Primordial Anisotropies in the Gravitational Wave Background from Cosmological Phase Transitions

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Phase transitions in the early universe can readily create an observable stochastic gravitational wave background. We show that such a background necessarily contains anisotropies analogous to those of the cosmic microwave background (CMB) of photons, and that these too may be within reach of proposed gravitational wave detectors. Correlations within the gravitational wave anisotropies and their cross-correlations with the CMB can provide new insights into the mechanism underlying primordial fluctuations, such as multi-field inflation, as well as reveal the existence of non-standard “hidden sectors” of particle physics in earlier eras.

With the recent observation of gravitational waves (GW) [1, 2] we have entered a new era of astronomy, which will illuminate the most mysterious objects in the sky such as black holes and neutron stars. Remarkably, with future improvements in GW detectors we will also enter a new era of observational cosmology. The early universe can readily contain a variety of GW sources - inflationary fluctuations [3], cosmological particle production [4, 5], (p)reheating [6], phase transitions [7–15], and cosmic strings [16, 17].

The case for phase transitions (PT) is particularly well motivated by theoretical and observational considerations. Many extensions of the Standard Model (SM) of particle physics, whether ambitious paradigms such as supersymmetry or particle compositeness, or relatively modest extensions (such as the addition of just one gauge singlet scalar field), can undergo strong first order PT at high temperatures of order the electroweak scale or higher, \( T \gtrsim \) TeV. The PT proceed through the nucleation and dynamics of large bubbles of the low temperature phase, acting as a strong source of long wavelength GW. (See Ref. [13] for a review.) This GW background is analogous to the cosmic microwave background (CMB) of 3K photons. Just as CMB radiation is emitted from the surface of last scattering, the GW are effectively emitted from a distant surface at the periphery of our universe at the time of the PT. These GW then travel to us undergoing a significant cosmological redshift en route, thereby obtaining frequencies, \( \omega_{GW} \sim \text{mHz} - \text{Hz} \), and signal strength that can be detected at proposed GW observatories such as LISA [18], BBO [19], MAGIS [20], DECIGO [21], ALIA [22]. While the fates of individual bubbles formed during the PT are essentially random, the resulting GW would be seen today as a diffuse background arriving from all directions, coarse-grained over an extremely large number of bubbles. The detailed frequency spectrum of this stochastic GW background would reflect the physics of the PT, during a cosmological era otherwise difficult to access. Furthermore, particle collider experiments, such as the CERN LHC or beyond, may provide a complementary view of the associated particle physics (for example, see Ref. [23]).

Stochastic GW generated by cosmological PT will appear as an approximately isotropic background with average energy density \( \bar{\rho}_{GW} \). In this paper we will argue that there necessarily are also anisotropies in this background, \( \rho_{GW} = \bar{\rho}_{GW} + \delta \rho_{GW} \), again analogous to the CMB, providing a unique window onto the physics generating primordial inhomogeneities, plausibly during an inflationary era well before the PT itself. (See Ref. [24] for a review of inflation.) We will show that such anisotropies may be accessible with sensitive directional detection at the proposed GW observatories. In this way while the GW frequency spectrum can teach us about the physics of the PTs at multi-TeV scales, the anisotropies can teach us about physics at vastly higher energy scales.\(^2\)

The key observable is given by the differential energy density of GW arriving from an infinitesimal solid angle of the sky:

\[
d\rho_{GW} = \rho_{GW}(\theta_2, \phi) \sin \theta d\theta d\phi.
\]

From this we can compute two-point correlations,

\[
C_{GW}(\theta) = \frac{\langle \rho_{GW}(1) \rho_{GW}(2) \rangle_\theta}{\bar{\rho}_{GW}^2},
\]

where we are averaging over all pairs of points on the sphere, \( 1, 2 \), separated by a fixed angle \( \theta \). It is standard

\(^1\) Even higher temperature PT are possible, say related to grand unification, but with GW signals outside the reach of proposed detectors [3].

\(^2\) Famously, anisotropic gravitational waves can be produced during high-scale inflation, expanded to very large “super-horizon” wavelengths and imprinted on the (polarized) CMB [25, 26] or re-enter the horizon directly as a very weak GW background [27]. This is quite distinct from the physics pursued here.
to expand anisotropies in Legendre polynomials,

$$C^{GW}(\theta) = \frac{1}{4\pi} \sum_l (2l + 1) C^l_{GW} P_l(\cos \theta).$$  \(3\)

As we will show, the GW background and the CMB can share the same primordial source of fluctuations or have quite different origins, and this will be visible in the cross-correlations with the CMB,

$$C^{\text{cross}}(\theta) \equiv \frac{\langle \rho_{GW}(1) \rho_{CMB}(2) \rangle_{\theta}}{\rho_{GW} \rho_{CMB}}.$$  \(4\)

Higher-point GW correlators may also be observable.

Anisotropic GW backgrounds have been considered earlier. Refs. \[28, 29\] have proposed anisotropic signals due to astrophysical sources, such as white dwarf mergers. Such anisotropies reflect both the inhomogeneous distribution of sources as well as the gravitational lensing of the GW by (dark) matter as they propagate to us. This would yield an independent measure of the matter power spectrum. Refs. \[30, 31\] have studied inflationary preheating as a source of very high frequency GW $\omega_{GW} \sim \text{MHz-GHz}$, although this is currently challenging for GW detection. Refs. \[28, 32, 33\] developed analytic frameworks for characterizing the anisotropies in GW detector data. Ref. \[29\] applied their formalism to the case of astrophysical mergers, while Ref. \[33\] generalized the framework to include cosmic string networks as the GW source.

The present paper makes four main points. (i) First-order PTs in multi-TeV extensions of the SM are a robust and plausible source of anisotropic GW. (ii) The anisotropies are almost completely primordial in nature, directly reflecting the era of inflation. (iii) The GW anisotropies can exhibit a variety of behaviors, including $(\delta \rho/\rho)_{GW} \gg (\delta \rho/\rho)_{CMB}$ and/or varying degrees of cross-correlation with the CMB, sensitive to the nature of inflation and reheating. (iv) Current cosmological data constrain the GW background, but nevertheless there is considerable scope for detecting a range of possible anisotropies at the next generation of detectors, including LISA.

Let us briefly sketch the origin of the GW signal in PT. The GW appear as a perturbation of the spacetime metric $\delta g_{GW}$. Einstein’s equations, linearized for this perturbation at the time of PT are schematically of the form

$$\omega^2_{GW} \delta g_{GW} \sim G_N \rho_{PT},$$  \(5\)

where $G_N$ is Newton’s constant, $\rho_{PT} \leq \rho_{\text{total}}$ is the energy density in the sector undergoing the PT, and $\omega_{GW}$ is the typical frequency of the GW,

$$\omega_{GW} \sim \frac{1}{\Delta t_{PT}} \sim \left( \frac{\Gamma}{\Gamma} \right)_{\text{PT}}.$$  \(6\)

Here $\Delta t_{PT}$ is the duration of the PT and $\Gamma$ is the bubble nucleation rate. In general, the field-energy in GW is given by

$$\rho_{GW} \sim \frac{1}{G_N} \omega^2_{GW} (\delta g_{GW})^2.$$  \(7\)

Therefore at the PT,

$$\rho_{GW} \sim \frac{\rho_{PT}^2}{\rho_{\text{total}}^2} (H_{PT} \Delta t_{PT})^2,$$  \(8\)

where $H_{PT} \sim \sqrt{G_N \rho_{\text{total}}}$ is the Hubble expansion rate at the time of the PT. The combination $(H_{PT} \Delta t_{PT})^2$ depends only on the microphysics of the PT, independent of details of gravity and cosmological history, varying in the literature between $10^{-4} - 10^{-1}$ for strongly first-order PT \[13\]. The smaller values in this range are natural but the larger values can arise with modest tuning of microphysical couplings.

After production, $\rho_{GW}$ undergoes cosmological redshift until today. Its final value can be conveniently expressed in terms of the energy in CMB photons today, $\rho_{\gamma}$,

$$\rho_{GW}^{\text{today}} \approx 0.1 \frac{\rho_{PT}^2}{\rho_{\text{total}}^2} (H_{PT} \Delta t_{PT})^2 \rho_{\gamma},$$  \(9\)

where the numerical prefactor is given by a more quantitative analysis (reviewed in Ref. \[13\]). We have assumed that the PT remnants consist of just SM + dark matter (DM) + GW, as motivated in extensions of the SM related to the electroweak hierarchy problem (reviewed in Ref. \[34\]) or electroweak scale baryogenesis (reviewed in Ref. \[35\]). Clearly the largest signal would arise if the entire contents of the universe undergoes the PT, $\rho_{\text{total}} \approx \rho_{PT}$,

$$\rho_{GW}^{\text{today}} \approx 0.1 (H_{PT} \Delta t_{PT})^2 \rho_{\gamma} \approx 10^{-5} - 10^{-2} \rho_{\gamma}.$$  \(10\)

The GW frequencies are also redshifted, roughly,

$$\omega_{GW}^{\text{today}} \sim \omega_{GW} \left( \frac{T_{\text{today}}}{T_{\text{CMB}}} \right) \gtrsim \text{mHz} - \text{Hz},$$  \(11\)

where we get ballpark detectable frequencies for PTs at temperatures $T_{PT} \gtrsim \text{TeV}$. Note that if $\omega_{GW} \sim \text{mHz}$, the signal strength is above the expected sensitivity of the LISA detector, $10^{-8} \rho_{\gamma}$ \[13\]. Quite model-independently from the above discussion, there is a CMB constraint on extra forms of radiation that applies to any GW background, $\rho_{GW} \lesssim 0.1 \rho_{\gamma}$.  

Let us now see how anisotropies arise in the GW signals in the simplest scenario when there is a single source

\[3\] This constraint usually is given in terms of the effective number of additional neutrino species, $\Delta N_{\nu} \lesssim 0.4$ (2$\sigma$) \[29\] \[67\].
of primordial adiabatic fluctuations. The CMB shows us that temperature $T$ is inhomogeneous in the universe, a redshifted reflection of inflationary quantum fluctuations. This implies that these fluctuations would also necessarily have been present during the PT as long as the “reheating” temperature at the end of inflation satisfies $T_{\text{reh}} > T_{\text{PT}}$. Hence the PT occurs at slightly different redshifts in different patches of the sky (see Fig. 1), generating a non-trivial power spectrum in Eq. (1). This GW anisotropy, $C_{\ell}^{\text{GW}}$, gives us a second copy of the CMB, reflecting their shared origins. However, these GW signals are created in pristine form: the production is long before matter domination and the non-linear growth of density perturbations. Therefore, $C_{\ell}^{\text{GW}}$ is simpler and less processed than $C_{\ell}^{\text{CMB}}$ because photons interact with the charged plasma thereby feeling effects such as baryon acoustic oscillations and Silk damping at high $\ell$. In principle then, $C_{\ell}^{\text{SI}}$ is proportional to the scale-invariant (SI) inflationary spectrum of fluctuations, expressed in spherical harmonics. For example, exact SI would correspond to

$$C_{\ell}^{\text{SI}} \propto \ell (\ell + 1)^{-1},$$

but there will necessarily be small deviations directly sensitive to the physics of inflation/reheating. The only other corrections to SI are created by the peculiar motion of the Earth, and subdominant gravitational effects of matter on GW en route to us - the “integrated Sachs-Wolfe effect” (ISW). Each of these is however calculable.

Because we have assumed a single source of primordial fluctuations, $(\delta \rho/\rho)_{\text{GW}} \sim (\delta \rho/\rho)_{\text{CMB}} \sim \delta T / T \sim 10^{-5}$. Furthermore the two anisotropies are strongly correlated in angle: “hot” and “cold” regions of the CMB directly overlap “hot” and “cold” regions in the GW background in the sky, as is captured by measuring $C_{\ell}^{\text{cross}}$. As with the CMB, the isotropic ($\ell = 0$) piece of the signal will be seen first. Turning to the anisotropy, Eq. (10) implies $\delta \rho_{\text{GW}} \sim 10^{-10} - 10^{-7} \rho_{\gamma}$. This range overlaps LISA sensitivity $\sim 10^{-8} \rho_{\gamma}$ for low $\ell \lesssim 10$ (within LISA angular resolution). It is important that the large isotropic component be cleanly subtracted from the signal in making the anisotropic measurements. Note that high $\ell > 10$ modes are more challenging, both because they require higher angular resolution and because of the asymptotic falloff $\sqrt{C_{\ell}^{\text{GW}}} \propto 1 / \ell (\ell > 1)$, Eq. (12). Even if the anisotropies are below sensitivity and only the isotropic component is detected at LISA, this would specify precisely what type of future detector sensitivity, angular resolution and frequency coverage are needed to fully exploit the valuable and robustly expected anisotropies. If this turns out to be the case, there is a known astrophysical foreground from white dwarf mergers in the LISA frequency range and at roughly LISA sensitivity, which would become relevant for a higher-sensitivity detector. However this should be subtractable from the signal in looking for primordial anisotropies in part because the foreground will be dominantly isotropic.

The simple scenario above, though well motivated, is not the only option. Observable GW anisotropies can
be completely uncorrelated with the CMB, and they can also have a larger contrast \((\delta \rho / \rho)_{GW} > 10^{-5}\). The current cosmological data constrain these different options, primarily through the \(N_{\text{eff}}\) and isocurvature bounds derived from the Planck satellite CMB data. These constraints are based on the gravitational backreaction on the CMB dynamics of the isotropic/anisotropic GW (as free-streaming dark radiation). Roughly, the isocurvature constraint is on the anisotropy,

\[
\delta \rho_{GW} \lesssim 10^{-6} \rho_{\gamma}, \tag{13}
\]

while the \(N_{\text{eff}}\) constraint is on the isotropic signal,

\[
\rho_{GW} \lesssim 0.1 \rho_{\gamma}. \tag{14}
\]

GW satisfying these limits can easily be above LISA’s sensitivity of \(10^{-8} \rho_{\gamma}\).

Detecting such GW backgrounds, either uncorrelated with the CMB and/or having a larger contrast, would be important because it would point to having two or more sources of primordial fluctuations. We illustrate this with a simple example within the inflationary paradigm, where the standard inflaton \(\phi\) is accompanied by an “axion-like particle” (ALP) (reviewed in Ref. [43]). \(a\), lighter than \(H_{\text{inflation}} > T_{\text{PT}}\).\(^5\) Each of these fields develops approximately SI quantum fluctuations during inflation, through repeated production, \(\delta \phi, \delta a, \omega_{\phi}, \omega_{a} \sim H_{\text{inflation}}\) followed by redshifting. But crucially the fluctuations in the two fields are completely uncorrelated. After inflation, \(\phi\) and \(a\) can decay and reheat lighter particles/fields. Consider the possibility that it is the ALP that reheats some extension of the SM + DM, which we will call the “visible sector” (VS), while the inflaton reheats another “hidden” sector (HS) of light particles which only couples to VS very weakly. Therefore, the VS inherits the quantum fluctuations of \(a\), and the HS inherits the quantum fluctuations of \(\phi\), so that the fluctuations in the two sectors are uncorrelated. Because the inflaton, by definition, carries the most (potential) energy until the end of inflation, it can naturally dominate the reheating process, so that \(\rho_{\text{HS}} > \rho_{\text{VS}}\), with the parameters of the inflaton/ALP dynamics and decay allowing a very large range of ratios \(\rho_{\text{VS}}/\rho_{\text{HS}}\).

Subsequently, the VS and HS redshift and cool, until the PT is reached in the VS and GW are released. At some time later, only bounded to be earlier than Big Bang Nucleosynthesis (as reviewed in Ref. [44]), \(T \sim\) MeV, we take the HS particles to decay entirely into the VS, via very weak couplings, leaving just the VS in thermal equilibrium. This cosmological history is illustrated in Fig. 2. Late-decaying HS decays to the VS have been discussed earlier as an attractive setting for generating the matter/antimatter asymmetry in baryons as well as for some variants of DM.

We now turn to the history of the primordial fluctuations. Standard single-field inflationary dynamics readily accommodates small fluctuations imparted by \(\phi\) decay to the HS, say compared to the CMB,

\[
\left(\frac{\delta \rho}{\rho}\right)_{\text{HS}} \leq 10^{-5}. \tag{15}
\]

However, the ALP can easily give rise to larger contrast, as follows. The field space of an ALP Goldstone boson is compact, its size characterized by its “decay constant”, \(f_{a}\). If \(f_{a} > H_{\text{inflation}}\), so that the associated spontaneous symmetry breaking has already taken place during inflation, then it is natural for \(a\) to have a background value of order \(f_{a}\), with quantum fluctuations \(\delta a \sim H_{\text{inflation}}\). This leads to

\[
\left(\frac{\delta \rho}{\rho}\right)_{\text{VS}} \sim \frac{\delta a}{a} \sim \frac{H_{\text{inflation}}}{f_{a}} \geq 10^{-5} \tag{16}
\]

being satisfied for a broad range of parameters. Once the VS undergoes its PT, it imparts its \(\delta \rho / \rho\) to its low-\(T\) phase as well as the GW,

\[
\left(\frac{\delta \rho}{\rho}\right)_{\text{GW}} \sim \left(\frac{\delta \rho}{\rho}\right)_{\text{VS}} \geq 10^{-5}. \tag{17}
\]

But after HS decay to VS, we have

\[
(\delta \rho)_{\text{VS}}^{\text{after}} \sim (\delta \rho)_{\text{VS}}^{\text{before}} + (\delta \rho)_{\text{HS}}^{\text{before}}, \tag{18}
\]

while \((\delta \rho)_{\text{GW}}\) is unaffected, implying that today the CMB has

\[
\left(\frac{\delta \rho}{\rho}\right)_{\text{CMB}} \simeq \frac{\delta \rho_{\text{VS}}^{\text{after}}}{\rho_{\text{HS}}^{\text{before}}} = \left(\frac{\rho_{\text{VS}}}{\rho_{\text{HS}}}ight)^{\text{before}} \left(\frac{\delta \rho}{\rho}\right)_{\text{GW}} + \left(\frac{\delta \rho}{\rho}\right)_{\text{HS}} \tag{19}
\]

Here, we have used that \(\rho_{\text{HS}} > \rho_{\text{VS}}\) originally. We see that we can easily reconcile \((\delta \rho / \rho)_{\text{CMB}} \sim 10^{-5}\) with larger \((\delta \rho / \rho)_{\text{GW}}\) by having \(\rho_{\text{HS}} > \rho_{\text{VS}}\) originally. Furthermore, we see two different patterns depending on whether the first or second term on the second line dominates: if the first term dominates then the CMB and GW backgrounds are completely correlated, even if \((\delta \rho / \rho)_{\text{GW}} > (\delta \rho / \rho)_{\text{CMB}},\) while if the second term dominates then the CMB and GW backgrounds are completely uncorrelated. For instance, in the uncorrelated case, if the effects of ISW were removed from the data, then the cross-correlator would vanish, \(C_{\text{cross}} = 0\).

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4 Refs. [30, 31] have also discussed larger contrasts appearing in high-frequency GW from inflationary preheating, but have not analyzed the cosmological constraints on this scenario.

5 An ALP is basically a pseudo Nambu-Goldstone boson associated to spontaneous breaking of an approximate \(U(1)\) global symmetry.
The separate evolution of the fluctuations in the VS and HS clearly requires that the two sectors are substantially decoupled. But minimally they must interact via gravity, as well as by the requirement that the HS decays into the VS after the PT. One danger is that large fluctuations in the VS, $\delta \rho_{\text{VS}}$, can source spacetime curvature fluctuations, in turn adding to $\delta \rho_{\text{HS}}$ beyond that inherited from the inflaton, threatening the ability to reconcile the small CMB anisotropy with large GW anisotropy in Eq. [19]. This danger is simply avoided by requiring that $\delta \rho_{\text{VS}} \leq \delta \rho_{\text{HS}}$ originally, even though $(\delta \rho/\rho)_{\text{VS}} \geq (\delta \rho/\rho)_{\text{HS}}$. The second danger is HS-VS interactions may equilibrate the two sectors before the PT. But the interaction rate per HS particle can naturally be as small as the decay rate, which is $< H_{\text{PT}}$, corresponding to decays after the PT. This then ensures that the interactions are also ineffective in equilibrating before the PT.

It is important to observe that the above mechanism for generating large $$(\delta \rho/\rho)_{\text{GW}} > 10^{-5}$$ implies that the GW signal strength is weaker than the minimal scenario of a single source of primordial fluctuations, first discussed. This can be seen from Eq. [4], which translates here to

$$\rho_{\text{GW}} \sim \frac{0.1 \rho_{\text{VS}}^2}{(\rho_{\text{HS}} + \rho_{\text{VS}})^2} (H_{\text{PT}}^2 t_{\text{PT}}^2)^2 \rho_{\gamma} \sim \left( \frac{\rho_{\text{VS}}}{\rho_{\text{HS}}} \right)^2 \rho_{\text{GW}}^{\text{single-source}}.$$  \tag{20}

With weaker signal, the $N_{\text{eff}}$ and isocurvature constraints, Eqs. [13][14], are automatically satisfied.

The weaker GW signal can still be above future detector sensitivity. For example, for a PT with $(H_{\text{PT}}^2 t_{\text{PT}}^2)^2 \sim 10^{-1}$, we can have $(\delta \rho/\rho)_{\text{GW}} \sim 10^{-5}$ but only partially correlated with the CMB, for $\rho_{\text{VS}} \sim \rho_{\text{HS}}$. This gives an anisotropic signal, $\rho_{\text{GW}} \sim 10^{-7} \rho_{\gamma}$, above LISA sensitivity (for $\ell \lesssim 10$), for $\omega_{\text{GW}} \sim \text{mHz}$. Alternatively, a PT with $(H_{\text{PT}}^2 t_{\text{PT}}^2)^2 \sim 10^{-2}$ can have fluctuations $$(\delta \rho/\rho)_{\text{GW}} \sim 10^{-4},$$ either correlated or uncorrelated with the CMB, for $\rho_{\text{VS}} \sim \rho_{\text{HS}}$. This gives an anisotropic signal, $\delta \rho_{\text{GW}} \sim 10^{-3} \rho_{\gamma}$, above the sensitivity of the proposed Big Bang Observatory (BBO) [11] of $10^{-11} \rho_{\gamma}$ (for $\ell \lesssim 100$, within BBO angular resolution [28]), for $\omega_{\text{GW}} \sim 0.1 - 1 \text{ Hz}$. An even weaker signal with $(H_{\text{PT}}^2 t_{\text{PT}}^2)^2 \sim 10^{-4}$ can have fluctuations $$(\delta \rho/\rho)_{\text{GW}} \sim 10^{-5},$$ correlated or not with CMB, with $\delta \rho_{\text{GW}} \sim 10^{-10} \rho_{\gamma}$ and would still be visible at BBO for $\ell \lesssim 10$. Again, there are astrophysical merger foregrounds relevant to LISA and BBO [11] but these should be predominantly isotropic or subtractable. But by frequencies $\omega_{\text{GW}} \gtrsim \text{Hz} (T_{\text{PT}} \gtrsim 1000 \text{ TeV})$, these foregrounds are essentially absent.

If the PT takes place in the VS, as we have so far considered, particle collider experiments in the future should be able to discover the associated extension of the SM, giving a corroborating microphysical view. But it is also possible that the PT takes place entirely in a HS [15]. This is straightforwardly accommodated by exchanging the roles of HS and VS above, and not having the final merging of the two sectors after the PT. In this case, colliders may miss the HS while GW and cosmological data provide our only view.

Beyond the two-point correlations in $C_{\text{GW}}$ and $C_{\text{cross}}$, higher-point correlators $(\rho_{\text{GW}}(1)\rho_{\text{GW}}(2)\rho_{\text{GW}}(3))$ may be visible, especially if the primordial fluctuations in the sector undergoing the PT are created by inflationary fields with substantial interactions. The full exploration of all of these GW correlators could give us deep insights into the dynamics of (multi-field) inflation and reheating, and hidden sectors to which we might otherwise be blind.

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