Self-Sustainability of Energy Harvesting Systems: Concept, Analysis, and Design

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Abstract

Ambient energy harvesting is touted as a low cost solution to prolong the life of low-powered devices, reduce the carbon footprint, and make the system self-sustainable. Most research to date have focused either on the physical aspects of energy conversion process or on optimal consumption policy of the harvested energy at the system level. However, although intuitively understood, to the best of our knowledge, the idea of self-sustainability is yet to be made precise and studied as a performance metric. In this paper, we provide a mathematical definition of the concept of self-sustainability of an energy harvesting system, based on the complementary idea of eventual outage. In particular, we analyze the harvest-store-consume system with infinite battery capacity, stochastic energy arrivals, and fixed energy consumption rate. Using the random walk theory, we identify the necessary condition for the system to be self-sustainable. General formulas are given for the self-sustainability probability in the form of integral equations. Since these integral equations are difficult to solve analytically, an exponential upper bound for eventual outage probability is given using martingales. This bound guarantees that the eventual outage probability can be made arbitrarily small simply by increasing the initial battery energy. We also give an asymptotic formula for eventual outage. For the special case when the energy arrival follows a Poisson process, we are able to find the exact formulas for the eventual outage probability. We also show that the harvest-store-consume system is mathematically equivalent to a $GI/G/1$ queueing system, which allows us to easily find the outage probability, in case the necessary condition for self-sustainability is violated. Monte-Carlo simulations verify our analysis.

Index Terms

Energy harvesting, self-sustainability, eventual energy outage, random walks, renewal theory, martingale

I. INTRODUCTION

In recent years, ambient energy harvesting and its applications have become a topic to great interest. Basically, a device is assumed to be able to harvest energy from random energy source in their surrounding

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environment for their future use. Such methods are useful when devices are low powered and energy constrained. They can help in extending the life time of a device and lower its maintenance cost. Furthermore, they can help lower carbon dioxide emissions and fight climate change. The readers are referred to [1]–[5] for general surveys of this field.

Apart from traditional sources of ambient energy such as solar, wind, and wave, in the past decade research has extended to energy scavenging techniques from diverse energy sources [6], [7] such as thermal [8], [9], pressure and vibrations [8], [10], ambient radio-frequency (RF) radiation [11], [12], bodily motions [13], magnetic field [14], ambient sound [15], and ambient light [16], [17]. Such energy harvesting techniques have been applied in the context of various technologies such as wireless sensor networks [18]–[21], telecommunications [22]–[24], cellular networks [25], cognitive radio network [26], [27], vehicular network [28], health care [29], [30], IoT (Internet of things) [31], [32], IoE (Internet of energy) [33], and smart grid [34]–[36] technology.

A. Nature of Energy Sources and General Architectures

The ambient energy sources can be broadly classified as steady energy source or stochastic energy source. These diverse sources of energy are converted into electricity, which is then either directly consumed, or stored in a rechargeable battery for future use. As with harvesting, consumption of energy can also be either steady or stochastic. Three general architectures of energy harvesting network are harvest-consume, harvest-store-consume, and harvest-store/consume [5].

In the harvest-consume model, the harvested energy is immediately consumed by the consumer. This model is appropriate when a steady supply of harvested energy can be guaranteed and a battery-free circuit is desired. The major problem with this approach is that, due to possible random nature of the energy source, when the harvested energy is less than the minimum operational energy required by consuming device, the device is disabled. We say that the consumer has experienced an energy outage.

In harvest-store-consume model, the harvested energy is first transferred to an energy storage facility or a rechargeable battery. The consumer then accesses the harvested energy from the battery. By this method the consumer can ensure a steady supply of energy, even though the harvested energy is randomly varying. This method is appropriate when the harvested energy is very small and is highly fluctuating, as in ambient RF energy harvesting.

We can also have a hybrid model given by harvest-store/consume. In this model the harvest energy is either stored for some future use or directly consumed. If the harvested energy is above minimum requirement of the consumer, then the excess energy can also be stored. This model is appropriate when the harvested energy does not fluctuate very rapidly.
B. Motivation: Energy Outage, Eventual Energy Outage, and Self-Sustainability

A lot of research attention has been paid either on the physical aspects of energy harvesting mechanism \([6] – [17]\) or on the consumption policy of the harvested energy \([18] – [36]\). Work dealing with former issue tend to focus on physical modeling and optimization of energy harvesting devices, with the goal of improving the efficiency of energy conversion process. Work dealing with the latter issue tend to focus on optimizing some performance metric, under energy constrains of the energy harvesting device. Clearly, the work of the latter category is affected by the general architecture assumed. Also, the exact nature of the metric depends on what the harvested energy is being used for. For instance, in the field of energy harvesting communication, when the harvest-consume model is used, a possible natural performance metric is the joint energy and information outage probability \([37], [38]\). If instead the harvest-store-consume model is used, a possible performance metric is the data rate of the communication system \([23]\).

While there have been plenty of work that have examined systems that have energy harvesting capability, very few work have focused on examining the concept of energy outage of the consumer. This is unfortunate because energy outage should be one of the key performance metric of any energy harvesting system. The consumer is said to experience energy outage if there is no energy available for its consumption. For the harvest-consume model, the energy outage is determined by the randomness of the harvester-to-consumer energy transmission channel or the inherent randomness of the energy source. In \([37], [38]\) the energy arrival process is assumed to be a stationary, ergodic process. As such, knowing the distribution of energy arrival allows us to calculate the energy outage probability. For the harvest-store-consume model, the harvested energy is modeled as discrete packets of possibly variable size \([23]\). The energy arrival process at the battery is generally modeled as a Poisson process, allowing the battery state to be modeled as a Markov chain \([39]\). However, research on energy outage is sorely lacking for harvest-store-consume architecture.

Another concept closely associated with energy harvesting systems is the concept of self-sustainability. Self-sustainability is commonly understood as the ability to supply one’s own needs without external assistance. In the context of energy harvesting systems, it would mean the ability of the system to provide the necessary energy for the consumer, without the need for external grid power. To the best of our knowledge, this concept is yet to be mathematically defined. In this paper, we define this concept as follows: The self-sustainability of an energy harvesting system is the probability that the consumer will not eventually experience an energy outage. Here we need to make a distinction between energy outage and eventual energy outage. We say that a consumer experiences an eventual energy outage if the consumer undergoes an energy outage within finite time. If the consumer has to wait for infinite amount of time to experience an energy outage, then we say that the energy harvesting system is self-sustainable. The
eventual energy outage and the self-sustainability are complementary in that the sum of their probabilities is unity.

Given these basic definitions, we can quickly point out that for harvest-consume system, if both the energy harvest and consumption rates are steady, then the system is self-sustainable if the rate of consumption is less than the rate of harvest. Likewise, if the energy harvest is stochastic and the consumption is steady (or vice versa), then the consumer will almost surely experience an eventual outage. Hence the harvest-consume system is not a self-sustaining architecture for stochastic energy harvest or consumption.

In general, we want the answers to the following questions:

1) Under what condition is self-sustainability possible?
2) Can we come up with a formula or a bound for self-sustainability probability?
3) How should we design a system, provided a constraint on the eventual energy outage probability?

C. Contributions

The main contribution of this work is the establishment of the concept of self-sustainability for energy harvesting system. We specifically study the harvest-store-consume architecture, which we analyze based on random walks, renewal theory, and martingales. In particular, we are able to adapt many ideas from the ruin theory of actuarial science, making our work cross-disciplinary. This leads to the following findings:

1) The necessary condition for harvest-store-consume system to be self-sustaining is simply that the rate of consumption be strictly less than the rate of harvest. In this paper, we refer this condition as the **self-sustainability condition**.
2) We provide general formulas for the self-sustainability probability of harvest-store-consume system. In particular, we demonstrate the relationship between the self-sustainability probability and the maximum of the underlying random walk using three different formulas. Two of these formulas are renewal type and Weiner-Hopf type integral equations.
3) Since finding the analytical solution to the integral equations is difficult, using the concept of martingales, we provide an exponential upper bound to eventual energy outage probability for harvest-store-consume system, provided that the self-sustainability condition is satisfied. We show that the eventual energy outage can be made arbitrarily small by simply increasing the initial battery energy. This leads to simple design guidelines, given the eventual energy outage constraint.
4) Using the renewal type integral equation for the self-sustainability probability, we provide an asymptotic formula for the eventual energy outage probability based on key renewal theorem for defective distributions.
5) For the special case when the arrival of energy packets is modeled as a Poisson process, we give exact formulas for the eventual energy outage probability.

6) We prove that harvest-store-consume system is mathematically equivalent to a GI/G/1 queuing system and draw parallels between the two systems. This allows us to import the results from queueing theory when the self-sustainability condition is not satisfied. While this is certainly not a new observation [23], we provide a systematic proof of the equivalence and connect it to the idea of energy outage.

From here on, without any ambiguity, we will simply refer to the energy outage as the outage and the eventual energy outage as the eventual outage.

D. Organization

The rest of the paper is organized as follows: Section II discusses the system model and assumptions, and also defines the probability of self-sustainability. Section III gives the random walk analysis of the energy surplus process and the self-sustainability probability. Section IV gives an exponential upper bound on eventual outage probability (which is the complement of self-sustainability probability). Section V gives an asymptotic approximation of the eventual outage probability and discusses the computation of the adjustment coefficient, while Section VI studies the special case when the energy arrivals is a Poisson process. Section VII examines the battery energy process, while Section VIII gives a numerical verification of the obtained formulas. Lastly, Section IX concludes the paper.

II. System Model, Assumptions, and the Concept of Self-Sustainability

A. Definitions and Assumptions

We will consider a harvest-store-consume (HSC) system. That is, all the harvested energy is first collected, before being consumed. Thus, the consumer obtains the harvested energy indirectly from the storage facility (or the rechargeable battery). The harvested energy can arrive into the storage in continuous fashion (as in solar or wind or grid power source) or in impulsive fashion. We will restrict our analysis to the case of impulsive energy arrivals. Thus, our main physical assumption is as follows:

**Assumption 1.** Harvested energy arrives as impulses into the storage system.

In other words, the harvested energy arrives in the form of packets into the storage system, and the size of each energy packets may vary randomly. As such, we can model the energy surplus process of the system at any time \( t \) as

\[
U(t) = u_0 - \int_0^t p(u, t) \, dt + \sum_{i=1}^{N(t)} X_i h(t - t_i).
\]  

(1)
Here $u_0 > 0$ is the initial battery energy and $p(u,t) > 0$ is the power consumption from the battery, i.e. \( \frac{du}{dt} = p(u,t) \). The $X_i \in \mathbb{R}_+$ is the amount of energy in an $i$-th energy packet that arrives at time $t_i$, while $N(t) = \max\{ i : t_i \leq t \}$ is the total number of energy packets that have arrived at the storage by time $t$. Lastly, $h(t)$ is the transient response of battery charging process. Clearly, if $U(t) > 0$, then the system is producing more energy than it is consuming. Likewise, if $U(t) < 0$, then the consumer takes in energy from the grid to compensate for energy deficit.

We will now define a few concepts that will be used in the paper:

**Definition 1** (Defective and proper distributions). A random variable with distribution $F$ is said to be **defective** if $\lim_{x \to \infty} F(x) < 1$, the amount of defect being $1 - F(\infty)$. The random variable is said to be **proper** if $\lim_{x \to \infty} F(x) = 1$.

**Definition 2** (Renewal process). A sequence $\{S_n\}$ is a renewal process if $S_n = X_1 + \cdots + X_n$ and $S_0 = 0$, where $\{X_i\}$ are mutually independent, non-negative random variables with common distribution $F_X$ such that $F_X(0) = 0$. When $F_X$ is a proper distribution, the renewal process is said to be a persistent renewal process. If $F_X$ is defective then the renewal process is said to be a transient or terminating renewal process.

The variables $X_i$ in Definition 2 is often interpreted as “interarrival time”. However, $X_i$ need not always be time, as we will see later in the paper. We will now make further mathematical assumptions required to simplify our analysis. We will be working with the resulting simplified model for the rest of the paper.

**Assumption 2.** The storage capacity of the battery is infinite.

**Assumption 3.** The rate of energy consumption is constant, i.e. $p(u,t) = p$.

**Assumption 4.** The transient response of battery charging process, $h(t)$, is a unit step function.

**Assumption 5.** The energy packets $\{E_i\}$ arrive at corresponding times $\{t_i\}$, for $i = 0, 1, 2, \ldots$, such that $0 = t_0 < t_1 < t_2 < \cdots$, where the energy packet $E_0$ arrives at time 0. The arrival times $\{t_i\}$ is (i) a renewal process. That is, the inter-arrival times $\{A_i\}$, where $A_i = t_{i+1} - t_i$, are such that $A_0, A_1, A_2, \ldots$ are independent and identically distributed. (ii) To avoid more than one energy arrivals at a time, we assume $F_A(0) = 0$. (iii) Lastly, the expected inter-arrival time is finite, $\mathbb{E}[A_i] < \infty$.

**Assumption 6.** The amount of energy $\{X_i\}$ corresponding to the energy packets $\{E_i\}$ for $i = 0, 1, 2, \ldots$, is (i) a non-negative, continuous random variable; (ii) $\{X_i\}$ are independent and identically distributed; (iii) to avoid energy packets of zero size, we assume $F_X(0) = 0$; and (iv) lastly, the expected energy in a packet is finite, $\mathbb{E}[X_i] < \infty$. 
Assumption 7. The inter-arrival times \( \{A_i\} \) and the energy packet sizes \( \{X_i\} \) are independent of each other.

Here Assumptions 2 – 4 essentially simplify the physical setup of our problem, whereas Assumptions 5 – 7 are essentially technical in nature to facilitate the mathematical analysis. Note that in Assumption 5, while not necessary, we have specifically assume that there is an arrival of an energy packet at time zero. Of all these Assumptions 2 – 7, Assumption 2 regarding the infiniteness of the battery capacity is perhaps be the most objectionable. However, for the time being, we shall keep it for preliminary analysis of the system.

Given these assumptions, our initial mathematical model of the energy surplus (1) simplifies to

\[
U(t) = u_0 - pt + \sum_{i=0}^{N(t)} X_i.
\]

This is an instance of a random walk on real line through continuous time with downward drift. As shown in Fig. 1, the graph of \( U(t) \) versus \( t \) will look like a sawtooth wave with descending ramps and with random jump discontinuities. The system experiences an outage just before \( t = 12 \).

B. Dual Model

It is also possible to have a dual model of the model given in (1). If instead of Assumption 1, the harvested energy arrives steadily and the energy consumption is impulsive, then we have the dual model given by

\[
U(t) = u_0 - \sum_{i=1}^{N(t)} X_i h(t - t_i) + \int_0^t p(u, t) dt.
\]
Notice that in this dual model there is only a change in the signs and an interchange in the interpretation of the symbols. Thus, the same results for our basic model can be shown to be valid for the dual model as well by a trivial variation in the arguments. For the rest of the work, we will focus on the first model given in [1].

C. Concept of Self-Sustainability

Let \( W(t) \) be the battery energy at time \( t \). We say that the system undergoes outage when \( W(t) = 0 \). In other words, the case \( W(t) = 0 \) represents the situation when the battery is empty, and as such, the consumer needs to fetch its required energy from the grid to sustain its consumption. We define the outage probability of the system as

\[
P_{\text{out}} = P(W(t) = 0),
\]

that is, probability of finding the battery empty at any time \( t \).

Now consider the first time that the battery is empty, \( \tau = \inf\{t > 0 : W(t) = 0, W(0) = u_0\} = \inf\{t > 0 : U(t) \leq 0, U(0) = u_0\} \). We will refer to \( \tau \) as the first time to outage. If \( \tau = \infty \), then the system become self-sustaining and the system will not face an eventual outage. The occurrence of eventual outage is equivalent to the event \( \{\tau < \infty\} \). However, since \( \tau \) itself is a random variable, we can only describe it probabilistically. Thus, we are interested in knowing the probability

\[
\phi(u_0, p, T) = P\left[ \sup_{0 \leq t \leq T} \left( pt - \sum_{i=0}^{N(t)} X_i \right) \leq u_0 \right],
\]

that the energy surplus \( U(t) \) will not fall below zero through time \( t \in [0, T] \) where \( T < \infty \), and

\[
\phi(u_0, p, \infty) = P\left[ \sup_{0 \leq t \leq \infty} \left( pt - \sum_{i=0}^{N(t)} X_i \right) \leq u_0 \right],
\]

of avoiding an eventual outage. Note that this is equivalent to the probability

\[
\phi(u_0, p, T) = 1 - P(\tau < T|u_0, p) = 1 - \psi(u_0, p, T),
\]

where \( \psi(u_0, p, T) \) is the probability of an outage occurring within a finite time \( T \).

**Definition 3.** We define the self-sustainability probability of an energy harvesting system as the probability that the first time to outage \( \tau \) is at infinity. Likewise, the eventual outage probability is the probability that the first time to outage \( \tau \) is finite. That is,

\[
\phi(u_0, p, \infty) = 1 - P(\tau < \infty|u_0, p) = 1 - \psi(u_0, p, \infty),
\]

where \( \phi(u_0, p, \infty) \) denotes the self-sustainability probability and \( \psi(u_0, p, \infty) \) denotes the eventual outage probability.
Since \( p \) is held constant and \( T = \infty \), from here on, we will drop these two parameters in the argument, and simply refer to the self-sustainability probability and eventual outage probability as a function of \( u_0 \) as given by \( \phi(u_0) \) and \( \psi(u_0) \).

### III. Energy Surplus Process and Evaluation of Self-Sustainability Probability

Let the expected inter-arrival time be \( \mathbb{E}[A_i] \) and the arrival rate of energy packets be defined as \( \lambda = \frac{1}{\mathbb{E}[A_i]} \). Also, let the average energy packet size be \( \mathbb{E}[X_i] = \bar{X} \).

Intuitively, if we want our system to be self-sustainable, then we would want the expected surplus energy to be positive \( \mathbb{E}[U(t)] > 0 \). For this to be true, it is sufficient that the consumption rate, \( p \), be less than the energy arrival rate, \( \lambda \bar{X} \). That is, \( \lambda \bar{X} > p \). We can state this condition rigorously as follows:

**Proposition 1.** If \( \lambda \bar{X} > p \), then \( \lim_{t \to \infty} \mathbb{E}[U(t)] > 0 \) almost surely.

**Proof:** The expected value of \( U(t) \) for any \( t \) is

\[
\mathbb{E}[U(t)] = u_0 - pt + \mathbb{E} \left[ \sum_{i=0}^{N(t)} X_i \right] \\
\quad \equiv u_0 - pt + \mathbb{E}[N(t)]\mathbb{E}[X_i] \\
\quad = u_0 + \left( \frac{\mathbb{E}[N(t)]}{t} \bar{X} - p \right) t.
\]

Here, the equality \((a)\) is due to Wald’s identity. From elementary renewal theorem, \( \frac{\mathbb{E}[N(t)]}{t} \to \lambda \) for large \( t \) almost surely \([40] \) Th 5.8.4). Thus, we have for large \( t \),

\[
\mathbb{E}[U(t)] = u_0 + (\lambda \bar{X} - p)t,
\]

almost surely. Given the assumption \( \lambda \bar{X} > p \), \( \lim_{t \to \infty} \mathbb{E}[U(t)] \to +\infty \), proving the statement.

It should be noted that when the condition \( \lambda \bar{X} > p \) is satisfied, the proposition does not tell us that an outage will never occur. Rather, it tells us that there is a chance of such non-occurrence of outage.

#### A. Random Walk Analysis

In general, it is difficult to determine the outage event. We therefore have to condition on the arrival process. We can tell that an outage has occurred if a new arrival finds the battery empty. The mathematical trick here is to reduce the continuous time process into a discrete time process by counting over the arrivals. In the analysis of random walks and ascending ladder process, we will basically follow the approach laid out by Feller in \([42]\). A modern treatment of the subject can be found in \([41]\).
Proposition 2. Let the sequences \( \{Z_i; i \geq 0\} \) and \( \{S_i; i \geq 1\} \) be defined as \( Z_i = pA_i - X_i \) and \( S_n = \sum_{i=0}^{n-1} Z_i \) where \( S_0 = 0 \). The surplus energy \( \{U_n\} \), observed immediately before the arrival of \( n \)-th energy packet, forms a discrete time random walk over real line, such that

\[
U_n = u_0 - S_n. \tag{6}
\]

**Proof:** Let us denote \( t_n^- = t_n - \epsilon \) where \( \epsilon > 0 \), as time immediately before the arrival of \( n \)-th energy packet at \( t_n \). Since \( t_n^- \sim t_n \) as \( \epsilon \to 0 \), we can decompose the arrival time \( t_n^- = \sum_{i=0}^{n-1} A_i \). If we follow the value of \( U(t) \) immediately before each arrival at \( t_n^- \), we have

\[
U_n = u_0 - pt_n^- + \sum_{i=0}^{N(t_n^-)} X_i
= u_0 - \sum_{i=0}^{n-1} (pA_i - X_i).
\]

Let \( Z_i = pA_i - X_i \) for \( n = 0, 1, 2, \ldots \). Here, \( Z_i \) is no longer a non-negative random variable; rather it can take any real value, \( Z_i \in \mathbb{R} \). Since both \( \{X_i\} \) and \( \{A_i\} \) are independent and identical to each other, \( \{Z_i\} \) are independent and identical to each other too. Our expression now becomes

\[
U_n = u_0 - \sum_{i=0}^{n-1} Z_i,
\]

which is a discrete time random walk over real line. Defining \( \{S_n; n \geq 1\} \) such that \( S_n = \sum_{i=0}^{n-1} Z_i \) with initialization \( S_0 = 0 \), gives us \( U_n = u_0 - S_n \), which is our desired result. \( \blacksquare \)

**Remark:** Here, the \( \{U_i\} \) records the troughs of the sawtooth wave \( U(t) \) and can be interpreted as the energy surplus that the \( n \)-th arrival “finds” the system in.

The existence of self-sustaining system can be directly proved with the help of the following theorem from the theory of random walk on real line:

**Theorem 1.** [41] Ch 8, Th 2.4] For any random walk with \( F_Z \) not degenerate at 0, one of the following possibilities occur:

1) (Oscillating Case) If \( \mathbb{E}[Z_i] = 0 \), then \( P(\lim \sup_{n \to \infty} S_n = +\infty) = 1 \), \( P(\lim \inf_{n \to \infty} S_n = -\infty) = 1 \);
2) (Drift to +\( \infty \)) If \( \mathbb{E}[Z_i] > 0 \), then \( P(\lim_{n \to \infty} S_n = +\infty) = 1 \);
3) (Drift to -\( \infty \)) If \( \mathbb{E}[Z_i] < 0 \), then \( P(\lim_{n \to \infty} S_n = -\infty) = 1 \).

The following propositions easily follow from the above theorem.

**Proposition 3.** The HSC system is self-sustaining only if it satisfies the self-sustainability condition given by \( \lambda \bar{X} > p \).
Proof: Consider the contra-positive of the statement: If $\lambda \bar{X} \leq p$, then the HSC system will experience eventual outage. Since $E[Z_i] = E[pA_i - X_i] = pE[A] - \bar{X}$, we have that $\lambda \bar{X} \leq p$ is equivalent to $E[Z_i] \geq 0$.

If $\lambda \bar{X} < p$, then this is equivalent to the condition $E[Z_i] > 0$. Hence, from Case 2 of Theorem 1, $S_n \to +\infty$ almost surely. Since $U_n = u_0 - S_n$, by the definition of limit,

$$P(\lim_{n \to \infty} S_n = +\infty) = P(\forall c, \exists n_0 : \forall n > n_0, S_n > c)$$

$$= P(\forall c, \exists n_0 : \forall n > n_0, u_0 - U_n > c)$$

$$= P(\forall c, \exists n_0 : \forall n > n_0, U_n < u_0 - c)$$

Taking $c = u_0$, we have $P(\exists n_0 : \forall n > n_0, U_n < 0) = 1$. That is, $U_n < 0$ almost surely. Thus, the HSC system will experience eventual outage.

Similarly, if $\lambda \bar{X} = p$, this is equivalent to the condition $E[Z_i] = 0$. Thus, according to the Case 1 of Theorem 1 $\limsup_{n \to \infty} S_n = +\infty$ and $\liminf_{n \to \infty} S_n = -\infty$ almost surely. It suffices to consider the latter case. From the definition of limit supremum, we have

$$P(\limsup_{n \to \infty} S_n = +\infty)$$

$$= P(\forall c, \exists n_0 : \forall n > n_0, \sup_{m \geq n} S_m > c)$$

$$= P(\forall c, \exists n_0 : \forall n > n_0, \sup_{m \geq n} (u_0 - U_m) > c)$$

$$= P(\forall c, \exists n_0 : \forall n > n_0, u_0 - \inf_{m \geq n} U_m > c)$$

$$= P(\forall c, \exists n_0 : \forall n > n_0, \inf_{m \geq n} U_m < u_0 - c).$$

Taking $c = u_0$, we have that $P(\exists n_0 : \forall n > n_0, \inf_{m \geq n} U_m < 0) = 1$. Here the $\inf_{m \geq n} U_m < 0$ implies the existence of a subsequence $U_{m_k}$ which is strictly less than zero.

Thus, combining the cases for $\lambda \bar{X} < p$ and $\lambda \bar{X} = p$, we conclude that the HSC system will experience eventual outage.

Remark: Since $\lambda \bar{X} > p$ is a necessary, but not sufficient, condition for self-sustainability, its satisfaction does not guarantee that outage will not occur. However, it provides an easily-to-check condition under which self-sustainability is possible. We will refer to this condition $\lambda \bar{X} > p$ (or equivalently $E[Z_i] < 0$) as the self-sustainability condition.

**Condition 1** (Self-sustainability). An HSC system is said to be self-sustainable when $\lambda \bar{X} > p$, or equivalently, $E[Z_i] < 0$.

In the following sections, we will investigate the two mathematical cases that arises when this condition is or is not satisfied. Below we give an immediate corollary of the Proposition 3.
Corollary 1. The HSC system will experience an eventual outage almost surely if the self-sustainability condition is not satisfied. If the self-sustainability condition is satisfied, then the probability of eventual outage will be less than unity. That is,
\[
\psi(u_0) \begin{cases} 
1, & \text{if } \lambda \bar{X} \leq p \\
< 1, & \text{if } \lambda \bar{X} > p.
\end{cases}
\]

B. Ascending Ladder Process

Now that we have succeeded in converting a continuous time process into discrete time process, consider the random walk \(\{S_n; n \geq 1\}\) as \(S_n = Z_0 + \cdots + Z_{n-1}\), or recursively as \(S_{n+1} = S_n + Z_n\), with initial value \(S_0 = 0\). Thus, we have \(U_n\) as \(U_n = u_0 - S_n\).

Consider the ascending ladder process defined by \(M_n = \max_{0 \leq i \leq n} S_i\), which is the partial maximum of partial sums. Since \(M_0 = S_0 = 0\), the \(\{M_i; i \geq 1\}\) is a positive non-decreasing sequence, hence the name ascending ladder process. We can also relate the values of \(M_n\) by the recursion \(M_n = \max(M_{n-1}, S_n)\). Also, let \(M = \sup_{i \geq 0} S_i\) be the maximum value attained by \(S_i\) through the entire duration of its run. We have the key observation that the eventual outage is equivalent to \(\{\tau < \infty\} \equiv \{M > u_0\}\). Hence,
\[
\phi(u_0) = F_M(u_0) \quad \text{and} \quad \psi(u_0) = 1 - F_M(u_0).
\] (7)

Given the sequence \(\{S_i; i \geq 1\}\), the first strict ascending ladder point \((\sigma_1, H_1)\) is the first term in this sequence for which \(S_i > 0\). That is, \(\sigma_1 = \inf\{n \geq 1 : S_n > 0\}\) and \(H_1 = S_{\sigma_1}\). In other words, the epoch of the first entry into the strictly positive half-axis is defined by
\[
\{\sigma_1 = n\} = \{S_1 \leq 0, \ldots, S_{n-1} \leq 0, S_n > 0\}.
\] (8)

The \(\sigma_1\) is called the first ladder epoch while \(H_1\) is called the first ladder height. Let the joint distribution of \((\sigma_1, H_1)\) be denoted by
\[
P(\sigma_1 = n, H_1 \leq x) = F_{H,n}(x).
\] (9)

The marginal distributions are given by
\[
P(\sigma_1 = n) = F_{H,n}(\infty)
\] (10)
\[
P(H_1 \leq x) = \sum_{n=1}^{\infty} F_{H,n}(x) = F_H(x).
\] (11)

The two variables have the same defect \(1 - F_H(\infty) \geq 0\).

We can iteratively define the ladder epochs \(\{\sigma_n\}\) and ladder heights \(\{H_n\}\) as
\[
\sigma_{n+1} = \inf\{k \geq 1 : S_{k+\sigma_n} > S_{\sigma_n}\},
\] (12)
\[
H_{n+1} = S_{\sigma_{n+1}} - S_{\sigma_n}.
\] (13)
The pairs \((\sigma_i, H_i)\) are mutually independent and have the same common distribution given in (9). These ladder heights are related to record maximum at time \(n\) by

\[
M_n = \sum_{i=1}^{\pi_n} H_i, \tag{14}
\]
where \(\pi_n\) is the number of ladder points up until time \(n\), i.e.

\[
\pi_n = \min\{k : \sigma_1 + \cdots + \sigma_k \leq n\}.
\]

An important observation related to the ladder points is that the sums of \(\{\sigma_i\}\) and \(\{H_i\}\), \(\sigma_1 + \cdots + \sigma_n\) and \(H_1 + \cdots + H_n\), form (possibly terminating) renewal processes with inter-renewal interval \(\sigma_i\) and \(H_i\). Clearly, we can have \(M < \infty\) if and only if the ascending ladder process is terminating. As per Definition 2, this renewal process is terminating if the underlying distribution \(F_H\) is defective.

Accordingly, let \(H\) be a defective random variable with \(F_H(0) = 0\) and \(F_H(\infty) = \theta < 1\), then the amount of defect given by \(1 - \theta\) represents the probability of termination. In other words, \(\theta\) represents the probability of another renewal, while \(1 - \theta\) represents the probability that the inter-renewal interval is infinite. Thus the termination epoch is a Bernoulli random variable with “failure” being interpreted as “termination” with probability \(1 - \theta\). The sum \(H_1 + \cdots + H_n\) has a defective distribution given by the \(n\)-fold convolution \(F_H^{(n)}\), whose total mass equals

\[
F_H^{(n)}(\infty) = F_H^{n}(\infty) = \theta^n. \tag{15}
\]

This is easily seen by re-interpreting the \(n\) ladder epochs as \(n\) “successes” of a Bernoulli random variable. The defect \(1 - \theta^n\) is thus the probability of termination before the \(n\)-th ladder epoch.

Let us define the renewal function for the ladder heights by the sum on \(n\)-fold convolutions of \(F_H\) as

\[
\zeta(x) = \sum_{n=0}^{\infty} F_H^{(n)}(x), \tag{16}
\]
where \(F_H^{(0)} = \zeta_0\) is a unit step function at the origin. The renewal function \(\zeta(x)\) is equivalent to the expected number of ladder points in the strip \([0, x]\), where the origin counts as a renewal epoch. Thus, \(\zeta(0) = 1\). For terminating renewal process, the expected number of epochs ever occurring is finite, as from (15), we have

\[
\zeta(\infty) = \sum_{n=0}^{\infty} F_H^{(n)}(\infty) = \sum_{n=0}^{\infty} \theta^n = \frac{1}{1 - \theta}.
\]

If the ascending ladder process terminates after \(n\)-th epoch, then \(H_1 + \cdots + H_n = M\), the all time maximum attained by the random walk \(S_n\). The probability that the \(n\)-th ladder epoch is the last and that \(\{M \leq x\}\) is given by

\[
P(M \leq x, \text{terminate after } n) = (1 - \theta)F_H^{(n)}(x). \tag{17}
\]
Using (15), the marginalization of (17) over $M$ shows us that the probability of the ladder process terminating after the $n$-th epoch follows a geometric distribution,

$$P(\text{terminate after } n) = (1 - \theta)F_H^{(n)}(\infty) = (1 - \theta)\theta^n.\)$$

Similarly, marginalizing (17) over $n$, we have

$$P(M \leq x) = (1 - \theta)\sum_{n=0}^{\infty} F_H^{(n)}(x) = (1 - \theta)\zeta(x). \tag{18}$$

We now need a criteria to determine whether the ascending ladder process terminates or not, as well as a method to find the value of $\theta$. The following proposition also immediately follows from Theorem 1 and our discussion about ascending ladder process:

**Proposition 4.** *Given the self-sustainability condition, $\lambda \bar{X} > p$, the ascending ladder height process $\{H_i\}$ of an HSC system is terminating almost surely. The probability of self-sustainability and eventual outage given in terms of the renewal function $\zeta$ is

$$\phi(u_0) = (1 - \theta)\zeta(u_0). \tag{19}$$

*Proof:* From Case 3 of Theorem 1 since the random walk $S_n$ drifts to $-\infty$ when $\mathbb{E}[Z_i] < 0$, the maximum $M < \infty$ almost surely, and thus $\{H_i\}$ terminates. Equation (19) is obtained from (7) and (18).

**Remark:** Proposition 4 can also be expressed in terms of Laplace transform as

$$\hat{\phi}(r) = (1 - \theta)\hat{\zeta}(r) = (1 - \theta)\sum_{n=0}^{\infty} \hat{F}_H^{(n)}(r).$$

Since $F_H$ is a positive function, $|\hat{F}_H(r)| < \hat{F}(0) = 1$. Thus, we have the geometric sum

$$\hat{\phi}(r) = \frac{1 - \theta}{1 - \hat{F}_H(r)}. \tag{20}$$

We now relate the self-sustainability probability with two convolution formulas.

**Proposition 5.** *Given the self-sustainability condition, the self-sustainability probability satisfies the following equivalent integral equations:

$$\phi(u_0) = (1 - \theta) + \int_0^{u_0} \phi(u_0 - x)f_H(x)dx, \tag{21}$$

$$\phi(u_0) = \int_0^{\infty} \phi(x)f_Z(u_0 - x)dx. \tag{22}$$
Proof: We begin with the fact that \( \phi(u_0) = P(M \leq u_0) \). The proofs of the two equations follow from the standard renewal type argument:

1. The event \( \{ M \leq u_0 \} \) occurs if the ascending ladder process terminates with \( M_0 \) or else if \( H_1 \) assumes some positive value \( x \leq u_0 \) and the residual process attains the age \( \leq u_0 - x \). So,

\[
\phi(u_0) = P(M = 0) + \int_0^{u_0} P(H_1 \leq u_0 | H_1 = x) f_H(x) \, dx
\]

\[
= (1 - \theta) + \int_0^{u_0} P(\text{age} \leq u_0 - x) f_H(x) \, dx.
\]

Since the ascending ladder process renews at \( H_1 \), the probability of the age of the residual process is \( P(\text{age} \leq u_0 - x) = \phi(u_0 - x) \). Thus \( \phi(u_0) = P(M \leq u_0) \) satisfies the renewal equation \( \phi(u_0) = (1 - \theta) + \int_0^{u_0} \phi(u_0 - x) f_H(x) \, dx \).

2. The event \( \{ M \leq u_0 \} \) occurs if and only if \( \max(Z_0, Z_0 + Z_1, Z_0 + Z_1 + Z_2, \ldots) \leq u_0 \). Conditioning on \( Z_0 = y \), this is equivalent to

\[
Z_0 = y \leq u_0 \quad \text{and} \quad \max(0, Z_1, Z_1 + Z_2, \ldots) \leq u_0 - y.
\]

Here, \( P(\max(0, Z_1, Z_1 + Z_2, \ldots) \leq u_0 - y) = \phi(u_0 - y) \). De-conditioning over all possible \( y \), we get \( \phi(u_0) = \int_{-\infty}^{u_0} \phi(u_0 - y) f_Z(y) \, dy \), which by change of variable \( x = u_0 - y \) becomes \( \phi(u_0) = \int_0^{\infty} \phi(x) f_Z(u_0 - x) \, dx \).

Equations (21) and (22) relates \( \phi \) with \( H \) (a defective random variable) and \( Z \) (a proper random variable) respectively. Equation (21) can be recognized as a renewal equation while (22) can be recognized as a Wiener-Hopf integral.

**Corollary 2.** Given the self-sustainability condition, \( \phi(u_0) \) is a proper distribution with (i) \( \phi(u_0) = 0 \) for \( u_0 < 0 \), (ii) \( \phi(0) = 1 - \theta \), and (iii) \( \phi(\infty) = 1 \).

Proof: (i) Follows from the fact that \( u_0 \) only takes non-negative values. (ii) By putting \( u_0 = 0 \) in (21). (iii) Putting \( u_0 = \infty \) in (21), we have \( \phi(\infty) = (1 - \theta) + \phi(\infty) \int_0^{\infty} f_H(x) \, dx \). Recalling that \( H \) is defective, with \( F_H(\infty) = \theta \), we have \( \phi(\infty) = 1 \).

Corollary 2 can also be proved as a consequence of Proposition 4, since \( \zeta(0) = 1 \) and \( \zeta(\infty) = 1/(1 - \theta) \). In the above corollary, it is remarkable that the system can be self-sustaining even when there is no initial battery energy. In other words, this is probability that the random walk \( U_n \) starting from the origin will always be positive. This also allows us to interpret \( \theta \) as the eventual outage probability when there is no initial battery energy, i.e. \( \psi(0) = \theta \). The corollary also guarantees that as \( u_0 \) becomes large, the eventual outage becomes zero. Thus, a possible strategy in reducing the eventual outage is to simply increase the initial battery energy. We will later show that the rate at which the eventual outage decreases with \( u_0 \) is exponential.
The exact relationship between $H$ and $Z$ is given by the well known Weiner-Hopf factorization identity \cite{42, Ch XII.3} \cite{41, Ch VIII.3} in terms of their moment generating functions (MGFs) as:

$$1 - \mathcal{M}_Z = (1 - \mathcal{M}_H)(1 - \mathcal{M}_{H_-}),$$

(23)

where $H_-$ is the descending ladder height, defined in a manner similar to the ascending ladder height process. The $H_-$ is defined over $(-\infty, 0]$. The above identity is also written in terms of convolution as

$$F_Z = F_H + F_{H_-} - F_H * F_{H_-}.$$

(24)

Likewise, the distributions of $Z$, $H$, and $H_-$ are related to the renewal function $\zeta$ by

$$F_H(x) = \zeta_+ * F_Z(x), \quad x > 0, \quad (25a)$$

$$F_{H_-}(x) = \zeta * F_Z(x), \quad x \leq 0, \quad (25b)$$

where $\zeta_+$ is the renewal function defined by $F_{H_+}$ in a manner similar to $\zeta$ given in (16).

When $E[Z] < 0$, while $F_H$ is defective, $F_{H_-}$ is proper. Since $H_-$ is defined over $(-\infty, 0]$, this gives us the condition that $F_{H_-}(0) = 1$. Lastly, since $H_-$ is a proper distribution, the descending ladder process is a proper renewal process.

The factorization (23) is in itself difficult to perform explicitly. As such, we will focus on obtaining bounds and asymptotic approximations of $\phi$ (or equivalently, $\psi$).

IV. Bound on Eventual Outage Probability

Thus far we have described the energy surplus process and related the various associated concepts to the eventual outage / self-sustainability probability. These probabilities can be evaluated by solving the formulas given in Propositions 4 and 5. However, doing so is not trivial. As such, we wish for a simple bound to estimate the eventual outage probability. Here we will establish a tight exponential bound for the eventual outage probability using the concept of martingales.

**Definition 4** (Adjustment Coefficient). The value $r^* \neq 0$ is said to be the adjustment coefficient of $X$ if 

$$E[\exp(r^*X)] = \int e^{r^*x}dF_X = 1$$

**Definition 5** (Martingale). A process $\{X_i\}$ is said to be a martingale if $E[X_{n+1}|X_n, \ldots, X_0] = X_n$.

**Proposition 6.** Let $\{Z_i\}$, $\{S_i\}$ be as before. Suppose there exists an adjustment coefficient $r^* > 0$ such that $E[\exp(r^*Z_i)] = 1$, then $\exp(r^*S_n)$ for $n = 0, 1, 2, \ldots$ is a martingale.
Proof: We have
\[
\mathbb{E}[\exp(rS_{n+1})|S_n, \ldots, S_1]
= \mathbb{E}[\exp(r(S_n + Z_{n+1})))|S_n, \ldots, S_1]
= \mathbb{E}[\exp(rZ_{n+1})] \cdot \mathbb{E}[\exp(rS_n)|S_n, \ldots, S_1]
= \mathbb{E}[\exp(rZ_{n+1})] \cdot \exp(rS_n).
\]
Since there exists a constant \( r^* > 0 \) such that \( \mathbb{E}[\exp(r^*Z_{n+1})] = 1 \), then \( \mathbb{E}[\exp(r^*S_{n+1})|S_n, \ldots, S_1] = \exp(r^*S_n) \), satisfying the definition of a martingale.

Remark: Note that since \( Z \) is a proper random variable, it is trivially true that \( \mathbb{E}[\exp(r^*Z_i)] = 1 \) if \( r^* = 0 \). In the following proposition, we show that there exists a non-trivial value of \( r^* > 0 \) for which this property holds true. We will now give the conditions under which the adjustment coefficient will exist.

Proposition 7. Suppose that \( \mathbb{E}[Z_i] < 0 \), which is the self-sustainability condition. Also, assume that there is \( r_1 > 0 \) such that the moment generating function (MGF) \( \mathcal{M}_Z(r) = \mathbb{E}[e^{rZ_i}] < \infty \) for all \( -r_1 < r < r_1 \), and that \( \lim_{r \to r_1} \mathcal{M}_Z(r) = \infty \). Then, there is a unique adjustment coefficient \( r^* > 0 \).

Proof: Since the MGF exists in a neighborhood of zero, derivative of every order exists in \((-r_1, r_1)\). By definition, \( \mathcal{M}_Z(0) = 1 \). Since \( \mathbb{E}[Z_i] < 0 \) by assumption, we have \( \mathcal{M}_Z'(0) = \mathbb{E}[Z_i] < 0 \), which means that \( \mathcal{M}_Z(r) \) is decreasing in the neighborhood of 0. Also, since \( \mathcal{M}_Z''(h) = \mathbb{E}[Z_i^2 e^{rZ_i}] > 0 \) (due to the fact that the expectation of positive random variable is positive), it follows that \( \mathcal{M}_Z \) is convex on \((-r_1, r_1)\). Again, by assumption, we have \( \lim_{r \to r_1} \mathcal{M}_Z(r) = \infty \). It now follows that there exists a unique \( s \in (0, r_1) \) such that \( \mathcal{M}_Z(s) < 1 \) and \( \mathcal{M}_Z'(s) = 0 \); and that on the interval \((s, r_1)\) the function \( \mathcal{M}_Z(\cdot) \) is strictly increasing to \(+\infty\). As such, there exists a unique \( r^* \in (0, r_1) \) such that \( \mathcal{M}_Z(r^*) = 1 \). Since the MGF does not exist on \([r_1, \infty)\), it follows that \( r^* \) is unique on \((0, \infty)\).

Proposition 8. Assume that the self-sustainability condition \( \mathbb{E}[Z_i] < 0 \) holds, and that the adjustment coefficient \( r^* > 0 \) exists. Then the eventual outage probability is bounded by
\[
\psi(u_0) = P(\tau(u_0) < \infty) \leq \exp(-r^*u_0), \quad u_0 > 0.
\] (26)

Proof: Put \( \tau(u_0) = \inf \{ n \geq 0 : S_n > u_0 \} \). The \( \tau(u_0) \) is the the first passage time that the random walk \( \{S_n\} \) exceeds \( u_0 > 0 \) in the positive direction. The event that \( \{\tau(u_0) \leq k\} \) is equivalent to the union of events \( \bigcup_{i=0}^{k} \{S_i > u_0\} \). Similarly, for \( a > 0 \), let \( \sigma(a) = \inf \{ n \geq 0 : S_n < -a \} \). Here \( \sigma(a) \) denotes the first passage time that the random walk \( \{S_n\} \) exceeds \(-a < 0 \) in the negative direction. Since \( \mathbb{E}[Z_i] < 0 \), by Case 3 of Theorem \([1] \) \( P(\sigma(a) < \infty) = 1 \). Hence, \( (\tau(u_0) \wedge \sigma(a)) \equiv \min(\tau(u_0), \sigma(a)) \) is a stopping
time with $P((\tau(u_0) \wedge \sigma(a)) < \infty) = 1$. Since $\exp(r^*S_n)$ for $n = 0, 1, 2, \ldots$ is a martingale, using optional sampling theorem, we now get

\[ 1 = \mathbb{E}[e^{r^*S_0}] \]
\[ = \mathbb{E}[\exp(r^*S_{(\tau(u_0) \wedge \sigma(a))})] \]
\[ = \mathbb{E}[\exp(r^*S_{\tau(u_0)})|\tau(u_0) < \sigma(a)] \]
\[ + \mathbb{E}[\exp(r^*S_{\sigma(a)})|\sigma(a) < \tau(u_0)] \]
\[ \geq \mathbb{E}[\exp(r^*S_{\tau(u_0)})|\tau(u_0) < \sigma(a)] \]
\[ \geq e^{r^*u_0}P(\tau(u_0) < \sigma(a)) \]

as $S_{\tau(u_0)} > u_0$. Since $P(\lim_{a \to \infty} \sigma(a) = \infty) = 1$, thus letting $a \to \infty$ we get $P(\tau(u_0) < \infty) = \lim_{a \to \infty} P(\tau(u_0) < \sigma(a)) \leq e^{-r^*u_0}$, as required.

**Remark:** This remarkable proposition guarantees that the eventual outage probability can be made arbitrarily small simply by increasing the initial battery energy. Thus, during the design of a system, where we are willing to tolerate an arbitrarily small eventual outage probability, our task is to determine the initial battery energy. We can use the above bound to roughly calculate the required initial battery energy. The following corollaries immediately follows.

**Corollary 3.** Assuming that the self-sustainability condition holds and $r^* > 0$ exists, then $\lim_{u_0 \to \infty} \phi(u_0) = 0$.

**Corollary 4.** Assuming that the self-sustainability condition holds and the adjustment coefficient $r^* > 0$ exists, for a given tolerance $\epsilon > 0$, if the eventual outage probability is constrained at $\psi(u_0) = \epsilon$, then the maximum initial battery energy required is $u_0 = \frac{1}{r^*} \log(\frac{1}{\epsilon})$.

**V. ASYMPTOTIC APPROXIMATION OF EVENTUAL OUTAGE PROBABILITY**

While the exponential bound given in the previous section is simple to use, we can sharpen our estimates using the key renewal theorem for defective distribution. The basic idea behind this approach is to transform a defective distribution into a proper distribution using the adjustment coefficient, and then apply the key renewal theorem for proper distribution.

**A. Asymptotic Approximation**

First we define the renewal equation and then give the related theorem.

**Definition 6.** The renewal equation is the convolution equation for the form $Z = z + F \ast Z$, where $Z$ is an unknown function on $[0, \infty)$, $z$ is a known function on $[0, \infty)$ and $F$ is a known non-negative measure.
on $[0, \infty)$. Often $F$ is assumed to be a probability distribution. If $F(\infty) = 1$, then the renewal equation is proper. If $F(\infty) < 1$, then the renewal equation is defective.

**Theorem 2.** [41] Ch. V, Prop 7.6, p. 164] [42] Ch. IX.6, Theo. 2, p. 376] Suppose that for defective distribution $F$, there exists an adjustment coefficient $r^* > 0$ such that $\tilde{\mu} = \int te^{r^*t}dF < \infty$ exists. If in the defective renewal equation $Z = z + F * Z$, $z(\infty) = \lim_{t \to \infty} z(t)$ exists and $e^{r^*t}(z(t) - z(\infty))$ is directly Riemann integrable, then the solution of the renewal equation satisfies

$$\tilde{\mu}e^{r^*}[Z(\infty) - Z(t)] \sim \frac{z(\infty)}{r^*} + \int_0^{\infty} e^{r^*}[z(\infty) - z(s)]ds. \quad (27)$$

**Proposition 9.** Given the self-sustainability condition, the eventual outage probability is asymptotically given by

$$\psi(u_0) \sim \frac{1 - \theta}{r^* \tilde{\mu}_H} e^{-r^*u_0}, \quad (28)$$

where $r^*$ is the adjustment coefficient of $H$ and $\tilde{\mu}_H = \int xe^{r^*x}f_H(x)dx < \infty$.

**Proof:** Recall that (21) is in the form of a renewal equation with the defective distribution of $H$. As such, in Theorem 2 we have $z(t) \equiv 1 - \theta$ and $Z \equiv \phi(u_0)$. Thus, the integral in (27) vanishes and

$$\tilde{\mu}_He^{r^*u_0}[\phi(\infty) - \phi(u_0)] \sim \frac{1 - \theta}{r^*}. \quad \text{(27)}$$

From Corollary 2, we know that $\phi$ is proper. Thus $\phi(\infty) - \phi(u_0) = 1 - \phi(u_0) = \psi(u_0)$. Hence, we have the desired result.

Note that the adjustment coefficient $r^*$ for $H$ is the same as that for $Z$. This is easily demonstrated, since $M_Z(r^*) = \mathbb{E}[e^{r^*Z}] = 1$ where $r^* > 0$, we have from the Weiner-Hopf factorization [23],

$$(1 - M_H(r^*))(1 - M_{H-}(r^*)) = 1 - M_Z(r^*) = 0.$$

Here since $H_-$ is defined over $(-\infty, 0]$, we have

$$M_{H-}(r^*) = \int_0^{\infty} e^{-r^*x}f_{H-}(-x)dx,$$

$$\leq \int_0^{\infty} f_{H-}(-x)dx,$$

$$\leq 1,$$

since in (a) $r^* > 0$ and $f_{H-}$ is positive, while in (b) $H_-$ is a proper distribution. Thus, $1 - M_{H-}(r^*) > 0$, and this implies that $M_H(r^*) = 1$, making $r^*$ the adjustment factor of $H$ as well.

The above result is asymptotic in the sense that higher values of $u_0$ gives more accurate results. However, without further modeling assumptions, this is as far as we can proceed, since explicit evaluation of $F_H$
is difficult. In Section VI we will explore the case when the energy packet arrival is modeled as Poisson process.

B. Computing the Adjustment Coefficient

The computation of the adjustment coefficient \( r^* \) is not trivial. As given by Definition 4, the adjustment coefficient must satisfy the condition \( \mathbb{E}[e^{r^*Z}] = \mathcal{M}_Z(r^*) = 1 \). For the computational purpose, it is more convenient to use the cumulant generating function (CGF) of \( Z \) rather than its MGF. The CGF of \( Z \) is defined as \( K_Z(r) = \log \mathcal{M}_Z(r) \). In terms of CGF, the adjustment coefficient satisfies the condition \( K_Z(r^*) = \log \mathcal{M}_Z(r^*) = \log 1 = 0 \). Hence, we see that the adjustment coefficient \( r^* \) is a real positive root of CGF \( K_Z(r) \), and can be found by solving the equation

\[
K_Z(r^*) = 0. \tag{29}
\]

Since \( Z_i = pA_i - X_i \), due to the linearity of CGFs, \( K_Z(r) = K_A(pr) + K_X(-r) \).

For the important case when the energy packet arrival is a Poisson process, the inter-arrival time \( A_i \sim \text{Exp}(\lambda) \). As such, \( \mathcal{M}_A(r) = \lambda/(\lambda - r) \), and \( K_A(r) = -\log(1 - \frac{r}{\lambda}) \). Thus for the Poisson arrivals, the equation (29) becomes

\[
-\log \left(1 - \frac{pr}{\lambda}\right) + K_X(-r^*) = 0,
\]

which after exponentiation can be expressed in fixed point form as

\[
r^* = \frac{\lambda}{p}(1 - \mathcal{M}_X(-r^*)). \tag{30}
\]

Given the Poisson arrival, some possible distributions for the energy packets sizes and their corresponding solutions are as follows:

1) If we further assume that size of the energy packets is exponentially distributed, \( X_i \sim \text{Exp}(1/\bar{X}) \), then we have \( \mathcal{M}_X(r) = 1/(1 - r\bar{X}) \). So, the solution to (30) is

\[
r^* = \frac{1}{\bar{X}} \left( \frac{\lambda \bar{X}}{p} - 1 \right). \tag{31}
\]

2) If instead we assume that the size of the energy packets is deterministic, \( X_i = c \), then we have \( \mathcal{M}_X(r) = e^{cr} \). So the solution to (30) is obtained by solving the equation

\[
e^{-cr} + \frac{pr}{\lambda} - 1 = 0. \tag{32}
\]

3) If instead we assume that the size of the energy packets is uniformly distributed, \( X_i \sim U(0, 2\bar{X}) \), then we have \( \mathcal{M}_X(r) = \frac{e^{2\bar{X}r/\lambda} - 1}{2\bar{X}r/\lambda} \). So the solution to (30) is obtained by solving the equation

\[
e^{-2\bar{X}r} - \frac{2p\bar{X}}{\lambda}r^2 + 2\bar{X}r + 1 = 0. \tag{33}
\]
A simple approximation of $r^*$ can be obtained by making a formal power expansion of $M_X(-r)$ in terms of the moments of $X_i$ up to second order term as $M_X(-r) \approx 1 - \bar{X}r + \frac{\lambda^2}{2} r^2$. Using this expression in (30) and solving for $r^* > 0$, we obtain

$$r^* \approx \frac{2p}{\lambda \bar{X}^2} \left( \frac{\lambda \bar{X}}{p} - 1 \right).$$

(34)

More generally, since $K_Z(r)$ can be expanded in terms of the mean and variance of $Z$ as $K_Z(r) \approx \mu_Z r + \frac{\sigma^2_Z}{2} r^2$, we have the approximate solution $r^*$ for (29) as

$$r^* \approx -\frac{2\mu_Z}{\sigma^2_Z}.$$

(35)

Since $\mu_Z = \mathbb{E}[Z] < 0$, the above approximation will correctly give $r^* > 0$. This value can be used as an initial point for a root finding algorithm. Better approximations can be found by including higher order terms in the expansion and reverting the series using Lagrange inversion.

VI. Special Case: Evaluation of Eventual Outage Probability for Poisson Arrivals

In general, the density of $Z_i$ is given in terms of the densities of $A_i$ and $X_i$ as

$$f_Z(z) = \frac{1}{p} \int_{\max(0, -z)}^{\infty} f_A \left( \frac{z + x}{p} \right) f_X(x) dx.$$  

(36)

When the energy packet arrival is assumed to be a Poisson process, the inter-arrival time $A_i$ is exponentially distributed, $A_i \sim \text{Exp}(\lambda)$. As such, the density of $Z_i = pA_i - X_i$ is

$$f_Z(z) = \frac{\lambda}{p} e^{-\frac{\lambda z}{p}} \int_{\max(0, -z)}^{\infty} e^{-\frac{\lambda x}{p}} f_X(x) dx.$$  

(37)

When $z \geq 0$, the density of $Z$ has the form

$$f_Z(z) = \frac{\lambda}{p} e^{-\frac{\lambda z}{p}} \int_{0}^{\infty} e^{-\frac{\lambda x}{p}} f_X(x) dx, \quad z \geq 0.$$

Since the above integral is independent of $z$, the right tail of the density is exponential. That is, $f_Z(z) = C e^{-\frac{\lambda z}{p}}$ for $z \geq 0$, where $C$ is some constant given by $C = \frac{\lambda}{p} \int_{0}^{\infty} e^{-\frac{\lambda x}{p}} f_X(x) dx$. Now, from (25a), we have

$$f_H(x) = \zeta_- * f_Z(x) = C \int_{-\infty}^{0} e^{-\frac{\lambda(x-s)}{p}} \zeta_-(s) ds.$$

Again we see that regardless of the expression for $\zeta_-$, $f_H$ takes an exponential form given by

$$f_H(x) = C e^{-\frac{\lambda x}{p}} \int_{-\infty}^{0} e^{\frac{s}{\lambda}} \zeta_-(s) ds,$$

where the integral on the right hand is some constant. Since we know that $H$ is defective, we can re-write $f_H$ in a manner similar to proper exponential distribution as

$$f_H(x) = \left( \frac{\lambda}{p} - r^* \right) e^{-\frac{\lambda x}{p}},$$
such that $\theta = F_H(\infty) = 1 - \frac{r^* p}{\lambda}$. Hence, the amount of defect is $1 - \theta = \frac{r^* p}{\lambda}$. When $f_H(x)$ is multiplied by $e^{r^* x}$ we have the proper distribution
\[
e^{r^* x} f_H(x) = \left(\frac{\lambda}{p} - r^*\right) e^{-(\frac{\lambda}{p} - r^*) x},
\]
and the mean of this proper exponential distribution is $\mu_H = \left(\frac{\lambda}{p} - r^*\right)^{-1}$. Thus, we have from (28),
\[
\psi(u_0) \sim \left(1 - \frac{r^* p}{\lambda}\right) e^{-r^* u_0}.
\]
(38)

When $X$ is also exponentially distributed, we have the exact value of $r^*$ from (31). Hence, we have
\[
\psi(u_0) \sim \frac{p}{\lambda X} e^{-r^* u_0}.
\]
(39)

In fact, the (38) and (39) are not just asymptotic approximations, but also exact formulas (see [42, Ch XII.5, Ex 5(b)]). From these arguments, we have the following proposition:

**Proposition 10.** Assume that the self-sustainability condition holds and the adjustment coefficient $r^* > 0$ exists. If the energy packets arrive into an HSC system as a Poisson process, then the eventual outage probability is given by
\[
\psi(u_0) = \left(1 - \frac{r^* p}{\lambda}\right) e^{-r^* u_0}.
\]
(40)

Furthermore, if the energy packet size is also exponentially distributed, then
\[
\psi(u_0) = \frac{p}{\lambda X} \exp\left\{-\frac{1}{X} \left(\frac{\lambda \bar{X}}{p} - 1\right) u_0\right\}.
\]
(41)

**VII. Battery Energy Process**

So far we have directed our attention to the energy surplus $U(t)$ and the case when the self-sustainability condition is satisfied. For the sake of completeness, let us now consider the battery energy $W(t)$ at time $t$ and the case when the self-sustainability condition is not satisfied.

**A. Equivalence with Queueing Systems**

We will first prove that the battery energy process is a Lindley process. Making this identification will then allow us to compare the HSC system to a $GI/G/1$ queue, which in turn will allow us to exploit the results from queueing theory, for which the Lindley process was first studied. When the self-sustainability condition is not satisfied, the battery energy process $W(t)$ is stationary and ergodic. Thus, it makes better sense to talk about the outage probability $P(W(t) = 0)$ of the system rather than the eventual outage probability which is always unity, i.e. $\psi(u_0) = 1$. 
Definition 7. [41] Ch 3.6 A discrete-time stochastic process \{Y_i\} is a Lindley process if and only if \{Y_i\} satisfies the recurrence relation

\[ Y_{n+1} = \max(0, Y_n + X_n), \quad n = 0, 1, \ldots \]  \tag{42} 

where \( Y_0 = y \geq 0 \) and \{X_i\} are independent and identically distributed. This recursive equation is called Lindley recursion.\(^1\)

Proposition 11. The battery energy process \( W(t) \) observed just before the energy packet arrivals is a Lindley process and satisfies the recursion \( W_{n+1} = \max(0, W_n + Z'_n) \), for \( n = 0, 1, \ldots \), where \( W_0 = u_0 \) and \( Z'_n = -Z_n \).

Proof: As with the energy surplus in the previous section, let \( W_n \) be the amount of battery energy immediately before the arrival of \( n \)-th energy packet. The initial battery energy immediately before the arrival of the first energy packet \( E_0 \) is \( W_0 = u_0 \). The amount of battery energy just before the arrival of next \((n + 1)\)-th energy packet, \( W_{n+1} \), is then the sum of \( W_n \) and \( X_n \), the amount of energy contributed by the \( n \)-th packet into the battery, minus the amount consumed during the inter-arrival period of the \((n + 1)\)-th packet, \( pA_n \). Thus we have \( W_{n+1} = W_n + X_n - pA_n \) if \( W_n + X_n - pA_n \geq 0 \). Likewise, the battery will be empty, \( W_{n+1} = 0 \), if \( W_n + X_n - pA_n \leq 0 \). Putting both of them together, we have

\[ W_{n+1} = \begin{cases} W_n + Z'_n, & \text{if } W_n + Z'_n \geq 0 \\ 0, & \text{if } W_n + Z'_n \leq 0. \end{cases} \]  \tag{43} 

where \( Z'_n = X_n - pA_n \). Since \{\( X_n \)\} and \{\( A_n \)\} are IID, the \{\( Z'_n \)\} is clearly IID. We can write (43) in compact form as \( W_{n+1} = \max(0, W_n + Z'_n) \), which is a Lindley recursion as given in (42). Since \{\( W_i \)\} satisfies the Lindley recursion, \{\( W'_i \)\} is a Lindley process.

The fact that \{\( W'_i \)\} is a Lindley process gives rise to a number of important consequences, as stated in the following propositions.

Proposition 12. The HSC system is equivalent to a GI/G/1 queueing system.

Proof: The proposition follows from the fact that the virtual waiting time (i.e. the amount of time the server will have to work until the system is empty, provided that no new customers arrive, or equivalently the waiting time of a customer in a first-in-first-out queueing discipline) of an \( n \)-th customer arriving into a GI/G/1 queueing system is a Lindley process [41] Ch 3.6, Ex 6.1]. Since HSC Consume system is also a Lindley process by Proposition 11, the equivalence is established.

\[ \text{Proposition 13. Suppose } 0 < \mathbb{E}[|Z_i|] < \infty. \text{ The HSC system is ergodic and stationary if and only if } \mathbb{E}[Z_i] > 0, \text{ i.e. } \lambda \bar{X} < p, \text{ when the self-sustainability condition is not satisfied. Under this condition, there} \]

\(^1\)The Lindley process is referred to as queueing process in [42].
TABLE I

| Parameter | Energy Harvesting | Queueing |
|-----------|-------------------|----------|
| -         | Consumer          | Server   |
| -         | Battery           | Buffer   |
| $E_n$     | $n$-th energy packet | $n$-th customer |
| $\lambda$| Packet arrival rate | Customer arrival rate |
| $W_n$     | Battery energy    | Virtual waiting time |
| $X_n$     | Packet size       | Service time |
| $pA_n$    | Energy consumed   | Inter-arrival time (scaled) |
| $P(W(t) = 0)$ | System outage | System idle |
| -         | Self-sustainability | System always busy |
| $\rho = \frac{\lambda X}{P}$ | Utilization factor | Utilization factor (traffic intensity) |

will exist a unique stationary distribution for $W_n$, independent of the initial condition $W_0$, which is given by the Lindley’s integral equation:

$$F_W(w) = \int_{0-}^{\infty} F_Z(w - x)dF_W(x), \quad w \geq 0$$  (44)

Proof: This is a standard result from queueing theory. See [41, Coro 6.6].

Remark: We see that when the self-sustainability condition is not satisfied, the battery energy process of the system is ergodic and stationary. This allows us to easily compute the outage probability, $P(W(t) = 0)$, for such case by invoking the ergodicity and stationarity of $W(t)$. The equation (44) is again a Weiner-Hopf integral; and except for some special cases, its general solution is difficult to obtain.

B. Discussion

1) By comparing the Lindley recursion for the energy harvesting system with that of a queueing system, we can translate the terms and concepts of one system into those of the other. The Lindley recursion for a $GI/G/1$ queueing system is given by $V_{n+1} = \max(0, V_n + X_n - T_n)$, where for an $n$-th customer $V_n$ is its virtual waiting time, $X_n$ is its service time, and $T_n$ is the inter-arrival time between customers $n$ and $n + 1$. In Table II, we compare the terminologies of our energy harvesting system with that of terminologies of a traditional queueing system.

2) This comparison also means that we are justified in using the Kendall notation to refer to the different arrival process, energy packet size distribution, and the number of consumers in the HSC system. Therefore, an energy harvesting system where the distribution of inter-arrival time is exponential, making the arrival process Poisson, can be expressed in Kendall notation as an $M/G/1$ system.
Furthermore, if the distribution of the energy packet size is exponential, then we have an \( M/M/1 \) system. Lastly, if the energy packet experiences no random fluctuations, then we have an \( M/D/1 \) system.

3) The consumer of our energy harvesting system is equivalent to the server of a traditional queueing system. Thus the figure of merit for an energy harvesting system, as given by the outage probability, corresponds to the probability of the server being idle. Likewise, the self-sustainability of the energy harvesting system is translated as the probability that the server always remains busy.

4) However, one crucial difference is that, unlike the customers in a queue, the energy packets are “anonymous”, in the sense that they lose their distinction once they enter into the battery. As such, it does not make any physical sense to talk about the number of packets in the system, in contrast to the number of customers in the system, except perhaps for mathematical convenience. Similarly, the queuing disciplines like first-in-first-out also loses its relevance in the energy harvesting framework.

5) In the context of the case when the system is unsustainable, it makes better sense to study the duration of outage and coverage rather than eventual outage. Under steady state, the distribution of the outage duration is given by the residual time distribution of the underlying renewal process:

\[
F_O(x) = \frac{1}{\mu_A} \int_0^x [1 - F_A(s)] ds. \tag{45}
\]

6) When the self-sustainability condition is not satisfied, if the packet arrival process is Poisson, then the outage probability is simply \( P(W(t) = 0) = 1 - \rho \).

**VIII. Numerical Verification**

In this section, we verify the obtained formulas and bounds for the eventual outage probability. Here \( \rho = \frac{\lambda X}{\mu} \) is the utilization factor, in analogy with the queueing theory (see Table I), which is a dimensionless number. For instance, \( \rho = 1.1 \) would mean that the energy harvest rate is 10\% higher than the energy consumption rate. In all the figures, Figs. 2 – 5, we assumed that the energy packets arrive as Poisson process. We have plotted the changes in eventual outage probability versus the initial battery energy, for varying \( \rho \) and varying energy packet size distribution.

The predicted eventual outage probability for Poisson arrival, as given by (40), is compared with the exponential upper bound and Monte-Carlo simulations. For the Monte-Carlo simulations, we are required to follow the evolution of \( U(t) \) until \( t = \infty \), in order to check whether \( U(t) \) becomes negative within finite time. However, this is certainly not possible. As such, we checked the evolution of \( U(t) \) through \( t \in [0, T] \) where \( T = 1000 \) time-units. For a given value of \( \rho \) and \( u_0 \), 50,000 simulations were run to produce a single data point.
Fig. 2. Eventual outage probability, $\psi(u_0)$, versus initial battery energy, $u_0$, when energy arrival is Poisson process and the energy packet size is exponentially distributed.

Fig. 3. Eventual outage probability, $\psi(u_0)$, versus initial battery energy, $u_0$, when energy arrival is Poisson process and the energy packet size is deterministic.

Figs. 2–4 plots the eventual outage probability, $\psi(u_0)$, versus initial battery energy, $u_0$, for cases when the energy packet size is exponential, deterministic, and uniform distributed, respectively. In all three figures, the energy arrival process was assumed to be a Poisson process. The eventual outage probability was predicted using (40). The adjustment coefficient $r^*$ was computed using (51) for exponential packet sizes; while for deterministic and uniform packet sizes, $r^*$ was obtained by numerically solving (32) and (33), respectively. The exponential upper bound was computed by using (26).

We observe that the trends in all three figures, Fig. 2–4, are similar. In the semi-log plots, the eventual
outage probability decreases linearly with respect to initial battery energy. Also, the slope of the lines tend to become steeper with increase in $\rho$. This means that we require smaller initial battery energy for a given eventual outage probability, when the value of $\rho$ is higher. The results from Monte-Carlo simulations agree closely with that predicted by (40). The upper bound given by (26) is observed to be tighter for smaller values of $\rho$. We also observe that the line for upper bound tends to run parallel to the exact line.

Fig. 5 compares how the eventual outage probability changes with initial battery energy for the exponential, deterministic, and uniform distributed energy packet size. In this figure, $\rho = 1.1$ for all three cases. We see that the deterministic packet size gives the best performance, while exponentially
distributed energy packet size perform the worst. This means that to achieve the same grade-of-service, say $\psi(u_0) = 0.01$, requires higher initial battery energy for exponential case than for deterministic case. Overall, the performance degrades with the randomness of the energy packet size.

IX. CONCLUSION

We have given a mathematical definition of the concept of self-sustainability of an energy harvesting system, based on the concept of eventual energy outage. We have analyzed the harvest-store-consume system with infinite battery capacity, stochastic energy arrivals, and fixed energy consumption rate. The necessary condition for self-sustainability has been identified, and general formulas have been given relating the eventual outage probability and self-sustainability probability to various aspects of the underlying random walk process. Due to the complexity of the resulting formulas, assuming the existence of an adjustment coefficient, an exponential upper bound as well as an asymptotic formula have been obtained. Exact formulas for Poisson arrival process has also been obtained. Lastly, a harvest-store-consume system have been shown to be equivalent to a $GI/G/1$ queueing system. Using the queueing analogy, outage probability could be easily found, in case the self-sustainability condition is not satisfied. Numerical results have been given to verify the results.

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