ABSTRACT

Using a recently proposed approximation for multiskyrmion fields based on rational maps we study the masses and baryonic radii of some strange multibaryons within the bound state soliton model. We find the tetralambda binding to be stronger than previously expected. In addition, the model predicts the existence of a “heptalambda” which is stable against strong decays.

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In recent years there has been a continuous effort in finding minimum energy skyrmion solutions for increasing winding number $B$. A decade ago, the so-called $B = 2$ torus configuration was found\[1\]. A few years later the lowest $B = 3$ and $B = 4$ energy solution were identified\[2\]. Finally, very recently some $B = 5$ to $9$ skyrmion configurations which are believed to represent global minima were discovered\[3\]. All these solutions could be obtained only after a sophisticated and demanding numerical work. The need to further investigate the physics behind these solutions has lead people to find good and algebraically simple approximations to them. For example, in Ref.\[4\] a variational approximation to the $B=2$ torus was introduced. Even more important for our purposes, less than a year ago Houghton, Manton and Sutcliffe \[5\] have exploited the similarities between the BPS monopoles and skyrmions to propose some ans"atze based on rational maps. They have shown that such configurations approximate very well the numerically found lowest energy solutions with $B \leq 9$ and conjectured they could also describe that with $B = 17$. The introduction of these approximations allows for the study of the properties of multibaryons with strangeness. These exotics have received quite a lot attention since Jaffe first speculated on the possible existence of a stable H-dibaryon twenty years ago\[6\]. Strange dibaryons have been extensively studied in different models including the Skyrme model in its various $SU(3)$ extensions \[7, 8, 9, 10\]. The tetralambda has been also considered using the $SO(3)$ ansatz for the skyrmion field\[11\]. In addition, experimental searches of these objects are in progress, particularly in heavy ion collisions at Brookhaven and CERN (see i.e. Refs.\[12\] and references therein). The purpose of the present work is to extend previous investigations by considering strange multiskyrmions with $B = 3$ to $9$ and $17$ in the bound state approach to the $SU(3)$ Skyrme model\[13\]. In this approach strange multibaryons can be obtained by binding kaons to a multisoliton background. To describe such multisolitonic field we will use the rational map ans"atze.

We start with the effective action for the simple Skyrme model with an appropriate symmetry breaking term, expressed in terms of the $SU(3)$–valued chiral field $U(x)$ as

$$\Gamma = \int d^4x \left\{ \frac{F_\pi^2}{16} Tr \left[ \partial_\mu U \partial^\mu U^\dagger \right] + \frac{1}{32\epsilon^2} Tr \left[ U^\dagger \partial_\mu U, U^\dagger \partial_\nu U \right]^2 \right\} + \Gamma_{WZ} + \Gamma_{SB} ,$$

where $F_\pi$ is the pion decay constant ($= 186 \text{ MeV}$ empirically) and $\epsilon$ is the so–called Skyrme parameter. In Eq.(1), the symmetry breaking term $\Gamma_{SB}$ accounts for the different masses and decay constants of the pion and kaon fields while $\Gamma_{WZ}$ is the usual Wess–Zumino action. Their explicit forms can be found in i.e. Ref.\[10\].

We proceed by introducing the Callan–Klebanov (CK) ansatz for the chiral field

$$U = \sqrt{U_\pi U_K} \sqrt{U_\pi} .$$

(2)

In this ansatz, $U_K$ is the field that carries the strangeness. Its form is

$$U_K = \exp \left[ \frac{2\sqrt{2}}{F_K} \left( \begin{array}{cc} 0 & K \\ K^\dagger & 0 \end{array} \right) \right] ,$$

(3)
where $K$ is the usual kaon isodoublet $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$.

The other component, $U_\pi$, is the soliton background field. It is a direct extension to $SU(3)$ of the $SU(2)$ field $u_\pi$, i.e.,

$$U_\pi = \begin{pmatrix} u_\pi & 0 \\ 0 & 1 \end{pmatrix}.$$  \hfill (4)

Replacing the ansatz Eq.(3) in the effective action Eq.(1) and expanding up to second order in the kaon fields we obtain the lagrangian density for the kaon–soliton system. In the spirit of the bound state approach this coupled system is solved by finding first the soliton background configuration. For this purpose we introduce the rational map ansätze\[5\]

$$u_\pi = \exp \left[ i \vec{\tau} \cdot \hat{\pi}_n F \right],$$  \hfill (5)

with

$$\hat{\pi}_n = \frac{1}{1 + |R_n|^2} \left( 2 \Re(R_n) \hat{i} + 2 \Im(R_n) \hat{j} + (1 - |R_n|^2) \hat{k} \right),$$  \hfill (6)

where we have assumed that $F = F(r)$ and $R_n = R_n(z)$ is the rational map corresponding to winding number $B = n$. Here, $r$ is the usual spherical radial coordinate whereas the complex variable $z$ is related to the other two spherical coordinates $(\theta, \phi)$ via stereographic projection, namely $z = \tan(\theta/2) \exp(i\phi)$.

In order to find the lowest soliton-kaon bound state we write the kaon field as\[9, 10\],

$$K(\vec{r}, t) = k(r, t) \vec{\tau} \cdot \hat{\pi}_n(z) \chi,$$  \hfill (7)

where $\chi$ is a two–component spinor.

Using Eqs.(5,7) the explicit form of the kaon-soliton effective lagrangian is

$$L = \int dr \, r^2 \left\{ f_n(r) \hat{k}^\dagger \hat{k} - h_n(r) \hat{k}^\dagger k' + i \lambda_n(r)(\hat{k}^\dagger \hat{k} - k^\dagger k) - k^\dagger k(m_K^2 + V_n^{eff}) \right\},$$  \hfill (8)

where

$$f_n(r) = 1 + \frac{1}{e^2 F_K^2} \left( F'^2 + 2n \frac{\sin^2 F}{r^2} \right),$$  \hfill (9)

and

$$h_n(r) = 1 + \frac{2n}{e^2 F_K^2} \frac{\sin^2 F}{r^2}. \hfill (10)$$

The term linear in time derivatives, whose coefficient is

$$\lambda_n(r) = -\frac{n N_c}{2\pi^2 F_K^2} F' \frac{\sin^2 F}{r^2},$$  \hfill (11)

is due to the Wess–Zumino action, and

$$V_n^{eff} = \left[ \frac{2n}{e^2 F_K^2 r^2} \left( \cos^4 (F/2) - 2 \sin^2 F \right) - \frac{1}{4} \right] F'^2.$$  \hfill (12)
\[
- \frac{2}{r^2} \left( \sin^2 F - 4 \cos^4 F/2 \right) \left( \frac{\mathcal{I}_n}{e^2 F_{K}^2} \frac{\sin^2 F}{r^2} + \frac{n}{4} \right) \\
- \frac{6n}{e^2 F_{K}^2 r^2} \frac{d}{dr} \left[ F' \sin F \cos^2 F/2 \right] - \frac{1}{2} \frac{F_{K}^2}{F_{K}^2} m_{\pi}^2 (1 - \cos F). \tag{12}
\]

Here, the angular integrals \( \mathcal{I}_n \) are defined as

\[
\mathcal{I}_n = \frac{1}{4\pi} \int \frac{2i \, dz \, d\bar{z}}{(1 + |z|^2)^2} \left( \frac{1 + |z|^2}{1 + |R_n|^2} \right) \left( \partial_r \frac{dR_n}{dz} \right)^4. \tag{13}
\]

The diagonalization of the Hamiltonian obtained from the effective Lagrangian Eq.(8) leads to the kaon eigenvalue equation

\[
\left[ -\frac{1}{r^2} \partial_r \left( r^2 h_n \partial_r \right) + m_{K}^2 + V_{n}^{eff} - f_n \epsilon_n - 2 \lambda_n \epsilon_n \right] k(r) = 0. \tag{14}
\]

In our numerical calculations we use two sets of parameters. Set A corresponds to the case of massless pions and Set B to the case where the pion mass takes its empirical value \( m_{\pi} = 138 \text{ MeV} \). In both cases, \( F_{\pi} \) and \( e \) are adjusted so as to reproduce the empirical neutron and \( \Delta \) masses. The ratio \( F_{K}/F_{\pi} \) and \( m_{K} \) are taken at their empirical values, \( F_{K}/F_{\pi} = 1.22 \) and \( m_{K} = 495 \text{ MeV} \). Once the parameters are fixed we obtain the soliton profiles using for each baryon number \( B \) the rational map given in Ref.[5]. Then, we proceed to solve the corresponding kaon eigenvalue equation. Results are shown in Table 1. There we list the soliton mass (per baryon number) and kaon eigenenergy for each baryon number for both sets of parameters.

To get some insight on the quality of the ansätze we first concentrate on \( B = 2 \). In this case, the exact numerical solution \[1\] corresponds to an axially symmetric soliton where the chiral angle \( F \) depends not only on \( r \) but also on \( \theta \), the angle with respect to the symmetry axis. A basic assumption of the rational map ansatz is, however, that \( F = F(r) \) for any winding number \( B \). In addition, within that particular class of solutions, this ansatz proposes a very particular dependence of \( \hat{\pi}_n \) as a function of \( \theta \) and \( \phi \). For \( B = 2 \), the best possible solution in which the chiral angle depends only on \( r \) was found in Ref.[4] and applied to strange dibaryons in Ref.[10]. Comparing with the results of those references, we observe that \( M_{sol} \) as predicted by the rational map ansatz is around 20 MeV larger (for both sets of parameters) than the one in Ref.[4] \[1\]. The effect on the kaon eigenenergy is even smaller: the values of \( \epsilon_2 \) in Table 1 are only 6 MeV larger than those reported in Ref.[10]. Therefore, possible improvements upon the rational map ansätze are not expected to change the values of the kaon eigenenergies in a significant way.

We turn now to the predictions for higher baryon numbers. As a general trend we see that the kaon binding energies \( D_{n}^{K} = m_{K} - \epsilon_n \) decrease with increasing baryon number. However, as in the case of energy required to liberate a single \( B = 1 \) skyrmion from

\(^{1}\)As compared to the exact numerically found minima, results in Table 1 are around 50 MeV larger.
the multisoliton background[2, 3], we observe some deviation from a smooth behaviour, namely, $D^K_4 > D^K_3$ and $D^K_7 > D^K_6$. Consequently, such deviations will be also present in the multiskyrmion mass per baryon. Interestingly, this kind of phenomena has been also observed in some MIT bag model calculations[14]. There they are due to shell effects.

Another magnitude of interest is the RMS baryonic radius of the strange multiskyrmions with strangeness $S$. It is related to the isoscalar electric radius and gives an idea of the size of these exotics. It is given by

$$
\left( r^B_S \right)^2 = r^2_{sol} + S \cdot r^2_K ,
$$

(15)

where $r^2_{sol}$ and $r^2_K$ are the solitonic and kaonic radii, respectively, defined by

$$
r^2_{sol} = -\frac{2}{\pi} \int dr \; r^2 F' \sin^2 F ,
$$

(16)

$$
r^2_K = 2 \int dr \; k^2 r^4 (f_n \epsilon_n + \lambda_n) .
$$

(17)

The corresponding results are given in Table 2 together with the radii of the $S = -1$ multiskyrmions. As expected, both the solitonic and kaonic radii increase monotonically as a function of $B$. For the radii of the $S = -1$ multiskyrmions such increase is much slower as a result of a compensation between both contributions. Actually, for Set A the radius for $B = 6$ is even smaller than that of $B = 5$. This effect, however, is very likely to be an artifact of this parameter set since it does not appear for Set B.

To obtain states with definite spin and isospin in the bound state model one should perform the $SU(2)$ collective quantization of the soliton-meson bound system. Such quantization has to be done taking into account all the symmetries of the bound system[10]. For $B > 2$, however, this is an intrincated problem which has not been fully understood even in the case of non-strange multiskyrmions. In this situation, we will only concentrate on estimating the centroid of the low-lying states assuming that, for those states, the rotational corrections are small. This approximation would become better as $B$ increases due to the associated increase of the moments of inertia. Using this assumption we can address the issue of the stability of the strange multibaryons against strong decays. For the $H$-dibaryon we obtain

$$
M_H - 2M_\Lambda = 12 \; MeV .
$$

(18)

Comparing with the results of Ref.[10] we note that improvements on the description of the soliton background together with the inclusion of rotational effects result in an extra binding of $\approx 40 \; MeV$. In any case, given the various uncertainties coming from i.e. Casimir effects, etc., it is clear that the present calculation basically leads to the same qualitative result: within soliton models the $H$ is predicted to be very close to threshold. The situation concerning the stability of the tetralambda with respect to the decay into two dilambda seems to be somewhat different, however. From Table 1 we have

$$
M_{4\Lambda} - 2M_{2\Lambda} = -176 \; MeV ,
$$

(19)
for both sets of parameters. Therefore, it is rather strongly bound. It should be noticed that in this case improvements on the ansatz will decrease the binding. For example, the solitonic contribution in the present approximation is $-200$ MeV to be compared with the exact value $-150$ MeV \cite{3}. Still, the present results indicate that, within the Skyrme model, the binding of the tetralambda might be stronger than previously expected \cite{11}.

There are some other states with vanishing hypercharge which might be stable against strong decays. To see that it is convenient to define the ionization energy $I_B$ as

\[ I_B = M_1 + M_{B-1} - M_B, \tag{20} \]

where $M_n$ is the mass of the state with $B = -S = n$. Within the present approximations $M_n$ can be calculated from Table 1 using $M_n = n(M_{sol} + \epsilon_n)$. Results for $B = 5$ to 9 are shown in Table 3. We readily see that $B = -S = 5, 8, 9$ are unstable, $B = -S = 6$ barely stable while $B = -S = 7$ is clearly stable against one hyperon emission. It is not hard to check that the stability of the $B = -S = 7$ strange multiskyrmion is not violated by any other strong decay. For example,

\[ M_{7\Lambda} - (M_{3\Lambda} + M_{4\Lambda}) = -177 \text{ MeV}. \tag{21} \]

Therefore, as it stands the present model predicts the existence of a stable “heptalambda”.

In conclusion, we have studied strange multibaryon configurations in the context of the bound state approach to the $SU(3)$ Skyrme model using for the soliton background fields some ans"atzes based on rational maps. We have obtained the solutions of the kaon eigenvalue equation for $B \leq 9$ and $B = 17$. With these solutions we have calculated the associated baryonic radii. As expected, for $S = -1$ they increase with increasing $B$. We have calculated the strange multiskyrmion masses within the adiabatic approximation. We have found that among the exotics with vanishing hypercharge and low baryon number only those with $B = 4$ and 7 are clearly stable against strong decays. This might be of interest for current experimental searches of strange exotics. Finally, we note that, in principle, it would be possible to study hypernuclei within the present framework. However, as wellknown from the case of the non-strange $B = 2$ system, one should definitely perform a quantum treatment of the $SU(2)$ multiskyrmion before a reasonable description can be made. Investigations along this line are currently underway.

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Table 1: Soliton mass per baryon unit $M_{sol}$ and kaon eigenenergy $\epsilon$ (both in $MeV$) as a function of the baryon number $B$. Set A corresponds to $F_\pi = 129 \ MeV$, $e = 5.45$, $m_\pi = 0$ while Set B to $F_\pi = 108 \ MeV$, $e = 4.84$, $m_\pi = 138 \ MeV$. The other parameters take their empirical values as it is explained in the text.

| $B$ | $M_{sol}$ | $\epsilon$ | $M_{sol}$ | $\epsilon$ |
|-----|-----------|-------------|-----------|-------------|
| 1   | 863       | 222         | 864       | 210         |
| 2   | 847       | 244         | 848       | 232         |
| 3   | 830       | 255         | 832       | 241         |
| 4   | 797       | 250         | 798       | 238         |
| 5   | 804       | 263         | 808       | 248         |
| 6   | 797       | 267         | 802       | 251         |
| 7   | 776       | 262         | 780       | 246         |
| 8   | 784       | 271         | 790       | 252         |
| 9   | 787       | 276         | 796       | 256         |
| 17  | 766       | 284         | 782       | 256         |

Table 2: Baryonic radii $r_B$ of the $S = -1$ multibaryons as a function of the baryon number. The solitonic $r_{sol}$ and kaonic $r_K$ contributions to those radii are also displayed. Parameter sets A and B are defined in the caption of Table 1. All the radii are in $fm$.

| $B$ | $r_{sol}$ | $r_K$ | $r_B$ | $r_{sol}$ | $r_K$ | $r_B$ |
|-----|-----------|-------|-------|-----------|-------|-------|
| 1   | 0.59      | 0.44  | 0.39  | 0.68      | 0.51  | 0.44  |
| 2   | 0.81      | 0.65  | 0.49  | 0.94      | 0.75  | 0.57  |
| 3   | 0.97      | 0.80  | 0.54  | 1.12      | 0.93  | 0.63  |
| 4   | 1.06      | 0.89  | 0.57  | 1.23      | 1.03  | 0.67  |
| 5   | 1.20      | 1.03  | 0.64  | 1.37      | 1.18  | 0.68  |
| 6   | 1.29      | 1.13  | 0.62  | 1.48      | 1.29  | 0.73  |
| 7   | 1.35      | 1.18  | 0.65  | 1.55      | 1.35  | 0.76  |
| 8   | 1.46      | 1.29  | 0.68  | 1.66      | 1.47  | 0.78  |
| 9   | 1.55      | 1.39  | 0.69  | 1.76      | 1.57  | 0.80  |
| 17  | 2.05      | 1.89  | 0.79  | 2.27      | 2.09  | 0.88  |
Table 3: Ionization energies $I_B$ as defined in Eq.(20) (in $MeV$) for $B = -S = 5$ up to 9 multibaryons. Parameter sets A and B are as in Table 1.

| B | Set A | Set B |
|---|-------|-------|
| 5 |  -62  |  -62  |
| 6 |  +36  |  +36  |
| 7 |  +203 |  +210 |
| 8 |  -59  |  -80  |
| 9 |  -42  |  -58  |