Stable Policy Optimization via Off-Policy Divergence Regularization

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Abstract

Trust Region Policy Optimization (TRPO) and Proximal Policy Optimization (PPO) are among the most successful policy gradient approaches in deep reinforcement learning (RL). While these methods achieve state-of-the-art performance across a wide range of challenging tasks, there is room for improvement in the stabilization of the policy learning and how the off-policy data are used. In this paper we revisit the theoretical foundations of these algorithms and propose a new algorithm which stabilizes the policy improvement through a proximity term that constrains the discounted state-action visitation distribution induced by consecutive policies to be close to one another. This proximity term, expressed in terms of the divergence between the visitation distributions, is learned in an off-policy and adversarial manner. We empirically show that our proposed method can have a beneficial effect on stability and improve final performance in benchmark high-dimensional control tasks.

1 INTRODUCTION

In Reinforcement Learning (RL), an agent interacts with an unknown environment and seeks to learn a policy which maps states to distribution over actions to maximise a long-term numerical reward. Combined with deep neural networks as function approximators, policy gradient methods have enjoyed many empirical successes on RL problems such as video games (Mnih et al., 2016) and robotics (Levine et al., 2016). Their recent success can be attributed to their ability to scale gracefully to high dimensional state-action spaces and complex dynamics.

The main idea behind policy gradient methods is to parametrize the policy and perform stochastic gradient ascent on the discounted cumulative reward directly (Sutton et al., 2000). To estimate the gradient, we sample trajectories from the distribution induced by the policy. Due to the stochasticity of both policy and environment, variance of the gradient estimation can be very large, and lead to significant policy degradation.

Instead of directly optimizing the cumulative rewards, which can be challenging due to large variance, some approaches (Kakade and Langford, 2002; Azar et al., 2012; Pirotta et al., 2013; Schulman et al., 2015) propose to optimize a surrogate objective that can provide local improvements to the current policy at each iteration. The idea is that the advantage function of a policy \( \pi \) can produce a good estimate of the performance of another policy \( \pi’ \) when the two policies give rise to similar state visitation distributions. Therefore, these approaches explicitly control the state visitation distribution shift between successive policies.

However, controlling the state visitation distribution shift requires measuring it, which is non-trivial. Direct methods are prohibitively expensive. Therefore, in order to make the optimization tractable, the aforementioned methods rely on constraining action probabilities by mixing policies (Kakade and Langford, 2002; Pirotta et al., 2013), introducing trust regions (Schulman et al., 2015; Achiam et al., 2017) or clipping the surrogate objective (Schulman et al., 2017; Wang et al., 2019b).

Our key motivation in this work is that constraining the probabilities of the immediate future actions might not be enough to ensure that the surrogate objective is still a valid estimate of the performance of the next policy and consequently might lead to instability and premature convergence. Instead, we argue that we should reason about the long-term effect of the policies on the distribution of the future states.

In particular, we directly consider the divergence between state-action visitation distributions induced by succes-
We define the discounted state visitation distribution $d^\pi_\rho(s,a)$ as:

$$d^\pi_\rho(s,a) \triangleq (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t \Pr^\pi(s_t = s, a_t = a \mid s_0 \sim \rho).$$

It is known (Puterman, 1990) that $d^\pi_\rho(s,a)$ is the expected sum of discounted rewards along the trajectory for policy $\pi$ after we execute action $a$ in state $s$ and that $\mu^\pi$ is characterized via: $V(\pi, s, a') \in S \times A$

$$\mu^\pi_\rho(s', a') = (1 - \gamma) \rho(s') \pi(a' \mid s') + \gamma \int \pi(a' \mid s') \Pr(s' \mid s, a) \mu^\pi_\rho(s, a) ds \ da.$$

### 2.2 CONSERVATIVE UPDATE APPROACHES

Most policy training approaches in RL can be understood as updating a current policy $\pi$ to a new improved policy $\pi'$ based on the advantage function $A^\pi$ or an estimate $\hat{A}$ of it. We review here some popular approaches that implement conservative updates in order to stabilize policy training.

First, let us state a key lemma from the seminal work of Kakade and Langford (2002) that relates the performance difference between two policies to the advantage function.

**Lemma 2.1** (The performance difference lemma (Kakade and Langford, 2002)) For all policies $\pi$ and $\pi'$,

$$J(\pi') = J(\pi) + \mathbb{E}_{s \sim d^\pi_\rho} \mathbb{E}_{a \sim \pi'(\cdot|s)} [A^\pi(s,a)].$$

This lemma implies that maximizing Equation (2) will yield a new policy $\pi'$ with guaranteed performance improvement over a given policy $\pi$. Unfortunately, a naive direct application of this procedure would be prohibitively expensive since it requires estimating $d^\pi_\rho$ for all $\pi'$ candidates. To address this issue, Conservative Policy Iteration (CPI) (Kakade and Langford, 2002) optimizes a surrogate objective defined based on current policy $\pi^i$ at each iteration $i$,

$$L_{\pi^i}(\pi') = J(\pi_i) + \mathbb{E}_{s \sim d^\pi_\rho} \mathbb{E}_{a \sim \pi'(\cdot|s)} [A^{\pi^i}(s,a)],$$

by ignoring changes in state visitation distribution due to changes in the policy. Then, CPI returns the stochastic mixture $\pi_{t+1} = \alpha_t \pi_t + (1 - \alpha_t) \pi_i$ where $\pi_t = \arg \max_{\pi'} L_{\pi_{t}}(\pi')$ is the greedy policy and $\alpha_t \in [0,1]$ is tuned to guarantee a monotonically increasing sequence of policies.

Inspired by CPI, the Trust Region Policy Optimization algorithm (TRPO) (Schulman et al., 2015) extends the policy improvement step to any general stochastic policy rather than just mixture policies. TRPO maximizes the same surrogate objective as CPI subject to a Kullback-Leibler (KL) divergence constraint that ensures the next
policy $\pi_{i+1}$ stays within $\delta$-neighborhood of the current policy $\pi_i$:

$$
\pi_{i+1} = \arg \max_{\pi} L_{\pi_i}(\pi')
$$

subject to $E_{s \sim d_\mu'} [D_{KL}(\pi'(. | s) || \pi_i(. | s))] \leq \delta,$

where $D_{KL}$ is the Kullback–Leibler divergence. In practice, TRPO considers a differentiable parameterized policy $\{\pi_\theta, \theta \in \Theta\}$ and solves the constrained problem (4) in parameter space $\Theta$. In particular, the step direction is estimated with conjugate gradients, which requires the computation of multiple Hessian-vector products. Therefore, this step can be computationally heavy.

To address this computational bottleneck, Proximal Policy Optimization (PPO) (Schulman et al., 2017) proposes replacing the KL divergence constrained objective (4) of TRPO by clipping the objective function directly as:

$$
L_{\pi}^{\text{clip}}(\pi') = \mathbb{E}_{(s,a) \sim \mu_{\pi'}} \left[ \min \left\{ A^{\pi}(s,a) \cdot \kappa_{\pi'/\pi_i}(s,a), \right. \\
\left. A^{\pi}(s,a) \cdot \text{clip}(\kappa_{\pi'/\pi_i}(s,a), 1-\epsilon, 1+\epsilon) \right\} \right],
$$

(5)

where $\epsilon > 0$ and $\kappa_{\pi'/\pi_i}(s,a) = \frac{\pi'(s,a)}{\pi_i(s,a)}$ is the importance sampling ratio.

### 3 THEORETICAL INSIGHTS

In this section, we present the theoretical motivation of our proposed method.

At a high level, algorithms CPI, TRPO, and PPO follow similar policy update schemes. They optimize some surrogate performance objective ($L_{\pi_i}(\pi')$) for CPI and TRPO and $L_{\pi}^{\text{clip}}(\pi')$ for PPO) while ensuring that the new policy $\pi_{i+1}$ stays in the vicinity of the current policy $\pi_i$. The vicinity requirement is implemented in different ways:

1. CPI computes a sequence of stochastic policies that are mixtures between consecutive greedy policies.

2. TRPO imposes a constraint on the KL divergence between old policy and new one ($\mathbb{E}_{s \sim d_\mu'} [D_{KL}(\pi'(. | s) || \pi_i(. | s))] \leq \delta$).

3. PPO directly clips the objective function based on the value of the importance sampling ratio $\kappa_{\pi'/\pi_i}$ between the old policy and new one.

Such conservative updates are critical for the stability of the policy optimization. In fact, the surrogate objective $L_{\pi_i}(\pi')$ (or its clipped version) is valid only in the neighborhood of the current policy $\pi_i$, i.e., when $\pi'$ and $\pi_i$ visit all the states with similar probabilities. The following lemma more precisely formalizes this:

**Lemma 3.1.** For all policies $\pi$ and $\pi'$,

$$
J(\pi') \geq L_{\pi}(\pi') - \epsilon^2 D_{TV}(d_\rho^\pi || d_\rho^\pi)
$$

(6)

$$
\geq L_{\pi}^{\text{clip}}(\pi') - \epsilon^2 D_{TV}(d_\rho^\pi || d_\rho^\pi),
$$

where $\epsilon^2 = \max_{s \in S} [\mathbb{E}_{a \sim \pi'(. | s)} \| A^\pi(s,a) \|]$ and $D_{TV}$ is the total variation distance.

The proof is provided in appendix for completeness. Lemma 3.1 states that $L_{\pi}(\pi')$ (or $L_{\pi}^{\text{clip}}(\pi')$) is a sensible lower bound to $J(\pi')$ as long as $\pi$ and $\pi'$ are close in terms of total variation distance between their corresponding state visitation distributions $d_\rho^\pi$ and $d_\rho^\pi$. However, the aforementioned approaches enforce closeness of $\pi'$ and $\pi$ in terms of their action probabilities rather than their state visitation distributions. This can be justified by the following inequality (Achiam et al., 2017):

$$
D_{TV}(d_\rho^\pi || d_\rho^\pi) \leq \frac{2\gamma}{1-\gamma} \mathbb{E}_{s \sim d_\rho^\pi} [D_{TV}(\pi'(s) || \pi(s))].
$$

(7)

Plugging the last inequality (7) into (6) leads to the following lower bound:

$$
J(\pi') \geq L_{\pi}(\pi') - \frac{2\gamma\epsilon^2}{1-\gamma} \mathbb{E}_{s \sim d_\rho^\pi} [D_{TV}(\pi'(s) || \pi(s))].
$$

(8)

The obtained lower bound (8) is, however, clearly looser than the one in inequality (7). Lower bound (8) suffers from an additional multiplicative factor $\frac{1}{1-\gamma}$, which is the effective planning horizon. It is essentially due to the fact that we are characterizing a long-horizon quantity, such as the state visitation distribution $d_\rho^\pi(s)$, by a one-step quantity, such as the action probabilities $\pi(. | s)$. Therefore, algorithms that rely solely on action probabilities to define closeness between policies should be expected to suffer from instability and premature convergence in long-horizon problems.

It follows from our discussion that $D_{TV}(d_\rho^\pi || d_\rho^\pi)$ is a more natural proximity term to ensure safer and more stable policy updates. Previous approaches excluded using this term because we don’t have access to $d_\rho^\pi$, which would require executing $\pi'$ in the environment. In the next section, we show how we can leverage recent advances in off-policy policy evaluation to address this issue.

### 4 OFF-POLICY FORMULATION OF DIVERGENCES

In this section, we explain how divergences between state-visitation distributions can be approximated. This is done

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1. The result is not novel, it can be found as intermediate step in proof of theorem 1 in Achiam et al. (2017), for example.
by leveraging ideas from recent works on off-policy learning (Nachum et al., 2019a; Kostrikov et al., 2019).

Consider two different policies π and π′. Suppose that we have access to state-action samples generated by executing the policy π in the environment, i.e. (s, a) ∼ µπ.

As motivated by the last section, we aim to estimate the total variation distance between state-action visitation distributions rather than the divergence between state visitation distributions. This is still a reasonable choice as DT(π∥π′) is upper bounded by DT(π∥π′) as shown below:

$$DT(\pi, \pi') = \int |(d\pi' - d\pi)(s)| ds$$

where $D\pi$ is the transition operator induced by π, defined as $D\pi(s, a, s')P(s' | s, a)g(s', a')$. $g$ may be interpreted as the action-value function of the policy π′ in a modified MDP where the states and actions are $P(s', s, a)$.

Here are some key points from the text:

- The total variation distance ($DT(\pi, \pi')$) is a broad class of divergences.
- The KL divergence ($KL(\pi, \pi') = \int \log(\frac{\pi}{\pi'}) \cdot \pi ds$) is a particular case of this class.
- Other possible divergence representations include the $\chi^2$-divergence and the $\log$-expected-exp divergence.

**Other possible divergence representations:** Using the variational representation of $\phi$-divergences was a key step in the derivation of Equation (12). But in fact any representation that admits a linear term with respect to $\mu_\pi$ (i.e. $E(s,a) \sim \mu_\pi[f(s,a)]$) would work as well. For example, one can use the the Donker-Varadhan representation (Donsker and Varadhan, 1983) to alternatively express the KL divergence as:

$$D_\phi(\mu_\pi || \mu_{\pi'}) = \sup_{f : \mathbb{S} \times \mathbb{A} \rightarrow \mathbb{R}} \left[ E(\mathbb{s},a) \sim \mu_{\pi}[f(s,a)] - \log(\mathbb{E}(\mathbb{s},a) \sim \mu_{\pi}[\exp(f(s,a))] ) \right].$$

where $\mathbb{P}$ is the transition operator induced by $\pi$, defined as $\mathbb{P}(s', s, a)P(s' | s, a)g(s', a')$. $g$ may be interpreted as the action-value function of the policy $\pi$ in a modified MDP where the states and actions are $P(s', s, a)$.

**5 A PRACTICAL ALGORITHM USING ADVERSARIAL DIVERGENCE**

We now turn these insights into a practical algorithm. The lower bounds in lemma 3.1, suggest using a regularized PPO objective: $L^{\text{clip}}_{\psi}(\pi) - \lambda DT(\pi || \pi')$, where $\lambda$ is a regularization parameter.

Both regularized $L^{\text{clip}}_{\psi}$ and $L_{\psi}$ are lower bounds on policy performance in Lemma 3.1. We use $L^{\text{clip}}_{\psi}$ rather than $L_{\psi}$ be-
Algorithm 1 PPO-DICE

1: **Initialisation**: random initialize parameters $\theta_1$ (policy), $\psi_1$ (discriminator) and $\omega_1$ (value function).

2: for $i=1, \ldots$ do

3: generate a batch of $M$ rollouts $\{s_1^{(j)}, a_1^{(j)}, r_1^{(j)}, s_1^{(j)}, \ldots, s_T^{(j)}, a_T^{(j)}, r_T^{(j)}, s_{T+1}^{(j)}\}_{j=1}^M$ by executing policy $\pi_{\theta_1}$ in the environment for $T$ steps.

4: Estimate Advantage function: $\hat{A}(s_t^{(j)}, a_t^{(j)}) = \sum_{t=1}^T (\gamma \lambda)^{t-1} (r_t^{(j)} + \gamma V_{\omega_1}(s_{t+1}^{(j)}) - V_{\omega_1}(s_t^{(j)}))$

5: Compute target value $y_t^{(j)} = r_t^{(j)} + \gamma \hat{V}(s_{t+1}^{(j)}) + \ldots + \gamma^{T+1} V_{\omega_1}(s_{T+1}^{(j)})$

6: $\omega = \omega_1; \theta = \theta_1; \psi = \psi_1$

7: for epoch $n=1, \ldots, N$ do

8: for iteration $k=1, \ldots, K$ do

9: \hspace{1em} // Compute discriminator loss:

10: $\hat{L}_D(\psi, \theta) = \frac{1}{MT} \sum_{j=1}^M \sum_{t=1}^T \phi^* (g_\psi(s_t^{(j)}, a_t^{(j)}) - \gamma g_\psi(s_{t+1}^{(j)}, a_{t+1}^{(j)})) - (1 - \gamma) g_\psi(s_1^{(j)}, a_1^{(j)})$ where $a_{t+1}^{(j)} \sim \pi_\theta(\cdot | s_{t+1}^{(j)}), a_t^{(j)} \sim \pi_\theta(\cdot | s_t^{(j)})$.

11: \hspace{1em} // Update discriminator parameters: (using learning rate $c_\psi \eta$)

12: $\psi \leftarrow \psi - c_\psi \eta \nabla_\psi \hat{L}_D(\psi, \theta)$

13: \hspace{1em} end for

14: \hspace{1em} // Compute value loss:

15: $L_V(\omega) = \frac{1}{MT} \sum_{j=1}^M \sum_{t=1}^T \left(V_\omega(s_t^{(j)}) - y_t^{(j)}\right)^2$

16: \hspace{1em} // Compute PPO clipped loss:

17: $L_{\text{clip}}(\theta) = \frac{1}{MT} \sum_{j=1}^M \sum_{t=1}^T \min \left\{ \hat{A}(s_t^{(j)}, a_t^{(j)}), \kappa_{\pi_{\theta_1}}(s_t^{(j)}, a_t^{(j)}), \hat{A}(s_t^{(j)}, a_t^{(j)}), \kappa_{\pi_{\theta_1}}(s_t^{(j)}, a_t^{(j)}) \right\}$

18: \hspace{1em} // Update parameters: (using learning rate $\eta$)

19: $\omega \leftarrow \omega - \eta \nabla_\omega L_V(\omega)$

20: $\theta \leftarrow \theta - \eta \nabla_\theta (L_{\text{clip}}(\theta) + \lambda \cdot \hat{L}_D(\psi, \theta))$ (if reparametrization trick applicable, else gradient step on Eq. 20)

21: \hspace{1em} end for

22: $\omega_{i+1} = \omega; \theta_{i+1} = \theta; \psi_{i+1} = \psi$

23: end for

A regularization coefficient. If in place of the total variation we use the off-policy formulation of $\phi$ divergence $D_\phi(\mu_\pi || \mu_\rho)$ as detailed in Equation (12), our main optimization objective can be expressed as the following min-max problem:

$$\max_{\pi'} \min_{g: S \times A \rightarrow \mathbb{R}} L_{\pi_{\theta_1}}^{\text{clip}}(\pi') - \lambda \left( (1 - \gamma) \mathbb{E}_{s \sim \rho, a \sim \pi'} [g(s, a) \nabla_\pi \log \pi'(a | s)] + \gamma \mathbb{E}_{(s, a) \sim \pi_{\theta_1}} \left[ \frac{\partial \phi^*}{\partial \ell} \left( (g - \gamma \nabla_\pi \log \pi'(a' | s')) \right) \right] \right).$$

When the inner minimization over $g$ is fully optimized, it is straightforward to show – using the score function estimator – that the gradient of this objective with respect to $\pi$ is (proof is provided in appendix):

$$\nabla_\pi L_{\pi_{\theta_1}}^{\text{clip}}(\pi') - \lambda \left( (1 - \gamma) \mathbb{E}_{s \sim \rho, a \sim \pi'} [g(s, a) \nabla_\pi \log \pi'(a | s)] + \gamma \mathbb{E}_{(s, a) \sim \pi_{\theta_1}} \left[ \phi^* \left( (g - \gamma \nabla_\pi \log \pi'(a' | s')) \right) \right] \right).$$

Furthermore, we can use the reparametrization trick if the policy $\pi$ is parametrized by a Gaussian, which is usually the case in continuous control tasks. We call the resulting new algorithm PPO-DICE, (detailed in Algorithm 1), as it uses the clipped loss of PPO and leverages the Distribution Correction Estimation idea from Nachum et al. (2019a).

In the min-max objective (14), $g$ plays the role of a discriminator, as in Generative Adversarial Networks (GAN) (Goodfellow et al., 2014). The policy $\pi'$ plays the role of a generator, and it should balance between increasing the likelihood of actions with large advantage versus inducing a state-action distribution that is close to the one of $\pi_{\theta_1}$.
As shown in Algorithm 1, both policy and discriminator are parametrized by neural networks $\pi_\theta$ and $g_\psi$ respectively. We estimate the objective (14) with samples from $\pi_i = \pi_\theta$ as follows. At a given iteration $i$, we generate a batch of $M$ rollouts $\{s_i^{(j)}, a_i^{(j)}, r_i^{(j)}, s_{i+1}^{(j)}, \ldots, s_T^{(j)}, a_T^{(j)}, r_T^{(j)}, s_{T+1}^{(j)}\}_{j=1}^M$ by executing the policy $\pi_i$ in the environment for $T$ steps. Similarly to the PPO procedure, we learn a value function $V_\omega$ by updating its parameters $\omega$ with gradient descent steps, optimizing the following squared error loss:

$$L_V(\omega) = \frac{1}{MT} \sum_{j=1}^M \sum_{t=1}^T (V_\omega(s_t^{(j)}) - y_t^{(j)})^2,$$  \hspace{1cm} (16)

where $y_t^{(j)} = r_t^{(j)} + \gamma r_{t+1}^{(j)} + \ldots + \gamma^{T+1-t} V_\omega(s_{T+1}^{(j)})$. Then, to estimate the advantage, we use the truncated generalized advantage estimate

$$A(s_t^{(j)}, a_t^{(j)}) = \sum_{j=1}^T (\gamma) t^{j-1}(r_t^{(j)} + \gamma V_\omega(s_{t+1}^{(j)}) - V_\omega(s_t^{(j)})).$$  \hspace{1cm} (17)

This advantage estimate is used to compute an estimate of $L_{\pi_i}$ given by:

$$L_{\text{clip}}(\theta) = \frac{1}{MT} \sum_{j=1}^M \sum_{t=1}^T \min \left\{ A(s_t^{(j)}, a_t^{(j)}) \kappa_{\pi_\theta/\pi_i}(s_t^{(j)}, a_t^{(j)}), A(s_t^{(j)}, a_t^{(j)}) \cdot \text{clip}(\kappa_{\pi_\theta/\pi_i}(s_t^{(j)}, a_t^{(j)}), 1 - \epsilon, 1 + \epsilon) \right\}.$$  \hspace{1cm} (18)

The parameters $\psi$ of the discriminator are learned by gradient descent on the following empirical version of the regularization term in the min-max objective (14):

$$L_D(\psi, \theta) = \frac{-1}{MT} \sum_{j=1}^M \sum_{t=1}^T (1 - \gamma) g_\psi(s_t^{(j)}, a_t^{(j)}) - \phi^*(g_\psi(s_t^{(j)}, a_t^{(j)}) - g_\psi(s_{t+1}^{(j)}, a_{t+1}^{(j)})),$$  \hspace{1cm} (19)

where $a_t^{(j)} \sim \pi_\theta(\cdot | s_t^{(j)})$ and $a_{t+1}^{(j)} \sim \pi_\theta(\cdot | s_{t+1}^{(j)})$. If the reparametrization trick is applicable (which is almost always the case for continuous control tasks), the parameters $\theta$ of the policy are updated via gradient ascent on the following objective:

$$\hat{L}_{\text{clip}}(\theta) - \lambda \frac{\lambda}{MT} \sum_{j=1}^M \sum_{t=1}^T (1 - \gamma) g_\psi(s_1^{(j)}, a_1^{(j)}) \log \pi_\theta(a_1^{(j)} | s_1^{(j)})$$

$$+ \gamma^{T+1-t} \frac{\partial^*}{\partial \theta} \left( g_\psi(s_t^{(j)}, a_t^{(j)}) - g_\psi(s_{t+1}^{(j)}, a_{t+1}^{(j)}) \right) - g_\psi(s_{t+1}^{(j)}, a_{t+1}^{(j)}) \log \pi_\theta(a_{t+1}^{(j)} | s_{t+1}^{(j)})),$$  \hspace{1cm} (20)

Note that the gradient of this equation with respect to $\theta$ corresponds to an empirical estimate of the score function estimator we provided in Equation 15.

We train the value function, policy, and discriminator for $N$ epochs using $M$ rollouts of the policy $\pi_i$. We can either alternate between updating the policy and the discriminator, or update $g_\psi$ for a few steps $M$ before updating the policy. We found that the latter worked better in practice, likely due to the fact that the target distribution $\mu_\pi^{(j)}$ changes with every iteration $i$. We also found that increasing the learning rate of the discriminator by a multiplicative factor $c_\phi$ of the learning rate for the policy and value function $\eta$ improved performance.

**Choice of divergence:** The algorithmic approach we just described is valid with any choice of $\phi$-divergence for measuring the discrepancy between state-visitation distributions. It remains to choose an appropriate one. While Lemma 3.1 advocates the use of total variation distance $\phi(t) = |t - 1|$, it is notoriously hard to train high dimensional distributions using this divergence (see Kodali et al. (2017) for example). Moreover, the convex conjugate of $\phi(t) = |t - 1|$ is $\phi^*(t) = t$ if $|t| \leq \frac{1}{2}$ and $\phi^*(t) = \infty$ otherwise. This would imply the need to introduce an extra constraint $\|g - P^{\infty} g\|_{\infty} \leq \frac{1}{2}$ in the formulation (12), which may be hard to optimize.

Therefore, we will instead use the KL divergence ($\phi(t) = t \log(t)$, $\phi^*(t) = \exp(t - 1)$). This is still a well justified choice as we know that $D_{TV}(\mu_\pi^{(j)} || \mu_\pi^{(j)}) \leq \sqrt{\frac{1}{2} D_{KL}(\mu_\pi^{(j)} || \mu_\pi^{(j)})}$ thanks to Pinsker’s inequality. We will also try $\chi^2$-divergence ($\phi(t) = (t - 1)^2$) that yields a squared regularization term.

### 6 RELATED WORK

Constraining policy updates, in order to minimize the information loss due to policy improvement, has been an active area of investigation. Kakade and Langford (2002) originally introduce CPI by maximizing a lower bound on the policy improvement and relaxing the greedification step through a mixture of successive policies. Pirotta et al. (2013) build on Kakade and Langford (2002) refine the
Our work is related to regularized MDP literature (Neu et al., 2017; Geist et al., 2019). Shannon Entropic regularization is used in value iteration scheme (Haarnoja et al., 2017; Dai et al., 2018) and in policy iteration schemes (Haarnoja et al., 2018). Note that all the mentioned works employ regularization on the action probabilities. Recently, Wang et al. (2019a) introduce divergence-augmented policy optimization where they penalize the policy update by a Bregman divergence on the state visitation distributions, motivated by similar techniques in Nachum et al. (2019b) for off-policy policy evaluation. They use the divergence between state-action visitations distribution, motivated the mirror descent method. While their framework seems general, it doesn’t include the divergences we employ in our algorithm. In fact, their method enables the use of the conditional KL divergence between state-action visitations distribution defined by \( \int \mu^\pi_s(s, a) \log \frac{\pi(a|s)}{\rho(a|s)} \) and not the KL divergence \( \int \mu^\pi_s(s, a) \log \frac{\mu^\pi(s, a)}{\mu^\rho(s, a)} \). Note the action probabilities ratio inside the log in the conditional KL divergence allows them to use the policy gradient theorem, a key ingredient in their framework, which cannot be done for the KL divergence.

Our work builds on recent off-policy approaches: DualDICE (Nachum et al., 2019a) for policy evaluation and ValueDICE (Kostrikov et al., 2019) for imitation learning. Both use the off-policy formulation of KL divergence. The former uses the formulation to estimate the ratio of the state visitation distributions under the target and behavior policies. Whereas, the latter learns a policy by minimizing the divergence.

The closest related work is the recently proposed AlgaeDICE (Nachum et al., 2019b) for off-policy policy optimization. They use the divergence between state-action visitation distribution induced by \( \pi \) and a behavior distribution, motivated by similar techniques in Nachum et al. (2019a). However, they incorporate the regularization to the dual form of policy optimization \( J(\pi) = E_{(s,a)\sim \mu^\pi}[r(s, a)] \) whereas we consider a surrogate objective (lower bound on the policy performance). Moreover, our method is online off-policy in that we collect data with each policy found in the optimization procedure, but also use previous data to improve stability. Whereas, their algorithm is designed to learn a policy from a fixed dataset collected by behaviour policies.

## 7 EXPERIMENTS AND RESULTS

We use the PPO implementation by Kostrikov (2018) as a baseline and modify it to implement our proposed PPO-DICE algorithm. We run experiments on a randomly selected subset of environments in the Atari suite (Bellemare et al., 2013) for high-dimensional observations and discrete action spaces, as well as on the OpenAI Gym (Brockman et al., 2016) MuJoCo environments, which have continuous state-action spaces. All shared hyper-parameters are set at the same values for both methods, and we use the hyperparameter values recommended by Kostrikov (2018) for each set of environments, Atari and MuJoCo.

### 7.1 IMPORTANT ASPECTS OF PPO-DICE

#### 7.1.1 Choice of Divergence

We conducted an initial set of experiments to compare two different choices of divergences, KL and \( \chi^2 \), for the regularization term of PPO-DICE. Figure 1 shows training curves for one continuous action and one discrete action environment. There, as in the other environments in which we run this comparison, KL consistently performed better than \( \chi^2 \). We thus opted to use KL divergence in all subsequent experiments.

#### 7.1.2 Effect of Varying \( \lambda \)

Next we wanted to evaluate the sensitivity of our method to the \( \lambda \) parameter that controls the strength of the regularization. We examine in Figure 2 the performance of PPO-DICE when varying \( \lambda \). There is a fairly narrow band for Hopper-v2 that performs well, between 0.01 and 1. Theory indicates that the proper value for \( \lambda \) is the maximum of the absolute value of the advantages (see Lemma 3.1). This prompted us to implement an adaptive approach, where we compute the 90th percentile of advantages within the batch (for stability), which we found performed well across environments. To avoid introducing an additional hyperparameter by tuning \( \lambda \), we use the adaptive method for subsequent experiments.
Table 1: Mean final reward and 1 standard error intervals across 10 seeds for Atari games evaluated at 10M steps.

| Game      | PPO            | PPO-DICE       |
|-----------|----------------|----------------|
| AirRaid   | 4305.0 ± 638.15 | 4217.5 ± 769.19 |
| Asterix   | 4300.0 ± 169.31 | 6200.0 ± 754.10 |
| Asteroids | 1511.0 ± 125.03 | 1653.0 ± 112.20 |
| Atlantis  | 2120400.0 ± 471609.93 | 3447433.33 ± 100105.82 |
| BankHeist | 1247.0 ± 21.36  | 1273.33 ± 7.89  |
| BattleZone| **29000.0 ± 2620.43** | 19000.0 ± 2463.06 |
| Carnival  | 3423.33 ± 369.51 | 3080.0 ± 189.81 |
| ChopperCommand | 566.67 ± 14.91 | 900.0 ± 77.46 |
| DoubleDuck| −6.0 ± 1.62     | −4.0 ± 1.26     |
| Enduro    | 1129.9 ± 73.18  | 1308.33 ± 120.09 |
| Freeway   | 32.33 ± 0.15    | 32.0 ± 0.00     |
| Frostbite | **639.0 ± 334.28** | 296.67 ± 5.96 |
| Gopher    | 1388.0 ± 387.65 | 1414.0 ± 417.84 |
| Kangaroo  | 4060.0 ± 539.30 | **6650.0 ± 1558.16** |
| Phoenix   | **12614.0 ± 621.71** | 11676.67 ± 588.24 |
| Robotank  | 7.8 ± 1.33      | **12.1 ± 2.91** |
| Seaquest  | 1198.0 ± 128.82 | 1300.0 ± 123.97 |
| TimePilot | 5070.0 ± 580.53 | **7000.0 ± 562.32** |
| Zaxxon    | **7110.0 ± 841.60** | 6130.0 ± 1112.48 |

7.1.3 **Importance of Clipping the Action Loss**

We earlier mentioned (see Footnote 2) two possible forms of our regularized objective: one with clipped action loss $L_{\text{clip}}$ and one without $L$. Clipping the action loss was an extra regularizing measure proposed in PPO (Schulman et al., 2017). For our algorithm also, we hypothesized that it would be important for providing additional constraints on the policy update to stay within the trust region. Figure 3 confirms this empirically: we see the effect on our method of clipping the action loss versus keeping it unclipped. Initially, not having the additional regularization allows it to learn faster, but it soon crashes, showing the need for clipping to reduce variance in the policy update.

7.2 **RESULTS ON ATARI**

Given our above observations we settled on using a KL-regularized $L_{\text{clip}}$ with the adaptive method for $\lambda$ that we explained Section 7.1.2. We run PPO-DICE on randomly selected environments from Atari. We tuned two additional hyperparameters, the learning rate for the discriminator and the number of discriminator optimization steps per policy optimization step. We found that $K = 5$.
For the OpenAI Gym MuJoCo suite, we also used $K = 5$ discriminator optimization steps per policy optimization step, and $c_\psi = 10 \times$ learning rate for the discriminator in all environments. We selected 5 of the more difficult environments to showcase in the main paper (Figure 4), but additional results on the full suite and all hyperparameters used can be found in Appendix B.1. We again see improvement in performance in the majority of environments with PPO-DICE compared to PPO and TRPO.

8 CONCLUSION

In this work, we have argued that using the action probabilities to constrain the policy update is a suboptimal approximation to controlling the state visitation distribution shift. We then demonstrate that using the recently proposed DIstribution Correction Estimation idea (Nachum et al., 2019a), we can directly compute the divergence between the state-action visitation distributions of successive policies and use that to regularize the policy optimization objective instead. Through carefully designed experiments, we have shown that our method beats PPO in most environments in Atari (Bellemare et al., 2013) and OpenAI Gym MuJoCo (Brockman et al., 2016) benchmarks.

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A Omitted Proofs

A.1 Proof of Lemma 3.1

According to performance difference lemma 2.1, we have

\[ J(\pi') = J(\pi) + \mathbb{E}_{s \sim d_\mu} \mathbb{E}_{a \sim \pi'(s)} [A^\pi(s, a)] \]

\[ = J(\pi) + \left( \mathbb{E}_{s \sim d_\mu} \mathbb{E}_{a \sim \pi'(s)} [A^\pi(s, a)] + \int_{s \in S} \mathbb{E}_{a \sim \pi'(s)} [A^\pi(s, a)] (d^\pi_\rho(s) - d^\rho(s)) ds \right) \]

\[ \geq J(\pi) + \left( \mathbb{E}_{s \sim d_\mu} \mathbb{E}_{a \sim \pi'(s)} [A^\pi(s, a)] - \int_{s \in S} |\mathbb{E}_{a \sim \pi'(s)} [A^\pi(s, a)]| \cdot |d^\pi_\rho(s) - d^\rho(s)| ds \right) \]

\[ \geq J(\pi) + \left( \mathbb{E}_{s \sim d_\mu} \mathbb{E}_{a \sim \pi'(s)} [A^\pi(s, a)] - \epsilon_\pi \int_{s \in S} |d^\pi_\rho(s) - d^\rho(s)| ds \right) \]

\[ \geq J(\pi) + \left( \mathbb{E}_{s \sim d_\mu} \mathbb{E}_{a \sim \pi'(s)} [A^\pi(s, a)] - \epsilon_\pi D_{TV}(d^\pi_\rho || d^\rho) \right) \]

\[ = L_\pi(\pi') - \epsilon_\pi D_{TV}(d^\pi_\rho || d^\rho) \]

where \( \epsilon_\pi = \max_s |\mathbb{E}_{a \sim \pi'(s)} [A^\pi(s, a)]| \) and \( D_{TV} \) is total variation distance. The first inequality follows from Cauchy-Schwartz inequality.

A.2 Score Function Estimator of the gradient with respect to the policy

\[ \nabla_{\pi'} \mathbb{E}_{s \sim P} [g(s, a)] = \nabla_{\pi'} \int g(s, a) \rho(s) \pi'(a | s) = \int g(s, a) \rho(s) \nabla_{\pi'} \pi'(a | s) = \mathbb{E}_{s \sim \pi} [g(s, a) \nabla_{\pi'} \log \pi'(a | s)] \]

\[ \nabla_{\pi'} \mathbb{E}_{(s, a) \sim \mu_\pi^\pi} [\phi^* ((g - \gamma \mathbb{P}^\pi g)(s, a))] = \mathbb{E}_{(s, a) \sim \mu_\pi^\pi} [\nabla_{\pi'} \phi^* ((g - \gamma \mathbb{P}^\pi g)(s, a))] \]

\[ = \mathbb{E}_{(s, a) \sim \mu_\pi^\pi} \left[ \frac{\partial \phi^*}{\partial t} ((g - \gamma \mathbb{P}^\pi g)(s, a)) \nabla_{\pi'} (g - \gamma \mathbb{P}^\pi g) \right] \]

\[ = -\gamma \mathbb{E}_{(s, a) \sim \mu_\pi^\pi} \left[ \frac{\partial \phi^*}{\partial t} ((g - \gamma \mathbb{P}^\pi g)(s, a)) \nabla_{\pi'} \int g(s', a') \mathbb{P}(s' | s, a) \pi'(a' | s')) \right] \]

\[ = -\gamma \mathbb{E}_{(s, a) \sim \mu_\pi^\pi} \left[ \frac{\partial \phi^*}{\partial t} ((g - \gamma \mathbb{P}^\pi g)(s, a)) \mathbb{E}_{a' \sim \pi'(s)} [g(s', a') \nabla_{\pi'} \log \pi'(a' | s')] \right] \]
B  Empirical Results

B.1  OpenAI Gym: MuJoCo

See Figure 5

B.2  Atari

See Figure 6

C  Hyperparameters

C.1  OpenAI Gym: MuJoCo

For the OpenAI Gym environments we use the default hyperparameters in Kostrikov (2018).

| Parameter name               | Value   |
|------------------------------|---------|
| Number of minibatches        | 4       |
| Discount $\gamma$            | 0.99    |
| Optimizer                    | Adam    |
| Learning rate                | 3e-4    |
| PPO clip parameter           | 0.2     |
| PPO epochs                   | 10      |
| GAE $\lambda$                | 0.95    |
| Entropy coef                 | 0       |
| Value loss coef              | 0.5     |
| Number of forward steps per update | 2048 |

Table 2: A complete overview of used hyper parameters for all methods.

C.2  Atari

For the Atari hyperparameters, we again use the defaults set by Kostrikov (2018).

| Parameter name               | Value   |
|------------------------------|---------|
| Number of minibatches        | 4       |
| Discount $\gamma$            | 0.99    |
| Optimizer                    | Adam    |
| Learning rate                | 2.5e-4  |
| PPO clip parameter           | 0.1     |
| PPO epochs                   | 4       |
| Number of processes          | 8       |
| GAE $\lambda$                | 0.95    |
| Entropy coef                 | 0.01    |
| Value loss coef              | 0.5     |
| Number of forward steps per update | 128 |

Table 3: A complete overview of used hyper parameters for all methods.
Figure 5: Our method with KL divergences in comparison to PPO and TRPO on MuJoCo, with 10 seeds. Standard error shaded.
Figure 6: Our method with KL divergences in comparison to PPO on Atari, with 10 seeds and standard error shaded.