Hemispherical Power Asymmetry from a Space-Dependent Component of the Adiabatic Power Spectrum

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The hemispherical power asymmetry observed by Planck and WMAP can be interpreted as being due to a spatially-varying and scale-dependent component of the adiabatic power spectrum. We derive general constraints on the magnitude and scale-dependence of a component with a dipole spatial variation. The spectral index and the running of the spectral index can be shifted from their inflation model values, resulting in a smaller spectral index and a more positive running. A key prediction is a significant hemispherical asymmetry of the spectral index and of its running.

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I. INTRODUCTION

The Planck satellite has observed a hemispherical asymmetry in the CMB temperature fluctuations at low multipoles [1], confirming the earlier observation by WMAP [2,3]. The asymmetry can be modelled by a temperature fluctuation dipole of the form [4]

\[
\frac{\delta T}{T}(\hat{n}) = \left( \frac{\delta T}{T} \right)_o (\hat{n}) [1 + A \hat{n} \cdot \hat{p}], \tag{1}
\]

where \(\left( \frac{\delta T}{T} \right)_o(\hat{n})\) is a statistically isotropic temperature fluctuation, \(A\) is the magnitude of the asymmetry and \(\hat{p}\) is its direction. Recent Planck results give \(A = 0.073 \pm 0.010\) in the direction \((217.5 \pm 15.4, -20.2 \pm 15.1)\) for multipoles \(l \in (2, 64)\) [1]. This asymmetry is unlikely to arise as a result of random fluctuations in a statistically isotropic model, with less than one out of a thousand isotropic simulations fitting the asymmetry observed by Planck [5]. Analyses and proposed explanations of the hemispherical power asymmetry are discussed in [6,13].

An important constraint on such models is the absence of an asymmetry at smaller angular scales. In particular, the asymmetry on scales corresponding to quasar number counts must satisfy \(A < 0.012\) at 95% c.l. [13].

A natural interpretation of these observations, which we discuss in this letter, is the existence of an additional space-dependent adiabatic component of the curvature power spectrum. This must be strongly scale-dependent in order to suppress the asymmetry on small angular scales. We will consider in the following the case of an additional adiabatic component with a dipole spatial variation.

II. HEMISPHERICAL ASYMMETRY FROM A DIPOLE COMPONENT OF THE ADIABATIC POWER SPECTRUM

We will consider a component of the adiabatic power spectrum whose magnitude is a function of angle \(\theta\) on the surface of last scattering,

\[
P_\text{asy} = P_\text{asy} + \Delta P_\text{asy} \cos \theta, \tag{2}
\]

where \(P_\text{asy}\) is the conventional inflaton power spectrum and \(\Delta P_\text{asy}\) is the additional scale-dependent adiabatic component responsible for the hemispherical asymmetry. \(P_\text{asy}\) consists of a mean value \(P_\text{asy}\) and a spatial variation about this mean of magnitude \(\Delta P_\text{asy}\),

\[
P_\text{asy} = \hat{P}_\text{asy} + \Delta P_\text{asy} \cos \theta, \tag{3}
\]

where \(\cos \theta = \hat{n} \cdot \hat{p}\). This corresponds to an adiabatic power spectrum component with a dipole term in the direction \(\hat{p}\).

To relate the asymmetry \(A\) to the curvature power spectrum, we will compute the mean squared temperature fluctuation as a function of \(\theta\). This is determined by the curvature power on the last-scattering surface at \(\theta\), which can be related to the corresponding multipoles via \(C_l(\theta) = P_{l}(k, \theta) \hat{C}_l\), where \(\hat{C}_l\) is the adiabatic perturbation multipole for a scale-invariant spectrum with \(P_{l} = 1\) [14] and \(C_l(\theta)\) are the modulated multipoles as a function of \(\theta\). Each multipole \(C_l\) receives contributions from a range of \(k\) around \(k = l/x_{ls}\), where \(x_{ls} = 14100\) Mpc is the comoving distance to the last-scattering surface. The range of \(k\) is sufficiently narrow that the effect of the scale-dependence of the power spectrum can be accurately estimated by setting \(k\) to \(l/x_{ls}\) in \(P_l(k, \theta)\). We define \(\bar{C}_l\) to correspond to \(\theta = \pi/2\) and \(\Delta C_l(\theta)\) to be the change as a function of \(\theta\). Then, for multipoles in the range \(l_{\text{min}}\) to \(l_{\text{max}}\), we obtain

\[
\frac{(\delta T)^2}{T}\bigg|_\theta = \sum_{l_{\text{min}}}^{l_{\text{max}}} (2l + 1) \Delta C_l(\theta) = \sum_{l_{\text{min}}}^{l_{\text{max}}} (2l + 1) \bar{C}_l. \tag{4}
\]

In practice, a binned power spectrum, which we will denote by \(\hat{C}_l\), is extracted from the temperature data, where \(l(l + 1) \hat{C}_l\) is a constant for each bin [16,17]. We therefore need to estimate \(\bar{C}_l\) from the true \(C_l\) for a given perturbation. To do this we match the mean squared temperature fluctuation calculated with \(C_l\) to that calculated with \(\hat{C}_l\). In this case \(\sum (2l + 1) \hat{C}_l = \sum (2l + 1) C_l\) for each bin. \(\bar{C}_l\) for the bin \(l = l_{\text{min}}\)
The observed asymmetry $A$ in a given bin is derived from the asymmetry in the corresponding $C_l$. We will therefore replace $C_l$ by $\tilde{C}_l$ in Eq. (4). To obtain $A$ we compare Eq. (4) with the value expected from the temperature fluctuation dipole Eq. (1).

$$\Delta \left( \frac{\tilde{P}_l}{T} \right)^2 = 2(\tilde{n} \cdot \hat{n}) A_l,$$

where we assume that $A \ll 1$. We define $\tilde{P}_l = P_{\text{inf}} + \tilde{P}_{\text{asy}}$ to be the adiabatic power at $\theta = \pi/2$. Then $\Delta C_l(\theta)/C_l = (\tilde{n} \cdot \hat{n}) \Delta P_{\text{asy}}(k)/P_{\tilde{\zeta}}(k)$ for $k$ corresponding to $l$. By comparing Eq. (4) with $A_l \to \tilde{C}_l$ and Eq. (6), we obtain

$$A = \sum_{l=1}^{l_{\text{max}}} \frac{2(2l+1)}{l(l+1)} \sum_{q'=\text{min}}^{l_{\text{max}}} \frac{\Delta P_{\text{asy}}(k')}{P_{\tilde{\zeta}}(k')} \tilde{C}_{l'},$$

where $k' = l'/x_{ls}$. In the following we will assume that $\xi \ll 1$, where $\xi = P_{\text{asy}}/P_{\text{inf}}$, and work to leading order in $\xi$. Then

$$\frac{\Delta P_{\text{asy}}}{P_{\tilde{\zeta}}} = \frac{\xi}{1 + \xi} \frac{\Delta P_{\text{asy}}}{P_{\text{asy}}} \approx \xi \frac{\Delta P_{\text{asy}}}{P_{\text{asy}}} .$$

In general, the scale-dependence of $\tilde{P}_{\text{asy}}$ may be different from the scale-dependence of the spatial change of the power $\Delta P_{\text{asy}}$. We will therefore introduce different spectral indices to parameterize these,

$$\tilde{P}_{\text{asy}} = \tilde{P}_{\text{asy}} 0 \left( \frac{k}{k_0} \right)^{n_\sigma - 1} ; \quad \frac{\Delta P_{\text{asy}}}{P_{\text{asy}}} = \left( \frac{\Delta P_{\text{asy}}}{P_{\text{asy}}} \right)_0 \left( \frac{k}{k_0} \right)^{n_{A} - 1} ,$$

where subscript 0 denotes values at the pivot scale $k_0$. If the space-dependence of the curvature power $\Delta P_{\text{asy}}$ has the same scale-dependence as $\tilde{P}_{\text{asy}}$ then $n_{A} = 1$. \footnote{This is true, for example, for the modulated reheating model of [12].} In the following we will use the Planck pivot scale, $k_0 = 0.05 \text{ Mpc}^{-1}$. In this case the corresponding multipole number is $l_0 = 700$. Setting $(k/k_0) = (l/l_0)$ in Eq. (9) then gives a good estimate of the scale-dependence. We will assume that the scale-dependence of the inflaton perturbation is negligible compared to that of $\tilde{P}_{\text{asy}}$. Eq. (7) then becomes,

$$A = \frac{\tilde{\xi}_0(\Delta P_{\text{asy}}/P_{\text{asy}})_0}{2} \times \sum_{l=\text{min}}^{l_{\text{max}}} \frac{2(2l+1)}{l(l+1)} \left( \frac{l}{l_0} \right)^{n_{A} - 2} \tilde{C}_l ,$$

where $n_{A} = n_{\sigma} + n_{A}$. In this we are assuming that $\tilde{C}_l$ is dominated by the inflaton perturbation, which can be considered to be scale-invariant here.

For $l$ from 2 to $l_{\text{max}} = 64$, $(l+1)\tilde{C}_l$ has only a small variation. We can therefore consider $(l+1)\tilde{C}_l$ to be approximately constant, in which case the large-angle asymmetry observed by Planck and WMAP, which we will denote by $A_{\text{large}}$, is given by

$$A_{\text{large}} \approx \tilde{\xi}_0(\Delta P_{\text{asy}}/P_{\text{asy}})_0 \times \frac{64(2l+1)}{2} \left( \frac{l}{l_0} \right)^{n_{A} - 2} \tilde{C}_l .$$

The non-observation of an asymmetry in quasar number counts implies that the small-angle asymmetry, $A_{\text{small}}$, satisfies $A_{\text{small}} < A_{\text{quasar}} = 0.012$ (95\% c.l.) on scales $k = (1.3 - 1.8) \text{ Mpc}^{-1}$, corresponding to $l = 12400 - 17200$. Over these scales, $(l+1)\tilde{C}_l$ and $(l/l_0)^{n_{A} - 2}$ do not vary much, therefore we can fix $l = l_{\text{small}} = 15000$ and so

$$A_{\text{small}} \approx \tilde{\xi}_0(\Delta P_{\text{asy}}/P_{\text{asy}})_0 \times \left( \frac{l_{\text{small}}}{l_0} \right)^{n_{A} - 2} .$$

### III. THE SPECTRAL INDEX AND ITS RUNNING

A general consequence of an additional scale-dependent adiabatic component of the power spectrum is that the spectral index and the running of the spectral index will be modified from their inflation model values. The power spectrum and spectral index are determined by the mean-squared CMB temperature fluctuations over the whole sky. This can be thought of as the average of the mean-squared temperature fluctuations at different $\theta$. Since from Eq. (6) the mean-squared temperature fluctuation at $\pi/2 + \Delta \theta$ cancels that from $\pi/2 - \Delta \theta$, the mean power from averaging over all angles $\theta$ will be equal to the power at $\theta = \pi/2$,

$$\tilde{P}_\tilde{\zeta} = P_{\text{inf}} + \tilde{P}_{\text{asy}} .$$

The spectral index as observed by Planck, $n_s$, is therefore given by

$$n_s - 1 = k \frac{d \tilde{P}_\tilde{\zeta}}{dk} = \left( n_s - 1 \right)_{\text{inf}} + \frac{\xi}{1 + \xi} (n_\sigma - 1) ,$$

where $\tilde{P}_\tilde{\zeta}$ is the mean-squared temperature fluctuation over the whole sky, and $n_\sigma$ is the scale-dependence of the inflaton perturbation.
therefore

\[ n'_{s} = \frac{dn_{s}}{d\ln k} = \frac{n'_{s,\text{inf}}}{(1 + \xi)} + \frac{\xi}{(1 + \xi)^2} (n_{s} - n_{s,\text{inf}})^2. \quad (15) \]

To leading order in \( \xi \) we therefore find that \( n_{s} - 1 \approx (n_{s} - 1)_{\text{inf}} + \Delta n_{s} \) and \( n'_{s} \approx n'_{s,\text{inf}} + \Delta n'_{s} \), where

\[ \Delta n_{s} = \xi \left( n_{s} - 1 \right) - (n_{s} - 1)_{\text{inf}} \quad (16) \]

and

\[ \Delta n'_{s} = \xi \left( n_{s} - n_{s,\text{inf}} \right)^2 - n'_{s,\text{inf}}. \quad (17) \]

IV. HEMISPHERICAL ASYMMETRY OF THE SPECTRAL INDEX AND ITS RUNNING

There is also a hemispherical asymmetry in the spectral index and the running of the spectral index, obtained by averaging the temperature fluctuations over each hemisphere. For the hemisphere from \( \theta = 0 \) to \( \theta = \pi/2 \), which we denote by +, the average power is

\[ \overline{P}_{\xi}^+ = \int_{0}^{\pi/2} \left( P_{\text{inf}} + \hat{P}_{\text{asy}} + \Delta P_{\text{asy}} \cos \theta \right) \sin \theta d\theta. \quad (18) \]

Therefore

\[ \overline{P}_{\xi} = \overline{P}_{\xi}^+ + \frac{1}{2} \Delta P_{\text{asy}}. \quad (19) \]

For the opposite hemisphere, \( \overline{P}_{\xi}^- = \overline{P}_{\xi} - \frac{1}{2} \Delta P_{\text{asy}} \). The spectral index from the average power in each hemisphere, \( n_{s} \pm \), is therefore

\[ n_{s} \pm - 1 = \frac{k}{\overline{P}_{\xi} \pm \Delta P_{\text{asy}}} \quad (20) \]

Assuming that \( \Delta P_{\text{asy}}/2 \ll \overline{P}_{\xi} \) and neglecting the scale-dependence of \( P_{\text{inf}} \), we find that \( n_{s} \pm \approx n_{s} \pm \delta n_{s} \), where

\[ \delta n_{s} = \frac{\xi_{0}(\Delta P_{\text{asy}}/\hat{P}_{\text{asy}})_{0}}{2} (n_{A} - 2) \left( \frac{k}{k_{0}} \right)^{n_{A} - 2}. \quad (21) \]

Similarly, for the running of the spectral index we find that \( n'_{s} \pm \approx n'_{s} \pm \delta n'_{s} \), where

\[ \delta n'_{s} = \frac{\xi_{0}(\Delta P_{\text{asy}}/\hat{P}_{\text{asy}})_{0}}{2} (n_{A} - 2)^2 \left( \frac{k}{k_{0}} \right)^{n_{A} - 2}. \quad (22) \]

V. RESULTS

In Table 1 we give the values of \( A_{\text{small}} \) and \( \xi_{0}(\Delta P_{\text{asy}}/\hat{P}_{\text{asy}})_{0} \) as a function of \( n_{A} = n_{\sigma} + n_{A} \), where have fixed \( A_{\text{large}} \) to its observed value 0.073 throughout. We find that \( n_{A} < 1.76 \) is necessary to have a strong enough scale-dependence to satisfy \( A_{\text{small}} < 0.012 \). \( \xi_{0}(\Delta P_{\text{asy}}/\hat{P}_{\text{asy}})_{0} \) decreases with \( n_{A} \) from a maximum value of 0.05 at \( n_{A} = 1.76 \).

We next consider the shift of the spectral index and the running of the spectral index from their inflation model values. We will consider the case where the scale-dependence is mostly due to \( P_{\text{asy}} \) rather than \( \Delta P_{\text{asy}}/\hat{P}_{\text{asy}} \) and therefore set \( n_{A} = 1 \), in which case \( n_{s} = n_{s} + 1 \). This gives the maximum shift of the spectral index and its running for a given value of \( n_{s} \) and \( \xi_{0} \), as seen from Eq. (16) and Eq. (17). We also set \( n_{s,\text{inf}} = 1 \) throughout. Table 1 gives the values of \( \Delta n_{s}/\xi_{0} \) and \( \Delta n'_{s}/\xi_{0} \) as a function of \( n_{A} \). The spectral index decreases relative to the inflation model value, while the running of the spectral index increases. The shift of the running of the spectral index imposes a strong constraint on \( \xi_{0} \). The Planck result is \( n'_{s} = -0.013 \pm 0.018 \) (Planck + WP) [18]. This imposes the 2-\( \sigma \) upper bound \( \Delta n'_{s} < 0.005 \), assuming that the running of the inflation model spectral index is negligible. For \( n_{A} = 1.76 \), this implies that \( \xi_{0} < 0.085 \), while for \( n_{A} = 1.5 \) we find that \( \xi_{0} < 0.020 \). Combined with \( \xi_{0}(\Delta P_{\text{asy}}/\hat{P}_{\text{asy}})_{0} = 0.05 \) for \( n_{A} = 1.76 \), this implies that \( (\Delta P_{\text{asy}}/\hat{P}_{\text{asy}})_{0} > 0.6 \) is necessary to account for the power asymmetry while keeping the running of the spectral index within the Planck 2-\( \sigma \) upper limit.

These constraints can be relaxed if \( n_{s} \) is increased for a given \( n_{A} \) by reducing \( n_{A} \). This will depend on the specific model responsible for the additional adiabatic component. For example, in the modulated reheating model of [12], \( n_{A} = 1 \).

It is also possible to achieve a significant shift of the spectral index relative to its inflation model value. For the case \( n_{A} = 1.76 \), \( \xi_{0} < 0.085 \) implies that \( |\Delta n_{s}| < 0.02 \), while for \( n_{A} = 1.5 \) we find that \( |\Delta n_{s}| < 0.01 \). Therefore the inflation model spectral index can be significantly reduced if \( n_{A} \) is close to the quasar upper bound and \( \xi_{0} \) is close to its upper bound from the running of the spectral index. This could bring some common inflation models into better agreement with the value of \( n_{s} \) observed by Planck.

We finally consider the hemispherical asymmetry of the spectral index and of its running. These are completely fixed by \( n_{A} \) and do not depend on whether \( n_{s} \) or \( n_{A} \) dominates the scale-dependence, as seen from Eq. (21) and Eq. (22). Therefore we can obtain an unambiguous range of possible values. From Table 1 we find that \( \delta n_{s} \) is in the range \(-6.2 \times 10^{-3} \) to \(-6.5 \times 10^{-4} \), and \( \delta n'_{s} \) is in the range \(6.5 \times 10^{-4} \) to \(1.9 \times 10^{-3} \), varying between 1.0 to 1.76. The asymmetry in the running of the spectral index appears to be particularly promising as a test of the model, with values between the present Planck 1-\( \sigma \) and 2-\( \sigma \) upper bounds on \( n'_{s} \). Observation of these asymmetries, possibly combined with the observation of a positive running of the spectral index, would therefore support the additional adiabatic component as the explanation of the hemispherical power asymmetry of the CMB.

Our analysis is model-independent, being based only on the scale-dependence and dipole variation of the additional adiabatic component of the power spectrum. These properties must be explained by specific models for the origin of the additional component. These models will also have to satisfy ad-
Additional constraints, in particular those from non-Gaussianity and the isotropy of the CMB temperature, which are beyond the model-independent analysis presented here.

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