DUALIZING COMPLEXES AND HOMOMORPHISMS
VANISHING IN KOSZUL HOMOLOGY

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Abstract. Let $C$ be a semidualizing complex over a noetherian local ring $A$. If there exists a local homomorphism with source $A$ satisfying some homological properties, then $C$ is dualizing.

1. Introduction

There is a number of characterizations of properties (of homological type) of noetherian local rings of positive characteristic in terms (of homological properties) of the Frobenius homomorphism. We start with [20].

Theorem (Kunz) Let $A$ be a noetherian local ring containing a field of characteristic $p > 0$, and let $\phi : A \to A, \phi(a) = a^p$ be the Frobenius homomorphism. We denote by $A$ the ring $A$ considered as $A$-module via $\phi$. The following conditions are equivalent:

(i) $A$ is regular
(ii) $\phi$ is a flat $A$-module.

Some years later, Kunz theorem was improved by Rodicio [25] as follows: if the flat dimension $fd_A(\phi) < \infty$, then $A$ is regular.

So we can think if similar characterizations for the properties complete intersection, Gorenstein and Cohen-Macaulay exist. For complete intersections the result was obtained in [12], characterizing complete intersections rings by the finiteness of the complete intersection dimension of its Frobenius homomorphism, and a similar characterization was also found for the Cohen-Macaulay property in [26].

We will examine now in more detail the case of the Gorenstein property. A first result was obtained by Herzog [17] (see also [15, Theorem 1.1] and [26, Proposition 6.1]):

Theorem (Herzog) Let $A$ be a noetherian local ring containing a field of characteristic $p > 0$, and let $\phi$ be its Frobenius homomorphism. Assume that $\phi$ is finite. The following conditions are equivalent:

(i) $A$ is Gorenstein
(ii) $\text{Ext}_A^i(\phi, A) = 0$ for all $i > 0$ and infinitely many $r > 0$.

This result was improved in [18], removing in particular the annoying finiteness hypothesis on $\phi$:

Theorem (Iyengar, Sather-Wagstaff) Let $A$ be a noetherian local ring containing a field of characteristic $p > 0$, and let $\phi$ be its Frobenius homomorphism.
The following conditions are equivalent:
(i) $A$ is Gorenstein
(ii) $G\text{-dim}_A(\varphi^* A) < \infty$ for some integer $r > 0$.

Here $G\text{-dim}$ denotes the Gorenstein dimension introduced by Auslander and Bridger in [2] (properly speaking, a modification of the original definition using Cohen factorizations [6, p.254], [18, Definition 3.3]).

Over the last years, some research was conducted in order to extend these results from the particular case of the Frobenius homomorphism to larger classes of homomorphisms. A first step was to consider contracting endomorphisms. An endomorphism $f$ of a noetherian local ring $(A, \mathfrak{m}, k)$ is contracting if for any integer $s > 0$ there exist an integer $r > 0$ such that $f^r(\mathfrak{m}) \subset \mathfrak{m}^s$. The Frobenius homomorphism is an example of contracting endomorphism. If $f$ is a contracting endomorphism on a noetherian local ring $A$, then $A$ must contain a field (of fixed elements), but unlike the case of the Frobenius homomorphism, it can be of characteristic zero. The above results for regularity were extended to contracting endomorphisms in [19, Proposition 2.6]. For the complete intersection property they were extended (even in an improved form) in [11], [9].

The Gorenstein case was studied first in [18]. In fact, they obtain the theorem stated above as a consequence of the more general:

**Theorem (Iyengar, Sather-Wagstaff)** Let $A$ be a noetherian local ring and $\phi : A \to A$ a contracting endomorphism. Then the following conditions are equivalent:
(i) $A$ is Gorenstein
(ii) $G\text{-dim}_A(\varphi^* A) < \infty$ for some integer $r > 0$.

Subsequently, in [24] this result was extended to the more general context of G-dimension over a semidualizing complex $C$ as defined in [13]. It is obtained in particular:

**Theorem (Nasseh, Sather-Wagstaff)** Let $A$ be a noetherian local ring, $C$ a semidualizing complex over $A$ and $\phi : A \to A$ a contracting endomorphism. The following conditions are equivalent:
(i) $C$ is a dualizing complex.
(ii) $G_C\text{-dim}(\varphi^* A) < \infty$ for infinitely many $r > 0$.

This result generalizes the “classical” case: the Gorenstein dimension of [2] is the particular case of $G_C\text{-dim}$ obtained by taking $C = A$ (which is always a semidualizing complex), and a ring $A$ is Gorenstein if and only if the semidualizing complex $A$ is dualizing.

A second step in the extension of these results to larger classes of homomorphisms was initiated in [21]. The purpose in that paper was not so much to extend the known results to larger classes of homomorphisms as to understand what a homomorphism must verify in order to be a “test homomorphism” for these properties of local rings. In that paper a new class of homomorphisms, the ones with the $h_2$-vanishing property, was introduced. A local homomorphism $f : (A, \mathfrak{m}, k) \to (B, \mathfrak{n}, l)$ of noetherian local rings is said to have the $h_2$-vanishing property if the canonical homomorphism between the first Koszul homology modules associated to minimal sets of generators of their maximal ideals

$$H_1(\mathfrak{m}) \otimes_k l \to H_1(\mathfrak{n})$$
is zero. Any contracting endomorphism has a power which has the \( h_2 \)-vanishing property, but \( h_2 \)-vanishing homomorphisms are not necessarily endomorphisms, and they can be defined on rings that do not contain a field. Moreover, unlike the class of contracting endomorphisms, the class of \( h_2 \)-vanishing homomorphisms contains at once the two main test homomorphisms: the Frobenius endomorphism and the canonical epimorphism of a local ring into its residue field.

In order to see, even in the case of an endomorphism, the difference between \( h_2 \)-vanishing and contracting, consider a complete local ring \((A, \mathfrak{m}, k)\) and a contracting endomorphism \( \phi \) of \( A \). We assume for simplicity that \( \phi(\mathfrak{m}) = \phi^1(\mathfrak{m}) \subset \mathfrak{m}^2 \). Take a regular local ring \((R, n, k)\) of minimal dimension such that \( A = R/I \) (i.e., \( \dim R = \text{emb.dim} A \)), and a contracting endomorphism \( \varphi \) of \( R \) making commutative the diagram \([23, 3.2.1, 3.2.4]\)

\[
\begin{array}{ccc}
R & \xrightarrow{\varphi} & R \\
\downarrow & & \downarrow \\
A & \xrightarrow{\phi} & A
\end{array}
\]

(details can be seen in \([21, \text{Example 3.ii}]\)).

Then the homomorphism induced by \( \phi \)

\[ H_1(\mathfrak{m}) \otimes_k \varphi k \to H_1(\mathfrak{m}) \]

can be identified with the canonical homomorphism induced by \( \varphi \)

\[ I/nI \otimes_k \varphi k \to I/nI. \]

Since \( \varphi \) is contracting, by the Artin-Rees lemma some power of it verifies \( \varphi^r(I) \subset nI \), and so \( \varphi^r \) has the \( h_2 \)-vanishing property. But the contracting property is not only a condition on the images of \( I \), but on the images of \( n \). For instance, any local homomorphism which factorizes through a regular local ring has the \( h_2 \)-vanishing property.

Our purpose in this paper is to extend the above result of Nasseh and Sather-Wagstaff to \( h_2 \)-vanishing homomorphisms. In order to achieve it, instead of working with \( G_C \)-dim, we consider a different definition, \( G^*_C \)-dim (see Definition \([1]\)). Both definitions are related in the same way that Gorenstein dimension \( G \)-dim is related to upper Gorenstein dimension \( G^* \)-dim \([27], [4, \S 8]\). They share the usual properties (see Propositions \([9]\) and \([3*]\)), but we do not know if the finiteness of \( G_C \)-dim is equivalent to the finiteness of \( G^*_C \)-dim.

We obtain:

**Theorem 6** Let \( \varphi : A \to B \) be a local homomorphism and \( C \) a semidualizing complex over \( A \). Assume that \( \varphi \) has the \( h_2 \)-vanishing property. The following conditions are equivalent:

(i) \( C \) is a dualizing complex.

(ii) \( G^*_C \)-dim\( (B) < \infty \).

A note on terminology. Since we are interested only in the finiteness of \( G^*_C \)-dim and not in its precise value, we use the terminology of derived \( C \)-reflexivity instead of finite \( G^*_C \)-dim.
2. Notation for complexes

All rings in this paper will be noetherian and local.

We will follow the conventions for complexes generally used in this context (see e.g. [13]). For convenience of the reader we will briefly recall some notation. Let \( A \) be a ring. A complex of \( A \)-modules will be a sequence of \( A \)-module homomorphisms

\[
X = \ldots \to X_{n+1} \xrightarrow{d_{n+1}} X_n \xrightarrow{d_n} X_{n-1} \to \ldots
\]

such that \( d_n d_{n+1} = 0 \) for all \( n \). If \( m \) is an integer, \( \Sigma^m X \) will be the complex with \( (\Sigma^m X)_n = X_{n-m} \), \( d^m_n = (-1)^m d^X_n \) for all \( n \).

The derived category of the category of \( A \)-modules will be denoted by \( \mathcal{D}(A) \). For \( X, Y \in \mathcal{D}(A) \), we will write \( X \simeq Y \) if \( X \) and \( Y \) are isomorphic in \( \mathcal{D}(A) \), and \( X \sim Y \) if \( X \simeq \Sigma^m Y \) for some integer \( m \). Sometimes we will consider an \( A \)-module as a complex concentrated in degree 0. The full subcategory of \( \mathcal{D}(A) \) consisting of complexes homologically finite, that is, complexes \( X \) such that \( H(X) \) is an \( A \)-module of finite type, will be denoted by \( \mathcal{D}^b(A) \).

The left derived functor of the tensor product of complexes of \( A \)-modules will be denoted by \( - \otimes^L_A - \), and similarly \( \text{RHom}_A(-, -) \) will denote the right derived functor of the Hom functor on complexes of \( A \)-modules. We say that a complex \( X \in \mathcal{D}(A) \) is of finite projective (respectively, injective) dimension if there exists a bounded complex \( Y \) (that is, \( Y_n = 0 \) for \( |n| \gg 0 \)) of projective (respectively, injective) modules such that \( X \simeq Y \). We will denote it by \( \text{pd}_A(X) < \infty \) (respectively, \( \text{id}_A(X) < \infty \)).

3. Derived reflexivity

Let \( X, C \in \mathcal{D}^b(A) \). We say that \( X \) is derived \( C \)-reflexive if \( \text{RHom}_A(X, C) \in \mathcal{D}^b(A) \) and the canonical biduality morphism

\[
X \to \text{RHom}_A(\text{RHom}_A(X, C), C)
\]

is an isomorphism in \( \mathcal{D}^b(A) \) [13, 2.7], [10, §2].

We will say that \( C \in \mathcal{D}^b(A) \) is a semidualizing complex [13, Definition 2.1] if \( A \) is derived \( C \)-reflexive, that is, if the homothety morphism

\[
A \to \text{RHom}_A(C, C)
\]

is an isomorphism in \( \mathcal{D}^b(A) \). If \( C \) is a semidualizing complex and \( X \in \mathcal{D}^b(A) \), then \( X \) is derived \( C \)-reflexive if and only if \( X \simeq \text{RHom}_A(\text{RHom}_A(X, C), C) \) [10, Theorem 3.3]. We will give precise references of all the results we need on derived reflexivity, but the reader may consult [13], [14] for a systematic study.

A dualizing complex is a semidualizing complex of finite injective dimension.

We now introduce a modification of derived \( C \)-reflexivity. We call it derived \( C \)-reflexivity*, since it is related to derived \( C \)-reflexivity in the same way that upper Gorenstein dimension \( G^* \)-dim is related to Gorenstein dimension \( G \)-dim [27, 4, §8].

A local homomorphism \( (A, m, k) \to (R, p, l) \) is weakly regular if it is flat and the closed fiber \( R \otimes_A k \) is a regular local ring. Let \( f : (A, m, k) \to (B, n, l) \) be a local homomorphism. A regular factorization of \( f \) is a factorization \( A \xrightarrow{j} R \xrightarrow{p} B \) of \( f \)
where \( i \) is weakly regular and \( p \) is surjective. If \( B \) is complete, then \( f \) has a regular factorization with \( R \) complete [7].

**Definition 1.** If \( C \) is a semidualizing complex over a ring \( A \), a \( C \)-defeormation of \( A \) will be a pair \((Q, E)\) consisting in a surjective homomorphism of (local) rings \( Q \rightarrow A \) and a semidualizing complex \( E, C \in D_b^f(A) \) such that \( \text{RHom}_Q(A, E) \sim C \). In this case, by [13, Theorem 6.1 and Observation 2.4], the \( Q \)-module \( A \) is derived \( E \)-reflexive.

Let \( C \) be a semidualizing complex over \( A \), and \( X \in D_c^f(A) \). We will say that \( X \) is derived \( C \)-reflexive if there exists a weakly regular homomorphism \( A \rightarrow A' \) and a \( C \otimes_A A'\)-deformation \((Q, E)\) of \( A' \) (note that \( C \otimes_A A' = C \otimes_{A'} A' \) is a semidualizing complex over \( A' \) [13, Theorem 5.6]) such that \( pd_Q(X \otimes_{A'} A') < \infty \).

Let \( \varphi : A \rightarrow B \) be a local homomorphism, \( C \) be a semidualizing complex over \( A \), \( X \in D_c^f(B) \). We will say that \( X \) is derived \( C-\varphi \)-reflexive if there exists a regular factorization \( A \rightarrow R \rightarrow \hat{B} \) such that the complex of \( R \)-modules \( X \otimes_R \hat{B} \) is derived \( C \otimes_A R \)-reflexive, where \( \hat{B} \) is the completion of \( B \).

**Proposition 2.** Let \( C \) be a semidualizing complex over \( A \), and \( X \in D_c^f(A) \). If \( X \) is derived \( C \)-reflexive, then it is derived \( C \)-reflexive.

**Proof.** Let \( A \rightarrow A' \) be a weakly regular homomorphism, \((Q, E)\) a \( C' := C \otimes_A A'\)-deformation of \( A' \) such that \( pd_Q(X \otimes_{A'} A') < \infty \). By [13, Proposition 2.9], \( X \otimes_{A'} A' \) is derived \( E \)-reflexive and then, by [13, Theorem 6.5], \( X \otimes_{A'} A' \) is derived \( C' \)-reflexive. Then faithfully flat base change [13, Theorem 5.10] gives that \( X \) is derived \( C \)-reflexive.

We do not know if the reciprocal of Proposition 2 is true, even in the (classical) case \( C = A \). However the usual characterization of dualizing complexes in terms of derived reflexivity of the residue field also remain valid for derived reflexivity (in the case \( C = A \) this is the theorem by Auslander and Bridger saying that a ring \( A \) is Gorenstein if and only if the Gorenstein dimension of any module of finite type is finite if and only if the Gorenstein dimension of its residue field is finite [2, Theorem 4.20 and its proof]; see also [13, Theorem 6.1]):

**Proposition 3.** [13, Proposition 8.4, Remark 8.5] Let \( C \) be a semidualizing complex over \( A \). The following are equivalent:

(i) \( C \) is dualizing.

(ii) Any \( X \in D_c^f(A) \) is derived \( C \)-reflexive.

(iii) The residue field \( k \) of \( A \) is derived \( C \)-reflexive.

**Proposition 3** Let \( C \) be a semidualizing complex over \( A \). The following are equivalent:

(i) \( C \) is dualizing.

(ii*) Any \( X \in D_c^f(A) \) is derived \( C \)-reflexive.

(iii*) The residue field \( k \) of \( A \) is derived \( C \)-reflexive.

**Proof.** By Propositions 2 and 5, we only have to show (i) \(\Rightarrow\) (ii*). Let \( \hat{A} \) be the completion of \( A \) and \( Q \rightarrow \hat{A} \) a surjection where \( Q \) is a regular local ring. Let \( D \) be a dualizing complex over \( Q \) (\( D \sim Q \)). Then \( \text{RHom}_Q(\hat{A}, D) \) is a dualizing complex over \( \hat{A} \) ([16, V.2.4] or [13, Corollary 6.2]). Also, \( C \otimes_{A'} \hat{A} \) is a dualizing complex over \( \hat{A} \) ([13, V.3.5]), so \( \text{RHom}_Q(\hat{A}, D) \sim C \otimes_{A'} \hat{A} \) by [16, V.3.1].
Therefore \((Q, D)\) is a \(C \otimes_A \hat{A}\)-deformation of \(\hat{A}\). Since \(Q\) is regular, for any \(X \in D_0^f(A)\) we have \(\text{pd}_Q(X \otimes_A \hat{A}) < \infty\), and so \(X\) is derived \(C\)-reflexive*.

\(\square\)

This result still holds for derived \(C\)-\(\varphi\)-reflexivity*:

**Proposition 4.** Let \(C\) be a semidualizing complex over \(A\). The following are equivalent:

(i) \(C\) is dualizing.

(ii) For any local homomorphism \(\varphi : A \to B\), any \(X \in D_0^f(B)\) is derived \(C\)-\(\varphi\)-reflexive*.

(iii) There exists a local homomorphism \(\varphi : A \to B\), such that the residue field \(l\) of \(B\) is derived \(C\)-\(\varphi\)-reflexive*.

**Proof.** (i) \(\Rightarrow\) (ii) Let \(\varphi : A \to B\) be a local homomorphism and let \(A \to R \to \hat{B}\) be a regular factorization with \(R\) complete. Since \(C\) is a dualizing complex over \(A\) and \(i\) is flat with Gorenstein (in fact regular) closed fiber, then \(C \otimes_A R\) is a dualizing complex over \(R\) \cite[Theorem 5.1, Proposition 4.2]{16]. Therefore the result follows from Proposition 3*.

(iii) \(\Rightarrow\) (i) Let \(A \overset{\lambda}{\to} R \overset{\varphi}{\to} \hat{B}\) be a regular factorization, \(R \to R'\) a weakly regular homomorphism, and \((Q, E)\) a \(C \otimes_A R'\)-deformation of \(R'\) such that \(\text{pd}_Q(l \otimes_R R') < \infty\). Since \(R \to R'\) is weakly regular, its closed fiber \(l \otimes_R R'\) is regular. Then \(Q\) is a regular local ring (it follows e.g. from the change of rings spectral sequence

\[E_p^2 = \text{Tor}_p^{l \otimes_R R'}(\text{Tor}_q^Q(l \otimes_R R', l'), l') \Rightarrow \text{Tor}_q^Q(l', l')\]

where \(l'\) is the residue field of \(Q\) and \(l \otimes_R R'\).

We deduce that \(\text{id}_Q(E) < \infty\), and so the semidualizing complex \(E\) is dualizing. Then \(C \otimes_A R' \sim \text{RHom}_Q(R', E)\) is also dualizing \cite[V.2.4]{16}. Since \(A \to R'\) is flat, it is easy to see that \(C\) is dualizing (or use the stronger result \cite[Theorem 5.1]{5}).

\(\square\)

**Definition 5.** \cite[Definition 1]{21} Let \(f : (A, m, k) \to (B, n, l)\) be a local homomorphism. Let \(H_\ast(m)\) (respectively, \(H_\ast(n)\)) be the Koszul homology associated to a minimal system of generators of the ideal \(m\) of \(A\) (respectively, the ideal \(n\) of \(B\)). We say that \(f\) has the \(h_2\)-vanishing property if the canonical homomorphism induced by \(f\)

\[H_1(m) \otimes_k l \to H_1(n)\]

vanishes.

By \cite[15.12]{11} (see \cite[2.5.1]{23}), this homomorphism between Koszul homology modules can be written in terms of Andrée-Quillen homology \cite{11} as the canonical homomorphism

\[H_2(A, k, l) \to H_2(B, l, l)\]

As we saw in the Introduction, a suitable power of any contracting endomorphism has the \(h_2\)-vanishing property (in fact, if \(f : (A, m, k) \to (A, m, k)\) is a contracting endomorphism, for any integer \(n\) there exists an integer \(s\) such that \(f^s\) has the \(h_n\)-vanishing property, in the sense that the morphism of functors \(H_n(A, k, -) \to H_n(A, k, -)\) vanishes \cite[Proposition 10]{22}).
Theorem 6. Let \( \varphi : A \to B \) be a local homomorphism and \( C \) a semidualizing complex over \( A \). Assume that \( \varphi \) has the \( h_2 \)-vanishing property. If (and only if) \( B \) is derived \( C \)-\( \varphi \)-reflexive\(^* \), then \( C \) is dualizing.

Proof. The “only if” part is a consequence of Proposition 4.
Assume then that \( B \) is derived \( C \)-\( \varphi \)-reflexive\(^* \). Consider a diagram of local homomorphisms

\[
\begin{array}{ccc}
Q & \xrightarrow{\rho} & R' \\
\downarrow{\alpha} & & \downarrow{\pi'} \\
A & \xrightarrow{\varphi} & B \\
\downarrow{\beta} & & \downarrow{\pi} \\
\hat{B} & \xrightarrow{\beta} & \hat{B} \\
\end{array}
\]

where \( \alpha \) and \( \rho \) are weakly regular, \( \pi \) is surjective and \((Q, E)\) is a \( C \otimes_A R'\)-deformation of \( R' \) such that \( \text{pd}_Q(\hat{B} \otimes R R') < \infty \). We will see first that \( Q \) is a regular local ring repeating an argument in the proof of [21, Proposition 6].

Let \( l \) be the residue field of \( Q \) and \( \hat{B} \otimes R R' \). The commutative square

\[
\begin{array}{ccc}
A & \xrightarrow{\alpha} & R \\
\downarrow{\varphi} & & \downarrow{\pi} \\
B & \xrightarrow{\beta} & \hat{B} \\
\end{array}
\]

induces a commutative square

\[
\begin{array}{ccc}
H_2(A, l, l) & \xrightarrow{\bar{\alpha}} & H_2(R, l, l) \\
\downarrow{\bar{\varphi}} & & \downarrow{\bar{\pi}} \\
H_2(B, l, l) & \xrightarrow{\bar{\beta}} & H_2(\hat{B}, l, l). \\
\end{array}
\]

We have \( \bar{\varphi} = 0 \) since \( \varphi \) has the \( h_2 \)-vanishing property (we have used that if \( k \to l \) is a field extension we have \( H_n(k, l, l) = 0 \) for all \( n \geq 2 \) [11, 7.4]; so if \( A \to k \to l \) are ring homomorphisms with \( k \) and \( l \) fields, from the Jacobi-Zariski exact sequence [11, 5.1] we obtain \( H_n(A, k, l) = H_n(A, l, l) \) for all \( n \geq 2 \); finally, \( H_n(A, k, k) \otimes_k l = H_n(A, k, l) \) for all \( n \) by [11, 3.20]).

Since \( \alpha \) is weakly regular, by [21, Lemma 5], \( \bar{\alpha} \) is an isomorphism, and so \( \bar{\pi} = 0 \). Consider now the commutative square

\[
\begin{array}{ccc}
H_2(R, l, l) & \xrightarrow{\bar{\beta}} & H_2(R', l, l) \\
\downarrow{\bar{\rho}} & & \downarrow{\bar{\pi'}} \\
H_2(\hat{B}, l, l) & \xrightarrow{\bar{\beta}} & H_2(\hat{B} \otimes_R R', l, l). \\
\end{array}
\]

Again by [21, Lemma 5], \( \bar{\rho} \) is an isomorphism, and then \( \bar{\pi'} = 0 \). So the composition

\[
H_2(Q, l, l) \to H_2(R', l, l) \xrightarrow{\bar{\pi}'} H_2(\hat{B} \otimes_R R', l, l)
\]
vanishes. But by [3], \( \text{pd}_Q(\hat{B} \otimes_R R') < \infty \) implies that
\[
H_2(Q, l, l) \to H_2(\hat{B} \otimes_R R', l, l)
\]
is injective. Therefore \( H_2(Q, l, l) = 0 \), and then \( Q \) is regular by [1, 6.26].

Now the proof finishes as the proof of Proposition 4: since \( \text{id}_Q(E) < \infty \), the semidualizing complex \( E \) is dualizing; then \( C \otimes_A R' \sim R\text{Hom}_Q(R', E) \) is also dualizing [16, V.2.4] and since \( \rho \alpha \) is flat, we deduce that \( C \) is dualizing.

\[ \square \]

Remark 7. If a homomorphism in a composition has \( h_2 \)-vanishing property, then so has the composition. Therefore Theorem 6 can also be stated as follows:

Let \( \varphi : A \to B \) be a local homomorphism and \( C \) a semidualizing complex over \( A \). Assume that \( \varphi \) has the \( h_2 \)-vanishing property. If there exists a local homomorphism \( \phi : B \to S \) such that \( S \) is derived \( C-\varphi \)-reflexive*, then \( C \) is dualizing.

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