A very simple statistical model to the quarks asymmetries

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Abstract. A simple statistical model is developed with the Fock states being the meson-hadron fluctuations. As expected, an insight about the violation of the Gottfried sum rule is obtained, and also a small difference between the strangeness amount in proton and neutron is explained.

1. Introduction

During the last decades, many collaborations, such as NMC,BCDMS,SLAC, have been performed to describe the nucleons structure function. These experiments have been executed with a large range of energy ($Q^2$) and from that works, the particle data functions (PDF) have been obtained, (see for instance the NNPDF collaboration at https://nnpdf.hepforge.org/ and MRS-MRST-MSTW collaborations at http://durpdg.dur.ac.uk/hepdata/mrs.html.)

Among the interesting results of the collaborations, is the violation of the Gottfried sum rule [1], that was not predict considering only the QCD phenomenology. One popular explanation is the meson cloud around the nucleon - there are more $\pi^+$ meson around the proton than $\pi^-$, because the fluctuation $udd(\bar{d}u)$ is more probable in the Fock space.

The observed violation of the GSR is depending on $Q^2$ [2][3], but there are few phenomenological works to explain such behavior. Abbate [2] shows a decrease of the difference $F_p^2 - F_n^2$ till $Q^2 = 10 GeV^2$ and a small increase at $Q^2 = 30 GeV^2$. Sohairy et al. [3] made a correlation among the scattering energy and the variables (Temperature, Volume, Chemical potential) of the statistical model by Zhang [4].

The polarized structure functions were also studied in the experimental way by E142,E143,E154,SMC,HERMES and COMPASS and it was found that there is a difference between the contribution of the strange polarized quarks to the total polarized structure function in proton and neutron. Let's consider

$$n_s^\uparrow - n_s^\downarrow + n_{\bar{s}}^\uparrow - n_{\bar{s}}^\downarrow = \Delta s$$

where $n_s^\uparrow$ means the amount of strange quark with positive spin, and so on, the result for the proton is:

$$\Delta s = -0.10 \pm 0.03$$
and for the neutron
\[ \Delta s = -0.06 \pm 0.04 \]

Here we adopted the same results of ref [5]. We studied the relation among all these facts and present a naive statistical model to provide a physical insight about it. This work is structured as follows: we review some concepts and facts about the structure function in the next section, after the statistical model is presented, the results are in section 4 and a discussion is in the last section.

2. Review on the structure function and quark sea asymmetries

On the present work, we use the following notation: \( q_p(x) \) is the quark \((u, d, s)\) distribution function inside the proton, and naturally, \( q_n(x) \) refers to the neutron case. We also suppose the symmetries below:

\[
u_p(x) = d_n(x) = u_v(x) + u_s(x) \\
d_p(x) = u_n(x) = d_v(x) + d_s(x)
\]

where the new index \( v \) and \( s \) refers to valence quark and sea quark, respectively.

The structure function \( F_2^p \) for the proton is given by:

\[
F_2^p(x) = \frac{4}{9} (u_p(x) + \bar{u}_p(x)) + \frac{1}{9} (d_p(x) + \bar{d}_p(x)) + \frac{1}{9} (s_p(x) + \bar{s}_p(x)) + S \tag{1}
\]

and, \( F_2^n \)

\[
F_2^n(x) = \frac{4}{9} (u_n(x) + \bar{u}_n(x)) + \frac{1}{9} (d_n(x) + \bar{d}_n(x)) + \frac{1}{9} (s_n(x) + \bar{s}_n(x)) + S \tag{2}
\]

where \( S \) is the contribution of another particles of the sea, without the strange quark. The strange quarks must remain with the index \( s \), because this is the crucial point of the present work. The antiquarks have the bar and dont need the index. Writing again \( F_2^p \) and \( F_2^n \) we have:

\[
F_2^p(x) = \frac{4}{9} (u_v(x) + u_s(x) + \bar{u}(x)) + \frac{1}{9} (d_v(x) + d_s + \bar{d}(x)) + \frac{1}{9} (s_p(x) + \bar{s}_p(x)) + S \tag{3}
\]

and

\[
F_2^n(x) = \frac{1}{9} (u_s(x) + u_s \bar{a}_n(x)) + \frac{4}{9} (d_v(x) + d_s(x) + \bar{d}(x)) + \frac{1}{9} (s_n(x) + \bar{s}_n(x)) + S \tag{4}
\]

Therefore, making the difference \((F_2^p - F_2^n)\), and integrating, the result is:

\[
\int_0^1 F_2^p(x) - F_2^n(x) = \frac{1}{3} + \frac{2}{3} \int_0^1 u_s(x) - d_s(x)dx + \frac{2}{9} \int_0^1 s_p(x) - s_n(x)dx \tag{5}
\]

The dependence with \( Q^2 \) was studied by Abate [2], that obtained the table 1:

In the Fock representation of hadron, besides the three quarks, the fluctuation with quark-antiquark may be represented as a state, and the combination of a valence quark with a antiquark may originates a meson-hadron state. Considering the creation of the pairs \( dd, u\bar{u} \) and \( ss \), one may have, for the proton, the states:

\[ |uud > = |uud > + |udd(u\bar{d}) > + |uds(\bar{u}s) > + |uuu(d\bar{u}) > + \text{other combinations} \]
Table 1. The amounts of the Gottfried sum rule versus different values of $Q^2$

| $Q^2 (GeV^2)$ | $S_G$ |
|---------------|-------|
| 1             | 0.2849 |
| 2             | 0.2548 |
| 3             | 0.2479 |
| 4             | 0.2415 |
| 5             | 0.2329 |
| 10            | 0.2279 |
| 30            | 0.2450 |

Table 2. The main fluctuations from proton that causes the violation of the Gottfried sum rule and have strangeness

| state       | energy (MeV)  | total    |
|-------------|---------------|----------|
| $|p>$         | 939           | 939      |
| $|n\pi^+>$   | 939+139.6     | 1078.6   |
| $|\Lambda K^+>$ | 1115.683 +493.7 | 1609.383 |
| $|\Sigma^0 K^+>$ | 1192.6 + 493.7 | 1686.3   |
| $|\Sigma^+ K^0>$ | 1189.3+497.7  | 1687     |
| $|p\pi^0>$   | 939+135       | 1074     |

with some possible configurations,

$$|p> = |p> + |n\pi^+> + |\Lambda K^+> + |\Sigma^0 K^+> + |\Delta^+ + \pi^- > + ...$$

and for the neutron

$$|udd> = |udd> + |uud(d\bar{u})> + |uds(d\bar{s})> + |dd(du \bar{d} > ... + oc$$

that may be

$$|n> = |n> + |p\pi^- > + |\Lambda K^0> + |\Sigma^- K^+ > + |\Delta^- \pi^+ > + ...oc$$

The difference in the probability of the state with $dd$ and $u\bar{u}$ originates the violation of the Gottfried sum rule. The model presented in the next section describe the behavior of such asymmetry and gives an insight about the difference of the amount of strangeness in the proton and neutron.

3. A simple statistical model

The phenomena describe above are analysed with a simple statistical model. Each Fock state has a energy level, that is the sum of masses of the hadron with the meson, therefore

$$p_i = \frac{1}{1 + \exp[(E_i - \mu_{p,n})/T]}$$

where $\mu_{p,n}$ is the chemical potential for the proton or neutron. As usual

$$\sum_{i=1}^{n} p_i = 1$$
Table 3. The main fluctuations from neutron that causes the violation of the Gottfried sum rule and have strangeness

| state   | energy (MeV) | total |
|---------|--------------|-------|
| $|n>$    | 939          | 939   |
| $|p\pi^->$ | 939+139.6    | 1078.6|
| $|\Lambda K^0>$ | 1115.683+497.7 | 1613.383|
| $|\Sigma^0 K^0>$ | 1192.6+497.7 | 1690.3|
| $|\Sigma^{-} K^+>$ | 1197.4+493.7 | 1691.1|
| $|n\pi^0>$   | 939+135      | 1074  |

Table 4. The dependence of states with the temperature.

| T       | $|n\pi^+>$          | $\sum$ states with $s_p$ - $\sum$ states with $s_n$ |
|---------|---------------------|--------------------------------------------------|
| 10      | 8.60928776E-05      | 2.52272171E-28                                   |
| 25      | 3.68270986E-02      | 3.71425330E-12                                   |
| 50      | 0.13331053          | 4.15262150E-07                                   |

and the thermal mass is defined as (if $n$ states are considered):

$$M_T = \sum_{i=1}^{n} p_i E_i$$

The thermal mass is a parameter for the energy system, that is, the mean energy. The comparison with the states energies help us to understand the physical meaning of the model.

4. Results

On the table 4 we show the dependence of the statistical weight and the temperature ($T$). The second column shows the probability of oscillation $n\pi^+>$, that is the more important contribution for the $\bar{u}$ and $\bar{d}$ asymmetry. Also, in the same table, the difference among the four states for neutron and the four states for proton that have some strange contribution. The strange contribution to proton and neutron structure function may have, therefore, different amounts, and the difference between them increases with the energy.

On despite that few states were considered, the main conclusion that comes from the model will not change with further improvements.

5. Discussion and conclusion

In the present work we review some features of the structure functions for proton and neutron, as the dependence with the energy in the scattering. The asymmetry $\bar{u}-\bar{d}$ increases, after has a small decrease. We also observe that must have a small difference in the amount of the strangeness in the proton and neutron and that this difference is more evident if the energy rises.

To explain these facts in a qualitative way, we implement a simple statistical model, with a Fermi-Dirac distribution for the Fock states. The physical explanation for the observed data, that comes from the model, is that with low scattering energy -that is, low temperature- , the states that generates the $\bar{d}$ and $\bar{u}$ asymmetry are more frequent, while, if the energy increases, the states with strange particles (or states that do not contribute to the $\bar{u}-\bar{d}$ asymmetry
gain statistical weight. Also, due the small difference of mass between the states with strange quarks in proton and neutron, there is a small difference on the strange contribution. Therefore, the model may explain two different phenomena. As a final remark, we notice that the small difference in the strangeness may be multiplied in a nuclear medium, where the number of protons and neutrons may be different, and this may affect some physical properties in the system.

References

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