Simplified Chaplygin Gas: Deriving $H_0$ From Ages of Old High Redshift Objects and Baryon Acoustic Oscillations

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Abstract

The discovery that the expansion of the Universe is accelerating is the most challenging problem of modern cosmology. In the context of general relativity, there are many dark energy candidates to explain the observed acceleration. In this work we focus our attention on two kinds of simplified Chaplygin gas cosmological accelerating models recently proposed in the literature. In the first scenario, the simplified Chaplygin gas works like a Quintessence model while in the second one, it plays the role of a Quartessence (an unification of the dark sector). Firstly, in order to limit the free parameters of both models, we discuss the age of high redshift objects with special emphasis to the old quasar APM 08279+5255 at $z = 3.91$. The basic finding is that this old high redshift object constrain severely the simplified Chaplygin cosmologies. Secondly, through a joint analysis involving the baryon acoustic oscillations (BAO) and a sample of old high redshift galaxies (OHRGs) we also estimate the value of the Hubble parameter, $H_0$. Our approach suggests that the combination of these two independent phenomena provides an interesting method to constrain the Hubble constant.

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I. INTRODUCTION

An impressive amount of different astrophysical data are suggesting that the observed Universe can be represented by an accelerating spatially flat cosmology driven by an exotic component sometimes called dark energy [1, 2]. In addition to the cosmological constant, the most interesting candidates for describing this dark energy component are: a vacuum decaying energy density, or a time varying $\Lambda(t)$ [3], the so-called “X-matter” [4], a relic scalar field rolling down its potential [5], and a Chaplygin Gas [6]. Some recent review articles discussing the history, interpretations, as well as the major difficulties of such candidates have also been published in the last few years [7].

On the other hand, some studies have pointed out that the age of old high redshift objects (at moderate and high redshifts) is a powerful physical constraint to cosmological models [8, 9, 10, 11, 12]. For old high redshift galaxies (OHRGs), for instance, their ages can be inferred by measuring the magnitude in different bands and then using stellar evolutionary codes to choose the model that reproduces the observed colors [8, 9]. In principle, for old high redshift objects, the derived ages can put tight constraints on the physical parameters or even rule out many kinds of dark energy models.

In this article, we discuss some observational constraints on two different classes of Chaplygin type accelerating cosmologies coming from the existence of the APM 08279+5255, an old quasar at high redshift located at $z = 3.91$ which has an estimated age of 2-3 Gyr [10]. As reported by many authors [11], the existence of this object is not compatible with many dark energy models unless the present accepted values of $\Omega_M$ and $H_0$ be further revised. We also derive new constraints on the Hubble constant $H_0$, by using a joint analysis involving different OHRGs samples and the current SDSS measurements of the baryon acoustic peak [13]. The BAO signature comes out because the cosmological perturbations excite sound waves in the relativistic plasma, thereby producing the acoustic peaks in the early universe. Actually, Eisenstein et al. [13] presented the large scale correlation function from the Sloan Digital Sky Survey (SDSS) showing clear evidence for the baryon acoustic peak at $100h^{-1}$ Mpc scale, which is in excellent agreement with the WMAP prediction from CMB data. We stress that the Baryon Acoustic Oscillations (BAO) method is independent of the Hubble constant $H_0$ although being heavily dependent on the value of $\Omega_M$. In this way, it con-
tributes indirectly to fix the value of $H_0$ since it breaks the degeneracy on the mass density parameter, $\Omega_M$. As we shall see, both set of data (OHRGs sample and BAO signature) provide an interesting method to constrain the Hubble constant.

II. THE MODELS

A. Simplified Chaplygin Quintessence Cosmology

The basic idea underlying the simplified Chaplygin gas concept (from now on SC-gas) either the Quintessence or Quartessence version, is to reduce the number of free parameters of the generalized C-gas, however, maintaining the physically interesting properties. In the Quintessence case it behaves as a pressureless fluid (nonrelativistic matter) at high-$z$ while, at late times, it approaches the quintessence behavior. The generalized C-gas has as equation of state (EOS):

$$p_C = -A/\rho_C^\alpha, \quad (1)$$

which leads to the following adiabatic sound speed:

$$v_s^2 = \frac{dp}{d\rho} = \alpha A/\rho_C^{1+\alpha}. \quad (2)$$

By using the definition $A_s \equiv A/\rho_{C0}$, the present C-gas adiabatic sound speed reads $v_{s0}^2 = \alpha A_s$. Thus, if we want to require the positiviness of $v_{s0}^2$ and also want to reduce the number of free parameters of the C-gas, the simplest way of doing that is imposing $A_s = \alpha$. With this we have that $v_{s0}^2 = \alpha^2$, thus assuring a natural positiviness for $v_{s0}^2$. Furthermore, in order to guarantee that $v_s \leq c$, the $\alpha$ parameter it will be restricted on the interval $0 < \alpha \leq 1$ (for more details see Lima, Cunha and Alcaniz). As a consequence, the simplified Quintessence Chaplygin gas is fully characterized only by two parameters: $\alpha$ and $\Omega_C$. As one may check, for a spatially flat Friedmann-Robertson-Walker geometry the dimensionless Hubble parameter is given by

$$E(z) = \left[\Omega_M(1+z)^3 + \Omega_C \left[\alpha + (1 - \alpha)(1 + z)^{3(\alpha+1)}\right]^{1/\alpha}\right]^{1/2}, \quad (3)$$

where $E(z) \equiv H(z)/H_0$, and $\Omega_C = 1 - \Omega_M$. In particular, this means that the $\alpha$ parameter is actually the unique unknown constant related to this SC-gas model. The free parameters of such cosmologies are reduced to $\Omega_M$, and the present rate of expansion, $H_0$. 

3
B. Simplified Quartessence Cosmology

In the Quartessence version of the SC-gas, the model maintain all the physical requirements of the Quintessence scenario, with the additional feature of unifying dark sector (dark matter and dark energy). In such a scenario, besides the SC-gas the unique nonvanishing contribution (radiation is important only at early times) comes from baryonic component which is tightly constrained by big-bang nucleosynthesis (BBN) and the temperature anisotropies of the cosmic background radiation (CMB).

In this case, it is readily seen that the dimensionless Hubble parameters is given by

\[ E(z) = \left[ \Omega_b(1 + z)^3 + \Omega_Q4 \left[ \alpha + (1 - \alpha)(1 + z)^3(\alpha + 1) \right]^{1/3} \right]^{1/2}, \]  

(4)

where \( \Omega_b \) is the baryon contribution, \( \Omega_Q4 \) stands for the SC-gas as Quartessence contribution. Note that \( \Omega_Q4 = 1 - \Omega_b \) because we have assumed spatial flatness. Therefore, since \( \Omega_b \) is quite well determined, one may conclude that only two parameters remain to be constrained in this unified dark matter/energy scenario, namely, \( \alpha \) and \( H_0 \).

III. OBSERVATIONAL CONSTRAINTS

A. Age-Redshift Test

Let us now discuss the age-redshift test in the above discussed backgrounds. For both cases, the expression for the age relation \( t_z \) takes the following form:

\[ H_0 t_z = \int_0^z \frac{dz'}{(1 + z')E(z')}, \]  

(5)

where \( E(z) \) is given by (3) or (4) for the Quintessence and Quartessence versions, respectively. Note that for \( \Omega_M = 1 \) the above expression reduces to the well known result for Einstein-de Sitter model (CDM, \( \Omega_M = 1 \)) for which \( t_z = \frac{2}{3}H_0^{-1}(1 + z)^{-3/2} \). As one may conclude from the above equation, limits on the cosmological parameters \( \Omega_M \) (or \( \Omega_b \)), \( \alpha \) and \( H_0 \) (or equivalently \( h \equiv H_0(\text{km s}^{-1}\text{Mpc}^{-1})/100 \)), can be derived by fixing \( t_z \) from observations. Note also that the age parameter, \( T_z = H_0 t_z \), depends only on the product of the two quantities \( H_0 \) and \( t_z \), which are usually estimated from completely independent methods \[18\]. The age-redshift test is given by the condition

\[ \frac{T_z}{T_q} = f(\Omega_{M,b}, \alpha, z) \left( \frac{1}{H_0 t_q} \right) \geq 1, \]  

(6)
FIG. 1: High-z Age Test. Panel a) Dimensionless age parameter as a function of redshift for some values of the pair \((\alpha, \Omega_M)\). Panel b) Dimensionless age parameter as a function of redshift for some values of \((\alpha, \Omega_{Q4})\). As explained in the text, all curves crossing the shadowed area yield an age parameter smaller than the minimal value required by the quasar APM 08279+5255, 2 Gyr, as reported by Hasinger et al. (2002) [10] (see main text).

where \(t_q\) is the age of an arbitrary object, say, a quasar or a galaxy at a given redshift \(z\) and \(f(\Omega_{M,b}, \alpha, z)\) is the dimensionless factor given by the \textit{rhs} of Eq. (5). For each object, the denominator of the above equation defines a dimensionless age parameter \(T_q = H_0 t_q\). In particular, for the 2.0-Gyr-old quasar at \(z = 3.91\) estimated by Hasinger et al. (2002) [10] yields \(T_q = 2.0 H_0 \text{Gyr}\). In addition, for the most recent determinations of the Hubble parameter, \(H_0 = 72 \pm 8 \text{ km}\text{s}\text{^{-1}}\text{Mpc}\text{^{-1}}\) as given by the Hubble Space Telescope Key Project [19], the age parameter take values on the interval \(0.131 \leq T_q \leq 0.163\). It follows that \(T_q \geq 0.131\). Therefore, for a given value of \(H_0\), only models having an expanding age bigger than this value at \(z = 3.91\) will be compatible with the existence of this object.

In Fig. 1, we show the dimensionless age parameter \(T_z = H_0 t_z\) as a function of the redshift for different values of \((\alpha, \Omega_M)\) in the Quintessence case, and, for some selected values of \(\alpha\) in the Quartessence version. The shadowed regions in the graphs were determined from the minimal value of \(T_q\). It means that any curve crossing the rectangles yields an age parameter smaller than the minimal value required by the presence of the quasar APM 08279+5255. At this point, in line with the arguments presented by Hasinger et al. (2002) [10], we recall that that X-ray observations show a Fe/O ratio for the quasar APM 08279+5255 compatible with an age of 2 Gyr. Naturally, we do not expect such results to be free of observational
FIG. 2: Panel a) Isochrones in the plane $\Omega_M$ versus $\alpha$, for the SC-gas Quintessence model, which match the estimated range of ages of the quasar. Panel b) Ratio $T_z/T_q$ at the redshift of the quasar as a function of the $\alpha$ parameter showing the range of this parameter allowed by the existence of the quasar. For $t_q = 2$ Gyr, we have $\alpha > 0.965$ and for $t_q = 3$ Gyr, $\alpha > 0.997$.

and/or theoretical uncertainties, but the analysis of the quoted reference has independently been confirmed by Friaça and collaborators [11].

In Fig. 2, we can see more clearly how the age of the quasar helps to constrain these models. In Panel (a), we show isochrones in the plane $\Omega_M - \alpha$, for the Quintessence scenario. Using the most conservative age estimate of the quasar (2 Gyr), the limits are $\Omega_M < 0.21$ and $\alpha > 0.947$. From Panel (b) we also see that the Quartessence scenario is compatible with the presence of the quasar only if $\alpha > 0.965$. Therefore, as happens with other dark energy candidates [11], the existence of APM 08279+5255 quasar constrains severely both formulations (Quintessence and Quartessence) of the Simplified Chaplygin Gas cosmology.

B. Age-Redshift and BAO: Joint Analysis

In order to break possible degeneracies in the simplified models, let us now consider a joint analysis involving a sample of 13 old galaxies [20, 21] and data from the large scale structure (LSS). For the LSS data, we consider the recent measurements of the BAO peak in the large scale correlation function as inferred by Eisenstein et al. [13] using a large sample of luminous red galaxies from the SDSS Main Sample. The SDSS BAO measurement provides $A = 0.469(n_s/0.98)^{0.35} \pm 0.017$. We shall fix the scalar spectral index as given by the the
FIG. 3: Contours in the parameter spaces using the Age Redshift and BAO joint analysis. The contours correspond to 68.3%, 95.4% and 99.7% confidence levels. Panel a) Quintessence simplified model. The best fit parameters are \( h = 0.42^{+0.25}_{-0.060} \) and \( \Omega_M = 0.38^{+0.092}_{-0.13} \), at 95.4% c.l. Panel b) Quartessence simplified model. The best fits parameters are \( \alpha = 0.743^{+0.073}_{-0.062} \) and \( h = 0.552^{+0.070}_{-0.057} \), at 95.4% c.l.

WMAP-5yr collaboration [22], who find \( n_s = 0.960^{+0.014}_{-0.013} \). For constraining the parameters with basis on the age-redshift test we consider the oldest galaxies in the samples observed by quoted authors [20] and previously employed by Lima, Jesus and Cunha [21] in the context of the cosmic concordance ΛCDM models. Actually, since the galaxy formation is a random process it is not need to consider the ages of all objects in order to constrain the model parameters. In other words, only the oldest ones are important for constraining a given cosmological model. It is also necessary to take into account the incubation time \( (t_{inc}) \), that is, the time elapsed from the beginning of structure formation until the formation of the galaxy in question. It is reasonable to assume that such a quantity does not vary too much and that its uncertainty is quantified by \( \sigma_{t_{inc}} \). In agreement with other works in the literature [23, 24], we estimate that \( t_{inc} = 0.8 \pm 0.4 \)Gyr.

Let us now perform a \( \chi^2 \) statistical analysis in order to constrain the free parameters of the models. In the Quintessence scenario me need to minimize the expression:

\[
\chi^2(\Omega_M, \alpha, h) = \chi^2_{Age}(\Omega_M, \alpha, h) + \chi^2_{BAO}(\Omega_M, \alpha) = \\
\sum_{i=1}^{13} \frac{(t_{obs,i} + t_{inc} - t_{th})^2}{\sigma_i^2 + \sigma_{inc}^2} + \left[ \frac{A - 0.469(n_s/0.98)^{0.35}}{0.017} \right]^2,
\]

(7)
while in the Quartessence we have to minimize:

$$\chi^2(\alpha, h) = \chi^2_{\text{Age}}(\alpha, h) + \chi^2_{\text{BAO}}(\alpha) = \sum_{i=1}^{13} \frac{(t_{\text{obs},i} + t_{\text{inc}} - t_{\text{th}})^2}{\sigma_i^2 + \sigma_{\text{inc}}^2} + \left[ \frac{A - 0.469(n_s/0.98)^{0.35}}{0.017} \right]^2,$$

since the baryon contribution can be fixed by the WMAP-5yr ($\Omega_b = 0.0462 \pm 0.0015$), and, as such, only the pair of parameters $(\alpha, h)$ can be considered in the Quartessence scenario.

To begin with let us consider the Quintessence case. Constraints on the free parameters are obtained by marginalizing the associated likelihood expression over $\alpha$ as follows:

$$\tilde{L}(\Omega_M, h) = \int_{-\infty}^{+\infty} \pi(\alpha) L(\Omega_M, \alpha, h) d\alpha = \int_{0}^{1} L(\Omega_M, \alpha, h) d\alpha,$$

where $L$ is the likelihood, given by $L \propto e^{-\chi^2/2}$, and $\pi(\alpha)$ is the prior on $\alpha$, assumed to be a top-hat on the physical region $0 < \alpha \leq 1$. In Panel (a) of Fig. 3 we display the constraints on the resulting 2-dimensional plane $\Omega_M - h$. Naturally, no marginalization is necessary to the Quartessence version since it has only two free parameters. In this case, the required constraints in the plane $\alpha - h$ are displayed in Panel (b) of Fig. 3.

In Tab. I we display some estimates of $H_0$. In the standard model of a flat $\Lambda$-dominated universe with CMB data alone, it is found $H_0 = 71.9^{+2.6}_{-2.7}$. On Sandage et al. (2006), the final result of the HST collaboration, ranging over 15 years, it is found $H_0(\text{cosmic}) = 62.3 \pm 1.3$ (random) $\pm 5.0$(systematic), based on 62 SNe Ia with $3000 < v_{\text{CMB}} < 20000\text{km s}^{-1}$ and on 10 luminosity-calibrated SNe Ia. Their local value of $H_0$ ($300 < v_{220} < 2000\text{km s}^{-1}$) is $H_0(\text{local}) = 60.9 \pm 1.3$(random) $\pm 5.0$ (systematic), from 25 Cepheid and 16 SNe Ia distances, involving a total of 34 different galaxies. This result is in disagreement with the result of Freedman et al. (2001), $H_0 = 72 \pm 8$, obtained from the HST Key Project, and this discrepancy is a matter of debate. In what concerns our analysis here, our principal results are, for the S-CG, $0.26 \leq \Omega_M \leq 0.47$, $0.36 \leq h \leq 0.67$ ($2\sigma$) and for the Q4-CG, $0.681 \leq \alpha \leq 0.805$, $0.495 \leq h \leq 0.622$ ($2\sigma$). These results depend weakly on the incubation time ($t_{\text{inc}} = 0.8 \pm 0.4$ Gyr). On Tab. II we may see that our result on the Hubble constant is consistent with the value of Sandage et al. (2006), where was found $H_0 = 62.3 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (statistical).

IV. RESULTS AND DISCUSSIONS

It is usually believed that old high-redshift objects may play an important role to the question related to the ultimate fate of the Universe. In a point of fact, their age estimates
| Method                    | Reference                  | $H_0$          |
|--------------------------|----------------------------|----------------|
| Age Redshift             | Jimenez et al. (2003) [25] | 69 ± 1.2       |
| S-Z effect               | Schmidt et al. (2004) [27] | 69$^{+8}_{-8}$ |
| S-Z effect               | Jones et al. (2005) [28]   | 66$^{+11+9}_{-10-8}$ |
| SNe Ia/Cepheid           | Freedman et al. (2001) [19] | 72 ± 8        |
| SNe Ia/Cepheid           | Sandage et al. (2006) [29] | 62 ± 1.3 ± 5  |
| CMB                      | Dunkley et al. (2008) [22] | 71.9$^{+2.6}_{-2.7}$ |
| Old Galaxies+BAO         | Lima et al. (2007) [21]    | 71 ± 4         |
| Old Galaxies+BAO         | Figure 3 a) (S-CG)         | 42$^{+25}_{-6}$ (95% c.l.) |
| Old Galaxies+BAO         | Figure 3 b) (Q4-CG)        | 55.2$^{+2.0}_{-1.7}$ (95% c.l.) |

TABLE I: Limits on $H_0$ (km s$^{-1}$ Mpc$^{-1}$) using different methods.

provide a powerful technique for constraining the free parameters in a given cosmological model [9, 11]. In particular, this means that the so-called high-redshift age crisis is now becoming an important complement to other independent cosmological tests.

In this work we have discussed some constraints in the so-called Simplified Chaplygin Gas (SC-gas) both in the Quintessence and Quartessence versions. Initially, we have investigated the dimensionless age parameter $T_z = H_0 t_z$. In the simplified Quintessence scenario, by assuming that the quasar has an age of at least a 2 Gyr, we have obtained $\alpha \geq 0.947$ while for the simplified Quartessence model the limit is $\alpha > 0.965$. We have also studied the constraints on the Hubble constant using a joint analysis involving different OHRGs samples and the current SDSS measurements of the baryon acoustic peak for both scenarios. In this case, we have found values of the Hubble constant moderately low, specially, to the Quintessence case (see Fig. 3 and Table 1).

As a conclusion, we would like to stress that our results point to at least 3 possibilities. The first one is that the simplified Chaplygin type gas models (Quintessence and Quartessence) do not provide a good fit to the Age+BAO data because the best fit predicted for $H_0$ is low. A second possibility is that the estimated ages of the old high redshift galaxies considered here (or even their incubation time) need to be revised, and this may increase the predicted value of the Hubble constant. Finally, as has been advocated by some authors [29, 30], there exist the possibility that we live in a Universe with Hubble constant
smaller than the one predicted by the HST Key Project [19].

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