Z₃ orbifold construction of SU(3)³ GUT with $\sin^2 \theta_W^0 = \frac{3}{8}$

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Abstract

It is argued that a phenomenologically viable grand unification model from superstring is $SU(3)^3$, the simplest gauge group among the grand unifications of the electroweak hypercharge embedded in semi-simple groups. We construct a realistic 4D $SU(3)^3$ model with the GUT scale $\sin^2 \theta_W^0 = \frac{3}{8}$ in a $Z_3$ orbifold with Wilson line(s). By two GUT scale vacuum expectation values, we obtain a rank 4 supersymmetric standard model below the GUT scale, and predict three more strange families.

[Key words: $Z_3$ orbifold, 4D superstring, trinification]

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A. Introduction and motivation

Supersymmetric standard models (SSM), if proven experimentally, need a theoretical explanation of why they become the effective theory below the Planck scale \( M_P \approx 2.44 \times 10^{18} \) GeV. A most probable scenario is that they result from compactifications of superstring models preserving one supersymmetry \( N = 1 \). The effective 4D \( N = 1 \) field theory models were extensively considered in this regard in the Calabi-Yau compactifications\[1\] and orbifold compactifications\[2, 3\]. Furthermore, the standard-like models initiated more than 15 years ago opened up the search for SSM directly from superstring\[4\].

The initial standard-like models \( SU(3) \times SU(2) \times U(1)^n \) were very attractive, in realizing the standard model (SM) gauge group and reasonable matter spectrum\[4, 5, 6\], with possible desirable physics on the strong CP problem\[7\] and cosmology with a hidden world\[8\]. Furthermore, the doublet-triplet splitting has been realized in some standard-like models\[4\]. However, these standard-like models failed because they generally do not predict correct weak mixing angle \( \sin^2 \theta_W \) at the string scale\[9\]. To predict the observed coupling constants at the electroweak scale successfully at least in \( \sim 2.2 \sigma \) level, the \( \sin^2 \theta_W \) at the unification scale \( \sim (2 - 3) \times 10^{16} \) GeV is required to be \( \approx \frac{3}{8} \). The reason is very simple.

In these standard-like models, the electroweak hypercharge group \( U(1)_Y \) is one combination out of \( n \) \( U(1) \)'s. Thus, the singlet representations of the standard-like gauge group, not belonging to the family structure of the fifteen (or sixteen if we include a heavy Majorana neutrino), can have nonvanishing \( U(1)_Y \) charges, which lowers the string scale weak mixing angle from the needed value of \( \frac{3}{8} \), because the string scale weak mixing angle \( \sin^2 \theta_W \) is expressed if we assume \( \alpha_2^0 = \alpha_1^0 \) at the string scale,

\[
\sin^2 \theta_W = \frac{Tr T_3^2}{Tr Q_{em}^2}.
\]

This \( \sin^2 \theta_W \) problem can be resolved if the standard model gauge group is unified in a simple group GUT, for example \( SU(5) \), where \( U(1)_Y \) is a subgroup of the GUT group. Then, the electroweak hypercharge generator is an \( SU(5) \) generator. Namely, \( SU(5) \) singlets do not carry nonvanishing electroweak hypercharges and we conclude that the string scale \( \sin^2 \theta_W \) is \( \frac{3}{8} \). To obtain a supersymmetric standard model in 4D, \( SU(5) \) must be broken by a VEV of an adjoint Higgs field\( (24_H) \). However, it is impossible to obtain an adjoint matter field at the level 1, i.e. \( k = 1 \). If simplicity is any guidance to the truth of nature, one must break the GUT group without an adjoint matter representation. This leads us to GUT groups.
with a $U(1)$ factor, notably $SU(5) \times U(1)$ which is now called flipped $SU(5)$. The flipped $SU(5)$ is an interesting rearrangement of a singlet field and fifteen chiral fields of $SU(5)$\[11]\]. The symmetry breaking of the flipped $SU(5)$ is particularly interesting in supersymmetric flipped $SU(5)$\[12]\]. In this regards, the string compactifications toward flipped $SU(5)$ is very interesting, since breaking of $SU(5) \times U(1)$ down to the standard model gauge group can be achieved without an adjoint Higgs representation\[12]\]. Indeed, the fermionic construction of 4D flipped $SU(5)$ was obtained already fifteen years ago\[13]\]. As shown in many subsequent papers, the flipped $SU(5)$ has many phenomenologically interesting features\[13]\].

However, the flipped $SU(5)$ generally fails in the aforementioned $\sin^2 \theta_W$ problem. The reason is the following. The flipped $SU(5)$ needs three $SU(5)$ singlet representations which carry +1 unit of the electric charge for the three singlet charged leptons of SSM. This implies, $SU(5)$ singlets can carry electromagnetic charges, or the electroweak hypercharge $Y$. Since there appear numerous $SU(5)$ singlets from string compactification, the charged singlets generally reduce dramatically $\sin^2 \theta_W$ from the needed value $\frac{3}{8}$, viz. (1). In the orbifold construction, this $\sin^2 \theta_W$ problem has been really serious. In the literature, one can find many models with $SU(5) \times U(1)$ groups\[14]\], and even it was claimed that there are flipped $SU(5)$s\[15]\], but as shown above these models ignored the $\sin^2 \theta_W$ problem. However, one may argue that even if the flipped $SU(5)$ contains a $U(1)$ factor, the $\sin^2 \theta_W$ problem goes away if the representations are embeddable in $SO(10)$. In this case, the $U(1)_Y$ generator belongs to $SO(10)$ and hence $SO(10)$ singlets do not carry the $U(1)_Y$ charge. Then, the singlets of the flipped $SU(5)$ carry only the needed electroweak hypercharges of the flipped $SU(5)$, and hence the string scale $\sin^2 \theta_W^0$ is $\frac{3}{8}$. However, this scenario is not realized generally in orbifold compactifications, which can be easily understood by remembering that orbifolds generally choose only part of the original complete representation. In fact, this property is the root for the solutions of the doublet-triplet splitting problem in the 4D orbifold compactifications\[4]\].

However, if it happens that the extra fields beyond the complete multiplets conspire to contribute to $TrT_3^2$ and $TrQ_{em}^2$ in the ratio 3/8, then we can obtain 3/8 as the string scale value of $\sin^2 \theta_W^0$. Therefore, the above argument is not a no-go theorem. It may be extremely difficult however, if not impossible, to find such a model with the electroweak hypercharge leaking to $U(1)$ at the GUT scale.

Before considering our 4D string model, let us comment on the recent field theoretic
orbifold breaking of grand unification group with extra dimensions\textsuperscript{16}. One interesting feature here has been family unification groups with $SO(2n)$ with $n \geq 7$ \textsuperscript{17}. In these extra-dimensional field theories, it is possible to allow fixed point fields as far as there are no anomalies, and hence it is not much achieved in the prediction of the matter representations at the orbifold fixed points. In this context, 6D string theoretic models were considered as an intermediate step toward a final 4D string theory construction\textsuperscript{18}. In this paper, however, we attempt to obtain a more ambitious 4D model.

B. $Z_3$ orbifold with Wilson line

In 4D, if a GUT group containing a $U(1)$, as in the $SU(5) \times U(1)$, is difficult to obtain, the next simple GUT groups to try are semi-simple groups. Therefore, we propose grand unified theories with the hypercharge embedded in a semi-simple group with no adjoint representation needed(HESSNA) as possible 4D string models toward a realistic SSM. For a realistic 4D superstring model, we must require that the factor groups of the HESSNA can be broken to SSM without an adjoint representation. In this regard, note that the Pati-Salam GUT group $SU(4) \times SU(2)_L \times SU(2)_R$ is not a HESSNA because it has the same problem as that in the $SU(5)$ model: one needs an adjoint representation. Therefore, the simplest HESSNA is $SU(3)^3$. The next simple HESSNA is $SU(3) \times SU(3) \times SU(4)$. If we find a realistic HESSNA, then it is a simple matter to find a SSM from this HESSNA, as the SU(5) model leads to the SM.

In the HESSNA also, the orbifold compactification is very much chiral, and may be too much chiral. But here at least it is easy to study the electroweak hypercharge concretely in a few steps.

At the phenomenological level, the group $SU(3)^3$ has been extensively considered\textsuperscript{19}. Our objective in this paper is to realize a string theory $SU(3)^3$. If we obtain such a model, it can be considered as a realistic superstring GUT.

We expect that one family in the $SU(3)^3$ HESSNA is composed of 27 chiral fields,

$$ (\bar{3}, 3, 1) + (1, \bar{3}, 3) + (3, 1, 3) $$

under $SU(3)^3$ group. It can be embeddable in 27 of $E_6$. Suppose, we assign the electroweak hypercharge in $E_6$ such that the two neutral members in 27 appear in the $SO(10)$ singlet
and $SU(5)$ 10, namely as in the flipped $SU(5)$ subgroup. If we do that in $E_6$, $E_6$ is completely broken down to the SM. Similarly, two neutral members in the Higgs representation, transforming like $[2]$, are given large HESSNA vacuum expectation values and a SSM can be obtained.

Only two possible $SU(3)^3$ groups can be found in the extensive tables of $Z_N$ orbifold models[14]. They appear in $Z_{12}$ orbifold models. However, the fermionic spectrums of these $Z_{12}$ compactifications are not the one required in $[2]$. This leads us to consider orbifold models with Wilson lines.[24] In a separate publication, we tabulate $Z_3$ orbifold models with one Wilson line.[20]

In the remainder of this paper, we present a $SU(3)^3$ model in a $Z_3$ orbifold compactification with one Wilson line. Let us denote the $Z_3$ shift vector as $v$ and the Wilson line as $a_1$. These must satisfy the conditions for the shift vectors,

$$v^2 = \frac{2}{3} \cdot \text{(integer)}, \quad a_1^2 = \frac{2}{3} \cdot \text{(integer)},$$

$$ (v_I)^2 = \frac{2}{9} \cdot \text{(integer)} \quad \text{for} \quad I = \{1, 2, \cdots , 8\} \text{ or } \{9, 10, \cdots , 16\}, \quad (3)$$

$$a_{1I}^2 = \frac{2}{9} \cdot \text{(integer)} \quad \text{for} \quad I = \{1, 2, \cdots , 8\} \text{ or } \{9, 10, \cdots , 16\}.$$  

The modular invariance condition requires in addition,

$$3 \ v \cdot a_i = \text{(integer)} \quad \text{for} \quad (i = 1, 3, 5),$$

$$3 \ a_i \cdot a_j = \text{(integer)} \quad \text{for} \quad i \neq j. \quad (4)$$

The notation is the same as those discussed in[21]. For an $SU(3)^3$ gauge group, we choose the following shift vector and a Wilson line,

$$v = (0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{2}{3}) \ (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

$$a_1 = (\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{5}{3}) \ (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad (5)$$

C. Untwisted sector

**Gauge group** : From the mass shell condition $\frac{p^2}{4} = \frac{v^2}{2} - 1$, we find the massless spectrum in the untwisted sector. For the gauge bosons, the $p^2 = 2$ root vectors, satisfying $p \cdot v = 0$ and $p \cdot a_1 = 0 \mod \text{integer}$, are the nonvanishing roots. These are presented for the first $E_8$ subgroup in Table[11]. The second $E_8'$ gauge group is not broken.
TABLE I: Root vectors $p_I$ in untwisted sector satisfying $p \cdot v = 0$ and $p \cdot a_1 = 0$. The underlined entries allow permutations. The $+$ and $-$ in the spinor part denote $\frac{1}{2}$ and $-\frac{1}{2}$, respectively. $I, V,$ and $U$ spin directions of $SU(3)$’s are also shown.

| vector                              | number of states | gauge group |
|-------------------------------------|------------------|-------------|
| $(1 - 1 0 0 0 0 0 0)$               | 6                | $SU(3)_1$   |
| $(0 0 0 1 1 0 0 0)I_+$              | 1                |             |
| $(0 0 0 -1 -1 0 0 0)I_-$            | 1                |             |
| $(+ + + + - - +)V_+$               | 1                | $SU(3)_2$   |
| $(- - - - + + -)V_-$               | 1                |             |
| $(+ + + - - + + -)V_+$             | 1                |             |
| $(- - - + + + + -)U_+$             | 1                |             |
| $(0 0 0 1 -1 0 0 0)I_+$             | 1                | $SU(3)_3$   |
| $(0 0 0 -1 1 0 0 0)I_-$             | 1                |             |
| $(+ + + + - + - +)V_+$             | 1                |             |
| $(- - - + - - + +)V_-$             | 1                |             |
| $(+ + + - - + - + -)U_+$           | 1                |             |
| $(0 0 0 0 0 1 -1 0)I_-$             | 1                | $SU(3)_4$   |
| $(0 0 0 0 0 0 -1 1)I_+$             | 1                |             |
| $(0 0 0 0 0 0 0 1)V_+$               | 1                |             |
| $(0 0 0 0 0 0 1 1)V_-$               | 1                |             |
| $(0 0 0 0 0 -1 0 1)U_+$             | 1                |             |
| $(0 0 0 0 0 0 0 1)U_-$               | 1                |             |

In Table I we use the convention that the underlined entries allow permutations. There are 6 winding states in the first row and adding two oscillators we have the 8 roots for the first $SU(3)_1$. Similarly, we obtain the rest $SU(3)$’s. Thus, we obtain the gauge group $SU(3)^3 \subset SU(3)^4$ with the corresponding nonvanishing root vectors explicitly shown. Note in passing that there is no $U(1)$ subgroup, which means that there is no anomalous $U(1)$ gauge group with the above orbifold. Thus, it is possible to realize the model-independent axion as a quintessential axion \footnote{22}. 6
TABLE II: Root vectors $p_i$ in untwisted sector satisfying $p \cdot v = \frac{2}{3}$ and $p \cdot a_1 = 0$. The underlined entries allow permutations. The notations are the same as in Table I, except that [] implies even numbers of sign flips. In the last column, we reverse the chirality to compare directly with the twisted sectors.

| Sector | From $E_8$ roots | $E_8$ root | $SU(3)^4$ |
|--------|------------------|------------|-----------|
| UT     | $(1 \ 0 \ 0 \ 0 \ 0 \ -1 \ 0 \ 0)$ | $(1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1)$ | $3(\bar{3}, 3, 1, 3)$ |
|        | $(+_--[+ +]++--)$ | $(+_--[+ +]++++)$ |           |

Matter from the untwisted sector: The matter fields from the untwisted sector satisfy the condition

$$p^2 = 2, \ p \cdot v = \frac{2}{3} \mod \text{integer}, \ p \cdot a_i = 0 \mod \text{integer}. \quad (6)$$

In Table II we present the root vectors satisfying these.

D. Matter from the twisted sectors

In $Z_3$ orbifolds, there are three fixed point on a 2-torus. Since we compactify six internal spaces via three 2-tori, there are 27 fixed points. These 27 fixed points look the same in every aspect if we do not introduce Wilson lines. If we allow the possibility to wrap the 2-torus by a Wilson line, then three fixed points on the torus can be distinguished by the gauge fields going around the torus. There are two directions to wrap the torus, but the modular invariance requires that they must be the same, i.e. $a_1 = a_2$. Similarly, if we wrap more tori, we have $a_3 = a_4$ and $a_5 = a_6$. Thus, we can consider at most three independent Wilson lines, $a_1, a_3$, and $a_5$. In this paper, we considered the simplest Wilson line, i.e. $a_1 \neq 0$, and $a_3 = a_5 = 0$. So the 27 fixed points are grouped into three classes: 9 trivial fixed points around which there is no Wilson line($v$), 9 positively wrapped fixed points($v + a_1$),
and 9 negatively wrapped fixed points \((v - a_1)\), which are denoted as T0, T1, and T2 twisted sectors, respectively.

In our model, the massless matter fields from the twisted sectors satisfy \((p + \tilde{v})^2 = \frac{4}{3}, \frac{4}{3}\), where \(\tilde{v} = v, v + a_1, v - a_1\), for T0, T1, and T2, respectively. Of course, the weights we present survive the GSO-like projection. For the vectors corresponding to \(\frac{4}{3}\), the multiplicity is 9 as described above, and for the vectors corresponding to \(\frac{2}{3}\), the multiplicity is 27 because of the three oscillator modes in this case.

In general, the matter fields from the twisted sectors make the theory extremely chiral which was the reason that we have not obtained yet any realistic SSM or flipped SU(5) model from orbifold compactification of the heterotic string. Since it is very chiral, there is a chance that the spectrum (2) can appear through orbifolding.

In Tables III, IV, and V, we list the massless spectrum from the twisted sectors. But note that the chirality of the twisted sector in the \(Z_3\) orbifold is the opposite of the chirality of the untwisted sector matter fields.

E. Electroweak hypercharge

In the \(SU(3)^3\) GUT, the color factor should not carry the electroweak hypercharge. To break \(SU(3)^4\) gauge group down to \(SU(3)^3\) another \(SU(3)\) should not carry the hypercharge. Let us break \(SU(3)^4\) completely by two independent vacuum expectation values of \((1,1,1,3)\). Thus the \(SU(3)^3\) group is \(SU(3)_1 \times SU(3)_2 \times SU(3)_3\). We identify \(SU(3)_2\) as the group containing the \(W^\pm\) bosons and \(SU(3)_3\) as QCD. Under the \(SU(3)_1 \times SU(3)_2 \times SU(3)_3\), we obtain the following chiral fermions,

\[
9 [(\bar{3}, 3, 1) + (1, \bar{3}, 3) + (3, 1, 3)]_a
+ 9 [(\bar{3}, 3, 1) + (1, \bar{3}, 3) + (3, 1, 3)]_b + \cdots
\] (7)

where \(\cdots\) represents 27 multiplets of the vectorlike combination \((3,1,1) + (\bar{3}, 1, 1) + (1, 3, 1) + (1, \bar{3}, 1) + (1, 1, 3) + (1, 1, \bar{3}) + 3(1, 1, 1)\). Eq. (7) realizes the representation given in (2).

The hypercharge (\(\equiv\) electroweak hypercharge) \(Y\) is a combination of generators of \(SU(3)_1\) and \(SU(3)_2\),

\[
Y = -\frac{1}{2} (-2I_1 + Y_1 + Y_2)
\] (8)
TABLE III: Root vectors $p_I$ in the T0 twisted sector satisfying $p \cdot \tilde{v} = \frac{2}{3} \cdot \frac{4}{3}$. The notations are the same as in Table II.

| sector | Weights | $SU(3)^4$ |
|--------|---------|-----------|
|        | vector  |           |
| T0     | (0 0 0 0 0 0 0 0) | 27(1,1,1,3) |
|        | (0 0 0 0 0 -1 0 -1) |           |
|        | (0 0 0 0 0 0 -1 -1) |           |
|        | (-1 0 0 0 0 0 0 -1) |           |
|        | (+ + - [-] - [-] - [-]) | 9(3,3,1,1) |
|        | (1 0 0 0 0 0 0 -1) |           |
|        | (+ - - [-] - [-] - [-]) | 9(3,1,3,1) |
|        | (0 0 0 0 0 -1 -1 0) |           |
|        | (0 0 0 1 0 0 0 -1) |           |
|        | (0 0 0 1 0 0 0 -1) |           |
|        | (+ + + [+ -] - [-] - [-]) | 9(1,3,3,1) |
|        | (- - - [+ -] - [-] - [-]) |           |

TABLE IV: Root vectors $p_I$ in the T1 twisted sector.

| sector | Weights | $SU(3)^4$ |
|--------|---------|-----------|
|        | vector  |           |
| T1     | (0 0 0 0 0 -1 -1 -2) | 27(1,3,1,1) |
|        | (- - - [+ +] - - $\frac{5}{2}$) |           |
|        | (0 0 0 0 0 -1 0 -3) |           |
|        | (0 0 0 0 0 0 0 -2) |           |
|        | (- - - [- -] - - $\frac{3}{2}$ $\frac{5}{2}$) | 9(1,1,3,3) |
|        | (- - - [+ -] - - $\frac{3}{2}$) |           |
|        | (-1 0 0 0 0 -1 -1 -3) |           |
|        | (-1 0 0 0 0 1 0 -2) |           |
|        | (-1 -1 0 0 0 -1 -1 -2) |           |
|        | (+ - - [- -] - - $\frac{5}{2}$) |           |
|        | (-1 -1 0 0 0 -1 -1 -2) |           |
TABLE V: Root vectors \( p_1 \) in the \( T2 \) twisted sector.

| sector | Weights | \( SU(3)^4 \) |
|--------|---------|---------------|
|        | vector  |               |
|        | \((1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1)\) | \(27(3,1,1,1)\) |
|        | \((+ \ + \ [+ \ -] \ + \ - \ +)\) | \(9(1,1,3,3)\) |
|        | \((0 \ 0 \ 0 \ 0 \ 0 \ -1 \ 0 \ 1)\) |               |
|        | \((+ \ + \ [+ \ -] \ + \ + \ +)\) | \(9(1,3,3,1)\) |
|        | \((1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1)\) |               |
|        | \((0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1)\) |               |
|        | \((0 \ 0 \ 0 \ 0 \ -1 \ 0 \ 0 \ 1)\) |               |
|        | \((+ \ + \ + \ + \ + \ + \ +)\) | \(9(1,3,1,3)\) |
|        | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1)\) |               |
|        | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) |               |
|        | \((+ \ + \ [+ \ +] \ - \ - \ +)\) |               |
|        | \((0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1)\) |               |
|        | \((0 \ 0 \ 0 \ -1 \ 0 \ 0 \ 0 \ 1)\) |               |
|        | \((+ \ + \ [+ \ +] \ - \ - \ +)\) |               |
|        | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) |               |
|        | \((+ \ + \ [+ \ +] \ + \ - \ +)\) | \(9(1,3,3,1)\) |
|        | \((0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1)\) |               |
|        | \((0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1)\) |               |
|        | \((+ \ + \ [+ \ +] \ - \ + \ +)\) | \(9(1,3,3,1)\) |
|        | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1)\) |               |

where \( I_1 \) is the third component \( (T_3)_1 \) of the isospin generators of the group \( SU(3)_1 \), and \( Y_i \) is the \( SU(3)_i \)(\( i = 1,2 \)) hypercharge \( \frac{2\sqrt{3}}{3}(T_8)_i \). The eigenvalues of \( I \) and \( Y \) are \( \{\frac{1}{2}, -\frac{1}{2}, 0\} \) and \( \{\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\} \), respectively. One can easily check that the model presented in (7) gives \( \sin^2 \theta_W^0 = \frac{3}{8} \), thus solving the string \( \sin^2 \theta_W \) problem. Since the hypercharge \( U(1)_Y \) does not leak to \( SU(3)_3 \)(=QCD), in counting the eigenvalues of the electroweak \( (T_3)_2^2 \) and \( Q_{em}^2 \), the
contributions from the first and the second [ ] brackets of Eq. (7) are exactly the same. We also checked that the vectorlike representation contributes in the same ratio. There unfamiliar particles such as lepton doublets with \( Y = \pm \frac{1}{6} \) appear, but they form a vectorlike representation, are removed at the GUT scale and do not alter \( \sin^2 \theta_{W}^{0} \). This miraculous prediction of \( \sin^2 \theta_{W}^{0} \) is based on the fact that everything appears in the multiples of 3. The model given in Eq. (7) gives 9 families. But note that there appear additional 9 families with the opposite colors. By adding more Wilson line(s) in the hidden sector \( E_{8}' \) part, the family number can be easily reduced to 3, not spoiling our precious spectrum obtained in (2). Below, we comment on six family models obtained by adding more Wilson line(s) at \( E_{8}' \).

The spectrum (7) has two villages, each having three families. The family mixing is allowed inside the village but is forbidden between different villages, predicting two CP phases, one in each village. To explain the three light families, the members of the strange village are required to be heavy at the electroweak scale. With 6 families, the QCD coupling is not asymptotically free, but still perturbatively unifiable at the GUT scale. The rank–6 \( SU(3)^{3} \) is directly broken down to the rank–4 SSM by two vacuum expectation values in \( (\bar{3},3,1) \), i.e. \( \langle (1_{\bar{3}},1_{3},1) \rangle = \text{(GUT scale)} \) and \( \langle (2_{\bar{3}}^{1},1_{3},1) \rangle = \text{(GUT scale)} \), where \( 2_{\bar{3}}^{1} \) is the \( (I_{\bar{3}})_{1} = -\frac{1}{2} \) member in \( \bar{3} \) of \( SU(3), etc. \) With some hypotheses on removing a set of vectorlike representations at the GUT scale, we obtain a realistic SSM in the present orbifold compactification with the help of the GUT scale VEV’s. Here, we assume that three SM singlets in our village are removed at high energy scale, but the three singlets of the stranger village are left light so that they can acquire Dirac masses at the electroweak symmetry breaking scale. Namely, we will have three light neutrinos.

The three strange village families presented in (7) may be considered as a drawback of the present construction. But remembering the enormous difficulties during the last two decades in obtaining a superstring derived SSM, in view of a model like (7) we may envision a Planck scale string theory verifiable through TeV–scale probing colliders.

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[1] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. B258 (1985) 46.
[2] L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261 (651) 1985; Nucl. Phys. B274 (1986) 285.
[3] L. Ibanez, H. P. Nilles, and F. Quevedo, Phys. Lett. B187 (1987) 25.
[4] L. Ibanez, J. E. Kim, H. P. Nilles, and F. Quevedo, Phys. Lett. B191 (1987) 282.
[5] L. Ibanez, J. Mas, H. P. Nilles, and F. Quevedo, Nucl. Phys. B301 (1988) 157.
[6] J. A. Casas and C. Munoz, Phys. Lett. B209 (1988) 214; Phys. Lett. B214 (1988) 63; A. E. Faraggi, Nucl. Phys. B403 (1993) 101, and references therein; M. Cvetic, G. Shiu and A. M. Uranga, Nucl. Phys. B615 (2001) 3 [hep-th/0107166]; L. E. Ibáñez, F. Marchesano and R. Rabadán, JHEP 0111 (2001) 002; C. Kokorelis, JHEP 0209 (2002) 029; M. Cvetic, P. Langacker and G. Shiu, Nucl. Phys. B642 (2002) 139 [hep-ph/0206115].
[7] J. E. Kim, Phys. Lett. B207 (1988) 434; E. J. Chun, J. E. Kim and H. P. Nilles, Nucl. Phys. B370 (1992) 105.
[8] J. E. Kim and H. P. Nilles, Phys. Lett. B263 (1991) 79.
[9] See, for example, L. Ibañez, Phys. Lett. B303 (1993) 55.
[10] Z. Kakushadze and S. H. H. Tye, Phys. Rev. Lett. 77 (1996) 2612.
[11] S. M. Barr, Phys. Lett. B112 (1982) 219.
[12] J.-P. Derendinger, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. B139 (1984) 170.
[13] I. Antoniadis, J. Ellis, J. S. Hagelin, and D. V. Nanopoulos, Phys. Lett. B205 (1988) 459. For more references, see, I. Antoniadis, J. Ellis, J. S. Hagelin, and D. V. Nanopoulos, Phys. Lett. B231 (1989) 65.
[14] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono, and K. Tanioka, Kanazawa Univ. preprint DPKU-8904 (1989).
[15] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono, and K. Tanioka, Nucl. Phys. B341 (1990) 611.
[16] Y. Kawamura, Prog. Theor. Phys. 103 (2000) 613 [hep-ph/9902423]; G. Altarelli and F.
Feruglio, Phys. Lett. B511 (2001) 257; A. Hebecker and J. March-Russel, Nucl. Phys. B613 (2001) 3; H. D. Kim, J. E. Kim and H. M. Lee, Europhys. J. C 24 (2002) 159; T. Asaka, W. Buchmüller and L. Covi, Phys. Lett. B523 (2001) 199.

[17] K. S. Babu, S. Barr, and B. Kyae, Phys. Rev. D65 (2002) 115008; K. Hwang and J. E. Kim, Phys. Lett. B540 (2002) 289; H. Georgi, Nucl. Phys. B156 (1979) 126; J. E. Kim, Phys. Rev. Lett. 45 (1980) 1916; Phys. Rev. D23 (1981) 2706.

[18] K.-S. Choi and J. E. Kim, Phys. Lett. B552 (2003) 81.

[19] S. L. Glashow, in Proc. Fourth Workshop(1984) on Grand Unification, ed. K. Kang et al.(World Scientific, Singapore, 1985), p. 88; G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D51 (1995) 2486.

[20] K.-S. Choi, K. Hwang and J. E. Kim, to be published.

[21] H. B. Kim and J. E. Kim, Phys. Lett. B300 (1993) 343.

[22] J. E. Kim and H. P. Nilles, Phys. Lett. B553 (2003) 1 [hep-ph/0210402].

[23] At higher level $k > 1$, it was shown that the adjoint representation $45$ of SO(10) can be obtained[10].

[24] Since there does not exists a complete table for $Z_M \times Z_N$ orbifolds, we are not sure whether HESSNA is possible for $Z_M \times Z_N$. 