Can Induced Θ Vacua be Created in Heavy Ion Collisions?

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The development of the early Universe is a remarkable laboratory for the study of most nontrivial properties of particle physics. What is more remarkable is the fact that these phenomena at the QCD scale can be, in principle, experimentally tested in heavy ion collisions. We expect that, in general, an arbitrary induced θ vacuum state (|θ_{\text{ind}}\rangle) would be created in heavy ion collisions, similar to the creation of the disoriented chiral condensate with an arbitrary isospin direction. It should be a large domain with a wrong θ_{\text{ind}} \neq 0 orientation which will mimic the physics of the early universe immediately following the QCD phase transition when it is believed that the fundamental parameter, θ_{\text{fund}} \neq 0. We test this idea numerically in a simple model where we study the evolution of the phases of the chiral condensates in QCD with two quark flavors with non-zero θ_{\text{ind}}-parameter. We see the formation of a non-zero θ_{\text{ind}}-vacuum with the formation time of the order of 10^{-23} seconds.

1. The realization of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven opens up some exciting doors in fundamental physics research. The possibility of producing and studying quark-gluon plasma is the most well publicized research area. But it is certainly not the only new field that RHIC makes possible. This letter is concerned with the possible creation of non-trivial θ_{\text{ind}}-vacua inside the high temperature fireball created at RHIC.

The idea is very similar to the creation of the Disoriented Chiral Condensate (DCC) in heavy ion collisions [1, 2, 3] (see also a nice review [4] for a discussion of DCC as an example of an out of equilibrium phase transition). DCC refers to regions of space (interior) in which the chiral condensate points in a different direction from that of the ground state (exterior), and separated from the latter by a hot shell of debris. As we shall see in a moment, for both cases (DCC and |θ_{\text{ind}}\rangle-state) the difference in energy between a created state and the lowest energy state is proportional to the small parameter m_q and negligible at high temperature. Therefore, energetically an arbitrary |θ_{\text{ind}}\rangle can be formed which is the crucial point for what follows.

Before going into details, we would like to recall some general properties of the DCC, which (hopefully) can be produced and seen at RHIC, with an emphasis on the analogy between the DCC and a misaligned |θ\rangle-state. If the cooling process is very rapid and, therefore, the system is out of equilibrium, there will be a large size of the correlated region in which the vacuum condensate orientation mismatches its zero temperature value. The absolute value of the chiral condensate right after the phase transition is expected to be close to the zero temperature magnitude we can parametrize Goldstone fields by matrix U in the following way:

\[ U = e^{i\phi(\bar{\theta}\tau)} , \quad UU^+ = 1 , \quad \langle \bar{\Psi}^j_L \Psi^j_R \rangle = -\langle \bar{\Psi}_L \Psi_R \rangle |U_{ij}\]

The energy density of the DCC is determined by the mass term:

\[ E_\phi = -\frac{1}{2} Tr(MU + M^\dagger U^\dagger) = -2m\langle \bar{\Psi} \Psi \rangle \cos(\phi) \quad (1) \]

where we put m_u = m_d = m for simplicity. Eq.(1) implies that any \( \phi \neq 0(\text{mod} 2\pi) \) is not a stable vacuum state because \( \frac{\partial E_\phi}{\partial \phi} |_{\phi \neq 0} \neq 0 \), i.e. the vacuum is misaligned. On the other hand, the energy difference between the misaligned state and true vacuum (\( \phi = 0 \)) is small and proportional to m_q. Therefore, the probability to create a state with an arbitrary \( \phi \) at high temperature \( T \sim T_c \) is proportional to \( \exp[-V(E_\phi - E_0)/T] \), where V is 3D volume, and depends on \( \phi \) only very weakly, i.e. \( \phi \) is a quasi-flat direction. Right after the phase transition when \( \langle \bar{\Psi} \Psi \rangle \) becomes non-zero, the pion field begins to roll toward \( \phi = 0 \), and of course overshoots \( \phi = 0 \). Thereafter, \( \phi \) oscillates. One should expect the coherent oscillations of the \( \pi \) meson field which would correspond to a zero-momentum condensate of pions. This is exactly what was found in [4]. Eventually these classical oscillations produce real \( \pi \) mesons which hopefully can be observed.

Now we turn to our main point when the \( U(1)_A \) phase of the disoriented chiral condensate is also non-zero and, therefore, the |θ\rangle-vacuum state could be formed[5]. The

*From now on we omit the label "ind" for the induced θ. We hope θ_{\text{ind}} will not be confused with θ_{\text{fund}} which is zero in our world and which can not be changed. The simplest way to visualize θ_{\text{ind}} is to assume that right after the QCD phase transition the flavor singlet phase of the chiral condensate
production of non-trivial $\theta$-vacua would occur in much the same way as discussed above. The new element is that in addition to chiral fields differing from their true vacuum values the $\theta$-parameter of QCD, which is zero in the real world, becomes effectively nonvanishing in the macroscopically large region.

In this letter we show that in a simplified numerical model non-trivial $\theta$-vacua can be realized. While this is a simplified model which differs somewhat from the real physics associated with high energy ion collisions, there is no reason to believe that the non-trivial $\theta$-vacua cannot be realized at RHIC. Our results also give a very rough estimate of the time it takes for these non-trivial $\theta$-vacua to be formed. In order to have a hope of observing them they must form within the time that the central region of the fireball is isolated from the true vacuum.  

2. To take into account the $U(1)_A$ phase associated with $|\theta|$-vacua we choose the matrix $U_{ij}$ in the form $U = \text{diag} (e^{i\phi_i})$ with $\sum_{i=1}^{N_c} \phi_i$ in general non-zero. The energy density of the misaligned vacuum is determined in this case by the following low-energy potential $\|$, $\|^2$:

$$V(\phi_i, \theta) = -E \cos \left( \frac{1}{N_c} \left( \sum_{i=1}^{N_f} \phi_i - \theta \right) + \frac{2\pi}{N_c} l \right) \sum_{i=1}^{N_f} M_i \cos \phi_i$$

$$l = 0, 1, \ldots, N_c - 1,$$

where $E = \langle b \phi_i / (32\pi)G^2 \rangle \sim 10^{-2}\text{GeV}^4$ is much larger than $M_i = -m_q \langle \bar{\Psi} \Psi \rangle \sim 10^{-3}\text{GeV}^4$. Here $b = 11/3N_c - 2/3N_f$, $m_q \sim 5\text{MeV}$, $\langle \bar{\Psi} \Psi \rangle \sim -(240\text{MeV})^3$. The crucial point is that the $\theta$ parameter appears only in the combination, $\sum \phi_i - \theta$. This is a direct consequence of the transformation properties of the chiral fields under $U(1)_A$ rotations. To convince the reader that $\|\|^2$ does indeed represent the anomalous effective low energy Lagrangian, three of its most salient features are listed below:

i) Eq. $\|\|^2$ correctly reproduces the Witten-Di Vecchia-Veneziano effective chiral Lagrangian $\|\|$ in the large $N_c$ limit;

ii) it reproduces the anomalous conformal and chiral Ward identities of QCD;

iii) it reproduces the known dependence in $\theta$ (i.e. $2\pi$ periodicity of observables) $\|\|$. As mentioned above, in the large $N_c$ limit the effective Lagrangian $\|\|$ takes the Witten-Di Vecchia-Veneziano quadratic form. We would like to remark here that the exact form is not important for our present purposes as long as the $\theta$-parameter appears in combination $\sum \phi_i - \theta$ and the parameter $E \gg M_i$.

The most important difference between Eqs. $\|$, $\|$ is the presence of the parametrically large term $\sim E \gg m_q \langle \bar{\Psi} \Psi \rangle$ in the expression for energy $\|$, describing the $U(1)_A$ phase of the disoriented chiral condensate. This term does not go away in the chiral limit and provides a non-zero mass for the $\eta'$ meson which is expressed in terms of the parameter, $E$. It was for exactly this reason that it was thought until recently $\|$ that the non-trivial $\theta$-vacua would involve too large an energy cost to be produced because of the large parameter associated with the $\theta$ parameter.

The key point is the following. For arbitrary phases $\phi_i$ the energy of a misaligned state differs by a huge amount $\sim E$ from the vacuum energy. Therefore, apparently there are no quasi-flat ($\sim m_q$) directions along $\phi_i$ coordinates, which would lead to the long wavelength oscillations with production of a large size domain. However, when the relevant combination $(\sum \phi_i - \theta)$ from Eq. $\|$ is close by an amount $\sim O(m_q)$ to its vacuum value, a Boltzmann suppression due to the term $\sim E$ is absent, and an arbitrary misaligned $|\theta\rangle$-state can be formed.

Indeed, in the limit $M_i \ll E$, because of the large parameter in the first term the vacuum state that is favored for non-zero $\theta$ is the solution $\phi_i \approx \theta/N_f$. Substituting this back in to the potential we reproduce the well-known result $\|\|$ for the vacuum energy as a function of $\theta$.

$$V(\theta) \approx -E - \sum M_i \cos(\theta/N_f)$$

This shows that the energy cost of creating a non-trivial $\theta$-vacuum goes like the much smaller parameter $M_i$ in agreement with a general theorem that the $\theta$ dependence appears only in combination with $m_q$ and goes away in the chiral limit.

At this point we can apply the same philosophy as for DCC. The chiral fields $\|\| \phi_i$ are allowed to take random values and after the phase transition begin to roll toward the true solution $\phi_i \approx \theta/N_f$ and of course overshoot it. The situation is very similar to what was described for the DCC with the only difference that in general we expect an arbitrary $|\theta\rangle$-disoriented state to be created in heavy ion collisions, not necessarily the $|\theta = 0\rangle$ state. The difference in energy between these states is proportional to $m_q$ from $\|\|$.

To be more quantitative, one should study the evolution of the chiral fields with time. For this paper we assume the following situation. The rapid expansion of

\footnote{If $\theta \neq 0$, the Goldstone fields are not exactly the pseudoscalar fields, but rather are mixed with the scalars; the mixing angle between the singlet and octet combinations also depends on $\theta$, see $\|$ for detail.}
the high energy shell leaves behind an effectively zero temperature region in the interior which is isolated from the true vacuum. The high temperature non-equilibrium evolution is very suddenly stopped, or “quenched”, leaving the interior region in a non-equilibrium initial state that then begins to evolve according to (almost) zero temperature Lagrangian dynamics. Starting from an initial non-equilibrium state we can study the behavior of the chiral fields using the zero temperature equations of motion. The equations of motions are non-linear and cannot be solved analytically but we can solve them numerically in order to determine the behavior of the fields. The production of a non-trivial $\theta$-vacuum is indicated by the fact that the chiral fields relax to constant and equal non-zero values on a time scale over which spatial oscillations of the fields vanish. This fact confirms the self consistency of our approach. First we assumed that the singlet phase of the chiral condensate is non-zero on a macroscopically large domain. We then calculated the evolution of the chiral phases in this background. Finally we closed the circle by realizing that the chiral fields relax to constant, equal, non-zero values as mentioned above. The formation of a non-perturbative condensate is also supported by observation of the phenomenon of coarsening (see below) and by a test of volume-independence of our results.

3. The equations of motion for the phases of the chiral condensate with two quark flavors consists of two coupled second order nonlinear partial differential equations:

\[ \ddot{\phi}_i - \nabla^2 \phi_i + \gamma \dot{\phi}_i + \frac{d}{d\phi_j} V(\phi_j, \theta) = 0 \quad i = 1, 2 \]  

(4)

where $\nabla^2$ is a three dimensional spatial derivative and the potential is given in $[\text{(3)}]$. Emission of pions and expansion of the domain will contribute to the damping, $\gamma$, as might other processes. We do not know exactly how they would contribute but we simulate these unknown effects by including a damping term with a reasonable value for the damping constant, $\gamma \sim \Lambda_{QCD} \sim 200 \text{ MeV}$.

The initial data for each of the chiral fields $\phi_i$ is chosen on a 3D grid of $16^3$ points. The initial data consisted of random values of $\phi_i$ and $\dot{\phi}_i = 0$. The initial data was evolved in time steps using a Two-Step Adams-Bashforth-Moulton Predictor-Corrector method for each grid point with the spatial laplacian approximated at each grid point using a finite difference method. We used periodic spatial boundary conditions.

The grid spacing was determined by the length of a side of the spatial grid which was varied in order to vary the volume. The size of the time step between successive spatial grids was much smaller than the spatial grid spacing and was fixed at about $10^{-5} \text{MeV}^{-1}$.

We evolved the data for 8000 time steps and then applied a Fast Fourier Transform $[\text{9}]$ to the spatial data at evenly spaced time steps. We then binned the data in small increments of the magnitude of the wave vector in order to obtain the angular averaged power spectrum.

This procedure was carried about for different volumes. In all cases the results were qualitatively the same. We saw an initial growth of long wavelength modes as in $[\text{2}]$ and subsequent damped oscillation of all modes. The $|k| = 0$ modes oscillate and approach the equilibrium values of the fields. They exhibit this behaviour in the same time frame in which the Fourier coefficients of the modes with non-zero wave vectors fall to a tiny fraction of the zero mode coefficient. This qualitative behavior occurs for different total volumes and grid sizes suggesting that this behavior is not due to finite size effects.

We should note that our $|k| = 0$ mode is really only a quasizero mode as it is obtained in a finite spatial volume with periodic boundary conditions. However, our quasizero mode approaches the same value irrespective of the total spatial volume indicating that this really is a condensate. If it were not we would expect the value of the coefficient to decrease when the volume of the system increases.

The evolution of the Fourier modes of the $\phi$ fields is shown in Fig.$[3]$ for the specific case of $\theta = 2\pi/16$ and a spatial grid of 10 fm on a side. The initial values of the $\phi$ fields were randomly chosen within the range $-7\pi/16$ to $7\pi/16$. The zero mode clearly settles down to a non-zero value. All higher momentum modes vanish extremely rapidly and are negligible long before the zero mode settles down to its equilibrium value.

The instantaneous distribution of Fourier modes for the evolution above is shown in Fig.$[2]$ at a few different times. This graph clearly shows the amplification of the zero mode as time increases. This phenomenon of coarsening and the formation of a nonperturbative condensate is very similar to earlier discussions in ref. $[4]$.

In Fig.$[2]$ we plot $|\phi_k|$ as a function of time for three different volumes. We chose $\theta = \pi/16$ and the volumes $(8 \text{fm})^3$, $(16 \text{fm})^3$, and $(32 \text{fm})^3$. For each volume, we plotted the zero mode and the same non-zero mode. Notice that the zero mode is independent of the volume of the system, while the magnitude of the non-zero mode decreases with increasing volume. This is the signature

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{\(\phi_k\) is shown for various $|k|$ as a function of time. Notice that the zero mode settles down to $\phi_i \approx \theta/N_f$.}
\end{figure}
that a real condensate has been formed. For a total volume of \((10 fm)^3\) and \(\theta = 2\pi/16\) the time for relaxation from the initial non-equilibrium state following the quench to the non-trivial \(\theta\)-vacuum is approximately 0.064 MeV\(^{-1}\) \(\approx 4 \times 10^{-23} s\). This seems to be of the same order of magnitude as the time which we might expect the central region of the fireball to be isolated from the usual vacuum. The volume we have used is just at the upper limit of what we would expect at RHIC. As well we have not attempted to correctly account for the collision geometry at RHIC either. However, our simplified calculation suggests the possibility of producing non-trivial \(\theta\)-vacua.

4. We have shown through a numerical calculation of the zero temperature equations of motion with a non-zero induced \(\theta\) parameter that the chiral fields \(\phi\) after a quench from high temperature go to a spatially constant non-zero value related to the \(\theta\)-parameter. This occurs on a time scale of the order of \(10^{-23}\) seconds. The fact that all other non-zero modes fall to negligible values long before indicates that we have formed a condensate or a non-trivial \(|\theta\rangle\) vacuum state.

The most intriguing question is: “What is the signature of the produced \(\theta\) state?” First of all, due to the fact that the \(\theta\)-vacua are odd under charge conjugation times parity, CP, their decay must produce some CP odd correlations suggested in ref. [10]. However, one should expect that the signal will be considerably (if not completely) washed out by the re-scattering of the pions and their interactions in the final states, which mimic true CP-odd effects. In practice it is quite difficult to overcome the problem of separating a true CP violation from its simulation due to the final state interactions. A more promising direction is to look at \(\eta'(\eta') \to \pi\pi\) decays which are strongly forbidden in our world, but nevertheless will be of order one if \(\theta \neq 0\). Indeed, one can show [11] that

\[
\Gamma(\eta \to \pi\pi) = \frac{2}{3\pi m_\eta(\theta)} \left( \frac{m_\eta \sin \frac{\theta}{2} |\langle 0 |\bar{q}q|0\rangle|^2}{f_\pi^3} \right)^2 \\
\approx 0.5 \text{ MeV} (\sin \frac{\theta}{2})^2, \tag{5}
\]

and therefore the effect could be quite noticeable. We should remark here that all masses in the \(\theta\) vacuum state are shifted in comparison with their values at \(\theta = 0\) [7]. Formula (5) reduces to the corresponding expression of reference [12] in the limit \(\theta \to 0\). An analogous calculation for \(\eta' \to \pi\pi\) leads to a similar numerical estimation for \(\Gamma(\eta' \to \pi\pi) \sim 2 \text{ MeV} (\sin \frac{\theta}{2})^2\) which is almost an order of magnitude larger than the full width of the \(\eta'\) meson in the \(\theta = 0\) world. More importantly, this width is exclusively due to the CP odd decay \(\Gamma(\eta' \to \pi\pi)\).

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