Hawking Radiation and Back-Reaction in a Unitary Theory of 2D Quantum Gravity

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**ABSTRACT**

We investigate the semiclassical limit and quantum corrections to the metric in a unitary quantum gravity formulation of the CGHS 2d dilaton gravity model. A new method for calculating the back-reaction effects has been introduced, as an expansion of the effective metric in powers of the matter energy-momentum tensor. In the semiclassical limit the quantum corrections can be neglected, and we show that physical states exists which contain the Hawking radiation. The first order back-reaction effect is entirely due to the Hawking radiation. It causes the black-hole mass to monotonically decrease, and it makes it unbounded from below as the horizon is approached. The second order quantum corrections have been estimated. Since the matter is propagating freely in this unitary theory, we expect that the higher order corrections will stabilize the mass, and the black hole will completely evaporate leaving a nearly flat space.

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The question of the back-reaction of the Hawking radiation is important for understanding the quantum fate of a black hole. Since a proper treatment of the problem requires a quantum gravity theory, not much progress has been made in 4d. However, in 2d, the CGHS dilaton gravity theory represents an excellent toy model for study of the backreaction problem \[1\]. Moreover, a quantum gravity formulation exists \[2, 3\], and the model can be thought of as a crude approximation of the BCMN model, which describes a spherically symmetric scalar field collapse in 4d \[4, 5\]. Hence, the CGHS model offers a possibility to study the backreaction problem in a quantum gravity context. Another interesting point is that the quantum gravity formulation of CGHS can be made unitary \[2, 3\]. The reason is that the theory can be deparametrized, and the corresponding Hamiltonian can be promoted into a Hermitian operator, because it is just a free-field Hamiltonian. Note that Hajicek has used exactly the same reasoning some time ago to argue that the quantum theory of the BCMN model could be made unitary \[6\]. However, the problem there is that the reduced Hamiltonian is a complicated nonlocal function of the matter fields, and hence the existence of a Hermitian extension is not guaranteed. Also the BCMN gauge does not penetrate the horizon, and hence a different slicing, which should be complete, is needed.

Therefore one can explore the problem of whether the evaporating black holes can exist within a unitary quantum gravity theory in 2d. In \[3\], a somewhat vague arguments were given in the support of the claim that the Hawking radiation exists in the semiclassical limit of the 2d quantum dilaton gravity. An argument of a different type has also been made in \[7\]. In this paper we present concrete calculations supporting the argument given in \[3\] for the existence of the Hawking radiation. Moreover, we present a method for calculating the backreaction corrections to the metric, which follows from the solvability of the classical theory. The metric is known as an explicit function of the matter energy-momentum tensor $T_m$. The corresponding nonlocal operator is then appropriately expanded into powers of the $T_m$ operator, and the effective quantum metric is calculated as an expectation value of the metric operator in a physical state. In the zeroth order one obtains the classical metric corresponding to the classical matter content of the quantum state, while the n-th order correction is proportional to the expectation value of $(T_m)^n$ in the initial quantum state.

We start from the CGHS action \[1\]

$$S = \int_M d^2x \sqrt{-g} \left[ e^{-\phi} \left( R + (\nabla \phi)^2 + 4\lambda^2 \right) - \sum_{i=1}^{N} (\nabla f_i)^2 \right], \quad (1)$$

where $\phi$ is a dilaton, $f_i$ are scalar matter fields, $g$, $R$ and $\nabla$ are determinant, scalar curvature and covariant derivative respectively, associated with a metric $g_{\mu \nu}$ on the
2d manifold \( M \). Topology of \( M \) is that of \( \mathbb{R} \times \mathbb{R} \). We make a field redefinition

\[
\tilde{g}_{\mu\nu} = e^{-\phi} g_{\mu\nu} \quad , \quad \tilde{f} = e^{-\phi} \quad ,
\]

so that

\[
S = \int_M d^2x \sqrt{-\tilde{g}} \left( \tilde{R} \tilde{f} + 4\lambda^2 - \sum_{i=1}^{N} (\tilde{\nabla} f_i)^2 \right) .
\]

Canonical analysis of (3) \[3\] gives

\[
S = \int dt dx \left( \pi_{\tilde{\rho}} \dot{\tilde{\rho}} + \pi_{\tilde{\phi}} \dot{\tilde{\phi}} + \pi_{\tilde{f}} \dot{\tilde{f}} - N_0 G_0 - N_1 G_1 \right) ,
\]

where \( N_0 \) and \( N_1 \) are the laps and shift, while the constraints \( G_0 \) and \( G_1 \) are given as

\[
G_0 = -\pi_{\tilde{\phi}} \pi_{\tilde{\phi}} - 4\lambda^2 e^{\phi} + 2 \dot{\phi}'' - \dot{\rho} \dot{\phi} + \frac{1}{2} (\pi_{\tilde{f}}^2 + \dot{f}^2)
\]

\[
G_1 = \pi_{\tilde{\phi}} \dot{\phi} + \pi_{\tilde{\rho}} \dot{\rho} - 2\pi_{\tilde{\rho}} \pi_{\tilde{\phi}} \dot{f} + \pi_{\tilde{f}} \dot{f} .
\]

The primes stand for the \( x \) derivatives, \( \tilde{\rho} \) is the conformal factor \( (\tilde{g} = e^{\tilde{\phi}}) \), and we have taken \( N = 1 \) for the simplicity sake. Now we fix the gauge

\[
\tilde{\rho} = 0 \quad , \quad \pi_{\tilde{\phi}} = 0 \quad ,
\]

which can be thought of as the 2d dilaton gravity analog of the BCMN gauge. However, unlike the BCMN gauge, (6) penetrates the horizon and corresponds to the classical CGHS solutions where the gauge functions are fixed to be \( x^\pm = t \pm x \) \[3\].

Solving the constraints gives

\[
\tilde{\phi} = a + bx + \lambda^2 x^2 - \frac{1}{4} \int dx \int dx (\pi_{\tilde{f}}^2 + \dot{f}^2) \quad , \quad \pi_{\tilde{\rho}} = c + \frac{1}{2} \int dx \pi_{\tilde{f}} \dot{f} \quad ,
\]

so that the independent canonical variables (or true degrees of freedom) are \( (\pi_{\tilde{f}}, f) \) canonical pairs. The reduced phase space Hamiltonian is a free-field Hamiltonian

\[
H = \frac{1}{2} \int_{-\infty}^{\infty} dx (\pi_{\tilde{f}}^2 + \dot{f}^2) .
\]

The dilaton and the original metric can be expressed in the gauge (6) as

\[
e^{-\phi} = -\lambda^2 x^+ x^- - F_+ - F_- \quad , \quad ds^2 = -e^{\phi} dx^+ dx^- \quad ,
\]

where

\[
F_\pm = \int_{-\infty}^{y} dy G(x^\pm - y) T_{\pm\pm}(y) .
\]

\( G \) is a 2d Green’s function \( (G''(x) = \delta(x)) \), and we take \( G(x) = x \theta(x) \), where \( \theta(x) \) is a step function. \( T_{\pm\pm} \) is the matter energy-momentum tensor

\[
T_{\pm\pm} = \frac{1}{2} \partial_\pm f \partial_\pm f .
\]
The formulas (9-10) can be derived from (7) by using $\pi_f = \dot{f}$.

Quantum theory is defined by choosing a representation of the quantum canonical commutation relations

$$[\pi_f(x), f(y)] = -i\delta(x - y) \; .$$

(12)

We take the standard Fock space representation, by defining the creation and annihilation operators $a^\dagger, a$ as

$$a_k = -i\pi_k + k \text{sign}(k)f_k \sqrt{2|k|} ,$$

(13)

where

$$f(x) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} e^{ikx} f_k \; , \; \pi_f(x) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} e^{ikx} \pi_k \; .$$

(14)

so that (12) is equivalent to

$$[a_k, a^\dagger_q] = \delta(k - q) \; .$$

(15)

The Fock space $\mathcal{F}(a_k)$ with the vacuum $|0\rangle$ is the physical Hilbert space of the theory. The Hamiltonian (8) can be promoted into a Hermitian operator acting on $\mathcal{F}$ as

$$H = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \omega_k a^\dagger_k a_k + E_0$$

(16)

where $\omega_k = |k|$ and $E_0$ is the vacuum energy. Therefore one has a unitary evolution described by a Schrödinger equation

$$i \frac{\partial}{\partial t} \Psi(t) = H\Psi(t) \; ,$$

(17)

where $\Psi(t) \in \mathcal{F}$. It will be convenient to work in the Heisenberg picture

$$\Psi_0 = e^{iHt}\Psi(t) \; , \; A(t) = e^{iHt} Ae^{-iHt} \; ,$$

(18)

so that

$$f(t, x) = e^{iHt} f(x) e^{-iHt} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\omega_k}} \left[ a_k e^{i(kx - \omega_k t)} + a^\dagger_k e^{-i(kx - \omega_k t)} \right] \; .$$

(19)

It is also useful to split (19) into left and right moving parts, so that $f = f_+ + f_-$ where

$$f_\pm(x^\pm) = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} \frac{dk}{\sqrt{2\omega_k}} \left[ a_\pm(k) e^{-ikx^\pm} + a^\dagger_\pm(k) e^{ikx^\pm} \right] \; .$$

(20)

The metric is given by the operator $e^\phi$, which can be defined as the inverse of the $e^{-\phi}$ operator. The $e^{-\phi}$ operator in the Heisenberg picture can be easily defined from the expressions (9) and (10), where now $f$ is the operator given by (19), while in the
expressions for $T_{\pm \pm}$ there will be a normal ordering with respect to some vacuum in $\mathcal{F}$, which can be different from $|0\rangle$.

Given a physical state $\Psi_0$, one can associate an effective metric to $\Psi(t)$ as

$$e^{\rho_{\text{eff}}(t,x)} = \langle \Psi(t) | e^{\phi(x)} | \Psi(t) \rangle = \langle \Psi_0 | e^{\phi(t,x)} | \Psi_0 \rangle \quad .$$

(21)

$e^{\rho_{\text{eff}}}$ can be interpreted as a metric if $\langle (e^\phi)^2 \rangle - \langle e^\phi \rangle^2$ is sufficiently small. This deviation cannot be zero, since if it was zero it would mean that $\Psi_0$ is an eigenvalue of the metric, whose spectrum is continuous while $\Psi_0$ is a normalisable state. In order to calculate (21), we use the following formal identity

$$(-\lambda^2 x^+ x^- - F)^{-1} = e^{\phi_0}(1 - e^{\phi_0} \delta F)^{-1} = e^{\phi_0} \sum_{n=0}^{\infty} e^{n\phi_0} \delta F^n \quad ,$$

(22)

where $F_0$ is a c-number function, $e^{-\phi_0} = -\lambda^2 x^+ x^- - F_0$ and $\delta F = F - F_0$. Then

$$\langle (-\lambda^2 x^+ x^- - F)^{-1} \rangle = e^{\phi_0} \sum_{n=0}^{\infty} e^{n\phi_0} \langle \delta F^n \rangle \quad .$$

(23)

We now want to choose $\Psi_0$ such that it is as close as possible to the classical matter distribution $f_0(x^+)$ describing a left-moving pulse of matter. The corresponding classical metric is described by

$$e^{-\rho_0} = \frac{M(x^+)}{\lambda} - \lambda^2 x^+ \Delta(x^+) - \lambda^2 x^+ x^-$$

(24)

where

$$M(x^+) = \lambda \int_{-\infty}^{x^+} dy y T^{0+}_0(y) \quad , \quad \lambda^2 \Delta = \int_{-\infty}^{x^+} dy T^{0+}_0(y)$$

(25)

and $T^{0+}_0 = \frac{1}{2} \partial_+ f_0 \partial_+ f_0$. The geometry is that of the black hole of the mass $M = \lim_{x^+ \to +\infty} M(x^+)$ and the horizon is at $x^- = -\Delta = -\lim_{x^+ \to +\infty} \Delta(x^+)$. The asymptotically flat coordinates $(y^+, y^-)$ at the past infinity are given by

$$\lambda x^+ = e^{\lambda y^+} \quad , \quad \lambda x^- = -e^{-\lambda y^-} \quad ,$$

(26)

while the asymptotically flat coordinates $(\sigma^+, \sigma^-)$ at the future infinity satisfy

$$\lambda x^+ = e^{\lambda \sigma^+} \quad , \quad \lambda (x^- + \Delta) = -e^{-\lambda \sigma^-} \quad .$$

(27)

Since the particle vacuum for the observer at the past infinity is defined with respect to the $y^\pm$ coordinates, we will introduce the in creation and annihilation operators $a_{in}^\dagger, a_{in}$ as

$$f_+(t, x) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \frac{dk}{\sqrt{2\omega_k}} \left[ a_{in}(k)e^{-iky^+} + a_{in}^\dagger(k)e^{iky^+} \right] \quad ,$$

(28)
and similarly for the right-moving sector. Hence we take for $\Psi_0$ a coherent state

$$\Psi_0 = e^{A} \left| 0^+_{in} \right\rangle \otimes \left| 0^-_{in} \right\rangle$$

(29)

where

$$A = \int_{0}^{\infty} dk [f_0(k)a_{in}^+(k) - f_0^*(k)a_{in}(k)]$$

(30)

and $f_0(k)$ are Fourier modes of $f_0(y^+)$. Before we start computing the effective metric, we have to choose the function $F_0$, and we have to fix the normal ordering in the $F$ operator. We take $F_0$ corresponding to the classical metric (24), since we want to compute the quantum corrections to it, and we take the normal ordering with respect to the out vacuum. From (26-27) follows that

$$y^+ = \sigma^+$$

so that

$$\left| 0^-_{in} \right\rangle = \left| 0^+_{out} \right\rangle$$

and hence in the left sector “out” normal ordering is the same as the “in” normal ordering.

This is not the case in the right sector, where the in and out vacuum are related by

$$\left| 0^-_{in} \right\rangle = \mathcal{S} \left( a_{out}, a_{out}^\dagger \right) \left| 0^-_{out} \right\rangle$$

(31)

and $\mathcal{S}$ is the generator of the Bogoliubov transformation [8]

$$a_{out}(k) = \mathcal{S}a_{out}(k)S^\dagger = \int_{0}^{\infty} dq \left( b_q \alpha_{qk} + \hat{b}_q \beta^*_{qk} + \hat{b}_q^\dagger \alpha_{qk} + \hat{b}_q^\dagger \beta^*_{qk} \right) ,$$

(32)

where $a_{out}$ are split into operators outside and inside the horizon, represented by $b$ and $\hat{b}$, respectively.

Now it is not difficult to show that

$$\langle F_+ \rangle = F_0,$$

while

$$\langle F_- \rangle = \int_{-\infty}^{x^-} dx_1 (x^- - x_1^-) \langle T_-(x_1^-) \rangle .$$

(33)

Note that

$$\langle T_-(x^-) \rangle = \left( \frac{\partial \sigma^-}{\partial x^-} \right)^2 \langle T_-(\sigma^-) \rangle ,$$

(34)

and from [1, 8] we have

$$\langle T_-(\sigma^-) \rangle = \frac{\lambda^2}{48} \left[ 1 - (1 + \lambda \Delta e^{\lambda \sigma^-})^{-2} \right] ,$$

(35)

so that

$$\langle F_- \rangle = \int_{-\infty}^{x^-} dx_1 (x^- - x_1) \frac{1}{48} ((x_1 + \Delta)^{-2} - x_1^{-2}) = -\frac{1}{48} \log \left| x^- + \Delta \right| .$$

(36)

This gives

$$e^{\rho_{eff}} \approx e^{\rho_0} (1 + e^{\rho_0} \langle F_- \rangle) ,$$

(37)

which is a good approximation if

$$| e^{\rho_0} \langle F_- \rangle | = \frac{1}{48} \log \left| \frac{x^- + \Delta}{x^-} \right| \ll 1 .$$

(38)
Eq. (38) implies
\[
\log \left| \lambda \Delta \right| + \frac{\lambda \sigma^-}{48} \ll \frac{M}{\lambda} + \lambda x^+ e^{-\lambda \sigma^-},
\]
so that if \( \frac{M}{\lambda} >> \frac{1 + \log |\lambda \Delta|}{48} \) then for a considerable time we can neglect the quantum corrections, and we effectively have a quantum field propagation on a black hole background. As is well known, this will produce the Hawking radiation, with the temperature \( T_H = \frac{\lambda}{2 \pi} \) and the flux at the future infinity given by (35) \([1]\). Hence the Hawking radiation is present in the semiclassical limit of our unitary quantum gravity theory.

Note that when \(|e^{\rho_0} (F_-)| << 1\), then
\[
e^{\rho_{\text{eff}}} \approx e^{\rho_0} (1 + e^{\rho_0} (F_-)) \approx e^{\rho_0} (1 - e^{\rho_0} (F_-))^{-1} = (e^{-\rho_0} - (F_-))^{-1} \quad . \tag{40}
\]
The last expression in eq. (40) can be interpreted as the semiclassical metric with the first order backreaction effect included. It can be written as
\[
e^{-\rho_1} = \frac{M(x^+)}{\lambda} - \lambda^2 x^+ \Delta(x^+) - \lambda^2 x^+ x^- + \frac{1}{48} \log \left| \frac{x^+ + \Delta}{x^-} \right| . \tag{41}
\]
As expected, the curvature singularity is still present at this order, but what is interesting is the behaviour of a local mass function, which can be defined as
\[
M_1(x^+, x^-) = \frac{1}{4\lambda} e^{-\phi_1} R_1 . \quad \tag{42}
\]
One then obtains
\[
M_1(x^+, x^-) = M(x^+) + \frac{\lambda}{48} \log \left| 1 + \frac{\Delta}{x^-} \right| + \frac{\lambda}{48} \frac{\Delta}{x^-} , \quad \tag{43}
\]
which grows from 0 to \( M \) for large negative \( x^- \) (corresponding to the formation of the black hole at early times), but then it starts to monotonically decrease, and it reaches zero at some \( x^-_0 < -\Delta \). As \( x^- \) is approaching the horizon \( -\Delta \) (or for very late late times), \( M_1 \) goes to \(-\infty\). Hence in the region of validity of our approximation (\( x^- \) not to close to \( -\Delta \), which is equivalent to not too late times) the backreaction effect is precisely what one expects, i.e. black hole mass decreases due to the Hawking radiation. As time approaches the future infinity, one expects that the higher order corrections will stabilize the mass. One can think of various possibilities for this stabilized mass function, but the one which fits most naturally with the fact that the matter is freely propagating, is that mass goes to zero, i.e. black hole completely evaporates end one ends up with a nearly flat space-time. Since the theory is unitary, the purity of the state is preserved through non-local quantum correlations \([3]\).

In order to see whether something like this will happen, we will have to calculate the higher order corrections. Note that if we use \( F_0 = \langle F_+ \rangle + \langle F_- \rangle \), the zero order...
metric will be the semiclassical metric $e^{\phi_1}$, and the expansion (23) will become more symmetric, with $n = 1$ term vanishing. Calculating the $\langle \delta F^n \rangle$ terms will require calculating $\langle T(x_1) \cdots T(x_2) \rangle$ and this will require a regularization. The singularity structure of such an expression is encoded in the OPE

$$T(x)T(y) = \frac{c/2}{(x-y)^4} + \frac{2T(y)}{(x-y)^2} + \frac{2\partial T(y)}{x-y} + \text{const.} + o(x-y) \quad ,$$

and the simplest thing one can do is to define

$$: T(x_1) \cdots T(x_n) := T(x_1) \cdots T(x_n) - \langle 0 \rangle T(x_1) \cdots T(x_n) |0 \rangle$$

where $|0\rangle$ is the relevant vacuum (i.e. $\langle 0 | T | 0 \rangle = 0$).

In the $n = 2$ case we will have

$$\langle T(x_1) T(x_2) : \rangle = \langle 0 | e^{-X} T(x_1) T(x_2) : e^X | 0 \rangle = \langle 0 | T_X(x_1) T_X(x_2) : | 0 \rangle$$

where $T_X = e^{-X} T e^X$ and $X = A$ for the left sector, while $X = \log S$ for the right sector. Then by using (44) and (45) we get

$$\langle 0 | T_X(x_1) T_X(x_2) : | 0 \rangle = \frac{2 \langle 0 | T_X(x_2) : | 0 \rangle}{(x_1 - x_2)^2} + \frac{2 \partial \langle 0 | T_X(x_2) : | 0 \rangle}{x_1 - x_2} + o(x_1 - x_2)$$

and since $\langle 0 | T_X : | 0 \rangle \neq 0$, the singularity is still present. However, in the left sector, one can calculate exactly (47) by using $T_X = T - [X, T] + \frac{1}{2} [X, [X, T]] + \cdots$, where it terminates after the third term, and one obtains

$$\langle 0 | : T_A(y_1) T_A(y_2) : | 0 \rangle - T_0(y_1) T_0(y_2) = \langle 0 | [A, T]_1 [A, T]_2 | 0 \rangle$$

$$= - \int_0^\infty dk k e^{ik(y_1 - y_2)} \frac{\partial f_0}{\partial y_1} \frac{\partial f_0}{\partial y_2} .$$

Since

$$\int_0^\infty dk k e^{ik(y_1 - y_2)} = - \frac{1}{(y_1 - y_2)^2} ,$$

one can get back to the OPE form, but we will not do that. Instead we write

$$\langle \delta F_+^{2} \rangle = - \prod_{i=1}^2 \int_{-\infty}^{y_i} dy_i (e^{\lambda(y_i - y_i)} - 1) \int_0^\infty dk k e^{ik(y_1 - y_2)} \frac{\partial f_0}{\partial y_1} \frac{\partial f_0}{\partial y_2}$$

$$= \int_0^\infty dk k |\mathcal{F}(k, y^+)|^2$$

where

$$\mathcal{F}(k, y^+) = \int_{-\infty}^{y^+} dy_e e^{ik(y^+ - y)} (e^{\lambda(y^+ - y)} - 1) \frac{\partial f_0}{\partial y} .$$

Then by choosing $f_0$ which falls quickly enough away from the centre of the matter pulse, we can get $\mathcal{F}(k)$ such that (49) is finite.
In the right sector one can apply a similar trick, although the actual calculation is more difficult. Calculating $\langle T_{-}(1) \cdots T_{-}(n) \rangle$ requires calculating

$$\langle 0_{in} \left| a_{out}^{\pm}(1) \cdots a_{out}^{\pm}(2n) \right| 0_{in} \rangle , \tag{52}$$

where $a_{out}^{\pm}$ stands for the creation or the annihilation operator, respectively. Expression (52) boils down to a sum of products of the following integrals

$$\int_{0}^{\infty} dp \beta_{\pm}^{\alpha} \beta_{pq}^{\pm}, \quad \int_{0}^{\infty} dp \beta_{kp}^{\pm} \beta_{pq}^{\pm}, \quad \int_{0}^{\infty} dp \alpha_{kp}^{\pm} \alpha_{pq}^{\pm} , \tag{53}$$

where $X^{\pm} = X$ or $X^{*}$, and $\alpha$ and $\beta$ are the coefficients of the Bogoliubov transformation (32). One can calculate the integrals in (53) for late times, since then

$$\int_{0}^{\infty} dp \beta_{kp}^{\pm} \beta_{pq}^{\pm} \approx \langle n_{k} \rangle \delta(k-q) = \frac{e^{-\beta k}}{1-e^{-\beta k}} \delta(k-q) \tag{54}$$

and

$$\alpha_{kp} \approx -e^{\beta k/2} \beta_{kp} , \tag{55}$$

where $\beta = \frac{2\pi}{\lambda}$, and consequently obtain a good estimate of (52). From (54) one can expect that the convergence will come from the expontenally damping factor $e^{-\beta k}$ in $\langle n_{k} \rangle$. For example

$$\langle F_{-}^{2} \rangle \approx \frac{1}{\lambda^{2}} \prod_{i=1}^{2} \int_{-\infty}^{\sigma_{-}} d\sigma_{i} (e^{-\lambda(\sigma_{-}-\sigma_{i})} - 1) \left[ \frac{c_{0}}{(\sigma_{1} - \sigma_{2})^{4}} + \frac{c_{1}}{(\sigma_{1} - \sigma_{2})^{2}} \right] , \tag{56}$$

where $c_{i}$ are certain numerical constants. Note that $c_{0} + \frac{4}{\lambda^{2}} c_{1} = 0$ because of the absence of $(\sigma - \sigma_{i})^{-4}$ term in the OPE. However, there is still a divergence due to $(\sigma - \sigma_{i})^{-2}$ terms, and hence we use eq. (49) to rewrite the first two terms in (56) as

$$-\frac{1}{\lambda^{2}} \int_{0}^{\infty} dk k^{2} \prod_{i=1}^{2} \int_{-\infty}^{\sigma_{-}} d\sigma_{i} \left[ e^{-\lambda(\sigma_{-}-\sigma_{i})} - 1 \right] e^{i k (\sigma_{1} - \sigma_{2})}$$

$$\left[ -\frac{1}{(\sigma_{1} - \sigma_{2})^{2}} + \frac{\lambda^{2}}{4} \right] , \tag{57}$$

which gives a finite expression.

In conclusion, we have find a way to incorporate the Hawking radiation and backreaction effects in a manifestly unitary 2d quantum gravity theory. It still remains to be investigated how to regulate the higher order corrections, and whether a cut-off in momentum will be needed. The cut-off would require a renormalization scheme, and since the CGHS theory is renormalisable, the hope is that will symplify
the analysis in our case. Related to that is the question of the sumability of the perturbative series in order to obtain the exact quantum metric. Another possibility is to define the effective quantum metric recursively as

$$e^{\rho_{n+1}} = e^{\rho_n} (1 + e^{\rho_n} \langle F - F_n \rangle + e^{2\rho_n} \langle (F - F_n)^2 \rangle) = (-\lambda^2 x^+ x^- - F_{n+1})^{-1}$$  \hspace{0.5cm} (58)

starting from $n = 1$. One would then obtain a series of geometries $G_n$, and the question then would be to find a meaningful limit of such a series. As far as the 4d black holes are concerned, our method could be applied to the BCMN model. However, a modification will be necessary, since the BCMN model is not classically solvable, and hence one will not have an explicit expression for the metric as a function of the matter fields, which is the basis of our approach.

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