Motivated by the recent lattice data that $J/\psi$ will survive up to $1.6T_c$, we calculate the thermal width of $J/\psi$ at finite temperature in perturbative QCD. The inputs of the calculation are the parton quarkonium dissociation cross sections at the NLO in QCD, which were previously obtained by Song and Lee, and a gaussian charmonium wave function, whose size were fitted to an estimate by Wong by solving the schrodinger equation for charmonium in a potential extracted from the lattice at finite temperature. We find that the total thermal width above $1.4T_c$ becomes larger than 100 to 200 MeV, depending on the effective thermal masses of the quark and gluon, which we take it to vary from 600 to 400 MeV.

1. Introduction

The original claim by Matsui and Satz [1], that $J/\psi$ suppression is a signature of quark gluon plasma, has witnessed a number of landmark developments, every aspect of which has to be taken into account consistently in confronting the recent RHIC data [2], and in predicting results for LHC. Among these theoretical developments are the phenomenologically successful statistical model for $J/\psi$ production [3, 4, 5], based on a coalescence assumption near $T_c$ [5], and the recent lattice calculations, showing strong evidence that the $J/\psi$ will survive up to $1.6T_c$ [6, 7, 8, 9]. While these two results seem at odd with each other, it only suggests that one still needs a more detailed understanding of the properties of heavy quark system in the quark gluon plasma, especially for temperatures between $T_c$ and $1.6T_c$, before a consistent picture of $J/\psi$ suppression in heavy ion collision is achieved.

In this respect, an important quantity to investigate is the effective thermal width, and/or the effective dissociation cross section of $J/\psi$ in the quark gluon plasma. Except for its existence, the present lattice results are far from making definite statements on the thermal width for Charmonium states above $T_c$. Hence, in this work, we will use the perturbative QCD approach to calculate the thermal width. So far, such calculations have been limited to dissociation processes by gluons to LO [11, 12, 13], because the elementary $J/\psi$-parton dissociation cross section was available only to that order [14, 15]. Recently, two of us have performed the dissociation cross section calculation to NLO in QCD [16]. Here, we will implement the NLO formula, to calculate the corresponding thermal width of Charmonium states in the quark gluon plasma.
The NLO calculation of $J/\psi$-parton dissociation calculations involves collinear divergence. When applying this elementary cross section to dissociation by hadrons, the collinear divergence is cured by mass factorization, which renormalizes the divergent part of the cross section into the parton distribution function of the hadron. Such complications disappear at finite temperature, as the thermal masses of the partons automatically renders the divergence finite.

2. Quarkonium hadron interaction in QCD

Let us begin with some introduction on the propagation of heavy quarks in the QCD vacuum. The propagation of a heavy quark can be approximated by a perturbative quark propagation with a perturbative gluon insertion, which probes the non perturbative gluon field configuration in the QCD vacuum. Hence, the full heavy quark propagator is,

$$iS^A(q) = iS(q) + iS(q)(-igA)iS(q) \cdots,$$

where, $iS(q) = i/(q - m)$ and $m$ is the heavy quark mass. The description in Eq. (1) is valid even for $q \to 0$, because $m \gg A \sim \Lambda_{QCD}$, where in the end only gauge invariant combination of the gauge field $A$ will remain after taking the vacuum expectation value.

Table 1. Physical processes involving two heavy quarks.

| $q^2$ | Process | Expansion parameter |
|-------|---------|---------------------|
| 0     | Photo production of open charm | $\Lambda_{QCD}^2$ |
| $-Q^2 < 0$ | QCD sum rules for heavy quark system | $\Lambda_{QCD}^2 / (4m^2 + Q^2)$ |
| $m_{J/\psi}^2 > 0$ | Dissociation cross section of bound states | $\Lambda_{QCD}^2 / (4m^2 - m_{J/\psi}^2)$ |

The propagation of a system composed of a heavy quark and an antiquark can also be approximated by combined perturbative heavy quark propagator with gluon insertions. However, since there are two heavy quarks involved, based on the operator product expansion, the propagation can typically be written in the following form.

$$\Pi(q) = \ldots + \int_0^1 dx \frac{F(q^2, x)}{(4m^2 - q^2 - (2x - 1)^2 q^2)^n} \langle G^n \rangle \cdots,$$

where, $F(q^2, x)$ is a function depending on the structure of the two quark system and $\langle G^n \rangle \sim \Lambda_{QCD}^{2n}$ denotes the typical gauge invariant expectation value of gluonic operator of dimension $2n$. The integration variable $x$ can be thought of as the momentum fraction carried by one of the heavy quark. Here, one notes that such perturbative expansion is valid when $4m^2 - q^2 \gg \Lambda_{QCD}^2$. The cases where this condition is satisfied and perturbative QCD treatments are possible are summarized in Table 1. In the last line of Table 1, $4m^2 - m_{J/\psi}^2 \approx (2m + m_{J/\psi})\epsilon_0$, where $\epsilon_0$ is the binding energy of the $J/\psi$. In QCD
if $m \to \infty$, the bound state becomes Coulombic and $\epsilon_0 = m[N_c g^2/(16\pi)]^2 \gg \Lambda_{QCD}$ in the large $N_c$ limit. Therefore, the expansion parameter becomes small and the dissociation cross can be calculated using perturbative QCD.

3. Thermal width of $J/\psi$

The LO invariant matrix element for the $J/\psi$ dissociation cross section by gluon was first obtained by Peskin[14] and rederived by one of us[17] using the Bethe-Salpeter equation. The NLO invariant matrix element was derived by two of us[16]. The dissociation by quarks comes in as part of the NLO calculation. The thermal width of $J/\psi$ is calculated by folding the elementary cross section with the thermal parton distribution function.

$$\Gamma_T = \text{deg} \int \frac{d^3k}{(2\pi)^3} n(k_0) v\sigma(s),$$

(3)

where $\text{deg}$ is the degeneracy of the partons, $n(k_0)$ is the thermal quark or gluon distribution function and $\sigma(s)$ is the elementary cross section, with $s$ being the square of the sum of the gluon and charmonium four momentum, and $v$ their relative velocity.

In both the LO and NLO calculations, $\sigma(s)$ is proportional to $|\partial \phi(p)/\partial p|^2$, where $\phi$ is the charmonium wave function and $|p|$ is related to the incoming four momentum that breaks the $J/\psi$ into $\bar{c}c$. As temperature increases, $n(k_0)$ favors higher $k_0$ or $|k|$, while the wave function favors smaller $|p|$, because the wave function becomes shallow and the momentum space wave function becomes smaller, for which we take the result given in ref[13].

For the LO calculation shown in Fig 1 (a), $k_0 = \epsilon_0 + p^2/(2m_c)$. Hence, larger $k_0$ means larger $p$. That means at higher temperature, although there are more energetic gluons, these are inefficient in breaking the charmonium states. Moreover, if one introduces thermal gluon mass, the LO width becomes even smaller.

For the NLO calculation, Fig 1 (b) and the corresponding diagram where the incoming quark is replaced by a gluon, have collinear divergences, which in the vacuum
is cured by mass factorization\cite{16}. At finite temperature, the problem can be avoided naturally by introducing thermal masses, which we take here to be from 600 MeV\cite{18} to 400 MeV, the smaller value giving an upper limit to the effective width. Moreover, even an energetic parton can radiate a gluon with small four momentum ($r$ in Fig 1 (b)), which can effectively dissociate the $J/\psi$ into open charms. Fig 2 shows the effective width of $J/\psi$ obtained by folding the elementary NLO quark (a) and the corresponding gluon (b) dissociation cross section obtained in ref\cite{16} by a massive thermal parton distribution. As can be seen from the figure, even with a upper (lower) limit of 600 (400) MeV thermal mass, the sum of thermal width due to quarks and gluons becomes larger than 100 (200) MeV above $1.4T_c$, where we take $T_c = 170$ MeV. Hence, although the $J/\psi$ might start forming at 1.6 $T_c$, the effective thermal width is very large and will not accumulate until the system cools down further\cite{5}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{effective_width.png}
\caption{Effective thermal width due to (a) quarks (b) gluons. Squares and circles are the results at temperatures where the $J/\psi$ radius is calculated in ref\cite{13}.}
\end{figure}

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