CONSCIOUSNESS AND THE WIGNER’S FRIEND PROBLEM

Bernard d’Espagnat

Laboratoire de Physique Théorique

Université de Paris XI, Bâtiment 210, 91405 Orsay, Cedex, France

It is generally agreed that decoherence theory is, if not a complete answer, at least a great step forward towards a solution of the quantum measurement problem. It is shown here however that in the cases in which a sentient being is explicitly assumed to take cognizance of the outcome the reasons we have for judging this way are not totally consistent, so that the question has to be considered anew. It is pointed out that the way the Broglie-Bohm model solves the riddle suggests a possible clue, consisting in assuming that even very simple systems may have some sort of a proto-consciousness, but that their “internal states of consciousness” are not predictive. It is, next, easily shown that if we imagine the systems get larger, in virtue of decoherence their internal states of consciousness progressively gain in predictive value. So that, for macro-systems, they may be identified (in practice) with the predictive states of consciousness on which we ground our observational predictions. The possibilities of carrying over this idea to standard quantum mechanics are then investigated. Conditions of conceptual consistency are considered and found rather strict, and, finally, two solutions emerge, differing conceptually very much from one another but in both of which the, possibly non-predictive, generalized internal states of consciousness play a crucial role.

Key words: measurement, decoherence, reality, consciousness, time.

1Unité Mixte de Recherche (CNRS) UMR 8627
1. INTRODUCTION

The central claim, in this paper, is that the Schrödinger-cat – or Wigner’s friend – paradox cannot be really solved without going deeply into a most basic question, namely: are we able to describe things as they really are or should we rest content with describing our experience? A priori, of course, we have a hope of doing both at once. We think that, by enlarging our experience, and reasoning on it, we shall progressively lift the veil of appearance and attain knowledge of reality. But historically things did not take this turn. It is well known that the attempts at imparting an ontological interpretation to modern physics, and particularly quantum physics, have met, and still meet, with difficulties. Admittedly, these obstacles are not insuperable, but still they imply that essentially ad hoc and quite artificial-looking changes should be made in the formalism. And this is one of the reasons why the alternative standpoint, centered on the description of communicable human experience, is also considered reasonable after all. Of course another reason – a historical one – is the fact that such a standpoint was taken up both by the “founding fathers” of quantum physics and, in the same period, by the Vienna Circle positivists. Unfortunately, it must be observed that, on the whole, neither the ones nor the others were explicit concerning the very existence of the dilemma in hand. Schlick, for example [1] had clearly stated the basic positivist axiom that the meaning of a scientific statement boils down to its method of verification, which obviously makes the notion of “mind” (the mind or minds that verify) prior to that of scientific reality. But still, in spite of this, in most of their writings the logical positivists implicitly seemed to suggest that the rules they claimed science should follow (the principle of verification for example) somehow led to some knowledge of a reality “out there”, in other words of mind-independent reality.

This ambiguity got transferred to quantum physics and, in fact, it is still with us. In a sense it should be considered beneficial. Thanks to it, most physicists do not think it necessary to take sides concerning two opposite and equally unpleasant views, the – seemingly aberrant – one that mind is actually the “basic stuff” and the technically disturbing one that words such as “observables”, “measurements” and so forth should be banned from basic physics... Which is fortunate since it turns out that explicitly opting in favor of one of these standpoints is not at all necessary for
doing research in that science. In it, the vague notion of “empirical reality” serves as a conceptual basis and is, for practically all purposes, a totally sufficient one. On the other hand however, it is a general rule of reasoning that, wherever they appear, ambiguities should be removed. And sometimes this indeed is even useful in practice. I claim that this is the case concerning the problem in hand and that, for solving it, the rule in question should be followed, even at the price of having to openly face one or both of the two just mentioned queer views. Here both of them will therefore be taken seriously. In fact, it will be shown that either of them constitutes a suitable framework for solving the Wigner’s friend paradox, provided that they are taken with all the consequences each one implies, disregarding their incompatibility with such and such deeply engrained “received views”.

In view of the foregoing it would not be entirely unreasonable to consider that going into such considerations somehow amounts to deviating from physics proper. It is therefore not surprising that most physicists turn away from such problems. But still, this disinclination is far from being fully general. In our times some first rate physicists do take great interest in the question of how quantum mechanics is to be interpreted, whether or not it is compatible with realism, and so on. And Asher Peres is distinctly one among them, as the number of papers in which he touched upon such matters convincingly shows. It is therefore a pleasure to dedicate this article to him. It is true that, in such a field, most physicists – Peres included – manage so as to avoid speculating, which renders their statements concise and, correspondingly, leaves a few questions open. Here I shall not be so careful. I shall quite avowedly – but, still, not wildly! – speculate.

The paper is divided into four parts. In the first one (Section 2) general facts are perused concerning the measurement problem and the extent is discussed to which decoherence theory may be considered to solve the latter, particularly in the case in which, besides instruments, the measurement process is explicitly assumed to involve also conscious beings. The second part (Section 3) describes a tentative solution to the just mentioned problem – often called the “Wigner’s friend problem” – based on an explicitly ontologically interpretable approach to quantum mechanics (in fact, on the Broglie-Bohm model). The third part (Section 4) explores the possibilities of building up a solution in line with the general ideas underlying the just
mentioned one but freed from the condition of ontological interpretability, hence essentially compatible with the general philosophy of standard quantum mechanics. It deals with criticisms of a logical-conceptual nature that might conceivably be raised against it, and shows they can be overcome. Finally, in the fourth part (Section 5) possible bearings of this approach on an age-old philosophical problem, the one of the nature of time, are briefly sketched.

2. DECOHERENCE AND MEASUREMENT

As is well known, the main difficulty with quantum measurement theory is that when the system $Q$ on which a quantity $B$ is to be measured (by means of an appropriate instrument) is not, initially, in an eigenstate of $B$, if the instrument is described quantum mechanically the Schrödinger time evolution leads, for the overall system composed of $Q$ and the pointer (or of $Q$ and the rest of the world if, along with the pointer, we take the environment into account, as we should), to a state that is a superposition of macroscopically distinct states or a combination thereof. This result seems hardly compatible with the often held view that the basic quantum mechanical symbols describe reality as it really is quite independently of us, since what is observed has no clear relationship with such superpositions. And, what is more, under the assumption that the quantum predictive rules are universal the superposition in question could be shown [2, 3] to be incompatible with the view that macroscopic objects always have definite localizations.

Then decoherence theory came in. As we all know, it is grounded on the fact that all macroscopic systems significantly interact with their environment (including their “internal” one) and on the remark that, in practice, most of the physical quantities pertaining to the environment cannot be measured. It is claimed by many that decoherence actually solves the measurement problem. This assertion however is far from being endorsed by all physicists, and the reason is that recognition of the universality of the said interaction is only one of the ingredients in the solution. Another one is a watering down of realism.

This point being quite crucial for what follows, some details are worth being recalled concerning it. In quantum mechanics it is usually and appropriately considered that the meaning of factual statements is directly tied to what we count as
evidence for them. More precisely: in it a factual statement, if true at all, can be true only in virtue of something of which we could know. In particular, statements concerning the physical state in which a physical system lies can have truth-values (be true or false) only in virtue of measurements that we could perform. Hence it is only by referring to the possible outcomes of the measurements of some observables pertaining to a physical system that we may define the state of the latter. A strict, somewhat Schlick-like, interpretation of these epistemological principles leads to the (strong) completeness hypothesis, according to which, referring to the measurement outcomes of a complete set of compatible observables entirely specifies the physical state in question. This argument corroborates the commonly accepted view that states of individual systems are specified by state vectors.

It is on this basis that, in one of the above referred to papers, Bassi and Girardi could build up quite a general proof that decoherence fails to solve the measurement problem. They proceeded as follows. Given the macroscopic configuration of a macro-object of any sort they considered the set $V$ of all the state vectors that may represent it. Next, they pointed out that, assuming the (strong) completeness hypothesis, the sets of vectors corresponding to two well-separated macroscopic configurations should be “almost orthogonal” in a mathematically well-defined sense. In particular this must hold true concerning the sets, $V_U$ and $V_D$, that, in a measurement process, contain the final overall state-vectors corresponding to two distinct values, $U$ and $D$, say, of the measured quantity. They then considered a generalized such process, such as the above-described one, in which the to-be-measured quantity $B$ may have one of the two values $U$ and $D$. And they could show that, in virtue of the quantum evolution law, the final state-vector of the overall system (including the instrument pointer as well as the environment) can then be neither in $V_U$ nor in $V_D$. (nor indeed in any other macroscopic position different from “$U$” and “$D$”).

In view of this, an upholder of the view that pointers always are at definite places (in short, a “realist”) may not consider that decoherence solves the measurement problem.

---

2It is true that the notion of “protective measurements” (in which the interaction Hamiltonian acts for a long time with low intensity) makes it possible, in principle, to impart an operationally defined meaning to mean values of observables of individual systems, and hence also to the notion of density matrices attached to such systems. To my knowledge the question whether this new idea might possibly serve as a basis for new attempts towards a realist approach of the measurement problem has not been examined.
riddle. This shows that for decoherence to be significant a further move, one of a philosophical nature, is necessary. Roughly speaking it consists in taking two “conceptual steps” successively. The first one is to consider, along the lines marked out by Plato, Descartes, Kant and others, that our senses may, to a great extent, be deceitful and that what we apprehend – the set of the phenomena – is liable to considerably differ from whatever may be said to “really exist”. Such a view makes it easier for us to grant that, after all, there might be a gap between what we see – pointers positions – and what we think of – state vectors. On the other hand, the view in question is still a purely negative one. It does not positively tell us what type of information mathematical entities such as state vectors may provide us with. Our second step must be a positive one in this direction. And it seems that there is no other one to be taken than just to turn to the (generalized) Born rule, otherwise said to the rule yielding the probabilities that, upon measurement of a physical quantity, such or such value should be obtained. Now, what is most important at this point is that the rule in question is essentially predictive of observations. It has no ontological significance. It does not describe objects and their properties (it is well known that attributing such a function to the Born rule would immediately raise a host of conceptual difficulties). It merely informs us of what we shall see – or of the chances we have of seeing this or that – if we perform such and such actions.

Hence there are some good grounds for considering quantum theory to be essentially – and exclusively – predictive of observations. Now, it is true that if quantum mechanics is considered universal this limitation partly deprives physics of its “explanatory power at first level”, grounded on the notion of physical events being explanatory causes of other physical events. It implies that ultimately such notions as those of cause and explanation should be either dropped or (as seems more rationally justified) transferred to the “higher realm” of some “veiled reality” lying outside the direct reach of science proper. It is therefore understandable that some of us should consider the limitation in question to be a pity. But it may be claimed that, in a strict scientific sense, it is not. After all, describing ultimate reality as it really is always was the role taken up by metaphysics, and it was often claimed that the success of science was due to just its parting with metaphysics. In this respect there is therefore much truth in Schlicks above cited axiom according to which the meaning of a statement is nothing else than its method of verification, which
is, indeed, tantamount to saying that statements that look descriptive are, in truth merely predictive of observations.

It follows that the fact quantum mechanical predictive methods are so good as to never have been found at variance with experimental tests may legitimately be felt to remove our conceptual qualms relative to this theory, including those concerning measurement. And this is especially true since it may then legitimately be claimed [6] that, at least when only inanimate instruments are involved (see below), decoherence solves the measurement problem. Basically this is due to the fact that, to repeat, if Schlick’s views are taken literally, when we say that “obviously”, in an ensemble $E$ of systems $S$ that include pointers, each individual pointer is in one definite scale interval, the word “is” should not be misunderstood. In fact, it does not describe a state of things. It merely means that if we look at $E$ we shall have the impression of seeing the pointers distributed as just said. True, we also have something else in mind. Accustomed as we are to the “descriptive” sense of the verb “to be”, by using this verb here we also express our expectation that, of all the predictions concerning practically feasible measurements that are normally inferred from its use, none will be at variance with the data. Fortunately, in virtue of decoherence this condition is fulfilled as we know.

It is important to observe that this “predictive” or “purely operational” interpretation of quantum mechanics happens to remove two well-known conceptual difficulties that, within the more realist interpretations, still beset the decoherence theory of measurement. The first one is tightly connected with what was just noted. It is known that, concerning some extraordinarily complex measurements also involving the environment, quantum mechanics unavoidably yields probabilities that are at variance with consequences of our usual “realist” way of picturing “outside macroscopic reality”. But here, and especially since (as has now been shown [7]) the predictive rules of classical mechanics are mere consequences of the quantum mechanical ones, we may completely drop any realist picture. We may center on observational predictions. And it is then clear that the discrepancy in question, exclusively bearing as it does on practically unfeasible measurements, is void of both theoretical and experimental significance.

The second difficulty we have in mind is the famous “and-or” one. Within the more current (and more “realist”) approaches, applying decoherence theory to indi-
vidual cases rather than ensembles raises quite serious problems. Let it be stressed that such a difficulty does not actually arise here. The reason is that, to repeat, quantum mechanics is now viewed as merely being a set of computational rules informing us, via the (generalized) Born formula, of the probabilities we have of observing this or that. And it is in the spirit of the approach in hand that these rules, including the Born one of course, should be considered truly primitive. As such, they do not have to be explained, or inferred from anything else. They are just given as they are. And in particular, the Born rule is not derived from ensemble considerations... It is given in its probabilistic form, that is, it yields the probabilities that we shall have such or such specific, “individual” impression. Admittedly, experimentally verifying the correctness of the thus computed probabilities necessitates considering ensembles. But this was already the case, in classical physics, concerning all the theories that involved probabilities. In other words the probabilities that, making use of decoherence, we calculate on the basis of the quantum rules apply to individual events in the same way as do those a card player attributes to the event of pulling out a given card.

So, upholders of the views of Schlick and the most thoughtful authors of philosophy of science books may well consider that everything is quite in order. On the other hand, considered under this light quantum mechanics obviously has the notion of consciousness, not the notion of reality, as a referent. It deals, not with what is but with what we perceive. More precisely, it involves two basic notions, one having to do with actuality and the other one with potentiality, both related to consciousness. The “actual” one is that of “state of consciousness” or “state of mind” (distinguishing these two notions will not be here necessary): When a human being has made some observation it is quantum mechanically meaningful to state that he or she “is” in the corresponding state of consciousness. As for the “potential” notion, it is the one of “probabilities of observation” following from applying the quantum rules to the pieces of knowledge the state of consciousness contains.

Now, it so happens that this state of affairs raises new questions. The point is as follows. True, quantum mechanics does not provide us with anything like a God’s Eye view on what is. That much we just saw. But still, in fact it gives us – or seems to give us – a point of view that somewhat partakes of a God’s Eye one. It does so in the sense that the predictions quantum mechanics makes are held to be true for
a whole abstract community of human beings. For, in fact, a kind of disembodied Experimentalist, who is supposed to be looking at an ensemble of measurement-performing instruments and whom these predictions inform about the proportion of the latter on which he or she will see the pointer at such and such a place on the dial. This would raise no problem if only one Experimentalist, or, say, one conscious being existed. Unfortunately (as Wigner quite appropriately remarked!) there are several ones. Unavoidably we therefore have to face the so-called “Wigner’s friend problem”, which is just the Schrödinger-cat one, only, with a cat explicitly endowed with consciousness.

This raises, as I said, a question. To explain what it consists of, let us consider again a “measurement process” (in the generalized sense) bearing on a microscopic quantum system $Q$ and let us assume that a physicist $P$ has prepared $Q$ in a state that is not an eigenstate of the to-be-measured observable $B$. Let then $S$ be the system composed of $Q$, the instrument and a friend $F$ of $P$ looking at the pointer. Let us think of $S$ at a time $t$ when the process is over but $P$ has not yet looked at the result (nor asked $F$ what he saw). Within a statistical ensemble $E$ of such $S$’s, each one of the friends then sees “his or her” pointer lying in a graduation interval corresponding to one definite eigenvalue of $B$. From this actual knowledge, and assuming quantum mechanics is exactly true, he/she may make definite (probabilistic) predictions concerning what results would be obtained in the future if such or such measurements were done. All these predictions may then be combined according to classical, standard probability rules, so as to yield the probabilities with which $P$ should herself obtain these results. As we know, there are physical quantities (involving the environment) concerning which these probabilities do not coincide with those $P$ directly obtains by means of a quantum-mechanical calculation based on the content of her own state of consciousness, without assuming the friends to be in definite consciousness states. True these quantities are, in virtue of decoherence etc., not measurable in practice. But, in the present context, this remark does not remove the difficulty. The reason is that, in fact, we here have to do with a question of logical consistency. Assuming both that quantum mechanics is a universal theory and that the states of consciousness of the friends are predictive same as the one of $P$ is (i.e. assuming the predictive quantum rules may be applied to them same as to it) would amount to putting forward a theory yielding different results according to
the way calculations are made. This is unacceptable, quite independently of whether or not the results are practically checkable.

Note that this objection cannot be raised against the general argument on the basis of which decoherence is said to “solve” the measurement problem (when the consciousness of the “friends” is not brought into the picture). The reason is that, to repeat, within the conception in hand the only data that lie in the realm of actuality (i.e. to which the verbs “to be” or “to have” may, strictly speaking, be applied) are states of consciousness. Now, in the just mentioned case, the only relevant state of consciousness is that of $P$, which merely contains potentialities of observations, informing her of the chances she would have of getting this or that result if she chose to measure such and such observable. True, $P$ likes to think of these results as referring to some empirical reality. She expects that, of all the predictions concerning practically feasible measurements that may be inferred from assuming this to be true, none will be at variance with what she directly derives from what she knows, without making that assumption. But, to repeat, decoherence provides her with proper insurance in this respect. Within the here taken up conception of what quantum mechanics is and describes it is therefore true that the above objection is restricted to the case in which conscious “friends” are involved.

3. THE “WIGNER’S FRIEND” PROBLEM, A MODEL

The just mentioned conception may be referred to as the “purely operationalist” one\textsuperscript{3}. But note that it does not in any way amount to rejecting – as meaningless – the notion of a basic mind-independent reality, as pure idealist thinkers do. It consists in observing that, whatever this entity “really is”, it presumably differs even more than previously thought from what it looks like. And that, consequently, the more secure standpoint is not to take sides, in an a priori manner, on the question whether or not it should be pictured and, if it should, how it should be. Observe in this respect that within this standpoint the completeness assumption should not be stated in the standard, strong form: “hidden variables do not exist” that, following Bassi and Ghirardi, we implicitly imparted to it in Section 2, since this is a metaphysical hypothesis, even though a negative one. Following Stapp \textsuperscript{9}, the assumption in

\textsuperscript{3}The content of this Section appeared in a preliminary form in Ref. \textsuperscript{8}.
question should be expressed as the hypothesis that “no theoretical construction can yield experimentally verifiable predictions about atomic phenomena that cannot be extracted from a quantum theoretical description”. This leaves open the possibility that hidden variables exist, provided that they should be “really hidden”. Otherwise said the operationalist conception is, as here understood, quite general. For example, it is compatible with both pure antirealism and Bohm’s ideas, especially when the latter are expressed via the conception of a real implicit order, differing very much from the explicit one that reflects but the appearance of mind-independent reality.

Now, it turns out that the latter remark is here of help, in that it provides us with some sort of a guiding line. For indeed, when faced with riddles such as the one described in Section 2 we, physicists, feel somewhat at a loss until we find some similarity between them and problems that are, to us, more familiar. In this respect, the old Louis de Broglie-Bohm (hereafter B.B.) hidden variable model (with pilot wave or quantum potential) may be useful. Not that we should necessarily believe it is true. Many arguments (e.g. those described in detail in [5] and [10]) speak against it. But it does reproduce all the observational quantum mechanical predictions; it yields, to the quantum measurement problem, a solution differing from the above one but fully consistent as well\(^4\); and it has the great advantage of being conceptually crystal-clear. It can therefore be used as a “theoretical laboratory”. If, within it, it proves possible to take explicitly into account the (unquestionable!) fact that Wigner’s friend is conscious, it is conceivable that the basic idea underlying this solution can be extended, outside the model, to the general theory. Incidentally, note that in the model the above-mentioned difference between implicit and explicit orders of course holds. The implicit order concerns the hidden variables that, together with the nonlocal pilot-wave, compose mind-independent reality. The explicit order is the order that is manifest in the appearances that compose the set of the observed phenomena, alias empirical reality.

According to the B.B. model, within a Young-type thought-experiment with two slits the particle is driven by a pilot-wave that passes through both slits at once;

\(^4\)In it, the representative point is, right from the start, determined to proceed into one or the other of the sectors of configuration space corresponding to the possible pointer positions. The reason why this solution is not at variance with the Bassi and Ghirardi proof is, as pointed out by these authors, that, in this model, the strong form of the completeness assumption is not assumed.
and this has the consequence that, in the model, while each particle passes through but one slit, fringes nevertheless appear. We can therefore say that, in the B.B. model, the particle is at any time at some well-defined place even though it takes part in a typically quantum phenomenon. In this respect it resembles the friend in Wigner’s apologue, who is at any time in some well-defined state of consciousness while he also is taking part in a quantum phenomenon. To strengthen the analogy it is then appropriate that, in the model, we should attribute to the particle some kind of a mentality (or, say, proto-mentality), the physical nature of which needs not be specified in detail. Within the experiment in question, each one of the involved particles then “observes” that, at a certain time, it passes through one, well specified, slit. This is an internal state of consciousness of the particle and since, in the model, the particle position is an element of mind-independent reality, this internal state of consciousness should also be considered as being an element of mind-independent reality. For the particle, this internal state of consciousness has no predictive power, since what will happen to the said particle is entirely governed by the pilot wave.

Now, it is often, and quite rightly, said that the very fact of knowing through which slit the particle passes prevents the fringes from being formed. At first sight this might seem to constitute a valid objection against any idea of attributing a state of consciousness to the particles. And in fact, so it would be if we assumed that the particle could communicate its knowledge to the world at large. So let us assume that it can’t. That, at least as far as micro-systems are concerned, “internal states of consciousness” are really private (since hidden variables do not act on the pilot wave, such an idea is quite consistent with the fact that, in the model, the “consciousness state” in question is a “hidden variable” just as the position itself). This being the case, an external observer such as our \( P \) above, even if we assume she knows of the existence of such internal states of consciousness, must explicitly ignore this existence when predicting what will be observed. She therefore predicts the fringes will appear. And this prediction agrees, as we know, with experiment.

On the other hand, imagining a category of states of consciousness that always remain totally hidden would obviously be quite pointless. Hence, we should ask whether circumstances exist in which statements bearing on such “internal” states may, after all, have some relationship with the public domain of shared experience (while remembering of course that this domain is the one of the explicit order, that
is, in the model, the one of “appearances that are the same for everybody”). Now, decoherence helps us here. For, in the Young-type experiment, call $\phi_1$ and $\phi_2$ the partial wave functions issuing from slits 1 and 2 respectively, and suppose we replace the micro-particles by corpuscles that are appreciably larger and whose interaction with the environment is, consequently, not negligible. The fringes then fade and, when the corpuscles are macroscopic enough, they practically disappear. For the purpose of predicting outcomes of future observations, the ensemble of the involved corpuscles may then be treated as a mixture of two “pure cases” described by the wave functions $\phi_1$ and $\phi_2$. Now $\phi_1$ ($\phi_2$) is just the wave function that an observer would attribute to a set of corpuscles known to have passed through slit 1 (2). This shows that, in such circumstances, the internal state of consciousness of the corpuscles passing through one particular slit may indeed be considered without harm to have the intersubjective predictive role normally attributed, in quantum mechanics, to pieces of knowledge obtained from measurements. Incidentally, note that this reasoning is fully consistent with the Broglie-Bohm model since, as far as mere predictions are concerned, this model yields the same ones as non-relativistic quantum mechanics.

If we now turn back to the Wigner’s friend problem and still consider it within the Broglie-Bohm model, we find that the foregoing views can quite naturally be fitted to it. Indeed, for the same reasons as above, we may assume without inconsistency that $S$ has an internal state of consciousness as long as we assume it has no predictive power. And we can also relax somewhat the latter assumption when we take into account the fact that $S$ is macroscopic and interacts therefore with its environment. In fact, for the purpose of predicting what will practically be observed, (forgetting about measurements that are conceivable only in principle) an ensemble $E$ of thus prepared $S$’s can be treated as a mixture, for decoherence is at work. And – just as above – the quantum states composing this mixture are the ones that the various possible internal states of consciousness of the friends would generate if these consciousness states were viewed as predictive. Hence these internal states of consciousness, which, related as they are to hidden variables, are basically ontological, still may be considered to also represent elements of empirical reality. More precisely they can be viewed as coinciding in nature with the predictive states of consciousness we normally refer to when we state that such and such a measurement
outcome has been observed.

To sum up, within this conception (or “model”) it is considered that even microsystems can be endowed with “internal states of consciousness” (or “proto-consciousness”, whatever this may be) that are elements of a basic, not publicly accessible, reality, rather than of empirical reality. In other words, they are hidden (remember we left open the possibility that hidden variables should exist, provided that they should be “really hidden”). It is only when the involved systems become macroscopic enough for their interaction with the environment to be appreciable that these internal states of consciousness obtain some degree of public significance. This means that they gain predictive power. More precisely (as we easily realize by thinking of intermediate cases in which “not quite macroscopic” systems are involved) they make it possible to correctly predict the outcomes of a certain class – call it $A$ – of observations whereas they yield incorrect ones concerning those of another class – call it $B$. Now, it is a fact that (due to the nature of these two classes) human beings perform observations of class $A$ much more easily than observations of class $B$. And indeed this is true to such an extent that when the involved systems are thoroughly macroscopic, observations of class $B$ are, as a rule, practically unfeasible, as we know. Moreover, it is also the case that the impressions corresponding to the outcomes of measurements of class $A$ may usually be described in a realist language, that is, as if they referred to objects existing per se. The set of such intersubjective appearances is what is called here “empirical reality”. It is thus meaningful to speak of a kind of “co-emergence” of, on the one side, “public”, states of consciousness that are practically predictive (although, concerning class $B$ observations, they are not), and, on the other side, empirical reality. In line with one of the foregoing remarks this co-emergence is to be thought of as (a-temporally) taking place out of a “mind-independent reality” that, itself, presumably lies beyond our intersubjective abilities at describing.

It is interesting to note that, surprisingly enough, this model shows a similarity with Whitehead’s views. For indeed the notion of internal states of consciousness is quite akin to the ones of “prehensions” and “occasions of experience” that, in Whitehead’s philosophy, play a basic role even at the elementary particle level.
Discussion

Before proceeding further we should, as a precaution, ask ourselves whether the model is, after all, fully consistent. The reason why this question arises is as follows. Although the Bohm theory on which the model rests is ontologically interpretable by construction, still, in it, the difference between the implicit and explicit orders is basic and unavoidable. Which, to put it bluntly, means that the explicit order (alias empirical reality) is, in a way, but an illusion of our senses. This raises a question since the here described model takes macroscopic systems to be elements of mind-independent reality. Now, by definition so to speak, macroscopic objects are localized in definite regions of space whereas Bell’s theorem shows mind-independent reality to be non-local. At first sight it would therefore seem that macroscopic objects couldn’t be elements of the latter (alias of implicit order). A moment reflection shows however that this objection has no real substance. The point is that the Broglie-Bohm theory is ontological and that, in it, complex systems are composed of corpuscles each of which has, at any time, quite a definite localization in space. In virtue of the interaction existing between them, many such corpuscles may then constitute a stable localized composite system. This does not violate the Bell theorem since, in the B.B. model, non-locality essentially takes the form of instantaneous interactions that do not decrease when distance increases. Admittedly the complex systems in question somehow interact this way with one another (through the non-local overall wave function or, equivalently, the quantum potential) but this in no way prevents them from existing as definite entities. And indeed Bohm himself forcefully claimed (Bohm and Hiley [11], ch. 8) that, in his model, macroscopic objects do exist, not only as elements of the “explicit order” but quite independently from us, that is, in an ontological sense.

Remark

Consider, within the B.B. model, a two photon correlation-at-a-distance experiment, of the type used for checking Bell’s theorem, and assume that two distant observers $A$ and $B$ successively make, in this time order, polarization measurements along one and the same direction. According to the foregoing their internal states of consciousness are elements of mind-independent reality. On the other hand, the results that $A$ and $B$ get are strictly correlated, but we know from the Bell theo-
rem that this correlation is not due to “common causes at the source”. John Bell ([Ref.12], ch.15) explicitly showed it follows from the fact that what takes place in A’s instrument directly influences – via the non-local pilot wave – the behavior of the photon in B’s instrument. In other words, contrary to what we naively expect, the outcome of B’s measurement does not depend at all on the hidden variables of the photon arriving to B. And therefore, since, in our model, B’s internal state of consciousness is strictly linked to the hidden variables affecting B we must consider the said internal state to be non-predictive. But of course both A and B have a natural tendency to take up the view according to which (i) their internal states of consciousness are predictive and (ii) the correlation they observe is due to common causes at the source. This view may be considered to be the embryo of a conception of empirical reality. In the case in hand such a conception is obviously deficient since all the tests of Bell’s inequalities violation prove it is wrong. But when photons are replaced by somewhat more complex corpuscles, the corresponding tests rapidly become extremely difficult to make. In fact they quite soon become practically unfeasible. Correspondingly the conception of empirical reality the embryo of which we just described becomes more and more credible. Finally, when macroscopic objects are involved in place of photons, it becomes established truth. This is all right, after all. But it is conceptually and logically all right, only provided we are aware that, when we express ourselves in this way, we merely refer to an “empirical reality” that crucially depends on human aptitudes.

Again, in this model it is legitimate to speak of a kind of “co-emergence” of empirical reality on the one hand and states of consciousness on the other hand. But it is so with the important reservation that the said states of consciousness are not those that are deepest in our mind (the internal ones, which alone are ontological) but just the predictive ones, which refer but to Bohm’s “explicit order”.

4. BACK TO STANDARD QUANTUM MECHANICS

In Section 3 we used the ontologically interpreted Broglie-Bohm theory as a theoretical laboratory and we constructed a model. It is true that along with distinct advantages, the said theory has in it quite unpleasant, well known features. But, to repeat, our idea has been that if, within it, it proves possible to take explicitly
into account the fact that Wigner's friend is conscious – as we have just found is indeed the case – it is conceivable that the basic idea underlying this solution may be extended to a much wider theoretical framework. It is the feasibility of this that is now to be examined.

Let us first observe that there exists a fully consistent and very simple way of doing this. It consists in observing that the features of the B.B. model we made use of are but general ones. In fact, they boil down to the idea that the notion of a basic, mind-independent reality is meaningful and that the quantum mechanical symbols do not necessarily yield the finest possible description of it, so that the completeness assumption may be taken up merely in the weak, Stapp, sense. Now there is, of course, no reason to believe that the B.B. model is the only possible ontological interpretation of the quantum observational predictive rules. Indeed, other models are available that also have the general features in question. Hence a fully reasonable standpoint is to assume what follows. The notion of reality does have an ontological significance even though we don’t really know what reality consists of. Objects have an ontological status. Minds, with also an ontological status, are attached to some of them. And quantum mechanics happens to provide minds, directly or indirectly, with reliable observational predictions in, as it seems, all the various domains of physics. Within such a conception space and time – or space-time, or cosmic time – also enjoy an ontological status. They are arenas in which quite real events take place. Clearly, such a world-view is general enough to allow for the possibility of developing within it considerations akin to those unfolded in the foregoing section. It may therefore be claimed that, from a quantum-mechanical point of view, it constitutes an acceptable metaphysics.

On the other hand, since the advent of quantum mechanics it was always considered imperative to avoid anything resembling mechanistic, or too realistic, models. And, to repeat, it was claimed that to this end one should abide to the basic epistemological principle that statements unrelated with anything we could get informed about through appropriate measurements are meaningless. According to this view, ontological commitments should be banned. As we saw, in quantum mechanics following such a line of thought results in attributing primeval importance to the Born rule. Which, since this rule is fundamentally predictive of observations, amounts to consider quantum mechanics to be essentially and exclusively predictive of ob-
servations. It is therefore appropriate that we should inquire whether or not the way of removing the Wigner’s friend paradox put forward in the foregoing section is susceptible of being transposed into the framework of such a conception of quantum mechanics.

A preliminary question arises at this point, namely: is there a risk that this purely observational predictive nature of quantum mechanics should jeopardize the validity of the whole decoherence-based quantum measurement theory? In this section it will first be shown that, concerning measurements that involve but inanimate instruments this, fortunately, is not the case. But it will also be shown that indeed, same as above, a problem is thereby raised as soon as animated participants are involved. It will however be proved that, when all is said and done, here also the problem in question may be solved by adopting, concerning minds, the views stated in Section 3.

The reason why, prima facie, we might wonder whether the standard quantum measurement theory is as self-consistent as we usually believe it to be is of the same general nature as the one that motivated the discussion towards the end of Section 3 ... although it “pulls in the opposite way” so to speak. While the difficulty came there from the fact that macroscopic objects seemed not to be “real” enough, here it comes from them looking too ontologically real. The point is this. In Section 2 we noted that for showing that decoherence does remove the measurement riddle we had to drop the idea that pointers are intrinsically in such and such states corresponding to definite places on the dial. We observed that such a descriptive view has to be replaced by an approach purely predicting observations. But on the other hand, in order to show that the measurement riddle is really removed, we eventually had to take into account the fact that instruments are macroscopic. And this was by no means just an observational prediction. It was a statement of a descriptive sort. Now, is the occurrence, in the theory, of a statement of such a nature compatible with the view that the said theory should be purely predictive and in no way “ontological”?

The question sounds debatable but it should be answered positively. To require that a theory should merely be predictive of observations does not mean that it should involve no descriptive assertions. It means that the constitutive statements of the theory should be of the form “in such and such circumstances we shall ob-
serve this or that”, and this very structure implies that the circumstances in question should be described. Now, the requirement that the theory should not be ontological implies in turn that these circumstances should be mere phenomena, that is, should be referred to human experience. But here this raises no problem for in standard (non-ontological) quantum mechanics this requirement is satisfied. The set of the quantum rules includes the time-independent Schrödinger equation, which predicts what types of objects we shall perceive; and this equation shows that among such objects there may be bound states involving a great number of particles. In other words, macroscopic objects, far from necessarily belonging to the “ontological” realm may consistently be considered to be mere phenomena, in a Kantian sense. Moreover, the same Schrödinger equation informs us that the energy levels of such objects must lie very close to one another, so that the said objects have non-negligible interactions with their environment. It follows that the decoherence-based measurement theory as it is reported on in Section 2 is indeed fully consistent with what we called the operationalist conception of quantum mechanics.

On the other hand the same cannot be said concerning the theory of measurements involving animated observers (cats or “friends”). As repeatedly noted above, this operationalist conception centers on the notion of consciousness, which (apart from basic unknowable reality, see below) indeed is, in it, (as, by the way, in logical positivism!) the only primitive one (objects and so on essentially being “what is perceived by consciousness”). According to it the whole of quantum mechanics deals with the impressions consciousness will get under such and such circumstances. And, since physics essentially deals with inanimate objects, in this science the multiplicity of conscious beings does not normally constitute a problem. As pointed out above, the predictions quantum mechanics makes are held to be true for a whole abstract community of human beings, otherwise said, for a kind of disembodied Experimentalist, assumed to be looking at instruments. But then, when it is assumed that a “friend” (or, for what we know, just even a cat) looks at the pointer, the situation radically changes. For, clearly, the consciousness of this individual may not be lumped together with the one of the disembodied Experimentalist, that is, in the present instance, with the one of the people who started the experiment. But, on the other hand it must obviously be considered to stand on the same “ontological level” as the latter, for Wigner’s friend surely is just as thoughtful a person as you
and I. If the consciousness of the experimentalist – call her $P$ again – who initiated the experiment is objectively real, the one of the “friend” must be real as well. Now, in the operationalist conception this raises a problem for, as we just noted, in it the instruments of observation are not considered to exist per se. They are mere “objects for us”, that is, in the case in hand they ultimately are referred to $P$’s consciousness. They are just parts of what $P$ perceives, otherwise said they are, in a sense, mere appearances. Hence the body of the “friend” is a mere “appearance to $P$” as well. And it seems absurd to assume that his consciousness, which, as we just pointed out, is just as real as $P$’s one, should be univocally bound to something that is a mere “appearance to $P$”. Is it conceivable that a mere “appearance to consciousness” be the bearer of consciousness?

To investigate this question let it first be reiterated that the operationalist conception differs from pure idealism. In it, it is assumed that some fundamental reality exists in an ontological sense; that it is basically unknowable; and that it is somehow endowed with (hidden) structures. And it is further postulated that the existence of these structures is what accounts for the regularities we observe within the phenomenal realm and synthesize in the form of rules enabling us to predict our future observations. (Whether or not these rules vaguely reveal us something concerning the said hidden structures is an open question that lies outside our present subject). Consequently, we have to do with two notions of an ontological nature, the said, hidden mind-independent reality and consciousnesses, alias minds (although, of course the latter may be taken to just be components, or emanations, of the former).

Now, normally intersubjective agreement holds between individual minds about what they see (“what they see” being an abbreviation for “what they have the impression of seeing”, for remember that objects are but appearances). Physicists had to account for it and the way they managed to do so was to invent universal predictive rules, at present remarkably synthesized in the form of the quantum predictive rules. Now, the important point concerning the question in hand is the (already mentioned) one that the said rules (mainly the Born one) intersubjectively predict the appearance (to the minds) of (phenomenal) macroscopic objects. And we may consider this observational prediction to be experimentally corroborated since we do see macroscopic objects. In particular, each individual mind has the impression of being associated to one particular such macroscopic object, called its own “body”,
as well as that of seeing other similar objects. Now, according to the conception in hand, for a long – indeed a very long! – time the fact that the realist language is by far the most practical one (without it we could hardly communicate as Bohr stressed) misled us into considering such bodies to be ontologically real. And we even took them to be the bearers, or supports, of our minds ... which nowadays directly leads to the above stated difficulty (how could what is just an appearance, or image, in our mind be the support, or seat, of a mind?). But within the operationalist conception here under study bodies, which are mere phenomena, are not in the least the supports of minds. Quite on the contrary, the fact that physics is basically a set of observational predictive rules indicates that the objects – human bodies included – essentially are intersubjective appearances to minds. We must then consider it as a given fact that each individual mind has the impression of being associated to one particular such macroscopic object, called its own “body”, and also has the impression of seeing other similar objects, bodies included.

Under these conditions, it is clear that the reasoning made near the end of Section 2 (the one bearing on a “measurement in the generalized sense”) holds good. At time $t$ both physicist $P$ and her friend $F$ have the impression of seeing, along with the instrument, two human bodies; and also have the impression that one of these bodies is their own. And, moreover, $F$ has the impression of seeing the pointer at one definite place on the scale. When ensemble $E$ is considered the same holds true concerning all the “friends” in $E$ and their respective pointers. According to standard quantum mechanics, from this knowledge they have, each of them infers definite probabilities concerning what results would be obtained in the future if this or that measurement were done. And, again all these possible observational predictions may – theoretically – be combined according to classical and standard probability rules, so as to yield the probabilities with which $P$ should herself obtain these results. Now, to repeat, there are (environment involving) physical quantities concerning which these probabilities do not coincide with those $P$ directly obtains by means of a quantum-mechanical calculation based on the content of her own state of consciousness, without assuming the friends to be in definite consciousness states.

Hence, to sum up, we found:

(i) That within the operational conception the received view according to which
bodies are (ontologically) the bearers of minds is inconsistent and must be dropped\textsuperscript{5}.

(ii) That to drop the said view changes nothing to the fact that, as long as observational outcomes are considered to be predictive, the Wigner’s friend riddle remains unsolved.

But then the considerations that were put forward in Section 3 obviously yield the solution. Here, just as there, it suffices to introduce the notion of “internal states of consciousness”, taken to be elements of a basic reality, and to assume that they are not publicly accessible in general. The friends \( F \) do really have the impression of seeing this or that, but this does not necessarily count as public knowledge. Hence, when \( P \) calculates her probabilities of getting such and such outcomes were she to perform such and such measurements (possibly involving the \( F \)’s as objects), she should, as a matter of principle, ignore the very existence of these private impressions the \( F \)’s have. As for the \( F \)’s, however, due to the fact that they feel themselves associated with macroscopic bodies it turns out that, in practice, they may use the private impressions in question in order to predict what they themselves will see.

Note moreover that, since \( P \) thinks of the \( F \)’s as being so associated, when directly calculating the above-mentioned probabilities she must, concerning all practically feasible measurements, take decoherence theory into account. This implies that if she chose to violate the above-stated prescription and take the existence of the said private impressions of the \( F \)’s into account, this choice, concerning the said measurements, would by no means lead her astray. She would make no detectable error as to the probabilities of their outcomes. This explains that, not only in our daily life but also in our scientific activity, we may do as if our states of consciousness relative to factual data were totally predictive (though, quite strictly speaking, we know they are not) and as if those of our colleagues yielded predictions fully consistent with our own.

\textsuperscript{5}It is well known that, in the line of Kant’s views, many philosophers split the “mind” notion into two. They consider, on the one hand a disembodied, non-personal so called “epistemic subject” or “first person subject”, to whom objects, bodies etc. appear, and on the other hand “third person subjects”, who are minds as emanating from bodies. We consider that the existence of the Wigners friend paradox seriously invalidates this conception and we strive here to avoid it.
5. THE CONSTRUCTION OF TIME

When a physicist speculates his guidelines include taking account of all relevant data and having strict regard for consistency requirements. They do not specify that commonsense and received views should be obeyed, be it only for the reason that quantum mechanics gave him abundant proofs of the frailty of such handrails. This remark, obviously, applies first of all to the “space” and “object” notions. While, according to commonsense corpuscles clearly are somewhere in space, quantum physicists take them to be at no definite place until we look. And, what is more, nonseparability suggests that space itself should be but an “a priori mode of our sensibility”, as Kant thought. But then, what about time? In contrast with space, up to now time resisted such an “idealization” process. Attempts at making it a quantum observable were, on the whole, unsuccessful. They were so, however, merely for technical reasons. Within a speculative theory such as the present one, in which, conceptually, minds are taken to be prior to objects and their location, it is therefore quite natural – be it only for curiosity sake! – that we should wonder whether, by any chance, they might be conceptually prior to time as well.

Well, it turns out that, in fact, it is possible to modify the foregoing model so as to make it compatible with such a view. For this purpose, let us start with the following set of ideas. Minds possess ontological existence, have definite impressions, may communicate with one another and observe that they agree about most impressions they have. Among these impressions there is the one of space and of perceiving objects of various sizes lying in space. There also is the impression of perceiving events taking place in that space, as well as that of perceiving two events as taking places either simultaneously or successively. Minds, moreover, are able to imagine events they don’t actually see and separate them in two classes, those they “believe in” and call “objective” and those that they call “purely imaginary”. Let it be added that minds primitively have the notions of “before” and “after” as well as the ones of longer, shorter or equal duration between two pairs of successive events. They also realize that if the duration between events $a$ and $b$ is strictly longer than the one between events $c$ and $d$ they are able to think of an event $d'$ taking place after $a$ and before $b$ and such that the duration between $a$ and $d'$ is equal to that between $c$ and $d$. Now, it can be shown (Montbrial [13], Kranz et al [14] as quoted by
Montbrial) that the fulfillment of this set of conditions (plus a few others, needed for mathematical strictness but trivial in this context) entails the existence of a mapping of all the objective events onto the set of real numbers. This mapping, then, is just (intersubjective) time.

Now, with the “time” notion at our disposal, we may essentially take over the reasoning developed towards the end of the foregoing section. Indeed, we may argue that the minds observe some events to regularly follow other ones. They take note of such regularities and quite naturally (let us admit they have a natural propensity to induction) they make use of them and build up observational predictive rules, enabling them to calculate probabilities of future events when past ones are known. In this way they eventually construct the, classical and quantum, observational predictive rules (now, in fact, unified since we know the former ones to be deducible from the latter). With the help of these rules they predict future instances of intersubjective agreement between them all. And all the rest of our Section 4 construction may then be taken over without change. Finally we therefore get a grand, somewhat Plotinus-like, model, according to which Ultimate Reality generates or contains (appropriate words are lacking!) minds that, just as Ultimate Reality itself, are prior to both space and time. But these minds have a great many impressions of all sorts, including the one of having bodies, of being in time (as we just saw), of living in a Universe endowed with events and laws, and so on.

This model is rational and – as it seems to me – truly consistent with what we know about basic physics. In view of the great amount of questioning that, since its advent, quantum mechanics raised with regard to its interpretation this is certainly a nontrivial point, and if the model is to be taken seriously I think it should be on this ground. Indeed, it cannot be on any other one since it is at variance with basic things we think we know! Hence it may be taken into consideration only provided we balance acceptation of it with due recognition of the empirical reality notion, and acknowledge the central role of the latter in, not only our activity but also our way of thinking. Empirical reality is the whole set of the phenomena. Otherwise said, it is an understandable and manageable mapping of everything that minds actually perceive and act on. So that in practice – in our daily life but also in pure science (interpretation of quantum mechanics is no real exception for strictly speaking it lies beyond science) – we must do and think as if the model it constitutes did
represent Reality-per-se. Concerning minds (alias consciousness) this, in particular, implies that no changes at all are necessary concerning the investigation procedures of neurologists. True, these procedures are grounded on the view that minds are on the dependence of bodies and more or less produced by them, which is quite at variance with the views propounded here and in the foregoing section. But such notions of dependence and causation are themselves of a purely phenomenal nature, and this is what, in a phenomenal World, makes them useful. They synthesize in a simple way some features of the observational predictive rules that the minds discovered, so that, clearly, we have to go on using them.

6. CONCLUSION

The main result reached in this article is that decoherence theory alone does not remove the Wigner’s friend problem but that the said problem can still be solved, by introducing the “internal state of consciousness” notion in the above described manner. More precisely, we found that this can be done in two ways. Admittedly both of them are quite obviously at variance with the two most common received views. This however is partly due to the fact that, upon inspection, the said views turn out to be inconsistent with one another. It is impossible to claim at the same time that the meaning of statements about the existence of objects of any kind (neurons included) boils down to their method of verification and that neurons are conceptually prior to the verifying agent. Looking for an approach to the Wigner’s friend problem made it necessary to clearly face this conceptual difficulty. And finally two solutions were put forward.

One of them consists in openly accepting the idea, discarded both by the quantum mechanics founding fathers and by the logical positivists, that the theory should be ontologically interpreted. This solution is self-consistent. It has the virtue of being compatible with the intuitive views of both the “man in the street” and the scientists at large, inasmuch as most of the latter take minds to emerge from matter. It may however be criticized on the basis that it is vague without being deliberately so. As a matter of principle, an ontologically interpretable theory should describe reality “as it really is” and such descriptions should match what is observed. The B.B. model does not fulfill this condition and there are good reasons for believing
(nonseparability foremost) that other models yet to be invented cannot fare better in this respect.

Conceptually, the other solution we found is the opposite of this one. The Ultimate Reality it refers to is in principle unknowable and is not what science deals with. Indeed, the subject matter of the latter is just communicable human experience. In other words it is the set of all the impressions human minds may have and communicate to others. There is no doubt that the initiators of this conception intended to place scientific knowledge on secure grounds by setting it apart from ill-defined problems and particularly those of an ontological nature. Quantum mechanics essentially developed along these lines and the difficulties realist models steadily meet with indicate that, indeed, they presumably constituted, for it, the most favorable conceptual framework. On the other hand, it is unquestionable that the conception in question implicitly sets minds in a privileged position, especially since not only matter but also space and time are, in it, elements of experience. It was therefore to be expected that its progressive development should, at some time or other, involve – not just implicitly but explicitly – the somewhat disturbing idea that minds are conceptually prior to matter, space and time.

Anyhow, it may be difficult to choose between the two solutions we found but it is worth stressing that the proto-mentality idea is independent of this choice since we showed it to be consistent within both conceptions.

Throughout Antiquity and the Middle Ages thinkers focused on purely existential questions. But during the last centuries, and in relation with the development of science, they were led to attach more and more importance, even within the realm of pure knowledge, to what can be achieved and to how it can be achieved. Maybe it is now time that between these two tendencies – existential and operational – some balance should be reached, and that the research tools of the latter should be applied to the former. The present article may be viewed as a step in this direction.
References

[1] M. Schlick, *Allgemeine Erkennislehre*, (Springer, Berlin, 1918).

[2] B. d’Espagnat, *Conceptual Foundations of Quantum Mechanics, 2d. ed.* (Addison-Wesley, Reading, Mass, 4th ed. Perseus Books, Reading, Mass., 1999).

[3] A. Bassi and G. C. Ghirardi, *Phys. Letters A275*, 373 (2000).

[4] Y. Aharonov and J. Anandan, *ArXiv*, quant-ph/9803018 (1998).

[5] B. d’Espagnat, *Veiled Reality* (Addison-Wesley, Reading, Mass, 2d. ed. Westview Press, Boulder, Colorado, 2003).

[6] B. d’Espagnat, *Phys. Letters A282*, 133 (2001).

[7] R. Omnès, *The Interpretation of Quantum Mechanics* (Princeton University Press, Princeton NJ, 1994).

[8] B. d’Espagnat, *ArXiv*, quant.ph.0301160 (2003).

[9] H. P. Stapp, “The Copenhagen interpretation”, *Am. J. Phys.* 40, 1098 (1972).

[10] B. d’Espagnat, *Traité de physique et de philosophie* (Fayard, Paris, 2002).

[11] D. Bohm and B. J. Hiley, *The Undivided Universe* (Routledge, London, 1993).

[12] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).

[13] T. de Montbrial, “Evénements et temps quasi leibnizien” in *Implications philosophiques de la physique contemporaine T.3 B. d’Espagnat ed.*, (P. U. F., Paris, 2003).

[14] D. Kranz, D. Luce, P. Suppes and A. Tversky, *Foundations of Measurement* (Academic Press, Vol. 7, 1971).