The FFLO state with angle-dependent gap in Asymmetric Nuclear Matter

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Abstract

We consider the FFLO and angle-dependent gap (ADG) states together with arbitrary angle $\theta_0$ between the directions of the Cooper pair momentum and the symmetry-axis of ADG for asymmetric nuclear matter. We find two kinds of locally stable states, i.e., the FFLO-ADG-Orthogonal and FFLO-ADG-Parallel states, which correspond to $\theta_0 = \frac{\pi}{2}$ and $\theta_0 = 0$, respectively. Furthermore, the FFLO-ADG-Orthogonal state settles at low asymmetry, whereas the FFLO-ADG-Parallel state is favored for large asymmetry. The critical isospin asymmetry $\alpha_c$, where superfluid vanishes, is enhanced largely by considering the Cooper pair momentum with ADG.

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I. INTRODUCTION

Neutron-proton (n-p) pair correlations are potentially important in a number of physical contexts, including the mechanism of the deuteron formation in heavy-ion collisions\cite{1} at intermediate energies and supernovas\cite{2–4}. In the nuclei context, as observed by the recent experiments\cite{5} on excited states in $^{92}$Pd, large nuclei may feature spin-aligned n-p pairs, and moreover the exotic nuclei with extended halos provide a locus for n-p pairing. In addition, the n-p pairing may play a major role in determining the cooling and rotation dynamics in the model of “nucleon stars” which permits pion or kaon condensation\cite{6}.

The occurrence of the n-p pairing crucially depends on the overlap between the neutron and proton Fermi surfaces. The paring correlation is suppressed when the system is driven out of the isospin symmetric state. At low temperature, the thermal smearing of the Fermi surfaces promotes the pairing, but, is ineffective when the separation between the surfaces is large compared to the temperature. However, a shift of neutron and proton Fermi spheres with respect to each other, which results in the nonzero total momentum of the Cooper pairs, is expected to enhance the overlap between the two Fermi surfaces. The overlap regions then provide the kinematical phase space for pairing phenomena to occur. In this configuration space such a condensate forms a periodic lattice with finite shear modulus. The resulting inhomogeneous superconducting state is called FFLO state\cite{7, 8}. Another possible mechanism to enhance the phase-space overlap between the driving apart Fermi surfaces of neutron and proton is the deformation of the two Fermi surfaces\cite{9, 10} which causes the formation of DFS (deformed Fermi surfaces) state. Both these two configurations imply an anisotropic quasiparticle spectrum. On the other hand, the previous studies of the DFS state\cite{10} and the FFLO state\cite{11} adopt the angle-averaging procedure which has been proved to be a quite good approximation in symmetry nuclear matter\cite{12} by considering the pairing gap as an isotropic one. In fact, our previous work\cite{13} indicates the angle dependence of the pairing gap should be taken into account when calculating the pairing gap in asymmetric nuclear matter at low temperature. In the Ref.\cite{13}, we propose an axi-symmetric angle-dependent gap (ADG) state which corresponds to the axi-symmetric deformation of the neutron and proton Fermi spheres. In the ADG state, the rotational symmetry is broken spontaneously and there exists a symmetry breaking axis. While in the FFLO configuration, both the rotational symmetry and translational symmetry are spontaneously broken and the
axis of the symmetry breaking is along the direction of the total Cooper pair momentum. To determinate the structure of the true ground state for bulk isospin-asymmetric nuclear matter, we should consider the FFLO and ADG state together within the same model as that in Ref. [11, 13].

The purpose of the present paper is a combined treatment of the FFLO and the axisymmetric angle-dependent gap (ADG) phases within the same model as that in Ref. [11, 13] for asymmetric nuclear matter. And the arbitrary angles between the two symmetry breaking axes are considered to determine the favored angle for the ground state. In the calculations we take into account only the \( ^3SD_1 \) partial-wave channel, which dominates the pairing interaction at low density [14–19]. The paper is organized as follows: In Sec. II we derive the gap equations, which include the effects of the finite momentum of the Cooper pairs and the axisymmetric angle dependence of pairing gap, from the Gorkov equations. The numerical solutions of these equations are shown in Sec. III, where we discuss the phase diagram of the combined FFLO and ADG state and the properties for different angles between the two symmetry breaking axes at finite temperature. We finish in Sec. IV with a discussion of our results and present our conclusions.

II. FORMALISM

At low density, the isospin singlet \( ^3SD_1 \) paring channel dominates the attractive pairing force. In this case we can consider \( SD \) channel alone, and the gap function can be expanded [12, 20] according to

\[
\Delta_{\sigma_1,\sigma_2}(k) = \sum_{l,m_j} \Delta_{l}^{m_j}(k)[G_{l}^{m_j}(\hat{k})]_{\sigma_1,\sigma_2},
\]

(1)

the elements of the spin-angle matrices are

\[
[G_{l}^{m_j}(\hat{k})]_{\sigma_1,\sigma_2} \equiv \frac{1}{2} \sigma_1 \frac{1}{2} \sigma_2 | 1\sigma_1 + \sigma_2 \rangle \langle 1\sigma_1 + \sigma_2, lm_l | 1m_j \rangle Y_{l}^{m_l}(\hat{k}),
\]

(2)

where \( m_j \) and \( m_l \) are the projections of the total angular momentum \( j = 1 \) and the orbit angular momentum \( l = 0,2 \) of the pair. The \( Y_{l}^{m_l}(\hat{k}) \) denotes the spherical harmonic with \( \hat{k} \equiv k/k \). The anomalous density matrix follows the same expansion. Moreover the time-
reversal invariance implies that

\[ \Delta_{\sigma_1, \sigma_2}(k) = (-1)^{1+\sigma_1+\sigma_2}\Delta^*_{-\sigma_1, -\sigma_2}(k). \]  

(3)

Namely, the pairing gap matrix \( \Delta(k) \) in spin space possesses the property

\[ \Delta(k)\Delta^\dagger(k) = ID^2(k), \]  

(4)

i.e., the gap function has the structure of a “unitary triplet” state [12]. \( I \) is the identity matrix and \( D(k) \) is a scalar quantity in spin space.

A. The quasiparticle spectrum

Once the isospin singlet SD channel has been selected, the gap is an isoscalar and the isospin indices can be dropped off. The proton/neutron propagators follow from the solution of the Gorkov equations are present in the form (\( \hbar = 1 \))

\[ G^{(p/n)}_{\sigma, \sigma'}(k, \omega_m) = -\delta_{\sigma, \sigma'}\frac{\omega_m + \xi_k \mp \delta \varepsilon_k}{(\omega_m + E_k^+)(\omega_m - E_k^-)}, \]  

(5)

where \( \omega_m \) are the Matsubara frequencies. The upper sign in \( G^{(p/n)}_{\sigma, \sigma'} \) corresponds to protons, and the lower to neutrons. The neutron-proton anomalous propagator matrix in spin space has the form

\[ F^\dagger(k, \omega_m) = -\frac{\Delta^\dagger(k)}{(\omega_m + E_k^+)(\omega_m - E_k^-)}. \]  

(6)

The quasiparticle excitation spectra are determined by finding the poles of the propagators in Gorkov equations,

\[ E_{k}^{\pm} = \sqrt{\xi_k^2 + \frac{1}{2}Tr(\Delta\Delta^\dagger) \pm \frac{1}{2}\sqrt{[Tr(\Delta\Delta^\dagger)]^2 - 4\det(\Delta\Delta^\dagger) \pm \delta \varepsilon_k}}, \]  

(7)

where \( \xi_k = \frac{1}{2}(\varepsilon_k^p + \varepsilon_k^n) \), and \( \delta \varepsilon_k = \frac{1}{2}(\varepsilon_k^p - \varepsilon_k^n) \). The free single particle spectra (in this paper we consider the free single particle spectrum) of neutron and proton are given by

\[ \varepsilon_k^{(n)} = \frac{(Q + k)^2}{2m} - \mu^{(n)}, \varepsilon_k^{(p)} = \frac{(Q - k)^2}{2m} - \mu^{(p)}, \]  

with the chemical potential for neutrons and protons \( \mu^{(n/p)} \) which are derived from the BCS theory self-consistently. The neutron and proton Fermi spheres are shifted with respect to
each other and the Cooper pairs have a total pair momentum $2Q$. Thus the symmetric and asymmetric parts of the spectrum (which are even and odd with respect to the time-reversal symmetry) are defined as
\[ \xi_k \equiv \frac{k^2}{2m} + \frac{Q^2}{2m} - \mu, \quad \delta \varepsilon_k \equiv \delta \mu - \frac{k \cdot Q}{2m}. \] (8)

Here $\mu = (\mu^{(n)} + \mu^{(p)})/2$, $\delta \mu = (\mu^{(n)} - \mu^{(p)})/2$ are the average and relative chemical potential. The form $\frac{kQ}{2m}$ due to the pair momentum can reduce the suppression from the mismatched Fermi surface $\delta \mu$ in certain directions. Using the “unitary” property Eq. (7), the quasiparticle spectra can be simplify to
\[ E^\pm_k \equiv E^\pm(k, Q) = \sqrt{\xi_k^2 + D^2(k)} \pm \delta \varepsilon_k. \] (9)

The limit $\delta \varepsilon_k \to 0$ corresponds to the BCS pairing in symmetric nuclear matter. While in asymmetric nuclear matter the spectra Eq.(9) splits into two branches due to the isospin asymmetry ($\delta \mu \neq 0$) and the finite-momentum of the Cooper pair ($Q \neq 0$).

B. The FFLO-ADG gap equations

In the present “unitary triplet” case, the gap equation at finite temperature can be written in the standard form
\[ \Delta_{\sigma_1,\sigma_2}(k, Q) = -\sum_{k'} \sum_{\sigma_1',\sigma_2'} \langle k\sigma_1, -k\sigma_2 | V | k'\sigma_1', -k'\sigma_2' \rangle \]
\[ \times \frac{\Delta_{\sigma_1',\sigma_2'}(k', Q)}{2\sqrt{\xi_{k'}^2 + D^2(k')}}[1 - f(E_{k'}^+) - f(E_{k'}^+)]. \] (10)

where $f(E) = [1 + \exp(\beta E)]^{-1}$ is the Fermi distribution function and $V$ is the interaction in the $SD$ channel. $\beta^{-1} = k_B T$, where $k_B$ is the Boltzmann constant and $T$ is the temperature.

It is worth noting that the orientation of $Q$ affect only the value of $\delta \varepsilon_k$ through the angle between $Q$ and $k'$ in Eq.(10). Using the properties of spherical harmonics, we can represent $\delta \varepsilon_k$ as
\[ \delta \varepsilon_k = \delta \mu - \frac{k' \cdot Q}{2m} \]
\[ = \delta \mu - \frac{k' Q}{2m} \cos(\hat{k}' \hat{Q}) \]
\[ = \delta \mu - \frac{k' Q}{2m} [\sin \theta_0 \sin \theta \cos(\varphi - \varphi_0) + \cos \theta_0 \cos \theta], \] (11)
where \((\theta_0, \varphi_0)\) and \((\theta, \varphi)\) are the directions of \(Q\) and \(k'\) in the spherical coordinate respectively. \(\varphi_0\) as a constant phase can be eliminated by choosing a special spherical coordinate in which the direction of \(Q\) represents as \((\theta_0, \varphi_0 = 0)\). Thus \(\theta_0\) as a parameter represent the direction of \(Q\).

Substituting the expansion Eq.(1) into Eq.(4) and Eq.(10), one gets a set of coupled equations for the quantities \(\Delta_{m,j}^{m,j}(k, Q, \theta_0)\)

\[
\Delta_{m,j}^{m,j}(k, Q, \theta_0) = -\frac{1}{\pi} \int_0^\infty dk' k'^2 \sum_{l'=0,2} (2l' + 1) \sum_{\mu} \Delta_{l',\mu}^{m,j}(k', Q, \theta_0) \\
\times \int d\Omega_{k'} Tr[G_{l'}^{m,j*}(\hat{k}') G_{l'}^{\mu*}(\hat{k})] \frac{1 - f(E_{k'}^+ - f(E_{k'}^-)}{\sqrt{\xi_{k'}^2 + D^2(k')}}
\]

with

\[
D^2(k) = \frac{1}{2} Tr[\Delta \Delta^\dagger] = \sum_{l' = 0,2} \sum_{m_j m_j'} \Delta_{l',m_j}^{m,j}(k, Q, \theta_0) \Delta_{l',m_j'}^{m,j'}(k, Q, \theta_0) Tr[G_{l'}^{m,j*}(\hat{k}) G_{l'}^{m,j'}(\hat{k})]
\]

(13)

where

\[
V_{l'}^{l1}(k', k) \equiv \langle k' | V_{l'}^{l1} | k > = \int_0^\infty r^2 dr j_{l'}(k'r) V_{l'}^{l1}(r) j_l(k r)
\]

(14)
is the matrix elements of the NN interaction in different partial wave \((\lambda = T, S, l, l')\) channels.

In the present calculation, \(\lambda\) corresponds to the coupled \(^3SD_1\) channel.

In the Ref. [13], we propose an axi-symmetric \(D^2(k)\) solution which corresponds to an axi-symmetric deformation of the neutron and proton Fermi spheres. The axi-symmetric \(D^2(k)\) corresponds to the \(m_j = 0\) gap components of \(\Delta_{l,m}^{m,j}(k)\) only. Moreover, we can obtain the relations \(\Delta_{l,m}^{m,j*}(k, Q, \theta_0) = -(-1)^{m_j} \Delta_{l,m}^{-m,j}(k, Q, \theta_0)\) from the Eq.(3), thus we can describe \(\Delta_{0}^{m,j}(k, Q, \theta_0)\) and \(\Delta_{2}^{m,j}(k, Q, \theta_0)\) as

\[
\Delta_{0}^{m,j}(k, Q, \theta_0) = i\delta_0(k, Q, \theta_0), \\
\Delta_{2}^{m,j}(k, Q, \theta_0) = i\delta_2(k, Q, \theta_0).
\]

(15)

Then we write the axisymmetric \(D^2(k)\)

\[
D^2(k) \rightarrow D^2(k, \theta, Q, \theta_0) = \frac{1}{2} f(\theta)[\delta_0(k, Q, \theta_0)]^2 \\
- g(\theta) \delta_0(k, Q, \theta_0) \delta_2(k, Q, \theta_0) + \frac{1}{2} f(\theta)[\delta_2(k, Q, \theta_0)]^2,
\]

(16)
where

\[
\begin{align*}
  f(\theta) &= Tr[G_0^0(\hat{k}^\prime)G_0^0(\hat{k}^\prime)] = \frac{1}{4\pi}, \\
  g(\theta) &= -Tr[G_0^0(\hat{k})G_0^0(\hat{k}^\prime)] \\
  &= -\frac{\sqrt{2}}{8\pi}(3\cos^2 \theta - 1), \\
  g(\theta) &= Tr[G_2^0(\hat{k}^\prime)G_0^0(\hat{k}^\prime)] = \frac{1}{8\pi}(3\cos^2 \theta + 1).
\end{align*}
\]

The \(\theta\) dependent \(D^2(k, \theta, Q, \theta_0)\) is independent of \(\varphi\) and maintains the rotational symmetry [in group theory the O(2) symmetry] along the axis \((\theta = 0, \varphi = 0)\) (the symmetry-axis of ADG), which is also the O(3) symmetry breaking axis. While the pair momentum \(2Q\) breaks both the rotational and translational symmetry and the symmetry breaking axis is \((\theta = \theta_0, \varphi = 0)\). The angle between the two symmetry breaking axes is \(\theta_0\) and the two axes are parallel/perpendicular to each other when \(\theta_0 = 0/\pi = \frac{\pi}{2}\).

Taking the normalization

\[
\Delta_0(k, Q, \theta_0) = \sqrt{\frac{1}{8\pi}}\delta_0(k, Q, \theta_0), \quad \Delta_2(k, Q, \theta_0) = -\sqrt{\frac{1}{8\pi}}\delta_2(k, Q, \theta_0),
\]

one get the \(m_j = 0\) components of the gap equations from Eq.(12) with finite Cooper pair momentum for the FFLO-ADG state

\[
\begin{align*}
  \begin{pmatrix}
    \Delta_0 \\
    \Delta_2
  \end{pmatrix}(k, Q, \theta_0) &= -\frac{1}{\pi} \int dk'k'^2 \begin{pmatrix}
    V^{00} & V^{02} \\
    V^{20} & V^{22}
  \end{pmatrix}(k, k') \\
  \times &\int d\Omega_k \frac{1 - f(E_{k'}^+) - f(E_{k'}^-)}{\sqrt{\xi_{k'}^2 + D^2(k', \theta, Q, \theta_0)}} \begin{pmatrix}
    f(\theta) & g(\theta) \\
    g(\theta) & h(\theta)
  \end{pmatrix}\begin{pmatrix}
    \Delta_0 \\
    \Delta_2
  \end{pmatrix}(k', Q, \theta_0),
\end{align*}
\]

where \(V^{00}, V^{02}, V^{20}, V^{22}\) are present in the Eq.(14) with \(l, l' = 0, 2\) and the quasiparticle spectrum

\[
E_{k}^{\pm} = \sqrt{\xi_{k}^2 + D^2(k, \theta, Q, \theta_0)} \\
\pm[\delta \mu - k'Q \sin \theta_0 \sin \theta \cos \varphi + \cos \theta_0 \cos \theta],
\]

with the axi-symmetric

\[
D^2(k, \theta, Q, \theta_0) = \Delta_0^2(k, Q, \theta_0) + \Delta_2^2(k, Q, \theta_0)[\frac{3\cos^2 \theta + 1}{2}] \\
\quad + \sqrt{2}\Delta_0(k, Q, \theta_0)\Delta_2(k, Q, \theta_0)[3\cos^2 \theta - 1].
\]
The two coupled components \( \Delta_0 \) and \( \Delta_2 \) represent the \( S \) and \( D \) channel gap respectively. Different from the discussion in the Ref. [11], the orientation of the Cooper pair momentum cloud affect the quasiparticle excitation and the system may become more asymmetric if the two symmetry breaking axis are not parallel to each other.

Following from Eq.(5), we can get the neutron and proton densities,

\[
\rho^{(p/n)} = \sum_{k,\sigma} n^{(p/n)}_{\sigma}(k),
\]

with the distributions

\[
n^{(p/n)}_{\sigma}(k) = \left\{ \frac{1}{2} \left( 1 + \frac{\xi_k}{\sqrt{\xi_k^2 + D^2(k, \theta, Q, \theta_0)}} \right) f(E^+_{k}) + \frac{1}{2} \left( 1 - \frac{\xi_k}{\sqrt{\xi_k^2 + D^2(k, \theta, Q, \theta_0)}} \right) [1 - f(E^-_{k})] \right\}.
\]

Summation over frequencies in Eq.(6) leads to the density matrix of the particles in the condensate,

\[
\nu_{\sigma_1, \sigma_2}(k, Q, \theta_0) = \frac{\Delta_{\sigma_1, \sigma_2}(k, Q, \theta_0)}{2 \sqrt{\xi_k^2 + D^2(k, \theta, Q, \theta_0)}} [1 - f(E^+_{k}) - f(E^-_{k})].
\]

For isospin-asymmetric nuclear matter, the coupled equations (19) and (22) should be solved self-consistently with the expressions (20) and (21) for the FFLO-ADG state. In the calculation the total Cooper pair momentum \( 2Q \) and the angle \( \theta_0 \) are treated as variational parameters to be determined from the ground state energy of the system.

C. Thermodynamics

For asymmetric nuclear matter at a fixed finite temperature and given neutron and proton densities, the net density \( \rho = (\rho^{(n)} + \rho^{(p)}) \) and isospin asymmetry defined as \( \alpha = (\rho^{(n)} - \rho^{(p)})/\rho \) are fixed. As a result, the essential quantity to describe the thermodynamics of the system is free energy defined as

\[
F|_{\rho, \beta} = U - \beta^{-1} S,
\]

where \( U \) is the internal energy and \( S \) is the entropy. A thermodynamically stable state minimizes the difference of the free energies between the superconducting and normal state,
\[ \delta f = F_s - F_n. \]

In the mean-field approximation, the entropy of the superfluid state is

\[
S_s = -2k_B \sum_k \left\{ f(E^+_k) \ln f(E^+_k) + \tilde{f}(E^+_k) \ln \tilde{f}(E^+_k) \right. \\
+ f(E^-_k) \ln f(E^-_k) + \tilde{f}(E^-_k) \ln \tilde{f}(E^-_k) \right\},
\]

(26)

where \( \tilde{f}(E^+_k) = 1 - f(E^+_k) \) and the sum is over the momentum states in quasiparticle approximation. Taking the limit \( \Delta \to 0 \), we can get the expression of the entropy \( S_n \) in the normal state. The mean-field internal energy of the superfluid state reads

\[
U = \sum_{\sigma k} \left[ \varepsilon^{(n)}_k n_{\sigma}(k) + \varepsilon^{(p)}_k n_{\sigma}(k) \right] \\
+ \sum_{k, k', \sigma_1, \sigma_2, \sigma_1', \sigma_2'} < k \sigma_1, -k \sigma_2 | V | k' \sigma_1', -k' \sigma_2' > \\
\times \nu_{\sigma_1, \sigma_2}^\dagger(k, Q, \theta_0) \nu_{\sigma_1', \sigma_2'}(k', Q, \theta_0).
\]

(27)

The second term of Eq.(27) includes the BCS mean-field interaction among the particles in the condensate and can be eliminated in terms of the gap Eq.(9). Finally, the internal energy is written as

\[
U = \sum_{\sigma k} \left[ \varepsilon^{(n)}_k n_{\sigma}(k) + \varepsilon^{(p)}_k n_{\sigma}(k) \right] \\
- \sum_k \frac{D^2(k, \theta, Q, \theta_0)}{\sqrt{\varepsilon^2_k + D^2(k, \theta, Q, \theta_0)}} \left[ 1 - f(E^+_k) - f(E^-_k) \right].
\]

(28)

The first term in Eqs.(27) and (28) includes the kinetic energy of the quasiparticles which is a functional of the paring gap. In the normal state it reduced to the kinetic energy of the neutrons and protons. Noting that the existence of the Cooper pair momentum \( 2Q \) can both enhance the paring energy and increase the kinetic energy. The competition between the two mechanisms can adjust the value and the direction of \( Q \).

III. NUMERICAL RESULTS

The nuclear FFLO states with ADG are studied numerically using the Argonne \( V_{18} \) potential. In the current paper we focus on the effects due to the finite momentum of the Cooper pairs in the axi-symmetric ADG state and investigate the favored angle between the directions of the Cooper pair momentum and the symmetry-axis of ADG. Several assumptions are adopted to simplify the calculations. Firstly, we adopt the free single-particle
(s.p.) spectrum, which may affect the level density of the state around Fermi surface. Secondly, the paring interaction is approximated by the bare interaction, i.e., ignoring the the screening effects of the paring interaction. This two approximations may affect the absolute magnitude of the paring gap. Moreover, we only study the coupled $SD$ channels which domain the pairing force at the considered density $\rho_0 = 0.17\, fm^{-3}$. The computations are carried out at the temperature $\beta^{-1} = 0.5\, MeV$ at which the effect of angle dependence of the pairing gap becomes important\cite{13}. Both the values of $\theta_0$ and $Q$ for the local stable state are to be determined via minimizing the free energy.

Fig.1 shows the pairing gap $\Delta_0(k_F)$ in the $^3SD_1$ partial-wave channel as a function of $\theta_0$ (the angle between the directions of the Cooper pair momentum and the symmetry-axis of ADG) and $Q$ (the half value of the Cooper pair momentum in units of the Fermi momentum $k_F$). The isospin asymmetries $\alpha$ are set to be 0.15, 0.25, 0.30, 0.40 in Fig.1a, Fig.1b, Fig.1c, Fig.1d, respectively. For the small asymmetry $\alpha = 0.15$ in Fig.1a, $\Delta_0(k_F)$ decreases monotonically as a function of $Q$ at $\theta_0 = 0$. However, for $0.4\pi \leq \theta_0 \leq 0.5\pi \, \Delta_0(k_F)$ is maximal at $Q \neq 0$, which may indicate the FFLO state can exist in the regime of $\theta_0$ around the direction of the Cooper pair momentum perpendicular to the symmetry-axis of ADG. At the two moderate isospin asymmetries $\alpha = 0.25$ and 0.30 in Figs. 1b and 1c, the maxima of $\Delta_0(k_F)$ locate at $Q \neq 0$ for any $\theta_0$ of $0 \leq \theta_0 \leq \frac{\pi}{2}$, indicating a FFLO state for any orientation of the Cooper pair momentum. Fig.1d shows that $\Delta_0(k_F)$ gets the maximum at $Q \neq 0$ for $0 \leq \theta_0 \leq 0.28\pi$, i.e., the FFLO state exists in narrow regime of $\theta_0$ near the the direction of the Cooper pair momentum parallel to the symmetry-axis of ADG. It is also shown in Fig.1d that the pairing gap can only exist with nonzero $Q$, which implies only the FFLO state can survive for quite large isospin asymmetry $\alpha = 0.40$.

In the $^3SD_1$ partial-wave channel, the pairing gap $\Delta_2(k_F)$ also plays an important role in the condensate. Therefore, we exhibit $\Delta_2(k_F)$ as a function of $\theta_0$ and $Q$ (in units of the Fermi momentum $k_F$) in Fig.2, where the parameters are set to be the same as those in Fig.1. The shapes of the $\Delta_2(k_F)$ surfaces closely resemble those of the pairing gap $\Delta_0(k_F)$ in Fig.1 except for Fig.2a. In Fig.2a, there exists a minimum of $\Delta_2(k_F)$ at a nonzero $Q$ for $\theta_0 = 0$, which may indicate the FFLO state can not exist for small asymmetry when the Cooper pair momentum is parallel to the symmetry-axis of ADG.

In order to find the ground state, we should calculate the free-energy difference $\delta f$ between the normal and the superconducting states with varying $\theta_0$ and $Q$. The results are shown
FIG. 1: The value of $\Delta_0(k_F)$ vs $\theta_0$ (the angle between the direction of the Cooper pair momentum and the symmetry-axis of ADG) and $Q$ (the half value of the Cooper pair momentum) in units of the Fermi momentum $k_F$. The temperature is set at $\beta^{-1} = 0.5$ Mev, and a, b, c, d are related to the select isospin asymmetry $\alpha = 0.15, 0.25, 0.25, 0.30, 0.40$, respectively.

in Fig. 3, where the parameters are set to the same values as those in Fig. 1. At $\alpha = 0.15$ in Fig.3a, the sole local minimum of $\delta f$ locates at $(\theta_0 = \pi/2, Q = 0.097)$, indicating that the FFLO state is stable when the directions of the Cooper pair momentum and the symmetry-axis of ADG are orthogonal for small $\alpha$. And we call this state FFLO-ADG-Orthogonal state below. In Figs.3b, there exist two local minima of $\delta f$ settled at $(\theta_0 = \pi/2, Q = 0.145)$ and $(\theta_0 = 0, Q = 0.18)$, respectively. The second local minimum is related to the case that the directions of the Cooper pair momentum and the symmetry-axis of ADG are parallel,
FIG. 2: The value of $\Delta_2(k_F)$ vs $\theta_0$ (the angle between the direction of the Cooper pair momentum and the symmetry-axis of ADG) and $Q$ (the half value of the Cooper pair momentum) in units of the Fermi momentum $k_F$. The temperature is set at $\beta^{-1} = 0.5$ Mev, and a, b, c, d are related to the select isospin asymmetry $\alpha = 0.15, 0.25, 0.25, 0.30, 0.40$, respectively.

and we call it FFLO-ADG-Parallel state below. For moderate asymmetries (for examples, $\alpha = 0.25, 0.30$), both the FFLO-ADG-Orthogonal state and FFLO-ADG-Parallel state are locally stable. Comparing Fig.3b and Fig.3c, the FFLO-ADG-Orthogonal state is more favored at smaller $\alpha$, while the FFLO-ADG-Parallel state is more favored for larger $\alpha$. Fig.3d shows only the FFLO-ADG-Parallel state can survive at large enough $\alpha$.

We can find from Fig.3 that the minimum of the free-energy difference between the normal and the superconducting states $\delta f$ settles either at $\theta_0 = 0$ or at $\theta_0 = \frac{\pi}{2}$. In fact, we
FIG. 3: The difference of the free energy of the normal and superconducting state $\delta f$ as a function of $\theta_0$ and $Q$, where $Q$ have been normalized to the Fermi momentum $k_F$ and $\theta_0$ is the angle between the direction of the Cooper pair momentum and the symmetry-axis of ADG. The temperature is set at $\beta^{-1} = 0.5$ Mev, and a, b, c, d are related to the select isospin asymmetry $\alpha = 0.15, 0.25, 0.25, 0.30, 0.40$, respectively.

have calculated $\delta f$ with varying $\theta_0$ and $Q$ for different asymmetries $\alpha (0.01 \leq \alpha \leq 0.47)$. The results show the minimum of $\delta f$ is related to either $\theta_0 = \frac{\pi}{2}$ or $\theta_0 = 0$, indicating only the FFLO-ADG-Parallel state and the FFLO-ADG-Orthogonal state are locally stable. Fig.4 displays the calculated values of the local minimum of $\delta f$ vs the asymmetry $\alpha$. As a comparison, the ADG state which corresponds to $Q = 0$ is also exhibited in Fig.4 (the red solid line). The black dashed line is related to $(\theta_0 = \frac{\pi}{2}, Q \neq 0)$, i.e., the FFLO-ADG-
Orthogonal state. And the blue dash-dotted line corresponds to \((\theta_0 = 0, Q \neq 0)\), i.e., the FFLO-ADG-Parallel state. For small isospin asymmetry \(\alpha\) \((0.025 < \alpha < 0.16)\), there exists only one kind of locally stable FFLO state, i.e., the FFLO-ADG-Orthogonal state. With increasing \(\alpha\), the FFLO-ADG-Parallel state emerges above \(\alpha = 0.16\), but it is not a favored state for \(\alpha < 0.286\). However, the FFLO-ADG-Parallel state becomes more stable than the FFLO-ADG-Orthogonal state when \(\alpha > 0.286\). At \(\alpha = 0.37\), the FFLO-ADG-Orthogonal state vanishes, whereas the FFLO-ADG-Parallel state can exist up to \(\alpha = 0.47\).

Since the contribution to superfluidity from the Cooper pairs around the average Fermi surface is dominant, for n-p pair correlation, a small separation of the neutron and proton Fermi surfaces \(\delta \mu\) may suppress the superfluidity strongly. In the ADG configuration, the neutron Fermi sphere possesses an oblate deformation along the symmetry-axis of ADG, whereas the proton Fermi sphere has a prolate deformation. These two different kinds of deformation enhance the correlation between neutrons and protons near their average Fermi
surface, i.e., the phase space near \((\theta = 0, \varphi = 0)\) and \((\theta = \pi, \varphi = 0)\). In the FFLO configuration, the shift of the neutron and proton Fermi spheres with respect to each other may enhance the overlap between the two Fermi surfaces. However, the influence of the Cooper pair momentum turns to be complicated if considering the angle dependence of the pairing gap. For example, in the weakly isospin-asymmetric case, the difference between the neutron and proton Fermi Surfaces \(\delta \mu\) is small, the deformation of the Fermi spheres in the ADG configuration is sufficient to compensate this differences. As a consequence, a shift of the two Fermi spheres with respect to each other along the symmetry-axis of ADG \((\theta_0 = 0)\) may even reduce the overlap of the phase-space for pairing near the average Fermi surface. Therefore, the FFLO-ADG-Parallel state could not exist for small \(\alpha\). Nevertheless, if the shift between the two Fermi spheres is perpendicular to the symmetry-axis of ADG, the pairing could be enhanced in three areas in the phase space, i.e., the area near \((\theta = 0, \varphi = 0)\), \((\theta = \pi, \varphi = 0)\) and \((\theta = \frac{\pi}{2}, \varphi = 0)\). Therefore, only the FFLO-ADG-Orthogonal state is stable for small asymmetry \(\alpha\).

When the asymmetry \(\alpha\) gets large, the splitting of the neutron and proton Fermi surfaces becomes quite large, so that the effect of the deformation of the two Fermi spheres in the ADG configuration could not compensate the splitting completely. The Cooper pair momentum along the direction of \(\theta_0 = 0\) is expected to reduce the remained splitting partially. Consequently, the the FFLO-ADG-Parallel state emerges and becomes local stable. However, the splitting is not large enough to destroy the mechanism of the FFLO-ADG-Orthogonal state totally. Hence, both the FFLO-ADG-Parallel state and FFLO-ADG-Orthogonal state are local stable for moderate \(\alpha\). At large asymmetry, the neutron and proton Fermi surfaces are split so much that the deformation of the two Fermi spheres in the ADG configuration can no longer guarantee the pairing around \((\theta = 0, \varphi = 0)\) and \((\theta = \pi, \varphi = 0)\) near the average Fermi surface. On this condition, the neutron and proton Fermi surfaces are much closer along the direction \((\theta = 0, \varphi = 0)\) than the direction \((\theta = \frac{\pi}{2}, \varphi = 0)\) in the ADG configuration. Thus, a shift between the two Fermi spheres along the direction \((\theta = 0, \varphi = 0)\) is easier to promote the phase overlap for pairing than that along the direction \((\theta = \frac{\pi}{2}, \varphi = 0)\), i.e., the FFLO-ADG-Parallel state is more favored than the FFLO-ADG-Orthogonal state for large \(\alpha\).

In Fig.4, the phase transition from the FFLO-ADG-Orthogonal state to the FFLO-ADG-Parallel state is of first order. In order to investigate in detail the phase transitions in Fig.4,
we calculate $\frac{\partial \delta f}{\partial \alpha}$ with varying $\alpha$ and the results are shown in Fig.5. At $\alpha = 0.025$, the transition from the ADG state to the FFLO-ADG-Orthogonal state is of second order. But at $\alpha = 0.16$, $\frac{\partial \delta f}{\partial \alpha} \mid_{ADG} \neq \frac{\partial \delta f}{\partial \alpha} \mid_{FFLO-ADG-Parallel}$, which indicates a first transition from the ADG state to the FFLO-ADG-Parallel state. In Fig.4, the three curves of ADG, FFLO-ADG-Orthogonal and FFLO-ADG-Parallel tend to zero gently near the transition points, indicating the three transitions from the superconducting states to the normal state are of second order. Fig.5 shows $\frac{\partial \delta f}{\partial \alpha} \to 0$ near the three phase transition points, which are according with the results in Fig.4.

IV. SUMMARY AND OUTLOOK

The fermionic condensation in asymmetric nuclear matter leads to superconducting states with anisotropic Fermi spheres (such as the FFLO and ADG states). In the ADG state, the angle dependence of the pairing gap results in the deformation of the neutron and proton

![Graph showing $\frac{\partial \delta f}{\partial \alpha}$ vs $\alpha$ with different colors for ADG, FFLO-ADG-Orthogonal, and FFLO-ADG-Parallel states. The graph includes a phase transition point at $\beta^{-1} = 0.5$ MeV.]
Fermi spheres. Moreover, the FFLO state corresponds to a shift of the two Fermi spheres with respect to each other. In this paper we consider these two mechanisms together and investigate the FFLO-ADG state with arbitrary angle between the directions of the Cooper pair momentum and the symmetry-axis of ADG. Two kinds of local stable states are found, one corresponds to the direction of the pair momentum perpendicular to the symmetry-axis of ADG (the FFLO-ADG-Orthogonal state), whereas the other is related to the direction of the pair momentum parallel to the symmetry-axis of ADG (the FFLO-ADG-Parallel state). It is shown the FFLO-ADG-Orthogonal state possesses the lowest free energy at small isospin asymmetries, while the FFLO-ADG-Parallel state is favored for large asymmetries. The transitions from both the ADG and FFLO-ADG-Orthogonal states to the FFLO-ADG-Parallel state are of first order. Whereas, it is second order transition from the ADG state to the FFLO-ADG-Orthogonal state. Moreover, the transitions from the three superconducting states (ADG, FFLO-ADG-Orthogonal and FFLO-ADG-parallel states) to the normal state are of second order.

In previous studies such as Ref.[10, 11, 16, 21], the effect of the angle dependence of the pairing gap is abandoned, i.e., supposing the neutron and proton spheres as isotropic ones. Thus, the FFLO states are degenerate for arbitrary directions of the Cooper pair momentum. In fact, for n-p pairing in asymmetric nuclear matter, the angle dependence of the pairing gap can compensate the effect due to the splitting of the neutron and proton Fermi surfaces[13]. In the axi-symmetric ADG configuration, one particular direction (the symmetry-axis of ADG) is selected. In this case, the FFLO state with angle-dependent gap is nondegenerate for the orientations of the Cooper pair momentum. Only two orientations are shown to be local stable, corresponding to the FFLO-ADG-Orthogonal and FFLO-ADG-Parallel states, respectively. As pointed out in Ref.[13] that the ADG configuration is related to the deformation of the neutron and proton Fermi spheres. Therefore, the FFLO-ADG-Parallel/Orthogonal state corresponds to more complicated deformation of the neutron and proton Fermi spheres. In this improved calculation, both the value of the pairing gap and the regime of $\alpha$ in which the superconducting state exists become large. Especially for the temperature $\beta^{-1} = 0.5$ MeV, the ADG state vanishes at $\alpha = 0.267$, whereas the FFLO-ADG-Parallel state vanishes at $\alpha = 0.467$. The combination of the FFLO and ADG states promotes the n-p pairing for quite large asymmetry. Here we only consider the nuclear saturation density $\rho_0 = 0.17 fm^{-3}$ at the low temperature of $\beta^{-1} = 0.5$ MeV, however,
the properties of FFLO-ADG-Parallel/Orthogonal state are expected to be similar for high densities at low temperature. At low densities and/or high temperatures the effect of the angle dependence of the pairing gap becomes weak, and the difference among the ADG, FFLO-ADG-Parallel and FFLO-ADG-Orthogonal states may become unobvious.

In ADG state, the the rotational symmetry is spontaneously broken [in terms of group theory, the O(3) symmetry breaks down to O(2)]. Moreover, the the translation and rotational symmetries are both broken in FFLO-ADG-Parallel state, however, the O(2) rotational symmetry is maintained. In the FFLO-ADG-Orthogonal state, the O(2) rotational symmetry is broken as well. As is well known, the continuous symmetry breaking leads to collective excitations with vanishing minimal frequency (Goldstone’s theorem). The symmetry breaking of the FFLO-ADG-Parallel/Orthogonal state may imply new complicated collective bosonic modes in asymmetric nuclear matter (except the collective motion of the Cooper pairs). We only consider the simplest FF state ($\Delta(r) = \Delta e^{i\mathbf{q}\cdot\mathbf{r}}$) with ADG in the paper. In fact, the FFLO state is much more complicated, the structure of the true ground state remains for the future work.

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