Analysis of field-angle dependent specific heat in unconventional superconductors: a comparison between Doppler-shift method and Kramer-Pesch approximation

Nobuhiko Hayashi\textsuperscript{a,b}, Yuki Nagai\textsuperscript{c,d}, Yoichi Higashi\textsuperscript{e}

\textsuperscript{a} Nanoscience and Nanotechnology Research Center (N2RC), Osaka Prefecture University, 1-2 Gakuen-cho, Sakai 599-8570, Japan
\textsuperscript{b} CREST(JST), 4-1-8 Honcho, Kawaguchi, Saitama 332-0012, Japan
\textsuperscript{c} Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
\textsuperscript{d} JST, TRIP, Chiyoda, Tokyo, 102-0075, Japan
\textsuperscript{e} Department of Mathematical Sciences, Osaka Prefecture University, 1-1 Gakuen-cho, Sakai 599-8531, Japan

Abstract
We theoretically discuss the magnetic-field-angle dependence of the zero-energy density of states (ZEDOS) in superconductors. Point-node and line-node superconducting gaps on spherical and cylindrical Fermi surfaces are considered. The Doppler-shift (DS) method and the Kramer-Pesch approximation (KPA) are used to calculate the ZEDOS. Numerical results show that consequences of the DS method are corrected by the KPA.

Key words:
Unconventional superconductor, Field-angle dependent measurement, Specific heat

PACS: 74.20.Rp, 74.25.Op, 74.25.Bt

In the last decade, experimental techniques for the magnetic-field-angle dependent measurements of the thermal conductivity and specific heat have been developed to investigate the superconducting gap anisotropy in various materials \cite{1,2}. The Doppler-shift (DS) method \cite{3} has been frequently utilized to analyze such measurements \cite{4,5}. Theoretical development has been in progress so far \cite{6,7,8,9,10,11,12,13,14}. Recently, we have developed a new method called the Kramer-Pesch approximation (KPA) \cite{13,15}, where the contribution of a vortex core neglected in the DS method is taken into account \cite{13}. Then, the KPA may be a useful tool that enables a more quantitative analysis.

In this paper, we show several numerical results for the zero-energy density of states (ZEDOS) obtained by the DS method and the KPA to demonstrate how much the KPA improves quantitative results of the DS method. The specific heat over the temperature, \(C(T)/T\), is proportional to the ZEDOS in the low temperature limit \(T \rightarrow 0\).

The expression for the density of states (DOS) \(N(E)\) in the DS method is given as \(3,12\)

\[
N(E) \propto \text{Re} \left( \int dS_F \frac{|E - \delta E|}{\sqrt{(E - \delta E)^2 - |\Delta|^2}} \right)_{\text{DS}},
\]

where \(\delta E = mv_F \cdot \nu_s\) is the DS energy. \(m, v_F,\) and \(\nu_s\) are the electron mass, the Fermi velocity, and the circulating superfluid velocity around a vortex, respectively. The field-angle dependence is taken into account via \(\nu_s\), which is perpendicular to the magnetic field \(H\) (i.e., \(\nu_s \perp H\)). Around a single vortex, \(|\nu_s| = h/2mr\) (\(r\) is the radial distance from the vortex center).

\[
\langle \cdot \cdot \rangle_{\text{DS}} = \int_{r_0}^{r_1} dr \int_0^{2\pi} d\alpha \cdot \cdot \cdot,
\]

is the spatial integration around the vortex in the cylindrical coordinates \((r, \alpha, z)\) with \(\hat{z} \parallel H\). Here, \(\xi_0\) is the coherence length and \(r_a\) is the cutoff length with \(r_a/\xi_0 = \sqrt{H_\perp/H_c2}\). \(H_\perp \equiv \Phi_0/\pi\xi_0^2\) and \(\Phi_0 = \pi r_c^2 H\), where \(H_c=\Phi_0/\pi\xi_0^2\) is the flux quantum. \(dS_F\) is an area element on the Fermi surface (FS) [e.g., \(dS_F = k_\parallel \sin \theta d\theta d\phi\) for a spherical FS in the spherical coordinates \((k, \phi, \theta)\), and \(dS_F = k_{ab} d\phi dk_c\) for a cylindrical FS in the cylindrical coordinates \((k_{ab}, \phi, k_c)\)]. The pair potential is \(\Delta = \Delta_0 \Lambda(k_F)\), where \(\Delta_0\) is the maximum gap amplitude and \(\Lambda(k_F)\) represents the gap anisotropy on the FS. \(\Lambda\) and \(v_F\) are functions of the position \(k_F\) on the FS.

On the other hand, the DOS in the KPA is given as \(13\)

\[
N(E) = \frac{v_F \eta}{2\pi^2 \xi_0} \left( \int \frac{dS_F}{|v_F|} \frac{\int |\cos(x/\xi_0)|^{-2\Delta/zh}}{(E - E_c)^2 + \eta^2} \right)_{\text{KPA}}.
\]
References

[1] Y. Matsuda, K. Izawa, I. Vekhter, J. Phys.: Condens. Matter 18 (2006) R705.

[2] T. Sakakibara, A. Yamada, J. Custers, K. Yano, T. Tayama, H. Aoki, K. Machida, J. Phys. Soc. Jpn. 76 (2007) 051004.

[3] G. E. Volovik, JETP Lett. 58 (1993) 469.

[4] I. Vekhter, P. J. Hirschfeld, J. P. Carbotte, E. J. Nicol, Phys. Rev. B 59 (1999) R9023.

[5] H. Won, S. Haas, D. Parker, S. Telang, A. Ványolos, K. Maki, AIP Conf. Proc. 789 (2005) 3.

[6] P. Miranović, N. Nakai, M. Ichioka, K. Machida, Phys. Rev. B 68 (2003) 052501.

[7] H. Kusunose, J. Phys. Soc. Jpn. 73 (2004) 2512; Phys. Rev. B 70 (2004) 054509.

[8] M. Udagawa, Y. Yanase, M. Ogata, Phys. Rev. B 71 (2005) 024511.

[9] A. B. Vorontsov, I. Vekhter, Phys. Rev. Lett. 96 (2006) 237001; Phys. Rev. B 75 (2007) 224501; Phys. Rev. B 75 (2007) 224502; I. Vekhter, A. B. Vorontsov, Physica B 403 (2008) 958.

[10] T. R. Abu Alrub and S. H. Curnoe, Phys. Rev. B 78 (2008) 104521.

[11] G. R. Boyd, P. J. Hirschfeld, I. Vekhter, A. B. Vorontsov, Phys. Rev. B 79 (2009) 064525; S. Graser, G. R. Boyd, C. Cao, H.-P. Cheng, P. J. Hirschfeld, D. J. Scalapino, Phys. Rev. B 77 (2008) 180514(R).

[12] Y. Nagai, Y. Kato, N. Hayashi, K. Yamauchi, H. Harima, Phys. Rev. B 76 (2007) 214514.

[13] Y. Nagai, N. Hayashi, Phys. Rev. Lett. 101 (2008) 097001.

[14] Y. Nagai, N. Hayashi, Y. Kato, K. Yamauchi, H. Harima, J. Phys.: Conf. Ser. 150 (2009) 052177.

[15] Y. Nagai, Y. Ueno, Y. Kato, N. Hayashi, J. Phys. Soc. Jpn. 75 (2006) 104701.