**Coherent destruction of Stark many-body localization**

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We study the phenomenon of many-body localization (MBL) in an interacting system subjected to a combined DC as well as square wave AC electric field. First, the condition for the dynamical localization, coherent destruction of Wannier-Stark localization and super Bloch oscillations in the non-interacting limit, are obtained semi-classically. In the presence of interactions (and a confining/disordered potential), a static field alone leads to “Stark many-body localization”, for sufficiently large field strengths. We find that in the presence of an additional high-frequency AC field, there are two ways of maintaining the MBL intact: either by resonant drive where the ratio of amplitude to the frequency of the drive ($A/\omega$) is tuned at the dynamical localization point of the non-interacting limit, or by off-resonant drive. Remarkably, resonant drive with $A/\omega$ tuned away from the dynamical localization point leads to a coherent destruction of Stark-MBL. Moreover, a pure (high-frequency) AC field can also give rise to the MBL phase if $A/\omega$ is tuned at the dynamical localization point of the zero DC field problem.

**Introduction.** Driven systems exhibit many counterintuitive features in comparison with their undriven counterparts. Striking examples include the Kapitza pendulum [1], kicked rotors [2, 3], dynamical localization [4, 5], and Floquet topological insulators [6–8]. Driving a topological insulator can dramatically change its nature, converting it from a trivial to a topologically non-trivial system [6–8]. A number of studies in the recent past have considered both periodic drives [6, 9–11] and aperiodic drives [12–15], leading to the discovery of many novel phenomena.

In the context of many-body physics where interactions play a crucial role, the phenomenon of many-body localization has grabbed much attention in the last decade or so [16–21]. While disorder-induced localization (Anderson localization [22]) is well known, a thorough understanding of the survival of localization in the presence of interactions (termed as “many-body localization (MBL)” [19–22]) is a work in progress. The phenomenon of MBL has been generalized to non-random potentials. Very recently, MBL-like signatures (Stark-MBL) have been observed in a clean interacting system subjected to a static electric field [23–26]. In this Letter, we consider the driven version of this model, replacing the static field by a combined DC and periodic time-dependent AC field and investigate the fate of the Stark-MBL phase under such periodic driving.

In general, driving a many-body system heats it up as a consequence of the energy absorption from the external drive [27–30]. Thus, the system has a tendency to reach a featureless infinite-temperature-like state in the long-time limit. One way to avoid this is by the inclusion of a strong disorder leading to a stable MBL phase in the presence of high frequency driving [31–35]. The MBL phase in this context has been observed experimentally [36]. Subjecting the system to a time-periodic electric field drive is special as it effectively suppresses the hopping strength [4, 5, 37, 38]. One recently reported [39] example of this kind is that of high-frequency electric field leading to the conversion of an ergodic phase into a stable MBL phase. In the noninteracting limit, a vast variety of phenomena are associated with electric field drive, ranging from dynamic localization [4, 5, 37, 38], coherent [40, 41] and incoherent [14] destruction of Wannier-Stark localization to super-Bloch oscillations [42–44].

An important open question has to do with the effect of a drive on a clean MBL system, which we address here. Specifically, we study the model resulting from an application of an AC field comprising of square wave pulses onto...
the clean MBL system of Schulz et al [23]. Remarkably, the drive is found to take the undriven system from an ergodic phase to a MBL phase and vice-versa when the parameters are set appropriately. In the non-interacting limit, for the case of a combined dc and square-wave driving, we obtain analytically the conditions for dynamical localization, coherent destruction of Wannier-Stark localization and super-Bloch oscillations.

Our main findings are captured schematically in Fig. 1. Keeping the dc field alone is equivalent to the undriven model, which exhibits a phase transition from an ergodic to a Stark MBL phase [23]. In the presence of a high-frequency drive, we obtain an intricate set of possibilities dependent on how the static electric field and the ratio of the amplitude to the frequency \((A/\omega)\) of the drive are tuned. The addition of a drive in the zero dc field limit, induces an MBL phase if the ratio \(A/\omega\) is tuned close to the dynamical localization point, analogous to the drive-induced MBL phase reported [39] in a conventional disordered MBL model. For a large dc field, where the undriven model yields the Stark MBL phase, the addition of resonantly driven drive leads to a destruction of Stark MBL for all values of the ratio \(A/\omega\) tuned away from the dynamical localization point. We refer to this as coherent destruction of Stark many-body localization.

However, Stark MBL is found to be robust against off-resonant drive. In the low-frequency limit of the drive, the nature of the phase obtained depends heavily on the choice of \(F\) and \(A\). All our results are supported by a study of the dynamics of entanglement entropy.

**Model Hamiltonian.** The system consists of a nearest-neighbor interacting model subjected to a time-dependent electric field. The Hamiltonian of the system can be written as

\[
H = -J \sum_{j=0}^{L-2} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - F(t) \sum_{j=0}^{L-1} j(n_j - \frac{1}{2}) \\
+ \alpha \sum_{j=0}^{L-1} \left( \frac{j^2}{L-1} \right)^2 (n_j - \frac{1}{2}) + V \sum_{j=0}^{L-2} (n_j - \frac{1}{2})(n_{j+1} - \frac{1}{2}),
\]

where \(F(t)\) is the time-dependent linear electric field, \(\alpha\) is the curvature term and \(V\) is the nearest neighbor interaction. The lattice constant is kept at unity and natural units \((\hbar = e = 1)\) are adopted for all the calculations. We have taken the time-dependent field as a combination of both dc and ac field. The electric field can be written as: 

\[F(t) = F + A \text{sgn}(\sin(\omega t))\],

where \(A\) and \(\omega\) respectively are the amplitude and frequency of the ac field, while \(F\)
is the static dc field.

Non-interacting case: Semi-classical Description. Let us consider the non-interacting case \((V = 0)\), and with zero curvature \((\alpha = 0)\). The quasi-momentum can be expressed as: \(q_k(t) = k + Ft + \int_0^t d\tau F_{ac}(\tau)\). Due to the dc part, the quasi-momentum is no longer a periodic function. However, for the resonance condition \(F = n\omega\), the quasi-momentum becomes a periodic function. Solving (as described in the supplementary section) for the one cycle average of quasi-energy, we get

\[
\epsilon(k) = -2J_{\text{eff}} \cos(k + \frac{n\pi}{2}),
\]

where \(J_{\text{eff}} = J \left\{ \frac{\sin(\frac{\pi s}{K} + \frac{k\pi}{n\pi})}{(K + n\pi)} + (-1)^n \frac{\sin(\frac{k\pi}{K} - \frac{n\pi}{n\pi})}{(K - n\pi)} \right\}\), and \(K = A/\omega\). In the limit \(F = 0\), this reduces to the well known quasi-energy dispersion for square wave driving:

\[
\epsilon(k) = -2J \text{sinc}(\frac{\pi s}{2K}) \cos(k),
\]

where \(\text{sinc}(z) = \sin(z)/z\). The quasi-energy band collapses at the zeros of the function \(\text{sinc}(\pi K/2)\), which occurs when \(K = K_c = 2\nu, \nu\) being any integer, which is the condition for dynamical localization [35].

For a finite \(F = n\omega\), the quasi-energy spectrum can be further simplified. For even and odd \(n\) respectively, we get

\[
\epsilon(k) = -2J_{\text{even}} \cos(k) \quad \text{and} \quad \epsilon(k) = -2J_{\text{odd}} \sin(k),
\]

where \(J_{\text{even}} = \frac{2JK \sin(\frac{K\pi}{2})}{(K^2 - n^2)\pi}; \quad J_{\text{odd}} = \frac{2JK \cos(\frac{K\pi}{2})}{(K^2 - n^2)^2}\). (3)

In the even and odd cases respectively, the band collapse occurs at \(K = K_c = 2\nu\) and \(K = K_c = 2\nu+1, \nu\) being any integer and \(K_c \neq n\). At these points an initially localized wave packet returns to its starting position. This gives the condition of dynamical localization. For other values of \(K\), and provided that the resonance condition holds, band formation takes place and the Wannier-Stark localization due to the static dc field is destroyed. A slight detuning from resonance \(F = (n + \delta)\omega\), results in super-Bloch oscillations with the time period given by \(T_{\text{SBO}} = \frac{2\pi}{\delta \omega}\) [42-44].

Interacting case. For any general time-periodic Hamiltonian, the time evolution operator (Floquet operator) over one cycle can be expressed as: \(U(T) = \mathcal{T} \int_0^T e^{-iT(t)} dt\), where \(\mathcal{T}\) represents the time ordering and \(T\) is the time period of the drive. The Floquet operator is related to the Floquet Hamiltonian by the expression: \(U(T) = e^{-iH_F T}\). For the square wave drive, defining \(H_+\) and \(H_-\) as the Hamiltonians for the first and second half of the driving period respectively, the Floquet operator can be simplified to \(U(T) = (e^{-iH_- T/2}e^{-iH_+ T/2})\).

The required quasi-energies and the Floquet eigenstates are then calculated by numerically diagonalizing the Floquet operator (upto \(L = 16\) at half-filling). The obtained quasi-energy spectrum (see supplementary section) leads support to the conclusion of Luitz et al [45] that many-body interactions destroy dynamic localization in a model that is purely driven (i.e. \(F = 0\)). Here we study the case where the drive has an additional static field, which as we will show later, leads to new possibilities. To contrast with the driven model, it is useful to plot the average level spacing ratio [46, 47] of the un-driven model as a function of the static field strength (Fig. 1). For small electric field strength an ergodic phase is obtained while the Stark-MBL phase is obtained for sufficiently large field strengths and a non-zero curvature term, which is added to break degeneracies [23].

We first consider the points where the choice of \(A/\omega\) yields dynamical localization in the non-interacting limit. Although interactions are inimical to dynamical localization, the presence of a non-zero curvature term can lead to the MBL phase. Fig. 2 shows the probability distribution of the quasi-energy gap-ratio parameter: \(r_n = \min(\delta_n/\delta_{n+1}, \delta_{n+1}/\delta_n)\), where \(\delta_n\) is the difference between the \(n^{th}\) and \((n - 1)^{th}\) quasi-energies, for a system of size \(L = 16\) and various values of the curvature term for a large driving frequency \(\omega = 5\). For all the cases \((F = 0, n\omega)\), the probability distribution agrees with the Poisson distribution: \(P(r) = 2/(1 + r)^2\) and suggests an MBL phase at these special points. The inset in Fig. 2(a) shows the level-spacing ratio as a function of \(A/\omega\) for zero dc field. Although the un-driven model \((F = 0, \text{Fig. 1})\) is in the ergodic phase, the application of drive leads to the MBL phase with a proper tuning of the ratio \(A/\omega\) to the dynamical localization point \((A/\omega = 2\nu\) with \(\nu\) being any integer\) of the non-interacting problem.

For the case where an additional static field is also present and satisfies the resonance condition: \(F = n\omega\), the condition for dynamical localization in the non-interacting limit depends on whether the integer \(n\) is odd or even (Eq. 3). We therefore explore both the cases set-
Figure 4. (a) Absence of dynamical localization in the presence of many-body interactions. A departure from perfect periodicity is observed. (b) Dynamics of the difference in entanglement entropy ($\Delta S$) between the interacting and the corresponding non-interacting limit, at the dynamical localization point. The plots are smoothed out by convolution with a Gaussian: $w(n) = e^{-(n/\sigma)^2/2}$, with $\sigma = 4$. (c) Coherent destruction of Stark-MBL phase at resonant driving and the robustness of Stark-MBL phase at off-resonant driving from the entanglement entropy dynamics. The fitting is done with the curve: $f(x) = a\log(x) + b$. The other parameters are: $L = 18, J = 1.0, V = 1.0$.

In the low-frequency regime, the nature of the phase crucially depends on the choice of the driving amplitude and the static field strength. The supplementary section carries some details of the low-frequency regime, and how it yields the bottom part of the schematic in Fig. 1.

Dynamics of entanglement entropy. For a system in a pure state, the entanglement entropy of a subsystem $A$ is defined as: $S_A = -\text{Tr}(\rho_A \ln \rho_A)$, where $\rho_A$ is the reduced density matrix of the subsystem $A$ obtained by tracing out the degrees of freedom of the other subsystem $B$. To study the dynamics of the entanglement entropy, we start with an initial product state (where all the particles occupy the even sites) and use an exact numerical approach based on the re-orthogonalized Lanczos algorithm [24, 48] for the time evolution. Due to the interactions in the many-body localized phase, a logarithmic growth of the entanglement entropy is expected [49, 50].

We first consider the limit $\alpha = 0$, and study the stability of dynamical localization in the presence of interactions. The dynamics of entanglement entropy for various interaction strengths is plotted in Fig. 4(a). In all the cases, the ratio $A/\omega$ is tuned at the dynamical localization point. It can be seen that in the presence of interactions, the entanglement entropy starts to grow in time as opposed to the non-interacting case where as a consequence of the band collapse, the entanglement entropy shows an oscillatory behavior and recurs to its initial value (zero due to the choice of the initial state) at times $t = mT$ with $m$ being an integer.

We now turn to the case with a finite curvature strength ($\alpha \neq 0$), where an MBL phase is found at sufficiently high frequencies. Here, we investigate the stability of the MBL phase from a dynamical perspective. We first consider the case where the MBL phase is obtained by tuning the ratio $A/\omega$ at the dynamical localization point. We define the quantity: $\Delta S = S(t, V) - S(t, V = 0)$ as the difference between the entanglement entropy

...
of the interacting and the corresponding non-interacting limit. Fig. 4(b) shows the dynamics of $\Delta S$ as a function of time for different sets of the frequency and the static field. In all the cases, a logarithmic behavior is observed which signifies an MBL-like phase at these points.

The dynamics of the entanglement entropy for the parameters tuned away from the dynamical localization point is shown in Fig. 4(c). It can be seen that for resonant drive ($F = n\omega$), the Stark-MBL phase is destroyed. The entanglement entropy in this case increases rapidly for smaller times followed by a slow growth for the intermediate times and finally saturates to its thermal value. This slow growth of entanglement entropy in the intermediate times is a signature of Floquet prethermalization [51–54], where the system prethermalizes before reaching an infinite-temperature-like state at high frequencies. For the off-resonant drive at high frequency, the entanglement entropy shows the usual logarithmic growth signifying the robustness of the Stark-MBL phase.

Summary and Conclusions. To summarize, we study a clean interacting system driven by a combined ac and dc electric field. The underlying non-interacting problem is itself of interest, and we semi-classically obtain the condition for dynamic localization, coherent destruction of Wannier-Stark localization and super Bloch oscillations. In the presence of interactions, generic clean many-body systems under a drive, reach a featureless infinite-temperature-like state. In contrast, we find that our system can avoid such ‘heat death’, under high-frequency drive. This is achieved either by tuning the system at the dynamical localization point of the corresponding noninteracting model, or by subjecting the system to off-resonant drive.

We further study the fate of the Stark-MBL phase in the presence of an additional drive. Observing that the effects of low-frequency driving are heavily dependent on the field strength and the amplitude of the drive, we focus on high-frequency driving, uncovering an intricate set of possible phases. One striking possibility is that of generating an MBL phase from the undriven ergodic phase by the application of a pure ac field. A second remarkable possibility appears for sufficiently large dc field, where it is possible to destroy Stark-MBL, by the application of a resonantly tuned drive provided that the ratio $A/\omega$ is tuned away from the dynamical localization point. We term this as ‘coherent destruction of Stark-MBL’. On the other hand, the Stark-MBL phase is found to be robust against off-resonant drive.

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[1] P. L. Kapitza, Collected papers of PL Kapitza 2, 714 (1965).
[2] G. Casati, B. Chirikov, F. Izrailev, and J. Ford, in Stochastic behavior in classical and quantum Hamiltonian systems (Springer, 1979) pp. 334–352.
[3] S. Fishman, D. Grempel, and R. Prange, Physical Review Letters 49, 509 (1982).
[4] D. Dunlap and V. Kenkre, Physical Review B 34, 3625 (1986).
[5] D. Dunlap and V. Kenkre, Physics Letters A 127, 438 (1988).
[6] A. Gómez-León and G. Platero, Phys. Rev. Lett. 110, 200403 (2013).
[7] J. Cayssol, B. Dóra, F. Simon, and R. Moessner, physical status solidi (RRL)-Rapid Research Letters 7, 101 (2013).
[8] M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Physical Review X 3, 031005 (2013).
[9] T. Mishra, T. G. Sarkar, and J. N. Bandyopadhyay, The European Physical Journal B 88, 231 (2015).
[10] A. Eckardt, C. Weiss, and M. Holthaus, Physical review letters 95, 260404 (2005).
[11] T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Demler, Physical Review B 84, 235108 (2011).
[12] S. Nandy, A. Sen, and D. Sen, Physical Review X 7, 031034 (2017).
[13] T. Cádež, R. Mondaini, and P. D. Sacramento, Physical Review B 96, 144301 (2017).
[14] D. S. Bhakuni, S. Dattagupta, and A. Sharma, Phys. Rev. B 99, 155149 (2019).
[15] S. Nandy, A. Sen, and D. Sen, Physical Review B 98, 245144 (2018).
[16] F. Alet and N. Laflorencie, Comptes Rendus Physique 19, 498 (2018).
[17] D. A. Abanin and Z. Papić, Annalen der Physik 529, 1700169 (2017).
[18] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, arXiv preprint arXiv:1804.11065 (2018).
[19] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Annals of physics 321, 1126 (2006).
[20] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. Lett. 95, 206603 (2005).
[21] R. Nandkishore and D. A. Huse, Annu. Rev. Condens. Matter Phys. 6, 15 (2015).
[22] P. W. Anderson, Phys. Rev. 109, 1492 (1958).
[23] M. Schulz, C. Hooley, R. Moessner, and F. Pollmann, Physical review letters 122, 040606 (2019).
[24] E. van Nieuwenburg, Y. Baum, and G. Refael, Proceedings of the National Academy of Sciences , 201819316 (2019).
[25] D. S. Bhakuni and A. Sharma, arXiv preprint arXiv:1909.10542 (2019).
[26] S. R. Taylor, M. Schulz, F. Pollmann, and R. Moessner, arXiv preprint arXiv:1910.01154 (2019).
[27] D. A. Abanin, W. De Roeck, and F. Huveneers, Physical review letters 115, 256803 (2015).
[28] L. DAlessio and M. Rigol, Physical Review X 4, 041048 (2014).
[29] A. Lazarides, A. Das, and R. Moessner, Physical Review E 90, 012110 (2014).
[30] A. Lazarides, A. Das, and R. Moessner, Physical review
letters 112, 150401 (2014).
31 P. Ponte, A. Chandran, Z. Papić, and D. A. Abanin, Annals of Physics 353, 196 (2015).
32 A. Lazarides, A. Das, and R. Moessner, Physical review letters 115, 030402 (2015).
33 D. A. Abanin, W. De Roeck, and F. Huveneers, Annals of Physics 372, 1 (2016).
34 L. DAlessio and A. Polkovnikov, Annals of Physics 333, 19 (2013).
35 P. Ponte, Z. Papić, F. Huveneers, and D. A. Abanin, Physical review letters 114, 140401 (2015).
36 P. Bordia, H. Lüschen, U. Schneider, M. Knap, and I. Bloch, Nature Physics 13, 460 (2017).
37 D. S. Bhakuni and A. Sharma, Phys. Rev. B 98, 045408 (2018).
38 A. Eckardt, M. Holthaus, H. Lignier, A. Zenesini, D. Ciampini, O. Morsch, and E. Arimondo, Physical Review A 79, 013611 (2009).
39 E. Bairey, G. Refael, and N. H. Lindner, Physical Review B 96, 020201 (2017).
40 M. Holthaus, G. Ristow, and D. Hone, EPL (Europhysics Letters) 32, 241 (1995).
41 M. Holthaus, G. H. Ristow, and D. W. Hone, Physical review letters 75, 3914 (1995).
42 K. Kudo and T. Monteiro, Physical Review A 83, 053627 (2011).
43 S. Longhi and G. Della Valle, Phys. Rev. B 86, 075143 (2012).
44 R. Caetano and M. Lyra, Physics Letters A 375, 2770 (2011).
45 D. J. Luitz, Y. Bar Lev, and A. Lazarides, SciPost Physics 3, 029 (2017).
46 V. Oganesyan and D. A. Huse, Physical review b 75, 155111 (2007).
47 Y. Atas, E. Bogomolny, O. Giraud, and G. Roux, Physical review letters 110, 084101 (2013).
48 D. J. Luitz and Y. B. Lev, Annalen der Physik 529, 1600350 (2017).
49 J. H. Bardarson, F. Pollmann, and J. E. Moore, Phys. Rev. Lett. 109, 017202 (2012).
50 M. Serbyn, Z. Papić, and D. A. Abanin, Phys. Rev. Lett. 110, 260601 (2013).
51 D. A. Abanin, W. De Roeck, W. W. Ho, and F. Huveneers, Physical Review B 95, 014112 (2017).
52 T. Mori, T. Kuwahara, and K. Saito, Physical review letters 116, 120401 (2016).
53 D. Abanin, W. De Roeck, W. W. Ho, and F. Huveneers, Communications in Mathematical Physics 354, 809 (2017).
54 D. J. Luitz, R. Moessner, S. Sondhi, and V. Khemani, arXiv preprint arXiv:1908.10371 (2019).
In this supplementary section, we have presented a detailed calculation of the semi-classical analysis carried out to obtain the conditions for the dynamical localization in the presence of a combined dc and ac (square wave) electric field. The effect of the many-body interactions on the dynamical localization and the low frequency analysis to obtain the schematic of the main paper is also provided.

**NON- INTERACTING CASE: SEMI-CLASSICAL DESCRIPTION**

The form of the combined dc and ac field can be written as

\[ F(t) = \begin{cases} F + A & \text{for } 0 \leq t < T/2 \\ F - A & \text{for } T/2 \leq t < T, \end{cases} \]

where \( A \) and \( T \) are the amplitude and the time-period of the drive respectively and \( F \) is the static field strength.

Firstly, we will consider the case where the static field is absent (\( F = 0 \)). The quasi-momentum in the presence of this time dependent field changes as

\[ q_k(t) = k + \frac{1}{\hbar} \int_0^t d\tau F(\tau). \]

For square wave driving (Eq. 1), the expression for the quasi-momentum can be solved as [1]:

\[ q_k(t) = \begin{cases} k + A(t - T/4)/\hbar & \text{for } 0 \leq t < T/2 \\ k + A(3T/4 - t)/\hbar & \text{for } T/2 \leq t < T. \end{cases} \]

The change in the quasi-momentum leads to a change in the dispersion, which is now time-dependent (\( E(k) = -2J \cos(q_k(t)) \)). Due to the absence of energy conservation, we focus on the one-cycle average of the quasi-energy, which is given by:

\[ \epsilon(k) = -\frac{2J}{T} \int_0^T \cos[q_k(t)] \, dt. \]

Substituting the expression for \( q_k(t) \) and solving the integral, we arrive at

\[ \epsilon(k) = -2J \sin c\left(\frac{\pi K}{2}\right) \cos(k), \]

where, \( \sin c(z) = \sin(z)/z \) and \( K = A/\omega \). The quasi-energy band collapses at the zeros of the function \( \sin c(\pi K/2) \), which occurs when \( K = K_c = 2\nu, \nu \) being any integer (Fig. 1(a)). This is the condition for dynamic localization.

For a combined ac and dc field the quasi-momentum can be expressed as

\[ q_k(t) = k + Ft + \int_0^t d\tau F_{ac}(\tau). \]

Due to the dc part, the quasi-momentum is no longer a periodic function. However, for the resonance condition \( F = n\omega \), the quasi-momentum becomes a periodic function. Solving for the one cycle average of quasi-energy, we get

\[ \epsilon(k) = -\frac{2J}{T} \int_0^T \cos[q_k(t)] \, dt \\
= -\frac{2J}{T} \int_0^{T/2} \cos(k + n\omega t + A(t - T/4)) \, dt + \int_{T/2}^T \cos(k + n\omega t + A(3T/4 - t)) \, dt \\
= -\frac{2J}{T} \int_{-T/4}^{T/4} \cos\left(k + \frac{3n\omega T}{4} + (-n\omega + A)t\right) \, dt. \]
FIG. 1: Quasi-energy spectrum in the non-interacting case (with $L = 6$ and half filling) for different strength of the static field. (a) for $F = 0.0$, the condition for dynamic localization occurs for an even integer value of $A/\omega$. (b,c) for a finite value of the static field $F = n\omega$. The corresponding condition for dynamic localization also changes depending whether $n$ is odd (b) or even (c). The other parameters are: $\omega = 1.0, t = 0.25$ in all the figures.

FIG. 2: Quasi-energy spectrum for the interacting case with different strengths of the static field. A finite interaction avoids the band collapse thus destabilizing dynamic localization in the presence of interactions. The other parameters are the same as in the non-interacting case.

The integral can be solved to yield:

$$\epsilon(k) = -2J_{\text{eff}} \cos\left(k + \frac{n\pi}{2}\right),$$

(8)

where we define $J_{\text{eff}} = J \left\{ \frac{\sin\left(\frac{k\pi}{2} + \frac{n\pi}{2}\right)}{(K\pi + n\pi)} + (-1)^n \frac{\sin\left(-\frac{k\pi}{2} + \frac{n\pi}{2}\right)}{(K\pi - n\pi)} \right\}$.

Case 1: Odd $n$

For odd $n$, Eqn. 8 can be simplified to

$$\epsilon(k) = -2J \left(\frac{2K \cos\left(\frac{K\pi}{2}\right)}{(K^2 - n^2)\pi}\right) \cos\left(k - \frac{\pi}{2}\right).$$

(9)

Here, the band collapses for $K = K_c = 2\nu + 1$, $\nu$ being any integer and $K_c \neq n$ (Fig. 1(b)). This gives the condition of dynamic localization whereas for other $K$ with the resonance condition destruction of Wannier-Stark localization occurs.

Case 2: Even $n$

For even $n$, Eqn. 8 can be simplified to

$$\epsilon(k) = -2J \left(\frac{2K \sin\left(\frac{K\pi}{2}\right)}{(K^2 - n^2)\pi}\right) \cos(k).$$

(10)
FIG. 3: Absence of dynamical localization (a) for zero DC field, (b) for $F = \omega \ (n\text{-odd})$, and (c) $F = 2\omega \ (n\text{-even})$.

For the non-interacting case $V = 0$, the entanglement entropy shows periodic oscillations with the same time period as of the drive, while for finite interaction the entanglement entropy starts to grow in time. The absence of perfect oscillations leads to the destruction of dynamical localization in the presence of interactions. The other parameters are: $L = 12$ and $\alpha = 0.0$. The filling fraction is kept at 0.5.

In terms of the $\sin c$ function we have

$$\epsilon(k) = -2J \left( \frac{\sin c\left(\frac{K\pi}{2}\right)}{1 - \frac{n^2}{K^2}} \right) \cos(k).$$

(11)

Again, the band collapse occurs at $K = K_c = 2\nu$, $\nu$ being any integer and $K_c \neq n$ (Fig. 1(c)). At these points an initially localized wave packet returns to its starting position. This gives the condition of dynamical localization. For other values of $K$, and provided that the resonance condition holds, band formation takes place and the Wannier-Stark localization due to the static dc field is destroyed.

The band collapses for zero dc field and both even and odd $n$ are shown in Fig. 1. The band collapse in Fig. 1(b) comes about because the quasi-energy is conserved modulo $\omega$, and therefore the zero level is the same as 0.5.

**Super-Bloch Oscillations**

Considering the case of a slight detuning from the resonant condition

$$F = (n + \delta)\omega,$$

(12)

the corresponding quasi-momentum can be written as:

$$q_k(t) = k + n\omega t + \delta\omega t + \int_0^t d\tau F_{ac}(\tau).$$

(13)

The quasi-momentum is no longer periodic due to the extra term. However for $\delta \ll 1$, we can approximately take $q_k(t)$ as periodic and can proceed further to calculate the quasi-energy by assuming $\delta\omega t$ as a constant. It can be easily verified that for both even and odd $n$, the cosine term $\cos(k)$, acquires an additional phase $\delta\omega t$, which is equivalent to a static dc field of magnitude $\delta\omega$. The dynamics shows oscillatory behaviour similar to Bloch oscillations. These oscillations are termed as super-Bloch oscillations. The time period is given by

$$T_{SBO} = \frac{2\pi}{\delta\omega}.$$

(14)

**DESTRUCTION OF DYNAMICAL LOCALIZATION IN THE PRESENCE OF INTERACTIONS**

The dynamical localization is known to be destroyed in the presence of interactions [2]. As can be seen from Fig. 2, the quasi-energy spectrum starts to avoid the band collapse as opposed to the non-interacting case where the band collapse happens at certain special points. The dynamics of the entanglement entropy also starts to grow in time as opposed to the case of dynamical localization where the entanglement is bounded and periodic in time (Fig. 3).
FIG. 4: The average level spacing ratio as a function of the static field $(F)$ for $A = 1.0$ (left) and $A = 2.0$ (right) for the low frequency drive ($\omega = 1.0$). The transition from ergodic to the MBL phase is found to be dependent on the parameters $A$ and $F$.

LOW FREQUENCY DEPENDENT PHASE

In this section, we discuss how a strong $F$ and $A$ dependence appears at low frequencies. The effective field in this case becomes $F + A$ and $F - A$ in the two half cycles respectively. Fig. 4 shows the variation of the average level spacing ratio as a function of the static dc field for different driving amplitudes. It can be seen that the MBL transition depends on the choice of driving amplitude. The driven model also yields an MBL phase when both $F + A$ and $F - A$ lie in the MBL phase of the undriven model. In this case the drive only mixes the localized eigenstates of the undriven system. On the other hand, when one or both of the parameters: $F + A$ or $F - A$ falls into the ergodic region of the undriven model, an ergodic phase is observed. We infer that if the drive mixes both localized and extended states, it is extended states that dominate.

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[1] A. Eckardt, M. Holthaus, H. Lignier, A. Zenesini, D. Ciampini, O. Morsch, and E. Arimondo, Physical Review A 79, 013611 (2009).

[2] D. J. Luitz, Y. Bar Lev, and A. Lazarides, SciPost Physics 3, 029 (2017).