Heat transfer in an unsteady vertical porous channel with injection/suction in the presence of heat generation

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ABSTRACT
This paper studies convective heat transfer in an unsteady vertical porous channel with suction/injection in the presence of heat generation. A semi-implicit finite difference method was employed to solve the Cartesian coordinate forms of the governing equations. Subsequently, the entropy generation number for varying values of intrinsic parameters, as well as the temperature and velocity profile, Bejan number and irreversibility of the system were successfully calculated. We observed that the convective cooling by suction is more effective on thermal properties than injection convective cooling. Hence, the process of entropy can be enhanced by convective heat transfer and viscous dissipation.

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Thermodynamics; Bejan number; semi-implicit finite difference; heat generation

1. Introduction
Heat transfer analysis of fluid flow in porous media has attracted rapid research interests due to the widespread applications of electrically and thermally conducting fluids (including nanofluids). Such industrial applications include magneto-hydrodynamic (MHD) flow and heat transfer employed in petroleum engineering, medical hyperthermia, geothermal reservoirs, aerodynamics, nuclear reactor cooling, marine propulsion and so on [1,2]. These numerous applications brings to bare the importance of understanding fluid flow in porous structure. Some interesting experimental studies of viscous fluid application in porous structures abound in literature; however, the first laboratory experiment on modern MHD flow was conducted by Hartmann and Lazarus [3]. Recently, several numerical investigation have been conducted to evaluate effect of different parameters such as Hall current, magnetic field, thermophoresis and entropy generation rate on fluid flow with convective boundary conditions [4–7].

In real-life processes, entropy generation provides explanations for the thermal equilibrium and properties of physical processes. The evolution of entropy as a result of irreversibility can manifest in terms of entropy generation which is always against all thermal systems, for instance, energy storage, insulation, and geothermal energy system [8]. Since the temperature difference of entropy is dissipation, it can therefore be studied via thermodynamics second law [9]. The pioneering work of Bejan introduced the study of entropy within a vertical channel for different applications [10–12]. The influence of buoyancy force on the entropy generation rate within vertical channels have been studied [13,14]. Likewise, it has been reported that entropy generation level rises along the suction wall in fluids in a slanted porous plate [15].

The possibility of a reduction of entropy generation in a viscous convection cooling system has been reported for a double face structure [9,14,16]. Several analyses have been conducted on the irreversibility of the entropy generation as a result of reactive flow in a channel [17–27]. Collocation method has likewise been used to obtain heat generation force of convective flow through permeable walls. It was observed that both velocity and temperature profile increased with an increase in heat generation [28]. Recently, heat transfer was systematically explained by an increase of entropic potential [29].

Our survey of literature indicates that while several methods including finite difference method have been employed in heat transfer analysis of fluid flow in porous media, heat generation parameter have not hitherto been considered in the semi-implicit finite difference method. Thus, in this study, the distribution of velocity and temperature profile, Bejan number and irreversibility rate in an unsteady vertical porous system was evaluated via the thermodynamics second law. A model built on semi-implicit finite difference method was used to evaluate the heat transfer in an unsteady porous channel with suction and injection is presented, with the introduction of heat generation term.
2. Mathematical formulation

We considered an unsteady incompressible flow of viscous fluid through a vertical wall under heat transfer of electrical conduction. The vertical channel wall is heated and the suction/injection is homogenous through the channel walls in the presence of a diagonally imposed external magnetic field $B_0$. The fluid is injected at the left side of the plate, i.e. at $y = 0$, at a constant velocity into the channel. While the suction takes place at the right side of the plate $y = H$, with the same velocity as the injection (Figure 1).

The buoyancy effect was taken into account in the momentum equation. The governing equations; continuity, energy and momentum equations describing the flow can be expressed as [13,16,28]

$$\frac{\partial \bar{u}}{\partial \tau} = 0$$

(1)

$$\frac{\partial \bar{u}}{\partial \tau} + V \frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \frac{\bar{u} \frac{\partial \bar{u}}{\partial x}}{\bar{\rho}} \right)$$

$$- \frac{\sigma B_0^2 \bar{u}}{\rho} + \alpha g (T - T_0).$$

(2)

$$\frac{\partial T}{\partial \tau} + V \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \left( \frac{\bar{u}}{\bar{\rho} c_p} \right) \left( \frac{\partial \bar{u}}{\partial y} \right)^2$$

$$+ \frac{\sigma B_0^2 u^2}{\rho c_p} + \frac{Q_0}{\rho c_p} (T - T_0)$$

(3)

The initial boundary conditions are taken as

$$\bar{u}(0, \bar{y}) = 0, T(0, \bar{y}) = T_0$$

(4)

$$\bar{u}(\bar{r}, 0) = 0, T(\bar{r}, 0) = T_w, \bar{u}(\bar{r}, H) = 0, T(\bar{r}, H) = T_w$$

(5)

where $\bar{r}$ is the time, $H$ is the channel thickness, $\rho$ is the fluid density, $p$ is pressure, $V$ is the uniform suction/injection velocity, $\alpha$ is the heat diffusivity, $\sigma$ is the electrical conductivity, $k$ is the thermal conductivity, $c_p$ is the isobaric specific heat, $T$ is the temperature within the walls, $T_w$ is the channel walls temperature, $\bar{u}$ is the axial velocity, $T_0$ is the fluid initial temperature, $g$ is the gravitational acceleration, $(\bar{x}, \bar{y})$ are the distance measured in the axial direction.

$$\theta = \frac{T - T_0}{T_w - T_0}, \rho = \frac{\bar{H} \bar{p}}{\mu \bar{V}}, \mu = \frac{\bar{u}}{\bar{V}}, \nu = \frac{\bar{y}}{\bar{V}}, u = \frac{\bar{u}}{\bar{V}},$$

(6)

$$x = \frac{\bar{x}}{\bar{H}}, \mu = \frac{\bar{H} \bar{u} \varepsilon (T - T_0)}{\bar{\rho} \bar{V} \bar{H}}, A = -\frac{\partial \bar{p}}{\partial x}, Re = \frac{\rho \bar{V} \bar{H}}{\mu_0},$$

$$y = \frac{\bar{y}}{\bar{H}}, t = \frac{\bar{t} \bar{V}}{\bar{H}}, Br = \frac{\mu_0 \bar{V}^2}{k (T_w - T_0)}, \beta = \gamma (T_w - T_0),$$

$$Gr = \frac{\alpha \bar{a} (T_w - T_0) \bar{H}^2}{\bar{V} \mu_0}, Pr = \frac{\bar{c}_p \bar{H}}{k},$$

$$M = \frac{\sigma B_0^2 \bar{H}^2}{\mu_0}, \delta = \frac{Q_0}{\bar{V}}.$$

(7)

where $M$, $Re$, $Gr$, $\beta$, $Pr$, $B_\Omega^{-1}$, $t$, $\delta$ are the magnetic field parameter, suction/injection Reynold’s number, Grashof number, viscosity variation parameter, Prandtl number, Brinkman number, unsteady parameter and heat generation parameter respectively. Applying the dimensionless properties in Equation (6) into Equation (2) and Equation (3), we obtain

$$Re \left( \frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} \right) = A + \frac{\partial}{\partial y} \left( \varepsilon - \beta \theta \frac{\partial u}{\partial y} \right) - Mu + Gr \theta$$

(8)

and the boundary condition as

$$u(0, y) = 0, \theta(0, y) = 0$$

(9)

$$u(t, 0) = 0, \theta(t, 0) = 1, u(t, 1) = 0, \theta(t, 1) = 1$$

(10)

2.1. Finite difference method

The set of coupled non-linear differential Equation (7) and Equation (8) subjected to the initial and boundary conditions Equation (9) and Equation (10) were solved by a semi-implicit finite difference scheme. The discretization form of the velocity equation is given as

$$Re \frac{u_j^{N+1} - u_j^{N}}{Dt} + Re \frac{u_{j+1}^{N} - u_j^{N}}{Dy}$$

$$= A + \varepsilon - \beta \theta^{N} \frac{u_{j+1}^{N+1} - \theta^N}{Dy} - Mu_{j+1}^{N+1} + Gr \theta_j^{N}.$$

(11)

Given that

$$\xi_j^{N+1} = \xi_j^{N+1} + (1 - \xi) \xi_j^{N}$$

and

$$u_{j+1}^{N+1} = \xi u_{j+1}^{N+1} + (1 - \xi) u_{j+1}^{N}.$$

(12)
Applying the second-order difference, and substituting Equation (12) into Equation (11) to get

\[
\frac{u_j^{(N+1)} - u_j^{(N)}}{Dt} + \frac{u_{j+1}^{(N+1)} - u_j^{(N+1)}}{Dy} = A + e^{-\beta(N)} \left( \frac{\xi}{Dy^2} (u_{j+1}^{(N+1)} - 2u_j^{(N+1)} + u_{j-1}^{(N+1)}) \right) + \frac{1 - \xi}{Dy^2} (u_j^{(N+1)} - 2u_j^{(N)} + u_{j-1}^{(N)}) \]

\[- \frac{\beta}{Dy^2} e^{-\beta(N)} ((u_{j+1}^{(N)} - u_j^{(N)})(\theta_j^{(N)} - \theta_j^{(N)})) \]

\[- M(\xi u_j^{(N+1)} - (1 - \xi)u_j^{(N)}) + Gr\theta_j^{(N)}. \tag{13} \]

Multiply Equation (13) by \(Dt\) then collect like terms (implicit = explicit)

\[U_j^{(N+1)} - U_j^{(N+1)} = U_j^{(N+1)} - U_j^{(N)} = U_j^{(N)} - U_j^{(N)} \tag{14} \]

Let \(r_1 = \xi \frac{Dt}{Dy^2} e^{-\beta(N)}, r_2 = -\frac{Dt}{Dy^2}Re, r_3 = \frac{Dt}{Dy^2} e^{-\beta(N)}.\)

The subscript \(j\) indicates the match point where \(0 \leq \xi \leq 1\). The initial value problem gives a solution of level \((N)\) i.e \(u\), the following in time level \((N+1)\) of velocity, the solution reduces to

\[\text{explicit term} = (r_2 + r_3(1 - \xi) - (\theta_j^{(N)} - \theta_j^{(N)})(u_j^{(N)})) \]

\[+ (r_2 + 2r_3(1 - \xi) + r_3\theta_j^{(N)} - \theta_j^{(N)}) \]

\[+ M(\xi u_j^{(N+1)} - (1 - \xi)u_j^{(N)}) + Gr\theta_j^{(N)} = \text{implicit terms.} \tag{15} \]

The discretization form of temperature equation is given as

\[Pr j^{(N+1)} - \theta_j^{(N)} + Pr \left( \frac{\theta_j^{(N+1)} - \theta_j^{(N)}}{Dy^2} \right) \]

\[= \theta_j^{y(N+1)} + \beta e^{-\beta(N)} ((u_{j+1}^{(N)} - u_j^{(N)})) - BrM(u_j^{(N)})^2 \]

\[+ Q_0\theta_j^{(N)}. \tag{16} \]

Applying and substituting the second-order difference into Equation (16) gives

\[Pr j^{(N+1)} - \theta_j^{(N)} = \left( \frac{\xi}{Dy^2} ((u_{j+1}^{(N+1)} - 2u_j^{(N+1)} + u_{j-1}^{(N+1)})) \right) \]

\[+ \frac{1 - \xi}{Dy^2} (u_j^{(N+1)} - 2u_j^{(N)} + u_{j-1}^{(N)})) \]

\[+ Br((\theta_j^{(N+1)} - \theta_j^{(N)})(u_{j+1}^{(N)} - u_j^{(N)})) - BrM(u_j^{(N)})^2 + Q_0\theta_j^{(N)}. \tag{17} \]

Multiplying Equation (17) by \(Dt\) and then rearranging the terms as implicit = explicit yields

\[U_j^{(N+1)} - U_j^{(N+1)} = U_j^{(N+1)} - U_j^{(N)} = U_j^{(N)} - U_j^{(N)} \tag{18} \]

Let \(r = \xi \frac{Dt}{Dy}, r_4 = -\frac{Dt}{Dy}, r_5 = Pr \frac{Dt}{Dy} \]

We obtain the equation \(\theta^{(N+1)}\) as

\[\text{explicit terms} = -r_5\theta_j^{(N)} + Pr - r_4 - r_5(u_{j+1}^{(N)} - u_j^{(N)}) \]

\[+ (r_6 + 2(1 - \xi) + r_6(u_{j+1}^{(N)} - u_j^{(N)})\theta_j^{(N)}) \]

\[+ BrM(u_j^{(N)})^2 \]

\[+ Q_0\theta_j^{(N)} = \text{implicit terms.} \tag{19} \]

The method was reduced to an inversion of tridiagonal matrices. We verified the consistency of the schemes in Equation (15) and Equation (19) using \(\xi = 1\). The first order is found exact in time, while the second order is correct in space, in agreement with the schemes reported in [30]. By using \(\xi = 0.5\), we were able to enhance the accuracy of the scheme in time up to second order. While the use of \(\xi = 1\) presents us with the freedom of selecting large time steps without diminishing the accuracy of the scheme. The semi-implicit finite difference method in this study was implemented with the mathematical symbolic package MATHEMATICA.

### 3. Entropy generation rate

The theoretical and practical understanding of entropy generation has been intensely considered in flow analysis [12,30]. Entropy generation is the amount of strength disoriented when performing the fundamental of convection heat. This is important for determining the flow and temperature. Entropy generation disorderliness is produced in the energy which also applies to power generation. The local volumetric rate of entropy can be expressed as

\[S_G = \frac{K}{T_w} \left( \frac{\partial T}{\partial y} \right)^2 + \frac{\mu}{T_w} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2 u^2}{T_w} \tag{20} \]

where the first term represents the irreversibility as a result of heat transfer, with the second term denoting entropy generation as a result of the dissipation of heat, and the third term representing the magnetic field. Here \(T_w\) represents the absolute temperature of a point where \(S_G\) is being evaluated using dimensionless form of Equation (20) given in Equation (21)

\[N_4 = \frac{T_w^2 H^2 S_G}{k(T_w - T_0)^2} = \left( \frac{\partial T}{\partial y} \right)^2 \]

\[+ \frac{Br e^{-\beta\theta}}{\Omega} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{MBru^2}{\Omega} \tag{21} \]

where \(\Omega = \frac{T_w - T_0}{T_0}\) is a non-dimensional temperature difference.

### 4. Irreversibility analysis

The study of irreversibility analysis in convective heat transfer is very important as it is useful for predicting the fluid friction and heat transfer. The impact of irreversibility as a result of the fluid’s viscous dissipation, friction,
and the applied magnetic force that governs the heat irreversibility analysis can be defined as

$$N_1 = \left( \frac{\partial \theta}{\partial y} \right)^2, \quad N_2 = \left( \frac{Br \Omega^{-1} \theta}{\Omega} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{MBru^2}{\Omega} \right),$$

$$\Phi = \frac{N_2}{N_1}. \quad (22)$$

The Bejan number can be expressed as $Be = \frac{N_1}{N_2} = \frac{1}{1+\Phi}$, while the irreversibility rate is given as $\Phi = \frac{N_2}{N_1}$.

The Bejan number $Be$ ranges from 0 to 1, where $Be = 0$ is the point of irreversibility which controls the fluid’s friction, $Be = 1$ is the heat transfer that gives equitability to the entropy generation. The numerical solutions of Equations (7)–(22) were obtained using semi-implicit first-order difference. The outcome of this study will
5. Results and discussion

MATHEMATICCA was used to execute the program for the mathematical schemes in Equations (7) and (8). The parameters $M = 0.5, Gr = 0.5, Re = 1, A = 1, \Delta y = 0.02, \Delta t = 0.5, t = 50$ are kept unchanged through the study unless otherwise stated. Figure 2(a) shows the effect of the magnetic field on the velocity profile. It was observed that the velocity reduces with the positive difference of $m$. While it decreases up to about 50% in the centreline as a result of increased resistance of the liquid to flow. Figure 2(b) displays the impact of the magnetic field ($M$) on the temperature profile. Increase in values of $M$ leads to a decrease in the temperature profile. Also, the fluid suction and convection losses bring about the presence of Lorentz heating, which serves as an additional heat source.

Figure 3(a) graphically shows the impact of the external magnetic field on the rate of entropy generation. At the point where $M$ increases, the corresponding value of $Ns$ increase in the direction of the channel along the centerline area and reduce at the wall, especially at the upper region. Figure 3(b) displays the effect of magnetic field on Bejan number ($Be$). Increasing values of magnetic field ($m$) is observed to result in decrease of the channel centerline region. This effect is more prominent than the Lorentz force as result of magnetic fluid friction which increases the heat transfer. Figure 3(c,d) displays the effects of Grashof number ($Gr$) on velocity and temperature profile. It can be observed from Figure 3(c) that increase in the number of $Gr$ indicates enhanced buoyancy force which is greater when related to the viscous force. This performance is more enhanced towards the momentum boundary layer. Hence, the fluid velocity increases with rising values of $Gr$. It is also shown in Figure 3(d) that the rising values of $Gr$ shows corresponding increase in the velocity profile.

Figure 4(a,b) shows the fluid velocity and temperature profile decrease as the unsteadiness parameter increase until it approaches the steady-state. It is vital to note that the rate of cooling is much faster for large values of unsteadiness parameter. Figure 4(c,d) illustrates the effect of the heat generation parameter. An increase in the heat generation parameter induces a corresponding increase in the fluid velocity and temperature. Therefore, the heat source produces an incremental heat transfer resulting to higher flow rate. Figure 5(a,b) shows that the outcome of heat generation inside the channel is negligibly close to the cool wall. The existence of heat source assists the fluid friction irreversibility closer to the heated channel wall.
Figure 5. (a) Entropy generation number, and (b) Bejan number for different values of $\delta$, when $Re = 1.0$, $Dt = 0.02$, $Dy = 0.001$, $t = 50$, $M = 2$, $\beta \Omega^{-1} = 0.5$, $Gr = 0.5$, (c) Entropy generation number, and (d) temperature profile for different values of $Re$, when $\delta = 1.0$, $Dt = 0.02$, $Dy = 0.001$, $t = 50$, $M = 2$, $\beta \Omega^{-1} = 0.5$, $Gr = 0.5$, (e) entropy generation number, and (f) Bejan number for different values of $\beta \Omega^{-1}$ when $\delta = 1.0$, $Dt = 0.02$, $Dy = 0.001$, $t = 50$, $M = 2$, $Re = 1$, $Gr = 0.5$.

Figure 5(c) displays the effect of suction Reynolds number on entropy generation. It can be seen that the suction Reynold’s number decreases towards the left side of the wall and increases towards the right side close to the upper plate.

Figure 5(d) shows the effect of suction Reynolds number ($Re$) on entropy generation rate. It is observed that increasing values of ($Re$) results in loss of heat at the lower plate, while more energy is gained at the upper plate region. The suction ($Re$) increase in the direction of the upper plate as a result of suction and reduce close to the low plate as a result of the injection. Figure 5(e) shows the effect of Brinkman number ($\beta \Omega^{-1}$) on entropy generation. As a result of viscous dissipation, the ($\beta \Omega^{-1}$) on the lower and upper wall region initially increases. Subsequently, it reduces when reacting towards the direction of the channel centre-line. Figure 5(f) shows the relationship between ($\beta \Omega^{-1}$) and Bejan number ($\Phi$). It is noticeable that an increase in $\beta \Omega^{-1}$ results to an increase in the fluid’s friction and magnetic field irreversibility $N_2$. Also, it is observed that it has no reaction on the heat transfer $N_1$, hence, the increase in $\Phi$ reduces the Brinkman number ($\beta \Omega^{-1}$).

6. Conclusion
In this work, we have successfully incorporated heat generation term to the analysis of fluid flow in an
unsteady vertical porous channel with suction and injection. The non-linear numerical equations were solved via a semi-implicit finite difference method. Our results reveal that the fluid velocity increases with increasing values of $Gr$, $Re$ and $\delta$, and decreases with increasing values of $M$. Similarly, the temperature profile of the fluid increases with increase in $Gr$, $Re$ and $\delta$, while decreasing with increasing values of $M$. The entropy generation rate increase with increasing parameters of $\delta^2$, $Re$, $\delta$. It is also observed that heat transfer irreversibility controls the fluid velocity in the centerline region of the channel. Also, the convective cooling by suction is more effective on the fluid’s thermal properties than injection convective cooling. Hence, the process of entropy can be enhanced by convective heat transfer and viscous dissipation. The presented analysis method could be extended to explore the case of nanofluid undergoing unsteady hydromagnetic flow in porous media in the presence of heat generation.

Nomenclature

| Symbol | Description |
|--------|-------------|
| $\tau$ | time |
| $H$ | channel thickness |
| $\rho$ | fluid density |
| $P$ | fluid pressure |
| $V$ | uniform suction/injection |
| $\alpha$ | heat diffusivity |
| $k$ | thermal conductivity |
| $c_p$ | isobaric specific heat |
| $T$ | fluid temperature |
| $T_w$ | channel walls temperature |
| $\bar{u}$ | axial velocity |
| $T_0$ | initial fluid temperature |
| $g$ | gravitational acceleration |
| $\sigma$ | electrical conductivity |
| $(\bar{x}, \bar{y})$ | distance measured in the axial. |
| $M$ | magnetic field parameter |
| $Re$ | suction/injection Reynold’s number |
| $Gr$ | Grashof number |
| $\beta$ | viscosity variation parameter |
| $Pr$ | Prandtl number |
| $B\Sigma^{-1}$ | Brinkman number |
| $t$ | unsteady parameter |
| $\delta$ | heat generation parameter |

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References

1. Waqas H, Khan SU, Hassan M, et al. Analysis on the bioconvection flow of modified second-grade nanofluid containing gyrotactic microorganisms and nanoparticles. J Mol Liq. 2019;291:111231.
2. Wahaab FA, Yahya N, Shafie A, et al. Determination of optimum frequency for electromagnetic-assisted nanofluid core flooding. Appl Sci. 2019;9(21): 4608.
3. Hartmann J. Theory of laminar flow of an electrically conductive liquid in a homogeneous magnetic field. København: Munksgaard; 1937.
4. Bhatti M, Ellahi R, Zeeshan A, et al. Numerical study of heat transfer and Hall current impact on peristaltic propulsion of particle-fluid suspension with compliant wall properties. Modern Phys Lett. 2019;33(35):1950439.
5. Bhatti M, Yousif MA, Mishra S, et al. Simultaneous influence of thermo-diffusion and diffusion-thermo on non-Newtonian hyperbolic target magnetised nanofluid with Hall current through a nonlinear stretching surface. Prama. 2019;93(6):88.
6. Riaz A, Bhatti MM, Ellahi R, et al. Mathematical analysis on an asymmetrical wavy motion of blood under the influence entropy generation with convective boundary conditions. Symmetry. 2020;12(1):102.
7. Bhatti M, Khaliq C, Bég TA, et al. Numerical study of slip and radiative effects on magnetic Fe 3 O 4 water-based nanofluid flow from a nonlinear stretching sheet in porous media with Soret and Dufour diffusion. Modern Phys Lett. 2020;34(02):2050026.
8. Ellahi R, Alamri SZ, Basit A, et al. Effects of MHD and slip on heat transfer boundary layer flow over a moving plate based on specific entropy generation. J Taibah Univ Sci. 2018;12(4):476–482.
9. Lawal KK, Jibril HM. Unsteady MHD natural convection flow of heat generating/absorbing fluid near a vertical plate with ramped temperature and motion. J Taibah Univ Sci. 2019;13(1):433–439.
10. Bejan A. A study of entropy generation in fundamental convective heat transfer. J Heat Transfer. 1979;101(4):718–725.
11. Bejan A. Second-law analysis in heat transfer and thermal design. Adv Heat Transfer. 1982;15(1):1–58.
12. Bejan A. Fundamentals of exergy analysis, entropy generation minimization, and the generation of flow architecture. Int J Energy Res. 2002;26(7):545–565.
13. Makinde OD, Chinyoka T. Numerical investigation of buoyancy effects on hydromagnetic unsteady flow through a porous channel with suction/injection. J Mech Sci Technol. 2013;27(5):1557–1568.
14. Agunbiade SA, Dada M. Effects of viscous dissipation on convective rotatory chemically reacting Rivlin–Ericksen flow past a porous vertical plate. J Taibah Univ Sci. 2019;13(1):402–413.
15. Makinde O, Osalusi E. Entropy generation in a liquid film falling along an inclined porous heated plate. Mech Res Commun. 2006;33(5):692–698.
(16) Adesanya SO, Falade J, Jangili S, et al. Irreversibility analysis for reactive third-grade fluid flow and heat transfer with convective wall cooling. Alex Eng J. 2017;56(1):153–160.

(17) Ibáñez G, Cuevas S, de Haro ML. Minimization of entropy generation by asymmetric convective cooling. Int J Heat Mass Transfer. 2003;46(8):1321–1328.

(18) Mahmud S, Fraser RA. The second law analysis in fundamental convective heat transfer problems. Int J Thermal Sci. 2003;42(2):177–186.

(19) Hooman K, Hooman F, Mohebpour SR. Entropy generation for forced convection in a porous channel with isoflux or isothermal walls. Int J Exergy. 2008;5(1):78–96.

(20) Makinde O. Entropy-generation analysis for variable-viscosity channel flow with non-uniform wall temperature. Appl Energy. 2008;85(5):384–393.

(21) Makinde O. Irreversibility analysis of variable viscosity channel flow with convective cooling at the walls. Can J Phys. 2008;86(2):383–389.

(22) Makinde OD, Bég OA. On inherent irreversibility in a reactive hydromagnetic channel flow. J Thermal Sci. 2010;19(1):72–79.

(23) Eegunjobi A, Makinde O. Entropy generation analysis in a variable viscosity MHD channel flow with permeable walls and convective heating. Math Prob Eng. 2013; 2013. Article ID 630798.

(24) Adesanya S, Oluwadare E, Falade J, et al. Hydromagnetic natural convection flow between vertical parallel plates with time-periodic boundary conditions. J Magnet Magn Mater. 2015;396:295–303.

(25) Jangili S, Adesanya SO, Ogunseye HA, et al. Couple stress fluid flow with variable properties: a second law analysis. Math Methods Appl Sci. 2019;42(1):85–98.

(26) Gbadeyan J, Oyekunle T, Fasogbon P, et al. Soret and Dufour effects on heat and mass transfer in chemically reacting MHD flow through a wavy channel. J Taibah Univ Sci. 2018;12(5):631–651.

(27) Jha BK, Samaila AK, Ajibade AO. Natural convection flow of heat generating/absorbing fluid near a vertical plate with ramped temperature. JEAS. 2012;2(4). Article ID 25976.

(28) Ajala O, Abimbade S, Obalalu A, et al. Existence and uniqueness of forced convective flow through a channel with permeable walls in presence of heat generation. IJEAST. 2019;4(7):194–197.

(29) Eger T, Bol T, Daróczy L, et al. Numerical investigations of entropy generation to analyze and improve heat transfer processes in electric machines. Int J Heat Mass Transfer. 2016;102:1199–1208.

(30) Herwig H. How to teach heat transfer more systematically by involving entropy. Entropy. 2018;20(10). Article ID 791.