Cosmic microwave background constraints on the epoch of reionization

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ABSTRACT
We use a compilation of cosmic microwave anisotropy data to constrain the epoch of reionization in the Universe, as a function of cosmological parameters. We consider spatially-flat cosmologies, varying the matter density \( \Omega_0 \) (the flatness being restored by a cosmological constant), the Hubble parameter \( h \) and the spectral index \( n \) of the primordial power spectrum. Our results are quoted both in terms of the maximum permitted optical depth to the last-scattering surface, and in terms of the highest allowed reionization redshift assuming instantaneous reionization. For critical-density models, significantly-tilted power spectra are excluded as they cannot fit the current data for any amount of reionization, and even scale-invariant models must have an optical depth to last scattering of below 0.3. For the currently-favoured low-density model with \( \Omega_0 = 0.3 \) and a cosmological constant, the earliest reionization permitted to occur is at around redshift 35, which roughly coincides with the highest estimate in the literature. We provide general fitting functions for the maximum permitted optical depth, as a function of cosmological parameters. We do not consider the inclusion of tensor perturbations, but if present they would strengthen the upper limits we quote.

Key words: cosmology: theory — cosmic microwave background

1 INTRODUCTION

The absence of absorption by neutral hydrogen in quasar spectra, the Gunn–Peterson effect (Gunn & Peterson 1965; see also Steidel & Sargent 1987; Schneider et al. 1991; Webb 1992; Giallongo et al. 1992,1994), tells us that the Universe must have reached a high state of ionization by the redshift of the most distant known quasars, around five. Several mechanisms for reionization, which requires a source of ultra-violet photons, have been discussed, and are extensively reviewed by Haiman & Knox (1999). In the two most popular models, the sources are massive stars in the first generation of galaxies, or early generations of quasars, and these models have seen quite extensive investigation (Couchman & Rees 1986; Shapiro & Giroux 1987; Donahue & Shull 1991 amongst others). Other possibilities are that the reionization is caused by the release of energy from a late-decaying particle, usually thought to be a neutrino (Sciama 1993), mechanical heating from supernovae driven winds (Schwartz, Ostriker & Yahil 1975; Ikeuchi 1981; Ostriker & Cowie 1981) or even by cosmic rays (Ginsburg & Ozernoi 1965; Nath & Bierman 1993) or by radiation from evaporating primordial black holes (Gibilisco 1996).

One of the most important consequences of reionization is the effect on the anisotropies in the cosmic microwave background (CMB), again reviewed by Haiman & Knox (1999). Before reionization, the microwave background photons have insufficient energy to interact with the atoms, but after reionization they can scatter from the liberated electrons. This leads both to a distortion of the blackbody spectrum and to a damping of the observed anisotropies. Typically, the number density of electrons after reionization is low enough that only a fraction of the photons are scattered, so that some fraction of the original anisotropy is preserved.

There has been continuing rapid progress in observations of microwave background anisotropies, and it is now well established that there is a rise in the spectrum around angular scales of one degree or so, which is where one expects to see the first acoustic (or Doppler) peak. While the issue of whether or not there is an actual peak, with the spectrum falling off again on yet smaller angular scales, remains somewhat controversial, the existence of significant perturbations on the degree scale already indicates that reionization cannot have occurred extremely early, as that would have wiped out the anisotropy signal. A detailed analysis of the current constraints on reionization is our purpose in this paper. The earliest analysis of this type was made by de Bernardis et al. (1997), and more recently Adams et al. (1998) made a specific application to the decaying neutrino model.
We will work within the class of generalized cold dark matter (CDM) models. We consider a subset, where the dark matter is cold and the spatial geometry flat, and we assume that the initial perturbations are Gaussian and adiabatic, with a power-law form, as predicted by the simplest models of inflation. Qualitatively, the COBE DMR detections (Smoot et al. 1992; Bennett et al. 1996) provided evidence supporting this class of models, by showing that large angular scale fluctuations have a spectrum close to a scale-invariant one. Comparison with a range of observations, including the galaxy cluster number density and the galaxy power spectrum, have led to several different recipes aiming at concordance, with the CDM models presently providing the best framework for understanding the evolution of structure in the Universe.

We allow the possible existence of a cosmological constant, as supported by recent observations of the magnitude–redshift relation for Type Ia supernovae (Garnavich et al. 1998; Perlmutter et al. 1998a,b; Riess et al. 1998; Schmidt et al. 1998). We fix the baryon density using nucleosynthesis (Schramm & Turner 1998). The parameters we vary are therefore the matter density \( \Omega_0 \), the Hubble parameter \( h \) and the spectral index \( n \) of the density perturbations. In this paper, we constrain the amount of reionization as a function of these parameters, by carrying out a goodness-of-fit test against a compilation of microwave anisotropy data. We do not consider the related question of finding the overall best-fitting parameters, and in particular of whether the favoured parameter regions are much altered by the inclusion of reionization, leaving that to future work.

## 2 REIONIZATION AND THE OPTICAL DEPTH

### 2.1 The optical depth

First we briefly review the relation between reionization redshift and the optical depth. The effect on the microwave background anisotropies is mainly determined by the optical depth to scattering, and doesn’t depend too much on the background anisotropies. The effect on the microwave background anisotropies is mainly determined by the optical depth. The effect on the microwave background anisotropies is mainly determined by the optical depth.

The shift and the optical depth. The effect on the microwave background anisotropies is mainly determined by the optical depth:

\[
\tau(z) = \int_0^z \frac{dz'}{H(z')} \chi(z'),
\]

where \( \chi(z) = n_e / n_p \), with \( n_e \) being the electron density and \( n_p \) the proton density. Assuming a 24% primordial helium fraction, \( n_p = 0.88 n_B \), where \( n_B \) is the baryon number density. The present value of which is related to the baryon density parameter by

\[
\Omega_B = \frac{8 \pi G m_p n_B}{3 H_0^2},
\]

with \( m_p \) being the proton mass and \( H_0 \) the Hubble parameter in the usual units. For simplicity, we will assume that helium is fully ionized as well as hydrogen; allowing for helium to be only singly ionized is a small correction (as indeed is allowing for the neutrons at all). Two useful relations are the redshift evolution of the electron number density

\[
n_e \propto (1 + z)^3,
\]

and the time–redshift relation

\[
dz = -(1 + z) H dt.
\]

They give

\[
\tau(z) = \int_0^z (1 + z')^3 \frac{dz'}{H(z')} \chi(z'),
\]

where the ‘0’ indicates the present value

As long as the dominant matter is non-relativistic

\[
\frac{\Omega(z) H^2(z)}{(1 + z)^3} = \text{const} = \Omega_0 H_0^2,
\]

and we can write

\[
\tau(z) = \tau^* \int_0^z (1 + z')^{1/2} \sqrt{\frac{\Omega(z')}{\Omega_0}} \chi(z') dz',
\]

where

\[
\tau^* = \frac{3 H_0 \Omega_B \sigma_T c}{8 \pi G m_p} \times 0.88 \simeq 0.061 \Omega_0 h,
\]

the last equality following simply by substituting in for all the constants, with the usual definition of the Hubble constant \( h \). A useful equation for the redshift dependence of \( \Omega \), again assuming only non-relativistic matter and a flat spatial geometry, is

\[
\Omega(z) = \Omega_0 \frac{(1 + z)^3}{1 - \Omega_0 + (1 + z)^3 \Omega_0}.
\]

For illustration we will assume instantaneous reionization at \( z = z_{\text{ion}} \), so that \( \chi(z) = 1 \) for \( z \leq z_{\text{ion}} \) and zero otherwise. Equation (7) can then be integrated to give

\[
\tau(z_{\text{ion}}) = \frac{2 \tau^*}{3 \Omega_0} \left[ (1 - \Omega_0 + \Omega_0 (1 + z_{\text{ion}})^3)^{1/2} - 1 \right].
\]

Sample curves are shown in Figure 1. Inserting the latest permitted reionization redshift, \( z_{\text{ion}} > 5 \) from the Gunn–Peterson effect, implies only that \( \tau \) exceeds a percent or

![Figure 1. Optical depth for instantaneous reionization at redshift \( z_{\text{ion}} \). From top to bottom the curves are \( \Omega_0 = 0.3, 0.6 \) and 1. We took \( \Omega_0 h^2 = 0.02 \) and \( h = 0.65 \).](image-url)
two for typical cosmological parameters. In order to give an
optical depth of unity, the epoch of reionization would be at
\[ z \sim 100 \left( \frac{h \Omega_b}{0.03} \right)^{-2/3} \Omega_0^{1/3}. \] (11)
Therefore, we should expect reionization to occur somewhere
between 5 < \( z_{\text{ion}} \) < 100.

2.2 Estimates of the reionization epoch

Estimating the reionization redshift theoretically remains an
uncertain business. Structure formation in the CDM framework is hierarchical, with the smallest gravitationally-bound systems forming first and the bigger ones appearing later, by merging of the smaller structures. When the first fluctuations enter the non-linear growth regime sometime after \( z \sim 100 \) (Peebles 1983), we expect the appearance of the first bound objects and therefore, the possible onset of reionization. In most reionization models, the assumed recipe is that baryons fall into the potential wells of the developing structures in the cold dark matter, forming stars and quasars which emit ultraviolet radiation. When this radiation escapes the galaxies, it will ionize and heat the intergalactic medium (IGM), and the usual calculations aim to estimate when sufficient radiation is available to complete the reionization. This is already a complex and uncertain calculation, made more so if one allows for inhomogeneities which can strongly affect the recombination rate (Carr, Bond & Arnett 1984). Further, we should note that other heating contributions are not currently excluded (Stebbins & Silk 1986; Tegmark, Silk & Blanchard 1994; Tegmark & Silk 1995) and may even be necessary. Indeed, it has been claimed from observations of the present UV background that it may have been insufficient to reionize the IGM (Giroux & Shapiro 1994), suggesting that collisional heating from supernovae-driven winds or cosmic rays could also contribute to early reionization.

Density perturbation growth slows down with time, and structures in low-density universes cease growing around 1 + \( z \sim 1/\Omega_0 \). Therefore, given the present observed matter power spectrum, this implies that galaxies formed much earlier in low matter density universes. Consequently, reionization is expected to occur earlier in low-density models and, given the bigger look-back time, the optical depth will be larger.

The most extensive theoretical calculations, based on the Press–Schechter mass function, tend to show that reionization occurred after \( z \sim 50 \), and that a good guess for most models would be \( z_{\text{ion}} \sim 10 - 40 \) (Fukugita & Kawasaki 1994; Tegmark et al. 1994; Tegmark & Lyth 1995; Tegmark & Silk 1995). Low-density models are towards the top of this range and critical-density ones towards the lower end (Liddle & Lyth 1995). These results have some corroboration from numerical simulations (Haiman & Loeb 1997). Specifically, for CDM models, Ostriker & Gnedin (1996) and Baltz, Gnedin & Silk (1997) show that reionization by population III stars should have sufficed to reionize the IGM by \( z \sim 20 \), although recently Haiman (1998) suggested a lower reionization redshift of around \( z_{\text{ion}} = 9 - 13 \) for a flat low-density model. If this is indeed the case, then besides the determination of \( z_{\text{ion}} \) via damping of the CMB anisotropies by CMB satellites MAP and Planck, the reionization redshift can be measured directly from the spectra of individual sources with the Next Generation Space Telescope (Haiman & Loeb 1998) or with 21cm “tomography” with the Giant Meterwave Radio Telescope (Madau, Meiksin & Rees 1997).

In summary, the theoretical uncertainties in estimating the reionization redshift are large, and the plausible range stretches from just above the Gunn–Peterson effect of \( z \sim 5 \) up to perhaps 40.

2.3 Spectral distortions from reionization

In the Thomson limit, where the incident photon energy in the
electron rest frame is much less than the electron rest mass-energy, the blackbody form is preserved by scattering. However, the spectrum is measured so accurately that one can hope to detect deviations (Zel’dovich & Sunyaev 1969). The best-known example is the Sunyaev–Zel’dovich effect in clusters, which is detectable because of the very high electron temperatures in clusters. The reionized intergalactic medium is much cooler, but there is much more of it. The amount of distortion of the CMB spectrum is defined through the Compton \( y \) parameter (Zel’dovich & Sunyaev 1969; Stebbins & Silk 1986; Bartlett & Stebbins 1991):

\[ y = \int \left( \frac{kT_e - kT_{\text{CMB}}}{m_e c^2} \right) n_e \sigma_T c \, dt. \] (12)

At the epochs of interest, the CMB temperature is negligible compared to the electron temperature, which we measure in units of \( 10^4 \) Kelvin, denoted \( T_e \). If the electron temperature is taken as constant, this is the same integral as that giving the optical depth, apart from the prefactor.

With the current limits on this distortion coming from the FIRAS experiment, \( y < 1.5 \times 10^{-5} \) (Fixsen et al. 1996), this implies, for typical parameters,

\[ z_{\text{ion}} < 400 T_4^{-2/3} \left( \frac{h \Omega_b}{0.03} \right)^{-2/3} \Omega_0^{1/3}. \] (13)

For the expected typical temperature evolution of the intergalactic medium, it is hard to say much solely from the spectral distortions about the reionization epoch, except that the Universe must have undergone a neutral phase. Stebbins & Silk (1986), Bartlett & Stebbins (1991), Sethi & Nath (1997) and more recently Weller, Battye & Albrecht (1998) show that almost no reasonable reionized cosmological model violates current spectral distortion constraints. Therefore, the information coming from the spatial damping of CMB anisotropies, rather than from spectral distortions, is crucial to determine the history of the reionization epoch, and from now we focus on the anisotropy power spectrum.

3 THE THEORETICAL MODELS

As stated in the introduction, our aim is to constrain the epoch of reionization for a range of spatially-flat cosmological models. We fix the baryon density at \( \Omega_b h^2 = 0.02 \) from nucleosynthesis [Schramm & Turner 1998]. This is at the high end of values currently considered, which makes it a conservative choice because decreasing the baryon density lowers the acoustic peak and hence permits less reionization. We also do not consider tensor perturbations, which contribute predominantly to the low multipoles. As with the
baryons, our constraints are conservative in that they would strengthen if tensors were included, because including them lowers the acoustic peaks relative to the low-\(\ell\) plateau.

The three parameters we vary are

- \(\Omega_0\) in the range (0.2, 1).
- \(h\) in the range (0.5, 0.8).
- \(n\) in the range (0.8, 1.2).

Our focus is directed towards obtaining upper limits on the amount of reionization, though in some parts of parameter space there are lower limits too.

The quantity to be compared with observation is the radiation angular power spectrum \(C_\ell\), which needs to be computed for each model. The spherical harmonic index \(\ell\) indicates roughly the angular size probed, \(\theta \sim 1/\ell\). The power spectrum is readily calculated using the CMBFAST program (Seljak & Zaldarriaga 1996), which allows the input of all the parameters we need. However, exploring a multi-dimensional parameter space is computationally quite intensive, and rather than run CMBFAST for every choice of the optical depth, it is more efficient and flexible to use an analytic approximation to the effect of reionization. This enables us to quickly and accurately generate \(C_\ell\) spectra for arbitrary amounts of reionization.

The first step is to obtain spectra for the case with no reionization, for each combination of our three parameters. We take our parameters on a discrete grid of dimensions \(9 \times 7 \times 9\). From these spectra without reionization, we can generate spectra including reionization using a version of the reionization damping envelope technique of Hu & White (1997). This procedure readily generates accurate enough spectra for comparison with the current observational data, as we will show. However, we do caution the reader that this approach will not work once data of improved accuracy becomes available. Indeed, as shown by the Herculean 8-parameter analysis of Tegmark (1999), the error bars on the cosmological parameters coming from the CMB don’t seem to change very much with the addition of more parameters (see also Lineweaver 1998) and in the near future the advent of better quality data will make them decrease, introducing the necessity for a refined treatment of reionization.

The underlying physics is the following, illustrated in Figure 2. Given an optical depth \(\tau\), the probability that a photon we see originated at the original last-scattering surface is \(\exp(-\tau)\), the exponential accounting for multiple scatterings. Those photons will still carry the original anisotropy, which we will denote by \(C_\ell^{\text{int}}\). The remaining fraction will have scattered at least once. Their contribution to the anisotropy depends on scale. On large angular scales, they will have rescattered within the same large region and will continue to share the same temperature contrast; this is simply a statement that causality prevents large-scale anisotropies being removed. On small scales, however, the rescattered photons come from many different regions with different small-scale temperature contrasts (the small circle in Figure 2), and their anisotropy averages to zero. Consequently we have two limiting behaviours for the observed anisotropy \(C_\ell^{\text{obs}}\)

\[
C_\ell^{\text{obs}} = C_\ell^{\text{int}} \quad \text{Small } \ell; \quad (14)
\]

\[
C_\ell^{\text{obs}} = \exp(-2\tau) C_\ell^{\text{int}} \quad \text{Large } \ell. \quad (15)
\]

The factor 2 in the latter is because the power spectrum is the square of the temperature anisotropy.

The reionization damping envelope (Hu & White 1997) is a fitting function which interpolates between these two regimes. For a given \(\tau\), we obtain the observed spectrum by

\[
C_\ell^{\text{obs}} = R_\ell^2 C_\ell^{\text{int}}, \quad (16)
\]

where the reionization damping envelope \(R_\ell\) is given in terms of the optical depth and a characteristic scale \(\ell_c\) by (Hu & White 1997)

\[
R_\ell^2 = \frac{1 - \exp(-2\tau)}{1 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4} + \exp(-2\tau), \quad (17)
\]

with \(x = \ell/\ell_c + 1\) and \(c_1 = -0.267, c_2 = 0.581, c_3 = -0.172\) and \(c_4 = 0.0312\).

The characteristic scale comes from the angular scale subtended by the horizon when the photons rescatter (i.e., that subtended by the circle in Figure 2). In order to obtain a highly accurate result, Hu & White (1997) compute the characteristic scale \(\ell_c\) via an integral which weights the horizon scale with the optical depth, but for our purposes the simple formula

\[
\ell_c = (1 + z_{\text{ion}})^{1/2} (1 + 0.084 \ln \Omega_b) - 1, \quad (18)
\]

gives sufficient accuracy, where \(z_{\text{ion}}\) is given by rearranging equation 17. This formula is a fit to the angular size of the horizon at reionization (Hu & White 1997, with a sign error in their paper corrected).

We are not quite finished yet, because while the reionization damping envelope accounts for the loss of anisotropy due to scattering, it does not allow for the generation of new anisotropies because of the peculiar velocities of the rescattering electrons. These create a new, but much less prominent, acoustic peak at smaller \(\ell\) than the original one. Because it is a minor feature, it can be modelled simply using a Gaussian, the amplitude, width and location of which...
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Figure 3. Generating a $C_\ell$ curve including reionization, illustrated for $n = 1$, $h = 0.5$, $\Omega_0 = 1$ and an optical depth $\tau = 0.4$. The top curve shows the spectrum without reionization. Applying the reionization damping envelope generates the dotted line, and the correction for the new acoustic peak, equation (19), then gives the lower solid line. This is to be compared with the exact result from cmbfast for this model, shown as the dashed line.

depend mildly on the cosmology. The form we choose gives the reionized spectrum as

$$C_{\ell}^{\text{obs}} = \frac{R^2 C_{\ell}^{\text{int}}}{1 - f(\ell)},$$

where

$$f(\ell) = A \exp \left( -\frac{1}{2\sigma^2} \ln^2 \frac{\ell}{\ell_{\text{max}}} \right),$$

$$A = \tau (\tau + 0.16),$$

$$\sigma = 0.85,$$

$$\ell_{\text{max}} = 33\Omega_0 + 21h + 12.5\tau.$$ (23)

The various numerical factors were fits from a comparison to cmbfast output in specific cases. This approach is easily accurate enough given the current data, especially as the data are given in $\delta T$ which corresponds to the square root of the $C_\ell$ curve. Figure 3 shows an example compared to an exact curve from cmbfast.

4 THE OBSERVATIONAL DATA

In recent years the detection of CMB anisotropies on different angular scales has become commonplace. At large scales, the COBE measurements (Smoot et al. 1992; Bennett et al. 1996) constrained the amplitude of the spectrum with high accuracy, and to some extent the slope. Since then, a plethora of ground-based and balloon-borne experiments probing medium and small scales have followed, providing increasingly accurate measurements. Although there is still quite a large scatter, there is very strong evidence for the existence of an acoustic peak at $\ell$ of a few hundred, as first claimed by Scott & White (1994) and by Hancock & Rocha (1997), and therefore a limit on the amount of reionization damping which is permitted.

Our data sample is shown in Figure 4, and Table 1 lists the data points and indicates the sources from which they were obtained. It is similar to the compilations described by Hancock & Rocha (1997) and Lineweaver et al. (1997), and several researchers have up-to-date compilations available on the World Wide Web. We use updated data and thus our sample includes:

- The 8 uncorrelated COBE DMR points from Tegmark & Hamilton (1997).
- The new calibration of the Saskatoon points (Leitch 1998); the shared calibration error of these points is small enough to be neglected.
- The new updated QMAP results (de Oliveira-Costa et al. 1998).
- The new OVRO Ring5M result (Leitch et al. 1998).

We use a $\chi^2$ goodness-of-fit analysis employing the data in Table 1 along with the corresponding window functions, following the method detailed by Lineweaver et al. (1997). In brief, the window functions describe how the anisotropies at different $\ell$ contribute to the observed temperature anisotropies. For a given theoretical model, they enable us to derive a prediction for the $\delta T$ which that experiment would see, to be compared with the observations in Table 1.

It has been noted that the use of the $\chi^2$ test can give a bias in parameter estimation in favour of permitting a lower power spectrum amplitude, as in reality there is a tail to high temperature fluctuations. Other methods have been proposed (Bond, Jaffe & Knox 1998; Bartlett et al. 1999) which give good approximations to the true likelihood, though they require extra information on each experiment which is not yet readily available for the full compilation. We do not use these more sophisticated techniques here, but do note that as these methods are less forgiving of power spectra with too low an amplitude, the results from the $\chi^2$ analysis give conservative constraints on the optical depth.

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Table 1. The data used in this study, plotted in Figure 4.

| Experiment | Reference | $\ell_{\text{eff}}$ | $\delta T_{\ell_{\text{eff}}} \pm \sigma_{\text{data}}$ ($\mu$K) |
|------------|-----------|---------------------|--------------------------------------------------|
| COBE1      | 1         | 2.1                 | $8.5^{+16}_{-16}$                                |
| COBE2      | 1         | 3.1                 | $28.0^{+19}_{-19}$                               |
| COBE3      | 1         | 4.1                 | $34.0^{+5.0}_{-5.0}$                             |
| COBE4      | 1         | 5.6                 | $25.1^{+5.2}_{-5.2}$                             |
| COBE5      | 1         | 8                   | $29.4^{+3.6}_{-3.6}$                             |
| COBE6      | 1         | 10.9                | $27.7^{+3.9}_{-3.9}$                             |
| COBE7      | 1         | 14.3                | $26.1^{+4.4}_{-4.4}$                             |
| COBE8      | 1         | 19.4                | $33^{+4.6}_{-4.8}$                               |
| FIRS       | 2         | 10                  | $29.4^{+7.5}_{-7.5}$                             |
| Tenerife   | 3         | 20                  | $32.6^{+8.3}_{-8.3}$                             |
| SP91       | 4         | 59                  | $30.2^{+8.6}_{-8.2}$                             |
| SP94       | 4         | 59                  | $36.3^{+13.6}_{-13.6}$                           |
| IAC1       | 5         | 33                  | $111.8^{+59.4}_{-59.4}$                          |
| IAC2       | 5         | 53                  | $54.6^{+17.6}_{-17.6}$                           |
| BAM        | 6         | 74                  | $55.6^{+15.5}_{-15.5}$                           |
| Pyth1      | 7         | 92                  | $60.0^{+15.3}_{-15.3}$                           |
| Pyth2      | 7         | 177                 | $66.0^{+15.6}_{-15.6}$                           |
| IAB        | 8         | 118                 | $94.5^{+41.8}_{-41.8}$                           |
| ARGO1      | 9         | 9                   | $39.1^{+8.2}_{-8.2}$                             |
| ARGO2      | 9         | 9                   | $46.8^{+9.3}_{-9.3}$                             |
| MAX        | 10        | 137                 | $46.9^{+9.2}_{-9.2}$                             |
| QMAP1      | 11        | 80                  | $49.0^{+7.7}_{-7.7}$                             |
| QMAP2      | 11        | 126                 | $50.0^{+6.6}_{-6.6}$                             |
| Sk1        | 12        | 86                  | $51.0^{+8.4}_{-8.4}$                             |
| Sk2        | 12        | 166                 | $72.0^{+7.7}_{-7.7}$                             |
| Sk3        | 12        | 236                 | $88.4^{+10.3}_{-10.3}$                           |
| Sk4        | 12        | 285                 | $89.4^{+12.5}_{-12.5}$                           |
| Sk5        | 12        | 348                 | $71.8^{+19.8}_{-19.8}$                           |
| MSAM       | 13        | 95                  | $35^{+15}_{-15}$                                 |
| MSAM       | 13        | 210                 | $49^{+10}_{-10}$                                 |
| MSAM       | 13        | 393                 | $47^{+7.6}_{-7.6}$                               |
| CAT1       | 14        | 396                 | $50.8^{+13.6}_{-13.6}$                           |
| CAT2       | 14        | 608                 | $49.1^{+13.7}_{-13.7}$                           |
| CAT3       | 14        | 415                 | $57.3^{+13.6}_{-13.6}$                           |
| OVRO       | 15        | 589                 | $50.6^{+5.6}_{-5.6}$                             |

(1) Tegmark & Hamilton 1997; (2) Ganga et al. 1994; (3) Gutiérrez et al. 1997; Hancock et al. 1997 (binned); (4) Gundersen et al. 1995; (5) Femenia et al. 1998; (6) Tucker et al. 1997; (7) Platt et al. 1997; (8) Piccirillo & Calisse 1993; (9) de Bernardis et al. 1994; (10) Masi et al. 1996; (11) Tanaka et al. 1996 (binned); (12) de Oliveira-Costa et al. 1998; (13) Netterfield et al. 1997; (14) Wilson et al. 1999; (14a) Scott et al. 1996 and Hancock & Rocha 1997; (14b) Baker et al. 1998 (15) Leitch et al. 1998.

5 CONSTRAINTS ON THE REIONIZATION EPOCH

A model is specified by four parameters, $\Omega_0$, $h$, $n$ and $\tau$. There is an additional hidden parameter, which is the normalization of the spectrum. We do not fix this by normalizing to COBE alone, but rather seek the normalization which gives the best fit to the entire data set. We then examine whether each model is a good fit to the data.

There are $N_{\text{data}} = 35$ data points. Because we are measuring absolute goodness-of-fit on a model-by-model basis, with one hidden parameter, the appropriate distribution for the $\chi^2$ statistic has $N_{\text{data}} - 1$ degrees of freedom. Nothing further is to be subtracted from this to allow for the main parameters, as they are not being varied in the fit. To be specific, the question we are asking is “If you are interested in particular values of $\Omega_0$, $h$ and $\tau$ for some reason other than the CMB data, will the predicted CMB anisotropies be an adequate fit to the observations?”

To assess whether a model is a good fit to the data, we need the confidence levels of this distribution. These are

$$\chi^2_{34} < 48.6 \quad 95\% \text{ confidence level} ;$$
$$\chi^2_{34} < 56.1 \quad 99\% \text{ confidence level} .$$

Models which fail these criteria are rejected at the given level. We will use the 95 per cent exclusion. Our main focus is on limiting reionization, so for each choice of $\Omega_0$, $h$ and $n$, we are interested in the largest value of $\tau$, $\tau_{\text{max}}$, which gives an acceptable fit.

Although we are not concerned with finding the overall best-fitting parameters (which would require variation of $\Omega_0$ and ideally the inclusion of tensor perturbations, as in Tegmark 1999), we note that the absolute best-fitting model in our set, $\Omega_0 = 0.4$, $h = 0.6$, $n = 1.15$ and $\tau = 0.3$ has a
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Figure 6. Contours of the maximum permitted optical depth, as a function of $\Omega_0$ and $H_0$ with $n = 1$.

Figure 7. Limits on the reionization redshift, for $\Omega_0 = 0.3$. Reionization must occur late than that indicated by the contour levels. This plot assumes instantaneous complete reionization.

$\chi^2$ of 32, agreeing remarkably well with expectations for a fit to 35 data points with five adjustable parameters (the four mentioned plus the amplitude). These $\chi^2$ values agree with other analyses of this type (Lineweaver 1998; Tegmark 1999), and the best-fit model has parameter values in excellent agreement with indications from other types of observation.

The upper limits on the optical depth are shown in Figures 5 and 6 for different slices across the parameter space. For $\Omega_0 = 1$, quite a large amount of otherwise-interesting parameter space is now excluded by the CMB data, namely the region beyond the $\tau_{\text{max}} = 0$ contour which will not fit the data for any value of the optical depth. For the preferred Hubble constant values of around $H_0 = 65 \text{ km s}^{-1}$, the lower limit on $n$ is now around $n = 1$, severely constraining any attempts to salvage critical-density CDM models through tilting the primordial spectrum. For critical density with $n = 1$, as commonly employed in mixed dark matter models, the optical depth is constrained below 0.3 or even 0.2, depending on one's preference for $H_0$ [note that the CMB anisotropies are hardly altered by introduction of some hot dark matter in place of cold (Dodelson, Gates & Stebbins 1996)].

In the low-density case, the constraints on the optical depth are weaker, because the first acoustic peak is predicted to be higher in the absence of reionization. However, as there is a greater optical depth out to a given redshift in low-density models, the constraints on the actual reionization epoch prove to be quite similar. For $\Omega_0 = 0.3$, this is shown in Figure 7, which was obtained from the optical depth, assuming sufficiently-instantaneous reionization, using equation (10). We see that for the most commonly discussed $n = 1$ paradigm, the current limit on the reionization redshift is around $z_{\text{ion}} = 35$, which is just about at the upper limit of the theoretically anticipated range discussed in Section 2.2. Future observations may well start to eat into that range.

In Figure 8 we show two typical models which fail to fit the data, as well as the absolute best-fitting model. As well as describing the results graphically, it is useful to having a fitting function for the maximum allowed optical depth. A good fit for two particular $\Omega_0$ values is given by the formulae

$$\tau_{\text{max}} = 0.03 - (2.9 - 1.5n)(h - 0.65) + 1.9(n - 1) \quad [\Omega_0 = 1];$$

$$\tau_{\text{max}} = 0.36 - (2.4 - 1.4n)(h - 0.65) + 1.7(n - 1) \quad [\Omega_0 = 0.3].$$

The second of these is illustrated in Figure 8. For general $\Omega_0$, a suitable interpolation between these is to interpolate the three coefficients linearly in $\sqrt{\Omega_0}$ (e.g. for the first coefficient take $0.76 - 0.73 \sqrt{\Omega_0}$ and so on).

There is no simple fitting function for the reionization redshift, but an analytic fit is obtained by rearranging equation (10) and putting in the fitting functions for $\tau_{\text{max}}$. 

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We have developed an analytic method of generating the $C_\ell$ spectra in reionized models from models without reionization, and confronted models with current observational data in order to place upper limits on the optical depth caused by reionized electrons. We stress that the constraint is best expressed on the optical depth, as the main physical effect is that rescattered photons lose their short-scale anisotropy and to a good approximation it doesn’t matter where the scattering took place. In general the optical depth is a function of the complete reionization history, as well as the cosmological model, but at least the first of these dependencies can be simplified if it is assumed that reionization happens completely and fairly rapidly, in which case the constraint can be re-expressed as an upper limit on the reionization redshift.

We considered only a single value of the baryon density, at the high end of the preferred range, and did not include tensor perturbations. The second of these is definitely conservative, and the first likely to be so, so our results can be regarded as rather safe upper limits. However, these quantities would in general have to be included if one undertakes the more ambitious task of trying to estimate best-fitting parameters from the data, rather than delimiting the allowed region. Several analyses have been carried out in recent years to use available information to constrain the cosmological parameters, with the majority neglecting the influence of reionization (Ganga, Ratra & Sugiyama 1996; White & Silk 1996; Bond & Jaffe 1997; Lineweaver et al. 1997; Bartlett et al. 1998; Hancock et al. 1997; Lineweaver & Barbosa 1998a, 1998b). Most closely related to this work are the papers of de Bernardis et al. (1997) and more recently Tegmark (1998), who investigated how reionization could affect cosmological parameter determination. Our results update and extend the former paper, by employing more up-to-date data and exploring a wider parameter space. Neither of those papers aimed at providing detailed constraints on the epoch of reionization, preferring instead to find best-fitting parameters. Although we have not made a serious attempt at parameter estimation, we do concur with those papers that the best-fitting models have a blue ($n > 1$) spectrum and significant reionization.

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