CMB two-point angular correlation function in the Ellipsoidal Universe

Paolo Cea

INFN - Sezione di Bari, Via Amendola 173 - 70126 Bari, Italy

Abstract

We suggest that the Ellipsoidal Universe cosmological model, proposed several years ago to account for the low quadrupole temperature-temperature correlation of the Cosmic Microwave Background, can also provide temperature-temperature two-point angular correlation function in reasonable agreement with Planck observations.

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1 Electronic address: paolo.cea@ba.infn.it
1 Introduction

The latest release of the Cosmic Microwave Background (CMB) anisotropy data by the Planck Collaboration has confirmed the $\Lambda$ Cold Dark Matter ($\Lambda$CDM) cosmological model to an unprecedented level of statistical significance (Planck results can be obtained by means of the Planck Legacy Archive [1]). However, at large angular scales there are several anomalous features in the temperature maps, such as the alignment of multipoles and the hemispherical power asymmetry. The most notable discrepancy resides in the low quadrupole moment which, indeed, signals an important suppression of power at large scales. In the standard CMB analysis the temperature fluctuations are expanded in spherical harmonic:

$$\frac{\Delta T(\vec{n})}{T_0} = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$  \hspace{1cm} (1)

where $\theta, \phi$ are the polar angles of the unit vector $\vec{n}$, and $T_0 \simeq 2.7255 K$ is the actual average temperature of the CMB radiation [2]. The properties of the CMB anisotropy are fully characterised by the angular power spectrum:

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{+\ell} \langle |a_{\ell m}|^2 \rangle.$$  \hspace{1cm} (3)

where the brackets denote the full-sky average. Indeed, assuming statistical isotropy and gaussianity, i.e. the coefficients $a_{\ell m}$ are independent gaussian random variables of zero mean, the parameter $C_\ell$ turns out to be the best estimator. Considering that there is only one sky there is an intrinsic uncertainty in the knowledge of $C_\ell$ given by the cosmic variance:

$$\sigma_{CV}^{C_\ell} = \sqrt{\frac{2}{2\ell + 1}} C_\ell.$$  \hspace{1cm} (4)

The quadrupole anisotropy refers to the multipole $\ell = 2$. Defining

$$\langle \Delta T_\ell \rangle^2 = T_0^2 \frac{\ell(\ell + 1)}{2\pi} C_\ell,$$  \hspace{1cm} (5)

the observed quadrupole anisotropy [1]

$$\langle \Delta T_2 \rangle^2 \simeq 225.9 \mu K^2,$$  \hspace{1cm} (6)

turns out to be much smaller than the quadrupole anisotropy expected according to the best-fit $\Lambda$CDM model to the Planck 2018 data [1]:

$$\langle \Delta T_2^{\Lambda\text{CDM}} \rangle^2 \simeq 1150 \mu K^2.$$  \hspace{1cm} (7)

The quadrupole anisotropy is affected by the largest uncertainty due to the cosmic variance, so that the best-fit $\Lambda$CDM quadrupole anisotropy Eq. (7) differs from the observed value Eq. (6) by less than two standard deviations. As a consequence, it could well be that the quadrupole anomaly is due to a mere statistical fluctuation. Nevertheless, several years ago in Refs. [3, 4] to account for the observed suppression
of power in the quadrupole temperature anisotropy it was proposed the Ellipsoidal Universe cosmological model. In the Ellipsoidal Universe model the flat Friedmann-Lemaître-Robertson-Walker metric is replaced by the following Bianchi I anisotropic metric with planar symmetry:

$$ds^2 = -c^2 dt^2 + a^2(t) \left( \delta_{ij} - e^2(t) n_i n_j \right) dx^i dx^j$$

(8)

where $e(t)$ is the ellipticity and the unit vector $\vec{n}$ determines the direction of the planar symmetry axis. More precisely, in Refs. [3, 4] it was assumed that the temperature fluctuations satisfied:

$$\Delta T \simeq \Delta T^I + \Delta T^A$$

(9)

where $\Delta T^I$ and $\Delta T^A$ were the temperature fluctuations induced by the cosmological scalar perturbations and by the spatial anisotropy of the metric. After that, the contributions to the temperature fluctuations due to the metric anisotropy were estimated by simple geometric arguments or, equivalently, by means of the integrated Sachs-Wolf effect. In this way it was suggested that a small ellipticity at decoupling could explain both the almost planarity and the suppression of power of the quadrupole moment. Subsequently, in a series of papers [5, 6, 7] we solved at large scales the Boltzmann equation for the photon distribution functions by taking into account the effects of the inflation produced primordial scalar perturbations and the anisotropy of the geometry. We showed that, in fact, at large scales one recovers Eq. (9). We, also, showed that the anisotropy of the spatial geometry contributes mainly to the temperature quadrupole anisotropy without affecting the higher multipoles since:

$$\ell (\ell + 1) C_\ell^A \sim \frac{1}{\ell^2} , \quad \ell \gtrsim 3 .$$

(10)

Moreover, we found that the ellipsoidal geometry of the universe induces sizeable polarisation signal only at large scales ($\ell \lesssim 10$) without invoking the reionization processes. Finally, in Ref. [7] we were able to fix the eccentricity at decoupling and the polar angles $\theta_n, \phi_n$ of the direction of the symmetry axis $\vec{n}$ such that the quadrupole temperature-temperature correlation matched exactly the Planck 2018 value, Eq. (6) obtaining:

$$e_{dec} = 8.32 \pm 1.32 \times 10^{-3} , \quad \theta_n \simeq 73^\circ , \quad \phi_n \simeq 264^\circ .$$

(11)

Another notable large-scale anomaly was displayed by the CMB temperature-temperature correlation function. In fact, it is now well established that at large angular scales the temperature two-point angular correlation function is found to be smaller than expected within the ΛCDM cosmological model. The main aim of the present paper is to show that the anomalies displayed by the two-point angular correlation function could be solved by the Ellipsoidal Universe cosmological model.

The remaining part of the paper is organised as follows. In Sect. 2 we critically discuss the large scale anomalies of the two-point angular correlation function focusing, for definiteness, on the Planck 2013 and Planck 2015 data. Sect. 3 is devoted to the two-point temperature correlation function within the Ellipsoidal Universe model. Finally, in Sect. 4 we, briefly, summarise the results presented in this paper and draw our conclusions.
2 Two-point temperature-temperature angular correlation function

The two-point temperature correlation function is defined as the average product of two temperatures measured in a fixed relative orientation on the sky:

\[ C(\theta) = \langle \Delta T(\vec{n}_1)\Delta T(\vec{n}_2) \rangle, \quad \vec{n}_1 \cdot \vec{n}_2 = \cos \theta \ . \quad (12) \]

Under the assumption of statistical isotropy, this correlation function does not depend on the particular position or orientation on the sky and, thereby, it depends only on the angle \( \theta \). The two-point angular correlation function is related to the angular power spectrum by:

\[ C(\theta) = T^2_0 \sum_\ell \frac{2\ell + 1}{4\pi} C_\ell P_\ell(\cos \theta) \, , \quad (13) \]

with the related cosmic variance:

\[ (\sigma_{CV}(\theta))^2 = T^4_0 \sum_\ell \frac{2\ell + 1}{8\pi^2} C^2_\ell P^2_\ell(\cos \theta) \ . \quad (14) \]

Interestingly, the 2-point angular correlation function shows clear evidence of a lack of structure for large separation angles. The lack of correlations at large angular scales was clearly detected by the observation of temperature anisotropies by the Cosmic Background Explorer (COBE) \[8\], by the Wilkinson Microwave Anisotropy Probe (WMAP) \[9, 10\], by the first release of the Planck data (Planck 2013) \[11\] and confirmed by the Planck 2015 \[12\] and Planck 2018 data \[13\]. This lack of large-angle correlations in the observed microwave background temperature fluctuations probably is related to the lowness of the temperature quadrupole. In particular, there is a strong correlation between the low quadrupole and the lack of correlation in the two-point correlation function of the CMB anisotropies \[14, 15\]. Nevertheless, it is believed that it is a different problem that, in principle, could challenge the assumed fundamental prediction of gaussian random, statistically isotropic temperature fluctuations. Indeed, this problem has been subjected to several studies \[16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28\].

To illustrate the problem we display in Fig. 1 (black) continuous lines, the two-point angular correlation function as observed with Planck. More precisely, Fig. 1 (left panel), adapted from Fig. 1 of Ref. \[23\], corresponds to the full-sky angular correlation function reported in Ref. \[23\] using the Planck 2013 data. On the other hand, in Fig. 1 (right panel), adapted from Ref. \[12\], it is shown the Planck 2015 measured angular correlation function with the UT78 mask. The UT78 mask has a usable sky fraction of approximately 78 % and it is the most conservative mask to omit foreground residuals. The Planck team presented the analyses of the angular two-point correlation function at low resolution for their four component separation methods (COMMANDER, NILC, SEVEM, SMICA). It turned out that the results of the Planck analyses by means of the COMMANDER, SEVEM, NILC and SMICA maps fell on top of each other. Therefore, without loss in generality, in Fig. 1 we restricted to angular correlation function extracted with the SMICA map. Moreover, it is useful to stress that the observed angular correlation function by WMAP and Planck 2013, 2015 and 2018 are perfectly consistent each other. The (red) dashed lines in Fig. 1 correspond to the expected two-point angular correlation based on comparison with 1000
realisations of the best-fitting ΛCDM cosmological model to the Planck data. We, also, display the 68% cosmic variance confidence interval (red dotted lines). Comparing left and right panels in Fig. 1 one can see how the temperature two-point correlation function depends on the Galactic mask. Looking at Fig. 1 one sees that what is most striking is the difference between the best-fitting ΛCDM model and the observed \( C(\theta) \). Indeed, there is an evident suppression of power in the angular correlation function above about 60 degree. Moreover, this feature seems to be a robust and statistically significant result. There are other puzzling aspects that it is worthwhile to mention. Firstly, the full-sky two-point angular correlation function (left panel in Fig. 1) seems to vanish at three angular scales, \( \theta_1 \simeq 31^\circ \), \( \theta_2 \simeq 96^\circ \) and \( \theta_3 \simeq 154^\circ \). As a consequence, for angular scales larger than 150 degree, the observed two-point angular correlation function is negative, while the expected one is positive. Even though at these large angular scales the best-fitted correlation function is affected by a sizeable cosmic variance, it is quite difficult to image a physical mechanism able to account for such discrepancy. It should be evident that this problem is intimately connected with the observed suppression of the quadrupole temperature anisotropy. In addition, we feel that the another puzzling discrepancy resides on the fact that the expected two-point angular correlation function does not track closely the observed \( C(\theta) \) for small angular scale \( \theta \lesssim 30^\circ \). Looking at Fig. 1 we see that the best-fitting two-point angular correlation function is systematically higher than the observed \( C(\theta) \), even though the difference in each angular bin is within one cosmic variance standard deviation. However, the cumulative effect of the deviations can hardly be due to statistical fluctuations. In this regards, however, it should be mentioned that the values of \( C(\theta) \) in different angular bins are correlated, so that the sizeable deviation between the expected ΛCDM and the observed curve could be not so significant at it may appear. On the other hand, from Fig. 1 one infers that the main effects of the Galactic mask is to shift the angular correlation function towards smaller angular separations together with a further reduction of the signal at large angular scales. Notwithstanding, also the masked two-point angular correlation function confirms the discrepancies between predictions and observations. In any case, if one believes that these anomalous features of the two-point angular correlation function cannot be ascribed to statistical fluctuations, then the resolution requires to identify the physics underlying the anomalies.
3 The Ellipsoidal Universe

Let us, now, consider the two-point angular correlation function in the Ellipsoidal Universe model. According to Eq. (9) we have:

\[ C_{\text{El}}(\theta) = T_0^2 \sum_\ell \frac{2\ell + 1}{4\pi} C_{\text{El}}^\ell P_\ell(\cos \theta) , \]  

To determine the coefficients \( C_{\text{El}}^\ell \) we should perform the best fits to the CMB anisotropy data within the Ellipsoidal Universe model. Unfortunately, we do not yet have at our disposal the best-fitting \( C_{\text{El}}^\ell \). Nevertheless, we can reconstruct the correlation function \( C_{\text{El}}(\theta) \) if we restrict to the angular region \( \theta \gtrsim 2^\circ \). In fact, in the \( \Lambda \)CDM model the two-point temperature-temperature correlation function can be approximated as:

\[ C_{\Lambda \text{CDM}}(\theta) \simeq T_0^2 \sum_{\ell=2}^{\ell_{\text{max}}} \frac{2\ell + 1}{4\pi} C_{\Lambda \text{CDM}}^\ell P_\ell(\cos \theta) \]  

where \( \ell_{\text{max}} \ll 200 \). Moreover, for \( \ell \lesssim \ell_{\text{max}} \) the power spectrum coefficients can be estimated by the Sachs-Wolf effect (see, eg, Ref. [29]):

\[ \ell (\ell + 1) C_{\Lambda \text{CDM}}^\ell \simeq \frac{8}{25} A_s , \]  

where \( A_s \) is the amplitude of the curvature power spectrum assuming a scale-invariant spectrum. From Eqs. (16) and (17) we get:

\[ C_{\Lambda \text{CDM}}(\theta) \simeq T_0^2 \frac{8}{100 \pi} A_s \sum_{\ell=2}^{\ell_{\text{max}}} \frac{2\ell + 1}{\ell (\ell + 1)} P_\ell(\cos \theta) . \]  

In Fig. 2 we compare the two-point angular correlation function Eq. (18) to the best-fitting \( \Lambda \)CDM model correlation function. We fixed:

\[ \ell_{\text{max}} \simeq 100 , \quad A_s \simeq 3.0 \times 10^{-9} \]  

such that \( C_{\Lambda \text{CDM}}(\theta) \) in Eq. (18) reproduces as closely as possible the best-fit \( \Lambda \)CDM correlation function. Indeed, from Fig. 2 we infer that the given approximations to evaluate...
Figure 3: (Color online) Comparison of the Ellipsoidal Universe model correlation function (blue continuous lines) as given by Eq. (21) to the two-point angular correlation function from full-sky Planck 2013 (left panel) and Planck 2015 UT78 mask (right panel) SMICA maps (black continuous lines).

the angular correlation function are quite adequate to our purposes for both the full-sky and masked Planck SMICA maps. This allows us to estimate the angular correlation function for the Ellipsoidal Universe cosmological model. In fact, according to our previous discussion and taking into account Eq. (10), we can write:

$$ C_{\ell}^{El} \simeq 0.26 C_{\ell}^{\Lambda CDM}, \quad C_{2}^{El} \simeq C_{2}^{\Lambda CDM}, \quad \ell \geq 3 $$

(20)

where we have taken into account that $(\Delta T_{2})^{2}/(\Delta T_{2}^{\Lambda CDM})^{2} \simeq 0.26$. Accordingly, we have:

$$ C^{El}(\theta) \simeq T_{0}^{2} \frac{8}{100 \pi} A_{s} \left( 0.26 \times \frac{5}{6} P_{2}(\cos \theta) + \sum_{\ell=3}^{\ell=\ell_{max}} \frac{2\ell + 1}{\ell(\ell + 1)} P_{\ell}(\cos \theta) \right). $$

(21)

The main results of the present paper are displayed in Fig. 3 where we contrast $C^{El}(\theta)$, Eq. (21), to the observed two-point angular correlation function from the Planck SMILCA maps. In the case of the full-sky angular correlation function we see that the Ellipsoidal Universe model correlation function is able to trace closely the observed correlation function. There are some small deviations at very high angular separations that, however, are well within the cosmic variance uncertainties. In any case, please note that our theoretical curve predicts correctly a negative angular correlation function for angular scales larger than 150 degree. Even for the masked angular correlation function the agreement between theoretical expectations and observations seems to be satisfying. In this case there are some small deviations even at small angular scales that, as already discussed, can be ascribed to the Galactic mask.

To quantify the lack of power on angular scales greater than 60° in Ref. [30] it was introduced the parameter:

$$ S_{1/2} = \int_{1}^{1/2} [C(\theta)]^{2} d(\cos \theta). $$

(22)

It is useful to determine this parameter for both $\Lambda CDM$ and Ellipsoidal Universe models. To this end we have evaluated numerically the integrals in Eq. (22) by using Eqs. (18) and (21). We get:

$$ S_{1/2}^{\Lambda CDM} \simeq 42527 \, (\mu K)^{4}, $$

(23)

$$ S_{1/2}^{El} \simeq 6837 \, (\mu K)^{4}. $$

(24)
For comparison we, also, display the value of the parameter \( S_{1/2} \) based on the WMAP 5-year anisotropy measurements as reported in Table 1 of Ref. [19]:

\[
S_{1/2} \simeq 8833 \, (\mu K)^4, \quad WMAP \ 5-\text{year}. 
\]  

(25)

Comparing this last equation with Eqs. (23) and (24) one sees that the Ellipsoidal Universe cosmological model seems to be in better agreement with observations with respect to the standard ΛCDM cosmological model. However, it should be keep in mind that the lack of correlations at large angular scales in the correlation function is due almost entirely to the suppression of power in the quadrupole temperature anisotropy and that the quadrupole \( C_2 \) is subject to a large intrinsic uncertainty given by the cosmic variance, \( \Delta C_2 = \sqrt{\frac{2}{5}} C_2 \).

Actually, if we allow the quadrupole coefficient to vary in the interval \( (C_2 - \Delta C_2, C_2 + \Delta C_2) \), then both the standard Λ Cold Dark Matter cosmological model and the Ellipsoidal Universe cosmological model are consistent with the observed \( S_{1/2} \), as given by Eq. (25), at the 68% confidence level.

It should, now, be evident that the Ellipsoidal Universe cosmological model compares rather well to the Planck observations. It is worthwhile to stress that to recover the anomalous features in the CMB angular correlation function one must admit that the temperature quadrupole suppression is a truly physical effect and not a mere statistical fluctuation. In this respect, it should be mentioned that there are also other models that are able to explain the low quadrupole. For instance, a fast roll phase of the inflation preceding the slow roll phase is an explanation often considered in the literature [31], or the introduction of a hard lower cutoff in the primordial power spectrum [32].

### 4 Summary and Conclusions

The latest results on the CMB anisotropies by the Planck Collaboration are confirming the standard Λ Cold Dark Matter cosmological model with an exquisite level of accuracy. Nevertheless, at large angular scales there are still anomalous features in CMB anisotropies. Actually, the most evident discrepancy resides in the quadrupole temperature correlation. It is conventional wisdom to believe that this quadrupole anomaly is due to a statistical fluctuation. However, there is also a persistent anomaly in the temperature two-point angular correlation function computed as an average over the full sky. We have shown that, if we consider the quadrupole suppression a truly physical effect, then we can account for the persistent lack of correlations at large angular scales in the two-point temperature angular correlation function. This last point implies that the standard cosmological model necessitates some changes. Remarkably, the Ellipsoidal Universe cosmological model, advanced several years ago to account for the CMB quadrupole anomaly, constitutes a viable alternative to the standard cosmological model. In fact, if one assumes that the large-scale spatial geometry of our Universe is slightly anisotropic, then the quadrupole amplitude can be drastically reduced without affecting higher multipoles of the angular power spectrum of the temperature anisotropies. At the same time, we showed in the present paper that the Ellipsoidal Universe two-point angular correlation function compares reasonable well to observations. On the other hand, at variance of the standard cosmological model, it is known since long time that anisotropic cosmological model could induce sizeable large-scale CMB polarisation [33, 34, 35, 36]. Indeed, we already argued in Refs. [6, 7] that in the Ellipsoidal Universe model there is a sizeable
polarisation signal at scales \( \ell \lesssim 10 \). Moreover, we showed that the quadrupole TE and EE correlations in the Ellipsoidal Universe are in reasonable agreement with the Planck 2018 data. Finally, quite recently, we suggested \(^{37}\) that the Ellipsoidal Universe model should also alleviate the tensions on the Hubble constant \( H_0 \) and the cosmological parameter \( S_8 \).

In conclusion, we have shown that the Ellipsoidal Universe cosmological model allows to explain several anomalous features in the CMB temperature anisotropies. Our results are suggesting that the Ellipsoidal Universe cosmological model is not only a viable alternative to the \( \Lambda \)CDM cosmological model, but also it seems to compare observations slightly better than the standard cosmological model. Finally, we would like to conclude the present paper by stressing that, due to the low statistical significance, by using only the CMB temperature anisotropies one cannot distinguish the \( \Lambda \)CDM cosmological model from eventual extensions. One way to overcome this problem might be to consider the large-scale polarisation. Unfortunately the polarisation Planck data at low \( \ell \) are not signal-dominated as in temperature. The future CMB experiments sensitive to the very low multipoles of the CMB polarisation, such as the LiteBIRD satellite, may provide us important information about it. Indeed, LiteBIRD represents the fourth generation of satellites dedicated to the CMB following its predecessors COBE, WMAP and Planck and it will be the first completely dedicated to the CMB polarisation. Actually, LiteBIRD’s primary goal is to map the microwave sky in polarisation on large angular scales with an unprecedented sensitivity \(^{38}\).

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