Thin Spherical Elastic-Plastic Shells under the Action of Concentrated Loads

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Abstract. Thin isotropic rotation shells of constant thickness taking into account the appearance of plastic deformations under the local loads are considered. The method of running with discrete orthogonalization is used for the numerical realisation of the present problem. Thin spherical shells with the angle 90 degrees and the relation of the radius to the thickness 100 under local load are considered.

1. State of the problem
With the calculations of dome covers and some sheet structures (such as reservoirs, gasholders), it is taken into account that surface loads and concentrated forces act on the shell.

In the present work, the method of calculation of thin isotropic rotation shells of constant thickness under the action of distributed and local loads beyond the elastic limit is presented. The shells may be long and short, withdrawn or ring. It can have different support conditions, for example, rigid attachment, hinged support, free margin. The solving differential equations system is based on the linear shells theory in consideration with Hirschhoff-Lave hypothesis and physical equations of small elastic-plastic deformations theory [1-5]. It is supposed that the median surface of revolution regarding axis $z$ without particular points apart from the boundary ones where such feature is permitted. Let’s define [6]:

$\theta$ – azimuthal angle in the horizontal plane,
$s$ – meridian arc length,
$\zeta$ – normal to the mid-surface of the shell.

2. The method of solution
Using elastic solutions method, internal efforts $N$ are presented as a difference between linear-elastic solutions $N^0$ and some corrections considering physical nonlinearity of material $\Delta$ [7-9]:

$$N = N^0 - \Delta$$

All functions of stress-stain state and loads Fourier transform in azimuthal coordinate $\theta$. The transformation of the whole system of differential equations in partial derivatives leads to the resolution system from eight ordinary differential equations of first order. It contains the next dimensionless values [6] that are the coefficients of Fourier series of unknown functions: $N_r/Eh$, $N_z/Eh$ – radial and axial efforts, $\tilde{S}/Eh$ – generalized tangential effort, $M_s/Eh^2$ – meridian bending moment, $u_r/h$, $u_z/h$ – radial and axial movements, $\zeta_{e}$ – angle of rotation of section. The right part of the resolution system of differential equations of first order (2) contains components of load $\tilde{f}(s)$ and functions that consider physical nonlinearity of material $\Delta$ [9-10]:
\[ \ddot{\mathbf{N}} = A(s)\dot{\mathbf{N}} + \dot{f}(s) + B(s)\Delta, \]  

(2)

where: \( \dot{\mathbf{N}} = \begin{pmatrix} N_x, N_z, \frac{\Delta}{EH} \end{pmatrix} \), \( A(s) = \|a_{ij}\|, \) \( B(s) = \|b_{ik}\|, \)

\[ \dot{f}(s) = (q_r, q_z, q_{\theta}, 0, 0, 0, 0) \]

\[ \Delta = \begin{pmatrix} \Delta N_x, \Delta N_z, \Delta M_x, \Delta M_z, \Delta S, \Delta \theta \end{pmatrix}. \]

Shell deformations are defined by formulas (2):

\[ \ddot{\mathbf{e}} = C(s)\dot{\mathbf{N}} + D(s)\dot{\Delta}, \]  

(3)

where: \( \dot{\mathbf{e}} = (\dot{\varepsilon}_s, \dot{\varepsilon}_\theta, \dot{\varepsilon}_{s\theta}, \dot{\varepsilon}_{n\theta}, \dot{\varepsilon}_{n\theta}, \dot{\varepsilon}_{s\theta} n). \) \( C(s) = \|c_{ij}\|, \) \( D(s) = \|d_{ik}\|, \)

\[ i, j = 1,2, ..., 8, \quad k, l = 1,2, ..., 6. \]

and \( A(s), B(s), C(s), D(s) \) – matrixes with variable coefficients.

3. Results

In the article, the results of calculation of stress-stain state of spherical shell under the action of local loads distributed the parallel are presented. The angle \( \theta_0 \) measuring from the zero meridian is 0,02° that allows to support that the action distributed along the parallel loads is equivalent to the action of concentrated force. The spherical shell with the one-half angle \( 90^\circ \) and the relation of radius to the thickness \( R/h = 100 \) of hard pitching along the contour under the action of concentrated force \( P/Eh = 0.3816 \cdot 10^{-3} \) applied to the point of the surface with coordinates \( \theta = 0^\circ, \phi = 0.5 \) (‘figure 1’) are studied. The diagram (stress intensity and intensity of deformation) \( \sigma - \varepsilon \) polyline is approximated. It is supposed to be \( U = 0.95, V = 2.1 \cdot 10^W, \) \( \varepsilon_s = 10^Q, \nu = 0.5 \) – Poisson coefficient [11].

![Figure 1](image)

Figure 1. The spherical shell is under the action of local loads.

The numerical realization of the present boundary value problem has been implemented by the run method with discrete orthogonalization [7, 11]. The research on the convergence elastic solutions method was doing with all the values of required functions [7].

The performed studies showed that, with the whole ring load \( P/Eh = 0.2862 \cdot 10^{-3} \), the fastest converging function is the horizontal movement \( u, \) it differs little from iteration to iteration with \( n_0 = 20-30, \) but the slowest converging function is meridian bending moment \( M_s. \) The convergence \( M_s \) is observed with \( n_0 = 80-90. \) Moreover, with growing of the coefficient of linear hardening of material \( \lambda, \) the convergence of elastic solutions method is becoming smaller. Thus, for the same load, with \( \lambda = 0.95, \) it takes 40 iterations, with \( \lambda = 0.992 \) – 100 iterations, with \( \lambda = 1.0 \) – 150 iterations [8]. In the case of action of local loads, the convergence of elastic solutions method is getting faster.
At the ‘figure 2’ and ‘figure 3’, the changing of meridian bending moment $M_s$ and axial longitudinal effort $N_z$ along the zero meridian and along the parallel at the level of application of the load $s = 0.5$ is presented. Solid line – the calculation by elastic solutions method. Totted line – the result of the calculation without consideration of the appearance of plastic deformations. From ‘figure 2’, it follows that the most bending moment stretching internal shell fibers is located under the point of application of load. For the zero meridian, the value of the moment about 7 times more than the maximum values of moments stretching top shell fibers and located on both sides from the place of application of load. Also it should be noticed that the changing of bending moment along the parallel, for example, with $s = 0.5$ has some vibrations of its value conditioned by little number of members of Fourier series $k = 10$ is observed. Also it should be noticed that the value of the moment at the points on the meridian $\theta = 180^\circ$ approximately ten times more lower than the corresponding value for the zero meridian.

The epure $N_z$, ‘figure 3’. The difference between elastic-plastic solution and elastic solution at the point of application the force is approximately 6-15%. Moving away from the zero meridian along the parallel, the value of effort $N_z$ is becoming smaller too.

The maximum values of all the components of the movements $u_r, u_\theta, v$ are located at the point $\theta = 0^\circ, s = 0.5$ and for $u_r, u_\theta$ they get values approximately $0.25h$, but for $v \approx 0.1h$. On the rest of surface of the shell the value of movements approximately ten times or more, are lower than a maximum one.
Figure 4. The areas enveloped by plastic deformations for the meridians $\theta = 0^\circ, 10^\circ, 20^\circ, 30^\circ$.

‘Figure 4’ shows the areas enveloped by plastic deformations for the meridians $\theta = 0^\circ, 10^\circ, 20^\circ, 30^\circ$. On the zero meridian elastic-plastic deformations appear on all thickness of the section within the limits $0.49 < s < 0.51$, i.e. in the closest surrounding points of application of force. The other sections are elastic-plastic with elastic area located on the compressed surface of the shell.

On the meridian $\theta = 10^\circ$, also it is possible to highlight only plastic area from $s = 0.47$ to $s = 0.53$ and elastic-plastic within the limits $0.43 < s < 0.47$ and $0.53 < s < 0.57$. On the meridian $\theta = 20^\circ$, the area elastic-plastic deformations is located within the limits $0.43 < s < 0.53$, but with $\theta = 30^\circ$ – within the limits $0.49 < s < 0.51$.

Figure 5. The changing of intensities of deformations:

- $a)$ on the internal surface of the shell,
- $b)$ on the external surface of the shell

The changing of intensities of deformations on the internal $\varepsilon_{i1}$ and external $\varepsilon_{i2}$ surface of the shell along the different parallels is presented at ‘Figure 5 a, b’. The biggest intensities of deformations, as expected,
achieved at the points of application of force. A bit smaller ones value intensities of deformations are located at the right and the left from $s = 0.5$ at the levels $s = 0.48$ and $s = 0.52$ too. Furthermore, on the internal surface of the shell, their values strongly differ from each other, but on the internal one, they are quite equal. However, in both cases the values $\varepsilon_{i1}$ and $\varepsilon_{i2}$ at the left from the application of force is more than at the right.

4. Conclusions
Thus, the studies shown that appearance of plastic deformations the shell essentially influences on the values of efforts and movements. With drifting apart from the point of application of force, the reduction of area of plastic and elastic-plastic deformations is clear. The presented results of calculations allow to compare changing of stress-stain state of shell under the action of the whole ring loads and local ring loads. They demonstrate the difference between the results of elastic solution and elastic-plastic solution.

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