Integration of Vibration Acceleration Signal Based on LabVIEW

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Abstract: In vibration measurement, the measurement of acceleration is more convenient than that of displacement, but the observation of vibration displacement is more intuitive than that of acceleration. In order to convert acceleration signal into displacement, the traditional method is time domain integration, but there is a trend term problem in time domain integration, while there is no trend term problem in frequency domain integration. Firstly, the algorithm of time domain integration and frequency domain integration is analyzed. Based on this algorithm, a program is programmed based on Labview 2015. The simulation results of time domain integration and frequency domain integration are compared by using this program. The results show that frequency domain integration can recover waveform better. In practical measurement and analysis, frequency domain integration is better than time domain integration.

1. Introduction
In engineering practice, it is sometimes necessary to measure the vibration displacement of components, but in most cases it is inconvenient or even impossible to measure the displacement. With the wide application of ICP or TEPE accelerometer, the velocity and displacement signals can be obtained by measuring the vibration acceleration signal and integrating the acceleration signal. Because of the zero drift, measurement error and low frequency interference of the vibration acceleration signal conditioning circuit, there are many difficulties in accurately integrating the signal, which is only in the dimension level at present. There are two main methods to integrate vibration acceleration signal at present, one is through hardware integration circuit, the other is through software integration. This paper mainly studies the software integration method of acceleration signal.

Software integration generally adopts time domain integration or frequency domain integration methods [1]. Time-domain integration is a direct quadratic integration of the measured acceleration signal after the DC component is removed. Generally, trapezoidal formula or Simpson summation formula is used. Because of the existence of trend term, the results are not ideal. Frequency domain integration is to transform the acceleration signal after DC component removal into FFT signal, then divide the FFT transform sequence in frequency domain, and finally IFFT the processed FFT sequence to get the displacement signal.

2. Time Domain Integration of Signals
Vibration signals generated by rotating machinery picked up by accelerometer are sampled as discrete digital signals by acquisition card. Considering the zero drift and noise of signal amplifier, if the signal is integrated directly, there will be a trend term, which will affect the integration results. Therefore,
before using time domain integration, it is necessary to remove DC component and eliminate noise. To remove the DC component, the usual method is to take the mathematical expectation of the signal, i.e. the mean value, as the estimation of the DC component, and remove it from the original signal.

2.1 DC Component of Signal
In theory, the DC component of the acceleration sensor signal should be 0, but in the actual measurement system, the acceleration signal needs to be adjusted. Charge amplifier is used to adjust the signal. Zero drift exists in all amplifiers. In addition, ambient noise and measurement error can also characterize the signal with DC component. The DC component must be removed before integrating the sampled N-point sampling sequence. Generally, the mathematical expectation \( \bar{a} = \frac{1}{N} \sum_{i=0}^{N-1} a_i \) of the sampled N-point sequence is used to estimate the DC component, and the discrete signal after DC removal is \( \bar{a}_i = a_i - \bar{a} \) (i = 0, 2, ...N - 1).

2.2 Time Domain Integration Algorithms
Assuming that the acceleration signal after DC removal is \( a_\infty (t) \), the sampling point of acceleration signal is \( a_i \) of a multiple of 1024, and \( t_0, t_1, t_2, ..., t_{N-1} \) is the base point, if the sampling period is sufficiently small, the accelerometer mass can be regarded as uniform acceleration motion on interval \([t_{i-1}, t_i]\), and the integral vibration velocity sequence is:

\[
v_i = v_{i-1} + \int_{t_{i-1}}^{t_i} \bar{a}(t) \, dt = v_0 + \frac{T_s}{2} (\bar{a}_i + \bar{a}_{i+1})
\]

\( T_s \) is the sampling period. The integral vibration velocity sequence is:

\[
v_1 = v_0 + \int_{t_0}^{t_1} \bar{a}(t) \, dt \approx v_0 + \frac{T_s}{2} (\bar{a}_0 + \bar{a}_1)
\]

\[
v_2 = v_1 + \int_{t_1}^{t_2} \bar{a}(t) \, dt \approx v_1 + \frac{T_s}{2} (\bar{a}_1 + \bar{a}_2)
\]

\[
v_n = v_{n-1} + \int_{t_{n-1}}^{t_n} \bar{a}(t) \, dt \approx v_{n-1} + \frac{T_s}{2} \sum_{i=0}^{n-1} (\bar{a}_i + \bar{a}_{i+1}) \quad (n = 1, 2, ..., N - 1)
\]

Vibration displacement time series can be obtained by integrating the velocity time series again with the same method. The time series of vibration displacement can be expressed as:

\[
s_n = \frac{T_s}{2} \sum_{i=0}^{n-1} (v_i + v_{i+1}) \quad (v_0 = 0, n = 1, 2, ..., N - 2)
\]

2.3 Problems in Time Domain Integral
It is assumed that the measured acceleration signal is \( a(t) = f(t) + C \), where C is measurement error, residual DC component or noise.

\[
v(t) = \int a(t) \, dt = \int (f(t) + C) \, dt = \int f(t) \, dt + C t + C_1
\]

\[
s(t) = \int v(t) \, dt = \int (\int f(t) \, dt) \, dt + \frac{1}{2} C_t^2 + C_1 t + C_2
\]

Formulas (3) and (4) show that the first integral contains the first trend term, and the second integral contains the second trend term. The calculation error increases with the increase of the number of integrals whether it is in the first integral or the second integral, and the integral trend term increases.

3. Frequency Domain Integration of Signal Based on Fourier Transform

3.1 Frequency Domain Integration Algorithms
In order to eliminate the influence of trend term in integration process, Fourier transform can be used to transform the signal into frequency domain, integrate in frequency domain, and then inverse Fourier transform.
transform can be used to get the inverse result. When the real part is transformed, the required

time-domain signal can be obtained.

For the first integral \( \int_{-\infty}^{\infty} x(\tau) d\tau \), combining the integral property \( \int_{-\infty}^{\infty} x(\tau) u(t - \tau) d\tau = \int_{-\infty}^{\infty} x(\tau) e^{-j\omega(t - \tau)} d\tau \) and the Fourier transform property \( \int_{-\infty}^{\infty} u(t - \tau) e^{-j\omega(t - \tau)} d\tau = \frac{1}{j\omega} + \pi\delta(\omega) \), it can be deduced that:

\[
F \left[ \int_{-\infty}^{\infty} x(\tau) d\tau \right] = \frac{X(\tau)}{j\omega} + \pi\delta(\omega)X(0) \quad (5)
\]

Similarly, it can be further deduced that:

\[
F \left[ \int_{-\infty}^{\infty} x(\tau) d\tau \right] = \frac{X(\tau)}{(j\omega)^2} + n\delta(\omega)X(0) \quad (6)
\]

Among them, \( \delta(\omega) \) is the unit impulse function and \( u(t) \) is the unit step function.

It can be seen from formula (5) and formula (6) that single and double integrals of functions can be
converted in the integral process in the integral process \([2][3]\).

### 3.2 Flow chart of frequency domain integration algorithm

The algorithm flow based on the frequency domain integration of formula (5) and formula (6) is as
follows:

1. Data are collected and time-domain arrays are obtained (In order to facilitate FFT calculation, N = 2^n data points are sampled);
2. Frequency spectrum \( X(\omega) \) is obtained by fast Fourier transform (FFT) for N data points;
3. Establish \( \omega \) vector:
   ① Frequency interval: \( df = f_s/N \). By calculating \( df \), the sampling time points in time domain and the frequency points in frequency domain are one-to-one corresponded.
   ② Angular frequency interval: \( d\omega = 2\pi df \);
   ③\( \omega \) Vector Computation
   Positive discrete angular frequency vector:
   \[
   \omega 1 = \left[ \omega 1_0, \omega 1_1, ..., \omega 1_{1/2}, ..., \omega 1_{N-1/2} \right]
   \]
   \[
   \omega 1_i = (d\omega \ast i)^n \quad i = (0, 1, \frac{N}{2} - 1)
   \]
   Negative discrete angular frequency vector:
   \[
   \omega 2 = \left[ \omega 2_0, \omega 2_1, ..., \omega 2_{1/2}, ..., \omega 2_{N-1/2} \right]
   \]
   \[
   \omega 2_i = \left[ -d\omega \ast \frac{N}{2} + d\omega \ast (i - \frac{N}{2}) \right]^n \quad i = (\frac{N}{2}, \frac{N}{2} + 1, ..., N - 1)
   \]
   \[
   \omega = \omega 1 \cup \omega 2
   \]
   \( n \) is the number of integrals.
4. integral frequency domain transformation (dividing the result sequence of FFT by the corresponding term of \( \omega \) vector in turn).
   \[
   X_i = \begin{cases} 
   \frac{X(\omega_i)}{\omega_i} & i = (0, 1, ..., N - 1) \\
   \frac{X(\omega_i)}{(\omega_i)^2} & i = (0, 1, ..., N - 1)
   \end{cases}
   \]
5. integral phase transformation
   Assuming that the real part of \( X_i \) is real and the imaginary part is imag.

One-time integral: \( X_i = -imag(X_i) + j \ast real(X_i) \)
Quadratic integral: \( X_i = -\text{real}(X_i) + j \cdot \text{imag}(X_i) \)

(6) filtering

If you want to filter out the low-frequency part and the high-frequency part, just set the complex numbers at the beginning and the end of the \( X_i \) array after phase transformation to zero directly. Assuming that the part below \( f_l \) needs to be filtered, the number \( m = f_l / df + 1 \) is calculated first, that is to say, the first \( m \) complex numbers of \( X_i \) array are set to 0, and the high frequency filtering is the same.

(7) The inverse Fourier transform (IFFT) returns to the time domain, and the integral result is obtained by multiplying the real part by the transform coefficients.

4. Programming and Analysis

According to the above time-domain integration and frequency-domain integration algorithm, the test program is compiled using LabVIEW2015 according to formulas (1), (2), (5) and (6). The test program includes three modules: simulation, actual measurement and trend elimination.

4.1 simulation

The simulation signal is generated by the Formula Waveform.vi function of LabVIEW. The parameters of the simulation signal are shown in Table 1.

| Simulated Signal Formula | a*sin(w*t) +0.3 | sampling rate | 1000 |
|--------------------------|----------------|---------------|------|
| Signal Amplitude         | 2              | Sampling Points | 512  |
| signal frequency         | 50             | Offset        | 0.3  |

Run the program with the parameters shown in Table 1, and the result interface is shown in Figure 1. From the operation results of Fig. 1, it can be seen that the trend baseline of velocity waveform after one integration is a straight line, which coincides with the trend term \( C_t + C_1 \) of Formula (1) and the trend baseline of displacement waveform after two integration is a second-order curve, which coincides with the trend term \( C_t^2 + C_1 t + C_2 \) of Formula (2). The displacement waveform of frequency domain integration is almost completely restored.

4.2 Elimination of Trend Term of Time Domain Integral

Time domain integration trend elimination steps:

Acceleration sequence is integrated once or twice to obtain the result sequence array \( x[N] \).

For the result sequence of the first and second integrals, the least square method is used to fit the corresponding polynomials as the trend baseline. The polynomials are: \( f[i] = \sum_{j=m} a_j (x[i])^j \), \( i = (0,1,2,...,N-1) \). \( f[i] \) is the output sequence of the best polynomial fitting, \( x[i] \) is the element of the input sequence \( X[N] \), \( a \) is the polynomial coefficient, \( m \) is the polynomial order.

Get the de-trending sequence, \( y[i] = x[i] - f[i] \), \( i = (0,1,2,...,N-1) \).
In the program, the general polynomial Fit.vi provided by LabVIEW is used to fit the elimination trend of higher order polynomials.

In theory, as shown in Fig. 1, for regular waveforms as sinusoidal waves, the first integration has only one trend term, and the second integration has only two trend terms. For the result sequence of the first integration in time domain, the first-order polynomial can be fitted as the trend baseline by using the linear least square method. For the result sequence of the second integration in time domain, the first-order polynomial can be fitted as the trend baseline. The second-order polynomial can be fitted by the least square method as the trend baseline.

The simulated velocity waveform in Fig. 1 is de-trended by first-order polynomial as shown in Fig. 2, and the displacement waveform is de-trended by second-order polynomial as shown in Fig. 3.

The simulation signal formula in Table 1 is set as $a \sin(\omega t) + 2 \cdot (\text{rand}(-1) - 0.5)$. Other parameters remain unchanged. Velocity is de-tended by first-order polynomial and displacement is de-tended by second-order polynomial. The simulation acceleration waveform is shown in Fig. 4. The velocity waveform after de-tending is shown in Fig. 5 and the displacement waveform after de-tending is shown in Fig. 6. The frequency domain integral waveform is shown in Fig. 7. From the results, it can be seen that for irregular waveforms, both the first integral and the second integral need to use higher order polynomial to eliminate the trend.
The waveforms of velocity and displacement are basically restored from lower to higher order of trend polynomial until the fifteenth order. The waveforms of velocity and displacement after the fifteenth order polynomial eliminates the trend are shown in figs. 8 and 9.

In fact, there always exist various kinds of noise in the acceleration signals collected in the field. Even after noise reduction processing, there are still many errors. The integral trend term formed by these errors after integration is generally a non-linear trend term. To eliminate the trend term of the first integral and the second integral, the least square method is needed. Hence high-order polynomials can be fitted as trend baseline, and the trend term can not be eliminated completely.

5. Case study
In the condition monitoring system of mechanical pumps, the acceleration sensor is used with a sampling rate of 2000. The vibration signal of the bearing of mechanical pumps is tested, and the test results are stored in the file of TDMS format. The program opens the data file and intercepts the waveform subset of 1024 sampling points for analysis. After repeated operation and selection, the polynomial order parameter of General Polynomial Fit.vi is 20, the fitting method parameters are set to least squares, and the SVD of rank H is selected by the algorithm. The subset of acceleration vibration waveform is shown in Fig.10, the velocity signal waveform after eliminating trend is shown in Fig.11, the displacement signal waveform after eliminating trend is shown in Fig. 12, and the displacement signal waveform obtained by quadratic integration of acceleration signal in frequency domain is shown in Fig.13. From figs.10, 11 and 12, it can be seen that the waveform recovery of velocity and displacement signals obtained by time domain integration is not ideal, even if the trend is removed by high-order polynomials of up to 20 orders, and the waveform recovery effect of frequency domain integration is ideal.
6. Conclusion
Through the above simulation and measurement analysis, we can see that in practical application, for
time domain integration, no matter the first integration or the second integration, only fitting the
higher order polynomial as the trend baseline can basically eliminate the trend term; frequency domain
integration has no trend term, so it can effectively restore the waveform; frequency domain integral meter The premise of calculation is $X(0) = 0$, which requires band-pass filtering and spectral leakage.
In short, in practical applications, it is more ideal to use frequency domain integration in order to
obtain displacement by acceleration.

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