Precise study of three-nucleon systems within a representation without explicit use of the isospin formalism

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Abstract

A representation without explicit use of the isospin formalism is developed for the precise study of few-nucleon systems, and the advantages of the proposed approach are demonstrated. Using the example of three-nucleon systems with central exchange NN-interaction potentials of the general type, the complete equivalence is shown for the both approaches with and without the isospin formalism. The new systems of equations contain a less number of components as compared to the commonly used ones and are more suitable for precise study of few-nucleon systems, in particular, within variational approaches. Optimal variational schemes are developed with Gaussian basis, and the binding energies, r.m.s. radii, density distributions, formfactors, and pair correlation functions are calculated with a high and controlled precision for $^3\text{H}$ and $^3\text{He}$ nuclei.

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1 Introduction

Noticeable progress in precise calculations of the few-nucleon binding energies within variational approaches \cite{1}, \cite{2}, \cite{3} stimulates the further development of optimal variational schemes and their possible refinement in order to achieve higher precision in calculations \cite{4}, \cite{5} of the main structural characteristics of the bound states of few-nucleon nuclei. The progress in this field enables one to obtain ”exact” results for various structural characteristics of the systems and to put forward the problem of precise fitting of the nucleon-nucleon potentials universal for all the light nuclei, as well as to carry out accurate variational calculations for heavier nuclei.

The achievement of a high accuracy in calculations of a few-particle quantum system is restricted both by the number of particles and the number of the wave function spatial components. At the
first stage of studies of a simple qualitative type, rather rough assumptions were used about nuclear interaction potentials and the wave function spin structure (see [6], [7], [8]), thus an account of all the wave function spatial components was considered to be not necessary. But precise calculations with controlled accuracy need to take into account all the factors contributing to the result of a given accuracy. Such a statement of the problem is faced with rather nontrivial difficulties even for comparatively simple three- and four-nucleon systems. In the standard commonly used scheme with the use of the isospin formalism, an additional symmetry of the wave function and the mixture of states with different total isospins due to a consistent account of the Coulomb interaction lead to the essential growth of the number of spatial components of the wave function and thus to a serious complication of calculations. As a result, to find the total wave function of the $^3\text{He}$ nucleus in the framework of the isospin formalism, one has to solve a system of six spatial equations, while in the case of $^4\text{He}$ nucleus one has already to deal with twelve equations.

Here, we present our approach proposed in [9] without explicit use of the isospin representation and, using the example of three-nucleon systems, show the complete equivalence of such a more convenient approach to the traditional isospin formalism. It is shown for the wave functions in both approaches to be connected by certain relations, while all the physical observables calculated within both approaches are shown to coincide. The obtained much more simple systems of equations for the spatial components of the wave functions of three nucleons in the bound state or scattering process (in the doublet or quartet spin states) enabled us to carry out precise calculations for three-nucleon nuclei and to analyze their structure.

2 Equivalence of the representations with and without isospin

In order to show the complete equivalence of the representation without explicit use of isospin (protons are not identical to neutrons) to the traditional isospin formalism, we recall the explicit form of equations for the three-nucleon system $(2p, n)$ within the isospin formalism with common assumptions about the central exchange nuclear two-particle potential and the Coulomb interaction between protons. In bound state $^3\text{He}$ nucleus), the system $(2p, n)$ has spin $S = 1/2$ and the isospin projection $T_3 = 1/2$, being in a mixed isospin state $(T = 1/2$ and $T = 3/2)$. Thus, the total antisymmetric wave function can be represented in terms of standard spin-isospin functions and consists of six spatial components:

$$\Psi^a = \psi^s \xi^a + (\psi' \xi'' - \psi'' \xi') + \psi^a \xi^s + (\varphi' \zeta'' - \varphi'' \zeta') \chi_{3/2}.$$  \hspace{1cm} (1)

After projecting the Schrödinger equation onto the basic spin-isospin functions, one has the system of six equations for the spatial components of the wave function:

$$\left[ \hat{K} - E + \frac{1}{2} (U^s_{31} + U^s_{13}) + \frac{1}{3} U^s_c \right] \psi^s - \frac{1}{2} \left[ (U''_{31} - U''_{13}) \psi' + (U''_{31} - U''_{13}) \psi'' \right]$$

$$+ \frac{1}{3} (U'_c \psi' + U'_c \psi'') + \frac{1}{3} (U''_c \varphi' + U''_c \varphi'') = 0,$$

$$\left[ \hat{K} - E + \frac{1}{4} (U^s_{33} + U^s_{31} + U^s_{13} + U^s_{11}) + \frac{1}{3} U^s_c \right] \psi'$$

$$- \frac{1}{4} \left[ (U''_{33} - U''_{31} - U''_{13} + U''_{11}) \psi'' + (U''_{33} - U''_{31} - U''_{13} + U''_{11}) \psi' \right]$$
\[
-\frac{1}{2} \left( U_{31}^t - U_{13}^t - \frac{2}{3} U_c^t \right) \psi^s + \frac{1}{2} \left( U_{33}^{s^*} - U_{11}^{s^*} + \frac{2}{3} U_c^{s^*} \right) \psi^a + \frac{1}{3} (U_c^t \varphi' + U_c^t \varphi'') = 0,
\]

\[
\left[ \hat{K} - E + \frac{1}{4} (U_{33}^s + U_{31}^s + U_{13}^s + U_{11}^s) + \frac{1}{3} U_c^s \right] \psi''
\]

\[
-\frac{1}{4} \left[ \left( U_{33}^{s^*} - U_{31}^{s^*} + U_{13}^{s^*} + U_{11}^{s^*} \right) \psi' - \left( U_{33}^{s^*} - U_{31}^{s^*} - U_{13}^{s^*} + U_{11}^{s^*} \right) \psi'' \right]
\]

\[
-\frac{1}{2} \left( U_{31}^{s^*} - U_{13}^{s^*} - \frac{2}{3} U_c^{s^*} \right) \psi^s - \frac{1}{2} \left( U_{33}^{s^*} - U_{11}^{s^*} + \frac{2}{3} U_c^{s^*} \right) \psi^a + \frac{1}{3} (U_c^t \varphi' - U_c^t \varphi'') = 0,
\]

\[
\left[ \hat{K} - E + \frac{1}{2} (U_{33}^s + U_{11}^s) + \frac{1}{3} U_c^s \right] \psi^a + \frac{1}{2} \left[ (U_{33}^{s^*} - U_{11}^{s^*}) \psi' - (U_{33}^{s^*} - U_{11}^{s^*}) \psi'' \right]
\]

\[
-\frac{1}{3} (U_c^t \psi'' - U_c^t \psi') + \frac{1}{3} (U_c^t \varphi'' - U_c^t \varphi') = 0,
\]

\[
\left[ \hat{K} - E + \frac{1}{2} (U_{33}^s + U_{11}^s) + \frac{1}{3} U_c^s \right] \varphi' + \frac{1}{2} \left[ (U_{33}^{s^*} - U_{11}^{s^*}) \varphi'' + (U_{13}^{s^*} - U_{33}^{s^*}) \varphi' \right]
\]

\[
+\frac{1}{3} (U_c^t \psi'' + U_c^t \psi' + U_c^t \psi^a) = 0,
\]

\[
\left[ \hat{K} - E + \frac{1}{2} (U_{33}^s + U_{11}^s) + \frac{1}{3} U_c^s \right] \varphi'' + \frac{1}{2} \left[ (U_{33}^{s^*} - U_{11}^{s^*}) \varphi' - (U_{13}^{s^*} - U_{33}^{s^*}) \varphi'' \right]
\]

\[
+\frac{1}{3} (U_c^t \psi' - U_c^t \psi'' + U_c^t \psi^a) = 0,
\]

where \( \hat{K} \) is the kinetic energy of three nucleons and

\[
U^s = V(r_{12}) + V(r_{13}) + V(r_{23}), \quad U' = \sqrt{\frac{3}{2}} (-V(r_{13}) + V(r_{23})), \quad U'' = -V(r_{12}) + \frac{1}{2} (V(r_{13}) + V(r_{23}))
\]

are the combinations of spatial components of the potentials (both nuclear and Coulomb ones) with a certain symmetry with respect to permutations of nucleons. The last two equations in \( (2) \) are present due to the mixing of different isospin states by Coulomb interaction. If we set \( U_c = 0 \) in \( (2) \), we get the standard system of four equations \( (3) \) for \(^3\)He nucleus in both the spin \( (S = 1/2) \) and isospin \( (T = 1/2) \) doublet states, as well as a separate system of two equations for the three-nucleon system with \( S = 1/2 \) and \( T = 3/2 \).

The system of six spatial equations \( (2) \) is the basic one for studying the \(^3\)He nucleus (as well as the scattering and reactions in the system \( (2p, n) \) ) within the isospin formalism. In practice, however, precise calculations with the use of \( (2) \) have some difficulties because of a considerable number of equations. We reformulate the problem in an equivalent form using the fact that the reason for an increase in the number of equations in the few-nucleon problem is the additional antisymmetrization of wave functions (the generalized Pauli principle for nucleons) connected with the introduction of isospin for treating a proton and a neutron as an isotopic doublet. It appears that not all the spatial components of the wave function in \( (1) \) are independent, and Eqs. \( (2) \) contain an implicit relation between them. The analysis of system \( (2) \) shows that it is suitable to introduce six new components \( \phi'_1, \phi'_2, \phi'_3, \phi''_1, \phi''_2, \phi''_3 \) instead of \( \psi^s, \psi', \psi'', \psi^a, \varphi', \varphi'' \):
with the account of the permutations of coordinates:

\begin{align*}
\begin{pmatrix}
\phi'_1 \\
\phi'_2 \\
\phi'_3 \\
\phi''_1 \\
\phi''_2 \\
\phi''_3
\end{pmatrix} = 
\begin{pmatrix}
-\frac{1}{2\sqrt{3}} & -\frac{1}{2} & \frac{1}{2\sqrt{3}} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{3}} \\
-\frac{1}{2\sqrt{3}} & -\frac{1}{2} & \frac{1}{2\sqrt{3}} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{3}} \\
-\frac{1}{2\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & 0 & 0 & -\frac{1}{\sqrt{3}} \\
\frac{1}{2} & \frac{1}{2\sqrt{3}} & \frac{1}{2} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{2} & \frac{1}{2\sqrt{3}} & -\frac{1}{2} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\
0 & -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0
\end{pmatrix}
\begin{pmatrix}
\psi^s \\
\psi' \\
\psi'' \\
\phi' \\
\phi'' \\
\varphi'
\end{pmatrix},
\end{align*}

(4)

Then, as a result of the unitary (orthogonal) transformation, we get a new form of system (2) with only two explicitly independent equations from the set of six ones. Respectively, only two spatial components of the wave function are independent, the rest being expressible through these two ones with the account of the permutations of coordinates:

\begin{align*}
\phi'_1 (123) &= -\frac{1}{2} \phi'_3 (321) + \frac{\sqrt{3}}{2} \phi''_3 (321), \\
\phi''_1 (123) &= \frac{\sqrt{3}}{2} \phi''_3 (321) + \frac{1}{2} \phi'_3 (321), \\
\phi'_2 (123) &= -\frac{1}{2} \phi'_3 (132) - \frac{\sqrt{3}}{2} \phi''_3 (132), \\
\phi''_2 (123) &= -\frac{\sqrt{3}}{2} \phi''_3 (132) + \frac{1}{2} \phi'_3 (132).
\end{align*}

(5)

We note an important fact that the unitary transformation (4) is of universal form and does not depend on the specific form of interaction potentials but only on the spin-isospin symmetry in (1), (2). To find all the wave function components and to obtain the total antisymmetric function (4) for the system \(2p, n\), it is sufficient to solve the system of only two equations (instead of six ones) for two independent components \(\phi'_3\) and \(\phi''_3\) (further we introduce notations \(\Phi_1 \equiv \phi'_3\) and \(\Phi_2 \equiv \phi''_3\)).

On the other hand, the same functions \(\Phi_1\) and \(\Phi_2\) can be found directly from the system of two equations formulated in an equivalent representation which does not use the notion of isospin and treats a proton and a neutron as different particles. In this case, the total antisymmetric (in protons) function of the system \(2p, n\) with total spin \(S = 1/2\) looks as follows:

\[\Psi(p_1, p_2, n_3) = \zeta' \Phi_1(p_1, p_2, n_3) + \zeta'' \Phi_2(p_1, p_2, n_3),\]

(6)

where \(\zeta'\) and \(\zeta''\) are the two known components of the spin function of three nucleons with total spin \(S = 1/2\). The spatial function \(\Phi_1(p_1, p_2, n_3)\) is symmetric, while \(\Phi_2(p_1, p_2, n_3)\) is antisymmetric with respect to permutations of identical protons. Within the representation free from the notion of isospin, the system of equations for \(\Phi_1\) and \(\Phi_2\) has the following form:

\[\begin{align*}
\left[\hat{K} + \frac{e^2}{r_{12}} + V_{s(pp)}^+(r_{12}) - E\right] \Phi_1 (123) + \frac{1}{8} \sum_{ij=13,23,+-} \sum \left[3V_{t(np)}^+(r_{ij}) + V_{s(np)}^+(r_{ij})\right] [1 \pm P (ij)] \Phi_1 (123) + \\
+ \frac{\sqrt{3}}{8} \sum_{ij=13,23,+-} (-1)^{i+j} \left[V_{s(np)}^+(r_{ij}) - V_{t(np)}^+(r_{ij})\right] [1 \pm P (ij)] \Phi_2 (123) &= 0,
\end{align*}\]

(7)
In the system of equations (7) for $^3\text{He}$ nucleus, we take into account the dependence of charge-dependent nuclear potentials on spin and parity in the orbital momentum of nucleon pairs. In the case of $^3\text{H}$ nucleus consisting of two neutrons and one proton with the total spin $S = 1/2$, one has also Eqs. (7), but with exchanged indices $n \leftrightarrow p$ and without the Coulomb potential. Thus, Eqs. (7) completely determine the properties of three nucleons in the doublet spin state ($S = 1/2$) both in the bound states and scattering processes.

The proposed approach is also applicable to the system of three nucleons in the quartet state ($S = 3/2$). All the properties of the system are determined in this case by only the triplet charge-dependent nuclear interaction potentials, and it is necessary to solve only one equation to find the spatial component of the wave function (the spin function of three nucleons is one-component in this case, and the spatial function of the system $(2p, n)$ is antisymmetric with respect to permutations of the two protons). The equation looks as follows:

$$\left[\hat{K} + \frac{e^2}{r_{12}} + V_{t(pp)}(r_{12}) - E\right] \Phi(123) + \frac{1}{2} \sum_{ij=13,23,+,-} \sum_{V_{t(np)}(r_{ij})} [1 \pm P(ij)] \Phi(123) = 0. \quad (8)$$

It should be noticed that the complete equivalence of the systems with different numbers of equations in different representations does not seem to be a miracle if one rewrites the nuclear interaction operator in an equivalent form in terms of the Majorana exchange operator without explicit introduction of isospin operators. It follows from the general form of the central exchange interaction potentials presented in terms of the Majorana exchange operators that the transition to the equivalent representation without use of the isospin formalism is also possible for heavier nuclei and is reasonable for simplifying numerical calculations without any approximations.

3 Results of calculations

To study the properties of three-nucleon systems using the system of Eqs. (7), we develop an optimal variational scheme [5] with the use of a Gaussian basis. Precise calculations are carried out for the basic characteristics of the bound states of $^3\text{H}$ and $^3\text{He}$ nuclei for various nuclear interaction potentials. Table 1 contains the results of calculation of the binding energies $B = -E_0$ and r.m.s. radii for the Afnan-Tang (ATS3) and Minnesota potentials ($\hbar^2/M = 41.47$ MeV·fm$^2$ in calculations with equal masses of a proton and a neutron, the Coulomb parameter being $e^2 = 1.44$ MeV·fm, and the parameters of potentials are taken from [3]). The more realistic Afnan-Tang potential with interaction only in even orbital states is denoted by AT-(S3)$^+$. The results for energies and radii are given with the accuracy of one unit in the last digit with high probability (the exact results for the binding energies are slightly greater, but within not more than 1 KeV).

High accuracy of the calculations and the convergence of the results with spreading the variational basis are obtained with the use of a comparatively small number of basis functions (about 60-100 ones for both the symmetric $\Phi_1$ and antisymmetric $\Phi_2$ functions). It is found that the optimal way to achieve a given accuracy is to take into account about three times more Gaussian components for $\Phi_1$ than those for $\Phi_2$. Due to the approach based on system (7) without explicit use of isospin, our numerical results are of higher precision than the known Varga-Suzuki ones [3], although we used a less number of Gaussian variational basis functions.

The greater the difference between the triplet and singlet interaction potentials and the greater the short-range repulsion, the more essential is the difference between the energies calculated exactly and those obtained in the spinless approximation. The complete calculation of energies with account of the difference in the neutron and proton masses ($M_p \neq M_n$) shows the binding energy of $^3\text{H}$ to increase a little and that of $^3\text{He}$ to slightly decrease, while r.m.s. radii are practically unchangeable.
For all used interaction potentials, the proton, neutron, and mass r.m.s. radii indicate the distinct specific structures of $^3$H and $^3$He nuclei. Namely, the proton radius in the $^3$H nucleus is essentially smaller than the neutron one (something like the neutron ”halo”), while the $^3$He nucleus reveals the inverse pattern (the proton ”halo”). The explanation of this effect lies in the fact that the attraction of a pair of nucleons in the singlet state is weaker than that in the triplet one.

The obtained wave functions, having a suitable Gaussian representation, enabled us to calculate the main structure functions of the three-nucleon nuclei for various nuclear potentials. Fig. 1 presents the proton density distribution $\rho_p(r)$ as well as the neutron one $\rho_n(r)$ for a $^3$H nucleus in the case of the AT-(S3)$^+$ potential (the nucleons are considered to be point-like particles). The peripheral neutron ”halo” effect is seen distinctly due to the neutron density distribution being somewhat more long-range than the proton one (see also the r.m.s. radii in Table 1). At the same time, the proton density is essentially higher than the neutron one at the center of the nucleus. We have almost the same density distributions for the $^3$He nucleus (even from the quantitative point of view), but with the exchanged protons and neutrons $p \leftrightarrow n$. The main reason for such a regularity lies in the difference between the interactions in triplet and singlet states, while the Coulomb repulsion plays a negligible role. Fig. 2 shows the charge formfactors for $^3$H and $^3$He nuclei calculated with the same potential. Note that the formfactor profile of $^3$He falls down already at small $q^2$ more rapidly as compared to that of $^3$H, due to the greater radius of the proton density distribution in $^3$He as compared with that in $^3$H. And the formfactor of $^3$He changes its sign at smaller $q^2$ (experimental value $q^2_{\text{min, exp}} = 11.6 \text{ fm}^{-2}$ for $^3$He) because of the essential role of short-range correlations between the protons. Fig. 3 presents the pair correlation functions of the $^3$H nucleus both for the pair of neutrons ($g_{2,nn}(r)$) and for the neutron-proton pair ($g_{2,np}(r)$) in the case of the AT-(S3)$^+$ potential. A noticeable difference between them is connected with a difference in the interactions in triplet and singlet states, and with the fact that, in the $^3$H nucleus, the proton and a neutron interact mainly in the triplet state, while the two neutrons do in the singlet one. For comparison, we give also the deuteron wave function squared in the same figure for the same interaction potential (i.e., the pair correlation function of two nucleons in the triplet state; see the curve $(np)_{D}$ in Fig.3). The high accuracy of the known approximation based on two-particle correlation functions for the few-nucleon nuclei is confirmed once again.

All the main conclusions are also valid for the Minnesota potential, in particular, those about the above-mentioned likeness of pair correlation functions of three- and two-particle systems. Moreover, for the both considered potentials, the pair correlation functions of $^3$H are similar to those of $^3$He (with the account of the substitution $n \leftrightarrow p$ like for density distributions). The structure of the nuclei is determined mainly by the triplet and singlet interaction potentials, while the role of the Coulomb interaction is small and leads to a little and almost proportional expansion of the $^3$He nucleus in comparison with $^3$H. But, in the case of the Minnesota potential having essentially less short-range repulsion than the AT-(S3)$^+$ one, the dips in the formfactors of $^3$H and $^3$He nuclei occur at greater $q^2$, and the pair correlation functions have an essentially less decrease at short distances.

4 Conclusions

Thus, using the example of three-nucleon systems with central interaction potentials, we show the complete equivalence of the isospin formalism (with the total wave function being antisymmetric in the space of spin, isospin, and coordinate variables) and the proposed representation without explicit use of isospin (with the total wave function being antisymmetric in identical nucleons in spin and spatial coordinate variables). The obtained equations in the representation without use of isospin are unitary equivalent to the standard approach using the isospin representation.
The proposed approach is much more suitable (due to the small number of spatial equations that is determined by the dimension of a spin Young scheme) and enables one to carry out a precise study (with given accuracy) of three-nucleon systems in the bound states or scattering processes with various central nuclear interaction potentials. Using certain optimization schemes for the variational method with Gaussian basis, we carried out the precise calculations of the bound states of three nucleons with several nucleon-nucleon interaction potentials. The results are of higher precision than those available in the literature. Density distributions, formfactors, and pair correlation functions are calculated as well.

The advantages of the proposed approach can reveal themselves in precise calculations of systems of four nucleons (when the total spin $S = 0$, there are two equations for spatial components), five nucleons (there are five equations at $S = 1/2$), and, perhaps, of six-nucleon systems (five equations for spatial components in the both cases of $^6$He and $^6$Li nuclei). The developed approach opens a real possibility to construct the realistic potentials describing completely, with reasonable accuracy, the main low-energy parameters of few-nucleon systems.

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Figure captions

Fig.1
Profiles of proton and neutron density distributions for $^3$H nucleus (for the AT-(S3)$^+$ interaction potential). Curves (1), (2) depict $\rho(r)$, while curves (3), (4) show $r^2\rho(r)$.

Fig.2
Charge formfactors for $^3$H and $^3$He nuclei (for the AT-(S3)$^+$ interaction potential).

Fig.3
Pair correlation functions $g_{2,np}(r)$ and $g_{2,nn}(r)$ for $^3$H nucleus (for the AT-(S3)$^+$ interaction potential). For comparison, the pair correlation function $g_{2,pp}(r)$ for $^3$He and the deuteron wave function squared are shown.
Table 1
Calculated binding energies $B = -E_0$ (MeV) and r.m.s. radii (fm) for $^3\text{H}$ and $^3\text{He}$ nuclei. $R_p$, $R_n$, $R_m$ are the proton, neutron, and mass density distribution radii, respectively. (Experimental values: $E_0 = -8.481$ MeV and $R_p = 1.57$ fm for $^3\text{H}$; $E_0 = -7.716$ MeV and $R_p = 1.70$ fm for $^3\text{He}$).

|               | $^3\text{H}$          | $^3\text{He}$          |
|---------------|------------------------|------------------------|
|               | $AT - (S3)^+$ $ATS3$ $\text{Minnesota}$ | $AT - (S3)^+$ $ATS3$ $\text{Minnesota}$ |
| $-E_0$        | 6.699 6.699 6.896      | 5.998 5.998 6.165      |
| spinless approx. | $R_p$ 1.738 1.738 1.730 | $R_p$ 1.772 1.772 1.766 |
|               | $R_n$ 1.738 1.738 1.730 | $R_n$ 1.756 1.756 1.750 |
|               | $R_m$ 1.738 1.738 1.730 | $R_m$ 1.766 1.766 1.760 |
| $\Phi_2 = 0$  | $-E_0$ 7.491 7.616 7.561 | $-E_0$ 6.833 6.963 6.882 |
|               | $R_p$ 1.614 1.591 1.613 | $R_p$ 1.790 1.784 1.796 |
|               | $R_n$ 1.758 1.753 1.761 | $R_n$ 1.632 1.608 1.632 |
|               | $R_m$ 1.712 1.701 1.713 | $R_m$ 1.739 1.727 1.743 |
| total         | $-E_0$ 8.491 8.765 8.386 | $-E_0$ 7.833 8.110 7.711 |
|               | $R_p$ 1.576 1.546 1.586 | $R_p$ 1.780 1.763 1.798 |
|               | $R_n$ 1.749 1.733 1.763 | $R_n$ 1.593 1.560 1.605 |
|               | $R_m$ 1.693 1.673 1.706 | $R_m$ 1.720 1.698 1.736 |
| total $(M_p \neq M_n)$ | $-E_0$ 8.495 8.769 8.389 | $-E_0$ 7.826 8.103 7.706 |
|               | $R_p$ 1.576 1.546 1.586 | $R_p$ 1.781 1.764 1.799 |
|               | $R_n$ 1.748 1.733 1.762 | $R_n$ 1.593 1.561 1.605 |
|               | $R_m$ 1.693 1.673 1.705 | $R_m$ 1.721 1.699 1.736 |
| $^3\text{He}$ | $-E_0$ – 8.753 8.380 | – – – |
|               | $R_m$ – 1.67 1.698 | – – – |
$r^2 \rho(r)$, \(\rho(r)\), \(\text{fm}^{-1}\), \(\text{fm}^{-3}\); $^3\text{H}$
$|F_{\text{ch}}(q^2)|$

$q^2, \text{ fm}^{-2}$

$^{3}\text{He}$

$^{3}\text{H}$
