Local Quantum Pure-state Identification without Classical Knowledge

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Suppose we want to distinguish two quantum pure states. We consider the case in which no classical knowledge on the two states is given and only a pair of samples of the two states is available. This problem is called quantum pure-state identification problem. Our task is to optimize the mean identification success probability, which is averaged over an independent unitary invariant distribution of the two reference states. In this paper, the two states are assumed bipartite states which are generally entangled. The question is whether the maximum mean identification success probability can be attained by means of an LOCC (Local Operations and Classical Communication) measurement scheme. We will show that this is possible by constructing a POVM which respects the conditions of LOCC.

INTRODUCTION

It is an extremely nontrivial problem to distinguish different states of a quantum system by measurement [1, 2, 3, 4]. First of all, this is because of statistical nature of quantum measurement, which destroys the state of the system to be measured and does not allow one to clone an unknown quantum state [5]. Another relevant issue is nonlocality of quantum mechanics. When the system to be measured is a composite, we can generally obtain more information of the system by the global measurement on the whole system than by a combination of local measurements on its subsystems [6, 7].

Let us focus on the problem of distinguishing two pure states of a composite system which is shared by two parties. Is it a fundamental question of quantum information theory whether the optimal distinguishment can be performed by means of local operations and classical communication (LOCC) scheme of the two parties.

Walgate et al. [8] showed that any two mutually orthogonal pure states can be perfectly distinguished by LOCC. This is rather surprising since their result holds regardless of entanglement of the states, which is a typical source of nonlocality in quantum information. It has also been shown that any two generally nonorthogonal pure states can be optimally discriminated by LOCC: the optimal success probability of discrimination by the global measurement can be attained by an LOCC protocol. This was shown for the two types of discrimination problems: the inconclusive discrimination problem where error is allowed and the conclusive (unambiguous) discrimination problem [9, 10, 11] where no error is allowed but an inconclusive guess can be made. These results can be interpreted that there is no nonlocality in the discrimination of two pure states.

We can consider a different setting for discrimination problem of two pure states. In the usual setting, it is assumed that perfect classical knowledge of the two states \( \rho_1 \) and \( \rho_2 \) is given to the two parties. The measurement scheme for the optimal discrimination naturally depends on the classical knowledge of the states. Instead, let us assume that no classical knowledge of the states \( \rho_1 \) and \( \rho_2 \) are given, but a certain number \( (N) \) of their copies are available as reference states. One’s task is correctly identify a given input state \( \rho \) with one of the reference states \( \rho_1 \) and \( \rho_2 \) by means of a measurement on the whole state \( \rho \otimes \rho_1^N \otimes \rho_2^N \). When the number of copies \( N \) is infinite, the problem is reduced to quantum state discrimination. This is because we can always obtain complete classical knowledge of a quantum state if we have infinitely many copies of the state. We call this problem ”quantum state identification”. The optimal success probability has been determined for the inconclusive [13] and conclusive (unambiguous) [14, 15] identification problems.

In this paper, we investigate the inconclusive pure-state identification problem of \( N = 1 \) where the two reference pure states \( \rho_1 \) and \( \rho_2 \) are bipartite. The input state \( \rho \) given to Alice and Bob is guaranteed to be one of the reference states \( \rho_1 \) and \( \rho_2 \) with an \( a priori \) probability \( \eta_1 \) and \( \eta_2 \). The two reference states are assumed to be independently distributed on the pure state space in a unitary invariant way. Each reference state generated this way is generally entangled. We will demonstrate that Alice and Bob can identify the input state by means of an LOCC protocol with the success probability given by the optimal global identification scheme.

PURE-STATE IDENTIFICATION PROBLEM WITHOUT LOCC CONDITIONS

In this section, we will precisely formulate the pure-state identification problem and derive the maximum mean success probability without the LOCC conditions for the case of \( N = 1 \) and an arbitrary \( a priori \) occurrence probability of the reference states. In the case of single-qubit system, the problem has been solved by Bergou et al. [16]. For the case of general \( N \) but with equal \( a priori \) occurrence probabilities, see Ref. [13].

We have three quantum systems numbered 0, 1, and 2, each on a \( d \) dimensional space \( \mathbb{C}^d \). The input pure state \( \rho = |\phi\rangle \langle \phi| \) is prepared in system 0 and the two reference
pure states $\rho_1 = |\phi_1\rangle \langle \phi_1|$ and $\rho_2 = |\phi_2\rangle \langle \phi_2|$ in system 1 and 2, respectively. The space which an operator acts on is specified by the number in the parenthesis. For example, $\rho_1(1)$ is a density operator on system 1. The input state $\rho$ is promised to be one of the reference states $\rho_1$ and $\rho_2$ with an \textit{a priori} probability $\{\eta_1, \eta_2\}$. The two reference states are independently chosen from the state space $C^d$ in a unitary invariant way. More precisely, the distribution is assumed uniform on the 2d - 1 dimensional unit hypersphere of 2d real variables $\{\text{Re} c_i, \text{Im} c_i\}_{i=0}^{d-1}$, where $c_i$ is expansion coefficients of the state in terms of an orthonormal base $\{|i\rangle\}_{i=0}^{d-1}$. The distribution does not depend on a particular choice of the base.

Our task is correctly identify the input state with one of the reference states $\rho_\mu (\mu = 1, 2)$ by measuring the whole system $0 \otimes 1 \otimes 2$. We denote corresponding POVM elements by $E_\mu (\mu = 1, 2)$. The mean identification success probability is then given by

$$p(d) = \sum_{\mu=1,2} \eta_\mu \langle \text{tr}[E_\mu \rho_\mu (0) \rho_1(1) \rho_2(2)] \rangle,$$

where the symbol $\langle \cdots \rangle$ represents the average over the reference states $\rho_1$ and $\rho_2$. Note that the POVM $E_\mu$ is independent of $\rho_1$ and $\rho_2$, since we have no classical knowledge on the reference states.

The average over the reference states can be readily performed by using the formula [17]:

$$\langle \rho \otimes^n \rangle = \frac{S_n}{d_n},$$

where $S_n$ is the projector on to the totally symmetric subspace of $(C^d)^\otimes n$ and $d_n$ is its dimension given by $d_n = n+d-1C_{d-1}$. Using $E_2 = 1 - E_1$, the mean success probability to be maximized is written as

$$p(d) = \eta_2 + \frac{1}{d_1d_2}\text{tr}[E_1(\eta_1 S(01) - \eta_2 S(02))],$$

where $S(01)$ and $S(02)$ are the projector onto the totally symmetric subspace of space $0 \otimes 1$ and $0 \otimes 2$, respectively.

The only restriction on the POVM element $E_1$ is $0 \leq E_1 \leq 1$. In order to maximize the mean success probability Eq. (3), we use the following result which holds for any Hermitian operator $\Delta$:

$$\max_{0 \leq E \leq 1} \text{tr}[E\Delta] = \text{sum of all positive eigenvalues of } \Delta,$$

where the maximum is attained when $E$ is the projector $P_+$ onto the subspace $V_+(\Delta)$ spanned by all eigenstates of $\Delta$ with a positive eigenvalue. Note that it does not matter whether the subspace $V_+(\Delta)$ includes eigenvectors with zero-eigenvalue. In our case, $\Delta$ is defined to be

$$\Delta = \eta_1 S(01) - \eta_2 S(02).$$

Let us decompose the total space into three subspaces according to the symmetry with respect to system permutations [13].

$$V = C^d \otimes C^d \otimes C^d = V_S \oplus V_A \oplus V_M.$$  (6)

Here $V_S$ is the totally symmetric subspace of dimension $\dim V_S = d_3 = d(d+1)(d+2)/6$ and $V_A$ is the totally antisymmetric subspace of dimension $\dim V_A = d(d-1)(d-2)/6$. And the remaining subspace $V_M$ is the mixed symmetric subspace of dimension $\dim V_M = 2d(d^2 - 1)/3$. The subspace $V_M$ contains the 2 dimensional irreducible representation of the symmetric group of order 3, $S_3$, with multiplicity $\dim V_M/2$. We will not exploit any representation theory of the symmetric group in the following arguments. We denote projectors onto $V_S$, $V_A$, and $V_M$ by $S_3$, $A_3$ and $M_3$, respectively.

It is clear that $\Delta = \eta_1 - \eta_2$ in $V_S$ and $\Delta = 0$ in $V_A$. To determine eigenvalues of $\Delta$ in $V_M$, it is convenient to introduce two operators $D$ and $A$ as

$$D \equiv S(01) - S(02) = \frac{1}{2} (T(01) - T(02)),$$

$$A \equiv S(01) + S(02) - 1 = \frac{1}{2} (T(01) + T(02)).$$

Here, $T(01)$ is the operator which exchanges system 0 and 1 and $T(02)$ exchanges system 0 and 2. Calculating $D^2$, we find

$$D^2 = \frac{1}{4} (2 - T(01)T(02) - T(02)T(01))$$

$$= \frac{3}{4} (1 - S_3 - A_3)$$

$$= \frac{3}{4} M_3,$$

which implies that eigenvalues of $D$ is $\pm \sqrt{3}/2$ in $V_M$ and 0 otherwise. It is also easy to show that

$$DA + AD = 0,$$

$$A^2 = 1 - D^2.$$  (13)

The anticommutability of Eq. (12) implies that if $|+\rangle$ is an eigenstate of $D$ with eigenvalue $\sqrt{3}/2$, then $A|+\rangle$ is also an eigenstate of $D$ with eigenvalue $-\sqrt{3}/2$. By Eq. (13), we find that $|-\rangle \equiv 2A|+\rangle$ is correctly normalized. Note that the positive and negative eigenvalues of $D$ have the same multiplicity. Thus we can choose the orthonormal base $\{|+, k\rangle, |-, k\rangle\}$ in $V_M$ such that

$$D|+, k\rangle = \frac{\sqrt{3}}{2} |+, k\rangle,$$

$$D|-, k\rangle = -\frac{\sqrt{3}}{2} |-, k\rangle,$$

$$A|+, k\rangle = \frac{1}{2} |-, k\rangle,$$

$$A|-, k\rangle = \frac{1}{2} |+, k\rangle,$$  (17)
where the index \( k \) runs from 1 to \( \dim V_M/2 \). In this base, \( D \) and \( A \) are block-diagonalized with respect to \( k \) and each block has the following \( 2 \times 2 \) matrix representation.

\[
D = \begin{pmatrix} \frac{\sqrt{\eta}}{2} & 0 \\ 0 & -\frac{\sqrt{\eta}}{2} \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1/\eta \\ 1/\eta & 0 \end{pmatrix}.
\] (18)

In terms of \( D \) and \( A \), the operator \( \Delta \) is written as

\[
\Delta = \frac{1}{2} \left( \eta_1 - \eta_2 + D + (\eta_1 - \eta_2)A \right).
\] (19)

The operator \( \Delta \) is also block-diagonalized with the same \( 2 \times 2 \) matrix representation which can be readily diagonalized. Two eigenvalues of \( \Delta \) are given by

\[
\lambda_\pm = \frac{1}{2} \left( \eta_1 - \eta_2 \pm \sqrt{1 - \eta_1 \eta_2} \right),
\] (20)

and we find that \( \lambda_+ \geq 0 \) and \( \lambda_- \leq 0 \).

Now we can calculate the maximum success probability. Let us assume \( \eta_1 \geq \eta_2 \) for the moment. The positive eigenvalues of \( \Delta \) are \( \eta_1 - \eta_2 \) in \( V_S \) with multiplicity \( \dim V_S \) and \( \lambda_+ \) in \( V_M \) with multiplicity \( \dim V_M/2 \). We thus obtain

\[
p_{\text{max}}(d) = \eta_2 + \frac{1}{d_1 d_2} \left( (\eta_1 - \eta_2) \dim V_S + \lambda_+ \frac{\dim V_M}{2} \right)
\]

\[
= \frac{1}{2} + \frac{d + 2}{6d} (\eta_1 - \eta_2) + \frac{d - 1}{3d} \sqrt{1 - \eta_1 \eta_2}.
\] (21)

If \( \eta_1 \leq \eta_2 \), the only positive eigenvalue of \( \Delta \) is \( \lambda_+ \) in \( V_M \), hence we obtain

\[
p_{\text{max}}(d) = \eta_2 + \frac{1}{d_1 d_2} \lambda_+ \frac{\dim V_M}{2}
\]

\[
= \frac{1}{2} - \frac{d + 2}{6d} (\eta_1 - \eta_2) + \frac{d - 1}{3d} \sqrt{1 - \eta_1 \eta_2}.
\] (22)

These two cases can be combined to yield a symmetric form of the maximum success identification probability for general magnitude relation between \( \eta_1 \) and \( \eta_2 \).

\[
p_{\text{max}}(d) = \frac{1}{2} + \frac{d + 2}{6d} |\eta_1 - \eta_2| + \frac{d - 1}{3d} \sqrt{1 - \eta_1 \eta_2}.
\] (23)

The maximum is attained when the POVM element \( E_1 \) is given by \( P_+ \), the projector onto the subspace of positive eigenvalues of \( \Delta \). The \( p_{\text{max}}(d) \) given by Eq. (23) reproduces the result for the case \( d = 2 \) obtained in Ref. [10] and the one for arbitrary \( d \) in Ref. [13] when \( \eta_1 = \eta_2 = 1/2 \).

**PURE-STATE IDENTIFICATION BY LOCC**

Let us assume that each of the three systems 0, 1, and 2, where the input state and the two reference states are prepared, consists of two subsystems. The state space of each system is represented by a tensor product \( C^d = C^{d_A} \otimes C^{d_B} \), which is shared by Alice and Bob. Their task is to identify a given input bipartite state with one of the two bipartite reference states by means of local operations and classical communication (LOCC). As in the preceding section, the two reference states are chosen randomly from the pure state space \( C^d \) in the unitary invariant way. Therefore, those bipartite states are generally entangled. The question is whether Alice and Bob can achieve the maximum mean identification success probability given by the global measurement scheme.

In this section, we will show that this is possible by explicitly constructing an LOCC protocol which achieves it.

The mean success probability is given by Eq. (13) in the preceding section. The optimal global POVM element \( E_1 \) is \( P_+ \), the projector onto the subspace of positive eigenvalues of \( \Delta \) defined by Eq. (9). The projector \( P_+ \) does not apparently satisfy the conditions of LOCC, since the operator \( \Delta \) is not of a separate form. However, it should be noticed that \( \text{tr}[E_1 \Delta] \) remains the same if the support of \( E_1 \) contains states with zero-eigenvalue of \( \Delta \). It is this freedom that we will exploit in order to construct a POVM element \( E_1 \) which satisfies the LOCC conditions.

We begin with rewriting the operator \( \Delta \) of Eq. (19) in terms of local operators of Alice and Bob. Note that the exchange operator \( T_{(01)} \), for example, can be written as \( T_{(01)} = T^{(a)}_{(01)} \otimes T^{(b)}_{(01)} \), where \( T^{(a)}_{(01)} \) is the operator which exchanges Alice’s part of system 0 and 1 and \( T^{(b)}_{(01)} \) is defined for Bob’s part in the same way. Hereafter, we use the suffix \((a)\) or \((b)\) for an operator to indicate which space of Alice or Bob the operator acts on. Since we have

\[
D = D^{(a)} \otimes A^{(b)} + A^{(a)} \otimes D^{(b)},
\]

\[
A = D^{(a)} \otimes D^{(b)} + A^{(a)} \otimes A^{(b)},
\]

the operator \( \Delta \) is expressed as

\[
\Delta = \frac{1}{2} \left( \eta_1 - \eta_2 + D^{(a)} A^{(b)} + A^{(a)} D^{(b)} \right)
\]

\[
+ (\eta_1 - \eta_2) (D^{(a)} D^{(b)} + A^{(a)} A^{(b)}) \right).
\] (26)

The task for Alice and Bob is to maximize \( \text{tr}[E_1^T \Delta] \) with a POVM element \( E_1^T \) which satisfies LOCC conditions. We first construct a separable POVM \( E_1^T \) which attains the maximum value \( \text{tr}[P_+ \Delta] \). This separable POVM \( E_1^T \) will then be shown to satisfy the LOCC conditions. Without loss of generality, we assume \( \eta_1 \leq \eta_2 \) throughout this section, since the problem is symmetric with respect to \( \rho_1 \) and \( \rho_2 \).

Suppose that Alice and Bob first determine the permutation symmetry of their systems by the projective measurement with projection operators \( \{ S^{(a)}_3, A^{(a)}_3, M^{(a)}_3 \} \) and \( \{ S^{(b)}_3, A^{(b)}_3, M^{(b)}_3 \} \), respectively. If one of them found
that his or her system is totally symmetric or antisymmetric, it is easy for the other party to find the best strategy. For example, assume that Alice found her system to be totally symmetric. Knowing Alice’s outcome, Bob performs a POVM measurement, which we denote by $x^{(b)}$. The contribution to $\text{tr}[E_1^t \Delta]$ is then given by

$$\text{tr}[S_3^{(a)} \otimes x^{(b)} \Delta] = \text{dim}(V_s^{(a)}) \text{tr}_b[x^{(b)} \Delta^{(b)}],$$

(27)

since $S_3^{(a)} D^{(a)} = 0$ and $S_3^{(a)} A^{(a)} = S_3^{(a)}$. It is clear that the best strategy for Bob is to take the projector $P_+^{(b)}$ onto the positive-eigenvalue space of $\Delta^{(b)}$. Note that the positive-eigenvalue space of $\Delta^{(b)}$ is a subspace of $V_M^{(b)}$, since the eigenvalue of $\Delta^{(b)}$ in $V_M^{(b)}$ is $\eta_1 - \eta_2 \leq 0$. In this case the contribution to $\text{tr}[E_1^t \Delta]$ is given by

$$\text{tr}[S_3^{(a)} \otimes P_+^{(b)} \Delta] = \frac{\lambda_+}{2} \text{dim} V_s^{(a)} \text{dim} V_M^{(b)}.$$ (28)

If Alice’s part is totally antisymmetric, the operator for Bob is given by

$$\text{tr}_a[A_3^{(a)}] = \text{dim}(V_A^{(a)}) V_A^{(a)} \Delta^{(b)},$$

$$\Delta^{(b)} = \frac{1}{2} \left( \eta_1 - \eta_2 - (\eta_1 - \eta_2) A^{(b)} \right).$$ (29)

The operator $\Delta^{(b)}$ differs from $\Delta^{(b)}$ only in the signs in front of $D^{(b)}$ and $A^{(b)}$. Its eigenvalues are 0 in $V_s^{(b)}$ and $\eta_1 - \eta_2 \leq 0$ in $V_A^{(b)}$. In $V_M^{(b)}$, the operator $\Delta^{(b)}$ has eigenvalue $\lambda_-$ in the positive-eigenvalue subspace of $\Delta^{(b)}$ and $\lambda_+$ in the negative-eigenvalue subspace of $\Delta^{(b)}$. This implies Bob’s best POVM element is $P_-^{(b)}$, the projector onto the $\Delta^{(b)}$’s negative-eigenvalue subspace in $V_M^{(b)}$. The contribution to $\text{tr}[E_1^t \Delta]$ in this case is given by

$$\text{tr}[A_3^{(a)} \otimes P_-^{(b)} \Delta] = \frac{\lambda_+}{2} \text{dim} V_s^{(a)} \text{dim} V_M^{(b)}.$$ (30)

The same argument also holds when Bob’s system is totally symmetric or antisymmetric. Therefore, when the total state does not belong to $V_M^{(a)} \otimes V_M^{(b)}$, the whole contribution to $\text{tr}[E_1^t \Delta]$ is given by

$$\text{tr}\left[ \left( S_3^{(a)} P_+^{(b)} + A_3^{(a)} P_-^{(b)} + P_+^{(a)} S_3^{(b)} + P_-^{(a)} A_3^{(b)} \right) \Delta \right] = \frac{1}{2} \lambda_+ \left( \text{dim} V_s^{(a)} \text{dim} V_M^{(b)} + \text{dim} V_s^{(b)} \text{dim} V_M^{(a)} \right).$$ (31)

When the total state belongs to $V_M^{(a)} \otimes V_M^{(b)}$, construction of the best strategy for Alice and Bob is rather involved. First we introduce the following operators $X_1$ and $X_2$ for each of Alice’s space and Bob’s space:

$$X_1^{(x)} = \frac{2}{\sqrt{3}} D^{(x)},$$

$$X_2^{(x)} = 2 A^{(x)}, \quad (\kappa = a, b).$$ (32)

Note that $X_1^{(x)}$ and $X_2^{(x)}$ anticommute and $(X_1^{(x)})^2 = (X_2^{(x)})^2 = 1$ in the mixed symmetric space $V_M^{(x)}$. The operator $\Delta$ in terms of $X_1^{(x)}$ is not diagonal with respect to the index $i$. We further define rotated $X_i$’s in order to diagonalize $\Delta$ with respect to the index $i$.

$$Y_i^{(x)} = \cos \theta X_1^{(x)} + \sin \theta X_2^{(x)},$$

$$Y_2^{(x)} = - \sin \theta X_1^{(x)} + \cos \theta X_2^{(x)}, \quad (\kappa = a, b).$$ (33)

We find that $\Delta$ takes the following "diagonal" form:

$$\Delta = \frac{1}{2} \left( \eta_1 - \eta_2 + \lambda_+ Y_2^{(a)} \right),$$

(34)

if we take

$$\cos 2 \theta = \frac{\eta_1 - \eta_2}{2 \sqrt{1 - \eta_1 \eta_2}},$$

(35)

$$\sin 2 \theta = \frac{\sqrt{3}}{2 \sqrt{1 - \eta_1 \eta_2}}.$$ (36)

Eigenvalues of $Y_i^{(x)}$ are 1 and -1 with multiplicity $\text{dim} V_M^{(x)}/2$ since we have

$$Y_1^{(x)} = 1, \quad Y_2^{(x)} = 1,$$

$$Y_2^{(x)} Y_2^{(x)} + Y_2^{(x)} Y_1^{(x)} = 0.$$ (37)

And the positive- and negative-eigenvalue subspaces of $Y_1^{(x)}$ are transformed to each other by the operation of $Y_2^{(x)}$ and vice versa. We should also notice that $|\lambda_-| \geq 0$ when $\eta_1 \leq \eta_2$. These considerations imply that the optimal separate POVM element is given by $Q_+^{(x)} \otimes Q^{(x)} + Q^{(x)} \otimes Q_+^{(x)}$, where $Q_+^{(x)}$ is the projector onto the positive- and negative-eigenvalue subspace of $Y_2^{(x)}$. The contribution to $\text{tr}[E_1^t \Delta]$ is found to be

$$\text{tr}\left[ \left( Q_+^{(x)} \otimes Q_-^{(x)} + Q_-^{(x)} \otimes Q_+^{(x)} \right) \Delta \right] = \frac{1}{4} \lambda_+ \left( \text{dim} V_M^{(a)} \text{dim} V_M^{(b)} \right).$$ (38)

where we used $\text{tr}[Q_+^{(x)} Y_i^{(x)}] = 0$.

Thus the whole POVM element is given by

$$E_1 = S_3^{(a)} P_+^{(b)} + A_3^{(a)} P_-^{(b)} + P_+^{(a)} S_3^{(b)} + P_-^{(a)} A_3^{(b)} + Q_+^{(a)} Q_-^{(b)} + Q_-^{(a)} Q_+^{(b)}.$$ (39)

Adding Eq. (31) and Eq. (35), we find that $\text{tr}[E_1^t \Delta]$ indeed attains the maximum value given by the global POVM element $E_1 = P_+$:

$$\text{tr}[E_1^t \Delta] = \frac{1}{2} \lambda_+ \text{dim} V_M = \text{tr}[P_+ \Delta].$$ (40)
To show the above equality, we used the relation
\[ \dim V_M = \dim V_S^{(a)} \dim V_M^{(b)} + \dim V_M^{(a)} \dim V_M^{(b)} \]
\[ + \dim V_M^{(a)} \dim V_M^{(a)} + \dim V_M^{(a)} \dim V_M^{(b)} \]
\[ + \frac{1}{2} \dim V_M^{(a)} \dim V_M^{(b)}, \]

which can be readily verified by a straightforward calculation. The factor 1/2 in front of \( \dim V_M^{(a)} \dim V_M^{(b)} \) reflects the fact that the inner product (Kronecker product) of two mixed symmetric representations contains the totally symmetric and antisymmetric representations in addition to the mixed symmetric representation.

On the other hand, we can show that the POVM element \( E_1 \) given in Eq. (31) can be implemented with an LOCC protocol. First Alice and Bob determine which permutation symmetries each one’s local state has; totally symmetric, totally antisymmetric, or mixed symmetric. If one of them finds that his or her state is totally symmetric, totally antisymmetric, or mixed symmetric, they conclude that the input state is totally symmetric, totally antisymmetric, or mixed symmetric representation.

In this paper we allowed Alice and Bob to make a mistake in identifying the input state with one of the reference states. Instead we can consider a different version of identification problem, unambiguous (conclusive) identification problem \[14, 15\], where one is not allowed to make a mistake. It is of interest to ask whether the unambiguous identification can be performed optimally by means of LOCC and the results on this issue will be discussed elsewhere \[15\].

CONCLUDING REMARKS

It has been known that two bipartite pure states can be optimally discriminated within LOCC scheme if classical knowledge on the two states are available. In this paper, we showed that this is also true in the identification problem of two bipartite pure states, where no classical knowledge on the two states is given but only a copy of the two states is available as reference states.

We assumed the number \( N \) of copies of each state is one. In the limit of large \( N \), the identification problem reduces to the standard discrimination problem. This is because one can obtain complete classical information on the reference states by performing a tomographical measurement on infinitely many copies of them. Therefore, it has been shown that the pure-state identification can be optimally performed by means of LOCC when \( N = 1 \) and \( N = \infty \). We conjecture that this is also true for arbitrary \( N \).

In this paper we allowed Alice and Bob to make a mistake in identifying the input state with one of the reference states. Instead we can consider a different version of identification problem, unambiguous (conclusive) identification problem \[14, 15\], where one is not allowed to make a mistake. It is of interest to ask whether the unambiguous identification can be performed optimally by means of LOCC and the results on this issue will be discussed elsewhere \[15\].

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