Evolutionary scenarios in the ghost-condensate dark energy model with self interaction

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A ghost-condensate model of dark energy with a generic self interaction is considered. The combined dynamics of the scalar field and gravitation is shown to impose non-trivial restriction on the self-interaction of the scalar field. This restriction is used to show that for the choice of a zero self-interaction certain evolution scenarios are absent. Also, using this restriction the generic self-interaction in the model has been expressed in terms of the measurable quantities. The conditions that the model goes over to the phantom regime have been derived. The model is then confronted with observational data assuming the phantom power law. Our results demonstrates the validity of the phantom regime assumption throughout the future evolution.

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I. INTRODUCTION

Recent cosmological observations indicate late-time acceleration of the observable universe. Why the evolution of the universe is interposed between an early inflationary phase and the late-time acceleration is a yet-unresolved problem. Various theoretical attempts have been undertaken to confront this observational fact. Although the simplest way to explain this behavior is the consideration of a cosmological constant, the known fine-tuning problem led to the dark energy paradigm. Here one introduces exotic dark energy component in the form of scalar fields such as quintessence, k-essence etc. Quintessence is based on scalar field models using a canonical field with a slowly varying potential. On the other hand the models grouped under k-essence are characterized by noncanonical kinetic terms. A key feature of the k-essence models is that the cosmic acceleration is realized by the kinetic energy of the scalar field. The popular models under this category include the phantom model, the ghost-condensate model etc.

It is well-known that the late time cosmic acceleration requires an exotic equation of state \( \omega_{DE} < -\frac{1}{3} \). Current observations allow \( \omega_{DE} < -1 \) which can be explained by considering negative kinetic energy with a field potential. The resulting phantom model is extensively used to confront cosmological observation. This model is however ridden with various instabilities as its energy density is unbounded. This instability can be eliminated in the so-called ghost-condensate models by including a term quadratic in the kinetic energy. In this context let us note that to realize the late-time acceleration scenario some self-interaction must be present in the phantom model. In contrast, in the ghost-condensate models the inclusion of self-interaction of the scalar field is believed to be a matter of choice. This fact, though not unfamiliar, has not been emphasised much in the literature.

Since very little is known about the nature of dark energy it may appear that the presence or otherwise of an interaction term in the ghost-condensate model may not be ascertained from any fundamental premise. However, in this paper we show that this issue can be approached by demanding a consistent scalar field dynamics. We establish here that this consistency requirement imposes non-trivial restriction on the self-interaction in the ghost-condensate model. Using this restriction we show that describing the general evolutionary scenario of the universe by ghost-condensate without self-interaction leads to a priori relation between the first and second derivatives of the scale factor of the universe \( a(t) \) with respect to time at certain evolutionary stages. Since \( a(t) \) is the dynamical solution of the Friedmann equation, such a priori relation automatically excludes some solutions which may be physically interesting. Further, that a real solution for the self-interaction potential is compatible with the ghost field dynamics has also been demonstrated. In the appropriate limit the action of the ghost-condensate model is known to go over to that of the phantom model. Reassuringly, the restriction obtained here, reproduces the phantom potential in the same limit.

The restriction obtained here also has a positive content. It enables us to express the generic self-interaction in the model in terms of measurable quantities once the evolution of the scale factor is chosen. From the seven year Wilkinson Microwave Anisotropy Probe (WMAP7) observations data, latest distance measurements from the BAO
and the Hubble constant measurements the current value of a constant EOS for dark energy has been estimated as \( \omega_{DE} = -1.10 \pm 0.14 \) (68%CL) for flat universe\(^2\). This value belongs to the well-known phantom regime. Recently this WMAP7+BAO+Hubble data has been confronted successfully with the phantom model\(^2\). As already mentioned the ghost-condensate model is a derivative of the phantom model where certain instabilities have been tamed\(^2\). It will thus be interesting to investigate how the ghost-condensate model conforms with the above mentioned data set in the phantom scenario. In other words we propose to use the phantom power law to investigate the adaptability of the ghost-condensate model in this scenario.

The conditions for the ghost-condensate model without interaction to cross the phantom limit are well-known\(^4\). Similar conditions have been worked out here for our model where we include a generic potential. It turns out that these involve the potential as well as the kinetic energy. At this point it is useful to remember that our interaction potential is generic and its time evolution is fixed once the time-variation of \( a(t) \) is chosen. It is thus mandatory to check whether the conditions for assuming the phantom power-law is satisfied during the whole course of future evolution.

The organisation of this paper is as follows. In section II the ghost-condensate model is introduced where we include an arbitrary self-interaction potential. The equations of motion for the scalar field and the scale factor are derived. These equations exhibit the coupling between the scalar field dynamics and gravity. Expressions for the energy density and pressure of the dark energy components are computed. These expressions are used in section III to demonstrate that the requirement of consistency between the Friedmann equations and the scalar field equations imposes nontrivial restriction on the self-interaction potential in the form of a quadratic equation. This restriction has been utilized in two ways. First, in the remaining portion of section III, we show that certain evolutionary pictures are excluded if no self-interaction is assumed. Next, in section IV, we keep the potential generic and show that our restriction can be utilized to express it in terms of measurable quantities once the scale-factor evolution is chosen. Choosing the phantom power law we confront our model with observation. The conditions for cross-over to the phantom regime are worked out and their validity throughout the future evolution is demonstrated. We conclude in section V.

We use mostly positive signature of the metric.

## II. THE GHOST-CONDENSATE MODEL WITH SELF-INTERACTION OF THE SCALAR FIELD

In this section we consider the ghost-condensate model with a self-interaction potential \( V(\phi) \). The action is given by

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2k^2} + \mathcal{L}_\phi + \mathcal{L}_m \right],
\]

where

\[
\mathcal{L}_\phi = -X + \frac{X^2}{M^4} - V(\phi)
\]

\[
X = -\frac{1}{2} g^\mu\nu \partial_\mu \phi \partial_\nu \phi
\]

\( M \) is a mass parameter, \( R \) the Ricci scalar and \( G = k^2/8\pi \) the gravitational constant. The term \( \mathcal{L}_m \) accounts for the total (dark plus baryonic) matter content of the universe, which is assumed to be a barotropic fluid with energy density \( \rho_m \) and pressure \( p_m \), and equation-of-state parameter \( w_m = p_m/\rho_m \). We neglect the radiation sector for simplicity.

The action given by equation (1) describes a scalar field interacting with gravity. Invoking the cosmological principle one requires the metric to be of the Robertson-Walker (RW) form

\[
ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2 d\Omega_2^2 \right],
\]

where \( t \) is the cosmic time, \( r \) is the spatial radial coordinate, \( \Omega_2 \) is the 2-dimensional unit sphere volume, \( K \) characterizes the curvature of 3-dimensional space and \( a(t) \) is the scale factor. The Einstein equations lead to the Friedmann equations

\[
H^2 = \frac{k^2}{3} \left( \rho_m + \rho_\phi \right) - \frac{K}{a^2}
\]

\[
\dot{H} = -\frac{k^2}{2} \left( \rho_m + p_m + \rho_\phi + p_\phi \right) + \frac{K}{a^2},
\]
In the above a dot denotes derivative with respect to \( t \) and \( H \equiv \dot{a}/a \) is the Hubble parameter. In these expressions, \( \rho_\phi \) and \( p_\phi \) are respectively the energy density and pressure of the scalar field. The quantities \( \rho_\phi \) and \( p_\phi \) are defined through the symmetric energy-momentum tensor

\[
T^{(\phi)}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} (\sqrt{-g})
\]

A straightforward calculation gives

\[
T^{(\phi)}_{\mu\nu} = g_{\mu\nu} \mathcal{L}_\phi + \left( -1 + \frac{2X}{M^4} \right) \partial_\mu \phi \partial_\nu \phi
\]

Assuming a perfect fluid model we identify

\[
\rho_\phi = -X + \frac{3X^2}{M^4} + V(\phi) \\
p_\phi = \mathcal{L}_\phi = -X + \frac{X^2}{M^4} - V(\phi)
\]

The equation of motion for the scalar field \( \phi \) can be derived from the action (1). Due to the isotropy of the FLRW universe the scalar field is a function of time only. Consequently, its equation of motion reduces to

\[
\left( 1 - \frac{3\dot{a}^2}{M^4} \right) \ddot{\phi} + 3H \left( 1 - \frac{\dot{a}^2}{M^4} \right) \dot{\phi} - \frac{dV}{d\phi} = 0.
\]

As is well known the same equation of motion follows from the conservation of \( T_{\mu\nu} \). Indeed under isotropy the equations (9) and (10) reduce to

\[
\rho_\phi = -\frac{1}{2} \frac{\dot{a}^2}{a^2} + \frac{3\dot{a}^4}{4M^4} + V(\phi) \\
p_\phi = -\frac{1}{2} \frac{\dot{a}^2}{a^2} + \frac{\dot{a}^4}{4M^4} - V(\phi)
\]

From the conservation condition \( \nabla_\mu T^{(\phi)}_{\mu\nu} = 0 \) we get

\[
\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0,
\]

which, written equivalently in field terms gives equation (11).

To complete the set of differential equations (5), (6), (14) we include the equation for the evolution of matter density

\[
\dot{\rho}_m + 3H(1 + w_m)\rho_m = 0,
\]

where \( w_m = p_m/\rho_m \) is the matter equation of state parameter. The solution to equation (15) can immediately be written down as

\[
\frac{\rho_m}{\rho_{m0}} = \left( \frac{a(t)}{a(t_0)} \right)^n,
\]

where \( n = 3(1 + w_m) \) and \( \rho_{m0} \geq 0 \) is the value of matter density at present time \( t_0 \). Now, the set of equations (5), (6), (14) and (15) must give the dynamics of the scalar field under gravity in a self-consistent manner. In the next section we demonstrate that this consistency requirement constrains the self-interaction \( V(\phi) \) in (1).

### III. RESTRICTION ON THE SELF-INTERACTION OF THE SCALAR FIELD

We start by constructing two independent combinations of the pressure and energy density of the dark energy sector in terms of the Hubble parameter \( H \), matter energy density \( \rho_m \), matter equation of state parameter \( w_m \) and curvature parameter \( K \) using (5), (6) and (15)

\[
\rho_\phi + p_\phi = A = -\frac{2H}{k^2} - \frac{n}{3} \rho_m + \frac{2K}{k^2 a^2}
\]

\[
\rho_\phi + 3p_\phi = B = -\frac{6\dot{a}}{k^2 a} - (n - 2) \rho_m
\]
Using equations (12) and (13), we rewrite these combinations in terms of the ghost-condensate field derivative \( \dot{\phi} \) and potential \( V(\phi) \):

\[
\rho_\phi + p_\phi = A = -\dot{\phi}^2 + \frac{\dot{\phi}^4}{M^4} \tag{19}
\]

\[
\rho_\phi + 3p_\phi = B = -2\dot{\phi}^2 + \frac{3\dot{\phi}^4}{2M^4} - 2V(\phi) \tag{20}
\]

Inverting these equations to write \( \dot{\phi}^2 \) and \( \dot{\phi}^4 \) in terms of \( A, B \) and \( V(\phi) \) and utilizing the algebraic identity \( (\dot{\phi}^2)^2 = \dot{\phi}^4 \) we obtain the following quadratic equation

\[
V^2(\phi) + B - 3A^2 + M^4 \left( \frac{B}{2} + \frac{M^4}{4} \right) V(\phi) + \frac{(3A - 2B)^2 - 4M^4 (A - B/2)}{16} = 0 \tag{21}
\]

This is the restriction on the choice of potential in the ghost-condensate model which has been indicated earlier. Note that the simple fact that \( V(\phi) \) has to satisfy a restriction of this form implies that care must be taken in asserting the absence of the self-interaction term. We will presently discuss this and other issues related to equation (21) in the following. Meanwhile, observe that in the limit \( M^4 \to \infty \) the equation (21) reduces to

\[
V(\phi) = A - B/2 \tag{22}
\]

Substituting for \( A \) and \( B \) in (22) and simplifying, we get

\[
V(\phi) = \frac{1}{k^2} \left( 3H^2 + \dot{H} + \frac{2K}{a^2} \right) + \frac{n - 6}{6} \rho_m \tag{23}
\]

where equations (17) and (18) have been used. This reproduces the result for the potential in the phantom model\(^{29}\).

Coming back to equation (21) let us first investigate the possibility of a vanishing self-interaction. Substituting \( V(\phi) = 0 \) we get

\[
\frac{(3A - 2B)^2}{4M^4} = (A - B/2) \tag{24}
\]

Since the left hand side is positive definite we immediately get the condition

\[
\left( A - \frac{B}{2} \right) = \frac{1}{k^2} \left( 3H^2 + \dot{H} + \frac{2K}{a^2} \right) + \frac{n - 6}{6} \rho_m \geq 0 \tag{25}
\]

Assuming matter in the form of dust \( (n = 3) \) in a universe with flat geometry \( (K = 0) \), this can be further simplified to

\[
3H^2 + \dot{H} \geq \frac{k^2}{2} \rho_m > 0 \tag{26}
\]

Using \( \dot{H} + H^2 = \dot{a}/a \) we reexpress this as

\[
H^2 > -\frac{1}{2} \frac{\dot{a}}{a} \tag{27}
\]

In the accelerating phase this condition is automatically satisfied. However, in the decelerating phase \( (\dot{a} < 0) \) it imposes a definite relation between \( \dot{a} \) and \( a \). Since the Friedmann equation is of second order in time, imposing such a priori condition will exclude a subclass of physically meaningful solutions in the process\(^{38}\). Note that the dark energy paradigm is introduced as a candidate solution to the fine-tuning problem. It is thus advisable to include appropriate interaction potential in the ghost-condensate model so as to tackle both the decelerating and accelerating phases of evolution.
At this point one may wonder whether the constraining equation (21) on \( V(\phi) \) at all allows a real solution. Solving (21) we get

\[
V(\phi) = \left( \frac{3A - 2B}{4} - \frac{M^4}{8} \right) \pm \left\{ \frac{M^4}{16} \left( \frac{M^4}{4} + A \right) \right\}^{\frac{1}{2}}
\]

(28)

The reality condition is thus

\[
\left( \frac{M^4}{4} + A \right) \geq 0
\]

(29)

That this condition is satisfied in general can be established explicitly if we substitute for \( A \) from equation (19) which gives

\[
\left( \frac{M^4}{4} + A \right) = \frac{1}{M^4} \left( \frac{\dot{\phi}^2 - M^4}{2} \right)^2 \geq 0
\]

(30)

Since from physical consideration the interaction potential is required to be real the above observation indicates the consistency of our formalism. Also, the solution (28) can be utilized to study the future evolution of the model as discussed in the following.

### IV. MODELING PHANTOM EVOLUTION AND CONFRONTATION WITH OBSERVATION

In this section we will use the ghost-condensate model with a generic self-interaction to study a specific evolutionary scenario, namely the phantom universe which has been allowed by current observations\(^{30}\). First we will identify the allowed range of the kinetic energy of the field in order to saturate the phantom limit in our model. This limit must be satisfied in order to assume phantom evolutionary scenario. We will begin with the assumption that it does and finally confront our model with the observations in order to check the validity of our assumption.

Next, assuming a phantom power law the time evolutions of both the potential and kinetic energies will be studied. To get explicit time variations of these quantities we require the values of various parameters appearing therein. These parameters include the phantom power law exponent, the big rip time as well as the present values of energy density etc. We use the combined WMAP7+BAO+Hubble as well as WMAP7 data\(^{30}\) as standard data set\(^{29}\). Also in our model there is a free parameter \( M \), the value of which will be estimated self-consistently using the same observational data.

#### A. Crossing the phantom divide

Now from equation (12) and (13), the equation of state parameter for the field \( \phi \) is obtained as

\[
\omega_\phi = \frac{P_\phi}{\rho_\phi} = \frac{-\frac{\dot{\phi}^2}{2} + \frac{3\dot{\phi}^4}{4M^4} - V(\phi)}{-\frac{\dot{\phi}^2}{2} + \frac{3\dot{\phi}^4}{4M^4} + V(\phi)}
\]

(31)

Defining \( f(\dot{\phi}) = \left( \frac{\dot{\phi}^2}{2} - \frac{3\dot{\phi}^4}{4M^4} \right) \) \(^{31}\) can be cast in the form

\[
\omega_\phi = -1 - \frac{\dot{\phi}^4}{V(\phi) - f(\dot{\phi})}
\]

(32)

This equation is more suitable to discuss the crossing of the phantom divide.

1. First assume that there is no self-interaction, i.e., \( V(\phi) = 0 \). For positive energy density we require \( \rho_\phi = -f(\dot{\phi}) > 0 \). Also, for \( \omega_\phi < -1 \) we require \( 1 - \frac{\dot{\phi}^2}{M^4} \) > 0. These conditions lead to the following restriction\(^4\)

\[
\frac{2}{3} M^4 < \dot{\phi}^2 < M^4
\]
so that the phantom limit is saturated.

2. Now suppose, \( V(\phi) \neq 0 \). We can always arrange matters so that

\[
V(\phi) > f(\phi)
\]

and the positive energy condition is automatically satisfied. The only restriction imposed is now \( \dot{\phi}^2 < M^4 \). Of course \( \phi \) is real so we now require

\[
0 < \dot{\phi}^2 < M^4
\]

It is clear that inclusion of appropriate self-interaction provides greater flexibility in crossing the phantom divide.

### B. Consequence of the phantom power law

Assuming that our model goes over to the phantom regime, we can now apply phantom power-law

\[
a(t) = a_0 \left( \frac{t_s - t}{t_s - t_0} \right)^\beta
\]

where \( t_0 \) and \( t_s \) are the present time and big-rip time respectively.

Our aim is to find explicit expressions of the potential and kinetic energies as functions of time. The potential is already given by equation (28). Substituting \( A \) and \( B \) from (17) and (18) respectively we get

\[
V(\phi) = \left[ \frac{3H^2}{8\pi G} + \frac{3\dot{H}}{16\pi G} - \frac{\rho_m}{4} - \frac{M^4}{8} \right] ^{\frac{1}{2}}
\]

Now solving equation (19) kinetic energy term is obtained as

\[
\dot{\phi}^2 = M^4 \left[ - \frac{\dot{H}}{4\pi G} - \frac{\rho_m + M^4}{4} \right] ^{\frac{1}{2}}
\]

The choice of signs in the equations (36) and (37) should be noted. This choice is done so as to satisfy (20).

Using the phantom power law we find

\[
H = -\frac{\beta}{t_s - t}
\]

\[
\dot{H} = -\frac{\beta}{(t_s - t)^2}
\]

Substituting these and restoring to S.I. units equation (36) and (37) become,

\[
V(t) = \left[ \frac{3\beta^2 c^2}{8\pi G(t_s - t)^2} - \frac{3\beta^2 c^2}{16\pi G(t_s - t)^2} - \frac{1}{2} \rho_m c^2 \left( \frac{t_s - t_0}{t_s - t} \right)^{3\beta} - \frac{M_{S.I.}^4}{8} \right] ^{\frac{1}{2}}
\]

\[
\dot{\phi}^2 = M_{S.I.}^4 \left[ - \frac{\beta c^2}{4\pi G(t_s - t)^2} - \frac{\rho_m c^2}{a_0^3} \left( \frac{t_s - t_0}{t_s - t} \right)^{3\beta} + \frac{M_{S.I.}^4}{4} \right] ^{\frac{1}{2}}
\]

where \( M_{S.I.} = M(in\ ev) \times 1.62 \times 10^{-2} \).

\[
\dot{\phi}^2 = M_{S.I.}^4 \left[ - \frac{\beta c^2}{4\pi G(t_s - t)^2} - \frac{\rho_m c^2}{a_0^3} \left( \frac{t_s - t_0}{t_s - t} \right)^{3\beta} + \frac{M_{S.I.}^4}{4} \right] ^{\frac{1}{2}}
\]

(40) and (41) are the desired time evolution equations of the potential and kinetic energies which will be confronted with the observational data in the following. Specifically our aim is to check the validity of the conditions that saturate the phantom regime in our model.
potential and kinetic energies. We find that $M$ should be chosen so that the quantity within square root in (40) and (41) is positive ensuring real values for the $b$ its baryonic and cold dark matter part, $\Omega_{b0}$ and the present cold dark matter density parameter $\Omega_{CDM0}$, for WMAP7 as well as for the combined fitting WMAP7+BAO+$H_0$. The values are taken from\(^{29}\).

| Parameter | WMAP7+BAO+$H_0$ | WMAP7 |
|-----------|-----------------|-------|
| $t_0$     | $13.78 \pm 0.11$ Gyr $[(4.33 \pm 0.04) \times 10^{17}$ sec] | $13.71 \pm 0.13$ Gyr $[(4.32 \pm 0.04) \times 10^{17}$ sec] |
| $H_0$     | $70.2^{+1.4}_{-1.3}$ km/s/Mpc | $71.4 \pm 2.5$ km/s/Mpc |
| $\Omega_{b0}$ | $0.0455 \pm 0.0016$ | $0.0445 \pm 0.0028$ |
| $\Omega_{CDM0}$ | $0.227 \pm 0.014$ | $0.217 \pm 0.026$ |

TABLE I: Observational maximum likelihood values in 1$\sigma$ confidence level for the present time $t_0$, the present Hubble parameter $H_0$, the present baryon density parameter $\Omega_{b0}$ and the present cold dark matter density parameter $\Omega_{CDM0}$, for WMAP7 as well as for the combined fitting WMAP7+BAO+$H_0$. The values are taken from\(^{29}\).

| Parameter | WMAP7+BAO+$H_0$ | WMAP7 |
|-----------|-----------------|-------|
| $\beta$   | $-6.51^{+0.24}_{-0.25}$ | $6.5 \pm 0.4$ |
| $\rho_{m0}$ | $2.52^{+0.25}_{-0.24} \times 10^{-27}$ kg/m$^3$ | $2.50^{+0.48}_{-0.42} \times 10^{-27}$ kg/m$^3$ |
| $\rho_{c0}$ | $9.3^{+0.3}_{-0.4} \times 10^{-27}$ kg/m$^3$ | $9.58^{+0.68}_{-0.66} \times 10^{-27}$ kg/m$^3$ |
| $t_s$     | $104.5^{+1.9}_{-2.0}$ Gyr$[(3.30 \pm 0.06) \times 10^{18}$ sec] | $102.3 \pm 3.5$ Gyr$[(3.23 \pm 0.11) \times 10^{18}$ sec] |

TABLE II: Derived maximum likelihood values in 1$\sigma$ confidence level for the power-law exponent $\beta$, the present matter energy density value $\rho_{m0}$, the present critical energy density value $\rho_{c0}$ and the Big Rip time $t_s$, for WMAP7 as well as for the combined fitting WMAP7+BAO+$H_0$.

### C. Confrontation with Observation

In the previous section we presented the cosmological scenario where dark energy sector is attributed to ghost-condensate scalar field and we have assumed a phantom power law for the time evolution of the scale factor. As has been done in\(^{29}\) we take into account the combined cosmic microwave background (CMB), baryon acoustic oscillations (BAO) and observational Hubble data ($H_0$) as well as the CMB-WMAP7 dataset only. The relevant results are tabulated in TABLE II Note that we quote the usual density parameter $\Omega_m = 8\pi G\rho_m/(3H^2)$, and we split $\Omega_m$ in its baryonic and cold dark matter part, $\Omega_b$ and $\Omega_{CDM}$ respectively ($\Omega_m = \Omega_b + \Omega_{CDM}$).Using the expression of the critical density $\rho_c = \frac{3H^2}{8\pi G}$, the matter density at the present time can be found as $\rho_{m0} = \Omega_m 0 \rho_0$. Where, as usual, we use the subscript 0 to denote the value of a quantity at present. We also set $a_0$ to 1.

From the phantom power law we get

$$\beta = -H_0(t_s - t_0) \tag{42}$$

Assuming a flat geometry and assuming that at late times the phantom dark energy will dominate the universe,$t_s$ can be expressed as\(^{17}\),

$$t_s \simeq t_0 + \frac{2}{3} \left[1 + w_{DE}|^{-1}H_0^{-1}(1 - \Omega_{m0})^{-\frac{1}{3}} \right] \tag{43}$$

In deriving the above formula it has been assumed that at late times the dark energy equation-of-state parameter $w_{DE}$ approaches a constant value. The values of the derived parameters are given in TABLE II.

We are now almost in a position to calculate numerical values of various quantities as function of time. But one last point is still missing. We require to fix the parameter $M$ in the ghost-condensate model. The value of this parameter should be chosen so that the quantity within square root in (40) and (41) is positive ensuring real values for the potential and kinetic energies. We find that $M = 1$ ev is a good choice. Also we choose the upper sign in (40) in order to ensure positive potential energy.

Using values from TABLE I and TABLE II in equations (40) we get expressions of $V(t)$ for the WMAP7+BAO+$H_0$ dataset as:

$$V(t) = \left[ \frac{6.823 \times 10^{27}}{(3.3 \times 10^{18} - t)^{2}} + \frac{5.24 \times 10^{26}}{(3.3 \times 10^{18} - t)^{2}} - \frac{0.661 \times 10^{-370.54}}{(3.3 \times 10^{18} - t)^{-19.53}} - 8.61 \times 10^{-9} \right] + \frac{1}{2} \left[ \frac{-6.987 \times 10^{26}}{(3.3 \times 10^{18} - t)^{2}} - \frac{2.642 \times 10^{-370.54}}{(3.3 \times 10^{18} - t)^{-19.53}} + 1.72 \times 10^{-8} \right] \tag{44}$$
and for the WMAP7 dataset as:

\[
V(t) = \left[ \frac{6.802 \times 10^{27}}{(3.23 \times 10^{18} - t)^2} + \frac{5.232 \times 10^{26}}{(3.23 \times 10^{18} - t)^2} - \frac{1.088 \times 10^{-370}}{(3.23 \times 10^{18} - t)^{-19.5}} - 8.61 \times 10^{-9} \right] \\
+ \frac{1}{2} \left[ \frac{-6.976 \times 10^{26}}{(3.23 \times 10^{18} - t)^2} - \frac{4.352 \times 10^{-370}}{(3.23 \times 10^{18} - t)^{-19.5}} + 1.72 \times 10^{-8} \right]^{\frac{1}{2}} \tag{45}
\]

Similarly for the kinetic energy term we get from equation (41)

\[
\dot{\phi}^2 = 3.444 \times 10^{-8} - \left[ \frac{-6.987 \times 10^{26}}{(3.3 \times 10^{18} - t)^2} - \frac{2.642 \times 10^{-370.54}}{(3.3 \times 10^{18} - t)^{-19.53}} + 1.72 \times 10^{-8} \right]^{\frac{1}{2}} \tag{46}
\]

and

\[
\dot{\phi}^2 = 3.444 \times 10^{-8} - \left[ \frac{-6.976 \times 10^{26}}{(3.23 \times 10^{18} - t)^2} - \frac{4.352 \times 10^{-370}}{(3.23 \times 10^{18} - t)^{-19.5}} + 1.72 \times 10^{-8} \right]^{\frac{1}{2}} \tag{47}
\]

for the WMAP7+BAO+$H_0$ and WMAP7 dataset respectively.

In Figs.\[1\] and [2] respectively the evolution of the potential energy and the kinetic energy term are shown graphically against time. As expected, these quantities shoot up as the big rip time is approached. Superposed on the plot of potential energy in Fig. [1] is the function $f(\dot{\phi})$. It can be clearly seen that the condition (33) is satisfied throughout the future evolution. Again from Fig. [2] we observe that the condition (34) is also satisfied because $\dot{M}$ always lies below $M = 6.89 \times 10^{-18}$. Thus we can say that the ghost-condensate model with a generic self-interaction is able to conform with the observational data in the phantom scenario. This is further illustrated by the evolution of the dark energy equation of state parameter $\omega_{\phi}$. This can be straightforwardly calculated from (29). The variation of $\omega_{\phi}$ against time is shown in Fig. [2]. It is observed that the calculated value of $\omega_{\phi}$ is everywhere less than $-1$ and approaches a constant limiting value.

V. CONCLUSION

We have considered the ghost-condensate model of dark energy with a self-interaction potential in a general FLRW universe. The combined dynamics of dark energy and gravity leads to coupled differential equations involving the universal scale factor $a(t)$ and the scalar field $\phi$. The standard barotropic matter equation of state is assumed. Two independent combination of the pressure and energy density of the dark energy are expressed in terms of the observable quantities from the normal matter and gravity sector. These combinations are then used to impose a consistency condition which leads to a quadratic equation for the self-interaction $V(\phi)$. This equation is shown to admit real roots. Also, in the appropriate limit it leads to the phantom model potential (29).

A very interesting consequence of the consistency condition follows when we examine the choice of zero selfinteraction. Using the quadratic equation satisfied by the self-interaction it has been demonstrated that this choice imposes an a priori relation between the first and second time derivatives of the scale-factor. Considering that the Friedmann equation is a second order differential equation in time, such a condition involving velocity and acceleration is mathematically restrictive. In particular, this excludes certain class of solutions of the scale-factor. Our analysis thus shows that in the class of ghost-condensate models, for general evolution of the universe, a self-interaction of the dark energy should be included.

The quadratic equation has been further used in the subsequent calculation to express the generic potential in terms of measurable quantities when a certain scale-factor evolution law is chosen. It may be recalled that the ghost-condensate model was introduced to remedy certain inadequacies of the phantom model. In the light of the recent observations that suggest the universe may be in the phantom regime, we have investigated the efficacy of the present model in realizing the phantom evolution. The conditions that our model resides within the phantom regime were worked out. These conditions have been expressed in terms of the potential $V(\phi)$ and kinetic energy term $\dot{\phi}^2$. It is noteworthy that inclusion of self-interaction widens the limits of the permissible kinetic energy. Assuming the phantom power law for the scale factor $a(t)$ we have found explicit expressions for $V(\phi)$ and kinetic energy term $\dot{\phi}^2$ as functions of time. These expressions contain characteristic parameters of the power law such as, the exponent $\beta$, big-rip-time $t_s$ and present time $t_0$. At this stage we have used the combined WMAP7+BAO+$H_0$ and WMAP7 observational datasets to evaluate these parameters. The time-evolutions thus constructed shows that the conditions for the applicability of the present model within the phantom landscape hold throughout the entire future evolution. The evolution of the potential energy and the kinetic energy term are plotted against time. Both the potential and


kinetic energy diverge as $t$ approaches $t_s$. Consequently the pressure and energy density of the dark energy fluid also diverges in this limit. However the equation of state parameter always remains below the phantom divide and approaches a constant value. Thus we have shown that the ghost-condensate model with a potential successfully confronts observational data to produce a self-consistent picture of the phantom evolution.

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38. Some particular type of scale-factor kinametics can be readily seen to be excluded. The ekpyrotic31-33 and other bouncing theories15-19 of the early universe require that spacetime bounce from a contracting to an expanding phase, perhaps even oscillating cyclically. Clearly, during the switch over from expanding to contracting phase, $\ddot{a} = 0$ but $\dot{a} < 0$ and thus the condition 127 is violated.
39. Note that we are assuming dust matter and flat geometry.
FIG. 1: The potential $V(\phi)$ plotted against time for the WMAP7 and WMAP7+BAO+$H_0$ data sets. The plot of the function $f(\dot{\phi})$ vs time is also shown.
FIG. 2: \( (d\phi/dt)^2 \) plotted against \( t \) for the WMAP7 and WMAP7+BAO+\( H_0 \) data set. Note that \( 6.89 \times 10^{-8} J/m^3 \) always lies above \( \phi^2 \).
FIG. 3: $\omega_\phi$ plotted against $t$ for the WMAP7 and WMAP7+BAO+$H_0$ data set.