Workspace analysis for a five degrees of freedom hybrid engraving plotter

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Abstract: At present, there is a little research on the workspace of the hybrid mechanism, and the further development of the engraving plotter is hindered due to the limitation of the workspace. In order to clearly understand the workspace of the hybrid engraving plotter, and verify whether the workspace meets the industrial production needs, the five degrees of freedom hybrid engraving plotter is analysed and studied. First, the structure of the engraving plotter is analysed and simplified, and then the direct equation is deduced by the analytic method. Second, according to the engraving plotter direct solution equation, the boundary of workspace is determined and the volume is calculated by using the optimization algorithm. In order to understand the change of workspace in different z planes, the workspace of the parallel part is intercepted by some planes, which are parallel to XOY; thus, the authors know that the cut plane area of the workspace increases with the increase of Z values. Finally, the influence of the mechanism parameters is discussed on the dexterity. These theoretical knowledge will make important research significance for guiding engineering practice.

1 Introduction

The hybrid mechanism is a combination of serial mechanisms and parallel mechanisms. It inherits the characteristics of the high strength and stiffness of parallel mechanisms and has advantages of a large workspace of serial mechanisms. The workspace is an important index to measure the performance of a robot, which refers to the set of all points by the end effector of the robot. It can be divided into the reachable workspace and flexible workspace [1, 2]. Owning to the small workspace and poor posture of the parallel mechanism with six degrees of freedom (6-DOF), the hybrid configuration with limited DOF has attracted the interest of the researchers [3], especially those mechanisms that can realise three-dimensional (3D) translational motion. For example, 3-PTT and 3-TPT mechanisms were designed by Tsai, and the Delta mechanism was proposed by Clavel, which are widely used in industrial production [4–6].

The workspace of serial mechanisms and parallel mechanisms have been studied at home and abroad. The analytic, graphic, and numerical methods can be used to solve the workspace of a robot [7, 8]. The analytic and graphic methods are not universal and complicated; however, the numerical method based on the polar axis theory and the optimisation method is used to calculate the robot workspace boundary point. At present, the research results of the robot workspace are relatively large, but the hybrid mechanism is still relatively small. Some scholars used the geometric superposition method to plan the dexterous workspace of the robot, which provides a theoretical basis for structural design and motion control [9]. Wang et al. analysed the workspace of the 3T2R redundant hybrid mechanism by the searching method, and obtained the performance index of the mechanism [10]. Other researchers proposed a closed method to solve inverse kinematics and calculate the workspace of the robot, which can quickly solve the workspace [11]. Liu et al. used the Monte Carlo method to solve the robot workspace, which can get more accurate workspace [12]. Xu et al. [13] found the workspace of the 3-PUU by using the Monte Carlo method. Stamper et al. [14] introduced the optimisation study of the global condition index for the workspace of a 3-DOF translational platform.

These scholars have made contributions to the workspace for parallel mechanisms and serial mechanisms to a certain extent; however, most of them are mainly concerned about the workspace of serial and parallel mechanisms. There is less research on the workspace of hybrid mechanisms, especially in the field of carving. So this article mainly discusses the workspace of the hybrid mechanisms.

2 Structure features of the engraving plotter

In order to clearly describe the structure of the hybrid engraving plotter, the 3D model of the engraving plotter is constructed with SolidWorks, as shown in Fig. 1.

The hybrid engraving plotter is usually divided into two parts: the parallel part and the serial part. The parallel part is mainly composed of the fixed base, the moving platform, three motors, and three links that include a slider and a parallelogram linkage. Each limb has six revolute pairs and one moving pair. The serial part is composed of a vertical rotating rod and a tool.

The simplification of the parallel part of the engraving plotter is shown in Fig. 2. The fixed coordinate system O – xoy is established at the center point of the fixed base, xo points of the intersection, xi point of the motor and the fixed base, P, P, P, form a circumcircle with radius R. The moving coordinate system O’ – xoy is established at the center point of the moving base, xo points of the intersection, x, points of the base, P, P, P, form a circumcircle with radius r. xo and zo are perpendicular to the fixed base, and yo determined by the right-hand rule. The intersection of the slider and parallelogram linkage are Qi (i = 1, 2, 3).

3 Forward algorithm for engraving plotter

For the engraving plotter, there is only 3D translational motion in the parallel part, so the forward solution of the parallel part is to solve the position of the moving platform center point G(u, v, w) when the displacement of the three sliders mi, mi, mi, is known. The forward solution of the parallel mechanism is usually obtained by the numerical iteration method. However, because of the simple structure, the analytic method is used to solve the forward solution of the engraving plotter.
In the fixed coordinate system \( O - xoy_0z_0 \), the space vectors to points \( B_1, B_2, B_3, Q_1, Q_2, Q_3 \) could be denoted as:

\[
Q_i = \begin{bmatrix} \cos \alpha_i \\ \sin \alpha_i \\ -m_i \end{bmatrix}, \quad \alpha_i = \frac{2\pi}{3}(i-1), \quad (i = 1, 2, 3)
\]

(1)

In the moving coordinate system \( O' - x_0y_0z_0 \), the space vectors to points \( B_1, B_2, B_3, N_1, N_2, N_3 \) could be denoted as:

\[
N_i' = \begin{bmatrix} \cos \alpha_i \\ \sin \alpha_i \\ 0 \end{bmatrix}, \quad \alpha_i = \frac{2\pi}{3}(i-1), \quad (i = 1, 2, 3)
\]

(2)

In the fixed coordinate system \( O - xoy_0z_0 \) it is assumed that the space vectors to point \( O' \) could be denoted as: \( O' = [u, v, w]^T \)

governed by geometrical relationship, the position of points \( N_i, N_{i'}, \) and \( N_i' \) could be obtained:

\[
N_i = \begin{bmatrix} \cos \alpha_i \\ \sin \alpha_i \\ u \end{bmatrix} + \begin{bmatrix} v \\ w \end{bmatrix}, \quad \alpha_i = \frac{2\pi}{3}(i-1), \quad (i = 1, 2, 3)
\]

(3)

According to the geometrical relationship, the following equations can be listed:

\[
L_i = |Q_i - N_i| (i = 1, 2, 3)
\]

\[
\begin{align*}
(r + u - R)^2 + v^2 + (w - m)^2 &= L_i^2 \\
\left(\frac{1}{2}r + u + \frac{1}{2}R\right)^2 + \left(\frac{\sqrt{3}}{2}r + v - \frac{\sqrt{3}}{2}R\right)^2 + (w - m)^2 &= L_i^2 \\
\left(-\frac{1}{2}r + u + \frac{1}{2}R\right)^2 + \left(\frac{\sqrt{3}}{2}r + v + \frac{\sqrt{3}}{2}R\right)^2 + (w - m)^2 &= L_i^2 \\
\end{align*}
\]

(4)

\[
v = \frac{(m_i - m)(2w_i - m_i)}{2\sqrt{3}(r - R)}
\]

(5)

\[
u = \frac{2(m_i - m)(2w_i - m_i) + (m_i - m)(2w_i - m_i)}{6(r - R)}
\]

(6)

Assuming that \( m_i - m = D, m_i - m = E, \) and \( r - R = F, \) and substituting (5) and (6) into (4), one can obtain (see (7)) \( w \) can be calculated by (7), and substituting \( w \) into (5)–(6), \( u \) and \( v \) will be obtained.

The serial part position and attitude can be solved directly when the position of the moving platform center point is known. The results can be obtained according to the following geometrical relationship:

\[
p_x = u + L_z \times \cos \theta_1 \times \sin \theta_0
\]

(8)

\[
p_y = v + L_z \times \sin \theta_1 \times \sin \theta_0
\]

(9)

\[
p_z = w - L_z \times \sin \theta_0
\]

(10)

where \( L_z \) is the length of the vertical rotating rod, and \( L_z \) is the length of the tool, \( \theta_1 \) and \( \theta_0 \) are the motor angles of the serial part, \( p_x, p_y, \) and \( p_z \) are the displacement components of the end point.
angle between the vectors $n_i$ and $n'_i$ could be denoted as $\theta_i$, and the angle between the vectors $n^i$ and $n^i_c$ could be denoted as $\theta^i_c$.

The angles should meet the following criteria:

$$\theta_i = \arccos \frac{n_i \cdot n^i_i}{|n_i| |n^i_i|}, \quad \text{for } i = 1, 2, 3$$

(12)

$$\theta^i_c = \arccos \frac{n^i_c \cdot n^i}{|n^i_c| |n^i|}, \quad \text{for } i = 1, 2, 3$$

(13)

### 4.1.2 Constraints of the serial part

The structure of the serial part is shown in Fig. 4.

(i) The limitation of the link length: the lengths $L_4$ and $L_5$ of the serial part will affect the size of the workspace, and the length of the rod should be increased as much as possible in the design.

(ii) The change of the kinematic pair angle: the range of the motor angle in the horizontal direction is $-180^\circ \leq \theta_i \leq 180^\circ$, and the range of the motor angle in the vertical direction is $-90^\circ \leq \theta_i \leq 90^\circ$.

#### 4.2 Workspace boundary in the parallel part

Considering the displacement of the three sliders and the constraint of the rotation pairs, a 1D search method (the polar coordinate search method) is used to search and analyse the workspace boundary. The specific solution steps are shown in Fig. 5.

(i) The basic parameters of the parallel part are as follows. The length of the parallelogram linkage is $L_4 = 750$, the radius of the fixed base platform is $R = 750$, the radius of the moving platform is $r = 350$, and the displacements of the three sliders are as follows:

$$0^\circ \leq \theta^i \leq 45^\circ, \quad -45^\circ \leq \theta^i_c \leq 45^\circ, \quad 0 \leq m_i \leq 500 \quad (i = 1, 2, 3)$$

(ii) Making $z_{\text{max}}$ increase to $z_{\text{max}}$, according to the step length size increment $\Delta Z$, when given a $Z$ valve, the polar coordinates of $X$ and $Y$ can be calculated in the $Z$ plane, so that $x = p \cdot \cos(\theta)$, $y = p \cdot \sin(\theta)$ where $p$ is the polar radius of the polar coordinates, $\theta$ is the angle between the $X$-axis and the polar radius.

(iii) Making $p$ and $\theta$ start from zero and increasing the step length size to $\Delta p$ of the polar radius. If the value of the solution reaches or exceeds the workspace boundary, the boundary point of the workspace is determined and recorded. At this time, making $\Delta p$ zero, and the angle $\theta$ is increased by the angle step length, which is directly increased to $2\pi$. After the plane search is completed, the angle $\theta$ becomes zero, and finally, $z = z_{\text{max}} + \Delta Z$, until $z = z_{\text{max}}$.

According to the above parameters and search conditions, the workspace of the parallel part can be obtained, as shown in Fig. 6.

In order to compare the size of the workspace at different planes, the workspace was intercepted by a different plane, which is parallel to the $XOY$ plane. Fig. 7 indicates the form process of the cut plane.

According to the results of the analysis, the following conclusions are drawn:

(i) It can be seen from Fig. 6 that the outer contour of the parallel part workspace is composed of a series of arcs, and the whole shape is like an inverted cone shape. As we can see from the shadow of the parallel part workspace in $YOZ$, the workspace is protruding outwards when $Z > -700$ mm; however, the workspace is internal protruding, with both sides sunken when $Z < -700$ mm.

(ii) We can understand from Fig. 6 that the workspace of the parallel part is symmetric about the $YOZ$ plane. Also, we can understand from the top view that the three convex corners are seen because three parallelogram linkages of the parallel part are showing a $120^\circ$ space symmetry. Therefore, it can be concluded...
whose radius is 300 mm and the height is 400 mm, and the height is 400 mm, and the
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Fig. 9 is the boundary of the workspace of the serial part.

(iii) We give the projection of the three planes in the XOY plane as shown in Fig. 8. It can be seen that the cross-sectional area of different XOY planes is different.

4.3 Workspace boundary in serial part

The basic parameters of the serial part are as follows. The length of the vertical rotating rod is \( L_4 = 400 \) mm, the length of the tool is \( L_5 = 300 \) mm, the workspace of the vertical rotating rod is a space ring whose radius is 300 mm and the height is 400 mm, and the workspace of the tool is a semi-circle with a radius of 300 mm. Fig. 9 is the boundary of the workspace of the serial part.

We can conclude from Fig. 9 that the workspace of the serial part is bowl shaped, which is affected by the length of the rod. We should ensure that under the condition of the reasonable and beautiful structure in the design, the length of the serial part is increased as much as possible.

4.4 Workspace analysis of the engraving plotter

Based on the parameters of the parallel and serial parts, the parameters of the engraving plotter are as follows: (see equation below) The workspace of the engraving plotter is composed of the workspace of the parallel part and that of the serial part. The program is written in MATLAB to gain the picture of the engraving plotter as shown in Fig. 10.

According to the analysis results, the following conclusions are drawn:

(i) The whole workspace of the hybrid engraving plotter shows the shape of a gyroscope, which can ensure the uniform change of the end point, and the vibration will not appear too much.

(ii) The workspace of the hybrid engraving plotter is composed of the workspace of the parallel part and that of the serial part. Therefore, the workspace is much larger than the single parallel mechanism or serial mechanism. There are many singular points in the spatial edge points for some hybrid mechanism, and the error of the edge points is greater than that of the internal points. However, since the effective points of the engraving plotter are more in the workspace, the engraving plotter has not only good continuity, but also less singularity.

(iii) The workspace of the hybrid engraving plotter is symmetric about the YOZ plane, and it can be seen from the top view that the workspace is symmetrical in space 120°.

4.5 Workspace volume of the hybrid engraving plotter

The volume of the workspace of the hybrid engraving plotter is used to measure its working range. As we can see from Fig. 10 workspace of the parallel part is irregular, so that the workspace of the engraving plotter cannot be calculated by the formula algorithm of a regular graph. Therefore, the workspace volume of the parallel part should be calculated by the numerical method and the workspace volume of the serial part can be accurately calculated by the analytic method. The workspace volume of the hybrid engraving plotter is the superposition of the serial part volume and serial part volume.

The basic idea of solving the parallel part workspace with the numerical method: by using the plane that is parallel to the XOY plane. The workspace is cut into a series of micro-molecular spaces, whose thickness is \( \Delta z \). If \( \Delta z \) is small enough, each micro molecular space can be regarded as a column with the height of \( \Delta z \). So that the volume of the parallel part workspace is obtained by superimposing the volume of all cylinders.

Any point on the bottom boundary of each micro-molecule space can be expressed in polar coordinates \( (p, \theta) \), where \( p = \sqrt{x^2 + y^2} \) and \( \theta \in (0°, 360°) \). First, assuming that \( m_i(p, \theta) \) is the bottom boundary point of the micro-molecule space, and then setting the step length increment \( \Delta \theta \), the next boundary point \( m_i(p, \theta + \Delta \theta) \) can be obtained. In turn, one can obtain all the boundary points on that surface.

According to these boundary points, the cylinder can be divided into several small differential regions, which are labelled as \( \Delta A_i \). When the step length increment is small enough, these small regions can be regarded as sectors, whose angle is \( \Delta \theta \). Each sector area \( \Delta A_i \) can be obtained by the following sector area formula:

\[
\Delta A_i = \frac{1}{2} p_i^2 \Delta \theta \quad (k = 0, 1, 2, \ldots)
\] (14)

Therefore, the bottom area of the cylinder can be obtained by the superposition of all the differential sectorial areas, and the volume

\[
L = 750, R = 750, r = 350, 0 \leq m_i \leq 500(i = 1, 2, 3), -180° \leq \theta_i \leq 180°, -90° \leq \theta_i \leq 90°;
\]
\[
L_5 = 300, 0° \leq \theta_4 \leq 45°, -45° \leq \theta_5 \leq 45°, -500 < h_i < 0, i = 1, 2, 3, L_4 = 400.
\]
The workspace of the engraving plotter is as follows:

\[ V_i = \sum \Delta V_i = \frac{1}{2} \sum \rho_i \Delta \theta \Delta z \]  

(16)

According to (16), the parallel part volume of the engraving plotter can be obtained \((V_i = 1.0473 \times 10^7 \text{mm}^3)\). It can be seen from (16) that the volume of the parallel part is related to the step length increment \(\Delta z\) and \(\Delta \theta\) of the division, and the smaller the step length increment is, the more accurate the volume is obtained. However, the smaller step length increment will lead to the increase of calculation, so that it should be selected according to the actual situation in the general engineering practice.

Due to the simple structure, the workspace of the serial part is not complicated, so that the volume of the workspace can be calculated according to the formula of the regular graph. Assuming that the serial part volume of the workspace is \(V_s\), the workspace of the vertical rotation rod is \(V_v\), and the workspace of the tool is \(V_t\), the workspace volume of the serial part is as follows:

\[ V_s = V_v + V_t = \pi L_1^2 \times 300 + \frac{2}{3} \pi L_3^3 \]  

(17)

Thus, we can obtain the workspace volume:

\[ V = V_s + V_l = 1.104268 \times 10^7 \text{mm}^3 \]  

(18)

From the volume of the workspace calculated, it can be seen that the workspace volume of the hybrid engraving plotter is large enough to the industrial and production needs.

### 4.6 Effect of the mechanism parameters on dexterity

The selection of the mechanism parameters directly affects the size of the workspace. Therefore, it is significant to discuss the influence of the mechanism parameters on the workspace. The volume is a parameter to measure the size of the workspace, however, the search method is complex for the determination of the volume. Therefore, it is significant to discuss the influence of the mechanism parameters on the workspace. The dexterity can reflect the comprehensive transmission performance of the mechanism, which can be used to determine the optimal structural parameters. The relationship between the workspace and the parameters of the mechanism can be determined by the influence of parameters on the dexterity. Since the moving platform has only three kinds of translational motion, and each branch is exactly the same, the parallel part has no posture change. So it is not necessary that to analyse the posture dexterity, we should also analyse the position dexterity.

For the first branch, the velocity relationship can be established as follows:

\[ V \cdot \vec{L}_1 = v_1 e_1 \cdot \vec{L}_1 \]  

(19)

where \(V\) is the velocity vector of the moving platform centre point. \(v_1\) is the velocity of the slider 1, and \(e_1(0, 0, 1)\) is the unit vector for the z-axis.

In the same way, the relationship between the moving platform velocity and the slider velocity is established according to other two branches:

\[ V \cdot \vec{L}_2 = v_2 e_1 \cdot \vec{L}_2 \]  

(20)

\[ V \cdot \vec{L}_3 = v_3 e_1 \cdot \vec{L}_3 \]  

(21)

This result can be obtained by merging the above velocity relations.

\[ V = Jv \]  

(22)

where \(v = (v_1, v_2, v_3)^T\) is the vector of the input velocity and \(J\) is the Jacobian matrix of the mechanism, which can be expressed as

\[ J = \begin{bmatrix} \vec{L}_1 & \vec{L}_2 & \vec{L}_3 \end{bmatrix} e_1 \]

Considering that the centre point of the moving platform is located in the origin, three branches of the mechanism are symmetrically distributed. According to the matrix theory, the condition number is identical when the two matrices are reciprocal matrix and the matrix is multiplied by a constant that never changes the condition number of the matrix \([15]\). According to the above formula, the Jacobian matrix condition number for the mechanism is equivalent to calculate the matrix \(A\) condition number:

\[ A = \begin{bmatrix} \vec{L}_1 & \vec{L}_2 & \vec{L}_3 \end{bmatrix} \]

(23)

According to (4), we have

\[ A = \begin{bmatrix} R - r & 0 & \sqrt{L_1^2 - (r - R)^2} \\ \frac{r}{2} (r - R) & \frac{\sqrt{3}}{2} (r - R) & \sqrt{L_1^2 - (r - R)^2} \\ \frac{2}{3} (r - R) & - \frac{\sqrt{3}}{2} (r - R) & \frac{\sqrt{3}}{2} (r - R) \end{bmatrix} \]

The matrix condition number is the ratio of the maximum singular value to the minimum singular value. According to the matrix theory, the singular value of the matrix \(A\) is identical with the eigenvalue of the matrix \(A^T A\). The following conclusions can be obtained by the previous analysis:

\[ A^T A - \lambda E = 0 \]  

(24)

Solving this, we obtain the following equation:

\[ \frac{3}{2} \lambda - \lambda^2 \cdot 3[L_1^2 - s^2] - \lambda = 0 \]  

(25)

where \(s = r - R\)

According to (24),

\[ \lambda_1 = \lambda_2 = \lambda_3 = 3(L_1^2 - s') \]

(26)

when \(\lambda_1 > \lambda_3\), the dexterity \(k\) of the mechanism can be expressed as

\[ k = \frac{\sqrt{2}}{\sqrt{3}} = \frac{s}{\sqrt{2(L_1^2 - s')}} \]

(27)

when \(\lambda_1 < \lambda_3\), we have the following result:

\[ s = \frac{\sqrt{2}}{\sqrt{3}} L_1 \]

(28)

The dexterity \(k\) of the mechanism can be expressed as

\[ k = 1 \]  

(29)
At this point, the dexterity of the mechanism reaches the optimal value, and the motion performance of the mechanism is isotropic. Therefore, the parallel part of the engraving plotter is isotropic when the mechanism meets certain conditions. Assuming that the radius of the moving platform is \( r \), the radius of the fixed base is \( R \), and the length of the parallelogram linkage is \( L \). We will adopt millimeter (mm) as a unit in the next analysis.

(i) The influence of \( r \) on the dexterity:
Fig. 11a indicates that the dexterity decreases with the increase of the radius of the moving platform, which shows that the mechanism can be improved with the increase of the radius of the moving platform. Therefore, on the analysis of the mechanism, the better the moving platform radius is, the better the mechanism performance will be.

(ii) The influence of \( R \) on the dexterity:
It can be seen from Fig. 11b that the dexterity of the engraving plotter increases with the increase of the radius of the fixed platform, which shows that the mechanism does not improve with the increase of the radius of the fixed platform. Therefore, on the analysis of the mechanism, the smaller the fixed platform radius is, the better the mechanism performance will be.

(iii) The influence of \( L \) on the dexterity:
It can be seen from Fig. 11c that the dexterity decreases with the increase of the length of the parallelogram linkage, which shows that the mechanism can be improved with the increase of the parallelogram linkage length. It can be concluded that the parallelogram linkage length should be increased to the maximum value as much as possible.

5 Conclusion
(i) According to the above research, we know that the workspace of the 5-DOF hybrid engraving plotter is relatively large, and the workspace is not only symmetrical about the \( YOZ \) plane, but also is symmetrical about the space \( 120^\circ \). The whole workspace of the hybrid engraving plotter shows the shape of a gyroscope, which can ensure the uniform change of the end point, and the vibration will not appear too much.

(ii) In order to understand the change of workspace in a different \( z \) plane, the workspace of the parallel part is intercepted by some planes, which are parallel to the \( XOY \) plane. The areas of three cut planes are calculated. Thus, we know that the area of the workspace increases with the increase of the \( Z \) value. According to search method, the volume of the workspace of the hybrid engraving plotter is calculated. Thus, we find that the engraving plotter has a large volume, which can make the engraving plotter have a wide range of applications.

(iii) It is necessary to investigate the influence of mechanism parameters on the dexterity because the mechanism parameters determine the size of the dexterity. Based on the above analysis, we can find that the mechanism parameters \( r, R, \) and \( L \) will affect the workspace and dexterity. In general, the maximum workspace can be obtained by changing the parameters of the mechanism, and the engraving plotter is suitable for the situation with a larger workspace.

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