The influence of the construction stiffness in the roller bearings arrangement with the line contact on their durability

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Abstract. This article deals with the arrangement stiffness of the roller bearings with the line contact in the bearing units with regard to their durability. It analyses stationary and flanged standardized bearing units with the roller bearing using finite-element analysis. The article describes Hertz, Lundberg, Palmgren and Stribeck theories and published evaluation methods of the stiffness for the bearing units. It also evaluates the stiffness influence (variable cross-section characteristics) and other construction influences using suitable procedures where the standard for durability calculation is not taken into consideration. On the basis of the finite-element analysis results there has been created the mathematical model of the stiffness influence on more accurate determination of the basic durability of the roller bearings with the line contact.

1. Introduction
The worldwide demand for high-quality bearings increases each year. The bearings manufacture requires precision in choice of semi-finished products, realization of construction and manufacturing itself, so that the bearings meet the requirements set by the international standards. But even the precise production and proper assembly does not often guarantee bearing operation it that way it corresponds with calculated durability according to the standard ISO 281. This condition is in most cases caused by the wrong choice of the bearing for a particular application, the wrong estimation of the transferred loads or the wrong assembly procedure. The bearing units are individually replaceable units consist of the bearing, the bearing body and the preparation for the placing on the shaft. They are commonly manufactured in the type series for industry and agriculture. Using the bearing units the user makes less serious mistakes in their choice and assembly itself. The bearing units with the established maintenance often exceed the standard set durability. Deflections appear when there is an excessive design of the bearing units for a particular use. This excessive condition is mainly caused by not including many influences in the standard ISO 281 which does not take into consideration such influences as stiffness arrangement, radial and axial clearance, geometry of rolling bodies and runways, loading distribution in the rolling bearing, etc. Inclusion of these factors in the rolling bearings durability calculation would mean the more effective design for a particular application.

2. Bearing power, tension and reshaping
The basic construction node of a bearing unit is a bearing which allows the mutual component movement and transfers loads. These loads affect in bodies contact points and they create the strain which is responsible for various reshaping. The contact of runways with the rolling bodies is called the
straight line (line) contact. The shape of the contact area depends on the surface curvature of both bodies in the contact point. The size of a contact area increases with the rising pressure. The formation of a contact area is always connected with the bodies flattening or bulging in the contact point where both bodies approach at the same time. The maximum load which has effect in the bearing is stated on Strubeck theory basis. To make the calculation valid, there are real figures which determine basic dynamic load bearing capacity according to the standard ISO 281. There is an assumption that load has effect only in a radial direction, which means that the half of the rolling bodies in the bearing is loaded. The standard is valid when the equivalent load is lesser or equal the half of dynamic load bearing capacity.

2.1. Hertz bodies contact theory
The theory is used for approximate calculation of a contact area with elliptical shape, also for present strains and flexible shift in two bodies contact. Rollers have a logarithmic profile which guarantees nearly even tension distribution along the whole length in the contact point with the rings runways in a standard direction. The pressure distribution is elliptical (Figure 1). The semi-minor axis $b$ is calculated using the formulas [1]. The highest contact tension is calculated by the formula [2]. If these parameters are known, it is possible to calculate roller bearings tension based on the relation [3] for the radial part and [4] for the axial part of the force:

$$
\sigma_{\text{max}} = 268.5 \cdot \sqrt{\frac{Q}{\frac{1}{l_{we}} + \frac{1}{F/E}}} ^{1/2}
$$

$$
Q = \frac{5F_r}{iz \cos \alpha}
$$

$$
Q = \frac{F_r}{iz \sin \alpha}
$$

Figure 1. Elliptical pressure distribution in the contact point.

2.2. Lundberg- Palmgren stress theory
According to this theory the determinative tension for the roller bearings fatigue is alternating shear stress $\tau_0$ inside the material which changes its direction. The highest stress is under the surface at the end of the semi-minor axis of the contact ellipse and in the depth $z_0$ (Figure 2) which equals
approximately the half of the small width of the contact area [1]. The cracks formed at these places are present because of the material fatigue and they extend to the surface. They cause the initial pitting and they end by the local peeling off the material surface layer.

3. **Stiffness calculation using finite-elements analyses**

The calculation of the static FEM analysis simulates idealized condition which is approaching to the real application of the working bearing unit. Analyses are running on the basis of the defined operating conditions which are replaced with suitably specified working energies, shifts in expected direction and attachment. The picture shows the selected analysed bearing unit.

4. **Evaluation of the finite-element analyses results**

The first result of the FEM analyses lies in reaction forces at the contact points of the rolling elements and the outer ring runways for various angles of the active equivalent load $F_r$. This load was specified by the standard ISO 281 on the maximum bearable limit for the roller bearing. The picture shows the distribution of the reaction forces between the rolling elements and the outer ring runway for the specified angle $\lambda$.

The graph in Figure 4 shows the forces distribution in the bearing during the activity of the same force but at the different angle $\lambda$. It clearly shows the load distribution on less than half of the rolling elements in the bearing. The bearing body was loaded by the outer force $F_r$ at the specified angle $\lambda$. The result of the static analyses is represented by the graph in Figure 5. It contains the data about the overall deformation at the point of acting force which runs at the angle $\lambda$ and the figure $1/u$. 

![Figure 2. Lundberg-Palmgren stressed volume V.](image)

![Figure 3. The selected analysed bearing unit.](image)
The graph of the distribution of the reaction forces in the selected bearing unit.

\[
\frac{1}{u_\lambda} = \frac{1}{\text{Total Deformation}_{FEM}}
\]  

(5)

The correlation of the maximum loaded elements \( Q_{\text{max}} \) for the specified angle \( \lambda \) and the figure \( 1/u \) is evaluated using the graph in Figure 5. The results of the overall deformation for a selected bearing body were evaluated for the whole volume and in the plane of the active outer force. The graph shows the correlation between the cross-section stiffness of the axle box and the intensity of the maximum loaded body. Consequently it can be stated that the figure of the maximum loaded bodies correlates with the figure \( 1/u \) during the changing angle \( \lambda \) in all bearing units.

5. Evaluation of mathematical results

The results gained by finite-elements analyses are sources for an analytical calculation. To simplify the calculation and to avoid the complicated finite-elements analysis there has been an effort to adjust the calculation of the maximum loaded body. According to Strieber this can be performed using the parameter \( \psi_\lambda \) in the mentioned relation. This parameter is a dimensionless quantity calculated in accordance with the formula (6) for all directions of the active load.
The parameter is the result of the forces ratio of the maximum loaded body according to Stribeck and finite-element analysis. It means that it includes all the influences present in the bearing unit model but also during its analysis. The effort lies in the creation of the general theory which would be common for all the standardized bearing units. This can be achieved by the transformation of the curve with the parameter figures $\psi$. The curve is the function of the direction change of the active load. The curve calculated by the interpolation of the parameter $\psi$ is described in formula (7). It is created by the substitution of the angle value $\lambda_n$ which is adjusted by the coefficients $s(n)$. These coefficients are different for each bearing unit.

$$
\psi_{\text{interpol}} = s(1) \cdot \lambda_n^4 + s(2) \cdot \lambda_n^3 + s(3) \cdot \lambda_n^2 + s(4) \cdot \lambda_n + s(5)
$$

Figure 6 shows the graph which represents the created curve using the calculation according the formula (7) and the points in the graph show the parameter $\psi$ figures calculated using the formula (6).

The calculated values $Q_{\text{max}}$ using the finite-element analysis and the analytical calculation were used for durability calculation according to Lundberg-Palmgren theory. From the dependency of this durability theory there is a derived relation for calculation of bearing cycles. This relation is consequently divided by defect frequencies $f_i$ for getting the value of the basic rating life of the bearing $L_{10}$. The results were calculated using the value $Q_{\text{max}}$ from the analytical calculation. The value $Q_{\text{maxStribeck}}$ was changed by the parameter $\psi_{\text{interpol}}$. Figure 7 shows the graph which compares the calculated durability according to the standard ISO 281, the durability calculated using FEM
analysis and the durability calculated using the analytical solution. The durability based on the standard ISO 281 standard is constant, while the durability $N_{MKP}$ and $N_{ANLT}$ change on the basis of the active load direction on the bearing unit.

6. Conclusion
This article demonstrates the influence of the construction stiffness in the arrangement of the roller bearings on their durability for the selected standardized bearing units. The bearing units contain the radial roller bearing with 13 elements. These elements have a logarithmic profile and they are loaded at the capacity limit which is specified by the standard as the value 0.5, Cr.

The results represent the value of the maximum load $Q_{max}$ in the bearing and the deformation of the bearing unit in the given direction of the active outer force. These values depending on the angle $\lambda$ correlate with each other. This means that there is a functional dependency among the $Q_{max}$, the body deformation and the angle $\lambda$ of the active outer force. The article presents the analytical solution using the parameter $\psi$ which is the function for the angle $\lambda$. The parameter also modifies the $Q_{max}$ calculation according to Stribeck and includes the influence of the construction stiffness of the roller bearings arrangement on the durability calculation according to Lundberg-Palmgren theory.

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