Adaptive Hierarchical Down-Sampling for Point Cloud Classification

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Abstract—While several convolution-like operators have recently been proposed for extracting features out of point clouds, down-sampling an unordered point cloud in a deep neural network has not been rigorously studied. Existing methods down-sample the points regardless of their importance for the output. As a result, some important points in the point cloud may be removed, while less valuable points may be passed to the next layers. In contrast, adaptive down-sampling methods sample the points by taking into account the importance of each point, which varies based on the application, task and training data. In this paper, we propose a permutation-invariant learning-based adaptive down-sampling layer, called Critical Points Layer (CPL), which reduces the number of points in an unordered point cloud while retaining the important points. Unlike most graph-based point cloud down-sampling methods that use k-NN search algorithm to find the neighbouring points, CPL is a global down-sampling method, rendering it computationally very efficient. The proposed layer can be used along with any graph-based point cloud convolution layer to form a convolutional neural network, dubbed CP-Net in this paper. We introduce a CP-Net for 3D object classification that achieves the best accuracy for the ModelNet40 dataset among point cloud-based methods, which validates the effectiveness of the CPL.

I. INTRODUCTION

Classification of unordered point cloud using neural nets has become a highly active research field with the introduction of many new methods, such as PointNet [17], PointNet++ [18], DGCNN [25], PointCNN [9], and SO-Net [8]. The effectiveness of these methods is obvious from various point cloud classification leader boards, such as ModelNet40 [26].

In practical scenarios, the number of points in the point cloud associated with an object may be large, especially with high density sensors such as Velodyne-64 [13]. A car zipping by, for example, may correspond to thousands of LiDAR points. Methods such as PointNet [17] and DGCNN [25] pass the same number of input points into deeper layers to extract complex features. Without reduction of input size, however, much redundant information must be processed by the deeper layers, resulting in extra delay in the classification pipeline.\footnote{A key constraint here is that the unordered point cloud is processed in a way such that the order of points has no effect. This differs from the image domain where pixels are densely ordered in space, which naturally leads to acceleration through GPU data parallelism. Due to order-agnosticism and data sparsity, the recent methods for classification of unordered point cloud can not take full advantage of GPU parallelism.}

One possible way to reduce computation is to down-sample the points in the point cloud as it gets passed through the network. A class of methods are proposed in which k-NN search [16] is used to find the neighbourhood for each point and down-sample according to these neighbourhoods. Such methods, however, trade one kind of expensive computation (neighbourhood search) for another kind (processing large point cloud).

What we need is a content-sensitive but fast way of down-sampling an unordered point cloud that can be easily integrated into a deep net – something similarly effective and efficient as max pooling in conventional CNN. In this paper, we introduce the Critical Points Layer (CPL), which meets these requirements.

Unlike previous down-sampling methods that generate new set of points, the CPL selects the important points to pass along – reducing the number of points without losing the critical ones. It is invariant to permutation of input points, i.e. order-agnostic. It is adaptive in that it learns to down-sample the points during training and it is a global method not limited to neighbourhood search, which makes it efficient.

The CPL can be used along with any graph-based point cloud convolution layer [14] to form larger neural nets, dubbed CP-Nets. Our experiments show that CP-Nets can achieve state-of-the-art classification accuracy with computational cost kept at a reasonable level.

II. RELATED WORK

A. Deep Learning on Point Clouds

Due to the natural sparsity of point clouds, deep learning methods for classification and object detection on point cloud data tend to partition the 3D space into regular voxels (voxelization) and then extend 2D CNNs to this data type [11], [27]. The main problem of these voxel-based networks is that network size and computational complexity grows quickly as spatial resolution increases. On the flip side, lower spatial resolution means larger voxel size and higher quantization error.

Other alternatives include octree-based [24] and kd-tree-based [6] neural networks. In [6], for example, a kd-tree for the input point cloud is built. This kd-tree is traversed by a hierarchical feature extractor from leaves of the tree to the root, utilizing the nesting of the point cloud at different spatial scale. Although such methods alleviate problems with spatial voxelization, they are still based on subdividing a bounding volume rather than exploiting local geometric structure of points themselves.
Point-based neural networks are a new class of architectures that eliminate the need for explicitly transforming point clouds to another data format (such as octrees and voxels). Using point-based NNs, the information loss due to any point cloud transformation and mapping can be avoided [28]. In [19], it is shown that given a symmetric function, such as max or summation, one can design a permutation-invariant layer in a deep neural network to feed an unordered point cloud for classification and segmentation purposes. PointNet [17] is one of the pioneering methods in this category for object classification and semantic segmentation. However, since this network treats points independently and uses global aggregation, it does not fully take advantage of the local structures (such as corners) of the point cloud, or any geometric dependency between neighbouring points.

In order to address this drawback, PointNet++ [18] partitions the point set into smaller overlapping clusters and then applies PointNet to each cluster independently. Although this method employs the local structure of points, the geometric relationship between points within each cluster is still ignored.

B. CNNs on Point Cloud Graphs

Modelling a point cloud as a graph of points is a promising way to exploit their local geometric information in a neural network. Recently, using graph representation in combination with CNNs to process unordered (non-euclidean) data has become a topic of interest [22]. The two main approaches in this category are based on spectral filtering and spatial filtering.

Spectral Methods The spectral methods use the spectral graph theory to redefine the spatial graph convolution as a multiplication in the spectral domain [22], [21]. The first proposed methods in this category suffer from the lack of spatial locality of filters, which is given by smoothness of filters in the spectral domain. However, by a parameterization of convolution filters as Chebyshev polynomials of eigenvalues and their approximate evaluation, a computationally efficient method is proposed in [4], which yields a localized spatial filter. Unfortunately, these filters are learned in the context of the spectrum of graph Laplacian [1] and therefore have to be the same for all the graphs in the dataset. This means that for the data types where the graph structure varies in the dataset, such as point clouds, a graph learnt on one shape cannot generalize to others.

Spatial Methods Local spatial filtering approach [22], [15], [7], [23], [29], [3], [10], [12] on the other hand, employs spatial rather than spectral filters. These methods have one thing in common, which is the notion of local patch on graphs, where an operator similar to convolution is applied to each local patch. Depending on how the correspondence between filter weights and nodes in each local patch is established, a number of techniques are proposed, such as MoNet [12], GCNN [5] and DCNN [2].

Although much work has been done to apply spatial filtering for deep learning on general graphs, only a few methods, such as KCNet [20], FoldingNet [28], ECC [22] and DGCNN [25] are proposed for using deep learning on point cloud graphs.

C. Point Cloud Down-Sampling in Deep Networks

Despite the fact that graph convolution on point clouds has recently received great interest, there is not much work done on down-sampling for it. There are several reasons why having point cloud down-sampling layers inside a graph convolutional network may be advantageous:

- Most existing point cloud graph convolution methods use k-NN search to find the neighbourhood of each point. Reducing the number of points can highly reduce the computational cost for subsequent convolution layers.
- Reducing the number of points in the network can also result in lower runtime memory usage.
- Down-sampling layers can boost the network robustness to certain perturbations in the input data.

In most of the point-based neural networks mentioned above, such as PointNet and DGCNN, the number of points in the point cloud is fixed throughout the network. PointNet++ however, down-samples the point cloud by using the farthest point sampling (FPS) algorithm to sample the points. It then generates overlapping partitions by finding the k nearest neighbour points around each sample point. Although the point cloud is hierarchically down-sampled, it has been shown in [18] that the computational complexity of the network has increased significantly relative to PointNet.

KCNet [20] and FoldingNet [28] down-sample the graph using a graph-based max-pooling that takes maximum features over the neighbourhood of each vertex using a previously built k nearest neighbour graph (KNNG).

While these methods down-sample the input points within the network, they provide no guarantee that the most important ones, which we call critical points (CP), will be passed to next layers. Consequently, an unimportant point with weak (less contributing) features may be selected or generated, while an important point may be removed from the point cloud or devalued.

In addition, the down-sampling methods used in some of these networks are static, where the sampling method relies only on the spatial locations of points in the input point cloud, but not their corresponding feature vectors that are learnt during training. On the other hand, methods that use feature space distance between points, such as PointNet++ [18], are computationally prohibitive due to the high cost of local nearest-neighbour search in high-dimensional feature space.

Moreover, all these methods generate a set of new points, instead of just selecting a subset of input points. This step makes it difficult to track the contribution of each input point on the output in spite of the added computational cost in the down-sampling process.

In this paper, we introduce a computationally efficient down-sampling layer, called Critical Points Layer (CPL), which learns to down-sample the points adaptively based on
Algorithm 1: Critical Points Layer (CPL)

1: function \((F_O, f_O, suidx) = CPL(F_S, F_r, k)\) 
2:  
3: \( \text{for } i = 0 \text{ to } \text{ncols}(F_S) - 1 \text{ do} \) \(\triangleright \) max pooling 
4:  
5: \( f_{\text{max}}[i] = \max(F_S[:, i]) \) 
6:  
7: \( \text{idx}[i] = \text{argmax}(F_S[:, i]) \) 
8:  
9: \( \text{end for} \) 
10: suidx = unique(idx) \(\triangleright \) set operation 
11:  
12: \( f_S[j] = \sum_{\text{idx}[i] = \text{uidx}[j]}^{} f_{\text{max}}[i] \) 
13:  
14: \( u_i = \text{sort}(f_S) \) \(\triangleright \) sorting 
15:  
16: \( \text{rsuidx} = \text{resize}(\text{uidx}, k) \) \(\triangleright \) nearest-neighbor resizing 
17:  
18: \( \text{for } i = 0 \text{ to } \text{ncols}(F_O) - 1 \text{ do} \) \(\triangleright \) point collection 
19:  
20: \( f_O[i] = \max(F_O[:, i]) \) 
21:  
22: \( \text{end for} \) 
23:  
24: \( \text{return } (F_O, f_O, \text{rsuidx}) \) 
25:  
26: \( \text{end function} \)

The block diagram of the proposed Critical Points Layer (CPL) is illustrated in Figure 1a. In order to elaborate more on the functionality of CPL, its pseudo-code is also provided in Algorithm 1. The steps of the algorithm are explained in more details as follows:

1) The input point cloud \( F_S \) is a matrix with \( n \) rows (corresponding to \( n \) input points) and \( d \) columns (corresponding to \( d \)-dimensional feature vectors).

2) In the first step (Operation 3), the maximum feature value is obtained for each column of the matrix \( F_S \). This is the same as the max-pooling operation in PointNet [17]. The resulting \( d \)-dimensional feature vector, denoted by \( f_{\text{max}} \), has the same dimension as input feature vectors and can be independently used for classification and segmentation tasks. However, we are interested in down-sampling the input points rather than generating a single feature vector out of them. To this aim, the index of each row with a maximum feature value is also saved in the index vector \( \text{idx} \). Vector \( \text{idx} \) contains the indices of all the points that have contributed to the feature vector \( f_{\text{max}} \). By definition, we call these points, the Critical Points (CP). These are the important points that should be preserved in the down-sampling process.

3) The index vector \( \text{idx} \) may contain multiple instances of the same point. To avoid these repetitions, unique indices are extracted from \( \text{idx} \), using the “set (unique)” function (Operation 9). The output set which has the unique indices is called the Critical Set (CS) and is denoted by \( \text{uidx} \). Beside finding the unique vector, we also add-up the feature values from \( f_{\text{max}} \) that correspond to the same point or index (Operation 7).

III. PROPOSED SOLUTION

In this section, we propose two new adaptive down-sampling methods that can be used in deep neural networks. These two new layers, named Critical Points Layer (CPL) and Weighted Critical Points Layer (WCPL), can efficiently down-sample unordered point clouds, while being permutation invariant. In this section, CPL, WCPL and a systemic approach to use these two layers in deep neural networks and more specifically classification neural networks will be explained in details.

A. Critical Points Layer (CPL)

Let’s assume the input to the CPL is an unordered point cloud with \( n \) points, each represented as a feature vector \( \mathbf{x} \in \mathbb{R}^d \), where \( \mathbb{R} \) is the set of real numbers and \( d \) is the dimension of the feature vector. The goal of CPL is to generate a subset of input points, called Critical Points (CP), with \( m \leq n \) points, each represented as a feature vector \( \mathbf{y} \in \mathbb{R}^l \), where \( l \) is the dimension of the new feature vector. The critical points of a point cloud are the points with maximum information that are needed to be preserved in a down-sampling (or pooling) process. These points may change based on the task and application.
The resulting feature vector $f_S$ will be later used to sort the input points.

4) Next, feature vector $f_S$ is sorted (in an ascending order). Corresponding indices in $uidx$ are also rearranged based on the sorting output (Operation 9), resulting in an index vector which is denoted by $suidx$. This step is necessary for the following sampling (resizing) operation. It also makes CPL invariant to the order of input points.

5) Number of elements in $suidx$ may differ for different point clouds in the input batch. For batch processing however, these numbers need to be the same. To address this, for each point cloud in the input batch, the index vector $suidx$ is up-sampled to a fixed size vector $rsuidx$ using an up-sampling method for integer arrays, such as nearest neighbor resizing (Operation 10).

6) As the final step, the up-sampled index vector $rsuidx$, which contains the indices of all the critical points, is used to gather points and their corresponding feature vectors. Since different feature vectors may correspond to a single point, and because of the information being filtered in hidden NN layers, we may want to gather the features from other layers (denoted by $F_1$) than those used for selecting the points (denoted by $F_S$). However, critical points are defined based on the contribution of each point in the maximum feature vector obtained from $F_S$ and it makes more sense to use $F_1 = F_S$.

One of the main requirements of any layer designed for point cloud processing is its invariance to input order (or input permutation). The proposed CPL, fulfills this requirement via following properties:

- Sorting the feature vector $f_S$ in step 4 is order independent, because sorting is based on feature values and not based on indices of the points.
- Nearest-neighbor resizing in step 5 is invariant to swapping the index of the input points, i.e.
  \[
  \text{resize}(\text{sort}(\text{swap}(uidx))) = \text{swap}(\text{resize}(\text{sort}(uidx))
  \]
  \[
  \text{(1)}
  \]
  where $\text{sort}$ is applied based on feature values, and $\text{swap}$ is applied on index only.

B. Weighted Critical Points Layer (WCPL)

In CPL, a point in the point cloud is counted as a critical point if any of its features contributes to the output maximum feature vector $f_{\text{max}}$, regardless of the number of its contributing features. For example, if a point contributes with two of its features, while another point has ten contributing features, both are treated the same in CPL. In other words, in CPL the “importance” of a point has a binary value: a given point is either important (critical) or unimportant (uncritical). In this section, we introduce a modified version of CPL, called Weighted Critical Points Layer (WCPL). The proposed WCPL assigns weights to points based on their level of contribution to $f_{\text{max}}$.

In this context, to increase the weight of a point by a factor of $C$, we repeat the point index $C$ times. By increasing the repetition frequency, the probability of selecting the point in the down-sampling process will also increase. From another point of view, in WCPL, the probability of missing a critical point in the output is lower than that in CPL. The pseudo-code of WCPL is given in Algorithm 2.

C. Critical Points Net (CP-Net)

In this section, we propose a hierarchical architecture to apply deep convolutional neural networks to point clouds, by systematically reducing the number of points using the proposed CPL/WCPL. In the proposed network model, named Critical Points Net (CP-Net), any graph convolution method, such as DCNN [2], GCNN [10], MoNet [12] or EdgeConv (from DGCNN [25]) can be used in convolution layers. The block diagram of CP-Net using EdgeConv as an example is shown in Figure 2.

The input in Figure 2 is an unordered point cloud of size $n$. In the first step, the point cloud is passed into a convolution layer of choice to filter the input into a richer set of features. The filtered output point cloud $F_{S0}$ is then used as an input to the first CPL/WCPL. Using a CPL/WCPL with downsampling factor $k_0$, the number of points in $F_{S0}$ is reduced to $n/k_0$ points. These steps are repeated for as many times as necessary to achieve a desired size of the point cloud (both in terms of number of points and feature vector size). Note that at $j$-th CPL/WCPL block, one can also benefit from using or concatenating features from all or some of the previous layers, i.e., $\{F_{I_{j0}}, F_{I_{j1}}, F_{I_{j2}}\}$, as long as they correspond to the same points. As a result, the number of output point will be $\frac{n}{k_0 k_1 k_{j+1}}$.

D. CP-Net for 3D Object Classification

Here we give an example of CP-Net application in the 3D object classification problem. Block diagram of the proposed

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**Algorithm 2** Weighted Critical Points Layer (WCPL)

1: function $(F_O, f_O, rsuidx) = WCPL(F_S, F_1, k)$

2: for $i = 0$ to ncols($F_S$) - 1 do \ comment pooling

3:   $\text{idx}[i] = \text{argmax}(F_S(:, i))$

4:   $f_{\text{max}}[i] = \text{max}(F_S(:, i))$

5: end for

6: $uidx = \text{unique}(\text{idx})$ \ comment operation

7: $f_S[j] = \sum_{\text{idx}[i] = uidx[j]} f_{\text{max}}[i]$

8: $f_S[j] = \{(i | \text{idx}[i] = uidx[j])\}$ \ comment frequency of $uidx[j]$ in $idx$

9: $-.1 = \text{sort}(f_S)$ \ comment sorting

10: $suidx = uidx[1]$

11: $midx = \text{repeat}(suidx, fr)$

12: $rmidx = \text{resize}(midx, k)$ \ comment nearest-neighbor resizing

13: $F_O = F_1[\text{rmidx}, :]$ \ comment collection

14: for $i = 0$ to ncols($F_O$) - 1 do \ comment pooling

15:   $f_O[i] = \text{max}(F_O(:, i))$

16: end for

17: return $(F_O, f_O, rsuidx)$

18: end function
Fig. 2: General block diagram of the proposed CP-Net.

network is illustrated in Figure 3. The network is composed of three subnets: 1) \( n \)-point feature extraction subnet, 2) \((n/4)\)-point subnet and 3) classification subnet. The detailed steps of the proposed network are as follows:

(a) The input to the network is an unordered point cloud of size \( n \times 3 \), where each point is a 3D vector.
(b) The input data goes through a spatial transformer network as explained in [17] and references therein, to make it robust against any rigid transformation, including rotation and translation. It is worth noting that instead of using the original input, a modified version of EdgeConv [25] edge feature is used for spatial transformation, as explained in the next step 2.
(c) The output of the spatial transform goes into a filtering CNN, here EdgeConv [25], to produce richer features. Unlike the original EdgeConv [25] operator which uses two kernels in the edge feature function, we use the triple-kernel version \( h(x_i, x_j - x_i, (x_j - x_i)^2) \), where \((x_j - x_i)^2\) is element-wise square operation between each point \( x_i \) and its neighbouring point \( x_j \). In the proposed network, applying the EdgeConv with 128 filters to the input point cloud of size \( n \times 3 \), results in a point cloud of size \( n \times 128 \).
(d) A multi-layer perceptron (MLP) layer expands the feature dimension from 128 to 1024 features, resulting in a point cloud of size \( n \times 1024 \).
(e) Next, CPL/WCPL is applied to find the critical points and to reduce the number of input points. As shown in Section IV this step reduces the computational complexity without any loss in the classification accuracy.

A down-sampling factor of \( 1/4 \) is chosen to reduce the number of points from \( n \) to \( n/4 \). Note that either CPL or WCPL can be used for this purpose.
(f) Another EdgeConv layer is used to filter the point cloud, this time by preserving the depth and size to further process the received point cloud. Note that reducing the number of points in the previous layer highly reduces the computational complexity of this layer.
(g) A reduce-max layer is used to generate a vector of size 1024, out of the point cloud of size \( n \times 1024 \).
(h) Finally, fully connected layers of size 512, 256 and 40 are applied to transform the feature vector of size 1024 to the number of classes in the ModelNet40 dataset [26], which is 40.

In the proposed 3D classification method, standard softmax cross entropy is used as the loss function. In addition, all layers include a ReLU activation function and batch normalization.

IV. EXPERIMENTS

A. Data preprocessing

We evaluate our model on ModelNet40 3D object classification dataset [26]. The dataset contains 12,311 meshed CAD models from 40 different object categories out of which 9,843 models are used for training and 2,468 models for testing. From each model mesh surface, 1024 points are uniformly sampled and normalized to the unit sphere. For data augmentation, we randomly scale, rotate and shift each object point cloud in the 3D space.

B. Training Details

To train the model, we use Adam optimizer with an initial learning rate 0.001 and exponentially decay it with a rate of 0.5 every 200,000 steps. The decay rate of batch
normalization starts from 0.5 and is increased to 0.99. The
Dropout with probability 0.5 is used in the last two fully-
connected layers. Training the network with TensorFlow on
an Nvidia P100 GPU with batch size 32, takes 9 – 10 hours
for 400 epochs.

C. Statistical Results

To evaluate the performance of a 3D point cloud classifica-
tion method, we use both overall accuracy and per-class
average accuracy, calculated over all the test samples.

The classification accuracy results for our proposed CP-
Net/WCP-Net are shown in Table I with comparisons against
the previously proposed methods. As seen, our CP-Net and
WCP-Net methods achieve the best overall classification
accuracy and mean class accuracy, respectively.

| Algorithm          | Overall Accuracy (%) | Mean Class Accuracy (%) |
|--------------------|----------------------|-------------------------|
| Vox-Net [11]       | 83.00                | 85.9                    |
| PointNet [17]      | 89.20                | 86.0                    |
| Pointnet++ [18]    | 90.70                | -                       |
| KD-Net [6]         | 91.8                 | -                       |
| ECC [22]           | 85.2                 | -                       |
| SO-Net [8]         | 89.16                | -                       |
| DGCNN[29] (1 vote) | 91.84                | 89.40                   |
| KCNet [20]         | 91.0                 | -                       |
| Ours (CP-Net)      | 92.35                | 89.90                   |
| Ours (WCP-Net)     | 92.41                | 90.53                   |

TABLE I: Classification accuracy results on ModelNet40
dataset [26], for input size 1024 × 3.

D. Qualitative Results

Figure 4 shows how the proposed CPL learns to down-
sample different point clouds. The original point clouds of
size 1024 for the object classes lamp, airplane, flower-pot,
laptop and car are shown in Figure 4(a). Figures 4(b-e) cor-
correspond to the outputs obtained using the down-sampling
ratio of 0.25, at epochs 1, 100, 200 and 300, respectively.
As seen in Figures 4(b), at the beginning of the training,
some important parts of the objects, such as lamp column,
flower leaves and laptop screen are partially lost as a result
of down-sampling. After 300 epochs of training however, CPL
learns to down-sample the point cloud such that the critical
points of the object are mostly retained. In the context of
point cloud classification, by important points of an object
we mean those points that contain the necessary information
discriminate between different objects in the dataset.

Figures 4(f) show the corresponding point clouds down-
sampled by the ratio of 1/16, after 300 training epochs.
As seen, the important points of each object for our classifica-
tion task are still preserved even in such small 64-point point
clouds. The lamp column, airplane wings and six corners
of the laptop are some examples of the preserved important
object parts.

E. Ablation studies

EdgeConv Kernels The effect of using two and three
kernels (in EdgeConv operator used in DGCNN [25]) on the
overall classification accuracy and execution time is shown
in Table II. For double-kernel version, we use the one used
in [25]. The triple-kernel version is defined in section III-
D. As seen, the triple kernel version is computationally
more complex than the double kernel version. In both cases,
not only the proposed CP-Net/WCP-Net outperforms the
DGCNN in classification accuracy, it is computationally less
complex, due to CPL/WCLP point cloud down-sampling.

| Method      | Double Kernel | Triple Kernel |
|-------------|---------------|---------------|
| DGCNN       | 91.84 (135ms) | 89.26 (141ms) |
| CP-Net      | 91.88 (115ms) | 92.33 (119ms) |
| WCP-Net     | 91.76 (116ms) | 92.41 (120ms) |

TABLE II: Effect of edge feature kernels on overall classifi-
cation accuracy (%) and execution time (in ms).

Effect of Bottleneck Dimension The effect of bottleneck
layer size (number of features in the output feature vector)
on classification accuracy is shown in Table III. Clearly,
increasing the bottleneck layer size improves the accuracy,
however it almost saturates at around 1024 features. Note
that even with the bottleneck size of 64, accuracy of the
proposed CP-Net (%89.35) is more than that of the PointNet
with bottleneck size of 1024 (%89.20).

| 64  | 128 | 256 | 512 | 1024 |
|-----|-----|-----|-----|------|
| CP-Net | 89.35 | 89.83 | 90.85 | 91.94 | 92.33 |
| WCP-Net | 89.16 | 89.71 | 90.73 | 91.54 | 92.41 |

TABLE III: Effect of bottleneck dimension on accuracy (%).

Effect of Down-Sampling Ratio Table IV shows the
effect of down-sampling ratio on classification accuracy. As
expected, the more the point cloud is shrunk, the lower the
accuracy. This is because some of the important information
about the object is lost as a result of down-sampling. The
proposed CPL and WCLP layers however preserve the important
points of the object as much as possible. This can be verified
from Table IV where the difference between the accuracy
values at down-sampling ratios 1 and 1/16 (corresponding to
point clouds of size 1024 and 64 points) is only %0.73. This
means that in the down-sampling process, CPL preserves the
most important information of each object, so that with such
small number of points, objects are still classified with high
accuracy.

| 1  | 1/2 | 1/4 | 1/8 | 1/16 |
|----|-----|-----|-----|------|
| CP-Net | 92.25 | 92.24 | 92.33 | 92.29 | 91.52 |
| WCP-Net | 92.09 | 92.15 | 92.41 | 92.03 | 91.81 |

TABLE IV: Effect of down-sampling ratio on accuracy (%).

Time and Space Complexity

Table V compares the execution time of running the proposed
CP-Net/WCP-Net with other state-of-the-art object classifi-
cation methods, on an Ubuntu machine with a single P100
GPU and an Intel 1.2 GHz CPU. Batch size of 32 1024-point
point clouds is used in all the experiments.

In terms of model size, the smallest models are generated by
KCNN. CP-Net and WCP-Net generate the largest models
due to their larger number of network parameters. The model
size can be reduced by decreasing the bottleneck dimension.
In terms of computational complexity, CP-Net and WCP-Net run faster than both DGCNN and PointNet++. However, they are slower than PointNet and KCNet. The lower computational complexity of CP-Net in comparison with DGCNN is due to the employment of CP layer in its network. Similarly, by a proper network design, the proposed CPL/WCPL is expected to accelerate other classification deep networks, such as KCNet and PointNet++.

| Method          | Model Size (MB) | Inference Time (ms) | Accuracy (%) |
|-----------------|-----------------|----------------------|--------------|
| PointNet        | 40              | 36.1                 | 89.20        |
| PointNet++      | 12              | 232.2                | 90.70        |
| KCNet           | 11              | 26.4                 | 91.0         |
| DGCNN           | 21              | 134.5                | 91.84        |
| CP-Net/WCPL-Net | 57              | 118.9                | 92.33/92.41  |

TABLE V: Model size (in MB), inference mode execution network. Similarly, by a proper network design, the proposed DGCNN is due to the employment of CP layer in its network. Similarly, by a proper network design, the proposed CPL/WCPL is expected to accelerate other classification deep networks, such as KCNet and PointNet++.

V. CONCLUSION

In this paper we proposed a new adaptive down-sampling method that can be trained to pass on the most important points (critical points), to the next layers in a neural network. Two down-sampling layers, the critical points layer (CPL) and its weighted version, weighted CPL (WCPL) are proposed here. As a systemic approach of using CPL/WCPL in deep neural networks, CP-Net, a hierarchical implementation of these layers, is also introduced. Finally, a deep classification network is designed based on the suggested CP-Net for 3D object classification. The results show the superiority of the proposed method in terms of classification accuracy in comparison with the previously proposed methods.

CPL is an adaptive layer that can adaptively down-sample the unordered data. This property can help researchers to design a new category of auto-encoders that can handle the unordered data such as point clouds. This approach is already under investigation and the results will be shown in a future work. Last but not least, we also intend to use the proposed down-sampling layers to design efficient deep networks for other applications such as semantic segmentation and object detection.

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Fig. 4: (From top to bottom and left to right) The original point clouds for the lamp, airplane, flowerpot, laptop and car object categories in ModelNet40 dataset [26], and their down-sampled versions (with ratio $\frac{1}{4}$) obtained after training the classification CP-Net, shown in Figure 3 for 1, 100, 200 and 300 epochs. (f) The result of training for 300 epochs with down-sampling ratio $\frac{1}{16}$. The images are color coded to reflect the depth information.