Study of Intelligent Reflective Surface Assisted Communications with One-bit Phase Adjustments

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Abstract—We analyse the performance of a communication link assisted by an intelligent reflective surface (IRS) positioned in the far field of both the source and the destination. A direct link between the transmitting and receiving devices is assumed to exist. Perfect and imperfect phase adjustments at the IRS are considered. For the perfect phase configuration, we derive an approximate expression for the outage probability in closed form. For the imperfect phase configuration, we assume that each element of the IRS has a one-bit phase shifter (0°, 180°) and an expression for the outage probability is obtained in the form of an integral. Our formulation admits an exact asymptotic (high SNR) analysis, from which we obtain the diversity orders for systems with and without phase errors. We show these are $N + 1$ and $\frac{1}{2}(N + 3)$, respectively. Numerical results confirm the theoretical analysis and verify that the reported results are more accurate than methods based on the central limit theorem (CLT).

Index Terms—Intelligent reflective surface, diversity order, phase error, performance analysis.

I. INTRODUCTION

Recently, the idea of using an intelligent reflective surface (IRS) located between the transmitting and receiving devices to create a “tunable” propagation environment has been proposed as a future enhancement to the physical layer of wireless systems [1]–[3]. An IRS consists of many reflective elements made of electromagnetic (EM) material, which can reconfigure the incident waves for different purposes, such as focusing (reflecting) the waves towards a prescribed target [4] or enhancing security [5]. Compared with other related technologies, such as relaying, IRS-assisted communication offers energy-efficiency, convenient deployment, and a full-band response [1].

Different implementations of IRSs – including smart reflect-arrays, software-defined hypersurfaces, and frequency-selective surfaces – have been investigated [2], [3], [6]. It has been shown that an IRS can improve communication performance between the source and the destination if the phase shifts are configured properly [7]. In [4], a beamforming technique was proposed for a multiple-input, single-output (MISO) system in the presence an IRS to maximize the power of the received signals. The authors in [8] discussed beamforming optimization for an IRS-assisted system when the elements of the IRS have discrete phase shifts. In [9] and [10], the performance of an IRS-aided communication link was compared with amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying, respectively. In [11], the authors derived the performance bounds of an IRS system without the presence of a direct link by using a central limit theorem (CLT) approximation. As a further step, phase errors were taken into consideration for an IRS system without the direct link in [12]. By using the CLT, the authors of that work showed that the channel distribution is Nakagami. It is known that the CLT is inaccurate when the number of elements in the IRS is small; however, it also turns out that the approximation error attributed to the CLT can be significant in the high SNR regime. The source of this inaccuracy results from the fact that the CLT is used to approximate a positive real random variable as a Gaussian, which has infinite support. To circumvent the CLT issue, a gamma distribution was used to model the channel fading of each reflecting path in [13]. This approach leads to more accurate results; however, most recent work that uses the gamma model ignores the direct link and the possibility of phase adjustment errors at the IRS.

In this paper, we shed further light on the performance of IRS-aided systems by analyzing the outage probability and calculating the diversity order of such systems when (1) a direct link is present and (2) the IRS performs a binary phase adjustment at each element. We also treat the benchmark case where perfect phase adjustments are made. Further, we do not employ a CLT approach.

Notation: The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable $X$ are denoted by $f_X(\cdot)$ and $F_X(\cdot)$. $\Gamma(a,x) = \int_0^\infty e^{-t} t^a e^{-x} \mathrm{d}t$ and $\Gamma(a,x) = \int_x^\infty e^{-t} t^a e^{-x} \mathrm{d}t$ represent the gamma function and the upper incomplete Gamma function. $K_n(\cdot)$ denotes the modified Bessel function of the second kind with order $n$, and $\lfloor x \rfloor$ is the integer closest to $x$. $P(\cdot)$ is the probability operator. $D(\cdot|\cdot)$ and $I(\cdot;\cdot)$ denote the relative entropy and mutual information functions.

II. SYSTEM MODEL

The system model considered in this paper includes a source (S) node, a destination (D) node and an IRS with $N$ reflecting elements operating in the far field of both S and D. Single, half-duplex antennas operate at the source and destination nodes. The reflecting elements, labelled $R_n, n \in \{1, ..., N\}$, can scatter the incident waves independently.

We consider the case where a direct link exists from S to D. It is assumed that the path losses of the S-to-R$_n$ channels
are the same for all $n$ and the path losses of the $R_n$-to-
D channels are also equal for all $n$. Denote the complex
attenuation coefficients corresponding the S-to-$R_n$, $R_n$-to-D,
and S-to-D channels by $h_{1n}^s$, $h_{2n}$, and $h_{sd}$, respectively. Under
this model, the received signal can be written as
\[
y' = \left( \sum_{n=1}^{N} \sqrt{\xi_n} h_{1n} e^{j\theta_n} \sqrt{\xi_n} h_{2n} + \sqrt{\xi_n} h_{sd}' \right) \sqrt{P} x + w'
\]
where $h_{1n}^s$, $h_{2n}$, and $h_{sd}'$ are circularly symmetric, complex
Gaussian random variables, each with zero mean and unit variance; $\xi_1$, $\xi_2$, and $\xi_d$ denote the path losses of the S-to-$R_n$,
$R_n$-to-D, and S-to-D channels, respectively; $\theta_n$ is the phase shift
induced by the $n$th reflecting element at the IRS; $P$ is the transmit power at $S$; $x$ is the information symbol with
unit power, i.e., $E[|x|^2] = 1$; and $w' \sim \mathcal{CN}(0, \sigma_w^2)$ represents
additive white Gaussian noise (AWGN) at the destination.

Dividing the left-hand and right-hand sides of the equation
above by $\sqrt{\xi_n}$, we have
\[
y = \left( \sum_{n=1}^{N} h_{1n} e^{j\theta_n} h_{2n} + h_{sd} \right) \sqrt{P} x + w
\]
where $h_{1n} \sim \mathcal{CN}(0, 2)$; $h_{2n} \sim \mathcal{CN}(0, 2)$; $h_{sd} \sim \mathcal{CN}(0, \frac{4 \sigma_d^2}{\xi_d})$;
and $w \sim \mathcal{CN}(0, \sigma_w^2)$. It follows that $|h_{1n}|$, $|h_{2n}|$, and $|h_{sd}|$ are
independent, Rayleigh distributed random variables. Furthermore,
$|h_{1n}|$ and $|h_{2n}|$ have unit scale parameters. We write
the scale parameter of $|h_{sd}|$ as $\sigma_d = \frac{2 \sigma_d}{\sqrt{\xi_d}}$. It should be
clear that it is assumed that all the channels are flat and slow
fading and mutually independent. For clarity, in the rest of
the paper, we will use (2) to analyze the performance of the
system.

A. Perfect Phase Alignment

Ideally, the phase shift of each reflecting element should
satisfy $\theta_n = \arg(h_{sd}) - \arg(h_{1n}) - \arg(h_{2n})$ to maximize
the received SNR [10]. In this case, the received signal at D is
\[
y = H e^{j\arg(h_{sd})} \sqrt{P} x + w
\]
where
\[
H = \sum_{n=1}^{N} |h_{1n}| |h_{2n}| + |h_{sd}| = S + R.
\]
Thus, the received SNR can be written as
\[
\gamma_t = \frac{P}{\sigma_w^2} H^2 = H^2 \gamma_t
\]
where $\gamma_t = P/\sigma_w^2$ is the transmit SNR.

B. One-bit Phase Adjustment

Although aligning the phases of the reflected paths to the
phase of the direct link can optimize the system performance,
the phases of the reflecting elements cannot be set precisely
in reality due to imperfect channel knowledge and the finite
precision inherent in the phase alignment operation. Here,
we focus on the latter issue, which manifests in phase
quantization at each element in the IRS. Mathematically, the
resulting phase adjustment at the $n$th element can be written
as $\theta_n = \arg(h_{sd}) - \arg(h_{1n}) - \arg(h_{2n}) + \phi_n$, where $\phi_n$
denotes the phase error introduced through quantization at the
$n$th element. We assume that each element of the IRS is a one-
bit phase shifter, either leaving the phase of the incident wave
unaltered or shifting it by $180^\circ$. It follows that the phase errors
$\phi_n$, $n \in \{1, ..., N\}$ are mutually independent and uniformly
distributed on the interval $[-\pi/2, \pi/2]$ [8 Prop. 1].

In this case, the composite channel, which we label $G$
instead of $H$ for notational clarity, can be written as
\[
G = \sum_{n=1}^{N} |h_{1n}| |h_{2n}| e^{j\phi_n} + |h_{sd}|
\]
and the corresponding received SNR for one-bit phase adjustment
can be written as
\[
\gamma_2 = \frac{P}{\sigma_w^2} |G|^2 = |G|^2 \gamma_t.
\]

III. OUTAGE PROBABILITY

The outage probability is equivalent to the received SNR
distribution, i.e., the probability that the received SNR falls
below a threshold $\gamma_{th}$:
\[
P_{out}(\gamma_{th}) = P(\gamma_t < \gamma_{th}) = F_{\gamma_t}(\gamma_{th})
\]
where $i \in \{1, 2\}$ denotes the perfect and one-bit phase
adjustment scenarios, respectively. The calculation of $P_{out}^{(i)}$
amounts to computing the CDF of the channel gain associated
with $\gamma_i$. For example, for the case of perfect phase alignment,
we have
\[
P_{out}^{(1)}(\gamma_{th}) = F_H \left( \sqrt{\frac{\gamma_{th}}{\gamma_t}} \right).
\]

Hence, in what follows, we treat the calculation of the channel
gain random variables for the cases of perfect and imperfect phase
adjustment.

A. Perfect Phase Alignment

Let $H_n = |h_{1n}| |h_{2n}|$. $H_n$ follows a double Rayleigh
distribution and the PDF of $H_n$ is given by [14]
\[
f_{H_n}(x) = x K_0(x).
\]
Unfortunately, continuing with this form of the calculation
will prove fruitless; hence, in accordance with [13], we adopt
a good approximation for the PDF of $H_n$ by using the gamma
distribution:
\[
f_{H_n}(x) \approx \frac{x^{k-1} e^{-\frac{x}{k}}} {\theta^k \Gamma(k)}
\]
where $k = \frac{\sigma_d^2}{16 \pi^2}$ and $\theta = \frac{10^{-3} \cdot 1.2}{\sigma_d}$. Since $H_n$
for $n \in \{1, ..., N\}$ are independent and identically distributed (i.i.d.),
we can write the PDF of $S = \sum_{n=1}^{N} h_{1n} |h_{2n}|$ as
\[
f_s(x) \approx \frac{x^{Nk-1} e^{-\frac{x}{Nk}}} {\theta^{Nk} \Gamma(Nk)}.
\]
Using this gamma-based framework, we can obtain the fol-

lowing approximation for the CDF of the composite chan-

nel $H$. 


Proposition 1 (CDF of $H$). The CDF of $H$ can be approximated as

$$F_H(t) \approx 1 - \frac{\Gamma([Nk], \frac{t}{\theta})}{\Gamma([Nk])} - A(t) \sum_{i=0}^{\lfloor Nk \rfloor - 1} \binom{\lfloor Nk \rfloor - 1}{i} B_i m(t)^{\lfloor Nk \rfloor - 1 - i}$$

where

$$A(t) = \frac{\exp\left(\frac{\sigma_d^2}{2\theta^2} - \frac{t}{\theta}\right)}{\Gamma([Nk]) \theta^{Nk}}$$

$$B_i = 2^{\frac{1}{2}i+1} \frac{\sigma_d}{\theta} i^{i+1} \left(\Gamma\left(\frac{i+1}{2}, \frac{m^2}{2\sigma_d^2}\right) - \Gamma\left(\frac{i+1}{2}, \frac{\sigma_d^2}{2\theta^2}\right)\right)$$

$$m(t) = t - \frac{\sigma_d^2}{\theta} \text{ and } \sigma_d = \sqrt{\frac{2\sigma_d}{\xi_1\xi_2}}.$$

Proof: See the appendix.

The approximation in Proposition 1 arises from two aspects: (1) the gamma distribution is used to approximate the double Rayleigh distribution, and (2) to obtain a simple, tractable expression, we induce the rounding operation and the use of the gamma distribution is quite high.

We can use (10) and (16) to obtain the PDF of $X$:

$$f_X(z) = \int_{-\infty}^{\infty} \frac{1}{z} f_{\cos(\phi_n)}(x) f_{\cos(\phi_n)}(\frac{z}{\theta}) \, dx = \exp(-z), \quad z \geq 0.$$  

Similarly, the PDF of $Y_n$ can be calculated to be

$$f_{Y_n}(y) = \frac{1}{2} \exp(-|y|), \quad y \in \mathbb{R}. \quad (18)$$

Since $X_n$ and $Y_n$ are i.i.d. for all $n$, the PDF of $X$ and $Y$ can be calculated to be [16 ch. 7] [17 ch. 2]

$$f_X(x) = \frac{\exp(-x) x^{N-1}}{(N-1)!}, \quad x \geq 0 \quad (19)$$

and

$$f_Y(y) = \frac{\exp(-|y|)}{2^N (N-1)!} \sum_{m=0}^{N-1} \frac{(N - 1 + m)! |y|^{N-1-m}}{2^m m! (N - 1 - m)!}, \quad y \in \mathbb{R}. \quad (20)$$
We are now in a position to state an approximation for the CDF of $|G|^2$; the outage probability follows from the relation
\[ P_{out}^{(2)} = F_{|G|^2} \left( \frac{2\ln}{{\gamma_t}} \right). \] (21)

**Proposition 2** (CDF of $|G|^2$). The CDF of $|G|^2$ satisfies the approximation
\[ F_{|G|^2}(t) \approx \int_0^t f_{\gamma^2}(y) F_X \left( \sqrt{t-y} \right) dy - \sum_{i=0}^{N-1} \frac{B_i}{(N-1-i)!} \]
\[ \times \int_0^t A(t-y)m(t-y)^{N-1-i} f_{\gamma^2}(y) dy \] (22)
where
\[ A(u) = \exp \left( \frac{\sigma_d^2 - 2\sqrt{u}}{2} \right) \]
\[ m(u) = \sqrt{u - \sigma_d^2} \]
\[ B_i = 2^{-i} \sigma_d^{i+1} \left( \Gamma \left( \frac{i+1}{2}, \frac{m^2}{2\sigma_d^2} \right) - \Gamma \left( \frac{i+1}{2}, \frac{\sigma_d^2}{2} \right) \right) \]
\[ F_X(u) = 1 - \sum_{n=0}^{N-1} \frac{u^n}{n!} e^{-u} \]
and
\[ f_{\gamma^2}(y) = \frac{1}{\sqrt{y}} f_Y(\sqrt{y}). \]

**Proof:** $F_{|G|^2}(t)$ can be written as
\[ F_{|G|^2}(t) \approx \int_0^t f_{\gamma^2}(y) F_X \left( \sqrt{t-y} \right) dy. \] (23)
Proposition 2 can be concluded by following a similar procedure as outlined in the proof of Proposition 1.

Note that the approximation in Proposition 2 does not arise from the use of a moment-matched gamma distribution, but rather from the assumption that $X$ and $Y$ are independent. It can be proved that $X_n$ and $Y_n$ are uncorrelated. Indeed, when the PDF of $\phi_n$ is even, we have
\[ E[\cos \varphi_n \sin \varphi_n] = \frac{1}{2} E[\sin 2\varphi_n] = 0 = E[\cos \varphi_n]E[\sin \varphi_n]. \]
(24)

IV. DIVERSITY ORDER

The diversity order of the system is defined as
\[ d_i = \lim_{\gamma_t \to \infty} -\frac{\log P_{out}^{(i)}}{\log \gamma_t}. \] (27)
From the analysis presented in the previous section, it is clear that the diversity order depends on the small-argument behavior of $F_H$ and $F_{|G|^2}$ for the systems studied herein. Below, we conduct an asymptotic analysis of the CDFs, which leads to diversity order expressions for the cases of perfect phase alignment and one-bit phase adjustments.

A. Perfect Phase Alignment

In the previous section, we invoked a gamma approximation to analyze the outage probability of the IRS-aided system under the assumption of perfect phase alignment. Here, we refrain from using this approximation. The asymptotic analysis that follows is exact.

We analyze $F_H(t)$ when $t$ approaches zero. The PDF of $H_n$ can be expanded about zero to yield $f_{H_n}(x) = -x \ln x + O(x)$. Performing an $N$-fold convolution and taking the leading term every time, we obtain the leading order of the PDF of $S$:
\[ f_s(x) = \frac{x^{2N-1}}{(2N-1)!} \left( \ln \frac{1}{x} \right)^N a \left( \frac{x^{2N-1}(\ln x)^N}{N} \right). \] (28)
Eq. (28) can be proved by induction. Similarly, the PDF of $R = |h_{cd}|^2$ can be written to leading order as $f_R(x) \approx x/\sigma_d^2$. Hence, the CDF of $H$, to leading order, is the integral of the convolution of $f_R$ and $f_s$:
\[ F_H(t) = \int_0^t \int_0^t f_s(x)f_R(r-s) dx ds \]
\[ = \frac{1}{4\sigma_d^2 N(N+1)(2N+1)(2N-1)!} \epsilon(t) \] (29)
where $\epsilon(t) = o(\epsilon^2(N+1)(\ln t)^N)$. Based on the definition of the outage probability in (28) and the definition of diversity order in (27), we find that the diversity order under perfect phase alignment is
\[ d_1 = N + 1. \]
(30)
This is entirely expected, since the assumption of the model we consider is that all $N+1$ spatial channels are independent.

B. One-bit Phase Adjustment

In a similar manner as was done for the case of perfect phase alignment, we start by analyzing $F_{|G|^2}(t)$ when $t$ approaches zero. The PDF of $X + R$ can be obtained by computing the convolution of the leading orders of the PDFs of $X$ and $R$. Note that in this case the PDF of $X$ has the reflecting path, it is worth noting that the dependence between $X = \sum_n X_n$ and $Y = \sum_n Y_n$ will decrease as the number of elements grows. Indeed, as the number of terms in each sum increases, central limit effects take hold and $X$ and $Y$ become approximately independent since the two random variables are (nearly) Gaussian and uncorrelated.
simple (exact) form given by [19]. Computing the convolution, retaining the leading order, and performing a transformation of variables, we arrive at the PDF of \((X + R)^2\), which is

\[
 f_{(X+R)^2}(x) = \frac{x^N}{2\pi^3(N+1)!} + o(x^{N/2}).
\]  

Similarly, we can write the leading order term of the PDF of \(Y^2\) in the succinct form

\[
 f_{Y^2}(y) = \frac{\Gamma(N - \frac{3}{2})}{2\Gamma(N)\sqrt{\pi}y^{N/2}} + O(1). \tag{32}
\]

Invoking the assumption that the real and imaginary parts of the channel gain are independent, we arrive at the following approximation for the leading order of the CDF of \(|G|^2\):

\[
 F_{|G|^2}(t) \approx \int_0^t \int_0^{t-y} f_{Y^2}(y) f_{(X+R)^2}(x) dx dy \approx \frac{t^{N+3}}{2\pi^3 \Gamma(N) \Gamma(N+3)} \frac{\Gamma(N - \frac{3}{2})}{\sqrt{\pi}y^{N/2}}. \tag{33}
\]

The inner integral above can be evaluated directly, and the outer integral results from [18, eq. 3.191.1].

From (21) and (27), we have that

\[
 d_2 = \frac{N + 3}{2}. \tag{34}
\]

This is a curious result, which requires further investigation. It is unclear exactly how the phase quantization leads to this asymptotic behavior. One might suspect that the independence assumption yields an erroneous result. We argue in the next section, by using numerical simulations, that this is not the case. It would be interesting to develop an exact asymptotic expression for the distribution of \(|G|^2\) where independence between the real and imaginary parts is not assumed. We refrain from doing so here, however, favoring a thorough numerical investigation instead.

V. NUMERICAL RESULTS

Denoting the distances of the S-to-R, \(n\), R-to-D, and S-to-D links as \(d_{SR}, d_{RD}\) and \(d_{SD}\), respectively, the path losses of the channels are calculated by using the 3GPP Urban Micro NLOS model with a carrier frequency of 5 GHz [19], which states that \(\xi = -40.9 - 36.7\log_{10}(d)\) dB, where \(d\) is the distance. In simulations, we set \(\gamma_{th} = 0\) dB.

The outage probability is plotted against the transmit SNR \(\gamma_t\) when there are no phase errors in Fig. 2. In particular, the outage probability is compared with the CLT method and Monte-Carlo (MC) simulations. We observe that the outage probability of the proposed approximation is more accurate than that based on the CLT method. Fig. 2 also illustrates that with an increasing number of reflecting elements, the outage probability decreases significantly for the same transmit SNR.

For example, when \(d_{SD} = 50\) m and \(\gamma_t = -25\) dB, the outage probability is 0.69 for \(N = 8\) and \(10^{-2}\) for \(N = 16\). To ascertain the influence of the direct link, we also present results for when the IRS is placed such that \(d_{SD} = 10\) m. Since \(d_{SD} \ll d_{SR} + d_{RD}\), this placement leads to a strong direct link. In this case, the diversity order emerges slowly as the transmit SNR increases, as one would expect. At high SNR, the curve becomes parallel to that corresponding to \(d_{SD} = 50\) m; however, a coding gain is observed in the SNR shift to the left, which is a result of the stronger direct link.

Fig. 3 illustrates the outage probability under the condition of one-bit phase adjustment. Similar to the case without phase errors, our analysis is in good agreement with the simulations. Comparing the results in Figs. 2 and 3, it can be seen that phase errors lead to a performance loss with the gap being about 5 dB. For example, when \(N = 16\), the transmit SNRs required to achieve an outage probability of \(10^{-2}\) are \(-25\) dB and \(-20\) dB for perfect and imperfect phase alignment, respectively. The asymptotic outage probability is plotted for \(N = 2\) in this example as well. This curve is included to illustrate that the approximate asymptotic analysis resulting from the independence assumption is reasonably accurate. Hence, we can be fairly confident that the diversity order is indeed \((N + 3)/2\) in this case.
VI. CONCLUSIONS

In this paper, the performance of IRS-assisted communication systems was analyzed. It was shown that the existence of a direct link slows the emergence of the diversity slope, and that one can expect a penalty of approximately 5 dB when a one-bit phase adjustment is made at each reflective element. We also demonstrated the importance of not using a CLT approximation to study high-SNR behavior. Through a more accurate asymptotic analysis (though still inexact), we conjectured that the diversity order of a one-bit phase adjustment system is $(N + 3)/2$. This result requires further investigation, and an exact understanding of the performance of systems that employ a $b$-bit phase adjustment at each element constitutes an important question for further study.

We plan to address this problem in future work.

APPENDIX

We begin with the following two lemmas.

Lemma 1. Let $Q$ be a non-negative random variable, and let $R$ be Rayleigh distributed with parameter $\sigma$ such that $Q$ and $R$ are independent. Let $Z = Q + R$. Then the distribution function of $Z$ satisfies the equation

$$F_Z(z) = F_Q(z) - \int_0^z f_Q(q) \exp\left(-\frac{(z-q)^2}{2\sigma^2}\right) dq.$$  \hspace{1cm} (35)

Proof: The proof follows by noting that

$$F_Z(z) = \int_0^z f_Q(q) \mathbb{P}(R^2 < (z-q)^2) dq$$ \hspace{1cm} (36)

and that $R^2$ is exponentially distributed with mean $2\sigma^2$. \hfill \Box

Lemma 2. Let $I$ be a positive integer, and let $a$ and $b$ be two real numbers. Then the following relation is true:

$$\int_0^t x^I \exp(-ax) \exp\left(-b(t-x)^2\right) dx = \frac{1}{2} \exp\left(\frac{a^2 - 4abt}{4b}\right) \times \sum_{i=0}^I \binom{I}{i} m^{I-i} \left(\frac{b^{i+1}}{2}\right) E_i.$$ \hspace{1cm} (37)

where $m = (2bt - a)/(2b)$ and

$$E_i = \Gamma\left(\frac{i+1}{2}, bn\right) - \Gamma\left(\frac{i+1}{2}, b(t-m)^2\right).$$ \hspace{1cm} (38)

Proof: The proof follows by noting that

$$\int_0^t x^I \exp(-ax) \exp\left(-b(t-x)^2\right) dx = \exp\left(\frac{a^2 - 4abt}{4b}\right) \times \int_{-m}^{t-m} (n+m)^I \exp\left(-bn^2\right) dn,$$ \hspace{1cm} (39)

where $n = x - m$, and the integral

$$\int n^I \exp\left(-bn^2\right) dn = -\frac{1}{2} b^{\frac{-i+1}{2}} \Gamma\left(\frac{i+1}{2}, bn^2\right).$$ \hspace{1cm} (40)

Since $H = S + R$ in Proposition 1, the CDF of $H$ satisfies (by Lemma 1)

$$F_H(t) = F_S(t) - \int_0^t f_S(r) \exp\left(-\frac{(t-r)^2}{2\sigma^2}\right) dr.$$ \hspace{1cm} (41)

The integral in (41) can be calculated with Lemma 2.

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