Thermodynamics of Regular Black Holes Inspired by Noncommutative geometry

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Abstract. We present a new, exact, solution of regular Schwarzschild black hole by introducing an anisotropic perfect fluid inspired by noncommutative geometry. The obtained solution is interpolated between two quantities which are the de Sitter space-time at little distance and the regular Schwarzschild geometry at extensive extent. The consequence of noncommutative geometry is that it alters the thermodynamical characteristics of the black hole. Hawking temperature is calculated and its graphical study affirms striking features with reference to the alteration of the Hawkins temperature with the petty radius of the black hole.

1. Introduction

Black holes behave as a thermodynamical object after the discovery of Bakenstein [1] and Hawking [2]. It is important to study black hole thermodynamics as the entropy and temperature of the black hole. Bardeen proposed the first non-singular (regular) black hole model [3, 4] which concludes that there are horizons but without singularity. The 4D regular black hole is described by the following metric [4, 5],

$$ds^2 = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega_2^2$$

(1)

Here

$$f(r) = 1 - \frac{2m(r)}{r}$$

where $m(r) = Me^{-k/r}$

(2)

and $d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2$ describes two-sphere. The $M$ and $k$ are the mass and deviation parameter respectively which are assumed to be positive. For specific value $k = 0$, the metric (2) gives to the well-known Schwarzschild solution and the metric (1) behaves as RN solution asymptotically $r \gg k$. This takes place only when the relation between charge $q$ and mass $M$ are related to the parameter $k$ by the relation $q^2 = 2Mk$, here $M$ is the integration constant associated to the black hole mass. Regular solution (2) for the spherically symmetric black hole was drawn out to incorporate gravity coupled with magnetic charge [6, 7, 8, 9, 10], rotating black holes [11, 12, 14, 13], cloud of string model [15], 4D EGB gravity [16, 17] and also to generalize to more familiar Lovelock gravity theory [18, 19, 20, 21, 22].

The paper is organized as follows. In the Sec II, we present the solutions of the charged rotating black string with noncommutative matter field including the physical properties. The thermodynamics of the noncommutative inspired charged rotating black string is given in Sec III and the summary of the paper is presented in the Sec IV.

2. Noncommutative inspired regular black hole

We apply the noncommutative methodology to find the solution of noncommutative geometry.
inspired regular black hole. A Gaussian distribution of minimal width $\theta$ is used in place of the Dirac delta function to familiarize the noncommutative corrections. In order to introduce the noncommutative corrections in the mass density, we replace the Dirac delta function by

\[ \sqrt{\theta} \]

Gaussian distribution of minimal width $\theta$. The expression for the mass density in term of $\theta$ is as follow [23, 24, 25]

\[ \rho_\theta = \frac{M}{(4\pi\theta)^{3/2}} e^{r^2/4\theta} \]  

(3)

It is inferred that mass of the black hole is smeared around in a region $\theta$ instead of being located at a point. As a result mass of the black hole is obtained by integrating (3) over a volume of radius $r$. As a result, we obtain

\[ M_\theta = \int_0^r \rho_\theta 4\pi r^2 dr = \frac{2M}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \]  

(4)

where $\Gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) = \int_0^{r^2/4\theta} t^{1/2} e^{-t} dt$ is not complete gamma function. In the limit $\theta \to 0$ the incomplete gamma function becomes conventional gamma function and $M_\theta(r) \to m(r)$. The mass $m(r)$ in term of the smeared mass distribution is given by the following relation,

\[ m(r) = M_\theta e^{-k/r} = \frac{2M}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) e^{-k/r} \]  

(5)

Substitute (5) in equation (2), we get the noncommutative regular Schwarzschild black hole,

\[ ds^2 = -\left(1 - \frac{2M}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) e^{-k/r}\right) dt^2 + \frac{1}{\left(1 - \frac{2M}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) e^{-k/r}\right)} dr^2 + r^2 d\Omega_2^2 \]  

(6)

The metric (6) is the solution of regular Schwarzschild black hole, which is characterized by the mass $M$, deviation parameter $k$ and the minimal width $\theta$. In the limit $r/\theta \to \infty$ the solution (6) reduces to regular Schwarzschild black hole.

The event horizon of the noncommutative regular Schwarzschild black hole can be found where $g^{tt}(r_h) = 0$, that is,

\[ r_h = -\frac{k}{W(-k/2M)} \Gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \]  

(7)

where $W$ is the Lambert function. The equation (7) can not be solved for $r_h$ in a closed form, but we can solve it for obtaining the mass in term of the horizon radius $r_h$

\[ M_h = \frac{2[\log f(\frac{r_h}{\sqrt{4\theta}}) - e^{-r^2/4\theta} \frac{r_h}{\sqrt{\pi\theta}}]}{e^{-r^2/4\theta} \frac{r_h}{\sqrt{\pi\theta}}} \]  

(8)
Figure 1. The plot of metric $f\sqrt{r}$ in term of $r$ for the different value of deviation parameter $k$ with fixed value of mass $M = 3\theta$.

where $Erf(n)$ is the Gaussian error function defined as $Erf(n) = \frac{2}{\sqrt{\pi}} \int_0^n e^{-x^2} dx$. The horizon of the noncommutative regular Schwarzschild black hole for the different value of $\sqrt{k}$ plotted in Fig. (1). The Fig. (1) shows that the black hole has two horizons when $\sqrt{k} < 1.53\theta$, no horizon for $k > 1.53\theta$ and extremal black hole when $k = k_c = 1.53\theta$. Also the horizon of the black hole decreases with increases the value of deviation parameter $k$.

3. Thermodynamics

The metric (6) has two types of horizons viz. inner horizon and outet horizons. The Killing horizon is a null surface whose null generator are tangent to a Killing field. It has been verified that a static black hole event horizon shall be a Killing horizon in 4-dimensions Einstein gravity. Even if our solution is static, the Killing vector $\xi^\mu = \partial_t$, (9) is the null generator of the event horizon. The temperature of NC inspired regular Schwarzschild black hole is calculated by this relation [11, 8, 9, 10, 20, 21, 22, 26] (10) The black hole temperature is written as

$$T_h = -\frac{e^{k/r}}{4\pi^3/2 r^3 \theta^3/2} \left( r^3 - 2k\theta + 2r\theta - 2\sqrt{\pi}e^{r^2/4\theta} \theta^{3/2}(r - k)Erf\left(\frac{r}{\sqrt{4\theta}}\right) \right).$$

The temperature versus horizon radius of the black hole is plotted in the Fig. (2). It shows that deviation parameter $\sqrt{k}$ and the peak of the temperature are inversely related to each others.

At large $r$ ($r/\theta \to \infty$), we recover the temperature of regular Schwarzschild black hole

$$T_h = \frac{1}{4\pi} f'(r)|_{r=r_h} = \frac{2M}{r_h^3}(r_h - k).$$
Figure 2. Noncommutative inspired regular Schwarschild black hole’s temperature as a function of the horizon radius for the different value of parameter k.

4. Results and Conclusion

We have studied the regular Schwarzschild black hole in the presence noncommutative matter field. The significant feature of the paper is the construction of regular Schwarzschild metric by taking into the consideration of noncommutative effects. Thermodynamic entities like the temperature and entropy are calculated. This result is analyzed in detail by using graphical depictions. Special attention has been given to the small scale behaviour of black hole temperature where the outcome of both noncommutative and back reactions are remarkably worthwhile.

Finally we conclude that although our work presents for the noncommutative inspired regular Schwarzschild metric, our framework is strong to explain other types of noncommutative black holes.

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