Comments on the Determination of the Neutrino Mass Ordering in Reactor Neutrino Experiments

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Abstract—We consider the problem of determination of the neutrino mass ordering via precise study of the vacuum neutrino oscillations in the JUNO and other future medium baseline reactor neutrino experiments. We are proposing to resolve neutrino mass ordering by determination of the neutrino oscillation parameters from analysis of the data of the reactor experiments and comparison them with the oscillation parameters obtained from analysis of the solar and KamLAND experiments.

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The establishment of the character of the neutrino mass ordering (normal or inverted?) is one of the major problem of future high precision neutrino oscillation experiments. This problem will be investigated via observation of matter effects in the accelerator T2K [1] and NOvA [2], atmospheric PINGU [3] and ORCA [4] and other neutrino experiments.

The measurement of the angle $\theta_{13}$ in the accelerator T2K [5] and in the reactor Daya Bay [6], RENO [7] and Double Chooz [8] experiments opened the way of the determination of the neutrino mass ordering by the investigation of the vacuum neutrino oscillations in reactor neutrino experiments.

Dependence of the probability of reactor $\nu_e$'s to survive on the neutrino mass ordering (NMO) was noticed in the paper [9], in which reactor CHOOZ data were analyzed in the framework of three-neutrino mixing, and in the paper [10].

The neutrino mass ordering can be revealed in experiments in which effect of both solar and atmospheric mass-squared differences ($\Delta m^2_{21}$ and $\Delta m^2_{31}$) can be observed. It was shown in [11, 12] that this condition can be realized in medium baseline reactor neutrino experiments with source-detector distance 20–30 km. Later in numerous papers (see [13–18]) a possibility to determine the neutrino mass ordering in reactor experiments was analyzed in details. It was established that the optimum baseline is about 50–60 km and energy resolution must be $\frac{3\%}{\sqrt{E}}$ (MeV) or better.

Two ambitious medium baseline reactor neutrino experiments JUNO [18] in China and RENO-50 [19] in Korea were proposed. The JUNO experiment is now under construction. In this experiment 20 kton liquid scintillator detector will be located at the distances ~53 km from two nuclear power plants. It is planned that the data taking in this experiment will be started in 2020.

Let us consider the three-neutrino mixing

$$\nu_{IL} = \sum_{i=1}^{3} U_{iL} \nu_{iL} (l = e, \mu, \tau),$$

where $\nu_{iL} (l = e, \mu, \tau)$ is the flavor neutrino field, $\nu_i$ is the field of neutrino (Dirac or Majorana) with mass $m_i$, and $U$ is a unitary $3 \times 3$ PMNS [20, 21] mixing matrix which is characterized by three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one $CP$ phase $\delta$. As it is well known, in this case two neutrino mass spectra are possible.

Usually, neutrino masses are labeled in such a way that for both spectra $m_2 > m_1$ and $\Delta m^2_{21} = \Delta m^2_{31} > 0$ is solar mass-squared difference.²

Possible neutrino mass spectra are determined by the $m_3$ mass. There are two possibilities

1. Normal ordering (NO) $m_3 > m_2 > m_1$.
2. Inverted ordering (IO) $m_3 > m_1 > m_2$.

² We will use the following definition of the mass-squared difference: $\Delta m^2_{ij} = m^2_i - m^2_k$. 

\[425\]
Following the standard procedure, for the probability of $\nu_e$ to survive in vacuum we have

$$P(\nu_e \to \nu_e) = 1 - 2 \sum_{i<k} |U_{ei}|^2 |U_{ek}|^2 (1 - \cos 2\Delta_{ki}). \quad (2)$$

Here

$$\Delta_{ki} = \frac{\Delta m_{ki}^2 L}{4E}, \quad (3)$$

where $L$ is the distance between neutrino source and detector and $E$ is the neutrino energy. Taking into account that

$$|U_{ei}|^2 = \cos^2 \theta_{13} \cos^2 \theta_{12}, \quad |U_{ei}|^2 = \cos^2 \theta_{13} \sin^2 \theta_{12}, \quad |U_{ei}|^2 = \sin^2 \theta_{13}, \quad (4)$$

we have

$$P(\nu_e \to \nu_e) = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_S$$

$$- \frac{1}{2} \sin^2 2\theta_{13} \left[ \cos^2 \theta_{12} (1 - \cos 2\Delta_{13}) + \sin^2 \theta_{12} (1 - \cos 2\Delta_{23}) \right]. \quad (5)$$

Only last term of this expression, proportional to the small parameter $\sin^2 2\theta_{13}$, depends on the neutrino mass ordering.

In the case of three neutrino masses, there are two independent mass-squared differences. Three neutrino mass-squared differences in (5) (for both mass spectra) satisfy the following identity

$$\Delta m_{13}^2 = \Delta m_{23}^2 + \Delta m_{12}^2. \quad (6)$$

From (6) we have

$$|\Delta m_{13}^2| = |\Delta m_{23}^2| \pm |\Delta m_{12}^2|, \quad (7)$$

where $\pm$ corresponds to NO (IO), respectively. Thus, we have

$$\cos 2\Delta_{13} = \cos 2|\Delta_{23}| \cos 2\Delta_S \mp \sin 2|\Delta_{23}| \sin 2\Delta_S. \quad (8)$$

From (5) and (8) for the $\nu_e$ survival probability we obtain the following expression [22, 23]

$$P(\nu_e \to \nu_e) = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_S$$

$$- \frac{1}{2} \sin^2 2\theta_{13} \left[ \cos^2 \theta_{12} (1 - \cos 2|\Delta_{23}| \mp \varphi) \right] \quad \text{(9)}$$

Here

$$\sin \varphi = \frac{1}{a} \cos^2 \theta_{12} \sin 2\Delta_S,$$

$$\cos \varphi = \frac{1}{a} (\cos^2 \theta_{12} \cos 2\Delta_S + \sin^2 \theta_{12}), \quad (10)$$

where

$$a = \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{12}} \quad \text{(11)}$$

and $\pm$ correspond to NO and IO, respectively.

The expressions (9)–(11) were used in several recent papers (see [22, 24–26]) to estimate the sensitivity of the JUNO experiment to the neutrino mass ordering.\(^3\) It was suggested in these papers that the only difference between NO and IO is the sign before the phase $\varphi$ in the expression (9).\(^4\) Let us notice, however, that the parameter $|\Delta m_{23}^2|$ also depends on the neutrino mass ordering. In fact, we have

$$|\Delta m_{23}^2| = |\Delta m_{4}^2| \text{ NO}, \quad |\Delta m_{23}^2| = |\Delta m_{4}^2 + \Delta m_{5}^2| \text{ IO}. \quad (12)$$

where atmospheric mass-squared difference $\Delta m_4^2$ is determined as follows

$$\Delta m_4^2 = \Delta m_{23}^2 \text{ NO, } \Delta m_4^2 = |\Delta m_{13}^2| \text{ IO}. \quad (13)$$

From our point of view neutrino oscillation parameters must be determined in a NMO independent way. Neutrino mixing angles, CP phase and $\Delta m_2^2$ are determined in such a way. The atmospheric mass-squared difference $\Delta m_4^2$, determined by (13), (but not the parameter $|\Delta m_{23}^2|$) also satisfies this criteria.

From (5) and (13) for the $\nu_e$ survival probability for the normal and inverted mass ordering we have, correspondingly,

$$P^{\text{NO}}(\nu_e \to \nu_e) = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_S$$

$$- \sin^2 2\theta_{13} \left[ \cos^2 \theta_{12} \sin^2 (\Delta_A + \Delta_S) + \sin^2 \theta_{12} \sin^2 \Delta_A \right], \quad (14)$$

and

$$P^{\text{IO}}(\nu_e \to \nu_e) = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_S$$

$$- \sin^2 2\theta_{13} \left[ \cos^2 \theta_{12} \sin^2 (\Delta_A + \Delta_S) + \sin^2 \theta_{12} \sin^2 (\Delta_A - \Delta_S) \right]. \quad (15)$$

There are several possibilities to choose NMO independent atmospheric mass-squared difference (see [33]). If $\Delta m_4^2$ is determined as in [30]

$$\Delta m_4^2 = \Delta m_{13}^2 \text{ NO, } \Delta m_4^2 = |\Delta m_{23}^2| \text{ IO.} \quad (16)$$

for the $\nu_e$ survival probability in the case of NO and IO we have

$$P^{\text{NO}}(\nu_e \to \nu_e) = 1 - \cos \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_S$$

$$- \sin^2 2\theta_{13} \left[ \cos^2 \theta_{12} \sin^2 \Delta_A + \sin^2 \theta_{12} \sin^2 (\Delta_A - \Delta_S) \right]. \quad (17)$$

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\(^3\) Similar expression was used in [27] for the estimation of the sensitivity of the RENO-50 experiment to NMO.

\(^4\) See, for example, [24]: “As shown in (9), neutrino mass ordering dependence comes solely through the phase shift $\varphi$ ...”
The value of the parameter $\tan^2\theta_{13}$ is known from analysis of the data of the KamLAND and solar neutrino experiments. From the latest three-neutrino analysis of the data it was found [35]

$$\tan^2\theta_{13} = 0.437^{+0.029}_{-0.026}$$

Thus, in order to reveal the neutrino mass ordering we need to determine from analysis of the data of future medium baseline reactor neutrino experiments the parameter $X$ and to check whether $X$ is equal to

$$\sin^2\theta_{12} = 0.304 \pm 0.014$$

or

$$\cos^2\theta_{12} = 0.696 \pm 0.014.$$  

In the first case neutrino mass ordering is the normal one and in the second it is the inverted one.

It was shown in [18, 24] that after six years of data taking the parameters $\sin 2\theta_{13}, \Delta m^2_S$ and $\Delta m^2_A$ will be determined in the JUNO experiment with accuracy better than 1%. From the latest result of the Daya Bay experiment it was found [36]

$$\sin^2 2\theta_{13} = 0.084 \pm 0.005.$$  

Apparently, the future high precision JUNO and RENO-50 experiments and future knowledge of the neutrino oscillation parameters will allow to distinguish the values in (25) and (26).

In conclusion I like to stress that I do not consider the method discussed here as a new method of the determination of NMO which can replace the existing ones. I would like only to draw attention to the possibility which was not considered before.

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