Measurement method of adjacent harmonics or interharmonics based on Gaussian radial basis function frequency domain approximation

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Abstract—Interpolated FFT method based on window function is a common method to measure harmonics or interharmonics in power systems. When the frequency resolution is insufficient or the main lobe of the window function is wide, the spectrum of adjacent harmonics or interharmonics may have aliasing of the main lobe, and it is difficult to accurately measure the parameters of harmonics or interharmonics. This paper proposes a modeling method for adjacent harmonics or adjacent interharmonics. The Fourier transform (FT) of the Gaussian function is still a Gaussian function. According to this feature, after adding a Gaussian window in the time domain and calculating the FFT spectrum, the real and imaginary parts of the aliased spectrum are decomposed into a series of linear superpositions of Gaussian radial basis functions (RBF). The center of the RBF is the frequency of the harmonic or interharmonic, and the amplitude and phase can also be calculated by the superposition coefficient of the RBF. The simulation verifies the effectiveness of the method.

1. Introduction

The windowed FFT method is the main method to measure harmonics and interharmonics. Since it is impossible to analyze an infinitely long signal, spectrum leakage is unavoidable. Accurate measurement of adjacent harmonics or interharmonics is a major difficulty. When the frequency resolution is not enough or the main lobe of the window function is too wide, the spectrum of adjacent harmonics or interharmonics interfere with each other.

To suppress the spectrum interference generated by the main lobe, a window function with a narrower main lobe is usually selected, such as rectangular window\cite{1} and hanning window\cite{2}. But the side lobe attenuation characteristics are not good, the first side lobe attenuation level of the hanning window is -31dB, and the rectangular window is only -13dB. To suppress the interference generated by the side lobes of the window function, a window function with a small side lobe attenuation level and a fast side lobe attenuation is usually selected\cite{3}, such as a combined cosine window, including Nutall window\cite{4} and Rife-vincent window\cite{5}, etc., The sidelobe attenuation level can be As low as -100dB. However, the main lobe of these window functions is wider than the rectangular window and the hanning window, and is not suitable for the measurement of adjacent harmonics or interharmonics. Therefore, it is necessary to compromise the selection of the window function.
Gaussian window is an adjustable window function[6], has the best time-frequency resolution capability, and is widely used in time-frequency signal analysis based on S transform. However, since the Gaussian window tends to be infinite, it needs to be cut when used, which affects the spectral characteristics of the Gaussian window, especially the sidelobe characteristics. The S transform is more used in the detection of power quality events[7-9], and is rarely used in the measurement of harmonics and interharmonics[10], especially when the window width is fixed. In recent years, there has been literature replacing the Gaussian window with the digital prolate spheroidal window used in the detection of time-varying harmonics[11], and good detection results have been achieved. The Gaussian function is also used in the RBF neural network as the kernel function of the RBF neural network. Literature[12, 13] applies Gaussian RBF neural network to the measurement of harmonics and interharmonics. This method is a time domain modeling method. The advantage is that the data cycle required for calculation is short, and the parameters can be measured with only half-cycle data, which is particularly suitable for the measurement of non-stationary harmonics and interharmonics. This method generally uses the clustering method to select the radial basis center, which has a large amount of calculation and is suitable for offline measurement. The FT of the Gaussian function is still a Gaussian function. If the signals of adjacent harmonics or interharmonics are multiplied by a Gaussian window in the time domain, the spectrum should theoretically be a linear superposition of a series of Gaussian functions, so it can also be expressed by Gaussian RBF. At this time, the center of the Gaussian RBF corresponds to the frequency of harmonics and interharmonics, which has obvious physical meaning. Moreover, usually signals containing harmonics and interharmonics are sparse in the frequency domain. If the peak spectral line of the amplitude frequency is searched in the frequency domain and used as the initial value of the center of the RBF, the search efficiency will be higher than the clustering method. There are still some problems to be discussed in this paper. First of all, it is necessary to clarify the correspondence between the parameters of the FT of the Gaussian window and the parameters of the Gaussian RBF in theory. Secondly, the frequency spectrum is a complex number, not an ideal superposition of real functions. It is necessary to distinguish the real part and the imaginary part and approximate them separately. Finally, this article is the measurement under the window width recommended by the standard IEC 61000-4-7[14] (the window width under 50Hz system is 10 fundamental wave periods, and under 60Hz system 12 fundamental wave periods), the influence of shape parameters of Gaussian window on the spectrum needs to be considered.

This paper is organized as follows. The second part discusses the phenomenon of spectrum interference, and the third part discusses how the Gaussian RBF measures adjacent harmonics or interharmonics, including the problems mentioned above, and gives the algorithm steps. The fourth part is the verification and discussion of the method, and finally the conclusion is given.

2. Spectrum interference of windowed FT
Assume that the multi-frequency signal $x(t)$ is

$$x(t) = \sum_{k=1}^{m} A_k \cos(\omega_k t + \varphi_k)$$

Where $m$ is the number of harmonics and interharmonics and $t$ is the time variable. $\omega_k$, $A_k$, and $\varphi_k$ are respectively constant angular frequency, amplitude and phase.

Suppose a symmetric window function $w(t)$ with width $T$, and the FT is $W(\omega)$. The FT of $x(t)$ multiplied by the window function is

$$X_w(\omega) = \mathcal{F}\{x(t)w(t-T/2)\}$$

$$= \sum_{k=1}^{m} \frac{A_k}{2} e^{-j(\omega+\omega_k)T/2} e^{j\omega_k} W(\omega + \omega_k)$$

$$+ \sum_{k=1}^{m} \frac{A_k}{2} e^{-j(\omega-\omega_k)T/2} e^{j\omega_k} W(\omega - \omega_k)$$

(2)
It can be seen that the spectrum is equivalent to the left and right translation $\omega_k$ of the spectrum of the window function, and the phase shift is $-(\omega \pm \omega_k)/2 + \phi_k$, and the weight of the superposition is $A_k/2$.

In the process of weighted superposition, if the main lobe of the window function is relatively wide or the frequency resolution is relatively low, adjacent harmonics or interharmonics may cause spectrum interference. Fig. 1 shows the spectrum interference phenomenon when the window function is a Gaussian window. The signal contains interharmonics 104Hz, 117Hz, 134Hz and 147Hz. The sampling rate is 1000 Hz, 10 cycles are recommended in accordance with IEC 61000-4-7, and the frequency resolution is 5 Hz. It can be seen that the four interharmonics are not integer multiples of 5 Hz. Due to the fence effect, these interharmonics cannot be directly measured from the discrete spectrum. At the same time, due to spectrum leakage, the four interharmonics and adjacent interharmonics all have spectrum interference. Therefore, the parameters of inter-harmonics are measured according to interpolating windowed FFT algorithm, which has a large error.

![Figure 1 Spectrum interference after FFT of the signal multiplied by Gaussian window.](image)

3. PROPOSED METHOD

3.1. Equivalent relation between Fourier transform of Gaussian window function and RBF

The symmetric discrete Gaussian window function $w(n)$ is defined as

$$w(n) = \exp \left\{ -\frac{1}{2} \left( \frac{n}{(N-1)/2} \right)^2 \right\}$$

Where $n \in [-{(N-1)/2}, (N-1)/2]$, $\alpha$ is the Gaussian window parameter and $N$ is the sequence length. The corresponding continuous Gaussian window function is

$$w(t) = \exp \left\{ -\frac{1}{2} (\alpha t)^2 \right\} \quad -1 \leq t \leq 1$$

And its FT is

$$W(\omega) = \mathcal{F} \{ w(t) \} = \sqrt{2\pi} \alpha \exp \left\{ -\frac{1}{2} \left( \frac{\omega}{\alpha} \right)^2 \right\}$$

In general, the most common Gaussian RBF[15] is the, as given by

$$\phi(f) = \exp \left\{ -\varepsilon \| f - f_j \|^2 \right\}$$

Where $\varepsilon$ is shape parameter of RBF. $f$ refers to the frequency and takes a value between -1 and 1. $f_j$ represents the $j$th center of $f$.

If the Gaussian function of (5) is equal to $\Phi(f)$ in (6), and the discretization into $N$ points is considered, the relationship between the shape parameter and the Gaussian window parameter is

$$\varepsilon = \frac{\sqrt{2\pi} N}{4\alpha}$$

3.2. Gaussian RBF Approximation of Spectrum

Take only the positive frequency part of (2), there are
\[ X_n(\omega) = \sum_{k=-\infty}^{\infty} A_k \cos \beta_k W(\omega - \omega_k) + j \sum_{k=-\infty}^{\infty} A_k \sin \beta_k W(\omega - \omega_k) \]
\[ = S_R + jS_I \]  

(8)

Where

\[ \beta_k = -(\omega - \omega_k)T / 2 + \phi_k \]  

(9)

It can be seen from (8) that the real and imaginary parts of the Gaussian windowed FT are linear combinations of Gaussian functions, which can be approximated by Gaussian RBFs. It can be defined as:

\[ S_{Rn} = \sum_{j=1}^{n} \lambda_R \phi(j) \| f - f_j \| \]  

(10)

\[ S_{In} = \sum_{j=1}^{n} \lambda_I \phi(j) \| f - f_j \| \]  

(11)

Among them, \( \lambda_R \) is the \( j \)th weight of the approximation of the real part, \( \lambda_I \) is the \( j \)th weight of the approximation of the imaginary part, and \( f_j \) is the frequency of the harmonic or interharmonic. Note that the angular frequency \( \omega_j \) and \( f_j \) satisfy the relationship of \( \omega_j = 2\pi f_j \).

The amplitude and phase can be calculated from (6) to (11), as shown below

\[ A = [A_1, A_2, \cdots, A_n]^T = \frac{4\alpha}{\sqrt{2\pi}} \lambda_R + j\lambda_I \]  

(12)

\[ \varphi = [\varphi_1, \varphi_2, \cdots, \varphi_n]^T = \text{arg}[\lambda_R + j\lambda_I] - \pi f_0 T \]  

(13)

Where \( \lambda_R, \lambda_I \) and \( f_0 \) are defined as

\[ \lambda_R = [\lambda_{R1}, \lambda_{R2}, \cdots, \lambda_{Rn}]^T \]  

(14)

\[ \lambda_I = [\lambda_{I1}, \lambda_{I2}, \cdots, \lambda_{In}]^T \]  

(15)

\[ f_0 = [f_1, f_2, \cdots, f_n]^T \]  

(16)

3.3. Gradient descent method

\( \lambda_R, \lambda_I \) and \( f_0 \) can be iteratively obtained by gradient descent.

The objective function can be defined as

\[ E = \frac{1}{2} (S_R - S)^2 \]  

(17)

In the formula, \( S_R \) and \( S_I \) of (8), and \( S_R \) takes the corresponding approximate values \( S_{GR} \) and \( S_{GI} \) of (10) and (11).

According to the gradient descent method, \( \lambda_R, \lambda_I \) and \( f_0 \) are adjusted as follows:

\[ \Delta \lambda(k) = -\eta \frac{\partial E(k)}{\partial \lambda(k)} = \eta \phi(k)(S_R - S) \]  

(18)

\[ \lambda(k) = \lambda(k-1) + \Delta \lambda(k) + \rho [\lambda(k-1) - \lambda(k-2)] \]  

(19)

\[ \Delta f_j(k) = -\eta \frac{\partial E(k)}{\partial f_j(k)} = 2\eta \omega^2 \lambda_j(k)(S_R - S)^T [\phi_j(k) \ast [f - f_j(k)]] \]  

(20)

\[ f_j(k) = f_j(k-1) + \Delta f_j(k) + \rho [f_j(k-1) - f_j(k-2)] \]  

(21)

Among them, \( k \) is the iteration number, \( \eta \in (0, 1) \) is the learning rate, \( \rho \in (0, 1) \) is the momentum factor, \( \lambda \) represents \( \lambda_R \) and \( \lambda_I \), \( \Phi_j \) is the \( j \)th column vector of \( \Phi \), and the operation symbol "\( \ast \)" represents the multiplication of elements in the same position of the two column vectors.
3.4. Algorithm
The algorithm is as follows.

Step 1: Apply FFT to the digital signal multiplied by the Gaussian window, and the spectrum can be obtained.

Step 2: Spectrum preprocessing. Phase correction, and then shift the frequency spectrum to make the amplitude-frequency even symmetrical, and zero-frequency component is to the center of spectrum.

Step 3: Search for the peak of the amplitude spectrum, and the corresponding frequency is used as the initial value of the center of the RBF.

Step 4: Iteratively calculate the center and weight of the RBF by the gradient descent method.

Step 5: Let the center of the RBF be the frequency of the harmonics and interharmonics, and calculate the amplitude and phase according to (12) and (13).

4. Application of the method
In order to verify the method in this article, the simulation signal is cited from IEEE Interharmonic Task Force[16] containing the adjacent interharmonics. The simulation signal includes the fundamental wave (50Hz), the 5th harmonic (250Hz), 104Hz, 117Hz, 134Hz and 147Hz. The waveforms of the four adjacent interharmonics are shown in Fig. 2. The parameters are shown in Table I.

| Harmonics/interharmonics | Frequency(Hz) | Amplitude (p.u.) | Phase (°) |
|--------------------------|---------------|-----------------|-----------|
|                          | 50            | 1.0             | 0         |
|                          | 104           | 0.3             | 0         |
|                          | 117           | 0.4             | 0         |
|                          | 134           | 0.2             | 0         |
|                          | 147           | 0.2             | 0         |
|                          | 250           | 0.5             | 0         |

4.1. Spectrum analysis using Gaussian windowed FFT
The sampling frequency is 1000Hz, the sequence length is 200, and the spectrum is calculated by Gaussian windowed FFT, where $\alpha$ is equal to 4, as shown in Fig. 1.

For a fixed data width, the shape parameters $\alpha$ of the Gaussian window in (3) should be selected reasonably to avoid changes in spectral characteristics due to truncation. Fig. 3 shows Gaussian windows with five different shape parameters and their amplitude spectra. It can be seen that when $\alpha$ is equal to 2, the sidelobe characteristics have changed significantly and are no longer similar to Gaussian functions. Looking at the Gaussian function in the time domain, its truncation effect is obvious. When $\alpha$ is greater than 2, the sidelobe characteristics change less, tending to a Gaussian function, and there is almost no truncation effect in the time domain. The value of $\alpha$ selected in this example is 4.
4.2. Spectrum preprocessing
Considering the phase angle of (9), when the time width is $T$, calculate $N$-point FFT, then

$$
\beta_k = -(2n\pi / T - \omega_0)T / 2 + \varphi_k = -n\pi + \omega_0 T / 2 + \varphi_k \quad n \in Z
$$

It can be seen that the adjacent spectrum samples are $180^\circ$ apart in phase, so phase correction is required. The method of correction is to multiply the discrete samples of (8) by $(-1)^n$. Fig. 4 shows the real and imaginary parts of the spectrum before and after correction.

As shown in Fig. 4, the spectrum has been shifted. Considering that there may be zero-frequency components, the zero-frequency components must be placed in the middle. At the same time, since the amplitude spectrum of the negative frequency and the positive frequency are even symmetric, the phase spectrum is odd symmetric, which can be used as a stopping condition for the iteration of the gradient descent method, eliminating the need for verification of the learning method.

4.3. Iteratively calculate the center and weight of the RBF by the gradient descent method.
The learning rate $\eta$ of the gradient descent method is set to 0.01, and the momentum factor $\rho$ is 0.2. It can be seen from Fig. 1 that the positive frequency component has 6 peaks, and the negative frequency component is also 6 peaks. The frequency corresponding to the 12 peaks together is used as the initial value of the center of the Gaussian RBF. The weight of the RBF is initialized to 0.

Fig. 5, Fig. 6 and Fig. 7 respectively show the relative frequency error, amplitude relative error and phase error with the number of iterations. It can be seen from the figure that the errors of each harmonic and interharmonic are all convergent, which proves that the method is correct and effective. Among them, the fundamental wave and the 5th harmonic can be iterated to a very low error very quickly, while
the 4 adjacent interharmonics require more iterations. The fundamental wave and the 5th harmonic are synchronously sampled, so that the exact frequency can be determined when searching for the peak spectrum. However, the interharmonics are not synchronously sampled, and it takes more time to iterate to the correct frequency.

Figure 5 The relative frequency error changes with iteration.

Figure 6 The relative amplitude error changes with iteration.

4.4. **Comparison with the calculation results of Hanning window interpolation algorithm.**

In order to verify the measurement effect, the calculation results of this method and the hanning window interpolation algorithm[17] are compared here, as shown in Table 2. The minimum distance between adjacent interharmonics in the simulated signal is 13 Hz, and the rectangular window and Hanning window are basically satisfying this measurement requirement. The side lobe interference of the rectangular window is relatively larger. When the frequency resolution is 5 Hz, the half width of the main lobe of the hanning window is 10 Hz, so the hanning window is selected. The measurement results of the method in this paper vary with the number of iterations. Table II shows the results of 3000 iterations. It can be seen from Table II that this method has a higher calculation accuracy than Hanning window interpolation after multiple iterations, especially for the measurement of four adjacent interharmonics.
Figure 7 The phase error changes with iteration.

Figure 8 Spectrum interference of Hanning windowed FFT interpolation

Fig. 8 is the spectrum analysis of the signal multiplied by the hanning window. It can be seen that the main lobes of the adjacent interharmonics are aliased. Although the adjacent peak spectral lines are not aliased, they are still interfered by the side lobes of the hanning window. A large measurement error occurred.

Table 2 Comparison of measurement results

| Harmonics/interharmonics | Actual Value | Hanning window | Method of this article |
|--------------------------|--------------|----------------|-----------------------|
| f (Hz)                   | 50           | 104            | 103.99                |
| Amplitude (p.u.)         | 1.0          | 0.3            | 0.3                   |
| Phase (°)                | 0            | 0              | 0.12                  |
| f (Hz)                   | 104          | 117            | 116.97                |
| Amplitude (p.u.)         | 0.4          | 0.4            | 0.4                   |
| Phase (°)                | 0            | 0              | 1.05                  |
| f (Hz)                   | 134          | 147            | 133.97                |
| Amplitude (p.u.)         | 0.2          | 0.2            | 0.2                   |
| Phase (°)                | 0            | 0              | 1.02                  |
| f (Hz)                   | 147          | 250            | 146.93                |
| Amplitude (p.u.)         | 0.5          | 0.5            | 0.5                   |
| Phase (°)                | 0            | 0              | 2.28                  |

5. conclusion
The measurement of adjacent harmonics or interharmonics is a very challenging problem. This paper proposes a modeling method for adjacent harmonics or interharmonics in the frequency domain. The simulation results show that this method has higher accuracy than the Hanning window interpolation algorithm, and is suitable for the measurement of adjacent harmonics or interharmonics.
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