On Some Types of Proximity $\psi$–set

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Abstract. Various forms of $\psi$- set corresponding to different cases of spaces were introduced in a previous studies and The relationships among them therefore the focus of the study in this paper is the nature of the effects that can be obtained by using proximity spaces when studying this type of set by investigated a new class of sets called $\psi\delta$-set and $F\psi$–set in $i$-topological proximity spaces.

Keywords. focal function, focal closure, $i$-topological proximity space, $\psi\delta$-set, $F\psi$–set

1. Introduction
These guidelines, The ideal topological space $(X, \tau, I)$ is a topological space $(X,T)$ with an ideal $I$ over $X$ and was defined by Kuratowski K. in 1933 [1], This topic has become important after the studies presented by the two scholars T.R. Hamlett and Jankovic D.[2] in these spaces, which were followed by applications in various fields such as fuzzy, nano, soft spaces and here some of these researches [3,4,5,6,7,8] and by using the local function which is relates to the end points of the set $A$ in ideal Topological space where it is indicated by $A^*$ is formed the basis for the definition of operator $\psi: P(X) \to T$ by the complement factor for this operator and it was defined by $\psi(A) = X / (X / A)^*$ which introduced by Natkaniec T. in [9]. After that Bandyopadhyay C, Modak S. A [10] introduce The Definition of $\psi$- set in $(X, \tau, I)$ of $A \subseteq X$ as the set satisfy $A \subseteq \text{Int(Cl}(A))$ and the collection of all $\psi$ -set in $(X, \tau, I)$ is indicated by $\tau^v$ also another definition of $\psi$- set introduced by M. Monirul Islam and S. Modak [11] which is indicated by $a^{\psi}-$set and this type of sets satisfy that $A \subseteq (\psi(A^*))$.

Therefore, it will be interesting to study previous concepts in i-topological proximity spaces and in this context, we introduce a sets that is called a $\psi\delta$-set and $F\psi$–set which based in its definition on the concept of focal function [12] and the $\psi\delta$-operator [13] in the i- topological proximity spaces.

Throughout this research, $(X, \tau, I, \delta)$ always means the i- topological proximity spaces which is defined in a previous paper [12], and it is depends on both the i- topological space presented by Z.Irina [14] and the proximity space that defined by the researcher Riesz and developed by the researcher V.A. Efremovich [15] the proximity theory was led to a new type of topological spaces and various types of studies were presented within this field in a different shapes [16,17,18,19,20,21,22,23,24,25].Also in this paper we will discuss a set of properties related to these sets $\psi\delta$-set and $F\psi$–set in the space $(X,T,I,\delta)$.

In this paper i-TS means that the space is i-topological space and i-TPS means the space is i-topological proximity space.

2. Basic definition

2.1 Definition [12]
Let \( (X, T, I) \) is i-TS, a subset \( A \) of \( X \) is termed focal set of a point \( x \) in \( X \) if there is i-open set \( U \) satisfy \( \text{U} \cap A \), where \( \text{U} \cap A \) means that \( U - A \in I \). we will use \( I_f(x) \) to stand for the set of all focal set of the point \( x \).

2.2 Definition [12]

Let \((X,T,I)\) is i-TS then a point \( x \) in \( X \) is named focal limit point (simply \( \mathcal{F} \)lp) if and only if for all \( U \subseteq I_f(x) \) such that \( x \in U \) then \((U \setminus {A})/\{x\} \neq \emptyset\) and the set of all \( \mathcal{F} \)lp points of \( A \) is named the focal derived set and indicated by \( \mathcal{F} \mathcal{D}(A) \). Also the focal closure (simply \( \mathcal{F} \text{cl}(A) \)) is indicated by \( \mathcal{F} \text{cl}(A) = \mathcal{A} U \mathcal{F} \mathcal{D}(A) \).

2.3 Definition [12]

Let \((X,T,I)\) is i-TS, for a subset \( A \) of \( X \) the \( i - \text{cl}(A) \) is the intersection of all i-closed sets of \( A \).

2.4 Definition [12]

Let \((X,T,I,\delta)\) is i-TPS, for a subset \( A \) of \( X \) the point \( x \) in \( X \) is termed occlusion point of \( A \) if for all \( M \) belong to \( I_f(x) \), \( x \in M \) satisfy that \( M \cap B \). We will use \( \text{\#}(A) \) to stand for the set of all occlusion points of the set \( A \).

2.5 Proposition

In the \( i \)-TPS \((X,T,I,\delta)\) we have the following for each subset \( A \) of \( X \)

1. \( \text{Fcl}(A) \subseteq \#(A) \)
2. \( \psi_{T(X)}(A) \subseteq \psi_{\delta}(A) \) [26]
3. \( A \subseteq \#(A) \).
4. \( A \subseteq \#e(A) \) for each i-open set \( A \) of \( X \).
5. \( \#(AUB) = \#(A)U\#(B) \). [12]
6. \( \psi_{\delta}(A) \cup \psi_{\delta}(B) \subseteq \psi_{\delta}(A \cup B) \). [26]
7. \( \psi_{\delta}(A) \subseteq \#(A) \) [26]
8. \( \#(\psi_{\delta}(A)) \subseteq I_f(x) \) for some \( x \) in \( X \).

2.6 Definition

Let \((X,T,I,\delta)\) is \( i \)-TPS then:

1. \( \psi_{\delta}(A) = \{x \in X: \exists U \in I_f(x) \text{ and } u \ll A \} \), where the relation \( u \ll A \) means that \( u \delta A \). [28]
2. \( \psi_{T(X)}(U) = \{x \in X: \text{ there exist } U \in T(x), u \ll U \} \).[27]

2.7 Definition [28]

A mapping \( f : (X, \delta_X) \rightarrow (Y, \delta_Y) \) is said to be proximity or \( \delta \) - continuous if \( A \delta_X B \) then \( f(A) \delta_Y f(B) \) for each \( A, B \subseteq X \).

2.8 Proposition

A mapping \( f \) from a proximity space \((X, \delta_X)\) into a proximity space \((Y, \delta_Y)\) is \( \delta \) - continuous iff for each \( P, Q \subseteq Y \) \( P \delta_Y Q \) implies \( f^{-1}(P) \delta_X f^{-1}(Q) \).

2.9 Definition [28]

If \( f : (X, \delta_X) \rightarrow (Y, \delta_Y) \) is bijective \( \delta \) - continuous mapping and \( f^{-1} : (Y, \delta_Y) \rightarrow (X, \delta) \) is \( \delta \) - continuous mapping then \( f \) is said to be proximally equimorph, proximally isomorphic or \( \delta \) - homeomorphism from \( X \) onto \( Y \).

2.10 Proposition

Let \( f : (X, T, I) \rightarrow (Y, T_Y, I_Y) \) is i-closed function then for any i-closed set \( A \) of \( X \)
\[ \text{Fcl}(f(A)) = f(\text{Fcl}(A)) \]

2.11 Definition [28]

Let \( f : (X, T, I) \rightarrow (Y, T_Y, I_Y) \) is any function then we say that \( f \) is focal function if and only if \( f(U_x) \in I_Y \delta_x (f(x)) \), for each \( U_x \in I_f(x) \).

2.12 Proposition

Let \( f \) is homeomorphism function from \( i - TS \) \((X, T, I)\) into \( i - TS \) \((Y, T_Y, I_Y)\) then the inverse image of any focal set in \( Y \) is a focal set in \( X \).

Proof:
Let $U_y \in I_Y(y) \therefore$ so since $f$ is onto there exist exist $x \in X$ such that $y = f(x)$, also there exist $H \in T_Y(y) \therefore$ satisfy that $H / U_y \in I_y$ and $f$ is homeomorphism we get that $f^{-1}(H) / f^{-1}(U_y) = f^{-1}(H / U_y) \in f^{-1}(I_y)$ hence $f^{-1}(U_y)$ is a focal set of the point $x$.

3- On $\psi_\delta$-set and $\mathcal{F} \psi$ – set

3.1 Definition

Let $(X, T, I, \delta)$ is $i$ – TPS then a subset $A$ of $X$ is said to be $\psi_\delta$ – set iff $A \in \hat{\delta} (\psi_\delta(A))$ and it is called $\mathcal{F} \psi_\delta$ – set $\delta \mathcal{F} cl \left( \psi_{T,X}(A) \right)$, the collection of all $\psi_\delta$ – set of $X$ indicated by $\psi_\delta (X, T, I, \delta)$ and $\mathcal{F} \psi(X, T, I, \delta)$ stand for the set of all $\mathcal{F} \psi$ – set.

The relation between $\psi_\delta$ – set and $\mathcal{F} \psi$ – set showed below

3.2 Proposition

Let $(X, T, I, \delta)$ is $i$ – TPS then every $\mathcal{F} \psi$ – set is $\psi_\delta$ – set but not conversely.

Proof:

By proposition (2-5) (1, 2) we get that $\mathcal{F} cl \left( \psi_{T,X}(A) \right) \subseteq \hat{\delta} (\psi_\delta(A))$ and then since $A \delta \mathcal{F} cl \left( \psi_{T,X}(A) \right)$ then $A \in \hat{\delta} (\psi_\delta(A))$ therefor $A$ is $\psi_\delta$ – set.

3.3 Example

Let $X = \{a, b, c\}$, $T = \{X, \emptyset, \{a, b\}, \{a, c\}\}$, $I = \{\emptyset, \{c\}\}$, $A \delta \mathcal{B}$ iff $A \cap \mathcal{B} \neq \emptyset$. If $A = \{a\}$ then

$\hat{\delta} (\psi_\delta(A)) = X \subseteq \mathcal{F} cl \left( \psi_{T,X}(A) \right)$ hence $A$ is $\psi_\delta$ – set but not $\mathcal{F} \psi$ – set.

3.4 Proposition

Let $(X, T, I, \delta)$ is $i$ – TPS then each of the following statement are exist:

1. $A$ is $\psi_\delta$ – set for each $i$ – open set $A$
2. $A$ is not $\psi_\delta$ – set neither $\mathcal{F} \psi$ – set for each $A \in I$
3. If $A, B \in \psi_\delta(X, T, I, \delta)$ then $A \cup B \in \psi_\delta(X, T, I, \delta)$
4. $X \in \psi_\delta(X, T, I, \delta)$ and $X \in \mathcal{F} \psi(X, T, I, \delta)$

Proof:

1. Let $A$ is $i$-open set so by proposition (2-5) (3,4) and by we get that $A$ is $\psi_\delta$ – set
2. Let $A \in I$, so $\psi_\delta(A) = \emptyset$ [30] and then $A \delta \hat{\delta} (\psi_\delta(A))$ also by proposition (2-5)(2) $A$ is not $\mathcal{F} \psi$ – set.
3. since $A \delta \hat{\delta} (\psi_\delta(A))$ and $B \delta \hat{\delta} (\psi_\delta(B))$ also we have $A \subseteq A \cup B$ and $B \subseteq A \cup B$, so by property of proximity space we get that $A \cup B \delta \hat{\delta} (\psi_\delta(A))$ and $A \cup B \delta \hat{\delta} (\psi_\delta(B))$ and $\mathcal{F} \psi(A \cup B)$.
4. Since $X \delta \hat{\delta} (\psi_\delta(A))$ for each $A \subseteq X$, so $X \in \psi_\delta(X, T, I, \delta)$.

Now the following proposition discuss the inclusion condition

3.5 Proposition

Let $(X, T, I, \delta)$ is $i$ – TPS and $A, B$ are subset of $X$ such that $A \subseteq B$ and $A$ is $\psi_\delta$ – set (resp., $\mathcal{F} \psi$ – set), then $B$ is $\psi_\delta$ – set (resp., $\mathcal{F} \psi$ – set).

Proof:

Because $A$ is $\psi_\delta$ – set, then $A \delta \hat{\delta} (\psi_\delta(A))$ and by the axiom of proximity we get that $B \delta \hat{\delta} (\psi_\delta(B))$, hence $B$ is $\psi_\delta$ – set. In a same way we get that $B$ is $\mathcal{F} \psi$ – set.

3.6 Proposition

Let $(X, T, I, \delta)$ is $i$ – TPS and $A, B$ are subset of $X$ such that $A \cap B$ is $\psi_\delta$ – set (resp., $\mathcal{F} \psi$ – set) then $A$ and $B$ are $\psi_\delta$ – set (resp., $\mathcal{F} \psi$ – set).

Proof:

By proposition (3-5) we get that $A, B$ are $\psi_\delta$ – set.

3.7 Proposition

Let $(X, T, I, \delta)$ is $i$ – TPS then every $\psi_\delta(A)$ is $\psi_\delta$ – set for every subset $A$ of $X$ such that $\psi_\delta(A) \neq \emptyset$.

Proof:
Since by proposition (3-5) and proposition (2-5)(7) we have that $\psi_\delta(A) \subseteq \mathcal{F}(\psi_\delta(A))$ hence $\psi_\delta(A) \delta \neq \psi_\delta(A)$ therefor $\psi_\delta(A)$ is $\psi_\delta$ – set.

3.8 Proposition

Let $(X, T, I, \delta)$ is i – TPS then $A \in \psi_\delta(X, T, I, \delta)$ for each $A \in I_{\delta(x)}$.

Proof:

By proposition (2-5)(8) $\mathcal{F}(\psi_\delta(A)) \in I_{\delta(x)}$ for some $x \in A$, and since $A$ is focal set we get that $A \delta \neq \mathcal{F}(\psi_\delta(A))$ hence $A \in \psi_\delta$ – set.

The following example showed that proposition (3-8) is not exist with respect to $F\psi$ – set.

3.9 Example

Let $X = \{a, b, c\}$, $T = \{X, \emptyset, \{a, b\}, \{a, c\}\}$, $I = \{\emptyset, \{c\}\}$, $A \delta B$ iff $A \cap B \neq \emptyset$. If $A = \{a\}$ is $I_{\delta(x)}$ but $A$ is not $F\psi$ – set.

3.10 Remark

- Since $\emptyset \neq \mathcal{F}(\psi_\delta(A))$ for each $A \subseteq X$, then $\emptyset \notin \psi_\delta(X, T, I, \delta)$ also $\emptyset \notin F\psi(X, T, I, \delta)$
- $\psi_\delta(X, T, I, \delta)$, $(\psi_\delta(X, T, I, \delta))$, is not an ideal and not a filter
- If $A, B \in \psi_\delta$ – set then $A \cap B$ is not $\psi_\delta$ – set. Also if $A, B \in F\psi$ – set then $A \cap B$ is not $F\psi$ – set.

3.11 Example

Let $f: (X, T, I, \delta) \rightarrow (Y, T_Y, I_Y, \delta_Y)$ is homeomorphism, focal function and $\delta$ – homeomorphism function then $A \in \psi_\delta$ – set iff $f(A)$ is $\psi_\delta$ – set.

Proof:

we will prove the first condition that $\psi_\delta(f(A)) = f(\psi_\delta(A))$ and for that let $y \in \psi_\delta(f(A))$ so there exist $x \in X$, such that $y = f(x)$ and there exist $U_y \in I_{\delta(x)}(y)$, satisfy $U_y \delta_Y(f(A)) = f(A)$, since $f$ is $\delta$ – homeomorphism and $f$ is homeomorphism then $f^{-1}(U_y) \delta_X f^{-1}(f(A)) = A \delta_X$, now since $f$ is homeomorphisms and by proposition (2-12) $f^{-1}(U_y) \in I_{\delta_X}(x)$ and then $x \in \psi_\delta(A)$, so $f(x) \in f(\psi_\delta(A))$.

Let $y \in f(\psi_\delta(A))$, so $f^{-1}(y) \in \psi_\delta(A)$ and then there exist $U_{\delta^{-1}(y)} \in I_{\delta(x)}(f^{-1}(y))$, $U_{\delta^{-1}(y)} \delta_X A \delta_X$ and then by onto condition of $f$, $U_{\delta^{-1}(y)} \delta_X A \delta_X$, and because $f$ is $\delta$ homeomorphism we get $f(U_{\delta^{-1}(y)}) \delta_Y f(A \delta_X)$, since $f$ is focal function then $f(U_X) \in I_{\delta_Y}(f(x))$, so we get $f(U) \delta_Y (f(A))$. Then $y \in \psi_\delta(f(A))$.

Now to prove the second relation that $\mathcal{F}(\psi_\delta(f(A)) = f(\mathcal{F}(\psi_\delta(A)))$.

Let $y \in f(\psi_\delta(f(A)))$ then foreach $U_y \in I_Y(y)$, $U_y \delta_Y \psi_\delta(f(A))$ and by the first condition $U_y \delta_Y \psi_\delta(f(A))$, since $f$ is $\delta$ – homeomorphism that is $f^{-1}(U_y) \delta_X f^{-1}(f(\psi_\delta(A)) = \psi_\delta(A)$, but by proposition (2-12) $f^{-1}(U_y) \in I_{\delta(x)}(y)$, hence $x \in f(\psi_\delta(A))$ and $f(x) \in f(\mathcal{F}(\psi_\delta(A)))$. Conversely, let $y \in f(\mathcal{F}(\psi_\delta(A)))$, then $f^{-1}(y) = x \in f(\psi_\delta(A))$. and for each $U_x \in I_{\delta(x)}$, $U_x \delta_X \psi_\delta(A)$ but $f$ is $\delta$ – homeomorphism, $f(U_x) \delta_Y \psi_\delta(f(A))$ and by first condition $f(U_x) \delta_Y \psi_\delta(f(A))$, we get that $y \in f(\psi_\delta(f(A)))$. Now we go back to the assumption $A \delta \neq \mathcal{F}(\psi_\delta(A))$ and by the second relation, $A \delta \neq \mathcal{F}(\psi_\delta(f(A)))$, hence $f(A)$ is $\psi_\delta$ set.

Conversely, let $f(A)$ is $\psi_\delta$ – set then $f(A) \delta_Y \neq \mathcal{F}(\psi_\delta(f(A)))$ and by the first condition $f(A) \delta_Y f(\mathcal{F}(\psi_\delta(A)))$ and since $f$ is $\delta$ – continuous then $f^{-1}(f(A)) \delta_X f^{-1}(f(\mathcal{F}(\psi_\delta(A))))$, hence $A \delta_X \neq \mathcal{F}(\psi_\delta(A))$ and we have $A$ is $\psi_\delta$ – set.

3.13 Proposition

For the homeomorphism, $\delta$ – homeomorphism and focal function $f: (X, T, I, \delta) \rightarrow (Y, T_Y, I_Y, \delta_Y)$ and for any $i$ – closed set $A$ of $X$, $A \in F\psi$ – set iff $f(A)$ is $F\psi$ – set.

Proof:
Let A ∈ Ψψ – set, so AδFcl(ψ_T(ψ) (A)) since f is δ-homeomorphism then f(A)δf(Fcl(ψ_T(ψ) (A)) .

Now we will prove that ψ_T(ψ) (f(A)) = f(ψ_T(ψ) (A)) and for that let y ∈ ψ_T(ψ) (f(A)) , hence there exist \( U_y \in T(y) \), \( U_y \overline{\delta}(f(A))^c \) but f is homeomorphism and f is δ – continuous, then f\(^{-1}\)\((U_y)\overline{\delta}A^c \). f is continuous function, then f\(^{-1}\)\((U_y)\in T(x)\) imply that \( x \in ψ_T(ψ) (A) \) and then f(\( x \)) ∈ f(ψ_T(ψ) (A)) so by the onto condition of f y ∈ f(ψ_T(ψ) (A)) .

Now , let y ∈ f(ψ_T(ψ) (A)) then f\(^{-1}\) (y) ∈ ψ_T(ψ) (A) , so there exist w ∈ T(x) , w\overline{\delta}A^c but f is δ – continuous f(w)\overline{\delta}(A)^c and since f is homeomorphism f(w)\overline{\delta}(f(A))^c , but f(w) ∈ T(y) , so y ∈ ψ_T(ψ) (f(A)) .

By above conversation we have that (A)\overline{\delta}f(Fcl(ψ_T(ψ) (A)) = Fcl(f(ψ_T(ψ) (A)) = Fcl(ψ_T(ψ) (f(A))) . hence f(A) is Fψ – set .

Conversely , let f(A) = Fψ – set , so f(A)\overline{\delta}Fcl(ψ_T(ψ) (f(A))) but Fcl(f(A)) = Fcl(f(A)) and ψ_T(ψ) (f(A)) = f(ψ_T(ψ) (A)) , also since f is δ – continuous we get that (f(A))\overline{\delta}f\(^{-1}\)(Fcl(ψ_T(ψ) (f(A)))) therefor A\overline{\delta}Fcl(ψ_T(ψ) (A)) imply that A is Fψ – set .

Conclusion

Through this research we note that the two sets ψ_δ-set and Fψ – set which defined in i- topological proximity spaces is independent in its definition and the nature of its properties than ψ-set that knowledge in the ideal topological spaces as this is evident through the set of characteristics that have been proven in this research

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