Magnetic monopoles and unusual transport effects in magnetoelectrics

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The quest for magnetic monopoles (MM) became a hot topic in condensed matter physics lately, both theoretically and experimentally [1, 2, 3]. Especially exciting is the suggestion of the appearance of MM on image charges in topological insulators (TI) [4]. However the experimental setup proposed for their observation in [4] is rather elaborate, requiring TI covered by a magnetic layer, using special STM setup, etc.

We propose below that MM should exist in much simpler and well studied systems – in solids with the linear magnetoelectric (ME) effect such as Cr$_2$O$_3$ [5], or in some multiferroics [6]. Their existence can lead to rather striking consequences, such as (magneto)electric Hall effect, magneto-photovoltaic effect etc., which can be observed experimentally. In addition, in contrast to the case of TI considered in [4], in ordinary ME materials not only MM can accompany the charge, but also more complicated local magnetic objects can be created, e.g. local toroics, which can also lead to unusual effects in transport and other properties of such systems.

The main idea is actually similar to that proposed in [4]: linear ME coupling of the type $\alpha E \cdot B$ can lead to the creation of MM (magnetic “hedgehog”) by the electric charge. In contrast to TI where this could happen only on the image charge, in the bulk ME materials the term of the type $\alpha E \cdot B$, or even more general $\alpha_{ij} E_i B_j$, exist in the bulk, with the ME tensor $\alpha_{ij}$ which can have both symmetric and antisymmetric components. In the simplest case, when $\alpha_{ij} = \alpha \delta_{ij}$, we recover the ME coupling $\alpha E \cdot B$ like in [4]. Then every charge placed in such media, such as an electron or charge impurity, would create a MM, see fig. 1: the electric field $E(r) = (e/r^2)(r/r)$ would create a radial magnetic polarization $M = \alpha E$ – exactly as for a

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1This idea was first presented at the Asian-Pacific Workshop on Multiferroics in Tokyo in January 2011.
magnetic monopole with magnetic charge \( g = \alpha e \). If an electron would move through such media, the magnetic monopole would move with it, which can lead to interesting consequences and could allow to observe such MM. The resulting equations describing the dynamics of corresponding objects will be the same as those for an electric charge, with the substitution (electric charge \( e \leftrightarrow \) magnetic charge \( g \); electric field \( E \leftrightarrow \) magnetic field \( H \)) and the linearity of the corresponding equations leads to the well-known principle of superposition, valid also for the magnetic charges.\(^2\)

![Fig. 1. Magnetic monopole (red arrows) created by a charge \( e \) in magneto-electric media with the coupling \( \alpha E \cdot B \).](image)

Some of the resulting consequences are more or less trivial. Thus, one could think that the motion of a MM in a magnetic field would be a special feature proving its existence. If we put such material with an extra electron in an external magnetic field \( H \), there will indeed be a force \( F = gH \) acting on the MM associated with the charge \( e \), with magnetic charge \( g = \alpha e \), i.e. the force will be \( F = \alpha eH \). But an external magnetic field creates in ME media an electric field \( E = \alpha H \), i.e. this force will be \( F = gH = (\alpha e)H = e(\alpha H) = eE \), i.e. it is the same force as that of an electric field acting on a charge. In this sense the very motion of an electron in a magnetic field is not a unique consequence of the appearance of MM.

The question is whether in this case both forces will act on the electron. Thus, if we apply an electric field to such a media, it will act on the electron by the force \( F = eE \), but this external field will also create a magnetic field \( H = \alpha E \), and this magnetic field will also act on the MM, with the force

\[ F = gH = \alpha eH = e(\alpha H) = eE. \]

\(^2\)Actually the radial magnetic field around the charge is created by the corresponding deviations of ordered spins forming the ME material. Therefore there will be no real singularity at the centre of MM. But the field outside the core will be exactly that of a MM, and all the dynamic effects will be described by the usual dynamics with the electric charge \( e \) replaced by the magnetic charge \( g \), and the electric field by \( H \).

There will be also no contradiction with the Maxwell equations: as in [1], the monopolar-like distribution of \( M \) will lead to a similar distribution of magnetic field \( H \), so that the field \( B = H + 4\pi M \) will satisfy the standard Maxwell equation \( \text{div} \, B = 0 \).
\[ F' = gH = e\alpha^2 E. \] In effect the effective force acting on the electron could be combined, \( F + F' \).

One can also think of other consequences. Thus, similar to the Lorentz force acting on a charge moving in a magnetic field, \( F \sim e[v \times H] \), which leads to the Hall effect, there will be a similar tangential force acting on a MM moving in an electric field, \( \sim F \sim g[v \times E] \). Consequently, if we put a stripe of such ME material with a current in a perpendicular electric field, the MM, together with the electron, would be deviated, exactly as in the Hall effect, see fig. 2. Such “electric Hall effect” should be observable in ME materials with diagonal ME coupling \( \alpha_{ij} = \alpha \delta_{ij} \) and with MM. Once again, this may not be surprising: in such media the electric field \( E \) perpendicular to the current will also create a magnetic field parallel to \( E \), so that one can also attribute the Hall effect depicted in fig. 2 to the ordinary Hall effect in this effective magnetic field \( H = \alpha E \). Still, this (magneto)electric Hall effect is an interesting special feature of such ME materials, which would be very interesting to study.

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![Fig. 2. Motion of a magnetic monopole \( g \), accompanying the electric charge \( e \), in a perpendicular electric field. The force \( g[v \times E] \) will bend the trajectory of this object, leading to an “electric Hall effect”.](image-url)
rearrangement of magnetic dipoles in the ME media, and this field will not exist outside of such sample; in this sense of course we do not create a real monopole. But, as we argued above, there will be nontrivial effects connected with the forces and dynamic of electrons in the bulk of such materials.

Yet another interesting effect may exist in such ME media under illumination. In ordinary ferroelectrics there exists the well-known photovoltaic effect [10]: the electron–hole pairs created under illumination move in the internal electric field, existing in ferroelectrics, in opposite directions, thus creating a voltage. Similarly, in ME media with, for example, a diagonal ME tensor \( \alpha_{ij} = \alpha \delta_{ij} \), there should exist a similar effect caused by the magnetic field (cf. [11]): photoexcited electrons and holes would be accompanied by the corresponding monopoles with magnetic charges \( \pm g \), which would move in opposite direction in an external magnetic field (magneto-photovoltaic effect). But here one can also explain this effect simpler: an external magnetic field will create an electric field inside the ME material, which will act as the usual electric field in ferroelectrics, causing the photovoltaic effect.

The origin of the effects considered above conceptually is similar to that proposed for TI [4], although the resulting effects have here a bulk nature and seem to be more general and easier to observe. However ME media open yet other possibilities. The ME tensor \( \alpha_{ij} \) can have not only a symmetric, but also an antisymmetric part, which can be described by the dual vector – the toroidal, or anapole moment \( T \). The resulting ME coupling can then be written as \( T \cdot [E \times B] \). Consequently, an electric charge placed in such media will create a local magnetic “toroid” – a magnetic vortex with the magnetic moment \( M(r) \sim T \times E \) (with \( E = (4\pi e/r^2)(r/r) \)), see fig. 3. The moving electron will carry with itself such a “magnetic donut”. To detect it would be less easy than MM: the toroidal moment couples not to \( H \), but to \( \text{curl } H \sim T \cdot [\nabla \times H] = T \cdot 4\pi j \). In effect we can have in such media for example a spontaneous Hall effect like that shown in fig. 2, but even without a perpendicular external electric field: if the toroidal moment is perpendicular to the current \( j \), such current would deviate so as to have a component parallel to \( T \), decreasing the corresponding interaction energy. This would create a Hall voltage, see fig. 4. Again one can explain this simpler by saying that the applied voltage and the corresponding electric field \( E \parallel j \) will create a magnetic field \( H \sim T \times E \), and in this magnetic field there will appear an ordinary Hall effect. Nevertheless, it is still an interesting effect, which would be interesting to study experimentally. There should be probably some other nontrivial effects connected with the presence of toroidal moments in the system, which are worth exploring.

Summarising, in analogy with the suggestion of [4], we propose that nontrivial local magnetic objects, such as magnetic monopoles or magnetic
Fig. 3. Local toroid (“magnetic donut”) created around charge $e$ in magnetoelectric media with antisymmetric magnetoelectric tensor and with magnetic moments $M(r) \sim E(r) \times T$ (close to the core $M$ should decrease again). Thin radial arrows show the electric field, and red arrows show the orientation of magnetic moments around the charge $e$.

Fig. 4. Spontaneous Hall effect in media with antisymmetric magnetoelectric tensor $\alpha_{ij}$, or with toroidal moment $T$.

toroids, should exist in magnetoelectric media in the bulk, associated with electrons or with charged impurities. Their presence can have nontrivial manifestations, such as an “electric Hall effect” or a magneto-photovoltaic effect – effects which could be observable experimentally. Thus the ME media can provide yet another possibility of modelling of magnetic monopoles in solids, more general and more easily detectable than e.g. in frustrated magnets [1, 3]. One may expect many interesting effects in such systems, some of which were proposed above. Most of these can be explained both in the picture of magnetic monopoles and by the more traditional concepts, but the monopole language is more transparent and helps to suggest new effects. One could also think of more special effects for which the monopole picture would be more appropriate.

Note added: Very recently the monopoles in magnetoelectric systems were also discussed from a different perspective in [12], also with the use of ab initio calculations for specific materials. I am grateful to N. A. Spaldin.
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