The endpoint of the electroweak phase transition

M. Gürtler\textsuperscript{a}, E.-M. Ilgenfritz\textsuperscript{b}, and A. Schiller\textsuperscript{a}

\textsuperscript{a}Institut für Theoretische Physik, Universität Leipzig, Augustusplatz 10-11, D-04109 Leipzig, Germany

\textsuperscript{b}Institut für Physik, Humboldt-Universität zu Berlin, Invalidenstr. 110, D-10115 Berlin, Germany

The 3\textsuperscript{d} SU\textsubscript{(2)}–Higgs model is used to find the critical Higgs mass above which the first order phase transition ends. One method is focused on the disappearance of the two-state signal of the scalar condensate (vanishing of the latent heat). Another method is based on the analysis of Lee–Yang zeroes of the partition function which allows to characterise the change from first order transition into an analytical crossover.

1. Introduction

Different approaches (see e.g. \cite{1}) predicted that there is a critical Higgs mass of $O(100)$ GeV above which there is no first order thermal phase transition anymore. The 3\textsuperscript{d} SU\textsubscript{(2)}–Higgs model is an effective model of the electroweak standard model at high temperature and describes the electroweak phase transition \cite{2}. In this contribution some results \cite{3} obtained within this framework are summarised to give a more precise characterisation of the end of the phase transition.

2. The model

The lattice action is

$$S = \beta_G \sum_p \left( 1 - \frac{1}{2} \text{tr} U_p \right) - \beta_H \sum x,\alpha E_{x,\alpha} + \sum x \left( \rho^2_x + \beta_R \left( \rho^2_x - 1 \right)^2 \right),$$

(1)

$$E_{x,\alpha} = \frac{1}{2} \text{tr} (\Phi_x^+ U_{x,\alpha} \Phi_{x+\alpha}), \quad \rho^2_x = \frac{1}{2} \text{tr} (\Phi_x^+ \Phi_x).$$

(2)

An approximate Higgs mass $M^*_H$ entering

$$\beta_R = \frac{\lambda_3}{g_3^2} \frac{\beta_H^2}{\beta_G} = \frac{1}{8} \left( \frac{M^*_H}{80 \text{ GeV}} \right)^2 \frac{\beta_H^2}{\beta_G}$$

(3)

is used to label our lattice data. $\lambda_3$ and $g_3$ are renormalisation group invariant dimensionful quartic and gauge couplings of the corresponding 3\textsuperscript{d} continuum model.\footnote{The mapping of the lattice couplings to 4\textsuperscript{d} continuum physics is summarised in \cite{4}.}

The continuum limit corresponds to $\beta_G \to \infty$. We use $\rho^2 = (1/L^3) \sum_x \rho^2_x$ as order parameter.

We express lattice sizes and 3\textsuperscript{d} results in mass units $g_3^2$ which allows to combine data obtained at different $\beta_G$. The quality of this scaling checks to what extent the continuum limit is reached. In the chosen units, the physical lattice size is

$$l g_3^2 = \log g_3^2 = 4L/\beta_G.$$  

(4)

The discontinuity of the quadratic scalar condensate is defined by

$$\Delta \langle \phi^+ \phi \rangle / g_3^2 = 1/8 \beta_G \beta_H \Delta \langle \rho^2 \rangle$$

(5)

through the infinite volume limit of the jump in $\rho^2$ measured at the pseudo–critical hopping parameter between the broken and symmetric phases. In our analysis we have widely used the Ferrenberg-Swendsen method where the reweighting is performed by double–histogramming in two parts of the action.

3. Latent heat criterion

A first order phase transition is characterised by non–vanishing latent heat which is proportional to $\Delta \langle \phi^+ \phi \rangle$. We define the end of the transition line in the $\beta_H$–$M^*_H$ plane (for $\beta_G \to \infty$) where this discontinuity vanishes.

The Ferrenberg–Swendsen method allows to interpolate between different $\beta_H$ and $M^*_H$. In

\cite{2} the mapping of the lattice couplings to 4\textsuperscript{d} continuum physics is summarised in \cite{4}.
Fig. 1 the condensate jump for three values of \( M_H^* \) and for \( \beta_G = 12, 16 \) is plotted as function of the inverse physical length squared. The respective pseudo-critical hopping parameters \( \beta_{Hc} \) have been defined at the minimum of the Binder cumulant and maximum of the susceptibility of \( \rho^2 \). The plot uses MC data taken in runs at \( M_H^* = 70, 74, 76 \) and 80 GeV on cubical lattices.

Figure 1. Thermodynamical limit for \( \Delta \langle \phi^+ \phi \rangle \) for three values \( M_H^* \)

Fig. 2 represents the infinite volume extrapolations of \( \Delta \langle \phi^+ \phi \rangle / g_3^2 \) as function of \( M_H^* \). The expected quadratic finite size scaling behaviour of \( \Delta \langle \phi^+ \phi \rangle / g_3^2 \) is delayed to lattice sizes not less than \( 80^3 \) for Higgs masses larger than 70 GeV. If one includes only the two largest lattices (\( 80^3 \) and \( 96^3 \) for \( M_H^* = 74 \) GeV) into the extrapolation the latent heat vanishes for

\[
\lambda_{3 \text{ crit}}/g_3^2 < 0.107 .
\]  

4. Lee–Yang zero criterion

A phase transition is characterised by non-analytical behaviour of the infinite volume free energy density. This is caused by zeroes of the partition function (extended to complex couplings \( \beta_H \)) approaching the real axis in the thermodynamical limit. At finite volume the zeroes nearest to the real axis cluster along a line. In the case of a first order phase transition the first

\[
\Im \beta_H^{(n)} = \frac{2\pi \beta_{Hc}}{L^3 (1 + 2 \beta_{Rc}) \Delta \langle \rho^2 \rangle} \left( n - \frac{1}{2} \right) \quad (7)
\]

\[
\Re \beta_H^{(n)} \approx \beta_{Hc} . \quad (8)
\]

The partition function for complex couplings is obtained by reweighting from measurements at real couplings. The first zeroes can be well localised (cf. Fig. 3) using the Newton-Raphson algorithm.

We fit the imaginary part of the first zero for each available physical length \( lg_3^2 \) according to (see Fig. 4)

\[
\Im \beta_H^{(1)} = C(l g_3^2)^{-\nu} + R . \quad (9)
\]

The scenario suggested is the change of the first order transition into an analytic crossover above
Figure 4. Logarithm of the imaginary part of first zeroes vs. logarithm of the physical length together with the fit $M^∗$. A positive $R$ signals that the first zero does not approach anymore the real axis in the thermodynamical limit. A similar investigation has been performed recently at smaller gauge coupling in [5].

We attempt a global fit according to (9) shifting the zeros as follows (cf. (7))

$$\ln \text{Im} \beta_H(1) \rightarrow \ln \text{Im} \beta_H(1) - \ln(c_1 c_2) \quad (10)$$

where $c_1 = \beta^2_{Hc}/(\beta^2_G(1 + 2\beta_{Rc}))$ and $c_2$ is an extra free constant (numerically between 1.028 and 1.095). The fitted $R$ crosses zero (Fig. 5) at $\lambda_3^{\text{crit}}/g^2 = 0.102(2) \, . \quad (11)$

5. Conclusions

We have compared two methods promising to give estimates for the critical Higgs mass. The criterion based on the correct thermodynamical limit of Lee–Yang zeroes gives the critical coupling ratio (11). The vanishing of the discontinuity of the quadratic condensate seems to predict a somewhat larger critical coupling ratio. Very near to the critical Higgs mass one needs data at larger volumes as our analysis has shown.

Using relations between $3d$ and $4d$ quantities we obtain from (11) the critical (zero temperature) Higgs mass $m_H$ and the corresponding critical temperature $T_c$ for two versions of the $4d \, SU(2)$–Higgs theory, without fermions and including the top quark as given in Table 1 ($g^2(m_W)$ is the renormalised $4d$ gauge coupling).

| $m_H$/GeV | $T_c$/GeV | $g^2(m_W)$ |
|-----------|-----------|------------|
| 67.0(8)   | 154.8(2.6)| 0.423      |
| 72.4(9)   | 110.0(1.5)| 0.429      |

Table 1

$m_H$ and $T_c$ at $\lambda_3^{\text{crit}}/g^2 = 0.102$; upper row without fermions, lower including top

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