Even-denominator Fractional Quantum Hall Effect at a Landau Level Crossing

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The fractional quantum Hall effect (FQHE), observed in two-dimensional (2D) charged particles at high magnetic fields, is one of the most fascinating, macroscopic manifestations of a many-body state stabilized by the strong Coulomb interaction. It occurs when the filling factor (ν) of the quantized Landau levels (LLs) is a fraction which, with very few exceptions, has an odd denominator. In 2D systems with additional degrees of freedom it is possible to cause a crossing of the LLs at the Fermi level. At and near these crossings, the FQHE states are often weakened or destroyed. Here we report the observation of an unusual crossing of the two lowest-energy LLs in high-mobility GaAs 2D hole systems which brings to life a new even-denominator FQHE at ν = 1/2.

A strong magnetic field B applied perpendicular to the plane of a 2D electron system quantizes the electron energies into a set of discrete LLs, separated by the cyclotron energy. With increasing B, the degeneracy of each LL, which is equal to eB/ℏ, increases so that the number of occupied LLs, the filling factor ν, decreases (e is the electron charge and ℏ is Planck’s constant). The discrete LL structure combined with the decreasing ν gives rise to the integral QHE, the formation of incompressible states signaled by a vanishing longitudinal resistance Rxx and a quantized Hall resistance Rxy, when an integer number of LLs are fully occupied and the Fermi level (EF) lies in a gap separating two adjacent LLs [1–3]. At very low temperatures, and if disorder is low, electron-electron interaction leads to new phenomena. An example is the FQHE, the condensation of 2D electrons into many-body incompressible states which are phenomenologically similar to the integral QHE, but are stable predominantly at odd-denominator fractional ν [1–3].

Now a 2D system can have extra electronic (pseudo-spin) degrees of freedom, such as spin, valley, layer, or electric subbands. These lead to additional sets of LLs with different energy separations which can be tuned, e.g., by tilting the sample in the magnetic field to enhance the spin Zeeman energy [4], or by applying uniaxial strain to manipulate the valley splitting energy [5]. The tuning causes the LLs to cross each other. When a crossing occurs at EF, in a single-electron picture there is no gap at EF and the QHE is destroyed. In an interacting system, however, depending on the pseudo-spins of the crossing LLs, one can have QHE ferromagnetism where a gap opens at the crossing and stabilizes a ferromagnetic QHE state [6, 7]. For example in a two-valley system, even when the LLs of the valleys are tuned to have the same energy, both the integral QHE at ν = 1 and the FQHE ν = 1/3 survive [8, 9]. In other systems, when LLs belonging to different electric subbands and with opposite spins cross, the FQHE disappear [10, 11]. Here we describe unexpected phenomena in 2D hole systems (2DHSs) confined to GaAs quantum wells (QWs). We observe an unusual crossing of the two lowest-energy LLs. The crossing leads to a weakening or disappearance of the commonly seen odd-denominator FQH states in the filling range 1/3 ≤ ν ≤ 2/3. But, surprisingly, a new FQH state at the even-denominator filling ν = 1/2 comes to exist at the crossing.

Our samples were grown on GaAs (001) wafers by molecular beam epitaxy. The 2DHS is confined to 30- and 35-nm-wide GaAs QWs, flanked by undoped Al0.3Ga0.7As spacer layers and carbon δ-doped layers, and have a very high mobility μ ≥ 200 m²/Vs at low temperatures. We made samples in a van der Pauw geometry, 4 × 4 mm², and alloyed In:Zn contacts at their four corners. We then fitted each sample with an evaporated Ti/Al front-gate and an In back-gate to control the symmetry of the charge distribution in the QW and 2D hole density, n, which we give throughout this paper in units of 10¹¹ cm⁻². Unless otherwise noted, all the data presented here were taken in QWs with symmetric charge distributions. The measurements were carried out in a dilution refrigerator with a base temperature of T ≈ 30 mK and a superconducting magnet up to 18 T.

Figures 1-3 capture the highlights of our work. In Fig. 1(a) we show Rxx traces for 2D holes confined to the 30-nm-wide-QW for several densities, from 1.20 to 1.72 [12]. The x-axis in Fig. 1(a) is the ν⁻¹ in order to normalize B and align the FQH states for the different traces. At the lowest p (top trace), Rxx minima are observed at numerous fillings such as ν = 2/3, 3/5, 4/7, ..., and 2/5, 3/7, 4/9, ..., attesting to the very high quality of the sample. These FQH states and their relative strengths resemble those typically seen in high-mobility 2D holes (or electrons) confined to GaAs QWs. More important, they are all at odd-denominator ν and there is no FQHE at ν = 1/2. As p increases, starting at p ≈ 1.32 [12], an Rxx minimum develops at ν = 1/2, and quickly deep-
FIG. 1. (a) & (b): Magneto-resistance ($R_{xx}$) traces for 2D holes confined to 30- and 35-nm-wide GaAs QWs at several densities. The inset in (a) is the measured $\nu = 1/2$ FQH energy gap. (c) $R_{xx}$ for 2D holes confined to the 35-nm-wide QW at density $p = 1.31 \times 10^{11} \text{ cm}^{-2}$ and different tilting angles, as indicated. The traces in (a)-(c) are shifted vertically for clarity. (a') - (c'): Results of self-consistently calculated (at $B = 0$) charge distribution (red curve) and potential (black curve).

FIG. 2. (a) A Color-scale plot of $R_{xx}$ vs $B$ for the 35-nm-wide QW as $p$ is changed from 0.85 to $1.35 \times 10^{11} \text{ cm}^{-2}$. The solid white curve is a guide to the eye, showing the observed LL crossing trajectory. The dashed white curve represents the calculated position of the high-$B$ LL crossing (see text). (b) Calculated LL fan diagram, showing a crossing of the two lowest-energy levels at high $B$ ($\approx 8.6$ T).

FIG. 3. Schematic figures showing: (a) $\Psi_{331}$ state with layer degree of freedom. The two layers are linear combinations of the symmetric ($\psi_S$) and antisymmetric ($\psi_A$) subbands, and the total charge distribution is bilayer-like. The state has $\nu = 1/3$ FQHE-like correlations in each layer, as well as inter-layer correlation (Eq. 1). (b) For the hole systems in our experiments, the charge distribution is essentially single-layer-like so that the two pseudo-spin species effectively reside in the same layer.

ens and turns into a zero-resistance plateau centered at $\nu = 1/2$ for $p = 1.47$. Concomitantly, the $R_{xy}$ trace exhibits a Hall plateau quantized at $2h/e^2$, signaling the formation of a strong FQH state at $\nu = 1/2$. At a slightly higher density, $p = 1.59$, the $\nu = 1/2$ $R_{xx}$ minimum becomes weak, but returns again at higher $p$ and the $\nu = 1/2$ FQHE persists up to the highest densities we can achieve in this sample. Data for another QW sample with a wider well width of 35 nm, shown in Fig. 1(b), exhibit a qualitatively similar evolution, albeit in a somewhat lower density range. In particular, the $\nu = 1/2$ FQHE is strongest at $p = 1.03$.

The behavior of $R_{xx}$ near $\nu = 1/2$, and in particular
the strengths of the nearby FQH states, provide clear hints that a LL crossing is occurring in Figs. 1(a,b) data sets. This crossing is better seen in the color-coded plot of Fig. 2(a), which condenses many $R_{xx}$ traces for the 35-nm-QW sample (see [12] for a similar plot of the 30-nm-QW data). On the flanks of $\nu = 1/2$ and for $0.9 < p < 1.1$, there is an overall rise of $R_{xx}$, seen as a red band in Fig. 2(a) which starts at low $p$ on the right side (high field side) of $\nu = 1/2$ and gradually moves to the left and past $\nu = 1/2$ with increasing $p$. Away from this band, the odd-denominator FQH states are strong, indicated by deep $R_{xx}$ minima seen as blue stripes. The solid white line in Fig. 2(a) is a guide to the eye and shows the trajectory of the LL crossing, when the odd-denominator FQH states become weak or disappear. Surprisingly, the $\nu = 1/2$ FQHE is very strong at this crossing.

The data presented so far indicate a crossing near $\nu = 1/2$ between the two lowest-energy LLs of the 2DHS. But how could there be such a crossing? In most 2D systems, e.g., electrons confined to a GaAs QW, the energy separation between the lowest two LLs at a given $\nu$ is determined by either the Zeeman or cyclotron energies, both of which increase with density, or by the electric subband separation (in sufficiently wide QWs) which decreases with increasing density [13] but this separation remains finite and the two lowest-energy LLs do not cross. However, in 2D hole systems confined to relatively wide GaAs QWs where the heavy-hole (HH) and light-hole (LH) subbands are close in energy, an unusual crossing of the two lowest-energy LLs does indeed occur. This crossing can be qualitatively understood in a simple picture. The HH and LH bands have, respectively, heavy and light effective masses for the (bound) out-of-plane motion. For the in-plane motion, however, the HH bands have a smaller mass compared to the LH bands [14]. The energy of the lowest LL of the HH bands therefore increases more rapidly with $B$, and eventually intersects the lowest LL of the LH bands at sufficiently high $B$.

In a more quantitative picture, besides crossings, there is significant mixing and repulsion between the LLs in a 2DHS so that a more complex, non-linear LL fan diagram ensues. We demonstrate this in Fig. 2(b) where we show the results of our calculations, based on an $8 \times 8$ Kane model [14], of the LL fan diagram for a 2DHS at $p = 1.05$ and confined to a 25-nm-wide GaAs QW [15]. At $B = 0$, the two lowest-energy subbands are HH-like. The application of $B$ leads to mixing and numerous crossings and anti-crossings between various LLs of the HH and LH subbands, as seen in Fig. 2(b). Of particular interest to us is the crossing between the two lowest-energy levels at $B \approx 8.6$ T which corresponds to $\nu = 1/2$ for $p = 1.05$ [16, 17]. We believe this is the crossing at which the $\nu = 1/2$ FQHE emerges in our samples.

Three qualitative features of this LL crossing favor our interpretation. First, for a fixed QW width, the calculated field position of the LL crossing moves from high $B$ to low $B$ as $p$ is increased; see the dashed white curve in Fig. 2(a). The calculated curve has a steeper dependence on $p$ compared to the crossing seen experimentally (solid white curve), but the behavior is qualitatively similar and explains the successive weakening or disappearance and reappearance of the nearby, odd-denominator FQH states as $p$ is varied. Second, our calculations show that for a narrower QW, the crossing at $\nu = 1/2$ moves to a larger $p$, again consistent with the experimental observations in Figs. 1(a,b): also, compare Fig. 2(a) with Fig. S2 of [12]. Third, for a fixed QW width and $p$, our calculations indicate that when the charge distribution in the QW is made asymmetric, the crossing turns into an anti-crossing, meaning that the two lowest-energy LLs are always separated by a finite energy gap. This is also consistent with our experimental data: Even for a very small ($\lesssim 5\%$) charge distribution asymmetry in the QW, which we induce by adjusting the front- and back-gate biases, the nearby FQH states no longer exhibit a pronounced weakening, consistent with the absence of a LL crossing, and the $\nu = 1/2$ FQHE disappears.

Having established that the $\nu = 1/2$ FQHE state we observe indeed emerges at a LL crossing, it is natural to interpret it as a two-component FQH state described by the Halperin-Laughlin $\Psi_{331}$ wavefunction [18]:

$$\Psi_{331} = \prod_{i,j}(z_i - z_j)^3 \prod_{s,t} (w_s - w_t)^3 \prod_{i,s} (z_i - w_s)^1, \quad (1)$$

which has strong correlations between the two pseudo-spin components ($z$ and $w$ denote the coordinates of electrons with different pseudo-spins). The $\Psi_{331}$ state is believed to describe the $\nu = 1/2$ FQHE seen in bilayer electron systems [13, 19–21] confined to either wide GaAs QWs [13, 19, 21] or to double-QWs [20, 22]. In these systems, the two components are the "layers" which can be constructed through linear combinations of the symmetric and antisymmetric states. In electron systems confined to wide GaAs QWs, the energy separation between the states can be substantial but if it becomes only a small fraction ($\lesssim 0.1$) of the in-plane Coulomb energy $E_C = e^2/4\pi \epsilon l_B$, a $\nu = 1/2$ FQHE emerges ($\epsilon$ is the GaAs dielectric constant and $l_B$ the magnetic length) [13, 21]. Returning to the 2DHSs in our study, it is natural to associate the two crossing LLs with the two pseudo-spins needed for a $\Psi_{331}$ state. Since the energy separation between these two LLs is zero at the crossing, $E_C$ certainly dominates over the pseudo-spin energy separation. The emergence of a strong $\nu = 1/2$ FQHE at the crossing is therefore plausible.

The above interpretation raises interesting questions. First, in the case of electrons which are confined to either a wide or double GaAs QWs, the $\nu = 1/2$ FQHE is observed only when the charge distribution is bilayer-like (Fig. 3(a)) [13, 19, 21]. In our hole system, however, the charge distribution is essentially single-layer-like at
$B = 0$ (Fig. 3(b); see also Figs. 1(a’,b’)). Although we do not know the exact charge distribution at $\nu = 1/2$, we expect it to be qualitatively similar to $B = 0$ because deviations would cost significant electrostatic energy. Moreover, the mixing of LH states [17], which have smaller out-of-plane mass, should favor a single-layer-like charge distribution. It appears then that we are observing a $\Psi_{331}$ FQHE state in an essentially single-layer system (Fig. 3(b)). This is very surprising. Although the $\Psi_{331}$ state was originally proposed for a 2D system with spin degree of freedom [18], a $\nu = 1/2$ FQHE has never been reported for systems with spin or valley [29] degrees of freedom where the particles with different pseudo-spins are essentially in the same layer [30, 31].

Second, is there a $\nu = 1/2$ FQHE in 2DHSs which is equivalent to the state seen in electron systems in wide GaAs QWs, i.e., is stabilized when the system is bilayer-like? We believe that the $\nu = 1/2$ FQHE we observe at high $p$, well past the LL crossing, is indeed such a state. As reported elsewhere [32], at very high $p$, the 2DHS has a bilayer charge distribution which stabilizes a $\nu = 1/2$ FQHE. But the data in Figs. 1 and 2 for both samples show that, as $p$ is increased so that the LL crossing is moved to $\nu$ larger than $1/2$, the $\nu = 1/2$ FQHE weakens and then reappears again at higher $p$. The weakening is readily seen as a rise in $R_{xx}$ from zero at $p = 1.59$ for the 30-nm and at $p = 1.14$ in the 35-nm-QW samples [see Figs. 1(a,b) and 2(a)]. For the 30-nm-wide QW, we also measured the $\nu = 1/2$ FQHE energy gap $^{1/2}\Delta$ from the Arrhenius plots of $R_{xx}$ minimum vs inverse temperature, for several densities. The data, shown in Fig. 1(a) inset, corroborate our conclusion: $^{1/2}\Delta$ is highest at the LL crossing ($p \approx 1.45$) and exhibits a minimum at $p \approx 1.60$ before becoming larger again at higher $p$. This weakening signals a transition between a $\nu = 1/2$ FQHE stabilized by the LL crossing, and one stabilized by the bilayer-like charge distribution at high $p$.

Finally, we present results from our preliminary study of the same 2DHSs in the presence of an additional parallel magnetic field ($B_{\parallel}$) applied in the 2D plane; we introduce this $B_{\parallel}$ by tilting the sample so that the total applied field makes an angle $\theta$ with respect to the normal to the 2D plane. The data for the 35-nm-QW at a fixed density of $p = 1.31$ are shown in Fig. 1(c). Note that this density is higher than $p = 1.03$ at which the two lowest-energy LLs cross at $\nu = 1/2$ at $\theta = 0$ (see Figs. 1(b) and 2(a)). Consistent with Figs. 1(b) and 2(a), we indeed observe a reasonably well-developed $\nu = 1/2$ FQHE at $\theta = 0$ in Fig. 1(c). As we tilt the sample, the $\nu = 1/2$ FQHE becomes weaker and in fact disappears at $\theta = 17.5^\circ$. Surprisingly, at $\theta = 25^\circ$, a very strong FQHE reappears at $\nu = 1/2$. Note that in the $\theta = 17.5^\circ$ and $25^\circ$ traces in Fig. 1(c), the surrounding odd-denominator FQHEs become weak, signaling a LL crossing near $\nu = 1/2$. As the sample is further tilted to higher $\theta$, the $\nu = 1/2$ FQHE becomes weak and disappears, while the other FQH states get strong. The trace at highest angle, $\theta = 40^\circ$, indeed has a strong resemblance to the trace taken at low $p$ in this sample at $\theta = 0$ (see Fig. 1(b) top trace).

The evolution in Fig. 1(c) as a function of $\theta$ is very different from what is seen for 2D electrons confined to wide GaAs QWs. There, the $\nu = 1/2$ FQHE either quickly disappears as the sample is tilted, or initially becomes slightly stronger before disappearing [19, 32, 33]. On the other hand, the evolution in Fig. 1(c) is qualitatively similar to what we observe in the 2DHSs at $\theta = 0$ as we decrease $p$ (see Figs. 1(a,b)), suggesting that $B_{\parallel}$ induces a LL crossing near $\nu = 1/2$. This is not unexpected. Previous measurements in similar 2DHSs have indeed revealed a crossing of the two lowest-energy LLs as the sample is tilted in $B$ [35]. In Fig. 1(c) it is particularly noteworthy that the $\nu = 1/2$ FQHE essentially disappears (at $\theta = 17.5^\circ$) before becoming very strong at the LL crossing ($\theta = 25^\circ$). This is similar to what happens at $\theta = 0$ at $p = 1.14$ in the same sample (Fig. 1(b)), providing further evidence that a compressible state appears to separate the $\nu = 1/2$ FQHE observed at high $p$ from the one seen at the LL crossing.

It is remarkable that the LL crossing induced by $B_{\parallel}$ causes a significant increase in $R_{xx}$ near $\nu = 1/2$. Note that the $\theta = 25^\circ$ trace in Fig. 1(c) is shown reduced by a factor of 10, meaning that although $R_{xx}$ at $\nu = 2/3, 1/2$, and 2/5 is close to zero, $R_{xx}$ at other fields far exceeds the values in traces taken at other angles. This rise in $R_{xx}$ is more pronounced compared to the rise seen near the LL crossing at $\theta = 0$ (Figs. 1(a,b)). Evidently, the addition of $B_{\parallel}$ adds yet another twist to the fascinating phenomena observed in clean, interacting 2DHSs.

The results presented here attest to the rich many-body physics of the 2DHSs confined to GaAs QWs. They demonstrate an unusual crossing of the two lowest-energy hole energy levels in a large perpendicular magnetic field and, more remarkably, the emergence of a very rare, even-denominator FQHE at the crossing. We have tentatively interpreted this FQHE as a two-component $\Psi_{331}$ state, although a detailed and quantitative understanding of its origin and properties await future research.

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SUPPLEMENTARY MATERIALS

In this Supplement, we present additional magneto-transport data for the two-dimensional (2D) holes confined to the 30-nm-wide GaAs quantum well sample as the 2D hole density ($p$) is changed, while the charge distribution is kept symmetric. In the lower part of Fig. S1 we show $R_{xx}$ traces for several densities, ranging from 1.20 to 1.72 (all densities are given in units of $10^{11} \text{cm}^{-2}$). At the lowest $p$ (top $R_{xx}$ trace), minima in $R_{xx}$ are observed at numerous odd-denominator fillings such as $\nu = 2/3$, 3/5, 4/7, ..., and 2/5, 3/7, 4/9, ..., signaling strong fractional quantum Hall effect (FQHE) states. As $p$ increases, $R_{xx}$ develops a minimum at $\nu = 1/2$, starting at $p \approx 1.32$. This minimum quickly deepens and turns into a zero-resistance plateau centered at $\nu = 1/2$ for $p = 1.47$. Concomitantly, the $R_{xy}$ trace, shown in the upper part of Fig. S1, exhibits a Hall plateau quantized at $2\hbar/e^2$, providing evidence for the formation of a strong FQH state at $\nu = 1/2$. At slightly higher densities ($p = 1.54$ and 1.59), the $\nu = 1/2$ $R_{xx}$ minimum becomes weaker, but then it turns deep again and the FQHE persists up to the highest densities we can achieve in this sample.

In Fig. S1, note the rise in $R_{xx}$ on the right (high field) side of $\nu = 1/2$ as the density is raised from $p = 1.27$ and 1.32. The rise is seen nearly symmetrically on both sides of $\nu = 1/2$ at $p = 1.44$ and 1.47, and then it moves to the left of $\nu = 1/2$ as density is further increased; e.g., see the $p = 1.54$ trace. At the highest densities (lowest two trances) the $R_{xx}$ rise has disappeared and the traces look qualitatively similar to the low-density trace except that now there is a strong FQHE at $\nu = 1/2$. The rise in $R_{xx}$ qualitatively resembles a “wave” that moves from right to left (of $\nu = 1/2$) as the density is raised and, in its passage, weakens or destroys the odd-denominator FQH states. We associate the rise in $R_{xx}$ with a crossing of the two lowest-energy Landau levels.

For a better view of this crossing we provide the color-coded plot of Fig. S2. The overall rise of $R_{xx}$ is seen as a red band, which starts at low $p$ on the right side of $\nu = 1/2$ and gradually moves to the left and past $\nu = 1/2$ with increasing $p$. Away from this band, the odd-denominator FQH states are strong, indicated by deep $R_{xx}$ minima seen as blue stripes. The white line in Fig. S2 is a guide to the eye and shows the trajectory of the Landau Level crossing, which causes the odd-denominator FQH states to become weak or disappear. Clearly, the $\nu = 1/2$ FQH state is very strong at this crossing, and becomes weaker on either side of the crossing.

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There is another crossing of the lowest two LLs in 2DHSs, namely a crossing of the two spin levels of the HH-S subband. This crossing, which has been studied previously [F. Fischer, R. Winkler, D. Schuh, M. Bichler, and M. Grayson, Phys. Rev. B 75, 073303 (2007)], occurs at very low fields (B < 2 T) in our samples (see Fig. 2(b)).

The exact nature of the two crossing LLs is complicated; they have HH-like character but with significant LH admixture. Nonetheless, these two levels have the same LL quantum number (N = 0), but opposite parity.

In the higher (N = 1) LL, there is of course the enigmatic even-denominator FQHE observed at ν = 5/2 [R.L. Willett, J.P. Eisenstein, H.L. Stormer, D.C. Tsui, A.C. Gossard and J. H. English, Phys. Rev. Lett. 59, 1776 (1987)], whose origin is still unknown, but it is more likely described by a one-component (Pfaffian) wavefunction rather than $\Psi_{3\text{II}}$ [C. Nayak, S.H. Simon, A. Stern, M. Freedman and S. Das Sarma, Rev. Mod. Phys. 80, 1083 (2008)].

As this manuscript was being completed, we learnt of experiments by J. Falson, D. Maryenko, D. Zhang, B. Friess, Y. Kozuka, A. Tsukazaki, J.H. Smet, and M. Kawasaki (unpublished), where they see a $\nu = 3/2$ FQHE in a ZnO 2D electron system at a LL crossing.

FIG. 5. (Fig. S2) A color-scale plot of $R_{xx}$ vs magnetic field for the 30-nm-wide GaAs quantum well as $p$ is changed from 1.3 to 1.6 $10^{11}$ cm$^{-2}$. The red band shows the trajectory of the Landau Level crossing as a function of $p$, and the white curve is a guide to the eye.

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Our calculations, although state-of-the-art, do not quantitatively match the experimental data. We used a QW width of 25 nm in order to match the field value of the calculated crossing ($B \approx 8.6$ T) to what we measure experimentally in a 35-nm-wide QW (Fig. 2). Despite this discrepancy, whose origin is unknown at the moment, we emphasize that all the features of the calculated crossing are qualitatively consistent with the experimental data.

There is another crossing of the lowest two LLs in 2DHSs, namely a crossing of the two spin levels of the HH-S subband. This crossing, which has been studied previously [F. Fischer, R. Winkler, D. Schuh, M. Bichler, and M. Grayson, Phys. Rev. B 75, 073303 (2007)], occurs at very low fields ($B < 2$ T) in our samples (see Fig. 2(b)).