The luminosity and stellar mass Fundamental Plane of early-type galaxies

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ABSTRACT

From a sample of ∼50 000 early-type galaxies from the Sloan Digital Sky Survey, we measured the traditional Fundamental Plane in the g, r, i and z bands. We then replaced luminosity with stellar mass, and measured the ‘stellar mass’ Fundamental Plane. The Fundamental Plane, \[ R \propto \sigma^a/I^B, \]

steepens slightly as one moves from shorter to longer wavelengths: the orthogonal fit has slope \( a = 1.40 \) in the \( g \) band and 1.47 in \( z \), with a statistical random error of \( \sim 0.02 \). However, systematic effects can produce larger uncertainties, of the order of \( \sim 0.05 \). The Fundamental Plane is thinner at longer wavelengths: it has an intrinsic scatter of 0.062 dex in \( g \) and 0.054 dex in \( z \). We have clear evidence that the scatter is larger at small galaxy sizes/masses; at large masses, measurement errors account for essentially all of the observed scatter (about 0.04 dex), suggesting that the Plane is rather thin for the very massive galaxies. The Fundamental Plane steepens further when luminosity is replaced with stellar mass to 1.54 or 1.63 when stellar masses are estimated from broad-band colours or from spectra, respectively. The intrinsic scatter also reduces further to 0.048 dex on average. Since colour and stellar-mass-to-light ratio are closely related, this explains why colour can be thought of as the fourth Fundamental Plane parameter. However, the slope of the stellar mass Fundamental Plane remains shallower than the value of 2 associated with the virial theorem. This is because the ratio of dynamical to stellar mass increases at large masses: \( M_{\text{dyn}}/M_* \propto M_{\text{dyn}}^{0.17\pm0.01} \). This scaling is the edge-on projection of the stellar mass \( \kappa \)-space. The face-on view suggests that there is an upper limit to the stellar density for a given dynamical mass, and this decreases at large masses: \( M_*/R_e^3 \propto M_{\text{dyn}}^{-4/3} \). All these trends can be used to constrain early-type galaxy formation models.

We also study how the estimated coefficients \( a \) and \( B \) of the Plane are affected by other selection effects, whether in apparent or in absolute quantities. For example, if low-luminosity objects are missing from the sample, and one does not account for this, then \( a \) and \( B \) are both biased low from their true values. If objects with small velocity dispersions are missing, then \( a \) is biased high, although this matters more for the orthogonal than the direct-fitted quantities. These biases are seen in Fundamental Planes which have no intrinsic curvature, so the observation that \( a \) and \( B \) scale with \( L \) and \( \sigma \) is not, by itself, evidence that the Plane is warped. On the other hand, we show that the Plane appears to curve sharply downwards at the small-size/mass end, and more gradually downwards as one moves towards larger sizes/masses. Whereas the drop at small sizes is real, most of the latter effect is due to correlated errors.

Key words: methods: analytical – galaxies: formation – galaxies: haloes – dark matter – large-scale structure of Universe.

1 INTRODUCTION

The Fundamental Plane has been a useful diagnostic of galaxy distances and galaxy evolution (e.g. Djorgovski & Davis 1987; Dressler et al. 1987; Jørgensen, Franx & Kjærgaard 1996; Pahre, Djorgovski & de Carvalho 1998; Bernardi et al. 2003; Jørgensen et al. 2007). If galaxies are virialized, then one expects observable signatures of the balance between potential and kinetic energies. For example,

\[ \sigma^2 \propto \frac{GM_{\text{dyn}}}{R} \propto \frac{M_{\text{dyn}}}{L} \frac{L}{R^2} \propto \frac{M_{\text{dyn}}}{L} I, \]  \( (1) \)

where \( \sigma \) is a velocity dispersion, \( I \) is a surface brightness, \( R \) is a scale and \( M_{\text{dyn}}/L \) is the mass-to-light ratio. This suggests that the
observed line-of-sight velocity dispersion $\sigma^2$ and surface brightness $I_e \equiv L/(2\pi R_e^2)$ at or within some fiducial radius $R_e$ should be correlated with one another. (In what follows, we will use the subscript ‘e’ to denote quantities estimated from de Vaucouleur’s (1948) fits to the surface brightness profiles.)

Velocity dispersion and surface brightness are distance-independent observables, whereas the size is not, so it is common to write

$$R_e \propto \sigma^\alpha I_e^{-\beta},$$

and to find that pair $(\alpha, \beta)$ for which the scatter either in the $R_e$ direction or in the direction orthogonal to the Plane is minimized. If the resulting values of $(\alpha, \beta)$ differ from the virial scalings $(2, 1)$, then the Fundamental Plane is said to be ‘tilted’. The tilt of the Fundamental Plane is interpreted as evidence that the mass-to-light ratio $(M_{\text{dyn}}/L)$ depends on some combination of the observables ($R_e$, $I_e$, $\sigma$) – the prejudice that galaxies are virialized sets what this combination must be $(M_{\text{dyn}}/L) \propto \sigma^{2-\alpha} I_e^{\beta-1}$. In low-redshift ($z \sim 0.1$) samples, $\alpha \sim 1.5$ and $\beta \sim 0.8$, so the implied $M_{\text{dyn}}/L$ is expected to increase slightly with $M_{\text{dyn}}$ or $L$. Jørgensen et al. (2007) find that $\alpha = 0.6, \beta = 0.7$ and $M_{\text{dyn}}/L \propto M^{0.54}$ at $z \sim 0.9$.

Contributions to the tilt of the Fundamental Plane can be further examined by expressing the mass-to-light ratio in terms of the dynamical mass, total mass, stellar mass and broad-band luminosity:

$$\left( \frac{M_{\text{dyn}}}{L} \right) = \left( \frac{M_{\text{dyn}}}{M_{\text{tot}}} \right) \left( \frac{M_{\text{tot}}}{M_*} \right) \left( \frac{M_*}{L} \right),$$

where $M_{\text{tot}}$ is the sum of dark matter and baryonic mass, $M_*$ is the stellar mass and $L$ is the observed broad-band luminosity. If any of these terms varies as a function of $M_{\text{dyn}}$ or $L$, the Plane will be tilted.

The assumption that $M_{\text{dyn}}/M_{\text{tot}}$ is constant is the assumption of homology. The validity of this assumption is supported by detailed kinematic modelling of galaxies observed with SAURON (Spectroscopic Areal Unit for Research on Optical Nebulae) integral-field spectroscopy. The combination of two-dimensional photometric and spectroscopic data allowed Cappellari et al. (2007) to account for differences in, e.g., $\rho(r), \sigma(r)$ and $V(r)$. They found that dynamical mass is a robust tracer of total mass. More recent work by Bolton et al. (2008) suggests that the de Vaucouleur-based quantity $R_e \sigma^2$ is linearly proportional to the mass estimated from the strong gravitational lensing effect, further reinforcing the homology assumption that $M_{\text{dyn}} \propto M_{\text{tot}}$.

For a given initial mass function (IMF), the stellar-mass-to-light ratio $(M_*/L)$ depends on the age and metallicity of the stellar population as well as on wavelength (e.g. Tinsley 1978; Worthey 1994). There is now growing evidence for correlations between mass and metallicity (e.g. Trager et al. 2000; Nelan et al. 2005; Thomas et al. 2005) and between age and velocity dispersion (e.g. Bernardi et al. 2005). Hence, stellar population models predict that $(M_*/L)$ will depend on total mass and luminosity, plausibly producing a tilt that will depend on wavelength. While there is evidence that the tilt does indeed depend on wavelength, the dependence is weak (Pahre et al. 1998; Bernardi et al. 2003; La Barbera et al. 2008), and so this effect alone cannot explain all of the tilt.

There is little discussion in the literature of the possibility that the IMF changes along the sequence in such a way as to make the dependence on $M_*/L$ weak. If there is no such effect, then stellar population effects and non-homology are weak, so the dominating contribution to the tilt of the Fundamental Plane is due to the systematic variation of $M_{\text{dyn}}/M_*$ with mass. In a hierarchical scenario of galaxy formation, the relative distribution of stars and dark matter in a galaxy depends on the roles of dissipational and dissipationless merging (e.g. Bender, Burstein & Faber 1992). Hydrodynamical simulations of galaxy mergers (Robertson et al. 2006; Hopkins, Cox & Hernquist 2008) suggest that the fractional gas content of merging galaxies determines the Fundamental Plane of their remnants. Further dissipationless mergers preserve the Fundamental Plane, but not its projections (e.g. Boylan-Kolchin, Ma & Quataert 2005). Observations of brightest cluster galaxies by Bernardi et al. (2007) support these conclusions.

The dissipational content in mergers sets the effective mass-to-light ratio of the remnant merger (Robertson et al. 2006; Hopkins et al. 2008). If the dissipational content varies with mass (in spirals the gas fraction decreases with mass, e.g. Bell & de Jong 2000), this would cause a systematic dependence of the mass-to-light ratio with mass. If this ratio varies as a power law with mass $(M/L \propto M^\gamma)$, then the Fundamental Plane would be tilted relative to the virial scaling. If the mass-to-light ratio varies in a more complicated manner, the Fundamental Plane could be warped or have a more complicated shape.

Therefore, in this work, we separate out the contribution from these effects by rewriting equation (1) as

$$\sigma^2 \propto \left( \frac{M_{\text{dyn}}}{M_*} \right) \left( \frac{M_*}{L} \right) \left( \frac{L}{R^2} \right) R.$$

We do not consider variation of $(M_{\text{dyn}}/M_{\text{tot}})$, since two rather different methods (Cappellari et al. 2007; Bolton et al. 2008) suggest that this ratio is a constant across the early-type population. (This is also why we do not include effects associated with fitting Sersic’s (1968) generalization of the de Vaucouleur profile to the images, even though there is a well-developed literature on this subject.) If the stellar population models which one uses to estimate $M_*/L$ are accurate, then multiplying the surface brightness by $M_*/L$ should eliminate most of the dependence on waveband. (There is a small remaining dependence which arises from the fact that the half-light radii are slightly but systematically smaller in redder bands, e.g. Hyde & Bernardi 2009). The remaining term $(M_{\text{dyn}}/M_*)$ is the ratio of the dynamical to stellar mass, which we will use as our proxy for structural differences. This is interesting because $M_{\text{dyn}}/M_*$ is not expected to be very much greater than unity – recent work suggests that $M_{\text{dyn}}/M_* \approx 1/0.7$ (Cappellari et al. 2007) – how it scales with mass is a quantity of great interest to galaxy formation models (e.g. Bower et al. 2006; De Lucia et al. 2006; Hopkins et al. 2008), and previous work in the Sloan Digital Sky Survey (SDSS) suggests that $M_{\text{dyn}}/M_*$ is not constant across the early-type population (Padmanabhan et al. 2004; Gallazzi et al. 2006).

Section 2 discusses how we select our sample and shows the result of fitting for the coefficients $(\alpha, \beta)$ of the traditional Fundamental Plane in the SDSS g, r, i and z bands. (Appendix A shows that our sample probably contains some non-early-type galaxies but they are too few to affect our conclusions.) We also study the possibility of detecting if the Plane is warped, or merely thick. This is prompted in part by recent works showing that the parameters of the fitted Fundamental Plane appear to depend on the luminosity and velocity dispersion range of the sample (D’Onofrio et al. 2008; Nigoche-Nieto et al. 2009). We show that such dependencies exist in thick Planes that are not warped – they are a consequence of not accounting for selection effects. We then discuss more reliable estimates of curvature along the Plane.

Section 3 describes our estimates of $M_*/L$ and the result of fitting for the coefficients $(\alpha, \beta)$ in

$$R_e \propto \sigma^\alpha \Sigma_e^{-\beta},$$

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where $\Sigma \equiv (M_*/L) I$. The difference between $(\alpha, \beta)$ and $(\alpha, B)$ is due to stellar population effects; the difference between $(\alpha, \beta)$ and the virial scalings $(2, 1)$ encodes information about $M_{\text{dyn}}/M_*$. If $(\alpha, \beta) < (2, 1)$, then $(M_{\text{dyn}}/M_*)$ increases with $M_{\text{dyn}}$ or $M_*$, for the same reason that $(\alpha, B) < (2, 1)$ implies that $(M_{\text{dyn}}/L)$ increases with $M_{\text{dyn}}$ or $L$.

A complementary analysis of the Fundamental Plane variables was introduced by Bender et al. (1992). This construction, which they termed as $\kappa$-space, was criticized as being an ‘obfuscation, not a simplification’, by Pfahre et al. (1998), primarily on the grounds that $R_e$ and $I_e$ depend on waveband. However, by replacing luminosity with stellar mass, what we will call $\kappa$-space, most of the wavelength dependence is removed; only the weak dependence of $R_e$ on waveband remains. Therefore it is interesting to examine how early types are distributed in $\kappa$-space. Section 4 presents the first analysis of $\kappa$-space. The scaling of $M_*/M_{\text{dyn}}$ with $M_{\text{dyn}}$ referred above is the edge-on view of the $\kappa$-Plane. The face-on view provides a relation between the maximum stellar density and dynamical mass of early-type galaxies.

A final section summarizes our findings and discusses some implications.

2 FP: THE TRADITIONAL FUNDAMENTAL PLANE

We use the sample of about 50 000 early-type galaxies assembled by Hyde & Bernardi (2009). The sample is based on the SDSS Data Release 4 (DR4), but with photometric and spectroscopic parameters updated from the SDSS DR6 data base. Briefly, objects have deVaucouleur magnitudes $14.5 < m_r < 17.5$, $\text{fracDev} = 1$ in both the $g$ and $r$ bands, and axis ratios $b/a \geq 0.6$. Appendix A shows that these cuts almost certainly do not yield a pure early-type galaxy sample (e.g. the face-on analogues of the objects with $\text{fracDev} = 1$ and $b/a \leq 0.6$ are still in our sample). However, the non-early-types in our sample are too few to affect our conclusions.

Photometric parameters for the best-fitting deVaucouleur surface brightness profiles are ‘corrected’ for known problems which arise from the SDSS sky-subtraction algorithm. In addition, there are some systematic differences between the velocity dispersions output by SDSS DR6 and the $\text{mdlspec2d}$ reduction (Hyde & Bernardi 2009). Our velocity dispersion estimates are simply the average of the two reductions (Section 2.1.2 discusses the dependence of the Fundamental Plane parameters on systematics). We also compute the velocity dispersion for objects for which the SDSS pipeline does not estimate $\sigma$ [due to low signal-to-noise ratio (S/N) or the presence of weak emission lines, i.e. the $\text{status}$ flag not equal to 4]. We select galaxies with velocity dispersions $60 < \sigma < 400 \text{ km s}^{-1}$. The velocity dispersions are then corrected to $R_e/8$. Stellar mass estimates, from Gallazzi et al. (2005), are available for all these objects. Since these are actually stellar-mass-to-light ratios multiplied by a luminosity, and we have corrected the luminosities for the sky-subtraction problems whereas Gallazzi et al. did not, we apply a correction to their stellar masses to account for this effect. Where necessary, distances were computed from redshifts assuming a Hubble constant of 70 km s$^{-1}$ Mpc$^{-1}$ in a flat $\Lambda$ cold dark matter model with $\Omega_0 = 0.3$.

2.1 Fitting the Plane

We fit the Plane as follows. We begin by writing the Fundamental Plane as

$$\log_{10} \left( \frac{R_e}{\text{kpc}} \right) = a \log_{10} \left( \frac{\sigma}{\text{km s}^{-1}} \right) + b \frac{\mu_*}{\text{mag}} + c,$$  

where $c = (\log_{10} R) - a(\log_{10} \sigma) - b(\mu_*)$ and $\mu_*$ is the mean surface brightness within the half-light radius, defined explicitly as follows:

$$\mu_* = -2.5 \log_{10}(I_v) = -2.5 \log_{10} \left( \frac{L}{2\pi R^2} \right) = m + 5 \log_{10}(r_e) + 2.5 \log_{10}(2\pi) - 10 \log_{10}(1 + z),$$  

where $m$ is the evolution, reddening and $k$-corrected apparent magnitude (refer to Hyde & Bernardi 2009 for details on magnitudes), and $r_e$ is the angular size in arcseconds. In this form, the virial scaling would follow $(\alpha, b) = (2, 0.4)$ because of the -2.5 term in the definition of $\mu_*$. Unless we say so explicitly, we work in logarithmic units. Therefore, following Bernardi et al. (2003), we will sometimes abuse notation by using $R$ to denote log$_{10}(R_e/\text{kpc})$, $V$ to denote log$_{10}(\sigma/\text{km s}^{-1})$ and $I$ to denote $\mu_*$. Thus, for example, $e_R$, $e_\sigma$ and $e_I$ denote the measurement errors on log$_{10} R_e$, log$_{10} \sigma$ and $\mu_*$. The shape of the Fundamental Plane is determined by estimating $a$ and $b$. This is done by minimizing residuals either in the $R_e$ direction or in the direction orthogonal to the fit. In general, the ‘direct’ and ‘orthogonal’ fit parameters are different combinations of the mean values of and covariances among the variables log$_{10} R_e$, log$_{10} \sigma$ and $\mu_*$. In practice, naive estimation of these means and covariances (e.g. simply summing over the data without including other weight terms) may lead to biases induced by measurement errors (these usually affect the covariances) or by selection effects (which bias the means and the covariances). The effects of both must be accounted for to estimate the intrinsic shape parameters $a$ and $b$ (e.g. Saglia et al. 2001). This is especially important when the FP is determined for galaxies in a magnitude-limited sample (Bernardi et al. 2003). We do this following method described in Sheth & Bernardi (in preparation). Analytic expressions which quantify the bias in the Fundamental Plane due to the magnitude limit of the survey may be found there.

Briefly, the effect of the magnitude limit is removed by weighting each object by $\frac{1}{V_{\text{max}}^2(L)}$, the inverse of the volume over which it could have been observed, and measurement errors are subtracted in quadrature from the measured covariances. The mean values of magnitude, size, surface brightness and velocity dispersion in the $r$ band are

$$\langle M_*/\text{mag} \rangle = -20.99, \quad \langle \log R_e/\text{kpc} \rangle = 0.39,$$

$$\langle \mu_* \rangle = 19.53, \quad \langle \log \sigma/\text{km s}^{-1} \rangle = 2.19.$$  

With typical measurement errors

$$\mathbf{E} = \begin{pmatrix} (e_R^2) & (e_R e_\sigma) & (e_R e_I) \\ (e_\sigma e_R) & (e_\sigma^2) & (e_\sigma e_I) \\ (e_I e_R) & (e_I e_\sigma) & (e_I^2) \end{pmatrix}$$

$$= \begin{pmatrix} 0.0542 & 0.0162 & 0 \\ 0.0162 & 0.0049 & 0 \\ 0 & 0 & 0.0016 \end{pmatrix},$$

we find that the intrinsic covariance matrix (i.e. corrected for measurement errors and the magnitude-limited selection effect) is

$$\mathbf{F} = \begin{pmatrix} C_{II} & C_{IR} & C_{IV} \\ C_{IR} & C_{RR} & C_{RV} \\ C_{IV} & C_{RV} & C_{VV} \end{pmatrix}$$

$$= \begin{pmatrix} 0.2947 & 0.0782 & -0.0096 \\ 0.0782 & 0.0552 & 0.0189 \\ -0.0096 & 0.0204 & 0.0187 \end{pmatrix}.$$
From this covariance matrix, we obtain
\[ a_{\text{dir}} = 1.170 \quad \text{and} \quad b_{\text{dir}} = 0.303 \]  
(11)
\[ a_{\text{ort}} = 1.434 \quad \text{and} \quad b_{\text{ort}} = 0.315. \]  
(12)

The observed rms scatter about the direct fit is 0.107, of which 0.096 is intrinsic. These quantities for the orthogonal fit are 0.066 and 0.058, respectively. The uncertainty on the coefficients \( a \) and \( b \) is dominated by systematic effects more than random errors, as described here below. Typical uncertainties on the coefficients due to random errors are \( \delta a < 0.02 \) and \( \delta b < 0.01 \). Systematic errors give \( \delta a \sim 0.05 \) and \( \delta b \sim 0.02 \).

### 2.1.1 Dependence of \( \delta a \) and \( \delta b \) on sample size

We analyse how the uncertainty on \( a \) and \( b \) depends on sample size using bootstrap resampling. Figs 1 and 2 show how \( \delta a \) and \( \delta b \) depend on sample size. They were obtained by dividing the total sample up into smaller subsamples, each containing \( N_{\text{gal}} \) galaxies. For a given \( N_{\text{gal}} \), we do not include any galaxy more than once, so there are a total of \( N = N_{\text{gal}}/N_{\text{gal}} \) subsamples. We then fit the FP in each subsample and compute the mean and rms values of \( a \), \( b \) and intrinsic scatter. Fig. 1 shows that the precision increases as the number of objects in the sample increases, as one might expect. However, Fig. 2 shows that sample sizes smaller than about 300 tend to result in underestimates of the intrinsic scatter, in good agreement with La Barbera, Busarello & Capaccioli (2000). For a sample of \( \sim 50,000 \) galaxies, the statistical random errors \( \delta a \) and \( \delta b \) are quite small: \( \delta a < 0.02 \) and \( \delta b < 0.01 \).

### 2.1.2 Dependence of \( \delta a \) and \( \delta b \) on systematics

When working with a large galaxy sample, systematic effects may be more important than random errors. The correction applied to the magnitudes, sizes and stellar masses for known problems, which arise from the SDSS sky-subtraction algorithm (Hyde & Bernardi 2009), results in small changes to \( a \) and \( b \) – the variation is less than 0.01 for both coefficients.

However, recall that our estimate of the velocity dispersion is the average of the DR6 and IDLSPEC2D values. If we only use the DR6 values, we find \( (a_{\text{dir}}-\text{DR6}, b_{\text{dir}}-\text{DR6}) = (1.189, 0.303) \) with intrinsic scatter 0.108 dex, and \( (a_{\text{ort}}-\text{DR6}, b_{\text{ort}}-\text{DR6}) = (1.464, 0.315) \) with scatter 0.058 dex. Using the IDLSPEC2D reductions instead gives \( (a_{\text{dir}}-\text{IDLSPEC2D}, b_{\text{dir}}-\text{IDLSPEC2D}) = (1.122, 0.304) \), with intrinsic scatter 0.100 dex and \( (a_{\text{ort}}-\text{IDLSPEC2D}, b_{\text{ort}}-\text{IDLSPEC2D}) = (1.401, 0.317) \) with scatter 0.061 dex, respectively. This suggests that we should assume as typical systematic errors \( \delta a \sim 0.05 \) and \( \delta b \sim 0.02 \). These values are larger than the random errors quoted above, showing that it is important to separate random from systematic errors.

### 2.2 Dependence on selection

If we do not account for selection effects, we find mean values \( \langle M_e \rangle = -21.94, \langle \log R_e \rangle = 0.62, \langle \mu_e \rangle = 19.71 \) and \( \langle \log \sigma \rangle = 2.30 \), and covariances \( C_{\mu e} = 0.2660, C_{\mu R} = 0.0488, C_{\mu V} = 0.0127, C_{RV} = 0.0820, C_{RV} = 0.0036 \) and \( C_{RV} = 0.0159 \). Although these covariances have changed, the coefficients of the Fundamental Plane change little: \( (a_{\text{dir}}, b_{\text{dir}}) = (1.1723, 0.2924) \) with intrinsic scatter 0.079, and \( (a_{\text{ort}}, b_{\text{ort}}) = (1.4235, 0.2914) \) with scatter 0.047. Note that the intrinsic scatter is smaller! This is largely because the distribution of luminosities in a magnitude-limited survey is narrower than in a volume-limited survey (because the faint objects are under-represented); if one does not account for this, then the magnitude-limited catalogue is more homogeneous, making for a tighter FP. The fact that the coefficients \( a \) and \( b \) hardly change is largely fortuitous (Sheth & Bernardi 2009 show why they are expected to change); it happens because the scaling relations are

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**Figure 1.** Precision of the FP coefficients \( a \), \( b \), and scatter as the number of galaxies used to fit the FP changes. The quantities \( \delta a \), \( \delta b \) and \( \delta \text{FP rms} \) show the standard deviation values of \( a \), \( b \) and the scatter around the FP, obtained from fitting the FP to \( N = N_{\text{gal}}/N_{\text{gal}} \) subsamples of the whole sample (i.e. each galaxy was chosen only once). Triangles and circles show results for the direct and orthogonal fits, respectively. For clarity, the triangles have been slightly shifted to the right.

**Figure 2.** As for Fig. 1, but now for the dependence of the estimated intrinsic scatter around the FP on sample size. Error bars show the \( \delta \text{FP rms} \) values from Fig. 1.
Table 1. Dependence of $r$-band Fundamental Plane coefficients on sample selection and parameters. Typical uncertainties on the coefficients due to random errors are $\delta a \sim 0.02$ and $\delta b \sim 0.01$; systematics are $\delta a \sim 0.05$ and $\delta b \sim 0.02$.

| Band        | $a$     | $b$     | $\text{rms}_{\text{int}}$ |
|-------------|---------|---------|---------------------------|
| Direct      | 1.1701  | 0.3029  | 0.0964                    |
| DR6 $\sigma$| 1.1892  | 0.3032  | 0.1081                    |
| spec2$\sigma$ | 1.1223  | 0.3041  | 0.1002                    |
| no-$V_{\text{max}}$ | 1.1703  | 0.3036  | 0.0792                    |
| All $b/a$   | 1.1674  | 0.2936  | 0.0913                    |
| Orthogonal  |         |         |                           |
| $r$         | 1.4335  | 0.3150  | 0.0578                    |
| DR6$\sigma$ | 1.4642  | 0.3151  | 0.0581                    |
| spec2$\sigma$ | 1.4013  | 0.3173  | 0.0613                    |
| no-$V_{\text{max}}$ | 1.4235  | 0.2914  | 0.0473                    |
| All $b/a$   | 1.4294  | 0.3052  | 0.0554                    |

Had we not removed objects with $b/a < 0.6$, then $C_{\text{II}} = 0.3756$, $C_{\text{IR}} = 0.0549$, $C_{\text{VV}} = 0.0171$, $C_{\text{IV}} = 0.0948$, $C_{\text{RR}} = 0.0132$ and $C_{\text{RV}} = 0.0171$. The resulting FP has $(a_{\text{direct}},b_{\text{direct}}) = (1.1674,0.2936)$ with intrinsic scatter 0.091, and $(a_{\text{orth}},b_{\text{orth}}) = (1.4294,0.3052)$ with scatter 0.055. Since objects with $b/a < 0.6$ are almost certainly not early types, the fact that $a$ and $b$ are almost the same as when these objects have been removed suggests that the presence of a few non-early-types in our sample has little effect on our findings.

Finally, selecting only galaxies with SDSS-DR6 velocity dispersions (i.e. excluding objects with low S/N spectra or presence of weak emission lines, i.e. the STATUS flag not equal to 4) changes $a$ and $b$ very slightly – the variation is less than 0.01 for both coefficients. Table 1 compares the $r$-band FP coefficients associated with these various selection and parameter choices.

2.3 Dependence on waveband

The Fundamental Planes associated with the orthogonal fits to the $g$-, $r$-, $i$- and $z$-band data are shown in Fig. 3. The coefficients actually slightly curved (see Section 2.4 and also Hyde & Bernardi 2009).
Table 2. Coefficients \((a, b)\) of the luminosity Fundamental Plane. Typical uncertainties on the coefficients due to random errors are \(\delta a \sim 0.02\) and \(\delta b \sim 0.01\). Systematic errors give \(\delta a \sim 0.05\) and \(\delta b \sim 0.02\).

| Band | \(a\)  | \(b\)  | \(c\)  | rms\(_{\text{obs}}\) | rms\(_{\text{stat}}\) |
|------|-------|-------|-------|----------------|----------------|
| Direct |       |       |       |                |                |
| \(g\) | 1.1154 | 0.2957 | -8.0463 | 0.1102         | 0.1005         |
| \(r\) | 1.1701 | 0.3029 | -8.0858 | 0.1074         | 0.0964         |
| \(i\) | 1.1990 | 0.3036 | -8.0481 | 0.1067         | 0.0950         |
| \(z\) | 1.2340 | 0.3139 | -8.2161 | 0.1052         | 0.0921         |

Orthogonal |       |       |       |                |                |
| \(g\) | 1.4043 | 0.3045 | -8.8579 | 0.0696         | 0.0617         |
| \(r\) | 1.4335 | 0.3150 | -8.8979 | 0.0664         | 0.0578         |
| \(i\) | 1.4572 | 0.3182 | -8.8914 | 0.0652         | 0.0563         |
| \(z\) | 1.4735 | 0.3295 | -9.0323 | 0.0635         | 0.0538         |

\((a, b)\) of these Fundamental Planes are shown in each panel; they are also reported in Table 2, as are the corresponding coefficients of the direct fits. Note that there is a small but systematic increase of \(a\) with wavelength. To estimate its significance, we require an estimate of the errors on \(a\) and \(b\). Although systematics associated with how \(\sigma\) was measured make \(\delta a \sim 0.05\) and \(\delta b \sim 0.02\), this additional systematic error is not relevant if we wish to compare the Planes at different wavelengths, because the same choice for \(\sigma\) is made for all wavelengths. What matters here is the typical uncertainty due to random errors on these best-fitting values. Section 2.1.2 and Fig. 1 show that these random errors are \(\delta a \sim 0.02\) and \(\delta b \sim 0.01\). Thus, we have a 3\(\sigma\) detection of the steepening of the Fundamental Plane with wavelength.

2.4 Evidence for curvature

Fig. 4 shows residuals

\[ \Delta_{FP} \equiv \log R_e - a \log \sigma - b \mu_e - c \]

from the orthogonal fit as a function of distance

\[ X_{FP} \equiv \frac{a \log R_e + \log \sigma + b \mu_e}{\sqrt{1 + a^2}} \]

along the Plane. Weak trends are seen in all bands.

Interpretation of these trends is complicated by the fact that the measurement errors are correlated, and they are larger at small sizes. To see what effect is expected, we must use the numbers from the covariance matrices \(\mathcal{F}\) and \(\mathcal{E}\) to estimate \(\langle \Delta_{FP} X_{FP} \rangle / \langle X_{FP}^2 \rangle\), the expected slope of the correlation shown in Fig. 4. In the \(r\) band, \(\langle \Delta_{FP} X_{FP} \rangle = 0 - 0.0007\), where the first term is the contribution from \(\mathcal{F}\) and the second from \(\mathcal{E}\). That is, there is no intrinsic correlation (essentially, by definition), but there is a contribution from the measurement errors. This contribution is easy to estimate because \(\Delta_{FP}\) is almost proportional to \(IP - aV\), where \(IP \equiv \log_{10}(R_e) - 0.3 \mu_e\), and the error in \(IP\) is well known to be negligible, at least for deVaucouleur-like profiles (Saglia et al. 1997; or see bottom panel of fig. 5 in Hyde & Bernardi 2009), so the error in \(\Delta_{FP}\) is dominated by the measurement errors.
by the error in $V$. On the other hand, the error in $X_{FP}$ is due to errors in $R$, $V$ and IP. Errors in $R$ and $V$ are almost uncorrelated, so the dominant contribution from correlated errors comes from the fact that $V$ appears with different signs in $X_{FP}$ and $\Delta_{IP}$.

A similar analysis of $\langle X_{FP}^2 \rangle$ shows that it is dominated by the contribution from $r$ rather than $e$. In the $r$ band, $\langle X_{FP}^2 \rangle = 0.087 + 0.008$. Thus, correlated errors are expected to produce a weak trend, with slope $-0.007$. The solid line in the figure shows this trend — it is similar to that observed, suggesting that correlated errors can account for most of the observed weak decline in $\Delta_{IP}$. On the other hand, correlated errors cannot account for the break downwards at small $X_{FP}$; this is genuine curvature.

A similar analysis in the other bands yields the solid lines shown in Fig. 4. This illustrates that sample sizes are now large enough that correlated errors produce systematic effects which must be accounted for when performing the fits.

At the low-size/mass end, the Plane is probably warped (Fig. 4, and see also Fig. 10). However, we view this with caution since uncertainties/systematics in the velocity dispersion/size measurements are also larger. The possibility of curvature induced by the presence of spirals in the sample is discussed in Appendix A. We argue that the location of spirals on the Fundamental Plane would not result in negative residuals at small $X_{FP}$, and that the spirals are too few to significantly bias our results.

### 2.5 Variation in thickness along the Plane

Note that the Plane is thinner at large $X_{FP}$. Fig. 5 shows that the typical measurement errors vary little with distance along the plane. Since the observed scatter is larger at smaller $X_{FP}$ (the width of the regions enclosed by the dashed and dotted curves in Fig. 4 increases at small $X_{FP}$), but the contribution from errors is approximately constant, we conclude that the intrinsic scatter around the Plane decreases dramatically as $X_{FP}$ increases. It is remarkable that, at $X_{FP} \gtrsim 1$, measurement errors account for essentially all of the observed scatter (about 0.04 dex), suggesting that the Plane is rather thin at large $X_{FP}$.

### 2.6 Potential biases from cuts in $L$

There has been recent interest in the fact that the coefficients of the FP depend on how the sample was selected, a point emphasized by Bernardi et al. (2003). For example, D'Onofrio et al. (2008) and Nigoche-Netro et al. (2008) show that $a$ decreases if faint galaxies are removed from the sample, and one does not account for the fact that they are missing. Donofrio et al. suggest that this may reflect the fact that luminous and faint galaxies had different formation histories. We show below that although the latter may be true, the dependence of $a$ on sample selection alone is not a reliable indicator.

We do so by constructing an FP relation using pairwise correlations with $\mu$ curvature — so one would have concluded that nothing special was happening to galaxies at either end of the sequence. We then show that cuts in $L$ change $a$ and $b$ in ways that are quite similar to that observed by D’Onofrio et al. In the following section, we show that similar effects also occur if cuts in $\sigma$ are made.

Before discussing the FP, it is easier to first consider the size–surface-brightness relation. At fixed $L$, $\langle \log_{10} R \mid \mu_e \rangle$ will be a line of slope 1/5 with no scatter, by definition. Changing $L$ moves this line to the left or right, but does not change the slope. However, because there is a correlation between $R_e$ and $L$, more luminous galaxies are bigger on average, the high-$L$ galaxies only populate the large $R$ part of their line; galaxies with lower $L$ only populate the lower part of their line. Thus, in general, the full log $R_e - \mu_e$ correlation, which is got by averaging over the full range in $L$, will have a different slope than 1/5, with the difference being determined by the strength of the log $R_e - \log L$ correlation.

Fig. 6 shows this explicitly using three narrow non-overlapping bins in $L$. In any one bin, the slope of the relation (shown by the solid lines) is 0.2. The slope obtained from combining two nearby bins will be slightly steeper; combining all three bins would yield an even steeper slope. This is a generic argument, but note that it also works when $\langle \log R_e \mid \log L \rangle$ increases linearly with $\log L$. Since the slope of the log $R_e - \mu_e$ correlation is 1/5 for a narrow bin in $L$, but something else when all $L$ are included, the slope of the log $R_e - \mu_e$ correlation will appear to depend on the range of $L$ included in the sample even though the fundamental underlying correlation is linear. This, essentially, is the origin of the behaviour seen by D’Onofrio et al. (2008) and Nigoche-Netro et al. (2008).

To show this explicitly, the upper of the two dot–dashed lines in the figure shows the result of averaging over the objects in the three luminosity bins shown in Fig. 6: it is steeper than the relation for any one of the bins. Of course, in a magnitude-limited survey,
one does not simply average over all the luminosity bins. Rather, when objects with the full range of \(L\) are used, then each object is weighted by \(V^{-1}_\text{max} \times (L)\). The lower dot-dashed line shows this relation; this shows that both the slope and the zero-point of this correlation are sensitive to this weighting.

Extending this argument to the Fundamental Plane is slightly more involved. Once again, at fixed \(L\), the scaling between \(R\) and \(\mu\) is straightforward. The inclusion of a \(\sigma\)-dependent term to the \(x\)-axis serves to shift the lines associated with different \(\sigma\) horizontally – the goal of the FP algorithm is to shift them so that they lie on the top of one another as much as possible. To see the effect this has, note that for a given \(L\), each object with size \(R_e\) is shifted by an amount which depends on its \(\sigma\), so the full Fundamental Plane consists of shifting each point horizontally in the \(\log R_e - \mu\) plane by \((\log \sigma) \log L, \log R_e\) on average. If the mean shift depends on \(R_e\), this will change the slope of the line of fixed \(L\) from 1/5 to something else. If there is scatter around this mean shift, then the line of fixed \(L\) will be broadened. The combination of change in slope and additional scatter both serve to make the parameters \(a\) and \(b\) of the Fundamental Plane depend on the range of \(L\) which are included in the fit, even though the underlying pairwise correlations are linear (i.e. their slope does not depend on \(L\)).

Fig. 7 shows all of this explicitly. The symbols show how the FP coefficients change as faint objects are excluded (symbols on the left) or as luminous objects are excluded (symbols on the right). Large symbols show the effect of accounting for selection effects (each object is weighted by the inverse of the volume over which it could have been observed, so luminous objects have smaller weights), and smaller symbols show results when all objects are weighted equally (selection effects are ignored). Note that \(a\) depends strongly on the luminosity cut, whereas \(b\) is less strongly affected. The smooth curves show the result of making similar measurements in a mock catalogue constructed following methods given in Bernardi et al. (2003). Note in particular that all scaling relations in the mock catalogue were linear; there was no curvature. Nevertheless, Fig. 7 shows that the FP coefficients in the mock catalogue depend on the value of the luminosity threshold similarly to how they do in the data. This demonstrates that a detection of dependence of \(a\) on luminosity threshold does not imply that the underlying scaling relations are curved.

The strong dependence of \(a\) on luminosity threshold is easily understood. At fixed \(L\), \(R\) and \(\sigma\) are anticorrelated; it is only when \(\sigma\) is less strongly affected. The smooth curves show similar measurements in a mock catalogue in which all underlying correlations were linear. The inclusion of a \(\sigma\)-dependent term to the \(x\)-axis serves to shift the lines associated with different \(\sigma\) horizontally – the goal of the FP algorithm is to shift them so that they lie on the top of one another as much as possible. To see the effect this has, note that for a given \(L\), each object with size \(R_e\) is shifted by an amount which depends on its \(\sigma\), so the full Fundamental Plane consists of shifting each point horizontally in the \(\log R_e - \mu\) plane by \((\log \sigma) \log L, \log R_e\) on average. If the mean shift depends on \(R_e\), this will change the slope of the line of fixed \(L\) from 1/5 to something else. If there is scatter around this mean shift, then the line of fixed \(L\) will be broadened. The combination of change in slope and additional scatter both serve to make the parameters \(a\) and \(b\) of the Fundamental Plane depend on the range of \(L\) which are included in the fit, even though the underlying pairwise correlations are linear (i.e. their slope does not depend on \(L\)).

2.7 Potential biases from cuts in \(\sigma\)

There is no fundamental reason why we should have restricted our study to how \(a\) and \(b\) depend on \(L\). This section shows how the FP coefficients change as the range of \(\sigma\) in the sample is varied. This study is potentially more interesting than that of the previous section, since few samples are selected on the basis of absolute magnitude, but many samples exclude objects with small \(\sigma\), simply because small velocity dispersions are difficult to measure.

Fig. 8 shows that \(b\) is (again) hardly affected. However, \(a_{\text{orth}}\) depends strongly on this selection, and the trend is opposite to that for luminosity, while \(a_{\text{direct}}\) is less affected (also see Bernardi et al. 2003; Nigoche-Netro et al. 2008). This is easy to understand (also see discussion in Bernardi et al. 2003): \(a_{\text{direct}}\) is close to the slope \(C_{\text{RV}}/C_{\text{VV}}\) of the \((\log R) \log \sigma\) relation. If this relation is linear, then excluding small or large values of \(\sigma\) should not change this slope. On the other hand, \(a_{\text{orth}}\) measures the orthogonal slope of this relation, and this must steepen as the bin in \(\sigma\) narrows; after all,

![Figure 7](https://example.com/fig7.png)

**Figure 7.** Dependence of FP coefficients on the luminosity range of the sample. The set of symbols on the left show how \(a\) and \(b\) change if low-luminosity objects are excluded from the sample (the symbols show the absolute magnitude of the faintest object which is kept in the sample); the symbols on the right show how small high-luminosity objects are excluded (symbols show the absolute magnitude of the brightest object in the sample). Large symbols show the result of accounting for the selection effect which comes from the apparent magnitude limit of the SDSS; smaller symbols show the result of ignoring this effect. Smooth curves show similar measurements in a mock catalogue in which all underlying correlations were pure power laws.

![Figure 8](https://example.com/fig8.png)

**Figure 8.** Dependence of FP coefficients on the range of velocity dispersions in the sample. The set of symbols on the left show measurements in samples in which objects with \(\sigma\) less than the value indicated were excluded; the set of symbols on the right show results when the sample excludes objects with \(\sigma\) larger than the indicated value. Large symbols show the result of accounting for the selection effect which comes from the apparent magnitude limit of the SDSS; smaller symbols show the result of ignoring this effect. Smooth curves show similar measurements in a mock catalogue in which all underlying correlations were pure power laws.
in the limit of a very narrow bin in $\sigma$, this orthogonal slope must become infinite. (Sheth & Bernardi 2009 show this analytically – $a_{\text{orth}}$ depends both on $C_{\text{VV}}/C_{\text{VV}}$ and on $C_{\text{VV}}$ itself. Reducing the range of $\sigma$ reduces $C_{\text{VV}}$, thus increasing $a_{\text{orth}}$.) Once again, similar measurements in the pure power-law mock produce similar trends, although in this case the agreement with the SDSS is not as good. Most of this (small) discrepancy can be attributed to the fact that the underlying pairwise scaling relations in the SDSS are slightly curved (see next section), an effect which we have deliberately removed from our mocks so as to illustrate that Figs 7 and 8 are not good diagnostics of curvature.

2.8 Comparison with previous work

Our fits differ slightly from those reported by Bernardi et al. (2003). This is not unexpected, because the SDSS improved its photometric reductions significantly between DR1 (on which Bernardi et al. 2003 was based) and DR6 (on which this analysis is based). In addition, Fig. 8 shows that simply removing $\sigma < 100$ km s$^{-1}$ increases $a_{\text{orth}}$ by about 15 per cent. This is interesting because Bernardi et al. (2003) removed objects with $\sigma < 90$ km s$^{-1}$ from their sample (due to the dispersion of the SDSS spectrograph). The distribution of velocity dispersions is relatively narrow, so this cut can have a non-negligible effect. Fig. 8 shows that this might bias $a$ high by about 10 per cent, which is about the level of discrepancy between the Bernardi et al. (2003) FP and this work (but note that the maximum-likelihood method of Bernardi et al. did try to account for the cut in $\sigma$).

While the absolute values of the best-fitting coefficients have changed, the relative values have not changed significantly: both $a$ and $b$ increase while the intrinsic scatter decreases in the redder bands. This is true whether the fit minimizes the scatter in the log $R_e$ direction or in the direction orthogonal to the Plane. In addition, it is thinner at the large log $R_e$ end. The weak but significant increase of $a$ with wavelength is consistent with that reported by Bernardi et al. (2003) and, more recently, by La Barbera et al. (2008).

3 FP*: THE STELLAR MASS FUNDAMENTAL PLANE

The previous section studied the traditional Fundamental Plane, FP. In this section, we show the result of replacing luminosities with stellar masses to produce what we will call FP*. To illustrate that our results are robust to changes in how one estimates the stellar mass, we show results based on two different estimates of $M_*/L$: one, from Gallazzi et al. (2005), which is based on a likelihood analysis of the spectra; another, from Bell et al. (2003), who use the $k$-corrected broad-band colour ($M_*/L_*$) = 1.097($g - r$) − 0.306. Fig. 9 compares these estimates with each other; there is an offset of about 0.15 dex and a small trend which shows that $M_{\text{Gallazzi}}/M_{\text{Bell}}$ tends to increase at larger stellar mass. Most of this offset (0.11 dex) is due to the difference in IMF used in the stellar population models by the two groups. (Gallazzi et al. used Bruzual & Charlot 2003 models which assume the Chabrier 2003 IMF, while Bell et al. assume a diet-Saltzpeper IMF.) The rms scatter between these two estimates increases significantly at small $M_*$. While the offset will affect the zero-point but not the slope of the FP*, the weak trend could actually introduce a small systematic effect in the slope.

Fig. 10 shows the associated Fundamental Planes, the coefficients of which are reported in Table 3. The uncertainties in $\delta a$ and $\delta b$ are similar to those described in Section 2.1.2 (i.e. random errors $\delta a \sim 0.02$ and $\delta b \sim 0.01$, with larger systematic uncertainties of $\delta a \sim 0.05$ and $\delta b \sim 0.02$). In this case also, understanding the errors is important. We have assumed that the error in $\mu_*$ is given by the error in the photometric quantity $\mu$ plus a contribution from the error in $\log M_*/L$. Gallazzi et al. (2005) report rms errors of about 0.06 dex, so we add 2.5 times 0.06 dex in quadrature. We further assume that the error in $\log M_*/L$ is uncorrelated with that in the size or velocity dispersion estimates. Whereas the first assumption is probably correct, the second is almost certainly incorrect for the stellar mass estimates which come from the spectra. (Gallazzi et al. 2005 do not show how the error on $M_*/L$ correlates with the error on other observables. They do show that the $M_*/L$ error does not correlate with spectral type.) This may explain some of the differences we see between the two stellar mass planes.

The important point is that in both cases, the slope of FP* is steeper than that of the FP in the $z$ band, but it is shallower than the virial scaling. This suggests that the ratio of dynamical to stellar mass, $M_{\text{dyn}}/M_*$, varies systematically across the population. Our results suggest that it is a weakly increasing function of $M_*$ for both data sets. Fig. 11 shows a direct comparison: $\langle M_{\text{dyn}}/M_* \rangle \propto M_0^{0.17\pm0.01}$. There were no scatter around this relation, then we would expect $\langle M_{\text{dyn}}/M_* \rangle \propto M_0^{0.17-0.83} \propto M_0^{0.62}$, because there is scatter, this scaling is shallower, $\propto M_0^{0.06\pm0.01}$. (Hyde & Bernardi 2009). Gallazzi et al. (2006) also measured the $M_{\text{dyn}}$, $M_*$ relationship using deVaucouleur radii of SDSS galaxies. They obtain $\langle M_{\text{dyn}}/M_* \rangle \propto M_0^{0.09 \pm 0.05}$ which is statistically equivalent to our result.

4 THE PLANE IN $\kappa$-SPACE

Bender et al. (1992) suggested that the combination of variables which makes up the Fundamental Plane could be combined in a more physically transparent way. They called this combination $\kappa$-space. The three axes of this space are

\[
\kappa_1 \equiv \frac{\log_{10}(R_e\sigma^2)}{\sqrt{2}}, \quad \kappa_2 \equiv \frac{\log_{10}(\sigma^2I_e^2/R_e)}{\sqrt{6}} \quad \text{and} \quad \kappa_3 \equiv \frac{\log_{10}(R_e\sigma^2/L)}{\sqrt{3}}.
\]

It is interesting to examine how early types are distributed in what we call $\kappa_*$-space, which is obtained by replacing luminosity with $M_*$ (Gallazzi et al. 2005) stellar mass in the expressions above.
Figure 10. The stellar mass Fundamental Plane associated with the two estimates of the stellar mass (from spectra, left-hand panels, and photometry, right-hand panels). Symbols and line styles same as for Figs 3 and 4. Typical uncertainties on the coefficients due to random errors are $\delta a \sim 0.02$ and $\delta b \sim 0.01$. Systematic errors give $\delta a \sim 0.05$ and $\delta b \sim 0.02$.

Table 3. Coefficients ($\alpha$, $\beta$) of the stellar mass Fundamental Plane. Typical uncertainties on the coefficients due to random errors are $\delta a \sim 0.02$ and $\delta b \sim 0.01$. Systematic errors give $\delta a \sim 0.05$ and $\delta b \sim 0.02$.

| $M*/L$ | $\alpha$ | $\beta$ | $c$ | $rms_{obs}$ | $rms_{int}$ |
|--------|----------|----------|-----|-------------|-------------|
| Direct Spectra | 1.3989 | 0.3164 | 4.4858 | 0.1160 | 0.0894 |
| Colour | 1.3501 | 0.3293 | 5.0015 | 0.1115 | 0.0835 |
| Orthogonal Spectra | 1.6287 | 0.3359 | 4.9300 | 0.0638 | 0.0466 |
| Colour | 1.5462 | 0.3449 | 4.9300 | 0.0638 | 0.0466 |

Fig. 12 compares the two most interesting projections of $\kappa$-(left-hand panels) and $\kappa_*$-space (right-hand panels), when all variables have been expressed in solar units. The top panels show the ‘edge-on’ view: $M_{\text{dyn}}/L - M_{\text{dyn}}$ and $M_{\text{dyn}}/M_\ast - M_{\text{dyn}}$. Note that the top-right panel is essentially the same as Fig. 11, except that now we show the distribution using contours (objects have been weighted by $1/V_{\text{max}}$ to account for selection effects); dashed lines show forward and inverse fits, and the solid line, which has slope 0.338 and 0.429 in the two panels, shows the bisector fit.

The bottom panels show the distribution of objects within the Plane. The dashed line in the panel on the left shows $\kappa_1 + \kappa_2 = \text{constant}$ (as suggested by Bender et al. 1992). In the solar units used in the plot, the dashed line shows

$$
\left( \frac{L_e}{10^{10} L_\odot \text{ kpc}^{-2}} \right) \left( \frac{M_{\text{dyn}}}{M_\odot} \right)^{1/3} = 1.02 \left( \frac{M_{\text{dyn}}}{10^{10} M_\odot} \right)^{-1/\sqrt{3}} \quad (16)
$$

whereas it is

$$
\left( \frac{M_e}{10^{10} M_\odot} \right) \left( \frac{M_{\text{dyn}}}{M_\ast} \right)^{1/3} = 2.04 \left( \frac{M_{\text{dyn}}}{10^{10} M_\odot} \right)^{-1/\sqrt{3}} \quad (17)
$$

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for the panel on the right. In the case of stellar masses, it is helpful to cube both sides of this expression, and then rearrange so that all the dependence on the dynamical mass is on the same side. This yields
\[ \left( \frac{M_*}{R^3} \right)^2 = 8.49 \left( \frac{M_{\text{dyn}}}{10^{10} M_\odot} \right)^{-3/4} \] ; (18)
evidently, the dashed line expresses a relation between stellar density and dynamical mass: the upper limit to the stellar density is approximately proportional to \( M_{\text{dyn}}^{4/3} \).

\section*{5 DISCUSSION AND CONCLUSIONS}

We showed that the slope of the traditional, luminosity-based Fundamental Plane, FP, depends weakly but systematically on waveband – it steepens towards the virial relation in the redder bands (Fig. 3 and Table 2). Replacing \( L \) with stellar mass \( M_* \) leads to coefficients which are slightly closer to the virial ones; this is true whether one estimates \( M_*/L \) from the spectra or simply from broad-band colors (Table 3 and Fig. 10).

The stellar mass FP is slightly thinner (with an average intrinsic scatter of \( \sim 0.048 \) dex) than the luminosity-based FP (which has a typical intrinsic scatter larger than \( \sim 0.055 \) dex). The intrinsic scatter also decreases with wavelengths from 0.062 dex in \( g \) to 0.054 dex in \( z \).

We also showed that the intrinsic scatter around both the luminosity-based and stellar mass Planes becomes significantly broader at low sizes/masses (Figs 4 and 10) and that measurement errors account for essentially all of the observed scatter (about 0.04 dex) at large sizes/masses, suggesting that the Plane is rather thin for the very massive galaxies.

The fundamental nature of the stellar mass Plane \( F_{\ast} \) and the fact that \( g-r \) colour is a good indicator of \( M_*/L \) (e.g. Bell et al. 2003) explain why residuals from FP correlate with colour (e.g. Bernardi et al. 2003), or, equivalently, why colour may be thought of as the fourth fundamental parameter in early-type galaxy scaling relations.

The fact that FP does not quite have the virial scalings suggests that the ratio of stellar to dynamical mass, \( M_*/M_{\text{dyn}} \), should vary across the population. Fig. 11 showed this was indeed the case: \( M_{\text{dyn}}/M_* \propto M_*^{0.17} \). This is in qualitative agreement with the results of the SAURON project (Cappellari et al. 2007), which is based on a very different analysis technique. At higher redshifts, the Fundamental Plane method is technically less challenging, so we expect it to provide a useful measure of the evolution of \( M_{\text{dyn}}/M_* \). This will also provide a useful check on the suggestion that the slope of the correlation between \( M_{\text{dyn}} \) and \( M_* \) does not evolve out to \( z \sim 1 \) (Bundy, Treu & Ellis 2007).
We also presented an analysis of $\kappa$-space (Bender et al. 1992), but after replacing luminosities with stellar masses (using values from Gallazzi et al. 2005). The plot of $M_*/M_{\text{dyn}}$ versus $M_{\text{dyn}}$ referred to above is also known as the edge-on view of $\kappa$-space. The face-on view of the Plane in this space (Fig. 12) showed that the maximum stellar density is smaller in the more massive objects: $M_*/R_e^2 \propto M_{\text{dyn}}^{-4/3}$. The $\kappa$-space scalings at low redshift, and an estimate of how they evolve, should provide interesting constraints on galaxy formation models.

Data sets are now sufficiently large that statistically significant curvature in most scaling relations has now been seen (e.g. Hyde & Bernardi 2009). However, illustrating that the Fundamental Plane is warped is more difficult. Previous work has shown that the coefficients of the FP can change dramatically as objects of different type (e.g. low luminosity or $\sigma$) are removed from the sample (e.g. Bernardi et al. 2003 for dependence on $\sigma$, and D’Onofrio et al. 2008 and Nigroche-Nieto et al. 2008 for dependence on $L$). This has led to the suggestion that these changes indicate that the FP is warped (e.g. D’Onofrio et al. 2008). We showed that similar changes arise even in samples where the underlying pairwise scaling relations are not curved – i.e. when the Plane is not warped (Sections 2.6 and 2.7). Thus, the dependence of the fit parameters on the range of $L$ or $\sigma$ in the sample is not, by itself, evidence for curvature. This dependence on the range of $L$ or $\sigma$ may explain some of the relatively wide range of Fundamental Plane coefficients in the literature.

A more robust measure of how warped the Plane was also discussed. We showed that a good understanding of the errors is necessary to interpret the results; else, correlated errors might lead one to conclude that there is curvature even when there is none (Fig. 4 and discussion in Section 2.4). We conclude that the Plane is warped at the low-size/mass end (Figs 4 and 10), and that it is also significantly broader at low sizes/masses. These conclusions are not affected by a low rate of spiral galaxy contamination, as discussed in Appendix A.

Our analysis of the bias introduced in the FP coefficients by cuts in $L$ and $\sigma$ raises an interesting puzzle. If so, then the conclusions about the origin of ‘tilt’ should be revised.

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APPENDIX A: SAMPLE SELECTION AND DISC CONTAMINATION

The main text describes how our sample was selected; the main cut is on the shape of the light profile $FRAC_{DEV} = 1$. However, a non-negligible fraction of these objects (about 20 per cent) have $b/a < 0.6$, which we remove because we believe that the axis ratio is caused by an edge-on disc component. Presumably, similar objects viewed face-on remain in our sample, so the question arises as to how they may have affected our results.

To address this, we divided the full sample in five bins in luminosity, randomly selected $\sim 200$ galaxies in each bin, and classified them, by eye, as E/S0, spirals and others (irregulars, low-surface-brightness objects, interacting systems, projections). The different panels in Fig. A1 show these classifications in four luminosity bins.

In each panel, the solid histogram (the one with the most counts) contains $\sim 200$ objects (a few objects were removed because of contamination of a bright star or misclassification). The dot–dashed histogram shows the subset with $FRAC_{DEV} = 1$ in both $g$ and $r$ bands, the dashed histogram shows the result of applying all our other cuts except the one on $b/a$, and the dotted shows the result of excluding objects with $b/a < 0.6$. This shows that $FRAC_{DEV} = 1$ removes most of the spirals and others – our further cuts help in reducing the left over contamination.

While our selection removes most of the later type galaxies, it is perhaps surprising that a significant fraction of the objects we classified as ellipticals by eye do not have $FRAC_{DEV} = 1$. A selection of these objects is shown in Fig. A2; it is evident that many of these objects are only marginally E/S0s, so we are confident that $FRAC_{DEV} = 1$ is a reliable cut. To illustrate the effect that such objects

Figure A1. Effect of selection cuts on the morphological composition in our sample, shown for four bins in absolute magnitude. Solid histogram shows everything, dot–dashed is the subset with $FRAC_{DEV} = 1$, dotted is the smaller subset which has spectra and satisfies our cuts in $ECLASS$ and dashed is for $b/a \geq 0.6$. © 2009 The Authors. Journal compilation © 2009 RAS, MNRAS 396, 1171–1185

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Figure A2. Selection of objects classified by eye as E/S0s, but which have FRACDev < 1. Objects such as these are shown by the diamond symbols in Fig. A3.
might have had on our results, the diamond-shaped symbols in Fig. A3 show their location relative to the FP defined in the main text. These objects are offset to higher sizes, and they have substantially higher scatter around the FP (see inset in top-left corner), suggesting that our decision to exclude them from the analysis in the main text is reasonable.

The filled circles and open squares in Fig. A3 show objects we classified as E/S0s and spirals, and which satisfied our selection cuts on FRACDEV and b/a (i.e. they make up the dotted histogram in Fig. A1). There are a handful of spirals and they tend to have larger $R_e$ – as one might expect from trying to fit a disc component with a single deVaucouleur profile. They are a slightly larger fraction of the total counts at small $\sigma + 0.21\mu_e$, however they are so few that they do not bias the slope, scatter or curvature properties of the FP. Additionally, the curvature described in Section 2.4 results in negative residuals at low $X_{FP}$. Any curvature introduced by spiral contamination would introduce positive residuals because of artificially large radii.