Adaptive Robust of RBF Neural Network Control Based on Model Local Approximation Method for Upper Limb Rehabilitation Robotic Arm

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Abstract. For the problem of patient spasm disturbance and random disturbance of external environment in rehabilitation process of upper limb rehabilitation arm, Considering the approximation ability of neural network for arbitrary functions, an adaptive robust of radial basis function (RBF) neural network control algorithm based on model local approximation is proposed. This control algorithm introduces robust term to reduce the approximation error of neural network and the robustness of the rehabilitation manipulator control system is improved. The system can also obtain good track tracking performance under the condition of patient spasm disturbance and random disturbance of external environment. The asymptotic stability of the control system is proved by the stability theory of Lyapunov. Simulation results show that the proposed control algorithm has good control performance.

1. Introduction
With the advancement of intelligence control and computer technology, upper limb rehabilitation robotic arm control has developed greatly at home and abroad. Traditional intelligent control systems such as iterative learning control, adaptive control systems and neural network control systems are widely used in various scientific fields and have also been well applied in upper limb rehabilitation training robots [1-2]. RBF neural network has a strong ability to approximate arbitrary nonlinear function in neural network control system. However, the Neural network weights should be readjust for each sample learning. The convergence speed of neural networks is slow and easy to the local minimum, which makes it difficult to satisfy the high real-time and rapidity demand of the control system [3]. In this paper, an adaptive robust of RBF neural network control (NNAR) algorithm based on model local approximation (MLA) method is proposed. In this way, the boundedness of weights can be guaranteed effectively and the convergence of neural networks weights can be solved.

Neural network adaptive control has been widely studied in robot control and obtained a lot of research results[4-9]. Literature [4] first introduced a filter tracking error and then the adaptive neural network controller is designed based on the dynamic of the robot, which guaranteed the final bounded of input and output signals in the control system. In literature [5], a linear observer is used to design an controller of adaptive neural network for a manipulator with unknown nonlinear dynamics, which guarantee that the system tracking error and observer error are uniformly final bounded and the neural network weights are bounded. Literature [6] combines neural network control with sliding mode control to obtain robust progressive tracking control of the mechanical arm. Literature [7] used neural network to design a force/position coordination control scheme for the joint motion space robot to
achieve the dual control effect. In literature [8], an adaptive controller based on RBF neural network is designed, which realizes the hybrid trajectory tracking control of machine force/position and proves that the output force and position error of the controller are finally uniformly bounded. In literature [9], the direct adaptive control method of neural network for underwater robot was studied. Using Lyapunov function, it is proved that the tracking error of the underwater robot control system is gradually stable and bounded under the condition of bounded external disturbance and bounded approximation error.

The RBF neural network is used to design a NNARC algorithm in joint space for a manipulator whose parameters are completely unknown. In the past, the neural network weight was adjusted by the gradient descent method, which was easy to local minimum and could not render certain the asymptotic convergence of the control system [10]. However, the online NNAC based on Lyapunov stability analysis can effectively solve this problem. The designed NNARC not only realizes the final uniform bounded of input and output signals of the control system, but also realizes the convergence of neural network weights and the local accurate approximation of unknown control system dynamics along the periodic or regression tracking track in the stable control process [11-12].

In this paper, considering the dynamic characteristics of the upper limb rehabilitation arm and the non-linear relationship between the actuator torque and the control input, a design method of NNAC based on the MLA is proposed. Compared with the traditional control method based on the model global approximation (MGA), the robust term of robust control system is introduced to decrease the approximation error and makes the system more robust and more practical, which makes the controller more effective and has a wider application range.

2. Problem statement
The research object of this paper is a three-degree-of-freedom upper limb rehabilitation robotic arm, whose 3D structure model is shown in figure 1. The three degrees of freedom of the exoskeleton rehabilitation manipulator can control the movement of the three main joints of the human body, which are: rotation of the elbow (joint 1), pitch of the wrist (joint 2) and pitch of the shoulder (joint 3). Each joint is equipped with a protection switch to ensure the user's safety in rehabilitation training. The length of the upper arm and forearm of the exoskeleton rehabilitation robotic arm can be adjusted, which can realize the right and left wearing to meet the training needs of patients.

In the course of patients' daily rehabilitation training, various unknown errors and disturbances caused by the uncertainty of the system model and patients' spasm disturbance are inevitable. Therefore, this paper considers various errors and disturbances into the dynamic model of the rehabilitation robot, so the dynamic model of the upper limb rehabilitation robot can be described as:

\[
H(q(t))\ddot{q}(t) + Q(q(t), \dot{q}(t))\dot{q}(t) + G(q(t)) + F(\dot{q}(t)) = \tau(t) + d(t)
\]  

(1)

Where \( q(t), \dot{q}(t), \ddot{q}(t) \in R^n \) are the displacement, velocity and acceleration of the joint respectively. \( H(q(t)) \in R^{n\times n} \) is the inertial matrix of the robot, \( Q(q(t), \dot{q}(t)) \in R^{n\times n} \) is a nonlinear coupling
matrix of centrifugal force and gothic force, \( G(q(t)) \in R^n \) is a gravity term, \( F(\dot{q}(t)) \in R^n \) is friction, \( \tau(t) \in R^n \) is the control torque, \( d(t) \in R^n \) represents various errors and disturbances.

**Property 1:** \( H(q) \in R^{n \times n} \) is a symmetric positive definite bounded matrix, namely

\[
0 < \lambda_{\text{min}}(H) \leq \|H(q)\| \leq \lambda_{\text{max}}(H)
\]

Where \( \lambda_{\text{min}}(H) \) and \( \lambda_{\text{max}}(H) \) are the minimum and maximum values of matrix \( H(q) \) respectively.

**Property 2:** \( \dot{H}(q)-2Q(q, \dot{q}) \) is a symmetric matrix, satisfies:

\[
\forall x \in R^n , x^T (\dot{H}(q)-2Q(q, \dot{q}))x = 0
\]

3. **Adaptive robust of RBF neural network controller design**

First, we introduce adaptive control to the neural network and then use adaptive control law to adjust the weights and estimates of the neural network, so as to ensure that the output \( q \) of the rehabilitation robot system can track the reference trajectory \( q_d \) and estimate the weights \( \hat{W}_{nn} \) of the RBF neural network to converge to the ideal weights \( \hat{W}_{nn}^* \). Finally, robust control is added to ensure the robustness of the system. Therefore, The schematic diagram of the proposed adaptive robust of RBF neural network control system is shown in Figure 2.

Tracking error is

\[
e(t) = q_d(t) - q(t)
\]

The error function is defined as follows

\[
s(t) = \dot{e}(t) + \Gamma e(t)
\]

Where matrix \( \Gamma = \Gamma^T > 0 \), then

\[
\dot{s} = -s + \dot{q}_d + \Gamma e
\]

\[
M\dot{s} = M(\dot{q}_d - \dot{q} + \Gamma \dot{e}) = -Cs - \tau + f(x) + d
\]

Where \( f(x) = H(\dot{q}_d + \Gamma \dot{e}) + Q(\dot{q}_d + \Gamma e) + G + F \).

Using RBF neural network to approximation arbitrary nonlinear function, the output of RBF neural network is

\[
\hat{f}_{nn}(x) = \hat{W}^T \Phi(x)
\]

Where \( \hat{W}^T \) is an estimate of the ideal weight \( W^T \), \( \Phi(x) \) is a Gaussian function.

Design the control law is

\[
\tau = \hat{W}^T \Phi(x) + K_s s - r
\]

Where \( r \) is a robust term used to reduce the approximation error.

The robust term \( r \) is defined as

\[
r = -(\alpha + \beta) \text{sgn}(s)
\]

Substituting the system control law equation (9) into equation (7) to get
\[ HS = -(K_p + Q)s + \hat{W}^T \Phi(X) + \sigma + d + r = -(K_p + C)s + \lambda \]  

(11)

Where \( \lambda = \hat{W}^T \varphi(x) + \sigma + d + r \), \( \sigma = f(x) - \hat{f}_m(x) \), \( \hat{W} = W - \hat{W} \).

According to equation (7), \( f(x) \) in the controlled object can be written as:

\[ f(x) = H(q)\xi_1(t) + Q(q, \dot{q})\xi_2(t) + G(q) + F(\dot{q}) \]  

(12)

Where \( \xi_1(t) = (\dot{q}_d + \Gamma \dot{e}), \xi_2(t) = \dot{q}_d + \Gamma e \).

The terms in \( f(x) \) are approximated respectively, namely

\[
\begin{align*}
\hat{H}(q) &= \hat{W}_H^T \varphi_h(q), \\
\hat{Q}(q, \dot{q}) &= \hat{W}_Q^T \varphi_Q(q, \dot{q}) \\
\hat{G}(q) &= \hat{W}_G^T \varphi_G(q), \\
\hat{F}(\dot{q}) &= \hat{W}_F^T \varphi_F(\dot{q})
\end{align*}
\]

(13)

Then the output \( \hat{f}_m(x) \) of the neural network is:

\[ \hat{f}(x) = \hat{W}^T \Phi(x) \]  

(14)

Where \( \Phi(x) = [\varphi_h \varphi_Q \varphi_G \varphi_F]^T \), \( \hat{W}^T = [\hat{W}_H^T \xi_1(t) \hat{W}_Q^T \xi_2(t) \hat{W}_G^T \hat{W}_F^T] \).

The adaptive rate is

\[
\begin{align*}
\dot{\hat{W}}_H &= F_h \varphi_h s^T - k_h F_h \parallel s \parallel \hat{W}_h, \\
\dot{\hat{W}}_Q &= F_Q \varphi_Q r^T - k_Q F_Q \parallel \hat{W}_Q \parallel, \\
\dot{\hat{W}}_G &= F_G \varphi_G s^T - k_G F_G \parallel \hat{W}_G \parallel, \\
\dot{\hat{W}}_F &= F_F \varphi_F s^T - k_F F_F \parallel \hat{W}_F \parallel
\end{align*}
\]

(15)

Where \( k_h > 0, k_Q > 0, k_G > 0, k_F > 0 \).

4. Stability analysis

Lyapunov function is defined as

\[ V = \frac{1}{2} s^T Hs + \frac{1}{2} tr(\hat{W}_H^T F_h^{-1} \hat{W}_h) + \frac{1}{2} tr(\hat{W}_Q^T F_Q^{-1} \hat{W}_Q) + \frac{1}{2} tr(\hat{W}_G^T F_G^{-1} \hat{W}_G) + \frac{1}{2} tr(\hat{W}_F^T F_F^{-1} \hat{W}_F) \]  

(16)

Deriving the Lyapunov function \( V \), substituting equation (11) into it, then

\[ \dot{V} = -s^T K_p s + \frac{1}{2} s^T (H - 2Q)s + s^T (\sigma + d) + s^T r + tr(\hat{W}_H^T F_h^{-1} \hat{W}_h + \varphi_h s^T) \\
+ tr(\hat{W}_Q^T F_Q^{-1} \hat{W}_Q + \varphi_Q r^T) + tr(\hat{W}_G^T F_G^{-1} \hat{W}_G + \varphi_G s^T) + tr(\hat{W}_F^T F_F^{-1} \hat{W}_F + \varphi_F s^T) \]  

(17)

Considering the robot characteristics and substituting the adaptive rate equation (15) into equation (17), then

\[ \begin{align*}
\dot{V} &= -s^T K_p s + \frac{1}{2} s^T Hs + s^T \sigma + d + s^T r + tr(\hat{W}_H^T F_h^{-1} \hat{W}_h + \varphi_h s^T) \\
+ k_h \parallel s \parallel \hat{W}_h^T (W_h - \hat{W}_h) + k_Q \parallel W_Q - \hat{W}_Q \parallel \hat{W}_Q^T (W_Q - \hat{W}_Q) \\
+ k_G \parallel \hat{W}_G^T (W_G - \hat{W}_G) + k_F \parallel \hat{W}_F^T (W_F - \hat{W}_F) \hat{W}_F^T (W_F - \hat{W}_F) + r^T (\sigma + d) + s^T r
\end{align*} \]

(18)

Because the \( W, \hat{W} \) satisfies inequality \( tr(\hat{W}^T (W - \hat{W})) \leq \parallel W \parallel_p \parallel W \parallel_F - \parallel \hat{W} \parallel_F^2 \) and the robust term (10) is considered, then
\[
\dot{V} \leq -\|s\| + k_H \|\dot{W}_r\| (\|\dot{W}_r\|-W_{H_{\text{max}}}) + k_Q \|\dot{W}_Q\| (\|\dot{W}_Q\|-W_{Q_{\text{max}}}) + k_G \|\dot{W}_G\| (\|\dot{W}_G\|-W_{G_{\text{max}}})
\]
\[+k_F \|\dot{W}_F\| (\|\dot{W}_F\|-W_{F_{\text{max}}}) \]

(19)

If \(s\) and \(W\) satisfy the following inequalities:
\[\|s\| > \frac{k_H W_{H_{\text{max}}}^2}{4 + k_Q W_{Q_{\text{max}}}^2 / 4 + k_G W_{G_{\text{max}}}^2 / 4 + k_F W_{F_{\text{max}}}^2 / 4}{K_{Q_{\text{min}}}}\]
\[\|\dot{W}_r\| > W_{H_{\text{max}}}, \|\dot{W}_Q\| > W_{Q_{\text{max}}}, \|\dot{W}_G\| > W_{G_{\text{max}}}, \|\dot{W}_F\| > W_{F_{\text{max}}}\]

(20)

(21)

Therefore, when \(K_{\rho}\) is selected to be large enough, the filter tracking error can be as small as possible for all time \(t > t_1\). The state tracking error \((e^T, \dot{e}^T)\) can converge to a small area of zero, then \(\dot{V} < 0\).

5. Simulation and Analysis

In order to verify the stability, rapidity and robustness of NNARC arithmetic for upper limb rehabilitation arm based on MLA, the control algorithm proposed in this article is compared with the control arithmetic based on the MGA in the presence of the same control system and disturbance. Select two-joint manipulator as control object.

Considering the two free upper limb degree rehabilitation manipulator, its dynamic equation is
\[
H(q(t))\ddot{q}(t) + Q(q(t), \dot{q}(t))\dot{q}(t) + G(q(t)) + F(\dot{q}(t)) = \tau(t) + d(t)
\]

(22)

Where \(H = [h_{ij}]_{2 \times 2}, h_{11} = 3.7 + 1.6 \cos q_2, h_{12} = h_{21} = 0.76 + 0.8 \cos q_2, h_{22} = 0.76\)

\[Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix}, q_{11} = -0.87 \tilde{q}_2 \cos q_2, q_{12} = -0.87(\tilde{q}_1 + \tilde{q}_2) \sin q_2, q_{21} = -0.87 \tilde{q}_1 \sin q_2, q_{22} = 0,\]

\[G(q) = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}, g_{11} = 3g \cos q_1 + 0.8g \cos(q_1 + q_2), g_{12} = 0.8g \cos(q_1 + q_2), \quad F(\dot{q}) = 0.4 \text{sgn}(\dot{q})\]

\[d(t) = \begin{bmatrix} d_m \sin t + \text{rand(size(t))} \\ d_m \sin t + \text{rand(size(t))} \end{bmatrix}, \quad \begin{bmatrix} q_{1d} \\ q_{2d} \end{bmatrix} = \begin{bmatrix} 0.1 \sin(0.2 \pi t) \cdot \sin(0.4 \pi t) \\ 0.1 \sin(0.2 \pi t) \cdot \sin(0.4 \pi t) \end{bmatrix}.\]

Take \(b = 0.20, z = [e \ \dot{e} \ q_d \ \dot{q}_d \ \ddot{q}_d], q(0) = [0.1 \ 0 \ -0.1 \ 0], \ K_{\rho} = \text{diag} \{60, 60\}, F = \text{diag} \{30, 30\}, \Gamma = \text{diag} \{10, 10\}, \ v = 0.20, b_y = 0.10, k_H = k_Q = k_G = k_F = k = 0.01.\]

Simulation of the controlled system was verified by MATLAB simulation software and the simulation results were shown in figure 3-12.

Given the uncertainty of model parameters and the existence of external random disturbance, the trajectory tracking simulation results of the proposed controller and the adaptive of NNAC based on the GAM are shown in figure 3 and figure 6. It can be seen from figure 3 that due to the NNARC based on MLA and the introduction of robust terms, the two joints of the upper limb rehabilitation robot can achieve fast and stable tracking of the desired trajectory. On the contrary, the NNAC based on the GAM in figure 6 has poor tracking effect and weak robustness.

Figure 4 and figure 7 respectively show the angular velocity tracking of the joint under two kinds of controllers. Since the global approximation is the approximation to the uncertain item \(f(x)\), it is not possible to approximate every weight vector in the function \(f(x)\) like the local approximation, so the NNARC based on MLA has a fast tracking speed. Figure 5 and figure 8 are the position tracking errors and velocity tracking errors of the system under two controllers, respectively. It can be seen from figure 5 that the position and velocity error of the system quickly converge to zero to reach a
stable state, which indicates that the NNARC based on MLA proposed in this paper has better tracking effect and good robustness.

Figure 9 and Figure 10 show the control inputs of the two controllers. The comparative analysis shows that The control process of NNARC based on local approximation is stable and has a strong ability to overcome disturbance. Therefore, the NNARC controller based on MLA has stable control input. Figure 11 and Figure 12 respectively show the system's approximation process of uncertain item $f(x)$ under two kinds of controllers. Since the local approximation neural network is adopted in this article and the robust item is introduced, Figure 9 can quickly approximate $f(x)$ to a state of asymptotic stability.

6. Conclusion
An NNARC algorithm is adopted for the upper limb rehabilitation robot with completely unknown parameters and random disturbance based on the MLA in this article. The local approximation method
of neural network is used to approximate the uncertain nonlinear term in the control system of upper limb rehabilitation robot. The adaptive law is designed to adjust the weight vector of neural network online and estimate the unknown bound of time-varying environmental disturbance in the process of rehabilitation training.

The robust term is introduced to decrease the approximation error of the neural network and the simulation example proves that the proposed control scheme has strong robustness to the model uncertainty and completely unknown disturbance of the external environment. Finally, the availability of the NNARC control algorithm is verified by MATLAB simulation under the influence of various random disturbances in the rehabilitation training of patients.

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