Controlled Dynamics of Interfaces in a Vibrated Granular Layer

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We present experimental study of a topological excitation, interface, in a vertically vibrated layer of granular material. We show that these interfaces, separating regions of granular material oscillation with opposite phases, can be shifted and controlled by a very small amount of an additional subharmonic signal, mixed with the harmonic driving signal. The speed and the direction of interface motion depends sensitively on the phase and the amplitude of the subharmonic driving.

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Despite their ubiquity and many practical applications, an understanding of the fundamental dynamical behavior of granular materials remains a serious challenge [1]. One of the main obstacles for the development of a continuous description of granular flow is the difficulty in performing quantitative experiments under controlled conditions. Testing of theoretical models of the basic excitations of the system is especially important. It has been shown recently [2–5] that thin layers of granular materials subjected to vertical vibration exhibit a diversity of patterns which may play the role of such fundamental excitations. The particular pattern is determined by the interplay between driving frequency \( f \) and the acceleration amplitude \( \Gamma = 4\pi^2 A f^2 / g \) of the cell, where \( A \) is the amplitude of oscillation and \( g \) is the acceleration due to gravity. Periodic patterns, such as squares and stripes, or localized oscillons vibrating with frequency \( f/2 \) appear at \( \Gamma \approx 2.4 \). At higher acceleration (\( \Gamma > 3.72 \)), stripes and squares become unstable, and hexagons appear instead.

Further increase of acceleration replaces hexagons with a non-periodic structure of interfaces separating large domains of flat layers oscillating with opposite phase with frequency \( f/2 \). These interfaces were called kinks in Ref. [3,4]. These interfaces are either smooth or “decorated” by periodic undulations depending on parameters \( \Gamma \). For \( \Gamma > 5.7 \), various quarter-harmonic patterns emerge. Several theoretical approaches including molecular dynamics simulations, order parameter equations and hydrodynamic-type models, have been proposed recently to describe this phenomenology, see e.g. [6,7].

In this Letter we present an experimental study of the dynamics of interfaces in a vibrated thin layer of granular material. We find that an additional subharmonic driving results in a controlled displacement of the interface. In the absence of subharmonic driving, the interface drifts toward the middle of the cell. When the subharmonic frequency \( f/2 \) is slightly detuned, the interface moves periodically about the middle of the cell. The present experimental results are in agreement with theoretical predictions [1].

Interfaces in a granular layer separate regions of granular material oscillating with opposite phases with respect to the bottom plate of the vibrating cell. These two phases are related to the period-doubling character of the flat layer motion at large plate acceleration. Since an interface separates two stable symmetric dynamic phases, it can be interpreted as a topological defect, similar to a domain wall in ferromagnets separating regions of opposite magnetization [10]. Interfaces can only disappear at the walls of the cell or annihilate with other interfaces. The existence of the interfaces can be understood from the following consideration. Since grains in a layer lose their kinetic energy in multiple inter-collisions during landing at the plate, the behavior of a granular layer can be compared with the dynamics of a fully inelastic ball bouncing on a plate vibrating with amplitude \( A \) and frequency \( f/ \). In this case, for driving with acceleration \( \Gamma \) less than \( \Gamma_0 \approx 3.72 \), the ball lifts to the same height at each cycle \( \Gamma \). Above \( \Gamma_0 \) the motion exhibits period-doubling, i.e. the heights of elevation alternate at each cycle. As a result, for \( \Gamma > \Gamma_0 \), the two states of the bouncing ball, differing by the initial phase, would coexist. If the analogous states of the bouncing flat layer co-exist in different parts of the cell, they have to be separated by an interface. These interfaces, found experimentally [9] and theoretically [8], are flat for high frequency drives and show transverse instability leading to periodic decoration at lower frequencies (see Fig. 1).

In an infinite system a straight interface is immobile due to the symmetry between alternating states: the motion of flat layers on both sides of the interface is identical with a phase shift \( \pi \). Additional driving at the subharmonic frequency \( f_1 = f/2 \) will break the symmetry between domains with opposite phases. Depending on the phase of the additional driving, \( \Phi \), with respect to the phase of the primary driving, the relative velocity of the layer and the plate at collision will differ on different sides of the interface, and the material on one side will be lifted to
a larger height than on the other side. As a result, the
direction of the interface motion can be controlled by the
phase Φ. The speed at a given Φ is determined by the
amplitude of the subharmonic acceleration γ.

In our previous work we have developed a phenomeno-
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of granular material [4]. On the basis of our order pa-
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We developed an alternative experimental technique which allowed us to measure simultaneously the relaxation time $\tau$ and the “asymptotic” velocity $V_0$. This was achieved by a small detuning $\Delta f$ of the additional frequency $f_1$ from the exact subharmonic frequency $f/2$, i.e. $\Delta f = f_1 - f/2 \ll f$. It is equivalent to the linear increase of phase shift $\Phi$ with the rate $2\pi \Delta f$. This linear growth of the phase results in a periodic motion of the interface with frequency $\Delta f$ and amplitude $X_m = V_0/\sqrt{\tau^{-2} + (2\pi \Delta f)^2}$ (see Eq (2)). The measurements of the “response functions” $X_m(\Delta f)$ are presented in Fig. 6. From the dependence of $X_m$ on $\Delta f$ we can extract parameters $V_0$, $\alpha$ and $\tau$ by a fit to the theoretical function. The measurements are in very good agreement with previous independent measurements of relaxation time $\tau$ and susceptibility $\alpha$. For comparison with the previous results, we indicate the values for $\tau$ and $\alpha$, obtained from the response function measurements of Figs. 2 and 5 (stars). The measurements agree within 5 %.

In summary, the position of a vertically vibrated granular layer can be controlled by a very small acceleration applied at the subharmonic frequency (of the order of 0.1% of the primary harmonic acceleration). The direction and magnitude of the interface displacement depend sensitively on the relative phase of the subharmonic drive. Our measurements confirm the theoretical predictions made on the basis of the order parameter model.

We observed that period-doubling motion of the flat layers produces subharmonic driving because of the finite ratio of the mass of the granular layer and the cell. This in turn leads to the restoring force driving the interface towards the middle of the cell.

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Venkataramani and E. Ott, Phys. Rev. Lett. 80, 3495 (1998).
[9] I. Aranson, L. Tsimring, and V.M. Vinokur, patt-sol/9802004
[10] D.J. Craik and R.S. Tebble, Ferromagnetism and ferromagnetic domains, NY, Wiley, 1965.
[11] E. Van Doorn and R.P. Behringer, Europhys. Lett. 40, 387 (1997)
[12] A. Mehta and J.M. Luck, Phys. Rev. Lett. 65, 393 (1990)
[13] The measured acceleration $\mu$ may differ from the applied subharmonic (sinusoidal) driving $\gamma$ since the granular material moves inside the cell. We measure $\gamma$ independently by removing the granular material from the cell.

![Figure 3](image3.png)

**FIG. 3.** Equilibrium position of interface for $\Phi = 80^0$ (a); $\Phi = 170^0$ (b); $\Phi = 260^0$ (c) for $\Gamma = 4.1$, $f = 40$ Hz, $f_1 = f/2$, and $\gamma = 0.6\%$ of $\Gamma$ in a rectangular cell.

![Figure 4](image4.png)

**FIG. 4.** (a) Equilibrium position $X$ (b) amplitude of measured subharmonic acceleration $\mu$ as functions of phase $\Phi$. Circular cell, $\Gamma = 4.1$, $f = 40$ Hz, $\gamma = 1.25\%$ of $\Gamma$.

![Figure 5](image5.png)

**FIG. 5.** Susceptibility $\alpha = V_0/\gamma$ vs $\Gamma$ at $f = 40$ Hz, rectangular cell. Inset: Displacement $X$ as function of $\gamma$ at $\Phi = 260^0$.

![Figure 6](image6.png)

**FIG. 6.** Maximum displacement $X_m$ from center of rectangular cell as function of frequency difference $\Delta f = f_1 - f/2$ for $f = 40$ Hz, and for $\Gamma = 3.97$ (circles) and $\Gamma = 4.1$ (diamonds). Dashed lines are fit to $X_m = V_0/\sqrt{\tau^2 + (2\pi \Delta f)^2}$. The values of $\alpha$ and $\tau$ obtained from the fit are also indicated in Figs. 2 and 5 (stars).