Comment on “An excess of massive stars in the local 30 Doradus starburst”

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Schneider et al. (Reports, 5 January 2018, p. 69) used an ad hoc statistical method in their calculation of the stellar initial mass function. Adopting an improved approach, we reanalyse their data and determine a power law exponent of $2.05^{+0.14}_{-0.13}$. Alternative assumptions regarding data set completeness and the star formation history model can shift the inferred exponent to $2.11^{+0.19}_{-0.17}$ and $2.15^{+0.13}_{-0.13}$, respectively.

Schneider et al. (1) use spectroscopic observations of young massive stars in the 30 Doradus region of the Large Magellanic Cloud to infer a shallower-than-expected stellar initial mass function (IMF) with a power-law exponent of $\alpha = 1.90^{+0.37}_{-0.26}$, in contrast to the Salpeter exponent of 2.35 (2). They estimate the ages and masses of individual stars with the BONNSAI Bayesian code (3), then obtain an overall mass distribution by effectively adding together the posterior probability density functions of individual stars. There is no statistical meaning to a distribution obtained in this way, which does not represent the posterior probability density function on the
Hierarchical Bayesian inference provides the statistically justified solution to this problem (4). Mandel (5) specifically considered inference on a mass distribution given a sample of uncertain measurements, and we use a similar methodology here. We interpret the Schneider et al. (1) inference on individual masses and ages as independent Gaussian likelihoods for the logarithm of the mass and the age, with parameters fixed by matching the mean parameter to the peak and the standard deviation parameter to the 68% width of the individual stellar distributions in the Schneider et al. (1) data.

For our fiducial analysis we model the star formation history as a truncated Gaussian distribution, and generally find a star formation history similar to that in Schneider et al. (1), with the star formation rate in 30 Doradus peaking about 4 million years ago. We impose broad priors on the power–law exponent and the mean and standard deviation of the star formation Gaussian. We use the Hamiltonian Monte Carlo sampler Stan (6) to efficiently address the high-dimensional hierarchical problem with free parameters for each star’s actual mass and age in addition to the IMF exponent and the mean and standard deviation of the star formation history.

Figure 1 shows the inferred power-law exponent of the IMF. We use the Schneider et al. (1) fit to stellar lifetimes and assume that their data set is complete above 15 solar masses; that is, we select only those stars whose observed mass is above \(15 \, M_\odot\) (7, 8). We find an exponent of \(\alpha = 2.05^{+0.14}_{-0.13}\) where the quoted value corresponds to the median of the posterior distribution and the range to the 16th and 84th percentiles (i.e. the symmetric 68% credible interval). Hierarchical Bayesian modelling steepens the preferred IMF slope; our median \(\alpha\) value lies about 1 \(\sigma\) above the preferred value from Schneider et al. (1). More significantly, this analysis narrows the uncertainty interval by more than a factor of 2.

The analysis above uses the same assumptions as Schneider et al. (1). Below, we consider the impact of three additional assumptions: the stellar lifetime fit, the choice of the completeness
limit, and the model for the star formation history.

We performed an independent fit to the main sequence lifetimes \( \tau_{MS} \) of non-rotating massive stars of mass \( M \) as modelled by Brott et al. (9) and Köhler et al. (10):

\[
\ln \frac{\tau_{MS}(M)}{\text{Myr}} = 9.1973 - 3.8955 \ln \frac{M}{M_\odot} + 0.6107 \left( \ln \frac{M}{M_\odot} \right)^2 - 0.0332 \left( \ln \frac{M}{M_\odot} \right)^3. \tag{1}
\]

Following Schneider et al. (1), we increased the “observable” lifetime of a star by 10% beyond its main-sequence lifetime to account for helium burning. We find that this alternative fit does not affect the inferred IMF, yielding the same power-law exponent \( \alpha = 2.05^{+0.14}_{-0.13} \).

The inferred power-law exponent is somewhat sensitive to the choice of the cutoff mass for survey completeness. The data of Schneider et al. (1) show a relative scarcity of stars between 15 \( M_\odot \) and 20 \( M_\odot \); changing the mass cutoff from 15\( M_\odot \) to 20\( M_\odot \) further steepens the inferred exponent to \( \alpha = 2.11^{+0.19}_{-0.17} \). However, these fluctuations are within the expected statistical variation based on the sample size, as confirmed with posterior predictive checking. In particular, there is no statistical evidence against the claim of Schneider et al. (1) that the survey is complete for \( M \geq 15 M_\odot \).

Finally, we considered an alternative star formation history model – a double exponential with three free parameters: the time of the peak of the star formation rate, and the (possibly different) decay constants before and after the peak. This model allows for a sharper peak and longer tails than a Gaussian. This star formation rate history model is consistent with the data, as tested with posterior predictive checking (see below). However, it yields a power-law exponent \( \alpha = 2.15^{+0.13}_{-0.13} \), almost 1 \( \sigma \) steeper than for our fiducial analysis. This indicates that the inferred IMF is sensitive to the systematics of the assumed star formation history model.

We also considered the possibility that the IMF power law has an additional break at higher masses, allowing for three free parameters: the mass at which the break happens and the exponent below and above the break. However, we find that the data do not constrain the parameters.
Figure 1: The posterior on the IMF power-law exponent $\alpha$ is inferred from the observations $d$. See text for details on the four models (top to bottom in the legend): Blue: Schneider et al. (I) [S+] stellar lifetimes [l.t.], survey completeness for $M \geq 15 M_\odot$, and Gaussian star formation history model; green: same, but with our lifetime fit; orange: as green, but with completeness for $M \geq 20 M_\odot$; pink: as blue, but with a double-exponential star formation history model. The Salpeter power-law exponent is $-\alpha = -2.35 (2)$, indicated by a vertical black line. The 68.3% range of power-law exponents derived by Schneider et al. (I) is shaded in grey.
of this more general model, and there is no statistical preference for a broken power-law model.

We confirmed the stability of our conclusions with posterior predictive checking. Figure 2 shows the distribution of observed masses and ages (i.e., the peak of the likelihood) from the Schneider et al. (1) data overlain on the range of mass and age distributions that would be observed from a large number of data sets drawn according to our fitted fiducial IMF model. The data are consistent with being drawn from our model. We have also confirmed that all of our models yield predictions for the numbers of stars heavier than 30$M_\odot$ and 60$M_\odot$ that are consistent with observations.

We find that we can substantially reduce the statistical uncertainty in the IMF by applying an improved statistical analysis to the observations of young massive stars in 30 Doradus. However, the systematics from modelling uncertainties, such as the assumed star formation history model, can potentially shift the inferred power-law exponent by more than the statistical uncertainty. Furthermore, we adopted the mass and age posteriors for individual stars directly from Schneider et al. (1). Imperfect stellar models or the inclusion of other complicating factors described by Schneider et al. (1) (rotation, mass transfer, mergers, etc.) introduce further systematic uncertainty that could again shift the inferred IMF exponent. The combination of these factors makes it very challenging to infer the precise shape of the IMF even when a data set as good as that obtained by Schneider et al. (1) is available.

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Figure 2: Posterior predictive checking demonstrates that the observed data are consistent with being drawn from our model. The observed distribution of (maximum likelihood) masses $M$ (A) and ages $t$ (B) are shown as black curves; distributions of mass and age from synthetic data drawn from our fitted model (i.e. the posterior predictive distribution) are shown as blue curves (median) and shaded blue regions (68%, and 95% credible intervals), respectively.
Data and materials availability: The code and \LaTeX source used to prepare this document are publicly available under an open-source MIT license at \url{https://github.com/farr/30DorIMF}.

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