In this paper, we introduce a framework to study local interactions due to the presence of herding behavior in a minority game. The idea behind this approach is to consider that some of the agents who play the game believe that some of their neighbors are more informed than themselves. Thus, in this way, these agents imitate their most informed neighbors. The notion of neighborhood here is given by a regular network, a random network or a small world network. We show that under herding behavior the cooperation between the agents is less efficient than that one which arises in the standard minority game. On the other hand, depending on the topology of the network, we show that that the well known curve volatility versus memory, which characterizes the minority game, is a monotone decreasing curve.

Keywords: Complex networks, econophysics, herding behavior, minority games, phase transition.

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I. INTRODUCTION

In these last years, one of the most interesting contributions of the statistical physics to the social sciences has been to study the dynamics and the collective behavior of populations of agents who compete for limited resources. In particular, the so-called minority game (MG) introduced in [1] as a simplification of the Arthur’s El Farol Bar [2] attendance problem is one of the simplest complex systems that belong to this class. This game can be described in the following way. At a given instant of time, an agent who belongs to the population chooses between two opposing actions namely \( a = \pm 1 \). Since the resources are limited, the objective of each agent is to choose the side shared by the minority of the population. The difficulty is that each agent does not know what the others will choose. The agent decides his/her next action based only on a global information, which is the sequence of the last \( M \) outcomes of the game, where \( M \) is said to be the memory of the agents. Therefore, there is no best solution for the problem, i.e., the agents do not know what is the best strategy to deal with the game. Since there are only two possible choices, the number of states is \( 2^M \) and there are at all \( 2^M \) strategies. In [1], each agent has a fixed number of strategies that do not change over time. Since agents have different beliefs, the strategies differ from agent to agent. At every turn of the game, the agents use their strategies with highest scores.

This standard MG presented above has been very well-studied – a revision of these attempts may be found for instance in [3, 4, 5]. One of the most surprising properties presented first in [1] is that if one plots the ratio \( \sigma^2/N \) as a function of \( \alpha = 2^M/N \), one may conclude: (1) For small values of \( \alpha = 2^M/N \), the agents would perform worse than if they had taken purely random decisions. (2) For large values of \( \alpha = 2^M/N \), the agents’ performance approaches the random decision. (3) There is a critical value of \( \alpha = \alpha_c \) where the resources of the game are used in the best way possible, i.e., the ratio \( \sigma^2/N \) is the minimal possible – which suggests a non-equilibrium phase transition from the so-called low-\( M \) phase to the high-\( M \) phase. The low-\( M \) phase is characterized by a decrease in \( \sigma^2/N \) as \( \alpha = 2^M/N \) increases and the high-\( M \) phase is characterized by an increase in \( \sigma^2/N \) as \( \alpha = 2^M/N \) increases. (4) The behavior of the MG does not depend on the number of strategies available for each agent.

In this paper, we introduce a version of the standard MG with local interactions and exchange of local information. Actually, this is not the first paper that presents the exchange of local information and local interactions in some version of the MG. As far as we know, the first attempt in this line was presented in [6] where was developed a version of the Kauffman network using some rules of the minority game and each agent receives input from a fixed number of agents in the system. Other formulations of MGs with local interactions may be found in [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. In our paper, differently of the others cited above, the exchange of local interaction emerge from the imitation of the most informed agents, i.e., some of the agents who play the game believe that some of their neighbors are more informed then themselves. This phenomenon called herding behavior happens when an agent blindly follows the decision of other agent. The economic theory says that it is rational even when the former agent’s signals suggest a different decision and it is ignored[20]. In the case here, an agent follows another agent strategy, if he/she believes that there is an agent more informed than himself/herself, i.e., this most informed agent is more likely to know the decision of the minority. Actually, one should notice that if not one but many agents follow the most informed agent of their neighborhood, then, in the future, the most informed agent will be in the majority and he will no be followed anymore. In this paper, we investigate how this kind of policy performed by the agents affect the dynamics of the standard MG.
II. THE MG WITH HERDING BEHAVIOR

The game considered here has a framework quite similar to the standard one presented in [1]. The difference is described below. In each time step, each agent looks for the most informed agent located in his/her neighborhood. The most informed agent here is the one that has the highest scored strategy. Then, each agent compares his/her highest scored strategy with the highest scored strategy of his/her most informed neighbor. If the agent's highest scored strategy is higher scored than the highest scored strategy of the most informed agent in his/her neighborhood, then he/she follows his/her own strategy. Otherwise, he/she follows his/her most informed neighbor highest strategy.

III. THE NOTION OF NEIGHBORHOOD

The notion of neighborhood is provided by one of the following networks: (1) a regular network; (2) a random network [18] or (3) a small world network [19, 20]. While the regular network and the random network are chosen to be used as references, the small world network is chosen since it presents a topology that is likely to happen in real situations of social interaction [21]. Then, using one of these network structures, each agent of the minority game is located in a node of the network.

IV. RESULTS

Figures 1, 2, 3 and 4 present the main results of this paper. In all figures, we have plotted $\sigma^2/N$ as a function of $\alpha = 2^M/N$ for the coin toss market, for the standard minority game and for the minority game with the presence of herding behavior. The difference among them is the structure of neighborhood where in each figure the neighborhood is provided by a different network. In these figures, all simulations used the number of agents $N = 101$, the number of strategies $S = 2$ and the time horizon $T = 10000$. In figures 1, 2 and 4, $K$ is the number of “regular” neighbors – a parameter that arises in regular and small world networks. In figures 2, 3 and 4, $p$ is the probability of two agents being connected – a parameter that arises in random graphs and small world networks.

First of all, one may notice that in the presence of herding behavior, the volatility of the system is much larger than the volatility of the standard MG. This happens because the presence of herding behavior generates a crowd in the MG, i.e., a large groups of agents using the same strategy. This fact has been studied in economic theory [22] and econophysics [23], which show that herding behavior may be a source of large price movements and also crashes.

In all figures, up to a certain point of the presence of herding behavior, the slope of the low-$M$ phase is not modified and this phase is clear in almost all simulations. This can be easily interpreted. In the original MG [6], the low-$M$ phase is characterized by a crowded phase where the number of strategies is small when compared to the number of agents. The presence of herding behavior only reinforces this fact. However, if the presence of herding behavior is too strong as, for instance, in figure 4 where there are simulations with a small world network with parameters $K = 16$ and $p = 0.5$, part of the standard low-$M$ phase is replaced a horizontal line. This means that the crowd is so strong that the additional number of strategies introduced are not enough to reduce it.

On the other hand, in all figures, when the phenomenon of herding behavior emerges, the high-$M$ phase is strongly affected. In spite of the number of strategies is huge, this happens because the number of strategies that the agents actually use is small when compared to the number of agents.

Moreover, one should also notice that the value of $\alpha_c$ is almost the same in all simulations when herding behavior is present. However, the value of $\alpha_c$ in this situation is larger than the value found in the standard MG.

V. FINAL REMARKS

In this note, we have presented a new version of the standard MG with local interactions that emerge due to the presence of herding behavior. The herding behavior here arises since the agents that play the game sometimes do not believe in their own strategies and prefer to follow the most informed agents that belong to their neighborhood. As it been already pointed out, this kind of behavior is rational and justified by the economic theory [24].

Using this framework, we have found that the presence of herding behavior may affect both the low-$M$ phase and the high-$M$ phase. In particular, if one thinks the minority game as a model of financial market [25], then the results of this modified model agrees with the results also found in the economics [22] and econophysics [23] literature. Finally, we show that that the well known curve volatility versus memory, which caracterizes the phases of a minority game, is a monotone decreasing curve.
FIG. 1: The ratio $\sigma^2/N$ as a function of $\alpha = 2^M/N$. We compare here the standard MG with the modified MG presented in this paper using regular networks.

FIG. 2: The ratio $\sigma^2/N$ as a function of $\alpha = 2^M/N$. We compare here the standard MG with the modified MG presented in this paper using random networks.

FIG. 3: The ratio $\sigma^2/N$ as a function of $\alpha = 2^M/N$. We compare here the standard MG with the modified MG presented in this paper using small world networks with the basic structure provided by a regular network with $K = 2$ neighbors.

FIG. 4: The ratio $\sigma^2/N$ as a function of $\alpha = 2^M/N$. We compare here the standard MG with the modified MG presented in this paper using small world networks with the basic structure provided by a regular network with $K = 16$ neighbors.
[1] D. Challet and Y. C. Zhang, Physica A 246, 407 (1997).
[2] W. B. Arthur, American Economic Review 84, 406 (1994).
[3] N. F. Johnson, P. Jefferies, and P. M. Hui, Financial market complexity (Oxford University Press, Oxford, 2003).
[4] A. C. C. Coolen, The mathematical theory of minority games (Oxford University Press, Oxford, 2005).
[5] D. C. and M. Marsili and Y. C. Zhang, Minority games (Oxford University Press, Oxford, 2005).
[6] R. Savit, R. Manuca, and R. Riolo, Physical Review Letters 82, 2203 (1999).
[7] M. Paczuski, K. E. Bassler, and A. Corral, Physical Review Letters 84, 3185 (2000).
[8] T. Kalinowski, H.-J. Schulz, and M. Briese, Physica A 277, 502 (2000).
[9] S. M. P. D. L. Rios, Physica A 303, 217 (2002).
[10] A. Gabstyan and K. Lerman, Physical Review E 66, 015103 (2002).
[11] H. J. Quan, B. H. Wang, P. M. Hui, and L. X. S, Physica A 321, 300 (2003).
[12] H. F. Chau, F. K. Chow, and K. H. Ho, Physica A 332, 483 (2004).
[13] Y. Li and R. Savit, Physica A 335, 217 (2004).
[14] E. Burgos, H. Ceva, and R. P. J. Perazzo, Physica A 337, 635 (2004).
[15] L. Shang and X. F. Wang, Forthcoming in Physica A (2005).
[16] I. Caridi and H. Ceva, Physica A 339, 574 (2004).
[17] M. Kirley, Forthcoming Physica A (2005).
[18] P. Erdős and A. Rényi, Bulletin of the International Statistical Institute 38, 343 (1960).
[19] D. J. Watts and S. H. Strogatz, Nature 393, 440 (1998).
[20] D. J. Watts, Small worlds: the dynamics of networks between order and randomness (Princeton University Press, Princeton, 1999).
[21] D. O. Cajueiro, Physical Review E 72, 047104 (2005).
[22] I. H. Lee, Review of Economic Studies 65, 395 (1998).
[23] D. Sornette and A. Johansen, Physica A 245, 411 (1997).
[24] A. Banerjee, Quartely Journal of Economics 107, 797 (1992).
[25] D. Challet, M. Marsili, and Y. C. Zhang, Physica A 299, 228 (2001).
[26] In a financial market, for instance, this means to buy or to sell an asset.
[27] A strategy defines which action to take in each state.
[28] The strategies with highest scores are those which were successful in the previous turns of the game.
[29] For details, see, for instance [24].