Abstract

Measurement of Lepton-Flavor Violation (LFV) in the minimal SUSY Standard Model (MSSM) at Large Hadron Collider (LHC) is studied based on a realistic simulation. We consider the LFV decay of the second-lightest neutralino, $\tilde{\chi}_2^0 \rightarrow \tilde{l}' \rightarrow ll'\tilde{\chi}_1^0$, in the case where the flavor mixing exists in the right-handed sleptons. We scan the parameter space of the minimal supergravity model (MSUGRA) and a more generic model in which we take the Higgsino mass $\mu$ as a free parameter. We find that the possibility of observing LFV at LHC is higher if $\mu$ is smaller than the MSUGRA prediction; the LFV search at LHC can cover the parameter range where the $\mu \rightarrow e\gamma$ decay can be suppressed by the cancellation among the diagrams for this case.
1 Introduction

The minimal supersymmetric (SUSY) standard model (MSSM) is one of the attractive extensions of the standard model (SM). Many experiments are searching for the possible evidence of the low-energy supersymmetry. Among those, Lepton-Flavor Violation (LFV) processes may be considered as major discovery modes of supersymmetry; they do not exist in the SM or very small even if the small neutrino masses are introduced.

In the MSSM the off-diagonal components of the slepton mass terms violate lepton-flavor conservation, and they are related to the origin of the SUSY breaking terms and interactions in physics beyond the MSSM. An approximated universality of the sfermion masses should be imposed at some energy scale so that the flavor-changing processes are suppressed below the experimental bounds. One of the candidates to realize the universality is the minimal supergravity (MSUGRA) model \([1]\). In this model, the LFV slepton masses are induced by interactions above the GUT scale \([2]\) or the right-handed neutrino scale \([3]\). It is desirable to discover LFV in different processes so that we can reconstruct the off-diagonal slepton mass parameters which probe such interactions at the higher energy scale.

Main constraints for the off-diagonal components of the slepton mass matrices come from the rare decay process searches at low energy, such as \(\mu \rightarrow e\gamma\), \(\mu N \rightarrow eN\), and \(\tau \rightarrow \mu\gamma\). The current experimental limits are

\[
Br(\mu \rightarrow e\gamma) \ < \ 1.2 \times 10^{-11}\ [4], \\
Br(\mu N \rightarrow eN) \ < \ 6.1 \times 10^{-13}\ [5], \\
Br(\tau \rightarrow \mu\gamma) \ < \ 1.0 \times 10^{-6}\ [6].
\] (1)

Once the approximate universality of the scalar masses is imposed, the branching ratios are suppressed due to the GIM mechanism, and they can be smaller than these experimental bounds for reasonable SUSY parameter space. Therefore, those limits are not too serious at present. In future, some of the proposed experiments aim to reach to

\[
Br(\mu \rightarrow e\gamma) \ \lesssim \ 10^{-14}\ [7],
\]

\[
Br(\mu N \rightarrow eN) \ \lesssim \ 10^{-16}\ [8].
\]
LFV may be searched for at future collider experiments through the production and decay of the slepton. While the rare lepton decay widths suffer the suppression of the order of \((\Delta m_l/m_l)^2\), the production and decay processes of the slepton merely receive the suppression of the order of \((\Delta m_l/\Gamma_l)^2\). Here, \(\Delta m_l\) is the mass difference between the sleptons, and \(m_l\) and \(\Gamma_l\) are the average mass and decay width of the sleptons, respectively. Thus, the future high-energy collider experiments could explore the region of the parameter space which may not be reached to by the rare decay searches. At future e\(^+\)e\(^-\) linear collider experiments, the \(\tilde{e}\) production cross section could be very large if the bino-like neutralino mass \(M_1\) is relatively light, and then the \(e-\mu(\tau)\) mixing may be discovered there \([12][13]\). Similarly, a muon collider has a potential to access to \(\mu-\tau\) mixing \([14]\).

For the LFV search at Large Hadron Collider (LHC), the useful mode is the LFV decay of the second-lightest neutralino \((\tilde{\chi}_2^0)\), \(\tilde{\chi}_2^0 \rightarrow \tilde{l}l \rightarrow \tilde{\chi}_1^0 ll'\), where the slepton oscillation effect leads to the different flavors for the leptons in the final state \((l\text{ and } l')\). This mode has an advantage to observe LFV compared with the direct Drell-Yang production of the sleptons, since \(\tilde{\chi}_0^0\) can be copiously produced through the cascade decay of the squarks and gluinos \([15]\). Typically 60\% of the first- and second-generation left-handed squark decays into the wino-like neutralino and chargino, and in various models the right-handed slepton masses are predicted to be smaller than the second-lightest neutralino mass. By using the above mode, LFV in \(\tilde{e}-\tilde{\mu}\) mixing has been investigated by Agashe and Graesser \([13]\), in which they chose the point 5 of ATLAS TDR study \([16]\) in the MSUGRA model. The \(\tilde{\tau}-\tilde{\mu}\) mixing has been recently studied by Hinchliffe and Paige \([17]\).

In this paper, we estimate the reach of LFV at LHC based on a realistic simulation in the MSUGRA as well as a more generic model. We assume that LFV comes from mixing of the right-handed smuon and selectron. The signal of LFV \(\tilde{\chi}_2^0\) decay is the two opposite-sign leptons \((e^+\mu^-\text{ or } e^-\mu^+)\) in the final state. The distribution of the lepton-pair invariant mass \((m_{e\mu})\) in the
LFV final state has an edge whose value is known because it is the same as the edge of the lepton-flavor conserving opposite-sign modes ($m_{ee}$ and $m_{\mu\mu}$). Since the distribution of the background processes does not have an edge, we can obtain a sizable $S/N$ ratio in the region near the edge of $m_{e\mu}$.

In the MSUGRA model, LHC can reach to $M \sim 400$ GeV where $M$ is the common gaugino mass, when the mass difference and mixing angle of the right-handed selectron and smuon are $\Delta m = 1.2$ GeV and $\sin 2\theta = 0.5$. Unfortunately, the broad parameter space in the MSUGRA, which can be probed by LHC, are already excluded by $\mu \rightarrow e\gamma$ constraint. We point out that this is only for a case $\mu \gg M$ where $\mu$ is the Higgsino mass. The $\mu$ parameter may be close to the gaugino masses in the more generic model, and in the case the reduction of $Br(\mu \rightarrow e\gamma)$ by cancellation among the diagrams is generic. We study a model of this kind (cMSSM) and we find that the experimental reach of LFV search at LHC extends up to $M \sim 500$ GeV for the same slepton oscillation parameter.

We organize this article as follows. In the next section we discuss prospect of the LFV searches at LHC in the MSUGRA and the cMSSM, and present sensitivity at LHC for generic parameters in Section 3. Section 4 is devoted to conclusions and discussion. In Appendix A we present formula we used in this paper, and in Appendix B the mass spectrum and the branching ratios for the SUSY particles are given for the sample parameter sets we adopt in this paper.

2 Prospect of LFV searches at LHC in the MSUGRA and the more generic models

In the MSUGRA model, the GUT scale scalar mass $m$ and trilinear coupling $A$ are universal among generations, and the gaugino masses in the MSSM are given by the SU(5) gaugino mass $M$. The universal scalar mass is predicted when the SUSY breaking sector couples to chiral multiplets equally. However, it is possible, and might be natural, to assume the different scalar masses between the matter fields and the Higgs fields in the GUT context. For example, in the most simple SO(10) model, we can set the following boundary
condition,
\begin{align*}
m_Q = m_{\tilde{q}_R} = m_{\tilde{l}_L} = m_{\tilde{l}_R} (\equiv m_{16}), \\
m_{H_1} = m_{H_2} (\equiv m_{10}).
\end{align*}
(2)

We call this choice as the cMSSM. The MSUGRA model predicts $\mu \gg M$. On the other hand, the cMSSM allows $\mu$ comparable to $M$. We will discuss the phenomenology of these models in the context of the LFV search at LHC.

2.1 The MSUGRA model

We summarize qualitative features of the MSUGRA predictions relevant to the LFV study at LHC.

• $\mu \gg M$

The Higgs masses at the GUT scale are common to the other scalar masses. The Higgsino mass parameter $\mu$ can be calculated as a function of $m$, $M$, and $\tan \beta$ so as to reproduce the correct electroweak symmetry breaking. Especially for $m_t \sim 175$ GeV, the dependence on $m$ effectively disappears when $m$ is of the order of $M$, which is sometimes referred to the focus point \cite{focus_point}. In such a case, $\mu/M$ is more or less fixed, and the LSP ($\tilde{\chi}^0_1$), the lighter chargino ($\tilde{\chi}^+_1$), and the second-lightest neutralino ($\tilde{\chi}^0_2$) are gaugino-like. The $\mu$ parameter is lighter than the gaugino masses only when $m \gg M$, while the decay of $\tilde{\chi}^0_2$ to $\tilde{l}$, which is relevant to our study, is closed in the case. Thus, one can safely assume $\tilde{\chi}^0_2$ and $\tilde{\chi}^0_1$ are wino- or bino-like in our study.

• Light $\tilde{\tau}_1$

The lighter stau $\tilde{\tau}_1$ is lighter than the other sleptons due to the left-right mixing proportional to $\tan \beta \times \mu$. Note that the lower limit of the Higgs mass now strongly constrain small $\tan \beta$ cases \cite{tan_beta}, and therefore the mixing is expected to be significant. The typical outcome is an increased branching ratio of $\tilde{\chi}^0_2$ to $\tilde{\tau}$, and reduced branching fractions to the first- and the second-generation sleptons $\text{Br}(\tilde{\chi}^0_2 \rightarrow \tilde{e}, \tilde{\mu})$. 

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The fact that the MSUGRA model tends to reduce the branching ratio of $\tilde{\chi}_2^0$ to the first- and second-generation sfermions directly limits potential to search for LFV at LHC. This can be seen in Fig. 1, where we plot contours of some constant branching ratios of $\tilde{\chi}_2^0$ for $\tan\beta = 10$ and $\mu = 1.5M_2$ in the cMSSM, where $M_2$ is the gaugino mass of SU(2)$_L$. The $\mu/M_2$ ratio is roughly what the MSUGRA model predicts for moderate $\tan\beta$ and $M \sim m$. Due to the Higgs mass constraint, we do not plot the region where $M < 300$ GeV. It is found from this figure that the branching ratios to $\tilde{e}_R$ or $\tilde{\mu}_R$ are less than 6% for $M > 420$ GeV. The reasons are the following. First, note that the decay into the Higgs boson opens for $M > 325$ GeV, and it quickly dominates over the other decay processes. Since $\tilde{\chi}_2^0$ is wino-like, the decay $\tilde{\chi}_2^0 \rightarrow h \tilde{\chi}_1^0$ is suppressed by only $N_H \propto 1/\mu$, while $\tilde{\chi}_2^0 \rightarrow \tilde{e}_R$ is suppressed by $N_B \propto 1/\mu^2$. Here $N_H$ and $N_B$ is the Higgsino and the bino components of $\tilde{\chi}_2^0$, respectively. As $\mu$ is relatively high in the MSUGRA model, the branching ratio to $h$ is significantly larger than that to $\tilde{I}_R$. Second, large $\mu \tan\beta$ induces non-negligible $\tilde{\tau}$ left-right mixing. Due to the left-right mixing, $\tilde{\chi}_2^0$ decays into $\tilde{\tau}_1$ through the $\tilde{\tau}_L$ component in $\tilde{\tau}_1$, and the decay branching ratio dominates over the branching ratios to $\tilde{e}_R$ or $\tilde{\mu}_R$. Third, if $m \ll M$, the decay into $\tilde{I}_L$ is open, and this quickly dominates over the other decay modes. Thus, the region where the decay of $\tilde{\chi}_2^0$ to $\tilde{e}_R$ or $\tilde{\mu}_R$ has a sizable branching ratio is limited. We will discuss the LFV signal in the next section.

2.2 The cMSSM with $\mu \sim M_2$

We now turn into the case where universality of the sfermion masses does not hold for the Higgs sector as Eq. (2). We first argue that this model predicts different phenomenology for the LFV search.

In Figs. 2 and 3, we show the $\tilde{\chi}_2^0$ branching ratios for $\mu = M_2$ and $\tan\beta = 10, 20$. Not only the decay to $\tilde{e}_R$ is kinematically open, the branching ratio is larger than that of the MSUGRA case. This is because the relatively large Higgsino-gaugino mixing makes the $\tilde{\chi}_2^0$ mass smaller than $M_2$ and the mass difference between $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ becomes smaller. This makes the decay of $\tilde{\chi}_2^0$ to $h$ close up to $M \sim 420$ GeV. At the same time, the decay into $\tilde{I}_L$ is not
The decay into $\tilde{\tau}_1$ is also suppressed due to the reduced left-right mixing. Thus, in the cMSSM with $\mu \sim M$, the prospect of finding LFV at LHC is considerably better compared with the MSUGRA model.

Varying the $\mu$ parameter also non-trivially changes the $\mu \rightarrow e\gamma$ branching ratio. In Fig. 4, we show the contours of constant $Br(\mu \rightarrow e\gamma)$ in $m_{16}$ and $\mu/M_2$ plane for the GUT scale boundary condition given in Eq. (2). Here we take $\tan \beta = 10$ and $M = 300$ GeV. We assume the only non-zero mixing mass term between $\tilde{e}_R$ and $\tilde{\mu}_R$ exists at the GUT scale. We take the difference of scalar masses at the GUT scale and the mixing as $\Delta m = 1.2$ GeV and $\sin 2\theta = 0.5$.

In the MSUGRA model, which corresponds to $\mu/M_2 \sim 1.5$, $Br(\mu \rightarrow e\gamma)$ is less than $10^{-11}$ for $m_{16} = m > 210$ GeV and becomes minimum for $m_{16} \sim 300$ GeV. The suppression around $m_{16} \sim 300$ GeV is due to cancellation among diagrams that will be discussed soon. Although LFV in the sfermion decays
does not suffer from such a cancellation among diagrams, the decay of \( \tilde{\chi}_2^0 \) to \( \tilde{l} \) is closed for \( m_{16} = m > 210 \text{ GeV} \). This shows that it is difficult to observe LFV at LHC in the MSUGRA model. The constraint from \( Br(\mu \rightarrow e\gamma) \) is strong to observe LFV at LHC in the MSUGRA model for generic slepton oscillation parameters as we see in the next section.

The constraint weakens in the region where \( \tilde{\chi}_2^0 \rightarrow \tilde{l}_R \tilde{l} \) is open, when \( \mu \) is smaller than the MSUGRA prediction. The parameter dependence of the cancellation can be explained by the mass-insertion formula \[20\], which is expressed by the off-diagonal component \( m_{\tilde{e}_R\tilde{l}_R}^2 \) in the right-handed slepton mass matrix. It occurs among the 4 different amplitudes where the chirality flip occurs on either in the external or internal lines. Among them two diagrams involving the left-right mixing or lepton-slepton-Higgsino vertex have a common overall factor \( m_{\tilde{e}_R\tilde{l}_R}^2/m_{\tilde{e}_R}^4 \times M_1 \mu \tan \beta/m_{\tilde{e}_R}^2 \) with opposite-sign coefficients. When \( \mu \tan \beta \) is large and the absolute values of the two amplitudes are larger than the others, a nearly complete cancellation could occurs when \( \mu/m_{16} \sim \mu/m_{16} \sim 1.5 \), as in Fig. \[4\]. When \( \mu < 1.2 M_2 \), the cancellation occurs for \( m_{16} < 200\text{GeV} \) where the decay of \( \tilde{\chi}_2^0 \) to \( \tilde{l}_R \) is kinematically open.

We have been discussing the case where LFV occurs due to the non-zero \( m_{\tilde{e}_R\tilde{l}_R}^2 \). We will comment on the other cases in the last section.

### 3 Potential of the LFV search at LHC in the cMSSM model

In the limit of lepton-flavor conservation, the process \( \tilde{\chi}_2^0 \rightarrow \tilde{l}l \rightarrow ll\tilde{\chi}_0^0 \) at LHC has been studied by many authors \[16\]. The distribution of the lepton-pair invariant mass (\( m_{ll} \)) in the final state is given by

\[
\frac{d\Gamma_{l^-l^+}}{dm_{ll}} \propto \begin{cases} 
 m_{ll} (0 \leq m_{ll} \leq m_{ll}^{max}) \\
 0 & (m_{ll}^{max} < m_{ll})
\end{cases},
\]

where the edge \( m_{ll}^{max} \) is expressed by the slepton mass \( m_{l} \) and the neutralino masses \( m_{\tilde{\chi}_1,2} \), as follows:

\[
(m_{ll}^{max})^2 = m_{\tilde{\chi}_2^0}^2 (1 - \frac{m_{\tilde{l}}^2}{m_{\tilde{\chi}_2^0}^2}) (1 - \frac{m_{\tilde{\chi}_2^0}^2}{m_{\tilde{l}}^2}).
\]

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Figure 2: Contours of constant branching ratios in the cMSSM model for \(\tan \beta = 10\) and \(\mu = M_2\). Solid lines are for \(Br(\tilde{\chi}^0_2 \to h\tilde{\chi}^0_1) = 0.5, 0.7\) (from left to right), dotted lines for \(Br(\tilde{\chi}^0_2 \to \tau\tilde{\tau}^0_1) = 0.6, 0.2\) (from bottom left), and thick solid lines for \(Br(\tilde{\chi}^0_2 \to e\tilde{e}_R) = 0.2\). The LSP is charged above the thick solid line at top left.

Figure 3: Contours of constant branching ratios in the cMSSM model for \(\tan \beta = 20\) and \(\mu = M_2\). Solid lines are for \(Br(\tilde{\chi}^0_2 \to h\tilde{\chi}^0_1) = 0.1, 0.3, 0.5\) (from bottom to top), dotted lines for \(Br(\tilde{\chi}^0_2 \to \tau\tilde{\tau}^0_1) = 0.9, 0.8, 0.6\) (from bottom to top), a thick solid line for \(Br(\tilde{\chi}^0_2 \to e\tilde{e}_R) = 0.1\). The LSP is charged above the thick solid line at left.
This decay process would be identified through the edge in Eq. (4). The main background comes from uncorrelated leptons from different squark or gluino decay chains. Fortunately, the background are estimated using the $e^\pm \mu^\mp$ distribution, and the distribution is smooth and decreases monotonically as $m_{e\mu}$ increases. Thus, the background can be subtracted, and the subtracted distribution has a canonical structure as $d\Gamma/dm_{ll} \propto m_{ll}$ and terminates at $m_{ll}^{\text{max}}$.

The signature of LFV on the process is the edge structure of $e^\pm \mu^\mp$ distribution on top of the accidental leptons from uncorrelated sources, since such an efficient subtraction method as above does not exist. The level of the signal and the background would be estimated if the production cross sections, acceptance of the signal and the background, and the background distribution could be estimated. This is of course possible by doing the MC simulation for each model parameters, however, we present semi-analytical approach in this section. In the following subsections, we will discuss the production cross section of the SUSY particles, level of the signal and the background, and the background distribution, and show the experimental reach of the LFV search at LHC at the end.
Table 1: Description of the set of parameters A)-D). We chose \( \tan \beta = 10 \) and \( A = 0 \) in the MSUGRA model. The GUT scale gaugino mass \( M \), the scalar mass \( m \), the squark and gluino masses are given.

|     | \( M(\text{GeV}) \) | \( m \) | \( m_{\tilde{q}} \) | \( m_{\tilde{q}_L} \) | \( m_{\tilde{g}} \) |
|-----|----------------------|---------|------------------|------------------|------------------|
| A)  | 300                  | 100     | 706              | 633              | 470              |
| B)  | 350                  | 125     | 818              | 729              | 549              |
| C)  | 400                  | 150     | 915              | 824              | 627              |
| D)  | 500                  | 175     | 1135             | 1012             | 781              |

Table 2: Production cross sections for the parameters A)-D).

|     | \( \sigma(\text{total}) \) | \( \sigma(\tilde{g}\tilde{g}) \) | \( \sigma(\tilde{g}\tilde{q}) + \sigma(\tilde{g}\tilde{q}^*) \) | \( \sigma(\tilde{q}\tilde{q}) \) | \( \sigma(\tilde{q}\tilde{q}^*) \) |
|-----|-----------------------------|-----------------------------|-------------------------------------------------|-----------------------------|-----------------------------|
| A)  | 25.0                        | 3.03                        | 13.04                                           | 4.70                        | 2.89                        |
| B)  | 11.2                        | 1.11                        | 5.67                                            | 2.41                        | 1.41                        |
| C)  | 5.74                        | 0.45                        | 2.87                                            | 1.44                        | 0.67                        |
| D)  | 1.60                        | 0.09                        | 0.76                                            | 0.51                        | 0.19                        |

3.1 Production cross section of the SUSY particles

We start our discussion from an estimation of the squark and gluino production cross sections. The second-lightest neutralino \( \tilde{\chi}_2^0 \) is produced through the cascade decays of \( \tilde{q}_L \) or \( \tilde{g} \). The signal rate of the LFV decay \( \tilde{\chi}_2^0 \to l'^{+}l\tilde{\chi}_1^0 \) depends on the production cross section significantly, because it reduces very quickly with increase of the squark and gluino masses.

We use the ISAJET version 7.51 \[21\] to estimate the production cross sections. We are interested in the region of parameter space where \( m \lesssim M \) so that the \( \tilde{\chi}_2^0 \) decay to \( \tilde{l} \) is open. In this range, we choose the four parameter sets A)–D) in Table 1 as samples and derive a fitting functions for the production cross sections in Table 2. The fitting functions will be used later to estimate the number of the signal and background \( e^\pm \mu^\mp \) events.

Since the squark and gluino masses are quite close to each other for those points, we fit the production cross sections by the following simple functions

\[\text{The parton distribution is given by CTEQ3L.}\]
\[\text{We omit the cross sections of the stop- and sbottom-pair production since they give negligible contributions to the }\mu e\text{ background.}\]
of $m_\tilde{g}$,

$$\sigma(\tilde{q} \text{ or } \tilde{g}) = a_1 (m_\tilde{g}/\text{TeV})^{-a_2} \text{ (pb)}. \quad (5)$$

The production cross sections are fitted very nicely by the parameters $a_1$ and $a_2$, listed in the Table 3. When the scalar masses are changed in a relevant region of the parameter space, the cross sections are changed within only 20%, which is within QCD uncertainties.

Note that the scaling parameter $a_2$ depends on the production modes substantially. Processes involving initial state gluon(s) such as $\sigma(\tilde{g}\tilde{g})$, $\sigma(q\tilde{g})$ reduces quickly compared with $\sigma(\tilde{q}\tilde{q})$ production when gluino and squark masses become heavier. This comes from a fact that the parton distributions of the gluon and sea quarks are generally softer than valence quarks.

The $\tilde{u}/\tilde{d}$ and $\tilde{q}_L/\tilde{q}_R$ ratios affect the level of $\mu e$ background. The $\tilde{u}_L\tilde{d}_L$, $\tilde{u}_L\tilde{u}_L^*$, and $\tilde{d}_L\tilde{d}_L^*$ production are likely to contribute to the signal compared with the other squark productions, because the up-type squark $\tilde{u}_L$ would decay into $\tilde{X}_1^+$ producing $l^+$ while $\tilde{d}_L$ producing $l^-$, and $\tilde{q}_R$ decay mostly into $\tilde{X}_1^0$ when $\tilde{X}_1^+, \tilde{X}_0^1, \tilde{X}_2^0$ are gaugino-like. Also, note that the production is dominated by valence $u$ and $d$ components of parton distribution function as in Table 2. Thus, we adopt a simple approximation that $\tilde{u} : \tilde{d} = 2 : 1$, $\tilde{u}^* : \tilde{d}^* = 1 : 1$, and $\tilde{q}_L : \tilde{q}_R = 1 : 1$ in any squark and/or anti-squark production cross sections.\footnote{In the MC simulation, the ratio of production cross sections, $\sigma(\tilde{u}\tilde{u}) : \sigma(\tilde{u}\tilde{d}) : \sigma(\tilde{d}\tilde{d})$, is 4.4:3.9:1 for point A) and 7.5:3.1 for point D). For $\tilde{q}\tilde{q}^*$ production cross sections, the ratio $\sigma(\tilde{u}\tilde{u}^*) : \sigma(\tilde{u}\tilde{d}^*) : \sigma(\tilde{d}\tilde{d}^*)$ is 5.7:4.2:1:3:9 for point A) and, $\sigma(\tilde{q}_L\tilde{q}_L) : \sigma(\tilde{q}_L\tilde{q}_R) : \sigma(\tilde{q}_R\tilde{q}_L) : \sigma(\tilde{q}_R\tilde{q}_R) = 1:1.4:1.2$ and $\sigma(\tilde{q}_L\tilde{q}_L^*) : \sigma(\tilde{q}_L\tilde{q}_R^*) : \sigma(\tilde{q}_R\tilde{q}_L^*) : \sigma(\tilde{q}_R\tilde{q}_R^*) = 1:0.93:0.90:1.15 for point A).}

|   | $\sigma(\text{total})$ | $\sigma(\tilde{g}\tilde{g})$ | $\sigma(\tilde{g}\tilde{g}) + \sigma(\tilde{q}\tilde{q})$ | $\sigma(\tilde{q}\tilde{q})$ | $\sigma(\tilde{q}\tilde{g}^*)$ |
|---|--------------------------|----------------|-------------------------------------|-------------------|-------------------|
| $a_1$ | 3.34                     | 0.23           | 1.74                               | 0.92              | 0.39              |
| $a_2$ | 5.78                     | 7.407          | 5.780                              | 4.677             | 5.733             |

Table 3: Results of the fit of the production cross sections to the functions given in Eq. (5).
3.2 Level of the signal and the background and the acceptance

Next, we will estimate the level of the signal and the background, and evaluate the acceptance. For this purpose, we use the MC data for the following parameters:

**point I)** $\mu = 497.87\text{GeV}$, $\tan\beta = 2.1$, $M = 300\text{ GeV}$, and $m = 100\text{ GeV}$ in the MSUGRA \[22\].

**point II)** $\mu = 199.85\text{GeV}$ $\tan\beta = 10$, $M = 250\text{ GeV}$, and $m_{16} = 90\text{ GeV}$ in the cMSSM \[23\].

To estimate level of the background, we have to calculate branching ratios $\text{Br}(\tilde{g} \rightarrow l^\pm X)$, $\text{Br}(\tilde{g} \rightarrow l^+l^-X)$, $\text{Br}(\tilde{q} \rightarrow l^\pm X)$, and $\text{Br}(\tilde{q} \rightarrow l^+l^-X)$. We find that $\tilde{g}$ has a large branching ratio into multiple leptons. For example, $\tilde{g} \rightarrow \tilde{t}t$ and $\tilde{g} \rightarrow \tilde{b}_1b$ followed by $\tilde{b}_1 \rightarrow t\chi^+_1$ or $\tilde{b}_1 \rightarrow \tilde{t}W^-$ have fractions 15%, 6%, and 4%, respectively, for point I). This results in a high probability to get multiple leptons in the final states; $\text{Br}(\tilde{g} \rightarrow (\tilde{t}, \tilde{b}_1, \tilde{b}_2) \rightarrow l) = 4%$ and $\text{Br}(\tilde{g} \rightarrow (\tilde{t}, \tilde{b}_1, \tilde{b}_2) \rightarrow ll or l\tau (\rightarrow l')) = 2%$. Thus, the $\tilde{g}$ production may be a significant source for the background for the LFV search compared with the squark-pair production processes, since the squark-pair production processes are required to have two cascade decays involving a lepton each so that they contribute to the background.

The chargino production is also a significant source of the background. A chargino may decay into $W, \tilde{\nu}$, and $\tilde{\tau}_1$, followed by decay into leptons in the final state.\footnote{The mass parameters and relevant branching ratios are given in the Appendix B, which are calculated by the ISAJET.} A produced tau lepton may further decays into $e$ or $\mu$, whose branching fraction is about 35%. The second-lightest neutralino $\tilde{\chi}^0_2$ may also decay into $\tau\tilde{\tau}_1$. While the momentum of $e$ or $\mu$ from the tau-lepton decay tends to be low, the acceptance of such leptons would not be negligible for the adopted lepton $p_T$ cut as low as 10 GeV. For the case where $\mu \sim M_2$, one should also take care of the decay of $\tilde{q}$ into $\tilde{\chi}^0_{3(4)}, \tilde{\chi}^\pm_2$. They are also calculated and added in our background estimation.

\footnote{For example, 89% of chargino decays into $\tilde{\tau}_1$ for point I).}
The total $e^\pm \mu^\mp X$ and $l^+l^- X$ events before cuts are shown in Table 4. The numbers will be compared with the MC results in order to derive the acceptance. They are obtained by calculating the gluino and squark branching ratios into the lepton(s) using the major branching modes mentioned above, and multiplying our fitted production cross sections described previously to the branching ratios. This semi-analytical calculation is checked by independent toy MC simulations which includes all decay steps. We include the contribution from $\tilde{q}\tilde{q}$, $\tilde{q}\tilde{q}^*$ and $\tilde{q}\tilde{g}$ production. $N_{\ell\nu}$ is the number of the $e^\pm \mu^\mp$ events coming from primary leptons from $W$ or $\tilde{l}$ decay. $N_{\tau}$ is the number of the events with one primary leptons and one lepton from $\tau \to e$ or $\mu$. $N_{\tau\tau}$ is the number of $e^\pm \mu^\mp$ events from the leptonic decay of two $\tau$'s; here we omit the contribution from $\tilde{\chi}^0_1 \to \tau^+ \tau^- \tilde{\chi}^0_1$, because the $m_{\nu\ell}$ distribution are substantially softer than expected LFV signal $\tilde{\chi}^0_2 \to \tilde{l}'\ell \to l'l\tilde{\chi}^0_1$. Note that our calculation does not include the probability that $3\ell$, or $4\ell$ events are accepted as two lepton events etc.

Point II) is substantially lepton rich compared with point I) due to the enhanced branching ratio into sleptons. This is typical for parameters with $\mu \sim M_2$ with light sleptons as shown in Fig. 4.

Now we can estimate the acceptance of events involving $l^+l^-$, $e^+\mu^-$, or $e^-\mu^+$. Here we use MC data produced and simulated by the ISAJET and the ATLFAST. $2 \times 10^6 (1 \times 10^7)$ events are generated for point I) (point II)) corresponding to 95(196)fb$^{-1}$ of the integrated luminosity. In the simulation, we adopted the cuts given in [24]

- $E_{\text{miss}}^T > \max (100\text{GeV}, 0.2M_{\text{eff}})$,
- $P^T_{j_1} > 100\text{ GeV}$, $P^T_{j_2,j_3,j_4} > 50\text{ GeV}$,
- $M_{\text{eff}} > 400\text{ GeV}$,
- a pair of isolated opposite sign leptons with $P^T_\ell > 10\text{ GeV}$.

Those cuts are chosen to reduce backgrounds from QCD processes to a negligible level. After the cuts, the $e^\pm \mu^\mp$ events are reduced to 3609 events for point I), and 42721 events for point II). (See $N(e\mu)(\text{MC})$ in Table 4.)
Table 4: Number of events with $e^{\pm}\mu^{\mp}$ before cuts from different decay processes $N(l'^l)$, $N(l\tau)$ and $N(\tau\tau)$ when lepton flavor is conserved. (See text.) The numbers are for lepton-flavor conservation. $N(e\mu)(MC)$ is the numbers of accepted events with $e^{\pm}\mu^{\mp}$ satisfying $0\text{GeV}<m_{e\mu}<200\text{GeV}$, obtained by the ISAJET+ATLFAST simulation.

| point   | $N(e\mu)$ (MC) | $N(l'^l)$ | $N(l\tau)$ | $N(\tau\tau)$ |
|---------|----------------|-----------|------------|--------------|
| point I | 3609           | $1.49 \times 10^4$  | $3.61 \times 10^4$  | $3.2 \times 10^2$  |
| point II| 42721          | $3.1 \times 10^4$   | $10.8 \times 10^4$ | $5.2 \times 10^4$  |
| point I b veto | $1.58 \times 10^4$  | $5.4 \times 10^2$   | $0.67 \times 10^2$  |
| point II b veto | $3.78 \times 10^3$  | $3.85 \times 10^4$  | $2.20 \times 10^4$  |

comparing $N(l'^l)$, $N(l\tau)$, $N(\tau\tau)$ with $N(e\mu)(MC)$, we estimate the acceptance of uncorrelated $e\mu$ events at the ATLAS detector; 19% for point I), and 22% for point II).

The acceptance of LFV signal may be also estimated by looking at the the acceptance of opposite-sign same-flavor leptons in the same simulation. Note that the LFV signal from the $\tilde{\chi}_2^0$ decay has exactly the same kinematics to the lepton-flavor-conserving decay. We list in Table 5 the result of the MC simulation of the same samples as that of Table 4 and the estimated $l^+l^-$ production from the $\tilde{\chi}_2^0$ decay before the cut. The acceptance of the process is 28%. This is higher than that of $e^{\pm}\mu^{\mp}$ events under the same cut. To be conservative, we adopt constant 25% acceptance for both the signal and the background.

To suppress the background furthermore, the $b$-jet veto may be efficient while we will not use it in this article. In the Tables 4 and 5, we show numbers of events which do not involve the third-generation squarks in the cascade decays. The events involving $\tilde{t}$ or $\tilde{b}$ could be removed by the $b$-jet veto. If the efficiency of the veto is ideal, $N(l'^l)$ is reduced by an order as in Table 4, while the number of the correlated leptons from $\tilde{\chi}_2^0 \rightarrow \tilde{l}$ decays does not change in Table 5 by the $b$-jet veto. However, the rejection factor depends on the decay patterns. For example, $\tilde{\chi}_1^+ \rightarrow \tilde{\tau}$ dominates the chargino decay for point II), and the decay $\tilde{q}_L \rightarrow \chi_1^+$ becomes the efficient source of $\tau$ lepton. This means that $N(l\tau)$ is not reduced so much by the $b$-jet vet in point II).
Table 5: The number of $\tilde{\chi}_0^2 \rightarrow l^+l^-\tilde{\chi}_1^0$ when LFV is absent. The accepted $l^+l^-$ events in MC simulations are also given in the table.

| point               | $l^+l^-$ events before cut | accepted $l^+l^-$ by MC |
|---------------------|----------------------------|-------------------------|
| point I) (95fb$^{-1}$) | $8.72 \times 10^4$         | $2.55 \times 10^4$      |
| point II) (196 fb$^{-1}$) | $8.97 \times 10^5$           | $2.55 \times 10^5$      |
| point I) $b$ veto   | $6.51 \times 10^4$             |                         |
| point II) $b$ veto  | $6.96 \times 10^5$             |                         |

3.3 Background distribution

The signal distribution from $\tilde{\chi}_2^0$ decay increases with $m_{\ell\ell}$ as in Eq. (3) and the distribution has an edge determined by the neutralino and the sfermion masses. On the other hand, backgrounds come from $t$, $W$, and $\tilde{\chi}_i^\pm$ decays and do not have the edge structure. They reduce rather quickly as $m_{\ell\ell}$ increases. Therefore it is better to use the data near $m_{\ell\ell}^{\text{max}}$ so that $S/N$ ratio maximizes.

We should note that the position of the $m_{\ell\ell}$ edge is known precisely from the same-flavor opposite-sign $m_{\ell\ell}$ distribution.

To estimate the number of the background events near the edge, we again use the MC data and fit it to the following fitting function,

$$\frac{d\Gamma}{dm_{\ell\ell}} = k(m_{\tilde{g}}) \exp \left( -c m_{\tilde{g}} \times m_{\ell\ell} \right).$$

The background distributions and the fitting curves are shown in Figs. 3 and 4. The data between 40 to 200 GeV and 100 to 200 GeV are used for the fit. The best fit is obtained $c = 10.4$ for point I) and $c = 13.7$ for point II) for $30 \text{ GeV} < m_{e\mu} < 200 \text{ GeV}$. The fitting function reproduces the large $m_{\ell\ell}$ region reasonably well, but it fails significantly in the small $m_{\ell\ell}$ region. It is natural that the distribution has a certain peak, which must be proportional to a typical momentum of leptons of the uncorrelated production process, such as the half of $W$ boson mass. The distribution beyond this peak must be more sensitive to typical momentums of $W$, $t$, or $\tilde{\chi}_1^+$, which may depend on the gluino and squark masses. This is the reason we choose Eq. (5) as a fitting function. The average value for $c$ of those two points, $c = 12.1$, is used for our background estimation. For the plot, $k(M)$ may be fixed so that overall normalization agrees for the region used for the fit.
Figure 5: The $m_{\ell\ell}$ distribution for point I). The data corresponds to integrated luminosity of 95 fb$^{-1}$ and standard cuts are applied (see text). The histogram shows the distribution without LFV, while bars are number of events and the error with $\tilde{\mu}$-$\tilde{e}$ mixing. In the plot, $1/30$ of $\tilde{\chi}^0_2 \rightarrow \tilde{\ell}^0 \ell$, $\tilde{\ell}^0 \rightarrow \tilde{\chi}^0_1 \ell'$ decay chain is assumed to go to the $e\mu$ channel. Two curves are fits to the background distribution in the region $m_{\ell\ell} = 40$–200 GeV(solid) and $m_{\ell\ell} = 100$–200(dashed then solid). We use $c = 12.1$.

Figure 6: Same as Fig. 5, but for point II). The integrated luminosity is 196 fb$^{-1}$, and $c = 13.7$. 
3.4 Significance of LFV at LHC

Having gone through all estimation needed, we now calculate significance of the LFV signal at LHC. Numbers of the signal \( N_{\text{sig}} \) and background \( N_{\text{bg}} \) are estimated by \( (\text{the fitted cross sections}) \times (\text{the branching ratios}) \times (\text{the overall acceptance 25%}) \times (\text{the integrated luminosity}) \). We assume the signal distribution in Eq. (3) and background distribution in Eq. (6). We determine the overall normalization factor of Eq. (6) so that number of background above \( m_{ll'} > 20 \text{ GeV} \) agrees with the estimation.

We define the \( \Delta \chi^2 \) using the estimated signal and background events between

\[
\max \left( 30\text{GeV}, \frac{2}{3} m_{ll}^{\text{max}} \right) < m_{ll} < m_{ll}^{\text{max}},
\]

and calculates

\[
\Delta \chi^2 = \sum_i \frac{(N_i^{\text{sig}})^2}{N_i^{\text{sig}} + N_i^{\text{bg}}}
\]

for the bin size \( 2n \text{ GeV} \), where integer \( n \) is determined so that \( \max(N_i^{\text{sig}}) > 10 \). Eq. (8) expresses the statistical significance of the signal after the subtraction of expected background.\(^6\) In the experimental situation, one may determine the background distribution from the real data, when the events above the \( m_{ll}^{\text{max}} \) may be used to make a simple extrapolation as suggested in Eq. (6).\(^7\)

In Fig. 7, we show the contours of constant \( \Delta \chi^2 \) in \( m_{16} \) and \( M \) plane. \( \Delta \chi^2 = 25 \sim 5\sigma \) contours correspond to 70 signal \( e^\pm \mu^\mp \) events in the signal region in Eq. (7). Here, the integrated luminosity is 100fb\(^{-1} \). The SUSY background is roughly of the same order as the signal. In Ref. [26], 120 total SM background events are expected for 30 fb\(^{-1} \) under the cuts 1) \( E_T^{\text{miss}} > 300 \)

\(^6\) We assume no error for the background shape.

\(^7\) Alternatively, one can constrain the MSSM parameters as model-independent as possible, so that the background distribution can be determined model-independently. Note that the nature of the third-generation sfermions is important since the substantial fraction of the \( e^\pm \mu^\mp \) events might come from \( \tilde{g} \to \tilde{t}_1 t \) or \( \tilde{b} \tilde{b} \) followed by their cascade decays to \( W \) etc. The sbottom mass may be reconstructed from the distribution of the events with \( b \text{ jet} \). Attempts to reconstruct stop decays may be found in [14, 23].
GeV, 2) $p_T^l > 10$ GeV, and 3) two jets with $p_T^{jet} > 150$ GeV. The level of the background in the signal region is of the same order as that of the SUSY background. The significance beyond this 5σ contour is therefore the subject of more careful MC simulations both for SUSY and SM backgrounds.

When $\mu \sim M_2$, the parameter space covered by LHC extends, compared with the MSUGRA model ($\mu \sim 1.5M_2$), due to the enhanced branching rates of $\tilde{\chi}^0_2$ to $\tilde{l}_R$ as we see in the previous section. We can see another qualitative difference in $m_{16} \ll M$ region. For the MSUGRA case, the small $m_{16}$ region cannot be reached because $\tilde{\chi}^0_2 \to \tilde{l}_L$ dominates. The search region is extended to this region for the cMSSM with $\mu \sim M$ because $m_{\tilde{\chi}^0_2} < M_2$ and $\tilde{\chi}^0_2$ could not decay into $\tilde{l}_L$ for $\mu \sim M$.

We also estimate the LHC reach for generic oscillation parameters. In Fig. 8, we plot the 5σ contour and the line of $Br(\mu \to e\gamma)=1.2 \times 10^{-11}$, $1.0 \times 10^{-12}$, and $1.0 \times 10^{-14}$ in the parameter space of $\sin 2\theta$ and $\Delta m$ at the GUT scale. We use the the MSUGRA model with $\tan \beta = 10$, $A = 0$, $m = 100$ GeV, and $M = 300$ GeV. We can see that the most part of the parameter range where LFV can be observed at LHC is already excluded by the $Br(\mu \to e\gamma)$ constraint, and the remained region will be covered by next-generation experiments. The situation changes for the cMSSM case with $\mu = M_2$. The corresponding figure is shown in Fig. 9. Because of the change of the decay kinematics, the wider range of the parameter space is covered by the LHC than that in the MSUGRA case. $Br(\mu \to e\gamma)$, in contrast, becomes small due to the cancellation between the diagrams. It follows that LHC might be more advantageous to observe LFV than the $\mu \to e\gamma$ decay search, especially in the cMSSM.

4 Conclusions and Discussion

In this paper we investigate the potential of LHC to find LFV in $\tilde{\chi}^0_2 \to \tilde{l}' l$, $\tilde{l}' \to l' \tilde{\chi}^0_1$. Here we studied it in a general model where the $\mu$ parameter is independent of the gaugino mass $M$ by allowing the non-universal GUT scale Higgs masses. An approximated universality of squark and slepton masses is imposed. We find LHC would be able to find the LFV mixing between the
Figure 7: $\sqrt{\Delta \chi^2} = 5$ contours for the LFV discovery. The thick solid line is for $\mu = 1.5M_2$ and $\tan \beta = 10$ in the cMSSM, the thick dashed line for $\mu = M_2$ and $\tan \beta = 20$, and the solid line for $\mu = M_2$ and $\tan \beta = 10$. We fix the $\tilde{e}_R\tilde{\mu}_R$ mixing angle $\theta$ as $\sin 2\theta = 0.5$ and the slepton mass difference $\Delta m = 1.2$ GeV at the GUT scale.

first and second generation in the right-handed slepton masses, $m_{\tilde{e}_R\tilde{\mu}_R}^2$, while the $Br(\mu \to e\gamma)$ is undetectably small.

LFV in left-handed slepton mass matrix as $m_{\tilde{\mu}_L\tilde{\tau}_L}^2$ might be more motivated when the data from atmospheric neutrino is considered [14][27]. We note that the cancellation among the LFV diagrams is unlikely when only the left-handed slepton mass is the source of LFV. We show the relation between $Br(\tau \to \mu\gamma)$ (the dashed line) and $Br(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tau\mu)$ normalized by $Br(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tau\tau)$ (the solid line) in Fig. [11] when $m_{\tilde{\mu}_L\tilde{\tau}_L}^2$ is the unique source of LFV. Hinchliffe and Paige stated that it is possible to observe LFV for $Br(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tau\mu)$ $\gtrsim$ 0.01 in this figure. We can see in Fig. [11] that the reach of LHC corresponds to $Br(\tau \to \mu\gamma)$ $\simeq$ 10$^{-6}$, which is also within the range of the $\tau \to \mu\gamma$ search at the KEKB experiment [11].

Finally when several off-diagonal scalar masses $m_{\tilde{l}\tilde{l}'}^2$ are non-zero, $Br(l \to l'\gamma)$ could show the very complicated structure, therefore negative results in the rare decay search at low energy do not necessary constrain the processes involving $l$ decays at the high energy colliders. Especially, the LFV processes,
Figure 8: The LHC reach and the line of the constant $Br(\mu \to e\gamma)$ in the MSUGRA model are shown. Here, $\tan \beta = 10$, $A = 0$, $m = 100$ GeV, and $M = 300$ GeV.

Figure 9: The LHC reach and the line of the constant $Br(\mu \to e\gamma)$ in the cMSSM are shown. Here, $\mu = M_2$, $\tan \beta = 10$, $A = 0$, $m_{16} = 100$ GeV, and $M = 300$ GeV.
Figure 10: \( Br(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \tau \mu) \) normalized by \( Br(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \tau \tau) \) (the solid line) and \( Br(\tau \rightarrow \mu \gamma) \) (the dashed line) as functions of \( m_{\tilde{\mu} \tilde{\tau}}^2 / m_{\tilde{\mu}}^2 \). Here, we take \( m = 130 \text{GeV}, M = 250 \text{GeV}, A_0 = 300 \text{GeV} \) and \( \tan \beta = 10 \).

\( \mu \rightarrow e \gamma \) and \( \mu N \rightarrow e N \), may be more sensitive to all LFV slepton masses, compared with those involving the third-generation sleptons, such as \( \tau \rightarrow \mu \gamma \). The size of \( m_{\tilde{\tau}}^2 \) and \( m_{\tilde{\mu}}^2 \) is less constrained compared with \( m_{\tilde{\tau}}^2 \), and then the LFV through \( \tilde{\tau} \) could overcome direct \( \tilde{\mu} \tilde{e} \) mixing. For example, even when only the left-handed sleptons have the LFV masses, there is a cancellation in \( \mu \rightarrow e \gamma \) among the diagrams in some specific parameter space [28].

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Appendix A: Formula for LFV processes

First, we present our formula for the LFV decay of the second-lightest neutralino, $\tilde{\chi}^0_2 \to l^- (\bar{l}^+ \to) l^+ \tilde{\chi}^0_1$ and $l^+ (\bar{l}^- \to) l^- \tilde{\chi}^0_1$. The invariant mass ($m_{l\tilde{l}}$) distribution of the leptons is given as

$$
\frac{d\Gamma}{dm_{l\tilde{l}}^2} = \sum_{XY} \int_{q^2 \geq 0} dq^2 \rho(q^2) A_{XY}(q^2)
$$

Here $\Gamma_{XY}(q^2)$, $B_{XY}(q^2)$, and $\rho(q^2)$ are

$$
\Gamma_{XY}^{(\ell)}(q^2) = \frac{g_2^2}{32\pi} \left\{ R_{i2X}^{(n)} R_{i2Y}^{(n)*} + L_{i2X}^{(n)*} L_{i2Y}^{(n)} \right\} m_{\tilde{\chi}_2^0} \left( 1 - \frac{q^2}{m_{\tilde{\chi}_2^0}^2} \right)^2,
$$

$$
B_{XY}^{(\ell)}(q^2) = \frac{g_2^2}{16\pi} \left\{ R_{i1X}^{(n)} R_{i1Y}^{(n)*} + L_{i1X}^{(n)*} L_{i1Y}^{(n)} \right\} \frac{q^2}{m_{\Gamma iX iY}^2} \left( 1 - \frac{m_{\tilde{\chi}_1}^2}{q^2} \right)^2.
$$

$$
\rho(q^2) = \begin{cases} 
\frac{1}{(m_{\tilde{\chi}_2}^2)^2} & (0 \leq m_{l\tilde{l}}^2 \leq (m_{\tilde{\chi}_2}^2)^2) \\
0 & (m_{l\tilde{l}}^2 > (m_{\tilde{\chi}_2}^2)^2)
\end{cases}
$$

where $[m\Gamma]_{XY} = (m_{lX} \Gamma_{lX} + m_{lY} \Gamma_{lY})/2$, and

$$
(m_{\tilde{\chi}_2}^2)^2 = m_{\tilde{\chi}_2}^2 (1 - \frac{q^2}{m_{\tilde{\chi}_2}^2})(1 - \frac{m_{\tilde{\chi}_1}^2}{q^2}).
$$

The interaction Lagrangian of fermion-fermion-neutralino is written as

$$
L_{int} = -g_2 \bar{\chi}^0_2 (R_{iAX}^{(n)} P_R + L_{iAX}^{(n)} P_L) i\gamma_5 l^i \chi^0 + h.c.,
$$

and the coefficients are

$$
L_{iAX}^{(n)} = \frac{1}{\sqrt{2}} \{ [-(O_{N})_{A2} - (O_{N})_{A1} t_W] U_{X,i} + \frac{m_{l_i}}{m_{W} \cos \beta} (O_{N})_{A3} U_{X,i+3} \},
$$

$$
R_{iAX}^{(n)} = \frac{1}{\sqrt{2}} \{ \frac{m_{l_i}}{m_{W} \cos \beta} (O_{N})_{A3} U_{X,i} + 2(O_{N})_{A1} t_W U_{X,i+3} \}.
$$

The function of slepton momentum $A_{XY}(q^2)$ is

$$
A_{XY}(q^2) = \frac{1}{1 + ix_{XY}} \frac{\delta(q^2 - m_{lX}^2) + \delta(q^2 - m_{lY}^2)}{2}
$$
\[ \frac{d\Gamma}{dm_{\ell l}^2} = \frac{2\Gamma_0 Br_0}{m_W^2 (m_{\tilde{\tau}}^2 - m_{\tilde{\chi}}^2)^2} \frac{x_{\ell l}^2}{2(1 + x_{\ell l}^2)} \sin^2 2\theta \quad (A.9) \]

for \( 0 \leq m_{\ell l}^2 \leq (m_{\tilde{\tau}}^2 - m_{\tilde{\chi}}^2)^2 \). Here,

\[ \Gamma_0 = \frac{g_2^2}{16\pi} m_{\tilde{\chi}}^2 \left( 1 - \frac{m_{\tilde{\tau}}^2}{m_{\tilde{\chi}}^2} \right)^2 [O_N]_{21}^2, \quad (A.10) \]

\[ Br_0 = \frac{g_2^2}{16\pi} \frac{m_{\tilde{\tau}}^2}{m_W^2} \left( 1 - \frac{m_{\tilde{\chi}}^2}{m_{\tilde{\tau}}^2} \right)^2 [O_N]_{11}^2, \quad (A.11) \]

and \( m_{\tilde{\tau}}^2 \) and \( \sin 2\theta \) are the average mass and the mixing angle for the sleptons.

Next, we present formula for the LFV lepton decays \( \mu \to e\gamma \) or \( \tau \to \mu\gamma \). Those rates are also written in similar Lagrangian though, this time, contributions from chargino loops are also important. The Lagrangian involving chargino-slepton-lepton is given as

\[ \mathcal{L} = -g_2 \bar{l}(L_{iAX}^{(c)} P_R + R_{iAX}^{(c)} P_L) \tilde{\chi}_{A}^+ \tilde{\nu}_X \quad (A.12) \]

where \( \tilde{\chi}_{A}(A = 1, 2) \) is a chargino mass eigenstate. The coefficients are

\[ L_{iAX}^{(c)} = (O_R)_{A1} U_{X,i}^\nu, \]

\[ R_{iAX}^{(c)} = \frac{m_{\tilde{\tau}}}{\sqrt{2} m_W \cos \beta} (O_L)_{A2} U_{X,i}^\nu. \quad (A.13) \]

Then

\[ \Gamma(l_j \to l_i\gamma) = \frac{e^2}{16\pi} m_{l_j}^5 (|A^L|^2 + |A^R|^2) \quad (A.14) \]

where

\[ A^R = A^{(n)R} + A^{(c)R}, \quad A^L = A^{(n)L} + A^{(c)L}. \quad (A.15) \]

The coefficients in above equations are given as

\[ A^{(n)R} = \frac{1}{32\pi^2} m_{l_j}^2 \left( \frac{R_{iAX}^{(n)} R_{iAX}^{(n)*}}{6(1 - x_{AX})^2} \right) \]
\begin{align}
A^{(c)\,R} &= - \frac{1}{32\pi^2} \frac{1}{m_{\tilde{c}}^2} \left[ \frac{R_{iAX}^{(c)} R_{jAX}^{(c)*}}{R_{iAX}^{(n)*} R_{jAX}^{(n)}} \frac{1}{m_{\tilde{c}j}^2} \right] \left[ (1 - 6x_{AX} + 3x_{AX}^2 + 2x_{AX}^3 - 6x_{AX}^2 \ln x_{AX}) + R_{iAX}^{(n)*} \frac{1}{m_{\tilde{c}j}^2} \frac{1}{(1 - x_{AX})^3} (1 - x_{AX}^2 + 2x_{AX} \ln x_{AX}) \right],
\end{align}

B. Sample points

In this paper, we used MC simulation data for two sample points to estimate the event distribution and the acceptance. We summarize the masses of SUSY particles and decay branching ratios here because they depend on choice of gauge couplings and so on.

First, we list mass parameters and relevant SUSY-particle masses in GeV for the point I), which is studied in this paper. The ISAJET \cite{21} is used to generate this spectrum.

| \( m \) | 100.0 | \( \tan \beta \) | 2.1 |
|\hline|
| \( M \) | 300 | \( \mu \) | 497.87 |
| \hline|
| \( m_{\tilde{c}_R} \) | 157.21 | \( m_{\tilde{c}_L} \) | 238.78 |
| \hline|
| \( m_{\tilde{\nu}} \) | 230.19 | \( m_{\tilde{\tau}_1} \) | 121.52 |
| \hline|
| \( m_{\tilde{\nu}_e} \) | 233.02 | \( m_{\tilde{\nu}_\mu} \) | 499.18 |
| \hline|
| \( m_{\tilde{\tau}_2} \) | 523.23 | \( m_{\tilde{\tau}_3} \) | 230.14 |
| \hline|
| \( m_{\tilde{t}_1} \) | 156.81 | \( m_{\tilde{b}_1} \) | 238.92 |
| \hline|
| \( m_h \) | 94.33 | \( m_P \) | 606.79 |

For the parameter gluino could decay into squarks, and especially, it has enhanced branching ratios to the third-generation SUSY particles. The first-generation SUSY particles would be also generated and decay into charginos or neutralinos to produce leptons. The major branching ratios in (\%) are following;
\[g \rightarrow b_1 b \quad 15 \quad \tilde{g} \rightarrow b_2 b \quad 9.7\]
\[g \rightarrow t_1 t \quad 15 \quad \tilde{u}_L \rightarrow \tilde{\chi}_3^0 u \quad 33\]
\[\tilde{u}_L \rightarrow \tilde{\chi}_1^+ d \quad 65 \quad \tilde{d}_L \rightarrow \tilde{\chi}_3^0 d \quad 31\]
\[\tilde{d}_L \rightarrow \tilde{\chi}_1^0 u \quad 64 \quad t_1 \rightarrow \tilde{\chi}_1^0 t \quad 23\]
\[t_1 \rightarrow \tilde{\chi}_1^0 b \quad 64 \quad b_1 \rightarrow \tilde{\chi}_2^0 b \quad 28\]
\[b_1 \rightarrow \tilde{\chi}_2^0 b \quad 27 \quad b_2 \rightarrow \tilde{\chi}_1^0 b \quad 67\]
\[b_2 \rightarrow \tilde{\chi}_1^+ t \quad 9.6 \quad b_2 \rightarrow t_1 W^- \quad 15\]
\[\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_1 \tilde{\tau} \quad 9.2 \quad \tilde{\tau}_2 \rightarrow \mu R \mu \quad 9.2\]
\[\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_1 \tilde{\tau} \quad 12.1 \quad \tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^0 W^- \quad 89.7\]
\[\tilde{\chi}_1^+ \rightarrow \tilde{\nu}_e e \quad 1.4 \quad \tilde{\chi}_1^+ \rightarrow \tilde{\nu}_\tau \tau \quad 0.5\]
\[\tilde{\chi}_1^+ \rightarrow \tilde{\tau}_1 \nu_\tau \quad 6.6\]

For point II), the input mass parameters and the SUSY particle masses are following:

| \(m_{16} \) | \(M \) | \(\tan \beta \) | \(M_1 \) | \(M_2 \) |
|----------------|-----|----------------|-------|-------|
| 90.0           | 250 | 10             | 103.9 | 208.75|
| \(m_{\tilde{\tau}_1} \) | 139.3 | \(m_{\tilde{\tau}_2} \) | 206.09 | \(m_{\tilde{\nu}_L} \) | 556.07 | \(m_{\tilde{\nu}_R} \) | 534.36 | \(m_{\tilde{d}_L} \) | 561.67 |
| \(m_{\tilde{\tau}_2} \) | 190.28 | \(m_{\tilde{\tau}_1} \) | 208.74 | \(m_{\tilde{\chi}_3^0} \) | 148.44 | \(m_{\tilde{\chi}_2^0} \) | 272.52 |
| \(m_{\tilde{\tau}_1} \) | 273.8 | \(m_{\tilde{\tau}_2} \) | 188.67 | \(m_{\tilde{\chi}_3^0} \) | 374.43 | \(m_{\tilde{\chi}_2^0} \) | 563.81 |
| \(m_{\tilde{\tau}_2} \) | 132.56 | \(m_{\tilde{\tau}_1} \) | 206.06 | \(m_{\tilde{\chi}_3^0} \) | 498.27 | \(m_{\tilde{\chi}_2^0} \) | 531.2 |
| \(m_{\tilde{\tau}_1} \) | 112.59 | \(m_{\tilde{\tau}_2} \) | 436.98 | \(m_{\tilde{\chi}_3^0} \) | 437.63 | \(m_{\tilde{\chi}_2^0} \) | 624.36 |

Relevant branching ratios in % for the sample parameter are
| Process | Rate |
|---------|------|
| $\tilde{g} \rightarrow b_1 b$ | 17.4 |
| $\tilde{g} \rightarrow t_1 t$ | 13.2 |
| $\tilde{u}_L \rightarrow \tilde{\chi}_3^0 u$ | 0.4 |
| $\tilde{u}_L \rightarrow \tilde{\chi}_3^+ d$ | 21.4 |
| $\tilde{d}_L \rightarrow \tilde{\chi}_3^0 d$ | 0.7 |
| $\tilde{d}_L \rightarrow \tilde{\chi}_2^+ u$ | 36.4 |
| $t_1 \rightarrow \tilde{\chi}_2^0 t$ | 10.1 |
| $b_1 \rightarrow \tilde{\chi}_2^+ t$ | 37.8 |
| $b_1 \rightarrow t_1 W^-$ | 10.0 |
| $b_2 \rightarrow \tilde{\chi}_2^+ t$ | 35.5 |
| $b_2 \rightarrow t_1 W^-$ | 10.5 |
| $\tilde{\chi}_4^0 \rightarrow \tilde{\nu}_L e$ | 1.1 |
| $\tilde{\chi}_2^2 \rightarrow \tilde{\nu}_L e$ | 9.7 |
| $\tilde{\nu}_L \rightarrow \tilde{\chi}_1^+ e$ | 40.1 |
| $\tilde{e}_L \rightarrow \tilde{\chi}_1^0 e$ | 19.8 |
| $\tilde{g} \rightarrow b_2 b$ | 10.1 |
| $\tilde{u}_L \rightarrow \tilde{\chi}_2^0 u$ | 20.7 |
| $\tilde{u}_L \rightarrow \tilde{\chi}_3^0 u$ | 12.2 |
| $d_L \rightarrow \tilde{\chi}_2^0 d$ | 14.5 |
| $d_L \rightarrow \tilde{\chi}_3^0 d$ | 15.4 |
| $t_1 \rightarrow \tilde{\chi}_1^0 t$ | 9.5 |
| $b_1 \rightarrow \tilde{\chi}_1^+ t$ | 29.9 |
| $b_1 \rightarrow \tilde{\chi}_2^0 t$ | 9.8 |
| $b_2 \rightarrow \tilde{\chi}_1^0 t$ | 14.1 |
| $b_2 \rightarrow \tilde{\chi}_2^0 b$ | 4.9 |
| $\tilde{\chi}_3^0 \rightarrow \tilde{\nu}_L e$ | 5.5 |
| $\tilde{\chi}_3^2 \rightarrow \tilde{\nu}_L e$ | 23.6 |
| $\tilde{\chi}_1^2 \rightarrow \tilde{\tau}_2 \nu_\tau$ | 89.1 |
| $\tilde{\nu}_L \rightarrow \tilde{\chi}_1^0 e$ | 38.2 |
| $\tilde{e}_R \rightarrow \tilde{\chi}_1^0 e$ | 100 |
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