THERMOHYDRODYNAMICS OF CIRCUMSTELLAR DISKS WITH HIGH-MASS PLANETS

GENNARO D’ANGELO
School of Physics, University of Exeter, Stocker Road, Exeter EX4 4QL, UK; gennaro@astro.ex.ac.uk

THOMAS HENNING
Max-Planck-Institut für Astronomie, Königstuhl 17, D-69121 Heidelberg, Germany; henning@mpia-hd.mpg.de

AND

WILLY KLEY
Computational Physics, Auf der Morgenstelle 10, D-72076 Tübingen, Germany; wilhelm.kley@uni-tuebingen.de

Received 2003 May 16; accepted 2003 August 13

ABSTRACT

With a series of numerical simulations, we analyze the thermohydrodynamic evolution of circumstellar disks containing Jupiter-sized protoplanets. In the framework of a two-dimensional approximation, we consider an energy equation that includes viscous heating and radiative effects in a simplified yet consistent form. Multiple nested grids are used in order to study both global and local features around the planet. By means of different viscosity prescriptions, we investigate various temperature regimes. A planetary mass range from 0.1 to 1 $M_J$ is examined. Computations show that gap formation is a general property that affects density, pressure, temperature, optical thickness, and radiated flux distributions. However, it remains a prominent feature only when the kinematic viscosity is on the order of $10^{15}$ cm$^2$ s$^{-1}$ or lower, although it becomes rather shallow for 0.1 $M_J$ perturbers. Around accreting planets, a circumplanetary disk forms that has a surface density profile decaying exponentially with distance and whose mass is 5–6 orders of magnitude smaller than Jupiter’s mass. Circumplanetary disk temperature profiles decline roughly as the inverse of the distance from the planet, matching the values measured in the gap toward the border of the Roche lobe. Temperatures range from some 10 to $\sim$1000 K. Moreover, circumplanetary disks are generally opaque, with optical thicknesses larger than 1 and aspect ratios around a few tenths. Nonaccreting protoplanets provide quite different scenarios, with a clockwise, i.e., reversed flow, rotation around low-mass bodies. Planetary accretion and migration rates depend on the viscosity regime, with discrepancies within an order of magnitude. Co-orbital torques increase as viscosity increases. For high viscosities, type I migration may extend to larger planetary masses. Estimates of growth and migration timescales inferred from these models are on the same orders of magnitude as those previously obtained with locally isothermal simulations, in both two and three dimensions.

Subject headings: accretion, accretion disks — hydrodynamics — methods: numerical — planetary systems: formation — radiative transfer

On-line material: color figures

1. INTRODUCTION

Two decades have passed since the very first attempt (Miki 1982) at modeling a protoplanet interacting with its primitive nebula. Seventeen years passed before this problem could be tackled with the strength of new and more powerful computational tools (Bryden et al. 1999; Kley 1999; Lubow, Seibert, & Artymowicz 1999; Miyoshi et al. 1999). As a consequence of the mounting interest in extrasolar planets, boosted by 8 years of uninterrupted new detections,$^2$ the scientific community has made an extraordinary effort over the past 4 years to analyze disk-planet interactions by means of numerical calculations.

Yet, regardless of the processes accounted for in the implemented numerical models, limitations and restrictions remain issues to be dealt with. As a matter of fact, numerical calculations still lag behind the present knowledge of the possible physical effects that occur in a circumstellar environment containing massive bodies. Thus, simplifying assumptions are always demanded. Nonetheless, steps forward have been made in many directions to reduce their number.

Three-dimensional disks containing Jupiter-sized objects have been investigated by Kley, D’Angelo, & Henning (2001) and, including also magnetic fields, by Papaloizou & Nelson (2003), Nelson & Papaloizou (2003), and Winters, Balbus, & Hawley (2003). D’Angelo, Henning, & Kley (2002, hereafter Paper I) carried out global two-dimensional high-resolution simulations of Jupiter-mass as well as Earth-mass planets, resolving also the flow structure within the planetary’s Roche lobe by applying a resolution enhancement strategy. This study has lately been extended to three-dimensional space by D’Angelo, Kley, & Henning (2003, hereafter Paper II) and by Bate et al. (2003), who used instead a single grid with variable step size. Nelson & Benz (2003a, 2003b) have performed two-dimensional computations, considering the effects due to disk self-gravity. Morohoshi & Tanaka (2003) have investigated the interactions between a planet and an optically thin disk with local simulations in the shearing sheet approximation, in
order to determine the one-sided gravitational torque acting on the planet.

With the present work we intend to relax the widely adopted local-isothermal approximation, i.e., that of treating a disk as a system having a fixed temperature distribution that depends neither on time nor on any other system variable. Yet, we will preserve the global viewpoint that we believe to be crucial because disk-planet interactions can be highly nonlocal.

Many of the sophisticated accretion disk studies adopt two-dimensional or one+one-dimensional schemes because of the difficulty of dealing with all the necessary ingredients in a fully three-dimensional space. Until very recently, in fact, radiation (e.g., Dullemen, van Zadelhoff, & Natta 2002), convection (e.g., Agol et al. 2001), magnetic fields (e.g., Matt et al. 2002), mixtures of gas and dust (e.g., Suttner & Yorke 2001), and chemical evolution (e.g., Markwick et al. 2002) have been treated via two-dimensional models. Furthermore, numerical simulations generally involve parameter studies, because many physical quantities are poorly known. Thereby, with a restricted number of dimensions, one is at least able to acquire a spectrum of possible approximate solutions.

In order to overcome the local-isothermal hypothesis and to maintain a global approach at the same time, we introduce an energy transport in two-dimensional (r-φ) models. We include all the supposedly major causes responsible for generation, transfer, and loss of energy in low-temperature circumstellar disk environments. We take advantage of the small aspect ratio of the disk and assume that all the radiation transport is effective only in the vertical direction. Such approximation works reasonably well in accretion disks, away from the boundary and surface layers (see, e.g., Pringle 1981). Although such an approach relies on the “thinness” assumption, it is appropriate in the local environment around a protoplanet as well. In fact, around there, the disk becomes even thinner when the planet’s gravitational action is accounted for. Hence, the amount of energy transported by radiation in the vertical direction still overwhelms that transported horizontally. Although this represents a simplistic kind of thermal description, it allows us to investigate processes that have not been considered thus far, i.e., the joint thermohydrodynamic evolution of disk-planet interactions. Furthermore, since these simulations benefit from a nested-grid numerical technique (Papers I and II), although we rely on a global approach we are capable of achieving resolutions large enough to accurately describe the system within the planetary’s Roche lobe.

The outline of the paper is as follows. In the next section we introduce the physical formulation of the problem, focusing on the equation for the thermal energy density and related issues. Section 3 concerns the numerical method utilized to solve the energy equation along with a series of tests. In § 4 we specify the adopted parameters and other various details. Sections 5 and 6 describe, respectively, global and local properties of models with different masses and viscosity regimes. In § 7 we report the results for mass accretion and orbital migration. Our conclusions are presented in § 8.

2. PHYSICAL FORMALISM

As mentioned before, we are going to tackle the problem of energy transfer in an accretion disk with an embedded planet. Since solving the full set of equations in three dimensions and simultaneously achieving sufficient resolution around the protoplanet is computationally difficult at the moment, we restrict ourselves to two-dimensional disk models.

The reason for this choice is twofold. On one hand, with affordable computational times and sufficiently high resolutions, it allows us to investigate the coupling between hydrodynamics and thermodynamics both in the disk and nearby the embedded body. On the other hand, it permits us to adopt a strategic assumption in writing the energy equation, i.e., that the horizontal energy transport can be neglected compared to the vertical one. In other words, in a two-dimensional disk one can assume that radiation transport is only effective along the direction perpendicular to the equatorial plane of the system.

Hence, let us describe the disk material through the Navier-Stokes equations for the surface density Σ, the linear momentum (density) Συ, and the angular momentum (density) rΣυg, where we choose as the working coordinate frame a cylindrical one {O: r, φ, z}. The origin is located at the center of mass of the star and the planet, while the disk lies in the plane z = 0. The hydrodynamics equations have been already stated, using the same notations, in Paper I.

As in most of the computations performed so far, we avoid dealing with the Poisson equation and assume rather that the gravitational field is determined only by the star and the planet, neglecting the self-gravity of the disk material. This is allowed by the circumstance that the Toomre Q-parameter is well beyond 1 in all of our models. Nelson & Benz (2003a, 2003b) found that some effects indeed arise when disk self-gravity is taken into account. However, we note that they consider disk masses 10 times as large as those considered here.

Thus, we rely on a smoothed point-mass gravitational potential, which reproduces well more realistic protoplanetary gravitational potentials (Paper II):

\[
\Phi = -\frac{G M_s}{|r - r_s|} - \frac{G M_p}{\sqrt{|r - r_p|^2 + \delta^2}},
\]

where \(r_s\) and \(r_p\) are the radius vectors indicating the positions of the star and the planet, respectively. As in Paper II, we set the parameter \(\delta\) as a constant length (see § 4 for details).

The thermal structure of the fluid has usually been fixed through a power law of the distance \(r\). The set of equations has been closed by adding an equation of state that connects the local gas (two-dimensional) pressure to the surface density by means of a local isothermal sound speed \(c_s^{iso}\):

\[
c_s^{iso} = \sqrt{\frac{G M_s}{r}}.
\]

Thus, the Mach number of the flow is considered throughout the system’s evolution, constrained solely by the disk aspect ratio \(h = H/r\) (e.g., Lubow et al. 1999; Miyoshi et al. 1999; Nelson et al. 2000; Tanaka, Takeuchi, & Ward 2002; Maset 2002; Nelson & Papaloizou 2003; Bate et al. 2003; Nelson & Benz 2003b). Alternatively, Kley (1999) and Tanigawa & Watanabe (2002) did some simulations using a polytropic equation of state of the type \(P = K\Sigma^{\gamma}\). However, the evolution of the thermal properties of the system have not been generally considered.
In the present calculations we use an ideal equation of state that directly ties the gas pressure \( P \) to the thermal energy density (energy per unit area) \( E \),

\[
P = (\gamma - 1)E \, ,
\]

where the adiabatic index \( \gamma \) is a constant. Furthermore, if we suppose that disk material behaves as a perfect gas, then the temperature is

\[
T = \left( \frac{\mu m_{\text{H}}}{k} \right) \frac{P}{\Sigma} \, ,
\]

where \( \mu \) is the mean molecular weight, \( m_{\text{H}} \) is the hydrogen mass, and \( k \) is the Boltzmann constant. The adiabatic sound speed is given by

\[
c_s = \sqrt{\frac{kT}{\mu m_{\text{H}}}} \, .
\]

Equation (3) implicitly assumes that radiation pressure \( P_{\text{rad}} \) is negligibly small compared to gas pressure. Such a hypothesis is connected to the relatively low temperature regimes we deal with (in fact \( P_{\text{rad}} \propto T^4 \)), and its validity was checked afterward. In all of the models presented here, the ratio \( P_{\text{rad}}/P_{\text{gas}} \) never exceeds \( 10^{-4} \).

2.1. Energy Equation

Equations for energy transport differ according to the processes that have to be included for a correct description of the energy budget of fluid elements. In our case, we suppose that a parcel of disk fluid can gain or lose thermal energy only because of flow advection, compressional work, viscous dissipation, and dissipative effects due to radiation transport. In this sense, rather than strictly treating the transfer of radiation in the disk, we account only for the cooling effects that radiation causes. Then, the energy equation takes the form

\[
\frac{\partial E}{\partial t} + \mathbf{v} \cdot (E \mathbf{u}) = -P \mathbf{v} \cdot \mathbf{u} + \Upsilon - \Lambda \, .
\]

In equation (6) we indicate with \( \Upsilon \) the dissipation function and with \( \Lambda \) the radiated energy. In two dimensions, either of these quantities is an energy flux. Both functions are always positive, and thus the former acts as a heating source whereas the latter acts as a cooling term. In this study we do not consider convection because we do not expect it to be a major energy source (D’Alessio et al. 1998). Furthermore, we neglect irradiation from the central star, which is presumably an effective heating mechanism only in the upper layers of circumstellar disks (see, e.g., D’Alessio et al. 1998), whose effects however are reduced by small disk scale heights. More importantly, as a consequence of the gap formation and the protoplanet’s gravity, circumplanetary material is shielded from stellar radiation, as is clarified below.

The function \( \Upsilon \) can be directly computed from the components of the viscous stress tensor \( S_{rr} \), \( S_{\phi\phi} \), and \( S_{r\phi} \) (Mihalas & Weibel Mihalas 1999; Collins, Helfer, & Van Horn 1998):

\[
\Upsilon = \frac{1}{2} \nabla \Sigma \left( S_{rr} + S_{\phi\phi} + 2S_{r\phi} \right) + \frac{2\nu \Sigma}{9} (\mathbf{v} \cdot \mathbf{u})^2 \, .
\]

The divergence term \( \mathbf{v} \cdot \mathbf{u} \) in the equation arises because a three-dimensional definition of stress tensor \( \mathbf{S} \) is adopted. In fact, the reduction from three to two dimensions is achieved merely by setting the vertical component of the flow field \( u_z \) and the derivative operator \( \partial / \partial z \) to zero.

In a three-dimensional disk, the last term on the right-hand side of equation (6) is equal to \( \mathbf{v} \cdot \mathbf{F} \), where \( \mathbf{F} \) is the frequency-integrated radiation flux

\[
F = -\frac{16\sigma_R}{3\kappa \rho} T^3 \nabla T \, .
\]

In the above equation \( \sigma_R \) is the Stefan-Boltzmann constant, \( \kappa \) is a frequency-integrated opacity coefficient, and \( \rho \) is the mass density. As mentioned before, we suppose that the amount of energy transported by radiation in the vertical direction is much larger than that transported horizontally, i.e., \( |F_x| \ll |F_z| \). The validity of this statement holds as long as the vertical extent of the disk remains very small compared to the disk characteristic size in the other directions. Thus, in a two-dimensional cylindrical disk we have

\[
\Lambda = \int_{-H}^{+H} \mathbf{v} \cdot \mathbf{F} \, dz \simeq \int_{-H}^{+H} \frac{\partial F_z}{\partial z} \, dz \, .
\]

Since the vertical disk structure is not meant to be accounted for, it is possible to equate the pressure scale height to the photospheric scale height and assume that all the radiation is liberated at \( z = \pm H \). Thus, equation (9) becomes

\[
\Lambda = \int_{-H}^{+H} \frac{\partial F_z}{\partial z} \, dz = F(H) - F(-H) = 2F(H) \, .
\]

In two-dimensional disks it is useful to measure the emitted flux by means of an effective temperature: \( F(H) = \sigma_R T_{\text{eff}}^4 \) and therefore

\[
\Lambda = 2\sigma_R T_{\text{eff}}^4 \, .
\]

The factor 2 indicates that in a disk, radiation escapes from both of its sides.

A simple relation between the midplane temperature and the emergent radiation flux can be found by writing

\[
\Lambda = 2 \int_0^H \frac{\partial F_z}{\partial z} \, dz = 2\left[F(H) - F(0)\right] \approx \frac{8}{3} \frac{\sigma_R}{\kappa \rho H} \left[ T_{\text{eff}}^4(0) - T_{\text{eff}}^4(H) \right] \, .
\]

In the inner parts of a circumstellar disk the inequality \( T_{\text{eff}}^4(0) \gg T_{\text{eff}}^4(H) \) generally holds (e.g., Bell et al. 1997; D’Alessio et al. 1998), and hence equation (12) yields

\[
\Lambda \approx \frac{2\sigma_R T_{\text{eff}}^4}{(3/4)\tau} \, ,
\]

in which we introduced the optical thickness \( \tau = \kappa \rho H = \frac{1}{3} \kappa \Sigma \) (we generally adopt a Rosseland mean opacity). We notice that with the previous definition the total disk optical thickness is \( 2\tau \). From now on, the quantity \( T \) refers to the disk midplane temperature.

Equation (13) represents a fairly good approximation when the medium is very optically thick, i.e., \( \tau \gg 1 \). This is indeed the case in those regions of unperturbed accretion disks that we simulate, because \( \Sigma \) is large enough. Yet because of the action of massive bodies, in our case deep density gaps form where material is very diluted. In
addition, there are zones where the disk spirals interact with the circumplanetary disk spirals, which can have very low densities (see Paper I, Figs. 12 and 13). Such conditions make equation (13) not always applicable. Hubeny (1990) found a more rigorous relation between the effective and midplane accretion disk temperature, which represents a generalization of the “gray model” of classical stellar atmospheres in local thermodynamic equilibrium. Following Hubeny’s theory, in a circumstellar disk we can write

\[ \Lambda = 2\sigma_r T^4 \left( \frac{3\tau}{8} + \sqrt{\frac{3}{4} + \frac{\varepsilon_H}{4\tau}} \right)^{-1}. \] (14)

According to equation (14), the emitted flux is inversely proportional to \( \tau \) in optically thick disk portions, whereas it becomes proportional to \( \tau \) in the opposite limit. The quantity \( \varepsilon_H \) is equal to the ratio between the Rosseland and the Planck mean opacities. This quantity can also be roughly interpreted as the ratio of total extinction (i.e., absorption plus scattering coefficient) to the pure absorption coefficient: \( \kappa_{\text{ext}} = \kappa_{\text{abs}} + \kappa_{\text{sc}} \). In these computations, we set \( \varepsilon_H = 1 \) because we have checked that we can neglect radiation scattering (see § 3.1). Thus, no distinction is made between the extinction and the absorption coefficients.

Equation (14) mimics the radiative losses of both optically thick and optically thin media and therefore is very suitable to our purposes. In fact, it has been successfully applied to many studies on accretion disks (Popham et al. 1993; Godon 1996; Collins et al. 1998; Huré, Richard, & Zahn 2001).

### 2.2. Opacity Table

In order to calculate the disk optical semithickness \( \tau \), we adopt the opacity formulas derived by Bell & Lin (1994) (see Fig. 1), as an improvement of those by Lin & Papaloizou (1985). Eight temperature regimes are identified according to the dominant processes active in each of them. Contributions from dust grains, molecules, atoms, and ions are accounted for. Since we simulate a distance range where disk material can be relatively cold, dust opacity is crucial for an accurate estimation of radiative losses (Lin 1981). Because of this, Bell’s opacity includes grain absorption as tabulated by Alexander, Augason, & Johnson (1989).

However, for the sake of comparison we ran some test cases (see § 3.1) with a new opacity coefficient developed by D. Semenov and collaborators (2002, private communication) and based on the improved grain opacity tables provided by Henning & Stognienko (1996). They aimed at coupling gas and dust opacities, focusing on the temperature range proper to protostellar disk environments.

In either case, the midplane temperature \( T \) and mass density \( \rho \) have to be provided in cgs units. In turn, \( \kappa = \kappa(T, \rho) \) has units of square centimeters per gram. Within the infinitesimally thin disk approximation, dynamics variables are vertically integrated. Thus, the mass density \( \rho \) is not directly available. It must be retrieved from the surface density and the disk semithickness \( H \). For this purpose, we assume that

\[ \rho = \frac{\Sigma}{2H}. \] (15)

In the next section we explain how the disk scale height \( H \) is calculated in a fashion such that the gravitational influence of the planet is also accounted for. This guarantees a more refined and consistent modeling of the system.

### 2.3. Disk Scale Height

Inside a circumstellar disk it is natural to assume that material is in hydrostatic equilibrium along the vertical direction (Pringle 1981; Frank, King, & Raine 1992). Let \( g_z \) be the gravitational acceleration in the \( z \)-direction within the disk; then the hydrostatic equilibrium reads

\[ \frac{P}{\Sigma} = 2 \int_0^H g_z \, dz. \] (16)

The factor 2 is necessary because the midplane pressure sustains both sides of the disk.

When the temperature distribution is not coupled to the rest of the dynamical variables, equation (16) is used to obtain the disk pressure from the density and the pressure scale height, assuming that \( g_z = GM_*/(r^2 + z^2) \) and omitting the planet’s gravitational attraction.

In the present study, we compute the pressure directly from the thermodynamic processes occurring in the gas (eq. [3]); thus we can use equation (16) to estimate the disk scale height \( H \) in a consistent fashion. This quantity, in fact, is needed to obtain the mass density from the surface density, as explained above. Since the left-hand side of equation (16) is equal to \( \kappa_T/\gamma \), we can integrate the right-hand side in order to get an implicit function of \( H \). Furthermore, the effects due to the gravitational field of the protoplanet can be included in this evaluation. If we set \( q = M_\ast/M_p \) and \( s = |r - r_p| \), equation (16) generates the implicit function

\[ \frac{H^2}{2r^2} - \frac{q}{\sqrt{(s/r)^2 + (H/r)^2}} = \frac{r}{2\gamma} \left( \frac{c_s^2}{GM_\ast} \right) - q \left( \frac{r}{s} \right). \] (17)
Although there are no constraints on the ratio $s/r$, because with nested-grid computations that ratio can be very small, the aspect disk ratio $H/r$ is smaller than 1 by working hypothesis; otherwise the two-dimensional geometry would not be consistent. Therefore, the second term on the left-hand side can be expanded in a binomial series in $H/r$ (up to its second power), obtaining $H$ as an explicit function

$$H^2 = \frac{r}{\gamma} \left( \frac{c_s^2}{GM_p} \right) \left[ \frac{1}{r^2} + q \left( \frac{r}{s} \right) \right]^{-1}. \quad (18)$$

We note that, in the limit $q \to 0$ or $s \gg r$, equation (18) yields $\sqrt{\gamma}H = c_s/\Omega_K$, as in regular accretion disks. Close to the planet, i.e., in the circumplanetary disk where $s \lesssim r$, the above relation reduces to $\sqrt{\gamma}H = c_s/(GM_p/s)^{1/2}$, which resembles the previous limit because the circumstellar Keplerian angular velocity is replaced by the circumplanetary one. Rearranging the terms in equation (18), a more compact expression can be written

$$H^2 = \frac{1}{\gamma} \left( \frac{c_s}{\Omega_K} \right)^2 \left[ 1 + q \left( \frac{r}{s} \right)^3 \right]^{-1}. \quad (19)$$

Equation (19) has a singularity at $s = 0$. This arises from the corresponding singularity in the gravitational potential $\Phi$, which is overcome by introducing the smoothing length $l$. In our computations, for consistency reasons, the distance $(s^2 + \delta^2)^{1/2}$ is substituted for $s$ in equation (19).

2.4. Artificial Viscosity

Nonlinear effects in wave propagation inevitably lead to shock formation. This is indeed the case in disk-planet simulations. In ideal fluids shocks are mathematical discontinuities. Therefore, in finite-differencing schemes, they must extend only over one or two grid points. In order to provide the correct jump conditions ahead and behind a shock front, dissipative terms have to be present in the equations. This is usually done by introducing a nonlinear viscous pressure, otherwise known as “artificial viscosity.” Since the pressure gradient is no longer proportional to the density gradient, shocks stronger than those observed in local-isothermal computations (see, e.g., Paper I) may develop. Therefore, the physical viscosity might not be sufficient to provide the correct jump conditions across shock fronts.

The most rigorous treatment of shocks in a multidimensional space with a generic metric tensor requires the definition of an isotropic viscous stress tensor $Q$ (Winkler & Norman 1986), which can be written as (Mihalas & Weibel Mihalas 1999; Stone & Norman 1992)

$$Q = \mu_Q \left[ \nabla u - \frac{1}{3} (\nabla \cdot u) I \right], \quad (20)$$

$I$ being the unit tensor. Here we choose to discard the off-diagonal, i.e., shear-tensor, components because they may lead to artificial angular momentum transport (Stone & Norman 1992). The coefficient of artificial viscosity is defined as

$$\mu_Q = -L^2 \Sigma \min(\nabla \cdot u, 0), \quad (21)$$

where $L$ represents the length over which the shock is smeared out. This is usually fixed to the maximum grid spacing. The coefficient $\mu_Q$ is positive only for compression and zero for expansion, so the artificial viscosity is large in shocks and negligibly small elsewhere.

Since the artificial viscous tensor acts as a pseudo-pressure, it affects both momentum and energy equations through the terms $\nabla \cdot Q$ and $Q \cdot \nabla u$ (meant as a tensor product), respectively. The aforementioned equations are updated by using the correct components $Q_{ij}$, as explained in Stone & Norman (1992). One severe side effect is that $Q$ can reduce the viscous time step by a factor that is proportional to $(\nabla^2 \cdot \nabla u)^{-1}$.

3. ENERGY EQUATION SOLVER

The general numerical method employed to solve the hydrodynamic equations on a hierarchy of nested grids, applied to simulations of disk-planet interactions, has been explained in Paper I. In contrast to previous calculations, thermal energy density $E$ now appears as an independent variable. Here we outline how the source terms (right-hand side of eq. [6]) are dealt with. In the framework of the nested-grid scheme, whenever required, $E$ is interpolated from a finer to a coarser grid and vice versa, according to the procedures outlined in Paper I (see also Paper II).

The energy equation (eq. [6]) is solved by means of a multistep operator-splitting method. The first step takes care of the energy advection, and this is done in the same fashion as the advection of the other dynamical quantities, i.e., through the van Leer’s algorithm. Viscous dissipation and radiative cooling are treated by means of a predictor-corrector scheme, which is second-order accurate in time, where heating and cooling terms are considered simultaneously for better accuracy and stability.

During the course of the first experiments, it was discovered that because of very high pressures and large values of the flow divergence (in the wake of the circumplanetary spirals), the compressional work could lower the thermal energy by a large amount. Consequently, the predictor-corrector procedure applied to this equation term occasionally was unstable, producing negative energy values. Therefore, we decided to take advantage of equation (3) and use an analytic solution for updating the energy.

In the framework of the operator-splitting approach, in order to correct the energy because of the gas compression/dilation, we have to solve numerically the equation

$$\frac{\partial E}{\partial t} = -P \nabla \cdot u. \quad (22)$$

With the aid of the equation of state, this becomes

$$\frac{\partial E}{\partial t} = -(\gamma - 1)E \nabla \cdot u. \quad (23)$$

Equation (23) has an analytic solution, given by $E = E_0 \exp[-(\gamma - 1)\Delta t \nabla \cdot u]$, where $\Delta t$ is a time interval over which the divergence $\nabla \cdot u$ has not appreciably changed. Hence, thermal energy density can be updated as follows:

$$E_{n+1} = E_n \exp[-(\gamma - 1)\Delta t_n \nabla \cdot u_n]. \quad (24)$$

We notice that, expanding the exponential of equation (24) in a Taylor series and keeping all the terms up to the
second order in $\Delta t$, one finds
\[
\frac{E_{n+1} - E_n}{\Delta t} = -P_n \nabla \cdot \mathbf{u}_n + \frac{1}{2} (\gamma - 1) P_n \Delta t (\nabla \cdot \mathbf{u})^2 ,
\]
where $P_n = (\gamma - 1) E_n$. It easy to prove that equation (25) is equivalent to a predictor-corrector scheme. The above equation also shows that the second-order correction always increase the thermal energy.

Although the procedure in equation (24) is not as general as the predictor-corrector algorithm, it is more accurate and unconditionally stable. Furthermore, contrary to what may seem to be at first glance, from the computational viewpoint it is faster than the other because it requires less mathematical operations.

3.1. Solver Test

We have tested the equation energy solver from both the numerical and the physical point of view. Here we present some of these tests.

The first test is intended to check whether the equation of energy furnishes physically consistent results by comparing the computed temperature to that derived from some analytical solutions of equation (6). If we deal with a stationary Keplerian disk, then the energy budget simplifies enormously. In fact, in a pure Keplerian flow both energy advection and compressional work are negligibly small. Therefore equation (6) reduces to (e.g., see Pringle 1981)
\[
\mathcal{T} - \Lambda = 0 .
\]

From the same hypotheses, it follows that the dissipation function can be written in a simple form
\[
\mathcal{T} = \Sigma \nu \left( \frac{d \Omega_K}{dr} \right)^2 = \frac{9}{4} \Sigma \nu \Omega_K^2 .
\]

In fact, in a Keplerian flow, the terms $(\partial u_r / \partial r)^2$ and $(u_r / r)^2$ are both much less than $(r \partial \Omega_K / \partial r)^2$. If we additionally assume that the disk is optically thick, then the emitted flux can be approximated to
\[
\Lambda = 2 \sigma_R T_{\text{eff}}^4 \approx \frac{2 \sigma_R T^4}{(3/8) \tau} ,
\]
as implied by equation (14), when $\tau \gg 1$. Recalling that $\tau = 1/\kappa \Sigma$, then, equating equations (27) and (28) yields
\[
T^4 = \frac{27}{128} \frac{\kappa \nu}{\sigma_R} \Sigma^2 \Omega_K .
\]

Now we assume that disk material has an opacity that we can be cast in the form
\[
\kappa = k_0 T^2 \text{ cm}^2 \text{ g}^{-1} ,
\]
with $k_0 = 2 \times 10^{-6}$ (Morfill, Tscharnuter, & Voelk 1985).

Placing equation (30) in equation (29), one finds
\[
T = \sqrt{\frac{27}{128} \frac{k_0 \nu}{\sigma_R} \Sigma \Omega_K} .
\]

Such expression allows a direct comparison with the temperature distribution obtained from simulations, whose setup involves equations (27) and (28) and the opacity law (eq. [30]).

In order to carry out a comparison of this kind, we simulated an unperturbed disk (i.e., without any embedded body), with borders at 1 and 20 AU, surrounding a solar-mass star. The disk mass is $M_D = 0.03 M_\odot$, and the kinematic viscosity is $\nu = 5 \times 10^{16} \text{ cm}^2 \text{ s}^{-1}$. Both the initial surface density and temperature are constant: $\Sigma(t = 0) = 197 \text{ g cm}^{-2}$ and $T(t = 0) = 352 \text{ K}$. Once the system has reached a stationary state, the surface density is expected to decay as $r^{-1/\sqrt{r}}$, because $\nu$ is constant (Lynden-Bell & Pringle 1974). Indeed, this is what one observes in Figure 2 (left),

![Figure 2](image-url)

**Fig. 2.—** Test simulation of an unperturbed disk in which eqs. (27), (28), and (30) are implemented. Left: Crosses represent the computed $\Sigma$, averaged along the azimuth. The solid line indicates the power law $\Sigma = \Sigma_0 (r/p)^{1/2}$ (see text). Right: The analytical formula for the midplane temperature (eq. [31]) is indicated in the plot by a solid line. Crosses represent the computed average temperature. [See the electronic edition of the Journal for a color version of this figure.]
where the azimuthally averaged, computed surface density \( \Sigma \) (crosses) is fitted by the power law

\[
\Sigma = 300 \left( \frac{5 \text{ AU}}{r} \right) \text{ g cm}^{-2}.
\]

(32)

On the other hand, equation (31) states that \( T \sim \Sigma \Omega_k \sim 1/\rho^2 \), or more precisely, \( T = 104 (5 \text{ AU}/r)^2 \text{ K} \). Figure 2 (right) shows how the calculated averaged temperature \( \langle T \rangle \) (crosses) fits this analytic estimation.

For the sake of completeness, we repeated the same type of test in which, instead of the Morfill et al. relation (eq. [30]), we chose the Kramers’s law \( \kappa = 6.6 \times 10^{-2}\rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1} \), as in Frank et al. (1992), obtaining the same good agreement between analytical and numerical results.

Although not reported here, we also tested the complete algorithm against convergence stability; i.e., we checked that it always converges to the same solution whatever the initial conditions are. Two simulations were set up in which the azimuthally averaged, computed surface density \( \Sigma \) is fitted by the power law

\[
\Sigma = 300 \left( \frac{5 \text{ AU}}{r} \right) \text{ g cm}^{-2}.
\]

(32)

Figure 2 (right) shows how the calculated averaged temperature \( \langle T \rangle \) (crosses) fits this analytic estimation.

For the sake of completeness, we repeated the same type of test in which, instead of the Morfill et al. relation (eq. [30]), we chose the Kramers’s law \( \kappa = 6.6 \times 10^{-2}\rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1} \), as in Frank et al. (1992), obtaining the same good agreement between analytical and numerical results.

Although not reported here, we also tested the complete algorithm against convergence stability; i.e., we checked that it always converges to the same solution whatever the initial conditions are. Two simulations were set up in which the azimuthally averaged, computed surface density \( \Sigma \) is fitted by the power law

\[
\Sigma = 300 \left( \frac{5 \text{ AU}}{r} \right) \text{ g cm}^{-2}.
\]

(32)

For the sake of completeness, we repeated the same type of test in which, instead of the Morfill et al. relation (eq. [30]), we chose the Kramers’s law \( \kappa = 6.6 \times 10^{-2}\rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1} \), as in Frank et al. (1992), obtaining the same good agreement between analytical and numerical results.

Although not reported here, we also tested the complete algorithm against convergence stability; i.e., we checked that it always converges to the same solution whatever the initial conditions are. Two simulations were set up in which the azimuthally averaged, computed surface density \( \Sigma \) is fitted by the power law

\[
\Sigma = 300 \left( \frac{5 \text{ AU}}{r} \right) \text{ g cm}^{-2}.
\]

(32)

Although not reported here, we also tested the complete algorithm against convergence stability; i.e., we checked that it always converges to the same solution whatever the initial conditions are. Two simulations were set up in which the azimuthally averaged, computed surface density \( \Sigma \) is fitted by the power law

\[
\Sigma = 300 \left( \frac{5 \text{ AU}}{r} \right) \text{ g cm}^{-2}.
\]

(32)

Although not reported here, we also tested the complete algorithm against convergence stability; i.e., we checked that it always converges to the same solution whatever the initial conditions are. Two simulations were set up in which the azimuthally averaged, computed surface density \( \Sigma \) is fitted by the power law

\[
\Sigma = 300 \left( \frac{5 \text{ AU}}{r} \right) \text{ g cm}^{-2}.
\]

(32)

Although not reported here, we also tested the complete algorithm against convergence stability; i.e., we checked that it always converges to the same solution whatever the initial conditions are. Two simulations were set up in which the azimuthally averaged, computed surface density \( \Sigma \) is fitted by the power law

\[
\Sigma = 300 \left( \frac{5 \text{ AU}}{r} \right) \text{ g cm}^{-2}.
\]

(32)

Although not reported here, we also tested the complete algorithm against convergence stability; i.e., we checked that it always converges to the same solution whatever the initial conditions are. Two simulations were set up in which the azimuthally averaged, computed surface density \( \Sigma \) is fitted by the power law

\[
\Sigma = 300 \left( \frac{5 \text{ AU}}{r} \right) \text{ g cm}^{-2}.
\]

(32)
\( \mu = 2.39 \) (Morfill et al. 1985) and the adiabatic index \( \gamma = 1.4 \).

Figure 4 shows characteristic averaged quantities of non-perturbed, stationary disks with the aforementioned characteristics. To distinguish among the different viscosity (and temperature) disk regimes, models will be referred to as “hot” (H), “warm” (W), and “cold” (C).

To further explore the parameter space, two models were executed applying the Shakura-Sunyaev viscosity prescription \( \nu = \alpha_{\text{gs}} H \), with \( \alpha = 10^{-2} \) and \( 10^{-3} \) (models A2 and A3). Hence, in such models \( \nu \) is a function of space and time.

An embedded planet is placed on a fixed circular orbit at \( r_p = 5.2 \) AU. The azimuthal position of the planet is kept fixed (\( \varphi_p = \pi \)) by the employment of a rotating grid. In order to obtain reliable outcomes in the two-dimensional geometry, we consider a planetary mass range extending from \( M_p = 33 M_\oplus \) to \( 1 M_J \), or in terms of mass ratios
we have models with $q = 1 \times 10^{-4}$, $2 \times 10^{-4}$, $5 \times 10^{-4}$, and $1 \times 10^{-3}$. It has been recently demonstrated (Paper II) that concerning gravitational torques and mass accretion, three-dimensional models agree quite well with two-dimensional ones in this mass range.

However, since the disk thickness is sufficiently small, even for low protoplanetary masses (see Fig. 4), we go so far as to simulate an additional C-model with $M_p = 20 M_J$. Figure 5 shows the disk semithickness in the planet’s neighborhood for three selected masses. Indicating with $R_H = r_p(q/3)^{1/3}$ the planet’s Hill radius, one can clearly see that the condition $H \lesssim R_H$ locally holds even in the smallest mass case. The figure also shows that protoplanets dwell in cavities and thus that radiation from the central star cannot directly reach the surrounding matter.

The smoothing length $\delta$ is set to $2 \times 10^{-2} R_H$ (see eq. [1]). All of the models are executed in an accreting and a non-accreting mode. When accretion onto the planet is allowed, the procedure outlined in Paper I is utilized. The accretion region has a radius $\kappa_{ac} = 0.1 R_H$, which should be short enough to let the whole accretion procedure be almost independent of the removal characteristic time, as proved by Tanigawa & Watanabe (2002).

Figure 6 illustrates the influence of the extent of the domain’s radial extent for Jupiter C-models. Simulations provide quite consistent outcomes over the matching domain. Differences exist only close to the inner border, because of inflow boundary conditions.

### 4.1. Initial and Boundary Conditions

Initial conditions require the assignment of the three functions $\Sigma(t = 0)$, $T(t = 0)$, and $u(t = 0)$ that we suppose to be axisymmetric because we start from a stationary disk.

The theoretical surface density profile of a stationary accretion disk, with constant $\nu$, is a power law of the radial distance $r$ (Lynden-Bell & Pringle 1974), as also found numerically in § 3.1. Thus, as starting density distribution for our simulations, we set

$$\Sigma(t = 0) = \Sigma_0 \sqrt{r_{fp} \over r} ,$$

where $\Sigma_0$ is determined by the disk mass value $M_D$.

Calculations carried out for different viscosity magnitudes imply different temperature regimes (see Fig. 4). According to them, the initial temperature distribution is prescribed. The behavior of the profiles in the top-right panel of Figure 4 can be roughly reproduced by the power law

$$T(t = 0) = T_0(\nu) \left( {r_p \over r} \right)^{1.8} .$$

The initial temperature at $r = r_p$ is given in Table 1 for three values of $\nu$. Finally, the initial circumstellar flow $u(t = 0)$ is a Keplerian one, corrected for the grid rotation.

As for the boundary conditions, all models are run with a partially open inner radial border and a reflective outer one. Thus, matter is free to flow out of the computational domain at $r = r_{min}$, but the opposite is not allowed. This is the same expedient invoked in Papers I and II to artificially mimic the mass accretion toward the central star and avoid spurious wave reflection at the inner domain edge, which is
the closer to the planet. The flow field is Keplerian both at 
\(r_{\text{min}}\) and \(r_{\text{max}}\): 
\[ u = \frac{1}{2} \left( \frac{r}{C_1} \right) \left( \frac{K}{C_0} \right) \left( \frac{p}{C_{10}} \right). \]

4.2. Numerical Specifications

The basic computational mesh is made of \(N_r \times N_z = 143 \times 423\) cells, while subgrid patches are \(64 \times 64\). All of the computations are based on a five-grid hierarchy. Thus, the finest resolution ranges from \(1.3 \times 10^{-2} R_H\) (if \(M_p = 1 M_J\)) to \(2.9 \times 10^{-2} R_H\) (if \(M_p = 33 M_J\)). Further details on numerical issues have been presented in Papers I and II.

Despite our not having implemented a sophisticated energy transport treatment, these simulations take between \(35\%\) and \(40\%\) longer than those discussed in Paper I, for equal-sized grid hierarchies.

5. GLOBAL MODEL PROPERTIES

We now examine the behavior of global quantities for some selected models because the large-scale look of disks with planets is, observationally, of extreme importance. Figure 7 shows both the averaged surface density and temperature in Jupiter-mass computations with different kinematic viscosities. We note that matter (left) is more uniformly distributed in the H- (solid line) and W- (short-dashed line) models, with respect to the C-model (long-dashed line). In these cases, the density gap is not very deep and actually looks like a depression, because of the larger viscosity coefficient and the depletion of the inner disk. On one hand, in fact, a high viscosity prevents the formation of wide and deep gaps. On the other hand, a large viscosity facilitates the inner-disk depletion, since \(M_{\text{disk}} \propto \nu R^2\) (Lyn- den-Bell & Pringle 1974). The disk depletion rate that we measure in these simulations ranges from a few times \(10^{-8}\) to \(10^{-7} M_\odot \text{ yr}^{-1}\).

TABLE 1

| Sphere | Initial Temperatures versus Viscosity and Disk Mass |
|--------|-----------------------------------------------------|
| \(\nu\) (cm\(^3\) s\(^{-1}\)) | \(T_0\) (K) | \(T_{\text{max}}\) (K) | \(T_{\text{min}}\) (K) |
| 1.0 \times 10^{15} | 50 | 180 | 13 |
| 5.0 \times 10^{15} | 110 | 235 | 22 |
| 1.0 \times 10^{16} | 150 | 295 | 26 |

Notes.—Values of the initial temperature as a function of the three values of the kinematic viscosity. Temperatures are sampled at \(r = r_p = 5.2 \text{ AU (}T_0\), \(r = r_{\text{max}} = 2.1 \text{ AU (}T_{\text{max}}\), \(r = r_{\text{min}} = 13 \text{ AU (}T_{\text{min}}\). Models with \(\alpha\)-viscosities A2 and A3 are initiated like the C-model.
height, as seen in Figure 8. Such a deep trough represents additional evidence that, as long as an inner disk exists, protoplanets are shaded against stellar radiation.

Global views of both density and temperature two-dimensional distributions are displayed in Figure 9. The top panels illustrate the surface density, while the bottom ones show the temperature structure. From the density maps one can see that disk spirals get closer to each other toward the outer border of the domain. In fact, circumstellar spirals propagate at a velocity equal to the sound speed, which is proportional to $\sqrt{T_p}$ (see eq. [5]). Thus, the propagation velocity reduces as $r$ increases, and therefore faster waves overtake slower ones. Past the gap region, the temperature decays more rapidly in the C-model (Fig. 7, right), and hence this phenomenon is more evident. From the bottom panels of Figure 9, one can also deduce that spiral perturbations in the thermal energy and mass densities are quite phase-coherent and thus disturbances in the thermal structure are rather weak.

Shifting toward smaller masses, the gap is gradually refilled. This is visible in Figure 10, where C-models of different mass are compared. At $M_p = 0.1 \, M_j$ (i.e., $M_p = 33 \, M_\oplus$), the gap is 1 order of magnitude as shallow as it is when $M_p = 0.5 \, M_j$. We further address the issue of the gap structure in § 5.2, since gaps are considered promising features for detecting embedded planets. As regards the circumstellar disk spirals, only feeble traces are left when $M_p = 0.1 \, M_j$, both in the temperature and density distributions. The absence of global disk features produced by an embedded body invalidates direct imaging observations as a detection tool in the intermediate-mass range, i.e., around a few times Uranus’s mass.

Concerning the emitted energy, we notice that the optical thickness of the models is generally larger than 1 throughout the computational domain (Fig. 11, left). The averaged $r$ stays above 1 also at the outer disk edge. This is in agreement with what was found for accretion disks around T Tauri stars (D’Alessio et al. 1998). The density and temperature gap is reflected in a similar feature in

![Figure 7](image-url)

**Fig. 7.**—Average surface density (left) and temperature (right) in Jupiter-mass models after 240 orbital periods. By this evolutionary time, models have settled on a quasi-equilibrium state. The different profiles refer to different values of $\nu$, as indicated in the legends. The density gap, in the C-model ($\nu = 10^{15}$ cm$^2$ s$^{-1}$), is more than an order of magnitude deeper (long-dashed line) than it is in the other models. The temperature gap follows the same trend, although differences are smaller.

![Figure 8](image-url)

**Fig. 8.**—Average pressure scale height (eq. [19]), normalized to the distance from the star, in Jupiter-mass models according to different viscosity values, as indicated in the legend (in cm$^2$ s$^{-1}$).
the distribution of $\tau$, as shown in Figure 11 (left). Nonetheless, even within the gap region material is on average optically thick ($\tau > 1$). The exception is represented by the C-model, where gap optical thickness reaches values around 0.01 at $\varphi = \varphi_p + \pi$ (see Fig. 11, right). The most diluted portions of the gap, situated where the circumplanetary disk merges into the gap, have values of $\tau$ on the order of $10^{-2}$ to $10^{-3}$.

Finally, we examine the emitted flux $\Lambda$, computed according to equation (14) (see Fig. 12). From the observational point of view, disk gaps could represent a probe for protoplanet detection. In fact, this is by far the most extensive imprint that a planet leaves on a circumstellar disks. Prospective studies on observability of gaps due to Jupiter-mass protoplanets have already been presented by Wolf et al. (2002) and Steinacker & Henning (2003), and speculations...
on a developing gap around the T Tauri star TW Hya have been reported by Calvet et al. (2002). Here we do not intend to address the issue of whether gaps are really observable or to what extent they are. However, we have to notice that a necessary but not sufficient condition is that they must be wide, deep, and with sharp edges in order for the flux emitted from this region to have the strongest contrast with respect to the surrounding environment (Rice et al. 2003).

From the bottom panels of Figure 12, it appears clear that low kinematic viscosities would favor this kind of investigation. Disk spirals are probably too elusive to be captured by present-day ground-based instruments, as shown by the flux maps. We also argue that it would seem rather unlikely to detect planetary masses smaller than Jupiter’s by means of these measurements. The right panels of Figure 12 display the emitted flux in the case of a $M_p = 0.1 M_J$ model. The
quantity \( \Lambda \) furnished by the high-viscosity model (top) looks quite smooth, and only some inhomogeneities appear in the C-model (bottom).

5.1. Models with \( \alpha \)-Viscosity

So far we have considered disks with a constant \( \nu \). However, we have also investigated the effects of a different viscosity prescription, namely, an \( \alpha \)-type viscosity (Shakura & Sunyaev 1973). Two Jupiter-mass accreting models were run in this mode, one with \( \alpha = 10^{-2} \) (model A2) and the other with \( 10^{-3} \) (model A3). Such values are representative of those recently estimated by Papaloizou & Nelson (2003) and Winters et al. (2003), who performed MHD disk simulations with embedded Jupiter-sized bodies and could therefore evaluate the stress parameter \( \alpha \) in a self-consistent fashion. Indeed, referring to the constant-viscosity models discussed above, by making the azimuthal mean \( 2\pi \langle \alpha \rangle = \left( 1/\nu \right) \int c_r H d\varphi \), we get a value of \( \langle \alpha \rangle \) ranging from \( 3 \times 10^{-2} \) in the inner disk to \( 4 \times 10^{-3} \) in the outer disk. Specifically, C-models yield an overall trend of \( \langle \alpha \rangle \) that is close to \( 10^{-2} \). And in fact outcomes from \( \alpha \)-viscosity model A2 are not much different from those obtained from the Jupiter-mass C-model. The quantity \( \langle \Sigma \rangle \) is very similar to that illustrated in left panel of Figure 7 (long-dashed line), whereas \( \langle T \rangle \) is only slightly different (near the inner border) from the one in the right panel of the same figure.

Some more important deviations from what has been seen thus far are instead encountered in the A3-model, as displayed in Figure 13. The averaged surface density shows a significant hump (top left, solid line) inside the gap region, not observed in the other viscosity regimes. This is not caused by the mass accumulation around the planet but is produced by material lingering around Lagrangian points L4 and L5, as appears from the ringlike traces in the middle of the density gap in the top right panel Figure 13. This also appears to be a persistent feature; in fact, no change is measured over the last 50 orbital periods. The L4 and L5 points are equilibrium locations for the restricted three-body problem. Hence, one can expect that the thermal energy is larger near such points than anywhere else in the gap. In turn, viscous torques, which are enhanced by the locally increased kinematic viscosity, are more efficient in contrasting gravitational torques. Thus, material is more likely forced to evolve on the much longer viscous timescale \( t_{\text{vis}} \approx r_p^2/\nu = r_p/(\alpha c_s h) \).

As for the temperature (Fig. 13, bottom), the largest discrepancies between models A2 and A3 occur toward the inner border.

5.2. Gaps as Planet Signatures

We have seen in the previous two sections that the gap structure drastically depends on the viscosity regime. The standard criterion to open a gap in a disk requires two conditions (Lin & Papaloizou 1985, 1993). The first is a thermal condition given by

\[
\left( \frac{M_p}{M_\ast} \right) > 3 \left( \frac{H}{r} \right) = 3h^3, \tag{35}
\]

i.e., the Roche lobe must be larger than the disk scale height. Because of planet’s gravity (see eq. [19]) and the low temperatures established in gap regions, this condition is always fulfilled when \( M_p \geq 0.1 M_\ast \) (see Fig. 8). The second condition concerns the requirement that tidal torques exceed viscous torques and reads

\[
\left( \frac{M_p}{M_\ast} \right) > \left( \frac{40}{\text{Re}} \right), \tag{36}
\]

where \( \text{Re} = r^2\Omega K/\nu \) is the Reynolds number. The right-hand side of equation (36) is proportional to \( \nu \) and in our case goes from \( \approx 4 \times 10^{-3} \) (H-models) to \( \approx 4 \times 10^{-4} \) (C-models). Thus, we should not obtain gaps in any of the...
H-models. Figure 7 (solid line) agrees well with this prediction, since only a trough is dug in the H-model. However, condition (36) is partly violated in the low-viscosity regime (Fig. 10), because a clear gap is visible for the \( M_p = 0.2 \ M_J \) case and a trough appears for \( M_p = 0.1 \ M_J \). In Table 2 we report the gap occurrence and depth for some of the models.

One reason why the above criterion could not properly apply to our calculations lies in the fact that it was derived for simple polytropic disks, while we simulate also the thermal evolution of the system. As implied by the profiles in the top left panel of Figure 13, it is even harder to predict the gap presence when the kinematic viscosity depends on the local fluid temperature.

6. CIRCUMPLANETARY DISKS

The first distinction about the circumplanetary flow has to be made according to whether the planet is accreting or
not. Therefore, we shall carry out separate discussions, based on H- and C-models, as well as $\alpha$-viscosity models A2 and A3.

6.1. Accreting Models

Figure 14 indicates that inside of the Roche lobe, the general aspect of the flow circulation around $1$ and $0.1 \, M_J$ protoplanets resembles that obtained with local isothermal models in two dimensions (Paper I). However, specific characteristics of the flow do differ. A circumplanetary disk, extending over a fair fraction of the Roche lobe, can be identified for both masses and viscosity regimes. The mass of such disks around $1 \, M_J$ protoplanets, within the 80% of the Hill radius, is $1.03 \times 10^{-3} \, M_J$ in the H-model and $5.36 \times 10^{-6} \, M_J$ in the C-model. As regards calculations with $M_p = 0.1 \, M_J$ planets, the mass measured in the circumplanetary disk (or subdisk, for brevity) of the C-model

---

Fig. 13.—Left: Averaged surface density and temperature in the Jupiter-mass $\alpha$-viscosity models A3 ($\alpha = 10^{-3}$; solid line) and A2 ($\alpha = 10^{-2}$; long-dashed line). Right: Two-dimensional distribution of the surface density (top) and temperature (bottom) from model A3. The density scales linearly and the temperature scales logarithmically. Conversion factors are $\Sigma = 10^{-4} \rightarrow 32.9 \, g \, cm^{-2}$ and $T = 10^{-2} \rightarrow 198 \, K$. [See the electronic edition of the Journal for a color version of this figure.]
TABLE 2

| $M_p/M_J$ | C-Models | H-Models |
|-----------|-----------|----------|
|           | Accreting | Nonaccreting | Not Fulfilled | Accreting | Nonaccreting | Not Fulfilled |
| 1.0       | <0.01     | <0.01     |             | 0.6       | 0.8         | X            |
| 0.5       | 0.06      | 0.2       | X           | 0.9       | NG          | X            |
| 0.2       | 0.3       | 0.4       | X           | NG        | NG          | X            |
| 0.1       | 0.6       | NG        | X           | NG        | NG          | X            |
| 0.06      | 0.8       | NG        | X           |          |             |              |

Notes.—Ratio of the minimum density in the gap to the density at the gap’s taller shoulder (rounded to the first significant digit). Values are recovered from the azimuthal average of $\Sigma$ around the star. The letters "NG" stand for "no gap." The symbol "X" appears whenever at least one of criteria (35) and (36) is not fulfilled.

TABLE 3

| $M_p/M_J$ | $\langle \Sigma \rangle_0$ (g cm$^{-2}$) | $a_1$ | Range               |
|-----------|-------------------------------------|------|---------------------|
| C-Models  |                                     |      |                     |
| 1.0       | 2.92 × 10$^2$                       | -4.4 | [0.2$R_H$, 1.0$R_H$]|
| 0.5       | 1.59 × 10$^2$                       | -2.0 | [0.2$R_H$, 1.0$R_H$]|
| 0.2       | 2.32 × 10$^2$                       | -1.5 | [0.1$R_H$, 1.0$R_H$]|
| 0.1       | 1.99 × 10$^2$                       | -0.9 | [0.1$R_H$, 1.0$R_H$]|
| 0.06      | 1.47 × 10$^2$                       | -0.4 | [0.2$R_H$, 1.0$R_H$]|
| H-Models  |                                     |      |                     |
| 1.0       | 2.14 × 10$^2$                       | -2.3 | [0.2$R_H$, 1.0$R_H$]|
| 0.5       | 1.35 × 10$^2$                       | -1.3 | [0.2$R_H$, 1.0$R_H$]|
| 0.2       | 8.57 × 10$^1$                       | -0.5 | [0.1$R_H$, 1.0$R_H$]|
| 0.1       | 6.75 × 10$^1$                       | -0.2 | [0.2$R_H$, 1.0$R_H$]|

A2-Model

| $M_p/M_J$ | $a_1$ | Range |
|-----------|------|-------|
| 1.0       | 2.36 × 10$^2$ -5.0 | [0.2$R_H$, 1.0$R_H$] |

A3-Model

| $M_p/M_J$ | $a_1$ | Range |
|-----------|------|-------|
| 1.0       | 1.80 × 10$^2$ -7.5 | [0.2$R_H$, 0.8$R_H$] |

Notes.—Parameters that enter eq. (37). This represents a linear best fit of the logarithm of the averaged density $\langle \Sigma \rangle$ over the specified interval.
at \( z = 0 \), i.e., in the disk midplane. The latter two models are quasi-isothermal in the sense that the temperature distribution is fixed and depends only on the distance from the star \( r \) according to the relation \( T \propto (\mu/\gamma) h^2 M_\star / r \). Thus, around the planet’s location \( r = r_p \), \( T \approx 90 \) K if the disk aspect ratio \( h \) is 0.05, \( \mu = 2.39 \), and \( \gamma = 1.4 \). From Figure 17 one can see that in the outer part of the Roche lobe \( (s \gtrsim 0.3 R_H) \), where spiral perturbations are strongest, the discrepancy between the two quasi-isothermal models lies between 20\% and 30\%, whereas the two-dimensional thermal model provides a specific angular momentum only 10\% larger than that of the three-dimensional quasi-isothermal model. This occurrence can be attributed to the better pressure structure achieved by the calculations that account for the thermal energy budget.

Circumplanetary disks should not suffer from gravitational instabilities since the Toomre \( Q \)-parameter is orders of magnitude larger than unity, as proved by Figure 18. This arises from the small value of the ratio between the mass of the circumplanetary disk and that of the planet.
Two-dimensional temperature distributions are rather symmetric with respect to the planet’s position, although they are marked by weak spiral perturbations. Figure 19 (top) shows the azimuthal averaged temperature around protoplanets with $M_p = 1 M_J$ (left) and $M_p = 0.1 M_J$ (right). In the Jupiter-mass case, maximum temperatures...
range from roughly 1500 to 1000 K, whereas \( T \) reaches the value of the ambient medium (\( \leq 50 \) K) toward the border of the Hill sphere. Comparing H- and C-models, one realizes that the average temperatures in the inner part of the Roche lobe differ by less than a factor of 2, whereas the \( \alpha \)-viscosity model A3 shows significantly lower temperatures. In the low-mass case (\( M_p = 0.1 \) \( M_J \)), the peak temperature is around 1000 K and between \( \approx 80 \) and \( \approx 40 \) K at \( s = R_H \) for the H- and C-models, respectively. The average temperatures differ by less than \( \approx 30 \) K (Fig. 19, top right).

Over the entire subdisk domain, \( \langle T \rangle \) can be well fitted by the curve

\[
\langle T \rangle \approx \langle T \rangle_s \left( \frac{s}{s_s} \right)^{\zeta},
\]

in which the length \( s_s \) is set to 0.1 \( R_H \) for convenience. The fitting parameters are reported in Table 4, together with the validity range of the fitting function. From the entries in the table, it turns out that the temperature generally falls off as \( 1/s \).

The bottom panels of Figure 19 display the average effective temperature \( \langle T_{\text{eff}} \rangle \) (see eq. [11]) in the vicinity of protoplanets. Figure 20 shows that the magnitude of the optical thickness \( \langle \tau \rangle \) is between 10 and 100; hence, equation (14) yields \( T_{\text{eff}} \approx T / s^{0.25} \), and thus there is, at most, a factor of 3 between the two temperatures. It should be noted that only model A3 presents optically thin regions toward the edges of the subdisk.

Accreted matter also contributes to the energy budget of the circumplanetary disk via dissipation of gravitational energy into heat. This source of energy is not treated explicitly in these simulations. However, it is considered implicitly as a compression work term arising from the removal of matter in the accreting region. Tanigawa & Watanabe (2002) assumed the dissipation of gravitational energy as the only heating source and derived temperature profiles of subdisks with different sound-speed regimes, performing local simulations with an isothermal equation of state. They found that the temperature would scale as \( 1/\sqrt{s} \), independently of the planetary mass.

We have to point out that since radial transport of radiation is not treated here, one should expect large temperature gradients to be smoothed out. But we have seen that temperature profiles are not extremely steep in the range of distances covered by these models; hence, correction by radial transfer should not be dramatically relevant, as we now demonstrate.

### 6.1.1. Radial Radiation Transfer in Circumplanetary Disks

Since the temperature gradient in the vicinity of a protoplanet may be quite steep, one of the hypothesis used in \( \S \) 2.1 might locally break down. We refer to the assumption that the radiative flux in the vertical direction overwhelms those in the horizontal direction. With a semianalytical analysis, it is possible to make an a posteriori check of that hypothesis and shed some light on this matter. For simplicity we suppose that the temperature distribution has a cylindrical symmetry around the planet, which is indeed what is observed in all of the models. Hence the total flux of energy transferred via radiation is

\[
\Lambda \approx \int_{-\infty}^{+\infty} \frac{\partial F_z}{\partial z} \, dz + \int_{-\infty}^{+\infty} \frac{1}{s} \frac{\partial}{\partial s} (sF_z) \, dz = \Lambda_z + \Lambda_r,
\]

where \( s \) is the distance from the planet. The two terms on the
right-hand side can be written as

\[ \Lambda_z \approx -\frac{128\sigma_R}{3\kappa \rho} \frac{T^4}{H}, \tag{40} \]

\[ \Lambda_\tau \approx -\xi^2 \frac{128\sigma_R}{3\kappa \rho} \frac{H}{s^2} T^4. \tag{41} \]

Equation (40) was obtained by setting \( \partial T/\partial z \approx T/H \) in equation (39), whereas equation (41) was derived by adopting equation (38), which we have checked to hold as long as \( s \leq s \leq R_H \). The validity of relation (9), namely that \( \Lambda \approx \Lambda_z \), depends on the ratio of the left-hand sides of equations (40) and (41):

\[ \frac{\Lambda_{\tau}}{\Lambda_z} = \xi^2 \left( \frac{H}{s} \right)^2. \tag{42} \]
According to equation (42), only when \( H < s \) is the radiative cooling (eq. [14]) included in the energy equation also locally a good approximation. As shown in Figure 21, this seems indeed to be the case in models involving Jupiter-sized planets. To cast the above ratio in a more explicit form, one can use the expression for \( H \) (eq. [19]) in the limit \( s \ll r \), which yields

\[
\left( \frac{H}{s} \right)^2 = \frac{c_s^2}{\gamma} \left( \frac{s}{GM_p} \right).
\] (43)

By using the form of the sound speed in equation (5), one gets

\[
\left( \frac{H}{s} \right)^2 = \left( \frac{k T}{\mu m_H} \right) \left( \frac{s}{GM_p} \right).
\] (44)

Therefore, because of the assumption made above on the temperature profile, equation (42) becomes

\[
\frac{\Lambda_s}{\Lambda_z} \approx \frac{\xi^2 \left( \frac{k T}{\mu m_H} \right) \left( \frac{s}{GM_p} \right) \left( \frac{s}{s} \right)^{\zeta - 1}}{\xi^2 \left( \frac{k T}{\mu m_H} \right) \left( \frac{s}{GM_p} \right) \left( \frac{s}{s} \right)^{\zeta - 1}}.
\] (45)

Since the distance \( s \) is constrained by the applicability range of equation (38), the ratio given by equation (45) is supposedly meaningful roughly for \( s \in [0.1 R_H, R_H] \), for the models considered here. With the appropriate values, the ratio (45) becomes

\[
\frac{\Lambda_s}{\Lambda_z} \approx 0.145 \xi^2 \left( \frac{T}{1000 \text{ K}} \right) \left( \frac{M_J}{M_p} \right)^{2/3} \left( \frac{s}{s} \right)^{\zeta - 1}.
\] (46)

Equation (46) is an evaluation of equation (45) at a distance \( s \) from the planet, whose length shortens as \( M_p^{1/3} \). Since the squared subdisk aspect ratio grows as \( 1/M_p \), the flux ratio goes as \( M_p^{2/3} \). From Table 4, we see that \( |\zeta - 1| \) is between 0.03 and 0.12 in the C-models and between 0.04 and 0.21 in the H-models. Therefore, the rightmost term in equation (46) can be considered as a unity term. Thus, around \( s \approx s = 0.1 R_H \), \( |\Lambda_s|/|\Lambda_z| = 0.13 \) for the Jupiter-mass H-model and is a factor of 2 as small for the C-model. Similar small numbers are obtained from both the A2 and A3 models. For \( M_p/M_J = 0.1 \), the ratio increases to 0.2 and

---

**Fig. 20.**—Average optical thickness around \( M_p = 1 \ M_J \) (left) and \( M_p = 0.1 \ M_J \) (right) protoplanets. The solid, short-dashed, and long-dashed lines reproduce the H-, C-, and A3-models, respectively. [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 21.**—Average circumplanetary disk aspect ratio around \( M_p = 1 \ M_J \) protoplanets. The solid, short-dashed, and long-dashed lines reproduce the H-, C-, and A3-models, respectively. [See the electronic edition of the Journal for a color version of this figure.]
respectively, which is consistent with the applicability limit of the two-dimensional approximation. Thus, apart from regions closer than $\sim 0.1 R_H$ to the accreting planet, which probably cannot be properly investigated by these computations, radiation transport in the radial direction does not play a major role in the energy budget of circumplanetary disk material. This is in agreement with the standard scenario of the late stages of proto-Jovian disks (e.g., Coradini et al. 1989; Canup & Ward 2002).

### 6.2. Nonaccreting Models

Accreting models assume that the planetary accretion rate can adjust to the rate at which matter can be supplied by the surrounding environment. We now investigate the opposite situation, namely, when no further accretion onto the protoplanet is allowed.

The overall flow structure and density distribution of nonaccreting protoplanets are drastically different from those of accreting ones. This can be clearly seen in Figure 22, where we show the surface density and the flow field for $M_p = 1 M_J$ (top) and $0.1 M_J$ (bottom) objects. The standard scenario of a Keplerian-like subdisk is completely altered. This is a consequence of the large pressure gradient, built up by the large density gradient, which allows matter to move on orbits not constrained by the centrifugal balance. Therefore, the two-arm spiral feature is replaced by a more complex system of multiple shock fronts across which material is first deflected toward the planet and then ejected outward, as also indicated by the sign of the in-fall velocity $w_{\text{in}}$ within the region.

The flow strongly diverges in a zone that has a radius equal to $\approx 0.4 R_H$ in the top left panel (Jupiter H-model) and to $\approx 0.2 R_H$ in the top right panel (Jupiter C-model). The average surface density inside of $R_H/3$ is between 1 and 2.5 orders of magnitude larger than that measured in accreting models. The mass collected in $s \leq 0.8 R_H$ is $1.97 \times 10^{-4}$ and $4.30 \times 10^{-5} M_J$ for the Jupiter-mass H- and C-models, respectively. The azimuthal average of the circumplanetary density around Jupiter-sized objects can be approximated to $\langle \Sigma \rangle \approx 1.2 \times 10^5 (0.1 R_H/s)^{-2} \text{g cm}^{-2}$ (H-model) and $\langle \Sigma \rangle \approx 1.5 \times 10^3 (0.1 R_H/s)^{-2.7} \text{g cm}^{-2}$ (C-model). The top panels of Figure 23 show the averaged surface density profiles around $M_p = 1 M_J$ (left).

A major difference can also be observed in the subdisk circulation of low-mass models (Fig. 22, bottom). In fact, the fluid rotates in a clockwise direction. In general, the direction of rotation in subdisks is determined by the balance among the Coriolis force, the pressure gradient, and the gravitational attraction by the planet. Basically, referring to the bottom right panel of Figure 22, the Coriolis force deflects fluid elements, orbiting at $r \lesssim r_p$, rightward. Therefore, it forces matter to cross the gap and to reconnect to the other side, while still moving in a position upstream of the perturber body. In contrast, the term $-dP/ds$, supposedly positive, opposes reconnection when fluid is upstream of the planet’s location but favors it when matter is downstream of the planet. The prograde rotation that has been encountered so far indicates that the Coriolis deflection overwhelms the pressure gradient. For this reason reconnection across the gap always occurs upstream of the planet. Evidently, in the low-mass models ($M_p = 33 M_\oplus$) displayed in the bottom panels of Figure 22, the pressure gradient drives material flowing from upstream past the planet’s position and forces it to circulate clockwise around the perturber. Consequently, matter entering the Hill sphere has an angular momentum antiparallel (in the planet’s frame) to that of the circumstellar disk. In the range from $0.1 R_H$ to $1 R_H$, the magnitude of the rotational velocity $\sqrt{|\langle \omega_{\text{rot}} \rangle|}$ increases roughly as $\sqrt{s}$. The behavior of $\langle \Sigma \rangle$ resembles that of a power law, as in Jupiter-mass cases, with powers 0.9 and 0.5 for the H- and C-models, respectively (Fig. 23, right). Interestingly, no large differences are seen in the two density profiles.

The high pressure gradient is also caused by the large temperature gradient. Parameters for analytic approximations of the average temperature $T$, according to equation (38), are reported in Table 5. Maximum temperatures at $s = 0.1 R_H$ reach 1900 and 1200 K in the $M_p = 1 M_J$ H- and C-models, respectively (Fig. 23, bottom left). At the limit of the Roche lobe the temperature is somewhat higher than that measured in accreting models and depends on the gap structure. It varies from $\sim 100$ to $\sim 200$ K. Low-mass models furnish temperatures that are very similar in both viscosity regimes.

It is worth noting that in either planetary-mass and viscosity case, the resulting temperature profiles are steeper than the ones shown in Figure 19 only when $s < 0.1 R_H$, but they are nearly as steep past this distance. Actually, from Table 5 it appears that toward low planetary masses, temperature profiles are even flatter. Performing the same check as done in the previous section, it turns out that $|\Lambda_\lambda|/|\Lambda_\Sigma|$ is at most 0.3. In fact, around Jupiter-mass planets, the ratio $H/s$ varies between 0.3 and 0.5 for distances larger than $0.1 R_H$. Hence, even in these circumstances the type of energy equation implemented here yields a reasonable description of the thermal structure of subdisks down to $\sim 0.1 R_H$.

### 7. ACCRETION AND MIGRATION

The evaluation of the mass accretion rate of protoplanets is carried out by reducing the surface density around the planet, according to a relation of the type $\Delta \Sigma/\Sigma = \Delta t/\tau_{\text{ev}}$ (see Paper I for details). This reduction process has a timescale $\tau_{\text{ev}}$, that is much smaller than the integration time step.
In Figure 24 the planetary accretion rate $\dot{M}_p$, evaluated in these computations is compared with that achieved with local-isothermal two-dimensional (Paper I) and three-dimensional (Paper II) models, where the disk mass was rescaled to match the values adopted here (see § 4).

In the context of the accretion history of protoplanets, it is worthwhile to stress that two-dimensional calculations are able to simulate only the late stages of the accretion process, i.e., when it mainly occurs via mass transfer in the circumplanetary disk. This is consistent with the parameterization adopted here to describe the circumstellar environment.

From Figure 24, one can see that C-models (filled squares) provide accretion rates quite similar to those obtained from models with a fixed temperature distribution (crosses). This occurrence is related to the quasi-Keplerian, circumplanetary flow observed in both kinds of computations. Jupiter-mass models with an $\alpha$-type viscosity (open
triangles) give rates that are similar to those of the C-models when \( \alpha = 10^{-2} \) but somewhat lower (\( M_p = 6 \times 10^{-3} M_J \) yr\(^{-1}\)) when \( \alpha = 10^{-3} \), consistent with the low subdisk mass measured in this case. H-models provide values of \( M_p \) (filled diamonds) that, in the case of Jupiter-sized planets, fall above the estimates from all of the other models. According to the accretion procedure, the larger the amount of matter contained in the accretion region \( s = \kappa_{ac} \), the larger the accreted mass is. Within \( s = 0.1R_H \) of a Jupiter-mass planet, the mean density computed in the H-models is roughly 3 times larger than that in the C-models (Fig. 14). This is related to the background gap density, which is much higher in the H-models, as reported in Table 2 (see also Fig. 7), because of the larger fluid viscosity.

Figure 25 shows the total gravitational torque \( T_D \) exerted by disk material onto accreting planets, as the
are rescaled to match the disk mass and three dimensions, respectively. These last two sets of values sent the rates furnished by local isothermal computations in two (Paper I) and three (Paper II) dimensions, respectively. These last two sets of values are rescaled to match the disk mass $M_D = 4.8 \times 10^{-3} M_\odot$ used in these simulations.

The quantity is computed from the gravitational force acting on the planet, as explained in Paper I. The length $\beta$ represents the radius of the circular region around the planet’s position, the contribution of which is not taken into account. In Figure 25, this length is set equal to $R_H$. Outcomes from C-models, and generally those from H-models, compare well to the local-isothermal computations. Models with $\alpha$-viscosity provide migration rates on the same magnitude as well. C-models show a rather constant value of $\tau_M$ as a function of the planet’s mass, whereas H-models present a minimum at $M_p = 0.2 M_\odot$. Migration timescales of H-models differ by less than a factor 2 from those of C-models. Only for $M_p = 33 M_\odot$ is $\tau_M$ appreciably longer. One may attribute this effect to the absence of a real gap in this model (see Table 2) and thus to an early onset of the type I drifting regime.

In Table 6 we separate the contributions of different disk regions to the total torque $\mathcal{D}_p$, introducing the inner disk ($\mathcal{A}$), the outer disk ($\mathcal{B}$), and the gap region ($\mathcal{G}$). Further details on the regions are reported in the table note. All contributions are normalized to $|\mathcal{D}_p|$. The width of the gap region is fixed to 5$R_H$ (see Fig. 7), and the zone behind the planet ($\mathcal{B}$) is distinguished from the one ahead of it ($\mathcal{G}$). Torques exerted from inside the Hill sphere are not accounted for. A large fraction of the contributions arising from zones $\mathcal{B}$ and $\mathcal{G}$ are actually generated in the co-orbital regions, within a few Hill radii from the planet. From the entries in Table 6 one can argue that co-orbital torques in H-models are larger in magnitude than they are in C-models. This is a consequence of the shallower gap of H-models. The situation is reversed in the $\alpha$-viscosity models because of the density humps at the L4 and L5 Lagrangian points. As for the influence of the parameter $\beta$ on the total torque, it turns out that the H-models are generally more sensitive than C-models. By examining the function $\mathcal{D}_p = \mathcal{D}_p(\beta)$, some more insight can be gained. For $M_p = 1 M_\odot$ models (Fig. 26, top left), no significant net

| Model | $\mathcal{A}$ | $\mathcal{B}$ | $\mathcal{G}$ | $\mathcal{D}$ |
|-------|--------------|--------------|---------------|-------------|
| 1.0 $M_1$ | -0.0987 | -1.9416 | 1.3707 | -0.5278 |
| 1.0 $M_2$ | 0.3247 | -13.538 | 12.828 | -0.6817 |
| 1.0 $M_3$ | 0.9857 | -4.7780 | 4.9240 | -2.1317 |
| 1.0 $M_4$ | 0.0913 | -2.1357 | 1.3905 | -0.3461 |
| 0.5 $M_2$ | 0.1962 | -3.6112 | 2.9909 | -0.4949 |
| 0.5 $M_3$ | 0.3301 | -14.940 | 14.804 | -0.4732 |
| 0.2 $M_1$ | 0.4280 | -13.852 | 13.176 | -0.7520 |
| 0.2 $M_2$ | 0.2406 | -14.827 | 13.840 | -0.2536 |
| 0.1 $M_1$ | 0.4070 | -17.989 | 17.211 | -0.6290 |
| 0.1 $M_2$ | 0.6315 | -36.823 | 35.768 | -0.5785 |
| 20 $M_1$ | 0.3471 | -17.656 | 16.800 | -0.4911 |

Notes.—Gravitational torques exerted by the inner disk ($\mathcal{A}$), $r \leq r_p - 2.5R_H$, and the outer disk ($\mathcal{B}$), $r \geq r_p + 2.5R_H$. The remaining domain ($r_p - 2.5R_H < r < r_p + 2.5R_H$) is further divided in the trailing gap region ($\mathcal{G}$), $\phi \leq \phi_g$, and the leading gap region ($\mathcal{G}$), $\phi > \phi_g$. The width of the gap region is chosen according to the average width observed in Fig. 7. All torques are normalized to the absolute value of the total torque $|\mathcal{D}_p|$. Furthermore, they exclude the contribution from a circle of radius $R_H$ centered on the planet.
torque arises from within the Hill sphere in the C-models (dashed line). The situation appears to be different for the H-models (solid line) because $\mathcal{F}_D$ decreases to a minimum around $\beta = R_H/2$ and then increases and changes sign around $\beta = 0.1R_H$. Jupiter-mass $\alpha$-viscosity models (Fig. 26, top right) display a smooth behavior since the total torque is rather constant down to $\beta = R_H/2$. When $\beta$ diminishes, $\mathcal{F}_D$ increases (i.e., it has smaller absolute values) and the model with $\alpha = 10^{-3}$ (dashed line) shows a change of sign at $\beta = 0.4R_H$. When $M_p = 0.1 M_J$ (Fig. 26, bottom left), in either case, $d\mathcal{F}_D/d\beta < 0$, because of the positive torques arising within the subdisk. The rate of change is more drastic in the H-models, where the total torque becomes positive and keeps increasing when $\beta \lesssim R_H/2$. Torques acting on the 20 $M_\oplus$ planet (Fig. 26, bottom right) are strongly dependent on $\beta$, as positive torques are exerted down to $\approx 0.4R_H$ and large negative torques arise between $\approx 0.2R_H$ and $\approx 0.4R_H$. 

Fig. 26.—Inverse of the migration timescale vs. the radius $\beta$ of the region excluded from the torque calculation. The quantity $\nu = \dot{\alpha}/|\dot{\alpha}|$ indicates the direction of the planet migration. The solid lines refer to models H or A2 while the dashed lines refer to models C or A3. Top left: Jupiter-mass models H and C. Top right: Jupiter-mass models with $\alpha$-viscosities A2 and A3. Bottom left: $M_p = 0.1 M_J$ models H and C. Bottom right: $M_p = 20 M_\oplus$ C-model.
8. CONCLUSIONS

In this paper we have presented two-dimensional simulations of disk-planet interactions in which we considered the joint thermohydrodynamic evolution of the system. In order to do that, we solved an energy equation that accounts for the major processes responsible for the energetic balance of fluid parcels. We restricted the treatment of the problem to a flat geometry in order to make some simplifying assumptions on the radiative part of the energy budget, i.e., to treat the radiation transfer as a cooling term. A nested-grid technique was used to resolve different length scales; therefore both the global and local features could be investigated.

The two-dimensional approximation has proved to work reasonably well as long as the planetary mass is larger than a tenth of the mass of Jupiter (Paper II). Because of this, only planets more massive than \( \approx 0.1 M_J \) have been simulated. In order to compromise with the lingering uncertainty on the viscosity magnitude and to inquire into different temperature regimes, we considered different constant viscosity and \( \alpha \)-viscosity models. Both accreting and nonaccreting planets were simulated.

From the global point of view, and roughly speaking, we can conclude that circumstellar disk models with a fixed temperature distribution provide a reasonably good description of the system, although details of the gap structure and shape of the disk spirals differ. Around Jupiter-mass protoplanets, only with kinematic viscosities as low as \( 10^{15} \text{ cm}^2 \text{s}^{-1} \) is a wide and deep gap carved in, whereas it reduces to a shallow density trough when the viscosity is 10 times larger. Mean temperatures in the gap can be as low as 20–50 K, depending on the viscosity regime. The most diluted and cold gap material is located where the circumplanetary disk merges into the gap. The aspect ratio appears smaller than 0.04, and it is also marked by a gaplike feature, which shades the protoplanet’s environment against stellar radiation. The models with \( \alpha = 10^{-3} \) show a ring of matter within the gap that evolves on a viscous timescale. Regarding smaller planetary masses (\( M_p \approx 0.1 M_J \)), both gap and spiral perturbations become quite faint features. It is likely that only Jupiter-sized bodies are capable of digging gaps deep enough to be observed by present-day instrumentation.

From the local point of view, we obtained the distribution of density, temperature, and other thermodynamic quantities in circumplanetary disks. Except for the \( \alpha \)-viscosity model with \( \alpha = 10^{-3} \), rather comparable outcomes are provided by models with different viscosity regimes. The density, averaged around the planet, shows an exponential radial decline. Subdisks have masses on the order of \( 10^{-5} \) to \( 10^{-6} M_J \), which makes them gravitationally stable. Temperatures range from several hundred kelvins at distances \( s \approx 0.1 R_H \) from the planet to values between 50 and 100 K at the border of the Hill sphere. Azimuthal averages yield profiles dropping nearly as the inverse of the distance from the planet. Circumplanetary disks have aspect ratios \( H/s \) around a few tenths, and they tend to be optically thick. All of these properties appear to be consistent with the “fast-inflow” analytical models of proto-Jovian disks recently developed by Canup & Ward (2002). By means of a semianalytical treatment, we showed that also in the subdisk, the neglect of radial radiation transfer should not crucially affect our results. Nonaccreting models furnish a rather different scenario. Around Jupiter-mass objects, circulation in the Roche lobe is not Keplerian-like because of the large pressure built up mainly by the density gradient and partly by the temperature gradient. Around \( M_p = 0.1 M_J \), clockwise rotation is established as a result of the weakening of the Coriolis force with respect to the pressure gradient term. Accretion and migration rates inferred in these simulations are comparable to those evaluated with previous purely hydrodynamic local-isothermal computations. An analysis of the zonal torques indicates that co-orbital torques increase as viscosity increases. Moreover, negative co-orbital torques, arising from material outside of the Roche lobe and lagging behind the planet, are generally larger than those from material ahead of the planet. We also found that in the two-dimensional approximation, the material lying inside of the Roche lobe is able to exert strong gravitational torques on the planet. Since the density gap is usually filled in H-models (see Table 2), the type I migration regime might extend to larger planetary masses (see Fig. 25).

In order to address the issue of how the third dimension affects the thermohydrodynamics of the system, both globally and locally around protoplanets, we are extending this study to three-dimensional simulations in which radiation transfer is treated in the flux-limited diffusion approximation (Levermore & Pomraning 1981).

We thank Udo Ziegler for having made available to us an early FORTRAN version of his code NIRVANA. G. D. is deeply grateful to all the friends at the INAF-OAC (Naples) and RSMAS/MPO (University of Miami) for their warm hospitality. The paper benefited from the useful comments of an anonymous referee. This work was supported by the German Science Foundation and by the UK Astrophysical Fluids Facility (UKAFF). The numerical computations reported here were carried out at the Computer Center of the University of Jena and at the UKAFF.

REFERENCES

Agol, E., Krolik, J., Turner, N. J., & Stone, J. M. 2001, ApJ, 558, 543
Alexander, D. R., Augason, G. C., & Johnson, H. R. 1989, ApJ, 345, 1014
Armitage, P. J., Clarke, C. J., & Tout, C. A. 1999, MNRAS, 304, 425
Bate, M. R., Lubow, S. H., Ogilvie, G. I., & Miller, K. A. 2003, MNRAS, 341, 213
Bell, K. R., Cassen, P. M., Klahr, H. H., & Henning, Th. 1997, ApJ, 486, 372
Bell, K. R., & Lin, D. N. C. 1994, ApJ, 427, 987
Bryden, G., Chen, X., Lin, D. N. C., Nelson, R. P., & Papaloizou, J. C. B. 1999, ApJ, 514, 344
Calvet, N., et al. 2002, ApJ, 568, 1008
Canup, R. M., & Ward, W. R. 2002, AJ, 124, 3404
Collins, T. J. B., Helfer, H. L., & Van Horn, H. M. 1998, ApJ, 502, 730
Coradini, A., Cerri, P., Magni, G., & Federico, C. 1989, in Origin and Evolution of Planetary and Satellite Atmospheres, ed. S. K. Atreya, J. B. Pollack, & M. S. Matthews (Tucson: Univ. Arizona Press), 723
D’Alessio, P., Cunto, J., Calvet, N., & Lizano, S. 1998, ApJ, 500, 411
D’Angelo, G., Henning, Th., & Kley, W. 2002, A&A, 385, 647 (Paper I)
D’Angelo, G., Kley, W., & Henning, Th. 2003, ApJ, 586, 540 (Paper II)
Dullemond, C. P., van Zadelhoff, G. J., & Natta, A. 2002, A&A, 389, 464
Frank, J., King, A., & Raine, D. 1992, Accretion Power in Astrophysics (Cambridge: Cambridge Univ. Press)
Godon, P. 1996, ApJ, 463, 674
Henning, Th., & Stanghellini, R. 1996, A&A, 311, 291
Hubeny, I. 1990, ApJ, 351, 632
Huré, J.-M., Richard, D., & Zahn, J.-P. 2001, A&A, 367, 1087
