$p$-branes from $(p - 2)$-branes
in the Bosonic String Theory

Nobuyuki Ishibashi

KEK Theory Group, Tsukuba, Ibaraki 305, Japan

ABSTRACT

We show that Dirichlet $p$-brane can be expressed as a configuration of infinitely many Dirichlet $(p - 2)$-branes in the bosonic string theory. Using this fact, we interpret the massless fields on the $p$-brane worldvolume as deformations of the configuration of the $(p - 2)$-branes. Especially we find that the worldvolume gauge field parametrizes part of the group of diffeomorphisms on the worldvolume.
1 Introduction

D-branes\[1\] play very important roles in the study of string theory dynamics. Especially, lower dimensional D-branes such as D-strings, D0-branes and D-instantons are used in such a way that various matrix models are constructed by taking them as the fundamental degrees of freedom\[2\]\[3\]\[4\]. Lower dimensional D-branes are useful because it is easy to quantize the worldvolume theory. In constructing a theory of such lower dimensional branes, it is important to check if higher dimensional branes exist in the spectrum. The theory lacks something if it does not have higher dimensional branes, since superstring theory possesses of branes with various dimensions as classical configurations.

In \[5\]\[2\], it was pointed out that D2-brane can be realized as a configuration of infinitely many D0-branes in Type IIA/M theory. The unbroken supersymmetry in this configuration and the relation to the light-cone gauge supermembrane Hamiltonian\[6\] confirm that this configuration really corresponds to D2-brane. In \[3\], the authors showed that D-string can be expressed in a similar way as a classical configuration of infinitely many D-instantons in the Type IIB case. Such relationships between D-branes are generalized so that Dirichlet $p$-brane can be expressed as a classical configuration of infinitely many Dirichlet $(p-2k)$-branes for $k = 1, 2, \cdots$. What we would like to do in this paper is to examine these facts in the framework of string perturbation theory.

The string theory in the presence of infinitely many D-branes corresponds to an open string theory with infinitely many Chan-Paton factors. The configuration in question can be realized as a background in the open string theory. We would like to check if such a background is really equivalent to the string theory in the presence of a D-brane with dimension higher by two. In this paper we will show such an equivalence for the bosonic string. The superstring case will be considered in a separate publication.

If one regards a Dirichlet $p$-brane as a configuration of Dirichlet $(p-2k)$-branes, fluctuations of the $p$-brane can be interpreted as fluctuations of the configuration of $(p-2k)$-branes. The worldvolume theory of a D-brane is expressed by a $U(1)$ gauge field and scalars. In the latter part of this paper we will study what these fields correspond to if one considers this D-brane as a configuration of lower dimensional branes. Using this correspondence, we can clarify the relationship between the gauge field on D-brane worldvolume and the reparametrization invariance on the worldvolume.

The organization of this paper is as follows. In section 2, we consider a configuration of infinitely many D-branes and show that it yields a higher dimensional D-brane in the bosonic string case. In section 3, we study the worldvolume theory of a D-brane regarding it as a configuration of lower dimensional D-branes. Section 4 is devoted to the discussions.
2  $p$-branes from $(p - 2)$-branes

In this section we will show that Dirichlet $p$-brane can be expressed as a configuration of infinitely many Dirichlet $(p - 2)$-branes in the bosonic string theory. We will do so for $p = 1$ for simplicity. It is straightforward to generalize the discussions to other $p$'s.

The configuration of infinitely many D-instantons can be expressed by the $\infty \times \infty$ hermitian matrices $M^\mu$ ($\mu = 1, \cdots, 26$). The one we consider is

\[
M^1 = P, \\
M^2 = Q,
\]

where $P, Q$ satisfy

\[
[Q, P] = k i.
\]

Here $k$ is a real number. What we will do is to consider the bosonic string theory corresponding to the configuration eq.(1) of D-instantons and show that it is equivalent to the bosonic string theory in the presence of a D-string.

2.1 The Boundary State

The interaction between bosonic string and D-brane is described by the boundary state. Let us first examine what is the boundary state corresponding to the above configuration.

A D-instanton at the origin corresponds to the boundary state:

\[
|B >_{-1} = |X = 0 > \otimes |B >_{gh}.
\]

Here we define $|X = f >$ to be the coherent state satisfying

\[
X^\mu(\sigma)|X = f > = f^\mu(\sigma)|X = f > \quad (\mu = 1, \cdots, 26),
\]

and $|B >_{gh}$ is the ghost boundary state satisfying

\[
(c_n + \tilde{c}_{-n})|B >_{gh} = (b_n - \tilde{b}_{-n})|B >_{gh} = 0.
\]

A configuration of $N$ D-instantons is expressed by $N \times N$ matrices $M^\mu$ and the boundary state for such a configuration is

\[
|B > = TrPe^{-\frac{i}{\kappa} \int_0^{2\pi} d\sigma p^\mu(\sigma)M^\mu} |B >_{-1},
\]

where $P^\mu(\sigma) = p^\mu + \sum_n \frac{1}{2}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu)e^{-i n \sigma}$.

\[\text{Here we use } M^\mu \text{ instead of } X^\mu \text{ in order not to be confused with the string coordinates } X^\mu(\sigma). \text{ We consider the string theory with the Euclidean signature because we are going to study D-instantons. Accordingly what we mean by D-string in the following is just a two dimensional wall.}\]
Now let us consider the boundary state corresponding to the configuration eq.(1). We should consider the case $N = \infty$. The trace of the path ordered product can be expressed by using the path integral representation as

$$|B > = \int [dpdq] e^{\frac{i}{\hbar} \int d\sigma (p(p + p) + q(q + q))} |B >_{-1}. \tag{4}$$

Notice that the normalization of $|B >$ is ambiguous in this representation unless we are careful about the path integral measure $\int [dpdq]$. Here we will not pay much attention to the normalization of the boundary state. More complete argument will be given in the next subsection.

In this form, it is easy to see that this boundary state coincides with the boundary state corresponding to a D-string worldsheet with the background $U(1)$ gauge field $F_{12} = \frac{4\pi}{k}$, up to normalization. Indeed from the identities

$$0 = \int [dpdq] \frac{\delta}{\delta p(\sigma)} e^{\frac{i}{\hbar} \int d\sigma (p(p + p) + q(q + q))} |B >_{-1},$$

$$0 = \int [dpdq] \frac{\delta}{\delta q(\sigma)} e^{\frac{i}{\hbar} \int d\sigma (p(p + p) + q(q + q))} |B >_{-1},$$

one obtains

$$[P^1(\sigma) - \frac{2\pi}{k} \partial_\sigma X^2(\sigma)] |B > = 0,$$

$$[P^2(\sigma) + \frac{2\pi}{k} \partial_\sigma X^1(\sigma)] |B > = 0.$$

Solving these equations, one can show that $|B >$ corresponds to a D-string worldsheet with $F_{12} = \frac{4\pi}{k}$.

We can check this fact also by actually performing the path integral in eq.(4). Since $|B >_{-1}$ can be obtained up to normalization by solving eqs.(4), eq.(4) becomes

$$|B > \propto \int [dpdq] e^{\frac{i}{\hbar} \sum_n nq_n p_n + \sum_{n>0} (\frac{1}{n} - \sum_{n>0} (\frac{1}{n^2} - \frac{1}{n} - i\alpha_n p_n - i\alpha_n \sum_{n>0} q_n - i\alpha_n \sum_{n>0} q_n - i\alpha_n \sum_{n>0} q_n - i\alpha_n \sum_{n>0} q_n)} \times |x^1 = p_0, \ x^2 = q_0, \ x^3 = \ldots = x^{26} = 0 > \otimes |B >_{gh},$$

where

$$p(\sigma) = \sum_n p_n e^{-i\alpha_n \sigma},$$

$$q(\sigma) = \sum_n q_n e^{-i\alpha_n \sigma}.$$

The technique to treat such a non-abelian background of open string theory was studied previously in [7].
The integrations to be done are the Gaussian integrations and after the usual procedure of the zeta function regularization, we obtain
\[ |B >_1 \propto \left( 1 + \frac{4}{k^2} \right)^{\frac{1}{2}} e^{\sum_{n>0} \left( \frac{1}{2} \alpha_{-n} \cdot \alpha_{-n} + \frac{1}{2} \alpha_{-2n} \cdot \alpha_{-2n} + \frac{1}{2} \beta \left( \alpha_{-n} \cdot \alpha_{-n} - \alpha_{-2n} \cdot \alpha_{-2n} \right) - \frac{1}{4} \alpha_{-n} \cdot \alpha_{-n} \cdot \mu \cdot \mu \right)} \times |p_1 = p_2 = 0, x^3 = \cdots = x^{26} = 0 > \otimes |B >_gh. \quad (5) \]

This is exactly the boundary state for the D-string worldsheet.

### 2.2 Equivalence of the Open String Theories

The arguments in the previous subsection can show that the boundary state for the background eq.(1) coincides with the one for D-string only up to normalization. The problem is that we need degrees of freedom parametrized by a continuous parameter (\( p \) or \( q \)) to express the background eq.(1). This makes the normalization of the boundary state with \( M^\mu = 0 \) ambiguous. The most obvious way to avoid such ambiguities is to look at things in the open string channel. In this subsection, we will construct the open string theory corresponding to the D-instantons in the background eq.(1) and show that it is equivalent to the open string theory corresponding to the D-string with \( F_{12} = \frac{4\pi}{k} \).

The bosonic string theory with a D-instanton at the origin can be described by an open bosonic string theory with the Dirichlet boundary conditions:
\[ X^\mu (\sigma = \pi) = X^\mu (\sigma = 0) = 0 (\mu = 1, \cdots, 26). \]

When there are \( N \) instantons, one should consider an open string theory with the Chan-Paton factors \( a = 1, \cdots, N \) and a state in the open string Hilbert space is labeled by using two Chan-Paton factors as \(| >_{ab} \). In order to express the background eq.(1), we need the Chan-Paton factors parametrized by a continuous parameter. Moreover, since a nontrivial commutation relation eq.(2) is involved, one should consider an open string theory with quantum mechanical degrees of freedom sitting on the boundaries. Namely we will start with the following action
\[ I = \frac{1}{8\pi} \int d^2\sigma (\dot{X}^\mu \dot{X}_\mu - \dot{X}^\mu \dot{X}'_\mu) + \frac{1}{k} \int d\tau (p_\pi \dot{q}_\pi - p_0 \dot{q}_0). \quad (6) \]

\( p_\pi \), \( q_\pi \) and \( p_0 \), \( q_0 \) are the degrees of freedom sitting on the ends \( \sigma = \pi \) and \( \sigma = 0 \) respectively. Canonically quantizing them, we obtain the commutation relations
\[ [q_\pi, p_\pi] = ki, \]
\[ [q_0, p_0] = -ki. \quad (7) \]

The difference in the orientation of the two boundaries \( \sigma = \pi \) and \( \sigma = 0 \) causes the difference in the sign of the above commutation relations. The basis of the states may

\[ 3 \text{In this subsection, ghosts are ignored because they play no role.} \]
be taken to be the eigenstates of $q_{\pi}$ and $q_{0}$. Then a state in the open string Hilbert space is of the form $|>X \otimes |q_{\pi}, q_{0}>, \text{ where } |>X$ is a state in the Fock space of $X$. Hence $q_{\pi}$ and $q_{0}$ behave as continuous Chan-Paton factors.

For the background eq.(1), we should consider the following action:

$$I = \frac{1}{8\pi} \int d^{2}\sigma (\dot{X}^{\mu} \dot{X}_{\mu} - X^{\mu} X_{\mu}')$$

$$- \frac{1}{4\pi} \int_{\sigma=\pi} d\tau (X'^{1} p_{\pi} + X'^{2} q_{\pi} + V_{\pi}(p_{\pi}, q_{\pi}))$$

$$+ \frac{1}{4\pi} \int_{\sigma=0} d\tau (X'^{1} p_{0} + X'^{2} q_{0} + V_{0}(p_{0}, q_{0}))$$

$$+ \frac{1}{k} \int d\tau (p_{\pi} \dot{q}_{\pi} - p_{0} \dot{q}_{0}).$$

(8)

with the boundary conditions

$$X'^{\mu}|_{\sigma=0, \pi} = 0. \quad \text{(9)}$$

Here $V_{0}$ and $V_{\pi}$ are divergent counterterms. We will see that these terms are necessary.

What we would like to show in this subsection is that this open string theory is equivalent to the open string theory corresponding to a D-string worldsheet with $F_{12} = \frac{4\pi}{k}$. The action for such an open string theory is

$$I = \frac{1}{8\pi} \int d^{2}\sigma (\dot{X}^{\mu} \dot{X}_{\mu} - X^{\mu} X_{\mu}')$$

$$- \frac{1}{4\pi} \int_{\sigma=\pi} d\tau \dot{X}^{1} X^{2} F_{12}$$

$$+ \frac{1}{4\pi} \int_{\sigma=0} d\tau \dot{X}^{1} X^{2} F_{12},$$

(10)

with the boundary conditions

$$\partial_{\sigma} X^{1}|_{\sigma=0, \pi} = \partial_{\sigma} X^{2}|_{\sigma=0, \pi} = 0,$$

$$X^{i}|_{\sigma=0, \pi} = 0 \quad (i = 3, \cdots, 26). \quad \text{(11)}$$

We will show that the two dimensional field theory with the action in eq.(8) and the boundary condition in eq.(9) is equivalent to the one with the action in eq.(10) and the boundary condition in eq.(11). Since the equivalence is trivially seen for the variables $X^{i}$ ($i = 3, \cdots, 26$), we will concentrate on $X^{1}$ and $X^{2}$ in the following.

Before doing so, we should remark on the treatment of the boundary terms. Dealing with the action in eq.(10), one usually considers that the boundary conditions in eq.(11) are modified because of the boundary terms (the second and the third terms in eq.(10)). The new boundary conditions are chosen so that the equations of motion is the free one. Thus one obtains

$$(\partial_{\sigma} X^{1} - F_{12} \partial_{\tau} X^{2})|_{\sigma=0, \pi} = (\partial_{\sigma} X^{2} + F_{12} \partial_{\tau} X^{1})|_{\sigma=0, \pi} = 0. \quad \text{(12)}$$
Alternatively one can use the boundary conditions eq.(11) and take the effects of the boundary terms into account as interaction terms in the equations of motion. In this case, the contributions of the boundary terms to the equations of motion are proportional to $\delta(\sigma)$, $\delta(\sigma - \pi)$, and we need to be careful about the treatment of these terms. In order to avoid subtlety, we should first shift the positions of the boundary terms to be $\sigma = \epsilon$ and $\sigma = \pi - \epsilon$, $(\epsilon > 0)$ and take the limit $\epsilon \rightarrow 0$ later. This ensures that the boundary terms contribute to the equations of motion, instead of changing the boundary conditions. Of course these two approaches give equivalent results. (We will comment on this fact later.)

Now let us analyze the 2D field theory in eq.(8). Since there are quantum mechanical degrees of freedom on the boundaries, only the latter approach in the above paragraph is possible. Let us provisionally assume that $V_0 = V_\pi = 0$. Defining the new variables

$$Z = X^1 + iX^2,$$
$$s_\pi = p_\pi + iq_\pi,$$
$$s_0 = p_0 + iq_0,$$

for convenience, the equations of motion become

$$-\ddot{Z} + Z'' + s_\pi \delta'(\sigma - (\pi - \epsilon)) - s_0 \delta'(\sigma - \epsilon) = 0,$$
$$\frac{i}{k} \dot{s}_\pi + \frac{1}{4\pi} Z'(\sigma = \pi - \epsilon) = 0,$$
$$\frac{i}{k} \dot{s}_0 + \frac{1}{4\pi} Z'(\sigma = \epsilon) = 0,$$

and their complex conjugates. The positions of boundaries are shifted by $\epsilon$ as was mentioned in the above remarks. From the first equation in eqs.(14), one can see that $Z$ has discontinuities like

$$Z = (s_0 + \frac{\sigma - \epsilon}{\pi - 2\epsilon}(s_\pi - s_0))\theta(\sigma - \epsilon)\theta(\pi - \epsilon - \sigma) + \cdots,$$

at $\sigma = \epsilon$ and $\sigma = \pi - \epsilon$. Such discontinuities make the vertex operators $Z'(\sigma = \epsilon)$ and $Z'(\sigma = \pi - \epsilon)$ divergent and make the latter two equations in eqs.(14) ill-defined.4

Since we did not encounter such divergences in the previous subsection, there should be discrepancy between the vertex operator $P^\mu$ in the closed string channel and the vertex operator $X^\mu$. Indeed $P^\mu$ in the closed string channel corresponds to $\lim_{\eta \rightarrow 0^+} X^\mu(\sigma = \epsilon + \eta)$ and $\lim_{\eta \rightarrow 0^+} X^\mu(\sigma = \pi - \epsilon - \eta)$ in the open string channel and the divergences coming from the discontinuities are avoided. Therefore we will modify the action so that the vertex operators coincide with those in the closed string channel. Accordingly the counterterms in eq.(8) should be

$$V_\pi = \frac{1}{2} p^2_\pi \delta(0) + \frac{1}{2} q^2_\pi \delta(0),$$
$$V_0 = -\frac{1}{2} p^2_0 \delta(0) - \frac{1}{2} q^2_0 \delta(0).$$

4We would like to thank H. Dorn for pointing out this problem.
With this modification, the second and the third equations in eqs. (14) become
\[
\frac{i}{k} \dot{s}_\pi + \frac{1}{4\pi} \lim_{\eta \to 0^+} \eta Z'(\sigma = \pi - \epsilon - \eta) = 0,
\]
\[
\frac{i}{k} \dot{s}_0 + \frac{1}{4\pi} \lim_{\eta \to 0^+} \eta Z'(\sigma = \epsilon + \eta) = 0.
\]
(17)

The boundary conditions \(Z|_{\sigma = 0, \pi} = 0\) implies that \(Z\) can be expanded as
\[
Z(\tau, \sigma) = 2\sqrt{2} \sum_{n>0} z_n(\tau) \sin n\sigma.
\]
(18)

In terms of \(z_n\), the equations of motion are
\[
\ddot{z}_n + n^2 z_n + \frac{n}{\sqrt{2}\pi} \left(s_\pi \cos n(\pi - \epsilon) - s_0 \cos n\epsilon\right) = 0,
\]
\[
\dot{s}_\pi = \frac{ik}{\sqrt{2}\pi} \sum_{n>0} n z_n \cos n(\pi - \epsilon - \eta),
\]
\[
\dot{s}_0 = \frac{ik}{\sqrt{2}\pi} \sum_{n>0} n z_n \cos (\epsilon + \eta),
\]
(19)

and the canonical commutation relations are
\[
[z_n, \dot{z}_m^\dagger] = 2i\delta_{n,m},
\]
\[
[z_n^\dagger, \dot{z}_m] = 2i\delta_{n,m},
\]
0, otherwise. (20)

On the other hand, the action in eq. (8) can be treated in the usual manner\[8\]. The boundary term modifies the boundary condition as in eq. (12), which is
\[
(Z' + iF\dot{Z})|_{\sigma = 0, \pi} = 0 \quad (F \equiv F_{12}),
\]
(21)
in terms of \(Z = X^1 + iX^2\). Then \(Z\) can be expanded as
\[
Z(\tau, \sigma) = 2\sqrt{2} \left\{ \frac{x_+ + p_+ [\tau - iF(\sigma - \frac{\pi}{2})]}{\sqrt{1 + F^2}} + i \sum_{n=1}^{\infty} \left[ \frac{a_n(\tau)}{\sqrt{n}} \cos(n\sigma + \gamma) - \frac{b_n^\dagger(\tau)}{\sqrt{n}} \cos(-n\sigma + \gamma) \right]\right\},
\]
(22)
where \(\tan \gamma = F\). In terms of the coefficients of this expansion, the equations of motion become
\[
(\partial_\tau + in)a_n = 0,
\]
\[
(\partial_\tau - in)b_n^\dagger = 0.
\]
(23)

The canonical commutation relations are
\[
[a_n, a_m^\dagger] = \delta_{n,m},
\]
\[
[b_n, b_m^\dagger] = \delta_{n,m},
\]
\[
[x_+, p_+] = [x_-, p_-] = i,
\]
0 otherwise, (24)
where \( x_\pm \equiv (x_+)^\dagger \), \( p_\pm \equiv (p_-)^\dagger \).

Now we will show that these two systems are equivalent if \( kF = 4\pi \). Actually we can show that there exist a transformation from the variables \((z_n, \dot{z}_n, s_\pi, s_0)\) to the variables \((a_n, b_n^\dagger, x_+, p_-)\) which turns eqs.\((7), (18), (19), (20)\) into eqs.\((22), (23), (24)\). In order to do so, one should notice the following fact. The boundary terms are treated quite differently in these two systems. In the former case, the positions of the boundaries are shifted a bit and the boundary terms contribute to the equations of motion. In the latter case, the positions of the boundaries are not shifted and the boundary conditions are changed. Therefore, the variable \( Z \) in eq.\((18)\) and eq.\((22)\) can be compared only in the region \( \epsilon \leq \sigma \leq \pi - \epsilon \). As a function in such a region, \( Z \) in eq.\((22)\) can be expanded by \( \sin n\sigma \) and we obtain the following invertible transformation from the variables \((z_n, \dot{z}_n, s_\pi, s_0)\) to the variables \((a_n, b_n^\dagger, x_+, p_-)\) as long as \( F \neq 0, k \neq 0 \):

\[
\begin{align*}
a_n &= \frac{1}{2\sqrt{n(1 + F^2)}}(\dot{z}_n - inz_n) - \frac{2}{F\pi} \sum_{m>0} \frac{m}{m^2 - n^2}(\dot{z}_m - inz_m)(1 - (-1)^{n+m}) \\
& \quad - \frac{i}{\sqrt{2}\pi}((-1)^n s_\pi - s_0), \\
b_n^\dagger &= \frac{1}{2\sqrt{n(1 + F^2)}}(\dot{z}_n + inz_n) + \frac{2}{F\pi} \sum_{m>0} \frac{m}{m^2 - n^2}(\dot{z}_m + inz_m)(1 - (-1)^{n+m}) \\
& \quad + \frac{i}{\sqrt{2}\pi}((-1)^n s_\pi - s_0), \\
x_+ &= \frac{1}{\pi\sqrt{1 + F^2}} \sum_{m>0} z_m \frac{1 - (-1)^m}{m} + \frac{Fi}{2\sqrt{1 + F^2}} \sum_{m>0} \dot{z}_m \frac{1 + (-1)^m}{m} \\
& \quad + \frac{F^2}{4\sqrt{2}(1 + F^2)}(s_\pi + s_0), \\
p_- &= \frac{1}{\pi\sqrt{1 + F^2}} \sum_{m>0} \dot{z}_m \frac{1 - (-1)^m}{m} - \frac{Fi}{2\pi\sqrt{2}(1 + F^2)}(s_\pi - s_0). \tag{25}
\end{align*}
\]

After tedious but straightforward calculations, one can show that eqs.\((7), (19), (20)\) are transformed into eqs.\((23), (24)\) by this transformation.

We conclude this section by the following remark. What we have shown in this section is that the background of the form eq.\((4)\) turns the Dirichlet boundary conditions of \(X^1, X^2\) into the Neumann boundary conditions with the background gauge field \( F_{12} = \frac{4\pi}{k} \). Using this background, it is straightforward to show that Dirichlet \(p\)-brane with a background gauge field can be obtained as a configuration of infinitely many Dirichlet \((p-2)\)-branes. By repeating this procedure, Dirichlet \(p\)-brane can be obtained as a configuration of infinitely many Dirichlet \((p-2n)\)-branes for \((n = 1, 2, \cdots)\).

\textsuperscript{5}If one expands eq.\((23)\) in the same spirit in terms of \( \cos n\sigma \), one can show the equivalence of the two approaches to deal with the boundary terms in eq.\((11)\).

9
3 Worldvolume Theory

Thus far, we consider a straight Dirichlet $p$-brane at rest and show that it can be realized as a configuration of infinitely many Dirichlet $(p-2)$-branes, by proving the equivalence of the string theories in the two backgrounds. Since the modes to deform these backgrounds are included in the open string spectrum, it is possible to regard a more general configuration of $p$-brane worldsheet as a configuration of $(p-2)$-branes. Hence everything about $p$-brane can be reinterpreted by regarding it as a configuration of $(p-2)$-branes. Namely there are two ways of looking at the same thing, either as a $p$-brane or as a configuration of $(p-2)$-branes. From now on, let us call these two points of view $p$-brane picture and $(p-2)$-brane picture respectively. In this section we will examine what the fields on the worldvolume of Dirichlet $p$-brane correspond to in the $(p-2)$-brane picture. We will do so for $p=1$ for simplicity but the results here can be easily generalized to other $p$’s.

The worldsheet field content of D-string consists of a worldsheet vector $A_\alpha (\alpha = 1, 2)$ and scalars $\phi^i (i = 3, \cdots, 26)$. $\phi^i$ can be interpreted as oscillations in the position of the D-string. Since the open string vertex operator corresponding to $\phi^i$ is $\partial_\sigma X^i \phi_i(X)$, the boundary state corresponding to the D-string worldsheet with the shape specified by the equations $X^i = \phi^i(X^\alpha)$ is

$$|B > = e^{-\frac{i}{2\pi} \int_0^{2\pi} d\sigma P_i \phi_i(X)} |B >_{1}$$

For our purpose, it is convenient to rewrite $|B >$ by using eq.(4) as

$$|B > = \int [dpdq] \exp \left[ \frac{i}{k} \int d\sigma p \partial_\sigma q - \frac{i}{2\pi} \int_0^{2\pi} d\sigma (P^1 p + P^2 q + P_i \phi^i(p,q)) |B >_{-1}, \quad (26)$$

where $\phi(p, q) \equiv \phi(X^1 = p, X^2 = q)$.

The part $e^{-\frac{i}{2\pi} \int_0^{2\pi} d\sigma (P^1 p + P^2 q + P_i \phi^i(p,q))} |X = 0 >$ in the integrand is the coherent state $|X^1 = p(\sigma), X^2 = q(\sigma), X^i = \phi^i(p(\sigma), q(\sigma)) >$. Therefore the fact that the fundamental string is emitted from the worldsheet $X^i = \phi^i(X^\alpha)$ is obvious in this form. Moreover since it is in the form of the $|B >_{-1}$ deformed by vertex operators, it is easy to see how scalars $\phi^i$ can be interpreted as deformations of the configuration eq.(4) of D-instantons. It is obvious that $|B >$ corresponds to a background

$$M^1 = P,$$
$$M^2 = Q,$$
$$M^i = \phi^i(P, Q), \quad (27)$$

with $[Q, P] = ki$. Here the operators are assumed to be Weyl ordered in the expression $\phi^i(P, Q)$. Therefore the scalars $\phi^i$ can be directly interpreted as deformations of the background eq.(4).

However $\phi^i$’s do not exhaust all the possible deformations either in the D-string picture or the D-instanton picture. In the D-instanton picture, the existence of the
vertex operator $\partial_{\sigma}X^\mu \phi_\mu(X)$ ($\mu = 1, \cdots, 26$) implies that the more general deformation to be considered is

$$M^\mu = \phi^\mu(P, Q).$$

(28)

Here $P, Q$ play the role of the coordinates on of the worldsheet of D-string. In the background in eq.(27), $X^1$ and $X^2$ are taken to be such coordinates. The background in eq.(28) corresponds to a more general parametrization. The boundary state corresponding to such a background is

$$|B> = \int [dy] \exp \left[ \frac{i}{\theta} \int d\sigma A_\alpha(y) \partial_\sigma y^\alpha - \frac{i}{2\pi} \int_0^{2\pi} d\sigma P_\mu \phi^\mu(y) \right] |B>_{-1},$$

(29)

where we have switched the notation so that the variables $p, q$ in eq.(26) correspond to $y^1, y^2$. Accordingly $y^\alpha$’s play the role of the coordinates on the worldsheet.

On the other hand, in the D-string picture, we have $A_\alpha$ in addition to $\phi^i$. Since the vertex operator for $A_\alpha$ is $\partial_{\tau}X^\alpha A_\alpha(X)$, the deformation in this direction modifies the boundary state in eq.(29) as

$$|B> = \int [dy] \exp \left[ \frac{i}{4\pi} \int d\sigma A_\alpha(y) \partial_\sigma y^\alpha - \frac{i}{2\pi} \int_0^{2\pi} d\sigma P_\mu \phi^\mu(y) \right] |B>_{-1},$$

(30)

which coincides with eq.(29) when one has $F_{12} = 4\pi/k$.

The problem is what $A_\alpha$ is in the D-instanton picture. The vertex operator $\partial_{\tau}X^\alpha A_\alpha(X)$ in the D-string picture becomes $A_\alpha(y) \partial_\tau y^\alpha$, which is not a vertex operator in the D-instanton picture. Since the background $A_\alpha$ does not correspond to any vertex operator in the D-instanton picture, it appears that $A_\alpha$ cannot be interpreted as a deformation of the configuration in eq.(31). However one can derive the following identities:

$$0 = \int [dy] \frac{\delta}{\delta y^\alpha} e^{\frac{i}{\theta} \int d\sigma A_\alpha(y) \partial_\sigma y^\alpha - \frac{i}{2\pi} \int_0^{2\pi} d\sigma P_\mu \phi^\mu(y)} |B>_{-1}$$

$$= \int [dy] \left[ \frac{i}{4\pi} F_{\alpha\beta}(y) \partial_\beta y^\alpha - \frac{i}{2\pi} P_\mu \partial_\alpha \phi^\mu \right] e^{\frac{i}{\theta} \int d\sigma A_\alpha(y) \partial_\sigma y^\alpha - \frac{i}{2\pi} \int_0^{2\pi} d\sigma P_\mu \phi^\mu(y)} |B>_{-1}.$$

Hence the operator $\int d\sigma \partial_{\tau}X^\alpha \phi_\alpha(X)$ which corresponds to variation $\delta A$ in the D-string picture is equivalent to

$$2 \int d\sigma \delta A_\alpha(F^{-1})^{\alpha\beta} \partial_\beta \phi^\mu(y) P_\mu,$$

(31)

when operating on $|B>$. Here we assume that the background $F_{\alpha\beta}(y)$ is invertible as a $2 \times 2$ matrix for all $y$.

Therefore, at least perturbatively, variations of the gauge field $A_\alpha$ can be interpreted as variations of $\phi^\mu$ as long as $F_{\alpha\beta}(y)$ is invertible. This condition for $F_{\alpha\beta}(y)$ has the following meaning. Let us take a look at the expression of the boundary state in eq.(31). Being a path integral over the variables $y^\alpha(\sigma)$, one can evaluate the right-hand-side of eq.(31) by canonically quantizing $y^\alpha(\sigma)$. In doing so, the symplectic form to be used can be read off to be $F_{\alpha\beta}$. Therefore the quantum mechanics of $y^\alpha(\sigma)$ is well-defined if $F_{\alpha\beta}$ is invertible. This condition is satisfied, for example, by the background in eq.(31).
Then what kind of deformation of $\phi^\mu$ does $A_\alpha$ correspond to? It is easy to see from eq.(31) that the gauge field is related to the reparametrization of the worldsheet and the variations $\delta A_\alpha$ are equivalent to the variations

$$\delta y^\alpha = (F^{-1})^{\alpha\beta}\delta A_\beta,$$

(32)
of the coordinates $y^\alpha$. This is consistent with the results in [9], when $F_{\alpha\beta}$ is constant.

Conversely, we can say that the gauge field should be transformed as

$$\delta A_\alpha = F_{\alpha\beta}\delta y^\beta,$$

(33)
under the reparametrization $y^\alpha \to y^\alpha + \delta y^\alpha$ in the D-instanton picture. Eq.(33) was considered for the global transformation in [10]. This transformation law can be rewritten as

$$\delta A_\alpha = -\partial_\alpha\delta y^\beta A_\beta - \delta y^\beta\partial_\beta A_\alpha + \partial_\alpha(\delta y^\beta A_\beta),$$

(34)
and the first two terms coincide with the usual variation of $A_\alpha$ under $y^\alpha \to y^\alpha + \delta y^\alpha$. Therefore eq.(33) gives the usual transformation law for gauge invariant quantities.

Now let us summarize. If one regards D-string as a configuration of infinitely many D-instantons, the deformation of such a configuration is parametrized by $\phi^\mu(y)$ ($\mu = 1, \cdots, 26$) in eq(28). If there existed invariance under diffeomorphism $y^\alpha \to y^\alpha + \delta y^\alpha$, the space of deformations would be

$$\text{space of } \phi^\mu(y)$$

(35)
where $Diff$ is the group of diffeomorphisms. However in order to regard D-string as a configuration of D-instantons (i.e. in order for eq.(34) to be well-defined), one should have $F_{\alpha\beta}(y)$ invertible for all $y$, which breaks the reparametrization invariance. Therefore the space of deformations become

$$\text{space of } \phi^\mu(y) \over Diff_F,$$

(36)
where $Diff_F$ denotes the group of diffeomorphisms which preserve $F_{\alpha\beta}(y)$. If one takes $F_{\alpha\beta}$ as the volume element on the worldsheet, $Diff_F$ corresponds to the group of area preserving diffeomorphisms.

On the other hand, the deformations of a D-string worldsheet are parametrized by $A_\alpha$ and $\phi^i$. The space of deformations is

$$(\text{space of } \phi^i(y)) \otimes \left( \text{space of } A_\alpha(y) \over G \right),$$

(37)
where $G$ denotes the group of gauge transformations. Eq.(36) is equivalent to eq.(37). Indeed eq.(36) can be rewritten as

$$\left( \text{space of } \phi^\mu(y) \over Diff \right) \otimes \left( Diff \over Diff_F \right).$$

(38)
The first factor can be shown to be equivalent to the first factor of eq.(37) by taking the static gauge $y^1 = \phi^1$, $y^2 = \phi^2$. The second factor is transformed to the second factor of eq.(37) via the relation eq.(32). Indeed the reparametrization which preserves $F_{\alpha\beta}(y)$ gives a variation $\delta A_\alpha$ which does not change $F_{\alpha\beta}(y)$ i.e. a gauge variation. Thus the D-string picture and the D-instanton picture are equivalent. These two pictures are related to each other by the nonlinear field redefinition in eq.(32).

We conclude this section by the following remarks. What we have done in this section can be done for any Dirichlet $p$-brane. As was mentioned at the end of the previous section, such a brane can be considered as a configuration of infinitely many Dirichlet $(p-2)$-branes. The worldvolume field content of Dirichlet $p$-brane consists of $A_\alpha (\alpha = 1,\ldots,p+1)$ and $\phi^i (i = p+2,\ldots,26)$. $A_\alpha (\alpha = 1,\ldots,p-1)$ and $\phi^i (i = p+2,\ldots,26)$ are common to the $p$-brane and the $(p-2)$-branes. Via the relation eq.(32), $A_\alpha (\alpha = p,p+1)$ on the $p$-brane worldvolume corresponds to part of the group of diffeomorphisms in the $p$-th and the $(p+1)$-th directions in the $(p-2)$-brane picture.

4 Discussions

In this paper we have proved that Dirichlet $p$-brane can be considered as a configuration of infinitely many Dirichlet $p$-branes. What was essential in the proof is the fact that the system eq.(8) with the Dirichlet boundary condition for $X^1$, $X^2$ is equivalent to the system eq.(10) with the Neumann boundary condition for $X^1$, $X^2$. This correspondence between Dirichlet and Neumann conditions is reminiscent of T-duality[1][11]. However there are several important differences. Firstly the correspondence discussed in this paper is between the open string with the Dirichlet condition and the open string with the Neumann condition with infinitely many Chan-Paton factors. Secondly the variables $X^1$, $X^2$ are transformed to themselves in the transformation eq.(25).

Since the open string theories are equivalent, everything about $p$-brane can be reinterpreted by regarding it as a configuration of $(p-2)$-branes. Particularly when $p$ is odd, Dirichlet $p$-brane can be recognized as a configuration of D-instantons. The D-instanton picture is convenient when one considers the worldvolume theory of the $p$-brane. The fields to be considered are $\phi^\mu$ and the spacetime Lorentz invariance is manifest. Moreover the locality of the interaction between D-branes and fundamental string is manifest in the form of the boundary state eq.(30). This form of the boundary state is useful in analysing such interactions. The gauge field on the worldvolume of the $p$-brane is related to the group of diffeomorphisms on the worldvolume via the relation eq.(32). This relation was derived when there exists one $p$-brane. If there are $N$ $p$-branes, the gauge field becomes a $U(N)$ gauge field. It is intriguing to consider the generalization of eq.(32) for this case. It seems that the $U(N)$ gauge field should correspond to the diffeomorphism group enlarged to include the statistics group on $N$ $p$-branes.
It is straightforward to supersymmetrize all the results in this paper. In the supersymmetrized version, we should generalize the notion of Chan-Paton factor a little bit. The relation eq.(32) also holds in the superstring case. We will discuss this in a separate publication.

Acknowledgements

We would like to thank H. Dorn, K. Hamada, H. Kawai, Y. Kazama, H. Kunitomo, K. Murakami, M. Natsuume and T. Oota for useful discussions and comments. This work was supported by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

References

[1] J. Dai, R. G. Leigh and J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073; J. Polchinski, hep-th/9510017, Phys. Rev. Lett. 75 (1995) 4724.

[2] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, hep-th/9610043, Phys. Rev. D55 (1997) 5112.

[3] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612113, Nucl. Phys. B498 (1997) 467.

[4] R. Dijkgraaf, E. Verlinde and H. Verlinde, hep-th/9703030, Nucl. Phys. B500 (1997) 43.

[5] P. K. Townsend, hep-th/9512062, Phys. Lett. B373 (1996) 68.

[6] B. de Wit, J. Hoppe and H. Nicolai, Nucl. Phys. B305[FS23] (1988) 545.

[7] H. Dorn and H.-J. Otto, hep-th/9603186, Phys. Lett. B381 (1996) 81; hep-th/9702018, Nucl. Phys. B(Proc. Suppl.) 56B (1997) 30; H. Dorn, hep-th/9612210, Nucl. Phys. B494 (1997) 105.

[8] A. Abouelsaood, C. G. Callan, C. R. Nappi and S. A. Yost, Nucl. Phys. B280[FS18] (1987) 599; V. V. Nesterenko, Int. Journ. Mod. Phys. A4 (1989) 2627.

[9] M. Li, hep-th/9612222, Nucl. Phys. B499 (1997) 149.

[10] M. Aganagic, C. Popescu and J. H. Schwarz, hep-th/9612080, Nucl. Phys. B495 (1997) 99.
[11] E. Alvarez, J. L. F. Barbon and J. Borlaf, hep-th/9603089, Nucl. Phys. B479 (1996) 218;
H. Dorn and H. J. Otto, hep-th/9603186 Phys. Lett. B381 (1996) 81;
S. Forste, A. A. Kehagias and S. Schwager, hep-th/9604013, Nucl. Phys. B478 (1996) 141, hep-th/9610062, hep-th/9611060;
J. Borlaf and Y. Lozano, hep-th/9607051, Nucl. Phys. B480 (1996) 239;
Y. Lozano, hep-th/9610024, Mod. Phys. Lett. A11 (1996) 2893.