The Case for Neutrino Oscillations

P. Ramond

Physics Department, University of Florida
Gainesville, FL 32611

The building of a machine capable of producing an intense, well-calibrated, beam of muon neutrinos is at present regarded by particle physicists with keen interest because of its ability of studying neutrino oscillations.

The possibility of neutrino oscillations has long been recognized, but it was not made necessary on theoretical or experimental grounds; one knew that oscillations could be avoided if neutrinos were massless, and this was easily done by the conservation of lepton number. The idea of grand unification has led physicists to question the existence (at higher energies) of global conservation laws. The prime examples being baryon number conservation which prevents proton decay and lepton number conservation which keeps neutrinos massless and therefore free of oscillations. The detection of proton decay and neutrino oscillations would therefore be an indirect indication of the idea of Grand Unification, and therefore of paramount importance.

Neutrino oscillations occur when neutrinos acquire mass in such a way that the neutrino mass eigenstates do not match the (neutrino) eigenstates produced by the weak interactions. In the following we shall study the ways in which neutrinos can get mass, first at the level of the standard $SU_2 \times U_1$ model, then at the level of its Grand Unification generalization.

We start by discussing neutrinos in the standard model. The left-handed electron-(muon or tau) neutrino is best described in terms of a two-component left-handed (Weyl)
spinor, $\nu_L$, which represents a left-handed particle and its right-handed antiparticle, thus conserving CP in first approximation. This is to be contrasted with a charged particle (such as the electron) which is described by two such fields, $e_L$ and $e_R$, conserving C and P separately. The left-handed fields $\nu_{eL}$ and $e_L$ form a weak isodoublet ($I_w = 1/2$) and $\nu_L$ has $I_w^3 = +1/2$. The standard model interactions involving neutrinos are of the form $\nu_L^+ e_L$, $\nu_L^+ \nu_L$, and $\nu_L^+ e_R$. Hence if we assign lepton number $L = 1$ to all the fields, these interactions conserve $L$. Note that the electron mass term $e_L^+ e_R + c.c$ conserves $L$ as well, and it violates weak isospin by $\Delta I_w = 1/2$, since $e_R$ is a weak singlet. The left-handed neutrino field can have a mass, the so-called Majorana mass of the form $\nu_L^T \sigma^2 \nu_L$ (in the Weyl representation); it violates weak isospin as $\Delta I_w = 1$. However this Majorana mass clearly violates lepton number $L$ by two units. Hence, no neutrino Majorana mass can develop in a theory with $L$-conservation. In the standard model, the Higgs particle is taken to be a weak doublet with $L = 0$, which couples the right-handed electron field to the weak doublet $(\nu_L, e_L)$. When it acquires a vacuum expectation value, it gives the electron its (Dirac) mass and gives the famous relation

$$\frac{M_w}{M_z \cos \theta_w} = 1$$  \hspace{1cm} (1)

relating the Weinberg angle to the W- and Z-boson masses. The standard model conserves $L$ and neutrinos cannot acquire masses. However one can easily generalize it in order to get a massive neutrino by breaking $L$ explicitly or spontaneously in the Lagrangian. The easiest is to add a Higgs field which is a weak isotriplet ($\Delta I_w = 1$) and has $L = -2$. The extra Yukawa coupling would then be of the form

$$\left( \nu_L^T e_L^T \right) \tau^2 \overline{\nu_L} \cdot \phi$$  \hspace{1cm} (2)

where $\overline{\phi} = (\phi^0, \phi^+, \phi^{++})$ is complex in order to preserve electric charge. If the field $\phi^0$ gets a vev, it is known to be very small in comparison to that of the Higgs doublet since the relation (1) is experimentally good to 3-5%. However it generates a Majorana mass for the
neutrino. An interesting signature of this coupling would be the appearance of a double charged (exotic) Higgs particle in the \( e^+ e^+ \) channel. However this is only one of many ways to obtain massive neutrinos in the standard model. For instance the introduction of explicit \( L \)-violating terms in the Hamiltonian will liberate the neutrino mass and induce it sooner or later in perturbation theory. (Remember that in the standard model \( L \)-conservation is the only symmetry that prevents a Majorana neutrino mass.) Hence to go further one has to blend in extra theoretical prejudices. We use those of Grand-Unification which loosely speaking says that at some scale one cannot tell a quark from a lepton, which means that there exist vector (gauge) particles which cause transition between leptons and quarks. Assign baryon number \( B = 1/3 \ (−1/3) \) for quark (antiquark) and \( B = 0 \) for lepton; also set \( L = 0 \) for quarks and antiquarks. Thus a vector boson which mediates lepton-antiquark transition has \( B = 1/3 \) and \( L = 1 \) and is a color triplet. Another vector boson color triplet changes an antiquark into a quark and has \( B = −2/3, \ L = 0 \). In the simplest Grand Unified theory these two vector bosons are the same, thus violating \( B \) and \( L \) separately. However since these two have the same value of \( B − L = −2/3 \), \( B − L \) is conserved, and since the neutrino Majorana mass has \( B − L = −2 \), neutrinos are still massless in the simplest Grand Unified Model, although it allows for proton decay, as it well known.

Grand Unified models beyond \( SU_5 \) introduce fermions not found in the standard model and these fermions pave the way for \( B − L \) violation. In fact a characteristic of all models beyond \( SU_5 \), such as \( SO_{10} \), \( E_6 \) is their extra neutral fermions.

In the following, without showing any particular model, we will analyze in terms of the standard model what happens in five different types of generalizations for the neutral lepton content of the theory.

The first type of generalization involves more of the usual neutrinos; the mass matrix
is now purely $\Delta_w = 1$:

$$\left( \nu_L^{\frac{1}{2}} \right) \left( \Delta I_w = 1 \right) \left( \nu_L^{\frac{1}{2}} \right),$$

(3)

where $\nu_L^{\frac{1}{2}}$ stands for the normal neutrinos (3 in the standard model). Then, as discussed earlier, these neutrinos can be made massless by imposing $L$-conservation.

In the second case we have an extra neutrino with $I_w^3 = -1/2$ such as would appear in a theory with $V + A$ currents. Then the most general mass neutrino in the neutral section looks like

$$\left( \nu_L^{\frac{1}{2}} N_L^{-\frac{1}{2}} \right) \begin{pmatrix} \Delta I_w = 1 & \Delta I_w = 0, 1 \\ \Delta I_w = 0, 1 & \Delta I_w = 1 \end{pmatrix} \begin{pmatrix} \nu_L^{\frac{1}{2}} \\ N_L^{-\frac{1}{2}} \end{pmatrix}$$

(4)

In the above, $N_L^{-\frac{1}{2}}$ for the new type neutrino with $I_w^3 = -1/2$. The off-diagonal elements contain the so-called Dirac mass and the diagonal elements are the Majorana masses. Since the mass matrix contains a $\Delta I_w = 0$ component, it has to be understood why it is of the same order of magnitude as the $\Delta I_w = 1$ component. Since the matrix has no zero eigenvalues, the neutrinos are naturally massive.

The third type of generalization involves adding neutral leptons which are mute ($I_w = 0$) under weak interactions. We denote them by $N_L^0$. The neutral lepton mass matrix now looks like

$$\left( \nu_L^{\frac{1}{2}} N_L^{0} \right) \begin{pmatrix} \Delta I_w = 1 & \Delta I_w = \frac{1}{2} \\ \Delta I_w = \frac{1}{2} & \Delta I_w = 0 \end{pmatrix} \begin{pmatrix} \nu_L^{\frac{1}{2}} \\ N_L^{0} \end{pmatrix}$$

(5)

Note the appearance of $\Delta I_w = 1/2$ entries in this mass matrix. Barring any global conservation laws, then entries will be of the order of the charged leptons and quark masses, say $\sim 1$ GeV. Hence the resulting Majorana mass for the garden variety neutrino will be unacceptably large!

The way out is to give the $\Delta I_w = 0$ entry a very large value $M$. The mass matrix will then look like

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix},$$

(6)
and will have a small eigenvalue
\[ m_\nu \sim \frac{mM}{m}, \]  
(7)
i.e. depressed from the usual mass by the ratio of the $\Delta I_w = 1/2$ to $\Delta I_w = 0$ scales.

In Grand Unified theories, such as $SO_{10}$, the $\Delta I_w = 0$ scale is $\sim 10^{15}$ GeV, yielding the requisite suppression.

The fourth kind of generalization involves both types of weakly interacting extra neutrinos $N_{L}^{\frac{1}{2}}$ and $N_{L}^{-\frac{1}{2}}$ (a self-conjugate fermion). The mass matrix now looks like
\[
\begin{pmatrix}
\nu_{L}^{\frac{1}{2}} & N_{L}^{\frac{1}{2}} & N_{L}^{-\frac{1}{2}} \\
\Delta I_w = 1 & \Delta I_w = 0, 1 & \Delta I_w = 1
\end{pmatrix}
\begin{pmatrix}
\nu_{L}^{\frac{1}{2}} \\
N_{L}^{\frac{1}{2}} \\
N_{L}^{-\frac{1}{2}}
\end{pmatrix}
\]  
(8)

In the absence of the $\Delta I_w = 1$ component, the new matrix becomes
\[
\begin{pmatrix}
0 & 0 & A \\
0 & 0 & B \\
A & B & 0
\end{pmatrix},
\]  
(9)
which, upon diagonalization, gives a massless left-handed neutrino and a massive Dirac neutral lepton of mass of the order of the $\Delta I_w = 0$ mixing.

Lastly one can have a combination of the last two cases, such as in the Grand Unified Theory based on $E_6$. In the above we have not included generalizations to neutral fermions with $I_w = 1, 3/2, ...$ assignments since they would involve exotic charge assignments for their (weak) partners.

Thus when we have in addition to the usual neutrinos a self-conjugate fermion (i.e. like $N_{L}^{\frac{1}{2}}$ and $N_{L}^{-\frac{1}{2}}$) it is more natural to preserve the masslessness of the neutrino; on the other hand when the extra fermions are non self-conjugate (i.e. an odd number of extra fermions) it becomes rather difficult to preserve neutrino masslessness.

Can we now offer some guesses as to the numerical value of neutrino masses and mixing angles? In general, after diagonalization of the charged and neutral lepton mass matrices,
the charged current density will look like

\[(\begin{array}{ccc}
e^+_L & \mu^+_L & \tau^+_L \\
\end{array}) UT \begin{pmatrix}
\nu_{RL} \\
\nu_{\mu L} \\
\nu_{\tau L} \\
\end{pmatrix} + \ldots \tag{10}\]

where \(U\) is a unitary 3x3 matrix coming from the diagonalization of the charged lepton mass matrix, and \(T\) is a 3x3 matrix (not necessarily unitary) obtained by diagonalizing the neutral lepton mass matrix. (The unwritten part of the density (10) involves transitions to other particles.) If we take the ansatz between mass and mixing angles

\[\tan^2 \theta_{ij} \sim \frac{m_i}{m_j}, \tag{11}\]

and

\[\frac{m_e}{m_\mu} \sim \frac{1}{200} ; \quad \frac{m_\mu}{m_\tau} \sim \frac{1}{20} ; \tag{12}\]

we see that the \(U\) matrix does not mix appreciably the electron into the other two leptons, and provides a Cabibbo-like mixing between \(\mu\) and \(\tau\). The form of \(T\) is much less definite since we do not know any neutrino masses. So we take an example based on \(SO_{10}\) (the third case discussed above). The neutral mass matrix is

\[\begin{pmatrix}
M^1 \\
M^\frac{1}{2} \\
M^0 \\
\end{pmatrix}, \tag{13}\]

where \(M^\Delta I_w\) are 3x3 matrices (for 3 families). Set the strengths for \(M^\Delta I_w\) as follows

\[\Delta I_w = 0 \sim m_x \]

\[\Delta I_w = -1/2 \sim m_w \equiv \epsilon m_x , \tag{14}\]

\[\Delta I_w = 1 \quad \epsilon^2 m_x \]

where \(\epsilon\) is the hierarchy parameter. We rewrite the matrix (13) as

\[\begin{pmatrix}
\epsilon^2 \hat{M}^1 \\
\epsilon \hat{M}^\frac{1}{2} \\
\hat{M}^0 \\
\end{pmatrix}, \tag{15}\]

where all \(M\) are of the same order. Then the neutral fermion mass matrix is given by

\[\hat{M}^1 + \hat{M}^\frac{1}{2} T \frac{1}{M^0} \hat{M}^\frac{1}{2} = T^T DT , \tag{16}\]
where $T$ is the matrix appearing in (10) and $D$ is a diagonal matrix with the neutrino masses as entries. The point of this exercise is to note that the physically relevant parameters (mixing parameters in $T$, mass parameters in $D$) are determined from the knowledge of $\hat{M}^1$, $\hat{M}^{3/2}$ and $\hat{M}^0$. Now $\hat{M}^{3/2}$ can, under some general assumptions, be related to the charge 2/3 mass matrix (this happens in $SO_{10}$), but $\hat{M}^1$ and $\hat{M}^0$ are not directly related to known physics. In some schemes (where $\hat{M}^0$ is a perturbation on the Grand Unified scale) it can be argued that $\hat{M}^1$ can be neglected in (16), but this still leaves the matrix $\hat{M}^0$. So, life is very complicated. Still one can make educated guesses based on specific Grand Unified models. One obtains, more often than not, a very light $\nu_e$ and much heavier but comparable $\nu_\mu$ and $\nu_\tau$:

$$\frac{m_{\nu_e}}{m_{\nu_\mu}} \sim 10^{-6}; \quad \frac{m_{\nu_\mu}}{m_{\nu_\tau}} \simeq 1.$$  \hspace{1cm} (17)

Furthermore one finds very little mixing between $\nu_e$ and $\nu_\mu$ or $\nu_\tau$, but large mixing between $\nu_\mu$ and $\nu_\tau$. None of these results are ironclad, but they seem to be easier to obtain, using the greatest naïveté. Hence, they seem to indicate that $\nu_e - \nu_\mu$ oscillations will be all but impossible to detect while $\nu_\mu - \nu_\tau$ oscillations would be more apparent.

Now with a “low energy” machine such results indicate that one should first look for the extinction of the $\nu$ beam, and then later for $\nu_\mu - \nu_e$ oscillations. Moreover these are just theories which are not directly coordinated with known phenomenology, and it is impossible to gauge their validity. For the moment one would be satisfied with the findings of $\nu$-oscillations irrespective of which way they occur. This would reinforce our theoretical beliefs that global conservation laws are not fundamental and as such would be as important as the discovery of proton decay.

**Acknowledgment**

I wish to thank Profs. F. Boehm and G. Stevenson for asking me to participate in the stimulating workshop. I also wish to thank the Aspen Center for Physics where the above was written.