Charmed Meson Spectroscopy and Matrix Elements with an $O(a)$-Improved Clover Fermion Action

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We present preliminary results for the spectrum and decay matrix elements for heavy-light and heavy-heavy mesons, obtained on the 64-node Meiko Computing Surface at the University of Edinburgh.

Quark propagators are computed with an $O(a)$-improved fermion action on $24^3 \times 48$ lattices at $\beta = 6.2$, using three values of the quark mass up to around the strange quark mass, and four values of the quark mass in the region of the charm quark mass. We compare results for the hyperfine splitting in charmonium with those obtained using the conventional Wilson fermion action and find that the splitting is $1.83(15)$ times larger with the improved action. Our measurements of $f_B$ indicate non-scaling corrections of the order of 20% to the Heavy Quark Effective Theory expectation. A comparison is made with results obtained on $16^3 \times 48$ lattices at $\beta = 6.0$.

1. INTRODUCTION

This contribution represents the talk “Latest Results from the UKQCD Collaboration” presented at Lattice 92.

As the masses of hadrons in lattice gauge simulations approach the inverse lattice spacing, $a^{-1}$, quantities are expected to become increasingly sensitive to discretisation errors. Thus it is in this regime, corresponding in present simulations to quarks with masses of the charm and above, that we expect differences between an $O(a)$-improved fermion action and the standard Wilson fermion action to be most apparent.

In this talk I shall describe the recent studies by the UKQCD Collaboration of mesons containing propagating charm quarks. The talk begins with a brief description of the computational details. Then a comparison is made of the hyperfine splitting in charmonium between the standard Wilson action, and the $O(a)$-improved clover action. For the clover action, this investigation is extended to the $D$ and $B$ systems. Finally, measurements are made of the pseudoscalar decay constant, $f_P$, for heavy-light systems. By determining the behaviour of $f_P\sqrt{M_P}$ as a function of the pseudoscalar mass, $M_P$, an estimate is made of the magnitude of the non-scaling terms in the Heavy Quark Effective Theory (HQET) in the region of the $B$-meson mass.

2. COMPUTATIONAL DETAILS

Clover quark propagators were generated on quenched $24^3 \times 48$ lattices at $\beta = 6.2$, using three values of the quark mass up to the region of the strange quark mass, with hopping parameters $\kappa_l = 0.14144, 0.14226$ and 0.14262. In addition, propagators were generated at four values of the quark mass in the region of the charm quark mass, with hopping parameters $\kappa_h = 0.133, 0.129, 0.125$ and 0.121; the charm quark mass corresponds to $\kappa_h \simeq 0.129$. The preliminary results presented in this write-up are based on an analysis of 33 configurations, the results in the talk being based on an analysis of 21 configurations.

The light quark propagators were generated using local sources and sinks (LL). The charm quark propagators were computed from smeared sources, to both local sinks (LS) and smeared sinks (SS), using a Jacobi-smearing algorithm with a smearing radius $r \simeq 5$.

The comparison of the hyperfine splitting in charmonium between the Wilson and clover fermion actions was obtained on a common sam-
ple of 18 configurations, using local sources and sinks. All fits to the correlators are performed taking into account the correlations between different time slices, with the error bars obtained using a bootstrap procedure.

3. HYPERFINE SPLITTING

Quenched calculations with the standard Wilson fermion action typically find values of vector-pseudoscalar mass splittings, \( m_{D^*} - m_D \) and \( m_{J/\psi} - m_{\eta_c} \), which are much too small, indeed typically by a factor of 2 and 4 respectively at \( \beta = 6.2 \). The clover fermion action is related to the standard Wilson fermion action through the addition of a term,

\[
S^C_F = S^W_F - i \frac{\kappa}{2} \sum_{x,\mu,\nu} \bar{q}(x) F_{\mu\nu}(x) \sigma_{\mu\nu} q(x). \tag{1}
\]

The hyperfine splitting is expected to be sensitive to the magnetic moment form of this additional term.

To compare the values for the hyperfine splitting in charmonium, we chose \( \kappa = 0.135 \) for the Wilson action, and \( \kappa = 0.129 \) for the clover action, yielding approximately the same value of the pseudoscalar mass, \( m_{\eta_c} \), for the two actions. The measured values of the vector and pseudoscalar masses are shown in Table 1. Setting the scale from the string tension gives \( a^{-1} = 2.73(5) \) GeV, and we find the values of the masses quoted in the table to be within a few percent of their physical values. However, the mass splitting quoted in the third column is very different for the two actions, and we find that

\[
\frac{(m_{J/\psi} - m_{\eta_c})^\text{clover}}{(m_{J/\psi} - m_{\eta_c})^\text{Wilson}} = 1.83 \pm \frac{3}{15} \tag{2}
\]

Using the value of \( a^{-1} \) quoted above, we obtain

\[
m_{J/\psi} - m_{\eta_c} = 28 \pm \frac{2}{2} \text{ MeV} \quad (\text{Wilson})
\]

\[
m_{J/\psi} - m_{\eta_c} = 52 \pm \frac{3}{4} \text{ MeV} \quad (\text{clover}). \tag{3}
\]

These values are to be compared to the experimental value of 117(2) MeV. The corresponding values found by El-Khadra et al. using a mean field calculation of the coefficient of the clover term in eq. 1, are: 51(3) MeV at \( \beta = 5.7 \), 62(4) MeV at \( \beta = 5.9 \), and 68(5) MeV at \( \beta = 6.1 \), where the value of the string tension is obtained from the 1P-1S mass splitting. Extrapolating linearly in \( a^2 \) to the continuum limit, they obtain

\[
m_{J/\psi} - m_{\eta_c} = 73 \pm 10 \text{MeV}. \tag{4}
\]

This is still far short of the experimental value. However, it has been argued that the remaining discrepancy is consistent with the corrections arising from the quenched approximation.

Figure 1. The hyperfine mass splitting \( M_V - M_P \) against \( 1/M_P \), in lattice units, is shown both for heavy-heavy mesons (squares), and for heavy-light mesons after extrapolation of \( \kappa_l \) to \( \kappa_{\text{crit}} = 0.1431 \) (crosses). The fits to the heavy-heavy and heavy-light data are indicated by the solid and dashed lines respectively.

Figure 2 shows \( M_V - M_P \) (where \( V \) and \( P \) represent the vector and pseudoscalar respectively) for both heavy-light and heavy-heavy mesons obtained using the clover action on the 33 configurations. Fits are over the range 10 to 16, using the LS charm propagators. For both the heavy-light and heavy-heavy systems, linear behaviour with \( 1/M_P \) is apparent; the non-zero intercept of the fits is presumably a manifestation of lattice artifacts. Setting \( a^{-1} = 2.73 \) GeV, we obtain

\[
m_{D^*} - m_D = 102 \pm 10 \text{MeV}
\]
Table 1
Masses (in lattice units) of the pseudoscalar and vector heavy-heavy mesons.

|      | m_{\eta c} | m_{J/\psi} | (m_{J/\psi} - m_{\eta c}) |
|------|------------|------------|---------------------------|
| Wilson | 1.066 ± 0.4 | 1.076 ± 0.5 | 0.0104 ± 0.7 |
| clover | 1.071 ± 0.4 | 1.088 ± 0.5 | 0.0190 ± 0.8 |

\[ m_{B^*} - m_B = 14 \pm \frac{14}{5} \text{ MeV}, \] (5)

to be compared to the experimental values of 146(1) MeV and 46(3) MeV respectively. An analysis using the clover action at \( \beta = 6.0 \), for \( \kappa_l \) in the region of the strange quark mass, yields

\[ m_{D^*} - m_D = 76 \pm \frac{17}{5} \text{ MeV}, \] (9)

clearly indicating a remaining sensitivity to the lattice spacing, \( a \).

4. DECAY CONSTANT \( f_P \)

We extract the heavy-light pseudoscalar decay constant \( f_P \) through a fit to the ratio,

\[
\frac{\sum \langle A_\mu^4(\vec{x},t)P^\dagger_S(0) \rangle}{\sum \langle P_S(\vec{x},t)P^\dagger_S(0) \rangle} \sim \frac{f_P}{Z_S} \tanh M_P(T/2-t) \] (6)

where \( A_\mu \) is the axial vector current, \( P = \bar{\psi}\gamma_5\psi \), and \( Z \) is defined through

\[ Z = \langle 0|P|P \rangle. \] (7)

\( S \) and \( L \) refer to quantities constructed from smeared and local quark propagators respectively. \( Z_S \) and \( M_P \) are determined from the fit to \( \langle P_S(\vec{x},t)P^\dagger_S(0) \rangle \). The fits both to eq. 6 and to the pseudoscalar correlator are over the range 10 - 16. The quality of the data for eq. 6, together with the fits to the data, are illustrated in Figure 2.

The expected behaviour from the HQET is

\[ f_P \sqrt{M_P} \sim A + O(1/M_P) \] (8)

where \( A \) is a constant, up to logarithmic terms. Figure 3 shows \( f_P \sqrt{M_P} \) against \( M_P \) for all \( \kappa_l \) and \( \kappa_h \), together with the value after the extrapolation of \( \kappa_l \) to \( \kappa_{crit} = 0.1431 \). \[ Z \]

We determine the behaviour of \( f_P \sqrt{M_P} \) with \( 1/M_P \), at \( \kappa_{crit} \), from a correlated fit to

\[ f_P \sqrt{M_P} = a_1 + \frac{a_2}{M_P} + \frac{a_3}{M_P^2} \] (9)

In lattice units, a linear fit to the three heaviest points yields,

\[
\begin{align*}
    a_1 &= 0.106 \pm 0.6 \\
    a_2 &= -0.029 \pm 0.3 \\
    a_3 &= 0.018 \pm 0.4
\end{align*}
\] (10)

with \( \chi^2/\text{dof} = 2.4/1 \), while a quadratic fit to all four points yields

\[
\begin{align*}
    a_1 &= 0.120 \pm 0.8 \\
    a_2 &= -0.053 \pm 0.9 \\
    a_3 &= 0.018 \pm 0.4
\end{align*}
\] (11)

with \( \chi^2/\text{dof} = 0.1/1 \). A linear fit to all four points produces a noticeably poor fit, with \( \chi^2/\text{dof} = 12/2 \).

Using the linear fit to the heaviest three points, we obtain

\[ f_D = 0.204 \pm 0.6 \left( \frac{a^{-1}}{2.73} \right) \text{ GeV} \]
Figure 3. $f_P \sqrt{M_P}$ against $1/M_P$ in lattice units is shown for all $\kappa_l$ and $\kappa_h$ (squares). Also shown are the values after linear extrapolation of $\kappa_l$ to $\kappa_{\text{crit}} = 0.1431$ (crosses). The solid and dotted lines represent a quadratic fit to all four points, and a linear fit to the heaviest three points respectively, at $\kappa_{\text{crit}}$.

\[ f_B = 0.174 \pm 0.02 \left( \frac{a^{-1}}{2.73} \right) \text{ GeV}, \quad (12) \]

while from the quadratic fit we obtain

\[ f_D = 0.198 \pm 0.02 \left( \frac{a^{-1}}{2.73} \right) \text{ GeV} \]
\[ f_B = 0.183 \pm 0.02 \left( \frac{a^{-1}}{2.73} \right) \text{ GeV}, \quad (13) \]

where we take $Z_A = 1 - 0.02g^2$, with $g$ the bare coupling. The renormalisation coefficient, $Z_A$, for the clover action is much closer to unity than that for the Wilson action ($Z_A^W \approx 1 - 0.132g^2$). Thus the uncertainty due to the choice of expansion parameter \[ [1] \] is only about 1% for the clover action, compared to about 10% for the standard Wilson action.

The determination of $f_B$ is clearly more sensitive to the form of the extrapolation than that of $f_D$. An analysis at $\beta = 6.0$ \[ [1] \], using a hopping parameter expansion of the heavy quark propagator, favours a quadratic fit, yielding similar coefficients (in physical units) to those quoted in eq. \[ [1] \]. The non-scaling corrections to $f_B$ are approximately 15% and 20% of the scaling expectation, obtained from the constant term $a_1$, for the linear and quadratic fits respectively.

5. CONCLUSIONS

The improvement in the measured value of the hyperfine splitting both for heavy-heavy and heavy-light mesons using an $O(a)$-improved action is very encouraging, and the remaining discrepancies with experiment may be due to the effects of quenching.

The preliminary results for $f_D$ and $f_B$ presented here need further refinement, in particular to take account of the uncertainties in the lattice spacing. Nevertheless, they indicate that non-scaling contributions to the HQET predictions are substantial, even at the mass of the $B$ meson. It is particularly important to measure the static results, with light quarks computed using the clover action, on the same set of configurations. A consistent extrapolation between the propagating quark results presented here and the static predictions would provide powerful evidence for the reliability of lattice studies of heavy quarks.

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