Photonic quantum memory in two-level ensembles based on modulating the refractive index in time: equivalence to gradient echo memory

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We present a quantum memory protocol that allows to store light in ensembles of two-level atoms, e.g. rare-earth ions doped into a crystal, by modulating the refractive index of the host medium of the atoms linearly in time. We show that under certain conditions the resulting dynamics is equivalent to that underlining the gradient echo memory protocol, which relies on a spatial gradient of the atomic resonance frequencies. We discuss the prospects for an experimental implementation.

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I. INTRODUCTION

Quantum memory for light is an essential element for the photonic implementation of quantum communication and information processing [1, 2]. In recent years there has been a lot of work both on theoretical proposals and on experimental implementations [3, 4]. Thus far optical control, using relatively strong laser pulses, has been exploited for electromagnetically induced transparency and off-resonant Raman-type storage [5, 6] in ensembles of three-level systems. More recently, a direct control of the transition dipole-moment has been proposed that emulates Raman-type quantum memories in a two-level atomic configuration [7].

In photon-echo based memories [8], the light-matter coupling is controlled in a more indirect way by exploiting the dephasing and rephasing of inhomogeneously broadened atomic ensembles. This includes the controlled reversible inhomogeneous broadening protocol [9], the atomic frequency comb protocol [10], and the gradient echo memory (GEM) protocol [11]. The GEM protocol has allowed the demonstration of the highest memory efficiency (in the quantum regime) so far [12].

Recently Ref. [13] proposed a quantum memory protocol based on controlling the refractive index. Considering an ensemble of three-level atoms inside a host medium (e.g. rare-earth ions doped into a crystal), which is located in a circular optical cavity, they showed that a continuous change of the refractive index of the host medium during an off-resonant Raman interaction between a single photon, a classical control pulse and the atomic ensemble allowed mapping the state of the single photon into a collective atomic excitation.

Here we consider storing quantum states of light in an ensemble of two-level atoms in a host medium, where the refractive index of the medium can be modulated during the interaction of the light with the atoms. In contrast to Ref. [13] here there is no optical control pulse (which is related to the fact that we consider two-level instead of three-level atoms) and no cavity. Interestingly, we find that under certain conditions the considered system leads to dynamics that are equivalent to those of the GEM protocol; controlling the refractive index of the host medium in time can mimic the effect of the spatial frequency gradient present in GEM.

This paper is organized as follows. In Sec. II we derive the dynamical equations for our system under certain conditions, which we discuss in detail. In Sec. III we compare these results to the GEM protocol. In Sec. IV we discuss a possible experimental implementation of our protocol. Sec. V contains our conclusions.

II. MAXWELL-BLOCH EQUATIONS

Here we study the propagation of the light and its interaction with two-level atoms inside a host medium whose refractive index varies in time. We show that in a certain parameter regime the time-dependent refractive index does not play a role in the propagation equation for the light. In contrast, it plays an essential role in the dynamics of the atomic polarization. For simplicity, we assume the field is propagating in a certain direction with a fixed linear polarization. (This is well justified for our choice of possible implementation in a waveguide, see below.) The wave equation for the electric field operator is analogous to the classical equation, namely

$$\frac{\partial^2 E}{\partial z^2} = \mu_0 \frac{\partial^2 D}{\partial t^2} = \mu_0 \frac{\partial^2}{\partial t^2} (\epsilon E + P), \quad (1)$$

where $E$ is the electric field, $z$ is the direction of propagation, $\mu_0$ is the vacuum permeability, $D$ is the electric displacement field, $\epsilon$ is the permittivity of the propagation medium and $P$ is the polarization of the embedded two-level atoms. There are thus two fundamentally different contributions to $D$. The $\epsilon E$ term is due to the permittivity of the host medium, whereas $P$ describes the polarization of the two-level atoms that are the actual memory system for the light.

Consider the case where $\epsilon$ is time-dependent. The permittivity of the medium is related to its refractive index as $\epsilon(t) = n^2(t) \epsilon_0$. We consider a medium with a linearly changing refractive index, $n(t) = n_0 + nt$. Based on this, only the first derivative of the refractive index remains
in the Eq. (1), giving

$$\left(\frac{\partial^2}{\partial z^2} - \frac{n^2(t)}{c^2} \frac{\partial^2}{\partial t^2}\right) E = \frac{1}{c^2}(2\dot{n}^2 + 4n(t)\dot{n}E) + \mu_0 \dot{P},$$  \hspace{1cm} (2)

where $c$ is the speed of light. We now introduce the slowly varying components of the signal field $E$ and the atomic polarization $P$, $E = \mathcal{E} e^{-i(\omega_0 t - k_0 z)}$ and $P = \mathcal{P} e^{-i(\omega_0 t - k_0 z)}$. Here the wave vector $k_0(t) = k_i + k_t = (n_i + \dot{n}_t) \frac{\omega_0}{c}$ is a function of time and $k = \frac{\omega_0}{c}$, where $\omega_0$ is the central frequency of the signal.

The wave equation can be greatly simplified provided that a number of (realistic) conditions are fulfilled (see also the appendix). The second-order spatial derivative of $\mathcal{E}$ is dropped provided that the field amplitude changes appreciably over the length of the medium (such that the derivative is comparable to $\mathcal{E}/L$) and $k_0(t) \gg 1/L$. Similarly, the second order time-derivative can be ignored if $\omega_0 \gg 1/\tau$, where $\tau$ is the duration of the pulse. The same conditions also allow one to drop the first and second order derivatives of the slowly varying polarization operator. We are interested in the regime where the extent of the pulse in space (outside the medium), $L \ll 1$. This allows one to drop the first-order time derivative of $\mathcal{E}$ compared to the first-order spatial derivative. Finally, by assuming $\Delta n \ll n_i$, where $\Delta n$ is the total change in the refractive index, and $kL \ll \frac{2c}{\tau}$ one obtains the simplified propagation equation

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{i \mu_0 \omega_0^2}{2k_i} \mathcal{P}. \hspace{1cm} (3)$$

This shows that, under the above conditions, the propagation equation remains unchanged compared to that of systems with constant refractive index ($k$ does not play an appreciable role in the propagation), see e.g. [10, 11, 14]. The derivation of Eq. (3) from Eq. (2) is discussed in detail in the appendix.

We now derive the dynamics of the polarization of the dopant atoms. The polarization of the $j$-th atom is $P_j = \langle g^j | d^+ | e \rangle \sigma_{ge}^j$, where $\sigma_{ge}^j = | g^j \rangle \langle e |$ and $\langle g^j | d^+ | e \rangle$ is the matrix element of the corresponding dipole moment component between the ground and excited states. The collective atomic polarization at a certain position $z$ is the sum over the individual atoms in a slice of width $\Delta z$.

The slow component of this collective operator is given by

$$\mathcal{P} = \frac{1}{A\Delta z} \langle g|d|e\rangle \sum_{j=1}^{N_z} \sigma_{ge}^j e^{i(\omega_0 t - k_0(t)z_j)} \equiv \langle g|d|e\rangle \frac{N}{V} \tilde{\sigma}_{ge}, \hspace{1cm} (4)$$

where we assume equivalent dipole moment for all of the atoms; $A$ and $V$ are the cross-section area and volume of the light-atom interface; $N$ is the number of the dopant atoms and $\tilde{\sigma}_{ge} = \frac{1}{N} \sum_{j=1}^{N_z} \sigma_{ge}^j e^{i(\omega_0 t - k_0(t)z)}$ is the average atomic polarization at position $z$. The Hamiltonian of the ensemble of the dopant atoms interacting with the light field can be written as

$$H = H_0 + H_{int} \hspace{1cm} (5)$$

$$= \sum_{j=1}^{N} \hbar \Omega \sigma_{ee}^j - \langle e|d|g\rangle \sum_{j=1}^{N} \sigma_{eg}^j E(z_j, t) + h.c.,$$

where we assume uniform excited state energy $\hbar \Omega$ for all of the atoms. We can now derive the dynamics of the slowly varying collective atomic polarization using

$$\frac{d\tilde{\sigma}_{ge}}{dt} = \frac{i}{\hbar} [\tilde{\sigma}_{ge}, H] + \frac{\partial \tilde{\sigma}_{ge}}{\partial t}. \hspace{1cm} (6)$$

For the next step we assume that all of the atoms are initialized in the ground state and the number of atoms $N \gg 1$. For weak (quantum) signals one can then ignore the change in the excited state population.

Using Eqs. (3, 6), the above definition of the slowly varying field, and including the atomic excited state linewidth $\gamma$, one finds the Maxwell-Bloch equations describing the interaction of the light with the collective atomic polarization in a medium with linearly time-varying refractive index,

$$\frac{d\tilde{\sigma}_{ge}(z,t)}{dt} = -(\gamma + i(\Delta + \kappa z))\tilde{\sigma}_{ge}(z,t) + i g \tilde{E}(z,t), \hspace{1cm} (7)$$

$$\frac{\partial \tilde{E}(z,t)}{\partial z} = i \frac{n N g}{c} \tilde{\sigma}_{ge}(z,t),$$

where $\tilde{E} = \sqrt{\frac{\hbar \omega_0}{2\epsilon_i V}} \mathcal{E}$ and $\Delta = \Omega - \omega_0$ is the detuning. The coupling constant $g = \langle e|d|g\rangle \sqrt{\frac{\omega_0}{2\epsilon_i V}}$, where $\omega_0$ is the central frequency of the pulse and $\epsilon_i = n_i^2 \epsilon_0$ is the initial refractive index of the medium. Note that the time dependence of the permittivity can be ignored in the definitions of $\tilde{E}$ and $g$ because we are interested in the regime where $\Delta n \ll n_i$.

The above set of equations shows the role of the linearly changing refractive index of the host medium in the regime that we have discussed. One sees that the linear change of the refractive index in time results in a space-dependent frequency shift given by the $k z$ term in Eq. (7), see also Fig. 1(b). The above Maxwell-Bloch equations are identical to those underlying the GEM quantum memory protocol [11]. In the next section we therefore discuss in detail how the present proposal compares to GEM.

### III. COMPARISON WITH GRADIENT ECHO MEMORY

The dynamical equations (8) derived in the previous section (under a number of realistic conditions) are exactly equivalent to those underlying the GEM quantum memory protocol [11]. In the GEM protocol, an initially narrow atomic absorption line is broadened by applying an external (longitudinal) field gradient. This longitudinal broadening allows one to accommodate all frequency
This implies that the optical depth of the system is 
\[ d = 2 \beta \pi = 2 \pi n N g^2 c \dot{k} \gamma \]. Here \( d \) is the optical 
depth that is associated with the effectively broadened 
line, with the initial optical depth \( d_{\text{in}} \equiv 2 \pi n N g^2 L c \gamma \). The 
retrieval efficiency is then given by \((1 - \exp(-d_{\text{in}} \gamma / kL))^2 e^{-2\gamma \tau}\), see Fig. 2. Here we have assumed that the decay of the 
excited state only has an effect during absorption and 
retrieval, but not during storage. As mentioned before, 
this can be achieved e.g. by transferring the excitation 
to a third, longer-lived state. Hyperfine ground states 
in rare-earth doped crystals can have coherence times of 
many seconds [15].

IV. POSSIBLE IMPLEMENTATION

We now discuss a potential experimental implementa-
tion of our proposed protocol. We propose to use thulium 
ions doped into a lithium niobate waveguide. This sys-
tem was used in a recent implementation of an atomic 
frequency comb memory [16]. Lithium niobate is an 
attractive host for the present proposal because of its 
electro-optic properties, see below. The thulium ions in-
teract with near-infrared light at a wavelength of 795

![Figure 1](image1.png)

FIG. 1: (a) In the GEM protocol, a longitudinal energy shift 
in the atoms (solid dots) allows one to cover all of the fre-
cquency components of the incoming light. (b) In the protocol 
proposed here, due to the linear change of the refractive index 
in time, the light experiences an effective position-dependent 
frequency shift \( k z \). This allows different frequency compo-
nents of the light to interact on resonance with a spectrally 
narrow line of atoms.

![Figure 2](image2.png)

FIG. 2: The efficiency of the proposed memory protocol based 
on refractive index modulation in terms of the ini-
tial optical depth \( d_{\text{in}} \). The efficiency is given by \( e^{-2\gamma \tau} (1 - \exp(-d_{\text{in}} \gamma / kL))^2 \). The figure shows the efficiency for differ-
cent pulse durations \( \tau \), relative to the excited state line width, \( \gamma \). We assume \( kL \tau = 2 \). Depending on the available optical 
depth, one can optimize the achievable efficiency by choosing 
an appropriate pulse duration.
nm. The transition is naturally inhomogeneously broadened. We propose to prepare an initial atomic linewidth of $\gamma = 10$ MHz by optical pumping, which is very realistic. We consider the case where $k_{1} \ell \tau = 2$. This assures that the memory bandwidth is large enough that it can accommodate the incoming pulse. Assuming $L = 1$ cm this leads to the requirement $\Delta n \approx 5 \times 10^{-5}$. We choose the pulse duration $\tau = 1/10\gamma$. The above values assure that $\omega_0 \gg 1/\tau$, $L \ll \ell \tau = 2c$, $\Delta n \ll n_i$ and $k_0(t) \gg 1/L$, as required for the derivation in section II. For the given parameter values one needs $d_{in} = 50$ to achieve about 70% efficiency, see also Fig. 2.

We consider the refractive-index modulation of the ordinary optical axis of lithium niobate by a fast varying electric field. We consider the case where the crystal is clamped (spatially confined) and the temperature is a few Kelvin. Under these conditions the refractive index of the ordinary axis $n_0 \approx 2.26$ [17]. The change in the refractive index through the electro-optical effect is governed by $\Delta n_0 = \sum_r r_{ik} E_k$, where $i, j = 1$ is associated with the refractive index change of the ordinary axis. This means that a time-dependent external field in a certain direction, $E_k$, can impose a time-dependent refractive index for the ordinary axis if there exists a non-zero linear electro-optical coefficient for that direction, $r_{1ik}$. For lithium niobate $r_{113} \approx 10 \times 10^{-12}$ m/V and $r_{112} \approx -3 \times 10^{-12}$ m/V [17, 18]. This leads to $\Delta n \approx 6 \times 10^{-5}$ under $0.85 \times 10^{6} - 2.5 \times 10^{0}$ V/m electric field, depending on the direction of the field in the 2-3 plane. This is equivalent to applying 8.5 – 25 V to a system that has 10$\mu$m thickness, comparable to the waveguide used in Ref. [16]. The maximum change in the refractive index in lithium niobate is expected to be $10^{-3}$, which is limited by the breakdown electric field [19].

Applying the external electric field to change the refractive index is potentially accompanied by level shifts, due to the linear Stark shift, for the atomic ground and excited states. On the other hand, for a certain type of dopant, by having the external electric field orthogonal to the difference between the permanent electric dipole moment of the ground and excited states, one can keep the resonant frequency between these states unchanged. In our proposed system the permanent dipoles are aligned with the 3-axis [20, 21], therefore the electric field should be applied along the 2-axis in order to avoid level shifts.

V. CONCLUSION

We have proposed a memory protocol based on varying the refractive index of the host medium in time, and shown that in a certain regime it is equivalent to the GEM protocol, even though the latter is based on a spatial frequency gradient. One may wonder why no similar spatial gradient was seen in the protocol of Ref. [13], which also considered a time-varying refractive index, but in a Raman-type system, in contrast to the two-level ensemble considered by us. This can be understood by noting that in the scheme of Ref. [13] the refractive index modulation causes a spatial gradient in the frequencies for both the signal and the control fields, such that the two-photon transition frequency remains unchanged. This holds for co-propagating signal and control. It might be interesting to consider counter-propagating signal and control fields in this context, for which no such cancellation would occur. This might result in a protocol similar to Raman-GEM [22].

We found that a relatively small modulation of the refractive index should be able to provide sufficient memory bandwidth. We proposed a potential implementation in lithium niobate waveguides doped with rare-earth ions. However, other implementations could also be considered. In general, it may be easier to control the behavior of the refractive index in time, compared to the spatial control of the atomic transition frequencies required in standard GEM experiments.

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VI. APPENDIX: WAVE EQUATION SIMPLIFICATION

Here we explain in more detail how to obtain Eq. (3) from Eq. (2). Substituting $E = E e^{-i(\omega_0 t - k_0(t)z)}$ and $P = P e^{-i(\omega_0 t - k_0(t)z)}$ into Eq. (2) and canceling the fast varying phase gives

$$\left(\frac{\partial^2}{\partial t^2} + 2ik_0(t)\partial_z - k_0^2(t)E\right)$$

$$- \frac{n^2(t)}{c^2}(\partial^2_t - 2i(\omega_0 - \hat{k}z)\partial_t - (\omega_0 - \hat{k}z)^2)E$$

$$= \frac{1}{c^2}(2\dot{n}^2 - 4in(t)\dot{n}\omega_0 - \dot{k}z) + 4n(t)\dot{n}\partial_t E$$

$$+ \mu_0(\partial^2_t - 2i(\omega_0 - \hat{k}z))\partial_t - (\omega_0 - \hat{k}z)^2)P.$$
One can rewrite this equation and use $k_0(t) = n(t)\omega_0/c$ to simplify it to

$$
(\partial_z + \frac{n(t)}{c}(\partial_t + ikz))\mathcal{E} = \left(-\frac{in^2}{c^2k_0(t)} - 2\frac{n(t)}{c} - \frac{2in(t)\dot{n}}{c^2k_0(t)}\right)\partial_t\mathcal{E} + \frac{im\omega_0^2}{2k_0(t)}\mathcal{P}.
$$

(10)

Provided that $\Delta n \ll n(t)$ (and using again $\omega_0 \gg 1/\tau$), one has

$$
\frac{n^3}{c^3k_0(t)}\mathcal{E} < \frac{2in(t)\dot{n}}{ck_0(t)}\partial_t\mathcal{E} < \frac{im\omega_0^2}{2k_0(t)}\mathcal{E}.
$$

Under the condition $\Delta n \ll n(t)$ also allows one to drop $\frac{2n(t)}{c}\mathcal{E}$ compared to $\frac{n(t)}{c}\partial_t\mathcal{E}$. This simplifies the wave equation to

$$
(\partial_z + \frac{n(t)}{c}(\partial_t + ikz))\mathcal{E} = \frac{im\omega_0^2}{2k_0(t)}\mathcal{P}.
$$

(11)

Provided that the extent of the pulse in space, $L = c\tau$, is much larger than the length of the medium, $L$, one can ignore $\frac{n(t)}{c}\partial_t\mathcal{E}$ in comparison with $\partial_z\mathcal{E}$, which is the dominant term for the slowly varying component of the field. Finally, assuming that $\vec{k}L \ll \frac{n(t)kz}{c}$ the term $\frac{n(t)kz}{c}\mathcal{E}$ also can be dropped compared to the dominant term, leading to

$$
\partial_z\mathcal{E} = \frac{im\omega_0^2}{2k_0(t)}\mathcal{P}.
$$

(12)

The coefficient of $\mathcal{P}$ can be rewritten as,

$$
\frac{im\omega_0^2}{2k_i} \approx \frac{im\omega_0^2}{2k_i}(1 - \frac{\vec{k}t}{k_i}).
$$

(13)

Under the condition $\Delta n \ll n_i$ this can be well approximated by $\frac{im\omega_0^2}{2k_i}$, which leads to

$$
\partial_z\mathcal{E} = \frac{im\omega_0^2}{2k_i}\mathcal{P},
$$

(14)

which is the standard wave equation under similar (realistic) conditions in the absence of a time variation of the refractive index.

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