Berezinskii-Kosterlitz-Thouless transitions in two-dimensional lattice SO($N_c$) gauge theories with two scalar flavors

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We study the phase diagram and critical behavior of a two-dimensional lattice SO($N_c$) gauge theory ($N_c \geq 3$) with two scalar flavors, obtained by partially gauging a maximally O($2N_c$) symmetric scalar model. The model is invariant under local SO($N_c$) and global O(2) transformations. We show that, for any $N_c \geq 3$, it undergoes finite-temperature Berezinskii-Kosterlitz-Thouless (BKT) transitions, associated with the global Abelian O(2) symmetry. The transition separates a high-temperature disordered phase from a low-temperature spin-wave phase where correlations decay algebraically (quasi-long range order). The critical properties at the finite-temperature BKT transition and in the low-temperature spin-wave phase are determined by means of a finite-size scaling analysis of Monte Carlo data.

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I. INTRODUCTION

Abelian and non-Abelian gauge symmetries appear in various physical contexts. For instance, they are relevant for the theories of fundamental interactions [1–3] and in the description of some emerging phenomena in condensed matter physics [2,4,5]. The main features of these theories, such as the spectrum, the phase diagram, and the critical behavior at thermal and quantum transitions, crucially depend on the interplay between global and local gauge symmetries.

These issues have been recently investigated in several two-dimensional (2D) lattice gauge models, considering: (i) the multicomponent lattice Abelian-Higgs model [6], characterized by a global SU($N_f$) symmetry ($N_f \geq 2$) and a local U(1) gauge symmetry; (ii) the multiflavor lattice scalar quantum chromodynamics [7], characterized by a global SU($N_f$) symmetry and a local SU($N_c$) gauge symmetry; (iii) lattice SO($N_c$) gauge models with $N_f \geq 3$ real scalar flavors [8], characterized by a non-Abelian O($N_f$) global symmetry. In agreement with the Mermin-Wagner theorem [9], all these 2D lattice gauge models do not have finite-temperature transitions. A critical behavior is only observed in the zero-temperature limit: for $T \to 0$, the correlation length increases exponentially, as in the 2D O($N$) $\sigma$ model with $N \geq 3$ and in the 2D CP$^{N_f-1}$ model with $N_f \geq 2$ [2]. The interplay of global non-Abelian symmetries and local gauge symmetries determines the large-scale properties of the system in the zero-temperature limit, and therefore, the field theory realized in the corresponding continuum limit.

The results for the above-mentioned lattice gauge models support the following general conjecture, originally put forward in Ref. [7]. The universality class of the asymptotic low-temperature behavior of lattice gauge models is the same as that of the 2D field theories defined on the symmetric spaces [2,10] that have the same global symmetry. According to this conjecture, the zero-temperature critical behavior of multiflavor Abelian-Higgs models and lattice scalar chromodynamics with $N_f$ scalar flavors belongs to the universality class of the 2D CP$^{N_f-1}$ model, as both models have the same global SU($N_f$) symmetry. Analogously, lattice SO($N_c$) gauge theories with $N_f \geq 3$ real scalar flavors have the same critical behavior as RP$^{N_f-1}$ models [11] with the same global O($N_f$) symmetry. These predictions have been numerically verified in Refs. [6–8]. We note that all cases considered so far involve systems with global non-Abelian symmetries, which are not expected to show finite-temperature transitions in two dimensions [9].

In this paper, we investigate 2D lattice non-Abelian gauge models that undergo a finite-temperature transition, and show that also in this case the conjecture holds. We consider a 2D lattice SO($N_c$) gauge model with two real scalar flavors, obtained by partially gauging a maximally O($2N_c$) symmetric scalar theory. For $N_c \geq 3$, this model is characterized by a global Abelian O(2) symmetry...
(for \(N_c = 2\) the global symmetry group turns out to be SU(2) \[8\], which is non-Abelian, and therefore we only expect a zero-temperature critical behavior). If the general conjecture extends to systems with global Abelian symmetries, we expect this model to have the same critical behavior as the O(2)-invariant XY lattice model. Therefore, for \(N_c \geq 3\), 2D lattice SO(\(N_c\)) gauge models with two scalar flavors may undergo a finite-temperature Berezinskii-Kosterlitz-Thouless (BKT) transition \[12−20\], between the high-temperature disordered phase and a low-temperature spin-wave phase characterized by quasi-long range order (QLRO) with vanishing magnetization. We recall that BKT transitions are characterized by an exponentially divergent correlation length \(\xi\) at a finite critical temperature. Indeed, \(\xi\) behaves as \(\xi \sim \exp(c/\sqrt{T - T_c})\) approaching the BKT critical temperature \(T_c\) from the high-temperature phase.

To verify the general conjecture for the lattice SO(\(N_c\)) gauge theory with two scalar flavors, we present finite-size scaling (FSS) analyses of Monte Carlo simulations for several \(N_c \geq 3\). We anticipate that the numerical results confirm the presence of a low-temperature QLRO phase, separated by a BKT transition from the high-temperature disordered phase. These results extend the validity of the conjecture to 2D lattice non-Abelian gauge theories with global Abelian symmetries.

The paper is organized as follows. In Sec. II, we define the lattice SO(\(N_c\)) gauge model with scalar fields and the gauge-invariant observables that we consider in our numerical study. We also describe the FSS analysis we use to investigate the phase diagram and to determine the nature of the critical behavior. Section III reports the numerical results for \(N_c = 3\), 4, 5. We show that QLRO holds in the low-temperature phase and that the transition between the high-temperature and the low-temperature QLRO phase is a BKT one, as in the standard XY model. Finally, in Sec. IV, we summarize and draw our conclusions.

II. 2D LATTICE SO(\(N_c\)) GAUGE MODELS

We consider a multiflavor lattice SO(\(N_c\)) gauge model defined on square lattices of linear size \(L\) with periodic boundary conditions. It is obtained \[21\] by partially gauging a maximally O(\(M\)) symmetric model with \(M = N_f N_c\), defined in terms of real unit-length matrix variables \(\phi_{af}\), with \(a = 1, \ldots, N_c\) and \(f = 1, \ldots, N_f\) (we will refer to these two indices as color and flavor indices, respectively), such that \(\text{Tr} \phi_a \phi_c = 1\). Using the Wilson approach \[1\], we introduce gauge fields associated with each link of the lattice. The Hamiltonian reads \[21\]

\[
H = -\sum_{x,\mu} \text{Tr} \phi^\dagger_x V_{x,\mu} \phi_{x+\mu} + \frac{\gamma}{N_c} \sum_x \text{Tr} \Pi_x,
\]

where \(V_{x,\mu} \in \text{SO}(N_c)\) and \(\Pi_x\) is the plaquette operator,

\[
\Pi_x = V_{x,1} V_{x+1,2} V_{x+2,1} V_{x,2}.
\]
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\[ U = \frac{\langle \mu_2^2 \rangle}{\langle \mu_2 \rangle^2}, \quad \mu_2 = \frac{1}{V L} \sum_{x,y} \text{Tr} Q_x Q_y, \]  
\( (7) \)

where \( V = L^2 \) (note that \( \chi = V \langle \mu_2 \rangle \)), and the ratio

\[ R_\xi \equiv \xi/L. \]  
\( (8) \)

In the FSS limit, we have (see, e.g., Ref. [6])

\[ U(\beta, L) \approx F_U(R_\xi), \]  
\( (9) \)

where \( F_U(x) \) is a universal scaling function that completely characterizes the universality class of the transition. In particular, universality is expected at BKT transitions and in the whole low-temperature spin-wave phase, see, e.g., Refs. [16,17,20,26–28].

Because of the universality of relation (9), we use the plots of \( U \) versus \( R_\xi \) to identify the models that have the same universal behavior. If the estimates of \( U \) for two different systems fall onto the same curve when plotted versus \( R_\xi \), the transitions in the two models belong to the same universality class. Therefore, we will compare the FSS curves for the lattice \( \text{SO}(N_f) \) gauge model with the analogous ones for the 2D \( \text{XY} \) model. If the data for the two models have the same scaling behavior, we will conclude that the gauge model undergoes a BKT transition as the \( \text{XY} \) model. The same strategy was employed in Refs. [6–8], to characterize the asymptotic zero-temperature behavior of 2D lattice gauge models with non-Abelian global symmetry group.

### III. NUMERICAL RESULTS

#### A. The conjecture for systems with \( \text{O}(2) \) global symmetry

We wish to verify numerically the general conjecture originally put forward in Ref. [7]. In the present case, it predicts that, for any \( N_c \geq 3 \), the lattice model with Hamiltonian (1) with two flavors undergoes a transition analogous to that of the paradigmatic 2D \( \text{O}(2) \) invariant \( \text{XY} \) model defined by the Hamiltonian

\[ H_{\text{XY}} = -\sum_{x,\mu} \text{Re} \psi_x^* \psi_{x+\hat{\mu}}, \]  
\( (10) \)

where \( \psi_x \) are complex phase variables, \( |\psi_x| = 1 \), associated with each site of the square lattice. This model undergoes a BKT transition at \( \beta_c = 1.1199(1) \) [16,19], with a low-temperature phase that shows QLRO with vanishing magnetization.

The correspondence can be justified using the arguments presented in Ref. [8]. If the conjecture holds, the lattice model (1) with \( N_f \) scalar flavors should be related to the 2D \( \text{RP}^{N_f-1} \) model, defined by the Hamiltonian

\[ H_{\text{RP}} = -\sum_{x,\mu} (\psi_x \cdot \psi_{x+\hat{\mu}})^2. \]  
\( (11) \)

where \( \psi_x \) is a unit-length \( N_f \)-component real field. Indeed, the \( \text{RP}^{N_f-1} \) space is a symmetric space that has the same global \( \text{O}(N_f) \) symmetry. The model has also a local \( \mathbb{Z}_2 \) symmetry, which effectively appears because the order parameter \( Q_x \) is invariant under the local \( \mathbb{Z}_2 \) transformations \( \phi_x \rightarrow s_x \phi_x, \ s_x = \pm 1 \). In the \( \text{RP}^{N_f-1} \) model, the order parameter is

\[ q_{\xi} = q_{\xi}^f = q_{\xi}^g - \frac{1}{N_f} \delta^{fg}, \]  
\( (12) \)

which is the counterpart of \( Q_x^{fg} \) defined in the lattice \( \text{SO}(N_c) \) gauge theory. In the two-flavor case, \( N_f = 2 \), one can easily show that, for the computation of \( \mathbb{Z}_2 \) gauge-invariant quantities, the \( \text{RP}^1 \) model can be mapped onto the \( \text{XY} \) model. Under this mapping, the order parameter \( q_{\xi}^{fg} \) (which has only two independent real components) is mapped onto the complex field \( \psi_x \) of the \( \text{XY} \) model. Therefore, the critical behavior of the correlation function of the operator \( Q_x \), defined in Eq. (5), is expected to correspond to that of the two-point function,

\[ G_{\text{XY}}(x,y) = \langle \psi_x^* \psi_y \rangle, \]  
\( (13) \)

in the \( \text{XY} \) model. Using \( G_{\text{XY}} \), one can then define the correlation length \( \xi \), the Binder parameter \( U \), and the ratio \( R_\xi \), using again Eqs. (6), (7), and (8), respectively.

#### B. Monte Carlo simulations

In the following, we report numerical results for the 2D lattice \( \text{SO}(N_c) \) gauge theories with two scalar flavors, cf. Eq. (1). We consider square lattices of linear size \( L \) with periodic boundary conditions. To update the gauge fields, we use an overrelaxation algorithm implemented à la Cabibbo-Marinari [29], considering the \( \text{SO}(2) \) subgroups of \( \text{SO}(N_c) \). We use a combination of Metropolis updates and microcanonical steps [30] in the ratio 3:7. In the Metropolis update, link variables are randomly generated, and then accepted or rejected by a Metropolis step [31], with an acceptance rate of approximately 30%. For the scalar fields, a combination of Metropolis and microcanonical updates is used, with the Metropolis step tuned to have an acceptance rate of approximately 30%. Errors are estimated using a standard blocking and jackknife procedure, to take into account autocorrelations, which are expected to increase roughly as \( L^2 \). Typical statistics of our runs, for a given value of the parameters and of the size of the lattice, are approximately \( 10^4 \) lattice sweeps (in a sweep we update once all lattice variables). For the larger lattice sizes, the autocorrelation times of the observables considered were of order \( 10^4 \) sweeps at most, even at \( T_c \).
thus obtaining a sufficiently large number of independent measures.

C. The low-temperature spin-wave phase

To gain evidence of the existence of a low-temperature QLRO phase, we show that spin-wave relations hold asymptotically for sufficiently low temperatures. The spin-wave theory is expected to describe the critical behavior of the $XY$ model along the line of fixed points that runs from $T = 0$ up to the BKT point $T_c$. Conformal field theory, see, e.g., Ref. [32], exactly provides the large-$L$ limit of the two-point function in the spin-wave model. In particular, it allows us to compute the universal asymptotic relation between the ratio $R_\xi$ and the exponent $\eta$. Results for square lattices with periodic boundary conditions are reported in Refs. [19,27] (see, in particular, the formulas reported in Appendix B of Ref. [27]). The exponent $\eta$ characterizes the temperature-dependent power-law decay of the two-point function in the QLRO phase,

$$G(x) \sim |x|^{-\eta(T)}.$$  

Alternatively, we can define it by considering the large-$L$ behavior of the susceptibility,

$$\chi(L, T) \sim L^{2-\eta(T)}.$$  

In the QLRO phase, $\eta(T)$ varies from $\eta(T_c) = 1/4$ to $\eta(T \to 0) \to 0$, and $R_\xi$ from $R_\xi(T_c) = 0.750691$... to $R_\xi(T \to 0) \to \infty$.

We recall that, at $T_c$, the RG theory appropriate for the BKT transition predicts the asymptotic large-$L$ behavior [19,20,27],

$$R_\xi(L, T_c) = R_\xi(T_c) + \frac{C_{R_\xi}}{w(L)} + O(w^{-2}),$$

$$w(L) = \ln \frac{L}{\Lambda} + \frac{1}{2} \ln \ln \frac{L}{\Lambda},$$

where $\Lambda$ is a model-dependent constant, and $R_\xi(T_c)$ and $C_{R_\xi}$ are universal. Using the spin-wave theory, one obtains $R_\xi(T_c) = 0.750691$... and $C_{R_\xi} = 0.212431$... Analogous results can be obtained for the Binder parameter $U$ [26], in particular $U(T_c) = 1.018192(6)$.

To study the low-temperature behavior, we have performed simulations for $N_c = 3$ at values of $\beta$ such that $R_\xi > R_\xi(T_c)$, using periodic boundary conditions. We have determined the large-$L$ extrapolations of $R_\xi$ and $\eta$, by fitting the data of $\chi$ and $R_\xi$ at fixed $\beta$ to the Ansätze,

$$\ln \chi(L) = a + (2 - \eta) \ln L + b L^{-\epsilon},$$

$$R_\xi(L) = R_\xi + a L^{-\epsilon},$$

respectively, where $\epsilon$ is the exponent associated with the expected leading corrections [27,33],

$$\epsilon = \text{Min}(2 - \eta, \omega), \quad \omega = 1/\eta - 4 + O[(1/\eta - 4)^2].$$

For $N_c = 3$ and $\gamma = 0$, the quality of the fits can be assessed from the results shown in Fig. 1. Fits to Eqs. (17) and (18) are very good, as it also supported by the values of $\chi^2$/d.o.f ($\chi^2$ is here the sum of the fit residuals and d.o.f is the number of degrees of freedom of the fit), which are smaller than 1, if a few results for the smallest lattice sizes are discarded. For $N_c = 3$ and $\gamma = 0$, we obtain the large-$L$ extrapolations $\eta = 0.195(1)$, $0.1659(8)$, $0.1464(4)$, and $R_\xi = 0.8630(5)$, $0.9439(2)$, $1.0080(2)$, for $\beta = 4.0$, $4.2$, $4.4$, respectively. We have also performed a detailed study for $\gamma = 1$ and $\beta = 4.4$. We obtain $\eta = 0.153(2)$ and $R_\xi = 0.9895(3)$. Note that $\omega$, see Eq. (19), is known precisely.

![Fig. 1. Data of $R_\xi$ (bottom) and ln $\chi$ (top) in the low-temperature spin-wave phase of the model with $N_c = 3$ and $\gamma = 0$, at $\beta = 4.2$ and $\beta = 4.4$. The dashed lines are obtained by fitting the data (results for the smallest lattice sizes have been discarded) to the Ansätze (17) and (18).](image-url)
only for $\eta$ close to $1/4$. In the fits, we use $\omega$ as obtained from Eq. (19), and therefore $\omega$ gives the leading correction-to-scaling exponent for $\eta \gtrsim 0.17$. In such cases, to estimate the error due to the uncertainty on $\omega$, we checked the variation of the results of the fits when varying $\omega$ in around the approximation obtained from Eq. (19), within a reasonable interval of about 10%. This has allowed us to estimate how $\eta$ and $R_\xi$ vary with changes of $\omega$. Such variation has been included in the final error.

In Fig. 2, we plot $R_\xi$ versus $\eta$ together with the universal curve computed in the spin-wave theory. The results for $R_\xi$ and $\eta$ are in excellent agreement with the spin-wave predictions. This shows the existence of a low-temperature phase with QLRO, analogous to that occurring in the XY model.

### D. FSS at the BKT transition

In Sec. III C, we showed that the SO(3) gauge model has a low-temperature phase with the same features of the low-temperature phase of the XY model. Now, we focus on the finite-temperature transition that ends the high-temperature phase, to check whether the FSS behavior is the same as that observed at the BKT transition of the XY model.

To begin with, in Fig. 3, we show the estimates of the correlation length $\xi$ versus $\beta$ for the lattice SO(3) gauge model (1) with $\gamma = 0$, for several values of $L$, up to $L = 128$. When the results for different values of $L$ agree, they can be considered as good approximations of the infinite-volume correlation length, within errors. The vertical lines indicate the interval of values of $\beta$ in which the BKT transition occurs.

We note that the approach to the asymptotic FSS behavior (9) is apparently quite fast in all lattice models considered, including the 2D XY model. In particular, the scaling corrections for the lattice SO($N_c$) gauge models appear to effectively decrease roughly as $L^{-1}$ in the limited range of $L$ that we consider, up to $L = 128$. At BKT transitions, logarithmic corrections are generally expected [16,17,20,26,27]. However, our range of values of $L$ is too small to allow us to detect logarithmic changes of the estimates. In the range we consider, power-law corrections effectively dominate. Significantly larger sizes are needed to allow us to perform fits that include both logarithmic and power-law corrections. Even though our analyses are not sensitive to the slowly decaying logarithmic corrections, we can argue that the systematic error they induce is small (we only refer here to the behavior of $U$ versus $R_\xi$; we are not claiming that logarithmic corrections are always...
Indeed, the coefficients of the logarithmic corrections are not universal, and therefore we expect different logarithmic corrections in the $XY$ model and in the gauge models we consider here. Thus, assuming that all models have a common universal asymptotic behavior, we can infer the size of the logarithmic correction by looking at the differences between the results obtained in the different models. As apparent from Figs. 4 and 5, differences are tiny, indicating that these elusive corrections play little role here.

Accurate estimates of the critical BKT temperatures are hard to obtain, since their determination is generally affected by logarithmic corrections; see Eq. (16). The problem of the logarithmic corrections can be overcome by the so-called matching method put forward in Refs. [16,17,19] (see also Refs. [27,28] for applications to some 2D quantum lattice gas models). Here, we do not pursue this analysis further, since we are not particularly interested in obtaining precise estimates of the critical temperatures. We only mention some rough estimates of the transition temperatures obtained by looking at the $\beta$-values where $R_\xi(\beta, L) \approx R_\xi(T_c) = 0.750691\ldots$. For $N_c = 3$, we find $\beta_c \approx 3.82$ for $\gamma = 0$, $\beta_c \approx 3.77$ for $\gamma = 1$, and $\beta_c \approx 3.92$ for $\gamma = -1$. 

The horizontal and vertical lines indicate the BKT values $U(T_c) = 1.018192(6)$ and $R_\xi(T_c) = 0.750691\ldots$, respectively [19,26].

The FSS curve of the $XY$ model is clearly approached by the data for the lattice $\text{SO}(N_c)$ models with increasing $L$. The horizontal and vertical lines indicate the BKT values $U(T_c) = 1.018192(6)$ and $R_\xi(T_c) = 0.750691\ldots$, respectively [19,26].

FIG. 4. We plot data of $U$ versus $R_\xi$ for $N_c = 3$, $\gamma = 1$ (top), $\gamma = 0$ (middle), and $\gamma = -1$ (bottom). We report analogous data for the 2D $XY$ model (10). We observe a nice agreement, supporting the conjecture that the lattice $\text{SO}(N_c)$ gauge model with two scalar flavors undergoes a finite-temperature BKT transition for generic values of $\gamma$. The horizontal and vertical lines indicate the universal values of $U$ and $R_\xi$ at the BKT transition, i.e., $U(T_c) = 1.018192(6)$ and $R_\xi(T_c) = 0.750691\ldots$, respectively [19,26].

FIG. 5. Plot of $U$ versus $R_\xi$ for $N_c = 4$ (bottom) and $N_c = 5$ (top), at $\gamma = 0$. We also report data for the 2D $XY$ model (10).
for \( \gamma = -1 \). Moreover, we estimate \( \beta_c \approx 4.80 \) for \( N_c = 4 \) and \( \beta_c \approx 5.76 \) for \( N_c = 5 \), at \( \gamma = 0 \).

In conclusion, the FSS analysis has allowed us to determine the nature of the finite-temperature transitions occurring in the lattice \( \text{SO}(N_c) \) gauge model (1) with two flavors. For \( N_c = 3, 4, 5 \), we find that the transition belongs to the BKT universality class, as in the classical \( XY \) model. This occurs at least in an interval of values of \( \gamma \) around the infinite gauge-coupling value \( \gamma = 0 \).

**IV. CONCLUSIONS**

We have studied a class of 2D lattice non-Abelian \( \text{SO}(N_c) \) gauge models with two real scalar fields, defined by the Hamiltonian (1). Such lattice gauge models are obtained by partially gauging a maximally \( O(2N_c) \)-symmetric multicomponent real scalar model, using the Wilson lattice approach. For \( N_c \geq 3 \), the resulting theory is locally invariant under \( \text{SO}(N_c) \) gauge transformations and globally invariant under Abelian \( \text{O}(2) \) transformations. This study extends previous work on 2D models with a local gauge invariance and a global non-Abelian symmetry [6–8], in which a critical behavior can only be observed in the zero-temperature limit. In the models considered here, instead, the global Abelian \( \text{O}(2) \) symmetry may allow finite-temperature BKT transitions between the disordered phase and the low-temperature QLRO phase.

The universal features of the transitions have been determined by performing FSS analyses of Monte Carlo data. We present results for the two-flavor lattice \( \text{SO}(N_c) \) gauge models (1) with \( N_c = 3, 4, 5 \). They show that these systems undergo a finite-temperature BKT transition that separates the disordered phase from the low-temperature phase. Moreover, we have verified that the low-temperature phase is characterized by spin waves, analogously to the standard \( XY \) model.

These results provide additional evidence in favor of the conjecture that the critical behavior of 2D lattice gauge models, defined using the Wilson approach [1], belongs to the universality class of the field theories associated with the symmetric spaces that have the same global symmetry. This conjecture assumes that gauge correlations are not critical and decouple in the critical limit. Therefore, the conjecture may fail when the gauge correlations are critical, giving rise to a more complex behavior. A similar phenomenon has been observed in the three-dimensional lattice Abelian-Higgs model with noncompact gauge fields; see, e.g., Ref. [34] and references therein.

We finally mention that the interplay between global and gauge symmetries has also been studied in three-dimensional models; see Refs. [21,34–36].

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