Proton Synchrotron Origin of the Very-high-energy Emission of GRB 190114C

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Abstract

We consider here a proton-synchrotron model to explain the MAGIC observation of GRB 190114C afterglow in the energy band of 0.2–1 TeV, while the X-ray spectra are explained by electron-synchrotron emission. Given the uncertainty of the particle acceleration process, we consider several variations of the model, and show that they all match the data very well. We find that the values of the uncertain model parameters are reasonable: isotropic explosion energy $\sim 10^{54.5}$ erg, ambient density $\sim 10^{-10}$ cm$^{-3}$, and the fraction of electrons/protons accelerated to a high-energy power law is of a few percent. All these values are directly derived from the observed teraelectronvolt and X-ray fluxes. The model also requires that protons be accelerated to observed energies as high as a few $10^{20}$ eV. Further, assuming that the jet break takes place after $10^5$ s gives the beaming-corrected energy of the burst to be $\approx 10^{53}$ erg, which is one to two orders of magnitude higher than usually inferred, because of the small fraction of electrons accelerated. Our modeling is consistent with both late time data at all bands, from optical to X-rays, and with numerical models of particle acceleration. Our results thus demonstrate the relevance of proton-synchrotron emission to the high-energy observations of gamma-ray bursts during their afterglow phase.

Unified Astronomy Thesaurus concepts: Gamma-ray bursts (629); Interstellar synchrotron emission (856)

1. Introduction

While gamma-ray bursts (GRBs) are well known to emit in the energy band of keV $\leq \epsilon_{\gamma} \leq$ GeV, in recent years there are a growing number of observations at even higher energies, in the range of several gigaelectronvolts to teraelectronvolts. Early detections in this band date back to the 1990s. The Energetic Gamma-Ray Experiment Telescope on board CGRO, was the first instrument to monitor a photon with an energy of $\sim 18$ GeV in coincidence with GRB 910503 (Schneid et al. 1992; Band et al. 1993; Hurley et al. 1994), followed by a $\sim 1$ TeV photon observation associated with GRB 970417a in the Milagrito field of view (Atkins et al. 2003). Successive detections of 14 bursts per year, on average, have been done by Fermi-LAT (Large Area Telescope) in a broad range of energies up to $\sim 100$ GeV (Nava 2018). Very-high sub-teraelectronvolt band detections were recently reported by the ground-based Cherenkov observatories, namely, the High Energy Stereoscopic System (H.E.S.S.) and the Major Atmospheric Gamma Imaging Cherenkov (MAGIC). H.E.S.S. is actively detecting GRBs at energies larger than 100 GeV, e.g., the 100–440 GeV photons reported in GRB 180720B (Abdalla et al. 2019). Similarly, MAGIC is also exploring the window of very-high-energy (VHE) emissions with the detection of GRB 190114C in the sub-teraelectronvolt band (Acciari et al. 2019).

These recent discoveries have called for much attention as GRB afterglow spectra were long predicted to have VHE emissions with its unprecedented precision (Knödlseder 2020).

Of particular interest is the recent detection of the VHE (0.2–1 TeV) emission from GRB 190114C by MAGIC. Thanks to its proximity, this burst turned out to be one of the brightest bursts ever detected. It was observed by diverse space observatories from near the Fornax constellation 4.5 billion lt-yr away with a redshift of $z = 0.4245 \pm 0.0005$ (Castro-Tirado et al. 2019; Selsing et al. 2019) in a dense environment right in the middle of the luminous galaxy. It was reported with the prompt phase isotropic equivalent energy $E_{iso} \sim 2.5 \times 10^{53}$ erg for a duration of $T_{90} \sim 25$ s. The afterglow of this burst was observed in an unprecedentedly large energy band for an unprecedented duration. The broadband afterglow spectrum can be split into a low-energy component ranging from optical to X-rays and a high-energy component (mega electronvolt to giga electronvolt) extended by a nonthermal VHE tail (at least up to the teraelectronvolt band) (Frail et al. 1997; van Paradijs et al. 1997; Lloyd & Petrosian 2000; Zhang 2018). The low-energy spectral component is usually observed to be a broken power law (Costa et al. 1997; van Paradijs et al. 1997; Wijers et al. 1997; Harrison et al. 1999; Böttcher & Dermer 2000). It can be interpreted in the framework of synchrotron emission from shock-accelerated relativistic electrons, which naturally leads to a broken-power-law spectrum (Paczynski & Rhoads 1993; Mészáros & Rees 1997; Sari et al. 1998). However, the high-energy emission is too bright to be explained within the framework of this basic synchrotron model (Panaitescu & Mészáros 1998; Pe’er & Waxman 2005). For this reason, it is required to extend the basic afterglow model based on electron synchrotron to explain the emission at the highest observed energies.

Various theoretical models have been proposed to explain the observed signal at the high-energy end of the spectrum. A leading suggestion is the synchrotron self-Compton (SSC) mechanism, where the low-energy synchrotron photons inverse-Compton (IC) scatter on the electrons that previously
emitted them (Ghisellini & Celotti 1998; Chiang & Dermer 1999; Dermer et al. 2000; Sari & Esin 2001; Nakar et al. 2009; Liu et al. 2013; Derishev & Piran 2016, 2021; Fraija et al. 2019). Alternatively, it has been suggested that the high and VHE emission might be produced by accelerated baryons. Relativistic hadronic-induced emission processes such as proton-synchrotron, photopion, and photopair processes and emission from their secondaries have previously been suggested as viable candidates; see, e.g., Böttcher & Dermer (1998), Asano et al. (2009), Razzaque et al. (2010), Gagliardini et al. (2022).

Direct indication of proton acceleration to high energies is given by the explicit detection of high-energy cosmic rays (Asakimori et al. 1998; Sanuki et al. 2000; Lipari & Vernetto 2020) and of astrophysical petaelectronvolt neutrinos by IceCube (Aartsen et al. 2013; Bykov et al. 2015; Mészáros 2017). Their potential association with blazars (Petropoulou & Mastichiadis 2012; Banik & Bhadra 2019) indicates that relativistic jets might harbor high-energy protons, further supporting hybrid and hadronic models for the high-energy emission; see, e.g., Mücke & Protheroe (2001), Aharonian (2002), Mücke et al. (2003), and Gasparian et al. (2022). Such high-energy protons could also be present in GRB jets, emitting high-energy synchrotron photons (Vietri 1997; Totani 1998; Zhang & Mészáros 2001; Gupta & Zhang 2007; Razzaque et al. 2010). As we show here, the spectral theoretical expectations from the proton-synchrotron model are consistent with the collective data available for GRB 190114C. We, therefore, suggest that proton synchrotron may be the leading emission process, and the isotropic equivalent kinetic energy is \( E = 10^{53} \) erg. Hereinafter, any quantity \( X_n \) is such that \( X = 10^n X_n \) and cgs units are adopted. In addition, here and in all numerical expressions below, we consider the redshift to be \( z = 0.4245 \), i.e., the observed cosmological redshift of GRB 190114C (Castro-Tirado et al. 2019; Selsing et al. 2019). The comoving shell-expansion time (the dynamical timescale) is \( t_{\text{dyn}} \approx \Gamma r \).

We further use the standard assumption that the magnetic field is generated by the shock, and carries an unknown fraction \( \epsilon_B \) of the internal energy density behind the shock front. As a consequence, in the comoving frame, the magnetic field strength is

\[
B = \sqrt{32 \pi \epsilon_B \Gamma^2 n m_p c^2} = 0.4 \, E_{53}^{1/8} c^{1/2} r_{\text{shock}}^{-3/8} t_{\text{day}}^{-1/8} n_0^{-1/4} \text{G},
\]

where \( n = 1 \times n_0 \text{ cm}^{-3} \), \( t_{\text{day}} \) is the observed time in days, and the isotropic equivalent kinetic energy is \( E = 10^{53} \text{ erg} \).

We assume that both electrons and protons are accelerated by the propagating shock wave, and attain a power-law distribution \( n(\gamma) \approx \gamma^{-p} \) in the range of \( \gamma_{\text{min}} \leq \gamma \leq \gamma_{\text{max}} \), as they reach the downstream region. Here, \( p \) is the post-shock spectral index, which we consider to be similar for both the proton and electron populations. Typically, \( p \) is assumed to be \( p > 2 \), which is in agreement with both theoretical (Sari et al. 1996; Bednarz & Ostrowski 1998; Kirk et al. 2000; Sironi & Spitkovsky 2011) and observational analysis of GRB afterglows (Shen et al. 2006). Let \( \gamma_p(\gamma_e) \) be the Lorentz factor of a single proton (electron). As the particles radiate, they cool. The radiative cooling time of a particle \( x \) (standing for protons or electrons) having Lorentz factor \( \gamma_x \) is

\[
t_{\text{cool}} = \frac{\gamma_x m_p c^2}{\frac{4}{3} \sigma_T x \gamma_x^{2/3} B^2} = \frac{6 \pi m_e c}{\sigma_T x \gamma_x B^2},
\]
where $\sigma_T = (m_e^2/\pi\epsilon)\sigma_{T,e}$ is the Thomson cross section (Rybicki & Lightman 1979). The energy dependence of the cooling time implies the existence of a characteristic Lorentz factor, denoted by $\gamma_c$, for which the cooling time is equal to the dynamical time. There are two categories of solution defined by the values of $\gamma_{\min}$ and $\gamma_c$: the fast-cooling regime, where $\gamma_c < \gamma_{\min}$, and the slow-cooling regime, where $\gamma_{\min} < \gamma_c$.

To find the value of $\gamma_{\min}$, we assume that a fraction $\xi_p(\xi_e)$ of the proton (electron) population is accelerated and injected to a power-law distribution. We further assume that the accelerated particles of an energetic (power-law distributed) particle is $\gamma = [(p-1)/(p-2)]\gamma_{\min}$ (assuming $p > 2$)\(^3\), one finds the minimum injection Lorentz factor of the accelerated protons and electrons to be

$$\gamma_{\min} = f(p) \left( \frac{m_p}{m_e} \right) \left[ \frac{\xi_e}{\xi_p} \right] = 2 \times 10^3 ~ \left( \frac{f(p)}{1} \right) \xi_e \xi_p E_{53}^{1/8} n_0^{-1/8} t_{\text{day}}^{-3/8} \epsilon_{\delta, -1},$$

(5)

where the first number refers to electrons, the number in parenthesis refers to protons, and $f(p) \equiv (p-2)/(p-1)$ (Sari et al. 1996, 1998; Granot & Sari 2002). Here and below, we consider the number density of the accelerated electrons, $n_e = \xi_e n$, and of the accelerated protons, $n_p = \xi_p n$.

By balancing the comoving dynamical time $t_{\text{dyn}}$ and the cooling time by synchrotron emission $t_{\text{cool}}$, the cooling Lorentz factors of the particle species are $\gamma_{p,e} = (6\pi\epsilon/m_e c^2)/(\sigma_T m_e^2 B^2 \Gamma^2)$ and $\gamma_{e,c} = (m_e/m_p)\gamma_{p,c}$. A third characteristic Lorentz factor is the maximum Lorentz factor achieved by the particles (de Jager et al. 1996). It can be estimated as follows. As the particles cool via the synchrotron process, their maximum achievable Lorentz factors are calculated by equating the energy loss time and the particle acceleration time, $t_{\text{acc}} = 2\pi\alpha\epsilon/qB^2$ where $\epsilon = \gamma mc^2$ is the particle’s energy, and $\alpha \geq 1$ is a numerical coefficient normalizing the acceleration time to the Larmor time. The maximum Lorentz factor is thus

$$\gamma_{\min} = \left( \frac{2\pi q}{\sigma_T \epsilon B} \right)^{1/2} \sim 2 \times 10^8 \left( 3.6 \times 10^{11} \right) \alpha^{-1/2} E_{53}^{-1/16} n_0^{-3/16} t_{\text{day}}^{3/16} \epsilon^{-1/4}.$$

(6)

In principle, the coefficient $\alpha$ for electrons can differ from that of the protons. In our analysis, the only constraining one is that of the protons. For the parameters of the model presented here, a large magnetic field is required and therefore the electrons are in the fast-cooling regime with $\gamma_{\min} > \gamma_c$. The resulting electron distribution is a broken power law with index 2 between $\gamma_c$ and $\gamma_{\min}$ and index $p$ between $\gamma_{\min}$ and $\gamma_{\max}$. On the other hand, the protons are in the slow-cooling regime. Therefore, their distribution function is a single power law with an index $(p-1)$ between $\gamma_{p,\min}$ and $\gamma_{p,\max}$ (for the parameters we find, $\gamma_{p,\min} < \gamma_{p,c}$, see below).

The observed synchrotron spectra from these particle distributions have the broken-power-law shape with three characteristic frequencies (Blumenthal & Gould 1970; Gruzinov & Waxman 1999; Granot et al. 2000; Granot & Sari 2002). The observed characteristics spectral peak frequency of photons emitted by the particles at $\gamma_{\min}$ is

$$\nu_{\min} = \frac{3}{4\pi} \frac{qB}{mc^2} \gamma_{\min} \left( \frac{\Gamma}{1 + \epsilon} \right)$$

$$= \sim 0.124 \times 10^{-11} f(p)^2 E_{53}^{1/2} t_{\text{day}}^{-3/2} \epsilon_{\delta, -1} \epsilon_{\delta, -1} \left( B_0^{-2} \right) \text{eV}.$$

(7)

The cooling frequencies of the electrons and of the protons are

$$\nu_c = \frac{3}{4\pi} \frac{qB}{mc^2} \gamma_c \left( \frac{\Gamma}{1 + \epsilon} \right)$$

$$= \sim 2.19 \times 10^{-3} (0.55 \times 10^{13}) E_{53}^{-1/2} t_{\text{day}}^{-3/2} \epsilon_{\delta, -1} \epsilon_{\delta, -1} n_0^{-1} \text{keV}.$$

(8)

Note that the proton cooling frequency $\nu_{p,c}$ is $\sim 10^{16}$ times larger than the electron cooling frequency $\nu_{e,c}$. As a result of their higher mass, protons cool slower than electrons.

The maximum synchrotron frequencies for electrons and protons are

$$\nu_{p,e} = \frac{3}{4\pi} \frac{qB}{mc^2} \gamma_{p,e} \left( \frac{\Gamma}{1 + \epsilon} \right)$$

$$= \sim 1.65 \times 10^{-3} (2.89 \times 10^{13}) E_{53}^{-1/2} t_{\text{day}}^{-3/2} \epsilon_{\delta, -1} \epsilon_{\delta, -1} n_0^{-1} \text{MeV}.$$

(9)

Consequently, for fiducial values of the parameters, the highest energy of photons produced by the electron-synchrotron process is limited to less than or equal to a few gigaelectronvolts. Therefore, photons having energies much higher than that must have a different origin. From the previous equation, the cooling and injection frequencies are similar. However, a model in which proton synchrotron explains the teraelectron-volt observations requires a large magnetic field, which translates into a fast-cooling regime for the electron $\nu_{e,c} \ll \nu_{e,\min}$. However, for protons, $\nu_{p,c} \gg \nu_{p,\min}$ implying that the protons are in the slow-cooling regime, and lose their energy inefficiently. In fact, protons satisfy $\nu_{p,c} > \nu_{p,\max}$. Therefore, the proton synchrotron spectrum is a power law with a high-energy cutoff at frequency $\nu_{p,\max}$.

The maximum observed radiative power from a single particle (proton or electron) at observed frequency $\nu_{\text{obs}} = \nu_{\text{peak}}/(1 + z)$ is given by

$$P_{p,e,\max} = (1 + z)^2 \frac{m_e^2 c^2 \sigma_T B \Gamma}{9 q m_p}$$

(10)

and $P_{p,e,\max} = (m_p/m_e) P_{p,\max}$ (Rybicki & Lightman 1979; Sari et al. 1996, 1998).

$$F_{\nu,\text{peak}} = \frac{N_p P_{p,\max}}{4\pi d_L^2} = \sim 100(0.06) E_{53}^{1/2} t_{\text{day}}^{1/2} n_0^{1/2} d_{28}^{-2} \text{mJy}.$$

(11)
where \( N_e = (4\pi/3)n_e r^3 \) is the number of particles swept by the blast wave, which are actively radiating, where \( n_e = \xi n_e \). Here, \( d_L \) is the luminosity distance.

For particles accelerated to a power law, \( N(\gamma) d\gamma \propto \gamma^{-p} \) above \( \gamma_{\text{min}} \) and below \( \gamma_{\text{max}} \), the expected photon spectrum thus is a broken-power-law shape, with \( F_\nu \propto \nu^{4/3}, \nu^{-p_1/2}, \nu^{-p/2} \) for \( \nu < \nu_{\text{min}}, \nu_{\text{min}} < \nu < \nu_c, \nu_c < \nu < \nu_{\text{max}} \) in the slow-cooling regime and \( F_\nu \propto \nu^{4/3}, \nu^{-p_1/2}, \nu^{-p/2} \) for \( \nu < \nu_c, \nu_c < \nu < \nu_{\text{min}}, \nu_{\text{min}} < \nu < \nu_{\text{max}} \) in the fast-cooling regime (Rybička & Lightman 1979; Sari et al. 1998).

### 3. Model Constraints Derived from the Data Available on GRB 190114C

We proceed to interpret the available data of GRB 190114C within the framework of a hybrid model, for which the low-energy component is explained by synchrotron radiation from electrons while the high-energy teraelectronvolt component is required to be proton synchrotron. GRB 190114C was a long-GRB with prompt energy released, \( E_{\text{iso}} \approx 2.5 \times 10^{53} \text{ erg} \) (Ajello et al. 2020). It is seen at the redshift \( z = 0.4245 \), as identified by the Nordic Optical Telescope (Selsing et al. 2019) and further established by the Gran Telescopio Canarias (Castro-Tirado et al. 2019). Its prompt phase was recorded over an energy band of 8 keV–100 GeV by the Swift-Burst Alert Telescope (BAT; Gropp et al. 2019), the Gamma-ray Burst Monitor (GBM; Hamburg et al. 2019), and the Large Area Telescope (LAT; Kocevski et al. 2019). The reported duration is \( T_{90} = 25 \text{ s} \) (Acciari et al. 2019; Gropp et al. 2019), although the GBM Collaboration reported a duration of \( T_{90} = 116 \text{ s} \) (Hamburg et al. 2019). This longer duration is explained by the observation of a weak second emission episode after the initial signal. This episode was interpreted as emission from the afterglow (Ajello et al. 2020), and in this paper, we make the same assumption.

The afterglow follow-up observations were carried out by many instruments. As a result, a good temporal and multiwavelength data set exists during the early afterglow phase when the burst was bright enough in the teraelectronvolt band to allow the MAGIC instrument to measure the spectrum in five time bins; 68–110, 110–180, 180–360, 360–625 and 625–2400 s (Acciari et al. 2019). The first two time intervals have spectral data in the kiloelectronvolt to gigaelectronvolt band (XRT, GBM, LAT) along with data in the teraelectronvolt band from MAGIC. Inspection of the available data reveals that the afterglow spectra during the first two time bins centered at 90 and 120 s are characterized by two peaks, with the lowest peak at energy around 10 keV, which we attribute to synchrotron emission from electrons, and a second peak between the gigaelectronvolt and the teraelectronvolt bands, which we interpret as synchrotron emission from protons. In this section, we derive the relations between the model parameters to satisfy these two assumptions.

According to the XRT online repository (Evans et al. 2009), the X-ray spectrum of GRB 190114C is fitted by an absorbed power law, characterized by a photon index of 1.7 ± 0.04. With the XRT online tool (Evans et al. 2009), we checked that this holds in the first and second time bins independently. We find the photon index to be 1.68 ± 0.12 between 68 and 110 s, and 1.58 ± 0.11 between 110 and 180 s. This power law can naturally be produced in the electron-synchrotron model. Since for the fiducial values of the model parameters discussed herein (see the discussion above) electrons are expected to be in the fast-cooling regime, the spectral slope can be obtained in two cases. The first one corresponds to \( \nu_c < \nu_{\text{XRT}} < \nu_{\text{min}} \), producing a spectral index of \(-1.5\) comparable to the spectral index found in the XRT repository. In this case, the electron index cannot be determined and we can take \( p_e = 2.2 \). We discuss the impact of this assumption below. In the other case, \( \nu_c < \nu_{\text{min}} < \nu_{\text{XRT}} \), the electron index is deduced to be \( p_e = 1.2 \) as the observed photon spectral index is 1.6. It also requires \( \nu_{\text{max}} \) to be close to 10 keV to produce the first hump. However, the inspection of Equation (9) reveals that this case is difficult to achieve as it requires the observed time, \( t_{\text{day}} \) to be long (which is in contrast with the observation at hundreds of seconds) and a large \( \alpha \), meaning that the constraints on the particle acceleration mechanism are loose.

The available multiwavelength data are now used to constrain the free model parameters. For this purpose, we use the available data at the two time bins centered at 90 and 120 s. Ajello et al. (2020) reported a break at 4.72 keV (at 68–110 s) and 5.6 keV (at 110–180 s), and therefore we assume that the injection frequency \( \nu_{\text{inj}} \) is equal to 5.5 keV consistent with those findings. The large magnetic field requirement in our model implies that the cooling frequency \( \nu_{\text{cool}} \) to be below the XRT band, i.e., smaller than 0.3 keV. Using Equation (7) for the value of the injection frequency yields

\[
E_{53}^{1/2} \epsilon B_{-2}^{1/2} e^{-1/2} \epsilon_e^{-2} = \begin{cases} 
53.65 & \text{at } t = 90 \text{ s}, \\
82.60 & \text{at } t = 120 \text{ s}, 
\end{cases}
\]

while using Equation (8) for the cooling frequency gives

\[
E_{53}^{-1/2} / B_{-2}^{3/2} n_0^{-1} \leq \begin{cases} 
4.42 & \text{at } t = 90 \text{ s}, \\
5.10 & \text{at } t = 120 \text{ s}. 
\end{cases}
\]

The maximum observed photon energies, \( \sim 1 \text{ TeV} \) (90 and 120 s) (Acciari et al. 2019), can be used to set an estimation of the maximum frequencies of photons radiated from the protons, see Equation (9). Using these observed maximum energies at 90 and 120s, we constrain the parameters. We get

\[
\frac{1}{\alpha} \left( \frac{E_{53}}{n_0} \right)^{1/8} \leq \begin{cases} 
2.63 \times 10^{-2} & \text{at } t = 90 \text{ s}, \\
2.93 \times 10^{-2} & \text{at } t = 120 \text{ s}. 
\end{cases}
\]

We now use the specific fluxes at \( \nu_{\text{min}} \) and \( \nu_{p,\text{max}} \) to constrain the parameters. We first assume a power-law index \( p = 2.2 \) for both electrons and protons, the fluxes corresponding to the observed energies at the time period of 90 s (\( E_{\text{iso}} = 5.3 \text{ keV} \approx 2.56 \times 10^{26} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \), and \( E_{\text{iso}} = 0.23 \text{ TeV} \approx 9.5 \times 10^{-34} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \)) and 120 s (\( E_{\text{iso}} = 5.5 \text{ keV} \approx 1.41 \times 10^{26} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \), and \( E_{\text{iso}} = 0.23 \text{ TeV} \approx 4.15 \times 10^{-34} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \)), and we can explain using the synchrotron fluxes \( F_\nu \) calculated from Equations (7), (8), and (11). Further using Equation (12) provides restrictions on the parameter space as follows:

\[
E_{53}^{3/4} / B_{-2}^{1/2} n_0^{1/4} \epsilon_e \sim \begin{cases} 
0.21 & \text{for } t = 90 \text{ s}, \\
0.12 & \text{for } t = 120 \text{ s}. 
\end{cases}
\]

and

\[
E_{53}^{13/10} / B_{-2}^{5/2} n_0^{1/2} / p_{-1}^{1/5} \epsilon_e^{-1/5} \sim \begin{cases} 
2.4 \times 10^{5} & \text{for } t = 90 \text{ s}, \\
1.3 \times 10^{5} & \text{for } t = 120 \text{ s}. 
\end{cases}
\]

Equation (15) is derived from the relation \( F_{\nu} = F_{\nu, \text{peak}} (\nu / \nu_{\text{min}})^{1/2} (\nu / \nu_{\text{e, min}})^{-p/2} \) for the synchrotron
emission of the fast-cooling electrons and Equation (16) is obtained from \( F_p = F_{v,\text{peak}} (\nu/\nu_{p,\text{min}})^{(p-1)/2} \) relevant for the synchrotron emission from the protons.

Using those two last equations and assuming the value \( p = 2.2 \), one can express \( \epsilon_B \) and \( \xi_e \) as

\[
\epsilon_{B, -2} = \begin{cases} 
5.3 \times 10^6 \, e^{1/4} \, E_{53}^{-13/8} \, e^{-3/2} \, n_0^{-5/8} & \text{for } t = 90 \, \text{s} \\
2.5 \times 10^6 \, e^{-2} \, E_{53}^{-1} \, n_0 & \text{for } t = 120 \, \text{s} 
\end{cases}
\]

and for \( \xi_e \):

\[
\xi_e = \begin{cases} 
10.08 \, e^{1/6} \, E_{53}^{-37/32} \, e^{-3/8} \, n_0^{-5/32} & \text{for } t = 90 \, \text{s} \\
4.76 \, e^{1/6} \, E_{53}^{-37/32} \, e^{-3/8} \, n_0^{-5/32} & \text{for } t = 120 \, \text{s} 
\end{cases}
\]

We further use Equations (17) and (18) with Equation (12) to obtain the value of \( \epsilon_e \):

\[
\epsilon_{e, -1} = \begin{cases} 
1.54 \, E_{53}^{-1} & \text{for } t = 90 \, \text{s} \\
1.09 \, E_{53}^{-1} & \text{for } t = 120 \, \text{s} 
\end{cases}
\]

In our analysis below, we also use the physical condition that the electrons, protons, and magnetic field energy are all obtained from the post-shock thermal energy, namely, \( \epsilon_B + \epsilon_e + \epsilon_p \leq 1 \).

Inspection of those equations reveals that the time bin centered at 90 s gives adequate parameter magnitudes. Therefore, hereinafter we will use the constraints obtained from this time bin only and we will show that the time evolution can be well reproduced by the afterglow dynamics alone.

It is clear that satisfying the condition \( \epsilon_B < 1 \) requires a combination of (i) a small \( \xi_p \), i.e., a small fraction of protons being accelerated into a power law, (ii) a large total kinetic energy \( E_{53} \), (iii) a large circumburst medium density \( n_0 \), and (iv) a large (but smaller than the unity) \( \epsilon_p \), i.e., a large fraction of energy given to accelerated protons. We note that this combination of parameters also leads to a small value of \( \xi_e \), namely, only a small fraction of the electron population is accelerated to a power law.

We show the constraints on the parameters in Figure 1, which displays the value of \( \epsilon_B \) as a function of \( n, E_{53} \), and \( \alpha \). Inspection of Equation (12) reveals that the dependence of \( \epsilon_e \) on \( E_{53} \), \( n_0 \), and \( \epsilon_p \) implies that the condition \( \epsilon_e < 1 \) is automatically satisfied when satisfying the limitation given by Equation (17). Finally, inspecting Equation (14), it becomes apparent that the dependence on \( E_{53} \) and \( n_0 \) requires \( \alpha \) to not be too large, with \( \alpha \sim 40 \) owing to the weak dependence on \( E_{53} \) and \( n_0 \).

### 4. Results: Explaining the Teraelectronvolt Observation of GRB 190114C with Proton Synchrotron

In this section, we present several solutions for a proton synchrotron model to explain the teraelectronvolt emission of GRB 190114C. In particular, we consider three assumptions based on the uncertainty of the particle acceleration process: (i) similar particle spectral indices, \( p_e = p_p \), and a similar fraction of accelerated particles, \( \xi_e = \xi_p \), i.e, the acceleration process is similar for protons and electrons in terms of the number of accelerated particles and the obtained spectral shape. (ii) Similar particle spectral indices, \( p_e = p_p \) but \( \xi_e \neq \xi_p \), namely, the acceleration process accelerates different proportions of electrons and protons, but produces a similar spectral shape. And (iii) different spectral indices, \( p_e \neq p_p \) with \( \xi_e = \xi_p \), meaning that the acceleration process produces a different spectral shape. We find that the effect of \( \xi_p \) on the resulting spectra is not significant; therefore, we did not have to assume an extra degree of freedom (see below).\(^5\)

The results are presented in Figures 2–4, respectively. The data are extracted from Acciari et al. (2019) and are presented here for convenience. In particular, we did not aim at producing the statistical best fit to the data, but only to demonstrate the ability of our model to reproduce the fluxes and characteristic breaks in the XRT and MAGIC bands. This allows us to constrain the values of the free parameters of our model.

#### 4.1. Case (i): \( \xi_e = \xi_p \)

We first examine the assumption of similarity between the injection fraction of electrons and protons in the acceleration process. We impose \( \xi_e = \xi_p \) in Equation (18) for \( t = 90 \, \text{s} \), which results in

\[
\xi_e = 11.76 \, E_{53}^{-37/30} \, e^{-6/15} \, n_0^{-1/6}.
\]

Also in order for \( \epsilon_B \) to be smaller than unity, the kinetic energy is required to be greater than the observed released prompt energy \( E_{\text{iso}} \) as seen explicitly from Equation (17). Here, we choose \( E = 4 \times 10^{54} \, \text{erg} \), \( \epsilon_p = 0.8 \), and ambient density \( n = 80 \, \text{cm}^{-3} \), then Equation (20) yields \( \xi_e = \xi_p = 0.026 \), for which \( \epsilon_B = 0.152 \) and \( \epsilon_e = 3.85 \times 10^{-3} \) and so the relation \( \epsilon_B + \epsilon_e + \epsilon_p \leq 1 \) is satisfied. For this set of parameters, only 2.6% of the electrons and protons injected attain the power-law

---

\(^5\) Since we normalize the flux to the observed flux, reducing \( \xi_p \) enforces an increase in the values of other free model parameters, such as the magnetization.
distributions behind the shock front and the rest assume a thermal distribution with temperature lower than $\gamma_{\text{min}}$. We do not attempt to model the radiation from these thermal particles, as their contribution is below the observed band. Equation (14) constrains the numerical coefficient that determines the acceleration efficiency to be $\alpha = 35$. Above $\gamma_{\text{p, max}}$, we assume an exponential cutoff representing the inability of the acceleration process to accelerate protons to energies above $\gamma_{\text{p, max}}$ (see the dashed line in Figure 2). For this set of parameters, the spectra obtained in the time bins centered at 90 and 120 s are displayed in Figure 2. This set of parameters results in spectra and flux consistent with the observed data in both time intervals. The large value of $E$ implies that the efficiency $\eta = E_{\text{iso}}/(E + E_{\text{iso}})$ of the prompt emission $\eta \approx 5.9\%$ is low, but not extremely low. Similarly, the required energy is large, but acceptable. We comment on the low efficiency in the discussion below in Section 6.

4.2. Case (ii): $\xi_e = \xi_p$

We next examine a model in which the electrons and protons are accelerated in different numbers, $\xi_e = \xi_p$. The parameters we consider, $E_{53} = 50$, $n_0 = 130$, $\epsilon_p = 0.8$, $\alpha = 34$, and $\epsilon_e = 0.2$ provides adequate spectra that are consistent with the observed data at both 90 and 120 s. Using Equation (17) we obtain $\epsilon_B = 0.13$, and from Equation (18) one finds $\xi_e = 0.02$ and $\epsilon_e = 3.08 \times 10^{-3}$.

Our results are presented in Figure 3 for the time intervals centered at 90 and 120 s. The parameters of this solution are nearly the same as the ones obtained for the case $\xi_e = \xi_p$. This is because the protons dominate the energetic requirements. In particular, it means that the total kinetic energy of the blast wave has to be large, and correspondingly, the efficiency is low. In this case, it is $\eta \approx 4.8\%$. Due to the large energy, the interstellar medium density is also large. Again we point out that all the values we obtain are in the range with estimates and uncertainties of GRB energetics and ambient densities.

4.3. Case (iii): $\xi_e \equiv \xi_p$ and $p_e \neq p_p$

The critical parameter that determines the energetic budget is the proton injection index $p_p$. We have assumed the electron index to be 2.2 in agreement with theory (Sari et al. 1998) and observations (Evans et al. 2009). The proton index was then assumed to be equal to the electron index. Here, we relax this assumption. Since the teraelectronvolt flux is modeled by emission from protons at $\gamma_{\text{p, max}}$, a high value of $p_p$ would require a higher energy budget, and lower efficiency. We, therefore, consider $p_p = 2.1 < p_e$ in order to decrease the energy budget and increase the efficiency. The solution we present here is given by the parameters $n = 15 \text{ cm}^{-3}$, $E = 3 \times 10^{54} \text{ erg}$, $\epsilon_p = 0.8$. This gives $\alpha = 41.45$, $\epsilon_e = 4.7 \times 10^{-3}$, $\epsilon_B = 0.12$, and $\xi_e = \xi_p = 0.023$. The resulting spectrum in the two time bins centered at 90 and 120 s is shown in Figure 4.

A consequence of the assumptions used here is that the prompt phase efficiency is increased to 7.7%, which is higher than in the two previous cases. Furthermore, the density of the interstellar medium is substantially lower than the value obtained in our previous scenarios.

In addition to the data in the two time bins $[T_0 + 68 - T_0 + 110]$ and $[T_0 + 110 - T_0 + 180]$, MAGIC data exists for three other time bins, namely, 180–360, 360–625, and 625–2400 s; see Acciari et al. (2019). As a consistency check of our model, we examined the evolution of the proton-synchrotron component with time, using the same physical parameters, and the self-similar solution to obtain the dynamical evolution of the Lorentz factor, energy density, and magnetic field, from which the evolution of the flux and characteristic frequencies are readily obtained. The resulting synchrotron emission from the proton component at later times is presented in Figure 5 alongside the MAGIC data. Further, the multi-band light curve of the model in comparison with the observed data of GRB 190114C is shown in Figure 6. Here, the emission in the X-ray and gigaelectronvolt bands are obtained for the electron-synchrotron process (which operates in the fast-cooling regime) while the emission in the MAGIC band is explained by the proton-synchrotron process (in the slow-cooling regime). The similarity between the model results and the data is another independent indication of the ability of our
5. Additional Radiative Processes

5.1. IC Scattering

The high-energy emission of several GRBs has been interpreted as originating from the SSC process (Dermer et al. 2000; Sari & Esin 2001; Nakar et al. 2009; Liu et al. 2013; Derishev & Piran 2016; Fraija et al. 2019; Derishev & Piran 2021). This process considers the IC interaction of synchrotron photons with the electrons that emitted them. Indeed, it is possible to interpret the observed teraelectronvolt data using electron-IC rather than proton synchrotron, if one assumes different values of the free model parameters, as follows.

The break frequencies of the electron-IC component can be obtained within the Thompson regime, since Klein–Nishina corrections are important only when $\gamma e B^{-1} < \Gamma m_e c^2$ see Sari & Esin (2001). The $\nu F_\nu$ spectral peak is achieved at a frequency of $\nu_{\text{IC,min}} = 2\gamma_{\text{min}}^2 \nu_{\text{e,min}}$ or $\nu_{\text{IC,c}} = 2\gamma_{\text{c}}^2 \nu_{\text{e,c}}$ in the slow- and fast-cooling regimes, respectively. For the asserted model specifications (Section 2.2), these frequencies are defined as

$$h\nu_{\text{IC,min}} = 7.24 \times 10^5 f(p)^2 E_{53}^{3/4} n_0^{-1/4} \nu_{\text{day}}^{-9/4} \epsilon_{\text{B,-2}}^{1/2} x \xi_{\epsilon}^{-4} \xi_{\epsilon_{-1}}^{-1} \text{eV},$$

and

$$h\nu_{\text{IC,c}} = 2.28 \times 10^8 E_{53}^{-5/4} n_0^{-9/4} \nu_{\text{day}}^{-1/4} \epsilon_{\text{B,-2}}^{-7/2} \text{eV}.$$  \hspace*{1cm} (22)

The peak flux of the electron-IC spectrum is estimated as (Sari & Esin 2001)

$$F_{\nu,\text{peak}} = \frac{1}{3} \sigma_T n_e r F_{\nu,\text{peak}} \approx 1.5 \times 10^{-5} \xi_{\epsilon}^2 E_{53}^{5/4} n_0^{5/4} \nu_{\text{day}}^{1/4} D_{28}^{-2} \epsilon_{\text{B,-2}}^{1/2} \text{mJy}.$$  \hspace*{1cm} (23)

We can now define the condition for which the proton synchrotron component overcomes the electron-IC component (at $t = 90$ s) as follows:

$$\frac{F_{p,\nu}}{F_{\nu,\text{IC}}} = 1.64 \times 10^{-6} E_{53}^{0.225} \epsilon_{\text{B,-2}}^{1.75} \epsilon_{p,-1}^{1.2} n_0^{0.525} \xi_{\epsilon}^{-0.4} \xi_{\epsilon_{-1}}^{-2.4} \epsilon_{\nu}^{-0.2} > 1.$$  \hspace*{1cm} (24)

In Figure 7, we show the regions in which proton synchrotron or electron-IC dominates the spectrum as a function of $\epsilon_{\nu}$ and $\epsilon_{\text{B}}$. The conditions given by Equation (24) are shown by the solid lines.

Equating the electron-IC flux to the MAGIC flux at 0.23 TeV gives the following constraints on the parameters of this model:

$$\frac{\xi_{\epsilon}^2 E_{53}^{5/4} n_0^{5/4} \nu_{\text{day}}^{1/4} D_{28}^{-2} \epsilon_{\text{B,-2}}^{1/2}}{\epsilon_{\nu}} = 0.236,$$  \hspace*{1cm} (25)

from which we get

$$\epsilon_{\text{B,-2}} = 3.17 E_{53}^{-5/4} n_0^{5/4} \nu_{\text{day}}^{1/4} D_{28}^{-2} \epsilon_{\nu}.$$  \hspace*{1cm} (26)

This shows that an electron-IC model for explaining the MAGIC observations requires a small $\epsilon_{\text{B}}$, in contrast to a proton synchrotron model.

Also as depicted in Figure 7, it is evident that the proton-synchrotron process gains significance when $\epsilon_{\nu}$ is large.

However, within the framework of the proton-synchrotron model, for the parameters we considered to explain the data of GRB 190114C in Section 4.3, the energy peak flux of the electron-IC spectrum is around $\sim 40$ TeV (90 s) and $\sim 21$ TeV (120 s) with the specific flux of $\sim 8.3 \times 10^{-10}$ erg cm$^{-2}$ (90 s) and $\sim 6.2 \times 10^{-10}$ erg cm$^{-2}$ (120 s), respectively. The expected contribution from the electron-IC scattering process is shown in Figures 2–4, corresponding to the three models we presented. These figures show that the electron-IC component is always subdominant with respect to the proton synchrotron component in the teraelectronvolt band. This was expected since our model requires $\epsilon_{\text{B}} \gg \epsilon_{\nu}$, resulting in a strong suppression of the SSC component. Thus, it states that our model presents an alternative to the electron-IC component models, as suggested by Derishev & Piran (2021).
The Astrophysical Journal, 955:70 (13pp), 2023 September 20

Another mechanism to produce photons with energy around 1 TeV is photon–pion interactions and their subsequent electromagnetic pair cascade. Pions (π⁰ and π⁻) originating from the collisions of the high-energy protons and the seed synchrotron photon field emitted from the electrons, produce gamma-ray photons accompanied by neutrinos, p⁺⁺γ → π⁰ + p⁺ and p⁺⁺γ → π⁺ + n. Neutral pions quickly decay to photons, π⁰ → γ + γ, and charged pions decay to positrons and neutrinos in the decay chain (π⁻ → μ⁻ + νμ, μ⁻ → e⁺ + νe + η). High-energy electrons and positrons (e⁺) generated from the muon (π⁻) decay as well as muons and charged pions can emit a substantial amount of synchrotron radiation. We seek here to compute the relevance of this process.

In the Appendix, we derive the expression for the cooling time by photodisruptive interaction, given by Equation (A16). This time is compared to the cooling time by synchrotron given by Equation (4):

\[
\frac{t_{\text{synch}}}{t_p} \sim 2.9 \times 10^{-8} \left( \frac{p-2}{p-1} \right) \times E_{53}^{8/15} B_{\nu_0}^{11/32} \epsilon_p^{15/8} n_0^{37/32} \eta e^{-5/16}, \tag{27}
\]

where we also use the constraints on \( \epsilon_p, \epsilon_n, \) and \( \xi_e \) given by Equations (17)–(19). Therefore, for the protons, the synchrotron cooling rate is orders of magnitude faster than the photodisruptive interaction, and most of the energy radiated by the protons is done via synchrotron radiation. Direct production of photons at 1 TeV by \( \gamma_0 \) decay is also subdominant since the energy deposition rate is dominated by the highest proton energy for our choice of proton power-law index. In other words, the contribution from the hadronic cascade in our scenario can be neglected and no modification of the teraelectronvolt component is expected. This also means that, for our model, the neutrinos’ fluence at peetaelectronvolt energy is expected to be small, challenging observations and constraints by IceCube and future neutrino instruments.

5.2. Photooion Production

As we show here, our proton synchrotron model is able to explain the VHE emission of GRB 190114C. Within the framework of this model, we find that the prompt phase signal has an energy conversion efficiency of a few percent. Indeed, our most favorable model with \( \xi_e = \xi_p \) and \( p_\nu = p_p \) requires a blast wave with energy \( 3 \times 10^{54} \text{erg} \), resulting in a prompt efficiency of around 8%. The other models we considered all have a few percent efficiency as well. Interestingly, this efficiency is comparable to the radiative efficiency predicted in the internal shock scenario used by many authors to explain the prompt phase; see, e.g., Kobayashi et al. (1997), Panaitescu et al. (1999), and Guetta et al. (2001).

At first, our model requirements on the efficiency seem inconsistent with previous findings, specifically for the burst observed by Fermi-LAT for which a high (around 50%) prompt radiative efficiency is usually determined, e.g., Cenko et al. (2011). This trend seems to also be retrieved for bursts observed by the Neils Gherel Observatory (Cenko et al. 2010), as well as older bursts (Yost et al. 2003). However, we note the following:

(i) The analysis presented in those papers heavily relies on the numerical coefficients chosen for the emission process. Using updated coefficients, Fan & Piran (2006) reevaluated the efficiency of several bursts, finding it to be lower, in the order of a few percent, so of similar magnitude to the requirements from our model.

(ii) Their analysis also further relies on assuming that all the electrons participate in the radiative process, namely, \( \xi_e = 1 \), resulting in an under-evaluation of the kinetic energy of the blast wave compared to the case \( \xi_e < 1 \) as can be seen by rearranging Equation (18),

\[
E_{53} \approx 7.38 \epsilon_\pi^{-1/2} \epsilon_{p_\nu}^{-1/2} n_0^{-1/2} \eta_e^{-1/4} \xi_e^{-1/4}, \tag{28}
\]

for the parameters that satisfy the relation (17) at \( t = 90 \text{s} \). Note that our proton synchrotron model forbids \( \xi_e = 1 \), as it would necessarily result in \( \epsilon_p > 1 \). Instead, our model requires \( \xi_e \) to be in the order of \( 10^{-2} \). Recent studies
analyzed the spectral effect of a thermal population resulting in an incomplete acceleration of particles, see, e.g., Warren et al. (2018, 2022). Yet, it remains to be understood how the modification to the SED impacts the recovery of the blast-wave parameters. Thus, one can conclude that currently there is still no reliable measurement of a high efficiency during GRB prompt emission.

6.2. Constraints on the Electron Injection Fraction

The strong constraint on the magnetic field equipartition parameter $e_B$, requires that the number of electrons participating in the radiation process be smaller than unity, of the order of $10^{-2}$, weakly sensitive to all parameters but the kinetic energy of the blast wave, $E_{53}$. The $5.5$ keV break and the low-energy slope below it, suggest that the thermal component made of the bulk of the electrons, should have a temperature much smaller than $\gamma_{\text{min}}$. If this was not the case, the low-energy slope below $\nu_{\text{min}}$ would be different and entailed the exact injection function.

In other words, we require an acceleration scenario in which the thermal component and accelerated particles are two clearly separate entities. Such an injection function was originally proposed by Eichler & Waxman (2005). In that paper, they also studied the effect of this assumption on the emission properties of GRB afterglow. In particular, they found that such a model can be constrained by early afterglow emission, underlying the necessity of multiwavelength observations. Multiwavelength observations of GRB 190114C afterglow, which include the temporal evolution in the radio and optical bands, was reported by Misra et al. (2021). And indeed, the constraint upon the parameter $e_c$ at a time interval of 65 s reported in that work, resulted in an electron injection fraction of 2%, which is comparable to the fraction $\xi_c$ obtained in our model. These consistent results, therefore, serve as independent support of our model.

The presence of a dominant (in number) thermal electron population would leave a visible footprint in the spectrum at low energies. In our analysis, since we require $\nu_{\text{min}} < \nu_{\text{XRT}}$ such a footprint could be found in the optical and radio bands. The emission of those thermal electrons is usually neglected (e.g., Misra et al. 2021). Such a hypothesis could in principle be tested and its parameters constrained by early X-ray and megaelectronvolt observations in long GRBs with duration >100 s. To the best of our knowledge, this has not been done yet.

6.3. Constrain on the Jet Energetic from the Jet Opening Angle

The X-ray light curve of GRB 190114C is well described with a power-law decay of the flux until at least $10^6$ s. From the derived model parameters, we can therefore infer a lower limit on the jet opening angle and subsequently on the jet energy. The opening angle ($\theta$) of the jet depends upon three quantities, which are the isotropic-equivalent kinetic energy, $E_k$, the jet break time, $t_{\text{break}}$, and the ambient number density, $n$ (Levinson & Eichler 2005). From our model (case iii), we derived $E_k = 30 \times 10^{53}$ erg and $n = 15 \text{ cm}^{-3}$. Hence, a lower limit on $\theta$ is given by

$$\theta > 7.2 \times 10^2 \left( \frac{t_{\text{break}}}{1 + z} \right)^{3/8} \left( \frac{n}{E_k} \right)^{1/8} \approx 0.24 \text{ rad.}$$

(29)

Therefore, the collimation-corrected energy of the jet is

$$E_{\text{jet}} = \frac{\theta^2}{2} E_k \approx 8.85 \times 10^{52} \text{ erg.}$$

(30)

For case (ii), when $E_k = 50 \times 10^{53}$ erg and $n = 130 \text{ cm}^{-3}$, we get $\theta > 0.3 \text{ rad}$ and $E_{\text{jet}} \gtrsim 2.23 \times 10^{53}$ erg. In the same way, for case (i) we derived $E_k = 40 \times 10^{53}$ erg and $n = 80 \text{ cm}^{-3}$, hence $\theta > 0.3 \text{ rad}$ and $E_{\text{jet}} \gtrsim 1.67 \times 10^{53}$ erg.

The obtained jet energy appears to be large when compared to other estimates that find a typical energy output in the order of $10^{51}$ erg (Beniamini et al. 2015; Wang et al. 2015, 2018). We note however that these estimates rely on afterglow modeling with the assumption $\xi_c = 1$, while the total energy of the burst scales $\propto \xi_c^{-1}$. On the other hand, relativistic jets from solar mass black holes (microquasars) transfer a significant amount of their kinetic energy, estimated to be $\sim 10^{51}$ erg, to the surrounding ambient medium (Dubner et al. 1998; Mirabel 2003). Our model, therefore, requires jet energy to be larger by a factor approximately equal to tens to hundreds than the energy released by microquasars jets.

6.4. Production of High-energy Protons

To explain the MAGIC observations with proton synchrotron, protons have to be accelerated to comoving energies around $10^{19}$ eV, which results in observed particle energies around $10^{19}$ eV. This requirement would make GRBs able to accelerate the highest-energy ultra-high-energy cosmic rays (UHECR) observed on Earth (though most of them would not

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6 The XRT light-curve repository reports a change of slope around $t = 5 \times 10^6$. Using this time as the jet break would result in lowering the energy inferred below by about 1 order of magnitude.
be detected due to the GZK cutoff). It is well known that GRBs satisfy the Hillas criterion (Hillas 1984) making them a plausible source of UHECRs. We further note that the acceleration mechanisms must result in a proton power-law distribution function formed over many orders of magnitude in energies.

If such a challenging acceleration can indeed take place or not is not certain. Sironi et al. (2013) performed particle-in-cell (PIC) simulations of particle acceleration in collisionless plasma and estimated the maximum proton Lorentz factor for an external shock in the context of GRBs. They found that this maximum Lorentz factor is $\gamma_{\text{max}} \approx 10^8$, weakly dependent on the parameters. This is three orders of magnitude lower than the proton Lorentz factor required by our parameters. This result is however based on the scaling law $\gamma_{\text{max}} \propto t^{1/2}$. More recent results tentatively obtained the scaling law $\gamma_{\text{max}} \propto t$ (Huang et al. 2023). For this temporal scaling, the production of the highest energy protons required by our model is possible.

7. Conclusion

The broken-power-law spectrum of the electron-synchrotron emission model is a standard prediction of GRB afterglow theories, which was successful in explaining the spectral shape at energy lower than a few mega-electronvolts. Yet, the source of the VHE afterglow component is still being investigated. GRB 190114C is one such burst with a high-energy ($0.2 \sim 1$ TeV) peaking afterglow spectral component as reported by MAGIC within the epoch of 60–2400 s. We argue here that the source of the VHE segment is of proton synchrotron origin, while the sub-mega-electronvolt component is explained by electron-synchrotron radiation within the framework of the classical fireball evolution scenario. According to the model discussed in this paper, protons are simultaneously accelerated with the electrons in the blast wave, allowing protons to radiate in the (sub-)tera-electronvolt band.

We provided the constraints that this model must satisfy under different conditions for the parameters describing the uncertainty of the particle acceleration process. We presented three models with different injection parameters characterizing the particle acceleration and injection: (i) our first model has similar fractions of electrons and protons accelerated to a high-energy power law, $\xi_e = \xi_p = 2.6 \times 10^{-2}$, (ii) the second one has $\xi_e = \xi_p$, and (iii) the last one has $\xi_e = \xi_p = 2.3 \times 10^{-2}$, but (slightly) different power-law indices, $p_e = 2.2$ and $p_p = 2.1$, that is to say, electrons and protons are described by a different injection index. All the scenarios in our model exhibit external medium density $n \sim 10–100$ cm$^{-3}$, which is somewhat higher than the fiducial value often assumed in explaining many GRB afterglows, $n \approx 1$ cm$^{-3}$, but not unreasonably high. In fact, a similar value was inferred from fitting the much later time radio and optical afterglow data (Misra et al. 2021). Similarly, a reasonably high blast-wave kinetic energy ($E \approx 3 \times 10^{54}$ erg), which gives the inferred energy from the observed photons of $\approx 2.5 \times 10^{53}$ erg during the prompt phase, leads to a considerable prompt radiative efficiency of $\approx 8\%$. We point out that this value of efficiency is similar to the efficiency expected in kinetic energy conversion by internal shocks. Finally, we note the degeneracy in determining the values of the explosion energy and the ambient density: for a given observed flux, a low ambient density can be compensated by a higher explosion energy for a fixed magnetization, $\epsilon_B$, thereby reducing the radiative efficiency of the prompt phase.

Misra et al. (2021) analyzed and interpreted the radio, optical, and X-ray data of GRB 190114C at both early and later times. Very interestingly, they concluded that the kinetic energy $E$ was at least one order of magnitude higher than the observed isotropic equivalent energy. This result is in excellent agreement with the constraints on the energy we obtained here. Further examination of our model requirements conveyed low requirements on the normalization of the acceleration rate with $\alpha > 10$. In addition, we find that in order to reproduce the tera-electronvolt band data via the proton synchrotron process, the fractions of both electrons and protons accelerated to a high-energy power law, $\xi_e$ and $\xi_p$, are at the order of few percent. It also requires a large magnetic field; hence, a large kinetic energy, from which, using the observed data at the Swift-XRT band, a constraint on the fraction of electrons accelerated, $\xi_e$ is imposed as well.
The injection fractions we derive, of a few percent, are similar to the ones derived using the radio, optical, and X-ray observational data in Misra et al. (2021). Furthermore, these values are consistent with the theoretical values found in PIC simulations of particle acceleration in relativistic shock waves (Spitkovsky 2008). Using the same values, we showed in Figure 5 that the proton-synchrotron mechanism is in agreement with the MAGIC data at later times as well, until the last existing MAGIC observation at 2400 s. Importantly, we showed that for the constraints set by the data on the model presented here, the proton synchrotron emission is dominant at the teraelectronvolt band over other emission processes such as SSC and photopion production mechanism. This makes this model a viable alternative to leptonic models.

Our model also suffers from a certain number of limitations. First, it is not clear if protons can be accelerated to the energies ($\sim 10^{20}$ eV in the observer frame), which are required to explain the teraelectronvolt emission. Second, the beaming-corrected energy deduced for this burst is one to two orders of magnitude larger than the energy inferred for GRBs, since protons energy density and magnetic field energy density are required to be large. We however point out that those estimates are model dependent and rely on the assumption $\xi_e = 1$. Finally, we discarded the radiation from the thermal electron, which should emit a synchrotron component around the optical band. SSC models do not suffer from these drawbacks but instead are challenged by the broadband modeling specifically in the optical (for a thorough summary, see Section 4.3 in Miceli & Nava 2022).

To conclude, we presented a proton synchrotron model to explain the VHE afterglow spectrum of GRB 190114C and found that it can accommodate the available observed MAGIC data set from the epoch of 68 s up until 2400 s. The parameters we found are consistent with both independent measurements, as well as theoretical predictions of particle acceleration. Furthermore, the uncertain values of the explosion energy and ambient density, are within one to two orders of magnitude from fiducial values often assumed in the literature. These results, therefore, point to the need for a more thorough investigation of the role of proton-synchrotron emission during the afterglow phase in GRBs.

The VHE band limit of the GRB afterglows is anticipated to reach higher magnitudes with CTA in the near future. Therefore, highlighting the role played by highly relativistic protons in the GRB afterglow theory is of crucial importance. Conversely, the acceleration mechanism and radiative efficiency of protons are expected to be further constrained by the observation of VHE emission ($\sim 1$ TeV) in the afterglow spectrum.

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**Appendix**

**Contribution of Photopion Production**

In this appendix, we compute the cooling rate by photopion interaction for parameters relevant to the proton synchrotron model presented herein. Since at frequencies larger than $\nu_{\text{min}}$, the photon numbers from the electron-synchrotron component fall off rapidly, we neglect their contribution to the photopionic interaction. Therefore, we compute the Lorentz factor $\gamma_p^{\text{th}}$ of the protons interacting with electron synchrotrons at the comoving frequency of the peak flux, $\nu_p^{\text{th}} = \nu_{\text{min}}/T$, where $\nu_{\text{min}} = 5.5$ keV, by using the threshold condition of photopionic interactions $2\gamma_p^{\text{th}}\nu_p^{\text{th}} = 132$ MeV:

$$\gamma_p^{\text{th}} = 1.2 \times 10^5 E_{53}^{1/2} n_0^{-1/2} t_{\text{day}}^{-1/2}.$$  \hspace{1cm} (A1)

Comparing the threshold Lorentz factor to the maximum proton Lorentz factor given in Equation (6), it is clear that $\gamma_p^{\text{th}} < \gamma_{p,\text{max}}$, meaning that the protons at $\gamma_{p,\text{max}}$ producing the teraelectronvolt component can also interact with low-energy
photons produced by electron synchrotron. Therefore, we now estimate the efficiency of the photohadronic process.

The proton cooling rate for the photopion production is given by Mannheim & Schlickeiser (1994), see also Begelman et al. (1990) and Waxman & Bahcall (1997) and reads as

\[
t_{\text{pp}}^{-1} = \frac{c}{2\gamma_p^2} \sigma_{\text{pp}}(\tilde{\epsilon}, \gamma_p) \int_{\tilde{\epsilon}_a}^{\infty} \frac{d\tilde{\epsilon}}{\tilde{\epsilon}} n(x) \frac{dx}{x^2}.
\]  
(2)

Here, \(x\) is the photon energy in units of the electron rest mass energy and \(n_x\) is the comoving photon spectral number density produced by the electron-synchrotron process. Since electrons are fast cooling in our model (the magnetic field needs to be large), we obtain \(n_x\) as (Sari et al. 1998)

\[
n_x = \frac{n_{\text{synch}}}{\hbar \omega} \begin{cases} \left( \frac{x}{x_c} \right)^\frac{1}{2} & x < x_c, \\ \left( \frac{x}{x_c} \right)^\frac{1}{2} - \left( \frac{x}{x_m} \right)^\frac{1}{2} & x_c < x < x_m, \\ \left( \frac{x}{x_m} \right)^\frac{1}{2} - \left( \frac{x}{x} \right)^\frac{1}{2} & x_m < x, \end{cases}
\]  
(3)

where

\[
u_{\text{synch}} \sim \frac{n_{\text{synch}}(\gamma_{p,e})}{\nu_{e,c}} I_{\text{dyn}} = 1.86 \times 10^{-15} E^{\frac{3}{5}}_{53} \epsilon_{B, -2} n_0 \frac{\text{erg cm}^{-3} \text{Hz}^{-1}}{\text{Hz}^{-1}}.
\]  
(4)

To simplify analytically the expression given in Equation (2), we follow Petropoulou & Mastichiadis (2015) and set

\[
\sigma_{\text{pp}} = \sigma_0 (\tilde{\epsilon} - \tilde{\epsilon}_{\text{th}}),
\]  
(5)

\[
\sigma_0 = 1.5 \times 10^{-4} \sigma_T,
\]  
(6)

\[
K_p = 0.2,
\]  
(7)

\[
\tilde{\epsilon}_{\text{th}} = 145 \text{ MeV},
\]  
(8)

where \(H\) is the Heaviside function. We simplify this expression for protons with Lorentz factor \(\gamma_{p,\text{max}}\) which are the ones responsible for producing the spectral component in the teraelectronvolt band. Moreover, for most of the parameter space, we have

\[
2\gamma_{p,\text{max}} \nu_{e,c} = 2.3 \times 10^{-8} E_{53}^{\frac{3}{5}} \epsilon_{B, -2} n_0 \text{day} \nu_{e,c} \approx 132 \text{ MeV},
\]  
(9)

where we used Equations (17)–(19). In this equation, \(\nu_{e,c} = \nu_{e,c}/T\) is the comoving cooling frequency. Therefore, the main contribution to the integral in Equation (2) is for photons with frequency between \(\tilde{\epsilon}_{\text{th}}\) and \(\nu_{e,c}\). We now turn to the computation of the integral

\[
\frac{2\gamma_p^2}{cK_p \sigma_0} t_{\text{pp}}^{-1} = I_2 + I_3,
\]  
(A10)

where

\[
I_2 = \int_{\tilde{\epsilon}_a}^{\infty} \frac{d\tilde{\epsilon}}{\tilde{\epsilon}} \int_{\tilde{\epsilon}}^{\infty} \frac{dx \nu(x)}{x^2}.
\]  
(A11)

In other words, the integral has two contributions for different values of \(\tilde{\epsilon}\). We have

\[
I_2 = \frac{u_{\text{pp}}}{h} \left( \frac{2\gamma_p^2}{\gamma_{p,\text{max}}^2} \right)^{\frac{1}{2}} \frac{p - 2}{10 + 5p} \left( \frac{\tilde{\epsilon}_{\text{th}}}{2\gamma_{p,\text{max}}} \right)^2 - 1
\]

\[
+ 4 \left( \frac{2\gamma_{p,\text{max}}}{\tilde{\epsilon}_{\text{th}}} - 1 \right) \frac{2\gamma_{p,\text{max}}}{\tilde{\epsilon}_{\text{th}}},
\]  
(A13)

which in the limit of \(\tilde{\epsilon}_{\text{th}} < 2\gamma_{p,\text{max}}^2\) reduces to

\[
I_2 \approx \frac{4 u_{\text{pp}}}{5} \left( \frac{2\gamma_p^2}{\gamma_{p,\text{max}}^2} \right)^{\frac{1}{2}} \frac{1}{\gamma_{p,\text{max}}} \left( \frac{2\gamma_{p,\text{max}}}{\tilde{\epsilon}_{\text{th}}} \right) - 1.
\]  
(A14)

Finally, for \(I_3\), we find

\[
I_3 = \frac{u_{\text{pp}}}{h} \left( \frac{2\gamma_p^2}{\gamma_{p,\text{max}}^2} \right)^{\frac{1}{2}} \frac{1}{\gamma_{p,\text{max}}} \left( \frac{2\gamma_{p,\text{max}}}{\tilde{\epsilon}_{\text{th}}} \right) - 1.
\]  
(A15)

Therefore, it is clear that \(I_3\) dominates the contribution to the integral. Numerically, for \(\gamma_{p,\text{max}}\), we obtain the cooling time by the photohadronic interaction

\[
t_{\text{pp}}^{-1} \approx 1.2 \times 10^{-8} \left( \frac{p - 2}{p - 1} \right)
\]

\[
\times \epsilon_{e,c}^{-\frac{1}{4}} \frac{16}{15} \frac{1}{n_0} \text{day} E_{53}^{\frac{3}{5}} \alpha^{-\frac{1}{2}} \epsilon_{B, -2}^{-\frac{1}{2}} \epsilon^{-1} s^{-1}.
\]  
(A16)

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