Lattice bosons in quartic confinement

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Abstract. We present a theoretical study ofbose condensation of non-interacting bosons in finite lattices in quartic potentials in one, two, and three dimensions. We investigate dimensionality effects and quartic potential effects on single boson density of energy states, condensation temperature, condensate fraction, and specific heat. The results obtained are compared with corresponding results for lattice bosons in harmonic traps.

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1 Introduction

Over last few years, bosons and fermions in optical lattices have emerged as important controllable systems for investigations into several properties of quantum many-particle systems [1]. These are clean systems in which experimentalists have achieved great control over a wide range of particle numbers, particle hopping, and strength and sign of inter-particle interactions. Experimental groups have conducted extensive studies [2–8] of several properties of many-boson systems in one, two, and three dimensional optical lattices. The many-boson system in these experiments is under the combined influence of the periodic lattice potential and an overall confining harmonic potential. Many theoretical studies [9–20] of such lattice bosons in harmonic confinement have appeared in recent years. Unlike the case of lattice bosons in harmonic confinement, the effect of anharmonic potentials on the properties of lattice bosons have begun to be explored only recently [21]. In that work Gygi and collaborators studied zero temperature properties of strongly interacting lattice bosons in a quartic trap. It has been suggested [21] that it is experimentally possible to create an optical lattice in a quartic trap employing a combination of red-detuned and blue-detuned Gaussian laser beams. Finite temperature properties of lattice bosons in a quartic trap is of considerable interest in this context.

In this paper, we present a theoretical study of lattice bosons in a quartic potential in one, two, and three dimensions (1d, 2d, and 3d). We consider non-interacting bosons in a periodic lattice in 1d in a 1d quartic potential, a square lattice in a 2d quartic potential, and a cubic lattice in a 3d quartic potential. We study the effects of the potential on one-boson density of energy states (DOS) and the temperature dependence of ground state occupancy and specific-heat. We compare the results obtained for lattice bosons in quartic traps to the corresponding results for lattice bosons in harmonic traps [20]. This work is presented in Sections 2 and 3, and conclusions are given in Section 4. In the work presented in the following sections, we have not included the effect of boson-boson interaction (U). Our results would approximately also hold in the weak interaction regime (U \ll t, where t is the boson hopping energy) where the interaction induced depletion effects are not significant [22]. A weakly interacting regime may be achieved by adjusting the lattice potential depth to a low value as has already been done for lattice bosons in quadratic traps [2,4].

2 Model and method

In this section, we give a brief presentation of the model of the system and the method followed in the calculations of various properties presented in later sections. The Hamiltonian of the many-boson system we consider is

\[ H = -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + c_j^\dagger c_i) + \sum_i V(i)n_i - \mu \sum_i n_i, \]  

(1)

where t is the kinetic energy gain when a boson hop from site i to its nearest neighbor site j in the optical lattice, c_i^\dagger is the boson creation operator, V(i) is the quartic potential at site i, n_i = c_i^\dagger c_i the boson number operator, and \mu the chemical potential. The forms of the quartic potentials

\[ V(i) = V_0 \sum_{i,j} \phi_{i-j}^4, \]

where \phi_{i-j} is the wave function of the optical lattice.
used are: $V(i) = Q x_i^4$ in 1d, $V(i) = Q(x_i^4 + y_i^4)$ in 2d, and $V(i) = Q(x_i^4 + y_i^4 + z_i^4)$ in 3d. We first obtain the matrix representation of the system Hamiltonian in a site basis. We numerically diagonalize it to obtain energy levels of a lattice boson. We have used open boundary conditions. We have chosen lattice sizes large enough so that finite size effects are absent in the results presented in the next section. The lattice sizes were fixed by finding the lattice size beyond which results remain unchanged with further increase of lattice size. This depends on the magnitude of the quartic potential strength since it decides the spread of the boson distribution in the lattice for a given value of $t$ and temperature. The energy levels ($E_i$) obtained for a boson in these large lattices are used in calculations of the DOS, ground state occupancy, and the specific heat. The chemical potential and boson populations in the various energy levels are calculated using the boson number equation: $N = \sum_{i=0}^{m} N(E_i)$, where $E_0$ and $E_m$ are the lowest and the highest single boson energy levels and $N(E_i) = 1/[\exp[\beta(E_i - \mu)] - 1]$ in which $\beta = 1/k_B T$ with $k_B$ the Boltzmann constant and $T$ the temperature. The specific heat is calculated from the temperature derivative of total energy ($E_{tot} = \sum_{i=0}^{m} N(E_i) E_i$). All energies are measured in units of $t$.

3 Results and discussion

3.1 One-boson density of energy states

In this section, in Figures 1–4, we present our results on the one-boson density of energy states (DOS) for a lattice bosons in a quartic trap and compare with the corresponding DOS for a lattice boson in a harmonic trap for which: $V(i) = K(x_i^2 + y_i^2 + z_i^2)$. In Figure 1, we have exhibited the DOS for bosons in optical lattices with harmonic and quartic confinement potentials for several values of $q = Q a^3$ and $k = K a^2$, where $a$ is lattice constant. We find that the confining potential has a significant effect on the DOS in both cases. In the case of quartic confining potential (Fig. 1a), the divergence in DOS at the band edges is found to be suppressed and eventually destroyed with increasing strength of the potential. Further, the quartic potential spreads the DOS over a wide energy scale. It is also to be noted that for small potential strengths shown in the figure, the low energy part of the DOS continues to show similarity to the case of the DOS of lattice bosons without confinement. In comparison, in the case of harmonic potential (Fig. 1b), divergence of the DOS at the lower edge of the band is destroyed while the one at higher edge is significantly suppressed even for a weak harmonic potential, and with increasing strength the DOS is flattened to a wide energy scale [23]. In Figure 2, we have shown our results on the DOS of a boson in a 2d square lattice in a 2d quartic potential (Fig. 2a) and the corresponding results for the harmonic case (Fig. 2b).

In the quartic potential case, we find that the Van Hove singularity is strongly suppressed by the confinement potential. In comparison, the Van Hove singularity is destroyed by the harmonic potential [23]. In both cases the confining potentials spread the DOS compared to the pure lattice case. Figure 3 shows the DOS of a boson in a cubic lattice in a quartic potential. Increasing the quartic potential strength clearly has a strong effect on the DOS. Similar results are obtained for the harmonic potential case as well, as shown in Figure 4. The dotted lines in Figure 4 are the single particle DOS.