Merger rate of charged black holes from the dynamical capture

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Abstract

We consider the dynamical capture of black holes carrying U(1) charge which can not only correspond to electric or magnetic charge but also have other physical interpretations such as dark or hidden charge. In the low-velocity and weak-field regime, we study gravitational and electromagnetic radiations from point masses with U(1) charges in a hyperbolic orbit, and we develop a formalism to derive the merger rate of charged black holes from the dynamical capture. We apply the formalism to find the effects of the charge-to-mass ratio on the merger rate for possible different cases and discover that the effects depend on the models.

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I. INTRODUCTION

Nearly one hundred years after the theoretical prediction of gravitational waves (GWs) in the general relativity (GR) by Einstein (GR) \[1, 2\], the first direct measurement of GWs \[3\] by the Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) detectors established gravitational wave astronomy. The LIGO detection opened a new window to observe the universe and marked the dawn of multi-messenger astronomy \[4–6\]. So far, the LIGO detection results have already provided extremely accurate confirmation that all gravitational merger events can be described by GR \[7–9\].

The prominent no-hair theorem of black holes (BHs) in GR states that BHs can be is completely described as a Kerr-Newman metric, which is expressed by the mass, spin and charge \[10–12\]. This paramount and non-trivial theorem allows people to reduce all BH solutions to just three physical quantities. Compared to Schwarzschild BHs, charged BHs emit not only gravitational radiation but also electromagnetic radiation, and have attracted much attention \[13–31\]. Recently, assuming that the influence of BH spins can be neglected, Bozolla et al. found that the charge-to-mass ratios of up to 0.3 are compatible with Binary BH merger event GW150914 \[21\]. Later, they also presented general relativistic simulations of the inspiral and merger of non-spinning charged binary BHs \[27\].

In the universe, dynamical capture is a fairly common and effective way to form the binary BH systems including astrophysical black holes (ABHs) and primordial black holes (PBHs) \[32, 33\]. Therefore, it is important and meaningful to work out the merger rate of charged BHs from dynamical capture. To do so, we work out gravitational radiation and electromagnetic radiation from a point mass binary with charges in a hyperbolic orbit, and calculate the merger rate of BHs from dynamical capture with charges and a general mass function by taking into account gravitational and electromagnetic radiation. The U(1) charge considered in this paper can correspond to the following physical interpretations: (1) electric charges, (2) magnetic charges \[34–36\], (3) hidden or dark charges interacting with dark electromagnetism \[37–40\], (4) modified theories of gravity with additional scalar or vector fields \[41–43\], (5) the fifth force \[44–46\].

The paper is organized as follows. In Section II, in the low-velocity and weak-field regime, we work out gravitational and electromagnetic radiations from point masses with U(1) charges in a hyperbolic orbit. In Section III, we develop a formalism to derive the
merger rate of charged BHs from dynamical capture via gravitational and electromagnetic radiation. In Section IV, we show the effects of the charge-to-mass ratio on the merger rate of ABH and PBH binaries from dynamical capture. Then we summarize our conclusions in the last section.

In this paper, we set $G = c = 4\pi \varepsilon_0 = \frac{\mu_0}{4\pi} = 1$ unless otherwise specified. Although the U(1) charges are intended to have different physical interpretations, we will refer to their quantities by using “electromagnetic” for the sake of simplification through this paper.

II. GRAVITATIONAL AND ELECTROMAGNETIC RADIATION FROM POINT MASSES IN A HYPERBOLIC ORBIT

We study the encounter of two massive charges and gravitational and electromagnetic radiations from the encounter. We assume that the orbit lies in $x$-$y$ plane, and that the coordinates of the point masses $m_1$ with charge $Q_1$ and $m_2$ with charge $Q_2$ are $(d_1 \cos \psi, d_1 \sin \psi)$ and $(-d_2 \cos \psi, -d_2 \sin \psi)$, respectively. Choosing the origin at the center of mass, we have

$$d_1 = \left(\frac{m_2}{m_1 + m_2}\right) d, \quad d_2 = \left(\frac{m_1}{m_1 + m_2}\right) d. \quad (1)$$

The equation for a hyperbolic orbit is

$$d = \frac{a (e^2 - 1)}{1 - e \cos \psi}, \quad (2)$$

where $a$ is the semi-major axis and $e$ is the eccentricity of the orbit. The angular velocity along the orbit and the total energy including the gravitational, electrostatic and kinetic energy are given by

$$\dot{\psi} = \frac{[(m_1 + m_2) a (e^2 - 1) (1 - \lambda)]^{1/2}}{d^2}, \quad (3)$$

$$E = \frac{m_1 m_2}{2a} - \frac{Q_1 Q_2}{2a} \equiv \frac{m_1 m_2}{2a} (1 - \lambda), \quad (4)$$

where $\lambda \equiv \frac{Q_1 Q_2}{m_1 m_2}$ is the ratio of the Coulomb force to the gravitational force.

Firstly, we calculate the gravitational radiation from the point masses $m_1$ with charge $Q_1$ and $m_2$ with charge $Q_2$ in the hyperbolic orbit. The non-vanishing second mass moment is a $2 \times 2$ matrix which is given by

$$M_{ab} = \mu d^2 \begin{pmatrix} \cos^2 \psi & \sin \psi \cos \psi \\ \sin \psi \cos \psi & \sin^2 \psi \end{pmatrix}, \quad (5)$$
where subscripts \((a, b = 1, 2)\) refer to indices in the \(x-y\) plane and \(\mu = m_1 m_2/(m_1 + m_2)\) is the reduced mass. Note that \(M_{ij}\) is reducible while the traceless quadrupole moment \(Q_{ij} \equiv M_{ij} - \frac{1}{3} \delta_{ij} M_{kk}\) is irreducible. Following [47], the rate of energy loss from a system by gravitational quadrupole radiation is given by

\[
\frac{dE^{\text{quad}}_{GW}}{dt} \equiv -\frac{1}{5} \left( \ddot{Q}_{ij} Q_{ij} \right) = -\frac{2}{15} \left[ (\ddot{M}_{11} + \ddot{M}_{22})^2 - 3(\ddot{M}_{11} \ddot{M}_{22} - \ddot{M}^2_{12}) \right].
\] (6)

The derivatives of the components of the second mass moment tensor can be calculated by the aid of Eqs. (2) and (3); putting those expressions into Eq. (6), we can arrive at

\[
\frac{dE^{\text{quad}}_{GW}}{dt} = -\frac{4(1 - \lambda)^3 m_1^2 m_2^2 (m_1 + m_2) (e \cos(\psi) - 1)^4 (11e^2 \cos(2\psi) + 13e^2 - 48e \cos(\psi) + 24)}{15a^5 (e^2 - 1)^5}.
\] (7)

The post-Newtonian formalism gives the rate of angular momentum emission due to gravitational quadrupole radiation as [48]

\[
\frac{dL^{\text{quad}}_{GW}}{dt} \equiv -\frac{2}{5} \epsilon^{ikl} \left( \ddot{Q}_{ka} Q_{la} \right) = -\frac{2}{5} \epsilon^{ikl} \left( \ddot{M}_{ka} M_{la} \right),
\] (8)

where \(\epsilon^{ikl}\) is the Levi-Civita symbol \((\epsilon^{123} = 1)\). As the orbit is assumed to lie in the \(x-y\) plane and the gravitational and Coulomb forces do not change the orbital plane, we have \(L^z = L\) and \(L^x = L^y = 0\). Substituting Eqs. (2) and (3) into the components of the second mass moment tensor for the unbound system, the non-vanishing rate of angular momentum emission due to gravitational quadrupole radiation is obtained by

\[
\frac{dL^{\text{quad}}_{GW}}{dt} = \frac{dL^z_{GW}}{dt} = -\frac{8(1 - \lambda)^5/2 m_1^2 m_2^2 \sqrt{m_1 + m_2}}{5a^{7/2} (e^2 - 1)^{7/2}} \sin^2(\psi) (e \cos(\psi) - 1)^2 \times (e(\cos(2\psi) + 3) - 4 \cos(\psi))(e(3 \cos(2\psi) + 4) - 8 \cos(\psi)).
\] (9)

Secondly, we compute the energy and angular momentum loss rates due to electromagnetic dipole radiation. In our reference frame, the electric dipole is given by

\[
p \equiv Q_1 d_1 + Q_2 d_2 = \frac{m_2 Q_1 - m_1 Q_2}{m_1 + m_2} d \cos \psi \hat{x} + \frac{m_2 Q_1 - m_1 Q_2}{m_1 + m_2} d \sin \psi \hat{y}.
\] (10)

Then, according to [49, 50], the energy and angular momentum loss rates due to electromagnetic dipole radiation are

\[
\frac{dE^{\text{dip}}_{EM}}{dt} = -\frac{2p^2}{3} = -\frac{2(1 - \lambda)^2 (m_2 Q_1 - m_1 Q_2)^2 (e \cos(\psi) - 1)^4}{3a^4 (e^2 - 1)^4},
\] (11)
\[
\frac{dL_{EM}^{dip}}{dt} = -\frac{2}{3}(\dot{p}_2\ddot{p}_1 - \dot{p}_1\ddot{p}_2) = -\frac{2(1 - \lambda)^{3/2}(m_2Q_1 - m_1Q_2)^2(1 - e \cos(\psi))^3}{3a^{5/2}(e^2 - 1)^{5/2} \sqrt{m_1 + m_2}}.
\] (12)

The total energy and angular momentum radiated due to gravitational quadrupole radiation and electromagnetic dipole radiation are then the integral of Eqs. (7), (9), (11) and (12) over the orbit. Notice the asymptotes of the hyperbolic orbit are \( y = \pm \tan(\psi_0)x \), where \( \psi_0 \) is determined by \( \cos(\psi_0) = 1/e \), i.e., \( \psi \in (\psi_0, 2\pi - \psi_0) \); thus we find

\[
\Delta E_{GW}^{quad} = \int_{-\infty}^{\infty} \frac{dE_{GW}^{quad}}{dt} dt = \int_{\psi_0}^{2\pi - \psi_0} \frac{dE_{GW}^{quad}}{dt} \left( \frac{d\psi}{dt} \right)^{-1} d\psi = -\frac{2(1 - \lambda)^{5/2}m_2^2m_1^2 \sqrt{m_1 + m_2}}{45a^{7/2}(e^2 - 1)^{7/2}}
\times \left( 3 \left( 37e^4 + 292e^2 + 96 \right) \left( \pi - \cos^{-1} \left( \frac{1}{e} \right) \right) \pi + \sqrt{1 - \frac{1}{e^2}} \left( 673e^2 + 602 \right) \right),
\] (13)

\[
\Delta L_{GW}^{quad} = \int_{-\infty}^{\infty} \frac{dL_{GW}^{quad}}{dt} dt = \int_{\psi_0}^{2\pi - \psi_0} \frac{dL_{GW}^{quad}}{dt} \left( \frac{d\psi}{dt} \right)^{-1} d\psi = \frac{8(1 - \lambda)^2m_1^2m_2^2}{5a^2(e^2 - 1)^2}
\times \left( 7e^2 + 8 \left( \pi - \cos^{-1} \left( \frac{1}{e} \right) \right) + \sqrt{1 - \frac{1}{e^2}} \left( 21e^4 - 10e^2 + 4 \right) \right),
\] (14)

\[
\Delta E_{EM}^{dip} = \int_{-\infty}^{\infty} \frac{dE_{EM}^{dip}}{dt} dt = \int_{\psi_0}^{2\pi - \psi_0} \frac{dE_{EM}^{dip}}{dt} \left( \frac{d\psi}{dt} \right)^{-1} d\psi = -\frac{(1 - \lambda)^{3/2}(m_2Q_1 - m_1Q_2)^2}{3a^{5/2}(e^2 - 1)^{5/2} \sqrt{m_1 + m_2}}
\times \left( \pi \left( e^2 + 2 \right) + 6e \sqrt{1 - \frac{1}{e^2}} + 2 \left( e^2 + 2 \right) \csc^{-1}(e) \right),
\] (15)

\[
\Delta L_{EM}^{dip} = \int_{-\infty}^{\infty} \frac{dL_{EM}^{dip}}{dt} dt = \int_{\psi_0}^{2\pi - \psi_0} \frac{dL_{EM}^{dip}}{dt} \left( \frac{d\psi}{dt} \right)^{-1} d\psi = -\frac{4(1 - \lambda)(m_2Q_1 - m_1Q_2)^2}{3a(e^2 - 1)(m_1 + m_2)}
\times \left( e \sqrt{1 - \frac{1}{e^2}} - \cos^{-1} \left( \frac{1}{e} \right) + \pi \right).
\] (16)

In the case of no charges \((Q_1 = Q_2 = \lambda = 0)\) or the same charge to mass ratio \((\frac{Q_1}{m_1} = \frac{Q_2}{m_2})\), we find \(\Delta E_{EM}^{dip} = \Delta L_{EM}^{dip} = 0\) and the corresponding expressions for the total energy and angular momentum radiated due to gravitational quadrupole radiation agree with the results given by Hansen [51]. Gravitational and electromagnetic radiations from binary BHs with electric and magnetic charges can be found in [52–54].

**III. MERGER RATE OF CHARGED BLACK HOLES FROM THE DYNAMICAL CAPTURE**

If two charged BHs get closer and closer, the energy loss due to gravitational and electromagnetic radiation can exceed the orbital kinetic energy, and thus the unbound system
cannot escape to infinity anymore and form a binary with negative orbital energy. This binary immediately merges through consequent gravitational and electromagnetic radiations. From such a process, we can estimate the cross section and calculate the rate coefficient.

Let us consider the interaction of two charged BHs with masses \( m_1, m_2 \) and charges \( Q_1, Q_2 \) and assume that the initial relative velocity is \( v \), the distance of periastron \( r_p \) and the impact parameter \( b \). We can approximate the trajectory of a close encounter by the hyperbolic with \( e \rightarrow 1 \) since when the two charged BHs pass by closely, the true trajectory is physically indistinguishable from a parabolic one near the periastron, in which gravitational and electromagnetic radiations dominantly occur. According to Sec. II and Appendix 1, the total energy loss due to electromagnetic dipole and quadrupole radiations, and gravitational quadrupole radiation by the close-encounter is evaluated by using \( e \rightarrow 1 \) and the definition of \( r_p \equiv a(e - 1) \),

\[
\Delta E = \Delta E^\text{dip}_{EM} + (1 + \Lambda) \Delta E^\text{quad}_{GW},
\]

(17)

where we denote \( \Lambda = \frac{\mu^2 (Q_1/m_1^2 + Q_2/m_2^2)^2}{4} \) for short and

\[
\Delta E^\text{dip}_{EM} = -\frac{\pi (1 - \lambda)^{3/2} (m_2 Q_1 - m_1 Q_2)^2}{2\sqrt{2}\sqrt{m_1 + m_2}r_p^{5/2}},
\]

(18)

\[
\Delta E^\text{quad}_{GW} = -\frac{85\pi (1 - \lambda)^{5/2} m_1^2 m_2^2 \sqrt{m_1 + m_2}}{12\sqrt{2}r_p^{7/2}}.
\]

(19)

Similarly, the total angular momentum loss due to electromagnetic dipole and quadrupole radiations, and gravitational quadrupole radiation is given by

\[
\Delta L = \Delta L^\text{dip}_{EM} + \left(1 + \frac{\Lambda}{2}\right) \Delta L^\text{quad}_{GW},
\]

(20)

where

\[
\Delta L^\text{dip}_{EM} = -\frac{2\pi (1 - \lambda) (m_2 Q_1 - m_1 Q_2)^2}{3(m_1 + m_2) r_p},
\]

(21)

\[
\Delta L^\text{quad}_{GW} = -\frac{6\pi (1 - \lambda)^2 m_1^2 m_2^2}{r_p^2}.
\]

(22)

Another way to estimate the total energy loss is to approximate the trajectory of the close encounter by taking the \( e = 1 \) limit of the elliptical orbit; thereby the total energy loss is given by \( \Delta E = T \left\langle \frac{dE}{dt} \right\rangle \) where \( T \) is orbital period and \( \left\langle \frac{dE}{dt} \right\rangle \) is the time-averaged energy loss rate of the binary in the Keplerian orbit which is given by [50]. From the definition, the relation between \( r_p \) and \( b \) is

\[
b^2 = r_p^2 + \frac{2(1 - \lambda)(m_1 + m_2)r_p}{\mu^2}.
\]

In the limit of the strong gravitational
and electromagnetic focusing (i.e. \( r_p \ll b \)), then the distance of closest approach \( r_p \) is given by

\[
r_p = \frac{b^2 \nu^2}{2 (1 - \lambda) (m_i + m_j)}.
\]

The condition for the two charged BHs to form a binary is that the total energy loss due to radiation is larger than the kinetic energy \( \frac{\mu \nu^2}{2} \), i.e.,

\[
\Delta E + \frac{\mu \nu^2}{2} < 0.
\]

Equations (17), (23) and (24) yield the maximum impact parameter \( b_{\text{max}} \) for the charged BHs to form a bound system, and then we obtain the merging cross section as \( \sigma = \pi b_{\text{max}}^2 \), where \( b_{\text{max}} \) is determined by

\[
2\pi (1 - \lambda)^4 (m_1 + m_2)^2 (m_2 Q_1 - m_1 Q_2)^2 + \frac{170\pi (1 - \lambda)^6 \Lambda m_1^2 m_2^2 (m_1 + m_2)^4}{3 b_{\text{max}}^{17} \nu^7} = \frac{\mu \nu^2}{2}.
\]

If dipole radiation is dominant (i.e. \( \Delta E_{EM}^{\text{dip}} > (1 + \Lambda) \Delta E_{GW}^{\text{quad}} \)), the merging cross section is

\[
\sigma_{\text{dip}} \approx \frac{2^{4/5} \pi^{7/5} (1 - \lambda)^{8/5} (m_1 + m_2)^{6/5} (m_1 Q_2 - m_2 Q_1)^{4/5}}{m_1^{2/5} m_2^{2/5} \nu^{14/5}},
\]

whereas if the total quadrupole radiation is dominant (i.e. \( \Delta E_{EM}^{\text{dip}} < (1 + \Lambda) \Delta E_{GW}^{\text{quad}} \)), the merging cross section is

\[
\sigma_{\text{quad}} \approx \frac{(85/3)^{2/7} 2^{4/7} \pi^{9/7} (1 - \lambda)^{12/7} m_1^{2/7} m_2^{2/7} (m_1 + m_2)^{10/7}}{\nu^{18/7} (1 + \Lambda)^{2/7}},
\]

which is consistent with [32]. Notice that the Schwarzschild radius \( \sigma_{\text{Sch}} \sim \pi (2Gm)^2 \sim (v)^{18/7} \sigma_{GW}, v \ll 1 \) and \( \sigma \geq \sigma_{GW} \); thereby the Newtonian approximation is sufficiently accurate no matter whether electromagnetic radiation is dominant or gravitational radiation is dominant. The differential merger rate of charged BHs per comoving volume reads

\[
dR = n(m_1, Q_1)n(m_2, Q_2) \langle \sigma v \rangle dm_1 dm_2 dQ_1 dQ_2
\]

where \( n(m_1, Q_1) \) and \( n(m_2, Q_2) \) are the comoving average number density of charged BHs with masses \( m_1, m_2 \) and charges \( Q_1, Q_2 \), and \( \langle \sigma v \rangle \) denotes the average over relative velocity distribution with \( \sigma = \pi b_{\text{max}}^2 \) given in Eq. (25). In this section, we have worked out the merging cross section of charged BHs without assuming the origin of those charged BHs.
IV. EFFECTS OF CHARGE-TO-MASS RATIO ON THE MERGER RATE

A. Astrophysical black hole

In astrophysics, there are two main astrophysical mechanisms for ABH binary formation. One channel is isolated (or ‘field’) binary evolution and the other is dynamical capture in dense stellar environments. Dynamical capture can become absolutely effective in deeply dense stellar environments. Especially, Ref. [56] finds that the vast majority (~80%) of the merger events detected by LIGO-Virgo comes from dynamical capture.

According to the initial mass function that describes the number distribution of stars [58], we assume the mass function of ABHs takes a truncated power-law form as [59]

\[ p(m) \propto m^\zeta \theta (m - m_{\text{min}}) \theta (m_{\text{max}} - m) \] (29)

with \( \zeta = -1.3 \) and the normalization of mass function \( \int p(m) dm = 1 \). Here, we take \( m_{\text{min}} = 3.0M_\odot \) which corresponds to the lower mass bound of ABHs [60], and \( m_{\text{max}} = 55M_\odot \) which corresponds to the beginning of pair-instability supernova (PISN) gap [61]. We consider the following two simple but different models

- model (a): all ABHs have the same charge-to-mass ratio \( \iota \) in magnitude and sign,
- model (b): a half of ABHs have the same charge-to-mass ratio \( \iota \) and the other half ABHs have a charge-to-mass ratio equal in magnitude but oppositely charged.

In the following calculation, we take velocity distribution of BHs as the Maxwellian velocity distribution \( P(v) \propto v^2 \exp \left(-v^2/v_0^2\right) \) and \( v_0 = 100 \text{ km/s} \).

To show the effects of charge-to-mass ratio \( \iota \) on the merger rate of ABH binaries from dynamical capture, we define

\[ \eta(\iota) \equiv \frac{R_{\text{ABH}}(\iota)}{R_{\text{ABH}}(0)} \] (30)

as a function of \( \iota \), where \( R_{\text{ABH}}(\iota) \) is the total merger rate of charged ABHs with charge-to-mass ratio \( \iota \) and \( R_{\text{ABH}}(0) \) is the total merger rate of ABHs for uncharged case.

\[ \begin{align*}
1 \text{ There are others channels like mergers in triple systems assisted by the Kozai-Lidov mechanism (See a recent review [55] for details).} \\
2 \text{ There are currently some disagreements about the contributions of different channels. Ref. [57] claims that ~90% is provided by isolated binary evolution channel.}
\end{align*} \]
In the model a, we have the number density of charged ABHs
\[ n(m, Q) \propto \delta \left( \frac{Q}{m} - \iota \right) \frac{p(m)}{m}, \tag{31} \]
where \( \delta \) is the Dirac delta function. In such a case, we have \( Q_1/m_1 = Q_2/m_2 = \iota \). Thus, the dipole radiation vanish and the energy loss due to total quadrupole radiation is given by \( \Delta E = (1 + \iota^2/4) \Delta E_{GW}^{\text{quad}} \). Therefore, according to Eqs.\((27), (28)\) and the definition of \( \eta \), we obtain
\[ \eta(\iota) = (1 - \iota^2)^{12/7} (1 + \iota^2/4)^{2/7}. \tag{32} \]

In the model b, we have the number density of charged ABHs
\[ n(m, Q) \propto \frac{1}{2} \left( \delta \left( \frac{Q}{m} - \iota \right) + \delta \left( \frac{Q}{m} + \iota \right) \right) \frac{p(m)}{m}. \tag{33} \]

Therefore, the merger events can be divided into two different cases. One half of merger events are that binary BHs have the same sign charges while the other half of merger events are that binary BHs have the opposite charges. In the case that binary BHs have the opposite charges, we have \( Q_1/m_1 = -Q_2/m_2 = \iota \) and \( \Lambda = \left( 1 + \frac{\iota^2 (m_1 - m_2)^2}{4(m_1 + m_2)^2} \right) \). The merging cross section in such case is \( \sigma \simeq \pi b_{\text{max}}^2 \), where \( b_{\text{max}} \) is the root of
\[ \frac{8\pi G \iota^2 (\iota^2 + 1)^4 m_1^2 m_2^2 (m_1 + m_2)^2}{b_{\text{max}}^5 \nu^5} + \frac{170\pi G^7 (1 + \iota^2)^6 \Lambda m_1^2 m_2^2 (m_1 + m_2)^4}{3b_{\text{max}}^7 \nu^7} = \frac{\mu \nu^2}{2}. \tag{34} \]

In the model b, from the Eqs. \((28)\) and \((30)\), we have
\[ \eta(\iota) = \frac{1}{2} (1 - \iota^2)^{12/7} (1 + \iota^2/4)^{2/7} + \frac{1}{2} \int \int \frac{P(v)\nu^v p(m_1)^m_1 p(m_2)^m_2 dvdm_1dm_2}{\int \int P(v)\nu^v p(m_1)^m_1 p(m_2)^m_2 dvdm_1dm_2} \tag{35} \]
where
\[ \sigma_0 = \frac{2^{4/7} \left( \frac{85}{3} \right)^{2/7} \pi^{9/7} m_1^{2/7} m_2^{2/7} (m_1 + m_2)^{10/7}}{\nu^{18/7}}. \tag{36} \]

represents the merging cross section for uncharged BHs. From the definition, no matter whether in the model a or b, we both have \( \eta(\iota) = \eta(-\iota) \) and \( \eta(0) = 1 \). Therefore, we only need to consider \( \iota \in [0, 1] \). In Fig. 1, we plot \( \eta(\iota) \) as the function of \( \iota \) in different models. In the model a, we find that \( \eta(\iota) \) decreases as \( \iota \) increases and \( \eta(1) = 0 \). For the model b, we exactly show that \( \eta(\iota) \) increases as \( \iota \) increases and reaches the maximum value of \( \eta(1) \approx 8.0 \).
FIG. 1. Left: The plots of $\eta(\iota)$ as the function of $\iota$ in the model a (red) and model b (purple). Right: The normalized merger rate of charged PBH $R_{\text{PBH}}(\iota)/R_{\text{PBH}}(0)$ as the function of $\iota$ in the model a (red) and model b (purple).

B. Primordial black hole

 Compared to ABHs, PBHs are those BHs which are formed in the radiation-dominated era of the early universe due to the collapse of large energy density fluctuations [62–64]. The topic of how to distinguish ABHs and PBHs has been studied in [65–68]. In recent years, PBHs have attracted a lot of attention and have been extensively studied, not only because they can be candidates of dark matter [69], but also because they can be the sources of LIGO/Virgo detections [33, 70] and the seeds for galaxy formation [71–73]. There are two primary mechanisms for PBH binary formation which can be classified by the cosmological epoch when they occur. One is that the PBH binary would have formed in the late universe [33, 74, 75] while the other is that PBH binary would have formed in the early universe [70, 76–88]. In this subsection, we will investigate the effects of charge-to-mass ratio $\iota$ on the merger rate of PBH binaries which formed in the late universe. Similarly to ABH, we also consider two special models by assuming all PBHs have the same mass $M_{\text{PBH}}$:

- model (a): all PBHs have the same charge-to-mass ratio $\iota$ in magnitude and sign,

- model (b): a half of PBHs have the same charge-to-mass ratio $\iota$ and the other half of PBHs have charge-to-mass ratio equal in magnitude but oppositely charged.

In the model a, the number density of charged PBHs is given by

$$n(m, Q) = \frac{f \rho_{\text{DM}}}{M_{\text{PBH}}} \delta \left( \frac{Q}{m} - \iota \right) \delta (m - M_{\text{PBH}}),$$

(37)
where $f$ is the fraction of PBHs in the dark matter and $\rho_{DM}$ is the dark matter energy density at the present time. According to Sec. III, the merger rate of the charged PBHs which would have formed in the late universe is

$$R_{PBH}(\iota) \approx 1.5 \times 10^{-8} f^2 (1 - \iota^2)^{12/7} \left(1 + \iota^2/4\right)^{2/7} \text{Gpc}^{-3} \text{yr}^{-1},$$

(38)

which is independent of the mass of the charged PBH. For the model b, the number density of charged PBHs is

$$n(m, Q) = \frac{f \rho_{DM}}{2 M_{PBH}} \left(\delta\left(\frac{Q}{m} - \iota\right) + \delta\left(\frac{Q}{m} + \iota\right)\right) \delta(m - M_{PBH}).$$

(39)

Following Sec. III again, the merger rate of the charged PBHs which would have formed in the late universe can be simplified as

$$R_{PBH}(\iota) \approx 0.75 \times 10^{-8} f^2 (1 - \iota^2)^{12/7} \left(1 + \iota^2/4\right)^{2/7} \text{Gpc}^{-3} \text{yr}^{-1} + \frac{f^2 \rho_{DM}^2}{2 M_{PBH}^2} \int P(v) \sigma_1 v dv,$$

(40)

where $\sigma_1 = \pi b_{max}^2$ represents the merging cross section for charged PBHs and $b_{max}$ is determined by

$$- \frac{2720 \pi (\iota^2 + 1)^6 M_{PBH}^8}{3 b_{max}^5 \iota^5} - \frac{32 \pi \iota^2 (\iota^2 + 1)^4 M_{PBH}^6}{b_{max}^5 v^5} + \frac{M_{PBH} v^2}{4} = 0.$$  

(41)

Notice that from Eq.(41), we can find $b_{max} \propto M_{PBH}$. Therefore, $\sigma_1 \propto M_{PBH}^2$, and we find that $R_{PBH}(\iota)$ in (40) is independent of $M_{PBH}$ and scales as $f^2$. So, $R_{PBH}(\iota)/R_{PBH}(0)$ is only a function of $\iota$ where $R_{PBH}(0) \approx 1.5 \times 10^{-8} f^2 \text{Gpc}^{-3} \text{yr}^{-1}$. In Fig. 1, we plot the normalized merger rate of charged PBH $R_{PBH}(\iota)/R_{PBH}(0)$ as the function of $\iota$ in different models. Similarly, we notice that $\eta(\iota)$ decreases as $\iota$ increases and $\eta(1) = 0$ in the model a while $\eta(\iota)$ increases as $\iota$ increases and $\eta(1) \approx 8.4$ in the model b.

V. CONCLUSION

In this work, we have calculated gravitational and electromagnetic radiation from point masses with U(1) charges in a hyperbolic orbit in the low-velocity and weak-field regime and applied the result to derive the merger rate of charged BHs from dynamical capture via gravitational and electromagnetic radiations. We then have shown the effects of the charge-to-mass ratio on the merger rate of ABH and PBH binaries from dynamical capture. We have also found that the effects of the charge-to-mass ratio on the merger rate depend on the models.
There are multiple possible extensions and applications of BHs with U(1) charges. Firstly, the U(1) charges can be interpreted as electric charges. As shown in [89, 90], the mergers of electrically charged BH might potentially offer an explanation for the still-mysterious fast radio bursts and gamma-ray bursts. In Refs. [91, 92], the coalescence of earth-mass primordial BH with charge-to-mass ratio $\sim 10^{-4}$ is proposed to be candidate sources for the observed fast radio bursts. Here, we have confirmed that the effect of the charge-to-mass ratio of $\sim 10^{-4}$ can be ignored. Secondly, the U(1) charges can be interpreted as magnetic charges. Magnetic charges, if they would exist in the universe, will offer a new window to explore fundamental physics. Especially, J. Maldacena extensively discussed the spectacular electroweak symmetry restoration of magnetic charged BHs [34]. Since no evidence of magnetic charges has been found yet from terrestrial experiments [93, 94], the merger events of binary BHs detected by LIGO may provide us with a new way to investigate whether these BHs indeed have magnetic charges. Finally, the U(1) charges can be interpreted as dark or hidden charges with an extremely weak coupling constant. Refs [20, 21] show that some merger events detected by LIGO are compatible with charged BHs while some are compatible with uncharged BHs. An interesting suggestion to this controversy is that those merger events would have originated from different scenarios. Therefore, a key question is how those BHs would carry dark or hidden charges. We leave all those interesting issues for future works.

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APPENDIX

1. Electromagnetic dipole and quadrupole radiations

In the case of the same charge to mass ratio ($Q_1/m_1 = Q_2/m_2$), the electromagnetic dipole radiation vanishes and quadrupole radiation becomes the leading term. This is true for any system of charges [49]. Here, we will calculate the energy and angular momentum loss rate due to the electromagnetic dipole and quadrupole radiations. From the definition
of traceless charge quadrupole, we find

$$D^{ab} = \sum_{i=1,2} Q_i d_i^a d_i^b - \frac{1}{3} \sum_{i=1,2} Q_i d_i^a d_i^c \delta_{ab} = \mu \left( \frac{Q_1}{m_1} + \frac{Q_1}{m_1} \right) Q^{ab}. \quad (42)$$

We extend the magnetic vector potential $A$ as the quadrupole term,

$$A_i = \frac{P_{ij}(n)}{r} (p^j + \frac{1}{2} \ddot{D}^j n_k), \quad (43)$$

where $P_{ij} = \delta_{ij} - n_i n_j$ is the projection operator that realizes the transversal gauge in the $n$-direction. According to [49], the total energy emitted due to the electromagnetic dipole and quadrupole radiations is

$$\frac{dE_{EM}}{dt} = -\frac{r^2}{4\pi} \int \hat{H}^2 d\Omega \quad (44)$$

where $H^i = \epsilon^i_{jk} \dot{A}^j n^k$. Adding Eqs. (11) and (43) and using

$$\int n^i d\Omega = \int n^i n^j n^k d\Omega = 0, \quad (45)$$

$$\int n^i n^j n^k n^l d\Omega = \frac{4\pi}{15} (\delta^{ij} \delta^{kl} + \delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk}), \quad \int n^i d\Omega = \frac{4\pi}{3} \delta^{dk}, \quad (46)$$

we find

$$\frac{dE_{EM}}{dt} = \frac{dE_{EM}^{dip}}{dt} + \frac{dE_{EM}^{quad}}{dt}, \quad (47)$$

where

$$\frac{dE_{EM}^{dip}}{dt} = -\frac{2\dot{p}^2}{3}, \quad (48)$$

$$\frac{dE_{EM}^{quad}}{dt} = -\frac{\dddot{D}_{ij} \dddot{D}_{ij}}{20}. \quad (49)$$

A passing remark is that the electric dipole can be written as

$$\mathbf{p} = \mu \left( \frac{Q_1}{m_1} - \frac{Q_2}{m_2} \right) \mathbf{d}. \quad (50)$$

Similarly, the loss rate of angular momentum due to electromagnetic radiation is given by

$$\frac{dL_{EM}^i}{dt} = -\frac{r^2}{4\pi} \int (-\epsilon^{ikl} (\partial_0 A_j) x^k \partial^l A_j + \epsilon^{ikl} A_k \partial_0 A_l) d\Omega. \quad (51)$$

After very careful calculation, we obtain

$$\frac{dL_{EM}^i}{dt} = \frac{dL_{EM}^{i,dip}}{dt} + \frac{dL_{EM}^{i,quad}}{dt}, \quad (52)$$

13
where

\[
\frac{dL_{EM}^{i,dip}}{dt} = -\frac{2\epsilon^{ikl}}{3}\ddot{p}_k\ddot{p}_l
\]  
(53)

\[
\frac{dL_{EM}^{i,quad}}{dt} = -\frac{1}{20}\epsilon^{ikl}\left(\ddot{D}_{ka}\ddot{D}_{la}\right)
\]  
(54)

Noting

\[
\frac{dE_{GW}^{quad}}{dt} \equiv -\frac{1}{5}\left(Q_{ij}^{ij}\right), \quad \frac{dL_{GW}^{i,quad}}{dt} \equiv -\frac{2}{5}\epsilon^{ikl}\left(Q_{ka}\ddot{Q}_{la}\right)
\]  
(55)

and using the definition of traceless charge quadrupole (42), we find the relations between quadrupole radiations:

\[
\frac{dE_{EM}^{quad}}{dt} \equiv \frac{\mu^2}{4}\left(Q/m_1^2 + Q_2/m_2^2\right)^2 \frac{dE_{GW}^{quad}}{dt},
\]  
(56)

\[
\frac{dL_{EM}^{i,quad}}{dt} \equiv \frac{\mu^2}{8}\left(Q/m_1^2 + Q_2/m_2^2\right)^2 \frac{dL_{GW}^{i,quad}}{dt}.
\]  
(57)

From the condition of gravitational attraction dominance over the electric repulsion, i.e \(|Q_1| \leq m_1\) and \(|Q_2| \leq m_2\), we always have \(\frac{dE_{EM}^{i,quad}}{dt} \leq \frac{1}{4}\frac{dE_{GW}^{quad}}{dt}\) and \(\frac{dL_{EM}^{i,quad}}{dt} \leq \frac{1}{8}\frac{dL_{GW}^{i,quad}}{dt}\).

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