It is shown that a circular dipole can deflect the focused laser beam that induces it, and will experience a corresponding transverse force. Quantitative expressions are derived for Gaussian and top hat beams, while the effects vanish in the plane-wave limit. The phenomena are analogous to the Magnus effect pushing a spinning ball onto a curved trajectory. The optical case originates in the coupling of spin and orbital angular momentum of the dipole and the light. In optical tweezers the force causes off-axis displacement of the trapping position of an atom by a spin-dependent amount up to $\lambda/2\pi$, set by the direction of a magnetic field. This suggests direct methods to demonstrate and explore these effects, for instance to induce spin-dependent motion.

A common practice in many branches of sports is to send a ball onto a curved trajectory by giving it a spin. In this famous example of the Magnus effect [1] the spinning ball deflects the stream of air around it, and is pushed sideways by the reaction force perpendicular to its forward velocity. In analogy, we may ask if a rotating dipole in an atom may similarly deflect a beam of light, and thereby be pushed by a force perpendicular to the light beam. C.G. Darwin already remarked that for circular dipoles “...the wave front of the emitted radiation faces not exactly away from the origin, but from a point about a wave-length away from it.” [2]. A recent experiment confirmed that an atomic circular dipole can indeed appear to be displaced from its true location, due to the emitted spiral-shaped wavefront [3]. Circular dipoles provide perhaps the simplest example of the intrinsic coupling of spin and orbital angular momentum (SAM and OAM, respectively) in non-paraxial light fields [4–12]. Such fields, in the form of tightly focused laser beams, are of central importance in a rapidly growing range of experiments involving (arrays of) optical tweezers [13–23]. These are developed as precise tools to hold and manipulate single atoms or molecules at the quantum level, in creating platforms for quantum simulation and computation, as well as for quantum sensing and atomic clocks [24, 25].

Here we predict that an atomic circular dipole can deflect the centered focused laser beam that induces it, and conversely, that the atom will experience a transverse force when on-axis [33]. An important consequence of this force can be seen in the off-axis displacement of the trapping potential created by an optical tweezer. Thus, rather than ‘seeing an atom where it is not’ [3 27 28], here we describe a different situation, of ‘trapping an atom where the focus is not’ [13 23]. We find that, as SAM of the atom is coupled to transverse OAM of light, different spin states will be displaced by different amounts up to $\pm \lambda = \pm \lambda/2\pi$. The exact displacement depends on the polarization of the tweezer beam and the direction of a magnetic field that sets the quantization axis, providing a tool for state-dependent manipulation of the atomic motion within the tweezer. While the deflection angle is only significant near atomic resonance, the off-axis displacement is independent of the laser detuning, and persists in the usual far off-resonant regime of optical tweezers.

We describe these effects in terms of interference between the focused incident beam with the wave scattered...
by the circular dipole, see Fig. 1. In the optical theorem, such interference is used to describe the attenuation of light in terms of the forward scattering amplitude [29]. In contrast, here we concentrate on beam deflection, as a consequence of the tilt of the spiral wavefront with respect to the incident wave. Two simple atomic level schemes serve as examples, a \( j = 0 \rightarrow j' = 1 \) and a \( j = 1 \rightarrow j' = 0 \) transition. The former is conceptually simpler though limited in the range of useful laser detunings. The latter allows far off-resonance operation and offers interesting extra opportunities.

Starting with the \( j = 0 \rightarrow j' = 1 \) transition, we focus a linearly polarized (\( \mathbf{E} \parallel x \)) monochromatic laser onto a single atom placed in the origin, see Fig. 1. A magnetic field \( \mathbf{B} \parallel y \) defines the quantization axis and splits the excited state into three \( |j,m_j\rangle \) sublevels, separated by the Zeeman shift \( \sim \mu_B B/\hbar \), with \( \mu_B \) the Bohr magneton [30]. We tune the laser close to the \( \Delta m_j = 1 \) transition, with a detuning \( \Delta = \omega_L - \omega_0 \) small compared to the Zeeman shift, so that the \( \Delta m_j = 1 \) transition can be neglected. The emission by the induced circular dipole has a spiral wavefront in the \( xy \) plane, tilted with respect to the forward \( z \) direction of the incident beam.

We represent the light fields by their angular spectrum [31, 32], using spherical \( k \)-space coordinates \( (k, \theta, \phi) \). For monochromatic light \( k = \omega/c \) is fixed, so that the incident field can be written as \( i\frac{1}{2} \mathbf{E}_{\text{in}}(\Omega)e^{-i\omega t} + \text{c.c.} \), with \( \Omega = (\theta, \phi) \). The total field is the sum of the incident and scattered waves. Writing only the positive frequency \( e^{-i\omega t} \) components, the total field reads

\[
\mathbf{E}(\Omega) = \mathbf{E}_{\text{in}}(\Omega) + \mathbf{E}_{\text{sc}}(\Omega)
\]

(1)

with \( \mathbf{E}_{\text{sc}}(\Omega) \) the scattered wave.

We define the radiant intensity,

\[
J(\Omega) = ||\mathbf{E}(\Omega)||^2/2Z_0,
\]

(2)

with \( Z_0 = 1/\epsilon_0 c \), so that \( J(\Omega)d\Omega \) is the power flowing out of an infinitesimal solid angle \( d\Omega = \sin \theta \ d\theta \ d\phi \) around

\[
\mathbf{u}_\Omega = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)
\]

(3)

Combining Eqs. (1) and (2), the total radiant intensity is the sum of three terms,

\[
J(\Omega) = J_{\text{in}}(\Omega) + J_{\text{sc}}(\Omega) + J_{\text{dif}}(\Omega)
\]

(4)

The interference term

\[
J_{\text{dif}}(\Omega) = \frac{1}{2Z_0} \left[ \mathbf{E}^{\text{in}}(\Omega) \cdot \mathbf{E}^{\text{coh}}(\Omega) + \text{c.c.} \right]
\]

(5)

contains only the coherent component of the scattered field. An incoherent component would contribute to \( J_{\text{sc}}(\Omega) \) but not to \( J_{\text{dif}}(\Omega) \). For simplicity we assume that the scattered field is entirely coherent, essentially restricting ourselves to the low-saturation limit [32].

The deflection of the light beam can be expressed as the change in average wave vector \( (\mathbf{k}) - (\mathbf{k})_{\text{in}} \) between the total (incident plus scattered) and the incident wave, using

\[
(\mathbf{k})_{\text{in}} = k \int \frac{\mathbf{u}_\Omega J_{\text{in}}(\Omega) d\Omega}{\int J_{\text{in}}(\Omega) d\Omega} = k \int \frac{\mathbf{u}_\Omega J_{\text{in}}(\Omega) d\Omega}{P_{\text{in}}}
\]

(6)

and similar for \( (\mathbf{k}) \), omitting the subscript. Assuming (again for simplicity) that non-radiative decay is absent, we shall write \( P_{\text{in}} = P \) throughout.

The deflection is entirely determined by the interference term \( J_{\text{dif}}(\Omega) \). The scattered light itself does not contribute, due to the symmetry of the dipole radiation pattern, \( \int \mathbf{u}_\Omega J_{\text{sc}}(\Omega) d\Omega = 0 \). For the deflection we therefore have

\[
\delta(\mathbf{k}) = (\mathbf{k}) - (\mathbf{k})_{\text{in}} = k \frac{P}{\omega} \int \mathbf{u}_\Omega J_{\text{dif}}(\Omega) d\Omega
\]

(7)

and for the force on the atom, by momentum conservation,

\[
\mathbf{F} = \frac{P}{\omega} \delta(\mathbf{k}) = \frac{1}{c} \int \mathbf{u}_\Omega J_{\text{dif}}(\Omega) d\Omega
\]

(8)

While this expression includes the forward radiation pressure force, in the cases of interest here the main force will be transverse to the optical axis, \( \mathbf{F} \approx F_x \hat{x} \). Then (approximately) \( \delta(\mathbf{k}) \perp (\mathbf{k})_{\text{in}} \) and with \( (\mathbf{k})_{\text{in}} \approx k \mathbf{u}_z \) the deflection angle is

\[
|\delta \theta| \approx \frac{|\delta(\mathbf{k})|}{k}
\]

(9)

We will choose \( \delta \theta > 0 \) if \( F_x < 0 \).

Let us now introduce specific field patterns to calculate \( J_{\text{dif}}(\Omega) \). We take the dipole to be circular, \( \mathbf{p} = pe^{i\alpha} \mathbf{u}_z \), with \( \mathbf{u}_k = (\hat{x} + i\hat{z})/\sqrt{2} \) denoting spherical unit vectors, and \( \alpha \) the phase of the \( p_x \) component of the dipole, relative to the local driving field. The field radiated by a coherent dipole [29], in angular coordinates, takes the form [32],

\[
\mathbf{E}_{\text{sc}}(\Omega) = \mathcal{E}_{\text{sc}} e^{i\alpha} (\mathbf{u}_0 \times \mathbf{u}_x) \times \mathbf{u}_\Omega
\]

(10)

with corresponding \( J_{\text{sc}}(\Omega) \) given by Eq. (2). Here \( \mathcal{E}_{\text{sc}} = p k^2/4\pi \epsilon_0 > 0 \) is a real-valued amplitude. Assuming the steady state of the optical Bloch equations for a two-level system, \( c \alpha = -\Delta/\gamma \), with \( \Delta = \omega - \omega_0 \) the detuning from the \( \Delta m_j = +1 \) transition, and \( \gamma = \omega_0^3 D^2/6\pi \epsilon_0 hc^3 \) the half width of the transition, with \( D \) the transition dipole moment.

For comparison, we consider two different types of incident beams, Gaussian (‘G’) and angular ‘tophat’ (‘T’), where the latter approximates the output of a uniformly illuminated focusing lens. The field for these two beams can be written as

\[
\mathbf{E}^{(\text{G})}_{\text{in}}(\Omega) = \mathcal{E}_0 e^{i\alpha} \exp[\!-\theta^2/w_0^2] \mathbf{u}_x(\Omega)
\]

(11)

\[
\mathbf{E}^{(\text{T})}_{\text{in}}(\Omega) = \mathcal{E}_0 \Pi(\theta/2r_0) \mathbf{u}_x(\Omega)
\]

(12)
with amplitudes $\varepsilon_0^{(G)}, \varepsilon_0^{(II)} > 0$. The Gaussian beam has an angular width $w_\theta$ which is related to the minimum waist $w_0$ \((1/e^2\) spatial radius of intensity) as $w_\theta w_0 = \lambda/\pi$. For the tophat beam, $\Pi(\theta/2r_\theta)$ is the rectangular function with angular half width $r_\theta$ and unit amplitude.

The polarization vector $u_x(\Omega)$ is transverse to $u_\Omega$; it is obtained by co-rotating $\hat{x}$ when rotating $\hat{z} \rightarrow u_\Omega$, i.e. rotating by $\theta$ around an axis $\hat{z} \times u_\Omega$.

$$u_x(\Omega) = \begin{pmatrix} \cos \theta \cos^2 \phi + \sin^2 \phi \\ (\cos \theta - 1) \sin \phi \cos \phi \\ -\sin \theta \cos \phi \end{pmatrix} \quad (13)$$

When combining Eq. \((10)\) with Eq. \((11)\) or \((12)\) in Eq. \((5)\), the interference term contains the amplitude product $\varepsilon_0^{(G)} \varepsilon_{sc}$ or $\varepsilon_0^{(II)} \varepsilon_{sc}$. In the low-saturation limit, the amplitude $\varepsilon_{sc}$ is proportional to $\varepsilon_0^{(G)}$ or $\varepsilon_0^{(II)}$. Their ratio can be obtained by requiring energy conservation \((22)\). Upon insertion of the resulting ratios $\varepsilon_{sc}/\varepsilon_0^{(G)}$ and $\varepsilon_{sc}/\varepsilon_0^{(II)}$ into Eq. \((5)\), the interference term $J_{\theta}(\Omega)$ becomes proportional to the total power; the deflection angle is then independent of power.

In Fig. \(2\) we show $J_{\theta}(\Omega)$ in the plane of the dipole ($\phi = 0$), together with the total radiant intensity $J(\Omega)$. For the Gaussian beam, the effect of $J_{\theta}(\Omega)$ is to shift the peak and the average of the direction of propagation away from $\theta = 0$. For the tophat beam, the interference leads to an intensity gradient across the angular width of the beam, whereas the edges of the tophat stay in place. In this case the intensity gradient leads to a change in average beam direction.

Finally, the deflection angle is obtained by integration as in Eq. \((7)\),

$$\delta \theta \approx \frac{3}{4} \frac{\gamma \Delta}{(\gamma^2 + \Delta^2)} \cdot \begin{cases} \frac{w_\theta^4}{r_\theta^2/4} \quad \text{Gauss} \\ \frac{r_\theta^4/4}{r_\theta^2/4} \quad \text{tophat} \end{cases} \quad (14)$$

and the reaction force as

$$F_x \approx -\frac{P}{c} \delta \theta \quad (15)$$

The results are given as the leading order in $w_\theta$ and $r_\theta$. The deflection angle reaches maximal values of $\delta \theta = \pm 3w_\theta^4/8$ and $\pm 3r_\theta^4/32$, respectively, for $\Delta = \pm \gamma$; it vanishes in the plane-wave limit. In this central result the detuning dependence shows that the force is essentially a dipole force \((33)\). The force can also be seen as arising from polarization gradients that occur near the focus of a linearly polarized light beam \((7,9,11,13,23)\).

We now address the question of how we can observe the calculated effects, either by directly observing the deflection of a laser beam, or by observing the effect of the reaction force on the atom. As shown by Eq. \((14)\), the angle of deflection by a single atom is small compared to the divergence angle, $|\delta \theta| \ll r_\theta, w_\theta$, and vanishes in the plane wave limit, $w_\theta, r_\theta \rightarrow 0$.

A direct observation will thus require sufficiently high signal-to-noise ratio, similar to what was achieved in the recent observation of apparent $\lambda$ displacement of an emitter \((3)\). In the above calculations, we have assumed the atom to be fixed in the origin, which can be achieved in an ion trap, or in a tight optical tweezer. One can then look for the deflection of a weak near-resonant ($\Delta \approx \pm \gamma$) probe beam. A larger deflection angle may be obtained if multiple atoms cooperate. For example, one may consider dense clouds of sub-wavelength size, containing tens of hundreds of atoms, that have been observed to show collective scattering properties \((34,35)\). Another possibility may be to use elongated, (quasi-) one-dimensional samples with tight ($\lesssim \lambda$) radial confinement, achievable, e.g., in optical lattices \((36,37)\) and on atom chips \((39)\).

The second mode of observation, via the force on the atom, provides some extra opportunities to manipulate the atomic motion in an optical tweezer, and to separate spin states of the atom. To see this, we consider an optical tweezer, trapping an atom with a $j = 1 \rightarrow j' = 0$ transition. The $|m_j = \pm 1\rangle_y$ states now couple to the $(\sigma^+)_y$ components of the light field, and therefore experience opposite forces $F_x$. Interestingly, this configuration allows the use of the far off-resonance light of the tweezer itself, without the need for a separate weak, near resonance probe beam \((23)\). Looking at the spiral wave of a $u_\pm$ dipole shown in Fig. \(1\) we can readily see that the relative tilt of the forward wavefronts will vanish if we displace the atom by $\lambda$ in the $x$ direction. By thus aligning the wave fronts, the transverse force should vanish.

An atom in the $|m_j = -1\rangle_y$ sublevel will therefore find an equilibrium position in the tweezer at a displaced off-axis location $x_{eq} = \lambda$. By the same reasoning, the $|m_j = +1\rangle_y$ sublevel will have the opposite displacement, so that for the $j = 1 \rightarrow j' = 0$ transition:

$$x_{eq} = -(m_j)_y \lambda \quad (16)$$

FIG. 2: Beam deflection: radiant intensities in the plane of the $u_+$ dipole, for (a) a Gaussian incident beam with $w_\theta = 0.6$, and (b) an angular tophat incident beam with $r_\theta = 0.6$. In both cases, the gray/dotted curve shows $J_{\theta}(\theta, \phi, \gamma = 0)$ of the incident beam, normalized to 1 for $\theta = 0$; red/solid and blue/dashed curves show the outgoing, or total $J(\theta,0)$, for $\Delta = -\gamma$ and $+\gamma$, respectively. For clarity, we identify $(\theta,0) \equiv (-\theta, \pi)$. Curves remain the same upon shifting simultaneously the signs of the detuning and the spin of the dipole.
The tweezer thus traps the atom off axis, ‘where the focus is not’, in a spin-dependent location. For the situation considered here the \( |m_j = 0 \rangle_y \) state would be untrapped, for a lack of \( \pi \) component in the laser polarization. This could be changed by rotating \( B \). In particular, setting the angle between \( E_{in} \) and \( B \) to \( \arctan(\sqrt{2}) \), the polarization components \( \sigma^- \), \( \pi \), and \( \sigma^+ \) would become equal. At this ‘magic angle’ all three spin components would be trapped with a Stern-Gerlach type separation [23, 30].

These intuitive arguments are backed up by a calculation [39], that shows that Eq. (14) for the beam deflection is multiplied by \( 1 \mp kd \), for a \( u_k \) dipole displaced by \( d \) in the \( x \) direction, to lowest order in \( d \). Thus the transverse force indeed vanishes for a transverse displacement of \( d = k^{-1} = \lambda \) in the \( x \) direction. Remarkably, the size of the displacement is independent of the detuning and independent of the beam divergence angle.

The off-axis trapping locations offer interesting opportunities to manipulate the motion of atoms in the tweezer, see Fig. 3. Let us imagine an atom trapped in the \( |m_j = 1 \rangle_y \) state. As we slowly rotate the magnetic field in the \( yz \) plane, the orientation of the atom will adiabatically follow the rotating quantization axis. After rotating the field \( y \to z \to y \), the spin will have maintained its orientation relative to \( B \), i.e. \( |m_j = 1 \rangle_B \to |m_j = 1 \rangle_B \). However, its orientation will have flipped in space, \( |m_j = 1 \rangle_y \to |m_j = -1 \rangle_y \), since \( B \) has changed direction. The space-referenced spin flip implies that the atom must have moved to the other side of the optical axis. Thus, by rotating the magnetic field in the \( yz \) plane at a frequency \( \omega_B \), we effectively shake the trap back and forth: \( z_{eq} = -(m_j)B \cos \omega_B t \). The \( m_j = \pm 1 \) levels are shaken with opposite phase.

Shaking the trap at an amplitude \( \lambda \) is equivalent to a harmonic driving force \( F_x = m\omega^2\lambda \cos \omega_B t \), with \( \omega \) the trap frequency. Resonant shaking, \( \omega \approx \omega_B \), will induce an oscillatory motion in the trap. For example, for a tweezer with a laser wavelength of \( \lambda \approx 0.8 \mu m \), a Gaussian waist of \( 2 \mu m \), holding an atom of mass \( m = 88u \) in a \( 20 \mu K \) deep trap, the trap frequency will be \( \omega \approx 2\pi \times 7 \) kHz. In a simple driven harmonic oscillator model only 3.5 drive cycles would impart enough energy to kick the atom out of the trap, corresponding to a velocity of \( \sim 6 \) cm/s. In reality one would of course need to take anharmonicity into account. The point here is that magnetic field modulation can easily induce oscillatory motion in the trap which can then be detected either as trap loss, or by using time-of-flight imaging methods. For the required magnetic field a few gauss should be sufficient, to ensure that the Larmor frequency is large compared to the trap frequency. Rotating the field at frequencies of \( \sim 10 \) kHz is well possible, being comparable to what is used in TOP traps [41].

Many available atomic level systems should be suitable to observe off-axis tweezer trapping. For example, in \(^{88}Sr\) the transition \( 3P_2 \to 3S_1 \) would provide a \( j = 2 \to j' = 1 \) transition. The outer \( (m_j)_y = 2 \) \((-2)\) state couples only to the \( \sigma^- \) \((\sigma^+)\) polarization component, so its spatial shift will be \(-\lambda \) \((+\lambda)\). Using \(^{87}Rb\) one could operate a tweezer red detuned to the \( D_1 \) line \((795 \) nm\), driving the two hyperfine lines \( F = 2 \to F' = 1, 2 \). Also in this case the outer state \( (m_F)_y = 2 \) \((-2)\) is displaced by \(-\lambda \) \((+\lambda)\), as long as the detuning stays small compared to the fine structure splitting of the \( D \) lines.

In summary, it is predicted that a circular dipole can deflect a focused laser beam, with a corresponding reaction force on the atom, transverse to the laser beam. While the beam deflection could be observed directly, the transverse force leads to spin-dependent, off-axis displacements of up to \( \pm \lambda \) for atoms trapped in an optical tweezer. The displacements could be used to perform Stern-Gerlach type analysis of the spin states of the atom, as well as to manipulate the motion of the atoms in the tweezer. A full investigation of Stern-Gerlach type splittings for all \( m_j \) states, different \( j \to j' \) transitions, as well as for arbitrary relative orientations of \( E_{in} \) and \( B \), is beyond the scope of this paper.

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SPATIAL VS. ANGULAR k-SPACE COORDINATES

In this paper we express all fields by their angular spectrum $E(\Omega) = E(\theta, \phi)$. This is usually defined for fields propagating out into a half space $z \geq 0$ (see Ch. 3.2 in [S1]), as is clearly the case for the incident beams. The relationship with the field in the plane $z = 0$ is given by

$$\tilde{E}(x, y, 0) = k^2 \int_{\theta \leq \pi/2} E(\Omega) \sin \theta d\theta d\phi$$

(S1)

with $k = \omega/c$ the laser wave vector. For the Gaussian beam with angular waist $w_\theta$ the above equation yields the familiar Gaussian beam cross section, with minimum waist $w_0 = \lambda/w_\theta$, see also Ch. 5 in [S1]. The angular tophat beam approximates the output of a uniformly illuminated circular lens, and Eq. (S1) yields the resulting Airy pattern in the focal plane $z = 0$.

Although the emission by a dipole is not confined to $z \geq 0$, the angular representation of the radiation pattern of a dipole $p$ in a direction $u_\Omega$ is well known to be given by Eq. (10) (main text), see for example Ch. 9 in [S2]. Only the radiating, or ‘far field’, terms ($\sim r^{-1}$) are relevant in our case, because one can evaluate the beam deflection at arbitrarily large distance of the dipole, where the near fields ($\sim r^{-2}, r^{-3}$) have become negligible.

In the plane ($\phi = 0$) of a $u_\pm$ dipole,

$$(u_\Omega \times u_\pm) \times u_\Omega = e^{\pm i\theta} \sqrt{2} (\cos \theta, 0, -\sin \theta)$$

(S2)

shows the spiral wave character in the prefactor $e^{\pm i\theta}$.

The factor $i$ in Eq. (10) (main text) is a crucial detail. It is a consequence of expressing the spherical waves $e^{ikr}/r$ of the dipole field as an angular spectrum of plane waves. The same factor $i$ can be recognized in the Weyl representation of a diverging spherical wave [S1]. In the case at hand, one can readily see that it also ensures that a resonant beam is attenuated (absorbed) in the forward direction, due to destructive interference of incident and scattered waves.

The phase factor $e^{i\alpha}$ in Eq. (10) (main text) follows from the steady state of the optical Bloch equations [S3]. In a two-level atom with states $e, g$, the induced dipole moment is given by the off-diagonal density matrix element $\rho_{eg}$. If the atom is driven at detuning $\Delta$ by a monochromatic field with (real) amplitude $E_0$ the steady state (for $s \ll 1$) is given by

$$\rho_{eg} = i \frac{DE_0}{\bar{\hbar}} \frac{\gamma - i\Delta}{2\gamma - i\Delta}$$

(S3)

which has a complex argument $\alpha = \text{arg} \rho_{eg}$ given by $\cot \alpha = -\Delta/\gamma$. Here, since we choose an $x$ polarized incident wave, $\alpha$ is the phase of the $p_x$ component of the dipole, relative to the incident field.

LOW SATURATION LIMIT

In the definition of the saturation parameter $s$ we include the detuning, following [S3],

$$s = \frac{I/I_0}{1 + \Delta^2/\gamma^2}$$

(S4)

with $I$ the intensity and $I_0 = 2\pi\hbar c\gamma/3\lambda^3$ the saturation intensity. In the low-saturation limit, characterized by $s \ll 1$, the scattered light is almost entirely coherent, with a small incoherent fraction equal to $s/(1 + s)$. In optical tweezer experiments, using far off-resonant laser beams, typical values for $s$ are in the range $10^{-6} - 10^{-8}$, so that $s \ll 1$ is indeed well fulfilled and the incoherent scattering rate is low.
FIELD AMPLITUDES

The peak amplitudes $E_0^{(G)}, E_0^{(Π)}$ are related to the total power in the incident beam by

$$P = \int J_{in}(\Omega) \, d\Omega = \left\{ \begin{array}{l}
\approx \frac{(E_0^{(G)})^2}{2Z_0} \times \frac{\pi w_\theta^2}{4} \\
\left( \frac{E_0^{(Π)})^2}{2Z_0} \times 2\pi (1 - \cos r_\theta) \right)
\end{array} \right.$$ (S5)

for the Gaussian and angular tophat beam, respectively. The integrals were performed using Mathematica software [S4]. For the Gaussian, the equality is only approximate, we give here the leading term of a power series in $w_\theta$. The above expressions have been written as a product of the forward ($\theta = 0$) radiant intensity $E_0^2/2Z_0$ and an effective solid angle.

The average wave vector of the incident beams is shorter than the corresponding value for a plane wave,

$$\langle k \rangle_{in} = k^z \times \left\{ \begin{array}{l}
1 - \frac{w_\theta^2}{4} + \mathcal{O}(w_\theta^4) \quad \text{(Gauss)} \\
\cos^2 \left( \frac{r_\theta}{2} \right) \quad \text{(tophat)}
\end{array} \right.$$ (S6)

The amplitude ratios $E_{sc}/E_0^{(G)}$ and $E_{sc}/E_0^{(Π)}$ can be obtained from the energy conservation condition

$$\int [J_{if}(\Omega) + J_{sc}(\Omega)] \, d\Omega = 0$$ (S7)

The scattering term $J_{sc}(\Omega) > 0$ would increase the outflowing power, which must be cancelled by the interference term $J_{if}(\Omega)$. As a result,

$$\frac{E_{sc}}{E_0^{(G)}} \approx \frac{3 \sin \alpha}{4\sqrt{2}} w_\theta$$ (S8)

$$\frac{E_{sc}}{E_0^{(Π)}} = \frac{3 \sin \alpha}{4\sqrt{2}} \sin^2 \left( \frac{r_\theta}{2} \right) (\cos r_\theta + 3)$$ (S9)

where in the Gaussian case the leading order in $w_\theta$ is given.

With these ratios the interference terms in the radiant intensity, Eq. (5) (main text) can be obtained as

$$J_{if}(\Omega) = -\frac{E_{sc}}{\sqrt{2}Z_0} f(\Omega, \Delta) \times \left\{ \begin{array}{l}
\sim \frac{E_0^{(G)}}{E_0^{(Π)}} e^{-\theta^2}/w_\theta^2 \\
\frac{e_0^{(Π)}}{E_0^{(Π)}} \Pi(\theta/2r_\theta)
\end{array} \right.$$ (S10)

with

$$f(\Omega, \Delta) = \gamma \frac{(\cos \theta \cos^2 \phi + \sin^2 \phi) - \Delta \sin \theta \cos \phi}{\sqrt{\gamma^2 + \Delta^2}}$$ (S11)

For the deflection, expressed as $\delta(k) = (k) - \langle k \rangle_{in}$ we evaluate the integral of Eq. (7) (main text) to obtain

$$\delta(k) = \frac{3k}{4} \frac{\gamma \Delta}{\gamma^2 + \Delta^2} \left( w_\theta^4 + \mathcal{O}(w_\theta^6), 0, -2\gamma \frac{\gamma}{\delta} w_\theta^2 + \mathcal{O}(w_\theta^4) \right)$$ (S12)

for the Gaussian beam, and

$$\delta(k) = \frac{3k}{16} \frac{\gamma \Delta}{\gamma^2 + \Delta^2} \left( r_\theta^4 + \mathcal{O}(r_\theta^6), 0, -4\gamma \frac{\gamma}{\delta} r_\theta^2 + \mathcal{O}(r_\theta^4) \right)$$ (S13)

for the angular tophat beam.

Note that in both cases the $y$ component is absent. To leading order, the deflection angle is just given by

$$\delta \theta \approx \frac{(\delta(k))_x}{k}$$ (S14)

which leads to Eq. (14) of the main text.
CALCULATION FOR A DISPLACED ATOM

When the atom is located at a position $d$ away from the origin, the angular components of the scattered wave are phase shifted by an amount $\exp(-iku_\Omega \cdot d)$, so that the interference term, Eq. (5) (main text), is modified to

$$J_{if}(\Omega) = \frac{1}{2Z_0} \left[ E^*_{in}(\Omega) \cdot E^{(coh)}_{sc}(\Omega)e^{-iku_\Omega \cdot d} + \text{c.c.} \right]$$  \hspace{1cm} (S15)

For a displacement along $x$, we have $d = d\hat{x}$ so that

$$ku_\Omega \cdot d = kd \sin \theta \cos \phi$$  \hspace{1cm} (S16)

In the integrals $\int J_{if}(\Omega) d\Omega$ and $\int u_\Omega J_{if}(\Omega) d\Omega$, we develop the integrand in a power series of $kd$, up to fourth order and integrate the terms separately.

For the amplitude ratios $E_{sc}/E^{(G)}_0$ and $E_{sc}/E^{(\Pi)}_0$ we find that their lowest order ($\sim w_\theta^2$ and $\sim r_\theta^2$) is not affected by $kd$. For the deflection angle the leading order in $w_\theta, r_\theta$ is still fourth order, and up to order $(kd)^4$ the angle is

$$\delta \theta \approx \frac{3}{4}(1 \mp kd) \frac{\gamma \Delta}{(\gamma^2 + \Delta^2)} \times \begin{cases} w_\theta^4 & \text{(Gauss)} \\ r_\theta^4/4 & \text{(tophat)} \end{cases}$$  \hspace{1cm} (S17)

for a $u_\pm$ dipole, respectively. This shows that the deflection angle, and thus also the transverse force, vanishes if the atom is displaced by an amount $d = \pm k^{-1} = \pm \lambda$.

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[S1] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, 1995) Ch. 3 and 5.
[S2] J. D. Jackson, *Classical Electrodynamics* 3rd ed. (Wiley, New York, NY, 1999) Ch. 9 and 10.
[S3] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions: Basic Process and Applications* (Wiley-VCH, New York, 1998).
[S4] Wolfram Research, Inc., Mathematica, Version 12.0, Champaign, IL (2020).