HyperSpectral classification with adaptively weighted $L_1$-norm regularization and spatial postprocessing

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Abstract
Sparse regression methods have been proven effective in a wide range of signal processing problems such as image compression, speech coding, channel equalization, linear regression and classification. In this paper we develop a new method of hyperspectral image classification based on the sparse unmixing algorithm SUNSAL for which a pixel adaptive $L_1$-norm regularization term is introduced. To further enhance class separability, the algorithm is kernelized using a RBF kernel and the final results are improved by a combination of spatial pre and post-processing operations. We show that our method is competitive with state of the art algorithms such as SVM-CK, KLR-CK, KSOMP and KSSP.

Introduction
Over the last fifteen years hyperspectral images (HSI), which record the electromagnetic spectrum in a few tens to a few thousands of spectral bands, have been used in a variety of tasks such as hyperspectral image classification [1, 2, 3], target detection [5, 6], analytical chemistry, astronomy, pharmaceutical process modelling and biomedical applications [7].

A few overview papers written on this subject are [7, 8, 9, 10], together with the March 2013 issue of the Proceedings of the IEEE and the January 2014 issue of the IEEE Signal Processing Magazine. For image classification purposes, the available methods can generally be subdivided into statistical, neural networks and sparse regression-based algorithms.

Traditionally, many supervised statistical classifiers use the training set data to build models of the underlying density in the feature space for each of the various classes in the training set (mostly by using a mixture of Gaussians) [11, 12, 13]. Besides the fact that the Gaussian assumption is often incorrect, this density estimation in high-dimensional spaces suffers from the Hughes effect: for a fixed amount of training data, the classification accuracy as a function of the dimension of the data (the number of hyperspectral bands) reaches a maximum and then declines with increasing dimension, because there is a limited amount of training data to estimate the larger and larger number of parameters needed to describe the model of the densities of each class. To deal with this, usually a feature selection step is first performed on the high-dimensional data to reduce its dimensionality. Thus statistical classifiers either require considerable pre-processing or may entirely fail to work correctly on high-dimensional data.

Another well-established method of supervised classification is the SVM (support vector machine) [11 ch.3, 14], which separates classes by a hyperplane whose parameters are established using the training data (dictionary). While it has proven itself immune to the Hughes effect, it needs to be retrained for every single change to its dictionary and, in order to achieve good classification results it needs to use spatial-spectral composite kernels [15]. However, the spatial information required here may not always be available as part of the provided dictionary. Other variations of the SVM have been proposed to further improve classification performance when the number of training samples is low such as the transductive SVM [16, 17, 18] which increases its original training set by iteratively including initially unlabeled samples in a semi-supervised manner, or LFDA-SVM [19].
Due to the success of sparse coding for image compression and face recognition applications [20], many sparse regression (SR) algorithms have been proposed for the domain of image and signal processing [21, 22, 23], including the hyperspectral pixel unmixing problem. Unlike the SVM which can be considered to be a discriminative method, SR algorithms can be seen as generative models, where the subspaces representing different classes compete with each other during the pixel unmixing process, leading to a vector of unmixing coefficients which has only a few non-zero, representative coefficients.

Recently, a fast and efficient method for spectral unmixing, the SUnSAL algorithm, has been proposed in [24, 25]. It has already been used with very good results for HSI classification in several papers [3, 4] and generalized to a spatial-spectral collaborative method, CL-SUnSAL, in [26, 27]. Given that the CL-SUnSAL method is slower converging than SUnSAL and not directly kernelizable, we propose an adaptively weighted L$_1$-norm SUnSAL algorithm together with spatial post-processing in order to achieve competitive classification results.

The rest of this paper is organized as follows: section A provides the necessary background concerning sparse unmixing via the SUnSAL algorithm, section B introduces our development of the adaptively weighted L$_1$-norm SUnSAL algorithm with spatial postprocessing for classification and section C summarizes our experimental results. A brief conclusion is presented in section D.

A. Background

Given the dictionary matrix $A \in \mathbb{R}^{k \times n}$ and the observed mixed hyperspectral pixel $y \in \mathbb{R}^k$, let $x \in \mathbb{R}^n$ be the vector of unknown unmixing coefficients of the columns of $A$. For classification purposes, there are several signatures (columns) of $A$ for each class, and therefore usually $n >> k$. Since it has been observed that a hyperspectral pixel is made up usually of a reduced number of spectral signatures (endmembers) compared to the total number of endmembers present in a given image [e.g. 7], we know that the solution vector $x$ is sparse, i.e. it contains only a few non-zero entries, and should be obtained as the result of a constrained sparse regression (CSR) problem such as the L$_1$-norm regularized optimization:

$$
P_{CSR} : \min_x \frac{1}{2} \|Ax - y\|_2^2 + \lambda \|x\|_1$$  \hspace{1cm} (1)

possibly subject to $x \geq 0$, where $\lambda$ is the parameter controlling the relative weight between the L$_2$ and L$_1$ terms.

To solve problems of the type shown in eq.(1) it has been observed that it is often easier and more efficient to solve problems of the type shown in eq.(2) via an Augmented Lagrangian method in which variable splitting has been introduced. To this effect, the ADMM (alternating direction method of multipliers) has been developed in [28] and further specialized in [24, 25]. It can briefly be described as a general optimization problem of the type:

$$
\min_{x \in \mathbb{R}^n} f_1(x) + f_2(Gx) = \min_{x \in \mathbb{R}^n, u \in \mathbb{R}^p} f_1(x) + f_2(u) \hspace{1cm} (2)
$$

where $f_1 : \mathbb{R}^n \to \overline{\mathbb{R}}$, $f_2 : \mathbb{R}^p \to \overline{\mathbb{R}}$ and $G \in \mathbb{R}^{p \times n}$, $\overline{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$. A description of the ADMM algorithm steps is shown in Listing 1.


Listing 1. ADMM algorithm [25, 28]
1. Set \( k=0 \), choose \( \mu > 0 \), \( u_0 \) and \( d_0 \).
2. Repeat
3. \[ x_{k+1} \in \arg\min_x f_1(x) + \frac{\mu}{2} \| Gx - u_k - d_k \|_2^2. \]
4. \[ u_{k+1} \in \arg\min_u f_2(u) + \frac{\mu}{2} \| Gx_{k+1} - u - d_k \|_2^2. \]
5. \( d_{k+1} \leftarrow d_k - (Gx_{k+1} - u_{k+1}) \).
6. \( k \leftarrow k + 1 \).
7. Until stopping criterion is satisfied.

B. Contribution

While \( \ell_1 \)-norm regularized cost functions such as (1) are convex and provide some measure of sparsity, they do not always provide sufficient sparsity and result in an incorrect recovery of the support of the signal \( y \) in terms of the columns of the dictionary matrix \( A \), which is a structural error [29]. On the other hand, it is known that \( \ell_p \)-norm regularized cost functions, \( 0 < p < 1 \), are much more effective at recovering the correct sparse support of the signal \( y \) as \( p \) approaches zero, but their solutions are increasingly prone to be caught in some local minimum as \( p \) decreases (convergence error), since they are not convex functions [29]. Many efforts have been deployed to obtain convex approximations to \( \ell_p \)-norm regularized cost functions with increasing success [30, 31, 32, 33].

In the context of the convex optimization problem \( P_{CSR} \) in eq.(1), we introduce a diagonal weighting matrix \( \Gamma \) meant to further enhance the sparsity of the solution:

\[ P_{CSR} : \min_x \frac{1}{2} \| Ax - y \|_2^2 + \lambda \| \Gamma x \|_1, \quad (3) \]

A diagonal entry in \( \Gamma \) is large if the corresponding column of \( A \) is far from the current hyperspectral pixel \( y \) and small if the two vectors are close together, thus representing how much importance the various columns of \( A \) and their corresponding regression coefficients in \( x \) should be given in the optimization problem. Many measures of closeness can potentially be used for the diagonal entries of \( \Gamma \), such as the Euclidean distance between \( y \) and each column of \( A \), the angle, or the Euclidean distance after using LFDA [34, 35].

Using the following definitions for \( f_1(x) \) and \( f_2(u) \) in eqs.(2) and (3)
\[ f_1(x) = \frac{1}{2} \| Ax - y \|_2^2 \]
\[ f_2(u) = \lambda \| u \| \]
\[ G = \Gamma, \quad u = \Gamma x. \]

Step 3. of the ADMM procedure requires solving a quadratic problem

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\(^1\) Note that introducing \( \Gamma \) in eq.(3) instead of just using the \( \ell_1 \)-norm of \( x \) could be seen as using an adaptive dictionary \( A \) for the unmixing of each hyperspectral pixel \( y \).
\[ x_{k+1} = \arg\min_x f_1(x) + \frac{\mu}{2} \| \Gamma x - u_k - d_k \|_2^2 = \]  
\[ \arg\min_x \frac{1}{2} \| A x - y \|_2^2 + \frac{\mu}{2} \| \Gamma x - u_k - d_k \|_2^2. \]  
(5)

with Hessian and linear term given by
\[ H = A^T A + \mu \Gamma^T \Gamma \]  
\[ f = A^T y + \mu \Gamma^T (u_k + d_k) \]  
and solution \( x_{k+1} = H^{-1} f \).

Step 4. of the ADMM procedure becomes
\[ u_{k+1} = \arg\min_u f_2(u) + \frac{\mu}{2} \| \Gamma x_{k+1} - u - d_k \|_2^2 = \]  
\[ \arg\min_u \frac{\mu}{2} \| \Gamma x_{k+1} - u - d_k \|_2^2 + \lambda \| u \|_1 = \]  
\[ \arg\min_u \frac{1}{2} \| u - v_k \|_2^2 + \frac{\lambda}{\mu} \| u \|_1. \]  
(7)

where we have used \( v_k = \Gamma x_{k+1} - d_k \).

The solution to (7) is the soft-threshold function [25, 36]
\[ u_{k+1} = \text{soft} \left( v_k, \frac{\lambda}{\mu} \right) \]  
(8)

To impose the positivity constraint on the solution one only needs to project the result in (8) onto the positive orthant:
\[ u_{k+1} = \max \left\{ 0, \text{soft} \left( v_k, \frac{\lambda}{\mu} \right) \right\} \]  
(9)

A summary of the SUnSAL algorithm steps with the adaptively weighted L1-norm is shown in Listing 2.

Listing 2. SUnSAL algorithm with adaptively weighted L1-norm regularization
1. Set \( k = 0 \), choose \( \mu > 0 \), \( u_0 \) and \( d_0 \).
2. Repeat
3. \[ H = A^T A + \mu \Gamma^T \Gamma, \quad f_k = A^T y + \mu \Gamma^T (u_k + d_k). \]
4. \[ x_{k+1} = H^{-1} f. \]
5. \[ v_k = \Gamma x_{k+1} - d_k. \]
6. \[ u_{k+1} = \text{soft} \left( v_k, \frac{\lambda}{\mu} \right). \]
7. \[ d_{k+1} = d_k - (\Gamma x_{k+1} - u_{k+1}). \]
8. \( k \leftarrow k + 1. \)
9. Until stopping criterion is satisfied.

Unlike the original SUnSAL [25] algorithm, the adaptively weighted L1-norm variant proposed in this section introduces an adaptive Hessian for each pixel which is unmixed. Since the singular value
decomposition used to invert the Hessian in the original paper [25] is too computationally complex to be used for every iteration, it has been replaced here for the solution of eq.(6) with the Cholesky decomposition of matrix $H$. Furthermore, we did not enforce the positivity contraint in eq.(9) as was done in [4, 25] since this more than doubled the computation time while not providing any advantage for the final classification accuracy.

To finish the classification task, the reconstruction residuals for each class are summed up for the closest (top) M neighbors in an NxN spatial window around each central pixel and the final decision is made in favor of the class which presents the minimum residual sum. Closeness to the central pixel in each window is measured based on the cosine of the angle between the central pixel and each neighbor. As opposed to the pre-lowpass filtering procedure method used in [4], such a post-processing approach is more selective, but it does not force all the pixels in the selected neighborhood to be unmixed with the same support, as CL-SUnSAL [26].

**C. Results**

The image used in our experiments is the Indian Pines scene from the Airborne Visible/Infrared Imaging Spectrometer (AVIRIS) [37] with a spatial resolution per pixel of 20 meters. It has 220 bands across the spectral range from 0.2 to 2.4µm, but in the experiments 20 water absorption bands have been removed (bands 104-108, 150-163, 220). There are 16 ground truth classes and 10% of the samples in each class were randomly chosen as training samples in each of our experiments as in [2, 4] for comparison purposes. Note that this method of sampling leads to undersampling of several classes (such as classes 1, 7, 9, 15, 16) and other works [3, 35] use only a reduced set of 9 classes for which an adequate number of samples can be provided.

The algorithm in **Listing 2.** was used in both the original data space as shown in column A and in its kernelized form, as shown in column B of **Table 1.** For the kernelized form all dot products between original data vectors occurring in products of the form $A^T A$, $A^T y$ have been kernelized using an RBF kernel with $\sigma =$2400. The cosine of the angle between vectors was used to select the top M closest neighbors of each central pixel. For column A, the Euclidean distance was used to set the weights in $\Gamma$, while for column B the cosine of the angle between vectors in the kernel induced high-dimensional Hilbert space was employed. The neighborhood window size was set to N=7 and M=35 nearest neighbors were used for each central pixel in the spatial post-processing phase. These choices were made based on cross-validation in order to maximize classification accuracy.

The classification performance of each of the 16 classes averaged over 20 random trials, the overall accuracy (OA) and the kappa coefficient of agreement $\kappa$ [38, 39] are shown in Table 1. The standard deviation of the OA is 0.63% for column A and 0.42% for column B. It was observed that our proposed algorithm was most effective in improving classification results when the rank of the Hessian $A^T A$ was considerably lower than its dimension. We see that our method performs better than the SVM and the logistic regression methods with spectral-spatial composite kernels, but slightly worse than KSOMP and KSSP. More results given by other classifiers can be found in [2, 4] for comparison purposes.
Table 1. Results for the Weighted $L_1$-norm SUnSAL algorithm compared with other classification methods$^2$

| Class | SVM-CK | KLR-CK | A original space | B kernelized | KSOMP | KSSP |
|-------|--------|--------|-----------------|--------------|--------|------|
|       |        |        | Weighted $L_1$-norm SUnSAL | Weighted $L_1$-norm SUnSAL |        |      |
| 1     | 95.83  | 75.00  | 81.39           | 91.76        | 97.92  | 91.67|
| 2     | 96.67  | 96.43  | 91.12           | 96.64        | 97.21  | 97.98|
| 3     | 90.93  | 95.47  | 85.90           | 95.49        | 96.67  | 97.73|
| 4     | 85.71  | 86.19  | 94.94           | 96.58        | 93.33  | 96.67|
| 5     | 93.74  | 96.42  | 96.47           | 96.87        | 95.75  | 94.85|
| 6     | 97.32  | 98.66  | 99.97           | 99.41        | 99.55  | 98.96|
| 7     | 69.57  | 82.61  | 66.15           | 86.92        | 60.87  | 17.39|
| 8     | 98.41  | 97.95  | 100             | 100          | 100    | 100  |
| 9     | 55.56  | 50     | 10.25           | 19.50        | 0      | 0    |
| 10    | 93.80  | 93.80  | 92.69           | 93.63        | 94.60  | 94.37|
| 11    | 94.37  | 95.54  | 99.10           | 98.14        | 99.28  | 98.33|
| 12    | 93.66  | 91.85  | 97.25           | 97.48        | 95.65  | 97.46|
| 13    | 99.47  | 100    | 99.72           | 99.48        | 100    | 100  |
| 14    | 99.14  | 96.56  | 99.97           | 98.43        | 99.83  | 99.91|
| 15    | 87.43  | 88.01  | 78.05           | 90.42        | 91.81  | 97.08|
| 16    | 100    | 88.24  | 98.00           | 97.79        | 91.76  | 94.12|
| OA    | 94.86  | 95.10  | 95.10           | 96.91        | 97.33  | 97.46|
| AA    | 90.73  | 89.55  | 86.93           | 91.16        | 88.39  | 86.03|
| k     | 0.941  | 0.944  | 0.944           | 0.965        | 0.970  | 0.971|

D. Conclusion

In this paper we have introduced a new sparse regression method for hyperspectral image classification based upon the SUnSAL algorithm [24, 25] which adaptively weighs the level of sparsity applied to each pixel. Unlike the SVM which can be considered to be a discriminative method, our method can be seen as a generative model similar to KSOMP and KSSP, where the subspaces representing different classes compete with each other during the pixel unmixing process, leading to a vector of unmixing coefficients which has only a few non-zero, representative coefficients. Such a method could also be used for spectral unmixing if the positivity constraint on the abundance fractions is enforced.

The results obtained in the experiments are competitive with state of the art algorithms such as SMV-CK, KLR-CK, KSOMP and KSSP, and we suspect they could be further improved by using as the adaptive weights in matrix $\Gamma$ the Euclidean distances between the pixel to be unmixed $y$ and the signatures in the dictionary matrix $A$ after applying LFDA [34]. Such a distance measure could also be used as a more accurate method of neighbor selection for the spatial post-processing phase of our classifier, as the new

$^2$ Note that our results for SVM-CK, KLR-CK, KSOMP and KSSP have been reproduced from [2, 4] since these papers used the same sampling methodology.
weights/distances would better reflect actual class membership. Further experiments are needed to test these hypotheses and also in order to validate our proposed method on a larger set of images.

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