Learning Models of Model Predictive Controllers using Gradient Data *

Rebecka Winqvist * Arun Venkitaraman * Bo Wahlberg *

* Division of Decision and Control Systems, EECS, KTH Royal Institute of Technology, Stockholm, Sweden. (e-mails: {rebwin, arunv, bo}@kth.se).

Abstract: This paper investigates controller identification given data from a Model Predictive Controller (MPC) with constraints. We propose an approach for learning MPC that explicitly uses the gradient information in the training process. This is motivated by the observation that recent differentiable convex optimization MPC solvers can provide both the optimal feedback law from the state to control input as well as the corresponding gradient. As a proof of concept, we apply this approach to explicit MPC (eMPC), for which the feedback law is a piece-wise affine function of the state, but the number of pieces grows rapidly with the state dimension. Controller identification can here be used to find an approximate lower complexity functional approximation of the controller. The eMPC is modelled with a Neural Network (NN) with Rectified Linear Units (ReLUs), since such NN can represent any piece-wise affine function. A motivation is to replace on-line solvers with neural networks to implement MPC and to simplify the evaluation of the function in larger input dimensions. We also study experimental design and model evaluation in this framework, and propose a hit and run sampling algorithm for input design. The proposed algorithm are illustrated and numerically evaluated on a second order MPC problem.

Keywords: Identification for control; data-driven control; neural networks relevant to control and identification; input and excitation design; model predictive control; modeling for control optimization.

1. INTRODUCTION

Controller identification concerns estimating a model of a feedback controller from observed input and output data. This is typically done in a feedback mode, where the controller interacts with a dynamical system. It is a well studied topic, see e.g. (Ljung, 1999), in particular when the system and the controller can be described by linear dynamical models. It is only recently that learning models based on Neural Networks (NNs) have been pursued for Model Predictive Controllers (MPCs) with constraints (Chen et al., 2018; Blanchini, 1999; Chen et al., 2019; Maddalena et al., 2019). A motivation behind these approaches is the recent result that NNs with rectified linear units (ReLUs) can represent piece-wise affine functions or functions with linear regions (Montufar et al., 2014). This naturally makes such approaches suited to the MPC learning problem (Chen et al., 2018), particularly in the case of explicit MPC, where the optimal control law is an affine function of the state vector (Bemporad, 2013).

Consider an MPC problem with the feedback law \( u = u^\ast(x) \) corresponding to the full state information \( x \). The goal of any learning based approach to MPC is then to learn a mapping \( \mu(x) \) that describes \( u^\ast(x) \) as good as possible. In the case of NN based approaches, \( \mu \) corresponds to the function learnt by a NN. Most existing NN based learning approaches proceed by learning \( \mu \) using only the input and output observations of \( x \) and \( u \), respectively. Like any learning approach, an MPC mapping learnt in such a manner could perform poorly when the training data is limited. In such a scenario, additional structure can often be of merit in aiding the training of the NN. For example, in the case of explicit MPC (eMPC), \( u \) is a piece-wise affine function in \( x \) and hence, the gradient of the optimal control law with respect to \( x \) is piecewise-constant and contains important structural information useful in learning \( \mu \).

Motivated by this observation, we propose a NN-based learning approach for MPC where we explicitly use structural information in the form of gradient data \( \frac{\partial u}{\partial x} \) for training the NN. While training data in the form of gradient information is typically unavailable or difficult to obtain, the special structure of the MPC problem and the use of recently proposed tools in differentiable convex optimization (Diamond and Boyd, 2016) helps us achieve our goal. The main contributions of the paper are:

- **Learning Models:** We design and evaluate algorithms to train ReLU based NNs that implements MPC using input and output data \( u_i = u^\ast(x_i) \) and corresponding gradient data \( u'_i = \frac{\partial u^\ast(x)}{\partial x} \bigg|_{x = x_i} \). We show that taking the gradient information into account can significantly reduce the number of training data needed to achieve a high accuracy. We use eMPC as a proof of concept, while the proposed algorithm can handle more general MPC problems.

- **Data generation:** It is not obvious how to efficiently generate training data when learning the control law as a mapping. Grid-based approaches for sampling the input space would work for small state dimensions, but become cumbersome for high-dimensional systems. Keeping this in mind, we study the use of an efficient and statistically motivated hit-and-run sampler that extends well to higher state dimensions.

---

* This work was partially supported by the Swedish Research Council and by the Wallenberg AI, Autonomous Systems and Software Program (WASP) funded by the Knut and Alice Wallenberg Foundation and the Swedish Research Council Research Environment NewLEADS under contract 2016-06079.
• **Evaluation of performance**: The performance of trained NN based MPC controller should be evaluated in closed loop feedback, taking into account both tracking, disturbance rejection and stability. Here, we evaluate the performance on test data in terms of different metrics.

1.1 A motivating example: Identification using Gradient Data

In order to illustrate the identification algorithms to be proposed, consider the following scalar linear feedback example with measurements

\[ u_k = l_1 x_k + l_2 + e_k, \quad k = 1, \ldots, N, \]

with control signal \( u_k \), scalar state signal \( x_k \) and additive white zero mean Gaussian noise \( e_k \) with variance \( \lambda_e \). Assume that it is possible to measure the derivative of \( u \) with respect to \( x \),

\[ u'_k = l_1 + v_k, \quad k = 1, \ldots, N, \]

where \( v_k \) is white zero mean Gaussian distributed noise with variances \( \lambda_v \). The maximum likelihood estimate of \( l_1 \) and \( l_2 \) given the data \( \{ x_k, u_k, u'_k \} \), \( k = 1, \ldots, N \) is found by solving the least squares problem

\[ V(l_1, l_2) = \sum_{k=1}^{N} \left[ \frac{u_k - l_1 x_k - l_2}{\lambda_e} \right]^2 + \left[ \frac{u'_k - l_1}{\lambda_v} \right]^2. \]

The error covariance matrix of the least squares estimate \((\hat{l}_1, \hat{l}_2)^T\) equals, see Ljung (1999),

\[ \lambda_e \frac{1}{N} \sum_{k=1}^{N} \left[ \begin{array}{c} (x_k^2 + \lambda_v / \lambda_e) x_k \\ x_k \end{array} \right]^{-1}. \]

For the choice \( x_k = 1 \) it is not possible to estimate \( l_1 \) and \( l_2 \) individually without the extra gradient data. For this case the estimation error covariance matrix equals

\[ \lambda_e \frac{1}{N} \sum_{k=1}^{N} \left[ \begin{array}{c} \lambda_v - \lambda_e \lambda_v / \lambda_e \\ -\lambda_e \lambda_v / \lambda_e, 1 \end{array} \right]^{-1}. \]

This result can also be found by analyzing

\[ u_k - u'_k = b_2 + e_k - v_k. \]

The least squares estimate of \( l_2 \) is just the average of this difference signal. Notice that the variance of \( \hat{l}_1 \) is lower than the variance of \( l_2 \), which is expected given the extra information on \( l_1 \) and the added noises when estimating \( l_2 \). Hence gradient information can be crucial in terms of estimation quality for low input excitation.

The structure of this paper is as follows: The MPC problem and Neural Networks are introduced in Section 2, while Section 3 describes the proposed training and evaluation framework. Data generation and testing are studied in Section 4 and the numerical examples are presented in Section 5. Finally, the conclusion and ideas for future work are given in Section 6.

2. PRELIMINARIES

In this section, we briefly review the basic Model Predictive Control (MPC) problem, the explicit MPC, followed by a review of neural networks.

2.1 Model Predictive Control

Consider a discrete-time linear time-invariant system which evolves in time as

\[ x(k+1) = Ax(k) + Bu(k), \]

where \( x(k) \) denotes the state vector and \( u(k) \) denotes the input or control action, at the \( k \)th time-step, respectively; \( A \) and \( B \) denote the system matrices. Model-predictive control (MPC) refers to the problem of steering the state of the system (1) from an initial value to the origin by minimizing a control objective, subject to the state and input constraints

\[ x(k) \in \mathcal{X}, \quad u(k) \in \mathcal{U} \]

where \( \mathcal{X} \subseteq \mathbb{R}^n \) and \( \mathcal{U} \subseteq \mathbb{R}^m \) are polyhedra representing the constraint sets for the state and the input, respectively. We consider the following fixed horizon MPC problem, see Borrelli et al. (2017):

\[
\begin{align*}
\min_{\mathbf{u}(0) = \mathbf{u}(N-1)} & \ x(N)^T \mathbf{Q}_N \ x(N) + \sum_{k=0}^{N-1} x(k)^T \mathbf{Q} x(k) + \mathbf{u}(k)^T \mathbf{R} \mathbf{u}(k) \\
\text{s.t.} & \ x(k+1) = A \ x(k) + B \ u(k), \\
& \ x(k) \in \mathcal{X}, \\
& \ u(k) \in \mathcal{U}, \\
& \ x(0) = \mathbf{x}
\end{align*}
\]

where \( \mathbf{Q} \) and \( \mathbf{R} \) are the positive semi-definite weight matrices, \( \mathbf{Q}_N \) the terminal cost matrix, and \( N \) denotes the finite time horizon length. The optimal solution to such a constrained quadratic program can be found by solving a set of linear equations once the active constraints are identified, see Wang and Boyd (2010) for results on fast online MPC implementations.

The MPC control law is the mapping from the current state \( x(0) = x \) to the first optimal control action \( u(0) = u \). We will use the notation \( u = u^*(x) \) for this mapping.

2.2 Feasibility

We now discuss how the feasibility constraints can be characterized in terms of set-invariance, which will form the basis of incorporating structure into neural network solutions for MPC. As defined by Borrelli et al. (2017), a set \( \mathcal{C} \subseteq \mathcal{X} \) is a control invariant set for the system (1) subject to the constraints (2) if

\[ x(k) \in \mathcal{C} \implies \exists u(k) \text{ s.t. } x(k+1) \in \mathcal{C}, \quad k = 1, 2, \cdots \]

In other words, for any initial state in \( \mathcal{C} \) there exists a controller that ensures all future states reside in \( \mathcal{C} \). The maximal control invariant set, \( \mathcal{C}_{\infty} \), is then defined as the control invariant set containing all control invariant sets contained in \( \mathcal{C} \). \( \mathcal{C}_{\infty} \) being a polytope is expressible as an intersection of halfspaces as

\[ \mathcal{C}_{\infty} = \{ x \in \mathbb{R}^n \ | \ C x \leq d \} \]

(Bemporad, 2013). To compute \( \mathcal{C}_{\infty} \) we use algorithm 10.2 "Computation of \( \mathcal{C}_{\infty} \)" provided in Borrelli et al. (2017) and the accompanying software.

2.3 Explicit MPC

The main challenge in solving the MPC problem lies in determining which of the constraints are active. Nevertheless, this entails solving an optimization problem at each time instant, which could quickly become a bottleneck when applying to systems repeatedly. One of the ways of circumventing the determination of active sets is through offline pre-computation of the control laws such that the problem is transformed into that of specifying a mapping or lookup table in the form of a piecewise affine function. This mapping then acts on the input to produce the optimal control law. This approach is known as the explicit MPC (Alessio and Bemporad, 2009; Goodwin et al., 2010; Bemporad, 2013).
2.4 Neural networks

Neural networks and deep learning approaches are now ubiquitous and find the crux of most learning-based approaches today (LeCun et al., 2015). Neural networks learn a mapping from the input to the output from known training examples, when the problem at hand has either no clear closed-form input-output mapping, or even if there is one, is intractable to work with (Bishop, 2006). Neural networks comprise concatenated processing units known as neurons that combine linear and non-linear transformations. Mathematically expressed, a neural network $\mu(x)$ learns a mapping from the input $x$ to output $u$ in the form

$$u = \mu(x, \theta) = \sigma(W_L \sigma(W_{L-1} \sigma(\cdots \sigma(W_1 x + b_1) \cdots) + b_L),$$

where $\sigma(\cdot)$ denotes the point-wise nonlinearity or activation function, and the matrices $\{W_i, b_i\}_{i=1}^L$ are the parameters $\theta$ (weights and biases) learnt by the network from the training data. $L$ denoting the number of neuron layers.

The learning is typically performed by the use of backpropagation that uses the gradients of an error or loss function with respect to the network parameters. For the reasons of computational complexity and stability, the rectified linear unit (ReLU) is the most commonly employed activation function (Bishop, 2006; LeCun et al., 2015). As discussed earlier, the use of ReLU as the activation function has been shown to be well motivated in learning functions with linear regions (Montufar et al., 2014), and particularly in the MPC setting due to its piece-wise linear nature (Chen et al., 2018). A schematic of a two-layer neural network is shown in Figure 2.

3. PROPOSED APPROACH

Let $x \in \mathbb{R}^n$ denote the initial state vector $x(0)$, and $u \in \mathbb{R}^m$ the corresponding optimal control law $x(0)$. Further, let $\mu’ \in \mathbb{R}^{n \times m}$ denote the true gradient of the optimal control with respect to $x$. Consider that we are given a set of $N_r$ samples of initial states, the corresponding optimal control laws, and their gradients, given by $\{x_i, u_i, \mu’(x_i)\}_{i=1}^{N_r}$. We note that the subscript $i$ here denotes the $i$th samples and is not the time-index. Let us define the sets $\mathcal{X} = \{x_1, \ldots, x_{N_r}\}$, $\mathcal{U} = \{u_1, \ldots, u_{N_r}\}$, and $\mathcal{U}’ = \{\mu’(x_1), \ldots, \mu’(x_{N_r})\}$. Our goal is to train a ReLU NN to predict the optimal control law using gradient information – we learn the mapping

$$\mu(x, \theta) : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^m,$$

where $\theta \in \mathbb{R}^D$ denotes the all the learnable weights of the NN, by using a training loss function $\mathcal{L}_t(\theta) : \mathcal{X} \times \mathcal{U} \times \mathcal{U}’ \rightarrow \mathbb{R}$ that explicitly uses gradient information.

Specifically, we propose to learn $\mu(x, \theta)$ by minimizing the following loss function with respect to the parameters $\theta$:

$$\mathcal{L}_t(\theta) = \sum_{i=1}^{N_r} \left( \|u_i - \mu(x_i, \theta)\|_2^2 + \gamma \left\| \mu’(x, \theta) \frac{\partial \mu(x, \theta)}{\partial x} \right\|_2 \right),$$

where the first term denotes the model error, and the second term denotes the fit of the gradients, $\gamma > 0$ being the regularization constant. We note that typically NN training is performed by minimizing only the model error

$$\sum_{i=1}^{N_r} \|u_i - \mu(x_i, \theta)\|_2^2.$$

It is well known that training the NN in such a manner makes it data-hungry, and the performance of the NN often suffers when the number of training samples is limited.

In many control problems, and particularly with the MPC, often one has access only to a limited number of training observations given the very large state space. As seen from our discussion in Section 2, an explicit incorporation of structural information often aids in the learning process when the number of training samples is limited. This forms the motivation behind our approach - the regularization explicitly enforces structural information on the learnt NN mapping $\mu$. In the case of eMPC, the set feasible inputs is a polytope, and therefore, the gradient of the optimal control law $\mu’$ is piecewise constant. Assuming the training samples are drawn randomly over the feasible input space (ideally one sample per region of the feasible space), if the samples are taken uniformly from the feasible set, the network is given information about a large subsets of the feasible set. In such a case, we would expect that our approach would learn with even limited training data due to active incorporation of this structural information. In contrast, a regular NN based MPC solver that uses only the model error in the training is agnostic to this information and must abstract such structure purely from the training samples.

Thus, our approach is a trade-off between completely data-driven and structurally aware MPC solver. A schematic of the proposed approach is given in Figure 2. We note that our approach requires the value of the true gradients of the MPC problem evaluated at the given $x$. As described in Section 5, we evaluate the true gradients using the recently proposed cvxpylayers (Agrawal et al., 2019), a framework for obtaining differentiable convex layers in pytorch. Since the cost function

![Fig. 1. Schematic of a two-layer neural network.](image-url)
\( \mathcal{L}_{tr}(\theta) \) is non-convex, the training proceeds by backpropagation as described next. The details of the dataset generation and training are described in the next Section. Let \( \hat{\theta} \) denote the NN parameters obtained after training. Once the network parameters \( \theta \) are learnt, they are used to predict the optimal control law for test data \( x \) as

\[ \hat{u} = \mu(x, \hat{\theta}). \]

We note that the gradient information is not required during the test phase, and is used only in the training of the NN.

Fig. 2. The structure of the proposed neural network.

4. TRAINING AND DATA GENERATION

We first discuss the systematic strategy that we propose for the generation of data sets for the MPC problems. Our approach for generating the training and testing data set involves sampling (using the Hit-and-Run sampler described next) a set of states \( \mathcal{S} = \{x_1, \ldots, x_{N_0}\} \) from the feasible region \( \mathcal{G}_{tr} \) described in Section 3.3. The corresponding optimal control input and the corresponding gradient sets, \( \mathcal{U} \) and \( \mathcal{U}' \) are then computed using a stand-alone cvxpylayers. We follow the same strategy to generate test data set with the difference that it does not contain gradient data.

4.1 Sampling

To sample from the maximal control invariant set \( \mathcal{G}_{tr} \) defined in Section 2.2 we use the Hit-and-Run sampler, which is a Markov chain Monte Carlo method for sampling uniformly from convex shapes (Mete and Zabinsky, 2012). Essentially, starting from any point in the convex set, the sampler generates a set of points, \( \mathcal{S} \), by walking steps of length \( \lambda \), in randomly generated (unit) directions. The steps involved in the Hit-and-Run sampler are detailed in Algorithm 1. We employ this approach since it ensures that the generated datapoints cover the feasibility region in a reasonably uniform manner (Mete and Zabinsky, 2012; Zabinsky and Smith, 2013). This in turn ensures that the network has observed training samples that span the entire feasible set on an average, thereby aiding its ability to generalize.

4.2 Training

Once the training and test datasets are generated, the corresponding training outputs and gradients are obtained from using CVXPY to solve the MPC problem. We then use a supervised learning method to train the networks on a data set \( \mathcal{D} = \{[x, u, u']\} \) of input-output pairs. During the training, we learn for the network parameters by minimizing the loss function \( \mathcal{L}_{tr}(\theta) \) defined in (5) with respect to \( \theta \).

In order to increase the training speed, we split the data into smaller subsets (mini-batches) and compute the training loss function (5) for each mini-batch. Each mini-batch consists of five training samples. We then use the gradient descent-based Adam optimizer (Kingma and Ba, 2014) to backpropagate the loss and update the parameters \( \theta \) following each batch. Once all the mini-batches have been iterated over, one training epoch is completed. We train the networks until \( \mathcal{L}_{tr}(\theta) \) is reduced to 0.01 or for a maximum of 50000 training epochs.

5. EXAMPLES

We now consider the application of the proposed concepts on a set of networks with different regularization constants \( \gamma \) trained on MPC problems for a two-dimensional system. We evaluate the networks in terms of two performance metrics:

1. The normalized mean square error (NMSE) which is defined as

\[ \text{NMSE} = \frac{E[\|\hat{u} - u\|^2_2]}{E[\|u\|^2_2]}, \]

where \( E \) denotes the average over all samples in the training or test dataset.

2. The normalized control cost \( J \), defined as the objective in Equation (3) normalized by \( x(0)^TQx(0) \):

\[ J = \frac{x(N)^TQx(N) + \sum_{k=0}^{N-1} \|k(k)TQx(k) + u(k)^TRu(k)\|}{x(0)^TQx(0)} \]

where \( x(0) \) is the initial state of the trajectory.

Both the metrics are evaluated on test data, previously unseen by the networks during the training. The NMSE helps evaluate the control law predicted by the network with respect to the ground truth, whereas the control cost measures how well the control law is in terms of minimizing the control objective: the smaller the \( J \), the better the control achieved.

5.1 Two-Dimensional System

We consider the example of a two-dimensional state vector with a scalar input under constraints. Despite being relatively low-dimensional, such a scenario occurs regularly in many real-life control applications. Let us then consider a two-dimensional system specified as follows (Borrelli et al., 2017):

\[
A = \begin{bmatrix} 1.0 & 1.0 \\ 0.0 & 1.0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}
\]
We generate training and testing data by first sampling states from $\mathcal{C}_{\text{init}}$ for system (6) subject to the constraints (7).

subject to the constraints

$$
\begin{bmatrix}
-5.0 \\
-5.0 \\
\end{bmatrix} \leq \mathbf{x}(k) \leq \begin{bmatrix}
5.0 \\
5.0 \\
\end{bmatrix}, \quad -2.0 \leq u(k) \leq 2.0, \quad k = 1, \ldots, N
$$

(7)

and with cost parameters

$$
Q_{\mathbf{S}} = Q = \begin{bmatrix}
1.0 & 0.0 \\
0.0 & 1.0 \\
\end{bmatrix}, \quad R = 10, \quad N = 3.
$$

(8)

We generate training and testing data by first sampling states from $\mathcal{C}_{\text{init}}$ for system (6) subject to (7) using the Hit and Run Algorithm 1, see Figure 3 for an example of 1000 sampled points. We then solve for the optimal controls and corresponding gradients using CVXPY with cost parameters given by (8).

In our experimental setup we consider three sets of ten neural networks for four different settings of the regularization constant $\gamma \in \{0, 0.1, 1, 10\}$. For each set of networks we (pseudo) randomly generate 10 sets of training data, one for each network in the set. The first set of networks is trained using 25 samples, the second set using 50 samples, and the third set using 100 samples. We use $S_{(25)}$, $S_{(50)}$ and $S_{(100)}$ to denote the three network sets.

For the NMSE evaluation, we use three separate testing datasets, one for each set of networks, all consisting of 100 samples. For the control cost evaluation, we sample sets of 100 initial states from $\mathcal{C}_{\text{init}}$ using Algorithm 1. Starting from these, we then simulate the networks in closed loop for three time steps to generate trajectories.

In Figure 4 we plot the NMSE averaged over the ten trained networks in each network set in the decibel [dB] scale. Table 1 compares the lowest overall NMSE for the cases $\gamma = 0$ and $\gamma \neq 0$. We see that $\gamma = 1$ results in the lowest NMSE for each network set. This suggests that including gradient information during the training phase improves the performance of the network. From a theoretical point of view, the choice of $\gamma$ should not be so important since we do not have measurement noise. However, from a numerical point of view it makes a difference, for example in Figure 4 we note an increase in the NMSE for $\gamma > 10$. Notice that the we only evaluate the NMSE in the control mapping and not in its gradient.

Figure 5 shows a comparison of the the control costs for the two cases $\{S_{(25)}, \gamma = 1\}$ and $\{S_{(100)}, \gamma = 0\}$ averaged over the ten trained networks in each respective set. Table 2 shows the control cost averaged over all trajectories. An interesting observation is that the performance of the setting $\{S_{(25)}, \gamma = 1\}$ is comparable to that of the setting $\{S_{(100)}, \gamma = 0\}$. In fact, a larger training dataset will in general lower the generalization error. The results then points to the richness of the gradient information.

Considering the proportions of the amount of training data to test trajectories, it is likely that there are clusters of samples (initial states) in regions that the network has not been exposed to during training, which might explain the peaks we observe in Figure 5. A possible explanation is that the network in those cases produces either very large control signals and/or steers the state away from the reference state (possibly outside the feasible region), which would result in large control costs. Chen et al. (2019) suggest a projection strategy for ensuring feasibility, i.e. constraint satisfaction, of the generated control inputs by projecting them onto a safe region. The approaches in (Winqvist, 2020) is also based on this idea. We do not employ any such safety measures in our network structure. Note also that we do not explicitly train the networks to optimize the control cost, which is another possible approach for improving the results.
| Set  | NMSE [dB] (no regularization) | NMSE [dB] (regularization) | Best γ |
|------|------------------------------|---------------------------|--------|
| S_{(25)} | 2.5285                      | -9.8901                   | 1      |
| S_{(50)} | -0.8165                     | -19.5597                  | 1      |
| S_{(100)} | 2.1549                      | -34.3113                  | 1      |

Table 1. NMSE evaluation for 2D example. The table presents the lowest NMSE when no regularization (γ = 0) was used during training, as well as the lowest overall NMSE and the corresponding regularization constant γ. For all sets the lowest NMSE was found for γ = 1.

| Set, γ | J (avg over 100 test traj.) |
|--------|---------------------------|
| S_{(25)}, 1 | 13.3483                  |
| S_{(100), 0} | 12.2070                  |

Table 2. Control cost evaluation for 2D example.

6. CONCLUSION AND FUTURE WORK

We have presented a framework for off-line training and evaluation of a neural network approach for implementing MPC using gradient data. The underlying question is if it is possible to replace on-line MPC optimization solvers with trained NNs. This would allow for very efficient and robust real-time implementations. At the same time, there is great progress in the area of embedded convex optimization for control. The idea is to approximate the MPC mapping from state to control input with constrained ReLU-based neural network. The main novel result is on how to include the gradient of the MPC controller with respect to the state input in the training of NNs. We have used CVXPY (Agrawal et al., 2019) and PyTorch (Paszke et al., 2019) to implement this framework. We also use CVXPY and cvxpylayers to generate training data and to evaluate the resulting controller.

A key factor is the generation of samples for the off-line training, which related to input design in system identification, (Annergren et al., 2017). This is a challenge when the dimension of the state space increases. Here we proposed to use a hit-and-run sampler, and evaluated the resulting controller based on trajectories and normalized cost-functions. The numerical tests showed the trade-off between the number of training data and the approximation properties of the resulting controller.

This paper is a first step towards controller identification of MPC using ReLU networks. It should be noted that we do not assume any model information other than from x, u, and gradient in training the network—so or approach is not restricted to eMPC or even time-invariant MPC problems. One can also use model parameters as inputs to the NN and gradients with respect to them as training data. It is also possible to train the NN to predict the u(k), k = 0, …, N−1, for the entire MPC horizon by giving just x(0) as input. In this way we are feeding information that we need to control to take N steps.

The next step is also to evaluate this approach on more advanced MPC control problems including nonlinear MPC. The numerical example studied here is for proof of concept. We also use very basic methods for evaluation. The approach would also benefit from training on trajectories instead of just on the control mapping. We will be pursuing these aspects in the future.

REFERENCES

Agrawal, A., Amos, B., Barratt, S., Boyd, S., Diamond, S., and Kolter, J.Z. (2019). Differentiable convex optimization layers. In Advances in Neural Information Processing Systems, volume 32, 9562–9574.

Alessio, A. and Bemporad, A. (2009). A Survey on Explicit Model Predictive Control, 345–369. Springer Berlin Heidelberg, Berlin, Heidelberg.

Annergren, M., Larsson, C.A., Hjalmarsson, H., Bombois, X., and Wahlberg, B. (2017). Application-oriented input design in system identification: Optimal input design for control applications of control. IEEE Control Systems Magazine, 37(2), 31–56. doi:10.1109/MCS.2016.2643243.

Bemporad, A. (2013). Explicit Model Predictive Control, 1–9. Springer London, London.

Bishop, C.M. (2006). Pattern Recognition and Machine Learning (Information Science and Statistics). Springer-Verlag, Berlin, Heidelberg.

Blanchini, F. (1999). Set invariance in control. Automatica, 35(11), 1747 – 1767.

Borrelli, F., Bemporad, A., and Morari, M. (2017). Predictive Control for Linear and Hybrid Systems. Cambridge University Press, USA, 1st edition.

Chen, S., Saulnier, K., Atanasov, N., Lee, D.D., Kumar, V., Pappas, G.J., and Morari, M. (2018). Approximating explicit model predictive control using constrained neural networks. In 2018 Annual American Control Conference (ACC), 1520–1527.

Chen, S.W., Wang, T., Atanasov, N., Kumar, V., and Morari, M. (2019). Large scale model predictive control with neural networks and primal active sets.

Diamond, S. and Boyd, S. (2016). CVXPY: A Python-embedded modeling language for convex optimization. Journal of Machine Learning Research, 17(83), 1–5.

Goodwin, G., Seron, M.M., and de Don, J.A. (2010). Constrained Control and Estimation: An Optimisation Approach. Springer Publishing Company, Incorporated, 1st edition.

Kingma, D.P. and Ba, J. (2014). Adam: A method for stochastic optimization. CoRR, abs/1412.6980.

LeCun, Y., Bengio, Y. and Hinton, G. (2015). Deep learning. Nature, 521, 436–44. doi:10.1038/nature14539.

Ljung, L. (1999). System Identification: Theory for the User. Prentice Hall International, Inc.

Maddalena, E.T., da S. Moraes, C.G., Waltrich, G., and Jones, C.N. (2019). A neural network architecture to learn explicit MPC controllers from data.

Metre, H.O. and Zabinsky, Z.B. (2012). Pattern hit-and-run for sampling efficiently on polytopes. Operations Research Letters, 40(1), 6 – 11.

Montufar, G.F., Pascanu, R., Cho, K., and Bengio, Y. (2014). On the number of linear regions of deep neural networks. In Z. Ghahramani, M. Welling, C. Cortes, N. Lawrence, and K.Q. Weinberger (eds.), Advances in Neural Information Processing Systems, volume 27, 2924–2932. Curran Associates, Inc.

Parisini, T. and Zoppoli, R. (1995). A receding-horizon regulator for nonlinear systems and a neural approximation. Automatica, 31, 1443–1451.

Paszke, A., Gross, S., Massa, F., Lerer, A., Bradbury, J., Chanan, G., Killeen, T., Lin, Z., Gimelshein, N., Antiga, L., Desmaison, A., Kopf, A., Yang, E., DeVito, Z., Raison, M., Tejani, A., Chilamkurthy, S., Steiner, B., Fang, L., Bai, J., and Chanan, G. (2019). PyTorch: An imperative style, high-performance deep learning library. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long Papers), 4271–4282.
and Chintala, S. (2019). Pytorch: An imperative style, high-performance deep learning library. In *Advances in Neural Information Processing Systems 32*, 8024–8035. Curran Associates, Inc.

Wang, Y. and Boyd, S. (2010). Fast model predictive control using online optimization. *IEEE Transactions on Control Systems Technology*, 18(2), 267–278.

Winqvist, R. (2020). *Neural Network Approaches for Model Predictive Control*. Master’s thesis, KTH, School of Electrical Engineering and Computer Science (EECS).

Zabinsky, Z.B. and Smith, R.L. (2013). *Hit-and-Run Methods*, 721–729. Springer US, Boston, MA.

Åkesson, B. and Toivonen, H. (2006). A neural network model predictive controller. *Journal of Process Control*, 16, 937–946.