Fluctuation current in superconducting loops

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Abstract. A superconducting loop that encloses noninteger flux holds a permanent current. On the average, this current is also present above $T_c$, and has been measured in recent years. We are able to evaluate the permanent current within the TDGL or the Kramer–Watts-Tobin models for loops of general configuration, i.e., we don’t require uniform cross section, material or temperature. We can also consider situations in which the width is not negligible in comparison to the radius. Our results agree with experiments. The situations with which we deal at present include fluctuation superconductivity in two-band superconductors, equilibrium thermal fluctuations of supercurrent along a weak link, and ratchet effects.

1. Introduction
Thermal fluctuations give rise to nonvanishing average currents in superconducting loops that enclose noninteger magnetic flux, even above $T_c$ (the critical temperature at zero field). In recent years it has become possible [1, 2, 3] to measure these currents and study their dependence on temperature, flux and loop parameters. An additional source of interest on fluctuation superconductivity in loops resides on the ideas raised by Kibble and Zurek [4, 5] that compare between symmetry breaking when a loop is quenched through $T_c$ and symmetry breaking in the early universe.

The theory used until now for the analysis of fluctuation current in rings is due to von Oppen and Riedel [6], and is based on the Ginzburg–Landau model. In the case of [1], there is strong disagreement between this theory and experiment. The results of [2] are well explained by this theory, but there are several cases in which the existing theory is not applicable: (i) rings with non negligible width to radius ratio (and therefore with a significant difference between the fluxes enclosed by the inner and the outer boundaries), (ii) rings with large coherence length to perimeter ratio, (iii) situations that present metastability and hysteresis and (iv) rings with nonuniform cross section. We have developed a method that can cope with all these limitations; it is essentially a numeric solution of the 1D time-dependent Ginzburg–Landau model (TDGL) with Langevin terms, by means of finite differences. The central features that characterize our method are: (i) the variance of the Langevin terms is obtained from the fluctuation-dissipation theorem and depends on the size of the computational cell; (ii) the evolution equations for the gauge-invariant order parameter can be written without direct influence of the electromagnetic potential, but nevertheless the fluctuations of this order parameter are influenced by the fluctuations of the electromagnetic potential; (iii) the enclosed flux can be naturally taken into account through the boundary condition of the gauge-invariant

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order parameter, and the instantaneous current naturally appears as a Lagrange multiplier; (iv) in appropriate situations, two dimensional problems may be reduced to 1D by integration of the free energy density along the radial direction.

2. Method
Our method is described in detail in [7] and [8]. In the case of TDGL, in a discretized form, the order parameter $\psi_k$ and the tangential component of the electromagnetic potential $A_k$ in the $k^{th}$ cell change during a macroscopically short period of time $\tau$ by

$$\Delta \text{Re}[\psi_k] = -\Gamma_{\psi,k} \frac{\partial G}{\partial \text{Re}[\psi_k]} \tau + \eta_k^{\text{Re}\psi},$$  \hspace{1cm} (1)

with an analogous expression for $\Delta \text{Im}[\psi_k]$, and

$$\Delta A_k = -\Gamma_{A,k} \frac{\partial G}{\partial A_k} \tau + \eta_k^{A},$$  \hspace{1cm} (2)

where $\Gamma_{\psi,k}$ and $\Gamma_{A,k}$ are known relaxation constants, $G$ is the Ginzburg–Landau energy, and $\eta_k^{\text{Re}\psi}$ (resp. $\eta_k^{A}$) is a Langevin term with gaussian distribution, zero average, and variance $2\Gamma_{\psi,k}k_B T \tau$ (resp. $2\Gamma_{A,k}k_B T \tau$).

In the case of a loop with negligible self inductance, the current during the period of time $\tau$ is given by

$$I = \frac{e k_B T \sum \eta_k^{A}/\tau - (C_A \xi_\beta/L) \text{Im} \sum w_k^{-1}(\delta_k + \delta_{k+1})}{C_A \sum w_k^{-1}},$$  \hspace{1cm} (3)

where $L$ is the perimeter of the loop, $\xi_\beta$ plays the role of the coherence length very close to $T_c$, $w_k$ is the cross section of the $k^{th}$ cell, $C_A$ is proportional to $\Gamma_{A,k}k_B T w_k$, $\text{Im}[\delta_k + \delta_{k+1}]$ is proportional to the supercurrent in the $k^{th}$ cell, and the sums are over all the cells in the loop. The first term in expression (3) is the Johnson noise.

In the case of the Kramer–Watts-Tobin model (KWT), the macroscopic part of Eq. (1) generalizes to

$$d|\psi_k|/dt = -h_i|\psi_k|\Gamma_{\psi,k} \partial G/\partial |\psi_k|,$$  \hspace{1cm} (4)

$$d\chi_k/\tau = -h_\chi(\psi_k)\Gamma_{\psi,k} \partial G/\partial \chi_k,$$  \hspace{1cm} (5)

where $|\psi_k| = |\psi_k|e^{i\chi_k}$, $h_i|\psi_k| = (1 + K|\psi_k|^2)^{-1/2}$, $h_\chi(\psi_k) = 1/(h_i|\psi_k|^2)$ and $K$ is proportional to the square of the electron-phonon inelastic scattering time. Langevin terms have to be added to Eqs. (4) and (5) to take fluctuations into account. As pointed out in [9], due to the $|\psi_k|$-dependence of $h_i|\psi|$ and of the Jacobian $W = \partial(\text{Re}[\psi_k], \text{Im}[\psi_k])/\partial(|\psi_k|, \chi_k)$, fluctuations of $|\psi_k|$ do not have zero average but rather $\langle \eta_k^{\psi} \rangle = |\partial \ln(h_i|\psi|)W|/\partial|\psi_k| [h_i|\psi|^2 \Gamma_{\psi,k}k_B T \tau].$

3. Results and significance
We have evaluated the average current $I$ as a function of the enclosed flux $\Phi$ for many samples and several temperatures for $T_c - T$ of the order of $T_{LP}$, where $T_{LP}$ is the difference between the temperatures for the onsets of superconductivity that would be obtained in the absence of fluctuations in the case that there is no magnetic flux and the case that there is half-integer flux. In all the cases we obtained good agreement with experiment.

Figures 1 and 2 show representative results. In both cases the width to radius ratio was of the order of 0.4. Figure 2 focuses on a region for which there would be no current in the absence of fluctuations. The results in Fig. 1 were obtained using TDGL; in Fig. 2 some results were obtained using TDGL and some using KWT.
Figure 1. Current as a function of the magnetic flux for a wide ring. The lines are experimental and the dots calculated. Inset: hysteresis region; the red curves (actually, dense set of points) are experimental data for \( T = T_c - 1.56 T_{LP} \) and the black lines are calculated.

Figure 2. Fluctuation region around \( \Phi = 0.5 \Phi_0 \) for \( T = T_c - 0.42 T_{LP} \). The nearly straight lines describe the currents that would be obtained without thermal fluctuations. • TDGL; × KWT, \( \tau_{ph} = 10^{-9} \sec \) (\( \tau_{ph} \) = electron-phonon scattering time); + KWT, \( \tau_{ph} = 10^{-8} \sec \).

Figure 3. Comparison among currents for systems with different coupling \( \gamma \), but with otherwise identical parameters.

Figure 4. Maximum current at \( T = T_c \) as a function of the scaled \( T_c \) for \( \gamma = 0 \) and for \( \gamma \to \infty \).

The range of temperatures for which our results reproduce the measured currents extends to the order of \( 10^{-1} \)K below \( T_c \), which is much lower than the temperature for which TDGL is justified by microscopic theory. This is probably due to the fact that the average current is an equilibrium quantity.

The advantage of our method over that of [6] is its flexibility, which permits application to a wide realm of 1D problems. The most obvious possible application is to rings where nonuniformity is an essential feature of the system rather than an imperfection, such as in the loops considered in [10] or [11]. Our method can also deal with nonequilibrium situations and with applied parameters that vary in time, thus being able to evaluate lifetimes and electric fields, as required in [1] or [12]. A particular problem in which nonequilibrium and nonuniformity can be important is that of a constriction of arbitrary shape or composition. In our method the temperature can be varied at will as an arbitrary function of time, so that it can naturally simulate quenching of a ring through \( T_c \), as required in the Kibble–Zurek scenario [4].
4. Loops with two order parameters
We have extended the method of [6] to the case of materials with two order parameters. We followed a transfer matrix approach as in [13]. The details are given in [14].

As an illustration of our results, Figs. 3 and 4 show the case of two bands with equal parameters. $b$ quantifies the size of the quartic coefficient of both order parameters in their contributions to the energy, whereas $\gamma$ stands for the interband coupling; $2\pi R$ and $w$ are the perimeter and cross section of the loop. Figure 3 compares the currents for several values of $\gamma$ and $\Phi = \Phi_0/4$ (the value of $\gamma$ is marked next to each curve). For every temperature, the size of the current decreases as $\gamma$ raises from 0, reaches a minimum, and then increases as $\gamma$ continues to grow.

Figure 4 shows the maximum current (maximum as a function of the flux) at $T = T_c$ for the limiting cases $\gamma = 0$ and $\gamma \rightarrow \infty$. The case $\gamma = 0$ corresponds to a situation in which the fluctuations of each order parameter are totally independent of the other, whereas in the opposite limit both order parameters are forced to be equal. For intermediate situations, the fluctuations of both order parameters are correlated. We note that for small values of $T_c$ the average current is larger for $\gamma = 0$, whereas for large $T_c$ the current is larger for $\gamma \rightarrow \infty$. This result is somewhat surprising, since the average size of the order parameters is always larger in the case $\gamma = 0$.

5. Supercurrent fluctuations in Josephson junctions
The standard theory for a Josephson junction asserts that the supercurrent is determined by the current-phase relation, whereas the normal current includes Johnson noise. However, close to $T_c$ we expect supercurrent fluctuations to be important. Since in this regime the Ginzburg–Landau relaxation time is much longer than $\hbar/k_B T$, supercurrent fluctuations should have a behavior that is qualitatively different from Johnson noise.

Representing the junction by a weak link, we intend to evaluate supercurrent fluctuations and their autocorrelations as functions of the relevant parameters. We also intend to study fluctuations of charge imbalance.

6. Summary
We have at our disposal a method that enables us to evaluate thermal fluctuations for TDGL-like formalisms in quasi-1D systems in a wide range of situations, where the relevant parameters such as applied flux or current, material and geometric parameters, temperature, etc., may be arbitrary functions of position and time.

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