Solar wind and dynamics of meteoroid

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ABSTRACT

The effect of solar wind on dust particle, meteoroid, is investigated. Rotation of the particle is also considered. Detail derivations of both equation of motion and Euler’s dynamical equations, are presented. The most simple form of the action of the solar wind is considered and the derived equation of motion reduces to “pseudo-Poynting-Robertson effect” for non-rotating spherical particle. The final equation of motion contains also angular velocity term and it may cause change of orbital plane of the meteoroid. Analysis of the Euler’s dynamical equations shows that rotational axis of the meteoroid changes in time and equilibrium state for the orientation of the axis of rotation does not exist for spherical particle.

Key words. solar wind, meteoroid, equation of motion, Euler’s dynamical equations, orbital motion

1. Introduction

At the beginning of the past century, Poynting (1903) has formulated a problem of finding equation of motion for a perfectly absorbing spherical particle under the action of electromagnetic radiation. Robertson (1937) derived a correct equation of motion for a perfectly absorbing spherical particle. This result has been applied to astronomical problems for several decades and is known as the Poynting-Robertson (P-R) effect. Relativistically covariant equation of motion for general properties of spherical particles was presented by Klačka (2000, 2004). Complete understanding of the action of the incoming electromagnetic radiation on the motion of the spherical dust particle was obtained by Klačka (2008a, 2008b) and Klačka et al. (2009a). Partly similar effect on interplanetary dust particles is caused not only by solar electromagnetic radiation, but also by stream of charged particles flowing from the Sun. This stream continuously emanating from the Sun’s surface consists mostly from electrons and protons, α— particles and heavier atoms. The stream is called also corpuscular solar radiation, or, shortly, solar wind. Dynamical effects of solar wind on motion of meteoroids are similar to dynamical effects of solar electromagnetic radiation and they are known also as “pseudo-Poynting-Robertson effect” (Dohnanyi 1978). Detail understanding of the effect of solar wind on motion of spherical interplanetary dust particle is presented in Klačka et al. (2009b).

All the mentioned equations of motion for dust particles under action of electromagnetic radiation and solar wind are correct for non-rotating dust grains. What is the effect of rotation? How does dust particle rotate? In order to be able to answer the questions, we will deal with the effect of rotation on the dust particle. We will obtain results not only for time evolution of rotation of the particle, but also equation of motion of the rotating particle.

2. Proper reference frame of the dust particle – stationary particle

We will use considerations and accuracy to the first order in $v/u$ and $\omega R/u$ ($v$ is heliocentric velocity of the spherical dust particle of radius $R$, $u$ is the speed of solar wind
with respect to the Sun and $\omega$ is angular velocity of the rotation of the particle about a rotational axis containing center of mass of the particle.

The term “stationary particle” will denote a particle which does not exhibit translational motion in a given inertial frame of reference. Primed quantities will denote quantities measured in the proper reference frame of the center of mass of the particle. The particle may be characterized with a rotational motion in the proper reference frame of the center of mass.

Let $S'$ be an incident flux density of solar wind energy (energy flow through unit area perpendicular to the ray per unit time) and geometrical cross section of the spherical particle is $\pi R^2$. We have $S' = n' m_1 |\mathbf{u}'|^2 c$, where $n'$ is concentration (number of particles per unit volume) of solar wind particles of mass $m_1$ (for simplicity of these considerations only one type of particles is considered), $c$ is the speed of light, $\mathbf{u}' = \mathbf{u}' S'$ is velocity vector of solar wind particles: unit vector $\mathbf{S}'$ denotes direction and orientation of the incident beam of solar wind particles measured in the proper reference frame of the center of mass of the dust particle. Relations (1) – (19) in Klačka and Saniga (1993) are significant for the effect of incident solar wind particle. Moreover, Lorentz transformation yields for elementary force acting on the elementary area of the spherical particle due to action of the incident solar wind $(dp'/dt)_{el;inc} = (dp'/dt)_{el;inc} + E_{inc} n_{rot;el} c^2$, where double primed quantities are measured in the frame of reference connected with the rotating elementary force $\mathbf{F}_{rot;el}$ acting on the elementary area of the sphere. We have $(dp'/dt)_{el;inc} = n_{el} [(dA')_{el} c \cos \alpha'] \mathbf{u}'_{el} m_1 \mathbf{u}_{el}, \ n'_{el} = n' = n$ (within the considered accuracy), $\mathbf{u}'_{el} = \mathbf{u}' - \mathbf{\omega} \times \mathbf{\xi}'$. If we take all these results into account, then we can write for an elementary force caused by the incident solar wind

$$\left( \frac{d \mathbf{p}'}{d t} \right)_{el;inc} = n' m_1 \mathbf{u}^2 [(dA')_{el} \cos \alpha'] \times$$

$$\left[ \mathbf{S}' - \left( \frac{\mathbf{\omega} \times \mathbf{\xi}'}{|\mathbf{u}'|} \cdot \mathbf{S}' \right) \mathbf{S}' - \frac{\mathbf{\omega} \times \mathbf{\xi}'}{|\mathbf{u}'|} \right] +$$

$$n' m_1 |\mathbf{u}'| [(dA')_{el} \cos \alpha'] \mathbf{\omega} \times \mathbf{\xi}' ,$$

(1)

where $\mathbf{\xi}'$ is position vector of the element on the surface of the spherical particle with respect to the center of mass of the particle, $(dA')_{el}$ is area of the element. Within the considered accuracy, Eq. (1) is equivalent to

$$\left( \frac{d \mathbf{p}'}{d t} \right)_{el;inc} = \frac{S' (dA'_{el} \cos \alpha')}{c} \left( 1 - \frac{\mathbf{\omega} \times \mathbf{\xi}'}{|\mathbf{u}'|} \cdot \mathbf{S}' \right) \mathbf{S}' ,$$

(2)

if the incident flux density of solar wind energy $S' = n' m_1 |\mathbf{u}'|^2 c$ is used: this enable to consider more general composition of solar wind, as for various types of solar wind particles.

Eq. (2) enables to find the total force of the incident solar wind on the dust particle, and, also the total torque acting on the particle:

$$\left( \frac{d \mathbf{p}'}{d t} \right)_{total;inc} = \sum_{el} \left( \frac{d \mathbf{p}'}{d t} \right)_{el;inc} ,$$

(3)

$$M'_{total;inc} = \sum_{el} \mathbf{\xi}' \times \left( \frac{d \mathbf{p}'}{d t} \right)_{el;inc} .$$

(4)

In order to find $(dp'/dt)_{total;inc}$ and $M'_{total;inc}$, we may perform calculations in some special frame of reference:

$$(dA')_{el} = [(R \sin \alpha') \ d \alpha'] R \ d \alpha' ,$$
Putting Eqs. (2) and (5) into Eqs. (3) and (4), using also \( \sum_{\text{el}} \) → ∫, one finally obtains
\[
\left( \frac{d p'}{d t} \right)_{\text{total;inc}} = \frac{S' \pi R^2}{c} S',
\]
(6)
\[
M'_{\text{total;inc}} = -\frac{1}{4} \frac{S' \pi R^4}{c |u'|} \{ \omega - (\omega \cdot S') S' \}.
\]
(7)

As for the “outgoing part” of solar wind, we will suppose that the solar wind does not impart any momentum to the corresponding element of the area on the surface of the dust particle. Thus, we have \( (dp'/dt)_{\text{el;out}} = 0 \) for elementary force acting on the elementary area of the spherical particle due to action of the “outgoing” solar wind. Using the relation \( (dp'/dt)_{\text{el;out}} = (dp'/dt)_{\text{el;out}} + E_{\text{out}} v_{\text{el;out}} / c^2 \), we can write
\[
\left( \frac{d p'}{d t} \right)_{\text{el;out}} = \frac{x'}{c} \frac{S'}{c} \left[ (dA')_{\text{el}} \cos \alpha' \right] \frac{\omega \times \xi'}{|u'|},
\]
(8)
where it is supposed that the \( x' \) th part of the incident energy per unit time is lost from the dust particle (see Eq. 20 in Klačka and Saniga 1993).

Eq. (8) enables to find the total force acting on the particle due to the “outgoing” solar wind, and, also the total torque acting on the particle (the sign minus is due to the conservation of momentum):
\[
\left( \frac{d p'}{d t} \right)_{\text{total;out}} = \sum_{\text{el}} \left\{ - \left( \frac{d p'}{d t} \right)_{\text{el;out}} \right\},
\]
(9)
\[
M'_{\text{total;out}} = \sum_{\text{el}} \xi' \times \left\{ - \left( \frac{d p'}{d t} \right)_{\text{el;out}} \right\}.
\]
(10)

Detail calculations in a special frame of reference, e. g. given by relations (5), yield
\[
\left( \frac{d p'}{d t} \right)_{\text{total;out}} = \frac{x'}{c} \frac{S'}{c} \frac{\pi R^2}{3} \frac{2 R}{|u'|} \omega \times S',
\]
(11)
\[
M'_{\text{total;out}} = -\frac{1}{4} \frac{x'}{c} \frac{S' \pi R^4}{c |u'|} \left\{ 3 \omega - (\omega \cdot S') S' \right\}.
\]
(12)

The total force and torque acting on the particle due to solar wind:
\[
\left( \frac{d p'}{d t} \right)_{\text{total}} = \left( \frac{d p'}{d t} \right)_{\text{total;inc}} + \left( \frac{d p'}{d t} \right)_{\text{total;out}},
\]
(13)
\[
M'_{\text{total}} = M'_{\text{total;inc}} + M'_{\text{total;out}},
\]
(14)
or, using Eqs. (6)-(7) and (11)-(12):
\[
\left( \frac{d p'}{d t} \right)_{\text{total}} = \frac{S' \pi R^2}{c} \left\{ S' + x' \frac{2 R}{3 |u'|} \omega \times S' \right\},
\]
(15)
\[
M'_{\text{total}} = -\frac{1}{4} \frac{S' \pi R^4}{c |u'|} \left\{ (1 + 3 x') \omega - (1 + x') (\omega \cdot S') S' \right\}.
\]
(16)
3. Stationary frame of reference – equation of motion

By the term “stationary frame of reference” (laboratory frame) we mean a frame of reference in which particle moves with a velocity vector $\mathbf{v} = \mathbf{v}(t)$. The physical quantities measured in the stationary frame of reference will be denoted by unprimed symbols.

Our aim is to derive equation of motion for the particle in the stationary frame of reference. We will use the fact that we know this equation in the proper frame of reference: Eq. (15). In order to find the equation in the stationary frame of reference, we will use the Lorentz transformation for force acting on the dust grain:

$$\frac{dp}{dt} = \frac{dp'}{dt} + \frac{E'_{\text{inc}} - E'_{\text{out}}}{c} \mathbf{v}.$$

(17)

Since

$$E'_{\text{out}} = x' E'_{\text{inc}},$$

$$E'_{\text{inc}} = n' m_1 |\mathbf{u}'| c^2 \pi R^2,$$

(compare with the right-hand side of Eq. 1: $E'_{\text{inc}} = \sum_{el} n' m_1 |\mathbf{u}'| c^2 [(dA')_{el} \cos \alpha']$), we have

$$\frac{dp}{dt} = \frac{dp'}{dt} + (1 - x') n' m_1 |\mathbf{u}'| \pi R^2 \mathbf{v}.$$

(19)

Since $S' = S (1 - 2v \cdot S/|\mathbf{u}|)$, $S' = (1 + v \cdot S/|\mathbf{u}|) S - v/|\mathbf{u}|$, Eqs. (15) and (19) yield

$$\frac{dp}{dt} = S \pi R^2 \frac{c}{3 |\mathbf{u}|} \left\{ \left(1 - \frac{v \cdot S}{|\mathbf{u}|}\right) S - \frac{v}{|\mathbf{u}|} + x' \frac{2}{3} \frac{R}{|\mathbf{u}|} \mathbf{\omega} \times \mathbf{S} \right\} +$$

$$+ (1 - x') S \pi R^2 \frac{c}{3 |\mathbf{u}|} \mathbf{v}.$$

(20)

The last term in Eq. (20) corresponds to the change of grain’s mass, in accordance with Eq. (18):

$$\frac{dm}{dt} = (1 - x') S \pi R^2 \frac{c}{3 |\mathbf{u}|}.$$

(21)

Since $dp/dt = d(mv)/dt = m \ dv/dt + v \ dm/dt$, Eqs. (20)-(21) lead to

$$\frac{dv}{dt} = \frac{S \pi R^2}{m c} \left\{ \left(1 - \frac{v \cdot S}{|\mathbf{u}|}\right) S - \frac{v}{|\mathbf{u}|} + x' \frac{2}{3} \frac{R}{|\mathbf{u}|} \mathbf{\omega} \times \mathbf{S} \right\}.$$

(22)

Mass of the homogeneous spherical particle is $m = (4\pi/3) \varrho R^3$, where $\varrho$ is mass density. Eq. (21) reduces to

$$\frac{dR}{dt} = - \frac{K}{r^2},$$

(23)

where $r$ is distance between the particle and the Sun, and

$$K = (x' - 1) \frac{r^2 S}{4 \varrho c |\mathbf{u}|}.$$

(24)

is a constant for a given material. According to Dohnanyi (1978) and Kapišinský (1984) we can take $K \approx 4 \times 10^{-9} \text{ cm year}^{-1}$ (see also Leinert and Grün 1991). The phenomenon presented in Eq. (23) is known as the “corpuscular sputtering”. The units used in Eq. (23) are: [R] = cm, [t] = year, [r] = AU, [K] = cm year$^{-1}$. Eq. (24) enables to find the value of the dimensionless factor $x'$: $x' = 1.9 \times 10^{11} K [\text{cm year}^{-1}] \varrho [\text{g cm}^{-3}] + 1$. As an example, for the case $K = 4 \times 10^{-9} \text{ cm year}^{-1}$ and $\varrho = 1 \text{ g cm}^{-3}$, we obtain $x' = 7.6 \times 10^2$. 
4. Rotational motion of spherical particle

On the basis of Eq. (16), we can immediately write Euler’s dynamical equations:

\[
\frac{d}{dt} \left[ \frac{(2/5) m R^2 \omega}{c |u|} \right] = -\frac{1}{4} S \frac{R^4}{m c |u|} \left\{ (1 + 3 x') \omega - (1 + x') (\omega \cdot S') S' \right\},
\]

where we have used \( S \) instead of \( S' \) and \( |u| \) instead of \( |u'| \), within the considered accuracy. The moment of inertia of a sphere of uniform density is \((2/5) m R^2\).

In order to solve Eq. (25), together with Eqs. (22)-(23), we use, as standardly, Euler’s angles \( \psi, \theta \) and \( \varphi \). We have

\[
\begin{align*}
\frac{d}{dt} \omega_1' &= -\frac{5 \pi}{8} \frac{S R^2}{m c |u|} \left\{ (1 + 3 x') \omega_1' - (1 + x') (\omega \cdot S) S_1' \right\} - \frac{5}{R} \frac{d}{dt} \omega_1', \\
\frac{d}{dt} \omega_2' &= -\frac{5 \pi}{8} \frac{S R^2}{m c |u|} \left\{ (1 + 3 x') \omega_2' - (1 + x') (\omega \cdot S) S_2' \right\} - \frac{5}{R} \frac{d}{dt} \omega_2', \\
\frac{d}{dt} \omega_3' &= -\frac{5 \pi}{8} \frac{S R^2}{m c |u|} \left\{ (1 + 3 x') \omega_3' - (1 + x') (\omega \cdot S) S_3' \right\} - \frac{5}{R} \frac{d}{dt} \omega_3',
\end{align*}
\]

where we have used the fact that scalar product satisfies \( \sum_{p=1}^{3} \omega_p S_p = \sum_{q=1}^{3} \omega_q S_q' \), and, \( S = S_1 i + S_2 j + S_3 k \) in the stationary frame of reference, \( S' = S_1' i' + S_2' j' + S_3' k' \) in the frame of reference rotating with the particle. Moreover, the well-known relations are:

\[
\begin{align*}
\omega_1' &= \dot{\psi} \sin \varphi \sin \theta + \dot{\vartheta} \cos \varphi, \\
\omega_2' &= \dot{\psi} \cos \varphi \sin \theta - \dot{\vartheta} \sin \varphi, \\
\omega_3' &= \dot{\psi} \cos \varphi + \dot{\varphi}, \\
\omega_1 &= \varphi \sin \psi \sin \theta + \dot{\varphi} \cos \psi, \\
\omega_2 &= -\dot{\varphi} \cos \psi \sin \vartheta + \dot{\vartheta} \sin \psi, \\
\omega_3 &= \dot{\varphi} \cos \vartheta + \dot{\psi}, \\
S &= S_1 i + S_2 j + S_3 k, \\
S &= \frac{x}{r} i + \frac{y}{r} j + \frac{z}{r} k, \\
r &= x i + y j + z k,
\end{align*}
\]

where \( r \) is position vector of the particle with respect to the Sun, source of solar wind, \( r = |r|, v \equiv dr/dt \), and,

\[
\frac{S_1'}{S_2'} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \frac{S_1}{S_2},
\]

\[
\begin{align*}
R_{11} &= \cos \psi \cos \varphi - \cos \theta \sin \psi \sin \varphi, \\
R_{12} &= \sin \psi \cos \varphi + \cos \theta \cos \psi \sin \varphi, \\
R_{13} &= \sin \vartheta \sin \varphi, \\
R_{21} &= -\cos \psi \sin \varphi - \cos \theta \sin \psi \cos \varphi, \\
R_{22} &= -\sin \psi \sin \varphi + \cos \theta \cos \psi \cos \varphi, \\
R_{23} &= \sin \vartheta \cos \varphi, \\
R_{31} &= \sin \vartheta \sin \psi, \\
R_{32} &= -\sin \vartheta \cos \psi, \\
R_{33} &= \cos \vartheta.
\end{align*}
\]
Eqs. (28) and (29) yield
\[
\omega \cdot S = \left( \dot{\phi} \cos \psi + \dot{\varphi} \sin \vartheta \sin \psi \right) \frac{x}{r} + \left( \dot{\phi} \sin \psi - \dot{\varphi} \sin \vartheta \cos \psi \right) \frac{y}{r} + \left( \dot{\varphi} \cos \vartheta + \dot{\psi} \right) \frac{z}{r}.
\]

(32)

We can take into account a non-dimensional parameter, for Solar System
\[
\beta = \frac{r^2 S_{\text{light}} \pi R^2}{G M_\odot m c} \equiv \frac{L_\odot \pi R^2}{4 \pi G M_\odot m c} \frac{Q_{pr}'}{\bar{Q}_{pr}} = 2.4 \times 10^{-3} \frac{R^2 [m^2]}{m [kg]} \frac{\bar{Q}_{pr}}{\eta},
\]

(33)

where \( L_\odot \) is the rate of energy outflow from the Sun, the solar luminosity, \( M_\odot \) is the mass of the Sun and \( G \) is the gravitational constant and \( \bar{Q}_{pr} \) is a dimensionless efficiency factor for radiation pressure (Klačka 2004). The non-dimensional parameter ("the ratio of radiation pressure force to the gravitational force") reduces to \( \beta = 5.7 \times 10^{-5} \frac{Q_{pr}'}{\eta} \frac{G M_\odot}{r^2} \). On the basis of Eq. (33), we can immediately write for the quantities of solar wind:
\[
\frac{d \omega_1}{dt} = - \frac{5}{8 c} \frac{\eta G M_\odot}{Q_{pr}'} \left\{ \frac{x' + 11}{2} \omega_1' - \left( 1 + x' \right) (\omega \cdot S) S_1 \right\},
\]
\[
\frac{d \omega_2}{dt} = - \frac{5}{8 c} \frac{\beta \eta G M_\odot}{Q_{pr}'} \left\{ \frac{x' + 11}{2} \omega_2' - \left( 1 + x' \right) (\omega \cdot S) S_2 \right\},
\]
\[
\frac{d \omega_3}{dt} = - \frac{5}{8 c} \frac{\beta \eta G M_\odot}{Q_{pr}'} \left\{ \frac{x' + 11}{2} \omega_3' - \left( 1 + x' \right) (\omega \cdot S) S_3 \right\}.
\]

(34)

The orbital and rotational motion of spherical dust particle under action of solar wind and gravity of the Sun is given by Eqs. (23), (27)-(34), together with equation
\[
\frac{d v}{dt} = - \frac{G M_\odot}{r^2} S + \frac{\beta \eta G M_\odot}{Q_{pr}'} \frac{R}{r^2} \left\{ \left( \frac{v}{c} - \frac{v \cdot S}{c} \right) S - \frac{v}{c} + \frac{2}{3} \frac{R}{c} \omega \times S \right\}.
\]

(35)

Equation of orbital motion represented by Eq. (35) depends on angular rotational velocity of the particle.

5. Conclusion

The action of solar wind on motion of arbitrarily shaped dust particle would produce planar motion for non-rotating meteoroid orbiting the Sun. If angular velocity of rotation of the meteoroid is large enough, then rotational term may change orbital plane (see Eq. 35 for spherical particle; see also Eq. 8). Euler’s dynamical equations show that orbital axis of the particle changes and no equilibrium for the orientation of the axis exists.

Equation of motion and quantities for Euler’s dynamical equations for arbitrarily shaped dust grain are given by Eqs. (2)-(4), (8)-(10), (13)-(14), (17) and the first part of Eq. (18).

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