Complex Korteweg-de Vries equation and Nonlinear dust-acoustic waves in a magnetoplasma with a pair of trapped ions

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The nonlinear propagation of dust-acoustic (DA) waves in a magnetized dusty plasma with a pair of trapped ions is investigated. Starting from a set of hydrodynamic equations for massive dust fluids as well as kinetic Vlasov equations for ions, and applying the reductive perturbation technique, a Korteweg-de Vries (KdV)-like equation with a complex coefficient of nonlinearity is derived, which governs the evolution of small-amplitude DA waves in plasmas. The complex coefficient arises due to vortex-like distributions of both positive and negative ions. An analytical as well as numerical solution of the KdV equation are obtained and analyzed with the effects of external magnetic field, the dust pressure as well as different mass and temperatures of positive and negative ions.

I. INTRODUCTION

Recently, there has been a renewed interest in investigating electrostatic disturbances in pair-plasmas and, in particular, plasmas with a pair of ions [1–7]. However, nonthermal pair plasmas may frequently occur not only in semiconductors in the form of electron and ion holes, but also in many astrophysical environments, e.g., pulsars, magnetars, as well as in the early universe, active galactic nuclei and supernova remnants in the form of electrons and positrons [8–11]. On the other hand, a number of experiments have been conducted to create ion holes, but also in many astrophysical environments, e.g., pulsars, magnetars, as well as in the early universe, active galactic nuclei and supernova remnants in the form of electrons and positrons [8–11]. On the other hand, a number of experiments have been conducted to create pair-ion plasmas using fullerene as ion source [12]. Furthermore, it has been observed that the dust particles injected into a pair-ion plasma (e.g., K+/SF6− plasmas) can become positively charged when the number density of negative ions greatly exceeds that of electrons (n_{n0} ∼ 500n_{0}) [13–14]. These pose some possibilities to investigate collective behaviors as well as the formation of nonlinear coherent structures in pair-ion plasmas under controlled conditions. The formation of phase space holes in pure pair-ion plasmas [5] as well as ion holes in dusty pair-ion plasmas [6] in the propagation of large amplitude electrostatic waves have been investigated in the recent past in which ions have been treated as trapped in self-created localized electrostatic potentials as prescribed by Schamel [15].

In this paper we present a theoretical study on the formation and the dynamics of small-amplitude solitary structures in a dusty plasma composed of charged dust particles and a pair of ions without electrons. In our theoretical model the massive charged dusts are described by a set of fluid equations, while the dynamics of both positive and negative ions are governed by kinetic Vlasov equations. Using the reductive perturbation technique we show that the evolution of small-amplitude electrostatic waves can be described by a Korteweg-de Vries (KdV)-like equation with a complex coefficient of the nonlinearity. A stationary as well as numerical solutions of the KdV equation are obtained and analyzed with the effects of external magnetic field, the dust pressure as well as different mass and temperatures of ions.

II. BASIC EQUATIONS

We consider the nonlinear propagation of dust-acoustic (DA) solitary waves in a magnetized dusty plasma which consists of positively or negatively charged mobile dusts and a pair of trapped ions with vortex-like distributions. The dust particles are assumed to have equal mass and constant charge. The collisions of all particles are also neglected compared to the dust plasma period. Furthermore, in dusty pair-ion plasmas the ratio of electric charge to mass of dust particles remains much smaller than those of positive and negative ions. We also assume that the size of the dust grains is small compared to the average interparticle distance. The static magnetic field is considered along the z-axis, i.e., B = B_0 \hat{z}. While the dynamics of massive charged dusts in the propagation of DA waves (v_{td} ≪ v_p ≪ v_{p,n}) where v_{tj} = (\sqrt{k_B T_j/m_j}) is the thermal velocity of j-th species particles and v_p is the phase velocity of the wave) is described by a set of fluid equations (1) and (2), the dynamics of singly charged positive and negative ions are described by the Vlasov equations (3).

\[ \frac{\partial n_d}{\partial t} + \nabla \cdot (n_d v_d) = 0, \]

\[ \frac{\partial v_d}{\partial t} + (v_d \cdot \nabla) v_d = \frac{q_d}{m_d} (E + v_d \times B_0) - \frac{\nabla P}{m_d n_d}, \]

\[ \frac{\partial f_j}{\partial t} + v \cdot \nabla f_j - \frac{q_j}{m_j} \nabla \phi \cdot \frac{\partial f_j}{\partial v} = 0. \]

The system of equations is then closed by the Poisson equation

\[ \nabla \cdot E = 4\pi e (n_p - n_n + \alpha Z_d n_d). \]
In Eqs. (1)-(4), \( q_j, n_j, v_j, f_j \) and \( m_j \) respectively, denote the charge, number density (with its equilibrium value \( n_{j0} \)), velocity, velocity distribution function, and mass of \( j \)-species particles. Also, \( q_d = \alpha z_d e \) with \( \alpha = \pm 1 \) denoting for positively/negatively charged dusts and \( z_d \) the charge state. Also, \( \mathbf{E} = -\nabla \phi \) is the electric field with \( \phi \) denoting the electrostatic potential and \( P \) is the dust thermal pressure given by the adiabatic equation of state \( P/P_0 = (n_d/n_{d0})^{\gamma} \). Here, \( \gamma = 5/3 \) is the adiabatic index for three-dimensional configuration and \( P_0 = n_{d0} k_B T_d \) is the equilibrium dust pressure with \( k_B \) denoting the Boltzmann constant and \( T_d \) the thermodynamic temperature of \( j \)-species particles. Furthermore, the ion densities are given by

\[
n_j = \int_{-\infty}^{\infty} f_j dv. \tag{5}
\]

In what follows, we recast Eqs. (1)-(4) in terms of dimensionless variables. To this end the physical quantities are normalized according to \( n_j \to n_j/n_{j0}, (v_j, v_d) \to (v_j, v_d)/c_d, \phi \to e\phi/k_B T_0 \), with \( e \) denoting the elementary charge, \( f_j \to f_j v_j n_j/n_{j0} \), where \( c_d = \sqrt{z_d k_B T_d/m_d} = \omega_{pd} / \lambda_D \) is the dust-thermal speed with \( \omega_{pd} = \sqrt{4\pi n_{d0} e^2 / m_d} \) and \( \lambda_D = \sqrt{k_B T_d / 4\pi n_{d0} e^2} \) denoting, respectively, the dust thermal speed and the plasma Debye length. The space and time variables are normalized by \( \lambda_D \) and \( \omega_{pd}^{-1} \) respectively. Thus, from Eqs. (1)-(5) we have following set of normalized equations.

\[
\frac{\partial \eta_d}{\partial t} + \nabla \cdot (n_d v_d) = 0, \tag{6}
\]

\[
\frac{dv_d}{dt} + \alpha \nabla \phi = \alpha \omega_c v_d \times \hat{z} - \frac{5}{3} \sigma_d n_d^{-1/3} \nabla \eta_d, \tag{7}
\]

\[
\delta \frac{\partial f_j}{\partial t} + v_j \cdot \nabla f_j - \zeta_j m_j \nabla \phi \cdot \frac{\partial f_j}{\partial v} = 0, \tag{8}
\]

\[
\nabla^2 \phi = \mu_n n_n - \mu_p n_p - \alpha n_d, \tag{9}
\]

\[
n_j = \int_{-\infty}^{\infty} f_j dv, \tag{10}
\]

where \( d/dt \equiv \partial/\partial t + v_d \cdot \nabla, \alpha = \pm 1 \) for positively/negatively charged dusts, \( \omega_c = |q_d| B_0 / m_d c_d \) is the dust-cyclotron frequency normalized by the dust plasma frequency, \( \sigma_d \equiv T_d / T_p z_d, \delta = \sqrt{z_d m_p / m_d}, \zeta_j = \pm 1 \) for positive/negative ions and \( \mu_j = n_{j0} / Z_j n_0 \) are the density ratios \( (j = p, n) \) which satisfy the following charge neutrality condition at equilibrium:

\[
\mu_p + \alpha = \mu_n. \tag{11}
\]

We neglect the ion inertial effects compared to the charged dusts, i.e., \( \delta \to 0 \) in Eq. (8). The distribution functions \( f_j \) for positive and negative ions, which are constant of motion of the Vlasov Eq. (3), are chosen for the excitation of localized solitary waves so that (i) they are continuous, and both the free particle distributions are Maxwellian distribution where \( \phi \to 0 \) at \( |\xi| \to \pm \infty \) and trapped particles are absent, (ii) both trapped particle distributions are Maxwellian (with also negative temperatures). Thus, \( f_j \) (for free and trapped particles) are (with a suitable choice of the normalization constants) \( \Phi \) for positive ions

\[
f_{pf}(v) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( v^2 + 2\phi \right) \right], \quad |v| > \sqrt{-2\phi}, \tag{12}
\]

and for negative ions

\[
f_{nf}(v) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( v^2 - 2\phi / m \right) \right], \quad |v| \leq \sqrt{2\phi / m}, \tag{15}\]

where \( m (= m_n / m_p \gtrsim 1) \) is the mass ratio, \( \sigma (= T_p / T_n \gtrsim 1) \) is the temperature ratio and \( \sigma_j \), for \( j = p, n \), measure the inverse of the trapped positive and negative ion temperatures which may be negative \( (\sigma_j < 0) \) corresponding to a depression in the trapped particle distribution. The case of \( \sigma_j \to 0 \) represents the plateau (constant or flat-topped) and \( \sigma_j \to 1 \) corresponds to the Boltzmann distribution of ions. Next, integrating the particle distribution functions (12)-(15) over the velocity space, i.e., using Eq. (10) we obtain the number densities \( n_j \) for positive and negative ions as

\[
n_p(\phi) = I(-\phi) + \frac{1}{\sqrt{|\sigma_p|}} \left\{ e^{-\sigma_p \phi} \operatorname{erf} \left( \frac{\phi}{\sqrt{\sigma_p \phi}} \right), \quad \sigma_p \geq 0, \right. \tag{16} \]

\[
n_n(\phi) = I(\sigma \phi) + \frac{1}{\sqrt{|\sigma_n|}} \left\{ e^{\sigma_n \phi} \operatorname{erf} \left( \frac{\phi}{\sqrt{-\sigma_n \phi}} \right), \quad \sigma_n \geq 0, \tag{17}\right.
\]

where \( I(x) = \exp(x) \left[ 1 - \operatorname{erf}(\sqrt{x}) \right] \). The error and Dawson functions \( \operatorname{erf}(x) \) and \( W(x) \) are, respectively, given by

\[
erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad W(x) = e^{-x^2} \int_0^x e^{t^2} dt. \tag{18}\]

In the small amplitude limit \( \phi \ll 1 \), so that \( \sigma \phi \ll 1 \), we obtain from Eqs. (16) and (17) the following expressions for the number densities \( n_p, n_n \) for positive and negative ions.

\[
n_p \approx 1 - \phi - \frac{4(1 - \sigma_p)}{3\sqrt{\pi}} (\phi)^{3/2} + \frac{1}{2} \phi^2, \tag{19}\]

\[
n_n \approx 1 + (\sigma \phi) - \frac{4(1 - \sigma_n)}{3\sqrt{\pi}} (\phi)^{3/2} + \frac{1}{2} (\phi)^2. \tag{20}\]
FIG. 1. Profiles of $|\Phi|$ given by Eq. (33) are shown with respect to $\xi$ (and with $\sigma \sim 1$) for different values of the plasma parameters as in the figure. The fixed parameter values for the subplots (a) to (d), respectively, are (a) $\alpha = 1$, $\mu_p = 0.2$, $l_z = 0.1$, $\omega_c = 0.5$, $\sigma_d = 0.06$ and $u_0 = 0.1$, (b) $\alpha = 1$, $\mu_p = 0.2$, $l_z = 0.1$, $\sigma_p = \sigma_n = 0.3$, $\sigma_d = 0.06$ and $u_0 = 0.1$, (c) $\alpha = 1$, $\mu_p = 0.2$, $l_z = 0.1$, $\sigma_p = \sigma_n = 0.3$, $\omega_c = 0.5$, $\sigma_d = 0.06$ and $u_0 = 0.1$, and (d) $\alpha = 1$, $\mu_p = 0.2$, $\sigma_p = \sigma_n = 0.3$, $\omega_c = 0.5$, $\sigma_d = 0.06$ and $u_0 = 0.1$.

FIG. 2. The space-time evolution of the soliton profile $|\Phi|$ [numerical solution of Eq. (30)] is shown at $\tau = 100$ [Subplots (a) and (b)] and $\tau = 160$ [Subplots (c) and (d)]. While the plots (a) and (c) show the soliton profiles with space, plots (b) and (d) are the corresponding contour plots. The parameter values are the same as for the dashed line in Fig. 1(d) and $\sigma \sim 1$.

### III. EVOLUTION EQUATION

In order to derive the evolution equation for the DA waves, we transform the space and time variables according to [16]

$$
\xi = \epsilon^{1/4} (l_x x + l_y y + l_z z - Mt), \quad \tau = \epsilon^{3/4} t,
$$

where $\epsilon$ is a small parameter measuring the strength of nonlinearity. The dependent variables are expanded as
ties order (\(\epsilon\)) that the dust gyromotion is a higher-order effect than the motion parallel to the magnetic field. Next, we substitute Eqs. \((21)\) and \((22)\) into Eqs. \((6)-(9)\), i.e., assuming the perturbations as oscillations with the wave frequency \(\omega\) and the wave number \(k\). We find that the phase speed \(M\) (normalized by the DIA speed \(c_d\)) can be larger or smaller than the unity depending on the choice of the parameter values. The value of \(M\) increases with increasing values of both \(l_z\) and \(\sigma_d\). However, its values can slowly decrease with increasing values of the density ratios \(\mu_j\) as well as the temperature ratio \(\sigma\). From Eq. \((6)\), collecting the coefficients of \(\epsilon^{7/4}\) we obtain

\[
M \frac{\partial n^{(2)}}{\partial \xi} = \frac{\partial n^{(1)}}{\partial \tau} + \sum_{j=x,y,z} l_j \frac{\partial v^{(2)}_{x,j}}{\partial \xi}. \tag{26}
\]

Similarly, equating the coefficients of \(\epsilon^{3/2}\) from the \(x\)- and \(y\)-components of Eq. \((7)\), and the coefficients of \(\epsilon^{7/4}\) from the \(z\)-component of Eq. \((7)\) we successively obtain

\[
\alpha v^{(2)}_{x,y} = \pm \frac{M}{\omega_c} \frac{\partial v^{(1)}_{y,x}}{\partial \xi} = \left[ M \frac{l_{x,y}}{\omega_c} + \frac{5}{3} \sigma_d (\mu_p + \sigma \mu_n) \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \right], \tag{27}
\]

\[
M \frac{\partial v^{(2)}_{z}}{\partial \xi} = \frac{\partial v^{(1)}_{z}}{\partial \tau} + \left( \alpha \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{5}{3} \sigma_d \frac{\partial n^{(1)}}{\partial \xi} \right). \tag{28}
\]

From the coefficients of \(\epsilon^{3/2}\) of Eq. \((9)\), we obtain an equation in which \(n^{(2)}\) is eliminated by the use of Eqs. \((26), (27)\) and \((28)\), and the coefficient of \(\phi^{(2)}\) vanishes by Eq. \((25)\). Thus, arranging the terms and using Eq. \((23)\) one obtains the following KdV-like equation

\[
\frac{\partial \Phi}{\partial \tau} + \left( A_p \sqrt{-\Phi} + A_n \sqrt{\Phi} \right) \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = 0, \tag{29}
\]

where \(\Phi \equiv \phi^{(1)}\). It follows that Eq. \((29)\) has a complex solution for \(\Phi\). Typically, for \(\Phi \sim r \exp(i \theta)\), where \(r\) is a real function of \(\xi\) and \(\tau\), and \(\theta\) is a constant, one can have \(\sqrt{-\Phi} = i \sqrt{\Phi}\). Thus, Eq. \((29)\) can be written as

\[
\frac{\partial \Phi}{\partial \tau} + A \sqrt{\Phi} \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = 0, \tag{30}
\]

where the coefficients of nonlinearity \((A = A_n + i A_p)\) and dispersion \((B)\) are given by

\[
A_j = \frac{\alpha}{\sqrt{\pi}} \frac{(1 - \sigma_j) \mu_j}{T_j} \left( \frac{T_p}{T_j} \right)^{3/2}, \tag{31}
\]

\[
B = \frac{l_z^2}{2M} \left[ 1 + \frac{M^4 (1 - l_z^2)}{\omega_c^2 l_z^2} (\mu_p + \sigma \mu_n)^2 \right]. \tag{32}
\]

The nonlinear coefficient \(A\) becomes complex due to vortex-like distributions of two oppositely charged particles. In absence of one of them, \(A\) becomes real, and one can then obtain solitary waves with positive or negative potential. A stationary soliton solution of Eq. \((30)\) can easily be obtained with its absolute value as \((\text{For details see Appendix A})\)

\[
|\Phi| = \Phi_0 \sech^4 \left[ (\xi - u_0 \tau) / W \right], \tag{33}
\]

where \(u_0\) is a constant, and \(\Phi_0 = (15 u_0 / 8 |A|)^2\) and \(W = \sqrt{16B/\Phi_0}\) are the amplitude and width of the soliton respectively.

**IV. RESULTS AND DISCUSSION**

We numerically analyze the solution \((33)\) with different plasma parameters as shown in Fig. \((1)\). Since \(\sigma_j (j = p, n)\) represents the reciprocal temperature of the trapped positive and negative ions, and can be allowed from their negative to positive values corresponding to different trapped particle distributions, we consider negative, zero as well as positive values of \(\sigma_j\).

From Fig. \((1a)\), it is seen that as \(\sigma_j\) increases from \(\sigma_j = 0\) (corresponding to a constant or flat-topped distribution of ions) to \(\sigma_j \sim 1\) (corresponding to the Boltzmann distributions of ions), both the amplitude and width of the soliton increase (See the solid and dashed lines). Note here that the values of \(\sigma_j > 1\), for which the influence of the trapped ions are inverted, may be physically unrealistic as those correspond to a more steepened wave which can become unstable due to more peaked bump of the ion distributions. However, as the absolute
value of $A$ starts increasing for $\sigma_j < 0$, which corresponds
to a depression in the trapped particle distribution, both
the amplitude and width of the soliton are reduced (See
the dotted line). The same can further be enhanced for
values of $\sigma_j$ satisfying $\sigma_p\sigma_n < 0$ (See the dash-dotted
line).

Figure 1(b) shows the soliton profile with the influence
of the external magnetic field. Since $\omega_c$ contributes
only to the dispersive coefficient $B$ of Eq. (30), the
effect of the magnetic field with increasing its intensity is
to reduce the width (without changing the amplitude)
of the soliton. Thus, the external magnetic field makes
the solitary structure more spiky. However, for stronger
magnetic fields with $\omega_c \gg 1$, the width remains almost
unaltered as in this case $B \sim l_z^2/2M$.

The thermal effects of charged dusts are shown in Fig.
1(c). It is found that the effect of the dust thermal pressure
$\sigma_d$ is to enhance both the amplitude and width of
the soliton. The enhancement is due to the fact that as $\sigma_d$
increases, the values of $|A|$ ($B$) decrease (increase),
and hence the increase in both the amplitude and width.
However, an opposite trend occurs by the effects of the
positive to negative ion temperature ratio $\sigma$ (not shown
in the figure). Typically, it reduces both the soliton am-
plitude and width significantly with a small increment of
its value.

Figure 1(d) exhibits the effects of the obliqueness of
propagation $l_z$ and the relative (to dusts) concentration
of positive ions $\mu_p$. We find that both the amplitude
and width of the soliton are greatly enhanced by a small
increment of $l_z$ [Since $A_j$ ($B$) is inversely (directly) pro-
portional to $l_z$]. However, as the positive ion concentra-
tion increases, the amplitude gets reduced but the width
increases.

Next, we numerically solve Eq. (30) by the Runge-
Kutta scheme with an initial condition of the form $\Phi(\xi) =
0.05 \text{sech}^4(\xi/10) \exp(-i\xi/15)$ and time step $\Delta \tau = 0.001$.
The development of the wave form $\Phi$ after a finite in-
terval of time is shown in Fig. 2. The parameter values
are considered as the same as for the dashed line in Fig.
1(d). It is seen that the leading part of the initial wave
steepens due to positive nonlinearity. As the time goes
on the pulse separates into solitons and a residue due to
the wave dispersion [See the subplots (a) and (b)]. It is
found that once the solitons are formed and separated,
they propagate in the forward direction without chang-
ing their shape due to the nice balance of the nonlinearity
and dispersion [See the subplots (c) and (d)].

V. CONCLUSION

We have investigated the nonlinear propagation of
dust-acoustic waves in a magnetized plasma which con-
sists of warm positively charged dusts and a pair of free,
as well as, trapped ions. We have shown that the evolu-
tion of small-amplitude DA waves can be described by
a KdV-type equation with a complex coefficient of the

nonlinearity. Such complex coefficient appears due to
t vortex-like distributions of both the ion species. The
KdV equation is solved both analytically and numeri-
cally. The properties of the absolute value of $\Phi$ are only
exhibited graphically. It is shown that while the external
magnetic field only influences the width of the soliton,
the trapped ion temperatures, the thermal pressures of
ions and dusts, the relative concentration of positive ions
as well as the obliqueness of propagation have significant
effects on both the amplitude and width of the solitons.
We stress that other solutions [19–22] than those pre-

tended here of the complex KdV equation could be of interest
but beyond the scope of the present work. To conclude,
the present results should be useful in understanding
the nonlinear features of electrostatic localized disturbances
in laboratory and space plasmas.

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Appendix A: Stationary solution of the KdV-like
equation

Equation (30) is recast as
\[
\frac{\partial \Phi}{\partial \tau} + A\sqrt{\Phi} \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = 0. \quad (A1)
\]

Next, we apply the transformation $\eta = \xi - u_0 \tau$ to obtain
from Eq. (A1)
\[
\frac{d}{d\eta} \left( B\Phi - u_0 \Phi + \frac{2}{3} A\Phi^{3/2} \right) = 0, \quad (A2)
\]

where the dot denotes differentiation with respect to $\eta$.
Integrating Eq. (A2) with respect to $\eta$ and using the
boundary conditions $\Phi, \dot{\Phi} \rightarrow 0$ as $\xi \rightarrow \pm \infty$ we get
\[
B\Phi - u_0 \Phi + \frac{2}{3} A\Phi^{3/2} = 0. \quad (A3)
\]

Multiplying Eq. (A3) by $2\dot{\Phi}$ and integrating once with
respect to $\eta$, we obtain
\[
B\dot{\Phi}^2 - u_0 \Phi^2 + \frac{8}{15} A\Phi^{5/2} = 0, \quad (A4)
\]

where we have used the boundary conditions $\Phi, \dot{\Phi} \rightarrow 0$.
From Eq. (A1) we have
\[
\dot{\Phi} = \pm \Phi \sqrt{\frac{u_0}{B} - \frac{8A}{15B} \Phi}, \quad (A5)
\]
or, \[ \int \frac{d\Phi}{\sqrt{u_0/B - (8A/15B) \sqrt{\Phi}}} = \pm \int d\eta, \] \hspace{1cm} (A6)

which gives \((a = u_0/B \) and \( b = 8A/15B)\)

\[ -4 \sqrt{a} \tanh^{-1} \sqrt{\frac{a - b \sqrt{\Phi}}{a}} = \pm \eta, \] \hspace{1cm} (A7)

or, \( \sqrt{\frac{a - b \sqrt{\Phi}}{a}} = \mp \tanh \left( \sqrt{\frac{a}{4}} \eta \right). \) \hspace{1cm} (A8)

Thus, we obtain a soliton solution of Eq. (30) as

\[ 1 - \tanh^2 \left( \sqrt{\frac{\sqrt{a}}{4}} \eta \right) = \frac{b}{a} \sqrt{\Phi}, \] \hspace{1cm} (A9)

or, \( \Phi = \left( \frac{15u_0}{8A} \right)^2 \text{sech}^4 \left( \sqrt{\frac{u_0}{16B}} \eta \right). \) \hspace{1cm} (A10)

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