Introducing low-order system frequency response modelling of a future power system with high penetration of wind power plants with frequency support capabilities

Matej Krpan1, Igor Kuzle1

1Department of Energy and Power Systems, University of Zagreb Faculty of Electrical Engineering and Computing, Unska 3, Zagreb, Croatia
E-mail: matej.krpan@fer.hr

Abstract: Wind power generation has reached a significant share in power systems worldwide and will continue to increase. As the converter-connected generation reduces the grid inertia, more and more interest has been given to exploiting the kinetic energy and controllability of variable-speed wind turbine generators (VSWTGs) for frequency support. Consequently, the grid frequency dynamics are changing. Thus, it is necessary to include the frequency response of wind power plants in the system frequency response (SFR) model. A novel approach to low-order SFR modelling of a future power system with a high share of frequency-support-capable VSWTGs has been presented. Low-order model of VSWTGs with primary frequency response and natural inertial response has been developed considering different wind turbine operating regimes and compared to the non-linear model for validation. Low-order model has been presented in a symbolic transfer function form. Model accuracy has been discussed and the impact of VSWTG parameters on frequency response has been analysed. The developed model facilitates studying power system frequency dynamics by avoiding the need for modelling complex VSWTG systems, while retaining a satisfying level of accuracy.

1 Introduction
To reduce the carbon footprint of the power and energy sector, many countries throughout the world introduced various measures encouraging the integration of renewable energy sources. The most popular energy sources for renewable power generation are photovoltaics (PVs) and wind power plants (WPPs): in 2016, installed PV capacity was 291 GW and installed WPP capacity was 467 GW in the world [1] and they will continue to rise. As converter-connected renewable generation replaces conventional units, the total grid inertia is reduced which negatively impacts the frequency stability of the power system: the grid becomes weaker and reduces the capability of the system to remain stable after the occurrence of faults or disturbances [2, 3]. Some transmission system operators (TSOs) have already implemented active power and frequency control requirements on wind farms in their respective grid codes [4-6], while it is to be expected that other TSOs will start to impose those requirements [7]. Accordingly, a significant amount of recent research has focused on utilising the controllability of variable-speed wind turbine generators (VSWTGs) to provide an inertial response (so-called virtual or synthetic inertia) and/or primary frequency response which can be found in an excellent state-of-the-art overview [8].

However, not a lot of research has investigated low-order system frequency response (SFR) modelling of VSWTGs. Low-order SFR models provide a simple framework for studying power system frequency changes by taking into account only the most significant system dynamics in the time scale of interest which is ≤ 30 s [9]. Although it is not the most detailed representation of power system dynamics, it provides a good enough estimate of grid frequency behaviour and it is usually used in both academy and industry for studying impact of system inertia [3], design of under-frequency load shedding schemes [10] or roughly estimating frequency behaviour of a large power system or an islanded portion thereof to sudden load disturbances [9]. As the share of VSWTGs and other converter-connected generation in the power systems worldwide increases and becomes a significant part of the total capacity connected to the grid, coupled with the introduction of virtual inertia and power electronics control for frequency support, the paradigm of what is a power system is changing. Thus, the existing SFR models must be updated to include the influence of VSWTGs with frequency support capabilities.

In [11], reduced-order modelling of a doubly fed induction generator (DFIG) has been exploited to develop a wind farm controller for primary frequency response by pitch-based deloading. Similar work has been carried out by the same authors in [12]. Although works [11, 12] present a quality model order reduction by taking into account the actual number of wind turbines in a wind farm, the emphasis was on controller design and prediction of wind farm output change, not on SFR modelling for studying power system frequency changes. Moreover, only primary frequency control by pitch-based deloading was taken into account; wind turbines operate and are controlled differently during different wind conditions. No analysis was done on the influence of wind turbine parameters on frequency response. Similarly, reduced order modelling for choosing parameters of composite inertial control has been carried out in [13]: here, the symbolic form of a transfer function relating the change in system frequency to the change in rotor speed has been developed and analysed, and how. Symbolic form of an SFR model of a DFIG for different wind turbine generator power output has been taken into account in [17]. However, different operating regimes were not taken into account and a single control strategy was assumed. Besides, the presented transfer functions were also given in a numerical form so it is not clear which parameters influence the frequency response and how. Symbolic form of an SFR model of a DFIG for different
operating regimes has been presented in [18], but no discussion on the impact of inherent wind turbine parameters on frequency response has been provided. Also, coordinated control of rotor speed and pitch angle has not been taken into account and there was little discussion on the model accuracy. Hu et al. [10] have proposed an SFR modelling approach of a DFIG with inertia controller by considering the internal voltage of the machine to characterise the power exchange with the grid for constant wind speed below the rated set-point. Research about the impact of DFIG virtual inertia on power system small-signal stability has been conducted in [19] and consequently model-order reduction to facilitate small signal stability studies has been carried out in [20]. However, the emphasis was on phase-locked loop and virtual inertia controller impact on electromechanical modes of the power system, not on SFR which is even more simplified. In [21], small-signal model of a DFIG with droop control has been developed to investigate microgrid frequency stability, comparison of torque-based and power-based droop and standalone operation of a DFIG to stabilise frequency in a microgrid. As presented in the literature survey, there is a lot of headroom for low-order SFR modelling of VSWTGs because the existing work is not comprehensive or has been focused solely on DFIGs. Little to no discussion has been done on the relation of wind turbine parameters to low-order model time constants.

In this paper, novel low-order SFR model of a future power system with a significant amount of VSWTGs with natural response and primary frequency support capabilities is developed. This paper is organised as follows: In Section 2, methodology behind developing low-order SFR model is presented. In Section 3, low-order SFR model of a VSWTG is developed by taking into account different operating regimes according to the instantaneous wind speed. The low-order model is presented in its symbolic form and compared to the non-linear model for validation. Section 4 discusses the model accuracy with accompanying analysis. In Section 5, the influence of different wind turbine parameters on frequency response is analysed. Finally, the paper is concluded followed by a note about potential future research.

The main contributions of this paper are as follows:

- Symbolic form of a low-order model of VSWTGs for different wind speed regions.
- Analysis of low-order VSWTG model accuracy with respect to the disturbance size and operating point.
- Analysis of influence of wind turbine parameters on SFR.

2 Methodology

In this paper, the phenomena of interest are the electromechanical transients, thus the models of wind turbine generating systems will include only fundamental frequency components, as is practice in power system dynamics simulation [22]. Modelling of VSWTGs for power system dynamics simulations has been thoroughly covered in [22–25] and the VSWT non-linear models which will be used for validation of the low-order model are developed accordingly. Generator equations describing the partially and fully decoupled wind turbines are given in [25]. The main assumptions [25] on which the non-linear model in this paper is developed for power system stability studies are as follows:

- Stator transients in the generator equations are neglected which is routinely done for power system stability studies.
- In a DFIG system, the rotor transients are neglected. Otherwise, detailed representation of power electronic converter is necessary which would significantly reduce the time step of the simulation and includes electromagnetic phenomena well outside the area of interest.
- Similarly, in a full-scale converter (FSC) system, the avoidance of modelling power electronic converter is achieved by neglecting stator transients as well as rotor transients.
- Power electronic converters are replaced by voltage and current control loops.
- Generator equations become algebraic equations.

Theoretical considerations as well as experiments have shown that in both types of VSWTGs, the power electronic converters act very fast and the new setpoint is reached almost instantaneously (within 10 ms or less) [25] and the dynamics of rotor-side and grid-side control can be neglected [10, 19]. In the SFR studies with low-order models (electromechanical time scale), we are interested in the power balances and governor responses and not in the fast electromagnetic transients in the power electronics. Following this assumptions, the power set-point reference from the speed controller is used instead of the actual generator power output later in the paper. By taking into account generator rotor speed control for different wind speeds, low-order SFR model of VSWTGs will be developed and compared to the non-linear model (Fig. 1) to validate its accuracy in the MATLAB-Simulink environment. Equations describing the mechanical power of a wind turbine, generator rotor speed controller and pitch angle controller are identical between Types III and IV wind energy conversion system (WECS), again as described in [22–25]. The only difference is in the equations that describe how the power is drawn from the generator. General non-linear model of a VSWT is shown in Fig. 1, which consists of aerodynamic part, lumped-mass mechanical model, generator rotor speed control, generator/converter model and pitch angle controller (see Table 1 for wind turbine data used in this paper).

3 Deriving the new low-order SFR model

In this section, low-order SFR model of a VSWTG will be developed for integration into existing power system SFR model while respecting the different operating modes of a wind turbine. The main goal is to incorporate the influence of WPPs with frequency support capabilities on SFR in a simple manner: capturing the most important dynamics of frequency response from WPPs by using a small number of parameters. This facilitates studying power system frequency changes without the need for modelling complex electro-mechanical structures like a wind turbine generator.

Wind turbines have different operating regimes depending on the instantaneous wind speed conditions, thus different controls and dynamics play into account. According to the literature, these operating regimes can be generally divided into four zones as

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**Fig. 1 General non-linear model of a VSWT**

**Table 1 Wind turbine data**

| Parameter                | Value   |
|--------------------------|---------|
| Rotor diameter           | 75 m    |
| Nominal power            | 2 MVA   |
| Minimal rotor speed      | 9 rpm   |
| Nominal rotor speed      | 18 rpm  |
| Nominal wind speed       | 12 m/s  |
| $C_p(\lambda, \beta)$ expression | from [25] |
| DFIG and FSC generator parameters | from [25] |
shown on the generator power versus rotor speed diagram in Fig. 2. The top curve in Fig. 2 corresponds to the normal operation, while the bottom curve corresponds to the deloaded operation which in this case ensures a 10% power margin. Zone boundaries can be chosen arbitrarily between the minimal and maximal speed, but the elementary idea behind the power versus speed control law stays the same. Different zones will be addressed in the following subsections.

The general structure of a low-order SFR model of a future power system is shown in Fig. 3 where conventional generation consists of steam reheat, hydraulic and gas turbine-governors. Simplified steam and hydraulic turbine-governor models are explained in [26], while the gas turbine is simplified by including only the valve positioning, fuel system and compressor discharge time constants ($T_{v}$, $T_{f}$ and $T_{d}$, respectively) and neglecting temperature and acceleration control. Capacity share of each turbine is described by factors $d_{d}$, $d_{h}$ and $d_{g}$, 1/$R_{WTG}$, and $K_{V1}$ are supplementary frequency support loop droop gain and virtual inertia constant, respectively. $G(s)$ is the VSWTG transfer function representing the VSWTG low-order model. Share of WPPs in the total capacity is $d_{w}$, while $d$ is the reduction of system inertia (both in p.u.).

### 3.1 Operation at low wind speeds – zone I

During low wind speeds, the generator rotor speed is usually held constant at its minimum to limit the slip (therefore the rotor voltage), as well as to prevent the excitation of resonant frequency of the tower [27]. To prevent power fluctuations around the minimal speed, the power curve characteristic shown in Fig. 2 was chosen as such that power changes linearly with speed. Hence, the generator power versus rotor speed curve is defined by a line between minimal rotor speed at cut-in wind speed ($\omega_{min}$) and the rotor speed $\omega_{mppt,max}$ where the maximum power point tracking (MPPT) or deloaded operation starts. It is not unreasonable to assume that VSWTGs will not participate in any frequency support in this region because extra active power injection to the grid will slow down the turbine rotor which could ultimately lead to stalling of the turbine. Therefore, in this case, the WPPs will provide some active power to the grid, but are not sensitive to changes in grid frequency ($G(s) = 0$). It can be argued that a portion of online wind turbines with frequency support capability during low wind speeds will have the same effect as the portion of online wind turbines without frequency support capability during any wind speed: the total amount of system inertia will be reduced due to the power electronic decoupling. The total grid inertia of a conventional power system can be calculated as

$$H_{SYS} = \frac{\sum_{i} n_{i} H_{S,i}}{\sum_{i} n_{i}} = \frac{\sum_{i} n_{i} H_{S,i}}{S_{SYS}} \tag{1}$$

where $H_{SYS}$ and $S_{SYS}$ are the total grid inertia constant and total system generation capacity connected to the grid, respectively. $H_{S,i}$ and $S_{i}$ are the inertia constant and base apparent power of $i$th conventional generator, respectively, and $n_{i}$ is the total number of conventional generators in the system. If an $x$ number of conventional generators (corresponding to some percentage $d$ of total kinetic energy connected to the grid and percentage $d_{w}$ of total capacity connected to the grid) gets displaced by an equal share of WPPs $S_{WPP}$, the grid inertia constant can be calculated as

$$H_{SYS} = \frac{\sum_{i} n_{i} H_{S,i}}{\sum_{i} n_{i} - d S_{SYS} + S_{WPP}} = \frac{1 - d}{(1 - d)S_{SYS} + S_{WPP}} \tag{2}$$

$$= \frac{1 - d}{S_{SYS} + S_{WPP}} \tag{3}$$

$$= (1 - d)\frac{\sum_{i} n_{i} H_{S,i}}{S_{SYS}} \tag{4}$$

Thus, it can be concluded that if a $d_{w}$ percentage of conventional units is displaced by an equal amount of WPPs, the grid inertia constant is reduced by the percentage $d$. Concurrently, the share of conventional generation is reduced by the amount $d_{w}$ as well. Displacement of kinetic energy $d$ does not have to correspond to the displacement of capacity $d_{w}$ because individual units in a grid can have different inertia constants and rated apparent powers. A more general case is when the share of WPPs $S_{WPP}$ is not equal to the amount displaced per ($\Delta d$). Therefore, since the VSWTGs do not participate in frequency support in this operating region, active power injection $\Delta P_{WPP}$ from Fig. 3 is 0. More precisely, the whole branch that represents the low-order SFR model of a VSWTG does not exist.

To prove this hypothesis, a modified IEEE 39-bus New England test system has been modelled in DlgSILENT PowerFactory (Fig. 4). All the synchronous generators are equipped with AVR's and turbine governors (see Table 2). Turbine-governors parameters are shown in Table 3. The response of a detailed system has been compared to the low-order SFR model created in MATLAB-Simulink for two cases: 0% wind penetration and ~20% wind penetration for a step load increase of 2.1% (on total installed capacity base). The displaced synchronous generators are G2, G4, G6 and G9 which account for 16.66% of kinetic energy in the grid. System damping $D$ is set to 0. Simulation results are shown in Fig. 5a.

Although the detailed model in DlgSILENT PowerFactory shows inter-machine oscillations and the impact of AVR’s, which are not captured by the low-order SFR model in MATLAB-Simulink, the low-order model adequately describes the frequency behaviour which can be seen if one compares the RoCoF nadir and steady-state error of the two curves.

### 3.2 MPPT operation at medium wind speeds – zone II

In this region, the wind speed becomes high enough and the rotor speed is controlled to maximise the aerodynamic efficiency by keeping an optimal tip-speed ratio. The generator power varies proportionally with the cube of the rotor speed. In this case, the upper cubic curve between rotor speeds $\omega_{mppt,min}$ and $\omega_{mppt,max}$ corresponds to the MPPT operation. It is assumed that the wind conditions are good enough for the provision of frequency support by VSWTG. If only inertial response is required, there is no need for deloaded operation because the generator can be temporarily overloaded and the rotor speed will recover to the initial value after RoCoF diminishes. However, it cannot be used for a constant power increase because the rotor speed will decrease permanently, and with that the generator power. A large enough disturbance could lead to the stalling of the turbine. Hence, if a constant power reserve is necessary, the VSWTG can be operated in deloaded mode which is shown by the bottom cubic curve in Fig. 2 (in this case, corresponding to the 10% power reserve).

For this purpose, the VSWTG will operate according to the deloaded curve to ensure a power reserve for primary frequency support alongside with the natural inertial response. Pitch angle is kept at $0^\circ$ to maximise aerodynamic efficiency. The VSWTG behaviour is described as

$$\frac{d\alpha_{g}}{dt} = \frac{P_{g} - P_{e}}{2H_{WTG}O_{g}^\beta} \tag{5}$$

$$P_{in} = \frac{0.5 \rho R^2 v_w^3 C_{p}(4,0)}{S_{WTG,t}} \tag{6}$$

$$P_{e} = P_{e,ref} + k_{f} \omega_{g}^\lambda + \delta P_{e} \tag{7}$$

where the variables are as follows: $\alpha_{g}$, $P_{g}$ and $P_{e}$ are the generator speed, mechanical and electrical power, respectively (all in p.u.). $H_{WTG}$ is the inertia constant of the VSWTG in s. $\rho$ is the air density of 1.225 kg/m³, $R$ is the rotor radius in m, $v_{w}$ is the wind speed in m/s, $C_{p}(4,0)$ is the dimensionless aerodynamic coefficient with $\lambda$ being the tip-speed ratio, $S_{WTG,t}$ is the rated generator apparent...
power in MVA, \( k_{\text{del}} \) is the coefficient of the deloaded power curve in p.u. and \( \Delta P_{\text{ref}} \) is the change of power set-point from the supplementary frequency response control circuit. The generator electrical power output \( P_e \) is equal to the generator power set-point from the rotor speed control \( P_{e, \text{ref}} \) (see Fig. 1) because the new set-point is reached almost instantaneously due to the fast action of power electronic converters.

For the moment, the wind speed will be considered constant \( (v_o) \) and (6) and (7) are linearised around the initial operating point \((\omega_0, \beta_0) = (0) \) with the Taylor series expansion neglecting the higher order terms. After setting the state variable \( x = \omega_g \), input variable \( u = \delta P_{\text{ref}} \), and output variable \( y = P_e \), the state-space model can be written as

\[
\Delta x = Ax + Bu
\]

\[
\Delta y = Cx + Du
\]

where coefficients \( A, B, C, D \) are as follows:

\[
A = \frac{\partial \omega_g}{\partial P_{\text{ref}}} \frac{\partial P_{\text{ref}}}{\partial \omega_g} \frac{\partial x}{\partial \delta P_{\text{ref}}} - 2k_{g \omega \delta} \omega_0
\]

\[
B = \frac{1}{2H_{\text{WTG}} \omega_0}
\]

\[
C = 3k_{g \omega \delta} \omega_0^2
\]

\[
D = 1.
\]

The low-order SFR model of a VSWTG during medium wind speeds in Zone II is described by the transfer function (14), where \( \omega_0 \) is the initial generator speed, \( \alpha_1 = \frac{\partial \omega_g}{\partial P_{\text{ref}}} \frac{\partial P_{\text{ref}}}{\partial \omega_g} \frac{\partial x}{\partial \delta P_{\text{ref}}} \). The nature of the frequency response depends on the VSWTG inertia constant, the gradient of the deloaded curve and the initial conditions (initial generator speed which depends on the wind speed and a constant which is equal to the derivative of the wind turbine torque at the initial operating point).

\[
G(s) = \frac{2H_{\text{WTG}} \omega_0 \delta}{2H_{\text{WTG}} \omega_0 \delta - (k_{g \omega \delta} \omega_0 + \alpha_0 \omega_0)}
\]

### 3.3 Operation at medium wind speeds – zone III

Generally, generator power cannot follow the MPPT curve all the way to the nominal power without violating the upper rotor speed limit which is usually around 1.2 p.u. of synchronous speed. Therefore, if abrupt changes of generator power were to be avoided around the maximal rotor speed, a linear law can be introduced between the arbitrary speed \( \omega_{\text{max}} \) and \( \omega_{\text{max}} \) to bring the generator smoothly to the nominal power output.

In this region, the desired deloading may not be possible to reach by over-speed alone. Hence, it is coordinated with the pitch angle control to bring the VSWTG to a stable operating point. However, neither the deloaded curve nor the normal curve from Zone III will be used to provide the deloaded power set-point. The actual control law will be defined by the slope between points A and B (see Fig. 2) which is what is usually done for primary frequency response. This control strategy has been well explained in [28, 29]. Points \( (P_A, P_B, \omega_A, \omega_B) \) are obtained from the current wind speed and desired amount of deloading. The generator output power is described as

\[
P_e = P_A + \frac{(P_B - P_A)}{\omega_B - \omega_A} (\omega_B - \omega_A) + \delta P_{\text{ref}}
\]

Finally, the new stable operating point is reached when the mechanical power is equal to the electrical power. Therefore, pitch angle control completes this action and the new operating point C is reached. Most common structure of a pitch angle controller is shown in Fig. 1, where \( \omega_{\text{ref}} \) is the reference generator speed, \( \beta_{\text{ref}} \) is the reference pitch angle in degrees, \( T_s \) is the pitch servomechanism time constant in s. The reference rotor speed to complete the coordinated control of rotor speed control and pitch angle control is obtained as [30]

\[
\omega_{\text{ref}} = \omega_A + \frac{(\omega_B - \omega_A)}{P_B - P_A} \delta P_{\text{ref}}
\]
Table 2  IEEE 39-bus New England test system generator data for Zone I simulations

| Generator ID | Power plant type | $H_i$, s | $S_{i,n}$, MVA | AVR model | Turbine-governor model |
|--------------|------------------|----------|----------------|-----------|-----------------------|
| G1           | steam            | 5.000    | 10,000         | IEEET1    | TGOV1                 |
| G2           | hydro            | 4.329    | 700            | IEEET1    | IEEEG3                |
| G3           | hydro            | 4.475    | 800            | IEEET1    | IEEEG3                |
| G4           | hydro            | 3.575    | 800            | IEEET1    | IEEEG3                |
| G5           | hydro            | 4.333    | 300            | IEEET1    | IEEEG3                |
| G6           | hydro            | 4.350    | 800            | IEEET1    | IEEEG3                |
| G7           | Gas              | 3.771    | 700            | IEEET1    | GAST2A                |
| G8           | Gas              | 3.471    | 700            | IEEET1    | GAST2A                |
| G9           | Gas              | 3.450    | 1000           | IEEET1    | GAST2A                |
| G10          | Gas              | 4.200    | 1000           | IEEET1    | GAST2A                |

Table 3  Simulation parameters

| Wind power plant | $H_{WT}$, s | $1/R_{WT}$ | $K_{VI}$ | $K_p$ | $K_i$ |
|------------------|-------------|-------------|----------|-------|-------|
|                  | 3           | 20          | 6        | 150   | 50    |

| Steam reheat power plant | $T_8$, s | $F_8$, s | $K_d$ | $T_g$, s |
|--------------------------|----------|----------|-------|---------|
|                          | 8        | 0.3      | 20    | 0.3     |

| Gas power plant | $T_{vp}$, s | $T_{w}$, s | $T_{cd}$, s | $K_d$ | $T_g$, s |
|-----------------|--------------|-------------|--------------|-------|---------|
|                 | 0.1          | 0.4         | 0.4          | 25    | 0.1     |

| Hydro power plant | $T_w$, s | $T_2$, s | $T_1$, s | $T_4$, s | $K_{ht}$ |
|-------------------|----------|----------|----------|----------|---------|
|                    | 0.75     | 0.204    | 10       | 61.25    | 25      |

| System | $H_{SYS}$, s | $D$ | $d_w$ | $d$ | $d_w$, $d_h$, $d_l$ |
|--------|--------------|-----|-------|----|-------------------|
|        | 4            | 0.3 | 0.2   | 0.15 | 0.8, 0, 0            |
The pitch angle controller introduces two new states to the model, which are described as
\[
\frac{d\beta_1}{dt} = \alpha_\beta - \alpha_{\text{ref}}
\]
\[
\frac{d\beta_2}{dt} = \frac{1}{T_s} \{K_p (\alpha_\beta - \alpha_{\text{ref}}) + K_i (\beta_1 - \beta)\}
\]
where \(\beta_1\) is the output of the integral part of the PI controller. (Fig. 1) in degrees, \(K_p\) and \(K_i\) are proportional and integral gain of the PI controller.

The wind turbine behaviour in Zone III is now completely described in (5), (6), (15)–(18). The state vector is now:
\[
x = \begin{bmatrix} \alpha_\beta \\ \beta \end{bmatrix}
\]
(19)

Input variable stays the same (\(\delta P_{\text{WTG}}\)) and output variable \(y\) is now defined in (15). After the first-order Taylor series expansion around initial operating point, the state-space model defined in (8) and (9) can be written. Equations (20)–(23) describe matrices \(A\), \(B\), \(C\) and \(D\)
\[
A = \begin{bmatrix} a_{11m} - a_{11e} & 0 & a_{13} \\ \frac{-1}{2H_{\text{WTG}}} & 0 & \frac{a_{13}}{2H_{\text{WTG}}} \\ 1 & 0 & 0 \end{bmatrix}
\]
(20)
\[
B = \begin{bmatrix} -\frac{b_1}{2H_{\text{WTG}}} \\ -b_2 \\ -\frac{K_p b_2}{T_s} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2H_{\text{WTG}} \alpha_\beta} \\ \frac{-a_{13} - a_0}{P_b - P_i} \\ \frac{K_p \alpha_0 - a_0}{P_b - P_i} \end{bmatrix}
\]
(21)
\[
C = \begin{bmatrix} \frac{1}{b_2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{P_b - P_1}{\alpha_0 - \alpha_0} \\ 0 \\ 0 \end{bmatrix}
\]
(22)
\[
D = 1.
\]
(23)

where coefficients \(a_{11m}, a_{11e}\) and \(a_{13}\) are defined in
\[
a_{11m} = \alpha_{\text{ref}} \left. \frac{\partial (P_m)}{\partial \alpha_\beta} \right|_{\alpha_0} \quad a_{11e} = \alpha_{\text{ref}} \left. \frac{\partial (P_m)}{\partial \alpha}\right|_{\alpha_0}
\]
(24)
\[
a_{13} = \alpha_{\text{ref}} \left. \frac{\partial (P_m)}{\partial \beta} \right|_{\alpha_0}
\]
(25)

The low-order SFR model of a VSWTG during medium wind speeds in Zone III is described by the transfer function (26). \(H_{\text{WTG}}\) is used instead of \(H_{\text{WTG}}\) for brevity. It can be seen that the symbolic form of the transfer function is significantly more complex in this case due to the increased number of states and the interactions between different VSWTG components. (see (26)).

\[
G(s) = \frac{2HT_s s^3 + (a_{11m} T_s - a_{11m} T_s + 2H - (h/b)T_s) s^2 + (a_{11m} - a_{11e} - a_{13} - 2a_i K_p - (h/b)) s - 2a_i K_i}{2HT_s s^3 + (a_{11m} T_s - a_{11m} T_s + 2H) s^2 + (a_{11m} - a_{11e} - a_{13} K_p) s - a_i K_i}
\]
(26)

\[
G(s) = \frac{2HT_s s^3 + (2H - a_i T_s - (T_{\text{ref}} \alpha_0) T_s) s^2 - (a_i + a_i K_p) s - a_i K_i}{2HT_s s^3 + (2H - a_i T_s) s^2 - (a_i + a_i K_p) s - a_i K_i}
\]
(33)

3.4 Operation at high wind speeds – Zone IV

Finally, in this region, the rotor speed has reached the upper speed limit \(\omega_{\text{max}}\) when operating according to the normal curve. Therefore, deloading cannot be achieved by increasing the rotor speed anymore. The only way to obtain a power reserve is solely by pitch angle control. Normally, the electrical torque is held constant at nominal value [27], but since the VSWTG operates in deloaded mode, the torque is held constant at the value \(T_{\text{e},0}\) that corresponds to the deloaded power set-point. Consequently, deloaded electric power set-point varies linearly with the rotor speed. The rotor speed is controlled to keep it at its upper limit because the rotor speed set-point is equal to the upper speed limit. When grid frequency drops, the generator power set-point will be increased the same way as described in the previous subsections. Then, the pitch angle controller will decrease the pitch angle to inject extra active power to the grid. This operating mode differs from the operation in Zone III only by the expressions for generator electrical power (15) and rotor speed reference (16), which are now defined as
\[
P_e = T_e \delta \omega_0 + \delta P_{\text{WTG}}
\]
(27)
\[
a_{\text{ref}} = a_{\text{max}}
\]
(28)

The wind turbine behaviour in Zone IV is now completely described in (5), (6), (17), (18), (27) and (28). State vector and input variable \(\delta P_{\text{WTG}}\) stay the same and output variable \(y\) is now defined in (27). State-space model matrices \(A\), \(B\), \(C\) and \(D\) are now:
\[
A = \begin{bmatrix} \frac{-1}{2H_{\text{WTG}}} & 0 & \frac{a_{13}}{2H_{\text{WTG}}} \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]
(29)
\[
B = \begin{bmatrix} \frac{-b_1}{2H_{\text{WTG}}} \\ -b_2 \\ -\frac{K_p b_2}{T_s} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2H_{\text{WTG}} \alpha_\beta} \\ \frac{-a_{13} - a_0}{P_b - P_i} \\ \frac{K_p \alpha_0 - a_0}{P_b - P_i} \end{bmatrix}
\]
(30)
\[
C = \begin{bmatrix} T_{\text{e},0} \\ 0 \\ 0 \end{bmatrix}
\]
(31)
\[
D = 1.
\]
(32)

Transfer function (33) represents the low-order SFR model of a VSWTG during high wind speeds in Zone IV (see (33)).

3.5 Validation of low-order SFR model

The developed low-order SFR model has been validated against the non-linear model for Zones II, III and IV in a steam-wind system \((d_0 = d_\delta = 0)\) single-machine representation from Fig. 3. The response from the non-linear model (Fig. 1) is compared to the linearised model in MATLAB-Simulink environment for a 2.5% load increase in a system with 20% WPP as shown in Fig. 3. Simulation parameters from Table 3 have been used. Simulation results are shown in Figs. 5b–d which show that the low-order SFR model adequately describes the behaviour of the non-linear model during the active power injection for frequency support. Since the operation in Zone I is characterised just as an inertia reduction,
On the other hand, the low-order VSWTG model depends on the initial conditions as well, i.e. the instantaneous wind speed. This can be noticed in (14), (26) and (33) in which the coefficients of both the numerator and denominator depend on the values that are obtained from the initial conditions (\(a\), \(b\), \(a_0\), \(T_{c}\)). Therefore, to analyse the impact of initial conditions on the dynamics of the active power injection and, consequently, the grid frequency response, low-order models have been calculated for a wide range of wind speeds from Zones II to IV. Fig. 7 shows the low-order model output power change for a 2.5% step increase in load for different initial wind speeds.

The following conclusions have been drawn: initial wind speed does not have a substantial effect on the frequency disturbance response while operating in Zone II. Nonetheless, it can be observed that less extra power is provided when the initial wind speed is higher. This can be explained by the fact that the lower initial wind speed translates to lower initial rotor speed. Consequently, the denominator in the ratio of the free terms from (14) \((k_{\phi}a_0^2 + a_0\omega_0)/(a_0\omega_0 - 2k_{\phi}\omega_0)\) falls quicker with the slower rotor speed than the numerator, thus increasing the overall transfer function gain. Similar behaviour can be observed for operation in Zone IV, but Fig. 7c shows that the response is effectively insensitive to the initial conditions. On the other hand, operation in Zone III shows the largest sensitivity to the initial conditions. The overshoot is higher for higher wind speeds, but it does not affect the steady state. This is a reflection of the implemented control action and not of any physical phenomenon. It can be seen from Fig. 2 that as the wind speed increases, the point B is sliding towards the nominal power. It is obvious that if the power margin is kept constant, the linear slope AB will increase. This slope can become really steep close to the nominal wind speed which will increase the gain of the transfer function and may even become unstable at some point if pitch control cannot complete the control action. This can be mitigated by keeping the slope constant through the whole region or lowering it when the operating point is getting closer to the rated power. However, the unavoidable trade-off then is the reduced power margin when approaching the nominal wind speed.

### 5 Impact of VSWTG parameters on frequency response

In this section, the influence of \(H_{\text{WTG}}\), \(R_{\text{WTG}}\), \(K_{\text{c}}\), and \(T_{c}\) on SFR will be analysed. Since there is no frequency support from VSWTGs in Zone I, the simulations are only conducted for Zones II, III and IV. For simplicity, only steam-wind system is considered \((d_0 = d_1 = 0)\) in a single-machine representation from Fig. 3. Simulation parameters used are according to Table 3 and will be held constant, except for the one that is currently being analysed. The SFR is observed for a 0.15 p.u. step increase in load at \(t = 0.5\) s.

#### 5.1 Impact of \(H_{\text{WTG}}\) on SFR

The impact of wind turbine inertia constant on SFR is shown in Fig. 8. It can be seen that the wind turbine inertia constant does not have a significant impact on SFR in Zones II and IV (due to the power electronic decoupling). However, in Zone III, the generator rotor speed control algorithm has such an effect that heavier wind turbines \((H_{\text{WTG}})\) will have a slower response which can be noticed by lower frequency nadir in Fig. 8b.

#### 5.2 Impact of \(R_{\text{WTG}}\) on SFR

The impact of droop gain \((1/R_{\text{WTG}})\) from supplementary frequency control on SFR is shown in Fig. 9. It can be seen that the greater the droop gain, the smaller the frequency nadir and steady-state error. However, the impact of droop gain on steady-state error is not as significant for operation in Zone II because the generator rotor speed control in this region is such that it will have a large power output immediately after a disturbance, but the steady-state...
power output will not be that much higher from the initial power output before the disturbance (as seen in Fig. 5b).

5.3 Impact of $K_{VI}$ on SFR

The impact of virtual inertia constant from supplementary frequency control on SFR is shown in Fig. 10. It can be seen that the greater the virtual inertia constant the smaller the RoCoF. The impact of virtual inertia constant on frequency nadir is significant only in Zone III. It does not have any effect on steady-state frequency error because RoCoF diminishes after a certain amount of time.

5.4 Impact of $T_s$ on SFR

The impact of pitch angle controller servomechanism time constant on SFR is shown in Fig. 11. Since pitch angle is only active during operation in Zones III and IV, SFR for only those two regions is shown. It can be seen that the pitch servo time constant has significant impact on frequency response in Zone III in the form that slower pitch control will have bigger oscillations. It does not, however, have any impact on operation in Zone IV. The reason for this is that the speed reference for pitch angle control in Zone III depends on the change in generator power set-point as defined in (16), while the speed reference in Zone IV is held constant as defined in (28).

6 Conclusion

In this paper, we have presented an approach for low-order modelling of a VSWTG with primary frequency control and natural inertial response. The developed model takes into account different operating regimes dependent on the initial wind speed. The model has been presented by the transfer functions in their symbolic form to clearly show which parameters determine the time constants of a WPP. It has been shown that the low-order model adequately describes the non-linear model for up to 10% reserve power margin. Simulations have shown that initial conditions do not have a significant impact on the frequency response, except in Zone III which is a consequence of the control design and keeping the power margin constant during higher wind speeds. The influence of the wind turbine parameters on the frequency response has been analysed and discussed. The influence of mechanical parameters such as wind turbine inertia and pitch servo time lag is insignificant due to the decoupling effect of power electronic interface. However, significant impact on SFR comes from auxiliary frequency controller and its parameters, as well as the converter-generator control. It can be deduced that these different wind speed regions are reflected to the WECS design as certain generator control actions which determine the generator power output. The main challenge of developing the low-order model of VSWTGs is the aforementioned variety of generator control actions for different operating conditions and the dependence on initial conditions which means that currently there is no unique, one-size-fits-all low-order model that can adequately capture VSWTG frequency response. To add to the fact, frequency support from VSWTGs is still an area with an ongoing research and there are other frequency support strategies, not taken into account in this paper (e.g. variable droop, constant inertial response etc.), that shape the frequency response and change the WECS equations from which the low-order model is derived. Therefore, there is still a lot of research potential in this area to try to capture the most significant dynamics of VSWTG frequency response in order to facilitate studying frequency dynamics of power systems with high penetration of WPPs. Finally, some potential future research includes: investigation of impact of converter control parameters and phase-locked loop parameters on system frequency response, developing low-order models for other frequency support strategies; a more in-depth analysis of transfer function coefficients and normalising them to obtain a numerical range of typical values independent of exact WECS design, introduce the wind speed perturbations as another input variable, applying more advanced techniques of aggregated modelling of wind farms instead of lumped representation.

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