Rotor Airfoil Design Optimization Based on Unsteady Flow*

Yingtao ZUO,1) Pingjian CHEN,2) Wei ZHANG,2) Chunhua LI,2) Jiechu JIANG,1) and Feng FAN2)

1)National Key Laboratory of Aerodynamic Design and Research, Northwestern Polytechnical University, Xi’an, Shaanxi 710072, China
2)AVIC China Helicopter Research and Development Institute, Jingdezhen, Jiangxi 333001, China

A rotor airfoil design optimization framework based on a surrogate model and unsteady flow is constructed. Two optimization models of unsteady design are founded. A study of these models and comparison of design results based on steady and unsteady flows are conducted using the aforementioned optimization framework. The first optimization model aims to simultaneously reduce the time-averaged drag and time-averaged pitching moment. The Pareto front embodies the relation of drag and pitching moment of the rotor airfoil in an unsteady flow, which shows that a slight relaxation of the drag restrictions may yield a remarkable decrease in pitching moment, thereby improving the synthetic aerodynamic performance of the rotor airfoil. Optimization results also show that an unsteady optimization model must give consideration to low speed performance, which has been neglected by other researchers. Therefore, the second optimization model is designed to improve the time-averaged lift-to-drag ratio and the maximum lift coefficient at low speed. An additional case was designed to validate that a well-designed airfoil in a steady flow may be unsatisfactory in an unsteady flow. The results from the last case show that the unsteady pitching moment characteristic is not always proportional to the static characteristic at high speed.

Key Words: Rotor Airfoil, Unsteady Design, Multi-objective Design optimization, Optimization Model

Nomenclature

- \( \hat{c}_{\text{max}} \): relative thickness of the airfoil
- \( C_{\text{in}} \): instantaneous pitching moment coefficient (about 0.25 chord)
- \( C_{\text{d}} \): instantaneous drag coefficient
- \( C_{\text{y}} \): instantaneous lift coefficient
- \( N \): time steps for each period in unsteady simulation
- \( \langle \cdot \rangle \) = \((\sum_{i=1}^{N} \cdot )/N\): average through a period
- \( C_{\text{f}} \): lift coefficient at 15° and Mach 0.30
- \( C_{\text{dlo}} \): lift coefficient at 11° and Mach 0.4
- \( C_{\text{dlo}} \): drag coefficient at a lift coefficient 0.6 and Mach 0.6
- \( C_{\text{dlo}} \): zero lift drag coefficient at Mach 0.74
- \( C_{\text{ylo}} \): pitching moment coefficient at a lift coefficient 0.6 and Mach 0.6
- \( C_{\text{ylo}} \): zero-lift pitching moment coefficient at Mach 0.74
- \( C_{\text{dav}} \): time-averaged drag coefficient of rotor airfoil during unsteady motion
- \( C_{\text{yav}} \): time-averaged pitching moment of rotor airfoil during unsteady motion
- \( K_{\text{av}} \): time-averaged lift-to-drag ratio of rotor airfoil during unsteady motion

1. Introduction

The rotor airfoil is a basic element of a rotor blade. It affects various performances of rotor-based aircraft, such as maximum speed, flight efficiency and so on. It is one of the key technologies of helicopter design. Rotor airfoil design has been developed since the 1970s. Many rotor airfoils have been designed, and the most famous include the series of OA in France, the series of VR by Boeing Company, etc. The applications of these airfoils greatly enhance the maximum speed and cruising range, and also contribute to the reduction of noise.

As the helicopter moves forward, the inflow Mach number and angle-of-attack of the blade profile varies periodically. The advancing blade profile has larger inflow speed and smaller angle-of-attack, and the retreating blade profile has smaller inflow speed and larger angle-of-attack. This procedure is always unsteady. However, the widely used rotor airfoil design methodology is based on steady aerodynamic performance analysis.1) Researchers believe that the dynamic aerodynamic performance of a rotor airfoil will be good if it is well suited for corresponding static aerodynamic performance. Multi-object optimization based on a surrogate model is the most common methodology used in rotor airfoil design. The objective and constraints include the maximum lift at low speed, the lift-to-drag ratio when hovering, the pitching moment at all speeds, and the drag divergence Mach number among others. The optimization system that entails all of these objectives and constraints is very complex. This type of optimization method does not agree with the actual flow status of the blade profile. Therefore, it hampers improving rotor airfoil performance.

To solve the above problem, researchers have tried to simulate the actual flow status of the blade profile to improve the optimization model. Kim et al. did very excellent work about rotor airfoil optimization based on unsteady simulation.2) A numerical analysis code that can handle the oscillatory pitch-
ing motion of an airfoil and the inflow Mach number variation was developed in their paper, and a response surface methodology was adopted for optimization. A simple algebraic turbulence model was used in the preceding code, and a rotor airfoil that worked below the static stall angle was designed. The most important contribution of Kim et al. is that a case was presented to prove that static performance is not always proportional to dynamic performance. Since then, airfoil optimization based on unsteady flow has been considered necessary, and some researchers devote themselves to designing airfoils for unsteady flow. Nadarajah and Jameson developed a gradient optimization methodology for unsteady flow based on a time-accurate continuous and discrete adjoining method in 2007. Euler equations were solved in that paper, and a RAE 2822 pitching airfoil at a fixed inflow Mach number was optimized. At the same time, an inflow Mach number variation was introduced into a non-linear frequency domain (NLFD) method, with which Nadarajah developed the NLFD-based adjoining optimization method. The advantage of NLFD-based adjoining is its computational efficiency. Later, some other researchers also developed a time-accurate continuous and discrete adjoining method and surrogate model-based optimization.

Generally, most of the above-mentioned research focused on the optimization of general airfoils in periodic pitching motion with a fixed inflow Mach number. As for the optimization of rotor airfoil, little attention has been given to the issue. The above research focused on two aspects: 1) the construction of an optimization model, and various weighting function are studied; 2) the construction of various aerodynamic analysis methodologies and the corresponding sensitivity analysis methodologies. It’s well known that the pitching moment of a rotor airfoil has a severe effect on the hinge moment and weight of a helicopter’s actuating system. Most of the above research simply treats the moment as a constraint, or even neglect it in the optimization model. No research has been aimed at investigating the relation of drag and pitching moment in an unsteady flow.

The maximum lift coefficient at low inflow speed is one of the most important performance characteristics, and this has been neglected in preceding research. So one goal of this paper is to construct an optimization model that simultaneously enhances the time-averaged lift-to-drag ratio and the maximum lift coefficient of the rotor airfoil at low inflow speed. The unsteady performance of airfoils obtained by optimization based on steady flow are also be investigated in detail.

For the above purposes, the remainder of this paper is organized as follows. The CFD solver and validation of the solver are described in Section 2. The design optimization framework is introduced in Section 3, and optimization examples are presented in detail in Section 4. Finally, the conclusions are summarized in Section 5.

2. Numerical Simulation Method

2.1. CFD Simulation Method

The governing equations are the compressible Navier-Stokes equations. These equations describe the conservation of mass, momentum and total energy for a viscous compressible flow, which can be written as

\[
\frac{d}{dt} \int_{\Omega} Q dV + \int_{\partial\Omega} F \cdot \mathbf{n} dS = \int_{\partial\Omega} G \cdot \mathbf{n} dS
\]

where \( Q \) is the set of conservative flow variables, \( F \) is the inviscid flux tensor, and \( G \) is the flux tensor associated with viscosity and heat conduction.

In this context, LMNS3D(11) a multi-block viscous flow solver for steady and unsteady turbulent flows under the finite volume frame, is employed. The non-dimensional Navier-Stokes equations are solved on a body-fitted structured mesh. All of the variables are stored and operated in the cell center. Several approximate Riemann solvers (e.g., Roe, Harten-Lax-van Leer with Contact discontinuities (HLLC) and Simple Low-dissipation AUSM (SLAU)(12)) are available for inviscid flux calculation. In order to obtain a high order, a fifth-order Weighted Essentially Non-Oscillatory (WENO) scheme is developed. Considering the turbulence model, the one-equation Spalart-Allmaras (SA) model and two-equation Menter’s \( k-\omega \) SST model are adopted for unsteady RANS simulations. These turbulence models are discretized and updated in a loosely coupled way from the mean governing equation.

Due to the efficiency issue, the Roe scheme and Monotone Upwind Schemes for Scalar Conservation Laws (MUSCL) scheme are applied to evaluate the inviscid flux, and the SA turbulence model is used for this study. It should be noted that, if the flow is separated at a high angle-of-attack, the solution accuracy or credibility of RANS may be degraded.

In order to predict the flow-field of the rotor airfoil under unsteady conditions, the dual time-stepping approach is employed. Equation (1) can be rewritten as a discrete form

\[
\frac{3Q^{n+1} - 4Q^n + Q^{n-1}}{2\Delta t} + R(Q^{n+1}) = 0
\]

where \( \Delta t \) is the global physical time step. Equation (2) can be rewritten as

\[
\frac{d(Q^{n+1}Q^*)}{d\tau} + R^*(Q^*) = 0
\]

where \( Q^* \) is the approximation for \( Q^{n+1} \), \( \tau \) represents a pseudo time variable, and the unsteady residual is defined as

\[
R^*(Q^*) = \frac{3Q^* - 4Q^n + Q^{n-1}}{2\Delta t} + R(Q^n)
\]

The LU-SGS implicit method is adopted(14) for pseudo time marching.

The flow profile surrounding the rotor blade is dominated by three factors: pitching, revolving and far-field flow. A two-dimensional flow case of a pitching airfoil with sinusoidal motion along the flow direction is a reasonable simplification. Assume the Mach number and the angle-of-attack of the rotor airfoil can be described as

\[ M(t) = M_0 - A \sin(\omega t) \quad \alpha(t) = \alpha_0 + \alpha_m \sin(\omega t) \]
where \( M_0 \) is the freestream Mach number (also the speed of the rotor profile compared to the rotor axis), \( A \) is the amplitude of inflow speed oscillation (also the speed of the helicopter), \( \alpha_o \) is the mean incidence, \( \alpha_m \) is the amplitude of pitching oscillation, and \( \omega \) is the angular frequency of the motion. The instantaneous horizontal position of the airfoil compared to the balance position in the fixed inflow can be described as

\[
x = -\frac{A}{\omega} \cos(\omega t) = -\frac{A}{\omega} \sin(\omega t - pt/2)
\]

To simulate the above movement, first, the grid of the rotor airfoil is rotated by \( \alpha_o \) around the 1/4 chord point, which is different from usual simulation. Then, the angle-of-attack is set to zero and the inflow Mach number is set to \( M_0 \) in the input file of the unsteady simulation solver. The airfoil undergoes both sinusoidal pitching angle variation and sinusoidal variation in the horizontal direction. The CFD grid is rigid and undergoes the same movements as the airfoil. The actual angle-of-attack of the airfoil varies between \( \alpha_o - \alpha_m \) and \( \alpha_o + \alpha_m \) during the movement. Finally, the aerodynamic force is integrated in the wind axis, which means the drag is along the inflow axis because the angle-of-attack is zero in the input file of the unsteady simulation solver.

### 2.2. Validation of CFD solver

To validate the accuracy of the unsteady numerical analysis code developed, a NACA0012 airfoil oscillating in the pitch degree of freedom is investigated. The periodic motion of the airfoil is defined by the angle-of-attack as a function of time as

\[
\alpha(t) = \alpha_o + \alpha_m \sin(\omega t) \quad \alpha_m = 2.44^\circ
\]

\[
\alpha_o = 4.86^\circ \quad k = \omega c/2U_{\infty} = 0.0810
\]

The freestream Mach number is 0.6 and the Reynolds number is set to \( 4.8 \times 10^6 \). A comparison of constant transonic inflow is made because experimental data are not available to deal with the simultaneous variation of inflow velocity and incidence.

The grid used for NACA0012 is shown in Fig. 1. The grid size is \( 2.9 \times 10^4 \). Four pitching cycles are simulated. The lift and pitching moment hysteresis loop are compared with the experiment in Figs. 2 and 3, respectively. The calculated lift and pitching moment coefficients agree well with the experiment. Figure 4 shows a comparison of the pressure distribution calculated and the experimental data for two angles during the up-stroke and two angles during the down-stroke. The pressure distribution achieved using CFD also agreed well with the experiment.

The rotor airfoil undergoes unsteady movement. The time-accurate unsteady simulation process is very time-consuming. Therefore, it’s important to decide the proper time step to obtain a balance between efficiency and precision. The OA212 airfoil is tested here. The grid shown in Fig. 5 has a grid size of \( 3.8 \times 10^4 \) and a chord length of 1 m. The Reynolds number is set to \( 3.0 \times 10^6 \). This airfoil undergoes both pitching angle variation and sinusoidal variation of the freestream Mach number, which can be defined as follows.

\[
M(t) = M_0 - A \sin(\omega t) \quad M_0 = 0.45
\]

\[
A = 0.209 \quad \omega = 12.25 \text{ rad/s}
\]

\[
\alpha(t) = \alpha_o + \alpha_m \sin(\omega t) \quad \alpha_m = 5.0^\circ \quad \alpha_o = 7.0^\circ
\]

To simulate the unsteady flow, \( \alpha(t) \) is divided into \( N \) discrete points or time steps for each period. A series of time steps \( N \) such as 120, 240, 480 and 600 are tested for the OA212 airfoil. Lift hysteresis, pitching moment hysteresis and drag hysteresis of various time steps are compared and listed in Figs. 6–8. The reference speed used to calculate the aerodynamic coefficient is set to Mach 0.45. As can be seen, 480 steps per period is a very good compromise between precision and efficiency for optimization.

### 3. Optimization Framework

Unsteady flow simulation is very time-consuming, even in the case of two-dimensional flows. Therefore, a surrogate model is necessary to accelerate the optimization. The widely used optimization framework based on the surrogate model and genetic algorithm (GA) is adopted.
Sampling methods include completely randomized design, orthogonal design, uniform design, Latin hypercube design, optimal Latin hypercube,\(^{17}\) etc. The optimal Latin hypercube method is used to select samples here. The design space for each factor is uniformly divided using this technique.

The Kriging model, which yields good fitting results of multi-peak problems, together with second-order polynomial response surfaces are used to model the observed responses accurately in this paper.

A loosely coupled surrogate management framework is built up as shown in Fig. 9. In this framework, most high-fidelity simulations are replaced by surrogate models. The optimization is organized as follows.
The airfoil can be represented by the following functions.

\[ y_{\text{new}}(x) = y_{\text{old}}(x) + \sum_{j=1}^{n_t} c_j g_j(x) \]

\[ y_{\text{new}}(x) = y_{\text{old}}(x) + \sum_{j=n_t+1}^{n} c_j g_j(x) \]

where \( y_{\text{old}}(x) \) represents the thickness distribution function of the initial airfoil, \( y_{\text{old}}(x) \) represents the camber distribution function of the initial airfoil, \( y_{\text{new}}(x) \) is the thickness distribution function of the deformed airfoil, \( y_{\text{new}}(x) \) is the camber distribution function of the deformed airfoil. \( c_j \) is the design variable, and \( n_t \) and \( n \) are the number of thickness design variables and total number of design variables, respectively. The Hicks-Henne shape function is adopted in this paper as follows.

\[ g_j(x) = \begin{cases} \sin^4(\pi x^{e(j)}) & j > 1 \\ x^{0.25}(1-x)e^{-20x} & j = 1 \end{cases} \]

where

\[ e(j) = \frac{\log 0.5}{\log x_j} \quad (x_j \in (0, 1)) \]

Thirteen design variables are chosen. Seven of them are camber variables and six of them are thickness variables.

The movement of the OA212 airfoil is described in Section 2.2. Three-hundred-and-ninety samples are generated. Cross-validation is adopted to validate the precision of these samples. The leave-one-out strategy is computationally expensive, and we use the \( k \)-fold strategy in this paper. All of these samples are randomly split into 15 equal subsets. Each of these subsets is removed, in turn, from the complete sample set and the model is fitted to the remaining data. Each subset removed is predicted using a surrogate model constructed by the remaining data. When all subsets have been removed, we get 390 predictions of 390 observed data points.

In order to compare the accuracy, a validation set of data points was evaluated using second-order polynomial response surfaces and the Kriging model. Their predictions are contrasted with the true response. The Relative Root Mean Square Error (RMSRE), Relative Average Absolute Error (RAAE), Relative Maximum Absolute Error (RMAE) and \( R^2 \) are chosen to assess the precision. The definitions of the above measurements are listed as follows.

\[
\text{RMSRE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left( \frac{f(x_i) - \hat{f}(x_i)}{f(x_i)} \right)^2}
\]

\[
\text{RAAE} = \frac{1}{M} \sum_{i=1}^{M} |f(x_i) - \hat{f}(x_i)|
\]

\[
\text{RMAE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left( f(x_i) - \hat{f}(x_i) \right)^2}
\]

\[
R^2 = 1 - \sum_{i=1}^{M} \left( f(x_i) - \bar{f}(x_i) \right)^2 / \sum_{i=1}^{M} \left( f(x_i) - \bar{f}(x_i) \right)^2
\]

where \( M \) is the number of samples, \( x_i \) is the \( i \)-th observed data point. \( f(x_i) \) is the value of the \( i \)-th observed data point predicted by surrogate model, \( \hat{f}(x_i) \) is the true function response, and \( \bar{f}(x) \) is the mean of the true function response. The lower the value for RAAE and RMSRE, the more accurate the surrogate model. A small value for RMAE is preferred. The larger the value of \( R^2 \), the more accurate the surrogate model.

The precision of the Kriging model and quadratic polynomial response surface are listed in Tables 1 and 2.

As can be seen from the above tables, the accuracy of the Kriging model is slightly superior to that of the response surface in this case. So we adopt the Kriging model for the following optimization.

### 4.2. Case 1

The optimization of airfoil OA212 is performed in this...
The objectives are to reduce the time-averaged drag and pitching moment while maintaining the time-averaged lift coefficient of the rotor airfoil. The unsteady motion of the rotor airfoil in this case and the following cases is identical to the motion described in Section 2.2. Additionally, the relative thickness of the optimized airfoil should be no less than that of the OA212 airfoil. Accordingly, the optimization model can be expressed as follows:

Minimize : \( \langle C_d \rangle \) \( \langle |C_m| \rangle \)

Subject to : \( \bar{C}_{\text{max}} \geq 0.12 \)

\[ \langle C_l \rangle = 0.8909 \]

To maintain the time-averaged lift coefficient of the rotor airfoil during optimization, the mean angle-of-attack should be adjusted. In this study, the mean angle-of-attack is shifted every four periods with an increment according to the slope of the average lift coefficient versus the average angle-of-attack. Usually at least eight periods are needed for an airfoil to satisfy the lift constraint. This technique multiplies the total cost at least twofold.

With the samples generated in Section 4.1, the OA212 airfoil is optimized using the above framework. Validations are conducted 20 times. Finally, a serial of airfoils is obtained, and the Pareto optimal solutions are presented in Fig. 10. Five typical airfoils, as shown in Fig. 10, were selected for further comparison. Comparisons of the lift, drag and pitching moment hysteresis of OA212, as well as the optimal airfoils, are shown in Figs. 11–13. The geometries of the optimized airfoils and the baseline are shown in Fig. 14. Steady performance comparisons of these typical airfoils at low speed are shown in Figs. 15–17. The time-averaged performance is shown in Table 3. The zero-lift drag and zero-lift pitching moment characteristics are compared in Figs. 18 and 19.

Table 1. Prediction accuracy of \( C_d \).

| Method      | RRMSE   | RAAE    | RMAE    | \( R^2 \) |
|-------------|---------|---------|---------|-----------|
| Kriging     | 0.101705| 0.188624| 1.319237| 0.978430  |
| Response surface | 0.104276| 0.192530| 1.32089| 0.978297  |

Table 2. Prediction accuracy of \( C_m \).

| Method      | RRMSE   | RAAE    | RMAE    | \( R^2 \) |
|-------------|---------|---------|---------|-----------|
| Kriging     | 0.117417| 0.172034| 1.312931| 0.961605  |
| Response surface | 0.118238| 0.173839| 1.312982| 0.961282  |
high inflow speed. The results also show the relationship between drag and pitching moment. All of these are explained as follows.

Figure 11 shows that the maximum angle-of-attack is about 12° in unsteady motion, which corresponds to an inflow Mach number of 0.24. It is evident in Figs. 11 and 12 that Foil 5 has the smallest lift coefficient and the largest drag coefficient among all of the optimized airfoils, which means the smallest lift-to-drag ratio. As shown in Figs. 15 and 17, the lift-to-drag ratio of Foil 5 at an angle-of-attack of 12° and inflow Mach number of 0.24 is the smallest among all of the airfoils. This means that the unsteady performance is

Table 3. Optimization results.

| Foil  | $C_{d\text{avg}}$ | $C_{m\text{avg}}$ | $K_{\text{avg}}$ |
|-------|------------------|------------------|------------------|
| OA212 | 0.015246         | 0.017371         | 58.43            |
| Foil 1| 0.014622         | 0.018613         | 60.90            |
| Foil 2| 0.014764         | 0.010501         | 60.39            |
| Foil 3| 0.014838         | 0.004086         | 60.03            |
| Foil 4| 0.015102         | 0.003195         | 58.98            |
| Foil 5| 0.015600         | 0.002531         | 57.10            |
consistent with that of the steady performance at low inflow speed in this case.

The maximum inflow Mach number is 0.659, and this corresponds to the minimum angle-of-attack according to the movement of the airfoil in the aforementioned unsteady motion. As shown in Fig. 13, among all of the airfoils, Foil 4 has the best pitching moment performance when the inflow Mach number is 0.659, and on the contrary, Foil 1 has the worst pitching moment performance. It can be seen from Fig. 19 that Foil 4 has the best pitching moment among all of the airfoils at an inflow Mach number of 0.659, and on the contrary, Foil 1 has the worst pitching moment performance. So the pitching moment performance of the steady and unsteady flows at high inflow speed is almost consistent in this case.

The time-averaged drag coefficients of Foil 1–Foil 3 make little difference according to Table 3. The maximum difference compared to Foil 1 is 1.48%. However, the time-averaged pitching moment difference of Foil 1–Foil 3 compared to Foil 1 is 78%, which is significant. This case shows that it’s not wise to minimize only the time-averaged drag of the rotor airfoil. A slight relaxation of drag restrictions will yield a remarkable decrease in pitching moment, thereby improving synthetic aerodynamic performance to the rotor airfoil.

4.3. Case 2

As depicted in Fig. 15, the maximum lift of the optimized airfoil is less than that of the original. The reason is that the maximum angle-of-attack is about 12° during unsteady motion, which is far from the stalling angle, and the optimization model above does not take the characteristic of high angle-of-attack into consideration.

There are two methods to solve this problem. One method is to calculate the maximum dynamic lift coefficient under a motion similar to the above motion in Case 1. The other is to calculate the maximum lift coefficient under steady motion. Since dynamic maximum lift is not widely validated, the lift coefficient of 15° at an inflow speed of Mach 0.3 is chosen as one of the objectives, and Foil 3 is chosen to be the baseline. The new optimization model can be expressed as follows:

Minimize : \( \langle C_d \rangle \)

\[ -C_f \]

Subject to : 
\[ \dot{c}_{\text{max}} \geq 0.12 \]
\[ \langle C_l \rangle = 0.8909 \]
\[ \langle |C_m| \rangle \leq 0.007 \]

The parameterization of Foil 3 is the same as the above case. Three-hundred-ninety samples were generated and 33 validations were conducted for optimization. The Pareto optimal solutions are listed in Fig. 20. One foil, Foil 6 in Fig. 20, is selected for further analysis against Foil 3. Comparisons of the lift, drag, and pitching moment hysteresis are demonstrated in Figs. 21–23. The geometries of the optimized airfoil and the baseline are shown in Fig. 24. Steady performance comparisons of these two airfoils are shown in Figs. 25–32, in which Figs. 31 and 32 show the zero-lift drag and zero-lift pitching moment characteristics. Table 4 lists the time-averaged performance and maximum lift coefficient at an inflow Mach 0.3, which shows the optimization is very successful.

It is evident from Figs. 26 and 32 that the steady flow pitching moment performance of Foil 6 is inferior to that of Foil 3 and superior to that of OA212. The time-averaged pitching moment performance of Foil 6 is also inferior to that of Foil 3 and superior to that of OA212, as shown in Table 4 and Fig. 23. This case also indicates that the unsteady aerodynamic performance is in accordance with the steady performance for overall pitching moment.

The unsteady lift-to-drag ratio is not in accordance with
the steady lift-to-drag ratio in this case. The lift-to-drag ratio of airfoil OA212 is larger than that of Foil 3 in the range of Mach 0.3 to 0.6, as depicted in Figs. 27–30. Furthermore, Fig. 31 shows that the zero-lift drag coefficient of Foil 3 at Mach 0.659 is slightly larger than that of airfoil OA212. The lift-to-drag ratio of Foil 6 is remarkably superior to that of airfoil OA212 in the range of Mach 0.5–0.6, and the zero-lift drag at Mach 0.659 is also less than that of airfoil OA212. This proves that the steady flow lift-to-drag ratios of Foil 3 and Foil 6 are all inferior to that of OA212. However, the time-averaged lift-to-drag ratios of Foil 3 and Foil 6 during unsteady motion were all notably superior to that of airfoil OA212 according to Table 4. This case indicates a contradiction in steady performance and unsteady performance for lift-to-drag ratio.

4.4. Case 3

To further demonstrate the necessity of unsteady flow-based optimization, steady flow-based optimization of OA212 is performed and compared with unsteady flow-based optimization. The optimization objectives are to improve maximum lift at Mach 0.4 and minimize the drag coefficient at a lift coefficient of 0.6 and inflow speed of Mach 0.6, as well as to minimize the zero-lift drag coefficient at Mach 0.74. Constraints include the pitching moment coeffi-
The coefficients of the last two design states (Mach 0.6 and 0.74), etc. Since we obtain the maximum lift coefficient of airfoil OA212 at 11° and inflow speed of Mach 0.4, one of the objectives becomes improving the lift coefficient at 11° and inflow speed of Mach 0.4. Therefore, the optimization model can be constructed as:

Minimize:

\[
\begin{align*}
\text{objective1} & \quad -C_{l_{11\text{m}0.4}} \times 400 + C_{d_{0.6c0.0}} \times 30000 \\
\text{objective2} & \quad C_{d_{0.74c0.0}} \times 12000
\end{align*}
\]
Subject to:

\[ \bar{C}_{\text{max}} \geq 0.12 \]

\[ C_{\text{mm}0.6cR0.6} \geq -0.007 \]

\[ C_{\text{mm}0.74cR0.0} \geq -0.008 \]

The parameterization of OA212 is the same as the above case. The number of samples is 390, and validation is performed 40 times for optimization. The Pareto optimal solutions are presented in Fig. 33. The aerodynamic performance of the airfoils corresponding to the Pareto front in the design state is shown in Fig. 34. Two points of the Pareto front are selected for further analysis and they are named Foil 7 and Foil 8, as can be seen in Fig. 33. The aerodynamic performance of OA212, Foil 7 and Foil 8 in the design state are listed in Table 5. Figures 35–37 and Table 6 are comparisons of the lift, drag, and pitching moment hysteresis of these airfoils and Foil 6 under the preceding motion with a fixed time-averaged lift coefficient of 0.8909. Figure 38 shows the geometries of the optimized airfoils and the baseline. The zero-lift drag and zero-lift pitching moment characteristics are compared in Figs. 39 and 40.

It is evident in Table 5 that the performances of Foil 7 and Foil 8 are superior to that of OA212 in the design state. Therefore, the design is successful.

Figure 36 shows that, during unsteady motion, the drag coefficient of Foil 6 is the smallest among all of these airfoils when they have the maximum inflow speed (Mach 0.659). However, Foil 6 has the maximum steady flow drag at Mach 0.659 according to Fig. 39. The unsteady flow pitching moment of Foil 6 is the best among all of these airfoils according
to Fig. 37. Figure 40 also indicates that Foil 6 has the best pitching moment performance. Therefore, the steady pitching moment is generally consistent with the unsteady pitching moment, except for the situation of high inflow speed in this case. It can be seen from Fig. 37 that Foil 7 and Foil 8 have almost the same unsteady flow pitching moment characteristics and they are notably inferior to Foil 6 at the minimum angle-of-attack, corresponding to an inflow of Mach 0.659. Figure 40 indicates that Foil 6, Foil 7 and Foil 8 have almost the same steady flow pitching moment with an inflow of Mach 0.659.

It is apparent that Foil 7 and Foil 8 are superior to Foil OA212 for every steady flow target, as presented in Table 5. Table 6 demonstrates that the time-averaged drag of Foil 7 is larger than that of airfoil OA212. This case indicates that a well-designed airfoil with a steady flow may perform poorly for a rotor in forward flight.

(3) Two optimization models based on unsteady flow were constructed. Optimization results show that the second optimization model is more suitable for the purpose of engineering, and multi-objective optimization the rotor airfoil based on unsteady flow is still very necessary.

5. Conclusions

A rotor airfoil design optimization framework based on unsteady flow and a surrogate model is constructed. The OA212 airfoil was optimized using the framework proposed. The following conclusions were obtained.

(1) Time-averaged pitching moment may be reduced dramatically by slightly relaxing the time-averaged drag. This will improve synthetic performance efficiency.

(2) The static performance is not always proportional to the dynamic performance for pitching moment at a high inflow speed, and the same goes as well for drag. In other words, excellent rotor airfoil design optimization results based on steady flow may perform poorly for a rotor in forward flight.

(3) Two optimization models based on unsteady flow were constructed. Optimization results show that the second optimization model is more suitable for the purpose of engineering, and multi-objective optimization the rotor airfoil based on unsteady flow is still very necessary.

Acknowledgments

This research was supported by a fund of the National Key Laboratory of Aerodynamic Design and Research of Northwestern Polytechnical University and the Aeronautical Science Foundation of China (20155753042). The authors would like to thank Dr. Ke Zhao and Baigang Mi of Northwestern Polytechnical University for their assistance with this paper.

Table 6. Time-averaged performance of airfoils.

| Foil   | C_{avg}  | C_{avg}  | K_{avg} |
|--------|----------|----------|---------|
| OA212  | 0.015246 | 0.017371 | 58.43   |
| Foil 6 | 0.014803 | 0.008618 | 60.15   |
| Foil 7 | 0.015383 | 0.008278 | 57.92   |
| Foil 8 | 0.015013 | 0.008294 | 59.35   |

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S. Saito
Associate Editor