On the physical inadmissibility of ILES for simulations of Euler equation turbulence

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We present two main results. The first is a plausible validation argument for the principle of a maximal rate of entropy production for Euler equation turbulence. This principle can be seen as an extension of the second law of thermodynamics. In our second main result, we examine competing models for large eddy simulations of Euler equation (fully developed) turbulence. We compare schemes with no subgrid modeling, implicit large eddy simulation (ILES) with limited subgrid modeling and those using dynamic subgrid scale models. Our analysis is based upon three fundamental physical principles: conservation of energy, the maximum entropy production rate and the principle of universality for multifractal clustering of intermittency. We draw the conclusion that the absence of subgrid modeling, or its partial inclusion in ILES solution violates the maximum entropy dissipation rate admissibility criteria. We identify circumstances in which the resulting errors have a minor effect on specific observable quantities and situations where the effect is major. Application to numerical modeling of the deflagration to detonation transition in type Ia supernova is discussed.

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I. INTRODUCTION

The solutions of the Euler equation for fluid dynamics are not unique. An additional law of physics, in the form of an entropy principle, is needed to ensure a physically meaningful solution. Wild and manifestly nonphysical solutions have been studied extensively [1, 2] and offer counter examples to studies of the Euler equation as a model for fully developed turbulence. This paper is concerned with the nonuniqueness for Euler equation solutions that are the limit of Navier-Stokes solutions as the viscosity tends to zero. We address common practice in the construction of numerical solutions for turbulent flows. Applications to type Ia supernova are discussed.

Nonuniqueness (both mathematical and numerical) of solutions to the Euler equation is well known in the study of shock waves and its resolution is also well known: a maximum rate of entropy production is imposed as a selection criteria to yield a unique and physically relevant solution. But nonuniqueness persists in solutions of the incompressible Euler equation, where shock waves do not occur. Again, a physical principle must be added to select the physically meaningful solution.

This paper poses a challenge to existing standards of verification and validation (V&V). We propose that if turbulence is present in the problem solved, standards of V&V should ensure the physical relevance of the solutions.

As with the shock wave example, inadmissible numerical solutions of turbulent phenomena are also possible. We identify three broad classes of numerical solutions to the problems of Rayleigh-Taylor (RT) turbulent mixing and compare them to experimental data [3]. one of these agrees with the data, while two do not. The second main result of this paper is to identify these other two, solutions that include no subgrid terms and those for which the subgrid terms are limited, i.e., the Implicit Large Eddy Simulation (ILES), as physically inadmissible solutions of the turbulent RT mixing data [3]. ILES and solutions which report a DNS status and lack subgrid terms. These latter two solutions do not agree with each other, further indicating nonuniqueness issues.

To account for observed discrepancies between ILES predictions and experimental data, it is common to add “noise” to the physics model. As noise increases the entropy, some discrepancies between simulation and measured data are removed.

The solution with noise is, however, not predictive. Not only can it be missing in the required amounts, but it is only a qualitative cure, with no defined noise level or noise frequency spectrum specified.

The maximum entropy rate is a clearly defined physics principle. We propose it as a solution to the Euler equa-
tion nonuniqueness problem.

Reynolds averaged Navier Stokes (RANS) simulations resolve all length scales needed to specify the problem geometry. Large eddy simulations (LES) not only resolve these scales, but in addition they resolve some, but not all, of the generic turbulent flow. The mesh scale, i.e., the finest of the resolved scales, occurs within the turbulent flow. As this is a strongly coupled flow regime, problems occur at the mesh cutoff. Resolution of all relevant length scales, known as Direct Numerical Simulation (DNS) is computationally infeasible for many problems of scientific and technological interest. As a consequence, an understanding of the problems and opportunities of LES is an important issue.

The subgrid scale (SGS) flow exerts an influence on the flow at the resolved level. Because this SGS effect is not part of the Navier-Stokes equations, additional modeling terms are needed in the equations. These SGS terms added to the right hand side (RHS) of the momentum and species concentration equations generally have the form

\[ \nabla \nu_t \nabla \text{ and } \nabla D_t \nabla. \] (1)

The coefficients \( \nu_t \) and \( D_t \) are called eddy viscosity and eddy diffusivity.

According to ideas of Kolmogorov \cite{Kolmogorov}, the energy in a turbulent flow, conserved, is passed in a cascade from larger vortices to smaller ones. This idea leads to the scaling law \cite{Kolmogorov}

\[ \langle |v(k)|^2 \rangle = C_K \epsilon^{2/3} |k|^{-5/3} \] (2)

for the Fourier coefficient \( v(k) \) of the velocity \( v \). Here \( C_K \) is a numerical coefficient and \( \epsilon \), the energy dissipation rate, denotes the rate at which the energy is transferred within the cascade. It is a measure of the intensity of the turbulence.

At the grid level, the numerically modeled cascade is broken. The role of the SGS terms is to dissipate this excess grid level energy so that the resolved scales see a diminished effect from the grid cutoff. This analysis motivates the SGS coefficient \( \nu_t \), while a conservation law for species concentration similarly motivates the coefficient \( D_t \).

Higher order compact schemes may omit any subgrid model in their study of RT mixing. As an example, \cite{Miranda} present a nominally DNS solution, which, however, is not validated by comparison to experiment. Moreover, the DNS characterization of the simulation is not documented, with \( D \) and \( \nu \) not specified. It appears from the text that DNS refers to globally defined solution parameters such as the globally defined Kolmogorov scale \( \eta \) in relation to the mesh spacing, with \( \nu \) and \( D \) defined on this basis. Such resolution misses local fluctuations in the turbulent intensity, which require dynamically defined SGS terms added to the equation. As \cite{Miranda} is focused on applications to supernova Ia, additional comments are placed in our SN Ia discussion.

ILES is the computational model in which the minimum value of \( \nu_t \) is chosen so that a minimum of grid level excess energy is removed to retain the \( |k|^{-5/3} \) scaling law, while the prefactor \( C_K \epsilon^{2/3} \) is not guaranteed. It thus depends on limited and not full use of the subgrid terms that correspond to the local values of the energy dissipation cascade. An ILES version of Miranda, a modern higher order compact scheme, given in \cite{FronTier}, details in the construction of the ILES version of this code and analyzes a number of scaling related properties of the RT solutions the algorithm generates. The subgrid terms are chosen not proportional to the Laplacian as in \cite{LES}, but as higher power dissipation rates, so that large wave numbers are more strongly suppressed. The SGS modeling coefficients \( \nu_t \) and \( D_t \) are chosen as global constants. The basis for the choice is to regard the accumulation of energy at the grid level as a Gibbs phenomena to be minimized \cite{Miranda}. Miranda achieves the ILES goal of an exact \(-5/3\) spectral decay, see Fig. 3 right frame in Ref. \cite{Miranda}.

FronTier uses dynamic SGS models \cite{FronTier, FronTierII}, and additionally uses a sharp interface model to reduce numerical diffusion. In this method, SGS coefficients \( \nu_t \) and \( D_t \) are defined in terms based on local flow conditions, using turbulent scaling laws, extrapolated from an analysis of the flow at one scale coarser, where the subgrid flow is known.

The philosophy and choices of the SGS terms are completely different among the compact schemes, ILES and FronTier, a fact which leads to differences in the obtained solutions. Solution differences between FronTier and ILES were reviewed in \cite{FronTierII}, with FronTier but not ILES showing agreement with the data \cite{Miranda}. The schemes totally lacking SGS terms are even further from the experimental data \cite{FronTierII}.

As shown in \cite{FronTierII}, long wave length noise in the initial conditions was eliminated as a possible explanation of the discrepancies between ILES simulations and experimental data for the RT instability growth rate constant \( a_b \). We also note the mixedness parameter measured in \cite{FronTierII}, is furthest from experiment in \cite{Miranda}, is improved in the Miranda simulation code \cite{Miranda} lacking subgrid terms but with improved modeling of experimental parameters, and further improved by the FronTier simulation \cite{FronTierII}.

\section{Scaling Laws Compared}

Here we focus on differences in the spectral scaling exponents. As \cite{Miranda, FronTier} employ a thinly diffused initial layers separating two fluids of distinct densities, the immiscible experiments of \cite{Miranda} are the most appropriate for comparison. \cite{Miranda} does not report velocity spectral scaling properties, but this reference does report the very large growth of the interfacial mixing area, \cite{Miranda}. Fig. 6, a phenomena which we have also observed \cite{FronTierII, FronTierII}. The scaling rate we observe, Fig. 1 from the late time FronTier simulations reported in \cite{FronTierII}, shows a strong decay rate in the velocity.
We summarize in Table I the major code comparisons of this paper, based on the RT instability growth rate $\alpha_b$. A compact, higher order scheme \[5\] has the smallest value $\alpha_b$. ILES \[6\] captures neither the expected turbulent intermittency correction to the decay rate nor any stirring correction beyond this.

TABLE I: Three types of RT simulation algorithms according to their treatment of SGS terms and their value for $\alpha_b$, compared to the data of \[3\].

| Code          | SGS terms  | solution properties | evaluation relative to \[3\] |
|---------------|------------|---------------------|-------------------------------|
| compact high  | No SGS     | $\alpha_b \sim 0.02$ | Inconsistent                  |
| order \[5\]   |            |                     |                               |
| Miranda       | Limited SGS| $\alpha_b \sim 0.03$ | Inconsistent                  |
| ILES \[6\]    | Dynamic    | $\alpha_b \sim 0.06$ | Consistent                    |
| FronTier \[9\]| SGS        |                     |                               |

spectrum, resulting from a combination of the turbulent fractal decay and a separate cascading process we refer to stirring. Stirring is the mixing of distinct regions in a two phase flow. It occurs in the concentration equation and is driven by velocity fluctuations. For stirring, the concentration equation describes the (tracked) front between the phases. Stirring fractal behavior is less well studied than turbulent velocity. It accounts for the very steep velocity spectral decay seen in Fig. 1. In contrast, ILES \[6\] captures neither the expected turbulent intermittency correction to the decay rate nor any stirring correction beyond this.

We observe that the energy occurs outside of the integrals, and that the term $(1 - x) \ln(1 - x)$ is missing from the entropy. To model a time dependent state which has

\[ E_{p, l} \int_{x_{p, l}} x dx \quad \text{and} \quad E_{p, l} \int_{x_{p, l}} x \ln x dx , \]

\[ (3) \]
not yet achieved equilibrium, and is still evolving in time, the only change to (3) is that the equations are multiplied by the fractional equilibrium part of the state.

The log Poisson model \cite{20} selects a fractal set to describe each order \( p \) of clustering. The choice, conditional on prior choices for smaller \( p \) values, is not defined by an exponential, i.e., a pure fractal, but a mixture of exponentials in the energy dissipation rates. As the mixture is not narrowly concentrated about its peak value, the applicability of hypothesis (Fractal) cannot be assumed. In the limit of large \( p \), however, the mixture of exponentials is narrow, so that (Fractal) is justified for physically realizable solutions of Euler equation turbulence. The peak values for finite \( p \) are not identified in the log Poisson analysis, which finds the mean of the mixture exponentials on the basis a universality hypothesis. This multifractal model, evaluated for large \( p \) is applied uniformly to all \( p \). From the excellent agreement of these predictions with multiple experiments and simulations (1% accuracy) \cite{20}, the log Poisson model is validated. A plausible principle to select the physically relevant solutions from among the multiple nonunique solutions of the Euler equation, suggested by this analysis, is the principle of maximal rate of energy dissipation. The analysis of \cite{20} maximizes the mean value of competing exponentials rather than their peak value. The mean and peak coincide in the limit of large \( p \), but the distinction between them for finite \( p \) is a gap remaining in any validation argument. The maximum energy dissipation rate is a viable candidate for the required selection principle among nonunique solutions of the Euler equation. Accepting this, our analysis will be complete with solutions lacking subgrid terms and ILES seen to be invalid physically. To the extent that some maximal entropy likelihood reasoning is applicable, for example such as (Fractal), a maximum entropy production rate principle for the selection of physically relevant solutions of the Euler equation for fully developed turbulence would follow.

We refer to the highly developed extensions \cite{21–24} of \cite{20}. The references \cite{22,24} extend the log Poisson model to continuous \( p > 0 \). These references do not resolve the issue of either a maximum entropy production rate or a maximum energy dissipation rate for fully developed turbulence, but they appear to offer a plausible route for possible validation of either of these.

The dynamic equations are of Fokker-Plank type. The dissipation operator is a sum of a conventional Laplacian, for the thermal diffusion and an integral over \( p > 0 \) of the order \( p \) clustering contribution, which is a fractal, or power law dissipation, expressed in powers of the length scale \( l \).

IV. THE SECOND MAIN RESULT

In view of these observations, we note three independent reasons for concluding that the absence of subgrid terms or their limited presence in ILES is problematic on physical grounds.

1. The two limited subgrid schemes do not satisfy the maximum entropy production rate principle.

2. The two limited subgrid schemes are in violation of incontrovertible experimental and simulation evidence that the true total spectral decay rate is more negative than \(-5/3\) \cite{19}.

3. These two schemes understate the dissipated energy and are thus unphysical.

These are logically independent statements. The order is decreasing in the fundamental nature of the statement and increasing in the simplicity of the assumptions. Any one of these points is sufficient to invalidate schemes lacking in subgrid terms or ILES, with limited subgrid terms. Point 1 is the most fundamental in nature, and it is the subject of the remainder of this paper. Point 2 rests on established laws of physics and assumes the relevance of Kolmogorov scaling laws with their intermittency corrections to RT mixing. Point 3 assumes nothing. Simulations, even ILES simulations, show a transfer of energy from large to small scales. Point 3 accepts this as a physical fact. The energy transfer is not a numerical feature to be minimized, but a property of the solutions to be modeled correctly. The grid level cutoff terminates this transfer, and point 3 notes that the transfer, from the grid level to the subgrid level is incorrectly modeled in both types of limited dissipation schemes.

For the reader satisfied with either points 2 or 3, the remainder of the paper can be ignored, and the discussion has been completed, independent of the remainder of the paper.

The problems with current computational paradigms are well summarized by Zhou \cite{25}, Sec. 6 regarding evaluation of the RT instability growth rate \( \alpha_b \), “agreement between simulations and experiment are worse today than it was several decades ago because of the availability of more powerful computers.” As our computational method depends on front tracking in addition to dynamic subgrid modes (which address items 1-3 above), we additionally quote from Zhou \cite{25}, Sec. 5.2, in discussing \cite{20}: “it was clear that accurate numerical tracking to control numerical mass diffusion and accurate modeling of physical scale-breaking phenomena surface tension were the critical steps for the simulations to agree with the experiments of Read and Smeeton and Youngs”.

We raise the possibility of ILES related errors in an analysis of the deflagration to detonation transition in type Ia supernova. In that these simulations depend on ILES, their predictive value may be questioned. We propose a simple simulation search method for rare events, in which a physics simulation code drives the turbulence modeling. In agreement with \cite{5}, we recommend a new class of turbulent combustion subgrid models. See Sec. V.
A. Rayleigh-Taylor turbulent mixing

We assess ILES in terms of the RT instability of acceleration driven instabilities, and the prediction of the growth rate $\alpha_0$ of this instability. We identify situations in which ILES is in near agreement with experimental measurement of the growth rate in this measure $\alpha_0$ and ones $28$ where its predictions differ by a factor of about $2$ from experiments $3$. The first case is characterized by

- (a) low levels of turbulence
- (b) high levels of long wave length perturbations ("noise") in the initial conditions and
- (c) diffusive parameters in the physics model.

Regarding item (c), we observe that the successful ILES simulations referenced above concerned hot-cold water, with a moderate Schmidt number of 7, whereas no results are reported for the very low diffusive fresh-salt water channel with a Schmidt number of 600.

B. Noise as an adjustable parameter

The postulate $29$ of noise in the initial data $3$ was shown to lead to agreement of the ILES predictions with experiment. In previous studies $9,30$, we have shown that this postulate is not valid. The long wave length noise is present, but with a sufficiently small amplitude that its influence on the instability growth rate is about $5\%$. Thus long wave length initial “noise” in the initial conditions for $3$ is not sufficient to account for the factor of 2 discrepancy between ILES and this data.

We regard “noise” as a palliative, and not a fundamental principle. The noise level is not specified, nor is its frequency spectrum, so that standards of predictive science are not met. As noted, “noise”, of the required intensity, is missing in some instances. We propose the maximum entropy production rate as a more satisfactory solution to the problem of Euler equation turbulence nonuniqueness.

IES simulations have been used in the study of incompressible turbulence, a problem with ample experimental data reviewed in $20$. In such simulations, “noise” is added to the initial conditions. In this case the high frequency component of the noise is important. Agreement with experiment is obtained. Pure ILES, with no added noise would not meet this test.

C. Outline of derivation

Our reasoning is based on three fundamental laws of physics:

- Conservation of energy, the first law of thermodynamics
- Maximum entropy production rate, an extension of the second law of thermodynamics
- Universality in the clustering and compound clustering of intermittency in fully developed turbulence.

The third item is formulated in $20$. In the multifractal description of turbulence, universality states that the compound clusters, that is the multiple fractals in the description of turbulent intermittency, must all obey a common law. There can be no new physical law or parameter in passing from one level of clustering to the next. The law is evaluated in closed form $20$ in the limit that the order of the clustering becomes infinite. It is a power scaling law. By universality, this law is then applied to clustering at all orders.

The universality theories are developed for single constant density incompressible turbulence. Our use in a variable density context is an extrapolation of these theories beyond their domain of strict validated applicability. Scaling laws are similarly extrapolated. Such extrapolations are widely used (and verified) in simulation studies. For convenience, the Reynolds stress analysis uses this approximation.

In shock wave modeling, the Euler equation shock wave introduces a Gibbs phenomena of overshoot. The instability resulting is removed by dissipation (artificial viscosity, and its modern variants) of the minimum amount to just prevent the overshoot. The turbulent cascade of energy is not a Gibbs phenomena. It is an observable fact and not a numerical artifact. Minimizing its magnitude is an error, as opposed to an accurate model of the mesh dissipated error in the dynamic SGS models.

We proceed in the following steps. Using the Reynolds stress, we express the SGS terms to be modeled as a truncated two point function. In this formulation, we identify the minimum (ILES) and maximum (dynamic SGS) alternatives.

We then proceed from velocity fluctuations to the energy dissipation rate $\epsilon$ and from the latter to the entropy production rate. At each step we are looking at truncated two point functions. At the end, we are looking at the entropy production rate and must choose the solution with maximum entropy production rate.

Each step is monotone and preserves the minimum-maximum choice. Reasoning backwards, we see that the maximum choice is needed at the outset, and so ILES is inadmissible.

The transition, from velocities to energy truncated two point functions, has two components. The first is a scaling analysis to show equivalence, but in the process the order of clustering changes. The second component in this transition is to apply universality: all orders of clustering must obey a common minimum-maximum choice.
D. From velocities to entropy

1. Reynolds stress

The Reynolds stress results from regarding the mesh values as cell averaged quantities. This creates an obvious problem for nonlinear terms of the Euler equation. From the momentum equation, the quadratic nonlinearity is replaced by the product of the cell mean values. The resulting error, transferred to the RHS of the momentum equation is the negative of the gradient of the Reynolds stress, defined as

\[ R = \overline{\nu^2} - \overline{\nu} \overline{\nu} \]  

(4)

in the case of constant density, with a more complex expression involving density weighted (Favre) averages in the variable density case.

The added force term \(-\nabla R\) on the right hand side (RHS) of the momentum equation is modeled as \(\nu_\ell \Delta \nu\). Thus we see that the minimum and maximum values for the energy dissipation rate \(\nu_\ell\) correspond to minimum and maximum values for models of \(-\nabla R\). \(R\), as a truncated two point function, vanishes as its argument becomes infinite and is peaked at the origin. Thus minimum and maximum values for \(-\nabla R\) correspond to minimum and maximum values for \(R\) itself.

2. Velocities to energy

As technical preparation for the analysis of this section, we define the structure functions. They make precise the intuitive picture of multiple orders of clustering for intermittency. There are two families of structure functions, one for velocity fluctuations and the other for the energy dissipation rate \(\epsilon\). The structure functions are the expectation value of the \(p\)th power of the variable. For each value of \(p\), they define a fractal and satisfy a power law in their decay in a scaling variable \(l\). The structure functions and the associated scaling exponents \(\zeta_l\) and \(\tau_l\) are defined as

\[ \langle \delta v_l^p \rangle \sim l^{\zeta_p} \quad \text{and} \quad \langle \epsilon_l^p \rangle \sim l^{\tau_p} \]  

(5)

where \(\delta v_l\) and \(\epsilon_l\) are respectively the averages of velocity differences and of \(\epsilon\) over a ball of size \(l\). The two families of exponents are related by a simple scaling law

\[ \zeta_p = p/3 + \tau_p/3 \]  

(6)

derived on the basis of scaling laws and dimensional analysis [31]. This would seem to accomplish the velocity fluctuation to energy dissipation rate step, preserving the minimum vs. maximum choice, but it does not, because the value of \(p\) to which it applies has changed.

To fill this gap, we turn to the assumption of universality formulated in terms of the \(\tau_p\) [20], and as explained with mathematical formalisms replacing some of the reasoning of a theoretical modeling nature, [22, 24, 32, 33]. As a function of \(p\), \(\tau_p\) is a fractional order cubic, defined in terms of a fractional order dissipative operator with a fractional order exponent \(\beta\). This relation is derived exactly in the limit as \(p \to \infty\), and in the name of universality, then applied to all values of \(p\).

As a monotone fractional order cubic, it follows that the minimum-maximum choice for any \(p\) is reflected in the same choice for all \(p\). We have thereby completed the velocity to energy dissipation rate step, and preserved the minimum vs. maximum choice.

From the modeling principle (Fractal), the energy dissipation rate is maximized exactly when the entropy production rate is maximized. The maximum choice for the entropy production rate is required and the minimum choice is inadmissible. Reasoning backwards to the original energy dissipation choices, the minimum rate of energy dissipation (ILES) is inadmissible.

V. SIGNIFICANCE: AN EXAMPLE

For simulation modeling of turbulent flow nonlinearly coupled to other physics (combustion and reactive flows, particles embedded in turbulent flow, radiation), the method of dynamic SGS turbulent flow models, which only deals with average subgrid effects, may be insufficient. In such cases, the turbulent fluctuations or the full two point correlation function is a helpful component of SGS modeling. Such a goal is only partially realized in the simplest of cases, single density incompressible turbulence. For highly complex physical processes, the knowledge of the domain scientist must still be retained, and it appears to be more feasible to bring multifractal modeling ideas into the domain science communities.

In this spirit, we propose here a simple method for the identification of (turbulence related) extreme events through a modification of adaptive mesh refinement (AMR), which we call Fractal Mesh Refinement (FMR). We propose FMR to seek a deflagration to detonation transition (DDT) in type Ia supernova.

FMR allows high levels of strongly focused resolution. The method is proposed to assess the extreme events generated by multifractal turbulent nuclear deflagration. Such events, in a white dwarf type Ia supernova progenitor, are assumed to lead to DDT, which produces the observed type Ia supernova. See [34, 35] and references cited there.

FMR refines the mesh not adaptively where needed, but only in the most highly critical regions where most important, and thereby may detect DDT trigger events within large volumes at a feasible computational cost.

The detailed mechanism for DDT is presumed to be diffused radiative energy arising from some local combustion event of extreme intensity, in the form of a convoluted flame front, embedded in a nearby volume of unburnt stellar material close to ignition. Consistent with
the Zeldovich theory \[36\], a wide spread ignition and explosion may result. FMR refinement criteria will search for such events. In this plan, the FMR search should avoid ILES. See \[37\].

There is a minimum length scale for wrinkling of a turbulent combustion front, called the Gibson scale. Mixing can proceed in the absence of turbulence, via stirring. Thus the Gibson scale is not the correct limiting scale for a DDT event. Stirring, for a flame front, terminates at a smaller scale, the width of the flame itself. The analysis of length scales must also include correctly modeled transport for charged ions \[38\], which can be orders of magnitude larger than those inferred from hydro considerations. The microstructure of mixing for a flame front could be thin flame regions surrounded by larger regions of burned and unburned stellar material (as with a foam of soap bubbles, with a soap film between the bubbles). Here again multifractal and entropy issues appear to be relevant, although not subject to theoretical analysis comparable to that of multifractal for turbulent flow. A multifractal clustering of smaller bubbles separated by flame fronts can be anticipated, and where a sufficient fraction of these bubbles are unburnt stellar material, a trigger for DDT could occur.

FMR, with its narrow focus on extreme events, will come closer to discovering such DDT triggers than will an AMR algorithm design. For this purpose, the astrophysics code should be based on dynamic subgrid SGS, not on ILES.

We return to the discussion of \[5\]. Our FronTier computations of a 2D interface surface length are in qualitative agreement with those of \[5\] for the surface area. Such models of interface area should be the basis for subgrid scale modeling of the turbulent flame intensity. Work is currently in progress to construct an experimentally validated subgrid scale microstructure to complement models of turbulent flame surface area. These subgrid models may play a role in reaching beyond length scales reachable by FMR.

VI. CONCLUSIONS

We have shown that the ILES algorithm for the solution of Euler equation turbulence is inadmissible physically. It is in violation of the physical principle of maximum rate of entropy production.

We have explained observations of experimental flows for which this error in ILES has only a minor effect. They are associated with high levels of noise in the initial conditions, low levels of turbulent intensity and diffusive flow parameters. Prior work, e.g., \[9\] \[20\] \[30\] pertain to simulation validation studies RT instability experiments with a stronger intensity of turbulence and for which such significant long wave length perturbations to the initial data are missing. In these experiments, the present analysis provides a partial explanation for the factor of about 2 discrepancy between observed and ILES predicted instability growth rates.

We have noted the potential for ILES related errors to influence ongoing scientific investigations, including the search for DDT in type Ia supernova.

We believe V&V standards should include an analysis of the physical relevance of proposed solutions to flow problems, specifically turbulent and stirring problems. The ILES simulations of the experiments of \[8\] fail this test by a factor of 2 in the RT growth rate \(\alpha_b\), and on this basis we judge them to be physically inadmissible.

We recognize that the conclusions of this paper will be controversial within the ILES and high order compact turbulent simulation communities. A deeper consideration of the issues raised here is a possible outcome. The issues to be analyzed are clear:

- Is the transport of energy and concentration, blocked at the grid level, to be ignored entirely \[5\]?
- Is it to be regarded as a Gibbs phenomena \[9\], and thus to be minimized?
- Is it a physical phenomena, to be modeled accurately \[7\] \[8\]?

If the response to this paper is an appeal to consensus (everyone else is doing it), the argument fails. Consensus is of course a weak argument, and one that flies in the face of standards of V&V. More significantly, there is a far larger engineering community using dynamic SGS models in the design of engineering structures tested in actual practice. This choice is backed by nearly three decades of extensive experimental validation. It is further used to extend the calibration range of RANS simulations beyond available experimental data. The resulting RANS, calibrated to dynamic SGS LES data, are widely used in the design and optimization of engineering structures; these are also tested in real applications. Consensus in this larger community overwhelms the ILES consensus by its shear magnitude, and ILES loses the consensus argument.

VII. ACKNOWLEDGEMENTS

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