Uncertainty quantification and global sensitivity analysis for composite cylinder shell via data-driven polynomial chaos expansion

Ming Chen¹, Xinhu Zhang*, Kechun Shen¹, Guang Pan¹

¹ School of Marine Science and Technology, Northwestern Polytechnical University, Xi’an, Shaanxi Province, 710072, China
*Corresponding author’s e-mail: xinhu_zhang@nwpu.edu.cn

Abstract. The mechanical properties of composite material exhibit inherent variation with uncertainty. Uncertainties in material properties propagate and result in uncertainties of mechanical performance of structure made of composite material. Polynomial chaos expansion (PCE) is implemented to carry out uncertainty quantification (UQ) and global sensitivity analysis (GSA) of cylinder shell made of composite material for this paper. A case study concerning eigenvalue buckling load of composite cylinder shell is investigated. Design of experiment (DOE) is conducted by utilizing Latin hypercubic sampling. Then data-driven PCE is established and later validated. Statistical moments (mean and standard deviation) and Sobol sensitivity indices of eigenvalue buckling load are obtained respectively. It is found that the PCE can serve as an efficient approach to handle UQ and GSA in engineering applications.

1. Introduction

Thanks to its prominent performance over traditional metal alloys, composite material is widely applied in aerospace, shipbuilding, automotive and civil engineering[1-3]. The passed years have witnessed a sharp increase of naval architecture made of composite material deployed due to the excellent performance of composite material, for instance high strength to weight ratio, resistance against corruption and flexible designability[4-5].

Underwater vehicle is increasingly used in sea applications[6-7]. The hull of underwater vehicle, typical cylinder shell is generally made of steel, with a relatively low load capacity[8-9]. By substituting the steel with composite material, the hull weight is reduced sharply[10]. And a big load capacity can be achieved by employing composite material. Unfortunately as a typical multi-phase material, mechanical property of composite material is variable and uncertain[11-13]. The variation of mechanical property leads to a relatively big variation of structural performance[14]. The variation of composite material’s mechanical property has an effect on the structural performance of hull consisting of composite material. That is to say, the uncertainty of composite material’s mechanical property propagates to the hull’s mechanical performance. When deployed underwater, the composite cylinder shell of vehicle withstands huge hydrostatic pressure and protects equipment inside. Hence it is necessary to conduct uncertainty quantification of hull’s mechanical performance considering uncertainty of composite material’s mechanical property.

PCE is a superior method in the field of scientific computation concerning UQ. PCE is beneficial when compared with other methods when conducting uncertainty analysis[15-18] as follows:

1) PCE is mathematically sound and can be easily implemented.
(2) PCE can handle plenty of probability distributions.
(3) Statistical metrics of system response can be got instantly right after the PCE surrogate being established.
(4) When dealing with problems with continuous and smooth properties, PCE converges fast.

PCE with full polynomial basis is computationally prohibited for the polynomial basis increases exponentially with the increase of dimensionality. Therefore sparse PCE with vital polynomial basis which contributes most to the whole PCE is usually utilized. Sparse PCE is computationally cheap, meanwhile with a desired convergence rate[19-21].

This paper is aimed at investigating the application of PCE in UQ and GSA for mechanical performance of underwater vehicle made of composite material. Theory for PCE is briefly presented in Section 2. The details concerning uncertainty of material properties of composite cylindrical shell for underwater vehicle are given. Then results and discussions and conclusions follow respectively for the last two parts.

2. Theory for polynomial chaos expansion

Take into account a stochastic system with a single uncertain input x and a single system output y. The basis for the theory of PCE lies in an orthogonal polynomial family related with the input probability density function (PDF). The well known Askey scheme[22-24] shows the relationship between orthogonal polynomial families and the most important probability distributions, see table 1.

The orthogonal polynomials are used to build a surrogate model or meta-model which is computationally cheap as an approximation of the system output[25-27], refer to equation (1). M represents the truncation order of the PCE, \( c_\ell \) is coefficient of orthogonal polynomial, \( \phi_\ell(x) \) is the i order orthogonal polynomial.

\[
y(x) \approx \hat{y}(x) = \sum_{\ell=0}^{M} c_\ell \phi_\ell(x)
\]  

There exists two approaches to build PCE, namely pseudo-spectral projection and point collocation method. Pseudo-spectral projection is conducted by quadrature rules to estimate the inner product of the coefficient \( c_\ell \), refer to equation (2). Gaussian quadrature is often used for pseudo-spectral projection[28-29].

\[
c_\ell = \langle y, \phi_\ell \rangle = \int \phi_\ell(x)y(x) \text{PDF}(x)dx \approx \sum_{i=0}^{N} \omega_i \phi_\ell(x_i)y(x_i)
\]  

Point collocation is implemented by fitting the orthogonal polynomial to a limited number of samples. Least squares method can be used for fitting. Point collocation is more robust when tackling high dimensional problem by establishing sparse PCE which is more efficient[30-31].

| Type of probabilistic distribution | Orthogonal polynomial basis | Domain          |
|----------------------------------|-----------------------------|-----------------|
| Uniform distribution             | Legendre                    | [-1,1]          |
| Exponential distribution         | Laguerre                     | [0, +\infty]    |
| Normal distribution              | Hermite                     | [-\infty, +\infty] |
| Beta distribution                | Jacobi                      | [-1,1]          |
| Gamma distribution               | Generalized Laguerre        | [0, +\infty]    |

3. Application in composite cylindrical shell for underwater vehicle

When it comes to the application of PCE in composite underwater vehicle, the composite underwater vehicle consists of a cylinder shell. Uniformly distributed lateral pressure is applied on the cylinder shell. Dimensions of composite cylindrical shell are as follows: a length of 1000mm, diameter 324mm and thickness 6mm respectively. The composite cylindrical shell has a stacking sequence of [0/45/-45/90/0/60/30/60/-45/90]s, total 20 plies with a symmetrically stacking sequence. The commercial finite element program Abaqus is used for numerical analysis. As is shown in figure 1, finite element model of composite cylinder shell is built. S4R element is used for modelling composite
cylinder shell. The left end is fixed for all six degrees of freedom. The right end is fixed except the axial displacement along the length.

Figure 1. Modeling composite cylinder shell in Abaqus

Composite cylinder shell consists of carbon fiber-epoxy composite material with mechanical properties presented in table 2, with a coefficient of variation 3%, all subject to normal distribution.

Table 2. Material mechanical properties for carbon fiber-epoxy

| Mechanical properties for carbon fiber-epoxy composite material | Symbols and values                     |
|---------------------------------------------------------------|----------------------------------------|
| Three elastic modulus                                        | E11 = 121e3 MPa, E33 = 8.6e3 MPa, E22 = 8.6e3 MPa |
| Three shear modulus                                           | G12 = 3.35e3 MPa, G23 = 2.68e3 MPa, G13 = 3.35e3 MPa |
| Three Poisson’s ratio                                         | Nu12 = 253e-3, Nu23 = 421e-3, Nu13 = 253e-3 |

As for data-driven polynomial chaos expansion, design of experiment (DOE) is to be performed first. Then the surrogate model, namely polynomial chaos expansion is established and meanwhile validated. Once the polynomial chaos expansion model is built, we can conduct UQ and GSA for composite underwater vehicle. UQLab[32] is adopted as the tool to implement the study.

4. Results and discussions
As for DOE, training set has a sample size of 100 (about 11 times the dimension), test set has a sample size of 50, Latin hypercube method is used as sampling strategy. Base on the got samples, data-driven polynomial chaos expansion is built and validated.

Data-driven PCE converges at polynomial degree 2. Validation error for test set is 1.8069305e-07. So the established data-driven PCE is accurate. The mean of eigenvalue buckling load of composite cylindrical shell is 3.03MPa. The detailed information of data-driven PCE is as follows:

Table 3. Data-driven PCE results

| Data-driven PCE results | Value                      |
|-------------------------|----------------------------|
| Maximal degree          | 2                          |
| Number of input variables | 9                         |
| Size of polynomial basis for full PCE | 55                      |
| Size of polynomial basis for sparse PCE | 17                     |
| Number of model computation | 100                     |
| Validation error        | 1.8069305e-07             |
| Mean                    | 3.0316                     |
| Standard deviation      | 0.0828                     |
| Coefficient of variation | 2.73%                     |

As depicted in figure 2, the established PCE model is accurate as true Y and prediction of PCE lines on y=x. The established PCE model has a size of sparse basis 17, which is efficient for the 9
dimension problem. What is more, training set has a sample size of 100 (about 11 times the dimension), and PCE model is effective to handle the problem with small sample data, meanwhile possessing desired accuracy.

![Figure 2. True Y and prediction of PCE](image)

For the purpose of evaluating influences of material properties on mechanical performance, GSA is implemented. First order Sobol indices got by GSA stand for influences of an individual random variate on system output alone. Total Sobol indices got by GSA stand for the total effect of a certain random variate on system output including its interaction with other variates. Sobol indices are shown in table 4, 5 and figure 3, 4. E1 namely elastic modulus along fiber direction has the most influence on eigenvalue buckling load. The detailed information of global sensitivity analysis is as follows:

| Mechanical properties | Value    |
|-----------------------|----------|
| E11                   | 0.996748 |
| E22                   | 0.001802 |
| E33                   | 0        |
| Nu12                  | 0.000223 |
| Nu23                  | 0        |
| Nu13                  | 0        |
| G12                   | 0.000735 |
| G23                   | 0.000287 |
| G13                   | 0.000206 |

![Figure 3. Total Sobol indices](image)
Table 5. First order Sobol indices

| Mechanical properties | Value   |
|-----------------------|---------|
| E11                   | 0.996747|
| E22                   | 0.001802|
| E33                   | 0       |
| Nu12                  | 0.000223|
| Nu23                  | 0       |
| Nu13                  | 0       |
| G12                   | 0.000735|
| G23                   | 0.000287|
| G13                   | 0.000205|

Figure 4. First order Sobol indices

5. Conclusions

PCE is carried out for UQ and GSA of composite underwater vehicle. It is found that PCE can be adopted as an efficient surrogate model by conducting a limited number of DOE, thus reducing the computational time. Meanwhile PCE provides results with a good accuracy. Results show PCE is a powerful tool in UQ and GSA of composite underwater vehicle and can be extendly applied for more complex scenes such as geometry uncertainty and load uncertainty.

Acknowledgments

This work was supported by Fundamental Research Funds for the Central Universities (Grant No.3102019JC006), and China Postdoctoral Science Foundation (Grand No.2020M673492).

References

[1] Giuliani, P. M., Giannini, O., Panciroli, R. (2022) Characterizing flax fiber reinforced bio-composites under monotonic and cyclic tensile loading. Composite Structures, 280, 114803.

[2] Donhauser, T., Kenf, A., Schmeer, S., Hausmann, J. (2022) Calculation of highly stressed components made of carbonfiber-reinforced polyamide-6. Composite Structures, 280, 114830.

[3] Deng, J., Li, J., Zhu, M. (2021) Fatigue behavior of notched steel beams strengthened by a prestressed CFRP plate subjected to wetting/drying cycles. Composites Part B: Engineering, 109491.

[4] Huang, Z., Xing, Y., Gao, Y. (2022) A new method of stiffness prediction for composite plate structures with in-plane periodicity. Composite Structures, 280, 114850.
[5] Yu, J., Zhang, B., Chen, W., Liu, H. (2021) Multi-scale analysis on the tensile properties of UHPC considering fiber orientation. Composite Structures, 114835.

[6] Liu, J., He, B., Yan, T., Yu, F., Shen, Y. (2021) Study on carbon fiber composite hull for AUV based on response surface model and experiments. Ocean Engineering, 239, 109850.

[7] Li, B., Pang, Y. J., Cheng, Y. X., Zhu, X. M. (2017) Collaborative optimization for ring-stiffened composite pressure hull of underwater vehicle based on lamination parameters. International Journal of Naval Architecture and Ocean Engineering, 9(4): 373-381.

[8] Fathallah, E., Qi, H., Tong, L., Helal, M. (2015) Numerical investigation of the dynamic response of optimized composite elliptical submersible pressure hull subjected to non-contact underwater explosion. Composite Structures, 121: 121-133.

[9] Cho, Y. S., Paik, J. K. (2019) An empirical formula for predicting the collapse strength of composite cylindrical-shell structures under external pressure loads. Ocean Engineering, 172: 191-198.

[10] Craven, R., Graham, D., Dalzel-Job, J. (2016) Conceptual design of a composite pressure hull. Ocean Engineering, 128: 153-162.

[11] Nguyen, H. X., Hien, T. D., Lee, J., Nguyen-Xuan, H. (2017) Stochastic buckling behaviour of laminated composite structures with uncertain material properties. Aerospace Science and Technology, 66: 274-283.

[12] Pouresmaeeli, S., Falzon, B. G. (2021) Uncertainty quantification of pure and mixed mode interlaminar fracture of fibre-reinforced composites via a stochastic reduced order model. Composite Structures, 278, 114683.

[13] Trinh, M. C., Nguyen, S. N., Jun, H., Nguyen-Thoi, T. (2021) Stochastic buckling quantification of laminated composite plates using cell-based smoothed finite elements. Thin-Walled Structures, 163, 107674.

[14] Hamdia, K. M., Ghasemi, H., Zhuang, X., Rabczuk, T. (2022) Multilevel Monte Carlo method for topology optimization of flexoelectric composites with uncertain material properties. Engineering Analysis with Boundary Elements, 134: 412-418.

[15] Dias, F. S., Peters, G. W. (2021) Option pricing with polynomial chaos expansion stochastic bridge interpolators and signed path dependence. Applied Mathematics and Computation, 411, 126484.

[16] Dréau, J., Magnain, B., Nyssen, F., Batailly, A. (2021) Polynomial chaos expansion for permutation and cyclic permutation invariant systems: application to mistuned bladed disks. Journal of Sound and Vibration, 503, 116103.

[17] Zhou, Y., Lu, Z., Cheng, K. (2022) Adaboost-based ensemble of polynomial chaos expansion with adaptive sampling. Computer Methods in Applied Mechanics and Engineering, 388, 114238.

[18] Santanoceto, M., Tiberga, M., Perkó, Z., Dulla, S., Lathouwers, D. (2021) Preliminary uncertainty and sensitivity analysis of the Molten Salt Fast Reactor steady-state using a Polynomial Chaos Expansion method. Annals of Nuclear Energy, 159, 108311.

[19] Qian, J., Dong, Y. (2022) Uncertainty and multi-criteria global sensitivity analysis of structural systems using acceleration algorithm and sparse polynomial chaos expansion. Mechanical Systems and Signal Processing, 163, 108120.

[20] Yang, T., Zou, J. F., Pan, Q. (2021) A sequential sparse polynomial chaos expansion using Voronoi exploration and local linear approximation exploitation for slope reliability analysis. Computers and Geotechnics, 133, 104059.

[21] Sun, Q., Dias, D. (2021) Global sensitivity analysis of probabilistic tunnel seismic deformations using sparse polynomial chaos expansions. Soil Dynamics and Earthquake Engineering, 141, 106470.

[22] Askey, R., Wilson, J. A. (1985) Some basic hypergeometric orthogonal polynomials that generalize Jacobi polynomials. American Mathematical Society.
[23] W. Schoutens. (2000) Stochastic Processes and Orthogonal Polynomials. Springer-Verlag, New York.

[24] Koekoek, R., Swarttouw, R. F. (1996) The Askey-scheme of hypergeometric orthogonal polynomials and its q-analogue. arXiv preprint math/9602214.

[25] Wang, L., Yang, G. (2021) An interval uncertainty propagation method using polynomial chaos expansion and its application in complicated multibody dynamic systems. Nonlinear Dynamics, 1-22.

[26] Yang, T., Zou, J. F., Pan, Q. (2021) A sequential sparse polynomial chaos expansion using Voronoi exploration and local linear approximation exploitation for slope reliability analysis. Computers and Geotechnics, 133, 104059.

[27] Man, J., Lin, G., Yao, Y., Zeng, L. (2021) A generalized multi-fidelity simulation method using sparse polynomial chaos expansion. Journal of Computational and Applied Mathematics, 397, 113613.

[28] Zhang, X., Pandey, M. D., Luo, H. (2021) Structural uncertainty analysis with the multiplicative dimensional reduction–based polynomial chaos expansion approach. Structural and Multidisciplinary Optimization, 64(4): 2409-2427.

[29] Wu, K., Xiu, D., Zhong, X. (2021) A WENO-Based stochastic Galerkin scheme for ideal MHD equations with random inputs. Communications in Computational Physics, 30(2): 423-447.

[30] Zhang, B. Y., Ni, Y. Q. (2021). A hybrid sequential sampling strategy for sparse polynomial chaos expansion based on compressive sampling and Bayesian experimental design. Computer Methods in Applied Mechanics and Engineering, 386, 114130.

[31] Papadopoulos, A. D., Tehrani, B. K., Bahr, R. A., Tentzeris, E. M., Glytsis, E. N. (2021) Uncertainty quantification of printed microwave interconnects by use of the sparse polynomial chaos expansion method. IEEE Microwave and Wireless Components Letters.

[32] Marelli S, Sudret B. (2014) UQLab: A framework for uncertainty quantification in Matlab. In: Vulnerability, uncertainty, and risk: quantification, mitigation, and management. Liverpool. pp. 2554-2563.