Feedback Bits Allocation for Interference Minimization in Cognitive Radio Communications

Mirza Golam Kibria, Fang Yuan and Fumihide Kojima

Abstract—This letter studies the limited feedback cognitive radio system, where the primary users (PU) are interfered by the secondary transmitter (ST) due to the imperfect beamforming. We propose to allocate the feedback bits among multiple PUs to minimize the maximum interference caused by the ST, by exploiting the heterogeneous average channel gains. In addition, we study the problem of minimizing the total feedback bits under a predefined interference threshold at the PUs. The solutions with low complexity are proposed for the studied problems, and the performances of bit allocations are analyzed. Simulation results validate our analysis and demonstrate that the proposed solutions work very well in terms of minimizing the maximum interference caused by the ST and minimizing the total feedback bits under predefined interference threshold at the PUs for limited feedback CR system.

Index Terms—Cognitive radio, Bit allocation, Interference.

I. INTRODUCTION

COGNITIVE radio (CR) system is known as one of the promising techniques to meet the ever-increasing demand on spectrum efficiency (SE) in the future wireless systems [1, 2]. Depending on spectrum sharing strategies, there are generally two operation modes in CR systems. One is overlay mode, where the transmission in secondary system is enabled only when primary system is not on transmission [1]. The other is underlay mode, where the transmission in secondary system is enabled if the interference to primary system can be tolerated [3]. Compared to the overlay mode, the underlay mode has more potential in improving the SE as it allows more chances for simultaneous cognitive transmissions [3]. Thus this work focuses on the underlay mode for CR system.

Among all, the CR system with multiple antennas at the secondary transmitter (ST) is capable of applying the beamforming at the ST to improve the secondary transmission while protecting the primary users (PU) [4]. In these CR systems, the channel direction information from the ST to the PU (CDIsp) must be acquired at the ST to assist the beamforming, and the acquisition of CDIsp results in additional signaling overhead between the primary and secondary systems.

To reduce the signaling overhead for the CDIsp in CR system, limited feedback techniques have been proposed, where the CDIsp is quantized at the PU and fed back to the ST [5, 6]. The obtained CDIsp at the ST is imperfect because of the quantization, and thus the interference is residual to the PU that can not be completely nulled after the beamforming. Theoretically, allocating more feedback bits for each PU can provide more accurate CDIsp and reduce the residual interference, which is however constrained by the finite feedback capacity between primary and secondary systems [6]. To avoid the severe interference for the PU, it is important to minimize the maximum interference in primary system caused by the ST.

In this letter, we study the problem of allocating the feedback bits among multiple PUs to minimize the maximum interference caused by the ST in limited feedback CR system. We also minimize the total feedback bits budget under a predefined interference threshold at the PUs. For both two problems, the solutions are provided in closed-form and of low complexity. Notation: we use ||.||, ()T, E[] to denote the absolute value, the Hermitian operator and the expectation operator, respectively.

II. SYSTEM MODEL

Consider the ST equipped with a number of N antennas serves a secondary user (SU) by beamforming while mitigating the interferences to a number of K PUs in the primary system. The received instantaneous interference at the PU k from the ST is expressed as

\[ I_k = |\sqrt{\lambda_k} h_k^H v_0 d_0|^2, \]

where \( k = 1, \ldots, K \), \( \lambda_k \) and \( h_k \) corresponds respectively to the average channel gain and instantaneous channel vector from the ST to PU \( k \), \( v_0 \) and \( d_0 \) are respectively the unit-norm beamformer and data symbol at the ST for the SU. We assume the channel vectors \( h_k \) are subject to identically independent distributed (i.i.d.) flat Rayleigh fading, and the PUs have perfect channel information about \( h_k \) after the channel estimation.

Denote the perfect CDIsp as \( \bar{h}_k = h_k/|h_k| \), which is unit norm and conveys only channel direction information [8]. In the limited feedback literature [8, 9], the CDIsp is firstly quantized through a given codebook with a proper size at each PU, and then the bits for the quantized CDIsp are fed back to the ST for beamforming. There are many protocols to support forwarding the CDIsp from the primary system to the secondary system, e.g., the S1 protocol in [7] when primary and secondary systems are deployed in macro and pico cells, respectively.

Let \( \tilde{h}_k \) be the quantized version of \( \bar{h}_k \). The relation between perfect CDIsp and quantized CDIsp is given as [9]

\[ \tilde{h}_k = \cos \theta_k \bar{h}_k + \sin \theta_k q_k, \]
where $q_k$ is the quantization error vector, and $\cos^2 \theta_k = |\hat{h}_k H \hat{h}_k|^2$ reflects the accuracy of CDIsp received at the ST.

The average CDIsp distortion due to quantization is defined as [8]

$$\delta_k = 1 - E[|\hat{h}_k H \hat{h}_k|^2] = E[\sin^2 \theta_k].$$

The value of $\delta_k$ depends on the employed codebooks. For the tractability, we apply the quantization upper bound method introduced in [9], which assumes each quantization cell is a Voronoi region of a small spherical cap. From Eq. (13) of [9], we find the average quantization error as

$$\delta_k = \frac{N-1}{N} \frac{1}{2} \frac{\pi}{h},$$

where $b_k$ is the number of bits allocated to user $k$ for quantizing its CDIsp $\hat{h}_k$. The work in [9] demonstrates via simulation that (1) provides a good approximation for any other codebooks designed for i.i.d. flat Rayleigh channels.

As in most limited feedback literature [8,9], we consider the zero-forcing (ZF) beamforming is applied at the ST to reduce the interference into the primary system, i.e., the beamforming vector is selected such that $|\hat{h}_k H \hat{v}_0|^2 = 0$ for all $k$ under perfect CDIsp. Denoting $\hat{h}_k$ as the channel vector from the ST to the SU, one well-known ZF beamforming scheme is from the pseudo-inverse by normalizing the first column of the matrix $V = H[H H H]^{-1}$ where $H = [h_0, h_1, \cdots, h_K]$. Under limited feedback, although the interference cannot be null completely due to imperfect CDIsp, the ZF beamforming has the merit of simplicity and robustness to imperfect CDIsp. In ZF scheme, when the elements in $\hat{h}_k$ are i.i.d. with a unit variance, it is shown in [10],

$$E[|\hat{h}_k H \hat{v}_0|^2] = \delta_k.$$

Then the average interference from the ST to the PU $k$ under limited feedback becomes

$$I_{av,k} = E[\delta_k] = P_0 \lambda_k \delta_k,$$

where the transmit power at the ST is $E[|\hat{v}_0|^2] = P_0$.

III. PROBLEMS AND SOLUTIONS

A. Interference Minimization Under Limited Feedback

Observing (2), allocating more feedback bits to the PU $k$ reduces its residual interference from the ST. Yet the feedback capacity from the primary system to secondary system is usually finite and shared among the users. The interferences in (2) are linear with average channel gains $\lambda_k$, which are heterogenous for individual PUs. To avoid the severe interference to the primary system, it is necessary to allocate the total available feedback bits among multiple PUs by considering the heterogeneity of average channel gains $\lambda_k$.

Motivated by this, the problem of allocating feedback bits among multiple PUs to minimize the maximum interference in limited feedback CR system under the sum-bit constraint can be described as

$$\min_{b_k} \max_{k \in \{1, \ldots, K\}} I_{av,k}$$

s.t.

$$\sum_{k=1}^{K} b_k \leq B,$$

$$b_k \geq 0, \quad \forall k,$$

where $B$ is the total number of feedback bits for all $K$ PUs in the available feedback capacity from the primary system to the secondary system.

The problem in (3) is convex after relaxing the integers $b_k$ into the continuous variables, since the second-order derivative of the objective function $I_{av,k}$ with regard to $b_k$ is positive and the constraints in (4) and (5) are linear. Then, the problem in (3) can be solved via standard convex optimization tools [11]. After obtaining the optimal solution to (3), the allocation results can be rounded into nearest integers as the number of bits allocated for each PU [12].

To provide an explicit solution with low complexity for the bit allocation in (3), we further consider a suboptimal problem. Inspired by the equation

$$\lim_{L \to \infty} \left( \frac{K}{K} \sum_{k=1}^{K} b_{av,k} \right)^{1/2} = \max_{k \in \{1, \ldots, K\}} I_{av,k},$$

we try to optimize the new objective $\left( \sum_{k=1}^{K} b_{av,k} \right)^{1/2}$ instead of $\max_{k \in \{1, \ldots, K\}} I_{av,k}$ under a sufficiently large positive integer $L$.

Then the new optimization problem becomes

$$\min_{b_k} \left( \sum_{k=1}^{K} b_{av,k} \right)^{1/2}$$

s.t.

$$\sum_{k=1}^{K} b_k \leq B,$$

$$b_k \geq 0, \quad \forall k.$$

(8)

(9)

The problem in (7) is identical to that in (3) when $L$ approaches infinity as guaranteed by (6). However, under a finite $L$, the problem in (7) is suboptimal to that in (3) since $\left( \sum_{k=1}^{K} b_{av,k} \right)^{1/2} \leq \max_{k \in \{1, \ldots, K\}} I_{av,k}$ always holds.

The problem in (7) is still convex, since $L$ is a constant and the inside term $I_{av,k}$ is convex on $b_k$. The Lagrangian function of the problem in (7) is

$$L(b_k, v_0) = \left( \sum_{k=1}^{K} b_{av,k} \right)^{1/2} + v_0 \left( \sum_{k=1}^{K} b_k - B \right),$$

where $v_0$ is the lagrangian multiplier. Thus, the optimal solution to the problem (7) should satisfy the Karush-Kuhn-Tucker (KKT) conditions

$$\frac{\partial L(b_k, v_0)}{\partial b_k} = -c \left( \sum_{k=1}^{K} b_k - B \right) = 0,$$

$$\frac{\partial L(b_k, v_0)}{\partial v_0} = \sum_{k=1}^{K} b_k = B = 0,$$

(11)

(12)

where $c = \ln \frac{(N-1)y - \frac{\log_2 \lambda_k}{\rho}}{N}$. By introducing a new positive parameter $\nu = \frac{(N-1)y - \frac{\log_2 \lambda_k}{\rho}}{N}$, the KKT conditions can be written more concisely as

$$b_k = \frac{N-1}{L} \left[ L \log_2 \lambda_k - \nu \right]$$

where $\nu > 0$ should satisfy $\sum_{k=1}^{K} b_k = B$, and $[x]^+ = \max(0, x)$. From (13), it is clear that the optimal solution to the problem (7) allocates more bits to quantify the CDIsp from the PU who has a larger average channel gain $\lambda_k$. 


It is known that the solution satisfying (13) can be found by the standard water-filling algorithm [13], which can be implemented with one-dimensional search over \( \nu \) and converges very fast with less than \( K \) iterations. After \( \nu \) is found, the bit allocation results can be immediately obtained from (13), which only need regular scalar operations. Therefore, the computational complexity of the solution to (7) is reduced from the solution of standard optimization tools to (3). The solution to (3) can serve as the performance upper bound for any bit allocation to minimize the maximum interference in limited feedback CR system. In practice, the suboptimal solution in (13) may be more desirable for CR system which is sensitive to the computational complexity. Note that although the optimal setting \( L \) should be infinity, it is sufficient to set a finite large \( L \) in the optimization for achieving a good result in practice as shown in the simulation section.

The suboptimal problem in (7) allows us to obtain the relationship between the minimized maximum interference and the number of total feedback bits under asymptotical analysis. To see this, we consider that the total number of feedback bits \( B \) is large such that the operation \([\cdot]^\dag\) can be removed in (13). In this case, it can be verified that the solution in (13) achieves the same optimization result as the algorithmic inequality \( \sum_{k=1}^{K} x_{k}/K \geq (\prod_{k=1}^{K} x_{k})^{1/K} \) given by

\[
\left( \sum_{k=1}^{K} I_{v,k}^{L} \right)^{\frac{1}{L}} \geq \prod_{k=1}^{K} I_{v,k}^{L} = \prod_{k=1}^{K} \lambda_{k} \delta_{k}^{\frac{1}{L}} = \prod_{k=1}^{K} \frac{N-1}{N} P_{0}\left( \prod_{k=1}^{K} \lambda_{k} \right)^{2^{-\frac{1}{2K}}},
\]

where the last equality is led by \( \prod_{k=1}^{K} \delta_{k} = \frac{N-1}{N} 2^{-\frac{1}{2K}} \) under the sum-bit constraint that \( \sum_{k=1}^{K} b_{k} = B \).

Since the problem in (3) and (7) are identical when \( L \) approaches infinity, by using \( \lim_{L \to \infty} K^{\frac{1}{L}} = 1 \), the optimized maximum interference in (3) under a large total number of feedback bits \( B \) satisfies

\[
I_{\text{opt}} = \lim_{L \to \infty} \left( \sum_{k=1}^{K} I_{v,k}^{L} \right)^{\frac{1}{L}} = \frac{N-1}{N} P_{0}\left( \prod_{k=1}^{K} \lambda_{k} \right)^{2^{-\frac{1}{2K}}},
\]

which is linear with the geometric mean of the average channel gains among the PUs, and degrades exponentially as the number of antenna \( N \) increases.

### B. Feedback Minimization Under Interference Threshold

Another challenge in the design for CR systems is to guarantee that interferences at the PUs caused by the ST is restricted to below a predefined interference threshold, \( I_{\text{max}} \).

In such systems, it is more desirable to minimize the total required feedback bits such that the interference to each PU is below a given threshold. The corresponding feedback bits budget optimization problem can be modeled as

\[
\min_{b_{k}} \sum_{k=1}^{K} b_{k} \quad \text{subject to} \quad I_{v,k} \leq I_{\text{max}}, \quad b_{k} \geq 0, \quad \forall k.
\]

The above problem can be solved easily. To satisfy the constraints in (17) and (18), according to (1) and (2), it requires that

\[
b_{k} \geq (N-1) \left| \log_{2} P_{0} \lambda_{k} - \log_{2} \left( \frac{NI_{\text{max}}}{N-1} \right) \right|.
\]

The optimal solution to (16) is to set \( b_{k} \) as the smallest integer satisfying (19) for each PU, and the minimum required total feedback bits obtained under the unrounded \( b_{k} \) is

\[
\sum_{k=1}^{K} b_{k} = \sum_{k=1}^{K} (N-1) \left| \log_{2} P_{0} \lambda_{k} - \log_{2} \left( \frac{NI_{\text{max}}}{N-1} \right) \right|,
\]

which means the number of required total feedback bits increases with the average channel gain \( \lambda_{k} \) but decreases with the interference threshold \( I_{\text{max}} \).

### IV. Simulation Results

The proposed bit allocation solutions are evaluated with simulations. We consider the case of three PUs in the primary system, i.e., \( K = 3 \). The average channel gains from the ST to three PUs are respectively \( \lambda_{1}, \lambda_{2}, \lambda_{3} \), and the transmit power at the ST is set as \( P_{0} = 1 \). The codebooks for quantizing the CDIs are adopted as given in [8, 9].

The minimized maximum interferences versus different value of \( B \) under \( N = 4, 8 \) are provided in Fig. 1 where \( \lambda_{1} = 100, \lambda_{2} = 10 \) and \( \lambda_{3} = 1 \). The optimal solution in (3) and suboptimal solution in (7) by setting \( L = 100 \) are investigated, and both the rounded and unrounded results are provided. As shown in the figure, we can find that the proposed suboptimal solution has almost the same performances as the optimal one for both rounded and unrounded cases. The minimized maximum interference drops to zero as the total number of feedback bits increases, while the descending rate is dominated by the number of antennas \( N \) at the ST as indicated by the result in (15).

The minimized maximum interferences versus different values of \( B \) under different average channel gain are provided in Fig. 2 where \( \lambda_{1} = 100 \) and \( \lambda_{3} = 1 \) are fixed, \( \lambda_{2} \) varies in three cases, i.e., \( \lambda_{2} = 90, \lambda_{2} = 50 \) and \( \lambda_{2} = 10 \). As shown in the
Fig. 2. The minimized maximum interference versus different value of $B$ under different average channel gain.

The minimized maximum interference under the optimal solution to (3) and the asymptotical analysis in (15) are further compared in Fig. 3, where the setting is similar to Fig. 2 but with larger number of total feedback bits. As shown in the figure, we can find as the total number of bits $B$ gets larger, the minimized maximum interference converges to the asymptotical analysis in (15) gradually under different average channel gains, which validates our analysis in (15).

Finally, we evaluate the performance of the proposed solution for the total feedback bits minimization under the interference threshold in Fig. 4 with $N = 2, 4, 8$ respectively. As shown in the figure, as the interference threshold becomes larger, the minimum required number of total feedback bits decreases. Moreover, the minimum required number of total feedback bits increases with the average channel gains under different number of antennas as we have analyzed.

V. Conclusions
We have studied the problem of allocating the feedback bits for minimizing the maximum interference in CR system. The solution is proposed with low complexity and analyzed in asymptotic regime. It has revealed the minimized maximum interference is linear with the geometric mean of the average channel gains, and drops to zero exponentially as the number of feedback bits increases, while the descending rate is dominated by the number of antennas at the ST. We have also studied the total feedback bits minimization problem under a predefined interference threshold at the PUs, where the minimum required number of total feedback bits decreases as the interference threshold becomes larger. Simulation results demonstrate that the proposed schemes work very well for CR systems.

References
[1] S. Haykin, “Cognitive Radio: Brain-empowered Wireless Communications,” IEEE J. Sel. Areas Commun., vol. 23, no. 2, pp. 201-220, Feb. 2005.
[2] G. Villardi, Y. Alemseged, C. Sun, C. Sum, N. Tran, T. Baykas, and H. Harada, “Enabling Coexistence of Multiple Cognitive Networks in TV White Space,” IEEE Wireless Commun., vol. 18, no. 4, pp. 32-40, Feb. 2011.
[3] R. Zhang, “On Active Learning and Supervised Transmission of Spectrum Sharing Based Cognitive Radios by Exploiting Hidden Primary Radio Feedback,” IEEE Trans. Commun., vol. 58, no. 10, pp. 2960-2970, Oct. 2010.
[4] A. Taherpour, M. Nasiri-kenari, and S. Gazor, “Multiple Antenna Spectrum Sensing in Cognitive Radios,” IEEE Trans. Wireless Commun., vol. 9, no. 2, pp. 814-823, Feb. 2010.
[5] K. Huang and R. Zhang, “Cooperative Feedback for Multiantenna Cognitive Radio Networks,” IEEE Trans. Signal Process., vol. 55, no. 2, pp. 747-758, Feb. 2011.
[6] X. Chen, Z. Zhang, and C. Yuen, “Adaptive Mode Selection in Multiuser MISO Cognitive Networks With Limited Cooperation and Feedback,” IEEE Trans. Vehic. Technol., vol. 63, no. 4, pp. 1622-1632, May. 2014.
[7] 3GPP E-UTRAN S1 Application Protocol, TS36.413, online, 2015.
[8] N. Jindal, “MIMO Broadcast Channels With Finite-Rate Feedback,” IEEE Trans. Info. Theory, vol. 52, no. 11, pp. 5045-5060, Nov. 2006.
[9] T. Yoo, N. Jindal, and A. Goldsmith, “Multi-Antenna Downlink Channels with Limited Feedback and User Selection,” IEEE J. Sel. Areas Commun., vol. 25, no. 7, pp. 1478-1491, Jul. 2007.
[10] F. Yuan, C. Yang, G. Wang, and M. Lei, “Adaptive Channel Feedback for Coordinated Beamforming in Heterogeneous Networks,” IEEE Trans. Wireless Commun., vol. 12, no. 8, pp. 3980-3894, Aug. 2013.
[11] S. Boyd and L. Vandenberghe, “Convex Optimization,” Cambridge University Press, 2004.
[12] L. Guo and Y. Meng, “Round-up of Integer Bit Allocation,” Electronics Lett., vol. 38, no. 10, pp. 466-467, Oct. 2002.
[13] D. P. Palomar and J. R. Fonollosa, “Practical Algorithms for a Family of Waterfilling Solutions,” IEEE Trans. Signal Process., vol. 53, no. 2, pp. 686-695, Feb. 2005.