Mass stability in classical Stueckelberg-Horwitz-Piron electrodynamics

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Abstract. It is well-known that the 5D gauge structure of Stueckelberg-Horwitz-Piron (SHP) electrodynamics permits the exchange of mass between particles and the electromagnetic fields induced by their motion, even at the classical level. This phenomenon presents two closely related problems: (1) Under what circumstances can real particles evolve sufficiently off-shell to account for mass changing phenomena such as flavor-changing neutrino interactions and low energy nuclear reactions? (2) What accounts for the stability of the measured masses of the known particles? To approach these questions, we first propose a toy model in which a particle evolving through a complex charged environment can acquire a significant mass shift for a short time. We then consider a classical self-interaction that tends to restore on-shell propagation.

1. Introduction

Stueckelberg-Horwitz-Piron (SHP) electrodynamics (see [1] to [20] and references therein) is a theory of interacting events that has been shown [8] to be the most general dynamical system consistent with the quantum commutation relations

\[
[x^\mu, x^\nu] = 0 \\
m[x^\mu, \dot{x}^\nu] = -i\hbar g^{\mu\nu}(x)
\]

defined on an unconstrained 8D phase space. The velocity \(\dot{x}^\mu(\tau)\) is the derivative of the event \(x^\mu(\tau)\) with respect to the evolution parameter \(\tau\). The absence of constraints among the four velocities has three immediate and closely related consequences. First, \(\tau\) cannot be identified with the proper time of the motion, but is proportional to it through

\[
ds^2 = -g_{\mu\nu}dx^\mu dx^\nu = -\dot{x}^2(\tau)\,d\tau^2
\]

where \(\dot{x}^2(\tau)\) is a dynamical quantity. The parameter \(\tau\) must be understood as a monotonically increasing chronological time, independent of the spacetime coordinates and thus similar to the external time in nonrelativistic Newtonian mechanics. Second, the system is invariant under \(\tau \rightarrow \tau + a\) for constant \(a\) but is not generally reparametrization invariant. Therefore, the dynamical particle evolution is not restricted to a mass shell. In particular, events and the gauge fields they induce may exchange mass, although the total mass of the particles and fields is conserved. Third, an event may reverse direction in coordinate time \(x^0 = ct\), implementing the Stueckelberg model in which a pair process is represented by a single worldline, generated
dynamically by the interaction of a spacetime event $x^\mu(\tau)$ with an electromagnetic field, as $\tau$ grows monotonically from $\tau = -\infty$ to $\tau = \infty$.

Pair creation/annihilation at the classical level is an example of a phenomenon that cannot occur without mass exchange. In the continuous evolution of an event $x^\mu(\tau)$ from a future trajectory $\dot{x}^0 \geq c$ to the past $\dot{x}^0 \leq -c$ the event must pass through the spacelike region around $\dot{x}^0 = 0$, where $\dot{x}^2$ changes sign. It was shown in [17] that under certain conditions SHP electrodynamics permits these classical pair processes, with no \textit{a priori} guarantee that the outgoing particles will have identical mass. Small changes in particle mass are similarly relevant to neutrino oscillations [16] and low energy nuclear reactions [21]. As a practical matter such shifts in the asymptotic mass will be small [19], but even small mass shifts might be observable in a way that poses empirical difficulties for the theory.

Horwitz has recently shown [20] that by modeling a particle as an ensemble of events, the total particle mass is determined by a chemical potential. Following collisions governed by a general class of interactions that includes pair processes, particles return to their equilibrium mass values. This result may be related to another mass-related feature of SHP electrodynamics. The invariance of the action under $\tau \to \tau + a$ leads to singularities in classical currents that have been treated by regarding the particle as an ensemble of events $x^\mu(\tau + \Delta \tau)$ along the worldline, where $\Delta \tau$ is distributed as $\exp(-|\Delta \tau|/\lambda)$ and $\lambda$ is parameter with dimensions of time. The low-energy interaction in the theory then takes the form of a Yukawa potential where the photon possesses a mass spectrum of width $m_\gamma \sim \hbar / \lambda c^2$.

In this paper we propose a specific model that suggests a deterministic relaxation of off-shell to on-shell propagation, along with restoration of the fixed mass. In section 2 we present a quick overview of SHP interactions, by way of field equations and the Lorentz force. In section 3 we motivate the discussion of mass shift by showing a toy model in which a particle evolving through a complex charged environment can acquire a significant mass shift for a short time. In section 4 we consider a classical self-interaction in SHP that tends to restore on-shell propagation.

2. Interactions in SHP

In its present form, SHP electrodynamics is a U(1) gauge theory in which the $\tau$-dependence of gauge transformations requires five compensation potentials. In analogy with the notation $x^0 = ct$ we associate a constant $c_5$ with the invariant time $\tau$, and adopt the formal designations

$$x^5 = c_5 \tau \quad \partial_5 = \frac{1}{c_5} \partial_\tau \quad g_{55} = g^{55} = \pm 1$$

and the index convention

$$\mu, \nu = 0, 1, 2, 3 \quad \alpha, \beta, \gamma = 0, 1, 2, 3, 5.$$  \hspace{1cm} (4)

Because the low energy Coulomb particle-particle interaction differs [19] from the particle-antiparticle interaction by a factor $1 \mp g_{55}(c_5/c)$ we take $c_5/c$ to be small.

A classical event $X^\mu(\tau)$ induces an instantaneous five-component current density

$$j^\mu_p(x, \tau) = c \int ds \, \varphi(\tau - s) \hat{X}^a(s) \delta^4[x - X(s)]$$

(5)
where
\[ \varphi(\tau) = \frac{1}{2\xi} e^{-|\tau|/2\lambda} \]
and \( \lambda \) is a parameter with units of time. These currents induce five gauge potentials \( a^a(x, \tau) \) and field strengths
\[ f^{\alpha\beta} = \partial^\alpha a^\beta - \partial^\beta a^\alpha \]
leading to \( \tau \)-dependent field equations
\[ \partial_\beta f^{\alpha\beta}(x, \tau) = \frac{e}{c} j^\alpha(\tau) \quad \partial_a f_{\alpha\gamma} + \partial_{a\gamma} f_{\alpha\beta} + \partial_{\alpha\gamma} f_{a\beta} = 0 \]
which are formally similar to Maxwell’s equations in 5D, and are called pre-Maxwell equations. The fields act on the events through the Lorentz force
\[ M \ddot{x}^\mu = \frac{e}{c} f^{\mu}_{a}(x, \tau) \dot{x}^a \]
which entails mass exchange through
\[ \frac{d}{d\tau}(-\frac{1}{2}M \dot{x}^2) = -M \ddot{x}^\mu \dot{x}_\mu = -\frac{e}{c} x^\mu (cS_{\mu\beta} + f_{\mu\nu} x^\nu) = \frac{ecS}{c} x^\mu f_{\mu\beta} = g_{55} \frac{ecS}{c} f_{\mu\beta} \dot{x}_\mu \ . \]

The pre-Maxwell equations in Lorenz gauge lead to the wave equation
\[ \partial_\beta \partial^\beta a^\alpha = (\partial_\rho \partial^\mu + \partial_\tau \partial^\tau) a^\alpha = (\partial_\rho \partial^\mu + \frac{g_{55}}{c^2} \partial_\tau^2) a^\alpha = -\frac{e}{c} j^\alpha(\tau, \tau) \]
for which the principal part Green’s function was found in [6] to be
\[ G_p(x, \tau) = -\frac{1}{2\pi} \delta(x^2) \delta'(\tau) - \frac{cS}{2\pi^2} \partial \theta(-g_{55}g_{a\beta}x^a x^\beta) \frac{1}{\sqrt{-g_{55}g_{a\beta}x^a x^\beta}} . \]

The term \( G_{\text{Maxwell}} \) imposes \( \tau \)-instantaneous lightlike causality in spacetime, while the support of \( G_{\text{Correlation}} \) depends on the sign of \( g_{55} \). If \( g_{55} = 1 \), \( G_{\text{Correlation}} \) has timelike support with respect to the event trajectory, opening the possibility of a self-interaction of a type not present in standard Maxwell theory. We take \( g_{55} = 1 \) in the remaining sections of this paper. Evaluating the derivative in (12) for this case,
\[ G_{\text{Correlation}}(x, \tau) = -\frac{cS}{2\pi^2} \left( \frac{1}{2} \frac{\theta(-x^2 - c^2 \tau^2)}{(-x^2 - c^2 \tau^2)^{3/2}} - \frac{\delta(-x^2 - c^2 \tau^2)}{(-x^2 - c^2 \tau^2)^{1/2}} \right) . \]

Although this expression appears highly singular, the two terms cancel when calculating potentials, as long as the subtraction is performed before applying the limits of integration.
The form of $G_{\text{Maxwell}}$ allows us apply standard techniques associated with the Liénard-Wiechert potential. Writing

$$a^a (x, \tau) = -\frac{e}{c} \int d^4x' d\tau' \, G_{\text{Maxwell}} \left( x - x', \tau - \tau' \right) j^a_\varphi (x', \tau')$$

and using the identity

$$\int d\tau f (\tau) \, \delta \left( (x - X (s))^2 \right) \theta^{\text{ret}} = \theta (x^0 - X^0 (\tau)),$$

provides

$$a^\mu (x, \tau) = \frac{e}{4\pi} \varphi (\tau) \frac{u^\mu}{|u \cdot z|}, \quad a^5 (x, \tau) = \frac{e}{4\pi} \varphi (\tau) \frac{c_5}{|u \cdot z|}$$

where we write the event velocity and line of observation as

$$u^\mu = \dot{X}^\mu (\tau), \quad z^\mu = x^\mu - X^\mu (\tau).$$

By a similar procedure we find the field strengths as

$$f^{\mu\nu} (x, \tau) = -\frac{e}{4\pi} \varphi (\tau - \tau_R) \left\{ \frac{(z^\mu u^\nu - z^\nu u^\mu) \, u^2}{(u \cdot z)^3} + \frac{\epsilon (\tau - \tau_R) \, z^\nu u^\mu - z^\mu u^\nu}{\lambda (u \cdot z)^2} \right\}$$

and

$$f^{5\mu} (x, \tau) = c_5 \frac{e}{4\pi} \varphi (\tau - \tau_R) \left\{ \frac{z^\mu u^2 - u^\mu (u \cdot z)}{(u \cdot z)^3} + \frac{\epsilon (\tau - \tau_R) \, z^\mu - u^\mu (u \cdot z)}{(u \cdot z)^2} \right\}$$

where we used

$$\frac{d}{d\tau_R} \varphi (\tau - \tau_R) = -\frac{1}{2\xi} \frac{d}{d\tau} e^{-|\tau - \tau_R|/\xi \lambda} = -\frac{\epsilon (\tau - \tau_R)}{\xi \lambda} \varphi (\tau - \tau_R)$$

and $\epsilon (\tau) = \text{signum}(\tau)$. The contribution from $G_{\text{Correlation}}$ is smaller than the $G_{\text{Maxwell}}$ contribution by $c_5 / c$ and drops off as $1/|x|^2$. It may thus be neglected at low energy [19] compared to the fields (19), (21) and (22).
3. A toy model for mass shift

We consider an event propagating uniformly on-shell as

$$x(\tau) = u\tau = (u^0, u)$$

$$u^2 = -c^2$$

until it passes through a dense region of charged particles that induce a small stochastic perturbation $X(\tau)$ such that

$$x(\tau) = u\tau + X(\tau).$$

If the typical distance scale between force centers is $d$ then the perturbation will be roughly periodic with characteristic period

$$d \approx \text{very short distance}$$

$$|u| \approx \text{moderate velocity}$$

$$\tau \approx \text{very short time},$$

fundamental frequency

$$\omega_0 = 2\pi \frac{|u|}{d} \approx \text{very high frequency},$$

and amplitude on the order of

$$|X^\mu(\tau)| \sim ad$$

for some macroscopic factor $a < 1$. We may expand the perturbation in a Fourier series

$$X(\tau) = \text{Re} \sum_n a_n e^{in\omega_0 \tau}$$

and write the four-vector coefficients as

$$a_n = ad s_n = ad (s^0_n, s^\mu_n) = ad (c s^0_n, s^\mu_n)$$

where the $s_n$ represent a normalized Fourier series ($s^0_0 \sim 1$). The perturbed motion

$$X(\tau) = ad \text{Re} \sum_n s^\mu_n e^{in\omega_0 \tau}$$

is seen to be of scale $d$, but the perturbed velocity

$$\dot{x}^\mu(\tau) = u^\mu + \dot{X}^\mu(\tau)$$

$$= u^\mu + ad \text{Re} \sum_n n\omega_0 s^\mu_n ie^{in\omega_0 \tau}$$

$$= u^\mu + ad \text{Re} \sum_n \left(2\pi \frac{|u|}{d}\right) s^\mu_n ie^{in\omega_0 \tau}$$

$$= u^\mu + a |u| \text{Re} \sum_n 2\pi n s^\mu_n ie^{in\omega_0 \tau}$$

is of macroscopic scale $a |u|$. The unperturbed on-shell mass is

$$m = -\frac{M\dot{x}^2(\tau)}{c^2} = M$$

(33)
and the perturbed mass is

\[ m = -\frac{M \dot{x}^2(\tau)}{c^2} = -\frac{M}{c^2} \left( u + \alpha |u| \Re \sum_n 2\pi n \ s_n \ e^{in\omega_0 \tau} \right)^2 \]

\[ = -\frac{M}{c^2} \left( u^2 + \alpha |u| \Re \sum_n 2\pi n \ s_n \ e^{in\omega_0 \tau} \right)^2 + 2\alpha |u| \Re \sum_n 2\pi n \ (u \cdot s_n) \ e^{in\omega_0 \tau} \]

\[ = -\frac{M}{c^2} \left( -c^2 + 2\alpha |u| \Re \sum_n 2\pi n \ (u \cdot s_n) \ e^{in\omega_0 \tau} \right) - (\alpha |u|)^2 \Re \sum_{n,m} (2\pi)^2 nm \ s_n \cdot s_m \ e^{i(n+m)\omega_0 \tau} \]

\[ m = M \left( 1 - \frac{2\alpha |u|}{c^2} \Re \sum_n 2\pi n \ (u \cdot s_n) \ e^{in\omega_0 \tau} + \frac{\alpha^2 u^2}{c^2} \Re \sum_{n,m} (2\pi)^2 nm \ s_n \cdot s_m \ e^{i(n+m)\omega_0 \tau} \right). \quad (34) \]

Evaluating the typical coefficients in the rest frame of the unperturbed motion

\[ \frac{2\alpha |u|}{c^2} 2\pi n \ (u \cdot s_n) = \frac{4\pi \alpha |u| n}{c^2} (c, 0) \cdot (cs_n, s_n) = -4\pi \alpha |u| \ s_n \]

\[ \frac{\alpha^2 u^2}{c^2} (2\pi)^2 nm \ s_n \cdot s_m = (2\pi)^2 \alpha^2 u^2 \ n s^i_n \ s^j_m \left( s^k_n s^l_m - s^i_n \cdot s^l_m \right) \]

and neglecting the \( \alpha^2 \) term, we find

\[ m \simeq M \left( 1 + 4\pi \alpha |u| \Re \sum_n n s^i_n \ s^j_m \ e^{i(n+m)\omega_0 \tau} \right) \]

which expresses a macroscopic mass shift

\[ m \rightarrow m \left( 1 + \frac{\Delta m}{m} \right) \quad \Delta m = 4\pi \alpha |u| \Re \sum_n n s^i_n \ s^j_m \ e^{i(n+m)\omega_0 \tau} . \quad (38) \]

Larger mass shifts can be observed if \( \alpha > 1 \) and the second order term in \( \alpha^2 \) becomes significant.

4. A self-interaction

It was seen in (10) that particles may exchange mass with the fifth electromagnetic field through

\[ \frac{d}{d\tau} \left( -\frac{1}{2} M \dot{x}^2 \right) = \frac{e}{c} f^{5\mu} \dot{x}_\mu \]

and despite the scaling of \( f^{5\mu} \) by \( c_5/c \) this effect cannot be assumed to be insignificant. In order to account for the observed stability of particle masses, we must find some mechanism that tends to enforce on-shell evolution, perhaps by damping off-shell behavior in the manner
of air friction producing a terminal velocity. If, for example, some circumstance were to provide a field of the form $f^5 = \sigma \dot{x}^5$ then

$$\frac{d}{dt}(-\frac{1}{2}M\dot{x}^2) = \frac{e}{c} \sigma \dot{x}^\mu \dot{x}_\mu = -\frac{2e\sigma}{Mc} (-\frac{1}{2}M\dot{x}^2)$$

(39)

producing mass decay.

In this section, we propose a model for a self-interaction between a moving event and its electromagnetic field that produces a mass decay but vanishes for on-shell propagation. Unlike the self-interaction between a particle and its radiation field, associated with the Abraham-Lorentz-Dirac equation, this model involves the influence of the field induced through $G_{\text{Correlation}}$ with retarded timelike support. The event experiences a force along its worldline produced by its earlier motion along that worldline.

4.1. Framework

We study the motion of an arbitrarily moving event $X^\mu(\tau)$ in a co-moving frame so that

$$X(\tau) = (ct(\tau), 0) \quad \dot{X}(\tau) = (c\dot{t}(\tau), 0).$$

(40)

In this frame

$$\dot{X}^2 = -c^2\dot{t}^2$$

(41)

and so off-shell propagation is characterized by $\dot{t} \neq 1$ in the rest frame. We are interested in the self-force on the event at time $\tau^*$ and write the observation point as a later event along the worldline

$$X(\tau^*) = (ct(\tau^*), x(\tau^*))$$

(42)

so that

$$X(\tau^*) - X(s) = (ct(\tau^*), x(\tau^*)) - (ct(s), x(s)) = c(\dot{t}(\tau^*) - \dot{t}(s), 0).$$

(43)

Because $G_{\text{Maxwell}} = 0$ on this timelike separation, the sole contribution comes from the previously neglected term $G_{\text{Correlation}}$. We approximate $\varphi(\tau' - s) = \lambda \delta(\tau' - s)$, which becomes exact in the limit $\lambda \to 0$, introduce the function $g(s)$ to express terms of the type

$$c^2g(s) = -\left((X(\tau^*) - X(s))^2 + c_3^2(\tau^* - s)^2\right) = c^2 \left((t(\tau^*) - t(s))^2 - \frac{c_3^2}{c^2}(\tau^* - s)^2\right),$$

(44)

and write

$$a^a(X(\tau^*), \tau^*) = \frac{\lambda e c_5}{2\pi^2 c^3} \int ds \ X^a(s) \left(\frac{1}{2} \theta\left(\frac{g(s)}{g(s)}\right)^{3/2} - \frac{\delta(g(s))}{(g(s))^{1/2}}\right) \theta_{\text{ret}}$$

(45)

for the self-field experienced by the event. We designate the two terms as

$$a^a(X(\tau^*), \tau^*) = a^a_0 + a^a_\delta$$

(46)
4.2. Uniform on-shell motion

For an event evolving uniformly on-shell we have

\[ t (\tau^*) = \tau^* \]
\[ g(s) = \left( 1 - \frac{c^2 s}{c^2} \right) (\tau^* - s)^2 \]  \hspace{1cm} (47)

and using identity (17) are led to

\[ a (X (\tau^*), \tau^*) = \frac{\lambda e c_5}{2\pi^2 c^3} (c, 0, c_5) \int ds \theta (\tau^* - s) \]
\[ \left( \frac{1}{2} \theta \left( \left( 1 - \frac{c^2 s}{c^2} \right) (\tau^* - s)^2 \right) - \delta \left( \left( 1 - \frac{c^2 s}{c^2} \right) (\tau^* - s)^2 \right) \right) \]
\[ \int_{-\infty}^{\tau^*} ds \left( \frac{1}{2 (\tau^* - s)^3} - \left( \frac{\theta (\tau^* - s)}{(\tau^* - s)^2} \right) \right) \].  \hspace{1cm} (48)

Since

\[ \int_{-\infty}^{\tau^*} ds \frac{1}{(\tau^* - s)^3} = \frac{1}{2 (\tau^* - s)^2} \bigg|_{-\infty}^{\tau^*} = \lim_{s \to \tau^*} \frac{1}{2 (\tau^* - s)^2} \]  \hspace{1cm} (49)

and

\[ \int_{-\infty}^{\tau^*} ds \frac{\delta (\tau^* - s) \theta (\tau^* - s)}{(\tau^* - s)^2} = \lim_{s \to \tau^*} \frac{\theta (\tau^* - s)}{(\tau^* - s)^2} = \lim_{s \to \tau^*} \frac{1}{2} \left( \frac{1}{(\tau^* - s)^2} \right) \]  \hspace{1cm} (50)

we find that for uniform on-shell motion

\[ a (X (\tau^*), \tau^*) = \frac{\lambda e c_5}{2\pi^2 c^3} (c, 0, c_5) \lim_{s \to \tau^*} \left( \frac{1}{2 (\tau^* - s)^2} - \frac{1}{2 (\tau^* - s)^2} \right) = 0 \]  \hspace{1cm} (51)

the self-force vanishes.

4.3. Field strengths

From \( \dot{X}^i = 0 \) and the form of (45)

\[ a^i = 0 \quad \partial_t a^0 = \partial_t a^5 = 0 \quad \Rightarrow \quad f^{\mu \nu} = f^{5i} = 0 \]  \hspace{1cm} (52)

and so the field reduces to

\[ f^{50} = \delta^5 a^0 - \delta^0 a^5 = \delta^{55} \frac{1}{c_5} \partial_t a^0 - \delta^{00} \frac{1}{c} \partial_t a^5 = \frac{1}{c_5} \partial_t a^0 + \frac{1}{c} \partial_t a^5 \]  \hspace{1cm} (53)

where the partial derivative \( \partial_{\tau^*} \) only acts on the explicit variable (not on \( t (\tau^*) \) or \( \theta^{ret} \)). Similarly, the velocity appears as \( \dot{X}^a (s) \) and is constant with respect to \( \partial_{\tau^*} \).

Working piece-by-piece

\[ \dot{a}^0 = \frac{\lambda e c_5}{4\pi^2 c^5} \frac{1}{c_5} \partial_{\tau^*} \int ds \left( \frac{\theta (t (\tau^*) - t (s))^2 - \frac{c^2}{c^2} (\tau^* - s)^2)}{[(t (\tau^*) - t (s))^2 - \frac{c^2}{c^2} (\tau^* - s)^2]^{3/2}} \right) \theta^{ret} \]  \hspace{1cm} (54)
contains
\[ \partial_{\tau^*} \theta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right) = -2 \frac{c^5}{c^2} \delta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right)(\tau^* - s) \] (55)

and
\[ \partial_{\tau^*} \frac{1}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{3/2}} = 3 \frac{c^5}{c^2} \frac{\tau^* - s}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{5/2}}. \] (56)

Similarly,
\[ \frac{1}{c} \partial_{\mu} a_{\mu}^5 = \frac{\lambda ec_5}{4\pi^2c^3} \partial_{\mu(\tau^*)} \int ds c_5 \frac{\theta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right)}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{3/2}} \theta^{ret} \] (57)

contains
\[ \partial_{\mu(\tau^*)} \theta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right) = 2(t(\tau^*) - t(s)) \times \]
\[ \delta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right) \] (58)

\[ \partial_{\mu(\tau^*)} \frac{1}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{3/2}} = -3 \frac{t(\tau^*) - t(s)}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{5/2}} \] (59)

and
\[ \partial_{\mu(\tau^*)} \theta^{ret} = \partial_{\mu(\tau^*)} \theta (t(\tau^*) - t(s)) = \delta(t(\tau^*) - t(s)) = 0 \] (60)

where the last expression vanishes because \( t(\tau^*) = t(s) \) makes the argument of \( \theta(g(s)) \) negative. Putting the pieces together we find
\[ \partial^5 a_{\theta}^0 - \partial^0 a_{\theta}^5 = \frac{3\lambda ec_5 c_5}{4\pi^2c^3} \int ds c_5 \frac{\theta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right)}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{3/2}} \theta^{ret} \Delta(\tau^*, s) \]
\[ - \frac{\lambda ec_5 c_5}{2\pi^2c^3} \int ds \frac{\delta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right)}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{3/2}} \theta^{ret} \Delta(\tau^*, s) \] (61)

where
\[ \Delta(\tau^*, s) = \dot{t}(s)(\tau^* - s) - (t(\tau^*) - t(s)) \] (62)

characterizes the energy acceleration in the rest frame, which will be associated with mass shift. Similarly, the derivatives of \( a_{\theta} \) produce
\[ \partial^5 a_{\theta}^0 - \partial^0 a_{\theta}^5 = - \frac{\lambda ec_5 c_5}{2\pi^2c^3} \int ds \frac{\delta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right)}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{3/2}} \theta^{ret} \Delta(\tau^*, s) \]
\[ - \frac{\lambda ec_5 c_5}{2\pi^2c^3} \int ds \frac{2\delta \left( (t(\tau^*) - t(s))^2 - \frac{c^5}{c^2}(\tau^* - s)^2 \right)}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^5}(\tau^* - s)^2\right]^{1/2}} \theta^{ret} \Delta(\tau^*, s) \] (63)
and combining terms we find
\[ f^{50} = f^{50}_\delta + f^{50}_\delta + f^{50}_\delta \]
where
\[ f^{50}_\delta = \frac{3\lambda e c_2}{4\pi^2 c^4} \int ds \frac{\theta \left( (t(\tau^*) - t(s))^2 - \frac{c^2}{c^2_\delta} (\tau^* - s)^2 \right)}{\left( (t(\tau^*) - t(s))^2 - \frac{c^2}{c^2_\delta} (\tau^* - s)^2 \right)^{3/2}} \theta^\text{ret} \Delta (\tau^*, s) \] (65)
\[ f^{\delta 50} = -\frac{\lambda e c_2}{\pi^2 c^4} \int ds \left( \delta \left( (t(\tau^*) - t(s))^2 - \frac{c^2}{c^2_\delta} (\tau^* - s)^2 \right) \right)^{3/2} \theta^\text{ret} \Delta (\tau^*, s) \] (66)
\[ f^{50}_\beta = -\frac{\lambda e c_2}{\pi^2 c^4} \int ds \left( \delta \left( (t(\tau^*) - t(s))^2 - \frac{c^2}{c^2_\delta} (\tau^* - s)^2 \right) \right)^{3/2} \theta^\text{ret} \Delta (\tau^*, s) \] (67)

Notice that if the particle remains at constant velocity (in any uniform frame), then
\[ x^0(\tau) = \mu^0 \tau \Rightarrow \Delta (\tau^*, s) = \frac{\mu^0}{c} (\tau^* - s) - \left( \frac{\mu^0}{c} \tau^* - \frac{\mu^0}{c} s \right) = 0 \] (68)
and so the self-force \( f^{50}_\delta \) vanishes. For any smooth \( t(\tau) \), we may approximate
\[ t(\tau^*) - t(s) = t(s) + \dot{t}(s)(\tau^* - s) + \frac{1}{2} \ddot{t}(s)(\tau^* - s)^2 + o \left( (\tau^* - s)^3 \right) - t(s) \]
\[ = \dot{t}(s)(\tau^* - s) + \frac{1}{2} \ddot{t}(s)(\tau^* - s)^2 + o \left( (\tau^* - s)^3 \right) \] (69)
so the function
\[ \Delta (\tau^*, s) = \dot{t}(s)(\tau^* - s) - (t(\tau^*) - t(s)) = -\frac{1}{2} \ddot{t}(s)(\tau^* - s)^2 + o \left( (\tau^* - s)^3 \right) \] (70)
is nonzero only when the time coordinate accelerates in the rest frame, equivalent to a shift in the particle mass.

4.4. Mass jump

As a first order example, we consider a small, sudden jump in mass at \( \tau = 0 \) characterized by
\[ t(\tau) = \begin{cases} \tau, & \tau < 0 \\ (1 + \beta) \tau, & \tau > 0 \end{cases} \Rightarrow \dot{t}(\tau) = \begin{cases} 1, & \tau < 0 \\ 1 + \beta, & \tau > 0 \end{cases} \] (71)
and calculate the self-interaction. Since \( \theta^\text{ret} \) enforces \( t(\tau^*) > t(s) \), it follows that
\[ \tau^* < 0 \Rightarrow s < 0 \Rightarrow \dot{t}(\tau^*) = \dot{t}(s) = 1 \Rightarrow \Delta(\tau^*, s) = 0 . \] (72)
Similarly,
\[ \tau^* > 0 \text{ and } s > 0 \Rightarrow \dot{t}(\tau^*) = \dot{t}(s) = 1 + \beta \Rightarrow \Delta(\tau^*, s) = 0 . \] (73)
But when $\tau^* > 0$ and $s < 0$,

$$\Delta(\tau^*, s) = \dot{t}(s)(\tau^* - s) - (t(\tau^*) - t(s)) = (\tau^* - s) - [(1 + \beta)(\tau^*) - s] = -\beta \tau^* \quad (74)$$

and $f^{50}$ can be found from the contributions (65) – (67). Writing

$$g(s) = (t(\tau^*) - t(s))^2 - \frac{c^2}{c^2}(\tau^* - s)^2 = ((1 + \beta)(\tau^* - s))^2 - \frac{c^2}{c^2}(\tau^* - s)^2 \quad (75)$$

and solving for $g(s^*) = 0$, we find

$$s^* = \left(1 + \frac{\beta}{1 - \frac{c^2}{c^2}}\right) \tau^* > \tau^* \quad (76)$$

so that $g(s) > 0$ in the region of interest $s < 0 < \tau^*$ and there will be no contribution from the terms (66) or (67). Thus,

$$f^{50} = f_\theta^{50} = (-\beta \tau^*) \frac{3\lambda e c^2}{4\pi^2 c^4} \int_{-\infty}^{0} ds \frac{1}{\left[(t(\tau^*) - t(s))^2 - \frac{c^2}{c^2}(\tau^* - s)^2\right]^{5/2}} = (-\beta \tau^*) \frac{3\lambda e c^2}{4\pi^2 c^4} \int_{-\infty}^{0} ds \frac{1}{\left[((1 + \beta)(\tau^* - s))^2 - \frac{c^2}{c^2}(\tau^* - s)^2\right]^{5/2}} \quad (77)$$

Shifting the integration variable as $x = \tau^* - s$ the integral becomes

$$\int_{-\infty}^{0} ds \frac{1}{\left[((1 + \beta)(\tau^* - s))^2 - \frac{c^2}{c^2}(\tau^* - s)^2\right]^{5/2}} = -\int_{\infty}^{\tau^*} dx \frac{1}{(C x^2 + B x + A)^{5/2}} \quad (78)$$

where

$$C = 1 - \frac{c^2}{c^2} \quad B = 2\beta \tau^* \quad A = (\beta \tau^*)^2 \quad (79)$$

This integral can be evaluated using the well-known form [22]

$$\int \frac{dx}{(C x^2 + B x + A)^{5/2}} = \frac{2(2C x + B)}{3q \sqrt{C x^2 + B x + A}} \left(\frac{1}{C x^2 + B x + A} + \frac{8C}{q}\right) \quad (80)$$

where

$$q = 4AC - B^2 \quad (81)$$
leading to

\[ -\int_{-\infty}^{\tau^*} \frac{dx}{(Cx^2 + Bx + A)^{5/2}} = -\frac{1}{3 (\beta \tau^*)^4} \times \]

\[
\begin{bmatrix}
2 \frac{c^4}{c_5^2} \left(1 - \frac{c_5^2}{c^2}\right)^{3/2}
\left(1 - \left(1 - \frac{c_5^2}{c^2}\right)^{1/2} \left(1 + \frac{\beta}{1 - \frac{c_5^2}{c^2}} \right)^{-1/2} \right)
\end{bmatrix}
\]

\[
+ \frac{\beta^2}{c_5^2} \left(1 - \frac{c_5^2}{c^2}\right)^{1/2} \left[1 + \frac{2\beta}{1 - \frac{c_5^2}{c^2}} + \frac{\beta^2}{1 - \frac{c_5^2}{c^2}} \right]^{3/2}
\]

(82)

and the field strength in the form

\[ f^{50} = \frac{\lambda e}{4\pi^2 c_5^2 (\beta \tau^*)^3} Q \left( \beta, \frac{c_5^2}{c^2} \right) \]

(83)

where \( Q \left( \beta, \frac{c_5^2}{c^2} \right) \) is the positive, dimensionless factor
Since the Lorentz force for the mass function which causes the 0-coordinate to decelerate. When the event returns to on-shell propagation by its mass shift will restore the event to on-shell propagation.

We notice that if \( f^{\mu} = 0 \), the Lorentz force induced by this field strength is then

\[
M \dot{x}^\mu = e f^{\mu \alpha} \dot{x}_\alpha = e f^{5 \alpha} \dot{x}_\alpha = -g_{55} e f^{5 \mu} \dot{x}_5 = -e f^{5 \mu} c_5
\]

and since \( f^{5i} = 0 \)

\[
M \dot{x}^i = 0
\]

\[
M \dot{x}^0 = -c_5 e f^{50} = \begin{cases} 0 & , \tau^* < 0 \\ -\frac{\lambda e^2}{4 \pi^2} \frac{1}{c_5 (\beta \tau^*)^3} Q \left( \beta, \frac{c_5^2}{c^2} \right) & , \tau^* > 0 \end{cases}
\]

which causes the 0-coordinate to decelerate. When the event returns to on-shell propagation the function \( \Delta(\tau^*, s) \) and field strength \( f^{50} \) again vanish. The mass decay can also be seen in the Lorentz force for the mass

\[
\frac{d}{d\tau} \left( -\frac{1}{2} M x^2 \right) = e f^{5 \mu} \dot{x}_\mu = e f^{50} \dot{x} = -e c f^{50} \dot{t} = -\frac{\lambda e^2}{4 \pi^2} \frac{c}{c_5^2 (\beta \tau^*)^3} Q \left( \beta, \frac{c_5^2}{c^2} \right) \dot{t}.
\]

We notice that if \( \beta < 0 \) then \( f^{50} \) changes sign so that the self-interaction results in damping or anti-damping to push the trajectory toward on-shell behavior. Although this model is approximate, it seems to indicate that the self-interaction of the event with the field generated by its mass shift will restore the event to on-shell propagation.
5. Summary

In SHP theory, the mass shell relation $\dot{x}\cdot x = 1$ is demoted from the status of a priori constraint on the phase space to that of first integral, identifying by way of Noether’s theorem a constant of the motion for interactions mediated by $\tau$-independent potentials. Exchange of particle mass with the electromagnetic field can occur in various elastic scattering scenarios, including pair processes and the stochastic interaction with a region of densely packed charges described in section 3. Although the mass exchange may be small and total mass of particles and fields is conserved, the theory must account for the empirical observation of fixed asymptotic masses.

In this paper we have considered a classical self-interaction that acts on an off-shell event trajectory in a way that tends to restore on-shell propagation. In this model, an event that undergoes a mass increase, equivalent to acceleration of the time coordinate in the particle rest frame, experiences a force as it passes through the electromagnetic field induced by its earlier motion along its worldline. Such an interaction is prohibited by the lightlike support of $G_{\text{Maxwell}}$, the Maxwell part of the SHP Green’s function, and by the spacelike support of $G_{\text{Correlation}}$ for $g_{55} = -1$. However, for $g_{55} = +1$ the Green’s function has timelike support and includes the particle’s own future worldline. We found that for uniform motion this self-interaction vanishes, so that on-shell propagation is unaffected. But the Lorentz force acting on a particle that undergoes a discrete jump in $\dot{x}^0$ in its rest frame was found to oppose the time acceleration and restore the particle to on-shell propagation (for which the interaction again vanishes). This self-interaction would appear to provide an underlying mechanism for the asymptotic on-shell behavior found by Aharonovich and Horwitz in numerical solutions [14], and perhaps an explanation of the observed mass stability of the known particles.

To complete the investigation begun in this paper involves solving the Lorentz force equations under realistic conditions, for example a uniformly evolving event approaching a charge sheet, which was shown in [15] to induce a small mass shift. Because the interaction itself is non-linear and the self-interaction term must be evaluated iteratively at each point along the worldline by integrating with the Green’s function up to that point on the worldline, these configurations require careful numerical solution. Such solutions will be presented in a subsequent paper.

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References

[1] Stueckelberg E C G 1941 Helv. Phys. Acta 14 322
Stueckelberg E C G 1941 Helv. Phys. Acta 14 388
[2] Horwitz L P and Piron C 1973 Helv. Phys. Acta 48 316
[3] Arshansky R L, Horwitz P and Lavie Y 1983 Found. of Phys. 13 1167
[4] Horwitz L P, Arshansky R I and Elitzur A C 1988 Found. of Phys. 18 1159.
[5] Saad D, Horwitz L P and Arshansky R I 1989 Found. of Phys. 19 1126
[6] Land M C and Horwitz L P 1991 Found. of Phys. 21 299
[7] Land M C and Horwitz L P 1991 Found. of Phys. Lett. 4 61
[8] Land M C, Shnerb N and Horwitz L P 1995 J. Math. Phys. 36 3263
[9] Land M C 1996 Found. of Phys. 27 19
[10] Land M C and Horwitz L P 1998 Land M C A239 135
[11] Land M C 2001 Found. of Phys. 31 967
[12] Land M C 2003 Found. of Phys. 33 1157
[13] Land M C 2011 J. Phys.: Conf. Ser. 330 012015
[14] Aharonovich I and Horwitz L P 2012 J. Math. Phys. 53 032902
[15] Land M 2013 J. Phys.: Conf. Ser. 437 012012
[16] Horwitz L P and Aharonovich I 2013 J. Phys.: Conf. Ser. 437 012021
[17] Land M 2015 J. Phys.: Conf. Ser. 615 012007
[18] Horwitz L P 2015 Relativistic Quantum Mechanics (Dordrecht: Springer Science+Business Media)
[19] Land M, to appear 2017 J. Phys.: Conf. Ser.
[20] Horwitz LP http://arxiv.org/abs/1607.03742v2
[21] Davidson M 2015 J. Phys.: Conf. Ser. 615 012016
[22] Pierce P O 1899 A short table of integrals (New York: Ginn and Company) 24.