Sparse MIMO planar array two-dimensional imaging based on IF-MMV-SBL algorithm

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Abstract. MIMO sparse array can achieve two-dimensional imaging of the target based on sparse recovery algorithm, but traditional 1D-CS algorithm will lead to the problem of cell migration, which will affect the imaging quality and its operation time is long. In order to solve the problem, the joint sparsity of signal received by MIMO is utilized and the IF-MMV sparse bayesian learning algorithm is introduced to reconstruct the signal in fractal dimension, which can inhibit the migration of cell caused by traditional 1D-CS algorithm. By the inverse free processing, the inverse operation of matrix is transformed into the inverse of diagonal matrix, and the inverse can be obtained quickly. Simulation results show that the proposed method has good applicability under different array sparsity and SNR. We get the high resolution image of the target, and cost less operation time.

1. Introduction

As a new system of radar, MIMO radar can obtain virtual array that is much more than the actual array by taking advantage of its structure characteristics of multiple transmission and multiple receiving. Literature [1] studied the sparse array MIMO imaging method based on compressed sensing, which effectively reduced the number of MIMO array elements. Literature [2][3] pointed out that the recovery of 2D signal by 1D-CS algorithm will lead to the problem of cell migration. Literature [4] proposed an MMV algorithm to improve the quality of ISAR imaging. In this paper, the two-dimensional imaging problem of sparse MIMO planar array is studied, and the structural sparse characteristics of MIMO echo signal are fully mined. In the fractal reconstruction processing, the MMV sparse bayesian learning algorithm is introduced to suppress the problem of cells migration in the reconstruction processing of 1D-CS and improve the imaging quality. What’s more, to further reduce the operation time, we replaced the evidence lower bound (ELBO) in MMV-SBL by a relaxed evidence lower bound (relaxed-ELBO) [5], which is computationally more amiable. Expectation-
maximization (EM) scheme is then employed to maximize the relaxed ELBO and a computationally efficient IF-MMV-SBL algorithm is got. We apply IF-MMV-SBL algorithm to MIMO imaging, further reducing the operation time.

2. Sparse MIMO planar array imaging model

2.1. Sparse MIMO planar array model

As shown in figure 1 a), it is a 4-transmitter and 25-receiver MIMO radar plane array, in which the transmitting and receiving array elements form a square plane array with the origin of coordinates as the center. The spacing between adjacent receiving and transmitting arrays is $2d \times 10d$. We assume that the target is located in the far-field area, according to the Phase Center Approximation (PCA) principle, and the above MIMO planar array can approximately equivalent to a transceiver square planar array. Therefore, the MIMO plane array of 4 send-receive and 25 send-receive in figure 1 a) can be equivalent to the $10 \times 10$ transceiver common square array, and the spacing of the equivalent transceiver array elements is $d$. On the basis of uniform MIMO planar array, let the transmitting array elements remain unchanged, and only some of the receiving array elements are used as the receiving array elements to form a sparse array. The schematic diagram of the array is shown in figure 1 (b). In this array, only all the array elements on some rows or columns are missing, while those other than these rows and columns are retained. Sparse degree is defined here. If there is $N$ element in the original plane array and there is $N'$ element after the array is sparse, then the sparse degree is defined as $\eta = \frac{N - N'}{N} \times 100\%$. From the perspective of signal processing, the data form of the sparse array is similar to that of the full array [6].

![Figure 1. MIMO planar array model](image)

2.2. Echo signal model

Suppose that the transmitting array transmits a set of Pulse Code Modulation (PCM) signals, and the form of the $m$ th transmitting signal (basic frequency) is:

$$\varphi_m(t) = \exp \{ j \phi_m(t) \}$$

Where $m = 0,1,\ldots,M^2 - 1$, $t$ is the fast time and $\phi_m(t)$ is the phase encoding function. Consider a far-field target containing $U$ scattering centers, set $O$ as a reference point on the target, and the unit direction vector from the center (coordinate origin) of MIMO array to the reference point is $n_0 = (n_x, n_y, n_z)^T$. Here, $n_z > 0$. It is assumed that the distance between the scattering point on the target and the transmitting element and the receiving element is $T_{mQ}$ and $R_{kQ}$ respectively. And in a single snapshot time, the target can be considered static relative to radar. Then, after the carrier frequency is removed, the target echo signal received by the $n$ th receiving array can be expressed as:
\[ y_n(t) = \sum_{q=0}^{U-1} \sum_{m=0}^{M-1} \sigma_q \phi_m \left[ t - (T_m Q + R_n Q) / c \right] \exp \left[ -j2\pi (T_m Q + R_n Q) / \lambda \right] \] (2)

Where \( n = 0, 1, \ldots, N^2 - 1 \), \( q = 1, 2, \ldots, Q \), \( c \) is the wave velocity, \( \lambda \) is the wavelength, \( f_c = c / \lambda \) is the carrier frequency, and \( \sigma_q \) is the scattering coefficient of the scattering point. After each echo signal passes through the matched filter bank, \( M \) signals will be separated, and the \( m \) th signal from the \( n \) th receiving array can be expressed as

\[ y_{mn}(t) = \sum_{q=0}^{U-1} \sigma_q p_m \left[ t - (T_m Q + R_n Q) / c \right] \exp \left[ -j2\pi (T_m Q + R_n Q) / \lambda \right] \] (3)

Where \( p_m(t) \) is the autocorrelation function of the \( m \) th transmitted signal. For the ideal PCM orthogonal signal, its autocorrelation function \( p(t) \) should have similar signal form. So, without loss of generality, let’s assume \( p_m(t) = p(t), \forall m \). It can be seen from (3) that a single snapshot of MIMO radar can obtain a \((MN)^2\) high-resolution signal. Based on the geometric relationship between the target and the array, we can easily calculate the distance \( T_m O \) and \( R_n O \) of each transceiver array element \((T_m, R_n)\) and the target reference point \( O \). Using the approximate relation

\[ T_m Q + R_n Q - T_m O - R_n O \approx 2OQ^\top n_0, \]

the echo of scattering point \( Q \) in different transmitting and receiving elements will be corrected to the same distance unit. Usually for a far field target, the above approximation can meet the precision requirement of envelope alignment. Use

\[ \exp \left[ -j2\pi (T_m O + R_n O) / \lambda \right] \] to make the phase correction of the target echo. Therefore, the target echo can be expressed as:

\[ \tilde{y}_{mn}(t) = \sum_{q=0}^{U-1} \sigma_q p \left[ t - 2q^\top n_0 / c \right] \exp \left[ j2\pi (T_m O + R_n O - T_m Q - R_n Q) / \lambda \right] \] (5)

According to the lemma introduced in literature [7], the distance term \( T_m O + R_n O - T_m Q - R_n Q \) in formula (5) can be further expressed as:

\[ T_m O + R_n O - T_m Q - R_n Q = \Delta \tilde{q} + \left[ q - (q^\top n_0) n_0 \right]^\top \left( T_m T_m + R_n R_n \right) / r \] (6)

Where, \( \Delta \tilde{q} = T_0 O + R_0 O - T_0 Q - R_0 Q \), \( r \) is the distance from the target to the center of the array. According to the array structure shown in figure 1, the position coordinates of the array element \( T_m \) and \( R_n \) can be expressed as \((2mN+2,d,2d,0)\) and \((n_1 N+2d,2mN,d,0)\), where, \( P_1, P_R \) is the position coordinates of the array element \( T_0 \) and \( R_0 \). And \( m \) \((m_2) = 0, 1, \ldots, M-1\) and \( n \) \((n_2) = 0, 1, \ldots, N-1\). So we can get \( T_0 T_m + R_0 R_n = (2mN+2,d,2mN+2d,0) \), \( a = mN + n_1 \), \( b = m_2 N + n_2 \). Where \( a = 0, 1, \ldots, MN-1 \), \( b = 0, 1, \ldots, MN-1 \), we can further obtain \( T_0 T_m + R_0 R_n = (2ad,2bd,0) \). Let \( \tau_q = q^\top n_0 \), \( \bar{q} = q - (q^\top n_0) n_0 \), \( \bar{q} = (\bar{x}_q, \bar{y}_q, \bar{z}_q) \), for fast time discrete \( t = i \cdot \Delta t, i = 0, \ldots, I-1 \), where \( \Delta t \) is sampling interval and \( I \) is sampling number. The discretization form can be expressed as:

\[ \tilde{y}_{mn}(i) = \sum_{q=0}^{U-1} \tilde{\sigma}_q(i) \exp \left[ j2\pi \left( a\omega_q^i + b\omega_q^i \right) \right] \] (7)

Where \( \tilde{\sigma}_q(i) = \sigma_q p(i \cdot \Delta t - 2\tilde{x}_q / c) \exp \left( j2\pi \Delta \tilde{x}_q / \lambda \right), \omega_q^i = 2d\tilde{x}_q / \lambda r, \omega_q^i = 2d\tilde{y}_q / \lambda r \).

3. Sparse MIIMO planar array imaging based on IF-MMV- sparse bayesian learning algorithm

3.1. MMV- sparse Bayesian learning algorithm inhibits cells migration
As can be seen from the observation formula (7), the target echo signal after matched filtering has been compressed in the distance. After envelope alignment and phase correction, it is assumed that there are \( N_1 \) rows and \( N_2 \) columns elements in the sparse planar array, and the equivalent array contains the transceiver elements shared by the \( K = MN_1 \) rows and \( L = MN_2 \) columns. Its echo (8) can be expressed as the structured sampling of two-dimensional signal received by the uniform planar array in formula [8].

\[
\tilde{y}_{\mathbf{m}}(i) = \sum_{q=0}^{L-1} \sigma_q(i) \exp \left\{ j2\pi \left[ G(\mathbf{p}) \phi_q^0 + H(\mathbf{B}) \phi_q^1 \right] \right\}
\]  

(8)

Where, \( \mathbf{p} = 0,1, \ldots, K-1 \), \( \mathbf{B} = 0,1, \ldots, L-1 \), \( G \), \( H \) are all subsets of the set \([0: MN-1]\), which are determined by the R/C-SA structure, and \( |G| = K \cdot |H| = L \).

Formula (9) is expressed as a matrix:

\[
\mathbf{Y} = \mathbf{\Phi}_1 \mathbf{X} \mathbf{\Phi}_2^T
\]  

(9)

Where

\[
\mathbf{Y} = \begin{bmatrix}
\tilde{y}_{00} & \tilde{y}_{01} & \ldots & \tilde{y}_{0(L-1)} \\
\tilde{y}_{10} & \tilde{y}_{11} & \ldots & \tilde{y}_{1(L-1)} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{y}_{(K-1)0} & \tilde{y}_{(K-1)1} & \ldots & \tilde{y}_{(K-1)(L-1)}
\end{bmatrix},
\mathbf{\Phi}_1 = \begin{bmatrix}
1 & \Omega_{G(0)} & \ldots & \Omega_{(MN-1)G(0)} \\
1 & \Omega_{G(1)} & \ldots & \Omega_{(MN-1)G(1)} \\
\vdots & \vdots & \ddots & \vdots \\
1 & \Omega_{G(K-1)} & \ldots & \Omega_{(MN-1)G(K-1)}
\end{bmatrix},
\mathbf{\Phi}_2 = \begin{bmatrix}
1 & \Omega_{H(0)} & \ldots & \Omega_{(MN-1)H(0)} \\
1 & \Omega_{H(1)} & \ldots & \Omega_{(MN-1)H(1)} \\
\vdots & \vdots & \ddots & \vdots \\
1 & \Omega_{H(L-1)} & \ldots & \Omega_{(MN-1)H(L-1)}
\end{bmatrix}
\]

Where, \( \Omega = \exp \left\{ j \frac{2\pi}{MN} \right\} \), \( \mathbf{Y} \in \mathbb{C}^{K \times L} \) represents the echo signal matrix, \( \mathbf{\Phi}_1 \in \mathbb{C}^{K \times MN} \), \( \mathbf{\Phi}_2 \in \mathbb{C}^{L \times MN} \) is the sparse dictionary determined by the sparse array structure, and \( \mathbf{X} \in \mathbb{C}^{MN \times MN} \) represents the target two-dimensional image.

Fractal dimension compression is performed for formula (9), and the reconstructed model is expressed as:

\[
\hat{\mathbf{Y}} = \mathbf{\Phi}_1 \mathbf{\Phi}_2 \mathbf{\Phi}_1^T
\]

(10)

The traditional 1D-SBL algorithm mainly solves the one-dimensional sparse reconstruction problem. Considering the restoration of the original sparse signal from the observation vector, the observation signal can be expressed as follows

\[
\hat{\mathbf{y}}_l = \mathbf{\Phi}_1 \mathbf{\Phi}_2 \mathbf{\Phi}_1^T
\]  

(11)

In fact, this method is to solve a series of single-observation vector problems in sequence, and the direct use of single-observation vector method will lead to the problem of cells migration [9].

From equation (10), it can be seen that the echo signal of uniform planar array is actually the result of the target image \( \mathbf{X} \) passing through the two-dimensional FFT. Therefore, \( \hat{\mathbf{Y}} \) should be the result of one-dimensional FFT for each row of \( \mathbf{X} \) ideally. If there is a non-zero value in the \( i \) th row, the non-zero value will be completely concentrated in the \( i \) th row after FFT. In other words, for any transformed column vector \( \mathbf{\hat{y}}_l, l = 0,1, \ldots, L-1 \), its non-zero value will be in the same cell. This property of the matrix \( \hat{\mathbf{Y}} \) is called joint sparsity [10], that means all column vectors of \( \hat{\mathbf{Y}} \) have the same non-zero row. Multiple sparse vectors with this feature are recovered simultaneously, which
called Multiple Measurement Vector (MMV). For the signal \( \tilde{y}_j \) and \( \hat{y}_j \) in the \( j \)th column, the MMV-SBL algorithm satisfies the following gaussian likelihood model:

\[
p(\tilde{y}_j | \hat{y}_j) = \left(\frac{1}{\sigma^2}\right)^\frac{1}{2} \exp\left(-\frac{1}{\sigma^2} |\tilde{y}_j - \Phi\hat{y}_j|_2^2\right)
\]

Under the MMV-SBL framework, the coefficients at the same positions in each vector with the same sparse structure correspond to the same superparameter. We assumed that signal is given the following gaussian prior distribution.

\[
p(\tilde{Y} | \alpha) = \prod_{i=1}^{MN} p(\tilde{y}^{(i)} | \alpha_i)
\]

Where \( p(\tilde{y}^{(i)} | \alpha_i) = \mathcal{N}(0, \alpha_i^{-1}I) \), \( \alpha \equiv \{ \alpha_i \} \) is the non-negative superparameter controlling the sparsity. It can be seen that in the MMV-SBL algorithm, each superparameter corresponds to the coefficient of a certain line. In accordance with the fractal reconstruction model, the joint sparsity can be used to suppress the cells migration and improve the imaging quality.

3.2. IF-MMV- sparse Bayesian learning algorithm

In the MSBL framework, \( \tilde{Y} \) is assigned a two-layer hierarchical prior. In the first layer, \( \tilde{Y} \) is assigned a Gaussian prior distribution characterized by \( \alpha \)

\[
p(\tilde{Y} | \alpha) = \prod_{n=1}^{MN} p(\tilde{y}^{(n)} | \alpha_n) = \prod_{n=1}^{MN} \mathcal{N}(\tilde{y}^{(n)} | 0, \alpha_n^{-1})
\]

where \( \alpha \equiv \{ \alpha_n \} \) are nonnegative hyperparameters controlling the prior variance of each row of \( \tilde{Y} \), \( \tilde{y}^{(n)} \) denotes the \( n \)th row of \( \tilde{Y} \). In the second layer, the Gamma distribution is applied to \( \alpha \) as follows

\[
p(\alpha) = \prod_{n=1}^{N} \Gamma(a_n | a, b) = \prod_{n=1}^{N} \Gamma^{-1}(a) b^a a_n^{-a} e^{-ba_n}
\]

where \( \Gamma(a) = \int_0^\infty e^{-a\gamma} d\gamma \) denotes the Gamma function, \( b \) is very small and \( a \) is very big. Covariance matrix is \( (1/\gamma)I \). We can learn \( \gamma \) by iteration.

\[
p(\gamma) = \Gamma(\gamma | c, d) = \Gamma(c)^{-1} d^c e^{-d\gamma}
\]

Let \( \theta \equiv \{ \tilde{Y}, \alpha, \gamma \} \) denote the hidden variable of the hierarchical model. Assume that the variational distribution can be expressed as \( q(\theta) = q_\tilde{y}(\tilde{y}) q_\alpha(\alpha) q_\gamma(\gamma) \).

Suppose that \( q_\tilde{y}(\tilde{y}) \) is updated according to a Gaussian distribution. Combining likelihood and prior, the posterior density of the \( j \)th column of \( \tilde{Y} \) can be expressed as

\[
p(\tilde{y}_j | \hat{y}_j, \alpha) = \frac{p(\tilde{y}_j, \hat{y}_j, \alpha)}{p(\hat{y}_j, \alpha) d\hat{y}_j} = \mathcal{N}(\mu_j, \Sigma)
\]

with its mean and covariance matrix given by

\[
\mu_j = \mathcal{D}\Phi_j^T\Sigma^{-1}\hat{Y}
\]

\[
\Sigma = \mathcal{D} - \mathcal{D}\Phi_j^T\Sigma^{-1}\Phi_j\mathcal{D}
\]

where \( \mathcal{D} \equiv \text{diag}(\alpha) \) and \( \Sigma_j = \gamma I + \Phi \Phi^T \).

It can be seen from (18) and (19) that the update of the posterior distribution requires computing the inverse of a \( K \times K \) matrix. Thus, the variational MSBL method has a computational complexity as
This high computational complexity prohibits its application to large datasets problems. To solve this problem, we try to maximize a relaxed ELBO. The ELBO for MSBL is of the form

$$L(q) = \int q(\theta) \ln \frac{p(Y, \theta)}{q(\theta)} d\theta = \int q(\theta) \ln \frac{p(Y|\theta, \gamma)p(\theta|\alpha)p(\gamma)}{q(\theta)} d\theta$$  \hspace{1cm} (20)

By the lemma is introduced [11]. $T(f)$ is Lipschitz continuous gradient. We obtain the relaxed ELBO,

$$L(q, Z) = \int q(\theta) \ln \frac{G(\tilde{Y}, \theta, Z)h(Z)}{q(\theta)h(Z)} d\theta = \int q(\theta) \ln \frac{G(\tilde{Y}, \theta, Z)}{q(\theta)h(Z)} d\theta - \ln h(Z)$$  \hspace{1cm} (21)

where, $G(\tilde{Y}, \theta, Z) \triangleq G(\tilde{Y}, \theta, Z)h(Z)$. $h(Z)$ can be express as

$$h(Z) \triangleq \frac{1}{\int G(\tilde{Y}, \theta, Z) d\theta d\tilde{Y}}$$  \hspace{1cm} (22)

We use the variational EM algorithm to maximize the relaxed ELBO $L(q, Z)$. The E-step updates the posterior distribution of the hidden variables $\tilde{Y}$, $\alpha$, and $\gamma$. Where $\langle \alpha_n \rangle = \frac{a}{b_n}$, $\langle \gamma \rangle = \frac{c}{d}$, $\langle \tilde{y}_m^2 \rangle = \mu_m^2 + \Sigma_{n,n}$. $\Sigma_{n,n}$ represents the $n$th diagonal element of $\Sigma$.

$$\mathcal{M} = \gamma \left[ \left( \Phi_0^T \Phi_0 \mathbf{Z} - \Phi_0^T \mathbf{Y} + \frac{1}{2} \mathbf{Z} \right) \gamma = \left( \frac{\gamma}{2} \mathbf{I} + A \right)^{-1} \right]$$  \hspace{1cm} (23)

$$q_\alpha(\alpha) = \prod_{n=1}^{MN} \text{Gamma}(\alpha_n; \alpha, \beta_n), \alpha = a + \frac{1}{2}, \beta_n = b + \frac{1}{2} \sum_{l=1}^{K} \tilde{y}_m^2$$  \hspace{1cm} (24)

$$q_\gamma(\gamma) = \text{Gamma}(\gamma; \tilde{c}, \tilde{d}), \tilde{c} = c + \frac{K}{2}, \tilde{d} = d + \frac{1}{2} \left\langle g(\tilde{Y}, \mathbf{Z}) \right\rangle$$  \hspace{1cm} (25)

In the M-step, $L(q, Z)$ is maximized with respect to $\mathbf{Z}$, given $q(\theta)$ fixed. Substituting $q(\theta, Z^{\text{old}})$ into $L(q, Z)$, we can find the estimate of $\mathbf{Z}$ via the following optimization.

$$\mathbf{Z}^{\text{new}} = \underset{\mathbf{Z}}{\arg \min} \left\{ \ln G(\tilde{Y}, \theta, Z) \right\}_{q(\theta, Z^{\text{old}})} = \mathcal{Q}(\mathbf{Z} | \mathbf{Z}^{\text{old}})$$  \hspace{1cm} (26)

By setting the gradient of (25) with respect to $\mathbf{Z}$ to zero, we yield

$$\frac{\partial \mathcal{Q}(\mathbf{Z} | \mathbf{Z}^{\text{old}})}{\partial \mathbf{Z}} = 0 \iff \mathbf{Z} = \mathcal{M}$$  \hspace{1cm} (27)

We get this result because that $T - 2\Phi_0^T \Phi_0$ is a positive-definite matrix and $= 2\lambda_{\max}(\Phi_0^T \Phi_0)$, where $T > T(f) = 2\lambda_{\max}(\Phi_0^T \Phi_0)$ denotes the largest eigenvalue of $\Phi_0^T \Phi_0$. the IF-MMV-SBL procedure can be summarized by the following collection of steps.

1. Initialize, $\gamma = 1$ or another non-negative random initialization.
2. Initialize, $\mathbf{Z} = \Phi_0 \mathbf{Y}$, where $\Phi_0^+$ denotes the Moore–Penrose pseudo-inverse of $\Phi_0$.
3. Compute the posterior moments $\mathcal{M}$ and $\Sigma$ according (23). Update the posterior approximation $q_\alpha(\alpha)$ and $q_\gamma(\gamma)$ according to (24) and (25).
4. Update the parameter $\mathbf{Z}$ according to (27).
5. Iterate (3) and (4) until $\|\mathcal{M}^{(i)} - \mathcal{M}^{(i-1)}\|_F \leq \delta$, where $\delta$ is a prescribed tolerance value.
4. Experimental results and analysis
In order to verify the effectiveness of the MMV sparse bayesian algorithm proposed in this paper, the imaging scene of sparse MIMO planar array radar under the condition of far field was simulated. The uniform planar array of $4^2$ senders and $40^2$ receivers was established, and 20 rows and 20 array elements were randomly extracted from the receiving array. Where $\eta=50\%$. The simulation target is a Boeing 747, as shown in figure 2. The array plane is 10km away from the target. It transmits a group of 40-bit coded signals with a carrier frequency of 10GHz and a sub-pulse time width of 2

![Figure 2. Boeing 747 scatter point model](image)

The sparse array imaging simulation results achieved by direct FFT, 1D-OMP algorithm, 1D-SBL algorithm, MMV-SBL algorithm and IF-MMV-SBL are as follows.

![Figure 3. FFT](image)

![Figure 4. 1D-OMP](image)

![Figure 5. 1D-SBL](image)

![Figure 6. MMV-SBL](image)

![Figure 7. IF-MMV-SBL](image)

Due to the sparse array, the imaging quality of FFT is greatly affected. In the Fig.4 1D-OMP reconstruction algorithm is adopted. But unknown sparsity and migration across cell made the imaging quality is still far from that of full-array imaging. In figure 5, 1D-SBL algorithm is adopted to overcome the problem caused by the unknown sparsity of 1D-OMP algorithm. However, as SBL algorithm processes each observation vector sequentially, it causes a serious migration problem of
cells. MMV-SBL and IF-MMV-SBL algorithm, because of the joint sparsity of the signal, make the migration of cells be well suppressed, and get the high resolution two-dimensional image of the target. The imaging effects under various sparsity degrees were verified by simulation, and the Mean Square Error (MSE) was used as the evaluation index to evaluate and compare the imaging effects.

$$\text{MSE} = \frac{\|S_i - \hat{S}_i\|_2}{\|S_i\|_2}$$

(28)

Where, $S_i$ is the echo signal obtained by uniform full array; $\hat{S}_i$ is the echo signal obtained by sparse array. The sparsity was set to change from 10% to 90% at an interval of 10%, and 100 monte carlo experiments were repeated to obtain the MSE statistical results of different algorithms, as shown in figure 6. The operation time of MMV-SBL and IF-MMV-SBL under different array sparsity is shown in Table 1.

![Figure 8](image_url)  
**Figure 8.** Curve of mean square error changing with array sparsity of each algorithm

| Array Sparsity (%) | MMV-SBL | IF-MMV-SBL |
|-------------------|---------|------------|
| 30                | 2.391   | 0.406      |
| 40                | 2.449   | 0.307      |
| 50                | 2.411   | 0.232      |
| 60                | 2.320   | 0.169      |
| 70                | 2.148   | 0.116      |
| 80                | 1.980   | 0.078      |
| 90                | 1.793   | 0.042      |

Table 1. Operation time under different array sparsity

It can be clearly seen from the figure 7 that the reconstruction errors of the three methods all show an upward trend with the increase of sparsity, but the reconstruction error line of the MMV-SBL and IF-MMV-SBL are always lower than that of other algorithms, which has a better reconstruction performance. As we can see from the Table 1, IF-MMV-SBL algorithm needs less time than MMV-SBL.

![Figure 9](image_url)  
**Figure 9.** Curve of mean square error changing with SNR of each algorithm

| SNR | MMV-SBL | IF-MMV-SBL |
|-----|---------|------------|
| 15   | 2.024   | 2.372      |
| 20   | 2.178   | 2.408      |
| 25   | 2.149   | 2.378      |
| 30   | 2.174   | 2.404      |
| 35   | 2.147   | 2.375      |
| 40   | 2.177   | 2.401      |

Table 2. Operation time under different SNR
It can be seen from the figure 8 that the reconstruction errors of the three methods all show a downward trend with the increase of SNR, but the reconstruction error line of the MMV-SBL and IF-MMV-SBL algorithm are always lower than that of other algorithms. So the MMV-SBL algorithm have better performance in different SNR.

5. conclusion
The paper analyzed the MMV sparsity of data received by MIMO and used the MMV-SBL algorithm to supress the problem of cell migration led by traditional 1D-CS algorithm. In order to reduce the operation time further, the paper applied the inverse free processing to MMV-SBL algorithm, which reduced the operation time while the imaging quality is guaranteed. Simulation results showed that the proposed method has good applicability under different array sparsity and SNR.

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