Modelling of magnetic elastomer deformation dependence on magnetic field

D A Solodkiy and L L Afremov

Department of Theoretical and Nuclear Physics, The School of Natural Sciences, Far Eastern Federal University, 8, Sukhanova St., 690950, Vladivostok, Russian Federation

E-mail: dm_solodkiy@mail.ru, afremov.ll@dvfu.ru

Abstract. The model of magnetic deformable system was developed. The modelling of magnetic elastomer deformation and magnetization dependence on external magnetic field was calculated within the method of movable cellular automata (MCA). It was shown that ordering of magnetic particles substantially effects on elastomer saturation deformation. The maximum orderly elastomer saturation deformation is twice higher the saturation deformation of system with chaotic ordering of magnetic particles.

1. Introduction
Magnetic elastomers are materials that mechanic properties are depended on external magnetic field. They are non-magnetic environment (matrix) including magnetic micro- and nanoparticles.

Magnetic elastomers can be used in engineering field due to their specific properties. For example, they can be used as the framework for synthesis of managed elements that controlling adsorption of impacts. Thus magnetic elastomers are used as adaptive stiffness elements in vibration absorbers [1]. Besides magnetic elastomers are applied in biomedicine [2,3], microelectronics and electrical engineering [4].

This work is aimed to model external magnetic field influence on magnetic elastomer magnetization and deformation.

The modelling of composite elastomers physical properties was calculated within the method of movable cellular automata (MCA).

2. Magnetic elastomer model
Magnetic elastomer is N interacting with each other elements (spherical particles); the particles size is much lower system size. Each system element is movable cellular automata [5,6]. Automata is homogenous system, thus the center of automata mass coincides with its geometrical center. Besides particle-automata describes by set of the physical parameters (density, elastic and shear modules, saturation magnetization and coercive force) that characterize real environments. Some groups of particles can be under special conditions, for example, surface layer (group of particles) is rigidly fixed in initial position while another group is being effected by magnetic or external forces. This approach allows to describe electromagnetic and mechanic properties of composite materials quite simply.

- The state of the automata \( i \) is set by its coordinates \( r^i \) and velocity \( v^i \).
The communication of pair adjacent automata is determined by overlap parameter \( h^{ij} = r^{ij} - r_0^{ij} \), where \( r^{ij} = |r^i - r^j| \) and \( r_0^{ij} = (d^i + d^j)/2 \) are current and equilibrium distances between automata \( i \) and \( j \), respectively. Where \( d^i \) is the diameter of the \( i \)-th automata. If \( h^{ij} \leq h_{\text{max}} \), the pair of automata is interconnected, otherwise \( (h^{ij} > h_{\text{max}}) \) automata are unconnected. Moreover, sign of \( h^{ij} \) determines sign of the interaction force between automata.

It’s supposed that \( N_m \) automata are magnetic and single domain particles (from total \( N \) automata).

The dynamics of automata is describing by differential equations of translational and rotational movements:

\[
\begin{align*}
    m^i \frac{d^2 r^i}{dt^2} &= \mathbf{F}^i, \\
    J^i \frac{d^2 \theta^i}{dt^2} &= \mathbf{K}^i,
\end{align*}
\]

(1)

where \( m^i, J^i, r^i, \theta^i \) are mass, moment of inertia, linear and angular coordinates of automata \( i \), respectively, \( \mathbf{F}^i \) is resulting force acting on automata \( i \), \( \mathbf{K}^i \) is moment of resulting force. The resulting force \( \mathbf{F}^i \) is set by superposition of short-range mechanical forces acting from adjacent automata \( \sum_j \mathbf{F}_{ij}^m \), long-range magnetic forces \( \mathbf{F}_{\text{magn}}^i \) and external forces \( \mathbf{F}_{\text{ex}}^i \): \( \mathbf{F}^i = \sum_j \mathbf{F}_{ij}^m + \mathbf{F}_{\text{magn}}^i + \mathbf{F}_{\text{ex}}^i \). In its turn force \( \mathbf{F}_{ij}^m \) is set by superposition of central force \( \mathbf{P}_{ij} \) and viscous force \( \mathbf{F}_{ij}^v \).

3. Algorithms of deformation and magnetization calculation

- Numerical integration of differential equations system (1) has been calculated by velocity Verlet algorithm:

\[
\begin{align*}
    r^i_{n+1} &= r^i_n + v^i_n \Delta t + \frac{\mathbf{F}^i_n}{2m^i} \Delta t^2, \\
    v^i_{n+1} &= v^i_n + \frac{\mathbf{F}^i_n + \mathbf{F}^i_{n+1}}{2m^i} \Delta t, \\
    \theta^i_{n+1} &= \theta^i_n + \omega^i_n \Delta t + \frac{K^i_n}{2J^i} \Delta t^2, \\
    \omega^i_{n+1} &= \omega^i_n + \frac{K^i_n + K^i_{n+1}}{2J^i} \Delta t,
\end{align*}
\]

(2)

where step size (time step) \( \Delta t = d/5c_p \) is determined by automata size \( d \) and sound velocity \( c_p \). With next timestep (iteration) automata change their states synchronically, depending on their and adjacent automata states on the previous timestep. It can be accompanied by changes automata relations, position and orientation in space under the influence of external and internal forces, and also change of automata magnetic moments, depending on the resulting magnetic field in the sample.

- According to spherical form of magnetic automata, their magnetic field are dipole ones. Thus magnetic automata \( i \) is influenced by external field \( \mathbf{H}_0 \) and, in addition, total field of other magnetic particles-automata:

\[
\mathbf{H}(r^i) = \mathbf{H}_0 + \sum_{j \neq i} \left( -\frac{\mathbf{r}^i}{(r^{ij})^3} + \frac{3(\mathbf{r}^i, r^{ij})r^{ij}}{(r^{ij})^5} \right),
\]

(3)

so acting on automata \( i \) magnetic force \( \mathbf{F}_{\text{magn}}^i = \nabla \left( \mathbf{H} (r^i) \right) \):

\[
\mathbf{F}_{\text{magn}}^i = \sum_{j \neq i \neq 1}^{N_m} \frac{3}{r_{ij}^5} \left( -\frac{\mathbf{r}^i, r^{ij}}{r_{ij}} \mathbf{r}^i + \frac{3(r^{ij})^2}{r_{ij}^3} \mathbf{r}^{ij} + \frac{3}{r_{ij}^3} (\mathbf{r}^i, r^{ij}) \mathbf{r}^{ij} \right).
\]

(4)
• For calculation of magnetic automata interaction force (4) fields of interaction (3) are recalculated on every integration step. Fields of interaction effect on magnetic moments orientation. However calculation of magnetic moments distribution in field $H(r^i)$ is non-trivial independent problem, in this work it is not considered. Thus it is supposed that
  1) automata magnetic moments can be oriented only along or against certain axis 0y, i.e. automata magnetic moment: $\mu^i = \{0, \mu^y_i, 0\}$;
  2) the magnetic particle, which chosen from the bunch of total magnetic particles, has maximum value of resulting magnetic field module;
  3) if magnetic particle field is more than critical one ($|H(r^i)|_{\text{max}} > h_{cr}$) and magnetic moment of particle $\mu^i$ is directed against the field, then $\mu^i$ will be oriented along the field.

4. Modelling results

Modelling was based on system corresponding environment consisting on 2500 particles-automata (including single domain ferrum particles) with elastic module equals $2.04*10^9 \text{ dyn/cm}^2$ and Poisson’s ratio equals 0.44.

The magnetic elastomer magnetization and deformation dependences on external magnetic field are shown in figure 1 (a) and (b), respectively (inset figures show chaotic distribution of magnetic particles).

![Magnetic elastomer reduced magnetization $M/\mu$ (a) and deformation $\varepsilon$ (b) dependences on external magnetic field. The model is 2500 automata with 20% single domain ferrum particles. Inset figures show non-magnetic particles-automata (clear) and particles-automata with directed oppositely magnetic moments (colored).](image)

Figure 2 shows saturation deformation of composite dependence on number of magnetic automata and magnetic particles distribution. It is seen that saturation deformation of composite with ordered distribution of particles is much more deformation in system with random distribution.

Thus distribution of magnetic particles in material has great effect on its elastic properties.
Fig. 2. Saturation deformation $\varepsilon$ dependence on volume fraction magnetic automata $\omega$ and distribution of magnetic particles. Red curve corresponds to ordered distribution, black curve corresponds to random one. The upper inset figure shows ordered distribution of non-magnetic particles-automata (clear) and particles with oppositely directed magnetic moments (colored). The bottom inset figure shows chaotic distribution of particles-automata.

5. Acknowledgments
The work was supported by financial support in the framework of state task 3.7383.2017/8.9

References
[1] Deng H and Gong X 2007 Journal of intelligent material systems and structures 18 1205-1210
[2] Falk M H and Issels R D 2001 Int J Hyperthermia 17 (1) 1–18
[3] Glöckl G, Hergt R, Zeisberger M, Dutz S, Nagel S and Weitschies W 2006 J Phys Condens Matter 18 (38) 2935–2949
[4] Zhou G and Wang Q 2005 Smart Materials and Structures 14 (4) 1154
[5] Psakhie S G, Ostermeyer G P, Dmitriev A I, Shilko E V, Smolin A Yu and Korostelev S Y 2000 Method of movable cellular automata as a new trend of discrete computational mechanics. I. Theoretical description Physical Mesomechanics 3 5–12
[6] Psakhie S G, Horie Y, Ostermeyer G P, Korostelev S Yu and Smolin A Yu 2001 Movable cellular automata method for simulating materials with mesostructure Theoretical and Applied Fracture Mechanics 37 311-334