Effect of scalar leptoquarks on the rare decays of $B_s$ meson

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Abstract

We study the effect of scalar leptoquarks on some rare decays of $B_s$ mesons involving the quark level transition $b \rightarrow s \ l^+l^-$. In particular we consider the decays $B_s \rightarrow \mu^+\mu^-$, $\bar{B}_d^0 \rightarrow X_s \mu^+\mu^-$ and $B_s \rightarrow \phi \ \mu^+\mu^-$. The leptoquark parameter space is constrained using the recently measured branching ratio of the $B_s \rightarrow \mu^+\mu^-$ process at LHCb and CMS experiments. Using such parameters we obtain the branching ratio, forward backward asymmetry and the CP asymmetry parameters in the angular distribution of $B_s \rightarrow \phi \ \mu^\mp\mu^-$ process.

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I. INTRODUCTION

The standard model (SM) of electroweak interaction is very successful in explaining the observed data so far and is further supported by the recent discovery of a Higgs-like boson in the mass range of 126 GeV. But still there are many reasons to believe that it is not the ultimate theory of nature, rather some low energy limit of some more fundamental theory whose true nature is not yet well understood. It is therefore an ideal time to test the predictions of the standard model more carefully and try to identify the nature of physics beyond it. If there would be new physics (NP) at the TeV scale associated with the hierarchy problem, it is natural to expect that it would first show up in the flavor sector and in this context the rare decays of $B$ mesons induced by flavor changing neutral current (FCNC) transitions play a very crucial role. The FCNC transitions are one-loop suppressed in the SM and thus provide an excellent testing ground to look for possible existence of new physics.

In this paper we would like to investigate some rare decay modes of $B_s$ meson using the scalar leptoquark (LQ) model. The study of $B_s$ meson has attracted a lot of attention in recent times as large number of $B_s$ mesons are produced in the LHCb experiment and this would open up the possibility to study the rare decays of $B_s$ meson with high statistical precision. The most important and sought after rare decay mode is the $B_s \rightarrow \mu^+\mu^-$ process mediated by the FCNC transition $b \rightarrow s$, has been recently observed by the LHCb [1] and CMS [2] collaborations. This mode is very interesting as it is theoretically very clean and highly suppressed in the standard model and hence well suited for constraining the new physics parameter space. Another important rare decay channel mediated by the quark level transition $b \rightarrow s \mu^+\mu^-$ is the inclusive decay process $\bar{B}_d^0 \rightarrow X_s \mu^+\mu^-$. The integrated branching ratio for this process has been measured by both Belle [3] and BaBar [4] collaborations. It is expected that in the low $q^2$ region ($1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$) as well as in the high $q^2$ region ($q^2 \geq 14.4 \text{ GeV}^2$) the theoretical predictions are dominated by perturbative contributions and hence a theoretical precision of order 10% is in principle possible [5]. We will use the measured branching ratios of these processes to constrain the leptoquark parameters and subsequently apply these parameters to study the semileptonic rare decay mode $B_s \rightarrow \phi \mu^+\mu^-$. 

Leptoquarks are color-triplet bosons that can couple to a quark and a lepton at the same time and can occur in various extensions of the SM [6]. Scalar leptoquarks are expected
to exist at the TeV scale in extended technicolor models [7] as well as in models of quark and lepton compositeness [8]. The general classification of leptoquark models are discussed in [9] and the phenomenology of scalar leptoquarks have been studied extensively in the literature [10–12]. Here, we will consider the model where leptoquarks can couple only to a pair of quarks and leptons and thus may be inert with respect to proton decay. In such cases, proton decay bounds would not apply and leptoquarks may produce signatures in other low-energy phenomena [11].

The paper is organized as follows. In section II we briefly discuss the effective Hamiltonian describing the process $b \rightarrow s l^+ l^−$. The new contributions arising due to the exchange of scalar leptoquark are presented in section III. We present the rare decay modes $B_s \rightarrow \mu^+ \mu^−$ and $\bar{B}_d \rightarrow X_s \mu^+ \mu^−$ in sections IV and V respectively and obtain the constraints on leptoquark parameters. The decay mode $B_s \rightarrow \phi \mu^+ \mu^−$ is discussed in Section VI and section VII contains the Conclusion.

II. EFFECTIVE HAMILTONIAN FOR $b \rightarrow s l^+ l^−$ PROCESS IN THE STANDARD MODEL

Within the standard model the effective Hamiltonian describing the quark level transition is given as [13]

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[ \sum_{i=1}^{6} C_i(\mu) O_i + C_7 \frac{e}{16\pi^2} \left( \bar{s} \sigma_{\mu\nu} (m_s P_L + m_b P_R) b \right) F^{\mu\nu} \right. $$

$$+ C_9^{\text{eff}} \frac{\alpha}{4\pi} (\bar{s} \gamma^\mu P_L b) \bar{l} \gamma_\mu l + C_{10} \frac{\alpha}{4\pi} (\bar{s} \gamma^\mu P_L b) \bar{l} \gamma_\mu \gamma_5 l \right],$$

(1)

where $G_F$ is the Fermi constant and $V_{qf}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $\alpha$ is the fine structure constant, $P_{L,R} = (1 \mp \gamma_5)/2$ and $C_i$’s are the Wilson coefficients. The values of the Wilson coefficients are calculated at the next-to-next-leading order (NNLO) by matching the full theory to the effective theory at the electroweak scale and subsequently solving the renormalization group equation to run them down to the b-quark mass scale i.e., $\mu_b = 4.8$ GeV [14].

The coefficient $C_9^{\text{eff}}$ contains a perturbative part and a resonance part which comes from the long distance effects due to the conversion of the real $c\bar{c}$ into the lepton pair $l^+ l^-$. Thus,
$C_9^{eff}$ can be written as

$$C_9^{eff} = C_9 + Y(s) + C_9^{res}, \quad (2)$$

where $s = q^2$ and the function $Y(s)$ denotes the perturbative part coming from one loop matrix elements of the four quark operators and is given in Ref. [13]. The long distance resonance effect is given as

$$C_9^{res} = \frac{3\pi}{\alpha^2} (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \sum_{V_i = J/\psi, \psi'} \frac{\kappa m_{V_i} \Gamma(V_i \rightarrow l^+l^-)}{m_{V_i}^2 - s - im_{V_i} \Gamma_{V_i}}, \quad (3)$$

where the phenomenological parameter $\kappa$ is taken as 1.7 and 2.4 for the two lowest two $\bar{c}c$ resonances $J/\psi$ and $\psi'$ [14].

III. NEW PHYSICS CONTRIBUTIONS DUE TO SCALAR LEPTOQUARK EXCHANGE

In the leptoquark model the effective Hamiltonian describing the process $b \to sl^+l^-$ will be modified due to the additional contributions arising from the exchange of leptoquarks. Here, we will consider the minimal renormalizable scalar leptoquark models [11], where the standard model is augmented only by one additional scalar representation of $SU(3) \times SU(2) \times U(1)$ and which do not allow proton decay at the tree level. It has been shown in [11] that there are only two models which can satisfy this requirement. In these models the leptoquarks have the representation as $X = (3, 2, 7/6)$ and $X = (3, 2, 1/6)$ under the $SU(3) \times SU(2) \times U(1)$ gauge group. Our aim here is to consider these scalar leptoquarks which potentially contribute to the $b \to s\mu^+\mu^-$ transitions and constrain the underlying couplings from experimental data on $B_s \to \mu^+\mu^-$ and $\bar{B}_d^0 \to Xs\mu^+\mu^-$. Although the decay modes $\bar{B}_d^0 \to \bar{K}^0\mu^+\mu^-$ and $\bar{B}_d^0 \to K^{*0}\mu^+\mu^-$ are also mediated by the same quark level transition $b \to s\mu^+\mu^-$, we do not consider the measured branching ratios of such processes to constrain the NP parameter space as these measurements involve additional uncertainties due to the form factors. However, we will comment on the recent observation of several anomalies on angular observables in the rare decay $B \to K^{*0}\mu^+\mu^-$ by the LHCb collaboration [16].

Now we consider all possible renormalizable interactions of such leptoquarks with SM matter fields consistent with the SM gauge symmetry in the following subsections.
A. Model I: \(X = (3,2,7/6)\)

In this model the interaction Lagrangian for the coupling of scalar leptoquark \(X = (3,2,7/6)\) to the fermion bilinears is given as \([11]\)

\[
\mathcal{L} = -\lambda^{ij}_u \bar{u}^i_R X^T \epsilon L^j_L - \lambda^{ij}_e \bar{e}^i_R X^T Q^j_L + h.c. ,
\]

(4)

where \(i, j\) are the generation indices, \(Q_L\) and \(L_L\) are the left handed quark and lepton doublets, \(u_R\) and \(e_R\) are the right handed up-type quark and charged lepton singlets and \(\epsilon = i\sigma_2\) is a \(2 \times 2\) matrix. More explicitly these multiplets can be represented as

\[
X = \begin{pmatrix} V_{\alpha} \\ Y_{\alpha} \end{pmatrix}, \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \text{and} \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .
\]

(5)

After expanding the \(SU(2)\) indices the interaction Lagrangian becomes

\[
\mathcal{L} = -\lambda^{ij}_u \bar{u}^i_R (V_\alpha \nu_L^j - Y_\alpha \nu_L^j) - \lambda^{ij}_e \bar{e}^i_R \left(V_{\alpha}^\dagger u^j_{\alpha L} + Y_{\alpha}^\dagger d^j_{\alpha L}\right) + h.c. .
\]

(6)

Thus, from Eq. (4), one can obtain the contribution to the interaction Hamiltonian for the \(b \to s \mu^+ \mu^-\) process after Fierz rearrangement as

\[
\mathcal{H}_{LQ} = \frac{\lambda^{23}_\mu \lambda^{22}_\mu}{8M_Y} [\bar{s} \gamma^\mu (1 - \gamma_5) b][\bar{\mu} \gamma_\mu (1 + \gamma_5) \mu] \equiv \frac{\lambda^{23} \lambda^{22}_\mu}{4M_Y} (O_9 + O_{10}) ,
\]

(7)

which can be written analogous to the SM effective Hamiltonian (1) as

\[
\mathcal{H}_{LQ} = -\frac{G_F^\alpha}{\sqrt{2}\pi} V_{tb} V^*_{ts} (C^{NP}_9 O_9 + C^{NP}_{10} O_{10})
\]

(8)

with the new Wilson coefficients

\[
C^{NP}_9 = C^{NP}_{10} = -\frac{\pi}{2\sqrt{2}G_F^\alpha V_{tb} V^*_{ts}} \frac{\lambda^{23}_\mu \lambda^{22}_\mu}{M_Y^2} .
\]

(9)

B. Model II: \(X = (3,2,1/6)\)

Analogous to the previous subsection the interaction Lagrangian for the coupling of \(X = (3, 2, 1/6)\) leptoquark to the fermion bilinear can be given as

\[
\mathcal{L} = -\lambda^{ij}_d \bar{d}^i_R X^T \epsilon L^j_L + h.c. ,
\]

(10)
where the notations used are same as the previous case. Expanding the $SU(2)$ indices one can obtain the interaction Lagrangian as

$$\mathcal{L} = -\lambda^{ij}_a \bar{d}_R (V_e \nu^j_L - Y_\alpha \nu^j_L) + h.c. .$$

(11)

After performing the Fierz transformation the interaction Hamiltonian describing the process $b \to s \mu^+ \mu^-$ is given as

$$\mathcal{H}_{LQ} = \frac{\lambda^{22}_s \lambda^{32*}_b}{4M_V^2} [s \gamma^\mu P_R b][\bar{\mu} \gamma_\mu (1 - \gamma_5) \mu] = \frac{\lambda^{22}_s \lambda^{32*}_b}{4M_V^2} \left( O''_{10}^{NP} - O''_{10}^{NP} \right) ,$$

(12)

where $O'_9$ and $O'_{10}$ are the four-fermion current-current operators obtained from $O_{9,10}$ by making the replacement $P_L \leftrightarrow P_R$. Thus, the exchange of the leptoquark $X = (3, 2, 1/6)$ gives new operators with the corresponding Wilson coefficients as

$$C''_{10}^{NP} = -C''_{10}^{NP} = \frac{\pi}{2\sqrt{2}} \frac{G_F \alpha V_{tb} V^*_{ts}}{M_V^2} \lambda^{22}_s \lambda^{32*}_b .$$

(13)

After obtaining the new physics contributions to the process $b \to s \mu^+ \mu^-$, we will proceed the constrain the new physics parameter space using the recent measurement of $B_s \to \mu^+ \mu^-$. 

IV. $B_s \to \mu^+ \mu^-$ DECAY PROCESS

The rare decay process $B_s \to \mu^+ \mu^-$, mediated by the FCNC transition $b \to s$ is strongly helicity suppressed in the standard model. Furthermore, it is very clean and the only nonperturbative quantity involved is the decay constant of $B_s$ meson which can be reliably calculated by the well known non-perturbative methods such as QCD sum rules, lattice gauge theory etc. Therefore, it is believed to be one of the most powerful tools to look for new physics beyond the standard model. This process has been very well studied in the literature and in recent times also it has attracted a lot of attention [17–22]. Therefore, here we will quote the important results.

The most general effective Hamiltonian describing this process

$$\mathcal{H}_{eff} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V^*_{ts} \left[ C^{eff}_{10} O_{10} + C'_{10} O'_{10} \right] ,$$

(14)

where $C^{eff}_{10} = C_{10}^{SM} + C_{10}^{NP}$ and $C'_{10} = C'_{10}^{NP}$. The branching ratio for this process is given as

$$\text{BR}(B_s \to \mu^+ \mu^-) = \frac{G_F^2}{16\pi^3} \tau_{B_s} \alpha^2 f_{B_s}^2 m_{B_s}^2 m_{\mu}^2 |V_{tb} V^*_{ts}|^2 \left| C^{eff}_{10} - C'_{10} \right|^2 \sqrt{1 - \frac{4m_{\mu}^2}{m_{B_s}^2}} .$$

(15)
Now using $\alpha = 1/128$, $|V_{tb}V_{ts}^*| = 0.0405 \pm 0.0008$, $f_{B_s} = 227 \pm 8$ MeV \cite{19}, $C_{10}^{SM} = -4.134$ \cite{18}, the particle masses and lifetime of $B_s$ meson from \cite{23} we obtain the SM branching ratio for this process as

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) = (3.29 \pm 0.19) \times 10^{-9}, \quad (16)$$

which is consistent with the latest SM prediction $\text{Br}(B_s \rightarrow \mu^+\mu^-) = (3.23 \pm 0.23) \times 10^{-9}$ \cite{19}.

The branching ratio for this mode has recently been measured by both LHCb \cite{1} and CMS \cite{2} collaborations. Analyzing the data corresponding to an integrated luminosity of 1 fb$^{-1}$ at $\sqrt{s}$ TeV and 2 fb$^{-1}$ at $\sqrt{8}$ TeV the LHCb collaboration obtained the time integrated branching ratio as

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) = (2.9^{+1.1}_{-1.0}) \times 10^{-9}. \quad (17)$$

The CMS collaboration \cite{2} also obtained analogous result

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) = (3.0^{+1.0}_{-0.9}) \times 10^{-9}, \quad (18)$$

where they have used the data samples corresponding to integrated luminosities of 5 and 20 fb$^{-1}$ at $\sqrt{s} = 7$ and 8 TeV. The weighted average of these two measurements yields

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) = (2.95 \pm 0.71) \times 10^{-9}, \quad (19)$$

which is consistent with the latest SM prediction \cite{19}, but certainly it does not rule out the possibility of new physics in this mode. While new physics can still affect this decay mode, but certainly its contribution is not the dominant one.

However, as discussed in Ref. \cite{17}, in the experiment the time integrated untagged decay rate is measured, whereas in the above theoretical calculation the effect of meson oscillation is not taken into account. Therefore, while comparing the SM prediction for $B_s \rightarrow \mu^+\mu^-$ decay rate with the experimental result one should take into account the sizable width difference $\Delta \Gamma_s$ between $B_s$ mass eigenstates. i.e.,

$$y_s \equiv \frac{\Gamma_L^{(s)} - \Gamma_H^{(s)}}{\Gamma_L^{(s)} + \Gamma_H^{(s)}} = \frac{\Delta \Gamma_s}{2 \Gamma_s} = 0.087 \pm 0.014, \quad (20)$$

where $\tau_{B_s}^{-1}$ denotes the average $B_s$ decay width. Hence, the experimental result is related to the theoretical prediction as

$$\text{BR}^{\text{th}}(B_s \rightarrow \mu^+\mu^-) = \left[ \frac{1 - y_s^2}{1 + A_{\Delta \Gamma} y_s} \right] \text{BR}(B_s \rightarrow \mu^+\mu^-)^{\text{exp}}, \quad (21)$$
where the observable $A_{\Delta \Gamma}$ equals +1 in the SM. Thus, using the experimental value of $y_s$ we obtain the branching ratio in the standard model

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{th}}|_{SM} = (3.60 \pm 0.21) \times 10^{-9}.$$  \hspace{1cm} (22)

We will now consider the effect of scalar leptoquarks in this mode. One can write the transition amplitude for this process from Eq. (14) as

$$A(B^0_s \rightarrow \mu^+\mu^-) = \langle \mu^+\mu^- | H_{\text{eff}} | B^0_s \rangle = -\frac{G_F}{\sqrt{2}} \frac{\alpha f_{B_s} m_{B_s} m_{\mu}}{\pi} C_{10}^{SM} P,$$  \hspace{1cm} (23)

where

$$P \equiv \frac{C_{10} - C'_{10}}{C_{10}^{SM}} = 1 + \frac{C_{10}^{NP} - C'_{10}^{NP}}{C_{10}^{SM}} = 1 + r e^{i\phi^{NP}},$$  \hspace{1cm} (24)

with

$$r e^{i\phi^{NP}} = (C_{10}^{NP} - C'_{10}^{NP})/C_{10}^{SM},$$  \hspace{1cm} (25)

denotes the new physics contribution and $\phi^{NP}$ is the relative phase between SM and the NP couplings. In general $P \equiv |P| e^{i\phi_P}$ carries the CP violating phase $\phi_P$. The phases $\phi_P$ and $\phi^{NP}$ are related to each other by the relation

$$\tan \phi_P = \frac{r \sin \phi^{NP}}{1 + r \phi^{NP}}.$$  \hspace{1cm} (26)

As discussed in section III, the exchange of the leptoquark $X(3,2,7/6)$ gives new contribution to $C_{10}$ and $X(3,2,1/6)$ gives additional contribution $C'_{10}$ the branching ratio in both the cases will be

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{th}} = \left[ \frac{1 + A_{\Delta \Gamma}}{1 - y_s^2} \right] \text{BR}^{SM} (1 + r^2 - 2r \cos \phi^{NP}).$$  \hspace{1cm} (27)

In the leptoquark model the observable $A_{\Delta \Gamma}$ becomes [17]

$$A_{\Delta \Gamma} = \cos 2\phi_P.$$  \hspace{1cm} (28)

In order to find the constrain on the combination of LQ couplings we require that each individual leptoquark contribution to the branching ratio does not exceed the experimental result. Now using the SM value from [16], we show in Fig. 1 the allowed region in $r - \phi^{NP}$ plane which is compatible with the 2$\sigma$ range of the experimental data. From the figure one can see that for $0 \leq r \leq 0.1$ the entire range for $\phi^{NP}$ is allowed, i.e.,

$$0 \leq r \leq 0.1, \quad \text{for} \quad 0 \leq \phi^{NP} \leq 2\pi.$$  \hspace{1cm} (29)
V. ANALYSIS OF $\bar{B}_d^0 \to X_s \mu^+ \mu^-$ MODE

Now we would like to constrain the NP couplings from the measured branching ratio of the inclusive decay $\bar{B}_d^0 \to X_s \mu^+ \mu^-$. The integrated branching ratio for this process has been measured by both Belle [3] and BaBar [4] collaborations and the average value of these measurements in the two regions are [5]

$$\begin{align*}
\text{BR}(\bar{B}_d^0 \to X_s \mu^+ \mu^-) &= (1.60 \pm 0.50) \times 10^{-6} \quad \text{low } q^2 \\
&= (0.44 \pm 0.12) \times 10^{-6} \quad \text{high } q^2,
\end{align*}$$

(30)

where the low-$q^2$ and high-$q^2$ regions correspond to $1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$ and $q^2 \geq 14.4 \text{ GeV}^2$, respectively. The decay mode has been very well studied in the literature and here we are presenting only the main results. The differential branching ratio for this process in the standard model is given as [24]

$$\begin{align*}
\frac{d\text{BR}}{ds_1}\bigg|_{\text{SM}} &= B_0 \frac{8}{3} (1 - s_1)^2 \sqrt{1 - \frac{4t^2}{s_1}} \times \left[ (2s_1 + 1) \left( \frac{2t^2}{s_1} + 1 \right) |C_{9\text{eff}}|^2 \\
&\quad + \left( \frac{2(1 - 4s_1)t^2}{s_1} + (2s_1 + 1) \right) |C_{10}|^2 + 4 \left( \frac{2}{s_1} + 1 \right) \left( \frac{2t^2}{s_1} + 1 \right) |C_7|^2 \\
&\quad + 12 \left( \frac{2t^2}{s_1} + 1 \right) \text{Re}(C_7 C_{9\text{eff}}^*) \right],
\end{align*}$$

(31)
where \( t = m_\mu/m_b^{\text{pole}} \) and \( s_1 = q^2/(m_b^{\text{pole}})^2 \). The normalization constant \( B_0 \) is related to \( \text{BR}(\bar{B} \to X_c e \bar{\nu}_e) \) through

\[
B_0 = \frac{3\alpha^2 \text{BR}(\bar{B} \to X_c e \bar{\nu}_e)}{32\pi^2 f(\hat{m}_c)\kappa(\hat{m}_c)} |V_{tb}V^*_{ts}|^2 ,
\]

where \( \hat{m}_c = m_c^{\text{pole}}/m_b^{\text{pole}} \). \( f(\hat{m}_c) \) is the lowest order phase space factor for the \( \bar{B} \to X_c e \bar{\nu} \) process, i.e.,

\[
f(\hat{m}_c) = 1 - 8\hat{m}_c^2 + 8\hat{m}_c^6 - 3\hat{m}_c^8 - 24\hat{m}_c^4 \ln \hat{m}_c ,
\]

and the function \( \kappa(\hat{m}_c) \) is the power correction to \( \text{BR}(\bar{B} \to X_c e \bar{\nu}) \), which includes both the \( O(\alpha_s) \) QCD corrections and the leading order \( (1/m_b^2) \) power corrections

\[
\kappa(\hat{m}_c) = 1 - \frac{2\alpha_s(m_b)}{3\pi} g(\hat{m}_c) + \frac{h(\hat{m}_c)}{2m_b^2} .
\]

Here the two functions are given as

\[
g(\hat{m}_c) = \left( \pi^2 - \frac{31}{4} \right) (1 - \hat{m}_c)^2 + \frac{3}{2} ,
\]

\[
h(\hat{m}_c) = \lambda_1 + \frac{\lambda_2}{f(\hat{m}_c)} \left[ -9 + 24\hat{m}_c^2 - 72\hat{m}_c^4 + 72\hat{m}_c^6 - 15\hat{m}_c^8 - 72\hat{m}_c^8 - 72\hat{m}_c^4 \ln \hat{m}_c \right] ,
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the kinetic energy and magnetic moment operators.

In the leptoquark model there will be additional contribution arising due to the exchange of leptoquarks which will introduce the new couplings \( C_9^{NP} \), \( C_{10}^{NP} \), \( C_9^{\prime NP} \) and \( C_{10}^{\prime NP} \) as discussed in section III. Including these NP contributions and neglecting the sub-leading terms which are suppressed by \( m_\mu/m_b \) and \( m_s/m_b \), the branching ratio can be given as

\[
\left( \frac{d\text{BR}}{ds_1} \right)_{\text{Total}} = \left( \frac{d\text{BR}}{ds_1} \right)_{\text{SM}} + B_0 \left[ \frac{16}{3} (1 - s_1)^2 (1 + 2s_1) [\text{Re}(C_9^{HF} C_9^{NP*} + \text{Re}(C_{10}^{NP} C_{10}^{NP*})] 
\right.
\]

\[
+ \frac{8}{3} (1 - s_1)^2 (1 + 2s_1) \left[ |C_9^{NP}|^2 + |C_{10}^{NP}|^2 + |C_9^{\prime NP}|^2 + |C_{10}^{\prime NP}|^2 \right] 
\]

\[
+ 32 (1 - s_1)^2 \text{Re}(C_7 C_{10}^{NP*}) \right] .
\]

For numerical evaluation we use the input parameters as \( \hat{m}_c = 0.29 \pm 0.02 \) GeV, \( \text{BR}(\bar{B} \to X_c e \bar{\nu}) = (10.1 \pm 0.4)\% \), \( |V_{tb}V^*_{ts}|/|V_{cb}| = 0.967 \pm 0.009 \) and the parameters \( \lambda_1 \) and \( \lambda_2 \) as \( \lambda_1 = -(0.1 \pm 0.05) \) GeV\(^2\) and \( \lambda_2 = 0.12 \) GeV\(^2\). With these parameters the branching ratio in the SM is found to be

\[
\text{BR}(\bar{B} \to X_s \mu^+ \mu^-) = (1.92 \pm 0.08) \times 10^{-6} \quad \text{low } q^2 \\
= (0.38 \pm 0.01) \times 10^{-6} \quad \text{high } q^2.
\]
These predicted branching ratios are in agreement with the corresponding experimental values within their 1-σ range. To constrain the new physics couplings coming from the exchange of scalar leptoquarks $X(3,2,7/6)$ and $X(3,2,1/6)$, we assume only one type of leptoquark will contribute at a time. As discussed in section III, in the presence of the leptoquark $X(3,2,7/6)$ only the NP couplings $C_{9}^{NP}$ and $C_{10}^{NP}$ will arise whereas for $X(3,2,1/6)$ the couplings $C_{9}^{NP}$ and $C_{10}^{NP}$ will contribute. Furthermore, as shown in Eqs. (9) and (13) the magnitudes of these couplings in each case will be same. With the additional assumption that these two couplings will have the same phase $\phi^{NP}$ and neglecting the small phase difference between $C_{9}^{eff}$ and $C_{10}^{NP}$ we obtain the constraint equations for these NP couplings from Eqs. (30), (36) and (37) as

$$C_{10}^{NP} \left[ (0.58 + 0.128 \; C_{10} + 0.596 \; C_{7}) \cos \phi^{NP} + 0.02 \sin \phi^{NP} \right] + 0.13 \; |C_{10}^{NP}|^2 = -0.32 \pm 0.51$$

(for low $q^2$)

$$C_{10}^{NP} \left[ (0.11 + 0.03 \; C_{10} + 0.07 \; C_{7}) \cos \phi^{NP} + 0.009 \sin \phi^{NP} \right] + 0.03 \; |C_{10}^{NP}|^2 = 0.06 \pm 0.12$$

(for high $q^2$) \hspace{1cm} (38)

The corresponding 1-σ allowed region in the $|C_{10}^{NP}| - \phi^{NP}$ plane is shown in the Figure-2 where the green region corresponds to the constraint coming from high-$q^2$ bound and the magenta region coming from the low-$q^2$ limit. From the figure one can see that the bounds coming from the high-$q^2$ measurement is rather weak. From the low-$q^2$ constraint one can infer that for the value $-1 \leq C_{10}^{NP} \leq 1$ the entire range of $\phi^{NP}$ is allowed. These bounds can be translated to the bounds on $r$ and $\phi^{NP}$ as done for $B_s \to \mu^+\mu^-$ process as

$$0 \leq r \leq 0.24 \; , \; \text{for} \; \; 0 \leq \phi^{NP} \leq 2\pi .$$ \hspace{1cm} (39)

Thus, from Eqns. (29) and (39) one can see that the bounds on NP couplings coming from $BR(\bar{B}_d^0 \to X_s\mu^+\mu^-)$ is slightly weak in comparison to $BR(B_s \to \mu^+\mu^-)$.

Next we will consider the contributions coming from the $X(3,2,1/6)$ exchange. In this case the new couplings $C_{9}^{NP}$ and $C_{10}^{NP}$ will come into picture. Proceeding in a similar fashion as done for $X(3,2,7/6)$ leptoquark case, we obtain the constraint equations for these parameters as

$$0.064 \left[ |C_{9}^{NP}|^2 + |C_{10}^{NP}|^2 \right] = (-0.32 \pm 0.51) \; \; \text{(low $q^2$)}$$

$$0.014 \left[ |C_{9}^{NP}|^2 + |C_{10}^{NP}|^2 \right] = (0.06 \pm 0.12) \; \; \text{(high $q^2$)} .$$ \hspace{1cm} (40)
FIG. 2: The allowed region in the $C_{10}^{NP} - \phi^{NP}$ parameters space obtained from the BR($\bar{B}_d \to X_s \mu^+\mu^-$), where the green (magenta) region corresponds to high-$q^2$ (low-$q^2$) limits.

The corresponding allowed region in $C_9'^{NP}$-$C_{10}^{NP}$ plane is shown in Figure-3, where the green region corresponds to the bounds coming from high-$q^2$ limit and magenta region corresponds to the low-$q^2$ bound. Thus, from the low-$q^2$ bounds one can obtain the limits on $C_9'^{NP}$ and $C_{10}^{NP}$ as $-1.5 \leq |C_9'^{NP}|, |C_{10}^{NP}| \leq 1.5$. Again translating the above bounds into the bound on $r$ one can obtain

$$0 \leq r \leq 0.36,$$  \hspace{1cm} (41)

which is again much weaker than the bounds coming from $B_s \to \mu^+\mu^-$ measurements. However, in our analysis we will use relatively mild constraint, consistent with both BR($B_s \to \mu^+\mu^-$) and BR($\bar{B}_d^0 \to X_s \mu^+\mu^-$) measurements as

$$0 \leq r \leq 0.35, \quad \text{with} \quad 60^\circ \leq \phi^{NP} \leq 270^\circ.$$  \hspace{1cm} (42)

This limit on $r$ can be translated to give us bound on leptoquark coupling using Eqs. (9), (13) and (25) as

$$\left| \frac{\lambda_2^2 \lambda_{2*}}{M_Y^2} \right| = \left| \frac{\lambda_5^2 \lambda_{3*}}{M_Y^2} \right| \leq 4.8 \times 10^{-9} \text{ GeV}^{-2}.$$  \hspace{1cm} (43)

If we use the values of the couplings as $|\lambda_{d,e}| \approx 0.1$, allowing the perturbation theory to be valid, we get the lower bound on the scalar leptoquark mass as

$$M_X \geq 1.4 \text{ TeV}.$$  \hspace{1cm} (44)
FIG. 3: The allowed region in the $C_9^{NP} - C_{10}^{NP}$ parameters space obtained from the BR($\bar{B}_d \to X_s \mu^+ \mu^-$), where the green (magenta) region corresponds to high-$q^2$ (low-$q^2$) limits.

It should be noted that the recent measurement by LHCb collaboration [16] shows several significant deviations on angular observables in the rare decay $B \to K^{*0} \mu^+ \mu^-$ from their corresponding SM expectations. In particular an anomalously low value of $S_4$ at high $q^2$ at 2.8$\sigma$ level and an opposite sign of $S_5$ at low $q^2$ region at 2.4$\sigma$ level. Although it is conceivable that these anomalies are due to statistical fluctuations or under estimated theory uncertainties [29], but the possible indication of new physics could not be ruled out. It has been shown in Ref. [30] that a consistent explanation of most of the anomalies associated with $b \to s$ rare decays can be obtained by NP contributing simultaneously to the semileptonic operator $O_9$ and its chirally flipped counterpart $O_9'$ with $C_9^{NP} \simeq -(1.0 \pm 0.3)$ and $C_9'^{NP} \simeq 1.0 \pm 0.5$. However, in the leptoquark model since $C_9^{NP}$ and $C_{10}^{NP}$ contribute simultaneously it may not be possible to explain these anomalies.

After obtaining the allowed range for the leptoquark coupling we will now proceed to study the semileptonic decay process $B_s \to \phi \mu^+ \mu^-$. 

VI. $B_s \to \phi \ l^+l^-$ PROCESS

Here we will consider the decay mode $B_s \to \phi \mu^+ \mu^-$. At the quark level, this decay mode proceeds through the FCNC transition $b \to s l^+ l^-$, which occurs only through loops in the
SM, and therefore, it constitutes a quite suitable tool of looking for new physics. Moreover, the dileptons present in this process allow us to formulate many observables which can serve as a testing ground to decipher the presence of new physics [28].

Recently the branching ratio of this decay mode has been measured by the LHCb collaboration [31] using the data corresponding to an integrated luminosity of 1.0 fb\(^{-1}\) collected at \(\sqrt{s}=7\) TeV as

\[
\text{BR}(B^0_s \rightarrow \phi \mu^+ \mu^-) = (7.07^{+0.64}_{-0.59} \pm 0.17 \pm 0.71) \times 10^{-7}.
\]

They have also performed the angular analysis and determine the angular observables  \(F_L\), \(S_3\), \(A_6\) and \(A_9\) which are consistent with the standard model expectations. This process has been very well studied in the literature, both in the SM and in various extensions of it [32].

The branching ratio predicted in the standard model is in the range \((14.5 - 19.2) \times 10^{-7}\) which is significantly higher than the present experimental value [45]. This deviation may be considered as a smoking gun signal of new physics in this mode or more generally in the processes involving \(b \rightarrow s\) transitions.

Using the effective Hamiltonian presented in Eq. (1) one can obtain the transition amplitude for this process. The matrix elements of the various hadronic currents between the initial \(B_s\) meson and the final vector meson \(\phi\) can be parameterized in terms of various form factors as [33]

\[
\langle \phi(k, \varepsilon)|(V - A)_{\mu}|B_s(P)\rangle = \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} P^\alpha k^\beta \frac{2V(q^2)}{m_B + m_\phi} - i \varepsilon^{* \nu}(m_B + m_\phi) A_1(q^2)
\]

\[+ i(P + k)_{\mu}(\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_\phi} + i q_{\mu}(\varepsilon^* q) \frac{2m_\phi}{q^2} \left[A_3(q^2) - A_0(q^2)\right],\]

\[
\langle \phi(k, \varepsilon)|\bar{s} \sigma_{\mu \nu} q' (1 + \gamma_5) b|B_s(P)\rangle = i \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} P^\alpha k^\beta 2T_1(q^2) + \left[\varepsilon^{* \mu}(m_B^2 - m_\phi^2) - (\varepsilon^* q)(P + k)_{\mu}\right] T_2(q^2) + \left[\varepsilon^{* \mu}(m_B^2 - m_\phi^2) \right] T_3(q^2),
\]

where \(V\) and \(A\) denote the vector and axial vector currents, \(A_0, A_1, A_2, A_3, V, T_1, T_2\) and \(T_3\) are the relevant form factors and \(q\) is the momentum transfer.
Thus, with eqs. (1) and (46) the transition amplitude for \( B_s \to \phi l^+l^- \) is given as

\[
\mathcal{M}(B_s \to \phi l^+l^-) = \frac{G_F \alpha}{2\sqrt{2\pi}} V_{tb}V_{ts}^\ast \left\{ i\gamma^\mu l \left[ -2A\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}k^\alpha q^\beta - iB\varepsilon_\mu^* \right. \right.
\]
\[ + \left. \left. iC(P + k)_{\mu}(\varepsilon^* \cdot q) + iD(\varepsilon^* \cdot q)q_\mu \right\} + i\gamma^\mu \gamma_5 l \left[ -2E\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}k^\alpha q^\beta \right. \right.
\]
\[ - \left. \left. iF\varepsilon_\mu^* + iG(\varepsilon^* \cdot q)(P + k)_\mu + iH(\varepsilon^* \cdot q)q_\mu \right\} \right\}, \tag{47}
\]

where the parameters \( A, B, \cdots H \) are given as \[34\]

\[
A = 2 \left( C^\text{eff SM} + C^\text{NP} + C^\text{NP} \right) \frac{V(q^2)}{m_B + m_\phi} + 4\frac{m_b}{q^2} C_7 T_1(q^2),
\]
\[
B = (m_B + m_\phi) \left( 2(C^\text{eff SM} + C^\text{NP} - C^\text{NP}^\ast) A_1(q^2) + 4\frac{m_b}{q^2}(m_B - m_\phi) C_7 T_2(q^2) \right),
\]
\[
C = 2(C^\text{eff SM} + C^\text{NP} - C^\text{NP}^\ast) \frac{A_2(q^2)}{m_B + m_\phi} + 4\frac{m_b}{q^2} C_7 \left( T_2(q^2) + \frac{q^2}{m_B^2 - m_\phi^2} T_3(q^2) \right),
\]
\[
D = 4(C^\text{eff SM} + C^\text{NP} - C^\text{NP}^\ast) \frac{m_\phi}{q^2} \left( A_3(q^2) - A_0(q^2) \right) - 4C_7 \frac{m_b}{q^2} T_3(q^2),
\]
\[
E = (C^\text{SM} + C^\text{NP} + C^\text{NP}^\ast) \frac{2V(q^2)}{m_B + m_\phi},
\]
\[
F = 2(C^\text{SM} + C^\text{NP} - C^\text{NP}^\ast)(m_B + m_\phi) A_1(q^2), \tag{48}
\]
\[
G = (C^\text{SM} + C^\text{NP} - C^\text{NP}^\ast) \frac{2A_2(q^2)}{m_B + m_\phi},
\]
\[
H = 4(C^\text{SM} + C^\text{NP} - C^\text{NP}^\ast) \frac{m_\phi}{q^2} \left( A_3(q^2) - A_0(q^2) \right).
\]

The differential decay rate is given as

\[
\frac{d\Gamma}{d\hat{s}} = \frac{G_F^2 \alpha^2}{214\pi^5} |V_{tb}V_{ts}^\ast|^2 m_B T_B \lambda^{1/2}(1, r_\phi, \hat{s}) \nu_1 \Delta,
\tag{49}
\]

where \( \hat{s} = q^2/m_B^2, r_\phi = m_\phi^2/m_B^2, \nu_1 = \sqrt{1 - 4m_f^2/s}, \lambda \equiv \lambda(1, r_\phi, \hat{s}), \) is the triangle function and

\[
\Delta = \frac{1}{3r_\phi} \left[ 8\lambda m_\phi^4 \hat{s}\left((3 - v_1^2)|A|^2 + (12r_\phi \hat{s} + \lambda)(3 - v_1^2)|B|^2 \right) \right.
\]
\[ + \lambda^2 m_B^4 (3 - v_1^2)|C|^2 + 16v_1^2 m_B^4 r_\phi \hat{s} \lambda |E|^2 + (24r_\phi \hat{s} v_1^2 + \lambda(3 - v_2^2)) |F|^2 \]
\[ + m_B^4 \lambda (6\hat{s}(1 + r_\phi)(1 - v_2^2) - 3\hat{s}^2(1 - v_1^2) + \lambda(3 - v_2^2)) |G|^2 \]
\[ + 3\lambda m_B^4 \hat{s}\left[(1 - v_1^2)|H|^2 + 2\text{Re}[FG^*] m_B^2 \lambda (r_\phi (3 - v_2) + v_1^2(1 + 2\hat{s}) - 3) \right. \]
\[ - 6\text{Re}[FH^*] m_B^2 \hat{s}(1 - v_1^2) \lambda + 6\text{Re}[GH^*] m_B^2 \hat{s} \lambda (1 - r_\phi)(1 - v_2) \]
\[ + 2\text{Re}[BC^*] m_B^2 \lambda(3 - v_2^2)(r_\phi + \hat{s} - 1) \left\] \right]. \tag{50}
Another observable is the lepton forward backward asymmetry \(A_{FB}\), which is also a very powerful tool for looking into new physics signature. In particular the position of the zero value of \(A_{FB}\) is very sensitive to the presence of new physics. The normalized forward-backward asymmetry is defined as

\[
A_{FB}(s) = \frac{\int_0^1 \frac{d^2\Gamma}{d\hat{s}d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}d\cos\theta} d\cos\theta}{\int_0^1 \frac{d^2\Gamma}{d\hat{s}d\cos\theta} d\cos\theta + \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}d\cos\theta} d\cos\theta},
\]

(51)

where \(\theta\) is the angle between the directions of \(l^+\) and \(B_s\) in the rest frame of the lepton pair. The forward-backward asymmetry can also be written in the form \[3 4\]

\[
A_{FB}(q^2) = -\frac{1}{\Delta} 8m_B^2 \sqrt{\lambda} v_1 \bar{s} \text{ Re}[A^* F + B^* E]
\]

(52)

As seen from \[31\], the actual decay being observed is not \(B_s \rightarrow \phi \mu^+ \mu^-\) but \(B_s \rightarrow \phi(\rightarrow K^+ K^-) \mu^+ \mu^-\). Thus, the angular analysis of the four body final state offers a large number of observables in the differential decay distribution \[35\]. The angular distribution of the decay process \(\bar{B}_s^0 \rightarrow \phi(\rightarrow K^+ K^-) \mu^+ \mu^-\) can be defined by the decay angles \(\theta_K, \theta_l\) and \(\Phi\), where \(\theta_K(\theta_l)\) denotes the angle of \(K^-(\mu^-)\) with respect to the direction of flight of the \(\bar{B}_s\) meson in the \(K^+ K^- (\mu^+ \mu^-)\) center-of-mass frame and \(\Phi\) denotes relative angle of the \(\mu^+ \mu^-\) and the \(K^+ K^-\) decay planes in the \(\bar{B}_s\) meson center of mass frame and is given as \[14\] as

\[
\frac{d^4\Gamma}{dq^2 \, d\cos\theta_l \, d\cos\theta_K \, d\Phi} = \frac{9}{32\pi} I_1^s \sin^2\theta_K + I_1^c \sin^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \sin^2\theta_K) \cos 2\theta_l \\
+ I_3 \sin^2\theta_K \sin^2\theta_l \cos 2\Phi + I_4 \sin 2\theta_K \sin 2\theta_l \cos \Phi \\
+ I_5 \sin 2\theta_K \sin \theta_l \cos \Phi + (I_6^s \sin^2\theta_K + I_6^c \sin^2\theta_K) \cos \theta_l \\
+ I_7 \sin 2\theta_K \sin \theta_l \sin \Phi + I_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \\
+ I_9 \sin^2\theta_K \sin^2\theta_l \sin 2\Phi.
\]

(53)

The corresponding expression for CP conjugate process \(B_s^0 \rightarrow \phi(\rightarrow K^+ K^-) \mu^+ \mu^-\) \((d^4\Gamma)\) can be obtained from \[53\] by the replacement of \(I_i^s\) by \(\bar{I}_i^s\) where these observables are related to each other through

\[
I_{1,2,3,4,7}^{(a)} \rightarrow \bar{I}_{1,2,3,4,7}^{(a)}, \quad I_{5,6,8,9}^{(a)} \rightarrow -\bar{I}_{5,6,8,9}^{(a)},
\]

(54)

with all weak phases conjugated. The angular coefficients \(I_i^{(a)}\), usually expressed in terms
of the transversity amplitudes which are given as [14]

\[ A_{\perp,L,R} = N \sqrt{2\lambda_1} \left[ \left( C_{9e}^{eff} + C_9^{NP} + C_9^{(NP)} \right) \mp (C_{10} + C_{10}^{NP} + C_{10}^{(NP)}) \frac{V(q^2)}{m_B + m_\phi} + 2m_bsC_7T_1(q^2) \right] , \]

\[ A_{\parallel,L,R} = -N \sqrt{2(m_B^2 - m_\phi^2)} \left[ \left( C_{9e}^{eff} + C_9^{NP} - C_9^{(NP)} \right) \mp (C_{10} + C_{10}^{NP} - C_{10}^{(NP)}) \frac{A_1(q^2)}{m_B - m_\phi} \right. \]

\[ + \left. \frac{2m_b}{s}C_7T_2(q^2) \right] , \]

\[ A_{0,L,R} = -N \frac{2m_\phi}{\sqrt{s}} \left[ \left( C_{9e}^{eff} + C_9^{NP} - C_9^{(NP)} \right) \pm (C_{10} + C_{10}^{NP} - C_{10}^{(NP)}) \right. \]

\[ \times \left. \left( m_B^2 - m_\phi^2 - s/m_B + m_\phi \right) A_1(q^2) - \frac{\lambda_1}{m_B + m_\phi} A_2(q^2) \right] \]

\[ + 2m_BC_7 \left( m_B^2 + 3m_\phi^2 - s \right) T_2(q^2) - \frac{\lambda_1}{m_B^2 - m_\phi^2} \right] , \]

\[ A_t = N \frac{\lambda_1}{s} \left[ 2(C_{10} + C_{10}^{NP} - C_{10}^{(NP)}) \right] A_0(q^2) \]

where

\[ N = V_{tb}V_{ts}^* \left[ \frac{G_F^2\alpha^2}{3 \cdot 210^2 \pi^5 m_B^3} \sqrt{\lambda_1} \right]^{1/2} , \]

with \( \lambda_1 = (m_B^2 + m_\phi^2 + s^2) - 4m_B^2m_\phi^2 \). With these transversity amplitudes the angular coefficients are given as

\[ I_1^s = \frac{2 + v^2}{4} \left[ |A_{\perp}^L|^2 + |A_{\parallel}^L|^2 + (L \rightarrow R) \right] + \frac{4m_B^2}{s} Re \left( A_{\perp}^L A_{\perp}^{R*} + A_{\parallel}^L A_{\parallel}^{R*} \right) \]

\[ I_1^c = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_B^2}{s} \left( |A_0|^2 + 2 Re(A_0^L A_0^{R*}) \right) \]

\[ I_2^s = \frac{v^2}{4} \left( |A_{\perp}^L|^2 + |A_{\parallel}^L|^2 + (L \rightarrow R) \right) , \]

\[ I_2^c = -v^2 \left( |A_0^L|^2 + (L \rightarrow R) \right) , \]

\[ I_3 = \frac{v^2}{2} \left( Re(A_0^L A_{\perp}^{R*} + (L \rightarrow R) \right) , \]

\[ I_4 = \frac{v^2}{2} \left( Re(A_0^L A_{\parallel}^{R*} - (L \rightarrow R) \right) , \]

\[ I_5 = \sqrt{2}v \left( Re(A_0^L A_{\perp}^{R*} - (L \rightarrow R) \right) , \]

\[ I_6^s = 2v \left( Re(A_0^L A_{\perp}^{R*} - (L \rightarrow R) \right) , \]

\[ I_7 = \sqrt{2}v \left( Im(A_0^L A_{\perp}^{R*} - (L \rightarrow R) \right) , \]

\[ I_8 = \frac{v^2}{\sqrt{2}} \left( Im(A_0^L A_{\parallel}^{R*} + (L \rightarrow R) \right) , \]

\[ I_9 = v^2 \left( Im(A_{\perp}^{R*} A_{\perp}^L) + (L \rightarrow R) \right) . \]
From these angular coefficients one can construct twelve CP averaged angular coefficients \( S_i^{(a)} \) and twelve CP asymmetries \( A_i^{(a)} \) as

\[
S_i^{(a)} = \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \left( I_i^{(a)} + \bar{I}_i^{(a)} \right), \quad A_i^{(a)} = \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \left( I_i^{(a)} - \bar{I}_i^{(a)} \right). \tag{59}
\]

All the physical observables can be expressed in terms of \( S_i \) and \( A_i \). For example the CP asymmetry in the dilepton mass distribution can be expressed as

\[
A_{CP} = \frac{d(\Gamma - \bar{\Gamma})}{dq^2} = \frac{3}{4}(2A_1^s + A_1^c) - \frac{1}{4}(2A_2^s + A_2^c). \tag{60}
\]

The \( q^2 \) average of these observables are defined as follows:

\[
\langle S_i^{(a)} \rangle = \frac{\int_{q^2}^{6 \text{ GeV}^2} dq^2 (I_i^{(a)} + \bar{I}_i^{(a)})}{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 d(\Gamma + \bar{\Gamma})}, \quad \langle A_i^{(a)} \rangle = \frac{\int_{q^2}^{6 \text{ GeV}^2} dq^2 (I_i^{(a)} - \bar{I}_i^{(a)})}{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 d(\Gamma + \bar{\Gamma})}. \tag{61}
\]

After getting familiar with the different observables associated with \( B_s \rightarrow \phi \mu^+ \mu^- \) decay process we now proceed for numerical estimation. For this purpose we use the form factors calculated in the light-cone sum rule (LCSR) approach \[33\], where the \( q^2 \) dependence of various form factors are given by simple fits as

\[
f(q^2) = \frac{r_2}{1 - q^2/m_{fit}^2}, \quad \text{for } A_1, T_2
\]

\[
f(q^2) = \frac{r_1}{1 - q^2/m_R^2} + \frac{r_2}{1 - q^2/m_{fit}^2}, \quad \text{for } V, A_0, T_1
\]

\[
f(q^2) = \frac{r_1}{1 - q^2/m_{fit}^2} + \frac{r_2}{(1 - q^2/m_{fit}^2)^2}, \quad \text{for } A_2, \tilde{T}_3. \tag{62}
\]

The values of the parameters \( r_1, r_2, m_R \) and \( m_{fit} \) are taken from \[33\]. The form factors \( A_3 \) and \( T_3 \) are given as

\[
A_3(q^2) = \frac{m_B + m_V}{2m_\phi} A_1(q^2) - \frac{m_B - m_\phi}{2m_\phi} A_2(q^2),
\]

\[
T_3(q^2) = \frac{m_B^2 - m_\phi^2}{q^2} \left( \tilde{T}_3(q^2) - T_2(q^2) \right). \tag{63}
\]

The particle masses and the lifetime of \( B_s \) meson are taken from \[23\]. The quark masses (in GeV) used are \( m_b=4.8, m_c=1.5 \), the fine structure coupling constant \( \alpha = 1/128 \) and the CKM matrix elements as \( V_{ub}V_{ts}^* = 0.0405 \). Using these values we show in Figure-4 the
FIG. 4: Variation of the differential branching ratio (in units of $10^{-7}$) (left panel) and the forward-backward asymmetry with respect to the momentum transfer $s$ (right panel) for the $B_s \rightarrow \phi \mu^+ \mu^-$ process.

variation of differential decay rate (left panel) and the forward backward asymmetry (right panel) in the standard model with respect to the di-muon invariant mass.

In the leptoquark model, this process will receive additional contribution arising from the leptoquark exchange. Hence, in the leptoquark model the Wilson coefficients $C_{9,10}$ will receive additional contributions $C'^{NP}_{9,10}$ as well as new Wilson $C'_{9,10}$ associated with the chirally flipped operators $O'_{9,10}$ will also be present as already discussed in Section III. The bounds on these new coefficients can be obtained from the constraint on $r$ extracted from the experimental results on $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ and $\text{BR}(\bar{B}^{0}_{d} \rightarrow X_{s} \mu^+ \mu^-)$. For the leptoquarks $X = (3, 2, 7/6)$ and $X = (3, 2, 1/6)$, we obtain the value of $r \leq 0.35$ for $\phi$ in the range $(60 - 270)\degree$. This constraint can be translated with eqns (9), (13) and (42) which gives the value of the new Wilson coefficients as

$$|C^{LQ}_{9}| = |C^{LQ}_{10}| \leq |r \ C'^{SM}_{10}| \quad \text{(for } X = (3, 2, 7/6))$$

$$|C'^{LQ}_{9}| = |C'^{LQ}_{10}| \leq |r \ C'^{SM}_{10}| \quad \text{(for } X = (3, 2, 1/6)) \ .$$

(64)

Using these values we show the variation of differential decay rate and forward-backward asymmetry for $X = (3, 2, 7/6)$ in Figure-5 and for $X = (3, 2, 1/6)$ in Figure-6. From these figures it can be seen that the branching ratio could have significant deviation from its SM value both in the upward as well as downward direction. However, the zero position of the forward-backward asymmetry does not have any significant deviation.

We now proceed to calculate the total decay rate for $B_s \rightarrow \phi \mu^+ \mu^-$. It should be noted that the long distance contributions arise from the real $\bar{c}c$ resonances with the dominant
FIG. 5: Same as Figure-4, where the red curves represent the SM values and the grey regions represent the results due to $X = (3, 2, 7/6)$ leptoquark contributions.

FIG. 6: Same as Figure-4, where the red curves represent the SM values and the grey regions represent the results due to $X = (3, 2, 1/6)$ leptoquark contributions.

ccontributions coming from the low lying resonances $J/\psi$ and $\psi'(2S)$. In order to minimize the hadronic uncertainties it is necessary to eliminate the backgrounds coming from the resonance regions. The resonant decays $B_s \to J/\psi \phi$ and $B_s^0 \to \psi'(2S)\phi$ with $\psi/\psi'(2S) \to \mu^+\mu^-$ are rejected by applying the vetos on the dimuon mass regions around the charmonium resonances, i.e., $(2946 < m(\mu^+\mu^-) < 3176) \text{ MeV}/c^2$ and $(3592 < m(\mu^+\mu^-) < 3766) \text{ MeV}/c^2$ [31]. Using these veto windows we obtain the branching ratio for the $B_s \to \phi \mu^+\mu^-$ decay
TABLE I: Allowed range of the CP violating observables (in units of $10^{-3}$), in the leptoquark model.

| Observables | Allowed range (in units of $10^{-3}$) | Observables | Allowed range (in units of $10^{-3}$) |
|-------------|--------------------------------------|-------------|--------------------------------------|
| $\langle A_s^1 \rangle$ | $(0.18 \rightarrow 0.27)$ | $\langle A_5 \rangle$ | $-(60 \rightarrow 110)$ |
| $\langle A_1^c \rangle$ | $(8 \rightarrow 12)$ | $\langle A_6^c \rangle$ | $(7.6 \rightarrow 8.0)$ |
| $\langle A_2^s \rangle$ | $(0.06 \rightarrow 0.09)$ | $\langle A_8 \rangle$ | $(3.8 \rightarrow 4.0)$ |
| $\langle A_2^c \rangle$ | $-(7.9 \rightarrow 41.8)$ | $\langle A_7 \rangle$ | $-(46 \rightarrow 67)$ |
| $\langle A_3 \rangle$ | $-(1.3 \rightarrow 1.9)$ | $\langle A_9 \rangle$ | $-(0.12 \rightarrow 0.84)$ |
| $\langle A_4 \rangle$ | $(2.3 \rightarrow 3.4)$ | $\langle A_{CP} \rangle$ | $(8.4 \rightarrow 12.0)$ |

mode as

$$BR(B_s \rightarrow \phi \mu^+ \mu^-) = 13.2 \times 10^{-7} \quad \text{(in SM)},$$

$$= (5.8 - 24.4) \times 10^{-7} \quad \text{(in LQ Model – I (X = 3, 2, 7/6))},$$

$$= (8.1 - 22.0) \times 10^{-7} \quad \text{(in LQ Model – II (X = 3, 2, 1/6))}. \quad (65)$$

Thus, one can see that the observed branching ratio (45) can be accommodated in the scalar leptoquark model.

Our next objective is to study the effect of leptoquark in the CP asymmetry parameters $A_i^{(a)}$. The $q^2$ variation of these observables in the low $q^2$ regime is shown in Figure-7. Here we have varied new weak phase between (60-90) degree and fixed the $r$ value at 0.35. The time integrated value in the low $q^2$ region is shown in Table-1. Some of these asymmetries are measured by the LHCb collaboration, which are almost in agreement with the standard model predictions but with large error bars. Future measurement with large data samples could possibly minimize these errors and help to infer the presence of new physics, if there is any from these observables.

VII. CONCLUSION

In this paper we have studied the effect of the scalar leptoquarks in the rare decays of $B_s$ meson. The large production of $B_s$ mesons at the LHC experiment opens up the possibility
to study the rare decays of $B_s$ meson with high statistical precision. We have considered the simple renormalizable leptoquark models which do not allow proton decay at the tree level. Using the recent results on $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ and the value of $\text{BR}(\bar{B}_d^0 \rightarrow X_s\mu^+\mu^-)$, the leptoquark parameter space has been constrained. Using such parameters we obtained the bounds on the product of leptoquark couplings. We then estimated the branching ratio and the forward backward asymmetry for the rare decay process $B_s \rightarrow \phi\mu^+\mu^-$. The SM prediction for $\text{BR}(B_s \rightarrow \phi\mu^+\mu^-)$ is found to be higher than the corresponding experimental observed value. We found that the branching ratio has been deviated significantly from the corresponding SM value and the observed branching ratio can be accommodated in this
model. However, the zero-position of the forward-backward rate asymmetry does not have significant deviation in the leptoquark model but there is a slight shifting towards right. We have also shown the variation of different CP asymmetry parameters $A_i^{(a)}$ in the low-$q^2$ region. The time-integrated values of some of the asymmetry parameters are found to be significantly large, the observation of which in the LHCb experiment would provide the possible existence of leptoquarks.

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