The Significance of the General Principle of Relativity

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In this note, we discuss the significance of the general principle of relativity for a physical theory that abandons the Newtonian concept of force and, hence, uses an entirely different conception for the “cause” behind motions of material bodies.

I. CONCEPTUAL PRELIMINARIES

To begin with, let us note that the foremost of the concepts behind Newton’s theory is, undoubtedly, (Galileo’s) concept of the inertia of a material body. We postulate that every material body has this inertia for motion.

The association of the inertia of a material body with the points of the Euclidean space is the first primary physical conception that is necessary for Newton’s theory to describe motions of physical bodies. Then, with this association, the Euclidean distance becomes the physical distance separating material bodies and the Euclidean space becomes the physical space for any further considerations of Newton’s theoretical scheme.

Next, a physical clock is a material body undergoing periodic motion or a periodic phenomenon. Essentially, in Newton’s theory, a physical clock is a set of points of the Euclidean space exhibiting periodic motion under a periodic transformation. Mathematically, in Newton’s theory, let \( A \) be the set of all points \( x_A \) of the Euclidean space making up the clock. Let \( T \) be the periodic transformation such that \( \forall x_A \in A, T^n x_A = x_A \), where \( n \) is the period of the transformation \( T \).

In Newton’s theory, an observer can observe the entire periodic motion of the material body of the clock (under the use of the transformation \( T \)) without disturbing the clock in any manner whatsoever. Then, the known state or the reading of the clock represents the physical time. Any “measurement” of the physical time gives the period or the part of the period \( n \) of the transformation \( T \). It is tacitly assumed in these considerations that the involved quantities are exactly measurable.

Now, consider a material point with an initial location \( \vec{x}_o \). In Newton’s theory, the trajectory of this material point is a (continuous) sequence of points of the Euclidean space. It is then a “curve” traced by the point \( \vec{x}_o \) under some transformation \( T_t \) of the Euclidean space where parameter \( t \) labels points of the sequence. Of course, the transformation \( T_t \) need not be periodic.

The label parameter \( t \) can then be made to correspond to the physical time in a one-one correspondence. This is theoretically permissible as the measurement of the physical time does not disturb the clock in any manner whatsoever in Newton’s theory. This correspondence is the physical meaning of the labelling parameter \( t \).

An observer can thus check the position of another material point against the state of a physical clock. Without this “correspondence,” the geometric curve of the Euclidean space has no physical sense for the path of a material point.

When such physical associations are carried out, we say that the material point is at “this” location given by the three space coordinates and the physical clock is simultaneously showing “this” time. This simultaneity is inherent in the physical associations of Newton’s theory. It is an important assumption of Newton’s theory.

Also, as a consequence of the fact that Newton’s theory treats material bodies as existing independently of the space, the state of a physical clock or its reading is assumed to be independent of the motion of another material point or points separately under considerations. Then, physical time is independent also of the coordination of the metrically-flat Euclidean space.

But, then, the motion of a material point in the “physical” space does not produce any change in that space. Clearly, this fact applies also to the periodic motion or the periodic phenomenon making up the physical clock.

The “physical” construction of the coordinate axes and clocks must also be using the material bodies, for example, coordinate axes could be constructed using “sufficiently long” material rods, say, of wood. Then, any material object, a road-roller, say, crossing the coordinate axis must “affect” the corresponding wooden rod.

But, in Newton’s theory, the coordinate axes of the Euclidean space do not get affected by the motions of other material bodies. Clearly, use of non-cartesian coordinates does not change this state of affairs with Newton’s theory.
Now, any difference of coordinates in the Euclidean space is a “measuring stick or rod” that can be used to “measure” the physical separation of material bodies.

Furthermore, in Newton’s theory, each observer has a coordinate system of such measuring rods and clocks. Then, when one observer is in motion (relative to another one), the entire system of coordinate axes and clocks is also carried with that observer in motion.

It needs to be adequately recognized that it is really the theoretical possibility of making a genuine physical measurement at any stage which is the issue involved herein.

Conceptually, the aforementioned physical situation is “acceptable” except in one case. Surely, we cannot have a material rod with one observer and simultaneously, another material rod, at the same place, with other observer in (uniform or not) motion relative to the first one.

But, in Newton’s theory, measuring stick of one observer does not collide with that of another observer in motion even when both these sticks arrive at the same place. Unacceptably, any two such measuring sticks just pass through each other without even colliding on their first contact.

The same situation does not arise for other material bodies which are supposed to collide on their first contact. Then, in Newton’s theory, the measuring rods of the physical space - Euclidean space - are treated separately than other material objects. But, measuring rods must also be made up of material objects. Then, their separate treatment is, theoretically, not appropriate one. Surely, this problem with Newton’s theory is, undeniably, of serious theoretical concern.

(This above issue would not have been relevant if the Euclidean distance were also not, simultaneously, the physical distance separating material bodies. Mathematically, the continuum \( \mathbb{R} \times \mathbb{R} \times \mathbb{R} \) can be assumed.)

Hence, Newton’s theory attempts to explain all phenomena as relations between objects existing in Euclidean space and time. It achieves this mainly by attributing absolute properties to the space and the time, thereby totally separating them from the properties and motions of matter.

Thus, limitations of Newton’s theoretical scheme (providing his famous three laws of motion) originate in its use of the Cartesian concepts related to the Euclidean space and the associations of properties of material bodies with the points of this metrically-flat space.

Now, the entire physical structure of Newton’s theory is woven around only two basic concepts, namely, those of the inertia and the force.

Clearly, the force, as a cause of motion, is another pivotal concept of Newton’s theory. Hence, consider the status of the concept of force, the cause behind motions of material bodies, within Newton’s theory of mechanics.

Firstly, we could ask: What is the cause of this force? Within Newton’s overall theoretical scheme, only a material point can be the source of force. A material point “here” acts on a material point located “there” with the specified force. Newton’s theoretical scheme is therefore an action at a distance framework.

Then, in Newton’s theory, we can consider a physical body as one material point and also other physical bodies as other material points. We vectorially add the forces exerted by each one on the first physical body to obtain the total force acting on it. It is this total force that is used by Newton’s second law of motion to provide the means of establishing the path followed by that physical body under the action of that total force.

In Newton’s second law of motion, we must first specify the force acting on a material body. Only then can we solve the corresponding differential equation(s) and obtain, subject to the given initial data, the path of the material point representing that material body.

Then, without the Law of Force, it is clear that the Law of Motion is empty of contents in Newton’s theoretical framework. This is an extremely important issue for a physical theory.

From our ordinary, day-to-day, observations, we notice that various objects fall to the earth when left “free” in the air. We then say that objects gravitate towards the Earth. This is, in a nutshell, the phenomenon of gravitation.

We then need to explain as to why the objects “ordinarily” gravitate to Earth, ie, why they have a tendency to come together or why the distance between them decreases with time.

In Newton’s theory, only the force “causes” the motions of material bodies. Then, the gravitating behavior of objects is “explainable” only by postulating a suitable force of gravity that makes material bodies fall to the Earth.

But, in Newton’s theory, a force acts between any two separate material points possessing the required source property by virtue of which the force in question is generated. Furthermore, for the internal consistency of Newton’s scheme, the force so generated by one material point on the second material point must also be equal in amplitude but opposite in direction to that generated by the second material point on the first material point. This is Newton’s third law of motion. This law also has the status of a postulate within the overall scheme of the newtonian mechanics.
Newton had assumed that the force of gravity is proportional to the inertias of the two material points under consideration because, following Galileo, he had postulated that inertia “characterizes” a physical body.

Such a force of gravity must then be generated by a chosen body (Earth) on all the other material bodies because, by postulate, every material body possessed inertia. The force of gravity must then be universal in character.

To explain many of the day-to-day observations involving the terrestrial bodies as well as planetary motions that were already known in details, Newton was therefore compelled to state a Law of Force - Newton’s Law of Gravity - to explain the phenomenon of gravitation.

In fact, in Newton’s theory, a material body has two independent attributes: the first, its inertial mass, is a measure of the opposition it offers to a change in its state of motion, and the second, its gravitational mass, is a measure of the property by virtue of which it produces the force of gravity on another material point.

Various observations, since Galileo’s times, then indicate [1] that the inertial and the gravitational masses of a material body are equal to a high degree of accuracy. However, this equality becomes an assumption of Newton’s theory.

Furthermore, the inverse-square dependence of the gravitational force on the distance separating two bodies is also an important assumption of Newton’s theory.

In relation to the inverse-square dependence of Newton’s force of gravity on distance separating two bodies, we could then always raise questions: Why not any other power of distance? Why should this force not contain time-derivatives of the space coordinates? Clearly, Newton’s theory offers no explanation for even the inverse-square dependence of the force of gravitation.

Hence, in Newton’s scheme, his law of gravitation has the status of a postulate about the force acting between two material particles separated by some spatial distance.

Also, Coulomb’s law from the electrostatics provides another, postulated, fundamental force. It is also assumed to exist universally between any two charged material bodies. It is an “additional” force, over and above that of gravity, which Newton’s theory postulates to explain the motions of charged material points.

Now, every object does not fall to the Earth. So, “something” opposes the attractive force of gravity. That “something” must also be another force. As an example, the force of electrostatic repulsion can balance the force of gravity between two charged material bodies.

Then, under the action of this “another force” the distance separating two material bodies can also decrease. As an example, the force of electrostatic attraction between two charged material bodies can cause the decrement in their separating distance. Therefore, not every decrement in distance between material bodies is due to the force of mutual gravitation.

Thus, in a nutshell, one force can oppose another force. But, every force must be an assumption of Newton’s theory.

Then, if we find that some physical body, for example, a star, is stable, we could, in Newton’s theory, explain its stability by postulating another suitable force which counterchecks the force of self-gravity of the star. On the other hand, if the star were unstable, existence of “unbalanced” forces in the star is implied.

In Newton’s theory, there are no fundamentally important issues involved here than those related to finding the nature of the force opposing the self-gravity of the star. It is essential to recognize this important aspect of Newton’s theory.

Nonetheless, in spite of it being an assumption of Newton’s theory, Newton’s inverse-square law of gravitation does possess certain experimental justification - it is this inverse-square dependence that is known to be consistent with various observations and experiments.

Still, it cannot be denied that Newton’s law of gravitation is an important assumption of the newtonian mechanics.

To reemphasize the status of the laws of the force in Newton’s theory, we note that every force is an assumption of this theory. Some forces are assumed to exist universally between any two material bodies. In particular, “the force of gravitation” is postulated by this theory.

In Newton’s theory, every “fundamental” notion of the force necessarily requires a source property to be attributed to material bodies. Then, the action-at-a-distance force has this important characteristic always.

Obviously, Newton’s theory cannot hope to “explain” the origin of any of such source attributes, each of these source attributes being an assumption of that theory. Evidently, the same applies to other action-at-a-distance theories. It is important to recognize this fact at this early stage of our present considerations.

Perhaps, we would have been satisfied even with these assumptions of Newton’s theory if it were not for the fact that Newton’s theory does not explain the phenomena displayed by Light. Moreover, various observations related to the wave-particle duality of light as well as matter are also unexplainable within the newtonian scheme.
Apart from various fundamental reasons of theoretical nature as discussed earlier, it is also for such experiments or observations which cannot be explained by Newton’s theory, that some suitable “new” theory becomes a necessity.

(Historically, when various experimental discoveries contradicted Newton’s theory, the emphasis of the theoretical research suddenly changed from that of working out the detailed implications of Newton’s theory to the quest for a “new theory.” However, various serious theoretical difficulties of this theory were, of course, quite well known since Newton’s own times.)

Of course, different results of Newton’s theory which successfully describe motions of material bodies must be obtainable within the new theory in some suitable way.

In view of the overwhelming experimental data unexplained by Newton’s theory, “new theory” is inevitable. Then, in order to formulate a suitable new theory, we need to, not just modify but, completely abandon some newtonian concepts at a fundamental level.

In the passing, let us also note here the principle of relativity in Newton’s theory. This principle states that: If a coordinate system $K$ is chosen so that Newton’s laws of motion hold good without the introduction of any pseudo-forces with respect to this frame then, the same laws also hold good in relation to other coordinate system $K'$ moving in uniform translation relatively to $K$. This principle is a direct consequence of the experiments conducted by Galileo and inferences that can be drawn from these experiments.

These issues then bring us to the question of the status of the principle of relativity in a theoretical framework that abandons the newtonian concept of the force. It is to this and other related issues that we now turn to.

II. THE GENERAL PRINCIPLE OF RELATIVITY

To incorporate the physical description of the phenomena displayed by Light, zero rest-mass object, Einstein modified the newtonian principle of relativity as: If a coordinate system $K$ is chosen so that physical laws hold good in their simplest form with respect to this frame then, the same laws also hold good in relation to other coordinate system $K'$ moving in uniform translation relatively to $K$. This is the special principle of relativity. As is well known, together with the principle of the constancy of the speed of light in vacuo, it leads to the special theory of relativity.

Einstein’s this special principle of relativity is essentially the same as the principle of relativity of Newton’s theory. The word “special” indicates here that the principle is restricted to the case of uniform translational motion of $K'$ relative $K$ and does not extend to non-uniform motion of $K'$ in relation to the system $K$.

But, even the special theory of relativity is not sufficiently general to offer explanations for various physical phenomena as observed. Not only gravity, but, as should be amply clear, the origin of inertia as well as the origin of electrostatic charge are also not explainable in special relativity.

Primarily, the special theory of relativity is an extension only of the newtonian laws to incorporate the laws of motion for material bodies with vanishing inertia. It achieves this extension by acknowledging the fact that, in our day-to-day experiences, we use Light to observe.

But, the special theory of relativity also rests on the metrically flat continuum and is, thereby, beset with the problems of treating the measuring rods and clocks separately from all other objects. There is therefore the need to extend the special principle of relativity.

Then, Einstein extended this principle on the basis of Mach’s reasoning as follows.

Mach’s reasoning concerns the following situation. Consider two identical fluid bodies so far from each other and from other material bodies that only the self-gravity of each one needs to be considered. Let the distance between them be invariable, and in neither of them let there be “internal motions” with respect to each other. Also, let either body, as judged by an observer at rest relative to the other body, rotate with constant angular velocity about the line joining them. This is, importantly, a verifiable relative motion of the two identical fluid bodies.

Now, using surveyors’ instruments, let an observer at rest relative to each body make measurements of the surface of that body. Let the revealed surface of one body be spherical and of the other body be an ellipsoid of revolution.

The question then arises of the reason behind this difference in these two bodies. Of course, no answer is to be considered satisfactory unless the given reason is observable. This is so because the Law of Causality has the “genuine scientific” significance only when observable effects ultimately appear as causes and effects.

As is well known, Newton’s theory as well as the special theory of relativity require the introduction of fictitious or the pseudo forces to provide an answer to this issue. The reason given by these two theories is, obviously, entirely unsatisfactory since the pseudo-forces are unobservable.
Any cause within the system of these two bodies alone will not be sufficient as it would have to refer to the absolute space only. But, the absolute space is necessarily unobservable and, consequently, any such “internal” cause will not be in conformity with the law of causality.

The only satisfactory answer is that the cause must be outside of this physical system, and that must be referred to the real difference in motions of distant material bodies relative to each fluid body under consideration.

Then, the frame of reference of one fluid body is equivalent to that of the other body for a description of the “motions” of other bodies. As Mach had concluded, no observable significance can be attached to the cause of the difference in their shapes without this equivalence.

The laws of physics must then be such that they apply to systems of reference in any kind of motion (without the introduction of any fictitious causes or forces). This is then the extended or the general principle of relativity.

Clearly, the reference frames must be constructed out of material bodies and any motions of “other” material bodies must affect the constructions of the reference frames. Therefore, the general principle of relativity also means that the laws of physics must be so general as to incorporate even these situations in their entirety.

Now, equally important is the fact that the notion of the physical time must undergo appropriate changes when the above is implemented. In particular, the correspondence of the labelling parameter of the “path” of a physical body with the time displayed by a physical clock must be different than that in Newton’s theory or in special relativity. Notably, the underlying continuum and the physical space are then different.

Einstein connected the general principle of relativity with the observation that a possible uniform gravitation imparts the same acceleration to all bodies. This insight leads us to Einstein’s equivalence principle. It arises as follows.

Let $K$ be a Galilean frame of reference relative to which a material body is moving with uniform rectilinear motion when far removed from other material bodies. Let $K'$ be another frame of reference which is moving relatively to $K$ in uniformly accelerated translation. Then, relatively to $K'$, that same material body would have an acceleration which is independent of its material content as well as of its physical state.

The observer at rest in frame $K'$ can then raise the question of determining whether frame $K'$ is “really” in an accelerated motion. That is, whether this is the only cause for the acceleration of bodies being independent of material content.

Now, let various bodies, of differing material contents and of differing inertias, fall freely under the action of Earth’s gravity after being released from the same distance above the ground and at the same instant of time. Galileo had, supposedly at the leaning tower of Pisa, observed that these bodies reach the ground at the same instant of time and had thereby concluded that these bodies fall with the same accelerations.

Hence, the decrement in distance between material bodies displaying only the phenomenon of gravitation is then uniquely characterized by the fact that the acceleration experienced by material bodies, occupying sufficiently small region of space near another material body of large spatial dimension, is independent of their material content and their physical state. Here, the gravitational action of the larger material body can then be treated as being that of uniform gravitation.

Therefore, the answer to the question raised by the observer at rest in the frame $K'$ is in the negative since there does exist an analogous situation involving the phenomenon of uniform gravitation in which material bodies can possess acceleration that is independent of their material content and the physical state.

Thus, the observer at rest in the frame $K'$ can alternatively explain the observation of the “acceleration being independent of the physical state or the material content of bodies” on the basis of the phenomenon of uniform gravitation.

The mechanical behavior of material bodies relative to the frame $K'$ is then the same as that in the frame $K$, supposedly being considered as “special” as per the special principle of relativity. We can therefore say that the two frames $K$ and $K'$ are equivalent for the description of the facts under consideration. Clearly, we can then extend the special principle of relativity to incorporate the “accelerated” frames.

Borrowing Einstein’s words on this issue, this above situation is then suggestive that the systems $K$ and $K'$ may both with equal right be looked upon as “stationary,” that is to say, they have an equal title as systems of reference for the physical description of phenomena. [Note the use of the word “suggestive” in this statement.]

Now, the equivalence of inertial and gravitational masses of a material body refers to the “equality” of corresponding qualities of a material body. But, this is permissible only in a theory that assumes the concept of a force as an external cause of motions of material bodies. The concept of the gravitational mass is, however, irrelevant when the concept of force is abandoned. Only the concept of the inertia of a material body is then relevant to the motions of physical bodies.
What then is the status of the general principle of relativity in a theory that completely abandons the concept of force? Does it hold in the absence of the concept of force?

From the above, it should now be evident that the general principle of relativity stands even when the concept of force is abandoned because it only deals with the observable concept of an acceleration due to gravity. It rests only on the observation that uniform gravity imparts the same acceleration to all the bodies. The concept of force is not at all essential to establish this fact.

Now, it is crucial to recognize that the equivalence principle establishes only the consistency of the phenomenon of gravitation with that of the general principle of relativity. Clearly, the equivalence principle is not logically equivalent to the general principle of relativity.

As noted earlier, Einstein had, certainly, been quite careful to use the word “suggestive” in stating the relation of these two different principles. He further wrote in [2] “… in pursuing the general theory of relativity we shall be led to a theory of gravitation, since we are able to “produce” a gravitational field merely by changing the system of coordinates.”

From the above discussion, it should be equally clear that the general principle of relativity can be reached from more than one vantage issues. Each such issue can then indicate only that some physical phenomenon related to that issue is consistent with this principle of relativity. The mutual consistency of the general principle of relativity and various physical conceptions then becomes the requirement of a satisfactory theory.

It must therefore be realized that the physical construction of the frames of reference, the physical coordination of the physical space using measuring rods, is the primary requirement of the satisfactory theory based on the general principle of relativity - the general theory of relativity.

Then, it should now be also clear that the general theory of relativity, a physical theory explicitly based on the general principle of relativity, will not be just a theory of gravitation but, of necessity, also the theory of everything. It is certainly decisive to recognize this.

Therefore, a theory which abandons the concept of force completely can “explain” the phenomenon of gravitation by demonstrating that the decrement of distance between material bodies is, in certain situations, independent of their material contents and physical state. By showing this, a theory of the aforementioned type can incorporate the phenomenon of gravitation.

Why is this above mentioned demonstration expected to hold only in certain situations?

To grasp the essentials here, let us recall that, in Newton’s theory, only the total force acting on a physical body is used by Newton’s second law of motion. We usually also decompose this force into different parts in the well known manner as the one arising due to gravity, the one arising due to electrostatic force etc.

We have become so accustomed to this classical newtonian thinking in terms of the aforementioned decomposition of the total force that we ignore the following important fact.

What matters in Newton’s theory for the motion of any physical body is the total force acting on it and not the decomposition of this total force into parts, the decomposition, strictly speaking, being quite irrelevant.

Thus, the phenomenon of gravitation is, then, “displayed” by material bodies, essentially, only in certain situations, those for which the total force is due to gravity.

This above is, in overall, the significance of the general principle of relativity.

III. GENERAL EXPECTATIONS FROM THE NEW THEORY

Now, what are our general expectations from any “new” theory then?

Clearly, this “new” theory is not the special theory of relativity or even the known quantum theory. As we have already seen before, the conceptual framework of special relativity is not sufficiently general. The quantum theory too has not the required general basis as its framework needs to specify inertia and charge.

The standard formalism of the quantum theory provides us, essentially, only the means of calculating the probability of a physical event involving physical object(s). It presupposes therefore that we have specified, either the lagrangian or the hamiltonian, i.e., certain physical characteristics of the problem under consideration. Evidently, this is necessary to determine Schrödinger’s Ψ-function using which we can then make (probabilistic) predictions regarding that physical phenomenon under consideration.

Therefore, the formalism of the quantum theory, leading us to the probability of the outcome of a physical experiment about a chosen physical object, cannot provide us the means of “specifying” certain intrinsic properties of that physical body. This fact, precisely, appears to be the reason as to why we have to specify by hand the values of the mass and the charge in various operators of the non-relativistic as well as relativistic versions of the quantum theory.
The “origins” of “intrinsic” properties of physical bodies cannot then be explainable on the basis of the quantum theory. Then, the formalism of quantum theory as well its inherently probabilistic considerations cannot provide the universal basis for physics, in general.

Now, the concept of the inertia of a material body is more fundamental than that of the force because the conception of gravitational force requires the introduction of gravitational mass which is conceptually very different but “equals” the inertia in value to a high degree of accuracy [1]. Hence, only the newtonian concept of force comes under scrutiny for modifications.

Therefore, it must be adequately recognized that the newtonian concept of “force” will have to be abandoned in the process of developing the new theoretical framework. In other words, the “cause” behind the motions of material bodies will have to be conceptually entirely different than has been considered by Newton’s theory.

Consequently, “agreements” of the results of the new theory with the corresponding ones of Newton’s theory can only be mathematical of nature. The physical conceptions behind the mathematical statements of the new theory will not be those of Newton’s theory.

Therefore, any explanation of the phenomenon of gravitation in the new theory will only involve the demonstration of the “decrement of distance” under certain situations involving material bodies. It must, of course, be shown that this decrement in distance is, for these situations, such that the “acceleration” of the bodies is independent of their material content and the physical state. It must also be shown that the “known” inverse-square dependence of this phenomenon arises in the new theory in some mathematical manner.

Any such “new theory” must then explain the “origin” of the inertia of material bodies. It must also incorporate the “physical” construction of the coordinate system that must, necessarily, change with the motions of material bodies in the “physical” space. Without the appropriate incorporation of these two issues, no theory can be considered to be physically satisfactory.

Any such “new” theory needs also “explain” the equality of the inertial and the gravitational mass of a material body. The equality of these two entirely different physical conceptions, even with experimental uncertainties, is a sure indication that the same quality of a material body manifests itself, according to circumstances, as its inertia or as its weight (heaviness).

But, it must be remembered that the concept of the gravitational mass owes its origin to the newtonian concept of the force. Gravitational mass is the source of the newtonian gravitational force. Also, electrostatic charge is the source of Coulomb’s electrostatic force.

But, the “source properties” cannot be basic to the new theory that abandons the concept of the force. Consequently, the gravitational mass of the material body will not be fundamental to the new theory, but the inertial mass will be. Some “entity” that replaces the electrostatic charge will also be basic to the new theory.

The equality of the gravitational mass and the inertial mass for a material body is then not really the issue for the new theory.

But, it must also follow from the mathematical framework of the new theory that the inertial mass can also be “naturally” considered as the “source” in the mathematical quantity that can be the newtonian gravitational force.

Similarly, the quantity that, in the new theory, replaces the electrostatic charge must also naturally appear as the “source” in the mathematical quantity that can be considered to be Coulomb’s electrostatic force.

Furthermore, we need to demand that the “new” theory must also not contain the law of motion which is “independent” of the law of the force. That is to say, the force as an external quantity, to be “specified” separately of the law of motion, must not occur in this new theory. In it, we can only have the law of motion.

Crucially, abandonment of the concept of force that is independent of the law of motion applies, at the same time, to “every kind of (fundamental) force” postulated to be acting between the material particles by Newton’s theory.

Therefore, the conceptual framework as well as the mathematical formalism or procedure by which we “replace” the concept of force (as an “external cause of motion” that is independent of the law of motion) will have to be applicable to every kind of (fundamental) force that Newton’s theory has to postulate or assume to explain the observed motions of material bodies.

The Principle of the Simplicity (of Theoretical Construction) dictates that this above must be the case for a satisfactory theory.

Replacing only the concept of the gravitational force is then unacceptable not only from this point of view of the simplicity but also because the resultant theory then cannot account for the entirety of charged material bodies within its hybrid framework. Charged material bodies will have to be the singularities of the electrostatic force but not of the gravitational force in the mathematical framework of such a hybrid theory. Any such hybrid framework is then bound to be physically inconsistent and, hence, unacceptable.
Then, the mathematical procedure by which we replace the notion of, say, Newton’s gravitational force cannot be expected to be entirely different than the one adopted, say, for replacing the notion of Coulomb’s electrostatic force.

Now, the concept of force is, in a definite mathematical sense [3], equivalent to that of certain transformations of the point of the (Euclidean) space in Newton’s theory. This is then suggestive that mathematical transformations of points of the (underlying continuum) space can, quite generally as well as naturally, “replace” the newtonian concept of force as a cause of motion.

It should then be evident that the mathematical laws obtainable in this way will be applicable to every reference frame, and, hence, this mathematical formalism will be in conformity with the general principle of relativity.

It should then be also clear that the phenomenon of gravitation is incorporated in this framework as the concept of transformation is “applicable” in all the relevant situations.

This then brings us to the important issue of the appropriate mathematical formulation for the new theory whose certain characteristics we have considered above.

**IV. QUANTUM ASPECTS AND THE MATHEMATICAL FOUNDATION FOR THE NEW THEORY**

Now, quantum theory acknowledges an important limitation of the classical newtonian ideas by recognizing that any observation of a physical system involves, necessarily, an uncontrollable disturbance of that physical system. [This is similar in spirit to special relativity acknowledging the fact that we ordinarily use electromagnetic radiation to observe material bodies.]

It is then important to recognize that an “act of observation” will, necessarily, involve the transformation of the points of the underlying space representing the physical system that is being observed. Therefore, transformations of the underlying space are the key to quantum aspects.

But, we must first realize that “material bodies” and the “physical geometry of space” aught to be indistinguishable.

Firstly, moving a material body from its given “location” should cause changes to the construction of the coordinate system. Motion of a material body will then change the “physical geometry” because the construction of the coordinate system is the basis of the “metric function” of the geometry. Hence, the physical geometry is determined by material bodies.

In turn, material bodies are also determined by the physical geometry in that “given the metric function of the geometry” we would know how the totality of all the material bodies are “located” relative to each other.

In this context, efforts of Rylov [4] at defining physical geometry as mutual dispositions of the totality of geometric objects are noteworthy. It is proved in [4] that the distance function determines completely the physical geometry, and one does not need any additional information for determination of physical geometry.

However, the issue remains of the physical characteristics of material bodies such as, for example, their inertia, electrostatic charge etc. These are the “qualities” of every geometric object which is to be viewed, necessarily, as a measurable set of the underlying measure space [3].

A chosen measure can be averaged over the geometrically well-defined size of the geometric object under considerations. This averaged measure provides the non-singular notion of a point-object with the physical characteristics.

But, the location of the point-object so defined is indeterminate within the limits of the geometric object. Consequently, when the measurement of any characteristics such as the location of this point-object is performed, an intrinsic indeterminacy is involved. This is expected to lead to various quantum aspects. See, for further related details, [6, 7, 8, 9, 10].

Evidently, these ideas are then fundamentally different from those of Newton’s theory as well as from those of special relativity. In these new considerations, the concept of “force” is abandoned and is replaced by suitable properties of the geometry - its transformations. Since “force” can be looked upon as a transformation of the points of the underlying geometry, results of Newton’s theory and those of the special relativity are obtainable [11] within the proposed formalism.

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