Simulation of shoreline development in a groyne system, with a case study Sanur Bali beach

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Abstract. The process of shoreline changes due to transport of sediment by littoral drift is studied in this paper. Pelnard-Considère is the commonly adopted model. This model is based on the principle of sediment conservation, without diffraction. In this research, we adopt the Pelnard-Considère equation with diffraction, and a numerical scheme based on the finite volume method is implemented. Shoreline development in a groyne system is then simulated. For a case study, the Sanur Bali Beach, Indonesia is considered, in which from Google Earth photos, the beach experiences changes of coastline caused by sediment trapped in a groyne system.

1. Introduction

Groyne is a hydraulic structure built along a seashore to control erosion. Commonly, a groyne is installed perpendicular or slightly oblique to the shoreline. If waves approach the coastline obliquely, fluid flow propagating along the coast will be generated. On a sandy coast, this flow together with the stirring action of breaking waves causes a longshore sand transport, also known as littoral drift.

Top view of the Sanur Bali beach in Indonesia is shown in Figure 1. The coastline changes form time to time in response to the installation of a breakwater groyne system. Although groyne system serves as a breakwater, the presence of groyne interrupts the water flow and reduces sand movement, thus reducing erosion. So that the presence of groyne also affects the changing of the coastline, as a result of the accumulation of sand sediments in the up current part of the groyne, as well as the erosion in the down current area of the groyne, see Figure 1 (top).

Shoreline development due to the newly constructed breakwater groyne will be studied here. The mathematical model of shoreline change is derived from mass conservation of sand. Without diffraction, the equation is quite simple, in literature it is known as the Pelnard-Considère equation $[1, 2, 3, 4, 5]$. In this article, shoreline change in groyne system is considered, and diffraction effect is incorporated. Further, we develop a finite volume numerical scheme for simulating shoreline change due to groyne installment. As recorded in $[6, 7]$, finite volume is a suitable method for solving problems based on conservation equation.

This paper is organized as follows, in Section 2 we derive a diffusion model based on the conservation of sediment, to describe coastline development in a groyne system. In the model,
the groyne is formulated as boundary condition. In Section 3, thefinite volume method is formulated. Several numerical experiments are given in Section 4. Finally, some conclusions and discussion are drawn in the last section.

2. The Shoreline Model with Diffraction Effect

Without going deeply into the physics, we can have insight of the consequences of constructing a groyne. Here, we adopt the approach by [2, 8] which we resumed here for the sake of clarity.

Let $y(x, t)$ denotes the shoreline at time $t$, whereas $S$ ($m^3/\text{sec}$) denotes the sediment discharge brought by the littoral drift, see Figure 2. Transport of sand is governed by the following balance of sand in the control interval $[x, x + \Delta x]$ as follows

$$S|_{x} \Delta t - S|_{x+\Delta x} \Delta t = (y|_{t+\Delta t} - y|_{t}) a \Delta x,$$

with $a$ denotes the water depth. Divide both sides with $\Delta x \Delta t$, followed by taking the limit $\Delta x \to 0$, $\Delta t \to 0$, will result the conservation of mass in terms of differential equations as
follows
\[ \frac{\partial S}{\partial x} + a \frac{\partial y}{\partial t} = 0. \] (1)

Further, sand discharge is given by
\[ S = S_0 - q \frac{\partial y}{\partial x}. \] (2)

An estimate formula from the CERC (Coastal Engineering Research Center) give us
\[ S_0 = Eh^2 \phi_x, \quad \text{and} \quad q = Eh^2, \] (3)

with \( E \) (m³/s) a coefficient proportional to incident wave energy flux, \( \phi_x \) the angle of wave crest with respect to the \( x \)-axis, and \( h \) is the ratio of wave height at \( x \) to wave height at \( x = \infty \).

Using (2), equation (1) becomes
\[ \frac{dS_0}{dx} - \frac{\partial}{\partial x} \left( q \frac{\partial y}{\partial x} \right) + a \frac{\partial y}{\partial t} = 0. \]

Next step, the physical shoreline is split into two parts: \( y_0 \) as the stationary effect from diffraction, and \( y'(x, t) \) as the non-stationary effect, to be precise
\[ y(x, t) = y'(x, t) + y_0(x). \] (4)

Substituting (3) into (1) gives us the following equation
\[ a \frac{\partial y'}{\partial t} - q \frac{\partial^2 y'}{\partial x^2} - \frac{dq}{dx} \frac{\partial y'}{\partial x} = -\frac{dS_0}{dx} + q \frac{\partial^2 y_0}{\partial x^2} + \frac{dq}{dx} \frac{dy_0}{dx}. \] (5)

Further, we choose \( y_0(x) \) such that it satisfies an ordinary differential equation
\[ \frac{dS_0}{dx} - q \frac{d^2 y_0}{dx^2} - \frac{dq}{dx} \frac{dy_0}{dx} = 0, \] (6)

whereas \( y'(x, t) \) satisfies the following partial differential equation
\[ a \frac{\partial y'}{\partial t} - q \frac{\partial^2 y'}{\partial x^2} - \frac{dq}{dx} \frac{\partial y'}{\partial x} = 0. \] (7)

In this new formulation, the stationary effect can be obtained by solving the ordinary differential equation (6), whereas the non-stationary effect from littoral sand drift is governed by a diffusion equation (7). Next, we formulate the boundary conditions. Far away at \( x \to \infty \) the breakwater groyne has no effect, hence
\[ \lim_{x \to \infty} y = 0, \quad \lim_{x \to \infty} y' = 0, \quad \text{and} \quad \lim_{x \to \infty} y_0 = 0, \lim_{x \to \infty} y'_0 = 0. \] (8)

At the breakwater \( x = 0 \), there is no discharge or \( S(0) = 0 \), then
\[ \phi_x(0) = \frac{\partial y}{\partial x}(0). \] (9)

Moreover, we can rewrite the Equation (6) as
\[ \frac{d}{dx} \left( S_0 - q \frac{dy_0}{dx} \right) = 0. \] (10)
Direct integration will give us

$$S_0 - q \frac{dy_0}{dx} = E h_\infty^2 \phi_\infty - E h_\infty^2 \frac{dy_0}{dx}|_\infty.$$  

Further, as $h$ be the ratio of wave height to wave height as infinity, obviously $h_\infty = 1$. By adopting boundary conditions (8) and relation (3) the equation above can be written as

$$\frac{dy_0}{dx} = \phi_x - \frac{\phi_\infty}{h^2}. \quad (11)$$

We note here that relation (11) holds in the general case, but in the case of no diffraction, wave height is constant which make the wave height ratio $h = 1$, and the equation (3) reduces to

$$\frac{dy_0}{dx} = \phi_x - \phi_\infty,$$  

which is exactly the Pelnard-Considère approximation.

Following Bakker [2], and under the assumption that $\phi_x$ and $\phi_\infty$ are small, the angle of wave crest $\phi_x$ can be approximated as

$$\phi_x = \phi_\infty - \frac{\lambda}{2\pi} \frac{d\theta}{dx}, \quad (12)$$

where $\lambda$ is the wave length and $\theta$ the phase difference. Substituting (12) into Equation (11) yields

$$\frac{dy_0}{dx} = - \frac{\lambda}{2\pi} \frac{d\theta}{dx} + \phi_\infty \left( \frac{h^2 - 1}{h^2} \right). \quad (13)$$

Integrating (13) give us

$$y_0(x/\lambda) = - \frac{\lambda}{2\pi} \theta(x/\lambda) + \phi_\infty \int_0^{x/\lambda} \frac{h^2 - 1}{h^2} \, dx, \quad (14)$$

after using $y_0(0) = 0$ and $\theta(0) = 0$. The first and second terms on the right hand side of (14) are due to diffraction and changing wave height, respectively. For computation, (14) is rewritten in normalized 'length' variables $x/\lambda, y_0/\lambda$ as

$$y_0/\lambda = - \frac{1}{2\pi} \theta(x/\lambda) + \phi_\infty \int_0^{x/\lambda} \frac{h^2 - 1}{h^2} \, d(x/\lambda). \quad (15)$$

The first term on the right hand side of (15) shows the influence of wave diffraction, whereas the second term is the influence of the changing in wave height.

Figure 3. Sketch and notations of shoreline development in the lee side of breakwater.
2.1. Computation of the stationary shoreline \( y_0 \)

Assuming the angle of wave incidence \( \phi_\infty \) is known, and the length of the groyne is \( l \), we first compute the phase difference \( w \) as a function of \( x/\lambda \)

\[
w = \frac{r - \eta}{\lambda} = \sqrt{\left(\frac{x}{\lambda}\right)^2 + \left(\frac{l}{\lambda}\right)^2 - \frac{l}{\lambda} \cos \phi_\infty - \frac{x}{\lambda} \sin \phi_\infty}
\]

(16)

The relative wave height \( h \) and the change in phase by diffraction \( \theta \) will be computed with diffraction theory. In order to calculate the integral in (15) the simplified theory by Putnam and Arthur [9] will be used. Let \( u = 2\sqrt{w} \), and define

\[
f(u) = -\frac{1}{\sqrt{2}} e^{i\pi/4} \int_{-\infty}^{u} e^{-i\pi v^2/2} dv.
\]

(17)

The wave height ratio \( h \) and phase difference \( \theta \) as functions of \( x/\lambda \) are obtained from the modulus and argument of \( f(u) \), respectively. With the help of Maple software, the curves of \( h(x/\lambda) \) and \( \theta(x/\lambda) \) can be obtained, and the results for \( \phi_\infty = 0.1 \) and \( l = 10\lambda \) are plotted in Figure 4.

![Figure 4](image-url)

**Figure 4.** The curves of \( w(x/\lambda) \) (red), \( h(x/\lambda) \) (blue) and \( \theta(x/\lambda) \) (green) plotted using \( l = 10\lambda \) and \( \phi_\infty = 0.1 \).

2.2. Computation of the non-stationary \( y' \)

Further we consider Equation (7) for the non-stationary shoreline \( y'(x, t) \) subject to boundary conditions \( \lim_{x \to \infty} y = 0 \) and \( \lim_{x \to \infty} y' = 0 \). Remember that \( y' = y - y_0 \), and since (9) we thus have

\[
\left( \frac{\partial y'}{\partial x} \right)_{x=0} = (\phi_x)_{x=0} - \left( \frac{dy_0}{dx} \right)_{x=0},
\]

\[
\left( \frac{\partial y'}{\partial x} \right)_{x=0} = \frac{\phi_\infty}{(h^2)_{x=0}}.
\]

(18)
Note that the last formulation is obtained after applying (11) for \( \left( \frac{dy_0}{dx} \right)_{x=0} \).

The following algorithm summarizes the procedure of calculating shoreline change at the lee side of the breakwater

**Algorithm 1** The algorithm for calculating shoreline change.

**Step 0.** Start.

**Step 1.** Given an angle of wave incidence \( \phi_{\infty} \) and breakwater length \( l \).

**Step 2.** Compute the phase difference \( w \) as a function of \( x/\lambda \) using (16).

**Step 3.** Compute \( \theta \) and \( h \) as functions of \( x/\lambda \) using (17).

**Step 4.** Compute the stationary condition \( y_0(x) \) using (15) and given the initial condition of \( y'(x,0) = -y_0(x) \).

**Step 5.** While (time ≤ final time) do Steps 6-7

- **Step 6.** Compute \( y'(x,t) \) using (7).

- **Step 7.** Update the real shoreline as \( y = y' + y_0 \).

**Step 8.** (The procedure was successful.) STOP

**Step 9.** End.

Note: in the computation, diffraction effect is only considered in the lee side of breakwater.

3. Finite Volume Method

In order to solve (7) numerically, here we present the finite volume method for one-dimensional unsteady follows as given in of Versteeg et. al. [10]. First step of this method is to define the domain into sub-domains in discrete control volumes. Let us dividing our domain \( \Omega = [0,L] \) into a mesh \( M = \{1,2,\ldots,N_x\} \) nodal of points. Here, we denote \( i \in M \) is the general nodal point in our discretization. Moreover, the interfaces of control volume \( C_i \) of nodal \( i \) are defined by \( i \pm \frac{1}{2} \) (see Figure 5). In this case, we assume that the distance between \( i \)-th node to the its neighbours nodal points is in uniform which is \( \Delta x \).

![Figure 5. The control volume \( C_i \) of nodal point \( i \) with uniform discrete space distance \( \Delta x \). The interfaces of \( C_i \) are defined at points \( i - \frac{1}{2} \) and \( i + \frac{1}{2} \)](image)

The sense of the finite volume method is the integration of the governing equation (7) over a control volume \( C_i \) and over a time interval \( t \) to \( \Delta t \) in nodal point \( i \). Then for all \( i \in M \) we have,
\[\int_{t}^{t+\Delta t} \int_{C_i} \frac{\partial y}{\partial t} \, dC_i \, dt = \int_{t}^{t+\Delta t} \int_{C_i} D \frac{\partial^2 y}{\partial x^2} \, dC_i \, dt, \tag{19}\]

\[= \int_{t}^{t+\Delta t} \int_{C_i} \frac{\partial}{\partial x} \left( D \frac{\partial y}{\partial x} \right) \, dC_i \, dt. \tag{20}\]

This can be written as

\[\int_{i-\frac{1}{2}}^{i+\frac{1}{2}} \left[ \int_{t}^{t+\Delta t} \frac{\partial y}{\partial t} \, dt \right] \, dC_i = \int_{t}^{t+\Delta t} \left[ \left( D \frac{\partial y}{\partial x} \right)_{i+\frac{1}{2}} - \left( D \frac{\partial y}{\partial x} \right)_{i-\frac{1}{2}} \right] \, dt. \tag{21}\]

In Equation (21), \(A\) is the face area of control volume \(C_i\), which is equal to \(\Delta V/\Delta x\), where \(\Delta V\) denotes the volume. Further, we denote \(y_{i+1}^n\) and \(y_i^n\) are the distance of shoreline in \(i\)th node at time \(t + \Delta t\) and \(t\) respectively. Thus, the left hand side of equation (21) can be written as

\[\int_{i-\frac{1}{2}}^{i+\frac{1}{2}} \left[ \int_{t}^{t+\Delta t} \frac{\partial y}{\partial t} \, dt \right] \, dC_i = (y_{i+1}^n - y_i^n) \Delta V. \tag{22}\]

If we approximate the diffusion terms on right hand side of Equation (21) by central difference, thus we have

\[(y_{i+1}^n - y_i^n) \Delta V = \int_{t}^{t+\Delta t} \left[ \left( D \frac{\partial y}{\partial x} \right)_{i+\frac{1}{2}} - \left( D \frac{\partial y}{\partial x} \right)_{i-\frac{1}{2}} \right] \, Adt. \tag{23}\]

where \(D_{i+\frac{1}{2}} = (D_{i+1} + D_i)/2\) and \(D_{i-\frac{1}{2}} = (D_i + D_{i-1})/2\).

By generalizing the integral of shoreline distance with respect to time with weighted parameter \(\theta \in [0, 1]\),

\[\int_{t}^{t+\Delta t} y_i \, dt = [\theta y_{i+1}^{n+1} + (1 - \theta)y_i^n] \Delta t, \tag{24}\]

hence we get

\[\left( \frac{y_{i+1}^n - y_i^n}{\Delta t} \right) \Delta x = \theta \left[ \frac{D_{i+\frac{1}{2}}(y_{i+1}^{n+1} - y_i^{n+1})}{\Delta x} - \frac{D_{i-\frac{1}{2}}(y_{i}^{n+1} - y_{i-1}^{n+1})}{\Delta x} \right], \tag{25}\]

\[+(1 - \theta) \left[ \frac{D_{i+\frac{1}{2}}(y_{i+1}^{n} - y_i^{n})}{\Delta x} - \frac{D_{i-\frac{1}{2}}(y_i^{n} - y_{i-1}^{n})}{\Delta x} \right]. \tag{25}\]

Note that in Equation (25), if \(\theta = 0\) is called fully explicit and \(\theta = 0.5\) is called the Crank-Nicolson scheme. As explained, in Chapter 5 and 8 in [10], in the case of explicit scheme, the scheme are conditionally stable. The numerical scheme should satisfy the positiveness of coefficients which are defined as

\[\Delta t \leq 0.5 \frac{(\Delta x)^2}{D}. \tag{26}\]

However, in the case of fully implicit (\(\theta = 1\), this condition is not necessary since implicit scheme is unconditionally stable.
4. The Results of Numerical Simulation

In this section, three scenarios of numerical simulation will be given. The simulation of simple coastline development model without diffraction effect will be elaborated in first simulation. In the second simulation, the coastline model which takes into account the diffraction effect will be given. Finally, simulation in study case of Sanur Bali beach will be elaborated in the third simulation.

4.1. Simple coastline development model

Here, we implement the numerical scheme to simulate shoreline change due to the installation of one groyne without diffraction. Omitting the diffraction effect, \( y_0 (x) = 0 \), the discharge \( q \) and the angle of wave crest \( \phi_x \) in (7) is assumed to be constant. Thus the shoreline development is governed by the diffusion equation below

\[
a \frac{\partial y'}{\partial t} - q \frac{\partial^2 y'}{\partial x^2} = 0.
\]

The finite volume scheme (25) as described in the previous section is adopted, with \( D = q/a = S_0 (a\phi_x)^{-1} \) after using the relation (3).

![Shoreline change development around a groyne as functions of time](image)

**Figure 6.** Shoreline change development around a groyne as functions of time \( t = 0, 0.6, 1.2, \) and 3.6 years.

Taking the computational domain as \( \Omega = [0, 10] \) km with \( \Delta x = 0.25 \) km. A groyne is located at the middle of domain, whereas the length of the groyne is assumed to be infinite. The
coefficient $D$ is computed using parameters $S_0 = 1.5 \times 10^6 \text{ m}^3/\text{year}$, water depth $a = 18 \text{ m}$, and $\phi_x = 0.2 \text{ rad}$. In this simulation, we use the explicit finite volume scheme (25) with $\theta = 0$, and we did not take into account the diffraction effect.

Simulation results at subsequent times $t = 0, 0.6, 1.2$ and $3.6 \text{ years}$ are shown in Figure 6. We can see clearly that on the up current part of the domain there is accumulation of sand, whereas on the downstream part erosion occurs. Here, we can observe that the accumulation and erosion of sand is in equilibrium. Indeed this equilibrium occurs since the diffraction effect is not considered.

Further simulation, we present the shoreline development due to the installation of a groyne system consisting of three groynes. Domain of the simulation is $\Omega = [0, 20] \text{ km}$. The first groyne is installed at $5 \text{ km}$ in domain of simulation. Moreover, the second and third groyne are installed at $10 \text{ km}$ and $15 \text{ km}$ of the domain respectively. The other parameters are the same with the previous simulation. Result of this simulation is given in Figure 7. Similar to the result using one groyne, the accumulation of sand is observed in the left side of groynes. Meanwhile, erosion of sand is occurred on the right side of groynes. Without diffraction, the simulation results of shoreline change were not physically relevant. Therefore in the next subsections, the simulation which takes into account the diffraction effect will be elaborated and validated by a real shoreline from Google Earth photo.

4.2. Shoreline change due to diffraction effect
Here, two numerical simulations with the diffraction effect will be conducted. In the first simulation, only the right part of the groyne is considered. Parameters used in this simulation are as as follows

- The length of groyne is $10\lambda$.
- The angle of wave incidence is $\phi_\infty = 5^\circ$.

Computational domain is taken to be $\Omega = [0, 10]/\lambda$ and it is discretized uniformly with a discrete space $\Delta x = 0.5$ and discrete time $\Delta t = \Delta x^2/8$. Using Algorithm 1, the results of this simulation can be found in Figure 8. We can see that simulation result is now physically relevant, at the groyne $x/\lambda = 0$ the transported sand is not as much as those at $x/\lambda = 0.87$. This is because of the wave incident is set to be $\phi_\infty = 5^\circ$, thus the point where $w = 0$ is calculated by $x/\lambda = 1/\lambda \tan \phi_\infty = 0.87$. Moreover, at $x \to \infty$, no sand is transported. This result is in a good agreement with the numerical simulation by Bakker, 1970 [2].

The second simulation is the case where the groyne is installed in the middle. The same computational domain and parameters as the previous simulation in Subsection 4.1 is used.
However, the only difference is, here, the wave crest is set to be $\phi_\infty = 5^\circ$. Moreover, here the step time space is set to be $\Delta t = \Delta x^2/(8D)$ to keep the stability of numerical scheme.

Simulation result is given in Figure 9. In the Figure 9 (right), we can see clearly that diffraction has an effect only on the right part of the groyne, whereas on the left part diffraction seems to have no effect. This is because here we take the angle of incident wave to be $\phi_\infty = 5^\circ$.

5. Simulation of Shoreline Change in Sanur Bali Beach
In this research, Sanur Bali beach, Indonesia has been chosen for validating the solution of purposed model. Sanur Bali beach is located at southeast of Denpasar city (capital city of Bali island). This location is one of the most famous touristic place for surfing water sport in Bali. Moreover, along behind the coastal of Sanur Bali beach, some big and luxury hotels are existed. The box area in Figure 10 shows the location of Sanur Bali beach and the coastal area where some groynes are constructed (see Figure 1).
The initial configurations of this simulation are given as follows

\[ a = 1, \quad \phi_{\infty} = 0.1 \text{ rad}, \quad S_0 = 1.5 \times 10^2. \]

Further, the water depth is assumed to be 1 meter on average. The wave incident and water depth can be obtained from the websites which provide the ocean waves and weather information for surfing sport. For our computations, here we use the the parameters which are obtained from magicseaweed.com.

The spatial domain of this computation is \( \Omega = [0, 500 \text{ km}] \). This domain is discretized with the spatial grid size \( \Delta x = 5 \text{ km} \). From Google Earth photo in 2009, we can get shoreline data, which is used as the initial condition. By using procedure in Algorithm 1, the results of our simulation in three years time simulation are given in Figure 11. Numerical results is compared with shoreline data from Google Earth photo in 2012, for the case of (top) without diffraction, (bottom) with diffraction. From Figure 11 (top) discrepancy are clearly observed in the simulation result without diffraction, especially on the right side of the groyne.

Nevertheless, by accommodating the diffraction term, the numerical result is shown to be in a good agreement with shoreline data, Figure 11 (bottom). After three years simulation time, the shape of shoreline on the left part of the groyne is shown to be in a good agreement with the shoreline data.

6. Conclusion
Shoreline development due to the installation of breakwater groyne have been studied in this paper. Our study adopted the Pelnard-Considère equation with diffraction effect. The finite volume method was adopted here, and physically relevant results were obtained when diffraction effect was incorporated. For the case of Sanur Bali beach, simulation result of the shoreline changes due to a groyne system presents a good agreement with Google Earth photos.

Acknowledgement
The second author would like to thank Institut Teknologi Bandung for the financial support from Riset KK with contract 007/II.C01/PL/2016.
Figure 11. Numerical simulations after three years computation, in comparison with shoreline data extracted from Google Earth photo, (top) without diffraction, (bottom) with diffraction.

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