X(3872): Hadronic Molecules in Effective Field Theory

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Abstract

We consider the implications from the possibility that the recently observed state $X(3872)$ is a meson-antimeson molecule. We write an effective Lagrangian consistent with the heavy-quark and chiral symmetries needed to describe $X(3872)$. We claim that if $X(3872)$ is a molecular bound state of $D^0$ and $\overline{D^0}$ mesons, the heavy-quark symmetry requires the existence of the molecular bound state $X_b$ of $B^0$ and $\overline{B^0}$ with the mass of 10604 MeV.

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I. INTRODUCTION

The past two years have seen several experimental observations of new heavy and light quark states. Most of those states contain a charm quark, so their observation rekindled interest in heavy-flavor spectroscopy [1]. The unusual properties of those states invited some speculations regarding their possible non-$q\bar{q}$ nature. Among those is the $X(3872)$ state which, being discovered in the decay $X(3872) \rightarrow J/\psi \pi^{+}\pi^{-}$, contains charm-anticharm quarks [2, 3]. While a traditional $c\bar{c}$ quarkonium interpretation of this state has been proposed [4], its somewhat unusual mass and decay patterns prompted a series of more exotic interpretations [5]. Since the mass of the $X(3872)$ state lies tantalizingly close to the $D^{*0}D^{0}$ threshold of 3871.3 MeV, it is tempting to assume that $X(3872)$ could be a $D^{*0}D^{0}$ molecular state [6, 7]. Recent Belle data appear to be consistent with this assignment, preliminarily confirming its $J^{PC} = 1^{++}$ quantum numbers [8]. Of course, states of different “nature” can mix, if they have the same quantum numbers, further complicating the interpretation of the experimental data [9].

An unambiguous identification of this state must be done with many different measurements of its decay and production patterns. Regardless of whether $X(3872)$ is identified to be a molecule or a regular $q\bar{q}$ charmonium, a theoretical analysis of heavy-meson molecular states should be performed. Until recently these studies were done mostly with the help of various quark models [6, 10]. In this paper we shall study those states using the techniques of effective field theories.

This study is possible due to the multitude of scales present in QCD. The extreme smallness of the binding energy

$$E_b = (m_{D^0} + m_{D^{*0}}) - M_X = -0.6 \pm 1.1 \text{ MeV} \quad (1)$$

suggests that this state can play the role of the “deuteron” (or “deuson,” see N. A. Törnqvist’s paper in [6]) in meson-meson interactions. The “deuteron-like” nature of this state allows us to use methods similar to those developed for the description of the deuteron, with the added benefit of heavy-quark symmetry. The tiny binding energy of this molecular state introduces an energy scale which is much smaller than the mass of the lightest particle, the pion, whose exchange can provide binding. Thus, for a suitable description of this state in the framework of effective field theory, the pion, along with other particles providing possible binding contributions (i.e. the $\rho$-meson and other higher mass resonances), must be integrated out. The resulting Lagrangian should contain
only heavy-meson degrees of freedom with interactions approximated by local four-boson terms constrained only by the symmetries of the theory. This approach is similar to the description of the deuteron in the effective theory without dynamical pions \[11\]. Nevertheless, we shall often appeal to the “exchange picture” to gain insight into the structure of the effective Lagrangian.

This approach provides a model-independent description of molecular states with somewhat limited predictive power. In particular, we would not be able to say whether the state \(X(3872)\) is a \(D^*D\) molecule or not. What we would be able to say is that if indeed \(X(3872)\) is a \(D^*D\) molecule, the heavy-quark symmetry makes a definite statement on the existence of a molecular state in the \(B^*B\) system. We also show that even though \(D\) and \(D^*\) are degenerate in the heavy-quark limit, the existence of a molecular state in \(D^*D\) channel does not necessarily imply a molecular state in the \(D^0D^0\) or \(B^*D^0\) channels.

This paper is organized as follows. In Sec. II we write the most general effective Lagrangian consistent with the heavy-quark and chiral symmetry. In Sec. III we obtain the bound-state energy by solving a system of Lippmann-Schwinger equations and relate the bound-state energies of \(D^*D\) and \(B^*D\) states. We present our conclusions in Sec. IV.

II. THE EFFECTIVE LAGRANGIAN

In order to describe the molecular states of heavy mesons we need an effective Lagrangian which contains two- and four-body interaction terms. The two-body effective Lagrangian that describes the strong interactions of the heavy mesons \(P\) and \(P^*\) \((P = B, D)\) containing one heavy quark \(Q\) is well known \[13\]:

\[
\mathcal{L}_2 = -i \text{Tr} \left[ \overline{H}^{(Q)} v \cdot D H^{(Q)} \right] - \frac{1}{2m_P} \text{Tr} \left[ \overline{H}^{(Q)} D^2 H^{(Q)} \right] + \frac{\lambda_2}{m_P} \text{Tr} \left[ \overline{H}^{(Q)} \sigma^{\mu\nu} H^{(Q)} \sigma_{\mu\nu} \right] + \frac{g}{2} \text{Tr} \overline{H}^{(Q)} H^{(Q)} \gamma_\mu \gamma_5 \xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger \right] + \ldots \tag{2}
\]

where the ellipsis denotes terms with more derivatives or including explicit factors of light quark masses, \(D^\mu_{ab} = \delta_{ab} \partial^\mu - (1/2) \left( \xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger \right)_{ab}\), and \(g\) is the \(P^*P\pi\) coupling. As usual, we introduce a superfield describing the combined doublet of pseudoscalar heavy-meson fields \(P_a = (P^0, P^+)\) and their vector counterparts with \(v \cdot P^*_{a} = 0\),

\[
H^{(Q)}_a = \frac{1+\gamma_5}{2} \left[ P^{*(Q)}_{a\mu} \gamma^\mu - P^{(Q)}_{a\mu} \gamma_5 \right], \quad \overline{H}^{(Q)a} = \gamma^0 H^{(Q)a}_5 \gamma^0 . \tag{3}
\]
These fields have the usual transformation properties under the heavy-quark spin symmetry and SU(2)_V flavor symmetry,

\[ H_a^{(Q)} \rightarrow S \left( H^{(Q)} U^\dagger \right)_a, \quad \bar{H}^{(Q)\dagger}_a \rightarrow \left( U \bar{H}^{(Q)} \right)^a S^{-1}, \]

and describe heavy mesons with a definite velocity \( v \). The third term in Eq. (2) is needed to account for the \( P - P^\star \) mass difference \( \Delta \equiv m_{P^\star} - m_P = -2\lambda_2/m_P \).

The pseudo-Goldstone fields are introduced as \( \xi = e^{i\tilde{M}/f} \), where \( \tilde{M} \) is the usual meson matrix,

\[ \tilde{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 \end{pmatrix}, \]

and \( f \approx 135 \text{ MeV} \) is the pion decay constant. Notice that since heavy quark-antiquark pair production is absent in this effective theory, the effective Lagrangian of Eq. (2) does not contain heavy antimeson degrees of freedom. Since we are describing the molecular states of heavy mesons, those fields should have to be explicitly added to the Lagrangian. The fields \( H_a^{(Q)} \) and \( H_a^{(Q)\dagger} \) that describe the propagation of heavy antimesons, i.e. containing the heavy antiquark \( \bar{Q} \), are introduced as

\[ H_a^{(Q)} = \left[ P_{a\mu}^{(*)} \gamma^\mu - \bar{P}_a^{(Q)} \gamma_5 \right] \frac{1-\not{\! v}}{2}, \quad \bar{H}^{(Q)\dagger}_a = \gamma^0 H_a^{(Q)\dagger} \gamma^0, \]

and transform as \( H_a^{(Q)} \rightarrow \left( U H^{(Q)} \right)_a S^{-1} \) and \( \bar{H}^{(Q)\dagger}_a \rightarrow S \left( \bar{H}^{(Q)} U^\dagger \right)^a \) under heavy-quark spin and SU(2)_V symmetry.

In order to write an effective Lagrangian describing the properties of \( X(3872) \), we need to couple the fields \( H_a^{(Q)} \) and \( H_a^{(Q)\dagger} \) so that the resulting Lagrangian respects the heavy-quark spin and chiral symmetries. Since the binding energy of \( X(3872) \) is small, the size of a bound state is rather large. This means that the particular details of the interaction of the heavy meson and antimeson pair (for example, a \( \rho \)-meson exchange contribution) are irrelevant for the description of such a bound state and can be well approximated by four-meson local interactions. One can write a Lagrangian describing \( X(3872) \) by first writing an effective Lagrangian above \( \mu = m_\pi \) and then matching it onto the Lagrangian for \( \mu < m_\pi \), i.e. integrating out the pion degrees of freedom.

The general effective Lagrangian consistent with heavy-quark spin and chiral symmetries can be written as

\[ \mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4, \]

1 A generalization of this discussion to the flavor SU(3) symmetry is rather straightforward.
where the two-body piece, consistent with reparametrization invariance, is given by Eq. (2) and the four-body piece is

$$- \mathcal{L}_4 = \frac{C_1}{4} \text{Tr} \left[ H^{(Q)} H^{(Q)} \gamma_\mu \right] \text{Tr} \left[ H^{(Q)} H^{(Q)} \gamma_\mu \right] + \frac{C_2}{4} \text{Tr} \left[ H^{(Q)} H^{(Q)} \gamma_\mu \gamma_5 \right] \text{Tr} \left[ H^{(Q)} H^{(Q)} \gamma_\mu \gamma_5 \right].$$ (8)

This Lagrangian, together with $\mathcal{L}_2$, describes the scattering of $P$ and $P^*$ mesons at the energy scale above $m_\pi$. Integrating out the pion degrees of freedom at tree level corresponds to a modification $C'_2 \rightarrow C_2 + (2/3) (g/f)^2$. Since in this paper we will not discuss matching at higher orders, the Lagrangian in Eq. (8) will be used for the calculation of the bound state properties of $X(3872)$.

By virtue of the heavy-quark spin symmetry, the same Lagrangian governs the four-boson interactions of all $P_a^{(*)} = D^{(*)}$ or $B^{(*)}$ states, while the flavor $SU(2)_V$ implies that there could be four such states for each $P_a^{(*)} P_b^{(*)}$. Indeed, not all of these states are bound. Here we shall concentrate on $X(3872)$, which is a bound state of two neutral bosons, $P_a \equiv P^0 \equiv P$, assuming the isospin breaking advocated in Ref. [6]. Notice that the most general Lagrangian involves two couplings, $C_1$ and $C_2$. Other Dirac structures are possible, but will yield the same Lagrangian for the $P P^*$ bound state.

In order to relate the properties of $P P^*$ molecules in the charm and beauty sectors we shall need to see how $C_i$ couplings scale as functions of the heavy-quark mass $M$. To see this, we recall that a system of two heavy particles requires a nonrelativistic $v/c$ expansion, not a $1/M$ expansion. This is necessary to avoid that the resulting loop integrals acquire pinch singularities [11]. Therefore, we must powercount $p^0 \sim \vec{p}^2 / M$, where $\vec{p}$ is a characteristic 3-momentum of a heavy meson in the $P P^*$ molecular bound state, which implies that the first and the second terms in Eq. (2) scale in the same way. Since the action $S = \int d^4x \, \mathcal{L}$ does not scale with the heavy-quark mass, this implies that $d^4x \sim M$ and the Lagrangian density $\mathcal{L} \sim 1/M$. The kinetic term in Eq. (2) then gives the expected scaling of the heavy-meson field $H \sim P \sim P^* \sim M^0$, which in turn implies from Eq. (8) that the couplings

$$C_i \sim 1/M.$$ (9)

This dimensional analysis, however, cannot be used to predict the relative contributions of other couplings in $\mathcal{L}_4$, say, relativistic corrections to Eq. (8), because of the fine-tuning which produces a molecular state close to threshold in the first place [11]. We will use it only to relate properties of $DD^*$ and $BB^*$ systems. Similar dimensional analysis was proposed for non-relativistic QCD [12].
Evaluating the traces yields for the $P\bar{P}^*$ sector

$$\mathcal{L}_{4,PP^*} = -C_1 P^{(Q)\dagger} P^{(Q)} P_{\mu}^{*(\bar{Q})\dagger} P^{*(\bar{Q})\mu} - C_1 P^{*(Q)\dagger} P^{*(Q)} P_{\mu}^{*(\bar{Q})\dagger} P^{*(\bar{Q})\mu}$$

$$+ C_2 P^{(Q)\dagger} P_{\mu}^{*(\bar{Q})\dagger} P^{*(\bar{Q})\mu} P^{(Q)} + C_2 P^{*(Q)\dagger} P_{\mu}^{*(\bar{Q})\dagger} P^{*(\bar{Q})\mu} P^{(Q)} + \ldots$$

(10)

Notice that this Lagrangian differs from the one used in [7], where the interaction strength is described by a single parameter $\lambda = -C_1 = C_2$. The difference can be understood in the “exchange” model, where $C_1$ and $C_2$ come from the exchanges of mesons of different masses and parity. In this language the model of Ref. [7] corresponds to the situation of degenerate parity states. In QCD, however, negative parity states are generally lighter than their positive parity counterparts. This is especially clear for the lightest octet of pseudoscalar mesons, where chiral symmetry forces their masses to be almost zero, while all the corresponding scalar mesons have masses of the order of 1 GeV. We will nevertheless show that the resulting binding energy still depends on a single parameter, a linear combination of $C_1$ and $C_2$.

Similarly, one obtains the component Lagrangian governing the interactions of $P$ and $\bar{P}$,

$$\mathcal{L}_{4,PP} = C_1 P^{(Q)\dagger} P^{(Q)} P^{(Q)\dagger} P^{(Q)}.$$  \hspace{1cm} (11)

Clearly, one cannot relate the existence of the bound state in the $P\bar{P}^*$ and $P\bar{P}$ channels, as the properties of the latter will depend on $C_1$ alone, not a linear combination of $C_1$ and $C_2$.

### III. Properties of Bound States

In order to describe bound states we shall modify the approach of S. Weinberg [11]. The lowest-energy bound state of $P$ and $\bar{P}^*$ is an eigenstate of charge conjugation. Here we carry out our calculation for $(P,\bar{P}^*) = (D,\bar{D}^*)$, but analogous considerations will apply to the $B$ system. The two eigenstates of charge conjugation will be given by

$$|X_\pm\rangle = \frac{1}{\sqrt{2}} \left[ |D^*\bar{D}\rangle \pm |D\bar{D}^*\rangle \right].$$

(12)

To find the bound-state energy of $X(3872)$ with $J^{PC} = 1^{++}$, we shall look for a pole of the transition amplitude $T_{++} = \langle X_+|T|X_+\rangle$.

We first define the following transition amplitudes,

$$T_{11} = \langle D^*\bar{D}|T|D^*\bar{D}\rangle, \quad T_{12} = \langle D^*\bar{D}|T|D\bar{D}^*\rangle,$$
FIG. 1: Transition amplitudes for the $D - \bar{D}^*$ scattering written in the form of Lippmann-Schwinger equations. Double lines indicate the vector $D^*$ or $\bar{D}^*$ mesons, solid lines pseudoscalar $D$ or $\bar{D}$ states.

\[ T_{21} = \langle D\bar{D}^* | T | D^*\bar{D} \rangle, \quad T_{22} = \langle D\bar{D}^* | T | DD^* \rangle, \quad (13) \]

which correspond to the scattering of $D$ and $D^*$ mesons. Clearly, at tree level, $T_{ii} \sim C_1$ and $T_{ij}, i \neq j \sim C_2$, since we consider only contact interactions. But we also have to include a resummation of loop contributions to complete the leading order [11]. These transition amplitudes satisfy a system of Lippmann-Schwinger equations depicted in Fig. 1.

\[ iT_{11} = -iC_1 + \int \frac{d^4q}{(2\pi)^4} T_{11} G_{PP^*} C_1 - \int \frac{d^4q}{(2\pi)^4} T_{12} G_{PP^*} C_2, \]

\[ iT_{12} = iC_2 - \int \frac{d^4q}{(2\pi)^4} T_{12} G_{PP^*} C_2 + \int \frac{d^4q}{(2\pi)^4} T_{12} G_{PP^*} C_1, \]

\[ iT_{21} = iC_2 + \int \frac{d^4q}{(2\pi)^4} T_{21} G_{PP^*} C_1 - \int \frac{d^4q}{(2\pi)^4} T_{22} G_{PP^*} C_2, \]

\[ iT_{22} = -iC_1 - \int \frac{d^4q}{(2\pi)^4} T_{22} G_{PP^*} C_1 + \int \frac{d^4q}{(2\pi)^4} T_{22} G_{PP^*} C_2, \quad (14) \]

where

\[ G_{PP^*} = \frac{1}{4} \frac{1}{(\vec{p}^2/2m_D + q_0 - \Delta - \vec{q}^2/2m_{D^*} + i\epsilon)(\vec{p}^2/2m_D - q_0 - \vec{q}^2/2m_{D^*} + i\epsilon)}, \quad (15) \]
\( \vec{p} \) is the momentum of one of the mesons in the center-of-mass system, and we canceled out factors of \( m_D m_{D^*} \) appearing on both sides of Eq. (14). Note that in Eq. (15) the vector propagator includes the mass difference \( \Delta \), as a consequence of the term proportional to \( \lambda_2 \) in the Lagrangian of Eq. (2). This choice of propagators is not unique, but our results will not depend on it, because it amounts to a choice of a finite phase multiplying the heavy-meson fields. This rephasing is equivalent to measuring energies with respect to the pseudoscalar mass, \( m_D \). Then, the position of the transition amplitude pole on the energy scale will be measured with respect to the “constituent mass” of the system, which is in our case twice the pseudoscalar mass \( m_D \). A different choice of phase will give different propagators but also a different “constituent mass”.

Since we are interested in the pole of the amplitude \( T_{++} \), we must diagonalize this system of equations rewritten in an algebraic matrix form,

\[
\begin{pmatrix}
T_{11} \\
T_{12} \\
T_{21} \\
T_{22}
\end{pmatrix} =
\begin{pmatrix}
-C_1 \\
C_2 \\
C_2 \\
-C_1
\end{pmatrix}
+ i \tilde{A}
\begin{pmatrix}
-C_1 & C_2 & 0 & 0 \\
C_2 & -C_1 & 0 & 0 \\
0 & 0 & -C_1 & C_2 \\
0 & 0 & C_2 & -C_1
\end{pmatrix}
\begin{pmatrix}
T_{11} \\
T_{12} \\
T_{21} \\
T_{22}
\end{pmatrix}.
\]

(16)

Notice that the matrix is in the block-diagonal form, which allows us to solve Eq. (16) in two steps working only with \( 2 \times 2 \) matrices. The solution of Eq. (16) produces the \( T_{++} \) amplitude,

\[
T_{++} = \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) = \frac{\lambda}{1 - i\lambda \tilde{A}},
\]

(17)

with \( \lambda = -C_1 + C_2 \), and \( \tilde{A} \) is a (divergent) integral

\[
\tilde{A} = \frac{1}{4} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{\left( \vec{p}^2/2m_{D^*} + q_0 - \Delta - \vec{q}^2/2m_{D^*} + i\epsilon \right) \left( \vec{p}^2/2m_D - q_0 - \vec{q}^2/2m_D + i\epsilon \right)}
\]

\[
= \frac{i}{4} \frac{\sqrt{2m_{D^*}}} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{\vec{q}^2 - 2\mu_{DD^*} (E - \Delta) - i\epsilon},
\]

(18)

where \( E = \vec{p}^2/2\mu_{DD^*} \), \( \mu_{DD^*} \) is the reduced mass of the \( DD^* \) system, and we have used the residue theorem to evaluate the integral over \( q_0 \). The divergence of the integral of Eq. (18), as usual, is removed by renormalization. We choose to define a renormalized \( \lambda_R \) within the \( MS \) subtraction scheme in dimensional regularization. In this scheme the integral \( \tilde{A} \) is finite, which corresponds to an implicit subtraction of power divergences in Eq. (18). Computing the second integral in
Eq. (18) by analytically continuing to $d - 1$ dimensions yields

\[
\tilde{A} = - \frac{1}{8\pi} \mu_{DD^*} |\vec{p}| \sqrt{1 - \frac{2\mu_{DD^*}\Delta}{\vec{p}^2}}.
\]  
(19)

This implies for the transition amplitude

\[
T_{++} = \frac{\lambda_R}{1 + (i/8\pi)\lambda_R \mu_{DD^*} |\vec{p}| \sqrt{1 - 2\mu_{DD^*}\Delta/\vec{p}^2}}.
\]  
(20)

The position of the pole of the molecular state on the energy scale can be read off Eq. (20),

\[
E_{\text{Pole}} = \frac{32\pi^2}{\lambda_R^2 \mu_{DD^*}^3} - \Delta.
\]  
(21)

This is the amount of energy we must subtract from the “constituent mass” of the system, determined above as $2m_D$, in order to calculate the mass

\[
M_X = 2m_D - E_{\text{Pole}} = 2m_D + \Delta - \frac{32\pi^2}{\lambda_R^2 \mu_{DD^*}^3}.
\]  
(22)

Recalling the definition of binding energy, Eq. (1), and that $m_{D^*} = m_D + \Delta$, we infer

\[
E_b = \frac{32\pi^2}{\lambda_R^2 \mu_{DD^*}^3}.
\]  
(23)

Assuming $E_b = 0.5$ MeV, which is one sigma below the central value in Eq. (2), and the experimental values for the masses [17], we obtain

\[
\lambda_R \approx 8.4 \times 10^{-4} \text{ MeV}^{-2}.
\]  
(24)

Note that the binding energy scales as $1/M$ in the heavy-quark limit. Thus, the smallness of the binding energy is implied in the heavy-quark limit. The small binding energy of the $X(3872)$ state implies that the scattering length $a_D$ is large and can be written as

\[
a_D = \sqrt{\left(2\mu_{DD^*} E_b\right)^{-1}} = \frac{\lambda_R \mu_{DD^*}}{8\pi},
\]  
(25)

yielding a numerical value $a_D = 6.3$ fm. Since the scattering length is large, universality implies that the leading-order wave function of $X(3872)$ is known,

\[
\psi_{DD^*}(r) = \frac{e^{-r/a_D}}{\sqrt{2\pi a_D r}}.
\]  
(26)
This can be used to predict the production and decay properties of \(X(3872)\). Once we establish the molecular nature of \(X(3872)\), its experimental mass gives us its binding energy. The latter is dependent on the coupling constant \(\lambda_R\), and may be used to predict the binding energies of hypothetical molecular bound states in the beauty sector, as well as to discuss its implications for the beauty-charm sector. Alternatively, the coupling constant \(\lambda_R\) can be fixed from the resonance-exchange model, in which case (model-dependent) predictions are possible for all the heavy sectors. Taking into account the scaling of \(\lambda_R\) with \(M\) given by Eq. (9), we obtain

\[
\lambda_R^B \sim \frac{\lambda_R^{DD^*} \mu_{DD^*}}{\mu_{BB^*}}.
\]  

Formula (23) can now be used for the \(B\) system. This implies the existence of an S-wave bound states with binding energy \(E_b = 0.18\) MeV, mass \(M_{X_b} = 10604\) MeV, and a scattering length \(a_B = a_D = 6.3\) fm, because of the lack of scaling of the scattering length with the heavy-quark mass. The above prediction for the binding energy is lower than the quark-model prediction of Wong in [1]. Similar considerations apply to \(D^0 \overline{D}^0\) and \(B^0 \overline{B}^0\) states: In their case the starting point is the Lagrangian term in Eq. (11). Since it involves only a single term, the calculations are actually easier and involve only one Lippmann-Schwinger equation. The resulting binding energy is then

\[
E_b = \frac{256\pi^2}{C_1^2 R m_B^3},
\]  

and the equivalent formula for the \(B^0 \overline{B}^0\) system may be obtained by rescaling the coupling constant \(C_1^2\) as we did with \(\lambda_R\) in Eq. (27). Examining Eq. (28) we immediately notice that the existence of a bound state in the \(D^* \overline{D}\) channel does not dictate the properties of a possible bound state in the \(D^0 \overline{D}^0\) or \(B^0 \overline{B}^0\) channels, since \(C_1\) and \(C_2\) are generally not related to each other.

The discussion of the beauty-charm system parallels what was done above. In this case the situation is a bit different, because there are two states, \(B^0 \overline{D}^0\) and \(D^0 \overline{B}^0\). The treatment of these states depends on how the heavy-quark limit is taken. They have different masses, since \(\Delta_{BB^*} \neq \Delta_{DD^*}\). This implies that these two states do not mix and have to be treated separately, so that their binding energies could be obtained by respectively substituting \(\lambda_R \rightarrow C_1 R\) and \(\mu_{DD^*} \rightarrow \mu_{BD^*}\) or \(\mu_{DB^*}\) in Eq. (28). But just like in the case of \(D^0 \overline{D}^0\) or \(B^0 \overline{B}^0\) channels, we cannot predict bound states in these channels from the heavy-quark symmetry arguments alone.
IV. CONCLUSION

We have used an effective field theory approach in the analysis of the likely molecular state $X(3872)$, by describing its binding interaction with contact terms in a heavy-quark symmetric Lagrangian. The flexibility of this description allows us to ignore the details of the interaction and to concentrate on its effects, namely a shallow bound state and a large scattering length. Taking into account the universality and the scaling of the effective coupling constants we are able to extend our description to the $B$ system. We found that if $X(3872)$ is indeed a molecular bound state of $D^{*0}$ and $\bar{D}^0$ mesons, the heavy-quark symmetry requires the existence of the molecular bound state $X_b$ of $B^{*0}$ and $\bar{B}^0$ with the mass of 10604 MeV.

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