Weak Decay Form Factors from QCD Sum Rules on the Light-Cone

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I present a compilation of results on $B \to$ light meson form factors from QCD sum rules on the light-cone.

Form factors of $B$ to light meson transitions are not only needed for the extraction of $|V_{ub}|$ from $B \to \pi e\nu$ or $B \to \pi e\gamma$ and $|V_{cd}|$ from $B \to K\gamma$ and $B \to \rho\gamma$, respectively (provided there is no new physics in these decays), but they also enter most prominently the calculation of $B \to$ charmless nonleptonic decays in BBNS factorisation [1]. It is hence of eminent importance to calculate them as precisely as possible. The most precise calculation of the form factors will undoubtedly finally come from lattice simulations; the present state of this art is summarised in [2]. Another, technically much simpler, but also less rigorous approach is provided by QCD sum rules on the light-cone (LCSRs) [3][4]. The key idea is to consider a correlation function of the weak current and a current sandwiched between the vacuum and the meson $M$, i.e. $\pi$, $K$, $\eta$, $\eta'$, $\rho$, $\omega$, $K^*$ or $\Phi$. For large (negative) virtualities of these currents, the correlation function is, in coordinate-space, dominated by distances close to the light-cone and can be discussed in the framework of light-cone expansion. In contrast to the short-distance expansion employed by conventional QCD sum rules à la SVZ [5], where nonperturbative effects are encoded in vacuum expectation values of local operators with vacuum quantum numbers, the condensates, LCSRs rely on the factorisation of the underlying correlation function into genuinely nonperturbative and universal hadron distribution amplitudes (DAs) $\phi$ that are convoluted with process-dependent amplitudes $T_H$, which are the analogues to the Wilson-coefficients in the short-distance expansion and can be calculated in perturbation theory, schematically

\[ \text{correlation function} \sim \sum_n T^{(n)}_H \otimes \phi^{(n)}. \]  

The sum runs over contributions with increasing twist, labelled by $n$, which are suppressed by increasing powers of, roughly speaking, the virtualities of the involved currents. The same correlation function can, on the other hand, be written as a dispersion-relation, in the virtuality of the current coupling to the $B$ meson. Equating dispersion-representation and the light-cone expansion, and separating the $B$ meson contribution from that of higher one- and multi-particle states, one obtains a relation (QCD sum rule) for the form factor describing the $B \to M$ transition.

The particular strength of LCSRs lies in the fact that they allow inclusion not only of hard-gluon exchange contributions, which have been identified, in the seminal papers that opened the study of hard exclusive processes in the framework of perturbative QCD (pQCD) [6], as being dominant in light-meson form factors, but that they also capture the so-called Feynman-mechanism, where the quark created at the weak vertex carries nearly all momentum of the meson in the final state, while all other quarks are soft. This mechanism is suppressed by two powers of momentum-transfer in processes with light mesons; as shown in [4], this suppression is absent in heavy-to-light transitions and hence any reasonable application of pQCD to $B$ meson decays should include this mechanism. LCSRs also avoid any reference to a “light-cone wave-function of the $B$ meson”, which is a necessary ingredient in all extensions of the original pQCD method to heavy-meson decays [1][8], including factorisation formulas obtained in SCET [9], but about which only very little is known. A more detailed discussion of the rationale of LCSRs and of the more technical aspects of the method can be found e.g. in [10].

LCSRs are available for the $B \to \pi, K$ form factor $f_+$ to $O(\alpha_s)$ accuracy for the twist-2 and part of the twist-3 contributions and at tree-level for higher-twist (3 and 4) contributions [11][12][13]. For the $B \to$ vector transitions, the sum rules are known to $O(\alpha_s)$ accuracy for the twist-2 contributions and at tree-level for twist-3 and 4 contributions [14][15]; ditto for $B \to \gamma$ [16].

Let us now properly define the form factors in question. For a pseudoscalar meson $P$ we have ($q = p_B - p$)

\[ \langle P(p)|\bar{q}\gamma_{\mu}b(B(p_B)) = f_+(q^2) \left( (p_B + p)_{\mu} - \frac{m_B^2 - m_P^2}{q^2}q_{\mu} \right) \right) + \frac{m_B^2 - m_P^2}{q^2}f_0(q^2)q_{\mu}, \]  

\[ \langle P(p)|\bar{q}\sigma_{\mu\nu}q'(1 + \gamma_5)b(B(p_B)) = i \left( (p_B + p)_{\mu}q^2 - q_{\mu}(m_B^2 - m_P^2) \right) \frac{f_1(q^2)}{m_B + m_P}, \]  

\[ f_1(q^2) \] is a current-current correlation function, expressed in terms of two-point functions, both to $O(\alpha_s)$ accuracy [17].

\[ f_2(q^2) \] is the same in $B \to \pi$ and $B \to K$, and is also known to $O(\alpha_s)$ accuracy [18].

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whereas for a vector meson $V$ with polarisation vector $\epsilon_\mu$:

\[
(V(p)\bar{q}\gamma_\mu(1-\gamma_5)b|B(p_B)) = \epsilon_{\mu\nu\rho\sigma}e^{\nu\rho}p_\mu^\nu p_\sigma^\nu \frac{2V(q^2)}{m_B + m_V} \\
- ie_q(m_B + m_V)\Lambda_i(q^2) + i\langle p_B + p \rangle \epsilon \langle p_B \rangle A_2(q^2) \\
+ iq_\mu \epsilon e^{\mu} \frac{2m_V}{q^2} \left( A_3(q^2) - A_0(q^2) \right) \\
\text{with } A_3(q^2) = \frac{m_B + m_V}{2m_V} A_1(q^2) - \frac{m_B - m_V}{2m_V} A_2(q^2).
\]

As for the distribution amplitudes (DAs), they have been parametrized by their partial wave expansion in conformal factorisation, i.e. that the only relevant degrees of freedom are carried by the partons in the $\pi$, and that transverse momenta can be integrated over. Hard infrared (collinear) divergences occurring in $T_{\mu\nu}^{(n)}$ should be absorbable into the DAs. Collinear factorisation is trivial at tree-level, where the quark mass effects as regulator, but can, in principle, be violated by radiative corrections, by so-called “soft” divergent terms, which yield divergences upon integration over $u$. Such terms break, for instance, factorisation in non-leading twist in the treatment of nonleptonic B decays à la BBN [1]. For the simpler case of the correlation function (11), on the other hand, where the convolution involves only one DA instead of up to three in $B \rightarrow \pi \tau$, it was shown in [13] that factorisation also works at one-loop level for twist-3 contributions and that there are no soft divergences.

As for the distribution amplitudes (DAs), they have been discussed intensively in the literature, cf. [12] [13]. For instance, to leading order in the twist expansion, there are three DAs for light mesons, which are defined by the following light-cone matrix elements ($x^2 = 0$):

\[
0|\bar{q}(x)\gamma_\mu\gamma_5d(-x)P(p)|0\rangle = if_{\mu} p_{\rho} \int_{0}^{1} du e^{i\rho x} \phi_{\rho}(u),
\]

\[
0|\bar{q}(x)\gamma_\mu d(-x)V(p)|0\rangle = f_{\mu V} p_{\rho} \epsilon_{\rho x} \int_{0}^{1} du e^{i\rho x} \phi_{\rho}(u),
\]

\[
0|\bar{q}(x)\gamma_\mu d(-x)\sigma_{\mu\nu\rho}d(-x)\sigma_{\mu\nu\rho}d(-x)\sigma_{\mu\nu\rho}d(-x)|0\rangle =
\]

\[
if_{T}^{(v)}(\mu)(\epsilon_\mu p_\nu - p_\nu \epsilon_\mu) \int_{0}^{1} du e^{i\rho x} \phi_{\rho}(u),
\]

where $\xi = 2u - 1$ and we have suppressed the Wilson-line $[x, -x]$ needed to ensure gauge-invariance. The sum rule calculations performed in [11] [12] [13] [14] [15] include all contributions from DAs up to twist-4. The DAs are parametrized by their partial wave expansion in conformal spin, which to NLO provides a controlled and economical expansion in terms of only a few hadronic parameters, cf. [17] for details.

Let us now derive the LCSR for $f_\pi$. The correlation function $\Pi_\pi$, calculated for unphysical $p_\pi^2$, can be written as dispersion relation over its physical cut. Singling out the contribution of the B meson, one has

\[
\Pi_\pi = f_\pi(q^2) \frac{m_B^2 f_B}{m_B^2 - p_\pi^2} + \text{higher poles and cuts},
\]

where $f_B$ is the leptonic decay constant of the B meson, $f_Bm_B^2 = m_B\langle B|\bar{b}b\gamma_5d|0\rangle$. In the framework of LCSRs one does not use (13) as it stands, but performs a Borel transformation, $1/(t - p_\pi^2) \rightarrow B 1/(t - p_\pi^2) = 1/M^2 \exp(-t/M^2)$, with the Borel parameter $M^2$; this transformation enhances the ground-state B meson contribution to the dispersion representation of $\Pi_\pi$ and suppresses contributions of higher twist to the light-cone expansion of $\Pi_\pi$. The next step is to invoke quark-hadron duality to approximate the contributions of hadrons other than the ground-state B meson by the imaginary part of the light-cone expansion of $\Pi_\pi$, so that

\[
\bar{B}\Pi_\pi^{\text{LCE}} = \frac{1}{M^2} \frac{m_B^2 f_B}{m_B^2 - p_\pi^2} f_\pi(q^2) e^{-m_\pi^2/M^2} \\
+ \frac{1}{M^2} \frac{1}{\pi} \int_{0}^{\infty} dt \text{Im}\Pi_\pi^{\text{LCE}}(t) \exp(-t/M^2)
\]
and \( \hat{B}_{\text{sub}}^{\text{LCE}} \) is the LCSR for \( f_+ \). \( s_0 \) is the so-called continuum threshold, which separates the ground-state from the continuum contribution. At tree-level, the continuum-subtraction in (14) introduces a lower limit of integration, \( u \geq (m_b^2 - q^2)/(s_0 - q^2) \equiv u_0 \), in (12), which behaves as \( 1 - \alpha_{\text{QCD}}/m_b \) for large \( m_b \) and thus corresponds to the dynamical configuration of the Feynman-mechanism, as it cuts off low momenta of the u quark created at the weak vertex. At \( O(\alpha_s) \), there are also contributions with no cut in the integration over \( u \), which correspond to hard-gluon exchange contributions. Numerically, these turns out to be very small, \( \sim O(1\%) \) of the total result for \( f_+ \). As with standard QCD sum rules, the use of quark-hadron duality above \( s_0 \) and the choice of \( s_0 \) itself introduce a certain model-dependence (or systematic error) in the final result for the form factor, which is difficult to estimate. To be on the conservative side, one usually adds a 10% systematic error to the final result for \( f_+ \).

Putting everything together, we obtain \( f_+(q^2) \) as plotted in Fig. 1. The form factor can be accurately fitted by

\[
f_+(q^2) = \frac{f_+(0)}{1 - a (q^2/m_b^2) + b (q^2/m_b^2)^2},
\]

with \( f_+(0) \), \( a \) and \( b \) given in Tab. 1, for different values of \( m_b \), \( s_0 \) and \( M^2 \). The above parametrization reproduces the actual values calculated from the LCSR, for \( q^2 \leq 14 \text{ GeV}^2 \), to within 2% accuracy.

Within the same method, also all the other form factors defined in Eqs. (2) to (5) can be calculated. In Figs. 2 and 3 we plot the results for \( B \to \rho \) and \( B \to K^* \) form factors and compare them with lattice calculations, which are available for large \( q^2 \) only. It turns out that all form factors are very well described by the three-parameter formula (15); we list the corresponding best fit values in Tabs. 1 to 3.

Progress in the accuracy of the LCSRs is possible, but not likely to reduce the uncertainties dramatically. It would have to come primarily from a reduction in the uncertainty of input parameters, i.e. the meson decay constants and distribution amplitudes, which could come either from lattice calculations or, in particular for strange mesons, from a re-evaluation of SU(3) breaking effects from QCD sum rules [19].

References

1. M. Beneke et al., Phys. Rev. Lett. 83 (1999) 1914 [arXiv:hep-ph/9905312]; Nucl. Phys. B 591 (2000)
313 [arXiv:hep-ph/0006124]; Nucl. Phys. B 606 (2001) 245 [arXiv:hep-ph/0104110].
2. T. Onogi, contribution to CKM03
3. I.I. Balitsky, V.M. Braun and A.V. Kolesnichenko, Nucl. Phys. B 312 (1989) 509.
4. V.L. Chernyak and I.R. Zhitnitsky, Nucl. Phys. B 345 (1990) 137.
5. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385; ibd. 147 (1979) 448.
6. V.L. Chernyak and A.R. Zhitnitsky, JETP Lett. 25 (1977) 510 [Pisma Zh. Eksp. Teor. Fiz. 25 (1977) 544]; Sov. J. Nucl. Phys. 31 (1980) 544 [Yad. Fiz. 31 (1980) 1053]; A.V. Efremov and A.V. Radyushkin, Phys. Lett. B 94 (1980) 245; Theor. Math. Phys. 42 (1980) 97 [Teor. Mat. Fiz. 42 (1980) 147]; G.P. Lepage and S.J. Brodsky, Phys. Lett. B 87 (1979) 359; Phys. Rev. D 22 (1980) 2157; V.L. Chernyak, A.R. Zhitnitsky and V.G. Serbo, JETP Lett. 26 (1977) 594 [Pisma Zh. Eksp. Teor. Fiz. 26 (1977) 760]; Sov. J. Nucl. Phys. 31 (1980) 552 [Yad. Fiz. 31 (1980) 1069].
7. S. Descotes-Genon and C.T. Sachrajda, Nucl. Phys. B 625 (2002) 239 [arXiv:hep-ph/0109260].
8. H.N. Li, [arXiv:hep-ph/0303116]
9. C.W. Bauer et al., Phys. Rev. D 63 (2001) 114020 [arXiv:hep-ph/0011336]; C.W. Bauer, D. Pirjol and I.W. Stewart, Phys. Rev. D 67 (2003) 071502 [arXiv:hep-ph/0211069].
10. P. Colangelo and A. Khodjamirian, hep-ph/0010175; A. Khodjamirian, hep-ph/0108205.
11. E. Bagan, P. Ball and V.M. Braun, Phys. Lett. B 417 (1998) 154 [arXiv:hep-ph/9709243]; A. Khodjamirian, R. Ruckl and C.W. Winhart, Phys. Rev. D 58 (1998) 054013 [arXiv:hep-ph/9802412]; A. Khodjamirian et al., Phys. Rev. D 62 (2000) 114002 [arXiv:hep-ph/0001297].
12. P. Ball, JHEP 9809 (1998) 005 [arXiv:hep-ph/9802394].
13. P. Ball and R. Zwicky, JHEP 0110 (2001) 019 [arXiv:hep-ph/0110115].
14. P. Ball and V.M. Braun, Phys. Rev. D 55 (1997) 5561 [arXiv:hep-ph/9701238]; A. Khodjamirian et al., Phys. Lett. B 402 (1997) 167 [arXiv:hep-ph/9702318].
15. P. Ball and V.M. Braun, Phys. Rev. D 58 (1998) 094016 [arXiv:hep-ph/9805322].
16. P. Ball and E. Kou, JHEP 0304 (2003) 029 [arXiv:hep-ph/0301135].
17. P. Ball and V.M. Braun, Phys. Rev. D 54 (1996) 2182 [arXiv:hep-ph/9602321]; P. Ball et al., Nucl. Phys. B 529 (1998) 323 [arXiv:hep-ph/9802299]; P. Ball and V.M. Braun, Nucl. Phys. B 543 (1999) 201 [arXiv:hep-ph/9810475]; P. Ball, JHEP 9901 (1999) 010 [arXiv:hep-ph/9812375].
18. V.M. Braun, G.P. Korchemsky and D. Mueller, hep-ph/0306057.
19. P. Ball, E. Boglione, V. Sanz, in preparation.

Figure 2. Comparison of the light-cone sum-rule predictions for the $B \to \rho$ form factors with lattice calculations. Lattice errors are statistical only. The dashed curves show a 15% uncertainty range of the sum rules results. Figure taken from Ref. [15].

Figure 3. Like Fig. 2, but for $B \to K^*$ form factors.