THE HOLOGRAPHIC QUANTUM

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Abstract We present a holographic–like map of standard quantum mechanics (the bulk theory) onto a dual theory, that of the classical thermodynamics of irreversible processes (the boundary theory).

1 Introduction

The holographic principle \cite{2, 28, 29} has permeated wide areas of theoretical physics over the last twenty years. Stepping outside its initial quantum–gravity framework, it reached string theory \cite{19, 31} as well as more established domains such as QCD \cite{16}, and condensed matter theory \cite{13}, to name but a few.

Another theoretical development of recent years is the recognition that gravity arises as an emergent phenomenon \cite{22, 23, 30}, a fact that has far–reaching consequences for our understanding of spacetime. Added to the dissipative properties already known to be exhibited by gravity \cite{14, 23, 26, 27}, this opens the gate to the application of thermodynamics to nonthermal physics. Indeed, thermodynamics is the paradigm of emergent theories. It renounces the knowledge of a vast amount of detailed microscopic information, keeping just a handful of macroscopic variables such as volume, pressure and temperature—sufficient to state robust physical laws of almost universal applicability. These macroscopic variables are coarse–grained averages over the more detailed description provided by some underlying, microscopic degrees of freedom. Which brings us to yet another theoretical breakthrough of recent times that is worthy of mention: the notion of emergence \cite{6}.

The property of emergence has been postulated not only of gravity, but also of Newtonian mechanics \cite{30} and of quantum mechanics \cite{8, 12, 15}: a key concept here is that of an entropic force. Equipped with thermodynamical tools as befits any emergent theory, we have in refs. \cite{9, 10} developed a framework that maps semiclassical quantum mechanics onto the classical thermodynamics of irreversible processes in the linear regime, the latter as developed by Onsager, Prigogine and collaborators \cite{21, 25}. Within this framework, the statement sometimes found in the literature, quantisation is dissipation \cite{3}, can be given a new interpretation.

Altogether we realise the simultaneous presence, in the quantum theory of a finite number of degrees of freedom, of all the elements mentioned above: dissipation, emergence and holography. It is on this latter element, holography in quantum mechanics, that we intend to concentrate on in this letter.
2 The thermodynamical theory on the boundary

Let a mechanical system be described by a Lagrangian function \( L = L(q_i, \dot{q}_i) \). For simplicity we assume our configuration space to be \( \mathbb{R}^D \); an additional \( \mathbb{R} \) stands for the time axis. The time variable \( t \), initially real, will be complexified presently. Rather than using natural units, we will explicitly retain Planck’s constant \( \hbar \) and Boltzmann’s constant \( k_B \) in our expressions, in order to better highlight the holographic properties of the concepts appearing here. In particular, the role that \( \hbar \) plays on the mechanical side of our correspondence will be played by \( k_B \) on the thermodynamical side. If we were to set \( \hbar = 1 = k_B \), the fact that they are counterparts under our correspondence [7][17] would be somewhat obscured.

We will equate certain spacetime concepts (on the left–hand side of the equations below) to certain thermodynamical quantities (on the right). To begin with, we observe that the two physical constants \( \hbar \) and \( k_B \) allow one to regard time \( t \) and temperature \( T \) as mutually inverse, through the combination

\[
\frac{1}{t} = \frac{k_B}{\hbar}T.
\]  

(1)

Admittedly, this observation is not new [4].

Corresponding to the mechanical system governed by the Lagrangian \( L(q_i, \dot{q}_i) \) there will be a thermodynamical system whose dynamics will be governed by an entropy \( S \). We postulate the following differential relation between the two of them:

\[
\frac{1}{\hbar}Ldt \leftrightarrow \frac{C}{k_B}dS, \quad C \in \mathbb{C}.
\]  

(2)

The symbol \( \leftrightarrow \) stands for correspondence, in the sense of the map to be established below between quantum mechanics and irreversible thermodynamics. In what follows we will replace the symbol \( \leftrightarrow \) with an equality sign. Again, dimensionality arguments basically fix the two sides of the above relation, but leave room for a dimensionless number \( C \), that we will soon see must be complex. Now Eq. (2) overlooks the fact that the right–hand side contains the exact differential \( dS \), while the differential \( Ldt \) on the left–hand side is generally not exact. In other words, while there exists a well–defined entropy function \( S = \int dS \), the line integral \( I = \int Ldt \) generally depends on the trajectory in \( \mathbb{R}^D \) being integrated along.

The mechanical action \( I \), however, can define a path–independent function of the integration endpoint if we restrict to a certain class of trajectories in \( \mathbb{R}^D \). Let us see how this comes about. Let \( V = V(q_i) \) be the potential function of the mechanical system under consideration. The equation

\[
V(q_i) = \text{const}
\]  

(3)

defines, as the constant on the right–hand side is varied, a family of \((D−1)\)–dimensional, equipotential submanifolds of \( \mathbb{R}^D \). An elementary example, when \( D = 3 \), is the case of the Newtonian potential generated by a point mass located at the origin \( O \). Then the above family of equipotential surfaces is a family of concentric spheres \( S_\rho \) of increasing radii \( \rho > 0 \), all centred at \( O \). This family of equipotentials, singular only at
defines a foliation of \( \mathbb{R}^3 - \{O\} \), so the latter space equals the union \( \cup_{\rho > 0} S_{\rho} \) of all leaves \( S_{\rho} \). This foliation can also be used to define a coordinate system on \( \mathbb{R}^3 - \{O\} \). Namely, one splits \( \mathbb{R}^3 - \{O\} \) into 2 tangential directions to the spheres of the foliation, and 1 normal direction. For example, the standard spherical coordinates \( \rho, \theta, \varphi \) centred at \( O \) qualify as such a coordinate system, \( \rho \) being the normal coordinate and \( \theta, \varphi \) the tangential coordinates.

Returning now to the general case when both \( D \) and \( V(q_i) \) are arbitrary, Eq. (3) defines, for each particular value of the constant on the right–hand side, one equipotential leaf \( L_n \) of a foliation \( \cup_n L_n \) of \( \mathbb{R}^D \). Here the subindex \( n \) stands for a certain (local) coordinate \( n \) on \( \mathbb{R}^D \) that is normal to all the leaves. The \( D - 1 \) tangential coordinates thus span the \( (D - 1) \)-dimensional leaves \( L_n \), each one of them being located at a specific value of the normal coordinate \( n \). We will assume that all the leaves \( L_n \) are compact.

Trajectories within \( \mathbb{R}^D \) that run exclusively along this normal coordinate \( n \), thus being orthogonal to the leaves, are such that the action integral \( I \) does defines a function \( I_n \) of the integration endpoint; the subindex \( n \) reminds us of the restriction to these normal trajectories. Independence of path is merely a consequence of the 1–dimensional normal directions to the equipotential leaves \( L_n \). This is the particular class of trajectories mentioned above: along them, \( L dt \) defines an exact differential, \( dI_n \). For these normal trajectories, the differential equation (2) makes perfect sense as an equality between two exact differentials. For these normal trajectories we can write

\[
\frac{1}{\hbar} I_n - \frac{C}{k_B} S = \text{const.}
\] (4)

Now the boundary thermodynamics cannot be the standard thermodynamics of equilibrium processes as presented in any standard textbook, say, ref. [5]. Among other reasons for this not being the case, standard equilibrium thermodynamics does not include time as one of its variables. We will present arguments in section 3, to the effect that quantum states arise through a dissipative mechanism. Which thermodynamics can it then be? We have in ref. [9] produced evidence that it must in fact be the explicitly time–dependent, classical thermodynamics of irreversible processes as developed by Onsager, Prigogine et al [21, 25]. Without reproducing all the arguments already presented in ref. [9], let us briefly sketch the main reasoning. In the boundary theory, the probability of small thermal fluctuations can be easily computed using Boltzmann’s distribution, \( \exp(-S/k_B) \) [18]. This distribution is readily obtained in our framework, by simply integrating and taking the exponential of the right–hand side of Eq. (2), with \( C = i \). On the left–hand side we then we recognise the Feynman integral kernel \( \exp(i \int L dt / \hbar) \), necessary to compute correlators in the quantum theory.\(^2\) The assumption that thermodynamical fluctuations in the boundary theory are small, or Gaussian, is described mathematically by the assumption of linearity between forces and fluxes [21, 25]. In the bulk theory, this corresponds to the Gaussian, or semiclassical, approximation to quantum mechanics.

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1 We will ignore the possibility of singular leaves in this foliation.

2 What quantum theorists call the Feynman path integral was independently developed in ref. [21] by Onsager and collaborators, who appear to have arrived at the notion of a path integral all by themselves, without previous knowledge of Feynman’s earlier work [11].
For completeness the boundary thermodynamics must be supplemented with the thermodynamical relation

\[ \frac{1}{T} = \frac{\partial S}{\partial U}, \]  

which must always be satisfied. So we take (5) to define the internal energy \( U \) of the boundary theory, given that \( T \) and \( S \) have already been defined.

3 The quantum theory in the bulk

A key consequence of using normal and tangential coordinates in \( \mathbb{R}^D \) is that quantum states \( \psi \), to be constructed presently, will factorise as

\[ \psi = \psi_t \psi_n, \]

or sums thereof. Here, the normal wavefunction \( \psi_n \) depends exclusively on the normal coordinate \( n \), while \( \psi_t \) is a function of the tangential coordinates. For example, in the case of the Coulomb potential, the wavefunction \( \psi_t \) would be a spherical harmonic \( Y_{lm}(\theta, \phi) \), while \( \psi_n \) would be a radial wavefunction \( R_{nl}(\rho) \). This construction contains some holographic elements that are very reminiscent of those present in ref. [30].

In this latter paper, "equipotential surfaces of the gravitational potential are identified as isoentropic surfaces." Our equipotential leaves are the counterpart of the holographic screens of ref. [30]. Now, in holographic theories, gravity (often supersymmetric) lives in the bulk, while the boundary theory is conformal.

Needless to say, our theory contains no supersymmetry. In our case gravity may, but need not, be present. Our boundary theory is a thermodynamical system on the leaves \( L_n \), characterised by a temperature \( T \), an entropy \( S \) and an internal energy \( U \) as defined by Eqs. (1), (2) and (5). Conformal invariance of the boundary theory just means, in our case, that the boundary thermodynamics is insensitive to the normal coordinate \( n \).

Moreover, the classical mechanics in the bulk exhibits a precise mechanism whereby different classical trajectories coalesce into a single equivalence class that can, following ref. [15], be identified as a single quantum state \( \psi \). So the presence of Planck’s constant \( \hbar \) in Eq. (2) obeys not just dimensional reasons—it is the sure sign of an information-loss mechanism, a dissipative process that is truly quantum in nature.

Let us see how this dissipation comes about. In order to do this we need to explain why many different classical trajectories coalesce into one single quantum state \( \psi \). A quantum of area on the leaf \( L_n \) measures \( L_p^2 \), where \( L_p \) denotes the Planck length. According to the holographic principle, at most 1 bit of information fits into this quantum of area \( L_p^2 \). One classical trajectory traversing this quantum of area corresponds to 1 bit of information. Classically one can regard the surface density of trajectories as being correctly described by a smooth distribution function: there fit some \( 1.4 \times 10^{69} \) classical trajectories into each square meter of area on the leaf \( L_n \) [2]. Although this is a huge number, it sets an upper limit on the potentially infinite number of classical trajectories that can traverse one quantum of area \( L_p^2 \).

The holographic principle alone would suffice to account for the lumping together of many different classical trajectories into one equivalence class. One equivalence
class, or quantum state, would be comprised by all those different classical trajectories crossing one given quantum of area $L_P^2$.

Of course, the actual number of quantum particles traversing one square meter of area on the leaf $L_n$ is much smaller than the above $1.4 \times 10^{69}$. The reason is simple: quantum effects become nonnegligible on matter well before quantum–gravity effects become appreciable on the geometry. Again, the existence of a (now particle–dependent) quantum of area is responsible for this. This can be seen as follows.

Let $m$ be the mass of the particle under consideration. Its Compton wavelength $\lambda_C = \hbar/(mc)$ imposes a fundamental limitation on its position, that we can call a quantum of length, denoted $Q_1$. This $Q_1$, which is particle–dependent, is of a fundamentally different nature than the geometric quantum of length $L_P$. On configuration space $\mathbb{R}^D$, this gives rise to a quantum $Q_{D-1}$ of $(D-1)$–dimensional volume within the leaf $L_n$, with measure (proportional to) $\lambda_C^{D-1}$, and to a quantum of length $Q_1$ along the normal coordinate.

In the presence of more than one particle species with different masses, each mass $m_i$ defines one value of the quantum $Q_{D-1}^{(i)}$. Then a quantum of volume that remains valid for all particles is the largest value of all those $Q_{D-1}^{(i)}$. This is the quantum of volume determined by the lightest particle.

Let us now elucidate how quantum states $\psi$ can arise as equivalence classes of different classical trajectories. By Eq. (6) we have to account for the appearance of the normal wavefunction $\psi_n$ and of the tangential wavefunction $\psi_t$.

Starting with $\psi_t$, let us consider all the different classical trajectories traversing any one quantum of volume $Q_{D-1}$ within a leaf $L_n$. The allowed values of the momentum carried by those trajectories are those compatible with the uncertainty principle. Since the particle has been spatially localised to an accuracy of $\lambda_C$ along each tangential coordinate, the corresponding momentum can be specified to an accuracy of $\hbar/\lambda_C$. Therefore, corresponding to a spatial quantum of volume $Q_{D-1}$ in the leaf, we have a quantum of volume $P_{D-1} = (\hbar/\lambda_C)^{D-1}$ in momentum space.

We are now in a position to state: all the different classical trajectories traversing any quantum of volume $Q_{D-1}$ in the leaf $L_n$, and simultaneously traversing a quantum $P_{D-1}$ in tangential momentum space, are to be regarded as different representatives of just one tangential state $\psi_t$.

An analogous statement for the normal coordinate reads: all classical trajectories traversing any quantum of length $Q_1$ along the normal coordinate $n$, and simultaneously traversing the corresponding quantum $P_1$ in normal momentum space, make up one normal state $\psi_n$.

Having described the dissipative mechanism whereby classical trajectories organise into quantum states, we go next to a counting of the number of quantum states. Since the leaf $L_n$ has been assumed compact, it encloses a finite number $N_n$ of volume quanta $Q_{D-1}$. Tentatively identifying this number $N_n$ with the (complex) dimension of the tangential Hilbert space $\mathcal{H}_t$, we immediately realise that the quantum of momentum $P_{D-1}$ is contained an infinite number of times within tangential momentum space (this is however a countable number of times). Indeed the momenta may grow to arbitrarily large values. Therefore, the tangential Hilbert space $\mathcal{H}_t$ is infinite–dimensional, and separable.
On the other hand, the dimension of the normal Hilbert space $\mathcal{H}_n$ is infinite already from the start (again a countable infinity, hence $\mathcal{H}_n$ is separable). The reason for this is the noncompactness of $\mathbb{R}^D$: the normal coordinate $n$ must cover an interval of infinite length. This implies that the normal coordinate encloses an infinite (though countable) number of length quanta $Q_1$. Multiplication by the number of independent momentum quanta $P_1$ does not alter this separable, infinite–dimensionality of $\mathcal{H}_n$.

Altogether, the complete Hilbert space $\mathcal{H}$ of quantum states is the tensor product $\mathcal{H}_t \otimes \mathcal{H}_n$. Its construction is manifestly holographic. However, because it singles out the normal coordinate $n$, one might worry that it depends on the particular choice of a leaf $L_n$ within the foliation. Now the only possible difference between any two leaves $L_{n_1}$ and $L_{n_2}$ is the value of their $(D-1)$–dimensional volume. Hence the numbers of volume quanta $N_{n_1}$ and $N_{n_2}$ they enclose may be different—but they are both finite. This possible difference is washed away upon multiplication by the (countably infinite) number of momentum quanta $P_{D-1}$ corresponding to each leaf. The dimension of $\mathcal{H}_t$ is therefore countably infinite regardless of the point, $n_1$ or $n_2$, along the radial coordinate—that is, regardless of which leaf is considered.

As explained in ref. [1], determining the tangential wavefunctions $\psi_t$ does not require a knowledge of the specific dynamics under consideration. Instead, this tangential dependence is univocally fixed by the geometry of the leaves $L_n$. In more technical terms, the wavefunctions $\psi_t$ must provide a complete orthonormal set for a unitary, irreducible representation of the isometry group of the leaves $L_n$. Moreover, as argued in ref. [1], the modulus squared $|\psi|^2$, evaluated at the value $n$, is proportional to the surface density of entropy flux across the leaf $L_n$.

4 Quantum uncertainty vs. the second law

Just as Planck’s constant $\hbar$ represents a coarse–graining of phase space into cells of minimal volume, or quanta of action, so does Boltzmann’s constant $k_B$ represent a quantum of entropy. This implies that any process must satisfy the condition

$$\Delta S = N k_B, \quad N \in \mathbb{N}. \quad (7)$$

The above expresses a quantised form of the second law of thermodynamics. The extreme smallness of the numerical value of $k_B$ in macroscopic units makes this quantisation macroscopically unobservable. In particular, unless $N = 0$, the second law becomes

$$\Delta S \geq \frac{k_B}{2}. \quad (8)$$

3 In case more than just one normal coordinate is needed, this statement is to be understood as meaning the sum of all the lengths so obtained.

4 We should remark that the assumption of compactness of the leaves $L_n$ can be lifted without altering our conclusions. A noncompact leaf encloses an infinite (yet countable) number of volume quanta $Q_{D-1}$. Upon multiplication by an infinite (yet countable) number of momentum–space quanta $P_{D-1}$, the dimension of the tangent Hilbert space $\mathcal{H}_t$ remains denumerably infinite. This form of holography in which the leaves are noncompact replaces the notion of inside vs. outside the leaf with the equivalent notion of one side of the leaf vs. the other side. One should not dismiss this possibility as unphysical: the constant potential, for example, can be regarded as having either compact or noncompact equipotential submanifolds.
In this form, the second law is actually a rewriting of the quantum–mechanical uncertainty principle for the canonical pair $E, t$:

$$\Delta E \Delta t \geq \frac{\hbar}{2}. \quad (9)$$

It is only in the limit $k_B \to 0$ that the second law (8) reduces to its classical formulation $\Delta S \geq 0$. The limit $k_B \to 0$ is the boundary–theory counterpart of the usual semiclassical limit $\hbar \to 0$ of the quantum mechanics in the bulk.

We conclude that the equivalence between Eqs. (8) and (9) is a consequence of our basic postulate (4). In other words, the second law (8) expresses, in the boundary theory, the same statement as the uncertainty principle (9) expresses in the bulk theory.

Holography implies that, while one needs two canonical variables $E, t$ in order to express the uncertainty principle in the bulk, just one variable $S$ is needed in order to write the second law on the boundary. An equivalent way of saying this is that entropy is a selfconjugate variable: one does not have to multiply it with a canonical variable (say, $\tau$) in order to obtain a product $\tau S$ carrying the dimensions of the quantum $k_B$. The variable $S$ already carries the dimensions of its corresponding quantum $k_B$.

5 Discussion

We can summarise our construction as follows. Let a quantum–mechanical system be given in configuration space $\mathbb{R}^D$. Let this latter space be foliated as per $\bigcup_n \mathbb{L}_n$, where each leaf $\mathbb{L}_n$ is an equipotential submanifold, in dimension $D-1$, of the given mechanical potential function $V(q_i)$. Assume that each leaf $\mathbb{L}_n$ encloses a finite $D$–dimensional volume $\mathcal{V}_n$, so $\partial \mathcal{V}_n = \mathbb{L}_n$. Then a holographic correspondence between the bulk theory within $\mathcal{V}_n$ and a boundary theory on $\mathbb{L}_n$ can be formulated. Quantum states in $\mathcal{V}_n$ are equivalence classes of different classical trajectories. These equivalence classes comprise all those classical trajectories that fit into one given quantum of volume in configuration space, with the corresponding momenta inside the corresponding quantum in momentum space. No quantum particle can be located to an accuracy better than its Compton wavelength.

Hence a physically reasonable unit for defining this quantum of length (and thus areas and volumes) is the Compton wavelength $\lambda_C$. Configuration space is subdivided into many such elementary volume quanta, each one of them (with the corresponding quanta in momentum space) defining one different quantum state.

The quantisation of phase–space area by Planck’s constant $\hbar$ proceeds along lines that are somewhat similar to ours, although not exactly identical. We recall that, semiclassically, the (symplectic) area element $dp \wedge dq$, divided by $\hbar$, gives the number of different quantum states fitting into that area element. However, the coordinate width $dq$ may be arbitrarily squeezed, provided the momentum $dp$ is correspondingly enlarged, and viceversa.

On the contrary, our construction makes use of the Compton wavelength $\lambda_C$ as a fundamental quantum of length (for the specific particle considered), below which

\textsuperscript{3}Unless, of course, one is willing to allow for pair creation out of the vacuum, thus quitting quantum mechanics and entering field theory.
no sharper localisation is possible: there is no squeezing the particle below this lower limit. This gives rise to an arrangement of different classical trajectories into equivalence classes that, following ref. [15], we identify with quantum states. This is an irreversible, dissipative mechanism that exhibits the emergent nature of quantum mechanics. The Hilbert space of quantum states is determined using the holographic technique described in section 3.

Under our correspondence, an irreversible thermodynamics can be defined on $L_n$, as per Eqs. (1) and (2) and (5). We may thus regard the latter Eqs. as providing a dictionary that allows one to switch back and forth between a quantum–mechanical picture and a thermodynamical picture of one and the same physics.

A key point to remark is the following. Thermodynamical approaches to quantum theory are well known [4, 20]. In particular, the link between (complex–time) quantum mechanics, on the one hand, and the equilibrium statistical mechanics of the Gibbs ensemble, on the other, has been known for long. We should stress that we have not dwelled on this long–established connection. Rather, the new correspondence explored here is that between (complex–time) quantum mechanics and the classical thermodynamics of irreversible processes in the linear regime. So the equilibrium hypothesis of the corresponding thermodynamics has been lifted, and replaced with linear, classical irreversibility. The linearity of the thermodynamics has a counterpart in the quantum theory, namely, the semiclassical approximation. Classicality of the thermodynamics means that $\hbar$ does not appear on the thermodynamical side of the correspondence, its role being played instead by Boltzmann’s constant $k_B$. Finally, irreversibility implies the existence of dissipation, as befits the presence of quantum effects.

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