Experimental sizing and assessment of two-phase pressure drop correlations for a capillary tube with transcritical and subcritical carbon dioxide flow

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Abstract. In the last years, CO₂ was proposed as an alternative refrigerant for different refrigeration applications (automotive air conditioning, heat pumps, refrigerant plants, etc.) In the case of low power refrigeration applications, as a household refrigerator, the use of too expensive components is not economically sustainable; therefore, even if the use of CO₂ as the refrigerant is desired, it is preferable to use conventional components as much as possible. For these reasons, the capillary tube is frequently proposed as expansion system. Then, it is necessary to characterize the capillary in terms of knowledge of the evolving mass flow rate and the associate pressure drop under all possible operative conditions.

For this aim, an experimental campaign has been carried out on the ENEA test loop “CADORE” to measure the performance of three capillary tubes having same inner diameter (0.55 mm) but different lengths (4, 6 and 8 meters). The test range of inlet pressure is between about 60 and 110 bar, whereas external temperatures are between about 20 to 42 °C.

The two-phase pressure drop through the capillary tube is detected and experimental values are compared with the predictions obtained with the more widely used correlations available in the literature. Correlations have been tested over a wide range of variation of inlet flow conditions, as a function of different inlet parameters.

1. Introduction
Carbon dioxide has been proposed as an alternative to HFCs in refrigeration systems for its almost optimal environmental features. For this, several possible CO₂ fields of application (automotive air conditioning, residential heating and cooling, water heating, commercial refrigeration, etc.) have been studied and a number of plants have been built. In many of these applications, in particular in the case of medium-large powers, the target to obtain high COPs led to the achievements of complex and expensive engineering solutions. Instead, in the case of lower power applications, the simplicity of installation and the containment of plant costs induce the adoption of technical solutions which are simpler and with lower economic impact.

One possible CO₂ small size application is related to the household refrigeration: this application requires obtaining of rather low evaporation temperatures (Tₑv), typically from -25 to -30 °C, with cooling capacities (Qₑv) of the order of 60-180 W. If we do not want to use complex regulation systems for the execution of the expansion phase in the refrigeration CO₂ cycle, it is still possible using a capillary tube. This cheaper option can be pursued as, in the case of transcritical cycle, the
capillary tube demonstrates an intrinsic ability to adjust the higher pressure of CO₂ to a value close to the optimal one to vary the outlet temperature of gas-cooler [1].

The aim of this work is the characterization and the experimental testing of a capillary tube to be used in a prototypic household refrigerator. For this purpose, an experimental campaign was carried out on the test loop “CADORE” to measure the performance of three capillary tubes having the same inner diameter (0.55 mm) but different lengths (4, 6 and 8 meters). In particular, we tried to reach the working conditions of a CO₂ household refrigerator using the three capillary tubes; in this way, the influence of the choice of capillary tube dimensions on the performances was investigated. The test range of inlet pressure was between about 60 and 110 bar, whereas external temperatures were between about 20 to 42 °C.

The two-phase pressure drop through the capillary tube is detected and experimental values compared with the predictions obtained with the more widely used correlations available in the literature. Correlations have been tested over a wide range of variation of inlet flow conditions, as a function of different inlet parameters.

Moreover, the influence of the values of the subcooling, mass flow-rate, pressure and temperature at the capillary inlet on the pressure drop is evaluated.

2. Experiment

The tests have been carried out on the ENEA test loop CADORE (Carbon Dioxide Refrigeration) [2].

Figure 1. “CADORE” CO₂ circuit layout

Figure 1 shows a schematic of the experimental plant, which is composed of a carbon dioxide refrigeration loop, two auxiliary systems devoted to control the secondary fluid and a measurement and data acquisition system. The refrigeration loop is basically equipped with two hermetic single-stage reciprocating compressors, three finned coil heat exchangers (evaporator, gas cooler and intercooler), a double-stage expansion system and a filter dryer. Bypass circuits could allow the testing of different plant configurations.
Two auxiliary PID-controlled closed loop systems are employed to regulate and control the heat transfer rate in both the evaporator and the gas cooler, by setting the desired temperature, mass flow-rate and air humidity value at the heat exchangers inlet.

Temperatures at the inlet and the outlet of each component of the refrigeration loop are measured using Cr/Al K-type and Fe/Co J-type thermocouples. In correspondence of each thermocouple, except for the internal heat exchanger inlet and outlet, pressure transmitters with an accuracy of ±0.08% FS is used. Absorbed power and mass flow-rate are measured by a digital wattmeter with an accuracy of ±0.5% of reading and a Coriolis mass flow meter placed downstream the gas cooler and the filter dryer, respectively. Secondary fluid conditions are measured both for the evaporator and the gas cooler loops with several thermocouples placed alongside the air circuit, absolute and differential pressure transducers, hot wire anemometers and humidity active transmitters.

The output signals from the transducers and thermocouples are recorded in real time through a computerized data acquisition system and elaborated by a software developed in LabVIEW environment.

From the measured data, we calculated the thermodynamic and thermophysical properties (enthalpy, specific heat, etc.) using the computer code REFPROP version 8.0 of the NIST.

An important feature of CO\textsubscript{2} is the great variability of these properties around the critical point, as shown in figure 2 and 3 for the isobaric specific heat and for dynamic viscosity [3].

The facility, as said before, can work in different configurations; in this research, it was used in the basic configuration of a vapour compression refrigerating machine. Three experimental campaigns (for a total of 129 tests) with capillary tubes of the same inlet diameter (0.55 mm) and different lengths (4, 6 and 8 m) were carried out.

Another important feature of the CO\textsubscript{2} transcritical cycle is the existence of an optimum value for the gas cooler pressure p\textsubscript{GC}, which leads to obtain the maximum COP, once the other operating conditions have been fixed [4]. Figure 4 shows that the COP theoretically obtainable is a function of p\textsubscript{GC} for every CO\textsubscript{2} gas cooler outlet temperature (T\textsubscript{CO2outGC}). A numerical simulation performed using the data reported in table 1, allows to obtain the COP trend as a function of gas cooler pressure (figure 5).

Table 1. Data for performance evaluation of CO\textsubscript{2} vapour compression refrigerating cycles

| Parameter                                | Value          |
|------------------------------------------|----------------|
| Temperature of isobaric evaporation T\textsubscript{ev} | -25°C          |
| Isentropic efficiency of adiabatic compression \( \eta_{is} \) | 0.7            |
| Vapour superheat at compressor inlet \( \Delta T_{surr} \) | 5°C            |
| Isenthalpic transformation in the capillary tube |                |
Figure 4. Transcritical cycle with different value of gas cooler pressure [4].

Figure 5. Theoretical performance evaluation of CO₂ vapour compression refrigerating cycle.

This figure depicts the importance of working with pressures close to that corresponding to the highest COP, especially for $T_{CO2_{outGC}}$ higher than 30°C. This behavior confirms the importance of the throttling system and the expansion devices.

The compressor used in these experimental campaigns is a prototype of 500 W nominal cooling capacity and the heat exchangers (gas cooler and evaporator) are those planned for a CO₂ household refrigerator prototype. In particular, the gas cooler works in all tests with efficiency very high, so the CO₂ temperature at the inlet of capillary tube is always very similar to the temperature of the environment external to the household refrigerator.

Furthermore, operating conditions of the heat exchangers and the refrigerant charge are regulated to obtain the required different circuit operating conditions (in table 2, the operative conditions at inlet and outlet of the capillary tubes are indicated).

Table 2. Test Matrix

| Capillary length | Number of test | $P_{in}$ bar | $T_{in}$ °C | $P_{out}$ bar | $G$ kg/h | $\Delta T_{sub}$ °C | charge g |
|------------------|----------------|--------------|-------------|--------------|----------|--------------------|--------|
| 4                | 41             | 55           | 105.5       | 17.7         | 42.4     | 12.8               | 20.2   |
|                  |                | max          | min         | max          | min      | max                | max    |
| 6                | 63             | 54.2         | 107         | 16           | 41       | 10.8               | 18.2   |
|                  |                | max          | min         | max          | max      | max                | 2.3    |
| 8                | 25             | 55.2         | 103.5       | 16           | 40.5     | 10.8               | 18     |
|                  |                | min          | max         | min          | max      | 2.1                | 4.7    |
|                  |                |              |             |              |          | 0.7                | 500    |
|                  |                |              |             |              |          | 15                 | 700    |

Assuming an isoenthalpic expansion from the inlet conditions we can obtain the saturation pressure and therefore the pressure drops in the single phase and in the two-phase. In the first zone, the fluid can be considered incompressible and the single phase length ($\Delta l$ in equation 1) can be calculated by the equation [5]:

$$\left(\frac{\Delta p}{\Delta l}\right)_{SP} = \frac{f_{SP}G^2}{2\rho}$$ (1)

The single phase friction factor $f_{SP}$ can be evaluated with the explicit equation proposed by Swamee and Jain [6] and used by Madsen et al. [5] for CO₂.
where “e” is the surface roughness and “d” the diameter of capillary tube. This equation shows a maximum deviation of 1% from the Colebrook equation for Re values between 5000 and 10^4 and “e/d” values between 10^{-6} and 0.01: all experimental values of Re and e/d are in these ranges.

To improve calculation precision in (1), Δp between the initial state and saturation state is divided into 10 parts: the total length of tube required for the single phase flow is therefore calculated and consequently also the length available for two-phase flow. The experimental two-phase length in this way obtained is divided in 10 parts and for each section the two-phase pressure drop is calculated using the main correlations proposed in literature.

In two-phase conditions, the pressure drop in horizontal tubes is the sum of the momentum pressure drop (acceleration) and the frictional pressure drop:

\[ \Delta p_{TP} = \Delta p_a + \Delta p_{fr} \]  

(3)

The momentum pressure drop can be evaluated, for each section, with the equation:

\[ \Delta p_a = G_s^2 (v_{out} - v_{in}) \]  

(4)

To calculate this term it is necessary to know the thermodynamic outlet conditions, which depend on the total pressure drop. For their evaluation an iterative method has been used: the values of the momentum pressure drop are, for our tests, between 2 and 4 bar.

Kim e Mudawar [7] provided an exhaustive list of main prediction methods for the frictional pressure drop calculation. Some methods are semi-empirical, whereas others proceed, as the famous model of Lockhart-Martinelli, to the definition of a two-phase multiplier in order to obtain the \( \Delta p_a \) using an equation like that:

\[ \left( \frac{dp}{dz} \right)_{fr} = \left( \frac{dp}{dz} \right)_l \phi^2 \]  

(5)

Among the best-known models, we tested the semi-empirical correlation of Friedel [8], a homogeneous equilibrium model and the method of Sun and Mishima [9], which, starting from the Lockhart-Martinelli model, changes the two-phase multiplier using data obtained by experimental tests on the expansion of various refrigerants (including CO₂) in tubes of diameter between 0.5 and 12mm.

The correlation of Friedel, among other, was proposed by Madsen [5] for the capillary tube sizing in a transcritical refrigeration system and it is considered by Cecchinato et al. [1,10] as one of the most accurate methods for sizing of capillary tubes with CO₂.

3. Results and discussion

The thermodynamic capillary inlet conditions can be very different as the boundary conditions change (e.g. ambient temperature). Figure 6 shows the CO₂ capillary inlet conditions in the performed tests; in the experimental test loop “CADORE”, with a refrigerant charge of about 500 g and varying the outside air temperature from about 20 to 40 °C, we can observe that the upper pressure, depending on gas cooler outlet temperature, increases from about 60 to 100 bar, moving from subcritical to transcritical (or supercritical) conditions. We know that CO₂ is in supercritical conditions when the pressure and the temperature are both higher than the critical pressure (\( p_c = 7377 \) kPa) and critical temperature (\( T_c = 30.98°C \)), in transcritical conditions when the pressure is higher than the \( p_c \) but the temperature is lower than the \( T_c \) and in subcooled state when the pressure is lower than the critical value and temperature is between \( T_c \) and the evaporation temperature.
In the next sections we present:

- the performance obtainable (cooling capacity, $Q_{ev}$, evaporation temperature, $T_{ev}$) with the three capillaries in our refrigerant circuit;
- the analysis of some process variables, with particular regard to the relation between pressure drops and other parameters;
- the prediction capacity of pressure drop of some correlations available in the literature.

### 3.1 Comparison of performance obtained with the three capillary tubes

The total pressure drop ($\Delta p_{cap}$) in the capillary tube depends mainly on the CO$_2$ inlet thermodynamic conditions and on the refrigerant charge: in particular, it is noted that the pressure drop increases approximately linearly with the inlet capillary temperature, $T_{inCAP}$ (figure 7, where all performed tests are plotted). We can also observe that, for the same CO$_2$ inlet temperatures and capillary length (figure 8, in the case of 8 m length), a considerable increase in the pressure drop may be obtained raising the refrigerant charge.

**Figure 6.** Experimental capillary inlet points on the CO$_2$ p-h diagram

- Enthalpy [kJ/kg]
- Pressure [bar]

- 4m
- 6m
- 8m
Figure 7. Total pressure drop in the capillary tubes vs $T_{inCAP}$; parameter capillary length

Figure 8. Total pressure drop in the 8 m capillary tubes as a function of $T_{inCAP}$

Figure 9. $T_{ev}$ as a function of $\Delta p_{cap}$; parameter capillary length

Figure 10. $T_{ev}$ as a function of $\Delta p_{cap}$ for 8 m capillary tube

Figure 9 shows the evaporation temperatures obtainable with the three capillary tubes versus the pressure drop for all the performed tests. The evaporation temperatures seem to show a dependence on the refrigerant charge (figure 10, where, for clarity, only the tests with capillary length of 8 m are presented), whose mechanisms will be investigated in a future work; figure 9 also shows that, for capillary pressure drops larger than 80 bar, corresponding to inlet temperatures above 35 °C, see figure 7, evaporation temperatures equal to or lower than the target of -25°C are obtainable only with capillaries longer than 6 m.
Figure 11 shows the cooling capacity obtained with the three capillary tubes as a function of the pressure drop for all the performed tests. For the cooling capacity, the dependence on the charge appears to be more evident (figure 12, where only data obtained with 8 m capillary length are shown). Figure 11 shows how the target of 150 W is achieved in almost all cases (except for few tests with the capillaries of 6 and 8 m and low pressure).

Figure 11. $Q_{ev}$ as a function of $\Delta p_{cap}$; parameter capillary length

Figure 12. $Q_{ev}$ as a function of $\Delta p_{cap}$ in 8 m capillary tube

Figure 13. $p_{GCopt}$ vs $p_{GCexp}$ for all data; parameter refrigerant charge

Figure 14. $p_{GCopt}$ vs $p_{GCexp}$ for 8m capillary tube; parameter refrigerant charge

Regarding to capillary tube capability to adjust the gas cooler pressure $p_{GC}$ towards optimal value, the figure 13 shows that, for all tests with a charge of 500g, the experimental "upper pressure" $p_{GCexp}$ is
always very close to optimum value $p_{GC_{opt}}$ (calculated assuming the experimental values of $T_{ev}$, $\Delta T_{surr}$ and $T_{CO2out_{GC}}$, isoenthalpic expansion and an isentropic efficiency $\eta_{is}$ of compressor of 0.7).

This data would seem to confirm that, by setting a suitable refrigerant charge, it is possible to work with upper pressures close to the optimum value even using a capillary tube as expansion device, for a large range of external conditions. In the case of our prototype, the best value of the charge seems to be 500g. For each tube length, increasing the charge, $p_{GC}$ increases with respect to optimal value (figure 14, for the 8 m capillary tube). This leads to greater COP penalizations when the $T_{CO2out_{GC}}$ is of the order of 20-30 °C (figure 5).

Quantitatively, the maximum deviation of the experimental pressure from the optimal one is 5% with a charge of 500g and can reach the 30% with a charge of 700g.

3.2 Evaluation of pressure drop trough the capillary tubes

The upper pressure varies from 54 to 107 bar, the inlet temperature ranges from 0.6 (with use of an internal heat exchanger) to 41°C while the outlet pressure is between 10 bar and 20.2 bar. These test conditions are chosen to check the typical operating conditions of household refrigerators, which usually adopt capillary tubes as throttling devices. Figure 15 shows that, in our tests, the choking condition is never achieved. In fact, choking pressure, $p_{ch}$, calculated using the correlation proposed by Zhang e Ding [11], ranges from 4 to 12 bar, values always lower than the achieved evaporation pressures $p_{ev}$.

![Figure 15. Evaluation of the choked flow: $p_{ch}$ vs $p_{ev}$](image)

The performance of the tested correlations can be evaluated using the statistical parameters, standard deviation $\sigma$ and root mean square RMS:

$$\sigma = \sqrt{\frac{\sum (\Delta_i - \Delta_{med})^2}{N-1}}$$  

$$\text{RMS} = \sqrt{\frac{\sum \Delta_i^2}{N-1}}$$
where $\Delta_i$ indicates the percentage deviation from the measured value for a single test, $\Delta_{med}$ is the average of $\Delta_i$, and N is the number of tests performed.

Table 3 shows the performance of the main tested methods, implemented as described in [7]. In the following, we will analyze in particular the best three correlations: the homogeneous equilibrium model, Friedel and Sun-Mishima.

**Table 3. Statistical reproductive accuracy of the tested methods**

| Correlation               | $\Delta_{i,\text{min}}$ (%) | $\Delta_{i,\text{max}}$ (%) | $\sigma$ (%) | RMS (%) |
|---------------------------|-------------------------------|-------------------------------|-------------|---------|
| HOMOGENEOUS MODEL         | -59.1                         | +22.3                         | 14.1        | 30.4    |
| FRIEDEL                   | -50.5                         | +26.9                         | 13.6        | 23.1    |
| MULLER-STEINHAGEN         | -92.0                         | +134.3                        | 33.1        | 54.4    |
| SUN-MISHIMA               | -51.1                         | +89.5                         | 23.9        | 24.2    |
| BEATTYE_WHALLEY           | -46.1                         | +212.9                        | 48.5        | 58.6    |

Figure 16 shows the comparison between the frictional pressure drops calculated with the equilibrium homogeneous model and the measured ones. This method has considerable underestimation for low $\Delta p$ and a few overestimations when the $\Delta p$ approaches 60 bar. The high value of RMS compared to the standard deviation indicates a high dispersion of the predictions.

**Figure 16. $\Delta p_f$ calculated with the homogeneous model vs measured $\Delta p_f$**

The predictions of the Friedel method have a trend similar to those of the homogeneous model, but are more centered around the measured value (figure 17). In fact, the RMS is lower, indicating a lower dispersion of predictions.

Figure 18 shows the quality of the prediction of the Friedel correlation as a function of saturation pressure. The capacity of prediction has a trend fairly stable up to pressures of the order of 70 bar with a tendency to underestimation of 20% - 40%. Only when the pressure is above 70 bar the method presents some overestimation, which, apart from one single experimental point, remains lower than 10%. We can not note any particular trend with respect to Reynolds number.
Figure 17. $\Delta p_f$ calculated with the Friedel correlation vs measured $\Delta p_f$

Figure 18. Friedel correlation $\Delta p_f$ predictions vs isenthalpic saturation pressure; parameter: Re at starting of two-phase flow

Figure 19 shows the predictions of the Sun and Mishima correlation. This correlation adapts the correlation of Lockhart-Martinelli, in particular the two-phase multiplier, also using experimental data obtained with CO$_2$ and pipes of diameter between 0.5 and 12mm. The correlation, implemented as reported in [7], provides dispersed predictions with high overestimation values, up to 90% for high saturation pressures (figure 20). On the other hand, for low saturation pressures the underestimation can even reach 50%.

Figure 19. $\Delta p_f$ calculated with the Sun-Mishima correlation vs measured $\Delta p_f$

Figure 20. Sun-Mishima correlation $\Delta p_f$ predictions vs isenthalpic saturation pressure; parameter: Re at starting of two-phase flow
Figure 20 shows the quality of the prediction of the Sun and Mishima correlation as a function of the saturation pressure. Prediction capability tends to grow linearly with the saturation pressure up to pressures of the order of 70 bar and then rapidly increase beyond the optimal values for higher pressures. Even in this case, we can not note any particular trend with respect to Re number.

4. Conclusions
This work shows the results of experimental tests, carried out on 3 capillary tubes of different lengths, to characterize and size a capillary tube that ensures the achievement of certain cooling performance within definite reference conditions. This capillary is a part of a prototype of a household refrigerator that uses CO_2 as refrigerant.

Regarding to the refrigeration performance, tests have shown that:

- with unfavourable environmental conditions (i.e. with outside temperatures higher than 32-35 °C and with capillary lengths of 4 and 6 m, the desired evaporation temperature (almost -25 °C) can not be reached;
- the cooling capacity obtained is often larger or equal to the desired one.

Regarding to operating conditions, experimental data seem to confirm that, by setting a suitable refrigerant charge, it is possible to work with upper pressures close to the optimum values even using a capillary tube as expansion device, for a large range of external conditions.

The evaluation of the pressure drop during the expansion is important, among other, for the definition of the characteristic curve of the capillary tube, which, coupled with that of the compressor, defines the operating points and the performance of the refrigeration circuit.

Among the tested methods for the calculation of two-phase pressure drop, only the Friedel and homogeneous methods provide satisfactory predictions. All models have a noticeable worsening of the predictions when the saturation pressure approaches the critical one. This may be due to the assumption of calculation values for thermodynamic and thermophysical properties very different from the actual measurements. Indeed, these properties are computed as a function of some measured parameters (for example, isobaric specific heat and dynamic viscosity are a function of temperature, figure 2 and 3). Then a small error in the measurement, possible despite the use of high-precision instruments, can lead to large errors in the calculation of the thermodynamic and thermophysical properties.

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