Energy Differential Structure and Exchange of A Micro Flux Increment of Charged Particles in Longitudinal Acceleration

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Differential energy structure of a micro multi-charged-particle system and the beam internal potential energy is derived with consequent property and necessary inference. Then by combining the energy differential structure with differential transfer relations of physical flux density between different time domains, we present a general explicit formula for energy exchange in the beam element’s transfer or accelerating process between source and observer, further give an analysis on relativistic beam’s mass-energy relation and beam’s energy up limits for resonant frequency (rf) and DC acceleration. Finally we discuss the property of elementary charge density field and consequences.

I. INTRODUCTION

Tracing the energy gain process of high energy electrons beam through rf acceleration, it would be known that the longitudinally internal charge density field’s affection or contribution to the total energy of the group of electrons should be considered as the electron number density in longitudinal axis increases progressively due to accumulated compression, especially when its average velocity of the group of electrons becomes almost independent to the energy after the velocity approaching light speed. However, their functions to energy flux density basically are neglected except treating them for beam instability and impedance [1, 2, 3, 4]. From general mechanics, the multi-particles system’s energy in complete inertial frame can be divided into two essential parts: mass center’s kinetic energy and system’s internal energy; the latter could be further divided into two constituents: relative mass center’s kinetic energy and internal potential energy whose rate equals the rate of work done by internal forces negatively. Upon number density’s instantaneous distribution in longitudinal axis and corresponding velocity distribution of the micro particle system, we set up the explicit formulas on mass-center kinetic energy, relative mass center kinetic energy, internal electrical potential energy and its changing rate. Then based on differential transfer relation of energy flux [5] and the energy differential structure, we analyze and discuss the affection and contribution of the internal charge field to energy flux and energy exchange as well as other consequences.

II. ESSENTIAL INTERACTIVE MANNER AND ENERGY CONSTITUENTS OF MULTI-PARTICLES SYSTEM

A. Interaction and Power Exchange in A complete Inertial Frame

In a complete inertial frame, where both \(\vec{f}_{neti}(\dot{t}) = \vec{f}_{oi}(\dot{t}) + \vec{f}_{ii}(\dot{t})\) and \(\vec{f}(\dot{t}) \cdot d\vec{r}(\dot{t}) = dE(\dot{t})\) exist simultaneously, and for a multi-particle system consisted of \(\Delta N\) particles, there is the system’s power exchange relation:

\[
\sum \left[ \vec{f}_{oi}(\dot{t}) + \vec{f}_{ii}(\dot{t}) \right] \cdot \dot{\vec{r}}_i(\dot{t}) = \vec{r}_{mc}(\dot{t}) \cdot \sum \vec{f}_{oi}(\dot{t}) + \sum \vec{f}_{oi}(\dot{t}) \cdot \dot{\vec{r}}_{mc}(\dot{t}) + \sum \vec{f}_{ii}(\dot{t}) \cdot \dot{\vec{r}}_{mc}(\dot{t})
\]

\[
\sum \left[ \vec{f}_{oi}(\dot{t}) + \vec{f}_{ii}(\dot{t}) \right] \cdot \dot{\vec{r}}_i(\dot{t}) = \sum m_i \cdot \dot{\vec{r}}_i(\dot{t}) \cdot \vec{r}_i(\dot{t}) = \frac{d}{dt} \left[ \sum \frac{m_i}{2} v_{mc}^2(\dot{t}) \right] + \frac{d}{dt} \left[ \sum \frac{m_i}{2} v_{mc}^2(\dot{t}) \right]
\]
III. ENERGY DIFFERENTIAL STRUCTURE OF MICRO MULTI-CHARGED- PARTICLES SYSTEM

A. Kinetic Energy

Ref. Fig. (1): longitudinal distribution of number density and corresponding velocity.

\[ \rho_S(t, r) = \rho_i(t) + \frac{\partial \rho_S}{\partial r}(r - r_i) \quad (4) \]

and

\[ \int_{r_s}^{r_h} \rho_S(r)dr = \Delta \ell (\rho_i + \frac{1}{2} \frac{\partial \rho}{\partial r} \Delta \ell) = \Delta \ell \dot{\rho}_S = \Delta N \quad (4) \]

\[ v(t, r) = v_i(t) + \frac{\partial v}{\partial r}(r - r_i) \quad (5) \]
From Eq. 4 and 4', there

\[ \rho_m(t, r) = m_q \rho_N(t, r) \]

\[ \int_{r_s}^{r_h} \rho_m(r) dr = m_q \Delta \ell \bar{\rho}_N = \Delta \ell \bar{\rho}_m = m_q \Delta N = \Delta M \]  \hspace{1cm} (6)

\[ \rho_q(t, r) = q \rho_N(t, r) \]

\[ \int_{r_s}^{r_h} \rho_q(r) dr = q \Delta \ell \bar{\rho}_N = q \Delta N = \Delta Q \]  \hspace{1cm} (7)

For \( \int_{r_s}^{r_h} (r - r_{mc}) dm(r) = 0 \)

\[ r_{mc} - r_t - \frac{\Delta \ell}{2} = r_{mc} - r_h + \frac{\Delta \ell}{2} = \frac{\Delta \ell^3}{12 \Delta N} \frac{\partial \rho_N}{\partial r} \]  \hspace{1cm} (8)

\[ v_{mc} - v_t - \frac{\Delta \dot{\ell}}{2} = v_{mc} - v_h + \frac{\Delta \dot{\ell}}{2} = \frac{\Delta \ell^2}{4 \Delta N} \frac{\partial \rho_N}{\partial r} \Delta \ell + \frac{\Delta \ell^3}{12 \Delta N} \frac{\partial^2 \rho_N}{\partial \dot{r} \partial r} \]  \hspace{1cm} (9)

and for \( \int_{r_s}^{r_h} [v(r) - v_{mc}(r_{mc})] dm(r) = 0 \) to reduce Eq. 5,

\[ v_{mc} - v_t - \frac{\Delta \dot{\ell}}{2} = v_{mc} - v_h + \frac{\Delta \dot{\ell}}{2} = \frac{\Delta \dot{\ell}}{12 \Delta N} \frac{\partial \rho_N}{\partial r} \Delta \ell^2 = \frac{\rho_h - \rho_t}{12 \bar{\rho}_N} \Delta \dot{\ell} \]  \hspace{1cm} (9')
\[
\frac{\Delta N m q v_m^2}{2} = \frac{\Delta M}{2} \left( v_h - \frac{\Delta \dot{\ell}}{2} + \frac{\Delta \dot{\ell} \Delta \ell^2}{12 \Delta N} \frac{\partial \rho}{\partial r} \right)^2
\] (10)

\[
\sum \frac{m q}{2} v_{imec}(t) = \int_{r_t}^{r_h} dm(r) [v(r) - v_{mc}]^2
\]

\[
= \frac{m q}{2} \left[ \Delta N (v_{mc} - v_h)^2 - (v_{mc} - v_t) \left( \Delta N + \frac{1}{6} \frac{\partial \rho_N}{\partial r} \Delta \ell^2 + \frac{1}{3} \left( \Delta N + \frac{1}{4} \frac{\partial \rho_N}{\partial r} \Delta \ell^2 \right) \Delta \ell^2 \right) \right]
\]

\[
= \frac{\Delta M}{24} \Delta \ell^2 \left[ 1 - \frac{1}{12 \rho_N} \left( \frac{\partial \rho_N}{\partial \ell} \right)^2 \right]
\] (11)

Eq. (10) and (11) is kinetic energy formulas that contain the information of the micro particles system. Eq. (11) is kinetic energy spread of the micro system.

B. Internal Potential Energy and Its Property

Along axis we assume average repulsive charge field force is proportion to the difference of compressed charge density and uncompressed charge density; or critical charge density \( \rho_{q0} \); beam transverse radius keep in constant.

\[
\bar{f}_q(\Delta \ell) = \mu \left[ \bar{\rho}_q(\dot{t}) - \rho_{q0} \right] = \mu q \left[ \bar{\rho}_N(\dot{t}) - \rho_{N0} \right]
\] (12)

where

\[
\rho_{q0} = q \rho_{N0} = q \frac{\Delta N}{\Delta \ell_0}
\]

\( \mu \) — compression coefficient of charge density \( [\mu] = \frac{Nm^3}{C} \)

\[
\bar{f}_q(\Delta \ell) = \begin{cases} 
\mu q \Delta N \left( \frac{1}{\Delta \ell} - \frac{1}{\Delta \ell_0} \right) & \Delta \ell < \Delta \ell_0 \\
0 & \Delta \ell \geq \Delta \ell_0 
\end{cases}
\]

\[
\Delta E_p[\Delta \ell(\dot{t})] = \int_{\Delta \ell}^{\Delta \ell_0} \bar{f}_q(\Delta \ell) d\Delta \ell = \int_{\Delta \ell}^{\Delta \ell_0} \mu q \Delta N \left( \frac{1}{x} - \frac{1}{\Delta \ell_0} \right) dx
\]

\[
= \mu q \Delta N \left[ \ln \frac{\Delta \ell_0}{\Delta \ell(\dot{t})} - \frac{\Delta \ell_0 - \Delta \ell(\dot{t})}{\Delta \ell_0} \right]
\]

\[
= \mu q \Delta N \ln \frac{\Delta \ell_0}{\Delta \ell(\dot{t})} + \frac{\Delta \ell}{\Delta \ell_0} - 1
\]

\[
\Delta \ell(\dot{t}) = \sum_{i} n_i \mu q \left[ \ln \frac{\lambda_0}{\lambda_i(\dot{t})} + \frac{\lambda_i(\dot{t})}{\lambda_0} - 1 \right]
\] (14)

\( \lambda_0 \) — individual charge field’s critical diameter; \( \lambda \) — compressed diameter of individual charge field; \( \lambda \in (0, \lambda_0) \). here define

\[
k(\dot{t}) = \frac{\lambda(\dot{t})}{\lambda_0} = e^{-\eta(\dot{t})}
\]

\( k(\dot{t}) \in (0, 1], \eta(\dot{t}) \in [0, +\infty) \)

\( k(\dot{t}) \) — ratio of compressed and intrinsic volumes of individual charge field.

\( \eta(\dot{t}) \) — potential energy index of individual charge field.
\[
\frac{d\Delta E_p[\Delta \ell(i)]}{dt} = \frac{d\Delta E_p}{d\ell} \Delta \dot{\ell} = -\mu q \rho_{S_0} \left( \frac{\Delta \ell_0}{\Delta \ell} - 1 \right) \Delta \dot{\ell}
\]  
(15)

Further
\[
\sum f_{ii}(\dot{t}) \cdot \dot{r}_{imc}(\dot{t}) = \int_{r_h}^{r_t} f_i(r) \frac{\partial v}{\partial r} dr = \int_{r_h}^{r_t} \mu q [\rho_S(r) - \rho_{S_0}] \frac{\partial v}{\partial r} dr = \mu q \rho_{S_0} \left( \frac{\Delta \ell_0}{\Delta \ell} - 1 \right) \Delta \dot{\ell}
\]  
(16)

Compare Eq. (15) and (16)

\[
\frac{d\Delta E_p(\dot{t})}{dt} = -\int_{r_h}^{r_t} f_i(r) \frac{\partial v}{\partial r} dr
\]  
(17)

From Eq. (17) suppose \( \frac{d\Delta E_{exc}(\dot{t})}{dt} = 0 \); there \( \frac{d\Delta E_i(\dot{t})}{dt} = 0 \), so

\[
\frac{d\Delta E_p(\dot{t})}{dt} = -\frac{d}{dt} \left[ \sum \frac{m_q}{2} v_{imc}^2(\dot{t}) \right]
\]  
(17')

or \( \int_{r_h}^{r_t} f_i(r) \frac{\partial v}{\partial r} dr = \frac{d}{dt} \left[ \sum \frac{m_q}{2} v_{imc}^2(\dot{t}) \right] \)

It is shown in Eq. (17), (17') that for internal force to do positive work will elongate the length, enlarge kinetic energy spread and exhaust or consume the internal potential energy; for the force to do negative work, the length will be compressed and kinetic energy spread reduced, internal potential energy increased. However once the internal force starts to do negative work, the process will be irreversible without external power’s joining.

C. Energy Differential Structure of \( \Delta E(\dot{t}) \) Carried by the Micro Multi-Particle System

\[
\Delta E(\dot{t}) = \frac{\Delta N m_S}{2} v_{imc}^2(\dot{t}) + \int_{r_h}^{r_t} \frac{dm(r)}{2} v_{imc}^2(r) + \Delta E_p(\dot{t})
\]

\[
= \frac{\Delta M}{2} \left( v_h - \frac{\Delta \dot{\ell}}{2} + \frac{\Delta \dot{\ell}}{12 \Delta N} \frac{\partial \rho_S}{\partial r} \Delta \ell \right)^2 + \frac{\Delta M}{24} \Delta \dot{\ell}^2 \left[ 1 - \frac{1}{12 \rho_S^2} \left( \frac{\partial \rho_S}{\partial r} \Delta \ell \right)^2 \right] + \mu \Delta Q \left( \ln \frac{\Delta \ell_0}{\Delta \ell} + \frac{\Delta \ell}{\Delta \ell_0} - 1 \right)
\]

\[
= \frac{\Delta M}{2} \left( v_h - \frac{\Delta \dot{\ell}}{2} + \frac{\Delta \dot{\ell}}{12 \rho_S^2} \frac{\partial \rho_S}{\partial r} \Delta \ell \right)^2 + \frac{\Delta M}{24} \Delta \dot{\ell}^2 \left[ 1 - \frac{1}{12 \rho_S^2} \left( \frac{\partial \rho_S}{\partial r} \Delta \ell \right)^2 \right] + \sum n_i \mu q \left[ \ln \frac{\lambda_0}{\lambda_i(\dot{t})} + \frac{\lambda_i(\dot{t})}{\lambda_0} - 1 \right]
\]  
(18)

Eq. (18) indicates that both kinetic and potential energy have possible distributions; for kinetic energy it depends on length, length’s rate and \( \rho_S \) distribution or different compression states due to internal charge field force in longitudinal dimension.
IV. ENERGY FLUX AND ENERGY EXCHANGE

A. Energy flux density and energy flux

At instant $\dot{t} \in [t, t']$ there is energy flux density $\rho_{\Delta E}[r_{is}(\dot{t})]$ on a crossing section located at $r_{is}(\dot{t})$

$$\rho_{\Delta E}[r_{is}(\dot{t})] = \rho_N[r_{is}(\dot{t})] \left\{ \frac{m_a}{2} v_{mc}(\dot{t}) + \mu q \left[ \ln \frac{\lambda_0}{\lambda(\dot{t})} + \frac{\lambda(\dot{t})}{\lambda_0} - 1 \right] \right\} \quad (19)$$

and energy flux

$$J_{\Delta E}(\dot{t}) = \rho_{\Delta E}[r_{is}(\dot{t})] v_{is}(\dot{t})$$

$$= \rho_N[r_{is}(\dot{t})] \left\{ \frac{m_a}{2} v_{mc}(\dot{t}) + \mu q \left[ \ln \frac{\lambda_0}{\lambda_i(\dot{t})} + \frac{\lambda_i(\dot{t})}{\lambda_0} - 1 \right] \right\} v_{is}(\dot{t})$$

$$= J[r_{is}(\dot{t})] \left\{ \frac{m_a}{2} v_{mc}(\dot{t}) + \mu q \left[ \ln \frac{\lambda_0}{\lambda_i(\dot{t})} + \frac{\lambda_i(\dot{t})}{\lambda_0} - 1 \right] \right\} \quad (20)$$

B. Differential transfer relation of energy flux density between two time domains and energy exchange

Ref. [5] there

$$\Delta E_{exc}(t') = \Delta E(t') - \Delta E_{so}(t)$$

$$\Delta N(t') = \Delta N(t)$$

to multiply both equations with $\frac{1}{\Delta t}$ or $\frac{1}{\Delta t'}$, then take limit as $\Delta t \to 0, \Delta t' \to 0$

$$J_{exc}(t') = J(t) \frac{dt}{dt'} \quad \text{or} \quad \rho_N[r_p(t')] v_{io}(t') = \rho_N[r_s(t')] v_{sis}(t) \frac{dt}{dt'} \quad (21)$$

Then combine Eq. (20) and (22) with Eq. (21)

$$J_{exc}(t') = J[r_s(t)] \frac{dt}{dt'} \left\{ \frac{m_a}{2} v_{mc}(\dot{t}) + \mu q \left[ \ln \frac{\lambda_0}{\lambda(\dot{t})} + \frac{\lambda(\dot{t})}{\lambda_0} - 1 \right] \right\} \bigg|_{t'}$$

$$= \rho_N[r_s(t)] v_{sis}(t) \left[ 1 - \frac{dT(t')}{dt'} \right] \left\{ \frac{m_a}{2} v_{mc}(\dot{t}) + \mu q \left[ \ln \frac{\lambda_0}{\lambda(\dot{t})} + \frac{\lambda(\dot{t})}{\lambda_0} - 1 \right] \right\} \bigg|_{t'}$$

$$= \rho_N[r_o(t')] v_{io}(t') \left\{ \frac{m_a}{2} v_{mc}(\dot{t}) + \mu q \left[ \ln \frac{\lambda_0}{\lambda(\dot{t})} + \frac{\lambda(\dot{t})}{\lambda_0} - 1 \right] \right\} \bigg|_{t'}$$

$$= J[r_o(t')] \left\{ \frac{m_a}{2} v_{mc}(\dot{t}) + \mu q \left[ \ln \frac{\lambda_0}{\lambda(\dot{t})} + \frac{\lambda(\dot{t})}{\lambda_0} - 1 \right] \right\} \bigg|_{t'}$$

$$= J[r_o(t')] \left\{ \frac{m_a}{2} v_{mc}(\dot{t}) + \mu q \left[ \ln \frac{\lambda_0}{\lambda(\dot{t})} + \frac{\lambda(\dot{t})}{\lambda_0} - 1 \right] \right\} \bigg|_{t'} \quad (23)$$
By using $\lambda(t') + \lambda_c(t') = \lambda(t)$ \[2\], $\lambda_c(t') = \int_{t'}^{t} \frac{d\lambda_c(t')}{dt} dt = \int_{0}^{\lambda(t')} \frac{d\lambda_c(t)}{dt} dt \begin{cases} > 0 & \text{compression} \\ < 0 & \text{elongation} \end{cases}$

$$J_{exc}(t') = \rho_N[r_s(t)]v_{sis}(t') \frac{dt}{dt'} \left\{ \frac{m_g}{2} \left[ v_{io}^2(t') - v_{sis}^2(t) \right] + \mu q \left[ \ln \frac{\lambda(t)}{\lambda(t')} + \frac{\lambda(t') - \lambda(t)}{\lambda_0} \right] \right\}$$

$$= J[r_s(t)] \frac{dt}{dt'} \left\{ \frac{m_g}{2} \left[ v_{io}^2(t') - v_{sis}^2(t) \right] + \mu q \left[ \ln \left[ 1 + \frac{\lambda(t)}{\lambda(t')} \right] - \frac{\lambda_c(t')}{\lambda_0} \right] \right\}$$

$$= J[r_o(t')] \left\{ \frac{m_g}{2} \left[ v_{io}^2(t') - v_{sis}^2(t) \right] + \mu q \left[ \ln \left[ 1 + \frac{\lambda(t)}{\lambda(t')} \right] - \frac{\lambda_c(t')}{\lambda_0} \right] \right\}$$

$$= \rho_N[r_o(t')]v_{io}(t') \left\{ \frac{m_g}{2} \left[ v_{io}^2(t') - v_{sis}^2(t) \right] + \mu q \left[ \ln \left[ 1 + \frac{\lambda(t)}{\lambda(t')} \right] - \frac{\lambda_c(t')}{\lambda_0} \right] \right\} \quad (24)$$

Above Eq. (23) and (24) is energy exchange relation between a micro beam element and external system during the accelerating process within which the micro beam element travel through the space between source and detector.

For $J_{exc}(t') > 0$ external system supplies energy to beam for contributing to either kinetic or potential energy or both as well as time affection factor. For $J_{exc}(t') < 0$ external system gains energy from beam system, may from either of kinetic or potential energy; in case the time factor determined by dynamical process can also affect energy’s output of micro beam element.

### C. Illustration or illustrative examples

#### 1. Selection of time domains

This is completely dependent to which beam’s acceleration or transportation process you are interested in. The flux source and observing location could be selected at entrance and exit point or cross section of a resonant cavity respectively where signal’s transfer time $T(t) = T(t')$ is just charged particle’s transit time. The flux source and observing location could also be selected at beam source (gun) and beam extracting point for tracing the whole process of the beam’s acceleration and transportation drift where signal’s transfer time $T(t) = T(t')$ can be consisted by various constituent $T_i(t_{i-1}) = T_i(t_i)$ and there is consequent relation $T(t) = T(t') = \sum T_i(t_{i-1}) \quad [7]$. Once the time domains are decided, then all differential transfer relations \[3\] between the time domains and energy differential information could be used to search exact solution.

#### 2. Energy exchange of relativistic electrons beam and inference

In this case, $v_{io}(t') \approx v_{sis}(t) \approx c$, and from Eq. (24):

$$J_{exc}(t') \approx \rho_N[r_s(t)]c \left\{ \ln \left[ 1 + \frac{\lambda_c(t')}{\lambda_0} \right] \right\}$$

$$= e \mu q \rho_N[r_s(t)] \left\{ \ln \left[ \frac{\lambda(t)}{\lambda(t')} \right] - \frac{\lambda(t') - \lambda(t)}{\lambda_0} \right\}$$

$$= e \mu q \rho_N[r_s(t)] \left\{ \ln \left[ \frac{\lambda(t)}{\lambda(t')} \right] - \frac{\lambda(t) - \lambda(t')}{\lambda_0} \right\}$$

$$= e \mu q \rho_N[r_s(t)] \left\{ \ln \left[ \frac{k(t)}{k(t')} \right] - \left[ k(t) - k(t') \right] \right\} \quad (25)$$
Thus there are consequent inferences below for constant state, elongation’s and compression’s.

A. Constant State
\[
\frac{dT(t')}{dt'} = 0, \quad \frac{dt}{dt'} = 1, \quad \ln \frac{\lambda(t)}{\lambda(t')} - \frac{\lambda(t) - \lambda(t')}{\lambda_0} = 0; \quad \mathcal{J}_{exc}(t') = 0 \quad \eta(t') = \eta(t) \quad k(t') = k(t)
\]

B. Elongation State
\[
\frac{dT(t')}{dt'} > 0, \quad 0 < \frac{dt}{dt'} < 1, \quad \ln \frac{\lambda(t)}{\lambda(t')} - \frac{\lambda(t) - \lambda(t')}{\lambda_0} < 0; \quad \mathcal{J}_{exc}(t') < 0 \quad \eta(t') < \eta(t) \quad k(t') > k(t)
\]

C. Compression State
\[
\frac{dT(t')}{dt'} < 0, \quad \frac{dt}{dt'} > 1, \quad \ln \frac{\lambda(t)}{\lambda(t')} - \frac{\lambda(t) - \lambda(t')}{\lambda_0} > 0; \quad \mathcal{J}_{exc}(t') > 0 \quad \eta(t') > \eta(t) \quad k(t') < k(t)
\]

Above inferences show that in constant state A, \(\rho_N(t') = \rho_N(t)\) and without energy exchange; in elongation state B, \(\rho_N(t') < \rho_N(t)\) and internal potential energy releases to external system; in compression state C, \(\rho_N(t') > \rho_N(t)\) and internal potential energy increases result from beam’s gaining energy from external system.

Further more, we can make inferences below:

(i) For relativistic electron or ion beam, it’s energy increment or increase basically results from the gain of itself’s potential energy through longitudinal compression, otherwise the beam’s energy will meet its limit, that is its kinetic energy limit.

This is main difference of rf and DC acceleration method on electron beam’s energy up limits, for rf method the bunch length can be compressed progressively further and further so that its energy will not meet up limit; whereas using DC acceleration the length of a micro beam element is longer than the critical length within which the beam’s internal potential energy can be existed or accumulated through further compression, in addition, DC method has no compression mechanism. Thus DC acceleration beam’s energy has its energy limit, its kinetic limit, even DC acceleration voltage has no technical limit.

Ref. 5:
\[
\frac{dt}{dt'} - \frac{v_{iso}(t')}{v_{sis}(t)} \frac{dt'}{dt} = \frac{c}{v_{sis}(t)} = 1 + \frac{1}{v_{sis}(t)} \int_t^{t'} \frac{\partial v_{is}(t, r_{is})}{\partial r_{is}} v_{is}(i, t) \frac{di(t)}{dt} dt \gg 1
\]

We know that in DC acceleration the longitudinal length of a micro beam element is progressively elongated.

(ii) The mass of relativistic beam, collectives’ or individual’s, is independent to its energy.

From Eq. 24 it is known that the energy exchange of relativistic beam essentially depends upon beam internal potential energy’s exchange with external system, the density of the particles number at observing location will fluctuate as the energy exchanger occur. And the number density’s change is synchronized with potential energy gain or loss of the beam, as beam’s potential energy increases the density increases, while the energy losses the density decreases. Since \(\rho_N[r_0(t')] = \rho_N[r_s(t)] \frac{dt}{dt'} = \rho_N[1 - \frac{dT(t')}{dt}]\) then it is clear that the density’s fluctuation is time function related due to the variation of signal transfer time.

Ref. 3 mass flux density transfer relation \(J_m(t') + J_{loss}(t') = J_m(t) \frac{dt}{dt'} + \rho_m(t') c = \rho_m(t) c \frac{dt}{dt'}\), here \(J_{loss}(t') = \frac{dm_{loss}}{dt'} = 0\).
differential relation on number of particles, mass conversation

\[ \Delta N(t) + \Delta N_{\text{loss}}(t) = \Delta N(t) \]  \hspace{1cm} (26)

multiply Eq. (26) with \( m_q \):

\[ m_q \Delta N(t) + m_q \Delta N_{\text{loss}}(t) = \Delta N(t)m_q \]

\[ \Delta M(t) + \Delta M_{\text{loss}}(t) = \Delta M(t) \] here \( \Delta M_{\text{loss}} = \int_t^{t'} \frac{d\Delta M_{\text{loss}}(t)}{dt} dt \geq 0 \) there only

\[ \Delta M(t) \leq \Delta M(t) \varepsilon c \rho m(t') dt' \leq c \rho m(t) dt \]

However in rf acceleration the corresponding energy increment \( \Delta E \) and energy flux \( J \)

\[ \Delta E(t') = \Delta E(t) + \Delta E_{\text{exc}}(t') - \Delta E_c(t') \gg \Delta E(t) \]

\[ J(t') dt' \gg J(t) dt \]

V. DISCUSSION

A. Charge field property

Charge is structure density field possessed; for a elementary charge \( q \) there \( \iiint_V \rho(r) dV = q \) and in uncompressed state or its intrinsic state

\[ \rho(r) = \begin{cases} 
\frac{2q(R_0-r)}{R_0^2 \pi r^2} & 0 < r \leq R_0 \\
0 & r > R_0 
\end{cases} \]

and with \( \nabla \rho = \frac{q}{2 \pi R_0^2} \frac{r-2R_0}{r^2} \) apply to \( \iiint_V (r-r_{cc}) dq = 0 \), there charge center \( r_{cc}(t), v_{cc}(t) \) etc.

\[ \iiint_V r \rho(r) dV = r_{cc} \int \int \int_V \rho(r) dV = qr_{cc} \]

Once charge’s electromagnetic interaction and charge’s potential energy are concerned its density field effect can not be neglected anymore while merely regarding charge as a point charge whose charge all be centralized on its charge center for approximatively simplified application.

Charge density field can be compressed. Based on this, its internal potential energy can exist. In addition the compression can bring about some electromagnetic consequences in magnetic dipole. Density’s change will produce internal force within charge density field. The force depends upon both density’s difference between compressed density and corresponding intrinsic density, and ratio of compression which is the intrinsic density related. So calculation of stored potential energy of charge density field precisely can only be done through corresponding relation \( \rho_i(\bar{t}) d\nu_i(\bar{t}) = \rho_{oi} d\nu_{oi} \); this may be uneasy to resolve since the density’s spatial distribution at any instant is not linear.

Fortunately in multi-particle system, a relative macro system, the density’s distribution at any instant comply to linear distribution as we select the longitudinal length of the micro element short enough so that the compression ratio or coefficient \( \mu \) can be treated as constant.

In addition, for the micro beam element, or \( \Delta N \) number of particles, within a cylindrical volume with constant radius \( a \), there

\[ \frac{\Delta N}{\pi a^2} = \Delta \ell(\bar{t}) \tilde{\rho}(\bar{t}) = \text{constant} \]
then \( \Delta \ell(t) = \frac{\Delta N}{\pi a^2} \lambda(t) \) and \( \Delta \ell_0 = \frac{\Delta N}{\pi a^2} \lambda_0 \), thus

\[
\frac{\Delta \ell_0}{\Delta \ell(t)} = \frac{\lambda_0}{\lambda(t)}
\]

So we can convert the ratio of the micro beam’s lengths to ratio of individual particles’. Furthermore, we define \( k \) and \( \eta, k(t) = e^{-\eta(t)} \), for indicating individual charge field’s compression and potential energy status.

B. Time function related factor and energy differential structure’s implication

A For charged particle’s longitudinal interaction, a particle behind with higher velocity than front particle’s can not pass over the ahead particle; therefore related time function is positive reversible \( \frac{dT}{dt} > 0, \frac{dT}{dt'} > 0 \), or \( \frac{dT}{dt} > -1 \) and \( \frac{dT}{dt'} < 1 \). Besides, \( \frac{dt}{dt'} \) factor can contribute to enhance or decrease output number flux, mass flux, charge flux, energy flux \( J_{exc}(t') \) (Ref. Eq. (24)), however its value is determined by energy exchange of external system and micro beam element.

B The energy differential structure and relevant inference emphasize beam’s itself property and nature. The work, combined with differential transfer relations \( \frac{dT}{dt} \) and other tools, can help us insight beams inner behavior and response and tracing a specified micro beam element whole physics process of beams’ energy components and exchange. With it we may evaluate some conventional dictations and interpretations in differential angle.

VI. CONCLUSION

Energy of a micro beam element is consisted of three parts, mass center and relative center mass kinetic energy and beam’s internal potential energy. Each of them is particle’s longitudinal velocity distribution and density (number, consequent mass and charge as well as internal charge field force) distribution related within the micro element. The internal potential energy exists until its length shorter than the critical length or individual charge density field’s diameter shorter than its intrinsic diameter results from accumulated compression of the beam length. And quantitative the potential energy comply with logarithm function with respect to the longitudinal length variable. For relativistic beam, its energy exchange essentially occur actually between external system’s energy and beam’s internal potential energy; at same time beam’s mass, collective’s or individual’s, is independent to its energy. Time function factor can affect various fluxes of the beam simultaneously and the factor is determined by designated dynamical process or energy exchange between external system and beam itself.

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