Geometrically activated thermal mass: wood vs. concrete

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Abstract. Designing for climate resilience and carbon neutrality implies low-emission structures that double as thermal mass. In this study, the effect of using geometry to maximize natural surface convection on an internal thermal mass is investigated. Wood and concrete thermal masses are optimized for both instantaneous heat transfer and transient heat storage, and compared. It is found that the addition of optimally-sized fins can double or triple the convection coefficient at the interior surface, provided the thermal conductivity of the fins is sufficiently high. Doubling or tripling the surface heat transfer translates to an equivalent increase in dynamic energy storage, so long as the mass thickness, wall area, and vent openings are recalibrated to maintain thermal synchrony at the building-level.

1. Introduction
In 2007, Holford and Woods characterized the harmonic coupling of an internal thermal mass with buoyancy ventilation, defining some ‘tuning’ ratios ($Q$, $\lambda$, and $F$) that relate transient heat storage, surface convection, and temperature-driven air-exchanges to each other [1]. In 2019, Craig defined the optimal balance of these tuning ratios for a target interior temperature damping [2]. The results suggested that a wood thermal mass could perform as well as a concrete thermal mass, so long as the thickness and surface area of the wood (which has inferior thermal mass properties) is adjusted to maintain thermal synchrony at the building-level. In related work, Friedman studied the impact of geometrically ‘extending’ the surface of a thermal mass to increase surface convection [3]. This analysis was recently updated by Fortin and Craig and presented in an open-source app, showing the heat transfer enhancement associated with fins and pins of optimal thickness, length, and spacing [4]. The purpose of the present study is to consolidate these two bodies of theory, to understand the physical and practical limits of using surface geometry to improve the dynamic heat storage by an internal thermal mass. The limits of enhancing instantaneous surface heat transfer are defined in §2, as applied to standard construction materials (wood and concrete). The knock-on effects on periodic heat storage, when the internal thermal mass is synchronized with buoyancy ventilation, are defined in §3. Results are discussed in §4 and conclusions are drawn in §5.

2. Optimization of extended surfaces
An extended surface is a solid surface that is geometrically modulated to increase its thermal contact with the fluid surrounding it, such as air. Typically, extended surfaces have features like fins or pins (see Fig. 1) that protrude into the fluid and virtually increase the convection coefficient of their base surface. The theory for extended surfaces is a common subject in heat transfer and it can be found in most heat transfer handbooks [5][6][7]. But it is generally used on engines, electronic devices, or other heat-
generating objects that risk overheating. Is it possible to use extended surfaces at a building scale, with common construction materials, to improve the performance of thermal mass?

\[ \text{Figure 1 Panel geometry definition.} \]

\[ \text{2.1 Effectiveness of Extended surface} \]

The effectiveness of an extended surface is measured through a comparison with an equivalent flat surface:

\[ \varepsilon = \frac{\text{heat transfer with fins}}{\text{heat transfer in the absence of fins}} = \frac{q}{q_0} \]  

(2.1)

The uniform heat flux over the area of the reference (bare) surface is:

\[ q_0 = h A_0 \Delta T \]  

(2.2)

where \( h \) is the convection coefficient, \( A_0 \) is the surface area and \( \Delta T \) is the temperature difference between the surface and the air. In the case of an extended surface, \( A_0 \) can be divided into the area of the surface covered by the bases of the fins (\( n A_c \)) and the area of the unfinned surface (\( A_{0,u} \)):

\[ A_0 = A_{0,u} + n A_c \]  

(2.3)

The fins are increasing heat fluxes over their cross-sectional area \( A_c \) and the rest of the panel surface \( A_{0,u} \) behaves like the reference flat surface. The overall heat flux for an extended surface is therefore:

\[ q = h A_{0,u} \Delta T + n q_b \]  

(2.4)

where \( n \) is the number of fins and \( q_b \) is the heat flux through the base of one fin defined in [6] as:

\[ q_b = \frac{\sqrt{k A_c p h}}{\text{tanh}(ml_c)} \Delta T \]  

(2.5)

where \( k \) is the conductivity of the material, \( p \) is the perimeter of the fins and \( h \) is the convection coefficient (which is assumed uniform and constant over the fins surface and the reference surface). The dimensionless fin parameter \( m = \sqrt{h A_c p k} \) combines with the corrected fins length \( l_c = l + A_c / p \) to approximate the heat transfer occurring at the tip. Incropera and Dewitt [7] give the length limit:

\[ l_{max} = 2.3 / m \]  

(2.6)

beyond which the temperature of the tip of the fins is almost equal to the temperature of the fluid and thus no longer exchange heat. This model is valid when:

\[ (hd / k)^{1/2} << 1 \]  

(2.7)

where \( d \) is the fins thickness. In physical terms, this limit is a thickness-based Biot number that denotes when the temperature gradient over the length of the fins is significantly greater than the gradient over its thickness, thus allowing for the heat transfer through the fins to be treated as one-dimensional.

\[ \text{2.2 Maximum surface heat transfer} \]

This paper focuses on maximizing convective heat transfer and ignores radiant heat transfer. The number of features in an extended surface is limited by the thickness of the boundary layer on each side of the features. If the spacing between the feature (\( s \)) is too small, the boundary layers overlap and the flow in the channel cannot be fully developed. If \( s \) is too big, part of the channel is left unaffected. The optimal spacing \( s_{opt} \) occurs when the heat emitted from two adjacent fins can fully fill the channel in between and be dissipated without obstruction. It is defined by Bejan & Lorente in [8] as:
The effective convection coefficient of the extended surface is then $h_e = h_0 \epsilon$.

The extended surface potential for different materials can be assessed by following these steps:
1. Determine the size of the panel ($H, W$), the average temperature difference between the surface of the panel and the environment ($\Delta T$) and the conductivity of the material ($k$)
2. Estimate an average baseline convection coefficient ($h$). For a vertical panel, the solution for pure boundary layer natural convection is $h_0 = (k_{air}/H) \times 0.517 \text{Ra}_H^{1/4}$ [8].
3. Find $l_{max}$ with (2.6). $l$ can also be set as a ratio of the thickness as in Table 1.
4. Find the optimal spacing $s_{opt}$ with (2.8) or (2.9)
5. Find $d_{opt}$ by optimizing (2.13) with (2.11) or (2.12). This step can be easily achieved graphically as in the left-hand plot of Fig. 2 or with the free-to-download app written by the authors [4]. $d$ can also be set with the limit (2.7) as in Table 1.
6. The effective convection coefficient of the extended surface is then $h_e = h_0 \epsilon$.

**Table 1.** Comparison of design options. The thermal properties of each material are taken from [9]

| Material          | $\Delta T$ | $k \ (W/m \cdot K)$ | $\rho c \ (J/m^2/K)$ | $a \ (m^2/s)$ | $l \ (cm)$ | $s_{opt} \ (cm)$ | $d \ (cm)$ | $\sqrt{Bi} \ (-)$ | $\epsilon \ (-)$ |
|-------------------|------------|----------------------|-----------------------|---------------|------------|------------------|------------|-------------------|------------------|
| Softwood (//)     | 3          | 0.26                 | 540x1680              | 2.87e-7       | 0.36       | 1.27             | 0.12       | 0.45              | 1.43             |
| Hardwood (//)     | 3          | 0.45                 | 940x1680              | 2.85e-7       | 0.63       | 2.26             | 0.21       | 0.45              | 1.72             |
| Concrete          | 3          | 2                    | 2400x950              | 6.14e-7       | 3          | 2.82             | 1          | 0.35              | 1.71             |

**Figure 2** Effectiveness of the extended surface as a function of fins thickness
Table 3 shows six design options for geometrically activated made of softwood (/\), hardwood (/\) and concrete. According to the model presented here, extended surfaces made of pins are not feasible in any of the studied materials because even the higher conductivity concrete does not allow to meet the Biot-based validity criteria. Similarly, softwood is not a suitable candidate because its low thermal conductivity requires unrealistically thin fins to reach the validity range.

Figure 3. Two panel options in (a) hardwood (b) concrete. The exact dimensions are listed in Table 1

3. Surface heat transfer and diurnal heat exchange

An internal thermal mass uses diurnal temperature fluctuations to regulate interior conditions by storing excess heat during the day and releasing that excess during the night. The heat exchange between the mass and the interior air is governed by the conductivity of the material (mass to surface) and the convection coefficient $h$ (surface to air). Two designs for thermally massive panels in hardwood and concrete shown in Fig. 3 are used to analyze the impact of increasing the surface heat transfer on the diurnal performance of an optimally proportioned thermal mass.

3.1 Optimal thermal mass tuning

Holford & Woods [1] give a shortcut to model an effective thermal mass as a lumped mass in a diurnal heat exchange by comparing the timescales for convection and diffusion to affect the mass temperature. The effective heat transfer coefficient $\lambda$ and the effective thickness are defined as:

$$\lambda = \frac{1}{1 + \frac{\eta(\sinh \sinh (2\eta) - \sin \sin (2\eta))}{\xi(\cosh \cosh (2\eta) - \cos \cos (2\eta))}}$$

(2.14)

and

$$Lr = \frac{\cosh \cosh (2\eta) - \cos \cos (2\eta)}{\eta(\sinh \sinh (2\eta) - \sin)}$$

(2.15)

where $\eta = L\sqrt{\alpha/2\alpha}$ is the ratio of the mass thickness to the depth of thermal penetration and $\xi = \omega p c_1 h$ is the ratio of heat storage compared to the rate of surface heat transfer where $\omega = 2\pi/86400$ is the angular frequency. A massing parameter $\Omega$ is defined as:

$$\Omega = \xi Lr/\lambda$$

(2.16)

and a ventilation heat exchange parameter $F$ is defined as:

$$F = \frac{Q p c_1}{S h}$$

(2.17)

where $Q$ is the ventilation rate, $p c_1$ is the volumetric heat capacity of air and $S$ is the exposed surface area of the thermal mass. Craig [2] gives the optimal relation between $\Omega$ and $F/\lambda$ as

$$(F/\lambda)_{\max} = \tan \left(\frac{1.07 4^4}{2} \Omega^4\right) - 1$$

(2.18)

For every attenuating temperature difference $\Delta T/1$ there is an optimal coupling of $\Omega$ and $F/\lambda$. A target value for each of these parameters can be reached by tuning the thickness and surface area of the thermal mass based on a ventilation rate and the thermal properties of the material. The region of interest for the massing and the ventilation parameter are $0.01 < (F/\lambda) < 10$ and $1.3 \leq \Omega < 2.1$. 
3.2 Surface heat transfer and rate of heat storage
Solving (2.16) for a given $\Omega$, allows to find a thickness $L$ for different material options specific to their thermal properties ($\alpha$ and $\rho c$). An increase in the surface heat transfer (through an increase in $\epsilon$) leads to an increase in the rate of heat storage because the thickness $L$ of the thermal mass (or $L \ Lr$ of its equivalent lumped mass) is increased to meet a target $\Omega$, as shown in Fig. 4. Fig. 5 compares the rate of heat storage $\omega pc LLr$ with the convection coefficient $h\epsilon$ for the panel options and three $\Omega$s.

![Figure 4](image)

**Figure 4.** $\Omega$ as a function the thickness $L$ for increasing $h\epsilon$. The gray filling indicates the region of interest for $\Omega$. (a) Hardwood panel. (b) Concrete panel

Combining $\Omega$ and $\lambda$ in (2.18) allows to find a target value for $F$. When aiming for a fixed $F$, (2.16) shows that an increase in $h\epsilon$ is tied to either an increase in the ventilation rate $Q$ or a decrease in the surface area $S$. Fig. 6 shows the relation between $F/\lambda$ and $h\epsilon$ for various ratios of $Q/S$. Note that lower $F/\lambda$ are necessary for more efficient thermal masses (higher $1-1/\text{Ai}$).

![Figure 5](image)

**Figure 5.** Rate of heat storage as a function of the convection coefficient

![Figure 6](image)

**Figure 6.** Ventilation parameter as a function of the convection coefficient

4. Results and discussion
Fig. 7 shows the influence of the heat transfer enhancement associated with both material options. The upper right point represents a flat thermal mass for which the thickness ($L$), surface area ($S$), and ventilation rate ($Q$) are adjusted to reach an attenuating temperature difference $1-1/\text{Ai}$ of 0.7 and an optimal balance of $F/\lambda$ and $\Omega$. The convection coefficient of the flat surface is $h = 3 \ W/m^2 K$. The lower left point shows a non-optimal thermal mass for which only $h$ is augmented by $\epsilon$, $1-1/\text{Ai}$ increases, but $F/\lambda$ dominates. The lower right point shows how $F/\lambda$ and $\Omega$ can meet the optimal balance from (2.18) again. The thickness of the hardwood mass increases, and the flow rate decreases. Both the thickness and the exposed surface area of the concrete mass increase. For a geometrically activated
thermal mass to perform optimally, the thickness of the mass must be adjusted to account for the decrease of $\Omega$; and either the flow rate or the surface area must be adjusted to maintain $F/\lambda$. For hardwood and concrete, $\Omega$ increases by factors of 1.63 and 2.88 between the suboptimal and the optimal geometrically activated masses. Those factor changes are close to, but slightly inferior to corresponding $\epsilon$. They denote the anticipated gains in dynamic energy storage (per unit area) for both material options.

**Figure 7.** Contour plot of the attenuating temperature difference $1-1/Ai$ showing the impact $h\epsilon$ on the pairing of $\Omega$ and $F/\lambda$.

5. Conclusions

The theory for extended surfaces provides a framework to improve thermal mass performance, but the use of optimally-sized fins is limited to materials with higher conductivity like concrete. The convection coefficient of a finned concrete panel can increase by a factor of 3.13. Biogenic materials generally have lower thermal conductivity; and even though hardwood is a viable option (potential $\epsilon = 1.71$), other new biogenic materials, such as engineered bamboo, could prove to be more relevant. When combined with an optimal thermal mass, an increase of the surface heat transfer is not sufficient. The thickness and wall area of the internal thermal mass, as well as the ventilation rate of the interior space, must be adjusted to fully benefit from the extended surface. With such adjustments, the dynamic energy storage per unit area can increases by factors of 1.63 and 2.88 for hardwood and concrete respectively. The application of extended surfaces at building scale awaits experimental validation. The results presented here introduce new potential couplings to be explored between optimal thermal mass design, surface heat transfer, and structural design (increased thickness).

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