Single Transverse-Spin Asymmetry in Large $P_T$ Open Charm Production at an Electron-Ion Collider

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Abstract

We discuss the single transverse-spin asymmetry (SSA) to be observed in the $D$-meson production with large transverse-momentum in semi-inclusive deep inelastic scattering, $e p^\uparrow \to eD X$. This contribution is embodied as a twist-3 mechanism in the collinear factorization, which is induced by purely gluonic correlation inside the transversely-polarized nucleon, in particular, by the three-gluon correlation effects. The complete formula for the corresponding SSA in the leading-order QCD is expressed in terms of the four independent gluonic correlation functions and reveals the five independent structures with respect to the dependence on the azimuthal angle for the produced $D$-meson. We present the numerical calculations of the SSA formula at the kinematics relevant to a future Electron Ion Collider.
1 Introduction

Charm productions in semi-inclusive deep inelastic scattering (SIDIS), as well as in the $pp$ collision, are known to be associated with the twist-2 gluon distributions in the nucleon, since the $c\bar{c}$-pair creation through the photon-gluon or gluon-gluon fusion is their driving subprocess. Similarly, the twist-3 contributions in the charm productions can be generated by the purely gluonic effects inside the nucleon, in particular, the multi-gluon correlations. Indeed, the observation of the single transverse-spin asymmetry (SSA) in the open charm productions allows us to probe the corresponding twist-3 effects [1, 2, 3, 4, 5].

The corresponding SSA arises as a naively $T$-odd effect in the cross section for the scattering of transversely-polarized nucleon off an unpolarized particle, observing a $D$-meson with momentum $P_h$ in the final state, and this requires, (i) nonzero transverse-momentum $P_{h\perp}$ originating from transverse motion of quark or gluon; (ii) nucleon helicity flip in the cut diagrams for the cross section, corresponding to the transverse polarization; and (iii) interaction beyond Born level to produce the interfering phase between the LHS and the RHS of the cut in those diagrams. In particular, for large $P_{h\perp} \gg \Lambda_{\text{QCD}}$, the contribution (i) arises perturbatively as the recoil from the hard (unobserved) final-state partons, and this leads us to the collinear-factorization framework, so that the other two contributions, (ii) and (iii), are generated by the twist-3 mechanism associated with the three-gluon correlation functions for the transversely-polarized nucleon [3]. This twist-3 mechanism may be considered as an extension of the corresponding mechanism for the SSA in the pion productions in the SIDIS [10], $pp$ collisions [11, 12], etc., based on the quark-gluon correlations in the nucleon, but it has been clarified [3] that a straightforward extension [1, 2] to the $D$-meson productions leads to missing many terms in the SSA. The complete leading-order (LO) QCD formulae for the corresponding SSA in the $D$-meson productions have been recently derived in [3] for SIDIS and in [5] for $pp$ collisions, based on the relevant twist-3 mechanism, i.e., the soft-gluon-pole mechanism, and those results revealed, for the first time, the entire nonperturbative gluonic degrees of freedom to induce the SSA and the whole structures of the asymmetries with respect to the azimuthal angle for the produced $D$-meson. Remarkably, those complete twist-3 formulae are related to the certain derivative of the twist-2 cross sections for the $D$-meson productions [4, 5], thanks to universal structure behind the SSAs in a variety of hard processes [6, 7].

The purpose of this paper is to present a numerical estimate of the SSA in the high-$P_{h\perp}$ $D$-meson production in SIDIS, $ep^+ \rightarrow eDX$, based on the LO QCD formula of [3]. We use the modeling [5] of the twist-3 three-gluon correlation functions in the nucleon, which is guided by the SSA for the $D$-meson production observed at RHIC [8], and demonstrate the influence of the nonperturbative behaviors of gluonic correlations. We calculate the SSA at the kinematics relevant to Electron Ion Collider (EIC) [9].

2 The LO QCD formula for the $D$-meson production

To describe the $D$-meson production in SIDIS, $e(\ell) + p(p, S_\perp) \rightarrow e(\ell') + D(P_h) + X$, we use, as usual, the kinematic variables $S_{ep} = (\ell + p)^2$, $q = \ell - \ell'$, $Q^2 = -q^2$, $x_{bj} = Q^2/(2p \cdot q)$, and
Figure 1: Feynman diagrams for partonic subprocess in $ep^\uparrow \rightarrow eDX$; mirror diagrams also contribute.

$z_f = p \cdot P_h/(p \cdot q)$. We work in a frame where the 3-momenta $\vec{q}$ and $\vec{p}$ are collinear, both moving along the z-axis as $q^\mu = g^{\mu3}Q$ and $p^\mu = g^{\mu-}Q/((\sqrt{2}x_{bj}))$, and define $q_T \equiv P_{h\perp}/z_f$ and the azimuthal angles around the z-axis $\phi$, $\Phi_S$, and $\chi$ of the lepton plane, the spin vector $S_{\perp}$, and the $D$-meson momentum $P_{h\perp}^\mu$, respectively [3, 4]. We take into account the masses $m_c$ and $m_h$ for the charm quark and the $D$ meson.

In the LO in QCD perturbation theory, the photon-gluon fusion subprocesses of Fig. 1 drive the SSA for large $P_{h\perp} D$-meson production; in Fig. 1, the above-mentioned contribution (i) is provided by the recoil from the hard unobserved $\bar{c}$ quark and the $c$ quark with the momentum $p_c$ fragments into the $D$-meson in the final state. The short bar on the internal $c$-quark line indicates that the pole part is to be taken from the corresponding propagator, to produce the interfering phase for the contribution (iii); we note that these pole contributions from Fig. 1 would cancel the similar contributions from the corresponding mirror diagrams, if the $c$ quark were unobserved in the final state as in the case of the $\bar{D}$-meson production. The external curl lines represent the gluons that are generated from the three-gluon correlations present inside the transversely-polarized nucleon, $\langle pS_{\perp}|A_\alpha(0)A_\beta(\eta)A_\gamma(\xi)|pS_{\perp}\rangle$, corresponding to the contribution (ii). The diagrams obtained by the permutation of the gluon lines in Fig. 1 also produce the contributions (i)-(iii), but the Bose statistics of the gluons in the above matrix element guarantees that we need not consider those diagrams separately. Thus, the SSA in the present context can be derived entirely as the contributions of soft-gluon-pole (SGP) type [10], leading to $k_2 - k_1 = 0$, by evaluating the pole part in Fig. 1.  

The twist-3 nature of those contributions are unraveled by the collinear expansion, as usual. The expansion produces lots of terms, each of which is not gauge invariant. Indeed, many of them vanish or cancel eventually, and the remaining terms can be organized into a gauge-invariant form. This can be demonstrated [3] by sophisticated use of the Ward identities for the contributions of the diagrams in Fig. 1. The resulting factorization formula of the spin-dependent, differential cross section for $ep^\uparrow \rightarrow eDX$ reads [3, 4]

$$\frac{d^3\Delta\sigma}{d\omega} = \frac{\alpha_{em}^2\alpha_s\epsilon_c^2 M_N}{16\pi^2 x_{bj}^2 s_{ep}^2 Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1,\ldots,4,8,9} A_k S_k$$

The other types of pole contributions participate in the SSA in other processes, see [10, 12, 13].
\[
\times \int_{x_{\text{min}}}^{1} \frac{dx}{x} \int_{z_{\text{min}}}^{1} \frac{dz}{z} \delta \left( \frac{Q^2}{Q'^2} - \left( 1 - \frac{1}{x} \right) \left( 1 - \frac{1}{z} \right) + \frac{m_c^2}{\tilde{z}^2 Q'^2} \right) \\
\times \sum_{a=c,\bar{c}} D_a(z) \left( \delta_s \left\{ \left[ \frac{dO(x, x)}{dx} - \frac{2O(x, x)}{x} \right] \Delta \hat{\sigma}^1_k + \left[ \frac{dO(x, 0)}{dx} - \frac{2O(x, 0)}{x} \right] \Delta \hat{\sigma}^2_k \right\} + \frac{O(x, x)}{x} \Delta \hat{\sigma}^3_k + \frac{O(x, 0)}{x} \Delta \hat{\sigma}^4_k \right) + \left\{ O(x, x) \rightarrow N(x, x), O(x, 0) \rightarrow -N(x, 0) \right\},
\]

where \([d\omega] \equiv dx dy dQ^2 dz d\phi d\chi\) denotes the differential elements, \(\hat{x} = x_{bj}/x\) and \(\hat{z} = z_f/z\) are the partonic variables associated with the usual momentum fractions \(x\) and \(z\), respectively, \(D_c(z)\) denotes the usual twist-2 fragmentation function for a \(c\)-quark to become the \(D\)-meson, and the quark-flavor index \(\gamma\) can, in principle, be \(c\) and \(\bar{c}\), with \(\delta_{c} = 1\) and \(\delta_{\bar{c}} = -1\), so that the cross section for the \(D\)-meson production \(ep^+ \rightarrow eD\) can be obtained by a simple replacement of the fragmentation function to that for the \(D\) meson, \(D_a(z) \rightarrow \bar{D}_a(z)\). \(O(x, x)\) and \(N(x, x)\) represent a complete set of twist-3 gluonic correlation functions, defined through the gauge-invariant lightcone correlation \(^2\) of three field-strength tensors [3],

\[
M^\alpha_{F(-)}(x_1, x_2) \equiv -g^3 \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\zeta(x_2-x_1)} \langle pS_\perp | d^{bca} F^{\beta\gamma n}_b(0) F^{\gamma \alpha n}_c(\zeta) F^{\alpha n}_a(\lambda) | pS_\perp \rangle \\
= 2i M_N \left[ O(x_1, x_2) g^{\alpha \beta} \epsilon^{\gamma mn S_\perp} + O(-x_2, x_1 - x_2) g^{\beta \gamma} \epsilon^{\alpha mn S_\perp} + O(x_2 - x_1, -x_1) g^{\gamma \alpha} \epsilon^{\beta mn S_\perp} \right],
\]

\[
M^\alpha_{F(+)}(x_1, x_2) \equiv -g^3 \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\zeta(x_2-x_1)} \langle pS_\perp | f^{bca} F^{\beta\gamma n}_b(0) F^{\gamma \alpha n}_c(\zeta) F^{\alpha n}_a(\lambda) | pS_\perp \rangle \\
= 2i M_N \left[ N(x_1, x_2) g^{\alpha \beta} \epsilon^{\gamma mn S_\perp} + N(-x_2, x_1 - x_2) g^{\beta \gamma} \epsilon^{\alpha mn S_\perp} + N(x_2 - x_1, -x_1) g^{\gamma \alpha} \epsilon^{\beta mn S_\perp} \right],
\]

with the nucleon mass \(M_N\) and a lightlike vector \(n\) satisfying \(n^2 = 0\) and \(p \cdot n = 1\); \(d^{bca}\) and \(f^{bca}\) are, respectively, the symmetric and anti-symmetric structure constants of the color SU(3) group, so that \(O(x_1, x_2)\) and \(N(x_1, x_2)\) are the \(C\)-odd and \(C\)-even functions satisfying

\[
O(x_1, x_2) = O(x_2, x_1) = O(-x_1, -x_2),
N(x_1, x_2) = N(x_2, x_1) = -N(-x_1, -x_2).
\]

In (1), \(\alpha_{em}\) is the fine-structure constant, \(e_c = 2/3\) is the electric charge of the \(c\)-quark, and the summation for the subscript \(k\) runs over \(k = 1, 2, 3, 4, 8, 9\), with

\[
A_1 = 1 + \cosh^2 \psi, \quad A_2 = -2, \quad A_3 = -\cos(\phi - \chi) \sinh 2\psi, \quad A_4 = \cos 2(\phi - \chi) \sinh^2 \psi, \quad A_8 = -\sin(\phi - \chi) \sinh 2\psi, \quad A_9 = \sin 2(\phi - \chi) \sinh^2 \psi,
\]

\[
S_1 = S_2 = S_3 = S_4 = \sin(\Phi_S - \chi), \quad S_8 = S_9 = \cos(\Phi_S - \chi),
\]

\(^2\)We suppress the gauge-link operators to be inserted in between the field strength tensors.
where \( \cosh \psi \equiv 2x_{bj}S_{ep}/Q^2 - 1 \). The delta function in (1) implies that the lower limits of the integrals are given by \([1, 3]\)

\[
z_{\text{min}} = z_f \frac{(1 - x_{bj})Q^2}{2x_{bj}m_c^2} \left( 1 - \frac{4x_{bj}m_c^2}{(1 - x_{bj})Q^2} \left[ 1 + \frac{x_{bj}q_f^2}{(1 - x_{bj})Q^2} \right] \right),
\]

and

\[
x_{\text{min}} = \begin{cases} x_{bj} \left[ 1 + \frac{z_f^2q_f^2 + m_c^2}{z_f(1 - z_f)Q^2} \right] & \text{for } z_f \left( 1 + \sqrt{1 + \frac{q_f^2}{m_c^2}} \right) > 1, \\ x_{bj} \left[ 1 + \frac{2m_c^2}{Q^2} \left( 1 + \frac{z_f^2q_f^2}{m_c^2} \right) \right] & \text{for } z_f \left( 1 + \sqrt{1 + \frac{q_f^2}{m_c^2}} \right) \leq 1. \end{cases}
\]

Partonic hard parts \( \Delta \hat{\sigma}_k \) depend on \( m_c \) as well as other partonic variables; for the explicit formulae of \( \Delta \hat{\sigma}_k \), we refer the readers to Eqs. (71)-(74) in \([3]\). The participation of the “derivative terms”, the terms with the derivatives of the three-gluon correlation functions \( \sigma \), is schematically given by

\[
\Delta \hat{\sigma} \frac{\partial \sigma}{\partial \phi} \text{ at } \phi \text{ in the SSA in various processes associated with twist-3 quark-gluon correlation functions [6, 7].}
\]

We now reexpress as

\[
\phi - \chi = \phi_h, \quad \Phi_S - \chi = \phi_h - \phi_S,
\]

in (5), where \( \phi_h \) and \( \phi_S \) represent the azimuthal angles of the hadron plane and the nucleon’s spin vector \( \vec{S}_\perp \), respectively, measured from the lepton plane. Then, (1) can be expressed as

\[
\frac{d^6 \Delta \sigma}{[d\omega]} = \sin(\phi_h - \phi_S) \left( F_1 + F_2 \cos \phi_h + F_3 \cos 2\phi_h \right) + \cos(\phi_h - \phi_S) \left( F_4 \sin \phi_h + F_5 \sin 2\phi_h \right),
\]

where \( F_i \equiv g_i^{\mu \nu} - p_i^{\mu} p_i^{\nu} \), and \( H_{\mu \nu}(xp, q, p_c) \) represents the partonic hard part for the \( 2 \rightarrow 2 \) Born subprocess, expressed by the diagrams in Fig. 1 with the soft \( (k_2 - k_1 = 0) \) gluon line removed. \(^3\) This reveals that \( \Delta \hat{\sigma}_k \) in (1) are related to the twist-2 hard parts \( H_{\mu \nu}(xp, q, p_c) \), similarly as in the SSA in various processes associated with twist-3 quark-gluon correlation functions [6, 7].

\(^3\)The color factors of the type, \( \text{Tr}[t^a t^b t^c] \) = \( (d^{bca} + i f^{bca})/4 \), implied by the diagrams in Fig. 1, are contracted with the three-gluon matrix elements, giving rise to the combination of (2) and (3) in (8).
with the corresponding structure functions $F_1, F_2, \ldots, F_5$, exhibiting the five independent azimuthal dependences similarly as in the twist-3 SSA for $ep \uparrow \rightarrow e\pi X$, generated from the quark-gluon correlation functions \[6\]. Thus, the complete LO QCD formulae (1), (10) for the high-$\frac{p_T}{D}$ D-meson production in SIDIS are expressed in terms of the four types of gluonic functions $O(x, x), O(x, 0), N(x, x),$ and $N(x, 0)$ of the relevant momentum fraction $x$, and generate five independent structures about the dependence on the relevant azimuthal angles. This is in contrast to the corresponding results in [1], which were expressed by only two types of gluonic functions and three independent azimuthal structures.

The formulae (1), (10) for the single-spin-dependent cross section should be compared with the corresponding LO QCD formulae of the twist-2 unpolarized cross section for the high-$\frac{p_T}{D}$ D-meson production in SIDIS, which is generated from the usual unpolarized gluon-density distribution $G(x)$, as [3]

$$
\frac{d^6\sigma^{unpol}}{[d\omega]} = \frac{\alpha_s^2\alpha_s \epsilon_e^2}{64\pi^2 x \beta_0^2 S_{ep} Q^2} \sum_{k=1}^4 A_k \int_{x_{min}}^1 \frac{dx}{x} \int_{z_{min}}^1 \frac{dz}{z} \delta \left( Q^2 - \left( 1 - \frac{1}{x} \right) \left( 1 - \frac{1}{z} \right) + \frac{m_c^2}{z^2 Q^2} \right)
\times \sum_{a=c, \bar{c}} D_a(z) G(x) \hat{\sigma}^U_k
$$

$$
= \sigma^U_1 + \sigma^U_2 \cos \phi_h + \sigma^U_3 \cos 2\phi_h,
$$

where the partonic hard cross sections are given in Eq. (81) of [3] and obey

$$
\Delta \hat{\sigma}^1_k = \frac{2q_T x}{Q^2 (1 - z)} \hat{\sigma}^U_k,
$$

as implied by (8).

3 SSA in the $D$-meson production at EIC

We evaluate the SSAs for the $D^0$ production, $ep \uparrow \rightarrow eD^0 X$, based on the QCD factorization formula (1). In particular, using (10) and (11), we calculate the asymmetries,

$$
\frac{F_1}{\sigma_1^U}, \quad \frac{F_2}{2\sigma_1^U}, \quad \frac{F_3}{2\sigma_1^U}, \quad \frac{F_4}{2\sigma_1^U}, \quad \frac{F_5}{2\sigma_1^U},
$$

(13)

to be observed at a future EIC. Because the partonic hard parts in (1) are common for the $C$-even and $C$-odd three-gluon functions, we show the contributions to (13) from the $C$-odd correlation functions $O(x, x)$ and $O(x, 0)$ in the following calculations. For the first estimate presented in this paper, we assume the two types of functional forms of those correlations, corresponding to the different small-$x$ behavior,

Model 1 : $O(x, x) = O(x, 0) = 0.004 x G(x)$, \hspace{2cm} (14)

Model 2 : $O(x, x) = O(x, 0) = 0.001 \sqrt{x} G(x)$, \hspace{2cm} (15)

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where the coefficients 0.004 and 0.001 are suggested in [5] by the comparison of the three-gluon contribution to the SSA for \( p^1p \to DX \) with the data observed at RHIC [8]. \(^4\) We use the CTEQ6L gluon-density distribution [14] for \( G(x) \) of (11), (14) and (15), and KKKSO8 fragmentation function [15] for \( D_a(z) \) of (1) and (11). For those nonperturbative functions, we associate the scale \( \mu^2 = Q^2 + m_c^2 + z_fq_{\perp}^2 \), and, for simplicity, we assume that the scale dependence of \( O(x, x) \) and \( O(x, 0) \) is determined by that of \( G(x) \) according to (14) and (15). We use \( m_c = 1.5 \) GeV for the charm quark mass.

At EIC kinematics with \( S_{ep} = 1000 \) GeV\(^2 \), \( Q = 2.5 \) GeV, \( P_{h\perp} = 2 \) GeV, and \( x_{bj} = 0.01 \), the SSAs in (13) are, respectively, shown as a function of \( z_f \) in the first five panels in Fig. 2, where the contributions due to using the models (14) and (15) in (1) are plotted: the solid and dashed curves show the contributions from \( O(x, x) \) and \( O(x, 0) \) of (14) (Model 1), respectively, to the asymmetries (13), so that the difference between those two curves reflect the difference between the relevant partonic hard parts in (1), i.e., between \( \Delta \hat{\sigma} \) and \( \Delta \hat{\sigma} \) \((\Delta \hat{\sigma} \) and \( \Delta \hat{\sigma} \)); similarly, the dotted and dot-dashed curves show the contributions from \( O(x, x) \) and \( O(x, 0) \) of (15) (Model 2), respectively, to the asymmetries (13). The behavior of \( \sigma \) arising in the denominator in the SSAs (13), as well as of the other coefficients \( \sigma \) in the unpolarized cross section (11), is shown as \( \hat{\sigma} \equiv (2\pi S_{ep}x_{bj}/z_f^2)\sigma \) in the last panel in Fig. 2 and demonstrates the \( D \)-meson production rate at EIC. \(^5\) We find that the contributions to \( \mathcal{F}/\sigma \) are several percent level and significant, while those for the other asymmetries are small. With high energy at EIC, the \( z_f \) dependence of the asymmetries is influenced by the different small-\( x \) behaviors between the two models (14) and (15). Similar features are observed also in the \( P_{h\perp} \) dependence of the SSAs (13) and the unpolarized cross section (11) for the fixed \( z_f = 0.3 \), as shown in Fig. 3. Here the overall behaviors of the SSAs obey the \( 1/P_{h\perp} \) falloff characteristic of the twist-3 effect. A remarkable point revealed as a function of \( P_{h\perp} \) is the growth of the asymmetry \( \mathcal{F}/(2\sigma) \), as well as of the unpolarized cross section (11), for decreasing \( P_{h\perp} \); in particular, \( \mathcal{F}/(2\sigma) \) could reach a few percent. The formulae (1) for the single-spin-dependent cross section tells us that the \( P_{h\perp} \) as well as \( z_f \) dependence of the SSAs measured at EIC provides the information to determine the \( x \) dependence of the gluonic correlation functions \( O(x, x), O(x, 0), N(x, x), \) and \( N(x, 0) \). Indeed, the different behaviors of the relevant partonic hard parts in (1) give rise to very different shape among the five asymmetries (13) as a function of \( z_f \) as well as of \( P_{h\perp} \). Some asymmetries have maximum and/or minimum, some asymmetries have a node, and some asymmetries are monotonic functions.

The following figures are same as Figs. 2, 3 but for EIC kinematics with higher energies: With \( S_{ep} = 2500 \) GeV\(^2 \), \( Q = 4 \) GeV, and \( x_{bj} = 0.01 \), Fig. 4 (Fig. 5) shows the results as a function of \( z_f \) for \( P_{h\perp} = 2 \) GeV (as a function of \( P_{h\perp} \) for \( z_f = 0.3 \)). With \( S_{ep} = 5000 \) GeV\(^2 \),

\(^4\)\( O(x, x) \) and \( O(x, 0) \) in (14) and (15) are twice as large as those in [5], by which we take into account the effect of \( N(x, x) \) and \( N(x, 0) \) contributing constructively as \( O(x, x) \) and \( O(x, 0) \).

\(^5\)Changing the variables as \( Q^2 \to y (= p \cdot q/|p \cdot \ell|) = Q^2/(x_{bj}S_{ep}) \), \( q_{\perp}^2 \to P_{h\perp} \), and performing the integration over \( \chi \) for a fixed \( \phi_h \), (11) becomes \( d^2\sigma_{\text{ampol}}/(dx_{bj}dydz_fP_{h\perp}d\phi_h) = \sigma_{h}^2 + \sigma_{h}^2 \cos \phi_h + \sigma_{h}^2 \cos 2\phi_h \). This choice is convenient for comparison with the results in [1]. The results of \( \sigma \) shown in the last panel in Figs. 4 and 5 are reduced compared with the corresponding numerical results in [1]. The reduction mainly comes from the use of the quark mass \( m_c = 1.5 \) GeV corresponding to the pole mass [15], which is larger compared with the value \( m_c = 1.3 \) GeV used in [1].
Figure 2: The individual contributions of (14) and (15) to the SSAs (13) with (10), (1) (the first five panels) and the individual coefficients of the unpolarized cross section (11) (the last panel), plotted as a function of $z_f$, for $D^0$ production in SIDIS at EIC kinematics with $S_{ep} = 1000 \text{ GeV}^2$, $Q = 2.5 \text{ GeV}$, $P_{h\perp} = 2 \text{ GeV}$, and $x_{bj} = 0.01$. 
Figure 3: The individual contributions of (14) and (15) to the SSAs (13) with (10), (1) (the first five panels) and the individual coefficients of the unpolarized cross section (11) (the last panel), plotted as a function of $P_{h\perp}$, for $D^0$ production in SIDIS at EIC kinematics with $S_{ep} = 1000 \text{ GeV}^2$, $Q = 2.5 \text{ GeV}$, $z_f = 0.3$, and $x_{bj} = 0.01$. 
Figure 4: Same as Fig. 2, but for EIC kinematics with $S_{ep} = 2500 \text{ GeV}^2$, $Q = 4 \text{ GeV}$, $P_{h\perp} = 2 \text{ GeV}$, and $x_{bj} = 0.01$. 
Figure 5: Same as Fig. 3, but for EIC kinematics with $S_{ep} = 2500 \text{ GeV}^2$, $Q = 4 \text{ GeV}$, $z_f = 0.3$, and $x_{bj} = 0.01$. 
Figure 6: Same as Fig. 2, but for EIC kinematics with $S_{ep} = 5000$ GeV$^2$, $Q = 4$ GeV, $P_{h\perp} = 2$ GeV, and $x_{bj} = 0.005$. 
Figure 7: Same as Fig. 3, but for EIC kinematics with $S_{ep} = 5000$ GeV$^2$, $Q = 4$ GeV, $z_f = 0.3$, and $x_{bj} = 0.005$. 

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$Q = 4$ GeV, and $x_{bj} = 0.005$, Fig. 6 (Fig. 7) shows the results as a function of $z_f$ for $P_{h\perp} = 2$ GeV (as a function of $P_{h\perp}$ for $z_f = 0.3$). Due to the large values of gluon distribution in the small-$x$ region, the cross sections are still sizable for those higher energies, and the features found in the SSAs in Figs. 2, 3 are observed also in Figs. 4-7, with some of them being even more pronounced. We note that all contributions to the asymmetries from the functions $O(x, x)$ and $O(x, 0)$, as presented in Figs. 2-7 for the $D$-meson production, change their signs for the $\bar{D}$-meson production as implied by the factor $\delta_a$ in (1), while the contributions from $N(x, x)$ and $N(x, 0)$ do not.

The present results with (14), (15) indicates that the derivative terms due to $dO(x, x)/dx$ and $dO(x, 0)/dx$ in (1) are the dominant contributions of similar size in the largest asymmetry $\mathcal{F}_1/\sigma_U^1$, to make its value several percent. On the other hand, for the other asymmetries $\mathcal{F}_{2,3,4,5}/(2\sigma_U^1)$, the derivative terms do not give dominant contributions, so that even a non-derivative term could give the largest contribution; in particular, the contribution of the nonderivative term associated with $O(x, 0)\Delta\hat{\sigma}_5^4$ in (1) is responsible for the above-mentioned growth of $\mathcal{F}_5/(2\sigma_U^1)$, which becomes more pronounced for higher energies.

For all cases treated in Figs. 2-7, $x_{\text{min}}$ in (1) and (11) is given by the formula in the first line in (7), so that the value of $x_{\text{min}}$ becomes the smallest at $z_f = 0.5$. This property, combined with the fact that the (dominant) derivative-term contributions would give $\mathcal{F}_1/\sigma_U^1 \sim 1/(1-x_{\text{min}})$, would explain the existence of the minimum in the solid and dashed curves around $z_f \simeq 0.5$ in the first panel in Fig. 2, as discussed in [1]. As demonstrated by the dotted and dot-dashed curves in the same figure, however, such behavior is affected by the small-$x$ behavior of (15) different from (14), such that the above-mentioned minimum around $z_f \simeq 0.5$ could be changed into the maximum. In Figs. 4, 6 with higher energies the contributions from the small-$x$ region are more important, and the corresponding minimum or maximum is less pronounced.

4 Conclusions

In this paper we have discussed the SSAs in SIDIS, $ep^+ \to eDX$, with the $D$ meson having large transverse momentum through the photon-gluon fusion mechanism at the twist-3 level, which is induced by three-gluon correlation inside the nucleon. In particular, we have presented a numerical estimate of those SSAs for the first time with the consistent leading-order accuracy in QCD, using the corresponding collinear factorization formula. Gauge invariance and permutation symmetry among the gluons require the four types of gluonic nonperturbative functions and their derivatives to represent the relevant twist-3 mechanism, as a consequence of the soft-gluon-pole contributions associated with the $3 \to 2$ photon-gluon fusion subprocesses or of the master formula with the Born-level ($2 \to 2$) photon-gluon fusion subprocesses. The corresponding SSAs receive the five independent azimuthal structures, and our calculation of them, using gluonic nonperturbative functions suggested by the RHIC data for $p^+p \to DX$, demonstrates good chance to access multi-gluon effects at an Electron Ion Collider, in particular, through the asymmetries with $\mathcal{F}_1$ and $\mathcal{F}_5$. The similar multi-gluon effects also contribute to the SSAs in $e p^+ \to e\pi X$, as well as in Drell-Yan
and direct-photon productions [16].

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**References**

[1] Z. B. Kang and J. W. Qiu, Phys. Rev. D 78, 034005 (2008).

[2] Z. B. Kang, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D 78, 114013 (2008).

[3] H. Beppu, Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D 82, 054005 (2010).

[4] Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D 83, 114014 (2011).

[5] Y. Koike and S. Yoshida, Phys. Rev. D 84, 014026 (2011).

[6] Y. Koike and K. Tanaka, Phys. Lett. B 646, 232 (2007) [Erratum-ibid. B 668, 458 (2008)].

[7] Y. Koike and K. Tanaka, Phys. Rev. D 76, 011502 (2007).

[8] H. Liu [PHENIX Collaboration], AIP Conf. Proc. 1149, 439 (2009).

[9] M. Anselmino et al., Eur. Phys. J. A 47, 35 (2011);
D. Boer et al., arXiv:1108.1713 [nucl-th].

[10] H. Eguchi, Y. Koike and K. Tanaka, Nucl. Phys. B 752, 1 (2006); B 763, 193 (2007).

[11] J. W. Qiu and G. F. Sterman, Phys. Rev. D 59, 014004 (1999).

[12] Y. Koike and T. Tomita, Phys. Lett. B 675, 181 (2009);
K. Kanazawa and Y. Koike, Phys. Rev. D 82, 034009 (2010);
K. Kanazawa and Y. Koike, Phys. Rev. D 83, 114024 (2011).

[13] Y. Koike, W. Vogelsang and F. Yuan, Phys. Lett. B 659, 878 (2008);
Y. Koike and K. Tanaka, in the proceedings of 17th International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS 2009), Madrid, Spain, 26-30 Apr 2009, http://dx.doi.org/10.3360/dis.2009.209 [arXiv:0907.2797 [hep-ph]];
K. Kanazawa and Y. Koike, Phys. Lett. B 701, 576 (2011).
[14] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky and W. K. Tung, JHEP 0207, 012 (2002).

[15] T. Kneesch, B. A. Kniehl, G. Kramer and I. Schienbein, Nucl. Phys. B 799, 34 (2008).

[16] Y. Koike and S. Yoshida, Phys. Rev. D 85, 034030 (2012).