Edge state preparation in a one-dimensional lattice by quantum Lyapunov control

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Abstract
Quantum Lyapunov control uses a feedback control methodology to determine control fields applied to control quantum systems in an open-loop way. In this work, we employ two Lyapunov control schemes to prepare an edge state for a fermionic chain consisting of cold atoms loaded in an optical lattice. Such a chain can be described by the Harper model. Corresponding to the two schemes, two types of quantum Lyapunov functions are considered. The results show that both the schemes are effective at preparing the edge state within a wide range of parameters. We found that the edge state can be prepared with high fidelity even if there are moderate fluctuations of on-site or hopping potentials. Both control schemes can be extended to similar chains \((3m + d, d = 2)\) of different lengths. Since a regular amplitude control field is easier to apply in practice, an amplitude-modulated control field is used to replace the unmodulated one. Such control approaches provide tools to explore the edge states of one-dimensional topological materials.

Keywords: lyapunov control, edge state, fermionic chain

(Some figures may appear in colour only in the online journal)

1. Introduction
Topological materials are thought to be candidates to help realize fault-tolerant quantum computation [1] due to their robustness against perturbations. Usually the topological character is indicated by the emergence of edge states for a bulk system. Recently, there has been a great deal of work to explore topological systems related to such states [2–10]. In two-dimensional systems such as the \(\text{Bi}_2\text{Te}_3\) nanoribbon, manipulation of edge state by modulating a gate voltage has been reported in experiments [8]. Similarly, for topological \(\text{Bi}(111)\) bilayer nanoribbon, through first-principle simulations, the desirable edge state engineering can be realized by chemical decoration [9]. In addition, in one-dimensional lattice systems, by coupling the atomic spin states to a laser-induced periodic Zeeman field, a novel scheme is proposed to manipulate the edge state [10]. This has attracted both theoretical and experimental interest. Thus manipulation to the edge state is a way to investigate topological systems.

Compared to traditional solid state systems, cold atoms trapped in optical lattices are excellent simulators to investigate various interesting physical topics such as topological insulators [11, 12]. One important reason is that such systems with tunable on-site and hopping potentials provide more controllable platforms to investigate the quantum signatures of many-body systems. Once such a system possesses a topological material structure, manipulating the edge state is feasible in light of quantum control methodology.

Time-dependence control on quantum systems is valid to dynamically realize specific control goals [13–20]. Among them, quantum Lyapunov control has been investigated widely and applied to realize various kinds of objectives [16–20]. It was used to design an open-loop controller by simulating the evolution for an artificial closed-loop quantum system. Namely it is a ‘closed-loop design and open-loop apply’ control strategy. The dynamics is governed by the Schrödinger equation. In this control, with a suitable external Hamiltonian, the control fields play an important role. They are designed by making the positive Lyapunov function decrease monotonously. In order to design the control fields, we adopt two quantum Lyapunov control schemes: state distance and state error schemes.
These two schemes are so called because the Lyapunov functions in them are based on state distance and state error. Note that the control fields vanish if the system is asymptotically steered to a target state located in a set specified by LaSalle’s invariant principle [21].

We adopt the Lyapunov control method to prepare the edge state for a topological system consisting of atoms loaded in an optical lattice. Such a lattice can be created by superimposing two standing waves generated by laser beams of different wavelengths [22, 23]. This system can be described by a Hamiltonian of the Harper model [24]. We assume all the cold atoms are loaded in the lowest band of the optical lattice to make the tight-binding limit available. For the open boundary lattice, energies corresponding to the edge states are located away from the energy subbands. In this work, we choose to prepare one edge state as the goal. In order to carry out such a proposal, a control Hamiltonian is needed. Inspired by the lattice shaking technique [25–27], which can be applied to quantum simulators in optical lattices with tunable structures, in this work, a trigonometrical modulation control Hamiltonian (time-dependent Hermitian) is introduced with a time-varying control field to prepare the edge state.

In order to indicate the effectiveness of both control schemes, the fidelity defined by the scalar product of the controlled and goal state is employed. We found that despite the existence of static fluctuations which perturb the on-site or the hopping potentials, the fidelity can reach a high value at the terminated time. This demonstrates the robustness of both control schemes against static fluctuations. To examine the generality of both control schemes, we apply them to chains with different lengths with the same 3m + d configuration [28]. High fidelity control results manifest the effectiveness of both control schemes in this work. Taking the controllability for operation in experiments into account, we use amplitude-modulated control fields in place of the unmodulated ones to prepare the edge state. The modulation functions are determined since the sign of the control fields is more crucial than the amplitudes in the control process.

This paper is organized as follows. In section 2, we specify the model, i.e. a controlled chain made of cold atoms loaded in an one-dimensional optical lattice. In section 3, we exhibit the general procedure to design the control fields in two quantum Lyapunov control schemes, which is exemplified with a trigonometrical modulation control Hamiltonian. Then we examine the control effects of the strategies in large control parameter intervals and with random initial states. In section 4, we explore the robustness of the two control schemes against the number and amplitudes of on-site energy and hopping fluctuations, application to chains of 3m + d configuration with different lengths and a feasible modulation for the control fields. Finally, we conclude in section 5.

2. Fermionic chain made of atoms loaded in an optical lattice

In this work we consider a system made of cold atoms trapped in an one-dimensional optical lattice. The lattice of straight line shape can be generated by superimposing two standing waves of laser beams. First, we show the procedure to obtain the fermionic chain from single particle Hamiltonian, which has the following form in the periodically modulated lattice

$$
\hat{H}_1 = \hat{H}_0 + \hat{H}_1,
$$

$$
\hat{H}_1 = \frac{p^2}{2M} + V_1 \sin^2(k_s x),
$$

$$
\hat{H}_2 = V_2 \cos^2(k_2 x + \delta),
$$

where $V_j = s_j E_{ij}$ and $k_j = 2\pi/\lambda_j (j = 1, 2)$ are the lattice depth and wavenumbers respectively. $x$ denotes the positions for the atoms on the chain. $s_j$ and $E_{ij} = h^2/(2M \lambda_j^2)$ denote the height of the lattices and recoil energies respectively. $h$ is the Planck constant and $M$ is the mass of the atoms in the lattice. $\delta$ is an arbitrary phase of the second laser beam. We assume all the atoms are trapped in the lowest band of the optical lattice to make the tight-binding approximation available. Then in virtue of the field operators $\Phi(x)$, the Hamiltonian reads

$$
\hat{H}_0 = \int dx \Phi^\dagger(x) \hat{H}_1 \Phi(x),
$$

In the basis of Wannier functions, the field operator can be expanded as $\Phi(x) = \sum_n \hat{c}_n \omega(x - x_n). \hat{c}_n$ here denotes the annihilation operator for the fermion at site $n$ while the spin freedom is not considered. Substitute this into (2), omitting constant terms, one gets the Hamiltonian

$$
\hat{H}_0 = -J \sum_{n=1}^N (\hat{c}_n^\dagger \hat{n}_{n+1} + \text{H.c.})
$$

$$
+ \sum_{n=1}^N V \cos(2\pi/\hbar n + \delta) \hat{n}_n,
$$

where $\beta = k_2/k_1 = p/q (p$ and $q$ are prime to each other), the hopping amplitude $J = \int dx \hat{c}_n \hat{c}_{n+1} \hat{H}_1 \omega(x - x_{n+1})$ and on-site energy $V = \frac{\beta}{2} \int dx \omega(x) \cos(2k_2 x) \omega(x). \hat{c}_n^\dagger \hat{c}_n$ are the creation (annihilation) operators for the atoms on-site $n$, and $\hat{n}_n = \hat{c}_n^\dagger \hat{c}_n$. $J$ is set to be the unit of energy in this work and we set $h = 1$. We choose $\beta = 1/3$, then the chain has a 3m + d configuration: $m$ here means the number of eigen-energies in a energy subband since the total energy band can be divided into $q$ subbands in the Harper model [2, 28] and we choose $d = 2$ in this work. ‘3’ represents the periodic character for the chain which results from the ratio of the two wave vectors of the two lasers in a standing-wave configuration. ‘d’ here is the remainder for the length divided by ‘3’ which is a character for the destruction of the translation symmetry of the chain. To get $J$ and $V$ mentioned above, estimations have been obtained by calculating the integrals in terms of maximally localized Wannier functions [29]. They are $J \approx 1.43\alpha \exp(-2.07\alpha^2) E_0$ and $V \approx \gamma \beta \alpha^2 E_0$, where $\alpha$ and $\gamma$ are determined by fitting the numerical evaluation of the integral of $V$. Roughly, a Gaussian approximation for the
Wannier function can also be used to estimate $J$ and $V$ [30]. According to these expressions for $J$ and $V$, one can see that several parameters can be adjusted to yield various ratio of $V/J$. Since we focus on the control procedure, moderate value of $V/J$ is chosen directly in this work.

To determine the controlled chain, now we numerically find out the spectrum for the eigenenergies as a function of the parameters in the Hamiltonian. The chain has the identical mathematical structure of a ring with the two ends joined together without changing the periodic character in the bulk. Thus we assume that the hopping strength at the joint (denoted by $J_f$) breaks the periodic character of the chain. Such a distinct hopping strength would affect the energy spectrum of the chain. Numerical results show that different strengths of $J_f$ induce different numbers of isolated states in the total band. In this work, the eigenenergies corresponding to the isolated states are located away from the three energy subbands (since we have set $\beta = 1/3$). Here, in order to define such isolated states, we assume the energy difference between the isolated one and their nearest neighbors is $E_d$ and the maximum energy difference between the neighbor eigenenergies in the corresponding neighbor subband is $E_b$. Here the ‘corresponding neighbor subband’ refers to the one that the nearest neighbor eigenenergy belongs to. If $E_d > \chi E_b$, $\chi = 2$, we refer it as the isolated state in this work. With respect to mentioned above, in figure 1, we numerically explore the number of isolated eigenenergies in the total energy band versus the distinct hopping strength $J_f$ and the phase $\delta$. It can be seen that there would be more than two isolated states appearing in some combinations of $J_f$ and $\delta$. The isolated states include the edge states belonging to topological phases. Besides the edge states, the other isolated ones are just located in the gap since the eigenstates are orthogonal to each other. Numerical simulation shows that the eigenenergies corresponding to the edge states are not always located in the gap between the subbands but may be larger or smaller than all the other eigenenergies. Here $E_d > 2E_b$ is obviously a rough criterion to ascertain an isolated state since the energy difference between the isolated one and its neighbor varies gradually with respect to $J_f$ and $\delta$. So further investigations may be needed on this rough spectrum.

As mentioned above, if the criterion for an isolated state $E_d > \chi E_b$ changes, one obtains a different spectrum from figure 1. To further determine a model chain, we next examine the character of an edge state which is chosen as the target. Numerical simulations show that the edge state may appear as the $m+1$ eigenstate in the energy band with respect to $J_f$ and $\delta$. Here the eigenstates have been arrayed according to their corresponding eigenenergies in a small-value to large-value manner. Thus $m+1$ refers to the order for the eigenstate in the array. Since a more local edge state is desirable, we next check the localization for the $m+1$ eigenstate. IPR can be used to indicate the degree of localization for a state [31]. If a state $|\phi_j\rangle = \sum_{n=1}^{N} \psi_j(n)|n\rangle$ ($N$ is the number of basis $|n\rangle$), IPR can be defined as $I_j = \sum_{n=0}^{N} |\psi_j(n)|^4$ ($j = m+1$ in this work). We can see that if $\psi_j(n)$ are distributed homogeneously over all basis $|n\rangle$, namely $|\psi_j(n)|^2 \sim 1/N$, then $I_j \sim 1/N$. Whereas, if $\psi_j(n)$ are localized over a range $\zeta$, namely $|\psi_j(n)|^2 \sim 1/\zeta$, then $I_j \sim 1/\zeta$. So for large $N$, the larger $I_j$ is, the more degree of localization of state $|\phi_j\rangle$. Then we show $I_{m+1}$ versus $J_f$ and $\delta$ in figure 2 to pick a combination of $J_f$ and $\delta$ with high IPR. According to this figure, without loss of generality, we choose $J_f = 0$ and $\delta = 2\pi/5$ mainly to exhibit the control results. According to the mentioned above, we specify the $m+1$ eigenstate as the target in this work. Then the ring retrogresses to an one-dimensional chain.

Since the model chain has been specified, we next check the energy spectrum for it. This gives us intuitive knowledge of the target state. Figure 3 shows the single-excitation energy spectrum while insets (a) and (b) exhibit the population of the edge state-2$(m+1)$ and $m+1$-state of Hamiltonian (3) when $\beta = 1/3$ in the open boundary condition. It can be intuitively seen that the population of the edge states both localize at the ends. They are the eigenstates of the natural Hamiltonian (3) with protected eigenenergies [3]. We next
describe the natural Hamiltonian and the edge state by using
\[ \hat{H}_0 \left| \tilde{\psi} \right\rangle = \left( \hat{H}_0 + \sum_n f_n(t) \hat{H}_n(t) \right) \left| \varphi \right\rangle, \]

where \( \hat{H}_0 \) describes the natural Hamiltonian and \( \hat{H}_n(t) \) are the control Hamiltonians with the corresponding real-valued \( f_n(t) \). \( f_n(t) \) represent the control fields that need to be designed by using the quantum Lyapunov control method. The system state is \( \left| \psi \right\rangle = \sum_k \xi_k |k\rangle \), where \( |k\rangle \) denotes that the atom on site \( k \) is excited while the others not, and \( \xi_k \) is the corresponding probability amplitude. \( L \) denotes the total number of sites on the chain. The control Hamiltonian \( \hat{H}_n(t) \) should not commute with the natural Hamiltonian \( \hat{H}_0 \), i.e. \( \{ \hat{H}_0, \hat{H}_n(t) \} \neq 0 \), otherwise its effect can be included in the natural Hamiltonian. Note that the target state \( \left| \varphi \right\rangle \) is usually an eigenstate of the natural Hamiltonian, namely \( \hat{H}_0 | \varphi \rangle = \lambda | \varphi \rangle \). Then the Lyapunov function \( V_L \) related to the controlled state is constructed to design the control field. Then the control fields are determined by making the first-order time derivative of the Lyapunov function \( V_L \) negative. Then assisted by the control Hamiltonians with the corresponding designed control fields, the system would be steered to a LaSalle invariant set asymptotically in which the states satisfy \( V_L = 0 \).

Usually, there are alternatives of Lyapunov functions that can be used. One of the candidates is based on Hilbert–Schmidt distance between the system state \( | \varphi(t) \rangle \) and the target state \( | \varphi_\text{f} \rangle \) [32] (we call it Lyapunov-A for short hereafter). It is
\[ V_A = \frac{1}{2} (1 - |\langle \varphi_\text{f} | \varphi(t) \rangle|^2), \]

where \( |\langle \varphi_\text{f} | \varphi(t) \rangle|^2 \) denotes the transition probability from \( | \varphi(t) \rangle \) to \( | \varphi_\text{f} \rangle \). According to the description above, the first-order time derivative for \( V_A \) is
\[ \dot{V}_A = - \sum_n f_{A_n}(t) \cdot |\langle \varphi(t) | \varphi_\text{f} \rangle| \times \text{Im} \left[ e^{i \arg(\varphi(t) | \varphi_\text{f})} \langle \varphi_\text{f} | \hat{H}_0 | \varphi(t) \rangle \right]. \]

where \( \text{Im}[\ast] \) denotes the imaginary part of \( \ast \). Thus there are different kinds of control fields \( f_{A_n}(t) \) that meet the requirement \( V_A \leq 0 \). For example, a succinct and valid choice is
\[ f_{A_n}(t) = T_n \text{Im} \left[ e^{i \arg(\varphi(t) | \varphi_\text{f})} \langle \varphi_\text{f} | \hat{H}_0 | \varphi(t) \rangle \right]. \]

where \( T_n > 0 \). When \( \langle \varphi(t) | \varphi_\text{f} \rangle = 0 \), the angle \( \arg(\varphi(t) | \varphi_\text{f}) \) is uncertain. Without loss of generality, we artificially set \( \arg(\varphi(t) | \varphi_\text{f}) = 0 \) in this situation.

Another Lyapunov function based on state error [17] can be described by (we call it Lyapunov-B for short hereafter)
\[ V_B = \frac{1}{2} \langle \varphi(t) - \varphi_\text{f} | \varphi(t) - \varphi_\text{f} \rangle - 1 - \text{Re} \left[ \langle \varphi_\text{f} | \varphi(t) \rangle \right]. \]

Re[\ast] denotes the real part of \( \ast \). The first-order time derivative for \( V_B \) is
\[ \dot{V}_B = - \lambda \text{Im} \left[ \langle \varphi_\text{f} | \varphi(t) \rangle \right] - \sum_n f_{B_n}(t) \cdot \text{Im} \left[ \langle \varphi_\text{f} | \hat{H}_0 | \varphi(t) \rangle \right]. \]

Distinguishing from the first Lyapunov function, here we would employ a simple \( \hat{H}_0 = I \) where \( I \) is the identity matrix in Hilbert space and \( f_{B_0} = - \lambda \) to cancel the first term in (9). Therefore, we can choose
\[ f_{B_n}(t) = T_n \text{Im} \left[ \langle \varphi_\text{f} | \hat{H}_0 | \varphi(t) \rangle \right], \]

where \( T_n > 0 \). In the next section, we apply both control schemes to generate the edge state for the fermionic chain.

To apply Lyapunov control, we next specify the control Hamiltonian. There may be various control Hamiltonians that can be adopted to prepare the edge state for the chain. Taking available techniques into consideration, a trigonometrical shaking Hamiltonian which is generated by electro-optic phase modulator [26] is employed, i.e.
\[ \hat{H}_c = V_c \cos^2 \left[ k_d (x - b \cos(\omega t)) \right]. \]

\( V_c \) is the constant amplitude and \( k_d \) denotes the laser wave vector. \( \omega \) reflects the shaking frequency. \( b \) indicates the shaking depth and should not be large otherwise it may induce heating effect leading to failure of this model. In the
second quantization form, the control Hamiltonian reads 
\[
\hat{H}_c = \int \! d\xi \, \hat{\Psi}^\dagger (x) \hat{H}_c \hat{\Psi} (x). \tag{12}
\]

As all atoms are loaded in the lowest band of the optical lattice, in terms of Wannier basis similar to (2), the control Hamiltonian has the matrix element
\[
\hat{H}_c (m, n) = \delta_{m,n} V_{cd} \cos \left[ 2 \frac{k_{d} b}{k_{1}} m + 2 k_{d} b \cos (\omega_{c} t) \right], \tag{13}
\]
where \( V_{cd} = \frac{V}{\sqrt{2}} \int \! d\alpha \omega^{\alpha}(x) \cos (2k_{d} x) \omega(x) \) and \( m, n \) are integers indicating the order for atoms on the chain. The parameters \( 2k_{d} b \) and \( \omega_{c} \) can be modulated in experiments [26]. The total Hamiltonian reads \( \hat{H} = \hat{H}_0 + f_{A}(t) \hat{H}_c \). Here \( f_{A}(t) \) is the time varying control field determined by using the quantum Lyapunov method. They can be tuned by changing the voltage on the electro-optic phase modulator. In this work, we denote the control fields as \( f_{A}(t) \) in the Lyapunov-A and \( f_{B}(t) \) in the Lyapunov-B schemes respectively.

To exhibit the control effect, we quantify it by the fidelity which is defined as
\[
\mathcal{F} (t) = | \langle \varphi (t) | \varphi_{\text{edge}} \rangle |^2 , \tag{14}
\]
where \( | \varphi_{\text{edge}} \rangle \) is the target state as is shown by the inset (b) in figure 3. We examine the fidelity at terminated time \( t = 3000 \) versus \( 2k_{d} b \) and \( \omega_{c} \) numerically in figure 4 for the two kinds of control schemes. In these simulations, we have set the initial state with equally projection to the basis \( |k \rangle \). It can be seen that both schemes can be used to yield high fidelity in a wide range of parameter interval.

To be more concrete, we show the dynamics of the control process in figure 5 for both the control schemes, when \( 2k_{d} b = 0.2 \) and \( \omega_{c} = 115 J \). Numerical simulation shows that the fidelity can reach more than 0.95 at time \( t = 3000 \) while each control field approximately vanishes. To intuitively compare the control results with the target, in figure 6, we plot the density distribution at time \( t = 3000 \) for the two control schemes and the goal match well.

In practice there may be various kinds of initial states. In consideration of this, we tested 100 random initial states for both kinds of control schemes to check their effectiveness for preparing the edge state in figure 7. It can be seen that both control schemes are effective to complete the control goal.

4. Discussions

In this section we provide discussions for both control schemes with respect to their robustness against fluctuations.
on the chain, expandability to chains of different lengths and modulation for the control fields.

Inevitably there exist types of fluctuations on the chain which mainly be classified as on-site energy and hopping types. These two kinds of fluctuations can be denoted as \( \delta \tilde{V} = \eta_0 \tilde{H}_{pe} \) and \( \delta \tilde{J} = \eta_h \tilde{H}_{ph} \). In matrix form, \( \tilde{H}_{pe}(m, k) = \delta_{m,k} \) and \( \tilde{H}_{ph}(m, l) = \tilde{H}_{ph}^\dagger(l, m) = \delta_{m,l+1} \), here \( m, k \in [1, L] \) and \( l \in [1, L-1] \) are random integers representing positions for the fluctuations. In the simulation, we set the strength of the fluctuations randomly distributed in the interval: \( [0, \eta_0(\eta_h)] \).

Namely, there are a number of sites (at random positions on the chain) with strength (distributed randomly within an interval) added to the on-site or hopping potential during the control process whereas the target state is still the edge state of the original chain without fluctuations. We should confirm that the positions and strength of these fluctuations are assumed to be fixed during the control process even when they are random. We examine numerically the robustness of both control schemes for these two kinds of fluctuations by fidelity at time \( t = 3000 \) versus the number and strength of these fluctuations in figure 8 for these two kinds of control schemes. From this figure, it can be seen that, with the increasing of the number and strength of fluctuations, fidelity would decline. This indicates the hindering effect of these fluctuations to the preparing procedures. However these static fluctuations would change the original energy spectrum since they destroy the structure of the chain. Thus too many or fluctuations with too large amplitude would destroy this model, which is beyond the scope of this work. Whereas in a range of the number and strength of these fluctuations, the fidelity can reach a high value which leads to both control schemes being robust against both kinds of fluctuations.

Since chains with different lengths may be the controlled objects, to examine the expandability of both control schemes, we apply them to such \( 3m + d \) chains with \( d = 2 \), within the length of \( [32, 35, \ldots, 95] \) sites. In this length scope, the fidelity can reach a high value at time \( t = 3000 \) even with slightly decrease with lengthening of the chain which is shown in figure 9.

Figure 7. The evolution of the fidelity for 100 site occupation random initial states in each control scheme: (a) The Lyapunov-A scheme while (b) is for Lyapunov-B scheme. The red solid lines represent the average for the blues.

Figure 8. Fidelity at time \( t = 3000 \) versus the number and the maximal strength of the random fluctuations averaged over 30 times for each value. (a) and (b) Fidelity for on-site energy and hopping fluctuations in Lyapunov-A scheme; (c) and (d): fidelity for on-site energy and hopping fluctuations in Lyapunov-B scheme. The parameters are same to those in figure 5 where \( \eta_0 \) and \( \eta_h \) are in units of \( J \). All the figures have the same color map.

Figure 9. Fidelity at time \( t = 3000 \) versus several lengths of the chains with the configuration \( 3m + d \) while \( d = 2 \), where the other parameters are same to those in figure 5. The nail graph with square heads denote the control results for Lyapunov-A scheme while those with circle heads denote the results for Lyapunov-B scheme.
since the sign of the control field determines the decreasing trend of the positive Lyapunov function but the amplitude determines the decreasing rate. An amplitude-invariant square wave has been used to replace the unmodulated control field \([33]\) to complete the control process. However, since the fidelity approaches 1, the evolution of the controlled system is more sensitive to the amplitude of control field, i.e. the sign of the control field would switch more frequently between ‘+’ and ‘−’ if its amplitude stays invariant. Indeed, in general, control fields with time-dependent envelopes can be used to realize a control goal. We use \(f_A(t) = \frac{A}{1 + \kappa t}, A, \kappa > 0\). (16)

By using these modulated control fields, the fidelity can reach 0.95 at time less than the results in the unmodulated cases as is shown in figures 10 and 11. Here \(A\) and \(\kappa\) in (16) can be tuned flexibly. Finally, even the control fields are obtained by closed-loop simulation, whereas in light of Lyapunov control strategy, control fields with the same profile may be used in open-loop control process to prepare the edge state.

5. Summary

The fermionic chain discussed in this paper can be created by cold atoms loaded in optical lattice. Such an optical lattice can be created by two standing waves formed by laser beams of different wavelengths. This chain can be mapped to a ring with the same periodic structure mathematically. And it has an attractive spectrum, in which the number of isolated eigenenergy depends on the hopping and the modulated phase \(\delta\) in the Hamiltonian. After specifying the fermionic chain defined in the text, we present proposals to prepare an edge state by quantum Lyapunov control with state distance (Lyapunov-A) and state error (Lyapunov-B) schemes. By both control schemes, an initial state with equal population on each site can be steered to the edge state with high fidelity in a wide range of control parameters. In the simulation, we find that both schemes are effective at preparing the edge state for 100 random initial states. And both schemes are robust against fluctuations in on-site energies and hopping potentials. To reduce the difficulty in realization, the control fields can be replaced with amplitude-modulated ones. This is because the sign plays a more crucial role than the amplitude of the control field to achieve a control goal. Such control methods provide ways to explore edge states for topological materials which possess novel properties.

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