Towards resolving the $^3_ΛH$ lifetime puzzle

A. Gal$^{a,*}$, H. Garcilazo$^b$

$^a$Racah Institute of Physics, The Hebrew University, 91904 Jerusalem, Israel
$^b$Escuela Superior de Física y Matemáticas
Instituto Politécnico Nacional, Edificio 9, 07738 México D.F., Mexico

Abstract

Recent $^3_ΛH$ lifetime measurements in relativistic heavy ion collision experiments have yielded values shorter by $(30\pm8)\%$ than the free $Λ$ lifetime $τ_Λ$, thereby questioning the naive expectation that $τ(^3_ΛH)\approx τ_Λ$ for a weakly bound $Λ$ hyperon. Here we apply the closure approximation introduced by Dalitz and coworkers to evaluate the $^3_ΛH$ lifetime, using $^3_ΛH$ wavefunctions generated by solving three-body Faddeev equations. Our result, disregarding pion final-state interaction (FSI), is $τ(^3_ΛH)=(0.90\pm0.01)τ_Λ$. In contrast to previous works, pion FSI is found attractive, reducing further $τ(^3_ΛH)$ to $τ(^3_ΛH)=(0.81\pm0.02)τ_Λ$. We also evaluate for the first time $τ(^3_Λn)$, finding it considerably longer than $τ_Λ$, contrary to the shorter lifetime values suggested by the GSI HypHI experiment for this controversial hypernucleus.

Keywords: light $Λ$ hypernuclei, hypertriton lifetime, Faddeev calculations

1. Introduction

$^3_ΛH$, a $pnΛ$ state with spin-parity $J^P = \frac{1}{2}^+$ and isospin $I = 0$ in which the $Λ$ hyperon is bound to a deuteron core by merely $B_{Λ(^3_ΛH)}=0.13\pm0.05$ MeV, presents in the absence of two-body $ΛN$ bound states the lightest bound and one of the most fundamental $Λ$ hypernuclear systems [1]. Its spin-parity $\frac{1}{2}^+$ assignment follows from the measured branching ratio of the two-body decay $^3_ΛH \rightarrow ^3\text{He} + π^-$ induced by the free $Λ$ weak decay $Λ \rightarrow p + π^-$ [2]. There is no experimental indication, nor theoretical compelling reason, for a bound $J^P = \frac{3}{2}^+$ spin-flip excited state, and there is even less of a good reason to assume that an excited $I = 1$ state lies below the $pnΛ$ threshold.

*corresponding author: Avraham Gal, avragal@savion.huji.ac.il
Given the loose binding of the Λ hyperon in ${}^{3}\Lambda H$ it is natural to expect, perhaps naively, that nuclear medium effects modify little the free Λ lifetime in such a diffuse environment. An updated compilation of measured ${}^{3}\Lambda H$ lifetime values is presented in Fig. 1. Note the world-average value (dashed) which is shorter by about 30% than the free Λ lifetime $\tau_\Lambda = 263 \pm 2$ ps (solid). In sharp contrast with the large scatter of bubble chamber and nuclear emulsion measurements from the 1960s and 1970s, the recent measurements of $\tau({}^{3}\Lambda H)$ in relativistic heavy ion experiments marked in the figure give values persistently shorter by $(30 \pm 8)\%$ than $\tau_\Lambda$ [14]. Also shown are ${}^{3}\Lambda H$ lifetime values from three calculations (dot-dashed) that pass our judgement, two of which [15, 16] using fully three-body ${}^{3}\Lambda H$ wavefunctions claim ${}^{3}\Lambda H$ lifetimes shorter than $\tau_\Lambda$ by only $(4 \pm 1)\%$. The third one [17], using a $\Lambda d$ cluster wavefunction, obtained a ${}^{3}\Lambda H$ lifetime shorter than $\tau_\Lambda$ by as much as 13%. These cited results include a small nonmesonic decay rate contribution of 1.7% [18]. Among calculations that claim much shorter ${}^{3}\Lambda H$ lifetimes, we were unable to reproduce the results of Ref. [19], nor to make sense out of a $\frac{3}{\Lambda H}$ decay rate calculation based on a nonmesonic $\Lambda N \rightarrow NN$ weak interaction hamiltonian [20]. We comment briefly on the calculations in Refs. [15, 16]:

Figure 1: Measured ${}^{3}\Lambda H$ lifetime values in chronological order, with (a)–(f) from emulsion and bubble-chamber measurements [3, 4, 5, 6, 7, 8], and from recent relativistic heavy ion experiments: STAR(I) [9], HypHI [10], ALICE(I) [11], STAR(II) [12], ALICE(II) [13], see text. We thank Benjamin Döringus for providing this figure [14].
(i) Rayet and Dalitz (RD) [15], using a closure approximation to sum over the final nuclear states reached in the $^3\Lambda$H weak decay, reduced the $^3\Lambda$H lifetime calculation to the evaluation of a $^3\Lambda$H exchange matrix element defined in Sect. 2 below. Variational $^3\Lambda$H wavefunctions of the form $f(r_{\Lambda p})f(r_{\Lambda n})g(r_{pn})$ were used, accounting for both short-range and long-range correlations in the diffuse $^3\Lambda$H. With a suitable choice of the closure energy, and including a questionable 1.3% repulsive pion FSI decay-rate contribution (see below), they obtained $\tau(^3\Lambda$H) $\approx$ 0.95 $\tau_{\Lambda}$. However, the RD decay rate expressions miss a recoil phase-space factor which, if not omitted inadvertently in print, would bring down their calculated $^3\Lambda$H lifetime to 85% of $\tau_{\Lambda}$ (see footnote b to Table 4 in Ref. [8] to support this scenario).

(ii) Kamada et al. [16] in a genuinely ab-initio calculation used a $^3\Lambda$H wavefunction obtained by solving three-body Faddeev equations with $NN$ and $YN$ Nijmegen soft-core potentials to evaluate all three $\pi^-$ decay channels: $^3$He + $\pi^-$, $d + p + \pi^-$ and $p + p + n + \pi^-$. The $\pi^0$ decay channels were related by the $\Delta I = \frac{1}{2}$ rule in a ratio 1:2 to the corresponding $\pi^-$ channels. Their calculated $^3\Lambda$H lifetime is 256 ps: shorter by 3% than the measured value of $\tau_{\Lambda}$, but shorter by 6% than their calculated value of 272 ps for $\tau_{\Lambda}$. Hence, we refer to their result as $\tau(^3\Lambda$H) $\approx$ 0.94 $\tau_{\Lambda}$.

In this Letter we study pion FSI which in accord with low-energy pion-nucleus phenomenology [21, 22] is generally considered repulsive, thereby increasing rather than decreasing $\tau(^3\Lambda$H). However, exceptionally for $^3\Lambda$H, pion FSI is attractive and potentially capable of resolving much of the $\tau(^3\Lambda$H) puzzle. A fully microscopic inclusion of pion FSI requires a four-body final-state model, a formidable project that still needs to be done. Instead, we study here $\tau(^3\Lambda$H) within a closure-approximation calculation in which the associated exchange matrix element is evaluated with wavefunctions obtained by solving $^3\Lambda$H three-body Faddeev equations. Disregarding pion FSI, our result $\tau(^3\Lambda$H) $\approx$ 0.90 $\tau_{\Lambda}$ differs by a few percent from that of the microscopic Faddeev calculation by Kamada et al. [16]. Introducing pion FSI in terms of pion distorted scattering waves results in $\tau(^3\Lambda$H) = (0.81$\pm$0.02)$\tau_{\Lambda}$, that is (213$\pm$5) ps, in the right direction towards resolving much of the $\tau(^3\Lambda$H) puzzle.

Finally, as a by-product of studying $\tau(^3\Lambda$H), we estimate for the first time the lifetime of $^3\Lambda$n assuming that it is bound. The particle stability of $^3\Lambda$n was conjectured by the GSI HypHI Collaboration having observed a $^3$H+$\pi^-$ decay track [23], but is unanimously opposed by recent theoretical works [24, 25, 26]. Our estimate suggests a value of $\tau(^3\Lambda$n) considerably longer than $\tau_{\Lambda}$, in strong disagreement with the shorter lifetime reported in Ref. [23].
2. Total decay rate expressions for $^3\Lambda H$ and $^3\Lambda n$

The $\Lambda$ weak decay rate considered here, $\Gamma_\Lambda \approx \Gamma_\Lambda^- + \Gamma_\Lambda^0$, accounts for the mesonic decay channels $p\pi^-$ (63.9%) and $n\pi^0$ (35.8%). Each of these partial rates consists of a parity-violating $s$-wave term (88.3%) and a parity-conserving $p$-wave term (11.7%), summing up to

$$\Gamma_\Lambda(q) = \frac{q}{1 + \omega_\pi(q)/E_N(q)}(|s_\pi|^2 + |p_\pi|^2 \frac{q^2}{q^2_\Lambda}), \quad \frac{|p_\pi|^2}{|s_\pi|^2} \approx 0.132, \quad (1)$$

where $\Gamma_\Lambda$ is normalized to $|s_\pi|^2 + |p_\pi|^2 = 1$, $\omega_\pi(q)$ and $E_N(q)$ are center-of-mass (cm) energies of the decay pion and the recoil nucleon, respectively, and $q \to q_\Lambda \approx 102$ MeV/c in the free-space $\Lambda \to N\pi$ weak decay. The $\approx 2:1$ ratio of $\pi^-:\pi^0$ decay rates, the so called $\Delta I = \frac{1}{2}$ rule in nonleptonic weak decays, assigns the final $\pi N$ system to a well-defined $I = \frac{1}{2}$ isospin state.

2.1. $^3\Lambda H$

For $^3\Lambda H$ ground state (g.s.) weak decay, approximating the outgoing pion momentum by a mean value $\bar{q}$ and using closure in the evaluation of the summed mesonic decay rate, one obtains $[15]$

$$\Gamma^{J=1/2}_{^3\Lambda H} = \frac{\bar{q}}{1 + \omega_\pi(\bar{q})/E_{3N}(\bar{q})}(|s_\pi|^2(1 + \frac{1}{2} \eta(\bar{q})) + |p_\pi|^2(\frac{\bar{q}}{q_\Lambda})^2(1 - \frac{5}{6} \eta(\bar{q})))]. \quad (2)$$

In this equation we have omitted terms of order 0.5% of $\Gamma(\bar{q})$ that correct for the use of $\bar{q}$ in the two-body $^3\Lambda H \to \pi + ^3Z$ rate expressions $[17]$. We note that applying the $\Delta I = \frac{1}{2}$ rule to the isospin $I = 0$ decaying $^3\Lambda H_{g.s.}$, here too as in the free $\Lambda$ decay, the ratio of $\pi^-:\pi^0$ decay rates is approximately 2:1. The quantity $\eta(q)$ in Eq. (2) is an exchange integral ensuring that the summation on final nuclear states is limited to totally antisymmetric states:

$$\eta(q) = \int \chi(r_\Lambda; \bar{r}_{N2}, \bar{r}_{N3}) \exp[i\bar{q} \cdot (\bar{r}_{N2} - \bar{r}_{N3})] \chi^*(\bar{r}_{N2}; \bar{r}_{\Lambda}, \bar{r}_{N3}) d^3\bar{r}_\Lambda d^3\bar{r}_{N2} d^3\bar{r}_{N3}. \quad (3)$$

Here $\chi(r_\Lambda; \bar{r}_{N2}, \bar{r}_{N3})$ is the real normalized spatial wavefunction of $^3\Lambda H$, symmetric in the nucleon coordinates 2 and 3. This wavefunction, in abbreviated notation $\chi(1; 2, 3)$, is associated with a single spin-isospin term which is antisymmetric in the nucleon labels, such that $s_\Lambda = \frac{1}{2}$ couples to $s_1 + s_2 = 1$ to give $S_{tot} = \frac{1}{2}$ for the ground state and $S_{tot} = \frac{3}{2}$ for the spin-flip excited state.
(if bound), and \( t_\Lambda = 0 \) couples trivially with \( \vec{t}_1 + \vec{t}_2 = 0 \). Eq. (2) already accounts for this spin-isospin algebra in \(^3\Lambda H\). For completeness we also list the total decay rate expression for \(^3\Lambda H\) if its g.s. spin-parity were \( J^P = \frac{3^+}{2} \):

\[
\Gamma_{^3\Lambda H}^{J=3/2} = \frac{\bar{q}}{1 + \omega_\pi(\bar{q})/E_{3N}(\bar{q})} \left[ |s_\pi|^2 (1 - \eta(\bar{q})) + |p_\pi|^2 (\frac{\bar{q}}{q_\Lambda})^2 (1 - \frac{1}{3} \eta(\bar{q})) \right].
\]

(4)

Since \( 0 < \eta(q) < 1 \), the dominant s-wave term here is weaker than in the free \( \Lambda \) decay, Eq. (1), implying that the \(^3\Lambda H\) lifetime would have been longer than the free \( \Lambda \) lifetime, had its g.s. spin-parity been \( \frac{3^+}{2} \).

2.2. \(^3\Lambda n\)

For \(^3\Lambda n\) \((I = 1, J^P = \frac{1^+}{2})\) weak decay, it is necessary to distinguish between decays induced by \( \Lambda \to p + \pi^- \) and those induced by \( \Lambda \to n + \pi^0 \). In the first case the spectator neutrons are ‘frozen’ to their s shell in both initial and final state, without having to recouple spins or consider exchange integrals for the final proton. This means that the \(^3\Lambda n \to (pn) + \pi^- \) weak decay rate will be given in the closure approximation essentially by the \( \Lambda \to p + \pi^- \) free-space weak-decay rate. In the other case of \( \Lambda \to n + \pi^0 \) induced decays, production of a low-momentum neutron is suppressed by the Pauli principle on account of the two neutrons already there in the initial \(^3\Lambda n\) state. To a good approximation this \(^3\Lambda n\) weak decay branch may be disregarded. Our best estimate for the \(^3\Lambda n\) weak decay rate is then given by

\[
\Gamma_{^3\Lambda n}^{J=1/2} \approx \frac{\bar{q}}{1 + \omega_\pi(\bar{q})/E_{3N}(\bar{q})} 0.641 \left( |s_\pi|^2 + |p_\pi|^2 (\frac{\bar{q}}{q_\Lambda})^2 \right),
\]

(5)

where the coefficient 0.641 is the free-space \( \Lambda \to p + \pi^- \) fraction of the total \( \Lambda \to N + \pi \) weak decay rate. Evaluating the ratio \( \Gamma(^3\Lambda n)/\Gamma_\Lambda \) for the choice \( \bar{q} = q_\Lambda \) one obtains

\[
\Gamma(^3\Lambda n)/\Gamma_\Lambda \approx 1.114 \times 0.641 = 0.714,
\]

(6)

where the factor 1.114 follows from the difference between \( E_{3N}(q_\Lambda) \) and \( E_N(q_\Lambda) \) in the recoil phase-space factors. Our predicted \(^3\Lambda n\) lifetime is then

\[
\tau(^3\Lambda n) \approx 368 \text{ ps},
\]

(7)

but likely not shorter than 350 ps upon assigning 5% contribution from the \( \pi^0 \) decay branch. This lifetime is way longer than the \( 181^{+30}_{-24} \pm 25 \text{ ps} \) or \( 190^{+47}_{-35} \pm 36 \text{ ps} \) lifetimes deduced from the \( nd\pi^- \) and \( t\pi^- \) alleged decay modes of \(^3\Lambda n\). Note that adding a potentially unobserved proton could perhaps reconcile these deduced lifetimes with \( \tau(^4\Lambda H)=194^{+24}_{-26} \text{ ps} \).
3. $^3\Lambda\bar{H}$ lifetime calculation disregarding pion FSI

A lesson gained from Ref. [17] is that as long as the binding energy of $^3\Lambda\bar{H}$ is reproduced, the lifetime calculation is rather insensitive to the fine details of the particular $\Lambda N$ interaction model chosen. The main uncertainty in lifetime calculations arises in fact from the imprecisely known value of $B_\Lambda(^3\Lambda\bar{H})$. Therefore, to have as simple input as possible to a three-body description of the weakly bound $^3\Lambda\bar{H}$ we constructed baryon-baryon $s$-wave separable interactions of Yamaguchi forms by fitting to the corresponding low-energy scattering parameters. In particular, the binding energy of the deuteron, limited to a $^3S_1$ channel, is reproduced. For the $\Lambda N$ interaction we followed Ref. [26] by choosing values of scattering lengths and effective ranges close to those used in Nijmegen models. Scaling slightly the $\Lambda N$ $^1S_0$ interaction we covered, by solving three-body Faddeev equations, a range of $\Lambda$ separation energy $B_\Lambda$ values to account for the given uncertainty in the experimental value $B_\Lambda(^3\Lambda\bar{H})=0.13\pm0.05$ MeV. In these calculations the Faddeev integral equations were solved using a momentum-space Gauss mesh of 32 points, with half the integration points satisfying $q < 1$ fm$^{-1}$, thereby taking good care of the small $q$ (large $r$) region which is of utmost importance for the diffuse $^3\Lambda\bar{H}$. The results prove numerically stable already upon using 20 Gauss mesh points.

To calculate the total decay rate, Eq. (2), we evaluated the exchange integral $\eta(q)$ of Eq. (3) by using a wavefunction $\Psi(\vec{p}_1, \vec{q}_1)$, derived by solving the appropriate Faddeev equations in momentum space in terms of two Jacobi coordinates, say $\vec{p}_1$ for the relative coordinate of the two nucleons and $\vec{q}_1$ for that of the $\Lambda$ with respect to the center of mass of the nucleons; for details see Eqs. (10–12) and related text in our arXiv:1811.03842v1 [nucl-th] version. Eq. (3) is thereby transformed to

$$\eta(q) = \int \Psi(\vec{p}_1, \vec{q}_1) \Psi^*(\frac{1}{2} \vec{p}_1 + \frac{3}{4} \vec{q}_1 - \frac{1}{2} \vec{q}, \vec{p}_1 - \frac{1}{2} \vec{q}_1 + \vec{q}) \, d^3\vec{p}_1 \, d^3\vec{q}_1.$$  

(8)

To evaluate this form of the exchange integral $\eta(q)$ one needs to express the Faddeev three-body wavefunction $\Psi$ as a function of the two variables $\vec{p}_1$ and $\vec{q}_1$. This requires careful attention since the Faddeev decomposition $T = T_1 + T_2 + T_3$ of the total $T$ matrix into three partial $T_j$ matrices coupled to each other by the Faddeev equations $T_j = t_j(1+G_0 \sum_{k \neq j} T_k)$ implies a similar decomposition of the bound-state wavefunction $\Psi$ into three components

$$\Psi = G_0 T\phi = G_0(T_1 + T_2 + T_3)\phi \equiv \Psi_1 + \Psi_2 + \Psi_3,$$  

(9)
where \( G_0 \) is the three-body free Green's function and \( \phi \) is a three-body plane wave. The natural momentum variables for each \( \Psi_j \) component are \( \vec{p}_j \) and \( \vec{q}_j \), so we need to switch in \( \Psi_2 \) and \( \Psi_3 \) from their respective momentum bases to \( \vec{p}_1 \) and \( \vec{q}_1 \). This naturally involves integration on the angle between \( \vec{p}_j \) and \( \vec{q}_j \), \( j \neq 1 \), so that the redressed \( \Psi_2 \) and \( \Psi_3 \) necessarily develop \( \ell > 0 \) partial waves in addition to their dominant s-wave component. We have omitted such unwanted \( \ell \neq 0 \) partial waves. The error incurred in this approximation may be estimated by evaluating the normalization integral of \( \Psi \) in two ways, first with each component \( \Psi_j \) in its natural coupling scheme, and then with all three components expressed in terms of the \( \vec{p}_1, \vec{q}_1 \) variables, thus giving rise to a normalization integral smaller by about 2.5%.

The \(^3\Lambda\)H exchange integral \( \eta(\bar{q}) \) of Eq. (8) was evaluated numerically for two values of the closure momentum \( \bar{q} \) discussed in Ref. [15] and for several values of \( B_{\Lambda}(\Lambda^3\H) \) suggested by its experimental uncertainty. Our results are listed in Table 1 compared with those of Congleton [17] who considered the same range of \( B_{\Lambda}(\Lambda^3\H) \) values.

| Model          | \( \bar{q} \) (MeV/c) | \( \eta(\bar{q}) \)    |
|----------------|------------------------|------------------------|
| \( \Lambda d \) cluster [17] | 96                     | 0.212±0.011            |
| \( \Lambda pn \) Faddeev [present] | 96                     | 0.146±0.021            |
| \( \Lambda pn \) Faddeev [present] | 104                    | 0.130±0.021            |

As argued by RD [15], and followed up by Congleton [17], the appropriate choice for the \(^3\Lambda\)H pionic decay closure momentum \( \bar{q} \) is the empirical peak value \( \bar{q} = 96 \) MeV/c in the \( \pi^- \) weak decay continuum spectrum. To study the sensitivity of \( \eta(\bar{q}) \) to a small departure from this accepted value of \( \bar{q} \), we also evaluated \( \eta(\bar{q}) \) for \( \bar{q} = 104 \) MeV/c, a value a bit larger than that for the free \( \Lambda \) decay which was used in calculations that preceded RD. The variation of \( \eta(\bar{q}) \) with \( \bar{q} \) over the momentum interval studied is quite weak. For \( \bar{q} = 96 \) MeV/c, our calculated value of \( \eta(\bar{q}) \) is about 70% of Congleton's value. This apparent discrepancy can be shown to arise from his use of a \( \Lambda d \) cluster model for \(^3\Lambda\)H: specifically by (i) limiting the full Faddeev wavefunction \( \Psi \), Eq. (9), to \( \Psi_1 \) which is the component most natural to represent a \( \Lambda d \) cluster, and (ii)
suppressing then in the three-body free Green’s function $G_0$ the dependence on the $\Lambda$ momentum, we obtain a value of $\eta(\bar{q} = 96 \text{ MeV}/c) = 0.238 \pm 0.038$, in good agreement with the value listed for $'\Lambda d$ cluster’ in Table 1.

Using $\eta(\bar{q} = 96 \text{ MeV}/c) = 0.146 \pm 0.021$ from Table 1 in Eq. (2) for $^3\Lambda H$, and noting Eq. (1) for the free $\Lambda$ decay, we obtain the following $^3\Lambda H$ mesonic decay rate:

$$\Gamma_{^3\Lambda H}^{J=1/2}/\Gamma_\Lambda = 1.09 \pm 0.01,$$

or $\tau(^3\Lambda H)/\tau_\Lambda = 0.92 \pm 0.01$. (10)

Adding a $\approx 1.7\%$ nonmesonic weak decay branch [18], our final result is

$$\tau(^3\Lambda H)/\tau_\Lambda = 0.90 \pm 0.01.$$ (11)

4. Pion FSI effects

The pion emitted in the $^3\Lambda H$ decay is dominantly $s$-wave pion, see Eq. (1). Optical model fits of measured $1s$ pionic atom level shifts and widths across the periodic table [21, 22] suggest that the underlying $\pi N$ $s$-wave interaction term in nuclei at low energy is weakly repulsive and that the attractive $\pi N$ $p$-wave term has negligible effect on $1s$ pionic states. The corresponding $s$-wave induced $\pi^-$ nuclear optical potential is given by

$$V_{\pi^-}^{\text{opt}} = -\frac{4\pi}{2\mu_{\pi N}} \left( b_0 [\rho_n(r) + \rho_p(r)] + b_1 [\rho_n(r) - \rho_p(r)] \right),$$ (12)

in terms of fitted real $\pi N$ scattering lengths: isoscalar $b_0 = -0.0325$ fm and isovector $b_1 = -0.126$ fm [29]. With these negative signs one gets $\pi^-$ repulsion in the majority of stable nuclei, those with $N \geq Z$. However, in the few $Z > N$ available nuclear targets like $^1H$ and $^3$He this repulsion is reversed into attraction owing to the isovector term flipping sign under $Z \leftrightarrow N$. This is confirmed by the attractive $1s$ level shifts observed in the $\pi^-1H$ and $\pi^-3He$ atoms [30]. One therefore expects attractive FSI in the $^3$He+$\pi^-$ decay channel of $^3\Lambda H$, and repulsive FSI in the $^3H+\pi^0$ decay channel where the $\pi N$ isovector term associated with $b_1$ vanishes while the weakly repulsive isoscalar term associated with $b_0$ remains in effect. Altogether we have verified that the sum of these two FSI contributions to the $^3\Lambda H$ decay

\footnote{We disregard here a weak two-nucleon absorptive term in order to retain the few percent unobserved decay branch $^3\Lambda H \rightarrow pnn$.}
rate is nearly zero. Pion FSI in the context of $^3\Lambda\text{H}$ decay has been considered elsewhere only by RD [15], who indeed found it weakly repulsive and lowering the $^3\Lambda\text{H}$ decay rate by 1.3%, and in Ref. [31] where the attraction in the $^3\text{He}+\pi^-$ decay channel was overlooked.

The preceding argumentation on the role of pion FSI in the $^3\Lambda\text{H}$ decay is incomplete, if not misleading. The $\Delta I = \frac{1}{2}$ rule in the $\Lambda \to N\pi$ weak decay implies that the $3N + \pi$ final states are good ($I = \frac{1}{2}, I_z = -\frac{1}{2}$) isospin states which are coherent combinations of $ppn\pi^-$ and $pnn\pi^0$ configurations. For $I = \frac{1}{2}$, and in the Born approximation applied to the optical potential Eq. (12), the pion-nuclear scattering length is given by $3b_0 - 2b_1$ which is considerably more attractive than the scattering length $3b_0 - b_1$ valid for $^3\text{He}+\pi^-$ alone. The difference between these two expressions arises from charge exchange transitions between the nearly degenerate charge states of $(I = \frac{1}{2}, I_z = -\frac{1}{2})$ good-isospin states, such as between $^3\text{He}+\pi^-$ and $^3\text{H}+\pi^0$.

To estimate the effect of pion FSI on the $^3\Lambda\text{H}$ lifetime we consider $^3\Lambda\text{H}$ decays to good-isospin states made of the corresponding two nuclear charge states. In the distorted-wave (DW) approximation, the $^3\Lambda\text{H}$ decay amplitude is given by a form factor

\[ F_{\text{DW}}(q) = \int \Phi_{3N}^*(\vec{r},\vec{\rho}) \tilde{j}_0(qr_N) \Phi_{^3\Lambda\text{H}}(\vec{r},\vec{\rho}) \, d^3r \, d^3\rho, \]  

(13)

where $\tilde{j}_0$ is a pion DW evolving via FSI from a pion plane-wave (PW) spherical Bessel function $j_0$. The vectors $\vec{r}$ and $\vec{\rho}$ are Jacobi coordinates: $\vec{r}$ stands for the $\Lambda \to N$ ‘active’ baryon relative to the cm of the spectator nucleons, and $\vec{\rho}$ denotes the relative coordinate of the spectator nucleons. In Eq. (13), $\vec{r}_N = \frac{2}{3}\vec{r}$ stands for the coordinate of the ‘active’ baryon with respect to the cm of the $3N$ final system. The $\Phi_\alpha$ are properly normalized $L = 0$ initial and final $A = 3$ wavefunctions. For this first evaluation we approximated each $\Phi_\alpha(\vec{r},\vec{\rho})$ by a product form $\psi_\alpha(r)\phi_\alpha(\rho)$, with single-baryon bound-state wavefunctions $\psi_\alpha(r)$ given by

\[ r\psi_\alpha(r) \sim \exp(-\kappa_\alpha r) - \exp(-\beta_\alpha r) \]  

(14)

as generated by Yukawa separable potentials. The choice of the inverse range parameters $\kappa_\alpha$ and $\beta_\alpha$ is discussed below following Table 2. With a product form of $\Phi_\alpha(\vec{r},\vec{\rho})$, the form factor (13) reduces to

\[ F_{\text{DW}}(q) = \gamma d \int \psi_N^*(r) \tilde{j}_0(qr_N) \psi_\Lambda(r) \, d^3r, \]  

(15)

9
where
\[ \gamma_d = \int \phi^*_{3N}(\rho)\phi^*_{\Lambda H}(\rho) d^3\rho \]  
(16)
is the overlap integral of the two \( \phi \)s and is the same for both PW and DW pions. For this reason, the choice of the ‘deuteron’ wavefunctions \( \phi_\alpha(\rho) \) is not discussed further here. For the pion DW \( \tilde{j}_0 \) we used a continuum wavefunction, also generated from a separable Yukawa potential:
\[ \tilde{j}_0(qr_N) = j_0(qr_N) + \frac{f(q)}{r_N} (\exp(iqr_N) - \exp(-\beta_\pi r_N)) , \]  
(17)
where \( f(q) \) is a \( \pi \)-nuclear s-wave scattering amplitude derived from SAID \[32\] \( \pi N \) partial-wave amplitudes, with values also listed in Table 2.

| Decay to          | \( \kappa_\Lambda \) | \( \kappa_N \) | \( \beta_\Lambda \) | \( \beta_N \) | \( \beta_\pi \) | \( f(q) \)          | \[|F_{DW}/F_{PW}|^2\] |
|------------------|----------------------|----------------|-------------------|---------------|----------------|----------------|-------------------|
| \( \pi^+{}^3\text{Z} \) | 0.068                | 0.420          | 1.2               | 1.2           | 0.806          | 0.180+i0.048   | 1.097             |
| \( \pi+N+d \)    | 0.068                | 0.068          | 1.2               | 1.2           | 1.626          | 0.225+i0.022   | 1.119             |

4.1. Two-body decay modes

The input parameters for the evaluation of pion FSI effects on the two-body decay modes \( {}^3\Lambda H \rightarrow \pi^+{}^3\text{Z} \) are given in the first row of Table 2. The listed values of each wave number \( \kappa_\alpha \), Eq. (14), follow from the separation energy of the corresponding baryon \( \alpha \) with respect to the deuteron in the initial hypernucleus and final nucleus. The choice of \( \beta_N \) corresponds to r.m.s. radius \( r^2 > \frac{1}{2} \psi_N = 2.631 \text{ fm} \) which for a spatially symmetric \( {}^3\text{He} \) wavefunction reproduces its matter r.m.s. radius \( r^2 > \frac{1}{2} \psi_{\Lambda H} = 1.754 \text{ fm} \). The choice of \( \beta_\Lambda \) hardly matters in reproducing the expectedly large r.m.s. radius of the loosely bound \( {}^3\Lambda H \), which is 10.4 fm for \( \Lambda \) relative to \( d \) using the asymptotic term \( \exp(-\kappa_\Lambda r) \) of \( \psi_\Lambda \). For convenience we chose \( \beta_\Lambda = \beta_N \). As for the pion DW, Eq. (17), \( \beta_\pi \) was determined by equating the r.m.s. radius of a Yukawa form factor \( g(r_N) = \exp(-\beta_\pi r_N) \), for a separable pion potential \( g(r_N)g(r'_{\pi}) \), to the \( {}^3\text{He} \) matter r.m.s. radius: \( \beta_\pi = \sqrt{2}/1.754 \text{ fm}^{-1} \). Finally, the value of the pion-nuclear scattering amplitude \( f(q) = 3b_0 - 2b_1 \) listed in the table.
was derived using values of $b_0$ and $b_1$ taken from SAID \cite{32} at 31 MeV pion kinetic energy, compared to 0.155 fm for the threshold values listed following Eq. (12). The variation of both $b_0$ and $b_1$ with energy in the SAID analysis is rather weak. In the actual calculation we dropped the small imaginary part of $f(q)$ so as to account also for the few percent pion absorption contribution $^3\Lambda H \rightarrow pnn$ to the $^3\Lambda H$ lifetime.

Evaluating the form factors $F_{\text{DW}}(q)$ and $F_{\text{PW}}(q)$, where the latter is obtained from the former by reducing $j_0$ to $j_0$, we find a pion FSI enhancement factor $|F_{\text{DW}}(q)/F_{\text{PW}}(q)|^2 = 1.097$ for the $^3\Lambda H$ decay rate to the two-body final states $^3\text{He}+\pi^-$ and $^3\text{H}+\pi^0$. With this enhancement, these two-body decay modes take roughly 35.7% of the total decay rate in the present $s$-wave calculation, in agreement with the branching ratio $0.35\pm0.04$ extracted from $\pi^-$ decays in helium bubble chamber measurements \cite{8}.

### 4.2. Three-body decay modes

Evaluating the pion FSI in the three-body $^3\Lambda H$ decays to $p+d+\pi^-$ and $n+d+\pi^0$ final states is more involved than done above for the two-body decay modes because the nuclear bound state wavefunction $\psi_N$ has to be replaced by continuum nucleon wavefunctions. At this stage we report on a rough approximation that still uses $\psi_N$ of the form (14), but for a loosely bound nucleon on its way to become unbound. We chose for convenience $\kappa_N = \kappa_\Lambda$, as listed in the second row of Table 2, having verified that the resulting enhancement factor $|F_{\text{DW}}(q)/F_{\text{PW}}(q)|^2$ hardly changes near threshold, although each of the separate form factors does. For the pion distorted wave we assumed it is dominated by FSI with the outgoing nucleon, since the low-energy $\pi d$ interaction is much weaker than its $\pi N$ counterpart. The pion-nucleon inverse-range parameter $\beta_\pi$ listed in the table corresponds to a nucleon r.m.s. radius of 0.87 fm, and its scattering amplitude listed in the table corresponds to a combination $f(q) = b_0 - 2b_1$ using values of $b_0$ and $b_1$ again derived from SAID \cite{32}. This rough estimate gives a 1.119 enhancement factor for the three-body decay modes.

Given the $^3\Lambda H$ decay rate enhancement factors $|F_{\text{DW}}(q)/F_{\text{PW}}(q)|^2$ listed in Table 2 for two- and three-body final states, which according to Ref. \cite{16} saturate more than 98% of the pionic decay modes, the total pion FSI enhancement factor is $(11\pm2)$%, where the 2% uncertainty was estimated by varying the $\Lambda$ and $N$ inverse-range parameters listed in the table within reasonable limits. Varying the pion inverse-range parameter $\beta_\pi$ introduces larger
uncertainty, and this will have to be studied more quantitatively in future four-body calculations.

5. Conclusion

In this work we evaluated the mesonic decay rate of $^3\Lambda H$ by considering the closure-approximation decay-rate expression Eq. (2). The $^3\Lambda H$ exchange integral $\eta(\bar{q})$, Eq. (3), which provides input to Eq. (2) was calculated within a fully three-body Faddeev equations model of $^3\Lambda H_{g.s.}$. For $\bar{q} = 96$ MeV/c, as suggested by RD [15], our calculated value of $\eta(\bar{q})$ listed in Table 1 leads to a (9±1)% increase of the $^3\Lambda H$ mesonic decay rate over the free $\Lambda$ decay rate. This result supersedes Congleton’s result [17] of 12% increase based on a $\Lambda d$ cluster model of $^3\Lambda H$ which gave a value of $\eta(\bar{q})$ about 50% higher than our Faddeev equations model value. Adding a 1.7% nonmesonic decay rate contribution [18] we get a (10±1)% decrease of the $^3\Lambda H$ lifetime with respect to the free $\Lambda$ lifetime, a stronger decrease than the 6% derived in the more microscopically oriented Faddeev calculation by Kamada et al. [16]. Neither of these calculations is sufficient on its own to resolve the $^3\Lambda H$ lifetime puzzle.

Perhaps more importantly, we considered the pion FSI effect on the $^3\Lambda H$ lifetime. This effect goes beyond any pion FSI effect already present in the empirical $\Lambda \to N + \pi$ decay parameters $s_\pi$ and $p_\pi$. Simple arguments rooted in low-energy pion-nucleon and pion-nucleus phenomenology were shown to imply an attractive FSI, an observation that escaped the attention of all previous works. This attractive pion FSI was evaluated here semi-quantitatively and found to shorten further the $^3\Lambda H$ lifetime down to (81±2)% of $\tau_\Lambda$. Further, although little reduction of $\tau(^3\Lambda H)$ could arise from attractive $p$-wave pion FSI contributions. More involved calculations going beyond three-body calculations are required to verify the overall substantial reduction owing to the pion FSI in this $A = 3$ system.

Last, as a by-product of our formulation of the $A = 3$ hypernuclear lifetime, we showed in simple terms that the lifetime of $^3\Lambda n$, if bound, is considerably longer than $\tau_\Lambda$, in disagreement with the shorter lifetime with respect to $\tau_\Lambda$ extracted from the HypHI events assigned to this hypernucleus. Pion FSI should be repulsive in this case, aggravating this disagreement by increasing further the $^3\Lambda n$ lifetime by a few more percents.
Acknowledgements

A.G. acknowledges useful discussions with, and advice from Eli Friedman on pion FSI. H.G. acknowledges support by COFAA-IPN (México).

References

[1] A. Gal, E.V. Hungerford, D.J. Millener, Rev. Mod. Phys. 88 (2016) 035004.
[2] M.M. Block, et al., Proc. Int’l. Conf. on Hyperfragments 1963, Ed. W.O. Lock (CERN 64-1, Geneva, 1964) pp. 63-74; see Fig. 6.
[3] R.J. Prem, P.H. Steinberg, Phys. Rev. 136 (1964) B1803.
[4] G. Keyes, et al., Phys. Rev. Lett. 20 (1968) 819.
[5] R.E. Phillips, J. Schneps, Phys. Rev. 180 (1969) 1307.
[6] G. Bohm, et al., Nucl. Phys. B 16 (1970) 46.
[7] G. Keyes, et al., Phys. Rev. D 1 (1970) 66.
[8] G. Keyes, J. Sacton, J.H. Wickens, M.M. Block, Nucl. Phys. B 67 (1973) 269.
[9] B.I. Abelev, et al. (STAR Collaboration), Science 328 (2010) 58.
[10] C. Rappold, et al. (HypHI Collaboration), Nucl. Phys. A 913 (2013) 170.
[11] J. Adam, et al. (ALICE Collaboration), Phys. Lett. B 754 (2016) 360.
[12] L. Adamczyk, et al. (STAR Collaboration), Phys. Rev. C 97 (2018) 054909.
[13] S. Trogolo, et al. (ALICE Collaboration), Nucl. Phys. A 982 (2019) 815.
[14] P. Braun-Munzinger, B. Döningus, Nucl. Phys. A 987 (2019) 144.
[15] M. Rayet, R.H. Dalitz, Nuovo Cimento 46 A (1966) 786.
[16] H. Kamada, J. Golak, K. Miyagawa, H. Witała, W. Glöckle, Phys. Rev. C 57 (1998) 1595.
[17] J.G. Congleton, J. Phys. G 18 (1992) 339.

[18] J. Golak, K. Miyagawa, H. Kamada, H. Witala, W. Glöckle, A. Parreño, A. Ramos, C. Bennhold, Phys. Rev. C 55 (1997) 2196, Erratum: Phys. Rev. C 56 (1997) 2892.

[19] G. Bhamathi, K. Prema, Nuovo Cimento 62 A (1969) 661, 63 (1969) 555.

[20] H.M.M. Mansour, K. Higgins, Nuovo Cimento 51 A (1979) 180.

[21] C.J. Batty, E. Friedman, A. Gal, Phys. Rep. 287 (1997) 385.

[22] E. Friedman, A. Gal, Phys. Rep. 452 (2007) 89.

[23] C. Rappold, et al. (HypHI Collaboration), Phys. Rev. C 88 (2013) 041001(R).

[24] H. Garcilazo, A. Valcarce, Phys. Rev. C 89 (2014) 057001

[25] E. Hiyama, S. Ohnishi, B.F. Gibson, Th.A. Rijken, Phys. Rev. C 89 (2014) 061302(R).

[26] A. Gal, H. Garcilazo, Phys. Lett. B 736 (2014) 93; for discussion of the $NN$ and $YN$ interaction input see also H. Garcilazo, J. Phys. G, 13 (1987) L63.

[27] T. Saito, et al. (HypHI Collaboration), Nucl. Phys. A 954 (2016) 199.

[28] H. Outa, et al., Nucl. Phys. A 639 (1998) 251.

[29] E. Friedman, A. Gal, Nuc. Phys. A 928 (2014) 128.

[30] D. Gotta, Prog. Part. Nuc. Phys. 52 (2004) 133, updated in D. Gotta, et al., Lect. Notes Phys. 745 (2008) 165.

[31] N.G. Kelkar, Mod. Phys. Lett. A 12 (1997) 511.

[32] SAID program, gwdac.phys.gwu.edu, see R.A. Arndt, W.J. Briscoe, I.I. Strakovsky, R.L. Workman, Phys. Rev. C 74 (2006) 045205.