Controlling the Size of Popcorn

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Abstract

We present a thermo-statistical model of popcorn production and propose a way to control the final size of the popcorn by monitoring only the chamber pressure.

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How can one control the size of popcorn? The food industry may have ample reason to find ways to double or triple the size of popcorn, because large popcorn may be pleasing to the eyes, possibly energy saving in its mass production, and thus profitable to producers and consumers alike. The purpose of this note is to examine the physics of popcorn production based on a few thermodynamic principles and recommend ways to control the size of the final product. Before going into detail, let us first examine the underlying physics of popcorn production. First, there is moisture inside the corn, which, upon being heated in a chamber (e.g. a microwave oven), is subject to expansion. When the temperature of the chamber exceeds the boiling temperature, the moisture turns into a gas with a sudden expansion in its
volume. But because of the hard shell that surrounds the corn, the moisture is trapped inside and its pressure rises. When the gas pressure reaches the yield point of the shell, the shell is broken and the trapped gas undergoes a rapid adiabatic expansion, because the process is so fast that the gas does not have enough time to exchange heat and equilibrate itself with the environment. This adiabatic expansion stops when the gas pressure $P$ reaches that of the chamber, $P_o$, which is, under normal circumstances, the atmospheric pressure. Since our primary interest here is to control the final size of the popcorn, and we know when the expansion stops, all we need do is write down the governing equations of the interface right after the rupture of the shell. Hence, this problem is reduced to that of interfacial instability and pattern formation, i.e. we are dealing with an interface advancing into the chamber.

To write the equations, we recognize that the dynamic variable is obviously the pressure $P$ inside the corn at time $t$, which should obey the law of adiabatic expansion:

$$P(t)V(t)\gamma = C_o = \text{const} \quad (1)$$

where $V(t)$ is the volume of the corn, and $\gamma$ is the ratio of the specific heats at constant pressure and at constant volume. For water vapor, $\gamma \approx 1.3 \,(1)$. The constant $C_o$ is determined by the yield pressure, $P_Y$, of the shell and the initial volume of the corn, $V_o$, i.e. $C_o = P_YV_o\gamma$. It is physical to assume that the advancing velocity of the interface, or the normal velocity, $v_n$, is proportional to the pressure gradient:

$$v_n = \kappa(P - P_o) \quad (2)$$
where $\kappa$ is a material constant. Eqs. (1) and (2) define the dynamics of the interface.

We next examine the stability of the interface in a standard way. We first scale the time in such a way that $\kappa = 1$. If the radius of the corn at time $t$ is $R(t)$, then its volume is simply given by $V(t) = \frac{4}{3} \pi R^3(t)$. The evolution of the sphere is then governed by Eq. (2):

$$v_n = \frac{dR(t)}{dt} = P - P_o = \frac{A}{R^{3\gamma}} - P_o$$

where $A = C_o(\frac{3}{4\pi})^{\gamma}$. Note that $\gamma$ is a non-integer, and thus Eq. (3) is highly nonlinear, and the exact solution is not available. However, the asymptotic form of the solution is easy to find. In the beginning, one may set $P_o = 0$, because $P_o << P$, and obtain an approximate solution,

$$R(t)^{1+3\gamma} \approx A(1 + 3\gamma)t + B$$

where $B = R_o^{1+3\gamma}$ with $R_o$ being the initial radius at $t = 0$. In the limit, $t \rightarrow \infty$, the asymptotic solution is obtained by setting the left hand side of Eq. (3) to zero, i.e.: $R(\infty) = (A/P_o)^{1/3\gamma}$. So, $R(t)$ initially increases as a power law, and then approaches $R(\infty)$. We have solved Eq. (3) numerically, and indeed checked this behavior. To examine stability of this solution, we imagine the sphere is perturbed slightly. Since any perturbations in three dimensions around the sphere can be expanded in spherical harmonics, $Y_{lm}$, let

$$r(t, \theta, \phi) = R(t) + A_{lm}(t)Y_{lm}(\theta, \phi)$$

If the amplitude $A_{lm}$ grows in time, then the interface is unstable to the perturbation. Otherwise, it is stable. We should now obtain the corrections
to the pressure and volume caused by this perturbation. Define $P(t) = P(t) + f(t)$ and $V(t) = V(t) + g(t)$, where $f$ and $g$ will be functions of $A_{lm}$ and $Y_{lm}$. We first obtain $g(t)$ in the case $l \neq 0$ by direct integration of the volume: $V(t) = \frac{1}{3} \int r^3(t) d\Omega = \frac{4}{3} \pi R^3 + RA_{lm}^2$. Hence, we find: $g(t) = RA_{lm}^2$ and

$$f(t) = -\frac{\gamma C_o}{V^{\gamma+1}} RA_{lm}^2$$

Now, in order to investigate the stability, we need a time dependent equation of motion for the amplitude $A_{lm}$. The normal velocity, $v_n$, is given by:

$$v_n = dr / dt \cdot n = P - P_o + f(t)$$

where $r = (x, y, z) = r(t, \theta, \phi) (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ with $r(t, \theta, \phi)$ given by (6) and $n$ is the normal vector to the surface, which is given by:

$$n = (-\partial z / \partial x, -\partial z / \partial y, 1) / \sqrt{1 + (\partial z / \partial x)^2 + (\partial z / \partial y)^2}$$

Canceling $\dot{R}(t) = P - P_o$ (Eq. (3)), and calculating the derivatives by chain rules yields:

$$f(t) = \dot{A}_{lm} Y_{lm}(\theta, \phi) - \frac{(\dot{R} + \dot{A}_{lm} Y_{lm}(\theta, \phi))((\partial_{\theta} Y_{lm})^2 \sin^2 \theta + (\partial_{\phi} Y_{lm})^2)}{2 R^2 \sin^2 \theta} A_{lm}^2 + ...$$

Since there is no angular dependence in $f(t)$, we conclude from Eq. (9) that $\dot{A}_{lm} = 0$, namely all higher order harmonic perturbations of $l \neq 0$ must be marginally stable. If we consider only the radial perturbation, then we may set, $r(t) = R(t) + \delta(t)$. Then it is easy to show that the growth rate $\omega$ is negative:

$$\omega = \frac{\delta}{\dot{\delta}} = -\frac{4\pi R^2 C_o}{V^{\gamma+1}} + O(\delta^2) < 0$$
Hence, the interface is stable against the radial perturbation. In summary, we have shown at the level of the linear stability analysis, that perturbations of high order harmonics are marginally stable, and the lowest radial mode of \( l = 0 \) decays exponentially. We now present a physical argument why the interface is indeed stable. Note that the volume correction due to the perturbation, \( g(t) \), is positive, while the correction to the pressure, \( f(t) \) is negative. Thus, when the perturbation arises, the inside pressure that drives the instability decreases, and the propagating speed decreases. Consequently, the instability is suppressed. Note that this problem is analogous to the problem of a solidifying interface advancing from a cold to a hot environment, which is always stable against perturbations.

Since we have established that the interface is stable against infinitesimal perturbations, we are now in a position to determine the approximate size of the popcorn at a given chamber pressure \( P_o \). Suppose the corn stops its expansion at a certain time \( t_f \), at which point \( P(t_f) = P_o \). We can easily find from Eq. (1) the maximum volume of the corn at \( t_f \):

\[
V(t_f) = \left(\frac{C_o}{P_o}\right)^{1/\gamma} \tag{11}
\]

Note that \( V(t_f) \) is a function of the initial chamber pressure \( P_o \), yield pressure \( P_Y \) and the initial volume, \( V_o \), because \( C_o = P_Y V_o^\gamma \). For a given corn that has fixed \( V_o \) and \( P_Y \), Eq. (11) enables us to control the size of the popcorn by monitoring only the chamber pressure \( P_o \). Also, from Eq. (11), we may have a rough estimate of the yield pressure. Examining typical popcorn, we note that the radius increases at least by a factor 4, and thus \( V_f/V_o \approx 60 \).
Hence,

\[ P_f/P_o = (V_f/V_o)^\gamma \approx 200 \]

We note that the yield strength of the shell at the rupture point is about the same as or greater than polyethylene (LDPE) at room temperature (2). We now define the volume expansion rate \( \Gamma \) as the ratio of the final volume over the initial volume \( V_o \):

\[ \Gamma(P_o) = V(t_f)/V_o = (C_o/P_o)^{1/\gamma}/V_o \]  

(12)

If our goal is simply to increase or decrease the size of the popcorn relative to a given reference point, then the more relevant quantity is the ratio \( \alpha \equiv \Gamma(P'_o)/\Gamma(P_o) \). From Eq. (12), we obtain:

\[ \alpha = (P_o/P'_o)^{1/\gamma} \]  

(13a)

or equivalently,

\[ P'_o = \alpha^{-\gamma}P_o \]  

(13b)

which is the central result of this paper. If we want to increase the size of the popcorn by a factor \( \alpha \), then we must reduce the pressure of the chamber by a factor \( \alpha^{1/\gamma} \). If \( \alpha = 2 \), one must reduce the chamber pressure by a factor \( 2^{1.3} \approx 2.5 \). However, we caution that the assumption of our model that the solid expansion of the corn via eversion closely follows the adiabatic expansion of the gas may be too simplistic, and thus the exponent, \( 1/\gamma \), in (13a), which characterizes the expansion rate, may be somewhat smaller and presumably saturate beyond the critical pressure. We will report the experimental results in the near future. Another way of controlling the size of the
popcorn may be to increase the pressure $P$ inside the corn and the chamber pressure $P_o$ simultaneously, and let the chamber undergo a very rapid free expansion, when the temperature of the chamber reaches the popping temperature. During this free expansion, the chamber pressure decreases, while the pressure inside remains constant. When the difference, $P - P_o$, reaches the yield point, $P_Y$, the shell breaks, and the corn undergoes the adiabatic expansion. By controlling the duration of the heating and the free expansion, one may control the size of the final product. This method may be in fact more promising than the previous one, because by not reducing the chamber pressure, it prevents moisture from being leaked to the chamber during heating, which in turn increases the pressure of the corn, and hence the pressure gradient of the solid-gas interface during the expansion.

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References

1. CRC Handbook of Physics and Chemistry (CRC Press, Florida, 1985), p. D-172.
2. N. E. Dowling, Mechanical Behavior of Materials (Prentice Hall, New Jersey, 1993), p. 158.