Spin-dependent hole quantum transport in Aharonov-Bohm ring structure: possible schemes for spin filter

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We study the Aharonov-Bohm (AB) effect in two-dimensional mesoscopic frame in hole systems. We show that differing from the AB effect in electron systems, due to the presence of both the heavy hole and the light hole, the conductances not only show the normal spin-unresolved AB oscillations, but also become spin-separated. Some schemes for spin filter based on the abundant interference characteristics are proposed and the robustness against the disorder of the proposed schemes is discussed.

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The aim of using not only charge but also spin degree of freedom of electrons and holes in semiconductor electronic devices leads to a new field: semiconductor spintronics.1 Spin filter is one of the basic devices in this field. Many schemes for spin filters, most in electron systems, have been proposed2 in order to inject spin-polarized current into semiconductors, by means of spin-selective barriers, stubs,3 weak periodic magnetic modulations4,5 and anti-resonance effects in a double-bend structure.6

In this paper, we study the AB effect5,7 in two-dimensional mesoscopic hole system. The interferences between the four spin states, i.e., the spin-up and -down heavy hole (HH) states and the spin-up and -down light hole (LH) states are more complicated than the electron system. Possible schemes for spin filter are proposed based on the abundant interference characteristics: When the Fermi energy of the lead is lower than the LH band edge of the frame, one can use the AB frame as a spin filter of HH by controlling the AB flux. When a suitable strain is applied on the frame to make the band edges of the HH and the LH close to each other, then if one injects a spin unpolarized HH current into the frame, a spin polarized LH (or HH) current can be obtained by controlling the AB flux.

We consider the AB flux $\phi$ introduced by a homogeneous magnetic field $B$ through a two-dimensional (2D) AB frame structure as shown in Fig. 1, which is grown in a (001) GaAs quantum well with a small well width ($a = \sqrt{10}$ nm). The momentum states along the growth direction ($z$) are therefore quantized and one only need to consider the lowest subband. In this system there is no spin correlation $\langle a_1^{\dagger} a_{k-\frac{1}{2}} \rangle$ between the spin-up and -down HH’s (LH’s). The spin-up HH’s (LH’s) are only coupled with the spin-down LH’s (HH’s). This can be seen from the Luttinger Hamiltonian $H_{L}$ in the momentum space with the matrix elements arranged in the order of $\frac{1}{2}$, $\frac{3}{2}$, $-\frac{1}{2}$ and $-\frac{3}{2}$:

$$H_L = \frac{1}{2m_0} \begin{pmatrix} P + Q & 0 & R & 0 \\ 0 & P - Q & 0 & R \\ R^\dagger & 0 & P - Q & 0 \\ 0 & R^\dagger & 0 & P + Q \end{pmatrix}. \quad (1)$$

In this equation $P \pm Q = (\gamma_1 \pm \gamma_2)(P^2_x + P^2_y) + (\gamma_1 \mp 2\gamma_2)\sqrt{2} |t| \gamma_3 P_x P_y$ and $R = -\sqrt{3} \gamma_2 (P^2_x - P^2_y) - 2i \gamma_3 P_x P_y$ with $\gamma_1 = 6.85$ and $\gamma_2 = 2.1$ representing the Luttinger coefficients. $m_0/(\gamma_1 \pm \gamma_2)$ are the effective masses of the HH and the LH in the $x$-$y$ plane with $m_0$ representing the free electron mass. Additionally, the Luttinger Hamiltonian can be separated into two independent parts: $H_\alpha(k_x,k_y) = \frac{1}{2m_0} \begin{pmatrix} P + Q & R \\ R^\dagger & P - Q \end{pmatrix}$, with the matrix elements arranged in the order of $\frac{3}{2}$ and $-\frac{1}{2}$ for the spin-up HH and the spin-down LH subsystem noted as $\alpha$, and $H_\beta(k_x,k_y) = \frac{1}{2m_0} \begin{pmatrix} P + Q & R^\dagger \\ R & P - Q \end{pmatrix}$, with the matrix elements arranged in the order of $-\frac{3}{2}$ and $\frac{1}{2}$ for the spin-down HH and the spin-up LH subsystem noted as $\beta$.

![FIG. 1: Schematic view of a AB frame structure in 2D hole system with arm width $d$, frame length $L$ and frame width $W$.](image-url)
In real space, the Hamiltonian with AB flux can be written in the tight-binding version as:

\[
H_{2D} = \sum_{i,j,\sigma=\pm \frac{1}{2}, \pm \frac{3}{2}} \epsilon_{i,j,\sigma} a_{i,j,\sigma}^{\dagger} a_{i,j,\sigma} + \sum_{i,j,\sigma=\pm \frac{1}{2}, \pm \frac{3}{2}, \delta=\pm 1} (\gamma_1 \pm \gamma_2) V_{ij',ij}[a_{i+\delta,j,\sigma}^{\dagger} a_{i,j,\sigma} + a_{i,j+\delta,\sigma}^{\dagger} a_{i,j,\sigma}] \\
+ \left\{ \sum_{i,j,\delta=\pm 1, \lambda=0,1} (-\sqrt{3}) \gamma_2 V_{ij',ij}[a_{i+\delta,j,\sigma}^{\dagger} a_{i,j,\sigma} - \lambda a_{i,j,-\frac{1}{2}-\lambda} - a_{i,j+\delta,\sigma}^{\dagger} a_{i,j,-\frac{1}{2}+\lambda}]
+ \sum_{i,j,\delta=\pm 1, \lambda=0,1} \frac{\sqrt{3}}{2} \gamma_3 V_{ij',ij}[a_{i+\delta,j,\sigma}^{\dagger} a_{i,j,\sigma} - \lambda a_{i,j,-\frac{1}{2}-\lambda} - a_{i,j+\delta,\sigma}^{\dagger} a_{i,j,-\frac{1}{2}+\lambda}] + \mathrm{H.C.} \right\}
\]

(2)

where \(i\) and \(j\) denote the coordinates along the \(x\)- and \(y\)-axes. \(t = -\hbar^2/(2m_0 a_0^2)\) is the energy unit with \(a_0\) standing for the “lattice” constant. With the vector potential \(A\) in the Landau gauge, i.e., \(A = (-\frac{1}{2} By, \frac{1}{2} Bx, 0)\), the hopping energy from \(r_{i,j}\) to \(r_{i',j'}\) is given by \(V_{ii',jj'} = t \exp[i e A \cdot (r_{i',j'} - r_{i,j})]/\hbar\). \(\epsilon_{\pm \frac{1}{2}} = (\gamma_1 \pm 2\gamma_2) \hbar^2 / 2 t|t| - (\gamma_1 + \gamma_2) 4t\) and \(\epsilon_{\pm \frac{3}{2}} = (\gamma_1 + 2\gamma_2) \hbar^2 / 2 t|t| - (\gamma_1 - 2\gamma_2) 4t\) with the first terms standing for the lowest subband energy in the \(z\) direction. The first and the second terms in \{\cdots\} are the nearest-neighbor and the next-nearest-neighbor spin-flip hopping terms. Obviously, there is not any direct or indirect spin flip between the spin-up and -down HH’s or between the spin-up and -down LH’s in the Hamiltonian. Additionally

\[
H_{\text{strain}} = \sum_{i,j,\sigma=\pm \frac{1}{2}, \pm \frac{3}{2}} \epsilon_{i,j,\sigma}^{\ast} a_{i,j,\sigma}^{\dagger} a_{i,j,\sigma}
\]

(3)

is the strain Hamiltonian where \(\epsilon_{i,j,\sigma}^{\ast}\) represents the strain-induced energy with \(\epsilon_{\pm \frac{1}{2}} \neq \epsilon_{\pm \frac{3}{2}}\). By adding strain, one may adjust the separation between the HH and the LH bands.

The Spin-dependent conductance is calculated using the Landauer-Büttiker formula\(^{10}\) with the help of the Green function method.\(^{11}\) The two-terminal spin-resolved conductance is given by \(G_{\sigma} = (e^2/\hbar) \text{Tr}[\Gamma_{\sigma} G_{\sigma}^{\text{LL}} + \Gamma_{\text{LL}}^{\dagger} G_{\sigma}^{\text{LL}} \Gamma_{\sigma}^{\dagger}]\) with \(\Gamma_i(\Gamma_{\text{LL}})\) representing the self-energy function for the isolated ideal leads.\(^{11}\) We choose the perfect ideal Ohmic contact between the leads and the semiconductor. \(G_{\sigma}^{\text{LL}}\) and \(G_{\sigma}^{\text{LL}}\) are the retarded and advanced Green functions for the conductor, but with the effect from the leads included.

We perform a numerical calculation with \(d = 10 a_0\), \(W = 40 a_0\) and \(L = 200 a_0\). A hard wall potential is applied in the transverse direction. In Fig. 2(a), we plot the conductances of the spin-up HH \(G^{\uparrow} = G_{\uparrow}^{\text{LL}} + G_{\uparrow}^{\text{LL}}\) and the spin-down HH \(G^{\downarrow} = G_{\downarrow}^{\text{LL}} + G_{\downarrow}^{\text{LL}}\) at the right lead against the AB flux \(\phi\). It is noted that as there is no spin flip between the spin-up and -down HH’s, \(G^{\pm} = G^{\mp} \equiv 0\). By choosing a suitable strain on the two leads, one is able to separate the HH and LH bands well apart and consequently \(G_{\uparrow}^{\text{LL}} = G_{\downarrow}^{\text{LL}} = 0\). We further align the HH band edges of the leads and the frame and choose a low Fermi energy which is \(1.4|t|\) above the HH band edge \(E_{HH}^0\). As there is no strain applied on the frame, the band edge of the LH is \(8.29|t|\) above the \(E_{HH}^0\). Therefore, the LH can not provide a real transport channel but only provides a virtual one. One can see from the figure that when \(B = 0\), the conductances of the spin-up and -down HH’s are identical. However when \(B \neq 0\), the conductances vary differently with the AB flux. The reason is understood as follows: When \(B = 0\), the phase of the subsystem \(\alpha\) which comes from the Luttinger spin-orbit coupling has the same magnitude and the same sign as that of the subsystem \(\beta\) when a hole travels through different arms as \(H_{\alpha}(k_x, k_y) = H_{\beta}(k_x, -k_y)\). For instance, if one considers a hole of subsystem \(\alpha\) traveling through the upper arm along an arbitrarily chosen
path $P_1$ which consists of a series of hopping, the phase that comes from the accumulation of the imaginary part of the next-nearest-neighbor hopping, such as the hopping in Fig. 3(a), is the same as the phase that comes from a hole of subsystem $\beta$ travelling through the mirror-symmetric path of $P_1$, i.e., $P_1^*$. This is because that the hopping of subsystem $\beta$ along $P_1^*$ consists of terms as shown in Fig. 3(b) and one has $t_1 = t_4$ and $t_2 = t_3$ with $t_1 = -t_2 = \frac{\sqrt{3}}{2} t \gamma_3$. Then it is obvious that the conductances of subsystems $\alpha$ and $\beta$ are exactly the same because the interferences of the two subsystems that are determined by the summation of all the paths are entirely mirror-symmetric. However, when $B \neq 0$, the phase shift from the AB effect destroys the above symmetry, i.e. $t_1 = \frac{\sqrt{3}}{2} t \gamma_3 V_{i+1,j+1;i,j} \neq t_4 = \frac{\sqrt{3}}{2} t \gamma_3 V_{i+1,j-1;i,j}$ and $t_2 = -\frac{\sqrt{3}}{2} t \gamma_3 V_{i+1,j-1;i,j} \neq t_3 = -\frac{\sqrt{3}}{2} t \gamma_3 V_{i+1,j+1;i,j}$. Then the conductances of subsystems $\alpha$ and $\beta$ can be different.

It is interesting to see that although there is no real LH channel in the frame and the leads available for the transport due to the low Fermi energy of the leads, the LH states still provide virtual channels which manifest different phases of the $\alpha$ and $\beta$ subsystems. It is due to the presence of these virtual channels that separate the HH’s of different spins. If one applies a strain to further increase the separation of the HH and LH in the frame of the above structure, then the contribution from the virtual channels is suppressed and the spin separation becomes smaller. This is demonstrated in Fig. 4 where we use the same conditions as those in Fig. 2(a) except the LH band edge is lifted by $50|t|$. One can see that the difference of the conductance $G^\pm$ and $G^-^\pm$ becomes much smaller. And when we lift the LH band edge even higher, we find that $G^\pm = G^-^\pm$ recovers the ordinary AB effect in electron systems.

It is further seen from Fig. 2(a) that by using the different conductances of the spin-up and -down HH’s at different AB flux, one is able to make a spin filter. For example, when $\phi \approx 0.4 \theta_0$, one can get spin-up polarization of the HH because $G^+/\phi = G^-/\phi \approx 0$ and $G^\pm = G^\pm^\pm \approx G^\pm$. When $\phi \approx 0.3$, one can get spin-down polarization of the HH as $G^\pm \approx 0$ and $G^-^\pm \approx G^\pm$. In order to check the robustness of this filter, we plot in Fig. 2(b) the spin polarization which is defined as $P = (G^+ - \frac{1}{2})/(G^+ + G^-)$ averaged over 100 random configurations for different disorder strengths versus the AB flux when Anderson disorder is considered. One can see that even when the disorder strength is $0.5|t|$, there is still large spin polarization.

![FIG. 5: Conductances of spin-up and -down HH’s v.s. the AB flux of 2D AB frame. Solid curve: $G^\pm$; Dashed curve: $G^-^\pm$.](image)

We further show that this structure can be used to generate spin polarized current of LH while driving a spin-unpolarized HH charge current into the 2D frame. This can be realized by applying a strain on the left lead
which separate the HH and LH far away from each other and a different strain on the frame to recover the Γ-point degeneracy. There is no strain on the right lead. Therefore the LH band edge of the right lead is 8.29|t| above the HH band edge \( E_{H}^{0} \) and the HH (and of course the LH) band edge of the frame is 13.6|t| above \( E_{H}^{0} \). By applying a gate voltage on the left lead, one may align the HH band edge of the left lead to be the same as \( E_{H}^{0} \). The Fermi energy is chosen to be 14.65|t| above \( E_{H}^{0} \), i.e., 1.05|t| above the HH (LH) band edge of the frame and 6.36|t| above the LH band edge of the right lead. Therefore, only a HH charge current can be injected from the left lead into the frame but both HH and LH bands of the frame and the right lead contribute to the transport. The conductances of the HH and the LH are plotted versus the AB flux in Fig. 5. It is seen from the figure that when \( \phi = 0.68\phi_{0} (1.37\phi_{0}) \), \( G_{\parallel}^{2} \approx 0, G_{\perp}^{2} \approx 0 \) and \( G_{\perp}^{2} = G_{\perp}^{2} = 0.294e^{2}/h \ (0.371e^{2}/h) \), \( G_{\parallel}^{2} = G_{\parallel}^{2} = 0.196e^{2}/h \ (0.143e^{2}/h) \). Hence one can obtain a spin polarized current of LH with polarization \( P = 20 \% \ (44 \%) \) where \( P \) is defined as \( P = (G_{\parallel}^{2} - G_{\perp}^{2})/(G_{\parallel}^{2} + G_{\perp}^{2}) \). Therefore, the AB effect of such a 2D frame provides us another scheme for spin filtering that a spin unpolarized HH can be changed to the spin polarized LH. Similarly by choosing \( \phi = 0.21\phi_{0} \) and 0.44\( \phi_{0} \), one may get spin polarized LH current with a spin-unpolarized HH charge injection. However, the energy dependence of this filter is very sensitive. When we include the effect of disorder, the spin polarized LH current is always accompanied by the HH current.

In conclusion, the AB effect in two-dimensional mesoscopic hole system is studied. We propose some schemes for spin filter based on the abundant interference characteristics. When the band edges of the HH and LH are separated due to the confinement and the Fermi energy is lower than the LH band edge but above the HH band edge, we show that the LH still provides a virtual channel which leads to different phases for the spin-up and -down HH and gives rise to the spin separation. Therefore one can use the frame as a spin filter of HH by controlling the AB flux. Another spin filter is proposed when a suitable strain is applied on the frame in order to make the band edges of the HH and the LH close to each other and the channels of the leads are tuned so that both the HH and LH of the right lead but only the HH of the left lead are below the Fermi energy. When a spin unpolarized HH current from the left lead is injected into the frame, a spin polarized LH (or HH) current can be obtained by controlling the AB flux. It is shown that the first scheme for spin filter is very robust against disorder whereas the second one is very poor against disorder.

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