Anomalous muon magnetic moment, supersymmetry, naturalness, LHC search limits and the landscape

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Abstract

The recent measurement of the muon anomalous magnetic moment $a_{\mu} = (g-2)_{\mu}/2$ by the Fermilab Muon $g-2$ experiment sharpens an earlier discrepancy between theory and the BNL E821 experiment. We examine the predicted $\Delta a_{\mu} \equiv a_{\mu}(\text{exp}) - a_{\mu}(\text{th})$ in the context of supersymmetry with low electroweak naturalness (restricting to models which give a plausible explanation for the magnitude of the weak scale). A global analysis including LHC Higgs mass and sparticle search limits points to interpretation within the normal scalar mass hierarchy (NSMH) SUSY model wherein first/second generation matter scalars are much lighter than third generation scalars. We present a benchmark model for a viable NSMH point which is natural, obeys LHC Higgs and sparticle mass constraints and explains the muon magnetic anomaly. Aside from NSMH models, then we find the $(g-2)_\mu$ anomaly cannot be explained within the context of natural SUSY, where a variety of data point to decoupled first/second generation scalars. The situation is worse within the string landscape where first/second generation matter scalars are pulled to values in the $10^{-50}$ TeV range. An alternative interpretation for SUSY models with decoupled scalar masses is that perhaps the recent lattice evaluation of the hadronic vacuum polarization could be confirmed which leads to a Standard Model theory-experiment agreement in which case there is no anomaly.
1 Introduction

The recent measurement of the muon anomalous magnetic moment \( a_\mu \equiv (g - 2)_\mu / 2 \) by the Fermilab Muon \( g - 2 \) Collaboration experiment E989 \(^1\) has confirmed previous results from the BNL E821 experiment \(^2\) which had found a 3.7\( \sigma \) discrepancy between the experimental observation and the theoretical prediction, taken from the Muon \( g - 2 \) Theory Initiative \(^3\).\(^4\). \n
The latter relies mainly on using dispersive techniques applied to \( e^+e^- \rightarrow \text{hadrons} \) data in the vicinity of \( \sqrt{s} \sim 1 \) GeV to evaluate the hadronic vacuum polarization (HVP) contribution to \( a_\mu \) which contributes the largest uncertainty to the overall \( a_\mu \) calculation. The uncertainty arises in part because the hadronic cross section lies in the non-perturbative/semi-perturbative regime where a variety of hadronic resonances lurk. The E989 result makes it more implausible that the experimental value is the result of systematic errors. By combining Fermilab and BNL results, then the quoted discrepancy is given by \(^1\)

\[
a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (25.1 \pm 5.9) \times 10^{-10}
\]

(1)

 corres ponding to a 4.2\( \sigma \) effect. While this latter result was obtained by comparing experiment to the Theory Initiative value for \( a_\mu \), we note here that recent lattice evaluations of the HVP contribution to \( a_\mu \) find a theoretical value which is in accord with the Fermilab/BNL measured values \(^6\).\(^7\).

An exciting possibility is to account for the muon \( g - 2 \) anomaly by positing the existence of weak scale supersymmetry \(^7\), wherein each Standard Model (SM) field is elevated to a superfield containing both fermionic and bosonic components. The additional Higgs fermion (higgsinos) would destroy the heralded triangle anomaly cancellation within the SM, so an additional Higgs doublet is needed as well. Then, the SM \( \mu - \mu - \gamma \) triangle diagrams which contribute to \( a_\mu \) are augmented by sparticle loops containing \( \tilde{\chi}_{1,2} - \tilde{\nu}_\mu \) and \( \tilde{\chi}_0^{1,2,3,4} - \tilde{\mu}_{L,R} \) pairs \(^8\).\(^9\).\(^2\)

The SUSY contribution is given roughly by \(^2\)

\[
\Delta a_\mu^{\text{SUSY}} \sim \frac{m_{\mu}^2 \mu M_i \tan \beta}{m_{\text{SUSY}}^4}
\]

(2)

where \( \mu \) is the superpotential \( \mu \) parameter, \( M_i \) is the gaugino mass for gauge group \( i \), \( \tan \beta \) is the ratio of Higgs field vevs and \( m_{\text{SUSY}} \) is a loop average of scalar muons, mu-sneutrinos and electroweakinos. By performing a rather general scan over weak scale Minimal Supersymmetric Standard Model (MSSM) parameters, then it is found that in order to explain the muon \( g - 2 \) anomaly, the mass of the lightest observable SUSY particles \( m_{\text{LOSP}} \) must be \( m_{\text{LOSP}} \lesssim 900 \) GeV \(^{20}\).\(^{24}\). This result brings in considerable tension with recent LHC search results which so far find no direct evidence for SUSY particle production at ATLAS or CMS \(^{25}\). At present, LHC searches require slepton masses \( m_{\tilde{\mu}_{L,R}} \tilde{\nu}_\mu \gtrsim 700 \) GeV from 139 fb\(^{-1}\) of data at \( \sqrt{s} = 13 \) TeV \(^{26}\).\(^{27}\). These mass limits are considerably and even entirely relaxed in the case of degeneracy where \( m_{\tilde{\mu}_{L,R}} \approx m_{\tilde{\chi}_1^0} \) (dark matter coannihilation region).

While it is possible to interpret the presence of the Fermilab/BNL \( (g - 2)_\mu \) anomaly in terms of light smuons and mu-sneutrinos along with light electroweakinos, such an explanation seems

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1 In Ref. \(^6\), it is emphasized that a shift in the SM value of HVP to gain accord with the \( (g - 2)_\mu \) measurement would worsen global fits to EW data.

2 For some older related references, see \(^9\).\(^{22}\).
increasingly implausible in light of the global picture of SUSY theory as compared with a large array of data from many experiments along with theoretical considerations.

- **LHC sparticle search limits:** We have already remarked that searches for slepton pair production require \( m_{\tilde{\mu}_{L,R}} \gtrsim 700 \text{ GeV} \) (unless one lives in the degeneracy region where \( m_{\tilde{\mu}_{L,R}} \sim m_{\tilde{\chi}_1^0} \)). But also with 139 fb\(^{-1}\) at \( \sqrt{s} = 13 \text{ TeV} \), ATLAS and CMS now require gluinos \( m_{\tilde{g}} \gtrsim 2.25 \text{ TeV} \) and top squarks \( m_{\tilde{t}_1} \gtrsim 1.1 \text{ TeV} \) [25]. While it is certainly possible that \( m_{\tilde{t}_{L,R}} \ll m_{\tilde{g}}, m_{\tilde{t}_1} \), such a scenario would imply large splitting in the slepton/squark sector. Such splitting seems especially unlikely when sfermions occupy the 16-dimensional spinor of \( SO(10) \) which provides a beautiful unification of all members of each generation.

- **Higgs boson mass:** The LHC measured value of the light Higgs scalar is \( m_h = 125.10 \pm 0.14 \text{ GeV} \). Such a large mass famously requires TeV-scale top-squarks and large mixing [28] (large trilinear soft terms \( A_t \)) or else small mixing but top-squarks in the tens-to-hundreds of TeV, far beyond any naturalness limits [29]. Again, a large squark-slepton mass splitting would be required to explain both the \( (g - 2)_\mu \) anomaly and the Higgs mass.

- **Higgs couplings:** LHC measurements of Higgs boson couplings find them to be increasingly SM-like [30, 31]. In contrast, if SUSY particles are light, then one expects deviations to occur, especially for light scalars \( H, A \) which would lead to large mixing in the Higgs sector [32]. While SM-like Higgs couplings can also be explained via alignment [33], the decoupling solution seems more plausible.

- **Flavor-changing B decays:** The decay \( b \rightarrow s\gamma \) is particularly interesting in that SM and SUSY loops contribute at the same level, so it is a natural place to look for BSM deviations. The decay \( b \rightarrow s\gamma \) takes place via \( tW^- \) loops in the SM and via \( \tilde{t}_i\tilde{\chi}_j^- \) and \( tH^- \) loops in SUSY [34] (other SUSY loops also contribute but typically with much smaller amplitudes). The SM value for this decay is found to be [35] \( BF(b \rightarrow s\gamma) = (3.36 \pm 0.23) \times 10^{-4} \) which is to be compared to the recent Belle measurement [36] that \( BF(b \rightarrow s\gamma) = (3.01 \pm 0.22) \times 10^{-4} \). The overlap between theory and experiment are in accord with top-squarks and charged Higgs bosons in the multi-TeV regime [37].

- **FCNCs from SUSY:** Flavor changing neutral current interactions (FCNC) are notably absent in the SM due to the GIM (Glashow-Iliopoulos-Maiani) mechanism, but generically ought to be present in SUSY extensions [38] unless there is 1. flavor universality in the scalar sector or 2. alignment between quark and squark mass matrices [39] or 3. decoupling due to sfermions in the 10-100 TeV regime [40,41]. In generic gravity-mediated SUSY breaking models, non-universality is the rule while very special circumstances must be realized for alignment. Meanwhile, the decoupling solution is found to be consistent with the electroweak naturalness measure \( \Delta_{EW} \) since first/second generation contributions to the weak scale are all suppressed due to the small Yukawa couplings. Models such as gauge-mediated SUSY breaking (GMSB) and minimal anomaly-mediation (mAMSB) that do conserve flavor seem excluded in the former case by the Higgs mass constraint and
naturalness \[42\] and in the latter case by the Higgs mass constraint, naturalness and the presence of a wino-like lightest SUSY particle (LSP) \[43\].

- **SUSY CP violation:** Likewise, SUSY soft terms are expected to be complex in general leading to contributions to low energy CP violating processes \[38\]. The offending CP violating processes are all suppressed by a scalar mass decoupling solution with scalar masses in the several-TeV regime.

- **Cosmological gravitino problem:** In gravity-mediation, the gravitino mass \(m_{3/2}\) sets the mass scale for the soft breaking terms, and hence the sparticle masses. But if the gravitino is too light—below the TeV scale—then it can be long-lived. Then gravitinos produced thermally in the early universe will decay after the culmination of Big Bang nucleosynthesis (BBN) and disrupt the successful BBN predictions of the hot Big Bang model. Increasing \(m_{3/2}\) above about \(m_{3/2} \gtrsim 5\) TeV is usually sufficient in order to shorten the gravitino lifetime so that it decays before BBN \[44,45\].

- **Cosmological moduli problem:** Light moduli fields—gravitationally coupled string remnant scalar fields from the compactification of extra dimensions—may also exist (where by light we mean of order the soft SUSY breaking terms). Any light modulus could be produced in the early universe due to coherent scalar field oscillations, and if they are too light—typically below about 30 TeV—then like gravitinos, they would disrupt the successful light element production in the early universe \[46\].

- **Naturalness allows heavy first/second generation matter scalars:** Heavy scalar fields in SUSY models seemingly violate older notions of naturalness which was the motivation for sparticles at or around the weak scale (which would provide large contributions to \((g-2)_\mu\)). However, naturalness based on tuning of parameters \(p_i\), \(\Delta = \max_i |\partial \log m_Z^2/\partial \log p_i|\), was typically based on numerous free soft term parameters in the low energy SUSY effective field theory (EFT). In string theory, in our universe, the soft terms are expected to be all correlated and calculable in the case of stabilized moduli. Alternatively, naturalness evaluations requiring logarithmic radiative corrections to the light Higgs mass to be small would oversimplify and neglect that the SUSY radiative corrections depend on the Higgs soft mass itself (radiative corrections not independent of tree-level masses) and further would have to extrapolate radiative corrections across the electroweak phase transition (which is not the case in the SM). By using the more conservative \(\Delta_{EW}\) measure \[47,48\], then few-TeV scale top-squarks and 10 – 50 TeV first/second generation matter scalars are allowed.

- **Weak scale SUSY from the string landscape:** The presence of a vast landscape of metastable string vacua now seems inescapable \[49\]. The idea has some experimental support in that it allows for a solution to the cosmological constant problem and indeed even predicts its value to within a factor of a few which is confirmed by dark energy measurements. The

\[3\] Note that natural AMSB models which include a bulk trilinear term and non-universal bulk Higgs masses are consistent with naturalness, Higgs mass and dark matter constraints \[42\]. They have a higgsino rather than a wino as LSP.
landscape is also expected to favor a power-law draw to large soft terms which are limited by the ABDS\cite{50} anthropic window of not-to-large a value of the weak scale in pocket universes which are compatible with atoms as we know them (atomic principle). In this case, statistical predictions can be made for Higgs and superparticle masses. Indeed, it is found that the landscape statistically favors a light Higgs scalar with mass $m_h \sim 125$ GeV with sparticles (save light higgsinos) generally well beyond LHC search limits\cite{51}. The landscape pulls first/second generation matter scalar masses into the $10-50$ TeV regime. This would lead to only tiny SUSY contributions to $(g-2)_\mu$.

Thus, to summarize the above bullet points, there are a variety of reasons from both theory and experiment to expect that nature is supersymmetric, but with SUSY particles, save light higgsinos, in the $\sim (1-50)$ TeV range.

In this paper, we examine whether it is possible to reconcile the latest measurement of the muon $(g-2)_\mu$ anomaly with weak scale supersymmetry in light of recent LHC search limits and while requiring weak scale naturalness (in that unnatural models, while logically possible, are highly implausible). We also examine $(g-2)_\mu$ in the recent context of the string theory landscape. While the above bullet points indicate TeV-scale soft terms, the muon $(g-2)_\mu$ anomaly requires sub-TeV muonic scalars\cite{24}. These requirements were reconciled long ago within the context of normal scalar mass hierarchy models wherein the first two generations of scalars were much lighter than third generation scalars\cite{52}. Such a situation is possible within the three-extra-parameter non-universal Higgs model (NUHM3)\cite{54,59}, which also allows for EW naturalness and which we adopt for much of our analysis. The NUHM3 model is embedded in the computer code Isajet 7.88\cite{60} which we use for sparticle spectra generation and for evaluation of $\Delta a_\mu^{\text{SUSY}}$.

A number of other papers have recently appeared regarding a SUSY interpretation of the Fermilab/BNL $(g-2)_\mu$ anomaly\cite{61,79}.

2 A benchmark SUSY point which can explain $(g-2)_\mu$ anomaly

In Table 1, we show the spectra from a NUHM3 benchmark point which fulfills 1. LHC Higgs mass and sparticle limits, 2. is natural ($\Delta_{\text{EW}} = 29.9$) 3. provides sufficient contribution to explain the Fermilab/BNL $(g-2)_\mu$ anomaly and 4. is dark matter allowed\cite{57}. It has a higgsino-like LSP so that there is a deficit of neutralino dark matter. This leaves room for axions (thus two dark matter particles, axions and WIMPs) which result from an assumed solution to the strong CP problem\cite{81}. This benchmark point has $m_{\tilde{g}} \sim 2340$ GeV and $m_{\tilde{t}_1} \sim 1348$ GeV but has a spectrum of light sleptons. The unusual pattern of slepton masses occurs due to a strong influence of the SUSY renormalization group $S$-term (see p. 206 of Ref.\cite{7}) which is zero in

\footnote{see also Ref.\cite{53}.

\footnote{This point is generated using a modified version of Isajet 7.78\cite{60} where a common scale choice $Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$ is used to evaluate the Higgs mass and sparticle self-energies\cite{80}. In an earlier version, the highly split spectrum lead to unstable radiative corrections due to very large logs in the self-energy corrections that arose due to the strong disparity in mass scales. We thank C. Wagner for his sharp eyes.}
models with scalar mass universality but is large in this case due to the large splitting between $m_{H_u}^2$ and $m_{H_d}^2$. This RGE term boosts up the right-slepton mass to over a TeV while the left-sleptons have masses below 300 GeV. In this case, the $\mu$ parameter is large enough that the mu-sneutrino lies in the quasi-degeneracy region of the LHC slepton search exclusion plot where visible slepton decay products become soft enough so as to evade the bounds. Furthermore, the dominant decay mode is actually to invisibles: $\tilde{\nu}_\mu L \rightarrow \nu_\mu \tilde{\chi}_1^0$. The left smuon lies precisely on the edge of the LHC slepton excluded region. However, its dominant decay mode is $\tilde{\mu}_L \rightarrow \tilde{\chi}_1^- \nu_\mu$ (where the $\tilde{\chi}_1^-$ visible decay products are extremely soft due to the small mass gap $m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^-} \sim 13$ GeV) which is radically different from the assumed simplified models used in the ATLAS/CMS exclusion plots. Thus, the point should at present be allowed by LHC slepton searches.

The point is perhaps artificial in that the first two generations are taken as light and degenerate whilst the third generation is far heavier. This point is an example of a normal scalar mass hierarchy model \[52\] which was introduced to reconcile $\Delta a_{\text{SUSY}}^\mu$ (which requires light smuons) with the near match in theory-experiment for $b \rightarrow s\gamma$ decays (which requires heavier top-squarks to suppress non-standard contributions). Nowadays, it also helps to avoid LHC top-squark search limits and to explain the large light Higgs mass $m_h$ which in SUSY requires TeV-scale highly mixed stops to lift its mass to $\sim 125$ GeV. Naively, one might expect such a point to lead to detectable amounts of lepton-flavor-violation (LFV) \[82,83\].

3 \((g - 2)_\mu\) from natural SUSY

In this Section, we show the expected value of $\Delta a_{\mu}^{\text{SUSY}}$ from SUSY with electroweak naturalness $\Delta_{EW} \lesssim 30$. This measure requires the various independent SUSY contributions to the weak scale to be comparable to or less than the weak scale:

$$m_Z^2/2 = \frac{m_{H_u}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u(\tilde{t}_1, \tilde{t}_2) - \mu^2$$

where

$$\Delta_{EW} = \text{max}|\text{terms on right hand side of Eq. 3}|/(m_Z^2/2).$$

The $\Sigma_d^d$ and $\Sigma_u^u$ contain over 40 1-loop corrections which are listed in Ref. \[48\].

To proceed, we adopt the two- and three-extra parameter non-universal Higgs mass models which allow the presence of small higgsino masses $\mu$ (typical SUSY spectrum generators will tune the value of $\mu$ to exactly the right large unnatural value so as to ensure a $Z$-boson mass of $m_Z = 91.2$ GeV). The NUHM2 parameter space is given by

$$m_0, m_{1/2}, A_0, \tan \beta, m_{H_u}, m_{H_d}$$

where the scalar potential minimization conditions allow one to trade $m_{H_u}$ and $m_{H_d}$ for the more convenient weak scale variables $\mu$ and $m_A$. The NUHM3 model goes a step further and allows a common mass $m_0(1, 2)$ for first/second generation matter scalars and an independent mass $m_0(3)$ for third generation scalars.
| parameter                      | NUHM3 |
|-------------------------------|-------|
| $m_0(1, 2)$                  | 368   |
| $m_0(3)$                     | 2955  |
| $m_{1/2}$                    | 1055  |
| $A_0$                        | -4370 |
| $\tan \beta$                | 26    |
| $\mu$                        | 230   |
| $m_A$                        | 5365  |
| $m_{\tilde{g}}$              | 2341.6|
| $m_{\tilde{a}_L}$           | 2148.8|
| $m_{\tilde{a}_R}$            | 1848.6|
| $m_{\tilde{e}_R}$, $m_{\tilde{\mu}_R}$ | 1124.9|
| $m_{\tilde{e}_L}$, $m_{\tilde{\mu}_L}$ | 297.6 |
| $m_{\tilde{\nu}_L}$, $m_{\tilde{\nu}_R}$ | 284.9|
| $m_{\tilde{t}_1}$           | 1347.5|
| $m_{\tilde{t}_2}$           | 2613.0|
| $m_{\tilde{b}_1}$           | 2647.6|
| $m_{\tilde{b}_2}$           | 3192.0|
| $m_{\tilde{\tau}_1}$       | 2648.3|
| $m_{\tilde{\tau}_2}$       | 2704.0|
| $m_{\tilde{\nu}_r}$        | 2695.6|
| $m_{\tilde{\chi}_2^\pm}$   | -864.7|
| $m_{\tilde{\chi}_1^\pm}$   | -240.0|
| $m_{\tilde{\chi}_1^0}$     | -875.4|
| $m_{\tilde{\chi}_2^0}$     | -460.7|
| $m_{\tilde{\chi}_3^0}$     | 239.7 |
| $m_{\tilde{\chi}_2}$       | -227.1|
| $m_{\tilde{\chi}_1}$       | 124.1 |
| $\Delta a^\text{SUSY}_u$    | $18.6 \times 10^{-10}$ |
| $BF(b \rightarrow s\gamma) \times 10^4$ | 2.8 |
| $BF(B_s \rightarrow \mu^+\mu^-) \times 10^9$ | 3.8 |
| $\Omega_{\tilde{\chi}_1^0}^\text{std} h^2$ | 0.015 |
| $\sigma^{SI}(\tilde{\chi}_1^0, p)$ (pb) | $2.7 \times 10^{-9}$ |
| $\sigma^{SD}(\tilde{\chi}_1^0, p)$ (pb) | $5.2 \times 10^{-5}$ |
| $\langle |\sigma v| \rangle_{v \rightarrow 0}$ (cm$^3$/sec) | $1.6 \times 10^{-25}$ |
| $\Delta_{\text{EW}}$         | 29.9  |

Table 1: Input parameters and masses in GeV units for a natural NUHM3 SUSY benchmark point with a normal scalar mass hierarchy and with $m_t = 173.2$ GeV.
We next scan over the following NUHM2,3 parameter space.

\[
\begin{align*}
  m_0 &: 0 - 13 \text{ TeV} \\
  m_{1/2} &: 0 - 3 \text{ TeV}, \\
  -A_0 &: 0 - 20 \text{ TeV}, \\
  m_A &: 0 - 10 \text{ TeV}, \\
  \mu &: 100 - 360 \text{ GeV} \\
  \tan \beta &: 3 - 60.
\end{align*}
\]

For NUHM3, we additionally scan on \(m_0(1,2) : 0 - 55 \text{ TeV}\). The scan upper limits are taken beyond the upper limits imposed by naturalness (\(\Delta_{\text{EW}} < 30\)) so are not artificial. For surviving scan points, we only require an appropriate breakdown of electroweak symmetry and the Higgs mass to lie within its measured range \(123 \text{ GeV} < m_h < 127 \text{ GeV}\) (allowing for \(\pm 2 \text{ GeV}\) theory uncertainty in the Higgs mass calculation).

Our first results are shown in Fig. 1, where we show our NUHM2,3 scan points in the \(\Delta_{\text{EW}}\) vs. \(\Delta a_{\mu}^{\text{SUSY}}\) plane. The region between the dashed lines denotes the combined BNL/Fermilab measurement (at 2\(\sigma\) significance). The gray points denote NUHM2,3 points with naturalness \(\Delta_{\text{EW}} < 30\) and light Higgs boson mass within the range \(m_h : 123 - 127 \text{ GeV}\) but which otherwise are excluded by LHC search limits. The orange points are from the NUHM2 model which forces degeneracy between the first/second generation matter scalars (second generation scalars mainly contribute to the \((g - 2)_\mu\) anomaly) and third generation scalars (whose contributions \(\Sigma^\mu_u(\tilde{t}_1,2)\) impact upon naturalness). The NUHM3 points, which relax this degeneracy, are denoted as blue. Both the orange NUHM2 points and the blue NUHM3 points satisfy in addition LHC search constraints that

- \(m_{\tilde{g}} > 2.25 \text{ TeV}\),
- \(m_{\tilde{t}_1} > 1.1 \text{ TeV}\) and
- \(m_{\tilde{\ell}} > 700 \text{ GeV}\).

Finally, the green points denote NUHM3 points which obey the above LHC constraints except the last one, where \(m_{\tilde{\ell}} < 700 \text{ GeV}\) is allowed if one lives in the quasi-degeneracy region where \(m_{\tilde{\ell}} \sim m_{\tilde{\chi}^0_1}\) and/or if the slepton branching fractions deviate significantly from the ATLAS/CMS simplified model assumptions. These points, like our benchmark point, can allow the anomalous muon magnetic moment to occupy the 2\(\sigma\) band while still respecting electroweak naturalness and LHC Higgs and sparticle mass limits.

While many of the gray points populate the BNL/Fermilab band, they are all excluded by LHC search limits. Also, the bulk of blue and orange points have \(\Delta a_{\mu}^{\text{SUSY}}\) values well below the anomalous region. When the Higgs mass constraint is applied to NUHM2 points, then sleptons are too heavy to generate the preferred anomaly band, so the orange points all lie well below the \((g - 2)_\mu\) 2\(\sigma\) band. However, if we relax generational degeneracy (blue and green points), then we can allow for the normal scalar mass hierarchy model. In this case, the blue points almost reach the BNL/Fermilab 2\(\sigma\) band; the green points are then the subset of NUHM3
Figure 1: Value of $\Delta a_\mu^{\text{SUSY}}$ from natural SUSY vs. naturalness measure $\Delta_{\text{EW}}$. Gray dots are from NUHM2,3 models with $123 \text{ GeV} < m_h < 127 \text{ GeV}$ but which violate LHC search limits. Orange (NUHM2) and blue (NUHM3) points are LHC-allowed. Green (NUHM3) points are LHC-allowed, natural, have the right Higgs mass and occupy the $(g - 2)_\mu$ 2\sigma band.

points which fulfill all LHC bounds, the naturalness condition and explain the BNL/Fermilab $(g - 2)_\mu$ anomaly.

In Fig. 2, we show the distribution of scan points in the $m_\tilde{g}$ vs. $\Delta a_\mu^{\text{SUSY}}$ plane. The color-coding is as in Fig. 1. The ATLAS/CMS limit that $m_\tilde{g} \gtrsim 2.25 \text{ TeV}$ is denoted by the vertical line. In order to obtain $m_h \sim 125 \text{ GeV}$ within the orange NUHM2 points, the soft breaking stop mass terms must be in the several TeV regime and also have large mixing from a large $A_0$ parameter. The first of these tends to pull the sleptons up to high masses which suppress $\Delta a_\mu^{\text{SUSY}}$. This effect can be avoided by allowing non-universal generations $m_0(1,2) \neq m_0(3)$ whereby heavier stops and $m_h \sim 125 \text{ GeV}$ can co-exist with lighter sleptons, as in the LHC-allowed blue and green points. Once the LHC sparticle and Higgs mass constraints are imposed, along with naturalness, then we see that the green points which lie within the BNL/Fermilab anomalous region allow for gluino masses up to 3.5 TeV. This is in contrast to naturalness-only upper bounds on $m_\tilde{g}$ which allow for $m_\tilde{g}$ to range up to $\sim 6 \text{ TeV}$.

In Fig. 3, we show $\Delta a_\mu^{\text{SUSY}}$ vs. light stop mass $m_{\tilde{t}_1}$. The color-coding is again as in Fig. 1. The EW natural points with the allowed $m_h$ value occur for $m_{\tilde{t}_1} \lesssim 3 \text{ TeV}$—well beyond HL-LHC projected reach values. For the orange NUHM2 points with universality, then the larger $m_{\tilde{t}_1}$ becomes, the lower is the allowed $\Delta a_\mu^{\text{SUSY}}$. But for the NUHM3 points which allow for $m_0(1,2) < m_0(3)$, then green points which fulfill naturalness, LHC Higgs and sparticle search bound are found which can explain the $(g - 2)_\mu$ anomaly. These points actually have a much
Figure 2: Value of $\Delta a_\mu^{\text{SUSY}}$ from natural SUSY vs. gluino mass $m_{\tilde{g}}$. Gray dots are from NUHM2,3 models with $123 \text{ GeV} < m_h < 127 \text{ GeV}$ but which violate LHC search limits. Orange (NUHM2) and blue (NUHM3) points are LHC-allowed. Green (NUHM3) points are LHC-allowed, natural, have $m_h : 123 - 127 \text{ GeV}$ and occupy the $(g - 2)_\mu$ $2\sigma$ band.

reduced upper bound on $m_{\tilde{t}_1}$ where $m_{\tilde{t}_1} \lesssim 1.7 \text{ TeV}$. This is good news for HL-LHC top-squark searches since their projected reach with $3000 \text{ fb}^{-1}$ is to $m_{\tilde{t}_1} \sim 1.7 \text{ TeV}$ [84].

In Fig. 4 we show the distribution of $\Delta a_\mu^{\text{SUSY}}$ versus the left smuon mass $m_{\tilde{\mu}_L}$. The vertical line denotes the LHC limit that $m_{\tilde{\ell}} > 700$ for the non-degeneracy region and assuming the simplified model decay $\ell \rightarrow \ell \tilde{\chi}^0_1$. The orange NUHM2 points almost always have far heavier slepton masses $m_{\tilde{\ell}} \gtrsim 700 \text{ GeV}$. Meanwhile, the blue and green NUHM3 points allow for much lighter sleptons while fulfilling LHC search constraints. The green points are still LHC-allowed because, even with $m_{\tilde{\ell}} < 700 \text{ GeV}$, they can lie in the quasi-degeneracy region where $m_{\tilde{\ell}} \sim m_{\tilde{\chi}^0_1}$ or they have decay branching fractions which deviate significantly from LHC simplified model assumptions. Overall, we see the well-known result that the $(g - 2)_\mu$ anomaly can only be explained in models with $m_{\tilde{\mu}_L} \lesssim 900 \text{ GeV}$ (as shown long ago by Feng and Matchev [24]). The bulk of models have far higher smuon and mu-sneutrino masses and consequently much tinier contributions to $\Delta a_\mu^{\text{SUSY}}$.

4 Expected value of $\Delta a_\mu^{\text{SUSY}}$ from the landscape

Next we move to the predictions of string theory for the muon anomalous magnetic moment. We will appeal to the string theory landscape [85], wherein string flux compactifications [49] lead to an enormous number of metastable string vacua, say $10^{500} - 10^{272,000}$, each with different
moduli vevs and hence each with varying $4-d$ laws of physics. The existence of the landscape is by now widely accepted in the string community and indeed it provides the only plausible solution to the cosmological constant problem via Weinberg’s anthropic reasoning \[86\].

Next, we will assume a fertile patch of $4-d$ landscape of vacua including the MSSM as the low energy EFT. For spontaneous SUSY breaking in such a patch via complex-valued $F$-terms $F_i$ and real values $D$-terms $D_\alpha$, then the overall SUSY breaking scale $m_{soft}$ is statistically favored for large values as a power-law \[87\] $f_{SUSY} \sim m_{soft}^n$ where the exponent $n = 2n_F + n_D - 1$. Then, even for the textbook case of SUSY breaking via a single $F$-term, one expects an $n = 1$ draw to large soft breaking. However, the draw to large soft breaking must be tempered by the anthropic requirement of not-to-large a value of the weak scale in each pocket universe of our fertile patch. The Agrawal et al. \[50\] (ABDS) anthropic window requires that the pocket universe weak scale be not-too-far from the measured weak scale value in our universe: $m_{\text{PU weak}} < (2-5)m_{\text{OU weak}}$. For too large a value of the weak scale, then complex nuclei are no longer allowed and atoms as we know them would not arise (atomic principle). To be specific, we take $m_{\text{PU weak}} < 4m_{\text{OU weak}}$ which corresponds to EW finetuning measure $\Delta_{\text{EW}} < 30$. Under such a scenario, one can make statistical predictions for Higgs and sparticle masses. For $n \sim 1-4$ \[88\] or even a log distribution \[89\] of soft parameters, one gains a peak for $m_h \sim 125$ GeV with sparticle masses lifted beyond present LHC search limits.

In Fig. 3 we show the string landscape prediction for $\Delta a_\mu^{\text{SUSY}}$ assuming that the low
energy EFT is the NUHM3 model. The different soft terms should scan independently of each other \[90\]. The first/second generation soft terms \(m_0(1,2)\) are pulled to very large values \(\sim 10 - 50\) TeV since they contribute to the weak scale via tiny Yukawa-suppressed terms or via two-loop RGEs. Then the smuons become correspondingly very heavy and the prediction is for a tiny SUSY contribution to the \((g-2)_\mu\) anomaly. From the Figure, we see a peak probability distribution around \(\Delta a_\mu \sim 5 \times 10^{-13}\) with almost no probability extending to the anomaly region. While this is bad for explaining the \((g-2)_\mu\) anomaly, it does provide a mixed decoupling/quasi-degeneracy solution to the SUSY flavor and \(CP\) problems \[91\].

5 Conclusions

In this paper, we have examined the possibility of natural SUSY and also landscape SUSY to explain the BNL/Fermilab \((g-2)_\mu\) anomaly. We focus on natural SUSY models since they fulfill the plausibility requirement of also providing a natural explanation for why the weak scale \(m_{W,Z,h} \sim 100\) GeV and not in the TeV regime (thus solving the Little Hierarchy Problem). We presented an example in Table 1 of a natural SUSY point which explains the \((g-2)_\mu\) anomaly while respecting LHC Higgs mass and sparticle search limits, while remaining natural. It is characterized by a normal scalar mass hierarchy where \(m_0(1,2) \ll m_0(3)\) leads to light sleptons. The light sleptons are still LHC-allowed because either they lie in the quasi-degeneracy region
where $m(\text{slepton}) \sim m(LSP)$ and/or their branching fractions deviate significantly from the ATLAS/CMS simplified model assumption.

Otherwise, for the natural models, it is very hard to explain the anomaly while simultaneously requiring the calculated value of $m_h \sim 123 - 127$ GeV and also obeying LHC constraints that $m_{\tilde{g}} > 2.25$ TeV, $m_{\tilde{t}_1} > 1.1$ TeV and $m_{\tilde{\ell}} > 700$ GeV.\(^6\) Basically, the LHC Higgs and sparticle mass constraints favor multi-TeV soft terms which then also should pull the smuon sector to large masses which then suppresses the $\Delta a_{\mu}^{\text{SUSY}}$ anomaly contribution. One can in principle explain the anomaly within SUSY by working in the pMSSM and taking light smuons and light LSP while the remaining sparticles are in the TeV-and-beyond region. But such models with uncorrelated spectra may be pathological (in that they disregard the RGEs and hence may miss out on gauge coupling unification and radiative electroweak symmetry breaking and a light Higgs mass determined by correlations between parameters) so it is unclear if they should be taken seriously.

By moving to landscape SUSY, matters get even worse. The landscape is expected to pull soft terms to large values via a power-law or logarithmic draw until they start providing too large of contributions to the weak scale. This picture predicts a Higgs mass $m_h \sim 125$ GeV and sparticles beyond present LHC bounds. But since first/second generation sfermions contribute to the weak scale only via Yukawa suppressed terms or two-loop RGE contributions, their masses get pulled into the $10 - 50$ TeV regime which results in only tiny contributions to the

\(^6\)For a related analysis of LHC constraints and $\Delta a_{\mu}^{\text{SUSY}}$, see Ref. [92].
$(g - 2)_\mu$ anomaly.

By adopting the big picture, LHC Higgs and sparticle mass limits coupled with naturalness make an interpretation within weak scale SUSY seem rather implausible (but not impossible, as shown by our benchmark point and the green points of Sec. [3]). Another possible resolution of the BNL/Fermilab $(g - 2)_\mu$ anomaly which allows for decoupled sleptons is that perhaps the lattice calculations [5]—which lead to agreement between SM theory and experiment—are correct and that perhaps strong interaction effects disrupt the dispersive techniques used to calculate the HVP which require detailed knowledge of $e^+e^- \rightarrow \text{hadrons}$ data at or below the QCD confinement scale.

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