zeta)}{y-1} - \frac{1}{4} \int_0^1 dy \frac{\partial \Phi_1(y, \zeta)}{y} ,
\]

(15)

which yields with our choice of the leading twist terms:

\[
\Phi_3^{WW}(u, \zeta) = 5(1-2u)^3 \frac{B_{11}(m_{\pi \eta}^2)}{2\beta}
\]

\[
\Phi_A^{WW}(u, \zeta) = -5((1-2u)^2 - 2u(1-u)) \frac{B_{11}(m_{\pi \eta}^2)}{2\beta} .
\]

(17)

III. AMPLITUDES OF \( \gamma^* \gamma \to H \) AND \( \gamma^* \gamma \to \pi \eta \) PROCESSES

The gauge invariant expression for the \( \gamma(q')\gamma^*(q) \to H(p) \) amplitude reads

\[
T_{\mu\nu}^{\gamma^* \gamma \to H} = \frac{e^2(Q_u^2 - Q_d^2)}{\sqrt{2}} \left[ \frac{1}{2} g_{\mu\nu} e^{(\lambda)} \cdot n A_1 + \frac{e^{(\lambda)}_{\mu} (p+q')_{\nu}}{Q^2} A_2 \right],
\]

(18)

with \( Q_u = 2/3 \) and \( Q_d = -1/3 \), and where the following short notations have been introduced:

\[
A_1 = \int_0^1 du E_-(u) \Phi_1(u), \quad A_2 = \int_0^1 du \left( E_-(u) \Phi_3(u) - E_+(u) \Phi_A(u) \right)
\]

\[
\Phi_{1,3,A}(u) = f_H M_H \phi_{1,3,A}^H(u), \quad E_{\pm}(u) = \frac{1}{1-u} \pm \frac{1}{u} .
\]

(19)

Such an amplitude allows calculating the production cross section and the helicity density matrix of the hybrid meson. The knowledge of this helicity matrix leads to a definite angular distribution for any particular decay channel. In particular, for the process leading to the two meson \( \pi \eta \) final state the straightforward generalization of the amplitude derived in [12] reads

\[
T_{\mu\nu}^{\gamma^* \gamma \to \pi \eta} = \frac{e^2(Q_u^2 - Q_d^2)}{\sqrt{2}} \left[ \frac{1}{2} g_{\mu\nu} A_1^{(\pi \eta)} + \frac{(\Delta_{\pi \eta}^T)_{\nu} (P_{\pi \eta} + q')_{\mu}}{Q^2} A_2^{(\pi \eta)} \right],
\]

(20)
where, following to the notations (19), we introduce:
\[
A^{(\pi\eta)}_1 = \int_0^1 du E_-(u) \Phi^{(\pi\eta)}_1(u) = -10 \left[ Ke^{i\alpha} + B_{11}(m^2_{\pi\eta}) \cos \theta \right],
\]
\[
A^{(\pi\eta)}_2 = \int_0^1 du \left( E_-(u) \Phi^{(\pi\eta)}_3(u) - E_+(u) \Phi^{(\pi\eta)}_A(u) \right) = -\frac{5}{3\beta} B_{11}(m^2_{\pi\eta}). \tag{21}
\]

Note that the amplitudes (18) and (20) satisfy the gauge invariance condition: \( q_\mu T^{\mu\nu} = q'_\nu T^{\mu\nu} = 0 \) provided we neglect terms proportional to the square of the meson mass, which is quite natural in the light-cone formalism in the twist 3 approximation.

IV. CROSS SECTIONS

The kinematics of \( \gamma^* \gamma \rightarrow \pi\eta \) process is illustrated in Fig. 1. We can now calculate the cross sections for the process \( \gamma^* \gamma \rightarrow \text{Hybrid meson} \). As for the usual treatment of a pseudoscalar meson [13], it may be expressed in terms of the transition form factor \( F_{H\gamma} \), which scales like \( 1/Q^2 \) up to logarithmic corrections due to the QCD evolution of the DA (which we consistently ignore in this work). For an easy comparison with the well measured cases, we define \( R \) as the ratio of squared amplitudes for unpolarized photons,
\[
R = \frac{T^{\mu\nu}(\gamma\gamma^* \rightarrow H)T^{*\mu\nu}(\gamma\gamma^* \rightarrow H)}{T^{\mu\nu}(\gamma\gamma^* \rightarrow \pi^0)T^{*\mu\nu}(\gamma\gamma^* \rightarrow \pi^0)}. \tag{22}
\]

As shown on Fig. 2 by the solid line, the twist 2 transition form factor is sizeable in the hybrid case, of the order of the corresponding quantity for \( \pi^0 \) or \( \eta \) meson production. We are thus confident that a good detector as the ones existing in the present \( e^+e^- \) colliders will be able to detect the hybrid signal if one of the decay channel has a fairly large branching ratio. To estimate the twist 3 effects, we now approximate the twist 3 part of the hybrid distribution amplitude in the Wandzura Wilczek way [11] as explained in Section II. The resulting ratio \( R \) with the twist 2 and twist 3 contributions to the hybrid transition form factor (but only the twist 2 contributions to the \( \pi^0 \) case) is shown on Fig. 2 by the dashed line. The twist 3 contribution is negative and of the order of 20% when \( Q \approx 1 \) GeV, but quite negligible when \( Q \geq 3 \) GeV. Let us stress that this estimate is in the WW approximation which is by no means a proven result. Calculating the order of magnitude of the genuine twist 3 hybrid DA is very model dependent, and to our knowledge, no estimate exists in the literature.

The cross section for the \( e^+e^- \) process
\[
e(k_1) + e(l_1) \rightarrow e(k_2) + e(l_2) + H(p). \tag{23}
\]
for unpolarized lepton beams is easily obtained when including the leptonic parts. Note that the positive \( C \) parity of the hybrid meson does not allow any contribution from a Bremsstrahlung process. Specifying a positive
To analyze the cross section, let us first define the integrated over angles

$$\int \cos \theta d\varphi$$

In Eq. (28) the coupling constant $g$ depends on experimental cuts. Without a precise knowledge of these cuts, we will present results for the $e^+e^-$ process. Its cross section reads

$$\frac{d\sigma_{e^+e^- \rightarrow \pi^0\eta}}{dQ^2 dW^2 d\cos \theta d\varphi} = \frac{\alpha^3}{16\pi} \frac{\beta}{s_{e^+e^-}} \frac{1}{Q^2} \frac{1}{2} (Q_u - Q_d)^2$$

where $W = m_{\pi^0\eta}$ and $Q_{\text{min}}^2$ and $Q_{\text{max}}^2$ are the minimal and maximal virtuality of the photon $q'$, respectively. For a given $e^+e^-$ collider energy, the variables $x_2 = q'p/l_1p = s_{e^+e^-}/s_{e^+e^-}$ and $y = q'k_1q'$ are not independent at fixed $Q^2$ and $W^2$, since $y x_2 = (Q^2 + W^2)/s_{e^+e^-}$. $\varphi$ is defined as being the angle between leptonic and hadronic planes. The lower kinematical limit $Q_{\text{min}}^2 = x_2^2 m_{\pi^0}^2/(1 - x_2)$ is determined by the electron mass $m_e$, whereas $Q_{\text{max}}^2$ depends on experimental cuts. Without a precise knowledge of these cuts, we will present results for the $e^+e^- \rightarrow \pi^0\eta$ process. Its cross section reads

$$\frac{d\sigma_{e^+e^- \rightarrow \pi^0\eta}}{dQ^2 dW^2 d\cos \theta d\varphi} = \frac{\alpha^3}{16\pi} \frac{\beta}{s_{e^+e^-}} \frac{1}{Q^2} \frac{1}{2} (Q_u - Q_d)^2$$

To analyze the cross section, let us first define the integrated over angles $\theta$ and $\varphi$ cross section $d\sigma(Q^2,W^2)/dQ^2dW^2$

$$\frac{d\sigma(Q^2,W^2)}{dQ^2dW^2} = \int d\cos \theta d\varphi \frac{d\sigma_{e^+e^- \rightarrow \pi^0\eta}}{dQ^2 dW^2 d\cos \theta d\varphi} = \frac{250^3 \Gamma_{\eta}}{72s_{e^+e^-}} \frac{1 + (1 - y)^2}{y^2} \left[ K^2 + \frac{1}{3} a^2(W) b^2(W) \right]$$

where we restrict ourselves to the twist 2 contribution and

$$a(W) = \frac{5}{3} g_{H \pi^0} f_H \beta M_H, \quad b(W) = \frac{1}{\sqrt{(W^2 - M_H^2)^2 + \Gamma_H^2 M_H^2}}$$

In Eq. (28) the coupling constant $g_{H \pi^0}$ is related to the branching ratio $BR = \Gamma_{H \rightarrow \pi^0}/\Gamma_H$ of the hybrid meson by

$$g_{H \pi^0}^2 = \frac{48\pi}{\beta^3 M_H^2} \Gamma_H BR.$$
Since nothing is known about the value of the $H \to \pi\eta$ BR, we choose in our numerical analysis as a reasonable estimate $BR = 20\%$. Let us also note that simultaneous rescaling of the background magnitude $K$ and the BR by a coefficient $c$ amounts to a rescaling of the cross sections by $c^2$, leaving the angular distribution $W(\theta, \varphi)$ (Eq. (30)) unchanged.

![Figure 3](image1.png)

**Figure 3:** The differential cross-section for $\pi\eta$ pair production as a function of $W$ for $Q = 3\text{ GeV}$, $y = 0.3$, for different background magnitudes $K = 0$ (solid curve), 0.5 (dashed curve) and 1 (dotted curve).

![Figure 4](image2.png)

**Figure 4:** The differential cross-section for $\pi\eta$ pair production as a function of $W$ for $Q = 5\text{ GeV}$, $y = 0.3$, for different background magnitudes $K = 0$ (solid curve), 0.5 (dashed curve) and 1 (dotted curve).

This cross section does not depend on the phase $\alpha$ appearing in our model of the $\pi\eta$–GDA, Eq. (14). The plots on Figs. 3 and 4 show its $W$ dependence for different magnitudes $K$ of the assumed background, for respectively $Q = 3\text{ GeV}$ and $Q = 5\text{ GeV}$. We observe that the presence of the hybrid peak around $W = 1.4\text{ GeV}$ is hardly affected when changing the magnitude $K$ of the background. The magnitude of this signal is comparable with what has been achieved by the L3 experiment at LEP [5]. As expected, the comparison of Fig. 3 and Fig. 4 shows that the magnitude of the signal decreases when $Q$ increases. However, this decrease is not so dramatic due to the scaling behaviour of the amplitude from twist two dominance. We thus expect that the $Q$ dependence may be experimentally studied up to a few GeV.
A more detailed test of the nature of an eventual signal may be accessed by a study of the angular distribution of the \( \pi \eta \) final state. Using (27) and (27) supplemented by (17, 20) and (21), we obtain

\[
W(\theta, \phi) = \left( \frac{d\sigma(Q^2, W^2)}{dQ^2 dW^2} \right)^{-1} \frac{d\sigma_{e^+ e^- \rightarrow \pi \eta}}{dQ^2 dW^2 d\cos \theta d\phi} = \frac{1}{4\pi} [A + B \cos \theta + C \cos^2 \theta + D \sin 2\theta \cos \phi + E \sin \theta \cos \phi],
\]

with the functions \( A(W), B(W), C(W), D(W, Q) \) and \( E(W, Q) \):

\[
A = \frac{K^2}{K^2 + \frac{3}{5}a^2b^2}, \quad B = \frac{2Kab \cos (\gamma - \alpha)}{K^2 + \frac{3}{5}a^2b^2}, \quad C = \frac{a^2b^2}{K^2 + \frac{3}{5}a^2b^2},
\]

\[
D = \frac{W(2-y)\sqrt{1-y}}{Q \left[ 1 + (1-y)^2 \right]} C, \quad E = \frac{W(2-y)\sqrt{1-y}}{Q \left[ 1 + (1-y)^2 \right]} B,
\]

and where \( \gamma \) is the phase of the Breit-Wigner form, i.e.

\[
\cos \gamma = (M_H^2 - W^2) b(W), \quad \sin \gamma = b(W) \Gamma_H M_H.
\]

In (31) we restrict ourselves to the twist 2 contributions for the coefficients \( A(W), B(W), C(W) \), which is why \( A, B, C \) are \( Q \) independent. Due to the normalization of the angular distribution \( W(\theta, \phi) \), \( \int d\cos \theta d\phi W(\theta, \phi) = 1 \), the functions \( A \) and \( C \) are not independent but they satisfy the relation \( A + C/3 = 1 \). The function \( A \) is displayed in Fig. 5. Its shape is dictated by the inverse shape of the integrated cross-section shown in Fig. 3, as seen from (31) and (27). This is intimately related to our main assumption that the background is described by a \( J = 0 \) contribution.

![Figure 5](image-url)

Figure 5: The \( A \) component of the angular distribution function as a function of the \( \pi \eta \) mass \( W \) for \( Q = 3 \text{ GeV}, y = 0.3 \), for different background magnitudes \( K = 0.1 \) (solid curve), 0.5 (dashed curve) and 1 (dotted curve).

The \( B \) coefficient is displayed in Figs. 6 and 7 for a background magnitude \( K = 1 \) and \( y = 0.3 \). It measures the interference between the background and the hybrid signal. It is thus quite dependent on the value of the phase of the background, but its \( W \) dependence always reveals a dramatic change around the mass of the hybrid meson. One may use it to determine more precisely this mass.

Since \( D \) and \( E \) vanish at the twist 2 level, this part of the angular distribution of the final mesons is sensitive to the strength of the twist 3 amplitude. We calculate them in the Wandzura-Wilczek approximation described in Section II. The coefficient \( D \) is independent of the phase of the background. We show on Fig. 8 its behaviour when the magnitude of the background varies, for \( Q = 3 \text{ GeV} \) and \( y = 0.3 \). On Fig. 8 we show its \( W \) dependence for \( K = 1 \) and \( y = .3 \) and for three different values of \( Q \). Because of its proportionality with \( C \), and thus its relation to \( A \), it is strongly peaked around the hybrid mass. It will only be measurable at fairly small values of \( Q \). \( E(W, Q) \) depends on the background phase \( \alpha \), as does \( B \), and on the magnitude \( K \). We show in Figs. 10 and
Figure 6: The $B$ component of the angular distribution function as a function of the $\pi\eta$ mass $W$ and of the background phase $\alpha$ for the background magnitude $K = 1$, with $Q = 3$ GeV and $y = 0.3$.

Figure 7: The $B$ component of the angular distribution function as a function of the $\pi\eta$ mass $W$ for $Q = 3$ GeV, $y = 0.3$ and $K = 1$, for different background phases $\alpha = 0$ (solid curve), $\pi/4$ (dashed curve) and $\pi/2$ (dotted curve).

Its $W$ dependence for $\alpha = 0$ and $K = 0.5$ and $K = 1$ respectively. This behaviour is similar to the one of $B$ displayed in Fig. 6 but its magnitude is much smaller when $Q$ is greater than 5 GeV.

V. SINGLE SPIN ASYMMETRY

We consider now the exclusive process where a longitudinally polarized lepton (with helicity $h$) scatters on an unpolarized photon to produce the lepton and the hybrid meson detected through its decay into a $\pi\eta$ pair. Such a process allows to define an asymmetry which is zero at the leading twist level but receives contributions from the interference of twist 2 and twist 3 amplitudes. This asymmetry is related to the azimuthal angular dependence of the polarized cross section and it is defined as
Figure 8: The $D$ component of the angular distribution function as a function of the $\pi\eta$ mass $W$ and of the background magnitude $K$ for $Q = 3$ GeV and $y = 0.3$.

Figure 9: The $D$ component of the angular distribution function as a function of the $\pi\eta$ mass $W$ for $y = 0.3$ and $K = 1$, for different values of $Q = 3$ GeV (solid curve), 5 GeV (dashed curve) and 7 GeV (dotted curve).

Figure 10: The $E$ component of the angular distribution function as a function of the $\pi\eta$ mass $W$ for $y = 0.3$ and $K = 0.5$, for different values of $Q = 3$ GeV (solid curve), 5 GeV (dashed curve) and 7 GeV (dotted curve).
Figure 11: The $E$ component of the angular distribution function as a function of the $\pi \eta$ mass $W$ for $y = 0.3$ and $K = 1$, for different values of $Q = 3$ GeV (solid curve), 5 GeV (dashed curve) and 7 GeV (dotted curve).

\[
A_1(s_{\gamma \gamma}, Q^2, W^2; \varphi) = \frac{\int d\cos\theta_{cm}(d\sigma^{(\rightarrow)} - d\sigma^{(\leftarrow)})}{\int d\cos\theta_{cm}(d\sigma^{(\rightarrow)} + d\sigma^{(\leftarrow)})},
\]

where we denote by $d\sigma^{(\rightarrow)}$ the differential cross section $d\sigma^{(\leftarrow)}/dW^2 dQ^2 d\cos\theta_{cm} d\varphi$.

Note that the denominator (we restrict ourselves to the dominant twist 2 component) can be expressed through the unpolarized differential cross section defined in Eq. (27):

\[
\int_0^{2\pi} d\varphi \int_{-1}^1 d\cos\theta_{cms} \left( d\sigma^{(\rightarrow)} + d\sigma^{(\leftarrow)} \right) = 2 d\sigma_{\gamma\gamma}(Q^2, W^2) dQ^2 dW^2.
\]

The asymmetry (33) reads:

\[
A_1(s_{\gamma \gamma}, Q^2, W^2; \varphi) = \frac{\int d\theta_{cm} \sin\theta_{cm} N'(\theta_{cm}, Q, W, \varphi)}{2 \int_0^{2\pi} d\theta_{cm} \sin\theta_{cm} D(\theta_{cm})}
\]

where

\[
N'(\theta_{cm}, Q, W, \varphi) = \frac{4}{Q^6} \epsilon_{k_1 k_2} \Delta_{\pi \eta}^{(\pi \eta)} \text{Im}(A_1^{(\pi \eta)} A_2^{\ast (\pi \eta)}), \quad D(\theta_{cm}) = \frac{1}{Q^2} \frac{1 + (1 - y)^2}{4y^2} |A_1^{(\pi \eta)}|^2.
\]

The crucial dynamical quantity probed by this asymmetry is thus $\text{Im}(A_1^{(\pi \eta)} A_2^{\ast (\pi \eta)})$. It depends on the phase structure of the amplitudes, and may be written as

\[
\text{Im}(A_1^{(\pi \eta)} A_2^{\ast (\pi \eta)}) = \frac{50}{3} \frac{a(W) b(W) K}{\beta} \sin(\alpha - \gamma),
\]

where $a(W)$, $b(W)$ and $\gamma$ are defined by equations (28) and (32). This quantity depends much on the unknown background phase $\alpha$. On Fig. 12, we present our result with the choice $\alpha = 0$. The resulting asymmetry is sizeable and should be measurable.

In order to get more statistics, one may define an integrated asymmetry:

\[
A_2(s_{\gamma \gamma}, Q^2, W^2) = \frac{2\pi \int d(\cos\theta_{cm}) \int_0^{2\pi} d\varphi \sin\varphi (d\sigma^{(\rightarrow)} - d\sigma^{(\leftarrow)})}{\int d(\cos\theta_{cm}) \int_0^{2\pi} d\varphi (d\sigma^{(\rightarrow)} + d\sigma^{(\leftarrow)})}.
\]
Figure 12: The single spin asymmetry $A_1$ as function of $\varphi = (0, 2\pi)$. Values of parameters: $W = 1.4$ GeV, $Q^2 = 5.0$ GeV$^2$, $s_{e\gamma} = 10$ GeV$^2$, $\alpha = 0$. The solid line corresponds to $K = 0.8$, the short-dashed line to $K = 1.0$, the long-dashed line to $K = 0.5$.

which may be used as a probe of the $\pi\eta$ invariant mass dependence. We show on Figs. 13 and 14 the resulting $W$ dependence for two choices of the background magnitude $K$.

Figure 13: The integrated single spin asymmetry $A_2$ as a function of the $\pi\eta$ invariant mass. Values of parameters: $Q^2 = 5.0$ GeV$^2$, $s_{e\gamma} = 10$ GeV$^2$, $K = 1.0$, the solid line corresponds to $\alpha = 0$, the short-dashed line to $\alpha = \pi/4$, the long-dashed line to $\alpha = \pi/2$.

CONCLUSION

This theoretical study shows that if a hybrid meson exists with $J^{PC} = 1^{-+}$ around 1.4 GeV and with a sizeable branching ratio to $\pi - \eta$, much can be learned about it from the experimental observation of $\gamma^*\gamma$ reactions and the precise study of the $\pi - \eta$ final state. The magnitude of the cross section that we obtain in our model of the $\pi - \eta$ Generalized Distribution Amplitude indicates that present detectors at current $e^+e^-$ colliders are able to get good statistics on these reactions, provided the tagging procedure is efficient.
Figure 14: The integrated single spin asymmetry $A_2$ as a function of the $\pi\eta$ invariant mass. Values of parameters: $Q^2 = 5.0$ GeV$^2$, $s_{e\gamma} = 10$ GeV$^2$, $K = 0.5$, the solid line corresponds to $\alpha = 0$, the short-dashed line to $\alpha = \pi/4$, the long-dashed line to $\alpha = \pi/2$.

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