More on the universality of the Volkov-Akulov action under

$N = 1$ nonlinear supersymmetry

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Abstract

We discuss further the universality of the Volkov-Akulov (V-A) action of a
Nambu-Goldstone (N-G) fermion for the spontaneous breaking of supersymmetry (SUSY). We show general relations between the standard V-A action
and nonlinear (NL) SUSY actions including apparently (pathological) higher
derivatives of the N-G fermion. Composite fields of the N-G fermions are
found, which transform homogeneously under NL SUSY transformations of
V-A. Consequently, we obtain NL SUSY invariant constraints which connect
our NL SUSY actions with the V-A action. The constraints are explicitly
solved and we show examples of the NL SUSY actions which are equivalent
to the V-A action.

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Nonlinear (NL) realization of supersymmetry (SUSY) and NL SUSY action given by Volkov-Akulov (V-A) [1] are described in terms of a Nambu-Goldstone (N-G) fermion [2] indicating the spontaneous SUSY breaking [3, 4]. The equivalence of the V-A model of NL SUSY to various linear (L) supermultiplets [5, 6] was shown by many authors [7]-[10]. In the relation between the NL and the L SUSY, basic fields in the L supermultiplets are expressed as composites of the N-G fermion in SUSY invariant way, and this fact gives deep insight towards the unification of spacetime and matter from the viewpoint of compositeness of matter as discussed in [11]. While, it is known that there exists a nontrivial NL SUSY higher derivative action of the N-G fermion as is exemplified in [12]. In order to understand the implications of the nontrivial higher derivative action of the N-G fermion, it is useful to investigate the relation among NL SUSY actions i.e., the universality of NL SUSY actions with the N-G fermion. Recently, this problem has been discussed in [13] in the viewpoint of the braneworld scenario. And by heuristic arguments we have discussed in [14] the relation between the standard V-A action and a NL SUSY action including apparently a (Weyl) ghost field which originates from pathological higher derivatives of the N-G fermion.

In this letter, by extending the arguments of [14] with respect to the universality of NL SUSY actions to the cases where the order of derivatives of the N-G fermion is higher than in [14], we discuss more general relations between the standard V-A action and NL SUSY actions including apparently (pathological) higher derivatives of the N-G fermion under \( N = 1 \) NL SUSY. By using the algorithmic procedure given in [15], we find composite fields of the N-G fermions which transform homogeneously under NL SUSY transformations of V-A. Consequently, we obtain NL SUSY invariant constraints which connect our NL SUSY actions with the standard V-A action. The constraints are explicitly solved and we show examples of the NL SUSY actions which are equivalent to the V-A action. We show that the arguments of [14] with respect to the universality of NL SUSY actions are derived as a solution of the constraints in the examples.

Let us begin with the brief review of the NL realization of SUSY by V-A [1]. In the \( N = 1 \) V-A model, the NL SUSY transformation law of a (Majorana) N-G fermion \( \psi \) generated by a constant (Majorana) spinor parameter \( \zeta \) is \(^{1,2}\),

\[
\delta_Q \psi = \frac{1}{\kappa} \zeta - i \kappa (\bar{\zeta} \gamma^a \psi) \partial_a \psi,
\]

(1)

where \( \kappa \) is a constant whose dimension is \((\text{mass})^{-2}\). The NL SUSY transformation (1) satisfies a closed off-shell commutator algebra, \([\delta_Q (\zeta_1), \delta_Q (\zeta_2)] = \delta_P (v)\), where \( \delta_P \)

\(^{1}\)In this letter Minkowski spacetime indices are denoted by \( a, b, ... = 0, 1, 2, 3 \), and we use the Minkowski spacetime metric \( \frac{1}{2} (\gamma^a, \gamma^b) = \eta^{ab} = (+, -, -, -) \) and \( \sigma^{ab} = \frac{1}{4} [\gamma^a, \gamma^b] \).
is a translation with a parameter \( v^a = 2i\bar{\zeta} \gamma^a \zeta_2 \). Based on the invariant one-form under Eq.(1), i.e., \( \omega^a = w^a_b dx^a = (\delta^a_b - i\kappa^2 \bar{\psi} \gamma^a \partial_b \psi)dx^a \), the NL SUSY V-A action \( S_{VA}(\psi) \) is given by

\[
S_{VA}(\psi) = -\frac{1}{2\kappa^2} \int d^4x |w| = -\frac{1}{2\kappa^2} \int d^4x \left[ 1 + t^a_a + \frac{1}{2}(t^a_b t^b_a - t^a_a t^b_b) \right. \\
\left. - \frac{1}{6} \epsilon_{abcd} \epsilon_{efgh} t^a_e t^b_f t^c_g t^d_h \right],
\]

(2)

where \( |w| = \det w^a_b \) and \( t^a_b = -i\kappa^2 \bar{\psi} \gamma^a \partial_b \psi \).

On the other hand, let us consider NL SUSY actions which include (pathological) higher derivatives of a N-G fermion in addition to the standard V-A action as exemplified in [14]. Namely, we denote \( \lambda \) for the (Majorana) N-G fermion which transforms into \( \psi \) in Eq.(2) through NL SUSY invariant constraints as will be shown later, and we propose the actions \( S(\lambda) \) including apparently nontrivial terms with (pathological) higher derivatives of \( \lambda \),

\[
S(\lambda) = S_{VA}(\lambda) + \left[ \text{higher derivative terms of } \lambda \right]
\]

(3)

which are invariant under the NL SUSY transformation of \( \lambda \),

\[
\delta_Q \lambda = \frac{1}{\kappa} \zeta - i\kappa (\bar{\zeta} \gamma^a \lambda) \partial_a \lambda.
\]

(4)

Note that the form of Eq.(4) is the same as Eq.(1).

In order to construct the NL SUSY invariant constraints between the N-G fermions \( \psi \) and \( \lambda \), we use the algorithmic procedure given by Ivanov [15] to pass to a relevant NL SUSY theory from another one. Indeed, first we introduce the fields,

\[
\lambda + \sum_{n \geq 1} c_n (i\kappa^2)^n \gamma^A \partial_A \lambda,
\]

(5)

as the most general form of \( O(\lambda^1) \) in terms of \( \lambda \) and its higher derivatives with the arbitrary coefficients \( c_n \), where \( \gamma^A \) and \( \partial_A \) are defined respectively as

\[
\gamma^A = \prod_{\alpha=1}^{n} \gamma^a = \gamma^a_1 \gamma^a_2 \cdots \gamma^a_n,
\]

\[
\partial_A = \prod_{\alpha=1}^{n} \partial_{a_\alpha} = \partial_{a_1} \partial_{a_2} \cdots \partial_{a_n}
\]

(6)
with \(a_1, a_2, ..., a_n\) being Minkowski spacetime indices, and \(\gamma^A \partial_A = \mathcal{G}^a\). Second we show explicitly the following finite transformations of the fields (5) for the simplified case of \(c_n = 1\),

\[
\tilde{\lambda}(\zeta) = \left( 1 + \delta_\zeta + \frac{1}{2!} \delta_\zeta^2 + \frac{1}{3!} \delta_\zeta^3 + \frac{1}{4!} \delta_\zeta^4 \right) \left\{ \lambda + \sum_{n \geq 1} (i \kappa \frac{n}{2})^n \gamma^A \partial_A \lambda \right\}
\]

\[
= e^{\delta_\zeta} \left\{ \lambda + \sum_{n \geq 1} (i \kappa \frac{n}{2})^n \gamma^A \partial_A \lambda \right\}
\]

(7)

which are generated by the NL SUSY transformations (4). Note that in Eq.(7) the terms higher than \(\delta_\zeta^4\) vanish by means of \(\zeta^n = 0\) for \(n \geq 5\). By replacing the spinor parameter \(\zeta\) in Eq.(7) by \(-\kappa \psi\), we finally define the composite fields,

\[
\tilde{\lambda}(\psi) = \tilde{\lambda}_0(\psi) + \tilde{\lambda}_1(\psi),
\]

(8)

where \(\tilde{\lambda}_0(\psi)\) and \(\tilde{\lambda}_1(\psi)\) are the fields for the finite transformation of \(\lambda\) and its higher derivatives, respectively, i.e.,

\[
\tilde{\lambda}_0(\psi) = \left( 1 + \delta_\zeta + \frac{1}{2!} \delta_\zeta^2 + \frac{1}{3!} \delta_\zeta^3 + \frac{1}{4!} \delta_\zeta^4 \right) \lambda \mid_{\zeta \to -\kappa \psi},
\]

(9)

\[
\tilde{\lambda}_1(\psi) = \left( 1 + \delta_\zeta + \frac{1}{2!} \delta_\zeta^2 + \frac{1}{3!} \delta_\zeta^3 + \frac{1}{4!} \delta_\zeta^4 \right) \sum_{n \geq 1} (i \kappa \frac{n}{2})^n \gamma^A \partial_A \lambda \mid_{\zeta \to -\kappa \psi}.
\]

(10)

The explicit form of \(\tilde{\lambda}_0(\psi)\) becomes

\[
\tilde{\lambda}_0(\psi)
\]

\[
= \lambda - \psi + \eta^a(\psi) \partial_a \lambda + i \kappa^2 \eta^a(\psi) \bar{\psi} \gamma^b \partial_a \lambda \partial_b \lambda + \frac{1}{2} \eta^a(\psi) \eta^b(\psi) \partial_a \partial_b \lambda
\]

\[
- \kappa^4 \eta^a(\psi) \bar{\psi} \gamma^b \partial_a \lambda \bar{\psi} \gamma^c \partial_b \lambda \partial_c \lambda + \frac{i}{2} \kappa^2 \eta^a(\psi) \eta^b(\psi) \bar{\psi} \gamma^c \partial_a \partial_c \lambda
\]

\[
+ i \kappa^2 \eta^a(\psi) \eta^b(\psi) \bar{\psi} \gamma^c \partial_a \partial_c \lambda + \frac{1}{6} \eta^a(\psi) \eta^b(\psi) \eta^c(\psi) \partial_a \partial_b \partial_c \lambda
\]

\[
- \kappa^4 \eta^a(\psi) \eta^b(\psi) \bar{\psi} \gamma^c \partial_a \lambda \bar{\psi} \gamma^d \partial_c \lambda \partial_d \lambda - \kappa^4 \eta^a(\psi) \eta^b(\psi) \bar{\psi} \gamma^c \partial_a \partial_c \lambda \partial_d \lambda
\]

\[
- \kappa^4 \eta^a(\psi) \eta^b(\psi) \bar{\psi} \gamma^c \partial_a \partial_c \lambda \partial_d \lambda - \frac{1}{2} \kappa^4 \eta^a(\psi) \eta^b(\psi) \bar{\psi} \gamma^c \partial_a \partial_c \lambda \partial_d \lambda
\]

\[
+ \frac{1}{2} \kappa^4 \eta^a(\psi) \eta^b(\psi) \bar{\psi} \gamma^c \partial_a \lambda \bar{\psi} \gamma^d \partial_c \lambda \partial_d \lambda + \frac{i}{2} \kappa^2 \eta^a(\psi) \eta^b(\psi) \eta^c(\psi) \bar{\psi} \gamma^d \partial_a \partial_c \lambda \partial_d \lambda
\]

\[
+ \frac{i}{2} \kappa^2 \eta^a(\psi) \eta^b(\psi) \eta^c(\psi) \bar{\psi} \gamma^d \partial_a \partial_c \lambda \partial_d \lambda + \frac{1}{6} \kappa^2 \eta^a(\psi) \eta^b(\psi) \eta^c(\psi) \eta^d(\psi) \partial_a \partial_b \partial_c \partial_d \lambda
\]

(11)
with \( \eta^a(\psi) = i\kappa^2 \bar{\psi} \gamma^a \lambda \). On the other hand, in order to derive the explicit form of \( \tilde{\lambda}_1(\psi) \), we use the Leibniz rule of derivatives for some products \( A_1 A_2 \cdots A_m \),

\[
\partial_A(A_1 A_2 \cdots A_m) = \prod_{\alpha=1}^n \partial_{a_\alpha} (A_1 A_2 \cdots A_m)
= \sum_{k_1+k_2+\cdots+k_m=n} \frac{n!}{k_1! k_2! \cdots k_m!}
\times \prod_{\alpha=1}^{k_1} \partial_{a_\alpha} A_1 \prod_{\beta=k_1+1}^{k_1+k_2} \partial_{a_\beta} A_2 \cdots \prod_{\gamma=k_1+k_2+\cdots+k_{m-1}+1}^{n} \partial_{a_\gamma} A_m,
\]

where \( k_j \geq 0 \) \((j = 1, 2, \ldots, m)\) and the indices \( a_\alpha, a_\beta, \ldots, a_\gamma \) are totally symmetrized. According to Eq.(12), \( \tilde{\lambda}_1(\psi) \) is given by

\[
\tilde{\lambda}_1(\psi)
= \sum_{n \geq 1} i^n \kappa^2 \bar{\psi} \gamma^A \partial_A \lambda
+ \sum_{n \geq 1} i^{n+1} \kappa^2 \bar{\psi} \gamma^A \sum_{k_1+k_2=n} \frac{n!}{k_1! k_2!} \prod_\alpha \bar{\psi} \gamma^b \partial_{a_\alpha} \lambda \prod_\beta \partial_{a_\beta} \partial_b \lambda
- \sum_{n \geq 1} i^n \kappa^2 \bar{\psi} \gamma^A \sum_{k_1+k_2+k_3=n} \frac{n!}{k_1! k_2! k_3!} \prod_\alpha \bar{\psi} \gamma^b \partial_{a_\alpha} \lambda \prod_\beta \bar{\psi} \gamma^c \partial_{a_\beta} \partial_b \lambda \prod_\gamma \partial_{a_\gamma} \partial_c \lambda
+ \frac{1}{2} \sum_\alpha \bar{\psi} \gamma^b \partial_{a_\alpha} \lambda \prod_\beta \bar{\psi} \gamma^c \partial_{a_\beta} \partial_b \lambda \prod_\gamma \partial_{a_\gamma} \partial_c \lambda
- \sum_{n \geq 1} i^{n+1} \kappa^2 \bar{\psi} \gamma^A \sum_{k_1+\cdots+k_4=n} \frac{n!}{k_1! \cdots k_4!} \prod_\alpha \bar{\psi} \gamma^b \partial_{a_\alpha} \lambda \prod_\beta \bar{\psi} \gamma^c \partial_{a_\beta} \partial_b \lambda \prod_\gamma \bar{\psi} \gamma^d \partial_{a_\gamma} \partial_c \lambda \prod_\delta \partial_{a_\delta} \partial_d \lambda
+ \frac{1}{2} \sum_\alpha \bar{\psi} \gamma^b \partial_{a_\alpha} \lambda \prod_\beta \bar{\psi} \gamma^c \partial_{a_\beta} \partial_b \lambda \prod_\gamma \bar{\psi} \gamma^d \partial_{a_\gamma} \partial_c \lambda \prod_\delta \partial_{a_\delta} \partial_d \lambda
+ \sum_\alpha \bar{\psi} \gamma^b \partial_{a_\alpha} \lambda \prod_\beta \bar{\psi} \gamma^c \partial_{a_\beta} \partial_b \lambda \prod_\gamma \bar{\psi} \gamma^d \partial_{a_\gamma} \partial_c \lambda \prod_\delta \partial_{a_\delta} \partial_d \lambda
+ \frac{1}{6} \sum_\alpha \bar{\psi} \gamma^b \partial_{a_\alpha} \lambda \prod_\beta \bar{\psi} \gamma^c \partial_{a_\beta} \partial_b \lambda \prod_\gamma \bar{\psi} \gamma^d \partial_{a_\gamma} \partial_c \lambda \prod_\delta \partial_{a_\delta} \partial_d \lambda
\]
By straightforward calculations we can prove that in all orders the fields \( \tilde{\lambda}_0(\psi) \) and \( \tilde{\lambda}_1(\psi) \) transform homogeneously under the NL SUSY transformations (1) and (4), i.e.,

\[
\delta_\xi \tilde{\lambda}_0(\psi) = \xi^a \partial_a \tilde{\lambda}_0(\psi),
\]

\[
\delta_\xi \tilde{\lambda}_1(\psi) = \xi^a \partial_a \tilde{\lambda}_1(\psi),
\]

so that \( \delta_\xi \tilde{\lambda}(\psi) = \xi^a \partial_a \tilde{\lambda}(\psi) \). Therefore, the constraints,

\[
\tilde{\lambda}(\psi) = \tilde{\lambda}_0(\psi) + \tilde{\lambda}_1(\psi) = 0,
\]

are NL SUSY invariant and those explicitly give NL SUSY invariant relations which connect our NL SUSY actions (3) including apparently (pathological) higher derivative terms of \( \lambda \) with the V-A action (2) described by \( \psi \). \(^\ddagger\)

\(^\ddagger\)Note that if we consider the NL SUSY invariant constraint, \( \tilde{\lambda}_0(\psi) = 0 \), then it gives the unique solution \( \psi = \lambda \) by means of Eq.(11). This is just the case, [ higher derivative terms of \( \lambda \] = 0 in Eq.(3), i.e., \( S(\lambda) = S_{VA}(\lambda) = S_{VA}(\psi) \).
Here we briefly discuss on a simple example of the NL SUSY invariant relations derived from the constraints (15). Indeed, for the case of \(n = 1\) in the above discussion, i.e., for the case where the terms up to the first-order derivative of \(\lambda\) are considered in the fields (5), the composite fields \(\tilde{\lambda}_1(\psi)\) of Eq.(10) are

\[
\tilde{\lambda}_1(\psi) = \left(1 + \delta_\zeta + \frac{1}{2!}\delta_\zeta^2 + \frac{1}{3!}\delta_\zeta^3 + \frac{1}{4!}\delta_\zeta^4\right) i\kappa \frac{1}{2} \partial_\lambda \mid_{\zeta \to -\kappa \psi},
\]

(16)

and the constraints (15) have the following form at leading orders,

\[
\tilde{\lambda}(\psi) = \lambda - \psi + i\kappa \frac{1}{2} \partial_\lambda + i\kappa^2 \bar{\psi} \gamma^a \lambda \partial_a \lambda \\
- \kappa \frac{2}{5} (\bar{\psi} \gamma^a \partial_b \lambda \gamma^b \partial_a \lambda + \bar{\psi} \gamma^a \lambda \partial_a \partial_b \lambda) \\
- \kappa^4 \left(\bar{\psi} \gamma^a \lambda \bar{\psi} \gamma^b \partial_a \lambda \partial_b \lambda + \frac{1}{2} \bar{\psi} \gamma^a \lambda \bar{\psi} \gamma^b \lambda \partial_a \partial_b \lambda\right) \\
- i\kappa^2 \left(\bar{\psi} \gamma^a \partial_c \lambda \bar{\psi} \gamma^b \partial_a \lambda \gamma^c \partial_b \lambda + \bar{\psi} \gamma^a \partial_c \lambda \bar{\psi} \gamma^b \lambda \gamma^c \partial_b \lambda \right. \\
\left. + \frac{1}{2} \bar{\psi} \gamma^a \lambda \bar{\psi} \gamma^b \lambda \partial_a \partial_b \partial_c \lambda \right) + \mathcal{O}(\kappa^6) = 0.
\]

(17)

Solving Eq.(17) with respect to \(\psi\) as a function of \(\lambda\) gives the NL SUSY invariant relation,

\[
\psi = \lambda - \psi + i\kappa \frac{1}{2} \partial_\lambda + i\kappa^2 \bar{\psi} \gamma^a \lambda \partial_a \lambda \\
+ i\kappa^3 \left(\bar{\lambda} \gamma^a \partial \lambda \gamma^b \partial_a \lambda + \bar{\lambda} \gamma^a \partial \lambda \partial_b \lambda\right) \\
+ i\kappa^4 \left(\bar{\lambda} \gamma^a \partial \lambda \bar{\lambda} \gamma^b \partial \lambda + \bar{\lambda} \gamma^a \partial \lambda \bar{\lambda} \gamma^b \partial \lambda \gamma^c \partial_b \lambda\right) + \mathcal{O}(\kappa^5).
\]

(18)

Substituting this into the V-A action (2), we have the following NL SUSY action,

\[
S(\lambda) = S_{VA}(\lambda) + \int d^4x \left[\kappa \frac{2}{5} \partial_\lambda \bar{\lambda} \partial^a \lambda + \frac{i}{2} \kappa \partial_\lambda \bar{\lambda} \gamma^a \partial_\lambda \right] + \mathcal{O}(\lambda^4)
\]

(19)

except for total derivative terms. It can be understood from the higher derivative terms in \(\mathcal{O}(\lambda^2)\) of the action that Eq.(19) includes apparently a (Weyl) ghost field in the higher derivative fermionic field theory (for example, see [16]). However, those terms in \(\mathcal{O}(\lambda^2)\) do not alter the pole structure of the N-G fermion for the on-shell amplitudes because Eq.(19) is equivalent to the standard V-A action \(S_{VA}(\psi)\) of Eq.(2) through the NL SUSY invariant relation (18). Eqs.(18) and (19) are just the relations as discussed in [14] by heuristic arguments.
In the same way, for more general cases of $n \geq 2$ where the terms up to the second-order and the higher derivatives of $\lambda$ are considered in the fields (5) and in $\tilde{\lambda}_1(\psi)$ of Eq.(10), the constraints (15) can be solved unambiguously with respect to $\psi$ as a function of $\lambda$ in all orders. By substituting the obtained general NL SUSY invariant relations as, at $O(\lambda^1)$,

$$\psi = \lambda + \sum_{n \geq 1} \{(i\kappa^{\frac{1}{2}})^n \varphi^n \lambda + O(\kappa^{\frac{3}{2}+2})\}$$

$$= \lambda + i\kappa^{\frac{1}{2}} \varphi \lambda - \kappa \varphi \varphi \lambda + \cdots + \sum_{n \geq 1} O(\kappa^{\frac{3}{2}+2})$$

(20)

into the V-A action (2), we have also the corresponding NL SUSY actions including apparently (pathological) higher derivative terms of $\lambda$,

$$S(\lambda) = S_{VA}(\lambda) + \int d^4x \left[ \frac{i}{2} \sum_{n \geq 1} (i\kappa^{\frac{1}{2}})^n \{ \tilde{\lambda} \varphi^{n+1} \lambda + (-)^n \varphi^n \tilde{\lambda} \varphi \lambda \} \right.$$  

$$+ \frac{i}{2} \sum_{n \geq 1, m=1, \ldots, n} (-)^n (i\kappa^{\frac{1}{2}})^{n+m} \varphi^n \tilde{\lambda} \varphi^{m+1} \lambda \right] + O(\lambda^4)$$

$$= S_{VA}(\lambda) + \int d^4x \left[ -\frac{1}{2} \kappa^{\frac{1}{2}} (\tilde{\lambda} \Box \lambda - \partial_a \tilde{\lambda} \gamma^a \varphi \lambda) \right.$$  

$$+ \frac{i}{2} \kappa (\partial_a \tilde{\lambda} \gamma^a \Box \lambda - \bar{\lambda} \Box \varphi \lambda - \Box \varphi \lambda)$$

$$- \frac{i}{2} \kappa^{\frac{1}{2}} (\varphi \lambda \Box \varphi \lambda - \Box \varphi \lambda) + \frac{i}{2} \kappa^{\frac{3}{2}} \Box \tilde{\lambda} \varphi \lambda + \cdots \right] + O(\lambda^4)$$

(21)

which are equivalent to the standard V-A action. Our results, e.g., Eqs.(20) and (21), are also valid for the arbitrary coefficients $c_n$ in Eq.(5).

To conclude, we have extended the arguments in [14] with respect to the universality of the NL SUSY actions with the N-G fermion to more general cases where $\psi$ in (2) is expanded in terms of $\lambda$ in (3) and its higher derivatives at $O(\lambda^1)$ as Eq.(20). In order to determine higher order terms of Eq.(20) in NL SUSY invariant way, according to the algorithmic procedure given in [15], we have found the composite fields $\tilde{\lambda}(\psi)$ defined by the sum of Eqs.(9) and (10) (or explicitly, Eqs.(11) and (13)) which transform homogeneously under the NL SUSY transformations (1) and (4). Consequently, we have obtained the constraints (15) which connect our NL SUSY actions (3) with the V-A action (2). In our previous work in [14], we have constructed a NL SUSY invariant relation by heuristic arguments for the case where $\psi$ is expanded up to the first-order derivative of $\lambda$ at $O(\lambda^1)$, i.e., $\psi = \lambda + i\kappa^{\frac{1}{2}} \varphi \lambda + O(\kappa^{\frac{3}{2}})$. We have proved in this letter that the NL SUSY invariant relation is derived in Eq.(18) as a solution of the constraint (17). We have also discussed in Eqs.(20) and (21)
that more general NL SUSY invariant relations and the NL SUSY actions including
apparently (pathological) higher derivatives of the N-G fermion can be obtained by
solving the constraints (15) with respect to $\psi$ as a function of $\lambda$ and by substituting
the solutions into the standard V-A action (2).
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