Harmonic analysis of annular sector sandwich plate using FEM

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Abstract. In present study, vibration and harmonic behaviors of annular sector sandwich plate of two orthotropic composite materials are analyzed using FEM technique. Sandwich plate made of two face sheet and one core sheet in middle has been modelled using an ANSYS APDL FEM tool. A suitable finite element model is developed and proposed based on first order shear deformation theory. The model has been discretized using appropriate elements i.e. 8-node solid 185 for core sheet and SOLSH190 for face sheet from the ANSYS element library. Effect of varying face sheet thickness is demonstrated on vibration as well as harmonic behaviors of sandwich plate and related findings are discussed briefly. Effect of sector angle has also been demonstrated. The authenticity of the proposed methodology has been verified by comparing the simulation results with available literature. It can be observed that the results obtained are in good propinquity and presently proposed model shows the results with adequate accuracy.

Keywords: Sandwich plate, Sector Angle, Harmonic Behavior, Mode Number, Resonance, FEM

1. Introduction

Now a days, sandwich plates are increasingly used in weight saving structure in replace of conventional composite materials. Two face sheets of one material are glued above and below on core sheet of different material to manufacture the sandwich plate. Among the square, rectangular and skew structure, annular sector are also the most important structural components. Many researchers have paid attention in studying static and vibration characteristics of sandwich structures.

Kulkarni and Kapuria [1] developed a new improved element to study the vibration behaviour of composite as well as sandwich plates by using third-order zigzag theory. A sandwich plate with rectangular geometry containing flexible core was analysed to check vibrational behavior using FEM technique through APDL by Malekzadeh and Sayyyidmousavi [2]. The effect of parametric dimension such as thickness ratio, size and stiffness of the attached mass, length-to-thickness ratio etc. has also been carried out. Abdennadher M. et al [3] studied dynamic characteristics of sandwich plate in presence of impact modelled by a fixed rigid solid and a contact between sandwich plates. Li X. et al [4] presented a newly developed optimum design solutions for sandwich structures to satisfy bending as well as torsion requirements using Lagrange multiplier method. Gopichand A. et al. [5] did static
analysis of sandwich panel made of two isotropic materials i.e. stainless steel as face sheet and mild steel as core sheet using ANSYS tool and also obtained the results experimentally using UTM. Shen C. et al. [6] analyzed sound vibration developed from stiffened plates with orthogonally lamination scheme. Arbaoui J. et al. [7] investigated the effect of intermediate layers and thickness of core on the mechanical properties of sandwich structure made of a honeycomb core three points bending and also developed a theoretical model to calculate the shear properties in multi-cores. Khan and Awari [8] analyzed harmonic behavior of Square Plate with and without uncertain parameters. Maha M. A. et al. [9] studied the vibrational characteristics of sandwich beam structure at different loading conditions. Wang Q. et al. [10] demonstrated a combined result for the dynamic analysis of laminated composite plate of different geometries applying general boundary conditions. Krzyhak A. [11] compared the effect of manufacturing method of sandwich structure on mechanical properties such as compressive strength, impact strength and flexural strength. Ijaz H. et al. [12] presented a simplified methodology for analysis of sandwich structures using the homogenization method based upon the strain energy criterion by the hexagonal core with a simple equivalent volume for FE analysis. Pagani A et al. [13] discussed classical and different finite elements for the analysis of sandwich as well as laminates. Yadav Y. S. et al. [14] examined the vibration and harmonic response of isotropic annular plates in order to determine the frequency parameters and resonance amplitude using FEM technique. Tounsi A. et al. [15] presented the buckling analysis on FG sandwich plates using a new quasi-3D hyperbolic plate theory adding stretching effect and found this new theory more accurate than other old theory. Narwariya M. et al. [16] demonstrated the dynamic behavior of circular sector plate with eccentric hole. Many researchers have analyzed the sandwich plate using different techniques to obtain static and vibration behaviour of the plate having different structures. After extensive review of available literatures, the results show that the harmonic behaviour of the annular sector sandwich plate has not been studied as yet. This paper fills this gap.

2. Modelling

2.1 Geometric modelling

Five geometries of five-layered $(0^\circ/90^\circ/\text{Core}/90^\circ/0^\circ)$ annular sector sandwich plate with each ply in the face sheets being of material 1 and core of material 2 are considered to analyze the results as listed in table 1.

| Geometry No. | Upper face sheet $(T_{f1})$ with $0^\circ/90^\circ$ cross ply lamination scheme | Core sheet $(T_c)$ | Bottom face sheet $(T_{f2})$ with $90^\circ/0^\circ$ cross ply lamination scheme |
|--------------|---------------------------------------------------------------------------------|-----------------|---------------------------------------------------------------------------------|
| 1            | 0                                                                               | 1.00            | 0                                                                               |
| 2            | 0.04 each ply                                                                   | 0.84            | 0.04 each ply                                                                   |
| 3            | 0.06 each ply                                                                   | 0.76            | 0.06 each ply                                                                   |
| 4            | 0.08 each ply                                                                   | 0.68            | 0.08 each ply                                                                   |
| 5            | 0.10 each ply                                                                   | 0.60            | 0.10 each ply                                                                   |

Figure 1 shows the Geometry of annular sector sandwich plate with following dimensions:

- $T_{f1}$ and $T_{f2} =$ Thickness of upper and lower face sheet varies from 0d to 0.2d.
- $\theta =$ Sector Angle varies from $30^\circ$ to $90^\circ$
- $r_1 =$ Outer radius of sector plate taken as 5d
3

\[ r_2 = \text{Inner radius of sector plate} \quad 0.5d \]
and \[ d = \text{Depth of the plate taking as unity.} \]

**Figure 1.** Geometry of Annular Sector sandwich plate

2.2 **Finite element modelling**
An 8-node solid 185 and solid-shell 190 elements are taken for core and face materials respectively as shown in Figures 2 (a & b). SOLID185 Structural Solid is appropriate for modelling general 3-D solid structures and SOLSH190 is used for simulating shell structures with a wide range of thickness.

![Finite element modelling elements](image)

(a) 8-node solid 185
(b) SOLSH190

**Figure 2.** Finite element modelling element

2.3 **Loading Condition**
All the edges of the plate are clamped to all degrees of freedom to conduct vibration analysis and a force of 1000 N is applied on every node of the surface of top face sheet in clamped plate. Figure 3 shows the loading condition of the plate.
Figure 3. Annular sector plate with (a) meshing (b) clamped boundary condition (c) force of 1000 N on every node on top surface.

2.4 Material property
Material property has been taken from published literature of Kulkari et al. (2008) for two sheets of sandwich plate i.e. face sheet and core sheet as listed in table 2.

| Property       | Face sheet Material (1) | Core sheet Material (2) | Reference          |
|----------------|-------------------------|-------------------------|--------------------|
| Youngs Modulus | $Y_{f1} = 276$ GPa      | $Y_{c1} = 0.5776$ GPa  | Kulkarni et al. [1]|
|                | $Y_{f2} = 6.9$ GPa      | $Y_{c2} = 0.5776$ GPa  |                    |
|                | $Y_{f3} = 6.9$ GPa      | $Y_{c3} = 0.5776$ GPa  |                    |
| Shear Modulus  | $G_{f12} = 6.9$ GPa     | $G_{c12} = 0.1079$ GPa |                    |
|                | $G_{f23} = 6.9$ GPa     | $G_{c23} = 0.22215$ GPa|                    |
|                | $G_{f31} = 6.9$ GPa     | $G_{c31} = 0.1079$ GPa |                    |
| Poisson’s Ratio| $\nu_{f12} = 0.25$     | $\nu_{c12} = 0.0025$   |                    |
|                | $\nu_{f13} = 0.25$     | $\nu_{c13} = 0.0025$   |                    |
|                | $\nu_{f23} = 0.3$      | $\nu_{c23} = 0.0025$   |                    |
| Density        | $\rho_f = 681.8$ kg/m$^3$ | $\rho_c = 1000$ kg/m$^3$ |                |

3. Result and Discussion

3.1 Verification of present analysis

3.1.1 Convergence study: To find the proper mesh size, frequency parameters has been obtained upto six mode number with reference to different mesh sizes for five layered ($0^\circ$/90°/Core/90°/0°) annular sector sandwich plate with clamped boundary condition taking geometry 2 with as shown in table 3. From the table, it can be concluded that mesh size of 20 × 20 is good enough to analyze response of the plate.

3.1.2 Comparison study

For Sandwich Plate: In order to accuracy of the present analysis, a comparison study has been done with published result of Kulkarni et al. (2008). Frequency parameters have been obtained for clamped ($0^\circ$/90°/Core/90°/0°) square plate up to sixth mode for different mesh size taking $r_1/r_2$ as 10 as shown in table 4 and found that results are in good proximity.
Table 3. Convergence study for a annular sector sandwich plate under CCCC

| $r_1/r_2$ | Mode No. | Non-dimensionalized fundamental frequency $[\sigma = 100\omega_n a/\sqrt{(\rho_c/Y_f1)}]$ |
|-----------|---------|----------------------------------|
|           |         | $8\times8$ | $12\times12$ | $16\times16$ | $20\times20$ | $24\times24$ | $28\times28$ |
| 10        | M_1     | 0.936932  | 0.905901   | 0.892049    | 0.882745    | 0.882745    | 0.882745    |
|           | M_2     | 1.490971  | 1.409279   | 1.392563    | 1.362193    | 1.362193    | 1.362193    |
|           | M_3     | 1.611902  | 1.535032   | 1.513909    | 1.487662    | 1.487662    | 1.487662    |
|           | M_4     | 2.096475  | 2.005422   | 1.982257    | 1.865865    | 1.865865    | 1.865865    |
|           | M_5     | 2.258535  | 2.126731   | 2.133728    | 1.980934    | 1.980934    | 1.980934    |
|           | M_6     | 2.258535  | 2.126731   | 2.133728    | 1.980934    | 1.980934    | 1.980934    |

Table 4. Comparison of frequencies parameter $[\sigma = 100\omega_n a/(\rho_c/Y_f1)]$

| MESH SIZE | Reference          | M_1         | M_2         | M_3         | M_4         | M_5         | M_6         |
|-----------|--------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 12×12     | Present            | 11.4851     | 18.7178     | 19.8173     | 25.185      | 28.2658     | 31.3145     |
|           | Kulkarni et al. (2008) | 12.0464         | 18.2701    | 20.5724     | 24.8738     | 26.4054     | 30.6442     |
|           | %error             | -4.66       | 2.45        | -3.67       | 1.25        | 7.05        | 2.19        |
| 16×16     | Present            | 11.4199     | 18.581      | 19.5772     | 24.8921     | 27.7253     | 30.978      |
|           | Kulkarni et al. (2008) | 12.578         | 19.4258    | 21.7796     | 27.5495     | 28.6914     | 32.9497     |
|           | %error             | -9.21       | -4.35       | -10.11      | -9.65       | -3.37       | -5.98       |
| 20×20     | Present            | 11.3906     | 18.5101     | 19.4695     | 24.7598     | 27.4758     | 30.5963     |
|           | Kulkarni et al. (2008) | 12.4404         | 19.1067    | 21.4421     | 26.6911     | 28.0439     | 32.2577     |
|           | %error             | -8.44       | -3.12       | -9.20       | -7.24       | -2.03       | -5.15       |
| 24×24     | Present            | 11.3761     | 18.4668     | 19.4128     | 24.686     | 27.3416     | 30.3959     |
|           | Kulkarni et al. (2008) | 12.3654         | 18.9369    | 21.2618     | 26.2432     | 27.7205     | 31.9096     |
|           | %error             | -8.00       | -2.48       | -8.70       | -5.93       | -1.37       | -4.74       |

For Sector plate: Non-dimensionalised fundamental frequencies circular sector plate are obtained and compared with published results by Wang (2016). The results are presented in graphs as shown in Figures 4.

Figures 4(a-c) present the comparison for first four non-dimensionalised frequency parameters of (0°/90°/0°/90°) circular sector plates with $T/r_1$ (thickness to outer radius ratio) as 0.2 for various sector angles ($\phi$) as 90°, 180° and 270° respectively.
3.2 Harmonic Analysis

In present study, vibration and harmonic response of annular sector sandwich plate are analyzed to examine the effect of thickness of face sheet of sandwich plate and effect of sector angle of sandwich plate.

3.2.1 Effect of Thickness of Face sheet: The weight of the sandwich plate is obtained for different thicknesses of the face sheets and core sheet while keeping the total thickness as unity. As the thickness of face sheet is increased, the weight of sandwich plate is reduced as shown in figure 5.
percentage reduction in weight is listed in table 5. Weight of Plate for different geometries is calculated as:

\[
\text{Weight of the Plate (N)} = [\rho_{f1} \times V_{f1} + \rho_c \times V_c + \rho_{f2} \times V_{f2}] \times g
\]

Where \( \rho_{f1}, \rho_{f2}, \) and \( \rho_c \) = densities of top, bottom face sheet and core sheet respectively. \( V_{f1}, V_{f2}, \) and \( V_c \) = volume of top, bottom face sheet and core sheet respectively

\[ g = 9.81 \text{ m/s}^2 \]

![Figure 5. Graph between thicknesses of face sheet v/s weight of plate](image)

**Table 5.** Percentage reduction of weight of plate for varying thickness of face and core sheets

| Geometry No. | Thickness of sheets taking plate’s depth (d) as unity | Weight of the plate (N) | % reduction in weight |
|--------------|------------------------------------------------------|-------------------------|-----------------------|
| 1            | 0          | 127128.47 | 0                      |
| 2            | 0.08       | 120656.11 | 5.09                   |
| 3            | 0.12       | 117419.93 | 7.63                   |
| 4            | 0.16       | 114183.74 | 10.18                  |
| 5            | 0.20       | 110947.56 | 12.72                  |

Annular sector sandwich plate has been analyzed to examine the response of varying thickness of sheets on harmonic behavior under damping ratio as 0.01. The results are listed in Table 6 which shows that on increasing the thickness of face sheets, the natural frequency (NF) increases and resonant amplitude (RA) decreases. The percentage reduction in resonance amplitude is increased by 47.53% even a thin face sheet with dimension of 0.08d has been added in sandwich plate.

Harmonic behavior for all the geometries has been drawn on FRF (function response function) as presented in Figures 6(a-e).
Table 6. Effect of varying thickness of sheets of sandwich plate on NF and RA under damping factor as 0.01.

| R | BC  | Geometry No. | NF (Hz) | RA (m)   | % reduction in RA |
|---|-----|--------------|---------|----------|-------------------|
| 5 | CCCC| 1            | 57.125  | 0.126775 | 0                 |
|   |     | 2            | 93.362  | 0.066518 | 47.53             |
|   |     | 3            | 112.32  | 0.0478859| 62.23             |
|   |     | 4            | 136.98  | 0.0334550| 73.61             |
|   |     | 5            | 165.55  | 0.0141971| 88.80             |

These FRF show the graph between frequencies versus amplitude and frequencies versus phase angle. Peak value represents the resonant amplitude. Phase change angles have also been computed and represented in this graph.

Figure 6(a). FRF for annular sector sandwich plate for geometry 1.

Figure 6(b). FRF for annular sector sandwich plate for geometry 2.
Figure 6(c). FRF for annular sector sandwich plate for geometry 3.

Figure 6(d). FRF for annular sector sandwich plate for geometry 4.

Figure 6(e). FRF for annular sector sandwich plate for geometry 5.
3.2.2 Effect of Sector Angle: The results are listed in Table 8 for different sector angle. Table 7 presents that the NF decreases and RA is increased while increasing the sector angle. The percentage increment in resonance amplitude is increased by 78.58% when sector angle increases from 30° to 45°. Further increment in resonance i.e 53.67%, 39.69% and 19.84% have been occurred with increment of sector angle from 45° to 60°, 60° to 75° and 75° to 90° respectively.

Table 7. Effect of varying sector angle (θ) of sandwich plate on NF and RA under damping factor as 0.01

| Geometry No. | r₁/r₂ | θ  | NF (Hz) | RA (m) | % increment in RA |
|--------------|-------|----|---------|--------|------------------|
| 3            | 10    | 30°| 205.78  | 0.00475000 | 0.00             |
|              |       | 45°| 142.43  | 0.0221849  | 78.58            |
|              | 75°   | 112.32 | 0.0478859 | 53.67 |
|              | 90°   | 95.44 | 0.0794031 | 39.69 |
|              |       | 85.073 | 0.0990676 | 19.84 |

Harmonic behaviors for all the plates having different sector angles are drawn on FRF as presented in figures 7(a-d). These FRF show the graph between frequencies versus amplitude. Peak value represents the resonant amplitude.

Figure 7(a). FRF for sandwich plate for sector angle 30°.
Figure 7(b). FRF for sandwich plate for sector angle 45°.

Figure 7(c). FRF for sandwich plate for sector angle 60°.
4. Conclusions

This paper presents the harmonic behaviour of annular sector sandwich plate. The analyses are carried out for various geometries of plate to examine the effect of varying thicknesses of face sheets. The effects of varying sector angle on the frequency parameter and resonance amplitude have also been determined. From the study, following conclusion has been made:

- It has been found that when the analysis has been conducted for isotropic plate, resonance amplitude found high as compared to sandwich plate. As even very thin sheet (with thickness of
0.08d) has been added to a plate, the resonance amplitude is reduced by 47.53%. In this sequence, percentage reduction for resonance amplitude are increased on increasing the thickness of face sheet. It can be concluded that plate having thick face sheet is lighter in weight and possesses more strength for minimizing vibration.

- The natural frequency (NF) decreases with increase in sector angle and increases on increasing thickness of face sheet.

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