Application of Optimal Kalman Filter to Improve the Accuracy of Aircraft Wing Vibration Parameters Measurement System

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Abstract. This paper discusses the advantages of constructing a vibration parameters measurement system of an aircraft wing using mems IMUs. In addition to mems IMUs, the system makes use of displacement sensor and navigation system as secondary measurements, along with the optimal Kalman filter estimation. The basic principles of system operation are described. The main algorithms of the system and its errors mathematical model are presented. The results of simulation are presented, demonstrating the expected measurement accuracy of the system as a whole.

1. Introduction

Nowadays, thin-walled beams are used as the main structural elements of aircraft wings. Their use becomes especially effective with the advent of fibrous composites [1-3]. However, such aircraft aside with relative lightness, are very flexible, demonstrating for example large wing deformations under normal operating loads, which significantly affects their aerodynamic and strength characteristics [1, 4]. Therefore, it becomes necessary to use a vibration measurement system to diagnose the state of the wing, monitor and predict the appearance and development of defects based on its vibration behavior [5, 6]. Aircraft wing vibration measurement systems are usually implemented using piezoelectric accelerometers, optical sensors or MEMS accelerometers [2, 7-14]. The proposed system is built essentially on mems IMU and sensors data fusion technology [15-17]. Accordingly, it will predominantly share common features with the existing vibration measurement systems like the ones built on MEMS accelerometers, as well as to some extent the ones built on piezoelectric accelerometers. Moreover, the proposed system is intended to overcome the main disadvantages of the already existing systems, in particular the low measurement accuracy associated with high errors of MEMS sensors. So, the main advantages of the proposed system include: 1) relatively high accuracy, due to the application of data fusion technology from different sources of information, for example, mems IMUs, displacement sensor (DS), and also the airplane onboard navigation system (NS), which allows the estimation and correction of basic system errors; 2) great information content, since the proposed system will be able to provide a complete set of information about the vibration parameters in the measurement points (vibration acceleration, vibration velocity, vibration displacement, etc.); 3) low cost and power consumption, small weight and dimensions of the system, which allows its use when measuring
vibration parameters of a relatively small mechanical structures, including thin-walled ones; 4) the system will be able to operate both in a laboratory tests of the wing (within design stage or for diagnostic purposes) and during its operation (including at an airfield or in-flight), since the proposed system continuously receives complete information about the orientation and navigation parameters of the measurements point of the wing; 5) The simplicity of equipping an aircraft with such system is primarily associated with the possibility of relatively simple installation of mems IMUs at the measurement points, in addition to the fact that most modern aircraft already have their own NS, and some promising aircraft are equipped with DS built into their wings.

2. Structure and hardware components of the proposed vibration measurement system

A scheme of hardware components of the proposed system is shown in figure. 1, XYZ is the right coordinate system associated with the wing, where the axis X – directed along its longitudinal axis, and axis Z – along the right wing. The right Earth equatorial (Greenwich) coordinate system (ECS) was chosen as a base coordinate system of measurement, with the origin in the center of Earth (O_E) and the unit vectors are $\xi, \eta, \zeta$, where $\xi$ lies at the intersection of the equator and Greenwich meridian planes, $\zeta$ directed along the axis of rotation of Earth. From the figure. 1 it can be seen that to measure the wing vibrations, a mems IMU2 is installed at its end, DS is located on the wing, and a mems IMU1 is installed on the base (for example, the fuselage) close to the beginning of the wing. In this case, the proposed system contains IMU1, IMU2 (both mems), DS and also onboard NS, so that the fusion and processing of the data provided by these sources of information using optimal Kalman estimation allows the system to determine the wing vibration parameters with increased accuracy. When measuring the vibrations of aircraft wings in flight mode, it is advisable to use the information from the onboard NS (as illustrated in figure. 1), whereas within the ground tests, other sources of information about the wing coordinates can be used. In addition, it should be mentioned here that one of the important advantages of the proposed system is the possibility of introducing additional points for measuring the vibrations of complex structures, for example an aircraft wing with a variable sweep. To do this, an additional mems IMU need to be installed at each of the points of interest, which can be also supplemented by a DS. In the other hand, in the simplified case, IMU1 can be excluded, and the NS is directly used instead.

Figure 1. A basic scheme of the hardware components of the proposed wing vibrations measurement system, where: $R_N$ – geocentric radius-vector of the point O, measured by NS; $R_{M1}$ – geocentric radius-vector of the point $O_1$, measured by IMU1; $R_{M2}$ – geocentric radius-vector of point $O_2$, measured by IMU2; $R_{K1}$ – a vector characterizing the position of $O_1$ relative to O; $R_{K2}$ – a vector characterizing the position of $O_2$ relative to $O_1$, measured by DS; $u$ – the vector of the angular velocity of the Earth's own rotation; $O_E$ – the center of Earth.

3. Orientation and navigation algorithm

The operation algorithm of the proposed system ensures the determination of orientation and navigation parameters of mems IMU in an autonomous operation mode. This task is solved in ECS. In vector-matrix form, the corresponding equations have the following form [18]:

$$
\begin{bmatrix}
R_N \\
R_{M1} \\
R_{M2} \\
R_{K1} \\
R_{K2} \\
u
\end{bmatrix} =
\begin{bmatrix}
R_{O_E} \\
R_{O_1} \\
R_{O_2} \\
\mathbf{R}
\end{bmatrix} + \mathbf{K}
$$
\[
\begin{align*}
\dot{\theta} &= -u \times \theta - A_{\text{O/E}} (\Delta \omega_{\text{sys}} + \Delta \omega_{\text{acc}} + \Delta k_{\text{acc}} \cdot \omega_0); \\
\dot{\delta U} &= -2u \times \delta U + A_{\text{O/E}} (\Delta n_{\text{sys}} + \Delta n_{\text{acc}} + \Delta k_{\text{acc}} \cdot \omega_0) - \theta \times (A_{\text{O/E}} \times n_0) - \omega_0^2 (\delta R - 3(\delta R \cdot 1_e) 1_e); \\
\dot{\delta R} &= \delta U; \\
\Delta \omega_{\text{sys}} &= 0; \quad \Delta n_{\text{sys}} = 0; \quad \Delta k_{\text{sys}} = 0; \quad \Delta \omega_{\text{acc}} = 0; \quad \Delta n_{\text{acc}} = 0; \\
\Delta R_{\text{DSsys}} &= 0; \quad \Delta k_{\text{DS}} = 0; \quad \delta R_{\text{DSG}} = \Delta R_{\text{DSsys}} + \Delta R_{\text{DSacc}} + \Delta R_{\text{DSloc}} + \Delta k_{\text{DS}} \cdot R_{\text{KO}},
\end{align*}
\]

where \( \delta X \) denotes the error of the corresponding quantity \( X \); \( \theta \) – vector of small rotations characterizing the inclination of ECS calculated position relative to its true position; \( \Delta \omega_{\text{sys}}, \Delta n_{\text{sys}}, \Delta R_{\text{DSsys}} \) – vectors of systematic components of AU, GU and DS errors, respectively; \( \Delta \omega_{\text{acc}}, \Delta n_{\text{acc}}, \Delta R_{\text{DSacc}} \) – vectors of their random components in the form of white noise; \( \Delta \omega_{\text{loc}}, \Delta n_{\text{loc}}, \Delta R_{\text{DSloc}} \) – vectors of their random (stochastic) autocorrelated components, which are a first order stationary random processes with correlation functions of the form \( K(\tau) = \sigma^2 e^{-\mu |\tau|} \), where \( \sigma^2 \) – variance of the corresponding error, \( \mu \) – damping coefficient of the correlation function, \( \tau \) – correlation time; \( \Delta k_{\text{acc}}, \Delta k_{\text{loc}} \) – matrix of scale factors errors for AU, GU and DS, respectively; \( 1_e = R(R \cdot R)^{0.5} \) – vector of geocentric vertical; \( \omega_0 \) – natural frequency of an inertial system when an object moves in the vicinity of the Earth surface, usually called Schuler’s frequency [21].

4. The proposed system’s error model

The errors mathematical model of the proposed system includes the error model of the orientation and navigation channel, and the error model of DS [21]. In this case, the errors mathematical model of the proposed system in vector form can be represented as follows:

\[
\begin{align*}
\dot{\theta} &= -u \times \theta - A_{\text{O/E}} (\Delta \omega_{\text{sys}} + \Delta \omega_{\text{acc}} + \Delta k_{\text{acc}} \cdot \omega_0); \\
\dot{\delta U} &= -2u \times \delta U + A_{\text{O/E}} (\Delta n_{\text{sys}} + \Delta n_{\text{acc}} + \Delta k_{\text{acc}} \cdot \omega_0) - \theta \times (A_{\text{O/E}} \times n_0) - \omega_0^2 (\delta R - 3(\delta R \cdot 1_e) 1_e); \\
\dot{\delta R} &= \delta U; \\
\Delta \omega_{\text{sys}} &= 0; \quad \Delta n_{\text{sys}} = 0; \quad \Delta k_{\text{sys}} = 0; \quad \Delta \omega_{\text{acc}} = 0; \quad \Delta n_{\text{acc}} = 0; \\
\Delta R_{\text{DSsys}} &= 0; \quad \Delta k_{\text{DS}} = 0; \quad \delta R_{\text{DSG}} = \Delta R_{\text{DSsys}} + \Delta R_{\text{DSacc}} + \Delta R_{\text{DSloc}} + \Delta k_{\text{DS}} \cdot R_{\text{KO}},
\end{align*}
\]

5. Formation of estimates, correction and calculation of vibration parameters algorithms

To determine vibration parameters more accurately, the proposed system makes use of optimal Kalman filter (OKF) estimation and correction of the orientation and navigation channels (mems IMU1 and mems IMU2), as well as DS. For the mems IMU2 channel, OKF uses a measurement vector generated by comparing a vector \( R_{\text{NS}} \), obtained according to the measurements of IMU2, and a vector \( R_{\text{NS}} \), obtained according to the measurements of DS and NS (see figure. 1):
\[ z_2 = R_{M2} - R'_{M2}; \]
\[ R_{M2}' = R_{NC} + A_{OENC} (R_{K2OC} + R_{K1OC}) = R_{M2} + A_{OENC} (\delta R_{DSO} + \delta R_{ISO} + \delta R_N), \]

where the index \( c \) indicates that the corresponding vector is calculated; \( R_{NC}, A_{OENC} \) – calculated by high-precision onboard NS; \( \delta R_{IS} \) – systematic error of previously measured and stored \( R_{K1} \), this error is similar to \( \Delta R_{DSyst} \).

Similarly, For the mems IMU1 channel, OKF uses the following measurement vector:

\[ z_1 = R_{MIC} - R'_{MIC}; \]
\[ R_{MIC}' = R_{M1} + \delta R_{M1}; \]
\[ R'_{MIC} = R_{NC} + A_{OENC} R_{K1OC} = R_{M1} + A_{OENC} \delta R_{ISO} + \delta R_N. \]

As a result of Kalman estimation, estimates of the main system errors are formed, which are then used to correct the measurements and calculated outputs. Correction of specific parameters of the DS, as well as mems IMU2 and mems IMU1 channels, occurs in a similar way as follows:

\[ \hat{\mathbf{U}} = \mathbf{U}_c - \hat{\mathbf{U}}; \hat{\mathbf{R}} = \mathbf{R}_c - \hat{\mathbf{R}}; \mathbf{A}_{OE} = (E + K_\theta) A_{O/E}; \]
\[ \hat{\mathbf{\omega}}_O = \mathbf{\omega}_{OC} - (\Delta \mathbf{\omega}_{sys} + \Delta \mathbf{\omega}_{stoc} + \Delta \mathbf{\kappa}_{sys} \mathbf{\omega}_{OC}); \]
\[ \hat{\mathbf{n}}_O = \mathbf{n}_{OC} - (\Delta \mathbf{n}_{sys} + \Delta \mathbf{n}_{stoc} + \Delta \mathbf{\kappa}_{sys} \mathbf{n}_{OC}); \]
\[ \hat{\mathbf{R}}_{KOC} = \mathbf{R}_{KOC} - (\Delta \hat{\mathbf{R}}_{DSyst} + \Delta \hat{\mathbf{R}}_{DSyst} + \Delta \mathbf{k}_{DS} \mathbf{R}_{KOC}); \]
\[ \hat{\mathbf{U}} = \mathbf{U}_c + 2 \mathbf{U} \times \Delta \hat{\mathbf{R}} - \mathbf{A}_{OE} (\Delta \mathbf{n}_{sys} + \Delta \mathbf{n}_{stoc} + \Delta \mathbf{\kappa}_{sys} \mathbf{n}_{OC}) + \]
\[ + \hat{\mathbf{\theta}} \times (A_{OE}^T \times \hat{\mathbf{n}}_O) + \omega_\theta^2 (\Delta \hat{\mathbf{R}} - 3(\Delta \hat{\mathbf{R}} \cdot \mathbf{1}_R) \mathbf{1}_R), \]

where \( \hat{\mathbf{X}} \) – estimate of \( \mathbf{X} \), estimated by OKF; \( K_\theta \) – skew-symmetric matrix composed by the elements of \( \hat{\mathbf{\theta}} \); \( \mathbf{E} \) – unit matrix.

Hence, using the corrected \( A_{OE} \) of mems IMU1, as well as the corrected values of \( \mathbf{R}, \mathbf{U}, \hat{\mathbf{U}} \) for mems IMU1 and mems IMU2, it’s easy to calculate the required vibration parameters (vibration displacement, vibration velocity and vibration acceleration) in ACS according to the following equations:

\[ \hat{\mathbf{R}}_{K2O} = \hat{\mathbf{A}}_{O/E}^T (\mathbf{R}_{M2} - \hat{\mathbf{R}}_{M1}); \]
\[ \hat{\mathbf{U}}_{K2O} = \hat{\mathbf{A}}_{O/E}^T (\mathbf{U}_{M2} - \hat{\mathbf{U}}_{M1}); \]
\[ \hat{\mathbf{U}}_{K2O} = \hat{\mathbf{A}}_{O/E}^T (\mathbf{U}_{M2} - \hat{\mathbf{U}}_{M1}). \]

6. Simulation results
To evaluate the expected accuracy characteristics of the proposed system, a simulation of its operation was carried out. The case of an aircraft parked with engines and equipment turned on was considered, there were a random and harmonic linear and angular wing vibrations with amplitudes up to 6 cm and up to 4 degrees, respectively. The errors of GU and A.U. of mems IMU (systematic component and RMS of random component) equal to 0,3 deg/hour, 0,001 m/s², respectively, the error of NS – in the order of 20 m, 0,1 m/s – only systematic. The error of DS (systematic component and RMS of random component) – in the order of 2 cm. The errors of initial alignment in terms of orientation and navigation parameters for mems IMU1,2 – in the order of 10² rad, 10 m, 0,05 m/s. The distance from NS to mems IMU1 – 1 m, and to mems IMU2 – 10 m. Obviously, the obtained results, illustrated in figures. 2 and 3,
are quite satisfactory, since for a 200-250 s period of time the errors estimations are settled at levels of order 1.5 cm and 0.002 m/s, which is quite acceptable for the considered measurement system.

![Figure 2. Estimation errors for the projections of vibration displacement errors and assessing their RMS in ACS.](image)

![Figure 3. Estimation errors for the projections of vibration velocity errors and assessing their RMS in ACS.](image)

7. Conclusion
In this paper, a rational version of hardware components and structure of an aircraft wing vibration parameters measurement system was proposed. This system built on the basis of sensors data fusion technology of mems IMUs, DS and onboard NS using OKF. Its main operational algorithms have been developed, including an algorithm for optimal estimation of the proposed system parameters and its correction, as well as an algorithm for determining the main vibration parameters, including vibration displacement, vibration velocity and vibration acceleration in ACS. The conducted simulation confirmed the operability of the proposed system, and the possibility of achieving acceptable accuracy characteristics to measure airplane wing vibrations.

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