Construction of an Instrumentation Kit for Identification and Control of DC Motors

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Abstract— This paper presents the development of an instrumentation kit of voltage and current measurement for identification of the dynamic model and control of direct current (DC) motors. In the methodology for the parameters identification is used the responses of input voltage and current, and angular velocity of the DC motor. The validation of the obtained dynamic model is done through the comparison of the simulated and experimental responses, and the application of a control system based on state feedback and complete eigenstructure assignment (tracking system). The responses are compared through the normalized root-mean-square error criterion.

Keywords— Control Systems, Parameters Identification.

I. INTRODUCTION

Direct current (DC) motors are widely used in engineering, e.g. in the position control of a robotic arm. The DC motor guarantees a good torque generation and is efficient for speed control. In some cases, for the design of control systems it is necessary to have an accurate dynamic model of the DC motor [1, 2, 3]. The use of experimental methods allows the determination of the dynamic model of a DC motor. An example of identification methodology is presented in [3]. Where the identification of the dynamic model parameters is done by measuring the motor speed response that results from the application of a constant voltage input.

In this context, this work presents the construction of an instrumentation kit for identification of the dynamic model and control of DC motors. Thus, allowing the study of different identification techniques, and the practical study of control techniques, such as those seen in [4, 5, 6, 7]. The methodology to identify the model of the DC motor recalls [8]. The effectiveness of the identification methodology is demonstrated by the comparison of the dynamic model and the real system responses. This paper also presents the control of a DC motor, using the instrumentation kit, through the tracking control system that uses state feedback and complete eigenstructure assignment. The normalized root-mean-square error (NRMSE) is used to analyze the responses.

The work is organized as follows. Section II presents the linear state space model for a DC motor. In Section III is presented the development of the instrumentation kit. In Section IV, the methodology for the identification is presented. Section V describes the design of the tracking system that uses state feedback and complete eigenstructure assignment. In Section VI is shown the configuration of the experiments and the results obtained. Section VII concludes this paper.

II. DC MOTOR DYNAMIC MODEL

A common linear state space model for the motor is given in (1) and (2), where the angular velocity \( \omega_m \) and the direct current \( I_a \) are the states. The variable \( J \) is the moment of inertia of the rotor, \( b \) is the viscous friction constant, \( K_e \) is the electromotive force constant, \( K_t \) is the motor torque constant, \( R_a \) is the armature resistance, \( L_a \) is the armature inductance and \( V_a \) is the supply voltage.

\[
\frac{d}{dt} \begin{bmatrix} \omega_m \\ I_a \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K_t}{J} \\ -\frac{K_e}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} \omega_m \\ I_a \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix} V_a \quad (1)
\]

\[
y = [1 \ 0] \begin{bmatrix} \omega_m \\ I_a \end{bmatrix} \quad \text{..............................................} (2)
\]

This state space model may vary depending on the work. In [9] is presented a model with a few differences. The work developed in [10] shows the mathematical model of a DC motor with the angular position as a state.

III. INSTRUMENTATION KIT DEVELOPMENT

The instrumentation kit was built inside the case of a broken function generator, reusing its linear voltage source that was still functional. This approach was important for reducing the cost, and contributed for the recycling of electronic waste.

To identify a DC motor it is necessary to acquire its input voltage and current, in addition to its angular velocity. Based on this, the voltage (LV 25-P) and current (LA 25-NP) sensors were established. An encoder with 100 counts per-revolution was used to measure the angular velocity. The function generator's linear voltage source was reused.
to power up the sensors, as it generates a voltage of -16 V to 16 V with 1 A of direct current.

Once the sensors were established, it was necessary to define the acquisition ranges. The DC motor used has a nominal voltage of 12 V and peak current of 1 A. Then, the acquisition range for the voltage sensor was defined from -12 V to 12 V and for the current sensor was defined from -5 A to 5 A. Both transducers require an input voltage of -12 V to 12 V to operate correctly, so a voltage regulator lowers the voltage of the power supply from the -16 V to 16 V range to the -12 V to 12 V range. Since the sensor's output is a very low current, it must be amplified. The INA 121 operational amplifier was used and the output range was normalized from -10 V to 10 V, according to the analog input range of National Instruments NI-PCI 6251 acquisition system.

The schematic of the designed voltage sensor circuit is shown in Fig. 1. It reads the input voltage range of -12 V to 12 V, resulting in a voltage normalized from -10 V to 10 V.

Through the datasheets of the voltage sensor and the INA 121, it was determined the theoretical equation of the voltage sensor circuit, presented in (3), where $V_{in}$ is the input voltage and $V_{out1}$ is the output voltage from the Fig. 1 circuit. The voltage sensor has a gain of 2.5 times the input current. The output current of the LV 25-P passes through a resistor of 150 $\Omega$. This voltage drop across the resistor is amplified by INA 121, which has a gain of $(1 + (50/33))$.

$$V_{out1} = 2.5 \times 150 \times \left(1 + \frac{50}{33}\right) \times \frac{V_{in}}{1000} \quad \ldots \ldots \ldots (3)$$

The design procedure of the current sensor circuit is analogous to that of the voltage sensor circuit, the only difference is that a resistor is not required at the input. The LA 25-NP has several settings for measured current, for this application, it was configured for up to 5A. The schematic of the current sensor circuit is shown in Fig. 2.

It is verified through the schematic that the theoretical equation of the current sensor circuit is practically the same as the voltage sensor circuit. The only difference is that the gain of the sensor is $5/1000$, as shown in (4), where $I_{in}$ is the input current and $V_{out2}$ is the output voltage from the Fig. 2 circuit.

$$V_{out2} = \frac{5}{1000} \times 150 \times \left(1 + \frac{50}{33}\right) \times I_{in} \quad \ldots \ldots \ldots (4)$$

The theoretical equations of the circuits in (3) and (4) present a first estimate of the values to be obtained. However, some simplifications were made in this process, such as neglecting the sensor induction coil. Therefore, it is necessary to carry out a calibration of the two circuits. The calibration process consists of applying a known voltage and current, respectively, to the circuits of the voltage and current sensors, and measuring the resulting output voltages. The 0 V to 10 V range is chosen since it is the default range and is supported by the acquisition board. Both sensors have current as input quantity. The voltage sensor circuit requires a fixed 1 k$\Omega$ resistor to transform the voltage applied to the motor into current, attending the input range of the sensor. A variable voltage source and a resistor of 0.4 $\Omega$ with 10 W were used in the calibration of the current sensor to generate a known input current. Figs. 3 and 4 show the calibration procedure, respectively, for the voltage and current sensors.
Figs. 5 and 6 show the data points obtained experimentally and the curves fitted by the least squares method, respectively, for the voltage and current sensors.

The equations of the fitted curves are presented in (5) and (6) for the voltage and current sensors, respectively. The variable $V_{in}$ is the motor input voltage, $V_{out1}$ is the output from the voltage sensor circuit, $I_{in}$ is the motor input current and $V_{out2}$ is the output from the current sensor circuit.

\[ V_{in} = 1.3017 V_{out1} - 0.0511 \]  
\[ I_{in} = 0.5369 V_{out2} + 0.0035 \]

The inside view of the kit is presented in Fig. 7, where the linear voltage source, voltage regulator, and voltage and current sensor circuits are highlighted. Fig. 8 shows the complete instrumentation kit.

IV. IDENTIFICATION METHOD

The identification method starts from the second-order model presented in Fig. 9, formed by two blocks of first-order transfer functions. The first block, from left to right, of first-order is the electric part and the second first-order block is the mechanical part of the motor. The input of the model is the voltage applied to the motor terminals and the output is the angular velocity of the motor.
The first step of the identification method is to obtain the armature resistance $R_a$. This procedure consists of applying a known low voltage $V_a$ directly to the motor terminals, locking the shaft so that it does not rotate, and measuring the motor current $I_a$. The armature resistance is obtained by (7).

$R_a = \frac{V_a}{I_a} \tag{7}$

After obtaining the value of $R_a$, it is calculated the electromotive force constant $K_e$. In this experiment a known voltage $V_a$ is applied to the motor, and the angular velocity $\omega_m$ and current $I_a$ are measured. The electromotive force constant is found by (8). For this method, the torque constant $K_t$ presented in Fig. 9 has the same value as $K_e$.

$K_e = \frac{(V_a - I_aR_a)}{\omega_m} \tag{8}$

In order to identify the parameters of the electric and mechanical blocks, it is necessary to create the input and output vectors shown in Fig. 10. In this experiment, the motor is subjected to a step input with amplitude equal to its nominal voltage. The angular velocity and current responses are acquired to create the input and output vectors shown in Fig. 10.

The parameters of the first-order functions are identified using the functions from the System Identification Tools of the MATLAB/Simulink software. This gives the values of $K_{pe}$, $T_e$, $K_{pm}$ and $T_m$, concluding the identification methodology for the proposed second-order model.

V. TRACKING SYSTEM

The tracking control system is illustrated in Fig. 11. For the tracking system design, a controllable open-loop system is represented by the $n$ th-order state and the $p$ th-order output presented, respectively, in (9) and (10) [11].

$\dot{x} = Ax + Bu \tag{9}$

$y = Cx = [E]x \tag{10}$

where $y$ is a $p \times 1$ vector and $w = Ex$ is a $m \times 1$ vector that represents the outputs required to follow a $m \times 1$ input vector $r$. As presented in [11] the design method consists of adding a vector comparator and an integrator, satisfying (11).

$\dot{z} = r - w = r - Ex \tag{11}$

According to [12] the state feedback control law that is used here is given in (12). This control law assigns, if and only if the matrices pair $\overline{(A, B)}$ is controllable, the desired closed loop eigenvalues spectrum [11].

$u = K_1x + K_2z = [K_1 \ K_2] \begin{bmatrix} x \\ z \end{bmatrix} \tag{12}$

The pair $(\overline{A, B})$ is controllable if it satisfies (13) and the pair $(A, B)$ satisfies the controllability condition in (14) [11].

$rank \begin{bmatrix} B & A \end{bmatrix} = n + m \tag{13}$

$rank M_e = rank [BA \ AB A^2B \ldots A^{n-m}B] = n \ldots \tag{14}$

When conditions in (13) and (14) are satisfied, it is guaranteed that the control law given in (15) can be synthesized, in a way that the command input is tracked by the closed-loop output. The closed-loop state equation, for this case, is given in (15) [11].

$\dot{\bar{x}} = \begin{bmatrix} \bar{x} \\ \bar{z} \end{bmatrix} = \begin{bmatrix} A + BK_1 & BK_2 \\ -E & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} r \tag{15}$

The gains $K_1$ and $K_2$ of the control system were found by the eigenstructure assignment methodology proposed in [11]. The eigenvalues are selected to assign performance characteristics over time. For the closed-loop plant matrix in (15), the eigenvalues must be in the left-half plane of the complex plane and belong to the null vector space $N$ (i.e. vectors that represent the solutions to the matrix $[A - \lambda I B]$), this is done by the selection of the feedback matrix [11].

In this way, the piecewise constant command vector $\mathbf{r}(t)$ is tracked by the outputs $\mathbf{w}(t)$, in the steady state. The ker $S(\lambda_a)$ imposes constraints on the eigenvector $\mathbf{v}_i$, this may be associated with the assigned eigenvalue $\lambda_i$. A specific subspace is identified by the ker $S(\lambda_i)$, and the
chosen eigenvectors $v_i$ must be located within this subspace [11].

VI. EXPERIMENTS AND RESULTS
The workbench where the experiments were performed consists of a computer with Windows XP operating system with the MATLAB/Simulink software running in Real-Time Windows Target mode. The National Instruments NI-PCI 6251 card is installed on this computer to perform the reading of the encoders, the acquisition of the voltage and current signals from the instrumentation kit sensors and the control of the motor, through a linear power drive. The DC motor identified and controlled is the Maxon F2140 motor, and is powered by the Maxon LSC 30-2 linear power drive. The complete experiment workbench is shown in Fig. 12.

The transfer functions resulting from the identification of the Maxon F2140 motor are presented in (16) and (17), respectively, for the models of current and angular velocity.

$$I_a = \frac{0.06309s + 0.09498}{0.001339s^2 + 0.0662s + 20.44}\hspace{1cm} (16)$$

$$\omega_m = \frac{0.001339s^2 + 0.0662s + 20.44}{699.4}\hspace{1cm} (17)$$

The comparison between the experimental and simulated results are shown in Figs. 13 and 14, respectively, for current and angular velocity. Tables 1 and 2 shows the comparison between the experimental and simulated results using the NRMSE.

It is verified that the model identified for the nominal motor voltage (11.5 V), which represents the current peak of 0.9 A and the angular velocity of 400 rad/s, is the closest to the real behavior of the DC motor. In order to validate the identified model, it was subjected to a voltage of 8.5 V...
and 5.6 V, as it is possible to analyze from Figs. 13 and 14, and Tables 1 and 2.

In Figs. 15 and 16 are presented the responses for the tracking control system with a square wave input, respectively, for the angular velocity and control action. Table 3 presents the NRMSE to analyze the accuracy of the control responses for the square wave input.

Table 3: Comparison between experimental and simulated control responses for square wave input.

| Response                  | NRMSE (%) |
|---------------------------|-----------|
| Angular Velocity (rad/s)  | 97.27     |
| Control Action (V)        | 85.68     |

Figs. 17 and 18 present the responses for the tracking control system with a sine wave input, respectively, for the angular velocity and control action. Table 4 presents the NRMSE to analyze the accuracy of the control responses for the sine wave input.

Table 4: Comparison between experimental and simulated control responses for sine wave input.

| Response                  | NRMSE (%) |
|---------------------------|-----------|
| Angular Velocity (rad/s)  | 98.82     |
| Control Action (V)        | 93.87     |

VII. CONCLUSIONS

This article presented the development of an instrumentation kit, which composes an experimental workbench, with the purpose of identifying the dynamic model and controlling DC motors. In general, all purposes of the developed instrumentation kit have been met. The methodology used in the identification of the dynamic model presented satisfactory results for the Maxon F2140 motor. As the dynamic model of this motor was identified from the input voltage and current responses resulting from the application of a step signal with the amplitude voltage of 11.5 V, it was expected that the simulated response from the model and the experimental response from the actual
system would be close. This was proved by the calculated NRMSE criterion presented in Tables 1 and 2. As the dynamic model was tested for input signals with amplitudes smaller than the nominal voltage, it was possible to observe that the proximity between the experimental and simulated responses decreases, as seen in Figs. 13 and 14 and proved by the calculated NRMSE presented in Tables 1 and 2. However, the loss of accuracy in this extrapolation is considered satisfactory when considering the actual behavior of a DC motor. This satisfactory variation in extrapolation is due to the planning of the instrumentation kit and especially the choice to use a linear power drive. The tracking control system with state feedback and complete eigenstructure assignment proved to be efficient for DC motor control, as evidenced by the responses and the NRMSE criterion for square and sine wave inputs. As seen from Tables 3 and 4 the square wave response was less accurate than the sine wave response. This was expected considering the sudden changes in the setpoint represented by a square wave. As advantages of the instrumentation kit, it is possible to present the easiness for installing in the experimental workbench and connecting with the acquisition system. In addition, the whole structure of the experimental workbench was interesting because it allowed the execution of identification and control in a single software, being it MATLAB/Simulink or LabVIEW. The reuse of electronic waste is also an interesting feature of the work developed. The main limitations of the instrumentation kit are related to the voltage and current acquisition ranges. The voltage sensor circuit is designed to measure a maximum of 12V, the change of this limit can be made by altering the input resistor and performing a new calibration. The current sensor circuit was designed to measure a maximum of 5A, changing this limit depends on changing the circuit design for a different configuration of the LA 25-NP sensor.

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