Analysis of Residual Stress Distribution in Rods after Drawing Process with Regard to Process Variables

E V Kuznetsova*, G L Kolmogorov and A V Evsina

Department Dynamics and strength of machines, Perm National Research Polytechnic University, Perm, 29, Komsomolsky, 614990, Russia

E-mail: * mellen75@mail.ru

Abstract. Currently, many researchers pay attention to studying the influence of process variables on the quality of metal products after manufacture. The most common methods of metalworking are forming processes, in which a high degree of deformation, uneven plastic deformation, and elevated temperatures are utilized. In most cases of manufacturing using these methods, a self-balanced system of residual stresses is formed that significantly affects the accuracy, strength, durability, reliability, corrosion resistance, and fatigue strength of machine parts after manufacture and processing. At the same time, the level of technological residual stresses depends on the main process variables. And in this regard, the analysis of the residual stress calculation for the deformation of rods will identify the effect of process variables, mechanical properties of the material, the geometry of the workpiece on the level and distribution of formed residual stresses. The object of the study is distributions of residual stresses formed after plastic deformation in rods and wire. In the work, experimental data on the distribution of residual stresses in rods after drawing were investigated. The residual stress calculation method takes into account parameters of technology, geometry and mechanical constants of material. Dependencies of axial and circumferential residual stresses in steel rods and wires after drawing were found.

1. Introduction
There exists a significant number of research papers [1-5] where the causes of formation and methods of determining residual stresses are considered, and moreover, the applied problems of impact assessment of residual stresses on strength, reliability, durability of materials and construction were solved.

All methods of determining residual stresses can be conveniently classified into theoretical [6] and experimental [7].

In the process of plastic deformation in the metal product manufacture by forming techniques, high degrees of plastic deformation are realized and adhere to the flow plasticity theory. Nevertheless, in the multistage manufacture of details the application of known methods of determining residual stresses is still difficult since it is hard to trace the history of residual stress formation.

2. Determination of residual stresses using the energy approach
In order to determine process-induced residual stresses in the work [6] a methodology was used based on an energy approach, in which a potential energy of elastic residual stresses is considered as a fraction of energy that transfers into a plastic deformation.

\[ U = \psi U_d \] (1)

\( U \) is the potential energy of elastic residual stresses; \( U_d \) is the energy of plastic deformation during the metal product manufacture; \( \psi \) is a parameter of deformation depending on process variables and mechanical properties of the material.

The potential energy of elastic residual stresses in its turn can be computed using the following formula:
\[U = \frac{1}{2} \iiint_V \sigma_{ij} \varepsilon_{ij} \, dV\]  \hspace{1cm} (2)

\(\sigma_{ij}\) and \(\varepsilon_{ij}\) are components of residual stress and corresponding elastic strain tensors respectively.

The energy of the plastic deformation of rods can be counted from the following formula:

\[U_d = S \int_0^\varepsilon \sigma_d \, d\varepsilon\]  \hspace{1cm} (3)

\(S\) is a cross-section area of the workpiece; \(\varepsilon\) is a degree of plastic deformation; \(\sigma_d\) is a plastic resistance of workpiece material and for the most metals and alloys can be determined as an empiric exponential function:

\[\sigma_d = \sigma_{0d} + m \varepsilon^n\]  \hspace{1cm} (4)

\(\sigma_{0d}\) is an initial plastic resistance of workpiece material; \(m, n\) are empirically determined coefficients characterizing a strain hardening of the material.

In order to determine the components of residual stress tensor and corresponding elastic strain in rods, it is necessary to solve an axisymmetric problem of elasticity in the cylindrical coordinate system [8].

\[
\begin{align*}
\sigma_r &= \frac{\bar{a}_1}{4\mu} (\bar{r}^2 - 1) \\
\sigma_\theta &= \frac{\bar{a}_1}{4\mu} (3\bar{r}^2 - 1) \\
\sigma_z &= \frac{\bar{a}_1}{4} (2\bar{r}^2 - 1)
\end{align*}
\]  \hspace{1cm} (5)

\(\sigma_r, \sigma_\theta, \sigma_z\) are radial, circumferential (tangential), and axial (longitudinal) residual stresses respectively;

\[
\bar{r} = \frac{r}{R_d}
\]

\(r\) is a dimensionless radial coordinate; \(R_d\) is the radius of rod after the deformation; \(\bar{a}_1\) is a parameter that is determined from the equation (1) and depends on a degree of plastic deformation and mechanical properties of workpiece material [6]:

\[
\bar{a}_1 = \left(24\psi E\sigma_{0d}\mu^2(1 - \mu^2)^{-1} \varepsilon(1 + m\varepsilon^n(n + 1)^{-1})\right)^{1/2}
\]  \hspace{1cm} (6)

\(E\) is Young’s modulus; \(\mu\) is Poisson’s ratio.

The equation (6) includes a deformation parameter \(\psi\) that requires a supplemental explanation and refinement for different materials and types of plastic deformation [9]. As stated above, the distribution of residual stresses is determined up to the deformation parameter \(\psi\) that is essentially a fraction of the energy of plastic deformation transferring into the formation of residual stresses and is included in the energy equation (1). Accordingly, it was necessary to develop a method of determining a complex deformation parameter, refined for cases of plastic deformation of rods.

3. Determination of the deformation parameter

The essence of the method consists in plotting functional relations of the parameter of deformation to process variables, geometry, and mechanical properties of workpiece material on the basis of known experimental data of technological residual stresses on the surface of rods after a drawing process.

According to the equations (5) components of residual stress tensor on the surface of rod \((\bar{r} = 1, \quad r = R_d)\) after a drawing process are determined from the following formulas:

\[
\begin{align*}
\sigma_r &= 0 \\
\sigma_\theta &= \frac{\bar{a}_1}{2\mu} \\
\sigma_z &= \frac{\bar{a}_1}{2}
\end{align*}
\]  \hspace{1cm} (7)

From the equations (7) you can see that the radial residual stresses on the surface of a rod after a drawing process are equal to zero, that is, it meets the traction free boundary condition. The circumferential and axial residual stresses on the surface of rod are positive (tensile), and the maximum stress on the surface
is the axial ones. Besides, the parameter $\bar{\alpha}_1$ determine their relation to the technological, mechanical, and geometric characteristics.

Having regard to the equations (6) these relations take the following forms:

$$
\begin{align*}
\sigma_\theta &= \left(6\psi\sigma_0 E\varepsilon(1 - \mu^2)^{-1}(1 + m e^n(n + 1)^{-1})\right)^{1/2} \\
\sigma_\varphi &= \left(6\psi\sigma_0 E\varepsilon(1 - \mu^2)^{-1}(1 + m e^n(n + 1)^{-1})\right)^{1/2}
\end{align*}
$$

(8)

Let us suppose that experimental values of circumferential residual stresses on the surface of the rod after a drawing process are known:

$$
\sigma_\psi\big|_{r=R_d} = \sigma_\psi
$$

(9)

When the circumferential residual stresses are known, from the equations (7, 9) the parameter $\bar{\alpha}_1$ can be determined as:

$$
\bar{\alpha}_1 = 2\mu\sigma_\psi.
$$

(10)

Having regard to the equations (9) the parameter of deformation can be determined from the equations (8) as:

$$
\psi = \frac{1 - \mu^2}{6\sigma_0 E}\left(6\sigma_0 E\varepsilon(1 + m e^n(n + 1)^{-1})\right)^{-1}.
$$

(11)

As it is seen from the equation (11) the parameter of deformation is determined on the strain-stress behavior, mechanical properties of material, and process variations.

4. Impact of process variables on residual stress distribution

Residual stresses significantly impact on the post-deformation strength of metal products [10-11]. It is acknowledged that tensile residual stresses reduce the crack growth resistance [12-13], as well as the fatigue strength of metal products. Compressive stresses, as it is widely accepted, are considered as favorable. However, it is important to note that if service stresses are also compressive the superposition of both residual and service stresses of the same sign may lead to the exceedance of a yield strength or even an ultimate strength and following irreversible deformations and failure [14-15]. Therefore, it is necessary to research and analyze factors and parameters having an effect on the distribution of residual stresses formed in the manufacture.

The research paper [16] estimated the axial, circumferential, and radial residual stresses in calibrated rods of steel 35 ($D_d = 36$ mm) depending on the angle of the die and the degree of the compression. The degree of compression $Q$ determined as:

$$
Q = \left(\frac{D_0^2 - D_d^2}{D_0^2}\right) \cdot 100\%
$$

$D_0$ and $D_d$ are diameters of rod workpiece before and after the deformation process respectively.

The Sachs boring-out method was used for experimental measurement of residual stresses. For realization of this method, a fractional four-factor experiment was planned and implemented. The authors claim that parameters of the calibration process have a significant impact on residual stresses, that not only vary in magnitude, but also alternate in signs. In the experiments the angle of the die ranges from 8° up to 24°, the axial and circumferential residual stresses simultaneously increased 1.8 and 2.3 times respectively. The maximum tensile axial and circumferential stresses were recorded on the surface of the rod, and the maximum compressive stresses were recorded in the center. The change of sign occurs at a depth of $\bar{r} = 0.5 - 0.8$. As for the radial stresses, they have a negative sign (compressive) inside of the rod while equal to zero on the surface.

At Fig. 1 is shown the distribution curves of the circumferential stresses depending on the compression degree when the angle of the die is 20°. Ranging the compression degree from 5% to 34%, the circumferential residual stresses simultaneously increase by 2.5 times, $\sigma_{\psi \text{max}} = 270 \text{ MPa}$ for $Q = 20\%$.

The experiments also established the influence of the main parameters of the calibration process on the value and nature of the distribution of axial, circumferential, and radial residual stresses in the cross-section of cylindrical rods. It was revealed that in the calibration modes used in production by cold working with pressure, the tensile axial and circumferential residual stresses are formed in the peripheral layers of rods, and the compressive ones are formed in the central layers. The radial residual stresses equal to zero on the surface, while in the rest of the volume of the workpiece are compressive.
Applying the energy approach determined the distribution of calculated technological residual stresses after the plastic deformation of the rods. At Fig. 2, 3 are shown the distribution curves of calculated axial and circumferential residual stresses along the radial coordinate $\bar{r}$ at various degrees of the compression and angles of the die.

![Figure 1](image1.png)

**Figure 1.** The distribution curves of the circumferential stresses depending on the degree of compression of the rod.

![Figure 2](image2.png)

**Figure 2.** The distribution curves of axial residual stresses depending on the degree of compression and the angle of the die.
The distributions of residual stresses, shown at Fig. 2, 3, were determined depending on the degree of compression and the angle of the die taking into account the parameter of deformation, that was found from the equation (10) knowing residual stresses on the surface (Fig. 1), for rods of steel 35.

The graphs show that the axial and circumferential residual stresses increase in magnitude by nearly 2 times as the degree of compression ranges from 4% up to 34%. As the angle of the die ranges from 8° up to 24°, both the axial and circumferential stresses increase by nearly 2.15 times. Thus, the value of the taper angle of the wire affects the circumferential residual stresses more than the axial stresses.

The change of signs occurs at a depth of $\bar{r} = 0.7$ for the axial stresses and $\bar{r} = 0.5 - 0.6$ for the circumferential ones. In the central layers of the rod, compressive (negative) residual stresses are observed, while on the surface – tensile (positive) residual stresses.

### Figure 3. The distribution curves of circumferential residual stresses depending on the degree of compression and the angle of the die

The graphs show that the axial and circumferential residual stresses increase in magnitude by nearly 2 times as the degree of the compression ranges from 4% up to 34%. As the angle of the die ranges from 8° up to 24°, both the axial and circumferential stresses increase by nearly 2.15 times. Thus, the value of the taper angle of the wire affects the circumferential residual stresses more than the axial stresses.

### 5. Conclusion

In the work the energy approach was applied to determine technological residual stresses. Obtained equations of the circumferential, axial, and radial residual stresses are found in the analytical form and depend on the main process variables, geometry, and mechanical properties of the workpiece material. There was proposed the method of determining the deformability parameter for the case of drawing rod metal products that allows refining decisions on determining residual stresses for specific deformation conditions and materials, analyzing and revealing the influence of manufacturing and processing on the formation of technological residual stresses in solid axisymmetric profiles.

The analysis of the distribution of residual stresses over the cross section of rod, the theoretical and experimental dependences on the deformation conditions is carried out. The analysis of the results showed that the proposed methods are confirmed by the experimental data known from the literature and the dependences of the distributions of technological residual stresses in the rods after drawing.
References

[1] Keller I E, Trofimov V N, Vladynkin A V, Plusnin V V, Petukhov D S and Vindokurov I V 2018 On the reconstruction of residual stresses and strains of a plate after shot peening J. Samara State Tech. Univ., Ser. Phys. Math. Sci. 22 pp 40–64

[2] Kuznetsova E V, Kolmogorov G L and Vavel N V 2016. Process residual stresses in the production of zirconium sheets Russian Journal of Non-Ferrous Metals pp 26–31

[3] Keller I E, Petukhov D S, Kazantsev A V and Trofimov V N 2018 The limit diagram under hot sheet metal forming. A review of constitutive models of material, viscous failure criteria and standard tests J. Samara State Tech. Univ., Ser. Phys. Math. Sci. 22 pp 447–486

[4] Trofimov V N , Shiryaev A A and Karmanov V V May 2018 Influence of the position of the collimator of an X-ray diffractometer on the value of measured residual stresses AIP Conference Perc., AIP Publishing 2053 040907

[5] Klemenz M, Schulze V, Vöhringer O and Löhe D 2006 Finite element simulation of the residual stress states after shot peening Materials Sci. Forum 524–525 pp 349–354

[6] Kolmogorov G L and Kuznetsova E V 2017 Method for calculating the limiting technological residual stresses in a tubular billet Russ. Metallurgy (Metally) 2017 pp 237–239

[7] Trofimov V N, Karmanov V V, Shiryaev A A and Zvonov S N 2019 Calibration of X-Ray Diffraction Instruments for Residual-Stress Measurement Russ. Engin. Res. 39 pp 276–278

[8] Timoshenko S P and Goodier J N 1975 Theory of Elasticity (Moscow: Nauka)

[9] Arutyunyan A R and Arutyunyan R A 2010 The fatigue fracture criterion based on the latent energy approach Engineering 2 pp 318–321

[10] Totten G E, Howes M A H and Inoue T 2002 Handbook of Residual Stress and Deformation of Steel (Ohio: ASM International, Materials Park)

[11] Kraus I and Ganey N 1999 X-ray Analysis of the Inhomogeneous Stress State // Defect and Microstructure Analysis by Diffraction, ed Snyder R, Fiala J and Bunge H-J (Oxford; New York: Oxford University Press) pp 367–401

[12] Guo X, Leung A Y T, Chen A Y, Ruan H H and Lu J August 2010 Investigation of non-local cracking in layered stainless steel with nanostructured interface Scripta Materialia, pp 403 – 406

[13] Noyan I C and Cohen J B 1987 Residual Stress Measurements by Diffraction and Interpretation (NY: Springer-Verlag)

[14] Nikulin S A, Fedin V M , Rozhnov A B et al. 2016 Effect of volume-surface hardening on the cyclic strength of fragments of solebars of freight bogies Metal Science and Heat Treatment 57 pp 678–683

[15] Nikulin S A , Oguenko V N, Rozhnov A B et al. 2016 Strength of freightbogie solebar fragments after volume-surface quenching Russ. Metallurgy (Metally) 2016 pp 996–1001

[16] Zaides S A and Nguyen Van Khuan 2017 Determination of residual stresses in the calibrated rod Izvestiya. Ferrous Metallurgy 60 pp 109–115.