Compositeness of the $\Delta(1232)$ resonance in $\pi N$ scattering

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We evaluate the $\pi N$ compositeness of the $\Delta(1232)$ resonance so as to clarify the internal structure of $\Delta(1232)$ in terms of the $\pi N$ component. Here the compositeness is defined as contributions from two-body wave functions to the normalization of the total wave function and is extracted from the $\pi N$ scattering amplitude. In this study we employ the chiral unitary approach with the interaction up to the next-to-leading order plus a bare $\Delta$ term in chiral perturbation theory and describe $\Delta(1232)$ in an elastic $\pi N$ scattering. Fitting the $\pi N$ scattering amplitude to the solution of the partial wave analysis, we obtain a large real part of the $\pi N$ compositeness for $\Delta(1232)$ comparable to unity and non-negligible imaginary part as well, with which we reconfirm the result in the previous study on the $\pi N$ compositeness for $\Delta(1232)$.

**KEYWORDS:** compositeness, chiral unitary approach, internal structure of $\Delta(1232)$

1. Introduction

The $\Delta(1232)$ resonance is one of the most fundamental hadrons to understand the underlying theory of strong interaction, QCD. The most important influence on strong interaction is that $\Delta(1232)$ as a $|u \uparrow u \uparrow u \uparrow\rangle$ state leads to an idea that quarks have color degrees of freedom [1]; otherwise, it breaks the Pauli principle with respect to the exchange of quarks. Moreover, $\Delta(1232)$ was found to belong to a decuplet in the flavor SU(3) symmetry together with the $\Sigma(1385)$ and $\Xi(1530)$ resonances and it predicted the existence and properties of the $\Omega^-$ baryon, which was followed by the experimental discovery. These excellent successes of the quark model for $\Delta(1232)$ and other decuplet states strongly indicate that the decuplet states are described as genuine $qqq$ states very well.

However, there are several suggestions that the effect of the meson–nucleon cloud for $\Delta(1232)$ seems to be large. For instance, the $M1$ transition form factor for $\gamma^* N \rightarrow \Delta(1232)$ shows that the meson cloud effect brings $\sim 30\%$ of the form factor at $Q^2 = 0$ [2]. In addition, the $\pi N$ component in $\Delta(1232)$ was studied in terms of the so-called compositeness extracted from the $\pi N$ scattering amplitude in a simple model [3]. As a result, the real part of the $\pi N$ compositeness is large and comparable to unity although its imaginary part is non-negligible, which implies large contribution of the $\pi N$ cloud to the internal structure of $\Delta(1232)$.

In this study we aim at examining whether the $\pi N$ compositeness is large or not in a more refined model for $\Delta(1232)$. For this purpose, we employ the so-called chiral unitary approach for the $\pi N$ scattering [4–10]. We take the interaction kernel from chiral perturbation theory up to the next-to-leading order plus a bare $\Delta$ term, and evaluate the loop function in a dispersion relation. We fit the model parameters to the solution of the partial wave analysis for the $\pi N$ scattering amplitude, and calculate the $\pi N$ compositeness for $\Delta(1232)$ from the $\pi N$ scattering amplitude.
2. Framework

2.1 Compositeness from scattering amplitude

Recently the compositeness has been introduced into the hadron physics so as to discuss the hadronic molecular component inside hadrons [11–14]. The compositeness is defined as contributions from two-body wave functions to the normalization of the total wave function for the resonance, and corresponds to unity minus the field renormalization constant intensively discussed in the 1960s [15, 16]. Although the compositeness is not observable and hence a model dependent quantity, it will be an important piece of information on the structure of the resonance.

First we consider the scattering amplitude and compositeness in the non-relativistic formulation, for simplicity. The scattering amplitude \( T(E; \mathbf{q}', \mathbf{q}) \), a solution of the Lippmann–Schwinger equation, is described with the energy \( E \) and relative momenta in the initial and final states, \( \mathbf{q} \) and \( \mathbf{q}' \), respectively, and has a pole at \( E = E_{\text{pole}} \), which coincides with the eigenenergy of the resonance state \( |\Psi\rangle \). Near the resonance pole, the scattering amplitude is dominated by the pole term in the expansion by the eigenstates of the full Hamiltonian, and hence we have

\[
T(E; \mathbf{q}', \mathbf{q}) = \langle \mathbf{q}'|\hat{\mathcal{V}}|\Psi\rangle \frac{1}{E - E_{\text{pole}}} \langle \Psi'|\hat{\mathcal{V}}|\mathbf{q}\rangle, \tag{1}
\]

where \( \hat{\mathcal{V}} \) is the operator of the interaction and \( |\mathbf{q}\rangle \) is the two-body state with relative momentum \( \mathbf{q} \). For the bra vector of the resonance we take \( \langle \Psi'| \) instead of \( \langle \Psi \rangle \), with which we can obtain the correct normalization \( \langle \Psi'|\Psi\rangle = 1 \) [13, 14]. Now we assume that the interaction is separable type in general \( L \)-wave scattering as done in Ref. [12], which is essential to the correct behavior of the amplitude near the threshold: \( T_{L\text{-wave}} = |q|^L|q'|^L T^L(E) \). Then the residue of the scattering amplitude becomes \( \langle \mathbf{q}'|\hat{\mathcal{V}}|\Psi\rangle = \langle \Psi'|\hat{\mathcal{V}}|\mathbf{q}\rangle = g|q|^L \) with the coupling constant of the resonance to the two-body state \( g \). As a result, the norm of the two-body wave function is calculated as

\[
X \equiv \int \frac{d^3 q}{(2\pi)^3} \langle \Psi'|\mathbf{q}\rangle \langle \mathbf{q}|\Psi\rangle = g^2 \int \frac{d^3 q}{(2\pi)^3} \frac{|q|^2}{E_{\text{pole}} - [M_{\text{th}} + |q|^2/(2\mu)]^2} = -g^2 \left[ \frac{dG_L}{dE} \right]_{E = E_{\text{pole}}}, \tag{2}
\]

where \( M_{\text{th}} \) and \( \mu \) are the threshold of the two-body state and the reduced mass, respectively, and we have used a relation \( \langle \mathbf{q}|\Psi\rangle = \langle \Psi'|\mathbf{q}\rangle = g|q|^L/(E_{\text{pole}} - [M_{\text{th}} + |q|^2/(2\mu)]) \) obtained from \( \langle \mathbf{q}'|\hat{\mathcal{V}}|\mathbf{q}\rangle = g|q|^L \). The \( L \)-wave loop function \( G_L(E) \) is defined as

\[
G_L(E) \equiv \int \frac{d^3 q}{(2\pi)^3} \frac{|q|^2}{E - [M_{\text{th}} + |q|^2/(2\mu)]}. \tag{3}
\]

2.2 \( \Delta(1232) \) in chiral unitary approach

Next let us formulate the \( \pi N \) scattering amplitude in the chiral unitary approach. In this study we solve the following scattering equation in an algebraic form for the elastic \( \pi N \) scattering:

\[
T_{\pi N}^< (w) = V_{\pi N}^< (w) + V_{\pi N}^> (w) G_{\pi N} (w) T_{\pi N}^> (w) = \frac{1}{1/V_{\pi N}^> (w) - G_{\pi N} (w)}, \tag{4}
\]

with the center-of-mass energy \( w \), the interaction kernel \( V_{\pi N}^< \) and full amplitude \( T_{\pi N}^< \) in isospin \( I, L \) wave, and total angular momentum \( J = L \pm 1/2 \), and the \( L \)-wave loop function \( G_L \). The interaction kernel is taken from chiral perturbation theory up to the next-to-leading order, i.e., the Weinberg–Tomozawa term \( V_{\text{WT}} \), the \( s \) - and \( u \) -channel nucleon \([N(940)]\) exchange terms \( V_{s+h} \), and the contact next-to-leading order term \( V_2 \), plus a bare \( \Delta \) term \( V_\Delta ; V = V_{\text{WT}} + V_{s+h} + V_2 + V_\Delta \). This is projected to the eigenstate \( I, L \), and \( J = L \pm 1/2 \) to be \( V_{\pi N}^> \), and then the momentum prefactor \( |q|^2L \) is picked out as \( V_{\pi N}^> = |q|^2L V_{\pi N}^> \). Now \( V_{\pi N}^> \) is a function only of the center-of-mass energy \( w \) and we use it as the
interaction kernel in the scattering equation (4). On the other hand, the loop function is evaluated in a dispersion relation with the relative momentum $|q|^{2L}$ inside the integral as

$$G_L(w) \equiv \int_{s_{th}}^{\infty} \frac{ds'}{2\pi} \frac{\rho(s')q(s')^{2L}}{s - s'} = i \int \frac{d^4q}{(2\pi)^4} \frac{|q|^{2L}}{(P - q)^2 - m^2} \frac{1}{|q^2 - M_N^2|}, \quad \rho(s) \equiv \frac{q(s)}{4\pi w},$$

where $s = w^2$, $P^\mu = (w, \mathbf{0})$, $m_\pi$ and $M_N$ are the pion and nucleon masses, respectively, $q(s)$ is the center-of-mass momentum, and $s_{th} \equiv (m_\pi + M_N)^2$. We note that we need two subtraction constants for the $p$-wave loop function. In this study we fix one of them so that the nucleon mass stays physical, for which we require $G_L(w = M_N) = 0$. From the $\pi N$ scattering amplitude, we can extract the $\pi N$ compositeness $X_{\pi N}$ for $\Delta(1232)$ and $N(940)$ with the formula (2) [12] with replacing the loop function $G_L$ with that evaluated in the dispersion relation (5).

In this construction we have seven model parameters for the $\pi N$ scattering amplitude: four from the low-energy constants in the next-to-leading order interaction, the bare $\Delta$ mass, the bare $\pi N$-$\Delta$ coupling constant, and one subtraction constant $\Lambda$ in $p$ wave, which enters as $G_{L=1}(w) = (s - M_N^2)\Lambda +$ (finite part). They are determined from the fitting to the $\pi N$ partial wave amplitudes $S_{11}$, $S_{31}$, $P_{11}$, $P_{31}$, $P_{13}$, and $P_{33}$ obtained in Ref. [17], which we refer to as “WI 08”, up to $w = 1.35$ GeV.

3. **Numerical results**

Now let us calculate the $\pi N$ compositeness of $\Delta(1232)$ in the chiral unitary approach. We fit the model parameters to the $\pi N$ scattering amplitude WI 08, and we can reproduce the $\pi N$ amplitude very well with $\chi^2/N_{d.o.f.} = 486.3/809$. The best fit for the $P_{33}$ amplitude is shown in Fig. 4 as red solid lines (Naive). From the $\pi N$ amplitude, we can extract the $\pi N$ compositeness with the formula (2). The result of the $\pi N$ compositeness as well as the pole position and coupling constant is shown in the second and fourth columns in Table I. As one can see, the $\pi N$ compositeness for $\Delta(1232)$ has large real part comparable to unity. Therefore, our refined model reconfirms the result in the previous study [3], and the result implies large contribution of the $\pi N$ cloud to the internal structure of $\Delta(1232)$. However, for $N(940)$, the $\pi N$ compositeness is real but negative and hence unphysical, because one

### Table I. Properties of $\Delta(1232)$ and $N(940)$.

|                  | $\Delta(1232)$   | $N(940)$      |
|------------------|------------------|---------------|
|                  | Naive            | Constrained   | Naive          | Constrained   |
| $w_{\text{pole}}$ [MeV] | 1209.8 - 47.6$i$ | 1206.9 - 49.6$i$ | 938.9          | 938.9         |
| $g$ [MeV$^{-1/2}$] | 0.383 - 0.053$i$ | 0.395 - 0.061$i$ | 0.560          | 0.516         |
| $X_{\pi N}$      | 0.69 + 0.39$i$   | 0.87 + 0.35$i$ | -0.18          | 0.00          |
cannot interpret it as a probability even for a stable state. This is because \( dG_{L=1}/dw(w = M_N) \) is positive, which should be negative as the derivative of the integrand in Eq. (5) becomes negative.

In order to resolve this, in addition to \( G_L(w = M_N) = 0 \) we constrain the loop function as \( dG_{L=1}/dw(w = M_N) \leq 0 \), and the fitted amplitude becomes the blue dashed lines (Constrained) in Fig. 1 with \( \chi^2/N_{\text{d.o.f}} = 1239.9/809 \). The properties of \( \Delta(1232) \) and \( N(940) \) are shown in the third and fifth columns of Table I. The properties of \( \Delta(1232) \) shift only slightly and the \( \pi N \) compositeness for \( N(940) \) is non-negative. Again we reconfirm the result for \( \Delta(1232) \) in the previous study [3].

Finally we note that there is ambiguity in calculating the \( \pi N \) compositeness \( X_{\pi N} \) with the loop function in the dispersion relation (5). Namely, as discussed in Ref. [18], we can consider a shift of the subtraction constant \( \tilde{A} \), which can be compensated by the corresponding shift of the interaction \( V \) so as not to change the full amplitude \( T \). This shift of the subtraction constant can change the value of \( dG_{L=1}/dw \) and hence that of \( X_{\pi N} \), since the subtraction constant survives when we differentiate \( G_L(w) = (s - M_N)\tilde{A} + \) (finite part). However, if we have a constraint \( dG_{L=1}/dw(w = M_N) \leq 0 \), such a shift of the subtraction constant is also constrained and \( dG_{L=1}/dw \) cannot be close to zero around the \( \Delta(1232) \) energy region. In particular, in the present calculation \( \tilde{A} \) takes the maximal value under the constraint \( dG_{L=1}/dw(w = M_N) \leq 0 \), as seen from \( X_{\pi N} = 0 \) for \( N(940) \), which means \( dG_{L=1}/dw(w = M_N) = 0 \). As a consequence, the present calculation would give a minimal value of \( |X_{\pi N}| \) for \( \Delta(1232) \) in our approach from the viewpoint of the shift of the subtraction constant.

4. Summary

In this study we have investigated the internal structure of \( \Delta(1232) \) in terms of the \( \pi N \) compositeness, which was extracted from the elastic \( \pi N \) scattering amplitude in the chiral unitary approach. Fitting the model parameters so as to reproduce the solution of the \( \pi N \) partial wave analysis, we have obtained the large real part of the \( \pi N \) compositeness comparable to unity for \( \Delta(1232) \) and non-negligible imaginary part as well. Therefore our refined model reconfirms the result in the previous study on the \( \pi N \) compositeness for \( \Delta(1232) \). This implies large contribution of the \( \pi N \) cloud to the internal structure of \( \Delta(1232) \). The details of the present study will be given in a forthcoming paper [19].

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