Free and bound spin-polarized fermions in the fields of Aharonov–Bohm kind

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The scattering of electrons by an Aharonov–Bohm field is considered from the viewpoint of quantum-mechanical problem of constructing a self-adjoint Hamiltonian for the Pauli equation. The correct domain for the self-adjoint Hamiltonian, which takes into account explicitly the electron spin is found. A one-parameter self-adjoint extension of the Hamiltonian for spin-polarized electrons in the Aharonov–Bohm field is selected. The correct domain of the self-adjoint Hamiltonian can contain regular and singular (at the point \( r = 0 \)) square-integrable functions on the half-line with measure \( r \, dr \). We argue that the physical reason of the existence of singular functions is the additional attractive potential, which appear due to the interaction between the spin magnetic moment of fermion and Aharonov–Bohm magnetic field. The scattering amplitude and cross section are obtained for spin-polarized electrons scattered by the Aharonov–Bohm field. It is shown that in some range of the extension parameter there appears a bound state. Since the Hamiltonian of the nonrelativistic Dirac–Pauli equation for a massive neutral fermion with the anomalous magnetic moment (AMM) in the electric field of a linear charge aligned perpendicularly to the fermion motion has the form of the Hamiltonian for the Pauli equation in the Aharonov–Bohm flux tube, we also calculate the scattering amplitude and cross section for the neutral fermion.

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I. INTRODUCTION

The quantum Aharonov–Bohm (AB) effect, predicted in [1], is an important physical phenomenon. Due to the cylindrical symmetry of Aharonov–Bohm field configuration, the relevant quantum mechanical system is invariant along the symmetry (z) axis and essentially two-dimensional in the xy plane. So, the models applied to describe AB effect can usually be reduced to the (2+1)-dimensional ones. Such models are studied by means of the Dirac equation in 2+1 dimensions and results obtained are applied to other problems. Solutions to the two-component Dirac equation in the AB potential were first obtained and discussed by Alford and Wilczek in Ref. [2] in a study of the interaction of cosmic strings with matter and the production of particle-antiparticle pairs by the vector potential of a moving cosmic string. The effect of vacuum polarization in the field of infinitesimally thin solenoid is recently investigated in [3] and a wonderful phenomenon is revealed: the induced current is finite periodical function of the magnetic flux.

The scattering of spinless charged particle by an infinitely long thin solenoid was examined by Tyutin in [4] from the viewpoint of quantum-mechanical problem of constructing of self-adjoint Hamiltonians. The Hamiltonian for the Aharonov–Bohm problem is essentially singular and hence it cannot be immediately defined in the domain \([0, \infty)\) for any differentiable and enough rapidly decreasing (at \(r \to \infty\)) functions in the Hilbert space of square-integrable functions on the half-line with measure \(r\,dr\). Usually, the Hamiltonians are symmetric operators in its natural domain. The problem of constructing a self-adjoint Hamiltonian is to find all self-adjoint extensions of given symmetric operator and then to select correct self-adjoint extension. The correctness of the known Aharonov–Bohm solutions for scattering problem was analyzed for spinless particles in [4], in which a self-adjoint extension of the Hamiltonian is selected by physical condition -“the principle of minimal singularity”: the Hilbert-space functions for which the Hamiltonian is defined must not be singular.

A one-parameter self-adjoint extension of the Dirac Hamiltonian in 2+1 dimensions in the pure AB field was constructed by means of acceptable boundary conditions in [5–8]. For some extension parameters a domain of the self-adjoint Hamiltonian was found to exist in which the Hamiltonian is self-adjoint and commutes with the helicity operator [9]. One- and two-parameter families of self-adjoint Dirac Hamiltonians in the superposition of the Aharonov–Bohm solenoid field and a collinear uniform magnetic field in the respective 2+1 and 3+1 are constructed in [10, 11].

Nevertheless, there remains a question about the correctness of solutions for the scattering of spin particles in the AB flux tube. Indeed, for this problem we need to use the Hamiltonian for the Pauli equation, which contains one more extremely singular (spin) term taking account of the interaction between the spin magnetic moment of electron and magnetic field.

An interesting and important corollary to the Aharonov-Bohm geometric phase is a phase acquired by the wave function of a neutral massive fermion with the magnetic moment when it propagates in an electric field of a uniformly charged long conducting thread aligned perpendicularly to fermion motion (such a field will be called the AC configuration). The fermion transport is influenced by the phase acquired by the fermion wave function and the resulting phase difference leads to a spin-field dependent effects in scattering (the Aharonov–Casher (AC) effect [12]). The Hamiltonian of nonrelativistic Dirac–Pauli equation for a massive neutral fermion with the anomalous magnetic moment in the electric field of a linear charge aligned perpendicularly to the plane of fermion motion has the same form with the Hamiltonian for the Pauli equation in the pure AB potential. The nonrelativistic quantum motion of an uncharged massive fermion with an anomalous moment interacting with the electric and magnetic fields produced by linear electric and magnetic sources in a conical spacetime was studied in [13].

Thus, the self-adjoint extensions for the nonrelativistic Hamiltonians with the particle spin (this term leads to additional singular \(\delta(r)\) potential in the Hamiltonian) have still not been constructed.

This paper is organized as follows. In Section II we find a one-parameter self-adjoint extension of the Hamiltonian for spin-polarized electrons in the Aharonov–Bohm field. In Section III we find the scattering amplitude and cross section as well as the wave function of bound state for spin-polarized electrons in the Aharonov–Bohm flux tube of zero radius. In Section IV we discuss the scattering problem for spin-polarized neutral massive fermion with AMM in the field of uniformly charged conducting thread of zero radius. In Section V we briefly discuss physical results.

We shall adopt the units where \(c = \hbar = 1\).
II. SELF-ADJOINT EXTENSIONS FOR THE PAULI HAMILTONIAN IN AN AB POTENTIAL

The Pauli equation for an electron of mass $m$ and charge $e < 0$ in an AB potential specified in cylindrical coordinates as

$$A^0 = 0, \quad A_r = 0, \quad A_\varphi = \frac{B}{r}, \quad A_z = 0, \quad B = \frac{\Phi}{2\pi},$$

$$r = \sqrt{x^2 + y^2}, \quad \varphi = \arctan(y/x)$$

is

$$i\frac{\partial}{\partial t}\Psi(t, r, z) = \mathcal{H}\Psi(t, r, z), \quad \mathbf{r} = (x, y),$$

where $\Psi(t, r, z)$ is a spinor and the Hamiltonian $\mathcal{H}$ is

$$2m\mathcal{H} = -\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} \left( \frac{\partial^2}{\partial \varphi^2} + 2|e|B \frac{\partial}{\partial \varphi} - e^2B^2 \right) - \frac{\partial^2}{\partial z^2} + |e|B\sigma_3 \pi \delta(r).$$

Here $\sigma_3$ is the Pauli spin matrix

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The last term in (3) describes the interaction of the electron spin with a magnetic field.

Equation (1) is the potential of an infinitely thin solenoid with a finite magnetic flux $\Phi$ in the $z$ direction in the range $r > 0$. The magnetic field $\mathbf{H}$ is restricted to a flux tube of zero radius

$$\mathbf{H} = (0, 0, H) = \nabla \times \mathbf{A} = B\pi \delta(r).$$

Therefore, we need to construct a self-adjoint Hamiltonian of Eq. (3) in cylindrical coordinates for the electron motion in the $xy$ plane. It is seen that the inclusion of particle spin in the AB problem leads to the additional $\delta(r)$ potential in the Hamiltonian [14].

We seek solutions to the Pauli equation for the electron motion in the $xy$ plane in the form

$$\Psi(t, r, \varphi) = \exp(-iEt) f(r, \varphi) \psi \equiv \exp(-iEt) \sum_{l=-\infty}^{\infty} F_l(r) \exp(il\varphi) \psi^s,$$

where $E$ is the electron energy, $l$ is an integer, and $\psi^s$ is a constant two-spinor

$$\psi^s = \begin{pmatrix} \psi^1 \\ \psi^{-1} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + s \\ 1 - s \end{pmatrix},$$

where $s = \pm 1$ characterizes the electron spin projection on the $z$ axis.

The radial part $F_l(r)$ of the wave function satisfies

$$\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{l^2}{r^2} - \frac{2|e|Bl}{r^2} - \frac{e^2B^2}{r^2} + 2Em - |e|Bs \frac{\delta(r)}{r} \right) F_l(r) = 0$$

and it depends explicitly on the number $s$, which selects a particular value of the spin projection on the $z$ axis.

At first, it is rewarding to remind the problem of constructing solutions for the linear differential Bessel expression for functions $F_l(r)$ or $f(r) = F_l(r)/\sqrt{r}$ of the forms

$$l(F) = \left( -\frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} + \frac{(l + \mu)^2}{r^2} \right) F_l(r),$$

or

$$l(f) = \left( -\frac{d^2}{dr^2} + \frac{(l + \mu)^2 - 1/4}{r^2} \right) f(r),$$

where $\mu = |e|B$. In the form [9] the quasiderivative $l(f)$ is self-adjoint [15, 16] and can be defined in the range $-\infty < x < \infty$, \; $x \equiv r$. 
In the Hilbert space $L_2(a,b)$, i.e. the space square integrable functions in the range $(a,b)$ (the interval $(a,b)$ may be infinite) the linear everywhere dense set $D'_0$ is considered. The set $D'_0$ consists of the infinitely differential functions that vanish outside some cut $[c,d]$ (the cut may be various for various functions) contained completely in the range $(a,b)$. On $D'_0$ the operator $L_0$ is defined by equality $L_0f = l(f)$ [15, 16]. In each cut $[c,d]$ the Lagrange identity

$$\int_c^d l(f)\bar{g}dr - \int_c^d f(l(\bar{g}))dr = \left(\frac{df}{dr} - \frac{d\bar{g}}{dr}\right)|_c^d \equiv [f, g]^d_c = 0$$

(10)

is valid for any function $\bar{g}$ being complex conjugate of $g$. So $L_0$ is a symmetric operator. The closure $A_0$ of operator $L_0$ will be also a symmetric operator [17, 18]. We need to construct self-adjoint extensions $A^\theta$ of operator $A_0$. The adjoint operator $A^\theta_0$ is given by equation

$$A^\theta_0 f = l(f).$$

(11)

$A^\theta_0$ is defined on the set $D(A^\theta_0)$ of all functions that are absolutely continuous with their first derivative in the range $(a,b)$ and the quasi-derivative $l(f)$ belonging to $L_2(a,b)$.

If Eqs. (8), (9) are considered in the range $(0, \infty)$ then they will be singular on both ends. In this case the domain $D(A^0_0)$ of operator $A_0$ contains all functions $f$ from $D(A^\theta_0)$ for which

$$[f, g]^b_a = 0$$

(12)

for all $g$ from $D(A^\theta_0)$. In the most important case $\mu$ is nonintegral and the corresponding operator $A_0$ will have the deficiency indices $(2,2)$ at $0 < |l + \mu| < 1$. The eigenfunctions $G^\pm(r) = g^\pm \sqrt{r}$ of the adjoint operator $A^\theta_0$ with the eigenvalues $\pm i$ satisfy equations

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{(l + \mu)^2}{r^2} \pm ip^2\right) G^\pm_l(r) = 0,$$

(13)

where $p^2 \neq 0$ is inserted for dimensional reasons. Square integrable normalized solutions of $A^\theta_0$ are expressed by the Bessel functions of third kind (see, for instance [17, 18])

$$G^+_l(r) = A^+_l H^{(1)}_{l\nu}(e^{i\pi/4} pr), \quad G^-_l(r) = A^-_l H^{(2)}_{l\nu}(e^{-i\pi/4} pr),$$

(14)

where $A^\pm_l$ are the normalization factors and

$$\nu = |l + \mu|.$$  

(15)

If $\mu$ is nonintegral then the operator $A^\theta_0$ has two eigenfunctions belonging to $L_2(0, \infty)$ for each eigenvalue $\pm i$. Indeed, writing $\mu$ as $\mu = [\mu] + \gamma$, where $[\mu]$ is the largest integer $\leq \mu$, it is seen that these functions belong to the critical subspace $0 < \gamma < 1$ (with $l = -[\mu] - 1$). The self-adjoint extensions of $A_0$ can be parameterized by the parameter $\theta$. The correct domain $D(A^\theta)$ for the self-adjoint extension $A^\theta$ of $A_0$ is then given by

$$D(A^\theta) = D(A_0) + C_1 H^{(1)}_{1-\gamma}(e^{i\pi/4} pr) + C_2 H^{(1)}_{\gamma}(e^{i\pi/4} pr) + B_1 H^{(2)}_{1-\gamma}(e^{-i\pi/4} pr) + B_2 H^{(2)}_{\gamma}(e^{-i\pi/4} pr),$$

(16)

where $B_k = C_k \exp(-i\theta)$, $C_k$ are the arbitrary complex numbers and $\theta$ is an arbitrary (but fixed for a given extension) parameter. The domain $D(A_0)$ contains absolutely continuous, square integrable on the half-line with measure $rdr$ and regular (at the point $r = 0$) functions. If $\mu$ is a non-integer then the operator $A^\theta_0$ has two eigenfunctions belonging to $L_2(0, \infty)$ for each eigenvalue $\pm i$. Let us write (see, [18])

$$H^{(1)}_{\nu}(z) = \frac{\nu [J_{\nu}(z) - J_{-\nu}(z)]}{\sin \nu \pi}, \quad H^{(2)}_{\nu}(z) = -\frac{i [J_{\nu}(z) - J_{-\nu}(z)]}{\sin \nu \pi},$$

(17)

where $J_{\nu}(z)$ and $J_{-\nu}(z)$ denote the regular and irregular Bessel functions. The asymptotic behavior of the Bessel function at $r \to 0$ is

$$J_{\nu}(x) \approx \frac{x^\nu}{2^\nu \Gamma(1 + \nu)}.$$  

(18)

where $\Gamma(z)$ is the gamma function of argument $z$. 
Applying (18) we derive

\begin{equation}
CH_{\nu}^{(1)}(e^{i\pi/4}pr) + Ce^{-i\theta}H_{\nu}^{(2)}(e^{-i\pi/4}pr) = \frac{2Ce^{-i\theta/2}}{\sin \pi \nu} \times \left[ \frac{2^\nu (pr)^{-\nu}}{\Gamma(1-\nu)} \sin \left( \frac{\theta}{2} - \frac{\pi \nu}{4} \right) - \frac{2^{-\nu} (pr)^{-\nu}}{\Gamma(1+\nu)} \sin \left( \frac{\theta}{2} - \frac{3\pi \nu}{4} \right) \right].
\end{equation}

in which \( \nu = \gamma, 1 - \gamma \).

For any function \( f(r) \) from the defect subspaces the Lagrange identity (10) has to be satisfied

\begin{equation}
[f, g]^\infty_{0} = 0.
\end{equation}

Since the functions from defect subspaces must be square integrable on the half-line \([0, \infty)\) with measure \( rdr \) (i.e., they, in fact, obey the Lagrange identity at \( r \to \infty \)) it is enough to put a boundary condition at the origin

\begin{equation}
\lim_{r \to 0} r \left( f \frac{dg}{dr} - \frac{df}{dr} g \right) = 0.
\end{equation}

A symmetric operator is self-adjoint if its domain coincides with that of its adjoint (see, for instance 2). So, one has to posit the same boundary condition at the origin for functions from the spaces \( G^\pm(r) \) defined on the half-line.

The correct domain for the self-adjoint extension of the Hamiltonian of spinless particles was selected in 4 by the physical condition of “minimal singularity” – the domain of the Hamiltonian should be composed by entirely regular functions. It follows from (19) that this requirement fixes the parameter \( \theta = \pi \gamma/2 \) for \( \nu = \gamma \) and \( \theta = \pi(1 - \gamma)/2 \) for \( \nu = 1 - \gamma \). Since \( 0 < \gamma < 1 \) the correct domain of the Hamiltonian is composed by entirely regular functions for \( 0 < \theta < \pi/2 \). Insisting on regularity of all functions at the origin forces one to reject irregular (but square integrable on the measure \( rdr \)) solutions in the defect subspaces, which can entail a loss of completeness in the angular basis. Indeed, the inclusion of particle spin in the AB problem leads to the presence of spin-dependent potential term \( (|e|sB/r)\delta(r) \) in the Hamiltonian, which can be taken into account also by means of the boundary condition at the origin. Let \( B \) be chosen, for definiteness, positive. Then this potential is repulsive for \( s = 1 \), and attractive for \( s = -1 \) what means that if \( s = 1 \) then functions from the defect subspaces defined on the real half-line should be regular at the point \( r = 0 \) but if \( s = -1 \) then singular (concentrated at the origin) square integrable functions have to appear. It should be emphasized that the appearance of concentrated solution at the point \( r = 0 \) is due to the physical reason, namely, by the presence of attractive potential \( (|e|sB/r)\delta(r) \). We see from (19) that the defect subspaces does not contain the regular functions with indices \( \gamma \) and \( 1 - \gamma \), respectively, at \( \theta = 3\pi\gamma/2 \) for \( \nu = \gamma \) and \( \theta = 3\pi(1 - \gamma)/2 \) for \( \nu = 1 - \gamma \). The range of the extension parameter in this case is \( \pi/2 < \theta < 3\pi/2 \).

It should be noted that the linearly-independent solutions of equation

\begin{equation}
\begin{align*}
\left( -\frac{d^2}{dr^2} + \frac{(l + \mu)^2 - 1/4}{r^2} \right)f(r) &= \lambda f
\end{align*}
\end{equation}

with the Bessel operator 9 are expressed as

\begin{equation}
f(r) = a_l \sqrt{r} J_{\nu}(\sqrt{r}) + b_l \sqrt{r} J_{-\nu}(\sqrt{r}),
\end{equation}

where \( a_l \) and \( b_l \) are the normalization factors and \( \lambda \) is a real positive number. The continuous part of spectrum of any self-adjoint extension of operator \( A_0 \) 22 considered in \([0, \infty)\) coincides with the positive semiaxis \( \lambda \geq 0 \).

The self-adjoint extensions of the operator \( A_0 \) are parameterized by 16. For \( s = 1 \) the domain \( D(A_0) \) is composed of regular (for example, the regular Bessel \( J_{\nu}(\sqrt{r}) \)) functions for the range of the extension parameter \( 0 < \theta < \pi/2 \). Because of the invariance of Eq. 7 under simultaneous transformations \( \mu \to -\mu, l \to -l \) and \( s \to -s \) we shall restrict ourselves to study of the case \( \mu > 0 \).

Therefore, for \( s = 1 \) the complete set of functions of the self-adjoint Hamiltonian is given by the regular functions defined on the real half-line

\begin{equation}
f(r, \varphi) = J_{l+\mu}(\sqrt{r}) \exp(il\varphi), \quad l = 0, \pm 1, \pm 2, \ldots.
\end{equation}

For \( s = -1 \) for the range of the extension parameter \( \pi/2 < \theta < 3\pi/2 \) we derive that the defect subspaces contain two singular Bessel functions \( J_{-\gamma}(\sqrt{r}) \) and \( J_{\gamma-1}(\sqrt{r}) \) defined on the real half-line but bearing
in mind that $0 < \gamma < 1$ we have to leave only one of them. We shall leave $J_{-\gamma}(\sqrt{\lambda}r)$. In order to the set of functions would be complete in $l$, we need to remove from the domain $D(A_0)$ the function $J_{\gamma}(\sqrt{\lambda}r)$ with $l = -|\mu|$. Hence, the complete set of functions of the self-adjoint Hamiltonian is given by

$$f(r, \varphi) = J_{l+|\mu|}(\sqrt{\lambda}r) \exp(il\varphi), \quad l = 0, \pm 1, \pm 2, l \neq -|\mu|, \ldots; J_{-\gamma}(\sqrt{\lambda}r) \exp(-i|\mu|\varphi). \quad (25)$$

Therefore, the complete set of functions of a self-adjoint Hamiltonian for the Pauli equation in the AB potential is a set (24) for $s = 1$ and a set (26) for $s = -1$, in which we must replace in the argument of Bessel functions $\sqrt{\lambda}$ by $k \equiv \sqrt{2mE}$.

In addition, if the self-adjoint extension of $A_0$ is selected by the range of the extension parameter $\pi/2 < \theta < 3\pi/2$ there appears a bound state, which is expressed through the MacDonald function $K_{(1-\gamma)}(z)$ (or $K_{\gamma}(z)$). The appearance of bound state “suggests that this range of parameters in the effective Hamiltonian parameterizes nontrivial physics in the core” [5]. One sees from our model the appearance of bound state is possible only if the additional spin term in Eq. (7) is attractive, i.e. $s = -1$ (see, also [14, 19]).

III. SCATTERING OF ELECTRONS BY AN AB FIELD

We shall consider in what follows the case

$$\mu = \lfloor \mu \rfloor + \gamma \equiv n + \gamma, \quad (26)$$

where $n$ is an integer and

$$1 > \gamma > 0. \quad (27)$$

The expansion of the spatial electron wave function is given by

$$\psi(r, \varphi) = \sum_{l=-\infty}^{\infty} N_l J_{\nu}(kr)e^{il\varphi}, \quad s = 1, \quad (28)$$

where $N_l$ are constants and

$$\psi(r, \varphi) = \sum_{l=-\infty}^{\infty} N_l J_{\nu}(kr)e^{il\varphi} + D_{l=-n}J_{-\gamma}(kr)e^{-in\varphi}, \quad s = -1. \quad (29)$$

Here the summation is carried out with the omission of the $l = -n$ term.

If we assume that the electron wave propagates from the left along the $x$ axis so as to incident wave is $\psi = e^{ikx}$, then $\varphi$ is the scattering angle measured from the positive $x$ axis. The asymptotic form of electron wave function at $r \to \infty$ is a superposition of ingoing plane wave and scattered outgoing cylindrical wave

$$\psi_k(r, \varphi) = e^{ikx} + \frac{f(\varphi)}{\sqrt{r}} e^{i(kr-\pi/4)}. \quad (30)$$

At $r \to \infty$ the plane wave is given by

$$e^{ikr \cos \varphi} \to \frac{1}{\sqrt{2\pi kr}} \sum_{l=-\infty}^{\infty} e^{il\varphi} \left( e^{i(kr-\pi/4) + (-1)^l e^{-i(kr-\pi/4)} \right) \quad (31)$$

and Eq. (28) is given with using the expansion of Bessel functions as follows:

$$\psi(r, \varphi) \to \frac{1}{\sqrt{2\pi kr}} \sum_{l=-\infty}^{\infty} N_l e^{il(\varphi+\pi/2)} \left( e^{i(kr-\pi/4)} e^{-i|\mu|l+\mu}/2 + e^{-i(kr-\pi/4)} e^{i|\mu|l+\mu}/2 \right). \quad (32)$$

Choosing the coefficients $N_l$ so as to the ingoing waves would be canceled in Eq. (30), one easily obtains

$$N_l = e^{-i(\pi/2)|l+\mu|}, \quad (33)$$

The scattering amplitude for solution (28) is then found to be given by the AB formula

$$f_1(\varphi) = \sin \pi \gamma \frac{e^{-i(n+1/2)\varphi}}{\sqrt{2\pi kr \sin(\varphi/2)}}. \quad (34)$$
Inserting in (29) the \((\pm)\exp(-i\pi(\mu - n)/2)J_{\gamma}(kr)\) terms we obtain the scattering amplitude in the AB form again

\[
f_{-1}(\varphi) = \sin \pi \gamma \frac{e^{-i(n-1/2)\varphi}}{\sqrt{2\pi k \sin(\varphi/2)}}
\]

Therefore, if electrons are polarized so as to their spins are oriented along the \(z\) axis the scattering cross section in an AB potential is given by the AB formula

\[
\frac{d\sigma}{d\varphi} = \frac{\sin^2 \pi \gamma}{2\pi k \sin^2(\varphi/2)}
\]

The cross section for unpolarized electrons is obviously described Eq. (36) too.

It is easily to see that when the electron spin is perpendicular to the \(z\) axis we must take the wave function in the form

\[
\psi(r, \varphi) = \sum_{l=-\infty}^{\infty} N_l J_{\nu}(kr)e^{il\varphi} + e^{-i\pi\varphi}(J_{\gamma}(kr)e^{-i\pi(\mu - n)/2} + J_{-\gamma}(kr)e^{i\pi(\mu - n)/2})/2,
\]

where the summation is carried out with the omission of the \(l = -n\) term.

Now the scattering amplitude has the form

\[
f_0(\varphi) = \sin \pi \gamma \left( \frac{e^{-i(n+1/2)\varphi}}{\sin(\varphi/2)} + i e^{-i\varphi} \right)
\]

and the cross section is

\[
\frac{d\sigma}{d\varphi} = \frac{\sin^2 \pi \gamma}{2\pi k} \left( \frac{1}{\sin^2(\varphi/2)} - 1 \right)
\]

in case the electron spin lies in the plane of scattering. In case \(|\gamma| < 1\) this formula describes the scattering cross section of spin-polarized electrons by a long cylindrical magnetic flux tube of small radius [14].

### A. On a bound electron state in the AB field

It was shown in Section II that in the range of the extension parameter \(\pi/2 < \theta < 3\pi/2\) there appears a bound state, which is expressed through the MacDonald function \(K_{1-\gamma}(z)\) (or \(K_\gamma(z)\)). The radial part of wave function of bound state is solution of Eq. (7) with \(s = -1\) and \(E < 0\). It is \(K_{1-\gamma}(z)\) (or \(K_\gamma(z)\)) of argument \(z = r\sqrt{2m|E|}\) defined on the real half-line where \(m\) is the electron mass and \(E < 0\) is the energy of bound state. One sees that a bound electron state may occur in the quantum system under consideration if the interaction of the electron spin with a magnetic field is included and it is attractive.

### IV. SCATTERING OF A MASSIVE NEUTRAL FERMION IN THE AC FIELD CONFIGURATION

If we put \(|e| = Ms\) in equation (7) it will describe the motion of a massive neutral fermion with the anomalous magnetic moment \(M\) in the electric field of a linear charge aligned perpendicularly to the fermion motion. Then, \(\nu\) is

\[
\nu = |l + s\gamma| \neq 0, \quad \gamma = MB, \quad k = \sqrt{2mE}.
\]

where \(B/2\) is the linear charge density, \(s = \pm 1\) is the fermion spin projection on the \(z\) axis, \(E\) is the fermion energy and the results obtained above are valid for a neutral fermion with AMM in the AC configuration.

The scattering amplitudes for \(\mu > 0, \quad \mu < 0, \quad s = \pm 1\) are respectively

\[
f_s(\varphi) = \frac{s}{\sqrt{2\pi k}} \frac{e^{i\varphi(n+1/2)}}{\sin(\varphi/2)}
\]
\[ f_s(\varphi) = -\frac{s}{\sqrt{2\pi k}} \frac{e^{i s(|n| - 1/2)\varphi + i |n| \pi \sin(\pi \gamma)}}{\sin(\varphi/2)}. \] (42)

Thus, if the spin of neutral fermion is oriented along the \( z \) axis the scattering cross section in an the AC configuration is given by

\[ \frac{d\sigma}{d\varphi}_{AC} = \frac{\sin^2(\pi \gamma)}{2\pi k \sin^2 \varphi/2}. \] (43)

The cross section for unpolarized neutral fermions with AMM in the AC configuration is also described by Eq. (43) [12]. Eqs. (41), (42) and (43) coincide respectively with Eqs. (34), (35) and (36).

If the spin of neutral fermion in the initial state lies in the scattering plane then the scattering amplitude and cross section are respectively

\[ f_0(\varphi) = (-1)^{n+1} \frac{\sin(\pi \gamma)}{\sqrt{2\pi k} \sin(\varphi/2)} \sin(n + 1/2)\varphi, \] (44)

and

\[ \frac{d\sigma}{d\varphi} = |f_0(\varphi)|^2 = \frac{\sin^2(\pi \gamma)}{2\pi k \sin^2(\varphi/2)} \sin^2(n + 1/2)\varphi. \] (45)

One can show that these formulas are described the case of negative \( \mu \) too, if \( n \) replaced by \( |n| - 1 + \theta(\mu) \), where \( \theta(x) = [1 + \text{sign}(x)]/2 \). It is seen if the particle spin in the initial state lies in the scattering plane the scattering amplitudes and cross sections for AB and AC effects are different.

V. DISCUSSION

If the dependence of cross section is to be studied upon the spin polarization of particles in the initial state, the detector should detect only those scattered particles for which the angles between the spin and momentum vectors are equal in initial and final states. In the AB configuration the magnetic field \( \mathbf{H} \) is localized inside infinitely thin solenoid, so the Hamiltonian (3) does not contain the vector \( \sigma \) matrix in the range \( r > 0 \) and the electron spin \( \mathbf{s} \) oriented along a unit vector \( \mathbf{n} \) is conserved. Since, the scattering occurs in the \( xy \) plane the spin vector of electron in initial state can effect the scattering amplitude and cross section only when it has a nonzero projection on the plane of scattering. So, in fact, the dependence on the (initial) spin of electron in the cross section determines and arises because the direction of motion of electron is changed as a result of scattering (see [19]).

In case a fermion with AMM moves in the \( xy \) plane, the Hamiltonian contains only the \( \sigma_3 \) matrix. Therefore, only the fermion spin projection on the \( z \) axis \( s \) is conserved. Like the AB case the spin vector of fermion in initial state can effect the scattering amplitude when it has a nonzero projection on the scattering plane but in the AC case the dependence on the spin vector in the initial state in the cross section determines not only by the change in momentum of fermion but also by the change in the spin orientation of scattered fermion (the quasiclassical precession of fermion spin).

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