Abstract

Novel control of the emission spectrum, directionality, and polarization from modern thermal infrared microdevices requires accurate measurement of direct emission, which is challenging due to the scaling-down footprints and the uncontrollable radiation noises from surroundings. We realize a significant improvement in signal-to-noise ratio by combining a microscope and a lock-in amplifier with a Fourier transform infrared spectrometer. Based on the nonlinear analytical model of the signal detection process, we explore both global heating and modulated Joule heating to fulfill the potential of our system for noise reduction. Furthermore, the system is ultrasensitive and fast enough to provide a spectral-resolved frequency response of the device. We believe the system together with its analysis of the signal process can be beneficial for next-generation thermal infrared material and device exploration, boosting the applications in infrared imaging, sensing, and energy harvesting.

Introduction

Thermal emission, physically originating from any object with a finite temperature, can be exploited as a fingerprint in extracting material properties and used to measure device temperature distributions based on Stefan-Boltzmann’s law. Physical characteristics of thermal radiation are typically isotropy and incoherence. With the advent of nanotechnology, thermal infrared microdevices, showing strong capabilities in controlling their emission spectrum, directionality, and polarization, can drastically modify these characteristics, and open new opportunities in applications in lighting, sensing, imaging, and energy harvesting. Obtaining thermal emission features of both the materials and the devices is crucial, and, hence, an accurate and precise measurement technique becomes indispensable. Unfortunately, conventional characterization techniques bear inherent disadvantages. An indirect reflectance/transmittance method that relies on Kirchhoff’s law for emissivity measurement, may be unreliable for highly scattering or absorptive samples. Moreover, invalidation of Kirchhoff’s law under non-equilibrium or non-reciprocal conditions leaves a direct emission measurement as the only method to obtain thermal emission properties. However, it becomes increasingly challenging to directly catch weak signals from modern compact devices with scaling-down footprints. This is especially true for signals arising from far-field thermal infrared emission buried in noises due to uncontrollable room-temperature radiation from surroundings. Thus, a technique with a proper noise reduction to increase the signal-to-noise ratio (SNR) is required.

The marriage of a lock-in amplifier (LIA) and a Fourier transform infrared spectrometer (FTIR) for noise filtering is well known in near-field direct thermal radiation measurement with a modulated dithering tip. For far fields, a strong global heating is preferred in boosting emission signal, but gradually becomes impractical since microdevices can suffer damages at such a high temperature or may contain some temperature-sensitive materials. The LIA-based noise reduction for far-field emission was once achieved by driving a piezoelectric stage displacing the emitter periodically at 20 Hz. This slow spatial modulation however still endured 1/f noise to some extent and lengthened data collection time. A recent work then directly measured a device emission spectrum based on a 10 kHz electrical modulation, which generated a full-wave rectified since wave of far-field emission requiring a 20 kHz reference frequency for LIA. However, the cosine waveshape and the double of reference frequency should not be the paradigms for such far-field direct emission measurements. The Joule heating, including both its bias and waveshapes, is critical in improving the SNR based on the nonlinearities during the signal collection process.
Together with the analysis on global heating, we extensively study the underlying physics for the excitation of modulated thermal infrared signal to fulfill the LIA potential in SNR improvement.

Achieving a fast modulation also allows us to characterize another important aspect of thermal emission, its dynamic control. Recently, many thermal infrared devices employed either emissivity or temperature for rapid modulation of thermal emission, whose speed can be up to GHz-level\textsuperscript{12–19}. Their dynamic characterization, however, kept repetitive with its modulation amplitude always as emission power. Often, modulation mechanisms can be different and require measuring their own modulation amplitudes, whose specific dynamic behaviors may be quite different from the emission power, such as a peak shift for emission resonance tuning\textsuperscript{15,20}, an overall spectrum variation for chemical sensing\textsuperscript{21,22}, etc. Our system, on the other hand, can measure the entire emission spectrum for each modulation frequency, and thus provides a spectral-resolved dynamic characterization of the thermal infrared devices. This platform that can simultaneously monitor spectra and dynamic changes of thermal infrared emission is then particularly beneficial for material and device exploration.

In this work, we demonstrate a microscopic lock-in FTIR system to measure the direct thermal emission from an electrically driven metasurface. A microscope is used to visually locate the active region of the metasurface and extract the modulated thermal infrared signal to a FTIR. A LIA then demodulates the signal to significantly reduce optical and electrical noises during the measurement. We first develop an equivalent thermal circuit model for the electrothermal metasurface to demonstrate the full signal detection process from emission generation to the measured output. Based on the system nonlinearities in signal detection, we then focus on the combination of global and Joule heating in boosting the LIA-based SNR improvement. This microscopic lock-in FTIR can then work as an ultrasensitive detector that is fast enough to record spectral-resolved responses of the metasurface in frequency domain.

**Measurement Setup and Signal Detection Processing**

As shown in Figure 1(a), a reflective objective collects the modulated optical signal emitted by a device under test (DUT), and directs the signal to a dichroic beamsplitter through an aperture. Here, the aperture helps to block the noises from structures surrounding the DUT and somehow provides some spatial-resolved capability when the DUT emission is inhomogeneous. The beamsplitter transmits white light (from an external illumination) reflected by DUT to help locate and focus on the active area of DUT using a microscope subsystem (under the charge-coupled device (CCD) camera). The infrared optical signal is at the same time reflected from the beamsplitter into FTIR for interferometry that is modulated by a moving mirror. Each mirror position causes an optical phase difference (OPD) for the light wave interference resulting in a specific light intensity output. The interferogram, the modulated light intensities for all mirror positions (or equivalently for all OPDs), is then recorded by a Mercury-Cadmium-Telluride (MCT) detector. A computer can eventually recover the spectrum based on the Fourier pair of OPD~wavelength after the LIA noise reduction. The interferometer must work under the step-scan mode, which makes each mirror position static allowing a full cycle of the LIA signal processing, to avoid the double modulation issue when coupling the FTIR and the LIA\textsuperscript{23}. The detected electrical signal (interferogram) is fed to the LIA, which works a dynamic noise filter centered at the reference frequency that can be the modulation frequency or its higher order harmonics. The choice of reference frequency is critical in signal detection and straightly related to the way we heat up the DUT. The overall SNR improvement can attribute to two main aspects: the signal increment from the microscope light extraction; the noise reduction from the LIA demodulation.

The corresponding signal processing is then illustrated in Figure 1(b). Starting from a modulation voltage $V(t)$ given by an arbitrary wave generator (AWG), the Joule heat $Q(t)$ generates a dynamic temperature $T(t)$ of the DUT. Following Planck’s law, the thermal radiation $I(\lambda, t)$ is emitted by the DUT and transformed by the MCT detector as an electrical input voltage $V_{\text{in}}(t)$ for the LIA. The finally filtered output voltage $V_{\text{out}}(t)$ represents the emission intensity of the DUT. It is important to check the linearity along the whole signal flow. The Joule heating is nonlinear
from the electrical voltage $V(t)$ to the generated heat $Q(t)$, but can be linear from $Q(t)$ to the DUT temperature $T(t)$ based on heat diffusion. The thermal radiation based on Planck’s law is also nonlinear but its modulated portion can be linearized and extracted from a Taylor expansion when the Joule heating is much lower than the global heating. The lock-in detection is nonlinear in the signal mixing although the following low-pass filtering is linear. These nonlinearities in fact can be beneficial and generate extra signal components that can be utilized for our direct thermal infrared emission measurements, based on which we can find the potential in using the combination of global heating and Joule heating (bias and waveshapes) in improving the SNR.

It is thus important to specify the mathematical formula to describe the signal detection processing. The signal from the modulation voltage $V(t)$ to the thermal emission $I(\lambda, t)$ depends on a specific design (optical, electrothermal) of the DUT. Our case is an electrically driven thermal infrared metasurface based on a gold nanorod array with center-contacted electrode lines, as shown in Figure 2(a), allowing a fast Joule heating to electrically excite the emission without undermining the localized surface plasmonic resonance. The metasurface has a narrowband emissivity resonance at around 5.24 $\mu$m, and its optical details can be found in our recent work$^{24}$. To obtain the temperature $T(t)$ of the metasurface under a given $V(t)$, we can ignore the top surface radiation and natural convection since most heat dissipates down to the substrate. Then the electrothermal design can be modeled as one-dimensional (assumed isothermal in lateral plane) heat diffusion from the metasurface to the substrate, and this vertical dimension can be further lumped as an equivalent thermal circuit, as shown in Figure 2(b). To include the transient thermal response, we separate the substrate into two control volumes, represented by two nodes $T(t)$ and $T_0$, via the dashed line that is approximated by the thermal penetration depth. $T_0$ is the global heating temperature provided by the thermal stage and is chosen as the ground reference temperature. The control volume with $T(t)$ can store energy with the thermal capacitance as $C_t$, or transfer heat to the control volume with $T_0$ (grounded) below via the thermal resistance $R_t$. The metasurface the serves as a heat source $Q(t)$. Via the Kirchhoff’s current law or energy conservation, we can have the dynamic equation

$$C_t \frac{d(T(t) - T_0)}{dt} + \frac{T(t) - T_0}{R_t} = Q(t) \tag{1}$$

where $Q(t) = \frac{V^2(t)}{R}$ is from Joule heating and $R$ is the electrical resistance of the metasurface. This linear first-order ODE can be solved by constructing an exact differential with an integrating factor $\mu(t) = \frac{1}{C_t} e^{R_tC_t t}$. The solution $T(t)$, with the initial condition $T(0) = T_0$, is given by

$$T(t) = T_0 + T_0 e^{-\frac{t}{R_tC_t}} + e^{-\frac{t}{R_tC_t}} \int_0^t e^{\frac{t'}{R_tC_t}} V^2(t') \, dt'$$

$$\tag{2}$$

With the derived temperature $T(t)$ from this simple thermal circuit model of our metasurface or from fully solving the differential heat equation of a general DUT, we can then compute the spectral thermal emission based on Planck’s law in a time-pointwise manner

$$I(\lambda, T(t)) = \epsilon(\lambda) I_{BB}(\lambda, T(t)) \tag{3}$$

where $I_{BB}(\lambda, T(t))$ is the blackbody emission spectrum at temperature $T(t)$ and $\epsilon(\lambda)$ is the emissivity of the metasurface. To simplify the process, we need to separate the time dependence of temperature $T(t)$ as
\[ T(t) = T_0 + T_{DC} + T_m(t) = T_{avg} + T_m(t) \]  

(4)

where \( T_{DC} \) is the time-independent part of temperature generated by the root-mean-square (RMS) of Joule heating power and \( T_m(t) \) is the time-dependent part of temperature for modulated thermal emission. We can further group \( T_0 \) and \( T_{DC} \) as the average temperature \( T_{avg} \) that can be measured from a thermal mapping system. Since \( T_m(t) \ll T_{avg} \), \( I_{BB}(\lambda, T) \) can be Taylor expanded with respect to \( T(t) \) with the linearization as

\[ I(\lambda, t) = \varepsilon(\lambda)I_{BB}(\lambda, T_{avg}) + \varepsilon(\lambda) \frac{\partial I_{BB}}{\partial T}(\lambda, T_{avg})T_m(t) \]  

(5)

This thermal emission (optical signal) together with the optical noises \( I_{ON}(\lambda) \) is then transformed into the electrical voltages by the MCT detector as

\[ V_{in}(t) = r(\lambda)(I(\lambda, T) + I_{ON}) + I_{EN} = r(\lambda)\left[ \varepsilon(\lambda)I_{BB}(\lambda, T_{avg}) + \varepsilon(\lambda) \frac{\partial I_{BB}}{\partial T}(\lambda, T_{avg})T_m(t) + I_{ON}(\lambda) \right] + I_{EN} \]  

(6)

where \( r(\lambda) \) is the response function whose wavelength dependence comes from both MCT responsivity and all the optical components along the optical path. \( I_{ON}(\lambda) \) is the thermal infrared (optical) background noise including the room-temperature environmental thermal background surrounding DUT. \( I_{EN} \) is the unavoidable electrical noise generated during the data collection process, which becomes critical when we are extracting weak signals from micro-devices.

**Global and Joule Heating**

To demonstrate noise reduction achieved by the LIA, we first measure the spectral density of the input \( V_{in}(t) \), shown in Figure 3(a), via a fast Fourier transform analyzer. We clearly observe the drastically increasing noises when the modulation frequency approaches to 0 Hz. However, with a modulation frequency of 2600 Hz, the signals are moved to a frequency band with only small white noise. After the signals are mixed with a reference of the LIA, the white noise can be further reduced by a low-pass filter. With increasing unmodulated (0 Hz) global temperatures, the signal intensities at 2600 Hz still increase because the global temperature \( T_0 \) makes \( I_{BB}(\lambda, T_{avg}) \), i.e., \( T_{BB}(\lambda, T_0 + T_{DC}) \) overlap better with \( \varepsilon(\lambda) \) resulting in a larger modulated emission, as indicated by Eq.(6) with the term \( \frac{\partial I_{BB}}{\partial T}(\lambda, T_{avg})T_m(t) \).

This increment can even overcome \( T_m(t) \) decrease due to the increase of \( R \) that depends on \( T_0 \), as shown in Eq.(2). The drawback of the global heating is obviously also increasing the non-modulated part of signal, which has no difference from the non-modulated noises \( I_{ON} \) or \( I_{EN} \) to the LIA. Fortunately, the dynamic reserve, i.e., the ratio of the largest tolerable noise to the full-scale signal, of modern LIAs typically can tolerate this amount of non-modulated global heating.

After clarifying the global heating mode, we then specify the Joule heating mode \( Q(t) \), which plays a fundamental role in our LIA signal detection. More specifically, \( Q(t) \) depends on the DC bias and AC waveshapes. To figure out the influence of the DC bias on the final detected signal, we can choose the simplest cosine waveshape with a DC bias as \( V(t) = V_p \cos(\omega t) + V_0 \). Then Joule heat is
where we use complex representation for convenience. Note we keep the complex conjugate terms to fully express the real physical quantity since the measurement system is not all linear, as discussed in Figure 1(b). The steady-state temperature of DUT is then given by Eq. (2) as

\[ T(t) = T_0 + \frac{2V_p V_0 R_t}{R} \frac{1}{1 + \omega^2 R_t^2 C_t^2} \left( \frac{e^{i\omega t} + e^{-i\omega t}}{2} + \omega R_t C_t \frac{e^{i\omega t} - e^{-i\omega t}}{2i} \right) \]

\[ + \frac{V_p^2 R_t}{2R} \frac{1}{1 + 4\omega^2 R_t^2 C_t^2} \left( \frac{e^{2i\omega t} + e^{-2i\omega t}}{2} + 2\omega R_t C_t \frac{e^{2i\omega t} - e^{-2i\omega t}}{2i} \right) + \left( \frac{V_p^2}{2R} + \frac{V_0^2}{R} \right) R_t \]

\[ (8) \]

After the substitution of \( T(t) \) into Eq. (6), the LIA input \( V_{in}(t) \) is then mixed with a reference \( V_r(t) = e^{-i(\omega_r t + \Delta \phi)} \) for a dual-phase demodulation, based on which the fixed phase difference \( \Delta \phi \) between \( V_{in}(t) \) and \( V_r(t) \) can be finally cancelled out29.

\[ V_{mix}(t) = V_{in}(t) \cdot V_r(t) \]

\[ = r \Re \frac{\partial I_{BB}}{\partial T} \left( \frac{2V_p V_0 R_t}{R} \frac{1}{1 + \omega^2 R_t^2 C_t^2} \left( \frac{e^{i(\omega - \omega_r) t} + e^{-i(\omega + \omega_r) t}}{2} + \omega R_t C_t \frac{e^{i(\omega - \omega_r) t} - e^{-i(\omega + \omega_r) t}}{2i} \right) e^{-i\Delta \phi} \right) \]

\[ + \left( \frac{V_p^2 R_t}{2R} + \frac{V_0^2}{R} R_t \right) e^{-i\omega t} e^{-i\Delta \phi} \]

\[ (9) \]

If we select \( \omega \) term as \( \omega_r = \omega \), the magnitude of the DC output after the low-pass filter is

\[ V_{out,\omega} = r(\lambda) e(\lambda) \frac{\partial I_{BB}}{\partial T} \left( \lambda, T_{avg} \right) \frac{V_p V_0 R_t}{R} \frac{1}{\sqrt{1 + (2\pi f)^2 R_t^2 C_t^2}} \]

\[ (10) \]

and if selecting \( 2\omega \) term as \( \omega_r = 2\omega \), we have the output magnitude

\[ V_{out,2\omega} = r(\lambda) e(\lambda) \frac{\partial I_{BB}}{\partial T} \left( \lambda, T_{avg} \right) \frac{V_p^2 R_t}{4R} \frac{1}{\sqrt{1 + 4(2\pi f)^2 R_t^2 C_t^2}} \]

\[ (11) \]

We can also observe that the unmodulated portion of thermal emission together with the noises are always at \( \omega_r \neq 0 \), and finally filtered out.

Therefore, it is interesting to find that we can use the first harmonic for the Joule heating signal detection, unlike the previous studies which always focus on the full wave rectified cosine wave with \( 2\omega \) for reference frequency. More importantly, the filtered output signal \( V_{out,\omega} \) is proportional to the bias \( V_0 \) and it means we can go beyond the power limit of the AWG by connecting a DC source in series to it when more power is needed to improve SNR. To examine the effect of increasing \( V_0 \) on \( V_{out,\omega} \) and \( V_{out,2\omega} \), we measure the emission peaks at 5.24 \( \mu \)m of the
metasurface using our system with $T_0 = 125 \, ^\circ C$ and $f = 2600\,Hz$. Since $T_0 \gg T_{DC}$ and $f \ll f_{3dB} = \frac{1}{2\pi R_tC_t}$ where $f_{3dB}$ is the DUT cutoff frequency and can be verified in our later frequency response measurements, we can approximate that $T_{av\_g} \approx T_0$ and $2\pi f R_tC_t = 0$. As shown in Figure 3(b), with increasing $V_0$, the emission power $V_{out,\omega}$ increases linearly while the $V_{out,2\omega}$ keeps approximately constant since $V_0$ is unchanged. The crossing point of equal emission peak with about $V_p = 1\,V$ and $V_0 = 0.25\,V$ is well verified by theoretical predictions calculated using Eq.(10) for $V_{out,\omega}$ and Eq.(11) for $V_{out,2\omega}$. The above analysis can also be verified by choosing two inputs: a cosine pulse train with $V_p = 1\,V$ and $V_0 = 1\,V$ when the LIA picks up the signal with $\omega$, and this should be the same as a cosine wave with $V_p = 2\,V$ and $V_0 = 0\,V$ when the LIA picks up the signal with $2\omega$, as their spectra shown in Figure 3(c). Although counterintuitive, the DC Joule heating is fundamentally different from the global heating since it contributes to the modulated signal based on its nonlinear electrical interference with the AC joule heating as estimated in Eq.(7).

The output signal can also benefit from the waveshape of the modulation. If we change the cosine pulse train to a square pulse train defined as

$$V(t) = V_sD + \sum_{n=1}^{N=\infty} \frac{2V_s}{n\pi} \sin(nD\pi) \cos(n\omega t)$$

(12)

where $D$ is the duty cycle and $V_s$ is the pulse height. We can follow the same process and obtain the DUT temperature $T(t)$ as

$$T(t) = T_0 + T_{DC} + T_\omega(t) + \ldots$$
$$= T_0 + \left(\frac{V_s^2D^2}{R} + \frac{2V_s^2}{\pi R} \sum_{n=1}^{\infty} \frac{\sin^2(nD\pi)}{n^2\pi^2}\right)R_t$$
$$+ \frac{R_t}{\pi} \left(\frac{4V_s^2D}{\pi} \sin(D\pi) + \frac{V_s^2}{R_t} \sin(Dn\pi) \sin(2Dn\pi)\right)$$
$$+ \frac{2V_s^2}{\pi^2} \sum_{n=2}^{N=\infty} \left[\frac{1}{n(n-1)} \sin(nD\pi) \sin((n-1)D\pi)\right]$$
$$+ \frac{1}{n(n+1)} \sin(nD\pi) \sin((n+1)D\pi) \right) \left[1 + \frac{1}{1 + \omega^2 R_t^2 C_t^2} \left(\frac{e^{i\omega t} + e^{-i\omega t}}{2} + \omega R_tC_t \frac{e^{i\omega t} - e^{-i\omega t}}{2i}\right)\right] + \ldots$$

(13)

and the magnitude of the $\omega_r = \omega$ output voltage as

$$S_{out,\omega} = r(\lambda)\varepsilon(\lambda) \frac{\partial I_{BB}}{\partial T} \left(\lambda, T_{av\_g}(D)\right) \frac{R_t}{2R} \left(\frac{4V_s^2D}{\pi} \sin(D\pi) + \frac{V_s^2}{\pi^2} \sin(Dn\pi) \sin(2Dn\pi)\right)$$
$$+ \frac{2V_s^2}{\pi^2} \sum_{n=2}^{N=\infty} \left[\frac{1}{n(n-1)} \sin(nD\pi) \sin((n-1)D\pi)\right]$$
$$+ \frac{1}{n(n+1)} \sin(nD\pi) \sin((n+1)D\pi) \right) \left[1 + \frac{1}{1 + (2\pi f)^2 R_t^2 C_t^2}\right]$$

(14)

where the average temperature $T_{av\_g}(D)$ is also a function of $D$ as
The importance here is that our dynamic response characterization is spectral resolved, i.e., for each modulation frequency, we can obtain the full spectrum of the emission. This can be particularly useful when the modulation depth is no longer the emission power. For example, some tunability may induce the emission peak shift, and our

\[ T_{\text{avg}}(D) = T_0 + \left( \frac{V_p^2 D^2}{R} + \frac{2V_p^2}{R} \sum_{n=1}^{N=\infty} \frac{\sin^2(nD)}{n^2\pi^2} \right) R_t \]  

(15)

To compare the output of the cosine pulse train with \( V_p = V_0 = 1.0 \text{ V} \), we can set \( D = 50\% \) and \( V_s = 2V_p = 2.0 \text{ V} \) for the square pulse train which has the same period and pulse height of the cosine pulse train except for the waveshape. A \( \frac{\pi}{4} \) factor increment in output signal is expected and is clearly observed in Figure 3(c) with the emission peak ratio \( \frac{S_{\text{out,lo}}}{V_{\text{out,lo}}} \) at 5.24 µm (at gray dashed line) about 1.2. More importantly, the square waveshape introduces the duty cycle \( D \), an additional control factor that can increase the SNR. Under a given global heating temperature \( T_0 = 125^\circ\text{C} \), a modulation frequency \( f = 2600\text{Hz} \), and a pulse height \( V_s = 3 \text{ V} \), with decreasing \( D \), the dropping speed of \( S_{\text{out,lo}} \) and \( T_{\text{avg}} \) are quite different. As shown by the normalized data points in Figure 3(d), the decreasing slope of \( S_{\text{out,lo}} \) is initially smaller than that of \( T_{\text{avg}} \), but gradually increases and finally surpasses the decreasing slope of \( T_{\text{avg}} \) that is approximately linear. The average temperatures \( T_{\text{avg}} \) are measured by the emissivity-compensated thermal mapping system. These behaviors are well predicted by our Eq. (14) and Eq. (15) as the red-dashed line \( S_{\text{out,lo}}(D, T_{\text{avg}}) \) and blue-dash-dot line \( T_{\text{avg}}(D) \), respectively. The curve of \( S_{\text{out,lo}}(D, T_{\text{avg}}) \) includes the measured \( T_{\text{avg}} \) for each duty cycle. Furthermore, the decreasing rate, when \( S_{\text{out,lo}}(D, T_{\text{avg}}) \) and \( T_{\text{avg}}(D) \) are the same, can be computed approximately as 35% (gray-dashed lines). Therefore, there is a window of duty cycle \( D \) reducing from 50% to 35% we can significantly reduce the DUT average temperature but keep the measured signal less reduced. It can be beneficial not only to protect the DUT from high power injection (because either of the burn-down or of the temperature-sensitive components) but also suppress the noise-equivalent unmodulated emission from \( T_{\text{DC}} \). It is also interesting to mention that we use \( N = 13 \) in our computation since the number of harmonics that can be included for a square pulse waveshape depends on the DUT response speed. More specifically, \( Nf \leq f_{3\text{dB}} \) where \( f_{3\text{dB}} = 33.275 \text{ kHz} \) is the cutoff frequency from our next dynamic measurements. Thus, the beneficial window of duty cycle we obtain also depends on the DUT response speed that can cause the deformation of the square waveshape. It is then possible to further extend this window by choosing an appropriate waveshape of the pulse train and lower the modulation frequency \( f \) for a device with a given cutoff frequency \( f_{3\text{dB}} \).

**Spectral-Resolved Frequency Response**

To characterize the frequency response of the DUT, we choose the square pulse train with the duty cycle \( D = 50\% \) based on which we can benefit from the \( \frac{\pi}{4} \) increment factor compared to a conventional cosine pulse train. At the same time, we can get rid of the summation of \( N \) harmonic terms from the waveshape deformation that can complicates the frequency response measurement, as discussed in Figure 3(d). The measured output voltage is then

\[ S_{\text{out,lo}} = r(\lambda)\epsilon(\lambda) \frac{\partial I_{\text{BB}}}{\partial T}(\lambda, T_{\text{avg}}) \frac{V_p^2 R_t}{\pi R} \frac{1}{\sqrt{1 + (2\pi f)^2 R_t^2 C_t^2}} \]  

(16)

At each \( f \), we record the entire emission spectrum based on our system, as shown in Figure 4(a). The dynamic response can be characterized if we keep increasing the modulation frequency \( f \) up to the DUT cutoff frequency \( f_{3\text{dB}} \). With the curve fitting by Eq. (16), as indicated by Figure 4(b), we can obtain the thermal time constant \( R_tC_t = 4.78 \mu\text{s} \) and the related cutoff frequency as \( f_{3\text{dB}} = 33.275 \text{ kHz} \).

The importance here is that our dynamic response characterization is spectral resolved, i.e., for each modulation frequency, we can obtain the full spectrum of the emission. This can be particularly useful when the modulation depth is no longer the emission power. For example, some tunability may induce the emission peak shift, and our
spectral-resolved response measurement can directly monitor the peak changing, which may not change simultaneously with the emission power. With the help of LIA, our lock-in FTIR system can be ultrasensitive, and its measurable frequency range is limited by the MCT detector (> 20MHz) and the LIA bandwidth (50MHz). The time constant of the low-pass filter, which is typically long for noise reduction, has nothing to do with this frequency-domain measurement since the signal at this stage is at 0 Hz after mixing. Furthermore, this frequency range can be extended to the GHz-level based on a high-speed circuit model\textsuperscript{15,20}. The electrothermal model of our metasurface is currently lumped as an equivalent thermal circuit model, and it is possible to include the spatial thermal analysis, such as the lateral temperature distribution and vertical thermal penetration depth. With the spectral-resolved dynamic thermal emission, we can inversely obtain key information in material properties and device performances of the thermal infrared microdevices.

Conclusion

In summary, with the combination of a microscope, LIA, and FTIR, we propose and demonstrate a measurement technique, microscopic lock-in FTIR, to measure the direct emission of an electrothermal metasurface. With an analysis of the nonlinear signal detection processing from the initial heat generation to the final measured signal output, our method exploits the combination global and modulated Joule heating for the SNR improvement. A higher global heating temperature tends to make the device emissivity overlapping better with the blackbody emission resulting in a higher emission. The bias and waveshapes of the Joule heating can be optimized to achieve a modulated emission whose first harmonic component can be used for LIA signal detection under a limited power injection. In particular, a window of duty cycle, introduced by a square pulse train modulation, allows us to reduce the device average temperature for the device protection and the noise-equivalent emission suppression while maintaining the measured signal output. Furthermore, our system can be applied for a spectral-resolved frequency-domain characterization of the device response. It is ultrasensitive and fast enough to monitor the entire spectra for each modulation frequency providing more key information about the device performance. We expect that our microscopic lock-in FTIR system will play a significant role in characterization of the modern thermal infrared microdevices.

References

1. Li, W. & Fan, S. Nanophotonic control of thermal radiation for energy applications [invited]. Opt. Express, OE \textbf{26}, 15995–16021 (2018).
2. Baranov, D. G. et al. Nanophotonic engineering of far-field thermal emitters. Nat. Mater. \textbf{18}, 920–930 (2019).
3. Li, Y. et al. Transforming heat transfer with thermal metamaterials and devices. Nat Rev Mater \textbf{6}, 488–507 (2021).
4. Xiao, Y. et al. Measuring thermal emission near room temperature using Fourier-transform infrared spectroscopy. Phys. Rev. Applied \textbf{11}, 014026 (2019).
5. Xiao, Y. et al. Precision measurements of temperature-dependent and nonequilibrium thermal emitters. Laser & Photonics Reviews \textbf{14}, 1900443 (2020).
6. De Wilde, Y. et al. Thermal radiation scanning tunnelling microscopy. Nature \textbf{444}, 740–743 (2006).
7. Jones, A. C. & Raschke, M. B. Thermal infrared near-field spectroscopy. Nano Lett. \textbf{12}, 1475–1481 (2012).
8. Huth, F. Nano-FTIR nanoscale infrared near-field spectroscopy. (Universidad del Pais Vasco - Euskal Herriko Unibertsitatea, 2015).
9. Schuller, J. A., Taubner, T. & Brongersma, M. L. Optical antenna thermal emitters. Nature Photon \textbf{3}, 658–661 (2009).
10. Au, Y.-Y., Skulason, H. S., Ingvarsson, S., Klein, L. J. & Hamann, H. F. Thermal radiation spectra of individual subwavelength microheaters. Phys. Rev. B \textbf{78}, 085402 (2008).
11. Li, C. et al. Near-field and far-field thermal emission of an individual patch nanoantenna. *Phys. Rev. Lett.* **121**, 243901 (2018).
12. Wojszvzyk, L. et al. An incandescent metasurface for quasimonochromatic polarized mid-wave infrared emission modulated beyond 10 MHz. *Nat Commun* **12**, 1492 (2021).
13. Liu, X., Li, Z., Wang, Z., Yun, H. S. & Shen, S. Design and analysis of electrothermal metasurfaces. Preprint at https://doi.org/10.48550/arXiv.2206.13315 (2022).
14. Yao, Y. et al. Electrically tunable metasurface perfect absorbers for ultrathin mid-infrared optical modulators. *Nano Lett.* **14**, 6526–6532 (2014).
15. Zeng, B. et al. Hybrid graphene metasurfaces for high-speed mid-infrared light modulation and single-pixel imaging. *Light Sci Appl* **7**, 51 (2018).
16. Shiue, R.-J. et al. Thermal radiation control from hot graphene electrons coupled to a photonic crystal nanocavity. *Nat Commun* **10**, 109 (2019).
17. Wang, Y. et al. Electrical tuning of phase-change antennas and metasurfaces. *Nat. Nanotechnol.* **16**, 667–672 (2021).
18. Zhang, Y. et al. Electrically reconfigurable non-volatile metasurface using low-loss optical phase-change material. *Nat. Nanotechnol.* **16**, 661–666 (2021).
19. Abdollahramezani, S. et al. Electrically driven reprogrammable phase-change metasurface reaching 80% efficiency. *Nat Commun* **13**, 1696 (2022).
20. Fan, K., Suen, J., Wu, X. & Padilla, W. J. Graphene metamaterial modulator for free-space thermal radiation. *Opt. Express, OE* **24**, 25189–25201 (2016).
21. Hinkov, B. et al. A mid-infrared lab-on-a-chip for dynamic reaction monitoring. *Nat Commun* **13**, 4753 (2022).
22. Lochbaum, A. et al. On-chip narrowband thermal emitter for mid-IR optical gas sensing. *ACS Photonics* **4**, 1371–1380 (2017).
23. Zhang, Y. G. et al. Fourier transform infrared spectroscopy approach for measurements of photoluminescence and electroluminescence in mid-infrared. *Review of Scientific Instruments* **83**, 053106 (2012).
24. Liu, X. et al. Electrically driven thermal infrared metasurface with narrowband emission. Preprint at https://doi.org/10.48550/arXiv.2208.10484 (2022).
25. Zurich Instruments. Principles of lock-in detection. https://www.zhinst.com/en/resources/principles-of-lock-in-detection (2019).
Figure 1: (a) The measurement setup of the microscopic lock-in FTIR system. (b) The signal detection processing from the heat generation to the measured output signal. The SNR improvement of the system mainly applies the microscope subsystem for signal extraction and the lock-in FTIR subsystem for noise reduction. The nonlinearities in Joule heating and lock-in detection can generate additional signal components that can be beneficial for the signal detection once we optimize the global and Joule heating.
Figure 2: (a) Front view of the electrothermal metasurface based on a gold nanorod array (zoom-in) for narrowband emission and center-contacted electrode lines (zoom-in) for Joule heating. (b) Cross-section view of the device layers with an equivalent thermal circuit model for its electrothermal response. The optical and electrothermal analyses are important to obtain the signal input $V_{in}$ for the LIA from the initial modulation voltage $V(t)$. 
Figure 3: (a) The spectral density of the input signal $V_{in}$ under different global heating temperatures $T_0$. The increment of $V_{in}$ with a higher $T_0$ results from the better overlapping of the metasurface emissivity and the blackbody emission. (b) The peak emission of 1st-harmonic signal $V_{out,ω}$ and 2nd-harmonic signal $V_{out,2ω}$ from the LIA under different DC bias $V_0$. The SNR of $V_{out,ω}$ can benefit from the increasing $V_0$. (c) The equal spectra of $V_{out,ω}$ and $V_{out,2ω}$ by choosing appropriate DC bias $V_0$ and peak voltage $V_p$. The 1st harmonic signal can be used for the thermal infrared direct emission measurement based on the nonlinear electrical interference in Joule heating. In addition, the waveshape can further boost the SNR, as indicated by the $\frac{4\pi}{\pi}$ factor increment at resonance (gray-dashed line) of $S_{out,ω}$ with a square pulse train modulation compared to $V_{out,ω}$ under a cosine pulse train. (d) The square pulse train also introduces the duty cycle $D$ in modulation. A window of $D$ reducing from 50% to 35% we can significantly reduce the DUT average temperature but keep the measured signal less reduced.
Figure 4: (a) The spectra of emission at each modulation frequency. (b) The emission peak under different modulation frequencies. A curve fitting based on the analytical model $S_{\text{out}, \omega}$ can obtain the device thermal time constant as 4.78 $\mu$s. This spectral-resolved frequency-domain dynamic characterization can provide more key information in material properties and device performances of the thermal infrared microdevices.