Turbine blade profile design method based on Bezier curves

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Abstract. In this paper, the technique of two-dimensional parametric blade profile design is presented. Bezier curves are used to create the profile geometry. The main feature of the proposed method is an adaptive approach of curve fitting to given geometric conditions. Calculation of the profile shape is produced by multi-dimensional minimization method with a number of restrictions imposed on the blade geometry. The proposed method has been used to describe parametric geometry of known blade profile. Then the baseline geometry was modified by varying some parameters of the blade. The numerical calculation of obtained designs has been carried out. The results of calculations have shown the efficiency of chosen approach.

1. Introduction
The increase of efficiency of turbines is extremely important in today's heat-and-power engineering. One of the ways of solving this problem is an optimization of aerodynamic characteristics of turbine blades. With the progress of computational methods, the new techniques are developed in order to automate the design processes of turbine blades [1]. One of the most perspective algorithms is a gradient-based adjoint solver method [2]. It is based on a CFD code which is used to compute flow parameters and the gradients of their variations while blade design points are changed. This approach offers a very rich design space but has drawback that need to be considered [3]. It works with the grid points of computational domain and have no any connections with CAD geometry. This means that an additional step is required to transform the optimal shape defined by grid points back to smooth CAD shape. Also, it is very difficult to set some limitations to the profile geometry that are connected with aerodynamical and mechanical requirements. So, the efficient blade parameterization method is needed which could explicitly represent a profile geometry by means of a list of geometric parameters.

One can formulate the requirements for a blade parameterization method. Geometry parameters that describe the profile shape have to be directly connected with gas-dynamics or mechanical properties of the blade. Method has to bring abilities of affecting on flow features in blade passages by means of varying the chosen geometry parameters. This technique has to provide the universal approach which would be suitable for different types of turbomachinery profile designs (turbine stator blades and rotor blades, compressor blades) operating in a broad range of flow conditions.

The main problem of these methods corresponds to the third requirement. A lot of different approaches have been presented [4]. But most of them are limited by the types of produced profile designs. In this paper, we present a parameterization method of blades profiles which meets all the requirements listed above. This approach uses Bezier curves in order to represent the geometry.

2. Blade parametrization method
In order to represent the profile design, 13 parameters are used (see figure 1). Method computes geometry in the coordinate system in which the blade profile is oriented and scaled, so the centers of circles of leading and trailing edges are situated at X = 0; Y = 0 and X = 1; Y = 0 respectively.

Figure 1. Blade parameters.

Chosen parameters are directly connected with gas-dynamic and mechanical properties of the turbine blade cascade. $R_{\text{max}}$, $X_{\text{max}}$ have a sufficient influence on the moment of inertia and the moment of resistance of the profile; $R_{\text{ut}}$, $X_{\text{ut}}$, $\alpha_{\text{ut}}$ define the throat and spacing of blade cascade; $\alpha_1$, $\alpha_2$ define the inlet and exit angle of flow correspondingly as well as their combination define the stagger angle; $\omega_1$ defines sensitivity of profile operation at non-design inlet angles of the flow. Parameters $X_{\text{max}}$, $Y_{\text{max}}$ are included on the basis of unification of design production procedure for different types of blades. These parameters have a significant influence on the curvature distribution of blade suction and pressure sides. As a result, the change of these values is very efficient from the point of view of controlling the pressure distribution on the blade sides.

Chosen parameters describe the design of a single aerodynamic profile. But they are directly connected with the main characteristics of a blade cascade. In order to compute the shape of blade passage it is necessary to define the following parameters: stagger angle, opening, flow inlet angle, flow exit angle, blade spacing, and blade chord. On the basis of these characteristics some profile parameters (which are described in figure 1) can be calculated by geometrical construction ($X_{\text{bend}}$, $R_{\text{bend}}$, $\alpha_{\text{bend}}$) and the other can be expressed using the statistical relationships [5, 6].

The described method uses parametric Bezier curves in order to represent a profile design. Its shape is defined by an array of control points (P). A general equation of Bezier of a degree n can be written as:

$$\vec{R}(t) = \sum_{i=0}^{n} \vec{P}_i B_i^n(t),$$

where $\vec{R}$ – a position of curve point (x, y) at a parametric coordinate (t); $\vec{P}_i$ – a coordinate of control point (their amount equals n+1) which forms a polygonal line; $B_i^n(t)$ – Bernstein coefficients

Bezier curves have several advantages that are important for the blade profile design process: they are smooth - their first ($\vec{R}'$) and second ($\vec{R}''$) derivatives exist and are continuous; these curves are situated in a convex hull, formed by control points; each Bezier curve with a degree n can be presented as Bezier curve with n+1 degree; they start (t = 0) in the first point of control polygon ($\vec{P}_0$) and ends (t = 1) at the last point ($\vec{P}_n$). At t = 0 the tangent line to the curve matches with segment $\vec{P}_0\vec{P}_1$ and at t = 1 with segment $\vec{P}_{n-1}\vec{P}_n$.

Camber line, pressure and suction sides are represented as Bezier curve in the current method. The process of their calculation is a fitting procedure. Bezier curve which describes the shape of one of them is bounded by endpoints, lying in the leading and trailing edges (marked as blue circles in figure 1). Also, it has to pass throw intermediate points (marked as red squares in figure 1) and the tangent
angle in each point is a given value ($\beta_i$ in figure 1). Coordinates $\vec{s}_i$ and angles $\beta_i$ are calculated on the basis of the blade parameters values. At the boundary points (blue circles in figure 1) tangent angles and curve curvature are given.

In order to fit a Bezier curve to listed conditions the multidimensional minimization procedure is used. It is based on L-BFGS (Limited-memory Broyden-Fletcher-Goldfarb-Shanno) method [7] and represents an adaptive approximation to the desired curve shape. The algorithm of this process is described in figure 2.

![Figure 2. Fitting algorithm.](image)

At the beginning the algorithm tries to fit a cubic Bezier curve ($n = 3$) to given geometric conditions using multidimensional minimization. After this step the method defines the quality of fitting process by computing the minimal distances between intermediate points and obtained curve. If the fitting error is under the maximum value defined by user the process ends. Otherwise the degree of Bezier curve increases (that is the number of control points increases) and the fitting procedure repeats. The algorithm stops if the fitting error allowable or if the maximum degree of Bezier curve is reached (in this case we can say that there is no a suitable Bezier curve for current geometric parameters). It is important to note that at multidimensional minimization process the curve is subjected by additional constraints: the distribution of curvature $k(t)$ along all curve length has to be negative or equals 0 (this is due to aerodynamic requirements [5]).

Thus, the developed method automatically determines the degree and the array of parametric points which are most suitable for the given profile parameters. And the process of turbine profile calculation using this algorithm is divided into several stages: 1 -Calculation of the profile camber line; 2 -Calculation of the profile suction side; 3 -Calculation of the profile pressure side.

The approbation of the developed method has shown that the degree of Bezier curves for camber line usually doesn’t exceed $n = 5$, and for pressure and suction sides $n = 9$. The proposed method allows generating blade profiles in a broad range of geometry and types. Figure 3 shows the examples of obtained designs.

![Figure 3. Examples of developed profile designs.](image)

3. Optimization of a blade profile

The developed method was used to produce an improved blade profile design. As a baseline geometry, the S9015A [8] nozzle profile was used. The main objective of current study is to examine the blade parameterization technique be available to effectively control parameters of the flow in a blade passage. Current method is suitable for automatically representing of blade design described as an
ununiform distribution of suction and pressure sides points coordinates. The details of this procedure are presented in [9]. The aim of the blade optimization was to reduce the kinetic energy losses. The mass flow rate and the pressure drop (theoretical Mach number $M_{1t}$ downstream the blade cascade) were keeping constant. The CFD code was used for current investigation. It solves the Reynolds averaged Navier-Stokes (RANS) equations which are closed by k-kl-$\omega$ turbulence model. It allows solving as a laminar as a turbulent boundary layer. The geometry parameters of studied nozzle blade cascade are shown in figure 4. Two values of flow initial turbulence intensity have been considered: $Tu = 5\%$ and $Tu = 15\%$. The detailed analyze for Tu=5% is presented below.

Figure 4. Blade cascade geometry parameters.

Several baseline design modifications have been considered. All geometries are presented in figure 5. The zone of interest for current study was the area of blade suction side where the diffuser flow takes place and the trailing edge.

Figure 5. Baseline and modified profiles designs.

The results of numerical investigation of baseline profile have shown the presence of the separation bubble on the suction side near the trailing edge (see zone A in figure 4). It is presented in figure 6a. For our opinion, it appears due to the error of baseline parameterization procedure. It can be connected with the low number of coordinates that describe the profile geometry in [8]. One can clearly see this while considering the curvature distribution $k(t)$ of the suction side for default design in figure 6b. In the point, upstream the beginning of boundary layer separation (at $t = 0.8$), there is a change of curvature distribution behavior. It leads to increasing of the pressure and destabilization of the boundary layer. At the same value of $\alpha_{bend}$, but different shapes of a suction side in a blade bevel cut, parameters of boundary layer may differ considerably. As described in [6], in this area the distribution of suction side curvature has to be smooth and it should approach to some fixed value.

So, the first step of the blade modification was to prevent the boundary layer separation process in considered area. For this propose, the Mod1 (see figure 5) profile was developed. As one can see from figure 6b, the curvature distribution for this geometry variant is virtually constant in this zone. This has prevented the separation process (see figure 6a).
It should be noted that the considered baseline profile design is known as efficient. Our study has shown that the profile losses connected with the friction in boundary layers could not be decreased without dramatic change of the blade design for chosen operation conditions. So, we decided to focus on a reduction of the losses derived in the trailing edge wake. For modern turbine blades the trailing edge contributes about 1/3 of the total profile losses [10]. Losses arising at a trailing edge depend on the value of a base pressure downstream the blade [11]. This parameter depends on the regime in boundary layers (Re number) and relative size of the trailing edge. There is a direct relation between base pressure coefficient and a value of kinetic energy losses ($\zeta$) for subsonic flows [6]: with the increase of $\Delta \bar{P}$ the $\zeta$ decreases. So, one of the efficient ways of the profile design optimization can be the modification of the trailing edge radius. The Mod2 profile design (see figure 5) is based on the geometry of baseline shape. As one can see it differs only by the radius of the trailing edge. It leads to increasing of suction side curvature near the trailing edge (see figure 6b) and its shape becomes not optimal for the development of boundary layer in this area. As a result, the separation bubble appears in the zone B (see figure 4). It is shown in figure 6a. So, while modifying the design of a trailing edge it is important to control the geometry of a suction side. For this purpose, the Mod3 blade design was developed (see figure 5). Its geometry is based on the Mod2 and the idea of modification is the same as for Mod1 profile. This helped preventing the separation of boundary layer on the suction side (see figure 6a).

The described optimization procedures were conducted by improvement of the suction side geometry in a narrow zone near the blade trailing edge. For the next step of the blade modification we decided to change the conditions of flow along the suction side. For this purpose, the $Y_{\text{max}}$ parameter (see figure 1) has been changed. As the basis geometry for Mod4 the Mod3 geometry was used. As one can see this contributed a significant change of the suction side shape in the whole cascade bevel cut (see figure 5) and the distribution of curvature became smoother (see figure 6b). It creates favorable conditions for the flow in the boundary layer [6].

As a result, the values of profile losses have been derived for each modification. The numerical results for initial turbulence intensity $Tu = 5\%$ and $Tu = 15\%$ have been considered. Comparison of profile losses levels between modified designs and baseline geometry is presented in figure 7. Here the value $\Delta \zeta$ was determined as follows:

$$\Delta \zeta = \frac{\zeta_{\text{bl}} - \zeta_{\text{mod}}}{\zeta_{\text{bl}}} \times 100,$$

where $\zeta_{\text{bl}}$ – profile losses of the baseline geometry; $\zeta_{\text{mod}}$ – profile losses for the modified design. The different influence of blades modification on the profile losses is observed for each level of initial turbulence intensity. For $Tu = 5\%$ and for $Tu = 15\%$ decreasing the trailing edge size leads to decrease of the profile losses. But for the low level of initial turbulence this effect is observed more significantly. Also, the prevention of boundary layer separation for regimes with $Tu=5\%$ increases the efficiency of the blade profile for $5 - 3\%$. This is not observed for the conditions with $Tu=15\%$ because the boundary layer separation doesn’t appear on the suction side. This is due to the fact that
for Tu=5% boundary layer is laminar (due to the laminarization process in the flow with negative pressure gradients) while at the conditions with Tu=15% it is turbulent and it better resists the separation (see figure 7b). The transition from Mod3 shape to Mod4 doesn’t lead to any significant increase of profile efficient for the condition with low initial turbulence level. While for Tu=15% positive effect is observable. It is connected with the fact that the length of Mod4 suction side is less than of Mod3. For Tu=5% the boundary layer thickness is small due to its laminar behavior, while at regime with Tu=15% turbulent boundary layer swells much faster. And for this condition the length of suction side plays an important role from the point of view of friction losses in boundary layer.

Figure 7. Profile losses for considered design modifications relative to baseline geometry (a) and velocity distribution in boundary layer for Mod3 geometry in Zone B (b).

4. Conclusions
The fast and effective blade parametrization method was presented. It is suitable to use in combination with adjoint solvers in order to automatically optimize aerodynamic characteristics of turbomachinery blade profiles. Current method offers a unified approach for designing geometry of different profiles types.

The optimization of known blade profile has been carried out in order to investigate the abilities of developed method to effect on the flow parameters. The results have shown the efficiency of chosen approach. It allowed to increase the efficiency of the blade profile by almost 30 % for low levels of inlet turbulence and by 15% for high levels of inlet turbulence.

5. References

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